Control Comparison of Inverted Pendulum System Based on PID

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Abstract. Inverted pendulum systems and PID control methods have been the two foundations of control theory long time. The inverted pendulum system is used as an important tool for detecting the control method, and the PID control theory is the core of the classical control theory. More and more control methods have been proposed, and most of them are based on the optimization of traditional PID method. Therefore, in order to better test the control ability of the new control methods, this article will compare three kinds of control methods like traditional PID, fractional PID and neural network fractional PID, for the inverted pendulum system's control ability and anti-interference ability.

1. Introduction
Inverted pendulum system is a typical multi-variable and non-linear system. Effective and powerful control measures must be taken to make the inverted pendulum system in a stable state. And in the case of certain external interference, it can still restore stability after a certain period of time. For a long time, the inverted pendulum system has been regarded as a typical controlled object to test the feasibility of the control theory proposed by scholars and the stability of the control system.

PID is the core of classical control theory and has been widely used in various experiments and actual production environments. However, PID control methods still have certain deficiencies for complex systems. Based on this, more and more scholars have put forward many optimization methods in recent years. Zheng Bo Liu (2016) proposed a fractional order PID control method in the study. At the same time, the Signal Constraint optimization module was used to optimize the parameters. According to comparison, the fractional PID control method was found to be superior to the traditional PID. Shujun Mao and Xianjun Sheng(2014) proposed a fractional-order PID control theory based on neural networks in their research, which optimizes the disadvantages of PID that are not conducive to control when the system is changing at any time.

In order to optimize the control system of an inverted pendulum system, this paper compares and analyzes three control methods: PID, fractional-order PID, and neural network fractional PID. Through the comparison of the three control methods, a more optimized control method of the inverted pendulum system is obtained.
2. Establishment of inverted pendulum system model

This article uses a straight-line inverted pendulum system, which consists of a linear motion module and an inverted pendulum assembly. For ease of study, this article ignores various resistances and frictions that may be caused by air and contact surfaces during modeling, and assumes that the rocker is a rigid body. The composition of the inverted pendulum system is shown in Fig. 1.

The parameters of the inverted pendulum system are set as follows: $\Phi$ is the angle between the pendulum rod and the vertical direction, $F$ is the force added to the cart, $x$ is the position of the cart, $M$ is the quality of the cart, $m$ is the mass of the pendulum rod, and $l$ is the distance between the center of the pendulum and the center of rotation. By analyzing the force of the inverted pendulum system, the two equations of motion of the trolley and the pendulum are input with $u$ as the controlled object:

$$\begin{align*}
\frac{4}{3}ml^2 \ddot{\phi} + mg\phi &= ml\ddot{x} \\
(M + m)x + b\dot{x} - ml\dot{\phi} &= u
\end{align*}$$

According to formula (2-1), the system state space equation can be obtained as:

$$\begin{bmatrix}
\dot{x} \\
\dot{\phi}
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & \frac{3g}{4l} & 0
\end{bmatrix}
\begin{bmatrix}
x \\
\dot{x} \\
\phi \\
\dot{\phi}
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
1 \\
0
\end{bmatrix} u$$

According to formula (2-2), the system state space equation can be obtained as:

$$\begin{bmatrix}
\dot{x} \\
\dot{\phi}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
x \\
\dot{x} \\
\phi \\
\dot{\phi}
\end{bmatrix}$$

Assume that the mass of the pendulum rod $m = 0.0426$kg, the distance from the center of the pendulum rod to the center of rotation $l = 0.152$m, gravity acceleration $g = 9.81$m/s$^2$. The actual system state space model is:
The mathematical equation of the inverted pendulum system of formula (2-3) can be obtained as its mathematical model:

\[ y = \frac{7 \times 10^{\frac{1}{2}} \times 483^{\frac{1}{3}} \times \sinh((10^{\frac{1}{2}} \times 483^{\frac{1}{3}} \times t)/10)}{690} \]  

(2-4)

3. controller design

3.1. PID controller design

The PID controller is a proportional-integral-derivative controller, which is a control strategy based on classical control theory. Because it doesn't require a mathematical model of the controlled object, it is widely used in industrial production process control. Traditional PID controllers can be described as:

\[ u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt} \]  

(3-1)

Where e(t) is the error after feedback of the entire system, it can be expressed as: \( e(t) = r(t) - y(t) \).

The PID controller obtains the control signal \( u(t) \) by weighting the error signal e(t) to drive the controlled object so that e(t) changes in the decreasing direction, so as to meet the control requirements. Therefore, the design of the traditional PID controller is as described above, and its control block diagram is shown in Figure 2:

Figure 2. Inverted Pendulum System Based on PID Controller

3.2. Fractional PID controller design

Fractional order is a theory proposed at the same time as integer order. At present, the definition of fractional order is not uniform. There are three definitions in the mainstream: Grünwald-Letnikov fractional calculus definition (abbreviated as GL definition), Riemann-Lowville fractional calculus definition (abbreviated as RL definition) and Caputo score. Order calculus definition. This article uses the RL definition, which is defined as follows:

\[ {}_a D_t^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} f(\tau) d\tau \]  

(3-2)
The definition of gamma function is: \[ \Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt \]. The essence of fractional order PID is to convert the differential and integral links in traditional PID control into fractional differential and fractional integrals. Therefore, two parameters need to be added to express the order of fractional differentials and fractional integrals, so it can be it is considered as an optimization made on the traditional PID. Based on the traditional PID controller (Equation 3-1), the fractional PID controller can be expressed as:

\[
u(t) = K_p e(t) + K_i \frac{\beta}{t} e(t) + K_d \frac{\alpha}{t} e(t)\tag{3-3}\]

According to the definition of RL fractional calculus (3-2), the control law of fractional PID can be obtained as: \( \Delta u(k) = u(k) - u(k-1) \) (3-4). Therefore, the design of the fractional-order PID controller is as described above, and its control principle block diagram is shown in Figure 3.

**Figure 3.** Inverted pendulum system block diagram based on fractional PID controller

### 3.3. Neural Network Fractional PID Adaptive Controller Design

At present, neural network is a commonly used method for designing an adaptive controller, which can automatically adjust the parameters of the PID according to actual conditions. This article selects the BP neural network in the neural network as the design object. Set up a 3-4-5 BP neural network, and its structure is shown in Figure 4.

**Figure 4.** Neural network structure

As shown in Fig. 4, the input signals are: set value \( r \), output value \( y \), and error value \( e \). The output signals are: proportional link coefficient \( K_p \), integral link coefficient \( K_i \), and differential link coefficient \( K_d \), integral index \( \alpha \), differential index \( \beta \). And they are represented by 1, 2, and 3 in the upper right corner of the variable, respectively: the input layer, the hidden layer, and the output layer. Therefore, the input and output relations of the input layer can be expressed as:

\[
I_i^j(k) = x_i^j(k) \tag{3-5}
\]

\[
O_i^i = I_i^j(k) \tag{3-6}
\]

i has 4 hidden layers, so i=1, 2, 3, and 4. The induced local domain and output of the network hidden layer are:
In formula (3-7), $w_{ji}^2$ is the synaptic weight of neurons in the hidden layer. At the same time, the activation function of the hidden layer neurons selects the hyperbolic tangent function.

$$\varphi(x) = \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

(3-9)

The induced local domain and output of the network output layer are:

$$v_h^1(k) = \sum_{i=1}^{N} w_{hi}^3(k)O_j^2(k)$$

(3-10)

$$O_h^3(k) = \varphi(v_h^3(k)), j = 1,2,...,5$$

(3-11)

Oh in the formula (3-11) is the output, so $O_1^3 = k_p, O_2^3 = k_i, O_3^3 = k_d, O_4^3 = \alpha, O_5^3 = \beta$. Since the five parameters of the fractional PID need to satisfy the non-negative condition, the sigmoid function is selected as the activation function of the output layer:

$$f(x) = \frac{1}{2}(1 + \tanh x) = \frac{e^x}{e^x + e^{-x}}$$

(3-12)

In order to enable rapid convergence of the search and make the neural network achieve better results, an improved BP learning algorithm is used, and a momentum term related to the size of the weight at the previous moment is added. The synaptic weight adjustment algorithm for the network output layer neuron h is:

$$\Delta w_{hi}^3 = \eta \delta_h^3(k)O_j^2(k) + \alpha \Delta w_{hi}^3(k - 1)$$

(3-13)

Where $\eta$ is the learning rate and $\alpha$ is the momentum factor. The local gradient of neuron h is:

$$\delta_h^3(k) = e(k) \text{sgn} \left( \frac{\partial y(k)}{\partial \Delta u(k)} \right) x_h(k) f'(v_h^3(k))$$

(3-14)

Similarly, the synaptic weight adjustment algorithm for neurons in the hidden layer and the local gradient for neuron j are:

$$\Delta w_{ji}^2 = \eta \delta_j^2(k)O_i^1(k) + \alpha \Delta w_{ji}^2(k - 1)$$

(3-15)

$$\delta_j^2(k) = \varphi'(v_j^2(k)) \sum_{h=1}^{5} \delta_h^2(k)w_{hi}^2(k)$$

(3-16)
The neural network fractional PID controller control law is the same as equation (3-4). Therefore, the design of the neural network fractional PID controller is as described above, and its control principle block diagram is shown in Figure 5.

**Figure 5.** Inverted pendulum system block diagram based on neural network fractional PID controller

4. controller comparison

The inverted pendulum system model proposed uses the three controller design methods proposed to simulate in MATLAB and compare the differences between the three controllers.

4.1. Comparison of initial simulation

Simulate three controllers, set the input as a step response, and compare the three controllers' control capabilities. The step response curves of the three controllers are shown in Figure 6:

**Figure 6.** Comparison of step response curves of three controllers
According to Figure 6, the three controllers have all achieved the goal of system stability control. In order to clearly show the advantages and disadvantages of the three controllers, the overshoot, the settle time, and the rise time statistics of the three step responses are shown in Table 1. The settle time is calculated as the time when the error output by the system reaches within 1% of the input signal.

**Table 1. Comparison of Fractional PID and Neural Network Fractional PID Controller.**

| Number                  | Overshoot | Settle time (seconds) | Rise time (seconds) |
|-------------------------|-----------|-----------------------|---------------------|
| Traditional PID controller | 0.0139    | 5                     | 3.5450              |
| Fractional PID controller   | 0.0052    | 0.41                  | 0.1750              |
| Neural Network Fractional PID Controller | 0.0037    | 0.02                  | 0.0050              |

According to Figure 6 and Table 1, the initial control capabilities of the three control theories are as follows: neural network fractional PID controller, fractional PID controller, and traditional PID controller.

### 4.2. Comparison of interference adjustment

In order to compare the anti-jamming capabilities of the three controllers in the face of interference. At T=3s, an interference is imposed on the controlled object. At T=4s, a similar interference is applied to between the controller and the controlled object. And compare the adjustment ability of the three. The adjustment curve is shown in FIG. 7.

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**Figure 7.** Comparison of interference control of three controllers
According to Figure 7, it can be seen that the traditional PID controller has the worst control capability for interference. Fractional-order PID controllers have the ability to control disturbances that occur behind the controlled object, but have no control over the interference that occurs between the controller and the controlled object. However, the neural network fractional PID controller has the best ability to control interference.

5. Conclusion
Based on the above analysis, we can find that the neural network fractional PID controller is better than other controllers in both initial control and anti-interference ability. However, its use in actual industrial production is far lower than the traditional PID controller. The reason is that the design of the traditional PID controller is simpler than the neural network fractional PID controller. Due to the simple design, the cost of traditional PID controllers is much lower than other controllers. How to simplify the design of neural network fractional PID controller and reduce the design cost can be used in industrial production is a problem that needs to be solved in the future.

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