Determining dynamic water level control boundaries for a multi-reservoir system during flood seasons with considering channel storage

Yu Gong¹,² | Pan Liu¹,² | Lei Cheng¹,² | Guiya Chen³ | Yanlai Zhou⁴ | Xiaoqi Zhang¹,² | Weifeng Xu¹,²

¹State Key Laboratory of Water Resources and Hydropower Engineering Science, Wuhan University, Wuhan, China
²Hubei Provincial Collaborative Innovation Center for Water Resources Security, Wuhan, China
³Office of Flood Control and Drought Relief, Changjiang Water Resources Commission, Wuhan, China
⁴Department of Geosciences, University of Oslo, Oslo, Norway

Abstract

The dynamic control of reservoir water level during flood seasons (WLFS) allows a trade-off between flood control and water resources conservation. In this process, the WLFS dynamic control boundaries are key parameters. The channel flood routing affects the dynamic control operations. However, flood routing is seldom considered in multi-reservoir aggregation (a popular method for tackling the curse of dimensionality in multi-reservoir issues). To address this knowledge gap, this study conceptualises channel flood routing as a virtual channel storage reservoir (CSR) based on Muskingum flood routing in the aggregation method. The dynamic control boundaries for the multi-reservoir WLFS are then derived using the following aggregation-decomposition method: (a) the aggregation reservoir is created by considering channel storages, and the flood risk is quantified using pre-discharge rules; and (b) the aggregated reservoir storages are decomposed by maximising hydropower generation with constraints. The Xiluodu, Xiangjiaba and Three Gorges reservoir system on the Yangtze River, China, were selected for a case study. The annual power generation is 175.40 billion kWh when using static operation rules. With dynamic operation, the output would be 177.05 billion kWh if CSRs were not considered and 177.53 billion kWh if CSRs were considered. Thus, the proposed method can efficiently determine the dynamic control boundaries for the multi-reservoir WLFS.

KEYWORDS
aggregation-decomposition, channel storage reservoir, dynamic control boundary, multi-reservoir operation, water level during flood seasons

INTRODUCTION

Reservoirs are one of the most important facilities for managing integrated water resources systems and contributes to flood control, power generation, navigation and water supply for industrial, municipal and agricultural uses (Dai et al., 2019; Lee, Kang, & Lee, 2017; Saher, Nasly, Kadir, Yahaya, & Ishak, 2015). Reasonable
reservoir operating rules are central to reservoir management in distributing water for various purposes and increasing overall benefits (Bayazit, Uuml, & Nal, 1990; Feng et al., 2016; Liu, Li, Chen, & Rheinheimer, 2014; Liu, Nguyen, Cai, & Jiang, 2012; Ming et al., 2017; Wang, Zhou, Liang, & Xu, 2015; Zhang et al., 2018). The flood limited water level (FLWL) is a critical characterisation value (Ding et al., 2015; Liu et al., 2015; Xie et al., 2017). China’s Flood Control Act specifies that the water level during flood seasons (WLFS) should generally not be allowed to exceed the FLWL. This operating rule allows for the static control of WLFS, and generally sets the WLFS at the FLWL for the whole flood season. The operating rule provides sufficient flood control storage (Chen, Guo, Li, Liu, & Zhou, 2012), but may lead to excessive amounts of spilled water and water shortages at the end of the flood season, thereby reducing flood water utilisation efficiency (Liu, Guo, Xiong, Li, & Zhang, 2006). The dynamic control of WLFS (regulating WLFS above or below the FLWL according to hydrological forecasts), is a powerful approach for obtaining compromises between flood control and water resources conservation (Diao & Wang, 2010; Long, Madsen, Dan, & Pedersen, 2008; Ouyang, Zhou, Li, Liao, & Wang, 2015; Yun & Singh, 2008).

The dynamic control boundaries for the WLFS (upper and lower boundaries) are generated prior to the dynamic operation (Tan et al., 2017). For the single reservoir operation, Li, Guo, Liu, and Chen (2010) set up a dynamic control model for WLFS by considering the uncertainty of the flood hydrograph shape and inflow forecasts. Their model consisted of three modules: a pre-release module to estimate the upper boundary of WLFS based on inflow forecast information, a refill module to retain recession floods, and a risk analysis module to assess the flood risk. Jiang, Sun, Ji, and Zhou (2015) took the forecasting error as a fuzzy variable and described it using credibility theory to optimise the dynamic control boundaries for the WLFS. For the multi-reservoir operation, Tan et al. (2017) proposed a model that considered the capacity compensation of two reservoirs and the spatial uncertainty of flood characterisation using a confidence level that was subjective. Liu, Guo, Li, Chen, and Li (2009); Liu, Lin, and Wei (2014) developed a two-stage risk analysis method that used a forecast horizon point to divide future times into two stages (hydrological forecast lead time and unpredicted time), and effectively calculated the flood risk for single reservoirs. However, the risk analysis was affected by the issue of dimensionality as the number of reservoirs in the multi-reservoir system increased.

The aggregation-decomposition method is one of the most promising methods for addressing the problem of dimensionality in multi-reservoir systems. This method generally aggregates a multi-reservoir system into one virtual reservoir, optimises the objectives of the virtual reservoir, determines decisions for the whole system, and then decentralises the overall system decisions into decisions for individual reservoirs (Liu, Guo, Xu, & Chen, 2011; Saad, Turgeon, Bigras, & Duquette, 1994; Terry, Pereira, Neto, Silva, & Sales, 1986; Turgeon, 1981). Archibald, McKinnon, and Thomas (1997) proposed an aggregate stochastic dynamic programming model of multi-reservoir systems that solved a series of three-dimensional problems, that is, states representing the current reservoir, aggregate states of upstream reservoirs and approximation states of the downstream reservoirs. Chen et al. (2012) developed a joint operation dynamic control model for WLFS of multi-reservoir systems that determined optimal real-time reservoir storages using aggregation-decomposition simulations. Zhou, Guo, Chang, Liu, and Chen (2018) extended Chen’s model by introducing the enhanced Muskingum equation and proposed a multi-objective dynamic control model for WLFS based on the non-dominated sorting genetic algorithm-II.

In previous studies, the reservoirs or hydropower units were aggregated (Valdes, Montbrun, Strzepek, & Restrepo, 1992). The flood routing processes of channel storage were often included (Birkhead & James, 2002; Chen et al., 2012; McCarthy, 1938), but were seldom conceptualised. To address this knowledge gap, this study aims to: (a) consider channel storage reservoirs (CSRs) as a part of the aggregation reservoir, (b) perform risk analyses of the multi-reservoir system based on the aggregation-decomposition method and (c) optimise dynamic control boundaries for WLFS for multi-reservoir systems by maximising annual power generation without increasing the flood risk. The highlights of this study are the conceptualization of CSR and the risk analysis based on the aggregation reservoir method.

The remainder of this paper is organised as follows. In Section 2, a dynamic control boundary model for WLFS, incorporating quantitative risk analysis, is developed. In Section 3, a multi-reservoir system, comprising the Xiluodu, Xiangjiaba and the Three Gorges Reservoirs of the Yangtze River, China, is selected as a case study. Finally, conclusions are drawn in Section 4.

## 2 | METHODOLOGY

A procedure flowchart is shown in Figure 1. The method used was as follows:
1. In the aggregation step, aggregation reservoirs were built, operation rules were developed, and risk analysis was performed (Section 2.1).

2. In the decomposition step, the annual power generation was maximised, subject to the above flood risk constraints (Section 2.2).

2.1 | Aggregation

In this step, aggregation reservoirs with multi-reservoir system characteristics were built up. Pre-discharge operation rules for the aggregation reservoirs were then developed. Finally, a two-stage risk analysis method was applied to the aggregation reservoirs.

Aggregation reservoirs were assigned according to flood control stations, with the number of aggregation reservoirs being equal to the number of flood control stations. An example of the aggregation and decomposition is illustrated in Figure 2. Two multi-reservoir scenarios are presented as follows:

1. Reservoirs in series: flood control Station I lies between Reservoir A and B, and flood control Station II is downstream of Reservoir B. The aggregation can be implemented twice. The aggregation reservoir for I is comprised of Reservoir A and the first CSR. The aggregated inflow for the aggregation reservoir for I is the sum of the inflow of Reservoir A and the inter-zone flow between Reservoir A and B. The aggregation Reservoir for II is comprised of the aggregation Reservoir for I, the second CSR, Reservoir B and the downstream CSR. The aggregated inflow for the aggregation reservoir for II is the sum of Reservoir A inflow, the inter-zone flow between Reservoir A and B, and the inter-zone flow between Reservoir B and flood control Station II.

2. Reservoirs in parallel: the aggregation and decomposition procedures are similar to those used in Scenario (1). Because there are three flood control stations, three aggregations were implemented.

The in-series multi-reservoir system is illustrated in this study, and the method is applicable to the in-parallel multi-reservoir system.

2.1.1 | Aggregation reservoir characteristics incorporating channel storage

Aggregated storage

The storage of the aggregation reservoir comprises two parts: (a) the storage of the reservoirs and (b) the storage of CSRs. In the $k^{th}$ aggregation reservoir, the storage of the $i^{th}$ reservoir during the time period $t$ is $V_i(k(t))$, and can be calculated during the reservoir operation. In the $k^{th}$ aggregation reservoir, the storage of the $j^{th}$ CSR is determined using the Muskingum flood routing method as follows:

$$W_{j,k}(t) = K_{j,k}x_{j,k}I_{c,j,k}(t) + (1-x_{j,k})O_{c,j,k}(t)$$  \hspace{1cm} (1)$$

where, $W_{j,k}(t)$ is the storage of the $j^{th}$ CSR during the time period $t$ in the $k^{th}$ aggregation reservoir, $K_{j,k}$ is the storage-time constant for the $j^{th}$ CSR in the $k^{th}$ aggregation reservoir, $x_{j,k}$ is the weighing factor for the $j^{th}$ CSR in the $k^{th}$ aggregation reservoir, $I_{c,j,k}(t)$ is the inflow of the $j^{th}$ CSR during the time period $t$ in the $k^{th}$ aggregation reservoir, and $O_{c,j,k}(t)$ is the outflow of the $j^{th}$ CSR during the time period $t$ in the $k^{th}$ aggregation reservoir.

The storage for the $k^{th}$ aggregation reservoir is given by
Scenario one: reservoirs in series

Conceptualization of channel storage reservoirs

Aggregation step 1

Aggregation reservoir for I

Aggregation step 2

Aggregation reservoir for II

Decomposition step

Aggregation reservoir for III

Scenario two: reservoirs in parallel

Conceptualization of channel storage reservoirs

Aggregation step 1

Aggregation reservoir for I

Aggregation step 2

Aggregation reservoir for II

Decomposition step

Aggregation reservoir for III

Legend

Streamflow
△ Reservoir
● Flood control station
○ CSR
□ Aggregation reservoir

FIGURE 2 Aggregation and decomposition process for the multi-reservoir system, incorporating channel storage reservoirs

\[ S_k(t) = \sum_{i=1}^{i_{\text{max}}} V_{i,k}(t) + \sum_{j=1}^{j_{\text{max}}} W_{j,k}(t) \]  

(2)

where, \( S_k(t) \) is the storage of the \( k^{\text{th}} \) aggregation reservoir during the time period \( t \), \( i_{\text{max}} \) is the maximum number of reservoirs in the \( k^{\text{th}} \) aggregation reservoir, and \( j_{\text{max}} \) is the maximum number of CSRs in the \( k^{\text{th}} \) aggregation reservoir.

It should be noted that \( W_{j,k}(t) \) reaches its maximum value \( W_{j,k}^{\text{max}} \) when \( I_{c,j,k}(t) \) is equal to \( O_{c,j-1,k}(t) \) because this indicates that the inflow and release of the reservoir between CSR \( j-1 \) and \( j \) are equal. \( I_{c,j,k}(t) \) varying with \( O_{c,j-1,k}(t) \) leads to an unstable water profile. When the inflow and release remain the same, \( S_k^U, S_k^F \) and \( S_k^L \) of the aggregation reservoirs are the storage corresponding to the upper boundaries of WLFS, FLWL and the lower boundaries of WLFS, respectively, for reservoirs of the \( k^{\text{th}} \) aggregation reservoir.

\[ S_k^U = \sum_{i=1}^{i_{\text{max}}} V_{i,k}^U + \sum_{j=1}^{j_{\text{max}}} W_{j,k}^{\text{max}} \]  

(3)

\[ S_k^F = \sum_{i=1}^{i_{\text{max}}} V_{i,k}^F + \sum_{j=1}^{j_{\text{max}}} W_{j,k}^{\text{max}} \]  

(4)

\[ S_k^L = \sum_{i=1}^{i_{\text{max}}} V_{i,k}^L + \sum_{j=1}^{j_{\text{max}}} W_{j,k}^{\text{max}} \]  

(5)

where, \( V_{i,k}^U, V_{i,k}^F \) and \( V_{i,k}^L \) are reservoir storage values corresponding to the upper boundaries of WLFS, FLWL,
and the lower boundaries of WLFS, respectively, for the $i^{th}$ reservoir in the $k^{th}$ aggregation reservoir.

**Aggregated pre-discharge storage**

The pre-discharge storage is the incremental flood control volume of reservoirs within the hydrological forecast lead time. The pre-discharge storage of the $i^{th}$ reservoir is given as follows:

$$V_{p,k}^i(t_k) = V_{i,k}(t_k) - V_{i,k}(t_k + T_{f,k})$$

(6)

where, $V_{p,k}^i(t_k)$ is the pre-discharge storage of the $i^{th}$ reservoir in the $k^{th}$ aggregation reservoir, $t_k$ is the time when the pre-discharge operation of the $k^{th}$ aggregation reservoir begins, and $T_{f,k}$ is the effective lead time of the $k^{th}$ aggregation reservoir. The effective lead time refers to the time period needed to conduct the pre-discharge operation of the aggregation reservoir (Chen et al., 2012).

The pre-discharge storage of the $j^{th}$ CSR is as follows:

$$W_{p,j,k}^j(t_k) = \left[ Q_{i,k}^j - x_{j,k} I_{c,j,k} (t_k + T_{f,k}) \right] K_{j,k}$$

(7)

where, $W_{p,j,k}^j(t_k)$ is the pre-discharge storage of the $j^{th}$ CSR and $Q_{i,k}^j$ is the safety streamflow of the $j^{th}$ CSR in the $k^{th}$ aggregation reservoir. By using the above equation, it is possible to keep the pre-discharge storage of the channel storage at a low level to deal with large future floods.

The pre-discharge storage of the $k^{th}$ aggregation reservoir is as follows:

$$S_k^p(t_k) = S(t_k) - S(t_k + T_{f,k}) = \sum_{i=1}^{i_{\text{max}}} V_{p,k}^i(t_k) + \sum_{j=1}^{j_{\text{max}}} W_{p,j,k}^j(t_k)$$

(8)

where, $S_k^p(t_k)$ is the pre-discharge storage of the $k^{th}$ aggregation reservoir.

**Aggregated inflow**

The inflow of the $k^{th}$ aggregation reservoir consists of the inflow of the first reservoir and the inter-zone flow into CSRs. The inflow is as follows:

$$I_{a,k}(t) = I_{1,k}(t) + \sum_{j=1}^{j_{\text{max}}} I_{a,j,k}(t)$$

(9)

where, $I_{a,k}(t)$ is the inflow of the $k^{th}$ aggregation reservoir during the time period $t$, $I_{1,k}(t)$ is the inflow of the first reservoir during the time period $t$ in the $k^{th}$ aggregation reservoir, and $I_{a,j,k}(t)$ is the inter-zone flow into the $j^{th}$ CSR during the time period $t$ in the $k^{th}$ aggregation reservoir. It should be noted that the step needed to aggregate inflow is different for in-series reservoirs and parallel reservoirs. For in-series reservoirs, the aggregation inflow is the sum of the first reservoir inflow and the inter-zone flow into channel storages in the aggregation reservoir; for parallel reservoirs, the inflow of the aggregation reservoir is the sum of parallel reservoirs inflow and the inter-zone flow into channel storages in the aggregation reservoir.

**Aggregated outflow**

The outflow of the $k^{th}$ aggregation reservoir equals the streamflow of the last flood control station in the $k^{th}$ aggregation reservoir, for the reason that the last flood control station is the only outlet for the $k^{th}$ aggregation reservoir. The outflow of the aggregation reservoir is determined by the operation rules in the next section. It should be noted that ‘the last flood control station’ refers to the flood control station downstream the last reservoir for in-series reservoirs (the flood control Station II in the scenario one of Figure 2), and refers to the flood control station downstream all the other flood control stations for parallel reservoirs (the flood control Station III in the scenario two of Figure 2).

**2.1.2 | Aggregation reservoir operation**

The aggregation reservoir operation is divided into two stages, as shown in Figure 3a. The first stage is the pre-discharge operation, and the second stage is the flood control operation.

**Pre-discharge operation**

In this case, $t_k$ is the time when the aggregated inflow in the lead time $T_{f,k}$ is higher than the standard for flood events. The pre-discharge operation begins at $t_k + K_{k_{\text{max}}}^{\text{max}}$ and ends at $t_k + T_{f,k} + K_{k_{\text{max}}}^{\text{max}}$. The pre-discharge operation rules, without and with channel storage considered, will now be introduced and compared.

1. The pre-discharge operation without considering channel storage is given in as follows. Storage and release in the lead time can be found in Figure 3b.

$$S_k(t) = \max \{ S_k^U + \left( I_{a,k}(t) - O_{a,k}^l \right) \Delta t, S_k^p \}$$

$$t_k + K_{k_{\text{max}}}^{\text{max}} < t \leq t_k + T_{f,k} + K_{k_{\text{max}}}^{\text{max}}$$

(10)

where, $O_{a,k}^l$ is the safety outflow of the $k^{th}$ aggregation reservoir and $\Delta t$ is the calculation period, which is equal
to the safety streamflow of the last flood control station. 

$K_{\text{max},k}$ is the storage-time constant of the last CSR for the $k^{th}$ aggregation reservoir.

2. The pre-discharge operation incorporating channel storage is as follows:

$$S_k(t) = \begin{cases} 
\max \left\{ S_k^U + (I_{a,k}(t) - O_{a,k}) \Delta t, S_k^L - W_{p,\text{max},k} \right\} & t_k + K_{\text{max},k} < t \leq t_k + T_{f,k} \\
\min \left\{ S_k(t_k + T_f) + (I_{a,k}(t_k) - O_{a,k}) \Delta t, S_k^L \right\} & t_k + T_{f,k} < t \leq t_k + T_{f,k} + K_{\text{max},k} 
\end{cases}$$

where, and $O_{a,k}$ is the minimal outflow (e.g., for ecology purposes) of the $k^{th}$ aggregation reservoir, which is equal to the minimal streamflow of the last flood control station.

The aggregated reservoir storage can be less than $S_k^L$ because there is a need to reduce the storage of the river

![FIGURE 3 Schematic of the $k^{th}$ aggregation reservoir operations](image-url)
channel. At the end of the pre-discharge operation, the aggregated reservoir storage can be less than \( S_k^l \) because the river channel has a smaller inflow.

A comparison between the pre-discharge operation with and without considering CSRs can be found in Figure 3b. The pre-discharge operation without considering CSRs reaches \( S_k^l \) and the storage remains unchanged. The pre-discharge operation considering CSRs divides the lead time into two periods: reservoir drawdown and CSR drawdown. The beginning of the drawdown period for the aggregation reservoir is given by \( t_k + K_{j\max,k} \) and this period ends at \( t_k + T_{j,k} \). The aim during this period is to reduce the storage of the reservoirs. The drawdown period for CSRs is \( t_k + T_{j,k} \) to \( t_k + T_{j,k} + K_{j\max,k} \). The aim during this period is to reduce the storage of the last CSR in the aggregation reservoir.

The storage in the lead time for the pre-discharge operation considering CSRs can be found in Figure 3c. The aggregated storage is the sum of the last CSR storage and storage of reservoirs and CSRs upstream of the last CSR. The aggregation reservoir storage is lower than \( S_k^l \) at the end of the forecast period because of the drawdown regulation in the last CSR.

Flood control operation
The flood control operation begins at \( t_k + T_{j,k} + K_{j\max,k} \) and ends when the flood event is over (i.e., the aggregated inflow in the lead time is below the standard for flood events and the aggregated storage is decreasing). The flood control rules of the aggregation reservoir can be developed based on the safety requirements of the downstream flood control station, and the safety requirements are reflected from individual reservoir operation rules. Generally, the individual reservoir operation rules are from the local departments according to the safety requirements.

### 2.1.3 Two-stage flood risk analysis

Flood risk can be calculated from flood control operations. For the aggregation reservoir, the risk is influenced by the aggregation reservoir storage and the strategy (\( \Phi \)) used to allocate the pre-discharge storage of aggregation reservoirs through reservoirs at the start of the reservoir operation. The flood risk can be written as \( R(S_k^P, P, \Phi) \). The aggregated flood risk, expressed as \( R(S_k^P, P) \), directly uses the allocation strategy generating the largest risk with the same \( S_k^P \) and \( P \) for safety. For example, the risk with inflows for a 100-year return period by regulating from \( S_k^P \) with the flood control operation rule is \( R(S_k^0, 0.01) = 1 \times 10^{-2} \).

The two-stage theory developed by Liu, Lin, and Wei (2014) is extended to the aggregation system. The two-stage flood risk analysis equation for aggregation reservoirs is as follows:

\[
R_k(P) = \int P(S_{\text{end},k}) R(S_{\text{end},k}, P) dS = \sum_{q=1}^{F} P(S_{\text{end},k,q}) R(S_{\text{end},k,q}, P) \tag{12}
\]

where, \( R_k(P) \) is the flood risk with a flood with frequency \( P \) in the \( k \)th aggregation reservoir, \( F \) is the number of flood events, \( S_{\text{end}, k, q} \) is the storage at the end of the pre-discharge period of the \( k \)th aggregation reservoir with the \( q \)th flood, \( P(S_{\text{end}, k, q}) \) is the storage probability at the end of the pre-discharge operation for the \( k \)th aggregation reservoir with the \( q \)th flood, and \( R(S_{\text{end}, k, q}, P) \) is the relationship between the storage and the flood risk of the \( k \)th aggregation reservoir with the \( q \)th flood.

It should be noted that \( S_{\text{end}, k, q} \) in the above equation is calculated by the forecast inflow, and thus it deviates from the actual storage \( S_{\text{a}, k, q} \). It could be considered that the average positive errors between \( S_{\text{a}, k, q} \) and \( S_{\text{a}, k, q} \) is the same as the average negative errors, under the assumption that the forecast inflow is unbiased Gaussian (Salamon & Feyen, 2010; Schaeffli, Talambo, & Musy, 2007). From the real-world operating experiences, the flood risk increases faster when the storage increases (i.e., the second derivative of risk with respect to storage is positive). The increased risk caused by the positive errors is larger than the decreased risk caused by the negative errors. To conclude, risk calculated from the above equation is larger than the actual risk when the forecast information is not perfect and the analysis method is a bit of conservative, which makes the reservoir operation safe.

#### 2.2 Decomposition

In this step, the aggregation reservoir was decomposed with hydropower generation as the selected objective to optimise. The reservoir constraints were: the risk constraint from aggregation reservoirs, the refill constraint and the physical constraint from individual reservoirs.

##### 2.2.1 Optimization of WLFS

The annual average hydropower generation was selected as the objective function, and the upper boundaries of WLFS were used as decision variables for the multi-reservoir system. In this way, the upper boundary of WLFS for each reservoir was calculated by optimization. The objective function is as follows:
where, \( E \) is the average annual hydropower generation, \( Y \) is the number of years, \( T \) is the number of time periods in each year, \( M \) is the number of reservoirs, \( V_i^U \) is the storage corresponding to the upper boundary of WLFS for the \( i \)th reservoir, and \( N_{i,y}(V_i^U) \) is the output of the \( i \)th reservoir in time period \( t \). Standard operating policy (Lund, 1997) is used to calculate the annual average hydropower generation.

\[
\max E = \frac{1}{Y} \sum_{y=1}^{Y} \sum_{t=1}^{T} \sum_{i=1}^{M} N_{i,y}(V_i^U) \Delta t \tag{13}
\]

where, \( E \) is the average annual hydropower generation, \( Y \) is the number of years, \( T \) is the number of time periods in each year, and \( M \) is the number of reservoirs, \( V_i^U \) is the storage corresponding to the upper boundary of WLFS for the \( i \)th reservoir, and \( N_{i,y}(V_i^U) \) is the output of the \( i \)th reservoir in time period \( t \). Standard operating policy (Lund, 1997) is used to calculate the annual average hydropower generation.

### 2.2.2 Reservoir constraints

#### Flood risk constraints

The risk should be no larger than the frequency for all aggregation reservoirs. The relationship is as follows:

\[ R_k(P) \leq P \quad k = 1, 2, \ldots, n_f \tag{14} \]

where, \( n_f \) is the number of flood control stations.

The risk constraints limit the storage during the decomposition process. During the pre-discharge period, the dynamic storage of reservoirs and CSR should be no greater than the storage of the aggregation reservoirs deduced from the aggregation reservoir operation \( (S_k^O(t)) \). Equations (14) can be replaced as follows:

\[
\sum_{i=1}^{n_f} V_{i,k}(t) + \sum_{j=1}^{n_f} W_{j,k}(t) \leq S_k^O(t) \quad t_k < t < t_k + T_f
\]

\[ k = 1, 2, \ldots, n_f \tag{15} \]

#### Refill constraints

The storage of reservoirs should be refilled to \( S_k^E \) from \( S_k^F \) after the flood control operation within the forecast period \( T_{f,k} \). The storage after refilling is as follows:

\[ S_k(t_e,k + T_{f,k}) = S_k^F + \int_0^{T_{f,k}} I_{a,k}(t) dt - \int_0^{T_{f,k}} O_{a,k}^i dt \tag{16} \]

where, \( t_e,k \) is the time when the flood control operation of the \( k \)th aggregation reservoir ends.

The average storage \( S_{e,k} \) after the refilling is as follows:

\[ S_{e,k} = \frac{1}{F} \sum_{q=1}^{F} S_{k,q}(t_e,k + T_{f,k,q}) \tag{17} \]

where, \( S_{k,q}(t_e,k + T_{f,k,q}) \) refers to the storage after refilling of the \( k \)th aggregation reservoir with the \( q \)th flood.

Thus, the average end storage (the average aggregation storage after refilling) is larger than the storage corresponding to the FLWL. The constraint is as follows:

\[
\sum_{i=1}^{n_f} V_{i,k}^E + \sum_{j=1}^{n_f} W_{j,k}^E \leq S_{e,k} \quad k = 1, 2, \ldots, n_f \tag{18} \]

where, \( W_{j,k}^E \) is the average channel storage of the \( j \)th CSR in the \( k \)th aggregation reservoir during the refilling process, and \( S_{e,k} \) is the average storage of the \( k \)th aggregation reservoir after refilling.

#### Physical constraints

Physical constraints are given by Equations 19–22, which are the water balance equation, and water storage, water release and power output constraints, respectively.

\[ V_{i,k}(t) = V_{i,k}(t-1) + (I_{i,k}(t) - O_{i,k}(t)) \Delta t \quad k = 1, 2, \ldots, n_f \tag{19} \]

where, \( I_{i,k}(t) \) is the inflow of reservoir \( i \) at time \( t \) and \( O_{i,k}(t) \) is the outflow of reservoir \( i \) at time \( t \).

\[ V_{i,k}^{min} \leq V_{i,k}(t) \leq V_{i,k}^{max} \quad k = 1, 2, \ldots, n_f \tag{20} \]

\[ O_{i,k}^{min} \leq O_{i,k}(t) \leq O_{i,k}^{max} \quad k = 1, 2, \ldots, n_f \tag{21} \]

\[ N_{i,k}^{min} \leq N_{i,k}(t) \leq N_{i,k}^{max} \quad k = 1, 2, \ldots, n_f \tag{22} \]

where, \( V_{i,k}^{min} \) and \( V_{i,k}^{max} \) are the minimal and maximal values, respectively, for the storage of the \( i \)th reservoir in the \( k \)th aggregation reservoir, \( O_{i,k}^{min} \) and \( O_{i,k}^{max} \) are the minimal and maximal values, respectively, for outflow of the \( i \)th reservoir in the \( k \)th aggregation reservoir, \( N_{i,k}^{min} \) and \( N_{i,k}^{max} \) are the minimal and maximal values, respectively, for the output of the \( i \)th reservoir in the \( k \)th aggregation reservoir.

### 3 CASE STUDY

#### 3.1 Study area and data description

The Yangtze River is the longest river in China, with a length of 6,300 km and a drainage area of 1.80 million km². A multi-reservoir system has been constructed...
along the river. The largest reservoir in this system is the Three Gorges Reservoir, which currently is the largest dam in the world in terms of installed capacity (22,500 MW). The Xiluodu and Xiangjiaba Reservoirs are located upstream of the Three Gorges Reservoir (Figure 4). Table 1 presents key data regarding these three reservoirs. The Lizhuang and Zhicheng hydrological stations are flood control stations downstream of the Xiangjiaba Reservoir and the Three Gorges Reservoir, respectively. The safety discharge for Lizhuang is 25,000 m³/s, and the safety discharge for Zhicheng is 56,700 m³/s.

Relative forecast errors for the Xiluodu, Xiangjiaba and the Three Gorges Reservoirs were assumed to obey unbiased normal distribution: \( N(\hat{I}, \sigma_r \hat{I}) \), where \( \hat{I} \) is the forecasted inflow and \( \sigma_r \) is the relative forecast error. Because the drainage areas of the Xiluodu and Xiangjiaba Reservoirs are near, the relative forecast errors of them are set as the same. The relative forecast errors are shown in Table 2. Daily streamflow data for the three reservoirs are available from 1966 to 2013, and typical flood event data with the time step of 6 hr are available for 1961, 1966, 1981, 1989, 1991 and 1998. The ensemble forecasts in the pre-discharge operation were sampled from the probability distribution \( N(\hat{I}, \sigma_r \hat{I}) \). For each typical flood, 1,000 forecast floods were generated. Thus, a total of 6,000 forecast floods were generated. The effective forecasting lead time of the aggregation reservoir for Lizhuang is set as 48 hr, and the effective lead time of the aggregation reservoir for Zhicheng is set as 72 hr.

### 3.2 Determining dynamic control boundaries for WLFS using risk analysis

#### 3.2.1 Formation of aggregation reservoirs

The reservoirs between Xiluodu Reservoir and Lizhuang, comprising Xiluodu and Xiangjiaba Reservoirs, the CSR between Xiluodu and Xiangjiaba Reservoirs, and the CSR between Xiangjiaba Reservoir and Lizhuang were aggregated. The aggregation reservoir formed in this step is ‘the aggregation reservoir for Lizhuang.’

The reservoirs between Xiluodu Reservoir and Zhicheng, comprising the aggregation reservoir for Lizhuang, the CSR between Lizhuang and the Three Gorges Reservoir, the Three Gorges Reservoir, and the CSR between the Three Gorges Reservoir and Zhicheng were aggregated. The aggregation reservoir formed in this step is ‘the aggregation reservoir for Zhicheng.’

#### Pre-discharge storage

Pre-discharge storage of reservoirs was calculated using reservoir operations. Pre-discharge storage of CSRs in the last \( K_{max} + 1 \) period of the lead time was estimated according to the difference between the minimal streamflow and the safe streamflow. Taking the CSR from Xiangjiaba Reservoir to Lizhuang as an example, the time-storage constant was 6 hr, the minimal release was 5,000 m³/s, and the safe streamflow was 25,000 m³/s. The weighing factor was 0.2. As a safety factor, the difference between the minimal release and the safe
streamflow was set at 10,000 m$^3$/s. The pre-discharge storage, calculated using Equation (7), was 0.216 billion m$^3$. The results are summarised in Table 3. The proposed method is conservative in the pre-discharge storage estimation for the CSRs, and the accuracy can be improved by using hydraulic models. With accurate channel storage estimation, the benefits gained from the channel storage can be further increased.

**Flood risk for the aggregation reservoirs**

The aggregation reservoir flood control rules are shown in Table 4. For example, the aggregation reservoir for Zhicheng is assigned for Zhicheng’s safety requirements, and, at the same time, Zhicheng is also the flood control station for the Three Gorges Reservoir. Therefore, the operation rule of the aggregation reservoir for Zhicheng can be prescribed based on the operation rule of the Three Gorges Reservoir: (a) The inflow requirements of the aggregation reservoir for Zhicheng can be calculated by incorporating the inter-zone flow between the Three Gorges Reservoir and Zhicheng, based on the Three Gorges Reservoir operation rules. The reason is that the sum of the drainage area of the Three Gorges Reservoir (the drainage area upstream the Three Gorges Reservoir in the Yangtze River watershed) and the drainage area between the Three Gorges Reservoir and Zhicheng equals the drainage area of the aggregation reservoir for Zhicheng (the drainage area upstream the flood control station Zhicheng in the Yangtze River watershed). (b) The outflow requirements can also be calculated by incorporating the inter-zone flow between the Three Gorges Reservoir and Zhicheng, based on the Three Gorges Reservoir operation rules. The reason is that the release of the aggregation reservoir for Zhicheng is Zhicheng streamflow (please referring to Section 2.1.1), and the difference between the release of the aggregation reservoir for Zhicheng and the release of the Three Gorges Reservoir is the inter-zone flow between the Three Gorges Reservoir and Zhicheng. (c) The storage requirements can be calculated based on the Three Gorges Reservoir operation rules, by adding the storage of CSRs and the storage of the Xiluodu, Xiangjiaba, and Three Gorges reservoirs.

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**TABLE 1** Multi-reservoir system data

| Parameter                  | Unit     | Xiluodu reservoir | Xiangjiaba reservoir | Three gorges reservoir |
|----------------------------|----------|-------------------|----------------------|------------------------|
| Crest elevation            | m        | 610               | 384                  | 185                    |
| Normal pool level          | m        | 600               | 380                  | 175                    |
| FLWL                       | m        | 560               | 370                  | 145                    |
| Flood control capacity     | Billion m$^3$ | 4.65             | 0.90                 | 22.15                  |
| Firm power output          | MW       | 3,850             | 2,009                | 4,990                  |
| Maximum power output       | MW       | 13,860            | 7,750                | 22,500                 |

**TABLE 2** Standard deviation of the relative inflow forecast error

| Forecast horizon (6 hr) | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  | 13  | 14  |
|-------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Relative forecast error of Xiluodu and Xiangjiaba reservoir | 0.045 | 0.053 | 0.062 | 0.070 | 0.078 | 0.085 | 0.092 | 0.099 | —   | —   | —   | —   | —   |
| Relative forecast error of The three gorges reservoir        | 0.037 | 0.044 | 0.051 | 0.058 | 0.066 | 0.073 | 0.081 | 0.088 | 0.096 | 0.104 | 0.111 | 0.118 | 0.126 | 0.135 |

**TABLE 3** Pre-discharge storage of the CSRs

| CSRs                                  | Storage-time constant $K$ (hr) | $^{a}$Streamflow difference (m$^3$/s) | Pre-discharge storage (billion m$^3$) |
|---------------------------------------|--------------------------------|--------------------------------------|---------------------------------------|
| Xiluodu Reservoir-Xiangjiaba Reservoir | Too small /                    | /                                    | /                                     |
| Xiangjiaba Reservoir-Lizhuang          | 6                              | 10,000                               | 0.216                                 |
| Lizhuang-Three Gorges Reservoir        | 12                             | 10,000                               | 0.432                                 |
| Three Gorges Reservoir-Zhicheng        | 6                              | 10,000                               | 0.216                                 |

$^{a}$Streamflow difference refers to the streamflow difference between the safe streamflow and minimal flow in the channel.
Reservoirs. Since there are no specific storage requirements for Xiluodu, Xiangjiaba, and the CSRs in the Three Gorges Reservoir operation rules, their values are estimated conservatively as the storages when the Xiluodu and Xiangjiaba are regulating at their FLWLs. Similarly, the operation rule of the aggregation reservoir for Lizhuang can be developed based on the operation rule of the Xiangjiaba Reservoir. The individual flood control rules of the Xiluodu, Xiangjiaba, and Three Gorges Reservoirs can be found in Zhou, Guo, Xu, Liu, and Qin (2015).

Because the flood risk varies with storage and pre-discharge storage, a frequency analysis was conducted for the relationship between flood risk and pre-discharge storage. Figure 5 shows the relationship when the aggregated inflow frequencies were 5 and 2%, that is, the return periods were 20 and 50 years, respectively.

Different aggregation reservoirs were built up for different flood control aims. For the aggregation reservoir of Zhicheng, the aim was to keep Zhicheng safe for risks of 5–0.1%. For the aggregation reservoir of Lizhuang, the aim was to keep Lizhuang safe for risks of 5–2%. Table 5 shows the pre-discharge storage for the aggregation reservoirs and the flood risks corresponding to different flood frequencies.

### Table 4: Flood control operation rules of aggregation reservoirs

| Aggregation reservoir for Lizhuang | Reservoir inflow $I$ ($\text{m}^3/\text{s}$) | Reservoir storage $S$ (billion $\text{m}^3$) | Reservoir release $O$ ($\text{m}^3/\text{s}$) |
|-----------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|
| $I \leq 12,000$                  |                                          |                                          |                                          |
| $12,000 < I \leq 15,000$         |                                          |                                          | $O = I$                                 |
| $15,000 < I \leq 25,000$         |                                          |                                          | $O = 13,000$                            |
| $25,000 < I \leq 35,000$         |                                          |                                          | $O = 20,000$                            |
| $I > 35,000$                     |                                          |                                          | $O = 25,000$                            |
| Aggregation reservoir for Zhicheng| $I \leq 79,000$                          | $S \leq 29.90$                          | $O \leq 42,700$                         |
| $79,000 < I \leq 85,000$         | $29.90 < S \leq 44.37$                  |                                          | $O = 42,700$                           |
| $85,000 < I \leq 100,200$        | $29.90 < S \leq 44.37$                  |                                          | $O \leq 56,700$                         |
| $100,200 < I \leq 115,500$       | $44.37 < S \leq 48.10$                  |                                          | $O \leq 80,000$                         |
| $I > 115,500$                    | $S > 48.10$                             |                                          | $O \leq I$                              |

**Figure 5** Relationship between the flood risk and the pre-discharge storage of the aggregation reservoirs

The relationship between flood risk and pre-discharge storage for different aggregation reservoirs is shown in Figure 5. The flood risk is plotted against the pre-discharge storage for aggregated inflow frequencies of 5% and 2%. The aggregation reservoir for Lizhuang is compared with the aggregation reservoir for Zhicheng in the figure.
3.2.2 Optimization results

The determination of the upper boundary for WLFS is an optimization issue and is subject to both risk and refill constraints. The final optimization results are as follows: the upper WLFS boundaries for the Xiluodu, Xiangjiaba and Three Gorges Reservoirs were 562.8, 370.8 and 148.8 m, respectively.

For Xiluodu and Three Gorges Reservoirs, the water level was allowed to be lower than the FLWLs. For Xiangjiaba Reservoir, the FLWL is its dead water level of 370 m. The optimization results for the lower WLFS boundaries for the Xiluodu, Xiangjiaba and Three Gorges Reservoirs were 555.0, 370.0, and 144.7 m, respectively.

3.3 Comparison of different schemes

Table 6 shows the annual average power generation in different schemes satisfying risk constraints. In this way, the annual hydropower generation can be quantified. The static operation denotes that the WLFSs are fixed at FLWLs when there are no floods. The dynamic operation without considering CSR uses the pre-discharge operation rule in Equation (10). The empirical dynamic operation denotes that the maximal pre-discharge storage of the aggregation reservoirs is allocated to the Three Gorges Reservoir directly without optimization. It is clear that incorporating CSR, and optimizing the dynamic control boundaries for WLFS, have noticeable effects on improving hydropower generation without increasing the flood risk. Compared with the static operation, the dynamic operation considering CSRs improve power generation by 2.13 million kWh, which is worth ~91 million dollars every year. The incorporation of the CSRs increases power generation by 0.48 billion kWh in the dynamic operation, and the increased power generation is worth around 21 million dollars every year.

3.4 Pre-discharge storage during the pre-discharge operation

Figure 6 shows the reservoir operation process in depth, encountered with the 1998 observed flood. The left-hand-side figures and the right-hand-side figures show the process of pre-discharge of the multi-reservoir system and aggregation reservoirs, respectively. It is clearly found that the pre-discharge operation provides risk constraints for the multi-reservoir operation.

Figure 7 illustrates the pre-discharge storage for the multi-reservoir system, encountered with the 1998 flood. The lead time for the aggregation reservoir for Zhicheng is 72 hr. The pre-discharge operation began at $t = 0$. The last 6 hr of the pre-discharge operation is the CSR

| TABLE 5 | Maximum pre-discharge storage for the aggregation reservoirs |
| Content | Unit | Aggregation reservoir for Lizhuang | Aggregation reservoir for Zhicheng |
|---------|------|----------------------------------|----------------------------------|
| Maximum pre-discharge storage | Billion m³ | 0.78 | 0.77 | 2.19 | 1.69 |
| Flood frequency | % | 5.00 | 2.00 | 5.00 | 2.00 | 5.00 | 2.00 | 1.00 | 0.10 | 5.00 | 2.00 | 1.00 | 0.10 |
| Flood risk | % | 4.99 | 2.00 | 4.96 | 2.00 | 5.00 | 1.94 | 0.99 | 0.10 | 5.00 | 1.95 | 0.99 | 0.10 |

| TABLE 6 | Hydropower generation results for different schemes |
| Scheme | Dynamic control boundaries of WLFS | Upper boundaries of WLFS (m) | Power generation (billion kWh) |
|---------|----------------------------------|-----------------|-----------------|-----------------|
| Static operation | 560.0 | 370.0 | 145.0 | 175.40 |
| Dynamic operation | Without considering CSRs | 560.0 | 370.0 | 148.8 | 176.99 |
| Considering CSRs | Empirical boundaries | 562.5 | 370.2 | 147.9 | 177.05 |
| Optimal boundaries | 562.8 | 370.8 | 148.8 | 177.53 |

Note: The lower boundaries of WLFS for the Xiluodu, Xiangjiaba, and the Three Gorges Reservoirs are 555, 370 and 144.7 m, respectively.
drawdown period because the flood travel time from the Three Gorges Reservoir to Zhicheng is ~6 hr. It was found that the storage of reservoirs decreases in the reservoir drawdown period, and that the storage of CSR decreases during the CSR drawdown period. The lead time for the aggregation reservoir for Lizhuang is 48 hr, and the pre-discharge operation began at $t = 4$. It should be noted that changes in the channel storage are possible to erode the riverbanks. The channel storage was decreased in the pre-discharge operation. After the pre-discharge operation, the channel storage would be increased during the flood control operation, since the flood would be released from the upstream reservoir. The drawdown and refill processes may influence the
riverbanks in certain geological situations. The influence was not quantified in this study, but the proposed method is still applicable after the drawdown and refill variation constraints are added. Besides, it is suggested that parts of the financial benefit gained from the proposed method should be used to reinforce the riverbank. The reinforcement would also help to prevent other kinds of geological hazards.

Reservoir operation processes were compared for the proposed dynamic operation and the static operation by using typical flood events from 1961, 1966, 1981, 1989, 1991 and 1998 with inflow frequencies of 0.1, 1, 2, and 5%. Table 7 shows the maximal values of reservoir water levels and the streamflows at the flood control stations in 1998. It can be seen that all the values of the proposed dynamic operations are not greater than those of the static operations. Thus, the comparison further validates the effectiveness of the proposed dynamic operation.

| Maximum values                  | Proposed dynamic operation | Static operation |
|---------------------------------|---------------------------|-----------------|
|                                 | 0.10% 1.00% 2.00% 5.00%  | 0.10% 1.00% 2.00% 5.00%  |
| Zhicheng streamflow (m³/s)      | 56,700 56,700 56,700 56,700 | 56,700 56,700 56,700 56,700  |
| Lixiang streamflow (m³/s)       | 25,000 25,000 25,000 25,000 | 25,000 25,000 25,000 25,000  |
| Xiluodu reservoir water level (m)| 577.6 563.0 560.3 560.0 | 577.6 563.0 560.3 560.0  |
| Xiangjiaba reservoir water level (m) | 370.0 370.0 370.0 370.0 | 370.0 370.0 370.0 370.0  |
| Three gorges reservoir water level (m) | 157.5 148.6 146.7 145.0 | 157.8 148.6 146.7 145.0  |

1. Incorporating channel storage in the aggregation reservoirs improved annual hydropower generation from 176.99 to 177.44 billion kWh without optimization of dynamic control boundaries for WLFS, and from 177.05 to 177.53 billion kWh with optimization of dynamic control boundaries for WLFS. Incorporating channel storage allows hydropower generation to be increased. The combined use of dynamic operation and CSRs further increases the finial benefits.

2. The flood risks were controlled within acceptable flood control standards by using the pre-discharge operation considering CSRs. The maximum allowable pre-discharge storage of the aggregation reservoir for Lixiang was 0.78 billion m³, and the pre-discharge storage of the aggregation reservoir for Zhicheng was 2.19 billion m³. The flood risks of individual reservoirs were controlled by complying with the storage constraints from the aggregation reservoirs.

3. The WLFS upper boundaries for the Xiluodu, Xiangjiaba, and Three Gorges Reservoirs were 562.8, 370.8 and 148.8 m, respectively; and the WLFS lower boundaries for the Xiluodu, Xiangjiaba, and Three Gorges Reservoirs were 555, 370 and 144.7 m, respectively. The optimal dynamic control boundaries for WLFS increased annual hydropower generation from 175.40 to 177.53 billion kWh (1.21%) when compared with the static operation. It was found that the proposed method for the dynamic control of boundaries for multi-reservoir WLFS allows for increased benefits equivalent to 91 million US dollars every year without increasing flood risks.

4 | CONCLUSIONS

The channel flood routing affects the reservoir operations. However, flood routing processes of channel storage have seldom been conceptualised in aggregation reservoirs. In this study, the aggregation-decomposition method was used to determine the dynamic WLFS control boundaries for a multi-reservoir system and to consider the effect of CSR. In the aggregation process, aggregation reservoirs with multi-reservoir characteristics were built up and the operation rules for aggregation reservoirs were developed. The two-stage flood risk analysis was applied to the aggregation reservoirs. In the decomposition process, an optimization model of the dynamic WLFS control boundaries was built, with annual hydropower generation as the optimization objective. Flood risk constraints, refill constraints and physical constraints were considered. After carrying out the aggregation and decomposition method, the following conclusions could be drawn:

Aggregation reservoirs were built and channel storages were incorporated when determining dynamic WLFS control boundaries for a multi-reservoir system from a holistic perspective. However, issues such as the determination of the effective hydrological forecast lead time of the aggregation reservoir, and the accurate estimation of pre-discharge storage for CSRs, should be further researched.
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DATA AVAILABILITY STATEMENT
The data that support the findings of this study are available from the corresponding author upon reasonable request.

NOMENCLATURE

ABBREVIATIONS

CSR channel storage reservoir
FLWL flood limited water level
WLFS water level during flood seasons

INDICES

\( i \) index of reservoirs
\( j \) index of CSRs
\( k \) index of aggregation reservoirs
\( q \) index of floods
\( t \) index of the time period
\( y \) index of years

PARAMETERS/VARIABLES

\( E \) the annual average hydropower generation (kWh)
\( F \) the number of flood events
\( f_k^{\text{max}} \) the maximum number of reservoirs in the \( k \)-th aggregation reservoir
\( I_{i, k}(t) \) the inflow of reservoir \( i \) at time \( t \) (\( m^3/s \))
\( I_{c, j, k}(t) \) the inflow of the \( j \)-th CSR during the time period \( t \) in the \( k \)-th aggregation reservoir (\( m^3/s \))
\( I_{a, k}(t) \) the inflow of the \( k \)-th aggregation reservoir during the time period \( t \) (\( m^3/s \))
\( I_{1, k}(t) \) the inflow of the first reservoir during the time period \( t \) in the \( k \)-th aggregation reservoir (\( m^3/s \))
\( I_{i, j, k}(t) \) the inter-zone flow into the \( j \)-th CSR during the time period \( t \) in the \( k \)-th aggregation reservoir (\( m^3/s \))
\( j_k^{\text{max}} \) the maximum number of CSRs in the \( k \)-th aggregation reservoir
\( K_{j, k} \) the storage-time constant for the \( j \)-th CSR in the \( k \)-th aggregation reservoir (hr)
\( K_{k, j, k}^{\text{max}} \) the storage-time constant of the last CSR for the \( k \)-th aggregation reservoir (hr)
\( M \) the number of reservoirs
\( n_f \) the number of flood control stations
\( N_{i, y}(V_i^U) \) the output of the \( i \)-th reservoir in time period \( t \) (kW)
\( N_{i, k}(t) \) the output of the \( i \)-th reservoir during the time period \( t \) in the \( k \)-th aggregation reservoir (kW)
\( N_{i, k}^{\text{min}} \) the minimal values for the output of the \( i \)-th reservoir in the \( k \)-th aggregation reservoir (kW)
\( N_{i, k}^{\text{max}} \) the maximal values for the output of the \( i \)-th reservoir in the \( k \)-th aggregation reservoir (kW)
\( O_{i, k}^{\text{min}} \) the minimal values for the outflow of the \( i \)-th reservoir in the \( k \)-th aggregation reservoir (m^3/s)
\( O_{i, k}^{\text{max}} \) the maximal values for the outflow of the \( i \)-th reservoir in the \( k \)-th aggregation reservoir (m^3/s)
\( O_{s, k} \) the minimal outflow (e.g., for ecology purposes) of the \( k \)-th aggregation reservoir (m^3/s)
\( O_c, j, k(t) \) the outflow of the \( j \)-th CSR during the time period \( t \) in the \( k \)-th aggregation reservoir (m^3/s)
\( O_{i, k}(t) \) the outflow of reservoir \( i \) at time \( t \) (m^3/s)
\( P(S_{\text{end, } k, q}) \) the storage probability at the end of the pre-discharge operation for the \( k \)-th aggregation reservoir with the \( q \)-th flood
\( Q_{f, j, k} \) the safety streamflow of the \( j \)-th CSR in \( k \)-th aggregation reservoir (m^3/s)
\( R(S_k^P, P) \) the flood risk with the allocation strategy generating the largest risk
\( R_q(P) \) the flood risk with the flood with frequency \( P \) in the \( k \)-th aggregation reservoir
\( R(S_{\text{end, } k, q}, P) \) the relationship between the storage and the flood risk of the \( k \)-th aggregation reservoir with the \( q \)-th flood
\( S_{\text{o}}(t) \) the storage of the \( k \)-th aggregation reservoir during the time period \( t \) (m^3)
\( S_{\text{end, } k, q} \) the storage at the end of the pre-discharge period of the \( k \)-th aggregation reservoir with the \( q \)-th flood (m^3)
\( S_k^P(t_k) \) the pre-discharge storage of the \( k \)-th aggregation reservoir (m^3)
\( S_k^P(t) \) the storages of the \( k \)-th aggregation reservoir during the time period \( t \) deduced from the aggregation reservoir operation (m^3)
\( S_k^U \) the aggregation storage corresponding to the upper boundaries of WLFS for reservoirs of the \( k \)-th aggregation reservoir when each reservoir’s inflow and release remain the same (m^3)
the aggregation storage corresponding to FLWL for reservoirs of the $k^{th}$ aggregation reservoir when each reservoir’s inflow and release remain the same ($m^3$) 

The actual storage at the end of the predischarge period of the $k^{th}$ aggregation reservoir with the $q^{th}$ flood ($m^3$) 

the aggregation reservoir storage corresponding to the lower boundaries of WLFS for reservoirs in the $k^{th}$ aggregation reservoir when each reservoir’s inflow and release remain the same ($m^3$) 

the average storage of the $k^{th}$ aggregation reservoir after refilling ($m^3$) 

the storage after refilling the $k^{th}$ aggregation reservoir with the $q^{th}$ inflow ($m^3$) 

the time when the flood control operation of the $k^{th}$ aggregation reservoir ends (hr) 

the time when the predischarge operation of the $k^{th}$ aggregation reservoir begins (hr) 

the number of time periods in each year $T$ 

The effective lead time of the $k^{th}$ aggregation reservoir (hr) 

the storage of the $i^{th}$ reservoir during the time period $t$ in the $k^{th}$ aggregation reservoir ($m^3$) 

the reservoir storage values corresponding to the upper boundaries of WLFS for the $i^{th}$ reservoir in the $k^{th}$ aggregation reservoir ($m^3$) 

the reservoir storage values corresponding to FLWL for the $i^{th}$ reservoir in the $k^{th}$ aggregation reservoir ($m^3$) 

the reservoir storage values corresponding to the lower boundaries of WLFS for the $i^{th}$ reservoir in the $k^{th}$ aggregation reservoir ($m^3$) 

the pre-discharge storage of the $i^{th}$ reservoir in the $k^{th}$ aggregation reservoir ($m^3$) 

the minimal values for the storage of the $i^{th}$ reservoir in the $k^{th}$ aggregation reservoir ($m^3$) 

the maximal values for the storage of the $i^{th}$ reservoir in the $k^{th}$ aggregation reservoir ($m^3$) 

the storage of the CSR during the time period $t$ in the $k^{th}$ aggregation reservoir ($m^3$) 

the pre-discharge storage of the $j^{th}$ CSR ($m^3$) 

the maximum storage of the CSR during the time period $t$ in the $k^{th}$ aggregation reservoir ($m^3$) 

the average channel storage of the $j^{th}$ CSR in the $k^{th}$ aggregation reservoir during refilling process ($m^3$) 

the weighing factor for the $j^{th}$ CSR in the $k^{th}$ aggregation reservoir 

the number of years 

the calculation period (6 hr) 

the strategy to allocate the pre-discharge storage of aggregation reservoirs through reservoirs

**ORCID**

Pan Liu 🌟 [https://orcid.org/0000-0002-3777-6561](https://orcid.org/0000-0002-3777-6561)

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