Electron density stratification in two-dimensional structures tuned by electric field

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A new kinetic instability which results in formation of charge density waves is proposed. A spatial period of arising space-charge and field configuration is inversely proportional to electric field and can be tuned by applied voltage. The instability has no interpretation in the framework of traditional hydrodynamic approach, since it arises from modulation of an electron distribution function both in coordinate and energy spaces. The phenomenon can be observed in thin 2D nanostructures at relatively low electron density.

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Recent progress in microelectronics is related to great success in controlled fabrication of low-dimensional semiconductor systems. That is why transport properties of semiconductor nanostructures, both of classical and quantum nature, lately attract wide attention. In this paper we discuss a new type of purely classical instability, which can be observed in 2D nanostructures. The instability results in formation of charge density waves (CDW). The main feature of the phenomenon is its kinetic nature. In contrast to usual current and density instabilities in semiconductors [1, 2, 3, 4, 5, 6, 7, 8], the electron dynamics in the kinetic instability can not be described on the basis of the local hydrodynamic parameters, such as electronic density, drift velocity, and temperature. Remarkably, the kinetic instability can develop even in the Ohmic regime, when the stationary value of electric current is proportional to applied voltage.

Closely related phenomena of the striations formation are widely known in the gas discharge physics. The striated discharge has been observed since M. Faraday and is regarded as one of the most typical discharge forms [1, 2, 3, 4, 5, 6, 7, 8]. In spite of this, a consistent theory of striation is up to now absent. In the last decades it was realized that for description of typical striated discharges the fluid approach fails. It was shown that the hydrodynamic description is valid only for very high electron densities when the collisions between electrons are frequent enough for the maxwellization of electron distribution function (EDF) [9, 10]. At lower electron densities, occurring in typical gas discharges, the phenomenon is of essentially kinetic nature. In this case, the EDF perturbation in the striations is varying both in space, and along the energy axis [1, 2, 3, 4, 5, 6, 7, 8, 9, 10].

At the order of Fermi energy, are small compared to $W_0$, as well as Fermi energy, are small compared to $W_0$ (in what follows for simplicity we put $T_0 = 0$). The condition b) requires that electrons be "hot", and their energies be of the order of $W_0$, i.e. the electron gas in this case is non-degenerate. We also assume that electron concentration is small and neglect electron-electron collisions.
LEM: A transport scattering time energy. There are three different time scales in our problem: a transport scattering time $\tau$, a characteristic time of electron heating by the electric field $\tau_0 \sim L_0^2/D_0$ (here $L_0 = W_0/F_0$, $D_0 = W_0\tau/M$ and $M$ is the electron effective mass), and a time $W_0/\kappa$ which characterizes a rate of energy loss due to the emission of acoustic phonons. We will assume that

$$\tau \ll \tau_0 \ll \frac{W_0}{\kappa}. \quad (1)$$

Inequality (1) provides that the acoustic phonon scattering may be considered as a small perturbation. Due to this scattering the "staircase" diffusive trajectories move slowly down (along axis $E$) with velocity $\kappa/F_0$ and is much larger than $s_0$ (since the inequalities (1) may be rewritten as $s_0 \ll v \ll \sqrt{W_0/M}$). The fact that the instability can be observed in the Ohmic regime indicates that the effect is purely kinetic and can not be described in terms of hydrodynamic parameters.

As far as the elastic collisions are dominant (see Eq. (1)), the EDF is almost isotropic \cite{19}, $f(z, W, t) \approx f_i(z, W, t) + f_a(z, W, t) \cos(\varphi)$. Here $f_i$ is an isotropic part of EDF, $f_a \cos(\varphi)$ is a small anisotropic correction, and $\varphi$ is an angle between electron velocity and applied field. Denote $J(z, W, t) = \sqrt{W/4M}f_a$. The equations for $f_i$ and $J$ can be written as follows \cite{1, 2, 19}

$$J = -D(W) \left( \frac{\partial f_i}{\partial z} + F \frac{\partial f_i}{\partial W} \right), \quad (2)$$

$$\frac{\partial f_i}{\partial t} + \frac{\partial J}{\partial z} + F \frac{\partial J}{\partial W} = \kappa \frac{\partial f_i}{\partial W}. \quad (3)$$

where $D(W) = W\tau/M$ is an energy dependent diffusion coefficient. For simplicity we assume that $\tau$ is energy independent and, consequently, $D(W)$ is proportional to $W$. The boundary conditions for Eqs. (2), (3) read

$$f_i|_{W=W_0} = 0, \quad (FJ - \kappa f_i)|_{W=0} = FJ_{|W=W_0}. \quad (4)$$

Here $F(z, t) = -\partial U(z, t)/\partial z$, and $U(z, t)$ is a potential energy, which includes both the self-consistent potential created by electrons and the external potential $U_0(z) = -F_0 z$. Condition $f_i|_{W=W_0} = 0$ corresponds to the limit of a very strong interaction with optical phonons ("black wall" condition). The second boundary condition is related to the conservation of the number of particles.
in inelastic collisions. Eqs. (2), (3), and (4) at \( \kappa = 0 \) have a homogeneous stationary solution

\[
J = J_0 = \frac{n_0 v}{W_0}, \quad f_i = f_{i0} = \frac{n_0}{W_0} \ln \left( \frac{W_0}{W} \right),
\]

where \( n_0 \) is the stationary electron concentration (we assume the following normalization \( J_0 W_0 f_{i0} dW = n_0 \)).

Since \( \int_{W_0}^{W_0} J_0 dW = n_0 v \), solution (2) corresponds to the Ohmic regime. According to (14) we rewrite Eqs. (2), (3) in variables \((E, z, t)\), where \( E = W + U(z, t) \) is a full energy of a particle. The result is given by

\[
J = -D(E - U) \frac{\partial f_i}{\partial z},
\]

\[
\frac{\partial f_i}{\partial t} + \frac{\partial J}{\partial z} = (\kappa - \frac{\partial U}{\partial t}) \frac{\partial f_i}{\partial E}.
\]

The motion of a particle in the space of new variables is restricted by the curves \( E = E_1(z, t) = U(z, t), \quad E = E_2(z, t) = W_0 + U(z, t) \) (See Fig. 1b). The boundary conditions (4) can be rewritten as

\[
f_i|_{E=E_2(z,t)} = 0,
\]

\[
(FJ - \kappa f_i)|_{E=E_1(z,t)} = FJ |_{E=E_2(z,t)}.
\]

Since the total energy of an electron changes slowly (with the characteristic time \( W_0/\kappa \)), it will be useful to introduce an electron density distribution over the \( E \) axis

\[
N(E, t) = \int_{z_1(E, t)}^{z_2(E, t)} dz f_i,
\]

where \( z_1(E, t), z_2(E, t) \) are the inverse functions of \( E_1(z, t), E_2(z, t) \), correspondingly, and the value \( N dE \) represents the number of electrons on “staircase” trajectories restricted by \( E \) and \( E + dE \) (see Fig. 1b). The stationary value of \( N \) is given by \( N_0 = n_0/W_0 \). Introducing the notation \( I_E(t) = J(z_2, E, t) \) (the stationary of \( I_E \) being equal to \( J_0 \)) we find from Eq. (4)

\[
J(z_1, E, t) = I_E(t) + \int_{z_1}^{z_2} dz \left( \frac{\partial f_i}{\partial t} - (\kappa - \frac{\partial U}{\partial t}) \frac{\partial f_i}{\partial E} \right).
\]

Taking into account that \( \partial z_1/\partial E = -1/F(z_1, t), \partial z_2/\partial t = (\partial U/\partial t)/F(z_1, t) \) and using Eqs. (4), (5) we obtain the continuity-like equation that governs the electron motion over the axis of total energy

\[
\frac{\partial N}{\partial t} - \frac{\partial N}{\partial E} \left( N \left[ \kappa - \frac{\partial U}{\partial t} \right] \right) = I_{E + W_0}(t) - I_E(t).
\]

Here the angle brackets mean averaging over \( z \)

\[
\left\langle \frac{\partial U}{\partial t} \right\rangle = \frac{1}{N} \int_{z_1}^{z_2} dz \frac{\partial U}{\partial t} f_i(z, E).
\]

Next we consider the deviations from the stationary solution in the linear approximation. A small periodic over coordinate modulation of the potential \( U - U_0 = \Delta U \) \( \exp(-i \omega t - i q z) \) induces the energy dependence of quantities \( I_E, N \) in forms \( I_E - J_0 = \Delta I \exp(-i \omega t - i q E/F_0), \quad N - N_0 = \Delta N \exp(-i \omega t - i q E/F_0) \). We will demonstrate that for \( q \approx q_m = \pm 2 \pi m/L_0 \) (where \( m = 1, 2, \ldots \)) the imaginary part of \( \omega \) is positive which implies that a stationary solution (3) is unstable. For \( q = q_m \) the solution is periodic function of energy with a period \( W_0/m \) and \( I_{E + W_0}(t) = I_E(t) \). Then linearization of Eq. (11) yields

\[
\omega_m = \frac{\kappa}{F_0 + \Delta F_m} q_m,
\]

where \( \Delta F_m = -i q_m N_0 \langle \Delta U_m \rangle /\delta N_m \). We see that the physics of the problem is governed by the only parameter \( \langle \Delta U_m \rangle /\delta N_m \) (the subscript \( m \) implies that all quantities are taken at \( q = q_m \)). This parameter has a transparent physical meaning of a response of the averaged potential with respect to a small variation of electron density in energy space \( \delta N_m \). The instability (Im(\( \omega_m \)) > 0) occurs, when \( Re \langle \Delta U_m \rangle /\delta N_m > 0 \). In order to find this parameter one should go beyond the averaged kinetic equation (11) and solve Eqs. (1), (7) together with Poisson equation. As long as Eq. (13) is already proportional to a small parameter \( \kappa \), one can simplify the solution of Eqs. (6), (7) assuming that \( \kappa = 0 \), and neglecting \( \partial f_i/\partial t \) and \( \partial U/\partial t \) (since Eq. (13) provides that \( \omega \sim \kappa \)). Then Eq. (6) reduces to \( \partial J/\partial z = 0 \), which implies that \( J(z, E, t) = I_E(t) \). As a result, Eq. (4) yields

\[
f_i(z, E, t) = I_E(t) \int_z^{z_2} \frac{dz'}{D(E - U(z', t))}.
\]

The small variation of the distribution function \( \delta f_i \) can be found by linearization of this equation with respect to \( \Delta U_m, \delta U_m \), the functions \( z_1 \) and \( z_2 \) being also linearized. The Poisson equation gives us a proportionality between \( \delta U_m \) and the Fourier transform \( \delta n_m \) of the variation of electron concentration

\[
\delta n(z, t) = \delta \int_{z_1}^{z_2} dz f_i(z, E).
\]

Here the variation includes the variation of \( \delta f_i \) as well as variation of the integration limits \( E_1(z, t) \) and \( E_2(z, t) \). In this paper we restrict ourselves to the case of 2D semiconductor quantum well, assuming that the dielectric constant \( \epsilon \) is the same both inside and outside the quantum well. For such structure

\[
\delta U_m = \frac{2 \pi e^2}{\epsilon q_m} \delta n_m.
\]

Using Eqs. (15), (16) and linearized Eqs. (1), (4) one can find the relation between \( \delta N_m \) and \( \delta U_m \). To calculate the parameter \( \Delta F_m \), one should also average the
variation of potential $\delta U_m \exp(iq_m z)$ with the stationary distribution function $f_0(E,z)$ (since we solve the problem in a linear approximation). After cumbersome but straightforward calculations finally we get

$$\omega_m = s q_m + \frac{i \alpha_m |\alpha_m|^2}{W_0} \frac{\lambda_m}{1 + \lambda_m \alpha_m} \left(1 + \frac{\lambda_m}{\alpha_m} \frac{D^*}{2} \frac{q - q_m}{m} \right)^2, \tag{17}$$

where $\alpha_m = \int_0^{L_0} dy (1 - \exp(iq_m y))/y$, $\lambda_m = e^2 n_0/m e F_0$. It is easy to check that for any $m$, $\Im(\omega_m) > 0$. Thus, for $q \approx q_m$ a stationary solution is unstable. For a weak field, $\lambda_m \gtrsim 1$, the increment is field independent, $\Im(\omega_m) \sim \kappa/W_0$. One can show that for $q \approx q_m$ the spectrum reads

$$\omega(q) = \omega_m + (q - q_m) v - i \frac{D^*}{4} (q - q_m)^2, \tag{18}$$

where $D^* = D_0 \left(1 + \frac{\lambda_m/\alpha_m}{1 + \lambda_m \alpha_m} \frac{q - q_m}{m} \right)$ (we neglected small corrections of the order of $\kappa$ to $v$ and $D^*$). This implies that instability exists only in a small vicinity of $q_m$ (see Fig. 2). This instability should lead to the formation of CDW with the periods $L_0/m$.

Next we discuss a possibility of observation of the effect. The instability increment is proportional to the rate of energy loss $\kappa$, which can be calculated for electrons in 2D quantum well in full analogy with the 3D case [2].

For the case of infinitely deep rectangular quantum well of width $a$, calculations yield

$$\kappa = \frac{C_0^2 \pi^2 M}{\rho a^3 \hbar}. \tag{19}$$

Here $C_0$ is a deformation potential constant, $\rho$ is density of the crystal. This result justifies our assumption that $\kappa$ does not depend on electron kinetic energy. Also we see that $\kappa$ rapidly increases with decreasing $a$. The law $\kappa \sim a^{-3}$ can be understood from simple estimates. The momentum transfer from electron to phonon in the direction perpendicular to the quantum well is of the order of $h/a$. Emission of such a phonon leads to the energy loss $\sim hS/a$, where $S$ is the sound velocity. The energy loss rate by the emission of longitudinal phonons may be neglected due to a small factor $k_0/a$, where $k_||$ is the in-plane wave vector of 2D electron. We find that $\kappa$ is proportional to the integral over $dq_2$ of the product of energy loss $hS/a$ by the squared matrix element $V_q^2 \sim q \sim 1/a$. The upper limit of the integral is of the same order, of $1/a$, yielding $\kappa \sim a^{-3}$. This implies that the instability is more likely to be observed in thin 2D structures. On the other hand, the instability is suppressed by the electron-electron collisions, which lead to maxwellization of the EDF. Thus, the instability condition is given by $\Im(\omega_m) > 1/\tau_{ee}$, where $\tau_{ee}$ is the characteristic time of the electron-electron scattering. Crude estimate of $\tau_{ee}$ for hot electrons with characteristic energy $W_0$ gives $\tau_{ee}^{-1} \sim e^4 n_0/e^2 \hbar W_0$. Having in mind Eqs. (1), (17), one can see that for low electron densities $e^4 n_0/e^2 \hbar < \kappa$, a certain field interval exists, in which the instability can be observed. Simple estimates for GaAs and GaN show that for thin quantum wells, $a \approx 30 \AA$, the electron concentration is restricted by small but quite reasonable value $\sim 10^{10} \text{cm}^{-2}$.

In conclusion, we have presented a self-consistent theory of kinetic stratification. We have shown that the spatial periods of strata equal to $W_0/F_0 m$ and can be tuned by applied voltage.

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