Casimir Force between two Half Spaces of Vortex Matter in Anisotropic Superconductors

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Abstract

We present a new approach to calculate the attractive long-range vortex-vortex interaction of the van der Waals type present in anisotropic and layered superconductors. The mapping of the statistical mechanics of two-dimensional charged bosons allows us to define a Casimir problem: Two half spaces of vortex matter separated by a gap of width $R$ are mapped to two dielectric half planes of charged bosons interacting via a massive gauge field. We determine the attractive Casimir force between the two half planes and show that it agrees with the pairwise summation of the van der Waals force between vortices.

1 Introduction

Vortices in isotropic type-II superconductors repel each other with the interaction strength decaying exponentially on the scale $R > \lambda$ ($R$ is the distance between the vortices, while $\lambda$ denotes the London penetration length). However, it has recently been shown [1,2] that in layered superconductors the thermal fluctuations of the flux lines give rise to a long-range attraction $V_{\text{vdW}} \sim -(T/d)(\lambda/R)^4$ of the van der Waals type on the scale $R > \lambda$, where $T$ denotes the temperature and $d$ is the interlayer distance. The equivalence of three dimensional statistical mechanics with the $2 + 1$ dimensional imaginary time quantum mechanics allows us to describe the flux lines as two dimensional charged bosons with an interaction mediated by a gauge field $a$. In the case of vortex matter the material properties are mapped to a dielectric permittivity for the gauge field. Geometric boundary conditions and dielectric permittivities then cause a shift in the zero-point energy of the gauge field known as the Casimir effect. It is subject of the present work to calculate the
Casimir force between two half spaces of vortex matter separated by a vortex free region as shown in Fig. 1. It is well known [3] that in special geometries (e.g. two dielectric half spaces) van der Waals forces can be related to the Casimir force via pairwise summation. This interpretation of the Casimir force allows us to derive the van der Waals force between flux lines in anisotropic superconductors via the Casimir approach.

\[ F = \frac{\epsilon_0}{2} \int d^3x \int d^3y \ j(x) e^{-\lambda|x-y|} j(y) \]  

(1)

with the current \( j = (J, \rho) = \sum_\mu (\partial_\sigma R_\mu, 1) \delta^2 (R - R_\mu) \) describing the vortex lines. Following a suggestion of Nelson [4], the statistical mechanics of vortices can be mapped to the imaginary time quantum mechanics of two-dimensional (2D) bosons. The \( c \) axis of the superconductor is mapped to the imaginary time of the bosons \( z \to \tau \), the temperature \( T \) becomes the Planck constant \( \hbar^B \), while the interaction between the vortices is mediated by a fake gauge field \( a \) with a coupling \( g^2 = 4\pi \varepsilon_0 \), where \( \varepsilon_0 = (\Phi_0/4\pi\lambda)^2 \) is the line energy [5]. The action of the 2D bosons becomes

\[ S[j, a] = \int d\tau \sum_\mu \left\{ \frac{m}{2} (\partial_\sigma R_\mu(\tau))^2 - \mu^B \right\} + \int d\tau d^2R \left\{ i a \cdot j + \frac{1}{2g^2\lambda^2} a^2 + \frac{1}{2g^2} (\nabla \times a)^2 + \frac{c^2}{2g^2} (\nabla \times a) \right\} \]  

(2)

with the bare boson mass \( m = \varepsilon_0 \). The self-interaction of the vortices via the gauge field leads to a mass renormalization \( m \to m^B = \varepsilon_l \) where \( \varepsilon_l \) is the
dispersive line tension [6] of the vortex line. The anisotropy $\epsilon$ between the $ab$ plane and the $c$ axis is introduced by $c = 1/\epsilon$ (the light velocity in the boson system).

**Material Properties:** We can map the material properties of the vortex matter to a dielectric permittivity $\epsilon_V$ by integrating over the currents $J$ of the 2D bosons, leading to a term $(\rho/m)a_{xy}^2$ in the action. Performing functional derivatives leads to the dispersion relation for the gauge field $a_{xy}$

$$\left[\frac{\omega^2}{c^2}\epsilon_V(\omega) + k^2 + \frac{\epsilon^2}{\lambda^2}\right]a_{xy} = 0 \quad \text{with} \quad \epsilon_V(\omega) = 1 + \frac{g^2\rho}{m_B\omega^2}. \quad (3)$$

The boundary conditions at the interface between the vortex matter and the vortex free region together with the dispersion relation and the dielectric permittivity define a Casimir problem [3].

**Casimir Force:** The Casimir force of the 2D dielectric system becomes

$$f = -\frac{\hbar^2}{\pi^2} \int_0^\infty dp \int_0^\infty d\omega \frac{sp\omega^2}{c^2\sqrt{p^2 - 1}} \times \left[\left(\frac{1 + (\lambda\omega)^{-2}}{1 + (\lambda\omega)^{-2}}\right)s_e + \frac{\epsilon_V + (\lambda\omega)^{-2}}{\epsilon_V + (\lambda\omega)^{-2}}s_s\right]^2 e^{2\omega/c} - 1 \right]^{-1}, \quad (4)$$

where $p^2 = 1 + c^2k^2\omega^{-2}$, $s_e^2 = \epsilon_V - 1 + p^2 + (\lambda\omega)^{-2}$ and $s_s^2 = p^2 + (\lambda\omega)^{-2}$. This expression is determined by three frequencies: $\omega_d = \pi/d$ is an upper cut-off due to the layered structure of the superconductor, while $\omega_R = 1/\epsilon R$ derives from the geometric length, and $\omega_\lambda = 1/\lambda$ describes the mass of the gauge field.

**Pairwise Summation:** In the following we present the Casimir force for the three different length scales. The pairwise summation (i.e. the summation of the van der Waals forces between the vortex lines in each half space) then provides the van der Waals force between flux lines. For intermediate distances $R < d\epsilon^{-1}, \lambda\epsilon^{-1}$ the cut-off $\omega_d$ is relevant and we obtain the result

$$f = -\frac{(\epsilon_V - 1)^2}{16\pi^2} \frac{\hbar^2}{R^2} \omega_d \quad \Rightarrow \quad V_{\text{vdW}} = -\frac{4\epsilon_0}{\ln^2(\pi \lambda d^{-1})d\epsilon_0} \left(\frac{\lambda}{R}\right)^4. \quad (5)$$

At larger distances $d\epsilon^{-1} < R < \lambda\epsilon^{-1}$ retardation of the gauge field becomes important and the frequency $\omega_d$ is replaced by $c/R$

$$f = -\frac{19 (\epsilon_V - 1)^2}{1024\pi} \frac{\hbar^2 c}{R^3} \quad \Rightarrow \quad V_{\text{ret}}^{\text{vdW}} = -\frac{(171\pi/256)\epsilon_0}{\ln^2(\pi \lambda(\epsilon R)^{-1}) \lambda \epsilon \epsilon_0} \left(\frac{\lambda}{R}\right)^5. \quad (6)$$
At very large distances the mass of the gauge field leads to an exponential decay

\[ f = -\frac{\hbar^2 c}{R \lambda^2} \frac{8\pi \lambda^4 p^2}{e^2} e^{-\frac{R}{\lambda c}} \quad \Rightarrow \quad V_{vdW} = -4\sqrt{\pi} \varepsilon_0 T c^4 \left( \frac{\lambda}{e R} \right)^\frac{3}{2} e^{-2\frac{e R}{\lambda}}. \tag{7} \]

The van der Waals forces at intermediate and large distances agree with the derivation by Blatter and Geshkenbein [1].

**Fig. 2.** Phase diagrams: \( T_{ce}^{vdW} \) denotes a critical endpoint, \( T_{ce}^{vdW} \) is a critical point, and \( T_{T}^{vdW} \) is a triple point.

**Phase Diagram:** The attraction between flux lines leads to interesting modifications of the \( B - T \) phase diagram of anisotropic type-II superconductors, see Fig. 2 for a schematic drawing. At low temperature \( T < T_{ce}^{vdW} \) a first order phase transition takes the Meissner state into the vortex solid (with the critical field \( H_{c1} \) lowered by the attraction), while at higher temperature \( T_{c} > T > T_{ce}^{vdW} \) the Meissner state goes into a vortex gas through a second order phase transition. In the temperature range \( T_{T}^{vdW} < T < T_{c}^{vdW} \) we can distinguish a low density vortex gas from a high density vortex liquid. Concurrent to the first order jump in the magnetization a phase separation is observed and the domains of vortex matter separated by vortex free regions interact via the Casimir force described in this paper.

**References**

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