Modelling Early Age Shrinkage of Concrete

Gandla Nanabala Sreekanth, Shaik.Ruksana

ABSTRACT- During past decades the concrete has been developed by low water-cement ratio, addition of superplasticizer etc to improve the properties and lifetime of a structure. Despite of the improved properties it introduces the shrinkage which leads to cracks and reduces serviceability and durability of structure or material. Therefore the study of shrinkage becomes a challenge. In this study, the autogenous shrinkage of concrete mixtures at early age is modelled using analytical models for different water-cement ratios for different ages. The main objectives of the study are review of literature on autogenous shrinkage of concrete, comparative study of different models for autogenous shrinkage of concrete using experimental data available in literature, Identification/development of an analytical model for prediction of autogenous shrinkage of concrete, and probabilistic analysis of the autogenous shrinkage are discussed.

Keywords: Shrinkage of concrete, early age shrinkage, hydration of cement, chemical shrinkage, autogenous shrinkage, probabilistic analysis.

I. INTRODUCTION

Shrinkage is a very important parameter which is to be considered while designing because it leads to cracking that undermines the serviceability and durability of material or structure; therefore there is a need of predicting the shrinkage while designing. Shrinkage is of two phases early age and later age. The following are the different types of shrinkages which affect both early age and later age shrinkage.

a. Plastic shrinkage
The volumetric change due to loss of moisture which occurs immediately after pouring of concrete in the forms while concrete is still in plastic stage is referred as plastic shrinkage.

b. Drying shrinkage
The loss of free water contained in hardened concrete does not result in any appreciable dimension change. The change in volume of concrete due to the withdrawal of water held in gel pores causes drying shrinkage.

c. Carbonation shrinkage
Reaction between calcium hydroxide present in the concrete and carbon dioxide present in atmosphere with existence of moisture results to carbonation shrinkage. Its magnitude is negligible compared to other types of shrinkage.

d. Autogenous shrinkage
The volume reduction occurring without moisture transfer to the environment is referred as autogenous shrinkage. It is purely a result of the internal chemical and structural reactions of concrete components. The autogenous shrinkage is fully attributed to chemical shrinkage during the very first hours after mixing.

Published By: Blue Eyes Intelligence Engineering & Sciences Publication

DOI: 10.35940/ijrte.B3047.078219

3530
are observed by semi-quantitative evaluation of the XRD patterns shows that the hydration of alite and aluminate phase are relatively fast and belite and ferrite phase hydration are significantly slower. An analytical thermodynamic model is developed to predict the rate of hydration of individual clinker phases by a set of equations which describes the progress of dissolution in OPC.

Tongsheng Zhang et al. (2013) [4] investigated on comparison of ASTM C 1608 and an improved method on measurement of chemical shrinkage of cement paste. An improved method is implemented by adding a magnetic stirring bar to prevent the formation of skeleton structure of cement paste. The influence of w/c ratio of the cement paste on the chemical shrinkage was discussed clearly with seven different w/c ratios. Compared with ASTM C 1608, the improved method has superior precision, repeatability, and correlation with Paulini equation, and can be regarded as a reliable test method for chemical shrinkage measurement.

Zdenek P. Bazant (2015) [5] presented the model B4 for creep, drying shrinkage and autogenous shrinkage of normal and high strength concretes with multi-decade applicability. The prediction models are provided to predict the temperature effect on curing and aging process and depend on factors like a/c ratio, w/c ratio, and time. A simplified prediction model with parameters is presented based on strength to predict the drying and autogenous shrinkage and the calculations are explained with some numerical examples.

### III. MODELLING APPROACH

**a) Modelling of hydration of cement**

Hydration of cement is the reaction of water with cement. The hydration of cement can be predicted using Parrot and Killoh [8] approach. The approach describes the rate of hydration of the individual clinker phases by a set of equations. The rate of hydration (R) of individual clinker phase can be predicted by using the thermodynamic model given below.

\[
R_{i}(t) = \frac{k_{i} \alpha_{i}(t) \log(1 - \alpha_{i}(t))^{(1-N_{i})}}{N_{i}}
\]

Where \(\alpha_{i}(t)\) is the degree of hydration at any given time (days)

\[
\alpha_{i}(t) = a_{i}t + \Delta \alpha_{i} \text{ or } \alpha_{i}(t,w/c) = a_{i}t + \Delta \alpha_{i}w/c
\]

The influence of w/c is evaluated according to the given equation is as

\[
f(w/c) = 1 + 4.444 \div (w/c) - 3.333 \times \alpha_{i}(t) \times 1.333 \div (w/c)
\]

The empirical expressions are given by Parrot and Killoh used are together with their values of K1, N1, K2, K3 and N3 as compiled in table-1.

Table-1 Parameters from Parrot and Killoh used to calculate the hydration of the individual clinker phases as a function of time.

| Parameter | Clinkers |
|-----------|----------|
|           | Alite (C3S) | Belite (C2S) | Alumina (C3A) | Ferrite (C4AF) |
| K1        | 1.5       | 0.5       | 1.0       | 0.37       |
| N1        | 0.7       | 1.0       | 0.85      | 0.7        |
| K2        | 0.05      | 0.006     | 0.04      | 0.015      |
| K3        | 1.1       | 0.2       | 1.0       | 0.4        |
| N3        | 3.3       | 5.0       | 3.2       | 3.7        |

The total degree of hydration is evaluated by using the above equation

\[
\alpha_{i}(t) = a_{i}t + \Delta \alpha_{i} \text{ or } \alpha_{i}(t,w/c) = a_{i}t + \Delta \alpha_{i}w/c
\]

Where \(a_{i}\) is the degree of hydration at any given time (days)

\(C(i)\) is the composition of the clinker compound

\(C\) is the total clinker compositions

**b) Modelling of chemical shrinkage**

The weighted sum of the chemical shrinkage of each of the phase composition of the cement is considered as chemical shrinkage (Powers 1935). Chemical shrinkage can be calculated based on the differences in volumes between initial/basic reactants (Vr), and final hydration/ reaction products (Vr). Power suggested an empirical method based on phase composition of cement to determine the chemical shrinkage at various ages and are expressed as

\[
V_{cs} = a_{3}C_{3}S + b_{3}C_{3}S + c_{3}C_{3}A + d_{3}C_{4}AF
\]

Where,

\[V_{cs} = \text{Chemical shrinkage of the cement paste}\]

\[a,b,c,d = \text{Absorption coefficients for chemical shrinkage of individual cement phases (Table 2)}\]

\[C_{3}S, C_{3}A, etc = \text{percentage of the respective compounds}\]

The chemical shrinkage is calculated using the stoichiometry calculations. The chemical shrinkage values are as given in table-2.

Table-2 Chemical shrinkage values (i.e absorption coefficients) for individual cement phases

| Chemical shrinkage (CS) (cm²/g) | C₃S | C₃S | C₃A | C₄AF |
|-------------------------------|-----|-----|-----|------|
|                               | 0.053 | 0.04 | 0.1113 |       |
| C₃A – Ettringite               | 0.2842 |     |       |      |
| C₃A – Monosulphate            | 0.0151 |     |       |      |
| C₃A – Extra                   | 0.1785 |     |       |      |

Therefore, the total chemical shrinkage is given by the following equation

\[V_{cs} = 0.053 \times (\% \text{ of } C_{3}S) + 0.04 \times (\% \text{ of } C_{3}A) + 0.1785 \times (\% \text{ of } C_{1}A) + 0.1113 \times (\% \text{ of } C_{4}AF)\]

The chemical shrinkage at any given time (days) can be calculated with the help of the below prediction model which is a function of degree of hydration. The total chemical shrinkage or volume of chemical shrinkage at any time (days) for individual clinker phase for different water-cement ratios is given by S.Zhutovsky and K.Kovler as

\[V_{CS}^{t} = \sum_{i} V_{CS}^{t}(i) = (CS) \times [C] \times \alpha_{t}(t,w/c) \]

\[V_{CS}^{t}(i) = \sum_{i} V_{CS}^{t}(i)\]

The above equation can be explained as

\[V_{CS}^{t}(i) = \{0.053 \times (\% \text{ of } C_{3}S) \times a_{CS3}(t)\} + \{0.04 \times (\% \text{ of } C_{3}S) \times a_{CS3}(t)\} + \{0.1785 \times (\% \text{ of } C_{1}A) \times a_{CS4}(t)\} + \{0.1113 \times (\% \text{ of } C_{4}AF) \times a_{CS4}(t)\}\]

From the above equation it is found that the chemical shrinkage varies with respect to clinker compositions irrespective of time and degree of hydration.

Where \(V_{CS}^{t}(i) = \text{Total chemical shrinkage of individual clinker phase at any time (days)}\)
$$V_{cs}(t) = \text{total chemical shrinkage at any time (days)}$$

$$(CS)_i = \text{Chemical shrinkage value for individual clinker phase (from table)}$$

$$[C] = \text{composition of individual clinker phase and}$$

$$i \text{ refers the individual clinker (i.e. } i = C_3 S, C_2 S, C_3 A, C_4 AF)$$

$$\alpha_i(t) = \text{degree of hydration of individual clinkers at any given age (days)}$$

$$\alpha_i(t, w/c) = \text{a function of time and water cement ratio which is discussed.}$$

c) Autogenous shrinkage modelling

The early age autogenous shrinkage is modelled by using different analytical prediction models. Several analytical prediction models exist to evaluate or predict the autogenous shrinkage. The following are some of the analytical prediction models to predict the autogenous shrinkage at early ages.

1. Eurocode model

Both autogenous and drying shrinkage can be calculated according to BS EN 1992-1-1: 2004.

$$E_{ca} = E_{cad} + E_{css}$$

Superposition of drying $E_{cad}$ and autogenous shrinkage strain $E_{css}$ gives total shrinkage strain $E_{ca}$ whereas the autogenous shrinkage can be calculated by using following equation

$$E_{css}(t) = \beta_{cs}(t) \cdot E_{css}(\infty)$$

Where

$$E_{css}(\infty) \text{ and } \beta_{cs}(t) \text{ depending on characteristic compressive strength and time are given as}$$

$$E_{css}(\infty) = 2.5 \ast (f_{cm} - 10) \ast 10^{-6}$$

$$\beta_{cs}(t) = 1 - \exp (-0.2 \ast t^{0.5})$$

Whereas $E_{css}(t)$ is the autogenous shrinkage at any given time (days)

$$f_{cm} \text{ is the characteristic compressive strength ( N/mm}^2)$$

$E_{css}(\infty)$ is the shrinkage factor calculated using the above equation (μm/m)

2. Eurocode model (erika. Holt, 2001)

Erika holt considered the following prediction model for prediction of autogenous shrinkage strains which are considered from the draft European Standard prEN 1992-1 [Eurocode 2001]. The draft European Standard prEN 1992-1 [Eurocode 2001] is the first to include a method for predicting long term autogenous shrinkage strain, as given by Equation. The strain is based on the cement type and compressive strength of the concrete.

$$E_{ca}(t) = \beta_{cs}(t) \ast E_{css}$$

Where $E_{css}$ is autogenous shrinkage strain,

$$\beta_{cs}(t) = \text{coefficient which depends on the age of concrete given by the below equation,}$$

$$\beta_{cs}(t) = \exp \{ s \ast [1 - (\frac{t}{t_1})^{0.5}] \}$$

Where $s = \text{coefficient depending on the type of cement given in table-3}$

$t = \text{age of concrete (days), and } t_1 = 1 \text{ day.}$

$$E_{css} = 2.5 \ast (f_{cm} - 10) \ast 10^{-6} \text{, and}$$

$$f_{cm} = \text{characteristic compressive cylinder strength of concrete at 28 days.}$$

Table-3 Coefficient value (s) depending on type of cement

3. Japan society of civil engineers (jsce) model

The 1996 Japan code gives a total shrinkage consisting only of drying shrinkage for normal strength concrete with compressive strength until 55 N/mm². Autogenous deformation is included in the 2002 Japan code, for $0.20 \leq w/b \leq 0.65$ and based on mineral composition (Tazawa, 1998). Prediction of autogenous shrinkage according to the JSCE Specification 2002 is as follows

$$E_a(t) = E_{as,0} \ast \beta(t)$$

Here $E_a(t)$ is the autogenous shrinkage at any time (days) and Water to cement ratio, w/c, through $E_{as,0}$ and $\beta(t)$ and time(t in days), and time of setting, $t_s$ in days (i.e equals to 0), through $\beta(t)$. $E_{as,0} = 3070 \ast \exp \{-7.2 \ast (w/c)\} \text{ 0.2 } \leq w/c \leq 0.5$

$$E_{as,0} = 80 \text{ 0.5 } < w/c$$

$$\beta(t) = 1 - \exp \{-a \ast (t-t_s)^b\}$$

The factors, $a, b$ are determined by using the following equations which are proposed by Miyazawa & Tazawa [2005].

$$a = 3.27 \ast \exp \{- 6.83 \ast (w/c)\}$$

$$b = 0.251 \ast \exp \{2.49 \ast (w/c)\}$$

JSCE prediction models have been verified using data from RILEM and JSCE, both for the total shrinkage as for the autogenous shrinkage.

4. Ceb-mc 90-99 model

With respect to the shrinkage characteristics of high-performance concrete, the new approach for shrinkage subdivides the total shrinkage into the components of autogenous shrinkage and drying shrinkage. While the model for the drying shrinkage component is closely related to the approach given in CEB MC90 (CEB 1993), for autogenous shrinkage, new relations had to be derived. The prediction model to predict the autogenous shrinkage using CEB-MC 90-99 model is considered from ACI 209.2R-08 code. The total shrinkage of concrete $E_{sh}(t,t_c)$ can be calculated from Equation

$$E_{sh}(t,t_c) = E_{cad}(t) + E_{cas}(t,t_c)$$

Where

$$E_{cad}(t,t_c) = \text{the total shrinkage, } E_{cad}(t) = \text{the autogenous shrinkage and } E_{cad}(t,t_c) = \text{the drying shrinkage at concrete age (t days) after the beginning of drying at } t_c \text{ (days).}$$

The autogenous shrinkage component $E_{cas}(t,t_c)$ is calculated from Equation

$$E_{cas}(t) = E_{cas0} \ast f_{cm28} \ast \beta_{as}(t)$$

Where $E_{cas0}$ is the notional autogenous shrinkage coefficient from equation and $\beta_{as}(t)$ is the function describing the time development of autogenous shrinkage from equation.
Modelling Early Age Shrinkage of Concrete

\[
\varepsilon_{\text{auto}}(t_{\text{aut}}) = -\alpha_{\text{au}} \left( \frac{t_{\text{f}}}{t_{\text{f}} + t_{\text{aut}}} \right)^{2.5} \times 10^6
\]

\[
\beta_{\text{au}}(t) = 1 - \exp \left( -0.2^* \left( \frac{t}{t_1} \right)^{0.5} \right)
\]

Where \( f_{\text{em,28}} \) is the mean compressive strength of concrete at an age of 28 days (MPa or psi)

\( f_{\text{em,28}} = 10 \text{ MPa} \) (1450 psi), \( t \) is the concrete age (days), \( t_1 = 1 \) day, and \( \alpha_{\text{au}} \) is a coefficient that depends on the type of cement (Table 4).

Table 4: Coefficients according to equation, CEB MC 90-99 model

| Type of cement according to EC2 | \( \alpha_{\text{au}} \) |
|---------------------------------|------------------|
| SL (slowly-hardening cements)   | 800              |
| N or R (normal or rapid hardening cements) | 700          |
| RS (rapid hardening high-strength cements) | 600          |

5. **B4 model**

The B4 model is a new prediction model for shrinkage is presented and autogenous shrinkage is predicted by using the above B4 model which is the modification of Bazant B3 model which includes the autogenous shrinkage along with drying shrinkage in total shrinkage. The B4 model for prediction of autogenous shrinkage is given by Zdenek P. Bazant (2015) [5] as follows:

\[
\varepsilon_{\text{au}}(\bar{t}, \bar{t}_o) = \varepsilon_{\text{auto}} = \left[ 1 + \left( \frac{t_{\text{f}}}{t_{\text{f}} + t_{\text{au}}} \right)^{\varepsilon_{\text{w}}} \right]^{\varepsilon_{\text{w}}},
\]

\[
\varepsilon_{\text{auto}} = \frac{w/c}{0.38} \frac{a/c}{0.6} \alpha_{\text{au}} \frac{w/c}{0.38} \tau_w
\]

\[
\tau_{\text{au}} = \frac{\tau_{\text{w}}}{0.27}
\]

Where \( \varepsilon_{\text{auto}} \) is the final autogenous shrinkage and \( \tau_{\text{au}} \) is the autogenous shrinkage halftime.

The parameters \( \varepsilon_{\text{au}}, \tau_{\text{au}}, \tau_{\text{w}} \), as well as the exponents \( r_{\text{w}}, r_{\text{au}}, \tau_{\text{au}} \), are taken from table-6.

\[
\tau_o = (t - t_o) \alpha_{\text{au}}, \beta_{\text{au}}, \beta_{\text{au}} = \exp \left( \frac{U_h}{R} \left( \frac{1}{293} - \frac{1}{T_{\text{em,28}+273}} \right) \right),
\]

\[
\bar{t} = (t - t_o) \beta_{\text{au}}, \bar{t}_o = \exp \left( \frac{U_h}{R} \left( \frac{1}{293} - \frac{1}{T_{\text{em,28}+273}} \right) \right)
\]

for any constant temperature \( T_{\text{em,28}} \) [20°C, 30°C]

\( T_{\text{em,28}} \) = temperature at curing (all the temperatures are here given in °C);

\( U_h \) is the activation energy of hydration and \( U_t \) is the activation energies of moisture diffusion and the duration of drying is \( t - t_d \).

\[
U_h / R = U_t / R = 4000 \text{ K and } R \text{ is the gas constant.}
\]

Note, temperature \( T \) in \( \beta_{\text{au}} \) and \( \beta_{\text{au}} \) corresponds to the average environmental temperature before and after load application respectively. When the temperature is 20°C, the equivalent times reduce to actual times and durations (i.e., \( \bar{t}_o = t_o \) and \( \bar{t} = t - t_o \)).

Table 5: Autogenous shrinkage parameters depending on cement type for B4

| Parameter | R | RS | SL |
|-----------|---|----|----|
| \( \tau_{\text{au,em}} \) (days) | 1.00 | 41.00 | 1.00 |

IV. RESULTS AND DISCUSSION

The results of degree of hydration chemical shrinkage, and autogenous shrinkage predictions for different water cement ratio for different ages are presented. The comparison of predicted values with experimental values of different researchers for different water-cement ratios and modeling uncertainty is also presented. The mineral composition considered for predictions are given in table-7.

Table 7: Composition of individual clinker phases

| Clinkers | C\(_3\)S | C\(_2\)S | C\(_3\)A | C\(_{3AF}\) |
|----------|---------|---------|---------|---------|
| Composition (%) | 60.74 | 10.65 | 7.94 | 8.13 |

Strength-based model for simplified design (B4 with fck)

To estimate shrinkage solely from the chosen required strength \( f_c \) of concrete to be used in the structure. An simplified variant of model B4 using the mean compressive strength \( f_c \) has been developed by means of statistical optimization of the fit of the new NU database (it should be noted that the average strength, \( f_c \), is significantly higher than \( f_0 \); typically, \( f_c = f_c' + 8.3 \text{ MPa or } f_c = f_c' + 8 \text{ MPa} \))

Autogenous shrinkage \( \varepsilon_{\text{au}}(\bar{t}, \bar{t}_o) \) is given by the above equation

\[
\varepsilon_{\text{au}}(\bar{t}, \bar{t}_o) = \varepsilon_{\text{auto}} \left[ 1 + \left( \frac{t_{\text{f}}}{t_{\text{f}} + t_{\text{au}}} \right)^{\varepsilon_{\text{w}}} \right]^{\varepsilon_{\text{w}}}
\]

Final autogenous shrinkage \( \varepsilon_{\text{auto}}(\bar{t}, \bar{t}_o) \) and Autogenous shrinkage halftime \( (\tau_{\text{au}}) \) are given by

\[
\varepsilon_{\text{auto}}(\bar{t}, \bar{t}_o) = \varepsilon_{\text{auto}} \left[ 1 + \left( \frac{t_{\text{f}}}{t_{\text{f}} + t_{\text{au}}} \right)^{\varepsilon_{\text{w}}} \right]^{\varepsilon_{\text{w}}}
\]

Where \( \varepsilon_{\text{auto}} \) is the final autogenous shrinkage and \( \tau_{\text{au}} \) is the autogenous shrinkage halftime.

The parameters \( \varepsilon_{\text{auto}}, \tau_{\text{au,em}} \), as well as the exponents \( r_{\text{w}}, r_{\text{au}}, \tau_{\text{au}} \), are taken from the table-6 and \( \bar{t}, \bar{t}_o \) are calculated by using the above given equations

| Parameter | R | RS | SL |
|-----------|---|----|----|
| \( \tau_{\text{au,em}} \) (days) | 2.26 | 0.27 | 78.2 \times 10^6 |
| \( r_{\text{w}} \) | 1.03 | 1.73 | -1.73 |

Table 6: Autogenous shrinkage parameters for B4 with fck

For regular cement (R), rapid hardening cement (RS), and slow hardening cement (SL)
The degree of hydration of individual clinkers is presented in figure-1. It shows that C₃S and C₃A is faster whereas C₄AF hydrates moderate and C₂S hydrates very slowly.

**a) Degree of hydration**

Figure-1 Degree of hydration of all individual clinker phases for w/c ratio of 0.4

**Total degree of hydration**

From the figure-2, it can be observed that the total degree of hydration increases with respect to water cement ratio and time. It hydrates around 35% for the first day only and gradually increases to 60% for w/c of 0.3, 65% for w/c of 0.4, and 75% for w/c ratio of 0.5 at age of 28 days and the results are given in table-8.

![Figure-9 Total degree of hydration for different w/c ratios](image)

Table-8 Total degree of hydration for different w/c ratios for different ages.

| Time (days) | W/c Ratio | Degree of Hydration (%) |
|------------|-----------|-------------------------|
| 1          | 0.3       | 35.87                  |
| 3          | 0.3       | 36.054                 |
| 7          | 0.3       | 36.054                 |
| 28         | 0.3       | 36.054                 |
| 1          | 0.4       | 46.7668                |
| 3          | 0.4       | 52.8972                |
| 7          | 0.4       | 54.4632                |
| 28         | 0.4       | 54.4632                |
| 1          | 0.5       | 51.966                 |
| 3          | 0.5       | 53.511                 |
| 7          | 0.5       | 53.511                 |
| 28         | 0.5       | 53.511                 |
| 1          | 0.6       | 45.839                 |
| 3          | 0.6       | 51.966                 |
| 7          | 0.6       | 53.511                 |
| 28         | 0.6       | 53.511                 |

**b) Chemical shrinkage values**

The total chemical shrinkage versus time is shown in figure-3 which shows as increase in water-cement ratio the total chemical shrinkage also increases gradually with respect to time (days) and total chemical shrinkage values are given in table-9.

![Figure-3 Total chemical shrinkage at different water-cement ratios](image)

Table-9 Total chemical shrinkage for different w/c ratios for different ages.

| Time (Days) | W/c Ratio | Chemical Shrinkage (%) |
|------------|-----------|------------------------|
| 1          | 0.3       | 5.574                  |
| 3          | 0.3       | 5.1114                 |
| 7          | 0.3       | 4.858                  |
| 28         | 0.3       | 5.1114                 |
| 1          | 0.4       | 5.839                  |
| 3          | 0.4       | 5.359                  |
| 7          | 0.4       | 5.359                  |
| 28         | 0.4       | 5.359                  |
| 1          | 0.5       | 6.389                  |
| 3          | 0.5       | 6.482                  |
| 7          | 0.5       | 6.482                  |
| 28         | 0.5       | 6.482                  |
| 1          | 0.6       | 6.8873                 |
| 3          | 0.6       | 7.4193                 |
| 7          | 0.6       | 7.4193                 |
| 28         | 0.6       | 7.4193                 |

**c) Autogenous shrinkage strains**

The autogenous shrinkage strains are predicted from the Eurocode prediction models and results are presented in table-10.

![Figure-4 Autogenous shrinkage strains at different water-cement ratios](image)

Table-10 Autogenous strain for different w/c ratios for different ages.

| Time (Days) | Water-cement ratio | Autogenous Shrinkage Strain (10^6) |
|------------|--------------------|-----------------------------------|
| 1          | 0.3                | 16.330                            |
| 3          | 0.3                | 62.349                            |
| 7          | 0.3                | 160.76                            |
| 28         | 0.3                | 280.26                            |
| 1          | 0.4                | 2.2288                            |
| 3          | 0.4                | 31.616                            |
| 7          | 0.4                | 98.026                            |
| 28         | 0.4                | 280.26                            |
| 1          | 0.5                | -5.1114                           |
| 3          | 0.5                | -31.616                           |
| 7          | 0.5                | 98.026                            |
| 28         | 0.5                | 280.26                            |
| 1          | 0.6                | -5.1114                           |
| 3          | 0.6                | -31.616                           |
| 7          | 0.6                | 98.026                            |
| 28         | 0.6                | 280.26                            |

From figure-4 it can be observed that the autogenous shrinkage gradually increases with respect to time. The water cement ratio has an influence on autogenous shrinkage; it increases with decrease of water cement ratio. As w/c decreases, the initial spacing between particles decreases, denser structures lead to increasing magnitude of autogenous stresses induced by the meniscus in partially filled capillaries. Expansive processes are also influenced by w/c, but are in case of lower w/c overwhelmed by the magnitude of the shrinkage.

**d) Comparison with prediction models**

The results predicted from the prediction models are compared with the experimental values with different prediction models. The experimental values are considered from the data sheets of the experimental works performed by the different researchers which are discussed earlier in literature. The autogenous shrinkage at early age for different water-cement ratios at different ages is predicted by using different analytical models like Eurocode, Eurocode (Erika), Jsce, CEB MC90-99, B4 models which are discussed.
Modelling Early Age Shrinkage of Concrete

Figure 5 shows the Eurocode model predicted values gives nearer values with experimental data. The experimental data is considered from the experiments conducted by researcher AIF.

Figure 6 shows that the B4 model with fck model predicted values coincides well with experimental data. The experimental data is considered from the experiments conducted by researcher LEE (2006).

Figure 7 shows that the B4 with fck and B4 model predicted values gives nearer values with experimental data. The experimental data is considered from the experiments conducted by researcher LEE (2006).

From the above table 11, 12, 13 and 14 gives the autogenous shrinkage strain for different models which are measured by different investigators. The modelling error values which can be used to select the prediction model for different w/c ratios are given in tables 11, 12, 13 and 14. It can be observed that for overall average, w/c 0.3-0.4 and w/c ratio>0.4 values the B4 model gives the less uncertainty values than other models. For w/c ratio<0.3 the Eurocode and Jsce models can be considered for prediction.

Table 11 Calculation of modelling error of autogenous shrinkage strain of different investigators with different models

| Sl. No | Investigator | T dry | Modelling Errors (overall average) |
|-------|--------------|-------|-----------------------------------|
|       |              |       | Euro(Erika) | Euro | CE B M C 90-99 | Jsc | B4 with Fck | B4 |
|       |              |       | 0.6069 | 1.0 | 0.3 | 0.9 | 0.7 | 0.6 | 0.9 |
| 1     | AIF          | 1 day |       |     |     |     |     |     |     |
| 2     | Lee(2003)    | 2 5 day | 2.6232 | 4.5 | 1.5 | 1.5 | 1.5 | 3.1 | 1.7 |
| 3     | Lee(2006)    | 1 day | 1.7593 | 3.0 | 0.8 | 3.9 | 1.6 | 1.8 | 873 |
| 4     | Eickshen     | 1 day | 0.3860 | 0.6 | 0.7 | 0.7 | 0.7 | 0.3 | 0.8 |
| 5     | Larran       | 1 day | 0.8296 | 1.3 | 0.6 | 0.5 | 1.0 | 0.3 | 0.4 |
| 6     | Zhang        | 1 day | 1.8218 | 1.1 | 0.4 | 0.3 | 0.6 | 0.1 | 859 |
| 7     | Assie        | 1 day | 2.1004 | 2.8 | 0.8 | 2.5 | 1.3 | 2.7 | 155 |

Table 12 Calculation of modelling error of w/c<0.3 of autogenous shrinkage strain of different investigators with different models

| Sl. No | Investigator | T dry | Modelling Errors (average for w/c<0.3) |
|-------|--------------|-------|--------------------------------------|
|       |              |       | Euro(Erika) | Euro | CE B M C 90-99 | Jsc | B4 with Fck | B4 |
| 1     | AIF          | 0.4652 | 0.38 | 0.1 | 0.4 | 0.0 | 984 |

Published By: Blue Eyes Intelligence Engineering & Sciences Publication

Retrieval Number: B3047078219/19©BEIESP
DOI: 10.35940/ijrte.B3047.078219
Kurtosis characterizes the relative peak or flatness of a distribution compared with the normal distribution. Positive kurtosis indicates a relatively peak distribution. Negative kurtosis indicates a relatively flat distribution. Figure 8 contains the kurtosis for autogenous shrinkage strain of different prediction models with respect to time (days). The B4 model has high peak value with high kurtosis and Jsce model have some variation of positive kurtosis. If the variation in peakness is constant then it shows normal distribution for Eurocode model.

Table-14 Calculation of modelling error for w/c>0.4 of autogenous shrinkage strain of different investigators with different models

### Table-13 Calculation of modelling error for w/c ratio (0.3-0.4) of autogenous shrinkage strain of different investigators with different models

| Sl. No | Investigator  | T dry | Modelling Errors (average for w/c of 0.3-0.4) |
|--------|---------------|-------|-----------------------------------------------|
|        |               |       | Euro( Erika) | Euro | CE B M C 90-99 | Jsc e | B4 with Fck | B4 |
| 1      | AIF           | 1 day | 0.2199       | 0.3 | 0.0238        | 0.1 | 0.2 | 0.0274 | 0.0 |
| 2      | Lee(2003)     | 0.25 day | 5.1525       | 5.1 | 0.02384       | 0.0 | 0.2 | 0.0274 | 0.0 |
| 3      | Lee(2006)     | 1.2 day | 1.3082       | 2.2 | 0.0628        | 0.0 | 0.2 | 0.0604 | 0.0 |

Table-14 Calculation of modelling error for w/c>0.4 of autogenous shrinkage strain of different investigators with different models

![Kurtosis for different autogenous prediction models](image)

**Figure 8** Kurtosis for different autogenous prediction models

| Sl. No | Investigator | T dry | Modelling Errors (average for w/c>0.4) |
|--------|--------------|-------|---------------------------------------|
|        |              |       | Euro( Erika) | Euro | CE B M C 90-99 | Jsc e | B4 with Fck | B4 |
| 1      | AIF          | 1 day | 1.6644      | 3.0 | 0.92384       | 0.0 | 0.2 | 1.7 | 4.2 |
| 2      | Lee(2003)    | 0.25 day | 1.2552      | 2.1 | 0.52384       | 0.0 | 0.2 | 1.7 | 4.2 |
| 3      | Lee(2006)    | 1 day | 1.3478      | 2.3 | 0.52384       | 0.0 | 0.2 | 1.7 | 4.2 |
| 4      | Eickshen     | 1 day | 0.4294      | 0.7 | 0.72384       | 0.0 | 0.2 | 1.7 | 4.2 |
| 5      | Larrand      | 1 day | 2.6295      | 3.4 | 0.892384      | 0.0 | 0.2 | 1.7 | 4.2 |

**e) Probabilistic analysis**

**(i) Statistical analysis**

Kurtosis characterizes the relative peak or flatness of a distribution compared with the normal distribution. Positive kurtosis indicates a relatively peak distribution. Negative kurtosis indicates a relatively flat distribution. Figure 8 contains the kurtosis for autogenous shrinkage strain of different prediction models with respect to time (days). The B4 model has high peak value with high kurtosis and Jsce model have some variation of positive kurtosis. If the variation in peakness is constant then it shows normal distribution for Eurocode model.
Coefficient of variance (CV) can be used to determine how much variance there is in the data. The CV is particularly useful to compare results from two different surveys or tests that have different measures or values. Figure 9 show that the Eurocode model has less variation than B4 and Jsce models.

Skewness characterizes the degree of asymmetry of a distribution around its mean. Positive skewness indicates a distribution with an asymmetric tail extending toward more positive values. Figure 10 contains the skewness of different prediction models in which Eurocode model prediction values zero skewness because the probability distribution of Eurocode prediction values has normal distribution whereas B4 and Jsce models have a positive skewness.

Figure 11, 12, and 13 gives the comparison of mean values of prediction values of B4 model, Jsce model and Eurocode models with experimental data.

(ii) Probabilistic analysis
The observed data of different prediction models are presented in the form of histograms as shown in figure-15, figure-16 and figure-17.

Figure-14 contains the comparison of mean values of different prediction models. B4 model prediction values have high mean values than Jsce and Eurocode models.
Whereas the distribution $D_n$ is considered as $D_n = \max [F(x) - S_n(x)]$

$F(x)$ is the Cumulative Distribution Function values of the model and

$S_n(x) = n/(n+1)$ ; $n$ is the number of observations (i.e $n = 10000$ ) and $D_{n,a} = 1.36/\sqrt{n} = 0.0136$

Table-15 Kolmorgov – smirnov goodness fit test values for different prediction models

| Types of distributions | Difference ($D_n$) | Eurocode model | JSCE model | K-S allowable value (1.36 / $\sqrt{N}$) |
|------------------------|-------------------|----------------|------------|----------------------------------------|
| B4 model | Time at 0.1 day | Time at 3 days | Time at 0.1 day | Time at 3 days |
| Normal | 0.19 | 0.00 | 0.35 | 0.01 | 36 |
| Log normal | 0.02 | --- | --- | 0.00 | 36 |
| Exponential | 0.16 | --- | --- | --- | 36 |
| Gamma | 0.09 | --- | --- | --- | 36 |

From table-15 it can be observed that for B4 model gamma distribution values are nearer to the goodness fit condition whereas for Eurocode model, the normal distribution and for Jsce model the log normal distribution fits the condition well.

(i) For B4 model

The goodness fit test is applied to the different probability distributions of B4 model and are presented in table-15 to select the probability distribution which fits the B4 model.
The gamma distribution values are less and fit the data in the best possible way and figure 18 contains the variation of gamma distribution of cdf values for different ages of B4 model. Figure 19 contains the comparison of different types of probability distributions of B4 model.

Figure 18 Gamma distribution of B4 model at different ages
Figure 19 Comparison of different types of distributions

(ii) For Eurocode model

The goodness fit test is applied to the different probability distributions of Eurocode model and are presented in table-15 to select the probability distribution which fits the Eurocode model. The normal distribution values fit the data in the best possible way and figure 20 contains the variation of normal distribution of cdf values for different ages of Eurocode model. Figure 21 contains the comparison of different types of probability distributions of Eurocode model.

Figure-20 Normal distribution of Eurocode model at different ages
Figure-21 Normal distribution of Eurocode model

(iii) For Jsce model

The goodness fit test is applied to the different probability distributions of Jsce model and are presented in table-15 to select the probability distribution which fits the Jsce model. The lognormal distribution values fit the data in the best possible way and figure-22 contains the variation of lognormal distribution of cdf values for different ages of Jsce model. Figure-23 contains the comparison of different types of probability distributions of Jsce model.

V. SUMMARY AND CONCLUSIONS

In the present research work, the early age shrinkage of concrete is modelled. From the model for early age shrinkage, the simulations are performed using Excel software. The following major conclusions can be drawn.
As w/c ratio increases the degree of hydration increases in a linear pattern, showing the importance of moisture for the hydration process.

The calculations of chemical shrinkage can be performed using the stoichiometry of the hydration reaction and the density of hydration products and reactants.

Specific chemical shrinkage depends on w/c ratio and degree of hydration: the lower w/c ratio the higher chemical shrinkage, and the higher the degree of hydration the higher is the chemical shrinkage.

The autogenous shrinkage is influenced by water cement ratio; it increases with decrease of water cement ratio. The initial spacing between particles decreases with decrease in w/c ratio, denser structures lead to increasing magnitude of autogenous stresses induced by the meniscus in partially filled capillaries.

The autogenous shrinkage strain of different prediction models are compared with the experimental data from the literature and the modelling uncertainty is presented which can be used to select the prediction model for different w/c ratios.

From the modelling error values, the following prediction models can be considered to predict the autogenous shrinkage strains based on w/c ratios; B4 model can be used for w/c of 0.3-0.4 and w/c >0.4 and the Eurocode and Jsce models for w/c ratio<0.3.

The probabilistic analysis is performed to estimate the effect of variations of input data and different types of probabilistic distributions like normal, lognormal, exponential, and gamma distributions are performed to the prediction models to explain the effect of variations.

The prediction models are computed for goodness of fit test to measure the variation or uncertainty with the help of kolmogrov-smirnov test using different probabilistic distributions and the values are presented.

For B4 model the kolmogrov-smirnov goodness fit test gives allowable values for gamma distribution when compared with normal, lognormal and exponential distribution and the values are presented.

For Jsce model the kolmogrov-smirnov goodness fit test gives allowable values for lognormal distribution when compared with normal distribution.

The normal distribution shows allowable values of kolmogrov-smirnov goodness fit test for Eurocode model.

ACKNOWLEDGMENT

I acknowledge the support of Dr. M.B.Anoop, Senior Principal scientist in CSIR- Structural Engineering Research centre, Chennai for his continued support and knowledge sharing.

REFERENCES

1. Ei-ichi Tazawa et.al (1995), Chemical shrinkage and autogenous shrinkage of hydrating cement paste, Cement and Concrete Research, Vol. 25, No. 2, pp. 288-292.
2. Erika E. Holt (2001), Early age autogenous shrinkage of concrete, V IT Publications.
3. Barbara Lothenbach and Frank winnefeld (2005), Thermodynamic modelling of the hydration of Portland cement, Cement and Concrete Research 36, 209 – 226.
4. Tongsheng Zhang, Peng Gao, Ruifeng Luo, Yiqun Guo, Jiangxiong Wei, Qiju Yu (2013), Measurement of chemical shrinkage of cement paste: Comparison study of ASTM C 1608 and an improved method, Construction and Building Materials 48, 662–669.
5. Zdenek P. Bazant (2015), Model B4 for creep, drying shrinkage and autogenous shrinkage of normal and high-strength concretes with multi-decade applicability, Materials and Structures, 48:753–770.
6. L.J. Parrot, D.C. Killoh, Prediction of cement hydration, Br. Ceram. Proc. 35 (1984) 41-53.
7. Japan Concrete Institute, Technical Committee on Autogenous Shrinkage of Concrete “Committee Report,” Autogenous Shrinkage of Concrete, edited by Eiichi Tazawa, E & FN Spon, London, 1999, pp. 1 - 62.
8. ACI 209-92, “Prediction of Creep, Shrinkage, and Temperature Effects in Concrete Structures,” Committee 209, American Concrete Institute, Michigan, 1997.

AUTHORS PROFILE

GANDLA NANABALA SREEKANTH is presently working as assistant professor in the school of civil engineering at Rajeev Gandhi Memorial College of Engineering and Technology. He has expertise in the Structural Dynamics.

SHAIK.RUKSANA is pursuing M.Tech in Structural Engineering from Rajeev Gandhi Memorial College of Engineering and Technology, NANDYAL, A.P. She completed her B.Tech in Civil Engineering from JNTUA in 2016.