A Optimal Dimension Parameters Design of Needle Roller Bearings Considering Multi Factors Affecting Life

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Abstract. Taking into many influencing factors, a Needle Roller Bearings life calculation model was established using the Influence coefficient method and ISO formula. Take this model as the optimization objective function, some optimization variables were determined, and these variables were added constraint conditions. Using dynamic nonlinear strategy to improve the inertia weight, the arccosine strategy to adjust the learning factor, function constraints was solved by the methods of ensure particle legitimacy. Nonlinear optimization design of cylindrical roller bearings was realized by Improved Particle Swarm Algorithm. The proposed method of bearing design was verified effective by test of the optimization results.

1. Introduction
The traditional method is based on obtain the maximum rated dynamic load of the bearing as target. As the literature [1], based on rated dynamic load as objective function, the design parameters include the bearing pitch diameter, the rolling element diameter, number of rolling elements. Not only the inner structure of bearing is very complex, but also the work environment of bearings is more complex, so the results obtained by this method may not be optimal. the literature [2], a multi objective optimization design has been actualized for radial ball bearing that the lubrication factor was considered, based on rated dynamic load, rated static load, minimum oil film thickness as objective function, the design parameters include the bearing pitch diameter, the rolling element diameter, number of rolling elements, inner and outer raceway curvature coefficient. The literature [3], an optimum design of roller bearing was actualized which consider the influence of roller edge stress and the objective function is LP life formula, the design parameters include the number of rolling elements, rolling element diameter, bearing pitch diameter, roller length, roller modification parameters. However, the bearing life is influenced many factors, it is not enough to consider only one factor. As mentioned in literature [4], the effects of different materials and contaminating particles on the bearing life are discussed emphatically. The influence of roller inclination for the contact stress of bearing were discussed in the literature [5], the impact of bearing surface roughness on bearing life were discussed in the literature [6,7]. In literature [8], when the roller was modified and inclined, the effect of the Hertz stress on the life of bearing were emphatically discussed. To effective design the bearings based on the working conditions which given by user, it is necessary to establish a bearing life model considering the various influence factors, rather than only consider a single influence factor. In this paper, an improved slice method is used to establish the quasi-static model considering the inclination, clearance, roller modification shape, load and speed, the calculation results are substituted into the bearing life formula of ISO for the calculation of bearing
life, the calculation result as the optimization objective function finally. The optimal initial clearance of
the bearing and the modification parameters of roller shape are obtained by using the improved particle
swarm optimization algorithm. Finally, the bearing life test shows that the optimization design method
is very effective.

2. Considering multi-factor of needle roller bearing life model

![Fig. 1 Roller Bearing Force Analysis](image1)

![Fig. 2 Section of modified roller](image2)

The figure 1 is a force diagram of cylindrical roller bearings under the inclination condition, $F_c$ is
the centrifugal force of the roller, $Q_{jn}$ and $Q_{jw}$ are the force on the $j$-th roller that were applied for
the inner and outer ring. According to mechanical balance, get the formula (1):

$$Q_{jn} - Q_{jw} - F_c = 0$$  \hspace{1cm} (1)

As shown in Figure 2, the roller is divided into $N$ sections, the force on the roller can be expressed
by the formula (2), $P_{in(iw)}$ is the contact stress for the $j$-th roller and $i$-th sections contact with the inner
and outer ring. $a_i$, $b_i$ and $a_w$, $b_w$ are half the length and width of the contact area rectangle for the $j$-th roller and $i$-th sections contact with the inner and outer ring. Roller elastic deformation formula is (3), $D_{ij}$ is the deformation influencing coefficient of the $i$-th sections contact stress to $j$-th, $Z_i(y_i)$ is the gap between the roller of $i$-th section, the inner and outer ring, it can be used to represent the shape of roller. $\delta_{in(w)}$ is the deformation of the $i$-th section relative to the inner and outer ring, its value can be calculated according to the method given by [9] when the tilt and the clearance value are taken into account. In the formula (4), $E_1$ and $E_2$ are the elastic modulus of the contact material. $V_1$ and $V_2$ are the Poisson's ratio of the two-contact material. According to the balance relation of the radial force of the bearing, the formula (5) is obtained.

\[
\pi \sum_{i=1}^{n} a_{in(u)} b_{in(w)} p_{in(w)} = Q_{j_{in(u)}}
\]  

(2)

\[
\frac{1}{\pi E'} \sum_{j=1}^{n} p_{in(w)} D_{ij} = \delta_{in(u)} - Z_i(y_i)
\]  

(3)

\[
E' = \frac{E_1}{1-V_1^2} + \frac{E_2}{1-V_2^2}
\]  

(4)

\[
F_r - \sum_{j=1}^{N} Q_{jw} \cos \phi_j = 0
\]  

(5)

Simultaneous equations (1), (2), (3). The contact stress of each section of roller are obtained. Set the iteration termination condition using Eq. (5). The contact stress distribution and the load of the needle roller were obtained by the Newton iteration method. Based on literature [10], calculation formula of bearing life:

\[
L_{num} = a_1 \left\{ a_{ISO} \left( \frac{e_{sw}}{P_{ks}}, \kappa \right) \right\}^{9/8} \left\{ \left( \frac{q_{kii}}{q_{kii}} \right)^{-4.5} + \left( \frac{q_{kee}}{q_{kee}} \right)^{-4.5} \right\}^{8/9}
\]  

(6)

In which

\[
q_{kii} = \left[ \bar{f} \sum_{j=1}^{N} \left( f_{j[k]} q_{j,k} \right)^{4.5} \right]^{1/4.5}
\]

(7)

\[
q_{kee} = \left[ \bar{f} \sum_{j=1}^{N} \left( f_{j[k]} q_{j,k} \right)^{4.5} \right]^{1/4.5}
\]

(8)

\[
f_{c,j[k]} = \left[ \left( \frac{P_{in(u)}}{271} \right)^2 D_{sw} (1 + \gamma) \frac{L_{kee}}{n_s} \right] / q_{j,k}
\]

(9)
\[ P_{ks} = 0.323Zn \left[ \frac{q_{kei}}{q_{ce}}^{9/2} + \left( \frac{1038q_{i}}{q_{ce}} \right)^{9/2} \right] \left( 1 + \left( \frac{1038q_{i}}{q_{ce}} \right)^{9/2} \right)^{2/9} \]

\[ \gamma = \frac{D_w}{D_m} \]  

\[ a_{ISO} \] is bearing integrated life correction factor, \( e_c \) is contamination coefficient, \( c_{fr} \) is fatigue load limit, the above coefficients can be calculated according to the literature [11]. \( a_1 \) is reliability correction factor, \( P_{ks} \) is equivalent dynamic load for each piece of roller, \( k \) is viscosity ratio that can be calculated by formula (12), the above coefficients can be calculated according to the literature [12],

\[ k = \left( \frac{h_{min}}{\sqrt{s_1^2 + s_2^2}} \right)^{1.3} \]  

\( h_{min} \) is the minimum film thickness, \( R_{s1}, R_{s2} \) are the surface roughness of two contact bodies, \( q_{i}, q_{ce} \) are the basic dynamic load rating for each section of the inner and outer rings, \( q_{kei}, q_{kee} \) are the equivalent dynamic load for each section of the inner and outer rings, \( f_{e,j,k} \) is stress correction function of bearing outer ring and inner ring considering edge load. \( q_{j,k} \) is load for the \( j \)-th roller and \( i \)-th sections contact with the inner and outer ring.

A multi-influencing factors of needle roller bearing life calculation model could be established by formulas (1) - (11), the main factors include internal and external tilt factors, needle shape modification factor, lubrication factor, pollution factor, speed factor, load factor, clearance factor, roughness factor, material factor.

3. Optimization model

3.1. the objective function

The traditional bearing design method is based on the maximum rated dynamic load as the objective function. However, needle roller bearing manufacturers more and more need to be based on customer needs to design the production of needle roller bearings with the increase of the application of needle bearing in recent years. But the design of the internal structure size of the bearing is related to tilt factors, needle shape modification factor, lubrication factor, contamination factor, speed factor, load factor, clearance factor, roughness factor, material factor and then the roller bearing life was affected, therefore the bearing must be designed according to the specific working conditions. In this paper, the bearing life under the influence of multiple factors is taken as the objective function to realize the optimal design. So the optimization model can be expressed as:

\[ \max f(x) = \max[L_{min}] \]  

(13)
3.2. optimal variables
The bearing life are not only related to the basic dimensions of the needle roller, but also the initial clearance \( \Delta_0 \) and modification parameters of needle roller. The modification parameters include Straight length \( L_s \) and Crowning. As the basic parameters of the bearing is generally the standard value, therefore the optimization variable can be determined as:

\[
X = [\Delta_0, L_s, \delta]^T
\]

### 3.2.1. In the bearing life calculation.
the clearance is generally referring to the working clearance, but in the bearing operation, the first known only the initial clearance. It is necessary to calculate the working clearance \( \Delta_0 \) according to the initial \( P_d \) clearance which can be calculated in accordance with formula:

\[
P_d = \Delta_0 - \delta_e - \delta_i - \delta_i
\]

\( \delta_i, \delta_e \) are the influence quantity of bearing assembly on bearing clearance. \( \delta_i \) is the influence quantity of the operating temperature on the bearing clearance.

Then

\[
\Delta_{0_{\text{min}}} \leq \Delta_0 \leq \Delta_{0_{\text{max}}}
\]

\[
g_9(X) = P_d + \delta_e + \delta_i - \Delta_{0_{\text{min}}} \geq 0
\]

\[
g_{10}(X) = \Delta_{0_{\text{max}}} - (P_d + \delta_e + \delta_i + \delta_i) \geq 0
\]

### 3.2.2. When the roller contacted with the inner and outer ring.
the edge stress will be generated, in order to reduce the edge stress, it is necessary to modify the linear roller to achieve uniform stress and long bearing life. According to the experience, the roller profile correction generally has the following selection range: \( 0 \leq \delta \leq 0.008 \) mm. Because of the limitation of the processing technology of needle roller, it is generally used to arc repair form, then:

\[
0 \leq \sqrt{R^2 - \left(\frac{L_{we}}{2}\right)^2} - \sqrt{R^2 - \left(\frac{L_e}{2}\right)^2} \leq 0.008
\]

That is:

\[
g_{11}(x) = \sqrt{R^2 - \left(\frac{L_{we}}{2}\right)^2} - \sqrt{R^2 - \left(\frac{L_e}{2}\right)^2} \geq 0
\]

\[
g_{12}(x) = 0.008 - \left(\sqrt{R^2 - \left(\frac{L_{we}}{2}\right)^2} - \sqrt{R^2 - \left(\frac{L_e}{2}\right)^2}\right) \geq 0
\]

The needle of straight line length generally adopts the following method:

\[
0 < L_i \leq L_{we}
\]
Then:
\[ g_{13}(x) = L_s > 0 \]  
\[ g_{14}(x) = L_{we} - L_s \geq 0 \]  

4. Optimization design process

When the bearing life were calculated, it is necessary to calculate the load distribution. This calculation requires an iterative method to solve the nonlinear equations, and then the minimum oil film thickness was calculated. Enter other parameters required for bearing life, and finally that need to cycle into the load distribution calculation data to calculate the bearing life. These calculation process are very complex and cannot be expressed by a dominant formula. The most common optimization algorithms are Newton method, conjugate direction method, variable scale method, coordinate rotation method and so on. These optimization algorithms require that the optimization function be an algebraic expression, and this expression usually is continuously differentiable. If the bearing life calculation was being used these commonly optimization algorithm, it is necessary to fitting the calculated life data into a dominant algebraic expression. But the accuracy of the fitting formula is very low, so these optimization algorithms are not suitable for the bearing design in this paper. Genetic algorithms, particle groups and other intelligent algorithms are a lot less restrictions for optimization model, it is only need a result. So the improved particle swarm optimization algorithm is used to optimize the bearing parameters in this paper. Particle swarm optimization algorithm is an intelligent optimization algorithm proposed by J. Kennedy and R. C. Eberhart in 1995. However, the standard particle swarm optimization algorithm tends to fall into local optimum or early-maturing. Many scholars have proposed an improved algorithm, so that the practicability of algorithm has been greatly improved. In this paper, the inertia weight improvement strategy is used to realize the fast convergence.

In the process of optimization, not only it is necessary to realize the global search quickly, but also need to realize the fine local search. The linear inertial strategy and the nonlinear dynamic inertia strategy are used as follows (14) and (15), they are tested separately to verify which the inertia weight is more suitable for the functions in this article.

\[ \omega(t) = \omega_{\text{start}} - \frac{\omega_{\text{start}} - \omega_{\text{end}}}{t_{\text{max}}} t \]  
\[ \omega(t) = \omega_{\text{end}} + (\omega_{\text{start}} - \omega_{\text{end}}) e^{(-k \frac{t}{t_{\text{max}}})} \]  

In equation (25) (26), \( t_{\text{max}} \) is the maximum number of iterations, \( t \) is the current number of iterations, \( \omega_{\text{start}} \) is the initial value of the inertia weight value, \( \omega_{\text{end}} \) is the inertia weight value for the end of the iteration, \( k \) is the control factor. Learning factors \( c_1 \), \( c_2 \) determine the impact of particle experience and group experience on particle trajectories, and they reflect the exchange of information between particles, \( C1 \), \( C2 \) values are set larger or smaller that are not conducive to particle search. The standard learning factor is generally chosen \( c1 = c2 = 2 \). In this paper, the variety of non-linear is used to control the local and global search.

\[ c_1 = (c_{1e} - c_{1s}) \frac{t}{t_{\text{max}}} + c_{1s} \]  
\[ c_2 = (c_{2e} - c_{2s}) \frac{t}{t_{\text{max}}} + c_{2s} \]
In the formula (27) (28), \( s_1 \) and \( s_2 \) are the initial iteration value of \( C_1 \) and \( C_2 \), \( c_{1e} \), \( c_{2e} \) are the iteration final value of \( C_1 \), \( C_2 \).

In general, the constraint problem in optimization can be solved by constructing the penalty function. However, the calculation of the optimal objective function in this paper is very complex and can't be expressed by an expression, it is need more parameters. The sensitivity of the function is strong. So the result is not satisfactory that the optimization results tend to fall into the local optimal solution. The method is proved to be suitable for the optimization objective function in this paper.

![Fig. 3 Program flow chart](image)

Figure 3. is a block diagram of the implementation of the optimization algorithm. After the program starts, it is necessary the bearing designers to input the basic dimensions of the bearing. The program calculates the limit value and the maximum speed for each dimension according to the constraint conditions. The next step is randomly initialize the particle velocity and the program determine the particle position whether in the feasible region, if it is without the region, then return to the previous step that reinitialize the particle velocity, otherwise go to the next step. Call the load distribution calculation program and the life calculation program. By judging the condition, it is judged whether or not update each best particle fitness function value and the best position, if it is judged to be in conformity with the conditions and continue to determine the particle position whether in the feasible domain again, in the event in the region then update each best particle fitness function value and the best position and otherwise do not update and go to the next step. The next step is judge that the result whether or not met with the iterative termination conditions. If the iterative condition is not satisfied, then the program calculate the inertia weight and learning factor of particle velocity, get the new speed and position of the particle, and return to call the load distribution calculation program and the life calculation program, otherwise end the iteration condition and output the result. In order to select an improved particle swarm optimization algorithm suitable for the optimization function in this paper, the four methods are tested. Method 1, linear inertial strategy, particle learning factor \( c_1 = c_2 = 2 \). Method 2, nonlinear dynamic
improvement inertia strategy, particle learning factor $c_1 = c_2 = 2$. Method 3 linear inertial strategy, using equation (27) (28) to calculate learning factor of particle velocity. Method 4 nonlinear dynamic improvement inertia strategy, using equation (27) (28) to calculate learning factor of particle velocity.

Fig. 4 is obtained by experiment, the ordinate in the graph is the best fitness function value for each iteration, the abscissa is the number of iterations. It can be seen that the fourth methods are stable after 30 iterations, and the optimal solution is the highest compared with the other three methods. But the optimal value will be stable while the first three methods of the number of iterations in the 130-150 times. So the fourth method is most suitable for optimizing the objective function.

![Fig. 4 Four methods of optimizing the](image)

5. Optimize the results

Five types of bearings are randomly selected to optimize design, the five models of the basic parameters of the bearing as shown in Table 1.

| Type    | Fw | D  | B  | Dw | Lw | Z  |
|---------|----|----|----|----|----|----|
| HK1010  | 10 | 14 | 10 | 1.5| 7.8|16  |
| HK1210  | 12 | 16 | 10 | 1.5| 7.8|19  |
| HK1616  | 16 | 22 | 16 | 2.0|12.8|18  |
| HK2012  | 20 | 26 | 12 | 2.0| 8.8|22  |
| HK3020  | 30 | 37 | 20 | 2.5|15.8|26  |

Enter the parameters: the number of needle slices is 50 pieces, Needle roller material GCr15, Outer ring material 10# high quality carbon steel, the elastic modulus are $E_1 = 206900$, $E_2 = 196000$ respectively, Poisson's ratio are $\nu_1 = 0.3$, $\nu_2 = 0.24$, Speed are set at 5000r / min, adopt oil lubrication mode, the kinematic viscosity is 26mm$^2$/s, lubricant density is 0.89g/mm$^3$, the bearings and solid shafts are assembled in accordance with the H5, steel bearing housings and outer rings of bearings are assembled in accordance with N6, the final dimension of solid shaft is grinding, steel bearing housings is achieved by turning, the Ra value of all surface roughness of the bearing is 0.2, the Ra value of all surface roughness of the bearing housings is 0.8, the degree of contamination is normal clean, the pollution factors $E_c$ and Viscosity ratio $k$ are calculated by formula, the number of particles is 50, the number of iterations is 200 times.
The contents of Table 2 are the optimization results when the bearing installation is normal and do not consider the tilt. From the seventh and eighth columns of the table, we can see that the life of the modified needle roller bearings is much higher than the unmodified needle roller bearings.

As in actual use, the bearing is installed with different shafts, bearing outer ring irregular or inclined, bearing inner ring cocked or tilt, shaft load deformation will cause the bearing axis relative tilt. The inclination of the bearing is difficult to avoid, However, the impact of the incline on the bearing life is very large. Therefore, in the design of bearings need to consider the tilt problem. In Table 3, the optimum bearing size is obtained which taking into account the existence of the inner and outer rings of the bearing. Take dip is 0.1 degrees and 0.06 degrees for optimization design. It can be seen that the bearing life used bearing size parameters optimized in the presence of the inclination is significantly higher than the life calculated by the optimized parameters in Table 2.

Tab. 2 the optimization results are considered the clearance and needle shape modification

| Type  | \( \Delta_0 \) | Ls  | \( \delta \) | load(N) | modified lifetime(r) | Rated life of needle roller without modification (r) |
|-------|----------------|-----|-------------|---------|----------------------|----------------------------------------------------|
| HK1010| 0.023          | 6.54| 0.0036      | 2574    | 10124460             | 3954767                                           |
| HK1210| 0.025          | 6.69| 0.0035      | 2904    | 11209726             | 4374232                                          |
| HK1616| 0.021          | 10.1| 0.004       | 6300    | 8516493              | 3179358                                           |
| HK2012| 0.028          | 7.47| 0.004       | 5052    | 11665206             | 4354648                                           |
| HK3020| 0.031          | 13.66|0.005       | 11400   | 8564385              | 5782919                                           |

Tab. 3 Optimum size of needle roller in inclined condition of inner and outer ring

| Type  | Inclination angle | \( \Delta_0 \) | Ls  | \( \delta \) | Load (N) | modified lifetime (L) | Rated life of needle roller without modification (r) |
|-------|-------------------|----------------|-----|-------------|----------|----------------------|----------------------------------------------------|
| HK1010| 0.1               | 0.025          | 3.76| 0.008       | 2574     | 2257612              | 235804                                             |
|       | 0.06              | 0.022          | 4.7 | 0.006       | 2904     | 3726375              | 686250                                             |
| HK1210| 0.1               | 0.027          | 3.55| 0.008       | 6300     | 2405351              | 229413                                             |
|       | 0.06              | 0.027          | 5.43| 0.006       | 4103711  | 4013711              | 681591                                             |
| HK1616| 0.1               | 0.031          | 7.35| 0.008       | 6300     | 758727               | 108288                                             |
|       | 0.06              | 0.025          | 7.7 | 0.007       | 2302089  | 2302089              | 334938                                             |
| HK2012| 0.1               | 0.042          | 4.91| 0.008       | 5052     | 2432313              | 294779                                             |
|       | 0.06              | 0.04           | 6.18| 0.007       | 4759604  | 4759604              | 846478                                             |
| HK3020| 0.1               | 0.03           | 9.6 | 0.008       | 11400    | 495962               | 94416                                              |
|       | 0.06              | 0.044          | 10.3| 0.008       | 1754224  | 1754224              | 297158                                             |

6. Experimental Verification
The test equipment is a self-restraint needle roller bearing testing machine, Install the structure diagram as shown in Figure 5. The power is input from one end of the coupling, use Shell TF0870 lubricants, input speed is 5000 r/min. Test 1 points 2 groups, each group uses four HK3020 needle roller bearings as test bearings. An equivalent load 11400N is applied to each bearing. The first group bearings are not modified shape. The second group is the machined and assembled bearings according to table 2 optimized parameter. As can be seen from table 2, the modified life of the bearing is 19.27 hours when needle without modification, and after it were modified shape the modified life is 37.8 hours. The test results are shown in Table 4, the actual life of the first group of bearings mostly exceeds the calculated life and there is a small difference from predicted life expectancy. The second group of bearing life is significantly higher than the first group of bearing life, but only No. 3 bearings exceed the calculated
life. It can be seen from Fig. 6 and Fig. 7 that the edges of the non-modified shape needle roller wear are very serious, but the modified shape needle roller wear is very light. So from the test results can be learned that the optimize design has achieved the basic effect, bearing life has significantly increased. But there is a certain gap between the actual life of the modified shape bearing and the predicted life, which is a great relationship with the accuracy of the needle modified shape.

Test 1 points 2 groups too, each group uses two HK2012 needle roller bearings as test bearings. An equivalent load 11400N is applied to each bearing. The first group bearings are not modified shape. The second group is the machined and assembled bearings according to table 3optimized parameter. The bearings of each group are respectively adjusted the verticality of the bearing end face and the installation shaft to determine the inclination angle of the inner and outer rings, this angle is probably adjusted between 0.05-0.07°. As can be seen from table 3, the modified life of the bearing is 2.82 hours when needle without modification, and after it were modified shape the modified life is 11.89 hours. The two bearings of first groups, respectively, when the machine has worked at 4 hours 45 minutes and 3 hours 10 minutes, there is abnormal sound phenomenon. As shown in Figure 8. When the bearing is removed, it is found that the bearing face is worn out and a small amount of flaking off the needle surface. When the two bearings of Second groups has worked at 4 hours 45 minutes and 3 hours 10 minutes, the Vibration exceeds the standard. As shown in Figure 9. while the bearing is removed, it is found that the bearing face is worn out and a small amount of flaking off the needle surface. From the test results, the bearing life has reached the predicted life.

Fig. 5 Test design and diagram
Fig. 6 The pictures of 2# bearing of first groups in test 1

Fig. 7 The pictures of 2# bearing of second groups in test 1

### Tab. 4 Results of Experiment 1

| Group | Number | Test duration | Test phenomenon |
|-------|--------|---------------|-----------------|
| 1     | 1      | 23 hours 57 minutes | Vibration exceeds the standard and shutdown, both the mandrel and the needle surface have fatigue flaking |
| 2     | 25 hours 37 minutes | Abnormal sound changes and shutdown, Both the mandrel and the needle surface have fatigue flaking |
| 3     | 15 hours 39 minutes | Needle stuck in the raceway, the left side of the cage is broken |
| 4     | 22 hours 07 minutes | Abnormal sound changes and shutdown, Both the mandrel and the needle surface have fatigue flaking |
| 2     | 1      | 32 hours 32 minutes | Abnormal sound changes and shutdown, Both the mandrel and the needle surface have fatigue flaking |
| 2     | 29 hours 36 minutes | Abnormal sound changes and shutdown, the shaft was worn, the needle have burr |
| 3     | 44 hours 26 minutes | Abnormal sound changes and shutdown, Both the mandrel and the needle surface have fatigue flaking |
| 4     | 30 hours 34 minutes | Abnormal sound changes and shutdown, the shaft was worn, the needle have burr |

### 7. Conclusion
The results show that the improved PSO algorithm is very effective for the optimization design of some bearings parameters. Especially, the specific parameters can be determined according to different working condition. Therefore, this method can replace the traditional design method and guide the designer to more accurately simulate the actual working conditions and design the bearings.

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