Evaluation of the reliability of the averaging over the reservoir thickness for the model with a fixed streamtube

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Abstract. The reliability of the hypothesis of the model with a fixed streamtube on the weak dependence of the shape of the streamlines in the petroleum reservoir on the vertical coordinate is estimated. This hypothesis makes it possible to construct streamlines and streamtubes on the basis of solution of a two-dimensional steady state problem in a reservoir averaged over its thickness. An estimate was made of the degree of difference in the streamlines along the vertical caused by the incomplete opening of the formation of one of the wells. Models of a two-layer formation with a low-permeable seal and a multilayered reservoir are considered. Equivalence of the streamlines and streamtubes obtained from the initial 3D and the averaged over the thickness of the reservoir 2D problem was also estimated.

1. Introduction
The Model with a Fixed Streamtube (MFST) allows both fundamentally reduce costs of calculation and use high-resolution grids for numerical simulation of petroleum reservoir [1]. Such features of the model are realized due to decomposition of three-dimensional problem into a series of two-dimensional problems in vertical sections of streamtubes that connect injection and production wells. However, the algorithms of the model with a fixed streamtube are based on some hypotheses and simplifications that require verification of their reliability. One of the main assumptions is the assumption of independence of a streamline shape on a vertical coordinate. It allows imagining lateral edges of streamtubes as vertical surfaces and investigating streamlines from the calculation of two-dimensional flow equations that are averaged over the reservoir thickness. The reasons of possible violation of the assumption could be vertical heterogeneity of the geological structure of the reservoir and imperfection of wells by the degree of reservoir opening. Since the average distance between the wells usually does not exceed the common scale of the zonal heterogeneity of the reservoir, in the first place it is required to evaluate the effect of well imperfection on the variability of streamlines in the vertical direction.

Two typical problems on the vertical variability of the streamline shapes near an imperfect well are presented in the paper: 1) in a reservoir consisting of two layers separated by a low-permeability seal; 2) in stratified multilayered reservoir. The correctness of the streamlines construction in the framework of two-dimensional steady state problem for averaged over thickness reservoir is estimated with considering of the imperfection of wells for opening degree.
2. Statement of the task in dimensionless variables

A three-dimensional steady state single-phase flow problem in a rectangular section of a reservoir of constant thickness is considered. The reservoir is opened by two vertical wells - injector (I) and producer (P) (figure 1). The roof and the base of the reservoir are impermeable. The injection well opens the entire thickness of the reservoir. The degree of opening of the reservoir by the production well and the thickness of the model reservoir are determined according to the variant of the problem.

The characteristic scales for the dimensionless formulation of the problem will be chosen in such a way that the distance between the wells will take the value as $L=1$, the pressure on the contour of the section removed from the wells by the half of distance between the wells is $p_0=0$, and the wells pressure are $p_I=+1$, $p_P=-1$.

Without the consideration of the gravitational forces and the compressibility of the reservoir as well as liquid, the problem is described by equations [2]:

$$\text{div} \ u = \delta(x-x_i) \ q(p_I, r_w) + \delta(x-x_p) \ q(p_P, r_w), \ x \in \Omega = [0, 2] \times [0, 1] \times [0, H],$$

$$u = -k(z) \nabla p,$$

$$z = 0, H : \partial p / \partial z = 0,$$

where $u$ – vector of the flux velocity; $p$ – pressure in the liquid; $\delta$ – Dirac delta function; $x=(x,y,z)$ – vector of coordinates; $x_I, x_P$ – coordinates of wells $I, P$; $q(p_w, r_w)$ – well rate of the well with radius $r_w$ and bottomhole pressure $p_w$; $k(z)=k_i$, $z \in [[(i-1)H/N, iH/N]]$, $i=1...N$ – piecewise-constant function of absolute permeability in a reservoir consisting of $N$ layers.

The problem (1) - (3) was solved numerically by the finite volume method with two-point approximation of the flow in the package of Matlab Reservoir Simulation Toolbox [3], the streamlines were constructed by the Pollock method [4].

3. Volatility of streamlines along the vertical

In this paper, the vertical variability of streamlines is understood as the variability of the projections onto the XY plane of streamlines started from points with different $z$ coordinate.

The difference $r_i$ between two streamlines having coordinates of the starting points as $(x_i, y_i, z_1)$ and $(x_i, y_i, z_2)$ is calculated by counting the hits of the compared streamlines in the cells of the virtual grid on the XY plane:

$$r_i = 1 - 2 \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\text{hit}_1(i,j) \cdot \text{hit}_2(i,j)}{\left( \sum_{i=1}^{N} \sum_{j=1}^{N} \text{hit}_1(i,j) + \sum_{i=1}^{N} \sum_{j=1}^{N} \text{hit}_2(i,j) \right)},$$

Figure 1. Location of wells in the simulated area.
$n, m$ – the number of virtual grid steps by the directions of $x$ and $y$; \( \text{hit}_{1,2}(i, j) \) – the hit sign (takes the values of 1 and 0) of the streamline of the layer of \( z_{1,2} \) in the cell of the grid centered in the point \( (x_i, y_j) \). The value \( r_k \in [0, 1] \), where 0 corresponds to the total overlap of streamline, and 1 corresponds to their total difference.

Only streamlines that both exit the injection well (I) as well as enter the producing well (P) and form the streamtube are compared. The unified degree of difference of the pair of streamlines is calculated as the mean:

$$R = \frac{\sum_{k=1}^{K} r_k}{K}. \quad (5)$$

\( K \) – the number of streamlines’ pairs, in which at least one streamline hit the well P. The value \( R \in [0, 1] \), where 0 corresponds to the complete overlap of all streamlines’ pairs, and 1 corresponds to their total difference.

3.1. Two layers with a low-permeability seal

Let’s consider an inhomogeneous reservoir with the thickness of \( H = 0.001 \) (the typical thickness of the layers) consisting of two layers with a permeability \( k = 1 \), separated by a continuous seal with low permeability \( \kappa \in (0; 1] \). The thickness of all layers is the same one. The producing well opens only the first layer.

The dependence of the difference in the streamlines shape in the upper and lower layers from the permeability of the seal is shown in figure 2. It can be seen that the total difference of streamlines is achieved only when \( \kappa = 10^{-6} \) and lower; starting from the value \( \kappa = 10^{-3} \) the difference of the streamlines becomes insignificant, moving to zero when \( \kappa = 1 \). The streamline shape in the lower and upper layers with representative values \( \kappa \) is shown in figure 3.

![Figure 2. The difference of streamlines under changing seal permeability.](image)

3.2. Multilayered reservoir

Let’s consider a stratified reservoir with the thickness \( H = 0.1 \) (the typical thickness of the reservoir) consisting of a large number of layers (for example, 100). The thickness of the layers is the same one. The permeability of the layers is calculated by the Kozeni formula through the porosity, which is given by the normal distribution law [5]. The values of \( M' \) permeability, in which the maximum and minimum permeability values are \( 10^i (i = 0.4) \) times different, are studied.
Figure 3. Streamlines in the first (red) and the second (blue) layers
(1 - $\kappa = 10^{-6}$, $R = 1$; 2 - $\kappa = 10^{-3}$, $R = 0.7$; 3 - $\kappa = 10^{-3}$, $R = 0.05$).

Denote the reservoir opening degree by production well as $\alpha = h/H \in (0,1]$, where $h$ – the depth of opening. The dependence $R(\alpha)$ is shown in figure 4. It could be seen that the opening degree does not significantly change the streamlines behavior for a homogeneous reservoir; and for permeability sets $M^1$ and $M^2$ can be considered insignificant to affect streamlines shape. Streamlines shape in the lower, the upper and the middle layers for three values of permeability is shown in figure 5. The complete coincidence of streamlines is observed for the sets $M^2$ and, therefore, for the values of $M^1$, $M^0$. The significant difference of streamlines is noticeable for the sets $M^3$, $M^4$.

Figure 4. The difference $R$ of streamlines dependence on the opening degree $\alpha$ under all values of permeability: 1) $M^0$ – homogeneous reservoir, 2) $M^1$, 3) $M^2$, 4) $M^3$, 5) $M^4$.

Figure 5. Streamlines in the lower (red), the middle (green), and the upper (black) layers for $\alpha = 0.3$ and all sets of permeability: 1) $M^2$, $R = 0.065$; 2) $M^3$, $R = 0.48$; 3) $M^4$, $R = 0.9$.

4. Reliability of the averaging of the filtration model for the thickness of the reservoir
Since streamlines and streamtubes in the MFST are constructed in the framework of a two-dimensional problem on the $XY$ plane, the averaging 3D model over thickness needs to taking into
account the imperfection of the wells and anisotropy of the reservoir permeability \( \chi = \sqrt{k_x / k_{xy}} \). For reservoir model described above the horizontal and vertical permeabilities are calculated as follows [6]

\[
k_{xy} = \frac{1}{N} \sum_{i=1}^{N} k_{ij}, \quad k_x = N \left( \sum_{i=1}^{N} j / k_i \right)^{-1}.
\] (6)

The calculation of imperfection of wells at reservoir opening degree is performed by introducing the effective radius of the well, calculated according to Veliyev's formula [9]:

\[
r_j = r_e e^{-C_i}, \quad C_i = (1/\alpha - 1) \cdot \ln \left( 4H \cdot \sqrt{X / r_i} \right) + (\alpha \cdot \ln(\alpha) + (1-\alpha) \cdot \ln(1-\alpha)) / \alpha^2.
\] (7)

The comparison of the rate of the production well with different opening degree of the reservoir calculated within the framework of 2D and 3D models is shown in figure 6. Starting from the variance of the permeability values of the layers in \( 10^3 \), the error of the method (7) becomes significant.

![Figure 6](image)

Figure 6. The dependence of \( Q(\alpha) \) \((1 - M^1, 2 - M^2, 3 - M^3)\) by 3D (blue) and 2D (black) solution.

The equations of MFST are written with using of width \( W(l) \) of streamtube from its length \( l \) calculated as a value that is inverse to the flux rate modulus along the shortest streamline in streamtube (Fig. 7).

![Figure 7](image)

Figure 7. The \( W(l) \) function for \( \alpha = 0.5 \) \((1 - M^1, 2 - M^2, 3 - M^3)\) by 3D solution (blue marker), and 2D solution: with (black) and without (red) taking into account well imperfection.

5. Conclusion

The influence of the seal permeability on the streamlines difference in the two horizontal layers, when only one layer opened by producing well, has been evaluated. It is shown that the difference becomes significant only when the permeability of the seal is below than the layers permeability more than four orders of magnitude.

Also the effect of the reservoir opening degree by a well on the streamlines vertically difference in a homogeneous and stratified reservoir has been estimated. The opening degree leads to significant differences of streamlines only since the moment when the minimum and maximum values of layers permeability differ by more than three orders of magnitude. Thus, the hypothesis of model of a fixed streamtube is reliable one with the typical properties of the petroleum reservoir, and well distance.
The equivalence of the 3D XYZ and the 2D XY model averaged over the thickness of the reservoir in the case of imperfect wells in a layered heterogeneous reservoir has been estimated. The difference in the streamlines in the 2D and 3D problems remains insignificant when the permeability values in the layers differ by more than three orders of magnitude, after which Veliev's formula for the effective well radius requires some refinement.

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References
[1] Mazo A, Potashev K, Baushin V and Bulygin D 2017 Georesources 19 15-20
Potashev K, Mazo A, Ramazanov R and Buligin D 2016 Oil. Gas. Novations 4 32-40
[2] Barenblatt G, Entov V and Ryzhik V 1990 Theory of Fluid Flow Through Natural Rocks (London: Kluwer Academic Publishers)
[3] Lie K-A 2014 User Guide for the Matlab Reservoir Simulation Toolbox (Norway: SINTEF ICT)
[4] Thiele M 2005 Streamline simulation (Italy: 8th Internat. Forum on Reserv. Sim.)
[5] Kozeny J 1927 Aufstieg, Versickerung und Anwendung auf die Bewaesserung. Sitzungsberichte der Akademie der Wissenschaften in Wien 136 271–306
Daigle H and Dugan B 2009 Marine and Petrol. Geol. 26 1419–1427
[6] Basniev K, Dmitriev N and Rosenberg G 2005 Oil and gas hydromechanics: a textbook for high schools (Moscow-Izhevsk: Institute of Computer Research) [in Russian]
[7] Veliev M and Mamedov G 1999 Proc. of AzNIPInef 18-20