German Bond Yields and Debt Supply: Is There a “Bund Premium”? 

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Abstract

Are Bunds special? This paper estimates the “Bund premium” as the difference in convenience yields between other sovereign safe assets and German government bonds adjusted for sovereign credit risk, liquidity and swap market frictions. A higher premium suggests less substitutability of sovereign bonds. We document a rise in the “Bund premium” in the post-crisis period. We show that there is a negative relationship of the premium with the relative supply of German sovereign bonds, which is more pronounced for higher maturities and when risk aversion proxied by bond market volatility is high. Going forward, we expect German government debt supply to remain scarce, with important implications for the ECB’s monetary policy strategy.

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I. INTRODUCTION

1. **Bunds are becoming increasingly scarce compared to other sovereign safe assets.** This is driven by a combination of fiscal policy and the ECB’s quantitative easing, which has reduced the available supply to the market. The public sector purchase program of the ECB divided purchases between countries according to the size of the country’s economy and population (capital key), which means that German sovereign debt accounted for the largest share of the program. Going forward, while the supply of U.S. Treasuries is increasing to fund the U.S. fiscal expansion, the supply of German sovereign securities is projected to shrink further given current issuance plans (IMF, 2018). This will have an impact on German bond yields and therefore on eurozone benchmark yields.

2. **The extent to which government debt supply affects bond yields will depend on the substitutability of sovereign safe assets.** If there is full substitutability with U.S. government bonds, for example, they will “add” to the supply of euro area safe assets, thereby mitigating the scarcity effects of German debt. Substitutability in turn depends on investor preferences. If there are significant unique clientele for long-term German bonds, there will be less substitutability compared to a situation in which there are risk-neutral investors who maximize their expected wealth and are prepared to switch between safe government bonds of different countries. The fact that German Bunds are now the most highly sought-after collateral, with financial institutions preferring them over cash for collateral purposes, suggests that preferred clientele are significant.

3. **This paper investigates whether there is a “Bund premium.”** The extent to which investors value the liquidity and safety of government bonds and accept a lower yield to hold them over other types of safe assets (e.g., AAA-rated corporate bonds) is referred to as the convenience yield (Krishnamurthy and Vissing-Jorgensen, 2012). We quantify the difference in the convenience yields of other (non-German) sovereign bonds and German government bonds by measuring the difference between other countries’ synthetic yields (i.e., the synthetic euro borrowing cost of swapping the cash flows of the countries’ foreign exchange sovereign bonds into euros) and German bond yields. This differential is then corrected for deviations from covered interest parity (CIP), sovereign credit risk differentials and differences in liquidity. We call this wedge the “German bond premium” and, in the case of bonds with 10-year maturity, the “Bund premium”. A larger wedge suggests that there is a price premium, equivalent to an interest rate discount, and hence less substitutability of sovereign bonds.

4. **We measure the German bond premium vis-à-vis sovereign bonds of other G11 currencies and vis-à-vis the largest euro area economies.** Australia, Canada, Denmark, Germany, Japan, New Zealand, Norway, Sweden, Switzerland, the U.K. and the U.S. are commonly referred to as the G11 currencies based on trading volume and turnover. Except for Japan, all of these countries have AAA or near-AAA sovereign credit ratings and are perceived as near default free by global investors. The euro area countries we include are France, Italy and Spain. The latter two have seen a downgrade of their sovereign credit ratings in the wake of the European debt crisis but their sovereign bonds are still extensively used as collateral and safe saving vehicles by banks and households.
5. **Panel regression analysis is used to shed light on the role of the relative government debt supply in explaining the “Bund premium.”** Based on a model in which there are preferred habitat investors (only buying bonds with a specific maturity from a specific country) and risk-averse arbitrageurs (who want to maximize the mean and minimize the variance of their wealth changes), we expect that an increase in the relative supply of German debt decreases the Bund premium. We also hypothesize that the effect is stronger for longer maturities (as the risk-averse arbitrageurs would ask for higher risk premia to be compensated for the higher interest rate risk and higher cost of hedging exchange rate risk from holding longer-term bonds). Finally, the effect should be stronger the more risk averse are the arbitrageurs.

6. **Our empirical results indicate that there has been an increase in the “Bund premium” and therefore the specialness of German bonds post crisis.** This is true vis-à-vis the other G11 currency countries overall, the euro area countries and also vis-à-vis the U.S.. The post-GFC sample shows a steady increase in the “Bund premium” vis-à-vis U.S. Treasuries. Prior to the crisis and at the beginning of the GFC, the German bond premium was negative, with U.S. Treasuries being more attractive. This is in line with many papers which have argued that there is a “specialness” of U.S. Treasuries since the U.S. dollar is the world’s most significant reserve currency. However, most recently, this has changed. The European debt crisis stands out, presumably reflecting redenomination risk in the euro area, with many investors preferring to hold German bonds.

7. **Using our computed Bund premium, we construct a Bund demand curve.** Borrowing from the seminal paper by Krishnamurthy and Vissing-Jorgensen, 2012, who plot the deviation of U.S. Treasury yields from triple-A corporate bond yields against the supply of U.S. Treasuries, we plot our “Bund premium” against the relative supply of German public debt vis-à-vis the other countries’ public debt in our sample. We show that there is a negative relationship of the premium with the relative supply of German debt.

8. **The slope of the curve is steeper for longer-dated German government bonds and risk aversion matters.** Following the literature, we proxy risk aversion by bond market volatility. As hypothesized, supply changes at the longer end of the yield curve have a larger effect on the German bond premium. Interacting volatility with relative German debt supply tightens the relationship. A threshold estimation in which we endogenously determine the best fitting threshold also confirms that the curve is steeper in times of higher volatility in the other country (vis-à-vis Germany). Therefore the effect of scarcity constraints on German bond yields will be even stronger when risk aversion is high.

9. **Our results have important monetary policy implications.** German sovereign debt is projected to shrink over the coming years, further compressing Bund yields. While these scarcity effects are beneficial in helping the ECB maintain low long-term yields the near term, they could complicate the process of monetary policy normalization over the longer run.

10. **The paper extends the existing literature in several directions.** First, to the best of our knowledge, we are the first paper to compute what we refer to as a “Bund premium” vis-à-vis the other G11 countries and also the largest euro area countries. Second, we take into
account more factors that could drive a wedge between yields than existing papers for the U.S. Third, we extend the regression analysis to account for threshold effects and explain our findings on the relationship between debt supply and the “Bund premium” through a modified version of an existing model by Greenwood and Vayanos (2014). Finally, we provide a scenario analysis to simulate the implications for the “Bund premium” going forward of different monetary policy normalization scenarios.

11. **The remainder of this paper proceeds as follows.** The next section briefly discusses the related literature on this topic. Section III outlines the data and methodology to calculate the “Bund premium” and describes the behavior of the German bond premium over time, currencies and maturities. Section IV examines the relationship between relative bond supply and the Bund premium, and includes a threshold analysis. Section V looks at the policy implications including under different normalization scenarios of the ECB. Section VI concludes.

**II. RELATED LITERATURE**

12. **Our work is related to a number of papers that examine the convenience yield of U.S. Treasuries** (Krishnamurthy and Vissing-Jorgensen, 2012; Nagel, 2016). Krishnamurthy and Vissing-Jorgensen (2012) find that when the supply of Treasuries is low, the value that investors assign to the liquidity and safety attributes offered by Treasuries is high, with the opposite applying when the supply is high. They present detailed econometric evidence of this relation using several alternative yield spread measures of Treasuries versus Aaa rated corporate bonds and controlling for corporate bond default risk. They conclude that for the period of 1926-2008, the value that investors have paid on average for the liquidity and safety attributes of Treasuries amounts to 46 basis points and 27 basis points respectively. The authors thus conclude that there are significant unique clienteles for long-term safe assets, whose presence lowers the yields of such assets.

13. **Du and others (2018) put the idea of convenience yields in an international context.** They quantify the difference in the convenience yield of U.S. Treasuries and government bonds of other developed countries by measuring the deviation from covered interest parity between government bond yields. They call this the “Treasury Premium”. They document a secular decline in this premium for medium and long maturities. Our analysis builds on their work and extends it in several directions, including by taking into account liquidity differences and considering non-linear threshold effects in our estimation.

14. **Our simulations of the impact of QE on the Bund Premium is related to a series of papers that evaluate the impact of QE.** Many of the papers evaluate this impact for the U.S. and euro area through event studies (Christensen and Rudebusch, 2012; Middeldorp 2015). In the same spirit as our paper, Krishnamurthy and Jorgensen (2011), evaluate the effect of the Federal Reserve’s purchase of long-term Treasuries on interest rates. They argue that one of the channels through which quantitative easing lowers yields is through the safety channel, which can be thought of as describing a preferred habitat of investors that applies to the space of safe assets. By reducing the supply of safe assets to the market and hence increase the equilibrium safety premium, QE decreases the nominal yields on Treasuries.
15. The analysis of relative swap spreads by Codogno and others (2003) in the euro area is also relevant to this paper. The authors provide evidence that the movements in yield differentials between euro area government bonds explained by changes in international risk factors – as measured by banking and corporate risk premiums in the United States – are more pronounced for bonds issued by Italy and Spain. Liquidity factors play a smaller role. The risk of default is a small but important component of yield differentials’ movements.

16. Our paper is also related to a strand of research that examines the impact of relative supply of long-term government debt on the term spread. Bernanke and others (2004) and Greenwood and Vayanos (2014) provide some descriptive results on how bond yield movements in the U.S. may be attributed to changes in the maturity structure of government debt. Strohsal (2017) examines the responsiveness of German bond spreads to changes in debt supply. He also examines whether the impact of debt supply changes in times of high risk aversion compared to times of low risk aversion. He finds that the empirical results support the theoretical predictions of the preferred habitat theory and that under high risk aversion, yields react much more strongly to changes in government bond supply. Our paper contributes to this literature by analyzing the impact of supply changes on the Bund premium vis-à-vis other countries.

III. IS THERE A “BUND PREMIUM”?

Methodology

17. We start with the simplest case, assuming that local currency government bond yields of developed countries are default-free and international financial markets are frictionless. For government bond yields to be different from the risk-free rate, they need to offer convenience benefits (Krishnamurthy and Vissing-Jorgensen, 2012). Following Du and others, (2018), this can be formalized as follows:

Let \( \mu_{i,t+1} \) denote the convenience benefit of government bond of country \( i \) realized at \( t+1 \), and \( y_{i,t}^{rf} \) the risk-free rate at time \( t \), then the price of government bond of country \( i \) at time \( t \) can be written as:

\[
P_{i,t}^{gov} = \exp(-y_{i,t}^{rf}) EXP_{i,t}[1 + \mu_{i,t+1}] \tag{1}
\]

where \( EXP_{i,t} \) is the risk neutral expectation in country \( i \) at time \( t \). In logs, this becomes

\[
\gamma_{i,t} = y_{i,t}^{rf} - y_{i,t}^{gov} \tag{2}
\]

where \( y_{i,t}^{gov} = -\log(P_{i,t}^{gov}) \) is the government bond yield and \( y_{i,t} = \ln(EXP_{i,t}[1 + \mu_{i,t+1}]) \) is the convenience premium.

If international markets are frictionless, covered interest party (CIP) holds for risk-free rates. Therefore, if we define \( y_{DEU}^{rf} \) as the German risk-free rate at time \( t \), then:

\[
\varphi_{i,t} = y_{i,t}^{rf} - y_{DEU,t}^{rf} \tag{3}
\]
where $\varphi_{i,t}$ is the market-implied forward premium defined as

$$\varphi_{i,t} = \log(F_{i,t+1}) - \log(S_{i,t})$$  \[4\]

with $F_{i,t+1}$ denoting the outright forward exchange rate and $S_{i,t}$ the spot exchange rate, measured in terms of units of currency $i$ per euro.

The n-year German bond premium can then be formalized as follows:

$$\rho_{i,n,t} = y_{i,n,t}^{Gov} - \varphi_{i,n,t} - y_{DEU,n,t}^{Gov}$$ \[5\]

Hence the n-year German bond premium vis-à-vis country $i$ is the deviation from CIP between government bond yields in these two countries. Substituting for $\varphi_{i,n,t}$, it can be shown that

$$\rho_{i,n,t} = y_{DEU,n,t} - y_{i,n,t}$$ \[6\]

That is the German bond premium measures the differential of convenience yields between Germany and other countries $i$. For maturities equal to ten years we will refer to the German bond premium as “Bund premium.”

18. **We can also introduce default risk, FX swap market frictions as well as liquidity differences.** The above equation can be adjusted to account for these factors. The details will be laid out further below.

First, we correct for FX swap market frictions that are at the origin of deviations from covered interest parity (CIP).

$$\rho_{i,n,t}^{CIP} = \rho_{i,n,t} - \tau_{n,i,t}$$ \[7\]

with $\tau_{n,i,t}$ denoting FX swap market frictions. This correction applies only to German bond premia vis-à-vis non-euro area countries.

Second, we correct for sovereign risk differentials by subtracting the sovereign CDS differential from the German bond premium.

$$\rho_{i,n,t}^{CIP,CDS} = \rho_{i,n,t}^{CIP} + [CDS_{n,DEU,t}^{gvt} - CDS_{n,i,t}^{gvt}]$$ \[8\]

with $CDS_{n,i,t}^{gvt}$ denoting the n-year sovereign CDS of country $i$ at time $t$.

Finally, we correct our premium for liquidity differentials. Here we use the bid-ask spread liquidity measure proposed by Corwin and Schultz (2012) to compute the liquidity differential between German sovereign bonds and the other country’s sovereign bonds, in percentage points. This liquidity measure builds on the links between bid-ask spreads and high/low prices over one-day and two-day periods. It allows to derive an estimate of a stock’s bid-ask spread as a function of the high-to-low ratio for a single two-day period and the high-to-low ratios for two consecutive single days.
\[ \rho_{i,n,t}^{CIP,CDS,liq} = \rho_{i,n,t}^{CIP,CDS} - [BA_{spread_{i,t}} - BA_{spread_{DE,t}}] \]  

9.

Data

19. **Data sources are summarized in Appendix 1.** We mainly use Bloomberg data and focus on central (federal) government bonds. For maturities greater than one year, the liquidity of outright forward contracts is limited. We instead use n-year interest rate swaps and cross-currency basis swaps as is conventional in the literature. Thus, the market implied forward premium is computed as follows:

\[ \varphi_{i,t} = irs_{i,n,t} + bs_{i,n,t} - irs_{DEU,n,t} \]  

where \( irs_{i,n,t} \) is the n-year interest rate swap that exchanges fixed currency i cash flows into the floating interbank interest rate benchmark (Libor interest rate swap) in country i, \( bs_{i,n,t} \) is the n-year cross-currency basis swap that exchanges the latter floating benchmark interbank rate in country i for Euro Libor, and \( irs_{DEU,n,t} \) is the Euro Libor interest rate swap rate that exchanges fixed euro cash flows into Euro Libor.

20. **We use a variety of data sources to account for deviations from covered interest parity, sovereign credit risk, and liquidity differences:**

- For FX market frictions, we use data from Bloomberg. We consider Libor interest rate swap rates as proxies for the risk-free rates. As the deviation from covered interest parity is equal to the difference between the risk-free rate differential and the market implied forward premium \( (\varphi_{i,t}) \), the correction applied to the German bond premium (i.e. the Libor-based swap market mispricing \( \tau_{n,i,t} \)) is equal to the negative of the cross-currency basis swap:

\[ \rho_{i,n,t}^{CIP} = \rho_{i,n,t} + bs_{i,n,t} \]  

- For sovereign credit risk, we use differences in EUR denominated CDS from Datastream. When these are not available for a specific country, we consider that there is no CDS differential vis-à-vis Germany\(^2\).

- To compute the liquidity measure developed by Corwin and Schultz (2012), we use high and low prices from Bloomberg and then calculate the bid-ask spreads in line with the analysis in IMF (2015), as described above. Daily bid-ask spreads are computed for each sovereign bond and then aggregated by maturity bucket and country. The maturity bucket considered for the 10-year maturity is [10 years +/- 1 year]. For data availability reasons, the liquidity measure is not computed for other maturities. To expand the coverage, we replace the country-maturity specific spread by its lagged value whenever it is nil.

\[ ^2 \text{This happens for Switzerland, Australia, New Zealand and Norway.} \]
To check the robustness of our bid-ask spread liquidity measure, we compare it to two other measures computed by the ECB. First, the execution-based liquidity measure is based on information provided by quotes for transactions under the ECB’s public sector purchase program (i.e. the spread between the two best quotes, divided by the duration of the bond). Second, the order book liquidity measure is based on bid-ask spreads and quoted volumes of sovereign bonds (sum of the five best quotes on the ask side minus sum of the best quotes on the bid side, divided by the sum of the corresponding quoted volumes). Both measures tend to co-move with our liquidity measure when aggregated at the euro area level (see Appendix 2.a.).

We follow the methodology by De Santis (2015) to compute redenomination risk for the euro area countries in our sample (France, Spain and Italy). This is estimated as the difference between the country’s “quanto” CDS and the German “quanto” CDS, with the quanto CDS being itself the difference between the CDS quotes denominated in U.S. dollar and euros. Euro denominated CDS are from Datastream, whereas USD denominated CDS are from Bloomberg, for better data coverage.

Results

Synthetic and actual yields

21. **The yields on sovereign safe assets have been steadily declining** (Figure 1, left chart). They have now reached historically low levels in many countries. Part of this decline likely reflects a stabilization of inflation expectations at low levels, but other factors have also contributed. Following the global financial crisis (GFC), there has been an increased demand for high-quality liquid and risk-free assets (Grandia and others, 2019). In addition, there has been strong demand from investors such as pension funds and insurance companies in the context of an ageing society and regulatory and accounting changes. The extremely accommodative monetary policy in many countries has put further downward pressure on yields. One effect of these unconventional measures has been to decrease the amount of safe sovereign assets available to buy for private investors.

22. **Swap implied euro yields track the yields on Bunds very closely, with less dispersion than country specific yields.** Figure 1 (right chart) shows that once exchange rate differences are accounted for, the dispersion between yields decreases significantly. The Bund premium in this case can be visualized as the difference between swap-implied euro yields on other sovereign bonds and Bund yields.

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3 We are grateful to Renate Dreiskena, Daniel Kapp and Julian von Landesberger (all ECB) for sharing their bond market liquidity measures with us.
Average Premium

23. The specialness of German bonds vis-à-vis other government bonds has increased after the global financial crisis. We first calculate the average premium vis-à-vis other countries (country specific premia can be found in Appendix 2 b). Figure 2 shows the benchmark premium (adjusted for exchange rates), and the adjusted premium that additionally corrects for swap market frictions, sovereign risk, and liquidity differentials. Both at the 5- and 10-year maturity, the premium on German bonds spiked during the European debt crisis in 2012/2013, and, while coming down, has remained elevated since. Even after accounting for the different factors that could drive a wedge between yields, we find that during the debt crisis and also most recently, there has been a positive German premium. Looking over a longer time-period, since 2005, there has been a clear upward trend in the premium.

Note that, for data availability reasons, only 10-year premia are corrected for liquidity differentials (Figure 2, right hand side chart).
Germany vis-à-vis the U.S.

24. **While the pre-crisis period was characterized by a negative premium vis-à-vis the U.S., this has changed post-crisis.** The post-GFC sample shows a steady increase in the Bund premium vis-à-vis U.S. Treasuries (Figure 3). Prior to the crisis and also at the beginning of the GFC, the German bund premium was negative, with U.S. Treasuries being more attractive. This is in line with many papers that have argued that there is a “specialness” of U.S. Treasuries since the U.S. dollar is the world’s most significant reserve currency. However, most recently, this has changed. The European debt crisis stands out, presumably reflecting redenomination risk in the euro area, with many investors preferring to hold German bunds. But the most recent evolution post crisis also points to a loss of specialness of long-term U.S. Treasuries, with German bunds becoming increasingly attractive.

Germany vis-à-vis other euro area countries

25. **Vis-à-vis the euro area, the premium was broadly flat in the pre-crisis period, but this has changed post-crisis.** The European debt crisis stands out as a period of extremely high Bund premia (Figure 4). Looking at the charts more closely, the magnitude of the premium differs across countries and is much higher vis-à-vis periphery countries.
Correlation with measures of redenomination risk and policy uncertainty

26. The Bund premium moves in tandem with our measure of redenomination risk and policy uncertainty. Figure 5 shows a strong correlation between our preferred measure of the Bund Premium (that corrects for liquidity, credit risk and swap market frictions) and our own constructed measure of redenomination risk as well as the global economic policy uncertainty index. This is intuitive, as one would expect that if perceived redenomination risk is high there will be a high demand for German Bunds, which are seen as liquid and safe. Moreover, when uncertainty goes up, investors will look for a safe haven.

OIS and KfW yields

27. We can compare the German bond premium with the OIS-German sovereign bond spread. An overnight index swap (OIS) is a financial contract between two counterparties to exchange a fixed interest rate against a geometric average of overnight interest rates (in the euro area, the EONIA) over the contractual life of the swap. Many studies (see for example ECB, 2014) have argued that OIS rates serve as a useful measure of risk-free rates with the OIS swap market providing an almost-risk-free rate for any desired term (the interest that would be paid on continually-refreshed overnight loans to borrowers in the overnight market).

28. Another interesting comparison is comparing the KfW-German sovereign bond spread with our estimated Bund premium. The Kreditanstalt für Wiederaufbau (KfW) is a German development bank. Bonds issued by KfW and the German Bund are both guaranteed by the German state and, therefore, carry the same credit risk. Thus, a positive spread would imply that there are certain convenience benefits (such as liquidity) of holding German government bonds rather than bonds issued by the KfW.

29. The German bond premium is positively correlated with both the OIS-German bond spread and the KfW-German bond spread. Figure 6 illustrates this for the premium vis-à-vis the U.S. This is what one would expect. When the Bund premium is positive vis-à-vis other countries, Bunds are earning a price premium and a yield discount relative to other safe sovereign assets. This specialness should also translate into higher German government bond prices and therefore lower yields vis-à-vis other risk-free rates, such as KfW issued bonds and OIS rates. In volatile times, such as those during the GFC, this may reflect an extreme aversion of investors against any bank credit risk (OIS) and flight to liquidity flows into high rated and liquid assets (KfW).
IV. A DEMAND CURVE FOR BUNDS

Theoretical Predictions

30. How does the relative supply of sovereign bonds affect the Bund premium? We follow the literature on the preferred habitat theory and assume that the yield of a bond in a given currency is driven by the interaction of three types of agents: government, preferred habitat investors with a preference for bonds with a given maturity and issued by a given government, and risk-averse arbitrageurs who would like to maximize the mean and minimize the variance of their wealth. Preferred habitat investors could for instance be pension funds or life insurers who have a clear preference for longer maturity debt of a specific country. Based on such a model we can derive some testable hypotheses (the details can be found in Appendix 4):

**Hypothesis 1**: An increase in German bond supply relative to that of another G10 country will lead to a decrease in the Bund premium.

**Hypothesis 2**: This decrease will be higher for longer dated bonds.

**Hypothesis 3**: When arbitrageurs are more risk adverse, the effect of a change in the relative supply of German bunds on the Bund premium will be stronger for all maturities.
31. **Intuitively, in the absence of arbitrageurs, if there is an increase in the supply of 10-year bunds with no change to other maturities or in other countries, the yield for 10-year bunds will increase.** Arbitrageurs can jump in to benefit from this higher yield and if they were risk neutral this would undo the increase in yields. However, the arbitrageurs risk profile has changed. If they are buying longer dated bonds to substitute for shorter dated bonds, they are subject to higher interest rate risk. If they exchange bonds of different currencies, they have to hedge, which comes at a cost. For both effects, arbitrageurs will ask to be compensated by a risk premium. This risk premium is increasing in the maturity of the bonds that arbitrageurs are buying and also in the risk aversion of arbitrageurs.

### Empirical Methodology

32. **To test these hypotheses empirically, we first construct a “Bund Demand Curve.**” Borrowing from the seminal paper by Krishnamurthy and Vissing-Jorgensen, 2012, who plot the deviation of U.S. Treasury yields from triple A corporate bond yields against the supply of U.S. Treasuries, we plot our 10-year Bund premium (corrected for swap market frictions, sovereign risk and liquidity) against the relative supply of German public debt vis-à-vis the other countries in our sample. We follow the previous literature in using the general government debt to GDP ratios. The data source is the IMF’s World Economic Outlook database. This is only an approximation of marketable debt. Moreover, ideally, one would use the long-term debt supply when looking at the 10-year Bund premium. By using total general government debt, we assume that the relative long-term debt moves in tandem with the relative total debt. We later relax this assumption in our robustness analysis when we construct data series for longer-term marketable debt.

33. **We run the following panel regressions:**

\[
\rho_{i,n,t}^{CIP,CDS,liq} = \alpha + \beta \cdot \log \left( \frac{\text{debt}_{GDP,I}}{\text{debt}_{GDP,EU}} \right) + \theta \cdot X_{i,t} + \varepsilon_{i,t} \quad [12]
\]

where debt/GDP is the general government debt ratio and \(X_{i,t}\) is a set of explanatory variables, motivated by the literature. It includes for instance the country’s bond volatility. We also run regressions where we exclude the central bank holdings of government debt. The regression is estimated with fixed effects using quarterly data, following a Hausman test that confirms the applicability of fixed effects rather than random effects.

34. **We then specify a panel threshold regression.**

\[
\rho_{i,n,t}^{CIP,CDS,liq} = \alpha + \beta_1 \cdot \log \left( \frac{\text{debt}_{GDP,I}}{\text{debt}_{GDP,EU}} \right) + \delta_1 + \theta \cdot X_{i,t} + \varepsilon_{i,t} \text{ if risk aversion} \leq \gamma \quad [13]
\]

\[\text{and}\]

---

5 Our baseline regression does not include control variables. But robustness checks in the appendix include additional variables such as various measures of uncertainty and volatility.
\[
\rho_{GIP,CDS,liq}^{CIP,CD,liq} = \alpha + \beta_2 \cdot \log \left( \frac{(\text{debt})_{GDP}}{(\text{debt})_{DEU,lt}} \right) + \vartheta \cdot X_{i,t} + \varepsilon_{i,t} \text{ if risk aversion} > \gamma
\]

where \(\gamma\) is the threshold value that splits the sample into two regimes and the coefficients \((\beta_1, \beta_2)\) are regime-dependent. We use an extension of the original panel threshold model that has been developed by Hansen (1996, 2000). We allow the intercept to be regime-dependent, with \(\delta_1\) being the additional constant term in the first regime (low risk aversion regime). As in the original model, the threshold value is determined endogenously, based on maximum likelihood methods. The error term is assumed to be independent and identically distributed with mean zero and finite variance. The paper by Bick (2010) has more details on this extension and provides Gauss codes used for the analysis.

35. **It is important to test whether the threshold is statistically significant.** In principle, the significance of the sample split could be established with conventional structural break tests (Chow test). However, Davies (1977) has shown that such a procedure is invalid in the context of our study since it assumes that the sample split value is known with certainty, whereas in this case it is estimated endogenously. Hansen (1996) therefore develops a Supremum F-, LM- or Wald-test, with a non-standard distribution dependent on the sample of observations. The critical values are then obtained by a bootstrap methodology. The results of our threshold models, including tests for the number of thresholds, are displayed in Table 2.

36. **The question is how to proxy risk aversion.** According to the literature on time-varying risk preferences (Campbell and Cochrane, 1999), risk aversion is high when marginal utility is also high and vice versa. Market volatility also has this counter-cyclical property, which is why some (Mele, 2007) have argued that time-varying risk aversion is a major driving force behind counter-cyclical volatility. Hence, following Strohsal (2017), we consider volatility to be a reasonable choice to proxy risk aversion.

37. **Our volatility series, computed on daily data, correspond to the rolling standard deviation of the country’s 10-year bond yield, over a window of one quarter.** We assume that a quarter corresponds to thirteen business weeks (i.e., 65 business days). Then, we pass from daily to quarterly data, taking the average of the volatility measure over the whole quarter. It means that the quarterly volatility is not the standard deviation within this same quarter but an average of the daily rolling standard deviations of this quarter. For the threshold regressions, we use the ratio of the bond volatility of the other country over the volatility of German bunds. For instance, if we are looking at the Bund premium vis-à-vis the U.S, this would be U.S Treasury yield volatility over the German Bund volatility. We also tested for other potential proxies of risk aversion, such as economic policy uncertainty, but did not find significant evidence of significant threshold effects with these variables.

38. **In addition to the panel regressions, we also perform time series analysis of the 10-year Bund premium vis-à-vis the U.S using monthly data.** The explanatory variable in this case is the ratio of long-term German debt supply over the U.S. debt supply, where long-term debt is defined as exceeding a maturity of 5 years (in line with Strohsal (2017)). We approximate long-term debt using Bloomberg data on amount issued, issue date, and maturity
date by bond and then summing the different bonds up, to arrive at an estimate for total traded debt at a given maturity for a given month.

39. **We estimate the following regression:**

\[ \rho_{US,10Y,t}^{CIP,CD,LIq} = \alpha + \beta \log \left( \frac{\text{debt}_{LT,DEU}}{\text{debt}_{LT,US,t}} \right) + \epsilon_t \quad [15] \]

We compute robust standard errors following Newey-West (1987), in line with Greenwood and Vayanos (2014), allowing for up to 18 lags in the adjustment. Since we have only about 129 observations, we do not test for threshold effects.

**Results**

40. **The “Bund Premium” varies with the relative supply of German debt.** As hypothesized, a decrease in German debt supply relative to other countries leads to a rise in the Bund premium. Figure 7 depicts the relationship between the Bund premium and relative debt supply. It plots quarterly Bund premia vis-à-vis all countries in our sample against the ratio of the corresponding debt-to-GDP ratios. The regression results in Table 1 confirm the significance of the negative relationship. One can think of this relationship as a global demand curve for Bunds. This is because the relationship depicts how exogenous supply changes affect yields and therefore the demand of relative habitat investors and arbitrageurs, with the premium being the relative price of bunds once a new equilibrium has been established.

![Figure 7. A Demand Curve for Bunds](image)

Sources: Bloomberg; Markit; WEO; and IMF staff calculations.

| Table 1. Fixed-Effects Panel Regression Results |
|-----------------------------------------------|
| **Dependent Variable: Bund Premium - 10Y**    |
| (all countries) | (G10) | (G10+FR) |
| Debt supply ratio (log) | -0.415*** | -0.455*** | -0.428*** |
|                      | (-7.19) | (-6.07)  | (-6.66)  |
| Constant             | 0.102*** | 0.0492   | 0.0459*  |
|                      | (11.31) | (2.17)   | (2.88)   |
| Observations         | 456     | 333      | 374      |
| Adjusted R2          | 0.111   | 0.210    | 0.180    |
| R2                   | 0.113   | 0.212    | 0.182    |

Note: Robust standard errors. \( t \) statistics in parentheses. Country fixed effects.

* \( p < 0.05 \), ** \( p < 0.01 \), *** \( p < 0.001 \)
41. **The slope of the demand curve is greater for longer-dated German bonds and risk aversion matters.** In line with the predictions of the theoretical model, the slope is greater for 10-year premia than 2-year German bond premia (Figure 8 and Table 2). This is because risk-averse investors will demand higher risk premia for holding longer-dated debt. To provide a rough idea of whether risk aversion may affect the slope of the Bund demand curve, we interact our chosen proxy for risk aversion with the relative debt supply. Interestingly, as Figure 9 shows, this tightens the relationship between the relative debt supply and our estimated premia.

42. **We find significant evidence of threshold effects.** The effect of the relative debt supply on the Bund premium differs significantly depending on whether bond volatility vs. Germany is above (the high-risk aversion regime) or below the estimated threshold (the low-risk aversion regime, see Table 2). In the high-risk aversion regime, the effect of relative supply changes on Bund premia is significantly higher than in the low-risk regime. We decided to include redenomination risk in the threshold regressions, for two main reasons. First, the model by Bick requires to add at least one additional regressor in the analysis. Second, redenomination risk is the additional regressor which leads to the highest R-squared in the robustness checks estimates of Table A3.1 in the Appendix.

43. **The results are robust to alternative specifications.** These robustness checks can be found in Tables A3.1–A3.3 in Appendix 3. The size, sign, and significance of the coefficients remain broadly the same when we include other explanatory variables in the regressions. We test for the presence of heteroscedasticity, autocorrelation and cross-sectional dependence. The latter implies that the error terms are correlated across countries since the units of observations are simultaneously affected by common but unobserved factors. Since we find evidence of all three, we perform the same regressions with Driscoll and Kraay standard errors as is custom in the literature. We also adjust the debt supply by the central bank holdings to account for the fact that bonds held by central banks are not available to the public. Again, the coefficient on relative debt supply remains almost unchanged.

44. **The results of the time series analysis of the 10-year Bund premium vis-à-vis the U.S. are broadly in line with the panel regression.** The relative long-term debt supply is an important determinant of the Bund premium vis-à-vis the U.S. The size of the coefficient (-0.39) is broadly similar to the result for the panel regressions with all countries (Table 3).
Table 2. Threshold Regressions

| Dependent Variable: Bund Premium – 10Y | (entire sample) | (balanced sample) | (threshold model) |
|---------------------------------------|----------------|-------------------|------------------|
| Test for the number of thresholds: p-value |                |                   | 0.010            |
| H0: No threshold                      |                |                   |                  |
| H0: At most one threshold             |                |                   | 0.971            |
| Threshold estimates and confidence intervals |            |                   |                  |
| Estimated $\gamma$                    | 2.01           |                   |                  |
| 95% confidence interval               | [1.85, 2.01]   |                   |                  |
| Average EA                            | 0.760**        | 0.830*            | 0.669***         |
| (2.35)                                | (2.08)         |                   | [0.13]           |
| Debt supply ratio                     | -0.321***      | -0.225            | -0.231***        |
| (regime 1)                            | (-3.19)        | (-1.60)           | [0.09]           |
| Debt supply ratio                     | -0.506***      |                   | -0.506***        |
| (regime 2)                            |                 |                   | [0.14]           |
| Constant                              | -0.0743        | -0.0861           | -0.425***        |
| (regime 1)                            | (-1.12)        | (-1.13)           | [0.07]           |
| Additional constant                   |                 |                   |                  |
| Fixed effects                         | Yes            | Yes               | Yes              |
| N                                     | 422            | 324               | 324              |
| Adjusted R2                           | 0.188          | 0.134             |                  |
| R2                                    | 0.192          | 0.139             |                  |

Note: Robust standard errors. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. In the threshold model (last column), the threshold variable is the ratio of the other country’s bond volatility over Germany’s bund volatility. Regime 1 (297 obs.) corresponds to the low volatility regime vis-à-vis Germany, whereas regime 2 (27 obs.) corresponds to the high volatility regime vis-à-vis Germany. We have to use a balanced sample for the threshold model estimations. It is estimated with Gauss, using a code developed by A. Bick (2010) that allows the constant to be regime specific (as opposed to the original panel threshold model developed by Hansen, 1996, 2000). 1000 bootstrap replications were used to obtain the p-values to test for the number of thresholds. By construction, the confidence intervals for the threshold estimates can be highly asymmetric. We also performed the same estimations with constant intercepts. The results were broadly comparable and can be obtained from the authors upon request.

Figure 9. The Bund Premium Against Unadjusted and Adjusted Relative Debt Supply

Sources: Bloomberg; Markit; WEO; and IMF staff calculations.
Note: In the right chart, the volatility ratio is the ratio of German bund volatility over the other country’s bond volatility (the inverse of the variable that is used in the threshold model), i.e. there is high volatility in the other country vs. Germany when this variable is close to zero.
We again perform various robustness checks. These are summarized in Table A3.4 in Appendix 3. One immediate concern in a time series regression is that the results could be driven by a simple time trend. The appendix shows that this is not the case. Another concern relates to the potential presence of endogeneity. The relative long-term debt supply could be endogenous, perhaps influenced by the size of the Bund premium. While we are not sure how plausible this is, we perform an instrumental variable regression. The most straightforward is to use lagged values of the relative long-term debt ratio and GMM estimation. Our results show that the relationship in fact strengthens under this approach.

V. POLICY IMPLICATIONS UNDER DIFFERENT NORMALIZATION SCENARIOS

Going forward, we expect the total supply of safe sovereign assets to increase, but there are important shifts in its composition. While we expect the supply of U.S. Treasuries to increase because of the U.S. fiscal stimulus, the supply of German bonds will further decrease (Figure 10), in relation with the expected fiscal policy in Germany. Moreover, central banks holdings of government debt will also determine the relative debt supply available to the public going forward. Given the recent decision by the ECB to restart an open-ended new asset purchase program, with purchases of €20 billion a month, the supply of Germany bonds available to the market will further shrink.

Using the results from Section III, we can perform scenario analysis of Bund premia going forward:

- We first assume that the debt supply going forward is given by the projected relative debt-to-GDP ratios from the IMF’s World Economic Outlook. We assume no change in central banks’ balance sheets and use the baseline coefficient from the panel regressions. We then add different degrees of risk aversion and the corresponding coefficients on debt supply to project the Bund premium. We simulate the path going forward both vis-à-vis the U.S. as well as for the average Bund premium.
• We then extend this analysis with two different normalization scenarios, focusing on Germany vis-à-vis the U.S.: (i) New net asset purchases by the ECB over the projection horizon, with German purchases in accordance to the capital key. We abstract from a discussion of feasibility within the current issuer limits. The Fed is assumed to keep its balance sheet constant over the projection horizon; (ii) the ECB starts to normalize policy and German debt holdings are halved over the projection horizon. In this case, we assume that euro system holdings of German debt will halve by 2024 with normalization starting in 2021. The Fed balance sheet will evolve as in (i). While scenario 2 is unlikely to materialize barring a significant positive surprise inflation and growth shock in the euro area, it serves nonetheless as an interesting illustration of how different central bank balance sheet paths influence the Bund premium.

• We also look at a case in which we use scenario 2 but with the coefficient on relative debt supply with low volatility ratio (the lowest coefficient obtained in our threshold regression analysis in the previous section).

• Importantly, in this analysis, we assume there is an element of surprise and that future developments are not already factored into market expectations and therefore today’s Bund premium. This is an important caveat.

48. With scarcity of German debt expected to persist, Bund premia will remain elevated in the period ahead. As shown in Figure 11, the size of the Bund premium will differ depending on the risk aversion of investors. If the next years turn out to be relatively tranquil, the further increase in the Bund premium will be limited compared to a scenario in which risk aversion is high. The shaded areas should not be interpreted as confidence bands but rather as the range of outcomes using the different coefficients of our previous estimations.

49. Central bank quantitative easing will have an impact on the Bund premium. By altering the relative supply of debt available to the market, central banks’ holdings and purchases will matter for the premia. As seen in Figure 12, the Bund premium is expected to
be much higher in a scenario of continued ECB QE (dark blue line) than in a no monetary policy change scenario (grey line). However, as also shown in Figure 12, the fiscal trajectory is a key factor in determining relative debt supply and thus Bund premia. Combining rapid unwinding of Eurosystem German debt holdings with a low relative volatility scenario, results in the lowest Bund premium relative to U.S. Treasuries going forward. However, even in this case, the specialness of Bunds will increase over the projection horizon.

VI. CONCLUSION

50. This paper finds that starting with the European debt crisis, Bunds have become special. We estimate the “Bund premium” as the difference in convenience yields between German government bonds and other sovereign safe assets adjusted for sovereign credit risk, liquidity and swap market frictions. A higher premium suggests less substitutability of sovereign bonds. We document a rise in the “Bund premium” in the post-crisis period. Bunds are now earning a significant convenience yield relative to other, comparably safe government bonds.

51. Part of this new specialness of Bunds seems to reflect their extreme scarcity. Borrowing from the seminal paper by Krishnamurthy and Vissing-Jorgensen, 2012, who plot the deviation of U.S. Treasury yields from triple A corporate bond yields against the supply of U.S. Treasuries, we construct a “Bund demand curve”, which plots our estimated Bund premium against the relative supply of German public debt vis-à-vis the other countries in our sample. We show that there is a significant negative relationship of the Bund premium with the relative supply of German government debt.

52. With scarcity expected to persist, Bunds will remain close to “bulletproof” in the period ahead. The projected decrease in German government debt supply compared to other large economies will put further downward pressure on German bonds. In all our simulated
scenarios, the Bund premium is expected to increase over the coming years. In line with a simple model of preferred habitat investors and arbitrageurs, the effect of further decreases in debt supply will be more pronounced for longer-term German debt and in times when risk aversion is high. Going forward, if the next years turn out to be relative tranquil, the further compression of Bund yields could be limited compared to a scenario in which risk aversion is high.

53. These findings have important implications for the ECB’s monetary policy strategy. German bond yields are eurozone benchmark interest rates. While their scarcity effects are beneficial in helping the ECB maintain low long-term yields the near term, they could complicate the process of monetary policy normalization over the longer run.
### VII. APPENDICES

#### Appendix I. Data Sources

| Country | Government Yields | IRS     | Basis Swaps | Euro Denominated CDS | USD Denominated CDS | OIS     | KfW  |
|---------|------------------|---------|-------------|----------------------|---------------------|---------|------|
| US      | C082#Y Index     | USSW#CURN CY | EUBS#CURN CY | USV#EAC              |                      | USSO#Curn CY |      |
| UK      | C110#Y Index     | BPBS#CURN CY | GBG#EAC     |                      | UK CDS USD SR #Y D14 Corp | BPSWS#Curn CY |      |
| CH      | C256#Y Index     | SFSW#CURN CY | SFBS#CURN CY | Switzerland CDS USD SR #Y D14 Corp | SFSWT# Curn CY |      |
| JP      | C105#Y Index     | JYSW#CURN CY | JYBS#CURN CY | Japan CDS USD SR #Y D14 Corp | JYSO#Curn CY |      |
| AU      | C127#Y Index     | ADSWAP#Q CURN CY | ADBS#CURN CY | Australia CDS USD SR #Y D14 Corp | ADSO#Curn CY |      |
| CA      | C101#Y Index     | CDSW#CURN CY | CDBS#CURN CY | Canada CDS USD SR #Y D14 Corp | CDSO#Curn CY |      |
| NZ      | C250#Y Index     | NDSWAP#CURN CY | NDDBS#CURN CY | New Zealand CDS USD SR #Y D14 Corp | NDSO#Curn CY |      |
| NO      | C266#Y Index     | NKSW#V3 CURN CY | NKBS#CURN CY | Norway CDS USD SR #Y D14 Corp |      |      |
| SW      | C259#Y Index     | SKSW#CURN CY | SKBS#CURN CY | Sweden CDS USD SR #Y D14 Corp | SKSWTN#Curn CY |      |
| DK      | C267#Y Index     | DKSW#V3 CURN CY | DKBS#CURN CY | Denmark CDS USD SR #Y D14 Corp |      |      |
| DE      | C910#Y Index     | EUSA#CURN CY | DEG#EAC     | Germany CDS USD SR #Y D14 Corp | EUSWE#Curn CY | IB#MKFW Index |
| FR      | C915#Y Index     | FRG#EAC |            | France CDS USD SR #Y D14 Corp |      |      |
| ES      | C902#Y Index     | ESG#EAC |            | Spain CDS USD SR #Y D14 Corp |      |      |
| IT      | C905#Y Index     | ITG#EAC |            | Italy CDS USD SR #Y D14 Corp |      |      |

Note: This table lists the Bloomberg tickers used to construct the German bond premium for each country. # denotes the maturity of the contract. IRS denotes interest rate swaps. CDS denotes Credit Default swaps. OIS denotes overnight index swaps. KfW denotes Kreditanstalt für Wiederaufbau.
Appendix II. Additional Tables and Charts

a. Alternative Liquidity Measures

Source: ECB.
Note: see ECB Economic Bulletin, Issue 2 / 2018 – Box: Euro area sovereign bond market liquidity since the start of the PSPP)
b. Bund Premia Across Other Countries in the Sample

Sources: Bloomberg; and IMF staff calculations. Note: Excludes Norway due to very short sample period.
# Appendix III. Robustness Checks

## Panel Regression

### Table A3.1

| Dependent variable: Bund premium - 10Y | Baseline (entire sample) | Additional variables (entire sample) |
|---------------------------------------|--------------------------|-------------------------------------|
| Debt supply ratio                     | -0.412***                | -0.369***                           |
|                                       | (-6.68)                  | (-5.59)                             |
| Economic policy uncertainty           | 0.000739*                |                                     |
|                                       | (1.99)                   |                                     |
| Redenomination risk (10Y)             | 0.760**                  |                                     |
|                                       | (2.35)                   |                                     |
| Global economic policy uncertainty    | 0.00169***               |                                     |
|                                       | (4.56)                   |                                     |
| 10Y yield volatility                  | 1.344*                   |                                     |
|                                       | (1.92)                   |                                     |
| Constant                              | 0.0971***                | -0.0154                             |
|                                       | (10.82)                  | (-0.27)                             |
| Observations                          | 456                      | 456                                 |
| R2                                    | 0.0900                   | 0.112                               |
| Adjusted R2                           | 0.0920                   | 0.116                               |

Note: Robust standard errors. t statistics in parentheses. Country fixed effects.
* p < 0.10, ** p < 0.05, *** p < 0.01

### Table A3.2

| Dependent variable: Bund premium - 10Y | Baseline | Driscoll-Kraay standard errors |
|---------------------------------------|----------|-------------------------------|
| Debt supply ratio                     | -0.412***| -0.412***                     |
|                                       | (-6.68)  | (-4.01)                       |
| Constant                              | 0.097*** | 0.097***                      |
|                                       | (10.82)  | (3.79)                        |
| Observations                          | 456      | 456                            |
| Adj. R2                               | 0.09     | 0.09                           |

Note: Robust standard errors in the baseline regression. t statistics in parentheses. Country fixed effects.
* p < 0.05, ** p < 0.01, *** p < 0.001
### Table A3.3

**Dependent variable: Bund premium - 10Y**

(Entire sample)

| Variable                                         | Basic       | Time trend  | GMM         |
|--------------------------------------------------|-------------|-------------|-------------|
| Debt supply ratio (adjusted for CB holdings)     | -0.415***   | (-7.19)     |             |
| Constant                                         | 0.102***    | (11.31)     |             |
| Fixed effects                                    |             |             |             |
| N                                                | 456         |             |             |
| Adj. R2                                          | 0.111       | 0.113       |             |
| R2                                               |             |             |             |

Note: Robust standard errors. *t* statistics in parentheses. Country fixed effects. *p* < 0.05, **p** < 0.01, ***p** < 0.001.

### Table A3.4: Time-Series Analysis

**Dependent variable: 10-year Bund Premium vis-à-vis the U.S.**

| Variable             | Basic      | Time trend | GMM         |
|----------------------|------------|------------|-------------|
| C                    | 1.41 (9.19)*** | 2.49(4.47)*** | 1.42 (12.59)*** |
| Relative Long-Term Debt | -0.39 (8.64)*** | -0.61 (-5.63)*** | -0.39 (-11.72)*** |
| Time trend           | -0.002 (-1.83) * |           |             |
| R2                   | 0.47       | 0.48       | 0.47        |
| Adjusted R2          | 0.46       | 0.47       | 0.46        |
| Durbin-Watson stat   | 0.28       | 0.31       | 0.28        |
| Observations         | 129        | 129        | 129         |

Notes: *t* statistics in parentheses. *p* < 0.10, **p* < 0.05, ***p* < 0.01. HAC standard errors & covariance (Prewhitening with lags = 18, Bartlett kernel, Newey-West fixed bandwidth = 5.0000).
Appendix IV. A Model with Arbitrageurs and Preferred Habitat Investors

This appendix presents a modified version of the model by Greenwood and Vayanos (GV, 2014). Contrary to GV (2014), we consider more than one country as we are interested in the effect of relative supply changes of German versus other countries’ sovereign debt.

The model is set in continuous time. The term structure for each country at time t consists of a continuum of zero-coupon bonds with maturities in the interval (0,T) and face value one.

Let $P_t^{\tau,DEU}$ denote the price of a German bond with maturity $\tau$ at time t. The yield, $y_t^{\tau,DEU}$ is related to its price through

$$y_t^{\tau,DEU} = -\frac{\log P_t^{\tau,DEU}}{\tau} \quad [1]$$

Similarly, for other G10 countries, i:

$$y_t^{\tau,i} = -\frac{\log P_t^{\tau,i}}{\tau} \quad [2]$$

The short rates $r_t^{DEU}$ and $r_t^{i}$ are the limit of $y_t^{\tau,DEU}$ and $y_t^{\tau,i}$ respectively when the maturity goes to zero. We assume that short rates are exogenous and assume they follow the Ornstein-Uhlenbeck process

$$dr_t^{DEU} = \kappa_r (\bar{r} - r_t^{DEU})dt + \sigma_r dB_t \quad [3]$$

Similarly, for each of the other G10 countries in our sample:

$$dr_t^{i} = \kappa_r (\bar{r} - r_t^{i})dt + \sigma_r dB_t \quad [4]$$

where $\kappa_r$, $\bar{r}$, and $\sigma_r$ are constants (which are assumed to be identical across countries) and $B_t$ is a Brownian motion.

The yield of a bond in given currency is given by interaction of three types of agents: government, investors with a preference for bonds with a given maturity $\tau$ and issued by a given government (preferred habitat investors) and arbitrageurs.

The gross supply of $\tau$-year German bonds through the government less the demand of preferred habitat investors results in a net supply, $N_t^{\tau,DEU}$ at that specific maturity.

The time t value of net supply of German bunds is assumed to be negatively correlated with the yield $y_t^{\tau,DEU}$:

---

1 This assumes that preferred habitat investors can substitute between bonds and other asset classes (e.g., real estate). Thus, an increase in yields could raise their bond demand.
\[ NS_t^{DEU} = \beta^{DEU}(\tau) - \alpha^{DEU}(\tau)\tau y_t^{DEU} \]  \[5\]

Similarly, for other G10 countries:

\[ NS_t^{i,l} = \beta^{i}(\tau) - \alpha^{i}(\tau)\tau y_t^{i,l} \]  \[6\]

where \( \alpha^{DEU}(\tau) > 0 \) and \( \alpha^{i}(\tau) > 0 \)

If there are no arbitrageurs, the market for \( \tau \) year bonds would clear for

\[ y_t^{\tau,DEU} = \frac{\beta^{DEU}(\tau)}{\alpha^{DEU}(\tau)\tau} \text{ and } y_t^{\tau,i} = \frac{\beta^{i}(\tau)}{\alpha^{i}(\tau)\tau} \]  \[7\]

In this case, the market would be completely segmented. Each maturity and country constitute a separate market, with yields being determined by local supply and demand.

For the market to clear if there are arbitrageurs, total net supply has to be absorbed by the demand of arbitrageurs, \( x_t \). They can invest in both German bonds or other currency bonds and across different maturities and generally will aim for higher mean and low variance of their wealth changes \( dW \):

\[
\max_{\left[ x_t^{\tau,j} \in 0; x_t^{\tau,j} \in 1; N \right]} \left[ E_t(dW_t) - \frac{\alpha}{2}Var_t(dW_t) \right]
\]  \[8\]

where \( x_t^{\tau,j} \) denotes the investment in a bond of country \( j \) (this could be Germany or another G10 country, \( i \), with \( N \) countries in total) and maturity \( \tau \) while \( \alpha \) refers to the degree of arbitrageurs’ short rate risk.

We assume that the only source of uncertainty is a country’s short-term interest rate. We also abstract from liquidity and credit risk and assume that exchange rate risk is the only factor driving a wedge between convenience yields of different countries. In this case, the German bond premium vis-à-vis another G10 country is given as follows:

\[
\rho_{i,\tau,t} = y_t^{\tau,i} - \varphi_{i,\tau,t} - y_t^{\tau,DEU}
\]  \[9\]

where \( \varphi_{i,t} \) is the forward premium defined as

\[
\varphi_{i,\tau,t} = \frac{1}{\tau} \left[ \log(F_{i,\tau,t}) - \log(S_{i,t}) \right]
\]  \[10\]

with \( F_{i,\tau,t} \) denoting the \( \tau \)-year outright forward exchange rate and \( S_{i,t} \) the spot exchange rate, measured in terms of units of currency of country \( i \) per euro.

For simplicity, we assume that exchange rate risk is a function of interest rate risk. The cost of hedging against exchange rate risk is therefore a function of the uncertainty about the respective short rates.
By focusing on exchange rate risk and the cost of hedging, the modeling approach followed here applies to the German bund premium vis-à-vis the other G10 currencies. To model the Bund premium vis-à-vis the other euro area countries (where there is no exchange rate risk), one would need to introduce another friction, such as for example redenomination risk. Since the sample used in this paper predominantly consists of non-euro countries, the paper leaves this to future work. If bond prices are affine functions of the short rate, the risk premium demanded by arbitrageurs in equilibrium will be a function of the sensitivity of the bond to short rate risk and the market price of risk (as derived in Greenwood and Vayanos, 2014):

$$\theta_{t}^{DEU}(a) = A(\tau,a)\gamma_{DEU}(a)$$  \[11\]

where $A(\tau,a)$ is bond sensitivity to short-rate risk and $\gamma_{DEU}(a)$ is the market price of risk in Germany. The demanded term premium is increasing in both the maturity of the bond and the degree of risk aversion of arbitrageurs.

Similarly, for other countries $i$:

$$\theta_{t}^{i}(a) = A(\tau,a)\gamma_{i}(a)$$  \[12\]

Based on this simple model, we can derive three hypotheses:

**Hypothesis 1:** An increase in German bond supply relative to that of another G10 country will lead to a decrease in the German bond premium.

**Hypothesis 2:** This decrease will be higher for longer dated bonds.

To see the intuition behind this prediction, suppose that the relative supply of German 10-year bonds increases (for example, Germany issues more 10-year bonds, while assuming that the supply of German short-term debt stays the same and the debt supply at all maturities of the other countries stays unchanged). According to Greenwood and Vayanos, this is modeled as an increase in $\beta^{DEU}(10)$. We assume that $\beta^{i}(10)\tau$ remains unchanged. If there are no arbitrageurs, the above clearing condition applies, which means that the 10-year Bund yield increases.

If there are arbitrageurs, they could exploit the differences in yields by (i) selling 10-year bonds of the other G10 country and buying 10-year Bunds and (ii) selling shorter-term debt to buy longer term debt (this could be both German and foreign shorter-term debt). This would reverse the initial changes in yields (and therefore the premium). However, the risk profile of arbitrageurs has changed. Those that have exchanged shorter dated debt with longer dated debt are subject to higher interest rate risk. This leads to a higher market price of risk and hence an increase in risk premia at all maturities, reducing prices and raising yields. Since the sensitivity of longer dated bonds is higher for longer maturities, the increase in premia is stronger for longer term bonds. Those that have exchanged a G10 bond for a German 10-year bond in this scenario also have a change in their risk profile, namely the exchange rate risk exposure.
We assume that exchange rate risk is a function of uncertainty about the short rate and therefore also increases with the maturity of the bond. Investors can hedge against exchange rate risk, but the cost of hedging is higher the greater the interest rate risk. Therefore, those arbitrageurs will also demand a greater risk premium.

The fact that arbitrage is risky and costly has been well documented in the literature. In an ideal model-based world, arbitrage is riskless. However, as Borio and others (2016) show, in reality, arbitrage typically requires the arbitrageur to enlarge its balance sheet, incur credit risk in both borrowing and investing, and possibly face mark-to-market and liquidity risk (given the need to transfer collateral or take paper gains or losses) in the valuation of the positions. We do not model the risks/costs of arbitrage here explicitly, but simply assume that they increase with the maturity of the bond and hence the exchange rate risk.

Putting both of these effects together means that a change in the relative supply of longer-dated German debt will lead to an increase in German bond yields, which is more pronounced at longer maturities. Therefore, the bond premium, which is a function of the difference between another country’s bond yield and German bond yields adjusted for various factors that could drive a wedge between the two (and which we abstract from in this simple model), will decrease.

Hypothesis 3. When arbitrageurs are more risk adverse, the effect of a change in the relative supply of German bunds on the Bund premium will be stronger for all τ.

Let’s look at the extreme cases. If a is zero, and arbitrageurs are risk neutral, then the market price of risk would be zero. Local supply changes would be completely offset by arbitrageurs. If on the other hand, arbitrageurs are infinitely risk averse, then changes in risk premia would go to infinity and they would not participate at all. Bond markets would be completely segmented. For intermediate cases, the change in arbitrageurs’ risk premia caused by a change in the riskiness of their portfolio is stronger when risk aversion is higher. For example, if arbitrageurs exchange the same maturity bonds of one country for another country, the extent to which they are willing to buy insurance against exchange rate risk (hedge) will depend on their risk aversion. Hence, the more risk averse arbitrageurs are, the more they will hedge and therefore the higher the cost of insurance and the risk premium they will demand.
VIII. REFERENCES

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