Research on comprehensive optimization of engineering project based on non-radial super-efficiency DEA

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Abstract. Duration, cost and quality are three important main indexes to measure the comprehensive benefit of the construction engineering project, the comprehensive equilibrium optimization of which is great of significance. With simple process and clear results, non-radial super-efficiency DEA is an algorithm for studying the input and output, which considers the resource constraints and decision-maker preferences. By introducing this algorithm into the field of comprehensive optimization of engineering project, the multi-objective comprehensive optimization model of quality, cost and duration is established and the solution is obtained by using non-radial super-efficiency DEA model, thus completing the order of different construction schemes and realizing comprehensive optimization of engineering projects. Finally, through one application example, the solution is obtained by using LINGO software, which has proved the effectiveness of the optimization method and provided support to the decision-making of the project.

1. Introduction
The management and control of cost, progress and quality is the priority of project management, but these three factors have opposite and uniform relationship. How to realize the maximization of quality with low cost and short schedule is still the focus of engineering field and academic circle.

In recent years, many scholars have made researches on the comprehensive optimization of quality, duration and cost from different aspects. Liu Jia [1] obtained comprehensive optimization model among duration, cost and quality based on fuzzy set theory by using non-dominant sort genetic algorithm. Wang Jian et al.[2] used genetic algorithm to solve the comprehensive optimization model of engineering project based on the multi-attribute utility function theory. Gao Xingfu et al.[3] adopted the specific numbers between 0 to 1 to quantitate quality and obtain the comprehensive optimization model of duration, cost and quality by genetic algorithm. Yang Yaohong et al.[4] used adaptive genetic algorithm to acquire the optimization model of engineering project by adaptive genetic algorithm based on the membership degree of objectives. Yin Qixin et al.[5] established the model of cost, quality and duration of each engineering activity under nonlinear relation and obtained the optimization model by using genetic algorithm. Zuo Jinsong et al. [6] used Standard Particle Swarm Optimization to obtain the model of duration, quality and cost after standardized treatment. Zhang Lianying et al.[7] applied improved Immune Genetic Particle Swarm Optimization algorithm to obtain optimization model of engineering project based on reliability of network system. Liu Xiaofeng et al. [8] introduced the principle, process and algorithm of particle swarm optimization algorithm and applied it to obtain the best construction scheme.
From the references above, it is found that the first kind of researches [1-6] introduce genetic algorithm into the multi-objective optimization problem of engineering projects, but solving steps of those are relatively cumbersome. After complex intersection and variation, the selective operation is carried out and most of the optimization techniques are easy to trap in local optimum, thus unable to ensure the scientific nature of optimization. The second type of researches [6-8] introduce particle swarm optimization to the multi-objective optimization problem of engineering projects without considering the binding of resource and preferences of decision-makers.

DEA is a rapidly developing method that appraises the input and output. Due to its large advantages in optimization problem, such as simple operation processes and clear result, DEA is widely used in the efficiency evaluation of system input and output in the field of environmental protection, construction, logistics engineering, investment and financing. This paper analyzes the input-output efficiency of different construction schemes by non-radial super-efficiency DEA improved from the traditional EDA model, thus completing the optimization of the construction scheme.

2. Non-radial super-efficiency DEA

It is assumed that one evaluation system has \( t \) decision units (DMU) and each DMU has \( m \) input indexes and \( n \) output indexes, within which \( x=(x_1, x_2, \ldots, x_m) \), \( y=(y_1, y_2, \ldots, y_n) \). \( x_{ij} \) represents input number of No. \( i \) of DMU\(_j\), while \( y_{rj} \) is output number of No. \( r \) of DMU\(_j\) (\( j=1, 2, \ldots, t; i=1, 2, \ldots, m; r=1, 2, \ldots, n \)). The response input and output data of DMU\(_{j_0}\) are represented by \( x_{0j}=x_{j_0}, y_{0j}=y_{j_0} \). When \( 1 \leq j_0 \leq t \), the formula of DMU\(_{j_0}\) based on the input radial C2R model is expressed as follows:

\[
\min \left[ \theta - \epsilon \left( \sum_{i=1}^{m} s_i^+ + \sum_{i=1}^{n} s_i^- \right) \right]
\]  
\[
s.t \quad \sum_{j=1}^{t} d_j x_{ij} + s_i^- = \theta x_{ij_0} \quad i = 1, 2, \ldots, m
\]  
\[
\sum_{j=1}^{t} d_j y_{rj} + s_r^+ = y_{rj_0} \quad i = 1, 2, \ldots, n
\]  
\[
d_j \geq 0, j = 1, 2, \ldots, t; \quad s_i^+ \geq 0, s_r^- \geq 0
\]  

(1)

Where \( s_i^+ \) and \( s_r^- \) represent residual variable and slack variable respectively. \( \epsilon \) is infinitesimal of Non-Archimedean Quantity i. In general, \( \epsilon=10^{-6} \). When \( \theta=1 \), the radial of DMU\(_{j_0}\) is effective; when \( \theta<1 \), the radial of DMU\(_{j_0}\) is ineffective. However, traditional radial model never considers the situation that in the real practice, there is few input variables and output variables of decision unit equal to each other and input variables are always influenced by the objective conditions, while the preference of each output variable of the decision-maker is different. Therefore, many scholars (such as Zhu [9], Seiford and Zhu [10]) have proposed non-radial DEA model based on the traditional radial DEA model. In the non-radial model, the degree of resource condition constraint or decision-maker's preference for the \( i \)-th input variable is taken into account, and the sum of them is equal to 1. However, the two models above have a common shortcoming that their evaluation results only distinguish the invalid decision-making units instead of effective decision-making units. Therefore, Anerson and Peterson [11] established the advanced model of traditional radial DEA model, super-efficiency model. With the purpose of completely listing all schemes on the basis of non-radial model, this paper applies super-efficiency idea in the non-radial model and constructs the non-radial super-efficiency DEA model.

\[
\min \left[ \frac{\sum_{i=1}^{m} a_i \theta_i}{\sum_{i=1}^{m} a_i} - \epsilon \left( \sum_{i=1}^{m} s_i^+ + \sum_{i=1}^{n} s_i^- \right) \right]
\]
3. Multi-objective optimization of engineering project

3.1 Duration of engineering project
During the construction process, when determining the target duration is shorter than the normal construction duration, the main influenced factors, such as workers, construction technique, material, machinery need controlling. To reduce the impact of adverse factors on progress, managers will always adopt some crashing measures, which have an influence on quality and cost. For instance, to catch up the construction period, artificially increasing the working time of both workers and machinery and even adopting night shift will result in increasing night construction cost, management fee and reduced construction quality standards. The duration of construction could be optimized as follows [12]:

\[ T_l \leq T_p \]  \hspace{1cm} (3)

where \( T_l \) is the calculated duration obtained by time optimization of network; \( T_p \) is the duration required by users or owners.

3.2 Composite cost of engineering
Operating cost of engineering includes direct cost (\( C_{i0} \)) and indirect cost (\( C_{i1} \)). If the duration of direct cost \( C_{i0} \) is shortened, the higher technology and higher-performance materials are required, thus increasing the cost of project. The higher the compression strength of the construction duration, the higher the working strength, the higher the requirements for materials, technical equipment and personnel, thus resulting in the faster speed of the increasing of direct cost (\( C_{i0} \)). On the contrary, if the construction duration is lengthened, the standards of material, technical equipment and staffing will be reduced. In general, the investment of indirect cost (\( C_{i1} \)) in early time is large and it becomes indistinctive with the increasing of construction duration. The combined cost of a single project operation is expressed as follows:

\[ C_i = a(T_l - T_{iN})^2 + c_{i0}^N + b(T_l - T_{iN})^2 + c_{i1}^N \]  \hspace{1cm} (4)

Where \( C_{i0} \) and \( C_{i1}^N \) are the direct cost and indirect cost in the normal duration of project \( i \); \( T_l \) is the actual time of work \( i \), \( T_{iN} \) is the normal time of work \( i \); \( a \) and \( b \) are adjustment coefficient.

3.3 Quality of engineering operation
Presently, in the quantitative study of the project quality, the relevant scholars managing domestic projects are divided into two types. Some think that construction quality has a linear relation with its duration, that is, the shorter of duration, the lower of engineering quality, while the other think that there is a non-linear relationship between the quality and duration of project. However, the gap between theory and reality exists. The quality of engineering not only concerns with duration, but also relates to the cost and resource investment of the project. Quickening the progress of engineering activities will result in the fatigue of worker, super work of machinery and material supply tension, thus reducing the quality of project. However, increasing the input of cost resources at that time, such as increasing the number of worker and applying higher-performance materials, will improve the quality. For the quality quantification problem that is difficult to adopt technical quantitative analysis, this paper employs expert scoring method to collect the opinions of experts involved in the management of similar engineering projects anonymously and makes a summary of the quality score and process quality weight \( w_l \) of experts based on different duration and cost of the construction. Since quality could not be measured by
standardized price measurement like cost, the important degree of the quality of each operation is different to the quality of the whole project. After weighting quality of all processes, the total quality of the project is obtained.

\[ Q = w_i * Q_i \]  

(5)

3.4 Comprehensive optimization model of quality, duration and cost of engineering projects

From the analysis above, the optimization model of quality, duration and cost of engineering project is established as follows, whose cost is low, duration is short and quality is high under the premise of the standard of quality and safety.

\[
\begin{align*}
\min T &= \sum_{i \in K} T_i \\
\min C &= \sum_{i \in M} C_i \\
\max Q &= \sum_{i \in M} w_i Q_i 
\end{align*}
\]

where K is the sets of engineering activities on critical path and M is the sets of all the engineering activities.

4.Comprehensive optimization of non-radial super-efficiency DEA for engineering projects

4.1 Application of optimization method of non-radial super-efficiency

Based on the efficiency of the input and output, non-radial super-efficiency DEA model could offer a good solution to the comprehensive equilibrium optimization among the duration, cost and quality of project. According to the common requirements of owner and construction units on the quality, cost and duration, the optimization model is established, and the problem of multi-objective is transformed into input-output efficiency by using the idea of objective programming. Regarding duration and cost as input quantity, the quality as output quantity, the program composition is finished by adopting non-radial super-efficiency DEA model and Linear Interactive and General Optimizer (LINGO) software, thus analyzing each construction scheme and obtaining the efficiency value. Developed by professor Linus Schrag in University of Chicago in around 1980 and rolled out by American LINDO systems company, LINGO could provide a system environment for human-machine interaction, whose optimization function can solve a series of optimization problems such as continuous optimization and integer optimization [13].

4.2 Application example

There is a project example[8], through preliminary statistical analysis, the table of the data of one engineering case is established (shown in Table 1). The owner makes the requirements that the duration of engineering should be within 19 months and the quality assurance rate should be above 93%. If the project is finished within 20 months, the quality assurance rate should be above 94%. The construction units require that the cost of the engineering is as lower as possible under the conditions of guaranteeing duration and quality. The quality value of engineering is assumed 100 in the normal completion of engineering. The data in Table 2 is the evaluation and quantification of the engineering with different duration and cost obtained by expert experience and the mutual relationship among the quality, duration and cost of the project.

| Processes | \( T_i^N \) | Ti | Front Closely Activity |
|-----------|-------------|----|------------------------|
| A         | 8           | 4  | /                      |
| B         | 7           | 40 | /                      |
Table 2. Project quality, duration and cost information

| Scheme | Project Quality | Duration (month) | Project Cost (tens of millions yuan) |
|--------|-----------------|------------------|---------------------------------------|
| 1      | 100             | 22               | 5.8                                   |
| 2      | 99              | 21.5             | 5.6                                   |
| 3      | 98              | 21               | 5.3                                   |
| 4      | 96              | 20.5             | 5.1                                   |
| 5      | 97              | 20               | 5.2                                   |
| 6      | 96              | 19.5             | 5.05                                  |
| 7      | 95              | 19               | 4.7                                   |
| 8      | 94              | 18.5             | 4.8                                   |
| 9      | 94              | 18               | 4.9                                   |

From the table above, it is found that if the project is finished within 19 months, the quality assurance rate is above 93%; if the duration of project is within 20 months, the quality assurance rate is above 94%, which meets the basic requirements of the owners. Under the condition of ensuring the quality of engineering, the construction units could obtain larger profit when the cost is as low as possible and duration is short. In this paper, lingo17.0 software is used to solve the non-radial super-efficiency DEA model. For further comparative analysis, radial, non-radial and radial super-efficiency models are also solved. The value of $a_i$ reflects the preference degree of decision-makers in two input variables including cost and duration. The assumption $a_i=1/2$ is made here and the decision-makers could set its value according to the resource constraint or personal preference.

Table 3. The results of assessment

| Scheme | Radial model | Non-radial model | Radial super-efficiency model | Non-radial super-efficiency model |
|--------|--------------|------------------|-------------------------------|----------------------------------|
| 1      | 0.8834       | 0.8810           | 0.8834                        | 0.8810                           |
| 2      | 0.8999       | 0.8978           | 0.8999                        | 0.8978                           |
| 3      | 0.9250       | 0.9241           | 0.9250                        | 0.9241                           |
| 4      | 0.9342       | 0.9339           | 0.9342                        | 0.9339                           |
| 5      | 0.9486       | 0.9464           | 0.9486                        | 0.9464                           |
| 6      | 0.9646       | 0.9626           | 0.9646                        | 0.9626                           |
| 7      | 1             | 1                | 1.0321                        | 1.0055                           |
| 8      | 0.9947       | 0.9925           | 0.9947                        | 0.9925                           |
| 9      | 1             | 0.9968           | 1.0278                        | 0.9968                           |

From the data results shown in Table 3, the following results are found:

(1) From the comparison of radial model and non-radial model, it is found that with the same input and output quantity, the efficiency value obtained by non-radial model is less than or equal to that of radial model due to the influence of non-radial relaxation variables on $DMU_{j0}$ efficiency.

(2) From comparison of super-efficiency radial model and super-efficiency non-radial model, it is known that the effective decision units of radial super-efficiency model could not judge the merits and demerits of scheme 7 and scheme 9.
(3) Non-radial super-efficiency model realizes the ordered list of each construction scheme. The total duration of scheme 7 reaches 19 months, its total fee is 47 million RMB and the efficiency value with 95 quality level is highest. If the preference on duration and cost of owner is the same, scheme 7 is the best scheme. The total duration of scheme 9 reaches 18 months, the total fee is 49 million RMB, quality level is 94 and its efficiency value is 0.9968 (close to value 1). If the owner has a high requirement in duration, this one could be selected. The total duration of scheme 8 is 18.5 months, total fee is 48 million RMB, quality level is 94 and efficiency value is 0.9925. With the same quality level, although the duration of scheme 8 is half a month longer than scheme, it total cost is less than scheme 9 (1 million RMB). If the owner has lower requirement on duration without rich funds, the scheme 9 could be chosen. In addition, the different preference on cost of owner could be realized by setting concrete value of $a$. With cost and duration as input variable, quality as output variable, the efficiency values of different construction schemes could be solved by using non-radial super-efficiency DEA model, thus providing the intuitive and correct decision support to managers and revealing the applicability and scientficity of this model and algorithm in the field of comprehensive optimization of construction cost and quality.

5. Conclusions and prospect

5.1 Conclusions of research

This paper has made the systematic research of multiobjective optimization problem of duration, cost and quality based on non-radial super-efficiency DEA model and the main conclusions are made as follows:

(1) The non-radial super-efficiency DEA model constructed in this paper could make up for the deficiency of traditional DEA Model. Despite tedious distinction of calculation processes between genetic algorithm and particle swarm optimization algorithm, the evaluation results with strong distinguishing ability are understood easily, which considers the preference of decision-makers in advance and obtains ordered list of comprehensive optimization of each construction scheme.

(2) Aiming at the characteristics that the objectivity of construction activity quality evaluation is poor and it is not easy to quantify concretely, the expert scoring method is adopted and the influence of duration and cost on the quality of engineering is considered, thereby making the evaluation results more scientific.

5.2 Limitations and future research direction

This paper puts forward new ideas on the aspect of solving the multiobjective comprehensive optimization problem of duration, cost and quality of engineering project. However, there are some limitations in the paper:

The empirical cases come from literature and if the sufficient data of engineering project could be collected in the future, the specific data would be combined to make the deep research, thus improving the reliability and practicability of the research.

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