Approximation of Cross-tier interference in HETNET using Stochastic Geometry

Y Tarriba-Lezama¹, L Valdez-Cervantes
¹, ² Antonio de Arevalo University Foundation
¹ University of Cauca
¹ yesid.tarriba@unitecnar.edu.co, yesid.tarriba@unicauca.edu.co
² decano.fadi@unitecnar.edu.co

Abstract. This work describes an approximation model to interpret in a heterogeneous two-tier network using stochastic geometry, the behavior of inter-tier interference or cross tier, and compares in terms of distributions the existing relationship between the intensity variables at each tier and the transmission powers of BS. The relationship between intensity functions of the distributions of each BS level and between the transmission powers will be considered.

1. Introduction
The performance of any wireless network is conditioned by many evaluations metrics, in the point of view the physical layer the limits of performance the interference show the important factor to describe how the network is working, the interference have two origins in a HETNET, cross-tier interference and co-tier interference so called inter-tier interference and intra-tier interference respectively[1]. The stochastic geometry is a mathematical tool used to puntual process for model how is the behavior of variable in the network[2]. In this work we will use this tool for an estimate the cross-tier with some considerate characteristic that simplify the model but is a viable approximation generalized. In an two tier network we will used the PPP (Puntual Process of Poisson) for model of distribution of BSM (Macro Base Station) and BSF (Femto Base station) y we will a compare the behavior the PPP and Power of transmition in each tier.

2. Model and Considerations
Below is described a multi-tier heterogeneous network where we define RF characteristics as frequency and distribution power segmented by each tier. We define this difference as the tier at which we assume conditions that favor to describe an approximation of interference between tier. We determine at the topology level the distribution the Base stations (BS) at each level are modelled by PPP with Lambda k in such a way that their condition fulfills that: \( \lambda_k > \lambda_M \) where \( \lambda_M, \lambda_F \) is the intensity function for the distribution of the BSM and BSF, respectively. Our model is a two-Tier network distributed both under the homogeneous PPP and which has as level variables \( \text{Tier}_k = (\lambda_k > P_t_k) \). The stochastic geometry can, depending on the distance between BS and coverage, establish the interference. This principle starts from the integral geometry as indicated in [3]. within the considerations to have for our heterogeneous network model, it is posed below:

- The noise power \( P_N = 0 \) is postulated.
- The effect on the download link is considered.
- We neglect the effects of multipath fading and shadowing.
- The possible additional tier will be considered iid.
- The Antenna pattern are circular and considered isotropic the propagation.
The metric that we are going to consider is the interference between BS of each level called cross-tier interference. This is aggravated by the fact that BSF are usually installed and manipulate by their end users\cite{4}. The term of cross-tier interference defines way on stochastic geometry. The download link for the base stations will present areas where the areas for BSM and BSF converge. This effect is debasing to the performance of the network, considering that the use of frequencies in the base stations coincide or share radioelectric resources.

\[
SIR_K = \frac{P_t_M}{P_t_F}
\]  

(1)

The equation (1) show the general shows the general description of the signal-to-interference ratio. Due to the distribution of the base stations PPP distribution generates SIR as a stochastic process. In Figure 1 show this model for description of HETNET of two-tier whit fixed power and BS distributed under PPP.

Figure. 1. Model of Tow-Tier HETNET considered under PPP

3. Interference Description
Interference as a performance metric establishes the relationship between the transmission power of a node at a level as a result of the sum of the powers of the remaining signals of the network in which it is considered that they share radio resources and have interference in the area of coverage \cite{5}, In equation (1) the generalization of the SIR is described, for the case of a multilevel network we would establish the relationship in terms of the sum of the signal power. In the description of the interference in HETNET, there are scenarios that increase, become more complex, the greater the number of tiers in the network. For the two level case study we could discover:

- Interference Co-Tier between BSM.
- Interference Co-Tier between BSF.
- Interference Cross-Tier between BSM and BSF.

The SIR (Signal Interference Ratio) seen from BSm is described below in which by means of the equation.
\[ SIR = \frac{P_{t_M}}{\sum_{k=1}^{K} I_k} \]  

(2)

Where \( I_k \) is the total interference of a given tier. We propose the deployment in \( R^2 \) of the BSM and BSF in such a way that the strategy to mitigate the interference would be the FFR efficiently [6] In this work, the estimation of cross tier interference was described, bearing in mind the previously announced considerations in works such as [7] establish the estimate of cross tier interference’s considering the low coverage model in coverage regions that have a dependence on the tier distributed function.

\[ I_{CT} = \frac{G_k L_k P_{CT} \ell}{\epsilon R_k} \]  

(3)

The equations (3) describes the model of Cross tier interference’s considering the fading at each tier and the circular coverage model of the BS. It is possible to consider that within the exceptions listed in this work it is possible to consider a good approximation of the results.

4. Simulations and Approximation

Next we will describe the procedure for calculating the interference areas between the coverage regions of each tier. Initially, it is to be considered that the interference in the HETNET network under the PPP distribution is in itself a stochastic process in which several dependency approaches are planned and among these we will consider that the distances between BS of each tier are random [8]. we propose the equation of the signal to interference relationship for the approximation.

\[ SIR_K = \frac{N_M P_{t_M}}{N_F P_{t_F}} \]  

(4)

\( N_M \) and \( N_F \) are number of BSM and BSF respectly

\[ N_M = \text{PoissonRND}(\lambda_M S) \]  

(5)

\[ N_F = \text{PoissonRND}(\lambda_F S) \]  

(6)

\( S \) is the area of net wok in Km\(^2\).According to our model, let's set the distribution in a plane \( \mathbb{R}^2 \)

\[ D = \sqrt{(X_M - X_F)^2 + (Y_M - Y_F)^2} \]  

(7)

In equation.

\[ D_{MF} = \left( (R_M + R_F)^2 - D^2 \right) * \left( D^2 - (R_F + R_M)^2 \right) \]  

(8)

In equation 7 \( X_M;Y_M \) and \( X_F;Y_F \) are the Cartesian position of any BS in the Area and equations 8 \( D_{MF} \) is the distance between BS_M and BS_F and \( R_M, R_F \) are the Radius of BSM and BSF respectly.

\[ I_{Cross} = R_M^2 \arccos\left(\frac{R_M^2 - R_F^2 + D^2}{2DR_M}\right) + R_F^2 \arccos\left(\frac{R_F^2 - R_M^2 + D^2}{2DR_F}\right) - \frac{1}{2} \sqrt{D_{MF}} \]  

(9)
In relation to the equation 4 the formal planet of the SIR is established together with the equation 9, we define:

$$SIR = \frac{N_M^{PTM}}{\sum I_{Cross} + \sum I_{Overlap}}$$

(10)

In the Figure 2 we can see the possibilities to extract the interceptions in the interference areas that will determine the total interference of the network.

4.1 Simulations Parameters

In this section we will describe the simulation process and the discussion of the results of the network simulation to describe the cross-tier interference shown in table 1. In which two BS distributions under the PPP distribution with different intensities and different powers generate the distribution of coverage as shown in Figure 3 that uses equation 8 to establish in \( \mathbb{R}^2 \) the distances between the nodes between Tier for and with Equation 9 calculates the intercepts, having an input variable in this process the distance that is random. The realizations of the graph generate the interference matrix where the options shown in Figure 2 are considered and equation 10 is applied from here we obtain that the representation of the SIR contaminating the interference between tier is possible to approximate it by means of a Gamma distribution function. The gamma distribution models the relationship of SIR VS the probability where in stages, compliance requires a minimum of up to 5 realizations to exercise the approximation. For our case we carry out 85 realizations and once the results of the realizations are obtained we proceed to adjust the distribution to which best is adjustment to the result this procedure is carried out for the estimation of greater likelihood and is verified with the goodness adjustment tests. In the Figure 3 show one realizations of distribution at HETNET two-tier. The sign \( L_e \) represent the downlink whit interference between tier. This \( L_e \) is considered by the nature of an signal, random and it is who is considered of study to determine in the equations? The results based on table 1. shows us the approximation in Figure 4 where in the discrete distribution or histogram it is performed in adjustment to the gamma distribution function as mentioned before using the Kolmogorov-Smirnov test as a goodness test, that with the data shown in table 2 it is shown in that the Cross-tier SIR is considered a good approximation through the gamma distribution. The CDF of the results are shown in Figure 5, up to here they described them with established initial values that match the characteristics of a HETNET, next we are going to carry out the variation of the HETNET parameters that describe the domain characteristics. Among which the intensity functions and the transmission power are noted.
Table 1. Parameters of simulation for HETNET

| Index | Parameter | Value | Variable |
|-------|-----------|-------|----------|
| 1     | Lambda<sub>M</sub> | 1     | 𝜆<sub>M</sub> |
| 2     | Lambda<sub>F</sub> | 4     | 𝜆<sub>F</sub> |
| 3     | Number of Tier | 2     | k        |
| 4     | Distribution | Homogeneous PPP |
| 5     | Power<sub>M</sub> | 25W   | 𝑃<sub>t</sub><sub>M</sub> |
| 6     | Power<sub>F</sub> | 8W    | 𝑃<sub>t</sub><sub>M</sub> |
| 7     | Area of Network | 1000Km<sup>2</sup> | S |

We will describe the distribution fit through the results. The modifications are proposed where the behavior of the interference is described as the relationships between the variables vary. Where the claim between increasing and decreasing lams is shown in Figure 6, and the relationship between the transmission powers also increasing and decreasing. This analysis starts with describing in general terms.

Table 2. Result Distribution Fitting

| Approximation Distribution | Shape | Scale |
|----------------------------|-------|-------|
| Gamma                      | 12,8772 | 11,8239 |

Table 3. Goodness-of-Fit Tests

|          | Gamma |
|----------|-------|
| Dplus    | 0,0870596 |
| Dminus   | 0,0739218 |
| DN       | 0,0870596 |

| P-Value   | 0,557139 |
Figure 3. Interference Cross for HETNET in $\mathbb{R}^2$.

Figure 4. PDF SIR Approximation whit parameter of Table 1.
how the PDF and CDF share of cross-tier interference is as the variables are modified.

5. Conclusions.
From the results obtained, it is valid to note that the gamma function is considered a good approximation to model cross-tier interference in a HETNET network. Considering the use of stochastic geometry to model the distribution of nodes, this function is present when describing the SIR of the network. and as it is shown in Figure 6, the CDF tends to coincide when the relationships between transmission power and intensity functions are similar. This indicates that the cross tier interference is a function dependent on the intensity functions and the power ratio.
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