Nonlinear waves at the free surface of flexible mechanical metamaterials

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In this letter we investigate the propagation of nonlinear pulses along the free surface of flexible metamaterials based on the rotating squares mechanism. While these metamaterials have previously been shown to support the propagation of elastic vector solitons through their bulk, here we demonstrate that they can also support the stable propagation of nonlinear pulses along their free surface. Further, we show that the stability of these surface pulses is higher when they minimally interact with the linear dispersive surface modes. Finally, we provide guidelines to select geometries that minimize such interactions.

Surface waves that propagate along the boundary of a medium play a key role in a variety of natural and man-made systems. Seismic surface waves cause the ground to shake11 and surface gravity waves can be observed on rivers, lakes and oceans12. Further, surface ultrasonic waves are harnessed in non-destructive testing to detect cracks or corrosion13,14 and surface acoustic waves are commonly used to realize electronic systems15. It is therefore important to investigate the physics of surface waves in order to better control natural events and to advance technology.

Ongoing advances in fabrication are enabling the realization of mechanical metamaterials capable of manipulating elastic waves in unprecedented ways. These have been used to enable the design of waveguides and filters16, energy absorbers17, energy harvesters18 and vibration isolators19. They have also provided a powerful platform to investigate and observe surface waves20 and topologically protected edge modes21,22. While most mechanical metamaterials operate in the linear regime, it has been recently shown that large deformations and instabilities can be exploited to manipulate the propagation of finite amplitude elastic waves23–25. However, to date most studies have focused on nonlinear pulses propagating in the bulk of these flexible metamaterials. The propagation of large amplitude pulses on their free surfaces has received little attention.

In this letter, we combine experiments and simulations to investigate the propagation of nonlinear waves on the free surface of a flexible metamaterial comprising a network of squares connected by thin and highly deformable ligaments. Recent studies focused on the propagation of vector solitons through the bulk of such metamaterials have hinted at the existence of large amplitude pulses with stable shape localized at their free surface26 (Fig. 1a). Motivated by these observations, we systematically investigate the propagation of large amplitude waves on the surface of a rectangular sample. We find that the system supports surface pulses with coupled displacements and rotations that retain their shape during propagation. Further, we numerically investigate the stability of these surface pulses and find that the less they interact with the excited linear surface dispersive modes, the more stable they are.

We consider a 32 × 24 array of squares fabricated out of polymethylsiloxane (PDMS) using direct ink writing27,28,10. The squares are rotated by offset angles of θ0 = 25° with a center-to-center-distance of a = 10.89 mm, and are connected to one another by flexible ligaments of approximately 4 mm in width (Fig. 1b). In our experiments, we use a customized polylactide (PLA) impactor to apply an impulse to the top left corner of the sample (Fig. 1c). To characterize the propagation of the excited pulses, we record the experiments with a high-speed camera (Photron FASTCAM Mini AX) and extract the displacement and velocity of each square unit.

Fig. 1a shows the contour plot of the vertical displacement (uθ) at t = 9.4 ms after impact. The impact excites a pulse with the energy mostly localized close to the top surface. To further characterize the propagation of this pulse, we generate the spatio-temporal map of uθ along the top row of the sample (Fig. 1d). This indicates that a single pulse is formed and propagates at a speed of c ≃ 28 m/s until it reaches the end of the specimen.

Next, we make use of numerical simulations to systematically explore the characteristics of the nonlinear pulses that propagate along the surface of the metamaterial. We model the system as an array of rigid squares with mass m = 0.4 g and moment of inertia J = 4.8 g mm². Each square has three degrees of freedom (displacements u_x and u_y and rotation θ) and is connected to the neighbors via a combination of linear...
We find that the applied input excites a pulse with a face, we add progressively increasing damping to the top left corner (highlighted in red in Fig. 2a), and to prevent reflections from the bottom surface modes, we calculate the band structure of the system comprising 25 squares to minimize boundary effects. In order to analyze the stability of such large amplitude pulses during propagation, in Fig. 2b we report the spatial-temporal map of $u_y$ along the top row of the sample. Further, in Fig. 2c we show the evolution of the vertical ($u_y$) and horizontal ($u_x$) displacement components of the pulse as a function of space along the top surface at $t = 6, 11.2$ and $16.3$ ms. While the former indicates that the pulse travels along the surface with a relatively constant velocity and width, the latter shows that its amplitude and shape vary during propagation. Since such variation could be due to an applied impact that results in a displacement signal far from that of a potentially supported solitary wave, we then use the numerical signal collected at the 25th unit (which we fit with derivatives of Gaussian functions – Fig. 2d) as new impact signals for both the $u_x$ and $u_y$ components. As shown in Fig. 2, this input initially results in a more stable propagation along the surface, closer to what one would expect from a solitary wave. However, when simulating a longer sample comprising 200×25 units we find that the pulse gets largely distorted after a propagation distance of ≥100 units (Fig. 2e) – likely because of interactions with the linear surface waves. To better quantify this distortion, we introduce the ratio

$$
\eta(t) = \frac{\sum_{\Delta\mu \in Set_p} [u^{[1]}(t)]^2}{\sum_{i=1}^{200} [u^{[1]}(t)]^2},
$$

(1)

where $u^{[1]}_y(t)$ is the displacement of the i-th square on the top surface along the y-direction at time $t$ and $Set_p$ denotes the set of squares on the top surface that are in the nonlinear pulse. This set comprises the squares for which $x_i \in [x_0 - 3W, x_0 + 3W]$, where $x_0$ and $W$ denote the position and width of the nonlinear pulse, which are identified by fitting $u^{[1]}_y(t)$ with a Gaussian curve (A sech((x − $x_0$)/W)). As shown in Fig. 2f, we find that $\eta$ is close to 1 at $t=20$ ms, confirming that the energy is initially concentrated in the nonlinear pulse. However, during propagation $\eta$ monotonically increases ($\eta$ = 0.58 and 0.47 at $t$ = 55 and 90 ms, respectively), indicating that the energy progressively leaks out of the nonlinear pulse.

Interestingly, our simulations also indicate that the distortion of the pulse is largely affected by the mechanical properties of the hinges. By changing the torsional stiffness from $K_\theta = 4k g \cos^2 \theta_0/(kla^2) = 0.073$ (Fig. 2f) to $K_\theta = 0.03$ (Fig. 2g), we obtain a surface pulse that seems to be able to propagate stably with nearly constant shape, amplitude and speed over 200 units. In this case we find that $\eta \approx 0.9$ during the entire propagation.

To verify our hypothesis that the distortion of the nonlinear pulses is caused by interactions with linear surface modes, we calculate the band structure of the system. To this end, we perform one-dimensional Bloch wave analysis on a supercell comprising 25×2 square units, assuming free boundary conditions for the top and bottom edges. As reported in Fig. 3, for a structure characterized by $K_\theta = 0.03$, the band structure shows both bulk modes with motion distributed over the entire supercell (see modes iii-vi) and surface modes localized at the free boundary (see modes i-iii). Such surface modes occur at lower fre-
Figure 2. (a)-(e) Numerical results for a model comprising 50×25 squares. (a) Contour plots of $u_y$ over the entire model at $t = 12.8$ ms after impact and in a region close to the top surface at $t = 12.0$, 12.8, and 13.6 ms. The pulse is excited by applying the experimentally extracted displacement signal shown in Fig. 1 to the four squares highlighted in red. (b) Spatio-temporal map of $u_y$ along the top surface. (c) Spatial displacement profiles along the top surface at $t = 6.0$, 11.2, and 16.3 ms. (d) The signal collected at the 25th unit (continuous lines) is fitted with derivatives of Gaussian functions (dashed lines). (e) Spatial displacement profiles along the top surface at $t = 6.0$, 11.2, and 16.3 ms when the model is excited by applying the signal shown in (b). (f)-(g) Numerical results for a model comprising 200×50 squares. Spatial displacement profiles along the top surface at $t = 20.0$, 55.0, and 90.0 ms with (f) $K_0 = 0.073$ and (g) $K_0 = 0.03$ when excited by applying the signal shown in (d).

In order to quantify the proximity between the linear and nonlinear surface modes, in Figs. 3 and 4, we report the group velocities as a function of the wavenumber of the linear (green lines) and nonlinear (red lines) pulses for $K_0 = 0.073$ and 0.03. As expected, we find that the group velocity is constant for the nonlinear modes ($v_g/v_0 \approx 0.41$ for $K_0 = 0.073$ and $v_g/v_0 \approx 0.38$ for $K_0 = 0.03$), whereas it varies as a function of the wave number for the linear one. Further, it appears that for the metamaterial with $K_0 = 0.03$ the group velocity of the nonlinear pulse is, for most wavenumbers, larger than the one of linear modes (Fig. 3), ensuring separation and weak interactions between them. By contrast, for the structure with $K_0 = 0.073$ the group velocity of the linear waves is larger than that of the nonlinear pulse over a wide range of wavenumbers (see area highlighted in red in Fig. 3). It follows that in this case the linear waves propagate faster than the nonlinear pulse for a wide range of wavenumbers and this promotes interactions between them that ultimately lead to distortion of the nonlinear pulse during propagation. To quantify such interactions, we calculate the area $A_{\eta_{xy}}$ of the region below $v_g$ of the linear surface modes, but above that of the nonlinear pulse (see regions highlighted in red in Figs. 3 and 4). For the two structures with $K_0 = 0.073$ and $K_0 = 0.03$ we find that $A_{\eta_{xy}} = 0.15$ and 0.009, respectively.

Finally, to confirm the connection between the distortion of the nonlinear pulses and proximity between the linear and nonlinear surface modes, we simulate 330 systems characterized by $K_0 \in [0.01, 0.1]$, $K_s = k_s/k_0 \in [0.1, 1]$, $\theta_0 \in [15^\circ, 40^\circ]$ and input amplitude $A \in [1\text{mm}, 4\text{mm}]$. From each simulation, we extract the mean value of $\eta$ (defined in Eq. 1 and averaged over 10 values calculated at 10 times between 55 ms and 90 ms), as well as $A_{\eta_{xy}}$. As shown in Fig. 4, we find that the smaller $A_{\eta_{xy}}$ (i.e., the more separation there is between the group velocities of the linear and nonlinear surface pulses), the higher is $\eta$ (i.e., the more energy is concentrated in the nonlinear pulses). This observation clearly confirms that, for a given metamaterial design, nonlinear pulses are more stable when they weakly interact with the linear modes - a condition that is achieved when the group velocity of the
amplitude waves minimally interact with the linear surface dispersive modes. In practice, this condition is realized when the nonlinear pulses possess a larger group velocity than the linear surface modes for most wavenumbers. Although our numerical simulations offer ample evidence of the existence of nonlinear surface pulses, we have not yet been able to derive analytical solutions to prove their solitary nature. Given their characteristic width of about 5 units, as well as their spatial shapes, the supported surface pulses could be either compactons or micropterons \cite{m07} but an analytical solution is needed to confirm this hypothesis - a challenge for future work.

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Data availability. The data that support the findings of this study are available from the corresponding author upon reasonable request.

REFERENCES

1. A. Ben-Menahem and S. J. Singh, Seismic waves and sources (Springer Science & Business Media, 2012).
2. B. Mélac, An Introduction to Hydrodynamics and Water Waves (Springer Berlin Heidelberg, 1976).
3. D. Dhital and J. R. Lee, “A fully non-contact ultrasonic propagation imaging system for closed surface crack evaluation,” Experimental mechanics 52, 1111–1122 (2012).
4. S. K. Dwivedi, M. Vishwakarma, and A. Soni, “Advances and researches on non destructive testing: A review,” Materials Today: Proceedings 5, 3690–3698 (2018).
5. D. Morgan, Surface acoustic wave filters: With applications to electronic communications and signal processing (Academic Press, 2010).
6. K. Länge, B. E. Rapp, and M. Rapp, “Surface acoustic wave biosensors: a review,” Analytical and bioanalytical chemistry 391, 1509–1519 (2008).
7. P. A. Deymier, Acoustic metamaterials and phononic crystals (Springer, London, 2013).
8. T. Frenzel, C. Findeisen, M. Kadic, P. Gumbsch, and M. Wegener, “Tailored buckling microactuators as reusable lightweight shock absorbers,” Advanced Materials 28, 5865–5870 (2016).
9. M. Carrara, M. R. Cacan, J. Toussaint, M. J. Leamy, M. Ruzzene, and A. Erturk, “Metamaterial-inspired structures and concepts for elastoaoustic wave energy harvesting,” Smart Materials and Structures 22, 055004 (2013).
10. J. Mei, G. Ma, M. Yang, Z. Yang, W. Wen, and D. Sheng, “Dark acoustic metamaterials as super absorbers for low-frequency sound,” Nature Communications 3, 756 (2012).
11. S. Brüel, E. Javelaud, S. Enoch, and S. Guenneau, “Experiments on seismic metamaterials: molding surface waves,” Physical review letters 112, 133901 (2014).
12. D. Mu, H. Shu, L. Zhao, and S. An, “A review of research on seismic metamaterials,” Advanced Engineering Materials 22, 1901148 (2020).
13. M. Miniaci, R. Pal, B. Morvan, and M. Ruzzene, “Experimental observation of topologically protected helical edge modes in patterned elastic plates,” Physical Review X 8, 031074 (2018).
Figure 4. (a) Relation between $\eta$ and $A_{\text{avg}}$ for the 330 simulated metamaterials. (b)-(d) Evolution of $\eta$ as a function of $K_\theta$, $K_s$, input amplitude $A_\text{in}$ and offset angle $\theta_0$. The green and yellow stars correspond to the two structures considered in Figs. 2f and 2g, respectively.

14X. Ni, C. He, X.-C. Sun, X.-p. Liu, M.-H. Lu, L. Peng, and Y.-F. Chen, “Topologically protected one-way edge mode in networks of acoustic resonators with circulating air flow,” New Journal of Physics 17, 053016 (2015).

15S. D. Huber, “Topological mechanics,” Nature Physics 12, 621–623 (2016).

16M. Serra-Garcia, V. Peri, R. Siisstrunk, O. R. Bilal, T. Larsen, L. G. Villanueva, and S. D. Huber, “Observation of a phononic quadrupole topological insulator,” Nature 555, 342–345 (2018).

17H. Fan, B. Xia, L. Tong, S. Zheng, and D. Yu, “Elastic higher-order topological insulator with topologically protected corner states,” Physical review letters 122, 204301 (2019).

18F. Fraternali, G. Carpentieri, A. Amendola, R. E. Skelton, and V. F. Nesterenko, “Multiscale tunability of solitary wave dynamics in tensegrity metamaterials,” Applied Physics Letters 105, 201903 (2014).

19J. R. Raney, N. Nadikarni, C. Daraio, D. M. Kochmann, J. A. Lewis, and K. Bertoldi, “Stable propagation of mechanical signals in soft media using stored elastic energy,” Proceedings of the National Academy of Sciences 113, 9722–9727 (2016).

20M. Hwang and A. F. Arrieta, “Solitary waves in bistable lattices with stiffness grading: Augmenting propagation control,” Physical Review E 98 (2018), 10.1103/PhysRevE.98.022205.

21H. Yasuda, Y. Miyazawa, E. G. Charalampidis, C. Chong, P. G. Kevrekidis, and J. Yang, “Origami-based impact mitigation via rarefraction solitary wave creation,” Science Advances 5, eaau2835 (2019).

22B. Deng, J. H. Raney, K. Bertoldi, and V. Tournat, “Nonlinear waves in flexible mechanical metamaterials,” J. Appl. Phys. 130, 040901 (2021).

23B. Deng, J. Raney, V. Tournat, and K. Bertoldi, “Elastic vector solitons in soft architected materials,” Physical Review Letters 118 (2017), 10.1103/PhysRevLett.118.204102.

24B. Deng, P. Wang, Q. He, V. Tournat, and K. Bertoldi, “Metamaterials with amplitude gaps for elastic solitons,” Nature Communications 9 (2018), 10.1038/s41467-018-05906-9.

25B. Deng, C. Mo, V. Tournat, K. Bertoldi, and J. R. Raney, “Focusing and mode separation of elastic vector solitons in a 2d soft mechanical metamaterial,” Physical Review Letters 123 (2019), 10.1103/PhysRevLett.123.024101.

26B. Deng, V. Tournat, and K. Bertoldi, “Effect of predeformation on the propagation of vector solitons in flexible mechanical metamaterials,” Physical Review E 98 (2018), 10.1103/PhysRevE.98.053001.

27H. Yasuda, L. M. Korpas, and J. R. Raney, “Transition waves and formation of domain walls in multistable mechanical metamaterials,” Physical Review Applied 13 (2020), 10.1103/PhysRevApplied.13.054067.

28J. Li, Y. Yuan, J. Wang, R. Bao, and W. Chen, “Propagation of nonlinear waves in graded flexible metamaterials,” International Journal of Impact Engineering 156, 103924 (2021).

29K. Pajunen, P. Johanns, R. K. Pal, J. J. Rimoli, and C. Daraio, “Design and impact response of 3d-printable tensegrity-inspired structures,” Materials & Design 182, 107966 (2019).

30J. A. Lewis, “Direct ink writing of 3d functional materials,” Adv. Funct. Mater. 16, 2193–2204 (2006).

31T. Dauxois and M. Peyrard, Physics of solitons (Cambridge University Press, 2006).

32P. Rosenau and J. M. Hyman, “Compactons: Solitons with finite wavelength,” Phys. Rev. Lett. 70, 564–567 (1993).

33P. Rosenau and A. Zilburg, “Compactons,” Journal of Physics A: Mathematical and Theoretical 51, 343001 (2018).

34M. Remoissenet, Waves Called Solitons (Springer-Verlag Berlin Heidelberg, 1999).