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Disorder influences the quantum critical transport at a superconductor-to-insulator transition

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We isolated flux disorder effects on the transport at the critical point of the quantum magnetic field tuned superconductor-to-insulator transition (BSIT). The experiments employed films patterned into geometrically disordered hexagonal arrays. Spatial variations in the flux per unit cell, which grow in a perpendicular magnetic field, constitute flux disorder. The growth of flux disorder with magnetic field limited the number of BSITs exhibited by a single film due to flux matching effects. The critical metallic resistance at successive BSITs grew with flux disorder contrary to predictions of its universality. These results open the door for controlled studies of disorder effects on the universality class of an ubiquitous quantum phase transition.

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Transport phenomena near quantum critical points receive ongoing scrutiny. Much attention comes from efforts to understand strange metal behavior in high $T_c$ [1,2] and heavy fermion compounds [3,4]. Others who are developing string theory based techniques to calculate many body properties of condensed matter in the strong coupling limit have focused on quantum critical transport as well [5–7]. An interesting case is the superfluid to insulator transition in charged two-dimensional systems such as superconducting films. This superconductor-to-insulator transition (SIT) appears in numerous thin film systems [8,9] as a change from superconducting to insulating transport when a physical parameter such as thickness [10,11], magnetic field [12], or a gating field [13–16] is tuned. At the critical value of the tuning parameter, film resistances asymptote to a constant value in the zero frequency, low temperature limit. The limiting critical resistance assumes a value near the quantum of resistance for electron pairs $R_c \approx \frac{h}{2e^2} = R_Q$ [8,9]. Despite the simplicity of this behavior, questions remain regarding its universality and the influence of disorder.

Indeed, the critical resistance near a superconductor-to-insulator transition is at the heart of studies, both theoretical and experimental. Fisher [17] and collaborators [18] provided the original arguments supporting the universality of $R_c$. They focused on bosonic systems presuming that fermionic degrees of freedom in the form of unpaired electrons play no role. They argued that $R_c$ is likely to be constant in ordered systems within a universality class [18] especially in magnetic field. Disorder appeared to lead to an increase in $R_c$. Numerical simulations [19,20] also implied that quenched disorder modifies the SIT and $R_c$. On the other hand, recent quantum Monte Carlo simulations of ordered [7] and disordered systems [21] produced nearly equivalent values of $R_c$, suggesting only a weak disorder dependence. Similarly, experiments produce conflicting results. For indium oxide films, $R_c$ hovers around $R_c \approx R_Q$ [12,22–24] for a large variation in microscopic disorder. Studies of homogeneous amorphous bismuth films [25], however, show that $R_c$ grows with the normal state resistance, which is a similar measure of disorder. In both cases $R_c$ assumes values that are about a factor of 2 higher than observed in the most ordered systems, microfabricated Josephson junction arrays (JJAs) [26,27]. Such an elaborate picture reflects the variety of approaches and systems employed to study $R_c$. It suggests that deriving a new method for controllably varying disorder and selecting a thin film system with a bosonic SIT is crucial to making further experimental progress.

Opportunities to carefully test quantum critical transport models of bosonic systems have arisen for thin film superconductors. There are clear indications that boson degrees of freedom dominate the SITs of a number of them [8,9]. Increasing resistivity or applying a magnetic field transforms superconducting transport into thermally activated insulating charge transport consistent with boson localization in for example InO$_x$ [28,29], TiN [30], and nanostructured amorphous Bi films [31,32]. Magnetoresistance measurements showing Little-Parks oscillations [24,33] indicate that the bosons, Cooper pairs of electrons, remain intact across the transition. Tunneling experiments imply that the gap in the quasiparticle density of states persists into the insulating phase [34,35] indicating that the fermionic degrees of freedom are not active at these SITs. Near the critical point, resistance data from the prototypical indium oxide film system show the expected scaling behavior [12,23,36]. Thus, recent work has made it possible to focus on bosonic phenomena using well chosen superconducting films.

There are challenges to isolating the effects of disorder as it can take many forms that simultaneously influence film behavior at the SIT [28,30,37–39]. Models consider disorder as random variations in the electron potential [17,37,38], or intersite coupling in lattice models [21], or in physical parameters of grains in granular models [40]. Each of these can lead to qualitative accounts of SIT phenomena, such as the transition [8], the emergence of granular structure in the Cooper pair distribution [34], and the appearance of the giant peak in the magnetoresistance of the insulating phase [28]. Distinguishing the influences of each type of disorder, however, has been problematic. The common disorder parameter, the normal state sheet resistance $R_N$, can reflect any of the above forms as it depends on carrier density, impurity potential, and film morphology [8]. An additional confounding effect is that Coulomb interactions also grow with $R_N$. These
interisland resistances increase through a single substrate [33,46]. Sheet resistances were measured resistance by depositing a series of amorphous Bi films on unit cell areas for two typical NHC substrate [Figs.1(b) and (d) and (h). Film resistances as a function of temperature at zero field (¯T=0.5, T=0)] show data for the less ordered array NHC2. (a) and (e) Electron micrographs of the substrates. The unit cells, which were determined using a triangulation algorithm, are highlighted by yellow polygons. (b) and (f) Unit cell area distributions with Gaussian fits (red curves). The average unit cell area is A = 8 × 10^4 nm^2 for the two substrates. (c) and (g) Magnetoresistance oscillations (f = B A/Φ_0) at 130, 170, and 250 mK. (d) and (h) Film resistances as a function of temperature at zero field (f = 0), near f_c, and at f = 1/2. The films on NHC1 and NHC2 have normal state resistances and thicknesses R_N = 17.9 kΩ/□ and d = 1.22 nm and R_N = 19.2 kΩ/□ and d = 1.08 nm, respectively.

We developed a method to study quantum critical transport of a bosonic system near the SIT that isolates the effects of one form of disorder from other forms and interaction effects. We achieved this control by creating systems with tunable flux disorder. The method employs thin films patterned with a disordered triangular array of holes. The multiply connected geometry of these so-called nanohoneycomb (NHC) films enables a single film to exhibit a series of magnetic field driven bosonic SITs [43] (see Fig. 1). The geometric disorder leads to variations in the number of flux quanta per unit cell. This flux disorder [44] grows with magnetic field so that successive SITs occur with increasing disorder. It limits the number of magnetic field tuned SITs that appear due to flux matching effects. More notably, rather than being universal, the R_c of these SITs increase with flux disorder from about 4 kΩ per square to plateau at about 6 kΩ/□. We discuss how this observation implies that geometric array disorder presumed to exist in microscopically disordered thin films [34,38] enhances R_c compared to ordered Josephson junction arrays.

We used anodized aluminum oxide substrates with a nearly triangular array of holes [45] as a template for NHC films [Figs. 1(a) and 1(e)]. All substrates had the same average hole spacing of 100 nm. The more strongly disordered were produced by wrapping the aluminum with teflon tape to perturb the normally laminar flows that set up during anodization [45]. The geometric disorder is apparent in histograms of the unit cell areas for two typical NHC substrate [Figs. 1(b) and 1(f)]. The substrates were mounted to a dilution refrigerator and held at 8 K during film deposition. We studied arrays with fixed geometric disorder and varying normal state sheet resistance by depositing a series of amorphous Bi films on a single substrate [33,46]. Sheet resistances were measured at low frequencies using four probes. Transverse magnetic fields B were applied using a superconducting solenoid and are specified by the average number of flux quanta per unit cell in the array f = B A/Φ_0, where A is the average unit cell area and Φ_0 is a flux quantum.

Flux disorder results from variations in the geometry of the network of the templated NHC films. We characterize the network disorder by the fractional variation in the unit cell areas δA = ΔA/A, where ΔA is the standard deviation calculated from Gaussian fits to the unit cell area histograms [Figs. 1(b) and 1(f)]. In a magnetic field there are variations in the local frustration δ f = f δa that constitute the flux disorder [44]. This linear growth of δ f with magnetic field is presumed to dominate any field induced changes in other forms of disorder. In particular, previous work by our group [46] demonstrated that NHC films are comprised of an array of islands connected by weak links. It is supposed that randomness in weak link coupling and island size varies little with magnetic fields well below the estimated upper critical magnetic field [47].

The effects of flux disorder can be seen in a comparison of the low temperature magnetotransport of two films with different geometric disorder but similar R_N [Figs. 1(c) and 1(g)]. In both cases, the magnetoresistance oscillates with a period of 1 between low values at integer f and high values at half-integer f [31]. The oscillations decay more rapidly with f for the more disordered sample. Investigations of multiple substrates [47] indicate that the number of visible oscillations decreases from about 5 to 1 as δa increases from 0.03 to 0.14 and does not depend on R_N [48]. Thus, the data imply that oscillations persist only up to fields such that the flux disorder δ f \sim 0.3. The maximum number of oscillations observed in the most ordered arrays appears to be limited by the rise in the magnetoresistance that develops beyond 1 T [31].

Insight into the origin of the oscillations described above comes from prior experiments by Forrester et al. [49,50] on microfabricated Josephson junction arrays with geometrical
disorder. Their magnetoresistances near the superconducting transition temperature $T_c$ oscillated with a period of 1 and decayed more rapidly in more disordered arrays. The oscillations completely disappeared above a critical field that was inversely proportional to the amount of geometrical disorder. The visibility of just three oscillations in Fig. 1(g) for $\delta a = 0.115$ is in rough accord with their results. In addition, they showed that the phenomenon results from $T_c$ variations caused by oscillations of the average of the Josephson coupling energy $E_J = -(J \cos(\theta_i - \theta_j - A_{ij}))$ between neighboring nodes in the array [49]. $J$ is proportional to the amplitude of the superconducting order parameter on the nodes, $\theta_i - \theta_j$ is the difference in the phases of the order parameter, and $A_{ij} = \frac{h}{2} \int_{\text{SIT}} A \cdot dl$ is the line integral of the magnetic vector potential between neighboring nodes. $E_J$ oscillates as $A_{ij}$ increases with magnetic field and the oscillation amplitude decays as the variations in $A_{ij}$ grow. The resemblance of the oscillations presented in Fig. 1 with the prior results indicates that a similar modulation of $E_J$ occurs in the NHC films. Its effect, however, appears more dramatic as near the SIT, $E_J$ controls the strength of quantum phase fluctuations that potentially drive NHC films into an insulating state.

Quantum superconductor-to-insulator transitions [51] are evident in the magnetoresistance oscillations in Figs. 1(c) and 1(g). They are identified by the crossing points in the $R(\Delta B)$ traces taken at different temperatures. At each critical point $(J_c, R_c)$, the slope $\frac{dR}{dT}$ changes sign [43] as depicted by the $R(T)$ in Figs. 1(d) and 1(h). Negative and positive slopes correspond to films on the insulating and superconducting sides of the SIT, respectively (cf. Ref. [8]). Changes in the slopes with field appear to be governed by smooth changes in an activation energy [43], which is consistent with the onset of boson localization. Also, the insulating film $R(T)$s show reentrant dips indicative of Cooper pairing fluctuations [33] prior to rising at low temperatures [Figs. 1(d) and 1(h)]. Seven crossing points are apparent in the more ordered sample, while only three appear in the less ordered sample.

It is also possible to observe a bosonic superconductor-to-insulator transition at fields beyond the oscillation regime as shown in Fig. 2. The overlay of $R(\Delta B)$ traces at different temperatures shows a crossing near 1.9 T. This critical field corresponds to $J_c \approx 8$ or $\delta f \approx 0.9$. Qualitatively, $R(\Delta T)$s at fixed magnetic fields develop tails at low temperatures that evolve into a flat dependence at the critical point with $R_c \approx 6 \text{ k}\Omega/\square$ and finally an upturn with increasing field. These rising resistances fit to an Arrhenius form, which implies an activation energy in these insulating films [43]. Also, these $R(\Delta T)$s show Cooper pairing reentrant forms [33] prior to rising at low temperatures [Figs. 1(d) and 1(h)]. Seven crossing points are apparent in the more ordered sample, while only three appear in the less ordered sample.

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The flux disorder dependence of $R_c$ in Fig. 3 provides an explanation for the difference in $R_c$ measured in films and fabricated JJAs. Films showing bosonic SIT characteristics exhibit $R_c \approx R_Q$ with no clear dependence on $R_N$. Most notably, data on many different InO$_x$ films indicate $R_c \approx 5.8$ kΩ [12,22,24]. Because these films lack clear geometrical structure, their BSITs presumably occur in the high flux disorder limit. Experiments on JJAs, on the other hand, yielded $2.5 < R_c < 4.5$ kΩ [26] and $1.2 < R_c < 2.45$ kΩ [27]. Clearly BSITs occur in the zero flux disorder limit in JJAs. Altogether these experiments point to $R_c$ increasing with flux disorder (cf. Fig. 3).

It is important to view the current results relative to previous experiments showing a relation between $R_c$ and disorder as parametrized by the normal state sheet resistance $R_N$. Investigations of homogeneous MoGe films [52] and uniform amorphous Bi films [25] showed a strong positive correlation between $R_c$ and $R_N$. This correlation could imply that $R_c$ increases with disorder [25]. Alternatively, it could be produced by fermion quasiparticles known to be present at these SITs [53]. These fermions can provide a parallel dissipative channel that alters the transport in the vicinity of a nominally bosonic quantum critical point [54]. Thus, the correlation between $R_c$ and $R_N$ could reflect changes in quasiparticle density rather than disorder [52]. We hasten to add that patterning these films with hole arrays can potentially yield insight into the interplay of bosonic and fermionic degrees of freedom at an SIT. It can reveal how dissipation effects on a bosonic SIT due to quasiparticles [54] evolve with increasing disorder.

Although there have been many calculations of $R_c$ at bosonic superconductor-to-insulator transitions including some that explicitly consider disorder effects [17,19–21,55–57], none appear to predict the behavior observed here. Uniquely, Kim and Stroud [20] treated flux disorder effects in quantum Monte Carlo simulations of disordered square arrays of Josephson junctions. Their results predict that $R_c$ decreases strongly with flux disorder from $3.5R_Q$ to $0.2R_Q$ for integer $\bar{f}$, which is in sharp contrast to our results at noninteger $\bar{f}$ in Fig. 3. Likewise, a comparison [7] of a pair of quantum Monte Carlo results [7,21] employing the latest methods for extracting $R_c$ from simulations suggests that $R_c$ only weakly depends on disorder. In addition, the vast majority of predictions of $R_c$ in the low disorder limit are higher than experimental values by a factor of more than 2 [6,7,19,21,58–61]. This discrepancy could suggest that the universality classes of the quantum rotor and Villain models employed in some of the most advanced approaches [7,21] do not match the experimental systems. Or, it could reflect a need to further refine methods for calculating the quantum critical conductivity in the zero frequency limit [7,21,62]. There is the further possibility that experiments have not probed the true quantum critical regime.

In conclusion, we have described experiments revealing that disorder influences quantum critical transport of a bosonic system. We find that the critical resistance at the superconductor-to-insulator transition depends on flux disorder. This result invites more theoretical attention to its universality while illuminating a difference between SITs in thin films and Josephson junction arrays. The results also highlight a lack of quantitative agreement between theoretical predictions and measurements of the critical resistance. The studies establish NHC films as uniquely suited for studying the effects of well defined disorder on quantum critical transport and the universality class of a prominent quantum phase transition. Furthermore, they invite comparisons with other condensed matter systems in which the interplay of quantum criticality and disorder can arise such as doped strongly correlated electron systems [63] or cold atoms in random optical lattices [64].

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[1] C. M. Varma, Z. Nussinov, and W. van Saarloos, Singular or non-Fermi liquids, Phys. Rep. 361, 267 (2002).
[2] J. Vučičevič, D. Tansković, M. J. Rozenberg, and V. Dobrosavljević, Bad-Metal Behavior Reveals Mott Quantum Criticality in Doped Hubbard Models, Phys. Rev. Lett. 114, 246402 (2015).
[3] G. R. Stewart, Non-Fermi-liquid behavior in d- and f-electron metals, Rev. Mod. Phys. 73, 797 (2001).
[4] T. Faulkner, Non-Fermi-liquid behavior in d- and f-electron metals, Rev. Mod. Phys. 73, 797 (2001).
[5] T. Faulkner, Non-Fermi-liquid behavior in d- and f-electron metals, Rev. Mod. Phys. 73, 797 (2001).
[6] S. Sachdev, What can gauge-gravity duality teach us about condensed matter physics?, Annu. Rev. Condens. Matter Phys. 3, 9 (2012).
[7] K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokofev, Universal Conductivity in a Two-Dimensional Superfluid-to-Insulator Quantum Critical System, Phys. Rev. Lett. 112, 030402 (2014).
[8] W. Witzczak-Krempa, E. S Sorensen, and S. Sachdev, The dynamics of quantum criticality revealed by quantum Monte Carlo and holography, Nat. Phys. 10, 361 (2014).
[9] V. F. Gantmakher and V. T. Dolgopolov, Superconductor–insulator quantum phase transition, Phys.-Uspekhi 53, 1 (2010).
[10] V. Dobrosavljevic, N. Trivedi, and J. M. Valles, Jr., Conductor Insulator Quantum Phase Transitions (Cambridge University Press, Cambridge, 2012).
[11] R. C. Dynes, J. P. Garno, and J. M. Rowell, Two-Dimensional Electrical Conductivity in Quench-Condensed Metal Films, Phys. Rev. Lett. 40, 479 (1978).
[12] D. B. Haviland, Y. Liu, and A. M. Goldman, Onset of Superconductivity in the Two-Dimensional Limit, Phys. Rev. Lett. 62, 2180 (1989).
[13] A. F. Hebard and M. A. Paalanen, Magnetic-Field-Tuned Superconductor-Insulator Transition in Two-Dimensional Films, Phys. Rev. Lett. 65, 927 (1990).
[14] K. A. Parenko, K. H. S. B Tan, A. Bhattacharya, M. Eblen-Zayas, N. E. Staley, and A. M. Goldman, Electrostatic Tuning of the Superconductor-Insulator Transition in Two Dimensions, Phys. Rev. Lett. 94, 197004 (2005).
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[14] A. D. Caviglia, S. Gariglio, N. Reyren, D. Jaccard, T. Schneider, M. Gabay, S. Thiel, G. Hammerl, J. Mannhart, and J.-M. Triscone, Electric field control of the LaAlO$_3$/SrTiO$_3$ interface ground state, *Nature* **456**, 624 (2008).

[15] A. T. Bollinger, G. Dubuis, J. Yoon, D. Pavuna, J. Misewich, and I. Božović, Superconductor-insulator transition in La$_{24-x}$Sr$_x$CuO$_4$ at the pair quantum resistance, *Nature (London)* **472**, 458 (2011).

[16] A. Allain, Z. Han, and V. Bouchiat, Electrical control of the superconducting-to-insulating transition in graphene–metal hybrids, *Nat. Mater.* **11**, 590 (2012).

[17] M. P. A. Fisher, G. Grinstein, and S. M. Girvin, Presence of Quantum Diffusion in Two Dimensions: Universal Resistance at the Superconductor-Insulator Transition, *Phys. Rev. Lett.* **64**, 587 (1990).

[18] M.-C. Cha, M. P. A. Fisher, S. M. Girvin, M. Wallin, and A. P. Young, Universal conductivity of two-dimensional films at the superconductor-insulator transition, *Phys. Rev. B* **44**, 6883 (1991).

[19] I. F. Herbut, Dual Superfluid-Bose-Glass Critical Point in Two Dimensions and the Universal Conductivity, *Phys. Rev. Lett.* **79**, 3502 (1997).

[20] K. Kim and D. Stroud, Quantum Monte Carlo study of a magnetic-field-driven two-dimensional superconductor-insulator transition, *Phys. Rev. B* **78**, 174517 (2008).

[21] M. Swanson, Y. L. Loh, M. Randeria, and N. Trivedi, Dynamical Conductivity Across the Disorder-Tuned Superconductor-Insulator Transition, *Phys. Rev. X* **4**, 021007 (2014).

[22] G. Sambandamurthy, A. Johansson, E. Peled, D. Shahar, P. G. Björnsson, and K. A. Moler, Power law resistivity behavior in 2D superconductors across the magnetic field-driven superconductor-insulator transition, *Europhys. Lett.* **75**, 611 (2006).

[23] M. A. Steiner, N. P. Breznay, and A. Kapitulnik, Approach to a superconductor-to-Bose-insulator transition in disordered films, *Phys. Rev. B* **77**, 212501 (2008).

[24] G. Kopnov, O. Cohen, M. Ovadia, K. H. Lee, C. C. Wong, and D. Shahar, Little-Parks Oscillations in an Insulator, *Phys. Rev. Lett.* **109**, 167002 (2012).

[25] N. Markovic, C. Christiansen, and A. M. Goldman, Thickness-Magnetic Field Phase Diagram at the Superconductor-Insulator Transition in 2D, *Phys. Rev. Lett.* **81**, 5217 (1998).

[26] H. S. J. Van der Zant, F. C. Fritschy, W. J. Elion, L. J. Geerligs, and J. E. Mooij, Field-Induced Superconductor-to-Insulator Transitions in Josephson-Junction Arrays, *Phys. Rev. Lett.* **69**, 2971 (1992).

[27] C. D. Chen, P. Delsing, D. B. Haviland, Y. Harada, and T. Claeson, Scaling behavior of the magnetic-field-driven superconductor-insulator transition in two-dimensional Josephson-junction arrays, *Phys. Rev. B* **51**, 15645 (1995).

[28] G. Sambandamurthy, L. W. Engel, A. Johansson, and D. Shahar, Superconductivity-Related Insulating Behavior, *Phys. Rev. Lett.* **92**, 107005 (2004).

[29] M. Steiner and A. Kapitulnik, Superconductivity in the insulating phase above the field-tuned superconductor-insulator transition in disordered indium oxide films, *Phys. C: Superconduct.* **422**, 16 (2005).

[30] T. I. Baturina, A. Y. Mironov, V. M. Vinokur, M. R. Baklanov, and C. Strunk, Localized Superconductivity in the Quantum-Critical Region of the Disorder-Driven Superconductor-Insulator Transition in Tin Thin Films, *Phys. Rev. Lett.* **99**, 257003 (2007).

[31] H. Q. Nguyen, S. M. Hollen, M. D. Stewart, Jr., J. Shainline, A. Yin, J. M. Xu, and J. M. Valles, Jr., Observation of Giant Positive Magnetoresistance in a Cooper Pair Insulator, *Phys. Rev. Lett.* **103**, 157001 (2009).

[32] Y.-H. Lin, J. Nelson, and A. M. Goldman, The role of mesoscopic disorder in determining the character of the field-induced insulating regime of amorphous ultrathin films, *Phys. C: Supercond.* **497**, 102 (2014).

[33] M. D. Stewart, A. Yin, J. M. Xu, and J. M. Valles, Superconducting pair correlations in an amorphous insulating nanohoneycomb film, *Science* **318**, 1273 (2007).

[34] B. Sacépé, C. Chapelier, T. I. Baturina, V. M. Vinokur, M. R. Baklanov, and M. Sanquer, Disorder-Induced Inhomogeneities of the Superconducting State Close to the Superconductor-Insulator Transition, *Phys. Rev. Lett.* **101**, 157006 (2008).

[35] D. Sherman, G. Kopnov, D. Shahar, and A. Frydman, Measurement of a Superconducting Energy Gap in a Homogeneously Amorphous Insulator, *Phys. Rev. Lett.* **108**, 177006 (2012).

[36] M. Ovadia, D. Kalok, B. Sacépé, and D. Shahar, Duality symmetry and its breakdown in the vicinity of the superconductor-insulator transition, *Nat. Phys.* **9**, 415 (2013).

[37] A. Ghosal, M. Randeria, and N. Trivedi, Role of Spatial Amplitude Fluctuations in Highly Disordered s-Wave Superconductors, *Phys. Rev. Lett.* **81**, 3940 (1998).

[38] Y. Dubi, Y. Meir, and Y. Avishai, Nature of the superconductor–insulator transition in disordered superconductors, *Nature (London)* **449**, 876 (2007).

[39] K. Bouadim, Y. L. Loh, M. Randeria, and N. Trivedi, Single-and two-particle energy gaps across the disorder-driven superconductor-insulator transition, *Nat. Phys.* **7**, 884 (2011).

[40] I. S. Beloborodov, V. F. Fominov, A. V. Lopatin, and V. M. Vinokur, Insulating state of granular superconductors in a strong-coupling regime, *Phys. Rev. B* **74**, 014502 (2006).

[41] K. L. Ekinci and J. M. Valles, Jr., Morphology of Quench Condensed Pb Films Near the Insulator to Metal Transition, *Phys. Rev. Lett.* **82**, 1518 (1999).

[42] R. Fazio and H. Van Der Zant, Quantum phase transitions and vortex dynamics in superconducting networks, *Phys. Rep.* **355**, 235 (2001).

[43] M. D. Stewart, Jr., A. Yin, J. M. Xu, and J. M. Valles, Jr., Magnetic-field-tuned superconductor-to-insulator transitions in amorphous Bi films with nanoscale hexagonal arrays of holes, *Phys. Rev. B* **77**, 140501 (2008).

[44] E. Granato and J. M. Kosterlitz, Quenched disorder in Josephson-junction arrays in a transverse magnetic field, *Phys. Rev. B* **33**, 6533 (1986).

[45] A. J. Yin, J. Li, W. Jian, A. J. Bennett, and J. M. Xu, Fabrication of highly ordered metallic nanowire arrays by electrodeposition, *Appl. Phys. Lett.* **79**, 1039 (2001).

[46] S. M. Holien, H. Q. Nguyen, E. Rudisaille, M. D. Stewart, Jr., J. Shainline, J. M. Xu, and J. M. Valles, Jr., Cooper-pair insulator phase in superconducting amorphous Bi films induced by nanometer-scale thickness variations, *Phys. Rev. B* **84**, 064528 (2011).

[47] H. Q. Nguyen, Experiments on a Cooper pair insulator, Ph.D. thesis, Brown University, 2010.

[48] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevB.92.140501 for discussion of the evolution of
the magnetoresistance oscillations with weak link coupling and of the dependence of $R_c(\bar{\theta}=0)$ on weak link coupling.

[49] S. P. Benz, M. G. Forrester, M. Tinkham, and C. J. Lobb, Positional disorder in superconducting wire networks and Josephson junction arrays, Phys. Rev. B 38, 2869 (1988).

[50] M. G. Forrester, H. J. Lee, M. Tinkham, and C. J. Lobb, Positional disorder in Josephson-junction arrays: Experiments and simulations, Phys. Rev. B 37, 5966 (1988).

[51] E. Granato, Resistive transition in frustrated Josephson-junction arrays on a honeycomb lattice, Phys. Rev. B 87, 094517 (2013).

[52] A. Yazdani and A. Kapitulnik, Superconducting-Insulating Transition in Two-Dimensional a-moge Thin Films, Phys. Rev. Lett. 74, 3037 (1995).

[53] S.-Y. Hsu, J. A. Chervenak, and J. M. Valles, Jr., Magnetic Field Enhanced Order Parameter Amplitude Fluctuations in Ultrathin Films Near the Superconductor-Insulator Transition, Phys. Rev. Lett. 75, 132 (1995).

[54] N. Mason and A. Kapitulnik, Dissipation Effects on the Superconductor-Insulator Transition in 2D Superconductors, Phys. Rev. Lett. 82, 5341 (1999).

[55] M.-C. Cha and S. M. Girvin, Universal conductivity in the boson Hubbard model in a magnetic field, Phys. Rev. B 49, 9794 (1994).

[56] Y. Nishiyama, Numerical analysis of the magnetic-field-tuned superconductor-insulator transition in two dimensions, Physica C 353, 147 (2001).

[57] M. Wallin, E. S. So, S. M. Girvin, A. P. Young et al., Superconductor-insulator transition in two-dimensional dirty boson systems, Phys. Rev. B 49, 12115 (1994).

[58] G. G. Batrouni, B. Larson, R. T. Scalettar, J. Tobochnik, and J. Wang, Universal conductivity in the two-dimensional boson Hubbard model, Phys. Rev. B 48, 9628 (1993).

[59] E. S. Sorensen, M. Wallin, S. M. Girvin, and A. P. Young, Universal Conductivity of Dirty Bosons at the Superconductor-Insulator Transition, Phys. Rev. Lett. 69, 828 (1992).

[60] M. Makivić, N. Trivedi, and S. Ullah, Disordered Bosons: Critical Phenomena and Evidence for New Low Energy Excitations, Phys. Rev. Lett. 71, 2307 (1993).

[61] K. J. Runge, Numerical study of the onset of superfluidity in two-dimensional, disordered, hard-core bosons, Phys. Rev. B 45, 13136 (1992).

[62] K. Damle and S. Sachdev, Nonzero-temperature transport near quantum critical points, Phys. Rev. B 56, 8714 (1997).

[63] A. Rosch, Interplay of Disorder and Spin Fluctuations in the Resistivity Near a Quantum Critical Point, Phys. Rev. Lett. 82, 4280 (1999).

[64] B. Deissler, M. Zaccanti, G. Roati, C. D’Errico, M. Fattori, M. Modugno, G. Modugno, and M. Inguscio, Delocalization of a disordered bosonic system by repulsive interactions, Nat. Phys. 6, 354 (2010).