An effective formulation on quantum hadrodynamics at finite temperatures and densities

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Abstract

According to Wick’s theorem, the second order self-energy corrections of hadrons in the hot and dense nuclear matter are calculated. Furthermore, the Feynman rules are summarized, and an effective formulation on quantum hadrodynamics at finite temperatures and densities is evaluated. As the strong couplings between nucleons are considered, the self-consistency of this method is discussed in the framework of relativistic mean-field approximation. Debye screening masses of the scalar and vector mesons in the hot and dense nuclear matter are calculated with this method in the relativistic mean-field approximation. The results are different from those of thermofield dynamics and Brown-Rho conjecture. Moreover, the effective masses of the photon and the nucleon in the hot and dense nuclear matter are discussed.

Key words: quantum hadrodynamics, Wick’s theorem, Debye screening masses
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1 Introduction

The quantum field theory, introduced to solve the many-body problems since the late fifties, has proved to be highly successful in studying the ground state, the equilibrium and non-equilibrium properties at finite temperatures[1, 2, 3, 4, 5, 6, 7, 8, 9]. These theories to solve many-body problems are based on the conjecture that all of the effects of the medium at finite temperatures and densities would change the propagators of particles. When the propagators in the mediums are obtained, the properties of particles and the equation of state of the medium can be calculated easily. The propagators of particles in the medium are different from those in vacuum correspondingly, so this conjecture means the redefinition of the vacuum, and the calculation procedures are all carried out in this new vacuum.

In a series of our previous papers[10, 11, 12], according to Wick’s theorem, we have evaluated an effective formulation on quantum hadrodynamics to solve the nuclear many-body problems. In this formulation, the Feynman propagators in vacuum are adopted, and the second order self-energies of particles in the nuclear matter are calculated, while the effects of the nuclear matter are treated as the condensations of nucleons and mesons. It shows the same results as the method of quantum hadrodynamics[1, 2, 3]. With this new method, we have studied the effective masses of the photon and mesons[10, 11], and constructed the density-dependent relativistic mean-field model[11]. In this paper we will generalize this new method to the situation at finite temperatures and densities.

The organization of this paper is as follows. The second order self-energy corrections of the nucleon and the meson in the nuclear matter are calculated from Wick expansion in the framework of quantum hadrodynamics 1 in Sec. 2, then the Feynman rules and self-consistency for this method at finite temperatures and densities are summarized in Sec. 3. The results of screening masses of mesons are presented in Sec. 4. In Sec. 5, the effective mass of the photon in the hot nuclear matter is discussed. The summary is given in Sec. 6.

2 The self-energies of hadrons in the hot and dense nuclear matter

According to Walecka-1 model, the Lagrangian density in nuclear matter can be written as

\[ \mathcal{L} = \bar{\psi} (i\gamma_\mu \partial^\mu - M_N) \psi + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_{\mu} \omega^{\mu} - g_\sigma \bar{\psi} \sigma \psi - g_\omega \bar{\psi} \gamma_\mu \omega^{\mu} \psi, \]  

where \( \psi \) is the field of the nucleon, \( M_N \) is the nucleon mass, and

\[ \omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \]  

is the field tensor of the vector meson.
In the hot nuclear matter, the expectations of normal products of the creation operator and corresponding annihilation operator are relevant to the distribution functions of particles.

\[ \langle N, \beta | A^\dagger_{\mu \lambda} A_{\mu \lambda} | N, \beta \rangle = n_F \delta^3(\vec{p}' - \vec{p})\delta_{\lambda' \lambda}, \]
\[ \langle N, \beta | B^\dagger_{\mu \lambda} B_{\mu \lambda} | N, \beta \rangle = \bar{n}_F \delta^3(\vec{p}' - \vec{p})\delta_{\lambda' \lambda}, \]

where \( \lambda' \) and \( \lambda \) denote spins of fermions, \( n_F \) and \( \bar{n}_F \) are the distribution functions of the nucleon and antinucleon, respectively.

\[ n_F = \frac{1}{\exp \left[ \frac{(E(p) - \mu)}{T} \right] + 1}, \]
\[ \bar{n}_F = \frac{1}{\exp \left[ \frac{(E(p) + \mu)}{T} \right] + 1}, \]

where \( E(p) = \sqrt{\vec{p}^2 + M_N^2} \), and \( \mu \) is the chemical potential of the nucleon. The relation of chemical potential and the number density of nucleons \( \rho_B \) is

\[ \rho_B = \frac{2}{(2\pi)^3} \int d^3p \left[ n_F - \bar{n}_F \right], \]

where \( \delta' \) and \( \delta \) denote the spins of the vector meson, \( n_\sigma \) and \( n_\omega \) are the boson distribution functions of the scalar and vector mesons, respectively.

\[ n_\alpha = \frac{1}{\exp \left[ \frac{\left| \Omega_\alpha \right|}{T} - 1 \right]}, \quad \alpha = \sigma, \omega, \]

where \( \Omega_\alpha = \sqrt{\vec{k}^2 + m^2} \). Since the meson number is not conserved in nuclear matter, there is not the contribution of the meson chemical potential in the boson distribution function of Eq. (10).

The finite-temperature state of nuclear matter could be understood as the state that there are a great number of coupling nucleons, antinucleons and mesons in the perturbation vacuum, so the noninteracting propagators in vacuum are used in the calculation of the self-energies of particles.

The momentum-space noninteracting propagators of the scalar meson, vector meson and nucleon in perturbation vacuum \( |0\rangle \) follow as:

\[ i\Delta_0(p) = -\frac{1}{p^2 - m_\sigma^2 + i\varepsilon}, \]
\[ iD^\mu_{\sigma\nu}(p) = \frac{g_{\mu\nu}}{p^2 - m_\sigma^2 + i\varepsilon}, \]
\[ iD^\mu_{\omega\nu}(p) = \frac{g_{\mu\nu}}{p^2 - m_\omega^2 + i\varepsilon}, \]

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\[ iD^\mu_{\omega\nu}(p) = \frac{g_{\mu\nu}}{p^2 - m_\omega^2 + i\varepsilon}, \]

3
\[ iG_0^{\alpha \beta}(p) = \frac{-1}{\gamma_\mu p^\mu - M_N + i\varepsilon}. \] (13)

Since the vector meson couples to the conserved baryon current, the longitudinal part in the propagator of the vector meson will not contribute to physical quantities\[3, 13\]. Therefore, only the transverse part in the propagator of the vector meson is written in Eq. (12).

According to Eq. (1), the interaction Hamiltonian can be expressed as

\[ \mathcal{H}_I = g_\sigma \bar{\psi}\sigma\psi + g_\omega \bar{\psi}\gamma_\mu\omega^\mu\psi. \] (14)

In the second order approximation, only

\[ \hat{S}_2 = \frac{(-i)^2}{2!} \int d^4 x_1 \int d^4 x_2 T[\mathcal{H}_I(x_1)\mathcal{H}_I(x_2)] \] (15)

in the S-matrix should be calculated in order to obtain the self-energy corrections of the nucleon and mesons.

The coupling constants \( g_\sigma, g_\omega \), the masses of mesons \( m_\sigma, m_\omega \) and the nucleon mass \( M_N \) are supposed to have been renormalized, then the contributions of the nucleon-loop diagrams are not needed to be considered\[13, 14\]. Only the terms with one contraction of two fields in the Wick expansions of \( \hat{S}_2 \) should be calculated. Thus with the similar procedure of Ref.\[10, 11, 12\], the scalar meson self-energy \( \Sigma_\sigma \), the vector meson self-energy \( \Sigma_\omega \), the nucleon self-energy \( \Sigma^+ \) and the antinucleon self-energy \( \Sigma^- \) in the hot and dense nuclear matter are obtained.

The second order self-energy of the scalar meson is

\[ \Sigma_\sigma = (-ig_\sigma)^2 \sum_{\lambda=1,2} \int \frac{d^3 p}{(2\pi)^3} \frac{M_N}{E(p)} \left[ n_F \bar{U}(p, \lambda) (iG_0(p - k) + iG_0(p + k)) U(p, \lambda) 
- \bar{n}_F \bar{V}(p, \lambda) (iG_0(-p - k) + iG_0(-p + k)) V(p, \lambda) \right] \]

\[ = g_\sigma^2 \sum_{\lambda=1,2} \int \frac{d^3 p}{(2\pi)^3} \frac{M_N}{E(p)} \left[ n_F \bar{U}(p, \lambda) \left( \frac{1}{\hat{p} - \hat{k} - M_N} + \frac{1}{\hat{p} + \hat{k} - M_N} \right) U(p, \lambda) 
- \bar{n}_F \bar{V}(p, \lambda) \left( \frac{1}{-\hat{p} - \hat{k} - M_N} + \frac{1}{-\hat{p} + \hat{k} - M_N} \right) V(p, \lambda) \right], \] (16)

where \( U(p, \lambda) \) and \( V(p, \lambda) \) are the Dirac spinors of the nucleon and the antinucleon, respectively, and \( \bar{U}(p, \lambda) \) and \( \bar{V}(p, \lambda) \) are their conjugate spinors, respectively.

\[ \sum_{\lambda=1,2} U(p, \lambda) \bar{U}(p, \lambda) = \frac{\hat{p} + M_N}{2M_N}, \] (17)
The second order self-energy of the vector meson is

\[
- g_{\mu \nu} \Sigma_\omega = \left( -i g_\omega \right)^2 \sum_{\lambda = 1, 2} \int \frac{d^3 p}{(2\pi)^3} \frac{M_N}{E(p)} \left[ n_F \tilde{U}(p, \lambda) \left( \gamma_\nu iG_0(p - k) \gamma_\mu + \gamma_\mu iG_0(p + k) \gamma_\nu \right) U(p, \lambda) \\
- \bar{n}_F \tilde{V}(p, \lambda) \left( \gamma_\nu iG_0(-p - k) \gamma_\mu + \gamma_\mu iG_0(-p + k) \gamma_\nu \right) V(p, \lambda) \right]
\]

\[
= \left( -i g_\omega \right)^2 \sum_{\lambda = 1, 2} \int \frac{d^3 p}{(2\pi)^3} \frac{M_N}{E(p)} \left[ n_F \tilde{U}(p, \lambda) \left( \gamma_\nu \frac{1}{\not{p} - \not{k} - M_N} \gamma_\mu + \gamma_\mu \frac{1}{\not{p} + \not{k} - M_N} \gamma_\nu \right) U(p, \lambda) \\
- \bar{n}_F \tilde{V}(p, \lambda) \left( \gamma_\nu \frac{1}{\not{p} - \not{k} - M_N} \gamma_\mu + \gamma_\mu \frac{1}{\not{p} + \not{k} - M_N} \gamma_\nu \right) V(p, \lambda) \right] \quad (19)
\]

The second order self-energy of the nucleon is

\[
\Sigma^+ = \sum_{s = \sigma, \omega} \left( \Sigma^+_{s,1} + \Sigma^+_{s,2} + \Sigma^+_{s,3} \right)
\]

in which

\[
\Sigma^+_{s,1} = \left( -i g_\sigma \right)^2 \sum_{\lambda = 1, 2} \int \frac{d^3 p}{(2\pi)^3} \frac{M_N}{E(p)} \left[ n_F \tilde{U}(p, \lambda) U(p, \lambda) - \bar{n}_F \tilde{V}(p, \lambda)V(p, \lambda) \right] \\
= -\frac{g_\sigma^2}{m_\sigma^2} \rho_S
\]

where \( \rho_S \) is the scalar density of protons or neutrons,

\[
\rho_S = \frac{2}{(2\pi)^3} \int d^3 p \frac{M_N}{\sqrt{p^2 + M_N^2}} \left( n_F + \bar{n}_F \right)
\]

\[
\Sigma^+_{s,2} = g_\sigma^2 \sum_{\lambda = 1, 2} \int \frac{d^3 p}{(2\pi)^3} \frac{M_N}{E(p)} \left[ n_F \tilde{U}(p, \lambda) \left( \gamma_\nu iG_0(k - p) \gamma_\mu + \gamma_\mu iG_0(k + p) \gamma_\nu \right) \tilde{V}(p, \lambda) \right] \\
= -g_\sigma^2 \int \frac{d^3 p}{(2\pi)^3} \frac{M_N}{E(p)} \left[ n_F \left( \frac{\not{p} + \not{M}_N}{2M_N} \frac{1}{(k - p)^2 - m_\sigma^2} - \bar{n}_F \frac{\not{p} - \not{M}_N}{2M_N} \frac{1}{(k + p)^2 - m_\sigma^2} \right) \right] \quad (23)
\]
\[ \Sigma_{\sigma,3}^+ = (-i g_\sigma)^2 \int \frac{d^3 k}{2 \Omega_{\sigma}(2\pi)^3} n_\sigma \left( i G_0(p - k) + i G_0(p + k) \right) \]
\[ = g_\sigma^2 \int \frac{d^3 k}{2 \Omega_{\sigma}(2\pi)^3} n_\sigma \left( \frac{1}{\hat{p} - \hat{k} - M_N} + \frac{1}{\hat{p} + \hat{k} - M_N} \right), \quad (24) \]
\[ \Sigma_{\omega,1}^+ = (-i g_\omega)^2 \sum_{\lambda=1,2} \int \frac{d^3 p}{(2\pi)^3} \frac{M_N}{E(p)} \gamma^\mu \iota D_0^{\mu\nu}(0) \]
\[ \left[ n_F \bar{U}(p, \lambda) \gamma_\nu U(p, \lambda) - \bar{n}_F V(p, \lambda) \right] \gamma_\nu V(p, \lambda) \]
\[ = \gamma_0 \frac{g_\omega^2}{m_\omega^2} \rho_B, \quad (25) \]

where \( \rho_B \) defined in Eq. (7) is the number density of protons or neutrons, and

\[ \Sigma_{\omega,2}^+ = g_\omega^2 \sum_{\lambda=1,2} \int \frac{d^3 p}{(2\pi)^3} \frac{M_N}{E(p)} \]
\[ \left[ n_F \gamma_\mu U(p, \lambda) - \bar{n}_F \gamma_\mu V(p, \lambda) \right] \gamma^\nu \iota D_0^{\mu\nu}(k + p) \]
\[ = g_\omega^2 \int \frac{d^3 k}{(2\pi)^3} \frac{M_N}{E(p)} \]
\[ \left( n_F \gamma_\mu \frac{\hat{p} + M_N}{2M_N} \gamma^\nu \frac{g^{\mu\nu}}{(k - p)^2 - m_\omega^2} - \bar{n}_F \gamma_\mu \frac{\hat{p} - M_N}{2M_N} \gamma^\nu \frac{g^{\mu\nu}}{(k + p)^2 - m_\omega^2} \right), \quad (26) \]
\[ \Sigma_{\omega,3}^+ = (-i g_\omega)^2 \sum_{\delta=1,2,3} \int \frac{d^3 k}{2 \Omega_\omega(2\pi)^3} n_\omega \gamma^\mu \varepsilon_\mu(k, \delta) \left[ i G_0(p - k) + i G_0(p + k) \right] \gamma^\nu \varepsilon_\nu(k, \delta) \]
\[ = g_\omega^2 \sum_{\delta=1,2,3} \int \frac{d^3 k}{2 \Omega_\omega(2\pi)^3} n_\omega \gamma^\mu \varepsilon_\mu(k, \delta) \left[ \frac{1}{\hat{p} - \hat{k} - M_N} + \frac{1}{\hat{p} + \hat{k} - M_N} \right] \gamma^\nu \varepsilon_\nu(k, \delta), \quad (27) \]

where \( \varepsilon_\mu(k, \delta) \) and \( \varepsilon_\nu(k, \delta) \) are the space-like orthonormalized vectors of the vector meson.

The second order self-energy of the antinucleon in the nuclear matter is

\[ \Sigma^- = \sum_{s=\sigma,\omega} \left( \Sigma_{s,1}^- + \Sigma_{s,2}^- + \Sigma_{s,3}^- \right), \quad (28) \]
in which

\[ \Sigma_{\sigma,1}^- = - \Sigma_{\sigma,1}^+ = \frac{g_\sigma^2}{m_\sigma^2} \rho_S, \quad (29) \]
\[ \Sigma_{\sigma,2}^- = -g_\sigma^2 \sum_{\lambda=1,2} \int \frac{d^3 k}{2 \Omega_\sigma(2\pi)^3} \frac{M_N}{E(p)} \]
\[ \left[ n_F U(p, \lambda) - \bar{n}_F V(p, \lambda) \right] \Delta_0(-k + p) \]
\[ = g_\sigma^2 \int \frac{d^3 p}{(2\pi)^3} \frac{M_N}{E(p)} \left( n_F \frac{\hat{p} + M_N}{2M_N} \frac{1}{(k - p)^2 - m_\sigma^2} - \bar{n}_F \frac{\hat{p} - M_N}{2M_N} \frac{1}{(k + p)^2 - m_\sigma^2} \right), \quad (30) \]
\[\Sigma_{\sigma,3} = -(ig_\sigma)^2 \int \frac{d^3k}{2\Omega_\sigma(2\pi)^3} \ n_\sigma \ (iG_0(-p-k) + iG_0(-p+k)) \]
\[= -g_\sigma^2 \int \frac{d^3k}{2\Omega_\sigma(2\pi)^3} \ n_\sigma \left( \frac{1}{-\hat{p} - \vec{k} - M_N} + \frac{1}{-\hat{p} + \vec{k} - M_N} \right), \quad (31)\]

\[\Sigma_{\omega,1} = -\Sigma_{\omega,1}^+ = -\gamma_0 \frac{g_\omega}{m_\omega^2} \rho_B, \quad (32)\]

\[\Sigma_{\omega,2} = -g_\omega^2 \sum_{\lambda=1,2} \int \frac{d^3p \ M_N}{(2\pi)^3} E(p) \]
\[\left[ n_F \gamma_\mu U(p, \lambda) \ iD_0^{\nu\mu}(-k-p) \ \bar{U}(p, \lambda) \gamma_\nu - n_F \gamma_\mu V(p, \lambda) \ iD_0^{\nu\mu}(-k+p) \ \bar{V}(p, \lambda) \gamma_\nu \right] \]
\[= -g_\omega^2 \int \frac{d^3p \ M_N}{(2\pi)^3} E(p) \]
\[\left( n_F \gamma_\mu \frac{\hat{p} + M_N}{2M_N} \gamma_\nu \frac{g^{\mu\nu}}{(-k-p)^2 - m_\omega^2} - n_F \gamma_\mu \frac{\hat{p} - M_N}{2M_N} \gamma_\nu \frac{g^{\mu\nu}}{(-k+p)^2 - m_\omega^2} \right), \quad (33)\]

\[\Sigma_{\omega,3} = -(ig_\omega)^2 \sum_{\delta=1,2,3} \int \frac{d^3k}{2\Omega_\omega(2\pi)^3} n_\omega \gamma_\mu \varepsilon_\mu(k, \delta) \ [iG_0(-p-k) + iG_0(-p+k)] \ \gamma^\nu \varepsilon_\nu(k, \delta) \]
\[= -g_\omega^2 \sum_{\delta=1,2,3} \int \frac{d^3k}{2\Omega_\omega(2\pi)^3} n_\omega \gamma_\mu \varepsilon_\mu(k, \delta) \left[ \frac{1}{-\hat{p} - \vec{k} - M_N} + \frac{1}{-\hat{p} + \vec{k} - M_N} \right] \gamma^\nu \varepsilon_\nu(k, \delta). \quad (34)\]

### 3 Feynman rules

With the propagators in vacuum as Eqs. (11), (12) and (13), the second order self-energies of particles in the hot nuclear matter are obtained. Therefore, the expectation values of observables built out of products of field operators are allowed to be calculated. For the present method, the Feynman rules are as follows:

1. Draw all topologically distinct, connected diagrams.
2. Include the following factors for the scalar and vector vertices respectively,
   \[-ig_\sigma; \quad scalar \quad -ig_\omega \gamma_\mu; \quad vector. \quad (35)\]
3. Include the factors for the scalar, vector, and baryon propagators as Eqs. (11), (12) and (13), respectively.
4. Conserve four-momentum at each vertex, and include a factor of \((2\pi)^4 \delta(\sum p)\) at each vertex.
5. Include a factor of
   \[\sum_{\lambda=1,2} \int \frac{d^3p \ M_N}{(2\pi)^3} E(p) n_F\]
for each pair of crosses for external lines of the nucleon.

6. Include a factor of

\[ \sum_{\lambda=1,2} \int \frac{d^3p}{(2\pi)^3} \frac{M_N}{E(p)} (-1) \tilde{n}_F \]

for each pair of crosses for external lines of the antinucleon.

7. Include a factor of

\[ \int d^3\tilde{k} n_\alpha \]

for each pair of crosses for external lines of the scalar or vector meson, where

\[ \int d^3\tilde{k} n_\alpha = \left\{ \begin{array}{ll}
\int \frac{d^3k}{2\Omega_\sigma (2\pi)^3} n_\sigma, & \alpha = \sigma, \\
\sum_{\delta=1,2,3} \int \frac{d^3k}{2\Omega_\omega (2\pi)^3} n_\omega, & \alpha = \omega,
\end{array} \right. \]

and \( \Omega_\alpha = \sqrt{k^2 + m_\alpha^2} \).

8. Take the Dirac matrix product along a fermion line.

9. The momentums and spins of external lines with a cross or without a cross take the same values with each other, respectively.

10. Change the momentum \( p \) of the external lines of the nucleon into \((-p)\), the Dirac spinor \( U(p, \lambda) \) and the corresponding conjugate spinor \( \bar{U}(p, \lambda) \) for the external lines of the nucleon into the Dirac spinor \( V(p, \lambda) \) and the corresponding conjugate spinor \( \bar{V}(p, \lambda) \) of the antinucleon, respectively,

\[ p \to -p, \quad \text{(36)} \]
\[ U(p, \lambda) \to V(p, \lambda), \quad \text{(37)} \]
\[ \bar{U}(p, \lambda) \to \bar{V}(p, \lambda). \quad \text{(38)} \]

Therefore, the results of corresponding topological diagrams with external lines of the antinucleon are obtained.

11. Include a factor of \((-1)\) for the second order self-energy of the antinucleon.

12. Include a factor of \((-1)\) in the calculation of exchange diagrams.

13. Include a factor of \( \delta_{ij} \) along a fermion line for isospin (here \( i, j = p, n \)).

If the masses of the scalar meson, vector meson, baryon, and all coupling constants are renormalized, the loop diagrams are not needed to be considered at all. Therefore, only the diagrams with crosses should be considered in the calculation of self-energies of particles.

In Fig. 1, the Feynman diagrams on the second order self-energy of the meson interaction with nucleons in the hot and dense nuclear matter are shown. The Feynman diagrams on the self-energy of the meson interaction with antinucleons in the nuclear matter can be obtained by changing the directions of all the fermion lines in each diagram, respectively.

The Feynman diagrams on the second order self-energy of the nucleon in the hot and dense nuclear matter are shown in Fig. 2. According to Eqs. (36 - 38), the second order self-energy of the antinucleon in the hot and dense nuclear matter can be obtained easily.
As the anti-commutation relations are considered, a factor of \((-1)\) should be added for the second order self-energy of the antinucleon.

Because of the strong interaction between nucleons, the coupling constants are very large and the perturbation calculation does not converge. Although the second-order self-energy of the particle can be summed to all orders with Dyson equation, this procedure is not self-consistent, however, since the propagating particle interacts to all orders, whereas the background particles are noninteracting. Self-consistency can be achieved by using the interacting propagators to also determine the self-energy\cite{3}.

In the relativistic mean-field approximation, the meson fields operators can be replaced by their expectation values in the nuclear matter \cite{3}:

\begin{align*}
\sigma & \to \langle \sigma \rangle = \sigma_0, \\
\omega_\mu & \to \langle \omega_\mu \rangle = \omega_0 \delta_\mu^0.
\end{align*}

Therefore, the off-shell part of the nucleon propagator can be written as\cite{3,11}:

\begin{equation}
G_{\alpha\beta}^H(p) = \frac{i}{\gamma_\mu p^\mu - M_N^* + i\varepsilon},
\end{equation}

where \(p = (E^*(p), \vec{p})\), and \(E^*(p) = \sqrt{\vec{p}^2 + M_N^{*2}}\). \(M_N^* = M_N + g_\sigma \sigma_0\) is the effective mass of the nucleon in the nuclear matter.

With the nucleon propagator in Eq. (41), the self-energies of particles can be calculated. It corresponds to the transformation of

\begin{equation}
E(p) \to E^*(p), \quad M_N \to M_N^*, \quad \mu \to \mu - g_\omega \omega_0
\end{equation}
in calculations of Sec. 2.

4 Debye screening effect in the hot and dense nuclear matter

The properties of hadrons in the nuclear matter have caused more attention of nuclear physicists in the past years\cite{15,16,17,18,19,20}. Suppose the momentum of the meson is zero, the screening masses of the scalar and vector mesons in the hot and dense nuclear matter are obtained from Eqs. (16) and (19), respectively.

\begin{equation}
m_\alpha^* = \sqrt{m_\alpha^2 + \Sigma'_\alpha}, \quad \alpha = \sigma, \omega
\end{equation}
in which

\begin{equation}
\Sigma'_\sigma = \frac{g_\sigma^2 (\rho_S^p + \rho_S^n)}{M_N^*}, \quad \Sigma'_\omega = \frac{g_\omega^2 (\rho_S^p + \rho_S^n)}{2M_N^*},
\end{equation}

where \(\rho_S^p\) and \(\rho_S^n\) are the scalar densities of protons and neutrons, respectively. In the limit of zero momentum of the meson, our results on the self-energies of the mesons at zero temperature are same as those in Ref.\cite{15}.
With the parameters fixed in Ref. [21], and supposing the masses of the scalar and vector meson have been fixed already,

\[ m_\sigma = 520.0 \text{MeV}, \quad m_\omega = 783.0 \text{MeV}, \quad (45) \]

the screening meson masses, the effective mass of the nucleon and the energy per nucleon in the nuclear matter under different densities and temperatures are calculated self-consistently in the framework of the relativistic mean-field approximation.

The screening masses of \( \sigma \) and \( \omega \) mesons \( m^*_\sigma, m^*_\omega \), the effective mass of the nucleon \( M^*_N \) and the energy per nucleon \( E/A \) in the nuclear matter as functions of the nucleon number density \( \rho_N \) with the temperature \( T = 10.0 \text{MeV} \) are displayed in Table 1. We find that the screening masses of \( \sigma \) and \( \omega \) mesons both increase with the nucleon number density, while the effective mass of the nucleon decreases at the fixed temperature. It implies that the Debye screening effect enhanced in the denser nuclear matter.

The properties of particles vs temperature at \( \rho_N = \rho_0 = 0.148 \text{fm}^{-3} \) are listed in Table 2, where \( \rho_0 \) is the saturation density of nuclear matter at zero temperature. The energy per nucleon increases with the temperature, in other words, the binding energy decreases when the temperature becomes higher. Meanwhile, the effective mass of the nucleon increases with temperature and the screening masses of the scalar and vector mesons both decrease when the temperature increases at the fixed density of nuclear matter. Obviously, when the temperature increases, the Debye screening effect decreases, and the interaction between nucleons becomes stronger. Our results are different from those with the thermo-field dynamics[20] and the familiar Brown-Rho conjecture[16].

Since the screening masses of \( \sigma \) meson, \( \omega \) meson and the effective mass of the nucleon are all relevant to the scalar density of nucleons, where the relativistic effect of the nucleon is included, it is not difficult to understand the different changes of them with the temperature and density of the nuclear matter. When the temperature of the nuclear matter is not too high, i.e. \( M_N >> T \), the distribution function of the nucleon is equal to 1 and the distribution function of the antinucleon is zero approximately,

\[ n_F = 1, \quad \bar{n}_F = 0. \quad (46) \]

Therefore, when the temperature is fixed, the scalar density of the nucleon increases with the number density of the nucleon, so the screening masses of \( \sigma \) meson and \( \omega \) meson increase, while the effective mass of the nucleon decreases; When the number density of the nucleon is fixed, the mean velocity of the nucleon increases with the temperature, and the scalar density of the nucleon decreases, so the screening masses of both \( \sigma \) meson and \( \omega \) meson decrease, while the effective mass of the nucleon increases.

### 5 Photon in the hot nuclear matter

If the electromagnetic interaction between protons in the nuclear matter is considered, the Lagrangian density on the electromagnetic interaction can be written as

\[ \mathcal{L}_{\gamma}^{\text{int}} = - e \bar{\psi} \left( \frac{1 + \tau_3}{2} \gamma_\mu A^\mu \right) \psi, \quad (47) \]
where $\tau_3$ is the Pauli matrix,
\[
\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\]
(48)
With the similar method in Sec. 2, the effective mass of the real photon in the hot and dense nuclear matter can be obtained as
\[
m_\gamma^* = \sqrt{\frac{e^2 \rho_S}{2 M_N^*}},
\]
(49)
with $e^2 = 4\pi\alpha$, and $\alpha$ is the fine structure constant. This result is same as that at zero temperature except that the scalar density of protons takes the form at finite temperature[10, 11].

The screening mass of the photon in nuclear matter is equal to the effective mass of the real photon. The effective mass of the photon corresponding to different densities when $T = 10 \text{MeV}$ and different temperatures when $\rho_N = 0.148 \text{fm}^{-3}$ are displayed in Table 1 and 2, respectively. From Table 1 and 2, it can be concluded that the effective mass of the photon in nuclear matter increases when the density of protons increases, while it decreases when the temperature of nuclear matter increases.

The value of scalar density of nucleons is similar to that of the number density of nucleons in the nuclear matter at zero temperature, $\rho_S^B \approx \rho_B$. In the hot nuclear matter, the distribution of antinucleons should be considered. Supposing the phase transition to quark gluon plasma is forbidden, in the state with high temperatures and zero number densities of protons and neutrons,
\[
\rho_p = \rho_n = 0, \quad \text{and} \quad T >> 0.
\]
(50)
According to Eq. (7), $n_F(p) = \bar{n}_F(p)$, and the scalar density of protons in Eq. (22) is not zero, so the photon gains an effective mass in the state at finite temperatures and zero number density of nucleons.

6 Summary

In this paper, according to Wick’s theorem, the second order self-energy corrections of hadrons in the hot and dense nuclear matter are calculated in the framework of quantum hadrodynamics. Furthermore, the Feynman rules are summarized and the effective method on quantum hadrodynamics is generalized to the situation at finite temperatures and densities. As the strong couplings between nucleons are considered, the self-consistency of this method is discussed in the relativistic mean-field approximation.

Debye screening masses of the scalar and vector mesons are calculated with this method in the relativistic mean-field approximation. These screening meson masses decrease when the temperature becomes higher, while increase when the density of the nuclear matter increases. In a conclusion, the results are different from those of thermofield dynamics and Brown-Rho conjecture. Moreover, the effective masses of the photon and the nucleon in the hot and dense nuclear matter are discussed.
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Table 1: The screening masses of $\sigma$ and $\omega$ mesons $m_\sigma^*$, $m_\omega^*$, the effective mass of the nucleon $M_N^*$, the energy per nucleon $E/A$ and the effective mass of the photon $m_\gamma$ in nuclear matter as functions of the nucleon number density $\rho_N$ at the temperature $T = 10.0 MeV$, where $\rho_N$ is in units of the saturation density of nuclear matter $\rho_0$, and $\rho_0 = 0.148 fm^{-3}$, and the others are in units of $MeV$.

| $\rho_N$ ($\rho_0$) | $m_\sigma^*$ | $m_\omega^*$ | $M_N^*$ | $E/A$ | $m_\gamma$ |
|---------------------|--------------|-------------|--------|-------|-----------|
| 0.200               | 546.96       | 798.80      | 848.72 | 7.40  | 2.45      |
| 0.400               | 578.10       | 817.62      | 759.73 | 0.45  | 3.65      |
| 0.600               | 614.28       | 840.22      | 672.89 | -5.29 | 4.73      |
| 0.800               | 656.26       | 867.36      | 589.54 | -9.48 | 5.79      |
| 1.000               | 704.44       | 899.59      | 511.66 | -11.61| 6.87      |
| 1.200               | 758.23       | 936.80      | 441.64 | -11.08| 7.98      |
| 1.400               | 815.74       | 977.87      | 381.57 | -7.43 | 9.09      |
| 1.600               | 874.12       | 1020.75     | 332.30 | -0.24 | 10.16     |
| 1.800               | 930.49       | 1063.15     | 293.26 | 10.80 | 11.16     |
| 2.000               | 983.56       | 1103.87     | 262.46 | 25.18 | 12.07     |
Table 2: The screening masses of $\sigma$ and $\omega$ mesons $m_{\sigma}^*$, $m_{\omega}^*$, the effective mass of the nucleon $M_N^*$, the energy per nucleon $E/A$ and the effective mass of the photon $m_\gamma$ in nuclear matter as functions of the temperature $T$ at the density of nuclear matter $\rho_N = \rho_0 = 0.148 fm^{-3}$. All are in units of $MeV$.

| $T$ (fm$^{-3}$) | $m_{\sigma}^*$ (MeV) | $m_{\omega}^*$ (MeV) | $M_N^*$ (MeV) | $E/A$ (MeV) | $m_\gamma$ (MeV) |
|----------------|----------------------|----------------------|--------------|------------|-------------|
| 0.00           | 706.53               | 901.01               | 508.10       | -15.73     | 6.92        |
| 10.00          | 704.44               | 899.59               | 511.66       | -11.61     | 6.87        |
| 20.00          | 700.08               | 896.62               | 518.06       | -1.21      | 6.78        |
| 30.00          | 694.85               | 893.09               | 525.88       | 12.49      | 6.67        |
| 40.00          | 689.52               | 889.49               | 534.05       | 28.12      | 6.55        |
| 50.00          | 684.33               | 886.00               | 542.17       | 45.06      | 6.43        |
| 60.00          | 679.40               | 882.70               | 550.08       | 62.98      | 6.32        |
| 70.00          | 674.73               | 879.58               | 557.71       | 81.68      | 6.22        |
| 80.00          | 670.34               | 876.66               | 565.04       | 101.04     | 6.12        |
| 90.00          | 666.22               | 873.93               | 572.06       | 121.00     | 6.02        |
| 100.00         | 662.37               | 871.38               | 578.72       | 141.58     | 5.93        |
| 110.00         | 658.85               | 869.06               | 584.93       | 163.05     | 5.85        |
| 120.00         | 655.79               | 867.05               | 590.40       | 186.16     | 5.78        |
Figure Captions

**Fig. 1** Feynman diagrams on the second order self-energy of the meson interaction with nucleons in the hot and dense nuclear matter. The wave lines denote the scalar or vector meson, 1 and 2 denote particles of the initial state, 3 and 4 denote particles of the final state.

**Fig. 2** Feynman diagrams on the second order self-energy of the nucleon in the hot and dense nuclear matter. The meanings of denotations are same as those in Fig. 1.
