First Order Sensitivity Calculation of Car Door Based on Combined Approximate Reanalysis Method

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**Abstract.** The reanalytic calculation method is a highly efficient method of reanalysing the modified structural performance, and initial structural information are used in the structural design process (optimization and modification). This method can avoid the complete analysis of the modified structure and significantly reduce the computational cost. In order to improve the efficiency of iterative calculation in the reanalysis of the door structure, this paper takes a car door as an example, and compares the sensitivity analysis with the combination of the approximate reanalysis algorithm and the current mainstream commercial software, and the calculation accuracy of the reanalysis method in this paper is verified by calculation examples.

**1. Introduction**

The structural reanalysis method can accelerate the calculation process of structural dynamic modification and optimization design, avoiding the complete and accurate analysis of the structure again. This method uses the initial information that obtained in the last analysis and solving process to perform fast solution calculation, which is an efficient solution method.

The reanalysis is designed to calculate the displacement vector of the modified structure using the initial parameters before modification. In order to meet the accuracy requirements and efficiency requirements at the same time and effectively solve the linear static re-analysis problem, Kirsch\(^[1, 2]\) proposed the combination approximation method (CA method), which is applied to sensitivity analysis, structural layout and topology optimization. Chen\(^[3, 4]\) et al. improved the CA algorithm and proposed a general iterative combination approximation (ICA), which can be applied to various types of topological modifications. Based on the change of degrees of freedom, a method of undetermined coefficients was proposed by Wu\(^[5]\) et al..

The modal problem is also called the feature problem. Earlier, Carey\(^[6]\) et al. proposed the Lanczos method to solve such problems, and Segura\(^[7]\) introduced the substructure method, which was applied to modal analysis. Bouazzouni\(^[8]\) also proposed the Ritz method for eigenvalue reanalysis. For complex eigenvalue problems, Ma et al. proposed a new reanalysis method based on CA method and second order perturbation. Later, Huang\(^[9]\) et al. introduced the Rayleigh-Ritz method to modal reanalysis and discussed the change in degrees of freedom. Subsequently, Chen et al. proposed a new modal reanalysis method in combination with the Ritz method and Guyan reduction. Su HC\(^[10]\) et al. introduced the Padé approximation and matrix perturbation method into the modal reanalysis, and proposed the modal reanalysis method to increase the variable range of structural parameter modification. Later, the CA method was also introduced into the modal reanalysis, and further developed to be extended to solve the problem of time domain reanalysis.
This paper combines the reanalysis method to reanalyze the structure of the vehicle door, and introduce the reanalysis method and its development so that readers can understand the reanalysis calculation method, and further introduces the static combination approximation algorithm and the modal combination approximation algorithm in the field of reanalysis.

2. Method

Modal reanalysis is the use of reanalysis techniques to analyze the modality of a structure (i.e., the vibration frequency of a structure) and is an important part of kinetic calculations. In the structural dynamic modification process, after the structural modification, we need to recalculate the structure. The calculation of large-scale practical engineering modal analysis process is huge. In this process, the use of re-analysis method helps to reduce the amount of calculation. In the field of modal reanalysis, for undamped systems, the equation can be reduced to solving eigenvalue problems:

\[
K_0 u_0 = M_0 \lambda_0 \\
K_{0r} = U_0^T U_0
\]

(1)

Where \( K_0 \) is the stiffness matrix of the initial structure, its size is \( n \times n \) order, \( M_0 \) is the mass matrix of the initial structure, its size is \( n \times n \) order, \( \lambda_0 \) is the eigenvalue matrix of the initial structure, and \( u_0 \) is its corresponding modal vector group. The sizes are \( m \times m \)-order and \( n \times m \) order, \( n \) is the degree of freedom of the structure, \( U_0 \) is the upper triangular matrix obtained by \( K_0 \) through Cholesky decomposition, and is obtained from the modal analysis of the initial structure of \( U_0 \).

When we modify the structure (increasing or decreasing the number of nodes or units, or changing the parameter settings of the unit), we can get the modified eigenvalue problem finite element equation:

\[
K u_0 = M u \lambda
\]

(2)

among them,

\[
K = K_0 + \Delta K \\
M = M_0 + \Delta M
\]

(3)

The above formula indicates the modified stiffness matrix of the model, \( M \) represents the modified mass matrix of the model, represents the modified amount of the stiffness matrix, \( \Delta M \) represents the modified eigenvalue matrix of the model, \( \lambda \) represents the corresponding model and \( u \) represents the corresponding modal vector group after the model is modified.

According to the method proposed by Kirsch, the modal vector of the modified structure can be approximated by the following linear combination of expressions:

\[
u = y_1 r_1 + y_2 r_2 + \ldots + y_s r_s = R y
\]

(4)

Among them, the linear combination of \( r_1, r_2, \ldots, r_s \) approximates that the size of \( s \) is much smaller than the degree of freedom of the structure \( n \), \( R \) is the matrix of these selected base vectors, and \( y \) is the vector of the undetermined coefficients.

\[
R \equiv \begin{bmatrix} r_1, r_2, \ldots, r_s \end{bmatrix} \\
y \equiv \begin{bmatrix} y_1, y_2, \ldots, y_s \end{bmatrix}
\]

(5)

The modified model equation can be approximated by a small-scale system eigenvalue problem for \( y \). Substituting equation (4) into equation (2) and multiplying by \( R^T \), you can get:

\[
R^T K_{0r} y_1 = \lambda_1 R^T M_{0r} y_1
\]

(6)

Use the following formula to simplify:

\[
K_{0r} = R^T K R \\
M_{0r} = R^T M R
\]

(7)

We can get the reduced characteristic equation:

\[
K_{0r} y_1 = \lambda_1 M_{0r} y_1
\]

(8)
Where $K_{\Delta}$ is the stiffness matrix after polycondensation, and its size is much smaller than the modified structural stiffness matrix $K$. Using this method, we can effectively avoid solving the large matrix of $n \times n$-order, and by solving the small matrix of $s \times s$-order, we can get the first-order feature pairs $\lambda_i$ and $y_i$, and we can calculate the original system by formula (4). The first order feature pair. Higher order feature pairs can also be solved by the same method.

2.1. Select basis vector
We choose the first-order feature pair, and we multiply the formula (2) to get $K_{\Delta}^{-1}$:

$$
(I + B)u = r_i
$$

from them

$$
B = K_{\Delta}^{-1} \Delta K
$$

$$
r_i = K_{\Delta}^{-1} \lambda_i M u_i
$$

We multiply the equation (9) by the left $(I + B)^{-1}$ and get it through the Newman series expansion.

$$
u = (I + B)^{-1} r_i
$$

$$
= (I - B + B^2 - .......) r_i
$$

The combination approximation algorithm performs modal analysis to solve the eigenvalue problem. The selection basis vector can be obtained from the above formula. $r_i$ Can be approximated as:

$$
r_i = K_{\Delta}^{-1} \lambda_i M u_i
$$

Since the multiplication between the vector and the scalar does not affect the calculation result, we ignore the $\lambda_i$ in the above formula. When $M = M_0$, we can get the value of the first base vector equal to the initial modal vector $r_i = u_0$

Other base vectors can be derived from the following recursive formula:

$$
r_i = Br_{i+1}, i = 2,3, ....s
$$

In the case where the model has a large amount of modification, that is, when $\Delta K$ takes a larger value, the value range of the base vector becomes very large, so we need to take normalization measures for the base vector.

2.2. Schmidt orthogonalization
In order to improve the calculation accuracy of higher-order modes, we use the Schmidt orthogonalization method to process the approximate mode. In the case of the known m-order modal vector $u_1,u_2,......,u_n$, we need to calculate the m+1 order modal vector $u_{n+1}$ such that it is orthogonal to the first m-order modal vector with respect to the mass matrix $M$. To this end, we first orthogonalize the base vector $r_i$ and the front m-order vectors $u_1,u_2,......,u_n$ with respect to the mass matrix $M$, and perform orthogonalization. The orthogonalization process is as follows:

$$
\tilde{r}_{i+1} = r_{i+1} - \sum \sigma_k u_k (k = 1,2,3,......,m)
$$

Among them, we obtain the $r_{i+1}$ by normalization through the formula (13), and the coefficient $\alpha_k$ is usually calculated by the following formula:

$$
\alpha_k = u_k^T M \tilde{r}_{i+1}
$$

In the formula, $\delta_{ij}$ is the Kronecker symbol. Substituting the above formula into equation (16):

$$
\tilde{r}_{i+1} = r_{i+1} - \sum (u_k^T M) u_k \delta_{ij} \quad (k = 1,2,......,m) \quad i = 1,2,....,s - 1
$$

Calculate the first-order mode to get the first base vector:
\begin{equation}
\tilde{r}_i = r_i \tag{17}
\end{equation}

From equation (7), the first base vector is taken in the calculation of the higher-order mode:
\begin{equation}
\tilde{r}_i = r_i - \sum (u_i^T M r_i) u_k \quad (k = 1, 2, \ldots, m) \tag{18}
\end{equation}

For large structural modification problems, faced with the defects of high-order modal precision, we use normalization and Schmidt orthogonalization to improve high-order modal accuracy.

\subsection*{2.3. Central difference method}

Using the central difference method to calculate the first order partial derivatives, For example, here any binary function is \( f(x, y) \), assuming that the derivative is \( t \), there is the following difference formula:
\begin{align*}
\frac{\partial f}{\partial x} &= \frac{f(x+t, y) - f(x-t, y)}{2t} \\
\frac{\partial f}{\partial y} &= \frac{f(x, y+t) - f(x, y-t)}{2t} \\
\frac{\partial^2 f}{\partial x^2} &= \frac{f(x+t, t) + f(x-t, t) - 2f(x, y)}{t^2} \\
\frac{\partial^2 f}{\partial y^2} &= \frac{f(x, y+t) + f(x, y-t) - 2f(x, y)}{t^2} \\
\frac{\partial^2 f}{\partial x \partial y} &= \frac{f(x+t, y-t) + f(x-t, y+t) - f(x+t, y+t) - f(x-t, y-t)}{4t^2}
\end{align*}

According to the above difference formula, we can calculate the sensitivity information.

\section*{3. Calculations}

The door model is shown in Figure 1 as an example, the design variables take the thickness of the outer panel of the door and the thickness of the reinforcing beam on the inside of the door, as shown in Figure 2 and Figure 3:

\begin{center}
\textbf{Figure 1: The processed door model}
\end{center}

\begin{center}
\textbf{Figure 2: Door panel} \quad \textbf{Figure 3: Door reinforcement beam}
\end{center}
The differential variable $t=0.01\text{mm}$ is selected, and the above two design variables separately is modified, and use Hypermesh and Matlab to solve the stiffness displacement and the first three modal values respectively, and substitute them into the difference formula of equation to find the required First-order and second-order sensitivity information, the First-order sensitivity is shown in Table 1:

| $\delta\omega_1$ | $\delta\omega_1$ | $\delta\omega_4$ | $\delta\omega_5$ | $\delta\omega_6$ |
|------------------|------------------|------------------|------------------|------------------|
| $\delta x_1$     | 0.0017           | 2.0599           | 21.9238          | -2.9343          | 5.8875           |
| $\delta x_2$     | 5.9E-5           | 0.6484           | 0.1762           | -0.0022          | 0.785            |

To run the reanalysis algorithm, we need the joint stiffness matrix $K_0$ and the eigenvector matrix $U_\omega$ (the solution of $U_\omega$ can be solved by using eig or eigs in MATLAB, in order to improve the computational efficiency, the matrix can be changed to a sparse matrix solution), and the structural dynamics can be modified. After the stiffness matrix $K_1$ and the mass matrix $M_1$, then the CA code can be run to quickly solve the modified eigenvalues. Using the eigenvalues, we can compare the first-order sensitivity with the second-order sensitivity. The calculated first-order sensitivity information is shown in the following Table 2:

| $\delta\omega_1$ | $\delta\omega_1$ | $\delta\omega_4$ | $\delta\omega_5$ | $\delta\omega_6$ |
|------------------|------------------|------------------|------------------|------------------|
| $\delta x_1$     | 0.005            | 3.1338           | 24.0205          | -2.9342          | 5.8875           |
| $\delta x_2$     | -0.1453          | -7.1473          | 0.0562           | -0.0022          | 0.785            |

Comparing the sensitivity results before and after the re-analysis, it can be seen that the structural modification amount is small, and the CA method can also achieve the accuracy requirements comparable to commercial software, and the calculation amount is greatly reduced.

4. Conclusions
This paper introduces the basic principle and derivation process of the reanalysis algorithm, using the central difference method to calculate the first order sensitivity of the model. And popular commercial software is used to complete the processing of the model, and modify the model with the calculation data of the commercial software. Through the reanalysis method, we quickly get the modified eigenvalue information, then use the eigenvalue to find the sensitivity information, and then complete the optimization design through the same method, and compare it with the commercial software results to prove that it can meet certain precision requirements. In this case, the amount of calculation is significantly reduced.

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