Ghost dark energy in $f(R)$ model of gravity

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Abstract

We study a correspondence between $f(R)$ model of gravity and a phenomenological kind of dark energy (DE), which is known as QCD ghost dark energy. Since this kind of dark energy is not stable in the context of Einsteinian theory of gravity and Brans-Dicke model of gravity, we consider two kinds of correspondence between modified gravity and DE. By studying the dynamical evolution of model and finding relevant quantities such as, equation of state parameter, deceleration parameter, dimensionless density parameter, we show that the model can describe the present Universe and also the EoS parameter can cross the phantom divide line without needs to any kinetic energy with negative sign. Furthermore, by obtaining the adiabatic squared sound speed of the model for different cases of interaction, we show that this model is stable.

Finally, we fit this model with supernova observational data in a non interaction case and we find the best values of parameter at 1σ confidence interval as: $f_0 = 0.958^{+0.07}_{-0.25}$, $\beta = -0.256^{+0.02}_{-0.1}$, and $\Omega_{m0} = 0.23^{+0.3}_{-0.15}$. These best-fit values show that dark energy equation of state parameter, $\omega_{de}$, can cross the phantom divide line at the present time.

Keywords: Modified gravity; dark energy; QCD ghost model.

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1 Introductions

It was indicated that our Universe is in a positive accelerating expansion phase [1, 2, 3], and the standard model of gravity couldn’t explain this phenomena. This is a shortcoming for Einsteinian theory of gravity and several attempts have been accomplished to justify it.

One way for explaining this positive accelerating expansion is modified gravity. In fact some people have believed this shortcoming of Einsteinian theory of gravity is coming from the geometrical part of Hilbert-Einstein action and try to modify it by replacing a function of Ricci scalar, $f(R)$, instead of $R$ in the Hilbert-Einstein action and so-called $f(R)$ model of gravity.\footnote{There are another modified gravity formalism which are completely different with $f(R)$ model of gravity.} The various aspect of $f(R)$ models of gravity is investigated in [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20].

On the other hand some people looking for the reasons of this accelerating expansion in the matter part of Hilbert-Einstein action. Based on this idea, an ambiguous component of matter so-called dark energy (DE) is introduced for describing the positive accelerating expansion of the Universe. The simplest model of DE is the cosmological constant which is a key ingredient in the $\Lambda$CDM model. The $\Lambda$CDM model, which is consistent with nearly all observational data, but it faces with the fine tuning problem [21]. Also some other DE models have been introduced and are studied in [22, 23, 24, 25].

Recently, a new kind of dark energy model, so called QCD ghost dark energy (QCDGDE) model has been proposed [26]. This model, although is un-physical in the usual Minkoski space-time, express important physical effects in dynamical space-time or space-time with non-trivial topology. Actually, introducers of this dark energy model, claim the vacuum energy of Veneziano ghost field in effective quantum field theory at low energy, is related to density of dark energy which is used for explaining the positive accelerating expansion of the Universe [27]. They have shown that in a curved space time, the ghost field gives rise to a vacuum energy density $H\Lambda^3_{QCD}$ of the right magnitude $\sim (10^{-3}ev)^4$, where $H$ is the Hubble parameter and $\Lambda^3$ is QCD mass scale [28, 29, 30, 31, 32].

There are two groups which study QCD ghost dark energy model. The first group which have introduced it have believed that in the ghost model of dark energy, one needs not to introduce any new degree of freedom or modify gravity and it is totally embedded in standard model of gravity.
The second group have accepted the relation of density of QCDGDE as a phenomenological model of dark energy and studied the dynamical evolution of it in Einstein and Brans-Dicke model of gravity [33, 34, 35, 36, 37, 38, 39, 40, 41]. The studies of the second group show that the dynamical evolution of this model is instable. This means that the QCD ghost dark energy model has to be study further. One can study the proposal of the first group and consider the gravitational effective field theory at low energy for some extra interaction or for some another cutoff for the size of the Universe (manifold). And the another possibility for studding this issue is based on the second group work with some extra degree of freedom or modify gravity. According to second group’s view of point, we study QCDGDE model as a phenomenological model of dark energy in \( f(R) \) model of gravity.

As was mentioned earlier, modified gravity and DE model of gravity, are used for explaining the positive accelerating phase of the Universe and it is reasonable there are some correspondence between them. Based on this idea we separate some extra terms of modified gravity and define correspondence relation between it and a new kind of DE. Moreover, since there are some shortcoming in these models when anybody consider one of them alone, we studied a combination of modified gravity and dark energy model here. Therefore we assume the matter component of the action is consist of cold dark matter (CDM)\(^6\) and dark energy. In fact, we combine the extra terms of modified gravity with dark energy part of matter component and equivalent it with QCD ghost dark energy. Finally, we numerically and analytically compute some quantities such as DE density parameter, squared adiabatic speed, equation of state and deceleration parameter of the model.

This paper is organized as follows. In Sec. 2, we rewriting field equations on the \( f(R) \) model of gravity and obtain the equation of motions and conservation relation for density energy. In Sec. 3, we make an equivalence relation between \( f(R) \) model and GDE and obtain the relevant quantities of model such as, density parameter, squared adiabatic speed, EOS parameter and deceleration parameters. At last we summarize our work and give some discussion in Sec. 4.

2 Description and general properties of the model

The action of \( f(R) \) gravity with general matter is given by

\[
S = \int \sqrt{-g} \, d^4x \left[ \frac{f(R)}{2} + L_m(\psi, g_{\mu\nu}) \right].
\]  

\(^6\)In fact cold dark matter and ordinary matter are denoted by index "m" for simplicity.
Where $f(R)$ is an arbitrary function of Ricci scalar, $R$, $L_m = L_m(\psi, g_{\mu\nu})$ is the matter Lagrangian, $\psi$ is the matter field, $g_{\mu\nu}$ is the metric of spacetime and $g$ is the determinant of metric. Here we have assumed $8\pi G = 1$. Variation of (1) with respect of $g_{\mu\nu}$ gives

$$R_{\mu\nu} f' - \frac{1}{2} f g_{\mu\nu} + (g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu) f' = T^m_{\mu\nu}, \quad (2)$$

where prime represents the derivative with respect to the curvature scalar $R$, $\Box$ is the covariant d’Alembert operator ($\Box \equiv \nabla_\alpha \nabla^\alpha$) and $T^m_{\mu\nu}$ is the stress-energy tensor of matter which is defined by

$$T^m_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_m)}{\delta g^{\mu\nu}}. \quad (3)$$

The stress-energy tensor is covariantly conserved, this means that

$$\nabla^\mu \left[ R_{\mu\nu} f' - \frac{1}{2} f g_{\mu\nu} + (g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu) f' \right] = \nabla^\mu T^m_{\mu\nu} = 0. \quad (4)$$

Now we consider a homogeneous and spatially-flat space-time with Friedmann-Lemaître-Robertson-Walker (FIRW) line element

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2), \quad (5)$$

where $a(t)$ is the scale factor. We will assume the Universe is filled out with perfect fluids. Then the stress-energy tensor may be given by

$$T_{\mu\nu} = (\rho_t + p_t) u^\mu u^\nu - g_{\mu\nu} p_t, \quad (6)$$

where $\rho_t$ and $p_t$ are the energy density and pressure of the fluids and $u^\mu = (1, 0, 0, 0)$ is its normalized four-velocity in co-moving coordinates in which $u_\mu u^\mu = 1$. So by substituting (5) in to the right hand part of (4) and making use of (6), one can arrive at

$$\dot{\rho}_t + 3H(1 + \omega_t)\rho_t = 0, \quad (7)$$

where we have used $p_t = \omega_t \rho_t$. To study the dynamics of DE model in the $f(R)$ gravity, we consider a flat Universe with only two energy components; cold dark matter (CDM) and dark energy (DE), in which baryon is include in the CDM part\(^7\). This means

$$\rho_t = \rho_m + \rho_{DE}, \quad p_t = p_m + p_{DE}, \quad (8)$$

\(^7\)In fact in this work we study a model which consist of $f(R)$ model of gravity and dark energy simultaneously.
so, one can decompose (7) as

\[
\begin{align*}
\rho_m + 3H\rho_m & = Q, \\
\rho_{DE} + 3H(1 + \omega_{DE})\rho_{DE} & = -Q.
\end{align*}
\] (9) (10)

where \(Q\) is the direct interaction between two different components of matter. One can show that the action (1) is equivalent with [42]

\[
S = \int \frac{1}{2} \left[ f(\phi)R + \Phi(\phi) + 2L_m \right] \sqrt{-g} \, d^4x,
\] (11)

where \(f(\phi)\) and \(\Phi(\phi)\) are the proper functions of a scalar field \(\phi\). So using \(f(R) = f(\phi)R + \Phi(\phi)\), one can rewrite (2) as

\[
G_{\mu\nu} = \tilde{T}_{\mu\nu},
\] (12)

where \(G_{\mu\nu}\) is the Einsteinian tensor and

\[
\tilde{T}_{\mu\nu} = \frac{1}{f(\phi)} \left[ T^m_{\mu\nu} + T^\phi_{\mu\nu} \right],
\] (13)

and

\[
T^\phi_{\mu\nu} = \left[ \frac{1}{2} g_{\mu\nu} \Phi(\phi) + (\nabla_\mu \nabla_\nu - g_{\mu\nu} \Box) f(\phi) \right].
\] (14)

It is well known that \(\nabla^\mu G_{\mu\nu} = 0\), then from (12) we have \(\nabla^\mu \tilde{T}_{\mu\nu} = 0\). Since, \(\nabla^\mu T^m_{\mu\nu} = 0\), so we can obtain

\[
\nabla^\mu T^\phi_{\mu\nu} = \left[ T^m_{\mu\nu} + T^\phi_{\mu\nu} \right] \nabla^\mu (\ln f),
\] (15)

The \(tt\) component of (15) and Eqs. (9) and (10) gives

\[
\begin{align*}
\dot{\rho}_m + 3H\rho_m & = Q, \\
\dot{\rho}_{DE} + 3H(\rho_\phi + p_{DE}) & = -Q, \\
\dot{\rho}_\phi + 3H(\rho_\phi + \tilde{p}_\phi) & = 0,
\end{align*}
\] (16) (17) (18)

where

\[
\tilde{p}_\phi = p_\phi - \frac{f}{3Hf} [\rho_m + \rho_\phi].
\] (19)

As already mentioned in the Introduction, we suppose \(\rho_\phi\) has the same as dark energy role in evolution of the Universe. So we can combine it with the dark energy part which is coming from the matter part (the right hand
side) of Einstein equation, (2). This means that we can rewrite (16), (17) and (18) as follows

\[
\begin{align*}
\dot{\rho}_m + 3H\rho_m &= Q, \\
\dot{\rho}_\Lambda + 3H(\rho_\Lambda + p_\Lambda) &= -Q, \\
\end{align*}
\]

where

\[
\begin{align*}
\rho_\Lambda &= \rho_\phi + \rho_{DE}, \\
p_\Lambda &= \tilde{p}_\phi + p_{DE},
\end{align*}
\]

The $tt$ component of the gravitational equations (12), for the metric (5), can be simplified to

\[
H^2 = \frac{1}{3f(\phi)}(\rho_t + \rho_\phi),
\]

where

\[
\rho_\phi = \frac{1}{2}\Phi(\phi) - 3H\dot{f}(\phi).
\]

Using (23) we arrive at

\[
\dot{H} = -\frac{1}{2f(\phi)} \left[ \rho_t + \rho_\phi + p_{DE} + p_\phi \right].
\]

3 Dynamics of ghost dark energy

In this work we want to consider a correspondence between $f(R)$ model of gravity and ghost dark energy. In the ghost model of dark energy, the energy density of dark energy is given by $\rho_d = \alpha H$ where $\alpha$ is a constant with dimension [energy]$^3$ roughly of order of $\Lambda_{QCD}^3$, where $\Lambda_{QCD} \sim 100$ MeV is QCD mass scale [26]. For this correspondences we have two choices as follows

- We define a correspondence relation as

\[
\begin{align*}
\rho_d &= \rho_\phi + \rho_{DE}, \\
p_d &= p_{DE} + \tilde{p}_\phi.
\end{align*}
\]

Here $\rho_d$ and $p_d$ are density energy and pressure of ghost dark energy respectively. In this case the relation of conservation for matter and dark energy is the same as (20) and (21).

\[\text{Note that the relevant physical quantities, } \omega_d, q, \Omega_{cd}, \epsilon_d, \text{ are independent of } \alpha.\]
Another possibility for a correspondence relation is

$$\rho_d = \rho_\phi + \rho_{DE}, \quad p_d = p_{DE} + p_\phi.$$  \hfill (27)

In this case the relation of conservation for matter and dark energy is as follows

$$\dot{\rho}_m + 3H\rho_m = Q, \quad (28)$$

$$\dot{\rho}_d + 3H(\rho_d + p_d) = \frac{f}{f'}[\rho_m + \rho_d] - Q, \quad (29)$$

in this case we have two different interactions. Q interaction is a direct interaction between CDM and DE and the other term is coming from the interaction between matter and geometry.

### 3.1 Dynamics of ghost dark energy for $p_d = \rho_\Lambda$

Some observational data such as, observational of the galaxy cluster Abell A586, supports the interaction between DE and CDM [43]. Therefore in this section we study the direct interaction between DE and CDM and study the dynamical evolution of the model. So in (20) and (21), we assume $Q \neq 0$. Then by $\rho_d = \omega_dp_d$ we have

$$\dot{\rho}_m + 3H\rho_m = Q, \quad (30)$$

$$\dot{\rho}_d + 3(1 + \omega_d)H\rho_d = -Q. \quad (31)$$

One can rewrite (23), (30) and (31), in terms of dimensionless quantities as

$$\Omega_{em} + \Omega_{ed} = 1, \quad (32)$$

$$\dot{\Omega}_{em} + \frac{2H}{f'}\Omega_{em} + (3 + \beta)H\Omega_{em} = \frac{Q}{3H^2f}, \quad (33)$$

$$\dot{\Omega}_{ed} + \frac{2H}{f'}\Omega_{ed} + (3 + \beta + 3\omega_D)H\Omega_{ed} = -\frac{Q}{3H^2f}, \quad (34)$$

where $\Omega_{em} = \rho_m/3H^2f(\phi)$, $\Omega_{ed} = \rho_d/3H^2f(\phi)$ are the dimensionless energy density of CDM and DE respectively and $f = \beta fH$. Using (32) and (33) we obtain

$$-\dot{\Omega}_{ed} + \frac{2H}{f'}(1 - \Omega_{ed}) + (3 + \beta)H(1 - \Omega_{em}) = \frac{Q}{3H^2f}, \quad (35)$$

and substituting (34) into (35) we have

$$\frac{2H}{H} + (3 + \beta)H + 3H\omega_d\Omega_{ed} = 0. \quad (36)$$
As well, based on the linear relation between $\rho_d$ and $H$, we have

$$H \Omega_{ed} f = \text{constant}, \quad (37)$$

Expressing (35) in terms of efolding-number $x \equiv \ln a$, and making use (37) we have

$$-\Omega'_{ed} \frac{2 - \Omega_{ed}}{\Omega_{ed}} = \frac{Q}{3fH^3} + (\beta - 3)(1 - \Omega_{ed}). \quad (38)$$

Here, the equation of state parameter of the model versus $\Omega_{ed}$ is

$$\omega_d = -\frac{1}{3} \left[ \frac{3 + 2\Omega_Q - \beta}{2 - \Omega_{ed}} \right], \quad (39)$$

where $\Omega_Q = Q/(3f\Omega_{ed}H^3)$ and deceleration parameter $q = -1 - \dot{H}/H^2$ is

$$q = \frac{\beta + 1 - (\Omega_Q + 2)\Omega_{ed}}{(2 - \Omega_{ed})}. \quad (40)$$

Also we can obtain the squared adiabatic sound speed of our model as

$$c_s^2 = \frac{dp_d}{d\rho_d} = \left[ 1 - \left\{ \frac{(3 - \beta) - (3 + \Omega_Q - \beta)\Omega_{ed}}{(3 + \beta) - (3 + \Omega_Q)\Omega_{ed}} \right\} \Omega_{ed} \frac{d}{d\Omega_{ed}} \right] \omega_d. \quad (41)$$

For getting better insight we consider the interacting GDE for three different forms of $Q$ below.

3.1.1 $Q = 3b^2 H \rho_d$

In this case $\Omega_Q = 3b^2$ and Eq. (38) reduces to

$$\Omega'_{ed} \frac{2 - \Omega_{ed}}{\Omega_{ed}} = (\beta - 3 - 3b^2) [\Omega_{ed} + K], \quad (42)$$

where

$$K = \frac{3 - \beta}{(\beta - 3 - 3b^2)},$$

and by integrating of it we find

$$2 \ln(\Omega_{ed}) - (2 + K) \ln(|\Omega_{ed} + K|) = (3 - \beta) x + C_1, \quad (43)$$

where $C_1$ is the constant of integration and it is given by

$$C_1 = 2 \ln(\Omega_{ed_0}) - (2 + K) \ln(|\Omega_{ed_0} + K|)$$
and $\Omega_{\text{ed}}$ is the effective fraction of dark energy at the present time.

Also Eqs. (39), (40) and (41) reduce to

$$\omega_d = -\frac{1}{2-\Omega_{\text{ed}}} - \frac{6b^2 - \beta}{3(2-\Omega_{\text{ed})}}, \quad (44)$$

$$q = \frac{\beta + 1}{2-\Omega_{\text{ed}}} - \frac{(3b^2 + 2)\Omega_{\text{ed}}}{(2-\Omega_{\text{ed}})}, \quad (45)$$

$$c_s^2 = \left[ \frac{(3 - \beta) - (3 + 3b^2 - \beta)\Omega_{\text{ed}}}{(3 + \beta) - (3 + 3b^2)\Omega_{\text{ed}}} \right] \frac{\Omega_{\text{ed}}}{(2 - \Omega_{\text{ed})}} - 1 \right] \frac{A_1}{(2 - \Omega_{\text{ed}})}, \quad (46)$$

where $A_1 = (3 + 6b^2 - \beta)/3$. For getting better insight, we compute $\omega_d$, $q$ and $c_s^2$ for $\beta = -0.2$, $b = 0.1$ and $\Omega_{\text{ed}} = 0.8$ and obtain $\omega_d = -1.15$, $q = -0.99$ and $c_s^2 = 0.06$. It is seen that for this special choice, the equation of state cross the phantom divide line ($\omega_d = -1$) and the adiabatic squared sound speed is positive. This means that this model can describe the positive accelerating expansion of the Universe and also it is stable. We solve numerically (43) and plot it in Fig. 1. Fig. 1a is the effective fraction of dark energy, $\Omega_{\text{ed}}$, versus efolding number, $x = \ln(a)$, and shows that at the early time $\Omega_{\text{ed}} = 0$ and the late time is saturated to 1. Also at the present time $\Omega_{\text{ed}} = 0.8$. Fig. 1b is the adiabatic squared sound speed versus $\Omega_{\text{ed}}$. It is obviously seen that at the present time namely for $\Omega_{\text{ed}} = 0.8$, $c_s^2 > 0$, and then for this kind of interaction the model is stable.

3.1.2 $Q = 3b^2 H \rho_m$

In this case $\Omega_Q = 3b^2 (1 - \Omega_{\text{ed}})/\Omega_{\text{ed}}$ and equation (38) reduce to

$$\Omega'_{\text{ed}} \frac{2 - \Omega_{\text{ed}}}{\Omega_{\text{ed}}} = \xi (1 - \Omega_{\text{ed}} - 1), \quad (47)$$

where

$$\xi = 3 - \beta - 3b^2.$$  

By solving (47) we have

$$\Omega_{\text{ed}}^2 \frac{2 - \Omega_{\text{ed}}}{|1 - \Omega_{\text{ed}}|} = c_2 e^{\xi x}, \quad (48)$$

where $c_2$ is the constant of integration and it is given by

$$c_2 = \frac{\Omega_{\text{ed}}^2}{|1 - \Omega_{\text{ed}}|}.$$  

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where $\Omega_{ed}$ is the value of effective density parameter of dark energy at the present time. And Eqs. (39) and (40) become

$$\omega_d = -\frac{\xi}{3(2 - \Omega_{ed})} - \frac{2b^2}{\Omega_{ed}(2 - \Omega_{ed})},$$

$$q = \frac{\beta + 1 - (2 - 3b^2)\Omega_{ed}}{(2 - \Omega_{ed})},$$

and the adiabatic square sound speed Eq. (41), is

$$c_s^2 = \left\{ \frac{\xi(1 - \Omega_{ed})[\xi\Omega_{ed}^2 + 4b^2(1 - \Omega_{ed})]}{[(3 + \beta - 3b^2) - (3 - 3b^2)\Omega_{ed}](2 - \Omega_{ed})} \right\} - (\xi\Omega_{ed} + 2b^2) \frac{1}{3\Omega_{ed}(2 - \Omega_{ed})}.$$  

We solved numerically Eq. (48) and plot it in Fig. 2. Fig. 2a indicates the effective fraction of dark energy, $\Omega_{ed}$, versus efolding number, $x = \ln(a)$ for $Q = 3b^2H\rho_m$. This subfigure shows that the behavior of effective dimensionless parameter of dark energy is the same as $\Omega_{ed}$ for $Q = 3b^2H\rho_d$. Fig. 2b indicate the adiabatic squared sound speed versus $\Omega_{ed}$ in the case of $Q = 3b^2H\rho_m$. It is obviously seen that at the present time namely $c_s^2 > 0$, and then for this kind of interaction the model is stable too.
3.1.3 \( Q = 3b^2 H \rho_t \)

In this case \( \Omega_Q = 3b^2/\Omega_{ed} \) and equation (37) reduces to

\[
\Omega_{ed}' \frac{2 - \Omega_{ed}}{\Omega_{ed}} = (3 - \beta)(K - \Omega_{ed}),
\]

where

\[
K = \frac{3 - \beta - 3b^2}{3 - \beta}.
\]

By solving (50) we obtain

\[
2 \ln \Omega_{ed} + (K - 2) \ln(|\Omega_{ed} - K|) = (3 - \beta)K x + c,
\]

where

\[
c = 2 \ln \Omega_0 + (K - 2) \ln(|\Omega_0 - K|),
\]

is the constant of integration.

\[
\omega_d = -\frac{1}{3} \left[ \frac{(3 - \beta)\Omega_{ed} + 3b^2}{\Omega_{ed}(2 - \Omega_{ed})} \right],
\]

\[
q = \frac{\beta + 1 - 3b^2 - 2\Omega_{ed}}{(2 - \Omega_d)},
\]
Figure 3: (a): This subfigure shows $\Omega_{ed}$ versus efolding number, $x = \ln(a)$. (b): This subfigure shows $c_s^2$ versus $\Omega_{ed}$. We have taken $\beta = -0.2$, $p_d = p_\Lambda$ and $Q = 3b^2H\rho_t$.

and

\[
c_s^2 = \left[ \left\{ \frac{(3 - \beta - 3b^2) - (3 - \beta)\Omega_{ed}}{(3 + \beta - 3b^2) - 3\Omega_{ed}} \right\} \frac{(3 - \beta)\Omega_{ed}^2 - 6b^2(1 - \Omega_{ed})}{(2 - \Omega_{ed})} \right. \\
- \left[ (3 - \beta)\Omega_{ed} + 3b^2 \right] \left\{ \frac{1}{3\Omega_{ed}(2 - \Omega_{ed})} \right. \\
\left. \right]. \tag{56} \]

We solved numerically Eq. (53) and plot it in Fig. 3. Fig. 3a indicates the effective fraction of dark energy, $\Omega_{ed}$, versus efolding number, $x = \ln(a)$ for $Q = 3b^2H\rho_t$. This subfigure shows that the behavior of effective dimensionless parameter of dark energy is the same as $\Omega_{ed}$ in the two pervious subsection. Fig. 2b indicate the adiabatic squared sound speed versus $\Omega_{ed}$ in the case of $Q = 3b^2H\rho_t$. This subfigure shows that at the present time namely $c_s^2 > 0$, and then for this kind of interaction the model is stable too.

### 3.2 Dynamics of Ghost dark energy for $p_d = p_\phi + p_{DE}$

In this subsection dark energy is assumed to be without direct interaction with the matter part namely $Q = 0$. So according to (29) dark energy interaction may be with geometry. However the conservation relation of
(a) This subfigure shows $\Omega_{ed}$ versus efolding number, $x = \ln(a)$. (b) This subfigure shows $c_s^2$ versus $\Omega_{ed}$. We have taken $\beta = -0.2$, $p_d = p_{DE} + p_\phi$ and $Q = 0$.

Energy is as (28) and (29), so

$$\dot{\rho}_m + 3H\rho_m = 0, \quad \dot{\rho}_d + 3H(\rho_d + p_d) = \beta H \left[ \rho_m + \rho_d \right].$$

In this case Eqs. (33) and (34) can be rewrite as

$$\dot{\Omega}_{em} + \frac{2\dot{H}}{H}\Omega_{em} + (3 + \beta)H\Omega_{em} = 0, \quad (59)$$

$$\dot{\Omega}_{ed} - \beta H r \Omega_{ed} + 3H\Omega_{ed} + 3\omega_d H \Omega_{ed} + 2\frac{\dot{H}}{H}\Omega_{ed} = 0. \quad (60)$$

Using Eqs. (32) and (59) one can obtain

$$-\dot{\Omega}_{ed} + \frac{2\dot{H}}{H}(1 - \Omega_{ed}) + (3 + \beta)H(1 - \Omega_{ed}) = 0. \quad (61)$$

By combining Eq. (60) with Eq. (61), we have

$$\frac{2\dot{H}}{H} + 3\omega_d \Omega_{ed} H + (3 + \beta)H - (1 + r)\beta H \Omega_{ed} = 0. \quad (62)$$

Hence one can achieve in an expression for $\Omega_{ed}$ in terms of efolding-number $x \equiv \ln a$, as

$$-\Omega'_{ed} \left( \frac{2 - \Omega_{ed}}{\Omega_{ed}} \right) + \left( 1 - \Omega_{ed} \right) \left[ 3 - \beta \right] = 0. \quad (63)$$
By solving (63) we have
\[ \frac{\Omega_{ed}^2}{|1 - \Omega_{ed}|} = ce^{\xi x}, \]  
(64)
where \( \xi = 3 - \beta \) and \( c \) is the constant of integration. Eventually, in this case, the expression of the EoS, \( \omega_d \), of DE versus \( \Omega_{ed} \) is specified as follows
\[ \omega_d = -\frac{1}{3} \left[ \frac{3 - \beta}{2 - \Omega_{ed}} - \beta(1 + r) \right], \]  
(65)
and deceleration parameter is as
\[ q = \frac{1 + \beta - 2\Omega_{ed}}{2 - \Omega_{ed}} \]  
(66)
At last, we can find the squared adiabatic sound speed of the model as follows
\[ c_s^2 = \frac{1}{3} \left\{ \frac{(3 - \beta)(1 - \Omega_{ed})}{(3 + \beta) - 3\Omega_{ed}} \right\} \frac{\Omega_{ed}}{2 - \Omega_{ed}} - \beta(1 + r) - \frac{3 - \beta}{2 - \Omega_{ed}}, \]  
(67)
The numerical results of Eq. (64) and Eq. (67) is plotted in Fig. 4. In this case we don’t have any direct interaction between CDM and DE. This figure shows that the effective dimensionless energy density of dark energy is started from 0 at the early time and increased to asymptotic value, 1 at late time. Fig. 1b indicates the adiabatic squared sound speed versus \( \Omega_{ed} \) and shows that at the present time, \( \Omega_{ed} = 0.8, c_s^2 \) is positive. So as was expected, the model is stable in this case too.

4 Data fitting
In this Section we use the 557 Uion II sample dataset of SnIa[], to find the best-fit forms of our model. We consider a model which includes dark energy, cold dark matter (CDM), baryon (B) and radiation (R) in a flat FLRW universe. For simplicity we write the energy density of baryon and cold dark matter together as \( \Omega_m = \Omega_{CDM} + \Omega_B \). Also, since at present time the dimensionless energy density of radiation is very small compared to \( \Omega_m \), we neglect \( \Omega_R \). Therefore in this case the Friedmann equation is
\[ H^2 = \frac{1}{3f(t)}(\rho_d + \rho_m). \]  
(68)
Using Eqs. (57) and $\rho_d = \alpha H$, one can rewrite (68) as

$$H^2 = \frac{1}{3 f(t)} (\alpha H + \rho_{m_0} a^{-3}). \quad (69)$$

We can use $\dot{f} = \beta f H$ ($f = f_0 a^\beta$), and Eq. (69) can be rewritten as

$$H(f_0, \beta, \Omega_{m_0}) = H_0 \left[ \frac{1}{2} \left( 1 - \frac{\Omega_{m_0}(1+z)^\beta}{f_0} \right) \right] \quad (70)$$

$$+ \sqrt{\frac{1}{4} \left( 1 - \frac{\Omega_{m_0}(1+z)^\beta}{f_0} \right)^2 + \frac{\Omega_{m_0}(1+z)^{3+\beta}}{f_0}}.$$ 

In this model we have three free parameters $f_0$, $\beta$, and $\Omega_{m_0}$. To obtain the best-fit for free parameters we have to compare the theoretical distance modulus, $\mu_{th}$, with observed $\mu_{ob}$ of supernova. The distance modulus is defined by

$$\mu_{th} = 5 \log_{10} \left[ D_L(z; f_0, \beta, \Omega_{m_0}) \right] + \mu_0, \quad (71)$$

where $\mu_0 = 43.3$, and $D_L(z; f_0, \beta, \Omega_{m_0})$ is given by

$$D_L(z; f_0, \beta, \Omega_{m_0}) = (1+z) \int_0^z \frac{H_0}{H(x; f_0, \beta, \Omega_{m_0})} dx, \quad (72)$$

For comparing $\mu_{th}$ with $\mu_{ob}$ we need to obtain $\chi^2_{sn}$ which is defined by

$$\chi^2_{sn}(f_0, \beta, \Omega_{m_0}) = \sum_{i=1}^{557} \frac{[\mu_{th}(z_i) - \mu_{ob}(z_i)]^2}{\sigma_i^2}. \quad (73)$$

A minimization of this expression leads to

$$\chi^2_{sn_{min}}(f_0 = 0.958, \beta = -0.26, \Omega_{m_0} = 0.23) = 543.583, \quad (74)$$

where implies $\chi^2_{sn}/\text{dof} = \chi^2_{sn_{min}}/\text{dof} = 0.981$ (dof = 554). This show that this model is clearly consistent with the data since $\chi^2/\text{dof} = 1$.

The 1$\sigma$ errors on the predicted value of free parameter, $a$ is found by solving the following equation

$$\chi^2(a_{1\sigma}, b_{1\sigma}, c_{1\sigma}) - b_{\text{min}}^2(a_{bs}, b_{bs}, c_{bs}) = 1, \quad (75)$$

where $b_{bs} \equiv \text{best - fit value}$ and $x_{bs}$’s are the values for free parameters which minimize $\chi^2$. Using Eq. (75), we found the best-fit
Table 1: The best-fit values with 1σ errors for $f_0$, $\beta$, and $\Omega_{m0}$ in the $f(R)$ GDE model.

| parameter | $f_0$       | $\beta$     | $\Omega_{m0}$ |
|-----------|-------------|-------------|---------------|
| best-fit 1σ | $0.958^{+0.27}_{-0.25}$ | $-0.256^{+0.2}_{-0.1}$ | $0.23^{+0.3}_{-0.15}$ |

Figure 5: The observed distance modulus of supernova (points) and the theoretical predicted distance modulus (red-solid line) in the context of $f(R)$ GDE model.

Note that according to the best-fit values of parameters the present dark energy equation of state, $\omega_{de} = -0.987$. In figure 5 we show a comparison between theoretical distance modulus and observed distance modulus of supernova data. The red-solid line indicates the theoretical value of distance modulus, $\mu_{th}$, for the best value of free parameters (table 1). This figure show a reasonable result.

5 Conclusion

We investigated two kinds of equivalence between $f(R)$ model of gravity and an phenomenological kind of dark energy so-called ghost dark energy whose energy density is proportional to Hubble parameter. We studied the interacting and non interacting case of model in a flat FLRW background. For getting better results, we consider three kinds of interaction between matter and dark energy. The obtained results for equation of state and
deceleration parameter, show that the model can describe the accelerating expansion phase of the Universe and also $\omega_d$ can cross the line $\omega_d = -1$ from quintessence to phantom model for all cases of model with/without interaction. Also by obtaining the numerical results for effective dimensionless energy density of dark energy, $\Omega_{ed}$, we find that $\Omega_{ed}$ is started from 0 at the early time and saturated to its asymptotic value, 1, at late time. We further studied the dynamical evolution of the model by considering the adiabatic squared sound speed. This quantity is obtained for all cases of interaction and without interaction. Our investigation show that the adiabatic squared sound speed can be positive and then the model can be stable by a suitable choice of parameters.

Also we fitted this model with supernova observational data in a non interaction case. In this case the best values of parameter of the model are $f_0 = 0.958^{+0.07}_{-0.25}$, $\beta = -0.256^{+0.2}_{-0.1}$, and $\Omega_{m0} = 0.23^{+0.3}_{-0.15}$. These best-fit values show that the present dark energy equation of state parameter, $\omega_{d0}$, can cross the phantom divide line.

At last, we conclude that the dynamical behavior of the model for two different correspondence relations, also for all interactions/non interaction cases have the same behavior. This model is stable and can describe the present Universe.

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