Comparative study of three robust observers for automotive damper force estimation

Thanh-Phong Pham\(^1,2\), Olivier Sename\(^1\) and Luc Dugard\(^1\)

\(^1\) Univ. Grenoble Alpes, CNRS, Grenoble INP, GIPSA-lab, 38000 Grenoble, France. Institute of Engineering Univ. Grenoble Alpes.
\(^2\) Faculty of Electrical and Electronic Engineering, The University of Danang - University of Technology and Education, 550000 Danang, Vietnam

Email: {thanh-phong.pham2; olivier.sename; luc.dugard}@gipsa-lab.grenoble-inp.fr

Abstract. This paper aims at comparing three robust observers used to estimate the damping force of electro rheological (ER) dampers in a vehicle suspension system. Firstly, a nonlinear quarter-car model, augmented with a first-order dynamical nonlinear damper model, is developed. The first two methods are designed considering the nonlinearity as an unknown input and minimizing the effect of the unknown input disturbances (including a nonlinearity term, the measurement noise and the unknown road profile) on the estimation errors, by using an \(H_2\) and \(H_\infty\) criterion, respectively. The latter method aims at minimizing only the effects of measurement noises and road profiles on the state variable estimation errors by using a \(H_\infty\) criterion, while the nonlinearity is bounded through a Lipschitz condition. For implementation issue, two low-cost sensors signals (two accelerometers data from the sprung mass and the unsprung mass) are considered as inputs for the observer designs. Then, the observers are implemented in real-time on the INOVE test bench from GIPSA-lab (1/5-scaled real vehicle) to assess and compare experimentally the performances of the approaches. Both simulations and experimental results demonstrate a better effectiveness of the latter observer in terms of the ability of estimating the damper force in real-time despite the nonlinearity, the measurement noises and the road disturbances.

Keywords: Semi-active suspension, nonlinear model, \(H_2\) observer, \(H_\infty\) observer, damper force estimation.

1. Introduction

It is now well known that semi-active suspensions are a key component of vehicles, with the aim of improving comfort and safety (road holding) for on-board passengers, see [1]. The design of control algorithms for automotive semi active suspension system has received a lot of consideration in the last 20 years. Many approaches are considering the damper force as the control input of the vehicle model, and then use, for implementation, an inverse damper model (or look-up tables) to compute the input current or voltage (see [2, 3]). Few papers use local force tracking control schemes in order to attain control objectives (see [4]). However, as damper force sensors are difficult to install and expensive setups, the real-time estimation of the damper force is crucial for suspension control (and reliability). To this aim, some specific methods have been developed to estimate the damper force (see [5–10]). The key challenges are to reduce the cost of the required sensors, to take the non linear dynamical behavior of the damper into account, in presence of unknown road disturbance and sensor noises.
This paper presents a comparative study of three robust control approaches for solving the above challenges in designing the damper force observer of the semi-active suspension systems. The aspects of comparison are a) dealing with nonlinearity, b) minimizing the effect of sensor noise and unknown road profile to estimation errors. Note that such a comparison is proposed here for the first time in the case of suspension system. All of the observers use only two accelerometers in order to estimate the damper force in the quarter-car vehicle equipped with ER suspension. The designs of the observers are based on a nonlinear suspension model consisting of a quarter-car vehicle model, augmented with a first order dynamical nonlinear damper model. To handle the nonlinearity in the damper model, the two first observers (H\textsubscript{2} and H\textsubscript{\infty} observers) consider the nonlinearity as a unknown disturbance and use the H\textsubscript{2} and H\textsubscript{\infty} norms to minimize the effect of three unknown input disturbances (nonlinearity, sensor noises, unknown road profile) on the estimation errors of the state variables, respectively. In the latter observer (H\textsubscript{\infty} Lipschitz observer), the nonlinearity is bounded by a Lipschitz condition, while the H\textsubscript{\infty} norm is utilized to minimize the effect of sensor noises and road profile disturbance on the estimation errors. To assess and compare experimentally the performances of the observers, they have been implemented on a real scaled-vehicle test bench, through the Matlab/Simulink real-time workshop.

The remainder of this paper is organized as follows. Section II presents semi-active suspension modeling and III the design of the observers. In section IV, the analysis and comparison of the observer designs are provided in the frequency and time domains. Section V shows the experimental validation and comparison. Section VI gives some concluding remarks.

2. Semi-active suspension modeling and quarter-car system description

![Figure 1. 1/4 car model with semi-active suspension](image)

This section introduces the nonlinear quarter-car model equipped with the semi-active ER suspension system. The quarter-car model equipped with the semi-active ER suspension system, depicted in Fig.1, is described in this section. The well-known model consists of the sprung mass (m\textsubscript{s}), the unsprung mass (m\textsubscript{us}), the suspension components located between m\textsubscript{s} and m\textsubscript{us} and the tire which is modelled as a spring with stiffness k\textsubscript{t}. From second law of Newton for motion, the vertical system dynamics around the equilibrium are given as:

\[
\begin{align*}
\{m_s\ddot{z}_s &= -F_s - F_d \\
m_{us}\ddot{z}_{us} &= F_s + F_d - F_t
\end{align*}
\]
where $F_s = k_4(z_s - z_{us})$ is the spring force; $F_t = k_4(z_{us} - z_r)$ is the tire force; the damper force $F_d$ is given as follows [10]:

$$
\begin{align*}
\dot{F}_d &= k_0(z_s - z_{us}) + c_0(\dot{x}_s - \dot{z}_{us}) + F_{er} \\
\dot{F}_{er} &= -\frac{1}{\tau} F_{er} + \frac{f_c}{\tau} u \tanh(k_1(z_s - z_{us}) + c_1(\dot{x}_s - \dot{z}_{us}))
\end{align*}
$$

(2)

where $k_0, k_1, c_0, c_1, f_c, \tau$ are constant parameters; $z_s$ and $z_{us}$ are the displacements of the sprung and unsprung masses, respectively; $z_r$ is road displacement input.

By selecting the system states as $x = [x_1, x_2, x_3, x_4, x_5]^T = [z_s - z_{us}, \dot{z}_s, z_{us} - z_r, \dot{z}_{us}, F_{er}]^T \in \mathbb{R}^5$, the variables to be estimated $z = [x_1, x_2, x_3, x_5]^T \in \mathbb{R}^4$, the measured variables $y = [\tilde{z}_s, \tilde{z}_{us}]^T \in \mathbb{R}^2$ and the control input $u \in \mathbb{R}$, the system dynamics in the state-space representation can be written as follows:

$$
\begin{align*}
\dot{x} &= Ax + B\Phi(x), u + D_1\omega \\
y &= Cx + D_2\omega \\
z &= C_2 x
\end{align*}
$$

(3)

where $\Phi(x) = \tanh(k_1x_1 + c_1(x_2 - x_4))$; $\omega = \begin{bmatrix} \dot{z}_r \\ n \end{bmatrix}$, in which $\dot{z}_r$ is road profile derivative, $n$ is measurement noises; the system matrices $A, B, C, D_1, D_2, C_2$ are known.

Note that the measured outputs $y = [\tilde{z}_s, \tilde{z}_{us}]^T$ can be obtained easily from on board sensors (accelerometers).

3. Observer design

In this section, three observers ($H_2$, $H_\infty$ and $H_\infty$ Lipschitz observers) are developed to estimate the damping force accurately. In the design step, the sensor noises and road profile are considered as the unknown input $\omega$. Therefore, $H_2$ and $H_\infty$ observers are designed to minimize the effect of the unknown disturbance $\omega$ and of the nonlinearity on the state estimation errors, while an $H_\infty$ Lipschitz observer is developed to minimize the effect of the unknown input $\omega$ on the state estimation errors bounding the nonlinearity by Lipschitz constant.

Firstly, $H_2$, $H_\infty$ and $H_\infty$ Lipschitz observers for the system (3) are defined in same form as follows:

$$
\begin{align*}
\dot{\hat{x}} &= A\hat{x} + L(y - C\hat{x}) + B\Phi(\hat{x}), u \\
\dot{\hat{z}} &= C_2 \hat{x}
\end{align*}
$$

(4)

where $\hat{x}$ is the estimated states vector of $x$. $\hat{z}$ represents the estimated variables of the variables $z$. The observer gain $L$ will be determined in the next subsections. The estimation error is given as:

$$
\begin{align*}
e(t) &= x(t) - \hat{x}(t)
\end{align*}
$$

(5)

Differentiating $e(t)$ with respect to time and using (3) and (4), leads to:

$$
\begin{align*}
\dot{e} &= \dot{x} - \dot{\hat{x}} = (A - LC)e + B(\Phi(x) - \Phi(\hat{x})), u + (D_1 - LD_2)\omega \\
e_z &= C_2 e
\end{align*}
$$

(6)

Remark 1: In order to distinguish among three methodologies, the observer gain $L$ is denoted as $L_2$ for the $H_2$ observer, $L_\infty$ for the $H_\infty$ observer and $L_{\infty L}$ for the $H_\infty$ Lipschitz observer.

3.1. $H_2$ observer design

As discussed previously, an $H_2$ observer is designed to minimize the effect of $\omega$ and $(\Phi(x) - \Phi(\hat{x}))u$ on the state estimation errors $e_z$.

From (6) and Remark 1, one obtains

$$
\begin{align*}
\dot{e} &= (A - L_2 C)e + D\omega_n \\
e_z &= C_2 e
\end{align*}
$$

(7)

where $D = (B(D_1 - L_2 D_2))$, $\omega_n = \begin{bmatrix} (\Phi(x) - \Phi(\hat{x})) \end{bmatrix}u$.

The transfer matrix between the state estimation error $e_z$ and the unknown disturbance $\omega_n$ is:

$$
T_{e_z\omega_n}(s) = C_2(sI - (A - L_2 C))^{-1} D
$$

(8)

The $H_2$ observer design objectives are the following ones:

- The system (7) is stable for $\omega_n = 0$. 

• \( \| T_{e_{x\omega}}(s) \| \) is minimized for \( \omega_n \neq 0 \).

The following theorem solves the above problem into an LMI framework [11].

**Theorem 1:** Consider the system model (3) and the observer (4). If there exist a symmetric positive definite matrix \( P_2 \) and a matrix \( Y_2 \) minimizing \( Y_2 \) such that:

\[
\begin{pmatrix}
P_2A + A^TP_2 - Y_2C - (Y_2C)^T & P_2B & P_2D_1 - Y_2D_2 \\
B^TP_2 & -I & 0 \\
D_1^TP_2 + (Y_2D_2)^T & 0 & -I \\
\end{pmatrix} < 0
\]

\[
\begin{pmatrix}
P_2 & C_z^T \\
C_z & Y_2I \\
\end{pmatrix} > 0
\]

(9)

then, the observer gain \( L_2 \) determined from \( L_2 = P_2^{-1}Y_2 \) ensures that the objectives are attained.

### 3.2. \( H_\infty \) observer design

Similarly, to subsection 3.1, a \( H_\infty \) observer is designed to minimize the effect of \( \omega \) and \( (\Phi(x) - \Phi(\hat{x})).u \) on the state estimation errors \( e_z \).

From (6) and Remark 1, one obtains

\[
\begin{align*}
\dot{e} &= (A - L_\infty C)e + D\omega_n \\
e_z &= C_z e
\end{align*}
\]

(10)

where \( D = (B(D_1 - L_\infty D_2)) \), \( \omega_n = \left(\frac{(\Phi(x) - \Phi(\hat{x})).u}{\omega}\right) \).

The transfer function between the state estimation error \( e_z \) and the unknown disturbance \( \omega_n \) is:

\[
T_{e_{x\omega}}(s) = C_z(sI - (A - L_\infty C))^{-1}D
\]

(11)

The \( H_\infty \) observer design objectives are the following ones:

- The system (10) is stable for \( \omega_n = 0 \).
- \( \| T_{e_{x\omega}}(s) \|_\infty \) is minimized for \( \omega_n \neq 0 \).

The following theorem solves the above problem into an LMI framework [11].

**Theorem 2:** Consider the system model (3) and the observer (4). If there exist a symmetric positive definite matrix \( P_\infty \) and a matrix \( Y_\infty \) minimizing \( Y_\infty \) such that:

\[
\begin{pmatrix}
M & P_\infty B & P_\infty D_1 - Y_\infty D_2 \\
B^TP_\infty & -Y_\infty^2I & 0 \\
D_1^TP_\infty - (Y_\infty D_2)^T & 0 & -Y_\infty^2I \\
\end{pmatrix} < 0
\]

(12)

where \( M = P_\infty A + A^TP_\infty - Y_\infty C - (Y_\infty C)^T + C_z^TC_z \). Then, the observer gain \( L_\infty \) determined from \( L_\infty = P_\infty^{-1}Y_\infty \) ensures that the objectives are attained.

### 3.3. \( H_\infty \) Lipschitz observer design

In this section, the damping force is estimated through an \( H_\infty \) Lipschitz observer whose objectives are to minimize the effects of unknown road profile derivative and measurement noises on the estimation errors of the state variables and to account for the nonlinearity through a Lipschitz assumption.

Firstly, the control input function \( \Phi(x) \) of the system (3) can be rewritten under the following form

\[
\Phi(x) = \tanh(\Gamma x)
\]

(13)

where \( \Gamma = [k_1, c_1, 0, -c_1, 0] \).

Therefore, \( \Phi(x) \) satisfies the Lipschitz condition in \( x \)

\[
\| \Phi(x) - \Phi(\hat{x}) \| \leq \| \Gamma(x - \hat{x}) \|, \forall x, \hat{x}.
\]

(14)

From (6) and Remark 1, the estimation error dynamics of \( H_\infty \) Lipschitz observer is obtained

\[
\begin{align*}
\dot{e} &= (A - L_\infty C)e + B(\Phi(x) - \Phi(\hat{x})).u + (D_1 - L_\infty D_2)\omega \\
e_z &= C_z e
\end{align*}
\]

(15)

Assuming the Lipschitz condition (14) for \( \Phi(x) \), the \( H_\infty \) Lipschitz observer design objective is stated

- The system (15) is stable for \( \omega = 0 \).
- \( \| e_z(t) \|_{L_2} < \gamma_{\omega L}\|\omega(t)\|_{L_2} \) for \( \omega \neq 0 \).
The following theorem solves the above problem into an LMI framework [10].

**Theorem 3**: Consider the system model (3) and the observer (4). If there exist a symmetric positive definite matrix $P_{\omega L}$ and a matrix $Y_{\omega L}$ minimizing $y_{\omega L}$ such that:

\[
\begin{bmatrix}
\Omega & P_{\omega L}B & P_{\omega L}D_1 - Y_{\omega L}D_2 \\
* & -\varepsilon I_d & 0_n, d \\
* & * & -y_{\omega L}^2 I
\end{bmatrix} < 0
\] (16)

where $M = P_{\omega L}A + A^T P_{\omega L} - Y_{\omega L}C - (Y_{\omega L}C)^T + \varepsilon I_d^T + C_d^T C_d$. Then, the observer gain $L_{\omega L}$ determined from $L_{\omega L} = P_{\omega L}^{-1} Y_{\omega L}$ ensures that the objectives are attained.

4. Analysis of the observer design: frequency and time domain simulations

In this section, the synthesis results of the $H_2$, $H_\infty$ and $H_\infty$ Lipschitz observers are presented and some simulation results are provided.

4.1. Synthesis results and frequency domain analysis

Solving theorem 1, theorem 2 and theorem 3 leads the following solutions: the gains $y_2 = 3.1917$, $y_{\infty} = 1.4142$, $y_{\omega L} = 1.0032$ and the observer gains $L_2$, $L_\infty$, $L_{\omega L}$.

![Figure 2. Measurement noises attenuation](image1.png)

![Figure 3. Derivative road profile disturbance attenuation](image2.png)
The resulting attenuation of the sensor noises and derivative road profile disturbance on the estimation error are illustrated in Figures 2 and 3. These figures emphasize the attenuation level of the measurement noises and unknown road profile effect on the 4 estimation errors. The largest sensor noise and derivative road profile disturbance amplification of the 4 errors, over the whole frequency range ([1, 107] Hz for sensor noise and [0.1, 108] Hz for derivative road profile disturbance), are 13dB at 1 Hz and -17.6dB at 0.1 Hz, respectively.

4.2. Simulation
To evaluate the effectiveness of the proposed algorithms, the following simulation is carried out with the initial conditions of the observer states: \( \dot{x}_0 = [0.01, 0.1, 0.001, 0.1, 4]^T \) and 0 for the simulated system. The following simulation scenario is considered to compare the performance of the proposed observers: road profile is a sequence of sinusoidal bumps (2Hz) and the control input \( u \) is 0.2. The simulation results of three observers are shown in the Fig. 4.

5. Experimental validation
To experimentally assess and compare the effectiveness of the proposed methodologies, the proposed observers are implemented on the 1/5 car scaled car INOVE available at GIPSA-lab, shown in Fig. 5.

This test-bench is equipped with 4 semi-active ER suspensions, which is controlled in real-time using Matlab real-time workshop and a host computer. The three observer system is implemented with the sampling period \( T_s = 0.005s \). Note that the experimental platform is fully equipped with sensors to measure its vertical motion. At each corner of the system, a DC motor is used to generate the road profile.
Figure 6. Block diagram for the implementation of the observers.

The observers are applied on the rear-left corner using two accelerometers: the unsprung mass $\ddot{z}_{us}$ and the sprung mass $\ddot{z}_s$. For validation purpose only, the damper force sensor is used to compare the measured force with the estimated one. The following block-scheme illustrates the experimental scenario of the observer (shown in Fig. 6). The experimental scenario is as follows: the road profile is a sequence of sinusoidal bumps and the control input $u$ is constant ($u = 0.2$).

The experiment results of the observer are presented in Fig. 7. The result illustrates the accuracy and efficiency of the proposed observers. To further describe this accuracy, Table 1 shows the normalized root-mean square errors, considering the difference between the estimated and measured forces in the experiment tests.

![Figure 7. Experimental results of the observers.](image)

Table 1. Normalized Root-Mean-Square Errors (NRMSE).

| Observer          | NRMSE |
|-------------------|-------|
| $H_2$ Observer    | 0.1104|
| $H_\infty$ Observer | 0.1032|
| $H_\infty$ Lipschitz Observer | 0.0896|

6. Conclusion

This paper developed and compared several observers to estimate the damper force, using a dynamic nonlinear model of the ER damper. For this purpose, the quarter-car system is represented in state-space form by considering a phenomenological model of the damper. Based on two accelerometers, the observers are designed, giving a good estimation result of the damping force. The estimation error is minimized, accounting for the effect of unknown inputs (road profile disturbance and measurement noises) and the nonlinearity term bounded by a Lipschitz condition. Both simulation and experiment results assess the ability and the accuracy of the proposed models to estimate the damping force of the ER semi-active damper.

7. References

[1] S.M. Savaresi, C. Poussot-Vassal, C. Spelta, O. Sename, L. Dugard (2010). *Semi-active suspension control design for vehicles*. Elsevier.

[2] Poussot-Vassal, C., Spelta, C., Sename, O., Savaresi, S. M., & Dugard, L. (2012). Survey and performance evaluation on some automotive semi-active suspension control methods: A comparative study on a single-corner model. *Annual Reviews in Control, 36*(1), 148-160.
[3] Poussot-Vassal, C., Sename, O., Dugard, L., Gaspar, P., Szabo, Z., & Bokor, J. (2008). A new semi-active suspension control strategy through LPV technique. *Control Engineering Practice, 16*(12), 1519-1534.

[4] Priyandoko, G., Mailah, M., & Jamaluddin, H. (2009). Vehicle active suspension system using skyhook adaptive neuro active force control. *Mechanical systems and signal processing, 23*(3), 855-868.

[5] Koch, G., Kloiber, T., & Lohmann, B. (2010, December). Nonlinear and filter based estimation for vehicle suspension control. In *49th IEEE conference on decision and control (CDC)*(pp. 5592-5597). IEEE.

[6] Vela, A. E., Alcántara, D. H., Menendez, R. M., Sename, O., & Dugard, L. (2018). H∞ Observer for Damper Force in a Semi-Active Suspension. *IFAC-PapersOnLine, 51*(11), 764-769.

[7] Tudon-Martinez, J. C., Hernandez-Alcantara, D., Sename, O., Morales-Menéndez, R., & Lozoya-Santos, J. D. J. (2018). Parameter-Dependent H∞ Filter for LPV Semi-Active Suspension Systems. *IFAC-PapersOnLine, 51*(26), 19-24.

[8] Rajamani, R., & Hedrick, J. K. (1995). Adaptive observers for active automotive suspensions: theory and experiment. *IEEE Transactions on control systems technology, 3*(1), 86-93.

[9] Reichhartinger, M., Falkensteiner, R., & Horn, M. (2018). Robust Estimation of Forces for Suspension System Control. *IFAC-PapersOnLine, 51*(25), 328-333.

[10] Pham, T. P., Sename, O., & Dugard, L. (2019, June). Design and Experimental Validation of an H∞ Observer for Vehicle Damper Force Estimation. In *9th IFAC International Symposium on Advances in Automotive Control (AAC 2019)*.

[11] Scherer, C., & Weiland, S. (2000). Linear matrix inequalities in control. *Lecture Notes, Dutch Institute for Systems and Control, Delft, The Netherlands, 3*(2).