Two talks on a tentative theory of large distance physics

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Abstract

These talks present an overview of a tentative theory of large distance physics. For each large distance $L$ (in dimensionless units), the theory gives two complementary descriptions of spacetime physics: quantum field theory at distances larger than $L$, string scattering amplitudes at distances smaller than $L$. The mechanism of the theory is a certain 2d nonlinear model, the $\lambda$-model, whose target manifold is the manifold of general nonlinear models of the worldsurface, the background spacetimes for string scattering. So far, the theory has only been formulated and its basic working described, in general terms. The theory’s only claims to interest at present are matters of general principle. It is a self-contained nonperturbative theory of large distance physics, operating entirely at large distance. The $\lambda$-model constructs an actual QFT at large distance, a functional integral over spacetime fields. It constructs an effective background spacetime for string scattering at relatively small distances. It is background independent, dynamically. Nothing is adjustable in its formulation. It is a mechanical theory, not an S-matrix theory. String scattering takes place at small distances within a mechanical large distance environment. The $\lambda$-model constructs QFT in a way that offers possibilities of novel physical phenomena at large distances. The task now is to perform concrete calculations in the $\lambda$-model, to find out if it produces a physically useful QFT.
These talks present an overview of a tentative theory of large distance physics [1]. This written version of the talks differs from the actual spoken version in its arrangement and in some of the content. The transparencies that were used in the actual talks are available at [2]. References can be found in [1]. No references will be given here.

The need to produce QFT

My goal is a theoretical machinery capable of producing a definite, specific spacetime quantum field theory.

Everything, or almost everything, that is presently known about the laws of physics is summarized in the Standard Model of particle physics combined with General Relativity. General Relativity is a classical field theory, but it can just as well be regarded as a cutoff quantum field theory which we observe only in its large distance, classical regime. From this point of view, almost everything that is presently known about the laws of physics is summarized in one particular quantum field theory. An enormous body of experimental data is summarized in this amazingly successful QFT.

Since the establishment of the Standard Model, theoretical speculation has lacked the guidance and correction of a rapid flow of new experimental results. Speculative exploration of large theoretical search spaces with many free parameters could drag on forever without such guidance and correction. We urgently need a theory that is definite and specific. We need a theory that gives a definite and specific explanation of the Standard Model combined with General Relativity, so its reliability will be supported securely by the body of existing experimental knowledge.

To have a chance of explaining the Standard Model and General Relativity, a theory must first be capable of producing a QFT. It is not enough merely to identify a QFT, say by matching to an S-matrix. An S-matrix is not a mechanics. An actual QFT must be constructed. The theoretical machinery must produce a quantum mechanical space of states and a hamiltonian operator or, equivalently, a functional integral over spacetime fields, cut off perhaps at an unnoticeably small spacetime distance.

My strategy has been to look first for a well-defined theoretical machinery that is capable of producing a specific spacetime QFT, without assuming in advance that QFT operates. The spacetime fields of the QFT should include the spacetime metric, some gauge fields, and some chiral fermion fields. The dynamics should be generally covariant and gauge invariant. But otherwise I have postponed worrying about the details of the QFT that is produced. Until now, I have only been concerned with the problems of formulating a theory. The task now is to figure out what the theory does, and eventually to check whether it does in fact produce the Standard Model plus General Relativity, in all known details. If it fails to do so, then of course the theory is wrong.
The general nonlinear model

The theory is based on the renormalization of the 2d general nonlinear model, which was the subject of my first talk at Cargèse, as a student in the summer of 1979. The general nonlinear model is a functional integral over maps $x(z, \bar{z})$ from the plane to a given compact, riemannian target manifold. The action is

$$A(x) = \int \frac{d^2z}{2\pi} h_{\mu\nu}(x) \partial x^\mu \partial x^\nu$$

where $h_{\mu\nu}(x)$ is a positive definite, riemannian metric on the target manifold. In local coordinates on the target manifold, the map $x(z, \bar{z})$ is described by coordinate maps $x^\mu(z, \bar{z})$. Expanding the metric coupling in Taylor series, the action takes the perturbative form

$$A(x) = \int d^2z \frac{1}{2\pi} \left( h_{\mu\nu} \partial x^\mu \partial x^\nu + h_{\mu\nu,\sigma} x^\sigma \partial x^\mu \partial x^\nu + \cdots \right)$$

which describes interacting massless scalar fields $x^\mu(z, \bar{z})$. The matrix $h_{\mu\nu}$ governs the gaussian fluctuations around the origin of coordinates $x^\mu = 0$. The propagator is

$$\langle x^\mu(z_1, \bar{z}_1) x^\nu(z_2, \bar{z}_2) \rangle = -h^{\mu\nu} \ln(\mu^2 |z_1 - z_2|^2)$$

where $\mu^{-1}$ is a reference 2d distance. The higher Taylor series coefficients of the metric are the perturbative couplings of the model. Free massless scalar fields are dimensionless in 2d, so all the couplings have naive scaling dimension 0. The general nonlinear model is a renormalizable 2d field theory with infinitely many naively renormalizable coupling constants.

I called it the general nonlinear model because the metric $h_{\mu\nu}(x)$ was not assumed to have any special symmetries and the renormalization of the model was generally covariant in the target manifold, and because the model contained all possible naively renormalizable couplings of the dimensionless scalar fields $x^\sigma(z, \bar{z})$.

Renormalize the model at 2d distance $\mu^{-1}$. The renormalized metric coupling follows the renormalization group equation

$$\mu \frac{\partial}{\partial \mu} h_{\mu\nu}(x) = \beta_{\mu\nu}(x) = 2R_{\mu\nu}(x) + O(R^2)$$

where $R_{\mu\nu}(x)$ is the Ricci tensor of the metric. The model is scale invariant when $h_{\mu\nu}(x)$ satisfies the fixed point equation $\beta = 0$,

$$0 = 2R_{\mu\nu} + O(R^2)$$

The appearance of Einstein’s equation (without matter) in this formal 2d setting, apparently unrelated to spacetime physics, seemed potentially a clue to an explanation of the physical Einstein equation. It became possible to imagine that spacetime physics might
be explained by some theoretical structure based on the renormalization of the general nonlinear model. The target manifold would be spacetime. But the equation $\beta = 0$ was only the fixed point equation of the renormalization group flow in a 2d model. It had the form of the spacetime field equation, but it was not the equation of motion of a mechanical system.

The coupling constants $\lambda^i$

The renormalization group flow of the general nonlinear model exposed a direct relationship between the short distance properties of the 2d model and the large distance geometry of its target manifold. Take a particular scale invariant model to serve as reference point. The metric $h^\text{ref}_{\mu\nu}(x)$ of the reference model solves the fixed point equation $\beta = 0$. A nearby model has metric $h_{\mu\nu}(x) = h^\text{ref}_{\mu\nu}(x) + \delta h_{\mu\nu}(x)$. The scaling behavior of the perturbation $\delta h_{\mu\nu}(x)$ at short 2d distances is determined by the linearization of $\beta$, a covariant second order differential operator

$$\beta_{\mu\nu} = (-\nabla^\sigma \nabla_\sigma + \cdots) \delta h_{\mu\nu}(x) + O(\delta h^2).$$

The perturbation can be expanded in eigen-modes

$$\delta h_{\mu\nu}(x) = \lambda^i \delta_i h_{\mu\nu}(x)$$

$$(-\nabla^\sigma \nabla_\sigma + \cdots) \delta_i h_{\mu\nu}(x) = \gamma(i) \delta_i h_{\mu\nu}(x)$$

which form a discrete set because the target manifold was taken compact and riemannian. The summation convention is used for indices $i, j$ which range over the discrete set of wave modes. Each eigen-mode corresponds to a local 2d scaling field

$$\phi_i(z, \bar{z}) = \mu^{-2} \delta_i h_{\mu\nu}(x) \partial x^\mu \bar{x}^\nu$$

whose scaling dimension is $2 + \gamma(i)$. The eigenvalue $\gamma(i)$ is the anomalous dimension of $\phi_i(z, \bar{z})$. The perturbation of the action is

$$\delta A = \int \! d^2z \, \mu^2 \frac{1}{2\pi} \lambda^i \phi_i(z, \bar{z}).$$

The perturbed model is made by inserting

$$e^{-\delta A} = e^{-\int \! d^2z \, \mu^2 \frac{1}{2\pi} \lambda^i \phi_i(z, \bar{z})}$$

into the reference model. The wave modes $\lambda^i$ of the metric become the coupling constants that parametrize the 2d model. The renormalization group equation, written in terms of the $\lambda^i$, is

$$\mu \frac{\partial}{\partial \mu} \lambda^i = \beta^i(\lambda) = \gamma(i) \lambda^i + \cdots$$
so the coupling constant $\lambda^i$ has dimension $-\gamma(i)$. The spectrum of dimensions $-\gamma(i)$ is discrete and bounded above, because the target manifold is compact and riemannian.

Distances in the target manifold, which are to be spacetime distances, are pure numbers. Implicitly, there is a basic unit of spacetime distance. A distance is large when it is a large number in dimensionless units. The target manifold is macroscopic when it is very large in some of its dimensions, containing large regions where spacetime curvature and topology are negligible in the macroscopic dimensions. Locally in a macroscopic target manifold, the wave modes $\lambda^i$ are approximated by plane waves with spacetime wave vectors $p(i)$. The anomalous dimensions, being the eigenvalues of a second order differential operator, are essentially

$$\gamma(i) = p(i)^2.$$  \hfill (13)

The eigenvalues $\gamma(i)$ become tightly packed near 0, approximating a continuous spectrum indexed by the wave vectors $p(i)$.

The wave modes of the spacetime metric are the coupling constants $\lambda^i$ of the general nonlinear model. The anomalous dimensions $\gamma(i)$ of the 2d scaling fields are geometrical quantities in spacetime, essentially the squares of the spacetime wave vectors $p(i)$. A dictionary between spacetime physics and the general nonlinear model begins to take form.

**At short 2d distances $\Lambda^{-1}$**

From now on, the reference 2d distance $\mu^{-1}$ at which the general nonlinear model is renormalized will be held fixed. Everything will happen at much shorter 2d distances $\Lambda^{-1}$, where the renormalized model is parametrized by running coupling constants $\lambda^i_r$,

$$e^{-\int d^2z \mu^2 \frac{1}{2\pi} \lambda^i_r \phi(z,\bar{z})} = e^{-\int d^2z \Lambda^2 \frac{1}{2\pi} \lambda^i \phi^L(z,\bar{z})}.$$ \hfill (14)

The $\phi^L(z,\bar{z})$ are the scaling fields normalized at the short 2d distance. The $\lambda^i_r$ run with $\Lambda^{-1}$ according to the renormalization group equation

$$\Lambda \frac{\partial}{\partial \Lambda} \lambda^i_r = \beta^i(\lambda_r) = \gamma(i) \lambda^i_r + \cdots$$ \hfill (15)

whose integrated flow

$$\lambda^i_r = (\Lambda/\mu)^{\gamma(i)} \lambda^i + \cdots$$ \hfill (16)

when expressed in terms of the large number

$$L^2 = \ln(\Lambda/\mu)$$ \hfill (17)

shows the exponential suppression

$$\lambda^i = e^{-L^2\gamma(i)} \lambda^i_r = e^{-L^2p(i)^2} \lambda^i_r.$$ \hfill (18)
When $\gamma(i)$ is positive and large on the scale set by $L^{-2}$, the short distance coupling constant $\lambda^i_r$ is irrelevant. It has no significant effects in the renormalized model. The corresponding $\lambda^i$ has dimension $-\gamma(i)$ which is significantly negative on the scale set by $L^{-2}$. Such coupling constants $\lambda^i$ are non-renormalizable. The non-renormalizable coupling constants play no role in the renormalized model. The renormalization puts them to zero. Only the renormalizable coupling constants matter in the renormalized model, whose dimensions $-\gamma(i)$ are either non-negative or only slightly negative, the latter meaning close to zero on the scale set by $L^{-2}$. All significant perturbations of the renormalized model at the short 2d distance $\Lambda^{-1}$ are parametrized by the renormalizable $\lambda^i$, or, equivalently, by the $\lambda^i_r$ modulo the irrelevant $\lambda^i_r$.

To be specific, say that $\lambda^i_r$ is irrelevant if $e^{-L^2\gamma(i)} < e^{-400}$, the number 400 being more or less arbitrary. This is equivalent to $\gamma(i) > 400/L^2$ or $p(i) > 20/L$. Then the renormalizable coupling constants $\lambda^i$ are those with $\gamma(i) \leq 400/L^2$, which is $p(i) \leq 20/L$. The renormalizable coupling constants at the short 2d distance $\Lambda^{-1}$ are the spacetime wave modes at spacetime distances larger than $L$. The number $L$ acts as an ultraviolet cutoff distance in spacetime. It is a strong UV cutoff in the sense that the spacetime wave modes at distances smaller than $L$ are decoupled from the renormalized 2d model. There is no significant dependence on the irrelevant $\lambda^i_r$. The decoupling is accomplished in the renormalization of the model, when the non-renormalizable $\lambda^i$ are forced to 0.

The large distance structure of spacetime is now translated into the short distance structure of the 2d model. Specifically, the general nonlinear model at 2d distances shorter than $\Lambda^{-1}$ represents the mechanical structure of spacetime at distances larger than $L$.

The distinction between renormalizable and non-renormalizable coupling constants is familiar. In a typical model, all the negative dimension coupling constants are non-renormalizable. Once the ratio $\Lambda/\mu$ is sufficiently huge, it becomes effectively infinite. Only the zero and positive dimension coupling constants are renormalizable. All effects of irrelevant coupling constants at short distance are absorbed into shifts of the renormalizable coupling constants. The renormalized model is parametrized entirely by the fixed set of renormalizable coupling constants. The general nonlinear model is unusual in that its spectrum of scaling dimensions $-\gamma(i)$ can crowd arbitrarily close to zero when its target manifold becomes arbitrarily large. The set of renormalizable coupling constants depends on the ratio $\Lambda/\mu$, no matter how huge that ratio becomes.

### The manifold of general nonlinear models

The renormalizable coupling constants $\lambda^i$ serve as local coordinates parametrizing the renormalized models in the vicinity of the reference model. The collection of all such local coordinate systems defines the manifold of renormalized general nonlinear models, $M(L)$. It depends on $L$ because the set of renormalizable $\lambda^i$ depends on $L$. Locally, $M(L)$ is the manifold of spacetime fields at spacetime distances larger than $L$. Locally, $M(L)$ is finite dimensional, because the spectrum of $\gamma(i)$ is discrete and bounded below, which is
because the target manifold, spacetime, is compact and riemannian. But the dimension of $M(L)$ grows without bound when the spacetime becomes large.

In the idealized limit $\Lambda^{-1} = 0, L = \infty$, the only coupling constants that survive are the relevant coupling constants (positive dimension) and the marginal coupling constants (zero dimension). The marginal coupling constants are the $\lambda^i$ that preserve scale invariance, at least infinitesimally. They parametrize the manifold $M(\infty)$ of scale invariant models. The marginal $\lambda^i$ are the zero modes, $\gamma(i) = 0$, in spacetime.

**The general nonlinear model in string theory**

The general nonlinear model found use in string theory, to construct the string worldsurface in a curved background spacetime. The general nonlinear model provides the local structure of the worldsurface. The target manifold is the background spacetime in which strings scatter. String theory uses somewhat elaborate versions of the general nonlinear model, incorporating 2d supersymmetry and 2d chiral asymmetry. The couplings of the model are the spacetime metric and also some additional spacetime fields, which include spacetime gauge fields, fermionic spacetime spinor fields and more. The $\lambda^i$ are the wave modes of this entire collection of spacetime fields, all the coupling constants of the general nonlinear model of the worldsurface.

The perturbative algorithm for calculating the string S-matrix needs the worldsurface to be scale invariant. The equation $\beta = 0$ again looks like a spacetime field equation, but it is still not the equation of motion of a mechanical system. It is only a consistency condition on the background spacetime. At low momenta and energies, the perturbative string S-matrix can be identified with the perturbative S-matrix of a spacetime QFT. The classical field equation of the QFT can be identified with the consistency condition $\beta = 0$. These correspondences strongly suggested that there should be a mechanical means of producing an actual QFT with $\beta = 0$ as its equation of motion, and that the means of production of the QFT should be related to the construction of the background spacetime for string scattering. To find such a means of producing an actual QFT has been an urgent problem for many years.

Take the heterotic string theory, to be specific. $M(L)$ is now the manifold of general nonlinear models of the heterotic worldsurface. The $\lambda^i$ are the coupling constants of the heterotic worldsurface (after GSO projection), which are the spacetime wave modes of the spacetime metric, gauge fields, scalar fields, antisymmetric tensor field, and chiral fermion fields. Some of the $\lambda^i$ are fermionic, so $M(L)$ is now a graded manifold. The absence of tachyons means that $\gamma(i) \geq 0$ for all $\lambda^i$. There are no relevant coupling constants. The renormalization group flows inward to the manifold $M(\infty)$ of exactly scale invariant models, the exact solutions of $\beta = 0$. $M(\infty)$ is essentially the manifold of 10d compact Calabi-Yau target spaces (with some additional geometric structure). These are the consistent background spacetimes for heterotic string theory.

Spacetime is kept compact and riemannian in order to keep the 2d model under control.
The issue of analytic continuation to real time is put off indefinitely. The continuation can certainly be done in some extreme limits of 10d compact riemannian spacetimes, where at least one dimension becomes infinitely large.

**The infrared failure of string theory**

String theory has failed as physics because of the manifold of consistent background spacetimes. String theory can make no definite predictions, because the background spacetime depends on continuously variable free parameters which have to be fixed by hand. Moreover, the existence of a manifold of possible background spacetimes implies that the low energy string modes are exactly massless particles. Few, if any, real particle masses are exactly zero. One of the most urgent problems in physics is to explain the precise small nonzero values of the particle masses in the Standard Model.

My purpose in pointing to the failure of string theory is diagnostic. The free parameters and the massless particles are signs of pathology, calling for a remedy. The pathology is in the infrared, its symptoms being the zero modes of the spacetime fields. The infrared properties of string theory are encoded in the background spacetimes. Something is wrong or missing in the specification of the possible background spacetimes. The need to select a background spacetime is the basic sign of deficiency. The background spacetime should not have to be selected by hand, from among a manifold of consistent possibilities. It should be produced mechanically. A mechanism is missing that constructs the background spacetime. One might hope that such a mechanism will remedy the infrared failure of string theory.

**Every handle diverges logarithmically**

Consider a handle in the worldsurface, as pictured in figure 1. The handle is made by cutting two disks of radius \( r \) from the worldsurface, then gluing the two boundary circles to each other. The positions \( z_1 \) and \( z_2 \) of the holes, and the radius \( r \), are integration...
variables in the formula for string scattering amplitudes. The effect of the handle is to make a non-local insertion in the worldsurface,

$$\frac{1}{2} \int d^2 z_1 \mu_2^2 \frac{1}{2\pi} \phi_i(z_1, \bar{z}_1) \int d^2 z_2 \mu_2^2 \frac{1}{2\pi} \phi_j(z_2, \bar{z}_2) \int \frac{2 d(\mu r) (\mu r)^{\gamma(i)+\gamma(j)-1}}{\mu \Lambda^{-1}} T g^{ij}$$

(19)

which expresses the handle as a sum over the string states $i, j$ flowing through the two ends. The state $i$ flowing through the end at $z_1$ appears in the worldsurface as the local field $\phi_i(z_1, \bar{z}_1)$. The state $j$ appears as $\phi_j(z_2, \bar{z}_2)$. Each pair of states $i, j$ is weighted by a gluing matrix element $T g^{ij}$. The form of the integral is dictated by 2d scale invariance. The 2d distance $\Lambda^{-1}$ acts as short distance cutoff on $r$. The handle gluing matrix $T g^{ij}$ is the inverse of the metric

$$T^{-1} g_{ij} = Z \langle \phi_i(z_1, \bar{z}_1) \phi_j(z_2, \bar{z}_2) \rangle$$

(20)

which is the unnormalized two point function of the scaling fields, evaluated at the reference distance $|z_1 - z_2| = \mu^{-1}$ (on the complex plane compactified to form the 2-sphere). The number $T^{-1}$ is the normalizing factor $Z \langle 1 \rangle$, the partition function of the 2-sphere. In a macroscopic spacetime of volume $V$, the number $T$ is related to the string coupling constant $g_s$ by

$$T^{-1} = g_s^{-2} V.$$  

(21)

The factor $V$ comes from the zero mode in the worldsurface functional integral over $x^\mu(z, \bar{z})$. The combination $V g_{ij}$ is properly normalized to be the usual continuum inner product on the spacetime wave modes.

The kernel of the non-local insertion, is the factor in equation 19,

$$\int \frac{2 d(\mu r) (\mu r)^{2\gamma(i)-1}}{\mu \Lambda^{-1}} T g^{ij}$$

(22)

where $\gamma(i) + \gamma(j)$ is replaced by $2\gamma(i)$, because $T g^{ij} = 0$ unless $\gamma(i) = \gamma(j)$ (another consequence of 2d scale invariance). The cutoff dependent part of the kernel is

$$T g^{ij} \left[ (\mu \Lambda_1^{-1})^{2\gamma(i)} - (\mu \Lambda^{-1})^{2\gamma(i)} \right]$$

(23)

where $\Lambda_1^{-1}$ is a second 2d distance, chosen arbitrarily, independent of $\Lambda^{-1}$, to be the upper limit in the integral over $r$.

The handle is logarithmically divergent in $\Lambda^{-1}$ if and only if there is at least one worldsurface field $\phi_i(z, \bar{z})$ with $\gamma(i) = 0$. The infrared failure of string theory, due to the existence of marginal coupling constants $\lambda^i$, now appears as a technical pathology of the worldsurface, a logarithmic divergence in every string channel due to the discrete zero modes flowing through every handle.
Again, the spectrum is discrete because spacetime is compact and riemannian. The target manifold of the general nonlinear model was originally taken to be compact and riemannian so that the 2d model would be strictly well defined. The action was then bounded below and the fluctuating field $x^\mu(z, \bar{z})$ could not wander off uncontrollably in the target manifold. The motivation is essentially the same now. The background spacetime is taken to be compact and riemannian in order to control the spacetime infrared behavior of the string worldsurface, to be sure that nothing is missed in the spacetime infrared.

**Potentially realistic string scattering**

When $\gamma(i)$ is small, the kernel of the handle insertion becomes

$$
T g^{ij} \left[ \frac{1 - e^{-2L^2\gamma(i)}}{\gamma(i)} \right]
$$

which is the string propagator, cutoff in the spacetime infrared at distance $L$. The large distance modes are cut off in the infrared at $p(i)^2 \approx L^{-2}$. This is the usual demonstration of the well known principle that a worldsurface short distance cutoff is equivalent to an infrared spacetime cutoff. The infrared cutoff acts systematically, in every string channel.

When the worldsurface is cut off at short 2d distance $\Lambda^{-1}$, the manifold of consistent background spacetimes is the manifold $M(L)$. It is not necessary to satisfy $\beta = 0$ exactly. The non-renormalizable coupling constants $\lambda^i$ are zero in $M(L)$, so the equation $\beta = 0$ is satisfied at spacetime distances smaller than $L$, which is all that is needed for consistent string scattering with infrared cutoff $L$. The spacetime wave modes at distances greater than $L$ can have $\beta \neq 0$, so they can act as sources and detectors for strings. They describe the physical environment, including the experimental apparatus and the observers.

Spacetime can be pictured as in figure 2, tiled by cells of size $L$, which are experimental regions for string scattering. The number $L$ now plays a double role. $L$ is the IR cutoff distance imposed systematically on string scattering. $L$ is also the UV cutoff distance in the background spacetime. The background spacetime is specified by the spacetime fields at distances larger than $L$. String scattering takes place within the background spacetime, at distances smaller than $L$. 

![Figure 2: Experimental regions of size $L$.](image)
This is a potentially realistic version of string scattering, in the sense that the scattering takes place at relatively small distance in a real physical environment. But the large distance mechanical environment is still described only by classical fields. The equation $\beta = 0$ is still only a consistency condition. It is now the tree-level condition for extending the string scattering amplitudes to spacetime distances larger than $L$. Beyond the tree-level approximation, $\beta = 0$ is not sufficient. The cutoff dependence of the handles gets in the way.

**Local handles**

The logarithmic divergence is interpreted as symptom of a deficiency in the specification of the background spacetime, a deficiency in the construction of the string worldsurface. A new mechanism is to be added to the worldsurface, designed to cancel the logarithmic divergence. But we will not try to remove the divergence in every string channel. We have seen that the large distance structure of the background spacetime is encoded in the short distance, local structure of the general nonlinear model. We limit attention, therefore, to the *local handles*, the handles that are local in the worldsurface. These are the handles that connect a local region of the worldsurface to itself. Both endpoints lie within the same local neighborhood, as in figure 3. The cutoff dependent effects of local handles contribute to the local structure of the worldsurface, and depend only on the local structure of the worldsurface. They contribute to the large distance structure of the background spacetime, and depend only on the large distance structure of the background spacetime. In limiting attention to the local handles, we keep the large distance physics, which is the physics of the background spacetime, independent from the small distance physics of string scattering. The principle of 2d locality is used to accomplish the separation systematically. A theory of large distance physics that does not depend on small distance physics is attractive because it seems unlikely that a theory of small distance physics could be reliable.

The strategy is to add a new local mechanism at short distance on the worldsurface, designed precisely to cancel the divergent effects of the local handles. By acting at short 2d distance, the new mechanism acts on the large distance structure of spacetime. Two dimensional locality is preserved, so the formulas for calculating string scattering ampli-
tudes will remain consistent at small spacetime distance even after the new mechanism is added to the worldsurface.

The cutoff dependence of a local handle can be extracted naturally, using the separation $|z_1 - z_2|$ between the two endpoints in place of the arbitrary second 2d distance $\Lambda^{-1}$ of equation \text{#23}. The cutoff dependent effect of the local handle is the insertion

$$
\frac{1}{2} \int d^2z_1 \mu^2 \frac{1}{2\pi} \phi_i(z_1, \bar{z}_1) \int d^2z_2 \mu^2 \frac{1}{2\pi} \phi_j(z_2, \bar{z}_2)
$$

$$
T g^{ij} \left[ (\mu |z_1 - z_2|)^{2\gamma(i)} - (\mu\Lambda^{-1})^{2\gamma(i)} \right].
$$

(25)

It is bi-local rather than non-local, in the sense that the insertion points $z_1$ and $z_2$ range over the same local neighborhood of the worldsurface.

**Cancel the cutoff dependence of local handles**

To cancel the cutoff dependence of the local handles, insert sources

$$
e^{-\int d^2z \mu^2 \frac{1}{2\pi} \lambda^i(z, \bar{z}) \phi_i(z, \bar{z})}
$$

(26)

in the worldsurface, then set them fluctuating with gaussian propagator

$$
\langle \lambda^i(z_1, \bar{z}_1), \lambda^j(z_2, \bar{z}_2) \rangle = T g^{ij} (\mu\Lambda^{-1})^{2\gamma(i)} \left[ \frac{1 - (\Lambda |z_1 - z_2|)^{2\gamma(i)}}{\gamma(i)} \right]
$$

(27)

which is the negative of the kernel in equation \text{#23}. The sources inserted in the worldsurface, when contracted with the $\lambda$-propagator, make bi-local insertions that cancel the cutoff dependence of the gas of local handles, in the independent handle approximation. Interactions will have to be introduced to account for handle collisions.

Notice that the $\lambda$-fluctuations take place at 2d distances $|z_1 - z_2| < \Lambda^{-1}$, at the 2d distances where the $\lambda$-propagator is positive. The new mechanism is acting at 2d distances shorter than the characteristic distance $\Lambda^{-1}$. On the other hand, the worldsurface is constructed at 2d distances longer than $\Lambda^{-1}$. The local handles make insertions at 2d distances $|z_1 - z_2| > \Lambda^{-1}$, at the 2d distances where the kernel in equation \text{#23} is positive.

Notice also that the insertions of the irrelevant scaling fields $\phi_i(z, \bar{z})$ are suppressed by the appropriate factor $\left(\mu/\Lambda\right)^{\gamma(i)}$. There is no need for the non-renormalizable $\lambda^i$ to fluctuate. Only the renormalizable $\lambda^i$ have to be turned into fluctuating local sources $\lambda^i(z, \bar{z})$. Thus only a finite number of sources $\lambda^i(z, \bar{z})$ are needed to fluctuate at every short 2d distance $\Lambda^{-1}$.

The $\lambda$-propagator at 2d distance $|z_1 - z_2| \approx \Lambda^{-1}$ is

$$
\langle \lambda^i(z_1, \bar{z}_1) \lambda^j(z_2, \bar{z}_2) \rangle \approx -T g^{ij} (\mu\Lambda^{-1})^{2\gamma(i)} \ln(\Lambda^2 |z_1 - z_2|^2)
$$

(28)
which is the propagator of massless scalar fields governed by a metric \((\Lambda \mu^{-1})^{2}g_{ij}\) that depends on the 2d distance \(\Lambda^{-1}\). After changing variables to the running coupling constants \(\lambda^{i}_{r} = (\Lambda / \mu)^{\gamma(i)} \lambda^{i}\),

\[\lambda^{i}_{r}(z, \bar{z}) = (\Lambda \mu^{-1})^{\gamma(i)} \lambda^{i}(z, \bar{z})\]  

(29)

the gaussian fluctuations of the \(\lambda^{i}_{r}(z, \bar{z})\) are governed by the same metric \(T^{-1}g_{ij}\) at every 2d distance \(\Lambda^{-1}\). They are produced by a functional integral over the \(\lambda^{i}_{r}(z, \bar{z})\) with action

\[\int d^{2}z \frac{1}{2\pi} T^{-1}g_{ij} \partial \lambda^{i}_{r} \bar{\partial} \lambda^{j}_{r}\]  

(31)

which has the same form at every 2d distance \(\Lambda^{-1}\). The gaussian \(\lambda\)-fluctuations are therefore invariant under change of the 2d distance \(\Lambda^{-1}\), when the change of scale is accompanied by a change of the variables \(\lambda^{i}_{r}(z, \bar{z})\), keeping the \(\lambda^{i}_{r}(z, \bar{z})\) constant.

**The \(\lambda\)-model**

Interactions have to be added to the gaussian action in order to account for collisions among the local handles. The form of the interactions is constrained by 2d scale invariance and locality. The fields \(\lambda^{i}_{r}(z, \bar{z})\) are dimensionless massless scalar fields. The most general dimensionless local interactions are products of an arbitrary number of fields, with exactly two derivatives. The action must have the form

\[S(\lambda_{r}) = \int d^{2}z \frac{1}{2\pi} \left( T^{-1}g_{ij} \partial \lambda^{i}_{r} \bar{\partial} \lambda^{j}_{r} + T^{-1}g_{ij,k} \lambda^{k} \partial \lambda^{i}_{r} \bar{\partial} \lambda^{j}_{r} + \cdots \right)\]  

(32)

which is that of a 2d nonlinear model.

The reference background spacetime loses its significance once the \(\lambda^{i}_{r}(z, \bar{z})\) are set fluctuating. Each local 2d region can be described just as well by fluctuations around a nearby background spacetime \(\lambda_{r}\). The gaussian fluctuations around \(\lambda_{r}\) will cancel independent local handles in \(\lambda_{r}\). So the gaussian fluctuations around \(\lambda_{r}\) have to be governed by the metric in \(\lambda_{r}\), \(T^{-1}g_{ij}(\lambda_{r})\). The interactions are determined completely, given the gaussian fluctuations around every \(\lambda_{r}\). The action has to be

\[S(\lambda_{r}) = \int d^{2}z \frac{1}{2\pi} T^{-1}g_{ij}(\lambda_{r}) \partial \lambda^{i}_{r} \bar{\partial} \lambda^{j}_{r}\]  

(33)

This argument avoids direct calculation in the gas of local handles. The \(\lambda\)-model is the functional integral

\[\int D\lambda_{r} e^{-S(\lambda_{r})} e^{-\int d^{2}z \Lambda^{2} \lambda^{i}_{r}(z, \bar{z}) \phi^{i}_{r}(z, \bar{z})}\]  

(34)

It is a 2d nonlinear model whose target manifold is \(M(L)\), the manifold of general nonlinear models of the worldsurface. The local sources \(\lambda^{i}_{r}(z, \bar{z})\) are the component fields of a map
The $\lambda$-model is a functional integral over the maps $\lambda_r(z, \bar{z})$ from the worldsurface to $M(L)$. It is inserted at short distance in the worldsurface to cancel the cutoff dependence of the local handles. The metric coupling $T^{-1}g_{ij}(\lambda_r)$ is a natural mathematical object determined by the local properties of the worldsurface. It is the inverse of the gluing matrix of a local handle. Equivalently, it is the unnormalized two point function of the short distance fields $\phi_i^\lambda(z, \bar{z})$, evaluated at the short 2d distance $\Lambda^{-1}$. The coupling strength of the $\lambda$-model is the string coupling constant, since $T$ is proportional to $g_s^2$.

The $\lambda$-model is scale invariant in the generalized sense. A change of $\Lambda^{-1}$ is merely equivalent to a change of variables in the functional integral. The action, written in terms of $\lambda_r(z, \bar{z})$, has the same form at every 2d distance $\Lambda^{-1}$. When written in terms of $\lambda(z, \bar{z})$, the action is invariant under an infinitesimal 2d scaling $\Lambda^{-1} \to (1 + \epsilon) \Lambda^{-1}$ when combined with the infinitesimal renormalization group transformation of the target manifold $M(L)$, $\lambda^i \to \lambda^i + \epsilon \beta^i(\lambda_r)$, which is only a change of variables in the functional integral defining the $\lambda$-model. The field $\lambda_r(z, \bar{z})$ is the natural variable in which to describe the fluctuations at the particular 2d distance $\Lambda^{-1}$. The field $\lambda(z, \bar{z})$ is the natural variable in which to describe the accumulation of fluctuations over a range of 2d distances $\Lambda^{-1}$, because the sources $\lambda^i(z, \bar{z})$ couple to fields $\phi_i(z, \bar{z})$ which are independent of $\Lambda^{-1}$.

The $\lambda$-model is a somewhat peculiar nonlinear model in that its target manifold $M(L)$ changes with the 2d distance $\Lambda^{-1}$, depending on $L^2 = \ln(\Lambda/\mu)$. The target manifold is parametrized by the coupling constants $\lambda^i$ that are renormalizable at 2d distance $\Lambda^{-1}$, which are the spacetime wave modes at spacetime distances larger than $L$. Equivalently, the target manifold $M(L)$ is parametrized by the running coupling constants $\lambda^i_\nu$, modulo the irrelevant coupling constants, which are the wave modes at distances smaller than $L$. The dependence of the target manifold on $\Lambda^{-1}$ is unfamiliar, but poses no essential difficulty. The target manifold can be held fixed over any range of 2d distances, because extra irrelevant $\lambda^i_\nu$, at distances somewhat smaller than $L$, can be included without noticeable effect. They are decoupled from the renormalized worldsurface. The dependence of the target manifold on $\Lambda^{-1}$ arranges itself automatically.

The a priori measure

A 2d statistical model, such as the $\lambda$-model, is made by accumulating fluctuations at longer and longer distances, starting at a very short cutoff distance $\Lambda_0^{-1} \approx 0$ and proceeding up to the characteristic distance $\Lambda^{-1}$. Two pieces of data characterize the model at each 2d distance $\Lambda^{-1}$. The metric coupling $T^{-1}g_{ij}(\lambda_r)$ governs the fluctuations at distances near $\Lambda^{-1}$. The a priori measure $d\rho_r(\Lambda, \lambda_r)$ is the net distribution of all fluctuations that have already taken place, at 2d distances up to $\Lambda^{-1}$. The a priori measure is a measure on the target manifold $M(L)$.

The a priori measure is a familiar construct of statistical mechanics. In the 2d Ising model, for example, the target manifold consists of two points, the two possible phases of each domain. The characteristic 2d distance $\Lambda^{-1}$ is the lattice spacing. The action
governs the correlations between nearby domains. The *a priori* measure is the statistical weight on the two possible phases at each site. The *a priori* measure summarizes the distribution of phase domains at 2d distances shorter than the lattice spacing. In models with internal symmetries, such as the Ising model, the *a priori* measure can be determined uniquely by the symmetry. In general, however, the *a priori* measure is produced by the accumulation of fluctuations over the range of short 2d distances up to $\Lambda^{-1}$. The *a priori* measure evolves under the renormalization group of the model as fluctuations at longer and longer 2d distances are included.

The *a priori* measure is determined by its expectation values, which are calculated as the one point expectation values in the $\lambda$-model,

$$\int_{M(L)} f(\lambda_r) \, d\rho_r(\Lambda, \lambda_r) = \langle f \rangle = \langle f(\lambda_r(z, \bar{z})) \rangle, \quad (35)$$

including the $\lambda$-fluctuations at 2d distances up to $\Lambda^{-1}$. The renormalization group of the $\lambda$-model acts as a diffusion process,

$$-\Lambda \frac{\partial}{\partial \Lambda} d\rho_r = \nabla_i \left[ T \, g^{ij}(\lambda_r) \nabla_j + \beta^i(\lambda_r) \right] d\rho_r \quad (36)$$

with diffusion “time” being the logarithm of the 2d distance. The measure diffuses in $M(L)$ because more and more fluctuations are included as $\Lambda^{-1}$ increases. At the same time, $M(L)$ flows under the renormalization group of the general nonlinear model, so the diffusion is driven inward along the vector field $-\beta^i(\lambda_r)$ towards the $\beta = 0$ manifold.

At tree-level, the coefficients of the diffusion process are the metric coupling $T^{-1}g^{ij}(\lambda_r)$ and the worldsurface $\beta$-function $\beta^i(\lambda_r)$. These are constant in “time,” because the $\lambda$-model is classically scale invariant, in the generalized sense. A diffusion process with constant coefficients tends to equilibrium. The *a priori* measure will diffuse to an equilibrium measure under the renormalization group of the $\lambda$-model. The conditions that determine the *a priori* measure are the conditions of equilibrium, rather than symmetry conditions. The equilibrium measure can be determined explicitly, because $\beta^i(\lambda_r)$ is the gradient of a potential function $T^{-1}a(\lambda_r)$ on $M(L)$,

$$\beta^i(\lambda_r) = T \, g^{ij} \, \partial_j \left( T^{-1}a(\lambda_r) \right) \quad (37)$$

When a diffusion process is driven by a gradient flow, the equilibrium measure solves the first order equation

$$0 = \left[ T \, g^{ij} \nabla_j + \beta^i \right] d\rho_r \quad (38)$$

whose solution is

$$d\rho_r(\lambda_r) = e^{-T^{-1}a(\lambda_r)} \, d\text{vol}(\lambda_r) \quad (39)$$

where $d\text{vol}(\lambda_r)$ is the metric volume element.
The \textit{a priori} measure of the $\lambda$-model is a measure on the spacetime wave modes $\lambda^i$, a functional integral over the spacetime fields. It is a quantum field theory in spacetime. Its equation of motion is $\beta = 0$, as expressed by the equilibrium condition, equation 38. Its action is the potential function $T^{-1}a(\lambda, r)$, as shown in equation 39. This QFT governs the large distance physics in spacetime, because the \textit{a priori} measure of the $\lambda$-model governs the short distance structure of the worldsurface. Thus the $\lambda$-model mechanically produces a QFT with equation of motion $\beta = 0$, which governs the large distance physics in spacetime.

The expectation values $\langle \lambda^i \lambda^j \cdots \rangle$ in the \textit{a priori} measure are the correlation functions of the QFT. Each index $i$ represents a spacetime wave vector $p(i)$, along with whatever other properties label the spacetime wave mode $\lambda^i$, so the expectation values in the \textit{a priori} measure are the momentum space correlation functions of the spacetime QFT. For example, at tree level,

$$\langle \lambda^i \lambda^j \rangle = \langle \lambda^i_r(z_1, \bar{z}_1) \lambda^j_r(z_2, \bar{z}_2) \rangle_{z_1 = z_2}$$

$$= Tg^{ij} \left[ 1 - \frac{\Lambda |z_1 - z_2|^2 \gamma(i)}{\gamma(i)} \right]_{z_1 = z_2}$$

$$= Tg^{ij} \frac{1}{\gamma(i)}$$

which is the tree level spacetime propagator. The expectation values in the \textit{a priori} measure can be transcribed directly into spacetime correlation functions by summing over the spacetime wave modes, as in

$$\langle h_{\mu_1 \nu_1}(x_1) h_{\mu_2 \nu_2}(x_2) \rangle = \langle \lambda^i \lambda^j \rangle \delta_{i_1, \mu_1 \nu_1} \delta_{i_2, \mu_2 \nu_2}(x_1 \mu_2 \nu_2) \delta(x_2).$$

All of this structure carries over from the tree-level approximation to the full quantum $\lambda$-model. The fluctuations in the $\lambda$-model correct the metric coupling $T^{-1}g_{ij}$ and the $\beta$-function $\beta^i$, producing an effective metric coupling $T^{-1}g^{\text{eff}}_{ij}$ and an effective $\beta$-function $\beta^{\text{eff}}_i$. I argue in \cite{1} that the quantum $\lambda$-model is scale invariant, in the generalized sense, and that $\beta^{\text{eff}}_i$ is the gradient of a potential function $T^{-1}a^{\text{eff}}$. The full \textit{a priori} measure of the quantum $\lambda$-model is generated by the diffusion process whose constant coefficients are $T^{-1}g^{\text{eff}}_{ij}$ and $\beta^{\text{eff}}_i$. The actual equilibrium \textit{a priori} measure is a QFT whose action is $T^{-1}a^{\text{eff}}$ and whose equation of motion is $\beta^{\text{eff}} = 0$.

**Nonperturbative 2d effects in the $\lambda$-model?**

The $\lambda$-model is merely a somewhat elaborate 2d nonlinear model. It seems reasonable to presume that it can be constructed nonperturbatively. Nonperturbative effects in the $\lambda$-model would be nonperturbative in the string coupling constant $g_s$, because $T \propto g_s^2$. 

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Nonperturbative 2d effects in the $\lambda$-model might make useful corrections to the spacetime equation of motion $\beta_{\text{eff}} = 0$ and to the spacetime action $T^{-1}a_{\text{eff}}$. The most obvious source of nonperturbative 2d effects are the nontrivial classical solutions of the 2d equation of motion

$$0 = \partial \left( T^{-1} g_{ij}(\lambda_r) \bar{\partial} \lambda^i \right).$$

These are the harmonic surfaces in the manifold of spacetime fields. Solutions on the 2-sphere would be 2d instantons in the $\lambda$-model. Harmonic surfaces passing through singular points in $M(L)$ would appear as defects in the worldsurface. Condensation of such 2d defects in the $\lambda$-model might have interesting consequences, among which perhaps making spacetime macroscopic and dynamically inducing spacetime topology change. The crucial immediate question is whether there are 2d nonperturbative effects in the $\lambda$-model can eliminate the continuous degeneracy in the manifold of background spacetimes, violate spacetime supersymmetry, and produce realistic small particle masses.

**General covariance and gauge invariance**

The renormalization of the general nonlinear model is generally covariant in its target manifold, which is spacetime. The reparametrizations of spacetime act on the wave modes of the spacetime fields, which are the coupling constants $\lambda^i$. The renormalization of the general nonlinear model is invariant, as long as no anomalies intervene. The renormalization is similarly invariant under spacetime gauge transformations.

In the $\lambda$-model, the reparametrizations of spacetime and the spacetime gauge transformations become symmetries of its target manifold, $M(L)$. These symmetries of $M(L)$ are made into local 2d gauge symmetries in the $\lambda$-model, by introducing auxiliary spin-1 $\lambda$-fields coupled to the spin-1 worldsurface scaling fields, which are the generators of the spacetime gauge transformations. The renormalization of the $\lambda$-model needs to preserve these symmetries of its target manifold. In the general nonlinear model, the renormalization of special symmetries of the target manifold followed from the general covariance of the renormalization. In the $\lambda$-model, the renormalization of the symmetries of its target manifold should follow from the general covariance of its renormalization. General covariance in the target manifold of the $\lambda$-model is reparametrization invariance in the manifold of spacetime fields, a sort of meta general covariance. It must be shown that no anomalies intervene. The same formal technology that was used in the renormalization of the general nonlinear model should apply to the $\lambda$-model. If the renormalization of the $\lambda$-model is generally covariant in its target manifold, then its a priori measure, the spacetime QFT, will be generally covariant and gauge invariant in spacetime.

**The effective worldsurface**

The $\lambda$-fluctuations act as fluctuating sources, making insertions in the worldsurface at 2d distances shorter than $\Lambda^{-1}$. These insertions produce corrections to the general nonlinear
model. They produce an effective model of the worldsurface, at 2d distances longer than \( \Lambda^{-1} \). It is invariant under an effective renormalization group, generated by the effective \( \beta \)-function \( \beta_{\text{eff}} \). The effective worldsurface, cut off at short 2d distance \( \Lambda^{-1} \), is used to calculate effective string scattering amplitudes at spacetime distances smaller than \( L \). This effective model of the worldsurface is the effective background spacetime for string scattering at spacetime distances smaller than \( L \).

The form of the theory

For each large value of the number \( L \), the \( \lambda \)-model produces two complementary effective descriptions of spacetime physics: the \textit{a priori} measure, a specific QFT at spacetime distances larger than \( L \), and the effective worldsurface, a specific effective background spacetime for string scattering at distances smaller than \( L \). The two descriptions agree where they overlap, at distances on the order of \( L \). There is no single description that applies at all spacetime distances.

The two complementary descriptions are produced by the \( \lambda \)-model working downwards in \( L \). The \( \lambda \)-model starts at some extremely short 2d distance \( \Lambda_0^{-1} \approx 0 \) and works outward to \( \Lambda^{-1} \). The characteristic spacetime distance slides downwards from \( L_0 \approx \infty \) to \( L \). There is no dependence on the 2d cutoff distance \( \Lambda_0^{-1} \) because the \( \lambda \)-model is renormalizable, as a 2d nonlinear model. The \( \lambda \)-model works entirely at large distance in spacetime. QFT is not derived from a microscopic, small distance mechanics. Rather, it is constructed from the largest distances \textit{downwards}. Such a method of producing QFT might turn out useful, given the need to explain the mysteriously small value of the cosmological constant, which is not natural in any microscopic QFT.

When the QFT is constructed downwards in distance, spacetime locality has to be demonstrated. I argue that spacetime locality is built into the \textit{a priori} measure of the \( \lambda \)-model by the diffusion process that produces it. Locality is tested in the QFT by integrating out the small distance wave modes. In the \( \lambda \)-model, small distance wave modes \( \lambda^i \) start as non-renormalizable coupling constants, decoupled from the wave modes at larger distances and forced to zero. As \( \Lambda^{-1} \) increases, as \( L \) decreases, the wave modes at spacetime distances near \( L \) become renormalizable couplings. They diffuse away from zero to their equilibrium distribution. Integrating out the small distance wave modes \( \lambda^i \) undoes that diffusion. The \textit{a priori} measure at larger values of \( L \) is recovered from the \textit{a priori} measure at smaller values of \( L \) by integrating out the small distance wave modes. This is spacetime locality in the QFT.

The renormalization of the general nonlinear model is valid only as long as \( L^2 = \ln(\Lambda/\mu) \) is large enough to be treated as a divergence. Therefore the spacetime distance \( L \) must stay large. It seems reasonable to suppose that \( L^2 > 10^{24} \) or perhaps \( L^2 > 10^{20} \) will be large enough. If so, and if the unit of distance is within a few orders of magnitude of the Planck length, then \( L \) can be taken smaller than the smallest distance experimentally observable in practice. The \textit{a priori} measure describes all spacetime physics at distances
larger than $L$, so it will describe all observable physics, if this theory turns out to be right.

**Dynamical background independence**

Figure 4 pictures schematically the production of the *a priori* measure and the effective background spacetime, which is written $\text{Back}(L)$. The process starts at $\Lambda_0^{-1} \approx 0$, $L_0 \approx \infty$, in a background spacetime arbitrarily chosen from the manifold $M(L_0)$. The initial choice of background spacetime does not matter, because the diffusion process forgets its initial condition. The $\lambda$-model is background independent, dynamically.

This formal argument rests on the intuition that the $\lambda$-model is merely a well-behaved scale invariant 2d statistical system. It is presumed to behave as if its target manifold $M(L)$ is globally finite dimensional, compact and connected, in the limit $L \to \infty$. The diffusion of the *a priori* measure will then go to a unique equilibrium. This is a very strong presumption. There will have to be nonperturbative 2d effects in the lambda model that regularize the singularities in $M(L)$, especially the places where spacetime becomes macroscopically large and the dimension of $M(L)$ grows without bound. There will have to be 2d effects in the lambda model that change the spacetime topology. It can be imagined that fluctuating 2d defects in the $\lambda$-model, associated with the singularities in $M(L)$, will regularize the singularities. But such effects are now only imagined.

**What the cancelling means**

The $\lambda$-model was introduced to cancel the dependence on $\Lambda^{-1}$ of the local handles. But the local handles act at 2d distances longer than $\Lambda^{-1}$, while the $\lambda$-fluctuations act at 2d distances shorter than $\Lambda^{-1}$. It cannot be that the $\lambda$-fluctuations actually cancel the local handles, leaving no net effect. Rather, the incremental effects of the $\lambda$-fluctuations are opposite to those of the local handles, as $\Lambda^{-1}$ changes. Consider a small change, $\Lambda^{-1} \to (1 + \epsilon)\Lambda^{-1}$. The $\lambda$-fluctuations at 2d distances between $\Lambda^{-1}$ and $(1 + \epsilon)\Lambda^{-1}$ act on $\text{Back}(L)$ to produce the effective background spacetime $\text{Back}(L - \delta L)$, where $\delta L = \epsilon/2L$. Schematically,

$$\text{Back}(L) + (\lambda\text{-fluctuations}) = \text{Back}(L - \delta L).$$  \hspace{1cm} (43)
The cancelling between the $\lambda$-fluctuations and the local handles at 2d distances between $\Lambda^{-1}$ and $(1 + \epsilon)\Lambda^{-1}$ is, schematically,

\[(\lambda-\text{fluctuations}) + (\text{local handles}) = 0.\] (44)

Therefore,

\[\text{Back}(L) = \text{Back}(L - \delta L) + (\text{local handles}).\] (45)

The local handles act on $\text{Back}(L - \delta L)$ to produce $\text{Back}(L)$. The local handles undo the work of the $\lambda$-model.

In a macroscopic spacetime, with $n$ macroscopic dimensions of volume $V$, the discrete sums over wave modes become continuous momentum integrals

\[\sum_{i,j} Tg^{ij} = \sum_{p(i) p(j)} g^2_s V \delta_{p(i), p(j)}\] (46)

\[= \int d^n p(i) \int d^n p(j) \ g^2_s \delta^n(p(i) - p(j))\] (47)

writing here only the wave vector $p(i)$ for $i = (p(i), \cdots)$, leaving out all the other labelling of the wave modes. The kernel of the handle insertion (the string propagator) becomes

\[\delta^n(p(i) - p(j)) \ g^2_s \left[ 1 - \frac{(\mu\Lambda^{-1})^{2p(i)^2}}{p(i)^2} \right]\] (48)

Take $\Lambda \partial / \partial \Lambda$ of this kernel to find that its 2d scale dependence

\[\delta^n(p(i) - p(j)) \ g^2_s \left[ 2 e^{-2L^2p(i)^2} \right]\] (49)

is entirely at spacetime distances larger than $L$.

As the infrared cutoff is relaxed, as $L$ is increased, more and more large distance string modes pass through the local handles, modifying the background spacetime at distances larger than $L$. As $L$ increases, the background spacetime evolves, running back through the effective background spacetimes that were produced by the $\lambda$-model. The diffusion process runs backwards. But a diffusion process can run backwards only on states that have first been produced by the forward process. Thus string scattering at finite distances $L$ in spacetime is consistent only in the effective background spacetime produced by the $\lambda$-model.

**Physics at finite spacetime distance**

The $\lambda$-model is, in principle, a realistic theory. By working at nonzero $\Lambda^{-1}$, it produces a mechanical description of spacetime at large finite distances $L$. It produces a QFT, an actual functional integral over spacetime fields, which describes the large distance physics.
It produces an effective background spacetime for string scattering at every large distance $L$, evolving consistently with increasing $L$. String scattering takes place at finite distances in the mechanical large distance environment produced by the $\lambda$-model. States of the QFT can describe mechanical string scattering experiments which probe small distance physics, at least hypothetically.

Pure S-matrix string theory, on the other hand, describes strings scattering at infinite distance in an infinitely large spacetime. It is an idealized theory of perfectly asymptotic states, exactly on-shell. It assumes an ideal experimenter at infinity in spacetime. It has no room for mechanics. It has no room for a mechanical description of the physical environment in which scattering takes place, no room for a mechanical description of the experimental apparatus or the observers. The worldsurface is in the idealized continuum limit, $\Lambda^{-1} = 0$, at an exact solution of $\beta = 0$. The logarithmic dependence on $\Lambda^{-1}$ in the local handles is pushed away to $L = \infty$, outside the idealized scattering region, invisible in the S-matrix. There are no discrete zero modes to produce logarithmic divergences. (Logarithmic divergences can appear only in tadpole diagrams, where a non-local handle is attached at one of its ends to a vacuum diagram, which can make a contribution at the end of the handle proportional to $\delta^n(p)$. These are not contributions to the background spacetime, since they come from non-local handles. They see the small distance string modes circulating in the vacuum diagram.)

The dependence of local handles on $\Lambda^{-1}$ is a significant difficulty only in a potentially realistic theory of string scattering, which strives to describe scattering at finite spacetime distances $L$ within a mechanical environment. Taking the background spacetime to be compact and riemannian distills that difficulty down to the logarithmic divergence in local handles, coming from spacetime zero modes. Treating this logarithmic divergence is a way to treat the real difficulty, which is the dependence of the local handles on $\Lambda^{-1}$, at finite $L$. That difficulty cannot be avoided if string theory is to have a relation to the physics of the real world, because the real world is mechanical at large distances. There has to be a mechanical background spacetime at large finite distances $L$, which evolves consistently with $L$. String theory needs the $\lambda$-model to prepare the effective background spacetime within which string scattering at finite spacetime distance is consistent.

The conventional background spacetimes for pure S-matrix string theory are suspect as guides to the properties of the effective background spacetime. The conventional background spacetimes, the exact solutions of $\beta = 0$, are idealized settings for pure string S-matrices. Properties such as exact spacetime supersymmetry might turn out to be artifacts of prematurely setting $\Lambda^{-1} = 0$, $L = \infty$. A potentially physical S-matrix should derive from a theory which can describe finite mechanical experiments. Only in the limiting case of infinitely large experimental apparatus does an S-matrix emerge. This is how the S-matrix is derived from ordinary quantum field theories, such as the standard model.

The $\lambda$-model starts at $L_0 \approx \infty$. It starts at a background spacetime in $M(L_0)$, which is a solution of $\beta = 0$ at distances smaller than $L_0$, so is essentially a conventional background spacetime. The renormalization group of the $\lambda$-model then runs, eliminating dependence
on $\Lambda^{-1}$, forgetting the conventional background spacetime. The \textit{a priori} measure reaches equilibrium by the time any finite value of $L$ is reached. Only after this process has taken place, only after 2d universality has set in, does it make sense to explore the limit $\Lambda^{-1} \to 0$, $L \to \infty$. The nature of that limit would become the most basic question, if this theory should turn out to be right.

**What needs to be done**

The $\lambda$-model is a certain 2d nonlinear model. It is formulated, and its structure described, in abstract, general terms. The task now is to figure out what it actually does. This is a familiar kind of challenge in theoretical physics, to figure out what is to be calculated in a model, and how to do the calculations.

The general arguments for how the $\lambda$-model works need to be checked in detail. A first check can be made by calculating the one loop corrections in the lambda model, the corrections to the effective metric, the effective $\beta$-function, the \textit{a priori} measure, and the effective worldsurface. It should be possible to check explicitly the generalized scale invariance of the $\lambda$-model, and the correspondence between the \textit{a priori} measure and the effective string scattering amplitudes. One particular technical point to check is the equation $R_{ij} = g_{ij}/4$ on the graded manifold of supersymmetric background spacetimes, which follows from these general principles.

The most urgent task is to figure out the leading nonperturbative effects in the $\lambda$-model. In particular, do nonperturbative 2d effects remove spacetime supersymmetry? Do they generate small nonzero particle masses? Do they make spacetime macroscopic? If these questions have positive answers, it will become worthwhile to look for a definite, detailed explanation of the Standard Model and the rest of our current knowledge of the real physical world, to find out if the $\lambda$-model is physically useful. For now, it is only a speculative new way to do theoretical physics.

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