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Metastable state in a shape-anisotropic single-domain nanomagnet subjected to spin-transfer-torque

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We predict the existence of a metastable magnetization state in a single-domain nanomagnet with uniaxial shape anisotropy. It emerges when a spin-polarized current, which delivers a spin-transfer-torque possessing a field-like component, is injected into the nanomagnet. At a metastable state, the internal torque due to nanomagnet’s shape anisotropy cancels the externally applied spin-transfer-torque and hence the net torque acting on the magnetization becomes zero. Therefore, it prevents spin-transfer-torque from switching the magnetization from one stable state along the easy axis to the other, even in the presence of room-temperature thermal fluctuations. © 2012 American Institute of Physics. [http://dx.doi.org/10.1063/1.4761250]

Spin-transfer-torque (STT) is an electric current-induced magnetization switching mechanism that is widely used to switch the magnetization of a nanomagnet with uniaxial shape anisotropy from one stable state to the other.1,2 A spin-polarized current is injected into the magnet to deliver a torque on the magnetization vector and makes it switch. This has now become the staple of nonvolatile magnetic random access memory (STT-RAM) technology.3 Recent experimental measurements of STT4,5 following its theoretical prediction6 in magnetic tunnel junctions (MTJs), which is of primary interest for technological applications, showed a significant amount of out-of-plane (or field-like) torque in addition to the traditional in-plane torque.1,2

In this letter, we show analytically that the spin polarized current can spawn a metastable state in the presence of field-like torque, which can trap the magnetization vector and impede it from switching. This can be prevented if the spin-polarized current is higher than a critical value or the demagnetization factor in the m direction (m = x, y, z),8 and Ω is the nanomagnet’s volume.

The magnetization \( \mathbf{M}(t) \) of the single-domain nanomagnet has a constant magnitude but a variable orientation, so that we can represent it by the vector of unit norm \( \mathbf{n}_m(t) = \mathbf{M}(t)/|\mathbf{M}| = \mathbf{e}_r \), where \( \mathbf{e}_r \) is the unit vector in the radial direction. The other two unit vectors are denoted by \( \mathbf{e}_\theta \) and \( \mathbf{e}_\phi \) for \( \theta \) and \( \phi \) rotations, respectively.

The torque acting on the magnetization within unit volume due to shape anisotropy is

\[
T_E(t) = -\n_\text{m}(t) \times \nabla E(\theta(t), \phi(t)) \\
= -2B(\phi(t))\sin(\theta(t))\cos(\theta(t))\mathbf{e}_\phi \\
- \{B_{0e}(\phi(t)) \sin(\theta(t))\} \mathbf{e}_\theta. \quad (3)
\]

where

\[
B_{0e}(\phi(t)) = \frac{\mu_0}{2} M_s^2 \Omega (N_{d-xx} - N_{d-yy}) \sin(2\phi(t)). \quad (4)
\]

Passage of a constant spin-polarized current \( I \) perpendicular to the plane of the nanomagnet generates a spin-transfer-torque that is given by

\[
B(\phi(t)) = \frac{\mu_0}{2} M_s^2 \Omega [N_{d-xx} \cos^2(\phi(t)) + N_{d-yy} \sin^2(\phi(t)) - N_{d-zz}]. \quad (2)
\]

Here, \( M_s \) is the saturation magnetization, \( N_{d-mm} \) is the demagnetization factor in the m direction (m = x, y, z),8 and \( \Omega \) is the nanomagnet’s volume.

Consider a single-domain nanomagnet shaped like an elliptical cylinder. The stable states are along the \( \pm z \) axes. The magnetization direction can be rotated with a spin polarized current.

\[
E(\theta(t), \phi(t)) = B(\phi(t)) \sin^2 \theta(t) + \text{constant term}, \quad (1)
\]

where

\*
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\[ T_{\text{STT}}(t) = s \{ c_z(V) \sin \theta(t) \hat{e}_\theta - b_z(V) \sin \theta(t) \hat{e}_\phi \}, \]  
(5)

where \( s = (\hbar/2e)\eta I \) is the spin angular momentum deposition per unit time and \( \eta \) is the degree of spin polarization in the current \( I \). The coefficients \( b_z(V) \) and \( c_z(V) \) are voltage-dependent dimensionless terms that arise when the nanomagnet is coupled with an insulating layer as in an MTJ.\(^4\) \(^-\)\(^6\) \(^9\) The strength of field-like torque, i.e., \( b_z(V) \) is significant in MTJs, whereas it is small in spin valve devices with metallic spacers.\(^4\) \(^8\)\(^,\)\(^9\) We will use constant values of \( b_z(V) = 0.3 \) \(|c_z(V)|\) and \( |c_z(V)| = 1 \) to be in agreement with the theoretical prediction\(^6\) and subsequent experimental results.\(^4\)\(^,\)\(^5\) For \( \theta = 180^\circ \) to \( 0^\circ \) switching, \( c_z(V) = +1 \), and for \( \theta = 0^\circ \) to \( 180^\circ \) switching, \( c_z(V) = -1 \), while \( b_z(V) = +0.3 \) for both cases.

We can immediately conjecture from Eqs. (3) and (5) that the existence of a field-like torque can render a state, at which these two torques \( \{ T_E(t) \text{ and } T_{\text{STT}}(t) \} \) balance each other, which we would discuss next. The dimensionless dynamics of the single-domain nanomagnet under the action of various torques is described by the Landau-Lifshitz-Gilbert (LLG) equation as

\[ \frac{d\mathbf{n}(t)}{dt} = -\frac{\gamma}{M_V} \mathbf{T}_{\text{eff}}(t), \]  
(6)

where \( \mathbf{T}_{\text{eff}}(t) = \mathbf{T}_E(t) + \mathbf{T}_{\text{STT}}(t) \). \( \gamma \) is the dimensionless phenomenological Gilbert damping parameter, \( \gamma = 2\mu_B\mu_0/B \) is the gyromagnetic ratio for electrons, and \( M_V = \mu_0M_0 \). Solving the aforesaid equation, we get the following coupled equations for the dynamics of \( \theta(t) \) and \( \phi(t) \):\(^7\)

\[ (1 + \alpha^2) \frac{d\theta(t)}{dt} = \frac{\gamma I}{M_V} \left( (-s c_z(V) + \alpha b_z(V)) + B_{0 \theta}(\phi(t)) \sin \theta(t) - 2\alpha B(\phi(t)) \sin \theta(t) \cos \theta(t) \right), \]  
(7)

\[ (1 + \alpha^2) \frac{d\phi(t)}{dt} = \frac{\gamma I}{M_V} \left( (s b_z(V) - \alpha c_z(V)) + 2\alpha B_{0 \theta}(\phi(t)) \right) + 2B(\phi(t)) \cos \theta(t), \]  
(8)

Note that when the magnetization vector is aligned along the easy axis (i.e., \( \theta = 0^\circ, 180^\circ \)), the torque due to shape anisotropy, \( \mathbf{T}_E(t) \) and the torque due to spin-transfer-torque, \( \mathbf{T}_{\text{STT}}(t) \) both vanish [see Eqs. (3) and (5)], which makes \( d\theta(t)/dt \) as well as \( d\phi(t)/dt \) equal to zero. Hence, the two mutually anti-parallel orientations along the easy axis become “stable”; however, thermal fluctuations can dislodge the magnetization from a “stable” state and enable switching.\(^7\) We will now show that there can be a third set of values \( (\theta_3, \phi_3) \) for \( \theta(t) \) and \( \phi(t) \) for which both \( d\theta(t)/dt \) and \( d\phi(t)/dt \) will vanish.

We determine the values of \( (\theta_3, \phi_3) \) as follows. From Eqs. (7) and (8), by making both \( d\theta(t)/dt \) and \( d\phi(t)/dt \) equal to zero, we get

\[ 2\alpha B(\phi_3) \cos \theta_3 = B_{0 \theta}(\phi_3) - s c_z(V) - 2\alpha b_z(V), \]  
(9)

From the above two equations, we get \( B_{0 \theta}(\phi_3) = sc_z(V) \). If we put \( B_{0 \theta}(\phi_3) = sc_z(V) \) in Eq. (9) or in Eq. (10), we get

\[ 2B(\phi_3) \cos \theta_3 = -2B_{0 \theta}(\phi_3) + 2sc_z(V) - s b_z(V). \]  
(10)

We immediately see that \( \mathbf{T}_{\text{eff}}(t) \) vanishes when \( \theta(t) = \theta_3 \) and \( \phi(t) = \phi_3 \). Hence, there is no net torque acting on the magnetization vector if it reaches the state \( \theta(t) = \theta_3 \) and \( \phi(t) = \phi_3 \) at the same instant of time \( t \). Unlike in the case of the other two stable states, where both shape-anisotropy torque and spin-transfer-torque individually vanish, here neither vanishes, but they are equal and opposite so that they cancel to make the net torque zero.

If the magnetization ends up in the orientation \( (\theta_3, \phi_3) \), then it will be stuck and not rotate further unless we change the switching current \( I \) to change the spin-transfer-torque. Since changing \( I \) can dislodge the magnetization from this state, it is not a “stable” state like the ones when \( \theta = 0^\circ, 180^\circ \). Hence, we call it a “metastable” state.

For numerical simulations, we consider a nanomagnet of elliptical cross-section made of CoFeB alloy which has saturation magnetization \( M_s = 8 \times 10^5 \) A/m (Ref. 11) and a Gilbert damping parameter \( \alpha = 0.01 \). We assume the lengths of major axis (\( a \)), minor axis (\( b \)), and thickness (\( l \)) to be 150 nm, 100 nm, and 2 nm, respectively. These dimensions (\( a, b, \) and \( l \)) ensure that the nanomagnet will consist of a single ferromagnetic domain.\(^12\)\(^,\)\(^13\) The combination of the parameters \( a, b, l, \) and \( M_s \) makes the in-plane shape anisotropy energy barrier height \( \sim 32 \) kT at room temperature. The spin polarization of the switching current is always assumed to be 80%.

We assume that the magnetization is initially along the \( +z \)-axis, which is a stable state. At room temperature, the thermal fluctuations will deflect the magnetization vector by \( \sim 4.5^\circ \) from the easy axis when averaged over time,\(^7\) so that we will assume the initial value of the polar angle to be \( \theta_{\text{int}} = 4.5^\circ \). We choose the initial azimuthal angle \( \phi_{\text{int}} = +90^\circ \) because it is the most likely value (along with \( \phi_{\text{int}} = -90^\circ \)) in the absence of spin transfer torque.\(^7\) Similar assumptions are made by others.\(^10\) We then solve Eqs. (7) and (8) simultaneously to find \( \theta(t) \) and \( \phi(t) \) as a function of
time. Once $\theta(t)$ reaches $175.5^\circ$, regardless of the value of $\phi(t)$, we consider the switching to have completed. The time taken for this to happen is the switching delay.

Fig. 2 shows the switching delays versus switching current for different values of $b_i(V)$. The switching delay is “infinity” in some current ranges when $b_i(V) = 0.3$ because switching failed [see Fig. 2(a)]. However, beyond the current of 24.51 mA, switching always occurs within a finite time, meaning that the magnetization never ends up at the metastable state. Simulation results show that if the value of $b_i$ is small enough ($\leq 0.05$), the metastable state does not show up [see Figs. 2(b) and 2(c)].

The important question is why switching fails only for certain ranges of the current $I$, i.e., why does the magnetization vector land at the metastable state for certain values of $I$ and not others? The answer is that starting from some initial condition ($\theta_{\text{init}}, \phi_{\text{init}}$), the angles $\theta(t)$ and $\phi(t)$ must reach the values $\theta_3$ and $\phi_3$ at the same instant of time $t$. This may not happen for any arbitrary $I$. Hence, only certain ranges of $I$ will spawn the metastable state. It is also clear from Eq. (11) that above a certain value of $I$, there will be no real solution for $\phi_3$ since the argument of the arcsin function will exceed unity. This value will be

$$ I_{\text{threshold}} = \left[ \frac{\mu_0 M_s^2 \Omega (N_{d_{\text{xx}}} - N_{d_{\text{yy}}})}{|n c_s(V)|} \right]. $$

By maintaining the magnitude of the switching current above $I_{\text{threshold}}$, we can ensure that the magnetization vector will never get stuck at the metastable state. For the nanomagnet considered, $I_{\text{threshold}} = 32.7$ mA, but switching becomes feasible at even lower current of 24.52 mA since in the range [24.52 mA, 32.7 mA], the coupled $\theta$ and $\phi$-dynamics expressed by Eqs. (7) and (8) do not allow $\theta(t)$ and $\phi(t)$ to reach $\theta_3$ and $\phi_3$ simultaneously starting from $\theta_{\text{init}}, \phi_{\text{init}}$.

Another important question is whether thermal fluctuations can untrap the magnetization from this state. To probe this, we solved the stochastic LLG equation in the presence of a random thermal torque. Fig. 3 shows the magnetization dynamics for a switching current of 24.51 mA at room temperature (300 K). We observe that the magnetization gets stuck at a metastable state with $\theta_3 = 97.58^\circ$ and $\phi_3 = 335.87^\circ$ (and fluctuates around the metastable state due to thermal agitation) $\sim 50\%$ of the time, which means that roughly one-half of the switching trajectories intersect the metastable state and terminate there. The values of $\theta_3, \phi_3$ are also the angles predicted by Eqs. (11) and (12), thereby confirming that the metastable state indeed has the origin described here. Increasing the temperature to 400 K helps...
only a little by decreasing the probability that a switching trajectory will intersect the metastable state. What is important, however, is that if the magnetization vector gets stuck at the metastable state and the current remains on, then thermal fluctuations cannot dislodge it. In other words, this state is stable against thermal perturbations.

It should be emphasized that if \( b_1(V) = 0 \), then \( \theta_3 = 90^\circ \) (x-y plane), however, magnetization cannot remain stuck to that metastable state since no field-like torque is there to balance the out-of-plane shape-anisotropy torque when thermal fluctuations would dislodge the magnetization from \( \theta_3 = 90^\circ \). Also, if we consider \( \theta = 180^\circ \) to \( 0^\circ \) switching \( (c_i(V) = +1) \), field-like torque aids the rotation of magnetization towards its destination, hence metastable states do not crop up.\(^7\)

We also notice oscillations before magnetization settles into the metastable state (see Fig. 3). This is due to coupled \( \theta_3 \)- and \( \phi_3 \)-dynamics governing the rotation of the magnetization vector, which causes some ringing. Such ringing signifies that the magnetization is attracted to the metastable state as it comes inside the range of the attractor. Thus, it can be intuitively conceived that if the initial conditions \( (\theta_{ini}, \phi_{ini}) \) are changed, the range of currents for which metastable states would emerge can change as well.\(^7\) In general, choosing different parameters for the nanomagnet (e.g., damping constant, saturation magnetization, shape anisotropy\(^7\)) can change the occurrence of metastable states in different current ranges.

Finally, one issue that merits discussion is what happens if the spin polarized current is turned off after the magnetization gets stuck. In that case, the torque due to shape anisotropy will take over and drive the magnetization to the easy axis. One expects that if \( \theta_3 > 90^\circ \), then switching should succeed because the nearer easy axis \( (\theta_3 = 180^\circ) \) is the desired orientation. Equation (12) dictates that \( \theta_3 > 90^\circ \) since \( b_1(V) \) is always positive. Unfortunately, these simple expectations are belied by the complex dynamics of magnetization. The out-of-plane excursion of the magnetization vector, i.e., deviating from magnet’s plane \( \phi = \pm 90^\circ \) causes an additional motion that depends on \( \theta_3 , \phi_3 \) [see the \( e_\theta \) component of torque in Eq. (3)]. This motion can oppose the in-plane motion due to damping [see the last term in Eq. (7)].\(^7\) As a result, even when \( \theta_3 > 90^\circ \), switching can fail since the magnetization reaches the wrong orientation \( (\theta \approx 0^\circ) \) along the easy axis (see Fig. 4).

In conclusion, we have predicted the existence of a metastable magnetization state in spin-transfer-torque switching of a shape-anisotropic single-domain nanomagnet in the presence of field-like torque, which is significant in magnetic tunnel junctions. Since the occurrence of metastable states must be avoided for feasible implementation of devices based on spin-transfer-torque mechanism, we hope that our studies would stimulate experimental research and further theoretical studies onwards.

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