The $\Psi'/\Psi$ ratio in Nucleus-Nucleus Collisions:
a Measure for the Chiral Symmetry Restoration Temperature?

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We argue that a decrease of the chiral scalar meson mass is responsible for re-creation of $\Psi'$ from $J/\Psi$ in ultrarelativistic nucleus-nucleus collisions. This causes the charmonium yields to freeze out at temperatures close to the chiral symmetry restoration temperature $T_c$. As a result $\Psi'/\Psi$ may serve as a thermometer for $T_c$ itself. Results in a detailed reaction model support the conjecture. They show good agreement with recent data of NA38 and NA50 for $J/\Psi$ and $\Psi'$ production in S on U and Pb on Pb collisions.

Recent measurements by the NA50 collaboration at CERN show a so-called anomalous suppression of $J/\Psi$ yields in non-peripheral Pb(158AGeV) on Pb collisions. The results have been interpreted as a hint for Quark-gluon plasma (QGP) formation in these reactions. The QGP can "ionize" a $J/\Psi$ analogously to the photo-effect or even screen it out of existence. Other explanations, dissociation in coherent gluon fields and collisions with secondary hadrons (comovers) have also been put forward. Generally speaking, the discussion is centered about the question whether the observed suppression of charmonia is an initial or a final-state effect. The issue cannot be considered as settled yet. The arguments expressed e.g. in [2-4] that the different trends in S+U versus Pb+Pb suggest the appearance of a strong energy density-dependent destruction mechanism for charmonium states could be tested by repeating the experiment at lower beam energies. Here we assume that the charmonium states are indeed destroyed very early [12].

Naively, the fate of the different charmonium states is independent from each other and is governed by their size and binding energy. The location of the $2s$ excitation $\Psi'(3686)$ is barely below the threshold to the $D\bar{D}$ continuum while the $J/\Psi$ with a mass of 3097 MeV is deeply bound. It is natural to assume that the $\Psi'$ is more easily destroyed. Indeed, NA38 has found that the $\Psi'/\Psi$ ratio goes down continuously in S on U collisions with increasing centrality. However, this ratio is leveling-off at around 4% in non-peripheral Pb on Pb reactions according to the preliminary NA50 measurements. Approximately the same value is seen in the most central S on U collisions. This value is roughly a factor of 4 lower than in $pp$ and $p\Lambda$ reactions.

Does the constancy of the $\Psi'/\Psi$ ratio result from an accidental fine-tuning of the parameters which govern the survival rates of each species? Here we answer otherwise. $\Psi'/\Psi$ is leveling off, because observed $\Psi'$ are re-created from $J/\Psi$. If so, $\Psi'/\Psi$ will therefore no longer decrease in more violent collisions (at RHIC and LHC). The value of the $\Psi'/\Psi$ ratio reflects on the temperature $T$ of the medium provided the equilibration mechanism is sufficiently robust. In equilibrium, $\Psi'/\Psi$ changes with $T$ exponentially as $\exp(-(M' - M)/T) (M'/M)^{3/2}$. In fact, $\Psi'/\Psi$ is very sensitive to small changes of $T$, because the mass difference $M' - M$ is much larger than the relevant temperatures. The measured $\Psi'/\Psi$ ratio then suggests a freeze-out temperature around 170 MeV. Such a temperature is close to the expected transition temperature $T_c$ for chiral symmetry restoration.

What may the nature of the interactions which equilibrate the $\Psi'/\Psi$ ratio be? In the vacuum $\Psi'$ decays predominantly into a $J/\Psi$ and two pions in s-wave. The decay width is tiny (141 KeV), and therefore the inverse process in the pion gas is simply too slow. Based on the matrix element for the decay process we can calculate charmonium (de-)excitation probabilities in collisions with pions. However, even those are not large enough on the time scale of nuclear collisions. When $\Psi' \rightarrow J/\Psi \pi\pi$ decay was studied first in the seventies Schwinger et al. advocated an interpretation of these decays through the intermediary of a scalar resonance. The transition matrix element is suppressed in the vacuum due to the magnitude of the scalar resonance mass. The presumably large transition rate between $J/\Psi$ and $\Psi'$ in nuclear collisions may then be a consequence of a dropping scalar mass. The emergence of a low mass sigma meson which becomes degenerate with the pions at $T_c$ is responsible for rapid transitions between $\Psi'$ and $J/\Psi$ around this temperature. The transition rates become very small at slightly lower temperatures due to the sensitivity of the transition rates to the sigma mass. This means in turn that the $\Psi'/\Psi$ ratio in heavy ion collisions may serve as a rather clean measure of $T_c$ itself.

In the remainder of this Letter we estimate the transition rates between $J/\Psi$ and $\Psi'$ in thermal matter close to $T_c$. Subsequently, we will assess the main corrections to the equilibrium limit. Finally, we implement the $T$ dependent transition rates into a simple model for nucleus-nucleus collisions and compare the calculated $J/\Psi$ and $\Psi'$ yields to the preliminary NA38 and NA50 data.

Let us consider first charmonium transitions in a heat bath based on the linear sigma model. At $T_c$ the quark condensate vanishes and chiral symmetry gets restored. Seen from the hadronic side, the sigma meson mass decreases with temperature $T$, because the sigma state has to become degenerate with the pion. The pion mass is expected not to change much with $T$. Its smallness is
protected by the spontaneous break-down of chiral symmetry. It may be more surprising that a sigma model describes the relevant physics if one approaches \( T_c \) from above \([13, 16]\). QCD matter at very large temperatures is supposedly composed of a weakly coupled gas of colored quarks and gluons. However, at temperatures \( T_c < T < 2T_c \), strong nonperturbative phenomena in the QGP may be present, since the QCD coupling constant \( g \) is of order one. The scalar-pseudoscalar modes are rather special as far as chiral symmetry is concerned. Condensation is about to occur in the \( \sigma \) channel while the pions will emerge as the Goldstone bosons of the broken phase at slightly lower temperatures. Calculations based on the instanton model support the idea that correlations due to strong attractive forces and perhaps bound states are formed in the QGP at \( T \sim T_c \) \([17]\).

FIG. 1. \( \sigma(\pi \pi \rightarrow \Psi' \pi) \) as a function of the out-going pion energy with the \( \Psi' \) at rest (left side). The straight (dashed) line represents the calculation in the symmetric phase (vacuum with \( m_s = m_s^0 \)). The right side shows the rate constant for \( \Psi' \) transitions into a \( J/\Psi \) due to collisions and decays (straight line with dropping \( s \) mass, dashed line with \( m_s^0 \)). The dotted line represents just the contribution from \( \Psi' \) decay in the chiral scenario.

Brown and Cahn pointed out that the two most important observations – isotropic \( \pi \) emission in their common rest frame and strong enhancement at large invariant \( \pi \pi \) masses – are reproduced by the linear sigma model at tree level \([13]\). In this model \( \sigma \) and \( \pi \) form a four-vector \( \bar{\sigma} = (\sigma, \bar{\pi}) \) which enters into the interaction terms of the Lagrange density

$$L_{\text{int}} = c\Psi'_{\mu} \bar{\Psi}^\mu \bar{\sigma}^2 - U(\bar{\sigma}^2) .$$

A suitably chosen potential \( U \) in eq. (1) gives rise to the spontaneous break-down of the symmetry in the isoscalar field acquires a nonvanishing expectation value, in the vacuum \( \langle \sigma \rangle_0 = f_\pi \). \( f_\pi \approx 92 \text{ MeV} \) is the pion decay constant. The effective potential \( U \) cannot be reliably calculated from QCD. The standard ‘Mexican hat’ potential was introduced by Gell-Mann and Lévy \([14]\) mainly for its simplicity and renormalization property. We are going to utilize the ‘freedom’ in the choice of \( U \) to study how a variation of \( T_c \) affects \( J/\Psi \) and \( \Psi' \) in \( AA \) collisions. In the broken phase the effective potential at finite \( T \) is minimized by some nonzero expectation value \( f \) of the \( \sigma \) field. With the field redefinitions \( \bar{\sigma} = (f + s) \cos \theta \) and \( \bar{\pi} = (f + s) \bar{\pi} \sin \theta \) (with \( \bar{\pi} \) a vector of unit length) all burden of the self-interactions induced by \( U \) is laid on the scalar-isoscalar field \( s \). \( s \) represents the fluctuations of the order parameter. The following interaction vertices which are determined from the Lagrange density enter into our tree-level calculation:

$$V_{\Psi' \Psi_s} = g M' \epsilon_{\mu} e^{\mu} , \quad V_{\Psi' \phi \phi} = 2c^2 \epsilon_{\mu} e^{\mu} ,$$

$$V_{s \pi \pi} = -\frac{1}{f} (2p_1 p_2 + m^2_\pi) , \quad V_{ss \pi \pi} = -\frac{2}{f^2} p_1 p_2 .$$

The Cartesian isospin indices of pions have been suppressed. The coupling constant \( gM' \) of \( \Psi' \) decay equals \( 2c^2 f \). The field \( \phi \) represents the \( s \) meson field in the broken phase and an arbitrary \( \bar{\sigma} \) component in the symmetric phase.

The transition matrix element \( T(\Psi' J/\Psi \pi \pi) \) depends at tree level on \( U \) only via the mass \( m_s \) of the \( s \) meson. We parametrize its temperature dependence by \( (m_s(T)^2 - m_s^0)^2/(m_s^0 - m_s^2) = (\langle \bar{q}q \rangle(T)/\langle \bar{q}q \rangle_0)^{2/3} \), because \( m_s \) is expected to track roughly the strength of the quark condensate \( \langle \bar{q}q \rangle \). (Other parametrizations are also possible, without affecting the main results.) The \( T \) dependence of the quark condensate has been inferred from chiral perturbation theory (\( \chi PT \) calculations to order \( T^6 \) \([21] \), with the result that the condensate vanishes around \( 190 \text{ (150) MeV} \) for the number of massless flavors \( N_f \) equal two (three). Several factors limit the applicability of \( \chi PT \) already below \( T_c \). However, the \( T_c \) values from this extrapolation are overall consistent with the lattice QCD data \([13]\). Later we will treat \( N_f \) as an effective number of degrees of freedom whose magnitude controls the value of \( T_c \). Above \( T_c \), the scalar and the pion mass are taken to be degenerate and \( T \)-independent, since only a narrow window of temperatures above \( T_c \) will be probed.

We fix the vacuum value of the \( s \) meson mass to be \( 1400 \text{ MeV} \). The particle data group lists a very broad state at \( 1400 \text{ (1500) MeV} \) for the number of massless flavors \( N_f \) equal two (three). The location of the mass of the chiral sigma is controversial in the literature. Here we merely note that in the framework of the linear sigma model at tree-level a scalar meson with low mass is strongly disfavored by experimental data on \( \pi \pi \) scattering \([24, 25]\). This still allows a wide range of values for the scalar meson mass above \( 1 \text{ GeV} \). (Note that the narrow scalar-isoscalar state just below \( 1 \text{ GeV} \), the \( f_0(975) \), is probably not the chiral partner of the pions but perhaps a \( K K \) “molecule”.) Threshold theorems are not well suited to constrain the \( s \) mass much further, because the scalar propagator gets effectively replaced by a contact term. We determined \( g^2/4\pi \approx 0.36 \) from the partial decay width \( \Gamma(\Psi' \rightarrow \Psi \pi \pi) \). A term \( im_s^2 T_s(m_{\pi \pi}) \)
containing the $s$ decay width has been added to the denominator of the free $s$ meson propagator to take rescattering corrections into account. Similarly, the $s$ acquires a nonzero width from collisions ($s \pi \leftrightarrow s \pi$) at finite temperature.

On the left hand side of Fig. 1 we compare the cross section $\sigma(\Psi \rightarrow \Psi')$ in the vacuum and in the chiral phase. Note that the cross section is plotted as a function of the out-going pion energy. The in-going pion has an energy larger by approximately 600 MeV which is all of its typical energy at relevant temperatures. It becomes apparent that the thermally averaged cross section is tiny if the vacuum values are used. On the right hand side the rate constant $\lambda = \Gamma_{\text{dec}}(\Psi') + \Gamma_{\text{coll}}(\Psi')$ for $\Psi'$ transitions into a $J/\Psi$ in the thermal heat bath is displayed. $T_c=170$ MeV has been assumed. The total rate shoots up at $T_c$ and increases smoothly afterwards. The most important contribution to the transition between the two charmonium states comes from the collisions with pions. For comparison we also show the contribution from the $\Psi'$ decay to the rate and the total rate but with the sigma mass at its vacuum value.

![Figure 2](image)

**FIG. 2.** Time evolution of $\Psi'/\Psi$ for 3 different initial energy densities 1.5-2.5/3 GeV/fm$^3$ and $T_c=170$ MeV. The equilibrium value at $T=T_c$ is indicated by an arrow.

Next we would like to study the corrections to the $\Psi'/\Psi$ ratio, mainly feed-down from $\chi$ states into $J/\Psi$ after the collision and non-equilibrium effects. For that we consider a simple model which, however, contains the essential physics. Specifically, we calculate the initial charmonium production based on the Glauber model for $AA$ collisions at a given impact parameter $b$. We relate $b$ to the experimentally measured transverse energy $E_t$ by employing the results in Ref. [2]. The initial production yields of $J/\Psi$, $\Psi'$ and $\chi$ in nucleon-nucleon collisions are taken as 23, 6.5 and 84.8 respectively. Here the two $\chi$ states are lumped together with average branching ratio 0.21 into $J/\Psi$. The total normalization allows direct comparison with NA38 and NA50 data referred to as $B_0(J/\Psi)/\sigma(DY)$ in [23]. A common primordial suppression of all charmonia depending on parameter $L$ (see [24]) is assumed as in [2]. We calculate the final yields by integrating the coupled rate equations for the local densities of the charmonium states, e.g. for the $\Psi'$

$$\frac{dN^i}{d\tau} = -\lambda(\Psi' \rightarrow \Psi)(N' - N'_{eq}) - \lambda_{\text{cons}}(\Psi')N'$$

Boost invariance is assumed with formation time $\tau_0=2\text{fm}/c$. As motivated in [2] the rate constants $\lambda_{\text{cons}}$ for charmonium dissociation are taken as zero below and infinity above some critical energy density $e_{cr}$. The $\Psi'$ equilibrium density $N'_{eq}$ is related to the $J/\Psi$ density $N$ via $N'_{eq}=N(M'/M)^{3/2}\exp(-(M'-M)/T)$. Thus we need to model the temperature evolution of the created medium. The initial energy densities are taken as proportional to the density of participants in the transverse plane as in [2], more precisely $e = d_\text{AA}(E_t/N_p)/\tau_0 dN_p/d\tau$. The choices $d_\text{AA}=2.2$ and $d_\text{pp}=4.0$ lead to maximum local energy densities of 2.3 and 3.5 GeV/fm$^3$ for the most central S+U and Pb+Pb collisions. The equation of state which governs the isentropic expansion simulates a resonance gas with energy density and temperature related by a power law $e = abT^{b+1}$ and parameters $a, b$ taken from [23]. The latter choices have been made with an eye on the space-time evolution according to RQMD calculations [25] which connects our simple model with successful phenomenology of $AA$ collisions. Evolution is stopped at temperature 140 MeV although the precise value is unimportant due to the small transition rates below $T_c$ (cf. Fig. 1).

The $e_{cr}$ values which determine the energy density above which a charmonium state immediately dissolves or may not be formed at all are the remaining parameters of the model. They have been adjusted to reproduce the centrality dependence of $\Psi'$ and $J/\Psi$ yields in S+U and Pb+Pb collisions ($e_{cr}=3.2/1.5/2.4$ GeV/fm$^3$ for $\Psi'/\Psi/\chi$). Fig. 2 illustrates the effect of $e_{cr}(\Psi')$ by displaying the time evolution of $\Psi'/\Psi$ for different initial conditions. At initial energy density just below the critical value of 1.5 GeV/fm$^3$ the $\Psi'/\Psi$ ratio drops sharply until the system cools down to the critical temperature (here $T_c=170$ MeV). However, the initial $\Psi'$ yield is so far off equilibrium that the $\Psi' \rightarrow \Psi$ transitions do not equilibrate the yield. What $e_{cr}$ effectively does is to define rather crudely a core and a corona region. The larger than equilibrium values of $\Psi'/\Psi$ in the corona result in increasing $\Psi'/\Psi$ values for more peripheral reactions. In contrast, starting with larger energy densities above $e_{cr}$ and thus zero initial $\Psi'$ yield we see that the reactions equilibrate the $\Psi'$ yield. So, $\Psi'/\Psi$ works well as a thermometer, provided the initial energy densities in collisions are clearly above 2 GeV/fm$^3$.

Fig. 3 contains a direct comparison of the calculated $\Psi'$ and $J/\Psi$ yields for S+U and Pb+Pb collisions with the NA38 and NA50 data. $\Psi'/\Psi$ is displayed for three different choices of $T_c$ (150, 160 and 170 MeV). We find
excellent agreement with the data for $T_c=170$ MeV. The $J/\Psi$ yield is only very mildly affected by the $T_c$ variations. The strong thermal $\Psi'$ suppression factor $N'_{eq}/N$ works against the $\Psi \rightarrow \Psi'$ transitions to be effective as a destruction mechanism for $J/\Psi$'s. On the other side, we provide an explanation why scaling from $p+A$ works so amazingly well for the $J/\Psi$ in $S+U$ although some of the final feed-down from $\Psi'$ is gone. As noted above the $\Psi'$ feeds the $J/\Psi$ yield in the corona of the reaction. Thus $\Psi'$ suppression is partially offset by enhancement of direct $J/\Psi$'s!

Let us shortly comment on feed-down from $\chi$ states. The feed-down corrections are sizable, even in central Pb on Pb collisions. The $\Psi'/\Psi$ ratio changes from 5.64 \% before feed-down to 4.07 \% after feed-down. We see that both surface emission and feed-down affect $\Psi'/\Psi$ by approximately 40 \%. However, they contribute with different sign and cancel almost each other. Furthermore, $T_c$ is rather well determined from $\Psi'/\Psi$ even if one takes the feed-down corrections as a measure of the uncertainty involved. To summarize, we have suggested that $\Psi'/\Psi$ levels off in ultrarelativistic nucleus-nucleus collisions, because $\Psi'$ is re-created from $J/\Psi$. Transitions between $\Psi'$ and $J/\Psi$ equilibrate the ratio around $T_c$ due to a dropping sigma meson mass. A chiral symmetry restoration temperature close to 170 MeV explains nicely why $\Psi'/\Psi$ approaches a ‘universal’ value of around 4 \%. The future dilepton experiments at RHIC and LHC will be important to test the suggested universality of the final $\Psi'/\Psi$ ratio.

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