Cosmology and two-body problem of D-branes

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In this paper, we investigate the dynamics and the evolution of the scale factor of a probe Dp-brane which move in the background of source Dp-branes. Action of the probe brane is described by the Born-Infeld action and the interaction with the background R-R field. When the probe brane moves away from the source branes, it expands by power law, whose index depends on the dimension of the brane. If the energy density of the gauge field on the brane is subdominant, the expansion is decelerating irrespective of the dimension of the brane. On the other hand, when the probe brane is a Nambu-Goto brane, the energy density of the gauge field can be dominant, in which case accelerating expansion occurs for \( p \leq 4 \). The accelerating expansion stops when the brane has expanded sufficiently so that the energy density of the gauge field become subdominant.

The rest of this paper is organized as follows. In section IV we review the p-brane solutions in supergravity as the background spacetime of the source D-branes. We consider the motion of a probe brane in this background spacetime in section V and follow the evolution of the scale factor on the probe brane in section VI. In section VII and VIII we give discussion and summary, respectively.

I. INTRODUCTION

By the discovery of D-brane, not only string theory but also cosmology have been activated significantly. The Randall-Sundrum braneworld model is the simplest cosmological model which was induced by the idea of D-brane. In this model, the action of the brane is assumed to be the Nambu-Goto action. Cosmology with Born-Infeld action has also been investigated and it is found that behavior of the gauge field confined to the brane is significantly different from that of a gauge field added to the Nambu-Goto brane. Interaction between D-branes by R-R charge, which is absence in the Randall-Sundrum model, has been studied as a potential energy source that inflate the brane by many authors and Kazuhide Ichikawa(2)

Since D-brane is a fundamental object in superstring theory, their two-body problem is also fundamental. Burgess et. al. studied motion of a probe brane in the background spacetime of source branes and found that there exist bound states of D6-brane and anti-D6-brane, which they called 'branonium'. Probe-brane dynamics was also discussed. Recently cosmology on the probe brane was studied in the context of bouncing universe.

In this paper, we investigate two-body problem and cosmology of D-branes. Basic approach is the same as but we take into account a gauge field confined to the probe brane, which was neglected. The motion of the brane causes the time evolution of the induced metric on it, which is seen as cosmological expansion or contraction by an observer living on the brane. In this sense, our picture is similar to that of 'mirage cosmology' . Thus, by following the motion of the brane, we can also follow the evolution of the scale factor. We show that the gauge field on the probe brane, which has not been studied rigorously, can affect the behavior of the scale factor.

II. BACKGROUND SPACETIME

We consider a system in which a probe D-brane (or anti-D-brane) moves within the background of \( N \) parallel source D-branes. In this section, we review the p-brane solutions in supergravity as the background spacetime of the source D-branes. Low-energy effective theories for superstring theories are given by supergravities, among which we consider only Type IIA and IIB here for simplicity. The effective actions include the metric, the 2-form potential and the scalar dilaton in the NS-NS sector, \((n-1)\)-form gauge potentials in the R-R sector and Chern-Simons terms. Here \( n \) is even for IIA and odd for IIB.

To obtain a tractable system to study, we shall make a consistent truncation (see and references therein) of the action down to a simple system comprising only the metric \( G_{MN} \), the scalar dilaton \( \phi \) and a single \((n-1)\)-form gauge potential \( A_{[n-1]} \) with corresponding field strength \( F_{[n]} \). Then the background spacetime of source Dp-brane are determined by the following action in the Einstein Frame,

\[
S = \int D^D x \sqrt{-G} \left[ R - \frac{1}{2} G^{MN} \partial_M \phi \partial_N \phi - \frac{1}{2n!} e^{a\phi} F_{[n]}^2 \right],
\]

where \( D = 10 \) and \( a = (5 - n)/2 \) is the dilaton coupling of the R-R field. Assuming asymptotic flatness and spherical symmetry in the transverse directions, flatness of the branes and an “electric” gauge field, the background spacetime and gauge field for \( p \leq 6 \) are given.
by,
\[ ds^2 = h_{\gamma(7-p)/8} \eta_{\mu\nu} dx^\mu dx^\nu + h^{(p+1)/8} \delta_{mn} dy^m dy^n, \]  
(2)
\[ e^\phi = h^{(3-p)/4}, \]  
(3)
\[ A_{M_1 M_2 \cdots M_{p+1}} = \ell M_4 M_2 \cdots M_{p+1} (1 - h^{-1}), \]  
(4)
where \( x^\mu (\mu = 0, 1, \ldots, p) \) and \( y^m (m = 1, 2, \ldots, D - p - 1) \) are the coordinates parallel and transverse to the branes, respectively. We define the radial coordinate transverse to the brane as \( r^2 \equiv \delta_{mn} y^m y^n \) and then,
\[ h(r) = 1 + \frac{k}{r^{p-6}}. \]  
(5)
Here \( k \) is an integration constant which represent the energy scale of the source branes:
\[ k = (2\sqrt{\pi})^{5-p} \Gamma \left( \frac{7-p}{2} \right) g_s l_s^{7-p} N, \]  
(6)
where \( g_s \) is the string coupling constant at infinity, \( l_s \) is the string length scale and \( N \) is the number of the source branes. It should be noted that these solutions are reliable only for \( r \gg l_s \). This is because supergravity is a good approximation of superstring theory only within this region, where the brane interactions are dominated by massless string states.

The asymptotic behaviors of the gravitational field and the gauge field potential can be understood in terms of Gauss’ law. Both behave asymptotically like \( \sim r^{-7-p} \), as expected from the Laplace equation,
\[ \nabla^2 f(r) = \left[ \frac{d^2}{dr^2} + \frac{8 - p}{r} \frac{d}{dr} \right] f(r) = 0. \]  
(7)
Thus, the potential produced by a D6-brane is, like that of a point particle in ordinary 4-dimensional spacetime, \( \sim r^{-1} \). For global structures of these solutions, see, for example, [27].

On the other hand, there is no asymptotically-flat solution for \( p \geq 7 \). Hereafter we concentrate on \( p \leq 6 \) cases.

### III. DYNAMICS OF PROBE BRANE

In this section we consider the motion of a probe brane, which is assumed to be parallel to the source branes, in the background spacetime discussed in the previous section. The dynamics of the probe brane which has “electric” charge is determined by the Born-Infeld action (in the String Frame),
\[ S_{\text{BI}} = -T_p \int d^{p+1}x e^{-\phi} \sqrt{- \det \left( g_{\mu\nu} + 2 l_s^2 F_{\mu\nu} \right)} \]  
(8)
and the interaction with the background gauge field \( A_{[p+1]} \),
\[ S_{\text{WZ}} = - q T_p \int A_{[p+1]}. \]  
(9)
Here \( g_{\mu\nu} \) is the induced metric on the probe brane, \( F_{\mu\nu} \) is the \( U(1) \) gauge field strength confined to the brane and \( q \) is the R-R charge of the brane, which equals to \( \pm 1 \) for D-brane and anti-D-brane, respectively. Note that the field strength \( F_{\mu\nu} \) should be thermal in nature in order not to break the isotropy of the brane. Therefore, we interpret that \( F_{\mu\nu} F^{\mu\nu} \rightarrow \langle F_{\mu\nu} F^{\mu\nu} \rangle \) etc [23]. The induced metric on the brane in the String Frame is written as,
\[ ds^2 = e^{4\phi/(D-2)} ds^2 \]  
\[ = -h^{-1/2}(1 - hu^2) dt^2 + h^{-1/2} \delta_{ij} dx^i dx^j \]  
(10)
where we took the static gauge, \( t = x^0 \) and \( i, j = 1, 2, \ldots, p \). Here we defined the velocity \( v \) of the brane as,
\[ v^2 \equiv \delta_{mn} \frac{dy^m}{dt} \frac{dy^n}{dt}. \]  
(11)
Thus the motion of the brane in the dimensions transverse to the brane is described in terms of the radial coordinate \( r \) and the velocity \( v \).

Due to the spherical symmetry in the transverse direction, the angular momenta of the brane are conserved. This shows that the motion is confined to the plane which is spanned by the initial position and momentum vectors. We will denote the polar coordinate in this plane by \( \theta \) and the velocity \( \dot{\theta} \). Therefore, due to technical difficulty, we treat the gauge field as a perturbation and consider the leading term. Then the total Lagrangian of the probe brane is,
\[ L = -m h^{-1} \left[ \sqrt{1 - h(\dot{r}^2 + r^2 \dot{\theta}^2)} (1 + l_s^2 F_{\mu\nu} F^{\mu\nu}) - q \right], \]  
(12)
where we have neglected an additive constant, \( m = T_p \int d^p x \) is the “mass” of the brane, and dot denotes a derivative with respect to \( t \). The independent variables are \( r, \theta \) and the gauge potential \( A_\mu \) on the brane. The canonical momenta associated with these variables are,
\[ p_r \equiv m^{-1} \frac{\partial L}{\partial \dot{r}} = \frac{\dot{r}}{\sqrt{1 - h(\dot{r}^2 + r^2 \dot{\theta}^2)}} (1 + l_s^2 F_{\mu\nu} F^{\mu\nu}), \]  
(13)
\[ l \equiv m^{-1} \frac{\partial L}{\partial \dot{\theta}} = \frac{r^2 \dot{\theta}}{\sqrt{1 - h(\dot{r}^2 + r^2 \dot{\theta}^2)}} (1 + l_s^2 F_{\mu\nu} F^{\mu\nu}), \]  
(14)
\[ p_A^i \equiv m^{-1} \frac{\partial L}{\partial A_i} = -4\sqrt{1 - h(\dot{r}^2 + r^2 \dot{\theta}^2)} \frac{F^{i0}}{h}, \]  
(15)
where \( p_A^i \) is also conserved as we can see from the Euler-Lagrange equation. Thus the “electric field” \( F^{io} \) can be written in terms of the other variables. On the other hand, the “magnetic field” \( F^{ij} \) are obtained from Bianchi identity,

\[
\partial_{\mu} F_{\nu \lambda} + \partial_{\nu} F_{\lambda \mu} + \partial_{\lambda} F_{\mu \nu} = 0, 
\]

as,

\[
F_{ij} = C_{ij} = \text{const}. 
\]

Combining the above results, it follows that,

\[
F_{\mu \nu} F^{\mu \nu} = \left( \delta_{ik} \delta_{jl} C_{ij} C_{kl} - \frac{\delta_{ij} p_A^i p_A^j}{8} \right) h 
\equiv C'h, 
\]

where \( C' \) is a constant which represents the energy scale of the gauge field.

From Eq. (13) and (14), we obtain the following useful relation:

\[
\dot{r}^2 + r^2 \dot{\theta}^2 = \frac{p_r^2 + l^2 / r^2}{(1 + Ch)^2 + h(p_r^2 + l^2 / r^2)}, 
\]

where \( C \equiv C' l_A^4 \) is a dimensionless constant which represents the energy scale of the gauge field in units of \( l_s^{-1} \). Then the Hamiltonian can be written as,

\[
E = p_r \dot{r} + l \dot{\theta} + p_A^i \dot{A}_i - m^{-1} L 
= \left\{ 1 + (4D + C)h \right\} (1 + Ch) + h(p_r^2 + l^2 / r^2) - \frac{q}{h}, 
\]

which gives the conserved energy. Here we took a gauge \( A_0 = 0 \) and \( D = \delta_{ij} p_A^i p_A^j / 16 \). Note that this agree with (2.22) of [18] in the limit of no gauge field, \( C,D \rightarrow 0 \). Hereafter, we set \( D = 0 \) for simplicity, which means that there is only magnetic field. From Eq. (20), we expect that the dynamics does not change so much even if there are both electric and magnetic fields.

Following [18], we define effective potential \( V_{\text{eff}} \) for the radial motion as,

\[
V_{\text{eff}}(r) = E(p_r = 0) = h^{-1} \left[ \sqrt{(1 + Ch)^2 + h l^2 / r^2} - q \right]. 
\]

The asymptotic behavior depends on the charge and the dimension of the brane. For \( p = 5 \),

\[
V_{\text{eff}}(r) \rightarrow \begin{cases} 
\frac{1}{\sqrt{h}} r^{5-1/2} & \text{for } r \rightarrow 0 \\
1 + C - q + k(q - 1) r^{-1} + \left[ \frac{l^2}{2(1+C)} - k(q - 1) \right] r^{-2} & \text{for } r \rightarrow \infty
\end{cases}
\]

For \( p \leq 4 \),

\[
V_{\text{eff}}(r) \rightarrow \begin{cases} 
\frac{1}{\sqrt{h}} r^{(5-p)/2} & \text{for } r \rightarrow 0 \\
1 + C - q + \frac{l^2 (1+C)}{2} r^{-2} & \text{for } r \rightarrow \infty
\end{cases}
\]

As is pointed out in [18], there exist stable bound orbits in the case of anti-6-brane.

Behavior of the effective potential is shown in Fig. 4 and 5. Fig. 1 and 2 shows the effective potential of p-brane and anti-p-brane for various \( p \), respectively. From this, we can see that there can be stable bound state in the case of anti-6-brane, as expected. Note that the position of the potential minimum, \( r_{\text{min}} \), depends on the angular momentum \( l \) and \( r_{\text{min}} \) can be much larger than \( l_s \) if \( l \) is sufficiently large. Fig. 4 shows the effective potential of 6-brane for various \( C \). As can be seen, qualitative feature does not depend on \( C \).

Using Eq. (13), (14) and (20), \( \dot{r} \) can be expressed in terms of \( r,E,l \):

\[
\dot{r}^2 = h^{-1} \left[ 1 - \frac{r^2 (1 + Ch)^2 + h l^2 (u^2 + u'^2)}{r^2 (Eh + q)^2} \right]. 
\]

We can follow the motion of the brane by integrating this equation. Since, as can be seen from Eq. (10) the scale factor on the brane is a function of \( r \), its evolution can also be calculated from this equation as we discuss in the next subsection. Note that this equation corresponds to Friedmann equation and that this reduces to the Friedmann equation in (21) in the limit of \( C \rightarrow 0 \).

The brane trajectory can be calculated as follows: define \( u \equiv 1 / r \) and then,

\[
u' \equiv \frac{du}{d\theta} = - r^2 \frac{dr}{d\theta} = - r^{-2} \frac{\dot{r}}{\dot{\theta}} = - \frac{p_r}{r}. 
\]

Eliminating \( p_r \) from Eq. (20) using this equation, we obtain,

\[
E = h^{-1} \left[ \sqrt{(1 + Ch)^2 + h l^2 (u^2 + u'^2)} - q \right], 
\]

from which the orbit is obtained as,

\[
\theta - \theta_0 = \int_{1/r_0}^{1/r} \frac{du}{\sqrt{A + Bu^2 - p - u'^2}}, 
\]

where,

\[
A = l^{-2} (E^2 + 2Eq - C^2 - 2C) \\
B = E^2 - C^2. 
\]

Thus the orbit of the probe brane is equivalent to that of a classical nonrelativistic particle in the central potential proportional to \( r^{p-7} \), even when there exists a gauge field on the brane. Especially for \( p = 6 \), the bound orbit is closed.
IV. COSMOLOGY ON PROBE BRANE

A. Evolution of Scale Factor

From the induced metric on the brane Eq. (10), the scale factor $a$ is given by,

$$a = h^{-1/4}.$$  \hfill (31)

On the other hand, the cosmological time $\tau$ on the brane is expressed as,

$$\tau = \int h^{-1/4} \sqrt{1 - h^{2} + 2\dot{h}^{2}} dt$$

$$= \int h^{-1/4} \frac{1 + Ch}{Eh + q} dr$$

$$= \int h^{1/4} \frac{1 + Ch}{\sqrt{(Eh + q)^{2} - (1 + Chh - h^{2}/r^{2})}} dr.$$  \hfill (32)

Here we used Eq. (19) and (20) in the second equation and Eq. (25) in the last equation. Thus, from Eq. (31) and (32), the scale factor $a$ can be obtained as a function of $\tau$.

Here we define two characteristic radii: gravitational radius $r_{g}$ and gauge-field radius $r_{c}$. The former corresponds to the Schwarzschild radius,

$$r_{g} \equiv k^{1/(7-p)}.$$  \hfill (33)

It should be noted that,

$$h(r) \approx \begin{cases} 
\frac{k}{r^{7-p}} & \text{for } r \ll r_{g} \\
1 & \text{for } r \gg r_{g}. 
\end{cases}$$  \hfill (34)

The latter represents the radius, below which the approximation of the Lagrangian (12) breaks down ($l_{s}^{2}F_{\mu\nu}F^{\mu\nu} = Ch(r_{c}) = 1$):

$$r_{c} \equiv \left( \frac{Ck}{1-C} \right)^{1/(7-p)} 
\approx (Ck)^{1/(7-p)}.$$  \hfill (35)

Hereafter we consider the case $r_{g} \gg l_{s}$ because otherwise the scale factor does not change so much in the region where the background solution is reliable ($r \gg l_{s}$).

Then let us consider a situation that the probe brane goes away from the neighborhood of the source branes (but $\tau = r_{0} \gg l_{s}, r_{c}$, of course) to infinity. When $r \ll r_{g}$, the relation between the scale factor and the cosmological time is simple. In this case Eq. (32) becomes, noting that $h \gg 1$ and $Ch \ll 1$,

$$\tau \approx \int_{r_{0}}^{r} h^{1/4} \frac{1 + Ch}{Eh} dr$$

$$\approx E^{-1}k^{-3/4} \int_{r_{0}}^{r} r^{3/(7-p)/4}(1 + Ckr^{p-7}) dr.$$  \hfill (36)

Note that this is independent on $q$ in this limit. First we consider the case $C = 0$. Then,

$$\tau = \frac{4}{25 - 3p} E^{-1}k^{-3/4} \left( r^{(7-p)/4} - r_{0}^{(7-p)/4} \right).$$  \hfill (37)

At late time ($r \gg r_{0}$), we obtain,

$$\tau \propto r^{(7-p)/4},$$  \hfill (38)

from which the evolution of the scale factor is obtained as,

$$a(\tau) = h^{-1/4} \approx (kr^{p-7})^{-1/4} \propto \tau^{(7-p)/(25-3p)}.\hfill (39)$$

Here $(7-p)/(25-3p) = 1/7, 1/5, 3/13, 1/4, 5/19, 3/11$ for $p = 6, 5, \ldots, 1$. Although the expansion becomes faster with smaller $p$, acceleration phase cannot be realized.

If $C \neq 0$, a correction term is added,

$$a(\tau) \propto (\tau^{(7-p)/(25-3p)} - ACa_{1}(\tau)),\hfill (40)$$

where $A$ is a constant which depends on $E, k, p$ and $a_{1}$ is, to leading term,

$$a_{1}(\tau) = \begin{cases} 
\tau^{-3/(7-p)/(25-3p)} & \text{for } p \geq 4 \\
\tau^{-3/4} \log \tau & \text{for } p = 3 \\
\tau^{-1}(r_{0}^{-(3-p)/4} - B\tau^{-16/(3-p)(25-3p)}) & \text{for } p \leq 2, 
\end{cases}$$  \hfill (41)

where $B$ is also a constant which depends on $E, k, p$. It should be noted that the effect of the gauge field decrease as the brane expands since its energy density decreases as $h \propto a^{-4}$.

When $r$ becomes much larger than $r_{g}$, the scale factor stops to evolve and becomes almost unity. The behavior of the scale factor in the case of no gauge field is shown in Fig. 4. As is expected, the scale factor evolves by power-law and then decelerate quickly to become unity. Fig. 5 shows the effect of the gauge field on the brane. As can be seen, the effect is very small even if $C$ is as large as possible and the late-time behavior is independent on the existence of the gauge field.

We can also see the evolution of the scale factor by the effective Friedmann equation which can be derived from Eq. (25):

$$a^{-2} \left( \frac{da}{d\tau} \right)^{2} = \frac{(7-p)^{2}}{16} k^{-\frac{3}{2}p} a^{-\frac{2(1-p)}{7-p}} (1 - a^{3})^{\frac{2(8-p)}{7-p}} 
\times (1 + Ca^{-4})^{-2}$$

$$\times \left[ (E^{2} - C^{2})a^{-4} + 2(Eq - C) - l^{2}k^{-\frac{7-p}{7-p}} a^{-\frac{7-p}{7-p}}(1 - a^{3})^{\frac{7-p}{7-p}} \right],$$  \hfill (42)

which agrees with (22) in the limit of $C \to 0$.

B. High Energy Limit

Here we consider the probe brane to be a Nambu-Goto brane with gauge field and the same R-R charge
as a D-brane, for which the Lagrangian \( \text{L} \) is exact. In this case, we can take a high-energy limit \((Ch \gg 1)\). Although, for a D-brane, this limit is in contradiction to the approximation which we used to derive \( L \), we could still obtain tendency of high-energy effect, as is often done in higher derivative theory. In this regime, Eq. 32 becomes, 

\[
\tau \approx \int_{r_0}^{r} \frac{C}{\sqrt{E^2 - C^2}} dr \\
\approx \frac{C k^{1/4}}{\sqrt{E^2 - C^2}} \int_{r_0}^{r} r^{p-7/4} dr.
\]

For \( p \geq 4 \), 

\[
\tau = \frac{4}{p-3} \frac{C k^{1/4}}{\sqrt{E^2 - C^2}} \left( r^{(p-3)/4} - r_0^{(p-3)/4} \right) \propto r^{(p-3)/4},
\]

then, 

\[
a(\tau) \propto \tau^{(7-p)/(p-3)},
\]

where \((7-p)/(p-3) = 1/3, 1, 3\) for \( p = 6, 5, 4 \). Thus accelerating expansion is realized for \( p = 4 \). Next, for \( p = 3 \), 

\[
\tau = \frac{C k^{1/4}}{\sqrt{E^2 - C^2}} \log \frac{r}{r_0},
\]

then, 

\[
a(\tau) = k^{-1/4} r_0 \exp \left( \frac{\sqrt{E^2 - C^2} \tau}{C k^{1/4}} \right).
\]

Thus the scale factor increase exponentially. Finally for \( p \leq 2 \), 

\[
\tau = \frac{4}{3-p} \frac{C k^{1/4}}{\sqrt{E^2 - C^2}} \left( r_0^{-(3-p)/4} - r^{-(3-p)/4} \right),
\]

then, 

\[
a(\tau) = k^{-1/4} \left( r_0^{-(3-p)/4} - \frac{3-p}{4} \sqrt{E^2 - C^2} C k^{1/4} \tau \right)^{-(3-p)/2}.
\]

It can be easily shown that the expansion is accelerating in this case. These analyses are confirmed in Fig. 5.

As stated in the previous subsection, the energy density of the gauge field decrease as the brane expands. With the parametrization in Fig. 6 the gauge field is dominant for all over the evolution since \( C \) is sufficiently large so that \( Ch \) at infinity is still large \((Ch(r = \infty) = C = 1)\). If \( C \) is smaller than unity, late phase will behave like that of the case discussed in the previous subsection, even if accelerating expansion occurs in early phase. In Fig. 4 we show the cases with intermediate \( C \). We can see the transition from accelerating phase to decelerating phase. Of course, the transition occurs earlier with smaller \( C \).

C. Einstein Frame

Finally, we give the evolution of the scale factor in the Einstein Frame. The procedure is almost the same as in the String Frame. The induced metric in the Einstein Frame is, 

\[
ds^2 = -h^{-(7-p)/8} (1 - hv^2) dt^2 + h^{-(7-p)/8} \delta_{ij} dx^i dx^j.
\]

Then the cosmological time is, 

\[
\tau = \int_{r_0}^{r} h^{-(7-p)/16} \sqrt{1 - hv^2} dr.
\]

With \( C = 0 \), the scale factor evolves as, for \( r \gg r_b \), 

\[
a(\tau) = h^{-(7-p)/16} \propto \tau^{(7-p)/(11-p)^2},
\]

where the index is \((7 - p)^2/(11 - p)^2 = 1/25, 1/9, 9/49, 1/4, 25/81, 9/25 \) for \( p = 6, 5, \cdots, 1 \). In high-energy limit \((Ch \gg 1)\), for \( p \neq 3 \), 

\[
a(\tau) \propto h^{(7-p)^2/(3-p)^2},
\]

where \((7-p)^2/(3-p)^2 = 1/9, 1, 9, 25, 9\) for \( p = 6, 5, 4, 2, 1 \). For \( p = 3 \), 

\[
a(\tau) \propto \exp \left( \frac{\sqrt{E^2 - C^2} \tau}{C k^{1/4}} \right).
\]

Thus, the condition that accelerating expansion occurs is the same as in the String Frame. It should be noted that the induced metric in the String Frame and the Einstein Frame coincide with each other for \( p = 3 \) because the dilaton \( \phi \) is constant in this case.

V. DISCUSSION

In the previous section, we dealt with a simple situation that a probe brane goes away from the neighborhood of the source branes to infinity. If the probe brane approach the source branes, the scale factor decreases as the inverse of that in the previous section. Then the other situations, for example, scattering and bound state of branes, are easy to imagine. In the former case, the brane contracts first, then bounces and finally expands. In the latter case, the brane continues to expand and contract periodically.

In this paper, we followed the dynamics of a probe brane, that is, we neglected the back-reaction. This is justified if the probe brane is light compared to the source branes. This means \( N \gg 1 \), which we assumed in the analyses in section IV. If \( N \sim 1 \), we have to treat both branes equally and the self-gravity of the branes must be taken into account \[29, 30\].

Our analysis assumes a stability of the probe brane. There are possible instabilities due to brane bending and
radiation from the brane \cite{18}. Ref. \cite{18} has given a preliminary analysis on such instabilities. They have found, for $p = 6$, that the brane is stable classically against bending and that the radiation is dominated by the one into the bulk dilation field, which can be made sufficiently small by appropriate choice of the string coupling constant. One of the key assumptions they made is to treat the brane motion as non-relativistic one. In other words, their results have been obtained in the large-separation limit: $k/r - p -1 \ll 1$ (or $r - r_g$ in our notation). We obtain some of the expressions for the scale factor for $r \ll r_g$ where the relativistic treatment is necessary in strict sense but we expect that the relativistic corrections do not change the stability discussed in \cite{18}. Recently it was shown in \cite{31} that time-variations in the background moduli fields generally preclude the existence of stable elliptical orbits.

Finally, although our study is based on the approximated Lagrangian \cite{12}, it would be quite interesting and important to study the exact Lagrangian. This will be our future work.

VI. SUMMARY

In this paper, we investigated the evolution of the scale factor of a probe Dp-brane which move in the background of source Dp-branes. When the probe brane move away from the source branes, it expands by power law, whose index depends on the dimension of the brane. If the energy density of the gauge field on the brane is subdominant, the expansion is decelerating irrespective of the dimension of the brane. On the other hand, when the probe brane is a Nambu-Goto brane, the energy density of the gauge field can be dominant, in which case accelerating expansion occurs for $p \leq 4$. The accelerating expansion stops when the brane has expanded sufficiently so that the energy density of the gauge field become subdominant. Although this is not the case with a probe D-brane, we could still obtain tendency of high-energy effect of the Born-Infeld action.

The system which is investigated in this paper is too simple to be our universe. However, further investigation will give understanding for the relation between superstring theory and our universe.

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FIG. 1: Effective potential $V_{\text{eff}}$ for the radial motion of the probe brane, varying its spatial dimension $p$. Other parameters are set as $k = l = 1$ and $C = 0$.

FIG. 2: Effective potential $V_{\text{eff}}$ for the radial motion of the probe anti-brane, varying its spatial dimension $p$. Other parameters are set as $k = l = 1$ and $C = 0$. 
FIG. 3: Effective potential $V_{\text{eff}}$ for the radial motion of the probe anti-6-brane, varying the energy scale $C$ of the gauge field on it. Other parameters are set as $k = l = 1$.

FIG. 4: Evolution of the scale factor $a(\tau)$ without the gauge field on the brane for various $p$. Other parameters are set as, $k = 10^8, E = 10^3, l = 10, q = -1, r_0 = 1$. 
FIG. 5: Evolution of the scale factor $a(\tau)$ with the gauge field on the brane for various $C$. Other parameters are set as, $p = 4, k = 10^8, E = 10^3, l = 10, q = -1, r_0 = 1$.

FIG. 6: Evolution of the scale factor $a(\tau)$ of the brane dominated by the gauge field for various $p$. Other parameters are set as, $k = 10^8, E = 10^3, l = 10, q = -1, r_0 = 1, C = 1$. 
FIG. 7: Evolution of the scale factor $a(\tau)$ of the brane dominated by the gauge field for various $C$. Other parameters are set as, $p = 3, k = 10^8, E = 10^3, l = 10, q = -1, r_0 = 1$. 