Alignment of the CMS silicon Tracker using Millepede II

Peter Schleper, Georg Steinbrück, Markus Stoye
University of Hamburg, Germany.
E-mail: markus.stoye@cern.ch

Abstract. The positions of the silicon modules of the CMS tracker will be known to \( O(100 \, \mu\text{m}) \) from survey measurements, mounting precision and the hardware alignment system. However, in order to fully exploit the capabilities of the tracker, these positions need to be known to a precision of a few \( \mu\text{m} \). Only a track-based alignment procedure can reach this required precision. Such an alignment procedure is a major challenge given that about 50,000 geometry constants need to be measured. Making use of the novel \( \chi^2 \) minimization program Millepede II an alignment strategy has been developed in which all detector components are aligned simultaneously and all correlations between their position parameters taken into account. Tracks from different sources such as \( Z^0 \) decays and cosmic ray muons, plus information about the mechanical structure of the tracker, and initial position uncertainties have been used as input for the alignment procedure. A proof of concept of this alignment strategy is demonstrated using simulated data.

1. CMS Tracker Alignment Challenge
A precise alignment of the silicon tracker of CMS [1, 2] is mandatory in order to fully exploit its physics capabilities. The initial uncertainties on the module positions from the mounting precision, survey measurements, and the laser alignment system will be of the order of 100 \( \mu\text{m} \). The effect of such a misalignment on the performance is expected to be significant. For example, the transverse momentum resolution in the central region decreases from about 1% to 5% for muons with a transverse momentum of 100 GeV. Only track-based alignment procedures will be able to improve on this initial situation. To avoid significant adverse effects due to misalignment, the positions of detectors should be known to the order of a few \( \mu\text{m} \), which is an order of magnitude smaller than the typical intrinsic resolution of the sensors. Given the size of the tracker, it has a volume of 24.4 m\(^3\) filled with 13300 individual modules, this is a challenging alignment task. Modules are called the detector units, which provide the individual hits for track reconstruction. Some of the modules (sterro modules) are constructed from two silicon strip sensors, which are rotated with respect to each other by 100 mrad, in order to obtain two dimensional measurements.

2. Alignment Strategy
The aim of a track-based alignment procedure is to reduce the bias and uncertainty of the fitted track parameters and to minimize the \( \chi^2 \) of the track fits by correcting the positions of...
Figure 1. Examples of $\chi^2$-invariant deformation which introduce biases on reconstructed track parameters $\phi$ and $\kappa$. The dashed line illustrates the true trajectory. The different layers are rotated by different amounts, as illustrated by the arrows for the last layer. The red line shows the reconstructed trajectory, if a shearing deformation is present, and the green line visualizes a reconstructed trajectory, if a bending deformation of the tracker occurred. In both cases all trajectories remain to be helices and hence do have minimal $\chi^2$.

the detector components. The minimization of the $\chi^2$ is important to ensure track and vertex recognition, because the $\chi^2$ of the track fit can be used to identify hits that make exceptionally large contributions to the $\chi^2$ and hence are likely to be incorrectly associated to tracks. However, even with the $\chi^2$ minimized and the pattern recognition working well, it is still possible to end up with biased measurements of track parameters due to misalignment. Correlated displacements of sensors that introduce a track parameter bias, but do not change the mean $\chi^2$, are clearly a major challenge. Example of such deformations are illustrated in figure 1. In this example the layers of the tracker are rotated by different amounts around the $z$ (beam) axis, leading to a bias in the $\phi$ (azimuthal) angle measurement, the impact point measurement, and the measurement of the curvature of the trajectory. Systematic studies about such deformations can be found in [6].

The most important ingredients for alignment are the tracks used. In addition to tracks from proton-proton collisions, muons originating from the proton beam halo and cosmic rays will be important, because they cross modules, which are otherwise unconnected by tracks. In addition, correlated misalignment can be reduced with the help of reconstructed particle decays such as $Z^0 \rightarrow \mu\mu$.

Another key ingredient are the known uncertainties (from survey measurements and mounting precision) on the sensor positions and the correlations between them, which are introduced by the mechanical support structures of the tracker.

To optimally use these pieces of information, an alignment algorithm should be able to fulfill the following demands:

- Inclusion of all correlations between position parameters.
- Incorporation of survey measurements.
- Fast turn around time and computational feasibility.

The algorithm used, Millepede, is briefly described in the next section. An overview of the alignment ingredients used is given in figure 2.

Figure 2. Illustration of the alignment strategy. All elements of this strategy have been used except the Laser Alignment System (LAS) and beam halo data which have not yet been available.

Alignment Algorithms  Most track-based algorithms are based on the $\chi^2$ minimization principle. In the CMS tracker a track typically consists of about 20 independent measurements (e.g. seven 2D hits and 6 single strip module hits (1D) in the barrel region) and such that the five parameters of a helix track are, in principle, overdetermined. These measurements, $u_{mj}$, are compared to predictions from the track model. The predicted measurements, $u_{pj}$, from the track model depend, for track $j$, on the vector of track parameters, $\tau_j$, and the parameters, $p$, that describe the position, orientation and deformation of the detectors. The normalized residual, $z_{ij}$, between the predicted hit position and the recorded measurement of hit $i$ is given by:

$$z_{ij} = \frac{u_{ij;m} - u_{ij;p}(\tau_j, p)}{\sigma_{ij}},$$

(1)

The uncertainties, $\sigma_{ij}$, for each module do not contain correlations between the hit measurements, which are, in any case, generally not significantly correlated. Exceptions occur only if measurements from different sensors are combined into a single measurement (stereo modules) or if particle interactions with material are a major source of uncertainty, which is only the case for low momentum tracks. For the high momentum tracks used in this study, these correlations can be neglected.

Requiring optimal agreement between the track model and the data means minimizing a function that depends on the normalized residuals. Most commonly the function

$$\chi^2(\tau, p) = \sum_j \left( \sum_i z_{ij}^2(\tau_j, p) \right),$$

(2)

is minimized with respect to all $\tau_j$ and $p$. Generally, all overdetermined parameters from objects that are reconstructed in the tracker, like vertices for example, can be used for alignment.

Millepede II
There are standard methods to minimize a $\chi^2$-function. If possible, the $\chi^2$ minimization problem is linearized [3]. This procedure leads in principle to optimal results in one iteration for quadratic
Linear equality constraints can be implemented via Lagrangian multipliers. Iterations are eventually needed to reduce errors due to linearization or to improve outlier rejection. All demands on an alignment algorithm presented in the beginning of this section are fulfilled. The problem of this procedure is, that all track parameters of each track and all alignment parameters are fitted simultaneously, which leads to millions of free parameters. The large number of parameters required the development of a scheme to reduce the matrix size. The Millepede algorithm incorporates a customized matrix reduction procedure. However the matrix size (rank) still equals the number of alignment parameters, which is in the case of CMS about 50,000. Hence the matrix equation cannot easily be solved. For this reason Millepede II has been developed, which includes several fast and memory saving methods to solve the matrix equation. The matrix equation solver used in the following study is GMRES with preconditioning, for details see [5, 6, 7].

3. Full Tracker Alignment Case Studies

In this section an alignment study using all tracker and pixel modules in both barrel and endcap are presented.

**Misalignment:** The first data scenario, as described in [8], is used as initial misalignment. This represents approximately the initial position uncertainties at startup. The correlated nature of displacements due to common support structures of modules is taken into account. Technically, the larger structures such as half barrels are misaligned first by moving all their modules in a correlated manner, afterwards the smaller support structures and finally the modules are misaligned. The hierarchy of the support structures is illustrated in figure 3. The pixel modules are assumed to be aligned to about 15 μm in this scenario.

**Alignment:** The full tracker is aligned, all of the pixel and strip detector of barrel and endcaps. This amounts to 13300 modules. 48 pixel modules at very high η are not aligned in this study since no track, which passes the selection criteria, hit them. The module alignment parameters are defined with respect to the center of the half barrels or the endcaps, respectively. This allows to take into account the correlated nature of their global displacements due to half barrel/endcap displacements. The half barrels and endcaps are given additional alignment parameters and equality constraints are applied to the modules parameters to suppress the movement of a half barrel/endcap due to a correlated movement of its modules. Each object is given four alignment parameters: the translation parameters \(u, v, w\), and the rotation parameter \(γ\). The parameters are illustrated in figure 4. The parameter \(v\) (parallel to strips direction) is skipped for single-sided strip modules. Altogether this amounts to 44432 alignment parameters.

**Datasets:** The \(Z^0 \rightarrow μμ\) decay is often called the “golden channel” for alignment. The high momentum of the muons leads to very small amounts of multiple scattering. In addition, pattern recognition and particle identification are relatively straight-forward for these isolated high momentum muons. The dataset used contains two million Drell-Yan \(Z^0 /γ^* \rightarrow μμ\) events produced with a pileup expected for nominal luminosity. The invariant mass of the muon pair is required to be at least 80 GeV. Each track is required to have at least 8 measurements and a transverse momentum of more than 15 GeV. Half a million events are used with a vertex constraint, forcing the two muon tracks to a common vertex. In addition the \(Z^0\) mass is used as additional input to the fit. A detailed description of this procedure can be found in [9]. Single muons, without any vertex or mass constraint, of 1.5 million \(Z^0\) events are used to mimic 3 million \(W \rightarrow μν\) events. These data roughly correspond to an integrated luminosity of 0.5 fb\(^{-1}\) of data taking. In addition a dataset of 25000 simulated cosmic ray muons with a momentum...
Figure 3. Illustration of the hierarchical support structures used for the (mis)alignment study. This software implementation of the support structures follows the physical support structures. However, small differences to the hardware structures do exist, e.g. the physical outer barrels do not have layers as support structures.

Figure 4. Schematic illustration of the alignment parameters. In brackets \((r, z, r\phi)\) are corresponding directions in the global CMS coordinate system in case of barrel modules.

of at least 50 GeV was used. All the cosmic ray muons transverse the central barrel region. The pattern recognition was assumed to work perfectly, in order not to suffer from a preliminary seeding procedure. Details of the simulation of cosmic muons can be found in [6, 10]. All cosmic ray tracks were required to consist of at least 18 measurements and have an initial \(\chi^2/\text{ndof}\) value below 10.
Numerical Stability: Undefined degrees of freedoms like the $\chi^2$-invariant deformation might affect the numerical precision of minimization procedures. Hence, initial uncertainties (presigmas [5]) are given to each alignment parameter. The uncertainty restricts the size of the change of the alignment parameter per iteration. Given that several iterations are needed, the final alignment parameter can be much larger than the assigned presigma. This procedure (also known as Levenberg-Marquardt method) stabilizes the numerical stability (improves the condition number) of the matrix. The applied presigmas for the alignment parameters of the modules are chosen to be a factor of 10 smaller than the initial uncertainties in the misalignment scenario. The presigmas for parameters of higher level structures are set to exactly equal the misalignment uncertainties. The applied presigmas are summarized in table 1.

| type               | u [\mu m] | v [\mu m] | w [\mu m] | $\gamma$ [\mu rad] |
|--------------------|-----------|-----------|-----------|---------------------|
| TPB half barrels   | 10        | 10        | 10        | 10                  |
| TIB half barrels   | 105       | 105       | 500       | 90                  |
| TOB half barrels   | 67        | 67        | 500       | 59                  |
| TPE endcap         | 5         | 5         | 5         | 5                   |
| TID layers         | 400       | 400       | 400       | 100                 |
| TEC endcap         | 57        | 57        | 500       | 46                  |
| TPB modules$^1$    | 13        | 13        | 13        | 10$^*$              |
| TIB modules$^1$    | 200       | 200       | 200       | 10$^*$              |
| TOB modules$^1$    | 100       | 100       | 100       | 10$^*$              |
| TPE modules$^1$    | 2.5       | 2.5       | 2.5       | 10$^*$              |
| TID modules$^1$    | 105       | 105       | 105       | 10$^*$              |
| TEC modules$^1$    | 20        | 20        | 20        | 10$^*$              |

Table 1. Initial uncertainties according to the first data scenario [8], as used for the alignment procedure. A * denotes uncertainties that are assumed for alignment, but are zero in the simulation of the first data misalignment scenario. The presigmas applied for larger support structures are identical the their initial uncertainty, while the presigmas of the modules ($^1$ rows) are a factor of ten smaller than their initial uncertainties.

Coordinate System: The coordinate system is defined by the constraint that the sum of the alignment parameter vectors of the pixel half barrels has to be zero. Hence the average position of the pixel modules defines the origin of the coordinate system.

Millepede II Options: In order to solve such a large alignment problem, the GMRES with precondition (see section 2) method in Millepede II is used. An outlier rejection method is applied, which down-weights hits which are significantly ($> 1.42 \sigma$) away from the hit position expected by the track model. Tracks with many down-weighed hits are completely rejected. More details can be found in [5, 6]. Given that for the initial fits many hits are down-weighed and tracks rejected due to misalignment, iteration for become necessary. With each iteration less hits are down-weighed due to misalignment, but the truly bad hits remain to be down-weighed. The number of the iterations is set to 5.

Results
The remaining displacements in the most sensitive ($r\phi$) direction are compared to the displacements prior to the alignment procedure in figure 5. The precision of the modules in the barrel region is about 10 $\mu$m and about 25 $\mu$m in the endcap region (figure 5 a,b).
positions of modules of the pixel barrel are determined to 1 \( \mu \text{m} \) and 2 \( \mu \text{m} \) uncertainty remains for the modules of the pixel endcaps. The small remaining displacements are dominated by correlated (\( \chi^2 \)-invariant) deformations. This can be seen in figure 6 a), where the displacement in the \( r\phi \) direction for the last layer of the tracker barrel is shown as a function of \( \phi \) and an oscillation is clearly visible. The correlated nature of the remaining misalignment becomes also obvious in figure 6 b), which shows the mean displacement in the \( y \) (vertical) coordinate as a function of the radial position of the module. A roughly linear increase of the mean displacement in the \( y \) is observed.

To study the effect on the physics measurements a \( \mu^+ \) sample with 100000 events and a transverse momentum of 100 GeV from a vertex positioned at (0,0,0) are used. The \( \chi^2/\text{ndof} \) of the track fits is shown in figure 7 a). The average \( \chi^2 \) values with the ideal geometry and after the alignment are very similar. The \( \chi^2 \) values with the initial misalignment are clearly much larger. The bias on the reconstructed transverse momentum is illustrated in figure 7 b). The initial biases are clearly almost completely removed. The reconstructed transverse momentum for all tracks is shown in figure 7 c). The relative error of the transverse momentum measurement at
100 GeV increases from $(1.68 \pm 0.008)\%$ to $(1.72 \pm 0.008)\%$ (statistical uncertainty) if the aligned geometry is used instead of the ideal geometry. A bias in the transverse momentum of $(0.1 \pm 0.01)\%$ is introduced. The impact of the remaining misalignment on the reconstruction of the $y$ coordinate from the point of closest approach to the beam line is shown in figures 7 d). A bias of only about 1 µm of the measurement in the $y$ coordinate is visible. Given that the resolutions remain similar to the resolutions with ideal geometry and the biases are also very small, it is not expected that this remaining misalignment would influence the physics results significantly.

The computing requirements of the alignment procedure are modest. The calculation of the alignment parameters (running of Millepede II) took only about 2h of computing time on a 64 bit computer and 2 MB of memory.

4. Conclusion

For the first time in CMS a full scale alignment procedure has been tested successfully with simulated data of 0.5 fb$^{-1}$ of integrated luminosity. The position precision reached is sufficient for precision physics analysis and the computing requirements are modest. Real data will lead to further challenges like eventually time dependent movements, uncertainties of the magnetic field, and other effects which are not yet simulated. These systematic differences between simulation and real data might influence the final result. On the other hand, more complementary datasets will be available in real data, such as informations from the hardware alignment system [11] or muons from the proton-beam halo [10]. Once these datasets are available, the implementation to Millepede is straightforward. This study marks an proof of principle for this alignment strategy.

Acknowledgment

We would like to thank Volker Blobel for many helpful discussions and for making Millepede II available.
Figure 7. Control plots which compare distribution obtained with ideal, aligned and misaligned geometry. 100 GeV $\mu^+$-track from a vertex of (0,0,0) have been used. a) $\chi^2$/ndof of the tracks fits. b) Reconstructed average transverse momentum as function of $\phi$. c) Reconstructed transverse momentum. d) Reconstructed impact point position in the y (vertical) coordinate.

References
[1] CMS Collaboration, “The CMS tracker system project technical design report,”, CERN/LHCC 98-6 CMS TDR 5, (1998).
[2] CMS Collaboration, “Addendum to the CMS Tracker TDR by the CMS collaboration,” CERN/LHCC 2000-16 CMS TDR Addendum 1, (2000).
[3] V. Blobel and E. Lohrmann, “Statistische und Numerische Methoden der Datenanalyse,” Teubner, ISBN 3-519-03242-0, (1998).
[4] V. Blobel and C.Kleinwort, “A new method for high-precision alignment of track detectors,” Contribution to the Conference on Advanced Statistical Techniques in Particle Physics, Durham, hep-ex/0208021, March 18-22 (2002).
[5] V. Blobel, “Millepede II manual DRAFT,” http://www.desy.de/~blobel (2007).
[6] Markus Stoye, PhD thesis at the University of Hamburg, CERN-Thesis-2007-026, ”Calibration and alignment of the CMS silicon tracker”, July (2007).
[7] R. Barrett et al. “Templates for the solution of linear systems: building blocks for iterative methods,” SIAM, 2nd Edition, (1994).
[8] I. Belotelov et al., “Simulation of misalignment scenarios for CMS tracking devices,” CMS-NOTE 2006-008.
[9] E. Widl et al., “Representation and estimation of trajectories from two-body decays” CMS-NOTE 2007-032 (2007).

[10] V. Drollinger, “Simulation of beam halo and cosmic muons,” CMS-NOTE 2005-012, (2005).

[11] CMS Collaboration, “Detector performance and software,” Physics TDR Volume I, CERN/LHCC 2006-001, CMS TDR 8.1, (2006).