Recently observed $P_c$ as molecular states and possible mixture of $P_c(4457)$

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Recently observed spectrum of $P_c$ states exhibits a strong link to $\Sigma_b D^{(*)}$ thresholds. In spite of successful molecular interpretations, we still push forward to wonder whether there exist finer structures. Utilizing the effective lagrangians respecting heavy quark symmetry and chiral symmetry, as well as instantaneous Bethe-Salpeter equations, we investigate the $\Sigma_b D^{(*)}$ interactions and three $P_c$ states. We confirm that $P_c(4312)$ and $P_c(4440)$ are good candidates of $\Sigma_c D$ and $\Sigma_c D^*$ molecules with spin-$\frac{1}{2}^+$, respectively. Unlike other molecular calculations, our results indicate $P_c(4457)$ signal might be a mixture of spin-$\frac{3}{2}^-$ and spin-$\frac{5}{2}^-$ $\Sigma_c D^*$ molecules, where the latter one appears to be an excitation of $P_c(4440)$. Therefore we conclude that, confronting three LHCb $P_c$ signals, there may exist not three, but four molecular states.

I. INTRODUCTION

The study of exotic states, especially XYZ and pentaquarks, has become a hot topic in recent years. Benefit from upgraded $r$-charm and $b$ factories such BESIII, LHCb and Belle, large amount of tetraquark candidates, as well as two pentaquark candidates were observed. These findings indeed extend our knowledges about non-perturbative QCD. However, the properties and inner structures of the states are still in debate, see Refs. [1–4] for reviews of experimental and theoretical status.

The great progress was made in 2015, when two pentaquark candidates $P_c(4380)$ and $P_c(4450)$ were reported by LHCb collaboration [5]. With an amplitude analysis, LHCb studied the process $\Lambda_b \to J/\psi K^- p$ and observed them in $J/\psi p$ final states. Both resonances have to be fulfilled with minimal quark content $c\bar{c}u\bar{u}d$, therefore they are good candidates of hidden-charm pentaquarks. Just after the discovery, different dynamics were applied to look into their nature: the molecular states of $\Sigma_c D^{(*)}$ [6–8], the compact pentaquark structures [9–12], the dynamical effects [13–15], and etc. Till now, people have made tremendous effects to clarify their constituents and quantum numbers [1, 3]. Besides, some predictions have already made before the observations of the $P_c$ states [16, 18–20, 66].

Recently, LHCb [21] re-examined the process $\Lambda_b \to J/\psi K^- p$ with nine-times larger decay samples compared to Ref. [5], and reveals a more sophisticated structure in $J/\psi p$ invariant mass spectrum than before: $P_c(4450)$ signal splits into two peaks $P_c(4440)$ and $P_c(4457)$, while a new pentaquark state $P_c(4312)$ shows up in the lower mass region. The parameters of these $P_c$ states are collected in Tab. 1. From Tab. 1, we notice that the masses of $P_c(4440)$ and $P_c(4457)$ are slightly below $\Sigma_c D^{(*)}$ threshold, while $P_c(4312)$ is quite close to $\Sigma_c \bar{D}$, therefore it is strongly believed that $\Sigma_c D^{(*)}$ interactions are responsible for the enhancements in the $J/\psi p$ invariant spectrum. So far, a number of papers have came out to interpret the new results, which carry different opinions such as the molecular states [22–43], pentaquarks [44–50], hadro-charmonium [51], and etc.

As indicated above, $\Sigma_c D^{(*)}$ molecular interpretations seem to be the most suitable option. Although many papers (such as Refs. [22, 35]) have confirmed their molecular nature, it is still necessary to examine it from a different approach. Furthermore, the old $P_c(4450)$ splits into $P_c(4440)$ and $P_c(4457)$, so are there any chances that three $P_c$ states may have finer structures considering the complexity of threshold interaction?

To answer the question, we will study $\Sigma_c D^{(*)}$ interactions and the three $P_c$ states. We first calculate the heavy-hadron interaction amplitudes within the chiral symmetry and heavy quark symmetry [61–66], then iterate the obtained interaction kernel into the Bethe-Salpeter equation (BSE) to explore the nature of the $\Sigma_c D^{(*)}$ heavy-hadron systems. The Bethe-Salpeter (BS) methods adopted here have been successfully applied to investigate the properties of the meson systems, including the mass spectra, hadronic transitions and weak decays [52–60], as well as the recent $\Xi_c$ study [67]. Therefore extending to the meson-baryon molecular systems is quite natural.

Besides, in the BS framework, the relativistic effects and the mixing of the different partial waves can be automatically

| State   | $M$ [MeV]                      | $\Gamma$ [MeV] |
|---------|-------------------------------|----------------|
| $P_c^+$ | $3111.5$ with 6.8$\pm 0.6$     | 9.8$\pm 2.7$   |
| $P_c^-$ | $4440.3$ with 1.3$\pm 1.4$     | 20.6$\pm 4.9$  |
| $P_c^0$ | $4457.3$ with 0.6+1.7$\pm 0.9$ | 6.4$\pm 2.0$   |

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involved, in spite of some approximations. It is worth mentioning that, to understand the near threshold phenomena and resonance formations in a two-hadron system, a non-perturbative resummation is quite crucial. Such resummation has been considered in chiral dynamics of nucleon-nucleon systems [68–70] and heavy meson systems [71], as well as the phenomenological studies of molecular states and XYZ exotics (see reviews [1, 2]). A non-perturbative resummation is to partially summit interactions to all order, which can be achieved by a proper iterating equation such as the Lippmann-Schwinger equation, the Bethe-Salpeter equation and etc. However, the studies mentioned above take simplified or non-relativistic equations. Therefore, in this article we also want to focus on the BS equation itself to push forward the resummation method.

In the present work, we will study $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$ states by investigating the effective Lagrangians with the heavy quark symmetry and chiral symmetry to calculate the transition amplitudes of the three systems [71], as well as the phenomenological studies of nucleon-nucleon systems [68–70] and heavy meson systems [63, 65, 66]. Such resummation has been considered in chiral dynamics of nucleon-nucleon systems [68–70].

The interactions between $S$-wave heavy baryon and light mesons are [63, 65, 66]

$$\mathcal{L}_{\mathcal{B}i} = +i\beta_2(l_\mathcal{B}3)\langle\bar{B}\mathcal{B}\rangle + i\beta_1(l_\mathcal{B}3)\sigma\mathcal{B},$$

$$\mathcal{L}_S = \frac{i}{2} g_1\varepsilon^{\mu\nu\lambda} v_\mu (\bar{S}_\mu v_\nu S_\lambda) + i\beta_3 (\bar{S}_\mu v_\nu (V^\mu) S^\nu) + \lambda_5 (\bar{S}_\mu S^\nu) + \lambda_5 (\bar{S}_\mu \sigma S^\nu).$$

Here, $S^{ab}_{\mu}$ is composed of Dirac spinor operators

$$S^{ab}_{\mu} = -\sqrt{\frac{1}{3}} (\gamma_\mu + v_\mu) \gamma^5 S^{ab}_{6\mu},$$

$$\bar{S}^{ab}_{\mu} = \sqrt{\frac{1}{3}} \bar{S}^{0\mu} (\gamma_\mu + v_\mu) + \bar{S}^{ab}_{6\mu},$$

with

$$\mathcal{B}_3 = \begin{pmatrix} 0 & \Lambda_+^+ & \Xi_+^+ \\ -\Lambda_+^{-} & 0 & \Xi_0^+ \\ -\Xi_0^- & -\Xi_0^0 & 0 \end{pmatrix},$$

$$\mathcal{B}_0 = \begin{pmatrix} \Sigma_+^{++} & \frac{1}{\sqrt{3}} \Sigma_+^{*+} & \frac{1}{\sqrt{6}} \Xi_+^{0} \\ \frac{1}{\sqrt{3}} \Sigma_+^{*+} & \Sigma_0^{*0} & \frac{1}{\sqrt{3}} \Xi_0^{*0} \\ \frac{1}{\sqrt{6}} \Xi_+^{0} & \frac{1}{\sqrt{3}} \Xi_0^{*0} & \Omega_0^+ \end{pmatrix}.$$  

We consider two isospin channels ($I = \frac{1}{2}, \frac{3}{2}$) of $\Sigma_c D^{(*)}$ in our work, which contain eigenstates:

$$\left| \frac{1}{2}, \frac{1}{2} \right\rangle = \sqrt{\frac{2}{3}} \left| \Sigma_+^{++}, D^{(*)} \right\rangle - \sqrt{\frac{1}{3}} \left| \Sigma_+^{*+}, D^{(*)} \right\rangle,$$

$$\left| \frac{3}{2}, \frac{1}{2} \right\rangle = \sqrt{\frac{1}{3}} \left| \Sigma_+^{++}, D^{(*)} \right\rangle + \frac{2}{3} \left| \Sigma_+^{*+}, D^{(*)} \right\rangle.$$

With preparations above, we are able to calculate the interaction kernels which will be iterated into the instantaneous Bethe-Salpeter equation later. These kernels represent tree-level one-meson-exchange diagrams, including $\sigma, \pi, \eta, \rho$ and $\omega$ exchanges.
The calculated interaction kernel for $\Sigma, \bar{D}$ is expressed as

$$K(s_{\perp}) = F^2(s^2_{\perp}) (V_1 + V_2 \xi), \quad (14)$$

where $F(s^2_{\perp})$ denotes the form factor for the interaction vertex.

In the following, we specify the $g_s, \ell_s, g, g_1, \beta, \beta_s, \lambda$ and $\lambda_5$ (in Eqs. (1), (4), (5) and (8)) to $\sigma_1, \sigma_2, \pi_1, \pi_2, \rho_1^T, \rho_1^T$ and $\rho_1^T$ respectively, for convenience. For $I = \frac{1}{2}$, potentials $V_1$ and $V_2$ read

$$V_1 = \frac{2\pi \sigma \sigma_2}{E_\rho} + \frac{\pi \pi \rho_1 \rho_2 \gamma_5}{2} \left( \frac{1}{E_\rho} + \frac{1}{E_{\omega}} \right), \quad (15)$$

$$V_2 = \frac{1}{3} \rho_1 \rho_2 \gamma_5 \left( \frac{2}{E_\rho} - \frac{1}{E_{\omega}} \right). \quad (16)$$

For $I = \frac{3}{2}$, $V_1$ and $V_2$ are

$$V_1 = \frac{2\pi \sigma \sigma_2}{E_\rho} + \frac{\pi \pi \rho_1 \rho_2 \gamma_5}{2} \left( \frac{1}{E_\rho} + \frac{1}{E_{\omega}} \right), \quad (17)$$

$$V_2 = -\frac{1}{3} \rho_1 \rho_2 \gamma_5 \left( \frac{1}{E_\rho} + \frac{1}{E_{\omega}} \right). \quad (18)$$

In the above, $E_\phi = \sqrt{s^2 + m_\phi^2}$ denotes the energy of the exchanged meson, with $\phi = \sigma, \eta, \rho, \omega$. Notice that in the $\Sigma, \bar{D}$ interaction, only $\sigma$ and $\rho(\omega)$ exchanges contribute.

The interaction kernel for $\Sigma, \bar{D}^*$ is written by

$$K^{\alpha \beta}(s_{\perp}) = F^2(s^2_{\perp}) \left\{ \kappa_1 g^{\alpha \beta} + \kappa_2 k \cdot l + \kappa_3 k \cdot l (\gamma^\rho s^\rho_{\perp} - \gamma^\rho s^\rho_{\perp}) ight\} + \kappa_4 \rho^\alpha \gamma^\beta. \quad (19)$$

For the isospin-$\frac{1}{2}$ states, we have potentials

$$\kappa_1 = -\frac{2\pi \sigma \sigma_2}{E_\rho} + \frac{\pi \pi \rho_1 \rho_2 \gamma_5}{2} \left( \frac{1}{E_\rho} - \frac{1}{6E_\eta} \right) + \frac{\pi \pi \rho_1 \rho_2 \gamma_5}{2} \left( \frac{1}{E_\rho} + \frac{1}{2E_{\omega}} \right), \quad (20)$$

$$\kappa_2 = \frac{\pi \pi \rho_1 \rho_2 \gamma_5}{2} \left( \frac{1}{E_\rho} - \frac{1}{2E_{\omega}} \right), \quad (21)$$

$$\kappa_3 = \frac{\pi \pi \rho_1 \rho_2 \gamma_5}{2} \left( \frac{1}{E_\rho} - \frac{1}{6E_\eta} \right) + \frac{2\pi \sigma \sigma_2}{E_\rho} \left( \frac{1}{E_\rho} - \frac{1}{2E_{\omega}} \right), \quad (22)$$

$$\kappa_4 = -\frac{\pi \pi \rho_1 \rho_2 \gamma_5}{2} \left( \frac{1}{E_\rho} - \frac{1}{6E_\eta} \right). \quad (23)$$

For the isospin-$\frac{3}{2}$ states,

$$\kappa_1 = -\frac{2\pi \sigma \sigma_2}{E_\rho} + \frac{\pi \pi \rho_1 \rho_2 \gamma_5}{2} \left( \frac{1}{E_\rho} + \frac{1}{3E_{\omega}} \right) - \frac{2\pi \sigma \sigma_2}{E_\rho} \left( \frac{1}{E_\rho} + \frac{1}{3E_{\omega}} \right) - \frac{1}{3} \rho_1 \rho_2 \gamma_5 \left( \frac{1}{E_\rho} + \frac{1}{E_{\omega}} \right), \quad (24)$$

$$\kappa_2 = \frac{\pi \pi \rho_1 \rho_2 \gamma_5}{2} \left( \frac{1}{E_\rho} + \frac{1}{E_{\omega}} \right). \quad (25)$$

$$\kappa_3 = -\frac{\pi \pi \rho_1 \rho_2 \gamma_5}{2} \left( \frac{1}{E_\rho} + \frac{1}{3E_{\omega}} \right) - \frac{2\pi \sigma \sigma_2}{E_\rho} \left( \frac{1}{E_\rho} + \frac{1}{3E_{\omega}} \right) - \frac{2\pi \sigma \sigma_2}{E_\rho} \left( \frac{1}{E_\rho} + \frac{1}{3E_{\omega}} \right), \quad (26)$$

$$\kappa_4 = \frac{\pi \pi \rho_1 \rho_2 \gamma_5}{2} \left( \frac{1}{E_\rho} + \frac{1}{3E_{\omega}} \right). \quad (27)$$

III. THE BETHE-SALPETER EQUATIONS OF THE $\Sigma, \bar{D}^*$ SYSTEMS

In this part, we further study the Bethe-Salpeter formalism of the meson-baryon system $\Sigma, \bar{D}^*$. Considering $J^P = 1^+$ or $0^-$ for $\bar{D}^*$ and $J^P = \frac{1}{2}^+$ for $\Sigma$, the corresponding Bethe-Salpeter equations and BS wave functions can be obtained.

A. $\Sigma(1^+)\bar{D}^*(1^-)$ system

FIG. 1: (color online) The Bethe-Salpeter equation of the meson-baryon system. The Greeks (red) denote the Lorentz indices, while the Romans (blue) represent the Dirac indices. $P, p_1(k_1)$ and $p_2(k_2)$ stand for the momenta of the pentaquark, meson component, and the baryon component respectively.

The Bethe-Salpeter equation for a meson-baryon system is schematically depicted in Fig. 1, which is written by

$$\Gamma^\alpha(P, q, r) = \int \frac{d^3k}{(2\pi)^3} (-i)K^{\alpha \beta}(P, k_{\perp}, q_{\perp}) [S(k_2)\Gamma^\beta(P, k, r)] \times D_{\beta}(k_1), \quad (28)$$

where $\Gamma(P, q, r)$ denotes the pentaquark (refered as $P_c$ below) vertex carrying total momentum $P$, inner relative momentum $q$, and spin state $r$. Here we have on-shell condition $P^2 = M^2$, with $M$ the $P_c$ mass. The inner relative momenta $q$ and $k$ are defined as

$$q = \alpha_2 p_1 - \alpha_1 p_2, \quad k = \alpha_2 k_1 - \alpha_1 k_2, \quad (29)$$

with $\alpha_1(2) \equiv \frac{M_{1(2)}}{M_{1(2)}}$. $k_{1(2)}$ and $M_{1(2)}$ are the momentum and mass of the meson component (baryon component) respectively. $S(k_2) = i\frac{1}{q_{\perp}-M}$ is the free propagator of the baryon, while the propagator $D^{\alpha \beta}$ (for the $J^P = 1^-$ meson) reads

$$D^{\alpha \beta}(p_1) = D(p_1) d^{\alpha \beta}(p_{1\perp}), \quad d^{\alpha \beta}(p_{1\perp}) = -g^{\alpha \beta} + \frac{p_{1\perp}^{\alpha} p_{1\perp}^{\beta}}{M^2},$$

where $D(p_1) = i\frac{1}{p_{1\perp}^2 - m^2}$. $p_{1\perp} = p_1 - p_1 \cdot v \cdot v$. 

We define a four-dimensional BS wave function and a three-dimensional Salpeter wave function below:

\[
\psi_\beta(P, q) = S(p_2)\Gamma^\gamma(P, q, r)D_{\beta\gamma}(p_1),
\]

\[
\varphi_\beta(P, q_\perp) \equiv -i \int \frac{dp_1}{2\pi} \psi_\beta(P, q),
\]

where \( q_\perp = q - v \) and \( q_\parallel = q - q_\perp v \).

Performing the contour integral over \( q_\perp \) on both sides of Eq. (30) (see appendix A for details), we obtain the Salpeter equation (SE),

\[
\varphi_\alpha(P, q_\perp) = \frac{1}{2\omega_1} \left[ \frac{\Lambda^+}{M - \omega_1 - \omega_2} + \frac{\Lambda^-}{M + \omega_1 + \omega_2} \right] d_{\alpha\beta}(p_{1\perp})
\times \Gamma^\beta(P, q_\perp).
\]

In the above, \( \omega_i = \sqrt{M_i^2 - p_{i\perp}^2} \) with \( i = 1, 2 \) stands for the kinematic energy of the constituent meson or baryon. We also have the projector operators,

\[
\Lambda^\pm(p_{2\perp}) = \frac{1}{2} \left[ 1 \pm \hat{H}(p_{2\perp}) \right] \gamma^0
\]

with \( \hat{H} = H/\omega_2 \), and the Dirac Hamilton \( H(p_{2\perp}) = (\hat{p}_{2\perp} + m_2)\gamma^0 \).

We further split \( \varphi_\beta \) into a positive and a negative energy wave functions:

\[
\varphi_\beta = \varphi_\beta^+ + \varphi_\beta^-, \quad \varphi_\beta^\pm(P, q_\perp) \equiv \Lambda^\pm\gamma^0 \varphi_\beta.
\]

Notice in the weak binding condition \( M \sim (\omega_1 + \omega_2) \), we have \( \varphi^+ \gg \varphi^- \), i.e. the positive energy wave function \( \varphi^+(q_{\perp}) \) dominates. Combined with Eq. (34), the SE of \( \Sigma_\gamma \tilde{D}_c \) system can be further simplified to a “Schrodinger-like” equation:

\[
M\varphi_\alpha = (\omega_1 + \omega_2)\hat{H}(p_{2\perp})\varphi_\alpha + \frac{d_{\alpha\beta}\gamma^0\Gamma^\beta(q_{\perp})}{2\omega_1},
\]

with \( \Gamma^\beta(q_{\perp}) \) the integral of Salpeter wave function and the kernel,

\[
\Gamma^\beta(q_{\perp}) = \int \frac{d^3k_{\perp}}{(2\pi)^3} K^{\beta\gamma}(s_{\perp})s_{\gamma}(P, k_{\perp}).
\]

Notice that the interaction kernel \( K(s_{\perp}) \) is assumed to be instantaneous, thus no dependence on the time component of the momentum transfer \( s = (k - q) \).

Let us focus on the three-dimensional BSE (35). We can see that, the Salpeter wave function \( \varphi_\alpha(q_{\perp}) \) in Eq. (32) is just transformed to an integral-type eigenvalue equation. In Eq. (35), the first term of the right side, which is determined by Dirac Hamiltonian \( H \), stands for the kinetic energy. The second term contains the interaction kernel \( K \), therefore represents the potential energy.

In general, the normalization of a BS wave function is expressed as

\[
- i \int \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \bar{\varphi}_\alpha(P, q, \vec{r}) \frac{\partial}{\partial p_0} P^\alpha(P, k, q)\psi_\beta(P, k, r)
\]

\[= 2M\delta_{\vec{r}}, \]

where

\[
P^{\alpha\beta}(P, q, k) = (2\pi)^4\delta^4(k - q)S^{-1}(p_2)D^{-1\alpha\beta}(p_1)
\]

\[+ iK^{\alpha\beta}(P, k, q).\]

In the above, we have the inverse of the vector propagator

\[
D^{-1\alpha\beta}(p_1) = \theta_{\alpha\beta}D^{-1}(p_1),
\]

with \( \theta_{\alpha\beta} = -g^{\alpha\beta} + \frac{p_{0\perp}^\alpha p_{0\perp}^\beta}{\omega_1} \) as well as the identity \( \theta_{\alpha\beta}d_{\gamma\beta} = \delta_\alpha^\gamma \).

As mentioned before, the interaction kernel is assumed to be no dependence on \( P^0 \) and \( q_\perp \), namely, \( K^{\alpha\beta}(P, k, q) \equiv K^{\alpha\beta}(s_{\perp}) \), therefore the normalization would only involve the term related to two inversed propagators. After some deduction, Eq. (37) can be further simplified to

\[
\int \frac{d^3q_{\perp}}{(2\pi)^3} 2\omega_1 \theta_{\alpha\beta} \bar{\varphi}_\alpha(q_{\perp}, \vec{r})\gamma^0\varphi_\beta(q_{\perp}, \vec{r}) = 2M\delta_{\vec{r}},
\]

which is just the normalization condition of Salpeter equation (31).

**B. \( \Sigma_c(\frac{3}{2}^+)\tilde{D}(0^-) \) system**

Similarly, the Bethe-Salpeter equation for \( \Sigma_c\tilde{D} \) system reads

\[
\Gamma(P, q, r) = \int \frac{d^3k}{(2\pi)^3} (-i)K(P, k_{\perp}, q_{\perp})[S(k_2)\Gamma(P, k, r)D(k_1)],
\]

where \( \Gamma(P, q, r) \) denotes \( P_c \) vertex. The BS wave function \( \psi \) and related Salpeter wave function \( \varphi \) are also defined as

\[
\psi(P, q) = S(k_2)\Gamma(P, q, r)D(q_1),
\]

\[
\varphi(P, q_{\perp}) \equiv -i \int \frac{dp_0}{2\pi} \psi(P, q).
\]

Performing the contour integral on \( q_\perp \) over both sides of Eq. (41), we obtain the Salpeter equation

\[
M\varphi = (\omega_1 + \omega_2)\hat{H}(p_{2\perp})\varphi + \frac{\gamma^0\Gamma(q)}{2\omega_1},
\]

with

\[
\Gamma(q_{\perp}) = \int \frac{d^3k_{\perp}}{(2\pi)^3} K(s_{\perp})s_{\gamma}(P, k_{\perp}).
\]

Applying the same strategy above, we obtain the normalization of \( \varphi \):

\[
\int \frac{d^3q_{\perp}}{(2\pi)^3} 2\omega_1 \bar{\varphi}(q_{\perp}, \vec{r})\gamma^0\varphi(q_{\perp}, \vec{r}) = 2M\delta_{\vec{r}}.
\]
C. The constructions of the Salpeter wave functions and further reductions

We first turn to $\Sigma_c(1^-)\bar{D}(0^-)$ system. Accounting the spin-parity and the Lorentz structures, the Salpeter wave function ($J^P = \frac{1^-}{2^-}$) can be constructed as

$$\varphi(P, q, \lambda) = A(q)\gamma^5 u(P, r) = \left(f_1 + f_2 \frac{q_+}{q}\right)\gamma^5 u(P, r), \quad (46)$$

where $f_{1,2}(|q|)$ only depends on $|q|$. It is worth mention that, the wave function above can be rewritten in terms of the spherical harmonics $Y_{jm}$:

$$\varphi(P, q, \lambda) = C_0 \left[f_1 Y_{0}^0 + c_1 f_2 \left(Y_{1}^{1}Y_{1}^{-} + Y_{1}^{-1}Y_{1}^{+} - Y_{0}^{0}\right)\right] \times \gamma^5 u(P, r), \quad (47)$$

where $C_0 = 2\sqrt{\pi}$ and $c_1 = \frac{1}{\sqrt{2}}; \gamma^+ = \mp \frac{1}{\sqrt{2}}(\gamma^1 \pm i\gamma^2)$. Therefore it is quite obvious that $f_1$ and $f_2$ represent $S$- and $P$-wave components, respectively.

By inserting the wave function into Eq. (45), we obtain the normalization

$$\int \frac{d^3q}{(2\pi)^3} 2\omega_1 \left(f_1^2 + f_2^2\right) = 1. \quad (48)$$

The $\frac{1^-}{2^-}$ Salpeter wave function composed of $\Sigma_c(\frac{1^-}{2^-})\bar{D}(1^-)$ can be written as

$$\varphi_a(P, q, \lambda) = A_a(q) u(P, r), \quad (49)$$

with

$$A_a = \left(g_1 + g_2 \frac{q_+}{q}\right)(\gamma_a - v_a) + \left(g_3 + g_4 \frac{q_+}{q}\right)\hat{q}_{\lambda a},$$

where $\hat{q}_{\lambda a} = \frac{q_\lambda}{|q|}$. $u(P, r)$ represents the Dirac spinor carrying momentum $P$ and spin state $r$. Notice that the radial wave function $g_i(|q|)$ ($i = 1, \cdots, 4$) only depends on $|q|$. It is clear that $g_1$ corresponds to $S$ wave, $g_{2,3}$ belongs to $P$ wave, and $g_4$ contributes both to $S$ and $D$ partial waves (see Ref. [67] for a further reading about different partial waves in terms of the spherical harmonics $Y_{jm}$).

Inserting Eq. (49) into Eq. (39), we obtain following normalization condition

$$\int \frac{d^3q}{(2\pi)^3} 2\omega_1 \left[3c_3 \left(g_1^2 + g_2^2\right) + c_1 \left(g_3^2 + g_4^2 - 2g_1 g_4 + 2g_2 g_3\right)\right] = 1, \quad (50)$$

where $c = -q^2/\omega_1^2, c_1 = 1 + c, c_3 = 1 + c/3$.

For the $\frac{5}{2}^-$ state with $\Sigma_c(\frac{5}{2}^-)\bar{D}(1^-)$, the Salpeter wave function is written by

$$\varphi_a(P, q, \lambda) = A_a q^5 u(P, r), \quad (51)$$

where

$$A_a(q) = \left(h_1 + h_2 \frac{q_+}{q}\right)g_a + \left(h_3 + h_4 \frac{q_+}{q}\right)(\gamma_a + v_a)\hat{q}_{\lambda a}.$$
We apply a monopole form factor in our work: 

$$F(s^2) = \frac{\Lambda^2}{-s^2 + \Lambda^2},$$

(58)

where $\Lambda$ is a parameter that characterizes the shape of the form factor, and usually set to the energy scale of the meson exchange. In our case, $\Lambda = 0.12$ GeV for $\Sigma_c \bar{\Sigma}$ and 0.18 GeV for $\Sigma_c \bar{D}$. Notice that in the limit $s^2 \to 0$, the heavy hadrons are treated as free and point-like particles, therefore the form factor is normalized to 1 at $s^2 = 0$.

First, we illustrate the results of the potential $V_i$ and $\kappa_i$ with $I = \frac{1}{2}$ appearing in Eqs. (14) and (19). Their $s$ (transferred momentum) dependences are depicted in Fig. 2.

![Fig. 2: The isospin-$\frac{1}{2}$ potentials $V_i \cdot F^2$ (i = 1, 2) and $\kappa_n \cdot F^2$ (n = 1, 2, 3, 4) for $\Sigma D$ and $\Sigma D^*$, respectively.](image)

Later solving the relevant eigenvalue equations, we find bound state solutions for $I = \frac{1}{2} \Sigma_3 \bar{D}^{(*)}$ systems. The obtained mass spectra and corresponding binding energy are listed in Tab. II. To see the sensitivities of the calculations, we also vary $\Lambda$ by ±5% as uncertainties.

For $\Sigma_3 \bar{D}$, with the reasonable parameter $\Lambda$ the mass of experimental $P_c(4312)$ can be well reproduced, i.e., we obtain the meson-baryon bound state with mass 4.313 GeV. Therefore $P_c(4312)$ is a good candidate of $I = \frac{1}{2} \Sigma_3 \bar{D}$ molecular state carrying $J^P = \frac{3}{2}^-$. 

For $\Sigma_3 \bar{D}^*$ system, we obtain two bound states: spin-$\frac{1}{2}$ with mass 4.440 GeV and spin-$\frac{3}{2}$ with 4.457 GeV. These two are consistent well with the experimental $P_c(4440)$ and $P_c(4457)$ respectively. We conclude $P_c(4440)$ can be treated as $I = \frac{1}{2} \Sigma_3 \bar{D}^*$ molecular state carrying $J^P = \frac{1}{2}^-$, while $P_c(4457)$ is $I = \frac{3}{2} \Sigma_3 \bar{D}^*$ molecular state carrying $J^P = \frac{3}{2}^-$. 

However, it is not the end of our story. Differed from other molecular calculations, our approaches indicate an additional state in the $I = \frac{1}{2} \Sigma_3 \bar{D}^*$ channel. This $P_c$ state is an excitation of $P_c(4440)$ which carries $J^P = \frac{1}{2}^-$. The most interesting thing is, it has mass $M = 4.456$ GeV, which is located right at $P_c(4457)$ mass region. Therefore we speculate that $P_c(4457)$ signal discovered by LHCb may contain two overlapped signals: spin-$\frac{1}{2}$ one and spin-$\frac{3}{2}$ one.

We now refer the spin-$\frac{1}{2}$ signal as $P_c'(4457)$. In a word, we totally determine four $\Sigma_3 \bar{D}^{(*)}$ molecular states: $P_c(4312)$, $P_c(4440)$, $P_c(4457)$ and $P_c'(4457)$, where the last two are mixed as observed signal in $J/\psi p$ mass spectrum [21].

The BS wave functions of $P_c(4312)$, $P_c(4440)$, $P_c(4457)$ and $P_c'(4457)$ are displayed in Fig. 3. We can see that, $f_2$ is dominant in $P_c(4312)$’s wave function, while $g_2$ is quite prominent in the wave functions of $P_c(4440)$ and $P_c'(4457)$. In general, we observe that the wave functions of the $P_c$ states are mixtures of $S$, $P$, $D$ waves and even radial exited components. Notice that in our framework, there only exists limited number (four in our case) of bound states.

$P_c'(4457)$ predicted in our work is mainly a first radial excitation, which means it has a similar property comparing to $P_c(4440)$. Furthermore the mass gap between $P_c(4440)$ and $P_c'(4457)$ are just ~ 17 MeV, we believe their widths are quite close. However, as a radial excitation, we prefer a smaller production ratio of $P_c'(4457)$, therefore it is reasonable that LHCb can describe 4457 MeV peak now without additional $P_c'(4457)$.

Indeed, with the limited informations in [21] we can not trace $P_c'(4457)$ for now. We expect that LHCb can further investigate quantum numbers and more decay channels, as well as add $P_c'(4457)$ in their amplitude analysis, to justify our predictions. For example, LHCb can include only spin-$\frac{1}{2}$ or both spin-$\frac{1}{2}$ and spin-$\frac{1}{2}$ at 4457 Mev for comparing. On the other hand, for providing more useful and specific informations, we will theoretically investigate the decay properties and produc-

### Table II: Calculated mass spectrum and binding energy $\Delta E$ (in MeV) of $\Sigma_3 \bar{D}^{(*)}$ system with $I = \frac{1}{2}$, as well as categorized $P_c$.

| $\Sigma_3 \bar{D}^{(*)}(J^P)$ | $M$ (MeV) | $\Delta E$ (MeV) | $P_c$ |
|-----------------------------|----------|------------------|-------|
| $\Sigma_3 \bar{D}^{(\frac{1}{2})}$ | 4313.2^1_2 | $-5.2^3_5$ | $P_c(4312)$ |
| $\Sigma_3 \bar{D}^{(\frac{3}{2})}$ | 4440.5^2_4 | $-20.5^5_3$ | $P_c(4440)$ |
| $\Sigma_3 \bar{D}^{(\frac{1}{2})}$ | 4457.2^3_1 | $-3.2^5_3$ | $P_c(4457)$ |
| $\Sigma_3 \bar{D}^{(\frac{3}{2})}$ | 4456.2^4_1 | $-4.2^5_3$ | $P_c'(4457)$ |

where $\Lambda$ is a parameter that characterizes the shape of the form factor, and usually set to the energy scale of the meson exchange. In our case, $\Lambda = 0.12$ GeV for $\Sigma_3 \bar{D}^*$ and 0.18 GeV for $\Sigma_3 \bar{D}$. Notice that in the limit $s^2 \to 0$, the heavy hadrons are treated as free and point-like particles, therefore the form factor is normalized to 1 at $s^2 = 0$.
tion mechanism in future work.
In addition, we did not find any $I = \frac{3}{2}$ partners of the $\Sigma \bar{D}^{(*)}$ systems.

V. SUMMARY

Recently, LHCb re-examined the process $\Lambda_b \to J/\psi K^- p$, and discovered three $P_c$ pentaquarks: $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$ [21]. Although their molecular nature has been confirmed by many papers, we still think there may emerge interesting structures among these $P_c$ signals considering the experience that old $P_c(4450)$ splits into $P_c(4440)$ and $P_c(4457)$.

Benefited from the effective lagrangians respecting the chiral and heavy quark symmetry, as well as the instantaneous BS formalism of the $\Sigma \bar{D}$ interactions (Fig. 2), we investigate the $\Sigma \bar{D}^{(*)}$ interactions according to the effective lagrangians, and study the behaviors of the potentials. Finally, we obtain molecular solutions as well as their BS wave functions (Fig. 3). Our calculations show that, $P_c(4312)$ can be treated as $I = \frac{1}{2} \Sigma \bar{D}$ molecular state with $J^P = \frac{1}{2}^-$, while $P_c(4440)$ is a good candidate of $I = \frac{1}{2} \Sigma \bar{D}^*$ molecule, which also carries $J^P = \frac{1}{2}^-$. Differed from other molecular calculations, our work indeed indicate not one, but two bound states in $P_c(4457)$ mass region: one is $\Sigma \bar{D}^*$ with $J^P = \frac{1}{2}^-$, another is $\Sigma \bar{D}^*$ carrying $J^P = \frac{1}{2}^-$, which is just an excitation of $P_c(4440)$. Therefore we conclude that $P_c(4457)$ signal discovered by LHCb might be a mixture of $J^P = \frac{1}{2}^-$ and $\frac{1}{2}^-$ states.

In a word, we totally determine four molecular states $P_c(4312)$, $P_c(4440)$, $P_c(4457)$ and $P'_c(4457)$, which matches to three LHCb signals. We speculate the existence of the additional excitation ($P'_{c}(4457)$) is necessary, because the relatively large excitation space of ground $P_c(4440)$. Also, $P'_c(4457)$ has very similar decay property with $P_c(4440)$.

Moreover, we did not support any existence of corresponding isospin-1/2 molecular solutions in our calculations.

We expect LHCb can perform amplitude analysis with more data samples, as well as search for other decay channels, to separate two states in 4457 MeV signal region. We hope our conclusions can be testified in the future.

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Appendix A: Some expressions for derivations of the Salpeter equation

To perform the contour integral, we rewrite the propagators:
\begin{equation}
S(p_2) = i \frac{1}{p_2 - m_2} = -i \left( \frac{\Lambda^+}{q_p - \xi_1^+ - i\epsilon} + \frac{\Lambda^-}{q_p - \xi_2^- + i\epsilon} \right),
\end{equation}
\begin{equation}
D(p_1) = i \frac{1}{p_1 - m_1} = i \frac{1}{2\omega_1} \left( \frac{1}{q_p - \xi_1^+ + i\epsilon} - \frac{1}{q_p - \xi_1^- - i\epsilon} \right),
\end{equation}

where $\xi_1^+ = -\alpha_2 M - \omega_2$, $\xi_1^- = -\alpha_1 M + \omega_1$.

Inserting above expressions into Eq. (30), then performing the contour integral over $q_p$, we obtain the three-dimensional Salpeter equation (32). Utilizing $\varphi_\alpha^+$ defined in Eq. (34), the equation further reduces two coupled equations:
\begin{equation}
(M - \omega_1 - \omega_2)\varphi_\alpha^+ = \frac{\Lambda^\dagger \Gamma_\beta}{2\omega_1},
\end{equation}
\begin{equation}
(M + \omega_1 + \omega_2)\varphi_\alpha = \frac{\Lambda^\dagger \Gamma_\beta}{2\omega_1},
\end{equation}

which can otherwise be simplified to Eq. (35).

The BS vertex can also be expressed by the Salpeter wave function as
\begin{equation}
\Gamma^\alpha(P, q) = S^{-1}(p_2)D^{-1}(p_1)\varphi_\beta \psi_\beta(P, q).
\end{equation}

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FIG. 3: The BS radial wave functions of (a) $P_c(4312)$, (b) $P_c(4440)$, (c) $P_c(4457)$ and (d) $P_c'(4457)$.
