Single-photon frequency conversion for generation of entanglement via constructive interference in Sagnac interferometers

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Abstract

A single-photon frequency conversion process with a Sagnac interferometer (Bradford et al 2012 Phys. Rev. Lett. 108 103902 and Bradford and Shen 2012 Phys. Rev. A 85 043814) is described by the input–output formalism. The result shows that a perfect frequency conversion can be obtained in an appropriate parameter regime via constructive interference. Based on the frequency-conversion process, an alternative scheme is provided for the generation of entanglement between distant particles including two-body entanglement and multi-body Greenberger–Horne–Zeilinger entanglement. We study the effect of the parameters’ deviation and spontaneous decay of the excited state and waveguide mode on the transmission amplitudes, and also on the frequency-conversion efficiency. In addition, the influence of the experimental parameters on the fidelity of the entangled state is discussed.

Keywords: frequency-conversion process, Sagnac interferometer, Greenberger–Horne–Zeilinger state

(Some figures may appear in colour only in the online journal)

1. Introduction

As a nonlinear optical process, quantum frequency conversion retains the quantum characters of the light field [1–7]. Because of this, quantum frequency conversion is widely used in high-sensitivity detection of weak optical signals [8–13] and quantum information processes [14]. To date, numerous schemes have been presented for quantum frequency translation [1–23], like multiwave mixing [1–3, 15–18] and cavity opto-mechanics [19, 20] and so on. Recently, through a real-space theoretical approach and a pseudospectral numerical method, a theory scheme with near unity conversion efficiency is also proposed for quantum frequency conversion at the single-photon power level [22, 23]. In the scheme, quantum frequency up- or down-conversion can also be realized by quantum interference of single-photon states at a Λ three-level quantum emitter coupled to a Sagnac interferometer.

In addition, various different schemes have been put forward to realize entangled states in a controlled way in numerous quantum systems [24–35]. Recently, based on cavity quantum electrodynamics systems, some schemes are reported for the generation of entangled states via selective absorption and emission of polarized photons [30–35]. In fact, these schemes can be replaced by our scheme without the requirement on the polarized photon. For instance, in [31], a Λ-type three-level quantum emitter is driven by a polarized photon, and a Greenberger–Horne–Zeilinger (GHZ) state [36] is realized by completely translating the polarization of the incoming photon. When the quantum emitter is excited...
to the excited state by absorbing a vertical polarization photon, the emitter will emit not only a vertical photon but also a horizontal polarization photon with equal probability. Thus, halfway translation of the polarization of the incoming photon will greatly influence the generation of the GHZ state with high fidelity. The constructive interference scheme in this paper, however, can ensure the entire translation of the polarization of a photon. We investigate a similar optical system [22, 23] (i.e., a Λ three-level quantum emitter coupled to a Sagnac interference loop which is linked with a waveguide through a 50 : 50 beam splitter (BS)) via the input–output formalism and obtain high frequency-conversion efficiency even in the presence of dissipation. In particular, an alternative approach is proposed for the implementation of entanglement with two distant Λ-type three-level quantum emitters via the frequency up- and down-conversion process, which is completed by constructive interference. We also extend the present scheme to realize the GHZ state, which cannot be transformed into each other under local operations and classical communication protocols, and has potential applications in quantum information processes [37–40].

2. Description of the model system

We first describe the configuration of the optical coupled system considered in this paper. Figure 1(a) shows a quantum emitter directly coupling with a single-mode waveguide. The quantum emitter is modelled as a Λ-type three-level structure, i.e., an excited state |e⟩, and two ground states |g⟩ and |f⟩. The quantum states |f⟩ and |e⟩ have the energy ωf and ωe, respectively (we set the energy origin ω0 = 0). The interaction between each transition of the emitter and the propagating photon with frequency ω is described by g1 and g2. By choosing a proper free Hamiltonian, the interaction Hamiltonian of the physical system considered here in the interaction picture can be written as (ℏ = 1) [41–43]:

\[
H_I = \int_{-\infty}^{+\infty} \sum_{k = L, R} a_{1k}^\dagger(\omega) a_{1k}(\omega) d\omega + \int_{-\infty}^{+\infty} \sum_{k = L, R} a_{2k}^\dagger(\omega) a_{2k}(\omega) d\omega - g_1 \int_{-\infty}^{+\infty} [a_{1L}(\omega) \sigma_{eg} + a_{1R}(\omega) \sigma_{eg} + h.h.] d\omega - g_2 \int_{-\infty}^{+\infty} [a_{2L}(\omega) \sigma_{ef} + a_{2R}(\omega) \sigma_{ef} + h.h.] d\omega,
\]

where \(a_{m} = |m\rangle \langle n| (m, n = e, f, g)\) are the transition operators between the quantum state |m⟩ and |n⟩ (m ≠ n). \(a_{1k}^\dagger(\omega)\) and \(a_{1k}(\omega)\) denote the creation operators for a left(right)-moving photon which drives the transitions |e⟩ ↔ |g⟩ and |e⟩ ↔ |f⟩ of the quantum emitter, respectively. The creation and annihilation operators of the waveguide mode satisfy the commutation relation \([a_{jk}(\omega), a_{jk}(\omega')] = δ(\omega - \omega')\), (j = 1, 2, k = L, R). \(\Delta_1 = \omega - \omega_0\) and \(\Delta_2 = \omega - (\omega_0 - \omega_f)\) are the corresponding frequency detunings. If the initial state of the system is in the state |g⟩|ω⟩ or |f⟩|ω⟩, the optical coupled system has an interesting invariant Hilbert subspace with the bases \(|g⟩|\omega⟩, |f⟩|\omega⟩, |e⟩|vac⟩\), where in the state |m⟩|n⟩ (m = e, f, g) m means the state of the Λ-type three-level quantum emitter and |n⟩ (n = vac, ω) means the number of photons in the waveguide, i.e., |vac⟩ means the vacuum state of the waveguide mode and |ω⟩ means the one-photon Fock state of the waveguide mode with frequency ω. In such a case, the evolution the whole system can be described by the wave function

\[
Ψ(t) = \int_{-\infty}^{+\infty} \left[ c_{1L} a_{1L}^\dagger(\omega) + c_{1R} a_{1R}^\dagger(\omega) \right] |g⟩ |vac⟩ d\omega + \int_{-\infty}^{+\infty} \left[ c_{2L} a_{2L}^\dagger(\omega) + c_{2R} a_{2R}^\dagger(\omega) \right] |f⟩ |vac⟩ d\omega + c_e |e⟩ |vac⟩,
\]

where cjk is the amplitude for left (k = L)- or right (k = R)-moving photons of the waveguide mode with the quantum emitter in the ground states and ce is the excitation amplitude of the quantum emitter in the state |e⟩. In the interaction picture, substituting equation (2) into the Schrödinger equation, the time evolution of the amplitudes for the waveguide mode and quantum emitter are given as follows:

\[
i\dot{c}_{1L} = δ_1 c_{1L} - g_1 c_e,
\]

\[
i\dot{c}_{1R} = δ_1 c_{1R} - g_1 c_e,
\]

\[
i\dot{c}_{2L} = δ_2 c_{2L} - g_2 c_e,
\]

\[
i\dot{c}_{2R} = δ_2 c_{2R} - g_2 c_e,
\]

\[
i\dot{c}_e = -g_1 \int_{-\infty}^{+\infty} (c_{1L} + c_{1R}) d\omega - g_2 \int_{-\infty}^{+\infty} (c_{2L} + c_{2R}) d\omega - iγ c_e / 2,
\]

where \(δ_1 = \Delta_1 - iκ/2\) and we have set the waveguide modes to have the same decay rate. \(γ\) is the excited state’s spontaneous decay and \(γ > γ_f\) (\(γ_f\) is the dissipation rate of the ground state |f⟩), so we ignored \(γ_f\) in the following discussion. \(κ\) is the decay rate of the waveguide modes. Combining equations (3) and (4), we can obtain a set of coupling equations for connecting the input and output pulses

\[
i\dot{c}_{1L} = -2π (g_1^2 + g_2^2) c_e + i\sqrt{2π} g_1 (c_{1L}^n + c_{1R}^n) + i\sqrt{2π} g_2 (c_{2L}^n + c_{2R}^n) - iγ c_e / 2,
\]

\[
i\dot{c}_{1R} = 2π (g_1^2 + g_2^2) c_e + i\sqrt{2π} g_1 (c_{1L}^n + c_{1R}^n) + i\sqrt{2π} g_2 (c_{2L}^n + c_{2R}^n) - iγ c_e / 2.
\]

In deriving equations (5a) and (5b), we have used the condition \(t_0 < t < t_1\). And \(c_{1L}^n, c_{1R}^n, c_{2L}^n, c_{2R}^n\) (j = 1, 2 and k = L, R) are the input (output) left- or right-moving pulse operators of the waveguide mode which are defined as cjk = \(\frac{1}{\sqrt{2π}} \int_{-\infty}^{+\infty} c_{jk}(\omega) e^{-i(\omega - \omega_0) t} d\omega\), and cjk = \(\frac{1}{\sqrt{2π}} \int_{-\infty}^{+\infty} c_{jk}(\omega) e^{-i(\omega - \omega_0) t} d\omega\), where c1L(0), c1R(0), c2L(0) and c2R(0) are the respective values of the amplitudes c1L, c1R, c2L and c2R at \(t = t_0\). According to the symmetry of the
waveguide mode \[44, 45\], we can describe the physical system under consideration by the following new operators:

\[
\begin{align}
 a_{je}(\omega) &= \frac{1}{\sqrt{2}} [a_{jL}(\omega) + a_{jR}(\omega)], \\
 a_{jL}(\omega) &= \frac{1}{\sqrt{2}} [a_{jL}(\omega) + a_{jR}(\omega)], \\
 a_{jR}(\omega) &= \frac{1}{\sqrt{2}} [a_{jL}(\omega) - a_{jR}(\omega)], \\
 a_{jR}(\omega) &= \frac{1}{\sqrt{2}} [a_{jL}(\omega) - a_{jR}(\omega)],
\end{align}
\]

(6a)

(6b)

(6c)

(6d)

where \(a_{je}(\omega)[a_{je}^\dagger(\omega)]\) and \(a_{jR}(\omega)[a_{jR}^\dagger(\omega)]\) \((j = 1, 2)\) denote the even and odd mode annihilation (creation) operator, respectively. In this case, the interaction Hamiltonian (1) and the wave function (2) can be rewritten as

\[
H_l = \int_{-\infty}^{+\infty} \Delta_1 \sum_{k=e,o} a_{1k}^\dagger(\omega) a_{1k}(\omega) \ d\omega \\
+ \int_{-\infty}^{+\infty} \Delta_2 \sum_{k=e,o} a_{2k}^\dagger(\omega) a_{2k}(\omega) \ d\omega \\
- \sqrt{2} g_1 [a_{1e}(\omega) a_{1o}(\omega)] \ d\omega \\
- \sqrt{2} g_2 [a_{2e}(\omega) a_{2o}(\omega)] \ d\omega,
\]

(7)

and

\[
\Psi_l(t) = \int_{-\infty}^{+\infty} [c_{le} a_{1e}(\omega) + c_{lo} a_{1o}(\omega)] |g\rangle \langle \text{vac}| \ d\omega \\
+ \int_{-\infty}^{+\infty} [c_{le} a_{2e}(\omega) + c_{lo} a_{2o}(\omega)] |f\rangle \langle \text{vac}| \ d\omega + c_v |e\rangle |0\rangle,
\]

(8)

where

\[
c_{je} = \frac{1}{\sqrt{2}} (c_{jL} + c_{jR}).
\]

(9a)

Hamiltonian (7) shows that only the even mode contributes to the interaction between the quantum emitter and incoming photon. In other words, only the even mode experiences frequency conversion and the odd mode remains unchanged. When the phase between the left- and right-moving photon is zero, i.e., \(\Theta = 0\), and the even mode is the only mode propagating in the Sagnac loop. Carrying out some algebra on equations (5) and (9), we can get the input–output formalism \[46, 47\]

\[
c_{jo} = \frac{1}{\sqrt{2}} (c_{jL} - c_{jR}), \ (j = 1, 2).
\]

(9b)

where \(c_{jo} = \frac{1}{\sqrt{2}} (c_{jL} + c_{jR})\). Making a Fourier transform on equation (5), and combining with equations (9) and (10), we can obtain the transmission coefficients as follows:

\[
T_1(\omega) = \frac{c_{2o}^{\text{in}}}{c_{1o}^{\text{in}}} = \frac{\Delta_1 + 2\pi i (g_1^2 - g_2^2) - i(\gamma + k)/2}{\Delta_1 - 2\pi i (g_1^2 + g_2^2) - i(\gamma + k)/2},
\]

(11a)

\[
T_2(\omega) = \frac{c_{2o}^{\text{out}}} {c_{1o}^{\text{out}}} = \frac{4\pi i g_1 g_2}{\Delta_1 - 2\pi i (g_1^2 + g_2^2) - i(\gamma + k)/2},
\]

(11b)

where \(T_j(\omega) \ (j = 1, 2)\) denotes the transmission coefficient for the propagating photon entering the Sagnac loop through the BS and leaking out of the Sagnac loop. Subscripts \(j = 1\) and \(j = 2\) mean that the photon moves out of the Sagnac loop and leaves the quantum emitter in the states \(|g\rangle\) and \(|f\rangle\), respectively. Here, we have chosen the initial state of the system as \(|g\rangle|0\rangle\). When the incoming photon resonantly drives the transition \(|g\rangle \leftrightarrow |e\rangle\) and we set the coupling strength as equal (i.e., \(\Delta_1 = 0\) and \(g_1 = g_2\)), the transmission coefficients are given as \(|T_1| = 0\) and \(|T_2| = 1\). In this case, the incoming photon undergoes the complete frequency down-conversion process. The above case can be summarized as \(a_{1jL}(g)|\text{vac}\rangle \leftrightarrow a_{1jL}(f)|\text{vac}\rangle\). Similarly, if the initial state of the quantum emitter is \(|f\rangle\) and the input photon...
resonantly couples the transition $|f\rangle \iff |e\rangle$, the frequency up-conversion process is $\hat{a}_{2L}^\dagger|f\rangle|\text{vac}\rangle \iff \hat{a}_{1L}^\dagger|g\rangle|\text{vac}\rangle$. When $\Theta = \pi$, we can obtain $T_1^r(\omega) = c_{\text{out}(1)}^L/c_{\text{in}}^L = 1$, $T_2^r(\omega) = c_{\text{out}(2)}^L/c_{\text{in}}^L = 0$, and $c_{\text{in}} = 1/2(c_{\text{out}(1)}^L - c_{\text{out}(2)}^L)(j = 1, 2)$. This implies that the incoming photon does not undergo any frequency translation process.

3. Generation of entanglement

In this section, through the frequency-conversion process, we first consider how to realize the entangled state with two particles, then how to realize the multiple entangled state such as the GHZ state. The detailed process is described in figures 2(a) and (b). Two A-type three-level quantum emitters are coupled to two Sagnac loops individually and each Sagnac loop is connected to the waveguide by a 50 : 50 BS. The left and right emitters are initialized in the ground states $|g\rangle$ and $|f\rangle$, respectively. A photon with frequency $\omega = \omega_c$ passes through the BS and takes two different paths with equal probability. As shown in figure 2(a), the photon moves into the left Sagnac loop. The incoming photon with frequency $\omega = \omega_c$ interacts with the emitter in the left Sagnac loop and leaks out of the loop with frequency $\omega = \omega_c - \omega_f$, i.e., the incoming photon is scattered by the emitter and experiences the frequency down-conversion $\omega = \omega_c \iff \omega_c - \omega_f$. Then the photon enters the right Sagnac loop and leaves out of the Sagnac loop with frequency $\omega_c$ after resonantly driving the emitter transition $|f\rangle \iff |e\rangle$. Finally, the photons taking two paths emerge at the last BS and are mixed by the BS which erases the which-path information. Through the above frequency-conversion process we implement the maximum entangled state with two particles as $\frac{1}{\sqrt{2}}(|g\rangle|f\rangle + |f\rangle|g\rangle)$.

The two-qubit entangled state is realized through the frequency-conversion process of a single photon, and the proposed scheme can be extended to the generation of the GHZ state with distant emitters. We will take the realization of the four-qubit GHZ state as an example. Four quantum emitters are coupled to four different Sagnac loops individually which line up in a similar order, as shown in figure 2(a). We assume that from left to right the first and third emitters are in the initial state $|g\rangle$, and the rest are in the state $|f\rangle$. After passing through the left-most BS, one branch of the light interacts with the emitter in each Sagnac loop one by one and reaches the far right BS at the same moment with the other one. The lights taking two paths are finally mixed by the far right BS and the four-qubit GHZ state is finished as $\frac{1}{\sqrt{2}}(|g\rangle|f\rangle|g\rangle|f\rangle + |f\rangle|g\rangle|f\rangle|g\rangle)$. Essentially, the incoming photon experiences frequency down-conversion twice and frequency up-conversion also twice, so that four distance emitters are entangled (i.e., as described earlier in this paper, the process of the generation of the two-qubit entangled state can repeat a couple of times to achieve a four-qubit GHZ state).

4. Analysis and discussion

In this paper, the entangled states are realized by a series of frequency-conversion processes, thus it is necessary to obtain entire frequency conversion to ensure the implementation of the entangled state with a high fidelity. The transmission coefficients, equation (11), reflect the frequency-conversion efficiency. The case without a Sagnac loop (see figure 1(a)) has been discussed in detail [22, 23]. In the following discussion, we focus on the case with a Sagnac loop. In figures 3(a) and (b), we plot the absolute value of the transition coefficients $T_1(\omega)$ and $T_2(\omega)$ for different proportional coefficients $g_2/g_1$ without spontaneous decay. For the case $g_2 = g_1$, the transmission coefficient $|T_1(\omega)| = 0$ (see figure 3(a)) and the transmission coefficient $|T_2(\omega)| \approx 1$ (see figure 3(b)), that is to say, the incoming photon is entirely changed from the frequency $\omega_c$ to $\omega_c - \omega_f$. Once the coupling proportional coefficient $g_2/g_1$ deviates from 1, the transmission coefficient $|T_1(\omega)|$ increases rapidly and the transmission coefficient $|T_2(\omega)|$ decreases quickly. When $|g_2| > |g_1|$, the incoming photon undergoes little frequency conversion and moves out from the Sagnac loop. Thus, in order to achieve prefect frequency conversion, the proportional coefficient $g_2/g_1$ should be controlled around 1. In addition, we plot the transmission coefficients $T_1(\omega)$ with frequency $\omega_c$ and $T_2(\omega)$ with frequency $\omega_c - \omega_f$ versus the frequency detuning $\Delta_1$ in figures 3(c) and (d), respectively. Under the condition of $g_1 = g_2$, it is easy to see that the incoming photon undergoes a completely frequency change from $\omega_c$ to $\omega_c - \omega_f$ at the resonant point. Once the frequency of the incoming photon deviates from the resonant point, part of the incoming photon...
Figure 3. For the case with a Sagnac loop, the absolute value of the transmission coefficients (a) $|T_1(\omega)|$ and (b) $|T_2(\omega)|$ versus the proportional coefficient of coupling strength $g_2/g_1$ under the resonance condition, i.e., $\Delta_1 = \omega - \omega_1 = 0$; the transmission coefficients (c) $|T_1(\omega)|$ and (d) $|T_2(\omega)|$ versus frequency detuning $\Delta_1 = \omega_e - \omega$ between the incoming impulse and the emitter transition $|e\rangle \leftrightarrow |g\rangle$ with $g_1 = g_2$; the spontaneous decay is chosen as $\kappa = \gamma = 0$.

Figure 4. The efficiency of the frequency conversion ($P_c$) versus (a) the parameter $T\Gamma$ for three kinds of ratios $g_2/g_1$; (b) the coupling strengths’ proportional coefficient $g_2/g_1$; (c) $T\Gamma$ for three spontaneous decay $\xi = (\kappa + \gamma)/2$ with $g_2/g_1 = 1$; (d) the proportional coefficient $g_2/g_1$ with $T\Gamma = 180$. 
will maintain the same frequency $\omega_e$. According to the above analysis, it is important to maintain the resonant coupling to realize the entangled state with a high fidelity.

In the next section, we will discuss the effect of coupling strengths $g_2/g_1$ and the decay of the excited state and waveguide modes on the efficiency ($P_c$) of the frequency-conversion process. According to [23], $P_c$ is defined as $P_c = \int_{-\infty}^{+\infty} |T_2(\omega)|^2 f_a(\omega)^2 d\omega$, where $T_2(\omega)$ is the Fourier transform of the input pulse. If the initial pulse is a Gaussian pulse $f_a(\omega) = A \exp\left[-(t-T/2)^2/(T/5)^2\right]$, where $A$ is the normalization factor and $T$ is the period of the input pulse, we obtain the corresponding frequency-conversion efficiency $P_c$:

$$P_c = \frac{4\sqrt{\pi} (g_2/g_1)^2}{[1 + (g_2/g_1)^2]^2/[1 + (\xi/\Gamma)^2]^2} \times \frac{T\Gamma(1 + \xi/\Gamma)\Gamma}{20\ln\frac{2}{1}} \text{erfcx} \left[\frac{1 + \xi/\Gamma}{\Gamma}\right],$$

where $\xi = (\kappa + \gamma)/2$, and $\Gamma = 2\pi (g_1^2 + g_2^2)$ means the decay rates of the two transitions. erfcx($x$) = $e^{-x^2} \int_x^{\infty} e^{-t^2} dt$ is the scaled complementary error function. When $\xi = 0$, $P_c \rightarrow \frac{4\sqrt{\pi} (g_2/g_1)^2}{[1 + (g_2/g_1)^2]^2} \text{erfcx} \left[\frac{1}{\Gamma}\right]$ as $\Gamma T \rightarrow \infty$. In order to discuss the effect of $\Gamma T$ on frequency-conversion efficiency $P_c$, we plot the frequency-conversion efficiency $P_c$ as a function of $\Gamma T$ for different $g_2/g_1$ in Figure 4(a). For a fixed $g_2/g_1$, the frequency-conversion efficiency $P_c$ tends to significantly increase when the parameter $\Gamma T$ becomes bigger. For the case $g_2/g_1 = 1$ and $\Gamma T = 200$, the frequency-conversion efficiency $P_c$ reaches nearly unity. The influence of the proportional coefficient $g_2/g_1$ on the efficiency $P_c$ is shown in Figure 4(b). When the coupling strength $g_2$ approaches $g_1$, the value of the efficiency $P_c$ tends to reach maximum. For ensuring the high efficiency $P_c$ of frequency conversion, it is necessary to control the proportional coefficient $g_2/g_1$ around 1 and make $\Gamma T \rightarrow \infty$.

Figures 4(c) and (d) show that the decay rate $\xi = (\kappa + \gamma)/2$ has a significant impact on the frequency-conversion efficiency $P_c$. For example, under the conditions of $g_2 = g_1$ and $\Gamma T \geq 180$, the frequency-conversion efficiency $P_c < 30\%$ when $\xi = \Gamma$, but $P_c$ can reach 80\% when $\xi = 0.1\Gamma$. Figure 4(c) clearly shows that the perfect frequency conversion can be obtained when $\xi \leq 0.01\Gamma$.

We begin to consider the effect of experimental parameters on the fidelity of generating an entangled state with two distant particles (see figure 2(a)). The fidelity is defined as $F = \langle \Psi_I | \Psi_{out} \rangle$, where $|\Psi_I \rangle = \frac{1}{\sqrt{2}}(|g\rangle |f\rangle + |f\rangle |g\rangle)$ and $|\Psi_{out} \rangle = \frac{1}{\sqrt{2}}(|g\rangle |f\rangle + T_2(\omega)^2 |f\rangle |g\rangle)$. Figures 5(a) and (b) show the fidelity $F$ as a function of the proportional coefficient $g_2/g_1$ under different frequency detunings and spontaneous decay conditions, and $F$ is sensitive to the proportional coefficient $g_2/g_1$. We also consider the effect of the decay rate from the excited state and waveguide mode on fidelity in figures 5(c) and (d). The fidelity $F$ will go down quickly as the decay rate increases. However, if we control the frequency detuning and proportional coefficient $g_2/g_1$ in the ideal region, we can obtain
the entangled state with high fidelity on the present scheme. For example, if the pulse couples the transition resonantly (i.e., \( \Delta_1 \approx 0 \)) and coupling strengths are nearly equal, we have the fidelity \( F > 90\% \) and \( F > 84\% \) under the corresponding conditions \( \xi \approx 0.1\Gamma \) and \( \xi \approx 0.2\Gamma \), respectively.

Before ending this section, we would like to analyze the experimental feasibility of our scheme. Firstly, frequency detuning \( \Delta_1 \) will adversely affect the fidelity of the entangled state. However, the energy interval can be adjusted by an additional laser [48]. Secondly, we take the GaAs quantum dot coupled to the waveguide as a candidate three-level quantum emitter. A recent experiment [49] shows the decay rate of the quantum dot’s excited state can be controlled in an ideal region, i.e., \( \gamma \approx 0.1\Gamma \). In addition, the factor \( \beta = 0.95 \) (\( \gamma \approx 0.1\Gamma \) corresponds to the factor \( \beta \approx 0.91 \)) has been theoretically predicted in a photonic crystal waveguide [50, 51]. If we choose \( \kappa \approx \gamma \), the above discussion shows the entangled state can be realized with fidelity \( F > 90\% \). Thirdly, based on the present scheme, frequency conversion is realized under the phase coherence condition, which can be satisfied by adjusting phase shifter or changing the position of the quantum emitter. Fourthly, in the A-emitter, the decay rate \( \gamma_f \) of the metastable state \( |f \rangle \) is far less than \( \gamma \) the excited state’s [52], thus we neglect the effect of \( \gamma_f \) on our scheme. Finally, the coupling strength is defined as \( -\frac{D_2}{\hbar} (\omega_0 \gamma_{eg} \hat{\mathcal{E}})^{1/2} \), where \( D_2 \) (\( D_2 = D_{eg} \hat{e} \hat{\mathcal{E}} \) is the polarization unit vector of the laser field) is the dipole moment for the transition between the levels \( |e \rangle \) and \( |g \rangle \), and \( \omega \) is the frequency of the input pulse and \( \nu \) is the volume of the resonator. Thus, the coupling strength can be adjusted by the corresponding parameters. In addition, recent work [53] shows that perfect frequency conversion can be obtained with the coupling strengths’ proportional coefficient and frequency detuning in a range.

5. Conclusion

In a word, a single-photon frequency-conversion process via constructive interference is studied through the input–output formalism. The complete frequency conversion can be obtained with proper parameters. We focus on the generation of the entangled state through a series of frequency-conversion processes and the present scheme can be extended to the realization of the GHZ state. In the present scheme, perfect frequency conversion is a guarantee of the successful realization of the entangled state with high fidelity. Thus, we consider the influence of the parameters’ deviation and spontaneous decay on the single-photon frequency-conversion process in detail. The result shows that frequency can be entirely translated in an appropriate parameters regime. In addition, we directly study the effect of parameters on the fidelity of the realization of the two particles’ entangled state. As a result, the entangled state, including the GHZ state, can be generated with high fidelity in the current experiment. The conclusion may be helpful for the implementation of entangled state and quantum information engineering.

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