Methodology of experimental determination of mechanical impedance of vibration-insulating branch pipes taking into account flexibility of test facility at lengthwise vibrations

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Abstract. In the problems of vibration isolation of hydraulic system units, such as pumps, an important role is played by the problem of oscillation propagation through unsupported connections (pipelines). In order to solve this problem, in particular, rubber-cord tubes are used, which vibration-insulating properties are determined by their mechanical impedances. The aim of the work is to obtain the theoretical basis for the development of methods for the experimental determination of mechanical impedance of vibration insulating branch pipes, taking into account the flexibility of elements of the experimental facility in the case of lengthwise vibrations.

1. Introduction
The task of reducing the level of vibration transmitted from vibro-active hydraulic systems is relevant in various industries and transport, for example, in shipbuilding. Vibration propagates through support and neoprene connections. Unsupported connections between hydraulic aggregates are pipelines, which in systems with high pressure have a high rigidity, which contributes to the spread of vibration. In order to improve the vibration isolation properties of hydraulic systems, flexible piping rates are used, e.g. rubber-corded spigots, which have a relatively low rigidity while maintaining the required load-bearing capacity.

Vibro-isolation characteristics of the branch pipes are estimated by the value of inlet and outlet mechanical impedance of the branch pipe, which is a function of the frequency of vibro-active force acting on the inlet flange of the branch pipe. Mechanical impedance is the ratio of force to speed and in practice is determined experimentally by dynamic tests.

At dynamic tests on an entrance flange of a branch pipe force from the side of a vibrator operates, and the exit flange is fastened to motionless elements of experimental facility. The internal cavity of the pipe is filled with a liquid under pressure equal to the nominal pressure specified in the technical characteristics of the pipe. Then the vibrating influence on the inlet flange is made by the harmonic force excited by vibrators. Frequency of influence changes in a given frequency range. At each frequency with the help of sensors the forces and accelerations are measured at the required points and the mechanical impedance of the branch pipe is calculated.

In order to increase the performance of the tests, instead of harmonic exposure to frequency changes, a white noise signal can be used in a given frequency range and spectral analysis of the signals from the force and acceleration sensors can be carried out.

2. Problem definition
The method of mechanical impedance measurement is indirect and is based on the use of the results of direct measurements of dynamic forces and vibrational accelerations in the artificial excitation of the tube vibrations with the help of vibrators.
Experimental determination of mechanical impedance of rubber cored branch pipe is carried out with the help of the facility, the scheme of which is shown in Fig. 1. Corners 2 and 9 are installed on the plate 1, between which the branch pipe 7 is fixed. Outlet flange B of the outlet pipe is fixed to the corner through the technological flange 10 and force sensors 4, which measure the force with which the outlet flange affects the corner 9. Camera 6 is rigidly attached to the inlet flange A, moving in relation to piston 5 connected to corner 2. The piston ensures that the inner cavity is sealed when the working pressure in the branch pipe is created. Vibrators 3 provide the creation of dynamic lengthwise force $F$, measured by sensors 4, installed between the vibrators and the flange of the A1 chamber. The flanges A and B are equipped with accelerometers 8 to measure the acceleration of the pipe flanges. Four force sensors and four accelerometers are installed on each flange. The force values in sections A1 and B are defined as the sum of the four sensors. Acceleration values are defined as the arithmetic mean of the four sensors.

\[ Z_{in} = \frac{F_A}{V_A|_{V_B=0}}; \quad Z_{tr} = \frac{F_B}{V_A|_{V_B=0}}. \]  
\[ Z_{in}^{in} = \frac{F_A}{V_A|_{V_B=0}} - j \omega m_A; \quad Z_{tr}^{tr} = \frac{F_B}{V_A|_{V_B=0}}. \]

where $m_A$ is the mass of chamber 6, fasteners and other facility elements moving together with the inlet flange of the branch pipe.

The following problem occurs when testing with the above facility: The boundary condition $V_B = 0$ is not met because the experimental setup is not absolutely rigid and the output flange B moves under the force $F_B$. If in all test modes the speed of the output flange is much lower than the speed of the input flange $V_B \ll V_A$, then it is possible to accept $V_B = 0$ (according to the existing method the difference between the vibration acceleration of the input and output flanges of the branch pipe should be not less than 10 dB). But with the increase in the diameter of the branch pipe or working pressure there is an increase in its mechanical impedance, which leads to an increase in speed $V_B$, violation of the limit on the value of the vibration acceleration of the output flange and an increase in the error of determining mechanical impedance in the calculation of the formulas (2). In addition, the movement of angle 9 and outlet flange of the pipe can also be associated with the spread of vibrations through the angle 2 and plate 1.

Due to these problems, it is possible to formulate the tasks of this work:

1) The derivation of formulas for the calculation of mechanical impedance of the branch pipe using force measurements $F$ and $F_B$, accelerations $a_A$ and $a_B$, taking into account the mobility of the flange B.

Figure 1. Test facility for measuring mechanical impedance in lengthwise direction

Input $Z_{in}$ and transfer $Z_{tr}$ of mechanical impedance of the branch pipe by definition represent the ratio of the corresponding force to the velocity $V_A$ of the input flange A movement, with the output flange B [2].
2) Verification of the obtained formulas by means of harmonic finite element analysis of the experimental facility.

3. Theory

In order to determine the mechanical impedance of the branch pipe taking into account the flexibility of the facility elements, the second Newton's law for chamber 6 and technological flange 10 was considered (Fig. 1):

\[ m_A \ddot{a}_A = F - Z_{in}' \cdot V_A + Z_{tr}' \cdot V_B, \]  
\[ m_B \ddot{a}_B = Z_{tr}' \cdot V_A - Z_{out}' \cdot V_B - F_{B1}, \]  
\[ (3) \]

where \( F_{B1} \) is force value measured by the force sensors in the section \( B_1 \);
\( F = F_{A1} \) is vibrator excitatory force measured by the force sensors in the section \( A_1 \);
\( z_A, \dot{z}_A = v_A, \ddot{z}_A = a_A \) is movement, speed and acceleration of the inlet flange of the branch pipe;
\( z_B, \dot{z}_B = v_B, \ddot{z}_B = a_B \) is movement, speed and acceleration of the outlet flange of the branch pipe;
\( Z_{in}' \) is inlet mechanical impedance of the branch pipe;
\( Z_{out}' \) is output mechanical impedance of the branch pipe due to symmetry of the branch pipe \( Z_{out}' = Z_{in}' \);
\( Z_{tr}' \) is mechanical transfer impedances of the branch pipe;
\( m_B \) is mass of the process flange moving together with the outlet flange of the pipe connection.

When compiling equations (3), (4) details 6, 10, as well as the flanges of the branch pipe are absolutely rigid.

According to the method of complex amplitudes, all variables in the equations (2), (3), (4) are complex values, the module of which is equal to the amplitude, and the argument - to the phase of the harmonic signal. The amplitude and phase of the variables depend on the frequency of harmonic influence.

Testing directly measures complex acceleration amplitudes. Complex amplitudes of velocities can be calculated using the following formulas:

\[ V_A = \frac{a_A}{j\omega}, \quad V_B = \frac{a_B}{j\omega}. \]  
\[ (5) \]

Formulas for the mechanical impedance of the pipe can be obtained from the equation system (3) and (4):

\[ Z_{in}'(\omega) = \frac{F_{B1} v_B + F v_A + j\omega(m_B v_B^2 - m_A v_A^2)}{(v_A^2 - v_B^2)}, \quad Z_{tr}'(\omega) = \frac{F_{B1} v_A + F v_B + j\omega v_A v_B(m_B - m_A)}{(v_A^2 - v_B^2)}. \]  
\[ (6) \]

As it follows from formulas (5) and (6), to calculate the mechanical impedance of the pipe taking into account the flexibility of the facility it is necessary to measure the complex amplitudes of lengthwise (in the direction of \( z \)-axis) acceleration of the flanges \( a_A(\omega), a_B(\omega) \) and the forces in the sections \( A_1, B_1 ; F(\omega), F_{B1}(\omega) \).

Mechanical impedances calculated by formulas (6) are complex values depending on the frequency of harmonic influence.

4. Mechanical impedance calculation

For the purpose of verification of the offered settlement formulas (6) for definition of mechanical impedance of a branch pipe on the basis of the experimental data instead of rubber-corded branch pipe the spring simulator which finite element model is shown on fig. 2 is used. The choice of the spring simulator in the quality of the object of study is due to the following. Dynamic calculation of the composite structure of the rubber corded branch pipe is a complex scientific and technical problem, the solution of which, in turn, will require verification and which goes beyond the scope of this work. Meanwhile, the validity of the dynamic calculation of the isotropic construction of the spring simulator by the finite element method is not in doubt.
Figure 2. The finite element model of the spring simulator

Weight of the spring simulator: number of turns: 7; average diameter of the spring: 0.101 m; diameter of the wire: 0.018 m; weight of the simulator: 17.8 kg; material: steel; material characteristics: Young's modulus E=2\cdot10^{11} \text{ Pa}, Poisson's coefficient \mu=0.3, density \rho=7850 \text{ kg/m}^3.

Calculations were performed using Harmonic Analysis System of ANSYS software package.

First, a harmonic analysis of the spring simulator was carried out under ideal conditions when flange B is stationary (Fig. 2) and flange A is subjected to the harmonic force \(F_A = F(\omega)\) with an amplitude of 1 N, evenly distributed over the flange side surface area. We will call this calculation a virtual dynamic test of the spring simulator under ideal conditions.

Harmonic analysis was performed for the frequency range from 0.5 to 800 Hz in 0.5 Hz increments. The virtual test resulted in the amplitude and phase as a function of the frequency of the following values: acceleration \(a_A\) of the flange A, the force \(F_B\) of the hard termination reaction. The input force \(F_A\) is a reference force and has a zero phase. The complex amplitudes of these values are calculated by the following formulas:

\[
\begin{align*}
|a_A(\omega)| &= |a_A| e^{i\phi_A}, \\
F_A(\omega) &= 1, \\
|F_B(\omega)| &= |F_B| e^{i\phi_B},
\end{align*}
\]

where \(|a_A|, |F_B|\) – acceleration modules \(a_A\) and forces \(F_B\), \(\phi_A, \phi_B\) – acceleration phases \(a_A\) and forces \(F_B\).

The mechanical impedance of the spring simulator is calculated using formulas (1). The mechanical impedances calculated in this case will be considered as true mechanical impedances (neglecting the error of the numerical calculation), because the boundary condition \(V_B = 0\) is observed during modeling. The results of the calculation are shown in Fig. 4 and 5, on which the modules of true mechanical impedances are marked \(Z_{0n}^{\text{fr}}\) and \(Z_{0t}^{\text{fr}}\).

Further the harmonic analysis of the model of the experimental facility constructed in accordance with the design documentation for the experimental facility for dynamic testing of nozzles was carried out. The finite element model of the facility with a spring simulator is shown in Fig. 3. The action of the vibrators is given by two forces \(F\), acting both on the chamber 6 and on the angle 2 (Fig. 1). Also set a rigid seal for the bottom surface of the plate 1.
In this case, the limit condition $V_B = 0$ is not fulfilled and the formulas (6) must be used to calculate the mechanical impedance of the spring simulator. Harmonic analysis of this model will be called a virtual dynamic testing of the spring simulator taking into account the flexibility of the facility. The virtual experiment, taking into account the flexibility of the facility, was also carried out for the frequency range from 0.5 to 800 Hz with a pitch of 0.5 Hz. As a result of the calculation, the amplitude and phase as a function of the frequency of the following values were obtained: acceleration of $a_A$ flange A, acceleration of $a_B$ flange B, force $F_{B1}$. The complex amplitudes of accelerations and forces will be equal to

$$a_A(\omega) = |a_A| e^{j\varphi_A}, \quad a_B(\omega) = |a_B| e^{j\varphi_B}, \quad F(\omega) = 1, \quad F_{B1}(\omega) = |F_{B1}| e^{j\varphi_{B1}},$$

(8)

Mechanical impedances are calculated using data from the virtual dynamic spring simulator test, taking into account the flexibility of the facility, both formulas (2) and formulas (6). Input and transfer mechanical impedances calculated by formulas (2) are indicated by $Z_{in}$ and $Z_{tr}$, respectively, and mechanical impedances calculated by formulas (6) are indicated by $Z_{in}^*$ and $Z_{tr}^*$. The results of calculation of mechanical impedance modules based on the data of virtual dynamic tests, both under ideal conditions and taking into account the flexibility of the facility are shown in Fig. 4 и 5.
5. Results analysis

Analysis of graphs in Fig. 4 has shown that the input mechanical impedances can be calculated with sufficient accuracy according to the existing formulas (2), the mobility of the output flange does not lead to a significant deviation of the dependence of the input mechanical impedance from the true values.

When analyzing the dependencies in Fig. 5 it is visible that calculation of transfer mechanical impedances on formulas (2) without taking into account flexibility of details of facility leads to essential deviation from true dependence. While the calculation by formulas (6) taking into account the flexibility of the experimental setup gives a much more accurate result.

In order to quantify the correspondence of the mechanical impedances obtained at the real facility to the true mechanical impedances, the mean-integrated relative errors of determination of the modules of $Z_{in}$ and $Z_{tr}$ values were calculated, which for discrete functions can be calculated by the following formulas:

$$J_{in} = \frac{1}{N} \sum_{i=1}^{N} \left| \frac{|Z_{in}|}{|Z_{0,in}|} - 1 \right|,$$

$$J_{tr} = \frac{1}{N} \sum_{i=1}^{N} \left| \frac{|Z_{tr}|}{|Z_{0,tr}|} - 1 \right|,$$

where $N = 1600$ – number of dots in a range of frequencies.

Values of mean-integrated relative errors of determination of input and transfer mechanical impedances by formulas (2) and (6) are resulted in table 1.

|                  | According to the formulas (2) | According to the formulas (6) |
|------------------|------------------------------|------------------------------|
| $J_{in}$         | 0.0181                       | 0.0143                       |
| $J_{tr}$         | 4.9676                       | 0.1288                       |

Thus, the account of the flexibility of the facility allows to reduce the error of calculations of the input mechanical impedance in 1.27 times, and the transfer mechanical impedance – in 38.6 times.

6. Conclusions and conclusions

In the given work design dependences for definition of mechanical impedance of pipes on the basis of the data of dynamic tests are received. The proposed dependencies differ from the existing test methods in that they take into account the flexibility of the elements of the experimental facility and there is no limitation on the value of acceleration of the outlet flange of the nozzle.
The use of the proposed formulas increases the accuracy of determination of both input and transfer mechanical impedance. Input mechanical impedance is specified insignificantly, as it can be determined quite accurately by the existing method without taking into account the flexibility of the facility elements.

Transfer impedance without taking into account the flexibility of the facility elements is determined with high error and at some frequencies in the range from 200 to 800 Hz may differ from the true values by several times. And the calculated values of the transfer mechanical impedance are overestimated.

Average integral relative error of calculation of transfer mechanical impedance by formulas (6) taking into account flexibility of facility elements does not exceed 13%. From the point of view of vibroisolation properties of a branch pipe the greatest interest is just its transfer mechanical impedance.

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