Surrogate-based optimization of a cratered cylindrical hole to enhance film-cooling effectiveness

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Abstract
A cratered film-cooling hole was investigated to determine the effect of shape parameters on film-cooling performance, and shape of the hole was optimized to improve film-cooling effectiveness. Numerical analyses were performed using three-dimensional Reynolds-averaged Navier-Stokes (RANS) equations. The numerical results were validated by comparison with experimental data for film-cooling effectiveness for cratered film cooling. The ratio of the major axis length of the crater to the diameter of the hole, the ratio of the minor axis length of the crater to the diameter of the hole, and the ratio of the depth of the crater to the diameter of the hole were chosen as the design variables. From the results of a parametric study, the effects of these three design variables on film-cooling effectiveness were evaluated. The spatially-averaged film-cooling effectiveness was defined as the objective function. The values of the objective function were numerically evaluated through a RANS analysis at the design points determined by Latin hypercube sampling to construct surrogate models, i.e., RSA, RBNN, and Kriging models. Sequential quadratic programming was applied to search for the optimal point from the constructed surrogate models. The result showed that the Kriging model gave best optimization result and the film-cooling effectiveness for a cratered hole was improved by about 20% through the optimization in comparison with the reference geometry.

Key words: Film-cooling, Crater, Gas turbine, RANS analysis, Film-cooling effectiveness

1. Introduction

A gas turbine engine operates at a high inlet temperature in the turbine stage in order to enhance thermal efficiency and reduce fuel consumption. Since the inlet temperatures are higher than the allowable metal temperature, components of the turbine blade are exposed to high thermal loads. Therefore, there is a need for the development of effective cooling techniques to prevent blade failure and to ensure smooth operation of the gas turbine. Film cooling is an active cooling method whereby internal coolant air is ejected through discrete holes drilled in the turbine blade's exterior to protect the surface of the turbine blade from hot gas. Film cooling has been widely used in modern turbine blades to increase the thermal protection of the material.

It is well known that film-cooling performance is significantly affected by the hole geometry. As a result, various film-cooling hole shapes have been proposed and studied in attempts to improve film-cooling performance (1998-2012). Gritsch et al. (1998) carried out an experimental study to find the effects of the cross-flow Mach number and film-cooling hole geometry on cooling performance for a cylindrical hole and two diffuser-shaped holes. Saumweber and Schulz (2012) investigated the effects of the inclination angle, expansion angle of the diffuser, and length of the cylindrical part at the entrance of fan-shaped and cylindrical holes on film-cooling performance. Gritsch et al. (2005) conducted an experimental investigation to find the effects of various geometric parameters on film-cooling performance of fan-shaped film-cooling holes. The parameters tested in their experiment are the ratio of the cross-sectional area at the hole exit to that at the inlet, the ratio of the hole exit width to the hole pitch, the ratio of the hole pitch to the hole diameter, and the compound angle. They reported that the pitch-to-diameter ratio has the largest effect on film-cooling effectiveness among the tested parameters. Reiss and Bolcs (2000) compared three
configurations in a showerhead arrangement of film-cooling holes on the leading edge of a blade, for two different free-stream conditions. Hyams and Leylek (2000) performed a numerical analysis to investigate the film-cooling effectiveness of cylindrical, forward-diffused, laterally-diffused, inlet-shaped, and cusp-shaped film-cooling holes. The laterally-diffused hole showed the best film-cooling performance. Nguyen et al. (2011) investigated the film-cooling performance of a conically-shaped hole through an experimental study, and reported that the conically-shaped hole yielded substantially improved film-cooling effectiveness in comparison with a cylindrical hole. Sargison et al. (2002a, 2002b) proposed a so-called console film-cooling hole (converging slot hole). They performed an experimental investigation of console holes located at flat plate and blade’s leading edge.

Although the use of shaped film-cooling holes provides substantially improved cooling performance in comparison with a cylindrical hole, difficulty remains in terms of manufacturing due to the delicate designs involved. Therefore, cratered and trenched film-cooling holes designed for easy manufacturing and high performance were proposed by Bunker (2002) and Fric and Campbell (2002), respectively. Bunker (2002) first studied so-called trenched film-cooling holes and reported that they showed enhanced film-cooling effectiveness compared to cylindrical holes, with very little variation of film-cooling effectiveness versus the blowing ratio. Lu et al. (2009a) investigated the effects of trench width and depth on the film-cooling effectiveness and the heat transfer coefficient. Fric and Campbell (2002) first introduced the cratered film-cooling hole, which has an elliptical exit in a shallow elliptical surface cup or depression. Lu et al. (2009b) performed experimental and numerical analyses to evaluate the film-cooling effectiveness and heat transfer coefficients for three different cratered hole shapes; cylindrical, trenched, and shaped diffuser holes. In this study, the cratered holes did not perform well compared to the shaped holes and trenched holes, but yielded higher film-cooling effectiveness at a lower blowing ratio compared to the cylindrical holes. Dorrington et al. (2007) studied the effects of 18 different cratered and trenched shapes on film-cooling performance. They suggest that a crater with film-cooling hole shifted to downstream end showed improved effectiveness over the other configurations that have an annular gap between the crater wall and the film-cooling hole edge. Kalghatgi and Acharya (2014) proposed contoured cratered hole with V-shaped protrusion. They reported that best performing crater design showed about 200% increase in the peak performance gain compared to the cylindrical hole and greater performance gain over the trenched holes. Beside these works, few studies on the shape of cratered film-cooling holes, and optimization of a cratered hole, have been reported in the literature.

Due to rapid development of computing power, design optimization recently became popular for the design of thermo-fluid devices (2008-2010). Surrogate modeling is being extensively used in design optimization to approximate the objective function, and greatly reduces the computing time for repeated evaluations of the objective function. Lee and Kim (2010) optimized the shape of a simple fan-shaped hole with three design variables using a weighted-average surrogate model. Lee and Kim (2011) used a Krigeing model (2005) for the optimization of a laidback hole for various blowing ratios.

In the present work, a design optimization of a cratered film-cooling hole was performed to enhance film-cooling effectiveness at a blowing ratio of 0.5. Three-dimensional (3-D) Reynolds-averaged Navier-Stokes (RANS) equations were used to analyze the fluid flow and heat transfer. A Krigeing model (2005) was used to approximate the objective function; i.e., the spatially-averaged film-cooling effectiveness.

2. Numerical Analysis

The computational domain and geometric variables of a cratered cylindrical film-cooling hole considered in the present work are shown in Fig. 1. Fric and Campbell (2002) suggested that the geometry shown in Fig. 1 exhibits good film-cooling performance. The computational domain is comprised of a main duct with a hot gas stream, a coolant plenum, a crater, and a cylindrical hole. The diameter of the cylindrical hole and the injection angle of the hole were kept constant: \( D=12.7 \text{ mm} \) and \( \alpha=30^\circ \). The height of the film-cooling hole, including the crater height, was fixed to 23.8 mm.

For the analysis of fluid flow and convective heat transfer, ANSYS CFX 15.0 (2013) computational fluid dynamics commercial software was used to solve the 3-D RANS equations. The shear stress transport (SST) (2001) turbulence model was used as a turbulence closure; the model combines the k-\( \varepsilon \) and k-\( \omega \) models with a blending function. The k-\( \omega \) model was used in the region near the wall, and the k-\( \varepsilon \) model was activated in the region of bulk flow.

Figure 2 shows the boundary conditions, which were determined to match the experimental conditions of Lu et al.
The working fluid was regarded to be an ideal gas. No-slip and adiabatic conditions were applied at the walls. A uniform normal velocity of 13.8 m/s and total temperature of 321 K were specified at the inlet of the main duct. The mainstream Reynolds number based on film cooling hole diameter was 11,000. At the inlet of the coolant plenum, a constant mass flow rate was set to M=0.5 and the static temperature of the coolant was 296 K. The blowing ratio, M, was defined as the ratio of the mass flux of coolant to that of mainstream. At the outlet of the main duct, a static pressure of 1 atm was assigned. Periodic boundary conditions were used at the side boundaries of the main duct. The free stream turbulence intensity in the main duct was 2%, and the density ratio of the coolant and mainstream was 1.0.

Unstructured tetrahedral meshes with prism layers were used to resolve the high velocity gradient near the walls by using ANSYS ICEM 15.0. An example of the tetrahedral grid system used in this work is shown in Fig. 3. The meshes were clustered near the wall and inside the hole. Additionally, considering the effect of the interaction of the coolant jet with the mainstream hot gas, very dense meshes were concentrated around the crater. The first grid points adjacent to the wall were placed at $y^+$ less than 1 to implement the low Re SST model.

Convergence was assumed when the root mean square residual values of all flow parameters fell below 1.0 E-6, and imbalances of mass and energy in the entire computational domain were less than 0.001%. An Intel i7 3.41 GHz PC was used for the computation. The solver terminated a single simulation within 9000 iterations, which took approximately 100 to 120 hr of computational time depending on the geometry and the convergence rate.

### 3. Objective Function and Design Variable

For the present optimization, three geometric parameters--the ratio of the major axis length of the crater to the diameter of the hole ($D_1/D$), the ratio of the minor axis length of the crater to the diameter of the hole ($D_2/D$), and the ratio of the depth of the crater to the diameter of the hole ($d/D$) were selected as the design variables (Fig. 1). The

| Design variable | Reference | Lower bound | Upper bound |
|-----------------|-----------|-------------|-------------|
| $D_1/D$         | 3.0       | 2.0         | 4.0         |
| $D_2/D$         | 2.0       | 1.25        | 2.5         |
| $d/D$           | 0.5       | 0.3         | 0.8         |
design space of the optimization was determined as shown in Table 1, based on the results of a preliminary parametric study.

The objective function to be maximized was defined as a spatially-averaged film-cooling effectiveness ($\eta_s$):

$$\eta_s = \frac{T_{aw} \left( \frac{x}{D} \frac{z}{D} \right) - T_h}{T_c - T_h}$$

(1)
\[ \eta_s \left( \frac{x}{D}, \frac{z}{D} \right) = \frac{1}{20 \times 3} \int_{-1.5}^{1.5} \int_{0}^{20} \eta \left( \frac{x}{D}, \frac{z}{D} \right) \sigma \left( \frac{z}{D} \right) \sigma \left( \frac{x}{D} \right) \]  

where \( T_{aw} \) is the adiabatic wall temperature, and \( T_h \) and \( T_c \) are the hot gas temperature in the main duct and the coolant temperature, respectively. The spatially-averaged film-cooling effectiveness was averaged over an area of 3D in width and 20D in streamwise length.

4. Optimization Methods

The optimization procedure used in this work is shown in Fig. 4. Initially, an objective function and three design variables were defined and the design space was determined. Design points within the design space were then generated using Latin hypercube sampling (LHS) (Sacks et al., 1989). The objective function values at these design points were evaluated by using RANS analysis. Finally, a surrogate model was constructed, and an optimal point was sought by a gradient-based search algorithm on this surrogate model. Sequential quadratic programming (SQP) was used for this search algorithm. In this work, a comparative analysis of different surrogate models such as the response surface approximation (RSA) (Myers and Montgomery, 1995), the radial-basis neural network (RBNN) (Orr, 2008) and the Kriging (Martin and Simpson, 2005) models, was performed to maximize the achievement of the optimization.

In the RSA model, an approximating response function was created by using a second-order polynomial function. The RBNN which is a two layer network, is composed of hidden layer of the radial basis function and a linear output layer. The spread constant and error goal are used as the parameters for fitting RBNN model. In MATLAB (2015), the function for RBNN design is \texttt{newrb}.

Fig. 7 Effects of three geometric variables on spatially-averaged film-cooling effectiveness.
The Kriging method (Martin and Simpson, 2005), which is an interpolating meta-modeling technique, is used to estimate an unknown value of a function (response) at several unsampled points. Kriging postulation is a combination of two components, the global model and a departure, as follows:

\[ y(x) = f(x) + Z(x) \]  

where \( y(x) \) denotes an unknown function, and \( f(x) \) is a known function (generally a polynomial) that approximates the trend of the design space, also referred to as the “global” trend model.

The trend model \( Z(x) \) provides a localized deviation such that the Kriging model interpolates the sampled data points by quantifying the correlation of points with a Gaussian correlation having a zero mean and nonzero covariance. The covariance matrix of \( Z(x) \) is given by

\[ \text{Cov} [Z(x_i), Z(x_j)] = \sigma^2 R \{R(x_i, x_j)\}, \quad i, j = 1, 2, \ldots, ns \]  

where \( R \) is a correlation matrix consisting of spatial correlation functions (SCFs) with \( R(x_i, x_j) \) as their elements. \( \sigma^2 \) is the process variance representing the SCF quantifying the correlation between any two \( ns \) sampled data points \( x_i \) and \( x_j \), and thereby controls the smoothness of the Kriging model and differentiability of the surface. The Gaussian function used in this work is the most preferable SCF when used with a gradient-based optimization algorithm as it provides a relatively smooth and infinitely differentiable surface.
Table 2 Values of design variables and objective function at design points

| Design | D1/D | D2/D | d/D | Objective function |
|--------|------|------|-----|--------------------|
| 01     | 3.23 | 2.31 | 0.38 | 0.2662             |
| 02     | 2.98 | 1.68 | 0.54 | 0.2697             |
| 03     | 2.46 | 2.07 | 0.67 | 0.3254             |
| 04     | 3.62 | 1.92 | 0.57 | 0.3664             |
| 05     | 3.15 | 2.12 | 0.56 | 0.3664             |
| 06     | 3.10 | 2.15 | 0.60 | 0.3664             |
| 07     | 3.31 | 2.26 | 0.55 | 0.3664             |
| 08     | 2.77 | 1.78 | 0.76 | 0.2599             |
| 09     | 3.15 | 1.92 | 0.49 | 0.2940             |
| 10     | 3.04 | 2.45 | 0.68 | 0.3374             |
| 11     | 2.31 | 2.5  | 0.63 | 0.3664             |
| 12     | 3.69 | 1.25 | 0.65 | 0.2409             |
| 13     | 2.38 | 2.4  | 0.43 | 0.3080             |
| 14     | 3.15 | 1.92 | 0.49 | 0.2940             |
| 15     | 3.04 | 2.45 | 0.68 | 0.3374             |
| 16     | 2.31 | 2.5  | 0.63 | 0.3664             |
| 17     | 3.69 | 1.25 | 0.65 | 0.2409             |
| 18     | 2.38 | 2.4  | 0.43 | 0.3080             |
| 19     | 3.15 | 1.92 | 0.49 | 0.2940             |
| 20     | 3.04 | 2.45 | 0.68 | 0.3374             |
| 21     | 2.31 | 2.5  | 0.63 | 0.3664             |
| 22     | 3.69 | 1.25 | 0.65 | 0.2409             |
| 23     | 2.38 | 2.4  | 0.43 | 0.3080             |

Fig. 10 Normalized y-velocity distribution at the exit of the hole (M=0.5).

Using the constructed surrogate model, SQP (using the function fmincon in MATLAB R2015a (2015) was used to find the optimum point. SQP is a robust algorithm for nonlinear continuous optimization, and represents a generalization of Newton’s method to multiple dimensions. A multiple initial guess method was used at multiple locations in the design space to avoid a local minimum. The best ten points among the experimental designs were used as the initial guesses.

5. Results and Discussion

To find the optimum grid, a grid dependency test was performed for the reference shape in order to verify that the numerical solution is grid-independent. Figure 5 presents the distributions of the laterally-averaged film-cooling effectiveness with different numbers of meshes in a range from 2.4 to 4.5 million. From the results shown in Fig. 5, the optimum number of grids was determined to be 3.5 million. Validation of the numerical results was performed for three different film-cooling holes, i.e., cylindrical, trenched and cratered holes, by comparison with experimental data measured by Lu et al. (2009b) as shown in Fig. 6, which shows the distributions of laterally-averaged film-cooling effectiveness at a blowing ratio of 0.5. The numerical results were calculated by using the SST model. The numerical results obtained with the SST model show reasonable agreements with the experimental data not only for cratered hole but also for cylindrical and trenched holes as shown in Fig. 6. Therefore, the accuracy of the present predictions...
Table 3 Results of optimization (M=0.5)

| Shapes | Variables | Objective function | Error (%) |
|--------|-----------|--------------------|-----------|
|        | D₁/D     | D₂/D   | d/D   | Surrogate Prediction | RANS Calculation |
| Reference | 3.000 | 2.000 | 0.50   | -       | 0.3007 |
| KRG    | 2.077 | 2.500 | 0.542  | 0.3665  | 0.3611 | 1.4 |
| RSA    | 2.077 | 2.500 | 0.515  | 0.3831  | 0.3546 | 7.4 |
| RBNN   | 2.630 | 2.470 | 0.629  | 0.3843  | 0.3465 | 9.8 |

Fig. 11 Development of streamlines and vectors on the y-z plane at x/D=5 (M=0.5).

was acceptable for the optimization. Before the optimization was performed, a parametric study was made using the three design variables (D₁/D, D₂/D, and d/D) as shown in Fig. 7. When one variable was varied, the other two variables were fixed at the reference values shown in Table 1. The spatially-averaged film-cooling effectiveness was found to be sensitive to all of the design variables. In addition, variations of the spatially-averaged film-cooling effectiveness with the three design variables show maximum values at D₁/D = 2.25, D₂/D = 2.25, and d/D = 0.6.

In the film-cooling technique used in a gas turbine, the coolant air bleeds from the compressor stage. Thus, the excessive use of coolant causes a reduction in the overall engine efficiency. Thus, the goal of film-cooling is to achieve high cooling efficiency by using less coolant. The cratered hole yields higher film-cooling effectiveness at a lower blowing ratio (2009b). Therefore, the present optimization was performed at a low blowing ratio of M = 0.5.

The cratered film-cooling hole was optimized to enhance spatially-averaged film-cooling effectiveness in terms of the design variables, D₁/D, D₂/D, and d/D, at blowing ratios of 0.5. LHS was applied to select 27 design points in the design space (Table 2). The objective function values at these design points were obtained using 3-D RANS equations, and these values were used for constructing a surrogate model. Finally, the optimum design was found using the SQP algorithm.

Table 3 shows results of the optimization in comparison with a reference design using three surrogate models, i.e., RSA, RBNN, and Kriging models. The geometry reported by Lu et al. (2009b) was selected as the reference design. The predicted optimum values obtained by the RSA and RBNN models, represent relatively large deviations from the values calculated by RANS analysis with relative errors of 7.4% and 9.8%, respectively. However, the optimum objective function value predicted by the Kriging model shows good agreement with the value calculated by RANS.
analysis with a relative error of 1.4%. Furthermore, the optimum design of Kriging model showed the most improved RANS calculated value of the objective function among the three different surrogate models. Therefore, the Kriging model was decided to be a best surrogate model in this optimization. Through the optimization, the objective function (i.e., the spatially-averaged film-cooling effectiveness) was successfully increased by 20.0% compared to that of the reference design.

Figure 8 presents a comparison of the film-cooling effectiveness distribution between the optimum and reference designs. The figure shows a distribution of local film-cooling effectiveness along the centerline for the optimum design, and shows increasing enhancement of the film-cooling effectiveness as x/D increases. However, in the distribution of laterally-averaged film-cooling effectiveness, the optimum shape shows maximum enhancement just downstream of the film-cooling hole compared to the reference design, and nearly uniform enhancement far downstream of the film-cooling hole.

Figure 12 Streamwise vorticity ($\omega_x$) contours on y-z plane at x/D=5 (M=0.5).

Figure 13 Spanwise distribution of local film-cooling effectiveness at x/D=5 (M=0.5).

Figure 14 Streamlines on x-z plane at exit of crater and on x-y plane at z/D=0.5 (M=0.5).
Fig. 15 Iso-surfaces of the coolant at T=305 K and 315 K (M=0.5).

Distributions of the local film-cooling effectiveness on the film-cooling surface are shown in Fig. 9. Both the reference and the optimum designs yield similar distributions in the streamwise direction, but the optimum geometry provides a wider lateral spreading of coolant. This results in a larger spatially-averaged film-cooling effectiveness.

Distributions of the velocity component in the direction normal to the film-cooling surface (i.e., the y-direction) on the exit plane of the crater are shown in Fig. 10. Both the reference and the optimum designs show that the highest y-velocity component occurs near the center of the leeward edge. The y-velocity component is small near the lateral leeward edge of the crater in the reference design, whereas the optimum design generates the flow in the y-direction in the same region. This contributes to the wide lateral spreading of coolant in the optimum design, as shown in Fig. 9.

Streamlines and vectors are shown in Fig. 11. The streamlines are colored based on the temperature. As shown in the figure, the recirculating flow is located in the crater and generates a couple of anti-vortices that acts counter to the main vortex pair; the vortex pair located at the center are kidney vortices and those located at the sides are anti-kidney vortices (Heidmann, 2008). The anti-kidney vortices were also reported in the study of contoured crater performed by Kalghatgi and Acharya (2014). The present study confirmed that the anti-kidney vortices can be also generated by the basic elliptical crater. While the anti-kidney vortices created by the contoured crater are located between the kidney vortices, the anti-kidney vortices are located outside of the kidney vortices in the case of elliptically cratered hole. Heidmann (2008) reported that this also happened in an anti-vortex hole. For reference geometry, the recirculating flow is mainly generated by the main stream, and excessive mixing occurs in the upstream part of the crater. On the other hand, in the optimum design, the recirculating flow is generated inside the crater by the coolant, and the hot gas of the main stream scarcely penetrates into the crater.

Distributions of the streamwise vorticity component ($\omega_x$) on the y-z plane at x/D = 5 are presented in Fig. 12. Similar vortices structures can be observed in both the reference and optimum designs, but the optimum design generates weaker kidney vortices with stronger anti-kidney vortices compared to the reference design. This explains the higher film-cooling effectiveness in the central region, and also the wider spread of the local film-cooling effectiveness in the optimum design compared to the reference design, as shown in Fig. 9. This is also confirmed by Fig. 13, which shows the spanwise distributions of local film-cooling effectiveness at x/D = 5. The reference and
optimum designs have similar values around the centerline. However, in both of the side regions apart from the centerline, the film-cooling effectiveness for the optimum design shows higher values than those for the reference design.

Figure 14 shows streamlines on the x-z plane at exit of the crater and on the x-y plane at z/D = 0.5. As shown in Figs. 14(a) and (b), both the reference and optimum designs show the occurrence of the vortices in the crater, which generate anti-kidney vortices downstream of the crater. However, in the reference design, a larger recirculating flow is generated compared with the optimum design, and excessive mixing of hot gas with coolant occurs in the upstream part of the crater as shown in Fig. 14(c). Thus, in the reference design, the relatively high temperature goes out from the sides of the crater compared to the optimum design. In the optimum design, the temperature of the flows issuing from the sides of the crater, which generates anti-kidney vortices, is much lower than that of the reference design. This causes an enhancement of film-cooling effectiveness in the optimum design.

Figure 15 shows iso-surface plots at T = 305 K and 315 K, where the iso-surface represents the surface at constant temperature. The plots effectively show the coolant regions in the crater. As shown in Fig. 15, the iso-surfaces of T = 315 K are located at the top of the crater in both cases. In the optimum design, the iso-surface of T = 315 K is more widely distributed on the cooling surface in comparison with the reference design. On the other hand, the iso-surfaces of T = 305 K are located inside the crater in both cases. In the reference design, the iso-surfaces of T = 305 K is formed from the exit of the cylindrical hole, while the optimum design shows the iso-surface similar to that of T = 315 K in the crater. However, downstream of the crater, both the iso-surfaces of T = 305 K are similar in width and height. Therefore, in the optimum design, the coolant with a temperature between 305 K and 315 K is more widely distributed on the cooling surface compared to the reference design.

Temperature distributions on the y-z plane (x/D = 1 and 5) are shown in Fig. 16. The low temperature regions of the reference and optimum designs are similar around the centerline. However, as previously discussed for Fig. 12, the coolant of relatively higher temperature is more widely distributed on the cooling surface in the optimum design compared to the reference design.

6. Conclusion

A cratered film-cooling hole was numerically analyzed through RANS analyses of fluid flow and heat transfer, and optimized using a Kriging model. The numerical results predicted by the SST model for film-cooling effectiveness were in good agreement with the experimental data. Three geometric variables (the ratio of the major axis length of the
crater to the diameter of the hole, $D_1/D$, the minor axis length of the crater to the diameter of the hole, $D_2/D$, and the
depth of the crater to the diameter of the hole, $d/D$) were selected as the design variables, and a parametric study
suggested that maximum values in the variations of spatially-averaged film-cooling effectiveness with $D_1/D$, $D_2/D$, and
$d/D$, were found at $D_1/D = 2.25$, $D_2/D = 2.25$, and $d/D = 0.6$, respectively. The spatially-averaged film-cooling
effectiveness was selected as the objective function and its surrogate models were constructed using the objective
function values evaluated at 27 design points selected by LHS. In this work, the optimization was performed with three
different surrogate models, i.e., RSA, RBNN and Kriging models, and the results were evaluated comparatively. Design
optimization was conducted for a blowing ratio of 0.5. The optimum objective function value predicted by the Kriging
model showed a relative error of only 1.4% compared to the value calculated by RANS analysis. Furthermore, among
the three different surrogate models, the optimum design obtained by Kriging model showed the most improved value
of the objective function. The optimum geometry for a blowing ratio of 0.5 improved the objective function values by
20.0% compared to the reference geometry. Analyses of velocity and temperature fields indicated that the kidney
vortices in the central region downstream of the film-cooling hole were suppressed and the anti-kidney vortices were
enhanced by optimizing the cratered hole shape providing wider lateral spreading of coolant, which resulted in a larger
spatially-averaged film-cooling effectiveness, compared with the reference geometry.

**Nomenclature**

| Symbol | Description |
|--------|-------------|
| D      | film-cooling hole diameter |
| $D_1$  | major axis length of crater |
| $D_2$  | minor axis length of crater |
| d      | depth of crater |
| M      | blowing ratio: $(\rho_c U_c)/(\rho_h U_h)$ |
| $T_{aw}$ | adiabatic wall temperature |
| $T_h$  | hot gas temperature in the main channel |
| $T_c$  | coolant jet temperature |
| $U_c$  | velocity of coolant at the hole exit |
| $U_{\infty}$ | velocity at the main stream inlet |
| $y^+$  | $y$ in law-of-the-wall coordinate |

**Greek symbols**

| Symbol | Description |
|--------|-------------|
| $\rho$ | density, kg m$^{-3}$ |
| $\alpha$ | injection angle of the hole |
| $\eta$ | film-cooling effectiveness: $(T_{aw}-T_{\infty})/(T_c-T_{\infty})$ |

**Subscripts**

| Subscript | Description |
|-----------|-------------|
| aw        | adiabatic wall |
| $\infty$  | main stream |
| c         | coolant |
| h         | hot gas |
| l         | laterally-averaged |
| s         | spatially-averaged |

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