The pion form factor is calculated using quenched twisted mass QCD with $\beta = 6.0$ and maximal twisting angle $\omega = \pm \frac{\pi}{2}$. Two pion masses and several values of momentum transfer are considered. The momentum averaging procedure of Frezzotti and Rossi is used to reduce lattice spacing errors, and numerical results are consistent with the expected $O(a)$ improvement.

1. INTRODUCTION

Lattice QCD with a twisted mass term (tmQCD) is a computationally efficient method for eliminating the exceptional configurations that plague light Wilson quarks.\[1\] In addition, at maximal twist, many quantities are $O(a)$ improved and others become $O(a^2)$ improved through a momentum averaging procedure.\[2\] The pion form factor provides an opportunity to explore this improvement procedure.

With only two valence fermions and no disconnected lattice diagrams\[3\], the pion form factor is an appealing laboratory for studies of the transition between the perturbative and nonperturbative regimes of QCD. Experimental measurements are available for comparison, and current experiments at Jefferson Lab are exploring higher momentum transfers.\[4\]

Recent lattice studies of the pion form factor have considered various lattice actions, with and without $O(a)$ improvement, and the effects of $O(a)$ terms are found to be non-negligible.\[5\]

In this work, we present results from pion form factor calculations using tmQCD and compare to existing lattice studies.

2. METHOD

Computations are performed with the $\beta = 6.0$ Wilson gauge action and the twisted mass action for a degenerate doublet of up and down quarks with no Symanzik improvement (clover) term.\[6\] A twisting angle of $\pm \frac{\pi}{2}$ is obtained by setting the hopping parameter to its critical value, $\kappa_c = 0.156911$.\[7\] The quark mass is then determined by the remaining parameter $\mu$ in the tmQCD action, and results are reported here for two options, $|\mu| = 0.030$ and 0.015, corresponding to pion masses near 660 and 470 MeV respectively. We use 100 configurations that are $16^3 \times 48$ with periodic boundary conditions. The GMRES-DR matrix inverter\[7\], which deflates the smallest eigenvalues and systematically improves them upon successive restarts of the standard GMRES iteration, was used throughout this work.

The pion form factor $F(Q^2)$ is defined by
\[
\langle \pi^+(\vec{p}_f)|j_\mu(0)|\pi^+(\vec{p}_i)\rangle = F(Q^2)(p_i + p_f)_\mu
\] (1)
where $j_\mu(0)$ is a conserved vector current evaluated at the spacetime origin, $p_i$ and $p_f$ are the initial and final pion (Euclidean) 4-momenta respectively, $\vec{p}_i$ and $\vec{p}_f$ are the corresponding 3-momenta, and $Q^2 = (p_f - p_i)^2$ is the 4-momentum transfer. To compute the above matrix element on a spacetime lattice, one can use the three point correlator displayed in Fig. 1. A source with pion quantum numbers is placed at $x_i$, a sink at $x_f$, and a vector current is inserted at $x$. The pion form factor is extracted from a simultaneous single exponential fit to the long time range of the two and three point correlators given by:
\[
G_{\pi\pi}(t, t_i, \vec{p}) = \sum_x e^{-i(\vec{x} - \vec{x}_i)\cdot\vec{p}} \langle 0|\phi(x)\phi^\dagger(x_i)|0\rangle
\] (2)
\[
\sum_{t \gg t_i} \frac{|Z|^2}{E} e^{-\frac{t - T}{2}E} \cosh \left( t - t_i - \frac{T}{2} \right) E,
\]
Figure 1. Three point correlator for the pion form factor.

\[ \Gamma_{\pi\mu\pi}(t_i, t, t_f, \vec{p}_i, \vec{p}_f) = \sum_{\vec{x}_i, \vec{x}_f} e^{-i(\vec{x}_f - \vec{x}_i) \cdot \vec{p}_f} e^{-i(\vec{x} - \vec{x}_i) \cdot \vec{p}_i} \langle 0 | \phi(\vec{x}_f) \mu \phi(\vec{x}) | \pi^+ \rangle \]

where

\[ \langle 0 | \phi(x) | \pi^+ (\vec{p}) \rangle = Z e^{i p \cdot x} \]

(4)

\( \phi(x) \) is a local interpolating field operator with \( \pi^+ \) quantum numbers, \( T \) is the temporal extent of the lattice and \( E_i, E_f \) denote pion energies. In practice, we compute one propagator from \( x_i \) to \( x_f \), and a double propagator from \( x_i \) to \( x_f \) to \( x \). Three different options for the double propagator were studied, corresponding to the local pseudoscalar operator at \( x_f \) having momentum \( \vec{p}_f = (0, 0, 0) \), \( (0, 0, p_{\text{min}}) \) and \( (0, 0, -p_{\text{min}}) \), where \( p_{\text{min}} = \frac{2\pi}{L} \) (\( L=16 \)). In all cases, \( x_i \) and \( x_f \) are separated by 15 time slices. The conserved vector current was used at \( x_f \) and a local pseudoscalar at \( x_i \). Smeared operators have been used routinely in pion form factor studies\[5\], but we chose local operators for this first consideration of tmQCD.

### 3. RESULTS

Analyzing the pseudoscalar-pseudoscalar and vector-vector two point correlators, we found that

#### Table 1

Pseudoscalar and vector meson masses.

| \(|\mu|\) | \(am_\pi\) | \(am_\rho\) |
|---|---|---|
| 0.015 | 0.238(5) | 0.453(37) |
| 0.030 | 0.331(3) | 0.496(19) |

![Figure 2. Pion dispersion relation at |\(\mu|\) = 0.015.](image2)

![Figure 3. Form factor at |\(\mu|\) = 0.030 compared to vector meson dominance (VMD).](image3)
a reasonable signal could be obtained for momenta $|\vec{p}|^2 \leq 4p_{\text{min}}^2$. Figure 2 shows the dispersion relation for the ground state pseudoscalar meson at $|\mu| = 0.015$. Table 1 lists results for the pseudoscalar and vector meson masses in lattice units.

To extract the form factor at higher $Q^2$, nonzero sink momentum is important, but then improvement requires momentum averaging over positive and negative momenta.[2] Figures 3 and 4 show our results for the pion form factor as a function of $Q^2$. A comparison with existing literature is shown in Fig. 5. Notice that unimproved Wilson results are systematically below experiment (which follows vector meson dominance in this region of $Q^2$). Decreasing $m_\pi$ decreases $F(Q^2)$ even further. However, the tmQCD results are consistent with experiment and with $O(a)$ improved actions.

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