Opinion dynamics model based on cognitive biases

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We present an introduction to a novel model of an individual and group opinion dynamics, taking into account different ways in which different sources of information are filtered due to cognitive biases. The agent based model, using Bayesian updating of the individual belief distribution, is based on the recent psychology work by Dan Kahan. Open nature of the model allows to study the effects of both static and time-dependent biases and information processing filters. In particular, the paper compares the effects of two important psychological mechanisms: the confirmation bias and the politically motivated reasoning. Depending on the effectiveness of the information filtering (agent bias), the agents confronted with an objective information source may either reach a consensus based on the truth, or remain divided despite the evidence. In general, the model might provide an understanding into the increasingly polarized modern societies, especially as it allows mixing of different types of filters: psychological, social, and algorithmic.

Keywords: Opinion change, motivated reasoning, confirmation bias, Bayesian updating, agent based model

I. INTRODUCTION

The actual processes through which individual people and groups of people evaluate information and form or change their opinions are very complex. Psychology offers many descriptions of these processes, often including multiple pre-conditions and influencing factors. The assumption that opinions form through a truth-seeking, rational reasoning is, unfortunately, not true in most cases. The list of the recognized cognitive biases that influence our mental processes (rational and emotional) is very long, covering over 175 named entries (Benson [8]). The situation becomes even more complex when we try to describe how the individual opinion changes combine to form dynamical social systems. In addition to the problems alluded to above, one has to consider the multiple forms of social interactions: personal (fact to face and, especially in recent years, those mediated by electronic media) and public (news, comments, rumours and other modes of information reaching an individual). These interactions vary with respect to their informative and emotional content, trust to the source of the information, its pervasiveness and strength and more. Taking these difficulties into account, the task of an accurate description of the individual and group opinion change dynamics appears insurmountable. Yet, the need to understand how and why our societies (especially democratic ones) arrive at certain decisions, how and why people change their beliefs (or why they remain unconfident in the light of ‘overwhelming evidence’), what are the mechanisms driving the increasing polarization of our societies and how to make people talk to and understand each other, is so great that despite the challenges, there is intense research on the topic.

For several years, group opinion change has been a fertile ground for sociophysics and Agent Based Modelling. The initial works have used many of the tools and ideas developed to describe magnetic phenomena and used the analogies between atomic spin states and opinions, magnetic field and external influences to derive statistical descriptions of global opinion changes. There are many approaches, for example the voter model [7, 20, 25, 35], the Sznajd model [10, 72, 74, 75, 76, 93, 94, 93, 98, 99, 100], the bounded confidence model Deffuant et al. [27, 28, 29, 101], Weisbuch et al. [101], Weisbuch [102], the Hegelsmann-Krause model [12, 13, 15, 16, 19, 20, 22, 23, 24, and its further modifications including the role of leaders [14, 15, 16, 17, 20, 22, 23, 24, and many more others. Historically, the initial focus was on the formation of consensus — treated as a form of a phase transition — but the later works focused on the role of minorities, with special attention given to the effects of presence of inflexible, extremist individuals.

The literature on numerical models of opinion dynamics has grown enormously in the past decade. For a relatively recent reviews we point out Castellano et al. [13, Castellano et al. [14, Galam [34]. While most of the early works were limited to studies of the models themselves (rather than specific social contexts), showing very interesting sociophysical results, but only weak, qualitative correspondence to any actual societies (Sobkowicz [75]), the recent years have changed this situation. Availability of large scale datasets, documenting opinions and interactions between people (derived mainly from the Internet and social media), has allowed, in principle, to attempt quantitative descriptions of specific opinion evolution processes. The number of sociophysical and ABM based works aimed at quantitative description of real societies remains limited. For example, in the case of political elections, only a few papers attempt such description (Caruso and Castorina [15], Fonseca and Louca [32], Fortunato and Castellano [33], Galam [34], Palombi and Toti [60, Sobkowicz [81]).

Despite the undoubted advances, the sociophysical models of the individual behaviour are still rather crude. Most of the sociophysical agents and descriptions of their individual behaviour are too simplistic, too...
much ‘spin-like’, and thus unable to capture the intricacies of our behaviours. This observation applies also to the descriptions of the interactions between the agents, or, in more general way, to the way that new information is treated in the process of adjusting currently held opinions. Most of the Agent Based Models assume relatively simple forms of such interactions, for example rules which state that if an agent is surrounded by other agents holding an opinion different than its own, it would change it opinion to conform to the majority. As experience with real life situation shows, such ‘forced’ conversion is rather unlikely among people (in contrast with atomic spins...). The differences between the model behaviour of spin-persons (spinsons, Nyczka and Sznajd-Weron [62] and our understanding of real people have forced the introduction of special classes of agents, behaving in a way that is different from the rest: conformists, anticonformists, contrarians, inflexibles, fanatics... Using appropriate mixtures of ‘normal’ and special agents it has been possible to make the models reproduce more complex types of social behaviour.

In the author’s opinion, such artificial division of the agents into separate classes with different, fixed internal dynamics, while improving the models’ range of results, is psychologically incorrect. In a specific situation any person may behave inflexibly or show contrarian behaviour. For this reason, the author has proposed a model in which opinion change results from a combination of agent’s informative and emotional state, coupled with the informative and emotional content of the message processed by the agent (which may originate from an interaction with another agent or from the media). The model, introduced in Sobkowicz [77] [78] has allowed a quantitative description of an Internet discussion forum [79] and even to predict the results of recent elections in Poland [81]. The model applies however only to situations in which the emotional component is very strong, determining the individual behaviour.

One of the most active discussions in psychology of belief dynamics is centred around apparently irrational processing of information: the operation of biases, heuristic shortcuts and other effects that stand in contrast with the classical tenets of the rational choice theory. Such ostensibly irrational behaviours have not received much attention within the ABM community so far, despite their presence in many social situations. Important examples are provided by strong opposition to well documented irrational choices (such as disbelief in well documented updating. Despite the recognized status of the Bayesian updating in risk assessment and other areas, it is rather seldom used by the ABM community. To mention a few examples, Suen [87] has considered the effects of information coarsening (due to the agents’ reliance on specialists for the relevant information) and the tendency to choose the sources which confirm their pre-existing beliefs; Martins [56] has studied the case of continuous opinion model under Bayes rules, looking for long term evolution of the opinions; Bullock [10] has studied the conditions in which peoples’ beliefs, updated using Bayesian rules could, in the short term, instead of converging on a true value, diverge or even polarize. Ngaampruetikorn and Stephens [59] have analysed the role of confirmation bias in consensus formation in a binary opinion model on a dynamically evolving network.

The flexibility offered by the Bayesian approach allows much greater complexity of the behaviour of the individual agents, and as such, offers potentially more relevant descriptions of social behaviours than the spin-based models. Of course, these benefits do not come without a price: there are many more degrees of freedom in the system, and therefore many more unknowns in properly setting up the ABM simulations. Still, the importance of social phenomena observed around the world, in particular various forms and effects of polarization, suggests the need for a deeper understanding of the underlying mechanisms, and makes the effort worthwhile.

A. Confirmation bias vs. PMRP

One of the best recognized biases in information processing is confirmation bias, defined by the Wikipedia as ‘a tendency to search for, interpret, favour, and recall information in a way that confirms one’s pre-existing beliefs or hypotheses’. Such definition stresses that the operation of the confirmation bias may be on various levels: selecting and preferring the information sources, giving different weight to different sources and internal mechanisms (such as memory preferences is storing/recall of information). When people communicate, the individual confirmation bias effects may be combined in a way that creates group effects such as echo chambers. As a result, even when faced with true information, and agent (or a group of agents) may form or maintain a false opinion due to the confirmation bias.

The motivated reasoning paradigm considers the ways in which goals, needs and desires influence the information processing (Jost et al. [46]). These goals may be related to the individual needs, but also to group or global ones, for example the goal of achieving or maintaining the person’s position within a social group. In such a case, the motivated reasoning may bias the information processing by substituting the goal of truth-seeking by the person’s desires to affirm the affiliation with the chosen in-group. Seen through the lens of these desires, the apparently irrational choices (such as disbelief in well documented
evidence and belief in unproven claims) become rational again. We believe and act accordingly in a way that is congruent with the perceived beliefs and actions of our preferred social group. As the goal of the person is shifted from truth seeking to strengthening of the position within the social group, the disregard for the truth becomes rational. Especially, when the consequences of a rejection from the group are more immediate and important than the results of ‘erroneous’ perception of the world. Kahan [50] has provided a very attractive Bayesian framework, allowing not only to describe the role of various forms of cognitive biases, but also the empirical evidence of the differing predictions of the different heuristics, such as confirmation bias or political predispositions. Experiments with manipulated ‘evidence’, described by Kahan, are very interesting.

While both mechanisms introduced above lead away from the truth-seeking behaviour, their predictions might differ, especially with respect to new information. While the confirmation bias favours evidence in agreement with already held views (priors), the politically motivated reasoning selects and favours information congruent with person’s political identity (defined by the in-group characteristics). The confirmation bias depends on internal agent states, while PMR involves perception of external characteristics.

The vision of information processing, comparing the two forms of bias, described by Kahan is simple enough to become a framework of an ABM. As we shall argue, the Bayesian filtering approach is very flexible and may be applied to a variety of situations, contexts and types of processing bias. Our present goal is to describe such framework and to provide simple examples of the types of information processing leading to consensus or polarization. The latter case is of special importance, as the current political situation in many democratic countries seems to be irrevocably polarized, with large social sections unable to find common ground on many extremely important issues. Our hope is to find, with the use of the model, any suggestions for the processes that may reverse this polarization and enable communication across the current divisions.

II. INDIVIDUAL INFORMATION PROCESSING MODEL

A. Overview of the model

The current work aims at a general, flexible model of the individual opinion dynamics. We base our concepts on the Bayesian framework. Figure 1 presents the basic process flow, modelled after Kahan [50]. For simplicity, we shall assume that the belief which we will be modelling may be described as a single, continuous variable \( \theta \), ranging from -1 to +1 (providing a natural space for opinion polarization). The agent holds some belief on the issue, described at time \( t \) by a distribution \( X(\theta, t) \). For example, if the agent is absolutely sure that the ‘right’ value of \( \theta \) is \( \theta_0 \), then the distribution would take the form of Dirac delta function centred at \( \theta_0 \). Less ‘certain’ agents may have a different form of \( X(\theta, t) \). This distribution is taken as a prior for a Bayesian update, leading to the opinion at \( t + 1 \). In the simplest case, the Bayesian likelihood factor would be provided by the new information input \( S_i(\theta) \). Here the index \( i \) corresponds to various possible information sources. Kahan has proposed that instead of this direct update mechanism (prior opinion + information → posterior opinion), the incoming information is filtered by the cognitive biases or predispositions of the agent. The filtering function \( F(S_i, \theta) \) transforms the ‘raw’ information input into the filtered likelihood \( FL(S_i, \theta) \), so that the posterior belief distribution \( X(\theta, t + 1) \) is obtained by combining the prior opinion distribution \( X(\theta, t) \) with the likelihood filter \( FL(S_i, \theta) \).

It is important to note that different sources of information may be filtered in different ways. Trust in the source, cognitive difficulty of processing the information, its emotional context, the agent’s dominant goals – they all may influence the ‘shape’ of the filter. Moreover, we have to consider the ways that the information pieces from various sources are treated. Two simple versions are shown in Figures 2 and 3. The first treats each source separately and in a sequential order. Such approach may be sensible in cases where new information arrives in well separated, time ordered units, e.g. daily newspaper editions or TV news programs. The second approach treats the sources in an integrative way: it accumulates the filtered likelihoods (each of which contains the information and its specific filter), with some weights, into a single total likelihood function. Such approach may be better when various sources of information coexist at the same moment, e.g. when a group of people discusses the TV news. The weights associated with the sources could be different for each information processing event, depending on the relative importance and strength of the sources and other circumstances. Both approaches can be used (and combined) in the case of advanced models of specific systems.

B. Information sources

The information that influences the beliefs of people comes from multiple types of sources. There are, of course, personal experiences, which may provide high impact information about specific facts and events, and, with the application of some cognitive processes, about trends, estimates, diversity and prognoses. The direct experiences may be thought of as direct and therefore trustworthy, but in many cases we rely on memories, which may provide false information. Some other cognitive biases are also relevant for the personal observations: we may fall for certain illusions, disregard a part of experience and put emphasis on other parts, even to a degree of actually inventing events that did not take place.
Figure 1. Basic model of information processing. An agent holds a prior belief about an issue, described by a distribution $X(\theta,t)$. We assume a simple, one-dimensional ‘opinion parameter’ $\theta$ ranging from -1 to 1. The information on the issue, coming from the source $S_i$, has a distribution $S_i(\theta)$. This information is filtered by a function $F(S_i)$, specific to the information source. The form of the filtering function may vary, depending on the focus of the model. For example, if we assume fully rational, truth-seeking agents, the filter would be centred around the ‘true’ value of the parameter $\theta$. On the other hand, in the case of the model based on cognitive bias, the filter function would be simply related to the prior beliefs of the agent. In the case of PMRP, the filter is related to the distribution of beliefs held and approved by the agent’s in-group, or, more precisely, to the agent’s perception of such distribution. Combining the information input with the filter function yields the filtered likelihood information $FL(S_i)$. Bayesian update of the agent’s belief $X(t)$ via $FL(S_i)$ leads to the changed, posterior distribution of beliefs $X(\theta,t+1)$.

The second source of the information is related to the group of people with whom a person identifies (the in-group). These inputs may come from in-group information exchanges, either in person or via electronic or traditional communication media. The latter has become increasingly important during the past decade, especially among the younger population. In addition to the interactions with specific individuals in the in-group, the in-group may influence agents beliefs via cumulative indicators. These would include the official or semi-official statements of the group’s views on specific issues, but also the unofficial and media information about the group norms, average opinions and trends. The latter are especially interesting, as they may come both from within the group and from outside. In such case the information about the in-group views and norms may be manipulated and distorted.

The last group of the sources is related to any source outside the in-group. This may include the interactions with people outside one’s own self-identification group and the media perceived as not associated with the in-group. In case of the media the information is prepared by someone, which includes both the selection and presentation of the information. The information which we use to fortify or to change our beliefs may be manipulated ‘at source’. In personal interactions with other people we may get the wrong impressions because people due to many forms of dishonesty or distortion. Traditional sources of news are also subject to misrepresentation. The ideal of the fair and balanced journalism – giving comparable attention to all contradicting views – may also, at times, be considered manipulative, especially when it results in undue attention and coverage given to a tiny minority of views. A example of negative consequences of such ‘balanced’ reporting may be provided by the case of the anti-vaccination movement (Betsch and Sachse [13], Nelson [58], Tafuri et al. [93], Wolfe et al. [105]).
Figure 2. Sequential model of information processing when multiple sources are present. As before, an agent holds a prior belief about an issue, described by a distribution $X(\theta,t)$. We assume a simple, one-dimensional ‘opinion parameter’ $\theta$ ranging from -1 to 1. The information on the issue, coming from the source $S_i$, has a distribution $S_i(\theta)$. A different distribution $S_k(\theta)$ may come from another source $S_k$. There filtering functions $F(S_i)$ and $F(S_k)$ may differ, and as a consequence, the likelihood functions $FL(S_i)$ and $FL(S_k)$ would also differ. Bayesian update of the agent’s belief $X(t)$ via $FL(S_i)$ and $FL(S_k)$ is applied sequentially, leading to the changed, posterior distribution of beliefs $X(\theta,t+1)$ and $X(\theta,t+2)$. In the case of many sources, their relative importance may be described by the number of times they are present in the chain of evaluations. In reality, however, much more frequent are the manipulations due to unbalanced reporting. The polarization of both the traditional channels (newspapers, radio, TV) and the Internet sources (WEB versions of the traditional channels and independent WEB pages, blogs, Facebook pages and tweets) is a well known phenomenon (Adamic and Glance [2], Campante and Hojman [17], Jerit and Barabas [15], Lawrence et al. [54], Prior [69], Stroud [86], Wojcieszak et al. [103]). Many people rely on a limited number of information sources, the spectrum of the information reaching him/her could be heavily distorted. Their selective attention/selective exposure may lead to the echo-chamber phenomenon, where a person sees and hears only the information supporting the ‘right’ beliefs.

The US presidential election in 2016, with its increasing role of social media as information sources, brought our attention to yet another form of ‘at source’ information manipulation: fake news. The relative ease to create false information, in some cases supported by manipulated images, voice and video recordings, to post it online and to create a web of self-supporting links allows the perpetrator to spread such news. The trust associated with social networks (for example Facebook or twitter links) makes spreading of such information faster – especially if the fake news are designed to pass through the most common information filters.

As examples, we propose three specific forms of the distributions of the source information $S(\theta)$, suitable for the ABM approach. The first, $S_T(\theta)$, corresponds to results of social efforts to describe the phenomenon as accurately and objectively as possible. Let’s assume that for the topic in question (where the beliefs are described by the parameter $\theta$) there is some specific value that corresponds to objectively discoverable optimum, $\theta_T$. This may correspond to an exact description of a situation, or an universally optimal solution to a problem or any other situation in which, through rational, communicative processes, it is possible to arrive at a ‘true’ value of $\theta_T$. This would mean, that eventually, all beliefs other than this value should be labelled ‘erroneous’. We assume that $S_T(\theta)$ takes a form of a Gaussian distribution centred around $\theta_T$. In the following simulations we shall use $S_T(\theta)$ as the source.

The second possible form of a popular information distribution, a ‘flat’ one $S_F(\theta)$, assumes that all possible values of $\theta$ are represented equally in the information stream (an extreme version of the ‘fair and balanced’ news).

The third form, $S_P(\theta)$, is designed to represent partisan bias in the news stream, taking a form of a sigmoidal function, favouring one of the alternatives (for example $\theta > 0$). The three forms are shown in Figure 4.

C. Types of information filters

The way in which information received from various sources is evaluated and used to form new beliefs, de-
Figure 3. Integrative model of information processing when multiple sources are present. As before, an agent holds a prior belief about an issue, described by a distribution \( X(\theta, t) \). The information on the issue, coming from the sources \( S_i \) and \( S_j \), has distributions \( S_i(\theta) \) and \( S_j(\theta) \). There filtering functions \( F(S_i) \) and \( F(S_k) \) may be different. As a consequence, so would the likelihood functions \( FL(S_i) \) and \( FL(S_k) \). Instead of a sequential application, the Bayesian update of the agent’s belief \( X(t) \) via a single application of a weighted combination of \( FL(S_i) \) and \( FL(S_k) \):

\[
FL(\text{TOT}) = W_i FL(S_i) + W_k FL(S_k),
\]

leading to the changed, posterior distribution of beliefs \( X(\theta, t+1) \). The weights determine the relative importance of the different information sources in a single update.

The model depends not only on the sources, but also on the goals of a person. These goals may allow us to construct rules that would create and update the information filters. In some cases they would be independent of the characteristics of the person, in other cases they would depend on them, which would make the process of belief modification self-referential. Below is a partial list of the filter types that could be used in our agent based modelling. The filters are distinguished by their origin (internal to the person or external), dependence on some objectively measurable characteristics, possibility of an orchestrated manipulation and, finally, normative value.

- **Truth seeking filter.** It corresponds, on an individual level, to the objective source of information. The truth seeking filter could take a form of a distribution localized around \( \theta_T \), so that the eventual, repeated application of the information processing would lead the agents to converge their beliefs on \( \theta_T \). An example of such filter would be a narrow Gaussian distribution centred on \( \theta_T \). This type of the filter is at the core of the ‘rational discourse’ and ‘objective reality’ assumptions, and while important from the philosophical and moral standpoints it seems to be an exception rather than a rule in social life. Because the value of \( \theta_T \) is independent of the agents, we categorize the truth seeking filter into the ‘external’ category. And because the discovery of the value is assumed to rely on well established processes (for example depending on scientific methods), we also assume that the truth seeking filter in its pure form is not liable to manipulation. Applied to the flat (balanced) information \( S_F(\theta) \) the truth seeking filter would create a filtered information resembling the objective, truth-related source \( S_T(\theta) \).

- **Confirmation bias filter.** The tendency to give more weight to information supporting a person’s current views and to disregard sources disconfirming these views is well known in psychology. Such filter is relatively easy to be introduced into the
information distribution has a rather broad shape. Lastly, values at 6:1 ratio.

θ

In this example, the true value is set at θ focusing on a ‘true’ value accepted by the whole community.

S

sources.

Figure 4. Graphical representation of examples of information sources. Sₚ(θ) – the flat distribution representing an extreme form of ‘balanced’ news. Sₚ(θ) – an example of a distribution focusing on a ‘true’ value accepted by the whole community. In this example, the true value is set at θ = 0.6 and the information distribution has a rather broad shape. Lastly, Sₚ(θ) represents partisan bias, in this case favouring positive θ values at 6:1 ratio.

- **Memory priming/availability filter.** It is another example of an internal filter, which, however, is much more easily manipulated than the confirmation bias filter. This is because the confirmation bias compares the new information with currently held beliefs, which may be quite deeply ingrained, especially if they depend on moral foundations (Haidt 28, 39, 40, Jost et al. 17). In contrast, the availability filter acts via additional attention given to facts that are quickly accessible to our minds. Thanks to various forms of priming, its effects may be effectively stimulated and steered by outside influence: our peers or the media (Sunstein 89, Tversky and Kahneman 96, 99). In terms of an ABM approach, such filter could be approximated, for example, by the shape of the previously encountered information source.

- **Politically Motivated Reasoning (PMR) filter.** The notion of the PMR filter advocated by Kahan [50], is based on an assumption of a perfectly rational behaviour – but with a re-defined personal goals. Instead of the focus on the exact description driving the truth-seeking filter, the rationality of a person’s actions is judged by their usefulness for the goal of preserving or improving the position within a specific social group (the *in-group*). In such case, the dominant processes would be those which facilitate alignment with the in-group acceptance criteria, which often include expression of specific beliefs. Thus the PMR filter would be based on the *perceived* in-group opinions. As such, the PMR filter is an example of an external filter, that is, in itself, based on some information source, rather than on the internal characteristics of the agent. For example it could be a Gaussian distribution centred at the average belief of the in-group. It is worth noting that in some cases it can be manipulated. The range of such manipulation depends on a specific social context. Even in the cases when the knowledge about the in-group beliefs comes from direct interactions between the members of the in-group, some external social pressures might limit the expression of these beliefs. For example, political correctness might prevent overt expressions of some opinions, leading to a departure of the perceived average value from the ‘true’ average of internally held, but not expressed, beliefs. Another possibility of manipulation is when the information about a large in-group (such as political party support base) is mostly available via some external media: press, TV, social networks... The medium may withhold some information, enhance some other and in such way distort the perceived in-group opinions and thus manipulate the agent’s PMR filter.

- **Simplicity/attention limit filter.** This is an internal filter, related to the culturally and technologically driven change in the way external information is processed. Due to the information deluge, there is an increasing dominance of short forms of communication, especially in the Internet based media: WEB pages, Internet discussions, social media (Djamasi et al. 29, 30). The simplification (or oversimplification) of important issues, necessary to fit them to the short communication modes, may act against beliefs that are not disposed to such simplification. This part of the filter acts at the creation side of the information flow. Decreasing attention span and capacity to process longer, argumentative texts act as another form of filter, this time at the reception end of the flow. There are numerous forms of psychological bias related to and leading to such filtering, from venerable and accepted heuristics (like Occam’s razor), through the law of triviality and bike-shed syndrome (Parkinson
to a total disregard for too complex viewpoints (Qiu et al. [70]). Together, these tendencies can create a filter favouring the information that is easily expressed in a short, catchy, memorizable form. There is no simple universal form of the filter for the ABM approach, because in different contexts different beliefs might be easier to be expressed in the most simple way.

- **Emotional filter.** Some topics, contexts and communication forms may depend, in their processing, on the affective or emotional content. This may create a processing filter, for example one that favours extreme views, as they are typically more emotional than the consensus oriented, middle-of-the-road ones. Emotionally loaded information elicits stronger response and longer lasting effects (Allen et al. [4], Barsade [6], Berger and Milkman [9], Boose et al. [15], Chmiel et al. [22, 23], Clore and Huntsinger [24], Haidt [37], Hatfield et al. [11], Nielek et al. [60], Reifen Tagar et al. [71], Sobkowicz [77], Sobkowicz and Sobkowicz [80], Thagard and Findlay [95]). The specific form of the filter depends on the mapping of the belief range and the associated emotional values. Furthermore, the emotional filter may depend on the current agent belief function, e.g. anger directed at information contrary to the currently held beliefs, or at a person who acts as the source of the information.

- **Algorithmic filters.** An increasing part of the information reaching us comes from the Internet services such as our own social media accounts, personalized search profiles etc. The service providers organize and filter the content that reaches us often without our knowledge that any filter exists; and even more often without the knowledge how it works. These external algorithmic filters, shaping our perception, not only skew the opinions but, more importantly, they often limit the range of topics we are aware of and the opinions related to them (Albanie et al. [3], Pariser [67]). In some cases the effect of an algorithmic filter is similar to the internal confirmation bias (e.g. the search engine prioritizes the results based on the already recognized preferences of the user). In other cases, the machine filter may deliberately steer the user away from certain information, based on decisions unrelated to the particular user, fulfilling the goals of some other party.

**D. The filtering process**

An interesting question, important for the practical ABM implementation, is: how should the various filters be applied to obtain the relevant filtering function $F(S, \theta)$? A sequential application of filters focuses on the parts of the information that are minimized or deleted by each filter. In contrast, parallel application focuses on the information that is allowed by each of the filters. In reality some of the filters are applied sequentially, that is a person considers only the information that conforms to all of these filters (e.g. it must be highly emotional and in agreement with person’s views). In other cases, some filters may be added, for example a person may accept the information that confirms his/her views (the confirmation bias) or the information that agrees with his perceived views of the in-group. As a result, the overall shape of the filter function may become quite complex. Moreover, we should remember that even if we treat some external filters as relatively stable, the ones associated with person’s own views or with the in-group compliance may evolve in time.

The Bayesian-like form of the filtering process, multiplying the incoming information $S_i(\theta)$ by the filter function $F(\theta)$ is very efficient: a single process may decisively change the shape of the information distribution. For this reason we introduce here a process control parameter, the filtering efficiency $f$. Its role is to determine the relative strength of the influence of the specific filtering function on the incoming information. In particular, the effective filter function is assumed to take the form $f F(\theta) + (1 - f) U$, where $U$ is a uniform function.

**E. Information processing and memory effects**

The Bayesian processing of information may lead to very quick and dramatic revisions of the individual beliefs. Some shapes of the likelihood distributions (especially those with a narrow maximum) transform prior belief distributions into completely different posterior. Yet, with some exceptions, our interactions with other people or with the media sources are rarely so transformative. Martins [55] has proposed a modification of the original Bayesian rules, relaxing the transformation speed. He has proposed that only a fraction $p$ of encounters with the information sources leads to informative processing, characterized by the likelihood function $F L(S_i)$. In the remaining $1 - p$ cases, Martins has considered that the source is treated as uninformative (characterized by a uniform distribution), and the resulting information source is a mixture of the two, in a way similar to our treatment of the filtering efficiency.

In our approach, in the remaining $1 - p$ cases, the encounter is ignored and the information is not processed. The simplest approach to describe such situation would be to leave the belief distribution unchanged, $X_j(\theta, t) = X_j(\theta, t + 1)$, which may be treated as the case of the agent’s perfect memory. However, as we shall show in the next section, repeated application the Bayesian updates leads to a narrowing of the agents’ belief distributions. Eventually, the individual beliefs would become more and more focused, which influences the
Figure 5. Graphical representation of selected filtering types and their sources. The mix of various forms of filtering may depend on the source of the information being evaluated, for example in certain situations the personal, truth-seeking filter might be dominant, while in other situations the focus on the in-group acceptance would favour the PMR filter. When media or the Internet are the source of the information, the personal filters may be modified by the effects of personalized filtering by the search/presentation algorithms of the content providers or intentional modifications in the news industry. To allow the possibility of imperfect application of the filters, in the final stage the 'pure' filtered likelihood may be combined with non-specific, uniform function $U$, via the filter effectiveness factor $f$. The resulting function would have the form of $fF + (1 - f)U$.

whole system dynamics. For this reason we will introduce an imperfect memory mechanism that restores some level of an individual belief indeterminacy, in which the agent reverts partially to its intrinsic value of the standard deviation of the $X(\theta,t)$ distribution. This is described as follows: in the case of ignoring the information event (probability $1 - p$) the agent’s belief distribution does not remain unchanged but becomes

$$X_j(\theta,t + 1) = mX_j(\theta,t) + (1 - m)N(\langle\theta\rangle_j(t),\sigma_{0j}), \quad (1)$$

where the memory fidelity parameter $0 \leq m \leq 1$ describes the ratio of preserving the current distribution intact, and $N(\langle\theta\rangle_j(t),\sigma_{0j}(t))$ is a Gaussian distribution centred at the current average belief of the agent $\langle\theta\rangle_j(t)$, but characterized by a fixed standard deviation $\sigma_{0j}$, char-
characteristic for each agent. Thus for the perfect memory \((m = 1)\) we recover the unchanged distribution condition and for \(m = 0\), an agent ‘left to itself’ preserves the current average value of the belief, but resets the indeterminacy of its beliefs to \(\sigma_{0j}\). The information processing is graphically presented in Figure 7.

The results of resetting the indeterminacy of an individual agent belief distribution is shown in Figure 8. While the ‘broadening’ admixture may seem comparatively small (at least for the depicted \(m = 0.5\) value), it plays an important role in shaping the evolution of the beliefs of the agents under the influence of information sources. The origin of such reset of the indeterminacy may be explained by numerous encounters with a range of beliefs, other than the main source considered in the simulations, which are too weak to significantly shift the agent’s average opinion, but introduce some degree of uncertainty.

III. BASIC SIMULATION ASSUMPTIONS

In actual situations both the information sources and the filters described in the previous section combine their effects in quite complex ways. We encounter, in no particular order, information sources of various type, content and strength, in some cases acting alone, in others - combined. To elucidate the model effects we shall initially focus on drastically simplified systems, in which we would show the effects of the repeated application of the same filter to the same information source distribution \(S_i(\theta)\), for a range of starting belief distributions \(X_j(\theta, 0)\) (where the index \(j\) denotes individual agents). The aim of this exercise is to show if particular filters (no filter, confirmation bias, PMR) lead to stable belief distributions, polarization, emotional involvement etc.

As noted, for the simulations shown in this paper, we shall be using the truth-related form of the information source, \(S_T(\theta)\), assumed to take a rather broad Gaussian form, centred at \(\theta_T = 0.6\) and with standard deviation equal to 0.4. This choice of the information source distribution is motivated by two reasons. The first is to check if the simulated society is capable of reaching consensus when the information source points to a well defined value. The second reason was to study the effects of the asymmetry. Obviously, it is much easier for the
Figure 8. Example of the indeterminacy reset (memory factor) of an individual agent $i$ belief distribution. The current (already quite narrow) belief distribution (thin black line), centred at $\langle \theta \rangle_i(t)$ is mixed with a normal distribution centred at the same value, but with the standard deviation $\sigma_0_j$ characteristic for the agent (blue line shows the original distribution, which is centred at $\langle \theta \rangle_j(0)$). The resulting distribution (red line) exhibits the central peak but also some ‘tails’ that allow the agent to accept beliefs further from the centre of its belief function.

Agents whose initial opinion distribution favours positive $\theta$ values to ‘agree’ with an information source favouring a positive $\theta_T$. In contrast, the agents starting with belief distributions preferring negative $\theta$ values, would have to ‘learn’ and to overcome their initial disagreement and to significantly change their beliefs.

Each of the agents is initially characterized by a belief function of a Gaussian form (bounded between $-1$ and $+1$ and suitably normalized). The standard deviation parameters for the agents $\sigma_0_j$ are drawn from an uniform random distribution limited between 0.05 and 0.2.

Three separate sets of agents are created and used in the simulations: leftists, centrists and rightists (we note here that these names have no connection with the real world political stances and refer to the position on the abstract $\theta$ axis). Each agent community is composed of $N$ agents (in the simulations we use $N = 1000$). The leftists have their initial Gaussian centre values $\langle \theta \rangle_j(0)$ drawn from an uniform random distribution bounded between $-1$ and $-0.5$. The centrist group is formed by agents with $\langle \theta \rangle_j(0)$ drawn from between $-0.5$ and 0.5, and the rightists have $\langle \theta \rangle_j(0)$ drawn from between 0.5 and 1. Figure 9 shows examples of the belief distributions of a few individual agents, differing in their initial centre of the belief $\theta_0_j$ and the width of the belief distribution $\sigma_0_j$. Thick lines: averaged distributions for each group.

The simulations discrete time steps. The time is measured in time units in which each agent in the current group has had a single chance to interact with the information source or to ignore it (with the respective probabilities of $p$ and $1 - p$). As we shall show in the following sections some effects become visible after just a few time steps, but some other become important after thousands or tens of thousands events. From the point of view of the possible application of model results to the real life phenomena we should consider the mapping of the ‘simulation time’ to real hours, days or weeks. In the current work we focus on the long term behaviour of the system, especially on the final stable conditions.

IV. MODEL RESULTS

A. Case 1: Unfiltered effects of true information

We shall start the description of the model results with a relatively simple case, with the aim of showing the ef-
effects of some of the simulation parameters. The first case is based on unfiltered processing of the ‘truth-related’ information, $S_T(\theta)$, as shown in Figure 4. This is, as we have noted, equivalent to the situation where the information flow is nonspecific (uniform) but the agents employ a truth-seeking filter of the same form as $S_T(\theta)$. As the $\theta_T$ value is positive (equal to 0.6), the most interesting question is how such information would influence the agents who initially hold the opposite views (the ‘leftists’).

The speed with which the agents converge at the true value consensus depends on the significant information processing probability $p$. Figure 10 presents the time evolution of individual agents’ average beliefs $\langle \theta \rangle_j$, thin lines) and the ensemble averages $\langle \theta \rangle_G$ for the three agent groups. We start with agents characterized by perfect memory ($m = 1$). The time evolution of the average $\langle \theta \rangle_j$ for $p < 1$ looks qualitatively different than in the case of $p = 1$. They exhibit a step-like structure, due to ‘freezing’ of beliefs if no processing takes place. However, the ensemble averages are quite similar for $p < 1$ and $p = 1$. Figure 11 shows the dependence of the time evolution of the average belief for the whole leftist group $\langle \theta \rangle_L$ on the value of the parameter $p$ (note the logarithmic scale of the time axis). In fact, a simple rescaling of the time axis to $t' = pt$ (shown in the inset) shows that the evolution is really a simple slowdown due to inactivity periods, when no information is processed. Thus, for the perfect memory (i.e. for $m = 1$), the role of $p$ is rather trivial. It becomes more important when the ‘idle’ times are used to partially reset the individual uncertainty.

The truth-focused information flow eventually convinces all the agents to believe in the ‘true’ value of $\theta_T = 0.6$, regardless of their initial positions. The process is the fastest for agents with relatively broadminded beliefs (high $\sigma_{0j}$). For the agents with initial very narrow belief distributions the transition is shifted to later times – but then it is almost instantaneous (typical for Bayesian update of single valued probabilities rather than distributions). The changes in the form of the belief distribution consist of a more or less gradual ‘flow’ of beliefs from the original form to a belief centred around the maximum of $S_T(\theta)$. This is well illustrated by Figure 12, which
Figure 12. **No filtering applied, perfect memory.** Time evolution of the ensemble averaged belief distribution over the groups of leftists and rightists due to ‘truth-related’ information stream. Time is measured in interactions per agent. Black-blue and black-green curves: averaged belief \( \langle \theta \rangle_L \). Red curve shows, for comparison, the original information distribution \( S_T(\theta) \). Simulations use \( p = 1 \) value. As the time passes, the beliefs transform from the starting polarized distributions (thick black lines) and converge to the ‘true’ value of \( \theta_T = 0.6 \).

presents the time evolution of the ensemble average belief for the leftist and the rightist groups.

To understand the effects of the memory parameter \( m \), it is illustrative to study the effects of the indeterminacy reset on the evolution of the individual opinion distributions \( X_j(\theta, t) \). As shown in Figures 13 and 14 the relaxation of the indeterminacy introduced by imperfect memory factor \( m < 1 \) leads to a qualitatively different final form of the individual belief distributions. Instead of a set of narrow, delta-like functions grouped close to the \( \theta_T \) value typical for \( m = 1 \), the existence of belief relaxation leads to distributions of width comparable to the original values of \( \sigma_{oj} \) centred exactly at \( \theta_T \). Thus, while the final ensemble average may be similar, the underlying structure of the individual beliefs is quite different.

In the case of lack of filtering, the effects of the indeterminacy reset on group averages are rather subtle. For a given value of \( p \), decreasing the memory factor leads to a small, but observable shortening of the time scale of reaching the truth-based consensus (Figure 15). It is interesting that even a very small admixture of uncertainty reset (\( m = 0.99 \) instead of \( m = 1 \)) significantly influences the evolution of the group averaged belief \( \langle \theta \rangle_L \).

B. **Case 2: Individual confirmation bias filter of true information, perfect memory (\( m = 1 \)).**

In the previous section we have shown that under the influence of the truth-related information, without filtering, all the agents eventually converge their beliefs on the true value suggested by the information source. This is not surprising, as the process is a simple, repetitive, application of a Bayesian belief modification. We turn now to the issue of the effects of filtering of the information sources.

We shall start with the individual agent based **confirmation bias filter**. There are two reasons for this choice. The first is that confirmation bias is widely recognized in psychological literature, so ‘deserves’ a thorough treatment in the ABM framework. The second reason is a relative simplicity of the filter effects. Suppose that the information flow on which the filter acts is non-specific (i.e. uniform). If the initial belief distribution is given by a Gaussian function with the standard deviation \( \sigma \), then the application of the same function acting as the likelihood filter would lead to the posterior belief in the Gaussian form, but with \( \sigma \) decreased by a factor of \( \sqrt{2} \). A repeated information processing would eventually lead to a Dirac delta-like belief distribution. In other words, repeated application of confirmation bias narrows and freezes one’s own opinions. Once should, therefore, expect that the confirmation bias filter should diminish the effects of specific information sources, such as the truth-related source \( S_T(\theta) \).

The simulation setup for the case of confirmation bias filtering of true information is relatively simple. At every time step, with probability \( p \) each agent uses its current belief \( X_j(t) \) as the pure filter. In this case the final likelihood function is defined by

\[
FL(\theta) = (fX_j(\theta, t) + (1 - f)U)S_T(\theta),
\]

where we use the filter effectiveness \( f \) as parameter. As before, with probability \( 1 - p \), the agent does not process the information. In this section we focus on situations \( m = 1 \), when the agent simply retains its previous belief. In what follows, we use a fixed value \( p = 0.3 \). The crucial parameter in Case 2 is the filter effectiveness \( f \).

Figure 16 presents four snapshots of the evolution of the individual belief distributions \( X_j(\theta) \) for the leftist group. The individual distributions change under the competing influences of the confirmation bias filter (progressively narrowing the belief distributions) and the information (shifting the beliefs towards higher values of \( \theta \)).

The relative importance of these two factors depends on the value of the filtering effectiveness factor \( f \). If a pure form of the filter is used (\( f = 1 \)) the individual beliefs coalesce to delta form in less than 10 time steps and the average beliefs of the three groups remain practically unchanged (left panel in Figure 17). Thus, despite the availability of true information, the centrist and leftist groups keep their beliefs. For much smaller, but still
Figure 13. **No filtering applied, perfect memory.** Snapshots of examples of the individual agent belief distribution functions for the leftist agents under the influence of the unfiltered $S_T(\theta)$ information source. For the simulations $p = 0.3$ and $m = 1$ (that is, perfect memory is assumed and no indeterminacy of beliefs occurs). As expected, the individual beliefs move towards the true value of $\theta_T = 0.6$, at the same time becoming increasingly narrow. There is only a partial overlap of the individual opinions. The snapshots are taken at $t = 5, 40, 160$ and $1280$ (shown in clockwise order).

It is only for very small values of $f$ that the final distributions of beliefs begin to converge towards the truth-related consensus. Even for $f = 0.02$ there is a sizeable gap between the rightists and the centrists and leftists. Figure 18 presents the shape of the ensemble averaged belief distributions for each of the three groups at a very late time $t = 10000$. 

A non-negligible value of $f = 0.2$ (corresponding to Figure 16), we observe some change (more pronounced for the leftist group, where the dissonance between the initial views and the true information is the largest, middle panel in Figure 17).
Figure 14. **No filtering applied, reset of belief indeterminacy.** Snapshots of examples of the individual agent belief distribution functions for the leftist agents under the influence of the unfiltered $S_T(\theta)$ information source and with the indeterminacy reset present. For the simulations $p = 0.3$ and $m = 0.5$. In this case, not only do the individual beliefs move towards the true value of $\theta_T = 0.6$, but they also become almost fully overlapping. The snapshots are taken at $t = 5, 40, 160$ and $1280$ (shown in clockwise order).

Figure 19 presents the dependence of the ensemble averaged values of the average belief for each of the three groups on the filtering effectiveness $f$. For $f$ close to 1, the truth-related information is almost totally filtered out by the confirmation bias, the agents quickly evolve to fixed, delta-like belief distributions. For medium values ($0.3 < f < 0.9$) the rightists and the centrists show no effects, but the leftists are gradually ‘convinced’ to shift their opinions somewhat towards positive value. For small values of the filtering effectiveness ($f < 0.1$) the opinions of the three groups begin to converge, but getting close to consensus requires very small values of $f$ (on
the fully effective case

treme narrowing of the individual belief distributions (for

from the ‘truth-related’ information source. Reset to more ‘broadminded’ values are more likely to learn

agents with belief distributions which are (however seldom) reset to more ‘broadminded’ values are more likely to learn from the ‘truth-related’ information source.

the order of 0.02 or less).

C. Case 2a: Individual confirmation bias filter of true information, reset of beliefs due to imperfect memory ($m < 1$).

The confirmation bias filter very quickly leads to extreme narrowing of the individual belief distributions (for the fully effective case $f = 1$ this happens after a few tens of interactions). This suggests that the inclusion of the broadening mechanisms might have more significant effect than in the case of unfiltered information processing. Indeed, setting $m = 0.5$ changes the evolution of the individual beliefs dramatically, as we can see from Figure 21 (which corresponds to the ‘perfect memory’, $m = 1$ case in Figure 16). In situation when the beliefs are affected by the indeterminacy reset (which, we remind, does not change the current individual average belief), they are much more modifiable by the information source.

In the case of the $S_T(\theta)$ information source, the effects of the memory imperfection are most clearly seen by the behaviour of the leftist group, because this group is the furthest from the ‘true’ value $\theta_T$. The change introduced by the indeterminacy reset is best visualized when we look at the time evolution of the group ensemble average beliefs $\langle \theta \rangle_L$ (Figure 21). The presence of the indeterminacy reset due to imperfect memory causes the individual opinion distributions to retain some component of broader beliefs and facilitates their shift due to the information source influence. As the process of reaching the consensus looks similar for all groups, this would lead to a global consensus centred at the $\theta_T$ value. The process $\langle \theta \rangle_L(t) \rightarrow \theta_T$ is quite fast, on the order of a few hundreds of time steps when $m \leq 0.3$, but slows down for higher values of $m$. Above $m \approx 0.9$ (i.e. for an almost perfect memory) the narrowing of the individual opinion distributions dominates and the group average remain close to their initial values. In plain words, when the agents are allowed to become extremely close-minded due to the confirmation bias, the truth-related information has limited effect, and the initial assumed polarization between the leftists, centrists and rightists remains unchanged.

The transition between the polarized state at large enough $m$ and the consensus, for smaller $m$ values, is rather abrupt. Figure 22 presents the dependence of the $\langle \theta \rangle_L(t)$ values on $m$, for two values of the filter effectiveness $f = 1$ and 0.5 for three time snapshots, $t = 1000$, 10000 and 50000. Increasing the time leads to a step-like transition between conditions preserving the polarization and those leading to the consensus.

We recall here the brief discussion of the topic of mapping the simulation time to the real world units. Obviously, if we consider as events the cases when a person encounters really new, significant information (e.g. listens to a candidate speech at a rally, or a debate, or reads an important article in the press), then 50000 events is obviously not realistic. Even a few hundred events (necessary to reach consensus for very imperfect memory, $m \approx 0$) may be questioned. On the other hand, if we treat the time ‘between the events’ - essentially the very time in which the memory imperfection and uncertainty reset would be expected to occur as single entities or, perhaps, a multitude of them. A partial answer could be provided by psychological research devoted to the issue of the existence of the indeterminacy of opinion resets and the associated conditions.

D. Case 3: Politically Motivated Reasoning filter

In contrast with the confirmation bias, the PMR filter is assumed to depend on the current beliefs of the in-group, treated as a whole. In the simplest version, we assume that any agent knows perfectly the ensemble averaged belief distribution of its in-group $X_G(\theta,t)$, and uses it as a filter for information processing. The filter is dynamical, because as the individual agents change their beliefs, so does the average for the group. As in the previous sections we focus on the truth-related information source $S_T(\theta)$ and assume that $p = 0.3$. Our focus is, therefore, the role of the filter effectiveness $f$ in the evolution of the group belief distributions. The current section considers the case of agents with perfect memory ($m = 1$).

We shall start with Figure 23 which corresponds directly to the results for the confirmation bias filter (Figure 19). For very small values of $f$ the averaged beliefs converge on the true value, as the information source ‘gets through’, thanks to the uniform part of the filter. On the other hand, for $f \approx 1$, the PMR filtering mechanism effectively freezes the group opinions. For the two groups which are initially closer to the true opinion $\theta_T$, namely the rightists $\langle \theta \rangle_R$ and centrists $\langle \theta \rangle_C$, the fixed
Figure 16. **Confirmation bias filtering, perfect memory.** Time evolution of individual belief distributions of selected leftist agents using confirmation bias filtering of truth-related information for $f = 0.2$, $p = 0.3$. The distributions are for a subset of randomly selected leftist agents. As the time progresses (clockwise) the individual belief distributions shift to higher $\theta$ values and become increasingly narrow. The first process is the influence of the new information favouring $\theta_T = 0.6$, the second is the result of the confirmation bias. Quite quickly (much faster than in the case of unfiltered information processing), all agents evolve to delta-like belief distributions (the $t = 80$ panel). For $f$ values greater than 0.05 the processes of narrowing of the individual beliefs dominates over their shift towards the true value for a large number of the agents.

Value remains unchanged as we lower $f$, and for very small values of $f$ it changes gradually, resembling the behaviour for the confirmation bias filter. For the leftists, however, instead of a continuous change observed in the confirmation bias case we observe a discontinuous transition at certain value $f_{\text{crit}} = 0.43$ (for the current set agents and $p = 0.3$).

To understand this discontinuity we have to look into
Figure 17. **Confirmation bias filtering, perfect memory.** Time evolution of average beliefs $\langle \theta \rangle_j$ (thin lines) and the averages $\langle \theta \rangle$ (thick lines) for the three groups of agents using confirmation bias, for three values of the filter effectiveness $f = 1.0, 0.2$, and $0.01$. The value of the information processing probability is $p = 0.3$. Decreasing the effectiveness of the confirmation bias filter delays the time at which the individual opinion distributions become fixed and delta-like, shown in the figure as think horizontal lines. In some cases we observe jumps in the opinion, typical for discrete Bayesian updates.

The details of the evolution of the individual belief distributions. The two following Figures (24 and 25) show examples of time snapshots of the individual belief distributions $X_j(\theta,t)$, collected for $f$ just above the transition value ($f = 0.43$) and below it ($f = 0.42$). The starting point is the same in the two cases. The initial evolution ($t < 10$) is driven by the interplay of the asymmetry of the information source (favouring positive values of $\theta$) and the PMR filter. It leads to formation of two attractors, around which the individual agents group: one close to the upper end of the original leftist domain (around $\theta = -0.5$) and the second, corresponding to partially ‘convinced’ agents, located around $\theta = 0.1$. The decrease of the filter effectiveness $f$ increases the number of agents in the latter group. Because the ensemble averaged belief distribution enters the process for the next iteration, for $f < 0.42$, a positive feedback mechanism leads to the eventual dominance of the convinced group. On the other hand, for $f > 0.43$ the size of the convinced group is too small to persist, and eventually all agents retain or revert to their leftist stance.

The results for the Politically Motivated Reasoning filter were obtained using an assumption that the composition of the group to which an agent looks for the belief guidance remains unchanged. The simulations assume that each agent considers the whole group, defined in the initial input files, to calculate the ensemble averaged belief distribution $X_G(\theta,t)$, which would be used as the filter. This leads to the case when the more flexible agents, who have shifted their opinion can eventually pull the whole group with them (for small enough $f$ values).

Such assumption might be criticized from a sociological point of view. In a situation, such as that depicted in $t = 50$ panels of Figures 24 and 25, where the belief systems of the agents flexible and inflexible agents have very little overlap, one could expect that each of the subgroups would **restrict their PMR filter to the group of the currently like-minded agents**. In other words, the flexibles, who have moved away from the initial group average, would be rejected by the less flexible agents, as traitors of the cause, and disregarded when calculating the PMR filter. The obvious result would be a split of the initial group, occurring within just a few filtered iterations (somewhere between $t = 25$ and $t = 50$). In such approach it would be useful to change the simulation measurements from the group averages of belief $\langle \theta \rangle_G$ to the numbers of the inflexibles, unconvinced by the information, and the agents who have shifted their beliefs. Such a dynamical group composition model variant shall be the topic of later works.
Figure 18. **Confirmation bias filtering, perfect memory.** Averaged distributions of agent beliefs in the three groups. Thick, smooth lines: initial distributions, thin lines: distributions after 10000 steps. Results for three $f$ values are shown $f = 1$, $f = 0.2$ and $f = 0.01$.

Figure 19. **Confirmation bias filtering, perfect memory.** Dependence of the final value of $\langle \theta \rangle_G$ for the three groups as functions of filtering effectiveness $f$ for the confirmation bias filter. Note that the convergence of opinions near the true value requires very weak filtering ($f < 0.5$).

E. **Case 3a: PMR filter with imperfect memory** ($m < 1$)

The discontinuous change in the system behaviour, described in the previous section, results from the extreme narrowing of the individual belief distributions, due to the repeated application of the filter. Guided by the analyses of the confirmation bias filter with imperfect memory, we expect that the reset of individual belief indeterminacy should significantly change the system behaviour. Figure 26, which presents the results for $m = 0.5$, confirms these expectations are true. Instead of the discrete jump seen for the leftist group in the unmodified $m = 1$ case (Figure 23), we observe smooth changes of all group averages of beliefs $\langle \theta \rangle_G$. Moreover, a full consensus is reached for finite (although small) values of $f$.

An additional difference in the simulations for the imperfect memory PMR filter from all cases considered so far, is that simulation runs converge to somewhat different configurations. We have indicated this as error bars in Figure 26.

The roughly linear dependence of $\langle \theta \rangle_L$ on $f$, for $f > 0.1$, results from the increased individual opinion flexibility introduced by the admixture of the broad-minded component of the individual beliefs treated as priors. To better understand this, we have studied the dependence of $\langle \theta \rangle_L$ on the memory factor $m$ for fixed values of $f$. The results are shown in Figure 27. In the case of relatively effective PMR filter ($f = 0.7$ and $f = 1.0$) there are two distinct regimes of system behaviour. Above certain threshold value $m_T(f)$, there is only a weak, linear dependence of $\langle \theta \rangle_L$ on $m$, mostly due to individual belief shifts during a few initial time steps, which quickly
Figure 20. **Confirmation bias filtering, reset of belief indeterminacy.** Snapshots in the evolution of individual belief distributions of selected leftist agents using confirmation bias filtering of truth-related information for $f = 0.2$, $p = 0.3$ with the memory imperfection factor $m = 0.5$. The distributions are for a subset of randomly selected leftist agents. As the time progresses (clockwise) the individual belief distributions shift to higher $\theta$ values but remain rather broad-shaped (as are the original distributions). Much greater number of agents move to the true belief $\theta_T$. Eventually all agents would reach consensus at this value.

become frozen. On the other hand, for $m$ smaller than $m_T(f)$, all agents shift their opinions in accordance with the information source, moving eventually to centrist and rightist positions. The value of $m_T(f)$ is only approximate, as a consequence of the differences between individual simulation runs, due to the finite size of the system.
Time dependence of the value of $\langle \theta \rangle_L$ for the leftist group for various values of the memory parameter $m$, for $f$ equal to 1. Reducing the value of $m$ changes the evolution of the individual beliefs, and, in consequence, the group average $\langle \theta \rangle_L(t)$: for $m$ smaller than certain value (significant broadening), all agents become ‘convinced’ by the information source and accept the $\theta_T$ as the centre of their belief distributions. The conviction process is the fastest for the lowest values of $m$. On the other hand, for $m > 0.9$ the agents’ belief distributions remain frozen, which means that the whole system would exhibit significant polarization.

On the other hand, for $m > 0.9$ the agents’ belief distributions remain frozen, which means that the whole system would exhibit significant polarization.

**Figure 23. Politically Motivated Reasoning filtering, perfect memory.** Dependence of the final value of $\langle \theta \rangle_G$ for the three groups, as functions of filtering effectiveness $f$ for the PMR filter. For $f \geq 0.43$ the averages are almost independent of $f$. At $f \approx 0.43$ (marked by the red ellipse), the $\langle \theta \rangle_L$ shows a large jump towards the $\theta_T$ value, the result effectively turns the leftists into centrists. For very small values of the filtering effectiveness ($f < 0.1$) opinions of all three groups converge on the true value $\theta_t = 0.6$.

**V. DISCUSSION**

**A. Time dependency considerations**

The choice of the right simulation-to-reality time scaling may depend on the way we define the information processing events. On one hand, we could consider only the major news and real world occurrences, such as the crucial election stories and events. In such a case, the number of the opinion shaping encounters could be treated as relatively small, certainly not in the range of tens of thousands or thousands per month. In such a view, the time periods between the information processing events are long enough to allow the uncertainty reset.

At the other end of the spectrum is the vision, in which our beliefs are shaped by a continuous stream of events, differing in their source type, intensity, repetition and many other characteristics. Some of these would originate from external sources, characterized by relatively stable views and opinions (biased or unbiased at the source), while other events could originate from more or less random encounters with other people or observations of ostensibly small importance. In such microscopic approach, the number of the events could be very large.

The focus of this work was on the long term effects of
Figure 24. Politically Motivated Reasoning filtering, perfect memory. Time snapshots of the individual belief distributions for the PMR filter for $f = 0.43$. The individual agent’s belief distributions at $t \approx 25$ are divided into the ‘inflexibles’ – with opinions centred around $\theta \approx -0.5$, and the agents who were influenced by the news source, with their distributions centred around $\theta \approx 0$. Thick dark line shows the average distribution of beliefs - which serves as a filter for the next time step. For $f$ greater than 0.43 the number of the influenced agents is too small, and repeated interactions diminish the influence of the $\theta \approx 0$ filter peak. At $t = 100$ all agents revert to the leftist positions.

A single type of an information source, interspersed with the periods when the individual belief structure may become less certain. The goal was to construct a Bayes based filtered information processing ABM and see if such approach can yield ‘reasonable results’, by which we mean, depending on the situation, conditions leading to a general consensus, or, for other conditions, a persistent disagreement and polarization. The results have shown that the model can, indeed, produce these results under simple manipulation of a few key parameters.
Figure 25. **Politically Motivated Reasoning filtering, perfect memory.** Time snapshots of the individual belief distributions for the PMR filter for $f = 0.42$. As before, the individual agent’s belief distributions at $t \approx 25$ are divided into the ‘inflexibles’ – with opinions centred around $\theta \approx -0.5$, and the agents who were influenced by the news source, with their distributions centred around $\theta \approx 0$. Thick dark line shows the average distribution of beliefs - which serves as a filter for the next time step. For $f$ smaller or equal 0.42 the number of the influenced agents becomes large enough to eventually dominate, and the repeated interactions move all agents to the centrist position. The jump observed in Figure 23 occurs when the number of the influenced agents passes the necessary threshold. Due to the positive feedback, once the peak $\theta \approx 0$ dominates the filtering, the repeated filtered information processes further increase its the size in the subsequent interactions.
Figure 26. Politically Motivated Reasoning filtering, reset of belief indeterminacy. Dependence of the final value of $\langle \theta \rangle_G$ for the three groups, as functions of filtering effectiveness $f$ for the PMR filter with imperfect memory $m = 0.5$. The broadening of the individual belief distributions due to the imperfect memory restores the almost linear dependence of the ensemble average value of opinion for the leftist group. The resulting opinion distribution for the leftist group $\langle \theta \rangle_L$, for $f > 0.6$, shows sizeable differences between the individual simulation runs, which are indicated by error bars.

The question of the ‘right’ timescale for opinion change can not be resolved by such qualitative, simplified model. Among the unknowns are the effectiveness of the Bayesian update process and the filtering, the memory imperfection related uncertainty reset scale, and the elements omitted in the current model, for example differences in the intensity of particular events. A more realistic model should be based on psychological studies - which would, hopefully, provide also suggestions as to whether we should focus on the effects of a few (few tens? hundreds?) information processing events or to look at the stable or quasi-stable states reached after thousands of microscopic events.

B. Manipulation of the Politically Motivated Reasoning Filter

The current political developments in many democratic societies show dramatically increasing levels of polarization, covering the general public and the media (PEW [1], Baldassarri and Bearman [5], Bernhardt et al. [11], Fiorina and Abrams [31], Prior [69], Stroud [89], Tewksbury and Riles [94]). In many countries the chances of reaching the state in which a rational discussions between conflicted groups (not to mention working out a sensible compromise) seems almost impossible. Recent US presidential elections provide an obvious example, but the seemingly irrevocable split exists in many other aspects, sometimes with division lines not parallel to political ones. A good example of such split is the existence and (in many countries) growth of the anti-vaccination movements (Betsch [12], Betsch and Sachse [13], Blume [14], Davies et al. [26], Hough-Telford et al. [44], Kata [53], Leask et al. [55], McKeever et al. [57], Nelson [58], Olpiński [61], Sterfland [85], Wolfe and Sharp [104]), which are not strictly ‘politically’ aligned. The efforts to convince vaccination opponents are quite unsuccessful, regardless of the approach used. Similar problems occur in more politicized issues. This applies to the cases where suitable evidence is available, for example in controversies over gun control policies, climate change, GMO, nuclear energy, and in cases where the beliefs and opinions are largely subjective, such as evaluations of specific politicians (e.g. Hillary Clinton or Donald Trump).

The difficulty in minimizing the polarization may be
partially attributed to the cognitive biases and motivated information processing described in this paper. Filtering-out of information may be very effective in keeping a person’s beliefs unchanged. In fact, some cognitive heuristics are evolved to provide this stability (e.g. the confirmation bias). This makes the task of bridging the gaps between polarized sections of our societies seem impossible. Still, as Kahan has noted, some filtering mechanisms may be more flexible than others.

A good example is provided by comparison of the confirmation bias and PMR. Kahan [50] notes that in some cases PMR may be confused with the confirmation bias: *Someone who engages in politically motivated reasoning will predictably form beliefs consistent with the position that fits her predispositions. Because she will also selectively credit new information based on its congeniality to that same position, it will look like she is deriving the likelihood ratio from her priors. However, the correlation is spurious: a ‘third variable’—her motivation to form beliefs congenial to her identity—is the “cause” of both her priors and her likelihood ratio assessment.* Kahan notes the importance of the difference: if the source of the filter is ‘internal’ (confirmation bias), we have little hope to modify it. On the other hand, if the motivation for filtering is related to perceptions of in-group norms, the opinions may be changed if the perception of these in-group norms changes. Re-framing the issues in a language that conforms to specific in-group identifying characteristics or providing information that certain beliefs are ‘in agreement’ with the value system of the in-group and/or majority of its members, would change the PMR filtering mechanism. Through this change, more information could be allowed through, changing the Bayesian likelihood function, and, eventually, changing the posterior beliefs.

C. Model extensions and further research directions

The simulations presented in the current work are based on drastically simplified assumptions: only a single source of information, with consistently repeated $S(\theta)$ distribution, only one type of the filter, our focus is on long term stable conditions. These simplifications directly indicate the directions of further work: dealing with conflicting information sources, combinations of different types of filters, transient phenomena to describe immediate reactions to the exposure of news. Another planned model extension is related to modelling the possible dynamical nature of the group norms based PMR filter, mentioned in Section [V.D]. When opinions within a group initially treated as homogeneous begin to diverge, it is quite likely that the very definition of the group would change. The agents could redefine the criteria who they count as the members of the in-group, treating those with sufficiently different belief distributions as outsiders (possibly with a negative emotional label of traitors). Such a move would dynamically redefine the perceived in-group standards and norms. The resulting change in the PMR filter could change the model dynamics from opinion shifts to changes in group sizes and identification.

The model proposed in this work may be characterised as a ‘reach feature agent’, in contrast to the simplified ‘spinson’ models. To examine the possibilities of the approach, we have focused on a system in which agents repeatedly react to an unchanging, single external information source. This has allowed to discover some regularities and to understand the roles of the model parameters.

The same general framework of biased processing of information may be used in more complex environments. It can cover the agents interacting among themselves in arbitrarily chosen social networks. In such scenario, the input information would be generated by one of the agents (a sender) and would be received and evaluated using the filtering mechanisms and biases by other agent or agents (recipient(s)). Each recipient would then update its opinion (as described by the belief distribution), and, if applicable for the bias type, also the filter function. Of course, it is possible to reverse the roles of the agents and to allow bidirectional communication. Because the filters used by the communicating agents may be different, the interaction process may be asymmetric. It is also possible to combine the agent-to-agent interactions with the influences of external information sources, and to create a truly complex model approximating a real society.

Lastly, especially in the case of the studies of short term, transient changes, the possibilities of manipulation of the filters by outside agencies, offer a very interesting and important future research direction. Such investigations should cover both the manipulations increasing polarization (partisan information sources and the reliance on emotional context of the information) as well as the efforts in the opposite direction – to detect and to combat the manipulative influences. The latter are especially important to enhance the chances of a meaningful dialogue in our already highly polarized societies.

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