I. COMMENTS

ORK [1] introduced a Generalized Adaptive Polynomial (GAP) window function and presented examples of how GAP-based window functions outperform existing window functions. Claims in [1] need clarification, and more importantly, do not hold in all cases.

The first example where the claims do not hold is for Hann, also known as Hanning, window function. Hann window belongs to a class of Generalized Cosine Window (GCW) functions of the form

$$w(t) = \sum_{m=0}^{M} (-1)^{m} a_m \cos(2\pi m t), \text{ for } 0 \leq t \leq T. \quad (1)$$

Hann window has only two coefficients $a_0 = a_1 = 0.5$, obtained by solving a system of equations

$$\begin{align*}
 a_0 + a_1 &= 1 \\
 a_0 - a_1 &= 0
\end{align*} \quad (2)$$

The first equation normalizes coefficients so that their sum is one, and the second one represents the condition for 18 dB/oct roll-off. Conditions for general case of $6 + 12 \cdot n$ dB/oct roll-off are given Eq. 10b of [2].

In [1] authors compare Hann window with "Optimized Hann" in Fig. 3, and present that main lobe width has been reduced from 1.46 bins to 1.25 bins, while keeping the sidelobe level at -31.5 dB. This claim is flawed in several ways. First of all, the "Optimized Hann" window does not have a roll-off of 18 dB/oct, which is a defining feature of Hann window. Giving up on a steeper roll-off rate is equivalent to removing all degrees of freedom to improve the sidelobe suppression - but it is not a Hann window anymore.

Secondly, the presented "Optimized Hann" window is not even optimal in terms of considered criteria of main lobe width and maximum sidelobe level. Two-term GCW with coefficients $a_0 = 0.560425$ and $a_1 = 0.439575$ has the same main lobe width of 1.25 bins, but lower maximum sidelobe of -37.54 dB. Another example is $a_0 = 0.53929$, $a_1 = 0.46071$ with main lobe width of 1.303 bins and maximum sidelobe level of -42.89 dB. The frequency response of these two windows is shown in Fig. 1.

Furthermore, coefficients of two-term GCW with desired main lobe width of $F_{3dB}$ bins are a solutions of:

$$2 F_{3dB} \sin(\pi F_{3dB}/2) \left( \frac{a_0}{F_{3dB}^2} - \frac{a_1}{F_{3dB}^2 - 4} \right) = \sqrt{2} \quad (3)$$

System (3) has been solved for $F_{3dB} = [1.1, 1.5]$, and maximum sidelobe level and scalloping loss versus main lobe width are shown in Fig. 2, where horizontal dashed line represents the maximum sidelobe level of Hann function.
It is obvious that there are solutions with narrower main lobe width and better sidelobe suppression than "Optimized Hann". What’s more, "better" solutions can be obtained without resorting to optimization.

Claims about "Optimized Nuttall" window are also questionable. Sidelobe suppression of a minimum 4-term Nuttall window is -98.17 dB, as can be seen from Tbl. II in [2], and not -93.8 dB as reported in [1]. Also, reported maximum sidelobe -102.8 dB of "Optimized Nuttall" is obtained only for a 64 point window, and degrades for windows with more points, as summarized in Tbl. 1. Rise of maximum sidelobe level for longer windows is likely to be the consequence of coefficient optimization for a window length of 64 samples.

Window functions are of finite length in the time domain, so their spectrum must be infinitely wide. Consequently, sampling theorem conditions are always violated, and there will always be aliasing. Given enough degrees of freedom, numerical optimization can utilize aliasing to reduce the maximum sidelobe level for a given number of samples. However, this effect will work only for a given number of samples, which is consistent with data from Tbl. 1. Therefore, it is questionable whether "Optimized GAP Nuttall" has a better maximum sidelobe than Nuttall.

Comparison of a four-term Nuttall with 10-term GAP is also questionable. Maximum sidelobe level of function resembling the minimum side lobe 4-term Nuttall window can be improved by increasing the number of terms, but it will not be a Nuttall window anymore. One example is shown in Fig. 3, where Nuttall window is compared to custom six-term GCW with coefficients: \( a_0 = 0.359545, a_1 = 0.487971, a_2 = 0.140569, a_3 = 0.0119011, a_4 = 1.8 \cdot 10^{-5}, a_5 = -3.98276 \cdot 10^{-6} \). Custom six-term window has been designed solely for a demonstration that a sidelobe level can be reduced by introducing additional terms, and is not optimal in any way.

Lastly, paper [1] does not report the scalloping loss of optimized flat-top windows. As their name suggests, flat-top windows are designed to have a maximally flat response in the first bin. If improvement of side lobe suppression comes at a cost of increased amplitude uncertainty then the optimized window cannot be compared to the original one.

REFERENCES

[1] J. F. Justo and W. Beccaro, “Generalized adaptive polynomial window function,” *IEEE Access*, vol. 8, pp. 187 584–187 589, 2020.
[2] A. Nuttall, “Some windows with very good sidelobe behavior,” *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 29, no. 1, pp. 84–91, 1981.