γ-ray polarimetry with conversions to \( e^+e^- \) pairs: polarization asymmetry and the way to measure it

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Abstract

We revisit the measurement of the polarization fraction, \( P \), and the measurement of the polarization angle of partially linearly-polarized gamma rays using their conversion to \( e^+e^- \) pairs in the field of a nucleus. We show that an inappropriate definition of the azimuthal angle, \( \varphi \), used to reference the orientation of the final state degrades the precision of the measurement of \( P \), by comparison to the optimal case where the bisector angle of the electron and of the positron momenta is used. We then focus on the lowest part of the energy spectrum, below \( \approx 10 \text{ MeV} \), where a large part of the statistics lie for a cosmic source. We obtain the value of the polarization asymmetry, \( A \), of pair conversion at threshold and we show that in the case where the correct expression is used for \( \varphi \), the measured value of \( A \) tends to the limit.

1 Introduction

1.1 γ-ray polarimetry and astrophysics

While at low energies (from radio waves to X-rays), polarimetry is a powerful tool to gain an insight into understanding the working mechanisms of cosmic sources, this diagnostic is missing for photon energies \( E > 1 \text{ MeV} \). Polarimetry would provide information on the magnitude, on the direction and on the structure, in particular on the turbulence, of the magnetic fields in the regions that are emitting high-energy \( \gamma \) rays, in particular of relativistic jets.

Blazars, for example, active galactic nuclei in which one jet points at a small angle with respect to our line of sight, show SEDs (spectral energy distributions) that have two broad maxima, both originating from non-thermal emission: The low-energy component is believed to proceed from synchrotron radiation from relativistic electrons, while the origin of the high-energy (X-ray through \( \gamma \)-ray) component is still under debate, as both leptonic and hadronic models produce acceptable fits to the SEDs of most blazars (a recent review can be found in...
this discussion of the 2010 flare of 3C 454.3 [1]). In the leptonic model, the high energy component has contributions from synchrotron-self-Compton (SSC) and from external Compton (EC) radiation whose seed photons are from direct accretion disk emission and from any other external radiation field and are unpolarized. In the hadronic model, the high-energy emission consists primarily of contributions from proton synchrotron emission and synchrotron emission from secondary pairs produced in cascade processes. In most blazar classes, hadronic and leptonic models show similar degrees of X-ray polarization which makes polarization a weakly discriminant diagnostic, while in the γ energy range hadronic models predict still a large polarization fraction (up to $P \approx 70\%$), in contrast with leptonic models for which the small polarized SSC “signal” is washed out by the huge unpolarized EC emission component: high-energy γ-ray polarimetry will yield the discriminating diagnostic [2, 3]. A demonstration that a hadronic mechanism is at work in blazars would make AGNs the preferred suspect for the long-sought acceleration site of ultra-high-energy cosmic rays (UHERC).

For pulsars, after Fermi-LAT’s study showed that high-altitude emission zones are favored by observations [4] thereby disfavoring polar-cap models [5], one cannot cleanly discriminate between the two-pole caustic “slot gap” [5] [8], the outer gap [5] [7] [8] and the striped wind [9] [10] models, in particular as the pulsar inclination and viewing angles are nuisance parameters.

In contrast, polarization can provide a handle on the emission model, as the polarization signatures differ for the different models [5] [9] [7] [8] [10] [6] [11] [12]. As the polarization of the synchrotron emission is perpendicular to that of curvature radiation and as the transition between synchrotron and curvature radiation is expected to take place in the range 1–100 MeV in Crab-like pulsars [11] [12], a phase swing is expected to be observed by MeV polarimeters, a signature for that transition. Theoretical prediction of the energy-dependent polarization fraction and polarization angle phasograms from optical to high-energy γ-rays will be needed to use these results for model discrimination, see, e.g., Fig. 6 of Ref. [7].

Note also the recent quantum electrodynamics (QED) treatment of curvature radiation in pulsar magnetospheres after which spin-flip radiation makes an important contribution in (Crab-like) young pulsars and it even becomes dominant (with respect to constant-spin (classical) emission) in magnetars, and contrary to constant-spin (classical) radiation, radiation is mostly unpolarized [13] [14].

1.2 γ-ray polarimetry and fundamental science

We have many reasons to think that the Standard Model (SM) could be the low-energy remnant of a broken high-energy symmetry. To parametrize this unknown, the Standard-Model Extension (SME) has been built [15] [16], an effective field theory (EFT) that contains the Standard Model, general relativity, and, ordered after their mass dimension $d$, all possible operators that violate Lorentz symmetry [Lorentz Invariance Violation, LIV] [1] be it a global [15] or a local [16] LIV. In the photon sector, SME operators produce a number of effects that can be tested experimentally [17]. Dispersion is the variation of the

\[ ^1 \text{Since } d = 1 \text{ operators are absent in a linear theory and } d = 2 \text{ operators are gauge-violating, the development includes operators with } d \geq 3. d\text{-even operators conserve the CPT symmetry, while } d\text{-odd operators violate it.} \]
speed of light with photon energy and is due to $d \neq 4$ operators. Birefringence arises from the fact that the operator parameters for left and right circular polarized photons can have opposite signs: any linear polarization is therefore rotated through an energy-dependent angle as photons propagate, which depolarizes an initially linearly polarized radiation consisting of a range of photon energies. Hence, from the observation of linearly polarized radiation coming from a distant source we obtain upper limits of the operator parameter values. Most birefringence tests are much more sensitive than dispersion tests, so only when no birefringence is present does dispersion provide useful information as is the case for $d = 6$ limits, obtained from the non observation of any time-of-flight/energy correlations in AGN flares (of Mrk 501 by MAGIC [18], of PKS 2155-304 by HESS [19]) nor in GRB090510 by the Fermi-LAT [20]. For $d = 3$, limits are based on cosmic microwave background (CMB) polarization studies, while for $d = 4$ and $d = 5$, birefringence limits are obtained from measurements of the polarization fraction of the soft-$\gamma$-ray emission of GRBs (in the 200–325 keV and in the 70–300 keV energy ranges for integral/IBIS and IKAROS, respectively) [17]. As none of these soft-$\gamma$-ray GRB polarimetry measurements, taken alone, is found to be statistically significant (see the review in Ref. [21]) and as the rotation angle is proportional to the square of the photon energy, $E^2$, the development of higher-energy polarimeters is eagerly needed.

Polarimetry of the radiation emitted by a far away source enables the search for hints of the axion [22], the putative pseudoscalar pseudo-Goldstone boson induced by the breaking of the $U(1)$ symmetry devised to solve the QCD CP problem. Due to the axion-photon coupling, the propagation of the photon through the magnetic field generated by the GRB would induce a birefringence, that is a rotation of the direction of polarization, that turns out to depend on the photon energy, so that the effective average polarization fraction after propagation of a photon beam having a sizable energy spread would be diluted. The observation of a non-zero polarization fraction then translates to an upper limit on the axion-photon coupling $g_{a\gamma\gamma}$. The limit is GRB-model dependent, but it is presently the best limit for an axion mass close to 1 meV [22] (compare to the present Fig. 1 of Ref. [23]). As the limit is proportional to $1/\sqrt{E}$, extending the polarization measurement to higher energies would lead to an improved value, or even to a detection of a possible axion-like-particle (ALP), one of these pseudoscalars that are not bound to the mass-to-coupling-constant relation that axions are subject to, and that may or may not be a component of dark matter.

### 1.3 $\gamma$-ray polarimetry with pairs: techniques

In contrast with the radio-wave and optical regimes for which polarimetry is performed by the measurement of electric fields or of light intensities, in the X-to-$\gamma$-ray regime photons are detected one by one: the polarization information is extracted from a sample of such conversion events, from the distribution of an angle that is a measure of the orientation of the final state particles with respect to the polarization direction of the photon, an azimuthal angle, and that is denoted generically $\varphi$ in this paper.

Whatever the process at work during conversion, be it photo-electric effect, Compton scattering or pair conversion, due to the $J^{PC} = 1^{--}$ quantum num-
bers of the photon, the differential cross section is described by the expression:

$$\frac{d\sigma}{d\phi} \propto (1 + A \times P \cos(2(\phi - \phi_0))).$$  

(1)

The modulation factor of the cosine, $A \times P$, is the product of the polarization asymmetry of the conversion process (here of pair conversion), $A$, and of the linear polarization fraction of the incoming radiation, $P$. Both $A$ and $P$ are defined to be in the range $[0, 1]$ and $\phi_0$ is the polarization angle of the incoming radiation. Experimental effects affect the measurement of $\phi$, which leads to an effective asymmetry, $A_{\text{eff}}$ that is lower than the QED asymmetry, $A$. Their ratio $D \equiv A_{\text{eff}}/A$ is named the asymmetry dilution due to the experimental effects, $0 \leq D \leq 1$.

Measuring $\gamma$-ray linear polarization by pair production was first suggested by Yang [24] and the full polarized differential cross section was given by Berlin [25] and May [26] (an exhaustive review can be found at Ref. [27]). Unfortunately, the multiple scattering of the two lepton tracks in the high-$Z$ material converter plates of the past (COS-B, EGRET) and present (Fermi-LAT) telescopes blurs the azimuthal information, to the extent that the measurement of the azimuthal angle is impossible [28, 29, 30, 31], the key point being that the lower multiple scattering undergone by higher-momentum tracks from the conversion of higher-energy photons is compensated for by the fact that higher-energy photons convert to pairs with a smaller average opening angle$^2$.

A way out has been sought with triplet conversions, in which the target electron recoils at a large polar angle, giving hope that measuring its azimuthal angle would be easier [32, 33, 34]. Unfortunately, the useable fraction of the cross section, that is, the cross section of triplet conversion with the electron recoil momentum large enough that the track can be reconstructed is extremely small (Fig. 6 of Ref. [36]), even for gas detectors, with the consequence that the sensitivity of a space polarimeter that would use triplet conversion is very low (section 5.3 and Figs. 25-26 of Ref. [36]).

Past theoretical works showed that the asymmetry is larger when the two leptons share the energy equally [37], that it varies with the (azimuthal) acoplanarity, $\varphi_+ - \varphi_-$ [38] and that it is larger for small pair opening angles [39]. Many attempts have been undertaken to increase the effective polarization asymmetry by applying a well-chosen event selection on the collected sample, in the hope that the sensitivity to polarization would be increased [40, 41, 42, 43, 44, 45, 46]. Actually, even though it is true that the asymmetry increases after selection, the gain is almost entirely lost by the reduction in the statistics of the sample, so that ultimately the gain in precision in the polarization fraction of the incident radiation, if any, is found to be small (section 4.1 of Ref. [36]).

Another way out was sought by removing the W converters and using pure silicon converter/trackers. For thick wafers ($\approx 500 \mu m$ [47, 48, 49, 50, 51, 52]), the effective polarization asymmetry, and therefore the dilution factor, are still low because multiple scattering remains an issue. It’s only if very thin wafers can be made, held and launched ($\approx 150 \mu m$ [53]) that there is some hope of a sizeable sensitivity to polarization (see Figs. 5 and 6 of Ref. [53], though).

$^2$In Si/W trackers, in the case the wafers are located downstream to the W converter, there seems to be some hope that the selection of conversion events that took place in the silicon enable the measurement of the azimuthal angle with some precision [54].
Another avenue is the use of a very high-resolution dense homogeneous active target such as an emulsion detector\cite{53}. GRAINE has demonstrated the detection of a linear polarization signal with pairs in a 0.8 - 2.4 GeV $\gamma$-ray beam at SPring8, at a $3\sigma$ significance level\cite{55}, but their ability to get below 100 MeV where most of the statistics lie, remains an issue\cite{3}. Being sensitive at low energies is critical to $\gamma$-ray polarimetry because the polarization asymmetry increases there and, more importantly, because the product of the conversion cross section and of the typical $1/E^2$ flux of a cosmic source peaks at $\approx 6$ MeV (Fig. 2 of Ref.\cite{36}).

The extreme solution to the multiple scattering hurdle is the use of low density, that is, of gas detectors. The HARPO project used a time projection chamber (TPC) in which the conversion takes place in a fast-electron-drift, low-diffusion argon-based gas to perform the first demonstration of polarimetry with pairs at low energy with an excellent dilution factor\cite{56}. It was demonstrated that a $\gamma$-ray polarimeter can be triggered and event reconstruction performed down to the lowest $\gamma$-ray energy of 1.74 MeV that the polarized $\gamma$-ray beam line BL01 of the NewSUBARU electron storage ring could provide\cite{57}.

There is good prospects that the rapid degradation of the TPC gas quality that the EGRET spark chambers have undergone and that motivated a refill per year, is well under control with TPC proportional gas amplifiers \cite{58, 59}. The possibility to perform astronomy, that is, photon-to-source assignment, on a single photon basis, down to so low an energy is uncertain especially due to the huge angular spread induced by the fact that the momentum of the recoiling ion cannot be measured (a parametrization of this contribution to the single-photon angular resolution can be found in section 3.1.2 of Ref.\cite{60}). For GRBs though, source assignment and trigger are taken care of globally for the burst, and the low energy $\gamma$ rays can be logged, reconstructed and analyzed. For pulsars, measuring the phase difference with respect to the pulsar ephemerides enables to subtract the non pulsed background.

A similar project, which uses a negative-ion slow-electron-drift gas, is under development\cite{61} in the hope to get a device with enough amplification gain in the gas later this year\cite{62}. The low electron drift velocity (more than three orders of magnitude lower than that in noble-gas-based mixtures) and the pile-up of stray tracks from proton cosmic rays and from scattering of low-energy photons might, however, make negative-ion TPCs inappropriate for use in space.

1.4 Kinematics

We name $E, \vec{k}$ the energy and the momentum of the incident photon. Its direction defines the $z$ direction. We name $E_i, \vec{p}_i$ the energy and the momentum of the particles in the final state, with $i = +$ for the positron, $i = -$ for the electron and $i = r$ for the recoiling particle, that is the target ion (nuclear conversion) or electron (triplet conversion). The $e^+e^-$ pair is denoted by the index $p$, so for example $p_\nu^- = p_\nu^+ + p^-$. We name $\varphi_i$ and $\theta_i$ the azimuthal and the polar angles, respectively, for $i = +, -, r, p$ (Fig. 1). The unitary vector that shows the direction of a particle is $\vec{u}_i \equiv \vec{p}_i/p_i$ and the fraction of the incident photon energy that is carried away by particle $i$ is $x_i \equiv E_i/E$.

3 Note that polarimetry with pairs has been demonstrated at high energy (1.5 - 2.4 GeV) but in a configuration appropriate for use on a $\gamma$-ray beam and not for a space polarimeter, in particular since it has an efficiency of only 0.02\% \cite{46}.
1.5 Paper layout

In this paper, we examine in detail the energy variation of the polarization asymmetry of $\gamma$-ray nuclear conversion to $e^+e^-$ pairs, using the polarized, fully 5-dimensional (5D), exact down to threshold event generator that was documented in Ref. [36].

1. It has been known since the last century that the asymmetry increases at low $\gamma$-ray energies. The high-energy (HE) asymptotic variation was obtained for triplet conversion [63] and as the same two Feynman diagrams dominate nuclear conversion and, at high energy, triplet conversion, the expression is also asymptotic for nuclear conversion. We examine how the simulated value tends asymptotically to the expression of Ref. [63].

2. From the Bethe-Heitler (BH) differential cross section, we determine the value of the polarization asymmetry $A$ at threshold energy. We examine how the exact value tends to the limit at threshold.

3. Various expressions have been used in the past as “the azimuthal angle” that references the azimuthal orientation of the final state with respect to the direction of polarization of the incident photon:

- The analytical expression [26, 64] (see also eq.5) used the bisector angle $\phi \equiv (\varphi_+ + \varphi_-)/2$ of the lepton pair.
- Later works used the azimuthal angle of the recoiling particle, $\varphi_r$, especially for triplet conversion for which this angle is directly measurable [33, 34, 35]. The azimuthal angle of the pair, that is, of the momentum of the pair, $\varphi_p$, is equivalent to $\varphi_r$, as the pair and the recoiling target are flying back-to-back in the center-of-mass (CMS) system, $\varphi_p = \varphi_r \pm \pi$.
- Experimentalists, [46, 36, 54, 56] following Wojtsekhowski [65, 43] use the pair plane azimuthal angle $\omega$ (Fig.1):

$$\omega = \arctan \left( \frac{u_{-z}u_{+x} - u_{+z}u_{-y}}{u_{-z}u_{+y} - u_{+z}u_{-x}} \right).$$  (2)
Polarimetry of the $\gamma$ emission of a cosmic source can be performed only after astronomy, that is, the photon-to-source assignment, has been performed, be it on a single photon basis (steady sources) or on a burst population basis (GRB). Therefore a putative photon direction can always be defined and the measurement of $\phi$, of $\omega$ and of $\varphi_\tau$ (for triplet) are straight-forward, while the measurement of $\varphi_p$ requires, in addition, a measurement of the magnitude of the momenta of the electron and of the positron. We examine the performance of these possible definitions of the azimuthal angle with the aim to maximize the obtained value of the polarization asymmetry and, therefore, of the precision of the measurement of $P$.

2 Bethe–Heitler polarization asymmetry at threshold

As the original work by Bethe–Heitler neglected the Feynman diagrams for which the incident photon has its vertex with the target particle (Feynman diagrams (c) and (d) of Fig.1 of Ref. [36]), we name here Bethe–Heitler (BH) the calculations performed under this hypothesis. Even though

- the ion mass is much larger than the electron mass,
- in the case of triplet conversion there are two electrons in the final state, so that there is a set of 4 additional “exchange” diagrams,

the two same “Borsellino” diagrams that dominate the differential cross section for nuclear conversion also dominate the triplet conversion asymptotically at high energy.

The full 5D, unpolarized, differential cross-section was calculated by Bethe and Heitler [66, 64]:

$$d\sigma = \frac{-\alpha Z^2 r_0^2 m^2}{(2\pi)^2 E^3} dE_+ d\Omega_+ d\Omega_- \frac{|p_-| |p_+|}{|q|^4}$$

(3)

where

$$\frac{p_+ \sin \theta_+}{E_+ - p_+ \cos \theta_+} \left( 4E_+^2 - q^2 \right) + \frac{p_- \sin \theta_-}{E_- - p_- \cos \theta_-} \left( 4E_-^2 - q^2 \right) + 2p_+p_- \sin \theta_+ \sin \theta_- \cos (\varphi_+ - \varphi_-) \left( E_+ - p_+ \cos \theta_+ \right) \left( E_- - p_- \cos \theta_- \right) \left( 4E_+ E_- + q^2 - 2E^2 \right)$$

$$- 2E^2 \frac{(p_+ \sin \theta_+)^2 + (p_- \sin \theta_-)^2}{(E_+ - p_+ \cos \theta_+)(E_- - p_- \cos \theta_-)}$$

The azimuthal information that takes part explicitly in the expression of the unpolarized differential cross section comes by the factor $\cos (\varphi_+ - \varphi_-)$ that expresses its dependence on the acoplanarity of the two leptons.

Berlin and Madansky obtained the expression for a linearly polarized photon [25], that was later expressed in Bethe-Heitler notation by May [26], and after a correction by a factor of 2 as indicated by [67], is:

$$d\sigma = \frac{-2\alpha Z^2 r_0^2 m^2}{(2\pi)^2 E^3} dE_+ d\Omega_+ d\Omega_- \frac{|p_-| |p_+|}{|q|^4} \times$$

(4)

The $1/|q|^4$ factor obviously contributes to keep the leptons close to back-to-back.
The differential cross section becomes:

\[
\sigma = \frac{2E_+ p_- \sin \theta_- \cos \varphi_- + 2E_- p_+ \sin \theta_+ \cos \varphi_+}{E_+ - p_+ \cos \theta_+} \]

\[
-q^2 \left( \frac{p_- \sin \theta_- \cos \varphi_-}{E_- - p_- \cos \theta_-} - \frac{p_+ \sin \theta_+ \cos \varphi_+}{E_+ - p_+ \cos \theta_+} \right)^2
\]

\[
-E^2 \left( p_+ \sin \theta_+ \right)^2 + \left( p_+ \sin \theta_- \right)^2 + 2p_+ p_- \sin \theta_+ \sin \theta_- \cos (\varphi_+ - \varphi_-) \right) \right] \] ,

with

\[
|q|^2 = |p_+ - p_+ - E|^2 . \tag{5}
\]

Polarimetrists re-cast the above expressions for a partially polarized beam with linear polarization fraction \( P \) as:

\[
d\sigma = \Phi(X_u + P \times X_p) dE_+ d\Omega_+ d\Omega_-, \tag{6}
\]

with:

\[
\Phi = -\frac{\alpha Z^2 r_0^2 m^2 |p_-||p_+|}{(2\pi)^2 E^3} |q|^4 , \tag{7}
\]

and

\[
X_u = \left[ \left( \frac{p_+ \sin \theta_+}{E_+ - p_+ \cos \theta_+} \right)^2 (4E^2_+ - q^2) + \left( \frac{p_- \sin \theta_-}{E_- - p_- \cos \theta_-} \right)^2 (4E^2_- - q^2) + \frac{2p_+ p_- \sin \theta_+ \sin \theta_- \cos (\varphi_+ - \varphi_-)}{(E_+ - p_+ \cos \theta_+)(E_- - p_- \cos \theta_-) (E_+ + q^2 - 2E^2) - 2E^2 \left( p_+ \sin \theta_+ \right)^2 + \left( p_- \sin \theta_- \right)^2 \right] \right) , \tag{8}
\]

\[
X_p = \cos 2\varphi_- (4E^2_+ - q^2) \left( \frac{p_- \sin \theta_-}{E_- - p_- \cos \theta_-} \right)^2 + \cos 2\varphi_+ (4E^2_- - q^2) \left( \frac{p_+ \sin \theta_+}{E_+ - p_+ \cos \theta_+} \right)^2 + 2 \cos (\varphi_+ \varphi_-) (4E_+ E_- + q^2) \frac{p_- \sin \theta_- p_+ \sin \theta_+}{(E_- - p_- \cos \theta_-)(E_+ - p_+ \cos \theta_+)} . \tag{9}
\]

Close to threshold \( E \approx 2m \gg p_\pm, q \approx E, E_\pm \approx m, E_\pm - p_\pm \cos \theta_\pm \approx m \) and the differential cross section becomes:

\[
d\sigma = \frac{\alpha Z^2 r_0^2}{\pi^2 64 m^5} p_- p_+ dE_+ d\theta_+ d\varphi_+ \sin \theta_- d\theta_- d\varphi_- \left( (p_+ \sin \theta_+)^2 + (p_- \sin \theta_-)^2 - 2P \cos (\varphi_+ \varphi_-) p_- \sin \theta_- p_+ \sin \theta_+ \right) , \tag{10}
\]

which, after integration over the electron and positron polar angles, becomes:

\[
d\sigma = \frac{\alpha Z^2 r_0^2}{\pi^2 64 m^5} dE_+ d\varphi_+ d\varphi_- \left( (p_+^2 + p_-^2) p_- p_+ \frac{8}{3} - P \cos (\varphi_+ \varphi_-)(p_- p_+)^2 \pi^2 \right) . \tag{11}
\]
Integrating over $E_+$ and remembering the expression for the bisector angle of the lepton pair, $\phi \equiv (\varphi_+ + \varphi_-)/2$, we finally obtain:

$$d\sigma = \frac{\alpha Z^2 r_0^2}{3\pi 16m^3} d\varphi_+ d\varphi_- (E - 2m)^3 \left(1 - P\frac{\pi}{4} \cos 2\phi\right). \quad (12)$$

- Further integration over the lepton azimuthal angles yields the well-known low-energy asymptote for the Bethe-Heitler total cross section

$$\sigma = \alpha Z^2 r_0^2 \left(\frac{E - 2m}{m}\right)^3 \frac{\pi}{12} \quad (13)$$

- From eq. 12 we obtain the low-energy asymptote for the $\gamma$-conversion polarization asymmetry

$$A = \frac{\pi}{4}. \quad (14)$$

- The high-energy asymptotic expression of the singly differential cross section is

$$2\pi \frac{d\sigma}{d\phi} \propto \alpha r_0^2 \left[\left(\frac{28}{9} \ln 2E - \frac{218}{27}\right) - P \cos 2\phi \left(\frac{4}{9} \ln 2E - \frac{20}{28}\right)\right], \quad (15)$$

from which

$$A \approx \frac{4}{9} \ln 2E - \frac{20 \cdot 28}{28}$$

$$= \frac{20}{9} \ln 2E - \frac{218}{27}, \quad (16)$$

which provides the high-energy asymptotic value of the polarization asymmetry $A = 1/7 \approx 14.3\%$.

### 3 The pair conversion event generator

We use here a polarized event generator based on the SPRING [69] event generator, the 5D differential cross section being either the Bethe-Heitler analytical expression, Eqs. [3] [9] (that includes only the two dominant Feynman diagrams) or a full diagram computation using the HELAS amplitude calculator [70]. In the present section we briefly summarize the documentation and the validation cross-checks of this generator that were published in Ref. [36].

The two possible computations of the differential cross section are exact down to threshold, which means that no high-energy approximation is made. The final state is determined by five variables that are chosen to be the polar and azimuthal angles of the electron and of the positron and the fraction of the incident photon energy that is carried away by the positron, namely $\theta_-, \varphi_-, \theta_+\varphi_+$ and $x_+$. Energy-momentum conservation in the conversion is strictly enforced both when using differential cross sections from HELAS or from Bethe-Heitler, even though the early uses of the Bethe-Heitler formalism assumed that the energy carried away by the target was negligible. Event generation was qualified by confrontation with analytical results on 1D projections published in the past (Figs. 4–6 and (supplementary data) Figs. 28 and 29 of Ref. [36]).
Screening of the electric field of the nucleus by the atomic electrons plays a role at high energies only ([36] and references therein) and is therefore not simulated in the present work. Note also that the Coulomb corrections due to the electrostatic interactions between the charged particles in the final state are not taken into account in this generator.

4 Polarization asymmetry measurement

To measure a polarization fraction, whatever the definition of the azimuthal angle $\varphi$, we can either fit its distribution (eq.1) or use a moments method. The average $\langle w \rangle$ of a function named weight $w(\varphi)$ of a variable $\varphi$ is computed over the event sample. If $\langle w \rangle$ depends on the value of the parameter to be measured (here $A \times P$), $A \times P$ can be obtained from the value of $\langle w \rangle$.

Here we extend the result of Ref. [36] to the case for which the polarization angle $\varphi_0$ is not known:

$$A = 2\sqrt{\langle \cos 2\varphi \rangle^2 + \langle \sin 2\varphi \rangle^2},$$

$$\varphi_0 = \frac{1}{2} \arctan \left( \frac{\langle \sin 2\varphi \rangle}{\langle \cos 2\varphi \rangle} \right).$$

The interest is that the weight can be chosen to be an optimal estimator for $A \times P$ ([36] and references therein). In the case of a weight built on a single variable (here $\varphi$) the moments method is therefore equivalent to a likelihood fit of eq.1 (that we confirm by comparing the results of the two methods).

When instead the weight is built on the whole set of variables that describe the final state, the moments method is not only optimal but also much simpler than an $n$-dimensional likelihood fit. In the following we also present the results obtained with such a weight (eq.11 of Ref. [36]), that we name here “5D”. Moments methods provide a simple and robust estimator, which is very welcome especially in the case of a highly-dimensional final state for which efficiency correction is made difficult by correlations between variables and/or the multidimensional distribution of the background(s) is unknown: the analyst just needs to simulate the moments related to the efficiency and/or to measure the moments related to the background, for correction, without having to accurately parametrize the $n$-dimensional shapes of the efficiency and/or of the background [72].

We have generated samples of $N = 10^5$ events each, for photon energies ranging from 1.1 MeV to 1 GeV, a polarization angle of $\varphi_0 = 0$ and with either $P = 1$ or $P = 0$. Figure 2 presents the $\varphi$ distributions (left), the $2\cos 2\varphi$ distributions (center), and the $2\sin 2\varphi$ distributions (right) for the various definitions of angle $\varphi$ (upper row: $P = 1$, bottom row: $P = 0$). The polarigrams show a clear modulation of the azimuthal angle for the polarized event sample, for which the amplitude varies depending on the expression that is used to estimate this angle. The distribution of the weight $2\cos 2\varphi$ shows a strong asymmetry for $P = 1$, as is expected for a radiation polarized along $x$, none for $P = 0$, and the distribution of the weight $2\sin 2\varphi$ doesn’t show any asymmetry as can be expected for these samples generated with $\varphi_0 = 0$.

The results of measurements of $A$ and of $\varphi_0$ using the above weights are shown in Figure 3. The calculation of the uncertainties for $A$ and for $\varphi_0$ are detailed in [A].

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Figure 2: Distributions of $\varphi$ (left), of $2 \cos 2\varphi$ (center), and of $2 \sin 2\varphi$ (right) for various definitions of angle $\varphi$ (squares: $\omega$, upward triangles: $\varphi_r$, downward triangles: $\phi$), for samples of $10^5$ events generated with an incident photon energy of $E = 1.2$ MeV. On the cos-based plot, the distribution of the 5D weight is also shown (circles). Upper row: $P = 1$, bottom row: $P = 0$.

The uncertainty of the measurement of $A \times P$ is (eq. 6 of Ref. [36])

$$\sigma_{A \times P} \approx \sqrt{\frac{2 - (A \times P)^2}{N}},$$

where $N$ is the number of events, so the precision of the measurement of $P$ for a cosmic source is

$$\sigma_P = \frac{\sigma_{A \times P}}{A} \approx \frac{1}{A} \sqrt{\frac{2 - (A \times P)^2}{N}},$$

that is, $\sigma_P$ depends on the definition of the azimuthal angle, $\varphi$, of the event only through the value of the polarization asymmetry at photon energy $E$ and measured with that definition of $\varphi$. Here we are “calibrating” the detector, that is, we simulate an event sample with a known value of $P$, and we measure $A$ with an uncertainty

$$\sigma_A = \frac{\sigma_{A \times P}}{P} \approx \frac{1}{P} \sqrt{\frac{2 - (A \times P)^2}{N}},$$

- The variation of $A$ with the total available kinetic energy in the final state (that is, with $E - 2mc^2$), plot (a), shows that the best performance is achieved with the variable $\phi$ that takes place in the expression of the differential cross section and that using other expressions only degrades the precision of the measurement.
• The low and high energy asymptotes for \( \varphi \equiv \phi \) are found to be in good agreement with that predicted by the low energy limit (eq.14) and the high energy expression (eq.16).

• Plot (c) shows that the uncertainty of the measurement of \( A \) does not depend on the definition for \( \varphi \) and that it is close to \( \sqrt{2/N} \). The decrease due to the \( A \times P \) dependence of \( \sigma_{A \times P} \) (eq.19) is visible at low energy.

• Plot (e) shows the measured uncertainty of the measurement of \( A \), normalized to the prediction of eq.19 and confirms that eq.19 is a good representation of the data.

• Plot (b) confirms that the measurement of the polarization angle is unbiased and plot (d) confirms the value of its uncertainty,

\[
\sigma_{\varphi_0} \approx \frac{1}{A\sqrt{2N}}.
\]  

The results presented in this section were obtained from simulations with the HELAS calculation of the differential cross section. (plots not shown).
Figure 3: Amplitude $A$ (left) and phase $\varphi_0$ (right) of the modulation of the distribution for various definitions of the azimuthal angle (squares: $\omega$, upward triangles: $r\varphi$, downward triangles: $\varphi$). In addition, the performance of the 5D estimator is shown (circles). Measured value (top row), and uncertainty (middle row). The dotted lines show: (a) the asymptotic values of $A = \pi/4$ at low energy; (c) the approximate value of the uncertainty $\sigma_A \times \sqrt{N} \approx \sqrt{2}$; (d) the approximate value of the uncertainty $\sigma_{\varphi_0} \times \sqrt{N} \approx 1/\sqrt{2}$; (e) the estimated value (unity) of the the uncertainty normalized to the expected value $\sigma_A \times \sqrt{N}/\sqrt{2} = (A \times P)^2$. The dashed curve shows the high energy asymptotic expression for $A$ from Ref. [63] and eq.16 and the dashed line shows the high-energy asymptotic limit $A = 1/7$ (a). These results were obtained using simulated samples with $P = 1$ and $N = 10^5$ events each. Please note that the same quantity (for example $A$) is measured for various definitions of the azimuthal angle using the same event sample at a given photon energy, so that their statistical fluctuations are correlated. The error bars in plot (a) amount to $\approx \sqrt{2/N} \approx 0.0045$ and are therefore not visible.
5 Conclusion

We re-examine the polarization asymmetry of $\gamma$ conversion to $e^+e^-$ pairs with a special focus on the lower part of the energy range where most of the statistics lies for cosmic sources and for which gas-detector-based polarimeters presently under development show very good prospects. We use an exact event generator of the 5D differential cross section, either in its analytic Bethe-Heitler form or computed including all the Feynman diagrams, to obtain event samples at given photon energies and for either fully polarized or unpolarized photon beams. We measure the polarization asymmetry of pair conversion as a function of photon energy from the polarized samples, with a moments method that use sample averages of trigonometric functions of the event azimuthal angle. We demonstrate that defining the azimuthal angle $\varphi$ of the final state as the bisector angle of the pair, $\phi$, provides the most precise 1D measurement of the linear polarization fraction of the incident $\gamma$ radiation. We find that the polarization asymmetry $A$ keeps on increasing at low photon energies and reaches a value of $\pi/4$ at threshold. When the measurement is performed with $\varphi \equiv \phi$, the value obtained from simulation tends towards that limit at low energies and is well represented by the high-energy asymptotic expression for $E \geq 30$ MeV. We confirm the expression of the uncertainty of the measurement of $P$ down to threshold where the value of $A$ is large enough that the $A$-dependence of $\times P$ becomes sizable. We show that the 5D moments method that makes use of the whole set of variables that describe the final state also yields an asymmetry that increases at low energies, and that the gain in precision with respect to the 1D methods is still sizeable down to threshold.

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A Measurement of $A$ and of $\varphi_0$: uncertainties

The statistical uncertainty of the measurement of the weight average $\langle w \rangle$ is obtained from the R.M.S. of the distribution of $w$: $\sigma(\langle w \rangle) = \text{RMS}(w)/\sqrt{N}$. Propagating the uncertainties using Taylor extensions we obtain:

$$\sigma(A) = 2 \frac{(\cos 2\varphi)\sigma(\cos 2\varphi) + (\sin 2\varphi)\sigma(\sin 2\varphi)}{\sqrt{(\cos 2\varphi)^2 + (\sin 2\varphi)^2}}$$

and

$$\sigma(\varphi_0) = \frac{1}{2} \frac{\sigma(\sin 2\varphi)(\cos 2\varphi) + \sigma(\cos 2\varphi)(\sin 2\varphi)}{(\cos 2\varphi)^2 + (\sin 2\varphi)^2}.$$
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