Explorations of Compositeness

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Abstract. Some of the motivations for quark and lepton compositeness, and some problems associated with present schemes, are noted. One model is discussed in which quarks and leptons are taken as composites of spin-1/2 fermions $F$ with charges $\pm 1/2$ and spinless bosons $\overline{S}$ with charges $1/6$ and $-1/2$.

I INTRODUCTION

The breaking of electroweak symmetry is one of the unsolved problems of the “Standard Model.” It can be parametrized in terms of vacuum expectation values of one or more Higgs boson fields, but there is undoubtedly a deeper structure underlying the Higgs sector of the theory. The Higgs field(s) could be elementary but with mass(es) protected from large quartic divergences by cancellations in loop diagrams among particles of different spins. Supersymmetry is a convenient scheme, possibly the only one, for implementing this idea. Alternatively, the Higgs field(s) could be composed of more elementary objects, perhaps fermions and antifermions. Such schemes are collectively referred to as “dynamical symmetry breaking,” since they involve a new strong dynamics to bind the constituents of the Higgs boson(s) to one another.

The simplest incarnation of dynamical electroweak symmetry breaking [1,2], known as “technicolor,” envisions the condensation of fermion-antifermion pairs under the influence of a QCD-like force, but one which becomes strong at about 2650 times the QCD scale. The corresponding “techni-pions” become the longitudinal components of $W^\pm$ and $Z^0$. (For reviews, see [3].)

Technicolor works adequately to induce gauge boson masses, but electroweak symmetry breaking is also manifested in quark and lepton masses. To explain these, technicolor must be “extended” [4]. So far, no scheme of extended technicolor has proved adequate to explain the pattern of quark and lepton masses, the magnitudes

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and phases of elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, or the suppression of flavor-changing neutral currents.

If the Higgs bosons are composite, why not quarks and leptons as well? The patterns of masses and electroweak transitions is reminiscent of a level structure. However, the difficulties in constructing light composite fermions [5] make either supersymmetry or technicolor appealing by comparison. One has no a priori reason to expect an almost pointlike object such as a quark or lepton to have a mass much less than the characteristic scale \( \Lambda \) of its structure. Present lower limits on \( \Lambda \) in various processes exceed one to several TeV. The near-masslessness of the observed fermions, on this scale, could be attributed to a nearly exact chiral symmetry, realized in the Wigner-Weyl sense. Using a criterion proposed by 't Hooft [5], one can test theories for this chiral symmetry by comparing gauge anomalies as realized by composite fermions and by their constituents. The results should be the same. The construction of realistic models satisfying this anomaly-matching condition has proved extraordinarily difficult.

What has happened in the 20 years since 't Hooft first proposed his condition? A good review of the status of the early model-building efforts is given by Peccei [6]. No other classes of theories have provided much insight into the pattern of quark and lepton masses and mixings (though some partial glimmers have emerged in the context of supersymmetry and string theory [7]). It may be time to re-examine the idea of compositeness as “one more layer of the onion.” In the past few years, some powerful techniques have emerged for the study of light composite fermions [8]. These are being exploited in model-building exercises by several groups (see, e.g., [9]).

The present talk is intended as a recapitulation of some previous efforts to construct and test models of composite fermions, and an inducement to consider some of the attractive features of quark and lepton compositeness. The compositeness of hadrons (made of quarks) seems beyond question now, even though it may not have provided a full blueprint to properties of the proton. For example, we still do not know how much of a proton’s spin is carried by quarks, but the classification of the proton and seven other spin-1/2 baryons into an octet of flavor SU(3) [10] is much more easily visualized in terms of the quark model. Similarly, we might hope to understand the apparent existence of three families of quarks and leptons without solving all the associated dynamical problems.

In Section 2 we recall some reasons for expecting quarks and leptons to be members of a level structure. We note in Section 3 the suggestion that a heavy top might be singled out as having a special role in dynamical electroweak symmetry breaking, and discuss an Adler-Weisberger sum rule which hints that the top quark may not be any more special than any other quark if the electroweak scale is set by TeV-scale physics. We outline in Section 4 the motivation for regarding electroweak symmetry breaking as a scaled-up version of QCD, and discuss an extension of such a scheme to the description of composite quarks and leptons in Section 5. Some experimental signatures of this specific model are mentioned in Section 6. The model has features (Section 7) which suggest that it might be made supersymmetric with
FIGURE 1. Patterns of charge-changing weak transitions among quarks and leptons. Direct evidence for $\nu_\tau$ does not yet exist. The strongest inter-quark transitions correspond to the solid lines, with dashed, dot-dashed, and dotted lines corresponding to successively weaker transitions.

II WHY EXPECT A “LEVEL STRUCTURE”?  

The most compelling suggestion that quarks and leptons form some sort of level structure is the pattern, illustrated in Fig. 1 [11], of their masses and charge-changing weak transitions. The relative strengths of these transitions are parametrized by CKM matrix elements $V_{ij}$, where $V_{ud} \simeq V_{cs} \simeq 0.975$, $V_{us} \simeq -V_{cd} \simeq 0.22$, $V_{cb} \simeq -V_{ts} \simeq 0.04$, $|V_{ub}| \simeq 0.0036 \pm 0.0006$, and $|V_{td}| \simeq 0.009 \pm 0.002$ [12]. The unitarity of this matrix implies that only four of these parameters are independent. Given their magnitudes, they are likely to have relative phases which explain the observed CP violation in the kaon system and imply observable effects for $B$ mesons [12].

The transition strengths in the CKM matrix are strongest for quarks in the same family, weaker for quarks in neighboring families (numbering them in order of increasing mass), and weakest for transitions between the first and third families. In this respect the pattern looks like that of electric dipole (E1) transitions in atoms [13] or in quarkonium, where similar selection rules have been noted [14]. The S-wave and P-wave $b\bar{b}$ levels are shown in Fig. 2 [15], together with diagonal
Figure 2. Patterns of electric dipole (E1) transitions between S-wave and P-wave b\bar{b} levels. Solid lines denote strongest matrix elements; dashed lines denote weaker matrix elements.

In atoms and quarkonia, transitions to “nearest-neighbor” states are favored, as a result of greater overlaps between wave functions. Thus, for example, the dipole matrix element $\langle 3S|r|2P \rangle$ has a much larger magnitude than $\langle 3S|r|1P \rangle$. (Here we denote states by their radial, not principal, quantum numbers.) Hence the intensities of the photon lines in $\Upsilon(3S) \rightarrow \gamma + \chi(2P)$ are much greater than those in $\Upsilon(3S) \rightarrow \gamma + \chi(1P)$, far outweighing the advantage of increased phase space in the latter decays. In the case of transitions between states of the three-dimensional harmonic oscillator, there is an exact selection rule, since each cartesian component $x_i$ of the dipole operator is a linear combination of creation and annihilation operators: $x_i = (a_i + a_i^\dagger)/\sqrt{2}$. Thus E1 transitions in a three-dimensional harmonic oscillator can only change the total excitation quantum number $N = N_x + N_y + N_z$ by one unit.

Any composite model of quarks and leptons must involve a mass scale $M_c$ larger than the experimental lower bound $\Lambda$ (of order one to several TeV) describing deviations from pointlike behavior (as, for example, in the production of jets at high transverse momenta in hadron-hadron collisions [16]). In principle $M_c$ could be as high as a grand unification scale. In that case there is not much point to describing the quarks as composites; there would then be no mass range in which the subunits would exist as distinct entities. However, we shall note in Sec. 4 that for Higgs bosons a compositeness scale of one to several TeV is the highest scale for which their composite nature is a useful concept. If the composite natures of Higgs bosons and fermions are related, as will be proposed in Sec. 5, there will necessarily be a large range of masses (from several TeV to a unification scale) for which it
makes sense to speak of subunits of quarks and leptons.

Other varieties of beyond-standard-model physics have mass scales intermediate between the electroweak scale and the unification scale, though some are more adept at concealing it [17]. The supersymmetry-breaking scale often lies at such an intermediate value, while extended technicolor involves a large mass scale (of order 100 TeV or greater) in order to avoid flavor-changing neutral currents. Thus the existence of a new mass scale above which quarks and leptons can appear composite is not *per se* any more excess baggage than accompanies other theories.

New opportunities for regarding fermions as composite [8] have arisen as a result of the dualities between various Yang-Mills theories of gauge fields and matter discovered by Seiberg and Witten [18]. In these theories, if the number of flavors and colors is chosen properly, there can exist zero-mass fermionic bound states of other fermions, thereby automatically fulfilling the ‘t Hooft [5] condition. Attempts (see, e.g., [9]) have been made to construct realistic models based on these ideas. A minimal goal for these attempts would be to explain why we see only the three families of light fermions illustrated in Fig. 1. The partial decay width of the $Z_0$ indicates there are just three light weak-isodoublet neutrinos [19]. If there are fermions heavier than those in Fig. 1, they must fall into different families (at the very least, without light isodoublet neutrinos), perhaps in the manner of the transition metals in the periodic table of the elements.

A final reason to believe in compositeness of our present-day “fundamental units of matter” is that it has happened previously at every scale at which we have been able to look. Starting with the subdivision of macroscopic matter into molecules, molecules into atoms, atoms into electrons and nuclei, nuclei into neutrons and protons (nucleons), and nucleons into quarks, structure continues to change. Typically such changes occur over length scales varying anywhere from a factor of 10 to a factor of $10^5$ (the ratio between a Bohr radius and the size of a proton). Extrapolation from the electroweak scale of 100 GeV to a grand unification scale of $10^{16}$ GeV or the Planck scale of $10^{19}$ GeV is a bold step, fraught with potential *hubris*. It might be correct. The belief in compositeness of quarks and leptons is a conservative “hedge” against such an attitude.

### III A SPECIAL ROLE FOR HEAVY TOP?

The top quark, by virtue of its large mass and consequent large Higgs boson coupling, could play a special role in electroweak symmetry breaking [20]. A key question in such schemes is where one can expect new physics beyond the standard electroweak picture. If it begins just above the top quark, the top quark is indeed special. It may be considerably less tightly bound and more easily excited than other quarks. On the other hand, if the scale $M$ of new physics is at least 1 to 2 TeV, there is nothing particularly special about the top.

An example is provided by an Adler-Weisberger sum rule [21] for the axial-vector coupling $g_A$ of the top quark. One wishes to examine whether deviations of
$g_A$ from unity can provide information about the top quark’s structure [22]. The characteristic scale for deviations from $g_A = 1$ is $\Gamma_T/G_F M_T^2$, where $\Gamma_T$ and $M_T$ are the width and mass of the first excited state in the scattering between a top quark and a longitudinal $W$ or $Z$. For $M_T = \mathcal{O}(2 \text{ TeV})$ and $\Gamma_T$ characterized by the square of the weak SU(2) coupling constant times $M_T$, this deviation is at most a few percent. This stands in sharp contrast to the role of the $\Delta^{++}$ resonance in pion-nucleon scattering, where the sum rule reads, in the SU(6) limit,

$$g_A^2 = \left(\frac{5}{3}\right)^2 = 1 + \left(\frac{4}{3}\right)^2.$$

(1)

The left-hand side is the nucleon pole contribution. On the right-hand side, the first term refers to the equal-time commutator of two axial charges, normalized by the current algebra, while the second term refers to the $\Delta^{++}$ contribution. The existence of a low-lying excited state of the nucleon significantly affects its axial-vector coupling. (An application of the Adler-Weisberger sum rule to $g_A$ of the quark in the context of large-$N_c$ QCD [23] is not devoted to the issue of quark compositeness in the sense we are discussing here, but rather to models in which quarks and pions are treated as separate degrees of freedom.)

**IV MINIMAL TECHNICOLOR: QCD $\times 2650$**

Imagine a world with zero-mass pions, coupling to the divergence of the axial-vector current with a constant $f_\pi = 93$ MeV. These then induce, via the Higgs mechanism, a mass of about 30 MeV in the $W$, through mixing with its longitudinal component [2]. In order to induce a mass of 80 GeV in the $W$, one needs an analogue of the zero-mass pion, but with a coupling to the axial-vector divergence of $v = 2^{-1/4}G_F^{1/2} = 246$ GeV = (80 GeV/30 MeV)$f_\pi$. This boson $H^\pm$ is “eaten” by the $W^\pm$ and its neutral partner $H^0$ is “eaten” by the $Z^0$.

Just as the pion is a quark-antiquark pair, bound to (nearly) zero mass by the QCD interaction, imagine $H$ to be the state of a new fermion $F$ and antifermion $\bar{F}$, bound with a “technicolor” interaction $v/f_\pi \simeq 2650$ times as strong as QCD. The typical scale of hadrons (e.g., the mass of the $\rho$ meson) is of order $2\pi f_\pi$, as one can see, for example, from a dynamical calculation based only on current algebra, unitarity, and crossing symmetry [24]. Thus, the typical scale of states characterized by the new strong interaction should be about $2\pi v$ or $vm_\rho/f_\pi = \mathcal{O}(2 \text{ TeV})$. We expect vector $F\bar{F}$ states, for example, to have such masses. If we classify them according to $I = I_L + I_R$, they consist of an $I = 1$ state $\rho_T$ and an $I = 0$ state $\omega_T$. The suffix stands for TeV, their expected mass scale [28,29].

The spinless composite $(U\bar{U} + D\bar{D})/\sqrt{2}$ remains a particle in the spectrum. In analogy with the fate of the $\eta$ in QCD [27], this particle could acquire a mass characteristic of the electroweak symmetry breaking scale.

The axial current associated with the neutral pion is anomalous, accounting for the decay of the pion to two photons [25] with an amplitude proprtional to $Q_u^2 - Q_d^2$. 
Here $Q_u = 2/3$ and $Q_d = -1/3$ are the electric charges of the $u$ and $d$ quarks. One wishes to avoid a corresponding anomaly in the $Z - \gamma - \gamma$ coupling mediated by triangle graphs involving the fermions $F$. This is most easily satisfied by having the neutral Higgs boson which is “eaten” by the $Z^0$ be of the form $(UU - DD)/\sqrt{2}$, where $Q_U^2 - Q_D^2 = 0$ and $Q_U = Q_D + 1$, or $Q_U = -Q_D = 1/2$. This “minimal” solution appears in the early technicolor \cite{1,2} and compositeness \cite{26} literature. However, the fermions $U, D$ are of no direct use in generating quark and lepton masses.

V FERMION COMPOSITENESS AS A SUBSTITUTE FOR “EXTENDED TECHNICOLOR”

In order to explain quark and lepton masses, one has to “extend” technicolor in some manner \cite{4}, typically by introducing a proliferation of techni-fermions (to implement anomaly cancellation) and new gauge bosons connecting the ordinary and techni-fermions. An alternative scheme can be motivated in the following way \cite{29}.

Let us take the simple solution mentioned above for the technifermion charges: $Q_U = 1/2$, $Q_D = -1/2$.\footnote{There exist tests for these charge assignments based on the production and decay of the spin-1 composites $\rho_T = (UU - DD)/\sqrt{2}$ and $\omega_T = (UU + DD)/\sqrt{2}$ \cite{29}. These tests would have been possible at the planned Superconducting Supercollider, but probably are out of reach of the lower-energy Large Hadron Collider (LHC) at CERN.} There is a simple expression for the electric charge $Q$ of any quark and lepton \cite{30}:

$$Q = I_{3L} + I_{3R} + (B - L)/2$$

where $I_{3L,R}$ are left-handed and right-handed isospin, $B$ is baryon number, and $L$ is lepton number. For all known quarks and leptons, $I_{3L} + I_{3R} = \pm 1/2$. Then let quarks and leptons contain a single subunit $U$ or $D$, where this subunit carries the entire contribution to the electric charge of $I_{3L} + I_{3R}$. Moreover, let the chirality of the quark and lepton be strictly linked to that of the $U$ or $D$. Then to build quarks and leptons incorporating a subunit $U$ or $D$ one merely needs another subunit which (a) provides the contribution $(B - L)/2$ to the electric charge, and (b) does not destroy the correlation between chiralities mentioned earlier. The simplest choice is a scalar $\bar{S}$: $\bar{S}_q$ with charge $1/6$ for a quark and $\bar{S}_\ell$ with charge $-1/2$ for a lepton. The index $i = 1, 2, 3$ denotes three colors; in accord with Pati and Salam’s suggestion, lepton number is the fourth color \cite{31}. We shall thus refer in what follows to the extended (Pati-Salam) color group, including lepton number, as $SU(4)_C$.

The $FS\bar{S}$ picture of quarks and leptons was proposed explicitly by Greenberg and Sucher \cite{32} in 1981, and is implicit in an earlier model \cite{26}, in which $S$ itself is a composite of two fermions, one carrying a horizontal (family) symmetry and
FIGURE 3. Diagram illustrating mass generation in composite model. Solid lines denote fermions $F$; dashed lines denote scalars $S$. The dotted rectangle encloses a condensate $\langle U\bar{U} + D\bar{D} \rangle \neq 0$.

the other carrying color or lepton number. It requires that one solve a dynamical problem to have both light $F\bar{F}$ and $F\bar{S}$ states. In any vectorlike theory for the force which binds constituents into quarks and leptons, the chiral symmetry with which one hoped to ensure small fermion masses [5], instead of being realized in a Wigner-Weyl sense, is expected to be spontaneously broken [33], so that the lightest states are Nambu-Goldstone bosons rather than fermions. The chiral symmetry is then unavailable for protection of fermion masses. One has to postulate an effective interaction between fermions and scalars which ensures that their lightest bound states are massless [34].

Fermion masses in the present model would arise from a condensate $\langle U\bar{U} + D\bar{D} \rangle \neq 0$ affecting quark and lepton masses in a manner which depended on the bound state wave functions. The exchange of scalars $S$ is seen to be a substitute for extended technicolor, as shown in Fig. 3. In order to learn about the $F\bar{S}$ Yukawa couplings to quarks or leptons, one would have to solve the $F\bar{S}$ binding problem.

VI EXPERIMENTAL SIGNATURES

A typical new interaction in the $F\bar{S}$ model described above involves a new contribution to the process $ud \to e^+\nu_e$ with a $UD$ intermediate state. (The $W^+$ also contributes to this process.) Here $u = U\bar{S}_q$, $d = D\bar{S}_q$, $\nu_e = U\bar{S}_\ell$, $e^+ = D\bar{S}_\ell$. The $UD$ intermediate state will typically have a mass of order 2 TeV. One may consider just the effect of a spin-1 state, coupling via any combination of left- and right-handed couplings. Effects will be visible in production of lepton pairs at high transverse momenta ($p_T$) and in high-energy $e^+e^-$ annihilations [35,36], and in low-energy processes such as $\mu \to e\gamma$ and $K^-\bar{K}$ mixing [37].

As one example, one may consider the effect of an intermediate $\rho^+_T$ with mass 2 TeV and Yukawa coupling $g_Y$ to a left-handed $ud$ or $e^+\nu_e$ pair [36]. In $p\bar{p}$ collisions at 1.8 TeV, when $\alpha_Y = g_Y^2/4\pi = 0.1$, the cross section for production of a charged
lepton above $p_T = 200$ GeV changes by about a factor of 2 up or down with respect to its nominal value of about 4 fb, depending on whether the interference between the virtual $\rho_T$ and the off-shell $W^+$ is constructive or destructive. The forward-backward asymmetry in $e^+e^- \rightarrow b\bar{b}$ is affected by direct-channel interference of a virtual $\rho_T^0$ with the virtual photon and $Z$. The effect, however, is only about $\pm 0.03$ for left-handed couplings with $\alpha_Y = 0.1$, in an asymmetry expected to be a bit more than 0.50 at $E_{c.m.} = 200$ GeV [36].

In view of these modest effects, the direct search for excited states of the Higgs boson at the LHC may be the best entree to a composite picture of quarks and leptons. If Higgs bosons are found to be composite, one may then consider a similar possibility for quarks and leptons.

\section{VIII EFFECTIVE SUPERSYMMETRY IN A COMPOSITE MODEL?}

In the $F\overline{S}$ model, the fermions $F$ consist of $U, D$, and their corresponding antifermions, for a total of 8 degrees of freedom when spin is included. This is the same number of degrees of freedom possessed by the three scalars $S_q, S_\ell$ and the corresponding antiscalars. One would be tempted to see a manifestation of supersymmetry in this spectrum, except that the charges of the fermions and scalars are not equal (as in many other realizations of "effective supersymmetry" in condensed matter and nuclear physics [38]). Does this really matter for the ultra-strong dynamics responsible for $F\overline{S}$ binding? If it does, could we have a phase transition to equal charges for fermions and scalars at high energies? For example, by choosing the charges for the scalars $S$ to be $\pm 1/2$, one obtains integrally-charged (Han-Nambu [39]) quarks, for which color and electric charge do not commute with one another.

It may be that supersymmetry provides the necessary ingredient to ensure light composite fermions. If so, we should either begin to see evidence of it in the particle spectrum, or understand how a badly broken version of supersymmetry can still be of use in this context.

\section{VIII OPEN QUESTIONS IN THE “F–S” MODEL}

If there is only a single family of scalars $\{S_q, S_\ell\}$, what degree of freedom supplies the family structure? Greenberg and Sucher [32] proposed that different families corresponded to different radial excitations, with orbital excitations lying significantly higher as a result of an effective interaction close to a potential behaving as $r^{-2}$ near the origin. One would then have to understand CKM mixing in terms of overlaps of wave functions. If, on the other hand, the scalars carry a family label, one loses the motivation for a supersymmetric theory mentioned above, and one has to introduce CKM mixing in an ad hoc manner.
The pattern of quark and lepton masses and CKM matrix elements can be described in terms of various hierarchical structures in mass matrices, of which a recent and compelling example is given in Refs. [7]. Are such hierarchies natural in composite models?

Do fermion masses arise from a condensate $\langle \bar{U}U + \bar{D}D \rangle \neq 0$, as mentioned above? What are the spectrum and wave functions of $F\bar{S}$ bound states? What about $SS\bar{S}$ bound states? Are they light? They could be a nuisance. If so, can we make them heavy?

Are there other distinctive signatures of a generic $F\bar{S}$ model? The usual signatures of compositeness (see, e.g., [40]) include:

- Contact terms, e.g., new 4-fermion interactions
- Excited states (at the compositeness scale)
- Additional states which are light compared to the compositeness scale

and one would expect them to be present here as well. In what follows we construct a specific model for families based on a superstrong SU(4), which we may call SU(4)$_T$. This exercise illustrates both the pitfalls of explicit models and their potential for predicting characteristic exotic states beyond the usual quarks and leptons.

**IX MODEL BASED ON A SUPERSTRONG SU(4)**

Let both $F = U, D$ and a scalar $S$ be members of the fundamental $4_T$ of SU(4)$_T$. Let the first family $f_1 \simeq u, d, \nu_e, e$ be $f_1 = F\bar{S}$. Let the scalars in the second and third families be composites of $S$. For example, if one chooses $S'$ to be a composite of three techni-antiquartets $\bar{S}$ coupled up totally antisymmetrically to an SU(4)$_T$ quartet, it will automatically be a “flavor” [SU(4)$_C$] quartet as well, i.e., it will correspond to three quarks and a lepton, just like $S$. Thus the second family would have the structure $f_2 = F\bar{S}' = FSS\bar{S}$.

The mixing of the first and second families can take place if there is an operator in the Lagrangian transforming as $(SSSS)_{1_T}$. This operator will also be an SU(4)$_C$ singlet if there are no relative angular momenta, by Bose statistics. In turn, it can mix the second family with a third of the form $f_3 \equiv FSS\bar{S}$, with the $SS\bar{S}$ state in a $(4^*_T, 4^*_T)$ representation. An operator transforming as $(S\bar{S})_{1_T}$ would mix $f_3$ with $f_1$.

The combination $SS\bar{S}$ contains two technicolor quartets rather than one, so this model – aside from having no known dynamical realization – has serious shortcomings if we wish it to describe just the usual three families of quarks and leptons. The technicolor quartets, furthermore, are not limited to be quartets of SU(4)$_C$. Moreover, there seems to be no inherent limitation to the number of additional $SS\bar{S}$ pairs that can make up a composite structure. It may be that one has to consider instead the possibility that the scalars themselves are not elementary and/or that
they carry family indices, as in Ref. [26]. In any case, a detailed calculation of $F\bar{S}$, $FSSS$, and $FS\bar{S}\bar{S}$ binding is required to see if the model can yield sufficiently light quarks and leptons. One could imagine performing such a calculation using lattice methods, for example, once one learns how to treat chiral fermions on a lattice.

The spinless $(SS)_1T$ states form exotic particles, consisting of a neutral color octet, a color triplet with charges $2/3$, a color antitriplet with charges $-2/3$, and two neutral color singlets. The charged colored scalars are leptoquarks, coupling to a charged lepton and a charge $\pm 1/3$ quark or a neutrino and a charge $\pm 2/3$ quark. They have to be quite heavy ($\geq 100$ TeV) in order to suppress decays like $K_L \rightarrow \mu e$. This is also true of the gauge bosons which become massive when SU(4)$_C$ breaks down to SU(3)$_C \times U(1)_{B-L}$, if SU(4)$_T$ is a gauged symmetry. If a condensate $(\langle S_1\bar{S}_1 + S_2\bar{S}_2 + S_3\bar{S}_3 - 3S_4\bar{S}_4\rangle/\sqrt{12}) \neq 0$ is responsible for breaking SU(4)$_T$ to SU(3)$_C \times U(1)_{B-L}$, the leptoquark $SS$ bosons could be absorbed into longitudinal components of SU(4)$_C/ SU(3)_C \times U(1)_{B-L}$ gauge bosons and thus be removed from the low-lying spectrum.

The absence of flavor-changing neutral currents at zero momentum transfer is guaranteed by the orthogonality of wave functions of the mixtures of $F\bar{S}$, $FSSS$, and $FS\bar{S}\bar{S}$ that make up the first three quark and lepton families. [A strong constraint on the compositeness scale may arise from the apparent suppression of the decay $\mu \rightarrow e\gamma$ [37], but a detailed calculation is required for the present model.] Since charge-changing weak transitions involve the interchange $U \leftrightarrow D$, any difference in residual interactions of a $U$ and $D$ with the scalars can lead to non-trivial angles in the Cabibbo-Kobayashi-Maskawa matrix elements. This behavior has been illustrated recently in a different context [41].

It is not clear how one generates light neutrino masses in the present model. The conventional picture involves a “seesaw” mechanism [42] with large Majorana masses $M_M$ for right-handed neutrinos. A suitable condensate would seem to require at least two scalar fields $S_4$ in order to give the required two units of lepton number violation, and a pair of fermions $F$ to compensate the charge of the scalars.

Additional exotic technicolor-singlet combinations are possible. For example, one should be able to form $F\bar{F}$, $FFSS$, $FFFS$, and $FFFF$ states in this model. The spin-zero $F\bar{F}$ states are the pseudo-Nambu-Goldstone bosons which constitute the longitudinal components of the $W$ and $Z$, and one massive state $(U\bar{U} + DD)/\sqrt{2}$. The spin-1 $F\bar{F}$ states would be the techni-vector mesons $\rho_T$ and $\omega_T$ with predicted masses of about 2 TeV.

In the $FFSS$ states, the $SS$ subsystem must be in a $6_T$ (antisymmetric) representation. Because of Bose statistics, the $SS_{6_T}$ pair must be antisymmetric in SU(4)$_C$, and so will consist of a color triplet with charge $-1/3$ and a color antitriplet with charge 1/3. The $FF$ pair is also in a $6_T$ representation, and so must be symmetric in its remaining degrees of freedom. Thus, it consists of $I = J = 1$ and $I = J = 0$ states, just as do the nonstrange quarks in the $\Sigma$ and $\Lambda$ hyperons. The $FSSS$ states then consist of spin-1 color triplets with charges $2/3$, $-1/3$, and $-4/3$ and antitriplets with charges $4/3$, $1/3$, and $-2/3$, and spin-0 triplets with charge $-1/3$ and antitriplets with charge 1/3. Since the $SSSS$ operator mentioned earlier
can mix $FFSS$ with $FF\bar{S}\bar{S}$ (for example), the $FFSS$ states have many features in common with diquarks. As a result, their signatures in high-energy hadron collisions may not be very distinctive.

In the $FFFS$ states, the subsystem $(FFF)^{4T}_{4}$ must be totally symmetric in the product of its isospin $\times$ spin $[(I, J)]$ variables. This is also true of the lowest baryonic quark-model states, which consist of $N(1/2, 1/2)$ and $\Delta(3/2, 3/2)$ (for nonstrange states). Thus, we expect the $FFF$ states to form an isospin doublet with charges $Q = \pm 1/2$ and spin $1/2$, and an isospin quartet with charges $Q = (3/2, 1/2, -1/2, -3/2)$ and spin $3/2$. The corresponding $FFFS$ states are then:

\begin{align*}
J &= 1/2: \\
& \quad \text{Quarks with } Q = (2/3, -1/3) \\
& \quad \text{Leptons with } Q = (0, -1)
\end{align*}

\begin{align*}
J &= 3/2: \\
& \quad \text{Quarks with } Q = (5/3, 2/3, -1/3, -4/3) \\
& \quad \text{Leptons with } Q = (1, 0, -1, -2)
\end{align*}

The $FFFF$ states are found, by arguments similar to those presented for the $FFF$ states, to consist of states with $(I, J) = (2, 2), (1, 1), \text{ and } (0, 0)$. Orbital excitations of the lowest-lying states are expected. If experience with ordinary hadrons is any guide, we expect them to lie about a TeV above the corresponding $S$-wave states.

**X FAMILIES AND SPIN CONFIGURATIONS**

An alternative picture of family structure is based on a simple example chosen from quark-model baryon spectroscopy [41,43]. The idea (in search of a dynamics) is that when a down-type quark ($d, s, b$), containing a subunit such as $D$, changes into an up-type quark ($u, c, t$), containing a subunit such as $U$ by virtue of a weak transition, the changed interaction of the subunit with the rest of the constituents gives rise to a rotation of the eigenstates of the dynamics in such a way that off-diagonal transitions are generated.

In the quark model one can see this behavior very transparently with baryons consisting of three unequal-mass quarks (such as the charmed-strange baryons $\Xi_{c}^{+}, 0$). The hyperfine interactions between the quarks, of the form $\sigma_i \sigma_j / m_i m_j$, prevent the mass eigenstates from being diagonal in the combined spins of any two of the quarks, though they are approximately diagonal in the combined spin of the two lightest quarks [44]. In the $\Xi_{c}^{+}, 0$ the strange quark and the $u$ or $d$ quark are in an approximately spin-0 configuration, with a small admixture of spin 1, while the excited $\Xi_{c}^{+}, 0$ states, lying approximately 107 to 108 MeV higher [45], are mostly spin 1 in the two lightest quarks, with a small admixture of spin 0.

If one changes a $u$ quark to a $d$ quark, thereby changing $\Xi_{c}^{+} = csu$ into $\Xi_{c}^{0} = csd$, the mass eigenstates will rotate slightly with respect to the basis states classified
by the spin of the light-quark pair. One thus generates off-diagonal weak transitions between the excited and ground states. The analogue of these transitions for composite quarks would be the off-diagonal CKM matrix elements $V_{ij}$ ($i \neq j$).

Using only three fermions, one can only construct two states with spin 1/2. However, if one adds a unit of orbital angular momentum, one can construct a model of three quark families, which, while artificial, illustrates the principle [41].

**XI SUMMARY**

We have reviewed some reasons for considering composite models of quarks and leptons. Foremost among these is the desire to understand the pattern of masses and transitions in Fig. 1, which may signify a deeper level of structure. Analogies with previous experience, in atomic and nuclear physics, may be misleading, but they strongly suggest that one may not need to solve all outstanding dynamical problems before discerning the next layer of complexity.

We discussed one model in which the building blocks of quarks and leptons are sets of fermions $F$ with charges $\pm 1/2$ and scalars $\bar{S}$ with charges $1/6$ or $-1/2$. Higgs bosons are spinless $F\bar{F}$ states, all but one of which are absorbed into longitudinal components of $W^+, W^-$, and $Z$. In a particular (and probably inadequate) example of this model based on a superstrong SU(4), the three observed families are combinations of $F\bar{S}, FSSS$, and $FSS\bar{S}$. A rich spectrum of exotic states is predicted to lie in the 1–3 TeV region, including spin-1 $F\bar{F}$ mesons and quarks and leptons with unusual charges.

Perhaps the pattern in Fig. 1 is incomplete. Indeed, my favorite nonstandard model involves not only these states but complete multiplets of the exceptional group $E_6$, which includes isosinglet quarks and vector-like leptons [46,47]. So far I have not found a corresponding composite model, but it would certainly be different from the one described above. It may be that just as in the case of the periodic table of the elements, it will be variations in patterns which will provide the clue to underlying structure.

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3) One could imagine a model based on constituents transforming non-trivially under different parts of the SU(3)$_L \times$ SU(3)$_R \times$ SU(3)$_C$ subgroup of $E_6$, for example.
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