Estimation and Tracking for Millimeter Wave MIMO Systems Under Phase Noise Problem

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ABSTRACT Millimeter wave (mmWave) system is a key technology for the fifth generation (5G). Precoding techniques can be applied in the radio frequency (RF) stage to achieve spatial multiplexing gain. Studying the problems of hardware imperfections in the RF stage for mmWave communication system is very important because it can affect the transmitted signals and overall system performance. In this article, mmWave MIMO system under phase noise problem is proposed. In addition, an optimization problem for the design of hybrid precoder under phase noise is formulated. An alternating minimization algorithm using manifold optimization is proposed to close the performance of the fully digital precoder. A realistic power consumption model is considered. To estimate the mmWave channel parameters and to track the phase noise parameters, a Bayesian Cramér-Rao lower bounds (BCRLBs), Kalman filter, least square (LS) and maximum a posterior (MAP) algorithms are proposed. Furthermore, the proposed system with ideal hardware mmWave system is compared with non-ideal system under amplified thermal noise and residual additive transceiver hardware impairments. Spectral efficiencies (SEs) of the proposed system are provided with different scenarios. The analytical and simulation tests show that the influence of phase noise problem may decrease SE performance, especially at higher phase noise.

INDEX TERMS MmWave, hardware imperfections, BCRLBs MAP, LS and SE.

I. INTRODUCTION

Recently, the rapid development of mobile communications technology and the proliferation of integrated communications services have almost filled the low frequency radio band, which needs a rapid transfer rate. The mmWave communication is considered as a promised technology to empower the fifth generation (5G) networks, providing multiple gigabits per second for users. It has several enforcements in processing internet of things (IoT), business and wireless fusion networks, and high communications mission [1]. The mm-wave spectrum covers the 30 - 300 GHz frequency range, ten times higher than the current range of existing radio access technology (RAT) for less than 6 GHz vehicles [2], [3]. With the mmWave range in the extremely high frequency (EHF) range, large channels can be used for communication. These broad channels when simplified with better modulation and coding scheme (MCS), beamforming, and MIMO schemes can help in reducing distraction, enhancing spectral reuse, and allowing higher data quality support. Efficient bandwidth utilization is possible by enabling device-to-device (D2D) connectivity to next-generation networks. Using D2D connectivity in mmWave mobile networks, many direct corresponding links can be supported, which has led to improved network capacity.

A. PRIOR WORK

The mmWave spectrum is large, which allows us to achieve high data rates. Large antenna arrays are employed at both mobile stations (MSs) and base station (BS) because of high path loss in the mmWave. This can also increase distance of communication links, can increase signal power and can achieve high quality links [4]–[9]. The challenge of the mmWave systems is the higher atmospheric absorption and pathloss. These challenging can be solved by using large antenna arrays for highly directional signaling. However,
application of directional beams can vary the features of the wireless communication systems [5]. These directional beams are affected by blockages and obstacles [7]. The performance of mmWave network systems is degraded by thermal noise (i.e., noise limited region) due to high attenuation, high bandwidth and highly narrow beams. In the interference-limited region, interference can degrade the performance of mmWave systems, which depends on probability of the transmission, obstacles density and beamwidth. Unlike classical wireless network systems, mmWave systems can transfer from the region of noise limited to interference limited or intermediate behavior between two regions [8]. The region in which mmWave systems are performed highly, can affects the strategies of resource allocation and the design of medium access control layer [9].

Massive MIMO systems can give high SEs, but this technology faces problems due to hardware imperfections. Another problem is the effects of channel aging and channel estimation, particularly at high mobility environments. These problems must be addressed to enhance system performance. The authors in [10] studied both the single-carrier frequency domain processing (SC-FDP) and the orthogonal frequency division multiplexing (OFDM) under these problems, which show that these problems can degrade system performance.

In [11], lens antenna arrays are employed in mmWave massive MIMO systems can decrease the number of power-hungry RF chains. However, works on this system have not studied the problem of power leakage in the channels, which decrease the system sum-rate and signal to noise ratio (SNR). To overcome the power leakage problem, a beam aligning precoding technique is introduced; each RF chain can choose many beams to collect the leakage power. Then, precoding technique with phase shifter network is built to adjust the channel gains of beams in the user direction with maximizing the signal-to-noise ratio. This technique can give optimal sum-rate performance and higher energy efficiency (EE).

The authors in [12] studied an analytical mmWave ad hoc network to calculate the distribution of interference-to-noise ratio. A narrowband channel model and a Poisson Point Process (PPP) distribution are considered for the transmitter positions. Large antenna arrays are employed to obtain line of sight/none line of sight directions and directional beamforming. Results showed that the interference power is higher than the noise power at high traffic of a network. As a result, the design of ad hoc mmWave network is required to deal with the problem of interference and obtain gigabit speeds.

Practically, hardware imperfections (i.e., impairments) from the RF chains should be considered in the design of mmWave systems. Hardware imperfections arise from low quality devices, amplifier, non-linearities and mutual coupling between antenna ports. It can cause many problems such as quantization errors, phase noise, inphase/quadrature imbalance, power losses and nonlinear power amplifier [13]–[19].

In [20], [21], authors measure the effect of hardware imperfections on the communication systems. The problem of inphase/quadrature imbalance may cause rotating the desired constellation phase and attenuating the amplitude. Also, the non-linear HPAs problem can cause constellation position distort in addition to an additive Gaussian noise. Some compensation algorithms are studied to reduce the impact of hardware imperfections. Hardware imperfections may result in a negative effect on the performance of the system, especially in high data rate [20], [21]. Studies in information theory have illustrated that hardware imperfections can affect the capacity of communication systems with multiple antennas [20]. Most of mmWave systems studies assume the hypothesis of ideal hardware devices.

These hardware imperfections can be compensated partially by compensation or pre-distortion methods, but there is always a remaining distortion that is not estimated and compensated [32]. The remaining distortion can reduce the channel estimation performance and can damage the training pilots. However, there are few studies to illustrate the hardware imperfections in mmWave systems. The authors in [13] studied the channel estimation algorithm for mmWave massive MIMO systems under ideal hardware devices. In [14], the channel estimation algorithm for massive MIMO systems is studied in the case of hardware imperfections, which did not study for mmWave systems.

The authors in [22] studied three sources of hardware imperfections, which are residual impairments, multiplicative impairments and thermal noise. The residual impairments can degrade the performance of the downlink transmissions than in uplink transmissions. In [23], massive MIMO is illustrated in the case of multiplicative impairments and the results showed that system degradation due to mobility of the user is increased with multiplicative impairments. The most motivation of the studies is done on the mmWave systems performance. For more realistic scenarios, it is essential to study the hardware imperfections for mmWave systems. The authors on [24] study the impact of residual impairments in mmWave systems using OFDM waveforms.

Many studies are concentrated on the design of precoder and how to reduce its complexity [25]–[28]. Precoding/beamforming schemes based on large-scale chemical composition can allow adequate advantages for overcoming free space losses and create enough SNR communications. For large MIMO configurations operating at mmWave frequencies, the transmitting antennas number can be high, so a fully digital configuration that allows high frequency replication benefits needs a separate RF for each antenna, which is impossible because of high power and extra problematic power [29]–[32]. For large scale MIMO construction, the hybrid precoding structure constructed with the high RF precoder and low cost digital precoder proposed in [30] is receiving much attention. There are two kinds of hybrid precoding frames that are mostly investigated in the literature; 1) fully connected scheme in which whole antennas are linked to each RF chain with a phase shifter, and b) lowly connected
scheme in which some of antennas are connected to the RF series.

The transceiver designs for hybrid sub-array architecture and hybrid MIMO architectures with compressive sensing estimation studied in [33] and [34], respectively. Although demonstrated that these transceiver design in mmWave systems can give high SE, numerous studies consider ideal hardware. Practically, mmWave systems have hardware imperfections, thus degrading their performance.

In [35], beamforming mmWave MIMO architectures under residual hardware impairments is studied. The hardware impairments are modeled as additional distortion noise. Results showed that there is a ceiling effect in the capacity performance, meaning that it cannot increase the system capacity with the higher signal-to-noise ratio.

The authors in [36] studied the problems of hardware imperfections, which are non-linear power amplifiers, analog-to-digital converter resolution and phase noise. The analysis is performed on OFDM and SC-FDP for mmWave communications. From results single-carrier frequency domain equalization is robust against analog-to-digital converter resolution and non-linear power amplifiers than orthogonal frequency division multiplexing. The channel coding is illustrated to mitigate the less robustness of orthogonal frequency division multiplexing to a large extent.

The authors in [37] studied hardware aspects at 60 GHz for the wireless personal area networks. Hardware problems such as non-linear power amplifiers and phase noise are considered. The OFDM and SC-FDP systems over an AWGN propagation channel beneath RF impairments using programming tools at 60 GHz are studied in [38]. Similar studies and comparisons are introduced for an indoor wireless personal area networks environment in [39]. Also, an outdoor mmWave case with 3D channel model [40] using a sectorized beamforming method [41] is illustrated.

The channel estimation problem is explained in [42] for mmWave systems in the case of transmitter hardware impairments. Results show that the performance of the classical pilots-based channel estimation method is degraded due to hardware impairments, which can damage training pilots. Using the property of mmWave channel sparsity, the authors in [42] studied Bayesian compressive sensing for channel estimation, which give better performance compared to the orthogonal matching pursuit algorithm. The impact of path loss can be compensated using hybrid precoding schemes [43]. There is a challenge task in the compensation of the effects of phase noise in mmWave systems.

Phase noise problem can give high frequency shifts and can rapidly vary in time. Many algorithms are employed to give constant frequency offsets, but these algorithms cannot do the same job in the case of higher phase noise [43]. This can lead to apply the multicarrier modulation for mmWave systems. At higher frequencies 72 GHz, the RF hardware imperfections will be amplified over the effects at classical cellular frequencies. At the mmWave band, one dominant source of hardware imperfections is the increased phase noise [44].

This problem can be large enough to need tracking from one data symbols to another. The authors in [45] illustrated the mmWave system in the case of phase noise, which it is created by the local oscillators. Due to large phase noise, there is unknown phase value, which varies from one block to another in block-processing systems. Blind phase noise compensation technique is addressed for the tracking of phase noise in single-carrier communication system, which brings better performance.

In [46], mmWave MIMO system under residual transceiver hardware impairments is addressed. Also, three sources (i.e., residual impairments, multiplicative impairments and thermal noise) is studied. The SE is deduced to demonstrate the effect of these problems on system performance. There is degradation in the SE performance and phase noise is the dominant impact. The hardware imperfections are a challenge task in the design of mmWave transceivers. Thus, the paper goal is to study the effectiveness of hardware imperfections on mmWave systems with hybrid precoding and how to mitigate it by proposing compensation and tracking algorithms. A summary of related works are listed in Table 1.

B. CONTRIBUTIONS

We illustrate a comprehensive analysis on the mmWave system performance by calculating the spectral efficiency. It facilitates the ability of determine the impact of hardware imperfections at any SNR value. The main contributions are:

- MmWave MIMO system under hardware imperfections is proposed.
- A realistic power consumption model is demonstrated.
- Formulate an optimization problem for designing analog and digital precoders.
- An alternating minimization algorithm is proposed to solve the optimization problem.
- Propose BCRLBs, Kalman filter, LS and MAP algorithms for tracking the phase noise parameters.
- Comparison between the traditional mmWave system and the proposed mmWave system is provided in terms of SE.

C. ORGANIZATION

The paper remainder is arranged as follows. Section II illustrates phase noise problem along with its model. Section III shows the proposed mmWave cellular system under hardware imperfections. Section IV explains the design of hybrid precoder and combiner. The proposed channel estimation techniques is derived in Section V. Section VI explains the tracking of phase noise. Section VII shows the model of power consumption. Simulation tests and concluded remarks are explored in Sections VIII and IX.

II. PHASE NOISE PROBLEM AND ITS MODEL

The parameter that describes the behavior of phase noise is illustrated in this section. Phase noise problem is the instabilities of the phase values in communication systems. It can degrade the performance of communication systems [23]–[26]. It can cause signal constellation rotation
TABLE 1. Comparison of proposed scheme with respect to related works.

| Ref.   | Problem                                           | System                        | Solution                          | Studies  |
|--------|---------------------------------------------------|-------------------------------|-----------------------------------|----------|
| [8]    | Power noise problem and interference             | mmWave systems                | NO                                | SE       |
| [10]   | Channel aging and channel estimation             | OFDM and SC-FDP               | NO                                | SE       |
| [11]   | Power leakage problem                            | mmWave massive MIMO system    | Beam aligning precoding technique | Sum-rate and and SNR |
| [13]   | Ideal hardware                                    | mmWave massive MIMO systems   | Compressed sensing                | SE       |
| [17]   | Nonlinear amplifier and phase noise               | m-QAM-OFDM system             | NO                                | BER      |
| [20]-[21] | Inphase/quadrature imbalance and non-linear high power amplifier | Communication system          | Compensation algorithms           | BER      |
| [22]-[23] | Residual impairments, multiplicative impairments and thermal noise | Massive MIMO system            | NO                                | BER      |
| [24]   | Residual impairments                              | mmWave system                 | NO                                | SE       |
| [33]-[34] | Ideal hardware                                    | Hybrid MIMO architectures      | NO                                | SE       |
| [35]   | Additional distortion noise                       | Beamforming mmWave MIMO       | NO                                | Capacity |
| [36]   | Non-linear power amplifiers, ADC resolution and phase noise | OFDM mmWave communications    | Channel coding                    | SE       |
| [37], [39] | Non-linear power amplifiers and phase noise    | Wireless personal area networks | NO                                | BER      |
| [42]   | Hardware impairments                              | mmWave system                 | Bayesian compressive sensing scheme | SE       |
| [43]-[44] | Phase noise                                       | mmWave system                 | NO                                | SE       |
| [45]   | Phase noise                                       | Single carrier communication   | . Blind phase noise compensation  | SE       |
| [46]   | Residual impairments, multiplicative impairments and thermal noise | mmWave MIMO                   | NO                                | SE       |

from symbol to another symbol. Consider the transmitted signal $x$ is influenced by a fading channel $h$ and an additive noise $v$. The received signal in cell $i$ at a given channel uses $t \in \{1, \ldots, T\}$ of RF transceiver system under phase noise can be expressed as:

$$y_i(t) = D_{\phi_i(t)} \sum_{l=1}^{L} H_{il} x_l(t) + v_i(t),$$

(1)

where $D_{\phi_i(t)}$ is the phase noise, which consists of phase drifts as:

$$D_{\phi_i(t)} \triangleq \text{diag} \left(e^{j\theta_1(t)}, \ldots, e^{j\theta_N(t)}\right).$$

(2)

where $j$ is the imaginary unit and $\theta_n$ is the phase-drift of $n$th receive antenna. The variable $\theta_n(t)$ in local oscillators (LOs) is considered as a Wiener process $\theta_n(t) \sim N(\theta_n(t-1), \delta)$, which equivalent to the previous realization $\theta_n(t-1)$ in addition to Gaussian innovation of variance $\delta$ [20]. The phase drifts are autonomous between the antennas and correlated. In distributed arrays of separated LOs, the phase drifts are autonomous and phase-drifts variance $\delta$ are equal. The distributed array is assumed in the proposed mm-wave system. The variance $\delta$ is one of the important parameters to analyze this problem.

III. THE PROPOSED mmWave MIMO SYSTEM UNDER HARDWARE IMPERFECTION

A. SYSTEM BLOCK DIAGRAM AND SIGNAL MODEL

In this section, we propose a mmWave MIMO system with the existence of phase noise problem as illustrated in Fig. 1. The BS has $N_t$ antennas and $L_t$ RF chains. It can communicate with a MS, which has $N_r$ antennas and $L_r$ RF chains.
The received signal in vector and matrix form can be written as:

\[ y(k) = \mathbf{H}\mathbf{\Theta}^{[r]}(k)\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}s(k) + n(k) \]

The baseband received signal vector, \( \mathbf{y}(k) = [y_1(k), \ldots, y_{N_t}(k)]^T \) after combing process is:

\[ y(k) = \mathbf{\Theta}^{[r]}\mathbf{W}_{\text{BB}}^H\mathbf{W}_{\text{RF}}^H\mathbf{H}\mathbf{\Theta}^{[r]}\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}s(k) + \mathbf{W}_{\text{BB}}^H\mathbf{W}_{\text{RF}}n(k) \]

The received signal in vector and matrix form can be written as:

\[ y(k) = \mathbf{W}_{\text{BB}}^H\mathbf{W}_{\text{RF}}^H\left(\mathbf{A}\odot\mathbf{e}^{j\mathbf{B}(n)}\right)\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}s(k) + \mathbf{W}_{\text{BB}}^H\mathbf{W}_{\text{RF}}n(k) \]

Let \( \mathbf{P}(n) = \mathbf{A}\odot\mathbf{e}^{j\mathbf{B}(n)} \), then

\[ y(k) = \mathbf{W}_{\text{BB}}^H\mathbf{W}_{\text{RF}}^H\mathbf{P}(n)\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}s(k) + \mathbf{W}_{\text{BB}}^H\mathbf{W}_{\text{RF}}n(k) \]

where

- \( \mathbf{s}(k) = [s_1(k), \ldots, s_{N_t}(k)]^T \) with \( s_m(k) \) is the M-ary modulated signal transmitted from \( m \)th antenna at time \( k \);
- \( \mathbf{F}_{\text{BB}} \) is the \( N_s \times L_s \) baseband precoder;
- \( \mathbf{F}_{\text{RF}} \) is the \( N_t \times L_t \) RF precoder;
- \( \mathbf{W}_{\text{BB}} \) is the \( N_s \times L_s \) baseband combiner;
- \( \mathbf{W}_{\text{RF}} \) is the \( N_t \times L_t \) RF combiner;
- \( \mathbf{H} \in \mathbb{C}^{N_t \times N_s} \) is the channel matrix with \( \mathbb{E}\left[||\mathbf{H}||^2_F\right] = N_rN_t \);
- \( \mathbf{n} \in \mathbb{C}^{N_t \times N_s} \) defines the noise vector with i.i.d \( \mathbb{C}\mathbb{N}(0,\sigma_n^2) \) elements;
- \( \mathbf{\Theta}^{[r]}(k) = \text{diag}\left(e^{j\theta_{1}^{[r]}(k)}, \ldots, e^{j\theta_{N_t}^{[r]}(k)}\right) \) and \( \mathbf{\Theta}^{[l]}(k) = \text{diag}\left(e^{j\theta_{1}^{[l]}(k)}, \ldots, e^{j\theta_{N_t}^{[l]}(k)}\right) \) are the \( N_t \times N_t \) diagonal matrices, respectively;
- \( \theta_{1}^{[r]}(k) \) is the \( k \)th phase noise sample at the \( i \)th receive antenna;
- \( \theta_{1}^{[l]}(k) \) is the \( k \)th phase noise sample at the \( j \)th transmit antenna;
- \( \mathbf{A} = [\alpha_1, \ldots, \alpha_{N_t}]^T \) is the \( N_t \times N_t \) channel gain matrix;
- \( \mathbf{B} (n) = [\beta_1(n), \ldots, \beta_{N_t}(n)]^T \) is the \( N_t \times N_t \) phase shift matrix;
- \( \alpha_{k,l} = |h_{k,l}| \) is the channel gain between \( i \)th and \( j \)th antenna for the transmitter and receiver, respectively;
- \( \beta_{k,l}(n) = \theta_{k}^{[l]} + \theta_{k}^{[l]} + \angle h_{k,l} \) is the total phase shift.

The model of phase noise in the above equation can be represented as a Wiener process [49]–[52]. The phase noise \( \theta_{1}^{[r]}(k) \) at the \( j \)th transmit antenna and \( \theta_{1}^{[l]}(k) \) \( i \)th receive antenna, respectively is given by:

\[ \theta_{1}^{[r]}(k) = \theta_{1}^{[r]}(k - 1) + \Delta_{1}^{[r]}(k) \]

\[ \theta_{1}^{[l]}(k) = \theta_{1}^{[l]}(k - 1) + \Delta_{1}^{[l]}(k) \]

where \( \Delta_{1}^{[r]}(k) \) and \( \Delta_{1}^{[l]}(k) \) are the phase noise rate and it is considered Gaussian Process with \( \Delta_{1}^{[r]}(k) \sim \mathcal{N}\left(0, \sigma_2^{[r]}\right) \) and \( \Delta_{1}^{[l]}(k) \sim \mathcal{N}\left(0, \delta_2^{[l]}\right) \), respectively.

The phase noise \( \mathbf{B} (n) \) and the channel gains \( \mathbf{A} \) are assumed to be unknown. The variations of \( \mathbf{A} \) are slow compared to the
variations of $\Theta$. So, the estimated data is used to estimate the values of $B$ and $A$.

**B. CHANNEL MODEL**

A clustered channel model (i.e., Saleh-Valenzuela model) characterizes the environment of mmWave propagation due to high free-space path loss. It applied for mmWave propagation system [47], [48]. The channel matrix $H$ can be represented by:

$$H = \sqrt{\frac{N_r N_t}{N_c N_{\text{ray}}}} \sum_{i=1}^{N_c} \sum_{l=1}^{N_{\text{ray}}} \alpha_{il} R_r (\phi_{il}^r, \theta_{il}^r) T_t (\phi_{il}^t, \theta_{il}^t)^H$$

(9)

where $N_{\text{ray}}$ and $N_{cl}$ are the beams number and the clusters number in each cluster, respectively and $\alpha_{il}$ is the $i$th ray gain for $th$ ray index. The parameter $\alpha_{il}$ is independent and identically distributed, $\mathcal{C}\mathcal{N}(0, \sigma_{\alpha}^2)$ and the normalization factor $\sum_{i=1}^{N_{cl}} \alpha^2_{il} = \hat{\gamma}$, satisfy $\mathbb{E}||H||_F^2 = N_t N_r$. The array response vectors for the receiver and the transmitter are $R_r (\phi_{il}^r, \theta_{il}^r)$ and $T_t (\phi_{il}^t, \theta_{il}^t)^H$, respectively. Also, the azimuth and elevation angles of arrival and departure are $\phi_{il}^r, \phi_{il}^t$ and $\theta_{il}^r, \theta_{il}^t$ respectively. Assuming that $\sqrt{N_t} \times \sqrt{N_r}$ uniform planar array antennas. The array response for the transmitter and the receiver, respectively are:

$$R_r (\phi_{il}^r, \theta_{il}^r) = \frac{1}{\sqrt{N_r}} \left[ 1, \ldots, e^{j \frac{2\pi}{\sqrt{N_r}} r (p \sin \phi_{il}^r \sin \theta_{il}^r + q \cos \theta_{il}^r)}, \ldots \right]^T$$

$$T_t (\phi_{il}^t, \theta_{il}^t)^H = \frac{1}{\sqrt{N_t}} \left[ 1, \ldots, e^{j \frac{2\pi}{\sqrt{N_t}} r (p \sin \phi_{il}^t \sin \theta_{il}^t + q \cos \theta_{il}^t)}, \ldots \right]^T$$

(10)

(11)

where $r$ is the spacing between antennas and $\lambda$ is the wavelength. For 2D-plane, the bounds of antenna indices $p$ and $q$ are $0 \leq p < \sqrt{N_r}$ and $0 \leq q < \sqrt{N_t}$, respectively. This model is applied in our simulations to make our hybrid precoder more applicable for general models.

**C. PERFORMANCE METRICS**

We consider that channel state information (CSI) is recognized for the BS and the MS. Practically, CSI is obtained effectively by channel estimation techniques. The spectral efficiency (SE) can be written as:

$$\text{SE} = \log \det \left( I_{N_t} + \frac{\delta^2}{\sigma_{\alpha}^2 N_s} (W_{BB} W_{RF})^H P F_{RF} F_{BB} \right)$$

$$\times F_{RF}^H F_{BB}^H P^H (W_{BB} W_{RF})$$

(12)

where $\delta^2$ is the phase noise variance. The $F_{RF}$ and $W_{RF}$ are implemented with phase shifters, which change the signal’s phase. So their entries must achieve the constraints for nonzero elements, $|F_{RF}^{i,j}| = |W_{RF}^{i,j}| = 1$.

**IV. DESIGN OF HYBRID PRECODER AND COMBINER**

**A. PROBLEM STATEMENT**

It observed in [27] that the minimization of the Euclidean distance among the hybrid and the fully digital precoders is tractable and efficient alternative aim for spectral efficiency maximization of mmWave systems. The precoders and combiners design can be divided into precoding sub-problems and combining sub-problems. The mathematical analysis is the same for the two sub-problems except that there is a power constraint in the optimization technique. Therefore, we analyze the precoder design in this article and the same analysis can be equally applied for the combiner design. The optimization problem for the design of hybrid precoder under phase noise in the proposed mmWave MIMO system is:

$$\text{minimize} \quad F_{RF}, F_{BB} \quad \frac{1}{F} \left\| F_{opt} - \delta^2 F_{RF} F_{BB} \right\|_F^2$$

Subject to

$$\left\| \delta^2 F_{RF} F_{BB} \right\|_F^2 = N_s$$

(13)

where $F_{opt}$ is the optimal digital precoder, $F_{RF}$ is the analog precoder and $F_{BB}$ is the digital precoder. The feasible set $\mathcal{A}$, is induced by the unit modulus constraints. It is observed in [26] that minimization of the optimization function results in maximizing the SE. We treat the problem in (13) as a matrix factorization problem and alternating minimization algorithm is used. Alternating minimization algorithm is applicable for many applications such as image reconstruction [53], deconvolution [54], matrix completion [55] and matrix factorization [56]. The optimization problem is a matrix factorization problem including two matrices $F_{RF}$ and $F_{BB}$. This technique can alternately solve for $F_{RF}$ and $F_{BB}$ while the other matrix is fixed.

**B. DESIGN OF DIGITAL PRECODER $F_{BB}$**

First, we solve the optimization problem for digital precoder, $F_{BB}$ while the matrix of analog precoder $F_{RF}$ is fixed. Thus, the problem in (13) becomes:

$$\text{minimize} \quad F_{BB} \quad \frac{1}{F} \left\| F_{opt} - \delta^2 F_{RF} F_{BB} \right\|_F$$

(14)

The ZF digital precoder is adopted with nearly optimal performance and $F_{BB}$ is given by:

$$G = (H)^H \left( (H (H)^H)^{-1} F_{BB} - \beta G \right)$$

(15)

where $\beta = \sqrt{\beta}/\|F_{RF}\|_F$ is the power normalized factor. The above solution in (15) has provided a near optimal solution to the digital precoder design.

**C. DESIGN OF ANALOG PRECODER $F_{RF}$**

The set $\mathcal{A}$ of RF precoder is expressed as $|F_{RF}^{i,j}| = 1$, since every antenna is linked to all the RF chain. With the alternating minimization concept, we fix $F_{BB}$ and solve the optimization problem for $F_{RF}$ as follows:

$$\text{minimize} \quad F_{RF} \quad \frac{1}{F} \left\| F_{opt} - \delta^2 F_{RF} F_{BB} \right\|_F^2$$

Subject to

$$|F_{RF}^{i,j}| = 1, \quad \forall i, j.$$  

(16)
The unit modulus constraints are intrinsically non-convex. In this section, we apply manifold optimization technique to obtain a sub-optimal solution of the above problem (16). The background on manifold optimization is illustrated in [57]–[59]. Also, application of this algorithm can be found in [60].

1) DEFINITIONS AND TERMINOLOGIES FOR MANIFOLD OPTIMIZATION

Let $\mathcal{M}$ be the manifold space and $T_x\mathcal{M}$ be the tangent space at a point $x$ on the manifold $\mathcal{M}$ as shown in Fig. 2. Tangent space consists of tangent vectors $\xi_x$ of the curves $\gamma$ through the point $x$.

\[ T_x\mathcal{M} = \{ z \in \mathbb{C} : \left< x^* z + z^* x \right> \} = \{ z \in \mathbb{C} : x^* z + z^* x = 2 \left< x , z \right> \} = \{ z \in \mathbb{C} : x^* z = 0 \} \]

At point $x \in \mathcal{M}_{cc}$, the tangent space can be expressed as:

\[ T_x\mathcal{M}_{cc} = \{ z \in \mathbb{C} : z^* x + x^* z = 2 \left< x , z \right> \} \]

Let $x = \text{vec}(\mathbf{F}_{RF})$ be the vector of the analog precoder. It forms a complex circle manifold

\[ \mathcal{M} = \{ x \in \mathbb{C}^m : |x_1| = |x_2| = \cdots = |x_m| = 1 \} \]

The gradient at point $x$ is the tangent vector, \( \nabla f(x) \), determined by the Euclidean gradient orthogonal projection, \( \nabla f(x)^\dagger \) onto the space, \( T_x\mathcal{M}_{cc} \) [35]:

\[ \text{grad} f(x) = \text{Proj}_{T_x\mathcal{M}_{cc}} \nabla f(x) \]

where the cost function gradient is

\[ \nabla f(x) = -2 \left( \mathbf{F}^*_{BB} \otimes \mathbf{I}_N \right) \left[ \text{vec}(\mathbf{F}_{opa}) - (\mathbf{F}^T_{BB} \otimes \mathbf{I}_N)x \right] \]

The next step is retraction. It transforms a vector from the tangent space into the manifold zone.

\[ \text{Retr}_{x} : T_x\mathcal{M}_{cc} \rightarrow \mathcal{M}_{cc} \]

\[ \mathbf{R} \mathbf{d} \longmapsto \text{Retr}_{x} (\mathbf{R} \mathbf{d}) = \text{vec} \left( \frac{(x + \alpha \mathbf{R} \mathbf{d})}{|(x + \alpha \mathbf{R} \mathbf{d})|} \right) \]

To design the analog precoder, a line search based conjugate gradient algorithm is applied [36]. This algorithm uses Armijo backtracking search. Also, it uses the Polak-Ribiere variable to assure the objective function in each iteration to be non-increasing [37]. The Transport operation of a tangent vector $\mathbf{d}$ from $x_k$ to $x_{k+1}$ into two tangent spaces $T_{x_k}\mathcal{M}_{cc}$ and $T_{x_{k+1}}\mathcal{M}_{cc}$ is described as:

\[ \text{Transp}_{x_k \rightarrow x_{k+1}} : T_{x_k}\mathcal{M}_{cc} \rightarrow T_{x_{k+1}}\mathcal{M}_{cc} \]

\[ \mathbf{d} \longmapsto \mathbf{d} - \delta \text{Proj}_{T_{x_k}\mathcal{M}_{cc}} \left( \mathbf{d} \circ x_{k+1}^* \right) \circ x_{k+1} \]

Algorithm 1 Alternating Minimization Algorithm

1: Input: $\mathbf{F}_{opt}, \mathbf{F}^{(0)}_{BB}, x_0 \in \mathcal{M}_{cc}$
2: Output: $\mathbf{F}_{RF} \text{ and } \mathbf{F}_{BB}$
3: Step 1: Analog Precoder Design $\mathbf{F}_{RF}$
4: $\mathbf{d}_0 = -\text{grad} f(x_0)$ and $k = 0$;
5: for
6: Put step size of Armijo backtracking $\alpha_k$;
7: Apply the retraction process to obtain $x_{k+1}$ as;
8: Calculate the gradient
9: Determine the transport vectors $\mathbf{g}_{k+1}$ and $\mathbf{d}_{k+1}^\dagger$ from $x_k$ to $x_{k+1}$;
10: Fix the parameter of Polak-Ribiere $\beta_{k+1}$;
11: Calculate $\mathbf{d}_{k+1} = -\mathbf{g}_{k+1} + \beta_{k+1} \mathbf{d}_{k+1}^\dagger$;
12: $k \leftarrow k + 1$;
13: end

14: Step 2: Digital Precoder Design $\mathbf{F}_{BB}$
15: Initial $\mathbf{F}^{(0)}_{RF}$ and let $k = 0$;
16: for
17: Determine $\mathbf{F}^{(k)}_{RF}$ and $\mathbf{F}^{(k)}_{BB} = \delta^{2\dagger} \mathbf{F}^{(k)}_{RF} \mathbf{F}^{(k)}_{opt}$;
18: Solve $\mathbf{F}^{(k+1)}_{RF}$ using Algorithm 1 when $\mathbf{F}^{(k)}_{RF}$ is calculated;
19: $k \leftarrow k + 1$;
20: end

21: Normalization, $\tilde{\mathbf{F}}_{BB} = \sqrt{\mathbf{N}} \mathbf{F}_{BB} / \| \mathbf{F}^2_{RF} \mathbf{F}_{BB} \|_F$

D. DESIGN OF HYBRID PRECODER

An alternating minimization algorithm is applied to design hybrid precoder. It is expressed in algorithm 1 by resolving
equations (14) and (16). The power constraint in (13) is achieved by normalizing the $F_{BB}$ at Step 21 of algorithm 1. The objective function in optimization problem (13) is minimized at Steps 17 and 18 in algorithm 1, each iteration will never increase it. The algorithm 1 may be utilized as a system performance benchmark in terms of SE.

V. THE PROPOSED CHANNEL ESTIMATION TECHNIQUES

To minimize the effects of channel and phase noise issues, we suggest the MAP Estimator, LS Estimator, Bayesian Cramér-Rao Lower Bound Estimator and Kalman filter schemes.

A. MAP ESTIMATOR

This MAP estimator can estimates phase noise values $\Theta$ by employing all received signals in the frame $y$. This technique can maximize the posterior PDF log as follows:

$$
\log p(\Theta | y) = \log \left\{ \frac{p(y | \Theta)p(\Theta)}{p(y)} \right\} = K \log \left\{ \frac{p(\Theta(1))}{(\pi \sigma^2)^{N_t/2} (2\pi)^N/2 (\det \sum)^{1/2}} \right\} - \frac{1}{2} [\Theta(k) - \Theta(k-1)]^T \sum_{\tilde{\Theta}(k)}^{-1} [\Theta(k) - \Theta(k-1)] - \log p(y) \tag{26}
$$

Since the two terms $\log \left\{ \frac{p(\Theta(1))}{(\pi \sigma^2)^{N_t/2} (2\pi)^N/2 (\det \sum)^{1/2}} \right\}$ and $\log p(y)$ in equation (26) are independent of $\Theta$, the phase noise estimation using MAP becomes:

$$
\Theta^{[\text{MAP}]} = \arg \min_\Theta \sum_{k=1}^K \left\{ \left[ y(k) - \Theta^{[r]}(k) \mathbf{H}\Theta^{[l]}(k) \bar{s}(k) \right]^T \right. \\
\times \left. \sum_{\tilde{\Theta}(k)}^{-1} \left[ y(k) - \Theta^{[r]}(k) \mathbf{H}\Theta^{[l]}(k) \bar{s}(k) \right] \\
\left. + \frac{1}{2} [\Theta(k) - \Theta(k-1)]^T \sum_{\tilde{\Theta}(k)}^{-1} [\Theta(k) - \Theta(k-1)] \right\} \tag{27}
$$

where

- $\bar{s}(k) \triangleq \mathbb{E}_{\mathcal{P}(k)}[\mathbf{S}(k)]$ represents the signal mean for the $k$th sampling instant;
- $\sum_{\tilde{\Theta}(k)} \triangleq \mathbb{E} \left[ \tilde{\Theta}(k) \tilde{\Theta}^H(k) \right]$

$$
= \Theta^{[r]}(k) \mathbf{H}\Theta^{[l]}(k) \sum_{\tilde{\Theta}(k)} \left( \Theta^{[l]}(k) \right)^H \\
\times H^H \left( \Theta^{[r]}(k) \right)^H + \sigma_n^2 \mathbf{I}_{N_t N_r}.
$$

The computational complexity of the exhaustive search in (27) can be reduced into a series of 1-D searches by applying the alternating projection technique [64]. Thus, to obtain a proper initialization, $\hat{\Theta}(k)$, is started by $0_{N_t \times 1}$, for $K = 1, 2, \ldots, K$, as the overall phase noise matrix and channel gains processes are evaluated at the beginning of each frame.

B. LS ESTIMATOR

This sub-section derives an LS estimator of phase noise and channels parameters. The main function is to reduce the square distance between the transmitted and received signals. The received signal likelihood function, $y$ in terms of the phase noise $\Theta$ and channel magnitudes $\mathbf{H}$ is described as:

$$
p(y | H, \Theta(n)) = \frac{1}{(\sigma_n^2)^{N_t/2} (2\pi)^{N_t} \det (\mathbf{C}_y)} \times \exp \left\{ -\left( y - \mu_{s y} \right)^T \mathbf{C}_y^{-1} \left( y - \mu_{s y} \right) \right\} \tag{28}
$$

where $\mu_{s y}$ is the mean and $\mathbf{C}_y$ is the covariance matrix. The joint LS estimates of phase noise $\hat{\Theta}(n)$ and channel gains $\hat{H}$ is:

$$
\hat{H}, \hat{\Theta}(n) = \arg \min_{\hat{H}, \hat{\Theta}} \left( y - \mu_{s y} \right)^T \mathbf{C}_y^{-1} \left( y - \mu_{s y} \right) \tag{29}
$$

Then, the cost function became:

$$
\hat{H}, \hat{\Theta}(n) = \arg \min_{\hat{H}, \hat{\Theta}} \left( y - \mathbf{P}(n) \mathbf{S} \right)^T \left( y - \mathbf{P}(n) \mathbf{S} \right) \tag{30}
$$

where $\mathbf{P}(n)$ is defined in (6) and $\mathbf{S} = [s_1, \ldots, s_{N_t}]^T$ is an $N_t \times L_t$ matrix. The LS estimate of $\mathbf{P}(n)$, $\hat{\mathbf{P}}(n)$ is:

$$
\hat{\mathbf{P}}(n) = y \mathbf{S}^H \left( \mathbf{S} \mathbf{S}^H \right)^{-1} y \mathbf{S}^H \frac{1}{L_t} \tag{31}
$$

Using (31), estimates of the phase noise and channel magnitude matrices, respectively are calculated by:

$$
\hat{\Theta}(n) = \angle \left\{ \frac{y y^H}{L_t} \right\} \tag{32}
$$

$$
\hat{H} = \frac{y y^H}{L_t} \tag{33}
$$

where $\angle \{ \} \text{ returns the phase of variable and } | \cdot | \text{ is the absolute value operator.}

C. BAYESIAN CRAMER-RAO LOWER BOUND

To recover the transmitted signal at time instant $n$, $s(n)$, the receiver with BCRLBs can estimates the phase noise parameters, $\Theta_k(n)$ for $k = 1, \ldots, N_t$. This subsection derives the BCRLBs for phase noise estimation. It is calculated by taking the inverse of Fisher’s information matrix [65]–[67]:

$$
\text{BCRLB} (\Theta) = \text{diag} \left( \text{FIM}^{-1} \right) \tag{34}
$$

where $\text{FIM}$ is the $N_t \times N_t$ Fisher’s information matrix (FIM) [67]. We first explain the Proposition 1 to calculate the FIM.
Proposition 1: The $N_t \times N_r$ FIM for the phase estimation is:

\[
\text{FIM}_{k,\tilde{k}} = \left[ \frac{\partial \mu(\Theta)}{\partial \Theta_k} \right]^T C^{-1}_y(\Theta) \left[ \frac{\partial \mu(\Theta)}{\partial \Theta_{\tilde{k}}} \right] + \frac{1}{2} \text{Tr} \left[ C^{-1}_y(\Theta) \frac{\partial C_y(\Theta)}{\partial \Theta_k} C^{-1}_y(\Theta) \frac{\partial C_y(\Theta)}{\partial \Theta_{\tilde{k}}} \right]
\]

where

- $\frac{\partial \mu(\Theta)}{\partial \Theta_k} \triangleq \left[ \begin{smallmatrix} 0 \, (k-1) \times N_r, N_r \end{smallmatrix} \right]^T$, is an $N_r L_x \times N_r$ matrix, which denotes derivative of the received signal mean with respect to phase noise;
- $s \triangleq [s^T(1), \ldots, s^T(N_t)]^T$ is an $N_t \times L_t$ matrix;
- $\frac{\partial C_y(\Theta)}{\partial \Theta_k}$ denotes to derivative of the covariance matrix with respect to phase noise;
- $C^{-1}_y$ is the inverse of received signal covariance matrix.

After taking the expectation and making some mathematical processing, the covariance matrix becomes:

\[
C_{\tilde{k},\tilde{k}} = \sum_{n=1}^{N_t} |h_{k,l}|^2 |h_{\tilde{k},l}|^2 \left[ \frac{\partial \mu(\Theta)}{\partial \Theta_k} + \frac{\partial \mu(\Theta)}{\partial \Theta_{\tilde{k}}} \right] \left[ \begin{smallmatrix} \sigma_{\tilde{k}}^2 \mathbf{T} + \sigma_{\tilde{k}}^2 \mathbf{T} \end{smallmatrix} \right]
\]

\[
\text{FIM}_{k,\tilde{k}} = \left[ \begin{smallmatrix} \sigma_{\tilde{k}}^2 \mathbf{T} + \sigma_{\tilde{k}}^2 \mathbf{T} \end{smallmatrix} \right]
\]

where $\mathbf{T}$ is an $L_t \times L_t$ matrix given by:

\[
\mathbf{T} = \left[ \begin{smallmatrix} L_t - 1 & L_t - 2 & \cdots & 0 \\ L_t - 2 & L_t - 2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{smallmatrix} \right]
\]

and

\[
T_{k,\tilde{k}} = \sum_{m=-L_t+2}^{n} \left[ \frac{\partial \mu(\Theta)}{\partial \Theta_k} \right] \left[ \begin{smallmatrix} \frac{\partial \mu(\Theta)}{\partial \Theta_{\tilde{k}}} \end{smallmatrix} \right]
\]

The partitioned matrix inverse of FIM in (34) is used to obtain the CRLB closed-form expression for specific $N_t$ and $N_r$ values. After taking the matrix inversion of FIM, its diagonal elements gives the BCRLB for the estimation of phase noise.

VI. TRACKING PHASE NOISE

To track the phase noise in mmWave MIMO system, we propose Kalman filter scheme. First, the state and observation are calculated for the KF. The overall phase noise is:

\[
\Theta_{k,l}(n) = \Theta_{k,l}(n-1) + \Delta_{k,l}(n)
\]

where $\Delta_{k,l}(n) = \Delta_{l}^{[r]}(n) + \Delta_{l}^{[i]}(n)$ is total phase noise rate. The unknown state vector is:

\[
\Theta(n) = \left[ \begin{smallmatrix} \Theta_{T}^T(n) \ldots, \Theta_{N_r}^T(n) \end{smallmatrix} \right] = \Theta(n-1) + \vartheta(n)
\]

In matrix form, Eq. (41) can be expressed as:

\[
\mathbf{Q} = \mathbb{E}\{\vartheta(n)\vartheta(n)^T\}
\]

where

\[
\mathbf{Q}_{k,\tilde{k}} = \left\{ \begin{array}{ll}
\sigma_{\tilde{k}}^2 \mathbf{I}_{N_r \times N_r} + \text{diag}(\sigma_{\tilde{k}}^2, \ldots, \sigma_{\tilde{k}}^2), & k = \tilde{k} \\
\text{diag}(\sigma_{\tilde{k}}^2, \ldots, \sigma_{\tilde{k}}^2), & k \neq \tilde{k}
\end{array} \right.
\]

From equation (5), the received signal for the Kalman filter is:

\[
y(n) = x(n) + w(n)
\]

The received signal in the above equation can be seen as a non-linear function of the unknown state vector $\Theta$. So, we apply KF scheme in order to linearize it [68]. So, the Jacobian matrix is determined through calculating the 1st order partial derivative of $x(n)$ with respect to the $\Theta(n)$ as:

\[
\mathbf{J} = \frac{\partial \mathbf{x}(n)}{\partial \Theta^T(n)} = \left[ \begin{smallmatrix} \frac{\partial x_1(n)}{\partial \Theta^T(n)} & 0_{1 \times N_r} & \cdots & 0_{1 \times N_r} \\ \frac{\partial x_2(n)}{\partial \Theta^T(n)} & 0_{1 \times N_r} & \cdots & 0_{1 \times N_r} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial x_{N_r}(n)}{\partial \Theta^T(n)} & 0_{1 \times N_r} & \cdots & 0_{1 \times N_r} \end{smallmatrix} \right]
\]

Let $\mathbf{M}(n | n-1)$ and $\mathbf{K}(n)$ are the prediction error covariance matrix and the Kalman gain. After initialization step, the KF scheme is processed as follows. The predicted state vector of phase noise is:

\[
\hat{\Theta}(n | n-1) = \hat{\Theta}(n-1 | n-1)
\]

and the prediction error covariance matrix is:

\[
\mathbf{M}(n | n-1) = \mathbf{M}(n-1 | n-1) + \mathbf{Q}
\]
Then, the Kalman gain can be calculated as:

$$
K(n) = M(n|N-1)J^H(\theta(n|N-1)) \\
\times \left( J(\theta(n|n-1)) M(n|n-1) J^H(\theta(n|n-1)) + \sigma^2 I \right)^{-1}
$$

(48)

After that, the proposed Kalman filter refines the scores by the backward-recursive equations, for phase noise and error covariance matrix:

$$
\hat{\theta}(n|n) = \hat{\theta}(n|n-1) + \mathcal{R}(K(n) \\
\times \left[ y(n) - 1 \left( \hat{A} \odot e^{\hat{B}(n)} \right) s(n-1) \right])
$$

(49)

$$
M(n|n) = \mathcal{R}(M(n|n-1) - K(n) \cdot \theta(n|n-1)) \\
\times M(n|n-1)
$$

(50)

where

- \( J(\theta(n|n-1)) = J(\theta(n)|\theta(n) = \theta(n|n-1)) \) is the Jacobian matrix and is given in equation (45).
- \( \hat{\theta}(n|n-1) \) is the predicted state vector for the \( n \)-th symbol.
- \( \hat{B}(n) = B(n)|\theta(n) = \theta(n|n-1) \) is the estimated phase shift.

### VII. MODEL OF POWER CONSUMPTION

Power consumption model is important for describing different components and devices in mmWave MIMO system and their power consumption. The total power consumptions of each component in mmWave MIMO system, \( P_T \), is:

$$
P_T = P_{PA} + N_r P_{LNA} + L_r (P_{RF \text{ chain}} + P_{ADC}) + P_{BB}
$$

(51)

where

- \( P_{PA} \) is the power dissipated in the passed array and is given by:
  
  $$
P_{PA} = N_r \times L_r \times P_{RF \text{ shifter}}
  $$
  
  (52)

where \( P_{RF \text{ shifter}} \) is the power consumption shifters. Its value is based on the kind and resolution of the quantized phases. In [73], a power consumption shifter \( P_{RF \text{ shifter}} \) of 20 mW is suggested for each phase shifter.

- \( P_{RF \text{ chain}} \) is the power consumed by the RF chain, which includes a local oscillator, a mixer, a base band amplifier and a low pass filter. The authors in [74] considered values of 14mW and 5mW for low pass filter and base band amplifier, respectively. An oscillator phase noise is inversely proportional to the power consumption. In [75], the oscillator power consumption is 60 mW for each element in the voltage controlled oscillator. Signal power from oscillator is used to drive the mixer, which consumes 10 mW. Then, the total power consumption for the oscillator is 70 mW. In [77], the power consumption of voltage controlled oscillator is 122 mW. It is measured under phase noise at the oscillation frequency varies from 26.46 to 28.90 GHz. It is calculated as follows:

$$
P_{RF \text{ chain}} = P_{LO} + P_{\text{mixer}} + P_{BB \text{ amp}} + P_{LPF}
$$

(53)

- \( P_{ADC} \) is the power consumed by single analog to digital converter. Power consumption of ADC versus sampling frequency, effective bits number, and bandwidth for various studies are listed in TABLE 1. We assume a value of 200 mW.

- \( P_{LNA} \) is the power consumed by a single low noise amplifier. Studies for the low power LNAs at 60 GHz band in CMOS technology and mmWave MIMO system illustrate that power consumption is in the range of 10 to 86 mW. In our work, we consider 20 mW for the proposed system.

- \( P_{BB} \) is the power consumption of baseband precoder/combiner. The baseband processor of precoder/combiner consumes 200 mW.

Finally, we compare the power consumption of an ideal system [14] and proposed scheme to show the impact of phase noise problem as in Fig. 3. In Fig. 3, the power consumption for ideal system [14] and proposed system is compared. When the number of antennas increases, the power consumption is decreased.

### TABLE 2. Values of ADC power consumption.

| Ref. | Sampling rate [Gs/s] | Bandwidth [GHz] | Eff. number of bits | Power consumption [mW] |
|------|----------------------|-----------------|---------------------|------------------------|
| [69] | 1.25                 | > 0.6           | 6.7                 | 207                    |
| [70] | 1.8                  | > 0.8           | 5.1                 | 30                     |
| [71] | 2.2                  | > 1             | 7                   | 27.4                   |
| [72] | 2.5                  | > 1             | 4.9                 | 98                     |

- \( P_{ADC} \) is the power consumed by single analog to digital converter. Power consumption of ADC versus sampling frequency, effective bits number, and bandwidth for various studies are listed in TABLE 1. We assume a value of 200 mW.

- \( P_{LNA} \) is the power consumed by a single low noise amplifier. Studies for the low power LNAs at 60 GHz band in CMOS technology and mmWave MIMO system illustrate that power consumption is in the range of 10 to 86 mW. In our work, we consider 20 mW for the proposed system.

- \( P_{BB} \) is the power consumption of baseband precoder/combiner. The baseband processor of precoder/combiner consumes 200 mW.

### FIGURE 3. Average power consumption vs. number of antennas \( (N_A) \) of ideal system [14] and non-ideal proposed system with phase noise, \( \delta = 10^{-3} \text{rad}^2 \).

### VIII. RESULTS

The SE performance of our proposed mmWave MIMO system is evaluated numerically and analytically. All resulted
The information are forwarded from the transmitter with well with the marker symbols extracted from the simulation. Figures are extracted from analytical equations that match the parameters of the channel are $N_{\text{clusters}} = 5$, $N_{\text{rays}} = 10$ and the average power is $\sigma^2_{\alpha} = 1$. We assume the distribution of the azimuth and elevation angles is Laplacian with mean angles and angular spread of 10 degrees. The elements of antenna in the array of uniform square planar array are disconnected by a half wavelength distance. In [23], the authors studied the devices for communication system under phase noise and make measurements at 2.8 GHz with $T_s = 10^{-9}$sec and variance of phase noise equals $\delta = 10^{-4}$ rad$^2$. Consider that $\delta = 10^{-4}$ rad$^2$ represents the lowest impact of phase noise problem and $\delta = 10^{-2}$ rad$^2$ is the greatest effect of phase noise problem. To estimate the phases and channel gains at time $n$, we apply the proposed LS algorithm at the beginning of data. Then, we initialize the state model of the Kalman filter algorithm using these estimates and calculate the next estimates of phase noise at time $n+1$ and so on. We examine the impact of the suggested mmWave MIMO system with MAP estimator, LS estimator, BCRLBs and Kalman filter algorithms in terms of SE performance.

**A. IMPACT OF CHANGING RF CHAINS**

Firstly, we show the spectral efficiency achieved by MAP estimator, LS estimator, BCRLBs and Kalman filter algorithms when the number of RF chains is $L_t = L_r = 3$ and $\delta = 10^{-2}$ rad$^2$ as shown in Fig. 4.

The authors in [14] studied the ideal scenario of mmWave system without hardware problems. The ideal mmWave is a reference to evaluate the proposed scheme performance. The proposed system with LS achieves lower spectral efficiency than the other algorithms. On the contrary, BCRLBs algorithm can give near-optimal performance over the whole SNR range and close to the ideal case [14]. At low number of RF chains, there is an enhancement in the SE performance with using BCRLBs algorithm. It approaches to the ideal mmWave system [14]. The proposed MAP estimator can provide substantial performance gains over LS and Kalman filter, especially at high SNRs as in Fig. 4. When the RF chains number increased (i.e., $L_t = L_r = 10$) with the same value of phase noise $\delta = 10^{-2}$ rad$^2$, there exists a gain in the system performance as illustrated in Fig. 5. The LS estimator decreases the effect of phase noise, but its performance is quite far from the ideal system [14]. At low SNRs, the MAP estimator gives the same performance of the Kalman filter. BCRLBs algorithm can give the better performance.

**B. IMPACT OF CHANGING DIFFERENT-PATH CHANNELS**

Fig. 6 and Fig. 7 show different path channel scenarios for three and ten paths, respectively. For the three paths scenario, the proposed system with estimation methods shows the same performance. MAP, LS, BCRLBs and Kalman filter algorithms can improve the system performance and closes to the ideal system performance with the range of 0dB to 5dB as in Fig. 6. These algorithms can decrease the impact of phase noise. At SNR = 5dB, the SE is 8.5 bps/Hz for LS estimator, which is close to SE = 9 bps/Hz for the ideal system as in Fig. 6. In Fig. 7, there exists a large performance gap among the estimation algorithms when the RF chains number is increased to 10 as shown in Fig. 7. This scenario is a real study case, for recent estimation campaigns in mmWave bands, which demonstrate that channels in a city environment may be evaluated using a pre-estimated paths cluster number [75], [76]. The proposed MAP, BCRLBs and Kalman filter outperform LS scheme. The performance of LS algorithm becomes worse in multipath environment, because of lacking in exploiting multipath channel gains. The performance of the mmWave system worsens when the system is operated under
FIGURE 6. SE vs. SNR of ideal system [14] and suggested scheme with phase noise problem at $N_t = 64$, $N_r = 16$, channel paths = 3 and $N_s = 4$ data streams.

FIGURE 7. SE vs. SNR of ideal system [14] and suggested scheme with phase noise problem at $N_t = 64$, $N_r = 16$, channel paths = 10 and $N_s = 4$ data streams.

When the phase noise is increased, $\delta = 10^{-3} \text{ rad}^2$, SE performance is much less as shown in Fig. 9. With $\text{SNR} = 0 \text{ dB}$, SE for ideal system is 19 bps/Hz, which is higher than 17, 16.5, 15 and 10 bps/Hz for the proposed system with BCRLBs, Kalman filter, MAP and LS, respectively, as shown in Fig. 9. We can conclude that SE performance of proposed system varies with changing the phase noise. It is obvious that performance of mmWave MIMO system is extremely affected. As a result, we must design devices at the least defect of phase noise to obtain good SE performance.

We also compare the proposed system with [46], which studies the system under non-ideal hardware as in Fig. 10. The phase noise problem gives very low SE compared to the amplified thermal noise, and the residual additive transceiver
impairments in [46]. The problem of residual additive impairments, which arises from the antenna coupling, the quantization noise in the ADCs and the DC offset has lower impact on the SE performance compared to amplified thermal noise. The ideal hardware scenario in [14] is taking a benchmark to measure SE performance. The proposed system with LS and BCRLBs solution can enhance the Wave system under hardware problems. The performance of mmWave system is affected by phase noise. So, the hardware comments of mmWave system should be designed with good quality.

IX. CONCLUSION

In this article, mmWave MIMO system under hardware imperfection has been proposed. A realistic power consumption model has been assumed. An optimization problem is formulated to design hybrid precoding using an alternating minimization algorithm. A BCRLBs, Kalman filter, LS and MAP algorithms have been proposed for estimating and tracking phase noise and channel gains parameters. Comparison between proposed system and non-ideal hardware mmWave system has been illustrated under different scenarios. Results confirmed that mmWave MIMO system with proposed algorithms can mitigate the problems of non-ideal hardware.

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