DAMPING RATE FOR TRANSVERSE GLUONS WITH
FINITE SOFT MOMENTUM IN HOT QCD

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Abstract

We calculate the damping rate for transverse gluons with finite soft momentum to leading order in perturbative hot QCD. The internal momenta of the one-loop contributing diagrams are soft. This means we have to use effective vertices and propagators which incorporate the so-called hard thermal loops. We expand the damping rate in powers of the incoming momentum and argue that the series ought to converge within a finite radius of convergence. We contrast such a behavior with the one obtained from a previous calculation that produced a logarithmic behavior, a calculation based on letting the gluon momentum come from the hard limit down towards the interior of the soft region. This difference in behavior may point to interesting physics around some 'critical' region.
1. Introduction

The quark-gluon plasma (qgp) is a phase of hot hadronic matter that we hope to see in very near-future experiments like RHIC and/or LHC. One paramount importance of this plasma is that it allows us to see quarks and gluons, if not completely free, at least in a deconfined plasma phase, and get clues on the mechanism of confinement. In this respect, many properties of QCD at (high) finite temperature \( T \) have been investigated [1].

One important quantity that is related to the stability of the qgp is the damping rate \( \gamma(p) \) of gluons* with momentum \( p \) in the presumed plasma. This quantity has raised a great deal of controversy in the past because, when calculated at \( p = 0 \) in a standard loop expansion at finite \( T \), it is plagued with gauge dependence both in magnitude and sign [2]. This problem has been solved when recognizing that in such conditions, the loop expansion is not necessarily an expansion in powers of the QCD coupling constant \( g \), and hence one has to reorganize the perturbative expansion so that one takes into account the resummation of the so-called hard thermal loops, which are loop diagrams with hard (i.e., \( \sim T \)) internal momenta [3]. One then argues that \( \gamma(0) \) is finite, gauge-independent and positive.

However, a previous calculation [4] suggests that when we let \( p \) run from the hard limit down towards the interior of the soft region, the boundary of which being physically set by \( m_g \), the inverse thermal gluonic correlation length or thermal gluonic mass for short (which is of order \( gT \)), the damping rate \( \gamma_t(p) \) for transverse gluons gets a logarithmic behavior \( \ln(1/g) \). This result is in contrast with the fact that \( \gamma_t(p = 0) \) is finite. It is also in contrast with the expectation that the qgp ought to be stable for at least very small but nonzero momenta. A similar interest was raised in [5] and a discussion in the context of scalar QED was carried.

In this work, we undertake the expansion in the soft region \( p \leq m_g \) of the damping rate \( \gamma_t(p) \) in powers of \( p/m_g \), i.e., we write:

\[
\gamma_t(p) = \frac{g^2 N_c T}{24\pi} \left[ a_{t0} + a_{t1} \left( \frac{p}{m_g} \right)^2 + a_{t2} \left( \frac{p}{m_g} \right)^4 + \ldots \right],
\]

where \( N_c \) is the number of colors. The quantity \( \frac{g^2 N_c T}{24\pi} a_{t0} \) is just \( \gamma_t(0) \) with \( a_{t0} = +6.63538\ldots \), determined in [2]. Our primary aim is to argue that such an expansion is valid within a finite radius of convergence that we denote by \( \mu \). We do this by explicitly calculating the second coefficient \( a_{t1} \) in the above expansion and suggesting that the other coefficients may be calculated in a similar manner. If indeed this expansion is a valid one, it would mean that the analytic behavior of the damping rate changes when we cross from the region below \( \mu \) to the region above. Such a result would suggest that there could be interesting physics to investigate in the ‘critical’ region around \( \mu \).

This paper is organized as follows. After this introduction, we set up the stage in the next section for the calculation of the transverse-ghou damping rate. In the third section, we carry the calculation of

* We discuss electric gluons only. The interesting magnetic sector is more intricate as it is well known.
the effective self-energy to order $p^2$, which is the essential new quantity entering the definition of $\gamma_t(p)$, see equation (2.9) below. We discuss our results in the last section and finish with few concluding remarks.

2. Preliminaries

In this section, we prepare the ground for the calculation of the transverse gluonic damping rate $\gamma_t(p)$. We work in the imaginary-time formalism in which the euclidean momentum of the gluon is $P^\mu = (p_0, \mathbf{p})$ such that $P^2 = (p_0)^2 + p^2$ where $\mathbf{p} = p \hat{\mathbf{p}}$ and $p_0 = 2\pi n T$ where $n$ is an integer. After we perform the intermediary steps, we obtain the real-time amplitudes via the analytic continuation $p_0 = -i\omega + 0^+$ where $\omega$ is the energy of the gluon. The convention we adopt is that a momentum is said to be soft if both $\omega$ and $p$ are of order $gT$; it is said to be hard if one is or both are of order $T$ [3].

We carry our calculation in the Coulomb gauge in which the complete inverse gluon propagator is given by:

$$D_{\mu\nu}^{-1}(P) = P^2 \delta^{\mu\nu} - P^\mu P^\nu - \Pi^{\mu\nu}(P) + \frac{1}{\xi_C} \delta^{\mu i} \delta^{\nu j} p^i p^j ,$$  

where $\Pi^{\mu\nu}(P)$ is the gluon self-energy and the last term is due to Coulomb-gauge fixing. In fact, it is most suitable to work in the strict Coulomb gauge $\xi_C = 0$. The gluon self-energy can be decomposed into:

$$\Pi^{\mu\nu}(P) = \delta\Pi^{\mu\nu}(P) + *\Pi^{\mu\nu}(P) ,$$

where $\delta\Pi$ is the hard thermal loop and $*\Pi$ is the effective self-energy. $P$ being soft, the hard thermal loop is of the same order of magnitude as the inverse free propagator, i.e., $\delta\Pi \sim (gT)^2$, while the effective self-energy is of an order of magnitude higher, i.e., $*\Pi \sim g(gT)^2$. Since the momentum running inside $*\Pi$ is soft, we have to use effective vertices and propagators instead of their bare* counterparts when calculating it. This ensures the correct expression for the $g(gT)^2$-correction to the inverse gluon propagator and, in particular, that this correction is independent of the gauge.

The hard thermal loop $\delta\Pi$ can already be found in the literature, see for example [2,3]. It is real and contributes to the determination of the spectrum of the soft gluonic excitations to leading order $gT$. More explicitly, we know that $\delta\Pi$ is gauge-invariant and satisfies the identity $P^\mu \delta\Pi^{\mu\nu}(P) = 0$. This means its components can be expressed in terms of only two independent scalar functions denoted by $\delta\Pi_l(P)$ and $\delta\Pi_t(P)$ such that:

$$\delta\Pi^{00}(P) = \delta\Pi_l(P) ; \quad \delta\Pi^{0i}(P) = -\frac{p_0 p^i}{p^2} \delta\Pi_l(P) ;$$

$$\delta\Pi^{ij}(P) = (\delta^{ij} - \hat{p}^i \hat{p}^j) \delta\Pi_l(P) + \hat{p}^i \hat{p}^j \frac{(p_0)^2}{p^2} \delta\Pi_t(P) .$$

The expressions of $\delta\Pi_l(P)$ and $\delta\Pi_t(P)$ read [3]:

$$\delta\Pi_l(P) = 3m_g^2 Q_1\left(\frac{i p_0}{p}\right) ; \quad \delta\Pi_t(P) = \frac{3}{5} m_g^2 \left[ Q_1\left(\frac{i p_0}{p}\right) - Q_1\left(\frac{i p_0}{p}\right) - \frac{5}{3}\right] ,$$

* ‘Bare’ refers here to the usual quantities one considers as dictated by the Feynman rules.
where the \( Q_n \) are Legendre functions of the second kind. As already mentioned, \( m_g \) is the gluon thermal mass and, to lowest order, is equal to \( \sqrt{N_c + (1/2) N_f} gT/3 \), where \( N_f \) is the number of flavors.

The effective propagator for soft gluons that intervenes in the calculation of the effective self-energy is obtained by inverting (2.1) while disregarding \( * \Pi \). In the strict Coulomb gauge, its nonzero components are \( * \Delta^{00}_C (P) = * \Delta_l (P) \) and \( * \Delta^ij_C (P) = (\delta^{ij} - \bar{P}^i \bar{P}^j) * \Delta_l (P) \), where \( * \Delta_l \) and \( * \Delta_l \) are given by:

\[
* \Delta_l (P) = \frac{1}{P^2 - \delta \Pi_t (P)} ; \quad * \Delta_l (P) = \frac{1}{P^2 - \delta \Pi_t (P)} .
\] (2.5)

After analytic continuation to real energies, the pole in \( \omega \) of \( * \Delta_{l(t)} \) yields the dispersion relation \( \omega_{l(t)} (p) \) for the transverse (longitudinal) gluons to order \( gT \). One finds for the soft transverse ones:

\[
\omega_l (p) = m_g \left[ 1 + \frac{3}{5} \left( \frac{p}{m_g} \right)^2 - \frac{9}{35} \left( \frac{p}{m_g} \right)^4 + \frac{704}{3000} \left( \frac{p}{m_g} \right)^6 - \frac{91617}{336875} \left( \frac{p}{m_g} \right)^8 + \ldots \right] .
\] (2.6)

As mentioned earlier, the hard thermal loop \( \delta \Pi \) is real above the light cone, and so the poles of (2.5) are real, which means the gluons are not damped to this order \( gT \); this is clear from (2.6). In order to get the leading order of the damping rates, we have to include in the dispersion equations the contribution from the effective self-energy. \( * \Pi \) has of course a more complicated structure than \( \delta \Pi \). It satisfies the less restrictive identity \( P^\rho * \Pi^{\rho \nu} (P) P^\nu = 0 \) [3]. This means that in general, \( * \Pi^{\rho \nu} (P) \) can be written in terms of three independent scalar functions, but in the strict Coulomb gauge, only two of these are relevant. The transverse dispersion relation including the self energy reads:

\[
- \Omega_l^2 + p^2 - \delta \Pi_t (-i \Omega_l, p) - * \Pi_t (-i \Omega_l, p) = 0 ,
\] (2.7)

where \( * \Pi_t \) is given by:

\[
* \Pi_t (P) = \frac{1}{2} (\delta^{ij} - \bar{P}^i \bar{P}^j) * \Pi^{ij} (P) .
\] (2.8)

The transverse gluon damping rate is defined by \( \gamma_l (p) \equiv - \text{Im} \ \Omega_l (p) \). Since it is \( g \)-times smaller than the energy \( \omega_l (p) \), we can write from (2.7):

\[
\gamma_l (p) = \frac{\text{Im} * \Pi_t (-i \omega, p)}{2 \omega + \frac{2}{5 \gamma} \delta \Pi_t (-i \omega, p) \left|_{\omega=\omega_l (p)+i0^+} \right.} .
\] (2.9)

The denominator in (2.9) is easy to get since we already have an expression for the hard thermal loop \( \delta \Pi_t \). Indeed, we have:

\[
2 \omega_l (p) + \partial_\omega \delta \Pi_t (-i \omega, p) \left|_{\omega=\omega_l (p)+i0^+} \right. = 2 \left[ 1 + \frac{1}{10} \left( \frac{p}{m_g} \right)^2 + \ldots \right] .
\] (2.10)

This means our main task is to calculate the imaginary part of \( * \Pi_t \). This we do in the sequel.

3. The imaginary part of the transverse effective self-energy

In the Coulomb gauge, the only diagrams that contribute to the imaginary part of the effective self-energy above the light cone are the three-gluon and four-gluon one loop-diagrams with soft internal momentum [3]. Hence we write:

\[
\text{Im} * \Pi^{\rho \nu} (P) = - \frac{g^2 N_c}{2} \text{Im} \ \text{Tr}_{\text{soft}} \left[ * \Gamma^{\rho \nu \lambda \sigma} (P, -P, K, -K) * \Delta^{\lambda \sigma} (K) \right. \\
\left. + * \Gamma^{\sigma \rho \lambda} (-Q, P, -K) * \Delta^{\lambda \nu} (K) * \Gamma^{\lambda \nu \sigma'} (-K, P, -Q) * \Delta^{\sigma'} (Q) \right] .
\] (3.1)
where $K$ is the internal loop-momentum, $Q = P - K$ and $\text{Tr} \equiv T \sum k_0 \int \frac{d^3k}{(2\pi)^3}$. The subscript ‘soft’ means that only soft values of $k$ are allowed in the integral. Eq (3.1) is what one would normally write for the three-gluon and four-gluon contributions to the imaginary part of the gluon self-energy, except that everywhere, bare quantities are replaced by the corresponding effective ones.

We have already given in the last section the expressions of the nonvanishing components of the effective propagators (see eq (2.5) and the text before it). The gluon effective vertices can be written as:

\[ \ast \Gamma^{(n)} = \Gamma^{(n)} + \delta \Gamma^{(n)} ; \quad n = 3, 4, \]

where the first term is the QCD gluon tree vertex and the second one sums up the contributions from hard thermal loops with $n$ external legs. In the case $n = 3$ it can be written as:

\[ \delta \Gamma^{\mu \nu \lambda}(-Q, P, -K) = -\delta \Gamma^{\mu \nu \lambda}(-K, P, -Q) = 3m_0^2 \int \frac{dQ_S}{4\pi} \frac{S^{\mu}S^{\nu}S^{\lambda}}{PS} \left( \frac{i\phi_0}{QS} - \frac{iK_0}{KS} \right), \]

where $S \equiv (i, \hat{s})$ and $\Omega_S$ is the solid angle of the unit vector $\hat{s}$. Also, $PS = i\phi_0 + p \cdot \hat{s}$, etc. In the case $n = 4$ we have:

\[ \delta \Gamma^{\mu \nu \lambda \sigma}(P, P, K, -K) = 3m_0^2 \int \frac{dQ_S}{4\pi} \frac{S^{\mu}S^{\nu}S^{\lambda}S^{\sigma}}{PS KS} \left( \frac{i\phi_0}{PS - KS} - \frac{i\phi_0 + iK_0}{PS + KS} \right). \]

To be complete, we give the expression of the three-gluon tree vertex:

\[ \Gamma^{\mu \nu \lambda}(-Q, P, -K) = -\Gamma^{\lambda \nu \mu}(-K, P, -Q) = (P + K)^\mu \delta^{\nu \lambda} + (Q - K)^\nu \delta^{\lambda \mu} - (P + Q)^\lambda \delta^{\mu \nu}, \]

and that of the four-gluon tree vertex:

\[ \Gamma^{\mu \nu \lambda \sigma}(P, P, K, -K) = 2 \delta^{\mu \nu} \delta^{\lambda \sigma} - \delta^{\mu \lambda} \delta^{\nu \sigma} - \delta^{\mu \sigma} \delta^{\nu \lambda}. \]

From eqs (2.8) and (3.1) above, we can write more explicitly the expression of the transverse effective self-energy:

\[ \text{Im} \ast \Pi_t(P) = -\frac{g^2 N_c}{4} \delta^{ij} - \hat{p}^i \hat{p}^j) \text{Im} T \sum_{k_0} \int \frac{d^3k}{(2\pi)^3} \left[ \ast \Gamma^{ij00}(P, P, -K, -K) \ast \Delta_t(K) \right. \]
\[ + \ast \Gamma^{ijmn}(P, P, -K, -K) (\delta^{mn} - \hat{k}^m \hat{k}^n) \ast \Delta_t(K) \]
\[ + \ast \Gamma^{0i0j}(Q, P, -K) \ast \Delta_t(K) \ast \Gamma^{ij00}(K, P, -Q) \ast \Delta_t(Q) \]
\[ + \ast \Gamma^{0i0m}(Q, P, -K) (\delta^{mn} - \hat{k}^m \hat{k}^n) \ast \Delta_t(K) \ast \Gamma^{0j00}(K, P, -Q) \ast \Delta_t(Q) \]
\[ + \ast \Gamma^{0i0m}(Q, P, -K) (\delta^{mn} - \hat{k}^m \hat{k}^n) \ast \Delta_t(K) \ast \Gamma^{0j00}(K, P, -Q) \ast \Delta_t(Q) \]
\[ + \ast \Gamma^{0i0r}(Q, P, -K) (\delta^{rs} - \hat{k}^r \hat{k}^s) \ast \Delta_t(K) \ast \Gamma^{0s0j}(P, -Q) (\delta^{mn} - \hat{q}^n \hat{q}^m) \ast \Delta_t(Q) \] .

There are six contributions: two from the four-gluon vertices (subscript 4g in the sequel) and four from the three-gluon vertices (subscript 3g). Each one has to be calculated separately. As an illustration, we show how we carry the calculation corresponding to the contribution from the three-gluon vertices where the two
effective propagators involved are both longitudinal. We denote this contribution by \( \text{Im} \, \Pi_{\text{3gl}}(P) \) and the others correspondingly. The other contributions are manipulated in a similar way, with varying difficulties that we comment on later in this section. Also, we will take henceforth \( m_g = 1 \) and all momenta and energies are in units of it. This simplifies considerably the expressions we write down and, if and when needed, the \( m_g \)-dependence can be recovered in the final results.

Using the expressions of the gluon vertices we gave in eqs (3.2)-(3.6), we write:

\[
\text{Im} \, \Pi_{\text{3gl}}(P) = \frac{g^2 N_c}{8\pi^2} \text{Im} T \sum_{k_0} \int \frac{d^3 k}{4\pi} \left[ (p - 2k)^2 - (p - 2k) \hat{p}^2 \right] 
+ 6 \int d\Omega_S \frac{(p - 2k)s - (p - 2k)\hat{p} \hat{s} \hat{p}}{PS} \left( i\frac{k_0}{KS} - i\frac{q_0}{QS} \right) 
+ 9 \int d\Omega_{S_1} d\Omega_{S_2} \frac{s_1 \hat{s}_2 - s_1 \hat{p} \hat{s}_2 \hat{p}}{PS_1 PS_2} \left( i\frac{k_0}{KS_1} - i\frac{q_0}{QS_1} \right) \left( i\frac{k_0}{KS_2} - i\frac{q_0}{QS_2} \right) \right] \, \Delta_l(K) \, \Delta_l(Q) .
\]

(3.8)

We first work on the term that does not involve a solid-angle integral. We use the relation \( (p - 2k)^2 - (p - 2k)\hat{p}^2 = 4k^2 \sin^2 \psi \), where \( \psi = (\hat{p}, \hat{k}) \), and integrate over the solid angle of \( h k \) after we expand the effective propagator at the momentum \( Q = P - K \) in the following manner:

\[
\Delta_l(Q) = \left[ 1 - p \cos \psi \partial_k + \frac{p^2}{2} \left( \frac{\sin^2 \psi}{k} \partial_k \cos \psi \partial_k^2 \right) + \ldots \right] \Delta_l(q_0, k) ,
\]

(3.9)

where \( \partial_k = \partial / \partial k \). We find:

\[
\text{Im} T \sum_{k_0} \int \frac{d^3 k}{4\pi} \left[ ((p - 2k)^2 - (p - 2k)\hat{p}^2) \Delta_l(K) \right] \, \Delta_l(Q)
= \frac{8}{3} \text{Im} T \sum_{k_0} \int_0^\infty k^4 dk \, \Delta_l(K) \left[ 1 + \frac{p^2}{5} \left( \frac{2}{k} \partial_k + \frac{1}{2} \partial_k^2 \right) + \ldots \right] \Delta_l(q_0, k) .
\]

(3.10)

The next step in to perform the sum over \( k_0 \), but we take care of that a little later.

We leave the expression in (3.10) as it is for the moment and turn to the term in (3.8) that involves one solid-angle integral. It is sufficient to concentrate only on the piece that contains the ratio \( i k_0 / K S \) because the other one that contains the ratio \( i q_0 / Q S \) is in fact equal to the first one. We need an expression for the solid-angle integral. For this purpose, it is best to measure the solid angle \( \Omega_g = (\theta, \phi) \) with respect to \( \hat{k} \) such that \( \theta = (\hat{k}, \hat{s}) \). Also, we expand \( 1 / PS \) in the following manner:

\[
\frac{1}{PS} = \frac{1}{i p_0} \left( 1 - \frac{p_\hat{s}}{i p_0} - \frac{p_\hat{s}^2}{p_0^2} + \ldots \right) ,
\]

(3.11)

an expansion valid as long as \( |p/\imath p_0| < 1 \), which is satisfied for soft gluons before and after analytic continuation. With this, we can write:

\[
\int d\Omega_S \frac{(p - 2k)s - (p - 2k)\hat{p} \hat{s} \hat{p}}{PS} \, \frac{ik_0}{KS}
= -2k \sin \psi \, \frac{ik_0}{ip_0} \int d\Omega_S \frac{\sin \psi \cos \theta + \cos \psi \sin \theta \sin \phi}{(ik_0 + k \cos \theta)} \left( 1 - \frac{p_\hat{s}}{ip_0} - \frac{p_\hat{s}^2}{p_0^2} + \ldots \right) .
\]

(3.12)
With the relation $\mathbf{p}\mathbf{s} = \cos \psi \cos \theta - \sin \psi \sin \theta \sin \phi$, the angular integrals are performed straightforwardly and we end up with:

$$
\int \frac{d\Omega_S}{4\pi} \frac{(\mathbf{p} - 2\mathbf{k})\mathbf{s} - (\mathbf{p} - 2\mathbf{k})\hat{\mathbf{p}} \hat{\mathbf{s}}}{PS} \frac{ik_0}{KS} = -\frac{ik_0}{ip_0} (1 - x^2) \left[ 2 \left( 1 - \frac{ik_0}{k} Q_{ok} \right) + \frac{p}{ip_0} x \left( 3\frac{ik_0}{k} \right. \right. \\
+ \left( 1 + 3\frac{k_0^2}{k^2} \right) Q_{ok} \right) - \frac{p^2}{p_0^2} \left( \frac{2}{3} (1 - 2x^2) + (1 - 5x^2) \frac{k_0^2}{k^2} \right) + \left. \frac{ik_0}{k} \left( 1 - 3x^2 + (1 - 5x^2) \frac{k_0^2}{k^2} \right) Q_{ok} \right] + \ldots \right],
$$

where $x = \cos \psi$ and $Q_{ok} = Q_0 \left( \frac{d\phi}{k} \right)$. We plug this expression back into the integral over $d^3k$ and use (3.9) to perform the integral over the solid angle of $\hat{\mathbf{k}}$. That is quite straightforward and we get:

$$6 \text{Im} T \sum \frac{d^3k}{4\pi} \int \frac{d\Omega_S}{4\pi} \frac{(\mathbf{p} - 2\mathbf{k})\mathbf{s} - (\mathbf{p} - 2\mathbf{k})\hat{\mathbf{p}} \hat{\mathbf{s}}}{PS} \frac{ik_0}{KS} - \frac{iq_0}{QS} \right) \Delta_i(K) \Delta_i(Q)
$$

$$= -16 \text{Im} T \sum \frac{d^3k}{4\pi} \int \frac{d\Omega_S}{4\pi} \frac{k^2}{k} \Delta_i(K) \left\{ \frac{ik_0}{ip_0} \left[ 1 - \frac{ik_0}{k} Q_{ok} \right] + \frac{p^2}{5p_0^2} \left[ -1 + \frac{ik_0}{k} Q_{ok} \right] \right. \\
+ \left. \frac{ip_0}{2} \left( 3\frac{ik_0}{k} + 1 + 3\frac{k_0^2}{k^2} \right) Q_{ok} \right\} \Delta_i(q_0, k).
$$

Here also we leave this expression as it is for the moment and turn our attention to the term in (3.8) involving two solid-angle integrals. This term is actually equal to

$$-18 \text{Im} T \sum \frac{d^3k}{4\pi} \Delta_i(K) \Delta_i(Q) \int \frac{d\Omega_{s1}}{4\pi} \int \frac{d\Omega_{s2}}{4\pi} \frac{\hat{s}_1\hat{s}_2 - \hat{s}_1\hat{p}\hat{s}_2\hat{p}}{PS_S PS_S} \left( \frac{k_0^2}{KS_S KS_S} - \frac{k_0 q_0}{KS_S QS_S} \right)
$$

and we first work out the piece that contains $k_0^2/KS_S KS_S$. In order to be able to carry forward, it is best to write the double solid-angle integral in the following manner:

$$\int \frac{d\Omega_{s1}}{4\pi} \int \frac{d\Omega_{s2}}{4\pi} \frac{\hat{s}_1\hat{s}_2 - \hat{s}_1\hat{p}\hat{s}_2\hat{p}}{PS_S PS_S} \frac{k_0^2}{KS_S KS_S} = k_0^2 \left[ \sin^2 \psi \left( \int \frac{d\Omega_S}{4\pi} \frac{\cos \theta}{PS KS} \right)^2 \right. \left. + \cos^2 \psi \left( \int \frac{d\Omega_S}{4\pi} \frac{\sin \theta \sin \phi}{PS KS} \right)^2 \right.
$$

$$+ 2 \cos \psi \sin \psi \left( \int \frac{d\Omega_S}{4\pi} \frac{\cos \theta}{PS KS} \int \frac{d\Omega_S}{4\pi} \frac{\sin \theta \sin \phi}{PS KS} \right).
$$

Each single solid-angle integral that is involved in the above expression can be worked out straightforwardly as before, using the expansion (3.11). Putting things together, we find:

$$\int \frac{d\Omega_{s1}}{4\pi} \int \frac{d\Omega_{s2}}{4\pi} \frac{\hat{s}_1\hat{s}_2 - \hat{s}_1\hat{p}\hat{s}_2\hat{p}}{PS_S PS_S} \frac{k_0^2}{KS_S KS_S} = -k_0^2 (1 - x^2) \left[ \left( 1 - \frac{ik_0}{k} Q_{ok} \right) \right.
$$

$$+ \frac{px}{ip_0} \left( 1 - \frac{ik_0}{k} Q_{ok} \right) \left( 3\frac{ik_0}{k} + (1 + 3\frac{k_0^2}{k^2} \right) Q_{ok} \right) - \frac{p^2}{p_0^2} \left( \frac{2}{3} (1 - 4x^2) + (1 - 7x^2) \frac{k_0^2}{k^2} \right) Q_{ok} \right] + \ldots \right].
$$
We put this expression back under $\int d^3k$ and perform the integral over the solid angle of $\hat{k}$. We get:

$$\text{Im} T \sum_{k_0} \frac{d^3k}{4\pi} * \Delta_1(K) * \Delta_l(Q) \int \frac{d\Omega_{S_1}}{4\pi} \int \frac{d\Omega_{S_2}}{4\pi} \frac{\hat{s}_1 \hat{s}_2 - \hat{s}_1 \hat{p} \hat{s}_2 \hat{p}}{PS_1 PS_2} \frac{k_0^2}{K S_1 K S_2}$$

$$= \frac{2}{3} \text{Im} T \sum_{k_0} \frac{k_0^2}{p_0^6} \int_0^\infty dk^* \Delta_1(K) \left[ \left(1 - \frac{i k_0 Q_{0k}}{k} \right)^2 - \frac{p_r^2}{5 p_0^2} \left[ \frac{1}{4} \left( \frac{i k_0 Q_{0k}}{k} + \left(1 + \frac{k_0^2}{k^2} \right) Q_{0k} \right)^2 \right. \right.$$

$$\left. + \left(1 - \frac{i k_0 Q_{0k}}{k} \right) \left(2 - \frac{2 k_0^2}{k^2} - \frac{k_0^2}{k^2} Q_{0k} \right) - i p_0 \left(1 - \frac{i k_0 Q_{0k}}{k} \right) \right. \right.$$

$$\left. \times \left(3 \frac{i k_0 Q_{0k}}{k} + \left(1 + \frac{k_0^2}{k^2} \right) Q_{0k} \right) \partial_k - \frac{p_0^2}{2} \left(1 - \frac{i k_0 Q_{0k}}{k} \right)^2 \left( \frac{4}{k} \partial_k + \partial_k^2 \right) \right] + \ldots \right] * \Delta_l(q_0, k). \quad (3.17)$$

Next we turn to the piece that contains $k_0 q_0 / K S_1 Q S_2$. To get a manageable expression for the double solid-angle integral, it is most suitable to measure the solid angles with respect to $\hat{p}$. Then we have:

$$\int \frac{d\Omega_{S_1}}{4\pi} \int \frac{d\Omega_{S_2}}{4\pi} \frac{\hat{s}_1 \hat{s}_2 - \hat{s}_1 \hat{p} \hat{s}_2 \hat{p}}{PS_1 PS_2} \frac{k_0 q_0}{K S_1 Q S_2} = k_0 q_0 \int \frac{d\Omega_S}{4\pi} \sin \theta_0 \sin \phi_0 \int \frac{d\Omega_S}{4\pi} \sin \theta_1 \sin \phi_1,$$

where $(\theta_0, \phi_0)$ is the solid angle of $\hat{s}$ with respect to $\hat{p}$. Each solid-angle integral can be carried through separately. We get the following result:

$$\int \frac{d\Omega_S}{4\pi} \sin \theta_0 \sin \phi_0 \frac{\sin \psi}{i p_0 k} \left[ 1 - \frac{i k_0 Q_{0k}}{k} + \frac{p_r^2}{2 i p_0^2} \left(3 \frac{i k_0 Q_{0k}}{k} + \left(1 + \frac{k_0^2}{k^2} \right) Q_{0k} \right) \right.$$

$$\left. - \frac{p_r^2}{2 i p_0^2} \left[ \frac{2}{3} \left(1 - 2 x^2 \right) + \left(1 - 5 x^2 \right) \frac{k_0^2}{k^2} - \left(1 - 3 x^2 + \left(1 - 5 x^2 \right) \frac{k_0^2}{k^2} \right) i k_0 Q_{0k} \right] \right. \right.$$ \ldots $$, \quad (3.19)$$

and a similar one for the other integral in (3.18) where one replaces $K$ by $Q$ and the angle $(\hat{p}, \hat{k})$ by $(\hat{p}, \hat{q})$. We then multiply the two obtained expressions and put back the result under $T \sum_{k_0} \int d^3 k / 4\pi$. But here the integral over the solid angle of $\hat{k}$ is not straightforward yet because the integrand still depends on $q$ and the angle $(\hat{p}, \hat{q})$. Hence a further expansion is necessary, but instead of expanding $Q(q_0, k)$ directly, it is most suitable to write it in terms of $\Delta_1^{-1}(Q)$ using (2.4) and (2.5) and expand the resulting expression. The calculation carries thereon straightforwardly and we get:

$$18 \text{Im} T \sum_{k_0} \int \frac{d^3k}{4\pi} * \Delta_1(K) * \Delta_l(Q) \int \frac{d\Omega_{S_1}}{4\pi} \int \frac{d\Omega_{S_2}}{4\pi} \frac{\hat{s}_1 \hat{s}_2 - \hat{s}_1 \hat{p} \hat{s}_2 \hat{p}}{PS_1 PS_2} \frac{k_0 q_0}{K S_1 Q S_2} = \frac{4}{3} \text{Im} T \sum_{k_0} \int_0^\infty k^2 dk$$

$$\frac{\Delta_1(K)}{-p_0^2} \left[ i k_0 q_0 k^2 + \frac{p_r^2}{5 p_0^2} \left[-2 i k_0 q_0 k^2 + \frac{1}{4} \left(3 + k^2 + 3 k_0^2 \right) \left(3 + k^2 + 3 k_0^2 \right) \right) \right.$$

$$\left. - i q_0 k^2 \left(3 + k^2 + 3 k_0^2 \right) \partial_k + i k_0 q_0 k^2 \left(\frac{2}{k} \partial_k + \frac{1}{2} \partial_k^2 \right) \right] + \ldots \right] * \Delta_l(q_0, k). \quad (3.20)$$

It is time now we get into performing the sum over $k_0$. Because of the complicated $k_0$–dependence of the expressions we work with, it is best to perform the sum using the spectral representation of the different quantities involved [6]. For example, we have for the effective propagators:

$$* \Delta_{l1}(K) = \int_0^{1/T} d\tau e^{i k_0 \tau} \int_{-\infty}^{+\infty} d\omega \rho_{l1}(\omega, k) \left[1 + n(\omega)\right] e^{-\omega \tau}, \quad (3.21)$$
where \( n(\omega) = 1/(\exp(\omega/T) - 1) \) is the Bose-Einstein distribution and the spectral function \( \rho_{t,l} \) is given by:

\[
\rho_{t,l}(\omega, k) = \frac{Z_{t,l}(k)}{2\omega_{t,l}(k)} \left[ \delta(\omega - \omega_{t,l}(k)) \right. - \left. \delta(\omega + \omega_{t,l}(k)) \right] + \beta_{t,l}(\omega, k) \Theta(k^2 - \omega^2),
\]

expression in which the residue \( Z_{t,l}(k) \) is given by:

\[
Z_{t}(k) = \frac{\omega(\omega^2 - k^2)}{3\omega^2 - (\omega^2 - k^2)^2} \bigg|_{\omega = \omega_t(k)}; \quad Z_{l}(k) = -\frac{1}{k^2} \frac{\omega(\omega^2 - k^2)}{(3 - \omega^2 + k^2)^2} \bigg|_{\omega = \omega_t(k)},
\]

and the cut \( \beta_{t,l} \) reads:

\[
\beta_{t}(\omega, k) = \frac{3\omega(k^2 - \omega^2)}{4k^3 \left[ (k^2 - \omega^2 + \frac{3\omega^2}{2k^2} \ln \frac{k^2 + \omega}{k^2 - \omega}) + (\frac{3\pi^2}{2k}(k^2 - \omega^2))^2 \right]};
\]

\[
\beta_{l}(\omega, k) = -\frac{3\omega}{2k \left[ (3 + k^2 - \frac{3\omega^2}{2k} \ln \frac{k^2 + \omega}{k^2 - \omega}) + (\frac{3\pi^2}{2k})^2 \right]}.
\]

After we replace the effective propagators and other similar quantities by their spectral representations, we perform the integrals over the imaginary times. One such integration yields a delta-function and the others energy denominators. Actually, we arrange our expressions in such a way that we have always only two imaginary-time integrals, which ensures that we get only one energy denominator. When this is performed, we analytically continue \( ip_0 \) to the on-shell real energy \( \omega_t(p) + i0^+ \). The imaginary part of the different contributions is obtained using the well known relation \( 1/(x + i0^+) = \text{Pr}1/x - i\pi \delta(x) \). We apply this technique to the expressions in (3.10), (3.14), (3.17) and (3.20) and sum up the results to obtain:

\[
\text{Im}^*\Pi_{gres}(P) = \frac{g^2 N_c T}{24\pi} \int_0^\infty dk \int_{-\infty}^{+\infty} d\omega_1 \int_{-\infty}^{+\infty} d\omega_2 \left[ \begin{array}{c}
\frac{18k^4}{\omega_1 \omega_2} \rho_{t1} \rho_{t2} + \frac{6\omega_1^2}{k\omega_2} \Theta_{t1} \rho_{t2} + \frac{p^2}{3} \frac{3k^2}{2\omega_1 \omega_2} \\
\times (3 + 44k^2 - 12\omega_1 \omega_2) \rho_{t1} \rho_{t2} + \frac{3k^2}{2\omega_2} \left( 1 - \frac{\omega_1^2}{k^2} \right) \left( 1 - 9\frac{\omega_1^2}{k^2} \right) \Theta_{t1} \rho_{t2} + \frac{4k^3}{\omega_1 \omega_2} (13 + 3k^2) \\
+ 10\omega_1 - 9\omega_1^2 \end{array} \right] \rho_{t1} \partial_k \rho_{t2} + \frac{6\omega_1}{\omega_2} \left( 1 + 2\frac{\omega_1}{k^2} - 3\frac{\omega_1^2}{k^2} \right) \Theta_{t1} \partial_k \rho_{t2} + \frac{2k^4}{\omega_1 \omega_2} (2 + 5\omega_1) \rho_{t1} \partial_k^2 \rho_{t2} \\
+ \frac{3\omega_1^2}{\omega_2 k} \Theta_{t1} \partial_k^2 \rho_{t2} - \frac{54k^4}{\omega_1 \omega_2} \rho_{t1} \rho_{t2} \partial_{\omega_1} - \frac{18k^2}{\omega_2 k} \Theta_{t1} \rho_{t2} \partial_{\omega_2} + \ldots \right] \delta(1 - \omega_1 - \omega_2),
\]

where \( \rho_{ti} \) denotes \( \rho_i(\omega_i, k) \) with \( i = 1, 2 \), and \( \Theta_{t1} = \Theta(k^2 - \omega^2) \). In (3.25), we have used \( n(\omega_i) \approx T/\omega_i \) since only soft values of \( \omega_i \) are to contribute to the integrals. Also, we note that many terms drop out in the end because they do not hold an imaginary part. The remaining integrals in (3.25) are to be carried through numerically, more on this in the last section.

As we said earlier, the other contributions to \( \text{Im}^*\Pi_i(P) \) in (3.7) are manipulated along the same lines. Concerning the \( 3g \)-contributions, the intermediary steps are much longer, and at times quite intricate. For example, one thing that doesn’t look obvious from the outset is that, when performing the sum over \( k_0 \), it is necessary to group the different terms in a certain manner such that expressions containing \( k_0 \) in a way that prevents us from going to the spectral representation do cancel out and the technique can then be used. Another grouping of terms is also necessary in order to ensure that we end up at each time with only one
energy denominator. Such subtleties do not occur in the $ll$ contribution we worked out above. It would be too long and quite cumbersome to report on the intermediary steps of these contributions, and so we content ourselves with giving the final results, similar to (3.25). For the $tt$ and $tl$ contributions, we find:

\[
\text{Im}^* \Pi_{3gltt}(P) = \text{Im}^* \Pi_{3gltl}(P) = \frac{g^2 N_c T}{24 \pi} \int_0^\infty dk \int_{-\infty}^{+\infty} \frac{d\omega_1}{\omega_1} \int_{-\infty}^{+\infty} \frac{d\omega_2}{\omega_2} \left[ -18 k^2 (k^2 - \omega_1^2)^2 \rho_{l1} \rho_{l2} \\
- \frac{3 \omega_1}{k^3} (k^2 - \omega_1^2)^2 \Theta_{l1} \rho_{l2} + \frac{6 \omega_1}{k} (k^2 - \omega_1^2)^2 \Theta_{l1} \rho_{l2} + \frac{p^2}{5} \left[ \frac{36}{k^2} - 20 - 12k^2 + 93k^4 + 50k^6 \\
+ \left( -\frac{108}{k^2} + 8 + 12k^2 + 78k^4 \right) \omega_1 + \left( \frac{72}{k^2} - 80 - 2k^2 - 186k^4 \right) \omega_1^2 + \left( \frac{72}{k^2} + 36 - 176k^2 \right) \omega_1^3 \\
+ \left( -\frac{84}{k^2} - 179 + 222k^2 \right) \omega_1^4 + \left( \frac{60}{k^2} + 98 \right) \omega_1^5 + \left( \frac{48}{k^2} - 86 \right) \omega_1^6 \right] \rho_{l1} \rho_{l2} + \left[ \left( \frac{3}{2k} + 3k \right) \omega_1 \\
- \frac{3 \omega_1^3}{k} - \left( \frac{15}{k} \right) \omega_1^3 - \frac{9}{k} \omega_1^4 - \left( \frac{57}{2k^3} - \frac{21}{k^3} \right) \omega_1^5 + \frac{12}{k^3} \omega_1^6 - \frac{9}{k^3} \omega_1^7 \right] \Theta_{l1} \rho_{l2} + \left[ -30k \omega_1 \\
+ \frac{72}{k} \omega_1^3 - \frac{42}{k^3} \omega_1^5 \right] \Theta_{l1} \rho_{l2} + \left[ -12k - 72k^5 + (-24k - 12k^3 + 3k^5 + (-54k + 8k^3 + 6k^5) \omega_1 \\
+ (2k + 2k^3) \omega_1^3 + (8k - 12k^3) \omega_1^4 - 11k \omega_1^5 + 6k \omega_1^6 \right] \rho_{l1} \rho_{l2} + \left[ -\frac{9}{2} \omega_1 + \omega_1^2 + \frac{3}{2k} \omega_1^3 \\
- \frac{18}{k^2} \omega_1^4 + \frac{3}{2k} \omega_1^5 + \frac{9}{k^3} \omega_1^6 \right] \Theta_{l1} \rho_{l2} + \left[ 9 \omega_1 - 18 \omega_1^2 - \frac{9}{k^2} \omega_1^3 + \frac{18}{k^2} \omega_1^4 \right] \Theta_{l1} \rho_{l2} \\
+ \left[ -8k^2 - 16k^6 - 16k^2 \omega_1 + (-8k^2 + 32k^4) \omega_1^2 - 16k^2 \omega_1^3 \right] \rho_{l1} \rho_{l2}^2 + \left[ 30k^2 + 4k^4 - 2k^6 \right] \\
+ (-24k^2 - 8k^4) \omega_1 + (-4k^2 + 4k^4) \omega_1^2 + 8k^2 \omega_1^3 - 2k^2 \omega_1^4 \right] \rho_{l1} \rho_{l2}^2 \\
- \frac{3 \omega_1}{k^3} (k^2 - \omega_1^2)^2 \Theta_{l1} \rho_{l2} + \frac{6 \omega_1}{k} (k^2 - \omega_1^2) \Theta_{l1} \rho_{l2}^2 + 54k^2 (k^2 - \omega_1^2)^2 \rho_{l1} \rho_{l2} \partial_{\omega_1} \\
+ \frac{9 \omega_1}{k^3} (k^2 - \omega_1^2)^2 \Theta_{l1} \rho_{l2} \partial_{\omega_1} - 12 \omega_1^2 \left( k^2 - \omega_1^2 \right) \Theta_{l1} \rho_{l2} \partial_{\omega_1} \right] + \cdots \right] \delta(1 - \omega_1 - \omega_2). \tag{3.26}
\]

The above expression is rather long, partly because these two contributions do not benefit from a possible symmetry between $\omega_1$ and $\omega_2$, which can lead to some useful simplifications. Indeed, though the algebra is by far the tedious for the $tt$ contribution, the use of this symmetry renders its final result relatively simpler. It reads:

\[
\text{Im}^* \Pi_{3gltt}(P) = \frac{g^2 N_c T}{24 \pi} \int_0^\infty dk \int_{-\infty}^{+\infty} \frac{d\omega_1}{\omega_1} \int_{-\infty}^{+\infty} \frac{d\omega_2}{\omega_2} \left[ 36(k^2 + \omega_1 \omega_2)^2 \rho_{l1} \rho_{l2} - \frac{6 \omega_1^3}{k^3} (k^2 - \omega_1^2)^2 \Theta_{l1} \rho_{l2} \\
+ \frac{p^2}{5} \left[ -18k^2 + 122k^4 + \frac{1}{5} (-728 + 854k^2) \omega_1 \omega_2 + \left( \frac{30}{7k^2} - \frac{706}{5} \right) \omega_1^2 \omega_2^2 - \frac{174}{k^2} \omega_1^3 \omega_2^3 \right] \rho_{l1} \rho_{l2} \\
+ \frac{3 \omega_1}{4k} \left( 1 - \frac{\omega_1^2}{k^2} \right) \left[ -5k^2 - 16 \omega_1 + 2 \omega_1^2 + \frac{21}{k^2} \omega_1^4 \right] \Theta_{l1} \rho_{l2} + \left[ 32k^3 + 20k^5 \\
+ (112k + 12k^3) \omega_1 + \left( \frac{48}{k} - 36k - 16k^3 \right) \omega_1^2 - \left( \frac{108}{k} + 232k \right) \omega_1^3 + \left( \frac{12}{k} + 156k \right) \omega_1^4 \\
+ \frac{108}{k} \omega_1^2 - \frac{60}{k^3} \omega_1^6 \right] \rho_{l1} \rho_{l2} - \frac{6 \omega_1}{k} \left( 1 - \frac{\omega_1^2}{k^2} \right) (k^2 + 3 \omega_1 - 3 \omega_2^2) \Theta_{l1} \partial_{\omega_2} \rho_{l2} \\
+ \left[ 8k^2 + 20k^4 (1 + k^2) \omega_1 + 12 (1 + k^2) \omega_1^2 - (12 + 32k^2) \omega_1^3 - 12 \omega_1^4 + 12 \omega_1^5 \right] \rho_{l1} \partial_{\omega_2} \rho_{l2} \\
- \frac{3 \omega_1}{k^3} (k^2 - \omega_1^2) \Theta_{l1} \partial_{\omega_2} \rho_{l2} - 108(k^2 + \omega_1 \omega_2)^2 \rho_{l1} \rho_{l2} \partial_{\omega_1} \\
+ \frac{18 \omega_1^3}{k^3} (k^2 - \omega_1^2) \Theta_{l1} \rho_{l2} \partial_{\omega_1} \right] + \cdots \right] \delta(1 - \omega_1 - \omega_2). \tag{3.27}
\]
The two 4g-contributions are easier to work out and can be cast in different ways depending on how we perform the expansion. One compact way is to write:

$$\text{Im}^*\Pi_{\text{full}}(P) + \text{Im}^*\Pi_{\text{full}}(P) = \frac{g^2 N_c T}{24\pi} \int_0^\infty dk \int_{-\infty}^{+\infty} d\omega_1 \int_{-\infty}^{+\infty} d\omega_2 \left[ -\frac{g^2}{\omega_2} \Theta_1 \rho_{t2} + \frac{6(k^2 - \omega_1^2)}{k\omega_2} \Theta_1 \rho_{t2} ight]$$

$$+ \frac{p^2}{5} \left[ -\frac{24k^3}{\omega_1\omega_2} \delta_1 \rho_{t2} + 12\frac{\omega_1}{\omega_2} \epsilon(\omega_1) \delta_1 \rho_{t2} + \frac{6(4\omega_1 - 1)}{k\omega_2} \Theta_1 \rho_{t2} ight]$$

$$+ \frac{6k\omega_1}{\omega_2^3} \epsilon(\omega_1) \partial_{\omega_1} [\delta(k - \omega_1) - \delta(k + \omega_1)] \rho_{t2} + \frac{18k}{\omega_2} \Theta_1 \rho_{t2} \partial_{\omega_1}$$

$$- \frac{18(k^2 - \omega_1^2)}{k\omega_2} \Theta_1 \rho_{t2} \partial_{\omega_1} + \ldots \right] \delta(1 - \omega_1 - \omega_2),$$

(3.28)

where $\delta_1$ denotes $\delta(k^2 - \omega_1^2)$ and $\epsilon(\omega_1)$ is the sign function.

4. Discussion

Recall that our aim in this work is to calculate to order $p^2$ the damping rate for soft transverse gluons in QCD at high temperature $T$. We have mentioned in the introduction that our main motivation is to suggest that the analytic behavior of this damping rate, and perhaps similar quantities, may change around some scale $\mu$, see the first section.

The damping rate $\gamma_t(p)$ is given by equation (2.9). The denominator is given to order $p^2$ in (2.10). The numerator $\text{Im}^*\Pi_t(P)$ is the sum of the expressions given in (3.25)-(3.28). As we said in the last section, its actual value is to be determined numerically, an issue we comment on a little later. But at least from an analytic point of view, our calculation shows that the expansion in powers of $p^2$ of the damping rate is quite feasible: we recover the order zero in $p^2$ given in [2] and find an expression for the next one. Furthermore, our steps show that the calculation of the higher orders proceeds in a straightforward, though practically a lot more tedious, manner.

Supposing for a moment that the numerics go smoothly, it is interesting to ask about the nature of this scale $\mu$. The first suggestion that comes to mind is that $\mu$ is related to magnetic effects which are believed to manifest themselves at the next natural order beyond $m_g$, i.e., $g^2 T$. If this is so, there are two interesting points to mention in this regard. First of all, it would be interesting to understand how the magnetic effects get into play with respect to other effects and how and why they make this damping rate and possibly other similar quantities change their analytic behavior. Also, in the case of this change being indicative of some sort of phase transition, it would be interesting to understand the nature of this transition and the ‘critical’ behavior of QCD around it.

However, it may be that the scale $\mu$ is due to some other effects. Though this maybe remote a possibility, it would certainly be very interesting to contemplate into the matter. One way to start looking into this is to notice that the coefficients $a_{ti}$ in the expansion (1.1) are all pure numbers. Thus, with more coefficients calculated and assuming that they are all finite, it is possible to determine approximately the radius of convergence of the series, using for example a Padé approximant technique, and hence determining $\mu$ with
respect to $m_g$. If by other independent means we have an expression for the running coupling constant $g$, we can compare the approximate value of $\mu/m_g$ to this latter and carry a discussion thereon.

But at this stage, all this is still speculative. Indeed, though, as we said, the analytic expression of the effective self-energy to order $p^2$ is quite clean, it is not apriori obvious it is so as far as its numerical value is concerned. It is true that the coefficient $a_{i0}$ we get from our calculation has a finite value, but this is not certain for $a_{i1}$. First of all, we have expressions involving products of derivatives of delta-functions we have to clarify precisely how to carry through with. But more importantly we think that we are not assured of an infrared safeness with regard to the integration over the internal soft momentum $k$. This issue needs careful and thorough investigation, which is beyond the scope of the present work.

From a physical standpoint, the fact that $a_{i0}$ is finite means that the quark-gluon plasma is stable against zero-momentum gluonic excitations. For practical purposes, it is sufficient to take $\gamma_i(p) \simeq g^2 N_c T a_{i0}$ for small enough gluonic momenta. This implicitly presupposes that for such small non-zero momenta, the plasma remains stable. But this means that $\gamma_i(p)$ ought to have a smooth analytic behavior for such small momenta, and hence the higher-than-zero coefficients in our expansion (1.1) should be expected to be finite. This is the basic motivation of our work. If on the contrary they happen to be not so, because of infrared divergences for instance, then one must look into the matter more closely and try to reconcile between what we expect and what we get.

After these issues are settled should come the discussion of the physical implications of these results. For example, the sign of $a_{i0}$ being positive, is the sign of $a_{i1}$ also positive? If not, then at what momentum instabilities start to appear? We think all these issues are worth pursuing.

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