Quark Correlations in Nucleons and Nuclei

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Abstract

A dynamical quark model of hadron and nucleus structure is proposed. In the frame of the model, called the Strongly Correlated Quark Model, quarks and nucleons inside nuclei are arranged in crystal-like structure.

How are nucleon properties modified inside nuclei and do quarks manifest themselves explicitly in ground state nuclei? These questions are the topics of this paper, where we show that the nuclear modification of nucleon properties is a result of correlations of quarks of bound nucleons.

The analysis has been performed in the frame of the so-called Strongly Correlated Quark Model (SCQM) [1]. The ingredients of the model are the following. A single quark of definite color embedded in the vacuum begins to polarize its surrounding resulting in the formation of quark and gluon condensate. At the same time it experiences the pressure of the vacuum because of zero point radiation field or vacuum fluctuations which act on the quark tending to destroy the ordering of the condensate. Suppose that we place the corresponding antiquark in the vicinity of the first one. Owing to their opposite signs color polarization fields of the quark and antiquark interfere destructively in the overlapped space regions eliminating each other maximally in the space around the middle–point between the quarks. This effect leads to a decrease in condensates density in that space region and over-balancing of the vacuum pressure acting on the quark and antiquark from outer space regions. As a result an attractive force between the quark and antiquark emerges and the quark and antiquark start to move towards each other. The density of the remaining condensate around the quark (antiquark) is identified with the hadronic matter distribution. At maximum displacement in the $\bar{q}q$— system corresponding to small overlapping of polarization
fields, hadronic matter distributions have maximum extent and densities. The closer they come to each other, the larger is the destructive interference effect and the smaller hadronic matter distributions around quarks and the larger their kinetic energies. In that way quark and antiquark start to oscillate around their middle–point. For such interacting $\bar{q}q$− pair located on the $X$ axis at a distance $2x$ from each other, the total Hamiltonian is

$$H = \frac{m_{\bar{q}}}{(1 - \beta^2)^{1/2}} + \frac{m_q}{(1 - \beta^2)^{1/2}} + V_{\bar{q}q}(2x),$$

(1)

where $m_{\bar{q}}, m_q$ are the current masses of the valence antiquark and quark, $\beta = \beta(x)$ is their velocity depending on displacement $x$, and $V_{\bar{q}q}$ is the quark–antiquark potential energy with separation $2x$. It can be rewritten as

$$H = \left[ \frac{m_{\bar{q}}}{(1 - \beta^2)^{1/2}} + U(x) \right] + \left[ \frac{m_q}{(1 - \beta^2)^{1/2}} + U(x) \right] = H_{\bar{q}} + H_q,$$

(2)

were $U(x) = \frac{1}{2}V_{\bar{q}q}(2x)$ is the potential energy of quark or antiquark. The quark (antiquark) with the surrounding cloud (condensate) of quark – antiquark pairs and gluons, or hadronic matter distribution, forms the constituent quark. It is natural to assume that the potential energy of the quark (antiquark), $U(x)$, corresponds to the mass $M_Q$ of the constituent quark:

$$2U(x) = C_1 \int_{-\infty}^{\infty} dz' \int_{-\infty}^{\infty} dy' \int_{-\infty}^{\infty} dx' \rho(x, r') \approx 2M_Q(x)$$

(3)

where $C_1$ is a dimensional constant and the hadronic matter density distribution, $\rho(x, r')$, is defined as

$$\rho(x, r') = C_2 |\varphi(x, r')| = C_2 |\varphi_Q(x' + x, y', z') - \varphi_{\bar{q}}(x' - x, y', z')|.$$ 

(4)

Here $C_2$ is a normalization constant, $\varphi_Q$ and $\varphi_{\bar{q}}$ are density profiles of the condensates around the quark and antiquark located at distance $2x$ from each other. We consider by convention the condensates around quark and antiquark having opposite color charges. They have properties similar to compressive stress and tensile stress (around defects) in solids. Generalization to three–quark system in baryons is performed according to $SU(3)_{\text{color}}$ symmetry: in general, pair of quarks have coupled representations

$$3 \otimes 3 = 6 \oplus \overline{3}$$

(5)
in $SU(3)_{\text{color}}$ and for quarks within the same baryon only the $\bar{3}$ (antisymmetric) representation occurs. Hence, an antiquark can be replaced by two correspondingly colored quarks to get a color singlet baryon and destructive interference takes place between color fields of three valence quarks (VQs). Putting aside the mass and charge differences of valence quarks we may say that inside the baryon three quarks oscillate along the bisectors of equilateral triangle. Therefore, keeping in mind that the quark and antiquark in mesons and the three quarks in baryons are strongly correlated, we can consider each of them separately as undergoing oscillatory motion under the potential (3) in $1+1$ dimension. Hereinafter we consider VQ oscillating along the $X-$ axis, with $Z-$ axis perpendicular to the plane of oscillation $XY$. Density profiles of condensates around VQs are taken in gaussian form. This choice is dictated by our semiclassical treatment of VQs motion. It has previously been shown [4] that the wave packet solutions of the time dependent Schrodinger equation for the harmonic oscillator move in exactly the same way as corresponding classical oscillators. These solutions are called "coherent states". This relationship justifies (partly) our semiclassical treatment of quantum objects.

We define the mass of constituent quark at maximum displacement as

$$M_Q(x_{\text{max}}) = \frac{1}{3} \left( \frac{m_\Delta + m_N}{2} \right) \approx 360 \text{ MeV},$$

where $m_\Delta$ and $m_N$ are masses delta–isobar and nucleon correspondingly. The parameters of the model, namely, maximum displacement, $x_{\text{max}}$, and parameters of the gaussian function, $\sigma_{x,y,z}$, for hadronic matter distribution around VQ are chosen to be

$$x_{\text{max}} = 0.64 \text{ fm}, \quad \sigma_{x,y} = 0.24 \text{ fm}, \quad \sigma_z = 0.12 \text{ fm}. \quad (6)$$

They are adjusted by comparison of calculated and experimental values of inelastic cross sections, $\sigma_{in}(s)$, and inelastic overlap function $G_{in}(s, b)$ for $pp$ and $\bar{p}p-$ collisions [2]. The current mass of the valence quark is taken to be 5 $MeV$. The behavior of potential (3) demonstrates the relationship between constituent and current quark states inside a hadron (Fig. 1). At maximum displacement quark is a nonrelativistic, constituent one (VQ surrounded by condensate), since the influence of polarization fields of other quarks becomes minimal and the VQ possesses the maximal potential energy corresponding
to the mass of the constituent quark. At the origin of oscillation, \( x = 0 \), the antiquark and quark in mesons and the three quarks in baryons, being close to each other, have maximum kinetic energy and correspondingly minimum potential energy and mass: they are relativistic, current quarks (bare VQs). This configuration corresponds to so-called "asymptotic freedom". In the intermediate region there is increasing (decreasing) of the constituent quark mass by dressing (undressing) of VQs due to decreasing (increasing) of the destructive interference effect. This mechanism meets the local gauge invariance principle. Indeed, destructive interference of color fields of the quark and antiquark in mesons and three quarks in baryons depending on their displacements can be treated as phase rotation of wave function of single VQ in color space \( \psi_c \) on angle \( \theta \) depending on displacement \( x \) of the VQ in coordinate space

\[
\psi_c(x) \rightarrow e^{ig\theta(x)}\psi_c(x).
\] (7)

Phase rotation, in turn, leads to VQ dressing (undressing) by the quark and gluon condensate that corresponds to the transformation of the gauge field

\[
A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu \theta(x).
\] (8)

Here we drop the color indices of \( A_\mu(x) \) and consider each quark of specific color separately as changing its effective color charge, \( g\theta(x) \), in color fields of other quarks (antiquark) due to destructive interference. Thus gauge transformations (7, 8) map internal (isotopic) space of the colored quark onto coordinate space. On the other hand this dynamical picture of VQ dressing (undressing) corresponds to chiral symmetry breaking (restoration). Due to this mechanism of VQs oscillations, the nucleon runs over the states corresponding to the certain terms of the infinite series of Fock space

\[
| B \rangle = c_1 | q_1 q_2 q_3 \rangle + c_2 | q_1 q_2 q_3 \bar{q} \bar{q} \rangle + c_3 | q_1 q_2 q_3 g \rangle ...
\] (9)

The proposed model has some important consequences. Inside hadrons quarks and their accompanying gluons, as well, are strongly correlated. Nucleons are nonspherical objects: they are flattened along the axis perpendicular to the plane of quarks oscillations. First, because VQs undergo plane oscillations and second, owing to the flatness of hadronic matter distributions around VQs (according to (6)).

From the form of the quark potential (Fig.1) one can conclude that the dynamics of VQ corresponds to a nonlinear oscillator and VQ with its surrounding can be treated as a nonlinear wave packet. Moreover, our quark –
antiquark system turns out to be identical to the so-called ”breather” solution of the sine–Gordon (SG) equation[3]. SG equation in (1+1) dimension in reduced form for scalar function $\phi(x, t)$ is given by

$$\Box \phi(x, t) + \sin \phi(x, t) = 0,$$

where $x, t$ are dimensionless. The breather is a periodic solution representing bound state of soliton–antisoliton pair which oscillates around their center of mass:

$$\phi_{br}(x, t) = 4 \tan^{-1} \left[ \frac{\sinh \left( ut/\sqrt{(1-u^2)} \right)}{u \cosh \left( x/\sqrt{(1-u^2)} \right)} \right],$$

(11)

where $u$ is 4–velocity. During the oscillations of the soliton–antisoliton pair their density profile

$$\varphi_{s-as}(x, t) = \frac{d\phi_{br}(x, t)}{dx}$$

(12)
evolves like our quark–antiquark system, i.e. at maximal displacement the soliton and antisoliton are maximal and at minimum displacement they ”an-
nihilate”. This similarity is not surprising because our quark–antiquark system was formulated in close analogy with the model of dislocation–antidislocation [5], which in its continuous limit is described by the breather solution of the SG equation. It can be shown that the soliton, antisoliton and breather obey relativistic kinematics, i.e. their energies, momenta and shapes are transformed according to Lorentz transformations. Thus the dynamics of our strongly correlated quark system can be described by a sine–Gordon equation. Since the above consideration of quarks as solitons is purely classical, an important problem is to construct quantum states around them. Although the soliton solution of the SG equation looks like an extended (quantum) particle, the relation between classical solitons and quantum particles is not so trivial. Techniques for quantization of classical solitons with usage of various methods has been developed by many authors. The most well known of them is semiclassical method of quantization (WKB) which allows one to relate classical periodic orbits (breather solution of SG) with the quantum energy levels [6].

So far we have dealt with the scalar polarization field around VQ. How can one include spin in the frame of these classical considerations? According to the prevailing belief, the spin is a quantum feature of microparticles and has no classical analog. However, Belinfante [7] as early as in 1939 showed that the spin of an electron may be regarded as an angular momentum generated by a circulating flow of energy, or a momentum density, in the wave field of the electron. Furthermore, a comparison between calculations of angular momentum in the Dirac and electromagnetic fields, performed by Ohanian [8], shows that the spin of the electron is entirely analogous to the angular momentum carried by classical circularly polarized waves. Inclusion of spin in our scheme brings us to spinning quarks–solitons or extended vortex representation of constituent quarks. As shown in a previous paper [3] the dominating contribution to proton spin comes from the orbital angular momentum of gluons and sea q–pairs circulating around the oscillating VQs. Gauge field $A_\mu(x)$ in (8) contains along with the scalar part, $\varphi$, vector components, $A$, as well.

Now let us proceed to the many–nucleon problem. What would happen with oscillating quarks if we place a proton and a neutron nearby? Suppose that they aligned occasionally in such a way that a pair of quarks (one quark from the proton and another one from the neutron) with different flavor and color are nearest. Vacuum pressure acting from outside on these quarks

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decreases because of mutual influence (destructive interference, again) of the
different color fields of these quarks. This effect results in attractive force
between the nearest quarks from the proton and neutron. This force is near
half in magnitude for that between the quark and antiquark in mesons and the
three quarks in baryons at the same displacements. An additional attractive
force comes from the flavor difference. To restrict these attractive forces
quarks need to have parallel spins. As a result the potential (3) for adjacent
quarks acquires an additional minimum at large quark displacements, small
compared with the primary one. In that way the proton and nucleon form
a bound state, namely, a deuteron. Now quarks of both nucleons oscillate
around the deuteron center of mass interchanging their positions, with the
quarks at free ends possessing maximal displacements. When all quarks pass
the deuteron center of mass the adjacent quarks (at proton–neutron linkage)
acquire angular orbital momentum \(l = 2\) that should be accompanied with
spin flip of both quarks to conserve the total angular momentum of the
deuteron.

Noting that three quarks inside nucleons are totally antisymmetric in the
color space and two quarks from different nucleons at linkage are in anti-
symmetric color state \(\langle \bar{3} \rangle\) having different flavors and parallel spins, we can
construct more complex nuclei. The three nucleon system is formed by link-
age of two quarks of each nucleon with quarks of two other nucleons according
to the above rules. Three nucleon nuclei, namely \(^3H\) and \(^3He\), represent a
triangular configuration with three quarks at free ends. There is a drastic
difference of quark oscillation pattern starting from the three nucleon sys-
tem. The decrease of external vacuum pressure on quarks in each nucleon
results in decreased attraction force between them that leads to the displac-
ment of the origins of oscillations of each quark to the nucleon periphery
and to the amplitude reduction of quarks oscillations, i.e. each quark having
constituent mass oscillate near its own origin of oscillation. And so the sup-
pression of current (bare) quark configurations inside the nuclear medium
occurs. Completion of a four–nucleon system, \(^4He\), from a three–nucleon
one, occurs by binding free quark ends in \(^3H\) (\(^3He\)) with three quarks of an
additional proton (neutron) again in accordance with the above rules. Since
each quark in each of the four nucleons is coupled in a pair with a quark of
an adjacent nucleon, current quark configurations are totally suppressed and
only constituent quark configurations are realized in \(^4He\). Starting from \(^4He\)
al nuclei possess 3D-crystal–like structure. Indeed, planes of oscillations of
two protons and two neutrons are located on opposite faces of an octahedron with common vertex. In this geometrical configuration four nucleons are in an $s-$ state that corresponds to the first $s$-shell of the shell model. Next, the $p-$ shell represents octahedron of bigger size with two $^3He-$ triangles instead of protons and two $^3H-$ triangles instead of neutrons. The triangles are located parallel to empty faces of the $^4He-$ octahedron, the free quark ends of these triangles coupled as in the $^4He-$ octahedron. This octahedron with the nested $^4He-$ octahedron represents the nucleus of $^{16}O$. The next shell with principal number $n = 2$ is constructed in the same manner, extending triangles beforehand by adding a row of three protons to the row of two neutrons in $^3H$ and a row of three neutrons to the row of two protons in $^3He$. Again, these triangles are located in couples on opposite faces of an octahedron parallel to unoccupied faces of the nested $p-$ octahedron. Construction of the next shells is performed in the same manner by extending triangles with new rows of neutrons and protons. When building the shells for $n > 2$ one needs to take into account the predominance of neutron number over proton number. It can be shown that at fixed distances between nucleon centers of mass ($\sim 2$ fermi) the nucleons are arranged into a face-centered cubic lattice. It turns out that at nucleonic degrees of freedom our quark model of nuclear structure is identical to the lattice model formulated by N. Cook and V. Dallacasa more than twenty years ago [10] and called the FCC (face-centered–cubic)–lattice model. They demonstrated that it brought together shell, liquid-drop and cluster characteristics, as found in the conventional models, within a single theoretical framework. Unique among the lattice models, the FCC reproduces the entire sequence of allowed nucleon states as found in the shell model. Manifestation of a crystalline structure of nuclei has been observed in the diffraction pattern of scattering of $\alpha$-particles on nuclei [11].

The above picture of quark rearrangements in multinucleon systems resulting in considerable suppression of current quark configuration leads to essential modifications of the nucleon properties inside nuclei. The suppression of current quark configurations inside nuclei manifests in various observable effects: the old "EMC-effect" [12], color transparency breaking in large angle quasielastic $pp-$ scattering off nuclei, suppression of high transverse momenta, jets and $J/\psi$s in heavy ion collisions at high energies. According to our model there are holes (depression of hadronic matter density distribution) in the centers of three- and four-nucleon systems. This effect has
been observed experimentally [13]. And one more important corollary: all nuclei are non-spherically symmetric, even nuclei with magic numbers. In conclusion, the following consequences of the proposed approach should be emphasized:

- quarks and gluons inside hadrons and nuclei are strongly correlated;
- there are no strings stretching between quarks inside hadrons;
- strong interactions of quarks inside hadrons and nuclei are nonlocal;
- nuclei possess crystal-like structure.

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