Non-diffracting states in one-dimensional Floquet photonic topological insulators
Matthieu Bellec, Claire Michel, Haisu Zhang, Stelios Tzortzakis, Pierre Delplace

To cite this version:
Matthieu Bellec, Claire Michel, Haisu Zhang, Stelios Tzortzakis, Pierre Delplace. Non-diffracting states in one-dimensional Floquet photonic topological insulators. EPL - Europhysics Letters, 2017, 119 (1), pp.14003 - 14003. 10.1209/0295-5075/119/14003. hal-01613751

HAL Id: hal-01613751
https://hal.univ-cotedazur.fr/hal-01613751
Submitted on 10 Oct 2017

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Non-diffracting states in one-dimensional Floquet photonic topological insulators

Matthieu Bellec, Claire Michel, Haisu Zhang, Stelios Tzortzakis, and Pierre Delplace

1 Institut de Physique de Nice, Université Côte d'Azur, CNRS, 06100 Nice, France
2 Institute of Electronic Structure & Laser, FORTH, Heraklion, Greece
3 Materials Science and Technology Department, University of Crete, 71003, Heraklion, Greece
4 Science Program, Texas A&M University at Qatar, P.O. Box 23874 Doha, Qatar
5 Laboratoire de Physique, École Normale Supérieure de Lyon, Université de Lyon, Lyon, France

(Dated: March 3, 2017)

One dimensional laser-written modulated photonic lattices are known to be particularly suitable for diffraction management purposes. Here, we address the connection between discrete non-diffracting states and topological properties in such devices through the experimental observation and identification of three classes of non-diffracting state. The first one corresponds to topologically protected edge states, recently predicted in Floquet topological insulators, while the second and third are both bulk modes. One of them testifies of a topological transition, although presenting topological features different from those of the edge states, whether the other one result from specific band structure engineering.

Manipulating the flow of light with unusual diffraction features has been enabled, during the last decades, by considering optical transport in structured photonic media [1]. The analogy between solid-states physics and light propagation in specifically engineered arrayed structures allows the control of dispersion relations, which present in general a band structure, and thus of light transport properties. In propagating geometries, where the propagation axis plays the role of time, optical devices can be envisaged both at short scales, using integrated photonic waveguide arrays [2], and at large scales, with multicore optical fibres [3]. In those so-called photonic lattices, the discrete diffraction, as opposed to continuous diffraction in homogeneous media, may exhibit uncommon behaviors as observed in various experimental realizations [4–12]. Importantly, when diffraction cancels, the associated non-diffracting states are of great importance since they offer the possibility to route information to specific regions on the lattice [2, 13]. In particular, a periodic modulation of the guides along the propagation axis was shown to exhibit striking non-diffracting modes that propagate with a well defined drift angle [14, 15].

Interestingly, periodic modulations were also recently employed in the context of topological phases. While the concept of Floquet topological insulators was first developed for irradiated semimetals and semi-conductors [16–19], it found a spectacular experimental manifestation in out-of-equilibrium cold atom physics [20], single-photon quantum walks [21] and photonic lattices [22–24]. In the latter system, a periodic modulation of the waveguides array along the propagation axis acts as a periodic driving. Topologically protected edge states may emerge in these driven systems and are non-diffracting by nature in photonic lattices. This suggests a deep link between the existence of non-diffracting states, the longitudinal periodic modulation and the topological properties. For instance, do the modes reported in Ref. [14, 15] possess a topological property? If so, are there non-diffracting modes in periodically modulated waveguide arrays that are not topological?

In this paper, we answer these questions observing and manipulating experimentally three kinds of non-diffracting modes in periodically modulated one-dimensional (1D) arrays of optical waveguides. Within the framework of the Floquet theory, we identify different mechanisms at the origin of these remarkable states. Two of them are found to be related to a topological property: i) the edge states, which emerge at the interface between two topologically distinct Floquet gapped phases, and ii) the drift bulk states analogous to those observed in the optical beam rectification context [14, 15]. The third ones propagate straight in the bulk and result from a flat dispersion relation of the Floquet spectrum.

Periodically modulated arrays of evanescently coupled waveguides offer the possibility to investigate light propagation in structures that combine both discrete and continuous periodicities. Consider the propagation of a scalar discrete optical field of the form \(\exp[i(k_x x + k_z z)]\) through such a two-fold periodic array. The discrete periodic structure in the transverse direction yields a band structure for the wave vector \(k_z(k_x)\), where the transverse quasi-momentum \(k_z\) lives in a 1D Brillouin zone of length \(2\pi/a\). In addition, the periodic modulation of period \(Z\) of the guides along the propagation direction (\(z\) axis in Fig. 1(a)) ensures a \(2\pi/Z\) periodicity of \(k_z\) itself. It follows that \(k_z(k_x)\) displays a two-fold periodic band structure analogous to the quasi-energy spectrum of periodically driven quantum systems [25]. As presented in this paper, this striking property gives rise to various diffraction properties in the array.

The typical layout consists of a network of directional couplers as depicted in Fig. 1(a). We fabricate this structure in a 10 cm long fused silica sample (Suprasil 311, Heraeus) using the second harmonic output (515 nm) of a Yb:KGW regenerative laser system (Pharos, Light Con-
properties have been recently investigated [26–29]. In signal travels in both directions and whose topological like other two-dimensional oriented lattices in which the noting that the signal is driven from bottom to top, un-
rows) and scattering events (colored circles). It is worth
as an oriented network with propagating links (gray ar-
As shown in Fig. 1(b), our photonic lattice can be seen
centers created during the waveguide fabrication.
array is monitored by laterally visualizing the visible flu-
for the experiments, the whole light propagation in the
account the extra coupling from the waveguide bending.
agation at 633 nm. By measuring the output intensity
200 kHz repetition rate. The laser beam is tightly focused
version) delivering 150 nJ pulses with 190 fs duration at
200 μm inside the glass using a 20× microscope objective with
NA=0.40. The sample is moved with high-precision
translation stages (Aerotech ANT series) at 0.5 mm/s.
and light in the inset of Fig. 1(a)). Then the evolution
the scattering over a period Z of the light field wavefunc-
this configuration, the scattering at each node is captured
by a 2 × 2 unitary matrix whose coefficients describe how
light in an incoming waveguide is reflected and trans-
mis occurs at \( k_z Z = 0 \) [dashed line in (d)] and \( k_z Z = \pi \) [full line in (d)]
for at \( k_z = 0 \) [red in (d)] or \( k_z = \pi \) [black in (d)]. (b) Lin-
ear dispersion at bi-critical points [full and dashed diagonal
lines crossings in (d)]. (c) Flat bands dispersion appearing
at critical points [white line in (d)]. (d) Colors correspond
to \((\nu_x, \nu_z) = (0, 0)\) (white), \((0, 1)\) (light blue), \((1, 0)\) (medium
blue) and \((1, 1)\) (dark blue).

FIG. 1. (a) Sketch of the experimental realization of the 1D
periodically modulated photonic lattice composed of two dis-
tinct arrays characterized by different phase-couplings \( A_{1,2} \)
(red) and \( A'_{1,2} \) (blue). For each lattice, dark and light col-
dors define the two sublattices separated by \( a \) and \( m \) labels the
unit cells. \( A_{in} \) is the phase-coupling at the interface. \( Z \) is the
period modulation. Gray arrows show how an incoming beam
is transmitted \((t)\) and reflected \((r)\) by coupler defined by its
separation \( d \) and its length \( z_c \). (b) Corresponding oriented
network model where gray arrows are propagating links and
colored nodes are scattering matrices \( S_i \). (c) Exponential fit
of the measured coupling \( J \) at 633 nm versus waveguide sepa-
aration \( d \). Right axis scale corresponds to the phase-coupling
\( A = J z_c \).

As shown in Fig. 1(b), our photonic lattice can be seen
as an oriented network with propagating links (gray ar-
rows) and scattering events (colored circles). It is worth
noting that the signal is driven from bottom to top, un-
like other two-dimensional oriented lattices in which the
signal travels in both directions and whose topological
properties have been recently investigated [26–29]. In

\[
S_i = \begin{pmatrix} \cos A_i & -i \sin A_i \\ -i \sin A_i & \cos A_i \end{pmatrix}
\]  

(1)

Light propagation in the bulk is studied by considering
the scattering over a period \( Z \) of the light field wavefunc-
tion \( \psi(z) \). This is encoded by the evolution operator \( U_0 \)
defined as \( \psi(z + Z) = U_0 \psi(z) \). The bulk optical field \( \psi \)
reads in the Bloch basis as a two-component wavefunc-
tion resulting from the two guides of the unit cell (dark
and light in the inset of Fig. 1(a)). Then the evolution
operator \( U_0 = U_0(k_z) \) consists of a sequence of scattering
matrices as
\[ U_0(k_x) = B^\dagger(k_x)S_2B(k_x)S_1 \tag{2} \]
where \( B(k_x) \) is a unitary matrix taking into account the Bloch phase \( \exp(i k_x a) \) accumulated when waveguides from different unit cells couple. Eqs. (1, 2) are obtained by computing the evolution operator within a stepwise time dependent tight-binding model. After length \( Z \) has been spanned, each \( \psi \) component has accumulated the same phase \( \phi = k_x Z \) so that the light field satisfies the eigenvalue equation
\[ U_0(k_x) \psi(k_x) = e^{i\phi(k_x)} \psi(k_x). \tag{3} \]
A direct diagonalisation of \( U_0(k_x) \) shows that the two solutions \( \phi_\pm(k_x) = k_x^\pm(k_x) \) of the two bands which generically do not touch for any \( k_x \) in the 1D Brillouin zone (Fig. 2(a)).

Such gapped Floquet systems can display nontrivial topological properties that differ from those of topological insulators at equilibrium. Indeed, they can develop anomalous topologically protected boundary states while all the topological invariants defined for the bands vanish. This requires to define new topological indices that correctly account for the full periodic evolution [29–32]. In particular, depending on both the dimension of the system and its symmetries, a bulk topological index \( \nu_e \) can be assigned to each gap, labelled by \( \kappa \), rather than to a band [29–33]. This topological index is directly related to the existence (and number) of protected boundary states in the gap \( \kappa \) in finite geometry. As long as there exists a symmetry axis \( z \rightarrow -z \) of the lattice of Fig. 1(a) with respect to some origin, the operator \( U_0(k_x) \) holds a chiral symmetry. Following previous theoretical works [32, 34], this allows us to define a bulk topological index \( \nu_e \) for each of the two gaps \( \kappa = 0 \) and \( \kappa = \pi \) (see Fig. 2(a)). Four distinct topological phases, characterized by different values of the couple \( (\nu_0, \nu_e) \), are found when varying the phase-couplings \( A_1 \) and \( A_2 \) as represented in the phase diagram of Fig. 2(d). Note that it is similar to a previous study for a single-photon version of the problem, but where a different topological characterization was proposed [21].

We now consider a system with two different bulk properties which are separated by an interface. When the difference between the bulk topological indexes of each side \( \nu_e - \nu_e' \) does not vanish, interface states are expected to emerge in the gaps \( \kappa = 0 \) or \( \kappa = \pi \), meaning that they carry a quantized phase \( \phi_e = \kappa \) when propagating over a distance \( Z \). In particular, for \( \nu_0 - \nu_0' = \nu_e - \nu_e' = 1 \), both 0 and \( \pi \)-phase anomalous modes are expected. In order to practically investigate such states, we now consider two finite chains of waveguides. The two arrays are characterized by a set of two phase-couplings \( (A_1, A_2) \) and \( (A'_1, A'_2) \) (red and blue parts in Fig. 1 respectively), such that their topological invariants, \( (\nu_0, \nu_e) \) and \( (\nu_0', \nu_e') \), can be different. The last waveguide of the blue chain is coupled to the first waveguide of the red chain by a phase-coupling \( A_{\text{int}} \) which defines an interface along \( z \). We consider here two arrays characterized by the phase-couplings \( (A_1, A_2) = (\pi/2, \pi/4) \) and \( (A'_1, A'_2) = (\pi/2, 3\pi/4) \) [respectively black and red circles in Fig. 2(d)], corresponding to \( \nu_0 - \nu_0' = \nu_e - \nu_e' = 1 \). Green corresponds to bulk states, blue and red to localized states at the interfaces. For clarity, the degeneracy is lifted by adding a small potential on one of the interfaces. (b) Intensity of the corresponding \( \pi \)-phase modes for various \( A_{\text{int}} \).

The experimental setup (Fig. 4(a)) presents two boundaries in addition to the interface between the two arrays. To get rid of the additional localized states that may appear at these edges, we experimentally built large enough arrays and we numerically coupled their two extremities with the same \( A_{\text{int}} \). The calculation of the boundary modes shows that their existence is independent of the coupling \( A_{\text{int}} \) between the two arrays, which illustrates their topological robustness. However, as shown in Fig. 3(b), their intensity profile oscillate from one side of the interface to the other when tuning \( A_{\text{int}} \).

Figure 4 shows the corresponding experimental images of the light intensity propagation for a single waveguide excitation at the vicinity of the interface (a–d), delimited by the dashed line, and in the bulk (e,f). In Fig. 4(a) [resp. (b)], we clearly observe a boundary state in the red (resp. blue) array for \( A_{\text{int}} = \pi/6 \) (resp. \( \pi \)). Although not predominant, the local excitation of only one waveguide necessarily excites bulk modes, with a non-vanishing relative weight. The comparison of Fig. 4(a) [resp. (b)] with Fig. 4(e) [resp. (f)] shows that the bulk mode is indeed qualitatively visible. Note that 0 and \( \pi \)-modes are degenerated in intensity and only differ with their phase profiles. In the actual configuration, both modes are excited and additional (ongoing) experiments are required to discriminate them. On the contrary, as shown
FIG. 4. Experimental images of the light propagation for a single waveguide excitation in the dark (resp. light) sublattice [top (resp. bottom) panels]. The image size is 100 mm × 1.1 mm. The dashed line delimits the interface between red and blue arrays. (a–f). Lattice parameters: $(A_1, A_2) = (\pi/2, \pi/2)$ (respectively black and red circles in Fig. 2(d)], corresponding to $\nu_0 - \nu_0' = \nu_2 - \nu_2' = 1$, with $A_{\text{int}} = \pi/6$ (a,d), $\pi$ (b,c). (e,f). Bulk excitation. (g,h). Lattice parameters : $(A_1, A_2) = (A'_1, A'_2) = \pi/2$ [green circle in Fig. 2(d)] with $A_{\text{int}} = \pi/2$. (i,j). Lattice parameters : $(A_1, A_2) = (A'_1, A'_2) = (\pi/2, \pi)$ [blue circle in Fig. 2(d)] with $A_{\text{int}} = \pi$.

in Fig. 4(d) [resp. (c)], for $A_{\text{int}} = \pi/6$ (resp. $\pi$), when excited in the blue (resp. red) array, only bulk mode propagate. Here the comparison with the bulk modes excitation in Fig. 4(e-f) is much more obvious.

These results present an experimental observation of anomalous boundary states in a 1D Floquet photonic topological insulator. This is the first class of modes mentioned in the introduction which establishes a clear link between the existence of topologically protected edge states and diffractionless propagation.

Besides, the phase diagram in Fig. 2(d) shows bi-critical points (at full and dashed diagonal lines crossings) between gapped phases with distinct bulk topological invariants. At these points, given by $(A_1, A_2) = (\pi/2(2p + 1), \pi/2(2p' + 1))$, the two gaps close simultaneously leading to degeneracy points at $\phi = 0$ and $\phi = \pi$ (Fig. 2(b)). Importantly, these transition points are accompanied by an additional sublattice symmetry: the evolution operator $U_0$ becomes diagonal and thus commutes with $\sigma_z$, which is not true in general. It follows that the two bulk modes belong to opposite sublattices and remain uncoupled while carrying opposite group velocities in the transverse direction. As a result, the excitation of an arbitrary waveguide necessarily always coincides with an eigenmode of the system as shown experimentally in Fig. 4(g,h). This is a remarkable property of the bi-critical points, since it generates diffractionless bulk states with a transverse drift angle whose sign is reversed when changing the sublattice to which the excited guide belongs. Note that similar states have been found numerically [15] and observed experimentally [14] in modulated waveguides arrays. They were interpreted in terms of optical beam rectification. This interpretation is not inconsistent with our results but we go further by identifying them as a signature of a critical gapless Floquet phase.

This striking behavior is independent of the specific excited site, once the sublattice is fixed. Moreover, the excitation of a single waveguide corresponds to a global probe in quasi-momentum space. This suggests that these drift diffractionless states may reflect another topological property of the system. Clearly the existence of these states lies on the periodicity of the phase spectrum in $k_z Z$. They are thus specific to unitary systems. This
allows us to define the winding number
\[
w_{\pm} = \frac{1}{2\pi} \int_{-\pi/a}^{\pi/a} dk_x \langle \psi_\pm | U_0^\dagger (\partial_x U_0) | \psi_\pm \rangle
\]
of the Floquet bulk state $\psi_\pm$ which is non-zero at the gapless transition points. This winding number, that can be rewritten as a first Chern number, is proportional to the average displacement in the transverse direction, over a period $Z$ [35]. This leads to a quantized transversal displacement, that is clearly observed in Fig. 4(g,h). This is known as a topological pumping process, and usually arises in 1D equilibrium gapped phase adiabatically modulated in time (or space) [35–38]. However, here, the periodic modulation is specifically non-adiabatic, since the frequency driving period is smaller than the coupling length. Besides, in the absence of time-reversal symmetry breaking, the Chern number vanishes, which is consistent with the fact that the winding of the two Floquet bands compensates each other, i.e. $w_+ + w_- = 0$. However, as explained above, the two different branches of the spectrum can be excited separately so that light propagates without diffracting in one direction only. This artificially breaks time-reversal symmetry at the input of the array, similarly to what is performed e.g. in two-dimensional coupled resonators optical waveguides arrays to simulate an optical analog of a quantum Hall phase [39].

Finally, the third kind of diffractionless mode can be excited e.g. at the interface of two lattices that carry the same bulk topological index. However, they do not benefit from any topological protection inherited from chiral symmetry: their existence depends on the phase-coupling $A_{\alpha\beta}$ at the interface, unlike what is shown in Fig. 3(a). In particular, they are maximally localized at the edge and possess a Floquet phase of exactly $\kappa$ when the transmission coefficient with one of the two adjacent waveguides vanishes, that is for either $A_1 = p\pi$ or $A_2 = p\pi$, with $p$ an integer [white lines in Fig. 2(d)]. This is actually not specific to boundary modes, and can be engineered in the bulk. There, it actually corresponds to the case where the two Floquet bands are flat, as shown in Fig. 2(c). As a consequence, the degree of diffraction $\partial^2 k_x / \partial k_x^2$ vanishes for every $k_x$. This is remarkable as it involves a non-diffracting behavior of the light field for any excitation of the array, and not only for wave packets centered around specific $k_x$ [5]. In particular, diffraction vanishes for single-guide excitations, that are not eigenmodes of the system in contrast with the drift diffractionless bulk modes. This behavior is shown experimentally in Fig. 4(i,j) where straight diffractionless bulk modes are indeed observed for various positions of the excitation.

To summarize, we have observed three kinds of non-diffracting modes in a 1D array of evanescently coupled optical waveguides. First the edge states, that necessarily emerge at the interface between two arrays whose scattering matrices, ruling the evolution of the optical field along the propagation axis, carry distinct bulk topological indices. Second the drift bulk modes, that arise at the double transitions of the Floquet phase diagram by restoring a sublattice symmetry. Third the straight bulk modes, that result from a flat dispersion relation of the Floquet spectrum. It worth noticing that while the drift modes can be understood as the manifestation of a (transverse) topological pumping, the straight modes can instead be seen as the consequence of a dynamical (transverse) localization [12]. Furthermore, while they are of different origin, both the drift modes and the straight modes are insensitive of the way to excite the array. However, their existence requires some fine-tuning of the phase couplings $A_{\alpha\beta}$. In contrast, the edge states, which are more tricky to excite precisely, do not require any fine-tuning of the couplings, neither in the bulk nor at the interface.

P.D. thanks J. Li and M. Fruchart for fruitful discussions. M.B. and P.D. thank J.M. Jarre for his support. This work was supported by the French Agence Nationale de la Recherche (ANR) under grant TopoDyn (ANR-14-ACHN-0031). H.Z. acknowledges support from the EU FP7-REGPOT-2012-2013-1 (no 316165).

\* bellec@unice.fr
\† pierre.delplace@ens-lyon.fr

[1] J. D. Joannopoulos, S. G. Johnson, J. N. Winn, and R. D. Meade, Photonic Crystals: Molding the Flow of Light (Princeton University Press, 2008).
[2] D. N. Christodoulides, F. Lederer, and Y. Silberberg, Nature 424, 817 (2003).
[3] U. Röpke, H. Bartelt, S. Unger, K. Schuster, and J. Kobelke, App. Phys. B 104, 481 (2011).
[4] R. Morandotti, U. Peschel, J. S. Aitchison, H. S. Eisenberg, and Y. Silberberg, Phys. Rev. Lett. 83, 4756 (1999).
[5] H. S. Eisenberg, Y. Silberberg, R. Morandotti, and J. S. Aitchison, Phys. Rev. Lett. 85, 1863 (2000).
[6] T. Pertsch, T. Zentgraf, U. Peschel, A. Bräuer, and F. Lederer, Phys. Rev. Lett. 88, 093901 (2002).
[7] R. Iwanow, D. A. May-Arrioja, D. N. Christodoulides, G. I. Stegeman, Y. Min, and W. Sohler, Phys. Rev. Lett. 95, 053902 (2005).
[8] I. L. Garanovich, A. A. Sukhorukov, and Y. S. Kivshar, Phys. Rev. E 74, 066609 (2006).
[9] S. Longhi, M. Marangoni, M. Lobino, R. Ramponi, P. Laporta, E. Cianci, and V. Foglietti, Phys. Rev. Lett. 96, 243901 (2006).
[10] I. L. Garanovich, A. Szameit, A. A. Sukhorukov, T. Pertsch, W. Krolikowski, S. Nolte, D. Neshev, A. Tiunermann, and Y. S. Kivshar, Opt. Express 15, 9737 (2007).
[11] A. Szameit, I. L. Garanovich, M. Heinrich, A. Minovich, F. Dreisow, A. A. Sukhorukov, T. Pertsch, D. N. Neshev, S. Nolte, W. Krolikowski, A. Tiunermann, A. Mitchell,
and Y. S. Kivshar, Phys. Rev. A 78, 031801 (2008).
[12] I. L. Garanovich, S. Longhi, A. A. Sukhorukov, and Y. S. Kivshar, Phys. Rep. 518, 1 (2012).
[13] T. Pertsch, T. Zentgraf, U. Peschel, A. Bräuer, and F. Lederer, Appl. Phys. Lett. 80, 3247 (2002).
[14] F. Dreisow, Y. V. Kartashov, M. Heinrich, V. A. Vysloukh, A. Tünnemann, S. Nolte, L. Torner, S. Longhi, and A. Szameit, Eur. Phys. Lett. 101, 44002 (2013).
[15] Y. V. Kartashov, V. A. Vysloukh, V. V. Konotop, and L. Torner, Phys. Rev. A 93, 013841 (2016).
[16] T. Oka and H. Aoki, Phys. Rev. B 79, 081406 (2009).
[17] J.-i. Inoue and A. Tanaka, Phys. Rev. Lett. 105, 017401 (2010).
[18] T. Kitagawa, T. Oka, A. Brataas, L. Fu, and E. Demler, Phys. Rev. B 84, 235108 (2011).
[19] N. H. Lindner, G. Refael, and V. Galitski, Nature Phys. 7, 490 (2011), 1008.1792.
[20] G. Jotzu, M. Messer, R. Desbuquois, M. Lebrat, T. Uehlinger, D. Greif, and T. Esslinger, Nature 515, 237 (2014).
[21] T. Kitagawa, M. A. Broome, A. Fedrizzi, M. S. Rudner, E. Berg, I. Kassal, A. Aspuru-Guzik, E. Demler, and A. G. White, Nat. Commun. 3, 882 (2012).
[22] M. C. Rechtsman, J. M. Zeuner, Y. Plotnik, Y. Lumer, D. Podolsky, F. Dreisow, S. Nolte, M. Segev, and A. Szameit, Nature 496, 196 (2013).
[23] L. J. Maczewsky, J. M. Zeuner, S. Nolte, and A. Szameit, Nat. Commun. 8, 13756 (2017).
[24] S. Mukherjee, A. Spracklen, M. Valiente, E. Andersson, P. Öhberg, N. Goldman, and R. R. Thomson, Nat. Commun. 8, 13918 (2017).
[25] H. Sambe, Phys. Rev. A 7, 2203 (1973).
[26] M. Pasek and Y. D. Chong, Phys. Rev. B 89, 075113 (2014).
[27] W. Hu, J. C. Pillay, K. Wu, M. Pasek, P. P. Shum, and Y. D. Chong, Phys. Rev. X 5, 011012 (2015).
[28] C. Tauber and P. Delplace, New J. Phys. 17, 115008 (2015).
[29] P. Delplace, M. Fruchart, and C. Tauber, arXiv:1612.05769 (2016).
[30] M. S. Rudner, N. H. Lindner, E. Berg, and M. Levin, Phys. Rev. X 3, 031005 (2013).
[31] D. Carpentier, P. Delplace, M. Fruchart, and K. Gawędzki, Phys. Rev. Lett. 114, 106806 (2015).
[32] M. Fruchart, Phys. Rev. B 93, 115429 (2016).
[33] D. Carpentier, P. Delplace, M., K. Gawędzki, and C. Tauber, Nuclear Physics B 896, 779 (2015).
[34] J. K. Asbóth, B. Tarasinski, and P. Delplace, Phys. Rev. B 90, 125143 (2014).
[35] T. Kitagawa, E. Berg, M. Rudner, and E. Demler, Phys. Rev. B 82, 235114 (2010), 1010.6126.
[36] D. J. Thouless, Phys. Rev. B 27, 6083 (1983).
[37] Y. E. Kraus, Y. Lahini, Z. Ringel, M. Verbin, and O. Zilberberg, Phys. Rev. Lett. 109, 106402 (2012).
[38] M. Lohse, C. Schweizer, O. Zilberberg, M. Aidelsburger, and I. Bloch, Nature Phys. 12, 350 (2015).
[39] M. Hafezi, E. A. Demler, M. D. Lukin, and J. M. Taylor, Nature Phys. 7, 907 (2011).