Minimum accelerations from quantised inertia

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Abstract – It has recently been observed that there are no disc galaxies with masses less than $10^9 M_\odot$, and this cutoff has not been explained. It is shown here that this minimum mass can be predicted using a model that assumes that 1) inertia is due to Unruh radiation, and 2) this radiation is subject to a Hubble-scale Casimir effect. The model predicts that as the acceleration of an object decreases, its inertial mass eventually decreases even faster stabilising the acceleration at a minimum value, which is close to the observed cosmic acceleration. When applied to rotating disc galaxies the same model predicts that they have a minimum rotational acceleration, i.e.: a minimum apparent mass of $1.1 \times 10^9 M_\odot$, close to the observed minimum mass. The Hubble mass can also be predicted. It is suggested that assumption 1 above could be tested using a cyclotron to accelerate particles until the Unruh radiation they see is short enough to be supplemented by manmade radiation. The increase in inertia may be detectable.

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Introduction. – It was first noticed by [1] that galaxies were too energetic to be held together by their visible matter and he proposed the existence of an invisible (dark) baryonic matter that provides the extra required gravitational pull. Dark matter is now assumed to be non-baryonic because of the tight constraints on existing baryonic matter from cosmological nucleosynthesis, and it is still the most popular explanation [2] for the galaxy rotation problem [3,4]. However, after decades of searching, dark matter has not been directly detected, though attempts to do so are ongoing, for example: DAMA/LIBRA [5], CDMS-II [6], and XENON10 [7] and intercomparisons [8].

An alternative explanation for the galaxy rotation problem was proposed by [9] who suggested that either 1) the force of gravity may increase or 2) the inertial mass ($m_i$) may decrease in a new way for the very low accelerations at a galaxy’s edge. His empirical scheme (Modified Newtonian Dynamics, MoND) fits galaxy rotation curves, and has the advantage of being less tunable than dark matter. However, it does require a tunable parameter, the acceleration $a_0$, and a tunable interpolation function, and it does not correctly model galaxy clusters. MoND is also a controversial theory since, in its gravitational variant it disagrees with Solar System dynamics for some values of its interpolation function [10], and in its inertial variant it violates the equivalence principle (although this is a lesser problem since this principle has not been tested for the low accelerations seen at the edges of galaxies, which are unobtainable on Earth [11]).

The Pioneer anomaly is similar to the galaxy rotation problem. It is an apparent anomalous acceleration towards a gravity source, in this case the Sun, of $8.7 \times 10^{-10}$ m/s\textsuperscript{2} and was first observed by [12] and [13]. It has not been detected in the acceleration of the planets and some of their satellites [14–20], excluding, perhaps, Neptune [21], Pluto [22] and trans-Neptunian objects ([23] and [24]) whose trajectory data may not yet be accurate enough. This implies that the explanation is not a simple modification of gravity, since this would also effect the planets.

The possibility that the Pioneer anomaly is a consequence of an anisotropic dissipation of onboard sources of thermal radiation has been investigated by [25–28]. They have found that this process could explain up to one-third of the anomaly, but it is difficult to see how this could explain the constant anomalous acceleration, and the possible onset of the anomaly at 10 AU.

Other suggestions have included an empirical modification of gravity [29], and the addition of an extra dimension within general relativity [30]. Many of these suggestions
are summarised in [31] and a space mission to test for the anomaly was recently suggested by [32].

In summary the anomalous acceleration seems to be non-gravitational, dependent on trajectory (the bound planets are unaffected at the same orbital radii), and so it makes sense to investigate inertia [33], also because it is not well understood.

Following the prediction of Hawking radiation from black holes [34], [35] suggested that an accelerating body may also see thermal radiation and [36] derived an inertia-like force from this Unruh radiation. The wavelength of Unruh radiation increases as the acceleration reduces and [37] pointed out that at the very low accelerations at which galaxies start to deviate from expected behaviour, the Unruh wavelength reaches the Hubble scale. He suggested that then the Unruh waves may not be observable, and that somehow this may abruptly reduce the Unruh inertia at these low accelerations. This idea is suggestive, but does not quantitatively fit any galaxy rotation curves, and cannot explain the Pioneer anomaly, since the Unruh waves these spacecraft may see are far shorter than the Hubble scale.

Building on Milgrom’s abrupt break, a new model for inertia was proposed by [38] and this model could be called a Modification of inertia resulting from a Hubble-scale Casimir effect (MiHsC) or Quantised Inertia. As above, MiHsC assumes that the inertial mass of an object is caused by a drag from Unruh radiation. The new assumption is that this Unruh radiation is subject to a Hubble-scale Casimir effect. This means that only Unruh wavelengths that fit exactly into twice the Hubble scale (harmonics with nodes at the boundaries) are allowed, so that a greater proportion of longer Unruh waves are disallowed, reducing inertia in a new, more gradual, way for low accelerations. This model predicts that the inertial mass \( m_I \) varies

\[
m_I = m_g \left( 1 - \frac{\beta \pi^2 \theta^2}{|a| \Theta} \right),
\]

where \( m_g \) is the gravitational mass, \( \beta = 0.2 \) (empirically derived by Wien for Wien’s law), \( c \) is the speed of light, \( \Theta \) is the Hubble diameter \((2.7 \times 10^{26} \text{ m}, [39]) \) and the acceleration \( a \) for the Pioneer craft was the magnitude of the acceleration of the Pioneer 10 and 11 spacecraft relative to their main attractor the Sun (see [38] for the derivation of eq. (1)). MiHsC predicted a reduction of inertial mass for the Pioneer craft of about 0.01% which made them more sensitive to the Sun’s gravity resulting in an extra predicted Sunward acceleration of \( 6.9 \times 10^{-10} \text{ m/s}^2 \), in agreement with the Pioneer anomaly. The advantages of MiHsC are that it is based on a physical model, is simple, and it needs no adjustable parameters. Its main disadvantage is its apparent agreement only with unbound trajectories.

[40] showed that MiHsC agrees quite well with the flyby anomalies: unexplained velocity changes seen in Earth flyby craft observed by [41] (also discussed in [42] and [43]), if the conservation of momentum is considered, and also if the acceleration in eq. (1) is not the acceleration with respect to a single background (as in Milgrom’s MoND or [44]), but instead the acceleration relative to the surrounding matter (following Mach’s principle). In [45] this was taken to its logical conclusion, and the acceleration relative to the fixed stars was included in eq. (1).

Returning to the galaxy rotation problem: this is also an unexpected acceleration towards a source of gravity. The problem is that galaxies are bound systems and MiHsC only seems to apply to unbound objects. This problem is avoided here by looking at galaxies at the edge of boundedness, i.e.: very low mass ones, in the hope that their characteristics may be determined by MiHsC.

Recently, [46] studied the baryonic mass of rotationally supported (disc) galaxies and showed that there are none with a baryonic mass less than \( 10^9 M_{\odot} \) (within the central 500 parsecs) and these results are unexplained so far. In this paper it is shown that MiHsC predicts minimum linear and rotational accelerations. The former are shown to be close to the acceleration attributed to dark energy, and the latter imply a minimum disc galaxy mass close to that observed.

**Method and results.** Starting with Newton’s second law and gravity laws for a star with gravitational mass \( m_g \) and inertial mass \( m_I \) orbiting a galaxy with gravitational mass \( M \) we get

\[
F = m_I a = \frac{GMm_g}{r^2}.
\]

Following [38,40,45] the inertial mass is replaced with eq. (1), in which \( |a| \) is the average mutual acceleration of every other mass in the universe. Simplifying the constants (\( \beta \pi^2 \sim 2 \), introducing a 1.5% error) and cancelling \( m_g \), we get

\[
\left( 1 - \frac{2c^2}{|a| \Theta} \right) a = \frac{GM}{r^2}.
\]

Rearranging we get

\[
a = \frac{GM}{r^2} + \frac{2c^2}{\Theta} \hat{a},
\]

where \( \hat{a} = a/|a| \), a unit vector. As shown in [38], this formula implies that, even if the gravitational mass in its vicinity is zero, MiHsC (the second term on the right-hand side) predicts that an object must still accelerate: there is a minimum acceleration in nature. It is interesting that this is close to the observed acceleration attributed to dark energy ([47] and [48]). Now considering smaller and smaller galaxies, the mass \( M \) in eq. (4) will decrease and the extra acceleration due to the new second term will become ever more important. The extra acceleration is inwards (in the direction of \( a \), i.e.: \( \hat{a} \)) so the apparent proportion of dark matter in the galaxy will seem to increase (term 2 divided by term 1). When \( M \rightarrow 0 \) the MiHsC acceleration will still be finite, as follows:

\[
a = \frac{2c^2}{\Theta} \hat{a}.
\]
If we are unaware of this extra MiHsC term, then the residual observed acceleration will be misinterpreted as being due to an apparent (or dark) mass $M_{\text{dark}}$ so that

$$a = \frac{GM_{\text{dark}}}{r^2} = \frac{2c^2}{\Theta}. \quad (6)$$

This apparent dark mass, within a radius $r = 500$ parsecs, is predicted by eq. (6) to be

$$M_{\text{dark}} = \frac{2c^2 r^2}{G\Theta} = 2.3 \times 10^{39} \text{ kg} = 1.1 \times 10^9 M_\odot. \quad (7)$$

This agrees with the minimum mass of disc galaxies of about $10^9 M_\odot$ observed by [46]. If MiHsC is right then it implies that these small galaxies have remained above the MiHsC minimum acceleration by spinning as if they contained the apparent mass predicted by eq. (7). The apparent dark matter in larger galaxies may have a similar cause, but this is a far more complex problem and outside the scope of this paper.

Discussion. – To explain in a more intuitive way: as we consider smaller and smaller galaxies the rotational acceleration reduces, eventually for very small galaxies the rotational acceleration is so small that the effect of MiHsC begins to reduce the stars’ inertia and this has the effect of increasing the rotation again. Since the stars have a very low inertial mass they can easily be bent into rotation by even a tiny amount of baryonic mass. A balance is reached at the minimum allowed rotational acceleration. If we are unaware of MiHsC then we interpret the residual rotation as being due to extra dark matter. The mass at the balance point has a definite value and has been predicted above. The fact that this agrees with the observed mass cutoff for disc galaxies is encouraging.

The behaviour of elliptical galaxies has not been considered here since they are pressure supported and so eq. (2) would be more complex, with an extra pressure term. According to [46] they can attain masses as low as $10^6 M_\odot$. MiHsC can explain this qualitatively, since the extra nonrotational accelerations in these systems make it possible for them to stay above the minimum acceleration, even with lower rotational accelerations.

As an independent test, on a much larger scale, eq. (7) can be applied to the observable universe (which has a radius of $\Theta/2$) so that

$$M_{\text{dark}} = \frac{2c^2 \left( \frac{\Theta}{2} \right)^2}{G\Theta} = \frac{c^2 \Theta}{2G} = 2 \times 10^{53} \text{ kg}. \quad (8)$$

This is the apparent total (dark) mass that the observable universe must have if it is to accelerate fast enough to satisfy the minimum acceleration of MiHsC. From the Friedmann cosmological equations ([49]) the mass that the observable universe must have to be closed is given by

$$M_{\text{closed}} = \frac{3H^2}{8\pi G} \times \text{volume} = \frac{3c^2}{2\pi G\Theta^2} \times \frac{4\pi r^3}{3} = \frac{c^2 \Theta}{4G}. \quad (9)$$

This mass is $1 \times 10^{53}$ kg and the mass of the observable universe including dark matter is thought to be close to this, although the reason is unknown (this is called the age, or flatness, problem). As shown above $M_{\text{closed}}$ is close to the minimum mass predicted by MiHsC, so MiHsC could provide an explanation for the flatness problem.

The MiHsC model has a number of problems, among these are: 1) There is no proven reason why bound and unbound trajectories should behave differently. 2) How do such long Unruh waves interact with matter and the Hubble scale? 3) Why is it that the large-particle accelerations within stars, and within atoms, do not need to be considered? 4) The modified inertia predicts dynamical effects due to both an increased sensitivity to external force, and changes in momentum and both of these need to be considered.

A suggested practical test. – Since astronomical tests as discussed above can be ambiguous, it is important to suggest a direct controllable experimental test. This is attempted here. The wavelength of the Unruh radiation seen by accelerated objects [35] is given by

$$\lambda = \frac{43\pi^2 c^2}{a}. \quad (10)$$

For terrestrial accelerations these wavelengths are too long to be detectable, or to be generated. For example an object accelerated at $9.8 \text{ m/s}^2$, sees Unruh radiation with $\lambda \sim 10^{16} \text{ m}$. It is suggested here that the first assumption of MiHsC (inertia is due to Unruh radiation) could be tested by accelerating a particle around, say, the 1 km ring at the CERN particle accelerator. If the particle’s speed is 0.9c then its acceleration would be $7.3 \times 10^{13} \text{ m/s}^2$. Since its acceleration is so high, the Unruh radiation it sees would be short enough (9.7 km) to be produced artificially (these are long radio waves). Some extra man-made Unruh radiation could now be applied to the particle. Since the particle is travelling at 0.9c, and because of special relativity, a radiation of wavelength 22 km would have to be used so that the moving particle would see a wavelength of 9.7 km. Then, if assumption 1 of MiHsC is right, the particle would see more Unruh radiation corresponding to its acceleration, its inertial mass would increase, and this change in inertia could be detectable in its trajectory.

Conclusions. – The cosmic acceleration attributed to dark energy, the observed minimum disc galaxy mass of about $10^9 M_\odot$ and the Hubble mass, can be predicted using a model that assumes that 1) inertia is due to Unruh radiation, and 2) this radiation is subject to a Hubble-scale Casimir effect.

It is proposed that assumption 1 of MiHsC can be tested using a particle accelerator to accelerate particles to the extreme, so that the Unruh radiation they see (and that may determine their inertial mass) is short enough to be supplemented using man-made radiation. This may allow control of the particles’ inertia, with detectable consequences.
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REFERENCES

[1] Zwicky F., Helv. Phys. Acta, 6 (1933) 110.
[2] Rubin V., Science, 220 (1983) 1339.
[3] Bosma A., Astron. J., 86 (1981) 1791.
[4] Rubin V., Ford W., Thonnard N. and Burstein D., Astrophys. J., 261 (1982) 439.
[5] Bernabei R., presentation at BEYOND Conference, 1–6 February, 2010, Cape Town, S. Africa, arXiv:1002.1028.
[6] CDMS Collaboration (Ahmed Z. et al.), Phys. Rev. Lett., 102 (2009) 011301.
[7] XENON10 Collaboration, Phys. Rev. D., 80 (2009) 115005.
[8] Khlopov M. Yu., arXiv:0911.5685, to be published in AIP Proceedings of the Invisible Universe International Conference, UNESCO-Paris, June 29–July 3, 2009.
[9] Milgrom M., Astrophys. J., 270 (1983) 365.
[10] Iorio L., J. Gravit. Phys., 2 (2006) 26.
[11] McGaugh S. S., Sci. Lett., 318 (2007) 568.
[12] Anderson J. D., Laing P. A., Lau E. L., Liu A. S., Nieto M. M. and Turyshev S. G., Phys. Rev. Lett., 81 (1998) 2858.
[13] Anderson J. D., Laing P. A., Lau E. L., Liu A. S., Nieto M. M. and Turyshev S. G., Phys. Rev. D., 65 (2002) 082004.
[14] Ptitjeva E. V., Limitations on some physical parameters from position observations of planets, paper presented at the 26th Meeting of the IAU, 22–23 August 2006, Prague, Czech Republic, Joint discussion 16, No. 55.
[15] Iorio L. and Giudice G., New Astron., 11 (2006) 600.
[16] Iorio L., Found. Phys., 37 (2007) 897.
[17] Iorio L., J. Gravit. Phys., 1 (2007) 5.
[18] Iorio L., Does the Neptunian system of satellites challenge a gravitational origin for the Pioneer anomaly?, to be published in Mon. Not. R. Astron. Soc. (2010) doi:10.1111/j.1365-2966.2010.16637.x.
[19] Standish E. M., Planetary and lunar ephemerides: testing alternate gravitational theories, in Recent Developments in Gravitation and Cosmology - 3rd Mexican Meeting on Mathematical and Experimental Physics, edited by Macias A., Lammerzahl C. and Camacho A., AIP Conf. Proc., 977 (2008) 254.
[20] Standish E. M., Proc. IAU Symp., 261 (2010) 179.
[21] Iorio L., Tests in the outer regions of the Solar System with planetary dynamics Int. J. Mod. Phys., 18 (2009) 94.
[22] Page G. L., Wallin J. F. and Dixon D. S., Astrophys. J., 697 (2009) 1226.
[23] Page G. L., Dixon D. S. and Wallin J. F., Astrophys. J., 642 (2006) 606.
[24] Wallin J. F., Dixon D. S. and Page G. L., Astrophys. J., 666 (2007) 1296.
[25] Bertolami O., Francisco F., Gil P. J. S. and Paramos J., Thermal analysis of the Pioneer anomaly: a method to estimate radiative momentum transfer, preprint (2008).
[26] Rievers B., Lammerzahl C. and Dittus H.-J., New J. Phys., 11 (2009) 303224, 24pp.
[27] Rievers B., Bremer S., List M., Lammerzahl C. and Dittus H.-J., Acta Astron., 66 (2010) 467.
[28] Toth V. T. and Turyshev S. G., Phys. Rev. D., 79 (2009) 043011.
[29] Brownstein J. R. and Moffat J. W., Class. Quantum Grav., 23 (2006) 3427.
[30] Gerrard M. B. and Sumner T. J., arXiv:0807.3158 (2008).
[31] Turyshev S. G. and Toth V. T., arXiv:1001.3686 (2010).
[32] Rathke A. and Izzo D., J. Spacecr. Rockets, 43 (2006) 806.
[33] Milgrom M., Phys. Lett. A, 253 (1999) 273.
[34] Hawking S., Nature, 248 (1974) 30.
[35] Unruh W. G., Phys. Rev. D., 14 (1976) 587.
[36] Haisch B., Rueda A. and Puthoff H. E., Phys. Rev. A., 49 (1996) 678.
[37] Milgrom M., Ann. Phys., 229 (1994) 384.
[38] McCulloch M. E., Mon. Not. R. Astron. Soc., 376 (2007) 338, arXiv:astro-ph/0612599.
[39] Freedman W. L., Astrophys. J., 553 (2001) 47.
[40] McCulloch M. E., Mon. Not. R. Astron. Soc.-Lett., 389 (2008) L57.
[41] Anderson J. D., Campbell J. K., Ekelund J. E., Ellis J. and Jordan J. F., Phys. Rev. Lett., 100 (2008) 091102.
[42] Nieto M. M. and Anderson J. D., Phys. Today, 62, issue No. 10 (2009) 76.
[43] Iorio L., Sch. Res. Esch., 2009 (2009) article ID 807695.
[44] De Lorenzi V. A., Faundez-Arbes M. and Pereira J. P., Astron. Astrophys., 503 (2009) L1.
[45] McCulloch M. E., EPL, 89 (2010) 19001.
[46] McGaugh S. S., Schombert J. M., de Blok W. J. G and Zagursky M. J., Astrophys. J., 708 (2009) L14.
[47] Perlmutter S. et al., Astrophys. J., 517 (1999) 565.
[48] Riess A. G., Astron. J., 116 (1998) 1009.
[49] Coles P. and Lucchin F., Cosmology: The Origin and Evolution of Cosmic Structure, 2nd edition (J. Wiley and Sons Ltd.).