Space–time inhomogeneity, anisotropy and gravitational collapse

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Abstract We investigate the evolution of non-adiabatic collapse of a shear-free spherically symmetric stellar configuration with anisotropic stresses accompanied with radial heat flux. The collapse begins from a curvature singularity with infinite mass and size on an inhomogeneous space–time background. The collapse is found to proceed without formation of an even horizon to singularity when the collapsing configuration radiates all its mass energy. The impact of inhomogeneity on various parameters of the collapsing stellar configuration is examined in some specific space–time backgrounds.

Keywords Gravitational collapse · Heat flux · Einstein’s field equations · Space–time inhomogeneity

1 Introduction

One of the most fundamental problems in general relativity is the construction of realistic models describing various evolutionary stages of a star collapsing under its own gravity. When a star exhausts all its thermonuclear fuel, it cannot withstand the gravitational pull and starts collapsing. As it contracts appreciably, the density increases and at a sufficiently high density it produces non-thermal pressure via degenerate fermions and particle interactions to support it against further collapse and becomes a ‘compact star’. Stable stellar configurations like neutron stars are the end
products of such radiative collapse processes. A massive star, however, cannot come
to a stable stage by such processes. In the absence of any mechanism that can withstand
the gravitational pull, the general relativistic prediction is that such a collapse must
terminate into a space–time singularity. In view of the Cosmic Censorship Conjecture
(CCC), such a singularity must be covered within its event horizon of gravity \[1\]. How-
ever, there are several counter examples where a naked singularity is more likely to be
formed. In fact, there is yet no established theory available governing the formation
of either a black hole or a naked singularity (for a recent review see \[2\] and references
therein). The physics of evolving dynamical systems, therefore, continues to generate
a great deal of interest in various fields of astrophysics and cosmology even today.

To understand the nature of collapse of a self gravitating body, one needs to pro-
vide an accurate description of the exterior and interior space–times of the collapsing
body. It is also necessary to find out the appropriate boundary conditions joining the
two regions. The theoretical understanding of gravitational collapse was first initiated
by Oppenheimer and Snyder \[3\] who considered the contraction of a highly ideal-
ized spherically symmetric dust cloud. The exterior space–time of the collapsing dust
cloud was described by the Schwarzschild metric and the interior space–time was rep-
resented by a Friedman–like solution. Later on, the Vaidya \[4\] metric corresponding
to the exterior gravitational field of a stellar body describing an outgoing null fluid
gave a tremendous impetus in this direction. Making use of the Vaidya \[4\] solution,
Santos \[5\] proposed a procedure to obtain the description of the interior space–time of
a spherically symmetric radially shrinking distribution of non-adiabatic fluid. Several
stellar models (see for example, \[6–34\] and references therein) have been obtained
and examined critically using Santos’s approach. The procedure has been found to be
useful to examine the impact of various factors such as shear, inhomogeneity, anisot-
ropy, electromagnetic field and various dissipative processes on the evolution. In the
absence of any established theory governing the nature of collapse, such investiga-
tions have been found to be very useful to understand the dynamic behaviour of a
gravitationally collapsing system.

The objective of the present work is to formulate a framework to study the nature of
collapse of an inhomogeneously distributed anisotropic source. The dynamical behav-
ior of a gravitating system is expected to be influenced by density inhomogeneity and
anisotropy. Amongst many other issues, these factors have been found to play crucial
role in our theoretical understanding of the nature of singularity and its genericity
(see \[1,2,35–39\] and references therein). Eardley and Smarr \[40\] have shown that
inhomogeneous distributions may lead to a naked singularity in contrast to a homo-
geogeneous distribution where a black hole is more likely to be formed. For a spherically
symmetric dust cloud, Mena et al. \[41\] have examined the role of inhomogeneity and
anisotropy in Lamaitre–Tolman–Bondi collapse. Hererra et al. \[16,18\] have examined
the role of various factors contributing to inhomogeneity of matter distribution, its
evolution and development of anisotropy. It has been observed that tidal forces tend
to make a gravitating system more inhomogeneous which may lead to the formation
of a naked singularity while for a homogeneously distributed body it is more likely to
form a black hole. Similar observations may be found in \[21\] where it has been shown
that if an initially static star undergoing collapse has a homogeneous distribution it
ends up with the formation of a black hole. However, if an anisotropic star contains an