Planck Scale Phenomena

B.G. Sidharth
Centre for Applicable Mathematics & Computer Sciences
Adarsh Nagar, Hyderabad - 500 063, India

Abstract

Though the Planck scale is encountered in Quantum SuperString Theory and Quantum Gravity, it is the Compton scale of elementary particles which is encountered in the physical world. An explanation for this is given in terms of Brownian processes and the duality relation.

1 Introduction

It is well known that in Quantum Gravity as well as in Quantum Super-String Theory, we encounter phenomena at the Planck scale. Yet what we encounter in the real world is, not the Planck scale, but the elementary particle Compton scale. The explanation for this is that the very high energy Planck scale is moderated by the Uncertainty Principle. The question which arises is, exactly how does this happen? We will now present an argument to show how the Planck scale leads to the real world Compton scale, via fluctuations and a modification of the Uncertainty Principle.

2 The Planck Scale

It is well known that the Planck Scale is defined by

\[ l_P = \left( \frac{\hbar G}{c^3} \right)^{\frac{1}{2}} \sim 10^{-33} \text{cm} \]
\[ t_P = \left( \frac{\hbar G}{c^3} \right)^{\frac{3}{2}} \sim 10^{-42} \text{sec} \quad (1) \]

(1) defines the absolute minimum physical scale [1, 2]. Associated with (1) is the Planck mass

\[ m_P \sim 10^{-5} \text{gm} \quad (2) \]

There are certain interesting properties associated with (1) and (2). \( l_P \) is the Schwarzschild radius of a black hole of mass \( m_P \) while \( t_P \) is the evaporation time for such a black hole via the Beckenstein radiation [3]. Interestingly \( t_P \) is also the Compton time for the Planck mass, a circumstance that is symptomatic of the fact that at this scale, electromagnetism and gravitation become of the same order [4]. Indeed all this fits in very well with Rosen’s analysis that such a Planck scale particle would be a mini universe [5, 6].

We will now invoke a time varying gravitational constant

\[ G \approx \frac{lc^2}{m\sqrt{N}} = (\sqrt{N}t)^{-1} \alpha T^{-1} \quad (3) \]

which resembles the Dirac cosmology and features in another scheme in which (3) arises due to the fluctuation in the particle number [7, 8, 9, 10, 4]. In (3) \( m \) and \( l \) are the mass and Compton wavelength of a typical elementary particle like the pion while \( N \sim 10^{80} \) is the number of elementary particles in the universe, \( \sqrt{N} \) the fluctuation in particle number and \( T \) is the age of the universe.

In this scheme wherein (3) follows from the theory, we use the fact that given \( N \) particles, the fluctuation in the particle number is of the order \( \sqrt{N} \), as noted by Hayakawa, while a typical time interval for the fluctuations is \( \sim \hbar/mc^2 \), the Compton time. We will come back to this point later. So we have

\[ \frac{dN}{dt} = \frac{\sqrt{N}}{\tau} \]

whence on integration we get,

\[ T = \frac{\hbar}{mc^2\sqrt{N}} \]

and we can also deduce its spatial counterpart, \( R = \sqrt{N}l \), which is the well known empirical Eddington formula. It is also possible to then deduce
the Hubble Law and a hitherto mysterious empirical relationship, noted by Weinberg, between the mass of the pion and the Hubble constant (Cf. refs. for details).

Equation (3) which is an order of magnitude relation is consistent with observation [11] while it may be remarked that the Dirac cosmology itself has inconsistencies.

Substitution of (3) in (1) yields

\[ l = N^{\frac{1}{4}} l_P , \]
\[ t = N^{\frac{1}{4}} t_P . \]  

where \( t \) as noted is the typical Compton time of an elementary particle. We can easily verify that (4) is consistent. It must be stressed that (4) is not a fortuitous empirical coincidence, but rather is a result of using (3), which again as noted, can be deduced from theory.

(4) can be rewritten as

\[ l = \sqrt{n} l_P , \]
\[ t = \sqrt{n} t_P . \]  

wherein we have used (3) and \( n = \sqrt{N} \).

We will now compare (5) with the well known relations, deduced earlier,

\[ R = \sqrt{N} l \]
\[ T = \sqrt{N} t . \]  

The first relation of (6) is the well known Eddington formula referred to while the second relation of (6) is given also on the right side of (3). We now observe that (6) can be seen to be the result of a Brownian Walk process, \( l, t \) being typical intervals between "steps" (Cf. [4, 12, 13]). We demonstrate this below after equation (8). On the other hand, the typical intervals \( l, t \) can be seen to result from a Nelsonian process themselves. Let us consider Nelson’s relation,

\[ (\Delta x)^2 \equiv l^2 = \frac{\hbar}{m} t \equiv \frac{\hbar}{m} \Delta t \]  

(Cf. [14, 15, 16, 17, 12]).

Indeed as \( l \) is the Compton wavelength, (7) can be rewritten as the Quantum Mechanical Uncertainty Principle

\[ l \cdot p \sim \hbar . \]
at the Compton scale (Cf. also [18]) (or even at the de Broglie scale).

What (7) shows is that a Brownian-Nelsonian process defines the Compton scale while (6) shows that a Random Walk process with the Compton scale as the interval defines the length and time scales of the universe itself (Cf.[13]).

Returning now to (5), on using (2), we observe that in complete analogy with (7) we have the relation

\[
(\Delta x)^2 \equiv t_P^2 = \frac{\hbar}{m_P} \Delta t = \frac{\hbar}{m_P} \Delta t \tag{8}
\]

We can now argue that the Nelsonian-Brownian process (8) defines the Planck length while a Brownian Random Walk process with the Planck scale as the interval leads to (5), that is the Compton scale.

To see all this in greater detail, it may be observed that equation (8)(without subscripts)

\[
(\Delta x)^2 = \frac{\hbar}{m} \Delta t \tag{9}
\]

is the same as the Nelsonian equation, indicative of a double Weiner process. Indeed as noted by several scholars, this defines the fractal Quantum path of dimension 2 (rather than dimension 1) (Cf.e.g. ref.[15]).

Firstly it must be pointed out that equation (9) defines a minimum space time unit - the Compton scale \((l, t)\). This follows from (9) if we substitute into it \((\Delta x)_{max} = c\). If the mass of the particle is the Planck mass, then this Compton scale becomes the Planck scale.

Let us now consider the distance traversed by a particle with the speed of light through the time interval \(T\). The distance \(R\) covered would be

\[
\int dx = R = c \int dt = cT \tag{10}
\]

by conventional reasoning. In view of the Nelsonian equation (9), however we would have to consider firstly, the minimum time interval \(t\) (Compton or Planck time), so that we have

\[
\int dt \to nt \tag{11}
\]

Secondly, because the square of the space interval \(\Delta x\) (rather than the interval \(\Delta x\) itself as in conventional theory) appears in (9), the left side of (11)
becomes, on using (11)

\[
\int dx^2 \int (\sqrt{n}dx)(\sqrt{n}dy)
\]

\[\text{(12)}\]

Whence for the linear dimension \(R\) we would have

\[
\sqrt{n}R = nct \quad \text{or} \quad R = \sqrt{nl}
\]

\[\text{(13)}\]

Equation (12) brings out precisely the fractal dimension \(D = 2\) of the Brownian path while (13) is identical to (4) or (6) (depending on whether we are dealing with minimum intervals of the Planck scale or Compton scale of elementary particles). Apart from showing the Brownian character linking equations (4) and (9), incidentally, this also provides the justification for what has so far been considered to be a mysterious large number coincidences viz. the Eddington formula (3).

There is another way of looking at this. It is well known that in Quantum SuperString Theory, at the Planck scale we have a non commutative geometry \[19, 20\]

\[
[x, y] \approx 0(l_P^2), [x, p_x] = \hbar [1 + 0(l_P^2)]etc.
\]

\[\text{(14)}\]

Indeed (14) follows without recourse to Quantum SuperStrings, merely by the fact that \(l_P, t_P\) are the absolute minimum space time intervals as shown a long time ago by Snyder [21].

The non commutative geometry (14), as is known is symptomatic of a modified uncertainty principle at this scale [24]-[28]

\[
\Delta x \approx \frac{\hbar}{\Delta p} + l_P^2 \frac{\Delta p}{\hbar}
\]

\[\text{(15)}\]

The relation (13) would be true even in Quantum Gravity. The extra or second term on the right side of (13) expresses the well known duality effect - as we attempt to go down to the Planck scale, in fact we are lead to the larger scale represented by it. The question is, what is this larger scale?

If we now use the fact that \(\sqrt{n}\) is the fluctuation in the number of Planck particles (exactly as \(\sqrt{N}\) was the fluctuation in the particle number as in (3), so that \(\sqrt{nm\cancel{p}c} = \Delta n\) is the fluctuation or uncertainty in the momentum for the second term on the right side of (13), we obtain for the uncertainty in length,

\[
\Delta x = l_P^2 \frac{\sqrt{nm\cancel{p}c}}{\hbar} = l_P \sqrt{n}
\]

\[\text{(16)}\]
We can easily see that (16) is the same as the first relation of (5). The second relation of (5) follows from an application of the time analogue of (15). Thus the impossibility of going down to the Planck scale because of (14) or (15), manifests itself in the fact that as we attempt to go down to the Planck scale, we in fact end up at the Compton scale. Interestingly while at the Planck length, we have a left time of the order of the Planck time, as noted above it is possible to argue on the other hand that with the pion mass and length of a typical elementary particle like the pion, at the Compton scale, we have a life time which is the age of the universe itself as shown by Sivaram [3, 29]. Interestingly also Ng and Van Dam deduce the relations like [30]

\[ \delta L \leq (Ll_P^2)^{1/3}, \delta T \leq (Tt_P^2)^{1/3} \]  

where the left side represents the uncertainty in the measurement of length and time for an interval \( L, T \). We would like to point out that if in (17) we use for \( L, T \), the size and age of the universe, then \( \Delta L \) and \( \Delta T \) reduce to the Compton scale \( l, t \).

In conclusion, Brownian-Nelsonian processes and the modification of the Uncertainty Principle at the Planck scale lead to the physical Compton scale.

References

[1] A. Kempf, in From the Planck length to the Hubble radius, Ed. A. Zichichi, (World Scientific, Singapore, 2000), pp.613ff.

[2] G. Veneziano in The Geometric Universe, Ed. by S.A. Huggett, et al., (Oxford University Press, Oxford, 1998), p.235ff.

[3] C. Sivaram, Am.J.Phys., 51(3), 1983, p.277.

[4] B.G. Sidharth, Chaotic Universe: From the Planck to the Hubble Scale, (Nova Science Publishers, New York, 2001), p.20.

[5] N. Rosen, Int.J.Th.Phys., 32(8), 1993, p.1435-1440.

[6] B.G. Sidharth, and A.D. Popova, Differential Equations and Dynamical Systems, 4 (3/4), 1996, 431-440.
[7] J.V. Narlikar, *Int. J. Th. Phys.*, 1983, pp.311-323.

[8] B.G. Sidharth, *Int. J. Mod. Phys. A*, 13 (15), 1998, p.2599ff.

[9] B.G. Sidharth, *Int. J. Th. Phys.*, 37 (4), 1998, p.1307ff.

[10] B.G. Sidharth, *Il Nuovo Cimento*, 115B (12), (2), 2000, pg.151.

[11] E.G. Norman, *Am. J. Phys.*, 544, 317, 1986.

[12] B.G. Sidharth, *Chaos, Solitons and Fractals*, 12(1), 2000, 173-178.

[13] B.G. Sidharth, ”Quantum Spacetime”, to appear in *Chaos, Solitons and Fractals*.

[14] E. Nelson, *Physical Review*, Vol.150, No.4, October 1966, p.1079-1085.

[15] L. Nottale, *Fractal Space-Time and Microphysics: Towards a Theory of Scale Relativity*, (World Scientific, Singapore, 1993), p.312.

[16] L. Smolin, in *Quantum Concepts in Space and Time*, Eds., R. Penrose and C.J. Isham, (OUP, Oxford, 1986), pp.147-181.

[17] A. Kyprianidis, *Int. J. Th. Phys.*, 1992, pp.1449-1483.

[18] B.G. Sidharth, *Concise Encyclopaedia of SuperSymmetry and Non Commutative Structures in Mathematics and Physics* Eds. J. Bagger, S. Duply, W. Sugel, (New York, Kluwer Academic Publishers, 2001).

[19] Y. Ne’eman, in Proceedings of the First International Symposium, *Frontiers of Fundamental Physics*, Eds. B.G. Sidharth and A. Burinskii, (Universities Press, Hyderabad, 1999), pp.83ff.

[20] B.G. Sidharth, *Il Nuovo Cimento*, 116B (6), 2001, pg.4 ff.

[21] H.S. Snyder, *Physical Review*, Vol.72, No.1, July 1 1947, p.68-71.

[22] D. Amati in *Sakharov Memorial Lectures*, Eds. L.V. Kaddys and N.Y. Feinberg, (Nova Science, New York, 1992), pp.455ff.

[23] R. Guida, et al., *Mod. Phys. Lett. A*, Vol.6, No.16, 1991, p.1487-1503.
[24] W. Witten, *Physics Today*, April 1996, pp.24-30.

[25] L.J. Garay, *Int.J.Mod.Phys. A*, Vol.10, No.2, 1995, p.145-165.

[26] M. Maggiore, *Phys.Lett.B.*, 319, 1993, p.83-86.

[27] M. Maggiore, *Phys.Rev.D.*, Vol.49, No.10, 1994, p.5182-5187.

[28] S. Doplicher, et al., *Phys.Lett., B*, 331, 1994, p.39-44.

[29] C. Sivaram, *Am.J.Phys.*, 50(3), 1982, p.279ff.

[30] H. Van Dam, et al., *Mod.Phys.Lett.,A*, Vol.9, No.4, 1994, 335-340.