Forecasting Non-Stationary Time Series with the Multiscale Autoregressive (MAR) Model Approach Using the Haar Wavelet Filter at the Rupiah Exchange Rate Against the Dollar

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Abstract. Time series data is a collection of a phenomenon that occurs based on a fixed or at the same time index. Time series phenomena often exhibit non-stationary behavior. One of the time series analyses for non-stationary data is the Multiscale Autoregressive Model (MAR). The MAR model chosen is a model that meets the assumptions of normality and white noise. The predictors used in MAR modeling are wavelet coefficients and scales which are the result of decomposition using Maximal Overlap Discrete Wavelet Transform (MODWT), MODWT functions to decompose data based on the level of each wavelet filter (family). The wavelet filters used in this study are Haar. In this study, the model generated from the data that was stationary first was more accurate than the data that was not stationary first.

1. Introduction

The exchange rate is the ratio between the currency values of a country compared to another country. The exchange rate reflects the balances of the demand and the supply of both domestic and foreign currencies. The decline of the Rupiah exchange rate reflects the decrease of public demand for the rupiah currency caused by the decrease of the national economy role or the increase of foreign currency demand as the international payment method. The enhancement of the rupiah exchange rate to a specific limit describes that the performance on the money market showing improvement. The impact of inflation rate enhancement is the domestic currencies will lower than foreign currencies. This is causing the decrease performance of a company and investment in the capital market is reduced.

The rupiah exchange rate against foreign currencies also has a negative impact on the economy and the capital market. The decline in the rupiah exchange rate against foreign currencies will causing an increase of importing raw materials for production cost and increase the interest rates. Though the decline in the exchange rate can also encourage companies to export. Time series data is a set of data obtained from observing a phenomenon that occurs based on a time index with a constant or the same time interval [5]. The classic Box-Jenkins method or known as the ARIMA model (Autoregressive Integrated Moving Average) This method is still used for non-stationarity time series analysis [6]. The research on non-stationarity time series analysis has been done a lot, one of them is wavelet transform, by forming a Multiscale Autoregressive (MAR) model. The predictors used in MAR modeling are wavelet coefficients and the scale as the result of decomposition by using the Maximal Overlap Discrete
Wavelet Transform (MODWT) method. MODWT functions to elaborate the data based on the level of each wavelet filter. The wavelet filter used in this study is Haar.

Thus, in this research the data will be stationarity in two ways. They are, detrending (separation of trends) and differencing (differentiation) and then it will decomposed by the MODWT method so that the MAR modeling can be done. This study examines the role of the wavelet model which contains the MAR model, to forecast non-stationarity time series data. The chosen MAR model is a model that fulfilled the assumptions of normality and white-noise. By forming a wavelet forecasting model, so the forecasting for the next several periods can be applied to the data on the rupiah exchange rate.

2. Literature Review

2.1. Time Series

A time series is a set of data obtained from the observation of a phenomenon that occurs based on a time index with a fixed or equal time interval [5]. In the analysis of the usual time series is decomposition, which is to identify the component of factors that affect the value in the data series, so that the time series data can be used for forecasting [4].

2.2. Non-Stationary Time Series Data Analysis Method

In this study, methods were carried out in analyzing non-stationary time series data, namely wavelet transformation.

2.2.1 Wavelet transformation

Wavelet transformation is one of the methods that can be performed on stationary and non-stationary time series data. This method can automatically separate a trend from a unstationary time series data. Besides, wavelet transformations can model irregular or non-linear patterned data [2]. In this method, if the data is a non-stationary time series then it can be decomposed immediately without being stationed first. But based on [13], the data of the stationed time series provides better results than unstationed time series data. Most non-stationary time series data is marked by a trend. Trends in the time series are two types, namely stochastic trends and deterministic trends. There are two methods for indexing time series data, including differencing and detrending. Stochastic trends are usually addressed by differentiation processes. While deterministic trends are usually overcome by doing trend separation [9].

According to [10], e.g. a non-stationary time series \( Z_t = \beta_1 + \beta_2 t + \beta_3 Z_{t-1} + e_t \). If \( \beta_1 \neq 0, \beta_2 = 0 \), \( \beta_3 = 1 \) then obtained a model \( Z_t = \beta_1 + \beta_{Z_{t-1}} + e_t \), written as

\[
Z_t - Z_{t-1} = \beta_1 + e_t
\]

\[\Delta Z_t = \beta_1 + e_t \] (1)

Then \( Z_t \) will show a positive trend \( (\beta_1 > 0) \) or negative trend \( (\beta_1 < 0) \), which is called a stochastic trend.

On the differencing method, \( \nabla Z_t = Z_t - Z_{t-1} \) formed as the first reference. The d-reference for time series stationation when denoted by \( W_t \), i.e. \( W_t = \nabla^d Z_t \), with \( d \) is an integer \( d \geq 1 \). A non-stationary time series is considered a deterministic trend when the mean function \( (\mu_t) \) can be explained by the polynomial of the k-order.

\[
Z_t = \sum_{j=0}^{k} \alpha_j t^j + e_t
\]

\[= \mu_t + e_t \] (2)
The instasioneran achieved by the construction of a new time series in residuals is known as the detrending method. If the data of a stationary time series is detrending symbolized by $Y_t$ then $Y_t = Z_t - \mu_t$, where $Z_t$ is the initial data of a time series and $\mu_t$ is a trend model in the form of polynomial [11]. Stationary time series data is decomposed with MODWT method to obtain wavelet coefficients and scale coefficients, so that Multiscale Autoregressive (MAR) modeling can be performed. For forecasting purposes, the model is restored to its original form called the wavelet forecasting model. For example, a time series data istationed by detrending and trend that has been obtained / separated ($\mu_t$) in the form of $\alpha_1 + \alpha_2 t$ then after obtaining the MAR model from $Y_t$, it is then returned to the original form $Z_{t+1} = \alpha_1 + \alpha_2 t + Y_{t+1}$ where $Y_{t+1}$ contains wavelet coefficients or scale coefficients. Multiscale Autoregressive (MAR) model found in wavelet forecasting models, the process follows AR through the convergence of optimal procedures and asymptotically will be equivalent to the best forecasting [12].

2.3. MAR
A MAR model is a model by performing a transformation process using a wavelet, which assumes each scale of a wavelet transformation follows an AR process. Determination of lag-lag that becomes an input variable for the MAR model uses wavelet coefficients and scale coefficients derived from wavelet transformation results [2]. The wavelet coefficient (detail) and scale coefficient of wavelet transformation results through MODWT decomposition are considered to have an influence on predictions at the time $t + 1$ will be shaped $w_{j,t-2'(k-1)}$ and $v_{j,t-2'(k-1)}$, or can be written as:

$$\hat{X}_{t+1} = \sum_{j=1}^{A_j} \sum_{k=1}^{A_j} \hat{a}_{j,k} w_{j,t-2'(k-1)} + \sum_{j=1}^{A_j} \hat{a}_{j+1,k} v_{j,t-2'(k-1)} + \epsilon_t$$

with MODWT decomposition level ($j=1,2,\ldots,J$), $A_j$ is the order of the MAR model ($k=1,2,\ldots,A_j$), $\hat{a}_{j,k}$, $\hat{a}_{j+1,k}$ is the coefficient value of the MAR model, $t$ is the time of the, $w_{j,t-2'(k-1)}$ is a wavelet coefficient and $v_{j,t-2'(k-1)}$ is a coefficient of scale.

The determination of mar model input in the forecasting of the $(t + 1)$ data is shown in Figure 1, the first input on each scale is the $t$-data, and the second input on each scale is the $(t - 2')$ data.

Figure 1. MAR Wavelet Modeling Illustration

Figure 1 is an illustration to predict the 21st data ($t+1$st data) with the MAR model of order 2 ($A_j=2$) and level $j=4$, then the input variables used are the level 1 wavelet coefficient at $t=20$ and $t=18$, the level 2 wavelet coefficient at $t=20$ and $t=16$, the level 3 wavelet coefficient at $t=20$ and $t=12$, wavelet coefficient level 4 at $t=20$ and $t=14$, as well as scale coefficients at $t=20$ and $t=4$. Supposing $J=6$ and $A_j=2$ MAR models based on equations (3) can be expressed as:
\[ \hat{X}_{j+1} = \sum_{j=1}^{2} \sum_{k=1}^{2} \hat{a}_{j,k} w_{j-2^{(j-1)}}, + \sum_{k=1}^{2} \hat{a}_{j,k} v_{j+1,2^{(j-1)}} \]

\[ \hat{X}_{j+1} = \hat{a}_{1,1} w_{1,t} + \hat{a}_{1,2} w_{1,t-2} + \hat{a}_{2,1} w_{2,t} + \hat{a}_{2,2} w_{2,t-4} + \hat{a}_{3,1} w_{3,t} \]

\[ + \hat{a}_{3,2} w_{3,t-8} + \hat{a}_{4,1} w_{4,t} + \hat{a}_{4,2} w_{4,t-16} + \hat{a}_{5,1} w_{5,t} + \hat{a}_{5,2} w_{5,t-32} \]

\[ + \hat{a}_{6,1} w_{6,t} + \hat{a}_{6,2} w_{6,t-64} + \hat{a}_{7,1} w_{7,t} + \hat{a}_{7,2} w_{7,t-64} \]

(4)

or can be written as \( s_j = A_j \alpha \).

The guessing of vector parameters can be solved by the smallest squared method that minimizes squared error \( [7] \), i.e.:

\[ \alpha = (A' A)^{-1} A' \tilde{s} \]

Calculation of the alleged value of the MAR model, used \( \alpha \) value that has been allegedly using equations \( [5] \). The \( \alpha \) value and the wavelet coefficient value and scale obtained through decomposition are incorporated into the MAR model as in the equation \( [3] \).

2.4. **MODWT**

The wavelet transformation seen as more suitable for time series data is MODWT because in each decomposition level there is a wavelet coefficient and a scale of as much data length \( [3] \). The determination of the level \( (J) \) for MODWT decomposition depends on the width of the decomposition filter \( (L) \) and the amount of data \( (n) \), with the formula:

\[ J < \ln \left( \frac{n}{L-1} + 1 \right) \]

The pyramid algorithm for MODWT is a calculation algorithm to calculate the scale coefficient and wavelet coefficient of MODWT at the \( j \)-level. If a data is decomposed with a wavelet filter and a scale filter, it will produce a wavelet coefficient and a scale coefficient. Figure 2 follows a pyramid algorithm for MODWT \( [8] \).

![MODWT Pyramid Algorithm Scheme](image)

**Figure 2. MODWT Pyramid Algorithm Scheme**

3. **Research Methods**

The time series data used in this study are daily data on the Rupiah exchange rate against the US Dollar from January 2019 to February 2020, totaling 437 observations, it is non-stationarity time series data. The type of wavelet used is the Haar wavelet. The steps taken in this research are:

**Step 1.** Determine the data to be analyzed, in the form of rupiah exchange rate data
**Step 2.** Checking data
**Step 3.** Determine the level according to the filter used, namely Haar with filter 2
Step 4. Performed MODWT decomposition at each level
Step 5. Selected the variables to be input model the MAR based on the significant PACF lag
Step 6. Performed the stepwise method to obtain significant variables
Step 7. Performed MAR modeling on wavelet coefficients and significant scales
Step 8. Formed the MAR model
Step 9. Checking the goodness of fit by selecting the MAR model that has the smallest RMSE.

4. Result and Discussion
In this study, the MAR model was applied to the daily data of rupiah exchange rate against US Dollar which is hereby called Exchange Rate from January 01, 2019 – August 1, 2020, consisting of 437 observations. The description of Exchange Rate data is used to find out an overview of the data, namely how big the average value, data spread, maximum and minimum values, and the amount of Exchange Rate data used in this study.

Table 1. Statistik Deskriptif

| Variabel | N  | Average | st. Deviation | Minimum | Maximum |
|----------|----|---------|---------------|---------|---------|
| Kurs     | 437| 14312   | 518.8394      | 13572   | 16575   |

Based on Table 1 it can be seen that the amount of Exchange Rate data used for modeling is 437 observations. The average exchange rate is 14312. For the spread of exchange rate data is 518.8394. The minimum exchange rate is 13572. While the maximum exchange rate is 16575.

The first step is to test the stationarity of the data. Stationary testing using dickey-fuller augmented unit root test (ADF) can be seen in Table 2. Following:

Table 2. ADF Exchange Rate Data Test

| Data | Value p before differencing | Value p after differencing | Conclusion         |
|------|-----------------------------|----------------------------|-------------------|
| Kurs | 0.1611                      | 0.01                       | Stationary after differencing to 1 |

Based on Table 2, it is known that after differencing 1 time obtained a value of p = 0.01. This means that the average awareness is achieved after differencing 1 time. The plot of exchange rate data after differencing 1 can be seen in Figure 3. Following:

Figure 3. Plot Data Exchange Rate Differencing 1 (left) and Plot Analysis Box-Cox (right)
The Box-Cox transformation shows that the Exchange Rate data returns a value of $\lambda=1$. It can be said that the differencing exchange rate data 1 has been stationary against the variety. Thus the results of stationary examination show that the Exchange Rate at differencing 1 has been stationary against average and variety. Average stationary testing is performed by looking at critical values at $\alpha=5\%$ compared to the statistical value of $t$ in the Augmented Dickey-Fuller (ADF Test) test. A one-time differencing that produces a critical value is 5% less than the ADF test statistical value, so it is concluded that the data is stationary to the average.

4.1. MAR Model Determination

Significant lag-lag retrieval is significantly limited to 0.2

**Table 3.** MAR model inputs based on significant PACF lag-lag in wavelet coefficients and scaling coefficients.

| Coefficient | Significant lag | n Input |
|-------------|----------------|---------|
| W1          | Lag 1, 3       | 2       |
| W2          | Lag 1, 2, 3, 4, 6 | 5       |
| W3          | Lag 1, 2, 3, 4, 5, 7, 9, 10 | 8       |
| W4          | Lag 1, 3, 5, 6, 9, 10 | 6       |
| W5          | Lag 1, 2, 3, 5, 10 | 5       |
| W6          | Lag 1, 3, 5 | 3       |
| V1          | Lag 1, 2, 3 | 3       |
| V2          | Lag 1, 3, 5, 10 | 4       |
| V3          | Lag 1, 3, 9, 10 | 4       |
| V4          | Lag 1, 2, 3, 5 | 4       |
| V5          | Lag 1, 3, 5 | 3       |
| V6          | Lag 1, 3 | 2       |

Form mar models on significant wavelet and scaling coefficients. Next, choose the forecasting model with the smallest RMSE, which is presented in Table 4. RMSE Model ARIMA and MAR Based on SIGNIFICANT PACF and Renaund Et al. Proposal. Check the accuracy of the model by selecting the MAR model that has the smallest RMSE.

**Table 4.** RMSE values at Levels 1-6

| Level | AIC      | RMSE   |
|-------|----------|--------|
| 1     | 4996.024 | 76.77699* |
| 2     | 4918.201 | 76.31107 |
| 3     | 4909.316 | 74.63827 |
| 4     | 4909.522 | 74.30664* |
| 5     | 4907.36  | 73.94456 |
| 6     | 4898.652 | 73.53704 |

*) based on stepwise level 1 and level 4 results all have significant variables at a rate of significance of 5%.

Thus, the RMSE value of the MAR model level 1 is 76.77699, while the RMSE value of the MAR model level 4 is 74.30664. So, the better MAR model used is the MAR model at level 4 because it has a smaller RMSE value.

5. Conclusion

Based on the results of the analysis can be concluded that for the problem of data Exchange rate the best model used is the mar model which is best is the MAR model in accordance with renaund et al
proposal with RMSE value. Based on PACF wavelet coefficient and significant scale with RMSE value in sample 74.30664. Thus, a more suitable model used on exchange rate data issues is the level 4 MAR model based on renaund et al. and significant PACF proposals.

6. References
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