Richtmyer-Meshkov instability and solid $^4$He melting driven by acoustic pulse

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Recent experiments have shown remarkable dynamics of solid $^4$He melting and growth, driven by the normal incidence of an acoustic pulse on the solid-liquid interface. The theory of solid growth/melting, driven by the radiation pressure of the acoustic pulse, accounts well for the temperature dependence of the measured data. There is however an observed source of extra, temperature-independent, melting. We here propose that this extra melting is due to solid-liquid mixing (and consequent melting) at the interface, in a process similar to the Richtmyer-Meshkov instability: Initial undulations of the rough interface, grow when accelerated by the acoustic pressure oscillations.

This model predicts a temperature-independent extra melting and its dependence on the acoustic power, which is in agreement with the measured data.

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Recent experiments have shown that the radiation pressure $P_\gamma$ of an acoustic pulse, incident on the solid-superfluid interface of $^4$He, can induce the local melting or growth of the solid. The general temperature dependence of the pulse-induced melting/growth is accounted for using the concept of acoustic radiation pressure. Nevertheless, a large deviation was found, which seems to be temperature independent. It is this extra melting which we shall describe in this paper.

We begin with the analysis of the experimental data in terms of a linear growth/melting coefficient (Eq.1-6 of [1]). The amplitude of melting/growth is linearly related to the radiation pressure $P_\gamma$ through the (temperature dependent) growth coefficient $K(T)$:

$$h = K(T) \frac{P_\gamma}{\rho_c} \tau$$

(1)

where $\tau = 1$ msec is the overall duration of the acoustic pulse. The radiation pressure is itself temperature dependent: $P_\gamma = E \left[ 1 - \frac{\omega_2}{c_2} + R^2 \left( 1 + \frac{\omega_2}{c_2} \right) \right]$, where the energy density of the incident wave is $E = I/c_1$, the power density $I = 144W/m^2$, and the reflection coefficient is

$$R = \frac{z_2 - z_1 - z_1 z_2 \rho_1 K(T) \left( \frac{\rho_1 - \rho_2}{\rho_1 \rho_2} \right)^2}{z_1 + z_2 + z_1 z_2 \rho_1 K(T) \left( \frac{\rho_1 - \rho_2}{\rho_1 \rho_2} \right)^2}$$

(2)

where $z_{1,2} = \rho_{1,2} c_{1,2}$. The growth coefficient $K(T)$ is given by

$$K(T)^{-1} = AT^4 + Be^{(\frac{c_1}{K})}$$

(3)

(with $A = 3 \times 10^{-2}, B = 2.4 \times 10^3, \Delta = 7.2K$). In these expressions $\rho_{1,2}, c_{1,2}$ are the density and sound velocities of the two phases on either side of the interface, with the index 1 corresponding to the phase on the side of the incident wave. In the experiment [1] we have $\rho_1 = 0.17gr/cc, c_1 = 365m/sec$ and $\rho_2 = 0.19gr/cc, c_2 = 475m/sec$. In Fig.1 we compare the experimental data with the calculated growth/melting amplitude $h$, given by Eq.1.

The overall agreement is good (dotted lines in Fig.1), but there is clearly an additional source of melting which appears for pulses incident from both the solid and liquid sides. This extra melting is not caused by the sound heating of the solid [3].

The contribution from the acoustic pressure leads to growth/melting that proceeds at a constant rate for the duration of the applied pulse [1]. In addition there is the inertial acceleration of the solid-liquid interface by the incident pressure oscillations. This motion should average to zero if no instability or mixing occurs at the interface. It is only due to mixing at the interface that this mechanism can contribute on average to the position of the interface at the end of the pulse. We assume here that the shape changes of the interface due to Richtmyer-Meshkov (RM)-mixing [2] do not change the average acoustic pressure and therefore can be approximately decoupled from each other. We therefore add the contribution from the acoustic pressure to the mixing contribution due to the RM-mixing. A complete description has to take into account possible interference effects of one mechanism with the other.

Let us now summarize the main results of this paper. We propose that the measured extra melting is the result of dynamic solid-liquid mixing at the interface. This mixing is due to a RM-instability [3], that typically occurs when two liquids/gases of different densities are accelerated one against the other by an incident shock-wave, causing initial perturbations of the interface to grow. In our case the interface is between two phases of the same material, but of different density, which is repeatedly accelerated to and fro by a series of rapid pressure oscillations. Under these conditions, and a rough initial interface, solid-liquid mixing should occur due to a RM-type instability [1], since we note that in the limit of weak shock-waves these behave as acoustic pulses [4]. Using the linear growth theory of the RM instability we predict a temperature-independent melting, with the correct dependence on the incident acoustic power. With a single free parameter we get good quantitative account of the
measured extra melting.

A RM instability will induce a layer of mixing at the interface between two materials of different densities (here the solid-liquid interface). The mixing occurs when initial perturbations of the interface between two liquids (or gases) grow after the interface is accelerated by a shock-wave \[ \text{(Fig.2)} \]. The linear single-mode theory describes the inertial growth of an initial interface undulation, due to an impacting shock-wave, gives \[ \frac{da}{dt} = k a_0 v A \Rightarrow a_{RM} = a_0 + k a_0 v A \tau \] (4)

where \( k \) is the wavevector of the interface undulation of initial amplitude \( a_0 \), \( v \) is the interface velocity due to the acceleration and \( A = (\rho_2 - \rho_1)/(\rho_2 + \rho_1) \) is the Atwood number. The sign of \( A \) relates to the phase-reversal process (Fig.2b) and is irrelevant here. Note that Eq. (4) is temperature-independent. Since in our case the solid-liquid interface is continuously accelerated to and fro by the pressure oscillations \[ \text{[10]} \], we therefore interpret the equation for the rate of growth of surface instabilities \[ \text{[4]} \], as giving the rate at which a continuous flux of solid is locally mixed with the liquid and melts.

Furthermore, unlike the usual RM scenario, we have a solid crystal on one side of the interface. Due to the rapid melting-solidification at the rough (i.e. un-faceted) solid-superfluid interface, this interface deforms (Fig.2). The linear single-mode theory describes the inertial growth of initial perturbations of the interface between two liquids \[ \text{(Eq. 4), over the entire pulse duration} \]. This means that following a single pressure oscillation, the amplitude growth described in \[ \text{[4]} \] would have been quickly damped out, and result in negligible solid-liquid mixing.

Nevertheless, in the experiment \[ \text{[1]} \], there is not a single pressure oscillation followed by inertial growth, but rather a long pulse of continuous acoustic waves. Each pressure oscillation (of \( \sim 0.1 \mu \text{sec} \) duration, frequency \( 9.9 \text{MHz} \)) will cause the solid-liquid interface to accelerate, and existing undulations to grow in amplitude. The solid-liquid interface is therefore continuously accelerated back and forth, throughout the pulse duration. The damping due to the surface tension is therefore not relevant, and we proceed to estimate the amount of solid-liquid mixing due to this continuously driven RM-instability, using the simplest linear growth theory \[ \text{[4]} \] (Eq. 4), over the entire pulse duration \( \tau \). In this case we treat the growth rate equation \[ \text{[4]} \] as a measure of the rate at which solid material is continuously mixed into the liquid, in a manner similar to the (highly exaggerated) ”spike” in Fig.2. This material then melts as it finds itself in a region which is liquid in equilibrium. The accumulated amplitude \( a_{RM} \) represents therefore the total thickness of solid that was mixed (and melted) by this process over the duration of the acoustic pulse. In particular, it \textit{does not} correspond to the final amplitude of individual interface undulations.

The interface velocity \( v \), due to the oscillating pressure of amplitude \( \delta P = \sqrt{E_{p1.2c}} \) (depending on the direction of the incident wave), is given in terms of the local material velocities at the interface, after the passing of the pressure wave \[ \text{[10]} \]: \( v = (\rho_1 v_{m,1} - \rho_2 v_{m,2})/(\rho_1 - \rho_2) \). The local material velocities following a pressure oscillation are given by \[ \text{[10]} \]: \( v_m = \delta P/\gamma_1 \). For power density \( I = 144 \text{W/m}^2 \), we therefore get: \( v \approx 0.23 \text{m/sec} \).

In Fig.1 we compare the calculated combined melting/growth \( h - a_{RM} \) with the experimental data. We find that a good agreement \[ \text{[12]} \] is achieved when we use an interface roughness aspect-ratio of \( k a_0 \approx 20 \) in Eq. (4). For a rough solid-liquid interface, which is the case in the experiments done so far \[ \text{[1, 3]} \], we would naively expect \( k a_0 \sim 1 \). This discrepancy indicates that the linear growth theory is not entirely adequate, especially as it usually deals with a single pressure pulse, while in our case the interface is continuously driven. The use of a constant amplitude \( a_0 \) in the rate equation \[ \text{[4]} \] is a rough approximation, as the amplitude of surface undulations grows between each pressure oscillation. We therefore note \( k a_0 \) as the free parameter in our model, and takes into account our ignorance of the details of the solid-superfluid interface dynamics. Note that the agreement is better for the case of melting with an acoustic pulse from the liquid side (Fig.1).

Our model further predicts that \( a_{RM} \propto \delta v_m \propto \sqrt{I} \), while the radiation-pressure driven growth \[ \text{[1]} \] predicts a linear dependence on the acoustic power. In the regime where the melting is dominated by the RM-instability, i.e. at relatively high temperatures, we therefore expect to find \( a_{RM} \propto I \). This prediction is indeed shown to be true in the experiments at 1.2K \[ \text{[3]} \] does indeed show such a power law (Fig.3).

To conclude, we have analyzed recent experiments where the solid-superfluid interface is accelerated by an acoustic-pulse, and a large source of unexplained melting was observed. It was suggested that some mechanism of energy dissipation at the interface is responsible for this unexplained melting \[ \text{[1]} \]. We proposed here a possible mechanism for such a dissipation; local solid-liquid mixing (and melting) occurs due to a dynamic instability of the interface, of the RM-type. The proposed model is in good agreement with the experimental data, and describes a specific mechanism by which the accelerated interface melts the solid.

It will be interesting to study this instability using simulations \[ \text{[4]} \], taking into account both the material flows and the solid-liquid melting/solidification dynamics. Experiments on faceted crystals, with an atomically smooth interface, should show less melting due to the RM-like
process described here. It could also be useful to repeat these experiments with a controlled initial undulation on the solid-liquid interface, of macroscopic size, and visualize its evolution when accelerated by an acoustic pulse. This initial undulation can be formed by a standing capillary wave at the interface [11].

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FIG. 2: An illustration of the evolution of a Richtmyer-Meshkov instability and the consequent material mixing (bottom line). An initial sinusoidal undulation on the interface (top line) is accelerated by an impacting shock-wave (horizontal line with the attached arrow), from either the (a) light (low density) or (b) heavy (high density) side. The inertial mixing shown here ("spike" in the bottom line), after an acceleration by a single pressure pulse, corresponds to the case of negligible surface tension, unlike the solid-superfluid $^4$He case.
FIG. 3: Measured melting depth as a function of the acoustic power (both normalized by their largest measured values) at $T=1.2\text{K}$ (squares). The straight line is a fit to illustrate the $\alpha_{RM}^2 \propto I$ behavior, as predicted by our model.