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Experiment- and computation-based identification of mechanical properties of fiber reinforced polymer composites

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Abstract. Based on testing angle-ply composite specimens with different lay-ups, the lamina elastic properties are defined by minimization of the error function determining the differences between the predicted and measured strains. The problem that a number of the measured values obtained as a result of testing exceeds that of the lamina elastic constants is resolved by using the linear algebra methods with assessment of stability of the solutions against random perturbations of the matrix and/or column of free terms. Based on the lamina elastic constants defined, an analysis of the lamina nonlinear and time-dependent properties is carried out and the lamina in-plane shear creep is described, the weak singular Abel’s operator and matrix algorithms being used. To obtain the explicit forms of stiffness and compliance matrices of laminated composites on the basis of classical lamination theory relations, the assumption that their physical nonlinearity and rheological characteristics are substantially determined by the corresponding lamina in-plane shear properties is used, and the corresponding constitutive equations are derived to describe the anisotropy of their nonlinear and time-dependent properties. Testing a number of angle-ply polymer composite specimens under time variable loading shows the applicability of the approach elaborated. The satisfactory agreement between experimental and predicted data is observed.

1. Introduction

When designing a composite material with the given structure, it is necessary to elaborate methods of calculation and prediction of mechanical properties, i.e. it is important to create models allowing us to take into account the effect of operational factors on load-carrying capacity of composite members. The classical lamination theory (CLT) is widely used for estimation of thin-walled structures [1]. In most papers [1], the lamina properties are a basis for designing and they can be defined by testing unidirectional or angle-ply specimens. For adequate description of mechanical behavior of the lamina in a layered specimen, the elastic properties of the lamina may be defined by using the test results obtained for multidirectional fiber reinforced materials [2]. Assessment of the lamina elastic constants is the first and most important stage. They are determined by minimization of the error function measuring the differences between the predicted and measured strains. When determining the elastic constants, a number of the measured values exceeds, as a rule, that of the constants. Such problem
may be correctly resolved by using the methods of linear algebra with assessment of stability of the solutions against random perturbations of the matrix and/or column of free terms. It should be noted that there are differences in errors of the measured values resulting in necessity of adjustment of the elastic constants by using the weighted least square method. Increased values and duration of external loads necessitate elaborating the methods of estimation of nonlinear and time-dependent properties of composite materials. Since the laminated composite properties are determined by the nonlinear and time-dependent properties of the lamina, analysis of the corresponding properties of the lamina is also carried out on the basis of test results for unidirectional or angle ply composite materials. Analysis of experimental data obtained for carbon fiber reinforced plastics showed that physical nonlinearity and rheological characteristics are substantially determined by the corresponding in-plane shear properties of the lamina. In this connection it is possible to generalize the lamina elasticity relations and to construct the constitutive equations for the laminated composite material allowing the regularities of its nonlinear and rheological behavior to be taken into consideration.

2. Experimental procedure (quasi-static tensile loading)

4-ply plates of carbon plastic (300x300x0.125 mm) were used to make 20±0.2 mm wide specimens for testing. Fiber reinforcement configurations of the specimens were 0\textdegree, ±20\textdegree, ±40\textdegree, ±50\textdegree, ±70\textdegree, 90\textdegree. Both axial and lateral strains were measured. The procedure of testing was as follows: at first, the specimens were subjected to relatively small loads (within elastic strains), then to unloading and finally to reloading up to their failure. Linear stress-strain response was observed for 0\textdegree and 90\textdegree configurations. Visco-plastic response took place for other specimens and even in case of unloading from relatively small loads, the nonlinear effect (hysteresis loops) and residual strains were evident. The most significant nonlinear response was observed for [±40\textdegree] lay up.

3. Identification of the lamina elastic constants

According to elasticity theory, stresses and strains are connected with each other through the stiffness and compliance matrices [3]:

\[
\{\varepsilon_{x,y}\} = [S] \{\sigma_{x,y}\}; \quad \{\sigma_{x,y}\} = [G] \{\varepsilon_{x,y}\}. \tag{1}
\]

In case of an angle ply laminate under uniaxial tensile loading, identification of the lamina elastic constants may be carried out by using the following relations between tensile stress \(\sigma_x\) and axial \(\varepsilon_x\) and lateral \(\varepsilon_y\) strains [2]

\[
g_{xx} \varepsilon_x + g_{xy} \varepsilon_y = \sigma_x; \quad g_{yx} \varepsilon_x + g_{yy} \varepsilon_y = 0 \tag{2}
\]

where \(g_{xx}, g_{xy}, g_{yx}\) are elements of the stiffness matrix of the laminate.

The stiffness matrix elements in (2) can be expressed in terms of elements of the lamina stiffness matrix [3]

\[
\begin{align*}
(c^4 S_{11}^0 + s^4 S_{22}^0 + 2s^2 c^2 S_{12}^0 + 4s^4 c^2 S_{66}^0) \varepsilon_x + (s^4 c^2 S_{11}^0 + s^2 c^4 S_{12}^0 + (s^4 + c^4) S_{66}^0 - 4s^2 c^2) \varepsilon_y &= \sigma_x \\
(s^4 c^2 S_{11}^0 + s^2 c^4 S_{22}^0 + (s^4 + c^4) S_{66}^0 - 4s^2 c^2) \varepsilon_x + (s^4 g_{11}^0 + c^4 g_{22}^0 + 2s^2 c^2 g_{12}^0 + 4s^4 c^2 g_{66}^0) \varepsilon_y &= 0
\end{align*}
\]

where \(s = \sin \theta, c = \cos \theta, \theta\) is the angle between the loading direction and fiber one (ply angle).

Taking into account that \(\varepsilon_x = \sigma_x E_x^{-1}\) and \(\varepsilon_y = -v_{xy} \sigma_x E_x^{-1}\), equations (3) can be rewritten in the following matrix form

\[
\begin{align*}
A_x(\theta, E_x, v_{xy}) \tilde{g}^0 &= 1 \\
A_y(\theta, E_x, v_{xy}) \tilde{g}^0 &= 0
\end{align*}
\]

\[
\begin{align*}
A_x(\theta, E_x, v_{xy}) \tilde{g}^0 &= 1 \\
A_y(\theta, E_x, v_{xy}) \tilde{g}^0 &= 0
\end{align*}
\]
where \( A_x(\theta, E_x, \nu_{xy}) = E_x \left[ c^4 - \nu_{xy} s^2 c^2; s^2 - \nu_{xy} s^2 c^2; 2s^2 c^2 - \nu_{xy} (s^4 + c^4); 4s^2 c^2 (1 - \nu_{xy}) \right] \) and 
\( A_y(\theta, E_x, \nu_{xy}) = E_x \left[ s^2 c^2 - \nu_{xy} s^2 c^2; s^2 c^2 - \nu_{xy} s^2 c^2; (s^4 + c^4) - 2\nu_{xy} s^2 c^2 - \nu_{xy}; -4s^2 c^2 (1 + \nu_{xy}) \right] \) are matrix rows dependent on the ply angle and corresponding experimental values of elastic modulus and Poisson’s ratio of each of the laminates, \( \vec{g}^0 = \{ g_{11}^0; g_{22}^0; g_{12}^0; g_{66}^0 \} \) is the vector of unknown elements of the lamina stiffness matrix, \( T \) is transposition sign.

Average values of the elastic modulus and Poisson’s ratio determined on linear parts of stress-strain curves are used in equation system (4) for each of the fiber reinforcement lay ups. So, we have full rank matrices on columns, the calculation was carried out on contracted matrices \( A_x \) and \( A_y \). For six fiber reinforcement configurations we have a system of twelve equations in unknown elements of the lamina stiffness matrix:

\[
\begin{bmatrix}
A_x \\
A_y
\end{bmatrix}
\begin{bmatrix}
\vec{g}^0
\end{bmatrix}
= 
\begin{bmatrix}
1 \\
0
\end{bmatrix}
\tag{5}
\]

The column of free terms consists of six sequential ones and zeroes. The solution of equation system (5) was found by using the pseudoinverse of the matrix \( A = \begin{bmatrix} A_x \\ A_y \end{bmatrix} \) designated as \( (A)^+ \)

\[
\begin{bmatrix}
\vec{g}^0
\end{bmatrix}
= (A)^+ \begin{bmatrix}
I
\end{bmatrix}
\tag{6}
\]

where \( \begin{bmatrix} I \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \) is the column of free terms.

The solution found by using (6) minimizes the error function of predicted and measured values. Such solution may not be recognized as acceptable because of poor accuracy for unknown values and this is particularly true for the longitudinal elastic modulus of the lamina. Condition number of \( (A) \) was found to be large and the angle between the vector of free terms and the columns projected by the found solution onto space was too big \( \approx 32^0 \). It is obvious that the deviation of \( g_{11}^0 \) is considerably different from that of other parameters and such difference may achieve two orders of magnitude. Therefore, it is necessary to use the weighted least square method implying that the initial system of linear equations is multiplied from the left by a diagonal weight matrix \( W \)

\[
(WA)\begin{bmatrix}
\vec{g}^0
\end{bmatrix}
= (W)\begin{bmatrix}
I
\end{bmatrix}
\tag{7}
\]

where the first element of \( W \) is to be dominant. For solving this problem, the following weight matrix was used: \( W = \text{diag}(100 2 2 2 1 1 3 1 2 1 2 2) \). Then the solution system of linear equations (7) does not coincide with solution (6) and is determined by the following relation

\[
\begin{bmatrix}
\vec{g}^0
\end{bmatrix}
= (WA)^+ (W)\begin{bmatrix}
I
\end{bmatrix}
\tag{8}
\]

Condition number of \( (WA) \) was equal to 2.7 and the angle between the vector of free terms and the columns projected by the found solution onto space was approximately \( 1.2^0 \). The lamina elastic constants were calculated by using their connections with the stiffness matrix elements [2]:

\[
E_1 = g_{11}^0 - \left( g_{12}^0 \right)^2 \left( g_{66}^0 \right)^{-1};
E_2 = g_{22}^0 - \left( g_{12}^0 \right)^2 \left( g_{66}^0 \right)^{-1};
\nu_{12} = g_{12}^0 \left( g_{22}^0 \right)^{-1};
G_{12} = g_{66}^0.
\]

Numerical values of the lamina elastic constants were as follows: \( E_1 = 158 \text{ GPa}, E_2 = 4.24 \text{ GPa}, \nu_{12} = 0.311, G_{12} = 2.94 \text{ GPa} \). A comparison between the predicted and experimentally defined values for \([\pm40]_2 \) lay-up is given in Figure 1.
4. Matrix algorithm of construction of the constitutive equations for laminates

In general case, the constitutive equations for the laminate can be represented as follows:

\(( P + Qf )^x = y \) (9)

where \( P \) is the matrix characterizing the elastic properties of the laminated composite plate, \( Q \) is the matrix taking into account the nonlinear or time-dependent properties and \( f \) is the function describing the nonlinear or time-dependent properties.

For convenient representation of the constitutive equations and their inverse, it is necessary to do the following transformations

\(( P + Qf ) = P(I + P^{-1}Qf) = PV_{\text{diag}}(1 + m_f)V^{-1} \) (10)

where the following matrix diagonalization: \( P^{-1}Q = V_{\text{diag}}(1 + m_f)V^{-1} \) is applied.

To inverse the matrix presented in equation (10), the following function of matrix is used

\( \psi[V_{\text{diag}}(1 + m_f)V^{-1}] = V\psi[\text{diag}(1 + m_f)]V^{-1} \). (11)

According to (11) the inverse can be represented as follows:

\( [V_{\text{diag}}(1 + m_f)V^{-1}]^{-1} = V_{\text{diag}}[(1 + m_f)^{-1}]V^{-1} \). (12)

If \( f \) is a resolvent operator \( R^*(\mu) \), then by its inversion \( \left[ 1 + m_fR^*(\mu) \right]^{-1} = 1 - m_fR^*(\mu - m) \), then equation (10) can be written as

\( \left[ P + QR^*(\mu) \right]^{-1} = \left[ P(1 + QR^*(\mu))^{-1} \right] = V_{\text{diag}}\left[ 1 - m_fR^*(\mu - m) \right]V^{-1}P^{-1} \). (13)

5. Hereditary elasticity of layered composites

An analysis of the stress-strain curves obtained for the angle ply carbon fiber reinforced plastics KMU-4L shows the presence of rheological and time-dependent properties. Moreover, such properties are induced by the corresponding in-plane shear characteristics of the lamina. It is particularly

Figure 1. Stress-strain curves of 3 specimens with ±40 configuration; the straight lines show how the predicted elastic constants are in accordance with the experimental data in the linear parts of the curves.
noticeable for the stress-strain curves of \([\pm40]_2\) lay-up specimens. It should be noted that when the stress increases the residual strain is also growing.

In [4], the analysis of time-dependent properties of angle ply carbon reinforced plastics was carried out and the possibility of construction of hereditary constitutive equations was shown. It was assumed that the laminate time-dependent properties are mainly defined by the lamina in-plane shear rheological response which can be described by the hereditary constitutive equation with Abel’s kernel. Using the corresponding operator form for shear elastic modulus and compliance and the classical lamination theory relations, a method for construction of constitutive equations for laminates with various lay-ups was elaborated. The constitutive equation parameters under in-plane shear were defined as a result of testing of \([\pm40]_2\) lay-up specimens and the instant shear modulus and the parameters of Abel’s kernel were subsequently determined. The hereditary constitutive equation for carbon fiber reinforced plastics under in-plane shear can be written as

\[
\gamma_{12} = G_{12}^{-1}(1 + kI_a)^{t_{12}} \tag{14}
\]

where \(I_a^{t_{12}} = \int_0^1 \frac{t - \xi}{\Gamma(1 + \alpha)} \tau_{12}(\xi)d\xi\) is Abel’s operator having the singularity parameter \(-1<\alpha<0\), \(k\) is the parameter of the constitutive equation.

After substituting representation (14) into the expression for compliance matrix, the latter can be written as a sum of the elastic compliance matrix and the matrix having the hereditary operator as a multiplier, i.e.

\[
S_{12} = S_{12}^0 + S_{12}^*I_a \tag{15}
\]

where \(S_{12}^0\) is the matrix of instant compliance coefficients.

Inverse of the compliance matrix (15) can be accomplished immediately or following schema (9). It is obvious that stiffness matrix can be defined as difference between stiffness matrix representing matrix of instant values of stiffness and matrix characterizing time-dependent properties

\[
G_{12} = G_{12}^0 - S_{12}^* \Xi (-k) \tag{16}
\]

As for Abel’s operator in (15), Rabotnov’s fractional exponential function is the multiplier in (16).

In equation (16), the fact that the resolvent of Abel’s operator is Rabotnov’s fractional exponential function \(\Xi_n(-k) = t^{1-n} \sum_{n} \left(-kt^{1-n}\right)^n\left\{\Gamma[1+(1+\alpha)(1+n)]\right\}^{-1}\) [5] is used.

The stiffness matrix of the laminate (in particular of angle ply specimen) can be defined with the help of the following relationship: \(G_{xy} = \sum_j T_j G_{12}^j T_j^T\) which represents herein the difference of matrices representing elastic instant and time dependent properties of the laminate accordingly

\[
G_{xy} = G_{xy}^0 - G_{xy} \Xi (-k) \tag{17}
\]

In [4], an algorithm of representation and inverse of the laminate stiffness matrix was elaborated. After diagonalization of matrix multiplication \(G_{xy}^{-1}G_{xy} = Q\text{diag}\left(\lambda_i\right)Q^{-1}\), the stiffness and compliance matrices can be represented as

\[
G_{xy} = G_{xy}^0 Q\text{diag}\left(1 - \lambda_i \Xi (-k)\right)Q^{-1} \tag{18}
\]

\[
S_{xy} = Q\text{diag}\left(1 + \lambda_i \Xi (-k - \lambda_i)\right)Q^{-1}S_{xy}^0 \tag{19}
\]

where \(\lambda_i\) are the eigenvalues dependent on the lay-up and independent on the coordinate system.

Multipliers for elastic instant matrix of stiffness and compliance may be referred to in terms of perturbation introduced by time dependent properties of the laminate. In case of absence of time
dependent properties the matrices degenerate into the corresponding matrices of stiffness and compliance defined according to the classical lamination theory equations.

It is convenient to obtain the lamina in-plane shear time-dependent properties on the basis of tensile testing readings of [±45] lay-up. In [4], the experimental data for [±40] specimens under time variable loading were treated to define the parameters of time dependent response. The value of the in-plane shear instant elastic modulus was equal to 4.6 GPa, Abel’s kernel parameters were as follows: 

\[ k = 1.0415 \min^{-1/2} \] and \( \alpha = -0.9 \). A comparison of the predicted (curve) and experimental (points) data is given in Figure 2.

![Figure 2. Creep curve of KMU-4L [±40] lay-up carbon fiber reinforced plastics](image)

In correspondence with the algorithm described above the construction of the hereditary constitutive equation can be obtained by substituting expressions (18) and (19) into equations (1). By using asymptotic properties of the resolvent operators, it is possible to evaluate the long-duration behavior of fiber reinforced polymer composites.

6. Conclusions

The basic stages and methods of defining the elastic constants and material functions allowing one to describe anisotropy of elastic and viscoelastic properties of layered carbon fiber reinforced plastics are presented. The methods for construction of constitutive equations are proposed. The approach elaborated can be used for design and prediction of mechanical behavior of layered composites under time variable loading.

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