Do medium range ensemble forecasts give useful predictions of temporal correlations?

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Abstract

Medium range ensemble forecasts are typically used to derive predictions of the conditional marginal distributions of future events on individual days. We assess whether they can also be used to predict the conditional correlations between different days.

1 Introduction

We consider the question of how to make probabilistic forecasts of future temperatures over the 1-10 day timescale. A complete specification of the distribution of future temperatures over this time period would consist of information about the marginal distributions of temperature on each day and about the dependencies between the temperatures on different days. Investigating these marginal distributions and the dependency structure in full generality is rather difficult, and so we will make the approximation that temperature is normally distributed. This simplifies the problem greatly since in this case the distribution of future temperatures is described completely by 10 means, 10 variances, and a 10 by 10 correlation matrix.

Ensemble forecasts can be used to derive predictions of the mean, and such predictions are better than predictions derived from single integrations of forecast models at the same resolution. Ensemble forecasts can also be used to derive predictions of the variance. There are a number of ways this can be done. We have analysed some of these in detail in previous articles, including using linear regression on the ensemble mean, and spread regression on the ensemble mean and spread (see Jewson et al. (2003), Jewson (2003a), Jewson (2003b) and Jewson (2003c)). We have only been able to show that the spread of the ensemble improves the skill of forecasts when the calibrated forecasts are evaluated in-sample (i.e. the calibration and evaluation are performed on the same data that is used to calculate the calibration parameters). For out of sample forecasts it seems that it is very hard to prove that one can beat linear regression on the ensemble mean as a calibration method. We argue that this is because the predictable part of the variability in the variance is small and because only short records of past forecasts are available for training calibration models.

We now address the question of whether ensemble forecasts can be used to predict the correlations between temperatures on different days of the forecast, or whether such predictions should be made using past-forecast error statistics. The potential advantage of deriving the prediction of correlations from the ensemble is that it will then be flow-dependent. It is highly plausible that as the atmospheric state changes the correlations between forecast errors on different days of the forecast should change, and ensembles have the potential to predict that effect whereas past forecast error statistics do not. On the other hand numerical model forecasts are very prone to biases and our experience with trying to predict the variance of forecast errors from the ensemble has taught us that nothing from an ensemble forecast should be taken for granted: everything needs careful analysis and calibration to extract the useful information.

The author became interested in the question of whether ensembles can predict temporal correlations because it arises in a simple weather derivative pricing situation, as described in Jewson and Caballero (2002). Consider a weather option based on December mean temperature. To calculate the fair value of such an option one has to estimate the distribution of the settlement index i.e. the distribution of December mean temperatures. When it is estimated many months prior to the start of the contract, this
distribution can be derived entirely from historical data, while when it is estimated immediately prior to or during the contract it should be derived from a combination of historical data and forecasts. For example, imagine that we are estimating this distribution on the 1st of December. The expectation of the settlement index is then given by the sum of two values. The first of these values is the contribution from Dec 1st to Dec 10th (which can be estimated from a forecast) and the second is the contribution from Dec 11th to Dec 31st (which can be estimated from historical data). Thus estimating the mean of the settlement index is rather straightforward. Estimating the variance of the index, however, is more complicated, and involves making estimates of the variances of temperatures on each day of the month (31 values) and the correlations between temperatures on different days of the month (a 31x31 element matrix). One part of estimating this 31x31 matrix is to estimate the correlations between the days of the forecast (a 10x10 element sub-matrix) and this is what motivates the question of whether those correlations can be derived from the ensemble. A more general discussion of how the rest of the correlations and variances in this problem can be calculated is given by Jewson and Caballero (2002).

2 Data

We will base our analyses on one year of ensemble forecast data for the weather station at London’s Heathrow airport, WMO number 03772. The forecasts are predictions of the daily average temperature, and the target days of the forecasts run from 1st January 2002 to 31st December 2002. The forecast was produced from the ECMWF model (Molteni et al., 1996) and downscaled to the airport location using a simple interpolation routine prior to our analysis. There are 51 members in the ensemble. We will compare these forecasts to the quality controlled climate values of daily average temperature for the same location as reported by the UKMO.

There is no guarantee that the forecast system was held constant throughout this period, and as a result there is no guarantee that the forecasts are in any sense stationary, quite apart from issues of seasonality. This is clearly far from ideal with respect to our attempts to build statistical interpretation models on past forecast data but is, however, unavoidable: this is the data we have to work with.

Throughout this paper all equations and all values are in terms of double anomalies (have had both the seasonal mean and the seasonal standard deviation removed). Removing the seasonal standard deviation removes most of the seasonality in the forecast error statistics, and partly justifies the use of non-seasonal parameters in the statistical models for temperature that we propose.

3 Models

There are potentially many ways that one could address the question of whether or not the ensemble can be used to predict temporal correlations. Since this question has not, apparently, been addressed before, we will take a simple and pragmatic approach, which works as follows. We will model the mean and variance of the forecast using the spread regression model of Jewson et al. (2003) and will perform in-sample calibration of all the forecasts using this model. We are happy to perform this mean-variance calibration entirely in-sample because we are addressing the question of whether the correlations contain useful information, rather than whether the mean and variance of the ensemble do. We note that this calibration does not affect the correlations between days.

We will model the correlation matrix between the days of the forecast as a weighted sum of the two matrices \( C_{\text{past forecast error statistics}} \) and \( C_{\text{ensemble forecast}} \) as follows:

\[
C_i = \lambda C_{\text{past forecast error statistics}} + (1 - \lambda)C_i\text{ensemble forecast}
\]

where \( C_i \) is the modelled correlation matrix on day \( i \), \( C_{\text{past forecast error statistics}} \) is a stationary matrix derived from past forecast error statistics, and \( C_i\text{ensemble forecast} \) is a time-varying matrix derived from the ensemble forecast. We will vary the weighting \( \lambda \) of these matrices from zero to one and derive the combination that gives the optimum probabilistic forecast, defined as that forecast which maximises the likelihood.

Our first test is an in-sample test that fits the past forecast error based correlation matrix on the whole year of data, and then tests it on the same year of data. One can argue that this test is not very useful, since we are fitting a very large number of parameters (the 56 independent elements of the correlation matrix) on a relatively small amount of data. There is a very large danger of over-fitting.

Our second test avoids this problem by fitting the past forecast error based correlation matrix on the first six months of data and testing it on the second six months of data, and vice versa.
The likelihood score that we will attempt to maximise by combining the two correlation matrices is given by the multivariate normal distribution over all forecast days and all leads. We will assume that forecasts are independent from day to day but not from lead to lead. The likelihood then becomes:

$$L = \prod_{i=1}^{i=N} \frac{1}{\sqrt{2\pi D_i}} \exp \left( -\frac{1}{2}(e_i^T C_i^{-1} e_i) \right)$$

where $N$ is the number of days of forecasts, $n$ is the number of forecast leads, $e_i$ is a vector of forecast errors on day $i$ (of length $n$), $C_i$ is the estimated forecast error covariance matrix on day $i$ (of dimension $n$ by $n$) as given by equation 1, and $D_i$ is the determinant of this matrix.

4 Results

Our in sample results are shown in figure 1. The horizontal axis shows the weight applied to the past forecast error based correlation matrix (the $\lambda$ in equation 1). We have plotted the log-likelihood, given by the log of the likelihood from equation 2. As we vary the correlation matrix, the likelihood changes. We see that the highest values for the likelihood are given when we weight the two correlation matrices roughly in the proportions 90% (for the past forecast error based matrix) to 10% (for the ensemble forecast based matrix). In spite of the caveats we have about the in-sample nature of this test, the results are somewhat interesting in that they certainly imply that the ensemble forecast based correlation matrix contains some useful information. Because it has been performed in sample we would expect this test to strongly favour the past forecast error based matrix, and hence to be biased towards higher values of $\lambda$ than out of sample tests.

The out of sample results, in which the data being predicted is different from the data used to calculate the past forecast error correlation matrix, are shown in figures 2 and 3. We see that in both cases the likelihood has a maximum at around 80% (in fact, the exact numbers are 77% and 78%). As we expected, the optimum combination is at a lower level for the weight than for the in-sample tests. The results for the two tests are remarkably consistent, giving us reasonable faith that sampling error is not playing too important a role.

If we were forced to use either the past forecast error based correlation matrix or the ensemble based correlation matrix, then we see clearly that the past forecast error based correlation matrix performs better i.e. gives higher values for the log-likelihood. However there is a wide range of linear combinations of the two matrices that performs better still.

In figure 4 we show an example of the correlation time series derived from the optimum values of lambda. This example is based on data from the first 50 days of the predicted data from the first of the out of sample tests, and shows the correlation between leads 2 and 3. The two solid lines show the correlations from the past forecast errors and the ensemble forecast, while the dotted line shows the weighted combination of these two correlations using the optimum value for lambda. We see that the ensemble based correlation is on average lower than the correlation based on past forecast errors, but that for some values it is higher. The optimum weighting of the two reduces the variability of the predicted correlation very dramatically.

5 Summary

We have investigated whether the temporal correlations derived from an ensemble forecast are useful predictors for the correlations between the distributions of possible temperatures on each day of the forecast. We find:

- when we compare probabilistic forecasts derived from a correlation matrix based on past forecast error statistics with forecasts derived from a correlation matrix based on the ensemble, those based on past forecast error statistics are better
- a linear combination of the two correlation matrices performs better than either matrix on its own
- the optimum proportions of the two matrices seem to be fairly robustly given by around 77% for the past forecast error correlation matrix and around 23% for the ensemble derived correlation matrix

We have deliberately used a very simple methodology. In particular, we have used just a single weight for the whole correlation matrix. One could also consider using different weights for the different elements of the matrix. This might give better results, since one could certainly imagine that the ensemble would give...
better correlations at shorter leads, while the past forecast error based correlations would perform better at longer leads. If having separate weights for each member of the matrix proves unwieldy (there would be 56 different weights) one could consider parametrising the structure of the weights with a smaller number of parameters.

In terms of further work, there are two other questions related to the predictability of correlations that immediately spring to mind. The first is to ask: do ensemble forecasts contain useful correlation information about the correlation between stations? One could look at both the instantaneous cross-correlations, and the lag cross-correlations. The second is: do ensembles contain useful information about the correlation between different variables i.e. between temperature and precipitation at the same location and the same lead time, between temperature and precipitation at a different location but the same lead time, or even between temperature and precipitation at a different location and at a different lead time.

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Figure 1: The log-likelihood for one year of forecast data versus the weighting used to derive the inter-lead correlation matrix.
Figure 2: The log-likelihood for 6 months of forecast data versus the weighting used to derive the inter-lead correlation matrix. In this case the past forecast error based component of the correlation matrix was calculated on a different 6 month data period from that used to calculate the log-likelihood.
Figure 3: As figure 2 but with the data periods exchanged.
Figure 4: An example of the values for the inter-lead correlations, showing the correlations between lead 2 and lead 3. The solid lines show the correlations based on an ensemble forecast (varying line) and on past forecast error statistics (constant values), and the dotted line shows the optimum combination of these two.