Singlet-to-triplet ratio in the deuteron breakup reaction 
$p d \rightarrow p n p$ at 585 MeV

Yu.N. Uzikov$^{a,b}$, V.I. Komarov$^a$,

$^a$JINR, LNP, Dubna, 141980, Moscow Region, Russia

$^b$Kazakh State University, Almaty, 480121 Kazakhstan

F. Rathmann$^c$, H. Seyfarth$^c$

$^c$Institut für Kernphysik, Forschungszentrum Jülich, 52425 Jülich, Germany

Abstract

Available experimental data on the exclusive $p d \rightarrow p n p$ reaction at 585 MeV show a narrow peak in the proton-neutron final-state interaction region. It was supposed previously, on the basis of a phenomenological analysis of the shape of this peak, that the final spin-singlet $p n$ state provided about one third of the observed cross section. By comparing the absolute value of the measured cross section with that of $p d$ elastic scattering using the Fäldt-Wilkin extrapolation theorem, it is shown here that the $p d \rightarrow p n p$ data can be explained mainly by the spin-triplet final state with a singlet admixture of a few percent. The smallness of the singlet contribution is compatible with existing $p N \rightarrow p N \pi$ data and the one-pion exchange mechanism of the $p d \rightarrow p n p$ reaction.

Key words: proton deuteron scattering, final state interaction

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1 Corresponding author: Yu.N. Uzikov, Laboratory of Nuclear Problems, JINR, Dubna, 141980, Moscow Region, Russia; e-mail address: uzikov@nusun.jinr.ru; FAX: 7 09621 66666
Recently, the $NN \to NN\pi$ reactions with the formation of a spin-singlet $NN$ pair in the final state have received a renewed interest. Analyzes of the experimental data obtained at COSY [1], CESLIUS [2] and LAMPF [3], employing the largely model-independent approach of Ref. [4], show that the singlet channel is strongly suppressed in the $pp \to pm\pi^+$ reaction at proton kinetic energies between 300 and 800 MeV [5–7]. Direct measurements of the singlet channel in the reaction $pp \to pp\pi^0$ at RCNP [8] and CESLIUS [9] at 300–400 MeV indicate a singlet-to-triplet ($s/t$) ratio of about 1% in collinear kinematics, which increases up to $\sim 10\%$ as the cm scattering angle approaches 90°. The dominance of the triplet state can be related to the excitation of a $\Delta$-isobar in the intermediate state [7].

The measured pion production cross section in $pp$ collision allows one to estimate qualitatively the $s/t$ ratio in the deuteron breakup reaction $pd \to \{pn\}p$, when the quasi-bound $\{pn\}$ pair is observed in the final state interaction (fsi) region and the second proton is detected at large cm scattering angle ($\theta^* > 90^\circ$). It is well known that in backward elastic $pd$ scattering $pd \to dp$ the triangle diagram of one-pion exchange with the subprocess $pp \to d\pi^+$ considerably contributes in the $\Delta$-region [10]. This mechanism describes well the energy dependence of the $pd \to dp$ cross section at $\theta^* = 180^\circ$ and, in addition, explains the qualitative agreement between the proton vector analyzing power $A_y$ from $pp \to d\pi^+$ and $pd \to dp$, observed in the $\Delta$-region [11]. If one assumes that the triangle diagram with one-pion exchange dominates in the break-up $pd \to \{pn\}p$ at large scattering angles, one would expect in this reaction a similar $s/t$ ratio of a few percent, as observed in $pp \to pm\pi^+$. For the $\Delta$ mechanism of the $pd \to pnp$ reaction, which dominates the one-pion exchange triangle diagram, the product of spin and isospin factors yields a $s/t$ ratio of $\frac{1}{27}$ [12]. In contrast, one should expect a higher $s/t$ ratio of about $\frac{1}{3}$ for the one-nucleon exchange mechanism of the deuteron breakup [12]. It was suggested in Refs. [12–14] to directly measure the singlet channel in the reaction $pd \to (pp)(0^\circ) + n(180^\circ)$ with a $pp$ pair of low relative energy $E_{pp} = 0–5$ MeV emitted in forward direction and a neutron going backward. Due to a consid-
erable suppression of the $\Delta$-mechanism in this reaction [12] other mechanisms, more sensitive to the short-range structure of the deuteron, are expected to become important [15].

Recent experimental data on the deuteron breakup reaction $dp \to pnp$ with two outgoing nucleons in the fsi region were obtained at Saclay [6] at $T_d = 1.6$ GeV in semi-inclusive kinematics and at Dubna [16] at $T_d = 2-5$ GeV. Earlier, a kinematically complete exclusive experiment had been performed at Space Radiation Effects Laboratory (SREL) in Virginia [17] at a proton beam kinetic energy of $T_p = 585$ MeV, covering a region of low relative neutron-proton energy $E_{np} = 0 - 5$ MeV outside of quasi-free $pN$-kinematic. A clear peak was observed in the five-fold cross section at $E_{np} \sim 0$. Using the Migdal-Watson approximation [18,19], the authors of Ref. [17] described the shape of the fsi peak by assuming a $s/t$ ratio of one third, which corresponds to the spin statistical weights of the singlet and triplet states. A smaller $s/t$ ratio of about 10% was obtained from the data of Ref. [6]. The difference is possibly related to the different cm scattering angles of protons ($\theta^* \sim 90^\circ$ in Ref. [17] and $\theta^* \sim 180^\circ$ in Ref. [6]).

However, the fitting procedure described in Ref. [17] is rather ambiguous since the absolute value of neither the triplet nor the singlet cross section is known and was arbitrarily introduced. The $s/t$ ratio can be deduced in principle from the data, taking into account only the strong difference in shape of the singlet and triplet peaks (see, for example, Ref. [1]). Unfortunately, the low resolution in $E_{np}$ and limited statistics in the peak do not allow one to effectively use this procedure for the data of Ref. [17]. In this case the knowledge of the absolute value of the triplet (or singlet) cross section is necessary in order to determine the $s/t$ ratio. The triplet cross section can be calculated in a model-independent way in terms of the large angle proton-deuteron elastic scattering. Here we employ the approach described in Refs. [4–7] to determine the triplet cross section and on this basis reanalyze the data of Ref. [17].

The SREL data are shown in Fig. 1 as a function of the detected proton
momentum. At energies $E_{np}$ of about 1 MeV the cross section is strongly influenced by the $np$ fsi. The shape of this peak is well described by the Migdal-Watson formulae [18,19], which take into account the nearby poles in the fsi triplet ($t$) and singlet ($s$) $pn$–scattering amplitudes

$$d\sigma_{s(t)} = FSI_{s(t)}(k) K |A_{s(t)}|^2. \quad (1)$$

Here $A_{s(t)}$ is the production matrix element for the singlet (triplet) state, $K$ is the kinematical factor, and $FSI_{s(t)}$ is the Goldberger-Watson factor [19]. The latter can be written in the form

$$FSI_i = \frac{k^2 + \beta_i^2}{k^2 + \alpha_i^2}, \quad (2)$$

where $i = s, t$. The relative momentum in the $pn$ system at the relative kinetic energy $E_{np} = k^2/m_N$ is denoted by $k$, $m_N$ is the nucleon mass. The parameters $\alpha$ and $\beta$ are determined by known properties of the on-shell $NN$-scattering amplitudes at low energies: $\alpha_t = 0.232 \text{ fm}^{-1}$, $\alpha_s = -0.04 \text{ fm}^{-1}$, $\beta_t = 0.91 \text{ fm}^{-1}$, $\beta_s = 0.79 \text{ fm}^{-1}$ [20]. Important new information on the mechanism of $pd \rightarrow pnp$ and off-shell properties of the $NN$ system is hidden in the matrix elements $A_{s(t)}$, in particular in the ratio

$$\zeta = \frac{|A_s|^2}{|A_t|^2}. \quad (3)$$

One can find from Eqs. (1) and (3) the following parametrization for the full singlet plus triplet cross section [7]

$$d\sigma_{s+t} = \left(1 + \zeta \frac{FSI_s}{FSI_t}\right) d\sigma_t, \quad (4)$$

where $d\sigma_t$ is the triplet cross section. The second term in the brackets of Eq. (4) corresponds to the singlet contribution.

Using the Fälldt-Wilkin extrapolation [4], which relates the bound and the scattering S-wave functions in the triplet state at short $pn$ distances $r < 1 \text{ fm}$, and by taking into account the short-range character of the interaction mechanism, one can find a definite relation between the matrix elements of
the \( pd \rightarrow \{pn\}p \) and \( pd \rightarrow dp \) reactions \([4,6]\). The triplet differential cross section in the laboratory system can then be written as

\[
\frac{d^5\sigma_t(pd \rightarrow pnp)}{dp_1 \, d\Omega_1 \, d\Omega_2} = \frac{1}{16\pi^3} \frac{p_0^2 p_3^2 s f^2(k^2)}{p_0 m_d E_1 [p_2^2 E_n - \mathbf{p}_2 \cdot \mathbf{E}_2]} \frac{d\sigma}{d\Omega^*}(pd \rightarrow dp), \tag{5}
\]

where

\[
f^2(k^2) = \frac{2\pi m_N}{\alpha_t(k^2 + \alpha_t^2)} \tag{6}
\]
is the Fäldt-Wilkin factor \([6]\), \( d\sigma/d\Omega^* \) is the \( pd \rightarrow pd \) cm cross section. In Eq. (5) \( s \) denotes the squared invariant mass of the \( pd \) system, \( m_d \) is the deuteron mass, \( p_0 \) is the beam momentum, \( E_i \) and \( \mathbf{p}_i \) \((i = 1, 2, n)\) are the laboratory energy and momentum of the \( i \)-th nucleon in the final state. The indices 1 and 2 refer to the protons and the neutron is referred as \( n \). The proton scattering angles in the \( pd \rightarrow \{pn\}p \) and \( pd \rightarrow dp \) processes can be related to each other, if the difference between the effective mass of the final \( \{pn\} \) system and that of the deuteron is disregarded, as suggested in Ref. \([7]\). The result presented by Eq. (5) should i) be valid at low relative energies \( E_{np} \), ii) be independent of the form of the \( NN \)-potential and details of the large-angle \( pd \)-scattering mechanism, and iii) it automatically includes the fsi effects in the triplet \( pn \) system. On the other hand, this method cannot be used for small-angle \( pd \)-scattering since the \( NN \)-scattering and bound-state wave functions are very different at large \( NN \) distances, i.e. at low transferred momenta.

The value of the differential cross section \( d\sigma/d\Omega^* \) in Eq. (5) at \( T_p = 590 \) MeV and \( \theta^* = 92.7^\circ \) amounts to \( (30.4 \pm 0.8(\text{stat.}) \pm 2.9(\text{syst.})) \) \( \mu \text{b/sr} \) \([21]\). The SREL experiment \([17]\) was carried out at almost the same scattering angle \( (\theta_2^* = 93.95^\circ \) for \( E_{np} = 0)\). Other available data \([22,23]\) give larger values for the \( pd \rightarrow dp \) cross section under similar kinematic conditions. Therefore, in order to estimate an upper limit for the \( s/t \) ratio we use here only the data from Ref. \([21]\). As one can see from Fig. 1a, the triplet cross section calculated using Eq. (5) (dashed line) overshoots the experimental points in the central region around \( E_{np} \sim 0 \), but agrees with the data for \( E_{pn} > 3 \) MeV. However, a sizable effect arises from averaging of the theoretical results over the experimental angular acceptance.
and resolution of the spectrometer. In order to take these into account, we have carried out a five-dimensional integration of the cross section from Eq. (5) with Gaussian distributions, where smearing parameters $\sigma_\theta = 2.55^\circ$ for the polar angles of and $\sigma_p/p = 0.015$ for the momentum $p$ were used in accordance with Ref. [17]. For the azimuthal angles $\phi_1$ and $\phi_2$ the averaging was carried out in the interval $\Delta \phi = \pm 0.4^\circ$ with a rectangular distribution.

After smearing we obtain good agreement both in the shape and in absolute value between one data set (Fig. 1a) and a pure triplet contribution of the final $pn$ pair with a $\chi^2 = 0.7$. A small singlet contribution, corresponding to $\zeta = 0.02$, does not contradict the data ($\chi^2 = 0.9$), whereas larger values $\zeta = 0.05 (\chi^2 = 1.8)$ and $\zeta = 0.10 (\chi^2 = 4.6)$ result in too large a cross section in the vicinity of $E_{np} = 0$. The other data set (Fig. 1b), obtained at a different magnetic field setting, shows also dominance of the triplet contribution and allows a small singlet fraction: $(\zeta, \chi^2) = (0.0, 2.4), (0.02, 2.1), (0.05, 2.3), (0.10, 4.0)$. However, in this case the $\chi^2$ becomes worse. Under assumption of $\zeta = \frac{1}{3}$, made in Ref. [17], the absolute value of the cross section in the region around $E_{np} = 0$ results by a factor 2.5 - 3 too high compared with the data.

The accuracy of the approximation by Eq. (5) is estimated in Refs. [4–7] and [14] to be better than 5% for $E_{np} \leq 3$ MeV. This error arises from variations of the bound and scattering $NN$ wave functions at short distances for low $E_{np}$. The error of the $pd \rightarrow dp$ input is $\approx 9\%$ [21]. The systematic uncertainties in the measured $d\sigma_{s+t}$ are not given in [17], here we assume them not to exceed 10%. Combining all uncertainties given above, the $d\sigma_{s+t}$ in Eq. (4) is uncertain within 15%. If the measured cross section given in Fig. 1a is scaled by factors ranging from 0.85 to 1.15, our $\chi^2(\zeta)$ analysis shows that the resulting $\zeta$’s for minimum $\chi^2$ range from $+0.035$ to $-0.030$ with the corresponding uncertainties $\Delta \zeta$ ranging from $+0.065$ to $+0.040$ to $-0.055$, respectively. This implies that $\zeta$ and $\Delta \zeta$ are both of the order of a few percent, and thus are substantially smaller than the spin-statistical factor of $\frac{1}{3}$ assumed in Ref. [17].

The matrix element squared $|M|^2$ shown in Fig. 3 was obtained in Ref. [17] by
dividing the raw data point by point by a Monte Carlo $E_{np}$ energy distribution, that includes the phase space factor. By this procedure the authors of Ref. [17] minimized the effects from averaging over the detector acceptance. In contrast to the production matrix element $|A|$, defined by Eq. (1), the complete matrix element $|M|$ contains the fsi. The authors of Ref. [17] found that the spin-statistical fraction of the singlet of $\frac{1}{3}$ describes the measured data. However, the experimental data contain considerable uncertainties. Therefore, according to our calculations, they do not constrain the singlet fraction strongly enough. As can be seen from Fig. 3, values $\zeta = 0.05$ and 0.30 allow one to fit the experimental data equally well ($\chi^2 = 1.4$ and $\chi^2 = 0.9$, respectively), if the absolute value of the matrix element $|M|^2$, not given in Ref. [17], is treated as a free parameter. The small value of $\zeta$, which we found from the cross section, is compatible with the value $\zeta = 0.19^{+0.32}_{-0.16}$, resulting from our analysis of the $\chi^2(\zeta)$ distribution for the $|M|^2$ data.

To improve the sensitivity to the $s/t$ ratio using the extrapolation theorem of Ref. [4,6], the ratio of the $pd \rightarrow pnp$ and $pd \rightarrow dp$ cross sections has to be established better by a measurement of both reactions in the same experiment. A new measurement of the $\vec{p}d \rightarrow pnp$ reaction at the ANKE spectrometer of the proton synchrotron COSY-Jülich will put more stringent limits on the $s/t$ ratio by detecting both protons in the forward-forward or forward-backward directions at beam energies $T_p = 0.5 - 2.5$ GeV [15].

In conclusion, by comparing the $pd \rightarrow pnp$ cross section at 585 MeV with that of $pd \rightarrow dp$ on the basis of scattering theory, we found that the final state spin-triplet contribution is dominant allowing a singlet contribution of a few percent. This result is in agreement with existing experimental data on the $s/t$ ratio in the reaction $pN \rightarrow pN\pi$ and supports the dominance of the triangle diagram with the subprocesses $pN \rightarrow pN\pi$ in the reaction $pd \rightarrow pnp$.

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Fig. 1. Experimental cross section (points) of the $pd \rightarrow pnp$ reaction from Ref. [17] at beam energy 585 MeV and proton laboratory scattering angles $\theta_1 = 41^\circ$, $\theta_2 = 61^\circ$ as function of the proton momentum in comparison with our calculations. 

\textbf{a)} The pure triplet contribution calculated with corrections taking into account the experimental resolution (full line) and without (dashed), as explained in the text. The upper scale shows the relative energy (in MeV) of the $pn$-pair for $\theta_1 = 41^\circ$. 

\textbf{b)} The same observable as in \textbf{a)} but for another magnetic field setting, compared with calculations including the corrections for different $s/t$ ratios $\zeta = 0.0$ (full line), 0.02 (dashed), and $\zeta = 0.05$ (dotted).
Fig. 2. The squared matrix element, as obtained in Ref. [17], for arbitrary normalization is well described by $\zeta = 0.05$, $\chi^2 = 1.4$ (full line) and $\zeta = 0.30$, $\chi^2 = 0.9$ (dashed).