Tailoring of arbitrary optical vector beams

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Abstract. We present a robust interferometric method to generate arbitrary vector beam modes by diffracting a Gaussian laser beam from a spatial light modulator consisting of a high-resolution reflective nematic liquid crystal display. Vector beams may have the same intensity cross-section as the more common scalar Laguerre–Gaussian (LG) or Hermite–Gaussian (HG) beams, but with a spatially modulated polarization distribution. Special cases are the radially or azimuthally polarized ‘doughnut’ modes, which have superior focusing properties and promise novel applications in many fields, such as optical trapping, spectroscopy and super-resolution microscopy. Our system allows video rate switching between vector beam modes. We demonstrate the generation of high quality Hermite–Gaussian and Laguerre–Gaussian vector beam modes of different order, of vectorial anti-vortices, and of mode mixtures with interesting non-symmetric polarization distributions.

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1. Introduction

So-called ‘vector beams’, consisting of pure laser modes with an additional spatial polarization modulation, have recently gained interest in a variety of application fields. For example, Laguerre–Gaussian beams of radial index 0 and azimuthal index 1 (i.e. LG\(_{01}\) beams), which are also known as ‘vortex’ or ‘doughnut’ beams with a helical charge of \( m = 1 \), have a better efficiency for cutting metals if they additionally have a radial polarization [1, 2]. In stimulated emission depletion (STED) microscopy which provides ‘super-resolution’ beyond the traditional optical resolution limit, doughnut beams are used to stimulate the emission of fluorescence photons in a ring-shaped area around a dark spot in its centre, such that the ring area does not lead to a background for the detected molecules in the middle [3]. There, an additional radial polarization of the doughnut beam promises a significantly increased resolution by providing a perfectly symmetric and sharper focus [4]–[6].

Radially and azimuthally polarized doughnut beams have unusual properties when focused. For example, a radially polarized LG\(_{01}\) beam possesses a nonzero axial electric field component (without any magnetic field component) in its dark core, whereas the same beam with an azimuthal polarization has an axial magnetic field component but no electric field in its centre. These features may have applications in polarization spectroscopy of single molecules or microscopic crystals [7, 8], they are suggested as linear accelerator mechanisms for electron beams (radially polarized LG\(_{01}\) beams) [9, 10], and they might be used as addressing/switching mechanisms for magnetic cores in novel data storage materials (azimuthally polarized LG\(_{01}\) beams) [11]. Furthermore, vector beams made of higher order LG-modes have controlled complex electric and magnetic field distributions in their focused core, corresponding for example to electric and magnetic dipole, quadrupole, and higher order fields. Such modes may find interesting applications in spectroscopy, in atom trapping, and in optical tweezers.

Due to these application areas, there have been several attempts to generate such vector-beams [12]–[16]. In some of the methods, the resonator of a laser is manipulated such that the beam is emitted in a desired vector mode structure [12]. Other methods transform a ‘normal’ (usually Gaussian) laser beam with interferometric methods, diffractive elements or lithographically produced sub-wavelength polarization gratings [4, 5, 14, 17, 18] into the desired vector modes. Interferometric approaches are typically based on the combination of different scalar modes, e.g. a superposition of a HG\(_{01}\) mode and a HG\(_{10}\) mode with orthogonal polarizations.
yields a radially or an azimuthally polarized beam when the phase between the two beams is properly adjusted [13, 19].

All of the above mentioned methods are designed to produce a particular vector beam mode, but they cannot be adapted to produce a variety of modes, or to switch between different modes. A more flexible way is to use a spatial light modulator (SLM) to generate the desired light fields [20]. Wilson et al [21] demonstrated an elegant way to use a ferroelectric liquid crystal SLM for the generation of a variety of high quality vector beams. There an incoming laser beam is split by a Wollaston prism into two orthogonally polarized beams propagating into slightly different directions. A hologram displayed at the SLM is designed such that it transforms the beams by diffraction into the desired beam modes, and simultaneously superposes the two beams in one of its diffraction directions. For this purpose, the SLM used by Wilson et al had to diffract the two incoming orthogonal polarizations with equal efficiency. This was possible with a ferroelectric liquid crystal system as used by the authors of [21], but there is the disadvantage that such an SLM can display only binary diffractive structures, resulting in a poor diffraction efficiency.

Our method is similar to the methods in [21], but it avoids this disadvantage and allows us to use a common nematic liquid crystal SLM, with a theoretically obtainable mode conversion efficiency of 100%. Furthermore it provides a greater flexibility in the laser mode control, particularly it allows to control the spatial amplitude distribution of the diffracted wavefronts in addition to the pure phase control in [21]. This is possible since the two incident beams are diffracted not just from one single, but from two adjacent holograms that are displayed at the same SLM. Therefore, it is possible to tailor the spatial amplitude distribution of the two diffracted beams independently by varying the contrast of the respective holograms. Another advantageous feature is that one can create controlled superpositions of vector beam modes of different orders with unusual properties. As an example, we demonstrate mode mixtures which are designed such that they undergo a mixed geometrical phase shift that depends on the spatial position within the wavefront when the global beam polarization is rotated (note: during the review period of our paper, we were informed that another paper describing a similar experimental method has been recently submitted [22]).

2. Experimental set-up

The basic set-up shown in figure 1. For the demonstration we use linearly polarized light from a helium–neon laser at 633 nm in the transverse electromagnetic (TEM) ground-state mode TEM_{00}. Using a first half-wave plate, the linear polarization of the laser beam is adjusted to an angle of 45°. The beam is then deflected by a 50/50 nonpolarizing beam splitter cube to a Wollaston prism. There the incoming beam is split into a horizontally and a vertically polarized beam with a mutual propagation angle of 2.5°. Then the two beams pass an arrangement of two lenses that act as a telescope, expanding the beams by a factor of 4 to a diameter of 6 mm, and reducing their propagation angle difference to 0.65°. Then the two beams are diffracted at two adjacent phase holograms that are displayed in two image windows on a high-resolution SLM. Since the nematic SLM is optimized only for diffraction of a vertically polarized beam, the horizontally polarized beam first passes a half-wave plate (arranged at a 45° angle) in front of the SLM, that rotates its horizontal polarization to the vertical state. We use a LC-R 3000 reflective-liquid-crystal SLM from HOLOEYE Photonics. The SLM is controlled by a computer in real time, using Matlab software routines to compute the mode-converting holograms. The
Figure 1. Experimental set-up: a linearly polarized laser beam is deflected by a non-polarizing beam splitter cube to a Wollaston-prism. The prism splits the input beam into two orthogonally polarized beams with equal intensities. A half-wave plate (HP) in one of the beams rotates its polarization axis, such that it matches the vertical polarization direction required by the SLM. Two adjacent holograms displayed at the SLM transform the modes of the beams and diffract them back to the Wollaston prism, where they are recombined. Behind the beam splitter cube a quarter-wave plate transforms the orthogonal linear polarizations of the two superposed beams into right- and left-circular polarizations, respectively.

SLM has a resolution of $1920 \times 1180$ pixels, each with a rectangular area of $10 \times 10 \mu m^2$. Each pixel acts as an individually programmable phase shifter in an interval between 0 and $2\pi$, which is addressable by the grey value of the corresponding image pixel. The total diffraction efficiency into the first order is about 30%. The holograms are blazed phase grating structures, designed such that they exactly reverse the propagation directions of the two incoming beams in their respective first diffraction orders (indicated in the figure). Additionally, the holograms are programmed such that they transform the incoming vertically polarized TEM$_{00}$ beams into selected beam modes (like a Laguerre–Gauss mode) of the same vertical polarization. In order to optimize the performance of the SLM, we have measured its spatial phase profile with an interferometric method (registering an aberration from ‘perfect’ flatness with an amplitude on the order of $1.5 \mu m$), and incorporated an appropriate correction function to the holograms. Each of the two beams is diffracted at its own hologram consisting of a $500 \times 500$ pixel hologram window.

The corresponding holograms are calculated by using the analytically known wavefront cross-section of the desired mode, $E(x, y)$, like that of equation (1) later in this paper, that consists of an amplitude and a complex phase angle. In order to produce an off-axis hologram that generates this mode in its first diffraction order in a direction of $k = (k_x, k_y)$, the complex field amplitude $E(x, y)$ is multiplied with $\exp(i(k_xx + k_yy + \Phi))$. $\Phi$ denotes an offset phase that is used to control the relative phase between the beams diffracted at the two adjacent SLM holograms. The values of $k_x$ and $k_y$ are interactively optimized such that the first order diffracted beam is back-reflected. Then the phase angle of the obtained complex function is calculated modulo $2\pi$ which results in a blazed grating structure that is displayed as a phase hologram at the SLM. Since this procedure neglects a possible modulation of the absolute value of the amplitude of the desired mode, it works only well for ‘simple’ modes like LG beams with a
radial index of zero. For modes with a more pronounced amplitude profile, the modulation of the absolute value of the amplitude has to be considered by spatially modulating the contrast of the displayed holographic grating structure (i.e. its local diffraction efficiency) with this amplitude profile [23]. Practically, this is done by creating for each hologram a spatially modulated ‘look-up’ table (which is used to translate grey-values of an image into corresponding phase values) that corresponds to the amplitude modulation of the desired wavefront. Although this method slightly reduces the overall diffraction efficiency by increasing the amount of light scattered into the undesired zeroth order, it greatly improves the mode quality of the generated beams. An example is shown later in this paper in figure 4.

The two back-diffracted beams then pass the same arrangement of telescope lenses such that they are narrowed again and recombine at the rear port of the Wollaston prism. There, an iris diaphragm is placed that removes undesired beams that are scattered by the SLM into other (i.e. mostly the zeroth) diffraction orders. Note that the originally horizontally polarized beam passes again through the half-wave plate in front of the SLM, such that its final polarization in front of the Wollaston prism is again horizontal. The effect of the Wollaston prism is then to recombine the two back-travelling beams into one single beam, conserving their individual horizontal and vertical polarization states, thus creating a vector beam with a controlled spatial polarization distribution. This vector beam then propagates from the front port of the Wollaston prism back to the 50/50 beam splitter cube, where the transmitted part of the vector beam is analysed. For creating the interesting class of linearly polarized vector beams (i.e. a beam with a wavefront that has a linear polarization at each local position, but with a spatially varying direction), a quarter wave-plate is placed at a $\pm 45^\circ$ angle in the beam path. It transforms the horizontal and vertical linear polarization components within the combined vector beam into a left- and a right-circular polarization state, respectively. In all (typical) cases, where the intensity cross-sections of the two orthogonally polarized beam components are equal, such an interference of a left- and right-circularly polarized beam leads to a locally linear polarization. However, the linear polarization direction now depends on the spatial position within the wavefront since the local relative phase difference between its two circular polarization components has a spatial variation. It can be controlled by manipulating the relative diffraction phase of the two beams at the SLM, which is done by a spatial shift of the underlying grating structure by fractions of a grating period. Finally, the vector beam is sent through a rotatable linear polarization filter to a CCD camera. Recording the transmitted intensity distribution in the far field at a variety of polarization angles allows one to investigate the vector mode structure of the generated beam.

Although our method is interferometrical, i.e. the phase between the two split and recombined beams determines the properties of the generated vector beams, the set-up is very stable against environmental noise like vibrations or air circulation. This is due to the fact that the two beams travel along almost the same paths between splitting and recombination. For the same reason, a coherence length of the incident laser light on the order of some microns—or, after careful alignment of the beam paths even on the order of the light wavelength—is sufficient, i.e. the set-up also works for rather broad-band light, or for pulsed laser beams.

To date the set-up provides only a low overall conversion efficiency (limited to maximally 25%), since 75% of the incident beam is lost at the 50/50 beam splitter cube. However, such an intensity loss can in principle be prevented with additional optical components by avoiding the double pass set-up, using two different beam paths for splitting and recombination of the laser beams. For example, the SLM could be tilted by 45° in order to deflect the two beams by 90° to another telescope and half-wave plate, and to recombine them at the second Wollaston prism.
Although such an implementation would use more optical components, and thus slightly reduce the interferometric stability of the system, it could achieve a theoretical conversion efficiency of 100%, since it is theoretically possible to obtain a 100% diffraction efficiency at blazed phase holograms that can be displayed at a nematic SLM, and there are no further unavoidable loss mechanisms.

3. Vector fields of first order Laguerre–Gaussian modes

As already mentioned, the quarter-wave plate in our experimental set-up generates a left- and a right-circular polarization state of the two beams which are then coherently superposed. Figure 2 sketches the basic mechanism how such a superposition produces a linearly polarized vector beam. In the example, we investigate the superpositions of two left- (corresponding to spin = 1) and right- (corresponding to spin = −1) circularly polarized Laguerre–Gaussian beams of helical indices \( m = 1 \) and \( m = -1 \), respectively, i.e. of a LG01 and a LG0−1 mode of different circular polarizations.

Within the figure the polarization states at each position of the circularly polarized doughnut beams (left two columns) are indicated by small circles with a clockwise or counterclockwise orientation specified by arrows. They are arranged around the ring-shaped intensity profile of the beam, which is denoted by a large central circle with an orientation (i.e. the sign of its helical charge) indicated by an arrow. The positions of the arrow tips at the small circles denote the local phase of the circular polarization, i.e. they have to be interpreted such that the actual direction of the electric field vectors point from the centre of the small circles to the arrow tips. Similarly, the position of the arrow tip at the large central circle indicates the ‘global’ phase offset of the doughnut. For example, the circular polarization phase of the beam mode drawn in the upper left edge of the figure (corresponding to a left circularly polarized LG01 beam) changes along the ring-shaped intensity profile, as indicated by the changing positions of the arrow tips of the small circles. This is caused by the fact that the local phase of a LG01 mode changes linearly from zero to \( 2\pi \) (counter-clockwise) on a circle around the doughnut core, and this phase acts as an offset for the circularly polarized field components around the ring.

The first row shows the superposition of a LG01 mode with left circular polarization, and a right circularly polarized LG0−1 mode with a relative phase of zero. The result is a linearly polarized vector beam, where the local polarization direction (indicated by double-ended arrows) corresponds to the polar angle of each spot. Such a beam is called a radially polarized vector beam.

The second row shows the superposition of the same two beams, however with a relative phase offset of \( \pi \), indicated by the changed arrow tip position within the central circle of the second beam, and by the correspondingly changed phases of the small circles. In the experiment, such a phase offset of \( \pi \) is simply obtained by recalculating the corresponding hologram at the SLM with a \( \Phi = \pi \) phase offset, resulting in a grating structure that is spatially shifted by half a period. In the resulting vector beam the local direction of the linear polarization is oriented tangentially around the ring of light. Such a mode is known as an azimuthally polarized vector beam.
Figure 2. Superposition ‘arithmetics’ of vector beams: the figure shows the resulting vector beam modes (right column) if two $LG_{0\pm1}$ modes with opposite helical indices ($m = \pm 1$, as indicated by the arrow tips in the large circle in the centre of each graph), and opposite left (lc-) and right (rc-)circular polarization states (indicated by the arrow tips within the small circles around the large central ones) are coherently superposed with different relative phases of zero and $\pi$. The examples in the first two rows correspond to the case where each individual beam has the same sign of its spin and orbital angular momentum. This results in the creation of linearly polarized vector modes (right column) with a radial or azimuthal polarization (indicated by the directions of the ‘double-arrows’), depending on their mutual phase shifts of zero (first row) or $\pi$ (second row), respectively. The combinations in the third and fourth row correspond to the case, where each beam has opposite sign in spin and orbital-angular momentum. There the superposition results in so-called anti-vortex modes, which are linearly polarized at each local position, but with a local polarization direction changing between radial and azimuthal. A relative phase shift of zero (third row) and $\pi$ (fourth row) between the two superposed components just produces a rotation of the mode by $45^\circ$. 
The next two examples correspond to cases where each of the two superposed $\text{LG}_{0\pm1}$ modes has a different sign of its spin- and orbital-angular momenta. In the experiment this situation can be achieved by exchanging the helical indices $m$ of the two adjacent holograms with respect to the first two cases. In this case it turns out that the resulting vector beam is still linearly polarized, but its local polarization direction changes periodically from radial to azimuthal. For example, when following the ring of light in a clockwise direction, the local field vector rotates counterclockwise, such that the vectors are twisted inside. Such vector modes are known as anti-vortices \[11\]. The last example (fourth row) shows the superposition of the same two beams, but with a $\pi$ relative phase shift. This again results in the creation of a polarization anti-vortex, which is similar to that of the previous example, but rotated by an angle of $45^\circ$.

In order to demonstrate our method, we have experimentally generated all of the vector modes sketched in figure 2. The holograms that were displayed at the SLM for transforming the incoming TEM$_{00}$ beam into a diffracted $\text{LG}_{0\pm1}$ beam correspond to grating structures with a fork-like dislocation in the centre \[24\]. The results are plotted in figure 3.

The first row of the figure shows the intensity distributions of the different vector beam modes as recorded by the CCD camera without an inserted polarization filter. As expected, all generated modes show an isotropic doughnut intensity distribution. For generating a radially polarized vector beam (first column), the signs of the helical indices of the two holograms at the SLM were chosen such that the helical charge of each back-diffracted beam had the same sign as its spin. The relative phase between the two superposed beams could be interactively adjusted under computer control by shifting the grating phases of the corresponding holograms. In order to switch between the radial and the azimuthal polarization modes (first and second columns), it was just necessary to shift the phase $\Phi$ (i.e. the phase of the grating) of one of the two SLM holograms by $\pi$. For switching to an anti-vortex mode (third row), the signs of the helical indices of the two SLM holograms were switched. Finally, for obtaining the anti-vortex state in the fourth column, the phase of one of the displayed hologram gratings was again shifted by $\pi$.

The states of the beams were investigated by inserting the polarization filter in front of the CCD camera and rotating it to four positions, with corresponding transmission directions indicated at the left of the figure. Behind a polarization filter, all vector beam states show an intensity distribution similar to a Hermite–Gaussian beam of first order (e.g. HG$_{01}$), consisting of two adjacent intensity maxima, however with different behaviour when the polarizer is rotated. Most remarkably, when rotating the polarization filter in front of the CCD camera, the transmitted intensity distributions of the radially and azimuthally polarized modes (first and second column) rotate in the same direction, whereas the intensity distributions of the anti-vortex modes (third and fourth row) rotate in opposite direction. The purity of the polarization states was verified by measuring the contrast of the intensity maxima and minima behind the polarization analyser. This was done by calculating the ratio $(I_\parallel - I_\perp)/(I_\parallel + I_\perp)$, where $(I_\parallel)$ and $(I_\perp)$ are the intensities measured parallel and perpendicular to the transmission axes of the polarization filter, respectively. For the radially and the azimuthally polarized modes the contrast ratio was 94%. From the intensity distributions transmitted at the different polarization filter orientations, the state of the vector beams could be found out. The results are plotted as vector fields in the lowest row of the figure. They correspond to the modes which are expected according to our considerations of figure 2.

The right column of the figure compares our experimental results with a simulation based on the theoretical behaviour of an ideal radially polarized $\text{LG}_{0\pm1}$ vector beam behind a rotating
Figure 3. A set of experimentally generated linearly polarized vector beam modes based on LG\(_{0\pm1}\) modes. The upper row shows the isotropic doughnut intensity profiles of the vector beams as imaged without the polarization analyser in front of the CCD camera. The next four rows show the intensity cross-sections behind the inserted polarization filter at different polarization angles (the orientation of the transmission axis of the polarization filter is indicated at the left). From these measured intensity distributions the polarization states of the beams were reconstructed (vector-field plots in the lowest row). The mode in the first column corresponds to a radially polarized vector beam, the second column to an azimuthally polarized beam, and the next two columns to ‘anti-vortex’ beams with a mutual rotation angle of 45°, as expected from the model in figure 2. The last column shows the results of an analytic calculation of the intensity distributions of an ideal, radially polarized beam LG\(_{0\pm1}\) beam behind a rotated polarization analyser, corresponding to the experimental results of the first column.

Polarization analyser. Analytically, a scalar Laguerre–Gauss mode is described by [25]

\[
E(\rho) = E_0 \rho^{|m|} e^{-(\rho^2/2)} L^{|m|}_l (\rho^2) e^{i m \phi}.
\]  

Here \(\rho = (\sqrt{x^2 + y^2}/w)\), where \(w\) is the waist of the beam, and \(L^{|m|}_l\{x\}\) denotes Laguerre polynomials of radial order \(l\) and azimuthal order (i.e. helical charge) \(m\). For circular polarization

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the electric field becomes a vector field

\[
\vec{E}_1 = E_0 \rho^|m| e^\left(-\frac{\rho^2}{2}\right) L_i^{\text{rad}} \left\{ \rho^2 \right\} \left( \begin{array}{c} e^{im\phi} \cos(\omega t) \\ -e^{im\phi} \sin(\omega t) \end{array} \right) .
\]

(2)

Similarly, the second beam with opposite circular polarization (spin) and opposite helical index \( m \) is given by

\[
\vec{E}_2 = E_0 \rho^|m| e^\left(-\frac{\rho^2}{2}\right) L_i^{\text{rad}} \left\{ \rho^2 \right\} \left( \begin{array}{c} e^{im\phi} \cos(\omega t) \\ e^{im\phi} \sin(\omega t) \end{array} \right) .
\]

(3)

The superposition of the two fields is then

\[
\vec{E}_{\text{ges}} = 2E_0 \rho^|m| e^\left(-\frac{\rho^2}{2}\right) L_i^{\text{rad}} \left\{ \rho^2 \right\} \left( \begin{array}{c} \cos(m\phi) \cos(\omega t) \\ \sin(m\phi) \cos(\omega t) \end{array} \right) .
\]

(4)

for the radial case.

The corresponding intensities of the superposed beam behind a linear polarizer at different rotation angles is then calculated by projecting the electric field vector on the corresponding linear polarization states, and calculating the corresponding intensity by squaring the absolute values of the electric field. A comparison between the theoretical and the experimental results shows very good agreement and thus demonstrates the high quality of the generated vector modes.

4. Vector beams by superpositions of Hermite–Gaussian modes

An alternative interferometric construction method for radially or azimuthally polarized Laguerre–Gaussian vector beams superposes two orthogonal Hermite–Gaussian beams (HG\(_{01}\) and HG\(_{10}\)) with orthogonal linear polarizations and a controlled relative phase [19]. In our experiment this can be achieved by programming the two holograms such that they transform the incident Gaussian beams into a HG\(_{01}\) and a HG\(_{10}\), respectively. In this case, the quarter-wave plate behind the out-coupling beam-splitter cube is removed in order to keep the polarizations of the two superposed beams linear (instead of circular, as before). The two SLM–holograms are sketched in the upper row of figure 4, and the corresponding HG\(_{01}\) and a HG\(_{10}\) modes (as recorded individually by the CCD camera) are displayed below. These modes consist of two vertically or horizontally separated amplitude maxima with a relative phase difference of \( \pi \). Consequently, these modes are generated by two holograms consisting of (blazed) gratings with a phase jump of \( \pi \) in vertical (HG\(_{01}\)) or horizontal (HG\(_{10}\)) directions. However, for a perfect generation of these modes, not only the phase but also the amplitude has to be controlled. For this purpose we also shaped the local diffraction efficiencies by controlling the modulation depths of the diffraction gratings in the regions of the phase jump. This results in the ‘blurred’ appearance of the two holograms of figure 4 along their vertical or horizontal axes.

The superposition of the two orthogonally polarized HG\(_{10}\) and HG\(_{01}\) modes results in the doughnut mode displayed at the lower right edge of the figure, if the intensity profile is well adjusted. Without the amplitude mask the superimposed intensity profile becomes a square. An analysis of this mode with an inserted polarizer at different polarization angles showed that it equals that of the left column in figure 3, i.e. to a radially polarized LG\(_{01}\) vector beam.

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The principle of the generation of differently polarized vector beam modes by superpositions of Hermite–Gaussian beams is sketched in figure 5. The first row indicates the coherent superposition of a horizontally polarized HG_{10} beam with a vertically polarized HG_{01} beam with a relative phase of 0 (meaning that the corresponding electric field amplitudes reach their corresponding maxima and minima at the same instant). The $\pi$ phase shift between the left and the right parts of the HG_{10} mode (as well as the upper and the lower parts of the HG_{01} mode) are indicated by the vector arrow tips. The result of the superposition is a radially polarized LG beam, as sketched at the right side.

The situation changes if the phase of the HG_{01} beam is shifted by $\pi/2$ (second row—phase shift indicated by multiplication of the corresponding field amplitude by the imaginary unit ‘$i$’). This results in the vector mode sketched at the right, with alternating linear and elliptical/circular polarization states. A further phase shift of $\pi/2$ of the HG_{01} mode (third row, indicated by the switched position of the polarization vector arrows with respect to the first row) then creates an anti-vortex mode, consisting of locally linear polarizations whose orientation changes in an opposite direction with respect to their angular coordinate. This is the same mode as demonstrated in the third row of figure 3.
Finally, the last row of the diagram demonstrates the effect of switching the polarizations of the two HG modes (which is indicated by a $90^\circ$ rotation of the polarization vectors). In this case, the superposition results in an azimuthally polarized LG mode.

An experimental verification of these considerations is demonstrated in figure 6. There the respective beam modes were generated with the same methods as explained in the diagram of figure 5, i.e. by introducing relative phase shifts between the two superposing HG beams, via control of the respective grating phases, and by controlling the respective polarization directions. With this method a radially, an azimuthally and a mixed polarization state of the resulting LG beam were produced. The polarization structures of the corresponding vector beam modes were verified in the same way as before, i.e. by rotating a polarization analyser in front of the CCD camera and recording the transmitted intensities at different polarization angles.

The first column of figure 6 displays the doughnut intensity distributions recorded without a polarization analyser in front of the CCD. The next four rows display the transmitted intensities...
Figure 6. Experimentally generated LG$_{01}$ vector modes by superpositions of a HG$_{01}$ and a HG$_{10}$ beam. The upper row shows the intensity distributions of a radially polarized, an azimuthally polarized, and a beam with mixed polarization. The beams were created with the methods described in figure 5. The next four rows show the respective intensity distributions of the three beams behind a polarizer at a polarization angle indicated at the left. The reconstructed vector fields of the three modes are sketched in the lowest row.

behind a polarization filter at polarization angles indicated at the left. The reconstructed local polarization structures of each of the three beams are sketched below. The result shows that a radially (left column) and a horizontally (second column) polarized beam were constructed with the methods described in figure 5. A beam with mixed linear and elliptical polarization structure is displayed in the third column. A peculiar feature of it is that at two orthogonal positions of the polarization analyzer the transmitted intensity is still a doughnut. An analysis of these two cases shows that the light fields behind the polarizer corresponds to the two linearly polarized LG$_{0\pm1}$-modes of opposite helical charges, respectively. This might be tested, for example, by investigating the corresponding angular momentum transfer of such a laser mode to a trapped particle in an optical tweezers experiment. The effect of switching between the two doughnut beams by rotating an inserted polarizer by 90° would then be to switch the rotational direction of particles circling around the intensity ring of such a mode.
5. Higher order Laguerre–Gaussian vector beams

Hermite–Gaussian or Laguerre–Gaussian modes of higher order can be generated in laser resonators, however typically as a mixture of many modes, and not in a pure state. Normally they are also in an uncontrolled (or spatially unmodulated) polarization state. In figure 7 we present an experimentally recorded ‘gallery’ of higher order pure LG vector modes, up to a helical index of 7 and a radial index of 2.

All modes were created by superposing two Laguerre–Gaussian modes with opposite circular polarizations (i.e. the quarter-wave plate was re-inserted at the position of figure 1), and with the same radial, but opposite helical indices. The first row shows the results of a simulation of...
Figure 8. Numerically calculated axial electric and magnetic field distribution within the cross-section of certain vector beams.

the intensities of certain vector beam modes (LG_{01}, LG_{02}, LG_{07}, LG_{11}, LG_{12}, LG_{22}), calculated according to equation (4). The next row shows the corresponding experimentally measured intensity distributions. For these measurements, the polarizer in front of the CCD camera was removed. The next four rows show the intensity distributions with the polarizer re-inserted, and at four positions of its polarization axis, as indicated at the left. The distribution of the electric field as reconstructed from these measurements is sketched below as a vector field, that indicates the local directions of the electric field vectors at a certain instant.

In order to improve the quality of the modes we used the advantageous feature of our set-up, that not only the local phase, but also the local amplitude of the two beams diffracted at the respective SLM-holograms can be controlled. Similar to the holograms plotted in figure 4, the local diffraction efficiency was modulated by reducing the contrast of the diffraction gratings in the regions of \( \pi \)-phase jumps.

An interesting feature of vector beams is their unusual axial electric and magnetic field distribution within their dark core. In figure 8 the axial field distributions of a few selected vector beam modes have been numerically calculated from the Maxwell equations in the paraxial approximation. Therefore, the results are in principle only valid for unfocussed beams, but the qualitative structure of the axial field components will be similar also for focussed beams if the numerical aperture is sufficiently low.

The upper row shows the calculated intensity cross-sections of radially polarized LG_{01}, LG_{02}, LG_{03}, LG_{11} modes, and of an anti-vortex LG_{01} mode, respectively. The next two rows show the absolute values of the corresponding axial electric and magnetic field amplitudes, respectively. Note that adjacent field maxima (white areas) within one mode have alternating signs of their amplitudes (not indicated in the figure).

The first column shows that a radially polarized LG_{01} beam has a strong axial electric field component (which oscillates with the light frequency), without any magnetic field component. The amplitude ratio between the axial and the transverse field components depends reciprocally on the size of the focus (measured in wavelength units). As an example, if a light intensity of 1 W is focussed at a 1 \( \mu \text{m}^2 \) spot, the total electric and magnetic field amplitudes within the spot are on the order of \( 3 \times 10^7 \) Vm\(^{-1}\), and 0.1 T, respectively, from which the major part would be
in the axial component. For pulsed lasers, these amplitudes are much larger, for example they would increase by a factor of $10^4$ for a Nd:YAG laser emitting a pulse energy of 1 J in 10 ns.

Higher order LG$_{lm}$ modes with zero radial index ($l = 0$) have symmetric patterns of alternating axial magnetic and electric field components, which correspond for ($m = 2$) to dipole fields (second column), for ($m = 3$) to quadrupole fields (third column) and for higher $m$ to corresponding multi-poles (not plotted any more).

Beams with higher radial indices $l$ have rotationally symmetric electric or magnetic field distributions, similar to annual growth rings in trees, arranged around a string-like maximum in the centre. In the fourth column of the figure we show an azimuthally polarized LG$_{11}$ mode. Whereas its axial electric field component equals zero, its axial magnetic field is similar to that of a coil, with a forward-pointing magnetic field vector in the central core, and an opposite field direction in a ring around that core. Note that for all of the modes presented so far, the axial field amplitudes have their maxima in the dark regions of the modes.

This changes for anti-vortex modes. The right column shows an anti-vortex based on a LG$_{01}$ mode. It has a quadrupole-like electric and magnetic field distribution in its centre, with zero amplitudes in the middle. The axial field distribution resembles that of the radially polarized LG$_{03}$ mode of the third column. However, the intensity doughnut (upper row) of the anti-vortex is much smaller, such that the maxima of the axial field amplitudes now coincide with the intensity maxima.

In any case the amplitudes of the electric and magnetic field components can be exchanged by shifting the relative phase between the two interfering left- and right-circularly polarized components by $\pi$. For example, this allows to switch between pure magnetic and pure electric fields along the axis of a vectorial LG$_{01}$ mode. The gallery of figure 7 gives only a few examples of higher order vector beam modes that can be realized with the presented method. These modes have complicated, but nevertheless completely controllable axial field distributions within their cores. It may be assumed that a variety of fundamental research experiments may be performed by a controlled interaction of such beams with dielectric or magnetic materials, as well as with polarizable molecules, or atoms in magneto-optical traps.

6. Asymmetric beam rotation of superposed vector modes

An interesting feature of vector modes with locally linear polarization is the fact that their intensity image that is transmitted through a rotating linear polarizer also rotates continuously, either in the same or in the opposite direction with respect to the rotational direction of the polarization filter (see figure 3). Interestingly, the rotation angle of the image scales reciprocally with the helical index $m$ of the vector beam mode. For example, a locally linearly polarized LG$_{07}$ mode (as that in the third column of figure 7) rotates with a frequency of $1/7$ Hz, when the polarizer is rotated at a rate of 1 Hz.

This behaviour may be interpreted as a manifestation of an interplay between two different geometric phases, namely a ‘Pancharatnam’ phase associated with polarization rotation, and a ‘rotational’ Doppler shift, or actually the corresponding phase shift, which arises when a LG beam is rotated around its longitudinal axis. Pancharatnam phases appear if the polarization of a beam is continuously rotated by polarization modulating components, such as specific combinations of half- and quarter-wave plates [26, 27]. For example, if an incident linear polarization is continuously rotated by $360^\circ$ the phase of the corresponding beam changes by $2\pi$. A rotational
Figure 9. Transmitted intensity image of a mixed vector beam mode behind a rotating polarization filter. The mode consists of a superposition of a radially polarized LG$_{01}$ with a LG$_{04}$ mode. The sequence of nine images is recorded at the polarization angles indicated below. The arrows within the images indicate the changing positions of an intensity maxima of the LG$_{01}$ (inner arrow), and the LG$_{04}$ mode (outer arrow), respectively. Note that the two inner intensity maxima rotate four times faster than the eight outer ones.

phase shift of a LG beam, on the other hand, appears if the whole beam is rotated around its longitudinal axis, as for example by an arrangement of optical components (Dove prisms) that would also perform an image rotation by the same angle in an imaging experiment [26], [28]–[32]. Interestingly, this rotational phase shift scales with the helical index of the LG beam, i.e. a rotation of 360° results in a $2\pi m$ phase shift of a LG$_{lm}$ beam (an additional circular polarization of the beam would manifest itself as an additional $\pm 2\pi$ phase shift associated with a ‘spin redirection phase’ [32]).

In our experiment we introduce a Pancharatnam phase shift by projecting the polarization of the vector beams on a rotating polarization axis, by revolving the polarization filter in front of the CCD camera with a frequency $\omega$. This shifts the frequency of the beam by the same amount of $\omega$. This frequency shift due to a polarization rotation manifests as an image rotation of the light intensity distribution transmitted through the polarizer. This is an inversion of the rotational Doppler effect of [29], where an image rotation creates a frequency shift. However, since the rotational Doppler effect generates a frequency shift of $m\omega$, when a LG$_{lm}$ beam is rotated with a frequency $\omega$, the corresponding inversion of the effect demands that the image of the beam is rotated with a frequency of only $\omega/m$ when the frequency of the beam is shifted by $\omega$. This behaviour corresponds to our above mentioned experimental observation that the image rotation angle corresponds to the rotation angle of the polarizer multiplied by $m^{-1}$.

This feature appears in an even more striking way, if a superposition of vector beam modes with different helical charges is observed [30]. In this case, the image rotation introduced by a rotating polarizer varies for the different LG components within the beam, allowing for example to program beams whose intensity images transmitted through a rotating polarizer behave similarly to a cogwheel ‘gear-mechanism’, or to the small and the large hands of a watch.

In figure 9 we demonstrate such a peculiar behaviour of a certain mode superposition. For this purpose each of the two holograms at the SLM was programmed to transform an incident Gaussian beam into a superposition of an ‘inner’ LG$_{01}$ mode and an ‘outer’ LG$_{04}$ mode. The corresponding intensity distributions of the two modes are spatially separated in two concentric doughnut rings of different diameters, since the LG$_{01}$ mode has a much narrower cross-section than the LG$_{04}$ mode. The superposition of the beams diffracted by the two holograms (with opposite circular polarizations) then results in a mixture of two radially polarized modes with helical indices of 1 and 4, respectively.
Figure 9 shows the transmitted intensity of such a mode mixture as recorded behind a rotating polarization analyser (polarization directions indicated in the lower row). The pattern consists basically of a mixture of the individual patterns of a radially polarized LG\(_{01}\) mode as in figure 3 (consisting of two intensity spots in the central part of the mode) and that of a LG\(_{04}\) mode (consisting of eight intensity maxima arranged on a ring around the central two ones). The figure shows a sequence of nine images recorded at equidistant rotational angles within a full revolution of the polarization filter in a range between 0 and 2\(\pi\). For better illustration, the positions of one of the eight intensity maxima at the outer ring, and one of the two maxima of the inner ring are indicated by arrows. Obviously, the rotating polarization filter produces an intensity pattern rotating in the same direction, and additionally changing its shape. It turns out that the 2\(\pi\) rotation of the polarization filter results in a complete 360° revolution of the inner LG\(_{01}\) mode pattern, whereas the outer LG\(_{04}\) mode only rotates by 90 degrees. More generally, any LG mode with a helical charge of \(m\) is rotated by an angle of \(2\pi/m\) after a full 2\(\pi\) revolution of the polarization filter.

This effect can have applications in optical trapping experiments (‘optical tweezers’). Intensity maxima like that in figure 9 can be used as traps for individual particles (like micro-beads, cells etc.) and their relative spatial arrangement can then be controlled by rotating a polarizer in front of the trap. The ‘gear-reduction’ factor of \(m^{-1}\) of the intensity pattern rotation angle with respect to the rotation angle of the polarization filter allows an exact fine-adjustment of the angular position of a trapped particle. Furthermore, if an arrangement like that of figure 9 was used for trapping different (distinguishable) particles, a set of eight outer particles would rotate around a set of two inner ones, and the initial starting position would reappear not until four full 2\(\pi\) revolutions of the polarizer. Such a processing can be generalized to an arrangement of cogwheel shaped intensity distributions that rotate with different controllable velocities and process micro-particles like a ‘clockwork’. In the case of mixtures of modes with various helical charges \(m_i\), the overall periodicity (i.e. the number of full 2\(\pi\) polarizer rotations required to obtain the starting situation) is given by the least common multiple of all helical charges \(m_i\), e.g. for \(m_1 = 4\) and \(m_2 = 7\) there would be a periodicity of 28 polarizer revolutions.

7. Conclusion

We have demonstrated a novel method to generate vector beams of arbitrary mode structure, tailored polarization distribution, and controlled superpositions of different vector modes. In principle the method can achieve a mode conversion efficiency of 100%. It requires only low longitudinal coherence length, thus it can be used with pulsed lasers. It is stable against environmental disturbances like vibrations, and the generated vector beams are of high purity. The method provides the possibility to switch electronically between different mode structures. This can happen at video rate by changing the holograms displayed at the SLM. However, by making use of the fact that the phase difference between the two superposed beams determines the polarization state of the vector beams (as seen e.g. in the transformation between an azimuthally and a radially polarized beam), the switching can be done much faster (on a MHz or GHz scale), as such a phase change can, for instance, be introduced by inserting two acousto-optic modulators into the two separated beam paths in front of the SLM. An alternative would be to substitute the Wollaston prism in the set-up by an acousto-optic modulator that is driven such that it separates an incident beam equally into the zero and first diffraction order. The flexible and controlled
generation of vector beams is of practical interest in many areas like material processing, STED microscopy, and optical tweezers. The beams are also of fundamental interest due to their strange axial electric and magnetic field distributions, promising interesting interactions with dielectric and magnetic materials, nonlinear materials, optically active substances, fluorescent molecules, and atoms in gases or magnetooptical traps.

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