Joint parameter estimation employing coherent passive MIMO radar

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Abstract: Passive multiple-input multiple-output radar is studied for the problem of joint estimation of target location and velocity. A general case is considered, where the signals of opportunity may be non-orthogonal and the clutter-plus-noise may be correlated. The maximum likelihood estimate is provided and the Cramer-Rao bound is derived. The authors show the performance gain obtained from coherent processing and analyse the performance variation induced by the effects of signal non-orthogonality and the correlation of the clutter-plus-noise components.

1 Introduction

Estimating target location and velocity is one of the important tasks of radar system. Multiple-input multiple-output (MIMO) radar [1–4] exhibits superiority in joint parameter estimation. To gain advantages of low cost and high stealth, passive MIMO radar is studied in recent years [5–8].

In passive MIMO radar systems, signals emitted by illuminators of opportunity (IOO) have employed enormous formats, such as FM signals [9], digital signals [10], satellite-born signals [11], and global system for mobile (GSM) communication signals [12]. While signals from many existing IOOs are orthogonal with each other, in the upcoming 5G communication system, non-orthogonal signals tend to be favourable for a higher spectrum efficiency [13, 14]. Furthermore, in contemporary electromagnetic environment, the clutter-plus-noise associated with different transmitter-to-receiver paths may be correlated. In our previous work [15], we investigated the estimation performance of a passive MIMO radar with non-coherent processing. In this work, assuming good enough time synchronisation and phase synchronisation, we consider passive MIMO radar with coherent processing and analyse its performance in jointly estimating the location and velocity under a general case of possibly non-orthogonal signals of opportunity (SOO) and spatially correlated clutter-plus-noise.

The organisation of this paper is as follows. Section 2 develops a signal model for coherent passive MIMO radar. The maximum likelihood (ML) estimates and the Cramer-Rao bounds (CRB) for target position and velocity estimation are given in Section 3, where a simplified case is also analysed. Numerical results are presented in Section 4.

Notation: Throughout the paper we use $c$ to denote the speed of light, $\text{Diag}$ the block diagonal operator, $\mathcal{C}(\mu, Q)$ the complex Gaussian distribution with mean $\mu$ and covariance matrix $Q$. $I_K$ a $K \times K$ identity matrix, $I_k$ a size-$K$ all one column vector, $1_{M \times N}$ an $M \times N$ all one matrix, $\delta(\cdot)$ the unit impulse function, $\otimes$ the Kronecker product, $\det(\cdot)$ the determinant operator, $(\cdot)^H$ the transpose operator, $(\cdot)^{vec}$ the column-wise vectorisation operator.

2 Signal model

Consider an $N$-receiver passive MIMO radar system employing SOOs from $M$ IOOs. In a two-dimensional Cartesian coordinate system, the IOOs and receive stations are supposed to be sufficiently widely spaced within the same target beam [16], so that they share the same target reflection coefficient. Assume all IOOs and receivers are well synchronised in time and phase, which

\begin{equation}
\begin{aligned}
\mathbf{r}_n[k] &= \sum_{m=1}^{M} \left[ \frac{P_m}{d_{mn}^2} \mathbf{e}^{j2\pi f_{c,n} k T_s} \right] + \mathbf{w}_n(kT_s),
\end{aligned}
\end{equation}

where $d_{mn}$ and $f_{c,n}$ represent the time delay and Doppler shift associated with the propagation from the nth IOO to the nth receiver, $\zeta \sim \mathcal{C}(0, \sigma^2)$ denotes the target reflection coefficient [16], $d_{mn}$ is the distance between the nth IOO and the target, and $d_{mn}$ is the distance between the target and the nth receiver. The received signal strength at the $n$th receiver at time instant $kT_s$ is [17]

Taking $K$ samples of $r_n[k]$, the observation vector at the $n$th receiver can be written as

\begin{equation}
\begin{aligned}
r_n = (r_n[1], \ldots, r_n[K])^\top = \mathbf{U}_n \mathbf{s}_n + \mathbf{w}_n,
\end{aligned}
\end{equation}

where $\mathbf{w}_n = [w_n(T_1), \ldots, w_n(KT_s)]^\top$, $\mathbf{U}_n = [u_n[1], \ldots, u_n[K]]^\top$, $\mathbf{s}_n[k] = (s_n[1], \ldots, s_n[K])^\top$, and $\mathbf{w}_n[k] = (P_n P_j d_{mn} d_{mn}^* s_n[kT_s - \tau_{mn}) \mathbf{e}^{j2\pi f_{c,n} k T_s}$. The overall observation vector from all $N$ receivers is

\begin{equation}
\begin{aligned}
\mathbf{r} = (r_1, \ldots, r_N)^\top = \mathbf{S} \mathbf{1}_{N \times M} + \mathbf{w},
\end{aligned}
\end{equation}

where $\mathbf{S} = \text{Diag}[U_1, \ldots, U_N]$ and $\mathbf{w} = [w_1, \ldots, w_N]^\top$ is assumed to be zero-mean complex Gaussian distributed $\mathbf{w} \sim \mathcal{CN}(0, \mathbf{Q})$. 

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3 ML and CRB for coherent processing

According to the signal model in (3), the log-likelihood function can be computed
\[
L(r|\theta) = -i^H C^\dagger r - \ln \det (C) - KN \ln \pi, \tag{4}
\]
and the ML estimate of \( \theta \) is
\[
\hat{\theta} = \arg \max_{\theta} \{-i^H C^{-1} r - \ln \det (C)\}. \tag{5}
\]
where
\[
C = \left[ \xi \sum_{m=1}^{N} E \left( S(w \otimes 1_{x,M})^H \right) + \xi E\left( w \otimes 1_{x,M} S^H \right) + E(ww^H) \right]
\]
is the covariance matrix of \( r \).

Based on the signal model developed in the previous section, for the estimation of the unknown \( \theta \), the Fisher information matrix (FIM) can be obtained from (4) as follows
\[
J(\theta) = E_{r \theta}(V_{\theta}(r|\theta)(V_{\theta}(r|\theta))^T), \tag{7}
\]
where \( V \) denotes the gradient operator. Define an intermediate parameter vector \( \theta = [\tau, \phi, d_1, d_2]^T \), where \( \tau = [\tau_1, \tau_2, \ldots, \tau_{NM}]^T, f = [f_1, f_2, \ldots, f_{NM}]^T, d_i = [d_{i1}, d_{i2}, \ldots, d_{iN}]^T \), and \( d = [d_1, d_2, \ldots, d_n]^T \). According to the chain rule, the FIM is given by [18]
\[
J(\theta) = (V_{\theta}(\theta))(V_{\theta}(\theta))^T, \tag{8}
\]
where
\[
V_{\theta}(\theta) = \begin{bmatrix}
V_{\theta}\tau & V_{\theta}\phi & V_{\theta}d_1 & V_{\theta}d_1 \\
0 & V_{\theta}\phi & V_{\theta}d_1 & V_{\theta}d_2 \\
0 & 0 & V_{\theta}d_2 & 0
\end{bmatrix}, \tag{9}
\]
and the \( i \)th element of \( J(\theta) \) is
\[
|J(\theta)|_{ij} = \text{Tr}\left(C^{-1} \frac{\partial C}{\partial \theta_i} C^{-1} \frac{\partial C}{\partial \theta_j} \right) = \left( \frac{\partial C_{\text{vec}}}{\partial \theta_i} \right)^T \left( C^{-1} \otimes C^{-1} \right) \left( \frac{\partial C_{\text{vec}}}{\partial \theta_j} \right), \tag{10}
\]
in which \( C_{\text{vec}} = \text{vec}(C) \).

The previous analysis applies for a general case, which does not require specific assumptions about the clutter-plus-noise covariance or the SOOs. Next we look at a simplified case to get insight.

3.1 Simplified case

To simplify analysis, assume the clutter-plus-noise is spatially white, whose covariance matrix can be written as \( Q = \sigma_0^2 J_{SK} \). The SOOs are assumed approximately orthogonal and maintain approximate orthogonality for time delays and Doppler shifts of interest, such that
\[
\sum_{k=1}^{K} r_m(kT_i - \tau_{mn}) s_m^*(kT_i - \tau_{mn}) \times \exp\{j2\pi \left( f_m(kT_i - \tau_{mn}) + (f_{mn} - f_{nm}) \right)\} \approx \delta(m - m'), \tag{11}
\]

Further assume \((E_{r\theta}(d_{nm}d_{nm}))^{1/2} = 1 \) for all paths. Then, we have \( S^H S = I_{NM} \).

Applying these simplifying assumptions and according to the matrix identity proposed by Kailath [19], we have
\[
C^{-1} = Q^{-1} - \frac{\sigma_0^2}{\sigma_0^2} I_{NM}\left(\frac{1}{\sigma_0^2\sigma^2 + NM}\right)^{1/2} S^H. \tag{12}
\]

Omitting the terms independent of \( \theta \), the ML estimator in (5) becomes
\[
\hat{\theta} = \arg \max_{\theta} \left\{ 1 - \frac{NM}{\sigma_0^2(\sigma^2 + NM)} \right\} (1_{NM})^S S^H \right\}. \tag{13}
\]

Using the orthogonal SOOs and spatially uncorrelated clutter-plus-noise assumptions again, we obtain
\[
(1_{NM})^S S^H = \sum_{n=1}^{N} \sum_{m=1}^{M_K} \sum_{k=1}^{K} r_m[k] s_m[k] \exp\{j2\pi f_m(kT_i - \tau_{mn})\} \times \exp\{j2\pi f_{mn}(kT_i - \tau_{mn})\}. \tag{14}
\]

Hence, the ML estimator can be rewritten as
\[
\hat{\theta} = \arg \max_{\theta} \sum_{n=1}^{N} \sum_{m=1}^{M_K} \sum_{k=1}^{K} \left\{ e^{-j2\pi f_m kT_i} \left[ e^{-j2\pi f_{mn} kT_i} \right] \right\} \tag{15}
\]

Clearly, the phase terms \( e^{-j2\pi f_m kT_i} \) in (14) imply to exploit the received signals from all transmit-to-receive paths in a coherent way.

4 Numerical analysis

Consider a reference plane centred at \((15,10)\) km. Suppose that a target presenting at \((15.15, 10.1275)\) km is moving with velocity \((50, 30)\) m/s. Assume that there are \( M = 2 \) non-cooperative IOOs located 7 km distant away from the origin with directions uniformly ranging from \( \pi/5 \) to \( 5\pi/18 \), where the angle is measured with respect to the horizon axis. The \( N = 3 \) receivers are also 7 km distant away from the origin but the angles range from \( \pi/6 \) to \( \pi/3 \). The system setup is plotted in Fig. 1.

We employ the GSM communications stations as the IOOs, so that the SOOs transmitted by the \( m \)th IOO is the GMSK signals given by [12]
\(s_{\text{out}}[k] = A_{\text{out}} \exp \left\{ \sum_{j=1}^{N_s} \sum_{m=1}^{L} z(t_{j} \pm iT_{p}) \right\} e^{j2\pi \Delta f T_{p} \tau_{j}}, \) (16)

where \(c_{\text{out}} \in \{ -1, 1 \} \) is the \(i\)th (\(i = 1, \ldots, N_s\)) binary data bit, \(N_{s}\) denotes the number of bits contained in the observation interval, \(A_{\text{out}} = 1 / \sqrt{2}\) is the normalisation factor, and \(\Delta f = f_{c} - f_{s}\) is the frequency offset between adjacent signals. The term \(z(t)\) is defined as

\[
z(t) = \frac{\pi}{\sqrt{T_{p}}} \left( \frac{2 \pi B}{\ln 2} \left( \sqrt{T_{p}} - T_{p} \right) - \theta(t) \right),
\] (17)

where \(\theta(t) = \int (1/\sqrt{2\pi}) e^{-t^2/2} dt, T_{p}\) is the bit duration, and \(B\) denotes the 3 dB bandwidth of the Gaussian prefilter used in the GMSK modulators.

Let \(N_{s} = 16\) to reduce computation complexity. The bits \(\{c_{\text{out}}\} = 1\) or \(-1\) are generated randomly with equal probability. Considering all possibility of the data bits, the expected CRB averaged over all bit sequences (ECRBOB) [15] is calculated. According to conventional GSM system configuration, we set \(T_{p} = 577\) μs, \(B_{p} = 0.3\), and the carrier frequency \(f_{c} = 900\) MHz. We set \(\Delta f = 300\) Hz for non-orthogonal SOOs and 3 kHz for orthogonal SOOs [15].

4.1 Non-orthogonal and orthogonal signals

Assuming that the clutter-plus-noise are spatially independent, the root-mean-squared error (RMSE) and root-ECRBOB (RECRBOB) for the estimates of target location \(x, y\) and velocities \(v_{x}, v_{y}\) are plotted versus the signal-to-clutter-plus-noise ratio (SCNR) in Fig. 2, where both orthogonal and non-orthogonal signals are considered. We see that the MSE of the ML estimates of the target location \(x, y\) and velocities \(v_{x}, v_{y}\) approach the corresponding RECRBOBs, which proves the correctness of the previous derivations. The threshold SCNR is observed, at which the RMSE jumps drastically to a lower value and becomes close to the corresponding RECRBOB. We find that in this example, compared with the non-orthogonal SOOs, orthogonal SOOs lead to smaller RECRBOBs and smaller thresholds which means better estimation performance.

4.2 Correlated and uncorrelated clutter-Plus-noise

Now we investigate the estimation performance when the SOOs are non-orthogonal and components of the clutter-plus-noise \(w\) are spatially correlated. Assume the covariance matrix \(\mathbf{w}\) is \(\mathbf{Q} = \sigma_{w}^{2} \mathbf{I}_{k} \otimes \mathbf{I}_{k}\), where \(\mathbf{Q}\) is an \(N \times N\) matrix, whose \(m\)th element is set to be \(\exp \{ -\gamma_{d} d_{mn} \}\), in which \(d_{mn} = \left| (x_{m} - x_{n}) + i(y_{m} - y_{n}) \right|^{2}\). The factor \(\gamma_{d}\) denotes the level of the correlation, and a smaller \(\gamma_{d}\) means stronger spatial correlation.

The example in Fig. 3 considers different levels of spatial correlation of the clutter-plus-noise components, where \(\gamma = \infty\) (Case 1), 0.0001 (Case 2), and 0.00001 (Case 3), respectively. It can be seen that both the RECRBOBs and the thresholds decrease with the decrease of \(\gamma\), which implies that bigger spatial correlations of the clutter-plus-noise components result in better estimation performance.

4.3 Coherent and non-coherent processing

In this example, the estimation performance obtained by coherent processing is compared with that of non-coherent processing. For non-coherent processing reference, assuming the phase synchronisation is unavailable, so the phase terms in the ML estimate in (5) are ignored. The other parameters used are the same as those in Section 4.1. The RMSE and the RECRBOB curves for both coherent and non-coherent processing are shown in Fig. 4. We see that in all the tested cases, coherent processing leads to much lower RMSEs and RECRBOBs compared with the non-coherent processing as expected. Intuitively, such performance gain is obtained by exploiting the phase information via coherent processing.

5 Conclusion

This paper investigated joint parameter estimation using passive MIMO radar system with coherent processing. The ML estimator for the general case with possibly non-orthogonal SOOs and correlated clutter-plus-noise was presented, followed by a simplified version for a special case. The corresponding CRB has been derived. Employing GSM signals as the SOOs, numerical examples have been provided to show the correctness of the theoretical derivation and to illustrate the effects of the orthogonality of the SOOs and the correlation of the clutter-plus-noise. It has also been shown that coherent processing can outperform non-coherent processing in terms of the joint estimation performance.

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Fig. 4 RMSE (in m for location and m/s for velocity) and RECRBOB versus SCNR for coherent and non-coherent processing

7 References

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