Experimental and numerical modeling of a viscous incompressible fluid flow with dispersed particles in a rectangular channel

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Abstract. In this paper, a laminar flow of a viscous incompressible fluid with suspended spherical particles in a channel of rectangular cross-section is considered. Physical and mathematical modeling of the investigated process is conducted. The physical model is a vertical channel of rectangular cross-section into which fluid with solid particles is injected. The mathematical model is written in the diffusion approximation and includes the Navier-Stokes system of equations, the continuity equation and the diffusion equation which written, taking into account the effect of gravitational forces on particles.

1. Introduction
Disperse systems play a significant role in nature and industry. The properties of dispersed phases can be different and unique as well as incredibly complex due to the hydrodynamic interaction between particles (liquid drops or gas bubbles) and the surrounding liquid.

The problem of transport of particles suspended in a liquid in various hydraulic systems takes place in many industrial applications. The oil and gas industry faces one of these problems in solving the problem of separating water-in-oil emulsions in a convective flow [1]. It is associated with the search of regimes in which the dispersion of a dispersed system occurs. The study of the influence of emulsion properties on the dynamics of flow and stratification in the channel contributes to the solution of problems arising during transportation through pipes. Pressure losses in pipelines in multiphase flow regimes depend on many parameters, such as fluid density, viscosity, temperature, the internal diameter of the pipe, the proportion of water and flow rate [2]. At the same time, the effective viscosity of the emulsion may vary depending on the diameter of the pipe [3]. Experimental and mathematical modeling of these processes will solve some of these problems.

Another problem arises when applying one of the popular stimulation technologies - hydraulic fracturing. The question of transporting proppant in a fracture is related to gravitational sedimentation of the proppant, using the correct fluid to transport it deep into the fracture, leakage of fluid from the fracture into the reservoir and plugging of cracks [4].

Proppant transport in a fracture has been widely studied using not only experimental [5, 6] but also numerical methods [7, 8]. There is a family of closely related multiscale multiphase flow models built to describe all stages of hydraulic fracturing technology [9] as well as many models of proppant transport and sedimentation during hydraulic fracturing: starting from simplified models and ending with more complex models based on granular kinetic theory [10]. So the joint experimental and mathematical
modeling of the processes occurring in dispersed systems during transportation in hydraulic systems is of interest [11].

In this paper, the flow of a dispersed system in a rectangular channel is investigated using the methods of physical and mathematical modeling. Solid dispersed particles suspended in a viscous incompressible fluid are considered.

2. Physical modeling

Experimental studies were carried out on a specially built laboratory setup (figure 1). The main element of the setup is an experimental cell, which is a vertical rectangular channel, made of polycarbonate. Geometrical dimensions of the channel: length – 500 mm, height – 40 mm, and width – 2 mm. The working agent in the form of a mixture of liquid with solid glass particles (d = 0.6±0.1 mm) is in an airtight container.

The glycerol is used as the working fluid. The rheological characteristics of the working agent were determined using a viscometer Brookfield DV-II+ Pro. The average dynamic viscosity of pure glycerol \( \mu = 625 \text{ mPa} \cdot \text{s} \), which is in good agreement with the previous results [12]. The dependence of the dynamic viscosity of the mixture on the concentration of solid particles was measured using an express analyzer EAK-2M (figure 2).

It is seen that with an increase in the concentration of solid particles, the viscosity of the mixture increases nonlinearly. Experimental points are described by dependence [11, 13]:

\[
[\mu] = \mu_0 (1 - C/C_0)^m
\]
\[ \mu(C) = \mu(0) \left(1 - \frac{C}{C_m}\right)^{-m}, \]  

where the empirical coefficients in this work are defined as follows: \(C_m = 0.64, m = 1.03\).

The geometrical characteristics of the solid particles were determined using an optical microscope Olympus IX71, and the obtained images were processed using ImageJ software. Overpressure is created in the tank using an air compressor Fubag OL231/24. The pressure is adjusted using the pressure controller Parker Bench Top. To prevent premature separation of the mixture and create a uniform concentration, a mixing device is installed in an airtight container. All elements of the setup are connected by a system of tubes with ball valves. The experiments were carried out at room temperature of 27°C.

The technique of the experiments is as follows: a constant overpressure is created in the tank with the help of a compressor; the liquid with suspended solid particles from the tank under pressure is fed to the input of the experimental cell. The results of experimental studies are recorded on a video camera.

3. Mathematical modeling

Mathematical modeling of the process is carried out according to the experimental conditions. Figure 3a shows a schematic of the computational domain. It is believed that the fluid is incompressible, solid particles have a spherical shape and the same size, and the flow is laminar. The mathematical model is written in the diffusion approximation and includes the Navier-Stokes system of equations, the continuity equation and the diffusion equation, written taking into account the effect of gravitational forces on the particles.

\[ \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho} \nabla p + \nabla (\nu \nabla \mathbf{v}), \]  

\[ \frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla \rho = 0, \]  

\[ \frac{\partial C}{\partial t} + \mathbf{v} \cdot \nabla C = D \Delta C + (\mathbf{v}_{\text{sed}} \cdot \nabla) C, \]

where \(\rho\) is the density of the mixture; \(\mathbf{v}\) is the flow rate of the mixture; \(\mathbf{v}_{\text{sed}}\) is the deposition rate of particles; \(p\) is the pressure in the fluid; \(D\) is the diffusion coefficient; \(\nu\) is the kinematic viscosity of the mixture; and \(C\) is the volume concentration of particles in the liquid.

![Figure 3. Schematic of the computational domain (a) and distribution of inlet concentration (b).](image-url)
The sedimentation rate of the particles is calculated by:

\[ \nu_{sed} = \frac{2}{9} r_0^2 g \frac{(\rho_1-\rho_2)(1-C)^{4.7}}{\mu_2}, \]  

(5)

where \( \rho_1 \) is the particle density; \( \rho_2 \) is the liquid density; \( r_0 \) is the particle radius; \( \mu_2 \) is the dynamic viscosity of the liquid; and \( g \) is the acceleration of gravity.

The dynamic viscosity of the suspension is determined by the formula (1). According to the experimental conditions, at the initial time, the cell was filled with pure liquid; the system was at atmospheric pressure at rest:

\[ \nu = 0; \; C = 0; \; p = p_{atm} \]  

(6)

The injection of fluid with suspended solid particles occurs at a given pressure drop \( \Delta p \), and a sticking condition is set at solid channel boundaries, a flow condition is set at the inlet and outlet:

\[ p_{in} = p_{atm} + \Delta p, \; p_{out} = p_{atm}, \; \nabla p_{wall} = 0, \]
\[ \nabla \nu_{in} = 0, \; \nabla \nu_{out} = 0, \; \nu_{wall} = 0, \]
\[ C_{in} = C_0(y), \; D \nabla C_{wall} - \nu_{sed} C_{wall} = 0, \; \nabla C_{out} = 0, \]  

(7)

where \( C_0(y) \) is the concentration of particles at the entrance to the experimental cell, which is specified as a function of height in accordance with the experimental data (figure 3b).

The numerical solution of equations (2–4) with closing relations (1), (5) - (7) was carried out using the control volume method in an open-source CFD software OpenFOAM.

4. Results

Experiments on fluid flow modeling with dispersed particles in the cell were carried out at various values of pressure with suspension of 5% vol. At the outlet of the cell, there was atmospheric pressure. Figure 4 shows the distribution of solid particles in the fluid flow in the cell at time of 30 s at pressure drops of 20, 40, and 80 kPa.

![Figure 4](image_url)

Figure 4. Particle distribution in the fluid flow in the cell at a pressure drop of 20 kPa (a), 40 kPa (b), 80 kPa (c) at temperature of 27°C.
At pressure drops less than 20 kPa, particles had time to settle in the inlet tube. Thus, an almost completely stratified hydraulic mixture entered the channel. The accumulation of solid particles in the first half of the channel was observed. The results of the experiment at a pressure drop of 20 kPa show that after 30 seconds a shaft of solid particles formed in the first half of the channel.

Its height practically doesn't change during the whole experiment, although the particles continue to flow into the channel. The motion of the slurry is observed in the form of sliding layers in which solid particles move along the bottom of the channel in the direction of flow. Only the expansion of the precipitation area to the right occurs. This is explained by the fact that the characteristic sedimentation velocity of solid particles \( v_{\text{sed}} = 1.2 \text{ mm/s} \) in this case is over than the characteristic flow rate in the channel \( 1.1 \text{ mm/s} \). The bulk of the particles settle in the inlet tube. At differential pressure of 40 kPa (figure 4b), the shaft is formed in the second half of the channel. The experimental conditions are set in such a way that the characteristic sedimentation velocity of solid particles is less than the characteristic velocity of fluid flow along the channel. As a result, most of the particles do not have time to settle, and a small layer of solid particles is formed at the bottom of the channel. The concentration of particles is less than in the first case. The case when the sedimentation velocity of particles is much less than the flow rate corresponds to the results obtained at a pressure drop of 80 kPa (figure 4c). It can be seen that the particles practically don't have time to settle at the bottom of the channel; the formation of shafts isn't observed - only a barely noticeable layer with a high concentration of particles forms at the bottom.

Numerical simulation was carried out based on experimental data. The results are presented in figure 5 as a distribution of particle concentration in the channel also at time of 30 s with pressure drops of 20, 40, and 80 kPa.

![Figure 5](image)

**Figure 5.** The distribution of the concentration of solid particles in a liquid at time of 30 s at a pressure drop of 20 kPa (a), 40 kPa (b), 80 kPa (c).

In the simulation, the concentration of particles at the entrance to the channel was set according to the data that were observed during the experiments. At a pressure drop of 20 and 40 kPa, a partially separated dispersion system entered the channel; accordingly, the concentration in the lower part of the inlet was set higher taking into account the condition of preservation of the volume of injected particles. From figure 5a, it is clear that the particles immediately rush to the bottom of the channel, forming a small layer, the size of which agrees with the results of experimental studies (figure 4a). As the pressure drop increases, the separation of the disperse system occurs closer to the exit of the channel (figure 5b). The difference in the results that occurs in the initial part of the channel to a distance of 0.1 m is explained by incomplete reproduction in the mathematical model of the geometry of the inlet of the channel (at the junction of the tube with the channel). This affects the hydrodynamics of the flow at the entrance to the channel. According to the simulation results with a pressure drop of 80 kPa, there is a small layer of solid particles at the bottom of the channel (figure 5c), which is hardly visible in figure 4c.
Conclusion
The results of experimental and numerical modeling of the dynamics of the distribution of glass spherical particles (d = 0.6 ± 0.1 mm) in a liquid flow (glycerol) and sedimentation of them along the channel at a pressure drop of 20, 40, and 80 kPa and a temperature of 27°C are presented. The considered pressure drops correspond to cases when the characteristic velocity of sedimentation of solid particles is higher, comparable, or lower than the characteristic flow rate in the channel. At the pressure drop of 20 kPa, the formation of a shaft of solid particles in the first half of the channel was observed. Solid particles moved along the bottom of the channel in the direction of flow. At the pressure drop of 40 kPa most of the particles don't have time to settle, and a small layer of solid particles forms in the second half of the channel at the bottom of the channel. In the case of a large flow rate of the liquid at the pressure drop of 80 kPa, the particles practically don't have time to settle at the bottom of the channel; shafts' formation isn't observed - only a barely noticeable layer with a high concentration of particles is formed at the bottom.

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