Predictive Information Accelerates Learning in RL

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Abstract

The Predictive Information is the mutual information between the past and the future, $I(X_{\text{past}}; X_{\text{future}})$. We hypothesize that capturing the predictive information is useful in RL, since the ability to model what will happen next is necessary for success on many tasks. To test our hypothesis, we train Soft Actor-Critic (SAC) agents from pixels with an auxiliary task that learns a compressed representation of the predictive information of the RL environment dynamics using a contrastive version of the Conditional Entropy Bottleneck (CEB) objective. We refer to these as Predictive Information SAC (PI-SAC) agents. We show that PI-SAC agents can substantially improve sample efficiency over challenging baselines on tasks from the DM Control suite of continuous control environments. We evaluate PI-SAC agents by comparing against uncompressed PI-SAC agents, other compressed and uncompressed agents, and SAC agents directly trained from pixels.

1 Introduction

Many Reinforcement Learning environments have specific dynamics and clear temporal structure: observations of the past allow us to predict what is likely to happen in the future. However, it is also commonly the case that not all information about the past is relevant for predicting the future. Indeed, there is common a Markov assumption in the modeling of RL tasks: given the full state at time $t$, the past and the future are independent of each other.

However, in general not all RL tasks are specified with a full state vector that can guarantee Markovity. Instead, the environment may be only partially observable, or the state may be represented in very high dimensions, such as an image. In such environments, the task of the agent may be described as finding a representation of the past that is most useful for predicting the future, upon which an optimal policy may more easily be learned.

In this work, we approach the problem of learning continuous control policies from pixel observations. We do this by first explicitly modeling the Predictive Information, the mutual information between the past and the future. In so doing, we are looking for a compressed representation of the past that the agent can use to select its next action, since most of the information about the past is irrelevant for predicting the future, as shown in [4]. This corresponds to learning a small state description that makes the environment more Markovian, rather than using the entire observed past as a state vector.

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This explicit requirement for a concise representation of the Predictive Information leads us to prefer objective functions that are compressive. Philosophically and technically, this is in contrast to other recent approaches that have been described in terms of the Predictive Information, such as Contrastive Predictive Coding (CPC) and Deep InfoMax (DIM), which do not explicitly compress.

Modeling the Predictive Information is, of course, insufficient to solve RL problems. We must also provide a mechanism for learning how to select actions. In purely model-based approaches, such as PlaNet, that can be achieved with a planner and a reward estimator that both use the model’s state representation. Alternatively, one can use the learned state representation as an input to a model-free RL algorithm. That is the approach we explore in this paper. We train a standard Soft Actor-Critic (SAC) agent with an auxiliary model of the Predictive Information. Together, these pieces give us Predictive Information Soft Actor-Critic (PI-SAC).

The main contributions of this paper are:

- **PI-SAC**: A description of the core PI-SAC agent (Section 3).
- **Sample Efficiency**: We demonstrate strong gains in sample efficiency on nine tasks from the DM Control Suite of continuous control tasks, compared to state-of-the-art baselines such as Dreamer and DrQ (Section 4.1).
- **Ablations**: Through careful ablations and analysis, we show that the benefit of PI-SAC is due substantially to the use of the Predictive Information and compression (Section 4.2).
- **Generalization**: We show that compressed representations outperform uncompressed representations in generalization to unseen tasks (Section 4.3).

## 2 Preliminaries

**Predictive Information.** The Predictive Information is the mutual information between the past and the future, $I(\text{past}; \text{future})$. From here on, we will denote the past by $X$ and the future by $Y$. shows that the entropy of the past, $H(X)$, is a quantity that grows much faster than the Predictive Information, $I(X; Y)$, as the duration of past observations increases. Consequently, if we would like to represent only the information in $X$ that is relevant for predicting $Y$, we should prefer a compressed representation of $X$. This is a sharp distinction with techniques such as Contrastive Predictive Coding (CPC) and Deep InfoMax (DIM) which explicitly attempt to maximize a lower bound on $I(X; Y)$ without respect to whether the learned representation has compressed away irrelevant information about $X$.

The Conditional Entropy Bottleneck. In order to learn a compressed representation of the Predictive Information, we will use the Conditional Entropy Bottleneck (CEB) objective. CEB attempts to learn a representation $Z$ of some observed variable $X$ such that $Z$ is as useful as possible for predicting a target variable $Y$, while compressing away any information from $X$ that is not also contained in $Y$:

$$
CEB \equiv \min_{Z} \beta (I(X; Z|Y) - I(Y; Z)) = \min_{Z} \beta (-H(Z|X) + H(Z|Y)) - I(Y; Z)
$$

To get a variational lower bound on the $I(Y; Z)$ term, we will use the CatGen formulation from, which is equivalent to the InfoNCE bound but reuses the backwards encoder:

$$
I(Y; Z) \geq \mathbb{E}_{x,y,z \sim p(x,y) \mid c(z|x)} \log \frac{b(z|y)}{\frac{1}{K} \sum_{k=1}^{K} b(z|y^k)}
$$
Figure 1: PI-SAC system diagram for a single minibatch example. To compute $J_{CEB}$ requires $K$ $b_{\psi_k}(\cdot)$ distributions from the minibatch, as described in Section 2. Colored edges show how gradients flow back to model weights.

We write the objective for a single example in a minibatch of size $K$ to simplify notation. The $K$ examples are sampled independently. Altogether, this gives us:

$$CEB \leq \min_Z \mathbb{E}_{x,y,z \sim p(x,y) \text{e}(z|x)} \beta \log \frac{e(z|x)}{b(z|y)} - \log \frac{b(z|y)}{1 \sum_{k=1}^{K} b(z|y_k)}$$

(6)

**Soft Actor-Critic.** Soft Actor-Critic (SAC) [14] is an off-policy algorithm that learns a stochastic policy $\pi_{\phi}$, a Q-value function $Q_\theta$, and a temperature coefficient $\alpha$ to find an optimal control policy. It maximizes a $\gamma$-discounted return objective based on the Maximum Entropy Principle [39, 38, 13, 22]. SAC has objectives for the critic, the actor, and the temperature parameter, $\alpha$. The critic minimizes:

$$J_Q(\theta) = \mathbb{E}_{s_t, a_t \sim D} \frac{1}{2} \left( Q_\theta(s_t, a_t) - (r(s_t, a_t) + \gamma \mathbb{E}_{s_{t+1} \sim p} V_{\bar{\theta}}(s_{t+1})) \right)^2$$

(7)

where $V_{\bar{\theta}}(s_t) \equiv \mathbb{E}_{a_{t} \sim \pi(a_t|s_t)} Q_\theta(s_t, a_t) - \alpha \log \pi(a_t|s_t)$ is a value function that uses an exponential moving average of the $\theta$ parameters, $\bar{\theta}$. The actor minimizes:

$$J_\pi(\phi) = \mathbb{E}_{a_t \sim \pi} \mathbb{E}_{a_{t} \sim \pi(a_t|s_t)} \alpha \log \pi(a_t|s_t) - Q_\theta(s_t, a_t)$$

(8)

Given $\mathcal{H}$, a target entropy for the policy distribution, the temperature $\alpha$ is learned by minimizing:

$$J_{\alpha}(\alpha) = \mathbb{E}_{a_t \sim \pi(a_t|s_t)} - \alpha \log \pi(a_t|s_t) - \alpha \mathcal{H}$$

(9)

### 3 Predictive Information Soft Actor-Critic (PI-SAC)

A natural way to combine a stochastic latent variable model trained with CEB with a model-free RL algorithm like SAC is to use the latent representation at timestep $t$, $z_t$, as the state variable for the actor, the critic, or both. We will call this Representation PI-SAC and define it in Section 6. However, any representation given to the critic cannot have a dependency on the next action, and any representation given to the critic can depend on at most the next action, since during training and evaluation, the actor must use the representation to decide what action to take, and the critic needs the representation to decide how good a particular state and action are. The CEB model’s strength, on the other hand, lies in capturing a representation of the dynamics of the environment multiple steps into the future. We may therefore hypothesize that using CEB as an auxiliary loss can be more effective, since in that setting, the future prediction task can be conditioned on the actions taken at each future frame. Conditioning on multiple future actions in the forward encoder allows it to make more precise predictions about the future states, thereby allowing the forward encoder to more accurately model environment dynamics. Consequently, PI-SAC agents are trained using CEB as an auxiliary task, as shown in Figure 1.

1 We give details and results for Representation PI-SAC agents in Section 6. Representation PI-SAC agents are also very sample efficient on most tasks we consider, but they don’t achieve as strong performance on tasks with more complicated environment dynamics, such as Cheetah, Hopper, and Walker.
We evaluate PI-SAC on the DeepMind control suite \cite{46} and compare with leading model-free and model-based approaches for continuous control from pixels: SLAC \cite{27}, Dreamer \cite{15}, and DrQ \cite{25}. Our benchmark includes the six tasks from the PlaNet benchmark \cite{16} and three additional tasks: Cartpole Balance Sparse, Hopper Stand, and Walker Stand.

The PlaNet benchmark treats action repeat as a hyperparameter. On each PlaNet task, we evaluate PI-SAC with the action repeat at which SLAC performs the best\footnote{The SLAC results that we report outperform the current version \cite{27} at the time of submission. The paper also made a typo in that SLAC actually takes 3 gradient updates per environment step instead of 1.} and compare with the best DrQ result. The choices of action repeat are listed in Section \ref{exp-repeat}. On Walker Walk (also in the PlaNet benchmark), Cartpole Balance Sparse, Hopper Stand, and Walker Stand, we evaluate PI-SAC with action repeat 2 and directly compare with D Renderer and DrQ results on the Dreamer benchmark. We

Algorithm 1 Training Algorithm for PI-SAC

\begin{algorithm}[h]
\caption{Training Algorithm for PI-SAC}
\begin{algorithmic}
\Require $E_{\text{step}}, \theta_e, \phi_e^1, \phi_e^2, \phi_a, \alpha, \psi_a, \psi_b$
\State $\theta_b \leftarrow \theta_e$
\State $D \leftarrow \emptyset$
\For {each initial collection step}
\State $a_t \sim \pi_{\text{random}}(a_t)$
\State $s_{t+1}, r_{t+1} \sim E_{\text{step}}(a_t)$
\State $D \leftarrow D \cup \{(s_{t+1}, a_t, r_{t+1})\}$
\EndFor
\State $s_1 \leftarrow E_{\text{step}}()$
\For {$t=1$ to $M$}
\State $a_t \sim \pi_e(a_t|s_t)$
\State $s_{t+1}, r_{t+1} \sim E_{\text{step}}(a_t)$
\State $D \leftarrow D \cup \{(s_{t+1}, a_t, r_{t+1})\}$
\For {each gradient step}
\State $\phi_e^i, \theta_e \leftarrow \phi_e^i, \theta_e - \lambda \nabla_{\phi_e^i, \theta_e} J_Q(\phi_e^i, \theta_e)$ for $i \in \{1, 2\}$
\State $\alpha \leftarrow \alpha - \lambda \nabla_{\alpha} J_\pi(\alpha)$
\State $\{\theta_e, \psi_e, \psi_b\} \leftarrow \{\theta_e, \psi_e, \psi_b\} - \lambda \nabla_{\theta_e, \psi_e, \psi_b} J_{CEB}(\theta_e, \psi_e, \psi_b)$
\State $\dot{\theta}_e \leftarrow \tau \theta_e - (1 - \tau) \dot{\theta}_e$
\State $\dot{\theta}_b \leftarrow \tau \theta_b - (1 - \tau) \dot{\theta}_b$
\EndFor
\EndFor
\end{algorithmic}
\end{algorithm}

PI-SAC uses the same three objective functions from SAC, described above. The only additional piece to specify is the choice of $X$ and $Y$ for the CEB objective. In our setting, $X$ consists of previous observations and future actions, and $Y$ consists of future observations and future rewards. If we define the present as $t = 0$ and we limit ourselves to observations from $−T + 1$ to $T$, we have:

$$J_{CEB}(\theta_e, \psi_e, \psi_b) = \mathbb{E}_{s_{-T+1:T}, a_{0:T-1}, r_{1:T} \sim D, z_0 \sim \epsilon(z_0)} \log \frac{e_{\theta_e, \psi_e}(z_0|s_{-T+1:T}, a_{0:T-1})}{b_{\psi_e}(z_0|s_{1:T}, r_{1:T})} + \log \frac{b_{\psi_b}(z_0|s_{1:T}, r_{1:T})}{\sum_{k=1}^K b_{\psi_b}(z_0|s_{1:T}, r_{1:T})}$$

The training algorithm for PI-SAC is in Algorithm\footnote{The SLAC results that we report outperform the current version \cite{27} at the time of submission. The paper also made a typo in that SLAC actually takes 3 gradient updates per environment step instead of 1.} where $E_{\text{step}}$ is the environment step function. $\theta_e$ is the weight vector of the convolutional encoder. $\theta_b = \text{EMA}(\theta_e, \tau_b)$ is the weight vector of the convolutional backwards encoder, where EMA($\cdot$) is the exponential moving average function. $\phi_e^1$, $\phi_e^2$, and $\phi_a$ are the weight vectors for two critic networks and the actor network, respectively. $\alpha$ is the SAC temperature parameter. $\psi_e$ and $\psi_b$ are the weight vectors of MLPs to parameterize the CEB forward and backwards encoders. $\tau$ and $\tau_b$ are exponents for EMA calls. $\lambda_Q$, $\lambda_\pi$, $\lambda_\alpha$, and $\lambda_{CEB}$ are learning rates for the four different objective functions. See Section \ref{method} for implementation details.

4 Experiments

We evaluate PI-SAC on the DeepMind control suite \cite{46} and compare with leading model-free and model-based approaches for continuous control from pixels: SLAC \cite{27}, Dreamer \cite{15}, and DrQ \cite{25}. Our benchmark includes the six tasks from the PlaNet benchmark \cite{16} and three additional tasks: Cartpole Balance Sparse, Hopper Stand, and Walker Stand.

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Figure 2: Performance comparison to existing methods on 9 tasks from DeepMind control suite. The upper 6 tasks are the PlaNet benchmark [16]. Dreamer* indicates that the other agents do not use Dreamer’s action repeat of 2. We additionally include the 3 lower tasks with a fixed action repeat of 2 to compare with Dreamer [15] and DrQ [25] results on the Dreamer benchmark. PI-SAC matches the state-of-the-art performance on all 9 tasks and is consistently the most sample efficient.

Figure 2 compares PI-SAC with SLAC, Dreamer, and DrQ. PI-SAC consistently achieves state-of-the-art performance and better sample efficiency across all benchmark tasks. We report our results on Reacher Easy with one gradient update per environment step, on Cheetah Run with four gradient updates, and the rest with two gradient updates. A comparison of PI-SAC agents with different numbers of gradient updates is available in Section 4.1. The comparison in this section is system-to-system as all baseline methods have their own implementation advantages: SLAC performs three gradient updates per environment step and uses a much larger encoder architecture; DrQ’s SAC differs substantially from the standard SAC [14], including having much larger actor and critic networks; Dreamer is a model-based method that uses RNNs and learns a policy in simulation.

4.1 Sample Efficiency

Throughout these experiments we mostly use the standard SAC hyperparameters [14], including the sizes of the actor and critic networks, learning rates, and target critic update rate. Unless otherwise specified, we set CEB $\beta = 0.01$. We report our results with the best number of gradient updates per environment step in Section 4.1 and use one gradient update per environment step for the rest of the experiments. Full details of hyperparameters are listed in Section A.2. We use an encoder architecture similar to DrQ [25]; the details are described in Section A.1.

4.2 Predictive Information

We test our hypothesis that predictive information is the source of the sample efficiency gains here.
Figure 3: The predictive information improves performance on Cartpole Swingup and Cheetah Run without any data augmentation. With data augmentation, it continues showing strong improvements over the SAC baseline on all three tasks. We perform 5 runs for PI-SAC and SAC without augmentation. More results are presented in Section E.

Figure 4: We learn a diagnostic deconvolutional decoder to predict future observations from CEB representations learned along with PI-SAC for Reacher. We show ground truth future observations and the predicted future observations from CEB representations. Left: CEB representations learned without data augmentation only capture positions of the target. Right: CEB representations learned with data augmentation capture both the target and the arm.

**Data Augmentation.** We follow [25] to train our models with image sequences randomly shifted by $[-4, 4]$ pixels. Without this perturbation, Figure 3 shows that learning the predictive information by itself still greatly improves agents’ performance on Cartpole and Cheetah but makes little difference on Reacher. Learning PI-SAC with data augmentation continues showing strong improvements over the SAC baseline with data augmentation and solves all benchmark tasks.

[25, 26] showed that input perturbation facilitates actor-critic learning, and we show that it also improves CEB learning. As described in Section 2, we use the contrastive CatGen formulation to get a variational lower bound on $I(Y; Z)$. Because of its contrastive nature, CatGen can ignore information that is not required for it to distinguish different samples and still saturate its bound. In our experiments without data augmentation, we found that it ignores essential information for solving Reacher. We train a deconvolutional decoder to diagnostically predict future frames from CEB representations (we stop gradients from the decoder). As shown in Figure 4, CEB representations learned without input perturbation completely fail to capture the arm’s pose. This is because CatGen can perfectly distinguish frame sequences in a minibatch sampled from the replay buffer by only looking at the position of the target, since that is constant in each episode but varies between episodes. In contrast, CatGen representations learned with randomly shifted images successfully capture both the target and the arm. This observation suggests that appropriate data augmentation helps CatGen to capture meaningful information for control.

**Compression.** As described in Section 2, we compress the residual information $I(X; Z | Y)$ out to preserve the minimum necessary predictive information. Figure 5 studies the trade-off between strength of compression and agents’ performance by sweeping $\beta$ values. Some amount of compression improves sample efficiency and stability of the results, but overly strong compression can be harmful. The impact of $\beta$ on the agent’s performance confirms that, even though the CEB representation isn’t being used directly by the agent, the auxiliary CEB objective is able to substantially change the agent’s weights. Sweeping $\beta$ allows us to explore the frontier of the agent’s performance, as well as the Pareto-optimal frontier of the CEB objective as usual [6]. For example, for Cheetah Run, the residual information at the end of training ranges between $\sim 0$ nats for $\beta = 1$, to $\sim 947$ nats for $\beta = 0$. For the top performing agent, $\beta = 0.01$, the residual information was $\sim 6$ nats.

**Comparison to Multiview Self-Prediction.** Multiview Self-Prediction (MVSP) is an auxiliary objective used by CURL [34]. CURL uses the InfoNCE bound to capture the mutual information
between two random crops $I(X_{\text{crop1}}; X_{\text{crop2}})$ as an auxiliary task for continuous control from pixels. This approach preserves information about the present, differing philosophically from PI-SAC which captures information about the future. By changing the CEB prediction target from the future ($Y$) to a random shift of the past observation, $X'$, we can achieve the equivalent multiview self-prediction in our framework and fairly compare the two approaches. Figure 6 compares PI-SAC to the MVSP auxiliary task. PI-SAC agents outperform MVSP agents at all levels of compression. For MVSP agents, compression also helps for tasks like Cheetah and Reacher. More results are in Section D. This empirical evidence suggests that, for RL agents, knowing what will happen in the future matters more than knowing what has happened in the past.

4.3 Generalization to Unseen Tasks

It is well-known that compressed representations can generalize better in many machine learning settings \cite{31 3 7}, including RL \cite{18 10 9}. In addition to sample efficiency, for more testing of generalization, we explore transferring representations to an unseen task with the same environment dynamics. Specifically, we learn a PI-SAC agent on a source task, freeze the representation $\psi_{\text{det}}$ (see Figure 1), and train a new agent for a target task using the frozen representation. Figure 7 shows that compressed representations generalize substantially better to unseen tasks than uncompressed representations. Especially when the target task is intuitively harder than the source task, i.e. Cartpole Balance to Swingup and Walker Stand to Walk, the performance differences between different levels of compression are more significant on the target task than on the original tasks. It is, however, less prominent when the target task is easier, i.e. Walker Walk to Stand. Our conjecture is that solving the original Walk task would require exploring a wider range of the environment dynamics that presumably includes much what the Stand task would need. On the other hand, transferring from Stand to Walk requires generalization to more unseen part of the environment dynamics. Note that in these settings it is still more sample efficient to train a full new PI-SAC agent on the target task. These experiments simply demonstrate that the more compressed predictive information models have representations that are more useful in a task transfer setting.

5 Related Work

Future Prediction in RL. Future prediction is commonly used in reinforcement learning in a few different ways. Model-based RL algorithms build world model(s) to predict the future conditioned on
We presented Predictive Information Soft Actor-Critic (PI-SAC), a continuous control algorithm that trains a SAC agent using an auxiliary objective that learns a compressed representation of the predictive information of the RL environment dynamics. We showed with extensive experiments that learning a compressed predictive information representation can substantially improve sample efficiency and training stability at no cost to final agent performance. Furthermore, we gave preliminary indications that compressed representations can generalize better than uncompressed representations at task transfer. Future work will explore variations of the PI-SAC architecture, such as using RNNs for environments that require long-term planning.

6 Conclusion

We presented Predictive Information Soft Actor-Critic (PI-SAC), a continuous control algorithm that trains a SAC agent using an auxiliary objective that learns a compressed representation of the predictive information of the RL environment dynamics. We showed with extensive experiments that learning a compressed predictive information representation can substantially improve sample efficiency and training stability at no cost to final agent performance. Furthermore, we gave preliminary indications that compressed representations can generalize better than uncompressed representations at task transfer. Future work will explore variations of the PI-SAC architecture, such as using RNNs for environments that require long-term planning.
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Appendices

A PI-SAC Implementation

Initial CEB training steps. After collecting initial experiences with a random policy (in Algorithm 1), we optionally pre-train with the predictive information CEB objective ($\theta_c$, $\bar{\theta}_c$, $\psi_e$, $\psi_b$ are updated). The amount of initial CEB steps are selected empirically for each task (listed in Table 1).

Observation Horizon and Frame Stacking. As described in Section 3, we limit our observations from $-T + 1$ to $T$ (following [34, 25], we set $T = 3$). We construct the observational input to the encoder and backward encoder as a $T$-stack of consecutive frames, where each frame is a $[84 \times 84 \times 3]$ RGB image rendered from the 0th DMControl camera. The pixel values range from $[0, 255]$, and we divide each pixel by $255.0$.

Evaluation Setups. We evaluate our agent at every evaluation point by computing the average episode return over 10 evaluation episodes. At test time, our policy is deterministic and uses the mean of the policy distribution. For most of the experiments, we evaluate every 2500 environment steps after applying action repeat for Cheetah, Walker, and Hopper tasks. For Ball in Cup, Cartpole, Finger, and Reacher tasks, we evaluate every 1000 environment steps after applying action repeat.

SAC Implementation. Our SAC implementation is based off of TF-Agents [11]. It follows the standard SAC implementation [14]. The performance and sample-efficiency match with the benchmark results reported in [14].

A.1 Network Architecture

Encoder Networks. The convolutional encoder architecture consists of four convolution layers with $3 \times 3$ kernels, 32 channels, similar to the encoder architecture being used in [37, 34, 25]. We use stride 2 at the first convolution layer and 1 in the rest. Filter Response Normalization and Thresholded Linear Unit [33] are applied after each convolution layer. The output of the last convolution layer is fed into a fully-connected layer which projects to a 50-d feature vector and followed by Layer Normalization [2]. This gives us the 50-d $z_{det_e}$. As shown in Figure 1, we stop gradients from the actor network, but allow the critic optimizer and the CEB optimizer to update the convolutional encoder.

Actor and Critic Networks. Implementations of actor and critic follow the standard SAC [14]. Both actor and critic are parameterized by MLPs with two 256-d hidden layers. The actor network outputs mean and covariance for a parametric Gaussian distribution. And we use tanh as an invertible squashing function to enforce the action bounds as in [14]. Inputs to the critic network is a concatenation of $z_{det_e}$ and action.

CEB Auxiliary Model. Our CEB forward and backward MLP encoders are parameterized by MLPs with two 128-d hidden layers. Each MLP outputs 50-d mean followed by Batch Normalization [19] for a multivariate Gaussian distribution, and we fix diagonal covariance at 1.0. Inputs to the forward MLP is a concatenation of $z_{det_e}$ and $T$ future actions. Inputs to the backward MLP is a concatenation of $z_{det_e}$ and $T$ future rewards.

A.2 Hyperparameters

Throughout these experiments we mostly use the standard SAC hyperparameters [14]. The hyperparameters that are fixed across all tasks are listed in Table 1. The size of the replay buffer is a smaller 10^5 due to high memory usage for storing image observations. The heuristic entropy target is set to $-\text{dim}(A)/2$, a default value used in TF-Agents SAC implementation [11], where $\text{dim}(A)$ is number of dimensions of action. In our experiments the results are similar to using $-\text{dim}(A)$, the default used in [14].

The amount of action repeat (described in Section 4), initial collection steps, and initial CEB steps for each task are listed in Table 1. The standard SAC takes 10,000 initial collection steps, but for some of the tasks we take fewer steps of 1,000 in favor of sample efficiency.
Table 1: **Left:** Global PI-SAC hyperparameters. **Right:** Per-task PI-SAC hyperparameters. PlaNet tasks are indicated with (P).

| Parameter          | Value                          |
|--------------------|--------------------------------|
| optimizer          | Adam [24]                      |
| batch size         | 256                            |
| actor learning rate | $3 \times 10^{-4}$             |
| critic learning rate | $3 \times 10^{-4}$             |
| alpha learning rate | $3 \times 10^{-4}$             |
| CEB learning rate   | $3 \times 10^{-4}$             |
| discount ($\gamma$) | 0.99                           |
| replay buffer size | $10^6$                         |
| entropy target     | $-\dim(A)/2$                   |
| target smoothing coefficient ($\tau$) | 0.005                     |
| target update interval | 1                        |
| initial $\log(\alpha)$ | 0.0                        |
| backward encoder   | 0.05                           |
| EMA update rate    | 0.05                           |
| observation horizon $T$ | 3                           |

| Task                  | Action | Initial Collection Steps | Initial CEB steps |
|-----------------------|--------|--------------------------|-------------------|
| Cartpole Swingup (P)  | 4      | 1000                     | 5000              |
| Cartpole Balance Sparse | 2     | 1000                     | 5000              |
| Reacher Easy (P)      | 4      | 1000                     | 5000              |
| Ball in Cup Catch (P) | 4      | 1000                     | 5000              |
| Finger Spin (P)       | 1      | 10000                    | 0                 |
| Cheetah Run (P)       | 4      | 10000                    | 10000             |
| Walker Walk (P)       | 2      | 10000                    | 10000             |
| Walker Stand          | 2      | 10000                    | 10000             |
| Hopper Stand          | 2      | 10000                    | 10000             |

**B PI-SAC and SAC at Different Numbers of Gradient Steps**

Figure 8: Comparison of PI-SAC and SAC with image augmentation at different numbers of gradient steps (gs) per environment step. We report results at 1 and 2 gradient steps, except that we show 1 and 4 gradient steps for Cheetah Run. PI-SAC consistently outperforms the SAC baseline.

Another way to improve sample efficiency in SAC and PI-SAC models is to increase the number of gradient steps taken per environment step collected. In Figure 8, we see that PI-SAC outperforms the SAC baseline while varying gradient steps, particularly on Ball In Cup Catch and Cheetah Run.
C Comparison to SAC from States

It is interesting to compare PI-SAC agents, which are trained from pixels, to SAC agents that have been trained from states. Generally, training from pixels is considered to be more challenging than training from states. However, we find that PI-SAC performs comparably to SAC from states on most tasks. On Cheetah Run and Finger Spin, PI-SAC significantly outperforms state-SAC, indicating that those tasks benefit strongly from representations that model what will happen next, rather than simply needing a precise description of the current state. In contrast, state-SAC has a noticeable sample efficiency advantage over PI-SAC on Ball In Cup Catch, indicating that a precise description of the current state that is stable throughout training is more important than learning to model what will happen next.
D Comparison to Multiview Self-Prediction

Figure 10: Learning the predictive information outperforms multiview self-prediction (MVSP), which is described in Section 4.2. We compare PI-SAC to using the MVSP auxiliary task on all 9 tasks. We use the default $\beta = 0.01$ for experiments in this figure. The MVSP curves show results of 5 runs.

In Figure 10 we present results on all nine tasks for our experiments in Figure 6. For these experiments, we don’t sweep $\beta$ for the MVSP models, and instead use the default $\beta = 0.01$ for all models. In all cases, the PI-SAC models achieve equal or better performance.
Figure 11: Comparison of PI-SAC and SAC both without image augmentation on all 9 tasks. We perform 5 runs for experiments without image augmentation. PI-SAC without image augmentation always matches or improves on the SAC baseline, but some tasks are only solved with the addition of image augmentation.

In Figure 11 we compare PI-SAC agents with SAC and PI-SAC agents trained without image augmentation on all nine tasks. This is the same setting as Figure 3. Learning the predictive information without image augmentation is sufficient to significantly improve SAC agents for some tasks, and is never detrimental compared to the SAC baseline. However, augmentation is essential to solving Reacher Easy, Walker Walk, and Hopper Stand. On all tasks, having both the predictive information and image augmentation performs the best.
Figure 12: Comparison of the standard contrastive version and PI-SAC and the generative version of PI-SAC which directly predicts future observations and future rewards. The contrastive version shows better sample-efficiency and performance than the generative version every task except Hopper Stand, where the two approaches are essentially indistinguishable. The wall time per gradient step of the contrastive models is about 30% faster than the generative models, even with the small number of frames being predicted (3 future frames).

As shown in [6], under an assumption of a uniform distribution over the training examples, the contrastive CatGen formulation (eq. (5)) approximates the decoder distribution:

$$\frac{b(z|y)}{\sum_{k=1}^{K} b(z|y^k)} \approx p(y|z)$$

(11)

Instead of using CatGen in CEB, we can alternatively learn to predict $y$, the future observations and future rewards. This gives a generative variant of PI-SAC. To predict the future observations for the generative PI-SAC, we use a decoder network consisting of four transposed convolution layers with features of $(12, 64, 32, 3)$, kernel widths of $(3 \times 3, 3 \times 3, 11 \times 11, 3 \times 3)$, and strides of $(2, 2, 1, 2)$. To predict the future rewards, we use an MLP with 50-d first hidden layer and 25-d second hidden layer.

Figure 12 compares the standard contrastive version of PI-SAC to the generative variant. It shows that the contrastive version is generally more sample-efficient and gives better performance. Additionally, the generative version is slower to train in terms of wall time. These observations lead us to prefer the contrastive CatGen formulation for PI-SAC.
The CEB representation, $Z$, can be used directly by either the actor, the critic, or both. Figure 13 show the system diagrams for actor and critic Representation PI-SAC models. For models that use CEB representations for both actor and critic, it suffices to combine those systems so that there are two separate CEB objectives, but with both objectives updating the same convolutional encoder parameters ($\theta_e$). In this setting, neither the actor nor the critic pass gradients back through the convolutional encoder parameters. This an important difference from PI-SAC, where both the CEB objective and the SAC critic objective contribute gradient information to the convolutional encoder parameters. As shown in Figure 13 the actor or the critic use the mean of the forward encoder’s $Z$ distribution ($z_{det}$ in the figures). It is also possible to train using samples from the distribution ($z_0$ in the figures). Empirically, we found that the results were qualitatively the same, but often with higher variance in evaluation performance early in training, so we only present results using the mean of the representation here.

Figure 14 shows results comparing Representation PI-SAC models to Dreamer on the same tasks from the paper. In these experiments, no hyperparameters are changed between the different tasks. All Representation PI-SAC models in Figure 14 are trained using the hyperparameters in Table 1, action repeat of 2, 1000 initial collection steps, and 0 initial CEB steps. This makes the results directly comparable to Dreamer, which also uses action repeat of 2 for all tasks and has no task-specific hyperparameters.

Using a CEB representation for the actor, the critic, or both still gives much better sample efficiency than Dreamer at 6 of the 9 tasks, but the lack of the full set of future actions as input to the forward encoder in these models appears to make it more difficult for the agents to solve locomotion tasks like Cheetah Run and Walker Walk. We also tested a critic Representation PI-SAC model variant that allowed critic gradients to flow to the convolutional encoder parameters (as in PI-SAC), and found that doing so improved performance on Cartpole Swingup and Reacher Easy, but substantially worse on Ball In Cup Catch, Cartpole Balance Sparse, and Cheetah Run, and comparable on the remaining four tasks. This indicates that the difficulty on the locomotion tasks we see here is not due to the lack of critic gradients.

On most tasks, we found that actor Representation PI-SAC had the strongest performance and best stability of the three Representation PI-SAC variants. In particular, its stability on Cartpole Balance Sparse was remarkable: from 80,000 environment steps through 480,000 environment steps, all five agents got perfect scores of 1,000 on all 10 evaluation trials that occurred every 10,000 environment steps, for a total of 2,000 perfect evaluations in a row. In contrast, the other two variants (and Dreamer) had substantial deviations from perfect scores throughout training. More exploration is necessary to understand why actor Representation PI-SAC has this stability advantage.
Figure 14: Representation PI-SAC models with action repeat of 2 and no task-dependent hyperparameter changes, making the experiments directly comparable to the Dreamer results. All curves are 5 runs.

H Discussion of Compression

As discussed in Section 4.3, compression is known to improve generalization [31, 3, 7]. For example, in [31], the authors show that every additional bit in a representation $Z$ requires four times as much training data to achieve the same generalization, which may explain PI-SAC sample efficiency gains:

$$|I(Y; Z) - I(\hat{Y}; Z)| \leq O\left(\frac{|Y|^2 I(X; Z)}{\sqrt{N}}\right)$$

(12)

Here, $X$ and $Y$ are the training observations and targets, $Z$ is the learned representation, $N$ is the number of training examples, and $X$ and $Y$ are the observations and targets in the full distribution the training examples are sampled from. Since we are trying to learn $Z$ that maximizes $I(Y; Z)$ while only observing $\hat{X}$ and $\hat{Y}$, we want $I(\hat{Y}; Z)$ to be as close to $I(Y; Z)$ as possible. This bound makes it clear that we can make the two close by either increasing $N$ or decreasing $I(\hat{X}; Z)$.

Of course, one way to make that bound tight is to make $Z$ independent of $\hat{X}$ and $\hat{Y}$. In that case, all of $I(\hat{X}; Z)$, $I(\hat{Y}; Z)$, and $I(Y; Z)$ converge to 0. To avoid this, we would like to learn the Minimum Necessary Information (MNI), introduced in [6]. The MNI is defined as the equality:

$$I(X; Y) = I(X; Z) = I(Y; Z)$$

(13)

Intuitively, this says that we are trying to find a representation $Z$ that captures exactly the information that is shared between $X$ and $Y$. Learning such a $Z$ corresponds to finding the maximum of $I(Y; Z)$ while also having $I(X; Z|Y) = 0$.

3And also having $I(Y; Z|X) = 0$, but that can be enforced by having the Markov chain $Z \leftarrow X \rightarrow Y$, which means that $Z$ is a stochastic function of $X$ only.