On some new analytical solutions for new coupled Konno–Oono equation by the external trial equation method

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Abstract
In this paper, the external trial equation method is employed to solve new coupled Konno–Oono (CKO) equation. By using this method, new exact solutions involving parameters, expressed by generalized hyperbolic and elliptic solutions are obtained. The current method presents a wider applicability for handling nonlinear wave equations. In addition, explicit new exact solutions are derived in different form such as dark solitons, bright solitons, solitary wave, periodic solitary wave, rational function, and elliptic function solutions of CKO equation. The movements of obtained solutions are shown graphically, which helps to understand the physical phenomena of this soliton wave equation.

1. Introduction
The nonlinear Konno–Oono equation system introduced by Konno and Oono [1] as the coupled integrable dispersionless system

\[\begin{align*}
q_{xt} - 2\alpha q_t q_x + 2\beta q_{xx} - \gamma (q^2)_x &= 0, \\
r_{xt} - 2\alpha r_t r_x + 2\beta (r^2)_x + s_x - 2\gamma r_x &= 0, \\
s_{xt} - 2\beta s_t s_x - 2\alpha r^2_x + s_x - 2\gamma s_x &= 0,
\end{align*}\]

(1.1)

where \(\alpha\), \(\beta\), and \(\gamma\) are constants. The Konno–Oono equation system has been investigated as applications for current-fed string interacting with an external magnetic field [1–3], and the parallel transport of each point of the curve along the direction of time where the connection is magnetic-valued [4]. Authors of [5] obtained trawling wave solution for coupled Konno–Oono (CKO) equation using the modified exponential function method. A special case of system (1.1) to be considered transformed into new Konno–Oono equation system which is a coupled integrable dispersionless equations given as form of:

\[\begin{align*}
\tau_t(x, t) + \alpha \tau(x, t) \phi_t(x, t) &= 0, \\
\phi_{xt}(x, t) - 2\tau(x, t) \tau_t(x, t) &= 0.
\end{align*}\]

(1.2)

Authors of [6] utilized the tanh–function method and extended tanh–function method for obtaining to construct the exact soliton solutions for equation (1.2). Also, Khan and Akbar [7] obtained the exact solutions including kink solutions and bell-shaped solutions by help of the modified simple equation method for equation (1.2). In addition, in Yel et al [8], the sine–Gordon expansion method has been employed for getting some properties of equation (1.2) and complex hyperbolic solutions were obtained by this method. There are many nonlinear physical phenomena in nature that are described by nonlinear equations of partial differential equations. Nowadays, with rapid development of symbolic computation systems, the search for the exact solutions of nonlinear equations of PDEs has attracted a lot of attention. Recently, a variety of approaches has been proposed and applied to the nonlinear equations of PDEs, including the Exp-function method [9], the generalized Kudryashov method [10], the extended Jacobi elliptic function expansion method [11, 12], the improve
tan(ϕ/2)-expansion method [13–15], the G'/G-expansion method [16, 17], the generalized G'/G-expansion method [18], the Bernoulli sub-equation function method [19–22], the sine-Gordon expansion method [23–25], the Ricatti equation expansion [26], the formal linearization method [27], the Lie symmetry [28–30], the Bäcklund transformations [31–33], the Darboux transformation [34], the Fokas method [35–37], the Hirota bilinear method [38–41, 43], and so on.

Extended trial equation method is one of the robust techniques to look for the exact solutions of nonlinear partial differential equations that has received special interest owing to its fairly great performance. For example, Mohyud-Din and Irshad [44] explored new exact solitary wave solutions of some nonlinear PDEs arising in electronics using the extended trial equation method. Mirzazadeh et al [45] adopted the extended trial equation method to obtain analytical solutions to the generalized resonant dispersive nonlinear Schrödinger’s equation with power law nonlinearity. Ekici et al [46] found the exact soliton solutions to magneto-optic waveguides that appear with Kerr, power and log-law nonlinearities using the extended trial equation method.

This paper will adopt an integration scheme that is known as the external trial equation method that will reveal soliton solutions as well as other solutions. The system for the model studied here to investigate exact solution structures. We note that this system has not yet been studied using the extended trial equation method.
The rest of the paper is ordered as follows: we present the extended trial equation method in section 2. In sections 3, application of the new coupled KO equation is given and derived exact solutions. The graphical illustrations of some solutions are given in section 4. Finally, the conclusion is given in section 5.

2. Extended trial equation method

The current method described here is the extended trial equation method utilized to find traveling wave solutions of LPD model which can be understood through the following steps:

**Step 1.** We assume that the given nonlinear PDE
\[ N(u, u_x, u_t, u_{xx}, u_{tt}, \ldots) = 0. \]  
(2.1)

Utilizing the wave transformation
\[ u(x_1, x_2, \ldots, x_N, t) = u(\eta), \quad \eta = k \left( \sum_{j=1}^{N} x_j - \lambda t \right), \]  
(2.2)
where $\lambda \neq 0$ and $c \neq 0$. Substituting equation (2.2) into (2.1) yields a nonlinear ordinary differential equation,

$$Q(u, ku', -k\lambda u', k^2u'', k^3\lambda^2 u'', \ldots) = 0. \quad (2.3)$$

**Step 2.** Take the transformation and trial equation as follows:

$$u(\eta) = \sum_{i=0}^{\delta} \tau_i \Gamma^i, \quad (2.4)$$

where

$$(\Gamma')^2 = \Omega(\Gamma) = \frac{\Phi(\Gamma)}{\Psi(\Gamma)} = \frac{\xi_0 \Gamma^0 + \cdots + \xi_1 \Gamma + \xi_0}{\zeta \Gamma^r + \cdots + \zeta_1 \Gamma + \zeta_0}. \quad (2.5)$$

Using the equations (2.4) and (2.5), we can find

$$(u')^2 = \frac{\Phi(\Gamma)}{\Psi(\Gamma)} \left( \sum_{i=0}^{\delta} i r_i \Gamma^{i-1} \right)^2, \quad (2.6)$$
\[
\eta^{\prime\prime} = \frac{\Phi'(\Gamma)\Psi(\Gamma) - \Phi(\Gamma)\Psi'(\Gamma)}{2\Psi^2(\Gamma)} \left( \sum_{i=0}^{s} i\Gamma^{i-1} \right) + \frac{\Phi(\Gamma)}{\Psi(\Gamma)} \left( \sum_{i=0}^{s} (i-1)\Gamma^{i-2} \right),
\]
(2.7)

where \( \Phi(\Gamma) \) and \( \Psi(\Gamma) \) are polynomials. Substituting these terms into equation (2.1) yields an equation of polynomial \( \Lambda(\Gamma) \) of \( \Gamma \):

\[
\Lambda(\Gamma) = \rho_0 \Gamma^s + \ldots + \rho_s \Gamma + \rho_0 = 0.
\]
(2.8)

By utilizing the balance principle on (2.8), we can determine a relation of \( \theta, \epsilon \) and \( \delta \). We can take some values of \( \theta, \epsilon \) and \( \delta \).

Step 3. Setting each coefficient of polynomial \( \Lambda(\Gamma) \) to zero to derive a system of algebraic equations:

\[
\rho_i = 0, \quad i = 1, 2, \ldots, s.
\]
(2.9)

By solving the system (2.9), we will obtain the values of \( \xi_0, \xi_1, \ldots, \xi_s, \zeta_0, \zeta_1, \ldots, \zeta_s \), and \( \eta_0, \eta_1, \ldots, \eta_s \).

Step 4. In the following step, we obtain the elementary form of the integral by reduction of equation (2.5), as follows
\[ \pm (\eta - \eta_0) = \int \frac{d\Gamma}{\sqrt{\Omega(\Gamma)}} = \int \sqrt{\frac{\Psi(\Gamma)}{\Phi(\Gamma)}} \, d\Gamma, \]  

(2.10)

where \( \eta_0 \) is an arbitrary constant.

3. Application of ETEM for coupled KOE

New CKO equation system which is a coupled integrable dispersionless equations is given as follows

\[ v_t(x, t) + 2u(x, t)u_x(x, t) = 0, \]  

(3.1)

\[ u_{xt}(x, t) - 2v(x, t)u(x, t) = 0. \]  

(3.2)

By utilizing \( \xi = k(x - ct), U(\xi) = u(x, t), \) and \( V(\xi) = v(x, t) \) equation (3.1) reduced to

\[ -ckV'(\xi) + 2kU(\xi)U'(\xi) = 0, \]  

(3.3)

\[ -ck^2U''(\xi) - 2V(\xi)U(\xi) = 0. \]  

(3.4)

By integrating equation (3.3) with respect to \( \xi \), we obtain

\[ V = \frac{1}{c} (U^2 + p), \]  

(3.5)

where \( p \) is the integration constant. Substituting equation (3.5) into (3.4), we get

\[ c^2k^2U''(\xi) + 2U'(\xi) + 2pU(\xi) = 0. \]  

(3.6)

By the same manipulation as illustrated in the previous section, we can determine values of \( \delta, \theta, \) and \( \epsilon \), by balancing \( U^3 \) and \( U'' \) in equation (3.6) as follows:

\[ 2\delta = \theta - \epsilon - 2. \]  

(3.7)

For different values of \( \delta, \theta, \) and \( \epsilon \), we have the following cases:

Case I: \( \delta = 1, \theta = 4, \) and \( \epsilon = 0 \).

If we take \( \delta = 1, \theta = 4, \) and \( \epsilon = 0 \) for equations (2.4) and (2.5), then we obtain

\[ U(\eta) = \tau_0 + \tau_1 \Gamma, \]  

(3.8)

\[ (U'(\eta))^2 = \frac{2\tau_1^2(\tau_0^2 + \tau_1^2 + \tau_0 \Gamma + \xi_0)}{\xi_0}, \]  

(3.9)

where \( \xi_0 = 0 \) and \( \xi_0 = 0 \). Solving the algebraic equation system (2.9) yields

**First set of parameters:**

\[ \tau_0 = \tau_0, \tau_1 = \tau_1, \epsilon = \epsilon, k = k, \xi_0 = \xi_0, \xi_1 = \frac{4\xi_4 \tau_0 (\tau_0^2 + p)}{\tau_1^3}, \xi_2 = \frac{2\xi_4 (3\tau_0^2 + p)}{\tau_1^3}, \xi_3 = \frac{4\xi_4 \tau_0}{\tau_1}, \xi_4 = \xi_0, \xi_0 = -\frac{c^2k^2 \xi_4}{\tau_1^2}. \]  

(3.10)

Substituting these results into equations (2.5) and (2.10), we get

\[ \pm (\eta - \eta_0) = \int \frac{\Pi \, d\Gamma}{\sqrt{\Gamma^4 + \frac{4\tau_0}{\tau_1} \Gamma^3 + \frac{2(3\tau_0^2 + p)}{\tau_1^3} \Gamma^2 + \frac{4\tau_0 (\tau_0^2 + p)}{\tau_1^4} \Gamma + \frac{\xi_0}{\xi_4}}} = \Pi \sqrt{\frac{c^2k^2}{\tau_1^2}}. \]  

(3.11)

Integrating (3.11), we obtain the solutions to the equations (3.1) and (3.2) as follows:

**First solution:**

\[ \pm (\eta - \eta_0) = -\Pi \sqrt{\frac{\Gamma^4 + \frac{4\tau_0}{\tau_1} \Gamma^3 + \frac{2(3\tau_0^2 + p)}{\tau_1^3} \Gamma^2 + \frac{4\tau_0 (\tau_0^2 + p)}{\tau_1^4} \Gamma + \frac{\xi_0}{\xi_4}}{\Gamma - \alpha_1}} = (\Gamma - \alpha_1)^4, \]  

(3.12)

in which conclude \( p = 0, \tau_0 = -\alpha_1 \tau_1, \tau_1 = \tau_1, \alpha_1 = \sqrt{\frac{\xi_0}{\xi_4}}. \) Therefore, the solution for the CKO equation will be as

\[ u_t(x, t) = -\frac{\sqrt{-c^2k^2}}{k(x - ct) - \eta_0}, v_t(x, t) = -\frac{ck^2}{(k(x - ct) - \eta_0)^2}, \]  

(3.13)

where \( c, k, \) and \( \eta_0 \) can be selected as free constants.
Second solution:
\[
\pm (\eta - \eta_0) = \frac{\Pi}{\alpha_1 - \alpha_2} \ln \frac{\Gamma - \alpha_1}{\Gamma - \alpha_2} \Rightarrow \Gamma = \alpha_2 + \frac{(\alpha_1 - \alpha_2)\Pi}{1 - \exp((\eta - \eta_0)(\alpha_1 - \alpha_2))},
\]
(3.14)
\[
\Gamma^4 + \frac{4\tau_0}{\tau_1} \Gamma^3 + \frac{2(3\tau_0^2 + p)}{\tau_1} \Gamma^2 + \frac{4\tau_0(\tau_0^2 + p)}{\tau_1} \Gamma + \frac{\xi_0 \xi_4}{\xi_4} = (\Gamma - \alpha_2^2)(\Gamma - \alpha_2^2) \Rightarrow
\]
\[
\tau = \tau_0, \alpha_1 = 1, \alpha_2 = \alpha_2, \tau_0 = -\frac{\tau_0}{2\alpha_2}, \alpha_2 = -\frac{\tau_0}{2\alpha_2}, p = \frac{\tau_0^2}{4\alpha_2^2 \xi_4^2} \left[ 2\xi_2 \alpha_2^2 \left( \frac{\xi_0}{\xi_4} - \alpha_2^2 \right) - \xi_0 \right].
\]
(3.15)
Therefore, the solution for the CKO equation will be as
\[
u_2(x, t) = -\frac{\tau_0}{2\alpha_2} \left( \alpha_2^2 + \frac{\xi_0}{\xi_4} \right) + \tau_2 \alpha_2 + \frac{1}{\alpha_2} \left( \frac{\xi_0}{\xi_4} - \alpha_2^2 \right) \sqrt{-e^{2k^2}} \frac{1}{1 - \exp \left[ k(x - ct - \eta_0) \left( \frac{1}{\alpha_2} \left( \frac{\xi_0}{\xi_4} - \alpha_2^2 \right) \right) \right]},
\]
(3.16)
\[
u_3(x, t) = \frac{1}{c} \left[ -\frac{\tau_0}{2\alpha_2} \left( \alpha_2^2 + \frac{\xi_0}{\xi_4} \right) + \tau_2 \alpha_2 + \frac{1}{\alpha_2} \left( \frac{\xi_0}{\xi_4} - \alpha_2^2 \right) \sqrt{-e^{2k^2}} \frac{1}{1 - \exp \left[ k(x - ct - \eta_0) \left( \frac{1}{\alpha_2} \left( \frac{\xi_0}{\xi_4} - \alpha_2^2 \right) \right) \right]} \right]^{2}
\]
\[
+ \frac{\tau_2^2}{4\alpha_2^2 \xi_4 c} \left[ 2\xi_2 \alpha_2^2 \left( \frac{\xi_0}{\xi_4} - \alpha_2^2 \right) - \xi_0 \right],
\]
(3.17)
where \( \alpha_2, \tau_2, c, \xi_0 \) and \( \xi_4 \) can be selected as free constants.

Third solution:
\[
\pm (\eta - \eta_0) = \frac{\Pi}{\sqrt{(\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3)}} \ln \frac{\sqrt{(\alpha_1 - \alpha_3)(\Gamma - \alpha_2)} - \sqrt{(\alpha_1 - \alpha_2)(\Gamma - \alpha_3)}}{\sqrt{(\alpha_1 - \alpha_3)(\Gamma - \alpha_2)} + \sqrt{(\alpha_1 - \alpha_2)(\Gamma - \alpha_3)}}
\]
(3.18)
\[
\Rightarrow \Gamma = \alpha_1 - \frac{(\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3)}{2\alpha_1 - \alpha_2 - \alpha_3 + (\alpha_3 - \alpha_2) \cosh \left( \frac{\sqrt{\alpha_1 - \alpha_2}(\alpha_1 - \alpha_3)}{\alpha_1 - \alpha_2} (\eta - \eta_0) \right)},
\]
(3.19)
\[
\Gamma^4 + \frac{4\tau_0}{\tau_1} \Gamma^3 + \frac{2(3\tau_0^2 + p)}{\tau_1} \Gamma^2 + \frac{4\tau_0(\tau_0^2 + p)}{\tau_1} \Gamma + \frac{\xi_0 \xi_4}{\xi_4} = (\Gamma - \alpha_2^2)(\Gamma - \alpha_2^2) \Rightarrow
\]
\[
\tau = \tau_0, \alpha_1 = 1, \alpha_2 = \alpha_2, \alpha_3 = \alpha_3, \tau_0 = -\frac{\tau_0}{2\alpha_2}, p = -\frac{\tau_0^2}{8} (\alpha_2 - \alpha_3)^2,
\]
\[
\alpha_2 = \frac{\Sigma^4}{3\alpha_2^3 \xi_4} + \frac{1}{3} \alpha_2^2 \xi_4 \Sigma^2, \Sigma = \xi_4 \alpha_2^4 + 54 \xi_0 + 60 \alpha_2^2 \xi_4 \sqrt{3} \xi_0 \xi_4 \alpha_2^4 + 27 \xi_0.
\]
(3.20)
Therefore, the solution for the CKO equation will be as
\[
u_3(x, t) = \frac{1}{4\pi} \left( \alpha_3 - \frac{\Sigma^4}{3\alpha_3 \xi_4} - \frac{1}{3} \alpha_3^2 \xi_4 \Sigma^2 \right) \sech \left( \frac{\alpha_3 - \frac{\Sigma^4}{3\alpha_3 \xi_4} - \frac{1}{3} \alpha_3^2 \xi_4 \Sigma^2}{2c} (x - ct - \eta_0) \right) \frac{\eta}{\eta_0},
\]
(3.21)
\[
u_3(x, t) = \frac{1}{16c} \tau_1^2 \left( \alpha_3 - \frac{\Sigma^4}{3\alpha_3 \xi_4} - \frac{1}{3} \alpha_3^2 \xi_4 \Sigma^2 \right)^2 \sech^2 \left( \frac{\alpha_3 - \frac{\Sigma^4}{3\alpha_3 \xi_4} - \frac{1}{3} \alpha_3^2 \xi_4 \Sigma^2}{2c} (x - ct - \eta_0) \right) \frac{\eta}{\eta_0}
\]
\[
- \frac{1}{8} \tau_1^2 \left( \alpha_3 - \frac{\Sigma^4}{3\alpha_3 \xi_4} - \frac{1}{3} \alpha_3^2 \xi_4 \Sigma^2 \right)^2,
\]
(3.22)
in which \( \Sigma = \xi_4 \alpha_2^4 + 54 \xi_0 + 60 \alpha_2^2 \xi_4 \sqrt{3} \xi_0 \xi_4 \alpha_2^4 + 27 \xi_0 \) and \( \alpha_3, \xi_0, \) and \( \xi_4 \) can be selected as free constants.

Fourth solution:
\[
\pm (\eta - \eta_0) = \frac{2\Pi}{\sqrt{(\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3)}} F(\varphi, l), \alpha_1 > \alpha_2 > \alpha_3 > \alpha_4,
\]
(3.23)
where

\[
F(\varphi, l) = \int_0^\varphi \frac{d\psi}{\sqrt{1 - l^2 \sin^2 \psi}}, \quad \varphi = \arcsin \left( \frac{(a_2 - a_4)(l - a_1)}{(a_1 - a_4)(l - a_2)} \right), \quad I^2 = \frac{(a_2 - a_3)(a_1 - a_4)}{(a_1 - a_3)(a_2 - a_4)},
\]

\[
\Rightarrow \Gamma = a_2 + \frac{(a_1 - a_2)(a_4 - a_2)}{a_4 - a_2 + (a_1 - a_4)sn^2 \left( \frac{\sqrt{(a_1 - a_2)(a_1 - a_4)}}{2\pi} \right)(\eta - \eta_0)},
\]

\[
\Gamma^4 + \frac{4\eta_0}{\tau_1} \Gamma^3 + \left( \frac{2(3\tau_0^2 + \eta_0^3)}{\tau_1^2} - \frac{4\tau_0(\tau_0^2 + \eta_0^3)}{\tau_1^3} \right) \Gamma + \frac{\xi_0}{\xi_4} = (\Gamma - a_1)(\Gamma - a_2)(\Gamma - a_3)(\Gamma - a_4) \Rightarrow
\]

\[
a_1 = \frac{\xi_0}{a_2a_3a_4\xi_4}, \quad a_3 = a_3, \quad a_4 = a_4, \quad \tau_0 = -\frac{1}{2}(a_3 + a_4)\tau_1, \quad p = \frac{\tau_1^2}{4a_3a_4\xi_4}(2\xi_0 - a_3a_4\xi_4(a_3^2 + a_4^2)),
\]

\[
\tau_1 = \tau_0, \quad a_2 = \frac{\xi_0}{2a_3a_4\xi_4}((a_3 + a_4)^2 - 4a_3a_4\xi_4\xi_0).
\]

Therefore, the solution for the CKO equation will be as

\[
u(t, x) = \frac{1}{c} \left\{ \pm \frac{\sqrt{a_3^2a_4^2\xi_4^2(a_3 + a_4)^2 - 4a_3a_4\xi_4\xi_0}}{2a_3a_4\xi_4} + \frac{\xi_0}{a_2a_3a_4\xi_4} \left( \frac{a_1 - a_2}{2} + \frac{\sqrt{a_3^2a_4^2\xi_4^2(a_3 + a_4)^2 - 4a_3a_4\xi_4\xi_0}}{2a_3a_4\xi_4} \right) \right\}^2
\]

\[
+ \frac{\tau_1^2}{4a_3a_4\xi_4c}(2\xi_0 - a_3a_4\xi_4(a_3^2 + a_4^2)),
\]

where \( T = (a_1 - a_3)(a_2 - a_4) = \frac{\xi_0}{a_2a_3a_4\xi_4} \left( \frac{a_1 - a_2}{2} + \frac{\sqrt{a_3^2a_4^2\xi_4^2(a_3 + a_4)^2 - 4a_3a_4\xi_4\xi_0}}{2a_3a_4\xi_4} \right), \quad \Pi = \sqrt{-\frac{2\tau_1}{\tau_1}}, \quad a_3, \]

\( \alpha_4, \xi_0 \) and \( \xi_4 \) can be selected as free constants.

Taking \( \eta_0 = 0 \), the Jacobi elliptic function solution (3.27) and (3.28) can be written as:

\[
\begin{align*}
u(t, x) &= \pm \frac{\sqrt{a_3^2a_4^2\xi_4^2(a_3 + a_4)^2 - 4a_3a_4\xi_4\xi_0}}{2a_3a_4\xi_4} \left( \frac{a_1 - a_2}{2} + \frac{\sqrt{a_3^2a_4^2\xi_4^2(a_3 + a_4)^2 - 4a_3a_4\xi_4\xi_0}}{2a_3a_4\xi_4} \right) \right\}^2 \]

\[
+ \frac{\tau_1^2}{4a_3a_4\xi_4c}(2\xi_0 - a_3a_4\xi_4(a_3^2 + a_4^2)),
\]
\[ v_5(x, t) = \frac{1}{c} \left\{ \frac{\sqrt{\alpha_1^2 \alpha_2^2 \xi_0^2 (\alpha_3 + \alpha_4)^2 - 4\alpha_3 \alpha_4 \xi_0 \xi_1}}{2\alpha_3 \alpha_4 \xi_4} + \frac{\xi_6 - \alpha_3 \alpha_4 \xi_4}{2\alpha_3 \alpha_4 \xi_4} \left( \frac{\alpha_4 - \alpha_3}{2} \pm \frac{\sqrt{\alpha_1^2 \alpha_2^2 (\alpha_3 + \alpha_4)^2 - 4\alpha_3 \alpha_4 \xi_0 \xi_1}}{2\alpha_3 \alpha_4 \xi_4} \right) \right\}^2 + \frac{\tau_1^2}{4\alpha_3 \alpha_4 \xi_4} (2\xi_0 - \alpha_3 \alpha_4 \xi_4 (\alpha_3^2 + \alpha_4^2)), \]  

(3.30)

**Remark 1.** If the modulus \( l \to 1 \), then the solution for the coupled KO equation can be reduced to the solitary wave solution

\[ u_6(x, t) = \pm \frac{\sqrt{\alpha_1^2 - \xi_0 \xi_4}}{\xi_4} \left( \frac{\xi_6 - \alpha_3 \alpha_4 \xi_4}{\alpha_2 \alpha_4 \xi_4} \right) \right\} + \frac{\xi_6 - \alpha_3 \alpha_4 \xi_4}{\alpha_2 \alpha_4 \xi_4} \left[ \frac{\eta}{\sqrt{2}} \left( \frac{\alpha_4 - \alpha_3}{2} \pm \frac{\sqrt{\alpha_1^2 \alpha_2^2 (\alpha_3 + \alpha_4)^2 - 4\alpha_3 \alpha_4 \xi_0 \xi_1}}{2\alpha_3 \alpha_4 \xi_4} \right) \right] \right\}^2 + \frac{\tau_1^2}{4\alpha_3 \alpha_4 \xi_4} (2\xi_0 - \alpha_3 \alpha_4 \xi_4 (\alpha_3^2 + \alpha_4^2)), \]  

(3.31)

\[ v_6(x, t) = \frac{1}{c} \left\{ \frac{\sqrt{\alpha_1^2 - \xi_0 \xi_4}}{\xi_4} \left( \frac{\xi_6 - \alpha_3 \alpha_4 \xi_4}{\alpha_2 \alpha_4 \xi_4} \right) \right\} + \frac{\xi_6 - \alpha_3 \alpha_4 \xi_4}{\alpha_2 \alpha_4 \xi_4} \left[ \frac{\eta}{\sqrt{2}} \left( \frac{\alpha_4 - \alpha_3}{2} \pm \frac{\sqrt{\alpha_1^2 \alpha_2^2 (\alpha_3 + \alpha_4)^2 - 4\alpha_3 \alpha_4 \xi_0 \xi_1}}{2\alpha_3 \alpha_4 \xi_4} \right) \right] \right\}^2 + \frac{\tau_1^2}{4\alpha_3 \alpha_4 \xi_4} (2\xi_0 - \alpha_3 \alpha_4 \xi_4 (\alpha_3^2 + \alpha_4^2)), \]  

(3.32)

where \( \alpha_3 = \alpha_4 \).

**Remark 2.** If the modulus \( l \to 0 \), then the solution for the CKO equation can be reduced to the solitary wave solution

\[ u_7(x, t) = \pm \frac{\sqrt{\xi_0 \xi_4 (\alpha_3 + \alpha_4)^2 - 4\xi_0 \xi_4}}{2\sqrt{\xi_0 \xi_4}} + \frac{\xi_6 - \alpha_3 \alpha_4 \xi_4}{\alpha_2 \sqrt{\xi_0 \xi_4}} \left( \frac{\alpha_4 - \alpha_3}{2} + \frac{\sqrt{\alpha_1^2 \alpha_2^2 (\alpha_3 + \alpha_4)^2 - 4\alpha_3 \alpha_4 \xi_0 \xi_4}}{2\sqrt{\xi_0 \xi_4}} \right) \right\} \left[ \frac{\eta}{\sqrt{2}} \left( \frac{\alpha_4 - \alpha_3}{2} \pm \frac{\sqrt{\alpha_1^2 \alpha_2^2 (\alpha_3 + \alpha_4)^2 - 4\alpha_3 \alpha_4 \xi_0 \xi_4}}{2\sqrt{\xi_0 \xi_4}} \right) \right] \right\}^2 + \frac{\tau_1^2}{4\sqrt{\xi_0 \xi_4}} (2\xi_0 - \alpha_3 \alpha_4 \xi_4 (\alpha_3^2 + \alpha_4^2)), \]  

(3.33)
\[ \nu_j(x, t) = \pm \sqrt{\frac{\xi_0 \xi_4 (\alpha_3 + \alpha_4)^2 - 4 \xi_0 \sqrt{\xi_0 \xi_4}}{2 \sqrt{\xi_0 \xi_4}}} \]

\[ + \frac{\xi_0 - \alpha_3 \xi_4}{\alpha_3 \sqrt{\xi_4}} \left( \frac{a_4 - a_\alpha}{2} \pm \frac{\xi_0 (a_4 - a_\alpha)^2 - 4 \xi_0 \xi_4}{2 \sqrt{\xi_0 \xi_4}} \right) \eta_1 \]

\[ + \frac{\eta_1}{4c \sqrt{\xi_0 \xi_4}} (2 \xi_0 - \sqrt{\xi_0 \xi_4 (\alpha_3^2 + \alpha_4^2)}), \]

where \( \alpha_2 = \alpha_3. \)

**Case II**: \( \delta = 1, \theta = 5, \) and \( \varepsilon = 1. \)

If we take \( \delta = 1, \theta = 5, \) and \( \varepsilon = 1 \) for equations (2.4) and (2.5), then we obtain

\[ U(\eta) = \eta_0 + \eta_1 \Gamma, \]

\[ (U(\eta))^2 = \frac{\tau_1^2 (\xi_5 \Gamma^5 + \xi_4 \Gamma^4 + \xi_3 \Gamma^3 + \xi_2 \Gamma^2 + \xi_1 \Gamma + \xi_0)}{\xi_0 + \xi_1 \Gamma} \]

where \( \xi_5 \neq 0 \) and \( \xi_0 \neq 0. \) Solving the algebraic equation system (2.9) yields:

- **First set of parameters**:

  \[
  \begin{align*}
  \eta_0 &= \eta_0, \quad \eta_1 = \eta_1, \quad c = c, \quad k = k, \quad \xi_0 = -\frac{\eta_0 \xi_0 (4 \xi_0^3 p \eta_0 + 4 \xi_0 \xi_1 \xi_2 k^2 c^2) \xi_2 \xi_0}{c^4 k^4 \xi_5}, \\
  \xi_2 &= \frac{2(-p \tau_1^3 \xi_0 + 2 \tau_1^4 c^2 k^2 \xi_5 - 3 \tau_1^4 \xi_0 + 2 \tau_1^5 c \xi_5)}{c^2 k^2 \tau_1^2}, \\
  \xi_3 &= \frac{2(-2 \tau_1^3 \xi_0 \eta_0 + \xi_2 k^2 c^2 + 3 \xi_5 k^2 c^2 \xi_0)}{c^2 k^2 \tau_1^2}, \\
  \xi_4 &= \frac{4 \eta_0 c^2 k^2 \xi_5 - \tau_1^3 \xi_0}{c^2 k^2 \tau_1^2}, \\
  \xi_5 &= \xi_5, \quad \xi_0 \neq 0, \quad \xi_1 = -\frac{\xi_2 k^2 c^2}{\xi_2}. 
  \end{align*}
  \]

Substituting these results into equations (2.3) and (2.10), we get

\[ \pm (\eta - \eta_0) = \int \frac{\sqrt{\xi_0 - \xi_2 k^2 c^2 \eta}}{\sqrt{\Gamma^5 + \xi_5 \Gamma^4 + \xi_4 \Gamma^3 + \xi_3 \Gamma^2 + \xi_2 \Gamma + \xi_1 \Gamma}} \, d\Gamma. \]

Integrating (3.38), we obtain the solutions to the equations (3.1) and (3.2) as follows:

**Set I**: If \( \Gamma^5 + \frac{\xi_4 \Gamma^4}{\xi_5} + \frac{\xi_3 \Gamma^3}{\xi_4} + \frac{\xi_2 \Gamma^2}{\xi_3} + \frac{\xi_1 \Gamma}{\xi_2} + \frac{\xi_0}{\xi_1} \) can be written in the following form:

\[ \Gamma^5 + \frac{4 \eta_0 c^2 k^2 \xi_5 - \tau_1^3 \xi_0}{c^2 k^2 \tau_1^2} \Gamma^4 + \frac{2(-2 \tau_1^3 \xi_0 \eta_0 + \xi_2 k^2 c^2 + 3 \xi_5 k^2 c^2 \xi_0)}{c^2 k^2 \tau_1^2} \Gamma^3 + \frac{2(-p \tau_1^3 \xi_0 + 2 \tau_1^4 c^2 k^2 \xi_5 - 3 \tau_1^4 \xi_0 + 2 \tau_1^5 c \xi_5)}{c^2 k^2 \tau_1^2} \Gamma^2 + \frac{4 \eta_0 \xi_0 (4 \xi_0^3 p \eta_0 + 4 \xi_0 \xi_1 \xi_2 k^2 c^2)}{c^2 k^2 \tau_1^2} \Gamma + \frac{\xi_2 \xi_0 \eta_0 (4 \xi_0^3 p \eta_0 + 4 \xi_0 \xi_1 \xi_2 k^2 c^2)}{c^2 k^2 \tau_1^2} = (\Gamma - \alpha_1)^5. \]

Then, we obtain the following results as

\[ k = k, \quad p = 0, \quad \eta_0 = -\frac{\eta_0 \xi_0}{5 \alpha_1^2 \xi_5}, \quad \eta_1 = \eta_1, \quad c = \frac{\eta_1}{k} \frac{5 \alpha_1^2 \xi_0}{\xi_1}, \quad \alpha_1 = \frac{\xi_1}{\xi_5}. \]
where \(\alpha_1\) is an arbitrary constant. Then, we have

\[
\pm(\eta - \eta_0) = \int \frac{\sqrt{\zeta_0 - \frac{k^2\zeta_0^2}{\zeta}} \Gamma}{(\Gamma - \alpha_1)^2\sqrt{\Gamma - \alpha_1}} \, d\Gamma = -\frac{2}{3} \left( \frac{\zeta_0}{\zeta} - \frac{k^2\zeta_0^2}{\zeta\alpha_1} \right)^2 \left( \zeta_0 \right) (\Gamma - \alpha_1)^2 ,
\]

(3.41)

or

\[
\Gamma = \frac{\zeta_0 + \alpha_1 \left[ -\frac{3}{2} \left( \frac{\zeta_0}{\zeta} - \frac{k^2\zeta_0^2}{\zeta\alpha_1} \right) (\eta - \eta_0) \right]^2}{\left[ -\frac{3}{2} \left( \frac{\zeta_0}{\zeta} - \frac{k^2\zeta_0^2}{\zeta\alpha_1} \right) (\eta - \eta_0) \right]^2 - \frac{\zeta_0}{\zeta} } .
\]

(3.42)

Substituting (3.42) into (2.4), we get the exact solution for the CKO equation in the form of:

\[
u_t(x, t) = -\frac{\eta_0}{\eta_1 \sqrt{\frac{\zeta_0}{\zeta_1}} \xi_5} + \eta_0 \left[ -\frac{3}{2} \left( \frac{\zeta_0}{\zeta} - \frac{k^2\zeta_0^2}{\zeta\alpha_1} \right) \left( kx - \frac{\sqrt{\zeta_0\eta_0}}{\sqrt{\zeta}} (t - \eta_0) \right) \right]^2 ,
\]

(3.43)

\[
u_v(x, t) = \frac{k}{\eta_1 \sqrt{\frac{\zeta_0}{\zeta_1}} \xi_5} - \frac{\eta_0}{\eta_1 \sqrt{\frac{\zeta_0}{\zeta_1}} \xi_5} + \eta_0 \left[ -\frac{3}{2} \left( \frac{\zeta_0}{\zeta} - \frac{k^2\zeta_0^2}{\zeta\alpha_1} \right) \left( kx - \frac{\sqrt{\zeta_0\eta_0}}{\sqrt{\zeta}} (t - \eta_0) \right) \right]^2 ,
\]

(3.44)

where \(\zeta_0, \zeta, \xi_5, k, \eta_1, \) and \(\eta_0\) are free constants.

Set II. If \(\Gamma^1 = \frac{\xi_4}{\xi_5} + \frac{\xi_1}{\xi_5} \Gamma^1 + \frac{\xi_7}{\xi_5} \Gamma^2 + \frac{\xi_7}{\xi_5} \Gamma^3 + \frac{\xi_7}{\xi_5} \Gamma^4 + \frac{\xi_7}{\xi_5} \Gamma^5 + \frac{\xi_7}{\xi_5} \Gamma + \frac{\xi_7}{\xi_5} \Gamma = (\Gamma - \alpha_1)\Gamma^1(\Gamma - \alpha_2),
\]

(3.45)

where \(\alpha_1\) and \(\alpha_2\) are arbitrary constants. Then, we have

\[
\pm(\eta - \eta_0) = \int \frac{\sqrt{\zeta_0 + \frac{\zeta_1}{\zeta_5} \Gamma \sqrt{\Gamma - \alpha_2}}}{(\Gamma - \alpha_1)^2\sqrt{\Gamma - \alpha_2}} \, d\Gamma = -\frac{1}{2} \left( \frac{\zeta_0}{\zeta_5} \right) \frac{1}{(\alpha_1 - \alpha_2) \Pi_0 \Pi_2 \sqrt{(\Gamma - \alpha_1)^3(\Gamma - \alpha_2)}} ,
\]

(3.46)

where

\[
\Pi_0 = \sqrt{(\alpha_1 - \alpha_2) \left( \frac{\zeta_0}{\zeta_5} + \frac{\zeta_1}{\zeta_5} \alpha_1 \right) }, \quad \Pi_1 = \frac{\zeta_1}{\zeta_5} \Gamma^1 + \left( \frac{\zeta_0}{\zeta_5} - \frac{\zeta_1}{\zeta_5} \alpha_1 \right) \frac{\zeta_0}{\zeta_5} \alpha_2 , \quad \Pi_2 = \frac{\zeta_0}{\zeta_5} \alpha_2 - \frac{\zeta_1}{\zeta_5} \alpha_2 - \frac{\zeta_1}{\zeta_5} \alpha_1 , \quad \Pi_3 = \frac{\zeta_0}{\zeta_5} \alpha_1 - \frac{\zeta_0}{\zeta_5} \alpha_2 , \quad \Pi_4 = \frac{\zeta_1}{\zeta_5} \Gamma^1 + \left( \frac{\zeta_0}{\zeta_5} - \frac{\zeta_1}{\zeta_5} \alpha_1 \right) \frac{\zeta_0}{\zeta_5} \alpha_2 , \quad \Pi_5 = \frac{\zeta_0}{\zeta_5} + \frac{\zeta_1}{\zeta_5} \alpha_1 , \quad \Pi_6 = \frac{\zeta_0}{\zeta_5} + \frac{\zeta_1}{\zeta_5} \alpha_2 ,
\]

(3.47)

\[
\Pi_7 = \left( \frac{\zeta_0}{\zeta_5} - \frac{\zeta_1}{\zeta_5} \alpha_1 \right) \sqrt{\frac{\zeta_0}{\zeta_5} \alpha_1 - \frac{\zeta_1}{\zeta_5} \alpha_2} , \quad \Pi_8 = \left( \frac{\zeta_0}{\zeta_5} + \frac{\zeta_1}{\zeta_5} \alpha_2 \right) , \quad \Pi_9 = \frac{\zeta_0}{\zeta_5} + \frac{\zeta_1}{\zeta_5} \alpha_1 , \quad \Pi_{10} = \frac{\zeta_0}{\zeta_5} + \frac{\zeta_1}{\zeta_5} \alpha_2 , \quad Y = \frac{\Pi_4 + 2\Pi_5 \Pi_6}{\Pi_1 - \Pi_2} .
\]

(3.48)

Set III. If \(\Gamma^5 + \frac{\xi_4}{\xi_5} \Gamma^4 + \frac{\xi_1}{\xi_5} \Gamma^3 + \frac{\xi_7}{\xi_5} \Gamma^2 + \frac{\xi_7}{\xi_5} \Gamma + \frac{\xi_7}{\xi_5} \Gamma = (\Gamma - \alpha_3)^3(\Gamma - \alpha_2)^2,
\]

(3.49)
where $\alpha_1$ and $\alpha_2$ are arbitrary constants. Then, we have
\[
\pm(\eta - \eta_0) = \int \frac{\sqrt{\left(\frac{\beta}{\xi} + \frac{\gamma}{\xi^2}\right)}}{\sqrt{(\Gamma - \alpha_3)^2(\Gamma - \alpha_3)}}\, d\Gamma
\]
\[
= - \frac{\sqrt{\frac{\beta}{\xi} + \frac{\gamma}{\xi^2}}}{(\Gamma - \alpha_3)^2\Pi_2\sqrt{\Gamma - \alpha_1}} \left[2(\alpha_2 - \alpha_3)\Pi_4 + (\Gamma - \alpha_1)\Pi_1\ln(Y)\right],
\]
(3.50)
where
\[
\Pi_1 = -\sqrt{\left(-\alpha_3\right)} \left(\frac{\beta_0}{\xi} + \frac{\gamma_1}{\xi^2}\right), \quad \Pi_2 = -\sqrt{\frac{\gamma_1}{\xi}} = \frac{\gamma_1}{\xi}, \quad \Pi_3 = \frac{\beta_0}{\xi} + \frac{\gamma_1}{\xi^2} - \frac{\gamma_0}{\xi^2} \alpha_1\alpha_2, \quad \Pi_4 = \Pi_3 + 2\Pi_1\Pi_2
\]
\[
\Pi_5 = \frac{\beta_0}{\xi} + \frac{\gamma_1}{\xi^2} - \frac{\gamma_0}{\xi^2} \alpha_1\alpha_2, \quad \Pi_6 = \alpha_1 - \alpha_2)(\alpha_1 - \alpha_3)(\alpha_2 - \alpha_3),
\]
(3.51)
(3.52)

Set IV. If $\Gamma^5 + \frac{\beta_0}{\xi} \Gamma^4 + \frac{\gamma_1}{\xi} \Gamma^3 + \frac{\gamma_0}{\xi} \Gamma^2 + \gamma_0 = \alpha_1^2(\Gamma - \alpha_2)^2(\Gamma - \alpha_3)$, we can write in the following form:
\[
\Gamma^5 + \frac{\beta_0}{\xi} \Gamma^4 + \frac{\gamma_1}{\xi} \Gamma^3 + \frac{\gamma_0}{\xi} \Gamma^2 + \gamma_0 = \alpha_1^2(\Gamma - \alpha_2)^2(\Gamma - \alpha_3),
\]
(3.53)
where $\alpha_1$, $\alpha_2$, and $\alpha_3$ are arbitrary constants. Then, we have
\[
\pm(\eta - \eta_0) = \int \frac{\sqrt{\left(\frac{\beta}{\xi} + \frac{\gamma}{\xi^2}\right)}}{\sqrt{(\Gamma - \alpha_3)^2(\Gamma - \alpha_3)}}\, d\Gamma
\]
\[
= - \frac{\sqrt{\frac{\beta}{\xi} + \frac{\gamma}{\xi^2}}}{\Pi_1\Pi_6} \left[(\alpha_2 - \alpha_3)\Pi_4\ln(Y_1) - (\alpha_1 - \alpha_3)\Pi_3\ln(Y_2)\right],
\]
(3.54)
where
\[
\Pi_1 = -\sqrt{\left(-\alpha_3\right)} \left(\frac{\beta_0}{\xi} + \frac{\gamma_1}{\xi^2}\right), \quad \Pi_2 = -\sqrt{\frac{\gamma_1}{\xi}} = \frac{\gamma_1}{\xi}, \quad \Pi_3 = \frac{\beta_0}{\xi} + \frac{\gamma_1}{\xi^2} - \frac{\gamma_0}{\xi^2} \alpha_1\alpha_2, \quad \Pi_4 = \Pi_3 + 2\Pi_1\Pi_2
\]
\[
\Pi_5 = \frac{\beta_0}{\xi} + \frac{\gamma_1}{\xi^2} - \frac{\gamma_0}{\xi^2} \alpha_1\alpha_2, \quad \Pi_6 = \alpha_1 - \alpha_2)(\alpha_1 - \alpha_3)(\alpha_2 - \alpha_3),
\]
(3.55)
(3.56)
(3.57)
(3.58)

4. Graphical illustrations of the solutions

The results obtained here are hyperbolic solitons, solitary wave, periodic solitary wave, rational function, and elliptic function solutions and for some appropriated parameters, we obtain exact solutions including kink soliton wave solutions, soliton wave solutions and periodic wave solutions. We plot some of the solutions to have an idea on the mechanism of the original equation (1.2). Specifically, we plot solutions as:

ETEM: Solutions (3.21), (3.22) (3.31), and (3.32) for the CKO equation by taking suitable values of the parameters obtained. The graphical representations of these solutions are shown in figures 1–10 respectively.

4.1. Discussion and remarks

By help of ETEM, we obtain solutions including exponential, hyperbolic, trigonometric and rational function forms of the new CKO equation, whereas in [5], Koçak et al acquired the traveling wave solutions including equations (25–32) in which obtained by using MEFM. Meanwhile, in this paper are new hyperbolic and complex function solutions in which agree with results of [5]. Moreover, in the current paper the better results are
obtained when we compare these traveling wave solutions with solutions obtained by, K Khan, M Ali Akbar [7]. These traveling wave solutions have been seen that they have verified the new CKO equation using the help of Maple 13. To the best of our knowledge, the application of ETEM to the equation (1.2) has not been submitted to literature in advance.

5. Conclusion

In this paper, the new CKO equation was successfully studied. The mentioned task was accomplished by adopting the ETEM to generate a series of exact traveling wave solutions. A comparison of our results with those obtained in [8] by using the sine-Gordon expansion method shows, that there are many new solutions in the present work. Some graphical figures were also portrayed to demonstrate the dynamic behavior of extracted solutions. The observations confirm that above method is efficient algorithms for analytic treatment of a wide range of nonlinear systems of PDEs which arising in nonlinear physics.
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