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State feedback impulsive therapy to SIS model of animal infectious diseases
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HIGHLIGHTS

• A state feedback impulsive model is constructed to analyze the treatment of animal epidemics.
• The existence of order-1 periodic solution to an impulsive system is proved.
• The stability of the order-1 periodic solution is proved with a novel method.

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ABSTRACT

Controlling animal infectious diseases and its related infra microbes is of great significance to public health, since a lot of infectious diseases originate from animal epidemics and they often threaten human health. A state feedback impulsive model is constructed to depict the transmission and treatment of animal epidemics. Basing on the impulsive model, the existence of order-1 periodic solution and its stability are proved with a novel method. The theoretical results indicate that the impulsive treatment triggered by the number of infectious is an efficient approach to control animal infectious disease from breakout. Numerical simulation is presented to support the theoretical conclusion in the end.

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1. Introduction

Livestock, poultry and wildlife help human in supplying food resource and keeping balance of ecosystem etc that people cannot subsist without getting support from animals. However, animal infectious throw mankind a deadly menace with infectious diseases initiating from animals [1–3]. Some diseases such as African swine fever and bovine leukemia only influence animals. But to some epidemic, those pathogenic bacteria can be spilled from domestic animals or wildlife to human beings, and the development and mutation during the transmission will make the pathogenic bacteria difficult to control [4–6]. A lot of diseases besetting human beings such as human immunodeficiency virus (HIV), SARS corona virus, Ebola virus and avian influenza viruses (AIV) all can trace their filiation from animals [7]. Taylor’s achievement showed that about 75% of human epidemics stem from an animal source, and these zoo-noses are quite difficult to control after its prevalence among a crowd of human [8]. So preventing the formation and spread of zoonoses is quite important to public health of human beings.

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In recent achievements, some scholars have got some useful results in impulsive dynamic system. Among them, Jianjun Jiao et al. established an SIR model with impulsive vaccination and impulsive dispersal to analyze the effect of impulsive vaccination and restricting infected individuals boarding transports on disease spread [9]. Linfei Nie et al. proposed a state-dependent impulsive model with two control threshold values to study two-languages competition and they found the order-1 periodic solutions using qualitative analysis method [10]. Shengqiang Zhang et al. proposed a stochastic non-autonomous Lotka–Volterra predator–prey model with impulsive effects and investigate its stochastic dynamics [11]. Meng Zhang et al. considered the preventive control of computer worm’s transmission through Internet [12]. Recently discussions regarding impulsive disturbance based on state feedback have dominated the research in field of control theory and optimization [13–18]. Enlightened by those research about impulsive model, we construct a state feedback impulsive SIS model to analyze the therapy control of animal infectious diseases. In practice, animal epidemic is hardly to be detected during its latent period because no obvious symptoms can be observed. Veterinarians can find the disease only after the eruption and then can treatment measures be carried out. Nevertheless, a certain amount of animals have been infected by the virus at that time. Thus, it is more appropriate to represent the treatment of animal epidemics as a state feedback impulse which depends on the variable of infected animals.

Without loss of generality, we assume that the disease can be detected when the amount of infected animals cumulate to a certain level $I^*$, and the treatment will be carried out at the same time. We also assume that the infected animals recover after treatment and become susceptible again. Then the model is established as follows:

$$\begin{align}
\frac{dS}{dt} &= S(r - \frac{S}{K}) - \beta SI - d_S S, & I < I^*, \\
\frac{dI}{dt} &= \beta SI - d_I I - d_d I, \\
\Delta I &= -\theta I^*, \Delta S = \theta I^*, & I = I^*,
\end{align}$$

(1.1)

where $S(t)$ denotes the susceptible animals; $I(t)$ represents the infected ones. According to the transmission feather, we make the following assumption: (1) animals increase in logistic pattern, and the environmental capacity is $K$; (2) only the healthy animals can propagate and the newborn animals are susceptible ones; (3) the infected rate of susceptible is linear with both susceptible and infected animals at $\beta$; (4) natural death rate is $d_d$ and death rate caused by the epidemic is $d$; (5) $I^*$ is the threshold that the infectious disease can be discovered, when the amount of infections is smaller than $I^*$, the infectious disease develops according to the common way which is expressed as the first two equations of model (1.1); (6) once the threshold $I^*$ is reached, the infectious disease can be spotted and efficient treatment will be performed immediately. A portion of infections, here we assume the ratio is $\theta$ ($0 < \theta < 1$), will be treated and turn into susceptible ones and can be infected again. Considering the practical significance, all the parameters in this paper are positive.

The sketch of this paper is organized as the following: some preliminaries about the state feedback impulsive dynamic system were presented in Section 2; the existence and stability of the order-1 periodic solution to system (1.1) were analyzed in Sections 3 and 4 separately. Finally, we examine the theoretical results with numerical simulation in Section 5.

2. Preliminaries

In this section some definitions and lemmas about state feedback impulsive dynamic system are listed. These preliminaries will be used in Sections 3 and 4.

**Definition 2.1 ([19–25]).** A typical state feedback impulsive differential model can be constructed as

$$\begin{align}
\frac{dx}{dt} &= P(x, y), & \frac{dy}{dt} = Q(x, y), & (x, y) \notin M(x, y), \\
\Delta x &= \alpha(x, y), & \Delta y = \beta(x, y), & (x, y) \in M(x, y).
\end{align}$$

(2.1)

The dynamic system defined by (2.1) is called a state feedback impulsive dynamic system, and it is denoted by $(\Omega, f, \varphi, M)$. The initial point $P$ of system (2.1) is defined anywhere in the plane $\mathbb{R}^2$ but $M(x, y)$, i.e. $P \in \Omega = \mathbb{R}^2 \setminus M(x, y)$. $f$ is a continuous mapping which satisfies $\varphi(M) = N$, and we called it impulse mapping. $M(x, y)$ and $N(x, y)$ are impulse set, and phase set respectively, and they are lines or curves in the plane $\mathbb{R}^2$.

The order-1 periodic solution and successor function is defined as follows.

**Definition 2.2 ([23,26,27]).** Suppose the impulse set and the phase set of system (2.1) are straight lines $M$ and $N$ respectively (see Fig. 1), and denote the intersection of phase set $N$ and $y$ axis by $Q$. To any point $A \in N$, define the coordinate of $A$, denoted by $a$, as the distance between $A$ and $Q$. The trajectory initiating from $A$ intersects impulse set $M$ at point $B$, then the impulse function $\varphi$ maps $B$ to $C$ in phase set $N$, $C$ is called the subsequent point of $A$, and we denote the coordinate of $C$ as $c$. If $C$ and $A$ coincide with each other, then the trajectories constitute an order-1 periodic solution. The successor function of $A$ is defined as a continuous function $F(A) = c - a$. 
Remark 2.1. The necessary and sufficient condition that the trajectory passing through point A forms an order-1 periodic solution is $F(A) = 0$.

Lemma 2.1 ([23, 26, 27]). In state feedback impulsive dynamic system $(\Omega, f, \varphi, M)$, there exist two points B and C in phase set N, if $F(B) > 0$ and $F(C) < 0$, then there must exist a point $A$ between B and C satisfying $F(A) = 0$, and $f(A, t)$ is an order-1 periodic solution.

3. Existence of the order-1 periodic solution

For practical meaning, we only consider system (1.1) in space $R_+^2 = \{(S, I) \mid S \geq 0, I \geq 0\}$. Once the trajectory reaches the impulse set $M = \{(S, I) \in R_+^2 \mid S \geq 0, I = I^*\}$, impulse mapping $\varphi : (S, I^*) \in M \rightarrow (S + \theta I^*, (1 - \theta) I^*)$ \in N, here phase set $N = \varphi(M) = \{(S, I) \in R_+^2 \mid S \geq 0, I = (1 - \theta) I^*\}$. Comparing (1.1) with the typical model (2.1), system (1.1) is a state feedback impulsive dynamic system, where $\Omega = R_+^2 = \{(S, I) \mid S \geq 0, I \geq 0\}$.

Without considering the impulse mapping, system (1.1) reduces into

\[
\begin{align*}
\frac{dS}{dt} &= S(r - S/K) - \beta SI - d_0S, \\
\frac{dI}{dt} &= \beta SI - dl - d_0I,
\end{align*}
\]

in which we only focus on the situation in the first quadrant. It is obvious that $O(0, 0)$ and $E_1(K(r - d_0), 0)$ are two marginal equilibrium of system (3.1). Following the theory of stability, $O(0, 0)$ is a saddle. Denote $R = \frac{(r - d_0)\beta K}{d + d_0}$, if $R < 1$, the Jacobin matrix of $E_1$ is

\[
J_{E_1} = \begin{pmatrix}
-(r - d_0) & -(r - d_0)\beta K \\
0 & (r - d_0)\beta K - (d + d_0)
\end{pmatrix},
\]

then $E_1$ is a stable node (see Fig. 2) and the infectious disease will eliminate automatically without any human intervention.
Then we have $T$ to system (1.1), on the point where the slope of direction vector is $-\frac{1}{\beta} \left( r - d_0 - \frac{d+d_0}{\beta K} \right)$ is a stable focus. Parameters used: $r = 0.2, K = 5, \beta = 0.7, d = 0.2, d_0 = 0.08$.

If $R > 1$, then $E_1$ is a saddle and system (3.1) has a unique positive equilibrium $E_2 \left( \frac{d+d_0}{\beta} \cdot \frac{1}{\beta} \left( r - d_0 - \frac{d+d_0}{\beta K} \right) \right)$, and the Jacobin matrix of point $E_2$ is

$$
J_{E_2} = \begin{pmatrix}
-\frac{d+d_0}{\beta K} & -(d + d_0) \\
\frac{1}{\beta} - \frac{d+d_0}{\beta K} & 0
\end{pmatrix}
$$

The characteristic equation is $\lambda^2 + \frac{d+d_0}{\beta K} \lambda + \left( (d + d_0)(r - d_0) - \frac{(d+d_0)^2}{\beta K} \right) = 0$. Assume $\lambda_1^*$ and $\lambda_2^*$ are two characteristic roots, then

$$
\lambda_1^* + \lambda_2^* = \frac{-(d + d_0)}{\beta K} < 0, \quad \lambda_1^* \cdot \lambda_2^* = (d + d_0)(r - d_0) - \frac{d + d_0}{\beta K} > 0,
$$

i.e. $\lambda_1^* < 0$ and $\lambda_2^* < 0$. So we can assert that $E_2 \left( \frac{d+d_0}{\beta} \cdot \frac{1}{\beta} \left( r - d_0 - \frac{d+d_0}{\beta K} \right) \right)$ is a locally stable focus or node (see Fig. 3). This is the certain case we focus on in the following of this paper.

To prove the existence of order-1 periodic solution of system (3.1), we need the following two lemmas.

**Lemma 3.1.** Suppose $\Gamma(t) = (S(t), I(t))$ is a periodic trajectory of system (3.1) and its period is $T$, then the periodic trajectory is stable.

**Lemma 3.2.** If $R > 1$, then the equilibrium $E_2 \left( \frac{d+d_0}{\beta} \cdot \frac{1}{\beta} \left( r - d_0 - \frac{d+d_0}{\beta K} \right) \right)$ is globally asymptotically stable.

The proofs of those lemmas are similar with the process in [12] and they are omitted here.

Assume $I^* \leq \frac{1}{\beta} \left( r - d_0 - \frac{d+d_0}{\beta K} \right)$, then we can prove that system (1.1) has an order-1 periodic solution.

**Theorem 3.1.** If $R > 1$, for any $\theta(0 < \theta < 1)$, system (1.1) has an order-1 periodic solution.

**Proof.** In system (1.1), impulse set and phase set are $M : l = I^*$ and $N : l = (1 - \theta)I^*$ respectively. Assume phase set $N$ intersects the horizontal isoclinic line $\frac{dl}{dS} = 0$ at point $B\left( \frac{d+d_0}{\beta} \cdot \frac{1}{\beta} \left( r - d_0 - \frac{d+d_0}{\beta K} \right) \right)$, and there must exist a trajectory $L_B$ passing through $B$ and being tangent with phase set $N : l = (1 - \theta)I^*$ (see Fig. 4). Denote the intersection point of the trajectory $L_B$ and the impulsive set $M$ as $M_B(x_{M_B}, I^*)$. For the impulsive set is below the equilibrium $E_2$, i.e. $I^* \leq \frac{1}{\beta} \left( r - d_0 - \frac{d+d_0}{\beta K} \right)$, so $x_{M_B} > \frac{d+d_0}{\beta}$ which means the horizontal ordinate of point $M_B$ is larger than that of point $B$. Then the impulsive function $\varphi$ maps $M_B(x_{M_B}, I^*)$ to a point $N_B(x_{M_B} + \theta I^*, (1 - \theta)I^*)$ in phase set. So the successor function of $B$ is $F(B) = x_{M_B} - x_B = x_{M_B} + \theta I^* - \frac{d+d_0}{\beta} > 0$.

In the following we will prove the existence of point $C$ satisfying $F(C) < 0$.

First, we prove that in the first quadrant the line whose slope is $-1$ cannot be tangent with any trajectory of system (1.1). To system (1.1), on the point where the slope of direction vector is $-1$ the coordinate must satisfy

$$
\frac{dl}{dS} = \frac{\beta SI - (d + d_0)I}{S(r - d_0 - \frac{1}{\beta}) - \beta SI} = -1.
$$

Then we have

$$
l = \frac{1}{d + d_0} S^2 + \frac{r - d_0}{d + d_0} S.
$$
Its graph is a parabola with the origin as vertex, downward opening and located under the x axis. So it is obvious that the line whose slope is $-1$ cannot be tangent with any trajectory of system (1.1) in the first quadrant.

Second, there exists a line $L$ whose slope is $-1$ and the trajectory of system (1.1) runs across line $L$ from right side to left side at the points whose vertical coordinate satisfying $I < I^*$. Assuming $V = S + I$, to system (1.1) we have

$$\frac{dV}{dt} = \frac{dS}{dt} + \frac{dI}{dt} = S(r - d_0 - \frac{S}{K}) - (d + d_0)I.$$ 

If the equation of line $L$ is $S + I - e = 0$, then on line $L$ we have

$$\frac{dV}{dt} \bigg|_L = (e - I)(r - d_0 - \frac{e - I}{K}) - (d + d_0)I.$$ 

If the parameter $e$ satisfying $e > (r - d_0)K + I^*$, there must exist $\frac{dV}{dt} < 0$. Then on the points of line $L$ satisfying $I < I^*$, the trajectories of system (1.1) pass line $L$ from right side to left side.

Assuming the line $L$ passes the impulsive set, the phase set and x axis at $D$, $C$ and $E$ respectively (see Fig. 5). There exists a trajectory $L_C$ of system (1.1) which passes the line $L$ from right side to left side at point $C$ and runs upward-left until reaching the impulsive set at $M_C$, and $M_C$ must be located on the left side of point $D$. Then the impulsive function $\varphi$ maps $M_C$ to $N_C$. For $M_CN_C$ is parallel to $L$, the phase point $N_C$ must be located on the left side of point $C$. So the successor function of point $C$ is $F(C) = x_{N_C} - x_C < 0$.

From the continuity of successor function, there must exist a point $A$ between $B$ and $C$ in phase set which satisfies $F(A) = 0$. Then the trajectory passing point $A$ forms the order-1 periodic solution of system (1.1) (see Fig. 6).

In the following part we focus on the uniqueness of the order-1 periodic solution to system (1.1).

**Theorem 3.2.** If $R > 1$, for any $\theta (0 < \theta < 1)$, the order-1 periodic solution of system (1.1) is unique.
Fig. 6. The case of $F(A) = 0$, the existence of order-1 limited cycle.

**Proof.** We denote the intersection of the vertical isoclinic line $\frac{ds}{dt} = 0$ and phase set as $Q$ with horizontal ordinate $q$, and the conclusion will be proved in the following 5 different cases.

**Case I:** Select two different points $I$ and $J$ in phase set $N$ satisfying $x_I < x_J$ and both $I$ and $J$ are located on the left side of horizontal isoclinic line $\frac{ds}{dt} = 0$ (see Fig. 7). Then, there exist trajectories $L_I$ and $L_J$ passing $I$ and $J$, and they cross the impulse set $M$ at $M_I$ and $M_J$ respectively. $M_I$ must be located on the right of $M_J$. The impulse function $\varphi : M_I \rightarrow N_I$, $\varphi : M_J \rightarrow N_J$, $x_{N_I} = x_{M_I} + \theta I^*$ and $x_{N_J} = x_{M_J} + \theta I^*$, we have $x_{N_I} > x_{N_J}$; here, $N_I$ and $N_J$ are subsequent points of $I$ and $J$ respectively. The successor functions of $I$ and $J$ are $F(I)$ and $F(J)$, then, $F(I) - F(J) = (x_{N_I} - x_I) - (x_{N_J} - x_J) = (x_{N_I} - x_{N_J}) + (x_J - x_I) > 0$.

**Case II:** Select two different points $I$ and $J$ in phase set $N$ satisfying $x_I < x_J < q$, and they are located on different sides of horizontal isoclinic line $\frac{ds}{dt} = 0$ (see Fig. 8). Then, $F(I) - F(J) = 0$. The prove is similar with that in case I.

**Case III:** Select two different points $I$ and $J$ in phase set $N$ satisfying $x_I < x_J < q$, and both $I$ and $J$ are located on right side of horizontal isoclinic line $\frac{ds}{dt} = 0$ (see Fig. 9). Then, there exist trajectories $L_I$ and $L_J$ initiating from $I$ and $J$, and they cross the vertical isoclinic line at $P_I$ and $P_J$ and reach the impulse set $M$ at $M_I$ and $M_J$ respectively. During the process from $I$ to $P_I$ and $J$ to $P_J$ on $L_I$ and $L_J$, condition $r - d_0 - \frac{\beta}{K} - \beta l > 0$ is satisfied. With the same vertical ordinate, the horizontal ordinate of the point in $L_I$ is smaller than the horizontal ordinate of the corresponding point in $L_J$, then the value of $r - d_0 - \frac{\beta}{K} - \beta l$ is larger on the point belonging $L_I$ than the corresponding point belonging $L_J$. Similarly, during the process from $P_J$ to $M_J$ and $P_I$ to $M_I$ on $L_I$ and $L_J$, we have $r - d_0 - \frac{\beta}{K} - \beta l < 0$. With the same vertical ordinate, the horizontal ordinate of the point in $L_I$ is smaller than that of the point in $L_J$, then the value of $r - d_0 - \frac{\beta}{K} - \beta l$ is bigger on a certain point belonging $L_I$ than that on the corresponding point belonging $L_J$. Then we have $x_{N_I} > x_{N_J} = x_{M_I} - x_{M_J}$ which results in $F(I) - F(J) > 0$.

**Case IV:** Select two different points $I$ and $J$ in phase set $N$ satisfying $q < x_I < x_J$, and point $I$ is located on right side of horizontal isoclinic line $\frac{ds}{dt} = 0$.

**Case V:** Select two different points $I$ and $J$ in phase set $N$ satisfying $q < x_J < x_I$.

The proofs of Case IV and Case V are similar to that of Case III and they are omitted here.

By all accounts above, the successor function $F(\cdot)$ is monotonically decreasing in the phase set. Thus there exists only one point $A$ satisfying $F(A) = 0$. Hence, the order-1 periodic solution to system (1.1) is unique. This completes the proof.

### 4. Stability of the order-1 periodic solution

In this section, we study the stability of the order-1 periodic solution to system (1.1). To the order-1 periodic solution of a differential dynamic system with state feedback impulsive, it is fantastic to prove the stability with the technique of ordinary differential dynamic system. To explain a novel method, we restate the concept of stability and establish an orthogonal coordinate before proving.

**Definition 4.1 ([28]).** Assume $\Gamma$ is an order-1 periodic solution of a certain state feedback impulsive system. If there exists a sufficiently small neighborhood $U(\Gamma)$ such that the $\omega$ limit set of any trajectory whose starting point $P \in U(\Gamma)$ is $\Gamma$, then $\Gamma$ is stable. Otherwise, it is defined unstable.

Following the consequence of theorems in part 3, for any $\theta (0 < \theta < 1)$, system (1.1) has an exclusive order-1 periodic solution denoted by $\Gamma$, and we assume $A$ is the intersection of $\Gamma$ and phase set $N$ (see Fig. 10). Then to any point $S_0$ satisfying $|AS_0| \rightarrow 0$, we define a point-series

$$\{S_0, S_1, \ldots, S_k, S_{k+1}, \ldots\}.$$
where $S_{i+1}$ is the subsequent point of $S_i$, $i = 0, 1, \ldots, k, \ldots$. Denote their coordinates with $s_0, s_1, \ldots, s_k, s_{k+1}, \ldots$, where

$$s_k = \begin{cases} -d_k, & S_k \text{ is on left side of } A, \\ d_k, & S_k \text{ is on right side of } A. \end{cases}$$

In this expression, $d_k$ is the distance between $S_k$ and $A$, then we can easily find $0$ is the coordinate of $A$. 

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**Fig. 7.** The monotonous feature of the successor function in case I.

**Fig. 8.** The monotonous feature of the successor function in case II.

**Fig. 9.** The monotonous feature of the successor function in case III.
Fig. 10. $S_1, S_2, \ldots, S_{k+1}, S_{k+2}, \ldots$ are the subsequent points of $S_0, S_1, \ldots, S_k, S_{k+1}, \ldots$ respectively.

Fig. 11. Establish coordinate system $(s, n)$ on point $A$.

**Lemma 4.1** ([28]). Denote the mapping of the abstract impulsive differential dynamic system (2.1) as $f$, and function $\bar{s} = f(s)$ is derivative at $s = 0$. Then $s = 0$ is stable (unstable) if

$$\left| \frac{d\bar{s}}{ds} \right|_{s=0} < 1 (> 1).$$

Without loss of generality, we assume that functions $P(x, y)$ and $Q(x, y)$ of system (2.1) have arbitrary order partial derivatives. We also assume the time period from $A$ to $C$ on the certain trajectory of (2.1) is $T$, then the period of the closed orbit $\Gamma$ is $T$ too.

Then we can set up a new coordinate system $(s, n)$ in adequately closed scope of orbit $\Gamma$. To any point in that field, there exists a certain point in the closed orbit $\Gamma$ and its normal line passes the point. We define the first coordinate $s$ as the length of the trajectory between the certain point and $A$ on $\Gamma$, while the second coordinate $n$ as the number of the normal vector (here we assume upside as the positive direction) (See Fig. 11). With the first coordinate $s$ we have set up, we can establish the parametric equation of curve $\hat{ABC}$

$$x = \phi(s), \quad y = \psi(s),$$

where $s$ is the parameter.

Then we can expressed the rectangular coordinate $(x, y)$ with the orthogonal coordinate $(s, n)$ as

$$x = \phi(s) - n\psi'(s), \quad y = \psi(s) + n\phi'(s),$$

Following (2.1) and (4.2), we have

$$\frac{dn}{ds} = \frac{Q\phi' - P\psi' - n(P\phi'' + Q\psi'')}{P\phi' + Q\psi'} \triangleq F(s, n).$$

(4.3)

It is obvious that $n = 0$ is a solution of (4.3) and the first-order partial derivative of $F(s, n)$ with respect to $n$ is continuous, then (4.3) can be expressed briefly as

$$\frac{dn}{ds} = F'_n(s, n)\big|_{n=0} \cdot n + o(n),$$

(4.4)

Following (4.3), we can calculate

$$F'_n(s, n)\big|_{n=0} = \frac{P_0^2Q_0 - P_0Q_0(P_{x0} + Q_{x0}) + Q_0^2P_{x0}}{(P_0^2 + Q_0^2)^2} \triangleq H(s),$$
where \( P_0 \) and \( Q_0 \) represent the values of \( P \) and \( Q \) at point \( A \), \( P_{x0}, P_{y0}, Q_{x0} \) and \( Q_{y0} \) are the partial derivatives of \( P \) and \( Q \) at \( n = 0 \) with respect to \( x, y \) respectively. It is obvious that \( P_{y0}'' + Q_{y0}'' = 0 \). Then the order-1 linear approximate equation of (4.4) is

\[
\frac{dn}{ds} = H(s)n,
\]

and its solution is

\[
n = n_0 e^{\int_{0}^{s} H(s')ds'}, \quad n_0 = n(0).
\]

**Theorem 4.1.** Assume \( h \) is the length of curve \( ABC \) which is a segment of the order-1 periodic solution \( \Gamma \) to system (2.1), then \( \Gamma \) is stable (unstable) if integration of \( H(s) \) along \( ABC \) is negative (positive), i.e.

\[
\int_{0}^{h} H(s)ds < 0(> 0).
\]

**Theorem 4.2.** Suppose the region closed by order-1 periodic solution \( \Gamma \) is convex. If \( \int_{0}^{T} (P_{x0} + Q_{y0})dt < 0 \) is satisfied, then \( \Gamma \) is stable.

The proofs of Theorems 4.1 and 4.2 are similar with that in [12], and omitted here.

Then we can get the corollary directly.

**Corollary 4.1.** Suppose the area closed by order-1 periodic solution \( \Gamma \) is convex. If \( P_{x0} + Q_{y0} < 0(> 0) \) holds, then \( \Gamma \) is stable (unstable).

**Theorem 4.3.** The order-1 periodic solution of state feedback impulsive system (1.1) is stable.

**Proof.** Define a Dulac function \( u(S, I) = \frac{1}{S} \) and we assume \( u(S, I) \) is continuous, differentiable and positive in the first quadrant. Then the stability of the order-1 periodic solution of system (1.1) is the same with the system

\[
\begin{aligned}
\frac{dS}{dt} &= u(S, I) \frac{dS}{dt} = \frac{1}{I} \left( r - d_0 - \frac{S}{K} \right) - \beta \equiv p(S, I), \\
\frac{dI}{dt} &= u(S, I) \frac{dI}{dt} = -\frac{d + d_0}{S} \equiv q(S, I).
\end{aligned}
\]

In this system, we can calculate

\[
P_{x0} + q_{y0} = -\frac{1}{IK} < 0,
\]

where \( P_{x0} \) and \( Q_{y0} \) are the partial derivatives of \( p \) and \( q \) at \( n = 0 \) with respect to \( S \) and \( I \) respectively. Inspired with Corollary 4.1, we can reach the verdict that the order-1 periodic solution of system (1.1) is stable. This completes the proof.

5. Numerical simulation and discussion

System (1.1) describes the development of animal epidemics and its impulsive treatments, it is the most advanced model which can be regarded as an analog of the procedure. We have proved that system (1.1) has a unique stable periodic solution which means the animal epidemics will not break out with factitious treatment basing on the flip-flop of the infectious' number. And in this part, we examine the theoretical results with real parameter values.

Let \( r = 0.2, K = 5, \beta = 0.7, d = 0.2, d_0 = 0.08, \theta = 0.7 \), the impulse set and phase set are \( M = \{(x, y)|x > 0, y = 0.0457\} \) and \( N = \{(x, y)|x > 0, y = 0.0137\} \) respectively, and the condition \( R > 1 \) is satisfied. Here, we choose two groups of initial values, \( S_0 = 0.4, I_0 = 0.01 \) and \( S_0 = 0.65, I_0 = 0.01 \). The numerical simulation results verify the conclusion of Theorems 3.1, 3.2 and 4.3, that means system (1.1) has a unique stable periodic solution (see Fig. 12).

From the simulation results, we can find that system (1.1) has a unique limited cycle, and it does not depend on the initial values. In practice, the stable periodic solution means the animal epidemic can be controlled by intermittent state feedback treatment. Apparently the number of infectious animals must be observed and recorded, since the treatment is activated by the value. Once the number of infectious animals accumulates to the threshold of impulsive set, human will carry out some treatment measures and the number of the infectious drop to the imagine set. That work costs huge manpower and material resources. In fact, since the periodic solution is unique and stable, the period of the periodic solution is a constant. If we observe the phenomenon that the interval between each time the amount of the infectious reaches the threshold is a constant, it forebodes the periodic solution has formed. In that case, we can carry out artificial intervention depending on the fix period instead of observing the infectious and record the amount. This theoretical result can improve the efficiency of animal epidemic's treatment and control.
Fig. 12. Time series and phase portrait of system (1.1). The three figures in the left column are trajectory, time series of $S$ and $I$ of system (1.1) with initial values $S_0 = 0.4$, $I_0 = 0.01$, and the right column are the corresponding figures with initial values $S_0 = 0.65$, $I_0 = 0.01$.

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