Infrared finite solutions for the gluon propagator and the QCD vacuum energy

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Abstract

Nonperturbative infrared finite solutions for the gluon polarization tensor have been found, and the possibility that gluons may have a dynamically generated mass is supported by recent Monte Carlo simulation on the lattice. These solutions differ among themselves, due to different approximations performed when solving the Schwinger-Dyson equations for the gluon polarization tensor. Only approximations that minimize energy are meaningful, and, according to this, we compute an effective potential for composite operators as a function of these solutions in order to distinguish which one is selected by the vacuum.

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During the last years there has been much effort in trying to obtain a non-perturbative form for the gluon propagator [1-8], and perhaps one of the most interesting results is the one where it is argued that the gluon may have a dynamically generated mass [3]. The existence of a mass scale, or the absence of a pole at $k^2 = 0$ is natural if one assumes that gluons do not propagate to infinity, i.e. these propagators describe confined gluons. From the phenomenological point of view, this possibility may shed light on several reactions where long distance QCD effects can interfere, and examples about the consequences of massive gluons can be found in the literature, see, for instance, Ref.[9-11].

The study of the infrared behavior of the gluon propagator was also performed numerically [12], and the last numerical lattice computations give strong evidence for an infrared finite gluon propagator [13]. The most recent results [13] seems to exclude a propagator of the form $1/k^4$ determined in several analysis of Schwinger-Dyson equations (SDE) and claimed to give confinement [1, 2, 4]. In particular, the results of Bernard et al. [13] are consistent with an infrared finite propagator, but can be fitted by the Cornwall's massive propagator [3] as well as by the one found by Stingl et al. [5]. These lattice computations could be criticized because of the gluon propagator gauge dependence. However, there is also a lattice study of the “gauge-invariant” two-point correlation function of the gauge field strengths by Di Giacomo et al. [14], where a finite correlation length for the gluons in the QCD vacuum is found, indicating the presence of a mass scale for the gluon propagation.

If the lattice results can be considered exciting, the same cannot be said about the analytic calculations. Looking at Ref. [1] to [8] we verify that infinite as well as infrared finite non-perturbative propagators have been found in the literature, although this difference can be credited to the fact that some authors discarded from the beginning an infrared finite solution for the gluon propagator. Even if we separate the solutions of SDE for the gluon self-energy, according to the existence or not of a pole at $k^2 = 0$, we still remain with different forms of non-perturbative
propagators, which reflect the different approximations involved in the calculations, for instance, different truncations of the integral equations, or the handling of higher-point functions which are present in the two-point one, etc...

In this work we propose to compute the effective potential for composite operators \[^{15}\] at stationary points as one method to distinguish among the different solutions, \( i.e. \) we will calculate the vacuum energy for some infrared finite non-perturbative Schwinger-Dyson solutions of the gluon propagator, and the vacuum will select the solution that leads to the deepest minimum of energy. Therefore, this will be a complementary tool to find solutions of the gluonic SDE, \( i.e. \) they should be tested by means of the effective potential calculation. As we are far from being able to solve the SDE without any simplification, only approximations that minimize the vacuum energy should be performed when solving these integral equations. We will also discuss how the SDE solutions can be constrained phenomenologically.

For a non-abelian gauge theory the effective potential has the form \[^{13}\]

\[
V(S, D, G) = -i \int \frac{d^4 p}{(2\pi)^4} Tr \left( \ln S_0^{-1} S - S_0^{-1} S + 1 \right)
- i \int \frac{d^4 p}{(2\pi)^4} Tr \left( \ln G_0^{-1} G - G_0^{-1} G + 1 \right)
+ \frac{1}{2} i \int \frac{d^4 p}{(2\pi)^4} Tr \left( \ln D_0^{-1} D - D_0^{-1} D + 1 \right)
+ V_2(S, D, G), \quad (1)
\]

where \( S, D \) and \( G \) are the complete propagators of respectively fermions, gauge bosons and Faddeev-Popov ghosts; \( S_0, D_0 \) and \( G_0 \) the respective bare propagators. \( V_2(S, D, G) \) is the sum of all two-particle irreducible vacuum diagrams, depicted in Fig.(1), and the solutions of the equations

\[
\frac{\delta V}{\delta S} = \frac{\delta V}{\delta D} = \frac{\delta V}{\delta G} = 0, \quad (2)
\]

give the SDE for fermions, gauge bosons and ghosts.
We can represent $V_2(S, D, G)$ analytically by

$$nV_2(S, D, G) = -\frac{1}{2} Tr(\Gamma S \Gamma SD) - \frac{1}{2} Tr(FGFGD)$$

$$+ \frac{1}{6} Tr(\Gamma^{(3)} D \Gamma^{(3)} DD) + \frac{1}{8} Tr(\Gamma^{(4)} D)$$

(3)

where $\Gamma$, $F$, $\Gamma^{(3)}$ and $\Gamma^{(4)}$ are respectively the proper vertex of fermions, ghosts, trilinear and quartic gauge boson couplings [16]. In Eq.(3) we have not written the gauge and Lorentz indices, as well as the momentum integrals.

The complete gauge boson propagator $D$ is related to the free propagator by

$$D^{-1} = D_0^{-1} - \Pi,$$

(4)

where $\Pi$ is the gluon polarization tensor, which is obtained from Eq.(1) and Eq.(2), and described by

$$\Pi = \Gamma S \Gamma S + FGF - \frac{1}{2} \Gamma^{(3)} D \Gamma^{(3)} DD - \frac{1}{2} \Gamma^{(4)} D.$$  

(5)

The diagrams contributing to $\Pi$ are shown in Fig.(2), and this self-energy is the one that has been solved in many different ways, leading to a series of non-perturbative forms for the gluon propagator in the infrared region [1-6]. As our intention is to compute the effective potential through a variational approach, the best ansatz for the complete propagator will be given by the approximate solution of $\Pi$.

The physically meaningful quantity we need to compute is the vacuum energy density given by

$$\Omega = V(S, D, G) - V(S_p, D_p, G_p),$$

(6)

where $D_p$ is the perturbative counterpart of $D$, and we are subtracting from the potential the perturbative part that does not contribute to dynamical mass generation, denoted by $V(S_p, D_p, G_p)$. As our interest is concentrated in the pure glue theory, we disregard all the quark contributions for the effective potential from now on, i.e. we drop the first contribution on the right-hand side of Eq.(3). It is opportune to
recall that for perturbative gluons $\Omega$ is plagued by infrared divergences, and only for the infrared finite gluon propagators it can be calculated without ambiguities [3].

Before we start calculating $\Omega$ it is convenient to discuss how the solutions of $\Pi$ are determined. One of the first calculations of the gluon polarization tensor is due to Mandelstam [1], where the ghosts and the diagram with quartic coupling are disregarded. The neglect of ghosts diagrams was shown to be correct, because their contribution is numerically small [1]. These approximations, with the use of the Landau gauge, were shown to be satisfactory in the lengthy and detailed work of Brown and Pennington [4]. With this approximation only the diagram with trilinear coupling should be considered in Eq. (3), i.e. the gluon polarization tensor is going to be given by

$$\Pi = -\frac{1}{2} \Gamma^{(3)} D \Gamma^{(3)} D. \quad (7)$$

More complex approximations can be used, but in practice only the diagram with the trilinear coupling has been considered in most of the works up to now. As far as we know, few papers have gone beyond these approximations, and one of these approaches is the Cornwall’s determination, through the use of the pinch-technique, of a set of diagrams leading to a gauge invariant SDE [3]. Another one is the Stingl et al. manipulation of the higher-point functions [5], although this calculation was done in Landau gauge.

The vacuum energy can be computed with the same approximations performed to obtain solutions of the gluon self-energy. However, a much better approximation to study the vacuum stability results when we plug the stationary conditions (Eq. (2) and Eq. (4)) into $\Omega$, obtaining an expression for the values of $\Omega$ at its minimum, which will be denoted by $\langle \Omega \rangle$. For example, with the Mandelstam approximation the 2PI diagrams contributing to $\Omega$ would be given by

$$\Omega_2 = \frac{-1}{6} Tr(\Gamma^{(3)} D \Gamma^{(3)} D) - (D \rightarrow D_p), \quad (8)$$
but applying the stationary condition Eq.(2) we obtain

\[ \langle \Omega_2 \rangle = \frac{1}{3} Tr(\Pi D) - (D \rightarrow D_p), \tag{9} \]

where the calculation is now reduced to an “one-loop” integration. It is important to stress that \( \langle \Omega \rangle \) is much less dependent on the ansätze we use to compute the vacuum energy. This has already been emphasized in calculations of the vacuum energy for chiral symmetry breaking \([17]\). Note that \( \langle \Omega \rangle \) also does not suffer from possible problems in the determination of the effective potential for composite operators \([18]\).

The vacuum energy at stationary points, within the approximation discussed above, is given by

\[ \langle \Omega \rangle = \frac{1}{2} Tr[\ln(D_0^{-1}D) - D_0^{-1}D + 1] \]

\[ - \frac{1}{3} Tr[(D^{-1} - D_0^{-1})(D - D_0)], \tag{10} \]

where we assumed \( D_p = D_0 \).

Displaying the momentum integration, using the Landau gauge expression for the complete gluon propagator

\[ D^{\mu\nu}(p^2) = -\frac{i}{p^2 - \Pi(p^2)}(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}), \tag{11} \]

and calculating the trace, Eq.(10) can be cast in the following form

\[ \langle \Omega \rangle = -3(N^2 - 1) \int \frac{d^4 P}{(2\pi)^4} \left[ \frac{\Pi}{P^2 + \Pi} - \ln(1 + \frac{\Pi}{P^2}) + \frac{2}{3} \frac{\Pi^2}{P^2(P^2 + \Pi)} \right], \tag{12} \]

where all the quantities are in Euclidean space \( (P^2 = -p^2) \), and \( N = 3 \) for QCD. Note that the computation of the full vacuum energy \( \Omega \) requires the use of nonperturbative vertices satisfying the Slavnov-Taylor identities, which give strong relations between the diagrams of Fig.(1) \([19]\), and we escaped from this difficulty in our \( \langle \Omega \rangle \) calculation.

To compute \( \langle \Omega \rangle \) we need only a reasonable ansatz for the gluon propagator or its self-energy \( \Pi \), which will be given by the SDE solutions to be presented in
the sequence. One of the infrared finite propagators found in the literature was
determined by Cornwall \cite{3}

\[ D_c^{-1}(P^2) = [P^2 + m_g^2(P^2)] \beta g_s^2 \ln \left[ \frac{P^2 + 4m_g^2(P^2)}{\Lambda^2} \right], \quad (13) \]

where \( m_g(P^2) \) is the dynamical gluon mass

\[ m_g^2(P^2) = m_g^2 \left[ \ln \left( \frac{P^2 + 4m_g^2}{\Lambda^2} \right) / \ln \frac{4m_g^2}{\Lambda^2} \right]^{12/11}, \quad (14) \]

\( m_g \) is the gluon mass scale, \( g_s \) the strong coupling constant, \( b = 33/48\pi^2 \) is the leading order coefficient of the \( \beta \) function of the renormalization group equation, and \( \Lambda = \Lambda_{QCD} \) is the QCD scale. This form for the propagator was obtained as a fit to the numerical solution of a gauge invariant set of diagrams for the gluonic SDE, which also imposes \( g_s \simeq 1.5 - 2.5 \).

Eq.(13) is consistent with the expected ultraviolet behavior of the gluon propagator

\[ D_{UV}^{-1}(P^2) \to P^2 \ln(P^2). \quad (15) \]

However, the dynamical gluon mass \( m_g(P^2) \) is expected to decrease faster asymptotically as predicted by the operator product expansion (OPE) \cite{20,8}. It should be remembered that the SDE for the gluon propagator is accurate in the determination of the infrared dynamical mass, but its ultraviolet behavior can be better determined by means of the OPE. The behavior of the dynamical gluon mass through the OPE was determined by Lavelle \cite{8}, and is equal to

\[ \Pi_{UV}(P^2) \sim - \frac{34N\pi}{9(N^2 - 1)} \frac{\langle \alpha_s G^2 \rangle}{P^2}, \quad (16) \]

where \( \langle \alpha_s G^2 \rangle \) is the gluon condensate. Note that as long as the gluon self-energy is infrared finite, and no matter its ultraviolet behavior is given by Eq.(14) or Eq.(16), it is easy to see that \( \langle \Omega \rangle \) is free of divergences \cite{3}.

Another infrared finite propagator has been found by Stingl \textit{et al.} \cite{5}, its form agrees with that derived by Zwanziger based on considerations related to the Gribov
horizon \([7]\), and is given by

\[
D_s(P^2) = \frac{1}{P^2 + \frac{\mu_s^4}{(P^2 + c^2)}},
\]

(17)

where \(\mu_s\) is a mass scale not determined in Ref. \([5]\). Eq. (17) with \(c = 0\) is the one found by Zwanziger \([7]\). As claimed in the last paper of Ref. \([5]\), the full propagator should contain a \(c \neq 0\) term, however, due to the complexity introduced by such factor, this case was not studied in detail. The term \(\mu_s^4/P^2\) is exactly what is expected by OPE analysis whenever a mass scale for the gluon is introduced, as shown by Eq. (16), therefore, its appearance is not surprising. With \(c = 0\) the propagator goes to zero at \(P^2 = 0\), however the gluon self-energy diverges, and we discard this possibility here assuming that \(c = \mu_s\), which could be expected as a natural mass scale leading to a finite gluon polarization tensor. The Bernard et al. lattice result for the gluon propagator \([13]\) can be fitted by Eq. (13) as well as Eq. (17).

Finally, Marenzoni et al. \([13]\) also performed a lattice study of the gluon propagator in the Landau gauge, obtaining for its infrared behavior the following fit

\[
D_m(K^2) = \frac{1}{m_t^2 + Z P^2 (\frac{P^2}{\Lambda^2})^\eta},
\]

(18)

where \(m_t\), \(Z\) and \(\eta\) are constants determined with the numerical simulation. \(m_t\) is of \(\mathcal{O}(\Lambda \simeq 160 \text{ MeV})\), \(Z \simeq 0.4\), and \(\eta \simeq 0.5\) what is slightly different from the previous propagators. The results of Bernard et al. also show the behavior \((P^2)^\eta\), but with a smaller value for \(\eta\). We will consider only the propagators discussed up to now. Although other propagators can still be found in the literature, the propagators that we presented above are the most discussed ones and enough for our purposes.

The result of Ref. \([3]\), which led to Eq. (13), was shown to be gauge invariant, and the others were obtained in the Landau gauge, which we assume here expecting that any gauge dependence should be negligible. In the above equations for the infrared propagator, except from Cornwall’s one, it is not clear how to connect these
expressions with their ultraviolet part, and this is an extra complication, because any, *ad hoc*, link between the ultraviolet and infrared propagator will introduce a new mass scale. It is obvious that the physics should not depend on this particular scale, but it does introduce some arbitrariness. We, therefore, will not consider the ultraviolet renormalization group logarithmic behavior of the full propagator, as described by Eq.(15), since it should be common to all propagators (although it is only present in Eq.(13)). In the specific case of the lattice propagator, Eq.(18), in which there is no clear separation between the infrared and ultraviolet regions, we shall simply neglect its peculiar form, assuming that the vacuum gluon polarization can be cast as a pole in the perturbative propagator factorized from Eq.(18).

We adopt the following ansatze to calculate $\langle \Omega \rangle$: a) $\Pi(P^2) = m_g^2(P^2)$, where $m_g^2(P^2)$ is given by Eq.(14); b) To be consistent with the massive Cornwall propagator and with OPE, we will consider a gluon self-energy that interpolates between the constant infrared behavior of Eq.(14) and the ultraviolet one of Eq.(16), which is given by

$$\Pi(P^2) = \mu^2 \theta(\mu^2 - P^2) + \frac{\mu^4}{P^2} \theta(P^2 - \mu^2);$$

(19)
c) $\Pi(P^2)$ consistent with Eq.(17), i.e.

$$\Pi(P^2) = \frac{\mu_s^4}{P^2 + \mu_s^2};$$

(20)
d) We assume that $\Pi(P^2)$ for the lattice inspired propagator is given by

$$\Pi(P^2) = m_l^2 \theta(\mu^2 - P^2) + \frac{m_l^2}{Z(P^2/\Lambda^2)^n} \theta(P^2 - \mu^2),$$

(21)
which follows from considering the generated gluon mass as a pole in the perturbative propagator, factorized from Eq.(18). Finally, since $m_g$ and $\mu_s$ are not determined by the solution of the SDE, we compute the vacuum energy as a function of these parameters.

In Fig.(3) we show the vacuum energy computed with our ansatze a) to d), as a function of the gluon mass scale ($m_g \equiv \mu_s \equiv m_l$). Note that once selected a
form for the gluon self-energy, $\Pi$, the $\langle \Omega \rangle$ calculations showed in Fig. (3) only give us the vacuum energy as a function of the gluon mass scale ($m_g/\Lambda$), and we do not have a criterion to fix a particular value of this scale parameter. We can only compare the different vacuum energy curves, corresponding to the different gluon self-energy behavior, assuming a common value for the gluon mass scale. From the behavior of the vacuum energy presented in this figure we verify that the Cornwall propagator, Eq. (13), leads to the deepest values of minimum of the vacuum energy, as long as $m_g > 1.6\Lambda$, compelling us to believe that this could be the most favoured form of the gluon propagator to match the one chosen by the QCD vacuum, for gluon masses obeying such relation. For smaller gluon masses ($m_g < 1.6\Lambda$), the lattice propagator is the one that best represents the actual gluon propagator. It is clear that apart from the lattice propagator (Eq. (18)), where all the constants are determined, we can only compare the different values for the vacuum energy, if we are able to constrain the parameters $m_g, \mu_s$. In principle they should be related to the scale $\Lambda$, but we are far from this achievement, and we discuss in the sequence some of the constraints that have been put forward on these parameters.

The phenomenological constraints on the dynamical gluon mass may come from the quarkonium decays into gluons \[9, 11\], potential models for glueballs \[21\], approximations to the QCD-Hamiltonian combined with a variational method \[22\], a possible relation between the gluon condensate and the gluon mass through QCD sum-rules \[23\], the two-gluon exchange contribution to the Pomeron \[10\], and the vacuum functional of SU(3) Yang-Mills theory \[24\]. The gluon masses obtained in most of these studies are of the order of $2\Lambda$, but one should be careful when considering this value because with few exceptions these calculations consider only a bare gluon mass, and the specific behavior of the dynamical mass with the momentum may modify the result. These calculations in general also involve many different (and some untenable) approximations. Therefore, we are in the safe side if we consider that the gluon mass may lay between $\Lambda$ and $3\Lambda$, and according to Fig.(3), the
propagator determined by Cornwall and the one obtained in the lattice simulation lead to the deepest vacuum energy, and approach the actual behavior of the confined gluon propagator. Finally, we can also use one relation found by Cornwall \[3\] between the gluon condensate and the vacuum energy

\[
\langle g^2 G^a_{\mu\nu} G^a_{\mu\nu} \rangle = -8b^{-1} \langle \Omega \rangle ,
\]

which allows a determination of the gluon mass. To arrive at this relation it has been assumed that the \(\beta\) function is given by its perturbative behavior \((\beta \simeq -bg^3)\). Although the \(\beta\) function can be different from this in the condensed phase, maybe this relation is the most trustable one to allow a determination of the gluon mass.

The left-hand side of Eq. (22) has been evaluated through QCD sum-rules \[25\] and is equal to 0.47 GeV\(^4\). Therefore, assuming \(\Lambda = 0.3\) GeV we obtain the following gluon masses in the cases we discussed above: a) 527 MeV, b) 627 MeV, c) 715 MeV, and d) 560 MeV. Considering all the gluon mass determinations in the literature we verify again that the propagators of Ref. \[3\] and Ref. \[13\] are the ones to provide the most consistent picture.

In conclusion, starting from the effective action for composite operators, we determined an expression for the vacuum energy of QCD without fermions in the Landau gauge. The vacuum energy calculation was used as a criterion to determine which, among a series of different forms determined for the gluon propagator, leads to the deepest minimum of energy. We verify that two expressions are clearly favored and, consistently, when we compare the vacuum energy with the gluon condensate, we obtain gluon masses for these propagators that are in the range of several determinations of this mass existent in the literature.
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\[ V_2(S, D, G) = -\frac{1}{2} \]

Figure 1: Two-particle irreducible vacuum diagrams contributing to the effective potential.

\[ \Pi = \]

Figure 2: Diagrams contributing to the gluon polarization tensor.
Figure 3: Result of the vacuum energy calculation as a function of the gluon mass for different ansatze for the gluon vacuum polarization. The curves are related to the ansatze assumed inside the text: a) solid line Eq. (14); b) dotted line Eq. (19); c) dashed-dotted line Eq. (20); d) dashed line Eq. (21).