Tunneling between parallel one-dimensional Wigner crystals

R. Méndez-Camacho¹,² & E. Cruz-Hernández²*

Vertically aligned arrays are a frequent outcome in the nanowires synthesis by self-assembly techniques or in its subsequent processing. When these nanowires are close enough, quantum electron tunneling is expected between them. Then, because extended or localized electronic states can be established in the wires by tuning its electron density, the tunneling configuration between adjacent wires could be conveniently adjusted by an external gate. In this contribution, by considering the collective nature of electrons using a Yukawa-like effective potential, we explore the electron interaction between closely spaced, parallel nanowires while varying the electron density and geometrical parameters. We find that, at a low-density Wigner crystal regime, the tunneling can take place between adjacent localized states along and transversal to the wires axis, which in turn allows to create two- and three-dimensional electronic distributions with valuable potential applications.

Semiconductor nanowires (NWs) are exciting components to study unique one-dimensional (1D) physics such as the Coulombic strength-dependent electronic charge fractionalization¹, quantized conductance²,³; or the formation of a periodic charge distribution along the wires, known as a Wigner crystal, first predicted by Wigner in 1934⁴ and recently observed⁵–⁷. In addition to being remarkable systems for basic research, NWs are also key building blocks in the fabrication of nanoscale electronic and optoelectronic devices⁸–¹¹. Some NW-based devices that has been widely explored includes field-effect transistors (FETs)¹²–¹⁵, diodes¹⁶, nano-logic gates¹⁷, and nanoprocessors¹⁸.

When NWs are synthesized by self-assembly, usually pillar or in-plane arrays of parallel NWs (PNWs) are produced. In such arrays, or in the integration into the existing planar electronic platforms¹¹, the NWs separation can be short enough to permit electron tunneling between them. Furthermore, with the continuous size reduction in the design of smaller and more powerful devices; for example in advanced sub-5 nm nodes in NW-based vertically or laterally stacked gate-all-around FETs¹⁹–²¹, the consequences of the closeness of adjacent NWs must be thoroughly analyzed to both prevent undesirable effects and to look for new architecture strategies²².

On the theoretical side, even when there exist a number of advanced approaches to investigate the electron-electron (e-e) interaction in nanostructures, usually such formalisms are limited to one or few charge carriers and to the nanometric scale²³–²⁷. On the other hand, the many-electron interaction in 1D Wigner molecules, and the close related Friedel oscillations, has been studied by using Luttinger liquid theory as well as quantum Monte Carlo and ab initio methods²⁸–³⁰. Far less works dealing with the electron interaction between coupled NWs has been published, even when it can be a simple and powerful approach to explore more complex problems such as the strongly interacting topological states in two and higher dimensions⁴. Still, modeling of micron-long semiconductor NWs and the e-e interaction for electronic densities (n) obtained from usual doping levels (10¹⁴–10¹⁹ electrons/cm³) is a very cumbersome task, impracticable to carry out in a direct way. For such reason, despite the importance of this subject, the interaction of adjacent PNWs with realistic characteristics remains largely unexplored.

Given the difficulty of directly including the many-body forces to calculate the many-body quantum states, we instead use a coarse-grained model that considers two-electron wave-functions while the presence of the other electrons is assumed to only affect the interaction between the two electrons by a Yukawa-like screening. Although simple, the use of a Yukawa-like potential to address the many-body problem has been extensively used in other branches of physics, such as soft matter and nuclear physics and, applied to electron distributions in nanostructures, this approach is able to correctly describe experimental results related to the Wigner molecule onset in semiconductor NWs³⁷,³⁸.

In this contribution, we theoretically investigate the electron tunneling between closely spaced micron-long semiconductor PNWs. The resultant electronic distributions are analyzed as a function of the NWs separation...
Results and discussion

The effective potential. Figure 1 provides a schematic view of one of the systems studied in this work. Such system consists of GaAs PNWs of cross-sectional area \( L_x \times L_y \) and length \( L_z \), which are embedded in an AlGaAs matrix that acts as a finite potential barrier of width \( L_b \). As described in detail in “Theoretical model”, we solve the time-independent Schrödinger equation for a spinless two-electron wave function, \( \Psi_{1,2}(r) \), considering a Yukawa-like e-e interaction. Then, by using Fourier transforms, we derive for the \( z \) component a real-space effective potential able to manage long \( L_z \) and high \( n \) values in an easy way. For the \( x \) and \( y \) components we consider the usual solutions for \( m \)-coupled finite quantum wells. The electronic density distributions shown hereafter are then obtained from the square of the calculated \( \Psi_{1,2}(r) \).

Two parallel nanowires. In Fig. 2a, we plot the \( z-x \) projection of the ground state distributions for two PNWs of size \( L_{x,y} = L_x = L_y = 50 \) nm and \( L_z = 1 \) \( \mu \)m, kept apart by a variable \( L_b \), for \( n = 10^{17} \) electrons/cm\(^2\) (left) and \( n = 10^{20} \) electrons/cm\(^2\) (right). The ground and first excited state for a similar system, with a smaller cross-section \( L_{x,y} = 15 \) nm, are presented in Fig. 2b,c, respectively. Each row in Fig. 2 corresponds to a different \( L_b \) separation. We can observe from the upper row in Fig. 2 (where there is not tunneling between NWs), that in agreement with previous reports on individual NWs\(^37,38\), the lower \( n \) concentration produces for the ground state well defined individual distributions along the NWs while for the higher concentration the electrons merge in a single distribution. This concordance is also observed in the first excited state. For the ground state, we can observe from Fig. 2 that tunneling is strongly dependent on the electron density. Such dependence is mainly due to the localized and extended states induced for the relative low and high densities, respectively. For the excited states, a localized distribution is always found. Given the energy difference between the confined levels (meV for \( x-y \), and \( \mu eV \) for \( z \)), the excited states plotted in Fig. 2 are those related to the \( z \)-component, while the \( x \) and \( y \) components remains in the ground state.

As shown in Fig. 2, the variation on the NWs cross-section affects the way the tunneling take place. For \( L_{x,y} = 15 \) nm, electrons are less contained by the AlGaAs barriers, so the tunneling takes place for a larger \( L_b \) separation as compared to the PNWs of cross-section \( L_{x,y} = 50 \) nm. According to our model, \( L_b \) and \( L_{x,y} \) are the main parameters to control the tunneling strength through the PNWs, while by modifying \( n \) one can control the electron distribution connection along each wire. The dependence of the tunneling strength on these parameters, for the ground state in an array of two PNWs, are presented in Fig. 3a,b in terms of the \( h_2/h_1 \) ratio where \( h_1 \) and \( h_2 \) are heights in the 2D density profiles taken as shown in the inset of Fig. 3a. From these figures, we can observe that an appreciable tunneling where \( h_2 \sim 0.1h_1 \) for such configuration is given for values of \( n \sim 6 \times 10^{18} \) electrons/cm\(^2\) (along the wires) and \( L_b \sim 5 \) nm for \( L_{x,y} = 15 \) nm or \( L_b \sim 2 \) nm for \( L_{x,y} = 50 \) nm (transversal to the wires). As we show further below, for 2D and 3D arrays of more than two PNWs, the tunneling strength is not uniform between the NWs, so there is not simple relationships for such cases as the shown in Fig. 3.
Figure 2. $z$–$x$ projections of the charge distribution probability for two 1 µm-long GaAs/AlGaAs PNWs. The plots correspond to: (a) ground state for $L_{x,y} = 50$ nm, (b) ground state for $L_{x,y} = 15$ nm, and (c) first excited state for $L_{x,y} = 15$ nm. Two different $n$-doping concentration are plotted: $n = 10^{17}$ electrons/cm$^3$ and $n = 10^{20}$ electrons/cm$^3$. Each row corresponds to a different $L_b$.

Figure 3. (a) $h_2/h_1$ dependence on the electron density $n$ for an array of $1 \times 2$ PNW with $L_{x,y} = 20$ nm and $L_b = 5$ nm. (b) $h_2/h_1$ dependence on $L_b$ for a similar array for three different cross sections $L_{x,y}$. We use the $h_2/h_1$ ratio as a parameter to describe the tunneling strength between localized distributions. Here $h_1$ and $h_2$ are heights taken from 2D electron distribution profiles as illustrated in the inset in (a). In (a), the profiles are taken along the $z$-axis from one of the NWs while in (b), the profiles are taken across the NWs along the $x$-axis.
Multiple parallel nanowires. Additional interesting properties comes out in systems composed of more than two PNWs. As an example, the electronic ground state distribution in a $1 \times 6$ GaAs/AlGaAs PNWs array, as the one depicted in Fig. 4b, is plotted in Fig. 4a. The relative low $n$ concentration ($1 \times 10^{18}$ electrons/cm$^3$) is used to induce the formation of localized electronic states along the NWs $z$-axis (a Wigner molecule). We set a small cross-section ($L_{x,y}=20$ nm) and a short distance between NWs ($L_b=5$ nm) to observe a strong tunneling between them. Profiles taken along the $x$-axis presents a notorious intensity modulation from wire to wire, as the shown in Fig. 4c (taken along the horizontal yellow line drawn in Fig. 4a). Even when this modulation is not observable in the two PNWs system (see Fig. 3), such kind of distributions where the intensity is maximal at the center of a finite number of quantum wells is usually found in common 1D superlattices (see for example reference39).

If, in the parameters used in Fig. 4a, $n$ is increased to $2 \times 10^{18}$ electrons/cm$^3$, a connection between the individual electronic distributions along the NWs axis can be also established (see Fig. 4d). As the lateral tunneling depopulate the more external NWs, the distribution along the $z$-axis is different from wire to wire. This is clear from Fig. 4e, where profiles taken along the $z$-axis from three consecutive NWs in Fig. 4d are plotted; the vertical yellow line in (d) indicates where the profile number (1) was taken.

Because confinement along the $z$ axis is very weak and the energy separation between quantized energy levels is very small, of the order of $\mu$eV, these states can be easily populated and then are particularly important to analyze. As an example, in Fig. 5a we plot the fourth excited state corresponding to a system with the same parameters that the plotted in Fig. 4a. A typical profile, taken along the $x$ direction where the density is maximal, is shown in Fig. 5b. By the nature of our model, this transversal profile is practically the same as the shown in Fig. 4c, with the only difference that its absolute intensity is smaller (as the electron density is distributed in more zones along the wires). In Fig. 5c, the profiles taken along the $z$ axis show a sharp distribution of the well separated electronic regions.
2D and 3D interconnected distributions. As discussed before, three main electronic distributions appear in the arrays of PNWs by modifying \( n \), \( L_{b} \), and \( L_{x,y} \). When \( L_{b} \) is large enough to block tunneling and \( n \) is low enough to trigger the Wigner crystallization, then a disconnected but ordered 2D charge distribution is established. However, if \( L_{b} \) is small enough to allow lateral tunneling, then a 2D array of superlattices interconnected along the \( x \) axis (as in Figs. 4a, 5a) is produced. Furthermore, if the \( n \) density value allows to connect the charge distributions along the \( z \) axis (as in Fig. 4d), then a fully connected 2D arrangement can be assembled. Because the NWs \( y \)-component is mathematically equivalent to the \( x \)-component considered in this work, the 3D distributions are a straightforward generalization of the 2D distributions previously discussed.

In Fig. 6, we present the ground state distributions for a 6 × 6 array of 1-micron long PNWs of cross section \( L_{x,y} = 20 \) nm. The NWs mutual separation \( L_{b} \) is modified from 8 to 3 nm to observe the gradual transition to a strongly interconnected 2D distribution. The distributions are obtained by considering only the \( x \) and \( y \) wave function components. In (a), the \( x-y \) boundary of one of the NWs is outlined as a visual reference.

One striking effect is the related to the \( x-y \) excited states, which involves the combined contribution of all the excited states.
NWs. In Fig. 7 we plot the first three x–y excited states of the 6 × 6 array presented in Fig. 6c (Lb = 3nm). As observed, singular electronic distributions are shaped in the NW cross sections, triggering drastic changes in its electronic population. This kind of remarkable PNW collective phenomena described by our model could be used, in analogy to the approach presented in Reference 49, to deal with more complex problems in two and higher dimensions.

We would like to highlight that such 2D and 3D distributions, together with the possibility to switch between them by means of an external gate, could be of great interest in practical issues as well as in the investigation of new physical phenomena. For example, these NW arrays could be a valuable alternative to usual approaches, such as the 3D arrays of quantum dots to build 3D superlattices40 or to substitute the use of different materials along NWs to produce a charge distribution control along the NWs axis41. Such well separated charge distributions, at the nanometric scale, could also have applications in the design of alternative 3D NW-based logic architectures42.

Theoretical model
We focus on direct wide bandgap semiconductors, so the small interaction between the conduction and valence bands can be neglected. The model can be easily applicable to others wide bandgap compounds by changing the material parameters. As we use a Yukawa approach, a minimal in the electronic concentration must be fulfilled so that the small interaction between the conduction and valence bands can be neglected. The model can be easily applicable to others wide bandgap compounds by changing the parameter \( n \) and \( \kappa \). As for the Al concentration \( r \), the Al concentration in the calculation.

The time-independent Schrödinger equation for a spinless two-electron wave function, \( \Psi_{1,2}(x,y,z) \equiv \Psi_{1,2}(x_1,y_1,z_1, x_2,y_2,z_2) \), is

\[
\left[ -\frac{\hbar^2}{2m_e^*} \nabla^2_{\perp} + V_{\text{eff}}(r) \right] \Psi_{1,2}(x,y,z) = E \Psi_{1,2}(x,y,z),
\]

(1)

where \( \hbar = \hbar/2\pi \), \( \hbar \) the Planck constant, \( m_e^* \) the electron effective mass, and \( V_{\text{eff}} \) is the effective potential. \( V_{\text{eff}} \), which is derived further below, includes the finite confinement potential in the transversal x-y plane, the infinite barrier potential at the NW z-edges, and the many-body e-e Yukawa-like interaction.

The Yukawa-like potential is given by

\[
V_Y(r) = \frac{e^2}{4\pi \varepsilon} \frac{e^{-\kappa r}}{r},
\]

(2)

where \( r = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \) is the e-e separation; \( \varepsilon = \varepsilon_0\varepsilon_r \) is the absolute permittivity, with \( \varepsilon_0 \) the vacuum permittivity and \( \varepsilon_r \) the relative permittivity of the material (\( \varepsilon_r = 12.9 \) for GaAs and \( \varepsilon_r = 12.247 \) for Al\(_x\)Ga\(_{1-x}\)As for the Al concentration \( x \approx 0.23 \) considered in the calculations). The screening parameter \( \kappa \) is given by \( \sqrt{\frac{2\pi n}{\varepsilon_r K_B T}} \), with \( n \) the electronic density, \( K_B \) the Boltzmann constant, and \( T (= 300 \text{ K}) \) the temperature.

For \( L_x, L_y \leq 55 \text{ nm}, L_b \leq 40 \text{ nm}, \) and \( L_z \approx 1 \mu\text{m}, V_Y(r) \) can be considered as a small perturbation in the x and y directions. Then, we can make an approximation in the left side of Eq. (1) by splitting the transversal (\( \perp \)) and parallel (\( \parallel \)) contributions as

\[
H_{\perp} = -\frac{\hbar^2}{2m_e^*} \left( \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial y_1^2} + \frac{\partial^2}{\partial z_1^2} \right) + V_{x,y}(x,y) \]

(3)

\[
H_{\parallel} = -\frac{\hbar^2}{2m_e^*} \left( \frac{\partial^2}{\partial z_2^2} \right) + \frac{e^2}{4\pi \varepsilon} \frac{e^{-\kappa r}}{r} \]

(4)
Considering that the wave function is separable in its transversal and longitudinal components as
\( \Psi_{1,2}(r) = \Psi_\perp(x_1, y_1) \Psi_\parallel(z_1, z_2) \). Then, the \( y \) component of the wave function for each of the two confined electrons, for an \( 1 \times m \) PNW array, can be directly calculated from Eqs. (1) and (3) as
\[
\psi_y(y) = N_y \cos(k_y y)
\]  
(5)

where
\[ k_y = \frac{2\pi}{\sqrt{m^*_{nw}}} E_y, \quad m^*_{nw} = 0.0665m_e \]

for GaAs (\( m_e \) the electron mass), and \( N_y \) is a normalization constant.

Along the \( x \)-direction we must solve a system of \( m \)-coupled quantum wells, composed of the wave functions in the AlGaAs barriers (\( \psi_b(x) \)) and the GaAs NW (\( \psi_{nw}(x) \)):
\[
\psi_b(x) = A_i e^{d_i x} + B_i e^{-d_i x}
\]  
(6)

\[
\psi_{nw}(x) = C_j \cos(d_j x) + D_j \sin(d_j x)
\]  
(7)

in which \( i = 1, 2, \ldots, (m+1) \), \( j = 1, 2, \ldots, m \), \( d_i = \sqrt{\frac{2m^*}{\hbar^2}} (V_0 - E_x) \), \( d_j = \sqrt{\frac{2m^*}{\hbar^2}} E_x \), \( V_0 = 187 \) meV, and \( m^* = 0.0857m_e \) for Al\(_{0.23}\)Ga\(_{0.77}\)As, \( A_i, B_i, C_j \) and \( D_j \) are additional normalization constants.

In order to derive the effective potential, we consider the Fourier transform for each of the cross-sectional wave functions for a single confined electron, in an \( 1 \times m \) PNW array, given by
\[
\psi_b^2(x) = \frac{1}{2\pi} \int dq_x G_b(q_x) e^{-iq_xx}
\]  
(8)

\[
\psi_{nw}^2(x) = \frac{1}{2\pi} \int dq_x G_{nw}(q_x) e^{-iq_xx}
\]  
(9)

\[
\psi_y^2(y) = \frac{1}{2\pi} \int dq_y G_y(q_y) e^{-iq_yy}
\]  
(10)

where,
\[
G_b(q_x) = \sqrt{\frac{2\pi}{3}} \left( \frac{A_i}{B_i} \delta(q_x - 2ik_b) + \frac{B_i}{A_i} \delta(q_x + 2ik_b) \right) + 2\sqrt{\frac{2\pi}{3}} \delta(q_x)
\]  

(11)

\[
G_{nw}(q_x) = \sqrt{\frac{2\pi}{3}} \left[ \frac{C^2 + D^2}{C^2 D_J} \delta(q_x) + \sqrt{\frac{2\pi}{3}} \left[ \frac{C^2 - D^2}{C^2 D_J} \right] \left[ \delta(q_x - 2k_{nw}) + \delta(q_x + 2k_{nw}) \right] \right]
\]  

(12)

\[ G_y(q_y) = \sqrt{\frac{2\pi}{3}} N_y \left[ \delta(q_y) + \frac{\delta(q_y - 2k_y)}{2} + \frac{\delta(q_y + 2k_y)}{2} \right]. \]

(13)

For the longitudinal component, the corresponding Fourier transform is
\[
\psi_z^2(z) = \frac{1}{(2\pi)^2} \int dq_z G_z(q_z) e^{-iq_z z}
\]  
(14)

Then, by replacing Eqs. (8–10) and (14) into the e–e interaction energy, in its integral representation:
\[
W = \int d^3r_1 d^3r_2 \frac{e^2}{4\pi \epsilon} \frac{e^{-r}}{r} |\Psi_{1,2}(r)|^2,
\]  

(15)

where
\[
e^{-r} = \frac{1}{(2\pi)^2} \int d^3q \frac{e^{-i(q_x x + q_y y + q_z z)}}{k^2 + q_x^2 + q_y^2 + q_z^2}.
\]  

(16)

We can now consider the contribution of both electrons and, defining \( G(q_x, q_y) = G_{nw/b}(q_x)G_y(q_y) \), we obtain
\[
W = \frac{e^2}{4\pi \epsilon_{nw/b} 2\pi^2} \int d^3r \int d^3q \frac{G^2(q_x, q_y, G_y(q_y))}{k^2 + q_x^2 + q_y^2 + q_z^2} \times e^{-i(q_x x + q_y y + q_z z)}
\]  

(17)

where \( d^3q = dq_x dq_y dq_z \) and \( d^3r = dx dy dz \); \( \epsilon_{nw/b} \) corresponds to the permittivity that is replaced for each corresponding region (NW or the barrier) and \( G_{nw/b} \) is defined analogous to the permittivity. Integrating over \( r \),
\[ W = \frac{e^2}{4\pi \varepsilon_{nw}/b} \int d^3q \frac{G^2(q_x, q_y) G_z(q_z)}{\kappa^2 + q_x^2 + q_y^2 + q_z^2} \]  

(18)

Now, as

\[ G_z(q_z) = \int dz \psi_z^2(z) e^{\pm i(q_z z)}, \]  

(19)

Equation (18) can be expressed as

\[ W = \frac{e^2}{4\pi \varepsilon_{nw}/b} \int dxdy dz G^2(q_x, q_y) \psi_z^2(z) e^{\pm i q_z z} \]  

(20)

Then, by using the equality

\[ \int dq_z e^{iq_z z} dq_z = \pi e^{-i|q_z|^2 + z^2} \]  

(21)

Equation (20) can be rewritten as

\[ W = \frac{1}{2\pi} \int dz \psi_z^2(z) V_{\text{eff}}(z), \]  

(22)

where \( V_{\text{eff}} \) is defined by

\[ V_{\text{eff}}(z) = \frac{e^2}{4\pi \varepsilon_{nw}/b} \int dq_x dq_y G^2(q_x, q_y) e^{-i|q_z|^2 + q_z^2 + \kappa^2} \]  

(23)

By replacing Eqs. (11–13) in (23), we can obtain the final expression for \( V_{\text{eff}} \), which must be evaluated in each part of the GaAs/AlGaAs NWs along the \( z \)-direction. Then, for the extreme left and right AlGaAs barriers,

\[ V_{\text{eff}} = \frac{[N_y \pi \varepsilon]^2}{8\pi \varepsilon_b} \left[ A_i + B_i \right] \left[ e^{-i|q_z|^2 + q_z^2 + \kappa^2} \right] \left[ \frac{4k^2_B + 4k^2_y + \kappa^2}{\sqrt{4k^2_B + 4k^2_y + \kappa^2}} \right], \]  

(24)

at the internal barriers,

\[ V_{\text{eff}} = \frac{[N_y \pi \varepsilon]^2}{8\pi \varepsilon_b} \left[ A_i^4 + B_i^4 \right] \left[ e^{-i|q_z|^2 + q_z^2 + \kappa^2} \right] \left[ \frac{4k^2_B + 4k^2_y + \kappa^2}{\sqrt{4k^2_B + 4k^2_y + \kappa^2}} \right], \]  

(25)

and, in the GaAs NWs,

\[ V_{\text{eff}} = \frac{[N_y \pi \varepsilon]^2}{64\pi \varepsilon_{nw}} \left[ C_j^2 + D_j^2 \right] \left[ e^{-i|q_z|^2 + q_z^2 + \kappa^2} \right] \left[ \frac{4k^2_B + 4k^2_y + \kappa^2}{\sqrt{4k^2_B + 4k^2_y + \kappa^2}} \right], \]  

(26)

To solve the Schrödinger equation by using Eqs. (24), (25), and (26), we use the finite difference method.

On the other hand, for a \( m_1 \times m_2 \) array of PNWs we can follow a similar procedure but, instead of Eq. (5) for the \( y \) component, we must consider Equations of the type (6) and (7). Following the procedure described before, the final effective potentials for these arrays are, at the external barriers,

\[ V_{\text{eff}} = \frac{[2\pi \varepsilon]^2}{4\pi \varepsilon_b} \left[ A_i + B_i \right] \times \left[ O_i^2 + P_i^2 \right] \left[ e^{-i|q_z|^2 + q_z^2 + \kappa^2} \right] \left[ \frac{4k^2_B + 4k^2_y + \kappa^2}{\sqrt{4k^2_B + 4k^2_y + \kappa^2}} \right], \]  

(27)

at the internal barriers,

\[ V_{\text{eff}} = \frac{[2\pi \varepsilon]^2}{8\pi \varepsilon_b} \left[ A_i + B_i \right] \times \left[ O_i^4 + P_i^4 \right] \left[ e^{-i|q_z|^2 + q_z^2 + \kappa^2} \right] \left[ \frac{4k^2_B + 4k^2_y + \kappa^2}{\sqrt{4k^2_B + 4k^2_y + \kappa^2}} \right], \]  

(28)

and, in the GaAs NWs,
where $O_\alpha$, $P_\alpha$, $Q_i$, and $R_j$ are the additional normalization constants related to the $y$ component.

### Conclusion

We present theoretical calculations of two-electron states under the influence of a variable electron screening in semiconductor parallel NWs, considering the effect of the system size, the $n$-doping level and the NWs separation. When a low-density Wigner crystal regime is considered, localized effects in the tunneling between adjacent quantum wires are observed. By modifying NWs parameters such as the cross-section, the $n$ concentration and the NWs separation, the charge distribution pattern in 2D and 3D PWN arrays can form interconnected distributions between the adjacent NWs. Such nanoscale localized charge distribution could be valuable in the design of new architectures for photonics and electronics applications.

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Author contributions
R.M.C. developed the theoretical model and did the calculations. E.C.H. wrote the main manuscript text and prepared figures. All authors reviewed the manuscript.

Competing interests
The authors declare no competing interests.

Additional information
Correspondence and requests for materials should be addressed to C.-H.

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