Compliance Control and Analysis for Equivalent Hydraulic Legs

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Abstract In this article, the compliant behavior of an equivalent hydraulic leg is achieved by combining active compliance control of hydraulic cylinder with a passive spring at its end effector. Firstly, a position-based active compliance control is proposed based on impedance control to reduce the contact impact. Secondly, the stability of proposed compliance controller and the range of impedances (Z-width) of compliance control are analyzed, and the design procedure of proposed compliance controller is given out. Finally, experimental results show that the contact impacts could be well handled by the proposed control method. This research provides an insight for the compliance control of hydraulic legged robots.

Index Terms Compliance control, active compliance, passive compliance, stability analysis, Z-width.

I. INTRODUCTION

Nowadays, legged robots have been a research hotspot and gained various attention, especial for biped robots and quadruped robots. As for biped robots, there exist many famous ones, such as Atlas [1], Cassie [2], Valkyrie [3], ASIMO [4], KHR [5], LOLA [6], and Walker [7] etc. As for quadruped robots, many famous ones are Bigdog [8], HyQ [9], HyQ2MAX [10], StarlETH [11], Anymal [12], MIT Cheetah 1~3 [13]~[15], Cheetah-cub [16], minitur [17] and AlienGo [18] etc. In the DARPA Robotics Challenge (DRC) in 2015, many legged robots including Atlas are developed to compete on finishing a series of challenging tasks for their relevance to disaster response [19]. The legged robots were required to drive cars, open doors, rotate valves, uses tools walk on sands, stairs and uneven terrains etc.

Generally, friendly dynamic interactions are the foundation for the above tasks. Dynamic interactions of robots with environments, humans and other robots are usually implemented through position and force. As thus, legged robots can be seen as Coexisting- Cooperative-Cognitive Robots (Tri-Co Robots) [20]. Tri-Co Robots are popular research in current years and will be one of future development trends. Tri-Co Robots are kinds of robots that can interact with environments, humans and other robots naturally/friendly, adapt to complex dynamic environments autonomously, and accomplish assignments cooperatively.

The dynamic interactions of Tri-Co Robots (legged robots) with environments, humans and other robots can be established as FIGURE 1. The dynamic interaction interfaces are commonly position \( q \) and force \( F \), where \( q \) is a position vector including displacements/angles and attitude message; \( F \) is a force vector including forces and torques message. \( q \) and \( F \) can be obtained by sensors, environmental perception system or estimating. However, it is a huge challenge to handle the dynamic interaction of robots with outside subjects including environments, humans and other robots. Then the diagram of dynamic interactions of Tri-Co Robots based on position control can be illustrated in FIGURE 2, where \( q_f, q, q_e \in \mathbb{R}^n \) donates the desired displacement, actual displacement, required displacement, and the displacement caused by the environment, respectively. The dynamic interaction model is utilized to shape the relationship between force \( F \) and corresponding nominal position modifications or output of target admittance \( \Delta q_f \in \mathbb{R}^n \). And there exist an inner position loop and an outer dynamic interaction loop.

Compliance control is an effective way to handle the dynamic interactions [21], [22]. As shown in FIGURE 3,
Compliance control contains passive compliance and active compliance. The passive compliance contains non-adaptive and adaptive counterparts. The passive non-adaptive compliance like a spring is a simple way to achieve the compliant behavior of robots, but passive compliance has many disadvantages: 1) it cannot handle variable environments/cases simultaneously since the passive part cannot be adjusted flexibly (fixed stiffness) even if the passive compliance is adaptive [23]; 2) extra assembly parts are needed and extra mass is added; 3) only a spring cannot dissipate impact energy due to its less damping. Even though there are mentioned disadvantages in the passive compliance, passive compliance has the most important advantage: it can reduce the first contact impact in time while the active compliance cannot because of the limit of response frequency of active actuator. In addition, passive compliance is added to make this research complete. On the contrary, the compliant performance of active compliance can be determined by the active control of actuator, such as cylinder or motor, in real-time [24]–[26]. The active compliance contains force control and impedance control [27]–[29]. The force control is also called admittance control, which contains unified position/force control [30] and hybrid position/force control [31]. The impedance control contains force-based [32], [33] and position-based [34], [35]. In this article, a position-based impedance control is proposed to tackle the active compliance control [29], [36]. Compared with force control and force-based impedance control, the benefits of position-based impedance control are: 1) only Jacobian matrix and kinematics are required, and accurate dynamics and high performance force control are not necessary; 2) easier to implement in real applications, even though the former two can have a higher flexibility on designing the contact impedance [37]. And two disadvantages of position-based impedance control are: 1) the approach cannot control the desired contact force even if the contact force can be reduced by the approach effectively; 2) the approach does not consider the impedance of environment. However, as for legged robots, advantages outweigh disadvantages. Furthermore, without passive stiffness, force-based impedance control will behave similar control performance with position-based impedance control. Actually, these two methods have the same problem. That’s the response frequency of active actuator (not faster than the first contact impact). In the force-based impedance control, the force track control is still achieved by the change of displacement of active actuator. As thus, both two method cannot handle the first contact impact well.

In the realm of legged locomotion, being compliant to external unexpected impacts is crucial when negotiating unstructured environments [38]. To be more friendly with the environment, robots should be set to be task-based and flexible in stiffness and damping because of the changing loads, environments, and tasks. Robots are expected to be employed for a variety of tasks involving interaction with dynamic environments, like material transportation, geographic expedition and disaster rescue etc [39]. In these cases, the environmental terrains are normally unstructured or unknown and robots should confront with impacts and crashes when negotiating the contact. Traditional position servo control is with high gain (stiffness) and usually in kinematic control, which makes the robots rigid and increases the contact force. A good solution to deal with the impact is making the end-effector of robots work like a spring-damper [40], while the stiffness and damping should be variable with respect to time, terrains and tasks [41], [42]. Unfortunately, how to choose the suitable impedance parameters to keep the system stable is seldom discussed [25], [43].

Quadruped robots have the advantages of fast motion speed, high flexibility, strong terrain adaptability and high stability, which results in their great application prospects in material transportation, engineering exploration, rescue and military investigation. Therefore, the research of quadruped robot has important theoretical significance and practical value. Actually, a legged robot can be taken as a hopper or bouncing system [44], [45], which is similar to a mass-spring-damper system [46]. It is complicated to analyze the impedance performance of legged robot directly, but it is simple to discuss that in a hopper [36]. Besides,
A compliance controller combining position-based active compliance control with a passive spring is employed on an equivalent hydraulic leg to achieve the compliant behavior.

- The stability of proposed compliance controller and the range of impedances (Z-width) of compliance control are analyzed, and the design procedure of proposed compliance controller is given out as well.
- The proposed compliance control and analysis are validated by comparative experiments, which provides an insight for the compliance control of hydraulic legged robots.

The paper is organized as follows. In Section 2, the hardware platform and model of an equivalent hydraulic leg are given out. In Section 3, the control scheme of position-based active compliance control is proposed. In Section 4, stability analysis, choice of compliance parameters and design procedure are addressed. In Section 5, the proposed method is validated by experiments. In Section 6, conclusions are drawn and future works are issued.

**II. PLATFORM AND MODEL**

The research platform of an equivalent hydraulic leg is shown in FIGURE 4 and its system parameters is shown in TABLE 1. Noting that the equivalent hydraulic leg is constrained to move in the vertical direction. The position controllable cylinder and built-in passive spring are employed to achieve the active and passive compliance, respectively.

The equivalent system and model of hydraulic leg in FIGURE 4 are shown in FIGURE 5(a) and 5(b), respectively. Especially, $K_c$, $D_c$ are the actual stiffness and damping of position controllable cylinder, respectively.

**III. CONTROLLER DESIGN**

Combining FIGURE 2 and 4, the position-based active compliance control of equivalent hydraulic leg can be drawn in FIGURE 6(a).

The dynamic operator that determines an input force ($F^*$) from an output velocity ($\dot{x}$) of the end-effector at the interaction port is defined as the mechanical driving-point admittance ($Y$), which is defined as the inverse of the impedance ($Z$) as well and can be written as

$$\dot{x} = Y(s)F^* = Z^{-1}(s)F^*$$  (1)

For simplicity, most of following analysis is based on a linearized diagram as shown in FIGURE 6(b), where $x_i$, $\dot{x}$, $x_f$, $x_e \in \mathbb{R}^n$ donates the desired displacement, actual displacement, required displacement, and the displacement.
caused by the environment, respectively; \( G_c(s) \) donates the position controller of hydraulic cylinder, which can be non-linear in FIGURE 6(a), such as ARC in [49]; \( Z_f(s) \) donates the controlled system model; \( Z_j(s) \) donates the impedance model of contact environment; \( Z_f(s) \) donates the desired impedance model or target admittance model, which is utilized to shape the relationship between contact force \( F^* \) and corresponding nominal position modifications or output of target admittance \( \Delta x_j \in \mathbb{R}^n \).

Generally, a second-order linear system (spring-damping-inertia system) [24] is adopted as a desired impedance \( Z_f(s) \), which can be written as

\[
Z_f(s) = \frac{1}{M s^2 + D s + K}
\]  

(2)

IV. ANALYSIS
A. STABILITY ANALYSIS
For a legged robot, motion stability is one of the most important problem that should be issued. To handle the dynamic reaction with environment friendly is similar to keep the motion control steady, especially the compliance control in FIGURE 6(b). For the system in FIGURE 5(b), the compliance consists of active compliance of hydraulic cylinder and passive compliance of spring. Due to the passive spring is a dissipative part, the stability of the whole system is based on the stability of active compliance control.

Considering (1), the necessary and sufficient conditions for passivity of a linear time invariant multi-port system are as follows [25]. That is, \( Y(s) \) is passive if and only if:

- **C1**: \( Y(s) \) has no pole in right-half plane \( \Re(s) \geq 0 \);
- **C2**: \( Y(s) + Y^*(s) \) is positive semi-definite in \( \Re(s) > 0 \), where \( Y^*(s) \) is the conjugate transpose of \( Y(s) \).

When \( Y(s) \) has no poles in \( \Re(s) \geq 0 \), then C2 can be simplified to:

**C2’**: The matrix \( Y(j\omega) + Y^*(j\omega) \) is positive semi-definite for all real \( \omega \).

Considering the control scheme in FIGURE 6(b), the closed-loop position behavior of system under compliance control during contacting with the environment can be expressed as:

\[
x = G_p(s)x_d - G_p(s)Z_f(s)F^* - E_p(s)G_s(s)F^* \tag{3}
\]

where \( G_p(s) \) is the transfer function of inner closed loop and \( E_p(s) \) is the transfer function of position error. They are respectively defined as

\[
G_p(s) = \frac{x(s)}{x_d(s)} = \frac{1}{[1 + G_c(s)G_e(s)]^{-1}G_c(s)G_e(s)} \tag{4}
\]

\[
E_p(s) = \frac{x_d(s) - x(s)}{x_r(s)} = 1 - G_p(s) = [1 + G_c(s)G_e(s)]^{-1} \tag{5}
\]

Define the position tracking error of system is defined as \( e = x_d - x \). Combining (3), (4) and (5) yields the relationship between contact impact \( F^* \) and the position tracking error of system as follow

\[
e = E_p(s)x_d + E_p(s)G_s(s)F^* + G_p(s)Z_f(s)F^* \tag{6}
\]

Actually, the former two items: \( E_p(s)x_d \) and \( E_p(s)G_s(s)F^* \), are determined by the bandwidth of hydraulic valve and the gain of position loop, respectively. From (6), the position tracking error of system is mainly affected by the impedance model \( Z_f(s) \), which changes the sensitivity of system with respect to \( F^* \). Combining FIGURE 6(b) and under a given closed-loop position bandwidth, lower position loop gains or higher valve bandwidths will produce larger stable ranges of virtual impedances. In other words, the Z-width, which is the range of achievable virtual impedances, can be enlarged by these two ways. The results are similar to [25]. Note that Z-width defines the combinations of stiffness and damping that can be passively rendered by a certain mechanism.

For a mechanical system with position servo control, its closed-loop system characteristic behaves with a very high stiffness (\( G_p(s) \approx 1 \)), which means the disturbance term of stiffness \( E_p(s)G_s(s) \approx 0 \) and can be neglected. Note that a high accurate disturbance rejection control like [49] can be adopted in the hydraulic position controller to guarantee the influence of disturbance is less. Then, the \( G_p(s)Z_f(s)F^* \) term, caused by the introduce of active compliance control, makes the position tracking error larger. Therefore, the intrinsic of active compliance control is sacrificing the position tracking error to reduce the contact impact. Furthermore, if the inner loop is an ideal stiffness servo (\( G_p(s) \approx 1 \), \( E_p(s) \approx 0 \)) with an enough high bandwidth, then equation (6) can be rewritten as

\[
e = Z_f(s)F^* \tag{7}
\]

which implies that the response of system on \( F^* \) equals to the response of \( F^* \) on the given impedance model \( Z_f(s) \).
Thus, based on the equivalent model in Figure 5(b) and (2), error of hydraulic cylinder is
\[ e = E_p(s)x_d + G_p(s)Z_f(s)F^* \]
which means there always exists an impedance error between the actual \( K_e \) and desired \( K \). In this article, the most important goal is to get a good and stable response to the contact impact rather than an accurate impedance model. Thus, the active compliance control in Figure 6(b) is satisfied for the requirement. Meanwhile, only an out-loop impedance model is added into a traditional position servo system without any effect on the inner-loop position servo control structure, as shown in Figure 6(a). It is easier to realize based on a position servo system than a force control system.

Specially, for the equivalent hydraulic leg in Figure 5(b) and the active compliance control in Figure 6(b), we have \( x \approx x_d \) or \( x \approx x_d - \Delta x_f \). Then the actual position tracking error of hydraulic cylinder is \( \Delta L_c = e = x_d - x \approx \Delta x_f \). Thus, based on the equivalent model in Figure 5(b) and (2), the actual impedance response of system can be written as
\[ F_c - F_e = K_c \Delta L_c + D_c \Delta L_c \]

Fortunately, \( x_d \) can be set to be fixed in most cases of environmental contacts and \( E_p(s)x_d \) in (8) is usually small and can be omitted. Hence, the actual impedance of hydraulic cylinder can be expressed as
\[ Z_c(s) = \frac{F^*}{e} = \frac{1}{G_p(s)Z_f(s)} \]

In the above analysis, the impedance model of contact environment \( Z_e(s) \) in Figure 6(b) is not considered, which is mainly determined by the passive spring attached on the robot’s end effector. Actually, the passive spring plays an important role in the system stability and dynamic locomotion. Define the spring compression as \( p_e \) or \( p \), and define the total compression of the whole system as \( \mathbf{p} \), we have
\[ e = x_d - x = \mathbf{p} - p_e \]

\[ p_e = \frac{F^*}{Z_c(s)} \]

Using (10), (11) and (12) yields
\[ p_e = \frac{\mathbf{p}}{1 + Z_c^{-1}(s)Z_e(s)} \]

Define the contact impedance of the whole system as
\[ Z(s) = \frac{F^*}{\mathbf{p}} \]

Combining (12), (13) and (14) yields
\[ Z(s) = \frac{Z_c(s)Z_e(s)}{Z_c(s) + Z_e(s)} \]

The above equation (15) is similar to the stiffness calculation of a cascade spring system [39], which can be used to design the required compliance behavior and performance during contacting with rigid terrains. Even though the equivalent model of hydraulic leg in Figure 5(b) is a passive system, it is intrinsically an active system since it employs the energy to simulate/imitate a spring-damper system. Thus, the active component and the passive spring are cascaded into a coupled system and the coupled impedance is given out in (15).

In order to analyze the stability of coupled system, the diagram of coupled system can be drawn in Figure 6(c) based on (10), (11) and (12). Due to the environmental impedance model \( Z_e(s) \) and desired impedance model \( Z_f(s) \) are both stable, if the closed-loop \( G_p(s) \) of position control is also stable, then the stability condition of coupled system [51] can be written as
\[ \|Z_c(j\omega)Z_f(j\omega)G_p(j\omega)\| < 1 \]

Generally, the coupled system is stable when the active system is stable. Therefore, the stability of coupled system will be ensured if the transfer function
\[ [1 + Z_c^{-1}(s)Z_e(s)]^{-1} \]

is stable. This condition is obtained from (13)(15). More concrete stability analysis was performed by Vukobratovic et al in [39] and they also pointed out that the active stiffness of active system should be limited by the magnitude of environmental stiffness and the position control sensitivity. For a SISO (Single Input Single Output) system [39] or the equivalent hydraulic leg, this imposes the condition
\[ K \geq \min(K_p, K_s) \]

where \( K \) and \( K_p \) donate the desired and actual stiffness of position control, respectively; \( K_s \) donates the stiffness of passive spring, which could be the environmental stiffness.

Furthermore, compared with the stiffness of passive spring \( K_s \), the stiffness of position control \( K_p \) is higher for the controlled system. Then, the sufficient conditions of stability of the compliance controller can be set as follows:

**Condition 1.** The inner position control system \( G_p(s) \) should be stable.

**Condition 2.** The desired stiffness \( K \) of active system should be larger than the stiffness of passive system \( K_s \).

The stability conditions lay a fundamental basis for the compliance controller design. **Condition 1** is easy to satisfy for a mechanical system, and **Condition 2** is loose as well.

**B. Z-WIDTH ANALYSIS**

In robotic locomotion, the contact with environment should become stable as soon as possible, which makes sense for the stability and mobility of the whole system. Hence, how long the system will come into a stable state without oscillations or the setting time is another important index to comment the performance of compliance control apart from the stability. Choosing proper compliance control parameters is one effective way to make the system stable quickly.

Define the compression of active and passive compliance in Figure 5(b) as \( \Delta L_c \) and \( \Delta L_s \), respectively. Then,
\[ p = \Delta L_c + \Delta L_s \] and the dynamics of the whole system in FIGURE 5(b) can be expressed as

\begin{align*}
Mg - K_c \Delta L_c - D_c \Delta \dot{L}_c &= M(\Delta \ddot{L}_c + \Delta \dot{L}_c) \\
K_s \Delta L_s - K_c \Delta L_c - D_c \Delta \dot{L}_c &= 0
\end{align*}
(19)

Let the lumped mass as the system input \( u = Mg \), then the Laplace transformation of (19) can be written as

\[ p(s) = \frac{K_s + K_c + D_c s}{M D_c s^3 + M (K_c + K_s) s^2 + K_s D_c s + K_c K_s} \cdot u(s) \] (20)

With initial conditions \( \Delta p|_{t=0} = 0, \Delta \dot{p}|_{t=0} = v_0 \), \( \Delta \ddot{p}|_{t=0} = g \), where \( v_0 \) is the initial velocity when the system touches the ground. Obviously, the equivalent system is globally stable for it is a completely passive system with dissipation component \( D_c \). Also, the dynamic response consists of two parts: the zero state response under a step input and zero input response under initial contact velocity \( v_0 \).

The system in (20) is a three-order system, and its analytic solution of setting time is hard to obtain. However, the response speed of system can be analyzed by getting the dominant poles, which is the main factor to determine the response speed. And the faster response speed, the smaller setting time. Besides, the stiffness and damping in (2) cannot be set directly. The actual stiffness of hydraulic cylinder \( K_c \) is close and proportional to the desired \( K_c \), as discussed in (9). Thus, \( K_c \) can be a guide to design \( K_c \). Similarly, the actual damping of hydraulic cylinder \( D_c \) and the desired \( D \) are with the same relationship. In most cases, if the damping is not specified, it mainly depends on the frictional damping and throttle damping in the hydraulic cylinder system. According to the experiments (FIGURE 9(d)) in Section 5, a rough calculation result of \( D_c \) can be obtained: \( D_c \approx 1Ns/mm \). Therefore, only the stiffness \( K_c \) and \( K_s \) are considered in compliance control design based on the transfer function (20).

The zero-pole plot for different \( K_c \) and \( K_s \) are illustrated in FIGURE 7(a)&7(b).

From FIGURE 7(a)&7(b), there always exists a real pole and a pair of conjugate complex poles in the equivalent system. And, the zero of system is close to the real pole and far away from the conjugate complex poles. As a consequence, the dynamic response of the system is dominated by the conjugate complex poles, which are usually called the dominant poles. The explicit distribution of real part of dominant poles for different \( K_c \) and \( K_s \) is depicted in FIGURE 7(c).

The purple bold curve in FIGURE 7(c) represents \( K_c = K_s \), which splits the distribution surface into two parts. Considering the stability Condition 2, the proper compliance parameters \( K_c \) and \( K_s \) should lay in the lower right part, in which the real part of dominant poles becomes larger with \( K_c \) increasing and smaller with \( K_s \) increasing, as shown in FIGURE 7(d). The larger (further away from the imaginary axis) real part of dominant poles, the smaller setting time. Thus, a small \( K_s \) and a larger \( K_c \) make sense for reducing setting time.

FIGURE 7. The stability and Z-width analysis.
However, $K_s$ couldn’t too small, which will over compress the spring to its deformation margin. In this case, the lumped load mass of system should be considered as a determinant of $K_s$. Meanwhile, $K_c$ couldn’t too large as well, which will decrease the effect on reducing the contact impact.

To get the proper compliance parameters, some experiments are done firstly, where the free falling height $H = 0.2m$, the lumped load mass $M = 20kg$, the length of foot is $L_f = 120mm$, the initial lengths of hydraulic cylinder and passive spring are $L_{c0} = 130mm$ and $L_{s0} = 100mm$, respectively.

- Passive compliance parameter: $K_s$

  Based on the compression of passive spring and the lumped load mass, the range of stiffness $K_s$ can be obtained [24]. In this range, experiments can be implemented to analyze the influence of different $K_s$ on the compliance performance without active compliance. FIGURE 8(a) shows out the relationships among the stiffness of passive spring, the setting time and the maximum contact impact when $K_s$ changes around $[20, 80]N/mm$. Without active compliance, the setting time will decrease when $K_s$ increases, but the contact impact will increase as $K_s$ increases. Therefore, synthesizing setting time, contact impact and Condition 2, a proper stiffness of passive spring is chosen as $K_s = 40N/mm$.

- Active compliance parameters: $K_c$ and $D_c$

  Under the chosen stiffness of passive spring $K_s = 40N/mm$, the relationships among the stiffness of active compliance $K_c$, the setting time and the maximum contact impact are similar to FIGURE 8(a). Then the stiffness of active compliance $K_c$ mainly depends on the total stiffness $K_t = K_s + K_c$ based on (15). The introduce of stiffness of active compliance $K_c$ makes the total stiffness $K_t$ of system controllable: $K_t \in (0, K_s)$. For example, $K_t$ should be set as $60N/mm$ to get the total stiffness $K_t = 24N/mm$ when $K_s = 40N/mm$. Furthermore, the relationships among the damping $D_c$, the setting time and the stiffness of active compliance $K_c$ are shown in FIGURE 8(b). When the damping $D_c$ is smaller than that when the minimum point of setting time occurs, the setting time is sensitive to the changing of damping. And, the sensitivity will be weaken greatly when the damping $D_c$ is larger than that when the minimum point of setting time occurs. Thus, the damping should be a little larger than that when the minimum point of setting time occurs to avoid a too long setting time caused by a small damping. Actually, the damping could vary around a fixed value we set in real application due to the system lag. From FIGURE 8(b), the smallest setting time occurs when damping $D_c = 1Ns/mm$. Then a good/requisite compliance performance of free falling when damping $D_c = 1Ns/mm$ is shown in FIGURE 8(c). The oscillation time and overshoot are both small.

Noting that the magnitude of the total/desired stiffness of system is usually related to the mass of load. It maybe different from that in robotic legs, but it should be set to determine the proper or self-defined contact stiffness. In this article, $K_s = 40N/mm$, $K_c = 40N/mm$, $K_t = 20N/mm$ with mass of load $M = 20kg$ are used to do the following analysis and validation.

**C. DESIGN PROCEDURE OF COMPLIANCE CONTROLLER**

To sum up, the design procedure of compliance controller can be given as follows:
**Step 1.** Design the structure of compliance controller based on FIGURE 6(a). Noting that the position servo controller is not specified and can be self-defined. Concretely, the range of impedances (Z-width) should be estimated firstly. And then, the maximum and stable closed loop position bandwidth should be given out. If the position gain can not small any more, then a faster actuator is needed.

**Step 2.** Choose the stiffness of passive spring $K_s$ according to the spring length, lumped load mass and experiments in FIGURE 8(a).

**Step 3.** Calculate the stiffness of active compliance $K_c$ based on (15). And, choose a proper damping $D_c$ by referring to FIGURE 8(b).

**Step 4.** Test the whole system and tune the parameters until a good/required compliance performance is obtained.

Noting that, with the stability-guaranteed prerequisite, the compliance characteristics or impedance parameters in different environments all can be tuned and optimized by trails according to the required performances, such as the least contact force or the desired contact force.

**V. EXPERIMENTS**

In this section, comparative experiments between active and passive compliance and comparative experiments of active compliance with different system parameters are both implemented on the research platform to validate the feasibility and efficiency of proposed method.

**A. COMPARATIVE EXPERIMENTS BETWEEN ACTIVE AND PASSIVE COMPLIANCE**

If the contact velocity $\dot{L}$ or the free falling height $H$ is fixed, then using theorem of momentum yields

$$I = (Mg - F_c)t_c = Mv_e - M\sqrt{2gH}$$

(21)

where $v_e$ is the velocity of environment when contact occurs. To reduce the contact impact force $F_c$, the only way is to extend the impact contact time $t_c$ until the system velocity is consistent with environment ($v_e = 0$). Thus, the compliance control in Section 3 is introduced.

FIGURE 9(a) and FIGURE 9(b) show out the contact impacts of system without compliance, with passive compliance only, and with both active and passive compliance when $H = 0.1m$, $M = 20kg$, $K_s = 40N/mm$, $K_c = 40N/mm$ and $D_c = 1Ns/mm$. FIGURE 9(a) tells that the impact contact time $t_c$ is about 10ms, which means the system energy should be released into 10ms. However, the response frequency of hydraulic cylinder system is about 100Hz, which is mainly determined by the response frequency of servo valve (120Hz). Hence, only using the active compliance of hydraulic cylinder is impossible to accomplish the energy release in the short time. And that’s why the passive spring is added. FIGURE 9(b) reveals that the impact contact time $t_c$ increases to about 100ms, which makes enough time for hydraulic cylinder to

**FIGURE 9.** The contact performance of compliance control.
achieve its active compliance. As thus, the contact impact could be reduced efficiently. The contact impacts of system without compliance, with passive compliance only, and with both active and passive compliance are about 3500N, 1000N and 800N, respectively. Moreover, the system with both active and passive compliance could reach the stable state most quickly. That’s, its setting time is the shortest because the intrinsic idea of active compliance of hydraulic cylinder is involving the damping to dissipate the impact energy during the contact process.

FIGURE 9(c) shows the displacement of cylinder without and with compliance. The results confirm that position control is not the first place in FIGURE 6(a) and it should keep a balance with contact impact.

As mentioned before, the idea behind compliance control is to make the system work like a spring-damper. In order to evaluate the compliant performance of the system, the total actual stiffness and damping are illustrated in FIGURE 9(d). The total actual stiffness and damping of hydraulic cylinder along the impact direction are assessed by force balance equation, (15) and FIGURE 6(b). From FIGURE 9(d), easy to find that the stiffness increases rapidly at the first moment and then converges to a relative stable value: about 22N/mm, which closes to the calculated value 20N/mm. And so does the damping, which is around the given value $D_c = 1Ns/mm$. This phenomenon is because the position-based compliance controller is a hysteretic system and its dynamic response is always lagging behind the contact impact. Despite of the weakness, the system still could achieve an appropriate compliant behavior. Note that the calculated stiffness and damping are based on the assumption that they are the same in the neighboring two control cycles. And the assumption makes sense because the stiffness and damping can not change drastically in the neighboring two control cycles in the active compliance, which is achieved by the active control of cylinder.

**B. COMPARATIVE EXPERIMENTS OF ACTIVE COMPLIANCE WITH DIFFERENT SYSTEM PARAMETERS**

To further validate the proposed method, more experiments are implemented with the basis: $H = 0.1m$, $M = 20kg$, $K_s = 40N/mm$, $K_c = 40N/mm$ and $D_c = 1Ns/mm$. FIGURE 10(a), FIGURE 10(b) and FIGURE 10(c) show out the compliance performance of system under different free falling height, lumped load mass and stiffness of passive spring, respectively. It can be found that the contact impact is proportional to the free falling height, lumped load mass and stiffness of passive spring, respectively.

**VI. CONCLUSION**

In this article, a compliance control method for an equivalent hydraulic leg, which combines the active compliance control of hydraulic cylinder and a passive spring, are addressed. The main contributions are concluded as follows:

- The position-based compliance control with a simple structure is proposed for an equivalent hydraulic leg to achieve the compliant behavior with environment.
- In theory, the impedance performance of the whole system is analyzed in detail. And two loose conditions are pushed out to maintain the system stability. To make the system stable quickly, how to choose proper compliance parameters ($Z$-width), including stiffness of passive spring $K_s$, active stiffness $K_c$ and damping $D_c$, involving the damping to dissipate the impact energy during the contact process.
is discussed. The design procedure of compliance control 
method is given out in 4 steps as well.

- The proposed methods are verified on the research 
platforms. Comparative experimental results show that 
the proposed control method could handle the contact 
impact well and the system has an appropriate compli-
ance behavior as required.

Actually, one robotic leg can be equivalent to an equivalent 
hydraulic leg. Thus, the research of this article provides an 
insight for the compliance control for legged robots. Moreover, 
the proposed method can be extended to the application 
on manipulators as well based on the detailed cases. Our 
future works will focus on the dynamic locomotion control 
for legged robots based on compliance control.

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