To the description of the temperature and pressure dependences of the thermal conductivity of sandstone and ceramics

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Abstract. Here we present the results of an experimental study of the temperature and pressure dependences of the heat conductivity of composite compounds. The thermal conductivity of sandstone was measured by the absolute stationary method for pressures up to 400 MPa in the temperature range 273-523 K. From these experimental data we have proposed the equation describing the dependence of the thermal conductivity from the pressure and temperature. We have found that under the action of hydrostatic pressure the intensive growth of the heat-conductivity of gas-saturated sandstone is mainly up to 100 MPa, and then seamlessly switches to saturation. A comparative analysis is carried out with the experimental dependences of the thermal conductivity of ceramics (lanthanum sulfide LaS1.48).

1. Introduction

The regularities of the effective thermal conductivity of dielectrics and rocks under high pressure and temperature conditions provide fairly accurate information about the processes of thermal waves propagation and scattering in complex, block and disordered solids.

The heat transfer in solids with an ordered crystal structure has a wave nature, which is described by the Eucken [1, 2] Debye [3] models. The temperature dependence of the thermal conductivity in this case obeys the law $\lambda \propto T^{-1}$. At the same time, heat transfer processes in solids with an amorphous structure have an activation nature (that is, heat transfer from the atom to the atom takes place), which leads to a temperature dependence of the thermal conductivity as $\lambda \propto T^{0.5}$ [4,5]. As a result, the process of heat transfer in disordered structures (such as rocks and ceramics) is complex.

Analysis of experimental and theoretical studies on the effect of temperature on the thermal conductivity of dielectrics and rocks [5-14, etc.] shows that the temperature dependence of the thermal conductivity of rocks is mainly in a narrow region from $\lambda \propto T^{-0.5}$ to $\lambda \propto T^{+0.5}$ (see Fig. 1). There are examples of such rocks as andesite, obsidian, pyroxene, basalt, etc. [6-8], which heat conductivity does not change with increasing temperature (i.e. $\lambda \propto T^{0}$) in the quite large temperature range 273-
523 K). In general, the temperature dependence of the thermal conductivity of a composite material can be represented as:

$$\lambda(T) = C \cdot T^n,$$

(1)

by the magnitude and sign (±) of the measure of the degree of equality (n), we can estimate the percentage of the mineral with the crystal structure.

![Graph showing the relative dependences of a thermal conductivity.](image)

**Figure 1 (color online).** The relative dependences of a thermal conductivity ($\lambda_T/\lambda_0$) of dielectrics and rocks as a function of temperature ($T$).

2. Experiment

For experimental measurements we have taken (i) natural samples of fine-grained sandstone (Russia, Dagestan Republic, depth 4495-4900 m, open porosity K=7%, density $\rho=2.28$ g/cm$^3$) and (ii) ceramics lanthanum sulfide (LaS$_{1.48}$) samples, which were obtained by the hot pressing method [15] at the Research and technological institute of optical materials all-Russia scientific center "S.I.Vavilov State Optical Institute" (St. Petersburg).

The measurements of the thermal conductivity of a sandstone sample as a function of the hydrostatic pressure up to 400 MPa in the temperature range 273-523 K and ceramics in the temperature range 273-523 K were carried out by an absolute steady-state method, described in detail in [8,9].

3. Results and discussion

The experimental results for determining the temperature-pressure dependence of the thermal conductivity of sandstone are presented in Table 1. We also have detected the absence of hysteresis in the results of measuring the thermal conductivity of sandstone for the increase in pressure up to 400 MPa and then decreasing to 0.1 MPa.

As we can see from this table:

1. The pressure leads to a nonlinear increase in the thermal conductivity of the sandstone, and the most intensive growth occurs in the region up to 100 MPa, and further the growth of thermal conductivity weakens.

2. Pressure affects the temperature dependence of the thermal conductivity of the sandstone, so if $\lambda(T) \propto T^{-0.32}$ for $P=0.1$ MPa, then $\lambda(T) \propto T^{-0.19}$ for $P=400$ MPa.
Table 1. Dependence of the thermal conductivity (W/m·K) of sandstone on pressure and temperature

| Pressure (MPa) | 0.1 | 50 | 100 | 150 | 200 | 250 | 300 | 350 | 400 |
|---------------|-----|----|-----|-----|-----|-----|-----|-----|-----|
| 273           | 2.06| 2.18| 2.2 | 2.24| 2.25| 2.26| 2.27| 2.28| 2.29|
| 323           | 1.91| 2.10| 2.13| 2.16| 2.17| 2.19| 2.20| 2.21| 2.22|
| 373           | 1.83| 2.03| 2.07| 2.10| 2.11| 2.13| 2.14| 2.15| 2.16|
| 423           | 1.76| 1.97| 2.01| 2.04| 2.06| 2.08| 2.09| 2.10| 2.10|
| 473           | 1.71| 1.92| 1.97| 2.00| 2.03| 2.04| 2.05| 2.06| 2.05|
| 523           | 1.66| 1.88| 1.94| 1.96| 1.97| 2.00| 2.01| 2.02| 2.02|
| n             | 0.322| 0.227| 0.198| 0.204| 0.204| 0.188| 0.187| 0.186| 0.192|
| C             | 0.988| 1.058| 1.075| 1.086| 1.092| 1.097| 1.102| 1.107| 1.109|

The dependence of the thermal conductivity of sandstone on pressure and temperature can be described in the form of the following equation:

\[
\lambda(P, T) = \lambda_0 C(P) \left( \frac{T}{T_0} \right)^{-n(P)} .
\]  

(2)

Here \( n(P) \) and \( C(P) \) are dimensionless coefficients depending on the pressure.

Let us denote \( \lambda_{i,j} = \lambda(T_i, P_j) \). According Eqn. (2), the values \( \lambda_{i,j} = \lambda(T_i) \) for fixed pressure satisfy to:

\[
\lambda_{i,j} = C_j T_i^{-n_j} .
\]  

(3)

Here \( C_j \) and \( n_j \) are the pressure-dependent coefficients, which can be found from the experimental data, are as follows. Let us denote

\[
r_i = \frac{\ln(\lambda_i / \lambda_{i-1})}{\ln(T_i / T_{i-1})} , \quad g_i = \frac{\lambda_i / \lambda_0}{(T_i / T_0)^{n_j}} , \quad i=1,2,... .
\]

Then:

\[
n_j = \frac{1}{N} \sum_i r_i , \quad C_j = \frac{1}{N} \sum_i g_i .
\]

Here \( N \) is the sample size.

Consequently, the coefficient of thermal conductivity can be represented in the form:

\[
C(P) = b \cdot \left( \frac{P}{P_0} \right)^a , \quad n(P) = c \cdot \left( \frac{P}{P_0} \right)^d .
\]  

(4)

Here \( a = 0.014, b = 0.988, c = -0.32, d = -0.062 \) are the numerical coefficients obtained according to the data of Table 1, \( \lambda_0 \) the coefficient of thermal conductivity for \( T_0 = 273 \) K and \( P_0 = 0.1 \) MPa.

The obtained values of the coefficients \( C_j \) and \( n_j \) are also given in the Table 1.

Figure 2 shows the example of graphs of the thermal conductivity versus pressure, constructed from the experimental data and according Eqn. (3)-(4) at a fixed temperature (523 K). As can be seen from the figure, the maximum error of approximation is at the point \( P = 50 \) MPa and the maximum relative error at this point is of the order of 3%.
Figure 2 (color online). Experimental data and the calculation of the thermal conductivity of sandstone for $T = 523\, \text{K}$.

Figure 3 (color online). Experimental data and the calculation of the thermal conductivity of ceramics as a function of temperature for $P=0.1\, \text{MPa}$.

If we denote the average total increment of thermal conductivity at a constant temperature by $\Delta\lambda_P$, the average total increment of thermal conductivity at constant pressure by $\Delta\lambda_T$, then equation (4) can be written in the form:

$$C(P) = \left(\frac{P}{P_0}\right)^{0.12\Delta\lambda_P} \cdot \frac{1.54 \cdot \Delta\lambda_T}{\lambda_0}, \quad n(P) = \left(\frac{P}{P_0}\right)^{0.65\ln\left(\frac{\lambda_0}{\lambda_0 + \Delta\lambda_T}\right)}.$$

Values $\Delta\lambda_P$ and $\Delta\lambda_T$ are obtained from experimental data. For sandstone: $\Delta\lambda_P = 0.22$ $\text{W/m} \cdot \text{K}$, $\Delta\lambda_T = 0.443$ $\text{W/m} \cdot \text{K}$, $\lambda_0 = \lambda(T_0, P_0) = 2.06$ $\text{W/m} \cdot \text{K}$ ($P_0 = 0.1\, \text{MPa}$, $T_0 = 273\, \text{K}$).
Figure 3 shows the experimental values of thermal conductivity obtained at a pressure \( P_0 = 0.1 \) MPa in comparison with the calculated data, made by the Eqn. (2) with factors \( n(P) \) and \( C(P) \) given by Eqn. (5). The total increment of thermal conductivity at constant pressure was chosen to be the same (\( \Delta \lambda_T = 0.443 \)) as for sandstone.

Setting \( \lambda_0 = \lambda(T_0, P_0) = 1.44 \) in the Eqn. (5), we can construct a table for the thermal conductivity of LaS\(_{1.48}\) ceramics as a function of pressure and temperature. These data are presented in the table 2.

**Table 2.** The thermal conductivity (W/m·K) of ceramics, obtained from experimental \( \lambda_0 = \lambda(T_0, P_0) = 1.44 \) W/m·K \( (T_0=273 \) K, \( P_0=0.1 \) MPa) according to equation (5).

| \( T \) (K) | \( 0.1 \) | \( 50 \) | \( 100 \) | \( 150 \) | \( 200 \) | \( 250 \) | \( 300 \) | \( 350 \) | \( 400 \) |
|---|---|---|---|---|---|---|---|---|---|
| 273 | 1.44 | 1.61 | 1.63 | 1.64 | 1.65 | 1.66 | 1.67 | 1.67 | 1.68 |
| 323 | 1.33 | 1.54 | 1.56 | 1.58 | 1.59 | 1.60 | 1.61 | 1.61 | 1.62 |
| 373 | 1.24 | 1.48 | 1.51 | 1.53 | 1.54 | 1.55 | 1.55 | 1.56 | 1.57 |
| 423 | 1.17 | 1.44 | 1.47 | 1.48 | 1.49 | 1.50 | 1.51 | 1.51 | 1.52 |
| 473 | 1.11 | 1.39 | 1.42 | 1.44 | 1.46 | 1.47 | 1.48 | 1.48 | 1.49 |
| 523 | 1.06 | 1.36 | 1.39 | 1.41 | 1.42 | 1.43 | 1.44 | 1.45 | 1.45 |

4. Conclusion

Thus, we proposed the analytical expressions (2)-(5), which describes the results of our experimental data on the effect of hydrostatic pressure up to 400 MPa in the temperature range 273-523 K on the thermal conductivity of sandstone samples. We have established that at a pressure of 0.1 MPa this expression brilliantly describes the temperature dependence of lanthanum sulfide LaS\(_{1.48}\) ceramics.

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