Testing axion physics in a Josephson junction environment

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We suggest that experiments based on Josephson junctions, SQUIDS, and coupled Josephson qubits may be used to construct a resonant environment for dark matter axions. We propose experimental setups in which axionic interaction strengths in a Josephson junction environment can be tested, similar in nature to recent experiments that test for quantum entanglement of two coupled Josephson qubits. We point out that the parameter values relevant for early-universe axion cosmology are accessible with present day’s achievements in nanotechnology. We work out how typical dark matter and dark energy signals would look like in a novel detector that exploits this effect.

I. INTRODUCTION

Research in nanotechnology is currently at a very advanced stage, and so is research in cosmology. The two scientific fields are proceeding independently of each other, and the two scientific communities don’t know each other — dealing apparently with very different subject areas. But are these two research areas really that far apart? At first glimpse, certainly they are. But a look at the equations of motions of Josephson junctions, SQUIDS (superconducting quantum interference devices), coupled Josephson qubits and similar superconducting devices used in nanotechnology on the one hand and axionic dark matter on the other indicates that it makes sense to think about common approaches in both areas. The equations of motion are very much the same (with a suitable re-interpretation of the symbols used) and hence it makes sense to translate known results from one area (nanotechnology) into possible results and phenomena for the other area (axion cosmology). There are many different candidates for dark matter in the universe, with extensive experimental searches, WIMPS (weakly interacting massive particles) and axions being the most popular ones. In contrast to WIMPS, axions are very light particles that behave similar to a cold quantum gas. Axions have been around as models for dark matter for quite a while, and there are several experiments that search for them directly in the laboratory, using e.g. cavities and strong magnetic fields, which trigger the decay of axions into two microwave photons. These microwave photons are in principle detectable if the cavity resonates with the axion mass (see e.g. for a formula for the expected power generated by axion-photon conversions). Searches have been unsuccessful so far, but it is necessary to scan a huge spectrum of cavity frequencies, for which SQUID amplifiers with very low noise levels are a very useful technological tool to reduce the noise level and to improve the scanning efficiency. Quasi-axionic particles play also an important role in topological insulators, new materials with exotic metallic states on their surfaces. These recent developments illustrate that it does make sense to look at axion physics in a much broader context than within the original model, which was concerned with the solution of the strong CP problem in the standard model of elementary particle physics.

The axion is characterized by a phase angle, the axion misalignment angle. There are different types of axion production mechanisms in the early universe, topological axion production and vacuum alignment (see e.g. for more details). On the other hand, a Josephson junction is also characterized by a phase, namely the phase difference of the macroscopic wave function describing the two superconducting electrodes of the junction. As will be shown in this letter, the equation of motion of the axion misalignment angle is identical to that of the phase difference of a resistively shunted Josephson junction, with a suitable re-interpretation of the currents involved. This opens up the theoretical possibility to connect both fields, and to make analogue experiments simulating axion cosmology using superconducting devices in the laboratory. Moreover, this novel approach also opens up the possibility to test for interaction strengths of incoming present-day axionic dark matter in a given resonant Josephson junction environment. Suggested future experiments of this type can be easily performed in the laboratory and may ultimately open up the route for novel methods of dark matter and dark energy detection based on modern nanotechnology.

II. EQUATIONS OF MOTION OF THE AXION AND OF JOSEPHSON JUNCTIONS

Consider an axion field $a = f_a \theta$, where $\theta$ is the axion misalignment angle and $f_a$ is the axion coupling constant. We consider such an axion in the early universe, described by a Robertson Walker metric, and neglect spatial gradients. The equation of motion of the axion misalignment angle $\theta$ is

$$\ddot{\theta} + 3H\dot{\theta} + \frac{m_a^2 c^4}{\hbar^2} \sin \theta = 0. \quad (1)$$

Here $H$ is the Hubble parameter and $m_a$ denotes the axion mass. The forcing term $\sin \theta$ is produced by QCD instanton effects. In a mechanical analogue, the above equation is that of a pendulum in a constant gravitational field with some friction determined by $H$. 
If strong external electric and magnetic fields \( \vec{E} \) and \( \vec{B} \) are present, then the axion couples as follows:
\[
\dot{\theta} + 3H\dot{\theta} + \frac{m_a^2 e^4}{\hbar^2} \sin \theta = \frac{g_\gamma}{\pi f_a^2} e^3 c^2 \vec{E} \vec{B}.
\] (2)

\( g_\gamma \) is a model-dependent dimensionless coupling constant describing the decay of the axion into two photons \( (g_\gamma = -0.97) \) for KSVZ axions (Kim-Shifman-Vainshtein-Zakharov axions [17, 18]), \( g_\gamma = 0.36 \) for DFSZ axions (Dine-Fischler-Srednicki-Zhitnitsky axions [19, 20]). The typical parameter ranges that are allowed for dark matter axions are [1, 8]
\[
6 \cdot 10^{-6} \text{eV} \leq m_a e^2 \leq 2 \cdot 10^{-3} \text{eV}
\] (3)
and
\[
3 \cdot 10^{18} \text{eV} \leq f_a \leq 10^{21} \text{eV}.
\] (4)
The product \( m_a e^2 f_a \) is expected to be of the order \( m_a e^2 f_a \sim 6 \cdot 10^{15} (\text{eV})^2 \).

Let us now compare this with the equations of motion of resistively shunted Josephson junctions (RSJs). A Josephson junction consists of two superconductors with a thin insulator inbetween [2]. Such a Josephson junction can be a very complex system, with all kinds of interesting effects, in particular for small junctions. However, in the simplest theoretical models the phase difference \( \delta \) of the macroscopic wave function of the two superconductors satisfies
\[
\ddot{\delta} + \frac{1}{RC} \dot{\delta} + \frac{2eI_c}{hC} \sin \delta = 0.
\] (5)
Here \( R \) is the shunt resistance, \( C \) is the capacity of the junction, and \( I_c \) is the critical current of the junction. The frequency
\[
\omega = \sqrt{\frac{2eI_c}{hC}}
\] (6)
is called the plasma frequency of the Josephson junction. The product
\[
Q := \omega RC
\] (7)
is the so-called quality factor of the junction.

If a bias current \( I \) is applied to the junction by maintaining a voltage difference \( V \) between the two superconducting electrodes, then the equation of motion becomes
\[
\ddot{\delta} + \frac{1}{RC} \dot{\delta} + \frac{2eI_c}{hC} \sin \delta = \frac{2e}{hC} I.
\] (8)
Remarkably, the equations of motions of axions and of RSJs are identical provided we make the following identifications in eqs. (2) and (8):
\[
3H = \frac{1}{RC}
\] (9)
\[
m_a^2 e^4 = \frac{2eI_c}{h}
\] (10)
\[
\frac{g_\gamma}{\pi f_a^2} e^3 c^2 \vec{E} \vec{B} = \frac{2e}{hC} I.
\] (11)

Let us now further work out this interesting connection. We note that it is possible to make analogue experiments with RSJs that simulate axion cosmology in the laboratory. To simulate an axion in a certain era of cosmological evolution, one builds up an RSJ which has its parameters \( R, C, I_c, I \) fixed by eqs. (9)-(11). The left-hand side are cosmological parameters, the right-hand side is nanotechnological engineering. Essentially the inverse Hubble constant \( H^{-1} \) (the age of the universe) fixes the product \( RC \), the axion mass fixes the critical current \( I_c \) and the axion coupling to external electromagnetic fields \( \vec{E} \vec{B}/f_a^2 \) is represented by the strength of the bias current \( I \).

III. RELEVANT PARAMETER VALUES

It is remarkable that the numerical values of the parameters for typical axionic dark matter physics and for typical Josephson junction experiments have similar order of magnitude. Let us start to illustrate this with some old experiments that were ground-breaking technology some 30 years ago, the Josephson junction experiments performed by Koch, Van Harlingen and Clarke in [3]. They built up four different samples of Josephson junctions with parameters values in the range \( R \sim 0.075 - 0.77 \Omega, C \sim 0.5 - 0.81 \text{pF}, I_c \sim 0.32 - 1.53 \text{nA} \).

According to eqs. (9)-(11), these experiments of Koch et al. thus simulate the dynamics of axion-like particles in a very early universe whose age is of the order \( H^{-1} = 3RC \sim 10^{-13} - 10^{-12} \) seconds and where the axion mass is in the range \( 0.98 \cdot 10^{-3} - 1.58 \cdot 10^{-3} \) eV. This simulated axion mass is just at the upper end of what is expected for dark matter axions, see eq. (3).

By increasing either the shunt resistor \( R \) or the capacity \( C \) to larger values, one can simulate axion cosmology at a later time, for example during or after the QCD phase transition time \( (H^{-1} \sim 10^{-8} \text{s}) \) where axion physics becomes most relevant. Current nanotechnology covers this range. For example, the recent experiment by Steffen et al. [4], which tests for quantum entanglement of coupled Josephson qubits, is classically well described by an RSJ model where the product \( 3RC \) is of the order \( H^{-1} = 3RC = 1.2 \cdot 10^{-8} \text{s} \). This experiment thus simulates the dynamics of weakly coupled axions after the QCD phase transition in the early universe.

It is quite interesting to look at recent experiments with Josephson junctions and to check which axion masses these correspond to, based on our identification in eq. (8). The experiment of Steffen et al. [4] corresponds to an axion mass of \( m_a e^2 = h \omega = 3.3 \cdot 10^{-5} \text{eV} \), much smaller than for the Koch et al. experiment [3] but again within the range expected for dark matter axions, see eq. (3). The experiments of Pentilla et al. [21], dealing with superconductor-insulator phase transitions, simulate \( m_a e^2 = 1.32 \cdot 10^{-4} \text{eV} \). Nagel et al. [22] report on negative absolute resistance effects in Josephson junctions, these experiments formally have
\[ m_a c^2 = 2.83 \cdot 10^{-5} \text{eV}. \]

Superconducting atomic contacts correspond to even smaller axion masses, namely \( m_a = 6.7 \cdot 10^{-6} \text{eV} \), at the lower end of what is allowed in eq. (3). Two-dimensional Josephson arrays, as built up in [24], correspond to arrays of coupled axions with \( m_a c^2 \) in the range \( 6.62 \cdot 10^{-5} - 1.52 \cdot 10^{-4} \text{eV} \). Remarkably, all these recent experiments are within the range of axion masses that are of interest from a dark matter point of view. They can thus be regarded as simulating axionic physics with realistic parameters. The main point of this letter, which we will work out in more detail in the following, is that a given Josephson junction environment may indeed serve for incoming cosmological axions as a kind of resonant medium. This opens up new possibilities of axionic dark matter detection.

**IV. POSSIBLE COUPLING MECHANISM OF AXIONS TO JOSEPHSON JUNCTIONS**

Inspired by the above qualitative and quantitative agreement of the relevant equations of motion, we may consider the possible existence of interaction phenomena for axionic dark matter that are inspired by known effects in Josephson junctions. For example, it is well-known that two Josephson junctions may couple in a SQUID-like structure. Can axions form a similar SQUID-like state? Moreover, there is the Josephson effect, of utmost importance in many technological applications. This effect does not only exist for superconducting devices connected by a weak link but also for Bose Einstein condensates (BEC) [32]. Given that axionic dark matter behaves similar to a BEC [2], it is natural to ask whether the analogue of the Josephson effect exists for axionic dark matter. The mathematical equations allow for such an effect.

Let us first work out the theoretical possibility that axions are able to form SQUID-like structures. We just sketch the main idea. It is well-known that if two Josephson junctions, the first one having phase difference \( \delta \) and the second one having phase difference \( \theta \), are put together in a SQUID-like configuration, then the two phases \( \delta \) and \( \theta \) start to synchronize, according to the general equation

\[
\delta - \theta = 2\pi \frac{\Phi}{\Phi_0} \mod 2\pi. \tag{12}
\]

Here \( \Phi \) is the magnetic flux enclosed by the SQUID, and \( \Phi_0 = \frac{\hbar}{2e} \) is the flux quantum. Eq. (12) is a simple consequence of the fact that the gauge-invariant phase of the SQUID must be single-valued. As a matter of fact, if the flux \( \Phi \) enclosed by the SQUID is much smaller than \( \Phi_0 \), then the above condition implies

\[
\delta = \theta, \tag{13}
\]

i.e. both phases are synchronized. A mechanical analogue would be that of two pendula in a gravitational field, with masses of similar order of magnitude (in our case represented by the Josephson plasma frequency). The pendula will ultimately synchronize their movements. We would expect that axions can form similar synchronized states. One could in fact think of entire clumps of dark matter axions (in analogy to arrays of Josephson junctions, well-known in nanotechnology) which are coupled and perform synchronized motion. As a whole, they look like a spatially extended dark matter particle of bigger mass.

A related interesting theoretical idea would be that axions could weakly couple to ordinary Josephson junctions if these possess similar parameter values. This would be of utmost interest for detection purposes. Classically, the coupling between two Josephson junctions with shunt resistance \( R \), capacity \( C \), and inductivity \( L \) is described by the following coupled differential equations [3]:

\[
\begin{align*}
\ddot{\delta} + \frac{1}{RC} \dot{\delta} + \omega^2 \sin \delta &= \gamma_a (\dot{\theta} - \dot{\delta}) + \frac{1}{CL} (\delta + 2\pi M_1) \\
\ddot{\theta} + \frac{1}{RC} \dot{\theta} + \omega^2 \sin \theta &= \gamma_a (\dot{\theta} - \dot{\delta}) + \frac{1}{CL} (\theta + 2\pi M_2)
\end{align*}
\tag{14}
\]

\( \delta \) is the phase difference of the first junction, \( \theta \) that of the second junction. \( M_1 = \Phi_1/\Phi_0 \) is the normalized flux enclosed by junction 1 (\( i = 1, 2 \)), and \( \gamma_a = C_a/C \) is a small dimensionless coupling constant, assuming both junctions are capacitively coupled by a capacity \( C_a \). For example, in the experiments of Steffen et al. dealing with coupled Josephson qubits [4], \( \gamma_a = 2.3 \cdot 10^{-3} \). Usually the damping term proportional to \( \delta \) and \( \dot{\theta} \) is neglected in the theoretical treatment of these types of experiments [5].

In our case, if a similar effect is to be exploited for dark matter detection purposes, then the phase \( \delta \) would describe an ordinary Josephson junction and the phase \( \theta \) an axion that passes through this Josephson junction. One of the simplest coupling schemes that could be experimentally tested would be

\[
\begin{align*}
\ddot{\delta} + \frac{1}{RC} \dot{\delta} + \omega^2 \sin \delta &= \gamma_a (\dot{\theta} - \dot{\delta}) \\
\ddot{\theta} + 3H \dot{\theta} + \frac{m_a^2 c^4}{\hbar^2} \sin \theta &= \gamma_a (\dot{\delta} - \dot{\theta}). \tag{15}
\end{align*}
\]

This corresponds to the case \( L \to \infty \) in eq. (14). For present-day axions, it certainly makes sense to neglect the friction term \( 3H \), just as the corresponding friction term was neglected in the theoretical treatment of [5] for two coupled Josephson qubits. Formally, present-day axions have a very high quality factor \( Q = \omega RC = m_a c^2/(3 \hbar \omega) \), because the universe is very old.

If the axion mass is at resonance with the Josephson plasma frequency, \( m_a c^2 = \hbar \omega \), then synchronization effects of the phases \( \delta \) and \( \theta \) will occur if \( \gamma_a \) is not too small, just as they occur for coupled Josephson qubits [4, 5]. One could also allow for an axion coupling to fluxes similar as in eq. (14), in this case the axion would have a small additional self-interaction potential given
by $V(a) = -\frac{1}{\hbar^2}(\frac{1}{2}a^2 + 2\pi M_2 f_0 a)$. Quantum mechanically, one could even speculate on the formation of entangled states between axions and Josephson qubits. But of course we are still far away from such novel types of detectors at the moment.

In fact, nothing is known on the size of the dimensionless coupling $\gamma_x$ describing the coupling of an axion to a given Josephson junction environment. While the trivial solution $\gamma_x = 0$ is certainly possible, $\gamma_x > 0$ is not forbidden by any first principle. Given the quantitative agreement between the parameters of axion physics and Josephson junction physics outlined in section 3, one should consider the possibility that $\gamma_x$ might again be of similar order of magnitude as in current nanotechnological experiments. This can be experimentally tested.

There are no astronomical constraints on the size of $\gamma_x$ since almost all of the matter in the universe is not in the form of Josephson junctions. Hence the only way to constrain $\gamma_x$ is to scan a range of plasma frequencies and look for the possible occurrence or non-occurrence of universal resonance effects, produced by axions of the dark matter halo that hit terrestrial Josephson junction experiments. The intensity of this effect should display small yearly modulations, just similar as in the DAMA/LIBRA experiments [22]. What corresponds to tuning the cavity frequency in the experiments [12] would correspond to tuning the plasma frequency $\omega$ in these new types of nanotechnological dark matter experiments.

It has been previously suggested [25–27] that Josephson junctions could also be used as a laboratory test ground for dark energy. The basic idea here is as follows: The fluctuation dissipation theorem (FDT) [28, 29] predicts in addition to deterministic bias currents $I$ the existence of noise currents $I_N$ with a universal power spectrum given by

$$S(\nu) = \frac{4}{R} \left( \frac{1}{2} \frac{h\nu}{e^{\frac{h\nu}{kT}} + 1} + \frac{h\nu}{e^{\frac{h\nu}{kT}} + 1} \right)$$

on the right-hand side of eq. [8]. But the FDT is very general, it also predicts noise currents with the same power spectrum on the right-hand side of eq. [8] and [14].

Following the line of arguments presented in [25–27] the linear term $\frac{h\nu}{e^{\frac{h\nu}{kT}} + 1}$ could be influenced by zeropoint fluctuations of a dark energy field, whatever its nature, that is coupled into the macroscopic quantum system via a phase synchronization mechanism [27]. For example, this could be vacuum fluctuations associated with an axion field, thus connecting dark energy and dark matter. A simple toy model would be that the dark energy vacuum fluctuations kick the Cooper pairs via phase synchronization effects while they tunnel. Whereas ordinary electromagnetic zeropoint fluctuations exist at any frequency, vacuum fluctuations associated with this dark energy field would only exist below a critical frequency in the THz region because the dark energy density of the universe is finite. If this hypothesis is true then a cut-off of the measured noise power spectrum at a critical frequency corresponding to dark energy density should exist, which can be experimentally tested [27]. Clearly, from a new physics point of view, it is very important to test the validity of eq. [10] at frequencies corresponding to the dark energy scale, similarly as it is very important to test gravitational forces at these scales [30] or the dependence of the Casimir effect on a postulated cutoff [31]. In this way Josephson junctions could provide suitable experimental setups for tests on both dark energy and (axionic) dark matter.

V. AXIONIC JOSEPHSON EFFECT

Given the similarity between axions and Josephson junctions, we may also consider the existence of an axionic Josephson effect, similar in spirit to what was experimentally observed for BEC in [32]. An RSJ biased with voltage $V$ generates Josephson radiation with frequency

$$\hbar \omega_J = 2eV.$$  \hspace{1cm} (17)

For such a biased junction the phase $\delta$ grows linearly in time, i.e.

$$\delta(t) = \delta(0) + \frac{2eV}{\hbar} t \hspace{1cm} (18)$$

and the relation between bias current $I$ and applied voltage $V$ is

$$V = R\sqrt{I^2 - I_c^2} \approx RI \hspace{1cm} \text{for} \hspace{0.2cm} I >> I_c.$$  \hspace{1cm} (19)

Josephson oscillations set in if

$$I > I_c,$$  \hspace{1cm} (20)

i.e. the bias current $I$ must be larger than the critical current $I_c$ of the junction. In the mechanical analogue, the pendulum rotates with large kinetic energy.

According to eq. [2] and [8], the axion misalignment angle $\theta$ also starts to increase linearly in time if it is being forced by very strong products of $\vec{E}$ and $\vec{B}$ fields. So from a formal mathematical point of view, an axionic Josephson effect is possible. We get

$$\hbar \omega_J = 2eV \approx 2eRI = \frac{g_\gamma}{\pi} \frac{1}{f_3^2} \frac{e^2}{3\hbar} \vec{E} \cdot \vec{B},$$  \hspace{1cm} (21)

where in the last step we used eq. [9] and [11]. Condition [20] translates to

$$\frac{g_\gamma}{\pi} e^2 \hbar^2 \vec{E} \cdot \vec{B} > \frac{f_3^2 m_a^2 c^4}{e^2} =: \Lambda^4.$$  \hspace{1cm} (22)

QCD-inspired models of axions require $\Lambda \approx 78$ MeV [3]. One can easily check that the strength of the $\vec{E}$- and $\vec{B}$-fields required to observe axionic Josephson oscillations must be much higher than anything that can be produced in the laboratory.
However, allowing the possibility of phase synchronization, due to a $\gamma_A \neq 0$ in eq. (15), there is another interesting possibility here. If we assume that phase synchronization sets in for some axions hitting a Josephson junction with resonant plasma frequency, then a moderate bias current $I$ in the Josephson junction environment would simulate for these synchronized axions the formal existence a huge product $\vec{E} \cdot \vec{B}$ according to eqs. (2), (5), (11). The huge magnetic field formally seen by these synchronized axions will make the axions decay into two microwave photons, which could then be detected, for example in form of Shapiro steps (Shapiro steps are well-known step-like structures in the $I-V$ curves of irradiated Josephson junctions [32]). Again this theoretical idea opens up the possibility to develop new detectors for axionic dark matter [34].

If the voltage $V$ is applied to a Josephson junction whose superconducting electrodes are separated by a distance $d$, then this generates the electric field strength $E = V/d$ between the electrodes. Synchronized axions would oscillate with the same Josephson frequency as the Josephson junction. They would thus formally see the magnetic field

$$ B = \frac{6\pi f_s^2 dH}{y_c c^3 e} $$

in the same direction as $\vec{E}$. This expression is obtained by putting $E = V/d$ in eq. (21). The result is independent of the applied voltage $V$. Note that although today the Hubble parameter $H$ is very small ($H \sim 2 \cdot 10^{-18} \text{ s}^{-1}$), eq. (23) formally generates large magnetic fields (for $d \sim 10^{-6} m$ we get $B \sim 10^5 T$). These large (virtual) magnetic fields immediately make the synchronized axion decay into microwave photons.

VI. SUGGESTION FOR EXPERIMENTAL SEARCHES

Let us now discuss how a future experiment to detect dark matter axions with Josephson junctions could look like. We consider a detector consisting of many coupled Josephson junctions. Assume a dark matter axion with mass $m_a$ enters the tunneling region of one of the Josephson junctions. All Josephson junction are driven by bias currents $I > I_c$. If the plasma frequency is at resonance with the axion mass, then with a certain probability phase synchronization between the axion phase $\theta$ and the Josephson phase $\delta$ will set in. The axion will then formally see a huge magnetic field $\vec{B}$ given by eq. (23), because for it the Josephson environment of the detector looks like a set of fellow axions driven by the left-hand side of eq. (11). This huge magnetic field does not exist in reality, it is just briefly seen by the axion as long its phase is synchronized with the resonant Josephson environment. The huge (virtual) magnetic field will make the axion decay into two microwave photons. These microwave photons can then be detected.

Microwave photons produced by axion decays will modify the $I-V$ curve of the Josephson junction, by producing Shapiro steps. The theory of irradiated Josephson junction (Josephson junctions subject to external electromagnetic radiation) is well-established [1]. One knows that in the $I-V$ curve of the Josephson junction discontinuities appear, at fractions of frequencies of the incoming photons. These are the so-called Shapiro steps [33]. The theory is well-established and we refer to the textbooks [1]. The new things in our case would be that Shapiro steps produced by axion decays should now also occur if no external radiation is applied, since photons are internally created by axion decay. We may call this 'spontaneous' Shapiro steps. The observation of spontaneous Shapiro steps would be a clear signature of axions. To exploit this effect, the detector should be carefully shielded against any external electromagnetic radiation.

Let us give a rough estimate of signal strengths. From astronomical observations, the dark matter halo density of galaxies is known to be about $\rho_{\text{halo}} = 0.17...1.7 \cdot 10^6 \text{ GeV/m}^3$. Suppose a large part of this is due to axions, i.e. $\rho_a \sim \rho_{\text{halo}}$. The number density $n$ of axions is then given by $n = n_a/m_a c^2$. Suppose the earth moves through a cloud of axions with relative velocity $\vec{v}$. Then the flux of axions through our detector (number of axions $N_a$ per time unit $T$) is given by

$$ \frac{N_a}{T} = n v A = \frac{\rho_a v A}{m_a c^2} \quad (24) $$

where $A$ denotes the total area of tunneling regions of all Josephson junctions perpendicular to $\vec{v}$. With a certain probability $\eta$ a given axion crossing the detector will resonate with the Josephson junction environment and decay into detectable microwave photons. The number $\eta$ is unknown but can be experimentally constrained in future experiments. The total power produced by decaying axions is

$$ P = \eta \frac{n_a m_a c^2}{T} = \eta \rho_a v A. \quad (25) $$

Suppose the detector is large, say $A \sim 1 m^2$, and that $v$ is given by the movement of the sun relative to the galactic centre,

$$ v = (2.3 \pm 0.3) \cdot 10^5 \frac{m}{s} \approx 10^{-3} c. \quad (26) $$

The term $\pm 0.3$ describes yearly modulations [9], $v$ has a maximum in June and a minimum in December. Then the dissipated power due to decaying axions is of the order $P \sim \eta 10 W$. This is detectable even for very small values of $\eta$. If the dissipated power is really due to axions from the galactic halo, then a yearly modulation of about 10%, similar as in the DAMA-LIBRA experiment [8], with a maximum in June and a minimum in December should be observed. All this can be easily checked in future experiments.
VII. CONCLUSION

Whatever the conditions were in the very early universe, Josephson junctions, SQUIDS, and spatially extended arrays of these superconducting devices can nowadays be built for a wide range of different parameters $R, C, I_c$, and it is very easy to tune the bias current $I$ to any value of interest. It is also very easy to adjust the plasma frequency of a Josephson junction to any value of interest. It is thus possible to simulate an axionic dark matter environment in the early universe by building up the corresponding Josephson junctions with parameters given by eqs. (9)-(11). In particular, it is possible to investigate more realistic models of spatially-temporally coupled axions, by building up arrays of Josephson junctions. If the coupling $\gamma_x$ is different from 0, then these Josephson junctions can form a resonant environment for incoming dark matter axions.

There is the prospect of developing new generations of detectors for dark matter axions that search for possible resonance effects and phase synchronization if the Josephson plasma frequency is close to the axion mass. In this way the size of the coupling $\gamma_x$ between axions and a given Josephson environment could be experimentally constrained. Noise measurements could also be made to constrain certain types of dark energy models [27]. As outlined above, the relevant dark matter and dark energy parameter range is accessible by modern technological developments in nanotechnology. An obvious advantage of these types of experiments is that the formal existence of extremely large products of electric and magnetic field strengths $\vec{E}\cdot\vec{B}$ acting on the axion can be simulated by a very simple experimental setup, an easily tunable bias current $I$, assuming that some axions hitting the Josephson junction will synchronize their phase due to a non-zero $\gamma_x$. This effect may be exploited in the future to develop new types of axionic dark matter and dark energy detectors based on modern nanotechnology.

[1] M. Tinkham, *Introduction to Superconductivity*, Dover (2004)
[2] B.D. Josephson, Phys. Lett. 1, 251 (1962)
[3] R.H. Koch, D. Van Harlingen, J. Clarke, Phys. Rev. B 26, 74 (1982)
[4] M. Steffen et al., Science 313, 1423 (2006)
[5] J.A. Blackburn, J.E. Marchese, M. Cirillo, N. Groenbech-Jensen, Phys. Rev. B 79, 054516 (2009)
[6] M.P. Hertzberg, M. Tegmark, F. Wilczek, Phys. Rev. D 78, 083507 (2008)
[7] P. Sikivie, Q. Yang, Phys. Rev. Lett. 103, 111301 (2009)
[8] L.D. Duffy, K. van Bibber, New J. Phys. 11, 105008 (2009)
[9] R. Bernabei et al., Eur. Phys. J. C 56, 333 (2008)
[10] CDMS II collaboration, Science 327, 1619 (2010)
[11] C.E. Aalseth et al., Phys. Rev. Lett. 106, 131301 (2011)
[12] S.J. Asztalos et al., Phys. Rev. Lett. 104, 041301 (2010)
[13] J.E. Moore, Nature 464, 194 (2010)
[14] F. Wilczek, Nature 458, 129 (2009)
[15] X.-L. Qi et al., Phys. Rev. B 78, 195424 (2008)
[16] R.D. Peccei and H. Quinn, Phys. Rev. Lett. 38, 1440 (1977)
[17] J.E. Kim, Phys. Rev. Lett. 43, 103 (1979)
[18] M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B 166, 493 (1980)
[19] M. Dine, W. Fischler, M. Srednicki, Phys. Lett. B 104,199 (1981)
[20] A.R. Zhitnitsky, Sov. J. Nucl. Phys. 31, 260 (1980)
[21] J.S. Penttilä et al., Phys. Rev. Lett. 82, 1004 (1999)
[22] J. Nagel et al., Phys. Rev. Lett. 100, 217001 (2008)
[23] M.L. Della Rocca et al., Phys. Rev. Lett. 99, 127005 (2007)
[24] Y. Takahide et al., Phys. Rev. Lett. 85, 1974 (2000)
[25] C. Beck, M.C. Mackey, Phys. Lett. B 605, 295 (2005)
[26] C. Beck, M.C. Mackey, Physica A 379, 101 (2007)
[27] C. Beck, M.C. Mackey, Int. J. Mod. Phys. D 17, 71 (2008)
[28] R. Kubo, Rep. Prog. Phys. 29, 255 (1966)
[29] C. Beck, M.C. Mackey, Fluct. Noise Lett. 7, C31 (2007)
[30] D.J. Kapner et al., Phys. Rev. Lett. 98, 021101 (2007)
[31] L. Perivolaropoulos, Phys. Rev. D 77, 107301 (2008)
[32] S. Levy, E. Lahoud, I. Shomroni, J. Steinhauer, Nature 449, 579 (2007)
[33] S. Shapiro, Phys. Rev. Lett. 11, 80 (1963)
[34] A strong resonance of unknown origin, observed in the experiments of Koch et al. [3] at 368 GHz for the 4th junction of their experimental series would point to an axion mass of $1.52 \times 10^{-3}$ eV if interpreted in this way.