Global textures and the formation of self-bound gravitational systems

A. L. B. Ribeiro and P. S. Letelier

Departamento de Matemática Aplicada – Instituto de Matemática Estatística e Computação Científica, Universidade Estadual de Campinas, 13083-970 SP, Brazil

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ABSTRACT

Using a Newtonian approximation we developed a quantitative criterion for the collapse of a spherical distribution of matter under an isolated texture field. In particular, we found the evolution of an overdense region is strongly determined by two parameters: the energy scale of symmetry breaking ($\eta$) and the initial radius of the system. Applying our collapse criterion to typical galaxy scales we verified the formation of $10^{11}\odot$ objects at $z \leq 9$ and $10^{12}\odot$ objects at $z \leq 5$.

Key words: cosmology: miscellaneous – large-scale structure of Universe.

1 INTRODUCTION

The emergence of structures in the Universe is one of the most important problems of modern cosmology. The difficulty is to understand how an initially homogeneous mass distribution evolved into its present clumpy state. Generally, it is accepted that structures were initiated by small density fluctuations formed at the early Universe, and that the subsequent clustering was produced by gravitational instability (e.g. Kolb & Turner 1990). At the moment, there are two theoretical approaches proposed to explain the origin of the primordial inhomogeneities. In the context of inflationary models, they result of quantum-induced Gaussian fluctuations amplified over an accelerated phase of cosmic expansion. On the contrary, in topological defect scenarios, non-Gaussian fluctuations are produced during spontaneous symmetry breaking at phase transitions in the early Universe (e.g. Vilenkin & Shellard 1994).

Several types of topological defects can be formed depending on the kind of symmetry that is broken. Here, we are interested in a specific topological defect formed during the symmetry breaking of non-Abelian groups: a global texture. This defect has been proposed as a possible source of galaxy formation in the Universe (Turok 1989). The development of structures based on this topological remnant is related to the evolution of textures knots. On scales larger than horizon, textures knots are regions where the scalar field winds around the vacuum manifold in a non-trivial way. As knots come inside the horizon, they collapse at the speed of light down to an infinitesimal scale where they unwind themselves emitting spherical waves of outgoing radiation. This collapse produces an overdense region onto which matter is attracted, possibly leading to the formation of non-linear objects (Turok & Spergel 1990).

The most desirable way to study structure formation produced by textures knots is through numerical simulations like those carried out by Park, Spergel & Turok (1989), Spergel et al. (1991) and Cen et al. (1991). All these studies indicate important effects of textures on the distribution of matter, like coherent large-scale flows and clustering of galaxies fairly consistent with the correlation function observed in the nearby Universe. However, we should keep in mind that the dynamics of textures is highly non-linear since the decoupling era (Albrecht, Battye & Robinson 1999), and that it is not simple to trace the evolution of the fluctuations over this long time interval. Thus, all these numerical works are strongly dependent on the choice of the initial conditions for the $N$-body simulations (Deruelle, Longlois & Uzan 1997).

Actually, this is not the only problem with the texture scenario. The work of Pen, Seljak & Turok (1997) shows that the texture model has strongly suppressed acoustic peaks in comparison to current observations of CMB anisotropies. Also, when $COBE$ normalized, the model produces a low value of the rms density variation on scales of $8\, h^{-1}\,\text{Mpc}$ ($\sigma_8 \sim 0.23$), while a higher value of this parameter (closer to unity) is necessary in order to get agreement with the observed galaxy clustering. These results clearly pose serious difficulties to the texture scenario. However, this does not mean that the model should be completely discarded at the present time, since a decisive, model discriminating test will be possible only after the analysis of the high resolution CMB maps which will be available in the next years from the $MAP$ and $PLANCK$ satellite missions. Hence, for the moment, it remains valid to work with texture knots as a viable way to form galaxies in the Universe.

Indeed, an alternative way to study structure formation via global textures is to develop simple analytic models to describe the general properties of the defect dynamics. For instance, Gooding, Spergel & Turok (1991) studied the formation of bound objects induced by a number of collapsing knots in a $\Omega_0 = 1$ CDM-dominated universe. An important result of this study is the prediction of early spheroidal formation in the Universe, suggesting that by $z \sim 50, \sim 3$ per cent of the mass of the Universe has formed non-linear objects with mass greater than $10^8\,\odot$ and that most objects larger than $10^{12}\,\odot$ formed by $z \sim 2-3$.

In the present work, we use a Newtonian approximation to
investigate the formation of self-bound gravitational systems seeded by global textures. However, instead of applying a statistical approach to the contribution of many individual knots, we are only interested in the local influence of a single texture knot on the process of galaxy formation. Our aim is just to obtain a quantitative criterion for the collapse of a spherical distribution of matter under a texture field.

2 THE COLLAPSE CRITERION

Distributed field gradients induced by a number of global textures are supposed to give a larger contribution to density inhomogeneities than that produced by single isolated knots (e.g. Vilenkin & Shellard 1994). If this is correct, then, the effort to probe the process of structure formation seeded by these defects will require large dynamic range simulations in computers of latest technology. However, the demand for such numerical works does not mean that analytical developments are unnecessary. On the contrary, they are particularly useful to give a supplementary and intuitive picture of the defect dynamical contribution on structure formation. Specifically, in this work, we study the process of accretion of matter onto a single texture knot. This problem has been studied by several authors over the years. But, the process of structure formation seeded by these defects will require a self-bound system is

\[ \delta_i > \delta_c \]

For a dust configuration \( (p = 0) \), we have

\[ \Omega_i = \frac{\Omega_0(1 + z_i)}{1 + z_i} \]

and so

\[ \delta_c = \frac{1 - \Omega_0}{\Omega_0(1 + z_i)} \]

(e.g. Börner 1988). Therefore, for the case of a flat universe, \( \delta_c = 0 \) and any overdense region with \( \delta_i > 0 \) will collapse. On the other hand, for the case of an open universe, only regions where \( \delta_i > \delta_c \) can produce self-bound gravitational systems. In this case, introducing definition (3) in equation (6) we directly find

\[ \delta_i = \frac{24\eta^2}{\rho_b(t_i)r_i^2} \]

Now, it is easy to foresee the destiny of a spherical distribution of matter under a texture field when \( \Omega_0 < 1 \). Introducing equation (11) in condition (8), it is simple to show that

\[ r_i < \left( \frac{24\eta^2\Omega_0}{\rho_b(t_i)(1 - \Omega_i)} \right)^{1/2} \]

Recalling that

\[ \rho_b(t_i) = \Omega_i\rho_c(t_i) \quad \text{and} \quad \rho_c(t_i) = \frac{3H^2}{8\pi G} \]

we reach

\[ r_i < \frac{8\eta}{H}\left( \frac{\sqrt{G\pi}}{1 - \Omega_i} \right) = R_c \]

where \( R_c \) is the critical radius for the collapse. Once more, assuming a Friedmann model with vanishing pressure and with no cosmological constant

\[ H_i^2 = \frac{8\pi G}{3}\left[ \Omega_0(1 + z_i)^3 + (1 - \Omega_0)(1 + z_i)^2 \right] \]
(e.g. Sandage 1961), we can finally express the collapse criterion for any overdense region under a texture field at any epoch $z_i$:

$$r_i < \frac{8\eta}{H_0} \sqrt{\frac{G\pi(1+z_i\Omega_0)}{(1-\Omega_0)(\Omega_0(1+z)^3 + (1-\Omega_0)(1+z)^2)}}.$$  \hspace{1cm} (16)

3 THE CHOICE OF THE ENERGY SCALE

From equation (16) we see that, assuming specific values for $H_0$ and $\Omega_0$, the critical radius for the collapse will depend basically on the energy scale of symmetry breaking. Generally, global textures arise after a phase transition during the break of the strong interaction is unified with the electroweak force. Current symmetry described by the Grand Unified Theory (GUT), when the energy scale of symmetry breaking. Generally, global textures and gravitational systems

In view of these arguments, the energy scale of $10^{16}$ GeV seems to be the natural choice of $\eta$. However, let us proceed here to find a new and independent argument for this choice. First of all, assuming $\Omega_0 = 0.2$ and $h = 0.75$ ($h = H_0/100$ km s$^{-1}$ Mpc$^{-1}$), we plot the behaviour of $R_\gamma$ as a function of the epoch $z_i$ using $\eta$ as a free parameter (Fig. 1). Note that for earlier epochs $R_\gamma$ is smaller (independent of $\eta$) indicating that the first bound objects to be formed via textures knots would have small sizes and masses. In fact, isocurvature fluctuations like those produced by topological defects can evolve only after the decoupling ($z \sim 10^{3}$), when the Jeans mass is $\sim 10^9 M_\odot$. Thus, clumps with this mass should form first and evolve into larger systems through gravity.

Obviously, the details of the evolution of clumps up to form typical galaxies and clusters is a rather complex process, involving the thermal history of the gas, star formation phenomena and the mutual interaction between the clumps. It is beyond our aim to address this problem as a whole. Instead, we proceed to simplify it by supposing that a protogalaxy just consist of a regular distribution of matter (possibly sub-galactic clumps) under a texture field where we can apply our collapse criterion.

That being the case, an important quantity to be found is the mass $M_\gamma$ related to a given radius $R_\gamma(z_i)$. For a homogeneous universe, the non-relativistic mass within a physical scale $\lambda$ is given by

$$M(\lambda) = 1.45 \times 10^{11}(\Omega_0 h^2)\lambda^3 M_\odot.$$  \hspace{1cm} (17)

(e.g. Padmanabhan 1993). This quantity is conserved during the expansion and corresponds to the uncollapsed mass of a physical scale $\lambda$. If we take $\lambda = R_\gamma$, it is possible to study the behaviour of $M_\gamma$ as a function of the epoch $z_i$ and the parameter $\eta$. In Fig. 2 we see that, for a same epoch $z_i$, the value of $M$ will strongly depend on $\eta$. From the comparison between $M_\gamma$ and the Jeans mass, we also note that the lowest energy scale which is able to form self-bound systems over all the astrophysical range ($10^6$–$10^{12} M_\odot$) at any post-recombination time $z_i$ is $\eta = 10^{16}$ GeV. At this energy scale, $R_\gamma$ is well inside $H^{-1}/\sqrt{2}$ (see Fig. 1), which defines regions where we can consider the effects of isolated texture knots. This result can be taken as a qualitative argument for the choice of $\eta = 10^{16}$ GeV and so we are assuming this value for the rest of the work.

4 GALAXY FORMATION

In order to apply our collapse criterion to the specific problem of galaxy formation, we should follow the evolution of overdense regions containing typical galaxy masses ($10^{11}$–$10^{12} M_\odot$). If these respect the condition (8), they will expand to a maximum radius, then collapse and eventually virialize. The dynamical properties of the resultant systems can be estimated from the spherical model for non-linear collapse. Thus, at the epoch

$$1 + z_{col} = 0.36\delta_1(1+z_i).$$  \hspace{1cm} (18)
the dissipative component of the system (i.e. baryons) reaches the virialization with the following approximated properties:

\[ r_{\text{vir}} = 163(1 + z_{\text{col}})^{-1} \Omega_0^{-1/3} (M/10^{12} M_\odot)^{1/3} h^{-2/3} \text{kpc} \]  

(19) and

\[ \sigma_{\text{vir}} = 126(1 + z_{\text{col}})^{1/2} \Omega_0^{2/3} (M/10^{12} M_\odot)^{1/3} h^{1/3} \text{km s}^{-1} \]  

(20)

(e.g. Padmanabhan 1993).

Before applying these expressions to galaxy scales, it would be interesting to assure that the initial density contrast \( \delta_i \) can form a self-bound system before the present epoch, \( z_{\text{col}} > 0 \). We directly find such a condition from equation (18):

\[ \delta_i > 2.78 (1 + z_i)^{-1}. \]  

(21)

For the case of a flat model this is a simple condition for the collapse. However, for an open universe, we also should have \( \delta_i > \delta_c \) in order to collapse the overdense region. Using equation (10) and taking the limiting case when \( \delta_i = 2.78 (1 + z_i)^{-1} \), we conclude that \( \Omega_0 < 0.26 \) is the necessary condition for the collapse with \( z_{\text{col}} > 0 \).

In Fig. 3, we plot \( \delta_i \) as a function of \( z_i \) for initial radii that correspond to typical galaxy masses. The dotted line is the limit 2.78(1 + \( z_i \))\(^{-1} \). Note that for \( M = 10^{11} M_\odot \), the collapse is only possible for \( z_i \leq 33 \) (\( z_{\text{col}} \leq 9 \)) if \( \Omega_0 = 0.2 \) and \( z_i \leq 25 \) (\( z_{\text{col}} \leq 8 \)) if \( \Omega_0 = 1.0 \); while for \( M = 10^{12} M_\odot \), \( z_i \leq 15 \) (\( z_{\text{col}} \leq 5 \)) if \( \Omega_0 = 0.2 \) and \( z_i \leq 11 \) (\( z_{\text{col}} \leq 4 \)) if \( \Omega_0 = 1.0 \). These ranges in \( z_i \) produce associated loci in the \( \sigma - r \) plane (see Fig. 4). These loci show that our simple model texture knot + spherical collapse is able to produce self-bound gravitational systems with dynamical properties quite similar to real galaxies. In fact, the comparison with data compiled from the literature shows a significant agreement between our predictions and typical galaxies, while it is also clear in Fig. 4 an important trend towards larger radii and lower velocity dispersions in our bound systems. At the same time, the epoch when such objects are presumed to reach the virial equilibrium (\( z_{\text{col}} \)) is consistent with the results of several studies which point out an epoch of galaxy formation by \( z \approx 5 \) (e.g. Peebles 1993; Pahre 1998).

Now, by extending the approach developed in this section to a wider astrophysical range (\( 10^9 - 10^{14} M_\odot \)), it is possible to depict an intuitive picture of the structure formation process when it is seeded by textures knots. The results for both open and flat universes are summarized in Tables 1 and 2, respectively, where the columns are as follows: (1) is the mass of the systems in solar units; (2) is the initial redshift we take; (3) is the initial radius of the region to be collapsed in Mpc; (4) is the initial average density contrast; (5) is the redshift at which the systems reach the virialization; (6) is the time interval of the collapse and virialization process (the dynamical time); finally, (7) and (8) are the radius in kpc and the velocity dispersion in km s\(^{-1} \) of the virialized systems.

A simple analysis of the data assembled in these tables reveals some points we should emphasize. First of all, note that structure formation ceases at earlier times in the low-\( \Omega_0 \) universe in comparison to the flat case. This is expected as long as matter has a smooth distribution in high-\( \Omega_0 \) universes so that it contributes to the expansion rate but does not contribute to the gravitational potential and so it slows down the growth of fluctuations. At the same time, note that the process of structure formation in our model begins at a much earlier epoch in comparison to standard CDM models, where most of the objects with \( 10^9 M_\odot \) are formed at the epoch \( z = 50/b \) (\( b \) is the biasing factor) (e.g. Peebles 1993). Therefore, even an unbiased CDM model would present a less advanced structure formation stage for a given redshift in comparison to our model.

Such an early self-bound system formation may have serious consequences. In particular, we form the first generation of \( 10^9 M_\odot \) objects at \( z = 135 \) (\( \Omega_0 = 0.2 \)) and \( z = 112 \) (\( \Omega_0 = 1.0 \)). These objects could be related to an early star formation phase in the Universe and its consequent reionization. At the same time, these early spheroids may develop massive black holes on to...
which matter would be accreted, possibly forming an early generation of active galaxies.

Still concerning to the chronology of structure formation, Gooding et al. (1991) found that by \( z \sim 50\), \( \sim 3\) per cent of the mass of the Universe has formed non-linear objects of mass greater than \( 10^9 \, M_\odot \) and that most of objects larger than \( 10^{12} \, M_\odot \) form by \( z \sim 2-3\). These results should be compared to our findings, while in the present work we have no statistics on the fraction of matter which is collected in bound objects of a specific mass in a given redshift. Note, however, that by \( z \sim 50\) we already formed systems of \( 10^9-10^8 \, M_\odot \) and by \( z \sim 3\) we form objects with \( 10^{15}-10^{14} \, M_\odot \). Thus, in some measure, our results are consistent with Gooding et al. (1991).

On the other hand, a disadvantage of our model refers to the velocity dispersions of the collapsed systems which are systematically lower than those observed in astrophysical objects of same mass. Up to galaxy scales this effect is accompanied with larger radii and so it can be taken as a simple consequence of the virial theorem. On the contrary, the lower velocity dispersion effect seems to be particularly serious in larger scales, like galaxy clusters, where the model predicts objects with radii similar to those found in real clusters. Actually, this is the opposite one could expect in a texture scenario, where the positive skewness of the mass fluctuations induces higher velocity dispersions for objects of a given mass than (as an example) in standard CDM models (e.g. Bartlett, Gooding & Spergel 1993). We think this trend is probably a result of the fact we are considering only the dissipative baryonic component of matter inside \( r_{\text{vir}} \). By adding dark matter within this radius we could recover values of velocity dispersion nearer to reality. For instance, in the case of a galaxy cluster scale (\( 10^{14} \, M_\odot \)), if the total mass (dissipative plus non-dissipative components) inside \( r_{\text{vir}} \) reached \( 10^{15} \, M_\odot \) the velocity dispersion of the collapsed system would be \( 583 \, \text{km s}^{-1} \) (\( \Omega_0 = 0.2\)) or \( 1475 \, \text{km s}^{-1} \) (\( \Omega_0 = 1.0\)), which could be taken as more ‘normal’ values. However, if this is the right solution to the problem, we would have to assume specific distributions for the luminous and dark matter components when considering different objects. At the same time, we cannot discard the possibility that the simplicity of our model texture knot + spherical collapse is not taking account all of the physics relevant to the structure formation process. Indeed, we are ignoring interactions between clumps, star formation and hydrodynamical effects. These physical mechanisms are probably important over the systems evolution and their inclusion would change (to some extent) the outcomes of the model.

### 5 Discussion and Summary

In this work, we developed a quantitative criterion for the collapse of a spherical distribution of matter under a single texture field. Further, we applied this criterion to form galaxies in an idealized smooth universe where texture knots can be taken as isolated sources for gravitational collapse. Despite the simplicity of the model, the properties of the systems which collapse and virialize respecting our criterion are similar to real astrophysical objects. A drawback of the model is the trend towards larger radii and lower velocity dispersions in our bound systems in comparison to typical values found in the literature for real objects of same mass. This trend is specifically significant for velocity dispersions and could be related to the quantity of dark matter inside the virialization radius. However, we would like to emphasize that the most important achievement in this work is the determination of a collapse criterion using simple arguments and the possibility of applying it in a direct way to predict astrophysical objects. This means that local effects of isolated knots may be important to the development of structures in the Universe and suggests that individual textures (not only their dynamical collective effect) should be taken account in galaxy formation models.

In the following, we summarize the main results of this work.

(i) We showed that, under specific conditions, the destiny of an overdense region under a single texture field will basically depend on its initial size and the energy scale of symmetry breaking \( \eta \) (if \( \Omega_0 < 1 \)). For those regions smaller than \( R_\odot \), the system will expand to a maximum radius and then inevitably collapse.

(ii) We found an additional argument for the choice of \( \eta \) as \( 10^{16} \, \text{GeV} \). This is simply the lowest energy scale which allows the formation of structures on scales relevant to astrophysical objects (\( 10^9-10^8 \, M_\odot \)) at any time \( z \leq 1000 \).

(iii) We showed the loci of objects with masses \( 10^{11}-10^{12} \, M_\odot \) in the \( c-r \) plane agree well with the observed properties of real galaxies.

(iv) Our results also indicate an epoch of galaxy formation fairly consistent with other independent studies which point out it should have been by \( z \sim 5 \).

(v) Our model produces a chronology of structure formation which begins at earlier times in comparison to other models, in particular CDM models.

(vi) The existence of spheroids by \( z \sim 100 \) suggests the possibility of associated phenomena like early star formation process, reionization of the Universe, production of black holes and active galaxies.

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