Toward a quantum repeater protocol based on the coherent state approach

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Abstract
The aim of this paper is to swap the entanglement between two separate long-distant locations. The well-known entangled coherent states as two-mode continuous-variable states are very interesting in quantum teleportation and entanglement swapping processes. To make our investigation more realistic, by using such entangled states as the building blocks of our quantum repeater protocol, the effect of decoherence on the swapped entanglement is also considered. We explicitly establish our model for four locations; moreover, we find that our model can be extended to $2^N$ locations, where $N = 3, 4, \cdots$. Consequently, we can introduce this model as a quantum repeater, which is helpful for entanglement swapping to long-enough distances.

Keywords: quantum repeater, entanglement production, entanglement swapping, quasi-bell state, beam splitter

(Some figures may appear in colour only in the online journal)

1. Introduction

The distribution of entangled states over long distances is an important point in quantum communications. However, due to the unavoidable photon loss in optical channels that leads to entanglement attenuation, distributing the entangled states is interrupted. Using the quantum repeater protocol is a useful way to overcome photon losses [1–4]. Producing and swapping the entanglement are two important bases of the quantum repeater protocol. In this protocol, long distances are usually divided into several short entangled parts where the states associated with these parts are separable from each other. These parts then become entangled by the well-known entanglement swapping process. The entanglement between the separated parts can be produced using a beam splitter [5–8]. The entanglement of these produced entangled states can also be swapped by the Bell state measurement method [9–11] and interactions [12–14].

The quantum repeater protocol has extended the distance of communication to the order of km ($10^3$–$10^6$ km) [15]. This protocol has been investigated based on Rydberg atomic ensembles [16, 17] and quantum dots [18–20]. The creation of entangled pairs, which are used in quantum repeater applications, has been considered in [21], in which optimizing the noise properties of the initially distributed pairs significantly improved the rate of generating long-distance Bell pairs. In [22], quantum repeaters with photon pair sources were experimentally considered. Recently, quantum repeater protocols based on photonic systems have been considered [23, 24]; purification and swapping the entanglement between eight photons have been experimentally investigated. A new control algorithm and system design for a network of quantum repeaters was presented in [25], where eight field qubits were considered. The entanglement-based quantum key distribution was discussed in [26] by using a linear chain of quantum repeaters employing photon pairs.

The quantum repeater protocol based on entangled coherent states has already been investigated [27]. In this work, the entangled coherent states were produced using a beam splitter, and the produced entanglement was swapped using linear optics and photon-number resolving detectors. One of the advantages of entangled coherent states as two-mode continuous-variable states [28] is that the quasi-Bell states (QBSs) are distinguishable by parity [29]. Heating that changes the vibrational quanta is associated with bit flip errors; this can be detected and also corrected with the help of appropriate circuits. Entanglement swapping based on QBS measurement (QBSM) has been investigated [30]. Quantum teleportation of
In this line of research, in our previous work we considered the quantum repeater based on atomic systems [39]. Eight two-level atoms (1, 2, · · · 8) were considered, where the pairs (i, i + 1) with i = 1, 3, 5, 7 were prepared in atomic Bell states. Then, by performing interaction between each pair of non-entangled adjacent atoms in single- and two-mode cavities, the target atoms were finally entangled. Because of the importance of entangled coherent states, we are now motivated to propose a quantum repeater protocol based on the entangled coherent states as a two-mode continuous variable state. In fact, in our present work the entanglement between two separate distant locations (A, B) as well as (C, D) is produced by a beam splitter. The beam splitter’s output state is a QBS, and the input state is the Schrödinger cat state. Using the QBSM method, the produced entanglements are swapped to two separable states at distant locations (A, D). Finally, our model is extended to 2 \( N \) locations, where \( N = 3, 4, \ldots \); consequently, this model can be used for a quantum repeater protocol. To make our model close to real physics, we also consider lossy effects in our proposed protocol for the quantum repeater.

This paper is organized as follows. The entanglement production and entanglement swapping are considered in section 2. The effect of decoherence on swapped entanglement is considered in section 4. Finally, the summary and conclusions are presented in section 4.

### 2. Entanglement production and entanglement swapping

In this section we consider entanglement production between two separate adjacent locations A and B. To do this we first introduce a few preliminary and prerequisite concepts and issues. First, notice that the odd coherent state is defined as:

\[
|\alpha\rangle_A = \frac{1}{\sqrt{N_{\alpha}}}(|\alpha\rangle_A - | - \alpha\rangle_A) = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty} \frac{(-\alpha)^n}{\sqrt{n!}} |n\rangle
\]

where \( N_{\alpha} = 2(1 - e^{-2|\alpha|^2}) \) and \( | \pm \alpha \rangle_A \) are the standard coherent states, i.e., \( | \pm \alpha \rangle_A = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty} \frac{\pm \alpha^n}{\sqrt{n!}} |n\rangle \). Such states can be generated using different schemes like propagation in a nonlinear medium [40–43], micromaser cavity experiments [44, 45], and quantum nondemolition experiments [46]. The above odd coherent state and the vacuum state \( |0\rangle_B \) are sent to the 50 : 50 beam splitter. The odd coherent state and vacuum state are input states of the beam splitter from locations A and B, respectively (see figure 1). The unitary transformation matrix of this beam splitter is introduced as follows [5]:

\[
U_{BS}(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix},
\]

where \( \cos^2 \theta \) (\( \sin^2 \theta \)) are the transmissivity (reflectivity) of the beam splitter. Using the unitary transformation matrix (2), all the quantum statistical properties of the output fields can be calculated in terms of the properties of the input fields. Furthermore, since the transformation of the beam splitter is unitary, there exists an effective Hamiltonian for it. Input modes \( \hat{a} \) and \( \hat{b} \) from locations A and B after passing the beam splitter are combined using unitary transformation (2):

\[
\hat{a}(\theta) = \hat{a}\cos \theta - \hat{b}\sin \theta, \quad \hat{b}(\theta) = \hat{a}\sin \theta + \hat{b}\cos \theta.
\]

It is clear from (3) that

\[
\frac{d}{d\theta} \hat{a}(\theta) = -\hat{b}(\theta), \quad \frac{d}{d\theta} \hat{b}(\theta) = \hat{a}(\theta).
\]

The effective Hamiltonian of the beam splitter can now be obtained from comparing equation (4) with the Heisenberg-like equation \( \frac{d}{d\theta} \hat{c}(\theta) = \frac{1}{\hbar} [\hat{c}(\theta), \hat{H}_{BS}] \) as below:

\[
\hat{H}_{BS} = \hbar \hat{a}\hat{b}^\dagger - \hat{a}^\dagger \hat{b},
\]

where \( \hat{c}(\theta) = \hat{a}(\theta), \hat{b}(\theta) \). It should be noted that the dimension of the eigenvalues of the effective Hamiltonian \( \hat{H}_{BS} \) is the same as the dimension of \( \hbar \). In fact, in our calculations as well as in the approach introduced by [5], the dimension of eigenvalues of the effective Hamiltonian does not coincide with energy. Now, using the unitary evolution equation \( \hat{u}_{BS}(\theta) = \exp(-\frac{i}{\hbar} \theta \hat{H}_{BS}) \) with the effective Hamiltonian (5), the beam splitter operator \( \hat{u}_{BS}(\theta) \) can be obtained as

\[
\hat{u}_{BS}(\theta) = e^{(\hat{a}\hat{b}^\dagger - \hat{a}^\dagger \hat{b})\theta},
\]

by which the following relation is established between equations (3) and the operator (6) [47]:

\[
\begin{pmatrix} \hat{a}(\theta) \\ \hat{b}(\theta) \end{pmatrix} = \hat{u}_{BS}(\theta) \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} \hat{u}_{BS}(\theta)^\dagger.
\]
This makes us sure that the suggested effective Hamiltonian in (5) is introduced correctly. Now, let $|\pm\rangle_B$ be the input state, the output state can be obtained using definition (1), and the beam splitter operator (6) is

\[
\hat{u}_B(\theta)|\pm\rangle_A B = \frac{1}{\sqrt{N}} \sum_{n=0}^{\infty} \left( \cos \theta \right)_A |\pm\rangle_B |\pm\rangle_A B \sin \theta |\pm\rangle_B |\pm\rangle_A B,
\]

(8)

where $|\pm\rangle_A B = |\pm\rangle_A B$. Consequently, the QBS-like equation (9) is obtained for locations A and B. Now, we consider two separate locations C and D (shown in figures 2 and 3), and the above procedure is repeated for them. Accordingly, the QBS-like equation (9) is obtained for locations C and D as follows:

\[
|\psi^-\rangle_{CD} = \frac{1}{\sqrt{N}} \sum_{n=0}^{\infty} \left( \cos \right)_D |\pm\rangle_D |\pm\rangle_D |\pm\rangle_D C.
\]

(10)

In the continuation of this paper, the produced entanglement associated with locations (A, B) and (C, D) is swapped to the separate far distant locations (A, D). To do this, we use the QBSM process. The state of combined locations (A, B, C, D) is

\[
|\psi^-\rangle_{AB,CD} = |\psi^-\rangle_{AB} \otimes |\psi^-\rangle_{CD}.
\]

We conduct QBSM with the following QBS [48]

\[
|\psi^-\rangle_{AB} = \frac{1}{\sqrt{2(1-e^{-2|\beta|^2})}} (|\beta\rangle_B |\beta\rangle_C - |\beta\rangle_B |\beta\rangle_C)
\]

(11)

and

\[
|\psi^-\rangle_{CD} = \frac{1}{\sqrt{2(1-e^{-2|\beta'|^2})}} (|\beta'\rangle_D |\beta'\rangle_D - |\beta'\rangle_D |\beta'\rangle_D),
\]

(10)

on the state $|\Psi\rangle_{AB,CD}$, where $|\pm\rangle_B = e^{-\frac{|\pm\rangle^2}{2}} \sum_{n=0}^{\infty} \left( \cos \theta \right)_n |\eta\rangle_B$.

After this operation, the state in locations (A, D) is converted into the following QBS:

\[
|\Psi^-\rangle_{AD} = \frac{1}{\sqrt{N}} \left( |\alpha\rangle_A D - |\alpha\rangle_A D \right).
\]

(12)

Without loss of generality, $\alpha$ and $\beta$ are assumed to be real. By paying attention to the similarity of QBSs (9), (10), and (12), our model can be generalized to $2^N$ locations with $N = 3, 4, \ldots$, where the two end locations are entangled by $2^{N-1}$ times operation of QBSM performed with QBS (11). In the next section, we consider the effect of decoherence on the entanglement swapping.

3. Effect of decoherence

To consider the influence of dissipation, the produced two-mode entangled states that correspond to the separate locations (A, B) and (C, D) are sent to a noisy channel. In this respect, the produced QBSs (9) and (10) are mixed with vacuum states related to the environmental modes. Therefore, the states

\[
|\psi^-\rangle_{AB} \otimes |0\rangle_{E_1} \otimes |0\rangle_{E_2},
\]

(13)

and

\[
|\psi^-\rangle_{CD} \otimes |0\rangle_{E_1} \otimes |0\rangle_{E_2},
\]

(14)

are sent into the channel, where the subscripts $E_1(2), E_1(2)'$ relate to the environmental modes. The states (13) and (14), after passing the channel characterized by [49-51]

\[
|\pm\rangle_{A(B)} \otimes |0\rangle_{E_1(2)} \rightarrow |\pm\alpha\rangle_{A(B)} |0\rangle_{E_1(2)} \pm |\pm\alpha \sqrt{1-\eta}\rangle_{A(B)} |\eta\rangle_{E_1(2)},
\]

\[
|\pm\rangle_{C(D)} \otimes |0\rangle_{E_1(2)} \rightarrow |\pm\alpha\rangle_{C(D)} |0\rangle_{E_1(2)} \pm |\pm\alpha \sqrt{1-\eta}\rangle_{C(D)} |\eta\rangle_{E_1(2)}.
\]

(15)

are converted to the following states, respectively:

\[
|\phi_+\rangle_{AB,E_1E_2} = \frac{1}{\sqrt{N}} \left( |\pm\rangle_{A(B)} |\sqrt{\eta}\rangle_{A(B)} |\alpha\sqrt{1-\eta}\rangle_{E_1} |\alpha\sqrt{1-\eta}\rangle_{E_2} \right.
\]

\[
- |\pm\rangle_{C(D)} |\sqrt{\eta}\rangle_{C(D)} |\alpha\sqrt{1-\eta}\rangle_{E_1} |\alpha\sqrt{1-\eta}\rangle_{E_2}.
\]

(16)
where \( \eta \) is the transmittance of the noisy channel\(^1\). As mentioned above, the parameter \( 0 \leq \eta \leq 1 \) is related to the transmittance of the beam splitter, i.e. for \( \eta = 0 \) (maximum noise) the entanglement between locations (A, B) as well as (C, D) is completely washed out. Consequently, the entangled state for separate distant locations (A, D) after operating the QBSM performed with the QBS (11) on the state \( |\phi^-\rangle_{ABE_1E_2} \otimes |\phi^-\rangle_{CD,E_1E_2} \) results in

\[
|\phi^-\rangle_{ADE_1E_2E_1E_2} = \frac{1}{\sqrt{N'_c}} \left( |\alpha'\rangle_{A} |\alpha'\rangle_{E_1E_2} |\alpha'\rangle_{D} |\alpha'\rangle_{E_1E_2} \right. \\
- |\alpha'\rangle_{A} |\alpha'\rangle_{D} |\alpha'\rangle_{E_1E_2} |\alpha'\rangle_{E_1E_2} \\
- \left. - |\alpha'\rangle_{A} |\alpha'\rangle_{E_1E_2} |\alpha'\rangle_{D} |\alpha'\rangle_{E_1E_2} \right),
\]

(17)

where \( N'_c = 2(1 - e^{4(\eta - 2)}|\alpha'|^2) \). The density operator associated with state (18) after tracing over environmental modes can be obtained as follows:

\[
\rho_{AD} = \frac{1}{N''_c} \left( |\alpha'\rangle_{A} |\alpha'\rangle_{D} \right. \\
- |\alpha'\rangle_{A} |\alpha'\rangle_{D} \\
- \left. - |\alpha'\rangle_{A} |\alpha'\rangle_{D} \right),
\]

(19)

At this stage, to evaluate the entanglement degree (entanglement between A and a collection including D and environmental modes) we calculate the entropy [52] corresponding to the above density state via the relation

\[
S = 1 - \text{Tr} \rho_A^2,
\]

(20)

where \( \rho_A = \text{Tr}_{D} \rho_{AD} \), which results in

\[
S = 1 - \frac{2}{N''_c} \left( 1 - 4e^{-4|\alpha|^2} e^{2\eta|\alpha|^2} + e^{-8|\alpha|^2} e^{4\eta|\alpha|^2} \right) + e^{-2|\alpha|^2} + e^{-8|\alpha|^2} e^{4\eta|\alpha|^2}.
\]

(21)

Furthermore, the fidelity of density operator (19) is calculated as follows:

\[\text{F} = \langle \Phi | \rho_{AD} | \Phi \rangle,\]

\[
= \frac{1}{LN''_c} \left[ e^{-|\beta - \sigma|^2} e^{-|\gamma - \sigma|^2} + e^{-|\beta + \sigma|^2} e^{-|\gamma + \sigma|^2} \right] \\
- 2 e^{-|\beta - \sigma|^2} e^{-|\beta + \sigma|^2} e^{-|\gamma - \sigma|^2} e^{-|\gamma + \sigma|^2} e^{-8(1-\eta)|\alpha'|^2} \\
+ 2 \cos \varphi \left[ e^{-|\beta - \sigma|^2} e^{-|\gamma - \sigma|^2} e^{-|\gamma + \sigma|^2} e^{-|\mu - \sigma|^2} - e^{-|\beta + \sigma|^2} e^{-|\gamma - \sigma|^2} e^{-|\gamma + \sigma|^2} e^{-|\mu - \sigma|^2} \right] \\
+ e^{-|\beta - \sigma|^2} e^{-|\gamma - \sigma|^2} e^{-|\gamma + \sigma|^2} e^{-|\mu + \sigma|^2} \\
- e^{-|\beta + \sigma|^2} e^{-|\gamma - \sigma|^2} e^{-|\gamma + \sigma|^2} e^{-|\mu + \sigma|^2} \\
+ e^{-|\gamma - \sigma|^2} e^{-|\gamma + \sigma|^2} e^{-|\mu - \sigma|^2} e^{-|\mu + \sigma|^2} \\
- e^{-|\gamma - \sigma|^2} e^{-|\gamma + \sigma|^2} e^{-|\mu - \sigma|^2} e^{-|\mu + \sigma|^2} e^{-8(1-\eta)|\alpha'|^2} \right].
\]

(22)

Here,

\[|\Phi\rangle = \frac{1}{\sqrt{L}} \left( |\beta\rangle_{A} |\gamma\rangle_{D} + e^{i\varphi} |\omega\rangle_{A} |\mu\rangle_{D} \right),\]

\[L = 2 \left( 1 + e^{-|\beta - \omega|^2} e^{-|\gamma - \alpha|^2} \cos \varphi \right),\]

(23)

\(^1\)The photon loss is usually modeled via mixing the light mode with a vacuum state at a beam splitter with transmittance \( \eta \), where \( 1 - \eta \) is related to the loss probability of a single photon. The parameter \( \eta \) is defined as \( \eta = e^{-\frac{1}{L}} \), where \( L \) and \( L_{\text{opt}} \) are the optical propagation distance and attenuation length of the channel, respectively [49-51].
where \( \sigma = \alpha' \sqrt{\eta} = \alpha \sqrt{\frac{T}{2}} \) and \( |\beta|, |\gamma|, |\omega|, \) and \( |\mu| \) are the standard coherent states, i.e. \( |\zeta\rangle = e^{-|\zeta|^2/2} \sum_{n=0}^{\infty} \frac{c_n}{\sqrt{n!}} |n\rangle \), where \( \zeta = \beta, \gamma, \omega, \mu \).

In this section we have plotted the variation of entropy (21) (entanglement between A and a collection including D and environmental modes) as a function of \( \alpha \) and \( \eta \) in figure 3. From figure 3, one can see that the entropy is increased by increasing transparency \( \eta \). In this figure, the state of locations (A, D) is converted to QBS (12) for \( \eta = 1 \).

Furthermore, the effects of \( \alpha \) and \( \eta \) on fidelity (22) are considered in figure 4 for different QBSs. In figure 4(a), fidelity has been increased by increasing \( \eta \), and for \( \eta = 1 \) the state of locations (A, D) is completely converted to QBS (12); however, in figures 4(b) and (c), the maximum fidelity has been decreased by increasing \( \eta \). In figures 4(b) and (c), fidelity has been decreased by increasing \( \alpha \). Moreover, the maximum fidelity for \( \beta = \mu = \alpha' \), \( \gamma = \omega = -\alpha' \), and \( \varphi = \pi \) was calculated, and we found that for these considered circumstances, the fidelity is zero.

4. Summary and conclusions

In this paper the QBSs for two separate distant locations was produced using the 50 : 50 beam splitter, where the input states were the odd coherent state and vacuum state. Then, the produced entanglement between locations (A, B) and (C, D) was swapped to two separable far distant locations (A, D) using QBSM. Furthermore, the effect of decoherence on the swapped entanglement was considered. The entanglement of the produced entangled states was calculated via entropy, showing that the entropy increases with increasing transparency \( \eta \). Moreover, the maximum fidelity decreased with increasing \( \eta \) in most cases. We found that because of the stability of the produced entangled states, our model can be generalized for \( 2^n \) separate locations, where \( N = 3, 4, \ldots \), and the entanglement is swapped to two end locations by \( 2^{N-1} - 1 \) operations of QBSM. Therefore, we can introduce this model as a quantum repeater protocol to distribute the entangled coherent states to long distances.

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