Two Loop Radiative Seesaw Model with Inert Triplet Scalar Field

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We propose a radiative seesaw model with an inert triplet scalar field in which Majorana neutrino masses are generated at the two loop level. There are fermionic or bosonic dark matter candidates in the model. We find that each candidate can satisfy the WMAP data when its mass is taken to be around the half of the mass of the standard model like Higgs boson. We also discuss phenomenology of the inert triplet scalar bosons, especially focusing on the doubly-charged scalar bosons at Large Hadron Collider in parameter regions constrained by the electroweak precision data and WMAP data. We study how we can distinguish our model from the minimal Higgs triplet model.

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I. INTRODUCTION

Recent several experiments require the serious modifications of the standard model (SM) in spite of the great success. For an example, the SM Higgs boson search in the diphoton mode $h \rightarrow \gamma\gamma$ at Large Hadron Collider (LHC) is shown that its signal strength is $1.65 \pm 0.24$ at ATLAS $^1$ and $1.6 \pm 0.4$ at CMS $^3$. For another example, the existence of non-baryonic dark matters (DMs), which cannot be included in SM, dominates about 23% from the CMB observation by WMAP $^4$. The fact is strongly supported by the cosmological observations such as the rotation curves of the galaxy $^5$ and the gravitational lensing $^6$ in our universe. In recent years, direct detection experiments of DM; XENON100 $^7$, CRESSTII $^8$, CoGeNT $^9$ and DAMA $^{10}$, show the scattering events with nuclei. XENON100 has not shown a result of DM signal but shown an upper bound with the minimal bound around 100 GeV. On the other hand, CoGeNT, DAMA and CRESSTII have reported the observations which can be interpreted as DM signals that favor a light DM with several GeV mass and rather large cross section. As far as we consider these experiments, the mass scale of DM should be $\mathcal{O}(1-100)$ GeV.

In order to explain the excess in the $h \rightarrow \gamma\gamma$ channel, a modified diphoton event has been discussed in a lot of paper. If a model contains charged new particles which couple to the SM-like Higgs boson such as charged Higgs bosons, the decay rate of $h \rightarrow \gamma\gamma$ can be enhanced due to loop effects of these charged particles. However, in a model with only one pair of singly-charged scalar bosons such as two Higgs doublet models, it is difficult to predict around 60% enhancement of the decay rate unless the mass of the charged scalar bosons is taken to be smaller than about 100 GeV$^1$. The minimal Higgs triplet model (HTM) motivated from the type II seesaw mechanism $^{12}$, introducing the isospin triplet scalar field $\Delta$, can easily explain the diphoton anomaly. Moreover, if $\Delta$ can be inert scalar, then its neutral component can be a promising DM candidate that can strongly correlate with neutrinos. This is like a radiative seesaw models $^{13-42}$. Hence the theory can be realized at TeV scale, so be well-tested at current experiments like LHC.

In this paper, we propose a two-loop induced neutrino model with gauged $B-L$ symme-

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$^1$ If the decay rate of the Higgs to $bb$ mode is sufficiently suppressed compared to the SM value, the branching ratio of the Higgs to diphoton mode can be enhanced without changing its decay rate. In that case, the branching fraction of the Higgs to $WW^*$ and $ZZ^*$ modes are also enhanced $^{11}$. 
try that is an extension of the HTM. In the bosonic sector, three scalar fields are introduced in addition to the SM particles; \( \Delta \) and \( \eta \) are an \( SU(2)_L \) triplet and doublet fields, respectively which do not have vacuum expectation values (VEVs), and \( \chi \) is \( SU(2)_L \) singlet which acquires a VEV after the spontaneous \( B - L \) symmetry breaking \(^2\). In the fermionic sector, we introduce an exotic vector-like lepton with \( SU(2)_L \) doublet \(^{44}\), and three right-handed neutrino with \( SU(2)_L \) singlet, both of which can contribute to the radiative neutrino mass. Due to the abundant extra fields, we have several DM candidates. Also we can discuss the testability of the Higgs sector, especially, the doubly-charged scalar boson, in which we could have a discrimination to the HTM, since it couples to the exotic lepton. As a notice, such a complicated model can be realized within non-Abelian symmetries. So we briefly show how to realize our model in the appendix.

This paper is organized as follows. In Sec. II, we show our model building including the Higgs potential, stationary condition, neutrino mass, and lepton flavor violation (LFV). In Sec. III, we analyze DM phenomenologies. In Sec. IV, we analyze Higgs phenomenology including electroweak precision observables, signatures of the doubly-charged scalar boson at LHC. We summarize and conclude in Sec. V. In appendices, some result of detail calculations are write down; gauge boson two point functions, decay rate of the doubly-charged scalar boson, and the assignments for each particles in non-Abelian symmetries.

### II. THE RADIATIVE SEESAW MODEL

#### A. Model setup

| Particle | \( Q \) | \( u^c \) | \( d^c \) | \( L \) | \( e^c \) | \( N^c \) | \( L' \) | \( L'^c \) |
|----------|--------|--------|--------|--------|--------|--------|--------|--------|
| \((SU(2)_L, U(1)_Y)\) | (2, \( \frac{1}{6} \)) | (1, -2/3) | (1, \( \frac{1}{3} \)) | (2, -1/2) | (1, 1) | (1, 0) | (2, -1/2) | (2, 1/2) |
| \(U(1)_{B-L}\) | \( \frac{1}{3} \) | -1/3 | -1/3 | -1 | 1 | 1 | 1 | -1 |
| \(Z_2\) | + | + | + | + | + | + | - | - |

**TABLE I:** The particle contents and the charges for fermions. \( L' \) and \( L'^c \) are exotic leptons.

\(^2\) There exists a \( B - L \) gauge boson, however we neglect through our analysis because it can decouple to the other particles from the LEP II \(^{43}\).
We propose a two-loop radiative seesaw model with $U(1)_{B-L}$ gauge symmetry which is an extended model of the minimal HTM motivated from the type II seesaw mechanism \cite{12}. The particle contents are shown in Table II. We add three right-handed neutrinos $N^c$, vector-like $SU(2)_L$ doublet leptons $L'$ and $L'^c$, an $SU(2)_L$ triplet scalar $\Delta$, an $SU(2)_L$ doublet scalar $\eta$ and $B-L$ charged scalar $\chi$ to the SM, where $\eta$ and $\Delta$ do not have VEV. The $Z_2$ parity is also imposed so as to forbid the undesired terms. As a result, the neutrino mass is obtained not through the one-loop level (just like Ma-Model \cite{13}) but through the two-loop level and the stability of DM candidates can be assured. Where we define $L' \equiv (N^c_4, E^c_4)$ [44].

The renormalizable Lagrangians for Yukawa sector and Higgs potential are given by

$$-\mathcal{L}_{\text{Yukawa}} = y^\alpha_\ell \Phi^\dagger e^\dagger_\alpha L \beta + y^\beta_\nu \eta^\dagger N^c_\gamma L^\prime + y^\delta_\chi \tilde{\eta^\dagger i}\tau_2 \Delta L' + y^\mu S^a \chi N^c_a N^c_b + M L' L^c + \text{h.c.},$$

$$-\mathcal{L}_{\text{Higgs}} = m^2_1 \Phi^\dagger \Phi + m^2_2 \eta^\dagger \eta + m^2_3 \chi^\dagger \chi + m^2_4 \text{Tr}[\Delta^\dagger \Delta] - \mu[\Phi^T i \tau_2 \Delta^\dagger \eta] + \text{h.c.} + \lambda_1(\Phi^\dagger \Phi)^2 + \lambda_2(\eta^\dagger \eta)^2 + \lambda_3(\Phi^\dagger \Phi)(\eta^\dagger \eta) + \lambda_4(\Phi^\dagger \eta)(\eta^\dagger \Phi) + \lambda_5/2[(\Phi^\dagger \eta)^2 + \text{h.c.}] + \lambda_6(\chi^\dagger \chi)^2 + \lambda_7(\chi^\dagger \chi)(\Phi^\dagger \Phi) + \lambda_8(\chi^\dagger \chi)(\eta^\dagger \eta) + \lambda_9 \text{Det}(\Delta^\dagger \Delta)
+ \lambda_{10}(\text{Tr}[\Delta^\dagger \Delta])^2 + a_1 \text{Tr}[\Delta^\dagger \Delta](\eta^\dagger \eta) + a_2 \text{Tr}[\Delta^\dagger \tau_i \Delta](\eta^\dagger \tau_i \eta) + b_1 \text{Tr}[\Delta^\dagger \Delta](\Phi^\dagger \Phi) + b_2 \text{Tr}[\Delta^\dagger \tau_i \Delta](\Phi^\dagger \tau_i \Phi) + c_1 \text{Tr}[\Delta^\dagger \Delta](\chi^\dagger \chi),$$

where $\alpha, \beta, \gamma, \delta, a, b$ are the flavor indices. In the scalar potential, the couplings $\lambda_1, \lambda_2, \lambda_6,$ and $\lambda_9$ have to be positive to stabilize the potential. Notice here that one straightforwardly probes the term $\Phi N^c L$, which derives neutrino mass at tree level, cannot be forbidden by any Abelian symmetries. If we introduce non-Abelian discrete symmetries \cite{45, 46}, one finds some successful groups such as $T_7$ \cite{47–50} and $\Delta(27)$ \cite{51–53} to forbid the term, remaining that $\eta$ and $\Delta$ are one generation. It implies that such an extension does not affect on analyses of dark matter and Higgs phenomenology focused on the inert scalar bosons. So

\begin{table}[h]
\begin{tabular}{|c|c|c|c|c|}
\hline
Particle & $\Delta$ & $\Phi$ & $\eta$ & $\chi$ \\
\hline
$(SU(2)_L, U(1)_Y)$ & $\mathbf{(3, 1)}$ & $\mathbf{(2, 1/2)}$ & $\mathbf{(2, 1/2)}$ & $\mathbf{(1, 0)}$ \\
$U(1)_{B-L}$ & 0 & 0 & 0 & $-2$ \\
$Z_2$ & $-$ & $+$ & $-$ & $+$ \\
\hline
\end{tabular}
\caption{The particle contents and the charges for bosons.}
\end{table}

\footnote{We show in the appendix C how to realize.}
we stay to analyze our model with flavor independent way hereafter for simplicity.

The scalar fields $\Phi$, $\chi$, $\eta$ and $\Delta$ can be parameterized as

$$
\Phi = \begin{pmatrix} G^+ \\
\frac{1}{\sqrt{2}}(\phi^0 + v + iG^0) \end{pmatrix}, \quad \chi = \frac{1}{\sqrt{2}}(\chi_R + v' + i\chi_I),
$$

(II.3)

$$
\eta = \begin{pmatrix} \eta \\
\frac{1}{\sqrt{2}}(\eta + W) \end{pmatrix}, \quad \Delta = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\
\Delta^0_W & -\Delta^0_W \end{pmatrix}, \quad \text{with } \Delta^0_W = \frac{1}{\sqrt{2}}[\text{Re}\Delta^0 + i\text{Im}\Delta^0],
$$

(II.4)

where $v$ is VEV of the doublet Higgs field $\Phi$ satisfying $v^2 = 1/\sqrt{2G_F} \simeq (246 \text{ GeV})^2$, $v'$ is that of the singlet Higgs field $\chi$, and subscript $W$ denotes weak eigenstates. In Eq. (II.3), $G^\pm$ and $G^0$ ($\chi_I$) are (is) the Nambu-Goldstone bosons (boson) which are (is) absorbed by the longitudinal component of $W^\pm$ and $Z$ (additional $U(1)_{B-L}$ gauge boson). The VEVs of $\Delta$ and $\eta$ are taken to be zero because of the assumption of the unbroken $Z_2$ symmetry.

In this model, the $Z_2$-even fields $\Phi$ and $\chi$ cannot be mixed with the $Z_2$-odd fields $\Delta$ and $\eta$, so that the mass matrices for the component scalar fields from $\Phi$ and $\chi$ and those from $\Delta$ and $\eta$ can be separately considered. By inserting the tadpole conditions; $m_1^2 = -\lambda_1 v^2 - \lambda_7 v'^2/2$ and $m_3^2 = -\lambda_6 v^2 - \lambda_7 v^2/2$ (which is exactly the same as the results in Ref. [28]), the mass matrix for the CP-even scalar bosons in the basis of $(\phi^0, \chi^0)$ is calculated as

$$
\mathcal{M}^2(\phi^0, \chi^0) = \begin{pmatrix} 2\lambda_1 v^2 & \lambda_7 v v' \\
\lambda_7 v v' & 2\lambda_6 v^2 \end{pmatrix}, \quad (II.5)
$$

The mass matrices of the $Z_2$-odd scalar bosons are respectively obtained in the basis of $(\Delta^0_W, \eta^0_W)$, $(\text{Re}\Delta^0, \text{Re}\eta^0)$ and $(\text{Im}\Delta^0, \text{Im}\eta^0)$ by

$$
\mathcal{M}^2(\Delta^0_W, \eta^0_W) = \begin{pmatrix} m^2(\Delta^0_W) & \mu v/2 \\
\mu v/2 & m^2(\eta^0_W) \end{pmatrix}, \quad \mathcal{M}^2(\text{Re}\Delta^0, \text{Re}\eta^0) = \begin{pmatrix} m^2(\text{Re}\Delta^0) & \mu v/\sqrt{2} \\
\mu v/\sqrt{2} & m^2(\text{Re}\eta^0) \end{pmatrix},
$$

$$
\mathcal{M}^2(\text{Im}\Delta^0, \text{Im}\eta^0) = \begin{pmatrix} m^2(\text{Im}\Delta^0) & \mu v/\sqrt{2} \\
\mu v/\sqrt{2} & m^2(\text{Im}\eta^0) \end{pmatrix}, \quad (II.6)
$$
where we define as follows:

\[ m^2(\Delta^\pm_W) = m_4^2 + \frac{1}{2}c_1v^2 + \frac{1}{2}b_1v^2, \quad (\text{II.7}) \]

\[ m^2(\text{Re}\Delta^0) = m^2(\text{Im}\Delta^0) = m^2(\Delta^\pm_W) + \frac{b_2}{2}v^2, \quad (\text{II.8}) \]

\[ m^2(\eta^\pm_W) = m_2^2 + \frac{1}{2}\lambda_3v^2 + \frac{1}{2}\lambda_8v^2, \quad (\text{II.9}) \]

\[ m^2(\text{Re}\eta^0) = m_2^2 + \frac{1}{2}\lambda_8v^2 + \frac{1}{2}(\lambda_3 + \lambda_4 + \lambda_5)v^2, \quad (\text{II.10}) \]

\[ m^2(\text{Im}\eta^0) = m_2^2 + \frac{1}{2}\lambda_8v^2 + \frac{1}{2}(\lambda_3 + \lambda_4 - \lambda_5)v^2. \quad (\text{II.11}) \]

The mass eigenstates for the \( \mathbb{Z}_2 \)-even Higgs bosons and those for the \( \mathbb{Z}_2 \)-odd scalar bosons can be defined by introducing the mixing angles \( \alpha, \beta, \gamma \) and \( \delta \) as

\[
\begin{pmatrix}
\phi^0 \\
\chi^0_R
\end{pmatrix} = R(\alpha) \begin{pmatrix} h \\ H \end{pmatrix}, \quad \begin{pmatrix} \Delta^\pm_W \\
\eta^\pm_W \end{pmatrix} = R(\beta) \begin{pmatrix} \Delta^\pm \\
\eta^\pm \end{pmatrix}, \\
\begin{pmatrix} \text{Re}\Delta^0 \\
\text{Re}\eta^0 \end{pmatrix} = R(\gamma) \begin{pmatrix} \Delta^0_R \\
\eta^0_R \end{pmatrix}, \quad \begin{pmatrix} \text{Im}\Delta^0 \\
\text{Im}\eta^0 \end{pmatrix} = R(\delta) \begin{pmatrix} \Delta^0_I \\
\eta^0_I \end{pmatrix},
\]

with

\[ R(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta \\
\sin \theta & \cos \theta \end{pmatrix}, \quad (\text{II.12}) \]

where \( h \) can be regarded as the SM-like Higgs boson. These mixing angles are expressed as

\[
\tan 2\alpha = \frac{\lambda_7v^2}{\lambda_1v^2 - \lambda_6v^2}, \quad \tan 2\beta = \frac{\mu v}{m^2(\Delta^\pm_W) - m^2(\eta^\pm_W)}, \\
\tan 2\gamma = \frac{\sqrt{2}\mu v}{m^2(\text{Re}\Delta^0) - m^2(\text{Re}\eta^0)}, \quad \tan 2\delta = \frac{\sqrt{2}\mu v}{m^2(\text{Im}\Delta^0) - m^2(\text{Im}\eta^0)}. \quad (\text{II.13})
\]

The mass eigenvalues are calculated by using the mixing angles given in Eq. (II.13) and the mass matrices given in Eqs. (II.5) and (II.6) as

\[
\begin{align*}
R^T(\alpha)M^2(\phi^0, \chi^0)R(\alpha) &= \begin{pmatrix} m_h^2 & 0 \\
0 & m_R^2 \end{pmatrix}, & R^T(\beta)M^2(\Delta^\pm_W, \eta^\pm_W)R(\beta) &= \begin{pmatrix} m_{\Delta^+_W}^2 & 0 \\
0 & m_{\eta^+_W}^2 \end{pmatrix}, \\
R^T(\gamma)M^2(\Delta^0_R, \eta^0_R)R(\gamma) &= \begin{pmatrix} m_{\Delta^+_R}^2 & 0 \\
0 & m_{\eta^+_R}^2 \end{pmatrix}, & R^T(\delta)M^2(\Delta^0_I, \eta^0_I)R(\delta) &= \begin{pmatrix} m_{\Delta^+_I}^2 & 0 \\
0 & m_{\eta^+_I}^2 \end{pmatrix}.
\end{align*} \quad (\text{II.14})
\]

The doubly-charged triplet scalar bosons do not mix with others, and those masses are given by

\[ m_{\Delta^{++}}^2 = m^2(\Delta^\pm_W) - \frac{1}{2}b_2v^2. \quad (\text{II.15}) \]
FIG. 1: Neutrino mass generation via two-loop radiative seesaw. The particles indicated by a red font have the opposite $Z_2$ charge to those by a black font.

We note that there is a characteristic relations for the mass spectrum among the triplet-like scalar bosons $\Delta^{\pm\pm}$, $\Delta^{\pm}$, $\Delta^0_R$ and $\Delta^0_I$ in the limit of $\mu \to 0$ as

\begin{align}
  m_{\Delta^{++}}^2 - m_{\Delta^+}^2 &= m_{\Delta^+}^2 - m_{\Delta^0}^2, \quad (\text{II.16}) \\
  m_{\Delta^0}^2 &= m_{\Delta_R}^2 = m_{\Delta_I}^2. \quad (\text{II.17})
\end{align}

Therefore, two of four mass parameters for the triplet-like scalar bosons are determined by using above two equations. The same relations also appear in HTM when the lepton number violating coupling constant is taken to be zero [54].

B. Neutrino mass matrix

The active neutrino mass matrix (depicted in Fig.1) through two loop contribution is given by

\begin{equation}
(M_\nu)_{\alpha\beta} = \frac{(y_\Delta)_\alpha (y_\nu)_\beta^2 (M_{N_k})_2}{4} M^2 (y_\Delta)_\beta \int \frac{d^4 p}{(2\pi)^4} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(p^2 - M^2)^2 (p + q)^2 - M^2_{N_k}} \left( \frac{\sin^2 \gamma}{p^2 - m^2_{\eta_R}} + \frac{\cos^2 \gamma}{p^2 - m^2_{\eta_I}} - \frac{\sin^2 \delta}{p^2 - m^2_{\eta_R}} - \frac{\cos^2 \delta}{p^2 - m^2_{\eta_I}} \right) \times \left( \frac{\cos^2 \gamma}{q^2 - m^2_{\eta_R}} + \frac{\sin^2 \gamma}{q^2 - m^2_{\eta_I}} - \frac{\cos^2 \delta}{q^2 - m^2_{\eta_R}} - \frac{\sin^2 \delta}{q^2 - m^2_{\eta_I}} \right), \quad (\text{II.18})
\end{equation}
where

$$\int \frac{d^4p}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \frac{1}{(p^2 - M^2)^2} \frac{1}{(p + q)^2 - M_{N_k}^2} \frac{1}{p^2 - m_a^2} \frac{1}{q^2 - m_b^2}$$

$$= \frac{1}{(4\pi)^4} \int_0^1 dx \int_0^{1-x} dz \int_0^1 d\rho \frac{x\rho}{(z^2 - z)^2 (1 - \rho) M_{N_a} - \rho q^2},$$  \hspace{1cm} (II.19)

$$q^2 = \frac{x}{z^2 - z} M^2 + \frac{1 - x - z}{z^2 - z} m_a^2 + \frac{1}{z - 1} m_b^2.$$  \hspace{1cm} (II.20)

As can be seen in the above equation, we can reproduce observed neutrino masses $\sim \mathcal{O}(0.1)$ eV and the mixing data because of many parameters.

### C. Lepton Flavor Violation

We investigate the LFV process $\ell_\alpha \rightarrow \ell_\beta \gamma$ ($\ell_\alpha, \ell_\beta = e, \mu, \tau$) as shown in Fig. 2. The experimental upper bounds of the branching ratios are $\mathcal{B}(\mu \rightarrow e\gamma) \leq 2.4 \times 10^{-12}$, $\mathcal{B}(\tau \rightarrow \mu\gamma) \leq 4.4 \times 10^{-8}$ and $\mathcal{B}(\tau \rightarrow e\gamma) \leq 3.3 \times 10^{-8}$. The branching ratios of the processes $\ell_\alpha \rightarrow \ell_\beta \gamma$ are calculated as

$$\mathcal{B}(\ell_\alpha \rightarrow \ell_\beta \gamma) = \frac{3\alpha_{em}}{64\pi G_F^2} \left| y_{\Delta_a}^+ y_{\Delta_\beta} \right|^2 \left[ F_2 \left( \frac{M^2}{m_{\eta^+}^2} \right) \frac{\cos^2 \beta}{m_{\eta^+}^2} + F_2 \left( \frac{M^2}{m_{\Delta^+}^2} \right) \frac{\sin^2 \beta}{m_{\Delta^+}^2} \right]^2 \mathcal{B}(\ell_\alpha \rightarrow \ell_\beta \nu_\beta \nu_\alpha),$$  \hspace{1cm} (II.21)

where $\alpha_{em} = 1/137$, $\mathcal{B}(\mu \rightarrow e\tau_e\nu_\mu) = 1.0$, $\mathcal{B}(\tau \rightarrow e\tau_e\nu_\tau) = 0.178$, $\mathcal{B}(\tau \rightarrow \mu\tau_\mu\nu_\tau) = 0.174$, $G_F$ is the Fermi constant and the loop function $F_2(x)$ is given by

$$F_2(x) = \frac{1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x}{6(1 - x)^4}.$$  \hspace{1cm} (II.22)
We discuss DM candidates in this section. We have six DM candidates in general; that is, \( \eta_R(\text{I}), \Delta_R(\text{I}) \), the lightest one of \( N^c \), and \( N_4 \). However \( N_4 \) cannot be the candidate because \( N_4 \bar{N}_4 \) annihilation via \( Z \) boson gives too large cross section to obtain the observed relic density \( \Omega h^2 \simeq 0.11 \) \[4\] as well as too large scattering cross section in the direct detection search \[7\] \footnote{In Ref. \[44, 57\], \( N_4 \) is considered as a DM.}. Moreover, we restrict ourselves that only the real part of \( \eta_R(\text{I}) \) and \( \Delta_R(\text{I}) \) are considered as a DM candidates, since the DM property of the imaginary part is more or less the same as the real one. Hence we analyze three DM candidates; \( \eta_R, \Delta_R \), the lightest one of \( N^c \), below. Hereafter we symbolize the DM mass as \( m_{\text{DM}} \).
A. Fermionic Dark Matter

We discuss a fermionic DM candidate $N^c$, assuming the following mass hierarchy $M_1 < M_2 < M_3$ for the right-handed neutrinos $N^c_i$. Notice here that the DM mass range be less than $M = 100 \text{ GeV}$ to avoid the too short lifetime of DM.

**WMAP**: At first, we analyze the DM relic density from WMAP. We have only the s-channel process via the Higgs bosons $h/H$ as shown in the upper panel of Fig. 3. The effective cross section to $f\overline{f}/W^+W^-/Z^0$ is given as

$$
\sigma_{\text{eff}}^N \nu_{\text{rel}} \simeq \frac{m_{\text{DM}}^2 \sin^2 \alpha \cos^2 \alpha}{8\pi v'^2 v^2} \left[ \frac{1}{4m_{\text{DM}}^2 - m^2_h + im_h \Gamma_h} - \frac{1}{4m_{\text{DM}}^2 - m^2_H + im_H \Gamma_H} \right]^2 v_{\text{rel}}^2
\times \sum_{i=f,V} \left( 1 - \frac{m_i^2}{m_{\text{DM}}^2} \right)^{1/2} \left[ 3m_i^2 m_{\text{DM}}^2 \left( 1 - \frac{m_i^2}{m_{\text{DM}}^2} \right) \delta_{i,f} + m_i^4 \left( \frac{3}{2} - 2 \frac{m_{\text{DM}}^2}{m_i^2} + 2 \frac{m_{\text{DM}}^2}{m_i^2} \right) \delta_{i,V} \right]
\approx \frac{m_{\text{DM}}^2 \sin^2 \alpha \cos^2 \alpha}{8\pi v'^2 v^2} \left[ \frac{1}{4m_{\text{DM}}^2 - m^2_h + im_h \Gamma_h} \right]^2 v_{\text{rel}}^2 \sum_{i=f,V} \left( 1 - \frac{m_i^2}{m_{\text{DM}}^2} \right)^{1/2} \left[ 3m_i^2 m_{\text{DM}}^2 \left( 1 - \frac{m_i^2}{m_{\text{DM}}^2} \right) \delta_{i,b} + m_i^4 \left( \frac{3}{2} - 2 \frac{m_{\text{DM}}^2}{m_i^2} + 2 \frac{m_{\text{DM}}^2}{m_i^2} \right) \delta_{i,V} \right],
$$

(III.1)

where $V = W^\pm, Z^0$ and we neglected the light quarks contributions and put only bottom quark contribution; that is, $f \simeq b$, and $v_{\text{rel}} \simeq O(0.2)$, is the relative velocity of incoming DM. Here we neglect the contribution of $H$, assuming the mass is enough heavy. The SM-like Higgs mass is fixed to $m_h = 125 \text{ GeV}$. Notice here that the channel of $VV$ is opened only if $m_V \leq m_{\text{DM}}$. In $m_b \leq m_h/2 \leq m_{\text{DM}}$, the total decay width of $h$ is $\Gamma_h = 4.1 \times 10^{-3} \text{ GeV}$ [59]. Moreover, in $m_{\text{DM}} \leq m_h/2$, the channel $h \rightarrow 2\text{DM}$ is also added and given by

$$
\Gamma_h(h \rightarrow 2\text{DM}) \simeq \frac{m_h}{8\pi} \left( \frac{m_{\text{DM}} \sin \alpha}{v'} \right)^2 \left[ 1 - 4 \left( \frac{m_{\text{DM}}}{m_h} \right)^2 \right]^{3/2},
$$

(III.2)

which is known as an invisible decay and recently reported by the LHC experiment that the branching ratio $B_{\text{inv}}$ is excluded to the region $0.4 \leq B_{\text{inv}}$ [60].

**Direct Detections**: Let us move on to the discussion of direct detections. Our DM interacts with quarks via Higgs exchange. Thus it is possible to explore DM in direct detection experiments like XENON100 [61]. The Spin Independent (SI) elastic cross section $\sigma_{\text{SI}}$ with nucleon $N$ is given by

$$
\sigma_{\text{SI}}^N \simeq \frac{\mu_{\text{DM}}^2}{m_h^2 \pi} \left( \frac{(y s)_{11} m_N \sin \alpha \cos \alpha}{\sqrt{2} v} \sum_q f_q^N \right)^2,
$$

(III.3)
where $\mu_{\text{DM}} = (m_{\text{DM}}^{-1} + m_N^{-1})^{-1}$ is the DM-nucleon reduced mass and the heavy Higgs contribution is neglected. The parameters $f_q^N$ which imply the contribution of each quark to nucleon mass are calculated by the lattice simulation \[61, 62\] as

\[
\begin{align*}
  f_p^p &= 0.023, & f_d^p &= 0.032, & f_s^p &= 0.020, \\
  f_u^n &= 0.017, & f_d^n &= 0.041, & f_s^n &= 0.020,
\end{align*}
\] (III.4)

for the light quarks and $f_Q^N = 2/27 \left(1 - \sum_{q \leq 3} f_q^N\right)$ for the heavy quarks $Q$ where $q \leq 3$ implies the summation of the light quarks. The recent another calculation is performed in Ref. \[63\].

Considering all the above constraints, we find that the allowed region is sharp at around $m_h/2$.

### B. Bosonic Dark Matters

We discuss bosonic DM candidates $\eta_R$ and $\Delta_R$ \[64\]. Notice here that the DM mass range be less than $M_W = 80$ GeV to satisfy the constraint of the antiproton no excess reported by PAMELA \[65\] as well as WMAP 5.

**WMAP.** At first, we analyze the DMs relic density from WMAP. We have two annihilation modes; $t$ and $u$ channel of $2\eta_R/\Delta_R \rightarrow N_4(E_4) \rightarrow 2\nu(\ell \bar{\ell})$ and $s$-channel of $2\eta_R/\Delta_R \rightarrow h/H \rightarrow f \bar{f}$ 6. However since the $t$ and $u$ channel does not have $s$ wave contribution, the dominant cross section is given only through the $s$ channel \[31\] as shown in the lower diagram of Fig. 3. The effective cross section to $f \bar{f}$ is given as

\[
\begin{align*}
  \sigma_{\text{eff}}^{\eta_R/\Delta_R} v_{\text{rel}} &\simeq \frac{3m_b^2}{4\pi v^2} \left| \frac{\lambda_{h}^{\eta_R/\Delta_R} \cos \alpha}{4m_{\text{DM}}^2 - m_h^2 + im_h \Gamma_h} \right|^2 \left(1 - \frac{m_h^2}{m_{\text{DM}}^2}\right)^{3/2}, \\
  \lambda_h^{\eta R} &\equiv \cos \alpha \left[c \cos^2 \gamma (\lambda_3 + \lambda_4 + \lambda_5) + \sin^2 \gamma (b_1 + b_2) - \sqrt{2}\mu \sin \gamma \cos \gamma \right] v \\
  &\quad - \sin \alpha (\cos^2 \gamma \lambda_8 + \sin^2 \gamma \lambda_{12}) v', \\
  \lambda_h^{\Delta R} &\equiv \cos \alpha \left[c \sin^2 \gamma (\lambda_3 + \lambda_4 + \lambda_5) + \cos^2 \gamma (b_1 + b_2) + \sqrt{2}\mu \sin \gamma \cos \gamma \right] v \\
  &\quad - \sin \alpha (\sin^2 \gamma \lambda_8 + \cos^2 \gamma \lambda_{12}) v'.
\end{align*}
\] (III.6)

\[5\] If the charged vector boson channel are open, the cross section is too large to satisfy the observed relic abundance.

\[6\] We neglect the $H$ contribution.
When our DMs are less than $m_h/2$, the invisible decay width is given by \[ \Gamma^{\eta_R/\Delta_R}(h \to 2\text{DM}) \approx \left(\frac{\lambda_{h_R}^{\eta_R/\Delta_R}}{16\pi m_h}\right)^2 \sqrt{1 - \frac{4m_{\text{DM}}^2}{m_h^2}}. \] (III.9)

**Direct Detections**: In the direct detection, the spin independent elastic cross section $\sigma_{SI}$ with nucleon $N$ is given by
\[
\sigma_{SI}^{\eta_R/\Delta_R} = \frac{\mu_{\text{DM}}^2}{\pi} \frac{m_N^2}{m_N^2 v^2} \left(\frac{\lambda_{h_R}^{\eta_R/\Delta_R} \cos \alpha}{m_h^2}\right)^2 \left(\sum_q f_q^N\right)^2,
\] (III.10)

where $\mu_{\text{DM}}$ and $f_q^N$ has been defined in the fermionic part. We here comment on the scenario based on the neutral component of the inert triplet field being the DM candidate, which has been discussed in Ref. [63]. Such a scenario is severely constrained by the direct detection experiments because of the Z boson exchanging contribution if the CP-even scalar boson and the CP-odd scalar boson from the triplet field are degenerate in mass. According to Ref. [63], the mass of the DM candidate has to be around 2.8 TeV in the model with $Y=1$ inert triplet field in order to satisfy both the WMAP data and the direct search data. However, this result cannot be applied to our model, because we can take a mass splitting between $\Delta^0_R$ and $\Delta^0_I$ due to the mixing between $\eta$ and $\Delta$. Thus, we can avoid the Z boson exchanging contribution to the direct search experiments.

Considering all the above constraints, we find that the allowed region is the same as the fermionic case; that is, around $m_h/2$. As a result, we assume to analyze that DM mass (especially $\Delta^0$) is $m_h/2$ in the next section.

**IV. HIGGS PHENOMENOLOGY**

**A. Electroweak precision observables**

We discuss the constraints of the Higgs parameters from the electroweak precision observables; i.e., the Peskin-Takeuchi $S$, $T$ and $U$ parameters [66]. If the mixing angle $\alpha$ is taken to be zero, the new physics contributions to the $S$ and $T$ parameters can be separated from those from the SM, so that we here assume $\alpha = 0$ for simplicity. Then the new physics
contributions to $S$ and $T$ are calculated as

$$S_{\text{new}} = 16\pi \left[ \frac{\Sigma^3_{T}(m_Z^2) - \Sigma^3_{T}(0)}{m_Z^2} - \frac{\Sigma^{33}_{T}(m_Z^2) - \Sigma^{33}_{T}(0)}{m_Z^2} \right], \quad (IV.1)$$

$$T_{\text{new}} = 4\sqrt{2}G_F \frac{\alpha_{\text{em}}}{\alpha_{\text{em}}} \left[ \Sigma^{33}_{T}(0) - \Sigma^{11}_{T}(0) \right], \quad (IV.2)$$

where $\Sigma^{11}_{T}$, $\Sigma^{3Q}_{T}$ and $\Sigma^{33}_{T}$ are obtained by calculating the 1PI diagrams of the gauge boson two-point functions at the one-loop level whose analytic expressions are given in Appendix A. The experimental bound for the deviations in the $S$ and $T$ parameters from the SM prediction with the Higgs boson mass to be 117 GeV are given by fixing $\Delta U = 0$ as

$$\Delta S = 0.04 \pm 0.09, \quad \Delta T = 0.07 \pm 0.08, \quad (IV.3)$$

with the correlation factor to be 88% \[56\]. The prediction in our model for $\Delta S$ and $\Delta T$ are calculated by $S_{\text{new}} - (S_{\text{SM}} - S_{\text{SM}}|_{m_h=117 \text{ GeV}})$ and $T_{\text{new}} - (T_{\text{SM}} - T_{\text{SM}}|_{m_h=117 \text{ GeV}})$, respectively, where $S_{\text{SM}}$ ($S_{\text{SM}}|_{m_h=117 \text{ GeV}}$) is the prediction for $S$ in the SM with the SM Higgs boson mass to be $m_h$ (117 GeV), and $T_{\text{SM}}$ ($T_{\text{SM}}|_{m_h=117 \text{ GeV}}$) is those corresponding $T$ parameter. We note that the exotic lepton $L'$ does not contribute to the $S$ and $T$ parameters, because both the masses of the component lepton fields ($N_4$ and $E_4$) are given by $M$, and this field is introduced as the vector-like, so that both the custodial symmetry and the chiral symmetry are not broken by this field.

In Fig. 4, $S$-$T$ plot is shown in the case where $m_{\eta^+} = m_{\eta^0} = m_{\eta^0} = 300 \text{ GeV}$, and all the mixing angles are taken to be zero, so that the masses of the scalar bosons from the triplet field are determined by fixing two parameters; i.e., $m_{\Delta^0}$ and $\Delta m(= m_{\Delta^+} - m_{\Delta^0})$. Each dotted curve shows the prediction of the $S$ and $T$ parameters in the cases with $m_{\Delta^0} = 63 \text{ GeV} (m_h/2)$, 100 GeV and 200 GeV. The interval of each dot indicates the increment of $\Delta m(= m_{\Delta^+} - m_{\Delta^0})$ to be 3 GeV from right to left. The inside (outside) ellipse indicates allowed region with the 68% (95%) confidence level by the electroweak precision data. It is seen that that the $T$ parameter is getting larger values when $\Delta m$ is taken to be larger values. When $m_{\Delta^0}$ is taken to be 63 GeV, 100 GeV and 200 GeV, the allowed maximum value for $\Delta m$ with 95% confidence level is about 15 GeV, 23 GeV and 30 GeV, respectively.
FIG. 4: Prediction of the $S$ and $T$ parameters for the case with $m_{\eta^+} = m_{\eta^0} (= m_{\eta_R} = m_{\eta_I}) = 300$ GeV. All the mixing angles ($\alpha$, $\beta$, $\gamma$ and $\delta$) are taken to be zero in this plot. Each dotted curve is shown the results in the cases with $m_{\Delta^0}$ to be 63 GeV, 100 GeV and 200 GeV, where the interval of each dot indicates the increment of $\Delta m (= m_{\Delta^+} - m_{\Delta^0})$ to be 3 GeV from right to left. The regions within the blue (red) ellipse are allowed at the 68% (95%) confidence level by the electroweak precision data.

B. Signature of $\Delta^{\pm\pm}$ at LHC

We discuss how our model can be tested at collider experiments. We focus on the signature from the doubly-charged scalar bosons $\Delta^{\pm\pm}$, because appearance of $\Delta^{\pm\pm}$ is one of the striking properties of the model. In order to focus on the phenomenology of the triplet-like scalar bosons, we assume that the mixing between $\Delta$ and $\eta$ are taken to be quite small ($\beta \simeq \gamma \simeq \delta \simeq 0$), and the masses of $\eta^\pm$ and $\eta^0$ are much heavier than those of the triplet-like scalar bosons.

There are an indirect way and a direct way to identify existence of $\Delta^{\pm\pm}$. The farmer way is measuring the deviations in the event rate for the Higgs boson decay channels from the SM values. In particular, the Higgs to diphoton mode $h \rightarrow \gamma\gamma$ is one of the most important channels for the SM Higgs boson search at the LHC because of the clear signature. The
The current signal strength for this channel is $1.65 \pm 0.24$ at the ATLAS [1, 2] and $1.6 \pm 0.4$ at the CMS [3]. The decay rate of $h \rightarrow \gamma\gamma$ can be modified by the loop effect of $\Delta^{\pm\pm}$. Contributions from doubly-charged scalar bosons to the $h \rightarrow \gamma\gamma$ mode have already been analyzed in the several papers in the HTM [67, 68]. The signal strength for $h \rightarrow \gamma\gamma$ can be larger than 1.6 in the case with $m_{\Delta^{\pm\pm}}$ to be smaller than about 200 GeV [68] without contradiction with the constraints from the vacuum stability [69] and the perturbative unitarity [69, 70]. The prediction of the deviation in the decay rate of $h \rightarrow \gamma\gamma$ in our model is almost the same as that in the HTM as long as the contributions from $\eta^\pm$ are neglected.

Discovery of $\Delta^{\pm\pm}$ can be a direct evidence of our model. We consider the case where a lighter neutral scalar boson from the triplet field ($\Delta_R$ or $\Delta_I$) is assumed to be DM. In that case, the neutral scalar boson should be the lightest of all the triplet-like scalar bosons; i.e., $m_{\Delta^0} \leq m_{\Delta^+} \leq m_{\Delta^{++}}$ to guarantee the stability of DM, and its mass is around the half of the Higgs boson mass $m_h$ to satisfy WMAP data and direct detection experiments. In addition, the mass differences among the triplet-like scalar bosons are restricted from the $S$ and $T$ parameter as shown in Fig. 4 e.g., $\Delta m$ is constrained to be less than 15 GeV in the case with $m_{\Delta^0} = 63$ GeV. This implies that the upper limit for $m_{\Delta^{++}}$ is about 90.5 GeV by using the mass relation given in Eq. (II.16).

A search for doubly-charged Higgs bosons has been done at LEP [71], Tevatron [72] and LHC [73]. All these searches have been done under the assumption where doubly-charged Higgs bosons decay into the same sign dilepton. The most stringent lower bound for the mass of doubly-charged Higgs bosons is about 400 GeV given at LHC [73]. In our model, $\Delta^{\pm\pm}$ cannot decay into the same sign dilepton associated without any other particles, because they are the $Z_2$ odd particles. In that case, the mass bound given at LHC cannot be applied to that in our model.

First, we discuss the decay of $\Delta^{\pm\pm}$ in the scenario based on the lighter neutral component of $\Delta$ assumed to be DM. Basically, there are two decay modes of $\Delta^{\pm\pm}$: (1) the same sign dilepton decay through the Yukawa coupling $y_\Delta$ ($\Delta^{\pm\pm} \rightarrow E_4^\pm \ell^\pm$) with $\ell^\pm$ to be $e^\pm$, $\mu^\pm$ or $\tau^\pm$, and (2) W boson associated decay through the gauge coupling constant ($\Delta^{\pm\pm} \rightarrow \Delta^\pm W^{\pm*}$). The decay branching fractions for (1) and (2) are determined by the magnitude of $y_\Delta$, $\Delta m$

\footnote{The magnitudes of the branching fractions of $\Delta^{\pm\pm}$ in the same sign dilepton modes: $E_4^\pm e^\pm$, $E_4^\pm \mu^\pm$ and $E_4^\pm \tau^\pm$ depend on the value of $y_\Delta^4$. In the following discussion for the collider phenomenology, we do not specify the flavor of $\ell^\pm$.}
and the mass of the exotic lepton $M$. The formulae for the decay rates for these channels are given in Appendix B.

In Fig. 5, the branching fraction of $\Delta^{\pm \pm}$ is shown as a function of $M$. We take $m_{\Delta^{++}} = 90.5$ GeV and $m_{\Delta^+} = 78$ GeV which correspond to the case with $m_{\Delta^0} = 63$ GeV and $\Delta m = 15$ GeV. The Yukawa coupling $y_\Delta$ is taken to be 0.1 and 0.01. It is seen that the main decay mode is changed from $\Delta^{++} \to E_4^+ \ell^+$ to $\Delta^{++} \to \Delta^+ W^+\ast$ when $M$ is getting larger values. For example, 50% of $\mathcal{B}(\Delta^{++} \to E_4^+ \ell^+)$ can be obtained in the case of $M \simeq 89$ GeV (84 GeV) and $y_\Delta = 0.1$ (0.01).

In the following, we discuss the case where $\Delta^{\pm \pm}$ mainly decay into the same sign dilepton ($\Delta^{\pm \pm} \to \ell^\pm E_4^{\pm}$). In that case, the exotic lepton mass $M$ should be between $m_{\Delta^{++}}$ and $m_{\Delta^+}$, otherwise there is no possible decay channel for $E_4^\pm$. As a benchmark scenario, we take the following mass spectrum and the coupling constant which is allowed from the electroweak precision data and also the LFV data:

\[
m_{\Delta^{++}} = 90.5 \text{ GeV}, \quad m_{\Delta^+} = 78 \text{ GeV}, \quad m_{\Delta^0} = 63 \text{ GeV}, \quad M = 85 \text{ GeV}, \quad y_\Delta = 0.1.
\]  

(IV.4)

In this parameter set, the branching fraction of $\Delta^{\pm \pm} \to E_4^+ \ell^\pm$ is about 99%, and that of
\[ \Delta^{\pm} \rightarrow W^{\pm}\Delta^{0} \] and \[ E_{4}^{\pm} \rightarrow \Delta^{\pm}\nu \] are 100%, where \( \Delta^{0} \) is \( \Delta_{R} \) or \( \Delta_{I} \). Thus, the decay process of \( \Delta^{++} \) is expected as follows:

\[ \Delta^{++} \rightarrow \ell^{+}E_{4}^{+} \rightarrow \ell^{+}\Delta^{+}\nu \rightarrow \ell^{+}W^{+}\Delta^{0}\nu \rightarrow \ell^{+}\ell^{+}\Delta^{0}\nu\nu, \] (IV.5)

so that the final state contains the same sign dilepton and the missing energy.

We then discuss the signal of \( \Delta^{\pm\pm} \) at LHC in the parameter set given in Eq. (IV.4). At LHC, \( \Delta^{\pm\pm} \) can be mainly produced via the Drell-Yan processes: \( q\bar{q} \rightarrow \gamma^{*}/Z^{*} \rightarrow \Delta^{++}\Delta^{--} \) and \( q\bar{q}' \rightarrow W^{\pm*} \rightarrow \Delta^{\pm\pm}\Delta^{\mp} \). Thus, the signal events are expected to be

\[ q\bar{q} \rightarrow \Delta^{++}\Delta^{--} \rightarrow (\ell^{+}W^{+}\nu\Delta^{0})(\ell^{-}W^{-}\nu\Delta^{0}) \rightarrow \ell^{+}\ell^{+}\ell^{-}\ell^{-} E_{T}, \] (IV.6)

\[ q\bar{q}' \rightarrow \Delta^{\pm\pm}\Delta^{\mp} \rightarrow (\ell^{\pm}W^{\pm}\nu\Delta^{0})(W^{\mp}\Delta^{0}) \rightarrow \ell^{\pm}\ell^{\pm} E_{T}. \] (IV.7)

The cross sections of \( \sigma(pp \rightarrow \Delta^{++}\Delta^{--}) \), \( \sigma(pp \rightarrow \Delta^{++}\Delta^{-}) \) and \( \sigma(pp \rightarrow \Delta^{-}\Delta^{+}) \) are respectively evaluated as 566 fb, 889 fb and 494 fb with the collision energy to be 8 TeV by using CalcHEP [74] and CTEQ6L parton distribution functions. The cross sections for the final states expressed in Eqs. (IV.6) and (IV.7) are obtained as 63 fb and 154 fb, respectively.

The same events can happen in HTM. The doubly-charged Higgs bosons \( H^{\pm\pm} \) in HTM can decay into the same sign diboson \( H^{\pm\pm} \rightarrow W^{\pm}W^{\pm} \) which is realized in the parameter regions where VEV of the triplet field \( v_{\Delta} \) is larger than about \( 10^{-4} \) GeV, and \( H^{\pm\pm} \) is the lightest of all the Higgs bosons from the triplet field. At the same time, singly-charged Higgs bosons \( H^{\pm} \) in HTM can decay into \( W^{\pm}Z \) in this case as long as the mass difference between \( H^{\pm} \) and \( H^{\pm} \) is not too large. Therefore, the same events as expressed in Eqs. (IV.6) and (IV.7) can appear in the following way:

\[ pp \rightarrow H^{++}H^{--} \rightarrow W^{+}W^{+}W^{-}W^{-} \rightarrow \ell^{+}\ell^{+}\ell^{-}\ell^{-} E_{T}, \] (IV.8)

\[ pp \rightarrow H^{\pm\pm}H^{\mp} \rightarrow W^{\pm}W^{\pm}W^{\pm}Z \rightarrow \ell^{\pm}\ell^{\pm}\ell^{\mp} E_{T}, \] (IV.9)

Furthermore, in the case where \( H^{\pm\pm} \) are the heaviest among the triplet like Higgs bosons such like our scenario, the cascade decay of \( H^{\pm\pm} \) can be dominant; i.e., \( H^{++} \rightarrow W^{\pm}H^{\pm} \rightarrow W^{\pm}W^{\pm}H^{0} \) (\( H^{0} \) is a neutral component field of the triplet Higgs). In addition, \( v_{\Delta} \lesssim 10^{-4} \) GeV, \( H^{0} \) mainly decays into neutrinos. In this case, the final states of the signal event can

\footnote{The heavier \( \Delta^{0} \); e.g., \( \Delta_{R} \) can further decay into \( Z^{*} \) and \( \Delta_{I} \) which is corresponding to DM.}
FIG. 6: Invariant mass distribution for the $\ell^+\ell^+$ system from the event $pp \rightarrow \ell^+\ell^+\ell^-\ell^- E_T$ in our model (red solid curve) and that in HTM with the diboson decay (black dotted curve) of $H^{\pm\pm}$ and the cascade decay of $H^{\pm\pm}$ (blue dashed curve). We take the mass of the doubly-charged scalar bosons to be 90.5 GeV and the collision energy to be 8 TeV. The solid, dashed and dotted curves are respectively shown the distributions for the dilepton decay in our model, cascade decay in the HTM and diboson decay in HTM. Number of events are assumed to be $10^4$ in each distribution.

$$pp \rightarrow H^{++}H^{--} \rightarrow (W^+W^+H^0)(W^-W^-H^0) \rightarrow \ell^+\ell^+\ell^-\ell^- E_T.$$  \hspace{1cm} (IV.10)

$$pp \rightarrow H^{\pm\pm}H^{\mp} \rightarrow (W^\pm W^\mp H^0)(W^\mp H^0) \rightarrow \ell^\pm\ell^\pm E_T.$$ \hspace{1cm} (IV.11)

The invariant mass distributions in the system of the same sign dilepton $M_{\ell^+\ell^+}$ can be useful to discriminate between our model and HTM. In Fig. 6, the distribution for $M_{\ell^+\ell^+}$ in the $\ell^+\ell^+\ell^-\ell^- E_T$ system is shown in which the number of event is assumed to be $10^4$ and collision energy to be 8 TeV. The mass of the doubly-charged scalar bosons in our model and also in HTM are taken to be 90.5 GeV. The red solid curve and the black dotted (blue dashed) curve are respectively represent the $M_{\ell^+\ell^+}$ distribution from our model and HTM with the diboson decay (cascade decay) of $H^{\pm\pm}$. It can be seen that the event number from the diboson decay of $H^{\pm\pm}$ is distributed in the wide region of $M_{\ell^+\ell^+}$, and it takes
the maximum value at around the half of the mass of \( H^{\pm\pm} \); i.e., \( M_{\ell^+\ell^+} \sim 45 \) GeV. On the other hand, the other two shapes of the distributions look like each other in which the event number is distributed in the small region of \( M_{\ell^+\ell^+} \). However, the cross sections for the final states are different between these two events. In our model, one of the two leptons with the same sign comes from the decay of \( \Delta^{\pm\pm} \), and the other one comes from the leptonic decay of the W boson, so that the cross section is calculated as 
\[
\sigma(pp \to \Delta^{++}\Delta^{-*-}) \times \mathcal{B}(W \to \ell\nu)^2.
\]
On the other hand, in HTM with the cascade decay of \( H^{\pm\pm} \), all the leptons in the final state are obtained from the leptonic decay of the W boson, so that the cross section is evaluated by 
\[
\sigma(pp \to H^{++}H^{--}) \times \mathcal{B}(W \to \ell\nu)^4.
\]
The original cross sections of \( \sigma(pp \to \Delta^{++}\Delta^{-*-}) \) and \( \sigma(pp \to H^{++}H^{--}) \) are the same each other as long as we take the masses of \( \Delta^{\pm\pm} \) and \( H^{\pm\pm} \) to be the same. Therefore, the event number in HTM with the cascade decay scenario is smaller than that in our model by the factor of \( \mathcal{B}(W \to \ell\nu)^2 = 1/9 \).

We then conclude that the four lepton events with the missing energy \( \ell^+\ell^+\ell^-\ell^-E_T \) from our model and HTM with the diboson decay and the cascade decay of \( H^{\pm\pm} \) may be able to be discriminated by using the \( M_{\ell^+\ell^+} \) distribution and the number of event.

\textbf{V. CONCLUSIONS}

We have constructed a two-loop radiative seesaw model that provides neutrino masses in a TeV scale theory. We have also studied DM properties, in which our model has fermionic (\( N^c \)) or bosonic DM (\( \eta^0, \Delta^0 \)) candidate with the same mass scale, which is at around \( m_h/2 \), from the constraint of WMAP and the direct detection search in XENON100. We have also discussed Higgs phenomenology at LHC, in which the neutral scalar boson from the triplet field is assumed to be DM. In that case, the mass of the doubly-charged scalar bosons \( \Delta^{\pm\pm} \) is constrained to be smaller than about 90.5 GeV by the electroweak precision data. In this scenario, \( \Delta^{\pm\pm} \) can mainly decay into the same sign dilepton with the missing energy where the exotic leptons \( E^\pm_4 \) appear the intermediate state of the decay process of \( \Delta^{\pm\pm} \). We then considered how we can discriminate the signature of \( \Delta^{\pm\pm} \) in our model from that of doubly-charged Higgs boson \( H^{\pm\pm} \) from HTM. We have found that the distribution of the same sign dilepton system and the magnitude of the cross section for \( \ell^+\ell^+\ell^-\ell^-E_T \) can be useful to distinguish between our model and HTM.
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Appendix A: Gauge boson two point functions

We list the analytic formulae for the 1PI diagram contributions to the gauge boson two point functions at the one loop level which are necessary to calculate the $S$ and $T$ parameters. We ignore the mixing between the $\mathbb{Z}_2$-even scalar bosons; i.e., $\alpha = 0$. In that case, new physics contributions to the gauge boson two point functions can be separated from the SM ones. The new physics contributions are calculated by

$$ \Sigma_{WW}^\gamma(p^2) = \frac{1}{16\pi^2} \left[ \frac{g^2}{4} \left[ 16p^2 B_3(p^2, M, M) ight. ight. $$
$$ + 4c_\beta^2 B_5(p^2, m_{\Delta++}, m_{\Delta+}) + 4s_\beta^2 B_5(p^2, m_{\Delta++}, m_{\eta+}) $$
$$ + (\sqrt{2}c_\beta c_\gamma + s_\beta s_\gamma)^2 B_5(p^2, m_{\Delta+}, m_{\Delta+}) + (\sqrt{2}c_\beta c_\delta + s_\beta s_\delta)^2 B_5(p^2, m_{\Delta+}, m_{\Delta+}) $$
$$ + (c_\beta c_\gamma + \sqrt{2}s_\beta s_\gamma)^2 B_5(p^2, m_{\eta+}, m_{\eta+}) + (c_\beta c_\delta + \sqrt{2}s_\beta s_\delta)^2 B_5(p^2, m_{\eta+}, m_{\eta+}) $$
$$ + (-\sqrt{2}c_\beta s_\gamma + s_\beta c_\gamma)^2 B_5(p^2, m_{\Delta++}, m_{\Delta++}) + (-\sqrt{2}c_\beta s_\delta + s_\beta c_\delta)^2 B_5(p^2, m_{\Delta++}, m_{\Delta++}) $$
$$ + (-\sqrt{2}s_\beta c_\gamma + c_\beta s_\gamma)^2 B_5(p^2, m_{\eta+}, m_{\eta+}) + (-\sqrt{2}s_\beta c_\delta + c_\beta s_\delta)^2 B_5(p^2, m_{\eta+}, m_{\eta+}) $$
$$ \left. \left. \right] \right), \tag{A.1} $$

$$ \Sigma_{Z\gamma}^\gamma(p^2) = \frac{e^2}{16\pi^2} \left[ 8p^2 B_3(p^2, M, M) ight. $$
$$ + 4B_5(p^2, m_{\Delta++}, m_{\Delta++}) + B_5(p^2, m_{\Delta+}, m_{\Delta+}) + B_5(p^2, m_{\eta+}, m_{\eta+}) $$
$$ \left. \right] \right). \tag{A.2} $$

$$ \Sigma_T^{Z\gamma}(p^2) = \frac{egZ}{16\pi^2} \left[ \left( \frac{1}{2} - s_W^2 \right) 8p^2 B_3(p^2, M, M) ight. $$
$$ + 2(1 - 2s_W^2) B_5(p^2, m_{\Delta++}, m_{\Delta++}) + \frac{1}{2}(s_\beta^2 - 2s_W^2) B_5(p^2, m_{\Delta+}, m_{\Delta+}) $$
$$ + \frac{1}{2}(c_\beta^2 - 2s_W^2) B_5(p^2, m_{\eta+}, m_{\eta+}) $$
$$ \left. \right] \right), \tag{A.3} $$

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\[ \Sigma_{T}^{ZZ}(p^2) = \frac{1}{16 \pi^2} \frac{g_Z^2}{4} \left[ 32p^2 \left( \frac{1}{2} - s_W^2 + s_W^4 \right) B_3(p^2, M, M) \right. \\
+ 4(1 - 2s_W^2) B_5(p^2, m_{\Delta}, m_{\Delta}) + (s_W^2 - 2s_W^2) B_5(p^2, m_{\Delta}, m_{\Delta}) \\
+ (c_\beta^2 - 2s_W^2) B_5(p^2, m_{\eta}, m_{\eta}) + 2c_\beta^2s_\beta B_5(p^2, m_{\Delta}, m_{\eta}) \\
+ (2c_\gamma c_\delta + s_\gamma s_\delta) B_5(p^2, m_{\Delta}, m_{\Delta}) + (c_\gamma c_\delta + 2s_\gamma s_\delta) B_5(p^2, m_{\eta}, m_{\eta}) \\
+ \left. (-2c_\gamma c_\delta + s_\gamma s_\delta) B_5(p^2, m_{\Delta}, m_{\Delta}) \right], \quad (A.4) \]

where \( c_\theta = \cos \theta \) and \( s_\theta = \sin \theta \). In the above equations, \( B_3(p^2, m_1, m_2) \) and \( B_5(p^2, m_1, m_2) \) functions are respectively expressed in terms of the Passarino-Veltman functions \( \gamma \) by

\[ B_3(p^2, m_1, m_2) = -B_1(p^2, m_1, m_2) - B_2(p^2, m_1, m_2), \quad (A.5) \]
\[ B_5(p^2, m_1, m_2) = A(m_1) + A(m_2) - 4B_2(p^2, m_1, m_2). \quad (A.6) \]

The functions \( \Sigma_{T}^{11} \), \( \Sigma_{T}^{3Q} \) and \( \Sigma_{T}^{33} \) are given in terms of above the gauge boson two point functions by

\[ \Sigma_{T}^{11} = \frac{1}{g^2} \Sigma_{T}^{WW}, \quad \Sigma_{T}^{3Q} = \frac{1}{g^2} \left[ \Sigma_{T}^{Z\gamma} + \Sigma_{T}^{\gamma\gamma} \right], \quad \Sigma_{T}^{33} = \frac{1}{g^2} \left[ \Sigma_{T}^{Z\gamma} + \Sigma_{T}^{\gamma\gamma} \right]. \quad (A.7) \]

### Appendix B: Decay rates

The decay rates for the doubly-charged scalar bosons \( \Delta^{\pm\pm} \) are calculated as

\[ \Gamma(\Delta^{\pm\pm} \to \ell_i^+ E_4^-) = \frac{m_{\Delta^{\pm\pm}}}{16 \pi} |y_i^j|^2 \left( 1 - \frac{m_{\ell_i}^2}{m_{\Delta^{\pm\pm}}^2} - \frac{M^2}{m_{\Delta^{\pm\pm}}^2} \right)^{1/2} \left( \frac{m_{\ell_i}^2}{m_{\Delta^{\pm\pm}}^2}, \frac{M^2}{m_{\Delta^{\pm\pm}}^2} \right), \quad (B.1) \]
\[ \Gamma(\Delta^{\pm\pm} \to \Delta^{\pm} W^{\pm}) = \frac{g^2}{16 \pi} \frac{m_{\Delta^{\pm}}^2}{m_W^2} \cos^2 \beta \lambda^{3/2} \left( \frac{m_{\Delta^{\pm}}^2}{m_{\Delta^{\pm\pm}}^2}, \frac{m_W^2}{m_{\Delta^{\pm\pm}}^2} \right), \quad (B.2) \]
\[ \Gamma(\Delta^{\pm\pm} \to \Delta^{\pm} W^{\mp}) = \frac{g^2}{128 \pi^3} \frac{m_{\Delta^{\pm}}^2}{m_W^2} \cos^2 \beta G \left( \frac{m_{\Delta^{\pm}}^2}{m_{\Delta^{\pm\pm}}^2}, \frac{m_W^2}{m_{\Delta^{\pm\pm}}^2} \right), \quad (B.3) \]
\[ \Gamma(\Delta^{\pm\pm} \to \eta^{\pm} W^{\mp}) = \frac{g^2}{16 \pi} \frac{m_{\Delta^{\pm}}^2}{m_W^2} \sin^2 \beta \lambda^{3/2} \left( \frac{m_{\eta^\pm}^2}{m_{\Delta^{\pm\pm}}^2}, \frac{m_W^2}{m_{\Delta^{\pm\pm}}^2} \right), \quad (B.4) \]
\[ \Gamma(\Delta^{\pm\pm} \to \eta^{\pm} W^{\pm}) = \frac{g^2}{128 \pi^3} \frac{m_{\Delta^{\pm}}^2}{m_W^2} \sin^2 \beta G \left( \frac{m_{\eta^\pm}^2}{m_{\Delta^{\pm\pm}}^2}, \frac{m_W^2}{m_{\Delta^{\pm\pm}}^2} \right), \quad (B.5) \]
where $\lambda(x, y)$ and $G(x, y)$ are the phase space functions which are given by

$$
\lambda(x, y) = 1 + x^2 + y^2 - 2x - 2y - 2xy,
\quad \text{(B.6)}
$$

$$
G(x, y) = \frac{1}{12y} \left\{ 2 (-1 + x)^3 - 9 (-1 + x^2) y + 6 (-1 + x) y^2
\right. 
+ 6 (1 + x - y) y \sqrt{-\lambda(x, y)} \left[ \tan^{-1} \left( \frac{-1 + x - y}{\sqrt{-\lambda(x, y)}} \right) + \tan^{-1} \left( \frac{-1 + x + y}{\sqrt{-\lambda(x, y)}} \right) \right]
\left. - 3 \left[ 1 + (x - y)^2 - 2y \right] y \log x \right\},
\quad \text{(B.7)}
$$

### Appendix C: Flavor symmetry

| Particle | $L$ | $e^c$ | $N^c$ | $L'$ | $L'^c$ | $\Delta$ | $\Phi_1$ | $\eta$ | $\chi_1^{(i)}$ | $\chi_2^{(i)}$ |
|----------|----|-------|------|------|-------|--------|-------|------|-------------|-------------|
| $T_7$    | 3  | 3     | 3    | 3    | 3     | 10     | 1     | 1    | 3           | 3           |

TABLE III: The particle assignments in $T_7$ symmetry. Here $i$ runs 0-2.

Here we show an example how to realize our model for the lepton sector by using non-Abelian discrete symmetry. The minimal extension is to introduce $T_7$ flavor symmetry \[47-50\]. Each of the field assignment is given in Table III, where the other assignments are same. The extended Lagrangian to Eq. (II.1) is modified as

$$
\mathcal{L}_{N^c} = \sum_{i=1}^{3} \Phi_i^i \left( y_{e}^a \omega^{2(i-1)} y_{e}^b + \omega^{i-1} y_{e}^c \right) e_i^L L_i^c + \sum_{i=1}^{3} \eta_{i}^1 N_i^c L_i^c + y_{c}^b \sum_{i=1}^{3} \tilde{L}_i^c \Delta \tilde{L}_i^c + M \sum_{i=1}^{3} L_i^c L_i^c
$$

\[ \chi_1 \equiv e^{2i\pi/3} \]. Here notice that all the terms except $\chi N^c N^c$ are diagonal. It suggests that the observed neutrino mass and lepton mixing can be obtained only through the $\chi N^c N^c$ term. After the $B - L$ spontaneously breaking; $\langle \chi_{1}^{(i)} \rangle = v_{1}^{(i)}$ and $\langle \chi_{2}^{(i)} \rangle = v_{2}^{(i)}$, the right-

\[ \text{The charged-lepton sector has to be improved in order to forbid the universal Yukawa coupling of the SM-like Higgs boson, since it has been ruled out by the current Higgs boson search data at LHC. Straightforward ways to solve it is to change the flavor symmetry group and/or introduce some additional Higgs fields.} \]
handed neutrino mass matrix is given by

\[
m_{N^c} = g_S^a \begin{pmatrix}
 v_1^{(2)} & 0 & 0 \\
 0 & v_1^{(3)} & 0 \\
 0 & 0 & v_1^{(1)}
\end{pmatrix} + g_S^b \begin{pmatrix}
 0 & v_2^{(3)} & v_2^{(2)} \\
 v_2^{(3)} & 0 & v_2^{(1)} \\
 v_2^{(2)} & v_2^{(1)} & 0
\end{pmatrix}.
\]

(C.2)

As a result, we can easily find the observed neutrino mass difference and their mixings by controlling each of the VEV. In the Higgs potential, the modifications are as follows:

\[
\Phi^\dagger \Phi \rightarrow \sum_{i=1}^3 \Phi_i^\dagger \Phi_i, \quad \chi^\dagger \chi \rightarrow \sum_{i=1}^2 \sum_{j=1}^3 \chi_i^{(j)} \chi_i^{(j)}.
\]

(C.3)

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