HECO: Automatic Code Optimizations for Efficient Fully Homomorphic Encryption

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Abstract

In recent years, Fully Homomorphic Encryption (FHE) has undergone several breakthroughs and advancements leading to a leap in performance. Today, performance is no longer a major barrier to adoption. Instead, it is the complexity of developing an efficient FHE application that currently limits deploying FHE in practice and at scale. Several FHE compilers have emerged recently to ease FHE development. However, none of these answer how to automatically transform imperative programs to secure and efficient FHE implementations. This is a fundamental issue that needs to be addressed before we can realistically expect broader use of FHE. Automating these transformations is challenging because the restrictive set of operations in FHE and their non-intuitive performance characteristics require programs to be drastically transformed to achieve efficiency. In addition, existing tools are monolithic and focus on individual optimizations. Therefore, they fail to fully address the needs of end-to-end FHE development. In this paper, we present HECO, a new end-to-end design for FHE compilers that takes high-level imperative programs and emits efficient and secure FHE implementations. In our design, we take a broader view of FHE development, extending the scope of optimizations beyond the cryptographic challenges existing tools focus on.

1 Introduction

Privacy and security are gaining tremendous importance across all organizations, as public perception of these issues has shifted and expectations, including regulatory demands, have increased. This has led to a surge in demand for secure and confidential computing solutions that protect data confidentiality while in transit, rest, and in-use. Fully Homomorphic Encryption (FHE) is a key secure computation technology that enables systems to preserve the confidentiality of data end-to-end, including while in use. Hence, allowing outsourcing of computations without having to grant access to the data. In the last decade, advances in FHE schemes have propelled FHE from a primarily theoretical breakthrough to a practical solution for a wide range of applications [1–3]. In recent years, we have seen FHE emerging in real-world applications, e.g., Microsoft’s Edge browser uses FHE to realize its privacy-preserving password monitor [1]. With upcoming dedicated hardware accelerators for FHE promising further speedup [4, 5], FHE will soon be competitive for an even wider set of applications. Today, FHE is promising to become a major driver in transforming privacy. For example, a recent report by Gartner, Inc. predicts [6, 7] that FHE “will be a core technology for many future SaaS offerings.”

Though promising in its potential, developing FHE applications, unfortunately, remains a complex and tedious process. Writing efficient FHE programs that deliver on their potential performance is currently a complicated and tedious task that even experts struggle with. FHE’s use in practice and at scale is today primarily hindered by its complexity rather than its performance. These complexities derive from the need to map applications to the unique programming paradigm imposed by FHE (c.f. § 2.3). Code that is properly optimized for this paradigm is often overwhelmingly more efficient than poorly adapted code, with speedups of several orders of magnitude being common. Therefore, optimizing applications and rewriting them to match FHE’s programming paradigm is essential for practical applications of FHE. Today, this remains an unresolved issue posing a major barrier to wider adoption.

In order to facilitate easier development of FHE-based privacy preserving technologies, the current FHE development ecosystem must better address the needs of non-expert developers. Existing tools fail to provide an easy-to-use, general-purpose, and flexible toolchain to transform high-level code written by non-experts into efficient FHE implementations that match the performance of those created by experts. Ideally, developers should not need to adapt to FHE’s unique paradigm, making only minimal changes to their code such as marking inputs as secret. Instead, tools should bridge the gap between traditional programming patterns and the restrictions of FHE development by automatically rewriting and optimizing programs to match FHE’s unique programming paradigm.
Existing domain-specific tools have partially addressed this for some scenarios [8–10], but a lack of flexibility prevents their use when developers’ needs are even slightly misaligned. General-purpose tools, on the other hand, either sacrifice performance for ease-of-use [11–13] or require the developer to manually map their application to the FHE paradigm [14, 15].

In order to overcome these limitations, we need to fundamentally rethink the architecture of FHE compilers and develop novel optimizations that address the limitations of existing tools. In this paper, we present HECO, a new FHE compiler and propose a holistic architecture for FHE compilers that, for the first time, provides a true end-to-end toolchain for FHE development. In addition, we propose novel transformations and optimizations that map standard imperative programs to the unique programming model of FHE.

**E2E Architecture.** Compilers need to to be able to accommodate a wide variety of optimizations required to develop efficient FHE applications. We identify three distinct phases that constitute the process of converting an application to an efficient FHE implementation: (i) Program Transformation, which restructures a high-level program to become efficiently expressible using native FHE operations, (ii) Circuit Optimization, which primarily focuses on changes that reduce noise growth in the FHE computation (which, at this point, is equivalent to an arithmetic circuit), and (iii) Cryptographic Optimization, which instantiates the underlying scheme as efficiently as possible for the given program. Based on these insights, we propose a set of Intermediate Representations (IRs) that match the requirements of each phase, allowing us to naturally and efficiently express optimizations for each phase.

Most existing tools focus on the second, and to some extent third, phase of this process. This is reflected in their design, using internal representation that map directly to native FHE operations. Therefore, they are limited to optimizations expressible at this level and cannot easily be extended to address the first phase of FHE development. In HECO, we extend compilation to support all phases and introduce novel optimizations targeting the first phase of development.

**Automated Mapping to Efficient FHE.** We expand the FHE compilation chain with support for the automatic transformation of high-level programs to FHE’s unique programming paradigm. Experts spend significant time considering how to best express an application in the FHE paradigm, only considering the other aspects once the program is efficiently expressible using native FHE operations. Existing tools, in contrast, nearly universally disregard this arguably most important phase of the FHE development process.

In HECO, developers are able to express their algorithms conveniently in the standard imperative paradigm, making use of, e.g., loops that access and manipulate vector elements by accessing individual elements. However, such programs do not align well with the restricted set of operations offered by FHE schemes, requiring the compiler to translate and restructure the application. We specifically focus on transformations targeting the Single Instruction, Multiple Data (SIMD) parallelism present in most modern schemes, which allows one to batch many (usually $2^{13} - 2^{16}$) different values into a single ciphertext and compute over all elements simultaneously. Batching is used by experts to drastically reduce ciphertext expansion and computational overhead, frequently improving runtimes by several orders of magnitude when compared to naive implementations.

While it is arguably the single most important optimization for many applications, unlocking the performance potential of batching is currently restricted to experts with significant expertise and experience in writing FHE applications. Existing algorithms must be translated to ones expressed solely as SIMD operations and rotations, as the only natively supported data movements in FHE are cyclical rotations of the elements inside a ciphertext.

HECO converts operations over vector elements into efficient SIMD operations in this paradigm. Emulating element-wise operations is highly inefficient in batched FHE, requiring expensive FHE operations. Instead, HECO employs a series of transformations and optimizations that aim to remove the need to operate over individual vector elements. We replace individual element-wise operations with SIMD operations, which requires introducing additional operations to maintain the consistency of the program. However, our transformations are carefully designed to allow us to optimize the resulting program further, eliminating the vast majority of introduced operations. In our evaluation, we show that HECO can match the performance of expert implementations, providing up to 3500x speedup over naive non-batched implementations.

Beyond the contributions we present in this paper, we make our system available as an open-source project and we hope that HECO will help to advance the FHE development ecosystem. HECO decouples optimizations from front-end and back-end logic, allowing it to easily be extended for different languages, FHE libraries or accelerators, and novel optimizations as they emerge. We build HECO on top of the open-source MLIR framework [16], which is rapidly establishing itself as the go-to tool for domain-specific compilers and opens up the possibility of exchanges of ideas and optimizations even beyond the FHE community.

## 2 Background

In this section, we briefly introduce the notion of Fully Homomorphic Encryption (FHE) and key aspects of modern FHE schemes.
2.1 Fully Homomorphic Encryption

In a homomorphic encryption (HE) scheme there exists a homomorphism between operations on the plaintext and operations on the ciphertext, so that

\[ \text{Dec}(\text{Enc}(x + y)) = \text{Dec}(\text{Enc}(x) \oplus \text{Enc}(y)). \]

This allows a third-party to compute on encrypted data without requiring access to the underlying data. For example, a client can privately outsource computation by encrypting the data before sending it to the cloud, where the function is computed over ciphertexts, as shown in Figure 1. The client can decrypt the returned result, while the server learns nothing about inputs, intermediate values or results. In addition to outsourcing a computation, HE can also be used for two-party computations, where the server provides either some of the inputs, the function, or both. For example, in Microsoft Edge’s password monitor [1], the client sends encryptions of stored logins to the server which compares them against its list of known leaked usernames and passwords.

While additively or multiplicatively homomorphic encryption schemes such as the Paillier encryption scheme [17] or textbook RSA [18] have been known for many decades, fully homomorphic encryption (FHE) schemes that support an arbitrary combination of both operations remained unrealized until 2009 when Craig Gentry presented the first feasible FHE scheme [19]. Since then, the performance of these schemes has continued to improve dramatically: While the initial FHE schemes took around half an hour to compute a single multiplication, modern schemes require only a few milliseconds. Nevertheless, this still represents a slowdown of seven orders of magnitude compared to a standard \texttt{imul} executed on a modern CPU. With the emergence of dedicated FHE hardware accelerators, this gap will be reduced by several orders of magnitude.

2.2 FHE Schemes

We briefly describe the BFV scheme [20, 21], focusing on aspects relevant to FHE application development and referring to the original papers for further details. Since most modern FHE schemes follow a similar pattern, the descriptions also mostly apply to the BGV and CKKS schemes [22, 23].

In BFV, plaintexts are polynomials of degree \( n \) (usually \( n > 2^{12} \)) with coefficients modulo \( q \) (usually \( q > 2^{60} \)). Encryption introduces a small amount of noise into the ciphertext. This is initially small enough to be rounded away during decryption, but grows during homomorphic operations. If left unmanaged, the noise will eventually corrupt the ciphertext, leading to failed decryptions.

BFV supports additions and multiplications over ciphertexts, with additions increasing noise only negligibly, while multiplications significantly increase the noise.

This limits computations to a (parameter-dependent) number of consecutive multiplications (multiplicative depth) before decryption fails. Using the Chinese Remainder Theorem (CRT), it is possible to encode a vector of \( n \) integers into a single polynomial, with addition and multiplication acting slot-wise (SIMD). Since the polynomial degree \( n \) is usually \( 2^{13} - 2^{16} \) in order to ensure security, this can significantly reduce ciphertext expansion and computation cost. This creates a significant opportunity for optimizing programs to fully exploit these large vectors. BFV also supports rotation operations over such batched ciphertexts, which rotate the elements inside the vector cyclically. Finally, BFV includes a variety of noise-management (or ciphertext maintenance) operations which do not change the encrypted message but can reduce noise growth during computations.

2.3 FHE Programming Paradigm

We analyze and discuss the programming challenges FHE imposes, highlighting their impact on the development process. Some of the challenges derive directly from the definition of FHE and its security guarantees, while others result from the restrictions and cost models of the schemes. Both developers and FHE tools need to map applications to the unique programming paradigm of FHE in order to achieve practical and efficient FHE solutions.

Restricted Set of Operations. FHE schemes offer a very limited selection of data types and operations, with additions and multiplications as the basic operations of FHE. FHE over binary plaintext spaces (\( \mathbb{Z}_2 \)) can emulate arbitrary computation. However, the best performance is usually achieved when using non-binary plaintext spaces (e.g., \( \mathbb{Z}_t \) for \( t \gg 2 \)). In this setting, computations are equivalent to arithmetic circuits, which can only compute polynomial functions. While it is possible to approximate many functions using polynomials, this is frequently prohibitively inefficient in the context of FHE. Instead, developers need to rethink their approach, e.g., using different algorithms that are purely polynomial or well-suited to low-degree polynomial approximations.
More fundamentally, FHE does not support branching based on secret inputs, as revealing even the single bit required to branch would violate the security properties of FHE. While it possible to emulate some forms of branching, this requires evaluating both sides of each branch. As a result, it is inefficient and results in programs that perform worse than the worst-case performance of the original program, even before considering FHE overhead.

While the requirement for branch-free programs is inherent in the definition of FHE, future schemes might eventually remove the need to choose between inefficient binary emulation of arbitrary functions or efficient integer-based evaluation of polynomials. Recent works have explored homomorphic conversions between different schemes [24, 25] or introduced the notion of programmable bootstrapping to approximate non-polynomial functions [26]. However, both approaches are not yet practical enough for widespread adoption.

**SIMD Parallelism.** Since FHE ciphertexts in most modern FHE schemes must be very large for security reasons, using them to encrypt, e.g., a single integer is inherently inefficient. However, these schemes support batching, which allows one to encrypt many messages into the same ciphertext, reducing ciphertext expansion. In this setting, FHE operations act in a Single Instruction, Multiple Data (SIMD) fashion, i.e., element-wise. Since operating over batched ciphertexts has the same cost as operating over individual values, batching is a key component of designing efficient FHE solutions.

While it is trivial to use SIMD batching to increase throughput by processing many inputs at the same time, this is often not feasible in practice since many applications either do not feature enough inputs to fill a large ciphertext vector or are primarily latency limited. The challenge, therefore, is to use batching to accelerate the computation on a single (or a small number of) input(s). Once a ciphertext has been encrypted, we are limited to cyclical rotations and element-wise SIMD operations acting over all slots, since there are no native scatter/gather operations. The challenge of this paradigm arises from the complexity of efficiently computing functions where values from many different slots need to interact with each other.

We discuss these challenges in further detail in § 4, demonstrating how expert-optimized batched solutions can be nearly unrecognizable transformations of the original program.

### 2.4 Security

Modern FHE schemes rely on post-quantum hardness assumptions that are widely believed to be secure for the foreseeable future. The community has developed a relatively stable estimation of the concrete hardness of many of these problems [27] and parameter choices for a number of FHE schemes have been standardized [28]. Nevertheless, some attention must be paid to security when using FHE.

Most importantly, it is worth noting that FHE does not provide integrity by default, i.e., a server might perform a different calculation than the client requested or even no calculation at all. There exist techniques to address integrity, this range from zero-knowledge-proofs to hardware attestation [29]. Additionally, FHE does not by default provide circuit privacy, i.e., a client might be able to learn information about the circuit the server applied. Different techniques, varying in practicality and level of protection, can be used to address this [30, 31]. Finally, there are some subtle issues that can appear when using approximate homomorphic encryption (e.g., Cheon-Kim-Kim-Song (CKKS)). Since these arise only in the somewhat unlikely case that a third party gains access to a large number of high-precision ciphertext decryptions for which it exactly knows the computation performed on the ciphertexts, we merely note their existence and refer to the paper [32] for further details.

### 3 End-to-End FHE Compiler Design

This section provides a system overview of HECO and presents its core components.
3.1 System Overview

In HECO, developers express their programs using high-level languages (Python or a C-like domain-specific language), which are translated to a high-level Intermediate Representation (IR) that retains information from the input program beyond what is directly expressible in FHE. This representation is then optimized and lowered through a series of passes and IRs, arriving at a low-level program expressed in a cryptographic IR that corresponds directly to native FHE operations. Code-generation then translates this to calls to the underlying FHE implementation (e.g., Microsoft’s SEAL library). An overview of this end-to-end approach can be seen in Figure 2.

In order to enable a more holistic view of FHE compilation than existing tools offer, we design HECO to support a range of different Intermediate Representations (IRs) that enable it to apply optimizations from all three stages of FHE development: (i) Program Transformation, which restructures a high-level program to become efficiently expressible using native FHE operations, (ii) Circuit Optimization, which primarily focuses on changes that reduce noise growth in the FHE computation (which, at this point, is equivalent to an arithmetic circuit), and (iii) Cryptographic Optimization, which instantiate the underlying scheme as efficiently as possible for the given program. HECO’s design allows it to implement optimizations that other tools cannot express, and its modular nature allows optimizations across different layers to co-exist. In the remainder of this section we describe HECO’s components, abstractions, and optimizations.

3.2 Front End & High-Level IR

HECO allows developers to express their programs in the same imperative paradigm they are used to, using high-level languages. In addition to a custom C-like Domain-Specific Language (DSL), we provide a Python-based front-end which allows developers to work with FHE in a familiar environment, using Python type hints to annotate which inputs are secret. In order to avoid improving ease-of-use at the cost of expressiveness, HECO also exposes advanced features like rotations and ciphertext maintenance operations through built-in functions that seamlessly integrate with the front-end language.

In order to acquire the desired high-level representation of the input program, HECO’s front-end differs fundamentally from other FHE compilers. Existing tools mostly rely on replacing secret variables with placeholder objects that record operations performed on them during execution of the host language program, which condenses an entire function into a single gigantic expression. This obscures the program structure, preventing efficient high-level program transformations. Therefore, HECO instead parses the input program, using the placeholder-object approach only for calls to externally defined functions. Each front-end maps the language-specific parsed Abstract Syntax Tree (AST) to a common AST representation, allowing language-independent aspects (e.g., translation to Static Single Assignment (SSA) form) to be shared among all front-ends.

3.3 Transformation & Optimization

During the core part of the compilation process, the program is progressively lowered to more concrete, lower-level representations until the high-level input program has been converted into a program from which FHE application code can be generated directly. These lowering do not need to apply to the entire program at once, in fact, programs will commonly consist of a mix of different levels of abstractions. This is realized using the MLIR compiler framework [16] where operations from several dialects representing different levels of abstraction can co-exist in a single intermediate representation. Figure 3 provides an overview over the various dialects used in HECO, showing where they are employed and what types of operations they provide.

Existing FHE compilers frequently use circuit-like IRs, as FHE computations are equivalent to arithmetic circuits. Some earlier work also chose circuit IRs in the hope of applying existing techniques from the hardware domain. However, this has not produced significant performance gains so far [33] and it seems likely that high-impact FHE optimizations will continue to require custom designs. Since HECO needs to process high-level input programs that are not yet equivalent to circuits, we use Static Single Assignment (SSA) form based IRs that can represent a wider set of operations than circuit-based IRs modeling native FHE operations. This approach is also used for low-level FHE scheme IRs, providing a consistent interface for optimizations across the entire toolchain.

The transformations and optimizations in HECO are grouped according to the three stages we identified, and we present them accordingly in the following. However, the modular nature of the passes which implement the various optimizations also allows advanced users to dynamically compose...
compilation pipelines for specific scenarios. The modularity also facilitates simple extensibility of the compiler as new optimization techniques are developed by the FHE community.

**Program Transformations.** The first phase of HECO maps the input program from the traditional imperative paradigm to FHE’s unique programming paradigm. Existing tools either require developers to perform this mapping manually or rely on trivial transformations that generate prohibitively inefficient code. Most of the focus in the development of FHE compiler optimizations has so far been on second-phase circuit optimization and noise management techniques. However, even state-of-the-art circuit optimizations [34,35] are likely to accelerate a program by only around 2x in practice [33]. In contrast, performance differences between the runtimes of well-mapped and naively-mapped implementations can easily reach several orders of magnitude [33].

In this mapping process, effective employment of batching is one of the main differentiators between expert-written efficient solutions and impractical naïve approaches. The vast majority of state-of-the art FHE results rely on the support for SIMD operations present in the BFV/BGV/CKKS family of R-LWE based schemes [20–23] to turn otherwise infeasible programs into efficient secure solutions. While batching can be used to trivially increase throughput, most FHE applications are constrained by latency. However, employing batching effectively to improve performance on a single input is non-trivial due to the restrictions imposed by FHE’s unusual programming paradigm.

A significant part of our focus in HECO is consequently on automated batching optimizations which we present in more detail in § 4. Note that our focus on batching does not preclude other optimizations. Due to FHE’s high per-operation cost, even a single redundant multiplication can have measurable performance impacts. As a result, state-of-the-art expert-written code frequently contains a significant boiler-plate code handling edge-case optimizations. We implement a wide variety of general (e.g., constant folding, common sub-expression elimination) and FHE-specific optimizations that allow developers to write code in a more natural fashion by removing the need for menial hand-optimization.

**Circuit Optimizations.** In the second phase, the program is conceptually equivalent to an arithmetic circuit of native FHE operations. Optimizations at this stage are mostly concerned with noise management, which requires a scheme-specific approach. Therefore, HECO includes scheme-specific IRs that model the native FHE operations offered by the schemes and the appropriate ciphertext maintenance operations. These IRs also model the variety of slightly specialized operations that the underlying libraries might offer and which can be more efficient than the generic version when applicable (e.g., squaring vs multiplying). Existing work, much of which has focused almost exclusively on this phase, has proposed a variety of techniques [12, 14, 15, 34–36], and our system’s modularity allows HECO to be easily extended with further optimizations as they emerge. We omit a detailed description of these techniques here, as they are not the focus of this paper.

**Cryptographic Optimizations.** Finally, in the third phase there are cryptographic optimizations focused on instantiating the underlying FHE scheme as efficiently as possible. The primary challenge is parameter selection, identifying the smallest (most efficient) parameters where the noise capacity is sufficient to perform the computation correctly. While different techniques have been proposed to estimate the expected noise growth of an FHE program [37, 38], these tend to significantly over-estimate noise growth [37], leading to unnecessarily large parameter choices. As a result, experts primarily rely on a trial-and-error process to experimentally determine the point at which noise invalidates the results.

HECO includes basic automatic parameter selection, but also allows experts to easily override these suggestions.

With dedicated FHE hardware accelerators on the horizon [4, 5], there are potential future optimizations at this level. While current software implementations generally use FHE operations as the smallest unit of abstraction, it is likely that FHE hardware will feature a lower-level interface based on the underlying ring operations. This opens up further optimization opportunities, e.g., by using intermediate results created during one FHE operation in a following operation. HECO’s modular nature and ease of extension via self-contained optimization passes should enable our system to easily integrate such optimizations as they emerge.

3.4 Code Generation & Back-End

HECO supports two different back-end modes, one aimed at general use and an ‘expert’ mode designed for advanced users. In general use, the majority of HECO is invisible to developers, who compile and run programs entirely from the front-end language (e.g., Python). The front end integrates with the compiler and the underlying FHE library to run the generated FHE application and transparently route inputs and outputs. In the ‘expert’ mode, HECO instead emits the C++ source code of the optimized FHE program designed to link directly against the FHE library, allowing advanced users to analyze and potentially modify the generated code. This addresses a limitation we observed with existing tools, whose rigid nature frequently prevented experts from taking advantage of them for more complex scenarios.

HECO is designed to rely on existing FHE implementations, such as those provided by Microsoft’s Simple Encrypted Arithmetic Library (SEAL) [39]. While there can be advantages to co-designing scheme implementations and compilers, these do not outweigh the downsides of using custom scheme
implementations. Such an approach requires not only significant re-implementation, but also effectively disconnects a tool from advancements in FHE research made in the wider community. For example, Cingulata [12] uses a custom BFV implementation that has been leapfrogged by BFV implementations in actively developed libraries like SEAL. Translating our scheme-specific IRs to the function calls of the various library APIs is mostly trivial, as there is a nearly 1:1 correspondence between the two. Beyond targeting libraries, HECO is also designed to easily allow adding support for generating code targeting future FHE-specific hardware accelerators [4].

4 Automatic SIMD Batching

HECO introduces new FHE-specific optimizations to automatically rewrite standard imperative programs to match the unique programming paradigm of FHE, harnessing the performance potential of the SIMD parallelism present in most modern schemes by automatically identifying and exploiting opportunities to apply batching. Due to the large capacity of FHE ciphertexts (usually $2^{13} - 2^{16}$ slots), effective use of batching is arguably the single most important optimization for many applications, drastically reducing ciphertext expansion overhead and computation time. However, unlocking the performance potential of batching currently requires significant expertise and experience in writing FHE applications: existing algorithms must be translated to ones expressed solely as SIMD operations and rotations, as the only natively supported data movements are cyclical rotations of the elements inside a ciphertext.

Enabling non-experts to write efficient and secure FHE applications requires abstracting away the complexities of FHE development. In HECO, developers are be able to express their algorithms conveniently in the standard imperative paradigm, making use of, e.g., loops that access and manipulate vector elements by accessing individual elements. However, such programs do not align well with the restricted set of homomorphic SIMD operations and rotations offered by FHE schemes, requiring the compiler to translate and restructure the application. In the following, we describe the core idea behind our automatic transformation, and then walk through the individual challenges and our solutions on how to efficiently identify and exploit batching opportunities.

4.1 Approach

There is a stark contrast between naively translated code and the expert-level FHE-native code which HECO aims to produce. Rewriting code to operate virtually exclusively on entire ciphertexts rather than individual elements, all while considering the restricted ability for data movement in FHE, usually requires making far-reaching changes across the entire program. In fact, it is not unusual for expert-optimized versions of FHE applications to be nearly unrecognizable when compared to a non-FHE or naive implementation of the same function, as can be seen in Listing 1.

While program synthesis can achieve such drastic transformations [40], these approaches are not always practical, taking minutes to optimize even comparatively simple applications. Instead, we want to avoid the excessive performance penalty of synthesis and achieve similar results in (fractions of) seconds through a combination of efficient local transformations. Note that traditional SLP vectorization algorithms [41, 42], which try to group operations into SIMD instructions, do not apply in our setting. This is because they generally rely on the

Listing 1: Both of these functions apply a simple sharpening filter to an encrypted image of size $n \times n = N$, by convolving a $3 \times 3$ kernel ($-8$ in the center, 1 everywhere else) with the image. The version on the left encrypts each pixel individually, and follows the textbook version of the algorithm, operating over a vector of $N$ ciphertexts. The version on the right batches all pixels into a single ciphertext and uses rotations ($\ll$) and SIMD operations to compute the kernel over the entire image at the same time. Designing batched implementations requires out-of-the-box thinking in addition to significant expertise and experience.
ability to efficiently scatter/gather elements into and out of vectors, something that is only possible at great cost in FHE.

Since we want to target a broad range of domains and applications, our transformations must be domain-independent and apply generically to a range of different applications. We want to target generic patterns that are independent of the function being computed, and do not consider optimizations that change the application, such as moving parts of the computation to the client-side.

Based on these requirements, HECO focuses its batching optimizations on operations featuring vector\(^1\) elements. These are a natural target, as well-written code featuring vectors is likely to be highly batchable, even if this not immediately recognizable. While theoretically possible, searching for batching opportunities across scalar variables is not only computationally intensive, but also likely to result in no (significant) improvement in overall performance: reasonably written code usually does not contain enough ‘free’ scalar variables to make batching worthwhile given the lack of efficient scatter/gather operations and the very large capacity (≥ 4096) of FHE ciphertexts.

**Strawman Approach.** Note that encrypting the elements of a vector into a single ciphertext instead of a vector of ciphertexts does not improve performance unless the algorithm is also modified to take advantage of this. Naively emulating operations over individual elements would instead introduce significant overhead: Since native FHE operations act equally on all ciphertext slots and values must be aligned in the same slots to interact, emulating, e.g., \(x[i] = x[i] + y[j]\) with native FHE operations introduces several additional steps. First, \(y[j]\) must be rotated so that it is moved from slot \(j\) to slot \(i\). Then, the addition operation must be applied to a copy of \(x\) to avoid overwriting the other slots. Finally, the updated copy and the original ciphertext must be multiplied with complementary binary masks, so that when they are added, only slot \(i\) is set to the updated value. Due to the high cost of rotations and multiplications, applying this trivial batching approach would be prohibitively inefficient.

**HECO’s Approach.** Instead of naively translating element-wise operations to the native FHE operations, HECO applies a series of transformations and optimizations that aim to remove the need to operate over individual vector elements. Here, we see a benefit of our high-level IR, which can contain ‘virtual’ operations such as extract/insert operations which do not map directly to native FHE operations. At its core, our optimizations replace each operation over vector elements with an equivalent SIMD operation acting on entire vectors, inserting ‘virtual’ operations as required to maintain the internal consistency of the program. While this might appear equivalent to the strawman emulation approach, our transformations are carefully designed to allow us to optimize the resulting program further. As a result, we can frequently completely eliminate all ‘virtual’ operations without ever having to materialize them to native FHE operations. Our batching optimization consists of several passes with distinct purposes, which we group into a pre-processing stage and a main stage for easier presentation.

**Notation & Conventions.** In the following, we assume vectors are batched into ciphertext slots continuously (i.e., a ciphertext encrypting \(x\) contains \(x[i]\) in slot \(i\) and scalars are stored in the first slot (index 0) of a ciphertext. Note that, for ease of presentation, we use high-level code (e.g., \(x[i] + y[i]\)) for examples, instead of following the structure of the SSA IR (e.g., \%e=extract(x, i); \%f=extract(y, i); \%z=\%e+\%f). The differences between the IRs and equivalent high-level code are not relevant to understand the ideas presented, but the SSA form is essential in enabling efficient implementations of transformations.

### 4.2 Preprocessing

Before applying our batching transformations, we preprocess the program with transformations that, while not directly batching-related, enable the efficient optimizations in the following steps. In addition to standard simplifications and canonicalizing the IR (i.e., bringing operations into a standardized ‘canonical’ form to reduce the complexity of the IR), we also apply two more specialized transformations.

**Vectorizing Plaintexts.** FHE applications frequently include operations between secret values and plaintext values that are known to the party performing the computation. For example, an ML prediction-as-a-service server where clients provide encrypted inputs still has access to the model parameters in plaintext form. Ciphertext-plaintext operations are significantly more efficient than ciphertext-ciphertext operations, and HECO exploits them whenever possible. In addition, the presence of plaintext values can also enable further batching optimizations. Hand-optimized FHE applications sometimes use unconventional data layouts in order to improve performance. For example, matrix-vector products can be significantly accelerated when the matrix elements are grouped by their (generalized) diagonals, rather than row- or column-wise.

While re-arranging the layout of encrypted data requires expensive rotations, masking, and combining, plaintext values can be freely (re-)arranged, opening up opportunities for optimization. HECO batches plaintext values appearing in ciphertext-plaintext operations involving vectors into the required slots. For example, for \(z[i] = x[i] + a\) and \(z[j] = x[j] + b\) where \(a\) and \(b\) are plaintext values, we replace them with vector elements by batching them into a vector.

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\(^1\) Vectors, arrays and (indexed) lists are interchangeable for our purposes.
p so that p[i]=a and p[j]=b. This allows later batching optimizations to combine both operation into a single SIMD operation. While this might not catch all optimization opportunities, it is frequently sufficient to match expert-designed approaches such as the diagonalized matrix-vector product example mentioned above.

Merging Arithmetic Operations. During the pre-processing stage, we combine chained applications of (associative and commutative) binary operations into larger arithmetic operations with multiple operands (e.g., merging x=a+b; y=x+c to y=a+b+c). While not an optimization in itself, it frequently enables other simplification as it removes chains of dependencies and replaces them with a single operation. It is also extremely useful in the context of ‘fold’-style operations such as computing the sum of all elements in vector (e.g., for i in 0..10: sum = sum + x[i]). This common pattern can be optimized in non-trivial ways when using batched FHE, and this pre-processing step is essential in allowing later stages to efficiently optimize such applications. Note that, in the next section, we will use binary operations as examples for easier presentation, but the techniques described readily generalize to operations with more operands.

4.3 SIMD-ification

The goal of the main stage of our batching transformation is to eliminate operations on individual vector elements, replacing as much of the input program as possible with SIMD operations over entire ciphertexts. Many traditional vectorization techniques targeting CPU vector extensions actively search for combinable operations. These approaches generally rely heavily on the ability to (re)-arrange vectors freely, making them ill-suited for FHE’s more restrictive programming paradigm. Instead, HECO translates individual operations over vector elements into equivalent SIMD operations over entire ciphertexts, inserting ‘virtual’ insert/extract operations to maintain the consistency of the IR. While this does not directly reduce the amount of work the program is doing, the way we perform this transformation is carefully designed to allow any inherent parallelism present in the input program to emerge in a way that the compiler can easily recognize and simplify. Therefore, the translation pass is followed by a ‘clean-up’ pass that realizes the opportunities exposed by the translation pass. Finally, any remaining ‘virtual’ operations are efficiently lowered to native FHE operations, completing the transformation of the program.

Our batching optimization takes as input a program, expressed in our high-level SSA-based IR, that has undergone preprocessing but still contains operations over elements of (secret) vectors. During the optimization, vector types tensor<secret<T>> are translated to batchedsecret<T>, a high-level scheme-independent abstraction of batched ciphertexts. After this pass completes, the program contains only SIMD operations over entire vectors, which can be realized using native FHE operations.

Translation Approach. At its core, this pass is a linear walk over all (arithmetic) operations in the program. For each operation, we (i) transform the operands into a form suitable for a SIMD-operation, (ii) issue the actual SIMD operation and (iii) ensure the result is still valid for use in situations where a scalar is expected (including uses in later operations that have yet to be transformed). Since our batching convention uses slot 0 to store scalar values, an obvious approach would be to rotate each operand (e.g. x[i]) so that the element of interest ends up in slot 0. Then, performing the SIMD operation over the rotated operands produces the result in the same slot, trivially ensuring the result is in ‘scalar’ form (i.e., value in slot 0). While this approach is correct and easy to implement, it is not a suitable approach for an optimizing compiler, as it does not set the program up for further simplification.

Operand Mapping. Our approach introduces a complexity not present in the naive approach: When translating a statement with elements from different positions in their vectors (e.g., x[i]+y[j]), we must first bring the elements of interest into alignment by issuing a rotation for one of the operands. Note that this is no worse than the naive approach, which always requires two rotations. Again, our approach enables further simplifications when considering such operations appearing inside a loop. While each iteration requires a rotation for one of the operands, these are also all easy-to-simplify duplicates of each other, as long as the relative offset between i and j is constant. In order to ensure that such simplifications are possible, the choice of rotation must be the same for all
equivalent operations. By setting the target slot to be the first operand’s index and rotating the other operand to match, in combination with having canonicalized the IR during pre-processing, we can ensure this is always the case.

As mentioned earlier, the rotations of the results can also frequently be optimized away. Specifically, when the result of an operation is assigned back into a vector (e.g., \(z[i] = x[i] + y[i]\)), the rotations to slot 0 and then back to \(i\) cancel out and can be removed. In order to ensure this also works in less straightforward cases (e.g., \(z[i] = x[i] + y[i]\)), \(HECO\)’s target slot calculation recognizes when the immediate uses of an operation include an insertion operation and gives preference to the insertion’s target slot.

**Realizing Fold Patterns Efficiently.** Applying our batching transformation to fold or accumulation patterns (e.g., for \(i\) in \(0..10\): \(\text{sum} = \text{sum} + x[i]\)) creates a linear number of rotations for the inputs and, if necessary, an output rotation. Realizing the many-operand arithmetic operation (c.f. § 4.2) using \(FHE\) also requires a linear number of (binary) native \(FHE\) operations. While this might seem reasonable or even optimal, we can significantly improve upon this when using batched \(FHE\): experts translate these fold patterns into **logarithmic** series of rotations and arithmetic operations by taking a copy of the current ciphertext, rotating it by half the current work-set size (half/quarter/etc) and adding that rotated ciphertext to the current ciphertext. The result then becomes the current ciphertext and the process continues until, after only \(\log n\) steps, each slot contains the sum of all elements, as shown in Figure 4.

Clearly, this optimization requires a holistic view of the operation, which is one of the reasons we merge operations during pre-processing. Given this expanded representation of an operation, we can efficiently determine if it is a candidate for a fold-style transformation. While we used a sum over all elements as an example here, the technique can be generalized to any subset with a consistent stride. When applied to multiplication, it additionally has the benefit of automatically reducing the multiplicative depth of the expression as a side-effect.

Note that the pre-processing combination of binary operations into larger operations **must** happen before the batching translations described above, as the batching pass would otherwise insert rotations between the different operations, making them no longer directly chained. The actual translation to a series of rotations and native binary operations, meanwhile, has to be performed **after** the batching transformation described above, since it requires the operands to be entire ciphertexts, rather than scalars. Additionally, the de-duplication simplifications that can take place after the batching transformation can widen the applicability of this transformation by reducing the number of distinct ciphertexts appearing in the program. This is an example of emergent optimizations that occur not due to any individual transformation, but from the combination of many different transformations and simplifications adding up to more than the sum of its parts.

After these passes, and the associated clean-up passes, the program has been transformed, as far as possible, into an optimized and efficient batched program. Any operations (e.g., rotations) we introduced for consistency that have not been simplified away, are likely to be fundamental components of the algorithm. In the ideal case, the program is now equivalent to what an expert developer would have written.

## 5 Implementation

We build \(HECO\) on top of the open-source MLIR framework [16], which is rapidly establishing itself as the go-to tool for domain-specific compilers and opens up the possibility of exchanges of ideas and optimizations even beyond the \(FHE\) community. \(HECO\) consists of roughly 15k LOC of C++, with around 2k LOC of Python for the python frontend. \(HECO\) uses the Microsoft Simple Encrypted Arithmetic Library (SEAL) as its \(FHE\) backend. SEAL, first released in 2015, is an open-source \(FHE\) library implemented in C++ that is thread-safe and heavily multi-threaded itself. SEAL implements the BFV and CKKS schemes, and a fork implementing the BGV scheme is available, too.

In contrast to existing, monolithic, compilers, \(HECO\) is highly modular and designed to be flexible and extensible. We decouple optimizations from front-end logic, allowing for a wide variety of domain-specific front-ends and the ability to easily replace back-ends to target different \(FHE\) libraries or hardware accelerators as they become available. The toolchain can easily be adapted to different needs, with certain optimizations enabled or disabled as required.

## 6 Evaluation

\(HECO\) is designed to compile high-level programs, written by non-experts in the standard imperative paradigm, into highly efficient batched \(FHE\) implementations that achieve the same performance as hand-crafted implementations by
experts. HECO achieves its usability goals through a well-integrated Python front-end and by requiring developers to alter their code only minimally (annotating variables as secret). However, ease-of-use becomes moot when the performance of the generated code is not competitive. Therefore, we focus our evaluation on the performance of HECO and the code it generates, trying to answer whether or not automatic optimizations can bring naive code to the same performance level as expert implementations.

6.1 Evaluation Setup

We compare the solutions generated by HECO to a naive non-batched baseline and an optimally-batched synthesized solution. We report the run-time of the computation and the speedups achieved. We also present some insights into memory usage, which can be a limiting factor for non-batched approaches. In addition to metrics about the generated programs, we report on the compile-time of HECO. Especially for non-experts, a fast development cycle that allows iterating on ideas and development in ‘real time’ has a significant effect on user experience.

Baseline. Our baseline is a naive implementation of the application without taking advantage of batching, as one might expect FHE novices to implement. In this setting, vectors of secrets are directly translated to vectors of ciphertexts. Since the underlying FHE libraries usually provide rather low-level APIs, even this ‘naive’ baseline is not trivial to implement. While this approach introduces significant ciphertext expansion and increases the memory required, it is actually preferable in terms of run-time over a solution that batches vector data into ciphertexts, but does not re-structure the program to be batching-friendly. This is because such a solution adds the overhead of rotations, masking, etc., to the base runtime of the non-batched solution.

Synthesis. Synthesis-based approaches explore the space of all possible programs, constrained by a reference specification describing input-output behavior. While this tends to be computationally expensive, it has the potential to find optimal solutions featuring highly non-intuitive optimizations. Porcupine [40] is a synthesis-based compiler for batched FHE, using synthesis to find an optimal batching pattern for a given application. In addition to a reference specification, it takes a developer-provided sketch of an initial possible batching approach. In some cases, their synthesized solutions outperform even the previously accepted state-of-the-art approaches used by experts. While the significant search time Porcupine requires to find these solutions (over 10 minutes to synthesis a program with 10 instructions) makes it less practical for day-to-day development, it provides a useful lower bound to compare our optimizations against.

Environment. All benchmarks are executed on AWS m5np.xlarge instances which provide 4 cores and 16 GB of RAM. We used Microsoft SEAL [39] as the underlying FHE library, targeting its BFV [20, 21] scheme implementation. All experiments are run using the same parameters which ensure at least 128-bit security. We only report the runtime of the computation itself, omitting client-side aspects such as key-generation or encryption/decryption. All results are the average of 10 iterations, discarding top and bottom outliers.

Applications. We compare the baseline, HECO and synthesis approaches on a set of nine benchmark applications that represent a variety of common code patterns relevant to batching. These programs range from having no inter-element dependencies (e.g., Linear Polynomial), to simple accumulator patterns (e.g., Hamming Distance) and complex dependencies across a multitude of different vector elements (e.g., Roberts Cross). As a result, they provide a useful benchmark especially for batching optimizations.

These benchmarks were first proposed by Porcupine, and our ‘synthesis’ solutions are based on pseudo-code made available in an extended version of the paper [40]. The synthesized dot-product code targets 8-element vectors, while those for Hamming Distance and L2 Distance were provided for 4-element vectors. For the other applications, we use vectors of length 4096, representing 64x64 pixel images.

In the following, we briefly describe two applications which will explore in more detail, as they are representative of common batching opportunities. The roberts cross operator is an edge-detection feature used in image processing. It approximates the gradient of an image as the square root of the sum-of-squares of two different convolutions of the image, which compute the differences between diagonally adjacent pixels. As in all other kernel-based benchmarks, wrap-around padding is used, which aligns well with the cyclical rotation paradigm of FHE. In order to enable a practical FHE evaluation, the final square root is omitted, since it would be prohibitively expensive to evaluate under encryption. The hamming distance, meanwhile, computes the edit distance between two vectors, i.e., the number of positions at which they disagree. Here, we consider two binary vectors of the same length, a setting in which computing (non-)equality can be done efficiently using the arithmetic operations available in FHE. Specifically, this makes use of the fact that $\text{NEQ}(a,b) = \text{XOR}(a,b) = (a - b)^2$ for $a, b \in \{0, 1\}$.

6.2 HECO’s Performance

We start by comparing the performance of HECO’s generated solutions against the baseline and synthesis approaches. Since the synthesis approach does not scale to larger settings, we then dive into a more detailed evaluation of how HECO performs across different sizes of inputs, comparing HECO
against the baseline in run-time and memory consumption, and also provide insights into compile times.

As we can see from Figure 5, HECO dramatically improves performance over the non-batched baseline approach. For example, for the Roberts Cross benchmark, our batched solution is over 3500 times faster than the non-batched solution, taking less than 0.04 seconds instead of over 2.35 minutes. In addition, our results are nearly equivalent to the optimally batched solutions synthesized by Porcupine, especially when considering the stark contrast between the non-batched and the two batched solutions. In some cases (e.g., Box Blur), Porcupine has an advantage because it finds non-intuitive solutions beyond traditional batching patterns. Interestingly, for some applications (e.g. Hamming Distance) HECO actually outperforms Porcupine. This is because porcupine provides an optimally batched solution, but does not necessarily handle ciphertext management optimally, inserting unnecessary relinearization operations.

In order to better demonstrate the effect of batching on application performance, we explore two of the benchmark applications in further detail, exploring how runtime, speed-up, memory usage, and compile time vary with the size of the input program.

Roberts Cross Operator. In Figure 6a/6c, we show the runtime and memory requirements of the Roberts Cross benchmark for varying instance sizes, comparing the non-batched baseline with the batched solution generated by HECO. While the runtime of the naive version increases linearly with the image size, the batched solution maintains the same performance (until parameters must be increased to accommodate even larger images). Instead, the runtime is more closely tied to the size of the kernel than to that of the image. This highlights the dramatic transformations achieved by HECO, fundamentally changing the structure of the program. As a result of these transformation, HECO achieves a speedup of 3454x over the
non-batched baseline for 64x64 pixel images, demonstrating the extraordinary impact that effective use of batching can have on FHE applications: while the non-batched solution is borderline impractical at over two minutes, the batched solution takes only a fraction of a second (0.04 s).

Hamming Distance. The runtime of the generated code in this case does depend directly on the vector length. However, due to the fold-style optimization (c.f. § 4.3), this dependence is only logarithmic. In Figure 6d/6b, we can see that, for realistic problem sizes, the performance advantage of batching becomes significant, resulting in a speedup of 934x for 4096-element vectors. We also see that, while the runtime of the batched solution does increase with the vector length, this is nearly imperceptible when compared to the non-batched baseline.

Compile Time. HECO achieves these fundamental transformations without requiring heavy-weight approaches such as synthesis. While Porcupine takes just over two seconds for the hamming distance (for 4-element vectors), it requires more than 10 minutes to synthesize a batched solution for the Roberts Cross benchmark [40]. In contrast, HECO’s compile time never exceeds two seconds for either of the two benchmarks, as can be seen in Table 1.

## 7 Related Work

In this section, we briefly discuss related work in the domain of FHE compilation.

The EVA compiler [15] offers a user-friendly high-level interface and automatically inserts ciphertext maintenance operations into the circuit. EVA uses a circuit-based IR and requires developers to manually map their program to the FHE programming paradigm. In order to ease this process, recent versions [14] include a library of expert-implemented batched kernels for frequently used patterns, e.g., summing all elements in a vector. In contrast, HECO automatically translates arbitrary non-batched applications into efficient batched applications, with no involvement of the user.

The Porcupine compiler [40] is closest to our work, in that it also considers translating imperative programs to FHE’s batching paradigm. However, their tool has a significantly different focus, using a heavy-weight synthesis approach that tries to identify optimal solutions that can outperform even state-of-the-art approaches used by experts. Since it explores a large state space in the search for an optimal solution, compile times tend to be long (up to many minutes) and programs can contain at most a handful of statements before the approach becomes infeasible. Additionally, Porcupine requires that developers provide a sketch of the structure of the batched program, making it less suitable for non-expert users.

| $n$  | 4   | 16  | 64  | 256 | 1024 | 4096 |
|------|-----|-----|-----|-----|------|------|
| rc   | 0.06| 0.07| 0.08| 0.12| 0.30 | 1.28 |
| hd   | 0.03| 0.04| 0.06| 0.09| 0.25 | 1.85 |

Table 1: Compile time (in seconds) of the Roberts Cross (rc) and Hamming Distance (hd) benchmarks for different problem sizes ($n$).

Tools like Cingulata [12], E3 [13], and Google’s Transpiler [11] translate their input programs into binary circuits, encrypting each bit of the input individually. This allows them to translate arbitrary programs, without the usual restrictions of FHE. However, programs translated in this way are virtually always too inefficient to be of practical use.

Finally, current domain-specific compilers [9, 43], e.g., targeting encrypted Machine Learning applications, rely on a large set of hand-written expert-optimized kernels for common functionality (mostly linear algebra operations). Since these tools rely on pre-determined mappings rather than automatically identifying optimization opportunities, they do not transfer to other domains, such as the general-purpose setting HECO targets.

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