D Physics

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Abstract
Recently a lot of new experimental results on open charm hadrons have appeared. In particular many D meson resonances have been discovered. We discuss strong decays of positive and negative parity charmed mesons within heavy meson chiral perturbation theory and study the impact of excited charm states on the determination of the effective meson couplings [1]. Motivated by recent experimental results we also reconsider semileptonic $D \to P l \nu_l$ and $D \to V l \nu_l$ decays within a model which combines heavy quark symmetry and properties of the chiral Lagrangian. Using limits of soft collinear effective theory and heavy quark effective theory we parametrize the semileptonic form factors. We include excited charm meson states in our Lagrangians and determine their impact on the charm meson semileptonic form factors. In some scenarios of new physics an up-like heavy quark appears, which induces FCNC at tree level for the $c \to u Z$ transitions. We investigate FCNC effects in D rare decays in particular the $c \to u l^+ l^-$ transition which might occur in $D^+ \to \pi^+ l^+ l^-$ and $D^0 \to \rho^0 l^+ l^-$. 

1 Strong decays of positive and negative parity charmed mesons

The strong and electromagnetic transitions of positive and negative parity charmed mesons have already been studied within a variety of approaches (see references [8] - [23] given in [1]). In ref. [2] the chiral loop corrections to the $D^* \to D \pi$ and $D^* \to D \gamma$ decays were calculated and a numerical extraction of the one-loop bare couplings was first performed. Since this calculation preceded the discovery of even-parity meson states, it did not involve loop contributions containing the even-parity meson states. The ratios of the radiative and strong decay widths, and the isospin violating decay

\footnote{talk given by S. Fajfer}
$D_s^* \to D_s\pi^0$ were used to extract the relevant couplings. However, since that time, the experimental situation has improved and therefore we consider the chiral loop contributions to the strong decays of the even and odd parity charmed meson states using HH$\chi$PT. In our calculation we consider the strong decay modes $D^{*+}$, $D^{*0}$, $D_0^+$, and $D_0^0$ (given in Table 1 of [1]).

The existing data on the decay widths enable us to constrain the leading order parameters: the $D^*D\pi$ coupling $g$, $D_0^*D\pi$ coupling $h$, and the coupling $\tilde{g}$ which enters in the interaction of even parity charmed mesons and the light pseudo-Goldstone bosons. Although the coupling $\tilde{g}$ is not yet experimentally constrained, it moderately affects the decay amplitudes which we investigate.

Due to the divergences coming from the chiral loops one needs to include the appropriate counterterms. Therefore we construct a full operator basis of the relevant counterterms and include it into our effective theory Lagrangian. The details of the Heavy hadron chiral perturbation theory HH$\chi$PT we use is given in [1]. First we determine wavefunction renormalization of the relevant heavy meson fields considering the effects of the chiral loops given by the left diagram in Fig. 1. Then we calculate loop corrections for the $PP^*\pi$, $P_0P_1^*\pi$ and $P_0P\pi$ vertexes. At zeroth order in $1/m_Q$ expansion these are identical to the $P^*P^*\pi$, $P_1^*P_1^*\pi$ and $P_1^*P\pi$ couplings respectively due to heavy quark spin symmetry (right diagram in Fig. 1).

Using known experimental values for the decay widths of $D^{*+}$, $D_0^{*+}$, $D_0^{*0}$ and $D_1^0$, and the upper bound on the width of $D_0^{*0}$ one can extract the values for the bare couplings $g$, $h$ and $\tilde{g}$ from a fit to the data. The decay rates are namely given by

$$\Gamma(P_a^* \to \pi^i P_b) = \frac{|g_{P_a^* P_b \pi^i}|^2}{6\pi f^2} |\vec{k}_{\pi}|^3,$$

and a similar expression (up to polarization averaging phase space factors) for $\Gamma(P_0 \to \pi P)$ and $\Gamma(P_1^* \to \pi P^*)$ with $g$ coupling replaced by $h$ and $|\vec{k}_{\pi}|^3$ replaced by $E_\pi^2|\vec{k}_{\pi}|$. Here $\vec{k}_{\pi}$ is the three-momentum vector of the outgoing pion and $E_\pi$ its energy. The

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Figure 1: Sunrise (left) and sunrise road (right) topology diagrams
Calculation scheme & $g$ & $|h|$ & $\tilde{g}$ \\
\hline
Leading order & 0.61 & 0.52 & -0.15 \\
One-loop without positive parity states & 0.53 \\
One-loop with positive parity states & 0.66 & 0.47 & -0.06 \\
\hline

Table 1: Summary of our results for the effective couplings as explained in the text. The listed best-fit values for the one-loop calculated bare couplings were obtained by neglecting counterterms’ contributions at the regularization scale $\mu \simeq 1$ GeV.

The renormalization condition for the couplings can be written as

$$g_{P_n^* P_b \pi^i}^{\text{eff}} = g \sqrt{Z_{2P_n}} \sqrt{Z_{2P_b^*}} \sqrt{Z_{2\pi^i}} = g Z_{P_n^* P_b \pi^i}^g$$

with similar expressions for the $h$ and $\tilde{g}$ couplings.

We perform a fit with a renormalization scale set to $\mu \simeq 1$ GeV [2] and we neglect counterterm contributions altogether. Our choice of the renormalization scale in dimensional regularization is arbitrary and depends on the renormalization scheme. Therefore any quantitative estimate made with such a procedure cannot be considered meaningful without also thoroughly investigating counterterm, quark mass and scale dependencies. We perform a Monte-Carlo randomized least-squares fit for all the three couplings in the prescribed regions [1] using the experimental values for the decay rates to compute $\chi^2$ and using values from PDG [6] for the masses of final state heavy and light mesons. In the case of excited $D_0^*$ and $D_1^*$ mesons, we also assume saturation of the measured decay widths with the strong decay channels to ground state charmed mesons and pions ($D_0^* \to D \pi$ and $D_1^* \to D^* \pi$).

Due to the rather large mass splitting between positive in negative parity states $\Delta_{SH}$, we find that the perturbative expansion holds for scales below $\mu \leq 1$ GeV, while these new strongly scale dependent corrections become large at higher renormalization scales. From the data for the four decay widths we obtain the best-fitted values for the bare couplings, which we summarize in Table 1. We are able to determine all the three couplings since the contributions proportional to the coupling $\tilde{g}$ appear indirectly, through the loop corrections.

Since we consider decay modes with the pion in the final state, we do not expect sizable contribution of the counterterms. Namely, the counterterms which appear in our study are proportional to the light quark masses, and not to the strange quark mass [2]. The effects of counterterm contributions in the decay modes we analyze, are nevertheless estimated by making the random distribution of the relevant counterterm couplings (see [1]). The counterterm contributions of order $\mathcal{O}(1)$ can spread the best fitted values of $g$, $|h|$ by roughly 15% and $\tilde{g}$ by as much as 60%. Similarly, up to 20%
shifts in the renormalization scale modify the fitted values for the $g$ and $|h|$ by less than 10% while $\tilde{g}$ may even change sign at high renormalization scales. Combined with the estimated 20% uncertainty due to discrepancies in the measured excited heavy meson masses, we consider these are the dominant sources of error in our determination of the couplings. One should keep in mind however that without better experimental data and/or lattice QCD inputs, the phenomenology of strong decays of charmed mesons presented above ultimately cannot be considered reliable at this stage.

A full calculation of the strong decay couplings should also contain, in addition to the calculated contributions, the relevant $1/m_H$ corrections as discussed in ref [3]. There, the next to leading terms ($1/m_H$) were included in the study of charm meson mass spectrum. Due to the very large number of unknown couplings the combination of $1/m_H$ and chiral corrections does not seem to be possible for the decay modes we consider. In addition, recent studies of the lattice QCD groups [4, 5] indicate that the $1/m_H$ corrections do not contribute significantly to their determined values of the strong couplings, and we therefore assume the same to be true in our calculations of chiral corrections.

Due to computational problems associated with the chiral limit, lattice QCD studies perform calculations at large light quark masses and then employ a chiral extrapolation $m_\pi \to 0$ of their results. Our analysis of such chiral extrapolation of the coupling $g$ shows that the full loop contributions of excited charmed mesons give sizable effects in modifying the slope and curvature in the limit $m_\pi \to 0$. We argue that this is due to the inclusion of hard pion momentum scales inside chiral loop integrals containing the large mass splitting between charmed mesons of opposite parity $\Delta_{SH}$ which does not vanish in the chiral limit. If we instead impose physically motivated approximations for these contributions - we expand them in terms of $1/\Delta_{SH}$ - the effects reduce mainly to the changes in the determined values of the bare couplings, used in the extrapolation, with explicit $h$ contributions shrinking to the order of 5%. Consequently one can infer on the good convergence of the $1/\Delta_{SH}$ expansion.

As a summary of our results, we point out that chiral loop corrections in strong charm meson decays can be kept under control, but they give important contributions and are relevant for the precise extraction of the strong coupling constants $g$, $h$ and $\tilde{g}$.

## 2 Charm meson resonances in $D$ semileptonic decays

The knowledge of the form factors which describe the weak heavy $\to$ light semileptonic transitions is very important for the accurate determination of the CKM parameters from the experimentally measured exclusive decay rates. Usually, the attention has been devoted to $B$ decays and the determination of the phase of the $V_{ub}$ CKM
matrix element. At the same time in the charm sector, the most accurate determination of the size of $V_{cs}$ and $V_{cd}$ matrix elements is not from a direct measurement, mainly due to theoretical uncertainties in the calculations of the relevant form factors’ shapes.

Recently, there have been new interesting results on $D$-meson semileptonic decays. The CLEO and FOCUS collaborations have studied semileptonic decays $D^0 \rightarrow \pi^- \ell^+ \nu$ and $D^0 \rightarrow K^- \ell^+ \nu$\cite{7,8}. Their data provide new information on the $D^0 \rightarrow \pi^- \ell^+ \nu$ and $D^0 \rightarrow K^- \ell^+ \nu$ form factors. Usually in $D$ semileptonic decays a simple pole parametrization was used in the past. The results of Refs.\cite{7,8} for the single pole parameters required by the fit of their data, however, suggest pole masses, which are inconsistent with the physical masses of the lowest lying charm meson resonances. In their analyses they also utilized a modified pole fit as suggested in\cite{9} and their results indeed suggest the existence of contributions beyond the lowest lying charm meson resonances\cite{7}.

In addition to these results new experimental studies of charm meson resonances have provided a lot of new information on the charm sector\cite{10,11,12,13} which we can now apply to $D$ and $D_s$ semileptonic decays.

The purpose of our studies\cite{14,15,16} is to accommodate contributions of the newly discovered and theoretically predicted charm mesons in form factors which are parametrized using constraints coming from heavy quark effective theory (HQET) limit for the region of $q^2_{\text{max}}$ and in the $q^2 \approx 0$ region using results of soft collinear effective theory (SCET). We restrain our discussion to the leading chiral and $1/m_H$ terms in the expansion.

The standard decomposition of the current matrix elements relevant to semileptonic decays between a heavy pseudoscalar meson state $|H(p_H)\rangle$ with momentum $p_H^\nu$ and a light pseudoscalar meson state $|P(p_P)\rangle$ with momentum $p_P^\nu$ is in terms of two scalar functions of the exchanged momentum squared $q^2 = (p_H - p_P)^2$ – the form factors $F_+(q^2)$ and $F_0(q^2)$. Here $F_+$ denotes the vector form factor and it is dominated by vector meson resonances, while $F_0$ denotes the scalar form factor and is expected to be dominated by scalar meson resonance exchange\cite{17,18}. In order that the matrix elements are finite at $q^2 = 0$, the form factors must also satisfy the well known relation $F_+(0) = F_0(0)$.

The transition of $|H(p_H)\rangle$ to light vector meson $|V(p_V, \epsilon_V)\rangle$ with momentum $p_V^\nu$ and polarization vector $\epsilon_V^\nu$ is similarly parameterized in terms of four form factors $V, A_0, A_1$ and $A_2$, again functions of the exchanged momentum squared $q^2 = (p_H - p_V)^2$. Here $V$ denotes the vector form factor and is expected to be dominated by vector meson resonance exchange, the axial $A_1$ and $A_2$ form factors are expected to be dominated by axial resonances, while $A_0$ denotes the pseudoscalar form factor and is expected to be dominated by pseudoscalar meson resonance exchange\cite{18}. As in previous case in order that the matrix elements are finite at $q^2 = 0$, the form factors must also satisfy the well known relation $A_0(0) + A_1(0)(m_H + m_V)/2m_V - A_2(0)(m_H -$.
\( m_V/2m_V = 0 \).

Next we follow the analysis of Ref. [9], where the \( F_+ \) form factor in \( H \to P \) transitions is given as a sum of two pole contributions, while the \( F_0 \) form factor is written as a single pole. This parametrization includes all known properties of form factors at large \( m_H \). Using a relation which connects the form factors within large energy release approach \([19]\) the authors in Ref. [9] propose the following form factor parametrization

\[
\begin{align*}
F_+(q^2) &= \frac{F(0)}{(1-x)(1-ax)}, \\
F_0(q^2) &= \frac{F(0)}{1-bx},
\end{align*}
\]

where \( x = q^2/m_{H^*}^2 \).

Utilizing the same approach we propose a general parametrization of the heavy to light vector form factors, which also takes into account all the known scaling and resonance properties of the form factors \([15, 16]\). As already mentioned, there exist the well known HQET scaling laws in the limit of zero recoil \([20]\) while in the SCET limit \( q^2 \to 0 \) one obtains that all four \( H \to V \) form factors can be related to only two universal SCET scaling functions \([19]\).

The starting point is the vector form factor \( V \), which is dominated by the pole at \( t = m_H^2 \), when considering the part of the phase space that is close to the zero recoil. For the heavy \( \to \) light transitions this situation is expected to be realized near the zero recoil where also the HQET scaling applies. On the other hand, in the region of large recoils, SCET dictates the scaling described in \([19]\). In the full analogy with the discussion made in Refs. [9, 21], the vector form factor consequently receives contributions from two poles and can be written as

\[
V(q^2) = \frac{V(0)}{(1-x)(1-ax)},
\]

where \( x = q^2/m_{H^*}^2 \) ensures, that the form factor is dominated by the physical \( H^* \) pole, while \( a \) measures the contribution of higher states which are parametrized by another effective pole at \( m_{\text{eff}}^2 = m_{H^*}^2/a \).

An interesting and useful feature one gets from the SCET is the relation between \( V \) and \( A_1 \) \([19, 22, 23, 24]\) at \( q^2 \approx 0 \). When combined with our result \((4)\), it imposes a single pole structure on \( A_1 \). We can thus continue in the same line of argument and write

\[
A_1(q^2) = \xi \frac{V(0)}{1-b'x}.
\]

Here \( \xi = m_H^2/(m_H + m_V)^2 \) is the proportionality factor between \( A_1 \) and \( V \) from the SCET relation, while \( b' \) measures the contribution of resonant states with spin-parity assignment \( 1^+ \) which are parametrized by the effective pole at \( m_{H^{'*}}^2 = m_{H^*}^2/b' \). It can be readily checked that also \( A_1 \), when parametrized in this way, satisfies all the scaling constraints.
Next we parametrize the $A_0$ form factor, which is completely independent of all the others so far as it is dominated by the pseudoscalar pole and is proportional to a different universal function in SCET. To satisfy both HQET and SCET scaling laws we parametrize it as

$$A_0(q^2) = \frac{A_0(0)}{(1 - y)(1 - a'y)},$$

where $y = q^2/m_H^2$ ensures the physical $0^-$ pole dominance at small recoils and $a'$ again parametrizes the contribution of higher pseudoscalar states by an effective pole at $m_{H'}^2 = m_H^2/a'$. The resemblance to $V$ is obvious and due to the same kind of analysis [9] although the parameters appearing in the two form factors are completely unrelated.

Finally for the $A_2$ form factor, due to the pole behavior of the $A_1$ form factor on one hand and different HQET scaling at $q_{max}^2$ on the other hand, we have to go beyond a simple pole formulation. Thus we impose

$$A_2(q^2) = \frac{A_2(0)}{(1 - b'x)(1 - b''x)},$$

which again satisfies all constraints. Due to the relations between the form factors we only gain one parameter in this formulation, $b''$. This however causes the contribution of the $1^+$ resonances to be shared between the two effective poles in this form factor.

At the end we have parametrized the four $H \to V$ vector form factors in terms of the six parameters $V(0), A_0(0), a, a', b'$ and $b''$ ($A_2(0)$ is fixed by the kinematical constraint).

In our heavy meson chiral theory (HM$\chi$T) calculations we use the leading order heavy meson chiral Lagrangian in which we include additional charm meson resonances. The details of this framework are given in [14] and [15]. We first calculate values of the form factors in the small recoil region. The presence of charm meson resonances in our Lagrangian affects the values of the form factors at $q_{max}^2$ and induces saturation of the second poles in the parameterizations of the $F_+(q^2)$, $V(q^2)$ and $A_0(q^2)$ form factors by the next radial excitations of $D^*_+(s)$ and $D(s)$ mesons respectively. Although the $D$ mesons mat not be considered heavy enough, we employ these parameterizations with model matching conditions at $q_{max}^2$. Using HQET parameterization of the current matrix elements [14, 15], which is especially suitable for HM$\chi$T calculations of the form factors near zero recoil, we are able to extract consistently the contributions of individual resonances from our Lagrangian to the various $D \to P$ and $D \to V$ form factors. We use physical pole masses of excited state charmed mesons in the extrapolation, giving for the pole parameters $a = m_{H^*}/m_{H'}, a' = m_{H^*}/m_{H''}, b' = m_{H^*}/m_{H'}, b'' = m_{H^*}/m_{H''}$. Although in the general parameterization of the form factors the extra poles in $F_+, V$ and $A_{0,1,2}$ we can parametrize all the neglected higher resonances beyond the ground state heavy meson spin doublets ($0^-, 1^-$), we are here
saturating those by a single nearest resonance. The single pole $q^2$ behavior of the $A_1(q^2)$ form factor is explained by the presence of a single $1^+$ state relevant to each decay, while in $A_2(q^2)$ in addition to these states one might also account for their next radial excitations. However, due to the lack of data on their presence we assume their masses being much higher than the first $1^+$ states and we neglect their effects, setting effectively $b'' = 0$.

The values of the new model parameters appearing in $D \to P l l \nu_l$ decay amplitudes [14] are determined by fitting the model predictions to known experimental values of branching ratios $\mathcal{B}(D^0 \to K^{-} l^+ \nu_l)$, $\mathcal{B}(D^+ \to \overline{K}^0 l^+ \nu_l)$, $\mathcal{B}(D^0 \to \pi^- l^+ \nu_l)$, $\mathcal{B}(D^0 \to \pi^0 l^+ \nu_l)$, $\mathcal{B}(D_s^+ \to \eta l^+ \nu_l)$ and $\mathcal{B}(D_s^+ \to \eta' l^+ \nu_l)$ [6]. In our calculations of decay widths we neglect the lepton mass, so the form factor $F_0$, which is proportional to $q^2$, does not contribute. For the decay width we then use the integral formula proposed in [25] with the flavor mixing parametrization of the weak current defined in [14].

Similarly in the case of $D \to V l l \nu_l$ transitions we have to fix additional model parameters [15] and we again use known experimental values of branching ratios $\mathcal{B}(D_0 \to K^*^- l^+ \nu_l)$, $\mathcal{B}(D_s^+ \to \Phi l^+ \nu_l)$, $\mathcal{B}(D^+ \to \rho^0 l^+ \nu_l)$, $\mathcal{B}(D^+ \to K^{*0} l^+ \nu_l)$, as well as partial decay width ratios $\Gamma_L/\Gamma_T(D^+ \to K^{*0} l^+ \nu_l)$ and $\Gamma_L/\Gamma_T(D^+ \to K^{*0} l^+ \nu_l)$ [6]. We calculate the decay rates for polarized final light vector mesons using helicity amplitudes $H_{+,-,0}$ as in for example [26]. By neglecting the lepton masses we again arrive at the integral expressions from [25] with the flavor mixing parametrization of the weak current defined in [15].

We first draw the $q^2$ dependence of the $F_+$ and $F_0$ form factors for the $D^0 \to K^-$, $D^0 \to \pi^-$ and $D_s \to K^0$ transitions. The results are depicted in Fig. 2. Our model results, when extrapolated with the double pole parameterization, agree well with previous theoretical [27, 28] and experimental [7, 8] studies whereas the single pole extrapolation does not give satisfactory results. Note that without the scalar resonance, one only gets a soft pion contribution to the $F_0$ form factor. This gives for the $q^2$ dependence of $F_0$ a constant value for all transitions, which largely disagrees with lattice QCD results [28] as well as heavily violates known form factor relations.

We also calculate the branching ratios for all the relevant $D \to P$ semileptonic decays and compare the predictions of our model with experimental data from PDG. The results are summarized in Table 2. For comparison we also include the results for the rates obtained with our approach for $F_+(q_{\text{max}}^2)$ but using a single pole fit. It is very interesting that our model extrapolated with a double pole gives branching ratios for $D \to P l l \nu_l$ in rather good agreement with experimental results for the already measured decay rates. It is also obvious that the single pole fit gives the rates up to a factor of two larger than the experimental results. Only for decays to $\eta$ and $\eta'$ as given in Table 2, an agreement with experiment of the double pole version of the model is not better but worse than for the single pole case.

We next draw the $q^2$ dependence of all the form factors for the $D^0 \to K^{*-}$,
Figure 2: $q^2$ dependence of the $D^0 \to K^-$ (upper left), $D^0 \to \pi^-$ (upper right) and $D_s^+ \to K^0$ (lower) transition form factors.

Table 2: The branching ratios for the $D \to P$ semileptonic decays. Comparison of our model fit with experiment as explained in the text.

| Decay          | $\mathcal{B}$ (Mod. double pole) [%] | $\mathcal{B}$ (Mod. single pole) [%] | $\mathcal{B}$ (Exp. PDG) [%] |
|----------------|-------------------------------------|--------------------------------------|-------------------------------|
| $D^0 \to K^-$  | 3.4                                 | 4.9                                  | 3.43 ± 0.14                   |
| $D^0 \to \pi^-$| 0.27                                | 0.56                                 | 0.36 ± 0.06                   |
| $D_s^+ \to \eta$| 1.7                                | 2.5                                  | 2.5 ± 0.7                     |
| $D_s^+ \to \eta'$ | 0.61                              | 0.74                                  | 0.89 ± 0.33                   |
| $D^+ \to \overline{K}^0$ | 9.4                            | 12.4                                  | 6.8 ± 0.8                     |
| $D^+ \to \pi^0$ | 0.33                              | 0.70                                  | 0.31 ± 0.15                   |
| $D^+ \to \eta$ | 0.10                               | 0.15                                  | < 0.5                         |
| $D^+ \to \eta'$ | 0.016                            | 0.019                                   | < 1.1                         |
| $D_s^+ \to K^0$ | 0.20                              | 0.32                                   |                               |

$\rho^-$ and $D_s \to \phi$ transitions. The results are depicted in Fig. 3. Our extrapolated results for the shapes of the $D \to V$ semileptonic form factors agree well with existing theoretical studies \cite{26, 27, 29, 30}, while currently no experimental determination of
Figure 3: $q^2$ dependence of the $D^0 \to K^{*-}$ (upper left), $D^0 \to \rho^-$ (upper right) and $D_s \to \phi$ (lower) transition form factors.

the form factors’ shapes in these decays exists.

We complete our study by calculating branching ratios and partial decay width ratios also for all relevant $D \to V \ell \nu_\ell$ decays. They are listed in Table 3 together with known experimentally measured values.

| Decay   | $\mathcal{B}$ (Mod.) [%] | $\mathcal{B}$ (Exp.) [%] | $\Gamma_L/\Gamma_T$ (Mod.) | $\Gamma_+/\Gamma_-$ (Mod.) |
|---------|--------------------------|--------------------------|---------------------------|---------------------------|
| $D_0 \to K^*$ | 2.2                      | $2.15 \pm 0.35$ [6]     | 1.14                      | 0.22                      |
| $D_0 \to \rho$ | 0.20                    | $0.194 \pm 0.039 \pm 0.013$ [31] | 1.11                      | 0.14                      |
| $D^+ \to K_{0}^*$ | 5.6                      | $5.73 \pm 0.35$ [6]     | 1.13                      | 0.22                      |
| $D^+ \to \rho_0$ | 0.25                    | $0.25 \pm 0.08$ [6]     | 1.11                      | 0.14                      |
| $D^+ \to \omega$ | 0.25                    | $0.17 \pm 0.06 \pm 0.01$ [31] | 1.10                      | 0.14                      |
| $D_s \to \Phi$ | 2.4                      | $2.0 \pm 0.5$ [6]       | 1.08                      | 0.21                      |
| $D_s \to K_{0}^*$ | 0.22                    |                       | 1.03                      | 0.13                      |

Table 3: The branching ratios and partial decay width ratios for the $D \to V$ semileptonic decays. Comparison of our model fit with experiment as explained in the text.
3 Search for new physics in rare D decays

At low-energies new physics is usually expected in the down-like quark sector. Numerous studies of new physics effects were performed in the $s \to d, b \to s(d), \bar{s}d \leftrightarrow \bar{d}s, \bar{b}d \leftrightarrow \bar{d}b$ and $\bar{b}s \leftrightarrow \bar{s}b$ transitions.

However, searches for new physics in the up-like quark sector at low energies were not so attractive. Reasons are following: a) flavor changing neutral current processes at loop level in the standard model suffer from the GIM cancellation leading to very small effects in the $c \to u$ transitions. The GIM mechanism acts in many extensions of the standard model too, making contributions of new physics insignificant. b) Most of the charm meson processes, where $c \to u$ and $c \bar{u} \leftrightarrow \bar{c}u$ transitions might occur are dominated by the standard model long-distance contributions \[33\] - \[41\].

On the experimental side there are many studies of rare charm meson decays. The first observed rare $D$ meson decay was the radiative weak decay $D \to \phi \gamma$. Its rate $\text{BR}(D \to \phi \gamma) = 2.6^{+0.7}_{-0.6} \times 10^{-5}$ has been measured by Belle collaboration \[42\] and hopefully other radiative weak charm decays will be observed soon\[43\].

In the standard model (SM) \[33\] the contribution coming from the penguin diagrams in $c \to u \gamma$ transition gives branching ratio of order $10^{-18}$. The QCD corrected effective Lagrangian \[44\] gives $\text{BR}(c \to u \gamma) \simeq 3 \times 10^{-8}$. A variety of models beyond SM were investigated and it was found that the gluino exchange diagrams \[45\] within general minimal supersymmetric SM (MSSM) might lead to the enhancement

$$\frac{\text{BR}(c \to u \gamma)_{\text{MSSM}}}{\text{BR}(c \to u \gamma)_{\text{SM}}} \simeq 10^2. \quad (8)$$

Within SM the $c \to u l^+ l^-$ amplitude is given by the $\gamma$ and $Z$ penguin diagrams and $W$ box diagram. It is dominated by the light quark contributions in the loop. The leading order rate for the inclusive $c \to u l^+ l^-$ calculated within SM \[39\] was found to be suppressed by QCD corrections \[34\]. The inclusion of the renormalization group equations for the Wilson coefficients gave an additional significant suppression \[10\] leading to the rates $\Gamma(c \to u e^+ e^-)/\Gamma_{D_s^0} = 2.4 \times 10^{-10}$ and $\Gamma(c \to u \mu^+ \mu^-)/\Gamma_{D_s^0} = 0.5 \times 10^{-10}$. These transitions are largely driven by virtual photon at low dilepton mass $m_{l l}$.

The leading contribution to $c \to u l^+ l^-$ in general MSSM with conserved R parity comes from the one-loop diagram with gluino and squarks in the loop \[34, 39, 45\]. It proceeds via virtual photon and significantly enhances the $c \to u l^+ l^-$ spectrum at small dilepton mass $m_{l l}$. The authors of Ref. \[34\] have investigated supersymmetric (SUSY) extension of the SM with R parity breaking and they found that it can modify the rate. Using most resent CLEO \[43\] results for the $D^+ \to \pi^+ \mu^+ \mu^-$ one can set the bound for the product of the relevant parameters entering the R parity violating $\lambda_{22k} \lambda_{211}^\ast \simeq 0.001$ (assuming that the mass of squark $M_{\tilde{D}_k} \simeq 100 \text{ GeV}$). This bound gives the rates $\text{BR}_R(c \to u e^+ e^-) \simeq 1.6 \times 10^{-8}$ and $\text{BR}_R(c \to u \mu^+ \mu^-) \simeq 1.8 \times 10^{-8}$.
Some of models of new physics (NP) contain an extra up-like heavy quark inducing flavor changing neutral currents at tree level for the up-quark sector\cite{32,46,17,49,50}. The isospin component of the weak neutral current is given in\cite{32} as

\[ J_{\mu}^{\nu} = \frac{1}{2} U^m_{L\gamma^\mu} U_L^m - \frac{1}{2} D_L^m \gamma^\mu D_L^m \]

with \( L = \frac{1}{2}(1 - \gamma^5) \) and mass eigenstates \( U_L^m = (u_L, c_L, t_L, T_L)^T \), \( D_L^m = (d_L, s_L, b_L)^T \). The neutral current for the down-like quarks is the same as in the SM, while there are tree-level flavor changing transitions between up-quarks if \( \Omega \neq I \). The elements of \( 4 \times 4 \) matrix \( \Omega \) can be constrained by CKM unitarity violations currently allowed by experimental data. Even more stringent bound on \( \Omega_{cu} \) coupling \( \Omega_{uc} \) comes from the present bound on \( \Delta m \) in \( D^0 - \bar{D}^0 \) transition. It gives \( |\Omega_{uc}| \leq 0.0004 \) and we use the upper bound to determine the maximal effect on rare \( D \) decays in what follows. In this case the dilepton mass distribution of the \( c \rightarrow ul^+l^- \) differential branching ratio can be enhanced by two orders of magnitude in comparison with SM (see Fig. 4).

A particular version of the model with tree-level up-quark FCNC transitions is the Littlest Higgs model\cite{51}. In this case the magnitude of the relevant \( c \rightarrow uZ \) coupling \( \Omega_{uc} \) is constrained to be smaller and the effects on rare \( D \) decays coming from a general class of models with tree level \( cuZ \) coupling at its upper bound \( |\Omega_{uc}| = 0.0004 \). We already pointed out that in the Littlest Higgs model, which is a particular version of these models, the coupling \( \Omega_{uc} \) is constrained to be smaller and the effects on rare \( D \) decays is insignificant\cite{32}.

The study of exclusive \( D \) meson rare decay modes is very difficult due to the dominance of the long distance effects\cite{33,38}. The inclusive \( c \rightarrow ul^+l^- \) can be tested in the rare decays \( D \rightarrow \mu^+\mu^- \), \( D \rightarrow P(V)l^+l^- \)\cite{34,39,35}.

The branching ratio for the rare decay \( D \rightarrow \mu^+\mu^- \) is very small in the SM. The detailed treatment of this decay rate\cite{34} gives \( Br(D \rightarrow \mu^+\mu^-) \approx 3 \times 10^{-13} \)\cite{34}. This decay rate can be enhanced within a study which considers SUSY with R parity breaking effects\cite{34,41}. Using the bound \( \lambda_{22k}^{(2k)} \lambda_{21k}^{(2k)} \approx 0.001 \) one obtains the limit \( Br(D \rightarrow \mu^+\mu^-) \approx 4 \times 10^{-7} \).

The \( D \rightarrow P(V)l^+l^- \) decays offer another possibility to study the \( c \rightarrow ul^+l^- \) transition in charm sector. The most appropriate decay modes for the experimental searches are \( D^+ \rightarrow \pi^+l^+l^- \) and \( D^0 \rightarrow \rho^0 e^+e^- \). In the following we present the possible maximal effect on these decays coming from a general class of models with tree level \( cuZ \) coupling at its upper bound \( |\Omega_{uc}| = 0.0004 \). We already pointed out that in the Littlest Higgs model, which is a particular version of these models, the coupling \( \Omega_{uc} \) is constrained to be smaller and the effects on rare \( D \) decays is insignificant\cite{32}.

The calculations of the long distance contributions in the decays \( D^+ \rightarrow \pi^+l^+l^- \) and \( D^0 \rightarrow \rho^0 l^+l^- \) are presented in Refs.\cite{32,38,39}. The contributions of the intermediate vector resonances \( V_0 = \rho^0, \omega, \phi \) with \( V_0 \rightarrow l^+l^- \) constitute an important long-distance contribution to the hadronic decay, which may shadow interesting short-distance contribution induced by \( c \rightarrow ul^+l^- \) transition.
new physics: tree–level $c \rightarrow u e^+ e^-$ coupling ($\Omega_{uc} = 0.0004$)

Figure 4: The dilepton mass distribution $dBr/dm_{ee}^2$ for the inclusive decay $c \rightarrow ul^+l^-$ as a function of the dilepton mass square $m_{ee}^2 = (p_+ + p_-)^2$.

Our determination of short and long distance contributions to $D^+ \rightarrow \pi^+ l^+ l^-$ takes advantage of the available experimental data [32]. This is a fortunate circumstance for this particular decay since the analogous experimental input is not available for determination of the other $D \rightarrow Xl^+l^-$ rates in a similar way. The rate resulting from the amplitudes (14) and (19) of [32] with $|\Omega_{uc}| = 0.0004$ are given in Fig. 5 and Table 4.

We are unable to determine the amplitude of the long-distance contribution to $D^0 \rightarrow \rho^0 V_0 \rightarrow \rho^0 l^+l^-$ using the measured rates for $D^0 \rightarrow \rho^0 V_0$ since only the rate of $D^0 \rightarrow \rho^0 \phi$ is known experimentally. We are forced to use a model [38], developed to describe all $D \rightarrow Vl^+l^-$ and $D \rightarrow V\gamma$ decays, and the resulting rates are presented in Fig. 6 and Table 4.

Therefore, the total rates for $D \rightarrow Xl^+l^-$ are dominated by the long distance resonant contributions at dilepton mass $m_{ll} = m_\rho, m_\omega, m_\phi$ and even the largest contributions from new physics are not expected to affect the total rate significantly [34, 39]. New physics could only modify the SM differential spectrum at low $m_{ll}$ below $\rho$ or spectrum at high $m_{ll}$ above $\phi$. In the case of $D \rightarrow \pi l^+l^-$ differential decay distribution there is a broad region at high $m_{ll}$ (see Fig. 5), which presents a unique possibility to study $c \rightarrow ul^+l^-$ transition [39, 32].

The non-zero forward-backward asymmetry in $D \rightarrow \rho l^+l^-$ decay arises only when $C_{10} \neq 0$ (assuming $m_t \rightarrow 0$). The enhancement of the $C_{10}$ in the NP models [32] is
new physics: tree-level $c_L \rightarrow u_L Z$ coupling ($\Omega_{u_L} = 0.0004$)

$$D^+ \rightarrow \pi^+ e^+ e^-$$

Figure 5: The dilepton mass distribution $dBr/dm_{ee}^2$ for $D^+ \rightarrow \pi^+ e^+ e^-$. 

due to the tree-level $\bar{u}_L \gamma_\mu c_L Z^\mu$ coupling and leads to nonzero asymmetry $A_{FB}(m_{ll}^2)$ shown in Fig. 7. The forward-backward asymmetry for $D^0 \rightarrow \rho^0 l^+ l^-$ vanishes in SM ($C_{10} \simeq 0$), while it is reaching $\mathcal{O}(10^{-2})$ in NP model with the extra up-like quark as shown in Fig. 7. Such asymmetry is still small and difficult to be seen in the present or planned experiments given that the rate itself is already small.

We have investigated impact of the tree-level flavor changing neutral transition $c \rightarrow u Z$ on the rare $D$ meson decay observables. However, the most suitable $D^+ \rightarrow \pi^+ l^+ l^-$ and $D^0 \rightarrow \rho^0 l^+ l^-$ decays are found to be dominated by the SM long distance contributions. Only small enhancement of the differential mass distribution can be seen in the case of $D^+ \rightarrow \pi^+ l^+ l^-$ decay at high dilepton mass and tiny forward

| Br                              | short distance contribution only | total rate $\simeq$ | experiment |
|----------------------------------|---------------------------------|---------------------|------------|
| $D^+ \rightarrow \pi^+ e^+ e^-$  | $6 \times 10^{-12}$             | $1.9 \times 10^{-6}$ | $<7.4 \times 10^{-6}$ |
| $D^+ \rightarrow \pi^+ \mu^+ \mu^-$ | $6 \times 10^{-12}$             | $1.9 \times 10^{-6}$ | $<8.8 \times 10^{-6}$ |
| $D^0 \rightarrow \rho^0 e^+ e^-$ | negligible                     | $1.6 \times 10^{-4}$ | $<1.0 \times 10^{-4}$ |
| $D^0 \rightarrow \rho^0 \mu^+ \mu^-$ | negligible                     | $1.5 \times 10^{-7}$ | $<2.2 \times 10^{-5}$ |

Table 4: Branching ratios for the decays in which $c \rightarrow ul^+ l^-$ transition can be probed.
new physics: tree-level $c_l \rightarrow u_L Z$ coupling ($\Omega_w = 0.0004$)

$$D^0 \rightarrow \rho^0 e^+ e^-$$

Figure 6: The dilepton mass distribution for $D^0 \rightarrow \rho^0 e^+ e^-$.  

new physics: tree-level $c_l \rightarrow u_L Z$ coupling ($\Omega_w = 0.0004$)

$$D^0 \rightarrow \rho^0 e^+ e^-$$

Figure 7: The forward-backward asymmetry for $D^0 \rightarrow \rho^0 e^+ e^-$.  

backward asymmetry can be induced by new physics in $D^0 \rightarrow \rho^0 l^+ l^-$ decay.
We conclude that the NP scenarios which contain an extra singlet heavy up-like quark, have rather small effects on the charm meson observables.

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