Can oscillating scalar fields decay into particles with a large thermal mass?

Jun'ichi Yokoyama
Research Center for the Early Universe (RESCEU),
Graduate School of Science, The University of Tokyo, Tokyo 113-0033, Japan
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We calculate the dissipation rate of a coherently oscillating scalar field in a thermal environment using nonequilibrium quantum field theory and apply it to the reheating stage after cosmic inflation. It is shown that the rate is nonvanishing even when particles coupled to the oscillating inflaton field have a larger thermal mass than it, and therefore the cosmic temperature can be much higher than inflaton’s mass even in the absence of preheating. Its cosmological implications are also discussed.

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I. INTRODUCTION

In our contemporary understanding, the origin of the primeval fireball whose existence was assumed in the conventional hot big bang cosmology is the reheating processes after inflation—an accelerated cosmic expansion which has made the universe homogeneous and spatially flat with small density fluctuations that eventually grow to the observed large-scale structure 1,2. The universe is reheated through the dissipation of coherent oscillation of the zero-mode of the inflaton, the scalar field whose potential energy drives inflation. While the initial stage of reheating could be rather complicated due to an explosive particle production induced by parametric resonance, which is dubbed as preheating 3, the final stage is dominated by perturbative decay. The latter process determines the reheat temperature, \( T_R \), the temperature at the outset of the radiation domination 4.

Note, however, that in general \( T_R \) is much lower than the highest temperature the universe has ever experienced after inflation even in the case only perturbative decay operates to reheat the universe 2. This means that in the late stage of the reheating processes, the inflaton decays not in a vacuum but in a thermal medium. About this point an interesting claim has been made in 6 that if the would-be decay products of the oscillating inflaton acquire a thermal mass larger than the inflaton mass in the thermal background, it cannot decay into these particles, and that reheating is suspended for some time, based on the observation that the phase space would be closed for the mass of the decay product being larger than half the inflaton mass. The decay width of the inflaton \( \phi \) with mass \( m_\phi \) into two massive particles with mass \( m \) reads

\[
\Gamma_\phi = \Gamma_{\phi 0} \left( 1 - \frac{4m^2}{m_\phi^2} \right)^{1/2},
\]

where \( \Gamma_{\phi 0} \) is the decay rate in the case \( m = 0 \). So if we simply replace \( m \) with a thermal mass \( m(T) \sim gT \) and if it is larger than \( m_\phi/2 \), the phase space is closed and inflaton decay is apparently forbidden. Here \( g \) is some coupling constant of the would-be decay product. Then thermal history after inflation would be drastically changed. That is, the highest temperature in this era cannot exceed \( \sim m_\phi/g \) if preheating is inoperative, and also the reheat temperature is bounded from above by \( m_\phi/g \) and is independent of the decay rate of the inflaton in case the conventional calculation gives a larger value. The former would change the abundance of supermassive particles and the latter affects the gravitino abundance 6, because the gravitino-entropy ratio after inflation is proportional to the temperature at the onset of radiation domination.

Furthermore, this situation is not specific to the reheating stage after inflation but may apply in any epoch when significant amount of entropy is produced out of the decay of oscillating scalar field with a relatively small mass. Indeed the above possibility was first pointed out by Linde 7 in the context of Affleck-Dine baryogenesis 8 where the Affleck-Dine scalar field oscillates with a mass of order of \( 10^{2-3} \) GeV in a medium with a much higher temperature. The final magnitude of baryon asymmetry changes if this suspension of decay is operative 7.

The above naive picture, however, may be too simplistic because a thermal mass is different from the intrinsic mass and because coherent field oscillation is different from a collection of particles. Thus it is very important to analyze this problem from a more fundamental point of view, for it has a profound implication not only to the cosmology of the early universe but also to particle physics in that it strongly affects various species of the particles produced in the early universe as mentioned above. In this paper, extending our previous work 9, we analyze this problem in terms of a nonequilibrium quantum field theory at finite temperature 10. Inclusion of thermal masses of the would-be decay products of the inflaton is achieved by adopting a resummed propagator when we calculate the effective action for \( \phi \). As a result of resummation the self energy of the decay product acquires not only real part, which appears as
a high-temperature correction to the mass, but also an imaginary part. The latter plays a crucial role in determining the dissipation rate of the inflaton. Consequently, we find that the inflaton can dissipate its energy even when its would-be decay products have a larger thermal mass than the inflaton itself.

II. MODEL AND EQUATION OF MOTION

For clarity we adopt a simpler model than [9], that is, we adopt the following Lagrangian.

\[ \mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{2} (\partial_\mu \chi)^2 - \frac{1}{2} m_\chi^2 \chi^2 - M \phi \chi^2 - \frac{1}{4} g^2 \chi^4. \] (2)

Here \( \phi \) is an oscillating real scalar field and \( \chi \) is another real scalar field. \( \phi \) can decay into a pair of \( \chi \) particles through the interaction \( M \phi \chi^2 \), if it is energetically allowed. The dimensionful coupling constant \( M \) may be written as \( M = h m_\phi \) in some supersymmetric inflation models where \( h \) is a Yukawa coupling [11]. In such models \( \phi \) can also decay into two fermions, which is suppressed by Pauli-blocking at finite temperature. On the contrary, the decay into two bosons is enhanced due to the induced emission. This is the reason we consider the latter decay process.

We analyze the behavior of the above system under the following assumption which mimic the cosmological situations we are interested in, such as the late reheating phase after inflation. First we neglect cosmic expansion since we are interested in the phenomena which occur in a shorter time scale than the expansion time. Second we assume \( \chi \) is in a thermal state with temperature \( T = \beta^{-1} \) and that it acquires a large thermal mass due to the self coupling. Note that \( \chi \) can easily be thermalized during the reheating stage since its thermalization rate, \( \sim g^4 T \), can naturally be much larger than the cosmic expansion rate. Finally the scalar field \( \phi \) is oscillating but we consider the situation the parametric resonance is already terminated with a field amplitude \( M |\phi| \ll m_\phi^2 \).

Due to its coherent nature, scalar field oscillation behaves nearly classically, but its decay is of course a quantum process. So we calculate an effective action for \( \phi \) and derive an equation of motion for its expectation value. For this purpose we should use the in-in or the closed time-path formalisms in which the time contour starting from the infinite past must run to the infinite future without fixing the final condition and come back to the infinite past again in calculating the generating functional [12]. This method has been applied to various cosmological problems by a number of authors [13, 14, 15]. The generating functional in the present model is given by

\[ Z[J, K] = \text{Tr} \left[ T_- \left\{ \exp \left[ i \int_{-\infty}^{\infty} dt \int d^3 x (J_- \phi_- + K_- \chi_-) \right] \right\} T_+ \left\{ \exp \left[ i \int_{-\infty}^{\infty} dt \int d^3 x (J_+ \phi_+ + K_+ \chi_+) \right] \right\} \rho \right] \equiv e^{iW[J,K]}, \] (3)

where \( X_\pm \) denotes a field component \( X \) on the plus-branch \((-\infty \rightarrow +\infty)\) and \( X_- \) that on the minus-branch \((+\infty \rightarrow -\infty)\). The symbol \( T_+ \) represents the ordinary time ordering, and \( T_- \) the anti-time ordering. \( J_\pm \) and \( K_\pm \) are the external fields for \( \phi \) and \( \chi \), respectively. \( \rho \) is the initial density matrix which is assigned according to the assumption above mentioned.

In terms of the components along the plus and the minus branches, the effective action reads

\[ \Gamma[\phi_+, \phi_-] = W[J_+, J_-, K_\pm = 0] - \int_{-\infty}^{\infty} dt \int d^3 x [J_+(x)\phi_+(x) - J_-(x)\phi_-(x)], \] (4)

with \( \phi_+(x) = \delta W[J_+, J_-]/\delta J_+(x) \) and \( \phi_-(x) = -\delta W[J_+, J_-]/\delta J_-(x) \).

![Fig. 1: One-loop Feynman diagram incorporated in the effective action. Solid line denotes \( \phi \), and broken line \( \chi \).](image)

Here we consider a one-loop correction depicted in Fig. 1 which includes the essential effect in our analysis for illustration. We also note that instead of \( \phi_+ \) and \( \phi_- \) it is more convenient to use \( \phi_\Delta \equiv (\phi_+ - \phi_-)/2 \) and \( \phi_\Delta \equiv \phi_+ - \phi_- \) and set \( \phi_\Delta \rightarrow 0 \) in the end because \( \phi_+ \) and \( \phi_- \) should be identified with each other eventually. Then the effective
action to this order is given by

\[
\Gamma[\phi_c, \phi_\Delta] = - \int d^4x \phi_\Delta(x) (\Box + m_\phi^2) \phi_c(x) - \int d^4x d^4x' C(x-x') \theta(t-t') \phi_\Delta(x) \phi_c(x') + i \frac{1}{2} \int d^4x d^4x' D(x-x') \phi_\Delta(x) \phi_\Delta(x'),
\]

(5)

\[
C(x-x') \equiv 4M^2 \text{Im} \left[ G^F_\chi(x-x')^2 \right], \quad D(x-x') \equiv 2M^2 \text{Re} \left[ G^F_\chi(x-x')^2 \right],
\]

(6)

where \( G^F_\chi(x) \) is the Feynman propagator at finite temperature. Its Fourier modes read [16, 17]

\[
G^F_\chi(p) = \int d^3x G^F_\chi(x,t) e^{-i\omega x} = \frac{i}{2\omega_p} \left\{ [1 + n_B(\omega_p)] e^{-i\omega_p t} + n_B(\omega_p) e^{i\omega_p t} \right\}, \quad \omega_p \equiv \sqrt{p^2 + m_\chi^2}.
\]

(7)

The resultant effective action is complex-valued as a manifestation of the dissipative nature of the system. We cannot obtain any sensible equation of motion by simply differentiating with respect to a field variable because we are dealing with a real scalar field and its equation of motion should be real-valued. The cure for this problem has been proposed by Morikawa [13], according which we introduce an auxiliary random Gaussian field, \( \xi(x) \), to rewrite the effective action as follows.

\[
\exp(i\Gamma[\phi_c, \phi_\Delta]) = \int D\xi P[\xi] \exp \left\{ i\Gamma_{\text{eff}}[\phi_c, \phi_\Delta, \xi] \right\},
\]

(9)

where

\[
\Gamma_{\text{eff}}[\phi_c, \phi_\Delta, \xi] \equiv \text{Re} \Gamma[\phi_c, \phi_\Delta] + \int d^4x \xi(x) \phi_\Delta(x).
\]

(10)

Here \( P[\xi] \) is a statistical distribution functional for \( \xi(x) \) which is a Gaussian with its dispersion given by the imaginary part of the effective action, \( \langle \xi(x) \xi(x') \rangle = D(x-x') \).

While the mathematical equivalence between the above decomposition and the original expression [9] can easily be confirmed by performing the path integral with respect to \( \xi(x) \), we keep it as it is to obtain a real-valued equation of motion through

\[
\left. \frac{\delta \Gamma_{\text{eff}}[\phi_c, \phi_\Delta, \xi]}{\delta \phi_\Delta} \right|_{\phi_\Delta=0} = 0.
\]

(11)

From (10), it reads

\[
(\Box + m_\phi^2) \phi_c(x) + \int_{-\infty}^t dt' \int d^3x' C(x-x') \phi_c(x') = \xi(x).
\]

(12)

Hereafter we omit the suffix \( c \). The solution to the above equation of motion can be readily found through Fourier transform,

\[
\phi_k(t) = \int d^3x \phi(x, t) e^{-i k \cdot x}, \quad \tilde{\phi}_k(\omega) = \int dt \phi_k(t) e^{i\omega t},
\]

(13)

etc. We find [12] is transformed as

\[
[-\omega^2 + k^2 + m_\phi^2 + S_k(\omega)] \tilde{\phi}_k(\omega) - i\omega \tilde{\Gamma}_k \tilde{\phi}_k(\omega) = \tilde{\xi}_k(\omega),
\]

(14)

where we have defined

\[
\tilde{\Gamma}_k(\omega) \equiv \frac{i \tilde{C}_k(\omega)}{2\omega}, \quad S_k(\omega) \equiv \int \frac{d\omega'}{2\pi} \frac{1}{\omega - \omega'} i \tilde{C}_k(\omega'),
\]

(15)
with
\[ \tilde{C}_k(\omega) = \int dt d^3x C(x)e^{i\omega t - ik \cdot x}, \] (16)

which is pure imaginary. Since \( S_k(\omega) \) is divergent, we subtract the divergence at \( \omega = 0 \) to renormalize mass \( m_\phi \). We can show that the renormalized part \( S_{\text{ren}}(\omega) = S_k(\omega) - S_k(0) \) is of order of \( \mathcal{M}^2/(4\pi^2) \). From now on we refer to \( m_\phi \) as the renormalized mass, and assume that \( \mathcal{M} < m_\phi \). Then one can neglect \( S_k(\omega) \) in (14) to yield the solution
\[
\phi_\phi(t) = \left[ \phi_\phi(t_1) \cos M_k(t-t_1) + \frac{\phi_\phi(t_1)}{M_k} \sin M_k(t-t_1) \right] e^{-\frac{i}{2} \tilde{\Gamma}_k(M_k)(t-t_1)}
+ \frac{1}{M_k} \int_{t_1}^{t} dt' e^{-\frac{i}{2} \tilde{\Gamma}_k(M_k)(t-t')} \sin M_k(t-t') \xi_k(t'), \quad M_k^2 \equiv m_\phi^2 + k^2. \] (17)

Here we have also assumed \( \tilde{\Gamma}_k(M_k) \ll M_k \), which is justified whenever perturbation theory applies. From the above solution we can read off that \( \tilde{\Gamma}_k(M_k) \) gives the dissipation rate of the field oscillation. In particular, the dissipation rate of the zero-mode condensate is given by
\[
\tilde{\Gamma}_0(m_\phi) = \frac{\mathcal{M}^2}{8\pi m_\phi} \left[ 1 - \left( \frac{2m_\chi}{m_\phi} \right)^2 \right]^{1/2} \left[ 1 + 2n_B \left( \frac{m_\phi}{2} \right) \right] = \frac{\mathcal{M}^2}{2\pi m_\phi^2} \left[ 1 - \left( \frac{2m_\chi}{m_\phi} \right)^2 \right]^{1/2}, \] (18)
for \( m_\phi > 2m_\chi \). Here the last approximate equality applies at high temperature \( T \gg m_\phi \), and we have used
\[
\tilde{C}_\phi(m_\phi) = -i\pi \mathcal{M}^2 \int \frac{dp}{(2\pi)^3} \frac{1}{\omega_p^2} \left[ 1 + 2n_p \right] \left[ \delta(m_\phi - 2\omega_p) - \delta(m_\phi + 2\omega_p) \right], \quad n_p \equiv n_B(\omega_p), \] (19)
for \( k = 0 \) mode. Here the delta functions appear as a result of time-integral of the form
\[
\int_0^\infty \cos(m_\phi \mp 2\omega_p) dt = 2\pi \delta(m_\phi \mp 2\omega_p), \] (20)
which arises in turn because the spatial Fourier mode of the Feynman propagator at finite temperature can be expressed as (14).

### III. INCLUSION OF THE THERMAL MASS OF THE DECAY PRODUCTS

So far, although we have taken into account the fact that the decay product \( \chi \) is thermally populated, we have not considered effects of the thermal environment on \( \chi \) itself which induces a thermal mass to \( \chi \) particles. If we simply replaced the intrinsic mass \( m_\chi \) with the thermal mass \( m_\chi(T) \) in (18) and (19), we would find the dissipation rate would vanish for \( m_\chi(T) > m_\phi/2 \) as was claimed in (14). Here we carefully examine how to incorporate the thermal mass of \( \chi \) in our analysis in order to see the validity of such a simplistic argument. To this end, we should not rely on the formula (15) but go back to the more fundamental equation (19) to reconsider how this equation was derived.

First we note that \( \phi \)'s mass in \( \omega_p \) of (19) is that appears in the denominator of the Feynman propagator (5) and the delta function emerges due to the infinitesimally small imaginary part \( i\epsilon \). The effect of the thermal environment on \( \chi \), which gives rise to finite-temperature correction to its mass, can be incorporated to the calculation of the dissipation rate of \( \phi \) if we apply resummation and use a resummed propagator of \( \chi \) instead of the finite-temperature bare Feynman propagator (5) to calculate the effective action. With the help of the Matsubara formalism (18), the denominator of the propagator acquires a self energy whose real part yields a finite-temperature correction to the mass as desired. This resummation procedure, however, generates appreciable magnitude of the imaginary part to the self energy at the same time, so that the delta function seen in (19) will no longer be present as we see below. Thus we expect that the use of a resummed propagator changes the result qualitatively, and that the simple observation that a large thermal mass would close the phase space of the decay rate of the inflaton (18) would not apply.

We write real and imaginary parts of \( \chi \)'s self energy, \( \Sigma(p) \), as \( \Sigma_R(p) \) and \( \Sigma_I(p) \), respectively. Then the spectral function reads
\[
\rho_\phi(p, \omega) = i \left[ \frac{1}{(\omega + i\epsilon)^2 - p^2 - m_\chi^2 - \Sigma_R - i\Sigma_I} - \frac{1}{(\omega - i\epsilon)^2 - p^2 - m_\chi^2 + \Sigma_R + i\Sigma_I} \right]
= i \left[ \frac{1}{(\omega + i\hat{\Gamma}_\phi p)^2 - \omega_p^2} - \frac{1}{(\omega - i\hat{\Gamma}_\phi p)^2 - \omega_p^2} \right], \] (21)
where
\[ \omega'^2 = p^2 + m^2 + \chi(p) + \Gamma^2 \chi(p) = p^2 + m^2 + \Sigma p, \quad \Gamma \chi(p) = -\frac{\Sigma p}{2\omega}. \] (22)

The full dressed propagator is then given by
\[
G_{\text{full}}(\mathbf{p}, t) = \int \frac{d\omega}{2\pi} \left[ (1 + n_B(\omega)) \theta(t) + n_B(\omega) \theta(-t) \right] \rho_p(p, \omega) e^{-i\omega t} = \frac{1}{2\omega^p} \left\{ (1 + n_B(\omega + i\Gamma_p)) e^{-i\omega |t| - \chi(p)|t| + n_B(\omega - i\Gamma_p)} e^{i\omega |t| - \chi(p)|t|} \right\},
\] (23)

which should be compared with the bare propagator \[17\]. Then since the propagator has a complex phase now, the simple cosine integral in our previous calculation, which gave rise to the delta function in \[19\], is replaced by
\[
\int_0^{\infty} dt \ e^{-2\Gamma_p t} \cos(m_\phi - 2\omega p)t = \frac{2\Gamma_p}{(m_\phi - 2\omega p)^2 + (2\Gamma_p)^2},
\] (24)

namely, the Breit-Wigner function, when we calculate \[19\] in terms of the dressed propagator \[28\].

Since \[24\] is finite even for \( m_\phi \ll 2\omega p \), we find the dissipation rate of \( \phi \) is nonvanishing even when \( m_\phi \ll 2m_\chi(T) \). To the first order in \( \Gamma_p \), the zero mode dissipation rate in such a regime is given by
\[
\tilde{\Gamma}_0(m_\phi) = \frac{M^2}{2m_\phi} \int \frac{d^3 p}{(2\pi)^3 \omega_p^2} \left\{ \frac{2 \Gamma_p (1 + 2n_p)}{4 \Gamma_p^2 + (m_\phi - 2\omega p)^2} - \frac{2 \Gamma_p (1 + 2n_p)}{4 \Gamma_p^2 + (m_\phi + 2\omega p)^2} \right\}
+ 2 \beta \Gamma_p n_p(n_p + 1) \left[ \frac{2m_\phi - 2\omega p'}{4 \Gamma_p^2 + (m_\phi - 2\omega p)^2} - \frac{m_\phi - 2\omega p'}{4 \Gamma_p^2 + (m_\phi + 2\omega p)^2} \right].
\] (25)

where \( n_p \equiv n_B(\omega p) \). All the terms in the second line vanish in the limit \( \Gamma_p \rightarrow 0 \), while those in the first line reduce to delta functions in the same limit and the previous result with the undressed propagator is recovered except for the replacement \( \omega_p \rightarrow \omega_p' \).

As mentioned above, the above result remain finite even if \( m_\phi \) is smaller than \( 2\omega p \). In particular, if thermal mass of \( \chi \) is much larger than the inflaton mass, \( \omega p' \gg m_\phi \), we obtain
\[
\tilde{\Gamma}_0(m_\phi) = \frac{M^2}{16\pi^2} \int_{m_\chi(T)}^{\infty} \frac{d\omega_p}{\omega_p^2} \sqrt{\omega_p'^2 - m_\chi^2(T)} \Gamma_p \left[ \frac{1}{\pi^2 m_\phi^2 T} \right] \int_{m_\chi(T)}^{\infty} \frac{d\omega_p'}{\omega_p'^2} \sqrt{\omega_p'^2 - m_\chi^2(T)} n_p(n_p + 1) \Gamma_p. \] (26)

In the present simple model, \( \chi \) thermalizes only through self interaction \( g^2 \chi^4/4 \), then \( \chi(p) \) is given by \( \chi(p) \equiv 3g^2 T^2/(128\pi^2) \) \[19\]. We find the second term yields the dominant contribution in \[20\] for \( m_\phi \ll m_\chi(T) \approx gT/2 \). As a result we obtain
\[
\tilde{\Gamma}_0(m_\phi) \approx \frac{M^2 T^3 3g^2}{2\pi m_\phi^2 24\pi^2}.
\] (27)

Here the former factor is identical to the dissipation rate to massless particles at high temperature \( T \gg m_\phi \), and the latter factor represents the suppression due to the large thermal mass of the decay product. One may wonder the suppression factor might be proportional to \( g^4 \) just as \( \chi(p) \). However, the result of integration of the second term in \[20\] yields \( m_\chi^2(T) \) in the denominator, which partially cancels \( g^4 \) to \( g^2 \). Although the numerical value of the suppression factor in \[27\] is specific to the present model, it is a generic feature that the suppression is proportional to some combination of coupling constants which is related to thermalization processes of the decay product, because it arises from \( \Gamma_p/\omega_p' \).

\[ IV. \ \text{DISCUSSION} \]

The resultant reheat temperature in the present model is given by
\[
T_R = \left( \frac{90}{\pi g_*} \right)^{1/2} \frac{M^2 M^4}{2\pi m_\phi^2 24\pi^2} = 1.0 \times 10^{11} g^2 \left( \frac{g_*}{200} \right)^{-1/2} \left( \frac{M}{10^{16}\text{GeV}} \right)^2 \left( \frac{m_\phi}{10^8\text{GeV}} \right)^{-2} \text{GeV},
\] (28)
where \( g_* \) is the effective number of relativistic degree of freedom and \( M_G \) is the reduced Planck scale. Again it is suppressed by the same factor, \( 3g^2/(24\pi^2) \), compared with the case the scalar field decays into massless particles in the high-temperature medium, see eq. (157) of [9]. The above result has been obtained under the assumption of 
\[
m_\phi < g T_R \quad \text{and} \quad \mathcal{M} < m_\phi.\]
The former condition reads
\[
m_\phi \ll 1.0 \times 10^9 g \left(\frac{g_*}{200}\right)^{-1/6} \left(\frac{\mathcal{M}}{10^6 \text{GeV}}\right)^{2/3} \text{GeV.} \tag{29}
\]
The above reheat temperature should also be compared with \( T_R \approx m_\phi/g \) which would apply in the case large thermal mass could forbid inflaton decay completely.

Our result has important implications to the abundances of supermassive dark matter particles and gravitinos as well as baryon asymmetry. First, in case a large thermal mass forbids inflaton decay completely, the temperature after preheating cannot be higher than \( \sim m_\phi/g \). Then the abundance of supermassive particles with mass \( m_X \) is exponentially suppressed as \( \propto (m_X/m_\phi)^2 e^{-m_X/m_\phi} \) for \( m_X \gg m_\phi \) and \( g = 1 \). Our result shows that such a suppression is absent and an appreciable amount of supermassive particles could be created after inflation. Second, the gravitino abundance is suppressed by a factor of \( 3g^2/(24\pi^2) \) in the present model compared with the conventional reheating scenario, because its abundance is proportional to the reheat temperature. Hence we can relax constraint imposed by the gravitino decay to this extent.

In summary, we have calculated the dissipation rate of an oscillating scalar field in a thermal bath such as the inflaton in the late reheating stage, and shown that it is nonvanishing even if the would-be decay products have a thermal mass larger than the mass of the oscillating field. This yields several important implications that has not been taken into account so far, as discussed above.

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