Neutron stars in a Skyrme model with hyperons

L. Mornas*
Departamento de Física, Universidad de Oviedo, Avenida Calvo Sotelo 18, 33007 Oviedo, Spain

March 30, 2022

Abstract

Available Skyrme parametrizations with hyperons are examined from the point of view of their suitability for applications to neutron stars. It is shown that the hyperons can attenuate or even remove the problem of ferromagnetic instability common to (nearly) all Skyrme parametrizations of the nucleon-nucleon interaction. At high density the results are very sensitive to the choice of the $\Lambda-\Lambda$ interaction. The selected parameter sets are then used to obtain the resulting properties of both cold neutron stars and hot protoneutron stars. The general features known from other models are recovered.

PACS numbers: 97.60.Jd, 21.65.+f, 21.30.-x
keywords: neutron stars, Skyrme forces, hyperons

1 Introduction

Skyrme parametrizations provide a simple tool for calculating the equation of state. Skyrme or Skyrme-inspired models are often used in applications to neutron stars with adapted versions which pay special attention to the asymmetry properties. The Skyrme Lyon series [1] is the most modern example of this approach and was designed to reproduce saturation properties, the main properties of nuclei as well as the results of simulations of pure neutron matter.

In neutron stars in chemical equilibrium, hyperons appear at a threshold density about twice the nuclear saturation density. A sizable hyperon fraction can be expected to be present in the core of the most massive neutron stars; the central density of the Skyrme Lyon series for example is of the order of seven times saturation density in the star with maximum mass and three to four times saturation density in a $1.4\, M_\odot$ star.

Original Skyrme parametrizations did not take hyperons into account. Balberg and Gal [2] have remedied the fact with their parametrization of the energy density. It could be argued however that the description of the nucleon sector provided by this parametrization is rather poor.

Rather than designing a new force in the hyperonic sector with the purpose of describing neutron stars, an other possibility would be to restrict oneself to the numerous available data sets which were fitted to nuclear properties. This is the approach taken by Rikovska Stone et al. [3], who recently tested 87 parameter sets for the NN force in neutron star calculations.

In the hyperonic sector there exist a few parametrizations of the Lambda-Nucleon and Lambda-Lambda interaction fitted to the properties of hypernuclei, including several well tested sets by Lanskoy et al. [4, 5] and an earlier set by Fernandez et al. [6]. The present work can be considered as an extension of the study of Rikovska Stone et al. including the $\Lambda$ hyperon.

*lysiane@pinon.ccu.uniovi.es
A general feature of Skyrme models in the np sector is that they show a ferromagnetic transition at rather low density \[7, 8\]. If present, such a transition, apart from modifying the equation of state and chemical equilibrium, could give rise to induced magnetic fields in rotating neutron stars. It also plays a crucial role in calculations of the neutrino mean free path in supernova and protoneutron stars. Calculations of this parameter in non relativistic (Skyrme or Gogny) models of nuclear matter \[9, 10, 11\] thus obtained that the cross section diverges and the mean free path drops to zero at the transition.

It remains an open issue whether this transition is genuine or an artefact of the Skyrme model. While a free Fermi gas eventually becomes ferromagnetic, the nuclear correlations are known to play a crucial role. Most non relativistic calculations which take correlations into account, e.g. by solving the Brueckner-Hartree-Fock equations with modern bare NN potentials \[12, 13\], concluded that spin ordered matter was not favored energetically. Relativistic mean field models predict no ferromagnetic transition; on the other hand relativistic calculations in the Hartree-Fock approximation by Bernardos et al. \[14\] find a transition, albeit as a rather large density \(4 n_{\text{sat}}\). A recent work by Maruyama and Tatsumi \[15\], although not putting forward a quantitative prediction, reaches a similar conclusion as to the importance of the Fock contribution.

In this work we will take the viewpoint that the ferromagnetic transition, especially when appearing at such low densities as 2–3 times saturation, is probably an artefact of the Skyrme model, and will select among the available Skyrme parametrizations those which as far as possible avoid or delay the transition to higher densities.

This work includes the effects of temperature and neutrino trapping and can also be applied in hot protoneutron stars.

A further motivation of the present study was to obtain a model which would allow to calculate the neutrino-baryon scattering rate in the hot protoneutron star formed shortly after the supernova collapse. It was therefore necessary to be able to calculate the Landau parameters in the spin \(S = 1\) channel. Besides determining the position of an eventual ferromagnetic transition, the Landau parameters in the spin \(S = 1\) channel play a central role when calculating the axial response function in the random phase approximation, which is the dominant contribution to the neutrino-baryon scattering rate. This application is presented in a separate paper \[16\].

After a presentation of the available Skyrme parametrizations in section §2, the following sections proceed to select a few among them according to their suitability for neutron star applications. Section §3 first examines the threshold density for hyperon formation under the conditions of \(\beta\) equilibrium. §4 presents a discussion of the issue of ferromagnetism. The selected parameter sets are used in section §5 to obtain the properties of the corresponding neutron stars. The effect of non vanishing temperature and of a trapped neutrino fraction are studied in §6. The main results are collected and discussed in the conclusion.

The formalism is kept to a minimum in the main text, while all relevant formulae are gathered in the Appendix.

### 2 Skyrme parametrizations of the hyperonic sector

The usual Skyrme model of nuclear matter introduces the nucleon-nucleon potential

\[
V_{NN}(r_1 - r_2) = t_0 (1 + x_0 P_\sigma) \delta(r_1 - r_2) + \frac{1}{2} t_1 (1 + x_1 P_\sigma) \left[ k'^2 \delta(r_1 - r_2) + \delta(r_1 - r_2) k^2 \right] + t_2 (1 + x_2 P_\sigma) k' \delta(r_1 - r_2) k + \frac{1}{6} t_3 (1 + x_3 P_\sigma) \rho_N^2 \left( \frac{r_1 + r_2}{2} \right) \delta(r_1 - r_2) \tag{1}
\]
It can readily be generalized to include nucleon-Lambda and Lambda-Lambda interaction potentials (see e.g. [4, 5])

\[ V_{NN}(r_N - r_\Lambda) = u_0 (1 + y_0 P_\sigma) \delta(r_N - r_\Lambda) + \frac{1}{2} u_1 \left[ k' \delta(r_N - r_\Lambda) + \delta(r_N - r_\Lambda) \right] k^2 \]

\[ + u_2 k' \delta(r_N - r_\Lambda) + \frac{3}{8} u_3 (1 + y_3 P_\sigma) \rho_N \left( \frac{r_N + r_\Lambda}{2} \right) \delta(r_N - r_\Lambda) \]

\[ V_{\Lambda\Lambda}(r_1 - r_2) = \lambda_0 \delta(r_1 - r_2) + \frac{1}{2} \lambda_1 \left[ k'^2 \delta(r_1 - r_2) + \delta(r_1 - r_2) \right] k^2 \]

\[ + \lambda_2 k' \delta(r_1 - r_2) k + \lambda_3 \rho_\Lambda \rho_N \]

The potentials normally also include spin orbit contributions. They are not explicited here since they will not be used in the remainder of this paper.

The energy density is then obtained in the Hartree-Fock approximation from

\[ \mathcal{E} = \langle \psi | H | \psi \rangle \quad \text{with} \quad H = \sum_{A=N,\Lambda} T_A + \frac{1}{2} \sum_{A,B=N,\Lambda} V_{AB} \]

\[ = \mathcal{E}_{NN} + \mathcal{E}_{NA} + \mathcal{E}_{\Lambda\Lambda} \]

In homogeneous matter the wave functions are formed from antisymmetrized plane wave states. The explicit expression of the energy density is given in the Appendix.

In our search for some suitable sets of Skyrme parametrizations we have tested 43 NN Skyrme forces in combination with 13 parametrizations of the NA and 4 options for the \( \Lambda \Lambda \) forces. The NN forces were chosen among: SHII, SkM*, ST, SHII', SV, RATP, SGI, SGII, SLy230a, SLy2, SLy4, SLy5, SLy6, SLy7, SLy9, SLy10, SO, SO', SkS3, SkI1, SkI2, SkI3, SkI4, SkI5, SK272, SK255, Skz-0, Skz-1, Skz-2, Skz-3, SkSc4, SkSc15, SKX. After demanding that these sets fulfill various constraints that will be discussed in detail in the next two sections, only four NN forces were sorted out: the SLy10 force from the Skyrme-Lyon series [1], the modern SkI3 and SkI5 forces by Reinhard and Flocard [17] and the older SV force by Beiner et al. [18].

For the nucleon-\( \Lambda \) interaction we have tried several parameter sets given by Lanskoy and Yamamoto [4, 19] and by Fernandez et al. [6]: The NA sets numbered from I to V in [4] are named LLY1-LLYV here, the sets numbered from 1 to 6 in [19] are named YBZ1-YBZ6, other choices are the SkSH1 and SkSH2 sets of [6] or to switch off the NA interactions. Finally, Lanskoy gives in [5] three sets SLL1, SLL2, SLL3 for the \( \Lambda \Lambda \) interaction, or we can switch it off.

The parametrization of Lanskoy and Yamamoto [4] was extracted from G-matrix calculations [20] performed with the Jülich and Nijmegen potentials and were tested on hypernuclei. Other NA and \( \Lambda \Lambda \) interactions were fitted directly to hypernuclei data. The values given by Lanskoy and Yamamoto [4] assume that the nucleon sector is parametrized by the SkM* or SHII interactions while the parameter set of the Salamanca group [6] was used together with the SkS3 interaction. Even though this is not fully consistent, we also used other more modern parametrizations in the nucleon sector like the Skyrme Lyon sets [1] in order to investigate the role of the ferromagnetic transition (see section §4)

Selected values of the parameter sets are listed in tables I, II and III. For other parametrizations we refer the reader to the exhaustive table published by Rikovska Stone et al. in the case of the NN forces and the papers of Lanskoy et al. and Fernandez et al. for the NA and \( \Lambda \Lambda \) forces.

As this model only includes the \( \Lambda \) hyperon, it appears to be less complete than the model of Balberg and Gal [2] which includes all the hyperons. As mentioned in the introduction, we fixed our choice on the parametrization by Lanskoy et al. because these authors provide the two-particle potential rather than only the unpolarized energy density. This leaves open the possibility of going.
beyond the Hartree-Fock approximation and to investigate the response functions of the matter at the RPA level.

It would be straightforward to extend the model to take into account other hyperons such as the $\Sigma^-$. Data about the $\Sigma^-$ in nuclear matter however is rather scarce and the author is not aware of an equivalent Skyrme parametrization including the $\Sigma$ at the same level of precision. One possibility would be to adjust Skyrme parameters to reproduce Brueckner calculations e.g. using density matrix expansion techniques. On the other hand the neglect of the $\Sigma$ hyperon is sometimes justified from the very lack of observation of $\Sigma$ hypernuclei, when this fact is interpreted as indicating that the $\Sigma$-nucleon force is in fact repulsive. In that case $\beta$-equilibrium equations would predict that the threshold for $\Sigma$ hyperon formation in neutron stars is shifted to very large densities (see e.g. [2, 21, 22, 23]).

We will use the model of Balberg and Gal [2] and another by Banik and Bandhyopadhyay [24] in order to estimate the error committed by neglecting other hyperons and in particular the $\Sigma^-$.

### Table I: Skyrme parameters for the NN interaction

| Model | $\alpha$ | $t_0$ | $t_1$ | $t_2$ | $t_3$ | $x_0$ | $x_1$ | $x_2$ | $x_3$ |
|-------|----------|-------|-------|-------|-------|-------|-------|-------|-------|
| SLy10 | 1/6      | 2506.77 | 430.98 | -304.95 | 13826.41 | 1.0398 | -0.6745 | -1.0  | 1.6833 |
| SkI3  | 1/4      | -1762.88 | 561.608 | -227.09 | 8106.2  | 0.3083 | -1.1722 | -1.0907 | 1.2926 |
| SkI5  | 1/4      | -1772.91 | 550.84 | -126.685 | 8206.25 | -0.1171 | -1.3088 | -1.0487 | 0.3410 |
| SV    | 1        | -1248.3 | 970.6 | 107.2 | 0. | -0.17 | 0. | 0. | 1. |

$t_0$ is given in MeV.fm$^3$, $t_1$ and $t_2$ in MeV.fm$^5$, $t_3$ in MeV.fm$^{3+3\alpha}$, the other parameters are adimensional.

### Table II: Skyrme parameters for the NΛ interaction [4, 19]

| Model | $\beta$ | $u_0$ | $u_1$ | $u_2$ | $u_3$ | $y_0$ | $y_3$ | $V_\Lambda$ |
|-------|--------|-------|-------|-------|-------|-------|-------|--------|
| LY-I  | 1/3    | -476.42 | 23.1514 | -0.0452 | -0.280 | -27.62 |
| LY-IV | 1/3    | -542.5 | 67.61 | 37.39 | 2000 | -0.108 | 0 | -26.52 |
| YBZ-1 | 1      | -349.0 | 67.61 | 37.39 | 2000 | -0.108 | 0 | -26.52 |
| YBZ-5 | 1      | -215.3 | 23.14 | -23.14 | 2000 | -0.109 | 0 | -28.50 |
| YBZ-6 | 1      | -372.2 | 100.4 | 79.60 | 2000 | -0.107 | 0 | -24.98 |
| SkSH1 | -      | -176.5 | 44.1 | 0 | 0 | - | -27.68 |

$u_0$ is given in MeV.fm$^3$, $u_1$ and $u_2$ in MeV.fm$^5$, $u_3$ in MeV.fm$^{3+3\beta}$, the other parameters are adimensional. $V_\Lambda$ is the potential felt by a $\Lambda$ hyperon in nuclear matter at saturation and is given in MeV.

### Table III: Skyrme parameters for the ΛΛ interaction [5]

| Model | $\lambda_0$ | $\lambda_1$ |
|-------|-------------|-------------|
| SLL1  | -312.6      | 57.5        |
| SLL2  | -437.7      | 240.7       |
| SLL3  | -831.8      | 922.9       |

$\lambda_0$ is given in MeV.fm$^3$ and $\lambda_1$ in MeV.fm$^5$.

### 3 Threshold for hyperon formation

In neutron star matter we impose that the conditions for $\beta$-equilibrium are fulfilled. The neutron, proton and Lambda hyperons are subject to the processes

$$\begin{align*}
p + e^- &\rightarrow n + \nu_e \\
p + e^- &\rightarrow \Lambda + \nu_e
\end{align*}$$
as well as their inverse
\[ n \rightarrow p + e^- + \bar{\nu}_e \quad ; \quad \Lambda \rightarrow p + e^- + \bar{\nu}_e \] (5)

We write the equality of the chemical potentials
\[ \hat{\mu} = (\mu_n + m_n) - (\mu_p + m_p) = \mu_e - \mu_\nu \quad ; \quad \mu_n + m_n = \mu_\Lambda + m_\Lambda \] (6)

We must also impose electric charge conservation
\[ n_e = n_p \] (7)

At a given baryonic density the electron, proton and hyperon fractions are determined by the solution of Eqs. (6,7). The chemical potentials are determined by deriving the Skyrme energy density functional with respect to the density of corresponding particle. Their explicit expression is given in the Appendix. The electrons are relativistic and their chemical potential is given by
\[ \mu_e = \sqrt{k_{Fe}^2 + m_e^2} \]. In protoneutron star matter with trapped neutrinos we have \([\mu_\nu = (6\pi^2 n_\nu)^{2/3}]\), while in colder neutrino-free neutron star matter \(\mu_\nu = 0\).

The equations should actually take into account the muons in the equation for charge conservation together with the condition \(\mu_e = \mu_\mu\). The muons appear namely around saturation density. The muons were neglected in this simplified model. Their effect is in fact not very important, especially in our case where the charged \(\Sigma^-\) is also absent from the model.

The threshold for \(\Lambda\) hyperon formation is determined by the condition
\[ \mu_\Lambda(\text{thr}) = \mu_\Lambda(k_{F\Lambda} = 0) = \mu_n + m_n - m_\Lambda \]
\[ = u_0 \left( 1 + \frac{y_0}{2} \right) \rho_N + \frac{3}{8} u_3 \left( 1 + \frac{y_3}{2} \right) \rho_N^{\beta+1} + \frac{1}{8} \left[ u_1(2 + y_1) + u_2(2 + y_2) \right] k \rho_N^{5/3} \] (8)

with \(k = (3/5) (3\pi^2)^{2/3}\). It depends on the strength of the \(\Lambda\)N force through the parameters \(u_0, u_1, u_2, u_3\) and indirectly on the \(NN\) force through the chemical potential of the neutron (taken in \(npe\) matter in \(\beta\) equilibrium) and is independent of the \(\Lambda\)-\(\Lambda\) force. Depending on the choice of the parameters it is found that the threshold generally occurs between 1.7 \(n_{\text{sat}}\) and 4 \(n_{\text{sat}}\) in agreement with other Brueckner-Hartree-Fock or relativistic mean field calculations. When numerical values are inserted one can see that the value of \(\mu_\Lambda(\text{thr})\) is the result of a delicate cancellation between the \(u_0\) and \(u_3\) terms, the contribution of the \(u_1, u_2\) term being of the order of the sum of the \(u_0\) and \(u_3\) terms. For example at \(\rho_N = 3 n_{\text{sat}}\) and \(n_{\text{sat}} = 0.16 \text{ fm}^{-3}\) we have for the LY-I parametrization of the \(\Lambda\)A force
\[ u_0 \left( 1 + \frac{y_0}{2} \right) \rho_N = -223.32 \text{ MeV} \quad , \quad \frac{3}{8} u_3 \left( 1 + \frac{y_3}{2} \right) \rho_N^{\beta+1} = 183.51 \text{ MeV} \]
\[ \frac{1}{8} \left[ u_1(2 + y_1) + u_2(2 + y_2) \right] k \rho_N^{5/3} = 27.46 \text{ MeV} \]

Let us also note that the value of \(\mu_\Lambda(k_{F\Lambda} = 0)\) as saturation density with equal number of neutron and protons is nothing but the single particle potential felt by a \(\Lambda\) impurity in nuclear matter, \(i.e.\) it should equal to the binding potential \(V_\Lambda \simeq -28 \text{ MeV}\) obtained from data on hypernuclei. The actual values of \(V_\Lambda = \mu_\Lambda(k_{F\Lambda} = 0, nB = 0.16 \text{ fm}^{-3})\) are reported in the last column of table II.

We may conclude this section by stating that, once the parametrization of the \(\Lambda\)A force has been chosen so that the potential felt by a \(\Lambda\) in nuclear matter reproduces the experimental value, the condition that the threshold density for \(\Lambda\) hyperons in nuclear matter in \(\beta\)-equilibrium should lay around 2 – 3 times saturation density does not severely constrain the admissible Skyrme NN and NA forces. We observe that stiffer equations of state (eos) and the eos allowing for a larger proton fraction have lower hyperon thresholds (see table V for a sample of the results).
4 Transition to ferromagnetic state

4.1 Criterion for a ferromagnetic instability

Previous studies on the neutrino mean free path in neutron matter \[9\] or \(npe^-\) matter in \(\beta\) equilibrium \[11\] found that a pole appears in the calculation of the axial structure function above a certain critical density. This feature is typical of Skyrme models and is related to a transition to a ferromagnetic state. In this section we will study how this critical density is affected by the presence of hyperons.

Let us define the magnetic susceptibilities \(\chi_{ij}\) where \(i, j \in \{n, p, \Lambda\}\):

\[
\frac{1}{\chi_{ij}} = \frac{\partial^2 \mathcal{E}}{\partial M_i \partial M_j}, \quad M_i = \mu_i (\rho_{i\uparrow} - \rho_{i\downarrow})
\]

where \(\mathcal{E} = \mathcal{E}(\rho_{n\uparrow}, \rho_{n\downarrow}, \rho_{p\uparrow}, \rho_{p\downarrow}, \rho_{\Lambda\uparrow}, \rho_{\Lambda\downarrow})\) is the polarized energy density functional, \(M_i\) are the magnetizations and \(\mu_i\) are the magnetic moments. The inverse susceptibilities are therefore proportional to the second derivatives of the energy density functional with respect to the polarizations:

\[
\frac{\mu_i \mu_j}{\chi_{ij}} = \frac{2\rho_i \rho_j}{\rho_i \rho_j} \Delta_{ij}, \quad \Delta_{ij} = \frac{1}{2} \frac{\partial^2 (\mathcal{E}/\rho)}{\partial s_i \partial s_j} \quad \text{with} \quad s_i = \frac{\rho_{i\uparrow} - \rho_{i\downarrow}}{\rho_{i\uparrow} + \rho_{i\downarrow}}
\]

On the other hand it can be shown that the \(\Delta_{ij}\) are related to the Landau parameters \(g_{ij}^0\) defined in the Appendix through

\[
\Delta_{ij} = \frac{2\rho}{\rho_i \rho_j} \frac{1}{\sqrt{N_0^i N_0^j}} G_{ij}^0 \quad \text{if} \quad i \neq j
\]

\[
\Delta_{ii} = \frac{2\rho}{\rho_i^2} \frac{1}{N_0^i} (1 + G_{ii}^0)
\]

\[
G_{ij}^0 = \sqrt{N_0^i N_0^j} g_{ij}^0, \quad N_0^i = \frac{m_i^* k_F i}{\pi^2 \hbar^2}
\]

A criterion for the appearance of the ferromagnetic phase is that the determinant of the inverse susceptibility matrix vanishes

\[
Det \begin{pmatrix}
\frac{1}{\chi_{nn}} & \frac{1}{\chi_{np}} & \frac{1}{\chi_{n\Lambda}} \\
\frac{1}{\chi_{pn}} & \frac{1}{\chi_{pp}} & \frac{1}{\chi_{p\Lambda}} \\
\frac{1}{\chi_{\Lambda n}} & \frac{1}{\chi_{\Lambda p}} & \frac{1}{\chi_{\Lambda\Lambda}}
\end{pmatrix} = 0
\]

(12)

and in terms of the Landau parameters:

\[
Det \begin{pmatrix}
(1 + G_{0n}^n) & G_{0p}^{pn} & G_{0n}^{n\Lambda} \\
G_{0n}^{pn} & (1 + G_{0p}^{pp}) & G_{0n}^{p\Lambda} \\
G_{0n}^{n\Lambda} & G_{0n}^{p\Lambda} & (1 + G_{0n}^{n\Lambda})
\end{pmatrix} = 0
\]

(13)

It can be shown that this quantity appears in the denominator of the static axial response (see \[16\]).

4.2 Dependence of the criterion on parameter sets

A few NN forces (MSk7, SkX, SkS3) had to be discarded as not convenient for obtaining the equation of state away from saturation, some more for not permitting the formation of hyperons below \(5 n_{\text{sat}}\), others again for undergoing a transition to pure neutron matter before the threshold for hyperon formation was reached (SIII, SI', SIII', Skz0–Skz4).
The remaining ones all display a transition to the ferromagnetic state at some critical density $n_{\text{ferro}}^{\beta-\text{ne}}$ when no hyperons are present, with the exception of the SV force. Usually the ferromagnetic transition occurs earlier in pure neutron matter (PNM) than in symmetric nuclear matter (SNM), and at an intermediate density for $npe$ matter in $\beta$ equilibrium. One exception is the case of the Lyon Skyrme forces where the reverse situation appears to occur. A closer look nevertheless reveals that only traces of protons are enough to drastically lower the critical density in very neutron rich matter so that the usual pattern is in fact recovered.

Some forces (SkO, SkO', SkI1, SkI4, SkT4, SkT5) again need to be discarded since the ferromagnetic transition occurs below $n_{\text{sat}}$ in symmetric nuclear matter, so that the nuclei would in fact be unstable with respect to spin fluctuations. This leaves us with forces for which the $n_{\text{ferro}}^{\beta-\text{ne}}$ occurs in the range $n_{\text{sat}} - 3 n_{\text{sat}}$, and the threshold for $\Lambda$ hyperon formation in the range $1.7 n_{\text{sat}} - 4 n_{\text{sat}}$. A key point is now whether $n_{\text{ferro}}^{\beta-\text{ne}}$ is greater or lower than $n^{\Lambda}_{\text{thr}}$. While the criterion for the onset of the ferromagnetic instability Eq. (13) decreases with increasing density in $npe$ matter in $\beta$ equilibrium, we noted that it always tends to increase again when enough hyperons are present. This then rejects the critical density $n_{\text{ferro}}^{\beta-\text{ne},\Lambda}$ for the onset of ferromagnetism in $npe\Lambda$ matter in $\beta$ equilibrium to higher densities. In a few cases the instability even disappears altogether. For the majority of NN parameter sets, the ferromagnetic transition occurs before the threshold for hyperon formation. In some cases (SLy4, SLy7, SGI) we have $n^{\Lambda}_{\text{thr}} \sim < n_{\text{ferro}}^{\beta-\text{ne}}$, and the ferromagnetic transition is only delayed by a tiny amount.

We finally selected four NN forces (SLy10, SkI3, SkI5, SV) as relevant for our purpose, namely to study neutron star matter with an hyperonic component before the ferromagnetic transition sets in. In Figure 1 (a–c) we have represented the criterion (13) for these parametrizations of the NN Skyrme force and various choices of the N$\Lambda$ and $\Lambda\Lambda$ forces. The modification of the slope at the hyperon threshold around $2 n_{\text{sat}}$ is clearly visible on Fig. 1-a.

The SV NN interaction is atypical as it has no $t_3$ term. This parametrization is especially interesting since, among all tested NN interactions, it was the only one which does not give a ferromagnetic instability in $npe$ matter in $\beta$ equilibrium. This parametrization is rather old (1975) but yields an acceptable description of the properties of nuclear matter and nuclei and it complies with all the conditions necessary for describing a viable neutron star. The SLy10, SkI3 and SkI5 are modern forces. The SLy10 set was designed to reproduce features of pure neutron matter as obtained from the variational calculations of Wiringa et al. in view of its application to neutron stars. The SkI3 and SkI5 sets were designed in order to improve isotope shifts and incorporate the dipole sum rule enhancement factor $\kappa = 0.25$. Let us notice that the SkI3, SkI5 and SV allow for larger proton fractions than the SLy10, in particular we can see from table IV that the criterion for opening of the direct URCA process with nucleons is reached before the threshold for hyperon production.

The behaviour of the criterion for ferromagnetic instability is qualitatively very similar for all choices of the N$\Lambda$ interaction. Forces yielding smaller $\rho_{\Lambda}(\text{thr})$ and softer eos (see §5) also tend to be more efficient in lifting the ferromagnetic criterion $\det[1 + G_{ij}]$ above the critical zero axis. This is especially the case for the $YBZ4$ parameter set which permitted to avoid the pole also in the SLy4 and SLy7 parametrizations of the NN force. However the YBZ4 parameter set was rejected by Lanskoy on the ground that it gives overbinding of the $\Lambda$ in hypernuclei so that we will not consider it further. Despite this overall similarity the actual presence and position of the pole is sensitive to the choice of the N$\Lambda$ force. This happens because the $\det[1 + G_{ij}]$ has already decreased considerably prior to $\rho_{\Lambda}(\text{thr})$ and the turnover due to the contribution of the $\Lambda$ therefore must take place in the vicinity of the zero axis.

We can also see on this figure that the $\Lambda - \Lambda$ interaction plays an important role in determining $n_{\text{ferro}}^{\beta-\text{ne},\Lambda}$. The set SLL3 (which also gives a stiffer eos, see next section) is less efficient in removing
the pole while the SLL1 set which gives the softest eos is also the most efficient in preventing the onset of ferromagnetism. The Λ-Λ force of Lanskoy [5] is very schematic (it is parametrized by \( \lambda_0 \) and \( \lambda_1 \) only, see the Appendix); moreover it was adjusted to the older value \( \Delta B_{\Lambda\Lambda} = -4.8 \) MeV instead of the value \( \Delta B_{\Lambda\Lambda} \approx -1 \) MeV recently extracted from the “Nagara event”. The case where a vanishing Λ-Λ interaction is assumed may in fact be closer to the true situation. A better knowledge of the Λ-Λ interaction at high density is needed.

The values of density and proton content at the threshold for hyperon formation as well as the critical values for the ferromagnetic transition for various models are gathered in tables IV a–d. All densities are quoted in units of the saturation density of the corresponding model. The mention “grazing” means that \( \det[1 - G_{ij}] \), while not actually crossing the zero axis, comes so near it that for all practical purposes the static axial response function will behave as if a pole were present.

a) NN force=SLy10

[threshold for ferromagnetism in PNM: 3.837 \( n_{\text{sat}} \), in SNM: 3.821 \( n_{\text{sat}} \), in npe matter in \( \beta \)-equilibrium: 3.055 \( n_{\text{sat}} \)]

| NA     | \( n_{\text{thr}} \) | \( Y_p \) | no Λ-Λ | SLL1 | SLL2 | SLL3 |
|--------|-----------------|---------|--------|------|------|------|
| LY-I   | 2.719           | 0.041   | grazing [4.3] | no pole | 6.183 | 3.898 |
| LY-II  | 2.868           | 0.040   | 3.160  | 3.212 | 3.236 | 3.328 |
| LY-IV  | 2.764           | 0.041   | 3.657  | no pole | 6.480 | 3.818 |
| YBZ5   | 3.319           | 0.037   | –      | –    | –    | –    |
| YBZ6   | 5.101           | 0.029   | –      | –    | –    | –    |
| SKSH1  | 2.294           | 0.044   | 4.577  | 5.297 | 4.729 | 4.153 |

Fig. 1 – Criterion for the transition to a ferromagnetic state.
b) NN force=SkI3  
[threshold for ferromagnetism in PNM: 2.298 \( n_{\text{sat}} \), in SNM: 5.73 \( n_{\text{sat}} \), in \( npe \) matter in \( \beta \)-equilibrium: 3.078 \( n_{\text{sat}} \)]

| NA   | \( n_{\text{thr}} \) | \( Y_p \) | no \( \Lambda \Lambda \) | SLL1   | SLL2   | SLL3   |
|------|-----------------|--------|-----------------|--------|--------|--------|
| LY-I | 1.864           | 0.126  | no pole         | 9.234  | 4.663  |
| LY-IV| 1.873           | 0.127  | no pole         | 8.900  | 4.548  |
| YBZ1 | 2.015           | 0.139  | no pole         | 8.753  | 4.893  |
| YBZ5 | 1.942           | 0.133  | no pole         | 7.550  | 4.080  |
| YBZ6 | 2.076           | 0.144  | no pole         | 9.194  | 5.633  |
| SKSH1| 1.739           | 0.116  | 8.225           | 9.495  | 6.765  | 4.425  |
| SKSH2| 1.681           | 0.111  | 18.875          | 15.250 | 8.253  | 4.212  |

c) NN force=SkI5  
[threshold for ferromagnetism in PNM: 1.772 \( n_{\text{sat}} \), in SNM: 2.659 \( n_{\text{sat}} \), in \( npe \) matter in \( \beta \)-equilibrium: 2.140 \( n_{\text{sat}} \)]

| NA   | \( n_{\text{thr}} \) | \( Y_p \) | no \( \Lambda \Lambda \) | SLL1   | SLL2   | SLL3   |
|------|-----------------|--------|-----------------|--------|--------|--------|
| LY-I | 1.727           | 0.150  | grazing [4.34]  | no pole| 4.707  | 3.166  |
| LY-IV| 1.734           | 0.150  | grazing [4.14]  | no pole| 4.627  | 3.095  |
| YBZ1 | 1.838           | 0.162  | no pole         | no pole| 8.656  | 3.622  |
| YBZ5 | 1.785           | 0.156  | no pole         | no pole| 5.404  | 3.239  |
| YBZ6 | 1.882           | 0.167  | no pole         | no pole| 9.118  | 3.950  |
| SKSH1| 1.629           | 0.138  | 3.337           | 3.914  | 3.541  | 3.103  |

d) NN force=SV  
[threshold for ferromagnetism in PNM: 4.850 \( n_{\text{sat}} \), in SNM: no pole, in \( npe \) matter in \( \beta \)-equilibrium: no pole]

| NA   | \( n_{\text{thr}} \) | \( Y_p \) | no \( \Lambda \Lambda \) | SLL1   | SLL2   | SLL3   |
|------|-----------------|--------|-----------------|--------|--------|--------|
| LY-I | 1.793           | 0.125  | no pole         | no pole| 9.330  | 4.576  |
| LY-IV| 1.800           | 0.126  | no pole         | no pole| 9.143  | 4.533  |
| YBZ1 | 1.906           | 0.133  | no pole         | no pole| 8.649  | 4.635  |
| YBZ3 | 1.672           | 0.116  | no pole         | no pole| 11.700 | 5.050  |
| YBZ5 | 1.851           | 0.129  | no pole         | no pole| 7.521  | 3.956  |
| YBZ6 | 1.951           | 0.136  | no pole         | no pole| 9.093  | 5.200  |
| SKSH1| 1.691           | 0.118  | no pole         | no pole| 8.920  | 4.250  |
| SKSH2| 1.635           | 0.114  | no pole         | no pole| 8.253  | 4.068  |

Tables IV a–d: thresholds for ferromagnetism in \( np\Lambda e \) matter in \( \beta \) equilibrium

5 Equation of state and neutron star structure

5.1 Equation of state, effective masses and the hyperon fraction

For neutron star matter in beta equilibrium without hyperons, the equations of state are in order of decreasing stiffness parametrized by the NN interaction SV > SkI3, SkI5 > SLy10. The parametrizations SkI3 and SkI5 yield nearly indistinguishable results, so that SkI5 will not considered further in the remainder of this work. When the hyperons are taken into account in the
calculation of the $\beta$ equilibrium, the equation of state softens as expected. For a given NN interaction, we have in order of decreasing stiffness $Y_{BZ6} > Y_{BZ1} > LY-I, LY-IV > Y_{BZ3} > SKSH2 > SKSH1$. Finally, when varying the $\Lambda\Lambda$ interaction for given NN and NA forces, we obtain $SLL3 > SLL2 > SLL1$. If the $\Lambda\Lambda$ interaction is set to zero, the equation of state is somewhat stiffer than $SLL2$ at $n_B < 5 n_{sat}$ and much softer above $5 n_{sat}$. We note the sizable effect of the $\Lambda\Lambda$ interaction on the equation of state, as well as in the previous section on the value of the Landau parameters.

The equation of state is shown on Fig. 2 for two of our preferred NN forces, and with the same choice $LY-I$ for the NA interaction and $SLL2$ for the $\Lambda\Lambda$ interaction. Even though SkI3 was stiffer that SLy10 without hyperons, the combination SkI3+$LYI+SLL2$ gives rise to a higher hyperon content at a given density (see also Fig. 4) than SLy10+$LYI+SLL2$, so that it is also subject to more softening. As a consequence, the eos with hyperons are very similar.

Figure 3 illustrates the behavior of the effective masses in the case of the parameter set SkI3+$Y_{BZ6}$+$SLL2$. The effective mass of the neutrons are lower than that of protons in neutron rich matter, a feature generally encountered in Skyrme models. The presence of hyperons cause both the neutron and proton mass to decrease less rapidly at high density.

Figure 4 shows the particle fractions $Y_i = n_i/n_B$ as a function of total baryonic density $n_B$. 

Fig. 2 – Typical results for the equation of state

Fig. 3 – Modification of the effective masses by the presence of hyperons.

Fig. 4 – Particle fractions from the SkI3+$Y_{BZ6}$+$SLL2$ and SLy10+$LYI$+$SLL2$ parametrizations. Left: in neutrino-free matter, right: with trapped neutrinos and lepton fraction $Y_L = 0.4$
for the SkI3+YBZ6+SLL2 and SLy10+LYI+SLL2 parameter sets. It can be seen on the left panel that the hyperons appear at higher density and are less numerous for a given density with SLy10+LYI+SLL2 than with SkI3+YBZ6+SLL2. The right panel shows the chemical composition of matter with a non vanishing number of trapped neutrinos at zero temperature. The case with trapped neutrinos and finite temperature relevant for protoneutron stars will be discussed further in §6

5.2 Solution of the Tolman-Oppenheimer-Volkoff equation

Our selected parameter sets still have to pass the test of causality in the density range of interest, and whether they can support neutron stars with maximum masses larger than the observed value 1.4 $M_\odot$.

The properties of neutron stars formed of $npe$ matter in $\beta$ equilibrium were calculated by Rikovska Stone et al [3] for a large number of Skyrme parametrizations. The four NN interactions that we singled out in section §4 all belong to the subset of interactions selected by these authors as giving viable neutron stars, with maximum masses of the order 2 $M_\odot$ to 2.4 $M_\odot$. The central density reached in stars with the maximum mass is slightly larger than the density at which the velocity of sound reaches the velocity of light for the SLy10 and SV parametrizations whereas the equation of state obtained with SkI3 and SkI5 always remains causal. In any case, stars with the fiducial mass 1.4 $M_\odot$ always fulfill $n_c(1.4 \ M_\odot) < n_B(\ c_s^2 = 1)$.

When hyperons are added, the equation of state being softer, the limit $c_s^2 = 1$ is reached for larger densities. It is pushed to $\sim 8 - 11$ times saturation density for the SLL3 parametrization of the $\Lambda-\Lambda$ interaction, to $\sim 13 - 16 \ n_{sat}$ for the SLL2 parametrization and is larger than 20 $n_{sat}$ for the SLL1 set. The case where a vanishing $\Lambda-\Lambda$ interaction is assumed always remains causal. In any case, such densities are beyond the range of validity of the model. As a consequence, although the softening of the eos with hyperons gives rise to higher compression rates and larger central densities in neutron stars than in stars made of $npe$ matter only, the criterion $c_s^2 < 1$ is always fulfilled up to the central density of the most massive stars in our neutron star models with hyperons, except a few instances involving the SLL3 parameter set.

We have solved the Tolman-Oppenheimer-Volkoff equation to obtain the (non-rotating) neutron star mass-radius relation. The equation of state is matched at lower densities with that of Negele and Vautherin [25] for $\rho \in [0.001 - 0.08]$ fm$^{-3}$ and with the Baym-Pethick-Sutherland (BPS) [26] equation of state for $\rho < 0.001$ fm$^{-3}$. The results$^1$ are summarized in Table V a-d and a sample of mass-radius curves is shown in Fig. 5 and 6. For each combination of the NN, NΛ and ΛΛ interaction we give the density at which the speed of sound becomes superluminal, the central density and radius of a 1.4 $M_\odot$ in case it can be formed, and the central density and radius of the star with maximum mass $M_{\text{max}}$.

As a rule of thumb it is known that the stiffer the equation of state is, the larger will be the maximum mass of the neutron star. Our results for $M_{\text{max}}$ follow accordingly to the classification in stiffness given at the beginning of this section. Thus the SKSH1 choice generally yields $M_{\text{max}} < 1.4 \ M_\odot$ whereas the YBZ6 always pass this test successfully. We see again that the choice of the $\Lambda-\Lambda$ interaction is crucial to determine whether or not the star is able to reach to 1.4 $M_\odot$ line. When the $\Lambda-\Lambda$ interaction is switched off or for the SLL1 choice, the equation of state is generally too soft to support a 1.4 neutron star. On the other hand the SLL3 always succeed in producing $M_{\text{max}} > 1.4 \ M_\odot$. The SLL2 choice barely makes it to 1.4 $M_\odot$ for intermediate choices of the NΛ interaction. This results in a plateau feature, where tiny additions of mass effect a large reduction of radius without actually reaching the instability, the $M = 1.4 \ M_\odot$ point being finally reached at

$^1$The parameters we obtained for the stars composed of $npe$ matter may differ very slightly from those quoted by Rikovska Stone et al., presumably due to a different matching at low density or the neglect of the muons.
unprobably high densities $\rho > 10 n_{\text{sat}}$ much beyond the validity of the model. A stiff equation of state on the other hand is not convenient since it favours ferromagnetism to appear earlier. The latter condition usually rules out the SLL3 choice.

For the SLy10 choice of the NN interaction ferromagnetism always sets in before the central density of a 1.4 $M_\odot$ star is reached. In particular, it must be reminded at this point that ferromagnetism is reached before the threshold of production of hyperons for the choices YBZ1, YBZ5, YBZ6. For the SkI3 choice, the criterion $n_c(1.4 M_\odot) < n_{\text{ferro}}$ is satisfied for the sets (SkI3 + LY-IV + SLL3), (SkI3 + YBZ5 + SLL3) and (SkI3 + YBZ1 or YBZ6 + any choice of $\Lambda\Lambda$). The star with the maximum mass also fulfills all requirements if it is described by (SkI3 + YBZ6 + no $\Lambda\Lambda$) or (SkI3 + YBZ6 + SLL2). The results obtained with the choice SkI5 are very similar. The choice SV represents an intermediate situation. The 1.4 $M_\odot$ star fulfills both criteria $n_c(1.4 M_\odot) < n_B(\gamma^2_s = 1)$ and $n_c(1.4 M_\odot) < n_{\text{ferro}}$ in the cases (SV + LY-I + SLL3), (SV + LY-IV + SLL2), (SV + YBZ1 + SLL2/3), (SV + YBZ6 + any choice of $\Lambda\Lambda$). Moreover the star with the maximum mass also fulfills both criteria for the choices (SV + YBZ6 + no $\Lambda\Lambda$) and (SV + YBZ6 + SLL2).
**a) NN force=SLy10**

[in npe matter in $\beta$-equilibrium: $n_B(c_s^2 = 1) = 7.308 \, n_{\text{sat}}, \, n_c(1.4 \, M_\odot) = 3.60 \, n_{\text{sat}},
R(1.4 \, M_\odot)=11.05 \, \text{km}, \, n_{\text{max}} = 7.69 \, n_{\text{sat}}, \, M_{\text{max}} = 1.99 \, M_\odot, \, R(M_{\text{max}}) = 9.52 \, \text{km}]

| NA  | $\Lambda\Lambda$ | $n_B(c_s^2 = 1)$ | $n_c(1.4 \, M_\odot)$ | $R(1.4 \, M_\odot)$ | $n_{\text{max}}$ | $M_{\text{max}}$ | $R(M_{\text{max}})$ |
|-----|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| LY-I| no $\Lambda\Lambda$ causal | - | - | 6.94 | 1.317 | 10.36 |
| LY-I| SLL1 $> 20$ | 16.032 | 9.38 | 9.0 | 12.38 | 1.425 | 8.11 |
| LY-I| SLL2 | 11.036 | 5.25 | 10.42 | 10.23 | 1.658 | 8.55 |
| LY-IV| no $\Lambda\Lambda$ causal | - | - | 6.82 | 1.338 | 10.43 |
| SKSH1| no $\Lambda\Lambda$ | - | - | 5.09 | 0.875 | 10.8 |
| SKSH1| SLL2 $> 20$ | 15.880 | 8.49 | 9.35 | 12.07 | 1.437 | 8.21 |
| SKSH1| SLL3 | 13.210 | 10.25 | 8.15 | 13.65 | 1.453 | 7.40 |
| YBZ5| no $\Lambda\Lambda$ causal | - | - | 5.51 | 1.377 | 10.98 |
| YBZ5| SLL1 $> 20$ | 13.825 | 7.11 | 10.04 | 11.64 | 1.456 | 8.34 |
| YBZ5| SLL3 | 8.440 | 4.14 | 10.97 | 9.41 | 1.789 | 8.92 |

**b) NN force=SkI3**

[in npe matter in $\beta$-equilibrium: $n_B(c_s^2 = 1) = 6.343 \, n_{\text{sat}}, \, n_c(1.4 \, M_\odot) = 2.27 \, n_{\text{sat}},
R(1.4 \, M_\odot)=13.21 \, \text{km}, \, n_{\text{max}} = 6.12 \, n_{\text{sat}}, \, M_{\text{max}} = 2.263 \, M_\odot, \, R(M_{\text{max}}) = 11.16 \, \text{km}]

| NA  | $\Lambda\Lambda$ | $n_B(c_s^2 = 1)$ | $n_c(1.4 \, M_\odot)$ | $R(1.4 \, M_\odot)$ | $n_{\text{max}}$ | $M_{\text{max}}$ | $R(M_{\text{max}})$ |
|-----|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| LY-I| no $\Lambda\Lambda$ causal | - | - | 4.34 | 1.339 | 12.57 |
| LY-I| SLL1 $> 20$ | 15.880 | 10.44 | 8.99 | 12.59 | 1.413 | 8.30 |
| LY-I| SLL2 | 10.514 | 3.96 | 12.24 | 9.50 | 1.723 | 9.19 |
| LY-IV| no $\Lambda\Lambda$ causal | - | - | 4.33 | 1.351 | 12.68 |
| YBZ5| no $\Lambda\Lambda$ | - | - | 4.94 | 1.234 | 11.11 |
| YBZ5| SLL1 $> 20$ | 13.293 | 2.75 | 13.13 | 9.53 | 1.545 | 9.51 |
| YBZ5| SLL3 | 7.764 | 2.68 | 13.17 | 7.93 | 1.907 | 9.85 |
| YBZ5| SLL2 | 13.969 | 9.57 | 9.27 | 12.56 | 1.427 | 8.22 |
| YBZ5| SLL3 | 8.269 | 3.43 | 12.69 | 8.74 | 1.822 | 9.37 |
| YBZ6| no $\Lambda\Lambda$ causal | - | - | 2.36 | 1.655 | 12.52 |
| YBZ6| SLL1 $> 20$ | 13.077 | 2.39 | 13.19 | 6.79 | 1.642 | 11.02 |
| YBZ6| SLL3 | 7.439 | 2.41 | 13.18 | 7.43 | 1.967 | 10.16 |
| SKSH1| no $\Lambda\Lambda$ | - | - | 3.32 | 1.116 | 13.02 |
| SKSH1| SLL2 $> 20$ | 12.46 | 7.25 | 9.91 | 12.08 | 1.544 | 8.20 |
| SKSH2| SLL2 | 15.497 | - | - | 3.16 | 1.041 | 13.0 |
| SKSH2| SLL3 | 9.971 | 5.81 | 10.55 | 10.83 | 1.658 | 8.46 |
c) NN force=SkI5

In npe matter in β-equilibrium: $n_B(c_s^2 = 1) = 6.377 \ n_{\text{sat}}, \  n_c(1.4 \ M_\odot) = 2.10 \ n_{\text{sat}}, \ R(1.4 \ M_\odot)=13.87 \ km, \ n_{\text{max}} = 6.08 \ n_{\text{sat}}, \ M_{\text{max}} = 2.273 \ M_\odot, \ R(M_{\text{max}}) = 11.36 \ km$

| NA   | ΛΛ   | $n_B(c_s^2 = 1)$ | $n_c(1.4 \ M_\odot)$ | $R(1.4 \ M_\odot)$ | $n_{\text{max}}$ | $M_{\text{max}}$ | $R(M_{\text{max}})$ |
|------|------|-----------------|-----------------------|---------------------|-----------------|-----------------|------------------|
| LY-I | SLL2 | 15.915          | -                     | -                   | 3.70            | 1.297           | 13.32            |
| LY-I | SLL3 | 10.544          | 3.84                  | 12.77               | 9.52            | 1.719           | 9.27             |
| YBZ6 | no ΛΛ causal | 2.20       | 13.85                | 4.44                | 1.627           | 12.96           |
| YBZ6 | SLL2 | 13.103          | 2.25                  | 13.84               | 7.64            | 1.615           | 10.65            |
| YBZ6 | SLL3 | 7.470           | 2.28                  | 13.85               | 7.48            | 1.958           | 10.22            |
| SKSH1| SLL2 | > 20            | -                     | -                   | 2.80            | 1.133           | 13.85            |
| SKSH1| SLL3 | 12.494          | 7.28                  | 10.08               | 12.09           | 1.541           | 8.26             |

Table V: Conditions for causality and neutron star properties

In Fig. 6 we compare the mass-radius relation obtained in several non relativistic models of matter in β-equilibrium with hyperons. As discussed above, among our Skyrme parametrizations the best are the SkI3+YBZ6+SLL2 and the SLy10+LYI+SLL2 sets. The SkI5 or SV forces again give results very similar to those of the SkI3 force. The maximum mass reached with the stiffer SkI3+YBZ6+SLL2 set is 1.64 $M_\odot$ comfortably above the Hulse-Taylor value 1.44 $M_\odot$ while the maximum mass permitted by the softer SLy10+LYI+SLL2 set, 1.42 $M_\odot$, falls a bit short. This is however not uncommon a feature in non relativistic models (see Brueckner-Hartree-Fock calculations [27, 28]). Among the parametrizations given by Balberg and Gal [2], that with intermediate compressibility corresponding to $\gamma = 5/3$ was chosen. Figure 6 also displays the results obtained with a density-dependent Seyler-Blanchard potential as parametrized by Banik and Bandhyopadhyay [24]. In contrast to our Skyrme forces the models of Balberg and Gal and of Banik and Bandhyopadhyay both take into account Σ hyperons. The threshold for hyperon formation is represented on the figure with circles for the Λ and crosses for the Σ.
The parametrization of Balberg and Gal is in fact unstable slightly above the Λ hyperon threshold. As a consequence, the $M$ vs. $R$ relation displays an extended plateau, then recovers, crosses the line $M = 1.4 \, M_\odot$ and finally reaches a maximum at $M = 1.53 \, M_\odot$.

Also represented on the figure are lines of constant gravitational redshift $z = (1 - 2GM/Rc^2)^{-1/2} - 1$.

To date two determinations exist from the observation of spectral lines in isolated neutron stars [29]. Sanwal et al. obtained a result with a large error bar $z = 0.12 - 0.23$ while Cottam et al. could extract a precise determination $z = 0.35$. All the neutron star models displayed would be compatible with the determination of Sanwal et al.. Almost all models are also in agreement with the value $z = 0.35$ of Cottam et al.. On the other hand, the mass-radius relation from the SkI3+YBZ6+SLL2 set is only marginally compatible with this value.

Figure 7 displays the density profile of a $1.4 \, M_\odot$ neutron star and its hyperonic content. The profiles corresponding to two of the Skyrme parameters sets studied in this work, SkI3+YBZ6+SLL2 and SLy10+LYI+SLL2 (top panels) are compared to a version of the models of Balberg and Gal and of Banik and Bandhyopadhyay (medium and bottom left panels) where all hyperons save the Λs are artificially switched off. The profiles corresponding to the latter two models when also Σ⁻ hyperons are taken into account is displayed on the medium and bottom right panels. We can see that additional hyperons make the equation of state softer and the neutron star more compact.
Fig. 7 – Density profile of a 1.4 $M_\odot$ neutron star for several non relativistic models of baryonic matter.

6 Finite temperature effects, neutrino trapping

According to proto-neutron star formation and cooling calculations (see e.g. [30]), temperatures of the order of 30 to 50 MeV can be reached in the late phases of the supernova collapse. We therefore investigate here the range $T \in [0 - 50]$ MeV.

The thermodynamical quantities are now to be written in terms of Fermi integrals. The expressions which are given in the Appendix at $T = 0$ are easily generalized: The densities $\rho_i$ and the quantities $\tau_i$ related to the kinetic energy should be replaced in the expressions for the baryonic contributions to the energy density and the effective masses by (in units $\hbar = c = k_B = 1$):

$$\rho_i = \frac{1}{2\pi^2} (2m_i^*T)^{3/2} I_{1/2}(\eta_i), \quad \tau_i = \frac{1}{2\pi^2} (2m_i^*T)^{5/2} I_{3/2}(\eta_i)$$

with

$$I_n = \int_0^\infty \frac{u^n du}{1 + e^{u - \eta_i}}$$
while the chemical potentials is related to the $\eta_i$ by $\mu_i = \eta_i T + U_i(\rho_i, \tau_i)$. The pressure is obtained from the derivative of the free energy

$$F = \mathcal{E} - TS, \quad \mathcal{E} = \sum_{A,B=N,\Lambda} \mathcal{E}_{AB} + \mathcal{E}_{\text{leptons}}, \quad P = \rho^2 \frac{\partial^2 (F/\rho)}{\partial \rho^2}$$

with the entropy

$$S = \sum_{i=n,p,\Lambda} S_i + S_{\text{leptons}}, \quad S_i = \frac{5\tau_i}{6m_i^* T} - \rho_i \eta_i$$

We used the GFD\_D3 code published by Gong et al. [31] to calculate the Fermi integrals and keep as before the leptons fully relativistic whereas the baryons are treated nonrelativistically in consistency with the use of the Skyrme interaction.

Fig. 8 – Composition of neutron star matter in $\beta$ equilibrium at finite temperature, calculated with the parametrization SkI3+YBZ6+SLL2 - (a) without neutrino trapping - (b) with neutrino trapping, $Y_L = 0.4$

The effect of temperature on the effective masses begins to be significant at $T \geq 30$ MeV; it tends to increase the neutron mass and decrease the proton effective mass so that their difference is reduced. The presence of trapped neutrinos, which tends to render the matter less asymmetric, also reduce the proton-neutron mass difference. In a warm protoneutron star, both effects are present and cumulate, the main contribution coming from $Y_\nu \neq 0$. The hyperon mass on the other hand is not significantly modified by temperature or neutrino trapping.

The chemical composition is plotted on Fig. 8, without neutrino trapping on the left panel and with neutrino trapping on the right panel, for four values of the temperature, $T=0$, 10, 20 and 50 MeV. We used in this section the parametrization SkI3 + YBZ6 + SLL2.

Let us first discuss the neutrino-free case. It can be seen on Fig. 8-a that finite temperature effects are more important at moderate densities $n_B < 3n_{\text{sat}}$. Until $T = 1$ MeV the results are undistinguishable from the $T=0$ case. When $T$ is increased from 1 to 50 MeV, the matter is more symmetric (i.e. the proton fraction increases). There is stricto sensu no threshold for $\Lambda$ hyperon production anymore, rather there always exist a vanishingly small number of $\Lambda$s for any value of
the baryonic density below the $T = 0$ threshold. Nevertheless, at the temperatures considered here, it is still possible to define a threshold density for practical purposes, which then moves to lower values as the temperature increases.

The equation of state is somewhat stiffer at finite $T$, but this hardly affects the structure of the star until $T = 20$ MeV. This result concern the case when no neutrinos are present in the matter. It is known however that high temperature shorten drastically the mean free path of the neutrinos so that a large amount of $\nu$ are trapped. A typical value resulting from supernova collapse calculations is that the lepton fraction is of the order of $Y_L \simeq 0.4$. Fig. 8-b was drawn assuming this value for $Y_L$.

As expected the matter is more symmetric when neutrinos are trapped and the threshold for hyperon production is shifted to higher density. For example with the SkI3+YBZ6+SLL2 parametrization and at $T = 0$ we have $(n_{\text{thr}} = 2.08, Y_{\nu}^{\text{thr}} = 0.14)$ for $Y_{\nu} = 0$ and $(n_{\text{thr}} = 2.84, Y_{\nu}^{\text{thr}} = 0.36)$ for $Y_L = 0.4$. For a given lepton fraction $Y_L$, varying the temperature has even less impact on the composition of neutrino-trapped matter than in neutrino-free matter. Fig. 9-a shows that the neutrino trapped matter with hyperons is stiffer than its neutrino-free counterpart, a known result which leads to interesting consequences regarding a possible category of metastable stars which would collapse to a black hole as they cool when the deleptonization phase is completed (see e.g. [32]). Fig. 9-b illustrates this feature for the parameter set SkI3+YBZ6+SLL2: A newly formed protoneutron star with $T = 30$ MeV and $Y_L = 0.4$ is metastable if its mass lies in the range $M \in [1.64 - 1.88] M_\odot$ (and radius in the range $R \in [12.08 - 14.07]$ km). For a lower mass, the star contracts as it looses its neutrinos and cools and its hyperonic content increases.

As a conclusion to this subsection, our results concerning the effects of temperature and neutrino trapping are in full agreement with those obtained with other models of the baryonic interaction. The influence of the temperature on the equation of state and chemical equilibrium comes mostly indirectly through the buildup of an important fraction of trapped neutrinos.

| SkI3+YBZ6+SLL2 | $n_{\text{ferro}}$ | $n_c^{2_{=1}}$ | $n_c(1.4 M_\odot)$ | $R(1.4 M_\odot)$ | $n_{\text{max}}$ | $M_{\text{max}}$ | $R(M_{\text{max}})$ |
|----------------|---------------------|-----------------|---------------------|------------------|-----------------|----------------|------------------|
| $T = 0$, $Y_{\nu} = 0$ | 9.19 | 13.08 | 2.39 | 13.19 | 6.79 | 1.64 | 11.02 |
| $T = 30$ MeV, $Y_L = 0.4$ | 11.37 | 12.02 | 2.66 | 15.11 | 6.38 | 1.88 | 12.08 |

As a conclusion to this subsection, our results concerning the effects of temperature and neutrino trapping are in full agreement with those obtained with other models of the baryonic interaction. The influence of the temperature on the equation of state and chemical equilibrium comes mostly indirectly through the buildup of an important fraction of trapped neutrinos.
7 Conclusion

The aim of this work was threefold:
- Use the most recent Skyrme parametrizations with hyperons existing on the market which are adjusted to reproduce the data on nuclei and hypernuclei and test them on neutron stars,
- Study the influence of the hyperons on the ferromagnetic transition,
- Inquire whether the hyperons are likely to affect the tail of the neutrino burst in supernova explosions and prepare the background for the calculation of the neutrino mean free path in protoneutron stars.

While the model considered in this work is still very schematic, it has led to several interesting results.

The first result is that the presence of hyperons generally delay the onset of the ferromagnetic instability, and in many cases they are even able to remove it completely.

An other advantage is that, by softening the equation of state, the hyperons remove the causality flaw and keep \( c_s^2 < 1 \) even up to very large densities.

It is rather encouraging that NN and NΛ interactions from different authors and apparently very dissimilar parameter sets (compare e.g. SV to SkI3 or SKSH1 to LY-I in Tables I, II), once a series of reasonable requirements are fulfilled, yield very similar results qualitatively and even quantitatively. The major incognita is the Λ-Λ interaction which was very poorly known at the time the interactions used in this work were designed.

After studying 43 NN forces in combination with 13 parametrizations of the NΛ force and 4 options for the ΛΛ forces, all taken from the literature and known to reproduce correctly the properties of nuclei and hypernuclei, four combinations of NN+NΛ parameter sets were selected:

- (SkI3+YBZ6), (SkI3+YBZ6+SLL2), (SV+YBZ6+SLL2) and (SLy10+LYI+SLL2). Replacing SkI3 by SkI5 or LYI by LYII in the above sets would only give slightly different results.

These sets fulfill all of the following conditions:

(i) The NN force belongs to the subset selected by Rikovska Stone et al. The effective masses behave smoothly in all the relevant density and temperature range, pure neutron matter is always stable, the asymmetry energy does not decrease so much with density that protons would disappear from the system, neutron stars formed from \( npe \) matter in \( \beta \) equilibrium can reach a mass at least equal to 1.4 \( M_\odot \).

(ii) The NΛ force belongs to the set preferred by Lanskoy et al. as best reproducing the properties of hypernuclei. The threshold for hyperon formation should lay between 1.7 and 4 times saturation

(iii) The neutron star formed of softer npΛe matter in \( \beta \) equilibrium should still reach a mass at least equal to 1.4 \( M_\odot \).

(iv) No ferromagnetic transition should be present As explained in the main text, this is, for a Skyrme parametrization, a strong requirement.

The second result of this work is that the selected sets are also found to reproduce all features observed in other models such as non-relativistic Brueckner-Hartree-Fock or relativistic mean field calculations, not only qualitatively but also quantitatively. The threshold for hyperon formation is in fact restricted to the narrower range \([2-2.5] n_{\text{sat}}\). The softening of the equation of state brings the maximum mass of the star from 2 – 2.5 \( M_\odot \) for a npe star down to 1.4 – 1.6 \( M_\odot \) for a npYe star. This has even lead to some speculation (see e.g. [27]) whether the clustering of known pulsar masses around the value 1.4 \( M_\odot \) would be due to this feature of the equation of state rather than from the circumstances of their formation in a supernova. The metastability of hot stars with trapped neutrinos when hyperons are present, as discussed e.g. in [32], is also recovered.

The selected sets are also applied in a companion paper [16] to the calculation of the scattering
rate of neutrinos in baryonic matter, for a finite temperature and non-vanishing number of trapped neutrinos. The possibility to access to the properties of polarized matter was a further motivation to this work: It was necessary to be able to calculate the Landau parameters in the spin $S = 1$ channel, since the axial channel is dominant for neutrino scattering.

As a protoneutron star is formed in a supernova explosion, the high amount of neutrinos trapped in its interior by high temperature and density gradually diffuse out in the first 50 seconds as the star cools. Hyperons begin to appear in the course of this deleptonization process and could affect the tail of the neutrino signal. Modern detectors would now be able to detect this effect. The idea was explored by Reddy et al [33] in the mean field approximation. The calculation is performed in the random phase approximation in [16]. The parameter sets selected in the present work predict a non-negligible hyperon content for $t \geq 20$ s after the collapse (see Figs 2 and 3 of [16]).

Let us examine again the main shortcomings of the present model and the way it could be improved in future work.

(i) The Skyrme interaction is long known for displaying a series of problems at high density: behavior of asymmetry energy, onset of ferromagnetism, causality ... While we were able to avoid these problems by carefully passing the existing parametrizations through the crib of these various constraints, we cannot avoid the feeling that we are pushing the Skyrme model much beyond its capacity. More refined models do not have these problems; for example the non relativistic Brueckner Hartree Fock calculations do not show a ferromagnetic transition. A possible answer to this point would be to develop a parametrization of Brueckner-Hartree-Fock calculations in terms of an energy density functional for polarized matter with a non vanishing hyperonic content. Some steps have already been performed in this direction by Vidaña et al. These authors studied the polarized neutron-proton matter system [12] and concluded to the absence of a ferromagnetic transition. They also obtained the equation of state of unpolarized npY matter in $\beta$ equilibrium and applied it to the calculation of neutron star mass radius relation [27] and parametrized the neutron-proton-Lambda system [34] for applications to hypernuclei. The parametrization of Brueckner-Hartree-Fock results with modern Nijmegen potentials for the full polarized np$\Lambda\Sigma$ are currently under way [35].

(ii) We have seen that the effect of the $\Lambda-\Lambda$ force at high density is very important, whereas the description of the hyperon-hyperon interaction in phenomenological formalisms is still very poor. Brueckner-Hartree-Fock calculations offer a coherent framework to calculate the hyperon-hyperon interaction in medium starting from bare potentials such as the Nijmegen one known from scattering data of free particles, and then cross checking the results with the data on double hypernuclei. This has actually been performed with the Nijmegen NSC97e model e.g. in [36].

(iii) Relativistic effects can be expected to play an important role at high density. Many of the problems encountered in the Skyrme parametrization mentioned above in this paragraph (behavior of asymmetry energy, onset of ferromagnetism and causality) are not present in the relativistic formulation. Relativistic mean field models with hyperons (see e.g. [37]) are so well under control that they have made their way into textbooks [38]. The parametrizations of the present work obviously do not pretend to compete with this line of work at the mean field level; rather they were developed in view of their application at the RPA level where they give rise to simpler Dyson equations than in the relativistic formulation. The relativistic extension at RPA level to hyperons would be completely straightforward but somewhat unwieldy.

While it could be argued that the physical foundations of the Skyrme model are disputable when applied in the context of neutron stars, the parametrizations presented in this paper should rather be considered as reliable phenomenological models for run-of-the-mill calculations. Once the order of magnitude of physical effects are ascertained with this simple model, they can refined by applying more complicated Brueckner-Hartree-Fock or/and relativistic models.
Acknowledgements

This work was supported by the spanish-european (FICYT/FEDER) grant number PB02-076. Part of it was realized during a stay at the Departament d’Estructura i Constituents de la Materia of Barcelona University. Several discussions with A. Polls are gratefully acknowledged.
Besides the usual parameterization of the nucleon-nucleon interaction

\[
V_{NN}(r_1 - r_2) = t_0 (1 + x_0 P_\sigma) \delta(r_1 - r_2) + \frac{1}{2} t_1 (1 + x_1 P_\sigma) [k'^2 \delta(r_1 - r_2) + \delta(r_1 - r_2) k^2] \\
+ t_2 (1 + x_2 P_\sigma) k' \delta(r_1 - r_2) k + \frac{1}{6} t_3 (1 + x_3 P_\sigma) \rho_N^\alpha \left( \frac{r_1 + r_2}{2} \right) \delta(r_1 - r_2),
\]

we use the following Lambda-nucleon and Lambda-Lambda potentials [4, 5]:

\[
V_{NL}(r_N - r_\Lambda) = u_0 (1 + y_0 P_\sigma) \delta(r_N - r_\Lambda) + \frac{1}{2} u_1 [k'^2 \delta(r_N - r_\Lambda) + \delta(r_N - r_\Lambda) k^2] \\
+ u_2 k' \delta(r_N - r_\Lambda) k + \frac{3}{8} u_3 (1 + y_3 P_\sigma) \rho_N^\beta \left( \frac{r_N + r_\Lambda}{2} \right) \delta(r_N - r_\Lambda)
\]

\[
V_{\Lambda\Lambda}(r_1 - r_2) = \lambda_0 \delta(r_1 - r_2) + \frac{1}{2} \lambda_1 [k'^2 \delta(r_1 - r_2) + \delta(r_1 - r_2) k^2] \\
+ \lambda_2 k' \delta(r_1 - r_2) k + \lambda_3 \rho_\Lambda \rho_\Lambda \gamma \n
\]

We have dropped in these expressions the spin-orbit terms which are not used in this paper. We obtain the energy density in homogeneous matter in the usual way. In spin saturated np\Lambda matter, the functional reads:

\[
\mathcal{E} = \langle \psi | H | \psi \rangle = \mathcal{E}_{NN} + \mathcal{E}_{NL} + \mathcal{E}_{\Lambda\Lambda}
\]

\[
\mathcal{E}_{NN} = \frac{k^2}{2m_N} \tau_N + t_0 \left[ \left( 1 + \frac{x_0}{2} \right) \rho_N^2 - \left( \frac{1}{2} + x_0 \right) \left( \rho_N^2 + \rho_p^2 \right) \right] \\
+ \frac{t_3}{12} \rho_N^2 \left[ \left( 1 + \frac{x_3}{2} \right) \rho_N^2 - \left( \frac{1}{2} + x_3 \right) \left( \rho_N^2 + \rho_p^2 \right) \right] \\
+ \frac{1}{8} \left( t_1 (2 + x_1) + t_2 (2 + x_2) \right) \rho_N \tau_N - \frac{1}{8} \left[ t_1 (2x_1 + 1) - t_2 (2x_2 + 1) \right] \left( \rho_N \tau_N + \rho_p \tau_p \right)
\]

\[
\mathcal{E}_{NL} = u_0 \left( 1 + \frac{y_0}{2} \right) \rho_N \rho_\Lambda + \frac{3}{8} u_3 \rho_N^\beta \rho_\Lambda \left( 1 + \frac{y_3}{2} \right) \\
+ \frac{1}{8} \left( u_1 (2 + y_1) + u_2 (2 + y_2) \right) \left( \rho_N \tau_\Lambda + \rho_\Lambda \tau_N \right)
\]

\[
\mathcal{E}_{\Lambda\Lambda} = \frac{k^2}{2m_\Lambda} \tau_\Lambda + \frac{\lambda_0}{4} \rho_\Lambda^2 + \frac{1}{8} \left( \lambda_1 + 3 \lambda_2 \right) \rho_\Lambda \tau_\Lambda + \frac{\lambda_3}{4} \rho_\Lambda^2 \rho_\Lambda \gamma
\]

with \( \rho_N = \rho_n + \rho_p, \tau_N = \tau_n + \tau_p \). At \( T = 0 \) we have \( \tau_i = (3/5) \rho_i k_F^i \) and \( \rho_i = k_F^i/(3\pi^2) \). At \( T \neq 0 \) these expressions should be replaced by Fermi integrals, see §6. At \( T = 0 \) we obtain the chemical potentials

\[
\mu_n = \frac{\partial \mathcal{E}}{\partial \rho_n} = \frac{h^2}{2m_n} k_F^2 + \mathcal{U}_n(\rho_i, \tau_i)
\]

\[
\mathcal{U}_n(\rho_i, \tau_i) = \frac{t_0}{2} \left[ \rho_n (1 - x_0) + \rho_p (2 + x_0) \right] \\
+ \frac{t_3}{24} k_F^2 \rho_n^2 \left( 2 + \alpha (1 - x_3) + \rho_p^2 [2 (2 + x_3) + \alpha (1 - x_3)] + 2 \rho_n \rho_p \left[ 3 + \alpha (2 + x_3) \right] \right) \\
+ \frac{1}{8} \left( t_1 (1 - x_1) + 3 t_2 (1 + x_2) \right) \tau_n + \frac{1}{8} \left( t_1 (2 + x_1) + t_2 (2 + x_2) \right) \tau_p + \frac{\lambda_3}{4} \gamma \rho_\Lambda^2 \rho_\Lambda \gamma^{-1} \\
+ u_0 \left( 1 + \frac{y_0}{2} \right) \rho_\Lambda + \frac{3}{8} u_3 (\beta + 1) \left( 1 + \frac{y_3}{2} \right) \rho_N \rho_\Lambda + \frac{1}{8} \left[ u_1 (2 + y_1) + u_2 (2 + y_2) \right] \tau_\Lambda
\]
In the Skyrme model the single particle energies
\[ U = \frac{\partial E}{\partial \rho} = \frac{\hbar^2}{2m^2\rho} k_F^2 + U_\Lambda(\rho, \tau) \]

The effective masses are given by
\[ \frac{\hbar^2}{2m^2\rho} = \frac{\hbar^2}{2m_N} + \frac{1}{8} \left[ t_1(2 + x_1) + t_2(2 + x_2) \right] \rho_N - \frac{1}{8} \left[ t_1(2x_1 + 1) - t_2(2x_2 + 1) \right] \rho_n \]
\[ + \frac{1}{8} \left[ u_1(2 + y_1) + u_2(2 + y_2) \right] \rho_\Lambda \]

The effective mass of the proton follows from replacing \( \rho_n \) by \( \rho_p \) in this expression) and
\[ \frac{\hbar^2}{2m^2\rho} = \frac{\hbar^2}{2m_N} + \frac{1}{8} \left[ \lambda_1 + 3\lambda_2 \right] \rho_N + \frac{1}{8} \left[ u_1(2 + y_1) + u_2(2 + y_2) \right] \rho_N \]

The Landau parameters in the spin \( S = 1 \) channel are obtained from
\[ f_{\tau_1 \tau_2 \tau_3 \tau_4} := \left( \frac{dU_{\tau_1 \tau_2 \tau_3 \tau_4}}{d\rho_{\tau_1 \tau_2 \tau_3 \tau_4}} \right)_{k = k_{F,\tau_1 \tau_2 \tau_3 \tau_4}} = f_{\tau_1 \tau_2 \tau_3 \tau_4} = f_{\tau_1 \tau_2 \tau_3 \tau_4} \quad, \quad \tau \in \{n,p,\Lambda\}, \quad \sigma \in \{\uparrow, \downarrow\} \]

In the Skyrme model the single particle energies \( U_{\tau \sigma}(k) \) are quadratic in the momentum \( k \)
\[ U_{\tau \sigma}(k) := U_{\tau \sigma} + \frac{\hbar^2}{m^2_{\tau \sigma}} k^2 \]
and related to the chemical potentials in polarized matter
\[ \mu_{\tau \sigma} := \left( \frac{d\mathcal{E}}{d\rho_{\tau \sigma}} \right)_{\rho(\tau \sigma', \bar{\tau} \sigma) = \text{cst}} = U_{\tau \sigma} + \frac{\hbar^2}{m^2_{\tau \sigma}} k_{F,\tau \sigma}^2 = U_{\tau \sigma}(k_{F,\tau \sigma}) \]

with \( \rho_{\tau \sigma} = k_{F,\tau \sigma}^3/(6\pi^2) \) at \( T = 0 \).

The Landau parameters in the spin \( S = 0 \) channel are obtained from
\[ f_{\tau_1 \tau_2} := \frac{1}{4} \left[ f_{\tau_1 \uparrow \tau_2 \uparrow} + f_{\tau_1 \downarrow \tau_2 \downarrow} + f_{\tau_1 \uparrow \tau_2 \downarrow} + f_{\tau_1 \downarrow \tau_2 \uparrow} \right] \text{unpolarized} \]

and in the spin \( S = 1 \) channel
\[ g_{\tau_1 \tau_2} := \frac{1}{4} \left[ f_{\tau_1 \uparrow \tau_2 \uparrow} + f_{\tau_1 \downarrow \tau_2 \downarrow} - f_{\tau_1 \uparrow \tau_2 \downarrow} - f_{\tau_1 \downarrow \tau_2 \uparrow} \right] \text{unpolarized} \]
The $f_{\tau_1\tau_2}$ are related to usual thermodynamical quantities, for example the compressibility and the asymmetry energy

$$K = \frac{3k_F^2}{m_N}(1 + F_0), \quad a_{\text{asy}} = \frac{k_F^2}{6m_N}(1 + F'_0)$$

with $k_F = k_{Fn} = k_{Fp}$, $f_{nn} = f_{pp}$, $f_0 = \frac{f_{pp} + f_{np}}{2}$, $f'_0 = \frac{f_{pp} - f_{np}}{2}$,

$$F_0 = N_0 f_0, \quad F'_0 = N_0 f'_0, \quad N_0 = \frac{2m^*_N k_F}{\pi^2}$$

in symmetric nuclear matter.

The $g_{\tau_1\tau_2}$ are related to the magnetic susceptibilities and are used to form the ferromagnetic criterion (Eqs. 11 – 13)

Their explicit expressions are

In the spin $S = 0$ channel

$$f_{nn} = \frac{1}{2} t_0 (1 - x_0) + \frac{1}{12} t_3 \rho_{3N}^2 (1 - x_3) + \frac{1}{3} \alpha t_3 \rho_{3N}^{-1} \left[ (1 + x_3)^2 \rho_{N} - \left( \frac{1}{2} + x_3 \right) \rho_{p} \right] + \frac{1}{12} \alpha (\alpha - 1) t_3 \rho_{3N}^{-2} \left[ (1 + x_3)^2 \rho_{N} - \left( \frac{1}{2} + x_3 \right) \rho_{p} + \rho_{p}^2 \right]$$

$$+ \frac{1}{4} \left[ t_1 (1 + x_1) + t_2 (1 + x_2) \right] (k_{F_n}^2 + k_{F_p}^2) + \frac{3}{8} u_3 (1 + \frac{y_3}{2}) \beta (\beta + 1) \rho_{N}^{-1} \rho_{\Lambda} + \frac{1}{4} \lambda_3 \gamma (\gamma - 1) \rho_{N}^{-2} \rho_{\Lambda}^2$$

$$f_{np} = t_0 \left( \frac{1}{2} \rho_{3N}^2 (1 - x_3) + \frac{1}{6} \alpha t_3 \rho_{3N}^{-1} \left[ (1 + x_3)^2 \rho_{N} - \left( \frac{1}{2} + x_3 \right) \rho_{p} \right] + \frac{1}{12} \alpha (\alpha - 1) t_3 \rho_{3N}^{-2} \left[ (1 + x_3)^2 \rho_{N} - \left( \frac{1}{2} + x_3 \right) \rho_{p} + \rho_{p}^2 \right]$$

$$+ \frac{1}{4} \left[ t_1 (1 + x_1) + t_2 (1 + x_2) \right] (k_{F_n}^2 + k_{F_p}^2) + \frac{3}{8} u_3 (1 + \frac{y_3}{2}) \beta (\beta + 1) \rho_{N}^{-1} \rho_{\Lambda} + \frac{1}{4} \lambda_3 \gamma (\gamma - 1) \rho_{N}^{-2} \rho_{\Lambda}^2$$

$$f_{pp} = \frac{1}{2} t_0 (1 - x_0) + \frac{1}{12} t_3 \rho_{3N}^2 (1 - x_3) + \frac{1}{3} \alpha t_3 \rho_{3N}^{-1} \left[ (1 + x_3)^2 \rho_{N} - \left( \frac{1}{2} + x_3 \right) \rho_{p} \right] + \frac{1}{12} \alpha (\alpha - 1) t_3 \rho_{3N}^{-2} \left[ (1 + x_3)^2 \rho_{N} - \left( \frac{1}{2} + x_3 \right) \rho_{p} + \rho_{p}^2 \right]$$

$$+ \frac{1}{4} \left[ t_1 (1 + x_1) + t_2 (1 + x_2) \right] (k_{F_n}^2 + k_{F_p}^2) + \frac{3}{8} u_3 (1 + \frac{y_3}{2}) \beta (\beta + 1) \rho_{N}^{-1} \rho_{\Lambda} + \frac{1}{4} \lambda_3 \gamma (\gamma - 1) \rho_{N}^{-2} \rho_{\Lambda}^2$$

$$f_{n\Lambda} = \frac{1}{2} u_0 (2 + y_0) + \frac{3}{16} u_3 (2 + y_3) (1 + \beta) \rho_{N}^\beta + \frac{1}{8} \left[ u_1 (2 + y_1) + u_2 (2 + y_2) \right] (k_{F_{n\Lambda}}^2 + k_{F_n}^2)$$

$$+ \frac{1}{2} \lambda_3 \gamma \rho_{N}^{-1} \rho_{\Lambda}$$

$$f_{p\Lambda} = \frac{1}{2} u_0 (2 + y_0) + \frac{3}{16} u_3 (2 + y_3) (1 + \beta) \rho_{N}^\beta + \frac{1}{8} \left[ u_1 (2 + y_1) + u_2 (2 + y_2) \right] (k_{F_{n\Lambda}}^2 + k_{F_p}^2)$$

$$+ \frac{1}{2} \lambda_3 \gamma \rho_{N}^{-1} \rho_{\Lambda}$$

$$f_{\Lambda\Lambda} = \frac{1}{2} u_0 + \frac{1}{2} \lambda_3 \gamma \rho_{N}^\gamma + \frac{1}{4} [\lambda_1 + 3 \lambda_2] k_{F_n}^2$$

In the spin $S = 1$ channel

$$g_{nn} = \frac{1}{2} t_0 (x_0 - 1) + \frac{1}{12} t_3 \rho_{3N}^2 (x_3 - 1) + \frac{1}{4} \left[ t_1 (x_1 - 1) + t_2 (1 + x_2) \right] k_{F_n}^2$$

(36)
\begin{align}
  g_{np} &= \frac{1}{2}t_0x_0 + \frac{1}{12}t_3x_3\rho_\alpha^N + \frac{1}{8}[t_1x_1 + t_2x_2](k_{Fn}^2 + k_{Fp}^2) \\
  g_{pp} &= \frac{1}{2}t_0(x_0 - 1) + \frac{1}{12}t_3\rho_\alpha^N(x_3 - 1) + \frac{1}{4}[t_1(x_1 - 1) + t_2(1 + x_2)]k_{Fp}^2 \\
  g_{n\Lambda} &= \frac{1}{2}u_0y_0 + \frac{3}{16}u_3y_3\rho_\beta^\Lambda + \frac{1}{8}[u_1y_1 + u_2y_2](k_{F\Lambda}^2 + k_{Fn}^2) \\
  g_{p\Lambda} &= \frac{1}{2}u_0y_0 + \frac{3}{16}u_3y_3\rho_\beta^\Lambda + \frac{1}{8}[u_1y_1 + u_2y_2](k_{F\Lambda}^2 + k_{Fp}^2) \\
  g_{\Lambda\Lambda} &= -\frac{1}{2}\lambda_0 - \frac{1}{2}\lambda_3\rho_\lambda^\Lambda + \frac{1}{4}[-\lambda_1 + \lambda_2]k_{F\Lambda}^2
\end{align}

In the limit where hyperons are absent these expressions coincide with those of Hernández et al. [40]
References

[1] E. Chabanat, P. Bonche, P. Haensel, J. Meyer and R. Schaeffer, Nucl. Phys. A635 (1998) 231.

[2] A. Balberg and A. Gal, Nucl. Phys. A625 (1997) 435.

[3] J. Rikovska Stone, J.C. Miller, R. Konciewicz, P.D. Stevenson and M.R. Strayer, Phys. Rev. C68 (2003) 034324.

[4] D.E. Lanskoy and Y. Yamamoto, Phys. Rev. C55 (1997) 2330.

[5] D.E. Lanskoy, Phys. Rev. C58 (1998) 3351.

[6] F. Fernández, T. López Arias and C. Prieto, Z. Phys. A (1989) 349.

[7] A. Vidaurre, J. Navarro and J. Bernabeu, Astron. Astrophys. 135 (1984) 361.

[8] J. Margueron, J. Navarro and Nguyen Van Giai, Phys. Rev. C66 (2002) 014303.

[9] J. Navarro, E.S. Hernandez and D. Vautherin, Phys. Rev. C60 (1999) 045801.

[10] S. Reddy, M. Prakash, J.M. Lattimer and J.A. Pons, Phys. Rev. C59 (1999) 2888.

[11] J. Margueron, Ph.D. Thesis, Orsay University, France (2001).

[12] I. Vidaña and I. Bombaci, Phys. Rev. C66 (2002) 045801.

[13] W. Zuo, U. Lombardo and C.W. Shen, Phys. Rev. C67 (2003) 037301.

[14] P. Bernardos, S. Marcos, R. Niembro and M.L. Quelle, Phys. Lett. B356 (1995) 175.

[15] T. Maruyama and T. Tatsumi, Nucl. Phys. A693 (2001) 710.

[16] L. Mornas, “Neutrino scattering rates in the presence of hyperons from a Skyrme model in the RPA approximation”, submitted

[17] P.G. Reinhard and H. Flocard, Nucl. Phys. A584 (1995) 467.

[18] M. Beiner, H. Flocard, N. Van Giai and P. Quentin, Nucl. Phys. A238 (1975) 29.

[19] Y. Yamamoto, H. Bandō and J. Żofka, Prog. Theor. Phys. 80 (1988) 757.

[20] Y. Yamamoto and H. Bandō, Prog. Theor. Phys. Suppl. 81 (1985) 42.

[21] J. Mares, E. Friedman, A. Gal and B.K. Jennings, Nucl. Phys. A594 (1995) 311.

[22] J. Schaffner-Bielich and A. Gal, Phys. Rev. C62 (2000) 034311.

[23] J. Dabrowski, Phys. Rev. C60 (1999) 025205; ibid. Acta Phys. Pol. B35 (2004) 971.

[24] S. Banik and D. Bandyopadhyay, J. Phys. G26 (2000) 1495.

[25] J. Negele and D. Vautherin, Nucl. Phys. A207 (1973) 298.

[26] G. Baym, C. Pethick and P. Sutherland, Astrophys. J. 170 (1971) 299.
[27] I. Vidaña, A. Polls, A. Ramos, L. Engvik and M. Hjorth-Jensen, Phys. Rev. C62 (2000) 035801.
[28] M. Baldo, G.F. Burgio and H.-J. Schulze, Phys. Rev. C61 (2000) 055801
[29] J. Cottam, F. Paerels and M. Mendez, Nature 420 (2002) 51.
D. Sanwal, G.G. Pavlov, V.E. Zavlin and M.A. Teter, Astrophys. J. 574 (2002) L61
[30] J.A. Pons, S. Reddy, M. Prakash, J.M. Lattimer and J.A. Miralles, Astrophys. J. 513 (1999) 780.
[31] Z. Gong, L. Zejda, W. Däppen and J.M. Aparicio, Comp. Phys. Comm. 136 (2001) 294.
J.M. Aparicio, Astrophys. J. Supplt 117 (1998) 627
[32] M. Prakash, I. Bombaci, M. Prakash, P.J. Ellis, J.M. Lattimer and R. Knorren, Phys. Rep. 280 (1997) 1.
[33] S. Reddy, M. Prakash and J.M. Lattimer, Phys. Rev. D58 (1998) 013009
[34] I. Vidaña, A. Polls, A. Ramos and H.-J. Schulze, Phys. Rev. 64 (2001) 044301.
[35] I. Vidaña, A. Rios, L. Mornas, A. Polls and A. Ramos, “Parametrization of Brueckner-Hartree-Fock calculations for polarized baryonic matter”, in preparation
[36] I. Vidaña, A. Ramos and A. Polls, nucl-th/0307096
[37] N.K. Glendenning, Astrophys. J. 293 (1985) 470.
J. Schaffner and I.N. Mishustin, Phys. Rev. C53 (1996) 1416.
F. Hofmann, C.M. Keil and H. Lenske, Phys. Rev. C64 (2001) 025804.
[38] N.K. Glendenning, “Compact stars”, Springer Verlag, N.Y. (1997)
[39] M. Bender, J. Dobaczewski, J. Engel and W. Nazarewicz, Phys. Rev. C65 (2002) 054322.
[40] E.S. Hernández, J. Navarro and A. Polls, Nucl. Phys. A627 (1997) 460.