Two-way Protocol for Quantum Cryptography with Imperfect Devices

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Abstract. The security of a deterministic quantum scheme for communication, namely the LM05 \cite{1}, is studied in presence of a lossy channel under the assumption of imperfect generation and detection of single photons. It is shown that the scheme allows for a rate of distillable secure bits higher than that pertaining to BB84 \cite{2}. We report on a first implementation of LM05 with weak pulses.

1. Introduction

Deterministic quantum schemes (DQS) for secure communication have recently gained interest and diffusion in the field of quantum cryptography \cite{3,1}, and their first experimental `proofs of principle' have been already completed \cite{4,5}. Despite the main achievement of a secure direct communication (A. Beige et al. in \cite{3}) is still quite far, DQS can provide better security and higher transmission rates in the quantum key distribution (QKD) process than traditional schemes like the BB84 \cite{2}.

One of these DQS, namely the LM05 \cite{1}, saturates the Holevo bound for QKD \cite{6}, and is quite practical to implement as it does not require entanglement to work \cite{5}. The security of LM05 against eavesdropping in case the users (Alice and Bob) are endowed with a perfect equipment was discussed in \cite{1}. Specifically LM05 results robust against a general eavesdropping on a noisy but lossless channel, and explicit thresholds were given in case of individual attacks by the eavesdropper (Eve); furthermore a particular strategy by Eve on a noisy and lossy channel as described in \cite{7} was also deemed as detectable by legitimate users.

In this work we relax the hypothesis of perfect equipment for Alice and Bob. We take as photon source an attenuated laser that produces weak pulses; these pulses can accidentally (and uncontrollably) contain more than one photon. Furthermore Bob’s detectors are avalanche photodiodes (APD) that either ‘click’ or
‘not click’, without counting the exact number of photons in the pulse, and have nonunitary quantum efficiency and nonzero dark counts probability. It has been shown that the conjunction of imperfect devices with a lossy channel jeopardizes the security of QKD [8]. The main threat is represented by a photon-number splitting attack (PNS) in which an almighty Eve exploits the multiphoton pulses to acquire information, whilst concealing her presence behind the expected loss-rate [9, 10, 11]. PNS attacks are currently the main limitation to a long-distance BB84 realized with weak pulses [12, 13].

The paper is organized as follows. In section I we review the LM05 protocol and describe the PNS attacks against it. In section II we theoretically study the security against these attacks in terms of the rate of distillable secure bits. In section III we describe the first experimental test of LM05 with weak pulses.

2. Theory

The LM05 protocol works as follows [11]. Bob prepares a photon in one of the four polarization states $|0\rangle$, $|1\rangle$, $|±\rangle = 1/\sqrt{2} (|0\rangle ± |1\rangle)$, with $|0\rangle$, $|1\rangle$ eigenstates of the Pauli operator $\hat{σ}_z$, and sends it to Alice. With probability $c$ Alice measures the photon (control mode, CM) as she would do in the BB84 protocol. This guarantees that the scheme is at least as secure as the BB84. Otherwise, with probability $1−c$, she uses the photon to encode a bit (message mode, MM) by flipping (logical value ‘1’) or not flipping (logical value ‘0’) it. After that she sends the photon back to Bob. To flip the photon without knowing its state Alice uses the operation $i\hat{σ}_y$, that acts as a universal ‘equatorial NOT’ gate [14]. Bob can deterministically decode Alice’s message by measuring the qubit in the same basis he prepared it, without demand for a classical channel. We point out that LM05 does not allow for a direct communication when the channel is noisy or lossy. As explained in [15] it is not possible so far to achieve both a reliable and secure delivery of a message: if one uses the error correction protocol [16, 17] to make the communication reliable Eve can capture a non negligible amount of information, while if one uses the privacy amplification protocol [18] to make the communication secure Bob has no means to reconstruct Alice’s original message. Whether a secure and reliable direct communication in presence of noise or losses is really possible is still an open question.

In the following we describe two PNS attacks, that provide full information to Eve while remaining completely undetected. Notwithstanding the analysis includes also strategies of the same kind that are only partially informative to Eve: with privacy amplification [18] Alice and Bob can remove any remaining information from Eve, according to what explained in [12].

When the photon source is a laser attenuated with an average photon number
per pulse \( \mu \), the probability to have \( n \) photons in a single pulse is given by [19]:

\[
P_n(\mu) = \frac{\mu^n}{n!} e^{-\mu}.
\]

(1)

A typical value used for \( \mu \) in the experiments is 0.1 that gives \( P_0 \approx 9 \cdot 10^{-1} \), \( P_1 \approx 9 \cdot 10^{-2} \), \( P_2 \approx 4.5 \cdot 10^{-2} \), and so on. This means that with a probability \( P_n(\mu) \) Bob prepares the state

\[
|\psi\rangle^{\otimes n} = |\psi\rangle \otimes ... \otimes |\psi\rangle \quad \text{n times}
\]

(2)

rather than the desired state \( |\psi\rangle \) (\( \psi \) indicates one of the four polarizations of the photon prepared by Bob).

It is known (Lutkenhaus’s suggestion in [20]) that when \( n = 3 \) it exists a measurement \( \mathcal{M} \) that provides a conclusive result about the absolute polarization \( \psi \) with (optimal) probability 1/2. Eve can exploit this fact to eavesdrop on LM05 protocol in the following way. She performs a quantum nondemolition measurement (QND) on the pulses as soon as they exit Bob’s station; this can be done without perturbing the polarization \( \psi \). When she finds \( n < 3 \) she blocks the pulses. On the pulses with at least three photons she executes \( \mathcal{M} \) and if the outcome is not conclusive she blocks these pulses as well. When \( n \geq 3 \) and the outcome of \( \mathcal{M} \) is conclusive she prepares a new photon in the right state \( \psi \) and forwards it to Alice.

Until here this attack is completely analogous to the ‘IRUD-attack’ described in [20]. The only variant is that Eve waits for Alice encoding and measures again the photon on the backward trip, to know whether it has been flipped (in this case she finds the orthogonal state \( |\psi^\perp\rangle \)) or not (she finds \( |\psi\rangle \)). Since Eve did know \( \psi \), she can extract Alice’s information without perturbing the state. After that she forwards the photon in the correct state to Bob. We call this first attack PNS\( _{\mathcal{M}} \).

A second attack is more peculiar to LM05. This time suppose that \( n = 2 \) and call the two photons in the pulse \( p_1 \) and \( p_2 \). As before Eve can know the number of photons per pulse through a QND measure. When \( n < 2 \) Eve blocks the pulses. When \( n = 2 \) she stores \( p_1 \) and forwards \( p_2 \) to Alice; this let her remain undetected during a possible CM on the forward path. On the way back Eve captures again \( p_2 \). To gain Alice’s information she must decide whether the polarizations of \( p_1 \) and \( p_2 \) are parallel or antiparallel: in the first case she would deduce the logical value ‘0’; in the second case she would deduce ‘1’. But the discrimination between parallel and antiparallel spins is not as simple as it appears at a first glimpse: while the parallel-spin-state \( |P\rangle = |\psi\rangle_{p_1}|\psi\rangle_{p_2} \) is symmetric, the antiparallel-spin-state \( |AP\rangle = |\psi\rangle_{p_1}|\psi^\perp\rangle_{p_2} \) is neither symmetric nor antisymmetric. Upon symmetrizing \( |AP\rangle \) we can realize that it is not orthogonal to \( |P\rangle \), and by consequence it is not perfectly distinguishable from it (we remand to [21], [22] for a complete treatment of this problem). Actually an optimal measurement \( \mathcal{M}' \) is a nonlocal one and gives Eve a conclusive result (between \( |P\rangle \) and \( |AP\rangle \)) with a
probability 1/4 \([21]\). Hence Eve can block all the ‘inconclusive’ pulses to gain full information and still remain undetected. However it remains open the question of which photon must Eve forward to Bob. The measurement \(\mathcal{M}'\) consists in a generalized measurement with projectors in the four-dimensional Hilbert space given by: \(\Pi_{A}^{p_1,p_2} = |\psi\rangle\langle\psi| - |\psi\rangle\langle\psi|/2\) and \(\Pi_{S}^{p_1,p_2} = I - \Pi_{A}^{p_1,p_2}\). The conclusive answer is related to the antisymmetric state \(\Pi_{A}^{p_1,p_2}\).

Upon obtaining this result Eve does not know whether to give Bob the state \(|\psi\rangle\) or the state \(|\psi\rangle\), because she ignores the absolute value of \(\psi\) prepared by Bob. This shows that two photons are not sufficient for a perfect eavesdropping with \(\mathcal{M}'\). Yet the complete attack can be accomplished with an additional photon \(p_3\): Eve should store \(p_3\), execute \(\mathcal{M}'\), and eventually encode \(p_3\) according to the conclusive outcome of \(\mathcal{M}'\); the photon prepared in this way can be forwarded to Bob without risk of detection.

The above analysis establishes that a perfect (i.e. with zero QBER) eavesdropping can be realized with at least three photons in a pulse. It also establishes that the measurement \(\mathcal{M}\) represents a more powerful resource for Eve than \(\mathcal{M}'\), for a number of reasons: it gives information on the complete polarization state \(\psi\) of the photons, not only on Alice’s operation; the probability of conclusive results is 1/2 rather than 1/4; Eve knows about the conclusiveness of her measurement immediately, rather than after Alice’s encoding, and can use this information to improve her strategy. For these reasons hereafter we only study the robustness of the scheme against the PNS\(_{\mathcal{M}}\) attacks. We do it following Lütkenhaus’s approach in [12].

Bob prepares photons with a phase-averaged weak-pulse laser; the statistics of the photons in each pulse is described by Eq. (1). Given a forward-and-backward lossy channel with transmissivity \(t_{\text{link}}\), and with reference to the MM runs of the protocol, we see that the encoded photons are revealed by Bob’s APDs with average probability:

\[
p_{\text{av}}^\text{sign} = 1 - e^{-\mu_B l_{\text{link}}} \approx \mu_B l_{\text{link}},
\]

where the approximation is valid for small values of the exponent. \(\eta_B\) is the quantum efficiency of Bob’s detectors; \(t_{\text{link}} = 10^{-\left(\alpha l + \Gamma_c\right)/10}\) is the transmissivity of the channel, where \(\alpha\) is the absorption coefficient [23], \(l\) is the distance (in Km) between the place in which the photon is prepared and the place in which it is detected, and \(\Gamma_c\) is a constant total loss-rate given by Alice’s encoding equipment and Bob’s measuring apparatus. To the signal revealed by Bob contribute also the dark counts per gating windows, \(d_B\), from his (two) detectors: \(p_{\text{av}}^\text{dark} = 2d_B\), for a total signal probability equal to

\[
p_{\text{av}} = p_{\text{av}}^\text{sign} + p_{\text{av}}^\text{dark} - p_{\text{av}}^\text{sign} p_{\text{av}}^\text{dark},
\]

where the last term represents the probability of a coincidence between a dark count and a true signal photon. Now we can write the necessary condition for
security against the PNS\(_M\) attacks as:

\[
p_{av} > P = \frac{1}{2} p_{n=3} (\mu) + p_{n>3} (\mu) = 1 - \left(1 + \mu + \frac{\mu^2}{2} + \frac{\mu^3}{6}\right) e^{-\mu} \tag{5}
\]

In Eq.(5) we conservatively assumed that the probability of a conclusive outcome from \(\mathcal{M}\) for more than three photons in a single pulse is 1. The meaning of the above formula is that when the loss-rate is too high the probability to detect a signal photon becomes smaller and smaller, eventually letting Eve conceal under the expected losses. In Fig.1 we plotted the logarithm of the difference \((p_{sign}^{av} - \tilde{P})\):

![Graph](image)

Fig. 1: Secure rate versus transmittance for the protocols LM05 (continuous line) and BB84 (dotted line) with the following parameters: \(\mu = 1, \eta_B = 1, \Gamma_c = 0\). The ‘secure rate’ is defined by \((p_{av}^{sign} - \tilde{P})\) for LM05 and by \((p_{av}^{sign} - P^*)\) for BB84. See text for the explicit expressions of \(p_{av}^{sign}, \tilde{P}\) and \(P^*\).

that defines the security region of LM05, versus the transmittance of the channel \(t_{link}\), after setting \(\mu = 1, \eta_B = 1\) and \(\Gamma_c = 0\). We also plotted the analogous curve for BB84 under the same settings. The purpose is to show for which values of \(t_{link}\) the two protocols are secure against PNS attacks. It can be noted that despite the quite high value of \(\mu\) the security of LM05 is attained for almost all the values of \(t_{link}\). In order to reduce the probability on the right side of Eq.(5) and increase the security of the scheme we could decrease the value of \(\mu\); however in this way also the probability to detect a signal (Eq.(3)) will decrease. So there must be a tradeoff between these two opposite requirements (security and signal rate) that defines an optimality region for the scheme.

The tradeoff can be studied through the ‘gain of secure bits’ defined in [12],
that we rewrite here for LM05 protocol:

\[ G_{\text{sec}} = p_{\text{av}} \left[ \beta (1 - \tau') - f_{\text{casc}} (\epsilon) \right] \quad ; \quad \tau' = \tau (\epsilon / \beta) \,. \]  

(6)

It represents the fraction of secure bits that can be distilled from the transmitted bits after the procedures of error correction \[16, 17\] and privacy amplification \[18\]. \( \beta = (p_{\text{av}} - \bar{P}) / p_{\text{av}} > 0 \) is the security parameter: until it is positive the protocol is secure against PNSM attacks. \( f_{\text{casc}} \) is a function defined in \[17\] that takes into account the imperfect (although efficient) error correction procedure performed with the Cascade protocol; \( h (\epsilon) \) is the Shannon entropy for the QBER \( \epsilon \); \( \tau \) is the fraction of the error-corrected key which has to be discarded during privacy amplification when only single-photon pulses are taken into account \[25\]; it is a function of the QBER and amounts to \[26\]: \( \tau (\epsilon) = \log_2 (1 + 4\epsilon - 4\epsilon^2) \) for \( 0 \leq \epsilon \leq 1/2 \) and \( \tau (\epsilon) = 1 \) for \( 1/2 < \epsilon \leq 1 \). Finally \( \tau' \) in Eq. (6) represents the fraction of bits to discard after taking into account multiphoton pulses: it amounts to \( \beta \) scaled with the security parameter \( \beta \). The QBER \( \epsilon \) is given by the experiment according to the following expression \( \epsilon = (n_{\text{err}} + n_D/2)/n_{\text{tot}} \), where \( n_{\text{err}} \) is the number of error bits in the sifted key, \( n_D \) is the number of ‘ambiguous’ double clicks in Bob’s APDs and \( n_{\text{tot}} \) is the total number of used bits. The importance of \( n_D \) is theoretical: usually \( n_D \ll n_{\text{err}} \), and it can be completely neglected.

In Fig. 2 the gain \( G_{\text{sec}} \) for both LM05 (continuous lines) and BB84 (dotted lines) is plotted as a function of the distance between Alice and Bob. We note that when Alice-Bob distance is \( l \) the total distance between the creation of the photon and its final detection is \( l \) for the BB84, and \( 2l \) for the LM05, due to the double usage of the quantum channel. The BB84 implementation we adopted for comparison with LM05 is the one reported in Ref. \[24\] for the first optical fiber communication window at wavelength around 0.8\( \mu \)m. It is worthwhile noting that the secure gain has a maximum in \( \mu \) for every fixed length \( l \). Hence the pictures in Fig. 2 have been obtained by fixing four values of \( l \) \((l_1 = 1.5, l_2 = 3, l_3 = 4.5, l_4 = 6 \) Km respectively plot a, b, c, d) both for LM05 and BB84, and finding the values \( \mu_i \) that provide a maximum for \( G_{\text{sec}} (\mu_i | l_i) \). We allowed \( \mu_i \) to be different in BB84 and LM05. Vertical lines have been drawn at the typical distances \( l_i \) \((i = 1, \ldots, 4) \). The maximum distance for both the protocols is between 6 and 7 Km for the parameters given in the caption of Fig. 2. It can be seen that in correspondence of the vertical lines \( l_1, l_2, l_3 \) (plots a, b, c) the LM05 curves are above the BB84 curves, while it is the opposite for \( l_4 \) (plot d). This means that for almost all the relevant distances between Alice and Bob, the LM05 allows for a better gain of secure bits, that directly reflects in higher distribution rates of secure bits between the users. The improvement on small and medium distances has two reasons: the first is the deterministic nature of the protocol, that doubles the rate by removing the basis reconciliation procedure; this reflects in a factor 2 for \( G_{\text{sec}} \) pertaining to LM05 respect to that pertaining to BB84 \[12\]. The second reason is the two-way
Fig. 2: Secure rate vs Alice-Bob distance given the PNS attacks described in the text. \( \lambda = 830 \text{ nm} \), \( \alpha = 2.5 \text{ dB/Km} \), \( \Gamma_c = 8 \text{ dB} \), \( d_B = 5 \times 10^{-8} \text{ counts/slot} \), \( \eta_B = 0.5 \).

channel, that provides the probability \( \tilde{P} \) of Eq.(5) considerably smaller than the analogous of BB84, given by \( P^* = P_{n \geq 2}(\mu) = 1 - (1 + \mu) e^{-\mu} \). It should be noticed that the same double channel also implies a higher total loss-rate for LM05; then the increased gain of secure bits is a non trivial result.

3. Experiment

The experimental test of LM05 for QKD is realized exploiting non-orthogonal polarization states of near infrared photons (see Fig.3).

The photon source is a pulsed diode laser (Picoquant PDL 808) at 810nm with a repetition rate of 20MHz, pulse width 88ps FWHM. A pulse generator is used as sync source for the laser diode and a detection circuit. The light pulses are first split in two, one half goes at Bob’s side for the initial state preparation for both CM and MM runs, the other half is sent to Alice for the CM runs.

The first stage of the protocol is the preparation at Bob’s side of the qubits, encoded using a \( \lambda/2 \) waveplate (\( P_1 \)), in four polarization linear states of the light pulses attenuated to an average number of photons per pulse of \( \mu = (0.118 \pm 0.002) \). The prepared photons are launched into 5m long single mode fibers at
810nm (Thorlabs P1-830A-FC) connecting Alice and Bob. Before every test the fiber was aligned using polarization control pads (Thorlabs FPC-560) so that any polarization input state exits almost unchanged [the fibers proved to remain stable for quite long periods (∼4h), enough for several runs after the alignment]. The second stage is at Alice’s side. The switch between CM and MM is passively realized via a 50/50 BS. Control Mode: The photons are polarization analyzed by a set composed by a λ/2-waveplate (WP2), a polarizing BS (PBS2) and two APD (PerkinElmer SPCM-AQR-13-FC) modules with quantum efficiency ηB ∼ 50% at 810nm and dark counts ∼ 300cps (A0 and A1). The counts rates are measured in a 8ns time window triggered by the sync source, giving a dark counts per gating windows dB ∼ 2.4 × 10⁻⁶. To complete the control mode, Alice injects in the BS, used for switching from CM to MM, a light pulse generated by the second half of the pulse originated from the diode laser. The pulses are polarization encoded in similar fashion as at Bob’s side (P2), and attenuated to a mean photon number per pulse almost 1/20 of the MM one [27]. A couple of λ/2 waveplates (WP2,3) are used to realize the I and the i\(\hat{\sigma}_y\) operators necessary for the Message Mode [5]. As last step, the photon travels back to Bob through a different fiber, 5m long too, with a polarization control pads.

The photons coming from Alice are eventually polarization-analyzed at Bob’s side by PBS3 and a λ/2 waveplate (WP4) set so that the photons are measured in the same basis as they were prepared. The photons are collected after the PBS3 into two multimode fibers and then detected by two APD modules, B0 and B1. Counts out of B0 and B1 in a 8ns time window triggered to the sync source, can
be associated to logical values ‘0’ and ‘1’ corresponding to Alice encoding in the MM runs. A typical result of a communication test is reported in the inset of Fig. 3 for different state preparations performed by Bob and different encodings by Alice (all the eight configurations of interest).

In our experimental tests we estimated the total QBER as \( e = \frac{\bar{n}_{\text{err}}}{n_{\text{tot}}} \), where \( n_{\text{tot}} \) is the total number of counts and \( \bar{n}_{\text{err}} \) the counts in the ‘wrong’ detector. The best value we obtained for \( e \) is \((0.0248 \pm 0.0001)\). We have estimated a probability of ‘ambiguous’ double clicks \( n_{D}/n_{\text{tot}} \sim 9 \times 10^{-4} \), a factor \( \sim 30 \) lower than the probability of error bits \( e \), i.e. we can approximate \( \bar{n}_{\text{error}} \sim n_{\text{err}} \) (see discussion before Fig. 2). The channel transmissivity is estimated to be \( t_{\text{link}} \sim 0.27 \) giving \( \Gamma_{c} \sim 5.7\text{dB} \). These parameters allow to estimate the ‘secure bits gain’, \( G_{\text{sec}} \), for LM05 and BB84 to be 0.018 and 0.006, respectively. With a 20MHz repetition rate laser this entails the possibility to distribute secret bits with LM05 at \( \sim 360 \) kbits/s, 3 times higher than BB84 (\( \sim 120 \) kbits/s). The mean photon number used in the experiment represents the optimal \( \mu \) for distances up to \( \sim 3 \) Km.

4. Conclusion

Our study shows the security of the LM05 protocol against a class of PNS attacks, based on imperfections of Alice and Bob’s equipment. As a byproduct we found that LM05 allows for higher distribution rates of secure bits respect to the BB84, for almost all the relevant distances between Alice and Bob. In our analysis we made the implicit assumption that Eve is clever enough not to alter the statistics of the losses counted by Alice and Bob \[32\]. This means that in the frame of PNS\(_{M}\) attacks Eve should distribute her ‘blocking action’ on both the paths (to and fro) between Alice and Bob, otherwise resulting more easily detectable.

Furthermore we have reported on the first experimental test of LM05 implemented with weak coherent state at 0.8\( \mu \)m. We have measured a QBER \( e \sim 0.024 \) for a communication distance of 5m, which for the parameters of our setup allows for a secure bit rate \( \sim 3 \) times higher than BB84 for distances up to \( \sim 3 \) Km.

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27. The difference in the photon mean number between CM runs and MM runs can be positively exploited by Alice and Bob to further increase the security of LM05 against PNS attacks. This result descends from a straightforward application of the so called ‘decoy state’ technique \textit{25, 29, 30, 31}, and will be investigated in a future work.
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