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Real Estate Bubbles and Urban Development
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ABSTRACT

Why are real estate bubbles so common? Can these bubbles actually do some good? Real estate booms have regularly occurred throughout the world leaving painful busts and financial crises in their wake. This paper suggests that real estate is a natural investment for more passive debt investors, including banks, because real estate’s flexibility makes it better collateral than specifically built production facilities. Passive capital’s preference for real estate will be particularly strong when agency problems bedevil equity investments. Consequently, passive capital may flow disproportionately into real estate and the errors of passive capital can generate real estate bubbles. The preference of banks for more fungible real estate assets can also explain why real estate is so often the source of financial crises. In principle, real estate bubbles can be welfare enhancing, if cities would otherwise be too small either because of agglomeration economies or building restrictions. But given reasonable parameter values, the large welfare cost of any financial crisis associated with a real estate bubble is likely to be much higher than the modest benefits of extra building. The benefits of real estate bubbles are welfare “triangles” while the costs of widespread default are welfare “rectangles,” which is why bubbles rarely appear to be benign events.

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I. Introduction

Housing prices have increased by 15.9 percent annually between 2003 and 2013 in China’s most successful cities in real terms (Fang et al., 2015). Between 2011 and 2014, China built 106.5 billion square feet of floor space (Glaeser et al, 2016). Inevitably, some have claimed that the Chinese property market is a bubble waiting to burst, for real estate crises have been common in the developing world. The late 1990s Asian financial crisis was associated with extensive building and declining real estate prices in Bangkok, Djakarta and even Hong Kong. Japan’s massive post-1990 real estate bust set the stage for that country’s lost two decades.

Glaeser, Huang, Ma and Shleifer (2016) argue that Chinese real estate prices might maintain their value if new supply is limited over the next 15 years, but divining the future of Chinese real estate prices is not the purpose of this paper. This paper addresses the larger question of why real estate booms and busts are so common and whether they can be good for growth. The urban history of the U.S. is replete with real estate speculation, and at least some of that speculation ended without catastrophe (Glaeser, 2013).

Section II of this paper provides brief discussions of past U.S. and Asian real estate booms. Typically, historically, bubble-like events have occurred at moments of extraordinary positive change, such as revolutions in transportation and building technology or widespread industrial growth. Consequently, high prices were plausibly sensible given reasonable beliefs about the future demand for space. High prices were also often associated with large booms in building both structures and infrastructure. New York’s traditional skyline owes its shape to that city’s late 1920s real estate boom.

Section III provides a simple model to explain why real estate booms are so common. Land has been the most common store of wealth throughout human history, both because it is harder to expropriate than other assets, both intrinsically and because western law was partially built around the process of defending land ownership (Glaeser, Ponzetto and Shleifer, 2016). Well-defined property rights for land make it suitable collateral. Moreover, buildings are typically less specific than factories, which means that it is easier for creditors to realize value out from
foreclosed real estate. The attraction of real estate to passive debt investors means that credit
booms have often enabled developers to make large and leveraged bets on real estate. Strong
property rights and the relative flexibility of buildings do not make bubbles inevitable, but they
do explain why extra optimism can often lead to real-estate related activity.

In the model, there are both passive and active investors, where active investors can manage their
investments and passive investors must trust managers. In the event of a default, passive
investors lose value, but they lose less value with real estate than with factories. Consequently,
active capital (which is in limited supply) specializes in industry while passive capital specializes
in buildings. If there is erroneous over-optimism, then the flow of passive capital into real estate
will rise and there will be a bubble. Since the supply of active capital is limited, over-optimistic
beliefs increase the level of real estate investment more than they increase investment in the
export industry.

Olivier (2000) argued that there were conditions under which rational asset bubbles were
growth-enhancing and the same is true about irrational real estate bubbles. Urban economists
have long believed that there are positive (as well as negative) externalities associated with
working in cities. These agglomeration economies can mean that cities are too small, in the
absence of appropriate government policy. Cities can also be too small if government policy
excessively restricts new construction. An irrationally optimistic belief about future demand for
a city can lead to the city being over-developed, relative to private profit maximization. Yet, that
over-development may be optimal from a social perspective if agglomeration economies are at
work, as they appear to be in China (Chauvin et al., 2016).

There are also negative externalities associated with urban living, such as congestion and
contagious disease, and these can be exacerbated by urban growth. Yet much of the new Chinese
construction appears to be replacing older, more dilapidated and perhaps more dangerous
structures with newer, perhaps safer buildings. Consequently, this new construction may be
reducing the downsides of density in urban China.

In theory, real estate bubbles can be welfare enhancing but there is no guarantee that any
particular bubble will produce such a positive outcome. Most importantly, real estate bubbles
lead to foreclosures and potentially widespread social costs from financial dislocation. In Section
IV, I show that given reasonable parameter values, the foreclosure-related losses from a burst
bubble are likely to be higher than the benefits from encouraging extra building. Given moderate agglomeration economies, the losses from foreclosure need to be tiny (under one percent of value) for even the most well-designed bubble to increase overall urban wealth. The upside of bubbles increases as agglomeration economies increase, but it is hard to imagine many settings in which bubbles are actually desirable. Fundamentally, the benefits of the bubble (correcting an externality) are second order, while the costs (value lost due to defaults and foreclosures) are first order.

Real estate bubbles can also lead to welfare losses because there is too much construction or because construction occurs at the wrong time. In countries dominated by an urban primate, such as Korea or Indonesia, building in the wrong place is unlikely to occur. Similarly, it is almost inconceivable that there is too much growth in China’s first tier cities, such as Beijing, Shanghai and Shenzhen. The risk of over-building is most real in lower-tier cities that have benefitted from general exuberance about China but do not ultimately have the fundamentals to produce demand that is greater than supply.

The risk of incorrect timing is also real. New York’s Empire State Building was not fully occupied until after World War II, more than 15 years after its completion. Today, large vacancy rates in many lower-tier Chinese cities seem to imply that delayed construction might have been optimal, from a purely building cost point of view. Yet the costs of construction are likely to be higher in the future, so it is hard to be confident about the costs of building too quickly.

History seems to support the view that the larger costs of real estate bubbles come from financial dislocation, not from over-building. Many have argued that the chaos surrounding the subprime mortgage meltdown of 2007 played a major role in the recession that followed. By contrast, the costs of the Los Angeles real estate boom of the 1880s, which largely did not involve banks, seem to have been trivial.

The implication of this discussion is that fighting investment in real estate directly can be difficult and even harmful. The larger welfare gains come from ensuring that the financial system faces less risk from potential real estate downturns. Yet political leaders who are faced with declining real estate prices or economic stagnation often see credit as a tool for juicing growth or at least stability. Encouraging lending into a high-price market creates the possibility
that a real estate bust will spread across the larger financial and ultimately real sectors of the economy.

II. Real Estate Bubbles in U.S. History and Asia Today

Throughout U.S. history, real estate has been a dominant speculative asset. The financial crises of 1797, 1819, 1837, 1857 and 2007, as well as the savings-and-loan crisis that began in 1989, were all closely tied to real estate speculation. The Japanese bust of 1990 and the Asian financial crisis of 1997 were also closely tied to real estate. This section of the paper briefly discusses this course and consequences of some of these events.

Early U.S. Real Estate Bubbles

Real estate companies, the Virginia Companies of London and Plymouth, founded the first English colonies in the Americas. These investors hoped to get rich by developing land on the eastern seaboard of North America. North America seemed to offer virtually limitless supplies of land that was only weakly defended by disease-burdened Native Americans. For centuries, speculators like George Washington, Benjamin Franklin and Robert Morris, tried to amass vast empires of farmland in the American interior. Morris, the pre-eminent financier of the revolutionary period, accumulated a great mountain of debt to fund his purchases of millions of acres, and his inability to service that debt touched off the Banking Crisis of 1797.

The Louisiana Purchase opened up the interior further, and public land sales set off a frenzy of purchasing in states like Georgia after the conclusion of the War of 1812. Cotton, not wool, had proven to be the ideal crop for the newly industrial mills of England, and America’s southern frontier had ideal conditions for growing cotton. If cotton prices stayed high, agricultural revenues would easily have justified the high prices paid in 1819 for Georgia cotton-growing land (Glaeser, 2013). But cotton can be grown throughout much of the warmer parts of the world, and prices did not stay high. Land prices crashed in 1819, and this set off another banking crisis.
Banks and real estate were intimately related because land could readily serve as collateral. The early industrial enterprises of England were largely self-financed through retained earnings, but the cotton farms that provided the raw materials for those companies could be bank financed, because banks could readily realize the value in land, whereas factories required skilled managers. The slaves that worked the land could easily be debt financed, because they were geographically transferable. There was an American slave bubble in 1819 that accompanied the property bubble (Baptist, 2014).

America’s geographic and economic expansion led to rising land, cotton and slave prices during the 1830s. The 1830s boom boosted prices not only for agricultural land, but also for urban land from New York to Chicago. State-chartered banks expanded when President Jackson refused to renew the charter of the Second Bank of the United States. They lent heavily to real estate speculators. When Jackson’s Specie Circular order of 1836 required public land to be purchased only with silver and gold, land prices began to fall. Ultimately, the financial chaos created by the bust would disrupt the U.S. economy for five years.

These booms had real consequences, and optimism about the value of western land helped encourage the development of American transportation infrastructure. Early canal builders, including George Washington, were partially motivated by a desire to unlock the value of their own western land. Riverboats and railroads were built in anticipation of increasing western agricultural output. John Jacob Astor didn’t just buy up New York land during this period; he also developed that land into usable urban space.

By the 1840s, railroad shares were starting to replace frontier land as the pre-eminent speculative investment. Railroads required so much capital, and created so much upside, that investors were willing to provide equity investment despite the considerable risks and reality of insider expropriation. But land never stopped serving as an object of over-optimism. The Panic of 1857 was precipitated by the failure of the Ohio Life Insurance and Trust Capital, which had been founded as a tool for investing eastern money in mortgages on western land (Haeger, 1979). Ohio Life itself had moved from land to railroads, and its own dealing may have also reflected mismanagement (Calomiris and Shweikart, 1991). Declining wheat prices during 1857, generated by a rich harvest and reduced European demand after the end of the Crimean war, lowered the appeal of western land and the railroads that served that land. America was an
overwhelmingly rural country in 1857, and falling land prices readily rocked eastern financial institutions.

The Boom of the Roaring 1920s and the 1929 Bust

The Great Depression is far more associated with the stock market collapse of 1929 than with a real estate market bust, but property values closely paralleled the shifts in the Dow Jones Industrial Average. During the 1920s, real estate prices boomed in New York City, and a bust began in 1929. Nicholas and Scherbina (2013) report that average nominal prices per square foot rose from $3.89 in 1922 to $6.91 per square foot in 1929 before falling to $2.39 by 1935. In real terms, the price fluctuations are less dramatic, but their quality-adjusted real price index still declined by over 30 percent between 1929 and 1931. Wheaton, Baranski and Templeton (2009) document a similar drop in the value of New York City commercial real estate after 1929.

Ebullient investors lent the funds for the buying and building of New York City real estate. Securities, backed by commercial real estate mortgages, boomed during the 1920s, and at their height in 1925, real estate-backed securities represent one-fifth of all newly issued corporate debt (Goetzmann and Newman, 2010). Naturally, much of this lending went sour in the 1930s. Nicholas and Scherbina (2013) document the enormous increase in the proportion of sales of foreclosed properties between 1929 and 1934, which captures the stress that collapsing real estate values put on the banking system.

Yet the Manhattan real estate boom of the 1920s was not without its upsides. New York City experienced a building boom that still shapes the city. Goetzmann and Newman (2010) show that New York built over 25 tall (over 70 meters high) buildings during each of the five years after 1926. In three of those years, New York built over 35 such buildings, a pace of new construction that the city has never repeated. In 1920, all of New York’s taller buildings were around Wall Street. By 1931, there was a great forest of skyscrapers in midtown surrounding Grand Central Terminal.

This great wave of city building culminated in the great skyscraper race of 1929 and 1930. The 792 foot high Woolworth Building had loomed as the tallest building in New York City and the world for 17 years, but in rapid succession it was replaced by 40 Wall Street (927 feet), the Chrysler Building (1046 feet) and the Empire State Building (1250 feet). The Empire State came
in considerably under budget because the Great Depression severely reduced building costs, but it was known as the Empty State Building for years because of its vacancies. It only became profitable in 1950.

Clark and Kingston’s (1930) *The Skyscraper* provides a remarkably detailed picture of the economics of construction during the late 1920s. In 1929, construction costs were sufficiently low that tall buildings looked enormously profitable even given the rising land prices. Clark and Kingston’s error, which was presumably the error made by most New York builders during this period, is that they failed to anticipate the pressure that abundant supply would put on office rents. Basic economics tells us that prices must ultimately be tied to supply costs, and so skyscraper building could never remain a source of boundless profits. Rents were sure to fall, with or without the stock market crash.

Yet the welfare effects of New York’s skyscraper boom are far from obvious, assuming that the post-1929 financial crisis would have occurred without the real estate bust. The 42nd Street epicenter of the building boom runs primarily through two zip codes (10017 and 10036) which collectively employed over 300,000 people who earned over $40 billion in payroll in 2014, according to County Business Patterns. That building boom created the central business district of New York City, which has been a primary example of agglomeration economies at work for decades. The over-confident over built and many lost money, but they also created an enduring economic dynamo.

*The Savings and Loan Crisis and the 2007 Mortgage Market Meltdown*

The more recent American real estate-related crises have both been strongly associated with problems in the credit market. The U.S. experienced a strong real estate cycle during the 1980s and early 1990s, but the savings-and-loan crisis largely preceded the decline in real estate prices. These entities invested in real estate, but the fundamental issue was with their incentives, not with the real estate market.

The standard narrative about the savings and loans is that in the late 1970s, the thrift industry’s cost of capital rose with interest rates, but its revenues did not, since these were tied to a stock of fixed-rate mortgages that paid low pre-inflation interest rates. In response to these financial troubles, Congress deregulated the savings and loans starting in 1980, but continued to insure
depositors through Federal Savings and Loan Insurance Corporation. These deregulated entities, whose cost of capital was independent of their actual risk of default, behaved in textbook fashion. They loaded up on risk, some of it in real estate, since shareholders and management would benefit if the bets paid off, and taxpayers would pay if the bets failed. In various cases, there was also insider expropriation and influence peddling.

This story is connected to real estate primarily because real estate was the major business of the savings and loans. These initially modest institutions came into existence to help middle-income individuals save and borrow to become homeowners. Naturally, when they started to over-lend, they expanded their core business. For example, many extended the duration of commercial real estate loans from short-term capital to builders to long-term capital that supported eventual owners. Real estate played a central role in the savings-and-loan crisis because it is such a basic asset, and these were relatively simple institutions. Unfortunately, even relatively simple institutions betting on basic assets can impose billions of dollars of costs upon taxpayers.

The U.S. real estate convulsion that lasted from 1996-2012 was both larger and more complicated. Between 1996 and 2006, prices rose dramatically in many markets. The boom began in the supply-constrained, high-income coastal markets and then gradually spread inland (Ferreira and Gyourko, 2014). An ocean of mortgage-backed securities and a significant increase in sub-prime lending accompanied this price boom. Mian and Sufi (2008) document that prices rose more in areas that gained access to subprime lending, but there is still debate about whether easy credit “caused” the larger boom. Glaeser, Gottlieb and Gyourko (2012) show that the downward shift in interest rates after 2000 appears too small to explain the massive rise in prices given reasonable rational models connecting prices to rents.

During the boom years, there was substantial momentum in housing prices: high-price growth in an area from 2004 to 2005 predicted even stronger price growth from 2005 to 2006. Over the longer time horizon, however, mean reversion was almost perfect. On average, price declines from 2006 to 2012 almost completely wiped out price growth from 2000 to 2006.

The 1980s real estate boom was focused almost entirely in areas with relatively inelastic housing supply (Glaeser, Gyourko and Saiz, 2008). In the 2000s, even elastically-supplied cities, such as Phoenix, experienced robust price growth, although there was still a strong connection between inelastic supply and price growth during the period. The later price declines were most severe in
cities that had price booms and elastic supply, presumably because prices were depressed in these areas because of the abundant new construction.

Between 2000 and 2007, the number of housing units in the U.S. increased by 12.8 million units, for an average of 1.8 million units per year. The number of vacant units in the U.S. rose by 4.2 million units in that same time period. This large building boom left America with a sizable housing inventory, which helped depress construction for next seven years. From 2007 to 2014, the number of homes in the U.S. only grew by 4.9 million units from 2007 to 2014.

There were certainly social costs from over-building in cities like Las Vegas and Phoenix, but these costs were orders of magnitude less important than the financial disruption that followed the mortgage-market meltdown (Glaeser, 2013). Starting with the collapse of Bear Sterns and Lehman Brothers, global financial institutions teetered for years after real estate prices fell. The impact of financial disarray on the larger economy is hard to accurately assess, but the total economic costs of the financial crisis are surely in the trillions.

Europe experienced a parallel real estate boom starting in the late 1990s. Spain, for example, experienced a particularly sharp construction and price boom between 1997 and 2007, followed by a similar bust. Bardhan, Edelstein and Tsang (2008) link this growth to globalization and greater integration into the European Union. Spanish banks were conservatively regulated but still suffered. The regional Caja saving institutions, which were more lightly regulated, were decimated because of the housing bust. Spain may have been the most extreme European case, but booms and busts also occurred in Austria, France, Germany, Italy and elsewhere.

The Japanese Asset Boom and Bust

In the 1980s, Japan was the global economic superstar. Its post-war economic rebirth was astonishing. Japanese cars and electronics were ubiquitous. In 1955, per capita GDP in Japan was about one-tenth of that in the U.S. By 1980, per capita GDP at current exchange rates was higher in Japan than in America.

Japan was not just an astonishingly successful economy, it also had a relatively compact geography and a single dominant metropolitan area. Japanese regulations have long constrained land development, and agriculture is strongly protected. Consequently, demand for space in
Tokyo has long been high. A significant amount of the optimism about Japan’s future translated into optimism about the value of land in the capital city.

Nationwide, commercial land prices rose by 53 percent in real value between 1985 and 1990, according the Japan Real Estate Institute. In years, Japan’s six largest cities, commercial land prices rose by 269 percent over the same five year period. Over those five years, real residential land prices increased by 149 percent in those six large cities.

This boom was gigantic by any measure and so was the subsequent bust. In 2015, commercial real estate in the six large cities was worth, in real terms, 14 percent of its value in 1990. Nationwide, commercial real estate in 2015 was valued at 22 percent its value in 1990. Across Japan, residential real estate had lost over 50 percent of its real value in the 25 years after the boom. The decline in Japan’s real estate values was quick in the six large cities, but slower for the nation overall. Both the 1990s and the 2000s were terrible years for Japanese real estate investors.

The upside to the real estate boom was considerable construction in Tokyo and elsewhere. The number of non-wooden housing units in Tokyo increased by 254,000 between 1986 and 1990, while the number of wooden houses declined by 100,000. Builders created 524 million more square feet of living space in non-wooden structures during those years, which was substantially more than they built in the years before 1985. Given the enduring strength of the Tokyo agglomeration, the boom-era new construction seems like a benefit from the boom.

The downside of the boom was that the subsequent bust set off two lost decades of stagnant growth for Japan. Many serious observers of Japan argue 20 years of slow growth were the result of balance sheet problems in banks and businesses that started with declining real estate and stock values. While this view is disputed, it seems certain that the welfare consequences of over-building are surely dwarfed by any macroeconomic consequences of the bust, even if the real estate bust is responsible for only one-fiftieth of the larger malaise in Japan’s economy.

The Asian Financial Crisis of 1997

The Asian financial crisis of 1997 was a seminal event in the recent economic histories of Hong Kong, Indonesia, Malaysia, South Korea and Thailand. These economies had been doing well during the 1990s. In 1997, investors quickly lost confidence in the currencies of these
economies, asset prices tumbled and the International Monetary Fund (IMF) stepped in. The macroeconomic consequences were typically significant, if relatively short-lived.

Much of the economic debate around these crises has focused on contagion in the currency markets and the role of the IMF. Yet Quigley (2001) and Semlali and Collyns (2002) both provide evidence suggesting that real estate lending was a major part of the lending boom and subsequent bust. Quigley documents the rise in prices, commercial office supply and vacancy rates in the years before the crisis. He notes that real estate was an enormously important asset, and that Bangkok real estate was apparently valued at almost 50 percent of Thailand’s G.D.P. He points out that non-performing real estate loans were a large part of the banking sector’s portfolio. Real estate loans appear to have comprised between 30 and 40 percent of total bank loans in Malaysia and Thailand before the crisis and 40 to 55 percent of total bank loans in Hong Kong.

In a sense, the 1990s Asian real estate boom was eminently justifiable. All of these economies were growing well and were, like Japan, centered on a single large metropolitan area. Investors thought that demand for Bangkok, Seoul, Jakarta, Kuala Lumpur and Hong Kong was likely to continue to grow. Real estate seemed like a far safer investment than most private companies because of the opaque governance structures that prevailed in these countries during that time period. As legal systems develop, real property becomes secure long before more complex forms of investment. Moments of optimism during early stages of growth, therefore, tend to turn into real estate booms.

The fact that these countries are dominated by a single major metropolitan area somewhat limits the ability for a real estate boom to produce extremely wasteful structures. Today, space in Seoul, Bangkok, Djakarta, Hong Kong and Kuala Lumpur is still in short supply. Once again, the supply effects of the boom appear to have been largely benign.

Real estate troubles preceded the full-blown crisis, and seem to have helped generate anxiety about the health of the financial sectors in these countries. Thailand appears to have been the hardest hit by the real estate bust, because it suffered from “a combination of the bursting of a large property price bubble and a weak financial system” (Semlali and Collyns, 2002). By contrast, the property bust was less important in Korea and Indonesia, partially reflecting stronger and better financial regulations. Malaysian banks weathered the storm particularly well.
Firmer bank regulation seems to have stopped a real estate bust from turning into a wholesale recession.

*The Chinese Real Estate Boom*

Chinese real estate today seems to be following the same script as these earlier property-price booms. Chinese prices have risen dramatically over the past decade (Fang et al., 2015). A massive construction boom has accompanied that rise. There has been large-scale lending to real estate developers, which leaves the banking system exposed to any future real estate bust. The government has reacted to a slowdown in housing-price growth by encouraging more lending to private home-buyers.

Optimism about Chinese real estate prices is entirely understandable. China’s economic growth has also been miraculous. The country is rapidly urbanizing and the number of rural Chinese who could potentially urbanize is enormous. Moreover, there are a few metropolitan areas, such as Shanghai, Shenzhen and Beijing, that are spectacularly productive. Agglomeration effects appear to be quite strong in China (Chauvin et al., 2016).

Because China has a large number of major metropolitan areas, the potential for wasteful investment is much higher in China than elsewhere. While it is almost impossible to imagine that there will be too much construction in Beijing or Shanghai, tales of ghost cities in the interior of China have circulated for years. Glaeser, Huang, Ma and Shleifer (2016) estimate vacancy rates of 20 percent in many Chinese cities. Vacant homes include both those that are owned by developers and those that are owned by ordinary investors who have chosen to leave them vacant rather than renting them out. It seems distinctly plausible that there has been some over-building in some of China’s third and fourth tier cities.

Chinese investment in real estate also seems to reflect relatively well defined property rights for real property. Between June 2001 and June 2016, the Shanghai Stock Exchange Composite Index increased only 32 percent in nominal terms, which means that it actually fell in real terms. Perhaps because of governance problems within Chinese firms, investing in Chinese equities has not been a high-return strategy. By contrast, owning apartments seems like a way to potentially benefit from China’s growth without having to trust corporate management.
China is unusual in that its governments have played an outsized role in nudging the real estate cycle along. Local leaders are rewarded based on economic performance, and real estate development represents a simple way of ensuring jobs and growth. Moreover, Chinese local governments lack access to a steady stream of annual property tax revenues and must instead depend on the revenues generated by the sale of local land to real estate developers. As the large Chinese banks are also public institutions, the public sector is also in a position both to free land for real estate development and to supply the credit for real estate developers.

The power of China’s public sector means that China’s government has an outsized ability to actually control the future of the country’s real estate markets. Glaeser, Huang, Ma and Shleifer (2016) simulate potential prices for Chinese real estate in 20 years under different supply scenarios. If supply continues at current levels, it seems likely that future prices will be significantly below current prices. But if the Chinese government sharply reduces the amount of new construction, which is well within its power, then prices could remain stable in the long run. Naturally, there would be significant costs from shutting down new supply, including reducing the rate of urbanization and less employment within the construction sector.

III. Why Real Estate? Property Rights and Asset Bubbles

In several of the previous examples, I have argued that real estate is a natural object of speculation because it is particularly well-suited to be collateral. In many developing world contexts, property rights are so much better defined for real estate than for other forms of investment. Even in China, where the government still technically owns all the land, apartments seem far safer than corporate equities.

For lending, the essential distinction between real estate and other assets is transferability. Typically, buildings are relatively general physical capital. Someone else can occupy a Tokyo office building or a Las Vegas home without a huge loss in value. Machinery that was custom built for a particular company is far less transferable, and consequently customized corporate assets make far worse collateral than real property. Moreover, in some contexts, property markets are thick, making it easier for banks and their regulators to assess the value of real
estate. This advantage is far from universal. Assessing the value of real estate in China’s third and fourth tier cities can be almost impossible.

*Urban Development with Agency Problems in Investment*

This section discusses investment in buildings versus export-oriented manufacturing, when corporate governance is imperfect. There is free entry into the export industry, where capital and labor is transformed into a numeraire output good with a Cobb-Douglas production technology: $AK^\alpha L^{1-\alpha}$. Capital is itself made from the output good, which serves as the numeraire with a price of one. In the next section, I will consider how investment changes when $A$ depends on the size of the city, but here I treat it as exogenous.

There is also free entry into the building sector, where housing is produced with land and the numeraire good, referred to as structure in this context, with a second Cobb-Douglas technology:

$$
Housing \text{ Space } = \frac{1}{1-\sigma} \text{Land}^\sigma Housing \text{ Capital}^{1-\sigma}.
$$

Housing capital is again produced with the numeraire output good. The price of housing space is endogenous and denoted $P_H$. The wage is denoted $w$. Total land available in the city equals $M$.

Worker utility is Cobb-Douglas in earnings and housing, meaning that if a worker has an income of $w$, she will spend a fraction of that income $\gamma w$ on housing and indirect utility will equal $w P_H^{-\gamma}$.

$$
\text{Workers can live in another locale, earn } w_0 \text{ and pay one per unit of housing. The spatial equilibrium ensures that } P_H = \left( \frac{w}{w_0} \right)^{\frac{1}{\gamma}}.
$$

The timing of the model is that in the first period, developers borrow, buy land and housing capital and produce housing space. In that same period, industrialists borrow and build manufacturing capital. At the start of the second period, industrialists hire workers who buy housing space. At the end of the second period, industrialists sell their output.

We consider two possibilities for worker mobility. Our primary assumption is that workers move to the city at the start of the second period. Our alternative is that workers commit to their

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2 Direct utility is defined as $\frac{c^{1-\gamma} h^\gamma}{(1-\gamma)(1-\gamma h)^\gamma}$, where $c$ refers to non-housing consumption and $h$ refers to housing consumption.
decision to move at the beginning of the first period. The timing is immaterial in this section when there are no aggregate shocks or surprises.

In a frictionless world without financing and agency problems \( w = \tilde{\alpha}A^{\frac{1}{1-\sigma}} \), and \( P_H = A^{\frac{1}{(1-\sigma)\gamma}} \left( \frac{\tilde{\alpha}}{w_0} \right)^{\frac{1}{1-\sigma}} \) where \( \tilde{\alpha} = \alpha^\frac{\alpha}{(1-\alpha)}(1 - \alpha) \). As workers receive their reservation utilities and firms earn zero profits, land value is the only measure of social surplus and in this benchmark case, it equals \( MA^{\frac{1}{(1-\sigma)\gamma}} \left( \frac{\tilde{\alpha}}{w_0} \right)^{\frac{1}{1-\sigma}} \).

I now introduce financing frictions into the model. I assume a supply \( \bar{I} \) of active investors, who can manage their own projects. The quantity \( \bar{I} \) combines the actual wealth of these investors and whatever other financing they may have acquired. Active investors may lose money, but they do not default. There is also an entirely elastic supply of passive investors who can only lend to external managers.

I rule out equity investing by passive investors, by assuming that profits are non-verifiable and then managers would always completely expropriate outside passive investors through self-dealing. I do however assume that these investors can ensure that loaned funds are spent entirely on capital or buildings, which can be seized in the case of default. Consequently, the only feasible financial contracts are collateralized loans.

I introduce a risk of default on such loans by passive investors by assuming that there is some uncertainty about the timing of new cash flow. All debt is payable at the end of period 1. With probability \( 1 - \rho \), real estate developers are able to presell homes to investors at that date. With the same probability, industrialists are able to presell their output to consumers at that date and repay the debt and hire labor with purchase order financing. With probability \( \rho \), both types of firms face a liquidity shock and must give their assets to the lender. I assume that lenders never grant forbearance, possibly because of financial regulations. In the even of a liquidity shock, lenders foreclose and acquire either the homes or the industrial capital.

\[^3\] In this case, total housing supply equals \( MA^{\frac{1-\alpha}{1-\sigma} \sigma \gamma} \left( \frac{\tilde{\alpha}}{w_0} \right)^{\frac{1}{1-\sigma}} \), total population equals \( MA^{\frac{1-\alpha}{1-\sigma} \sigma \gamma} \left( \frac{\tilde{\alpha}}{w_0} \right)^{\frac{1}{1-\sigma}} \left( \frac{1}{1-\gamma} \right) \), and total output equals \( MA^{\frac{1-\alpha}{1-\sigma} \sigma \gamma} \left( \frac{\tilde{\alpha}}{w_0} \right)^{\frac{1}{1-\sigma}} \left( \frac{1}{1-\gamma} \right) \).
There are social losses from defaults on debt contracts, which reflect both legal costs and the manager-specific nature of investment. When a manager defaults and the lender acquires the asset, a fraction $\delta$ of manufacturing capital is destroyed and a fraction $\theta \delta$ of housing is destroyed, with $\theta < 1$. The parameters $\delta$ and $\theta$ can be interpreted as reflecting the quality of the legal system.

This difference between manufacturing capital and real estate provides the fundamental asymmetry of the model. Real estate is more fungible and less specific than factories. It is easier for a bank to resell apartments than to resell an industrial plant. Throughout the world, the connection between the banking system and the real estate sector is particularly strong, because real estate is relatively transferable collateral. The market for real estate in Third Tier Chinese cities may be thin, but I suspect that reselling manufacturing plants in such cities is also difficult.

Since default costs are higher for manufacturing, active investors will have a comparative advantage in that industry. Depending on parameter values, active investors may operate in both industries while passive investors lend only in real estate, or passive investors may lend in both industries while active investors operate only in manufacturing or the two types of investors may specialize completely.

The shocks in this model are essentially liquidity shocks, not shocks to real output. Consequently, without the losses from asset reallocation, there would be no social losses from these shocks. As active capital is essentially taking an equity stake in their investments, there face no losses from these shocks. The key problem is that the debt contract must specify repayment before the good is finally sold, so that the some firms are forced to repay before they acquire cash flow.

In real estate, both active and passive investors will try to minimize the cost of a unit of structure, which implies a structure to land ratio of $\frac{P_L(1-\sigma)}{\sigma}$, where $P_L$ refers to land costs. The nominal interest rate “r” will ensure that X dollars invested yields X dollars in expected returns, since investors are assumed to be risk neutral. Given optimal investment and a price of land $P_L$, one unit of investment in real estate will generate returns of $P_H \left( \frac{\sigma}{P_L(1-\sigma)} \right)^{\sigma}$ (which equals 1+r) if there is no default and $1 - \theta \delta$ times that amount otherwise.
In the appendix, I prove:

**Proposition 1:** There exist four values \( \bar{I}_1 > \bar{I}_2 > \bar{I}_3 \), where \( \left( \frac{1 - \rho \delta}{1 - \rho \theta \delta} \right)^{1 + \frac{\alpha}{\sigma \gamma (1 - \alpha)}} \bar{I}_2 = \bar{I}_3 \), such that if \( \bar{I} > \bar{I}_1 \) then only active capital invests in either sector, if \( \bar{I}_1 > \bar{I} > \bar{I}_2 \), then active capital invests in both sectors and passive capital also invests in real estate, if \( \bar{I}_2 > \bar{I} > \bar{I}_3 \), then only active capital invests in manufacturing and only passive capital invests in real estate and \( \bar{I}_3 > \bar{I} \), then passive capital invests in both sectors and active capital only invests in manufacturing.

The values of \( \bar{I}_1, \bar{I}_2, \) and \( \bar{I}_3 \) are all rising with \( A \) and \( M \) and falling with \( \rho, \theta, \delta \) and \( w_0 \).

Whenever passive capital invests in a sector, then the level of total investment in that sector is also rising with \( A \) and \( M \) and falling with \( \rho, \theta, \delta \) and \( w_0 \). The population level is always rising with \( A \) and falling with \( w_0 \), and whenever there is any passive investment in either sector, population is also rising with \( M \) and falling with \( \rho, \theta, \) and \( \delta \).

Proposition 1 characterizes the basic structure of the model, and provides the basic explanation for why real estate has a particular appeal to passive investors. Since real estate is good collateral, debt lending orients itself particularly towards real estate. Consequently, there end up being greater social losses from defaults and foreclosures in real estate than in the export sector of the economy.

Whenever active capital is abundant, then only active capital invests in either sector, since it accrues fewer losses from default. When active capital is slightly less abundant, then passive capital moves into real estate. When active capital is even less abundant, then there is a segregation of capital, where active capital invests in manufacturing and passive capital invests in real estate. When active capital is truly rare, then passive capital invests in both sectors and active capital only invests in manufacturing.

The cutoffs for the level of active capital are always increasing in \( A \) and \( M \), meaning that as the city becomes more productive or has more land, the incentive for passive capital to move into the city increases. Higher values of \( w_0 \) make the city relatively less appealing and reduce the inflow of passive capital. The three values that increase the default costs that passive capital potentially faces (\( \rho, \theta, \) and \( \delta \)) make it less likely that passive capital will enter the market.
These same parameters also determine the amount of capital invested in the city, as long as there is any passive capital, and the size of the city. When A and M are larger, the city’s population increases. Unsurprisingly, city size depends on city productivity and available land. When \( \rho, \theta, \) and \( \delta \) are larger, then the city shrinks. If foreclosure is associated with substantial reductions in productive capacity, either in the real estate sector or in the export sector, the city will be smaller. Somewhat surprisingly, neither \( \theta \) nor A impacts the ratio of export capital to real estate structures, because foreclosure costs in real estate impact both types of capital in exactly the same way.

Later, I will focus on the case, which seems to be most relevant for the developing, where active capital is relatively rare and invests only in manufacturing and passive capital invests only in real estate. The range of values for which the segregated equilibrium exists is dependent on \( \frac{(1-\rho\delta)}{(1-\rho\theta\delta)} \), which implies that the range of parameters for which this equilibrium occur is larger when \( \theta \) is lower. This segregation of capital across sectors will be particularly common when the asymmetry in foreclosure costs between real estate and factories is particularly large.

Embedding Bubbles into the Model

I model a bubble as simply an erroneous belief about the future state of the economy. In principle, this erroneous belief could be about any parameter, but “A,” the level of productivity, makes a particularly natural source of error. I will not model where this mistaken belief comes from, but rather simply assume that the error exists and examine the consequences. I assume that the error exists only in the first period, and by the second period true productivity has been revealed. The most natural assumption about the bubble is that it occurs during the first period, when building and capital investment occurs, but that reality reasserts itself before hiring occurs in the second time period.

At this point, it begins to matter whether workers decide to move during the first period, with overoptimistic beliefs, or at the start of the second period. If workers move based on bubbly beliefs, then they will earn less than their outside option in the second period, and workers end up paying for some of the cost of the bubble. An alternative assumption is that firms precommit to workers’ wages, which is easiest to imagine if industrialists have active capital. If workers
make their moving decision in period one, then the bubble impacts all actors symmetrically and Proposition 2 results:

**Proposition 2:** The bubble causes $\bar{I}_1, \bar{I}_2, \text{and } \bar{I}_3$ to increase. If only active capital invests, then the bubble has no impact on investment. If passive capital invests only in real estate and active capital invests in both sectors, then causes investment in both sectors $t$ to rise by a factor \[
\left(\frac{A}{\bar{A}}\right)^{(1-\alpha)\gamma \sigma}.
\]
If active capital invests only in manufacturing and passive capital invests only in real estate, then the bubble again has no impact on manufacturing but causes real estate investment to increase by a factor \[
\left(\frac{A}{\bar{A}}\right)^{\frac{1}{\alpha+\gamma-\alpha \gamma}}.
\]
If passive capital is in both sectors, then investment in both sectors increases by a factor \[
\left(\frac{A}{\bar{A}}\right)^{(1-\alpha)\gamma \sigma}.
\]

The bubble increases the appeal of the city to passive capital. Active capital will also anticipate higher returns, but since active capital is inelastically supplied, this has no impact on investment or city size. The added appeal caused by overoptimism makes it more likely that passive capital will be present in one or both of the sectors.

When active capital is present in both sectors, then investment will increase in both sectors, because passive capital flows into the real estate sector, which then leads active capital to flow into manufacturing. A bubble therefore leads the real estate sector to be especially vulnerable to defaults.

When there is segregation of the types of capital, then there is no impact on manufacturing investment, but the size of the real estate sector increases. The growth of the real estate sector is somewhat muted in this scenario because manufacturing is not growing with the bubble. This fact implies that prices and wages will end up being lower in equilibrium, because the bubble has made the city real estate heavy.

When passive capital is in both sectors, then again the bubble leads to balanced growth of both sectors. In this case, there will be a wave of defaults in both manufacturing and real estate, although the defaults will be rarer in manufacturing because of the presence of active capital. The growth due to the bubble will be larger than in the case where the two types of capital are segregated.
This model draws a distinction between real estate capital and export industry capital based on the ease of reallocating that capital in the event of default. The greater ease of managing real estate means that arms-length lending is easier in real estate. Consequently, over-optimism leads to more lending to real estate developers and more real estate busts. This logic perhaps explains why real estate busts play such an outsized role in the developing world financial crises.

IV. Can Real Estate Bubbles Be Good?

In the previous section, I addressed only the positive implications of asset bubbles for investment in the export industry and real estate. In this section, I turn to the welfare consequences of such bubbles. I will first assume a standard market failure: agglomeration economies. In the absence of liquidity effects, this market failure will imply that modest over-optimism is welfare enhancing. I will then reintroduce welfare costs of illiquidity which can mean that real estate busts have large, negative social costs.

Throughout this section, I will assume that only active capital invests in manufacturing and only passive capital invests in real estate. I will also compare the case in which workers move in the first period with the case in which workers move in the second period, but I will assume that manufacturing firms have committed to provide wages which deliver the workers’ reservation utility. I will assume that $\rho = 0$ so that there are no exogenous liquidity shocks.

Welfare-Enhancing Real Estate Bubbles and Agglomeration Economies

To create a positive impact of over-optimism, I assume an agglomeration economy: productivity rises with the city size. Formally, the productivity parameter $A$ equals $A_0N^\varepsilon$, with $\varepsilon > 0$. This type of agglomeration externality will mean that decentralized city size will be too small relative to the first best. Excessive land use restrictions will also lead to under building, and just as bubbles can help builders internalize agglomeration economies, they encourage builders to undo the unfortunate effects of too much regulation.

As in many spatial equilibrium models, worker utility will not be impacted by the externality, since worker utility is also pinned down by the reservation locale. As long as passive capital
receives their expected returns of one, then all benefits of internalizing the externality would go to active capital and land owners. I adopt the definition of social surplus as the total amount of manufactured good created minus the costs of building housing minus the costs of providing sufficient consumption so that workers receive their reservation utility.

As I will discuss in the next sub-section, real estate bubbles will always lead to liquidity crises in this model. Builders borrowed at rates so that borrowers will be repaid exactly their costs of capital based on the bubbly beliefs. When reality strikes, then no one can cover their debts and all real estate is foreclosed upon. Manufacturing capital doesn’t face foreclosure since it is entirely owned by active capital equity investors.

The structure of this model does not allow bubbles to be self-fulfilling, but that could occur given different assumptions about beliefs and technology. For example, assume that beliefs were not about some fundamental technology, but rather about housing prices. Given the right assumptions about technology, it would certainly be possible for optimistic beliefs to lead to enough building for the agglomeration economies to propel housing demand upwards and for pessimistic beliefs to lead to economic weakness and low housing demand.

To illustrate the upsides of the bubbles, I first assume that $\theta = 0$. The passive investors are surprised that the bubble turns out to be incorrect, and there will be defaults, but there will be no costs from default. When worker hiring occurs in the first period, manufacturing firms believe that $A_0$ can equal either $\hat{A}_0$, so the manufacturing firms share the same beliefs as the builders. When worker hiring occurs in the second period, manufacturing firms correctly perceive $A_0$. The difference in beliefs reflects the possibly that hiring decisions may be made later than decisions about construction.

Proposition 3 discusses how incorrect beliefs influence the surplus maximizing population and building stock, where surplus maximizing is defined as maximizing the total amount of rents potentially shared by the industrialists and the owners of land.

**Proposition 3**: Total surplus is maximized if hiring occurs in the first period and both builders and manufacturing firms believe that $\hat{A}_0 = \frac{1+\epsilon-\alpha}{1-\alpha} A_0$. If hiring occurs in the second period, but real estate developers believe $\hat{A}_0 = \frac{1+\epsilon-\alpha}{1-\alpha} A_0$, then the building stock will equal the surplus
maximizing level, but the city population will be lower than the surplus maximizing level. If hiring occurs in the second period, then some builder over-optimism is still surplus increasing, but the optimal level of builder over-optimism is less than when hiring occurs in the first period. Proposition 3 illustrates the upside of over-optimism. Agglomeration economies mean that the city is too small. Over-optimism on the part of builders leads to more construction. Over-optimism on the industrialists leads to more hiring. If hiring occurs in the first period, and the bubble sets $\hat{A}_0 = \frac{1+\epsilon-\alpha}{1-\alpha} A_0$, then the error will exactly offset the agglomeration externality and the city’s size will maximize total surplus.

If the builders and industrialists have different beliefs, which will happen if hiring occurs in the second period, then the city cannot have the optimal mix of housing stock and labor. In particular, if the builders make decisions optimistically, but the industrialists have learned the truth before they hire, then the city will have an abundance of housing stock relative to its population. This will create the classic overbuilt urban areas that follow the burst of a bubble.

Yet even when the industrialists are totally rational, some over-optimism on the part of builders is desirable from the perspective of increasing total surplus. The developers’ extra construction will still help boost city size, which yields beneficial externalities. The optimal bubble is smaller in this case than in the case of universal irrational exuberance, which suggests that the errors are complementary.

For my calibration, I assume that $1 - \alpha = .65$ (labor’s share of output), $\gamma = .3$ (housing’s share of consumption) and $\epsilon = .05$ (the agglomeration effect). Labor’s share of output is based on the share of labor compensation relative to GDP in the U.S., which was 60.4 percent in 2014, but has typically been closer to 65 percent. Housing share of expenditure is based loosely on the current expenditure survey, which found that in 2015, the average household had total expenditures of $55,978 and spent $18,409 on housing.\footnote{As such, housing is 32.8 percent of total expenditure, but I reduced the share slightly because some housing expenditures reflect movable household furnishing.} The usual agglomeration estimates are based on cross-section regressions connecting the logarithm of wages with the logarithm of area population or density and .05 is a standard number (Chauvin et al., 2016). Few results are strongly sensitive to moderate changes in these parameter values.
Based on these parameter values, when hiring occurs in the first period, then the optimal amount of over-optimism is 7.7 percent, meaning that ideally $\hat{A}_0 = 1.077A_0$. This suggests some value for ebullient expectations, but as over-optimism during a boom seems to run far above that level (Case, Shiller and Thompson, 2012), I take this as suggesting that bubbles rarely get the development level right. Lower values of labor share will make this estimate higher, so for example if $1 - \alpha = .55$, the optimal amount of over optimism is 9.1 percent. Higher agglomeration economies will also increase the optimal size of the bubble.

When hiring occurs in the second period, these same parameter values imply that the optimal bubble sets $\hat{A}_0 = 1.074A_0$, so the optimal bubble falls only slightly when hiring is based on accurate assessments of productivity. The benefits of the bubble fall far more dramatically than the optimal bubble when hiring occurs in the second period.

**Foreclosures and Over-building**

I assumed previously that there would be no costs from foreclosures and illiquidity in the event of a burst bubble. This is empirically false and does not even sit well within the basic assumptions of the model. The natural assumption is that if $A_0$ reveals itself to be lower than expected, then builders – all builders in the model—will go into default. They had no expected equilibrium profits, even under bubbly beliefs. When reality is darker than the bubble, they have negative net equity and the only course is default and foreclosure. I will focus entirely on the equilibrium where industry is funded entirely by active investors, and so there will be no default costs in that industry, even if they share the same erroneous beliefs with the builders.

In reality, of course, real estate companies still have equity cushions. Not every developer will go into default. I will let $\theta\delta$ continue to reflect the lost housing stock from default and foreclosure, but recognize that this represents an average of major and minor losses in the real world. This means that if there is a bubble, $\frac{x}{1-\theta\delta}$ units of pre-crisis space must be built to provide $x$ units of post-crisis space. This reduction in usable space, due to foreclosures, is the fundamental cost of the bubble.

Under what conditions can a bubble be welfare improving if the bubble’s burst generates a liquidity crisis? Proposition 4 compares an optimal bubble, which is defined as the bubble that
maximizes social surplus given profit-maximizing and fully informed behavior by the manufacturing sector, with outcomes without a bubble. I use the notation, \( \mu = \gamma (1 - \alpha + \epsilon) \),

**Proposition 4:** The optimal bubble will set \( \frac{\tilde{A}_0}{A_0} = \left( \frac{1 - \alpha + \epsilon}{\mu + (\mu + \alpha) + \epsilon \alpha - \epsilon} \right)^{\mu + (\alpha - \epsilon)} (1 - \theta \delta)^\mu \), and this will only increase surplus relative to the decentralized no bubble equilibrium if

\[
\left(1 + \frac{\epsilon}{(\sigma \mu + (\alpha - \epsilon))(1 - \alpha)}\right)^{\sigma \mu + (\alpha - \epsilon)} \left(1 + \frac{\epsilon}{(\mu + (\alpha - \epsilon))(1 - \alpha)}\right)^{-\mu + (\alpha - \epsilon)} < (1 - \theta \delta)^\mu.
\]

The proposition highlights how hard it is for a bubble to be socially optimal. The first part of the proposition notes the optimal bubble involves two terms. The first term
\[
\left(1 - \alpha + \epsilon \right) \left(1 - \alpha \right) + \epsilon \left(\mu + (\mu + \alpha) + \epsilon \alpha - \epsilon\right)
\]
pushes towards over-optimism to internalize the agglomeration externality. The second term \((1 - \theta \delta)^\mu\) is less than one. This term pushes towards pessimism because housing investment is actually less productive with a bubble because of the losses due to default. Unless these two terms multiply to be greater than one, there is no optimal building involving overbuilding. When they multiply to less than one, the “optimal bubble” is no bubble at all.

Yet even if the optimal bubble involves over-optimistic beliefs, it may still be worse than having no bubble. The positive effect of the bubble, internalizing the externality, must offset the harm from defaults. I now show what the model implies given reasonable parameter values, and ask what is the maximum value of \( \theta \delta \) that permits an “optimal bubble” to exist and to be welfare enhancing.

I continue to assume \( 1 - \alpha = .65 \) (labor’s share of output), \( \gamma = .3 \) (housing’s share of consumption), \( \sigma = .33 \) (land’s share of usable space) and \( \epsilon = .05 \) (the agglomeration effect).

The parameter \( \sigma \) reflects the National Association of Home Builders’ estimates that for much of the last twenty years, the costs of finished lots are approximately one-half the size of non-lot related construction cost.\(^5\) In this case, an optimal bubble only exists if \( \theta \delta < .315 \), which seems plausible in most settings. If, for example, \( \theta \delta = .2 \), then the optimal bubble will set \( \frac{\tilde{A}_0}{A_0} = 1.025 \), so expectations are not too far from reality.

\(^5\) In 2015, the ratio fell to one-third, presumably because building has become particularly concentrated in low-cost areas.
However, given these parameter values, the optimal bubble yields relatively little overbuilding benefit and unless $\theta \delta$ is less than .01, it is not optimal to have the bubble at all. The losses from defaults will overwhelm the benefits of a slightly larger housing stock. When agglomeration effects are modest, the model predicts that bubbles will be counter-productive given a reasonable range of parameter values.

The agglomeration effect of .05 is well in line with U.S. estimates, but it is at least possible that agglomeration effects can be much higher in some developing world contexts. Chauvin et al. (2016) found a density agglomeration effect (the coefficient of log earnings on log prefecture density) of .2 in China. The coefficient on prefecture population was much smaller. If $\epsilon = .2$, and the other parameter values stay the same, then the optimal bubble equals $1.326 * (1 - \theta \delta)^{2.55}$, and if $\theta \delta = .2$, this would set $\frac{\bar{a}}{a_0} = 1.25$. Consequently, the pre-default housing stock would be 50 percent larger and the post default housing stock would be 19 percent larger.

Yet even in the case of such massive agglomeration externalities, the optimal bubble will only be surplus enhancing if $\theta \delta < .17$. This constellation of parameters is not impossible, but it highlights that bubbles can only be welfare enhancing if the losses from default are modest and the agglomeration effects are extremely large.

What are reasonable values for $\theta \delta$? Typical estimates of foreclosures on ordinary apartments are quite high, and some estimates claim that almost fifty percent of the value of a home is lost through the foreclosure process. Ciochetti (1997) documents gross foreclosure recovery ratios, the ratio of value recovered relative to average loan balance, that average 82 percent. When total legal costs are included, the recovery ratio falls to 69 percent, but this also reflects reductions in market price relative to time of the loan. Another way of benchmarking these costs is to compare them with annual depreciation costs for housing which are typically thought to be about one percent per year. An alternative is to assume a fixed period of vacancy as a result of the transfer, perhaps six months to a year, which could mean that loss of five to ten percent of the value of the property if capitalization rates are ten percent.

Our calculations represent a thought experiment that assumes that the bubble gets building right, and there is little reason to be so confident. An actual bubble might over-build substantially, and consequently produce losses from both extra construction and from defaults. A bubble might not
lead to any extra building at all and in that case, the bubble does nothing but create losses. This might occur, for example, in places where supply restrictions are severe. I now turn to a more verbal discussion of the costs of real estate bubbles.

The fundamental asymmetry between the benefits of bubbles—internalizing agglomeration economies—and the costs of bubbles—defaults—is that the benefits are classic welfare triangles and the costs are rectangles. The gain from the bubble is to internalize an externality, and that generates benefits that are proportional to the change in the quantity times the gap between the price of housing and the true benefit of building. The cost of the bubble is to wipe out a proportion of the entire stock of housing being built.

The upsides of the bubble are limited because it only impacts the building stock, not the beliefs of the manufacturing firms at the point of hiring. This assumption is perhaps appropriate given this paper’s focus on real estate bubbles, but it is true that bubbly beliefs in the manufacturing firm would be more beneficial, especially when there are no defaults in manufacturing. If manufacturing capital was also wiped out in a crash, then the case for an export-industry bubble would also weaken.

*The Real and Financial Downsides of Bubbles*

In the model, a bubble created a “real sector” advantage—the production of extra housing—and a “financial sector” cost—the losses due to defaults. In reality, bubbles may cause costs in all three sectors. As discussed above, if agglomeration effects are modest, then even mild over-optimism can lead to too much building. A bubble can also lead to building too soon as well. These costs can be significant, but they are still likely to be smaller than the costs associated with financial disruptions. In a sense, all of these other costs are welfare triangles, while the costs from defaults can be rectangles.

This claim about rectangles and squares was certainly true in the previous model. All builders received a shock in their costs because prices fell and they had to default. The loss equals $\theta$ times all of the construction in existence. If there had been over-building, the cost from this over-building is a triangle. On the margin, the extra housing is just worth slightly less than its social value.
Does this logic about triangles and rectangles hold in a more general model or in the real world? In a world with linear supply and demand curves, the social cost of over-building should be equal to one-half times the amount of over-building times the distance between marginal cost of construction and marginal benefit. In the U.S. at least, this gap appears to be relatively small (Glaeser, 2013). Even after the bust in Las Vegas, prices didn’t fall far below construction costs and they seem to have recovered to roughly the costs of building new housing. My attempt to calibrate these quantities found them to be quite marginal relative to the larger wreckage of the economy.

The costs of default, however, can be extremely large and hit a large fraction of the population. They can spread beyond the real estate sector and cause the entire financial sector to freeze, which can have larger implications for industry as a whole. I chose not to model those effects above, but they certainly seem to exist.

In reality, price declines hit both builders and investors. The builders may represent a relatively small slice of the economy at any one time and some of them may be shielded by equity. Investors can represent a share of the stock of real estate, not just the flow, and consequently their financial difficulties can be extremely large. Their financial troubles then get sent through the system because banks fail to collect the full outstanding value of loans. As the banks get into trouble, the system as a whole freezes up. The losses from the financial side of a real estate bust are not Harberger triangles, but potentially a large implicit tax on the entire economy.

China could conceivably prove an exception to my claim that finance-related losses from real estate busts are far more severe than the costs of over-building. The over-building in Third and Fourth tier cities does represent a quite significant amount of economic activity. In 20 years, this building could be valued significantly less than today, at least if building continues. This poses the possibility that Chinese over-building will prove to have far larger economic consequences than the over-building of Las Vegas or Phoenix.

Both China and the U.S. share a feature that makes over-building much more likely than in Korea or Thailand: they have a large number of cities. When there is a single urban giant, then over-building typically means over-building that city. If demand for urbanization continues, then demand for this city will continue. In a country with many cities, it is quite possible that demand for any particular city will be low, even if the demand for urban space is strong. America has
robust demand for urban space in New York and San Francisco, but not in Detroit or Cleveland. The heterogeneity of China also means that it is possible that China is building up the wrong cities.

China’s large vacancy rates also create the possibility that China is building “too soon” rather than “too much.” I did not have costs of vacancy or technological change in the model described above, but building to leave vacant is surely sub-optimal. Yet Glaeser, Huang, Ma and Shleifer (2016) estimate vacancy rates of as much as one-fifth in Chinese cities. If depreciation runs at 1-2 percent per year and if vacancy lasts for 20 years, then the waste from over-building could run as high as 40 percent of the value of the construction.

These potential losses should also be framed against technological change in the construction industry. If building is improving, then waiting could have yielded advantages of being able to use newer technology. Naturally, it is also possible that rising labor costs could mean that future construction is more expensive, which will reduce the costs of building vacant homes today.

The view that the real costs of over-building in China could rival or exceed the financial costs of a real estate downturn also rests on assuming that those financial costs will be small. The resources of the public sector make it conceivable that the financial sector will remain capitalized even if real estate prices drop significantly. If we are sufficiently optimistic about the ability of the public sector to eliminate financial market distress, then China could be an exception. It could be the first place where over-building is more costly than financial distress.

While this is conceivable, it seems unlikely to me. If there is a Chinese bust, which is far from certain, it seems far more likely that it will fall in line with history. If that is the case, then the big risk is financial sector distress, not too many structures.

V. Policies Toward Real Estate Bubbles

At this point, I turn to a discussion about policies toward real estate booms. I separate out the policy discussion into two separate subsections. The first subsection deals with policies when a bust has already begun. This corresponds to the 2008 to 2010 period in the U.S., when financial
markets were falling and banks were facing insolvency, or at least illiquidity, problems. The second subsection deals with policies during a price boom. Broadly speaking, I will suggest that the right approach is to be soft on banks during the bust and tough during the boom.

I will not discuss other urban policies, but it is worth noting that if the cities adopt policies that lead builders to internalize agglomeration economies, then bubbles are always counter-productive. The only positive role for bubbles in the model was to counter a tendency to build too little. If that tendency is eliminated, then bubbles lose their upside.

Addressing a Financial Market Meltdown

At the point of a bust, when prices have begun to fall, the public sector has several possible policy approaches. They can try to artificially buoy prices and hope to ride out the storm. They can follow the tough path charted by former Treasury Secretary Andrew Mellon during the Great Depression and let the banks go bust. They can extend credit to the banks and try to avoid major financial market dislocation.

The first path is essentially impossible in the West. The U.S. government did not have the power to keep housing prices up in 2008. The downward trend was just too great. Some observers suggested that with easy enough credit, reasonable estimates of the impact of interest rates on prices suggest that this is unlikely. Indeed, interest rates did fall dramatically over the course of the great recession, with no observable impact on housing prices. Glaeser, Gottlieb and Gyourko (2012) estimate a semi-elasticity of prices with respect to interest rates of approximately six.

Consequently, a drop in interest rates of 100 basis points will lead to a nine percent increase in housing prices. If the logic of Himmelberg, Mayer and Sinai (2005) is correct, then this effect is likely to be weaker during times of low expected growth rates in future prices. Republican proposals to massively reduce interest rates through subsidies to homeowners were never likely to do much to reverse the enormous slump in housing prices.

In China, public capacity to boost prices is greater than in the U.S., because the Chinese government has far more ability to actually take housing off the market. Large scale public purchases are a far more direct and effective means of boosting prices than tinkering with the interest rate. Whereas, the U.S. government could not spend $600 billion dollars to buy two
million $300,000 homes, the Chinese government could do just that and more. If those homes were then used for social housing, this could remove excess supply and help keep prices afloat.

The big problem with this strategy is that if the government uses its purchasing power to set a price floor, and if that price floor is higher than construction costs, then builders will continue to supply new homes. Eventually, even the Chinese public sector’s ability to absorb excess stock would be overwhelmed by a flood of newly supplied housing. This public purchasing strategy can only work if it is coupled with some other intervention that will limit the supply of new housing. In China, this could be done by banning land sales for new construction for a period of five or ten years.

The costs of such an intervention would be significant. Buying millions of homes and then using them for new purposes will waste a great deal of the value of the real estate. Shutting down the construction sector will create large scale unemployment and reduce the benefits that will come from continued economic growth. While China could keep prices up, unlike the U.S., this strategy seems to carry costs that outweigh the benefits, as long as China takes other steps to reduce the financial sector dislocations from a real estate bust.

The U.S. didn’t really have the ability to keep housing prices high, but it did have a choice between supporting the banking sector or letting banks fail. The case for tough love was moral hazard. The pro-failure argument is that unless banks bear the costs of their mistakes then they will keep on making new mistakes. Andrew Mellon himself also believed that the failure of banks would lead to a benevolent process where bad banks fail and get replaced by good banks.

The opposing view is that the benefits of reducing moral hazard are vastly outweighed by costs of financial market chaos. Large scale bank failures can have horrendous consequences for the larger economy, as the Great Depression’s example makes particularly clear. I have little to add to the large financial literature that focuses on the downsides of banking failures (e.g., Reinhardt and Rogoff, 2009), but there are two points about the moral-hazard argument that are worth emphasizing.

Errors were made by individuals and errors can presumably be reduced if individuals suffer sharply when they make mistakes. This view lies at the heart of the economics of crime and punishment. Bank failures, however, are a poorly targeted means of punishing the bankers who
erred. Much of the costs of those failures will be paid by people who had little to do with the mistakes of the boom. The bankers themselves who made the biggest mistakes are likely to go into comfortable retirement despite the institutional failures. Good punishments tightly target malefactors. Bank failures aren’t tightly targeted at all.

The U.S. legal system makes it impossible to impose draconian punishments on over-optimistic bankers, but elsewhere such punishments are more conceivable. It is particularly easy to imagine tough but fair punishments towards public officials who unwisely encouraged over-development. In systems where targeted punishments are possible, it makes particularly little sense to risk financial chaos by getting tough on troubled financial institutions.

*Policy During the Boom*

While supporting the financial sector during a bust seems like an appropriate path to reduce system-wide risks, tougher government policies seem more appropriate during a boom. When a financial bust occurs, then failed banking institutions impose costs on everyone. Most directly, the government will have to pay the costs of recapitalizing these institutions. The externalities associated with the bust make it appropriate to impose tougher regulations during a boom.

Both regulations and Pigouvian taxes are potential tools to internalize the external costs of real estate speculations by financial institutions, such as banks. The standard regulatory response is to mandate minimum capital levels. The capital cushion makes it less likely that the financial institution will eventually default on its debts and spread failure throughout the system. Basel III requirements focus on the ratio of capital to risk-weighted assets, and real estate impacts the risk-weighting of assets. During the boom, real estate-related assets were being treated as relatively safe, while in reality those investments had considerable downside risk.

In the U.S., the mean reversion of housing prices is well established empirically. Consequently, during a boom the risks of future downward movement are considerable and real estate becomes far riskier. Housing price mean reversion implies that real estate-related investments are riskier when prices have recently risen. Regulations could treat them as such. An alternative is to focus on the gap between prices and construction and to increasingly treat real estate investments as being riskier when that gap widens. Either approach would have pushed banks to add more capital during the recent boom, at least if they had considerable exposure to real estate.
These policies are not free. They would induce banks to hold less real estate-related capital, which could put a damper on the real estate market. But leaning against a booming market may be a perfectly reasonable role for financial regulators.

A secondary approach is to impose Pigouvian taxes on banks, almost assuredly in addition to maintaining capital requirements. These payments are essentially insurance payments against a bailout and again they need to be calibrated against the risks of needing a bailout. The payment needs to rise with the risk of a bank’s portfolio. Consequently, the insurance fees should be higher when banks are holding real-estate related assets during a boom. As in the case of risk-weighting assets, the downside risk can be measured either by assuming mean reversion in prices or by using the gap between prices and construction.

There are other public policies that encourage booms and encourage financial participations in booms, that are also worth rethinking given the risks of financial distress. Recently, the Chinese government has responded to the housing boom by actually encouraging more, rather than less, lending to would-be homebuyers. Arguably, the U.S. followed a similar strategy in the years before 2006 to create an “ownership society.” These pro-lending strategies may temporarily boost prices, but they also increase the exposure of the banking system to real estate downturns. The goal of policies during the boom should be to reduce the exposure of the system, not to amplify the risks.

In particular countries, other policies may inadvertently exacerbate the impacts of real estate bubbles. For example, in China, the incentive of local governments to fund their operations through land sales is thought by many to artificially increase the amount of real estate development. One natural alternative is for these governments to switch to a standard annual property tax system as a means of revenue generation. A related policy change would be to ensure that there are better safeguards on abuse of eminent domain. Better land value assessment could achieve that end, especially if it was enforced at the national level. Alternatively, current residents could be given greater ability to reject offers collectively. While giving every resident a veto is an invitation to hold-out problems, requiring super-majority approval is much less likely to engender abuse.

Finally, the Chinese system of rewarding local officials for increasing G.D.P. growth may deserve reappraisal. Encouraging large amounts of new construction is a simple means of
increasing G.D.P. New construction can also overstate growth if the apartments are valued at artificially high prices. One potential reform would be to treat G.D.P. in the construction sector as distinct from export-related G.D.P. Providing stronger rewards for exported-related growth than for construction growth may work against any tendency to over-build.

VI. Conclusion

Real estate bubbles have been pervasive throughout American history. Recent Asian experience has also illustrated how a boom and bust cycle in real estate can set off greater financial distress. China has experienced a great boom, but it is still unclear whether there will also be a great bust.

The tendency of speculation to center on real estate reflects, partially, the flexible nature of built space and the strong history of property law in the west. Lenders often see real estate as safer collateral than industrial investment. Consequently, passive capital is more likely to flow into real estate and the enthusiasm of passive capital is more likely to create real estate bubbles.

Both U.S. and Asian history suggests that optimistic beliefs about real estate can have upsides. City-building benefits from enthusiasm about property values. When there are positive externalities from urban bigness, then a real estate bubble can create social benefits. Over-optimism can lead builders to deliver the socially optimal level of space, which they would not produce given more accurate beliefs.

Yet the financial costs of real estate bubbles are likely to overwhelm any such benefits. When foreclosures destroy value, then real estate bubbles have widespread consequences. In some well-known cases, in the U.S. in 2007 and in the Asian financial crisis, banking troubles began in real estate and spread to the wider economy. The potential downside of real estate speculation makes it an important topic for policy-makers.

The policy discussion in this paper supported the common view that supporting the financial system during a boost was likely to generate more benefits than costs. Bailing out banks does create moral hazard, but letting banks fail is an extremely socially expensive way to encourage discipline. Individual bankers pay only a small fraction of the actual costs imposed by a failure.
But if the public sector and the public are going to pay significant costs in the event of a bust, then pro-active policies during the boom may be beneficial. In particular, real estate is particularly risky after a large run-up in prices. Consequently, it may be appropriate for regulations to treat real-estate investments as being particularly risky during those periods. It may also be appropriate to impose Pigouvian taxes on financial institutions that invest heavily in real estate during boom periods.

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Appendix: Proofs of Propositions

Proof of Proposition 1: In all equilibria, the wage and housing price and population will be a function of the quantities of effective housing space and capital. If $H_S$ denotes housing space at
the start of period one (after default losses) and \( K_M \) denotes effective capital at the start of period one (again after default losses, then \( w = (1 - \alpha)AK_M^\alpha N^{-\alpha} \) and \( P_H S = \gamma(1 - \alpha)AK_M^\alpha N^{-\alpha} \).

Using \( P_H = \left(\frac{w}{w_0}\right)^{\frac{1}{\alpha + \gamma - \alpha \gamma}} \), we have \( H_S = \gamma(1 - \alpha)AK_M^\alpha N^{-\alpha} \) or

\[
((1 - \alpha)AK_M^\alpha)^{\frac{1 - \gamma}{\alpha + \gamma - \alpha \gamma}} (\frac{H_S}{w_0})^{\frac{\gamma}{\alpha + \gamma - \alpha \gamma}} w^{\alpha + \gamma - \alpha \gamma} = N, \quad ((1 - \alpha)A)^{\frac{\gamma}{\alpha + \gamma - \alpha \gamma}} (\gamma K_M)_{H_S}^{\frac{\gamma}{\alpha + \gamma - \alpha \gamma}} w^{\alpha + \gamma - \alpha \gamma} = w
\]

and \((1 - \alpha)A)^{\frac{1}{\alpha + \gamma - \alpha \gamma}} (\gamma K_M)_{H_S}^{\frac{\alpha}{\alpha + \gamma - \alpha \gamma}} w^{\alpha + \gamma - \alpha \gamma} = P_H.\]

In real estate, the supply of housing services satisfies: \( H_S = \frac{1 - S_P \rho \theta \delta}{1 - \sigma} M^\sigma K_H^{1 - \sigma} \), where \( S_P \) refers to the share of housing-related capital that is passive and \( K_H \) is the total stock of housing-related capital (before default related losses).

This implies that

\[
((1 - \alpha)A)^{\frac{\gamma}{\alpha + \gamma - \alpha \gamma}} (\frac{1 - \sigma}{1 - \sigma})^\gamma K_M^{\frac{\alpha}{\alpha + \gamma - \alpha \gamma}} M^{\frac{\sigma}{\alpha + \gamma - \alpha \gamma}} w^{\frac{\alpha + \gamma - \alpha \gamma}{\gamma}} - (1 - \sigma)^{\frac{\gamma}{\alpha + \gamma - \alpha \gamma}} K_H^{\frac{\alpha}{\alpha + \gamma - \alpha \gamma}} w^{\frac{\alpha + \gamma - \alpha \gamma}{\gamma}} = w
\]

and

\[
((1 - \alpha)A)^{\frac{1}{\alpha + \gamma - \alpha \gamma}} (\frac{1 - \sigma}{1 - \sigma})^\gamma K_M^{\frac{\alpha}{\alpha + \gamma - \alpha \gamma}} M^{\frac{\sigma}{\alpha + \gamma - \alpha \gamma}} w^{\frac{\alpha + \gamma - \alpha \gamma}{\gamma}} - (1 - \sigma)^{\frac{1}{\alpha + \gamma - \alpha \gamma}} K_H^{\frac{\alpha}{\alpha + \gamma - \alpha \gamma}} w^{\frac{\alpha + \gamma - \alpha \gamma}{\gamma}} = P_H.\]

The returns to one unit of active capital in manufacturing are

\[
\frac{\alpha}{1 - \alpha} (1 - \alpha)A^{\frac{\sigma(1 - \alpha)}{1 + \alpha + \gamma - \alpha \gamma}} M^{\frac{(1 - \sigma)(1 - \alpha)}{1 + \alpha + \gamma - \alpha \gamma}} K_H^{\frac{\alpha}{1 + \alpha + \gamma - \alpha \gamma}} w^{\frac{\alpha + \gamma - \alpha \gamma}{\gamma}}
\]

and the returns to passive capital would be \((1 - \rho \theta \delta)\) times that amount.

The returns to active capital in real estate are

\[
((1 - \alpha)A)^{\frac{\gamma}{\alpha + \gamma - \alpha \gamma}} (\frac{1 - \sigma}{1 - \sigma})^\gamma K_M^{\frac{\alpha}{\alpha + \gamma - \alpha \gamma}} M^{\frac{\sigma}{\alpha + \gamma - \alpha \gamma}} w^{\frac{\alpha + \gamma - \alpha \gamma}{\gamma}} - (1 - \sigma)^{\frac{\gamma}{\alpha + \gamma - \alpha \gamma}} K_H^{\frac{\alpha}{\alpha + \gamma - \alpha \gamma}} w^{\frac{\alpha + \gamma - \alpha \gamma}{\gamma}} = w
\]

and the returns to passive capital are \((1 - \rho \theta \delta)\) times that amount.

As \( \theta < 1 \), it is impossible to have both types of capital be indifferent between the two sectors. Active capital can be indifferent, in which case, passive capital will strictly prefer real estate. Passive capital can be indifference, and in that case, active capital will strictly prefer manufacturing. Consequently, it is only possible to have one type of capital investing in both sectors at once.

This logic implies that there are four possible equilibrium outcomes: only active capital is used in both sectors and in that case \( K_M + K_H = I \) and \( S_P = 0 \), active capital is used in both sectors and passive capital is also in real estate so \( K_M + K_H > I \), active capital is used in manufacturing and passive capital is used in real estate and in that case \( K_M = I \), \( S_P = 1 \), and active capital is used in manufacturing and passive capital is used in both industries and in that case \( K_M > I \).
**Equilibrium 1: Only Active Capital**

In this case, the returns to active capital investment must be equalized between the two sectors, which implies that \( \alpha K_H = (1 - \alpha)(1 - \sigma)\gamma K_M \), and consequently that \( K_M = \frac{\alpha}{\alpha + (1 - \alpha)(1 - \sigma)\gamma} \bar{I} \) and \( K_H = \frac{(1 - \alpha)(1 - \sigma)\gamma}{\alpha + (1 - \alpha)(1 - \sigma)\gamma} \bar{I} \). The returns to passive capital investing in real estate would be:

\[
(1 - \rho \theta \delta) A^{\alpha + \gamma - \alpha \gamma} \alpha^{\alpha + \gamma - \alpha \gamma} \left( \frac{1 - \alpha}{w_0} \right)^{1 - \alpha} \left( \frac{\alpha + (1 - \alpha)(1 - \sigma)\gamma}{(1 - \alpha)(1 - \sigma)\gamma} M \right)^{\frac{\sigma (1 - \alpha)}{\alpha + \gamma - \alpha \gamma}}
\]

For this to be an equilibrium, this must be less than one, which implies that

\[
I > \frac{\alpha + (1 - \alpha)(1 - \sigma)\gamma}{(1 - \alpha)(1 - \sigma)\gamma} M \left( 1 - \rho \theta \delta \alpha^{\alpha + \gamma - \alpha \gamma} A \alpha^{\alpha} \left( \frac{1 - \alpha}{w_0} \right)^{1 - \alpha} \right)^{\frac{1}{\sigma (1 - \alpha)}}
\]

I let the value of \( \bar{I} \) at which this hold with equality denoted \( \bar{I}_1 \), differentiation then gives us that \( \bar{I}_1 \) is rising with \( A \) and \( M \) and falling with \( \rho, \theta, \delta \) and \( w_0 \).

**Equilibrium 2: Active Capital in Both Sectors, Passive Capital in Real Estate**

Once again, the returns to active capital investment must be equalized between the two sectors, which now implies that \( \alpha (1 - S_P \rho \theta \delta) K_H = (1 - \alpha)(1 - \sigma)\gamma K_M \), and the returns to passive capital investing in real estate must equal one. These two equalities imply that:

\[
K_H = M \left( (1 - \rho \theta \delta)^{\alpha + \gamma - \alpha \gamma} A \alpha^{\alpha} \left( \frac{1 - \alpha}{w_0} \right)^{1 - \alpha} \right)^{\frac{1}{\sigma (1 - \alpha)}}
\]

One condition for this equilibrium to exist is that this quantity is greater than \( \frac{(1 - \alpha)(1 - \sigma)\gamma}{\alpha + (1 - \alpha)(1 - \sigma)\gamma} \bar{I} \), which is equivalent to requiring that \( \bar{I}_1 > \bar{I} \). The second requirement is that the returns be equalized for the active investors, without any passive capital investing in manufacturing. The extreme of this range equilibrium occurs when there is essentially no active investment in real estate, but the returns are still equalized and in that case, \( S_P = 1, K_M = \bar{I}, \) and this defines \( \bar{I}_2 \) which satisfies:

\[
\frac{\alpha (1 - \rho \theta \delta)^{1 + \frac{1}{\sigma} \frac{\alpha}{\sigma (1 - \alpha)}} M \left( 1 - \alpha \right)^{\frac{1}{\sigma}} \left( \frac{1}{w_0} \right)^{\frac{1}{\sigma}} A^{\frac{\alpha}{\sigma (1 - \alpha)}} \left( \frac{\alpha}{\sigma (1 - \alpha)} \right) = \bar{I}_2
\]

As long as \( \bar{I} > \bar{I}_2 \), then the returns in the real estate sector (with only active investment) will be low enough so that the returns can be equalized across the two sectors, but when \( \bar{I} < \bar{I}_2 \), then the returns will be higher in real estate than in manufacturing for the active investors and hence the two types of capital will specialize. As \( \alpha (1 - \rho \theta \delta) < \alpha + (1 - \alpha)(1 - \sigma)\gamma \), \( \bar{I}_2 < \bar{I}_1 \).

Differentiation also gives us that \( \bar{I}_2 \) is rising with \( A \) and \( M \) and falling with \( \rho, \theta, \delta \) and \( w_0 \).
Equilibrium 3: Segregation of Passive and Active Capital

When $\bar{I} < \bar{I}_2$, then the returns for active capital cannot be equalized across the two sectors. When $\bar{I}$ is still relatively abundant, then only active capital invests in manufacturing and only passive capital invests in real estate.

The return to passive capital in real estate is 1, which implies that

$$\left( (1 - \rho \theta \delta)^{\gamma(1-\alpha)} (1 - \alpha) AM^{\sigma \gamma (1-\alpha)} \left( (1 - \sigma) \bar{I} \right)^{\alpha} \right)^{\frac{1}{\alpha + \sigma \gamma - \alpha \sigma \gamma}} = K_H,$$

and this is increasing in $M$, $\bar{I}$ and $A$ and decreasing in $w_0$, $\rho$, $\theta$ and $\delta$.

This implies that the returns to active capital in manufacturing equal:

$$\alpha \left( (1 - \rho \theta \delta)^{\gamma(1-\alpha)} (1 - \alpha) (1-\alpha)(1-\gamma)AM^{\sigma \gamma (1-\alpha)} \left( (1 - \sigma) \bar{I} \right)^{\sigma \gamma (\alpha-1)} \right)^{\frac{1}{\alpha + \sigma \gamma - \alpha \sigma \gamma}}$$

This must be higher than $\frac{1}{1-\rho \delta}$, which implies that $\bar{I} < \bar{I}_2$ and it must be lower than $\frac{1}{1-\rho \delta}$ (so that passive capital doesn’t want to invest in manufacturing) which implies that

$$\frac{\alpha \left( 1-\rho \delta \right)^{\gamma \left(1-\alpha\right)} \left( 1-\alpha \right) \left( 1-\gamma \right) AM^{\sigma \gamma \left(1-\alpha\right)} \left( \left( 1-\sigma \right) \bar{I} \right)^{\sigma \gamma \left(\alpha-1\right)} \right)^{\frac{1}{\alpha + \sigma \gamma - \alpha \sigma \gamma}} < \bar{I},$$

which means that $\bar{I}$ must be greater than

$$\frac{\left( 1-\rho \delta \right)^{\gamma \left(1-\alpha\right)} \left( 1-\alpha \right) \left( 1-\gamma \right) M \alpha}{\left( 1-\sigma \right) \left( 1-\alpha \right) \bar{I}} = \bar{I}_2.$$ 

Differentiation gives us that $\bar{I}_3$ is rising with $A$ and $M$ and falling with $\rho$, $\theta$, $\delta$ and $w_0$.

Equilibrium 4: Passive Capital in Both Sectors

In this region, passive capital is active in both sectors and hence the expected returns to investing in both sectors for passive capital must equal one. Active capital will only be in manufacturing.

Equality of returns implies that

$$\frac{\alpha \left( 1-\rho \delta \right)^{\gamma \left(1-\alpha\right)} \left( 1-\alpha \right) \left( 1-\gamma \right) AM^{\sigma \gamma \left(1-\alpha\right)} \left( \left( 1-\sigma \right) \bar{I} \right)^{\sigma \gamma \left(\alpha-1\right)} \right)^{\frac{1}{\alpha + \sigma \gamma - \alpha \sigma \gamma}}}{\left( 1-\sigma \right) \left( 1-\alpha \right) \bar{I}} = K_M$$

and

$$(1 - \rho \theta \delta)^{\gamma(1-\alpha)} (1 - \alpha) (1-\alpha)(1-\gamma)AM^{\sigma \gamma (1-\alpha)} \left( (1 - \sigma) \bar{I} \right)^{\sigma \gamma (\alpha-1)} \right)^{\frac{1}{\alpha + \sigma \gamma - \alpha \sigma \gamma}} < K_H.$$ 

Hence both types of capita are rising with $A$ and $M$ and falling with $\rho$, $\theta$, $\delta$ and $w_0$. The value of $K_M$ must be greater than $\bar{I}$ in this equilibrium hence $\bar{I}$ must be less than $\bar{I}_3$.

I have also shown that whenever there are is passive capital investing in a sector, then the investment in that sector is rising $A$ and $M$ and falling with $\rho$, $\theta$, $\delta$ and $w_0$. Since the population level is increasing in both types of capital and also rising with $A$ and falling with $w_0$, then it follows that the population is always rising with $A$ and falling with $w_0$, and that when there is passive investment in either sector, population is rising $A$ and $M$ and falling with $\rho$, $\theta$, $\delta$ and $w_0$.

Proof of Proposition 2: In this case, I simply replace $A$ with $\hat{A}$ in all the formulas and the proposition immediately follows.
Proof of Proposition 3: I assume that the stock of manufacturing capital is fixed at $I$, and that employment in manufacturing will be competitively determined and based on the true level of productivity. I will allow potentially different errors at the stage of building and at the stage of hiring, so that builders believe that $A_0 = \hat{A}_0$ and manufacturers believe that $A_M = \hat{A}_M$. Builders also expect that manufacturers will have a value of $\hat{A}_M$ equal to $\hat{A}_0$.

Consequently, if the supply of housing is $H_S$, the city population will be

\[
\left(1 - \alpha\right)\hat{A}_M I^{\alpha} \frac{1 - \gamma}{\gamma + (1 - \gamma)(\alpha - \epsilon)} \left(\frac{H_S}{\gamma}\right)^{-\gamma(1 - \gamma)(\alpha - \epsilon)} w_0^{\frac{-1}{\gamma + (1 - \gamma)(\alpha - \epsilon)}}, \]

the wage will be

\[
\left(1 - \alpha\right)\hat{A}_M I^{\alpha} \frac{\gamma}{\gamma + (1 - \gamma)(\alpha - \epsilon)} \left(\frac{H_S}{\gamma}\right)^{-\gamma(1 - \gamma)(\alpha - \epsilon)} w_0^{\frac{\alpha - \epsilon}{\gamma + (1 - \gamma)(\alpha - \epsilon)}}, \]

and the price of housing will be

\[
\left(1 - \alpha\right)\hat{A}_M I^{\alpha} \frac{1}{\gamma + (1 - \gamma)(\alpha - \epsilon)} \left(\frac{H_S}{\gamma}\right)^{-\gamma(1 - \gamma)(\alpha - \epsilon)} w_0^{\frac{\alpha - \epsilon - 1}{\gamma + (1 - \gamma)(\alpha - \epsilon)}}. \] This implies that capital invested in housing will equal \(\left(1 - \alpha\right)\hat{A}_0 I^{\alpha - \epsilon} \frac{1}{w_0^{1 + \epsilon - \alpha}} \frac{\sigma^\gamma + (1 - \sigma)^\gamma(\alpha - \epsilon)}{\sigma^\gamma + (1 - \sigma)^\gamma(\alpha - \epsilon)} \frac{\sigma^\gamma(1 - \epsilon)(\alpha - \epsilon)}{M^\sigma(1 - \gamma)(\alpha - \epsilon)}\).

The total supply of housing will equal

\[
\left(1 - \alpha\right)\hat{A}_0 I^{\alpha - \epsilon} \frac{1 - \sigma}{\sigma^\gamma + (1 - \sigma)^\gamma(\alpha - \epsilon)} \frac{1}{w_0^{1 + \epsilon - \alpha}} \frac{\sigma^\gamma + (1 - \sigma)^\gamma(\alpha - \epsilon)}{\sigma^\gamma + (1 - \sigma)^\gamma(\alpha - \epsilon)} \frac{\sigma^\gamma(1 - \epsilon)(\alpha - \epsilon)}{M^\sigma(1 - \gamma)(\alpha - \epsilon)} \frac{1}{(1 - \sigma)^\gamma}. \]

The population level will equal

\[
N = \left(\frac{M}{(1 - \sigma)\gamma}\right)^{\sigma^\gamma} \left(\frac{1 - \alpha\gamma}{\gamma + (1 - \gamma)(\alpha - \epsilon)} \hat{A}_0 \right) \frac{1}{w_0} \frac{\sigma^\gamma(1 - \gamma)(\alpha - \epsilon)}{\sigma^\gamma + (1 - \sigma)^\gamma(\alpha - \epsilon)} \frac{\sigma^\gamma(1 - \epsilon)(\alpha - \epsilon)}{M^\sigma(1 - \gamma)(\alpha - \epsilon)} \frac{1}{(1 - \sigma)^\gamma}. \]

Maximizing social surplus involves choosing $N$ and $K_S$ and a level of consumption for workers (denoted $c$) to maximize: $\hat{A} I^{\alpha} N^{1 + \epsilon - \alpha} - K_H - Nc$ subject to the constraint that workers receive their outside option or $c^{1 - \gamma_h(\gamma)(1 - \gamma)} = w_0$, which implies that $c = (1 - \gamma)\frac{\gamma}{1 - \gamma} w_0^{1 - \gamma} \left(\frac{H_S}{N}\right)^{1 - \gamma}$.

Hence surplus maximization requires that maximizing:

\[
A_0 I^{\alpha} N^{1 + \epsilon - \alpha} - K_H - (1 - \gamma)\frac{\gamma}{1 - \gamma} N^{1 - \gamma} w_0^{1 - \gamma} \left(\frac{M^\sigma K_H^{1 - \sigma}}{(1 - \sigma)^\gamma}\right)^{1 - \gamma}, \]

which implies that $(1 + \epsilon - \alpha) A_0 I^{\alpha} N^{\epsilon - \alpha} = \gamma^{1 - \gamma} N^{1 - \gamma} w_0^{1 - \gamma} \left(\frac{M^\sigma K_H^{1 - \sigma}}{(1 - \sigma)^\gamma}\right)^{1 - \gamma}$, and $1 = \frac{1}{K_H} N^{1 - \gamma} w_0^{1 - \gamma} \left(\frac{M^\sigma K_H^{1 - \sigma}}{(1 - \sigma)^\gamma}\right)^{1 - \gamma}$. In the
absence of externalities, this outcome can be decentralized with \( P_H = \gamma^{1-\gamma}N^{1-\gamma}w_0^{1-\gamma} \left( \frac{M^\sigma K^1 H^{1-\sigma}}{(1-\sigma)} \right)^{-1} \), which then implies an investment decision of \( P_H M^\sigma K_H^{1-\sigma} = 1 \), which then implies \( \gamma^{1-\gamma}(1-\sigma) N^{1-\gamma}w_0^{1-\gamma} \left( \frac{M^\sigma K_H^{1-\sigma}}{(1-\sigma)} \right)^{-1} = 1 \), which is the same as the first best. That value of \( P_H \) implies that \( P_H^\gamma w_0 = w \), which implies a hiring first order condition of \( (1-\alpha)\tilde{A}^\alpha N^{-\alpha} = \gamma^{1-\gamma}N^{1-\gamma}w_0^{1-\gamma} \left( \frac{M^\sigma K_H^{1-\sigma}}{(1-\sigma)} \right)^{-1} \), which is also the same as the first best.

The surplus maximizing population level will equal

\[
N = \left( (1+\epsilon-\alpha)A_0 I \right)^{\frac{1-\sigma\gamma}{\sigma\gamma+(1-\sigma\gamma)(\alpha-\epsilon)}} \left( \frac{M^{\sigma\gamma+(1-\sigma\gamma)(\alpha-\epsilon)}}{(1-\sigma)\gamma^{\sigma\gamma+(1-\sigma\gamma)(\alpha-\epsilon)}} \right)^{\frac{1}{\sigma\gamma+(1-\sigma\gamma)(\alpha-\epsilon)}} \left( \frac{w_0^{\sigma\gamma+(1-\sigma\gamma)(\alpha-\epsilon)}}{1+\gamma^{\alpha-\gamma}} \right)^{\frac{1-\gamma}{\sigma\gamma+(1-\sigma\gamma)(\alpha-\epsilon)}} w_0^{-\frac{1}{\sigma\gamma+(1-\sigma\gamma)(\alpha-\epsilon)}} ,
\]

and the surplus maximizing capital invested in housing will be

\[
\left( \frac{1+\epsilon-\alpha}{1-\alpha} A_0 I \right)^{\frac{1-\sigma\gamma}{\sigma\gamma+(1-\gamma)(\alpha-\epsilon)}} \left( \frac{M^{\sigma\gamma+(1-\gamma)(\alpha-\epsilon)}}{(1-\sigma)\gamma^{\sigma\gamma+(1-\gamma)(\alpha-\epsilon)}} \right)^{\frac{1}{\sigma\gamma+(1-\gamma)(\alpha-\epsilon)}} \left( \frac{w_0^{\sigma\gamma+(1-\gamma)(\alpha-\epsilon)}}{1+\gamma^{\alpha-\gamma}} \right)^{\frac{1-\gamma}{\sigma\gamma+(1-\gamma)(\alpha-\epsilon)}} \left( \frac{1}{\sigma\gamma+(1-\gamma)(\alpha-\epsilon)} \right)^{\frac{1}{\sigma\gamma+(1-\gamma)(\alpha-\epsilon)}} ,
\]

As the surplus maximizing capital invested in housing is equal to \( \left( \frac{1+\epsilon-\alpha}{1-\alpha} A_0 I \right)^{\frac{1-\gamma}{\sigma\gamma+(1-\gamma)(\alpha-\epsilon)}} \) times the decentralized housing stock, then if \( \tilde{A}_0 = \frac{1+\epsilon-\alpha}{1-\alpha} A_0 \), the socially optimal level of housing will be built. That level of error can also produce the socially optimal level of population if \( \tilde{A}_M = \tilde{A}_0 \), but if \( \tilde{A}_M < \tilde{A}_0 \), then the housing stock will be at the surplus-maximizing level, but the level of population will be too small.

If manufacturing employment and the property market are decentralized, with firm beliefs equaling \( \tilde{A}_M \), then total surplus will equal

\[
\frac{A_0 - (1-\gamma)(1-\alpha)\tilde{A}_M}{(1-\alpha)\tilde{A}_M \gamma} \left( \frac{1-\gamma}{\sigma\gamma+(1-\gamma)(\alpha-\epsilon)} \right)^{\frac{1}{\sigma\gamma+(1-\gamma)(\alpha-\epsilon)}} \left( \frac{w_0^{\sigma\gamma+(1-\gamma)(\alpha-\epsilon)}}{1+\gamma^{\alpha-\gamma}} \right)^{\frac{1}{\sigma\gamma+(1-\gamma)(\alpha-\epsilon)}} K_H^{\gamma+(1-\gamma)(\alpha-\epsilon)} - K_H ,
\]

and surplus maximizing housing capital will satisfy
\[
\left(\frac{\left(A_0 - (1-\gamma)(1-\alpha)\hat{A}_M\right)(1+\epsilon - \alpha)}{(1-\alpha)(\gamma + (1-\gamma)(\alpha - \epsilon))\hat{A}_M}\right)^{\frac{\gamma + (1-\gamma)(\alpha - \epsilon)}{\sigma \gamma + (1-\gamma)(\alpha - \epsilon)}} \left(\frac{(1-\alpha)(\gamma + (1-\gamma)(\alpha - \epsilon))\hat{A}_M}{w_0^{1+\epsilon - \alpha}}\right)^{\frac{1}{\sigma \gamma + (1-\gamma)(\alpha - \epsilon)}} = K_H.
\]

If \(\hat{A}_M = A_0\), then constrained optimal investment will equal
\[
\left(1 + \frac{\epsilon}{(1-\alpha)(\gamma + (1-\gamma)(\alpha - \epsilon))}\right)^{\gamma + (1-\gamma)(\alpha - \epsilon)} \left(\frac{A_0}{\hat{A}_0}\right)^{\frac{1}{\alpha \gamma + (1-\gamma)(\alpha - \epsilon)}}
\]
times decentralized investment. Consequently, the optimal bubble is
\[
\frac{\hat{A}_0}{A_0} = \left(1 + \frac{\epsilon}{(1-\alpha)(\gamma + (1-\gamma)(\alpha - \epsilon))}\right)^{\gamma + (1-\gamma)(\alpha - \epsilon)} < \frac{1 + \epsilon - \alpha}{1 - \alpha}.
\]
The last inequality follows because \((1 + \frac{\gamma X}{X})\) is always increasing with \(X\) when \(X > 0, A > 0\).

**Proof of Proposition 4:**

The surplus without a bubble equals
\[
\left(\frac{\alpha(1-\sigma \gamma) + \sigma \gamma}{(1-\alpha)(\gamma + (1-\gamma)(\alpha - \epsilon))}\right)^{\gamma + (1-\gamma)(\alpha - \epsilon)} \left(\frac{1-\alpha)\hat{A}_M \sigma \gamma (1+\epsilon - \alpha)}{w_0^{1+\epsilon - \alpha}}\right)^{\frac{1}{\gamma + (1-\gamma)(\alpha - \epsilon)}}.
\]

In the case of any bubble, housing supply is reduced to \(1 - \theta \delta\) times the housing stock because of defaults and foreclosures. Even the slightest bubble causes this loss. As discussed in the text, I assume that manufacturers and workers actually observe \(A_0\), which implies that total surplus will equal
\[
\left(\frac{\alpha + \gamma - \alpha \gamma}{(1-\alpha)(\gamma + (1-\gamma)(\alpha - \epsilon))}\right)^{\gamma + (1-\gamma)(\alpha - \epsilon)} \left(\frac{1-\alpha)\hat{A}_M \sigma \gamma (1+\epsilon - \alpha)}{w_0^{1+\epsilon - \alpha}(\gamma + (1-\gamma)(\alpha - \epsilon))\gamma + (1-\gamma)(\alpha - \epsilon)}\right)^{\frac{1}{\gamma + (1-\gamma)(\alpha - \epsilon)}}
\]
and surplus maximizing housing capital will equal
\[
\left(\frac{\alpha(1-\sigma \gamma) + \sigma \gamma}{(1-\alpha)(\gamma + (1-\gamma)(\alpha - \epsilon))}\right)^{\gamma + (1-\gamma)(\alpha - \epsilon)} \left(1 - \theta \delta\right)^{\gamma + (1-\gamma)(\alpha - \epsilon)} - K_H.
\]

Decentralized investment will equal optimal investment if
\[
\frac{\hat{A}_0}{A_0} = \left(\frac{\alpha + \gamma - \alpha \gamma}{(1-\alpha)(\gamma + (1-\gamma)(\alpha - \epsilon))}\right)^{\gamma + (1-\gamma)(\alpha - \epsilon)} \left(1 - \theta \delta\right)^{\gamma + (1-\gamma)(\alpha - \epsilon)}.
\]

This will only be greater than one if
\[
\left(\frac{\alpha(1-\alpha)(\gamma + (1-\gamma)(\alpha - \epsilon))}{(\alpha + \gamma - \alpha \gamma)(1+\epsilon - \alpha)}\right)^{\gamma + (1-\gamma)(\alpha - \epsilon)} < 1 - \theta \delta.
\]

The term \(\frac{(1-\alpha)(\gamma + (1-\gamma)(\alpha - \epsilon))}{(\alpha + \gamma - \alpha \gamma)(1+\epsilon - \alpha)} = 1 - \frac{\epsilon}{(\alpha + \gamma - \alpha \gamma)(1+\epsilon - \alpha)}, \) so if \(\theta \delta\) is low enough, a bubble will actually increase the housing stock.
Assuming that this condition is met so that any over-optimism is desirable, then the total surplus given the optimal bubble equals 

$$\frac{\sigma y + (1 - \sigma y)(\alpha - \epsilon)}{1 - \sigma y(1 + \epsilon - \alpha)}$$

times the capital stock.

This will be higher than the decentralized surplus without a bubble if and only if

$$\frac{(\alpha(1 - \sigma y) + \sigma y)(1 + \epsilon - \alpha)}{\sigma y + (1 - \sigma y)(\alpha - \epsilon)(1 - \alpha)} \frac{(1 - \alpha)(y + (1 - y)(\alpha - \epsilon))}{(\alpha + y - \alpha y)(1 + \epsilon - \alpha)} < 1 - \theta \delta.$$

The term

$$\frac{(\alpha(1 - \sigma y) + \sigma y)(1 + \epsilon - \alpha)}{\sigma y + (1 - \sigma y)(\alpha - \epsilon)(1 - \alpha)} = 1 + \frac{\epsilon}{(\sigma y + (1 - \sigma y)(\alpha - \epsilon)(1 - \alpha)},$$

so the cutoff required for a bubble to be welfare improving is higher than the cutoff for a bubble to lead to more real estate investment.

Using the notation

$$\mu = y(1 + \epsilon - \alpha),$$

the condition can be rewritten as:

$$\left(1 + \frac{\epsilon}{(\sigma \mu + (\alpha - \epsilon))(1 - \alpha)}\right)^{\sigma y + (1 - \sigma y)(\alpha - \epsilon)} \left(1 + \frac{\epsilon}{(\mu + (\alpha - \epsilon))(1 - \alpha)}\right)^{-\mu - (\alpha - \epsilon)} < (1 - \theta \delta)^\mu.$$ 

As the function

$$\left(1 + \frac{\epsilon}{x(1 - \alpha)}\right)^x$$

is increasing with x for x>0,

$$\left(1 + \frac{\epsilon}{(1 - \alpha)(\gamma + (1 - \gamma)(\alpha - \epsilon))}\right)^{y + (1 - y)(\alpha - \epsilon)} > \left(1 + \frac{\epsilon}{(\sigma y + (1 - \sigma y)(\alpha - \epsilon))(1 - \alpha)}\right)^{\sigma y + (1 - \sigma y)(\alpha - \epsilon)}$$

and consequently

$$\left(1 + \frac{\epsilon}{(\sigma y + (1 - \sigma y)(\alpha - \epsilon))(1 - \alpha)}\right)^{\sigma y + (1 - \sigma y)(\alpha - \epsilon)} \left(1 + \frac{\epsilon}{(1 - \alpha)(\gamma + (1 - \gamma)(\alpha - \epsilon))}\right)^{-\gamma - (1 - \gamma)(\alpha - \epsilon)} < 1,$$

so that there must always exist a value of \(\theta \delta\) at which the bubble outcome is exactly as beneficial as the no-bubble outcome.