Lot-Sizing Decisions in Manufacturer-Retailer Inventory System under Carbon Emissions Reduction

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Abstract. This paper develops a mathematical model for supply chain inventory system of a manufacturer and a retailer under stochastic demand and carbon reduction policy. The objective of the model is to obtain the decisions regarding inventory management in supply chain system by minimizing the joint total cost. The model contributes to the current stochastic joint lot-sizing literature by allowing the investigation of the impact of carbon cap policy to inventory decisions. The carbon mainly emitted from several activities in the supply chain system, including the production, transportation and storage activities. The regulator, for example the government, uses a carbon cap policy to restrict the amount of emissions generated from the system. As we use strict carbon cap, the carbon emissions generated from the system must not exceed the carbon emission level set by regulator. A list of procedure is proposed to obtain the optimal solutions, which are ordering lot, safety factor, number of shipments and production rate. A numerical instance is provided to show the applicability of the model and to explore the impact of different levels of carbon cap on inventory decisions.

1. Introduction

Environmental responsibility has become an important factor that must be considered by manager for doing the modern business. The environmental regulations issued by government and the increased of customer’s awareness regarding environmental problems are encouraging the supply chain parties to lessen the generations of carbon emissions from their operations. Operational managements, such as inventory, shipment and ordering decisions, have proven to be best way to lessen the carbon emissions influence [1]. Thus, scholars have considered carbon emissions in developing the inventory management for supply chain system. The developed models not only concern to minimize total inventory cost but also try to minimize the negative impact of carbon emissions generated from the supply chain operations.

An inventory model dealing with carbon emissions was firstly developed by Wahab et al. [2]. They developed an integrated inventory model for domestic and international supply chain systems and investigated the impact of carbon emissions on inventory decisions. Bonney and Jaber [3] developed an inventory model and proved that the carbon emissions reduction can be gained through increasing the delivery lot size and reducing the frequency of deliveries. Konur [4] investigated heterogeneous truck types and carbon cap mechanism in an EOQ model. Battini et al. [5] studied the impact of carbon emissions on inventory and transportation decisions. Schaefer and Konur [6] developed inventory model addressing the continuous review policy, transportation and carbon emissions with two different objectives. Benjaafar et al. [7] and Chen et al. [8] studied joint economic lot sizing problem (JEL under carbon emission reduction. They considered some mechanisms, which are carbon emission tax, inflexible cap and cap-and-trade, to restrict the carbon emitted from the systems. Jaber et al. [10] addressed emission tax and penalty in a manufacturer-retailer system. Jauhari et al. [11] included carbon emission costs in their JELP model. They
proposed single-retailer single-manufacturer inventory model in which the deliveries are made under unequal-shipment policy and the demand is deterministic.

Some scholars have also investigated the carbon emissions reduction in stochastic inventory models. Wangsa [12] proposed a supply chain inventory model containing a manufacturer and a retailer under probabilistic demand and carbon reduction. An incentive and penalty is proposed to lessen the carbon levels emitted from industrial and transportation operations. Jauhari (2018) considered imperfect items and carbon tax in developing a mathematical model for two-level inventory system. The Manufacturing process is flawed and the lead time is formulated under stochastic function. Jauhari et al. (In press) studied a JELP with imperfect items, fuzzy demand, inspection errors and service level constraint. They adopt the incentive and penalty mechanism proposed by Wangsa (2017) to cut down the amount of carbon emitted from manufacturing and transportation operations.

Our review to the JELP literature shows that the research concerning with the impact of strict carbon policy, adjustable production rate and imperfect production on inventory decisions is rarely discussed in the paper. The published papers mostly focused on discussing the carbon tax and fixed production rate under perfect production. Therefore, this paper propose a mathematical model considered a situation in which the emissions generated by supply chain is restricted by a carbon cap regulated by government. In addition, defective items are accounted in holding cost and emission cost.

2. Problem description

In this research, a mathematical inventory model for a manufacturer and a retailer system is proposed. The retailer faces a stochastic demand, which follows normal distribution, from end customers and uses a continuous review policy to control his inventory level. The retailer makes an order of $nQ$ units from the manufacturer and incurs ordering cost $A$ for each order. The unsatisfied demand is considered to be partially backordered. To satisfy the demand of retailers, the manufacturer produces a batch of size $nQ$ units with a rate of $P$ ($P>D$) per setup and incurs a production setup cost $K$ for each manufacturing cycle. The delivery of $Q$ units is done from the manufacturer to the retailer over $n$ times. The lead time is assumed to be dependent on lot size, setup time and transportation time.

The manufacturer’s production process is flawed, hence, generating a portion of defective items during production time. The lot received by the retailer will be inspected to categorize the quality of items. The defective items obtained during inspection process will be sent back to the manufacturer at the end of delivery cycle. The manufacturer can determine the production capacity flexibly. It is assumed that the production rate can be adjusted between $P_{\text{min}}$ and $P_{\text{max}}$. The emissions are investigated in the model and computed from production, delivery and storage activities. The regulation to reduce the carbon emission, namely strict carbon cap, is applied in the system.

To formulate the mathematical model, we employ the following notations

- $D$ end customer’s demand (units/year)
- $\sigma$ standard deviation of end customer’s demand (units/year)
- $A$ replenishment cost per order ($/order$)
- $F$ delivery cost per shipment ($/shipment$)
- $h_b$ retailer’s carrying cost ($/unit/year$)
- $h_m$ manufacturer’s carrying cost ($/unit/year$)
- $\pi$ backorder cost per unit backordered ($/unit$)
- $\pi_0$ marginal profit per unit for retailer ($/unit$)
- $\theta$ percentage of backorder items
- $Y$ the defect rate
- $w$ warranty cost per unit item ($/unit$)
- $s$ inspection cost per unit item ($/unit$)
- $x$ inspection rate (units/year)
- $S_R$ fixed carbon emission for retailer (kg CO$_2$)
3. Model formulation

As mentioned above, we propose a JELP for manufacturer-retailer system under stochastic environment, flexible production rate and carbon reduction. The objective of the proposed mathematical model is to search the optimal solutions which are, delivery lot size, safety factor and number of shipments so the total cost is minimized. The total cost of supply chain consists of two costs, which are retailer cost and manufacturer cost. The development of each cost is provided in the sub section below.

3.1 Retailer cost

Consider the customer’s demand occurred at the retailer follows a normal distribution with mean $DL(Q)$ and standard deviation $\sigma \sqrt{L(Q)}$. The lead time of the first delivery is constructed by addressing setup and transportation time and the next lead time is only transportation time considered in the model (Hsiao, 2008). Therefore, the safety stock for first shipment and next shipment can be formulated by the following equations

$$S_i = k_i \sigma \sqrt{\frac{Q}{P} + T_w}$$

$$S_s = k_s \sigma \sqrt{T_w}$$

Equations (3) and (4) express the expected shortage of first shipment and the expected shortage of next shipment, respectively

$$ES_i = \sigma \sqrt{\frac{Q}{P} + T_w} \psi(k_i)$$

$$ES_s = \sigma \sqrt{T_w} \psi(k_s)$$

where,

$$\psi(k_i) = f_s(k_i) - k_i \left[ 1 - F_s(k_i) \right]$$

$$\psi(k_s) = f_s(k_s) - k_s \left[ 1 - F_s(k_s) \right]$$
Figure 1 demonstrate the behavior of retailer’s inventory pattern. By considering this figure, the expected retailer’s carrying cost per unit time is formulated as follows

\[ h_b \left( \frac{Y}{1-Y} \right) \frac{QD}{x} \frac{1}{2} \left( 1 - E(Y) \right) \sqrt{\frac{Q}{P}} + T_w + (1 - \theta) \sigma \sqrt{\frac{Q}{P}} + T_w \psi(k_i) \]  

The expected retailer’s shortage cost per unit time is presented by the following expression

\[ E \left( \frac{1}{1-Y} \right) \frac{D \sigma}{nQ} (\sigma + \pi_o (1-\theta)) \left( \sqrt{\frac{Q}{P}} + T_w \psi(k_i) + (n-1) T_w \psi(k_i) \right) \]  

The expected retailer’s total cost per unit time, which is consisted of replenishment cost, delivery cost, inspection cost, carrying cost and backorder cost, is calculated by

\[ ETCR = E \left( \frac{1}{1-Y} \right) \frac{D \sigma}{nQ} (A + nF) + E \left( \frac{1}{1-Y} \right) sD \]
\[ + h_b \left( \frac{Y}{1-Y} \right) \frac{QD}{x} \frac{1}{2} \left( 1 - E(Y) \right) \sqrt{\frac{Q}{P}} + T_w + (1 - \theta) \sigma \sqrt{\frac{Q}{P}} + T_w \psi(k_i) \]
\[ + E \left( \frac{1}{1-Y} \right) \frac{D \sigma}{nQ} (\sigma + \pi_o (1-\theta)) \left( \sqrt{\frac{Q}{P}} + T_w \psi(k_i) + (n-1) T_w \psi(k_i) \right) \]  

3.2 Manufacturer cost

The manufacturer’s inventory level is computed the accumulated deliveries minus the accumulated manufacturer’s production. The expected manufacturer’s carrying cost per unit time is given by
The emissions generated from storage activity are formulated by the expression below:

$$h_s \frac{Q}{2} \left[ n - E \left( \frac{1}{1 - Y} \right)^D \right] - 1 + E \left( \frac{1}{1 - Y} \right)^{2D/P}$$

The production cost is also included in the model and formulated by following Khouja and Mehrez’s (1994) formula, which is

$$E \left( \frac{1}{1 - Y} \right) \left( a_1 P + a_2 P \right) D$$

The expected total cost of manufacturer, which is composed of production setup cost, production cost, warranty cost, carbon emission cost and carrying cost is expressed by the following equation:

$$ETCM = E \left( \frac{1}{1 - Y} \right) \left( \frac{1}{nQ} DK \right) + E \left( \frac{1}{1 - Y} \right) wD + h_u \frac{Q}{2} \left[ n - E \left( \frac{1}{1 - Y} \right)^D \right] - 1 + E \left( \frac{1}{1 - Y} \right)^{2D/P} + E \left( \frac{1}{1 - Y} \right) \left( a_1 P + a_2 P \right) D$$

(12)

3.3 Joint total cost

As we described above that the proposed model considers carbon emission reduction policy, namely strict carbon cap. This means that the carbon emitted from supply chain activities (transportation, production, storage) is restricted by carbon emission level (carbon cap) determined by the regulator. For transportation activity, the emissions are calculated from delivering the items from manufacturer to retailer and delivering defective items from retailer to manufacturer. Here we adopt two types of emission, namely fixed emission and variable emissions. The emissions generated from transportation activity are given by

$$EM_{trans} = E \left( \frac{1}{1 - Y} \right) \left( \frac{1}{Q} DS_{x2} + E \left( \frac{1}{1 - Y} \right) z_s D + E \left( \frac{1}{1 - Y} \right) DS_{w2} + E \left( \frac{1}{1 - Y} \right) z_u D \right)$$

(13)

The emissions generated from production activity are given by

$$EM_{prod} = E \left( \frac{1}{1 - Y} \right) \left( y_1 P^2 - y_2 P + y_3 \right) D$$

(14)

The emissions generated from storage activity are formulated by the expression below

$$EM_{stor} = W \left( \frac{1}{1 - Y} \right) \left( QD \right) + E \left( \frac{1}{1 - Y} \right) \left( \frac{1 - E(Y)Q}{2} \right) + k_s \sqrt{Q/P + T_u + (1 - \theta) \sigma \sqrt{Q/P + T_u} \psi(k_3)}$$

$$+ W \frac{Q}{2} \left[ n - E \left( \frac{1}{1 - Y} \right)^{D/P} \right] - 1 + E \left( \frac{1}{1 - Y} \right)^{2D/P}$$

(15)

Therefore, by using equation $M = E \left( \frac{1}{1 - Y} \right)$, the proposed problem can be written as

$$EJTC(n, Q, k_1, P) = \frac{MD}{nQ} (A + nF) + MsD$$

$$+ h_s \left( \frac{MQD}{x} - \frac{QD}{x} \right) + \frac{1 - E(Y)Q}{2} + k_s \sqrt{Q/P + T_u + (1 - \theta) \sigma \sqrt{Q/P + T_u} \psi(k_3)}$$

$$+ \left( \frac{MD \sigma}{nQ} (\pi + \pi_0 (1 - \theta)) \right) \sqrt{Q/P + T_u \psi(k_3) + (n - 1) \sqrt{T_u \psi(k_2)}} + \frac{MDK}{nQ}$$

$$+ wDM - wD + h_u \frac{Q}{2} \left[ n - E \left( \frac{MD}{P} \right)^{D/P} \right] - 1 + E \left( \frac{MD}{P} \right)^{2D/P}$$

Subject to

$$EM_{trans} + EM_{prod} + EM_{stor} \leq C$$

$$P_{min} \leq P \leq P_{max}$$

(16)
4. Solution procedure

To obtain the proposed mathematical model’s solution, we search the first partial derivatives of \( EJT(T, Q, k_1, P) \) with the respect to \( k_1, Q \) and \( P \), respectively and set the results to zero, we then obtain the equations below

\[
\frac{1 - F_i(k_i) + (n - 1) \left[ 1 - F_i \left( \frac{Q}{p + T_u} \right) \right]}{F_i(k_i) + \theta \left[ 1 - F_i(k_i) \right]} = \frac{h_i \sigma Q}{MD(\pi + \pi_0(1 - \theta))}
\]

\[
Q = \left\{ \begin{array}{ll}
2MD \left[ \frac{A + K}{n} + \frac{(\pi + \pi_0(1 - \theta))\sigma}{n} \right] + \frac{2h_i MD}{x} + \frac{h_i k_i \sigma}{x} + \frac{h_i (1 - \theta) \sigma \psi(k_i)}{P} \sqrt{\frac{Q}{p + T_u}} & \\
\frac{MD(\pi + \pi_0(1 - \theta))\sigma \psi(k_i)}{nPQ} &
\end{array} \right.
\]

\[
P = \left\{ \begin{array}{ll}
\frac{a_i MD + MQh_i D - \frac{MQh_i nD}{2} + \frac{h_i k_i \sigma Q}{2\sqrt{\frac{Q}{p + T_u}}} + \frac{h_i (1 - \theta) \sigma \psi(k_i)Q}{2\sqrt{\frac{Q}{p + T_u}}} + \frac{MD \sigma}{n} (\pi + \pi_0(1 - \theta)) \psi(k_i)}{a_i MD} \right.
\]

If \( \lambda \) is negative, expression (19) is not feasible. Thus, we use \( P = P_{\text{min}} \) if \( \lambda < 0 \). The equation \( P \) should be reformulated as

\[
P = \max(P_{\text{max}}, \frac{MD}{a_i MD} \psi(k_i))
\]

An iterative procedure, depicted in figure 2, is suggested to obtain the model’s solutions.

5. Numerical example

To show the applicability of the proposed model, we provide the following data for parameters: \( D = 1,000 \) units/year, \( \sigma = 25 \) units/year, \( F = 50 \) delivery, \( A = 100 \) order, \( K = 400 \) setup, \( h_M = 1 \) unit/year, \( h_S = 3 \) unit/year, \( \pi = 50 \) unit, \( \pi_0 = 150 \) unit, \( \theta = 0.5 \), \( P_{\text{min}} = 1,500 \) units/year, \( P_{\text{max}} = 5,000 \) units/year, \( x = 7,000 \) units/year, \( a_j = 2,500 \), \( a_2 = 0.0004 \) unit, \( S_i = 1,300 \) kgCO\(_2\), \( z_2 = 1,300 \) kgCO\(_2\), \( z_0 = 10.4 \) kgCO\(_2\) unit, \( z_1 = 10.4 \) kgCO\(_2\) unit, \( y_1 = 0.003 \) kgCO\(_2\) year\(^2\) unit\(^3\), \( y_2 = 1.2 \) kgCO\(_2\) year\(^2\) unit\(^3\), \( y = 140 \) kgCO\(_2\) unit, \( C = 170,000 \) kgCO\(_2\), \( w = 2 \) unit, \( s = 0.5 \) unit, \( T_u = 0.05 \), \( T = 0.04 \). The pdf for the defect rate is taken to be

\[
f(y) = \begin{cases} 25, & 0 \leq y \leq 0.04 \\ 0, & \text{otherwise} \end{cases}
\]

Thus, we can formulate the expected value of \( Y \), which is

\[
E(Y) = \int_0^{0.04} 25y dy = 0.02 \quad \text{and} \quad E\left( \frac{1}{1-y} \right) = \int_0^{0.04} \frac{25}{1-y} dy = 1.02055.
\]
Set $n=1$ and $EJTC(n, Q_1, k_{1n}, P_n) = \infty$.

For $i=1$, set the values of $Q_i$ and $P_i$ with equation (18) and equation (20), respectively.

Compute $k_{1i}$ by substituting $Q_i$ into equation (17).

Compute $P_{i+1}$ from equation (20).

For a given previous value of $k_{1i}$ and $P_{i+1}$, compute $Q_{i+1}$ from equation (18).

$Q_i = Q_{i+1}$, $P_i = P_{i+1}$, $k_{1i} = k_{1i+1}$?  

Set $Q_n = Q_i$, $k_{1n} = k_{1i}$ and $P_n = P_i$ and compute $EJTC(n, Q_n, k_{1n}, P_n)$ from equation (16).

$EJTC(n, Q_n, k_{1n}, P_n) \leq EJTC(n-1, Q_{n-1}, k_{1n-1}, P_{n-1})$?  

Compute $EJTC(n', Q', k_{1n'}, P_n') = EJTC(n, Q_n, k_{1n}, P_n)$, if $n' = (n, Q_n, k_{1n}, P_n)$ are the optimal solution.

Set $n = n + 1$

$\text{Carbon emissions} < \text{Carbon Cap}$?  

Yes

Stop

No

Set $i = i + 2$

Yes

No

By applying the above iterative procedure, the solutions of the numerical example are as follows: the number of deliveries is 7, production rate is 2297 units/year, safety factor 1.89, delivery quantity 190 units.
and the total cost is $3912.6. The emissions generated by vendor and buyer are 151,460 kgCO₂ and 18,395 kgCO₂. Figure 3 shows the influence of carbon cap changes on cost and carbon emissions. It is observed that when the cap is increased the carbon emissions are increased as well. However, the parties cost is relatively decreased due to the increase in carbon cap. The influence of the changes of carbon cap on $n$ and $Q$ is presented in figure 4. It shows that when cap increases, $n$ sharply increases while $Q$ sharply decreases. It can be concluded that the impact of cap changes on decision variables is more significant than its impact on total cost.

6. Conclusion

In this paper, a JELP containing a manufacturer and a retailer is introduced to comply with carbon emission reduction regulated by a government. The manufacturing process is unreliable, thus producing some defective items during production run. Adjustable production rate is allowed in the proposed model which finally gives an opportunity to the vendor’s manager to set its value flexibly to optimize the production capacity. Generally, the manufacturing, warehousing and transportation activities will produce a certain level of carbon emissions. A strict carbon cap is selected by the regulator as a mechanism to reduce the emission generated from the vendor system and buyer system. A mathematical inventory model is built to minimize total cost charged by supply chain and to ensure that the emission levels are lower than carbon cap. A numerical instance is presented to show the application of the model. Future research may focus on...
discussing the impact of another carbon policies regulated by government on inventory decisions. An extension to multi supply chain structure, such as multi-vendor multi-buyer, is also interesting to be investigated.

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