Method for splitting the feasible region to increase the variability of continuous optimization test problems

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Abstract. The solution of optimization problems is essential for the design and control of technical systems. The optimization problem arising in practice has a high dimension, nonlinear criteria, and constraints. There are a lot of continuous optimization tasks for testing and research of optimization algorithms performance. These tasks have a convex range of acceptable values limited to a specified range for each parameter. The problem of generating test multidimensional continuous optimization tasks with nonlinear constraints and splitting the feasible region is considered. A method was proposed for splitting the feasible region by separate domains using the multidimensional grid of forbidden solutions. As a result, the problem acquires properties closer to optimizing technical systems with complex constraints. The method allows creating an unlimited number of test optimization problems, which can be used to research and develop optimization algorithms. The method is simple to implement, and the impact on the computational complexity of tasks is insignificant. Research has been carried out on four widely used continuous single-objective optimization test functions, with the Genetic algorithm and the Particle Swarm Optimization algorithm. It is shown that the proposed method has an influence on the process of solving multidimensional continuous optimization problems by population algorithms and on the dependence of the accuracy of the algorithm on its heuristic coefficients.

1. Introduction
The solution of optimization problems is essential for the design and control of technical systems. The optimization problem arising in practice has a high dimension, nonlinear criteria, and constraints. One of the most effective methods for solving such problems is population algorithms [1-3]. Genetic algorithm (GA) and Particle Swarm Optimization algorithm (PSO) are widely used. An overview of GA application in the optimization of technical systems by the case of electric power industry is given in [4]. Reviews of PSO applications are given in [5, 6]. There are also a large number of hybrid algorithms combining GA and PSO [7-10]. The heuristic nature of these algorithms does not have strict mathematical justification. Therefore, it becomes possible to make modifications to each of the stages of work. For example, there are various GA options for selection, crossover and mutation [11].
Optimization algorithms are often researched on the number of continuous optimization test problems [12-14]. These tasks have a convex feasible region, limited by a specified range for each parameter. Test functions with more complex feasible region are rare, for example, in [15].
The feasible region can be non-convex and divided into several areas in optimizing technical systems. In the general case, the problem statement can be written as follows:
2. Division of the feasible region of optimization problems

In this work, a multidimensional grid of forbidden solutions (MGFS) use as an easy-to-implement method for dividing the feasible region into several. For ease of implementation, the initial range of permissible values is scaled into a hypercube, each side of which is delimited from 0 to 1. For a problem of dimension \( n = 2 \), it is the square of the unit area, of dimension 3, the cube of unit volume, and so on. For a two-dimensional case, the MGFS is shown in figure 1.

![Figure 1. Grid of forbidden solutions (r = 0.5; d = 5; wd = 0.0732).](image)

The width of the border between the allowed areas along each axis (\( wd \)) is defined as:

\[
wb = \frac{1 - \sqrt[4]{r}}{d - 1}
\]  

where \( r \) is the fraction of the original feasible region that must remain admissible (from 0 to 1), \( d \) is the number of admissible subdomains for each of the parameters.

It should be taken into account that the solution is within the scope of the MGFS. As a rule, penalty functions are used to account for internal constraints. The value of the penalty function should be as close to zero as possible in the feasible region and take high values outside it, so that the sum of the objective function and the penalty function at any point outside the feasible region is significantly higher than inside. Although in the general case, the choice of the penalty function is a nontrivial problem [16, 17], for many population algorithms, it is sufficient to use a penalty function of the form:

\[
\text{penalty}(X) = \begin{cases} 
0, & G(X) = \text{True} \\
P, & G(X) = \text{False}
\end{cases}
\]

(3)
where $G(X)$ is the predicate that shows whether the solution $X$ falls within the admissible range, $P$ is a constant, obviously much larger than the possible values of the objective function in the feasible region. This method is suitable for population algorithms that do not use the difference in fitness functions, but only their ratio "more-less".

3. Computational experiments
Computational experiments were carried out with four continuous optimization functions (figures 2–4) and PSO and GA algorithms for evaluation of affection of proposed modification to operation of population algorithms.

3.1. Basic optimization functions
- Ackley function is defined by [12, 13]:

\[
f(X) = -20 \exp \left(-0.2 \sqrt{0.5 \sum_{i=1}^{n} x_i}\right) - \exp \left(0.5 \sum_{i=1}^{n} \cos(2\pi x_i)\right) + e + 20
\]

-32.768 < $x_j$ < 32.768, $j = 1, ... n$ (4)

- Rosenbrock function is defined by [12, 13]:

\[
f(X) = \sum_{i=1}^{n} \left(100(x_{i+1} - x_i^2)^2 + (1 - x_i)^2\right)
\]

-2.048 < $x_j$ < 2.048, $j = 1, ... n$ (5)

- Griewonk function is defined by [13, 14]:

\[
f(X) = 1 + \frac{1}{4000} \sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} \cos \left(\frac{x_i}{\sqrt{i}}\right)
\]

-600 < $x_j$ < 600, $j = 1, ... n$ (6)

- Rastrigin function is defined by [12, 13]:

\[
f(X) = 10n + \sum_{i=1}^{n} \left(x_i^2 - 10\cos(2\pi x_i)\right)
\]

-5.12 < $x_j$ < 5.12, $j = 1, ... n$ (7)

\[\text{Figure 2. Ackley function (a), Rosenbrock function (b).}\]
Figure 3. Griewonk function in the area from -600 to 600 (a), from -6 to 6 (b).

Figure 4. Rastrigin function.

All of the listed functions have a unique global extremum $f(X^{opt}) = 0$, and for all, except for the Rosenbrock function, extremum is attained when all $x_i = 0$. The Rosenbrock function has $X^{opt} = \{1, 1, \ldots, 1\}$. Therefore, for the Rosenbrock function, MGFS was shifted in all coordinates by 1 so that the global extremum would not be inside the invalid area. The dimensions $n = 2, 4, \text{and } 8$ were used in the calculations. MGFS divided the feasible region for each coordinate into 5 subregions ($d = 5$). The coefficient $r$ varied from 0.25 to 1.0 with a step of 0.25.

3.2. Optimization algorithms

The general description of the algorithms used is omitted: the variants of both GA and PSO were used that described in detail in domestic and foreign literature. The study used the GA implementation [18]. The selection is carried out using the roulette method, and the probability of selecting an individual from an unacceptable area is zero. Crossover from two parents, single point, probability of crossover 50%. The mutation changes the value of the gene to a new random one, not related to the previous one, the probability of mutation of each gene in the chromosome is 1% or 5% (calculations were performed separately with both values). The probability of crossover is 50%.

The PSO algorithm is implemented in the classical version [19] with an additional rate limitation [8]. Heuristic coefficients: influence of the collective best decision 1.5; influence of the individual best solution 1.5; inertia 0.7; speed limit $\beta = 0.3$ or no limit ($\beta = 1.0$). The population size for both algorithms was 25 (for problems of dimension 2) or 50 (for problems of dimension 4 and 8). The number of iterations was limited to 1000.

3.3. Results

The results of computational experiments are shown in tables 1 and 2. The cells of the tables show the averaged values of the objective function obtained in solving problems. Averaging was carried out over 1000 runs with a random initial distribution of chromosomes or particles. Table 2 results for the Ackley
problem are excluded, since in all experiments the same result, 0, was obtained, the algorithm always found the global optimum of the problem.

Since the tasks differ in complexity and scale of the objective function, visualization of the results was performed for clarity. The averaged result obtained by the algorithm for a specific task without MGFS \((r = 1)\) was used as a baseline. For the results with \(r = 0.25–0.75\), the relative deviation was calculated according to the following formula (the division is performed not by \(f_i\), but by the mean \(f_i\) and \(r\), since a situation is possible when \(f_i\) is several orders of magnitude closer to zero than \(f_i\):

\[
\Delta_r = \frac{f_r - f_i}{0.5(f_r + f_i)}, r = 0.25, 0.5, 0.75 \quad (8)
\]

The obtained results are shown in figure 5 for GA and in figure 6 for PSO. In most cases, at \(r = 0.25\), the obtained solution is significantly worse than without MGFS \((\Delta_r < 0)\). This shows that wide boundaries between subdomains of feasible region complicate the operation of population algorithms under partitioning. At the same time, this effect is not observed for PSO in the problems of Ackley and Grivonk. The surface of the objective function of the Ackley problem is smooth, the surface of the Givonk problem is also close to smooth when considered at a small scale (figure 2a), therefore, the particles of the algorithm in these problems pass through the boundaries of the MGFS, moving by inertia. In GA, the property of inertia is not present, which makes it difficult to cross the wide boundaries of the MGFS. At \(r = 0.5\), this effect persists, but the deterioration in the quality of solutions can be less than at \(r = 0.25\), as, for example, can be seen in figure 5a for Rosenbrock problem. For \(r = 0.75\), depending on the problem and its dimension, there can be both a deterioration of the result relative to the original problem and an improvement, as, for example, in figure 6b for the Rastrigin problem.

| \(f(X)\) | \(n\) | \(p_{ms},\%\) | The average result of the solution for the specified \(r\) |
|---|---|---|---|
| Ack. | 2 | 1 | 1.010 | 1.010 | 1.068 | 1.010 |
| Ack. | 2 | 5 | 0.00230 | 0.00201 | 0.00201 | 0.00201 |
| Ack. | 4 | 1 | 0.116 | 0.140 | 0.198 | 0.255 |
| Ack. | 4 | 5 | 0.0307 | 0.00425 | 0.00221 | 0.00331 |
| Ack. | 8 | 1 | 0.504 | 0.499 | 0.506 | 0.519 |
| Ack. | 8 | 5 | 2.015 | 0.568 | 0.144 | 0.0771 |
| Gr. | 2 | 1 | 0.131 | 0.128 | 0.127 | 0.132 |
| Gr. | 2 | 5 | 0.0106 | 0.0102 | 0.0101 | 0.0110 |
| Gr. | 4 | 1 | 0.0750 | 0.0749 | 0.0793 | 0.0781 |
| Gr. | 4 | 5 | 0.0607 | 0.0593 | 0.0506 | 0.0382 |
| Gr. | 8 | 1 | 0.337 | 0.3218 | 0.328 | 0.324 |
| Gr. | 8 | 5 | 0.720 | 0.668 | 0.476 | 0.286 |
| Ras. | 2 | 1 | 0.212 | 0.201 | 0.209 | 0.213 |
| Ras. | 2 | 5 | 2.50E-06 | 2.42E-06 | 2.43E-06 | 2.42E-06 |
| Ras. | 4 | 1 | 0.00940 | 0.0163 | 0.0203 | 0.0297 |
| Ras. | 4 | 5 | 0.00253 | 0.00106 | 3.35E-05 | 9.97E-06 |
| Ras. | 8 | 1 | 0.155 | 0.181 | 0.184 | 0.226 |
| Ras. | 8 | 5 | 2.104 | 1.192 | 0.303 | 0.0270 |
| Ros. | 2 | 1 | 0.472 | 0.285 | 0.212 | 0.205 |
| Ros. | 2 | 5 | 0.0434 | 0.0233 | 0.0151 | 0.0198 |
| Ros. | 4 | 1 | 1.134 | 1.012 | 0.967 | 1.032 |
| Ros. | 4 | 5 | 0.969 | 0.847 | 0.768 | 0.720 |
| Ros. | 8 | 1 | 5.257 | 4.491 | 4.484 | 4.614 |
| Ros. | 8 | 5 | 4.299 | 4.429 | 4.379 | 4.295 |
The paper proposes a modification of continuous optimization problems, named as "Multidimensional Grid of Forbidden Solutions," which divides a feasible region into a set of subdomains. The method allows creating an unlimited number of test problem modifications. The computational experiments were carried out on the problems of Exley, Grivonk, Rastrigin, and Rosenbrock, the Genetic algorithm, and the Particle Swarm Optimization algorithm. It was found out that the proposed method has a strong influence on the process of solving optimization problems by population algorithms. The imposition of the MGFS on the test optimization problems can change the obtained criterion value of solutions in the of 2–7 times when the ratio of admissible solutions is 50%. Application of the method in the study of optimization algorithms can increase their efficiency in problems with a complex system of constraints. The degree of efficiency improvement depends on the problem's dimension, the topology of its solution space, and the optimization algorithm. The analysis of these dependencies is supposed to be performed at the subsequent stages of the study.

Table 2. Results of computational experiments PSO.

| $f(X)$ | $n$ | $\beta$ | The average result of the solution for the specified $r$ |
|--------|-----|--------|-------------------------------------------------------|
| Gr. 2  | 2   | 0.3    | 0.00207 | 0.00201 | 0.00193 | 0.00193 |
| Gr. 2  | 2   | 1.0    | 0.00214 | 0.00201 | 0.00196 | 0.00196 |
| Gr. 4  | 4   | 0.3    | 0.0125  | 0.0127  | 0.0121  | 0.0121  |
| Gr. 4  | 4   | 1.0    | 0.0126  | 0.0128  | 0.0121  | 0.0127  |
| Gr. 8  | 8   | 0.3    | 0.0638  | 0.0638  | 0.0652  | 0.0644  |
| Gr. 8  | 8   | 1.0    | 0.0659  | 0.0641  | 0.0633  | 0.0622  |
| Ras. 2 | 2   | 0.3    | 0.0159  | 0.0159  | 0.00398 | 0.00298 |
| Ras. 2 | 2   | 1.0    | 0.0199  | 0.0199  | 0.0119  | 0.00398 |
| Ras. 4 | 4   | 0.3    | 0.537   | 0.438   | 0.0212  | 0.193   |
| Ras. 4 | 4   | 1.0    | 0.517   | 0.310   | 0.0744  | 0.171   |
| Ras. 8 | 8   | 0.3    | 7.813   | 4.864   | 3.581   | 3.313   |
| Ras. 8 | 8   | 1.0    | 7.443   | 4.941   | 3.621   | 3.464   |
| Ros. 2 | 2   | 0.3    | 0.0298  | 0.0196  | 0.00137 | 1.76E-31|
| Ros. 2 | 2   | 1.0    | 0.0479  | 0.0224  | 0.00108 | 1.93E-30|
| Ros. 4 | 4   | 0.3    | 1.174   | 0.921   | 0.525   | 0.368   |
| Ros. 4 | 4   | 1.0    | 1.300   | 1.0186  | 0.585   | 1.0186  |
| Ros. 8 | 8   | 0.3    | 4.177   | 3.861   | 2.291   | 1.017   |
| Ros. 8 | 8   | 1.0    | 7.466   | 5.006   | 3.579   | 3.382   |

At $r = 0.75$, the boundaries are not so wide as to hinder the transition of agents (particles or chromosomes) of population algorithms between the subdomains of feasible solutions, but at the same time, the MGFS removes some of the local extremum.

From table 1, it can be seen that for GA without the use of MGFS, the probability of mutation is 5%, other things being equal, is better than the probability of mutation of 1% for all 4 problems and for all dimensions used (2, 4, 8). At the same time, on problems with MGFS with wide boundaries ($r = 0.25$) and dimension 8, the results obtained with a mutation probability of 5% for the problems of Ackley, Grivonk, and Rastrigin turned out to be significantly worse than with 1%. This suggests that in the case when the share of feasible solutions is small (25%), a higher probability of mutation too often leads to the fact that the chromosome is in the unacceptable region and is eliminated from the process of natural selection. As a result, the population can rapidly degenerate. According to the table 2 and from the coincidence of the characters of the graphs in the upper and lower frequent in figure 6, it can be concluded that the effect of particle velocity limitation in PSO on the results is much less dependent on the parameters of the MGFS than the effect of the probability of mutation in GA.

4. Conclusion

The paper proposes a modification of continuous optimization problems, named as "Multidimensional Grid of Forbidden Solutions," which divides a feasible region into a set of subdomains. The optimization problem acquires properties that bring it closer to optimizing technical systems with complex internal constraints. The method allows creating an unlimited number of test problem modifications.
Figure 5. Influence of MGFS on GA results with a mutation probability of 1% (upper) and 5% (lower) at $n = 2$ (a) $b = 4$ (b), $n = 8$ (c).

Figure 6. Influence of MGFS on PSO results with speed limit $\beta = 0.3$ (upper) and without speed limit (lower) at $n = 2$ (a) $b = 4$ (b), $n = 8$ (c).
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