NONLINEAR EFFECTS DUE TO THE COUPLING OF LONG-WAVE MODES

BHUVNESH JAIN, EDMUND BERTSCHINGER
Department of Physics, MIT, Cambridge, MA 02139 USA.

Abstract

The cosmological fluid equations are used to study the nonlinear mode coupling of density fluctuations. We find that for realistic cosmological spectra there is a significant contribution to the nonlinear evolution on scales of interest to large-scale structure from the long-wave part of the initial spectrum. A consequence of this mode coupling is that at high redshift, \( z \), the nonlinear scale defined by \( \sigma(z) = 1 \) can be significantly larger than a linear extrapolation would indicate. For the standard CDM spectrum with a \( \sigma_8 = 1 \) normalization the mass corresponding to the nonlinear scale at \( z = 20, 10, 5 \) is about 100, 10, 3 times (respectively) larger than the linearly extrapolated value.

We also investigate the possibility of divergent contributions to the density field from long-wave modes if the spectral index of the power spectrum \( n < -1 \). Using an approximate non-perturbative approach we find that for \( n > -3 \) the divergent contribution appears only in the phase. This can be related to the large-scale bulk velocity, and clarifies previous results from N-body simulations.

1 Power Spectrum at Second Order

Perturbative studies of the Newtonian cosmological fluid equations have been used to study weakly nonlinear effects in the evolution of density fluctuations (e.g., [1]). We have used second-order perturbation theory to study the enhancement of power and changes in the time-dependence of characteristic length scales. We assume that vorticity and pressure are negligible. For the numerical results we have used the standard CDM spectrum with \( \Omega = 1, H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1} \) and \( \sigma_8 = 1 \).

The Fourier transform of the cosmological fluid equations can be written as:

\[
\frac{\partial \hat{\delta}}{\partial \tau} + \dot{\theta} = -\int d^3 k_1 \frac{\vec{k} \cdot \vec{k}_1}{k_1^2} \hat{\theta} (\vec{k}_1, \tau) \hat{\delta} (\vec{k} - \vec{k}_1, \tau),
\]

\[
\frac{\partial \hat{\theta}}{\partial \tau} + \frac{\dot{a}}{a} \hat{\theta} + \frac{6}{\tau^2} \hat{\delta} = -\int d^3 k_1 k^2 \frac{\vec{k} \cdot (\vec{k} - \vec{k}_1)}{2 k_1^2 |\vec{k} - \vec{k}_1|^2} \hat{\theta} (\vec{k}_1, \tau) \hat{\theta} (\vec{k} - \vec{k}_1, \tau),
\]

where \( \vec{k} \) is the comoving wavevector, \( \tau \) is conformal time and \( a(\tau) [= 1/(1 + z)] \) is the expansion factor. \( \hat{\delta} \) and \( \hat{\theta} \) are, respectively, spatial Fourier transforms of \( \delta = \delta \rho/\bar{\rho} \) and \( \theta = \vec{\nabla} \cdot \vec{v} \). Following the standard perturbative approach, we expand \( \hat{\delta} \) and \( \hat{\theta} \) in a perturbation series as: \( \hat{\delta} (\vec{k}, \tau) = a \hat{\delta}_1 (\vec{k}) + a^2 \hat{\delta}_2 (\vec{k}) + \ldots \).

Given the initial power spectrum, and assuming the initial fields to be Gaussian random fields, the formal solutions for \( \hat{\delta}(\vec{k}, \tau) \) can be used to obtain its power spectrum to second order:

\[
P(k, \tau) = a^2(\tau) P_1(k) + a^4(\tau) P_2(k),
\]
where \( P_1(k) = \langle \hat{\delta}_1 \hat{\delta}_1^* \rangle \) is the initial (linear) spectrum and \( P_2(k) = \langle \hat{\delta}_2 \hat{\delta}_2^* + 2 \hat{\delta}_1 \hat{\delta}_3^* \rangle \) is the second-order contribution. \( P_2(k) \) is an integral over wavevectors involving products of \( P_1(k_1) \) evaluated at different \( k_1 \). We have calculated \( P(k, \tau) \) at different values of \( a \) for the standard CDM spectrum and estimated the relative contributions from different parts of the initial spectrum.

We find that the second-order contribution provides a significant enhancement of power in the weakly nonlinear regime. It is in excellent agreement with results from a high resolution N-body simulation for \( z > \sim 2 \), but shows a larger enhancement for smaller \( z \). Due to the effect of long-wave mode coupling, the nonlinear evolution responds to the shallower slope of the spectrum at long waves. This slows the growth in time of characteristic nonlinear scales, i.e., causes them to be larger at early times. We find that the mass corresponding to the scale at which the r.m.s. fluctuation \( \sigma(z) = 1 \) at \( z = 20, 10, 5 \) is about \( 100, 10, 3 \) times (respectively) larger than indicated by the standard practice of extrapolating the nonlinear scale today using the linear spectrum. This result has significant implications for nonlinear structures at high \( z \).

2 Coupling of Long-Wave Modes

An alternative to the perturbative approach is to approximate the nonlinear terms on the right-hand side of the fluid equations shown above so as to include the dominant contribution from the long-wave modes. In the limit that the wavevector \( k_1 \to 0 \) one finds that the mean square of the nonlinear terms takes the form \( P_1(k) \int dk_1 P_1(k_1) \). This integral is divergent as \( k_1 \to 0 \) if \( P_1(k_1) \propto k_1^n \) with \( n < -1 \), even while the density variance is finite for \( n > -3 \). Indeed some studies of N-body simulations indicate that the evolution of \( n < -1 \) spectra is qualitatively different from that of \( n > -1 \) spectra, and is strongly influenced by the size of the box (the small-\( k \) cutoff) [2], [3].

The nonlinear terms described above can be approximated by making a Taylor series expansion in \( (k_1/k) \) so as to include the dominant contribution from small \( k_1 \). On further approximating the small-\( k_1 \) density and velocity fields by the linear fields, the equations reduce to linear partial differential equations. We have separated the equations for the phase and amplitude of the density field retaining the first order Taylor series term, and have obtained a closed form solution for the phase.

The divergent nonlinear terms drop out of the equation for the density amplitude implying that the power spectrum does not diverge for \( -3 < n < -1 \) in this approximation. The solution for the phase is divergent for \( n < -1 \). This divergence is related to the divergence of the linear mean square bulk velocity, and corresponds to translational motion of the fluid past the fixed coordinate frame. It agrees exactly with the deviation of the phases from their initial values found in N-body simulations. However this deviation does not seem to indicate dynamically interesting nonlinear evolution as it does not affect the amplitude. This conclusion appears to differ from the results of some \( n = -2 \) simulations. We are currently investigating the source of disagreement and also analyzing the long-wave limit of perturbative contributions to the power spectrum to verify the above results.

Acknowledgements. We are grateful to Alan Guth for several useful discussions.

References

[1] Peebles, P. J. E. 1980. The Large-Scale Structure of the Universe, Princeton University Press

[2] Ryden, B. S., & Gramann, M. 1991. Astrophys. J. 383, L33

[3] Gramann, M. 1992. Astrophys. J. 401, 19