Oscillations of the variable cross beams under seismic and technogenic influences (part II)

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Abstract. The forced transverse oscillations of the variable cross section compressed beams under seismic and technogenic impacts are considered. The source of oscillations is the kinematic perturbations of the beam ends in the form of a harmonic or random vector process. The mathematical model of oscillations is presented as a boundary value problem from the basic partial differential equation of the fourth order hyperbolic type in spatial coordinate and the second order in time, which is supplemented by the boundary conditions. The beam fluctuations are interpreted as the spatiotemporal random fields inhomogeneous in space and stationary in time. In the deterministic case, the amplitudes of the forced oscillations are determined, the influence of the kinematic disturbances’ frequency and the beam oscillations’ shift of their phases is studied. In the stochastic problem, the standard deviations for the deflections are calculated, the influence of the characteristic frequency and correlation of the components of the vector perturbation process on the deflections’ output function is studied. The problem is solved by the methods of the variables’ separation, finite differences and the theory of random processes. The examples are considered, the conclusions are drawn from the calculations results in the Matlab computing complex environment.

Introduction
This work is the continuation of the first part article with the same name written by the authors, which gives a general mathematical model and free oscillations of the beam. Here we consider the forced oscillations in deterministic and stochastic settings.

The horizontal beams’ transverse oscillations are dangerous during earthquakes caused by seismic and man-caused causes. The existing standards for the calculation and design of the building structures take into account only horizontal displacements of the base, while the disturbing process is vector, random and contains the horizontal, vertical and angular components. The influence of the horizontal component in this case is insignificant, the vertical and angular components and their correlation degree are more significant. Such a statement of the problem is very relevant for the structures located near the earthquakes’ epicenter, where the vertical component of displacements significantly exceeds the horizontal component. The need to take it into account is included in the design standards of the earthquake-resistant nuclear plants [1].

The simple variants of the beams’ random oscillations of are considered in the publication [2].

Mathematical Model of Oscillations
Without repeating the statement of the problem and the course of its mathematical modeling, we start with Figure 1 and the final simulation result. As it turned out, the main equation has the form of a homogeneous differential equation of hyperbolic type in partial derivatives with variable coefficients [3, 4].

\[ (b(x)u'')'' + Pu'' + r(x)u + \mu m(x)u = 0, \quad x \in (0, l), \quad t > -\infty, \]
\[ b(x) = EJ(x), \quad r(x) = m(x) + q. \]

Here \( u(x, t) \) - are the desired function of the beam deflection during movement, \( b(x) \) - is the bending stiffness, variable along the beam length, \( J(x) \) - defines the axial moment of inertia of the cross section, \( r(x) \) - is the total linear mass of the beam, \( \mu \) - is the specific coefficient of linear-viscous internal friction, defined in the first part of the article, \( m(x) \) - is the linear beam mass. The dashes in the upper index indicate the differentiation of the deflection function by the coordinate \( x \), the dots above the symbols – denote the time differentiation \( t \).

The boundary conditions are added to the main equation (1)
\[ u(0, t) = f_1(t), \quad [b(l) u''(l, t)]' - cu(l, t) = f_2(t), \quad u'(0, t) = f_3(t), \quad u''(l, t) = 0, \quad t > -\infty, \]
\[ c \] is the stiffness coefficient of the elastic support. There is no need for initial conditions, since the steady-state oscillation modes that last for a long time and pose the greatest danger will be considered.

The equation (1) and boundary conditions (2) form a mathematical model that makes it possible to determine the function \( u(x, t) \) for the forced oscillations.

**Forced Harmonic Oscillations**

The harmonic oscillation problem is considered for two reasons:

1. They can actually occur during the man-made disturbances of the beam supports;
2. The results of solving this problem greatly simplify the solution of the random oscillations problems during seismic and man-made impacts.

In the design scheme according to Fig. 1 we will now assume that
\[ f_1(t) = a_1 e^{j(\omega t + \varphi_1)}, \quad f_2(t) = a_2 e^{j(\omega t + \varphi_2)}, \quad f_3(t) = a_3 e^{j(\omega t + \varphi_3)}. \]

Here \( a_1 \) - denotes the disturbance amplitudes, \( \varphi_1 \) - shows the initial disturbances phases.

In this case, the boundary conditions (2) by virtue of (3) take the form:
\[ u(0, t) = a_1 e^{j(\omega t + \varphi_1)}, \quad u'(0, t) = a_3 e^{j(\omega t + \varphi_3)}, \quad [b(l) u''(l, t)]' - cu(l, t) = a_2 e^{j(\omega t + \varphi_2)}, \quad u''(l, t) = 0. \]

Since the system under consideration is linear, and the input processes \( f_1, f_2, f_3 \) have the same frequencies when considering oscillations, we use the principle of superposition, which states that the result of autonomous effects on the beam of three sources \( f_1, f_2, f_3 \) can be summed, i.e. \( u(x, t) \) there is a functional
\[ u(x,t) = L \{ f_1(t), f_2(t), f_3(t) \}, \]

where \( L \) is the linear operator mapping functions \( f_1(t), f_2(t), f_3(t) \) output function \( u(x,t) \).

The mathematical model of oscillations in this case takes the form

\[
\begin{align*}
(b(x)u^*)'' + Pu^* + r(x)u + \mu_0(x)u &= 0, \quad x \in (0, l), \quad t > -\infty, \\
u(0,t) &= a_0 e^{i(\omega t + \phi_0)}, \quad u'(0,t) = a_2 e^{i(\omega t + \phi_2)}, \\
[b(l)u''(l,t)]' - cu(l,t) &= a_3 e^{i(\omega t + \phi_3)}, \quad u''(l,t) = 0, \quad t > -\infty.
\end{align*}
\]

Let us consider the steady-state forced oscillations, which are undamped and harmonic, and their initial phase does not play a role. Therefore, the corresponding function has the form in the method of the variables’ separation [5]

\[ u(x,t) = X(x)e^{j\omega t}. \quad (6) \]

Here \( X(x) \) defines the amplitude function. Similarly to the case of free oscillations, using the methods of separation of variables and finite differences [6], we obtain a system of linear algebraic equations, but it is already heterogeneous:

\[ B(x, \omega) \ Y = d, \quad (7) \]

where \( B(x, \omega) \) – is the square order matrix \( n \), \( d \) – defines the column vector, \( Y \) – defines the discrete argument column vector \( Y = \{ y_1, y_2, ..., y_n \} \), \( y_i \approx X(x_i) \). We write out (10) in the expanded form:

\[
\begin{bmatrix}
1 \\
-3 & 4 & -1 \\
v_3 & \alpha_3 & \beta_3 & \gamma_3 & \xi_4 \\
v_4 & \alpha_4 & \beta_4 & \gamma_4 & \xi_4 \\
... & ... & ... & ... & ... \\
v_{n-2} & \alpha_{n-2} & \beta_{n-2} & \gamma_{n-2} & \xi_{n-2} \\
-1 & 4 & -5 & 2
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2 \\
... \\
y_n
\end{bmatrix}
= 
\begin{bmatrix}
d_1 \\
d_2 \\
... \\
d_{n-1}
\end{bmatrix}.
\]

Here the zero elements of the matrix and the vector \( d \) are not written out, the notations are introduced:

\[
\begin{align*}
\nu_i &= 1 - b_i h / b_i, \quad \alpha_i = -4 + 2b_i h / b_i + Ph_i^2 / b_i, \quad \beta_i = 6 - 2Ph_i^2 / b_i + e_i h_i^2 / b_i, \\
\gamma_i &= -4 + 2b_i h / b_i + Ph_i^2 / b_i, \quad \zeta_i = 1 + b_i h / b_i, \quad \varepsilon_i = \rho_i^2 + j\lambda_i m_i, \quad b_i = b(x_i), \quad b_0 = b'.
\end{align*}
\]

\[
\begin{align*}
\delta &= -2hb_0 / b_n - 14, \quad \beta_0 = -211 \cdot 10^{-8} E, \quad \sigma = 8hb_0 / b_n + 24, \quad \rho = -10hb_0 / b_n - 18, \\
\kappa &= (4hb_0 - 2h^3 c) / b_n + 5, \quad d_1 = a_1 e^{j\omega t}, \quad d_2 = 2ha_3 e^{j\omega t}, \quad d_{n-1} = 2h^3 c a_2 e^{j\omega t} / b_n, \quad d_n = 0.
\end{align*}
\]

Let us perform the calculations for the specific beam of variable cross section.

**Example 1.** The initial data:

\[
\begin{align*}
l &= 6 \text{ m}, \quad n &= 601, \quad m(x) = 21 - 1.2x \quad \text{kg/m}, \quad q = 300 \text{ kg/m}, \quad P = 50 \text{ kN}, \\
J(x) &= (1840 - 211x)10^8 \quad \text{m}^4, \quad c = 10 \text{ kN/m}, \quad \mu = 0.01 \text{ s}^{-1}, \\
a &= \{ 10 \text{ cm} \ 10 \text{ cm} \ 0.012 \text{ rad} \}, \quad \phi = \{ 0 \ 0 \ 0 \} \text{ rad}
\end{align*}
\]

The dependence of the oscillations’ amplitude \( Y(x_i, \omega) \) from the common-mode seismic actions’ frequencies at angular frequencies is under consideration:

\[ \omega = \{ 1 \ 3.5 \ 25 \ 35 \ 100 \} \text{ s}^{-1}. \]

The results obtained are shown in the form of curves in Fig. 2. The numbers indicate the components of the frequency vector.
Let us analyze the evolution of the curves with an increase in the perturbations’ frequency, taking into account the eigenvalues of free oscillations found in Part I

$$\omega_0 = [11.01 \ 54.57 \ 147.52] \text{ s}^{-1}.$$

It can be seen that for the small frequency values (curves 1, 2), the beam remains almost straight, similar to the first eigenfunction. But as the perturbation frequencies approach the second, third eigenfrequencies, the undulation of the lines and their large curvature appear (lines 3-5), which indicates the influence of the corresponding eigenfunctions.

![Figure 2. The amplitudes of in-phase harmonic oscillations.](image)

A significant dependence of the oscillation amplitudes on the perturbation frequencies is revealed, both in numerical deviations’ values and in the curvature of the axial lines, on which the strength and reliability of the beams during their operation depend. The frequency fluctuations 100 s\(^{-1}\) (curve 5) have the greatest curvature and therefore will be the most dangerous, since they will be accompanied by large deformations and stresses. The amplitudes of oscillations at the right end reach a large value (≈ 25 cm), which may be unacceptable. In this case, it is necessary to take the constructive measures to reduce them.

This conclusion gives an opportunity to predict, a significant effect of the frequency composition of seismic effects (spectral density) on the probabilistic dynamic characteristics of beams for the random oscillations studied below.

Further, we consider the dependence of the beam oscillations on the phase shift of harmonic disturbances with the same data. These oscillations are not in phase, but are coherent.

**Example 2.** The initial data of example 1 are saved in addition to the initial phases of disturbances:

$$\phi = \{ 0, \ \pi, \ \pi/2 \} \text{ rad.}$$

![Figure 3. The effect of the disturbances’ phase shift (8) on oscillations](image)
The results are shown in Figure 3, where the curve numbers correspond to the numbers component of the frequency vector. Here the oscillations are antiphase. The comparison of Figure 2 and Figure 3 shows that with low-frequency coherent oscillations (curves 1, 2), the difference in deviations is significant, in-phase oscillations are larger. The forced oscillations’ forms are similar to the corresponding intrinsic forms of free oscillations. At high perturbation frequencies (curves 3–5), the differences are insignificant, the oscillations’ forms are similar to the free oscillations’ eigenfunctions.

The repetition of the calculations with a different combination of initial disturbance phases:
\[ \phi = \{ 0, \pi/2, \pi \} \text{ rad.} \]  
(9)
gave a significant change in the graphs (Fig. 4) compared to previous graphs of Figure 3. For example, there were significant decreases in the amplitudes of the oscillations and their sign at low perturbation frequencies (curves 1, 2). At the same time, the amplitudes of high-frequency oscillations did not change (curves 3–5).

Such conclusions require a more extensive study of the disturbances phase shift effect on the oscillations’ amplitude than in this example. This is also important because the stochastic analogs of such effects are likely to be detected during random oscillations. The additional arguments are the results of several calculations with other values of the disturbances’ phase shift, which revealed the significant changes in the amplitudes. It can ultimately be argued that the phase shift of the kinematic effects on the supports should be taken into account in the calculations when designing beams.

\[ \text{Figure 4. The effect of the disturbances’ phase shift (9) on the oscillations} \]

**Forced random oscillations**

The results of solving the deterministic problems considered above show that a change in such an unimportant impact parameter as a phase shift is capable of leading to fundamental changes in the oscillation parameters. All the more volatility should be expected in the behavior of dynamic systems under the random perturbations’ action. The ever-increasing requirements for evaluating the reliability and efficiency of the designed buildings and structures continue to cause increased interest in probabilistic methods of calculation. Their use brings the design schemes of beams and the existing loads to real situations as close as possible. As a consequence of such requirements, interest has arisen and continues to oscillate the beams during impacts of a stochastic nature.

There are several methods for solving the stochastic problems on the linear distributed systems: the method of differential equations for the moment functions, the method of Green functions, the method of spectral representations, the method of generalized coordinates, etc. An attempt to use them will lead to the need for large transformations of the mathematical model of this problem and the creation of complex computer programs. It is possible to offer a simple and transparent way to solve this problem, using the problem of forced harmonic oscillations of a beam already solved above.

Beams are most at risk during the long-lasting stationary phase of earthquakes and man-made impacts. Therefore, we consider the case when the perturbations are presented as the stationary random processes.
In the mathematical model for harmonic oscillations given above, we introduce the necessary changes for random oscillations. They will concern only boundary conditions, namely, the input vector \( f(t) = \{ f_1(t), f_2(t), f_3(t) \} \). Its components will now be interpreted as the components of a centered stationary vector random process, represented by their spectral densities in the form of a symmetric matrix

\[
S_f(\omega) = \begin{pmatrix}
s_{11} & s_{12} & s_{13} \\
s_{21} & s_{22} & s_{23} \\
s_{31} & s_{32} & s_{33}
\end{pmatrix}, \quad s_{ij}(\omega) = s_{ji}(\omega).
\]

Then in steady state process \( u(x,t) \) will be a centered spatiotemporal random field, stationary in time and inhomogeneous in spatial coordinate. The task is to find the variances (or standard deviations) of the output process from the given spectral matrix of the input random process. The matrix elements \((7)\) have the form for the most frequently occurring earthquakes:

\[
s_{ij}(\omega) = \frac{2\alpha_{ij}\theta_{ij}^2k_{ij}\sigma_i\sigma_j}{\pi[\theta_{ij}^2 - \theta_{ij}^2]^2 + 4\alpha_{ij}^2\omega^2}, \quad \theta_{ij}^2 = \alpha_{ij}^2 + \beta_{ij}^2, \quad i, j = 1, 2, ..., 5. \quad (10)
\]

Here \( \alpha_{ij} \) and \( \beta_{ij} \) – are the broadband and characteristic frequency parameters, \( \sigma_i \) – is the process standard deviations \( f_i(t) \). \( k_{ij} \) – defines the elements of the normalized correlation matrix

\[
K_f = \begin{pmatrix}
k_{11} & k_{12} & k_{13} \\
k_{21} & k_{22} & k_{23} \\
k_{31} & k_{32} & k_{33}
\end{pmatrix}.
\]

This matrix is symmetric, non-negative definite, the elements of the main diagonal are equal to unity, i.e. \( k_{ii} = 1, i = 1, 2, 3 \).

Such a model makes it possible to artificially make the random perturbations sufficiently close to the harmonic perturbations. Thus, it becomes possible to repeat the solution of the harmonic oscillation problem given above, but with the help of the random functions’ theory apparatus, i.e., the deterministic problem can be considered as a special case of the stochastic problem. Two tasks become a test for each other.

Now we consider the abstract and simple case where the kinematic perturbation \( f(t) \) will be a scalar random function. Let us find the variance of displacements \( D_0(\chi) \) in the rod. It is known that a continuous random centered stationary process can be represented as a Fourier series [7]

\[
f(t) = \sum_{i=1}^{\infty} (A_i \cos \omega_i t + B_i \sin \omega_i t). \quad (11)
\]

Here \( A_i \) and \( B_i \) – are the uncorrelated random variables. The simple transformations that take into account the relationship between the dispersion and the correlation function of a stationary random process and the uncorrelated coefficients of the series (11) give the reason to write the random process variance as the total:

\[
D[f(t)] = \sum_{i=1}^{\infty} D_i.
\]

This result is shown graphically in Figure 5 in the form of the so-called discrete line spectrum of dispersions. According to Figure 5 it turns out that each discrete dispersion \( D_i \) is assigned to a specific frequency \( \omega_i \). It is easy to see that the limit transition from the discrete values \( \omega_i \) to the continuum \( \omega \) will lead to a continuous spectral density function (Figure 6). Moreover, the variance of the input random process \( f(t) \) seems to be an improper integral of the spectral density:

\[
\]
\[ D[f(t)] = \int_{-\infty}^{\infty} S(\omega)\,d\omega. \tag{12} \]

Now each discrete frequency \( \omega_i \) is associated with an elementary variance \( S(\omega)\,d\omega \), hatched in Fig. 6.

Let us return to the original problem in full for the generalized conclusions. If elementary variance is taken instead of the input harmonic process amplitude \( f(t) \) then at the output of the problem an elementary variance of deviations \( D_u(\omega_k, x) \) will be obtained. The subsequent summation (integration) over \( \omega_k \) gives a variance of deviations at a discrete point \( D_u(x_i) \).

We take into account that the dispersion of the input perturbations is distributed on the axis \( \omega \) frequency according to Fig. 6 and set the continuous spectral density to the discrete one calculated on an infinite countable set of positive frequencies \( S_i = S(\omega_i), \omega_i > 0 \).

Then the entire variance of the input process (12) can be represented as the sum of the harmonics elementary variances

\[ D_f \approx 2h_\omega \sum_k S_{k}. \]

Here \( h_\omega \) — is the step of dividing the frequency axis, the factor 2 takes into account the negative half-axis of the frequencies. Further, the calculations are carried out as in example 2, namely, instead of the harmonics \( f \), the elementary variance of disturbances is inserted into the algorithm and the calculation program \( S_k \).

The results are summarized as the output displacement process variance. One of the reasons that led to the inclusion of harmonic oscillations in this article is the simplicity of the above-mentioned method for calculating the variances. It has already been tested in a number of problems on the vertical rods’ oscillations [8, 9, 10].

**Example 3.** Initial data as in example 1. The parameters of random actions were added:

\[
\begin{align*}
\ell &= 6 \text{ m}, \quad n = 601, \quad m(x) = 21 - 1.2x \text{ kg/m}, \quad q = 300 \text{ kg/m}, \quad P = 50 \text{ kN}, \\
J(x) &= (1840 - 211x)\times 10^8 \text{ m}^4, \quad c = 10 \text{ kN/m}, \quad \mu = 0.01 \text{ s}^{-1}, \\
\sigma &= \{10 \text{ cm} \quad 10 \text{ cm} \quad 0.012 \text{ rad}\}, \\
\alpha &= 0.1, \quad \sigma = \{10 \text{ cm} \quad 10 \text{ cm} \quad 0.012 \text{ rad}\}, \quad \beta = \{1 \quad 3.5 \quad 25 \quad 35 \quad 100\} \text{ s}^{-1}. \end{align*}
\]

\[
1. \quad K_f = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad 2. \quad K_f = \begin{bmatrix} 1 & 0.7 & 0.2 \\ 0.7 & 1 & -0.4 \\ 0.2 & -0.4 & 1 \end{bmatrix}. \]

The whole variety of matrices cannot be considered. Therefore, we consider the most characteristic cases. For example, we simplify the spectral matrix (8) to the form:
3.2, 1, \alpha, \beta, \theta, \alpha \omega, \sigma_i, \sigma_j, k_{ij}, s_{ij}^f(\omega) = \frac{2 \alpha}{\pi \left(\omega^2 - \theta^2\right)} k_{ij} \sigma_i \sigma_j, \quad \theta^2 = \alpha^2 + \beta^2, \quad i, j = 1, 2, 3.

For the first calculation, we take a stochastic analogue of example 1 (Fig. 2) in-phase oscillations. The “phase” of random oscillations from three external influences will consist in the fact that the correlation matrix $K_f$ has the form 1 with equal frequencies in the spectral densities. All three components are independent of each other. The results are shown in the graphs of Figure 7. They have many similarities with the graphs in Figure 2.

Figure 7. RMS deviations from uncorrelated effects 1.

The parameters of random influences are selected so that the random fluctuations are as close to harmonic as possible. The goal was to be able to verify simultaneously the two results by comparison. A simple visual review of Figure 2 and Figure 7 reveals many similarities in the oscillations. For example, the magnitudes of the amplitudes and standard deviations are approximately the same. The waviness of the curves with random deviations is greater than that of the amplitudes. The logic of this fact is that the random disturbances’ spectrum contains an infinite number of frequencies, including those that coincide with the eigenfrequencies, and therefore the resonant oscillations appear.
Figure 8. RMS deviations for correlated effects 2.

Apparently, the standard deviations of curve 3 Figure 7 significantly exceeded the amplitudes of curve 3 of Figure 2 for this reason. In general, as the growth dominant exposure frequency $\beta$ there is a change in the oscillations’ forms. First, they occur in the first proper form, then in the second, third, etc. Such coincident dynamics of oscillations observed during free, forced harmonic and random oscillations is a kind of verification of mathematical models, algorithms, and programs for all these types of oscillations.

We consider the more general case of random oscillations, when all three perturbations are correlated with each other, which corresponds to the correlation matrix 2 of the problem conditions while maintaining the remaining input data. The calculation results are shown in Figure 8. The first conclusion from the analysis of the graphs is that there was a strong decrease in the standard deviations compared to the previous case. The obvious reason is the components’ correlation of the perturbations vector random process, which leads to mutual interference, “competition” between them. As in the previous case, curve 3 has the large ordinates caused by the resulting resonant oscillations.

Summary
1. The horizontal component of seismic and man-made impacts should be taken into account in the calculations for the beams design.
2. A significant dependence of the oscillation amplitudes on the perturbation frequencies was found, both in the numerical values of the deviations and in the curvature of the axial lines, on which the strength and reliability of the beams during their operation depend.
3. A more extensive study of the action components’ phase shift influence on the oscillation amplitudes is required, since they should be taken into account in the calculations when designing beams.
4. The correlation of the vector random process components very significantly affects the beam’s deviations during oscillations. Therefore, it should be determined by the results of experimental data and taken into account in beam calculations.

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