In compensated two-component systems in confined, two-dimensional geometries, vorticity of the electronic flow and the associated nonlocality may appear due to external magnetic field. Within a phenomenological two-fluid framework, we demonstrate the evolution of charge flow profiles and the emergence of a giant vortex-like pattern dominating charge transport in magnetic field. Applying our approach to the specific case of intrinsic graphene, we suggest a simple physical explanation for the experimental observation of giant nonlocality. Our results provide an intuitive way to predict the outcome of future experiments exploring the rich physics of many-body electron systems in confined geometries as well as to design possible applications.

The trend towards miniaturization of electronic devices requires a deeper understanding of the electron flow in confined geometries. In contrast to the electric current in household wiring, charge flow in small chips with multiple leads may exhibit complex spatial distribution patterns depending on the external bias, electrostatic environment, chip geometry, and magnetic field. Traditionally, such patterns were detected using nonlocal transport measurements \cite{11,17}, i.e. by measuring voltage drops between various leads other than the source and drain. Devised to study ballistic propagation of charge carriers in mesoscopic systems, these techniques were recently applied to investigate possible hydrodynamic behavior in ultra-pure conductors \cite{8,12}, where the unusual behavior of the nonlocal resistance is often associated with viscosity of the electronic system \cite{13,17}.

Nonlocal resistance measurements have also been used to study edge states accompanying the quantum Hall effect \cite{18,23}. While the exact nature of the edge states has been a subject of an intense debate, the nonlocal resistance, $R_{NL}$, appears to be an intuitively clear consequence of the fact that the electric current flows along the sample edges and not through the bulk. Such a current would not be subject to exponential decay \cite{24}, exhibited by the bulk charge propagation leading to a much stronger nonlocal resistance.

In recent years the focus of the experimental work on electronic transport has been gradually shifting towards measurements at nearly room temperatures \cite{6,8,11,21}. A particularly detailed analysis of the nonlocal resistance in a wide range of temperatures, carrier densities, and magnetic fields was performed on graphene samples \cite{21}. Remarkably, the nonlocal resistance measured at charge neutrality remained strong well beyond the quantum Hall regime, with the peak value $R_{NL} \approx 1.5 \text{k} \Omega$ at $B = 12 \text{T}$ and $T = 300 \text{K}$, three times higher than that at $T = 10 \text{K}$.

In this Letter, we argue that giant nonlocality observed in intrinsic graphene at high temperatures can be attributed to the presence of two types of charge carriers (electrons and holes): at the neutrality point, the two bands (the conductance and valence bands) touch creating a two-component electronic system. Physics of such systems is much richer than in their single-component counterparts. Observed phenomena that are directly related to the two-band structure of the neutrality point include giant magnetodrag in graphene \cite{25,26} and linear magnetoresistance \cite{27,28}. Both effects have been explained within a phenomenological framework \cite{26,27} allowing for a two-component (electron-hole) system coupled by the external magnetic field. We generalize this approach to investigate evolution of the spatial distribution of the electron current density in the experimentally relevant Hall bar geometry. In a sufficiently strong magnetic field, the current density forms a giant vortex-like pattern where the current is flowing not only in the bulk, but also along the boundaries leading to strong nonlocal resistance, see Fig. 1. Such patterns can be directly observed in laboratory experiments using the modern imaging techniques \cite{29,31}. Tuning the model parameters to the specific values available for graphene, we arrive at a quantitative estimate of the nonlocal resistance \cite{21}.

To highlight the difference between the one- and two-component systems, we briefly recall the macroscopic description of electronic transport in the standard (former) case. Allowing for nonuniform charge density, the linear...
relation between the electric current $\mathbf{J}$ and the external fields $\mathbf{E}, \mathbf{B}$ could be formulated as 

$$r_0\mathbf{J} = \mathbf{E} + r_H \mathbf{e}_B \times \mathbf{J} + (e\nu_0)^{-1}\nabla n,$$  

(1a)

where $e > 0$ is the unit charge, $\nu_0$ is the density of states (DoS), $n$ is the carrier density, $\mathbf{e}_B$ is the unit vector in the direction of the magnetic field, and $r_0$ and $r_H$ are the longitudinal and Hall resistivities. Within the Drude-like description, $r_H = \omega_c\tau r_0$ ($\omega_c$ is the cyclotron frequency and $\tau$ is the mean free path). The relation Eq. (1a) is applicable to a wide range of electronic systems from simple metals \cite{34, 35} to doped graphene \cite{11, 36}. The transport coefficients $r_0$ and $r_H$ could be treated as phenomenological or could be derived from the underlying kinetic theory \cite{11, 32, 37}.

In addition to Eq. (1a), the electric current satisfies the continuity equation, which for stationary currents reads

$$\nabla \cdot \mathbf{J} = 0.$$  

(1b)

Charge density inhomogeneity induces electric field, so that Eq. (1a) should be combined with the corresponding electrostatic problem. Most recent experiments were performed in gated structures, where the relation between the electric field and charge density simplifies \cite{27, 38}. In two-dimensional (2D) samples

$$\mathbf{E} = \mathbf{E}_0 -(e/C)\nabla n,$$  

(1c)

where $C = e/(4\pi d)$ is the gate-to-sample capacitance per unit area, $d$ is the distance to the gate, $e$ is the dielectric constant, and $\mathbf{E}_0$ is the external field.

In a two-terminal (slab) geometry, solution of Eqs. (1) is a textbook problem. In the absence of magnetic field, the resulting electrochemical potential is governed by the relation of the mean free path to the system size, exhibiting either a flat (in short, ballistic samples) or linear (in long, diffusive samples) spatial profile. Most recently, these solutions were used as benchmarks in the imaging experiment \cite{29} and the numerical solution of the hydrodynamic equations in doped graphene \cite{17}. In external magnetic field, the system exhibits the classical Hall effect, which in short samples is accompanied by nontrivial current flow patterns \cite{39}.

In a four-terminal Hall bar geometry, the electric current still fills the whole sample, but decays exponentially \cite{24} away from the direct path between source and drain. The resulting flow pattern was calculated (in the context of doped graphene) in Refs. \cite{14, 15, 17}. In magnetic field, the pattern gets skewed due to the classical Hall effect, but exhibits no qualitatively new features, see Fig. 2.

Let us now extend the transport equations (1) to a two-component system. Keeping in mind applications to graphene, we re-write Eq. (1a) for the quasiparticles in the conduction band ("electrons") in the form

$$- j_e = eD\nu_e \mathbf{E} + \omega_c\tau j_e \times \mathbf{e}_B + D\nabla n_e,$$  

(2a)

and

$$- j_h = -eD\nu_h \mathbf{E} - \omega_c\tau j_h \times \mathbf{e}_B + D\nabla n_h.$$  

(2b)

Here the electric current carried by the holes is $\mathbf{J}_h = e\mathbf{j}_h$ and DoS may differ from that of electrons, $\nu_h \neq \nu_e$. For simplicity, we assume that the the cyclotron frequency, mean free time, and diffusion constant for the two bands coincide (a generalization is straightforward, but doesn’t lead to qualitatively new physics).

The total electric current in the two component system is given by $\mathbf{J} = -e\mathbf{j}$, where $\mathbf{j} = j_e - j_h$. Introducing also the total quasiparticle flow $\mathbf{j}_f = j_e + j_h$, we find (cf. Ref. \cite{37})

$$\mathbf{j} + eD(\nu_e + \nu_h) \mathbf{E} + \omega_c\tau j_f \times \mathbf{e}_B + D\nabla n = 0,$$  

(3a)

$$\mathbf{j}_f + eD(\nu_e - \nu_h) \mathbf{E} + \omega_c\tau j_f \times \mathbf{e}_B + D\nabla \rho = 0,$$  

(3b)

where $n = n_e - n_h$ is the carrier density per unit charge (the charge density being $-en$) and $\rho = n_e + n_h$ is the total quasiparticle density. The transport equations have

FIG. 2: Classical Hall effect in a one-component electronic system. The current density (shown by the arrows) and the electrochemical potential (shown by the color map) were obtained from Eqs. (1) for a sample of the width $W = 1 \mu m$ and length $L = 4 \mu m$ with the carrier density $n = 10^{12} \text{cm}^{-2}$ at the temperature $T = 240 \text{K}$ and in magnetic field $B = 2 T$.

FIG. 3: Charge flow in compensated semimetals. Top: Ohmic flow in the absence of magnetic field. Bottom: emergent vortex in weak magnetic field $B = 0.4 \text{mT}$. The vortex grows with the increasing field filling the whole sample, see Fig. 4 for the pattern at $B = 10 \text{mT}$. Stronger fields expel the current from the bulk such that it flows along the boundary.
to be supplemented by continuity equations reflecting the particle number conservation. The electric current satisfies Eq. (1b), but the total number of quasiparticles can be affected by electron-hole recombination processes leading to a weak decay term in the continuity equation
\[ \nabla \cdot j = -\delta \rho / \tau_R, \quad (3c) \]
where \( \delta \rho \) is the deviation of the quasiparticle density from its equilibrium value and \( \tau_R \) is the recombination time.

Under the assumption of electron-hole symmetry (e.g., at the charge neutrality point in graphene), \( \nu_e = \nu_h \), we recover the phenomenological model of Ref. [27]. In the two-terminal geometry this model yields unsaturating linear magnetoresistance in classically strong fields [25].

Now we analyze the behavior of the phenomenological model [2] in the four-terminal Hall bar geometry. In the absence of the magnetic field, the system exhibits a typical Ohmic flow [14, 15, 17], see the top panel in Fig. 3. Applying the field we find a qualitative change in the flow pattern – the emergence of a vortex at the end of the Hall bar. Increasing the field leads to the vortex growing until it fills the whole sample, see Figs. 3 and 4. Stronger fields essentially expel the current from the bulk with the charge flow being concentrated along the sample boundaries, which leads to strong nonlocal resistance.

The vortex-like pattern emerging in magnetic field, see Figs. 3, 4, and 5 has to be contrasted with the vortices appearing in the viscous hydrodynamic flow (e.g., in doped graphene [14, 15, 17, 11]). In the latter case, vorticity appears due to the constrained geometry of the flow and the particular boundary conditions [15, 17, 12]: neglecting Ohmic effects, the solution of the hydrodynamic equations can be obtained by introducing the stream function, which obeys a biharmonic equation independent of viscosity, which however affects the distribution of the electrochemical potential. In contrast, within the model [3] the “Ohmic” scattering represents the only source of dissipation and hence cannot be omitted. One can still introduce the stream function, but now it is determined not only by the sample geometry, but also by the Ohmic scattering and magnetic field. As a result, the flow pattern exhibits a single vortex, unlike the multiple vortices suggested recently for the hydrodynamic flow in intrinsic graphene [11] (in the absence of magnetic field).

Nonlocal resistance in graphene subjected to external magnetic field was studied experimentally in Ref. [21]. At high enough temperatures where signatures of the quantum Hall effect are washed out, strong (or “giant”) nonlocality was observed at the neutrality point. The effect vanishes in zero field as well as with doping away from neutrality. Both features are consistent with the model [3]: in zero field the model exhibits usual Ohmic flow patterns, see Fig. 3, while at sufficiently high doping levels the effects of the second band are suppressed – the two equations (3a) and (3c) become identical showing the response typical of one-component systems, see Fig. 2.

Having discussed the qualitative features of the charge flow in two-component systems, we now turn to a quantitative calculation of nonlocal resistance in graphene. Although the model [3] is applicable to any semimetal, graphene is a by far better studied material with readily available experimental values for model parameters. Here we use the data measured in Refs. [8, 9, 21, 26, 43] and theoretical calculations of Refs. [11, 12, 26, 37, 41].

DoS of the quasiparticles in graphene has been evaluated in, e.g., Refs. [11, 12, 26, 37], and has the form
\[ \nu_e + \nu_h = 2T / (\pi v_g^2), \quad \nu_e - \nu_h = 2\mu / (\pi v_g^2), \quad (4) \]
where \( \mu \) is the chemical potential, \( v_g \) is the quasiparticle velocity in graphene, and \( T = 2T \ln[2\cosh(\mu/2T)] \). The generalized cyclotron frequency is \( \omega_c = eBv_g^2/(\mu T) \) and the diffusion coefficient has the usual form \( D = v_g^2 \tau / 2 \). At charge neutrality, \( \mu = 0 \) and \( T = 2T \ln 2 \), while in the degenerate regime \( T(\mu \gg T) = \mu \). The latter confirms that all coefficients in Eqs. (3a) and (3c) become identical with doping. Similarly, the continuity equations (1b) and (3c) should coincide in the degenerate regime. In graphene this happens by means of the fast decay of the recombination rate [20]. Close to neutrality we assume
\[ \tau_R^{-1} = g^2 T / \cosh(\mu/T), \quad (5) \]
where \( g \) is determined by the corresponding matrix element. The above expression [20] reflects the exponential
decay of the two-band physics away from charge neutrality, which is responsible for the fast decay of $R_{NL}$ as a function of carrier density [21], see Fig. 5. Finally, the mean-free time, $\tau$, in graphene is a non-trivial function of temperature and carrier density [11, 12, 86, 43, 44], which strongly depends on the model of the impurity potential [45]. However, these dependencies are typically not exponential and hence do not affect the exponential decay of the nonlocal resistance.

In Fig. 5 we demonstrate the decay of $R_{NL}$ for two impurity models – the Coulomb scatterers and short-ranged impurities – showing nearly identical behavior. Such robustness of the model [3] with respect of the functional dependence of the mean free time justifies the inaccuracy of our description of electronic transport in graphene, where close to charge neutrality the resistivity is strongly affected by electron-electron interaction. The data shown in Fig. 5 were obtained by solving Eqs. [3] in the Hall bar geometry of Fig. 4 using the estimate [20] for the recombination length scale, $\ell_R = v_t \tau R \approx 1.2 \mu m$. A more recent calculation of Ref. [41] puts it at a larger value 14 $\mu m$, which leads to similar results for the nonlocal resistance, but with a smaller peak value at charge neutrality.

The results for $R_{NL}$ shown in Fig. 5 are extremely similar to those reported in Ref. [21] with the exception of the peak value at neutrality, which is grossly exaggerated. There are several reasons for this behavior. Firstly, by ignoring the effects of electron-electron interaction, we strongly underestimate the usual resistivity of intrinsic graphene. Secondly, we ignore viscous effects. Finally, DoS in real graphene never really vanishes at neutrality due to electrostatic potential fluctuations [51]. As a result, the minimal carrier concentration is often as high as $10^{10} cm^{-2}$, essentially cutting off the lower density range around the peak in Fig. 5.

To conclude, we have argued that the observed giant nonlocality in neutral graphene in non-quantizing magnetic fields at relatively high temperatures observed in Ref. [21] is a direct consequence of the two-band nature of the quasiparticle spectrum in graphene. As such, this effect is not specific to graphene and should be observable in any compensated two-component system. Our theory does not involve spin-related phenomena including the effect of Zeeman splitting invoked in Ref. [21]. The latter should be independent of the field direction, however, the effect was not observed in the nearly parallel field studied in Ref. [51]. Assuming the $g$-factor to be equal to 2, we estimate the Zeeman splitting as $E_z = 0.35 \text{meV} \approx 4 \text{K}$ at $B = 10 \text{T}$. The corresponding residual quasiparticle density (at $T = 0$) is given by $\rho_{Q} = E_z^2 / (4 \pi v_F^2) \approx 2.2 \times 10^6 \text{cm}^{-2}$. As a result, we expect the effects of Zeeman splitting to be observable at temperatures and carrier densities much lower than those typical to nonlocal measurements discussed here.

With material-specific parameters, our phenomenological model is capable of a quantitative description of the effect. In the case of graphene, a more precise calculation involving solution of the full system of hydrodynamic equations near charge neutrality is required to reach perfect agreement with the data, however the present approach shows that the effect is more general and does not require additional assumptions necessary for electronic hydrodynamics.

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