Flavoured Leptogenesis in the CTP Formalism

Martin Beneke, Björn Garbrecht, Christian Fidler, Matti Herranen

Institut für Theoretische Teilchenphysik und Kosmologie,
RWTH Aachen University,
D–52056 Aachen, Germany

and

Pedro Schwaller

Institut für Theoretische Physik,
Universität Zürich, CH–8057 Zürich, Switzerland

Abstract

Within the Closed Time Path (CTP) framework, we derive kinetic equations for particle distribution functions that describe leptogenesis in the presence of several lepton flavours. These flavours have different Standard-Model Yukawa couplings, which induce flavour-sensitive scattering processes and thermal dispersion relations. Kinetic equilibrium, which is rapidly established and maintained via gauge interactions, allows to simplify these equations to kinetic equations for the matrix of lepton charge densities. In performing this simplification, we notice that the rapid flavour-blind gauge interactions damp the flavour oscillations of the leptons. Leptogenesis turns out to be in the parametric regime where the flavour oscillations are overdamped and flavour decoherence is mainly induced by flavour sensitive scatterings. We solve the kinetic equations for the lepton number densities numerically and show that they interpolate between the unflavoured and the fully flavoured regimes within the intermediate parametric region, where neither of these limits is applicable.
1 Introduction

Calculations of the baryon asymmetry of the Universe require the description of $CP$-violating processes in a finite-density background. Conventionally, semi-classical Boltzmann equations are employed, where one computes the evolution of classical particle number densities in the presence of scattering processes, that are given by matrix elements from the in-out formalism. Since $CP$ violation in these matrix elements is a higher order effect that is typically induced by loop diagrams, where particles in the loop are kinematically allowed to be on shell, this approach bears notorious difficulties concerning the correct counting: Is a certain process higher order or is it a combination of two leading order processes, that are already accounted for in the Boltzmann equations? For the $CP$-violating processes in baryogenesis, a way of addressing this caveat is the method of real intermediate state (RIS) subtraction [1].

A more direct and systematic way to deal with this counting problem is to avoid the detour via the in-out matrix elements and instead to formulate the problem directly in terms of Green functions. A method that achieves this is given in terms of the Closed Time Path (CTP) or in-in formalism [2,3], which has been successfully applied to various models of baryogenesis including also a realistic scenario of leptogenesis [4–10]. The set of Green functions that are computed in the CTP framework encompasses both, the spectral and the statistical information of the system. To leading accuracy in a weakly interacting situation, it is often sufficient to approximate the spectral functions by on-shell $\delta$-functions, while the statistical contributions encode the distribution functions of quasi-particles. In this way, the kinetic equations that govern leptogenesis have been recovered systematically and additional corrections due to the full quantum statistical distributions of right-handed neutrinos and the leptons and Higgs boson of the Standard Model have been derived [9].

The discussion in Ref. [9] is for a single flavour of Standard Model leptons $\ell$, which is appropriate in those situations where the different Standard Model Yukawa couplings of these flavours are negligible, because an asymmetry is only produced for one particular linear combination of the lepton flavours. However, at smaller temperatures, when the lepton Yukawa couplings $h$ of the Standard Model approach equilibrium, the flavour degeneracy is broken and effects of flavour have to be taken into account [11–14]. The derivation of a set of kinetic equations from non-equilibrium quantum field theory, which covers both the unflavoured and fully flavoured regimes, and is valid in between, is still missing up to now, and is the subject of this paper.

A simple model that encompasses the salient features of flavoured leptogenesis and which we consider in the present work is specified by the Lagrangian

$$\mathcal{L} = \frac{1}{2} \bar{\psi}_{Ni}(i\not{\partial} - M_i)\psi_{Ni} + \bar{\psi}_{la}\not{\partial}\psi_{la} + \bar{\psi}_{Rb}\not{\partial}\psi_{Rb} + (\partial^\mu \phi^\dagger)(\partial_\mu \phi)$$

$$- Y_{ia}\bar{\psi}_{ta}\phi^{\dagger}\psi_{Ni} - Y_{ia}\bar{\psi}_{Ni}\phi^{\dagger}\psi_{la} - h_{ab}\phi^{\dagger}\bar{\psi}_{Ra}P_L\psi_{lb} - h^{*}_{ab}\phi\bar{\psi}_{lb}P_R\psi_{Ra},$$

where $\psi_{Ni}$ is the four-component Majorana spinor representing the right handed singlet neutrino $N_i$, $\psi_{la}$ is the spinor for the SU(2)$_L$ doublet of left handed Standard Model
leptons $\ell_a$, where $a$ is the flavour index, and $\psi_{R_a}$ are the corresponding charged right handed leptons. The SU(2)$_L$ doublet of Higgs fields is represented by $\phi$, and we define $\tilde{\phi} = (\epsilon \phi)\dagger$ with $\epsilon$ the antisymmetric 2 matrix in the SU(2)$_L$ indices with $\epsilon_{12} = 1$. For simplicity, we assume $i = 1, 2$. The generalisation to more than two right handed neutrinos $N_i$ is straightforward.

Single-flavour calculations for leptogenesis can be easily generalised to the multi-flavour situation, provided there is a flavour basis in which one can express the lepton-number densities of the several flavours and at the same time neglect possible correlations between the different flavours. When the lepton Yukawa couplings are fully in equilibrium, the appropriate flavour basis is where these couplings $h$ are diagonal. In contrast, in the unflavoured regime the appropriate basis is determined by the linear combination, in which the lepton asymmetry is produced.

These considerations raise the following questions: First, is there a kinetic equation that is manifestly covariant under the choice of the flavour basis? And second, can such an equation also deal with the intermediate regime where the couplings $h$ are not yet in full equilibrium but are non-negligible at the same time? The latter point is of particular importance since leptogenesis is a process that takes a finite amount of time while temperature is decreasing, such that the flavour-sensitive interactions may be out of equilibrium initially, but equilibrate at later times. Clearly, in order to address this question, one has to promote the set of lepton number distributions within a certain flavour basis to a matrix, that also allows for off-diagonal flavour correlations. In a heuristic approach, by appealing to the Hamiltonian evolution of a density matrix, such a set of equations has been proposed [13], and numerical solutions to these equations have been obtained [15].

On the other hand, it is clear that the Green functions within the CTP formalism allow for the possibility of off-diagonal correlations in a straightforward way. Indeed, once the appropriate Kadanoff-Baym equations for unflavoured leptogenesis are known [9], the multi-flavour generalisation is easily written down. However, the Kadanoff-Baym equations are a coupled set of integro-differential equations that needs to be subjected to some analytic approximations before solving it numerically. Key simplifications are first the gradient expansion, which effectively is an expansion in terms of time derivatives, in deviations of the distribution functions from equilibrium, and in the small coupling constants. Second, one makes use of a separation of time scales between the interactions induced by the small Yukawa couplings $Y$ and $h$ and the faster gauge interactions, that maintain kinetic equilibrium and induce lepton-antilepton pair creation and annihilation processes. This allows to describe the distribution functions by generalised chemical potentials.

The application of these strategies of simplification to the multi-flavour case constitutes the main body of the present work. In Section 2, we set up the multi-flavour Kadanoff-Baym equations and show that the flavour-dependent dispersion relations for the leptons, that are induced by the right handed neutrino and Yukawa couplings $Y$ and $h$, respectively, give rise to commutator terms in the kinetic equations which are also characteristic of flavour oscillations in the presence of tree-level mass terms. In
Section 3, we make use of the short time scale for kinetic equilibration in order to express the lepton densities in terms of a generalised chemical potential. We show that the gauge interactions in addition to enforcing kinetic equilibrium damp the flavour oscillations and that parametrically, flavoured leptogenesis is in the overdamped regime. The processes that turn out to dominate the decay of the off-diagonal flavour correlations are the flavour-sensitive three-body scatterings through the couplings \( h \).

Let \( q_{\ell ab} \) denote the Hermitian matrix of lepton charge densities, including flavour off-diagonal correlations, which is defined more precisely below. The main result of the present work is the kinetic evolution equation

\[
\frac{\partial q_{\ell ab}}{\partial \eta} = \sum_c \left[ q_{\ell ac} \Xi_{cb} - \Xi_{ac} q_{\ell cb} - W_{ac} q_{\ell cb} - q_{\ell ac} W_{cb} \right] + 2 S_{ab} - \Gamma_{\ell ab}. \tag{2}
\]

In this equation, \( \eta \) denotes the conformal time, which is related to the physical comoving time through \( dt = a(\eta) d\eta \), where \( a(\eta) \) is the scale factor of the expanding Universe. The matrices \( W, S \) and \( \Gamma_{\ell}^{\text{fl}} \) are all Hermitian. Lepton number and \( CP \)-violating source terms are represented by \( S \), \( W \) encompasses the washout rates for the various lepton flavours, and \( \Gamma_{\ell}^{\text{fl}} \) is the matrix of flavour-sensitive damping rates. In case we impose that \( q_{\ell ab} \) is evaluated in the basis of mass eigenstates of leptonic quasi-particles, the anti-Hermitian matrix \( \Xi \) compensates for possible time-dependent flavour rotations. Numerical solutions to this equation are presented in Section 4 and it is shown that it indeed interpolates between the fully flavoured and the unfavoured regimes.

We conclude in Section 5. Appendix A contains a discussion of the constraint and mass-shell equations including finite width effects, and the contribution of the right-handed neutrino to the thermal mass of the leptons is calculated in Appendix B.

2 Flavoured Leptons

The usual strategy for deriving kinetic equations is to decompose the Schwinger-Dyson equations on the CTP into equations for the retarded and advanced propagators and the Kadanoff-Baym equations. In the weak coupling limit, one can solve the latter when approximating the particle densities as purely on-shell. This corresponds to taking the particle width to zero.

In this Section, we reiterate these arguments for the left-handed leptons. As an extension of earlier approaches [16–18], where mass terms are introduced at tree-level, we describe here the dynamics that arises from thermal one-loop corrections to the dispersion relations, which are mediated by flavour-blind gauge interactions as well as flavour-sensitive Yukawa interactions.

2.1 Schwinger-Dyson Equations

We employ here the notations and conventions for the CTP formalism and the gradient expansion that are explained in more detail within Ref. [9, 19, 20]. The Schwinger-Dyson
equation for the flavoured left-handed lepton propagator is
\[ i \partial_u S^f_{\ell ab}(u,v) = f \delta^f \delta_{ab} \delta^4(u-v) P_R + \sum_h \int d^4 w \Psi^{fh}_{\ell ac}(u,w) S^{hg}_{\ell cb}(w,v), \] (3)
which can be more compactly written as \cite{19, 20}
\[ i \partial_u S^f_{\ell} = f \delta^f \delta P_R + \sum_h \Psi^{fh}_{\ell} \odot S^{hg}_{\ell}, \] (4)
where the symbol \( \odot \) denotes a convolution and \( \delta \) the delta function, \( f, g, h = \pm \) are the CTP indices and \( a, b, c \) flavour indices. The CTP structure can be decomposed into equations for the retarded and advanced propagators and a Kad anoff-Baym equation,
\[ i \partial_u S^A_{\ell} = \delta P_R + \Sigma^H \odot S^A_{\ell} \pm i \Sigma^A \odot S^A_{\ell}, \] (5a)
\[ i \partial_u S^{<>}_{\ell} = \Sigma^H \odot S^{<>}_{\ell} + \Sigma^{<>} \odot S^H_{\ell} + \frac{1}{2}(\Sigma^> \odot S^<_\ell - \Sigma^<_\ell \odot S^>_\ell) . \] (5b)
Here and in what follows, we make use of the definitions
\[ G^< = G^{+-}, \quad G^> = G^{-+}, \quad G^T = G^{++}, \quad G^T = G^{--} \] (6)
and the combinations
\[ G^A = G^T - G^> = G^< - G^\bar{T} \quad \text{(advanced)}, \]
\[ G^R = G^T - G^< = G^> - G^\bar{T} \quad \text{(retarded)}, \]
\[ G^H = \frac{1}{2}(G^R + G^A) \quad \text{(Hermitian)}, \]
\[ G^A = \frac{1}{2i}(G^A - G^R) = \frac{i}{2}(G^> - G^<) \quad \text{(anti-Hermitian, spectral)}, \]
where \( G \) may stand for any two-point function on the CTP, in particular the propagators and self-energies.

The Wigner transformation is defined as a Fourier transformation of a two-point function with respect to the relative coordinate, while keeping the average coordinate fixed:
\[ G(k, x) = \int d^4 r e^{ik \cdot r} G(u, v), \quad \text{where } r = u - v \text{ and } x = \frac{u + v}{2}. \] (8)
It provides a separation between the typical energy scale (which is the temperature \( T \) in the present case) and the small macroscopic inverse time-scale, that governs the evolution of state parameters (particle number and charge distributions in the present case). The
ratio of these two time scales is then used to define the gradient expansion. In Wigner space, Eqs. (9) take the form
\[
e^{-i\phi} \left\{ k - \Sigma^H = \Sigma^A \right\} \left\{ S^{A,R}_\ell \right\} = P_R , \tag{9a}
\]
\[
e^{-i\phi} \left\{ k - \Sigma^H \right\} \left\{ S^{<,>}_\ell \right\} - e^{-i\phi} \left\{ \Sigma^{<,>}_\ell \right\} \left\{ S^H \right\} = \frac{1}{2} e^{-i\phi} \left( \left\{ \Sigma^{>}_\ell \right\} \left\{ S^<_\ell \right\} - \left\{ \Sigma^<_\ell \right\} \left\{ S^>_\ell \right\} \right) . \tag{9b}
\]
where
\[
\phi \left\{ A(k, x) \right\} \left\{ B(k, x) \right\} = \frac{1}{2} \left( \left[ \partial^\mu (A(k, x)) \partial_{(k)} \mu (B(k, x)) - \left[ \partial_{(k)} \mu (A(k, x)) \right] \partial^\mu \mu (B(k, x)) \right) . \tag{10}
\]
Eqs. (9) correspond to an infinite tower of integro-differential equations. Approximate solutions can be obtained within the scheme of gradient expansion [21]. In the context of leptogenesis, gradients occur due to the deviation of particle number distributions from equilibrium, which is induced by the Hubble expansion of the Universe. At the same time, for a sizeable lepton asymmetry to occur, it is crucial, that during leptogenesis, the rate \(|Y_{1a}|^2 T\) is not very different from expansion rate \(H\). Similarly, within this paper we are particularly interested in the parametric region where the \(\tau\)-lepton Yukawa coupling \(h_\tau\) relates to \(H\) as \(h_\tau^2 T \sim H\), since otherwise we are either in the fully flavoured or in the unflavoured regime, which can be described by conventional approaches. In the present context, gradient expansion is therefore understood not only as an expansion in time-derivatives, but also as a perturbative expansion in \(Y\) and \(h\).

Performing the expansion of Eqs. (9) up to first order in gradients, we obtain
\[
\left( k - \Sigma^H = \Sigma^A \right) S^{A,R}_\ell = P_R , \tag{11a}
\]
\[
\frac{i}{2} \partial S^{<,>}_\ell + \left( k - \Sigma^H \right) S^{<,>}_\ell - \Sigma^{<,>}_\ell S^H = \frac{1}{2} \left( \Sigma^{>}_\ell S^<_\ell - \Sigma^<_\ell S^>_\ell \right) . \tag{11b}
\]
The derivative term \(\frac{i}{2} \partial S^{A,R}_\ell\) would contribute to Eq. (11a) only at second order in gradients, since the retarded and advanced propagators do not depend on the particle distribution functions at tree level. Damping occurs explicitly through the collision term on the right hand side of Eq. (11b) (for a detailed discussion, see Ref. [22]). At the same time, damping is contained in the \(\Sigma^A\) term in Eq. (11a), as it is well known from linear response theory.

In order to take account of the dilution of particles due to the expansion of the Universe, we follow Ref. [9]. We first observe that after appropriate field redefinitions and the rescaling \(M_i \rightarrow a(\eta) M_i\), the Lagrangian (1) describes the fields in the background of a flat Friedmann-Robertson-Walker Universe in conformal coordinates defined by the metric
\[
g_{\mu\nu} = a^2(\eta) \text{diag}(1, -1, -1, -1) .
\]
We then take all explicit momentum variables within the equations for the Wigner transformed quantities as conformal. The relation to a physical momentum is given

\[\text{5}\]
by \( k_{\text{ph}} = k/a(\eta) \) and the time coordinate is understood as the conformal time \( \eta \). Likewise, \( T \) denotes a comoving temperature that is related to the physical temperature as \( T_{\text{ph}} = T/a(\eta) \), and \( \beta = 1/T \). The masses of the right-handed neutrinos \( M_i \) are the physical masses. When they occur explicitly in the Wigner transformed equations, they are accompanied by the scale factor \( a(\eta) \) to give the conformally rescaled mass \( a(\eta)M_i \).

The Hermitian and anti-Hermitian parts of the Kadanoff-Baym equations \([11,13]\) lead us to the constraint and the kinetic equations

\[
2k^0 i_{\gamma^0 S^<_{\ell}} - \left\{ k \cdot \gamma^0 + H_\ell^0, i_{\gamma^0 S^<_{\ell}} \right\} - \left\{ i\Sigma^{<,>}_{\ell}, \gamma^0 S^H_{\ell} \right\} = -\frac{1}{2} \left( iC_\ell - iC^I_\ell \right),
\]

\[
i\partial_\gamma i_{\gamma^0 S^<_{\ell}} - \left[ k \cdot \gamma^0 + H_\ell^0, i_{\gamma^0 S^<_{\ell}} \right] - \left[ i\Sigma^{<,>}_{\ell}, \gamma^0 S^H_{\ell} \right] = -\frac{1}{2} \left( iC_\ell + iC^I_\ell \right),
\]

with the collision term

\[
C_\ell = i\Sigma^{<,>}_{\ell} - i\Sigma^{<,>}_{\ell}.
\]

### 2.2 Thermal Self Energies

We now specify the form of the Hermitian self energy \( \Sigma^H_\ell \), which determines the thermal corrections to the dispersion relation. For the purpose of this discussion, we assume that the deviation of the right handed neutrino distribution from thermal equilibrium is small, so that the self energy, being proportional to coupling constants, can be evaluated with thermal propagators to first order in the gradient expansion. The self energy receives contributions from the interactions specified in Eq. \([\text{I}]\), but also from \( SU(2)_L \times U(1)_Y \) gauge interactions. We parametrise this as

\[
\Sigma^H_\ell = P_R \left[ \gamma^0 \left( \varsigma^{bl} + \varsigma^{fl} \right) + \frac{k \cdot \gamma}{|k|} \left( \varsigma^{bl} + \varsigma^{fl} - \text{sign}(k^0) \left[ \varsigma^{bl} + \varsigma^{fl} \right] \right) \right] P_L,
\]

where we have decomposed the contributions to the self energy into a flavour blind part that originates from \( SU(2)_L \times U(1)_Y \) gauge interactions

\[
\varsigma^{bl}_{ab}(k^0, k) = \delta_{ab} \varsigma^{bl}(k^0, k), \quad \varsigma^{fl}_{ab}(k^0, k) = \delta_{ab} \varsigma^{bl}(k^0, k),
\]

and a flavour dependent part that receives contributions from the charged lepton and the singlet neutrino Yukawa couplings. To one loop order, these can be parametrised as

\[
\varsigma^{fl}_{ab}(k^0, k) = h_{ac}^{\dagger} h_{cb} \varsigma^{fl,h}(k^0, k) + \sum_i Y_{ia}^{*} Y_{ib} \varsigma^{fl,Y}_{i}(k^0, k),
\]

\[
\varsigma^{fl}_{ab}(k^0, k) = h_{ac}^{\dagger} h_{cb} \varsigma^{fl,h}(k^0, k) + \sum_i Y_{ia}^{*} Y_{ib} \varsigma^{fl,Y}_{i}(k^0, k).
\]

In the hierarchical mass limit (\( M_1 \ll M_2 \)), which we assume within this paper, we may restrict the sum to \( i = 1 \). Note that \( \gamma^0 \Sigma^H_\ell \) is Hermitian, such that \( \varsigma^{fl} \) and \( \varsigma^{fl} \) are Hermitian matrices in flavour space.
The fact that two independent functions $\varsigma(k^0, \mathbf{k})$ and $\bar{\varsigma}(k^0, \mathbf{k})$ occur is because the self energy $\Sigma^{\mathcal{H}}$ acquires contributions that are proportional to $\mathbf{k}$ and $\hat{\mathbf{k}}$, where $u^\mu = (1, 0, 0, 0)^T$ is the plasma vector. The relation between $\varsigma(k^0, \mathbf{k})$, $\bar{\varsigma}(k^0, \mathbf{k})$ and the terms proportional to $\mathbf{k}$ and $u$ can be easily established, see e.g. Eqs. (B.2). The motivation for our parametrisation is that it corresponds to a decomposition into a correction $\varsigma$ for the dispersion relation and a correction $\bar{\varsigma}$ that leaves the dispersion relation unaltered [see Eqs. (26) and (A.8) below]. Besides, $\varsigma$ and $\bar{\varsigma}$ exhibit useful symmetry properties under the exchange $k^0 \rightarrow -k^0$ [see Eq. (35) below].

The matrix $\varsigma_{\mathbb{D}}$ can be diagonalised through a unitary transformation $U$ and we define
\[ \varsigma_{\mathbb{D}} = U \varsigma_{\mathbb{D}} U. \] (17)

Note that in general, $U$ is momentum- and time-dependent. At temperatures $M_{2,3} \gg T/a(\eta) \gg M_1$ and momenta $|\mathbf{k}| \sim T$, both $\varsigma^\mathcal{H}$ and $\bar{\varsigma}^\mathcal{H}$ are approximately proportional to $T^2/|\mathbf{k}|$, such that the diagonalisation matrix $U$ is constant in time. When the temperature $T/a(\eta)$ drops below $M_1$, the distribution of $N_1$ becomes Maxwell suppressed. We consider this situation in Appendix B. The function $\varsigma_{\mathbb{D}}$ then falls toward zero as $\sim [T/(aM_1)]^4$, cf. Eq. (B.11), and $U$ may generically undergo a change. Afterwards, at temperatures $T/a(\eta) \ll M_1$, $U$ becomes constant again and the matrix $U^\dagger h^\dagger h U$ is diagonal.

All quantities that carry left-handed flavour indices transform under the basis transformation defined by $U$. We denote matrices evaluated in the flavour-diagonal basis by a subscript $\mathbb{D}$. For example,
\[ \Sigma^{\mathcal{H}}_{\mathbb{D}} = U^\dagger \Sigma^{\mathcal{H}} U, \] (18a)
\[ iS_{\mathbb{D}} = U^\dagger iS U. \] (18b)

Note that unlike $\varsigma_{\mathbb{D}}$, these matrices are in general not diagonal in flavour-space.

Inserting the definitions (18a) and (18b) into (12b) and multiplying with $U^\dagger$ from the left and with $U$ from the right, we obtain the kinetic equation in the lepton thermal mass basis,
\[ i\partial_\eta \gamma^0 S^{<,>}_{\mathbb{D}} + i [\Xi, i\gamma^0 S^{<,>}_{\mathbb{D}}] - [\Sigma^{\mathcal{H}}_{\mathbb{D}}, \gamma^0 S^{<,>}] = \frac{1}{2} \left(iC_{\mathbb{D}} + iC_{\mathbb{D}}^\dagger\right), \] (19)
where $C_{\mathbb{D}} = U^\dagger C U$ and
\[ \Xi = U^\dagger \partial_\eta U. \] (20)

is the compensation matrix for time-dependent flavour rotations. At first order in gradients, the only consequence of a time dependent $U$ is the additional term involving $\Xi$. Therefore, we switch to the diagonal basis and drop the subscript $\mathbb{D}$ from all subsequent expressions.

---

1 The terms $\gamma^0$ and $\gamma^0 S^{<,>}_\ell$ commute when using the ansatz (21) below.
2.3 Kinetic and Constraint Equations

The Weyl fermion propagator can be parametrised through the vector and pseudovector functions

\[ i \gamma^0 S^{<,>}_\ell = \frac{1}{2} \sum_{h=\pm} \left[ g_{h_0}^{<,>} \left( 1 + \hat{k} \cdot \gamma^0 \gamma^0 \gamma \right) + g_{h_3}^{<,>} \left( \gamma^5 + \hat{k} \cdot \gamma^0 \gamma \right) \right], \tag{21} \]

where \( \hat{k} = k / |k| \). When compared to the case of a Dirac fermion, there are no scalar and pseudoscalar contributions. This is because gauge symmetry prevents the dynamical generation of scalar, pseudoscalar and tensor densities, provided the gauge symmetry is neither broken spontaneously nor through initial conditions, as we assume here. Thus, Eq. (21) is the most general form of the lepton propagator compatible with isotropy and chiral symmetry. Besides, from the fact that the leptons \( \ell \) are left-handed, we immediately obtain

\[ g_{h_0}^{<,>} = g_{h_3}^{<,>} \equiv g_h^{<,>} \tag{22} \]

We furthermore see that \( k \cdot \gamma^0 \) and \( i \gamma^0 S^{<,>}_\ell \) commute, such that the constraint and kinetic Eqs. (12) simplify to

\[
\begin{align*}
2(k^0 - k \cdot \gamma^0) i \gamma^0 S^{<,>}_\ell - \{ \Sigma^{H,0}_\ell \gamma^0, i \gamma^0 S^{<,>}_\ell \} & - \{ i \Sigma^{H,0}_\ell, \gamma^0 S^{H}_\ell \} = -\frac{1}{2} \left( iC_\ell - iC_\ell^\dagger \right), \\
\tag{23a}
\end{align*}
\]

\[
\begin{align*}
\text{i} \partial_\eta \gamma^0 S^{<,>}_\ell - \left[ \Sigma^{H,0}_\ell \gamma^0, i \gamma^0 S^{<,>}_\ell \right] - \left[ i \Sigma^{H,0}_\ell, \gamma^0 S^{H}_\ell \right] & = -\frac{1}{2} \left( iC_\ell + iC_\ell^\dagger \right) . \\
\tag{23b}
\end{align*}
\]

To zeroth order, the constraint equation (23a) reduces to the simple form

\[ \{ k, iS^{<,>} \} = 0 . \tag{24} \]

Substituting the ansatz (21) leads us to

\[
\begin{align*}
g_{h_0}^{<,>} k^0 + h|k|g_{h_3}^{<,>} & = 0 , \\
g_{h_3}^{<,>} k^0 + h|k|g_{h_0}^{<,>} & = 0 . \tag{25a-b}
\end{align*}
\]

The constraint (22) then implies that \( g_h^{<,>} (k^0, k) \) is non-vanishing only when \( k^0 = -h|k| \), which corresponds to the singular zero-mass shell. In particular \( h = -\text{sign}(k^0) \), that is for leptons \( (k^0 > 0) \) the helicity \( h = -1 \) is negative, while for anti-leptons \( (k^0 < 0) \) the helicity \( h = 1 \) is positive, as expected. This relation is weakly broken for momenta \( |k| \sim T \) when including thermal corrections, because hole modes exhibit an opposite connection between frequency and helicity. For momenta \( |k| \ll T \), the hole modes couple to the plasma at a similar strength as the particle modes. However, this region
only corresponds to a small portion of the available phase space, such that we may neglect it here.

Substituting the parametrisations (14), (21) and the constraint (22) into the kinetic equations (23b) and taking the trace leads us to

\[ i\partial_t g_h^{<,>} + i [\Xi, g_h^{<,>}] + h [\zeta^\parallel, g_h^{<,>}] = -\frac{1}{4} \text{tr} \left( iC_\ell + iC_\ell^\dagger \right), \quad (26) \]

where the trace is taken only in Dirac space, and \( \zeta^\parallel \) and all other objects are evaluated in the mass-diagonal basis. As explained above, the helicity is determined from the zeroth order constraint equation by the relation \( h = -\text{sign}(k^0) \). Eq. (26) is accurate up to first order in gradients, because \( \zeta^\parallel \) itself is of first order. We note that the term \( [i\Sigma^{<,>\gamma^0}, \gamma^0 S_\ell^H] \) in Eq. (23b) does not contribute to Eq. (26) at first order, since first, \( S_\ell^H \) can be evaluated at zeroth order, where it is independent of the particle distribution functions and therefore proportional to the unit matrix in flavour space; and second, in Dirac space the commutator reads

\[ [i\Sigma^{<,>\gamma^0}, \gamma^0 S_\ell^H] \propto \frac{1}{4} \left( \text{tr} \left[ iC_\ell + iC_\ell^\dagger \right] \right) \quad (27) \]

Since we work in the flavour-diagonal basis, where \( \zeta^\parallel \) is diagonal, the commutator involving \( \zeta^\parallel \) in Eq. (26) can be explicitly evaluated, which yields

\[ i\partial_t g_h^{<,>} + i [\Xi, g_h^{<,>}]_{ab} + h (\zeta_{aa}^\parallel - \zeta_{bb}^\parallel) g_h^{<,>} = \frac{1}{4} \left( \text{tr} [iC_\ell + iC_\ell^\dagger] \right)_{ab}. \quad (28) \]

Hence, the thermal dispersion relations have the same impact on the equation of motion for the lepton density as explicit Dirac masses would [16–18].

An important feature of Eq. (26) is the sign change of the commutator term involving the thermal dispersion relation through \( \zeta^\parallel \) when \( h \to -h \) or, alternatively, \( k^0 \to -k^0 \). We now show that at the one-loop level, the elements of \( h\zeta^\parallel = -\text{sign}(k^0) \zeta^\parallel(k^0, k) \) are indeed odd functions in \( k^0 \). We define charge and parity conjugation through

\[ \psi^C(x) = C\bar{\psi}^T(x) \quad (29) \]
\[ \psi^P(x) = P\bar{\psi}(\bar{x}) \quad (30) \]

where \( \bar{u} \equiv (u_0, -u) \) and where in the Weyl representation, the conjugation matrices are given by \( C = i\gamma^0\gamma^2 \) and \( P = \gamma^0 \). Thereby, we fix possible \( CP \) phases that can arise in the definition of these conjugations to zero. The charge and parity conjugate propagators are

\[ iS^{C,fg}_{\ell ab}(u, v) = \langle \psi^C_{\ell a}(u^f)\bar{\psi}^C_{\ell b}(v^g) \rangle = C \left[ iS^{g,f}_{\ell ba}(v, u) \right]^T C^\dagger \quad (31a) \]
\[ iS^{P,fg}_{\ell ab}(u, v) = \langle \psi^P_{\ell a}(u^f)\bar{\psi}^P_{\ell b}(v^g) \rangle = PS^{f,g}_{\ell ab}(\bar{u}, \bar{v})P^\dagger \quad (31b) \]
The transposition acts here only on the Dirac indices, which in contrast to the flavour and CTP indices are not written explicitly. The CP conjugate of the Hermitian self energy is then given by (we suppress the average coordinate in the argument of \( \varsigma \) and \( \bar{\varsigma} \))

\[
\Sigma_{CP,H}^{\ell ab}(k, x) = CP\left[\Sigma_{H}^{\ell ba}(-\bar{k}, \bar{x})\right]^{T} (CP)^{\dagger} = PR\left[-\gamma^{0}(\varsigma_{ba}^{\dagger}(-\bar{k}) + \varsigma_{ba}(-\bar{k})) + \bar{k} \cdot \gamma(\varsigma_{ba}^{\dagger}(-\bar{k}) + \varsigma_{ba}(-\bar{k}))
\right.
\]
\[
+ \text{sign}(k^{0})[\varsigma_{ba}^{\dagger}(-\bar{k}) + \varsigma_{ba}(-\bar{k})]\left] P_{L}, \right.
\]

On the other hand, we may calculate this self energy from the CP-conjugate Lagrangian, within which the coupling constants are complex conjugated, as

\[
\Sigma_{CP,H}^{\ell ab}(k, x) = \Sigma_{H}^{\ell ab}(k, x) \bigg|_{h \rightarrow h^{*}}, \tag{33}
\]

provided the initial conditions preserve CP symmetry (no primordial asymmetry). To the one-loop order, the coupling constants appear as the prefactors \( h^{\dagger}h \) and \( Y^{\dagger}Y \) within \( \Sigma_{H}^{\ell ab} \), cf. Eq. (16). The effect of CP conjugation therefore amounts to the replacements \( [h^{\dagger}h]_{ab} \rightarrow [h^{\dagger}h]_{ab}^{*} = [h^{\dagger}h]_{ba} \) and \( [Y^{\dagger}Y]_{ab} \rightarrow [Y^{\dagger}Y]_{ab}^{*} = [Y^{\dagger}Y]_{ba} \), and it follows that to one-loop order

\[
\Sigma_{CP,H}^{\ell ab}(k, x) = \Sigma_{H}^{\ell ba}(k, x). \tag{34}
\]

Comparing this to the relation (32) and substituting Eq. (14), we find that

\[
\varsigma_{ab}^{\dagger}(-k^{0}, k) = -\varsigma_{ab}^{\dagger}(k^{0}, k) \quad \text{and} \quad \varsigma_{ab}^{\dagger}(-k^{0}, k) = \varsigma_{ab}(k^{0}, k), \tag{35}
\]

and accordingly for \( \varsigma_{ab}^{\dagger} \) and \( \varsigma_{ab}^{\dagger} \), which implies the result to be shown for the elements of \( \varsigma_{ab}^{\dagger} \). These symmetry properties with respect to \( k^{0} \) are in accordance with the equilibrium results from Ref. [23] and Appendix [B]. The present argument shows moreover that they also hold under out-of-equilibrium conditions.

For calculating the momentum integrals in the collision terms, we use on-shell conditions that we obtain from the constraint equations. In order to achieve accuracy to first order in gradients, it again suffices to solve the constraint equations to zeroth order, since the collision term is suppressed by higher orders in the coupling constants. A similar approximation is applied in Ref. [18], where in contrast to the present work, within the constraint equations, small tree-level mass differences rather than differences between one-loop dispersion relations and finite widths are neglected. A general solution to the zeroth-order constraint equation (24) is given by the Kadanoff-Baym ansatz

\[
iS_{\ell ab}^{<} = -2S_{\ell}^{A}[\vartheta(k^{0})f_{\ell ab}^{+}(k) - \vartheta(-k^{0})(1_{ab} - f_{\ell ab}^{-}(-k))], \tag{36a}
\]
\[
iS_{\ell ab}^{>} = -2S_{\ell}^{A}[\vartheta(k^{0})(1_{ab} - f_{\ell ab}^{+}(k)) + \vartheta(-k^{0})f_{\ell ab}^{-}(-k)]. \tag{36b}
\]

10
where
\[ S^A_\ell = \pi P_L k P_R \delta(k^2) , \] (37)
and \( f^{\pm}_{\ell ab} \) are the distribution function matrices of leptons and anti-leptons. Comparing Eqs. (36) to Eq. (21) we identify
\[ g^<_- = -2\pi \delta(k^2) \vartheta(k^0) |k| f^+_\ell(k) , \] (38)
\[ g^>_+ = 2\pi \delta(k^2) \vartheta(k^0) |k| (1 - f^+_\ell(k)) , \]
and for the present work, these zeroth-order solutions to the constraint equations are sufficient, we show in Appendix A how to extend them to first order in consistency with the equations for the retarded and the advanced propagators (11a).

Note that if we identify \( f^{\pm}_{\ell ab} \) with expectation values of number density operators then it follows from Eqs. (36) and the operator definition of \( iS^{<,>}_\ell \) that \( f^+_{\ell ab} \sim \langle a^\dagger_b a_a \rangle \) corresponds to the lepton density matrix, while \( f^-_{\ell ab} \sim \langle b^\dagger_a b_b \rangle \) corresponds to the transpose of the anti-lepton densities. This may also be seen when relating the CP conjugate to the original propagator as
\[ \text{CP}[iS^{<,>}_f \ell (k, x)] = \text{CP}[iS_{\ell ba}(k, x)]^T \] (39)
which is consistent with \( h = -\text{sign}(k^0) \) and \( k^0 = \pm |k| \). While for the present work, these zeroth-order solutions to the constraint equations are sufficient, we show in Appendix A how to extend them to first order in consistency with the equations for the retarded and the advanced propagators (11a).

Note that if we identify \( f^{\pm}_{\ell ab} \) with expectation values of number density operators then it follows from Eqs. (36) and the operator definition of \( iS^{<,>}_\ell \) that \( f^+_{\ell ab} \sim \langle a^\dagger_b a_a \rangle \) corresponds to the lepton density matrix, while \( f^-_{\ell ab} \sim \langle b^\dagger_a b_b \rangle \) corresponds to the transpose of the anti-lepton densities. This may also be seen when relating the CP conjugate to the original propagator as
\[ \text{CP}[iS^{<,>}_f \ell (k, x)] = \text{CP}[iS_{\ell ba}(k, x)]^T \] (39)
which is consistent with \( h = -\text{sign}(k^0) \) and \( k^0 = \pm |k| \). While for the present work, these zeroth-order solutions to the constraint equations are sufficient, we show in Appendix A how to extend them to first order in consistency with the equations for the retarded and the advanced propagators (11a).

Note that if we identify \( f^{\pm}_{\ell ab} \) with expectation values of number density operators then it follows from Eqs. (36) and the operator definition of \( iS^{<,>}_\ell \) that \( f^+_{\ell ab} \sim \langle a^\dagger_b a_a \rangle \) corresponds to the lepton density matrix, while \( f^-_{\ell ab} \sim \langle b^\dagger_a b_b \rangle \) corresponds to the transpose of the anti-lepton densities. This may also be seen when relating the CP conjugate to the original propagator as
\[ \text{CP}[iS^{<,>}_f \ell (k, x)] = \text{CP}[iS_{\ell ba}(k, x)]^T \] (39)
which is consistent with \( h = -\text{sign}(k^0) \) and \( k^0 = \pm |k| \). While for the present work, these zeroth-order solutions to the constraint equations are sufficient, we show in Appendix A how to extend them to first order in consistency with the equations for the retarded and the advanced propagators (11a).

3 Kinetic Equations for Lepton Number Densities

In this Section, we perform simplifications of the kinetic equations (28), such that they attain a form which can be solved numerically. The key simplification arises from the separation of the time scales of kinetic equilibration and flavour-sensitive interactions.
Because the former is much faster, the distribution functions are driven to kinetic equilibrium, such that they can be approximated by the Bose-Einstein or Fermi-Dirac form, parametrised through a matrix of chemical potentials. An integration over the momentum then allows to express the kinetic equations in terms of charge densities. We shall often use spatial homogeneity to set $f_{\ell ab}(-k) = f_{\ell ab}(k)$.

### 3.1 Matrices for Lepton Number Densities

The lepton number density matrices are defined as

$$ n_{\ell ab}^+ = \int \frac{d^3 k}{(2\pi)^3} f_{\ell ab}^+(k) = -\int \frac{d^3 k}{(2\pi)^3} \int_0^\infty \frac{dk^0}{2\pi} \text{tr} \left[ i \gamma^0 S_<^{\ell ab} \right], \quad (40a) $$

$$ n_{\ell ab}^- = \int \frac{d^3 k}{(2\pi)^3} f_{\ell ab}^-(k) = \int \frac{d^3 k}{(2\pi)^3} \int_{-\infty}^0 \frac{dk^0}{2\pi} \text{tr} \left[ i \gamma^0 S_>^{\ell ab} \right]. \quad (40b) $$

Due to the presence of the fast gauge interactions, we may assume kinetic equilibrium for the leptons, and denote $\delta n_{\ell ab}^\pm = n_{\ell ab}^\pm - n_{\ell ab}^{\pm \text{eq}}$ and $\delta f_{\ell ab}^\pm = f_{\ell ab}^\pm - f_{\ell ab}^{\pm \text{eq}}$. Then, we can introduce a matrix of generalised chemical potentials $\mu_{\ell ab}^\pm$ for particles and antiparticles, and write

$$ f_{\ell ab}^\pm(k) = \left( \frac{1}{e^{\beta |k|} - \beta \mu_{\ell ab}^\pm} + 1 \right)_{ab}, \quad (41) $$

such that the number density matrices are, to first order in the chemical potentials,

$$ \delta n_{\ell ab}^\pm = \mu_{\ell ab}^\pm \frac{T^2}{12}. \quad (42) $$

This allows to relate the lepton number densities to the distribution functions:

$$ \delta f_{\ell ab}^\pm(k) = 12 \delta n_{\ell ab}^\pm \frac{\beta^3 e^{\beta |k|}}{(e^{\beta |k|} + 1)^2}. \quad (43) $$

We also introduce the deviation of the $<,>$ propagators from equilibrium,

$$ i\delta S_{\ell ab} = -2S_A^\ell \left[ \partial(k^0) \delta f_{a5}^+(k) + \partial(-k^0) \delta f_{a5}^+(-k) \right]. \quad (44) $$

Besides, we use corresponding approximations and expressions for the right-handed leptons of the Standard Model by replacing $\ell \to R$.

The ansatz (41) is valid provided the interactions that establish kinetic equilibrium, which are the pair creation and annihilation processes and scatterings with gauge bosons, are faster than processes that distinguish between the particle flavours, in particular flavour oscillations and flavour-sensitive damping rates. Provided these assumptions hold, which is verified in Section 3.3, the time scales for kinetic equilibration and for flavour effects separate. Since $\mu_{\ell}^{\pm}$ is Hermitian, it is always possible to bring Eq. (41) to diagonal form by a flavour rotation. Due to the separation of time scales, kinetic
equilibrium in this diagonal basis is then attained at a rate that is faster than the flavour effects that may change the flavour orientation of $\mu_{ab}$.

Using the ansatz (36) and the spectral function (37) together with the decomposition (21) and the constraint (22) gives the relations

$$f_{\pm}^\ell(k) =\mp2\int_{-\infty}^{0}\frac{dk_0}{2\pi}g_{\pm}^{<,>}(k_0, k).$$

Performing a $k_0$-integration of Eq. (28) and using Eqs. (40) then yields

$$\partial_\eta f_{+}^{\pm,ab}(k) = [\Xi, f_{\pm}^\ell(k)]_{ab} \mp i(\varsigma_{aa} - \varsigma_{bb}) f_{\pm}^\ell(k) \pm \frac{1}{2} \text{tr} \int_{-\infty}^{0} \frac{dk_0}{2\pi} (C_{\ell ab} + C_{\ell ab}^\dagger).$$

Integrating over three momenta and applying the expansions explained above, we find

$$\partial_\eta \delta n_{+}^{\pm} = [\Xi^{\text{eff}}, \delta n_{\ell}^{\pm}]_{ab} \mp i\Delta\omega_{\ell ab}^{\text{eff}} \delta n_{+}^{\pm} \pm \frac{1}{2} \text{tr} \int_{-\infty}^{0} \frac{dk_0}{2\pi} \int \frac{d^3k}{(2\pi)^3} (C_{\ell ab} + C_{\ell ab}^\dagger),$$

where we have used that the equilibrium distributions are diagonal in flavour, $n_{+}^{\pm,\text{eq}} = \delta_{ab}n_{+a a}^{\text{eq}}$. We have defined here the thermally averaged frequencies of flavour oscillations and the compensation matrix as

$$\Delta\omega_{\ell ab}^{\text{eff}}(\eta) = \int \frac{d^3k}{(2\pi)^3} \frac{12\beta^3e^{\beta|k|}}{(e^{\beta|k|} + 1)^2} (\varsigma_{aa}(|k|, k, \eta) - \varsigma_{bb}(|k|, k, \eta)),$$

$$\Xi^{\text{eff}}(\eta) = \int \frac{d^3k}{(2\pi)^3} \frac{12\beta^3e^{\beta|k|}}{(e^{\beta|k|} + 1)^2} \Xi(|k|, k, \eta),$$

and used that $\varsigma_{aa}$ and $\Sigma$ are symmetric in their first argument, cf. (35). The dominant contributions to the phase space integrals originate from regions where $|k| \sim T$, where $\varsigma_{ab}$ can be approximated by (23)

$$\varsigma_{ab}^{\text{fl}}(k_0, k) = \frac{h_{ar}h_{eb}T^2}{16|k|} + \sum_i Y_{ia}^{*}Y_{ib}\varsigma_{i}^{\text{fl,Y}}(k_0, k).$$

While the form of $Y_{i}^{*}Y_{i}\varsigma_{i}^{\text{fl,Y}}$ is more complicated in general, as we discuss in Appendix B it is of the same order or smaller than $h_{r}^{\text{fl}}$ when flavour effects are important. We can therefore estimate

$$\Delta\omega_{\ell ab}^{\text{eff}} \approx O(h_{r}^{2}T),$$

where $h_{r}$ is the $\tau$-lepton Yukawa coupling.
We decompose the collision term\(^2\) as
\[
\mathcal{C}_\ell = \mathcal{C}_\ell^Y + \mathcal{C}_\ell^\text{fl} + \mathcal{C}_\ell^\text{bl}.
\]
The term \(\mathcal{C}_\ell^Y\) describes decays and inverse decays of \(N_1\) and hence the washout and the \(CP\)-asymmetric source of the lepton densities. Flavour sensitive interactions mediated by the Standard Model Yukawa couplings are encompassed in \(\mathcal{C}_\ell^\text{fl}\), while flavour-blind interactions mediated by gauge couplings are taken account of within \(\mathcal{C}_\ell^\text{bl}\). In the following, we show that these particular contributions can be cast into the form
\[
\frac{\partial \delta n^n_{\ell ab}}{\partial \eta} = \Xi_{ac} \delta n^n_{\ell cb} - \delta n^n_{\ell ca} \Xi_{cb} + i \Delta \omega_{\ell ab} \delta n^n_{\ell ab} \tag{52}
\]
\[-\sum_c [W_{ac} \delta n^n_{\ell cb} + \delta n^n_{\ell ca} W^*_{bc}] \pm S_{ab} - \Gamma^\text{fl} (\delta n^n_{\ell ab} + \delta n^n_{\ell ba}) - \Gamma^\text{fl}.
\]

### 3.2 Source and Washout Term

The washout term for \(\delta n^n_{\ell ab}\) is given by
\[
-\sum_c W_{ac} \delta n^n_{\ell cb} = \frac{1}{2} \text{tr} \int \frac{d^3k}{(2\pi)^3} \int_0^\infty \frac{dk_0}{2\pi} \mathcal{C}_\ell^Y \tag{53}
\]
\[-\sum_c \int \frac{d^3k}{(2\pi)^3} \int_0^\infty \frac{dk_0}{2\pi} \left[ i \Sigma_{\ell ca}(k)i S_{\ell cb}(k) - i \Sigma_{\ell ac}(k)i S_{\ell cb}(k) \right],
\]
evaluated to order \(Y_2^2\) (certain higher order terms are accounted for by the source term). Close to equilibrium, we can write
\[
\Sigma_{\ell ca}(k)i S_{\ell cb}(k) - i \Sigma_{\ell ac}(k)i S_{\ell cb}(k) = -i \left( \Sigma_{\ell ca}(k) - \Sigma_{\ell ac}(k) \right) i \delta S_{\ell cb}(k),
\]
and we note that
\[
\Sigma_{\ell ca}(k) - i \Sigma_{\ell ac}(k) = -Y_1^* Y_{1c} \int \frac{d^3k'}{(2\pi)^3} \frac{d^3k''}{(2\pi)^3} \frac{d^3k'''}{(2\pi)^3} \frac{(2\pi)^4 \delta^4(k' - k - k'')} (2\pi)^4 \delta^4(k' - k - k'') \times \text{sign}(k_0) P_R(k' + a(\eta) M_1) P_L (f_{N_1}(k') + f_\beta(k'')) \tag{55}
\]
Substituting Eqs. \([13]\) and \([14]\), we can identify
\[
W_{ac} = \frac{1}{2} Y_1^* Y_{1c} \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \frac{d^3k''}{(2\pi)^3} \frac{(2\pi)^4 \delta^4(k' - k - k'')} (2\pi)^4 \delta^4(k' - k - k'') \times 2k \cdot k' (f_{N_1}(k') + f_\beta(k'')) \frac{12 \beta^3 e^{|k|}}{(e^{\beta |k|} + 1)^2}. \tag{56}
\]
\[^2\text{The definition of the collision terms } \mathcal{C} \text{ in Ref. [13] differs from the present ones by an additional integration } \int dk^0/(2\pi).\]
The washout term for $\delta n_{\text{lab}}^-$ follows correspondingly. In straightforward generalisation of the single flavour case, the $CP$-violating source term for $\delta n_{\text{lab}}^+$ is

$$S_{ab} = \frac{3}{2} i \sum_c \left[ Y_{1a}^* Y_{1c} Y_{2c} Y_{2b} - Y_{2a}^* Y_{2c} Y_{1c} Y_{1b} \right]$$

$$\times \left( - \frac{M_1}{M_2} \right) \int \frac{d^3k'}{(2\pi)^3} \frac{\Sigma_N(k') \Sigma_N^\mu(k')}{g_w} \delta f_{N1}(k'),$$

where

$$\Sigma_N^\mu(k) = g_w \int \frac{d^3p}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \frac{(2\pi)^4 \delta^4(k - p - q) p^\mu \left( 1 - f^{\text{eq}}_\ell(p) + f^{\text{eq}}_\nu(q) \right)}{\Sigma_N^\mu(k)} \int \frac{d^3k'}{(2\pi)^3} \frac{\Sigma_N^\mu(k')}{g_w} \delta f_{N1}(k'),$$

with $g_w = 2$, and where the source for the anti-leptons $\delta n_{\text{lab}}^-$ is $-S_{ab}$. The deviation of the distribution function of the right-handed neutrinos from equilibrium is denoted by $\delta f_{N1}(k) = f_{N1}(k) - f_{N1}^{\text{eq}}(k)$. This source term is understood to include both, wave-function and vertex contributions in the hierarchical limit, $M_1 \ll M_2$. Note that there is an additional wave-function contribution that violates lepton flavour but conserves total lepton number $[11, 14]$. Within flavoured models of leptogenesis, this may contribute to the final lepton asymmetry. Compared to the total lepton-number violating contributions, it is however suppressed by a factor $M_1/M_2$ since one picks up the $k \sim M_1$ rather than the $M_2$ term from the numerator of the intermediate neutrino propagator in the wave-function diagram. Hence, we do not account for this term here.

### 3.3 Flavour Blind Interactions

The flavour-blind contribution to the lepton self-energy that is mediated by gauge interactions can be expressed as

$$i\Sigma_{\text{bl}}^{\gamma g} = g^2 \int \frac{d^4k'}{(2\pi)^4} \frac{d^4k''}{(2\pi)^4} (2\pi)^4 \delta^4(k - k' - k'') \gamma^\nu iS_{\text{lab}}^{f_g}(k') \gamma^\mu i\Delta_{A\mu\nu}^{f_g}(k''),$$

where $i\Delta_{A\mu\nu}^{f_g}$ is the gauge boson propagator on the CTP. The corresponding collision term is then

$$C_{\text{lab}}^{\text{bl}}(k) = i\Sigma_{\text{lab}}^{\gamma g}(k) iS_{\text{lab}}^{\gamma g}(k) - i\Sigma_{\text{lab}}^{\gamma g}(k) iS_{\text{lab}}^{\gamma g}(k)$$

$$= g^2 \int \frac{d^4k'}{(2\pi)^4} \frac{d^4k''}{(2\pi)^4} (2\pi)^4 \delta^4(k - k' - k'') \left[ \gamma^\nu iS_{\text{lab}}^{\gamma g}(k') \gamma^\mu i\Delta_{A\mu\nu}^{f_g}(k'') iS_{\text{lab}}^{\gamma g}(k) \right.$$}

$$- \gamma^\nu iS_{\text{lab}}^{\gamma g}(k') \gamma^\mu i\Delta_{A\mu\nu}^{f_g}(k'') iS_{\text{lab}}^{\gamma g}(k) \right].$$

To check the consistency of the generalised chemical potential ansatz $[11]$, we now verify that the collision term $[60]$ vanishes provided the leptons and anti-leptons have
opposite chemical potentials, \( \mu^- = -\mu^+ \). To see this, we first note that \( \mu^- = -\mu^+ \equiv -\mu_{ab} \) implies the generalised KMS relation \( S^>_{ab}(k) = -\left(e^{\beta k_0 - \beta \mu}\right)_{ac} S^<_{ceb}(k) \). Using this and the fact that the gauge bosons are in thermal equilibrium, which implies \( \Delta^>_{A\mu
u}(k) = e^{\beta k_0} \Delta^<_{A\mu
u}(k) \), in Eq. (59) yields the analogous relation \( \gamma^bl_{ab} = -\left(e^{\beta k_0 - \beta \mu}\right)_{ac} \gamma^b_{ceb} \) for the lepton self energy. This allows us to write

\[
C^b_{\ell}(k) = \left[i\gamma^b_{\ell}(k), e^{\beta k_0 - \beta \mu}\right] iS^<(k) \tag{61}
\]

Inserting Eq. (59) leaves the commutator \( [i\gamma^b_{\ell}(k), e^{\beta k_0 - \beta \mu}] \) in flavour space, which is easily seen to vanish upon use of the ansatz (36) together with Eq. (41) and \( \mu^- = -\mu^+ \). The vanishing of the collision term under the conditions \( \mu^- = -\mu^+ \) means that the kinetic equilibrium distribution (11) with opposite chemical potentials is indeed a stationary solution of the kinetic equation in the limit when only the fast flavour-blind gauge interactions are present.

Furthermore, when the gauge bosons are in equilibrium and the equilibrium deviation of the leptons is small and parametrised as in Eq. (43), we can approximate the collision term (60) as

\[
C^b_{\ellab}(k) \approx g^2 \int \frac{d^4k'}{(2\pi)^4} \frac{d^4k''}{(2\pi)^4} (2\pi)^4 \delta^4(k - k' - k'') \frac{\gamma'}{iS'_{\ellab}(k')\gamma' i\Delta^>_{A\mu
u}(k'')iS^<_{\ellab}(k) - i\Delta^<_{A\mu
u}(k')iS^>_{\ellab}(k) - i\Delta^<_{A\mu
u}(k')i\Delta^<_{A\mu
u}(k'')i\delta S_{\ellab}(k)}.
\tag{62}
\]

For this expression, we note that the terms in square brackets are odd under a change of sign of the momenta, since to leading order in deviations from equilibrium, we may substitute the equilibrium distributions and make use of the fact that \( i\Delta^>_{A\mu
u}(k) = i\Delta^<_{A\mu
u}(-k) \) and \( iS^<_{\ellab}(k) = iS^<_{\ellab}(-k) \). Hence, after performing the \( k^0 \) integration of the collision term, the same sign contributions to the equations (47) and (52) for \( \delta n^+_{\text{e}} \) and \( \delta n^-_{\text{e}} \) occur due to a cancellation of the relative sign in Eq. (17).

However, if we substituted tree-level propagators in Eq. (62), this collision term would vanish, since all the three particles involved are massless. It is therefore necessary to account for thermal masses and for finite width effects, that relax the zero-temperature on-shell conditions. When employing finite width propagators, analytical simplifications of the collision integral due to on-shell \( \delta \)-functions no longer apply. In the present work, we therefore do not perform collision integrals that vanish for zero-temperature propagators explicitly. Rather, we discuss their general form and give estimates, while relegating more precise numerical evaluations to future studies.

Of particular interest within the collision term (62) are contributions for which \( \text{sign}(k'^0) = -\text{sign}(k''^0) = \text{sign}(k^0) \). These are allowed when we account for the finite width in the spectral functions (for both, \( \ell \) and the gauge fields \( A \)) and they correspond to lepton anti-lepton pair creation and annihilation processes. After performing
the integrations and the Dirac trace, lepton- and antilepton contributions are identical. Therefore, we may parametrise the flavour blind contribution to the collision term in Eq. (47) by

\[ \pm \frac{1}{2} \operatorname{tr} \int_{0,-\infty}^{\infty,0} \frac{dk_0}{2\pi} \int \frac{d^3k}{(2\pi)^3} (\epsilon^{\text{bl}}_{\ell ab} + \epsilon^{\text{bl}*}_{\ell ab}) = -\Gamma^{\text{bl}} (\delta n_{\ell ab}^+ + \delta n_{\ell ab}^-) , \quad (63) \]

which leads to the corresponding term in Eq. (52). Here \( \delta n_{\ell ab}^\pm \) has been factored out by substituting Eqs. (43) and (44). Note that by use of Eqs. (43) and (44) this defines \( \Gamma^{\text{bl}} \), which therefore may readily be evaluated within a more detailed numerical study. For the purposes of the present work, we estimate \( \Gamma^{\text{bl}} \sim g_2^4 T \), where \( g_2 \) is the SU(2)\(_L\) gauge coupling and where the additional factor of \( g_2^2 \) compared to the tree-level matrix element arises from the finite-width effects \[22\].

The fact that the flavour-blind collision terms for \( \delta n_\ell^+ \) and \( \delta n_\ell^- \) are of the same sign also implies that in the absence of additional flavour-sensitive effects, \( \delta n_\ell^+ - \delta n_\ell^- \) is conserved, as it is required for the ansatz of generalised chemical potentials \[41\] to be valid.

### 3.4 Flavour Sensitive Interactions

We now turn to the active lepton Yukawa couplings. These contribute to the self-energy of the left-handed leptons as

\[ i\gamma^\text{fl}_{\ell ab}(k) = h_{ac}^\dagger h_{db} \int \frac{d^4k'}{(2\pi)^4} \frac{d^4k''}{(2\pi)^4} (2\pi)^4 \delta^4(k' - k'' - k''') i S^{fg}_{\text{Red}}(k') i \Delta^g_f(k'') \quad (64) \]

To linear order in deviations from equilibrium, the collision term is

\[ C^{\text{fl}}_{\ell ab}(k) = i\gamma^\text{fl}_{\ell ac}(k) i S^{<}_{\ell cb}(k) - i\gamma^\text{fl}_{\ell ac}(k) i S^{<}_{\ell cb}(k) \approx \int \frac{d^4k'}{(2\pi)^4} \frac{d^4k''}{(2\pi)^4} (2\pi)^4 \delta^4(k - k' - k'') \]

\[ \times \left\{ h_{ac}^\dagger h_{de} i \delta S^{\text{Red}}_{\ell cb}(k') \left[ i \Delta_{\phi}^{>}(k'') i S^{<}_{\ell cb}(k) - i \Delta_{\phi}^{<}(k'') i S^{<}_{\ell cb}(k) \right] + h_{ac}^\dagger h_{de} \left[ i S^{>}_{\text{Red}}(k') i \Delta_{\phi}^{>}(k'') - i S^{<}_{\text{Red}}(k') i \Delta_{\phi}^{<}(k'') \right] i \delta S^{<}_{\ell cb}(k) \right\} \]

\[ = \int \frac{d^4k'}{(2\pi)^4} \frac{d^4k''}{(2\pi)^4} (2\pi)^4 \delta^4(k - k' - k'') \]

\[ \times \left\{ h_{ac}^\dagger h_{de} i \delta S^{\text{Red}}_{\ell cb}(k') \left[ i \Delta_{\phi}^{>}(k'') i S^{<}_{\ell cb}(k) - i \Delta_{\phi}^{<}(k'') i S^{<}_{\ell cb}(k) \right] + h_{ac}^\dagger h_{de} \left[ i S^{>}_{\text{Red}}(k') i \Delta_{\phi}^{>}(k'') - i S^{<}_{\text{Red}}(k') i \Delta_{\phi}^{<}(k'') \right] i \delta S^{<}_{\ell cb}(k) \right\} . \]

Again, the leading thermal corrections to the propagators should be employed, since at tree-level, this integral is vanishing for kinematic reasons. We have to distinguish
two relevant kinematic situations: First, when \( \text{sign}(k^0) = -\text{sign}(k'^0) \), the collision term corresponds to pair creation or annihilation of a left- and a right-handed Standard Model lepton. Second, when \( \text{sign}(k^0) = \text{sign}(k'^0) \) the left- and right-handed leptons scatter from a Higgs boson. Again, both configurations are only possible due to the finite width of the spectral functions of \( \ell \), \( R \) and \( \phi \). We summarise both contributions to Eq. (47) by writing

\[
\Gamma_{\ell ab}^{\pm} = \pm \frac{1}{2} \text{tr} \int_{0, -\infty}^{\infty, 0} \frac{dk^0}{2\pi} \int \frac{d^3k}{(2\pi)^3} \left( C_{\ell ab}^{\pm}(k) + C_{\ell ab}^{\pm}(k) \right) \]

\[
= \Gamma_{\ell ab}^{\text{an}} \left( [h\bar{h}]_{ac} \delta n_{\ell cb}^+ + \delta n_{\ell ac}^{\pm\dagger} [h\bar{h}]_{cb} + h_{ac}^\dagger \delta n_{Rcd}^+ h_{db} + h_{ad}^\dagger \delta n_{Rdc}^+ h_{cb} \right) + \Gamma_{\ell ab}^{\text{sc}} \left( [h\bar{h}]_{ac} \delta n_{\ell cb}^+ + \delta n_{\ell ac}^{\pm\dagger} [h\bar{h}]_{cb} + h_{ac}^\dagger \delta n_{Rcd}^+ h_{db} + h_{ad}^\dagger \delta n_{Rdc}^+ h_{cb} \right).
\]

For later use we note that for the right handed leptons, we have the corresponding flavour sensitive scattering rate

\[
\Gamma_{R ab}^{\pm} = \Gamma_{\ell ab}^{\text{an}} \left( [h\bar{h}]_{ac} \delta n_{R cb}^+ + \delta n_{R ac}^{\pm\dagger} [h\bar{h}]_{cb} + h_{ac}^\dagger \delta n_{Rdc}^+ h_{db} + h_{ad}^\dagger \delta n_{Rdc}^+ h_{cb} \right) + \Gamma_{\ell ab}^{\text{sc}} \left( [h\bar{h}]_{ac} \delta n_{R cb}^+ + \delta n_{R ac}^{\pm\dagger} [h\bar{h}]_{cb} + h_{ac}^\dagger \delta n_{Rdc}^+ h_{db} + h_{ad}^\dagger \delta n_{Rdc}^+ h_{cb} \right).
\]

We estimate the factors \( \Gamma_{\ell ab}^{\text{an}} \) and \( \Gamma_{\ell ab}^{\text{sc}} \) as \( \sim g^2 T \), which is again due to the finite width of \( \ell \) and \( \phi \) at finite temperature. (The U(1)_Y contribution is smaller because of the smaller gauge coupling and the smaller number of gauge bosons). We recall that within these expressions, \( \delta n_{\ell cb}^+ \) and the second index of the coupling \( h \) transform under left-handed flavour rotations, while \( \delta n_{R cb}^+ \) and the first index of \( h \) remain without change. By a unitary transformation of the right-handed flavour basis, we may choose the matrix \( h\bar{h} \) to be diagonal, which is what we assume here.

This concludes the derivation of the kinetic equation (52) for the number densities. An analogous equation (without washout and source terms) holds for the right-handed Standard Model leptons.

### 3.5 Suppression of Flavour Oscillations

The largest collision term within the kinetic equations (52) is \( \Gamma_{bl} = O(g^4 T) \). Close to equilibrium, it imposes the constraint

\[\delta n_{ab}^+ = -\delta n_{ab}^-\]  

(68)

This is expected, since in the flavour-blind limit where \( h_{ab} \to 0 \), this condition is manifestly invariant with respect to flavour rotations and it reduces to the assumption that the lepton charge density of leptons is the same as the lepton charge density of antileptons. Therefore, this condition is implicitly employed in Ref. [9] as well as in many
other kinetic-theory approaches to various problems. Now, due to the ± in the first term on the right hand side of the kinetic equations (52), a large $\Gamma^{\text{bl}}$ effectively inhibits flavour oscillations, which would be present in the absence of collisions. To see this in more detail, consider the toy system of differential equations

$$\frac{d}{dt}\delta g^+(t) = -i \Delta \omega \delta g^+(t) - \Gamma [\delta g^+(t) + \delta g^-(t)] ,$$

(69a)

$$\frac{d}{dt}\delta g^-(t) = +i \Delta \omega \delta g^-(t) - \Gamma [\delta g^-(t) + \delta g^+(t)].$$

(69b)

The relevant parameters for flavoured leptogenesis can be estimated as

$$\Gamma = \Gamma^{\text{bl}} \sim g_2^4 T , \quad \Delta \omega \sim h_\tau^2 \tau T \ll \Gamma ,$$

(70)

where $h_\tau$ denotes the $\tau$-lepton Yukawa coupling. Since $g_2^4 \gg h_\tau^2$, the solutions are linear combinations of two eigenmodes with short $\tau_s = 1/(\Gamma + \sqrt{\Gamma^2 - \Delta \omega^2}) \approx 1/(2\Gamma)$ and long $\tau_l = 1/(\Gamma - \sqrt{\Gamma^2 - \Delta \omega^2}) \approx 2\Gamma/\Delta \omega^2$ decay times, respectively. The corresponding eigenvectors are given by

$$\delta g_{s,l} = \delta g^+ + \frac{-i \Delta \omega \pm \sqrt{\Gamma^2 - \Delta \omega^2}}{\Gamma} \delta g^- \approx \delta g^+ \left( 1 \mp \frac{i \Delta \omega}{\Gamma} \right) \delta g^- ,$$

(71)

with

$$\delta g_{s,l} = (\delta g_{s,l})_0 e^{-t/\tau_{s,l}} .$$

(72)

The short mode $\delta g_s \approx \delta g^+ + \delta g^-$ is thus damped to zero very rapidly by pair annihilations, implying an effective constraint

$$\delta g^+ \sim - \left( 1 - i \frac{\Delta \omega}{\Gamma} \right) \delta g^- .$$

(73)

Note that the different sign of $\Delta \omega$ terms in Eq. (69) is decisive, since it implies that the driving term for oscillations in

$$\frac{d}{dt} (\delta g^+(t) - \delta g^-(t)) = -i \Delta \omega \left( \delta g^+(t) + \delta g^-(t) \right)$$

(74)

is damped away, while in the case of same sign $\delta g^+ - \delta g^-$, could have freely oscillated. As explained in Section 2.3, the opposite sign of the $\Delta \omega$ term in Eq. (52) is a consequence of $CP$ invariance at leading order.

Within the gradient expansion, the first order correction to Eq. (68) therefore is of order $\Delta \omega^{\text{eff}}/\Gamma^{\text{bl}}$. Since the source terms for the off-diagonal correlations are already of first order in gradients, it is justified to use the zeroth order constraint Eq. (68) within our approximations. The long-lived mode describes the damping of flavour coherence in the lepton charge density matrix due to flavour-blind interactions. It
is much slower compared to the damping rate due to flavour sensitive interactions, $\Delta \omega_{\text{eff}}^2 / \Gamma^{\text{bl}} \sim h_g^3 g_2^{-4} T \ll \Gamma^{\text{fl}} \sim g_2^2 h_r^2 T$ since $h_r \ll g_2^3$. Therefore, we may neglect the damping due to flavour-blind interactions, while we keep the direct damping due to flavour sensitive processes. While in the case of leptogenesis, we conclude that because of $\Delta \omega \ll \Gamma$, flavour oscillations are overdamped and effectively frozen, we note that for $\Delta \omega > \Gamma$, there are damped flavour oscillations. It is interesting to note that even though we assume flavour blind interactions, the off-diagonal flavour coherence functions are decaying. Such a behaviour, in particular in the oscillatory regime, has been observed numerically in Ref. [18].

We emphasise that the conclusion that the oscillations induced by $\Delta \omega$ are overdamped for $\Gamma \gg \Delta \omega$ does not depend on the choice of the flavour basis. To see this, we extend $g^\pm$ to a vector of functions and consider the system of matrix equations

\[
\frac{d}{dt} \delta g^+(t) = -i [\omega, \delta g^+(t)] - \Gamma [\delta g^+(t) + \delta g^-(t)], \quad (75a)
\]

\[
\frac{d}{dt} \delta g^-(t) = +i [\omega, \delta g^-(t)] - \Gamma [\delta g^-(t) + \delta g^+(t)]. \quad (75b)
\]

Here, $\Gamma$ is proportional to the unit matrix and $\omega = \omega^{\text{fl}} + \omega^{\text{bl}}$, where $\omega^{\text{bl}}$ is proportional to the unit matrix and $\omega_{ab}^{\text{fl}} \ll \Gamma_{cc}$ for all $a, b, c$. It then follows that $[\omega, \delta g^+(t)] = [\omega^{\text{fl}}, \delta g^+(t)]$. By taking the sum of Eqs. (75), we again conclude that $\delta g^+(t) + \delta g^-(t) \sim e^{-2\Gamma t}$. Consequently, the difference of Eqs. (75) yields

\[
\frac{d}{dt} \left[ \delta g^+(t) - \delta g^-(t) \right] = 0 + \left[ \delta g^+(t) - \delta g^-(t) \right] \times \mathcal{O} \left( \frac{\omega_{ab}^2}{\Gamma_{cc}} \right), \quad (76)
\]

where the right hand side is estimated as the eigenvalues of a matrix with large diagonal and small off-diagonal elements. Alternatively, this can be seen by substituting in the right hand side of the difference of the system of matrix equation

\[
[\omega, \delta g^+(t) + \delta g^-(t)] = (\delta g^+(t) - \delta g^-(t)) \times \mathcal{O} \left( \frac{\omega^2}{\Gamma} \right), \quad (77)
\]

where an estimate according to Eq. (73) is made. This confirms the suppression of the effect of $\zeta^{\text{fl}}$ by $\Gamma^{\text{bl}}$ in a general flavour-basis.

We can also generalise this discussion to the case of a time-dependent mass basis. In order to model this situation, consider the system

\[
\frac{d}{dt} \delta g^+_{ab}(t) = -i \Delta \omega_{ab} \delta g^+_{ab}(t) + \Xi_{ac} \delta g^+_{cb} - \delta g^+_{ac} \Xi_{cb} - \Gamma [\delta g^+_{ab}(t) + \delta g^-_{ab}(t)], \quad (78a)
\]

\[
\frac{d}{dt} \delta g^-_{ab}(t) = +i \Delta \omega_{ab} \delta g^-_{ab}(t) + \Xi_{ac} \delta g^-_{cb} - \delta g^-_{ac} \Xi_{cb} - \Gamma [\delta g^-_{ab}(t) + \delta g^+_{ab}(t)]. \quad (78b)
\]

In the limit $\Gamma \gg \Delta \omega_{ab}$, we may find an approximate solution by imposing $\delta g^+ = -\delta g^-$. This leads to

\[
\frac{d}{dt} \left( \delta g^+_{ab}(t) - \delta g^-_{ab}(t) \right) = \left[ \delta g^+_{ab}(t) - \delta g^-_{ab}(t), \Xi \right], \quad (79)
\]
which is solved by the unitary evolution
\[
\delta g_{ab}^+(t) - \delta g_{ab}^-(t) = (T e^{-\Xi t}) \left( \delta g_{ab}^+(t = 0) - \delta g_{ab}^-(t = 0) \right) (\bar{T} e^{\Xi t}),
\]
where $T$ implies the time-ordered exponential. Therefore, the freezing of flavour oscillations also persists when we account for the time dependence of the mass basis. From above equation, we recover Eq. (76) by undoing the flavour rotation, that is by left multiplication by $U$ and right multiplication by $U^\dagger$.

### 3.6 Kinetic Equations for Left and Right Handed Number Densities

We now define the charge number density matrix as
\[
q_{\ell ab} = \delta n_{\ell ab}^+ - \delta n_{\ell ab}^-.
\]
Imposing that fast (compared to the interactions accounted for in $W_{ab}$ and $S_{ab}$) pair creating and annihilating interactions enforce the constraint
\[
\delta n_{\ell ab}^+ = -\delta n_{\ell ab}^-,
\]
we can take the linear combinations from Eq. (52) that solve for the charge density matrix (81). For the flavour-sensitive interactions, define
\[
\Gamma_{\ell ab}^\text{fl} = \Gamma_{\text{an}} \left( [h^\dagger h]_{ac} q_{\ell cb} + q_{\ell ac} [h^\dagger h]_{cb} - h^\dagger_{ac} q_{Rcd} h_{db} - h^\dagger_{ad} q_{Rdc} h_{cb} \right)
\]
\[
+ \Gamma_{\text{sc}} \left( [h^\dagger h]_{ac} q_{\ell cb} + q_{\ell ac} [h^\dagger h]_{cb} - h^\dagger_{ac} q_{Rcd} h_{db} - h^\dagger_{ad} q_{Rdc} h_{cb} \right).
\]
Using this and the results of the previous sections, we obtain the kinetic equations (2) which we repeat here for completeness:
\[
\frac{\partial q_{\ell ab}}{\partial \eta} = \sum_c \left[ q_{\ell ac} \Xi_{cb} - \Xi_{ac} q_{\ell cb} - W_{ac} q_{\ell cb} - q_{\ell ac} W_{cb} \right] + 2 S_{ab} - \Gamma_{\ell ab}^\text{fl}.
\]
Note that similar to the toy system of equations the flavour-blind term drops out in the equation for $q_{\ell ab}$, while it is consistent to neglect the $\Delta \omega_{\ell ab}^\text{eff}$ term, which would multiply $\delta n_{\ell ab}^+ + \delta n_{\ell ab}^-$ in this equation, which is strongly damped. This holds in an arbitrary, time-independent basis in flavour space, where the $\Xi$ terms are absent. In a time-dependent basis such as the basis where $\varsigma_{ab}$ is diagonal, the $\Xi$ terms are introduced to account for the time-dependent basis rotation.

For the right-handed leptons, there is the analogous equation
\[
\frac{\partial q_{Rab}}{\partial \eta} = -\Gamma_{Rab}^\text{fl}.
\]
with
\[ \Gamma_{Rab}^{fl} = \Gamma_{Rab}^{an} \left( [h h^\dagger]_{ac} q_{Rcb} + [h h^\dagger]_{cb} - h_{ac} q_{Rcd} h_{db}^\dagger - h_{ad} q_{Rdc} h_{cb}^\dagger \right) \] (86)
\[ + \Gamma_{Rab}^{sc} \left( [h h^\dagger]_{ac} q_{Rcb} + [h h^\dagger]_{cb} - h_{ac} q_{Rcd} h_{db}^\dagger - h_{ad} q_{Rdc} h_{cb}^\dagger \right) . \]

Similar results have been obtained earlier within an approach that makes use of the density matrix in an occupation number basis. In its details, the equation for the difference between lepton and anti-lepton densities in Ref. [13] exhibits however differences to our kinetic equation (84). It is not clear whether the lepton charge densities in Ref. [13] should correspond to our \( q_{\ell ab} \) (which is the difference of the lepton density and the transpose of the anti-lepton density) or to the difference of the lepton density and the anti-lepton density. In the former case, the flavour oscillations frequencies in Ref. [13] should have opposite signs for particle and transposed antiparticle modes, if they were to agree with our result obtained within the CTP formalism. This is apparently not the situation within the equation for the lepton charge density in Ref. [13]. In the latter case, as it follows from Eq. (39) and the discussion at the end of Section 2.3 within the CTP formalism the washout and source matrices for the lepton and anti-lepton densities are transposed (or complex conjugated, as these matrices are Hermitian) with respect to each other, which is apparently not the case in Ref. [13]. Furthermore, the same conclusions on damping of coherence would result from our equations if the charge density matrix were defined as \( \delta n_{\ell ab}^+ - \delta n_{\ell ba}^- \). Hence, with either interpretation, there is a difference between the occupation number formalism result that is derived in Ref. [13] and the kinetic equation (2) derived within the CTP formalism. The phenomenological consequence of this can be seen when comparing the present work with Ref. [13], where the kinetic equations from Ref. [13] are solved numerically. While in the present work, we conclude that flavour oscillations effectively freeze out due to fast pair creation and annihilation processes, the results in Refs. [13, 15] imply that the flavour oscillations are important and in particular faster than the flavour-sensitive damping processes.

4 Solutions to the Flavoured Kinetic Equations

We are considering a scenario with two lepton flavours and assume that there is one dominant Standard Model Yukawa coupling \( h_\tau \). In the basis where the lepton Yukawa coupling matrix is diagonal, the matrix \( h \) is therefore simply
\[ h = \begin{pmatrix} h_\tau & 0 \\ 0 & 0 \end{pmatrix} . \] (87)
Provided the \( \mu \) and \( e \) Yukawa-couplings are negligible \( h_{\mu,e}^2 T/a(\eta) \ll H \), the realistic case with three lepton flavours can be reduced to the present case by separating out a linear combination of lepton flavours, for which no asymmetry is produced. This corresponds to an unflavoured approximation for the \( e \) and \( \mu \) flavours, cf. the discussion
of the unflavoured limit below. We note that Eq. (2) is manifestly invariant under flavour rotations induced by $U$, while Eqs. (46) and (47) are not, because the term that describes flavour oscillations is given in the diagonal basis. In our approximation, we can drop this term, because we have shown in Section 3 that due to the constraints from kinetic equilibrium, the time scale for flavour oscillations is suppressed when compared to the time scale of decoherence from flavour-sensitive scatterings. We use this freedom of choice of a lepton flavour basis and perform the discussion in this section within the time-independent basis of charged lepton flavours, which is more transparent than the time-dependent basis of the leptonic quasi-particles, that is determined by the diagonalisation of $\zeta^\text{fl}$. Likewise, we present all numerical results in the basis of charged lepton flavours.

In the charged lepton basis, the flavour-sensitive collision terms read

$$\Gamma^\text{fl}_\ell = (\Gamma^\text{an} + \Gamma^\text{sc}) h^2 \tau \left[ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} q_\ell + q_\ell \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} - 2 \begin{pmatrix} q_{R11} & 0 \\ 0 & 0 \end{pmatrix} \right], \quad (88a)$$

$$\Gamma^\text{fl}_R = (\Gamma^\text{an} + \Gamma^\text{sc}) h^2 \tau \left[ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} q_R + q_R \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} - 2 \begin{pmatrix} q_{\ell11} & 0 \\ 0 & 0 \end{pmatrix} \right]. \quad (88b)$$

In the fully flavoured limit, which we define by the requirement $(\Gamma^\text{an} + \Gamma^\text{fl}) h^2 \tau \gg H$ (note that for the present purposes, “fully flavoured” refers to the situation where $h_\mu$ and $h_e$ are still assumed to be out-of-equilibrium), we see that within this setup, the flavour sensitive collision terms enforce

$$q_{\ell11} - q_{R11} = 0, \quad q_{\ell12} = q_{\ell21} = q_{R12} = q_{R21} = 0. \quad (89)$$

This agrees with the expectation that when Standard Model Yukawa couplings are in equilibrium, the lepton asymmetries are projected onto the charged lepton basis.

The processes $\ell + R \leftrightarrow \phi^*$ (annihilation) and $\ell + \phi \leftrightarrow R$ (scattering) are kinematically forbidden when all the three particles involved are massless. At finite temperature, this holds no longer true due to effects that at leading order can be either thought of as thermal masses and finite widths or as radiation of gauge bosons. The latter point of view is taken in Ref. [25] to calculate $\Gamma^\text{sc}$. However, important $t$-channel diagrams are not included there and a calculation of $\Gamma^\text{an}$ is not provided. A systematic calculation of these rates may be performed along the lines of Ref. [24], which is however beyond the scope of the current work. Motivated by the partial result of Ref. [25], we take here for numerical definiteness the estimate

$$\Gamma^\text{an} + \Gamma^\text{sc} \approx 0.7\alpha_W T/a(\eta) = 1.75 \times 10^{-2} T/a(\eta), \quad (90)$$

which should be accurate up to a factor of order unity. Besides, we take here $\alpha_W = 1/40$ as the weak coupling constant at the scale of about $10^{12}$ GeV. The precise value depends on the particular extension of the Standard Model.

Yet, we have used the requirement that computations in both bases must yield the same results as a consistency check on the numerical results.
To obtain numerical solutions, we first solve the kinetic equations for the distribution of the right-handed neutrinos $N_1$. They are given in Ref. [9], and the generalisation from the single flavour to the two-flavour case follows by the straightforward replacement $|Y_1|^2 \rightarrow \sum_a |Y_{1a}|^2$. We employ this distribution to calculate the washout and the source terms within Eq. (2). To be specific, we choose thermal initial conditions for $N_1$. For the singlet neutrino masses, we choose $M_1 = 10^{12}$ GeV and $M_2 = 10^{14}$ GeV. For the Yukawa couplings of the right handed neutrinos, we consider two scenarios

$$Y = \begin{pmatrix} 1.4 \times 10^{-2} & 1 \times 10^{-2} \\ i \times 10^{-1} & 10^{-1} \end{pmatrix}, \quad \text{Scenario (A)}, \quad (91)$$

$$Y = \begin{pmatrix} 1.4 \times 10^{-2} & 3 \times 10^{-3} \\ i \times 10^{-1} & 10^{-1} \end{pmatrix}, \quad \text{Scenario (B)}.$$

We vary the Yukawa coupling $h_\tau$, since this will directly exhibit the dependence of the results on the flavour effects, while of course, for a phenomenological study, it would be more pertinent to vary the unknown parameters $Y$ and $M_{1,2}$.

In Figure 1 we show the interaction rates $\Gamma^\text{fl} = h^2_\tau (\Gamma^\text{an} + \Gamma^\text{sc})$ for different values of $h_\tau$ and compare them to the expansion rate of the universe $H$ and to the inverse decay rate for the individual flavours $a$, $\Gamma^\text{ID}_a = 2W_{aa}$ as a function of the ratio of $M_1$ to the physical temperature, $z = a(\eta)M_1/T$. Scenario (A) exhibits moderate to strong washout in both flavours, where the dominant contributions to the lepton asymmetry are generated between $z \approx 3$ and the point when the lepton asymmetry freezes out, $\Gamma_{1D} \approx H$. We expect flavour effects to be negligible when $\Gamma^\text{fl} \lesssim H \approx \Gamma^\text{a}_{1D}$ during these times before freeze out [26], i.e. for $h_\tau$ significantly smaller than $4 \times 10^{-3}$ by inspection of Figure 1. On the other hand a fully flavoured description should be applicable when
Figure 2: Results for Scenario (A) with $h_r = 3 \times 10^{-2}$, $7 \times 10^{-3}$, $4 \times 10^{-3}$, $2 \times 10^{-3}$, $10^{-3}$, 0, from top left to bottom right. The key is $Y_{\ell 11}$ (dark blue, solid), $Y_{\ell 22}$ (light blue, solid), Re[$Y_{\ell 12}$] (dark red, dotted), Im[$Y_{\ell 12}$] (light red, dashed). The densities are evaluated in the flavour eigenbasis, which means that the larger the flavour effects are (the larger $h_r$ is), the smaller are the off-diagonal densities.
Figure 3: Results for Scenario (B) with $h_\tau = 3 \times 10^{-2}, 7 \times 10^{-3}, 4 \times 10^{-3}, 2 \times 10^{-3}, 10^{-3}, 0$ from top left to bottom right. The key is $Y_{\ell 11}$ (dark blue, solid), $Y_{\ell 22}$ (light blue, solid), $\text{Re}[Y_{\ell 12}]$ (dark red, dotted), $\text{Im}[Y_{\ell 12}]$ (light red, dashed).
\(\Gamma^\text{fl} \gtrsim \Gamma^\text{id}_\text{ID}\) during the times when the quantitatively relevant contributions to the lepton asymmetry are produced, i.e. for \(h_\tau\) significantly larger than \(7 \times 10^{-3}\). The numerical solutions to the kinetic equations for Scenario (A) are displayed in Figure 2. Shown are the absolute values of the entropy normalised asymmetries

\[ Y_{\text{lab}} = 2g_\text{w} \frac{q_{\text{lab}}}{2\pi^2} \frac{T^3}{g_\ast} , \]  

where we use \(g_\ast = 106.75\). Since \(Y_\ell\) is Hermitian, we plot the real and imaginary parts of \(Y_{\text{lab}}\). The results confirm our expectations about the validity of the fully flavoured and the unflavoured descriptions of leptogenesis. For \(h_\tau \lesssim 2 \times 10^{-3}\) the total lepton asymmetry \(\text{tr}[Y_\ell] = Y_{\ell 11} + Y_{\ell 22}\) is almost independent of \(h_\tau\), and the off diagonal densities decay away after freeze out \((z \approx 10)\) only. (Only the initial stages of this decay are visible in the plots for \(h_\tau = 2 \times 10^{-3}\), \(10^{-2}\) in Figure 2 due to the cut off at \(z = 20\). For \(h_\tau = 0\), the off-diagonal densities do not decay.) On the other hand in the fully flavoured regime, \(h_\tau \gtrsim 3 \times 10^{-2}\), the off diagonal densities are strongly suppressed before freeze-out. This confirms that neglecting the off-diagonal densities, an approximation that is commonly used in the fully flavoured regime, is indeed justified in this regime. In the intermediate regime, where neither of these approximations is valid, the correct lepton asymmetry is obtained by solving the full kinetic equation (2).

Scenario (B) is a situation with strong washout due to \(Y_{11}\) and weak washout for \(Y_{12}\). The numerical solutions are displayed in Figure 3. Since \(\Gamma^\text{id}_\text{ID}\) is now significantly smaller, it takes also smaller values of \(h_\tau\) before the unflavoured description may be expected to be valid, cf. Figure 1. In the fully flavoured regime, we observe that one of the lepton flavours suffers from strong washout while the other one is only weakly washed out. On the other hand, in the basis where the source term is diagonal, the flavours apparently mix in such a way that both lepton flavours are strongly washed out when flavour effects are turned off. This importance of flavour effects for washout is well known [12–14], and it can be easily understood when we recall how the fully flavoured and the unflavoured regimes are described.

First, in the fully flavoured case, densities that are off-diagonal in the flavour basis undergo fast damping through flavour-sensitive interactions. As a result, there are two washout rates that are proportional to \(|Y_{11}^2|\) and \(|Y_{12}^2|\), respectively. Second, in the unflavoured regime, it is convenient to bring \(Y\) to a triangular form through

\[ Y^\Delta_{ia} = Y_{ib}V_{ba}, \]  

where \(V = \frac{1}{\sqrt{|Y_{21}|^2 + |Y_{22}|^2}} \begin{pmatrix} Y_{22} & Y_{21}^* \\ -Y_{21} & Y_{22}^* \end{pmatrix}. \)  

(94)

In the triangular basis, a lepton asymmetry is only produced for the linear combination

\[ \frac{1}{\sqrt{|Y_{21}|^2 + |Y_{22}|^2}} (Y_{21} \ell_1 + Y_{22} \ell_2). \]  

(95)
The washout rate for this linear combination is proportional to

\[ |Y_{12}|^2 = \frac{1}{|Y_{21}|^2 + |Y_{22}|^2} (|Y_{11}|^2|Y_{21}|^2 + |Y_{12}|^2|Y_{22}|^2 + 2\text{Re} [Y_{11}Y_{21}^*Y_{12}Y_{22}^*]) \quad (96) \]

The change of the effective flavour basis in the transition from the unflavoured to the flavoured regime therefore explains the apparent change in the washout rates for the individual flavours for Scenario (B), that are visible in Figure 3.

To obtain an estimate of the extent of the intermediate regime in terms of the parameter \( M_1 \), we now fix the \( \tau \)-Yukawa coupling to \( h_\tau = 0.007 \), close to its physical value, and vary \( M_1 \rightarrow \alpha M_1 \) instead. To keep the effect of the washout term and the source term constant, we also scale \( Y_{11} \rightarrow \sqrt{\alpha} Y_{11} \) and \( Y_{12} \rightarrow \sqrt{\alpha} Y_{12} \) as well as \( M_2 \rightarrow \alpha M_2 \).

This scaling behaviour can be seen when recasting Eq. (2) into the form

\[ zH \frac{\partial q_{lab}}{\partial z} = \frac{1}{a} \left\{ \sum_c \left[ q_{lac} \Xi_{ecb} - \Xi_{ac} q_{ecb} - W_{ac} q_{ecb} - q_{ac} W_{cb} \right] + 2S_{ab} - \Gamma_{lab}^\text{fl} \right\} \quad (97) \]

The terms on the right hand side now correspond to the physical instead of conformal interaction rates per unit volume. Since at fixed \( z \), \( T/a(\eta) \rightarrow \alpha T/a(\eta) \), \( H \rightarrow \alpha^2 H \) and \( q_\ell \rightarrow \alpha^3 q_\ell \), both sides scale as \( \alpha^5 \), except for the term \( 1/a \times \Gamma_{\ell}^\text{fl} \), which scales as \( \alpha^4 \). Therefore, all the scale-dependence is isolated within the flavour-dependent damping rate.

We solve the kinetic equations for \( 10^{10} \text{ GeV} < M_1 < 2 \times 10^{14} \text{ GeV} \). Parametrically this brings us from a regime where flavour effects are maximal to the unflavoured regime [26]. For comparison we also calculate the lepton asymmetry over this parameter range using first the unflavoured approximation \( (h_\tau = 0) \) and then using the fully flavoured approximation, where the off-diagonal number densities are set to zero throughout the calculation.

The results are shown in Figure 4. We find that both the fully flavoured approximation and the unflavoured approach lead to accurate predictions of the total lepton asymmetry within their expected ranges of validity. The intermediate regime where the full kinetic equation needs to be solved ranges from around \( 5 \times 10^{11} \text{ GeV} - 10^{13} \text{ GeV} \) for Scenario (A) and even further for Scenario (B) where the unflavoured behaviour is only recovered for \( M_1 \gtrsim 10^{14} \text{ GeV} \). This is because the condition for the unflavoured description to be valid, \( \Gamma_{\ell}^a \gtrsim h_\tau^2 (\Gamma_{an} + \Gamma_{sc}) \) for \( \alpha = 1, 2 \) is only fulfilled for larger values of \( M_1 \) within Scenario (B). Besides, within the flavoured approximation of Scenario (B), the flavour \( \alpha = 2 \) is only weakly washed out. Therefore, quantitatively relevant contributions to the lepton asymmetry arise at earlier times, where \( \Gamma_{\ell}^a / [h_\tau^2 (\Gamma_{an} + \Gamma_{sc})] \) is enhanced compared to this ratio close to freeze-out. As a consequence, the fully flavoured description of Scenario (B) requires smaller values for \( M_1 \) when compared to Scenario (A). Note that the absolute limits for the validity of the unflavoured or fully flavoured description may vary by up to order one factors due to the uncertainty in the overall prefactor of \( \Gamma_{\ell}^\text{fl} \). We identify this also as a probable source of the numerical difference between the present work and Ref. [26], where it was found using \( \Gamma_{an} + \Gamma_{sc} \approx 5 \times 10^{-3} T/a(\eta) \) that the unflavoured description is valid already for \( M_1 \gtrsim 5 \times 10^{11} \text{ GeV} \).
Figure 4: Shown is the total lepton asymmetry $\text{tr}[Y_\ell] = Y_{\ell 11} + Y_{\ell 22}$ as a function of the righthanded neutrino mass $M_1$, for parameters corresponding to Scenarios (A) and (B). The result of the full kinetic equations including flavour effects (solid blue line) is compared to the results of the unflavoured approximation $h_{\tau} = 0$ (red, dotted) and to the results from a fully flavoured approximation that neglects off-diagonal flavour excitations (green, dashed).
5 Conclusions

Using the CTP formalism, we have derived and solved kinetic equations that describe flavoured leptogenesis. Our results allow for systematic calculations of the lepton asymmetry within the intermediate regime, that is neither fully flavoured nor unflavoured, and where off-diagonal correlations between the lepton densities are of importance. The CTP framework proves particularly suitable for this problem, since off-diagonal densities are straightforwardly implemented within the two-point Green functions.

So far, kinetic equations that describe flavoured leptogenesis within the intermediate regime have only been available as extrapolations from a toy model description of number density matrices in the occupation number formalism [13]. The importance of a more systematic derivation of these equations has been emphasised for example in Ref. [26]. Our main result, Eq. (2), that is derived within the CTP framework with the Lagrangian (1) as the starting point, turns out to resemble the corresponding equation of Ref. [13] in many details of its flavour structure, but it also exhibits qualitative differences. The main improvements provided with the present work may be summarised as follows:

- We derive the impact of the thermal dispersion relations on the kinetic equations for the left-handed leptons. This confirms the expectation that the effects of these dispersion relations share some qualitative features with flavour oscillations induced by tree-level mass terms and allows for quantitative predictions. Our results might also be useful for describing the dynamics of neutrino flavours in interacting backgrounds.

- We find that fast pair creation and annihilation processes through gauge interactions effectively overdamp flavour oscillations. This is a qualitatively distinct feature from the results of Refs. [13, 15].

- The washout, source and damping terms in the kinetic equations are derived from first principles. They correspond to collision terms which we explicitly present in the form of integrals. Some of these integrals crucially depend on finite width effects, which make their evaluation difficult. For the purpose of our numerical examples, we make only estimates for those collision terms that strongly depend on finite-width effects. Yet, these collision terms are well-defined, and their quantitatively accurate evaluation may be the subject of future work, possibly using the methods of Ref. [24].

For a more accurate prediction of the lepton asymmetry, the present analysis has to be supplemented by a number of improvements, which vary in how detailed they have been discussed in the literature yet and in how straightforwardly the present work can be generalised to include them. First, there are the so-called spectator processes [27, 28], which are transitions induced by Yukawa couplings and by strong and weak sphalerons and which transfer charges between the ℓ_a and φ to other particles of the Standard Model. Note that also the scatterings induced by h, that transfer charges to the charged
right-handed leptons belong to this category. Depending on whether the additional interactions are fully equilibrated, out-of equilibrium or in an intermediate regime, the kinetic equations have either to be supplemented by algebraic constraints for the various charges, or the network of equations has to be extended in a way that is similar to how we account for the charged right-handed leptons. Conceptually more interesting and challenging is the systematic inclusion of thermal effects. In the present work, we have noted that the finite width and thermal mass effects allow for certain three-body processes that are kinematically forbidden in the vacuum. Again, the CTP formalism bears the potential to account for thermal effects more systematically and may hence serve to confirm or to extend earlier results on these effects \[29\]. Therefore, important improvements remain to be incorporated in order to calculate the lepton asymmetry of the Universe to a good accuracy and with a quantifiable account of the theoretical uncertainty. In order to achieve this goal, the systematic computation of the flavour effects from first principles as presented in this work may serve as a building block.

**Acknowledgements**

This work is supported by the Gottfried Wilhelm Leibniz programme of the Deutsche Forschungsgemeinschaft and by the Schweizer Nationalfonds.

## A Pole-Mass Equation and Finite Width Propagators

In this Appendix, we show how close to equilibrium, the constraint equation \([12a]\) and the equations for the retarded and advanced propagators \([11a]\) can be solved consistently to first order in gradients. The first order corrections account for modified dispersion relations and for finite widths.

In thermal equilibrium, the collision term is vanishing on the right-hand side of the constraint equation \([12a]\). Since the collision term is already first order in gradients, we may therefore neglect it within the constraint equations when being close to equilibrium. Furthermore, the propagators and self-energies are approximately flavour-diagonal, such that we can express the constraint equation \([12a]\) in the simple form

\[
\left\{ \frac{L}{\Sigma^H}, iS^{<<H} \right\} - \left\{ \Sigma^{<<H}, iS^{H}_L \right\} = 0 .
\]  

The spectral function \( S^{<<A}_L \) is defined through Eq. \([7]\). In order to solve the pole-mass equation \([11a]\), it is useful to introduce

\[
\tilde{\Sigma}^H_L = \begin{pmatrix} 0 & \Sigma_L \cdot \sigma \\ \Sigma_L \cdot \bar{\sigma} & 0 \end{pmatrix} ,
\]  

\[(A.2)\]
where $\Sigma^\mu_\ell = \frac{1}{2} \text{tr} \gamma^\mu \Sigma_\ell$. This facilitates the inversion of propagators within the four component formalism in analogy with the doubling of degrees of freedom within the Weyl-fermion propagators that is familiar from the in-out framework \cite{30}. We obtain (cf. \cite{22})

\[
S^A_\ell = P_L \left( \frac{1}{2} \left( k - \tilde{\Sigma}^H_\ell / 2 \right) \Sigma^A_\ell \cdot (k - \Sigma^H_\ell) - \tilde{\Sigma}^A_\ell \left( k - \tilde{\Sigma}^H_\ell / 2 \right)^2 + \Sigma^{A3}_\ell \right) P_R. \tag{A.3}
\]

Note again that we have used simplifications due to the flavour-diagonal structure in equilibrium. Similarly, we find the Hermitian propagator

\[
S^H_\ell = P_L \left( \frac{1}{2} \left( k - \tilde{\Sigma}^H_\ell / 2 \right) \Sigma^H_\ell \cdot (k - \Sigma^H_\ell) + 2 \tilde{\Sigma}^A_\ell \Sigma^H_\ell \cdot (k - \Sigma^H_\ell) \right) P_R. \tag{A.4}
\]

Now, if we use that in equilibrium

\[
\vartheta(k^0) f_{\ell ab}^{eq+}(k) - \vartheta(-k^0)(1_{ab} - f_{\ell ab}^{eq-}(k)) = \delta_{ab} \frac{1}{e^{\beta k^0} + 1} \tag{A.5}
\]

and the KMS relation to substitute

\[
\tilde{\Sigma}^A_\ell = -\frac{i}{2} (e^{\beta k^0} + 1) \tilde{\Sigma}^<_\ell, \tag{A.6}
\]

we find that Eq. (36a) with $S^A_\ell$ given by Eq. (A.3) indeed solves the constraint equation (A.1) in equilibrium. In a similar way, the same observation holds for Eq. (36b). A related discussion can be found in Ref. \cite{19}. Note that relation (A.6) establishes the connection between the finite width and the collision term \cite{13}, that controls how fast a small perturbation in the lepton density relaxes to its equilibrium value. The zero-width approximation \cite{37} follows from the result (A.3) in the limit $\Sigma^A_\ell \cdot (k - \Sigma^H_\ell) \to 0$.

Close to the poles, where $(k - \tilde{\Sigma}^H_\ell_{\ell a a})^2 = 0$, the Hermitian propagator $S^H_\ell$ is suppressed compared to the spectral function $S^A_\ell$ by an additional factor $O(\Sigma^A_\ell / k)$. To first order in the gradient expansion and in the narrow width limit, where $\Sigma^A_\ell \cdot (k - \Sigma^H_\ell) \ll k^0$, we can therefore neglect the terms involving $S^H_\ell$ in Eqs. (12).

Plugging the ansatz (21) into the constraint equation (A.1) and neglecting the term $\{\Sigma^<_\ell, iS^H_\ell\}$ leads us to

\[
\begin{align*}
g_{h_0}^{k^0} k^0 + h |k| g_{h_0}^{k^0} - \frac{1}{2} \{\bar{\epsilon}^b + \bar{\epsilon}^a, g_{h_0}^{k^0} \} - \frac{h}{2} \{ \text{sign}(k^0)(\bar{\epsilon}^b + \bar{\epsilon}^a) - \bar{\epsilon}^b - \bar{\epsilon}^a, g_{h_3}^{k^0} \} &= 0, \tag{A.7a} \\
g_{h_3}^{k^0} k^0 + h |k| g_{h_3}^{k^0} - \frac{1}{2} \{\bar{\epsilon}^b + \bar{\epsilon}^a, g_{h_3}^{k^0} \} - \frac{h}{2} \{ \text{sign}(k^0)(\bar{\epsilon}^b + \bar{\epsilon}^a) - \bar{\epsilon}^b - \bar{\epsilon}^a, g_{h_0}^{k^0} \} &= 0. \tag{A.7b}
\end{align*}
\]
When neglecting the hole modes, for leptons \((k^0 > 0)\) the helicity \(h = -1\) is negative, while for anti-leptons \((k^0 < 0)\) the helicity \(h = 1\) is positive. In conjunction with the constraint \((22)\), this implies within the flavour-diagonal basis the dispersion relations

\[
k^0 = \pm \left[ |k| + \zeta^{bl} + \frac{1}{2}(\zeta^{fl}_{aa} + \zeta^{fl}_{bb}) \right]
\]

for \(g_{hab}^{<,>}\). Note that in the present case, \(\zeta^{bl} \sim g_2^2\), while \(\zeta^{fl}_{aa} \sim [h^\dagger h]_{aa}\). Since \(g_2^2 \gg [h^\dagger h]_{aa}\), the expressions for the dispersion relations \((A.8)\), that are accurate to order \(g_2^2\), are not reliable to order \([h^\dagger h]_{aa}\) in case \(g_2^4 \sim [h^\dagger h]_{aa}\). However, since the flavour-blind terms are universal, the differences between the dispersion relations for different \(i,j\) are nonetheless accurate to order \([h^\dagger h]_{aa}\).

### B Thermal Lepton Dispersion Relation Induced by the Right Handed Neutrino

In this Appendix, we calculate the thermal correction to the dispersion relation of the left handed lepton \(\ell\) induced by its Yukawa coupling \(Y\) to the heavy right handed neutrino \(N_1\) and the Higgs boson \(\phi\). The leading order contribution comes from the one-loop wave-function correction where \(N_1\) and \(\phi\) are running in the loop. A similar calculation has been performed in Ref. \([23]\) for a massless fermion in the loop and in Ref. \([31]\) for the light massive case. In the latter calculation, instead of the Higgs field, there is a massless gauge boson in the loop, but the intermediate results before taking the limit of small fermion mass can be straightforwardly applied to the present situation.

#### B.1 Decomposition of the Self Energy

The structure of the self energy and the dressed propagator is restricted by Lorentz invariance, which allows us to parametrise both by two invariant functions \(a\) and \(b\). These can be determined by an explicit calculation of the self energy as performed in detail in Ref. \([31]\), and they can be used to find the poles of the propagator and thereby the dispersion relation. We define \(a\) and \(b\) by:

\[
\Sigma^H_\ell = \frac{1}{2} \left[ \Sigma^T_\ell + \gamma^0 (\Sigma^T_\ell)^\dagger \gamma^0 \right] = P_R \left[ -a(k) - b(k) \right] P_L .
\]

Using Eqs. \((14,15,16)\) and \((B.1)\) in the rest frame of the thermal bath with \(u = (1, 0, 0, 0)\), we can identify:

\[
\zeta^{fl} = -k^0 a - b , \quad (B.2a)
\]

\[
\zeta^{bl} = -(|k|^0 - |k|)a - \text{sign}(k^0)b . \quad (B.2b)
\]

The poles of the propagators \((A.3)\) and \((A.4)\) are given by the equation

\[
(k - \Sigma^T_\ell)^2 = 0 , \quad (B.3)
\]
which determines the flavoured dispersion relation of the lepton \( \ell \). Alternatively, the dispersion relation is given by the solution of the constraint equations (A.7) leading to Eq. (A.8) to first order in gradients (in particular, when assuming \( a \ll 1 \), which is justified by perturbatively small coupling constants and loop suppression factors). Substituting the relations (B.2) we then obtain the following first order dispersion relation in the flavour-diagonal basis:

\[
k^0 = \pm |k| - \frac{1}{2} (b_{Da} + b_{Db}) ,
\]

where \( b_D \equiv U^\dagger b U \). In accordance with Eqs. (14,15,16), \( b \) can be decomposed into contributions from flavour blind gauge interaction and flavour sensitive (\( h \)- and \( Y \)-) Yukawa interactions:

\[
b_{ab} = \delta_{ab} b^{bl} + \delta_{ac} \delta_{cb} b^{bl,h} + \sum_i Y_{ia}^* Y_{ib} b^{fl,Y}_i ,
\]

with an analogous decomposition for \( a \). We see that according to Eqs. (B.4), (B.5) different types of contributions to the full dispersion relation are simply additive. There is a caveat, however, if the contributions have a nontrivial hierarchy. For instance, in the heavy massive case with \( M_1 \gg T \), we find that \( b^{fl,Y}_1 \sim (T/M_1)^4 \) while \( a^{fl,Y}_1 \sim (T/M_1)^2 \), cf. Eq. (B.10) below. Then the term mixing gauge- and \( Y \)-induced contributions, \( b^{fl,Y}_{1a} \sim T^4/(k^2 M_1^4) \), would be of leading order in the \( Y \)-induced dispersion relation (B.4) instead of \( b^{fl,Y}_1 \), if \( k^2/M_1^2 \ll 1 \), which implies \( |k| \sim T \). This condition reduces to \( M_1/T \gg 10 \). The momentum region \( |k| \sim T \) is of particular relevance for \( \ell \)-leptogenesis, since this is where most of the leptons \( \ell \) are present. For \( z = M_1/T \gg 10 \), the process of leptogenesis has typically already completed. Thus we will not consider the case when these mixed contribution dominate in this Appendix. Instead, we use the additive dispersion relation (B.4) and consider only the \( Y \)-induced contributions in what follows. We therefore subsequently also drop the superscript “fl,Y”.

The one-loop contribution to the thermal lepton self-energy induced by the couplings \( Y \) is given by

\[
\Sigma_{\ell ab}^{TY}(k) = -i Y_{1a}^* Y_{1b} \int \frac{d^4 p}{(2\pi)^4} i \Delta^T_\phi(p) P_R i S_{N_1}^T(p+k) ,
\]

where \( i \Delta^T_\phi(p) \) and \( i S_{N_1}^T(p+k) \) are the time-ordered thermal propagators for the Higgs field and the heavy right handed neutrino, respectively (cf. Ref. [23]). We have restricted the sum over the right-handed neutrino flavours to \( i = 1 \), since in the hierarchical case only the lightest right handed neutrino leads to a relevant contribution during leptogenesis.\(^4\)

\(^4\)In order to simplify the notation, in this Appendix, we denote by \( T \) the physical temperature, in contrast to the comoving temperature throughout the remainder of this paper.

\(^5\)More precisely, this restriction is justified below where we find that in the heavy massive case the leading order correction to the dispersion relation is proportional to \( (T/M_1)^4 \).
In Ref. [31], only the light massive case is discussed where $T \gg M_1$. For leptogenesis however, we also need a calculation that is valid at late times, when $T \ll M_1$ as well as in the intermediate regime which can be treated only numerically. Since our calculation is very close to the one in Ref. [31] we do not repeat the first steps and continue evaluating the integrals given in Ref. [31] for the case of a heavy massive fermion in the loop.

### B.2 Heavy Massive Case

In the heavy massive case $T \ll M_1$, we perform an expansion of the logarithmic functions $L_{\pm}^{B,F}(|p|)$ defined in Ref. [31] and of the neutrino number density $n_F(E) = 1/(e^{E/T} + 1)$ in powers of $T/M_1$. The energy of the heavy neutrino, $E(p) = \sqrt{p^2 + M_1^2}$, is of order $M_1$, and thus the distribution function $n_F(E)$ is exponentially suppressed for $T \ll M_1$. For this reason the fermionic contributions involving $L_{\pm}^F(|p|)$ need not be considered. Within the functions $L_{\pm}^B$, we count $p^0$, $|p|$ and $k^0$, $|k|$ as order $T$, such that the expansion up to order $(T/M_1)^2$ yields:

$$L_+^B(|p|) = -\frac{8|p||k|}{M_1^2} \left(1 + \frac{k^2}{M_1^2}\right),$$

$$L_-^B(|p|) = 0.

(B.7)$$

Inserting Eqs. (B.7) into the expressions for the self-energy (in accordance with Ref. [31]), the remaining integrations reduce to basic integrals and we obtain:

$$\text{tr}(k^{H,Y}_{\tau ab}) = -Y^*_a Y_{1b} \frac{k^2 T^2}{6M_1^2},$$

(B.8a)

$$\text{tr}(u^{H,Y}_{\tau ab}) = -Y^*_a Y_{1b} \frac{k^0 T^2}{6M_1^2}.

(B.8b)$$

It is easy to relate these expressions to the $Y$-induced contributions to the Lorentz invariant functions $a$ and $b$ decomposed as Eq. (B.5) to get

$$a_1 = \frac{T^2}{12M_1^2},$$

(B.9a)

$$b_1 = 0.

(B.9b)$$

Up to order $(T/M_1)^2$ the dispersion relation (B.3) then simplifies to the massless dispersion, $k^0 = |k|$. The calculation up to order $(T/M_1)^4$ requires a more tedious expansion.

6The difference in the calculations affects only the prefactors but not the structure of the integrations.

7 $L_+^B(|p|)$ has to be expanded one order further than $L_0^B(|p|)$ to extract the correct result for $\text{tr}(k^{H,Y}_{\tau ab})$ up to a given order.
For the functions \( a \) and \( b \) we find
\[
a_1 = \frac{T^2}{12M_1^2} \left( 1 + \frac{15k^2 - 4\pi^2T^2}{15M_1^2} \right),
\]
(B.10a)
\[
b_1 = \frac{4\pi^2k^0T^4}{45M_1^4}.
\]
(B.10b)

At this order, the non-zero \( b \) implies the following dispersion relation for \( g_{hab} \) in the flavour-diagonal basis:
\[
k^0 = \pm |k| \left( 1 - \tilde{Y}_{ab}^2 \frac{4\pi^2T^4}{45M_1^4} \right),
\]
(B.11)
where \( \tilde{Y}_{ab}^2 \equiv \frac{1}{2} \left( U_{ac}^\dagger Y_1^* Y_{1d}^* U_{da} + U_{bc}^\dagger Y_1^* Y_{1d}^* U_{db} \right) \), and + applies to negative, − to positive helicity \( h \). Furthermore, we note that to the leading order \((T/M_1)^2\), Eqs. (B.2) and (B.9) imply that
\[
\varsigma_{fl,Y}^\prime = -4\pi^2 \frac{|k| T^4}{45M_1^4}
\]
and \( \bar{\varsigma}_{fl,Y}^\prime = -\frac{k^0 T^2}{12M_1^2} \).
(B.12)

Note also that \( a \) is even in \( k^0 \) and \( b \) is odd, such that \( \varsigma_{fl,Y}^\prime \) and \( \bar{\varsigma}_{fl,Y}^\prime \) have the correct symmetry properties. We emphasize that these results are valid for \(|k| \lesssim T \ll M_1\).

In the limit of \(|k| \gg M_1 \gg T\), we find another analytical expansion with
\[
\text{tr}(k^H_{Y_{tab}}) = -Y_{1a}^* Y_{1b} \frac{T^2}{12},
\]
(B.13)
while the contribution from \( \text{tr}(k^H_{Y_{tab}}) \) to the dispersion relation is vanishing in the leading order. The dispersion relation is then given by
\[
k^0 = \pm |k| \left( 1 + \tilde{Y}_{ab}^2 \frac{T^2}{24k^2} \right),
\]
(B.14)
from which an effective thermal mass for leptons \( \ell \) with large momentum may be extracted:
\[
m_{heavy}^Y = \frac{|\tilde{Y}_{ab}| T}{2\sqrt{3}}.
\]
(B.15)

### B.3 Light Massive Case

We consider next the light massive case \( M_1 \ll T \). Assuming \(|k| \gtrsim T\) and further that the thermal corrections to the dispersion relation are small, \(||k^0| - |k|| \ll T\), we find that to the leading order, \( b \) is given by
\[
b_1 = \frac{T^2}{16k^2} \left( \frac{k^2}{2|k|} \ln \left( \frac{k^0 + |k|}{k^0 - |k|} \right) - k^0 \right)
\]
(B.16)

---

8 Within this Appendix, we imply by \( A \lesssim B \) that \( A \) is either of order of \( B \) or much smaller.
Then, by using the assumption of small thermal corrections (in particular $a \ll 1$) we can use Eq. (B.4) to find the dispersion relation

$$k^0 = \pm |k| \left(1 + \tilde{Y}^2_{ab} \frac{T^2}{16k^2}\right),$$

(B.17)

where we have neglected a logarithmic correction proportional to \((\tilde{Y}^2_{ab} \frac{T^2}{32k^2})\).

Now we can extract an effective thermal mass for $|k| \gtrsim T$ by using the relation $m_Y^2 = k^0 - k^2$ to find

$$m_Y^\text{light} = \frac{|\tilde{Y}_{ab}| T}{2\sqrt{2}}.$$  

(B.18)

For leptogenesis, where most of the leptons are within the momentum region $|k| \sim T$, the results (B.11) and (B.17) imply that the matrix $U$ which diagonalises the lepton mass will be approximately constant for $T \gg M_1$ as all contributions to the dispersion relations of $\ell$ are proportional to $T^2$. Then it is time dependent around $T \approx M_1$ and again approximately time independent when $T \ll M_1$, when the contributions induced by the couplings $Y$ are suppressed by $(T/M_1)^4$.

### B.4 Numerical solutions

In this Section we solve the $Y$-induced dispersion relation numerically and compare it to the analytical approximations (B.11), (B.14) and (B.17). For comparison, we also derive the dispersion relation and the thermal mass for the light massive case using hard thermal loop (HTL) approximation when $k^0, |k| \ll T$. In this Section we consider only the single flavour case and set $\tilde{Y}_{ab}^2 \equiv |Y_1|^2 = 1$ in all plots. Note also that we do not take account of relevant contributions from gauge couplings, such that the numerical results presented in this Section should be considered as checks of the analytic limits and as a study of a toy system in the absence of gauge interactions.

For the purpose of comparison to the numerical result, we give the analytical expression for the dispersion relation for the light massive case $M_1 \ll T$ within the HTL approximation $k^0, |k| \ll T$. Following Ref. [31] we find

$$\text{Tr}(k^2 Y_{1a}^H Y_{1b}^H) = Y_{1a}^* Y_{1b} \frac{T^2}{8},$$

(B.19a)

$$\text{Tr}(k^0 Y_{1a}^H Y_{1b}^H) = Y_{1a}^* Y_{1b} \frac{T^2}{16k^2} \ln \left(\frac{k^0 + |k|}{k^0 - |k|}\right),$$

(B.19b)

which leads to

$$a_1 = \frac{T^2}{16k^2} \left(1 - \frac{k^0}{2|k|} \ln \left(\frac{k^0 + |k|}{k^0 - |k|}\right)\right),$$

(B.20a)

$$b_1 = \frac{T^2}{16k^2} \left(\frac{k^2}{2|k|} \ln \left(\frac{k^0 + |k|}{k^0 - |k|}\right) - k^0\right).$$

(B.20b)
Figure 5: Shown is the single flavour dispersion relation with $k^0 - |k|$ as a function of $|k|$ in the light massive case $M_1 \ll T$. The solid black line corresponds to the numerical solution, while the dashed red line to the approximate dispersion relation (B.21) and the dotted red line to Eq. (B.17).

From this expansion, we see that the suppression of $a$ due to small coupling constants and loop suppression factors breaks down in the limit $|k| \to 0$. We notice that $b$ is given by the same expression (B.16) as in the light massive case for $|k| \gtrsim T$. This is not the case for $a$, however. To extract the thermal mass we expand $a$ and $b$ to the lowest order in $|k|/|k^0|$ to get $a_1 = -\frac{T^2}{48k^0}$ and $b_1 = -\frac{T^2}{24k^0}$. Now we cannot use the dispersion relation (B.4), since it has been derived assuming that $a \ll 1$, which is not the case when $|k| \to 0$. (Using Eq. (B.22) below, we find that $a = 1/3$ for $|k| = 0$.) In the single flavour case, we find an identical dispersion relation to the one in Ref. [31]: $(k^0 \mp |k|)(1 + a) + b = 0$, which now gives

$$k^0 - |Y_1|^2 \frac{T^2}{16k^0} = \pm|k| \left(1 - |Y_1|^2 \frac{T^2}{48k^0}\right).$$ (B.21)

The thermal mass in the limit $|k| \to 0$ is then given by

$$m_{HTL}^Y = \frac{|Y_1|T}{4}.$$ (B.22)

This is a well-known result within the HTL approximation, see Ref. [23].

In Figure 5, we compare the approximate dispersion relations (B.17) and (B.21) for the light massive case $M_1 \ll T$ to the exact numerical solution. We confirm that the HTL approximation agrees with the exact numerical solution only for small $|k|$, while
Figure 6: Shown is the dispersion relation for the lepton $\ell$ with $|k^0 - |k||/|k|$ as a function of $M_1/T$ for $|k|/T = 0.7, 1.0, 1.3$ from dark red (top at left) to light red (bottom at left). The dashed line correspond to the analytic approximation (B.11).

Figure 7: Shown is the dispersion relation with $k^0 - |k|$ as a function of $|k|$ in the heavy massive case with $M_1/T = 8$. The solid black line corresponds to the numerical solution, while the red dashed and dotted lines are the approximate dispersion relations (B.11) and (B.14). Also shown is the light cone $k^0 = |k|$ (horizontal line).
approximating the dispersion by the thermal mass (B.18) is reasonably accurate in the region $|k| \gtrsim T$.

We next turn to heavy massive case $M_1 \gg T$. In Figure 4 we compare the large $M_1/T$ limit of the approximate analytical dispersion relation (B.11) to the numerical solutions to find a reasonably good agreement for $M_1/T \gtrsim 10$. In addition, these plots clearly confirm the asymptotic $(M_1/T)^4$ behaviour.

Moreover, we notice that the approximate dispersion relations (B.11) and (B.14) give $|k^0| < |k|$ for small $|k|$ and $|k^0| > |k|$ for large $|k|$, so we conclude that in between their regions of validity the dispersion curve must cross the light cone $k^0 = |k|$, implying an intermediate region of superluminal group velocity $v_g = \frac{d(k^0)}{d|k|} > 1$, which might indicate dissipative effects in this momentum region. This interpolation is indeed confirmed by the exact numerical solution of the dispersion relation. In Figure 7 we show the approximate dispersion relations together with exact numerical solution for $M_1/T = 8$. We see that the analytic approximations are fairly accurate in the regions $|k| \lesssim T$ and $|k| \gg T$ and the crossing of the light cone takes place at the momentum $|k| \sim 10T$.

The region in between large and small $M_1/T$ can be treated only numerically. In Figure 8 we show the numerical solution for the dispersion relation for several values of $M_1/T$. We observe that for $M_1/T \geq 0.9$ the dispersion curves cross the light cone at larger $|k|$ for larger $M_1$. It would be of interest to investigate how finite-width effects and thermal masses for the neutrino and the Higgs fields affect the present analysis.
References

[1] E. W. Kolb and S. Wolfram, “Baryon Number Generation In The Early Universe,” Nucl. Phys. B 172 (1980) 224 [Erratum-ibid. B 195 (1982) 542].

[2] J. S. Schwinger, “Brownian motion of a quantum oscillator,” J. Math. Phys. 2 (1961) 407.

[3] L. V. Keldysh, “Diagram technique for nonequilibrium processes,” Zh. Eksp. Teor. Fiz. 47 (1964) 1515 [Sov. Phys. JETP 20 (1965) 1018].

[4] W. Buchmüller and S. Fredenhagen, “Quantum mechanics of baryogenesis,” Phys. Lett. B 483 (2000) 217 [arXiv:hep-ph/0004145].

[5] A. De Simone and A. Riotto, “Quantum Boltzmann Equations and Leptogenesis,” JCAP 0708 (2007) 002 [arXiv:hep-ph/0703175].

[6] M. Garny, A. Hohenegger, A. Kartavtsev and M. Lindner, “Systematic approach to leptogenesis in nonequilibrium QFT: vertex contribution to the CP-violating parameter,” Phys. Rev. D 80 (2009) 125027 [arXiv:0909.1559 [hep-ph]].

[7] M. Garny, A. Hohenegger, A. Kartavtsev and M. Lindner, “Systematic approach to leptogenesis in nonequilibrium QFT: self-energy contribution to the CP-violating parameter,” Phys. Rev. D 81, 085027 (2010) [arXiv:0911.4122 [hep-ph]].

[8] A. Anisimov, W. Buchmüller, M. Drewes and S. Mendizabal, “Leptogenesis from Quantum Interference in a Thermal Bath,” Phys. Rev. Lett. 104, 121102 (2010) [arXiv:1001.3856 [hep-ph]].

[9] M. Beneke, B. Garbrecht, M. Herranen and P. Schwaller, “Finite Number Density Corrections to Leptogenesis,” arXiv:1002.1326 [hep-ph].

[10] M. Garny, A. Hohenegger and A. Kartavtsev, “Quantum corrections to leptogenesis from the gradient expansion,” arXiv:1005.5385 [hep-ph].

[11] T. Endoh, T. Morozumi and Z. h. Xiong, “Primordial lepton family asymmetries in seesaw model,” Prog. Theor. Phys. 111 (2004) 123 [arXiv:hep-ph/0308276].

[12] A. Pilaftsis and T. E. J. Underwood, “Electroweak-scale resonant leptogenesis,” Phys. Rev. D 72 (2005) 113001 [arXiv:hep-ph/0506107].

[13] A. Abada, S. Davidson, F. X. Josse-Michaux, M. Losada and A. Riotto, “Flavour Issues in Leptogenesis,” JCAP 0604, 004 (2006) [arXiv:hep-ph/0601083].

[14] E. Nardi, Y. Nir, E. Roulet and J. Racker, “The importance of flavor in leptogenesis,” JHEP 0601, 164 (2006) [arXiv:hep-ph/0601084].
[15] A. De Simone and A. Riotto, “On the impact of flavour oscillations in leptogenesis,” JCAP 0702 (2007) 005 [arXiv:hep-ph/0611357].

[16] T. Konstandin, T. Prokopec and M. G. Schmidt, “Kinetic description of fermion flavor mixing and CP-violating sources for Nucl. Phys. B 716, 373 (2005) [arXiv:hep-ph/0410135].

[17] T. Konstandin, T. Prokopec, M. G. Schmidt and M. Seco, “MSSM electroweak baryogenesis and flavour mixing in transport equations,” Nucl. Phys. B 738 (2006) 1 [arXiv:hep-ph/0505103].

[18] V. Cirigliano, C. Lee, M. J. Ramsey-Musolf and S. Tulin, “Flavored Quantum Boltzmann Equations,” Phys. Rev. D 81 (2010) 103503 [arXiv:0912.3523 [hep-ph]].

[19] T. Prokopec, M. G. Schmidt and S. Weinstock, “Transport equations for chiral fermions to order h-bar and electroweak baryogenesis,” Annals Phys. 314 (2004) 208 [arXiv:hep-ph/0312110].

[20] T. Prokopec, M. G. Schmidt and S. Weinstock, “Transport equations for chiral fermions to order h-bar and electroweak baryogenesis. II,” Annals Phys. 314 (2004) 267 [arXiv:hep-ph/0406140].

[21] E. Calzetta and B. L. Hu, “Nonequilibrium Quantum Fields: Closed Time Path Effective Action, Wigner Function and Boltzmann Equation,” Phys. Rev. D 37 (1988) 2878.

[22] B. Garbrecht and T. Konstandin, “Separation of Equilibration Time-Scales in the Gradient Expansion,” Phys. Rev. D 79 (2009) 085003 [arXiv:0810.4016 [hep-ph]].

[23] H. A. Weldon, “Effective Fermion Masses Of Order Gt In High Temperature Gauge Theories With Exact Chiral Invariance,” Phys. Rev. D 26 (1982) 2789.

[24] P. B. Arnold, G. D. Moore and L. G. Yaffe, “Transport coefficients in high temperature gauge theories: (I) Leading-log results,” JHEP 0011 (2000) 001 [arXiv:hep-ph/0010177].

[25] M. Joyce, T. Prokopec and N. Turok, “Nonlocal electroweak baryogenesis. Part 1: Thin wall regime,” Phys. Rev. D 53 (1996) 2930 [arXiv:hep-ph/9410281].

[26] S. Blanchet, P. Di Bari and G. G. Raffelt, “Quantum Zeno effect and the impact of flavor in leptogenesis,” JCAP 0703 (2007) 012 [arXiv:hep-ph/0611337].

[27] W. Buchmüller and M. Plümacher, “Spectator processes and baryogenesis,” Phys. Lett. B 511 (2001) 74 [arXiv:hep-ph/0104189].

[28] E. Nardi, Y. Nir, J. Racker and E. Roulet, “On Higgs and sphaleron effects during the leptogenesis era,” JHEP 0601, 068 (2006) [arXiv:hep-ph/0512052].
[29] G. F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, “Towards a complete theory of thermal leptogenesis in the SM and MSSM,” Nucl. Phys. B 685 (2004) 89 [arXiv:hep-ph/0310123].

[30] A. Denner, H. Eck, O. Hahn and J. Kublbeck, “Compact Feynman rules for Majorana fermions,” Phys. Lett. B 291 (1992) 278.

[31] E. Petitgirard, “Massive fermion dispersion relation at finite temperature,” Z. Phys. C 54 (1992) 673.