Local and 2-local derivations of simple $n$-ary algebras

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Abstract: In the present paper we prove that every local and 2-local derivation of the complex finite dimensional simple Filippov algebra is a derivation. As a corollary we have the description of all local and 2-local derivations of complex finite dimensional semisimple Filippov algebras. All local derivations of the ternary Malcev algebra $M_8$ are described. It is the first example of a finite-dimensional simple algebra which admits pure local derivations, i.e. algebra admits a local derivation which is not a derivation.

INTRODUCTION

The study of local derivations started with Kadison’s article \cite{8}. A similar notion, which characterizes non-linear generalizations of derivations, was introduced by Šemrl as 2-local derivations. In his paper \cite{15} was proved that a 2-local derivation of the algebra $B(H)$ of all bounded linear operators on the infinite-dimensional separable Hilbert space $H$ is a derivation. After these works, appear numerous new results related to the description of local and 2-local derivations of associative algebras (see, for example, \cite{11}). The study of local and 2-local derivations of non-associative algebras was initiated in some papers of Ayupov and Kudaybergenov (for the case of Lie algebras, see \cite{2,3}). In particular, they proved that there are no pure local and 2-local derivations on semisimple finite-dimensional Lie algebras. In \cite{5} it is also given examples of 2-local derivations on nilpotent Lie algebras which are not derivations. After the cited works, the study of local and 2-local derivations was continued for Leibniz algebras \cite{4} and Jordan algebras \cite{11}. Local automorphisms and 2-local automorphisms, also were studied in many cases, for example, they were studied on Lie algebras \cite{2,6}. The present paper is devoted to the study local and 2-local derivations of $n$-ary simple algebras, such that Filippov algebras and the ternary Malcev algebra $M_8$. Early, some certain types of generalized derivations of these algebras were described in \cite{2,10}.

Our brief introduction finishes with two principal definitions.
**Definition 1.** Let $A$ be an $n$-ary algebra. A linear map $\nabla: A \to A$ is called a local derivation, if for any element $x \in A$ there exists a derivation $\mathfrak{D}_x: A \to A$ such that $\nabla(x) = \mathfrak{D}_x(x)$.

**Definition 2.** A (not necessary linear) map $\Delta: A \to A$ is called a 2-local derivation, if for any two elements $x, y \in A$ there exists an derivation $\mathfrak{D}_{x,y}: A \to A$ such that $\Delta(x) = \mathfrak{D}_{x,y}(x), \Delta(y) = \mathfrak{D}_{x,y}(y)$.

1. **Local and 2-local derivations of Filippov algebras**

1.1. **Preliminaries.** A Filippov algebra, whose definition appeared in [7], is defined as an algebra $L$ with one anticommutative $n$-ary operation $[x_1, \ldots, x_n]$ satisfying the identity

$$[[x_1, \ldots, x_n], y_2, \ldots, y_n] = \sum_{i=1}^{n} [x_1, \ldots, [x_i, y_2, \ldots, y_n], \ldots, x_n].$$

An example of an $(n+1)$-dimensional $n$-ary Filippov algebra is the algebra with the basis $\ell = \{e_1, \ldots, e_{n+1}\}$ and the multiplication table

$$[e_1, \ldots, \hat{e}_i, \ldots, e_{n+1}] = (-1)^{n+i+1} e_i,$$

where $\hat{e}_i$ denotes the omission of the element $e_i$ from the $n$-ary product. We denote this algebra by $A_{n+1}$. We will consider $(n-1)$-ary $n$-dimensional algebras $A_n$ for $n \geq 4$. As mentioned in [12], the algebras of type $A_n$ exhaust all simple finite-dimensional $(n-1)$-ary Filippov algebras over an algebraically closed field of characteristic zero. Thanks to [9] we have the following description of the matrix of a derivation of the $n$-ary algebra $A_n$.

**Proposition 3.** A linear map $\mathfrak{D}: A_n \to A_n$ is a derivation of the $(n-1)$-ary algebra $A_n$ if and only if the matrix of $\mathfrak{D}$ has the following matrix form:

$$[\mathfrak{D}]_{\ell} = \begin{pmatrix}
0 & x_{12} & x_{13} & \ldots & x_{1k} & \ldots & x_{1n-1} & x_{1n} \\
-x_{12} & 0 & x_{23} & \ldots & x_{2k} & \ldots & x_{2n-1} & x_{2n} \\
-x_{13} & -x_{23} & 0 & \ldots & x_{3k} & \ldots & x_{3n-1} & x_{3n} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
-x_{1k} & -x_{2k} & -x_{3k} & \ldots & 0 & \ldots & x_{kn-1} & x_{kn} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\
-x_{1n-1} & -x_{2n-1} & -x_{3n-1} & \ldots & -x_{kn-1} & \ldots & 0 & x_{n-1n} \\
-x_{1n} & -x_{2n} & -x_{3n} & \ldots & -x_{kn} & \ldots & -x_{n-1n} & 0 
\end{pmatrix},$$

that is, $x_{ii} = 0$ and $x_{ij} + x_{ji} = 0$ to $i \neq j$.

1.2. **Local derivations of semisimple Filippov algebras.** In the present subsection, we proved that each local derivation of the complex finite-dimensional simple Filippov $n$-ary $(n > 2)$ algebra is a derivation. As a corollary, jointed with the results from [3][12], we have the same statement for all complex finite-dimensional semisimple Filippov $n$-ary $(n > 1)$ algebras.

**Theorem 4.** Each local derivation of $A_n$ is a derivation.
Proof. Let $\nabla$ be an arbitrary local derivation of $A_n$, by definition we have
$$\nabla(x) = D_x(x).$$
Let us consider $B$ and $A_x$ the matrix of the linear operator $\nabla$ and $D_x$ respectively. Thus $B(x) = A_x(x)$,

$$B(x) = \begin{pmatrix}
    b_{11} & b_{12} & b_{13} & \cdots & b_{1k} & \cdots & b_{1n-1} & b_{1n} \\
    b_{21} & b_{22} & b_{23} & \cdots & b_{2k} & \cdots & b_{2n-1} & b_{2n} \\
    b_{31} & b_{32} & b_{33} & \cdots & b_{3k} & \cdots & b_{3n-1} & b_{3n} \\
    \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
    b_{k1} & b_{k2} & b_{k3} & \cdots & b_{kk} & \cdots & b_{kn-1} & b_{kn} \\
    \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\
    b_{n-11} & b_{n-12} & b_{n-13} & \cdots & b_{n-1k} & \cdots & b_{n-1n-1} & b_{n-1n} \\
    b_{n1} & b_{n2} & b_{n3} & \cdots & b_{nk} & \cdots & b_{nn-1} & b_{nn}
\end{pmatrix}
\begin{pmatrix}
    x_1 \\
    x_2 \\
    x_3 \\
    \vdots \\
    x_k \\
    \vdots \\
    x_{n-1} \\
    x_n
\end{pmatrix} = A_x(x),$$

for all $x \in A_{n+1}$. Taking $\Xi_k = (0, \cdots, 0, \frac{1}{k}, 0, \cdots, 0)^T$ for all $k = 1, \ldots, n$ we get $b_{kk} = 0$ for
$k = 1, \ldots, n$. Hence we obtain
$$B(\Xi_k + \Xi_l) = A_{\Xi_k + \Xi_l}(\Xi_k + \Xi_l).$$
Hence, $A_{\Xi_k + \Xi_l}$ is an antisymmetric matrix, it gives that $b_{kl} = -b_{lk}$ and the matrix $B$ is antisymmetric. The last gives that $\nabla$ is a derivation.

1.3. 2-Local derivations of semisimple Filippov algebras. In the present subsection, we prove that each local derivation of the complex finite-dimensional simple Filippov $n$-ary ($n \geq 2$) algebra is a derivation. As a corollary, jointed with the results from [5,12], we have the same statement for all complex finite-dimensional semisimple Filippov $n$-ary ($n > 1$) algebras.

Theorem 5. Each 2-local derivation of $A_n$ is a derivation.

Proof. Let $\Delta$ be an arbitrary 2-local derivation of $A_n$. Then, by the definition, for every element $a, b \in A_n$, there exists a derivation $D_{a,b}$ of $A_n$ such that
$$\Delta(a) = D_{a,b}(a), \quad \Delta(b) = D_{a,b}(b).$$
By Proposition 3, the matrix $A^{a,b}$ of the derivation $D_{v,a}$ has the following matrix form:

$$
A^{a,b} = \begin{pmatrix}
0 & x^{a,b}_{12} & x^{a,b}_{13} & \ldots & x^{a,b}_{1k} & \ldots & x^{a,b}_{1n} \\
x^{a,b}_{12} & 0 & x^{a,b}_{23} & \ldots & x^{a,b}_{2k} & \ldots & x^{a,b}_{2n} \\
x^{a,b}_{13} & -x^{a,b}_{23} & 0 & \ldots & x^{a,b}_{3k} & \ldots & x^{a,b}_{3n} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
x^{a,b}_{1k} & -x^{a,b}_{2k} & -x^{a,b}_{3k} & \ldots & 0 & \ldots & x^{a,b}_{kn} \\
x^{a,b}_{1n} & -x^{a,b}_{2n} & -x^{a,b}_{3n} & \ldots & -x^{a,b}_{kn} & \ldots & 0
\end{pmatrix}
$$

where

$$
T^{a,b} = \begin{pmatrix}
0 & T_{a,b} \\
-T_{a,b} & 0
\end{pmatrix}
$$

Let $a = \sum_{i=1}^{n} \lambda_i e_i$ be an arbitrary element from $A_n$. For every $v \in A_n$ there exists a derivation $D_{v,a}$ such that

$$
\Delta(v) = D_{v,a}(v), \quad \Delta(a) = D_{v,a}(a).
$$

Then from

$$
D_{v_n} e_n = D_{v_n,a}(e_n), \quad v \in A_n,
$$

it follows that $T^{e_n,v}_i = T^{e_n,a}_i$ for $i = 1, \ldots, n$. Then we can write

$$
A^{e_n,a} = \begin{pmatrix}
0 & T \\
-T & \ldots
\end{pmatrix}
$$

where $T^{e_n,v} = \begin{pmatrix}
x^{e_n,v}_{1n} \\
x^{e_n,v}_{2n} \\
\vdots \\
x^{e_n,v}_{n-1,n}
\end{pmatrix}$. Hence,

$$
\Delta(a) = D_{e_n,a}(a) = \sum_{i=1}^{n-1} \mu^{e_n,a}_i e_i + \sum_{i=1}^{n} (-x^{e_n,v}_{in} \lambda_i)e_n,
$$

for some elements $\mu^{e_n,a}_i \in F$. Similarly, taking $e_j$ for each $j = n-1, n-2, \ldots, 2$ we have from

$$
D_{e_j,v}(e_j) = D_{e_j,a}(e_j), \quad v \in A_n,
$$

we have the following $T^{e_j,v}_i = T^{e_j,a}_i$ for each $j = n-1, n-2, \ldots, 2$ and $T^{e_j,v}_j = -T^{e_j,v}_j$. Hence,

$$
\Delta(a) = D_{e_i,a}(a), \quad \text{for each } i = 1, \ldots, n.
$$

Note that

$$
\Delta(a) = \sum_{i=1}^{n} \sum_{j=1}^{n} (-T^{e_i,v}_{ij} \lambda_j) e_i, \quad v_j \in A_{n+1}, \quad j = 1, \ldots, n.
$$
Therefore the mapping $\Delta$ is linear and it is a local derivation. By Theorem 4 we get that $\Delta$ is a derivation. This completes the proof. □

2. Local derivations of the ternary Malcev algebra $M_8$

2.1. Preliminaries. The idea of introducing a generalization of Filippov algebras comes from binary Malcev algebras and it was realized in a paper of Pojidaev [13]. He defined $n$-ary Malcev algebras, generalizing Malcev algebras and $n$-ary Fillipov algebras. For construction of the most important example of $n$-ary Malcev (non-Filippov) we denote by $A$ a composition algebra with an involution $\bar{\cdot}: a \mapsto \bar{a}$ and unity $1$. The symmetric, bilinear form $\langle x, y \rangle = \frac{1}{2}(xy + y\bar{x})$ defined on $A$ is assumed to be nonsingular. If $A$ is equipped with a ternary multiplication $\lbrack \cdot, \cdot, \cdot \rbrack$ by the rule

$$[x, y, z] = (xy)z - \langle y, z \rangle x + \langle x, z \rangle y - \langle x, y \rangle z,$$

then $A$ becomes a ternary Malcev algebra [13], which will be denoted by $M(A)$. If dim $A = 8$ then $M(A)$ is simply ternary Malcev (non-3-Lie) algebra and we denote it by $M_8$.

Let $A$ be that above mentioned composition algebra and assume that $1, a, b, c$ are orthonormal elements in $A$. Choose the following basis of $M_8$ (see [14]):

$$e_1 = 1, \ e_2 = a, \ e_3 = b, \ e_4 = ab, \ e_5 = c, \ e_6 = ac, \ e_7 = bc, \ e_8 = abc.$$

Further we need to the following properties of basis elements (see [14]). For each $i \in \{2, \ldots, 8\}$, it is possible to choose $j, k, l, m, s, t$, all depending on $i$, such that

$$e_i = e_1e_i = e_je_k = ete_m = e_se_t \text{ and } e_re_m = e_t.$$

Thanks to [14] we have the following description of the basis of the algebra of derivations of $M_8$:

$$\mathcal{B} = \left\{ \Delta_{23} - \Delta_{14}, \Delta_{24} + \Delta_{13}, \Delta_{25} - \Delta_{16}, \Delta_{26} + \Delta_{15}, \Delta_{27} + \Delta_{18}, \Delta_{28} - \Delta_{17}, \Delta_{34} - \Delta_{12}, \right. \left. \Delta_{35} - \Delta_{17}, \Delta_{36} - \Delta_{18}, \Delta_{37} + \Delta_{15}, \Delta_{38} + \Delta_{16}, \Delta_{45} - \Delta_{18}, \Delta_{46} + \Delta_{17}, \Delta_{47} - \Delta_{16}, \right. \left. \Delta_{48} + \Delta_{15}, \Delta_{56} - \Delta_{12}, \Delta_{57} - \Delta_{13}, \Delta_{58} - \Delta_{14}, \Delta_{67} + \Delta_{14}, \Delta_{68} - \Delta_{13}, \Delta_{78} + \Delta_{12} \right\},$$

where $\Delta_{ij} = e_{ij} - e_{ji}$ and $e_{ij}$ are the ordinary matrix units.

**Proposition 6.** A linear map $\mathcal{D} : M_8 \to M_8$ is a derivation of the algebra $M_8$ if and only if the antisymmetric matrix of $D$ has the following matrix form:

$$[\mathcal{D}]_\mathcal{B} = \begin{bmatrix}
0 & -\gamma_1 & -\gamma_2 & -\gamma_3 & -\gamma_4 & -\gamma_5 & -\gamma_6 & -\gamma_7 \\
\gamma_1 & 0 & -\alpha_1 & -\alpha_2 & -\alpha_3 & -\alpha_4 & -\alpha_5 & -\alpha_6 \\
\gamma_2 & -\alpha_1 & 0 & -\alpha_7 & -\alpha_8 & -\alpha_9 & -\alpha_{10} & -\alpha_{11} \\
\gamma_3 & -\alpha_2 & -\alpha_7 & 0 & -\alpha_{12} & -\alpha_{13} & -\alpha_{14} & -\alpha_{15} \\
\gamma_4 & -\alpha_3 & -\alpha_8 & -\alpha_{12} & 0 & -\alpha_{16} & -\alpha_{17} & -\alpha_{18} \\
\gamma_5 & -\alpha_4 & -\alpha_9 & -\alpha_{13} & -\alpha_{16} & 0 & -\alpha_{19} & -\alpha_{20} \\
\gamma_6 & -\alpha_5 & -\alpha_{10} & -\alpha_{14} & -\alpha_{17} & -\alpha_{19} & 0 & -\alpha_{21} \\
\gamma_7 & -\alpha_6 & -\alpha_{11} & -\alpha_{15} & -\alpha_{18} & -\alpha_{20} & -\alpha_{21} & 0
\end{bmatrix},$$

where
\[\gamma_1 = -\alpha_7 - \alpha_{16} + \alpha_{21}, \quad \gamma_2 = \alpha_2 - \alpha_{17} - \alpha_{20}, \quad \gamma_3 = -\alpha_1 - \alpha_{18} + \alpha_{19},\]
\[\gamma_4 = \alpha_4 + \alpha_{10} + \alpha_{15}, \quad \gamma_5 = -\alpha_3 + \alpha_{11} - \alpha_{14},\]
\[\gamma_6 = -\alpha_6 - \alpha_8 + \alpha_{13}, \quad \gamma_7 = \alpha_5 - \alpha_9 - \alpha_{12}.\]

2.2. **Local derivations of** \(M_8\). In the present subsection, we shall give a description of all local derivations of \(M_8\). As the principal result, we have that \(M_8\) admits local derivations that are not derivations. Which gives the first known example of a simple finite-dimensional algebra admitting pure local derivations. The quotient space of the space of local derivation by the space of derivations of \(M_8\) is of dimension 7.

Recall \([14]\) that, if \(1, u, v, w \in A\) are orthonormal, then

\[uvu = -v, \quad (uv)w = -(uw)v, \quad u(vw) = -v(uw).\]

Note that \(u^2 = v^2 = w^2 = -1\). Thus using Moufang identity \((uv)(wu) = u((vw)u)\) for composition algebra and the above first identity we get that

\[(uv)(wu) = vw.\]

**Proposition 7.** Let \(x, y \in M_8\) be the elements such that \(x^2 = -1, y \in \{x\}^\perp, y^2 = -1\). Then there exists \(\Phi \in \text{Aut}(M_8)\) such that

\[\Phi(e_2) = x \text{ and } \Phi(e_3) = y.\]

**Proof.** Since \(x^2 = -1, y \in \{x\}^\perp, y^2 = -1\), it follows that \(\{e_1, x, y, xy\}\) is an orthonormal system. Take an element \(z \in \{e_1, x, y, xy\}^\perp\) such that \(z^2 = -1\). Using (3) we can infer that

\[\{e_1, x, y, xy, z, xz, yz, xyz\}\]

is an orthonormal system, in particular, for any three different elements \(u, v, w\) from the above system identities from (3) are true.

Define a linear mapping \(\Phi\) on \(A\) on basis elements as follows:

\[\Phi(e_1) = e_1, \quad \Phi(e_2) = x, \quad \Phi(e_3) = y, \quad \Phi(e_4) = xy,\]
\[\Phi(e_5) = z, \quad \Phi(e_6) = xz, \quad \Phi(e_7) = yz, \quad \Phi(e_8) = xyz.\]

Using identities (1), (3), (4) we obtain that \(\Phi\) is an automorphism of the composition algebra \(A\). Since any automorphism of \(A\) commutes with the involution and hence, it preserves bilinear form \(\langle \cdot, \cdot \rangle\). It follows that \(\Phi\) is also an automorphism of the ternary algebra \(M_8\).

**Theorem 8.** A linear mapping \(\nabla\) on \(M_8\) is a local derivation if and only if its matrix is antisymmetric. In particular, the dimension of the space \(\text{LocDer}M_8\) of all local derivations of \(M_8\) is equal to 28.

**Proof.** Let \(\nabla\) be a local derivation on \(M_8\). By a similar argument as in the proof of Theorem 4 we obtain that the matrix of \(\nabla\) is antisymmetric.

Let \(\nabla : M_8 \to M_8\) be an arbitrary linear mapping with the corresponding antisymmetric matrix \((\nabla_{ij})_{1 \leq i, j \leq 8}\). Let us show that \(\nabla\) is a local derivation.
For any $i \in \{1, \ldots, 21\}$ denote by $D_i$ derivation of $M_8$ defined as in (2) with the coefficients $\alpha_i = 1$ and $\alpha_j = 0$ for all $j \neq i$. In fact, $\mathcal{B} = \{D_i : 1 \leq i \leq 21\}$.

Let $x = \sum_{k=1}^{8} x_k e_k \in M_8$ be a fixed non zero element. We need to find a derivation $D_x$ such that $\nabla(x) = D_x(x)$.

Set $\nabla(x) = \sum_{i=1}^{8} y_i e_i$. Note that

$$\sum_{i=1}^{8} x_i y_i = \sum_{i=1}^{8} \sum_{j \neq i} \nabla_{ij} x_i x_j = 0,$$

because $\nabla_{ij} = -\nabla_{ji}$ for all $1 \leq i, j \leq 8$. This means that

$$\nabla(x) \in x^⊥ = \left\{ z = \sum_{i=1}^{8} z_i e_i \in M_8 : (x, z) = \sum_{i=1}^{8} x_i z_i = 0 \right\}.$$

Take a derivation $D_{e_1}$ of $M_8$ such that $\nabla(e_1) = D_{e_1}(e_1)$. If necessary replacing $\nabla$ with $\nabla - D_{e_1}$ we can assume that $\nabla(e_1) = 0$. Then $\nabla_{1i} = \nabla_{i1} = 0$ for all $1 \leq i \leq 8$. Thus $\nabla$ maps $M_8$ into $e_1^⊥$, that is,

$$\nabla(x) \in e_1^⊥$$

for all $x \in M_8$.

Let us consider the following possible two cases.

Case 1. Let $x = x_1 e_1$. Then

$$\nabla(x) = x_1 \nabla(e_1) = 0 = D_x(x),$$

where $D_x$ is a trivial derivation.

Case 2. Let $x = \lambda_0 e_1 + \lambda x_1$, where $\lambda \neq 0$ and $x_1 \in e_1^⊥$ with $x_1^2 = -1$.

Since $\nabla(e_1) = 0$, it follows that

$$y = \nabla(x) = \nabla(\lambda_0 e_1 + \lambda x_1) = \lambda \nabla(x_1).$$

Combining (6) and (7) we obtain that

$$y \in \{e_1, x\}^⊥.$$

Thus $y$ represents as $y = \mu y_1$, where $y_1^2 = -1$. By (8) we obtain that

$$y_1 \in \{e_1, x_1\}^⊥.$$

By Proposition 7 there exists an automorphism $\Phi$ such that

$$\Phi(e_2) = x_1 \text{ and } \Phi(e_3) = y_1.$$
Take a derivation $D = \frac{\mu}{\lambda} D_1$, where $D_1$ is a derivation from the list (5). Note that $D_1(e_1) = 0$ and $D_1(e_2) = e_3$. Then the following mapping

$$D_x = \Phi \circ D \circ \Phi^{-1}$$

is a derivation. We have that

$$D_x(x) = \Phi \circ D \circ \Phi^{-1}(\lambda_0 e_1 + \lambda x_1) = \Phi(D(\Phi^{-1}(\lambda_0 e_1 + \lambda x_1))) = \Phi(D(\lambda_0 e_1 + \lambda e_2)) = \Phi(D(\lambda e_2)) = \mu \Phi(e_3) = \mu y_1 = y = \nabla(x).$$

The proof is completed. \qed

At the end we formulate Problem concerning 2-local derivations of $M_8$. Likewise as in the proof of Theorem 5 we can obtain that any 2-local derivation of $M_8$ is linear. In particular, it is a local derivation. In this regard, the following question arises.

**Problem 9.** Is any 2-local derivation of $M_8$ a derivation?

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