Constraining the Nonextensive Mass Function of Halos from BAO, CMB and X-ray data

L. Marassi*, J. V. Cunha†, J. A. S. Lima‡

Instituto de Astronomia, Geofísica e Ciências Atmosféricas,
USP, CEP 05508-090, São Paulo, SP, Brasil

(Dated: July 2, 2008)

Abstract

Clusters of galaxies are the most impressive gravitationally-bound systems in the Universe and its abundance (the cluster mass function) is one important statistics to probe the matter density parameter ($\Omega_m$) and the amplitude of density fluctuations ($\sigma_8$). The cluster mass function is usually described in terms of the Press-Schechter (PS) formalism where the primordial density fluctuations are assumed to be a Gaussian random field. In previous works we have proposed a non-Gaussian analytical extension of the PS approach with basis on the $q$-power law distribution (PL) of the non-extensive kinetic theory. In this paper, by applying the PL distribution to fit the observational mass function data from X-ray highest flux-limited sample (HIFLUGCS) we find a strong degeneracy among the cosmic parameters, $\sigma_8$, $\Omega_m$, and the $q$ parameter from the PL distribution. A joint analysis involving recent observations from baryon acoustic oscillation (BAO) peak and Cosmic Microwave Background (CMB) shift parameter is carried out in order to break these degeneracy and better constrain the physically relevant parameters. The present results suggest that the next generation of cluster surveys will be able to probe the quantities of cosmological interest ($\sigma_8, \Omega_m$) and the underlying cluster physics quantified by the $q$-parameter.

PACS numbers: 98.80.Es; 95.35.+d; 98.62.Sb

* luciomarassi@astro.iag.usp.br
† cunhajv@astro.iag.usp.br
‡ limajas@astro.iag.usp.br
I. INTRODUCTION

The recent astronomical observations are strongly suggesting that the expansion of the Universe is speeding up and not slowing thereby revealing the presence of some form of repulsive gravity. The basic set of experiments includes: observations from SNe Ia[1], cosmic microwave background (CMB) temperature anisotropies[2], large scale structure[3, 4], X-ray data from galaxy clusters[5], age estimates of globular clusters and old high redshift galaxies[6]. In the present cosmic concordance ΛCDM model the Universe is formed of \( \sim 26\% \) matter (baryonic + dark matter) and \( \sim 74\% \) of a smooth vacuum energy component. The thermal CMB component contributes only about 0.01%, however, its angular power spectrum of temperature anisotropies encode important information about the structure formation process and other cosmic observables.

On the other hand, the number density of collapsed objects for a given mass at a certain time, named the mass or multiplicity function, is a key quantity in the analysis of cosmic structures such as clusters of galaxies. The simplest successful approach to analytically describe this quantity was developed more than three decades ago by Press and Schechter[7] (hereafter PS). This Gaussian PS formalism is extensively adopted to derive the mass function, \( F(M) \), of bounded objects in the observed Universe[8, 9, 10, 11, 12, 13].

The PS formalism was adopted by Reiprich and Boringer[11] in their construction of the cluster mass function based on the X-ray flux-limited sample of galaxy clusters (HIFLUGCS) selected from the ROSAT All-Sky Survey. As a result, the best fit parameters, \( \Omega_m = 0.12 \) and \( \sigma_8 = 0.96 \), were obtained in their analysis. These values are, respectively, very low and very high when compared with the nowadays independent CMB results[2], thereby leading to some skepticism about the usefulness of clusters as sensitive cosmological probes. However, several authors have recently claimed that there is no tension between cosmological constraints from CMB and clusters[12, 13]. In particular, Rines and collaborators[13] argued that the dynamical determination of cluster masses were overestimated and even modest values of velocity segregation between galaxies and dark matter are sufficient to match the mass function with the WMAP results.

In this article we discuss a different possibility. We advocate here that such a discrepancy comes out because the matter density fluctuations should be described by an intrinsically non-Gaussian random field (due to the action of the long-range gravitational interaction)
and, therefore, the PS approach should be somewhat modified.

In previous papers [14, 15], inspired by the Tsallis $q$-nonextensive statistics [16] and kinetic theory [17, 18, 19, 20], we have proposed a simple extension of the PS analytical formalism. Instead the Gaussian function of the original PS approach, it was adopted the power law (PL) Tsallis distribution for describing the fluctuations of the density field. A basic attractive feature of the PL distribution is that the resulting model is analytically tractable and the standard Press-Schechter formalism is recovered as particular case. In this way, a detailed comparison between the two approaches is immediate.

As we shall see, by applying the PL distribution to fit the observational mass function data from X-ray highest flux-limited sample (HIFLUGCS) we find a strong degeneracy among the pair of cosmic parameters ($\sigma_8$, $\Omega_m$), and the $q$ parameter from PL distribution. Through a joint analysis involving recent observations from baryon acoustic oscillation (BAO) peak [4] and Cosmic Microwave Background (CMB) shift parameter [21, 22, 23] the degeneracy is broken and the tension between independent determinations is alleviated.

II. THE PRESS-SCHECHTER APPROACH AND THE $Q$-POWER LAW

It is widely known that in the PS original approach the primordial density fluctuations $\delta \equiv \delta \rho / \rho$ for a given mass $M$ is described by a random Gaussian field

$$P(\delta) = \frac{1}{\sqrt{2\pi \sigma^2(M)}} \exp \left(-\frac{\delta^2}{2\sigma^2(M)}\right),$$

(1)

where $\sigma^2(M) \equiv \langle \delta^2_M \rangle$ is the mean squared fluctuation. When the amplitude of the density contrast grows above a critical value ($\delta_c$), a bound object is formed and the fraction $F(M)$, at a given time, can be written as

$$F(M) = \int_{\delta_c}^{\infty} P(\delta) d\delta = \frac{1}{\sqrt{2\pi \sigma^2(M)}} \int_{\delta_c}^{\infty} \exp \left(-\frac{\delta^2}{2\sigma^2(M)}\right) d\delta,$$

(2)

while the distribution of bound objects with masses between $M$ and $M + dM$ reads

$$\frac{dF(M)}{dM} = + \frac{1}{\sqrt{2\pi \sigma^2(M)}} \left( \frac{\partial \sigma(M)}{\partial M} \right) \exp \left(-\frac{\delta_c^2}{2\sigma^2(M)}\right).$$

(3)

Now, if instead of Gaussian initial fluctuations, we consider that the amplitudes are described by a class of $q$-parameterized power law distributions, we have the follow
expression 14, 15, 16, 17, 18

\[ P(\delta)_{PL} = \frac{B_q}{\sqrt{2\pi}\sigma(M)} \left[ 1 - (1 - q) \cdot \left( \frac{\delta}{\sqrt{2}\sigma(M)} \right)^2 \right]^\frac{1}{(1 - q)}, \]  

(4)

where the factor \( B_q \) is a one-dimensional normalization constant given by

a) \( B_q = (1 - q)^{\frac{3}{2}} \frac{\Gamma\left(\frac{2}{2} + \frac{1}{(1 - q)}\right)}{\Gamma\left(\frac{1}{1 - q}\right)} \), (if \( 0 < q \leq 1 \))

b) \( B_q = (q - 1)^{\frac{1}{2}} \frac{\Gamma\left(\frac{1}{1 - q}\right)}{\Gamma\left(\frac{1}{q + 1} - \frac{1}{2}\right)} \), (if \( 1 \leq q < 2 \))

In this framework, the non-extensive multiplicity function of bound objects with masses between \( M \) and \( M + dM \) reads

\[ \frac{dF_{M,PL}}{dM} = + \frac{B_q}{\sqrt{2\pi}\sigma^2(M)} \left( \frac{\partial\sigma(M)}{\partial M} \right) \cdot \left[ 1 - (1 - q) \cdot \left( \frac{\delta_c}{\sqrt{2}\sigma(M)} \right)^2 \right]^\frac{1}{(1 - q)}. \]  

(5)

Note that in the limit \( q \to 1 \) the above PL expressions reduce to the ones of the standard Gaussian approach.

III. POWER LAW DISTRIBUTION AND COSMIC PARAMETERS

As remarked earlier, by performing a \( \chi^2 \) statistical procedure with basis on the X-ray HIFLUGCS data sample, Reiprich and Boehringer (RB)\[11\] determined the statistical confidence contours for the pair of parameters, \( \sigma_8 \) and \( \Omega_m \).

In Figure 1 we show the contours in the \( \Omega_m - \sigma_8 \) plane obtained by using the standard PS and PL approaches. The first panel is the RB result. The legend of this panel show the parameter \( q = 1.0 \), and, as already explained, when the \( q \) parameter from the PL distribution tends to the unity the Gaussian results are recovered. In this case, the best-fits of the PS method are also shown, namely, \( \Omega_m = 0.12 \) and \( \sigma_8 = 0.96 \). The results from PL distributions (\( q \neq 1 \)) can be seen in the panels below (\( q = 1.10, q = 1.15, \) and \( q = 1.20, \) from top to bottom). Note that for each panel of Fig\[11\] the solid vertical and horizontal lines shows the minimum and maximum independent WMAP limits for \( \Omega_m \) and \( \sigma_8 \), respectively\[2\]. The important point here is that the contours using the Gaussian distribution (first panel) does not intercept the independent best-fit values from WMAP even at 99% confidence level. In
FIG. 1: The First panel shows the results from the Gaussian PS approach. The results based on the PL distribution appear in the panels below ($q = 1.10$, $q = 1.15$, and $q = 1.20$). The solid vertical and horizontal lines show the minimum and maximum WMAP limits for $\Omega_m$ and $\sigma_8$, respectively. Note that the contours using the Gaussian distribution (the first panel) does not intercept the WMAP values while the ones based on the PL distribution are intercepting them for a wide range of $q$ values ($1 < q < 1.2$).

in the other panels, we see that the $q$ free parameter of the PL distribution, however, permit the contours to intercept the WMAP results in a wide range of $q$ values. Roughly estimates show that the range between $q = 1.06$ and $q = 1.2$ is allowed, and, in principle, such a degeneracy need to be removed.

IV. JOINT ANALYSIS AND DISCUSSION

In principle, the above results suggest that in the non-extensive framework there are many possibilities for the theoretical mass function. Still more important, some of them are working in the right direction and may help for reconciling the independent estimates of the
cosmic parameters.

In this section we discuss how the parametric 2-dimensional spaces $\sigma_8 - \Omega_m$, $\Omega_m - q$ and $\sigma_8 - q$ can be further constrained by applying a statistical analyses involving different cosmological observations. To this end we consider the current estimates of the baryon acoustic oscillations found in the SDSS data \cite{4}, as well as, the shift parameter from WMAP observations \cite{2}. The basic aim is to break the degeneracy between the $\Omega_m$, $\sigma_8$ and $q$ parameters in order to better constrain the PL distribution that fits the HIFLUGCS data.

A. BAO

The Baryon Acoustic Oscillations (BAO) in the primordial baryon-photon fluid, leave a characteristic signal on the galaxy correlation function, a bump at a scale $\sim 100$ Mpc, as observed by Eisenstein and coworkers\cite{4}. This signature furnishes a standard rule which can be used to constrain the following quantity:

$$A \equiv \frac{\Omega_m^{1/2}}{H(z_\ast)^{1/3}} \left[ \frac{1}{z_\ast} \Gamma(z_\ast) \right]^{2/3} = 0.469 \pm 0.017,$$

where $H = H(z)/H_0$ is the normalized Hubble parameter of the $\Lambda$CDM model, $z_\ast = 0.35$ is a typical redshift of the SDSS sample, and $\Gamma(z_\ast)$ is the dimensionless comoving distance to the redshift $z_\ast$. This quantity can be used even for more general models which do not present a large contribution of dark energy at early times \cite{24}.
FIG. 3: Contours on the $\Omega_m$-$q$ plane with $\sigma_8 = 0.81$. In the left panel the contour permits almost all values for the $q$ parameter. The right panel shows the joint analysis with the BAO and the Shift Parameter: the $q$ parameter is restricted in the range $1.0 < q < 1.10$, and we have a high constraint on the $\Omega_m$ parameter as well.

B. CMB shift parameter

A useful quantity to characterize the position of the CMB power spectrum first peak is the shift parameter. For a flat Universe it is given by\textsuperscript{22, 25}:

$$R = \sqrt{\Omega_m} \int_0^{z_r} \frac{dz}{H(z)} = 1.70 \pm 0.03,$$

where $z_r = 1089$ is the recombination redshift and the value for $R$ above is calculated from the MCMC of the WMAP 3-yr in the standard flat $\Lambda$CDM model.

C. Results

Let us now discuss the main results of our statistical analyses. In Fig. 2 by fixing $q = 1.11$, we show the contours on the $\Omega_m$-$\sigma_8$ plane. The left panel shows that we have poor constraints on these cosmological parameters. However, when a joint statistical analysis is performed using the BAO the shift parameter (right panel) the available space parameter is considerably reduced. Note that the contour becomes small and shifted to higher and lower values of $\Omega_m$ and $\sigma_8$, respectively.

By applying the same procedure (now fixing $\sigma_8 = 0.81$) and plotting the contours on the $\Omega_m$-$q$ plane, the left panel of Fig. 3 is obtained. One may see that almost all values for the
FIG. 4: The left panel shows the contour on the $\Omega_m$-$\sigma_8$ plane when we marginalize over all the allowed values of $q$. In the right panel we apply the BAO and the Shift Parameter signatures. The cosmological parameters are in good agreement with the WMAP limits.

$q$ parameter is allowed, however, a joint analysis with BAO and shift parameter restricts severely the $q$ parameter ($1 \leq q \leq 1.1$). Note also that the $\Omega_m$ parameter becomes tightly constrained.

Finally, we discuss the contours on the $\Omega_m$-$\sigma_8$ plane when we marginalize over all possible values of $q$. The result is shown on the left panel of Fig. 4. In the right panel we display the result of the joint analysis. It is interesting that by applying BAO and shift parameter signature the space parameter is extremely reduced and displaced as occurred in Fig. 2 (see also right panel there). We stress that we have marginalized over $q$ and performed a joint analysis. The main consequence is that the cosmological parameters are now in agreement with the latest WMAP results.

V. CONCLUSIONS

In the precision era of cosmology it is very important to show the compatibility of the cosmological parameters as determined from independent observations. The current standard cosmological model, i.e., a flat, accelerating Universe composed of $\simeq 1/3$ of matter (baryonic + dark) and $\simeq 2/3$ of a dark energy component in the form of the vacuum energy density ($\Lambda$) seems to be fully consistent with a variety of observational data. However, some tension has recently been detected between the determinations of the pair $(\sigma_8, \Omega_m)$ from galaxy clusters and CMB. In principle, many temporary solutions are possible, but,
hopefully, only one will survive to future analyses based on the next generation of cluster surveys.

In this article we have discussed an alternative route which seems interesting to be investigated from a theoretical and observational viewpoint. Initially, inspired by the non-extensive kinetic theory, we have extended the original Gaussian distribution of the primordial density field\[14, 15\]. By using this PL distribution to fit the observational mass function from the HIFLUGCS data we have identified a degeneracy involving the cosmic parameters $\sigma_8$ and $\Omega_m$ with the $q$ parameter from the PL distribution.

Finally, a joint analysis involving the baryon acoustic oscillation signature and the CMB shift parameter have been applied. As a result, we have shown that the non-extensive PL distribution may alliviate the tension underlying the independent determinations of the cosmic parameters (see right panel of Fig. 4).

Acknowledgements

The authors are grateful to S. H. Pereira and J. M. Silva for helpful discussions. LM and JVC are supported by FAPESP No. 07/00036-4 and 05/02809-5, respectively, and JASL is partially supported by CNPq and FAPESP grant No. 04/13668.

[1] P. Astier et al., Astron. Astrophys. 447, 31 (2006); A. G. Riess et al., Astrophys. J. 659, 98 (2007); T. M. Davis et al., Astrophys. J. 666, 716 (2007).

[2] D. N. Spergel et al., Astrop. J. Suppl. 148, 175 (2003); D. N. Spergel et al., Astrophys. J. Suppl. 170, 377 (2007); J. Dunkley et al.,[arXiv:0803.0586][astro-ph]; E. Komatsu et al.,[ArXiv:0803.0547v1].

[3] R. G. Carlberg et al., ApJ. 462, 32 (1996); A. Dekel, D. Burstein and S. D. M. White, In Critical Dialogues in Cosmology, edited by N. Turok World Scientific, Singapore (1997); P. J. E. Peebles, in Formation of Structure in the Universe, edited by A. Dekel and J. P. Ostriker, Cambridge UP, Cambridge (1999).

[4] D. J. Eisenstein et al., Astrophys. J. 633, 560 (2005).

[5] S. W. Allen, R. W. Schmidt and A. C. Fabian, MNRAS 334, L11 (2002); S. Ettori, P. Tozzi
and P. Rosati, A&A 398, 879 (2003); J. A. S. Lima, J. V. Cunha and J. S. Alcaniz, Phys. Rev. D 68, 023510 (2003), astro-ph/0303388; J. V. Cunha, L. Marassi and R. C. Santos, IJMPD 16, 403 (2007).

[6] J. Dunlop et al., Nature 381, 581 (1996); Y. Yoshii, T. Tsujimoto and K. Kawara, ApJ 507, L133 (1998); J. S. Alcaniz and J. A. S. Lima, ApJ 521, L87 (1999), astro-ph/9902298; ibdem, ApJ 550, L133 (2001), astro-ph/0101544; J. S. Alcaniz, J. A. S. Lima and J. V. Cunha, MNRAS 340, L39 (2003), astro-ph/0301226; J. V. Cunha and R. C. Santos, IJMPD 13, 1321 (2004), astro-ph/0402169; A. Friaca, J. S. Alcaniz and J. A. S. Lima, Mon. Not. R. Astron. Soc. 362, 1295 (2005), astro-ph/0504031; J. F. Jesus, Gen. Rel. Grav. 40, 791 (2008), astro-ph/0603142; J. V. Cunha, L. Marassi and J. A. S. Lima, MNRAS 379, L1-L5 (2007), astro-ph/0611934.

[7] Press W. H., Schechter P., ApJ 187, 425 (1974).
[8] N. Katz et al., Mon. Not. R. Astron. Soc. 270, L71-L74 (1994).
[9] M. S. Longair “Galaxy Formation”, Springer-Verlag, Berlin (1998).
[10] M. Nagashima, Astrophys. J. 562, 7 (2001).
[11] T. H. Reiprich and H. Boehringer, Astrophys. J 567, 716 (2002).
[12] T. H. Reiprich and D. S. Hudson, astro-ph/0611514.
[13] K. Rines, A. Diaferio and P. Natarajan, “WMAP5 and the Cluster Mass Function”, arXiv:0803.1843v1 [astro-ph].
[14] L. Marassi and J. A. S. Lima, IJMP D13, 1345 (2004).
[15] L. Marassi and J. A. S. Lima, IJMP D16, 445 (2007).
[16] C. Tsallis, J. Stat. Phys. 52, 479 (1988).
[17] R. Silva, A. R. Plastino and J. A. S. Lima, Phys. Lett. A 249, 401 (1998); J. A. S. Lima, R. Silva and A. R. Plastino, Phys. Rev. Lett. 86, 2938 (2001), cond-mat/0101030; J. A. S. Lima, R. Silva and J. Santos, Astron. Astrophys. 396, 309 (2002), astro-ph/0109474.
[18] Hansen S. H., New Astron. 10, 371 (2005); J. D. Vergados, S. D. Jiulin, Eur. Lett. 67, 893 (2004); A. R. Plastino and H. G. Miller, Astrop. Space Sci. 290, 275 (2004); J. A. S. Lima and R. E. de Souza, Physica A350, 303 (2005), astro-ph/0406404; Hansen S. H. et al., New Astron. 10, 379 (2005); H. Hansen, and O. Host; Phys. Rev. D 77, 023509 (2008).
[19] R. Silva, J. A. S. Lima and J. Santos, Phys. Rev. E61, 3260 (2000); F. Valentini, Phys. Plasmas 12, 072106 (2005).
[20] E. K. Lenzi and R. S. Mendes, Eur. Phys. J. B201, 401 (2001); J. R. Bezerra, R. Silva and J. A. S. Lima, Physica A 322, 256 (2003); J. A. S. Lima, J. R. Bezerra and R. Silva, Physica A 289, 296 (2002); F. Q. Potiguar and U. M. S. Costa, Physica A303, 457 (2002).

[21] T. M. Davis et al., Astrophys. J. 666, 716 (2007), astro-ph/0701510.

[22] O. Elgarøy and T. Multamäki, Astron. Astrophys. 471, 65E (2007).

[23] Y. Wang and P. Mukherjee, Phys. Rev. D 76, 103533 (2007).

[24] M. Doran, S. Stern and E. Thommes, JCAP 0704, 015 (2007).

[25] G. Efstathiou and J. R. Bond, Mon. Not. R. Ast. Soc. 304, 75 (1999).