Some universal properties of the string breaking

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In gauge systems coupled to matter, the static potential flattens out at a scale where the confining string breaks by formation of a dynamical pair of particles. Surprisingly, such a breaking is invisible in Wilson loops even when the inter-charge separation is much larger than the flattening scale. Observing string breaking also requires using different operators. A known string mechanism provides us with a simple explanation, leading to the area law for large Wilson loops, as observed in most gauge models coupled to whatever kind of matter. It is also pointed out that in a simple 3D $Z_2$ gauge-Higgs model, once reformulated in terms of Fortuin Kasteleyn clusters, this peculiar behaviour of the Wilson operator can be ascribed to non-trivial linking of a percolating cluster to the Wilson loop. Some numerical tests on this model are also presented.

1. INTRODUCTION

In gauge theories coupled to matter (quarks or Higgs fields in the fundamental representation) the string breaking is the phenomenon of static potential flattening at large distances due to the screening of the sources produced by pair creation. A similar screening is expected also in pure Yang-Mills theory for sources in the adjoint representation, where the role of charged matter is played by the gluons (adjoint string).

The coupled systems of this kind studied up to now reveal a surprising, general phenomenon: the string breaking is invisible in the Wilson loops, namely, the area law continues to prevail in full QCD \cite{1,2} and in pure Yang-Mills theory with adjoint sources \cite{3} even at distances where the static charges are completely screened. In other terms there is no possibility to extract the true large distance behaviour of the static potential using only the Wilson loops. On the contrary, neat descriptions of potential flattening have been obtained in studies where the basis of the operators has been enlarged in order to gain a better ground state overlap. In this way it has been observed the breaking of the confining string between colour sources in fundamental representation in Higgs models \cite{3}, in QCD \cite{4} and the breaking of the adjoint string \cite{5}.

A possible explanation for this unexpected finding is that the available range of the Wilson loops of present studies is not wide enough and that for larger sizes the overlap of the Wilson operators to the broken string state could become visible. This however would imply that the presently observed area law obeyed by the Wilson loops, which has been tested for a wide range even in QCD \cite{2}, should be broken at a larger scale of doubtful physical significance.

There is another explanation, based on the effective string picture of confinement and supported by a well known string mechanism \cite{6}, which points in the opposite direction\cite{8}. According to such a picture, the area law describes the asymptotic, universal behaviour of the Wilson loops in most (perhaps all) gauge theories in the confined phase, coupled to whatever kind of matter. Any observed non-vanishing overlap of the Wilson operator to the broken string state should be a decreasing function of the loop size, going eventually to zero in the IR limit. If this prediction will pass the numerical tests of the future lattice simulations, one can envisage using the asymptotic area law of the Wilson loops for an unambiguous definition of the confining phase and string tension even when the static potential flattens because of sea quarks or other dynamical matter. This will be discussed in section 2.
There is one case, at least, where the area law of the Wilson loops can be proved without appealing to any effective theory: In the 3D $Z_2$ gauge Higgs model it can be shown that the very confining mechanism which is operating in pure $Z_2$ gauge theory survives to the addition of $Z_2$ charged matter. In particular, for a simple topological reason the Wilson loop continues to obey the area law. This will be discussed in section 3, where a very efficient numerical method is used to test the behaviour of large Wilson loops.

2. EFFECTIVE STRING BREAKING

The lack of visible effects of string breaking in the Wilson loop and its manifestation in other operators have a simple explanation in terms of the effective string picture of the confining phase of any gauge theory. According to such a description, the expectation value of a rectangular Wilson loop can be represented as

$$\langle W(R, T) \rangle \propto e^{-F_o},$$

where $F_o$ is the free energy of a 2D model describing the normal modes of vibration of the string world-sheet bounded by the rectangle $R \times T$. This assumption yields, in the IR limit,

$$e^{-F_o} \sim e^{-\sigma RT-p(R+T)} \left[ \frac{\sqrt{R}}{q^{\frac{d-1}{2}}} \right] \prod_{n=1}^{\infty} (1-q^n)^{1/2}$$

where $q \equiv \exp(-2\pi T/R)$, $\sigma$ is the string tension and $d$ the space-time dimension. Adding dynamical matter fields induces the formation of holes of any size in the world-sheet, hence eq.(1) is replaced by a loop expansion:

$$\langle W \rangle \propto e^{-F_o} + \sum_{\text{one hole}} e^{-F_1} + \sum_{\text{two holes}} e^{-F_2} + \ldots$$

where the loops inside the Wilson rectangle represent pair created particles. What we can say about the sum of this series? A simple solvable matrix model, describing a free string with holes of any size, suggests that there are two phases:

In the normal phase this sum is dominated by configurations with small holes. Their size does not depend on that of the Wilson loop: Larger Wilson loops have more holes. These holes do not influence the area law and their effect can be absorbed into a renormalization of the string tension, according to a well known string mechanism.

In the other (tearing) phase the mean size of the holes increases with that of the Wilson loop. In this case asymptotic screening should be visible also in the Wilson loops.

The observed poor overlap of the Wilson operator with the broken string state suggests that in all the known cases the confining string belongs to the normal phase. In this phase large Wilson loops behave exactly like in the pure Yang Mills theory. There are however finite size effects which are worth mentioning. As a matter of fact, different phases are completely separated only in the thermodynamic limit. The expectation value of finite Wilson loops in the normal phase receives a non-vanishing contribution of configurations typical of the tearing phase, of course. Thus the overlap of a finite Wilson loop to the broken string state cannot be exactly zero, but should be a decreasing function of its size, going to zero in the infinite size limit. In order to gain an intuitive understanding of what is going on, imagine to cut a rectangular Wilson loop $R \times T$ at a given “time” (the dashed line in the following drawing)

![Diagram of Wilson loop with holes](image)

The above naive picture of the string world-sheet riddled with holes suggests that the Wilson operator has an important overlap with a multi-meson state $|n\rangle$ where the mean number $n$ of particles is a growing function of the distance $R$ between the static sources.
The above reasoning does not imply that there is no flattening in the static potential. The observed screening of sources is a mixing phenomenon. It is simply due to the possibility, in presence of charged matter, to construct other operators with a sufficient overlap to the ground state even for large separations between the static source \( \frac{1}{\epsilon} \). Applying the loop expansion \( \frac{3}{\epsilon} \) to operators made of disconnected pieces leads to a contribution with a large overlap with the broken string state. This very argument also explains why \( \frac{3}{\epsilon} \) in the QCD at finite temperature the string breaking is neatly visible already in the Polyakov correlator \( \frac{3}{\epsilon} \).

### 3. 3D \( Z_2 \) GAUGE-HIGGS MODEL

The action of a 3D \( Z_2 \) gauge theory coupled to a charged matter field can be written as

\[
S(\beta_G, \beta_I) = \beta_I \sum_{ij} \sigma_i U_{ij} \sigma_j + \beta_G \sum_{\text{plaq.}} U_\square
\]

where both the link variable \( U_{ij} \equiv U_{\ell} \) and the matter field \( \sigma \) take values \( \pm 1 \) and \( U_\square = \prod_{\ell \in \square} U_\ell \). This model is self-dual: its partition function

\[
Z(\beta_G, \beta_I) = \sum_{\{\sigma_i = \pm 1, U_\ell = \pm 1\}} e^{-S(\beta_G, \beta_I)}
\]

fulfills, in the thermodynamic limit, the functional equation

\[
Z(\beta_G, \beta_I) = (\sinh 2\beta_G \sinh 2\beta_I)^{\frac{N}{2}} Z(\tilde{\beta}_I, \tilde{\beta}_G)
\]

with \( \tilde{\beta} = -\frac{1}{2} \log(\tanh \beta) \).

The phase diagram of this model has been studied long ago \( \frac{10}{\epsilon} \). There is an unconfined region surrounded by lines of phase transitions toward the Higgs phase and its dual. These lines are second order until they are near each other and the self-dual line, where first order transition occurs. We want to show that large Wilson loops in the dual Higgs phase obey the area law.

Using the method of Fortuin and Kasteleyn (FK) \( \frac{11}{\epsilon} \) we can map this system into a percolation model, rewriting \( Z \) as

\[
Z = e^{-\beta_I N} \sum_{\{U_\square\}} e^{\sum_{U_\square}} \sum_{G} e^{n(G) v(G)}
\]

with \( v = e^{2\beta_I} - 1 \), \( G \) denotes a subgraph of the lattice made of \( n(G) \) links, called active bonds, and \( c(G) \) is the number of its connected components, called FK clusters. The only difference between the pure gauge theory and the one coupled to matter is that in the former the \( G \) subgraphs are arbitrary, while in the latter the allowed \( G \)'s are subjected to a constraint: the number of negative links \( (U_{ij} = -1) \) along every circuit made with active bonds must be even, as it is easily checked. Put differently, no frustration is allowed in \( G \), hence no \( Z_2 \) magnetic flux can pass through the circuits of \( G \). This can be rephrased by saying that the FK clusters behave as pieces of superconducting matter. On the other hand, inserting a Wilson loop \( W(R, T) \) in the vacuum corresponds to creating an unit of \( Z_2 \) flux in the dual version. Thus, owing to the above superconducting property, no FK cluster can be linked with it. Actually, using the methods of ref.\( \frac{12}{\epsilon} \), one easily gets the following exact identity

\[
\langle W(R, T) \rangle_{\beta_G, \beta_I} = \langle \varpi(R, T) \rangle_{\tilde{\beta}_I, \tilde{\beta}_G}
\]

where \( \varpi(R, T) \) is a projector on the \( G \)'s defined as follows

\[
\varpi(R, T) = \begin{cases} 0 & \text{if some FK cluster is linked to } W \\ 1 & \text{if no FK cluster is linked to } W. \end{cases}
\]

Thus the Wilson loop is a sort of cluster counter. It should be mentioned the surprising analogy of the circuits of the FK clusters with the center vortices \( \frac{13}{\epsilon}, \frac{14}{\epsilon} \); note however that they have little to do with the thin vortices associated to the sign of the plaquettes; they are rather describing new degrees of freedom related to the gauge variables through a highly non-local duality transformation and live on links of the dual lattice. They play a relevant role in providing us with a very efficient disordering mechanism which leads to an area law for Wilson loops.

For a large Wilson loop, finite clusters can link with it only along the loop perimeter. Therefore they contribute only to the perimeter term of eq.\( \frac{3}{\epsilon} \). However, owing to the duality transformation involved in eq.\( \frac{3}{\epsilon} \), the Wilson loop in the dual Higgs phase is dominated by an infinite, percolating, FK cluster characterizing the
Higgs phase. It is precisely this percolating cluster which leads to the area law for large Wilson loops, as we wanted to show.

Figure 1. Universal shape effects in Wilson loops.

Thus, in the dual of the Higgs phase, even if the ground state potential necessarily flattens out, the area law behaviour of large Wilson loops is unavoidable. As a consequence we can give a precise definition of confinement phase: it is the one where the area law holds true. In that phase the string tension $\sigma$ has an unambiguous definition as the coefficient of the area term of eq.(2).

As a check of these results, we have estimated $\sigma$ in this gauge-Higgs model in a lattice of size $L^3 = 40^3$ at $\beta_G = 0.75245$ and $\beta_I = 0.16683$. We measured all the squared Wilson loops $\langle W(R, R) \rangle$ with $10 < R \leq 20$ applying a very powerful algorithm already used in the pure gauge theory [15] and based on eq.(8). We found $\sigma = 0.00828(26)$.

It also interesting to uncover the universal shape effects produced by the quantum fluctuations of the string. Actually the quantity

$$R(n, L) = \frac{\langle W(L+n, L-n) \rangle}{\langle W(L,L) \rangle} e^{-\sigma n^2}, \quad (9)$$

according to the asymptotic form (3), is only a (known) function of the ratio $n/L$, does not contain any adjustable parameters, nor it depends on the gauge group or on the nature of the matter fields. The function $R(n/L)$ is plotted in Fig.1 (dotted line) along with the data of the present study and those of pure $Z_2$ gauge theory taken form ref. [16].

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