Adaptive chattering-free terminal sliding-mode control for full-order nonlinear system with unknown disturbances and model uncertainties

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Abstract
This study investigates an adaptive chattering-free sliding-mode control method for n-order nonlinear systems with unknown external disturbances and uncertain models. The proposed method takes the advantage of finite-time fast convergence to avoid singularity problem and ensure its robustness against system uncertainty and unknown disturbance. To achieve fast convergence from any initial condition to system origin, a full-order terminal sliding-mode controller containing differential terms is proposed based on the property of n-order nonlinear systems. Then the continuous and smooth actual control law is obtained by integrating the differential control law containing the discontinuous sign function to realize chattering free. Meanwhile, instead of evaluating the fixed upper bound of system uncertainty and interference in practical implementations, an adaptive method is utilized for its unknown upper bound estimation. The convergence of the adaptive terminal sliding-mode controller in finite time is verified based on Lyapunov stability theory. Finally, two simulation results demonstrate the effectiveness of the proposed control method.

Keywords
Full-order system, adaptive method, chattering free, singularity, sliding-mode control, finite-time stability

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Introduction
Nonlinear dynamical systems suffer from the performance degradation caused by uncertainties and external disturbances.¹ Various nonlinear control methods, such as adaptive method,² fuzzy control method,³–⁷ and sliding-mode control (SMC) method,⁸,⁹ have been successively applied to guarantee the nonlinear system stability. Among all of these methods, SMC has wide acceptance in practical implementations and achieves many research results because of its relatively simple design, rapid response, and robustness to unknown interference and model uncertainty. The sliding-mode technique is designed to drive the system state variables to equilibrium using a discontinuous feedback control law,⁹ which is widely used in the fields such as motor control,¹⁰–¹³ mechanical arm control,¹⁴,¹⁵ and multi-agent system.¹⁶–¹⁹

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Slide mode control is divided into linear slide mode (LSM) control and terminal slide mode (TSM) control. The sliding-mode surfaces selected by LSM are linear functions containing the state of the system, which are suitable for systems with low requirements on state accuracy. However, it has poor control effect and cannot converge in finite time for a complex nonlinear system such as robot. Compared with LSM, TSM,\textsuperscript{1,20–23} a nonlinear sliding-mode surface, has a series of advantages such as convergence in finite time and high control precision. By introducing the nonlinear part, the trajectory of the system reaches the sliding-mode surface in finite time and generates sliding-mode dynamics on the sliding-mode surface. Moreover, it has strong robustness to the uncertainty of the system. For the robot manipulators with uncertainties and external disturbances, an adaptive terminal SMC method was proposed\textsuperscript{24} to eliminate the chattering problem of the controller. In Feng et al.,\textsuperscript{25} a robust adaptive end-sliding-mode controller was applied to n-link rigid robotic manipulators, and the unknown parameters of the upper bound of the system uncertainty were estimated by adaptive techniques. These estimates are used as controller parameters to eliminate the effects of uncertain kinetics and to guarantee that the error of the TSM converges in finite time. However, singularity problem, which is caused by the fact that the output of the controller may approach infinity when the system state converges to the equilibrium point, is a serious problem for LSM and TSM. Many research studies have been conducted to overcome the singularity of TSM control system. A full-order terminal-SMC designed in Yi and Zhai\textsuperscript{26} is suitable for full-order nonlinear systems but fails to consider the uncertainty of the model and unknown interference. A chattering-free second-order fast nonsingular terminal SMC scheme was proposed\textsuperscript{27} in combination with nonlinear observers. Although it can converge in fixed time, the result of the controller is complex and fails to be applied to high-order system. A nonsingular TSM control based on the anti-step method proposed by Jianqing and Zibin\textsuperscript{28} is only applicable to specific higher order systems. Other methods such as integrated TSM control\textsuperscript{29} and TSM\textsuperscript{30} combined with homogeneous methods are only applicable to second-order or specific high-order systems, although unknown interference and model uncertainty are considered in the process of solving singularity problem.

Since the chattering phenomenon triggering by discontinuous characteristic of symbol function itself, singularity problem is not the only problem of TSM. In nonlinear system, discontinuous characteristic will activate its high-frequency characteristic, weaken the control effect, or even result in uncontrollable state directly, which are inevitable for the SMC. To avoid chattering phenomenon, the key is to transmit the discontinuous sign function to a bounded continuous function so as to keep the continuity of the SMC law.\textsuperscript{31} The boundary layer method was adopted\textsuperscript{32} to reduce chattering, whereas it leads to the steady-state error of wide boundary layer and unobvious chattering effect reduction of narrow boundary layer. Several studies\textsuperscript{33–35} verify the application of high-order SMC. However, the finite-time convergence system based on high-order SMC may give rise to stronger chattering effect\textsuperscript{36} compared with asymptotic convergence system. A reaching law method was proposed\textsuperscript{37,38} to eliminate or reduce chattering by controlling the parameters of the reaching law. A terminal SMC based on disturbance observer was used to avoid chattering.\textsuperscript{39,40} Although these methods can eliminate or reduce chattering effect, they are not applicable to all-order systems.

After in-depth study of full-order terminal SMC system based on the above analysis, an adaptive chattering-free full-order sliding-mode controller is proposed. By integrating the differential control law with sign function, a continuous and smooth control law is obtained to avoid the influence of chattering. An adaptive method is applied to estimate the unknown upper bound of system uncertainty and unknown disturbance. The algorithm stability is analyzed and proved by Lyapunov stability theory.

The remainder of this article is organized as follows. The second section introduces the adaptive chattering-free full-order sliding-mode controller designed for the second-order system. For high-order systems, the adaptive chattering-free full-order sliding-mode controller with Lyapunov stability analysis is presented in the third section. Simulation studies performed on the second-order system and the third-order system are provided in the fourth section. Finally, conclusions are offered in the fifth section.

Adaptive chattering-free full-order sliding-mode controller of the second-order systems

Consider the following second-order nonlinear systems with unknown disturbances and model uncertainties

\begin{equation}
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f(x) + \Delta f(x) + d(x) + bu
\end{aligned}
\end{equation}

where $x = [x_1 \ x_2]^T \in \mathbb{R}^2$ is the measurable state variable of the nonlinear system, $u \in \mathbb{R}$ is the system input, $f(x) \in \mathbb{R}$ represents the known function about the state $x$ of the system, $f(x)$ is a smooth nonlinear function, $b \in \mathbb{R}$ is a known constant and $\Delta f(x) \in \mathbb{R}$ and $d(x) \in \mathbb{R}$ are the uncertainty of the system and the unknown disturbance term, respectively.

**Assumption 1.** The first derivative $\Delta f(x)$ of the system uncertainty and the first derivative $d(x)$ of the unknown disturbance are bounded.\textsuperscript{41,42} which satisfy the following inequality

\begin{equation}
|\Delta f(x, t) + d(x, t)| < h_0 + h_1|x_1| + h_2|x_2|^2
\end{equation}

where $h_0$, $h_1$, and $h_2$ are unknown positive constants. Due to the impact of the practical engineering environment, the
parameters of the system model are perturbed. Therefore, these parameters are difficult to obtain.

In this section, the control goal is to design an adaptive chattering-free sliding-mode controller to stabilize the nonlinear system (equation (1)) from any position \( x(0) \neq 0 \) to the origin \( x = [0 \ 0]^T \) under the condition of satisfying Assumption 1. The design of adaptive chattering-free sliding-mode controller is as follows: Firstly, a TSM manifold is defined according to the nonlinear system (equation (1)). By establishing the suitable TSM manifold, the derivatives of TSM manifold contain the differential form of system input, and the control law is obtained by integrating the differential form of control input. The actual control law obtained is smooth, thus avoiding chattering problem.43–45 Adaptive law is utilized to solve the problem of unknown upper bound of model uncertainty and unknown disturbance.

For system (equation (1)), the TSM manifold can be designed as follows

\[
s = \bar{x}_1 + C_2 \text{sign}(\bar{x}_1)|\bar{x}_1|^{\alpha_2} + C_1 \text{sign}(\bar{x}_1)|\bar{x}_1|^{\alpha_1}
\]

where \( C_1, C_2 \) and \( \alpha_1, \alpha_2 \) are constants. \( C_1, C_2 \) can be obtained by the polynomial \( p^2 + C_2 p + C_1 \). The polynomial is Hurwitz, which means that its coefficient is positive real number, and its roots are on the left half of the complex plane or on the imaginary axis, namely, the real part of the root is zero or negative. The values of \( \alpha_1, \alpha_2 \) can be selected by the following conditions46

\[
\begin{align*}
\alpha_2 &= \alpha \\
\alpha_1 &= \frac{\alpha_2 \alpha_3}{2 \alpha_3 - \alpha_2}
\end{align*}
\]

where \( \alpha_3 = 1, \alpha \in (1 - \varepsilon, 1), \) and \( \varepsilon \in (0, 1) \).

When the sliding-mode surface \( s \) is reached, \( s = 0 \), the nonlinear system (equation (1)) is equivalent to the following nonlinear differential equations

\[
\dot{x}_1 + C_2 \text{sign}(x_1)|x_1|^{\alpha_2} + C_1 \text{sign}(x_1)|x_1|^{\alpha_1} = 0
\]

and

\[
\dot{x}_2 = -C_2 \text{sign}(x_1)|x_1|^{\alpha_2} - C_1 \text{sign}(x_1)|x_1|^{\alpha_1}
\]

The parameters \( \alpha_1, \alpha_2 \) can be selected by equation (4), and the values of \( C_1, C_2 \) are determined by the condition that \( p^2 + C_2 p + C_1 \) is Hurwitz polynomial. If the ideal sliding mode \( s = 0 \) for system (equation (1)) is established, which represents the establishment of equation (5) or (6), the system (equation (1)) will converge from any position \( x(0) \neq 0 \) to \( x = [x_1, x_2]^T = [0, 0]^T \) in finite time.

Theorem proves that the actual control law is continuous and smooth without any high-frequency switching term by integrating the differential control input containing the sign function, so that the proposed adaptive chattering-free sliding-mode controller is completely free from chattering. The unknown upper bound of external disturbance and model uncertainty term is obtained by designing an adaptive tuning method.

The flowchart of adaptive chattering-free sliding-mode controller is shown in Figure 1.

**Theorem 1.** Considering the uncertain second-order system (equation (1)) will reach the sliding-mode surface \( s = 0 \) within finite time and then converge to zero along the sliding-mode surface in finite time, if equation (3) is selected as the sliding-mode surface, the design of the control law is as follows

\[
u = b^{-1}(u_{eq} + u_n)
\]

where

\[
u_{eq} = -f(x, t) - C_2 \text{sign}(x_1)|x_1|^{\alpha_2} - C_1 \text{sign}(x_1)|x_1|^{\alpha_1}
\]

\[
u_n = -\int_{0}^{\phi} \left[ \dot{h}_0 + \dot{h}_1|x_1| + \dot{h}_2|x_1|^2 \right] \text{sign}(s) + \beta \delta \, d\phi
\]

where \( \dot{h}_0, \dot{h}_1, \) and \( \dot{h}_2 \) are the estimates of \( h_0, h_1, \) and \( h_2 \), respectively, which are updated by the following adaptive laws

\[
\dot{h}_0 = B_0|s|
\]

\[
\dot{h}_1 = B_1|s||x_1|
\]

\[
\dot{h}_2 = B_2|s||x_2|^2
\]

where \( B_0, B_1, \) and \( B_2 \) are positive constants.

**Proof**

\[
s = f(x, t) + \Delta f(x, t) + d(x, t) + bu + C_2 \text{sign}(x_1)|x_1|^{\alpha_2} + C_1 \text{sign}(x_1)|x_1|^{\alpha_1}
\]

Substituting equation (7) into the above expression gives

\[
s = f(x, t) + \Delta f(x, t) + d(x, t) + u_{eq} + u_n + C_2 \text{sign}(x_1)|x_1|^{\alpha_2} + C_1 \text{sign}(x_1)|x_1|^{\alpha_1}
\]

\[
s = \Delta f(x, t) + d(x, t) + u_n
\]

Differentiating \( s \) with respect to time yields

\[
s = \Delta f(x, t) + d(x, t) + u_n
\]
Define the adaption error as \( \hat{h}_i = \hat{h}_i - h_i \) (\( i = 0, 1, 2 \)), and consider the following Lyapunov function candidate:

\[
V = \frac{1}{2} s^T s + \sum_{i=0}^{2} \frac{\gamma_i}{B_i} (\hat{h}_i - h_i)^2
\]

(13)

Differentiating \( V \) with respect to time and using equation (12) yields

\[
\dot{V} = s^T \dot{s} + \sum_{i=0}^{2} \frac{\gamma_i}{B_i} (\hat{h}_i - h_i) \dot{h}_i
\]

\[
= s^T (\Delta f(x, t) + \dot{d}(x, t) + u_n) + \sum_{i=0}^{2} \frac{\gamma_i}{B_i} (\hat{h}_i - h_i) \dot{h}_i
\]

(14)

Applying equation (8) yields

\[
\dot{V} = s^T \left( \Delta f(x, t) + \dot{d}(x, t) - [(\hat{h}_0 + \hat{h}_1 |x_1| + \hat{h}_2 |x_2|^2) \text{sign}(s) - \dot{h}_i] \right)
\]

\[
+ \sum_{i=0}^{2} \frac{\gamma_i}{B_i} (\hat{h}_i - h_i) \dot{h}_i
\]

(15)

\[
\dot{V} \leq s^T (\Delta f(x, t) + \dot{d}(x, t) - (\hat{h}_0 + \hat{h}_1 |x_1| + \hat{h}_2 |x_2|^2) \text{sign}(s))
\]

\[
+ \sum_{i=0}^{2} \frac{\gamma_i}{B_i} (\hat{h}_i - h_i) \dot{h}_i
\]

(16)

Considering the update laws (equation (9)) yields

\[
\dot{V} \leq |s| \cdot |\Delta f(x, t) + \dot{d}(x, t)| - |s| \cdot |(\hat{h}_0 + \hat{h}_1 |x_1| + \hat{h}_2 |x_2|^2)|
\]

\[
- |s| \cdot |(\hat{h}_0 + \hat{h}_1 |x_1| + \hat{h}_2 |x_2|^2)| + |s| \cdot |(\hat{h}_0 + \hat{h}_1 |x_1| + \hat{h}_2 |x_2|^2)|
\]

\[
+ \gamma_0 |\hat{h}_0 - h_0| \cdot |s| + \gamma_1 |\hat{h}_1 - h_1| \cdot |s| \cdot |x_1| + \gamma_2 |\hat{h}_2 - h_2| \cdot |s| \cdot |x_2|^2
\]

\[
\leq -\sqrt{2A_1} \frac{|s|}{\sqrt{2}} - \sqrt{2A_2} \frac{|\hat{h}_0|}{\sqrt{2}} + \sqrt{2A_3} \frac{|\hat{h}_1|}{\sqrt{2}} + \sqrt{2A_4} \frac{|\hat{h}_2|}{\sqrt{2}}
\]

(17)

\[
\dot{V} \leq |s| \cdot \left( |(\hat{h}_0 + \hat{h}_1 |x_1| + \hat{h}_2 |x_2|^2)| - |\Delta f(x, t) + \dot{d}(x, t)| \right) - (|s| - \gamma_0 |s|) \cdot |\hat{h}_0|
\]

\[
- (|s| |x_1| - \gamma_1 |s| |x_1|) \cdot |\hat{h}_1| - (|s| |x_2|^2 - \gamma_2 |s| |x_2|^2) \cdot |\hat{h}_2|
\]

\[
\leq -\sqrt{2A_1} \frac{|s|}{\sqrt{2}} - \sqrt{2A_2} \frac{|\hat{h}_0|}{\sqrt{2}} + \sqrt{2A_3} \frac{|\hat{h}_1|}{\sqrt{2}} + \sqrt{2A_4} \frac{|\hat{h}_2|}{\sqrt{2}}
\]

(18)

\[
\dot{V} \leq -A_5 |s|^{1/2}
\]

(20)

Hence

\[
\dot{V} \leq -A_5 |s|^{1/2}
\]

(21)

where

\[
A_5 = \min \left\{ \sqrt{2A_1}, \sqrt{2A_2}, \frac{1}{\sqrt{2A_3}}, \frac{1}{\sqrt{2A_4}} \right\}
\]

(22)

where \( A_5 > 0, \gamma_i < 1 \). Therefore, based on the Lyapunov stability criterion, the TSM surface \( s = 0 \) will hold in finite time and will remain at zero under the control law (equation (7)) and the adaptive law (equation (9)). Once \( s = 0 \) and \( \dot{s} = 0 \) are established, the system (equation (6)) will converge from any initial condition \( x(0) \neq 0 \) to \( x = [x_1, x_2]^T = [0, 0]^T \) along the sliding-mode surface in finite time, and the adaptive error will also converge to zero. This completes the proof.

**Remark 1.** If the condition of \( |s| = 0 \) holds, the proposed method is applicable. However, due to measurement noise, switching delay and nonlinearities, \( |s| \) can only be infinitely close to zero but not reach zero. Therefore, the values of the adaptive parameters \( \hat{h}_0, \hat{h}_1, \) and \( \hat{h}_2 \) will increase slowly and infinitely. By dividing the domain of \( |s| \), the adaptive parameters are set to constant and \( \hat{h}_i = 0 \) in the interval of \( |s| \) approaching zero, namely, the dead zone technique is adopted to improve the adaptive law (equation (9)) as

\[
\hat{h}_0 = \begin{cases} B_0 |s|, & \text{if } |s| \geq \varepsilon \\ 0, & \text{if } |s| < \varepsilon \end{cases}
\]

\[
\hat{h}_1 = \begin{cases} B_1 |s| |x_1|, & \text{if } |s| \geq \varepsilon \\ 0, & \text{if } |s| < \varepsilon \end{cases}
\]

\[
\hat{h}_2 = \begin{cases} B_2 |s| |x_2|^2, & \text{if } |s| \geq \varepsilon \\ 0, & \text{if } |s| < \varepsilon \end{cases}
\]

(23)

where \( \varepsilon \) is a small positive constant.
Remark 2. The parameter $\varepsilon$ is quite important for the adaptive law, because it will affect the convergence rate of the sliding-mode surface. The large value of $\varepsilon$ will drive the required control input to be large, which is infeasible in the actual control. Moreover, $|s|$ is evolving around $\varepsilon$, which will affect the accuracy of the controller. If the value of $\varepsilon$ is too small, then $|s|$ will always be greater than $\varepsilon$, and lead to the increase of $\dot{h}_i$, which results in TSM manifold oscillation. Therefore, the value of $\varepsilon$ cannot be too large, and it needs to be comprehensively selected based on the control input and response speed.\textsuperscript{10}

Remark 3. The parameter $B_i$ in the adaptive law (equation (9)) determines the convergence rate of $\dot{h}_i$. A large value of $B_i$ will speed up the convergence of $\dot{h}_i$ and quickly reduce the adaptive error $\dot{h}_i$.

Remark 4. In the process of obtaining the actual control law, the $C_i\text{sign}(x_i)|x_i|^{\alpha_i}$ in the terminal sliding-mode surface (equation (3)) will not be differentiated. Consequently, the terminal sliding-mode surface can avoid the singularity problem.

Adaptive chattering-free full-order sliding-mode controller of the high-order systems

Consider the following high-order nonlinear systems with unknown disturbances and model uncertainties

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
& \vdots \\
\dot{x}_{n-1} &= x_n \\
\dot{x}_n &= f(x) + \Delta f(x) + d(x) + bu
\end{align*}
\] (24)

where $x = [x_1 \ x_2 \ \cdots \ x_n]^T \in \mathbb{R}^n$ is the state vector and the definition of $u \in \mathbb{R}$, $f(x) \in \mathbb{R}$, and $b \in \mathbb{R}$ is the same as equation (1). $\Delta f(x) \in \mathbb{R}$ and $d(x) \in \mathbb{R}$ are the uncertainty of the system and the unknown disturbance, respectively.

Remark 5. The uncertainty $d(x)$ and the unknown interference $\Delta f(x)$ in the system (equation (24)) still obey Assumption 1

\[|\Delta f(x,t) + d(x,t)| < h_0 + h_1|x_1| + h_2|x_2|^2\]

where $h_i$ is the positive unknown constant.

The TSM for the nonlinear system (equation (24)) drives the system to the terminal sliding-mode surface from any initial state $x(0) \neq 0$ and ensures that the system reaches the system origin $x = [0 \ 0 \ \cdots \ 0]^T$ along the terminal sliding-mode surface in finite time. Compared with the system (equation (1)), the system order of the nonlinear system (equation (24)) is increased, which results in the increase in complexity. Accordingly, more factors need to be considered before designing TSM for system (equation (24)).

To make the sliding-mode variable converge to the origin in finite time and drive the state variable to converge to the origin in finite time, TSM manifold of system (equation (24)) can be designed as follows\textsuperscript{22}

\[s = \dot{x}_n + C_n\text{sign}(x_n)|x_n|^{\alpha_n} + \cdots + C_1\text{sign}(x_1)|x_1|^{\alpha_1} \] (25)

where $C_i$ and $\alpha_i (i = 1, 2, \ldots, n)$ are constants. The parameter $C_i$ can be selected based on the condition of Hurwitz polynomial $p^n + C_np + \cdots + C_2p + C_1$. The value of $\alpha_i$ can be obtained according to the following conditions\textsuperscript{43}

\[
\begin{align*}
\alpha_1 &= \alpha, & n &= 1 \\
\alpha_{i-1} &= \frac{\alpha_i\alpha_{i+1}}{2\alpha_{i+1} - \alpha_i}, & i &= 2, \ldots, n & \forall n \geq 2
\end{align*}
\] (26)

where $\alpha_n = \alpha$, $\alpha_{n+1} = 1$, $\alpha \in (1 - \varepsilon, 1)$, and $\varepsilon \in (0, 1)$.

When the terminal sliding-mode surface is established, $s = 0$, the high-order system (equation (24)) can be rewritten as follows

\[
\begin{align*}
\dot{x}_n &= -C_n\text{sign}(x_n)|x_n|^{\alpha_n} - \cdots - C_1\text{sign}(x_1)|x_1|^{\alpha_1}
\end{align*}
\] (28)

$C_i$ is chosen based on the Hurwitz polynomial $p^n + C_np + \cdots + C_2p + C_1$ and $\alpha_i$ in TSM manifold (equation (25)) is determined by formula (equation (26)). If $s$ converges to zero in finite time and the sliding-mode manifold $s = 0$ is established, equation (27) or (28) is established. The state variable will approach to the sliding-mode manifold from any initial condition $x(0) \neq 0$ in finite time and converge to $x = [0 \ 0 \ \cdots \ 0]^T$ along the sliding-mode surface.

The adaptive chattering-free SMC law is expressed as follows

\[
u = b^{-1}(u_{eq} + u_n) \] (29)

\[
u_{eq} = -f(x,t) - C_n\text{sign}(x_n)|x_n|^{\alpha_n} - \cdots - C_1\text{sign}(x_1)|x_1|^{\alpha_1} \] (30)

\[
u_n = -\int_0^s [(\dot{h}_0 + \dot{h}_1|x_1| + \dot{h}_2|x_1|^2)\text{sign}(s) + \beta s]d\phi \] (31)
The definition of adaptive law is consistent with equation (9). Although the control law \( u_{eq} \) contains switching terms, there are no such terms in the actual control law. Therefore, the proposed control law is chattering free.

For nonlinear high-order system (equation (24)) and control law (equation (29)), the TSM manifold (equation (25)) can be rewritten as follows

\[
\begin{align*}
    \dot{x} &= C_n \text{sign}(x_n) |x_n|^\alpha + \cdots + C_1 \text{sign}(x_1) |x_1|^\alpha \\
    &= f(x, t) + \Delta f(x, t) + d(x, t) + u_{eq} + u_n \\
    &\quad + C_n \text{sign}(x_n) |x_n|^\alpha + \cdots + C_1 \text{sign}(x_1) |x_1|^\alpha \\
\end{align*}
\]

Substituting the control (equation (30)) into equation (32) gives

\[
    \dot{s} = \frac{1}{2} s^T \dot{s} + \sum_{i=0}^{2} \frac{\gamma_i}{B_i} \dot{h}_i^2 
\]

Using equation (34), the time derivative of \( V \) is

\[
\begin{align*}
    \dot{V} &= s^T (\dot{\Delta f}(x, t) + \dot{d}(x, t) + \ddot{u}_n) + \sum_{i=0}^{2} \frac{\gamma_i}{B_i} \dot{h}_i \dot{h}_i \\
    &= s^T (\dot{\Delta f}(x, t) + \dot{d}(x, t) + \ddot{u}_n) + \sum_{i=0}^{2} \frac{\gamma_i}{B_i} \dot{h}_i \dot{h}_i \\
\end{align*}
\]

Using equation (31) yields

\[
\begin{align*}
    \dot{V} &\leq s^T (\dot{\Delta f}(x, t) + \dot{d}(x, t) - (\dot{h}_0 + \dot{h}_1 |x_1| + \dot{h}_2 |x_1|^2) \text{sign}(s) - \beta s) \\
    &\quad + \sum_{i=0}^{2} \frac{\gamma_i}{B_i} \dot{h}_i \dot{h}_i \\
\end{align*}
\]

Hence, a conclusion similar to the proof of the stability of the second section can be drawn as

\[
\begin{align*}
    \dot{V} &\leq -A_s \left[ |s| + \sqrt{\gamma_0} |\dot{h}_0| + \sqrt{\gamma_1} |\dot{h}_1| + \sqrt{\gamma_2} |\dot{h}_2| \right] \\
    &\leq -A_s V^{1/2} \\
\end{align*}
\]

where

\[
A_s = \min \left\{ \sqrt{2} A_1, \sqrt{\gamma_0} A_2, \sqrt{\gamma_1} A_3, \sqrt{\gamma_2} A_4 \right\} 
\]

\[
\begin{align*}
    A_1 &= |(h_0 + h_1 |x_1| + h_2 |x_1|^2) - |\Delta f(x, t) + \dot{d}(x, t)| \\
    A_2 &= (1 - \gamma_0) |s| \\
    A_3 &= (1 - \gamma_1) |s| |x_1| \\
    A_4 &= (1 - \gamma_2) |s| |\dot{x}_1|^2 \\
\end{align*}
\]

where \( A_s > 0, \gamma_i < 1 \). The TSM manifold \( s \) will converge to zero in finite time, and it will be stable at zero. With the establishment of \( s = 0 \) and \( \dot{s} = 0 \), the system (equation (28)) will converge from any initial position \( x(0) \neq 0 \) to \( x = [x_1, x_2, \ldots, x_n]^T = [0, 0, \ldots, 0]^T \) along the TSM surface in finite time. This completes the proof.

Practically, \( |s| \) cannot reach zero accurately in finite time due to measurement noise, switching delay, and non-linearity. This problem is solved by adding dead-zone technology to adaptive law

\[
\begin{align*}
    \dot{h}_0 &= \begin{cases} 
        B_0 |s|, & \text{if } |s| \geq \varepsilon \\
        0, & \text{if } |s| < \varepsilon 
    \end{cases} \\
    \dot{h}_1 &= \begin{cases} 
        B_1 |s| |x_1|, & \text{if } |s| \geq \varepsilon \\
        0, & \text{if } |s| < \varepsilon 
    \end{cases} \\
    \dot{h}_2 &= \begin{cases} 
        B_2 |s| |x_1|^2, & \text{if } |s| \geq \varepsilon \\
        0, & \text{if } |s| < \varepsilon 
    \end{cases}
\end{align*}
\]

Remark 6. To calculate \( \text{sign}(s) \) in equation (31), a function \( G(t) \) is defined as follows

\[
G(t) = \int_0^t s(t) \, dt = x_n + \int_0^t C_n \text{sign}(x_n) |x_n|^\alpha + \cdots + C_1 \text{sign}(x_1) |x_1|^\alpha \, dt
\]

\( \text{sign}(s) \) can be derived as

\[
\text{sign}(s) = \text{sign}(G(t) - G(t - \eta))
\]

where \( \eta \) is the time delay. By reason of

\[
\begin{align*}
    s(t) &= \lim_{\eta \to 0} \frac{G(t) - G(t - \eta)}{\eta} \\
\end{align*}
\]

an appropriate sample time can be chosen as \( \eta \).

**Simulations**

In this section, the performance of the proposed algorithm is demonstrated, and the following two simulation examples are given.
**Example 1 (TSM control of a second-order system)**

\[
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_1^3 + \cos(2t) + 0.5\sin(400\pi t) + u
\end{aligned}
\]

A TSM manifold for equation (3) is designed as follows

\[
s = x_1 + 7\text{sign}(x_1)x_1^{9/16} + 10\text{sign}(x_1)x_1^{9/23}
\]

\[
= x_2 + 7\text{sign}(x_1)x_1^{9/16} + 10\text{sign}(x_1)x_1^{9/23}
\]

where the parameters \(C_1 = 10, C_2 = 7\) in equation (4) are selected by the Hurwitz polynomial \(p^2 + 7p + 10 = (p + 2)(p + 5)\); \(\alpha_1 = 9/23\) and \(\alpha_2 = 9/16\) are determined to guarantee that equation (4) is established.

The control law (equations (7) and (8)) can be rewritten as

\[
u = u_{eq} + u_n
\]

\[
u_{eq} = -f(x, t) - 7\text{sign}(x_1)x_1^{9/16} - 10\text{sign}(x_1)x_1^{9/23}
\]

\[
u_n = -\int_0^t [(\hat{h}_0 + \hat{h}_1)x_1 + \hat{h}_2x_1^2]\text{sign}(s) + \beta s \, d\phi
\]

where \(\text{sign}(s)\) and \(s\) can be calculated using equations (43) and (44).

The adaptive tuning laws are given as

\[
\dot{\hat{h}}_0 = \begin{cases} 
1.47|x|, & \text{if } |x| \geq 1 \\
0, & \text{if } |x| < 1 
\end{cases}
\]

\[
\dot{\hat{h}}_1 = \begin{cases} 
1.03|x||x_1|, & \text{if } |x| \geq 1 \\
0, & \text{if } |x| < 1 
\end{cases}
\]

\[
\dot{\hat{h}}_2 = \begin{cases} 
1.84|x||x_2|^2, & \text{if } |x| \geq 1 \\
0, & \text{if } |x| < 1 
\end{cases}
\]

Taking the second-order system as the control object and the simulation time as a fixed step of 0.01 s, the simulation was carried out in the MATLAB R2016a-Simulink platform. The initial conditions of the adaptive parameters are \(\hat{h}_0(0) = 1, \hat{h}_1(0) = 1\), and \(\hat{h}_2(0) = 1\), and the state of the system is \(x = [x_1, x_2]^T = [1, -1]^T\), control input \(u(0) = 0\) in the simulation.

The simulation results are shown in Figures 2 to 5. The sliding-mode variable \(s\) of the second-order system is depicted in Figure 2, which shows that the numerical change of the sliding-mode variable \(s\) is relatively reasonable and smooth, and converges to zero in a finite time. Figure 3 shows that the curve of the control law is relatively smooth and there is no high-frequency oscillation phenomenon though the control law (equations (7) and (8)) contains discontinuous sign functions. Hence, the algorithm effectively eliminates chattering. The convergence of the adaptive parameters \(\hat{h}_0, \hat{h}_1, \text{and } \hat{h}_2\) are plotted in Figure 4, where the adaptive law can converge, stabilize, and accurately estimate the parameters in finite time under complex unknown disturbances and model uncertainties. The convergence of the state variables of the second-order system is shown in Figure 5. The system state \(x = [x_1, x_2]^T\) converges in finite time, which verifies the effectiveness of the algorithm.

**Example 2 (TSM control of a third-order system)**

\[
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= x_1^3 + \cos(2t) + 0.5\sin(400\pi t) + u
\end{aligned}
\]

Based on equation (41), a TSM manifold is established as follows

\[
s = x_1^{10} + 15\text{sign}(x_1)x_1^{7/10} + 66\text{sign}(x_1)x_1^{7/13} + 80\text{sign}(x_1)x_1^{7/16}
\]

\[
= x_1 + 15\text{sign}(x_1)x_1^{7/10} + 66\text{sign}(x_1)x_1^{7/13} + 80\text{sign}(x_1)x_1^{7/16}
\]

where the parameters \(C_1 = 80, C_2 = 66, \text{and } C_3 = 15\) in equation (25) are chosen by Hurwitz polynomial \(p^3 + 15p^2 + 66p + 80 = (p + 2)(p + 5)(p + 8)\); \(\alpha_1 = 7/16, \alpha_2 = 7/13, \text{and } \alpha_3 = 7/10\) are determined by the establishment of equation (26).

Based on equations (29) to (31), the control law for the third-order system can be given as
Figure 4. ((a) to (c)) Adaptation of parameters of the second-order system.

Figure 5. System states of the second-order system.

Figure 6. Sliding manifold of the third-order system.

Figure 7. Control input with the uncertainty and disturbance of the third-order system.

Figure 8. ((a) to (c)) Adaptation of parameters of the third-order system.

Figure 9. System states of the third-order system.
\[ u = u_{eq} + u_a \]
\[ u_{eq} = -f(x, t) - 15\text{sign}(x_3)|x_3|^{7/10} - 66\text{sign}(x_2)|x_2|^{7/13} - 80\text{sign}(x_1)|x_1|^{7/16} \]
\[ u_a = -\int_0^t \left([\hat{h}_0 + \hat{h}_1|x_1| + \hat{h}_2|x_1|^2]\text{sign}(s) + \beta s\right) \text{d}\phi \]

where \( \text{sign}(s) \) and \( s \) can be adopted using equations (43) and (44).

The adaptive tuning laws are given as

\[
\begin{align*}
\dot{\hat{h}}_0 &= \begin{cases} 
0.042|s|, & \text{if } |s| \geq 1.8 \\
0, & \text{if } |s| < 1.8
\end{cases} \\
\dot{\hat{h}}_1 &= \begin{cases} 
0.01|s||x_1|, & \text{if } |s| \geq 1 \\
0, & \text{if } |s| < 1
\end{cases} \\
\dot{\hat{h}}_2 &= \begin{cases} 
0.47|s||x_2|^2, & \text{if } |s| \geq 1 \\
0, & \text{if } |s| < 1
\end{cases}
\end{align*}
\]

The simulation is carried out in the MATLAB R2016a-Simulink platform with the third-order system as the control object and the simulation time as a fixed step of 0.005 s. The initial conditions of the adaptive parameters are \( \hat{h}_0(0) = 1 \), \( \hat{h}_1(0) = 1 \), and \( \hat{h}_2(0) = 1 \), and the initial state is \( x = [x_1, x_2, x_3]^T = [0.5, 0, -1]^T \). Control input \( u(0) = 0 \) applies to the simulation.

The simulation results are shown in Figures 6 to 9. The convergence of the sliding-mode variable \( s \) for the third-order system is shown in Figure 6, which guarantees that the states converge to zero in finite time. The actual control \( u \) of the third-order system is depicted in Figure 7. The figure shows that the control law curve is relatively smooth, and the algorithm can effectively avoid chattering accordingly. The convergence of the adaptive parameters \( \hat{h}_0, \hat{h}_1, \) and \( \hat{h}_2 \) are plotted in Figure 8, where the adaptive law accurately estimates the parameters in finite time without the prior knowledge of unknown interference and model uncertainty. Three state variables of the third-order system are plotted in Figure 9. It is obvious that the convergence time of the simulation results of the third-order system is about 0.5 s slower than that of the state variables of the second-order system. Based on the simulation debugging experience, the convergence time will be reduced as \( |x_3| \) approaches zero.

**Conclusion**

An adaptive chattering-free full-order sliding-mode controller method is proposed for \( n \)-order in this article. The continuous actual control law is obtained by integrating the constructed differential control law containing discontinuous symbol function to avoid chattering. Without the demand of prior knowledge concerning the upper bound of system uncertainty and unknown disturbance, an adaptive law is introduced to estimate the upper bound. The stability principle of Lyapunov demonstrates that the algorithm can guarantee the finite-time convergence of sliding-mode variable and the adaption error. Two simulation examples are utilized to verify performance of the proposed algorithm. The simulation results indicate that the control strategy can drive the system state to converge to zero in finite time under the premise that the parameters satisfy the simulation conditions of the second-order and third-order systems.

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