The Objective Space and the Formulation of Design Requirement in Natural Laminar Flow Optimization

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Abstract: Design requirement is as important in aerodynamic design as in other industries because it sets up the objective for the samples in design space to approach. Natural Laminar Flow (NLF) optimization belongs to the type of aerodynamic design problems featured by the combination of distinct aerodynamic performance, where the design requirement is often formulated in form of summation of laminar-related performance and pressure drag performance with different weight assignment according to different perspectives. However, the formulations are rather experience-oriented and are decided non-quantitatively. Inspired by data manipulation approaches in design space (spanned by design variables of geometrical representation parameters) in many aerodynamic designs, this paper proposes new formulations of design requirement in NLF optimization via consideration of objective space (projection of design space through aerodynamics) and shows the impact of the corresponding formulation of design requirement to the result of NLF optimization in cases of transonic airfoil and aero engine compressor blade design from two perspectives: Pareto front convergence and improving effect of accessory performance. The paper uses Principal Component Analysis (PCA) to obtain the eigenvectors of objective space to extract the intrinsic information about specific problem. The method is realized in two cases with satisfactory result.

Keywords: design requirement; objective space; design objectives; engineering design; natural laminar flow; aerodynamic performance; principal component analysis

1. Introduction

The formulation of design requirements is assumed critical for effective optimization in all design disciplines [1], since they characterize the design process through design space exploration for the best solution via evaluation and analysis [2–5]. This paper is focused on providing new perspective from objective space via formulating problem-oriented design requirement in the field of aerodynamic design such as Natural Laminar Flow optimization of a transonic airfoil and aero engine compressor blade.

The destructive impact of carbon emissions and the enduring bad economic situation of airlines makes it necessary for aircraft manufacturers to minimize fuel burn in aircraft design, one solution to which has been drag reduction [6,7]. The skin friction is important part of drag composition (it occupies nearly 50% of total aircraft drag in some cases [6]). Therefore, Natural Laminar Flow (NLF) optimization has been worked on for skin friction drag reduction by extending laminar area over aircraft surface, on account of lower skin friction drag magnitude in laminar than in turbulence [6,8,9].

In many aerodynamic designs, numerical optimization algorithms based on geometrical parameterization are used for the samples in design space to be iterated towards a pre-defined design requirement. The classification of skin friction drag and pressure drag comes from their different
generation mechanisms and the fact that they can be easily calculated without involving the other’s information. The formulation of design requirement in NLF optimization usually reduces to the summation of pressure drag \( C_p \) and laminar-related performance (i.e., skin friction drag \( C_f \)) as in Equation (1)

\[
\min (w_1 \cdot C_p + w_2 \cdot C_f), \text{ s.t. geometrical constraints, etc.} \tag{1}
\]

where \( w_1 \) and \( w_2 \) are the assigned weights that represents the importance attached to the aerodynamic performance behind variables given by designers. Wang et al. and Iuliano et al. put \( w_2 \) under major consideration (solely minimize \( C_f \)) in design requirement formulation with \( C_p \) being kept under a passive constant constraint \([8,10]\). Chen and Tang and Tang et al. added wave drag (a subset of pressure drag) in design requirement formulation where they put much consideration on shock wave at transonic regime \([11,12]\). Han et al. and Rashad and Zingg simply put total drag as the design requirement \([13–15]\). Wang et al. set 80% to \( w_2 \) in Equation (1) for increasing the effect of laminarization \( C_f \) \([16]\).

The above settings represent different perspectives in design process. From industrial view, making total drag of a geometry alone as the design requirement is ultimate, but this would hinder the effort of methodological manipulation in emphasizing NLF’s influence \([17–20]\). Usually, it is not how much importance should be adequately given to laminarization that matters in such cases, but whether laminarization is considered in drag reduction design. In fact, these studies are more of verification of transition models in the prediction of laminar transition with coupling of ordinary drag force computation. Adjusting the corresponding weights in Equation (1) is a solution to the challenge of reasonable design requirement formulation. The problem, however, is that this is usually done according to experience. For example, simply singling out laminarization in the design requirement formulation ignores the fact that the change of geometry is also linked to the change of pressure drag force as well. Simple collocation of skin friction drag and pressure drag in multi-objective problem of \( \min C_p, C_f \) is also not a perfect solution as well in that it ignores the factor of impact of each objective. However, it can be used as a good measurement of optimization progress, and it will be used in the first case in this paper.

This paper proposes to formulate design requirement of NLF optimization from the perspective of objective space via data manipulation approach. For an aerodynamic design with given algorithm and geometric representation method (i.e., a given design space), the objective space is the projection from design space via aerodynamics or CFD (Computational Fluid Dynamics) models. This paper believes that objective space contains the intrinsic information of the ‘optimization potentials’ of samples in terms of aerodynamic performance: thus, it can be injected in the design requirement formulation as an aide to capture the nature of the problem at hand. In the two proof-of-concept studies in this paper, the improved formulation of concerned design requirement injected with objective space information is compared with traditional design requirement formulation with evenly distributed weight distribution in terms of design efficiency and comprehensive aerodynamic performance, respectively.

The solution proposed in this paper is done via Principal Component Analysis (PCA). PCA is used to strengthen preprocessing of data to improve manipulation efficiency \([21,22]\), dimension reduction \([23]\), and in the sample selection of initialization \([24]\), etc. The methods for data manipulation have been utilized in aerodynamic designs, but mostly in design space than in objective space. In fact, apart from extending the subjects of aerodynamic design to complex aircraft components \([25–27]\) and strengthening the link between geometry and aerodynamic performance \([21,28–30]\), aerodynamic designers deployed data preparation in design space for preprocessing, e.g., dimensional reduction by parameter filtering according to physical intuitiveness (e.g., cutting out geometry parameters with less impact on ultimate aerodynamic design goal \([31]\)) and feature extracting from flowfield for economical use of data \([32,33]\), etc. It is noted that the intention of PCA in this paper is not simply for variable reduction but for extracting the main modes from objective space considered to be the essential information that may help establish a more reasonable formulation of design requirement in aerodynamic designs.
The rest of the paper is organized as follows: Section 2 studies the CFD solver used in the paper, which is validated by wind-tunnel experiment. First, grid sampling is applied to prepare for the creation of objective space, which offers data to be processed by PCA so that a more problem-oriented formulation of design requirement can be obtained. The eigenvectors of the result are projected onto the principal axis generated via PCA and thus forms the formulation of design requirement. Secondly, the approach of objective space analysis is applied to an NLF optimization of transonic airfoil under fixed lift via Differential Evolution algorithm, the result of which shows that the information of objective space help improve the optimization route in aerodynamic design (cf. Section 3). Thirdly, a reformulated design requirement is also applied to an NLF aero engine compressor blade design, showing that a third aerodynamic performance objective’s concern can be accessorially reflected in a two-objective problem if an objective space of three aerodynamic performances (i.e., the pressure drag and laminar length related drag as well as pressure loss in the case of blade design) is created and influence design requirement formulation (cf. Section 4).

2. CFD Solver and Its Validation

A wind-tunnel experiment was conducted in the continuous transonic wind tunnel. Its sectional area is \(0.6 \times 0.6\) m\(^2\). The turbulence intensity is less than 0.2% when freestream Mach number is less than 0.8 under 1 atm pressure. The Mach number in the experiment is 0.785 with Reynold number at \(3.4 \times 10^6\) (viscosity \(1.7894 \times 10^{-5}\)). A pair of transonic airfoils (#1 and #2, hereinafter) have been tested whose result is compared with CFD result in terms of:

- the absolute laminar/turbulence transition position on the upper surfaces of the two airfoils, respectively
- pressure distribution on the surface of airfoil #1

The turbulence intensity is less than 0.2% and turbulent viscosity ratio 10%. The airfoils are of 0.2 m long. The photos of the experimental set-up are shown in Figure 1. The laminar transition is detected via infrared camera based on the fact that the convective heat exchange coefficients of laminar is different from that of turbulence [34]. Through infrared photos, the laminar flow region appears white and the turbulent flow region appears grey, which forms the judgement of laminar transition position. More detailed description of the experimental set-up and post-processing method can be found in another work of the authors accomplished in the same wind tunnel [30]; the infrared camera was also used by the authors for laminar/turbulence detection in a validation of three-dimensional nacelle design without thrust [8].

Figure 1. Photos of airfoil experimental set-up.
2.1. Mesh

The flow solver is ANSYS Fluent, where the Transition SST (Shear Stress Transport) model is picked as the turbulence transition model. The no-slip conditions are applied on the surface, and Neumann boundary conditions on the pressure of farfield (99,614 Pa). The air is treated as ideal gas with temperature at 310 K at farfield boundary. A C-Mesh is used in this paper, with radius of approximately 30 times chord length. A higher mesh dense is placed in boundary-layer and near-wake regions where dictates high orthogonality and \( y^+ \approx 1.0 \) \([30,35] \). The mesh topology is shown in Figure 2. The mesh independence is finished on three grid formats, as shown in Table 1. The Adopted mesh is shown to balance the grid number and CFD accuracy; therefore it is chosen in the paper.

![Figure 2. An illustration of the adopted mesh topology.](image)

**Table 1.** Comparison of mesh parameters and their CFD simulation.

| Mesh Type    | Lift Coefficient % | Drag Coefficient % | Laminar Transition Position % |
|--------------|--------------------|--------------------|-------------------------------|
| Coarse (201 x 101) | 0.10278            | 0.007801           | 20.1                          |
| Adopted (501 x 150) | 0.11907            | 0.007427           | 23.0                          |
| Fine (701 x 160)  | 0.11906            | 0.007427           | 22.9                          |

2.2. Aerodynamic Performance Prediction vs. Experiment

There are some criteria for judging the laminar length over airfoil (e.g., distribution of intermittency, turbulence intensity over airfoil surface and near wall region) \([30] \). This paper selects to observe the distribution of intermittency over the surface above the airfoil wall by \( 5 \times 10^{-5} \) m (approximately 10 grids away from the wall) from the leading edge: the first position that bears intermittency factor bigger than 0.5 is referred to as the transition point which determines the laminar length. This operation is selected also for the convenience in editing automatic script applied later in optimization. Figure 3 shows that the simulated laminar transition position over the two airfoils; quantitative comparison of laminar transition position is displayed in Table 2, which shows that the two aerodynamic performances that are the concerns of this paper can be obtained with confidence. Figure 3 also shows that the pressure distribution of the first airfoil are simulated well compared with the wind-tunnel experiment. For determining aerodynamic performance under fixed lift force value, 18 CPU (Central Processing Unit)’s are used with time cost at approximately 40 min for an airfoil sample.
Figure 3. Comparison of laminar transition position and pressure distribution between CFD (Computational Fluid Dynamics) simulation and EXP (experiment) result.

Table 2. Comparison of laminar length: wind-tunnel experiment vs. CFD simulation.

|                        | Transition Position Detection | Transition Position Improvement |
|------------------------|------------------------------|--------------------------------|
| Experiment             | 60.1 and 63.5                | 3.4                            |
| Computational Fluid Dynamics | 60.14 and 63.74              | 3.6                            |

3. Proof-of-Concept Study 1: A Differential Evolution Optimization with Conventional and Improved Design Requirement Formulation

3.1. NLF Optimization of RAE2822 Airfoil in Transonic Flow under Fixed Lift

The AIAA Aerodynamic Design Optimization Discussion Group (ADODG) has defined benchmark optimization problem of airfoil RAE2822 in viscous (Reynolds number is 6.5 million), transonic flow (freestream Mach number is 0.734), subject to lift (0.824), pitching moment (not smaller than $-0.092$), and area (decrement is not allowed) constraints [36]. The problem has been studied by many scholars, which has significance with respect to engineering applications [37–40].

This paper intends to introduce NLF concerns (i.e., skin friction drag) to the original problem. For better illustration of the effect of laminar design, the fixed value of lift force is therefore modified form 0.824 to 0.5 to prevent the angle of attack being too high which goes against the favorable direction of laminar design. In recognition of the above, the authors formulate an optimization problem as follows:

$$\text{minimize } w_1 \cdot C_p + w_2 \cdot C_f$$

w.r.t. $y, \alpha$

s.t. $C_l = 0.500$

$A \geq A_{\text{baseline}}$

where $C_l$, $C_p$, $C_f$ are the lift, pressure drag and skin friction drag applied on the airfoil samples; $\alpha$ is the angle of attack; $y$ is the vertical coordinates of airfoil; $A$ is the area occupied by the airfoil samples while $A_{\text{baseline}}$ is the area occupied by RAE2822. The farfield flow comes at Mach number 0.734. The lift coefficient is fixed (hence the angle of attack is determined by dichotomy method in this paper).

$w_1$ and $w_2$ are the weights assigned to problem. This paper will set up two optimizations with different design requirement. In the basic case, the weights will be assigned as 0.5 and 0.5 for $w_1$
and \( w_2 \); in the case with modified design requirement formulation, the weights will be calculated via analysis in the following section with consideration of objective space information.

3.2. Design Variables in Geometry Description

The parametric geometric representation approach Class-Shape Transformation (CST) conquered many aerodynamic designs with its smoothness and convenience in shape description [31,41–43]. Conventional CST (Equation (5)) imposes shape functions (Equation (3)) on specific class function (Equation (4)) with accumulated Bernstein polynomials assuring shape smoothness [44–46], where \( n \) is the scale of the method; \( u \) is the non-dimensional position, \( u_{TE} \) is the trailing edge thickness correction (equal to zero in a sharp edge case); \( A_i \) are the parameters. For ordinary airfoil surfaces, two parameters: \( N_1 \) and \( N_2 \) take the value of 1 and 0.5, respectively.

\[
\text{Shape}(u) = CF(u) \cdot \left( \sum_{i=0}^{n} A_i \cdot BP^n_i(u) \right) + u_{TE}u \tag{3}
\]

\[
\text{Class}(u, N_1, N_2) = u^{N_1}(1 - u)^{N_2} \tag{4}
\]

\[
\text{Class}_\text{Shape}(u) = (u^{N_1}(1 - u)^{N_2}) \cdot \left( \sum_{i=0}^{n} A_i \frac{n!}{i!(n-i)!} u^i(1 - u)^{n-i} \right) + u_{TE}u \tag{5}
\]

The difference in form of \( L_2 \)-norm of vertical coordinates between Class\_Shape\( (u) \) function and original data can be minimized until the parameters of \( A_i \) are obtained via least square method:

\[
\sqrt{\sum_{i} (C(u_i)_{\text{original}} - C(u_i)_{\text{fitted}})^2} \tag{6}
\]

A CST method of 11 scale is applied to get airfoil RAE2822’s shape variation, the Bernstein polynomial decomposition of which is shown in Figure 4. The x-y plot represents coordinates of different Bernstein polynomials. For given x position, the summation of local values of Bernstein polynomials is the actual y coordinate of airfoil. The difference between the original RAE2822 airfoil surface and fitted one via CST is shown in Figure 5, which meets the demand of accuracy of model tolerance of \( 1 \times 10^{-7} \) m recommended by [35].

The design space is created based on the variation of the first 4 CST parameters representing top, front region of the airfoil RAE2822. Each valid airfoil sample needs to meet detailed geometric constraints: not only the parameter is variated within bounding limits, the generated shapes have to be verified later in terms of curvature distribution, area check, and monotonicity check (cf. Figure 6):

- The curvature \( (\kappa \equiv \frac{y''}{(1+y'^2)^{3/2}}) \) distribution has to be within \( \pm 0.1 \) based on that of the original airfoil surface.
- The monotonicity must be consistent with that of the original airfoil surface to avoid twisted shape which is definitely harmful to the approaching towards the target via Differential Evolution algorithm.
- And constraint of maximum area of 125% of the initial area is enforced to avoid shape with too high drag before CFD calculation.

These constraints affect the data domain and its distribution in design space as well as objective space, which has impact on the final formulation of design requirement.
Principal Component Analysis as Data Manipulator

PCA is introduced to reduce the number of variables or design space dimensions on the premise of maintaining/capturing intrinsic information (e.g., the dominant physics of a system) in the original empirical data, when the problem requires expensive high-fidelity simulations [47–49]. PCA is also referred to as Karhunen–Loève transformation, Proper Orthogonal Decomposition, singular
systematic analysis, singular value decomposition under different circumstances, which has other variations in methodologies, e.g., kernel PCA, probabilistic nonlinear PCA/Gaussian process latent variable models, etc. [50]. The method is proposed against traditional variable screening that reduce dimensionality by simply removing variables; the latter often fails because there may be correlations between each variable in high-dimensional data [51]. PCA can solve that problem by providing new linear combinations of the original variables representing orthogonal eigenvectors of principal component axes in space, each of which with different variances [23,24]. The first principal component is associated with the largest variance, and the following ones are perpendicular to the previous ones [52].

In some aerodynamic designs, the application of PCA is mainly on the data processing, e.g., classifier. Some examples can be seen in reduced-order models for simulations balancing computation cost and flowfield accuracy in numerous fields [22,53–56].

The data-oriented technique has been studied to accelerate the process of capturing the dominant modes of a system [57]. This type of PCA applications is focused on the idea that if samples of similar features are clustered, then the methods can be applied to them more effectively.

Beyond that, PCA is not often applied to extract useful information linked with objective space. One similar example is found in [58] where PCA is used to guide the application of evolution operators in Genetic Algorithm where the front of non-dominated solutions is divided into sub-fronts. Its information was obtained from the calculated aerodynamic performance distribution; then, the result was fed back to the design space to further manipulate the samples. This guidance from objective space is no longer simply an aerodynamic performance progress of individual samples as in many traditional genetic algorithm; on the contrary, the extracted global information of objective space alleviates the dependence of randomness in the design space, which brings benefits to their study.

In such spirit, this paper is also focused on studying objective space information by PCA via the formulation of design requirement, which constitutes the novelty of this paper.

3.3. Information Extracted from Objective Space

If there is some information that represents some features to be deployed in optimization process, then it should produce result that is irrelevant of sampling strategy. With the approach of enumeration, a grid strategy is used for sampling of different scales. An index \( n \) indicates the evenly distribution of sampling seeds in every CST parameter, or sampling scale, as is shown in Table 3. The total number of samples easily exceeds reasonable calculation limitation if the value of \( n \) is large enough because of iteration through all the parameters. This traditional sampling strategy is for revealing the intrinsic nature hidden in objective space. The huge amount of calculation corresponding to different values of \( n \) is not necessary in the real practice: if the relevant data is enough, then the eigenvectors can be estimated within approximation.

| \( n \) Value | Normalized Location of Sample Seeds for Each Parameter |
|--------------|-------------------------------------------------------|
| 2            | 0.250, 0.750                                         |
| 3            | 0.166, 0.500, 0.833                                   |
| 4            | 0.125, 0.375, 0.625, 0.875                           |
| 5            | 0.100, 0.300, 0.500, 0.700, 0.900                     |

For airfoil samples corresponding to given \( n \) value, their aerodynamic performance of \( n \) airfoils obtained through CFD constitute a database represented by array (7), where the two columns are skin friction drag and pressure drag force, respectively.
\[ \mathbf{A} = \begin{bmatrix} a_{1,1} & a_{1,2} \\ \vdots & \vdots \\ a_{n,1} & a_{n,2} \end{bmatrix} \] (7)

The data from objective space systematically forms the covariance matrix \( \mathbf{P} \) in Equation (8). The spectral decomposition is shown in Equation (9) where \( \mathbf{A} \) represents diagonal matrix of eigenvalues and \( \mathbf{U} \) represents agglomeration of eigenvectors.

\[
\mathbf{P}_{n \times n} = \frac{1}{n} \mathbf{A} \mathbf{A}^T \tag{8}
\]

\[
\mathbf{P}_{n \times n} = \mathbf{U} \Lambda \mathbf{U}^T \tag{9}
\]

The objective space and eigenvectors are obtained for each \( n \) in Table 3, the result of which is shown in Figure 7 where green, solid lines represent first principal component and the red, dashed lines represent second principal component. Since the design requirement in this case involves two aerodynamic performances, only two components are required. The boundary of the objective space can be clearly seen, which is caused by the various geometrical constraint posed by the problem. Similar to the ‘convergence’ of mesh independence check, the directions of eigenvectors are also ‘converged’ to a stable direction with the increasing of \( n \) (i.e., the directions of eigenvectors for objective space for \( n \) equals 4 and 5 are close), which shows the intrinsic information in the objective space determined by the problem description can be captured. The paper thinks that the understanding of the objective space and eigenvectors is directly associated with a more reasonable and problem-oriented formulation of design requirement. With the sampling, a comprehensive pattern that the data is arranged in objective space can be perceived, where the eigenvectors can be a quantitative description of it.

\[(a) \ n = 2 \quad (b) \ n = 3 \quad (c) \ n = 4 \quad (d) \ n = 5\]

**Figure 7.** Global view of objective space for each \( n \) and their eigenvectors.

Therefore, if the formulation of design requirement adopts such information, the optimization can be better guided by leading every sample to move in the direction that is parallel to the best direction.
decided by the nature of the problem. Also, the eigenvectors has engineering meaning in that a better optimization criterion can be proposed in industrial aerodynamic design process as a balanced solution to the ultimate effect of total drag reduction and the perspective of emphasizing part of drag reduction via methodological guidance.

In the next section, a newly formulated design requirement is proposed to compete with conventional design requirement in the Differential Evolution NLF optimization to the transonic airfoil RAE2822. A more reasonable, problem-oriented weight is determined by the PCA analysis in this section, where $w_1$ is 0.878 and $w_2$ is 0.122, while the conventional design requirement is $w_1 0.5$ and $w_2 0.5$.

3.4. Evolutionary Optimization and Its Significance

The problem description and data manipulation in the previous sections are the preparation for the Differential Evolution optimization in this section. As a variant of an evolutionary algorithm, Differential Evolution (DE) iteratively improves a candidate solution based on floating-point value representation rather than the binary strings [59]. It has been deployed in many aerodynamic designs [8,60,61].

The flow chart in Figure 8 is a possible application of extracting information from objective space in current evolutionary types of optimization. A better formulation of design requirement encourages initialized samples to move in the directions that are intrinsic to the nature of the problem.

![Flow chart](image)

**Figure 8.** An improved optimization process with the new formulation of design requirement applied in the optimization.

3.5. Realization of Algorithm

Since the purpose of this paper is to study the impact of objective formulation on the result of aerodynamic design, the realization of optimization algorithm in different cases must be kept in similar configuration. The algorithm starts with a randomly populated initial generation of $NP D$-dimensional parameter vectors representing candidates with considerations of aforementioned constraints in the design and objective space, where $NP = 15$ is the number of parents in a population and $D = 4$ is the number of parameters in this paper. After initializing the population, each target vector of candidate in generation undergoes mutation and crossover operation before producing a sample to be tested. The perturbations on the parameters are the factors to be multiplied by the original values (instead of the values to be treated as profile description parameters) thus assuring the uniformity in random seeding. Only when the newly generated candidate is within the various constraint and has better aerodynamic performance measured by conventional/improved optimization criterion
(i.e., design requirement) than its parent, then the old candidate is updated with the new one. In this paper, surrogate models are not used to avoid inaccuracy in description of design and objective space.

The two algorithms with different design requirement are realized in two cases. As is shown in Algorithm 1, \( x \) represents candidate, \( v \) represents guided possible marching step, \( r_i, i = 1, 2, 3 \) represent random candidate in current generation; candidates, constant \( F \) represents scaling factor in amplifying differential steps.

Algorithm 1: Differential Evolution in this paper.

**Input:** Relevant CST parameters \( A_0, A_1, A_2, A_3 \)

**Output:** History of candidate’s evolution in aerodynamic performance

1. for \( g = 0 : \) Generations do
2.   for \( i = 0 : \) Candidates do
3.     Mutation: \( \vec{v}_{i,g} \leftarrow x_{1,g}^* + F \times (x_{2,g}^* - x_{3,g}^*) \);
4.     Crossover: elementwise: abandon vs. adopt varied parameters (probability of 3:7);
5.     Performance check (where the comparison takes place);
6.   end
7. end
8. Return ();

The randomness in the evolutionary marching of both cases is minimized to illustrate the comparison of the effect of design requirement, which is the core of this paper. Therefore, the mutation ratio and crossover operation is kept the same for the same serial number of candidates in the same generation in both cases; the selection of samples for obtaining differential vector is the same in terms of the serial number of samples (e.g., Nos. 3, 7, 11 are picked for generating differential vector in both cases, although they represent totally different locations in the objective space.).

3.6. Result Analysis

The optimization process took about 120 h with each CFD simulation occupying approximately 45 min, considering the fact that the evolution has to be processed in non-parallel way for each candidate of each generation. When the iteration with improved design requirement comes to generation no. 79, there has been no updates of candidates in objective space for the last 10 generations. Therefore, a ‘convergence’ of Differential Evolution is considered to be obtained. The result in Figure 9 shows that the improvement in design requirement formulation takes effect in enhancing the optimization by applying the intrinsic nature of the problem via objective space analysis. With the improved design requirement formulation, the Pareto front can be obtained faster than conventional configuration. The cost for extracting information from objective space brings rewards in the final optimization process.

The Pareto front represented as thick, purple dashed line shows that due to the various constraint in the problem description, most of the candidates are in the region relatively lower than the corner of the line. Therefore, most of the optimization is done mainly in the aspect of pressure drag reduction (e.g., candidate no. 2), while some in the aspect of laminar length increasing (e.g., candidate no. 12). The candidate no. 5 is selected as the optimized airfoil to be compared with original airfoil RAE2822 because it is located in the corner of the obtained Pareto front (therefore possessing good performance in two aspects). The comparison of aerodynamic performance is displayed in Figure 10 in terms of pressure coefficient contour and Figure 11 in terms of two-dimensional comparison of pressure distribution and laminar length in form of skin friction drag coefficient, respectively. From Figure 10, a shock with less strength and more rearward position is shown in optimized case that gives explanation of the better pressure drag performance. The quantitative improvement in laminar-related performance is also shown in Figure 11.
Figure 9. The comparison of the two optimization processes when the case of improved design requirement formulation has converged to the Pareto front.

Figure 10. The pressure coefficient contour of the initial and optimized airfoil (i.e., candidate no. 5 in Figure 9).

Figure 11. The two-dimensional pressure distribution and skin friction drag coefficient plot of the initial and optimized airfoil (i.e., candidate no. 5 in Figure 9).
4. Proof-of-Concept Study 2: Improvement of Accessory Aerodynamic Performance via Formulation of Design Requirement with Information of Objective Space

4.1. NLF Optimization Adopting Information of Objective Space of Laminar-Related Performance, Pressure Drag, and Pressure Loss

In this case of aero engine blade design, the effect of NLF optimization can not only reduce skin friction drag by laminar design, but can also enhance comprehensive performance in case of aero engine blade design with a preview from three-aerodynamic-performance objective space information. The idea of natural laminar flow blade is feasible by modifying the shape even in the coming flow with high temperature and high pressure \[62\]. The shape modification in the leading-edge region is responsible for the alleviation of separation of peak pressure profile \[63\], and the curve tendency in the mid-chord also needs reconsideration for maintaining laminar flow.

The CFD simulation in this case is to analyze the flow through the tunnel established between blades, where the aerodynamic performance and its optimization is considered (cf. Figure 12). On the boundary of pressure inlet (total pressure of 122,800 Pa) and outlet (static pressure of 101,325 Pa), the Mach number of coming flow is kept as 0.89. The total temperature is 293 K, and the turbulent intensity is 0.2%. The solver is Fluent by using its SST four-equation model. For each blade, a mesh is created where the farfield is approximately 4.5 chord length away. The mesh is of 1301 × 157 grids and first boundary layer grid height \(1.46 \times 10^{-6}\) m, which is shown in Figure 13. The mesh configuration in this case also follows the requirement in Section 2.

![Figure 12. Illustration of flow between blades.](image)

![Figure 13. The mesh for aero engine compressor blade CFD simulation.](image)

4.2. PCA Analysis to Three-Aerodynamic-Performance Objective Space

In this paper, Hicks–Henne parameterization (bump function) is used to modify the profiles of blades by adding wavy perturbation that guarantees the smoothness of products in form of:

\[ y(x) = y_{\text{init}}(x) + \sum_{i=1}^{N} a_i b_i(x) \]
with the supplementary geometry represented as:

\[ b_i(x) = \sin^\omega(\pi x^{\frac{\alpha}{\ln\left(\ln x_i\right)}}) \]

where \( \omega \) is the width of bump, \( \alpha \) controls its amplitude, \( x_i \) controls its position.

To create design space as preparation for objective space, three bump functions are added to the original profile with locations, perturbation base value, and perturbation variance shown in Table 4, where the numeric values are all normalized. An example of application of bump functions on original profile is shown in Figure 14.

**Table 4.** Perturbation of Hicks–Henne bump functions.

|               | Leeward: Chordwise Positions | Windward: Chordwise Positions |
|---------------|------------------------------|------------------------------|
| Base Value    | 0.05 0.4 0.7                | 0.05 0.4 0.7                 |
| Variance      | 0.0577 0.0577 0.0577        | 0.0577 0.0577 0.0577         |

**Figure 14.** An example of application of bump functions on original profile.

Similar to previous transonic NLF airfoil design, samples of different sampling echelons are gathered and analyzed via PCA. Three aerodynamic performances are listed including pressure drag, skin friction drag, and pressure loss for the blades; therefore, three components of PCA analysis are required to be calculated, as is shown in Figure 15.

The eigenvectors are converged, leading to a modified formulation of aerodynamic design as \( 0.59966 \cdot C_p + 0.40034 \cdot C_f \), which has the information of a third aerodynamic performance (pressure loss).

The idea is to use the new formulation in the following optimization case so that not only the concerned the objective of pressure drag and laminar-related performance will be obtained, but also will bring supplementary benefit for the pressure loss performance.
4.3. Result Analysis

With DE being applied as is shown in Table 5, the result from conventional design formulation \((0.5 \cdot C_P + 0.5 \cdot C_f)\) (hereafter ‘Opt Blade 1’) is compared with that from new design formulation \((0.59966 \cdot C_P + 0.40034 \cdot C_f)\) (hereafter ‘Opt Blade 2’). The comparison of their shapes is shown in Figure 16, where the pressure sides of Opt blade 1 is lower than that of Opt blade 2 while the curvatures of the suction sides of the two optimized results are different. The shapes of the two optimized results as well as their apparent difference from the shape of the original blade profile illustrates that the main direction of drag reduction is reflected in both optimizations. The comparisons in terms of skin friction drag coefficient (i.e., laminar-related performance) and pressure distribution of the two optimized blade profiles are shown in Figures 17 and 18.

Since the information of a third aerodynamic performance (i.e., pressure loss) is introduced in the new formulation, a better pressure loss performance is observed in the result of Opt blade 2 than Opt blade 1, as is shown in Table 6. This illustrates that the impact of objective space analysis to the formulation of design requirement can be transmitted to a two-objective problem with consideration of third aerodynamic performance.

| Table 5. Configuration of Differential Evolution algorithm in the blade optimization case. |
|----------------------------------|---------------------------------------------------|
| Population type | double vector |
| Population size | 50 |
| Mutation | constraint dependent |
| Crossover | augmented Lagrangian |
Figure 16. The shape comparison of original blade profiles and two optimized blade profiles with different design objective formulations.

Figure 17. The skin friction drag coefficient and pressure distribution comparison of the two optimized blade profiles with different design objective formulations.

Table 6. Optimization results comparison of the two optimized blades.

| Blade Profiles | Laminar Length: Suction Side | Laminar Length: Suction Side | Pressure Drag   | Pressure Loss |
|----------------|-------------------------------|-------------------------------|-----------------|---------------|
| Opt blade 1    | 0.0088                        | 0.0144                        | 3127.8409       | 0.0156        |
| Opt blade 2    | 0.0092                        | 0.0143                        | 3136.0074       | 0.0154        |

Figure 18. Global view of pressure contour in the domains of the two optimized blades.
5. Conclusions

The paper illustrates an approach that uses the importance in extracting information from objective space and applying it to improve optimization process via the formulation of design requirement. NLF optimization often meets with challenge of the treatment to more than one objective in terms of formulation of design requirement, which has been rarely done in a problem-oriented way by many studies. The study of new weight combination has its direct significance in NLF-related engineering problems. It is equivalently the problem of distributing weight among all the concerned aerodynamic performances. The analysis is mainly on the comparison between improved weight distribution and traditional weight distribution (i.e., 50-50 format of design requirement).

The method proposed by this paper uses PCA as the data manipulator that verifies the eigenvectors of objective space that is irrelevant to sampling strategy and has significance in understanding the nature of the problem at hand. Application of this information is finished in two cases of the paper:

proof-of-concept study 1 analyzes the objective space of skin friction and pressure drag force and compared the final effect brought by conventional and improved design requirement in terms of Pareto front formation. proof-of-concept study 2 analyzes the objective space of skin friction, pressure drag, and a third aerodynamic performance of pressure loss. The case illustrates that the introduction of a third aerodynamic performance into the creation of objective space can guide the formulation of two-objective optimization design requirement and get accessory improvement in the comprehensive aerodynamic performance.

In conclusion, the reward for the extraction and use of objective space information not only influences the acceleration of Pareto front marching (as in proof-of-concept study (1)), but also provides a formulation of design requirement that reflects the nature of the problem at hand (as in proof-of-concept study (2)). The reconsideration of design requirement formulation in objective space analysis in this paper has potentials to be extended to other multi-objective optimizations. The significance of the paper is that the information of objective space can be injected in the optimization and its result will inspire the discovery of certain pattern hidden behind the nature of the problem.

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