Damage Identification in Beam Structure using Spatial Continuous Wavelet Transform

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Abstract. In this paper the applicability of spatial continuous wavelet transform (CWT) technique for damage identification in the beam structure is analyzed by application of different types of wavelet functions and scaling factors. The proposed method uses exclusively mode shape data from the damaged structure. To examine limitations of the method and to ascertain its sensitivity to noisy experimental data, several sets of simulated data are analyzed. Simulated test cases include numerical mode shapes corrupted by different levels of random noise as well as mode shapes with different number of measurement points used for wavelet transform. A broad comparison of ability of different wavelet functions to detect and locate damage in beam structure is given. Effectiveness and robustness of the proposed algorithms are demonstrated experimentally on two aluminum beams containing single mill-cut damage. The modal frequencies and the corresponding mode shapes are obtained via finite element models for numerical simulations and by using a scanning laser vibrometer with PZT actuator as vibration excitation source for the experimental study.

1. Introduction

Modern engineering structures, including but not limited to civil buildings, bridges, dams, automotive and aerospace facilities have to maintain their integrity and functionality under severe environmental conditions. Their failure often leads to tragic consequences and therefore structures have to undergo regular inspections. Rigorous inspection of structures on a regular basis is usually a time-consuming and costly procedure; therefore non-destructive structural health monitoring (SHM) methods have become an important research area in civil, mechanical and aerospace engineering communities.

In the recent years, various vibration-based damage detection methods have been proposed for SHM. Many of them use various transformations of measured dynamic response of a structure. Dynamic responses, which in many cases can be easily obtained, offer damage information such as the...
location and severity. These methods are based on the fact that dynamic characteristics, i.e., natural frequencies, mode shapes and modal damping, are directly related to the stiffness of the structure. Therefore, a change in natural frequencies or a change in mode shapes will indicate a loss of stiffness. Extensive literature reviews of the state-of-the-art in the methods for detecting, localizing, and characterizing damage by examining changes in the dynamic response of a structure can be found in [1,2]. Many studies [3-9] have shown that mode shapes and corresponding mode shape transformations are highly sensitive to damage and can be used for detecting and quantifying damage. One of the most promising techniques of processing dynamic response for damage identification is a wavelet transform.

Wavelet theory has been developed since the beginning of the 20th century by Haar, but it did not result in a unified theory. A current wavelet analysis as we-know-it was introduced by Grossman and Morlet in 1984. Later, in 1989 Mallat founded wavelet theory for signal decomposition [10]. Wavelet transform technique originated in the 1990’s and was mainly used for signal singularity detection [10], signal denoising, image compression [11]. Later in the 1990’s wavelet transform technique was used on vibrational data for damage detection for the first time by Newland to study the effects of building vibrations, caused by underground trains and road traffic [10, 12, 13].

Many researchers have been studying damage detection in beams using spatial wavelet transform technique [12-18]. Studies in [12] showed that local perturbations in beam deflection profile are induced by crack and amplified through use of wavelet transform. Researchers in [18] suggested that wavelet transform is also capable of detecting two cracks in beams. Interesting studies in [13] proposed to classify wavelet-based methods for damage detection into 3 groups: 1) variation of wavelet coefficients [14, 15], 2) local perturbation of wavelet coefficients in space domain, 3) reflected wave caused by local damage. The first of these methods is used to find the existence and severity of damage and is based on the fact that variation of wavelet transform coefficients is caused by change of modal parameters of the structure. The second method is used to localize the damage in structures by detection of irregularity of wavelet transform coefficients near the damage site. The third method is used to measure the severity as well as location of the damage. It is based on the analysis of the wave reflected from local damage in the structure. Work similar to the third method was done in [19]. Authors suggested wavelet transform technique for signal processing to distinguish between the changes in wave propagation, caused by local discontinuities and cracks in beams and rods.

In this paper, the applicability of spatial continuous wavelet transform (CWT) technique for damage identification in the beam structure is analyzed by application of different types of wavelet functions and scaling factors. To examine limitations of the method and to ascertain the sensitivity of the method to noisy experimental data, several sets of numerically simulated data were tested. A broad comparison of ability of different wavelet functions to locate damage in beam structure is given. Effectiveness and robustness of the proposed algorithms are demonstrated experimentally on two aluminum beams containing single mill-cut damage.

2. Continuous wavelet transform

Wavelets are special functions, which are generally used to analyze different signals in time domain. Wavelet transform is simply a mathematical method to transform the original signal into a different domain where additional data analysis becomes possible, therefore damage-affected signal portion is revealed. Spatial analysis is obtained by simply replacing time with a spatial coordinate, say, x. This gives rise to signal f(x) [12]. If ψ(x) is a wavelet function, also called mother wavelet, which, in

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general, is complex-valued, then the set of wavelet family functions $\psi_{s,t}(x)$ is created by shifting (translating) (parameter $t$) and scaling (parameter $s$) the $\psi(x)$ [14, 20].

$$\psi_{s,t}(x) = |s|^{-1/2} \psi\left((x-t)s^{-1}\right)$$

(1)

Parameter $s$ is essential in noise reduction [16], and it is a real number. If $0 < s < 1$, the function is expanded, if $s > 1$, it is compressed [21]. In time domain $s$ represents the signal’s frequency content and $t$ – location of wavelet in time.

### 2.1. Benefits with respect to Fourier Transform

In contrast to Fourier transform (FT), wavelet function has time locality as well as frequency locality [20]. If $N$ is the data to be transformed, the required number of operations for CWT with orthogonal wavelets mentioned in [20] is $O(N)$ instead of $O(N \cdot \ln(N))$ for fast FT, which leads to much faster data transformation.

### 2.2. Wavelet properties

**Compact support** – wavelet function’s average value is 0 [14, 20] and it has a finite length [14], thus

$$\int_{-\infty}^{\infty} \psi_{s,t}(x) dx = 0$$

(2)

therefore wavelet function has at least some oscillations [20]. Another wavelet property is continuity. Continuous wavelet transform (CWT) is given by [14, 20]

$$W_{s,t} = \int_{-\infty}^{\infty} f(x) |s|^{-1/2} \psi^*(x-t)s^{-1} dx = \int_{-\infty}^{\infty} f(x) \psi_{s,t}^*(x) dx$$

(3)

where asterisk denotes complex conjugation. We will deal only with real-valued wavelets, which are used in case of low wavelength signals [21]. Using this formula, CWT coefficients are calculated, which are extremely sensitive to any discontinuities and singularities of signal $f(x)$, therefore damage due to a sudden loss of stiffness can be detected in mode shapes that have large amplitude wavelet coefficients. It is a damaged location. This perturbation of wavelet coefficients due to this damage is clearer in the finest scales of the CWT [10].

### 3. Damage detection algorithm

CWT for mode shapes is depicted as follows (recall equation (3))

$$D_{i}^{n} = W_{i,s,t}^{n} = \int_{L} w_{i}^{n} \times \psi_{s,t}^{*}(x) dx$$

(4)

where $L$ is the length of the beam, $w_{i}^{n}$ is transverse displacement of the structure, $n$ is a mode number, $i$ is number of grid point in $x$ direction, respectively and $\psi_{s,t}(x)$ is a particular wavelet function, which was chosen to be Daubechies, Gaussian and Symlet of orders 2, 4 and 8 (Db2, Db4, Db8, Gs2, Gs4, Gs8), Coiflet of orders 2 and 4 (Cf2, Cf4), Morlet (Morl), Meyer, Haar, and Mexican hat (Mex). $D_{i}^{n}$ is a damage index for each mode.

In practice, experimentally measured mode shapes are inevitably corrupted by measurement noise causing local perturbations into the mode shape, which can lead to false peaks in damage index...
profiles. These peaks could be mistakenly interpreted as damage or they could mask the peaks induced by real damage and lead to false or missed detection of damage. To overcome this problem, it is proposed to summarize results for all modes. The summarized damage index then is defined as the average summation of damage indices for all modes $N$, normalized with respect to the largest value of each mode

$$DI_i = \frac{1}{N} \sum_{n=1}^{N} \frac{DI_i^n}{DI_{i,max}}$$

(6)

4. Numerical simulation

To validate the effectiveness of the proposed algorithm, numerical modal analysis based on finite element (FE) method was conducted by using the commercial FE software ANSYS. An aluminum beam (Beam 1) with single mill-cut damage is examined. The geometrical configuration of the beam is shown in figure 5. Mill-cut damage with a depth of 2 mm and width of 50 mm is introduced at a distance of 750 mm from one edge of the beam. FE model of the beam consists of 2D beam elements. Each node has 3 degrees of freedom, namely translations along the X and Y axes and rotation along the Z axis. Beam 1 was constructed by means of 125 equal length elements ($i = 126$ nodes). The elastic material properties are taken as follows: Young’s modulus $E = 69$ GPa, Poisson’s ratio $\nu = 0.31$, and the mass density $\rho = 2708$ kg/m$^3$. For the healthy beam, a constant flexural stiffness $EI$ is assumed for all elements, while the damaged beam is modelled by reducing the flexural stiffness of the selected elements. Reduction of flexural stiffness is achieved by decreasing the thickness of elements in the damaged region of the beam. The modal frequencies and corresponding mode shapes for the first 15 flexural modes were calculated.

In order to compare the sensitivity of the algorithm to noisy experimental data, an artificial white Gaussian noise was added to the numerically simulated mode shapes

$$w'_{ni} = w_{ni} \cdot (1 + \delta \times (2r - 1))$$

(7)

where prime indicates noisy mode shapes, $\delta = 0 \%, 1 \cdot 10^{-3} \%, 1 \cdot 10^{-2} \%, 1 \cdot 10^{-1} \%, 1 \%$, 5 \% is the level of random noise and $r$ are uniformly distributed random values in the range (0, 1).

In order to study situations from real life when it is either not possible or not needed to equip the structure with many sensors, different numbers of data points were examined – simulated data with 149 entries was divided by $p = 1, 2, 5$ and 10. Based on these considerations, the following test cases were studied (table 1):

| Noise level (%) | reduction of used data |
|-----------------|------------------------|
| $\delta = 0$    | Case 1.1. Case 1.2. Case 1.3. Case 1.4. |
| $\delta = 0.001$| Case 2.1. Case 2.2. Case 2.3. Case 2.4. |
| $\delta = 0.01$ | Case 3.1. Case 3.2. Case 3.3. Case 3.4. |
| $\delta = 0.1$  | Case 4.1. Case 4.2. Case 4.3. Case 4.4. |
| $\delta = 1$    | Case 5.1. Case 5.2. Case 5.3. Case 5.4. |
| $\delta = 5$    | Case 6.1. Case 6.2. Case 6.3. Case 6.4. |
4.1. Results and discussion

According to [22], the damage indices, determined for each element are then standardized and a concept of statistical hypothesis testing is applied to classify damaged and healthy elements and to localize damage depending on the pre-defined damage threshold value. In hypothesis testing, the null hypothesis and alternate hypothesis are defined as $H_0$ (element $i$ of the structure is healthy); $H_1$ (element $i$ of the structure is damaged), respectively. In order to test the hypothesis, the summarized damage index, given in equation (6), is standardized

$$
Z_i = \frac{D_i - \mu_{D_i}}{\sigma_{D_i}}
$$

where $\mu_{D_i}$ and $\sigma_{D_i}$ are mean value and standard deviation of damage indices in equation (6), respectively. The decision for the localization of damage is established based on the level of significance used in the hypothesis test, which can be determined from a pre-assigned classification criterion: choose $H_0$ if $Z_i < C_r$ or choose $H_1$ if $Z_i \geq C_r$, where $C_r$ is a threshold value. Typical values of $C_r$, widely used in literature are 1.28, 2 and 3 for 90 %, 95 % and 99 % confidence levels for the presence of damage.

To quantify the reliability of wavelets to identify damage location, a new parameter, called “damage estimate reliability” (DER) is introduced and calculated as follows:

The whole interval along the axis of the beam (x axis) is split into 3 parts: the first part is the one in which $x < 750$ mm (part $a$), the second part is $750$ mm $< x < 800$ mm (part $b$) and the third part is $x > 800$ mm (part $c$).

In each of these parts SDI values from equation (8) of a respective wavelet are summed and divided by the number of data points in this particular interval, giving average amplitude of SDI ($Z_{i}$) in a respective part. DER is equal to average SDI in middle part (the damaged area or part $b$ in our case) divided by average SDI in all parts combined. It is expressed in percent as:

$$
DER_i = \frac{Z_{i}(b)}{Z_{i}(a) + Z_{i}(b) + Z_{i}(c)} \times 100\%
$$

By calculating average DER for each wavelet over all the cases, the best and the worst wavelets were determined (table 2). The best wavelets in each case are shown in boldface, whereas the worst wavelets are underlined.
Table 2. DER calculation results (in %) for Beam 1 with CBC on numerical data for the chosen wavelets at different noise levels and the number of data points used (bold – best, underlined – worst).

| Cj2 | Cj4 | Db2 | Db4 | Db8 | Gs2 | Gs4 | Gs8 | Haar | Meyer | Mxer | Morl | Sym4 | Sym8 |
|-----|-----|-----|-----|-----|-----|-----|-----|------|-------|------|------|------|------|------|
| 1.1 |     |     |     |     |     |     |     |      |       |      |      |      |      |      |
| 1.2 | 84.6| 80.0| 80.9| 83.9| 85.3| 77.5| 86.9| 80.1| **87.7**| 72.3 | 72.2 | 80.9 | 85.6 | 79.4 |
| 1.3 |     |     |     |     |     |     |     |      |       |      |      |      |      |      |
| 2.1 | 84.6| 79.9| 80.9| 83.9| 85.3| 77.5| 86.9| 80.0| **87.7**| 72.3 | 72.2 | 80.8 | 85.6 | 79.3 |
| 2.2 | 85.3| 83.8| 80.1| 79.5| 68.8| 70.2| 82.5| 84.0| 86.5 | 79.3 | 65.0 | 83.5 | **86.9**| 84.1 |
| 2.3 | 77.6| 76.6| 76.7| 30.2| 41.8| 83.3| 81.3| 77.0| 54.5 | 74.0 | 81.7 | 75.7 | 79.0 | 77.0 |
| 2.4 | 69.4| 67.8| 59.9| 27.3| 46.7| **73.5**| 69.3| 65.9| 24.1 | 67.8 | 72.4 | 66.2 | 65.9 | 69.8 |
| 2.5 |     |     |     |     |     |     |     |      |       |      |      |      |      |      |
| 3.1 | 84.4| 80.4| 80.8| 83.8| 85.4| 77.5| 86.9| 80.3| 87.7 | 72.4 | 72.2 | 81.1 | 85.7 | 80.0 |
| 3.2 | 85.3| 83.8| 80.1| 79.5| 68.8| 70.2| 82.5| 84.0| 86.5 | 79.3 | 65.0 | 83.1 | 86.9 | 84.0 |
| 3.3 | 77.6| 76.6| 76.7| 30.2| 41.8| 83.3| 81.3| 77.0| 54.5 | 74.0 | 81.7 | 75.7 | 79.0 | 77.0 |
| 3.4 | 69.4| 67.8| 59.9| 27.3| 46.7| 73.5 | 69.3| 65.9| 24.1 | 67.8 | 72.4 | 66.2 | 65.9 | 69.8 |
| 3.5 |     |     |     |     |     |     |     |      |       |      |      |      |      |      |
| 4.1 | 84.6| 76.5| 79.8| 83.5| 84.2| 77.6| 87.6| 77.3| 87.8 | 71.8 | 72.2 | 76.5 | 85.3 | 76.1 |
| 4.2 | 84.5| 82.8| 79.9| 78.9| 65.4| 70.2| 82.4| 83.1| 86.5 | 79.0 | 64.9 | 82.4 | 86.3 | 83.3 |
| 4.3 | 77.5| 76.4| 76.5| 30.2| 41.2| 83.3| 81.3| 76.9| 54.3 | 73.6 | 81.7 | 75.0 | 78.8 | 76.9 |
| 4.4 | 69.3| 67.5| 59.8| 27.7| 46.3| 73.5 | 69.3| 65.5| 24.1 | 67.9 | 72.4 | 66.1 | 66.0 | 69.5 |
| 4.5 |     |     |     |     |     |     |     |      |       |      |      |      |      |      |
| 5.1 | 82.8| 75.7| 79.7| 69.8| 74.9| 74.9| 76.6| 85.8| 76.3 | 88.3 | 77.3 | 72.0 | 72.4 | 84.1 | 75.3 |
| 5.2 | 84.8| 83.8| 77.0| 79.5| 67.5| 68.6| 81.0| 83.5| 86.4 | 79.2 | 64.4 | 84.3 | 86.4 | 84.2 |
| 5.3 | 79.0| 78.6| 77.0| 23.4| 48.0| 83.3| 81.6| 78.9| 54.5 | 75.7 | 81.7 | 77.0 | 79.3 | 78.5 |
| 5.4 | 68.1| 64.6| 60.0| 31.8| 40.3| 73.5 | 68.6| 61.7| 23.1 | 66.5 | 72.7 | 64.9 | 66.2 | 66.7 |
| 5.5 |     |     |     |     |     |     |     |      |       |      |      |      |      |      |
| 5.6 | 78.7| 75.7| 73.4| 51.1| 57.7| 75.5| **79.3**| 75.1| 63.1 | 74.7 | 72.7 | 74.7 | 79.0 | 76.2 |
| 6.1 | 50.1| 34.2| 65.9| 40.8| 33.3| 74.7| 72.6| 33.9| 87.8 | 38.7 | 34.7 | 71.7 | 35.0 | 55.6 | 34.5 |
| 6.2 | 81.0| 81.2| 75.0| 76.4| 57.3| 65.0| 76.9| 81.4| 86.3 | 76.0 | 64.2 | 75.4 | 83.5 | 81.6 |
| 6.3 | 79.0| 80.7| 74.4| 23.6| 47.8| 83.4| 80.9| 80.7| 53.9 | 77.0 | 81.6 | 79.4 | 78.7 | 80.8 |
| 6.4 | 63.3| 63.1| 59.8| 39.2| 46.0| 74.3 | 69.3| 60.4| 19.3 | 59.6 | 72.0 | 56.5 | 63.1 | 66.0 |
| 6.5 |     |     |     |     |     |     |     |      |       |      |      |      |      |      |
| 6.6 | 68.4| 64.8| 68.8| **45.0**| 46.1| 74.3| **74.9**| 64.1| 61.8 | 61.8 | 72.4 | 61.6 | 70.2 | 65.7 |

- The best wavelet over all of the analyzed cases turned out to be Gs4 with an average DER value of 79 %. It is important to note that peaks at random positions with relatively large amplitudes appeared in cases, corresponding to noise level of 5 %. Also different results were obtained, performing the analysis over several repetitions, which was due to the random nature of noise. Therefore a relative error of 5 % was attributed to average values of DER, depicting that these values are changing, conducting the experiment several times (table 2). As a result, the wavelets with best average DER that lies in the interval of 75 % - 83 % (79 % ± 4 %), apart from Gs4, are also all other wavelets, except for Db4, Db8 and Haar.

Aver | 77.3 | 74.6 | 73.2 | 52.8 | 57.5 | 75.7 | **79.0** | 74.2 | 62.9 | 71.6 | 72.7 | 73.5 | 77.7 | 75.2 |

Δ (±) | 3.9 | 3.7 | 3.7 | 2.7 | 2.9 | 3.8 | 4.0 | 3.7 | 3.1 | 3.6 | 3.7 | 3.9 | 3.8 |
• Haar wavelet dominated the damage detection at larger data sets. It was tightly located in the zone of damage and even increasing noise levels did not affect it significantly. As the number of the examined data points was reduced, Haar wavelet shifted to the larger x values – just out of the damaged zone.

• Db2 and Sym2 are equal at capturing damaged location, therefore Sym2 was not included in table 2. Noticeable tendency for Db4 is to deteriorate the damage localization effect as data set shrinks, which also makes it the overall least valuable wavelet in damage detection.

• Only finer scaling parameters s are appropriate at damage localization (s = 2) – at larger s values wavelets are performing poorly (s = 32) (not shown in this study).

SDI, using the best and worst wavelets – Gs2 and Db4, respectively, was plotted for original data amount and noise levels 0.1 % and 5 % (figures 1 – 4). The location and size of the introduced damage is indicated by red dotted lines.

As one can see in these figures, data set with more points is more sensitive to noise – there are many peaks with random amplitude due to noise, scattered all over the x axis in figure 1 (right), but, as data amount shrinks, cases with 0.1 % and 5 % noise levels look practically the same (this is especially true for case 6.4. in figure 4 (right)). Also, as one might expect, the reduction of the examined data points deteriorates damage identification. This is true by looking at these figures – for original data the best and worst wavelets both manage to capture that location of damage and as the examined data are reduced, the worst wavelet shifts out of the damage zone, and also the largest peaks for both wavelets become less localized in the zone of interest.

The conclusion is that there is a trade-off between sensitivity to noise and damage detection accuracy. We suggest that the best case corresponds to data, reduced by a factor of 2 (figure 2). Damage localization is almost as good as for original data and corruption due to noise is far less. As data is reduced further, damage localization worsens.

Figure 1. Beam 1, CBC, SDI. left: Case 4.1., right: Case 6.1. (numerical data).
Figure 2. Beam 1, CBC, SDI. left: Case 4.2., right: Case 6.2. (numerical data).

Figure 3. Beam 1, CBC, SDI. left: Case 4.3., right: Case 6.3. (numerical data).

Figure 4. Beam 1, CBC, SDI. left: Case 4.4., right: Case 6.4. (numerical data).
5. Experiment
Specimens under test were 2 aluminum beams with the following dimensions – length₁ = 1250 mm, length₂ = 1500 mm, width₁,₂ = 50 mm, height₁,₂ = 5 mm. A single mill-cut damage (width₁ = 50 mm, width₂ = 100 mm, depth₁,₂ = 2 mm) was introduced in a location at 750 mm from one edge of a Beam 1 and at 950 mm from one edge of Beam 2 (figure 5 (a)).

The modal frequencies and corresponding mode shapes of the beams are measured by a POLYTEC PSV-400-B scanning laser vibrometer. General experiment set-up consists of a PSV-I-400 LR optical scanning head equipped with a highly sensitive vibrometer sensor (OFV-505), OFV-5000 controller, PSV-E-400 junction box, a Bruel & Kjaer type 2732 amplifier, and a computer system with a data acquisition board and PSV software (figure 5 (b)).

PSV system requires defining the outer edges and geometry of the object and setting up a scanning grid, which consists of a number of points where vibrations are measured. To match the finite element model, 126 and 151 equally spaced scanning points are equally distributed to cover the area of Beam 1 and Beam 2, respectively. Free-free (all edges free) boundary conditions (FBC) are simulated by suspending the beam with two thin threads under a frame. In order to simulate clamped-clamped (two ends fixed) boundary conditions (CBC) experimentally, two vices are used to fix the ends of the beam (10 mm from both sides) with the clamped torque of 20 Nm. The beams are excited by a periodic input chirp signal generated by the internal generator with a 1600 Hz bandwidth through a piezoelectric actuator discs. After the frequency response function and resonant frequencies have been obtained, the beam is excited by a periodic sine wave signal with a frequency corresponding to each resonant frequency in order to obtain the corresponding mode shape. When the measurement is performed in one point, the vibrometer automatically moves the laser beam to another point of the scan grid, measures the response using the Doppler principle and validates the measurement with the signal-to-noise ratio. The procedure is repeated until all scan points have been measured. The mode shapes are obtained by taking the Fast Fourier Transform of the response signal. The first 15 mode shapes were measured in total.

![Figure 5. Left: geometry (in mm) of tested beams, right: experimental set-up.](image)

5.1. Results and discussion
Experimental results for DER for both beams are shown in table 3. Differences between results for different boundary conditions were observed. While there was no correlation between best-worst wavelet sets of CBC case (Gs2-Morlet, respectively) and FBC case (Haar-Db2, respectively) for Beam 1, there was some correlation between different boundary conditions for Beam 2 in terms of best-worst wavelet, namely for both BC cases best wavelet was Mexican hat and worst was Cf2 (CBC) and Meyer (FBC).
Analysis for experimental data refers to case 1.1. for numerical simulations, therefore, for the purposes of objective comparison, only 1st row of table 2 should be compared to the experimental results. Note the fact that for both numerical simulation case 1.1 and the experimental results wavelet that performed the best in damage localization was Haar, only numerical simulation was done for Beam 1 with CBC, but the corresponding experimental result was obtained for FBC. As for Beam 2, Mexican hat performed the best, but this particular wavelet was the poorest in case 1.1. These opposite results were obtained for beams with only noticeable differences in their length, damage width and location.

Table 3. DER results (in %) on experimental data for both beams at different BC.

| Wavelet   | Beam 1       |             | Beam 2       |             |
|-----------|--------------|-------------|--------------|-------------|
|           | DER (CBC)   | DER (FBC)   | DER (CBC)   | DER (FBC)   |
| Coif2     | 41.72        | 37.57       | 22.36        | 32.21       |
| Coif4     | 36.20        | 37.34       | 26.76        | 26.24       |
| Db2       | 70.90        | 32.08       | 71.50        | 60.59       |
| Db4       | 45.46        | 36.85       | 25.82        | 32.08       |
| Db8       | 42.43        | 37.97       | 32.50        | 28.27       |
| Gs2       | 82.55        | 58.95       | 80.21        | 69.26       |
| Gs4       | 60.86        | 32.87       | 63.58        | 43.19       |
| Gs8       | 38.02        | 37.91       | 28.70        | 25.67       |
| Haar      | 69.51        | 75.89       | 66.09        | 55.65       |
| Mexican hat | 82.08     | 60.97       | **80.58**    | **69.87**   |
| Meyer     | 38.92        | 36.86       | 39.68        | 24.83       |
| Morlet    | 33.83        | 37.53       | 25.65        | 29.38       |
| Sym4      | 43.85        | 37.63       | 32.04        | 29.35       |
| Sym8      | 37.21        | 38.01       | 26.24        | 27.75       |

Based on table 3, results for the best (thick continuous black line) and the worst (thick continuous green line) wavelets were selected and plotted (figures 6-9). The best wavelets with the corresponding threshold are shown in the pictures to the right. Although the peak values occur at the pre-determined location, the large values also emerged at the boundaries of beams. The boundary distortion problem is caused by discontinuity of mode shapes at their ends due to boundary conditions. Commonly, two methods are used to reduce the boundary effect [5]. One method is to extend the mode shape data beyond the original boundary by cubic spline extrapolation based on points near the boundaries. The other method is simply to ignore those values near the boundaries by cutting them off or setting them to zeros. In this study the latter method is adapted – values of data points, corresponding to edges were set to zero for both beams. It should be noted that neither of both methods is capable of detecting damage close to boundaries, since both of them smooth out the information near the boundaries.

As one observes figures 6-9, it can be seen, that the algorithm works – CWT with the selected wavelet is an appropriate technique for damage detection – peaks with largest amplitudes are located between two red vertical lines (zone of damage). In figures with SDI threshold only, one peak remains in the zone of damage, indicating that statistical hypothesis approach (refer to equation (8)) is working.
Figure 6. Left: SDI (Beam 1, CBC), right: SDI with threshold 3 for the best wavelet.

Figure 7. Left: SDI (Beam 1, FBC), right: SDI with threshold 2 for the best wavelet.

Figure 8. Left: SDI (Beam 2, CBC), right: SDI with threshold 3 for the best wavelet.
6. Conclusions

In this paper a numerical and experimental study on the applicability of mode shape based spatial continuous wavelet transform (CWT) technique for detection and localization of damage in beam structure is presented. The advantage of the proposed method is that it requires mode shape information only from the damaged state of the structure. Success of damage detection was demonstrated using statistical hypothesis approach, exploiting CWT. The obtained results show that the spatial CWT technique provides reliable information about the location of damage in two aluminum beams containing different length single mill-cut damage. To examine the limitations of the method, several sets of simulated data were analyzed. Overall, 15 different wavelets were tested at scale parameters 2, 8 and 32, of which scale 2 was suggested as the best. This analysis was performed at different noise levels, namely 0.001 %, 0.01 %, 0.1%, 1% and 5 % and different amount of data points, reduced by a factor of 2, 5 and 10.

Not all of the wavelets were equal at damage detection. Numerical simulation results were validated by experiment. Basing on the aforementioned results, the best and worst wavelets for both experimental and numerical cases are selected.

- At noise level 0 % and original data amount, Haar wavelet performed the best and it was only barely affected by increasing noise levels, on the other hand, data reduction affected its performance dramatically, so at the least amount of data Haar wavelet did not succeed in locating the damage. As for the experimental results, Haar wavelet, on the other hand, was the best at damage localization for Beam 1 with free – free boundary conditions (FBC).

- Gs4 was overall the best wavelet in terms of damage localization in numerical simulation, carried out for shorter beam – it possessed the best average DER, although several more wavelets came close. Its performance did deteriorate while working on fewer data points, but not significantly, and it performed satisfactory even at higher noise levels. As for experimental results, Mexican hat did the best job to locate the damage for longer beam for both clamped and free boundary conditions.

- Db4 was the worst wavelet for damage detection in terms of average DER for numerical simulation and also operated poorly on experimental data, however, the worst wavelets for experiment turned out to be Morlet (Beam 1, CBC), Db2 (Beam 1, FBC), Cf2 (Beam 2, CBC) and Meyer (Beam 2, FBC).
As expected, higher noise levels corrupt data. However, as data amount is reduced, noise affects data to a lesser extent, meaning that “the more data points, the better” statement is not always true. On the other hand, data reduction deteriorates damage localization quality, therefore the optimal conditions (data amount-noise level) are suggested as investigated data decrease by a factor of 2, where due to fewer data points good damage localization still holds and is barely influenced by increasing noise levels.

Haar wavelet is suggested for damage detection if there are sufficient data points available even for noisy signals. If not Haar, then Mexican hat can also be used. On the other hand, Daubechies of order 4 is not recommended to use at all.

7. References
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