Estimation of the Number of Cracks Generated by Compressive Stress in Polycrystalline Graphite using Ultrasonic Method

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Abstract. A theory was developed to estimate the change in the apparent sonic velocity in porous ceramics by solving Lavrov's integral equation of the cylindrical inclusion problem. It was shown that the change in apparent sonic velocity was caused by phase shift of the sonic wave. To extend the theory to the case of multiple inclusions, the superposition principle was applied. The theory was verified by analyzing the apparent sonic velocity during a compression test of polycrystalline graphite, and the number of cracks that formed during loading was estimated to be 700, when the dimensions of crack length and width were assumed to be 1mm and 10μm, respectively.

1. Introduction

Elastic inhomogeneity affects propagating waves in materials. This phenomenon has been utilized in the ultrasonic pulse echo method to detect the location of a crack [1]. However, it is difficult to use this method if a material contains multiple cracks. In addition, damage evolution itself is one of the most important factors in materials characterization now. In the ultrasonic pulse echo method, parameters such as apparent sonic velocity and attenuation coefficient have been used to estimate damage processes [2-4]. However, the quantitative relationship between these parameters and microstructural change (or micro damage process) has not been fully clarified yet. So, we need to analyze the relationship between the propagating wave change and the number of cracks for quantitative understanding of the damage processes.

Lavrov et al. studied the change in a plane elastic wave caused by a cylindrical inclusion [5]. This report suggests that it is possible to estimate the relationship between the wave change and the microstructural change in a material. To calculate the wave change due to multiple cracks, an assumption that the crack is a cylindrical inclusion must be introduced.

In this paper, we solved the wave equation suggested by Lavrov et al. for phase shift in a plane elastic wave due to a single cylindrical inclusion located in an isotropic elastic material. Then, the phase shift was superimposed into the frequency space in order to estimate the total phase shift due to multiple cracks. This value was converted into the change in apparent sonic velocity that was a parameter in the ultrasonic pulse echo method. From these results, we can calculate the number of cracks necessary to change the apparent sonic velocity.

The goal of this study is to propose a model for estimating the number of cracks generated in a material, that results in a change in the apparent sonic velocity. To verify this model, uni-axial compression tests were conducted in polycrystalline graphite, and the number of cracks was estimated using an arbitrary stress.
2. Theory of the wave change

We consider the system where plane elastic longitudinal wave propagates through an elastic matrix and impinges on a cylindrical inclusion. Figure 1 shows the configuration of the wave and the inclusion. The propagating direction of the plane wave is vertical to that of the longitudinal axis of the inclusion. The length and diameter of the inclusion are 2L and 2h, respectively, assuming that L is much larger than h. Both the matrix and the inclusion are linear elastic bodies with density \( \rho \), \( \rho_1 \), respectively.

After the wave passes through the inclusion, its direction is maintained if scattering of the wave is neglected by the inclusion. Therefore, we have one non-zero component of the displacement parallel to the \( x_3 \) axis in figure 1. This component can be calculated from an integral equation proposed by Lavrov et al. [6].

The relationship between the displacement \( u_3 \) in the material containing one inclusion and that \( u_0 \) without inclusion is formulated as below.

\[
u_3 = R \exp(i\theta) \cdot u_0 \tag{1}
\]

where \( R \) and \( \theta \) is the wave amplification ratio and the phase shift due to the single inclusion, respectively. Equation (1) results from the wave change by propagating through the inclusion, where \( u_3 \) is the complex number. In Eq. (1), \( R \) and \( \theta \) can be expressed as follows,

\[
R = \left[ \frac{\text{Re}(u_3/u_0)}{\sqrt{2}} \right] + \left[ \frac{\text{Im}(u_3/u_0)}{\sqrt{2}} \right]
\tag{2}
\]

\[
\theta = \tan^{-1} \left( \frac{\text{Im}(u_3/u_0)}{\text{Re}(u_3/u_0)} \right)
\tag{3}
\]

where \( R \) and \( \theta \) are both functions of frequency.

Generally, the incident wave has multiple frequencies. So, the pulse wave \( x(t) \) is separated into waves with a single frequency by a Fourier series expansion as follows

\[
x(t) = \frac{a_0}{2} + \sum_{n=1}^{N} (a_n \cos(2\pi f_n t) + b_n \sin(2\pi f_n t))
\tag{4}
\]

where \( a_n \) and \( b_n \) denote Fourier expansion coefficients corresponding to cosine and sine waves with frequency \( f_n \), respectively.

For the case where the pulse wave propagates through multiple inclusions, the calculations are simplified for \( K \) cylindrical inclusions by assuming vertical alignment to the wave propagation axis (the \( x_3 \) axis).
axis in figure 1) and each center of gravity is located along the $x_3$ axis. If the interactions among inclusions are neglected, the total phase shift by multiple inclusions is obtained by superimposing the phase shift due to the single inclusion in the frequency space. The output wave $y(t)$ after passing through these inclusions is obtained in the following form,

$$y(t) = \frac{d_0}{2} + \sum_{s=1}^{N} \left( R_s \right)^s \left[ a_n \cos(2\pi f_s + K0_s) + b_n \sin(2\pi f_s + K0_s) \right]$$

(5)

3. Experimental

Polycrystalline graphite (TOYO TANSO Co. Ldt.) specimens with dimensions of 30 mm in width ($L_1$), 30 mm in height ($L_2$) and 10 mm in thickness ($L_3$) were prepared for uni-axial compressive tests. Figure 2 shows an illustration of our experiments. The maximum applied stress was set to 83.3MPa, which is 90% of the mean fracture stress of the graphite. The loading axis was parallel to the direction of the height of the specimen and crosshead speed was 0.1 mm/min. During the loading-unloading process, ultrasonic measurements were carried out simultaneously. A transducer for transmitter-cum-receiver of longitudinal sonic waves (TOSHIBA TUNGALOY Co. Ldt.) was attached to the 30 $\times$ 30 mm face. The nominal frequency of the incident pulse wave was set to 1.0 MHz.

During the ultrasonic measurements, some echo peaks were observed. From the time interval of these peaks and thickness of the specimen, sonic velocity $V_A$ was calculated. As an acoustic parameter for damage evaluation, we calculated the relative change in sonic velocity $V_R$ as follows,

$$V_R = \frac{V_A - V_{A0}}{V_{A0}}$$

(6),

where $V_{A0}$ denotes sonic velocity before the first loading.

4. Experimental results

Figure 3 shows a stress-strain curve and figure 4 shows the change in apparent sonic velocity as a function of compressive stress. In the loading process, the relative change in apparent sonic velocity $V_R$ decreases with an increase in stress, which corresponds to the nonlinearity of the stress-strain curve. These results suggest that cracks were generated during the loading step, because both $V_R$ and Young’s modulus must decrease due to crack generation [7-8]. During the unloading step, $V_R$ increases and then decreases with stress. Since the unloading curve has an inflection point at
around 20 MPa, the curve changes from concave to convex. This change must be consistent with a slight decrease in $V_R$ while unloading below 20 MPa in figure 4. In this region, a secondary damage process has taken place.

From the above results, we propose a damage model for polycrystalline graphite as shown in figure 5. We assume two types of cracks in a polycrystalline graphite specimen; one is aligned almost parallel to the loading axis and the other is aligned vertical to the loading axis. We define the former cracks as longitudinal cracks and the latter as lateral cracks as shown in figure 5 (a). In the loading process, new cracks form at the interface between graphite filler grains. These longitudinal cracks open and extend but lateral cracks close under the load. The opening and growth of cracks causes a decrease in $V_R$, and vice versa. In this case, the two effects are competing. However, the effect of longitudinal cracks dominates the change in $V_R$, because ultrasonic wave propagates in the direction vertical to the loading axis (viz. longitudinal cracks also). Therefore, $V_R$ decreases during the loading process. For the unloading process, on the other hand, longitudinal cracks retract, and then lateral cracks begin to open again under 20 MPa. This mechanism contributes to the initial increase and the subsequent decrease in $V_R$ in the unloading step.

5. Estimation of the number of cracks

Equation (5) provides a relationship between the number of cracks and the total phase shift in the pulse wave. Therefore, the number of cracks is obtained as the function of the change in $V_R$ in figure 6. This figure shows that the number of cracks is almost proportional to the relative change in sonic velocity in each case. If the decrease in $V_R$ during the loading process is caused by an increase in the number of cracks, the increase in $V_R$ during the unloading process must attribute to the decrease in the length of cracks. Quantitatively, the aspect ratio of the crack is assumed to be constant ($L/h=100$), and we propose that the damage of the specimen must be explained by the generation of 700 cracks with a length of 1 mm and width of 10 μm during the loading step. If a smaller crack size is selected, then the number of cracks increases. After unloading, the cracks may be partly closed, and their length and width are estimated to be 800 μm and 8 μm to fit the experimental data. These changes need to be verified by direct measurement of cracks, however, up to now, this has yet to be undertaken. This topic will require further work.

6. Conclusions

In this paper, a model is proposed for estimating the number of cracks generated by compressive loading of graphite. The total phase shift of the incident wave was an important factor to determine the velocity change by crack generation. Based upon this theoretical framework, the number of cracks was estimated in a polycrystalline graphite specimen subjected to uni-axial compressive loading. The damage model assumes that crack generation only occurs during loading, whereas, the longitudinal cracks close while unloading.

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