Research of The Calculation Method for round-end concrete-filled steel tube under the eccentric load

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Abstract. Round-end concrete-filled steel tube (RCFST) columns have been applied to bridge structures. There is not the formula of RCFST under the eccentric load in code. Based on the equation of axial force and moment, the basic correlation equations of the strength and stability capacity of concrete-filled steel tube under the eccentric load are obtained through theoretical derivation. Formula results compared with finite element simulation results show results are in good agreement.

1. Introduction
Round-end concrete-filled steel tube (RCST) column is the concrete-filled steel tube with rectangle section in the middle and semicircle at ends. Compared with concrete-filled steel tube, the RCST column has advantages of high bearing capacity, satisfactory anti-seismic performance, remarkable space saving, etc. There is a paucity of research about the performance under load of the RCST columns, without corresponding design calculation theory provided by code. Therefore, it is of great necessity to study the performance under the eccentric load of the RCST columns, in order to provide a convenient and reliable calculation method, and thereby promoting engineering application of the RCST columns.

2. Theory of the eccentric load
RCST begins to flex laterally due to the unevenly distributed stress on the cross section when the eccentric load is applied. If column is slender and eccentricity ratio is small, it tends to exhibit characteristic of strength failure, and the full section becomes plasticized before reaching ultimate bearing capacity. For column under eccentric compression with a relatively long and slender length, its bearing capacity is determined by the stability.

Without considering instability failure, the CFST columns under the eccentric load mainly consist of two failure modes. The first type is CFST columns see failure at the dangerous section of the tensile zone when ultimate bending moment value under the combined action of axial compression and bending moment is larger than that under pure bending stress within a certain eccentricity ratio. Specifically, when bending moment is too large due to excessive eccentricity, compressive resistance generated by the axial compressive force in compressive zone is opposite in direction to the tensile stress generated by bending moment, so that a fraction of the bending tensile stress is offset by the compressive stress. In addition, the axial compressive force and tensile stress don’t reach the failure strength of the columns until the steel tubes in the compressive zone reach ultimate tensile strength.
The second type of failure is similar to failure caused by eccentric compression, i.e., concrete is crushed and then the steel tubes flexed at the section with the largest bending moment, ultimately leading to failure of the columns [1]. To be specific, when the axial compressive load is large, the whole section does not occur tensile zone due to the superposition of the compressive resistance generated by the axial compressive force and the tensile stress generated by bending moment as well as the excessive axial compression.

When the CFST columns fail in instability, stress distribution at the dangerous section is different with the difference of slenderness ratio and relative eccentricity. In the case of small slenderness ratio and eccentricity, full section is compressed; otherwise, both the tension zone and the compression zone become plasticized.

The CFST columns under eccentric compression have a more complicated working mechanism than those under axial compression. The working characteristics can be summarized as follows [2]:

1. When strength failure of CFST, the section will be fully plasticized, and the concrete in the tension zone doesn’t come;
2. When stability failure of CFST, stress distribution on the dangerous section has both a plastic zone and an elastic zone, and the stretching strain of the concrete that doesn’t come into play exceeds ultimate tensile strain of concrete;
3. Because Compression on dangerous section is unevenly and locally distributed, the confinement force of steel tubes to the concrete is also unevenly distributed.
4. The two materials on the dangerous section vary in the modulus of deformation not only with the position on the section, but also with the lengthwise direction.

The compression and bending moment acting on the compression-flexure columns can be caused by different loads; that is, compression and bending moment can be two independent variables. For eccentric compression, considering the proportional increase of axial compression \( N \) and bending moment \( M \), stress condition for most columns under eccentric compression in engineering practice falls into this category.

3. Derivation of calculation formula for revolving-short-axis flexural capacity

The theoretical study on CFST columns under eccentric compression starts from ultimate analysis, and the calculation theories include collapse theory [3], empirical coefficient method [4], axial force-bending moment correlation equation [5] and increased eccentricity ratio method [6]. The research, based on the axial force-bending moment correlation equation and combining the finite element modeling data collected, works out the practical and simplified theoretical calculation formula for the eccentric compression capacity of CFST columns through fitting.

When the CFST columns reach anti-bending ultimate state, stress distribution and sectional geometric dimension are shown in Fig. 1, where \( x_0 \) is the distance between sectional centroidal axis and neutral axis.

![Figure 1. Stress distribution and sectional geometric dimension of CFST in anti-bending ultimate state](image-url)
When the columns reach the bending limit state, the sectional stress distribution is as shown in Fig. 1. According to the static equilibrium condition, the position of neutral axis \( x_0 \) can be determined. According to the static equilibrium condition: \( \sum N = 0 \):

\[
A_{st} f_y - A_{scc} f_{y,c} = 0
\]

\[
A_{st} = 2at + \frac{2 \arccos \frac{r - c}{r}}{2\pi} \cdot t
\]

\[
A_{scc} = \frac{2 \arccos \frac{x_0 - \left( \frac{a}{2} + c - r \right)}{r}}{2\pi}
\]

\[
2 \arccos \frac{x_0 - \left( \frac{a}{2} + c - r \right)}{r}
\]

\[
A_{scc} = \frac{2 \arccos \frac{x_0 - \left( \frac{a}{2} + c - r \right)}{r}}{2\pi} - \left[ x_0 - \left( \frac{a}{2} + c - r \right) \right] \left( r^2 - \left[ x_0 - \left( \frac{a}{2} + c - r \right) \right]^2 \right) \frac{1}{3}
\]

\[
x_0 = \frac{a}{2} + c - r
\]

\[
+ \frac{2 \arccos \frac{x_0 - \left( \frac{a}{2} + c - r \right)}{r}}{2\pi} \cdot t
\]

In which, \( A_{st} \) is Area of steel tube; \( A_{scc} \) is area in compression of composite section.

According to the position of neutral axis calculated by formula (1), the moments against the neutral axis for the pulling stress and crushing stress are obtained respectively to primarily obtain the anti-bending ultimate bearing capacity.

\[
M_0 = M_z + M_{scc}
\]

\[
M_z = 2 \int_0^{\arccos \frac{r - c}{r}} r^2 \theta d\theta + 2 \int_0^{\arccos \frac{r - c}{r}} r^2 \theta d\theta + 2 \int_0^{\arccos \frac{r - c}{r}} r^2 \theta d\theta + 2 \int_0^{\arccos \frac{r - c}{r}} r^2 \theta d\theta + 2 \int_0^{\arccos \frac{r - c}{r}} r^2 \theta d\theta
\]

\[
M_{scc} = \int_0^{\arccos \frac{x_0 - \left( \frac{a}{2} + c - r \right)}{r}} r^2 \theta d\theta + 2 \int_0^{\arccos \frac{x_0 - \left( \frac{a}{2} + c - r \right)}{r}} r^2 \theta d\theta + 2 \int_0^{\arccos \frac{x_0 - \left( \frac{a}{2} + c - r \right)}{r}} r^2 \theta d\theta
\]

The above theoretical assumption ignores the tensile strength of concrete in the tensile region, so the actual anti-bending limit capacity is larger than the result calculated by formula (4). Through finite element analysis, the flexural capacity of the CFST columns can be determined:

\[
M = 1.12 M_0
\]

As illustrated by formulas (4), (5) and (6), the factors that influence the anti-bending limit capacity of the CFST columns mainly include steel strength, concrete crushing strength, and column sectional shape.
4. Correction of beam-column

The main factors that influence relation curve $N/N_u - M/M_u$ of the CFST beam-column members include steel and concrete strength, steel ratio and specimen slenderness ratio. There is one equilibrium point A on the typical CFST strength relation curve $N/N_u - M/M_u$, which is similar to the mechanical property of CFST beam-column members.

![Figure 2. Typical curve of $N/N_u - M/M_u$ Strength](image)

As shown in Fig.2, the abscissa and ordinate values for point A are $\xi_0$ and $\eta_0$, respectively. Assume all other conditions are equal, the larger the yield strength $f_y$ and steel ratio $\alpha$, the further inside the point A, showing the decreasing trend of both $\xi_0$ and $\eta_0$. Besides, the higher the concrete strength $f_{cu}$, the further outside the point A, showing the increasing trend of both $\xi_0$ and $\eta_0$. Larger $f_y$ and $\alpha$ indicates larger contribution of steel to the capacity of the whole column and less contribution of concrete. Higher $f_{cu}$ means larger contribution of concrete to the whole column.

Parameter analysis results show that the abscissa and ordinate values $\xi_0$ and $\eta_0$ for the equilibrium point A in Fig.2 can be approximately expressed as the function for confinement coefficient $\xi$. Through regressive analysis of the finite element calculation results, the calculation formula for CFST columns’ $\xi_0$ and $\eta_0$ can be derived:

$$\xi_0 = 1 + 0.16\xi^{-0.12}$$

$$\eta_0 = 0.1 + 0.14\xi^{-0.82} \quad (\xi > 0.4) \quad (8)$$

To reasonably derive $N/N_u - M/M_u$ correlation equation for CFST columns, extensive theoretical calculation has been performed. Analysis results show that the typical $N/N_u - M/M_u$ strength relation curve of CFST columns in Fig.2 can be generally divided into two parts and described with mathematical expression:

① CD section ($N/N_u \gg 2\eta_0$), it can be approximately described with straight line function, namely:

$$\frac{N}{N_u} + a \frac{M}{M_u} = 1 \quad (9)$$
② CAB section ($N / N_u \geq 2\eta_0$), it can be described with parabola function, namely:

$$-b \left( \frac{N}{N_u} \right)^2 - c \left( \frac{N}{N_u} \right) + \frac{M}{M_u} = 1$$

(10)

In which:  

$$a = 1 - 2\eta_0, \quad b = \frac{1-2\xi_0}{\eta_0^2}, \quad c = \frac{2(\xi_0-1)}{\eta_0}$$

In which $N_u$ is Bearing capacity of concrete –filled tube under axial load; $M$ is Flexural capacity.

Considering the influence of slenderness ratio of columns, $N_u / N_u - M / M_u$ correlation equation for CFST beam-column can be finally derived:

$$\begin{cases}
\frac{1}{\varphi} \times \frac{N}{N_u} + \frac{a}{d} \left( \frac{M}{M_u} \right) = 1 & (N / N_u \geq 2\varphi^3\eta_0) \\
\frac{b}{N_u} \left( \frac{N}{N_u} \right)^2 - c \left( \frac{N}{N_u} \right) + \frac{1}{d} \left( \frac{M}{M_u} \right) = 1 & (N / N_u < 2\varphi^3\eta_0)
\end{cases}$$

(11)

In which,  

$$a = 1 - 2\varphi^2\eta_0, \quad b = \frac{1-\xi_0}{\varphi^2\eta_0^2}, \quad c = \frac{2(\xi_0-1)}{\eta_0}$$

$1 / d$ is considered as the amplification coefficient of bending moment as a result of the two-order effect, $\varphi$ is the axially loaded stability coefficient.

$N / N_u - M / M_u$ correlation equation is corrected by taking CFST column as one kind of composite material and sectional developmental plasticity criterion as strength calculation standard. In the process, the finite element model is used [7], and round-ended finite element modeling analysis is adopted, so as to obtain the correlation equation for revolving-minor-axis eccentric compression of RFST columns:

$$\begin{cases}
\frac{1}{\varphi} \times \frac{N}{N_u} + \frac{a}{d} \left( \frac{M}{M_u} \right) = 1 & (N / N_u \geq 2\varphi^3\eta_0) \\
\frac{b}{N_u} \left( \frac{N}{N_u} \right)^2 - c \left( \frac{N}{N_u} \right) + \frac{1}{d} \left( \frac{M}{M_u} \right) = 1 & (N / N_u < 2\varphi^3\eta_0)
\end{cases}$$

(12)

In which,  

$$d = 1 - 0.30 \left( \frac{N}{N_E} \right)$$

Regressive analysis of calculation results is conducted by referring to literature [2] to obtain the expression of flexural bearing capacity calculation coefficient $\gamma_m$:

$$\gamma_m = 1.06 + 0.48 \ln(\xi + 0.1)$$

5. Verification of calculation results

To verify the effectiveness of the formula, theoretical calculation results and test data obtained by our research team [7] are compared with the finite element simulation results, as shown in Table 1. Results show that the average is 0.990 and the variance is 0.0004, suggesting that the correlation equation has basically reflected the mechanical property of revolving-minor-axis eccentric compression of CFST columns.
Table 1. Compare finite element model analysis data with formula calculation data

| No. | a   | b   | c   | r   | t   | $f_y$ | $f_{ck}$ | $e_0$ | $N_c$ | $N_{EEM}$ | $N_c/N_{EEM}$ |
|-----|-----|-----|-----|-----|-----|--------|----------|-------|-------|-----------|---------------|
| 1   | 60  | 240 | 120 | 120 | 4   | 235    | 20.1    | 90    | 577.77 | 577.191   | 1.001         |
| 2   | 240 | 240 | 120 | 120 | 4   | 235    | 20.1    | 90    | 1149.42| 1123.58   | 1.023         |
| 3   | 360 | 240 | 120 | 120 | 4   | 235    | 20.1    | 90    | 1428.64| 1385.69   | 1.031         |
| 4   | 60  | 240 | 120 | 120 | 4   | 235    | 23.4    | 90    | 605.50 | 599.50    | 1.010         |
| 5   | 60  | 240 | 120 | 120 | 4   | 235    | 26.8    | 90    | 663.08 | 669.78    | 0.990         |
| 6   | 60  | 240 | 120 | 120 | 4   | 345    | 20.1    | 90    | 647.50 | 667.52    | 0.970         |
| 7   | 240 | 240 | 120 | 120 | 4   | 235    | 23.4    | 90    | 1287.14| 1347.79   | 0.955         |
| 8   | 240 | 240 | 120 | 120 | 4   | 235    | 26.8    | 90    | 1429.03| 1470.20   | 0.972         |
| 9   | 240 | 240 | 120 | 120 | 4   | 235    | 20.1    | 130   | 1031.37| 1052.42   | 0.980         |
| 10  | 240 | 240 | 120 | 120 | 4   | 235    | 20.1    | 170   | 851.925| 866.66    | 0.983         |
| 11  | 240 | 240 | 120 | 120 | 4   | 345    | 20.1    | 90    | 1310.32| 1328.92   | 0.986         |
| 12  | 185 | 240 | 120 | 120 | 4   | 310    | 34.8    | 90    | 1104   | 1124.24   | 0.982         |
| 13  | 185 | 240 | 120 | 120 | 4   | 310    | 34.8    | 130   | 942    | 963.19    | 0.978         |
| 14  | 185 | 240 | 120 | 120 | 4   | 310    | 34.8    | 170   | 836    | 853.93    | 0.979         |
| 15  | 360 | 240 | 60  | 150 | 4   | 235    | 23.4    | 90    | 1178.93| 1193.25   | 0.988         |
| 16  | 360 | 240 | 60  | 150 | 4   | 235    | 26.8    | 90    | 1329.66| 1343.09   | 0.990         |
| 17  | 360 | 240 | 60  | 150 | 4   | 345    | 20.1    | 90    | 1098.87| 1107.73   | 0.992         |

6. Conclusion
According to the unified theory and CFST column-related calculation theories, the research proposes a calculation formula of eccentric compression capacity that is suitable for the CFST columns. Furthermore, the results obtained by the proposed formula are compared with the finite element modeling results, suggesting that the formula proposed is effective and feasible.

References
[1] Zhong Shantong. Concrete-filled steel tube[M].Beijing: Tsinghua University Press. 2003
[2] Liu Xichao. Research on basic property of elliptical concrete –filled tube [D]. Harbin: Harbin institute of technology,2010: 26-28.
[3] Zhou Guangshi. Research for stable bearing capacity of concrete –filled tube under eccentric load[J].Journal of Harbin Institute of Civil Engineering and Architecture,1982,15(4):29-46.
[4] Cai Shaoheai. Modern steel tube confined concrete structures [M].Beijing: China communications Press, 2003.
[5] Tang Guanzuo, Zhao Bingquan, Zhu Huixian. Study on the fundamental structural behavior of concrete filled steel tubular columns [J].Journal of building structures, 1982, 3(1):13-31.
[6] Hou Xiaoqing, Li Tian, Wang Hua. Comparison of Calculation Methods for Bearing Capacity of CFSTC under Eccentrically Loading Journal of Zhengzhou university (natural science edition), 2002, 34(3):91-94.
[7] Wang Erlei. Research on Compressive Behavior and Reliability of Round-ended Steel Tube-Filled Concrete Column [D]. Wuhan: Wuhan University of Technology, 2012:17-29.