BILINEAR R-PARITY VIOLATION IN RARE MESON DECAYS

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Abstract. We discuss rare meson decays \( K^+ \rightarrow \pi^- \ell^+ \ell^\prime \) and \( D^+ \rightarrow K^- \ell^+ \ell^\prime \) \((\ell, \ell' = e, \mu)\) in a supersymmetric extension of the standard model with explicit breaking of \( R \)-parity by bilinear Yukawa couplings in the superpotential. Estimates of the branching ratios for these decays are given. We also compare our numerical results with analogous ones previously obtained for two other mechanisms of lepton number violation: exchange by massive Majorana neutrinos and trilinear \( R \)-parity violation.

In the standard model (SM), the lepton \( L \) and baryon \( B \) numbers are conserved to all orders of perturbation theory due to the accidental \( U(1)_L \times U(1)_B \) symmetry existing at the level of renormalizable operators. But the \( L \) and \( B \) nonconservation is a generic feature of various extensions of the SM [1]. That is why lepton-number (LN) violating processes have long been recognized as a sensitive tool to put theories beyond the SM to the test. One of the most well known process of such type is neutrinoless double beta decay \((A, Z) \rightarrow (A, Z + 2) + e^- + e^-\) that has been searched for many years (see, e.g., [2] and references therein).

In Refs. [3, 4] rare decays of the pseudoscalar mesons \( K, D, D_s \), and \( B \) of the type

\[
M^+ \rightarrow M'^- \ell^+ \ell'^+ \quad (\ell, \ell' = e, \mu)
\]

mediated by light \((m_N \ll m_\ell, m_\ell')\) and heavy \((m_N \gg m_M)\) Majorana neutrinos were investigated. The indirect upper bounds on the branching ratios for the decays (1) have been derived taking into account the limits on lepton mixing and neutrino masses obtained from the precision electroweak measurements, neutrino oscillations, cosmological data, and searches of the neutrinoless double beta decay. These bounds are greatly more stringent than the direct experimental ones [5].

In Refs. [6, 7] we considered another mechanism of the \( \Delta L = 2 \) decays (1) based on the minimal supersymmetric extension of the SM with explicit \( R \)-parity violation (\( R \)MSSM, for a review, see [8]). \( R \)-parity is defined as

\[
R = (-1)^{B - \sum L + 2S},
\]

where \( B \), \( L \), and \( S \) are the baryon and lepton numbers and the spin, respectively. The SM fields, including additional Higgs boson fields appearing in the extended models, have \( R = 1 \) while \( R = -1 \) for their superpartners.

The most general form for the \( R \)-parity and lepton number violating part of the superpotential is given by [8]

\[
W_R = \varepsilon_{\alpha\beta} \left( \frac{1}{2} \lambda_{ijk} L_i^\alpha L_j^\beta \bar{E}_k + \lambda'_{ijk} L_i^\alpha Q_j^\beta \bar{D}_k + \epsilon_i L_i^\alpha H_u^\beta \right).
\]

Here \( i, j, k = 1, 2, 3 \) are generation indices, \( L \) and \( Q \) are \( SU(2) \) doublets of left-handed lepton and quark superfields \((\alpha, \beta = 1, 2 \) are isospinor indices),

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$\bar{E}$ and $\bar{D}$ are singlets of right-handed superfields of leptons and down quarks, respectively; $H_u$ is a doublet Higgs superfield (with hypercharge $Y = 1$); $\lambda_{ijk}(= -\lambda_{jik})$, $\lambda'_{ijk}$ and $\epsilon_i$ are trilinear and bilinear Yukawa couplings, respectively.

We assumed in [6, 7] that the bilinear couplings are absent at tree level ($\epsilon_i = 0$ in Eq. (2)). As well known they are generated by quantum corrections [8] but the dominant contribution of the tree-level trilinear couplings to the phenomenology is expected. In the present report, we investigate the case of tree-level bilinear couplings: $\epsilon_i \neq 0$ with $\lambda = 0$, $\lambda' = 0$. For this case, trilinear couplings cannot be generated via radiative corrections.

The bilinear terms in the superpotential (2) induce mixing between the SM leptons and the MSSM charginos and neutralinos $\tilde{\chi}_n^\pm$ in the mass-eigenstate basis and lead to the following $\Delta L = \pm 1$ lepton-quark operators [9, 10]:

$$\mathcal{L}_{LH} = -\frac{g}{\sqrt{2}} \kappa_n W^\mu_\ell \bar{\ell} \gamma^\mu P_L \tilde{\chi}_n + \sqrt{2} g \left( \beta^d_k \bar{\nu}_k P_R d_R + \beta^u_k \bar{\nu}_k P_R c_R + \beta^c \bar{\ell} P_R \bar{\ell} \right) + \text{H.c.}$$

Here $P_{L,R} = (1 \mp \gamma^5)/2$, $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$; the constants $\kappa_n$ ($n = 1, 2, 3, 4$), $\beta^d_k$, $\beta^u_k$, $\beta^c_k$ ($k, i = 1, 2, 3$) and $\beta^c$ depend on the elements of the mixing matrices diagonalizing the neutrino-neutralino and the charged lepton-chargino mass matrices.

The Lagrangian describing the bilinear mechanism of the decays (1) is

$$\mathcal{L} = \mathcal{L}_{LH} + \mathcal{L}_{SM} + \mathcal{L}_g + \mathcal{L}_\chi.$$  

(4)

In addition to the LN violating part (3), it includes the SM charged-current interactions

$$\mathcal{L}_{SM} = \frac{g}{\sqrt{2}} W^{\mu+}(\sum_\ell \bar{\nu}_\ell(x)\gamma_\mu P_L \ell(x) + \sum_{q,q'} \bar{q}(x)\gamma_\mu P_L V_{qq'} q'(x)) + \text{H.c.},$$  

(5)

where $\ell = e, \mu, \tau$; $q = u, c, t$; $q' = d, s, b$; $V_{qq'}$ is the CKM matrix, and the MSSM gluino-quark-squark and neutralino-quark(lepton)-squark(slepton) interactions [11]

$$\mathcal{L}_g = -\sqrt{2} g \frac{\lambda_r}{2} \left( \bar{q}_a L \bar{g}^r q_L - \bar{q}_a R \bar{g}^r q_R \right) + \text{H.c.},$$  

(6)

$$\mathcal{L}_\chi = \sqrt{2} g \sum_{r=1}^4 \epsilon_{Ln}(\psi) \bar{\psi} L \chi_n^0 \psi_L + \epsilon_{Rn}(\psi) \bar{\psi} R \chi_n^0 \psi_R + \text{H.c.}$$  

(7)

Here $(\lambda_r)^a_b$ are the $3 \times 3$ Gell-Mann matrices ($r = 1, \ldots, 8$) with color indices $a, b = 1, 2, 3$; the neutralino coupling constants are defined as

$$\epsilon_{Ln}(\psi) = -T_3(\psi) N_{n2} + \tan \theta_W (T_3(\psi) - Q(\psi)) N_{n1}, \epsilon_{Rn}(\psi) = Q(\psi) \tan \theta_W N_{n1},$$

where $Q(\psi)$ and $T_3(\psi)$ are the electric charge and the third component of the weak isospin for the quark (lepton) field $\psi$, respectively, and $N_{n1}$ is the $4 \times 4$ neutralino mixing matrix.

The leading order amplitude of the decay (1) is described by 9 diagrams shown in Fig. 1. The hadronic parts of the decay amplitude are calculated with the use of a simple model for the Bethe–Salpeter (BS) amplitudes for mesons as bound states of a quark and an antiquark [12] (see also [3, 4]). In
Figure 1: Feynman diagrams for the decay $M^+ \rightarrow M'^- + \ell^+ + \ell'^-$ in the bilinear $\bar{R}$MSSM. Bold vertices correspond to Bethe–Salpeter amplitudes for mesons. There are also crossed diagrams with interchanged lepton lines.

addition, taking into account that the meson mass $m_M \ll m_W, m_{SU SY}$, where $m_W$ is the $W$ boson mass and $m_{SU SY} \gtrsim 100$ GeV is the common mass scale of superpartners, we neglect momentum dependence in the propagators of heavy particles (see Fig. 1) and use the effective low-energy current-current interaction. In this approximation the decay amplitude does not depend on the specific form of the BS amplitude and is expressed through the known decay constants of the initial and final mesons, $f_M$ and $f'_M$.

Finally, for the total width of the decay (1) we have obtained

$$\Gamma_{\ell\ell'} \equiv \Gamma(M^+ \rightarrow M'^- + \ell^+ + \ell'^-) = (1 - \frac{1}{2} \delta_{\ell\ell'}) \frac{g^2 f_M^2 f'_M m_M^7}{2 m^4_\pi} \Phi_{\ell\ell'}^b,$$

$$\times \sum_{n=1}^4 \frac{g^2}{2 m_{n\pi}} \left[ \frac{n^2}{m_W} (V_{12} V_{43} + V_{14} V_{32}) + \frac{\kappa \beta}{2 m_W} \left( \frac{V_{12} (q_2) V_{43} + V_{14} (q_3)}{m_{41}^2 m_{32}^2} \right) \right] - \frac{2 \kappa \beta}{\sqrt{N_c}} \frac{g^2}{m_{41}^2 m_{32}^2 m_{42}^2} \Phi_{\ell\ell'}^b.$$  \( \text{(8)} \)

Here $\Phi_{\ell\ell'}^b = \int_{l_+}^{h_+} dz \frac{z}{2 (2z)^{1/2}} \left[ 1 - (h_+ + h_-)(2z)^{-1} \right]^{1/2} \left[ 1 - (l_+ + l_-)(2z)^{-1} \right]^{1/2}$

$$\times [(h_+ - z)(h_- - z)(l_+ - z)(l_- - z)]^{1/2} \text{ (9)}$$

is the reduced phase space integral with $h_\pm = (1 \pm m_{M'}/m_M)^2$ and $l_\pm = [(m_\ell \pm m_{\ell'})/m_{\ell'}]$. $N_c = 3$ is the number of colors.

For the numerical estimates of the branching ratios (BRs), $B_{\ell\ell'} = \Gamma_{\ell\ell'}/\Gamma_{total}$, we have used the known values for the SM couplings $g$ and $g_s$, meson decay constants, meson and lepton masses [5], and a typical set of supersymmetric parameters [13]: a) the MSSM parameters: $m_0 = 70$ GeV, $\mu = 500$ GeV, $M_2 = 200$ GeV, tan $\beta = 4$; b) the RPV parameters: $|\Lambda| \equiv \left( \sum_{i=1}^3 |\Lambda_i|^2 \right)^{1/2} = 0.1$ GeV$^2$ with $10 \Lambda_1 = \Lambda_2 = \Lambda_3$, $|\epsilon| = 1$ GeV$^2$ with $\epsilon_1 = \epsilon_2 = \epsilon_3$;
$M_3 = (g^2 / g^2) M_2$ at the electroweak scale. Using the MSSM mass formulas [11] with the gluino mass $m_{\tilde{g}} = M_3$, the masses of squarks and neutralinos and the elements of the neutralino mixing matrix were calculated numerically for the above set of parameters. The results of the calculations with the use of Eq. (8) are shown in the fourth column of Table 1. In the second and third columns of the table, the present direct experimental bounds on the BRs [5] and the indirect bounds for the Majorana neutrino mechanism of the rare decays are shown, respectively. We see that the BRs for the bilinear RPV mechanism are much smaller than these upper bounds. For comparison, the trilinear RPV mechanism leads to the upper limits on the BRs of order $10^{-22}$ $(10^{-24})$ for $K(D)$ rare decays with the use of conservative bounds on the trilinear couplings $|\lambda'_{ijk}\lambda'_{j'k'}| \lesssim 10^{-3}$ [6,7]. But for more stringent bounds $|\lambda'\lambda| \lesssim 5 \times 10^{-6}$ [8,14], $B^{\ell\ell}(triMSSM) \lesssim 10^{-28}$.

| Rare decay | Exp. upper bound on $B^{\ell\ell}$ | Ind. bound on $B^{\ell\ell}$ ($\nu_{\ell\ell}M_{SM}$) | $B^{\ell\ell}$ (biRMSSM) |
|------------|----------------------------------|-------------------------------------------------|---------------------|
| $K^+ \rightarrow \pi^+ e^+ e^-$ | $6.4 \times 10^{-40}$ | $5.9 \times 10^{-42}$ | $3.6 \times 10^{-43}$ |
| $K^+ \rightarrow \pi^- \mu^+ \mu^-$ | $3.0 \times 10^{-39}$ | $1.1 \times 10^{-41}$ | $1.0 \times 10^{-43}$ |
| $K^+ \rightarrow \pi^- e^+ \mu^+$ | $5.0 \times 10^{-40}$ | $5.1 \times 10^{-42}$ | $4.2 \times 10^{-43}$ |
| $D^+ \rightarrow K^+ e^+ e^-$ | $4.5 \times 10^{-40}$ | $1.5 \times 10^{-41}$ | $1.6 \times 10^{-43}$ |
| $D^+ \rightarrow K^+ \mu^+ \mu^-$ | $1.3 \times 10^{-39}$ | $8.9 \times 10^{-41}$ | $1.5 \times 10^{-43}$ |
| $D^+ \rightarrow K^- e^- \mu^+$ | $1.3 \times 10^{-39}$ | $2.1 \times 10^{-41}$ | $3.1 \times 10^{-43}$ |

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