Rotating sonic black hole from Spin-orbit coupled Bose-Einstein condensate

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We show that a sonic analogue of rotating BTZ type of black hole can be realised in a quasi-two-dimensional spin-orbit coupled BEC without any external rotation. The corresponding equation for phase fluctuations in total density mode that describes phonon field in hydrodynamic approximation, is described by a scalar field equation in $2+1$ dimension whose space-time metric can be identified with the space-time metric of rotating black hole of BTZ type. By time evolving the condensate in a suitably created laser induced potential, we show that the the moving condensate forms such rotating black hole in an annular region bounded by inner and outer event horizon as well as elliptical ergo surfaces. We discuss the self amplifying density modulations as well as the distribution of supersonic and subsonic zones in such rotating black hole that strongly depends on the spin-orbit coupled anisotropy that can be tested in experiments. We also calculate the density-density correlation in such analogue rotating black hole and the distribution of the analogue Hawking temperature on the event horizon.

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The Hawking radiation [1, 2] from a black hole (BH), formed at a particular stage of stellar evolution, is believed to hold the key to the understanding of compatibility of Quantum Mechanics to the Einstein’s General Theory of Relativity (GTR). However, a direct measurement of real BHs to resolve such issues is less likely even in a foreseeable future. Analogue systems in quantum fluids that can kinematically simulate such behaviour [3] are therefore very important. Recent observation of stimulated Hawking radiation from sonic black hole (SBH) in ultracold atomic superfluid of a Bose-Einstein condensate (BEC) of $^{87}$Rb [4, 5] is a major step towards this direction. This was followed by another experiment where a sonic analogue of expanding universe was realised in a supersonically expanding ring-shaped $^{23}$Na BEC [7]. Ultracold atomic systems thus emerged as a frontier candidate to test phenomena related to Gravitation and Cosmology through analogue experiments.

Anologue SBH in ultracold superfluid exists due to the fact that the hydrodynamic equation of phonons, which are the low energy quasiparticles of such atomic superfluid, takes a covariant form with a curved space-time metric, mimicking the curved space-time near a BH [8, 9]. The background induced metric of such SBH realized in recent experiments [4–6] is described by a $1+1$ dimensional generalisation of static singular Schwarzschild metric [10], that describes the simplest curved space-time in GTR. Consequently, such SBH does not contain a very important quantity that characterises a real BH generated from a spinning star, namely the angular momentum of a BH. Such BHs with angular momentum are called rotating black holes (RBH) [11]. The space-time ripples called gravitational waves, created through the collisions of such RBHs was first time detected in LIGO [12] recently and recharged the interest in black-hole physics enormously. The background space-time metric that characterizes such RBH has a different expression as compared to the Schwarzschild metric [13, 14]. It is therefore legitimate to ask what type of ultracold BEC can serve as the vacuum for phonons whose hydrodynamic equation governs the metric of a curved space-time around a RBH? In this paper we show that spin-orbit coupled (SOC) can realises a sonic RBH (SRBH) in $2+1$ dimension, which is the sonic analogue of well known BTZ (Bañados, Teitelboim and Zanelli) black holes in $2+1$ dimensional gravity [14, 15]. We show that for a two component SOC-BEC, in the limit where polarisation density $s_z$ is much less than the total density $n_d$, the phonon field is described by

$$\frac{1}{\sqrt{-g}}\partial_\mu (\sqrt{-g}g^{\mu\nu}\partial_\nu \hat{\theta}_d) = 0 \quad (1)$$

where $\hat{\theta}_d$ is the fluctuation in the phase of the superfluid BEC in the total density mode and $g^{\mu\nu}$ is the space time metric. Eq. (1) is massless scalar field equation in an analogue curved space-time like a single-component (henceforth called scalar) BEC, but with a fundamentally different $g^{\mu\nu}$. In a scalar BEC [6], $g^{\mu\nu}$ is a function of the condensate speed $v$ and the sound velocity $c$ both of which are isotropic. The velocity field $\mathbf{v} = \frac{\dot{\mathbf{r}}}{m} \frac{\Phi}{\nabla \Phi}$ is irrotational, where $\Phi$ is phase of the superfluid order parameter [16, 17] and $m$ is the atomic mass. Thus an appreciable azimuthal flow of such superfluid is not expected, unless they are rotated externally at sufficient angular velocities to introduce vorticity [18] that can be achieved in several analogue mediums [19–23].

In contrast to irrotational behavior of a scalar BEC superfluid without external rotation, in a SOC-BEC superfluid it is possible to introduce synthetic rotation through internal properties [24, 25]. The anisotropic expansion of such SOC BEC in free space [25], gives an anisotropic velocity profile. Thus $g^{\mu\nu}$ in Eq. (1) of such SOC-BEC promises a more exotic analogue space-time. This paper shows that such analogue space-time interval of a SOC-BEC has the same form as the well known BTZ solution for RBH in $2+1$ dimensional gravity [14].
The subsequent simulation shows that even though less in magnitude compared to the radial velocity, expanding condensate indeed have a finite azimuthal component of the velocity which changes with its anisotropy. Thus for such RSBH, apart from an event horizon like the case of a scalar condensate [4, 5], an elliptically shaped ergoregion [9], also exists. We subsequently discussed stimulated Hawking radiation from such RSBH.

To characterise such RSBH we need to calculate the phonon (sound) velocity in the SOC-BEC and the condensate velocity that violates irrotationality. We start with following spinorial time dependent Gross-Pitaevskii (GP) equation:

\[
\imath \hbar \frac{\partial \Psi}{\partial t} = \left[ \frac{\hbar^2}{2m} (-i\partial_x - \kappa \frac{\eta \eta'}{\hbar})^2 - \frac{\hbar^2}{2m_y} \partial_y^2 + V_{2D}(r,t) \right] \Psi
\]

\[
+ g_{2D} (|\psi_+|^2 + |\psi_-'|^2) \Psi, \quad \kappa = \pm
\]  

(2)

Eq. (2) is derived by considering \(^{87}\text{Rb}\) atoms with spin-orbit interactions and treating the inter and intra-species interaction strengths to be equal [26]. The corresponding single particle Hamiltonian, \(\hat{h} = \frac{p^2}{2m} - \eta \eta' \hat{\sigma}_y - \eta' \hat{p}_y \hat{\sigma}_z \), with non-abelian gauge potential \(A = (m \eta \hat{\sigma}_y, \eta' \hat{\sigma}_z, 0)\) (details in [29]). \(r = (x,y)\), \(m_y = \frac{\hbar}{m (1 - \frac{\eta'^2}{\eta^2})^2}\), \(\hat{\sigma}\) and \(\hat{I}'\) are Pauli and Identity matrices respectively, \(\eta, \eta'\) are SO couplings with \(\eta' < \eta\), and the external potential \(V_{2D}\) (Fig.1) includes the harmonic confinement. Also, \(g_{2D} = \frac{\sqrt{\omega_{2D}}}{\sqrt{2\pi n_d}}\), \(V_{int} = \frac{\hbar^2 \alpha^2}{2m}\), \(\alpha_\perp\) is the interatomic scattering length, \(a_\perp = \sqrt{\frac{\hbar}{m \omega_\perp}}\) is the transverse harmonic oscillator length.

Diagonalisation of the single particle dispersion, subsequent projection into the lowest energy band and finally Bogoliubov approximation in this lowest band including interaction yields Bogoliubov dispersion for the total density, \(n_d = n_+ + n_-\) and polarization density, \(s_z = n_+ - n_-\) given as: \(\hbar \Omega_\perp = \sqrt{E_-(p)}[E_-(p) + 2g_\perp n_d]\) and \(\hbar \Omega_\perp = E_-(p)\) where \(E_-(p) = \frac{p^2}{2m} + \frac{\eta^2}{2m_y} + \frac{\eta'^2}{2m_y}\), \(\bar{n}_d\) is the background density [27, 29]. In the long wavelength limit, the sound velocities for density and polarisation \((\_\_\_\_\_\_)\) modes can be calculated as \(c^2_{r,y} = \frac{\hbar \partial E_-(p)y}{\partial \psi_{x,y} \psi_{x,y}}\) [29].

For our case \(c^2_{r,y} = 0\), with same inter and intra species interaction strength and we set \(c^2_{r,y} = c^2_{r,y}\).

Denoting spinor order parameter as \(\Psi(r) = [\psi_+(r), \psi_-(r)]^T\), the components of current \(j\) that include the effect of anisotropy and the gauge field (details in [29]) are given by

\[
j^r = \frac{\hbar}{2\imath m} \left( \Psi^\dagger \partial_r \Psi - \Psi^T \partial_r \Psi^\star \right) - \eta \Psi^\dagger \partial_z \Psi
\]

\[
j^y = \frac{\hbar}{2\imath m_y} \left( \Psi^\dagger \partial_y \Psi - \Psi^T \partial_y \Psi^\star \right)
\]  

(3)

Hydrodynamic formalism: We write \(\psi_{\pm}(r,t) = \sqrt{n_\pm(r,t)} e^{i\theta_{\pm}(r,t)}\) in terms of density \(n_\pm\) and phase \(\theta_{\pm}\) in Eq.(2) and then consider \(n_+(r,t) \rightarrow \tilde{n}_+(r,t) + n_+(r,t), \quad \theta_+(r,t) \rightarrow \tilde{\theta}_+(r,t) + \tilde{\theta}_+(r,t)\). In the limit \(\tilde{s}_z \ll \tilde{n}_d [24-26, 28]\) through a straightforward but lengthy algebra, the linearised hydrodynamic equations yield Eq.(1) (details in [29]), a second order differential equation for \(\tilde{\theta}_d = \tilde{\theta}_+ - \tilde{\theta}_-\), that describes phonon field with

\[
g^\mu_\nu = \frac{mn_y}{n_d^2} \begin{bmatrix}
-1 & -v^x & -v^y \\
v^x & \frac{c^2 s + v^2}{\alpha} + \frac{v^2}{\alpha} & -v^y \\
v^y & -v^y & \frac{c^2 s + v^2}{\alpha} + \frac{v^2}{\alpha}
\end{bmatrix}
\]  

(4)

Here, \(v^r = \frac{\hbar m_y}{\bar{n}_d n_d} (\tilde{n}_+ \tilde{\partial}_r \tilde{\theta}_+ + \tilde{n}_- \tilde{\partial}_r \tilde{\theta}_- ) - \frac{n^2_+}{\bar{n}_d}, \quad v^y = \frac{\hbar m_y}{\bar{n}_d n_d} (\tilde{n}_+ \tilde{\partial}_y \tilde{\theta}_+ + \tilde{n}_- \tilde{\partial}_y \tilde{\theta}_- ) - \frac{n^2_+}{\bar{n}_d}\) and \(c^2_s = \sqrt{\alpha c^2 s} + c^2 \). Anisotropy makes metric \(g^\mu_\nu\) different from a scalar condensate [30] and gives an elliptic event horizon with \(c_s = \sqrt{c^2 + c^2_s}\).

BTZ metric: The acoustic metric determines the invariant acoustic interval, \(d\tilde{s}^2 = g_{\mu\nu} dx^\mu dx^\nu\) of sonic SOC-BEC black hole (for details [29]):

\[
d\tilde{s}^2 = \frac{n_d^2 (1 + \alpha)}{m m_y c^2_s} \left[ - \left( \frac{c^2_s}{1 + \alpha} + \left[ v^2 + \frac{v^2}{\alpha} \right] \right) dt^2 - 2 \left( v^x dx + \frac{v^y dy}{\alpha} \right) dt + (dx^2 + dy^2) \right]
\]  

(5)

with \(\alpha = 1 - \frac{L^2}{R^2}\).

To write \(d\tilde{s}^2\) in Eq.(5), as BTZ metric for RBH in 2+1 dimensions, we note that the corresponding line element of such a metric in \((R, \phi)\) coordinates where \(R = x(\alpha, \frac{\eta}{\alpha})\), is given as [14, 31]: \(d\tilde{s}^2 = g_{R\phi} dR^2 + 2g_{R\phi} dtd\phi + g_{RR} d\phi^2\). Accordingly, in polar coordinates \((R, \phi)\) and with transformations \(dt \rightarrow dt + \frac{R^2}{v^2 + c^2_s} dR, d\phi \rightarrow d\phi + \frac{R^2 v^2}{v^2 + c^2_s} dR\), Eq.(5) becomes

\[
d\tilde{s}^2 = \frac{n_d^2}{m m_y c_s^2} \left[ - \left( \frac{c^2_s}{1 + \alpha} + \left[ v^2 + \frac{v^2}{\alpha} \right] \right) dt^2 - 2 v^2 R dt d\phi + \frac{dR^2}{1 - \frac{v^2}{\alpha c^2_s}} + R^2 d\phi^2 \right]
\]  

(6)

similar to BTZ RBH metric where \(v^2 = |v| \cos \phi, \quad v^x = |v| \sin \phi\). The angular momentum per unit mass is given by \(g_{R\phi}\) component observed from the outside of a rotating SBH in analogy with the actual BH, at a radial distance \(R\), is \(J = v^2 R\). Schwarzschild metric coresponds to static case where the cross-terms of space and time (and hence, \(g_{t\phi}\)) does not appear. The “Acoustic Ergosurface” and “Acoustic Horizons” respectively can be defined from (more details in [29]) “\(g_{R\phi}\)’ and \(‘g_{RR}\)’ components which gives,

\[
\frac{\sqrt{v^2 R^2 + v^2 \phi^2}}{c_s} \leq \frac{c_s}{\sqrt{1 + \alpha}} \implies \quad |v| R \leq \frac{c_s}{\sqrt{1 + \alpha}}\]
\]  

(7)
the circular step, regulate 1 + 1 dimensional SBH [4–6]. This type of model densates as compared to the waterfall potential that sim- symmetric potential to accelerate a two dimensional con- initial position. The above potential is an azimuthally speed with which aperture of the step potential closes de- neous position, where \( v \) represents the location of the step. To simulate the RSBH through the Eq.(2) we now (1+)G−2≤(v^R^2+v^φ^2) defines the ergoregion as \( g_\eta \geq 0 \) for such a region. The non-zero \( g_\phi \) implies a local angular velocity for a non-rotating test particle [32, 33], \( \omega = -\frac{g_\phi}{g_\eta} = -\frac{\omega}{R} \).

To simulate the RSBH through the Eq.(2) we now time evolve this SOC-BEC in a two-dimensional time-dependent potential, (see Fig.1(top)) \( V_{2D}(r, t) = V(r) + V_{\text{step}}(r, t) \). Here \( V(r) = \frac{1}{2}m(\omega^2_r x^2 + \omega^2_\phi y^2) \) and \( V_{\text{step}}(r, t) = -V_s \Theta(r_s(t) - r) \), with \( V_s \sim 5 k_B \) nK is the circular step, \( r_s(t) = -v_s t + r_s(0) \) is its instanta- neous position, where \( v_s \sim 0.21 \text{ mm s}^{-1} \) is the constant speed with which aperture of the step potential closes decreasing its cross-sectional width, \( w(t) \) and \( r_s(0) \) is the initial position. The above potential is an azimuthally symmetric potential to accelerate a two dimensional condensate as compared to the waterfall potential that sim- ulate 1 + 1 dimensional SBH [4–6]. This type of model potential can be experimentally realized with the cur- rent available masking techniques [34] where dynamical potential to be experienced by the atoms is written on a digital micromirror device (DMD) (described in [7]). For our simulations, we considered SOC-BEC at the centre of the harmonic trap and then adiabatically closed the circular aperture width with time. Also \( \omega_r = \omega_\eta = 2\pi \times 4.5 \text{ Hz}, \omega_\phi = 2\pi \times 123 \text{ Hz}, N \sim 6000 \). For \(^{87}\text{Rb} \) the scattering length \( a \) is \( 5.1 \times 10^{-9} \), mass \( m = 1.44 \times 10^{-25} \text{Kg} \), characteristic length, \( x_s = 0.3407 \times 10^{-5} \text{m} \).

Time evolution of \( n_d \) for \( \frac{\omega_\eta}{\eta} = (a)0.4, (b)0.78 \) is shown in Fig.1(bottom) when width of the circular step potential is decreased. In either cases, the density modulation forms a set of concentric ring like fringes with alternating maxima and minima in an annular region with more anisotropy with the increase in \( \frac{\omega_\eta}{\eta} \). For demonstration purpose we also plotted the phase of one particular case in [29]. The sonic analogue of the BTZ black hole is formed in this annular region.

The density modulations in this region gets self amplified with time that is reminiscent of the similar density modulation in one-dimensional SBH in recent experimental and theoretical studies [4–6, 35, 36]. However, the relation of this self amplifying density modulation to self-amplifying Hawking radiation [29] and associated sonic black hole lasing requires a careful mode analysis [4, 36–38] that we plan to do in future [39]. For the present case even though the density modulation is mostly radial for the values chosen in our simulation, for higher anisotropy (Fig.1(b)), longer time simulation (80 ms) shows clear formation of interference fringes along the azimuthal direction. To investigate further we also plot the regions of subsonic and supersonic velocity corresponding to this density modulation (Fig.2(b1,b2)) using Eq.(3) [29]. We observed a stratified structure of subsonic and supersonic zones (Fig.2(b1,b2)) apart from the outermost and innermost boundary’s due to the modulated density pattern. These structures gets substantially modified with increasing anisotropy.

Fig.2(a1,a2) shows the plot of components of current vector superposed with the event horizon and ergosurface given by Eq.(7). For the present study \( v_\phi/v_R \ll 1 \), thus these two regions almost coincide within our numerical accuracy. However a closer inspection of Fig.2(a2) for the higher value of anisotropy shows the presence of azimuthal flow that leads to some separation between the the outer boundary of the ergo-region and the event hori- zon, indicating a relatively larger azimuthal velocity. The direction of the current vectors in the same figure also shows effect of anisotropy.

**Correlation function and Hawking temperature:** Motivated by the effect of anisotropy on the density modulation that shows strong azimuthal dependence at higher \( \frac{\omega_\eta}{\eta} \), we calculate the two-point, equal time connected density-density correlation function, \( G^{(2)}(r, r'; \zeta) = \langle \delta n_d(r, \zeta) \delta n_d(r', \zeta) \rangle \) along different azimuthal directions,
FIG. 2: (color online). (a1,a2) To illustrate the azimuthal flow, we plot $j$ for $\eta'/\eta=0.4, 0.78$ respectively at 80 ms. Length of arrows represent the magnitude of the current, $|j|$. Figures are superposed with the boundaries of the event horizon (red) and ergorsurface (black). Inset shows slight demarcation between the two. (b1,b2) shows the division of supersonic and subsonic regions for the parameters in (a1,a2); (c) shows the Hawking temperature and correlation function for various marked angles, $\zeta$ at 80 ms.

where $\delta n_d = n_d - <n_d>$ is the fluctuation in the density. The expectation values in the correlation functions are computed by taking an ensemble, of about 100 by considering variations in the number of atoms in the condensate which can always occur in BEC experiments (For detailed methodology, see [29, 35]) and $\zeta$ is angle in x-y plane. Whereas, in usual analogue black hole models, the correlation behaviour is isotropic in space at a given time, here at increased anisotropy $\eta'/\eta = 0.78$, we see a distinct azimuthal variation of the correlation function (Fig.2), that can experimentally characterise such analogue BTZ BH in a SOC-BEC.

The Hawking temperature $T_H$, that is calculated from the surface gravity $g_H$ (see [29]) at the outer and inner horizon of this analogue BTZ black hole can be calculated by generalising the methodology used for sonic black hole in scalar condensate [4, 9] in recent experiment, namely

$$T_H = \frac{\hbar}{2\pi k_B} \left( \frac{d}{dn} \left[ \frac{c_s}{\sqrt{1+\alpha}} - \mathbf{v'} \cdot \hat{n} \right] + \frac{d}{dn} \left[ g(\eta, \eta') \cdot \hat{n} \right] \right) \bigg|_{h_z}$$

where $k_B$ is the Boltzmann’s constant and ‘n’ corresponds to spatial coordinate normal to the horizon ($h_z$). For an arbitrary flow, the “surface gravity” $g_H$ of an acoustic horizon depends on the normal derivative of the local speed of sound and the normal component of the flow velocity at the horizon [9]. Here, $\mathbf{v'} = \frac{\hbar}{2imn_d} \left( \Psi^\dagger \nabla \Psi + \Psi^T \nabla \Psi^* \right)$ and $g(\eta, \eta') = \frac{\hbar}{m_d} \Psi^\dagger \partial_z \Psi + \frac{\hbar}{2imn_d \eta} \left( \Psi^\dagger \partial_\eta \Psi + \Psi^T \partial_\eta \Psi^* \right)$. $T_H$ and $g_H$ are expected to be direction dependent due to anisotropic behaviour of the sound and the flow velocities. This is demonstrated in Fig. 2(c) by plotting the spatial distribution of Hawking temperature at 80 ms over the inner and outer horizon (details in [29]).

In conclusion we show that a SOC-BEC in a suitable laser driven potential can realise an analogue rotating SBH of BTZ type, in the absence of any external rotation. The features of such analogue BTZ black hole includes strong anisotropy in the self amplifying density modulation, inner and outer horizon as well as ergo-surface and a stratified transition from supersonic to subsonic region through self amplifying density modulations that strongly depends on the spin-orbit coupling parameters. We expect that our studies will lead to interesting experiments in ultra cold atomic system to explore analogue RBH and their behaviour in future.

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[29] Supplementary information contains intermediate steps of certain derivations used in the main paper. Sec I provides the system description and a detailed derivation of Eq. (2) in the main paper. Section II provides the derivation of Eq. (3) in the main paper and the velocity expressions. Section III provides the details of hydrodynamic formalism that leads to Eq. (1) in the main paper. Section IV provides a discussion on the correlation and Hawking temperature plotted in Fig. 2(c).
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