Design of the control law of longitudinal attitude for Tail-sitter UAV

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Abstract: The longitudinal attitude control of a type of Tail-sitter UAV is studied in this paper. In order to solve the singular problem of pitch angle during the tilting process of fuselage, the double Euler angle algorithm is used for modeling. Fuzzy self-tuning PID control method and L₁ adaptive control method are used to design the longitudinal attitude controller respectively to counteract the susceptibility to disturbance during the flight, especially in tilting process. The numerical simulation is carried out through MATLAB, and the simulation results show that: This controller can overcome the model parameter changes and other uncertain disturbances to the Tail-sitter UAV during the flight.

1. Introduction

Fixed-wing UAV has the characteristics of high flight efficiency, high speed, simple system structure, but it needs a runway or launch and recovery units during the process of both takeoff and landing[1]. By introducing vertical takeoff and landing technology, fixed-wing UAV can have the characteristics of vertical takeoff, spot hover, vertical landing of multi-rotor UAV and high-speed level cruise of fixed-wing UAV at the same time [2]. The Tail-sitter UAV realizes the transformation between vertical flight mode and horizontal flight mode through large angle pitch maneuver and pull control. Its structure is similar to that of fixed-wing aircraft [3].

The flight speed, pitch angle, angle of attack and aerodynamic parameters of the Tail-sitter UAV will change greatly during the large angle pitch maneuver. As a result, it is difficult to control and maintain the flight attitude of the UAV stably [3].

Considering the singular problem of pitch angle in vertical flight, two SDOF dynamic models of UAV based on Euler angle are established. Fuzzy parameter self-tuning PID control method and L₁ adaptive control method are used to design longitudinal attitude controllers in vertical and near-vertical flight phases as well as horizontal and near-horizontal flight phases respectively. Finally,
the flight simulation of the designed flight control system is made in MATLAB, and the results show that the control effect is good.

2. Flight principle and dynamic model of Tail-sitter UAV

In this paper, the axisymmetric configuration mode is used for the Tail-sitter UAV. Specifically, a propeller is installed on each wingtip of four wings. Two rudder control surfaces are installed at the bottom of the wing. The aircraft is controlled by the vertical flight mode controller when the pitch angle is greater than 45 degrees, and the aircraft is controlled by the horizontal flight mode controller when the pitch angle is less than or equal to 45 degrees.

Table 1. UAV Control Strategy

| Movement mode | Vertical flight mode control strategy | Horizontal flight mode control strategy |
|---------------|--------------------------------------|----------------------------------------|
| Pitching movement | Motor differential | Linkage of two rudder control surfaces |
| Rolling movement | Motor differential | Differential of two rudder control surfaces |
| Yawing movement | Motor differential | Coordinate bank-to-turn |

Two reference frames are set up in this paper: geographic coordinate system (North East Down) $F_{e1}$, vertical geographic coordinate system (Up East North) $F_{e2}$, body axis system $F_b$, and air path axis system $F_{aero}$ are used in a way not different from normal use. The horizontal Euler angle is determined by the relationship between $F_{e1}$ and $F_b$ to describe the attitude of aircraft in the horizontal and near-horizontal flight phases; the vertical Euler angle is determined by the relationship between $F_{e2}$ and $F_b$ to describe the attitude of aircraft in the vertical and near-vertical flight phases. The mutual transformation relationship of vertical Euler angle and horizontal Euler angle is[4]:

$$\theta = \sin^{-1}(\cos \psi, \cos \theta); \phi = \sin^{-1} \left( \frac{\sin \psi, \cos \theta}{\cos \theta} \right)$$

$$\varphi = \sin^{-1} \left( \cos \psi, \sin \theta \sin \phi + \sin \psi, \cos \theta \right)$$

(2.1)

Based on the vertical Euler angle, the SDOF mathematical model of UAV in the body axis system $F_b$ is established as follows:

$$\dot{\phi} = p \sec \theta \cos \phi \psi - q \sec \theta \sin \phi \psi$$

$$\dot{\psi} = p \sin \phi \psi + q \cos \phi \psi$$

$$\dot{\psi}_e = -p \tan \phi \psi \cos \psi + q \tan \phi \psi \sin \psi + r$$

(2.2)

$$\dot{u} = rv - qw + g \cos \theta \cos \psi + \frac{1}{m} \left( T_{\text{motor}} + T_{\text{aero}} \right)$$

$$\dot{v} = pw - ru + g \cos \theta \sin \psi + \frac{1}{m} \left( T_{\text{motor}} + T_{\text{aero}} \right)$$

$$\dot{w} = qu - pv - g \sin \theta + T_{\text{aero}}$$

(2.3)

$$\dot{\rho} = \frac{(I_{y} - I_{z}) \rho - f_{x}^{2}}{I_{x} - I_{z}} \rho + \frac{1}{T_{e}} (M_{\text{motor}} + M_{\text{aero}})$$

$$\dot{\theta} = \frac{I_{z} - I_{y}}{I_{y}} \frac{1}{T_{e}} (M_{\text{motor}} + M_{\text{aero}})$$

$$\dot{\psi} = \frac{I_{z} - I_{y}}{I_{y}} \frac{p_{x}}{T_{e}} (M_{\text{motor}} + M_{\text{aero}})$$

(2.4)
Based on the horizontal Euler angle, the SDOF mathematical model of UAV in the body axis system \(F_b\) can also be derived from the equation (2.1) and equation (2.2–2.4).

3. Design of flight controller
In the transition flight phase, the fuzzy parameter self-tuning PID method and \(L1\) adaptive control method are used for the control of pitch angle respectively [5].

3.1 Design of longitudinal attitude controller for vertical flight mode
The control system is mainly composed of fuzzy parameter self-tuning controller and PID controller[6]. The inputs of the fuzzy parameter tuning part consist of the deviation \(e\) and the deviation change rate \(e_c\), and the outputs are the corrections, \(\triangle K_p\), \(\triangle K_i\), \(\triangle K_d\) of the PID parameters[7].

Referring to the fuzzification method in reference [8] and combining with the actual situation of the Tail-sitter UAV in this paper, a fuzzy language set \{negative large, negative medium, negative small, zero, positive small, positive medium, positive large\} is set up, and the universe is divided into seven parts evenly to describe. Since the pitch angle of UAV is limited to 90 degrees to - 90 degrees in radians, the universes of fuzzy variables \(e\) and \(e_c\) are both set to \{-3,3\}, and the fuzzy universes of \(\triangle K_p\), \(\triangle K_i\), \(\triangle K_d\) are \{-10,10\}, \{-3,3\}, \{-1,1\} respectively. In fuzzification, it is required to multiply the input variable by the corresponding quantization factor. In this paper, the quantization factors of the fuzzy parameter self-tuning PID controller are selected as \(k_{e}=1\), \(k_{ec}=1\), and the output scaling factors are \(k_{pu}=1.1\), \(k_{iu}=0.1\), \(k_{du}=0.1\).

Using the fuzzy control rule in reference[6], the triangle membership curve is used to describe each fuzzy subset in the universe. By mamdani reasoning, each output control variable has 49 fuzzy rules. The center-of-gravity method is used to solve the fuzziness as below: 
\[
k = \frac{\sum_{i=1}^{n} u_i \lambda(u_i)}{\sum_{i=1}^{n} \lambda(u_i)}.
\]

The initial PID parameters are obtained by adjusting by critical ratio method. The parameter self-tuning PID controller flow chart is shown in Figure 1.

![Figure 1. Flow chart of fuzzy parameter self-tuning PID controller](image)

3.2 Design of longitudinal attitude controller in horizontal flight mode
The simplified results of the pitch channel model for the horizontal Euler angle SDOF model based on the method in reference [9] are as follows:
\[
\begin{align*}
\dot{\theta} &= -a_{\theta\theta}\dot{\theta} - a_{\theta\varphi}\dot{\varphi} + a_{\theta\rho}\delta + d_{\theta}, \\
\dot{a}_{\theta\theta} &= -a_{\theta\varphi}\dot{\varphi} - a_{\theta\rho}\delta + \dot{d}_{\theta}, \\
\dot{a}_{\theta\varphi} &= -a_{\theta\rho}\delta + \dot{d}_{\theta}, \\
\dot{a}_{\theta\rho} &= -q_s s_c C_{J\alpha} - \frac{1}{I_s} - \frac{1}{I_s} pr + q_s s_c \left( C_{m\varphi} - C_{m\varphi\gamma} \right) - \frac{c}{2V_s} d_{\theta}, \\
\dot{d}_{\theta} &= -r \sin \phi + q (\cos \phi - 1).
\end{align*}
\]

(3.2)

This paper designs the L1 adaptive controller by Referring the L1 adaptive control theory in reference [10]. Taking \( x_\theta = [\theta, \dot{\theta}] \), write the pitch channel model in formula (3.2) as the standard form of L1 adaptive control:

\[
\begin{align*}
\dot{x}_\theta(t) &= A_\theta x_\theta(t) + b_\theta \left[ u(t)x_\varphi(t) + \theta^\top(t)x_\rho(t) + \delta(t) \right]
\end{align*}
\]

(3.3)

Taking: \( A_\theta = \begin{bmatrix} 0 & 1 \\ -a_{\theta\varphi} & -a_{\theta\rho} \end{bmatrix} \), \( b_\theta = \begin{bmatrix} 0 \\ a_{\theta\rho} \end{bmatrix} \), \( c_\theta = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \) as the system matrix, input matrix and output matrix. \( w(t) \) stands for input gain; \( \theta^\top(t)x_\rho(t) \) stands for unknown part related to state variable; \( \delta(t) \) stands for unknown part independent of state variable. In this paper, the flight speed \( V_s = 25m/s \) is taken as the operating point, then: \( A_\theta = \begin{bmatrix} 0 & 1 \\ -299.6973 & -14.7633 \end{bmatrix} \), \( b_\theta = \begin{bmatrix} 0 \\ -171.481762 \end{bmatrix} \), \( c_\theta = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \). According to the parameters of the Tail-sitter UAV, the projection boundary of unknown parameters are selected as follows: \( w = [0.5, 1.5] \); \( \delta = 4.6 \); \( \theta = [\theta_1, \theta_2]^\top \in \mathbb{R}^2 \); \( \theta_1 = [-0.08, 0.08], \theta_2 = [-0.6, 0.6] \). So: \( L = \max_{\theta \in \mathcal{B}} \| \theta \| = 1.3 \). The input of L1 adaptive controller contains two parts: state feedback input and adaptive control input: \( u_\varphi(t) = u_{\varphi}(t) + u_{\varphi\rho}(t) \cdot u_\rho(t) = -K_m^\top x(t) \), where \( K_m \in \mathbb{R}^2 \) makes \( A_{\theta \varphi} = A - bK_m^\top \) be Hurwitz matrix. In this paper, the feedback gain matrix \( K_m = [-9.3452, -7.6673] \) is obtained by using the LQR method in the optimal control theory, and then the closed loop system matrix is obtained:

\[
A_{\theta \varphi} = A - bK_m^\top = \begin{bmatrix} 0 & 1 \\ -1902.2 & -1329.6 \end{bmatrix}
\]

(3.4)

The pitch channel control model is converted to:

\[
\begin{align*}
\dot{x}_\theta(t) &= A_{\theta \varphi} x_\theta(t) + b_\theta \left[ u(t)x_\varphi(t) + \theta^\top(t)x_\rho(t) + \delta(t) \right]
\end{align*}
\]

(3.5)

The state estimator for pitch channel:

\[
\begin{align*}
\dot{\hat{x}}_\theta(t) &= A_{\theta \varphi} \hat{x}_\theta(t) + b_\theta \hat{w}(t)x_\varphi(t) + \hat{\theta}(t)x_\rho(t) + \delta(t), \\
\dot{\hat{\theta}}(t) &= c_\theta^\top \hat{x}_\theta(t),
\end{align*}
\]

(3.6)
Where: \( \hat{x}_a(t), \hat{u}(t), \hat{\theta}(t), \hat{\delta}(t) \) are the estimated values of \( x_a(t), u(t), \theta(t), \delta(t) \) respectively, and the parameter adaptive law based on projection operator is:

\[
\begin{align*}
\hat{\theta}(t) &= \Gamma \text{Proj} \left[ \hat{\theta}(t), -\hat{x}_a^T(t) P_b \hat{x}_a(t) \right], \hat{\theta}(0) = \hat{\theta}_0 \\
\hat{\sigma}(t) &= \Gamma \text{Proj} \left[ \hat{\sigma}(t), -\hat{x}_a^T(t) P_b \hat{\sigma}(t) \right], \hat{\sigma}(0) = \hat{\sigma}_0 \\
\hat{\phi}(t) &= \Gamma \text{Proj} \left[ \hat{\phi}(t), -\hat{x}_a^T(t) P_b \hat{\phi}(t) \right], \hat{\phi}(0) = \hat{\phi}_0
\end{align*}
\]

(3.7)

Where: \( \hat{x}_a(t) = \dot{x}(t) - x(t) \) is the tracking error of state observer and dynamic system; \( \hat{\theta}_0, \hat{\sigma}_0, \hat{\phi}_0 \) are initial values of \( \hat{\theta}(t), \hat{\sigma}(t), \hat{\phi}(t) \) respectively; \( P = P^T > 0 \) is the solution of lyapunov equation 

\[ A_x^T P + PA_x = -Q, \quad Q = Q^T > 0 \]

\[
\text{Proj}(\theta, y) = \begin{cases} 
0, & f(\theta) < 0 \\
y, & f(\theta) \geq 0, \nabla f(\theta)^T y > 0 \\
y, & f(\theta) \geq 0, \nabla f(\theta)^T y \leq 0
\end{cases}
\]

(3.8)

Where: \( f(\theta) = (\varepsilon_\theta + 1)\theta^T \theta - \theta^2_{max} \), \( \theta_{max} \) is the normative constraint of vector \( \theta \); \( \varepsilon_\theta \) is the projection boundary; \( \nabla f(\theta) \) is the gradient vector at \( \theta \). In this paper, we set \( Q \) to be the identity matrix \[ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \], calculate and obtain \( P = \begin{bmatrix} 1.0652 & 0.00002628 \\ 0.00026285 & 0.00037625 \end{bmatrix} \); set \( \Gamma = 1000.00 \). Thus the form of adaptive control input is obtained as follows: \( u_{ad}(S) = C(S)(\bar{\eta}(S) - k_r r(S)) \), where:

\[
k_r = \frac{-1}{c_\phi A_{ad} b_0} = -11.0927, \quad \bar{\eta}(S) = \hat{\eta}(S) u_{ad}(t) + \hat{\delta}(S) x_a(S) + \hat{\sigma}(S)
\]

\( C(S) \) is the low pass filter. The low pass filter as \( C(S) = \frac{\omega KD(S)}{1 + \omega KD(S)} \), \( \forall \omega \in \Omega_\omega \), where: \( K \) is the adaptive feedback gain and greater than zero; \( D(S) \) is a strictly regular transfer function. At the same time, it is required that the low pass gain of low pass filter \( C(0) = 1 \). In order to simplify the design, take \( D(S) = \frac{1}{S} \), then the low pass filter is \( C(S) = \frac{\omega K}{S + \omega K} \). According to L1 adaptive stability theory [10], in order to ensure the stability of the system, we need to satisfy: \( \lambda = \left\| G(S) \right\|_{\infty} L < 1 \), where \( G(S) = (SI - A_{ad})^{-1} b_0 \frac{S}{S + \omega K} \). To ensure the stability of the system, here we set \( K = 20 \). Thus the design of L1 adaptive controller is completed.

4. Simulation results and analysis

Initial conditions of simulation: \( V_0 = 0 \text{ m/s} \), \( h = 0 \), \( \theta_0 = 0 \), \( \phi_0 = 90^\circ \), \( \phi_0 = \psi_0 = \phi_0 = \psi_0 = 0^\circ \). Vertical
takeoff begins at 0 seconds and transits to horizontal flight mode. At 15 seconds, the command of transformation from level flight to vertical flight mode is given.

It can be seen from the simulation results that the tail-sitter UAV completes the transition from vertical to horizontal flight in 6 seconds, and cruises at a speed of 25m/s. It completes the transition from horizontal to vertical flight in 18 seconds, after which the airspeed gradually reaches to zero. Under the change of model parameters, the change of pitch angle, airspeed and altitude during the whole flight process is relatively stable.

5. Conclusion
In this paper, the fuzzy parameter self-tuning PID controller and $L_1$ adaptive controller are designed respectively for the longitudinal attitude control of the Tail-sitter UAV in vertical and horizontal flight modes. The simulation results show that the longitudinal attitude controller designed in this paper can overcome the model parameter change problem and uncertain disturbances caused by the flight condition changes, and the control effect is good.

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