Applications of a nonlinear evolution equation I: the parton distributions in the proton

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Abstract

The parton distributions in the proton are evaluated dynamically using a nonlinear QCD evolution equation - the DGLAP equation with twist-4 (the GLR-MQ-ZSR) corrections - starting from a low scale $\mu^2$, where the nucleon consists of valence quarks. We find that the negative nonlinear corrections can improve the perturbative stability of the QCD evolution equation at low $Q^2$. Our resulting parton distributions of the proton with four free parameters are compatible with the existing databases. We show that the sea quark distributions exhibit a positive and flattish behavior at small $x$ and low $Q^2$. This approach provides a powerful tool to connect the quark models of the hadron and various non-perturbative effects on them at scale $\mu^2$ with the measured structure functions at high scale $Q^2 >> \mu^2$.

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1 Introduction

The parton distributions in nucleon are the important knowledge of high energy physics. Currently available parton distributions were extracted from the existing experimental data with the linear QCD evolution equation—the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation [1]. The solutions of the QCD evolution equations depend on the initial parton distributions at a low starting scale $\mu^2$. There are two different choices for the input distributions: (1) In the global analysis, such as MRST [2] and CTEQ [3], the starting point is fixed at an arbitrary scale $Q_0^2 > 1\text{ GeV}^2$ and the corresponding input parton distributions are parameterized by comparing with the measured data at $Q^2 > Q_0^2$. These input distributions are irrelevant to any physical models, even allow negative input gluon distributions; (2) In dynamical models, the parton distributions at $Q^2 > 1\text{ GeV}^2$ are QCD radiatively generated from an imaginary intrinsic parton distributions at an optimally determined $Q_0^2 < 1\text{ GeV}^2$ according to a nucleon model. For example, it is well known that the nucleon is consisted of three constituents at very low $Q^2$. A natural attempt, which was firstly proposed in 1977, is to assume that the nucleon consists of valence quarks at a low static point $\mu^2$ (but is still in the perturbative region $\alpha_s(\mu^2)/2\pi < 1$ and $\mu > \Lambda_{\text{QCD}}$), and the gluons and sea quarks are radioactively produced at $Q^2 > \mu^2$ [4–6]. This input distribution allows us to construct a complete QCD picture of the proton [7]. However, such natural input is failed due to too steep behavior of the predicted parton distributions at small $x$. Instead of the natural input, Reya, Glück and Vogel (GRV) [8] added the valance like sea quark and gluon distributions to the input parton distributions at a little larger $Q^2$ scale. The predictions of the GRV model are compatible with the data at $Q^2 > 4\text{ GeV}^2$ and $x > 10^{-4}$.

All the above mentioned approaches determining the parton distributions use the DGLAP equation. Comparing with the natural input distributions, the valence-like distributions of the gluon and sea quarks in the GRV-model can slow down the evolution of the DGLAP equation at low $Q^2$ and reach the experimental results, since the evolution region of the valence like distributions is sizeably larger. On the other hand, it is well known that the contributions of the higher twist corrections become important at $Q^2 < 1\text{ GeV}^2$, which are neglected in the DGLAP equation. The correlation among initial partons can not be neglected towards small $x$ and low $Q^2$. The negative corrections of the parton recombination also slow down the partons evolution. These nonlinear effects can be calculated in the perturbative QCD. The nonlinear corrections of the gluon recombination to the DGLAP equation were firstly derived by Gribov, Levin and Ryskin [9] and by Mueller and Qiu [10] in the double leading logarithmic (DLL) approximation. This evolution equation was re-derived to include all parton recombination at whole $x$ region by Zhu, Shen and Ruan [11,12] in leading logarithmic (LL($Q^2$)) approximation. We refer to this last version of the nonlinear corrections as the Gribov-Levin-Ryskin-Mueller-Qiu-
The success of the GRV model inspires us to use the natural input to replace the valence like input in the GRV model since we consider the nonlinear corrections of the parton recombination. Our main results are: (i) we find that the GLR-MQ-ZSR equation suppresses the fast increase of the sea quark- and gluon-densities using the natural input and has similar results as the GRV(98LO) \cite{13} at $x > 10^{-4}$ and $Q^2 > 4 \text{ GeV}^2$; (ii) we predict that the parton distributions at $Q^2 < 1 \text{ GeV}^2$ are positively defined, particularly, the sea quark distributions appear a plateau at small $x$ and low $Q^2$, indicating Pomeron-like behavior \cite{14}; (iii) our input quark distributions are compatible with the valence quark distributions predicted by effective chiral quark model \cite{15}; (iv) this evidence of the parton recombination existing in the standard QCD evolution provides a possible dynamical way to explore the nuclear shadowing effects.

The organization of this paper is as follows. We present the GLR-MQ-ZSR corrections to the DGLAP equation in Sec. 2. Using the nonlinear QCD evolution equation and the natural input distributions, we calculate the evolution of the parton distributions in the proton, and the corresponding parameters are discussed in Sec. 3. Our resulting parton distributions comparing with the experimental data and the GRV (LO)-database are presented in Sec. 4. In Sec. 5 the proton structure functions are extended to very low $Q^2$. Section 6 is discussions and summary, where the applicability of the GLR-MQ-ZSR evolution equation at the low $Q^2$ is discussed.
2 The nonlinear QCD evolution equation

We denote \( f_{v_j}(x, Q^2) \) (j=u,d) for valence quark distributions, \( f_{q_i}(x, Q^2) \) (i=u,d,s) for sea quark distributions, \( f_{\bar{q}_i}(x, Q^2) \) (i=u,d,s) for anti-sea quark distributions and \( f_g(x, Q^2) \) for gluon distribution. We define \( \Sigma(x, Q^2) \equiv \sum_j f_{v_j}(x, Q^2) + \sum_i f_{q_i}(x, Q^2) + \sum_i f_{\bar{q}_i}(x, Q^2) \) and \( \Sigma_{sea}(x, Q^2) \equiv \sum_i f_{q_i}(x, Q^2) + \sum_i f_{\bar{q}_i}(x, Q^2) \).

The QCD evolution equation including the GLR-MQ-ZSR corrections arisen from the parton (gluon and quarks) recombination in LL\((Q^2)\) approximation reads [11]

\[
Q^2 \frac{dx f_{v_j}(x, Q^2)}{dQ^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} P_{qq}(z) x f_{v_j}(y, Q^2)
\]

\[- \frac{\alpha_s(Q^2)}{2\pi} x f_{v_j}(x, Q^2) \int_0^1 dz P_{qq}(z)
\]

\[- \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_x^{1/2} \frac{dy}{y} x P_{qq\to qg}(z)[y f_g(y, Q^2) y f_{v_j}(y, Q^2)]
\]

\[+ \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^x \frac{dy}{y} x P_{qq\to qg}(z)[y f_g(y, Q^2) y f_{v_j}(y, Q^2)]
\]

\[- \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_x^{1/2} \frac{dy}{y} x P_{qq\to qg}(z)[y \Sigma_{sea}(y, Q^2) y f_{v_j}(y, Q^2)]
\]

\[+ \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^x \frac{dy}{y} x P_{qq\to qg}(z)[y \Sigma_{sea}(y, Q^2) y f_{v_j}(y, Q^2)],\]

for valence quarks, where \( z = x/y \), the factor \( 1/(4\pi R^2) \) is from normalizing two-parton distribution, \( R \) is the correlation length of two initial partons,

\[
Q^2 \frac{dx f_{\bar{q}_i}(x, Q^2)}{dQ^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} P_{qq}(z) x f_{\bar{q}_i}(y, Q^2)
\]

\[- \frac{\alpha_s(Q^2)}{2\pi} x f_{\bar{q}_i}(x, Q^2) \int_0^1 dz P_{qq}(z)
\]

\[- \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_x^{1/2} \frac{dy}{y} x P_{gg\to q\bar{q}}(z)[y f_g(y, Q^2)]^2
\]

\[+ \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^x \frac{dy}{y} x P_{gg\to q\bar{q}}(z)[y f_g(y, Q^2)]^2\]
\[- \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_x^{1/2} \frac{dy}{y} xP_{q\pi\rightarrow q\pi}(z) y f_{q_i}(x,Q^2) y f_{\pi_i}(y,Q^2) \]
\[+ \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^x \frac{dy}{y} xP_{q\pi\rightarrow q\pi}(z) y f_{q_i}(x,Q^2) y f_{\pi_i}(y,Q^2) \]
\[- \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_x^{1/2} \frac{dy}{y} xP_{qg\rightarrow qg}(z) y [\Sigma(x,Q^2) - f_{q_i}(x,Q^2)] y f_{\pi_i}(y,Q^2) \]
\[+ \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^x \frac{dy}{y} xP_{qg\rightarrow qg}(z) y [\Sigma(x,Q^2) - f_{q_i}(x,Q^2)] y f_{\pi_i}(y,Q^2) \]
\[- \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_x^{1/2} \frac{dy}{y} xP_{gg\rightarrow gg}(z) [y f_g(y,Q^2) y f_{\pi_i}(y,Q^2)] \]
\[+ \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^x \frac{dy}{y} xP_{gg\rightarrow gg}(z) [y f_g(y,Q^2) y f_{\pi_i}(y,Q^2)], \]

for sea quark distributions and

\[Q^2 \frac{dx f_g(x,Q^2)}{dQ^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} P_{qg}(z) x \Sigma(y,Q^2) \]
\[+ \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} P_{gg}(z) x f_g(y,Q^2) \]
\[- f \frac{\alpha_s(Q^2)}{2\pi} x f_g(x,Q^2) \int_0^1 dz P_{gg}(z) \]
\[+ \frac{\alpha_s(Q^2)}{2\pi} x f_g(x,Q^2) \int_0^1 dz P_{gg}(z) \]
\[- \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_x^{1/2} \frac{dy}{y} xP_{gg\rightarrow gg}(z) [y f_g(y,Q^2)]^2 \]
\[+ \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^x \frac{dy}{y} xP_{gg\rightarrow gg}(z) [y f_g(y,Q^2)]^2 \]
\[- \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_x^{1/2} \frac{dy}{y} xP_{qg\rightarrow qg}(z) \sum_{i=1}^f [y f_{\pi_i}(y,Q^2)]^2 \]
\[+ \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^x \frac{dy}{y} xP_{qg\rightarrow qg}(z) \sum_{i=1}^f [y f_{\pi_i}(y,Q^2)]^2 \]
\[- \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_x^{1/2} \frac{dy}{y} xP_{gg\rightarrow gg}(z) [y \Sigma(y,Q^2) y f_g(y,Q^2)] \]
\[+ \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^x \frac{dy}{y} xP_{gg\rightarrow gg}(z) [y \Sigma(y,Q^2) y f_g(y,Q^2)], \]  

(1)
for gluon distribution. In the above equations the contributions of the negative nonlinear (shadowing) terms vanish in $0.5 < x < 1$ due to momentum conservation. The un-
regularized DGLAP splitting kernels in the linear terms are [16]

$$
P_{gg}(z) = 2C_A \left[ z(1-z) + \frac{1-z}{z} + \frac{z}{1-z} \right],
$$

$$
P_{gq}(z) = C_F \frac{1 + (1-z)^2}{z},
$$

$$
P_{qq}(z) = C_F \frac{1 + z^2}{1-z},
$$

$$
P_{qg}(z) = T_R \left[ z^2 + (1-z)^2 \right],
$$

where $C_A = N_c = 3$, $C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}$, $T_R = \frac{1}{2}$. The recombination functions in the nonlinear terms are [12]

$$
P_{gg\rightarrow gg}(z) = \frac{9}{64} \frac{(2y-x)(72y^4 - 48y^3x + 140y^2x^2 - 116yx^3 + 29x^4)}{y^5x},
$$

$$
P_{gg\rightarrow gq}(z) = \frac{1}{48} \frac{(2y-x)^2(18y^2 - 21yx + 14x^2)}{y^5},
$$

$$
P_{gq\rightarrow gg}(z) = \frac{2}{9} \frac{(2y-x)^2}{y^3},
$$

$$
P_{gq\rightarrow qq}(z) = \frac{1}{27} \frac{(2y-x)^2(3y^2 + yx + 3x^2)}{y^5},
$$

$$
P_{qq\rightarrow gg}(z) = \frac{1}{72} \frac{(2y-x)(16y^2 + 20yx + 25x^2)}{y^4},
$$

$$
P_{qq\rightarrow qg}(z) = \frac{8}{27} \frac{(2y-x)(-10y^2 + 5yx + 2x^2)}{y^3x}.
$$

(3)
3 Natural initial distributions and free parameters fitting

The solutions of the QCD evolution equations for the parton distributions depend on
the initial parton distributions at a low scale \( Q^2 = \mu^2 \). An ideal and simple assumption
is that the nucleon consists entirely of three valence quarks at \( \mu^2 \) and the gluon and sea
distributions at \( Q^2 > \mu^2 \) are generated radioactively. This naive model was firstly pro-
posed in Refs. [4–6] and it successfully predicts that at low \( Q^2 \sim 1 \text{ GeV}^2 \) about 50% of
the nucleon momentum is already carried by gluons, which agrees with the experimen-
tal results. Unfortunately, the distributions of the sea quarks and gluon predicted by
this natural input and the DGLAP equation are too steep. Instead of the simple input
distributions, Glück, Reya and Vogt (GRV) [8] assumed that the nucleon has valance
quarks and special valence-like sea quarks and valence-like gluon at a starting point at
\( Q^2 \simeq 0.2 \sim 0.3 \text{ GeV}^2 \). The GRV input in the DGLAP equation nicely predicts the parton
distributions in a broad kinematical range.

Now let us go back to the natural input distribution but considering the GLR-MQ-
ZSR equation. Using Eqs. (1-3) for the second moments of the distributions and the
measured momentum of the valence quark distributions at a higher \( Q^2 \), we obtain the
starting point \( \mu^2 = 0.064 \text{ GeV}^2 \) (with \( \Lambda_{QCD} = 0.204 \text{ GeV} \) for \( f=3 \) flavors). This value is
similar to the previous estimation in Refs. [4–6], where the DGLAP equation is used. The
reason is that the contributions of the nonlinear (shadowing and antishadowing) terms to
the \( Q^2 \)-evolution of the momentum fractions in the GLR-MQ-ZSR equation vanish due
to the momentum conservation [17]

\[
-\frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_0^{1/2} dx \int_x^{1/2} dy \int_0^{1/2} dy y x P_{gg \rightarrow qg}(z) \left[ y f_g(y, Q^2) y f_v(y, Q^2) \right]
\]

\[
+ \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_0^{1/2} dx \int_x^{1/2} dy \int_0^{1/2} dy y x P_{gg \rightarrow qg}(z) \left[ y f_g(y, Q^2) y f_v(y, Q^2) \right]
\]

\[
+ \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_{1/2}^{1} dx \int_{x/2}^{1/2} dy \int_0^{1/2} dy y x P_{gg \rightarrow qg}(z) \left[ y f_g(y, Q^2) y f_v(y, Q^2) \right] = 0. \tag{4}
\]

According to the natural input distribution,

\[
f_g(x, \mu^2) = 0, \quad f_{q_i}(x, \mu^2) = f_{\bar{q}_i}(x, \mu^2) = 0. \tag{5}
\]

We choose a minimum free parameter scheme for the typical valence quark distributions

\[
x f_{v_u}(x, \mu^2) = A_u x^{B_u} (1 - x)^{C_u},
\]

\[
x f_{v_d}(x, \mu^2) = A_d x^{B_d} (1 - x)^{C_d}, \tag{6}
\]
which satisfy the momentum sum rule

\[ \int_0^1 dx \left[ f_{v_u}(x, \mu^2) + f_{v_d}(x, \mu^2) \right] = 1. \quad (7) \]

and the normalization conditions

\[ \int_0^1 dx f_{v_u}(x, \mu^2) = 2, \quad \int_0^1 dx f_{v_d}(x, \mu^2) = 1. \quad (8) \]

Fitting to experimental data, we obtain following values for the proton \( A_u = 30.39, \ B_u = 2.07, \ C_u = 2.28, \ A_d = 12.42, \ B_d = 1.45, \) and \( C_d = 4.0. \)

The corresponding input distributions are shown in Fig. 1. The free parameter \( R \) in Eq.(1) depends on the geometric distributions of partons inside the proton. \( R \approx 5 \text{ GeV}^{-1} \) (or \( R < 5 \text{ GeV}^{-1} \)) when the partons distribute uniformly (or non-uniformly) in the proton. We take \( R = 4.17 \text{ GeV}^{-1} \) for best fitting to the data.

![Figure 1: The natural input distributions in the proton at the initial scale \( \mu^2 = 0.064 \text{ GeV}^2 \).](image)
4 Parton distributions in the proton

Since the four free parameters are fixed, now it is straightforward to calculate the evolution of the parton distributions in the proton using Eqs. (1-3). The $x$ and $Q^2$ dependence of our predicted structure functions and comparisons with the data [18-22] are shown in Figs. 2 and 3. Our results are generally lower than the experimental data at $x < 10^{-4}$. It implies that the Balitsky- Fadin-Kuraev-Lipatov (BFKL) correction [23] is not negligible in such small $x$ range.

![Graph showing the comparison of predicted $x$-dependence of $F_2^p(x, Q^2)$ with HERA data at small $x$.]

Figure 2: Comparisons of our predicted $x$-dependence of $F_2^p(x, Q^2)$ (solid curves) with HERA data [18, 19] at small $x$.

The comparisons of two dynamically generated parton distributions of the proton (i.e., using the GLR-MQ-ZSR equation with the natural input and using the DGLAP equation with the GRV(98LO) input) are shown in Figs. 4, 5, 6, and 7. In Figs. 6 and 7, we plot the parton distributions in the linear DGLAP equation with the natural input. One can find that the valence like input distributions in the GRV model are equivalent to the effective description of the nonlinear parton recombination.

We try to explore the parton distributions down to $Q^2 < 1$ GeV$^2$ at small $x$ since the GLR-MQ-ZSR equation includes the contributions of the higher twist corrections. Our predicted proton structure functions are shown in Fig. 8 where we plot the contributions of the Regge part in the Donnachie-Landshoff (DL) model [24]. The prediction of gluon and sea quark distributions at low $Q^2$ based on the GLR-MQ-ZSR equation is presented in Fig. 9. Although the predicted structure functions are lower than the ZEUS data [20],
Figure 3: Comparisons of our predicted $Q^2$-dependence of $F_2^p(x, Q^2)$ (solid curves) with various experimental data [18–22].
Figure 4: Comparisons of the $x$-dependence of our predicted $F_2^P$ (solid curves) with the GRV(98LO) results (dashed curves) [13].

Figure 5: Comparisons of the $Q^2$-dependence of our predicted $F_2^P$ (solid curves) with the GRV(98LO) results (dashed curves) [13]. For a better display, the structure function values are scaled at each $x$ by the factors shown in brackets.
Figure 6: Comparisons among our predicted gluon distribution (solid curves), the GRV(98LO) (dashed curves) and the results using the DGLAP evolution with the natural input (broken curves).

Figure 7: Similar to Fig. 6, but for the sea quark distributions.
the plateau-like shapes allow us to establish a smooth connection between the partonic and non-partonic pictures of the nucleon’s structure functions in low $Q^2$ and small $x$ range \[25\]. It will be studied in the next section.

\[Q^2 = 0.11 \text{ GeV}^2\]

\[Q^2 = 0.3 \text{ GeV}^2\]

\[Q^2 = 0.5 \text{ GeV}^2\]

\[Q^2 = 0.65 \text{ GeV}^2\]

Figure 8: Our predicted proton structure function (solid curves) in low $Q^2$ range. The contributions of Regge part in the Donnachie-Landshoff model (broken curves) \[24\], GRV(98LO) results (dashed curves) and the ZEUS data \[19\] are presented.

The difference between our model and GRV model becomes obvious near the GRV starting point $Q^2 \approx 0.26 \text{ GeV}^2$ (Fig. 10). The CTEQ distributions are based on the global QCD analysis with the DGLAP equation. The gluon distribution becomes flat at $Q^2 = 1.4 \text{ GeV}^2$ in the CTEQ6. It means that the gluon density will be negative at $Q^2 < 1.4 \text{ GeV}^2$ according to the DGLAP equation. The comparisons of our model and GRV predictions with the CTEQ6 at $Q^2 = 1.4 \text{ GeV}^2$ are presented in Fig. 11.

The predicted gluon distributions of the different databases exhibit large difference. The reason is that the lepton probes can not directly measure the gluon distribution. The parameters of input gluon distribution are determined by higher order QCD processes, such as the scaling violation and longitudinal structure function $F_L$. Because only limited amount of data are available, the constraints on the input gluon distribution are much looser than for quarks. Different from those global databases, both gluon and sea quarks are dynamically generated from the input valence quarks based on the GLR-MQ-ZSR
Figure 9: Our predicted gluon (Left) and sea quarks (Right) distributions in the low $Q^2$ range.
Figure 10: Comparisons of our predicted parton distributions (solid curves) with the GRV input (dashed curves) at the GRV-starting point $Q^2 = 0.26$ GeV$^2$. 
Figure 11: Comparisons of our predicted parton distributions (solid curves) with CTEQ input (broken curves) and the GRV-distributions (dashed curves) at the CTEQ-starting point $Q^2 = 1.4 \text{ GeV}^2$. 

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equation in this work. Once the valence quark input is fixed by the observed quark dis- tributions, the gluon distribution is also determined in the leading order. The comparisons of the $x$-dependence of the gluon distributions at given $Q^2$ are plotted in Figs. 12, 13 and 14. Our predicted gluon distribution of the proton is different from CTEQ and GRV.

Figure 12: Comparisons of our predicted $x$-dependence of gluon distribution (solid curves) with CTEQ (broken curves) and GRV results (dashed curves) at different $Q^2$.

Figure 13: Ratios of our predicted gluon distribution to CTEQ (Left) and GRV (Right) at $Q^2 = 10$ and 100 GeV$^2$, respectively.
Figure 14: Comparisons of our predicted gluon distribution (solid curves) at $Q^2 = 20$ GeV$^2$ with CTEQ (broken curves), GRV results (dashed curves) and HERA data.
5 Hadronic components in proton structure functions at low $Q^2$

In this section we combine the GLR-MQ-ZSR equation with a simple non-perturbative model to explore the proton structure functions at the low $Q^2$ transition region. It should shed light on the research how the pQCD description of $F_2^p$ gradually transits to the non-perturbative region.

According to the standard partonic picture, the proton structure functions are only related to the parton (quarks and gluons) distributions inside a proton, which are evolved by the perturbative QCD evolution equations. However, there are vector mesons with photon-like quantum numbers and low mass ($m_V \sim 700$ MeV), which may couple with virtual photon at $Q^2 < 1$ GeV$^2$ as long lifetimes of hadronic fluctuations. The strong interaction of these vector mesons with proton will lead to a larger DIS cross section, which covers the point-like interaction of the bare photo with the partons at low $Q^2$. This is so called the VMD model [26]. In the following we will use the duality picture of the structure function (Fig. 15), which was first proposed by Badelek and Kwiecinski in [27], i.e.,

$$F_2^p(x, Q^2) = F_2^{\text{parton}}(x, Q^2) + F_2^{\text{hadron}}(x, Q^2),$$

where

$$F_2^{\text{parton}}(x, Q^2) = x \sum_j e_j^2 f_{V^p_j}(x, Q^2) + x \sum_i e_i^2 [f_{q_i^p}(x, Q^2) + f_{\bar{q}_i^p}(x, Q^2)],$$

is evolved with $Q^2$ in QCD evolution equation (in this work, it is the GLR-MQ-ZSR equation).

Figure 15: The schematic representation of the dual model Eq. (9): (a) $F_2^{\text{hadron}}(x, Q^2)$ at target rest frame; (b) $F_2^{\text{parton}}(x, Q^2)$ at target rest frame (Left) and infinite momentum frame (Right).

The structure function $F_2^{\text{hadron}}$ is described by the VMD model. Here we choose its naive form, i.e.,
\[ F_{2}^{\text{hadron}}(x, Q^2) = \frac{Q^2}{4\pi \gamma_{\rho}} \frac{m_{\rho}^4 \sigma_{\rho p}}{(Q^2 + m_{\rho}^2)^2}. \] (11)

where \( \gamma_{\rho} \) is the coupling constant of \( \rho \) meson and proton; \( x \) is a variable defined by \( x = \frac{Q^2}{s + Q^2 - m_{\rho}^2} \) rather than the momentum fraction of parton. The contribution of \( \omega \) meson is similar to that of \( \rho \), while the contribution of \( \phi \) meson is small at \( Q^2 < 1 \text{ GeV}^2 \) and hence can be neglected. At high energy according to the Regge theory [14], the cross section \( \sigma_{\rho p} \) can be parameterized as the functions of energy \( W \): \( \sigma_{\rho p}(W) = A_1 \cdot (W^2)^{1-\alpha_P} \), where \( \alpha_P = 1.0808 \) is the intercept of the Pomeron and \( A_1 \) denotes its coupling [24]. It is equivalent to \( \sigma_{\rho p}(x) = A_2 x^{1-\alpha_P} \) at small \( x \). In consequence,

\[ F_{2}^{\text{hadron}}(x, Q^2) = B \frac{m_{\rho}^4 Q^2}{(Q^2 + m_{\rho}^2)^2} x^{1-\alpha_P} (1 - x)^7. \] (12)

Because the cross section for the absorption of real photons \( \sigma_{\gamma p} \sim F_2^p/Q^2|_{Q^2=0} \) is finite, it requires \( B = \beta (Q^2/m_{\rho}^2)^{\alpha_P-1} \), where \( \beta \) is a free parameter and we take \( \beta = 0.61 \text{ GeV}^{-2} \) for best fitting. The last factor in Eq. (12) is due to the spectator counting rules at high \( x \) [28], which require the Pomeron contributions to behave as \( (1 - x)^7 \) near \( x \approx 1 \). Note that the four free parameters in \( F_{2}^{\text{parton}}(x, Q^2) \) have been fixed by the data \( Q^2 > 4 \text{ GeV}^2 \), only one free parameter \( \beta \) is added. The electromagnetic current conservation requires that the structure function \( F_2^p \) must vanish in the limit \( Q^2 \to 0 \), which is satisfied in our work.

Comparisons of our predicted \( F_2^p \) with experimental data [18–20, 22] can be seen in Fig. [16 (at \( Q^2 < 1 \text{ GeV}^2 \)) and in Fig. [17 (at \( Q^2 > 1 \text{ GeV}^2 \)). We also calculated the \( Q^2 \)-dependence of \( F_2^p \) at various \( W \) values in Fig. [18. Although factorization has not been proved at low \( Q^2 \), the perfect fitting indicates that it is possible to define the parton distributions in the low-\( Q^2 \) region when the contributions of the non-partonic components are excluded from the measured proton structure functions.

Through the above discussions, we conclude that the parton distributions predicted by the GLR-MQ-ZSR equation with a natural input are compatible with the present databases and data of the proton structure functions.
Figure 16: The proton structure function $F_2^p(x, Q^2)$ (solid curves) as a function of $x$ at various $Q^2$ values ($< 1$ GeV$^2$) compared with experimental data. The contributions of $F_2^{\text{hadron}}(x, Q^2)$ (broken curves) and $F_2^{\text{parton}}(x, Q^2)$ (dashed curves) are indicated.
Figure 17: Similar to Fig.16 but at $Q^2 > 1$ GeV$^2$.

Figure 18: The proton structure function $F_2^p(x,Q^2)$ as a function of $Q^2$ at various values of fixed c.m. energy $W$ comparing with data [18–20, 22]. From bottom to top: $W = 20 \times 1$, 60 $(\times 2)$, 100 $(\times 3)$, 200 $(\times 4)$ GeV. The data points and curves are scaled by the numbers in brackets.
6 Discussions and Summary

An unavoidable question is whether Eq. (1) can be used at \( Q^2 \geq 0.064 \text{ GeV}^2 \). Now let us discuss this question.

One may think that a large value of \( \alpha_s \) at low \( Q^2 \) leads to perturbative expansion diverges. Refs. [4–6] have emphasized that the DGLAP evolution is still in the perturbative region even at low static point \( \mu^2 \sim 0.064 \text{ GeV}^2 \) since the expansion factor is \( \alpha_s(\mu^2)/2\pi < 1 \) in the DGLAP equation and the evolution kernels are non-singular at low \( Q^2 \). The GRV model was extended to the next leading order and even to the next-next leading order approximation at \( Q^2 < 1 \text{ GeV}^2 \) [29]. Its results indicate that the higher order corrections to the DGLAP evolution are small. A similar result was also obtained by Steffens and Thomas in the bag model [30].

We will show that these conclusions are still valid for the DGLAP equation with the GLR-MQ-ZSR corrections. The perturbative approximation may be invalid if the net (shadowing and antishadowing) effect is positive. As we have pointed out that the size of the GLR-MQ-ZSR correction is not only dependent on the magnitude of \( \alpha_s(Q^2) \), but also is related to the shape of the parton densities in the range of \( x \) to \( x/2 \) [31]. The net nonlinear correction will be positive if the parton distribution \( \sim x^{-\lambda} (\lambda > \lambda_{BFKL} \sim 0.5) \). However, the radioactively generated partons by the DGLAP evolution at small \( x \) behave as \( \sim x^{-\lambda} (\lambda < \lambda_{BFKL}) \) even through it is steep. In this case the shadowing effect in the evolution process will be weakened by the antishadowing effect, and the net effect of the parton fusions always keeps the negative correction. The parton distributions will asymptotically approach to a finite value by the action of the net shadowing in the leading order level. On the other hand, the contributions of the higher order recombination (i.e., the multi-parton recombination) is negligible since the parton densities are low at \( Q^2 < 1 \text{ GeV}^2 \). We also notice the possibility that the nonperturbative dynamics of QCD generates an effective gluon mass at very low \( Q^2 \) region [32], or gluons become Abelian gluons when \( Q^2 \rightarrow \Lambda_{QCD}^2 \) [33]. A simple phenomenological way can be used to estimate the above corrections to the evolution of the parton distributions: the QCD running coupling constant is frozen at an infrared value, i.e., \( \alpha_s(Q^2) \leq \kappa \), \( \kappa \) is an undetermined parameter. It is interesting that Deur extracted an effective strong coupling constant using low-\( Q^2 \) data and found that \( \kappa \sim \pi \) [34]. Using this restriction we obtain the similar results after adjusting the input distributions. Our numerical calculations also show that the resummations \( \sum_n[\alpha_s/(2\pi)\ln(Q^2/\Lambda_{QCD})]^n \) and \( \sum_n[\alpha_s^2/(4\pi R^2Q^2)]^n \) converge quickly, i.e., the perturbative evolution of Eq. (1) is stable at low \( Q^2 \).

We have examined that the parton recombination plays an important role at small \( x \) and low \( Q^2 \). Although any power correction including parton recombination will disappear at high \( Q^2 \), the contributions of the nonlinear terms in Eq.(1) at low \( Q^2 \) will be "remembered" in the parton distribution evolution process and observed at high \( Q^2 \). For
example, with the same natural input distribution there is a big difference between the parton distributions from the DGLAP equation and the ones from the GLR-MQ-ZSR equation, as shown in Figs. 6 and 7.

The input valence quark distributions parameterize the interactions of constituents of the proton at scale $Q^2 \rightarrow \Lambda_{QCD}$ in the strong coupling regime. One can use the chiral effective field theories to model the input valence quark distributions as Ref. [15]. It is interesting to notice that these distributions are compatible with our parameterized input shown in Fig. 19. Therefore, we think that the GLR-MQ-ZSR equation is a powerful tool to connect the effective QCD quark model of hadron at scale $\mu^2$ with the observed structure function at high scale $Q^2 \gg \mu^2$.

![Figure 19: Comparisons of our input quark distributions (solid curves) with the valence quark distributions in the Nambu-Jona-Lasinio model (dashed curves) [30].](image)

The parton distributions of the proton in this work can be generalized to nuclear target, where the nuclear shadowing effect is a natural result of the parton recombination among different bound nucleons, while the nuclear environment deforms the input valence quarks. We will discuss them in our following work.

In this work, all sea quarks are generated perturbatively by gluon splitting. Therefore, the sea quark distributions have isospin symmetry. However, experiments questioned these naive expectations [35]. These effects require nonperturbative explanations. In this aspect, the GLR-MQ-ZSR equation provides an effective ways to test various models which may modify the input distributions.

In summary, the parton distributions in the proton are evaluated dynamically starting from three valence quarks input at the low scale $\mu^2$ by the GLR-MQ-ZSR equation. Our results show that the negative nonlinear corrections improve the perturbative stability of
the QCD evolution equation at low $Q^2$. Our predicted parton distributions of the proton with four free parameters are compatible with the existing databases. We show that the sea quark distributions exhibit a positive and plateau-like behavior at small $x$ and low $Q^2$. This approach provides a powerful tool to connect the quark models of the hadron and various non-perturbative effects on them at scale $\mu^2$ with the measured structure functions at high scale $Q^2 >> \mu^2$.

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