Weakened Random Oracle Models with Target Prefix

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Abstract. Weakened random oracle models (WROMs) are variants of the random oracle model (ROM). The WROMs have the random oracle and the additional oracle which breaks some property of a hash function. Analyzing the security of cryptographic schemes in WROMs, we can specify the property of a hash function on which the security of cryptographic schemes depends. Liskov (SAC 2006) proposed WROMs and later Numayama et al. (PKC 2008) formalized them as CT-ROM, SPT-ROM, and FPT-ROM. In each model, there is the additional oracle to break collision resistance, second preimage resistance, preimage resistance respectively. Tan and Wong (ACISP 2012) proposed the generalized FPT-ROM (GFPT-ROM) which intended to capture the chosen prefix collision attack suggested by Stevens et al. (EUROCRYPT 2007).

In this paper, in order to analyze the security of cryptographic schemes more precisely, we formalize GFPT-ROM and propose additional three WROMs which capture the chosen prefix collision attack and its variants. In particular, we focus on signature schemes such as RSA-FDH, its variants, and DSA, in order to understand essential roles of WROMs in their security proofs.

Keywords: Weakened random oracle model · RSA-FDH · DSA · Chosen prefix collision attack.

1 Introduction

1.1 Background

A hash function is an important primitive for many cryptographic schemes. The security of these cryptographic schemes is often proved in the random oracle model (ROM) [1]. In the ROM, a hash function is regarded as an ideal random function. Instead of computing a hash value, all the parties can query the random oracle \( RO \) to get the hash value. Compared to the standard model, it is easier to design efficient cryptographic schemes with the provable security.

In an implementation of a cryptographic scheme, the random oracle is replaced by a hash function. However, when a hash function is attacked, it is not clear a problem whether the security of the cryptographic scheme proven in the ROM is guaranteed.

To explain this fact, we consider two signature scheme RSA-Full-Domain-Hash (RSA-FDH) [3] and RSA-PFDH [4] which is a variant of RSA-FDH. Both schemes satisfies the existential unforgeability against chosen message attacks (EUF-CMA) security in the ROM. Now, we give an observation of the EUF-CMA security of these scheme where these scheme are implemented by a hash function \( h \) whose collision resistance property is broken.

In RSA-FDH, if a collision \( (m, m') \) satisfying \( h(m) = h(m') \land m \neq m' \) is found, then the adversary makes a signing query on a message \( m \), gets a valid signature \( \sigma = h(m)^d \mod N \) where \( d \) is a signing key and \( N \) is a RSA modulo, the adversary can generate a valid forgery \( (m', \sigma) \). This example shows that the EUF-CMA security of RSA-FDH depends on the collision resistance property of a hash function.

By contrast, it seems that the attack mentioned above does not works for RSA-PFDH. In RSA-PFDH, a signature \( \sigma \) on a message \( m \) is computed as \( \sigma = (r, x) \) where \( r \) is randomly chosen by the signer and \( x = h(m || r)^d \mod N \) where \( m || r \) is the concatenation of two strings \( m \) and \( r \). Since randomness \( r \) is chosen by a signer, even if a collision \( (m || r, m' || r') \) satisfying \( h(m || r) = h(m'|| r') \land m || r \neq m' || r' \) is found, the adversary seldom obtains a signature \( \sigma \) on a message \( m \) formed as \( \sigma = (m || r) \) by making a signing query on a message \( m \). Intuitively, the EUF-CMA security of RSA-PFDH still holds even if the collision resistance property of the hash function is broken. However, we cannot prove this fact in the ROM.

⋆ This is a full version of a paper [18] in Innovative Security Solutions for Information Technology and Communications - 11th International Conference (SecITC 2018).
Weakened Random Oracle Models. Liskov [10] introduced the idea of weakened random oracle models (WROMs). Each model has the additional oracle that breaks the specific property of a hash function. For instance, assuming that the oracle which returns the collision is added, we can capture the situation where the collision is found.

Pasini and Vaudenay [12] used the Liskov’s idea to consider the security of Hash-and-Sign signature schemes in the random oracle model with the additional oracle which returns the first preimage.

Numayama, Ishi, and Tanaka [11] formalized the Liskov’s idea as three types of WROMs: The collision tractable ROM (CT-ROM), the second preimage tractable ROM (SPT-ROM), and the first preimage tractable ROM (FPT-ROM). Each model has the additional oracle CO, SPO, and FPO, respectively:

- CO(): It picks x randomly and uniformly returns a collision (x, x′) such that x ≠ x′ and h(x) = h(x′).
- SPO(x): Given an input x, it uniformly returns x′ such that x ≠ x′ and h(x) = h(x′).
- FPO(y): Given an input y, it uniformly returns x such that h(x) = y.

Since Liskov’s models considered only compression functions, these oracles are different from that of Liskov’s model in following respects. CO does not provide a collision if there is no x′ such that x ≠ x′ and h(x) = h(x′) for x. SPO (resp. FPO) does not provide x′ (resp. x) if there is no x′ (resp. x) such that h(x) = h(x′) (resp. h(x) = y).

Numayama et al. analyzed the EUF-CMA security of RSA-FDH and its variant in WROMs. There result showed that RSA-PFDH is EUF-CMA secure in CT-ROM. As described before, we give intuition that the EUF-CMA security of RSA-PFDH still holds even if the collision resistance property of the hash function is broken but we cannot prove this fact in the ROM. Thus, by using WROMs, we can prove the security of cryptographic schemes when a hash function is attacked.

Kawachi, Numayama, Tanaka, and Xagawa [8] analyzed the indistinguishability against adaptive chosen ciphertext attacks (IND-CCA2) security of RSA-OAEP [2], the Fujisaki-Okamoto conversion (FO) [5] and its variants in WROMs. They showed RSA-OAEP encryption scheme is IND-CCA2 secure even in the FPT-ROM.

Chosen Prefix Collision Attack. Stevens, Lenstra and de Weger [15] proposed the chosen prefix collision attack for a hash function. In this attack, we decide a pair (P, P′) of prefixes beforehand and find a collision (P||S, P′||S′). Using this attack against MD5 [14], an adversary can find a target collision by making roughly 2^{49} calls to the internal compression function.

Moreover, Stevens, Sotirov, Appelbaum, Lenstra, Mohar, Osvik and de Weger [16] succeeded in reducing the number of calls to the internal compression function to find a collision to 2^{16} by setting P = P′. However, this attack was not captured by WROMs by Numayama et al.

To capture the chosen prefix attack, Tan and Wong [17] proposed the generalized FPT-ROM (GFPT-ROM). This model has the additional oracle GFPO.

- GFPO(y, r): Given an input (y, r), it uniformly returns x = m||r such that h(m||r) = y.

They showed RSA-PFDH^B [11] is not EUF-CMA secure in the GFPT-ROM. Moreover, they proposed a generic transformation of Hash-and-Sign signature schemes. If the original scheme is secure in the ROM, the converted scheme is secure in the GFPT-ROM. They proposed RSA-FDH^+ by using this transformation for RSA-FDH.

1.2 Our Contributions.

Thanks to transformation proposed by Tan and Wong [17], constructing a secure scheme in WROMs is not a serious problem. But the security against the chosen prefix collision attack for
standard signature schemes have not been clarified. Moreover, GFPT-ROM captures a strong variant of the chosen prefix collision attack than that of attack proposed by Stevens et al and existing WROMs do not exactly capture the chosen prefix collision attack by Stevens et al. Furthermore, like the work by Tan and Wong, we can consider other variants of the collision attack.

In this work, we introduce new WROMs which captures the chosen prefix collision attack and its variants. Then, we analyze the EUF-CMA security of standard signature schemes against chosen prefix collision attacks by using our WROMs. Our analysis of these schemes in WROMs provides a more precise security indication against chosen prefix collision attacks.

**WROMs for the Chosen Prefix Collision Attack.** In order to analyze the security against chosen prefix collision attacks in more detail, we extend the idea of [17] and obtain new models. Consequently, we obtain the common-CP-CT-ROM and CP-CT-ROM from the CT-ROM. We also obtain the CP-SPT-ROM from the SPT-ROM. The common-CP-CT-ROM captures the case of \( P = P' \) in the chosen prefix collision attack, CP-CT-ROM captures the chosen prefix collision attack, and CP-SPT-ROM captures a variation of this attack that we decide \((P||S, P'||S')\) beforehand and find a collision \((P||S, P'||S')\).

**Security Analysis in WROMs.** We analyze RSA-FDH and its variants in our new WROMs. The analysis results in these WROMs are given in Fig. 1. In this table, models become weaker as it goes right of the table. The security in a weaker model indicates the scheme is secure against stronger attacks to hash functions. Our result indicates RSA-PFDH and RSA-PFDH\(\oplus\) are secure in the CP-CT-ROM, but not secure in the CP-SPT-ROM. Surprisingly, the analysis result of RSA-PFDH\(\oplus\) is interesting in that even if a cryptographic scheme is secure in the SPT-ROM or FPT-ROM, it may not be secure in the CP-SPT-ROM.

In 2018, Jager, Kakvi and May [6] prove that RSASSA-PKCS-v1.5 [7] is EUF-CMA secure in the ROM under the RSA assumption. RSASSA-PKCS-v1.5 and DSA [9] are not analyzed using WROMs in previous works [11,17]. We show that both signature schemes are not EUF-CMA secure in both the CT-ROM and common-CP-CT-ROM.

**1.3 Related Works**

Unruh [19] proposed the ROM with oracle-independent auxiliary inputs. In this model, an adversary \( \mathcal{A} \) consists of \((\mathcal{A}_1, \mathcal{A}_2)\). The first step, computationally unbounded \( \mathcal{A}_1 \) can full access to \( RO \) and store information (e.g collisions) in the string \( z \). In the second step, \( z \) is passed to the bounded running time adversary \( \mathcal{A}_2 \) who can access to \( RO \). Unruh showed that RSA-OAEP encryption scheme is IND-CCA2 secure in the ROM with oracle-independent auxiliary inputs.

One may think that the ROM with oracle-independent auxiliary inputs already encompasses WROMs, but it is not clear. In the ROM with oracle-independent auxiliary inputs, \( \mathcal{A}_2 \) cannot query additional oracles that breaks hash functions after given the public key. By contrast, in WROMs, an adversary can query additional oracles that breaks hash functions after given the public key. Situations captured by WROMs and the ROM with oracle-independent auxiliary inputs are different and relevance between WROMs and the ROM with oracle-independent auxiliary inputs is not still clear. In particular, even if a cryptographic scheme is insecure in the ROM with oracle-independent auxiliary inputs, it is not clear whether it is secure in WROMs or not. Hence it is worth analyzing cryptographic schemes with WROMs.

**1.4 Road Map**

In Section 2, first, we review a digital signature scheme, its security notions. Next, we review the ROM and WROMs proposed by Numayama et al. [11]. In Section 3, we propose WROMs which
capture the chosen prefix collision attack and its variants and give an intuition for simulation method in WROMs. This simulation technique is needed to prove security in our WROMs. In Section 4, we analyze the EUF-CMA security against chosen prefix collision attacks for several signature schemes.

In this full version, we provide missing materials in [18]. In Appendix A, we provide a simulation method for our WROMs. In Appendix B, we provide missing security proofs for signature schemes.

2 Preliminaries

Let \( k \) be a security parameter. A function \( f(k) \) is negligible in \( k \) if \( f(k) \leq 2^{-\omega(\log k)} \). PPT stands for probabilistic polynomial time. For strings \( m \) and \( r \), \( |m| \) is the bit length of \( m \) and \( m|r \) is the concatenation of \( m \) and \( r \). For a finite set \( S \), \( s \leftarrow A \) denotes choosing an element from \( S \) uniformly at random. For a distribution \( D \), \( x \leftarrow D \) denotes that \( x \) is sampled according to distribution \( D \) and \( f_D(x) \) is the probability function of distribution \( D \). Let \( B(N, p) \) be the binomial distribution with \( N \) trials and success probability \( p \). The statistical distance of two distributions \( P \) and \( Q \) over \( S \) is defined as \( \Delta(P, Q) = \frac{1}{2} \sum_{s \in S} |P(s) - Q(s)| \). Let \( y \leftarrow A(x) \) be the output of algorithm \( A \) on input \( x \).

2.1 Digital Signature Scheme

We review a digital signature scheme and the EUF-CMA security.

**Definition 1 (Digital Signature Scheme).** A digital signature scheme \( II \) over the message space \( M \) is a triple \( II = (\text{Gen}, \text{Sign}, \text{Verify}) \) of PPT algorithms:

- **Gen**: Given a security parameter \( 1^k \), return a key pair \((sk, vk)\).
- **Sign**: Given a signing key \( sk \) and a message \( m \in M \), return a signature \( \sigma \).
- **Verify**: Given a verification key \( vk \), a message \( m \) and a signature \( \sigma \), return either 1 (Accept) or 0 (Reject).

**Correctness.** For all \( k \in \mathbb{N} \), \((sk, vk) \leftarrow \text{Gen}(1^k), m \in M \), we require

\[
\text{Verify}(vk, m, \text{Sign}(sk, m)) = 1.
\]
Definition 2 (EUF-CMA). The EUF-CMA security of a digital signature scheme $\Pi$ is defined by the following EUF-CMA game between a challenger $C$ and PPT adversary $A$.

- $C$ produces a keypair $(sk, vk) \leftarrow \text{Gen}(1^k)$, and gives the $vk$ to $A$.
- $A$ makes a number of signing queries $m$ to $C$. Then, $C$ signs it as $\sigma \leftarrow \text{Sign}(sk, m)$, and sends $\sigma$ to $A$.
- $A$ outputs a message $m^*$ and its signature $\sigma^*$.

A digital signature scheme satisfies the EUF-CMA security if for all PPT adversaries $A$, the following advantage of $A$:

$$\text{Adv}_{EUF-CMA}^\Pi_A := \Pr[\text{Verify}(vk, m^*, \sigma^*) = 1 \land m^* \text{ is not queried to signing}]$$

is negligible in $k$.

2.2 Security Notions of Hash Functions

Let $h : X \rightarrow Y$ be a hash function. Security notions of a hash function are as follows.

- Collision resistance:
  It is hard to find a pair $(x, x')$ of inputs such that $h(x) = h(x') \land x \neq x'$.
- Second preimage resistance:
  Given an input $x \leftarrow X$, it is hard to find a second preimage $x' \neq x$ such that $h(x) = h(x')$.
- First preimage resistance:
  Given a hash value $y \leftarrow h(x)$ where $x \leftarrow X$, it is hard to find a preimage $x'$ such that $h(x') = y$.

2.3 WROMs Proposed by Numayama et al. [11]

Let $\ell$ be polynomial in $k$, $X = \{0,1\}^\ell$, $Y = \{0,1\}^k$, $h : X \rightarrow Y$ a random function, and $T_h = \{(x, h(x)) \mid x \in X\}$ the table which defines the correspondence between the inputs and outputs of $h$. To make a more rigorous discussion, we introduce subscript $(\ell,k)$ in the definition, which make explicit that the length of the input of $h$ is $\ell$ and the length of the output of $h$ is $k$.

Now, we define the random oracle model ROM$(\ell,k)$.

Definition 3 (ROM$(\ell,k)$). The random oracle model ROM$(\ell,k)$ is the model that all parties can query the random oracle $RO^h$.

- Random oracle $RO^h(x)$
  Given an input $x$, the random oracle returns $y$ such that $(x, y) \in T_h$.

Now, we review the CT-ROM$(\ell,k)$, SPT-ROM$(\ell,k)$ and FPT-ROM$(\ell,k)$ which are defined by Numayama et al. [11].

Definition 4 (CT-ROM$(\ell,k)$ [11]). The collision tractable random oracle model CT-ROM$(\ell,k)$ is the model that all parties can query $RO^h$ and the collision oracle $CO^h$.

- Collision oracle $CO^h()$
  The collision oracle picks one entry $(x, y) \in T_h$ uniformly at random. If there is any other entry $(x', y) \in T_h$ then it picks such an entry $(x', y)$ uniformly at random and returns $(x, x')$. Otherwise, it returns $\bot$.

In the CT-ROM$(\ell,k)$, we can capture the situation where the collision resistance property is broken.
Definition 5 (SPT-ROM\textsubscript{(ℓ,k)} \cite{11}). The second preimage tractable random oracle model SPT-ROM\textsubscript{(ℓ,k)} is the model that all parties can query \textit{RO}^h and the second preimage oracle \textit{SPO}^h.

- Second preimage oracle \textit{SPO}^h(x)
  
  Given an input \(x\), let \(y\) be the hash value of \(x\) (i.e., \((x, y) \in \mathcal{T}_h\)). If there is any other entry \((x', y) \in \mathcal{T}_h\) such that \(x' \neq x\), then second preimage oracle returns \(x'\) uniformly at random. Otherwise, it returns \(\bot\).

In the SPT-ROM\textsubscript{(ℓ,k)}, we can capture the situation where the second preimage resistance property is broken.

Definition 6 (FPT-ROM\textsubscript{(ℓ,k)} \cite{11}). The first preimage tractable random oracle model FPT-ROM\textsubscript{(ℓ,k)} is the model that all parties can query \textit{RO}^h and the first preimage oracle \textit{FPO}^h.

- First preimage oracle \textit{FPO}^h(y)
  
  Given an input \(y\), if there is any entry \((x, y) \in \mathcal{T}_h\) then the first preimage oracle returns such \(x\) uniformly at random. Otherwise, it returns \(\bot\).

In the FPT-ROM\textsubscript{(ℓ,k)}, we can capture the situation where the preimage resistance property is broken.

3 WROMs against Chosen Prefix Collision Attacks

In this section, we propose WROMs which capture the chosen prefix collision attack and its variants. Let \(\ell\) and \(t\) be polynomials in \(k\), \(M = \{0,1\}^{\ell}\), \(R = \{0,1\}^t\), \(X = M \times R\), \(Y = \{0,1\}^k\), \(h : X \rightarrow Y\) a random function, and \(\mathcal{T}_h = \{(x, h(x)) \mid x \in X\}\) the table which defines the correspondence between the inputs and outputs of the function \(h\). To make a more rigorous discussion, we introduce subscript \((\ell, t, k)\) in the definition, which make explicit that the length of the input of the function is \(\ell\), the length of the prefix is \(t\), and the length of the output is \(k\).

Definition 7 (common-CP-CT-ROM\textsubscript{(ℓ,t,k)}). The common chosen prefix collision tractable random oracle model common-CP-CT-ROM\textsubscript{(ℓ,t,k)} is the model that all parties can query \textit{RO}^h and the common chosen prefix collision oracle \textit{COMMON-CP-CO}^h.

- Common chosen prefix collision oracle \textit{COMMON-CP-CO}^h(r)
  
  Given an input \(r\) (\(|r| = t\)), the common chosen prefix collision oracle picks one entry \((m||r, y) \in \mathcal{T}_h\) uniformly at random. If there is any other entry \((m'||r, y) \in \mathcal{T}_h\) then it picks such an entry \((m'||r, y)\) uniformly at random and returns \((m||r, m'||r)\). Otherwise, it returns \(\bot\).

In the common-CP-CT-ROM\textsubscript{(ℓ,t,k)}, we can capture the case of \(P = P'\) of the chosen prefix collision attack.

Definition 8 (CP-CT-ROM\textsubscript{(ℓ,t,k)}). The chosen prefix collision tractable random oracle model CP-CT-ROM\textsubscript{(ℓ,t,k)} is the model that all parties can query \textit{RO}^h and the chosen prefix collision oracle \textit{CP-CO}^h.

- Chosen prefix collision oracle \textit{CP-CO}^h(r, r')
  
  Given an input \((r, r')\) (\(|r| = |r'| = t\)), the chosen prefix collision oracle first picks one entry \((m||r, y) \in \mathcal{T}_h\) uniformly at random. If there is any other entry \((m'||r', y) \in \mathcal{T}_h\) then it picks such an entry \((m'||r', y)\) uniformly at random and returns \((m||r, m'||r')\). Otherwise, it returns \(\bot\).

In the CP-CT-ROM\textsubscript{(ℓ,t,k)}, we can capture the chosen prefix collision attack.
Definition 9 (CP-SPT-ROM(ℓ,t,k)). The chosen prefix second preimage tractable random oracle model CP-SPT-ROM(ℓ,t,k) is the model that all parties can query RO^h and the chosen prefix second preimage oracle CP-SPO^h.

- **Chosen prefix second preimage oracle CP-SPO^h(x,r')**
  Given an input (x,r') (|x| = ℓ + t, |r'| = t), let y be the hash value of x (i.e., (x,y) ∈ T_h). If there is any other entry (m||r',y) ∈ T_h such that m'||r' ≠ x, then the chosen prefix second preimage oracle returns m'||r' uniformly at random. Otherwise, it returns ⊥.

In the CP-SPT-ROM(ℓ,t,k), we can capture a variation of the chosen prefix collision attack that we decide (P||S, P') beforehand and find a collision (P||S', P'||S').

In order to treat the name of GFPT-ROM [17] in a similar manner to above models name, we rename it to the CP-FPT-ROM(ℓ,t,k).

Definition 10 (CP-FPT-ROM(ℓ,t,k)). The chosen prefix first preimage tractable random oracle model CP-FPT-ROM(ℓ,t,k) is the model that all parties can query RO^h and the chosen prefix first preimage oracle CP-FPO^h.

- **Chosen prefix first preimage oracle CP-FPO^h(y,r)**
  Given an input (y,r) (|y| = k, |r| = t), if there is an entry (m||r,y) ∈ T_h then the chosen prefix first preimage oracle returns such m||r uniformly at random. Otherwise, it returns ⊥.

By Definition 7, 8, 9, 10, following relations among WROMs hold.

- If security of a cryptographic scheme is proven in the CP-CT-ROM(ℓ,t,k) (resp., CP-SPT-ROM(ℓ,t,k), CP-FPT-ROM(ℓ,t,k)), then the scheme satisfies this security in the common-CP-CT-ROM(ℓ,t,k) (resp., CP-CT-ROM(ℓ,t,k), CP-SPT-ROM(ℓ,t,k)).

**Intuition of Simulation for WROMs.** In the ROM, the reduction algorithm simulates RO^h using a table T which has entries (x, y) representing that the hash value of x is y. In WROMs, the reduction algorithm must simulate the additional oracle. In the CT-ROM(k), SPT-ROM(k) and FPT-ROM(k), the behavior of the additional oracle depends on the number of preimages. The reduction algorithm uses T and a table L. The table L has entries (y, n) representing that y has n preimages. In the common-CP-CT-ROM(ℓ,t,k), CP-CT-ROM(ℓ,t,k), CP-SPT-ROM(ℓ,t,k) and CP-FPT-ROM(ℓ,t,k), the behavior of the additional oracle depends on not only the number of preimages but also prefixes. The reduction algorithm simulates the additional oracle by adding an entry for prefix r to tables T and L. Concretely, T has entries ((m, r), y) representing that a hash value of m||r is y and L has entries ((y, r), n) representing that y has n preimages which have the prefix r. We will describe technical details in Appendix A.

## 4 Security of Signature Schemes in WROMs

In this section, we argue the EUF-CMA security of signature schemes in WROMs. We will describe the proof of security analyses of signature schemes in Appendix B. Before analyzing RSA based signature schemes, we recall the RSA assumption.

**Definition 11 (RSA Generator).** The RSA generator RSAGen, which on input 1^k, randomly chooses distinct k/2-bit primes p, q and computes N = pq and φ = (p − 1)(q − 1). It randomly picks e ∈ ∗ Zφ(N) and computes d such that ed = 1 mod φ(N). The RSA generator outputs (N, e, d).

**Assumption 1 (RSA Assumption).** A polynomial-time machine A is said to solve the RSA problem if given an RSA instance (N, e, z) where N, e are generated by RSA(1^k) and z ∈ ∗ Z_N^∗, it outputs z^{1/e} mod N with non-negligible probability. The RSA assumption is that there is no PPT adversary that solves the RSA problem.
4.1 RSA-FDH

Let $\ell$ be a polynomial in $k$, $M = \{0,1\}^\ell$ the message space, and $h : \{0,1\}^\ell \to \{0,1\}^k$ a hash function. RSA-FDH [3] is described in Fig. 2.

Theorem 1. In the common-CP-CT-ROM$_{\ell_1,t_1,k}$ ($\ell_1 + t_1 = \ell$), there exists a PPT adversary $A$ that breaks RSA-FDH by making queries to the signing oracle and COMMON-CP-CO$^h$ with probability at least $1 - e^{(1-2^\ell)/2^k}$.

Security proof of Theorem 1 is given in Appendix B.1.

4.2 RSA-PFDH

Let $\ell$ and $k_1$ be polynomials in $k$, $M = \{0,1\}^\ell$ the message space, and $h : \{0,1\}^{\ell+k_1} \to \{0,1\}^k$ a hash function. RSA-PFDH [4] is described in Fig. 3.

Theorem 2. In the CP-CT-ROM$_{\ell_1,t_1,k}$ ($\ell_1 + t_1 = \ell + k_1$), for all PPT adversaries $B$ that break RSA-PFDH with probability $\epsilon_{\text{euf}}$ by making $q_{\text{sign}}$, $q_h$ and $q_{\text{sc}}$ queries to the signing oracle, RO$^h$, and CP-CO$^h$ respectively. There exists a PPT adversary $A$ that solves the RSA problem with $\epsilon_{\text{rsa}}$ such that

$$
\epsilon_{\text{euf}} \leq \epsilon_{\text{rsa}} + \frac{1}{2^{k}} + \frac{q_{\text{sign}}Q_2}{2^{k_1}} + Q_1 \times P_{\text{prefixRO}}^h
$$

where $Q_1 = q_{\text{sign}} + q_h + q_{\text{sc}} + 1$, $Q_2 = q_{\text{sign}} + q_h + 2q_{\text{sc}} + 1$ and

$$
P_{\text{prefixRO}}^h \leq \left\{ \begin{array}{ll} ln \frac{2^{2^k}}{ln 2^k} & \frac{1}{2^{k}} + \frac{Q_2}{2^k} \end{array} \right\} (\ell_1 \geq k)
$$

$$
P_{\text{prefixRO}}^h \leq \left\{ \begin{array}{ll} ln \frac{2^{2^k}}{ln 2^k} & \frac{1}{2^{k}} + \frac{Q_2}{2^k} \end{array} \right\} (\ell_1 < k).
$$

Security proof of Theorem 2 is given in Appendix B.2.

Theorem 3. In the CP-SPT-ROM$_{\ell,k_1,k}$, there exists a PPT adversary $A$ that breaks RSA-PFDH by making queries to the signing oracle and CP-SPO$^h$ with probability at least $1 - e^{(1-2^k)/2^k}$. If $\ell \geq k \geq 2$, $A$ outputs a valid forgery with probability at least $1 - e^{-1/2}$.

Security proof of Theorem 3 is given in Appendix B.3.
4.3 RSA-PFDH\(^\oplus\)

Let \(\ell\) be a polynomial in \(k\), \(M = \{0,1\}^\ell\) the message space, and \(h : \{0,1\}^{\ell+k} \to \{0,1\}^k\) a hash function. RSA-PFDH\(^\oplus\) [11] is described in Fig. 4.

| Gen\(^{1}\) | Sign\(^{i}(sk, m)\) | Verify\((vk, m, \sigma)\) |
|------------|-----------------|-----------------|
| \((N, e, d) \leftarrow \text{RSAGen}(1^k)\) | \(r \leftarrow \{0,1\}^k\) | parse \(\sigma\) as \((r,x)\) |
| \(vk \leftarrow (N, e)\) | \(w \leftarrow h(m)\) | \(y = x^e \mod N\) |
| \(sk \leftarrow (N, d)\) | \(y \leftarrow w \oplus r\) | \(w \leftarrow h(m)\) |
| return \((vk, sk)\) | \(x \leftarrow y^d \mod N\) | if \(w \oplus r = y\) |
| | \(\sigma \leftarrow (r, x)\) | return 1 |
| | return \(\sigma\) | else |
| | | return 0 |

Fig. 4. RSA-PFDH\(^\oplus\)

**Theorem 4.** In the CP-CT-ROM\(\ell_t, t_1, k\) \((\ell_t + t_1 = \ell + k)\), for all PPT adversaries \(B\) that break RSA-PFDH\(^\oplus\) with probability \(\epsilon_{\text{uf}}\) by making \(q_{\text{sign}}\), \(q_h\) and \(q_{\text{sc}}\) queries to the signing oracle, RO\(^h\), and CP-CO\(^h\), respectively. There exists a PPT adversary \(A\) that solves the RSA problem with \(\epsilon_{\text{rsa}}\) such that

\[
\epsilon_{\text{uf}} \leq \epsilon_{\text{rsa}} + \frac{1}{2^k} + \frac{q_{\text{sign}}Q_2}{2^{k_1}} + Q_1 \times p_{\text{prefixRO}^h}
\]

where \(Q_1 = q_{\text{sign}} + q_h + q_{\text{sc}} + 1\), \(Q_2 = q_{\text{sign}} + q_h + 2q_{\text{sc}} + 1\) and

\[
p_{\text{prefixRO}^h} \leq \begin{cases} \frac{\ln^{2k} 10Q_2}{\ln 2} + \frac{1}{2^{k_1}} + \frac{Q_2}{2^{k_1}} & (\ell_1 \geq k) \\ \frac{\ln^{2k} 10Q_2}{\ln 2} + \frac{1}{2^{k_1}} + \frac{Q_2}{2^{k_1}} & (\ell_1 < k) \end{cases}
\]

Security proof of Theorem 4 is given in Appendix B.4.

**Theorem 5.** In the CP-SPT-ROM\(\ell_t, k, k\), there exists a PPT adversary \(A\) that breaks RSA-PFDH\(^\oplus\) by making queries to the signing oracle and the chosen prefix second preimage oracle for \(h\) with probability at least \(1 - e^{(1-2^k)/2^k}\).

Security proof of Theorem 5 is given in Appendix B.5.

4.4 RSASSA-PKCS-v1.5

We discuss RSASSA-PKCS-v1.5 [7]. To simplify the discussion, exclude detailed settings and treat octet strings as binary strings. Let \(\ell\), \(k_1\), and \(k_2\) be polynomials in \(k\), and \(c\) a constant which is determined by the type of hash function when implementing RSASSA-PKCS-v1.5. Let \(M = \{0,1\}^\ell\) be the message space and \(h : \{0,1\}^{\ell} \to \{0,1\}^{k_2}\) a hash function. Let \(s\) be a fixed binary string of length \(k_1\). The HashAlgID represent the type of hash function when implementing RSASSA-PKCS-v1.5 in a specific binary string of length \(c\). The HashAlgID and the string \(s\) are published in advance. Let \(k = k_1 + c + k_2\). RSASSA-PKCS-v1.5 is described in Fig. 5.

**Theorem 6.** In the CT-ROM\(\ell_t, k_2\), there exists a PPT adversary \(A\) that breaks RSASSA-PKCS-v1.5 by making queries to the signing oracle and CO\(^h\) with probability at least \(1 - e^{(1-2^k)/2^{k_2}}\).

**Theorem 7.** In the common-CP-CT-ROM\(\ell_t, t_1, k_2\) \((\ell_t + t_1 = \ell)\), there exists a PPT adversary \(A\) that breaks RSASSA-PKCS-v1.5 by making queries to the signing oracle and COMON-CP-CO\(^h\) with probability at least \(1 - e^{(1-2^{k_1})/2^{k_2}}\).

Theorem 6 and 7 can be proven similar way as in Theorem 1.
and 9 can be proven similar way as in Theorem 1 and common-CP-CT-ROM.

PFDH and RSASSA-PKCS-v1.5 also showed that

We discuss 4.5 DSA

queries to the signing oracle and CO

more precise security indication against chosen prefix attacks. We showed

lision attacks by defining three WROMs. Our analysis of these schemes in WROMs provides a

In this paper, we analyze the security of standard signature schemes against chosen prefix col-

5 Conclusion

-10

4.5 DSA

We discuss DSA [9]. We recall a group generator algorithm GrGen.

Definition 12 (Group Generator). The group generator algorithm GrGen, which on input

1k, randomly chooses a k-bit prime q and a j-bit prime such that q|(p − 1) (j is polynomial in k). It chooses an x ∈ {1, ..., p − 1} such that x^{(p−1)/q} ≠ 1 mod p and sets g = x^{(p−1)/q} mod p. The DSA setup algorithm outputs (p, q, g).

Let ℓ be a polynomial in k, M = {0, 1}^ℓ the message space, and h : {0, 1}^{ℓ+k} → {0, 1}^k a hash function. DSA is described in Fig. 6.

\begin{align*}
\text{Gen}(1^k) &
\quad \text{Sign}(sk, m) &
\quad \text{Verify}(vk, m, σ) \\
(N, e, d) &
\quad w ← h(m) &
\quad μ^* ← (σ^*)^μ \mod N \\
vk &
\quad y ← s||\text{HashAlgID}||w &
\quad \text{parse } y^* \text{ as } s^*||\text{HashAlgID}^*||w^* \\
sk &
\quad x ← y^d \mod N &
\quad \text{if } s^*||\text{HashAlgID}^* ≠ s||\text{HashAlgID} \\
\text{return } (vk, sk) &
\quad σ ← x &
\quad \text{return } 0 \\
\text{return } σ &
\quad \text{if } h(m^*) = w^* &
\quad \text{return } 1 \\
\text{else } &
\quad \text{else } &
\quad \text{return } 0 \\
\end{align*}

Fig. 5. RSASSA-PKCS-v1.5

| Gen(1^k) | Sign(sk, m) | Verify(vk, m, σ) |
|----------|-------------|------------------|
| (N, e, d) ← RSAGen(1^k) | w ← h(m) | w ← (σ^*)^μ \mod N |
| vk ← (N, e) | y ← s||HashAlgID||w | \text{parse } y^* \text{ as } s^*||HashAlgID^*||w^* |
| sk ← (N, d) | x ← y^d \mod N | \text{if } s^*||HashAlgID^* ≠ s||HashAlgID |
| return (vk, sk) | σ ← x | return 0 |
| | \text{return } σ | \text{if } h(m^*) = w^* |
| | | \text{return } 1 |
| | | \text{else } |
| | | \text{return } 0 |

Fig. 6. DSA

Theorem 8. In the CT-ROM(ℓ+k,k), there exists a PPT adversary A that breaks DSA by making queries to the signing oracle and CO^h with probability at least 1 − e^{(1−2^{k+1})/2^k}.

Theorem 9. In the common-CP-CT-ROM(ℓ1,k1,k) (ℓ1 + k1 = ℓ + k), there exists a PPT adversary A that breaks DSA by making queries to the signing oracle and COMMON-CP-CT-ROM with probability at least 1 − e^{(1−2^{k1})/2^k}.

Theorem 8 and 9 can be proven similar way as in Theorem 1.

5 Conclusion

In this paper, we analyze the security of standard signature schemes against chosen prefix collision attacks by defining three WROMs. Our analysis of these schemes in WROMs provides a more precise security indication against chosen prefix attacks. We showed RSA-PFDH and RSA-PFDH® are EUF-CMA secure in the CP-CT-ROM, but not secure in the CP-SPT-ROM. We also showed that RSASSA-PKCS-v1.5 and DSA are not EUF-CMA secure in both the CT-ROM and common-CP-CT-ROM.
When discussing the security in the CP-CT-ROM\(_{(\ell,t,k)}\), we fixed the length of the prefix \(\ell\). Studying the case of variable length is a future work. There are practical signature schemes and encryption schemes which have not been analyzed in WROMs. The security analysis of these schemes in WROMs is also an interesting future work.

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A Simulation Method in the CP-CT-ROM

We propose a simulation method in the CP-CT-ROM_{ℓ, t, k} based on a simulation method of the CP-FPT-ROM_{ℓ, t, k} which is proposed by Tan and Won [17]. We describe the algorithm prefix-RO_{h} and CP-CO_{h} which simulate RO_{h} and CP-CO_{h} in the CP-CT-ROM_{ℓ, t, k}. Since a behavior of CP-CO_{h} depends on the number of preimages and prefixes, we use two tables.

prefix-RO_{h} and CP-CO_{h} share two tables T, L, which are empty in their initial state. The table T has entries ((m, r), y) representing that a hash value of m|r is y. The table L has entries ((y, r), n) representing the number of m such that m|r is a preimage of y for some m. The two algorithms perform the simulation while synchronizing the table T and L so that there is no inconsistency between them. Let #T be the number of entries of T, #L the number of entries of L, #T((r′)) the number of entries of ((m, r′), y) ∈ T for some m and y, #T((r′), y) the number of entries of ((m, r′), y′) ∈ T for some m, and #L((r′)) the number of entries of ((y, r′), n) for some y and n.

Lemma 1. Let X = M × R, h : X → Y a hash function, #Y ≥ 2, and n_{y,r} represent the number of preimages of y that satisfy the conditions of h(x) = y and x = m|r for some m ∈ M under a function h. Let BAD_{r} be the event that there is some y such that n_{y,r} > L where if #M ≥ #Y, L = \frac{5\ln #Y}{\ln \ln #Y} \cdot \frac{M}{#Y}, or otherwise L = \frac{5\ln #Y}{\ln \ln #Y}.

Pr[BAD_{r}] < \frac{1}{(#Y)^2}

Lemma 1 is obtained by letting X of Lemma 1 in [8] correspond to M × \{r\}.

Lemma 2. The distribution on the outputs of RO_{h} is equal to the distribution on the outputs of the algorithm prefix-RO_{h}.
Lemma 2 is an extension of Lemma 1 in [11].

**Lemma 3.** The distribution on the outputs of \( \mathcal{R}_0^h \) and \( \mathcal{C}_{\text{P-CO}}^h \) is equal to the distribution on the outputs of algorithms prefix-\( \mathcal{R}_0^h \) and \( \mathcal{C}_{\text{P-CO}}^h \).

Lemma 3 is an extension of Corollary 1 in [11].

**Lemma 4.** Let \( X = M \times R \) and \( h : X \rightarrow Y \) a hash function in \( \mathcal{C}_{\text{P-CT-ROM}}(\ell,t,k) \). Let \( \mathcal{A} \) be a PPT oracle query machine that queries \( q_h \) and \( q_{sc} \) times for \( \mathcal{R}_0^h \) and \( \mathcal{C}_{\text{P-CO}}^h \) respectively and \( q = q_h + 2q_{sc} \). Let \( H_{A,h}(x) \) be a random variable that represents a hash value \( \mathcal{R}_0^h(x) \), where \( x = m || r \leftarrow A^{\mathcal{R}_0^h,\mathcal{C}_{\text{P-CO}}^h} \) and the correspondence \( (x, h(x)) \) is not returned by the two oracles. Then any \( \mathcal{A} \), the following inequality holds where \( 2q \leq \#M \) and \( 2q \leq \#Y \):

\[
\Delta(H_{A,h}(x), U_Y) \leq \begin{cases} 
\frac{1}{\#M} \left( 5q + 1 + \frac{4q^2}{\#Y} + 20q \frac{\ln \#Y}{\ln \ln \#Y} \right) & (\#M \geq \#Y) \\
\frac{1}{\#M} \left( 5q + 1 + \frac{4q^2}{\#Y} + 20q \frac{\ln \#Y}{\ln \ln \#Y} \right) & (\#M < \#Y).
\end{cases}
\]

Lemma 4 is an extension of Lemma 2 in [8].

**Lemma 5.** Let \( h \) be a hash function in \( \mathcal{C}_{\text{P-CT-ROM}}(\ell,t,k) \) and \( \mathcal{C}_{\text{P-CO}}^h \) the part of prefix-\( \mathcal{R}_0^h \) of the algorithm changed to prefix-\( \mathcal{R}_0^h \). Let \( q_h \) and \( q_{sc} \) be the number of queries to \( \mathcal{R}_0^h \) and \( \mathcal{C}_{\text{P-CO}}^h \) respectively and \( q = q_h + 2q_{sc} \). Except for the following probability \( p_{\text{prefix-RO}} \), the distributions of two tables \( L \) and \( T \) used in the simulation \( \mathcal{R}_0^h \) and \( \mathcal{C}_{\text{P-CO}}^h \) are identical to two tables \( L \) and \( T \) used in the simulation prefix-\( \mathcal{R}_0^h \) and \( \mathcal{C}_{\text{P-CO}}^h \):

\[
p_{\text{prefix-RO}} \leq \begin{cases} 
\frac{\ln \#Y}{\ln \ln \#Y} \frac{10q}{\#Y} + \frac{1}{\#M} & (\#M \geq \#Y) \\
\frac{\ln \#Y}{\ln \ln \#Y} \frac{10q}{\#M} + \frac{1}{\#Y} & (\#M < \#Y).
\end{cases}
\]

**Algorithm 1 prefix-\( \mathcal{R}_0^h(x) \)**

1: Parse \( x \) as \( m || r \).
2: If there is an entry \( ((m, r), y) \in T \) for some \( y \), then return \( y \).
3: Compute the following value:

\[
p = \sum_{(\tilde{r}, \tilde{n}) \in L} \frac{n - \#T(r, \tilde{n})}{\#M - \#T}.
\]

4: Flip a biased coin with \( \Pr[\alpha = 0] = p \).
5: If \( \alpha = 0 \), then return \( y \) as follows.
   (a) Pick \( y \sim D \) according to the following distribution:

\[
f_D(y) = \frac{n - \#T(r, y)}{\sum (\tilde{r}, \tilde{n}) \in L (\tilde{n} - \#T(r, \tilde{n}))} \text{ for } ((y, r), n) \in L.
\]

(b) Insert \( ((m, r), y) \) in \( T \) and return \( y \).
6: If \( \alpha = 0 \), then return \( y \) as follows.
   (a) Pick \( y \sim Y \setminus \bigcup_{(\tilde{r}, \tilde{n}) \in L} \tilde{y} \) uniformly at random.
   (b) \( n' \sim B \left( \frac{\#M - \sum (\tilde{r}, \tilde{n}) \in L \tilde{n}}{\#Y - \#T(r)}, 1 \right) \).
   (c) \( n \leftarrow n' + 1 \).
   (d) Insert \( ((y, r), n) \) in \( L \), insert \( ((m, r), y) \) in \( T \), and return \( y \).
Algorithm 2 CP-CO$^h(r, r')$

1: Pick uniformly $m \leftarrow M$.
2: $x \leftarrow m||r$.
3: Run the algorithm prefix-RO$^h(x)$ and get the hash value $y = h(m||r)$.
4: If there is no entry $((y, r'), n) \notin \mathbb{L}$ for any $n$, then
   (a) $n' \leftarrow \mathbb{B}\left(\frac{\#M - \sum_{((\tilde{y}, r), n) \in \mathbb{L}} \tilde{n}}{\#Y - \#\mathbb{L}(r)}\right)$.
   (b) Insert $((y, r'), n')$ in $\mathbb{L}$.
5: If $((y, r'), 0) \in \mathbb{L}$, then return $\bot$.
6: If $r = r'$
   (a) If $((y, r'), 1) \in \mathbb{L}$, then return $\bot$.
   (b) Compute the following value:
      $$q_{((y, r), n)} = \frac{\#\mathbb{T}(r, y) - 1}{n - 1}.$$ 
   (c) Flip a biased coin with $\text{Pr}[\beta = 0] = q_{((y, r), n)}$.
   (d) If $\beta = 1$
      (i) Pick uniformly one entry $((m', r), y) \in \mathbb{T}$ satisfying $m \neq m'$.
      (ii) Return $(m||r, m'||r)$.
7: If $r \neq r'$
   (a) Compute the following value:
      $$q_{((y, r), n)} = \frac{\#\mathbb{T}(r', y)}{n}.$$ 
   (b) Flip a biased coin with $\text{Pr}[\beta = 0] = q_{((y, r), n)}$.
   (c) If $\beta = 0$
      (i) Pick uniformly one entry $((m', r), y) \in \mathbb{T}$ satisfying $m \neq m'$.
      (ii) Return $(m||r, m'||r)$.
   (d) If $\beta = 1$
      (i) Pick uniformly $m' \leftarrow M$ such that there is no entry $((m', r), \tilde{y}) \in \mathbb{T}$ for any $\tilde{y} \in Y$.
      (ii) Insert $((m', r), y)$ in $\mathbb{T}$ and return $(m||r, m'||r)$.

Algorithm 3 prefix-RO$^h(x)$

1: Parse $x$ as $m||r$.
2: If there is an entry $((m, r), y) \in \mathbb{T}$ for some $y$, then return $y$.
3: $y \leftarrow Y$.
4: If $y \in \bigcup_{((\tilde{y}, r), n) \in \mathbb{L}} \tilde{y}$ then abort.
5: $n' \leftarrow \mathbb{B}\left(\frac{\#M - \sum_{((\tilde{y}, r), n) \in \mathbb{L}} \tilde{n} - 1}{\#Y - \#\mathbb{L}(r)}\right)$.
6: $n \leftarrow n' + 1$.
7: Insert $((y, r), n) \in \mathbb{L}$, insert $((m, r), y)$ in $\mathbb{T}$, and return $y$. 

14
Proof. Let $E$ be the event that step 5 in $\text{prefix-RO}^h$ does not occur and $F$ be the event that step 3 in $\text{prefix-RO}^h$ is $y \notin \bigcup_{(\tilde{y},r) \in \mathcal{L}} T(y)$. If both $E$ and $F$ occur, the behavior of $\text{prefix-RO}^h$ and $\text{prefix-RO}^h$ is identical.

\[
\Pr[\neg E] = \frac{\sum_{(\tilde{y},r) \in \mathcal{L}} (\tilde{n} - \#T(r, \tilde{y}))}{\#M - \#T(r)} \leq \frac{\sum_{(\tilde{y},r) \in \tilde{\mathcal{L}}} \tilde{n}}{\#M - \#T(r)}
\]

Consider the case where $\text{BAD}_r$ does not occur.

\[
\Pr[\neg E | \neg \text{BAD}_r] = \frac{qL}{\#M - \#T(r)} = \frac{qL}{\#M} \left(1 + \frac{\#T(r)}{\#M}\right) \leq \frac{2qL}{\#M}.
\]

where if $\#M \geq \#Y$, $L = 5 \ln \frac{\#Y}{\ln \ln \#Y}$, otherwise $L = \frac{5 \ln \#Y}{\ln \ln \#Y}$.

The probability $\Pr[\neg F]$ evaluates follows.

\[
\Pr[\neg F] \leq \frac{q}{\#Y}.
\]

By Lemma 1, we have

\[
\Pr[\neg (E \land F)] \leq \Pr[\neg E] + \Pr[\neg F]
\]

\[
\leq \Pr[\neg E | \neg \text{BAD}_r] + \Pr[\text{BAD}_r] + \Pr[\neg F]
\]

\[
\leq \frac{2qL}{\#M} + \frac{1}{(\#Y)^2} + \frac{q}{\#Y}.
\]

Hence

\[
p_{\text{prefixRO}^h} = \Pr[\neg (E \land F)]
\]

\[
\leq \Pr[\neg E | \neg \text{BAD}_r] + \Pr[\text{BAD}_r] + \Pr[\neg F]
\]

\[
\leq \begin{cases} \frac{\ln \#Y}{\ln \ln \#Y} \frac{10q}{\#Y} + \frac{1}{(\#Y)^2} + \frac{q}{\#Y} & (\#M \geq \#Y) \\ \frac{\ln \#Y}{\ln \ln \#Y} \frac{10q}{\#Y} + \frac{1}{(\#Y)^2} + \frac{q}{\#Y} & (\#M < \#Y). \end{cases}
\]

\[
\Box
\]

B Security Proof for Signature Schemes

B.1 Proof of Theorem 1

Proof. We construct an algorithm $\mathcal{A}$ as follows.

1. Query $\text{COMMON-CP-CO}^h$ with $p$ and obtain $\xi$.
2. If $\xi = \perp$ then abort, otherwise parse $\xi$ as $(m||p, m'||p)$, where $h(m||p) = h(m'||p)$.
3. Query the signature of $m||p$ to the signing oracle, and obtain a signature $\sigma$.
4. Output $(m'||p, \sigma)$ as a valid forgery.

If $\mathcal{A}$ does not abort, then $\mathcal{A}$ can output a valid forgery. Let $\text{abort}$ be the event that $\mathcal{A}$ aborts.

\[
\Pr[\text{abort}] = \Pr[\xi = \perp]
\]

\[
= \Pr[\#T_h(y, r) \leq 1]
\]

\[
= \Pr[\#T_h(y, r) = 1]
\]

\[
= \Pr[n' = 0 | n' \xrightarrow{\text{B}} 2^{\ell_1} - 1, \frac{1}{2^k}]
\]

\[
= \left(1 - \frac{1}{2^k}\right)^{2^{\ell_1} - 1} \leq e^{(1 - 2^{\ell_1})/2^k}
\]

Therefore, $\mathcal{A}$ can output a valid forgery with probability at least $1 - e^{(1 - 2^{\ell_1})/2^k}$. \(\Box\)
B.2 Proof of Theorem 2

Proof. Assume that a PPT algorithm $B$ breaks the EUF-CMA security with $\epsilon_{euf}$ which is non-negligible in $k$. To prove the theorem, we first describe a sequence of games. Let Game 0 be the original EUF-CMA game in the CP-CT-ROM$(\ell_1,t_1,k)$ and Game 6 be directly related to solve the RSA problem. Let $S_i$ be the event that an adversary outputs a valid forgery in the Game $i$.

- **Game 0**: The original EUF-CMA game in the CP-CT-ROM$(\ell_1,t_1,k)$.
  $\Pr[S_0] = \epsilon_{euf}$

- **Game 1**: We replace $RO^h$ and $CP-CO^h$ by algorithms $prefix-RO^h$ and $CP-CO^h$ respectively. Let tables $T$ and $L$ be simulation tables commonly used in algorithms $prefix-RO^h$ and $CP-CO^h$. By Lemma 3, we have
  $$|\Pr[S_0] - \Pr[S_1]| = 0.$$  

- **Game 2**: We replace algorithms $prefix-RO^h$ and $CP-CO^h$ by algorithms $prefix-RO^h$ and $CP-CO^h$. $CP-CO^h$ is the part of $prefix-RO^h$ in $CP-CO^h$ changed to $prefix-RO^h$. $Q_1$ is the total number of queries to $prefix-RO^h$. By Lemma 5, we have
  $$|\Pr[S_1] - \Pr[S_2]| \leq Q_1 \times p_{prefixRO^h}.$$  

- **Game 3**: When the signing algorithm runs $prefix-RO^h$ on input $m||r$, parse $m||r$ as $b||c$ ($|b| = \ell_1, |c| = t_1$). If there is an entry $((b,c), y) \in T$ for some $y$ already, then $prefix-RO^h$ aborts. $Q_2$ is the bound of the number of entries recorded in the table $T$.
  $$|\Pr[S_2] - \Pr[S_3]| \leq \frac{q_{sign}Q_2}{2k_1}.$$  

- **Game 4**: For the setting of a hash value of $prefix-RO^h$, fix $z \leftarrow Z_N^*$ and change it as follows.
  - If the hash value is queried by the signing algorithm, then $prefix-RO^h$ chooses $x \in Z_N$ and outputs $y = x^e \mod N$.
  - If the hash value is queried by the adversary or $CP-CO^h$, then $prefix-RO^h$ chooses $x \in Z_N$ and outputs $y = zx^e \mod N$.
  $$|\Pr[S_3] - \Pr[S_4]| = 0$$

- **Game 5**: We modify the signing algorithm in the computation $y^d$ to search $(x, y)$ such that $x = y^d$ instead of using the signing key $d$.
  $$|\Pr[S_4] - \Pr[S_5]| = 0$$

- **Game 6**: When receiving the output forgery $(m^*, \sigma^*)$ from the adversary, parse $\sigma^*$ as $\sigma^* = r^*||x^*$. If $m^*||r^*$ is not queried to $prefix-RO^h$, then aborts.
  $$|\Pr[S_5] - \Pr[S_6]| \leq \frac{1}{2k}$$

We construct the algorithm $A$ which breaking the RSA assumption using the algorithm $B$. The operation of $A$ for the input RSA instance $(N, e, z^*)$ is changed to the $z$ in Game 6 to $z^*$. Suppose $A$ do not abort receiving a forgery $(m^*, \sigma^*)$ from $B$. When parsing $\sigma^*$ as $r^*||x^*$, then $h(m^*||r^*) = y^* = (x^*)^e$ holds. When $A$ computes $(z^*)^1/e = x^*/x$ using $x$ chosen by $prefix-RO^h$ for the query of the hash value of $m||r$ from $B$ then $(x^*/x)^e = z^*$ holds. Hence, $A$ can output the solution $(z^*)^1/e$ of the RSA instance $(N, e, z^*)$. We can bound the probability $Pr[S_6] \leq \epsilon_{rsa}$.

$$\epsilon_{euf} \leq \epsilon_{rsa} + \frac{1}{2k} + \frac{q_{sign}Q_2}{2k_1} + Q_1 \times p_{prefixRO^h}$$

Therefore, $A$ breaks the RSA assumption with non-negligible probability $\epsilon_{rsa}$. $\square$
B.3 Proof of Theorem 3

Proof. We construct an algorithm $\mathcal{A}$ as follows.

1. Query the signature of $m$ to the signing oracle, and obtain a signature $\sigma$.
2. Parse $\sigma$ as $(r, x)$.
3. Query $CP-SPO^h$ with $((m||r), r)$, and obtain $\xi$.
4. If $\xi = \bot$ then abort, otherwise parse $\xi$ as $m'||r$ where $h(m||r) = h(m'||r)$.
5. Output $(m', \sigma)$ as a valid forgery.

If $\mathcal{A}$ does not abort, then $\mathcal{A}$ can output a valid forgery. Let abort be the event that $\mathcal{A}$ aborts.

$$\Pr[\text{abort}] = \Pr[\#T_h(y, r) = 1] \leq e^{(1-2\ell)/2k}$$

Therefore, $\mathcal{A}$ can output a valid forgery with probability at least $1 - e^{(1-2\ell)/2k}$. ∎

B.4 Proof of Theorem 4

Proof. We modify the Game 4 in the proof of Theorem 2 as follows.

Game 4: For the setting of a hash value of $\text{prefix-RO}^h$, fix $z \leftarrow \mathbb{Z}_N^*$ and change it as follows.

- If the hash value is queried by the signing algorithm, then $\text{prefix-RO}^h$ chooses $x \in \mathbb{Z}_N$ and $y = xe \mod N$, then outputs $w = y \oplus r$.
- If the hash value is queried by the adversary or $\text{CP-CO}^h$, then $\text{prefix-RO}^h$ chooses $x \in \mathbb{Z}_N$ and $y = zx e \mod N$, then outputs $w = y \oplus r$.

It can be shown in a similar way as in Theorem 2. ∎

B.5 Proof of Theorem 5

Proof. We construct an algorithm $\mathcal{A}$ as follows.

1. Query the signature of $m$ to the signing oracle, and obtain a signature $\sigma$.
2. Parse $\sigma$ as $(r, x)$.
3. Query $CP-SPO^h$ with $((m||r), r)$, and obtain $\xi$.
4. If $\xi = \bot$ then abort, otherwise parse $\xi$ as $m'||r$ where $h(m||r) = h(m'||r)$.
5. Output $(m', \sigma)$ as a valid forgery.

If $\mathcal{A}$ does not abort, then $\mathcal{A}$ can output a valid forgery. Let abort be the event that $\mathcal{A}$ aborts.

$$\Pr[\text{abort}] = \Pr[\#T_h(y, r) = 1] \leq e^{(1-2\ell)/2k}$$

Therefore, $\mathcal{A}$ can output a valid forgery with probability at least $1 - e^{(1-2\ell)/2k}$. ∎
# Table of Contents

1 Introduction ........................................................................................................... 1
   1.1 Background ....................................................................................................... 1
   1.2 Our Contributions ............................................................................................ 2
   1.3 Related Works .................................................................................................. 3
   1.4 Road Map ......................................................................................................... 3
2 Preliminaries ................................................................................................................. 4
   2.1 Digital Signature Scheme .................................................................................. 4
   2.2 Security Notions of Hash Functions .................................................................... 5
   2.3 WROMs Proposed by Numayama et al. [11] ....................................................... 5
3 WROMs against Chosen Prefix Collision Attacks ......................................................... 6
4 Security of Signature Schemes in WROMs ................................................................. 7
   4.1 RSA-FDH .......................................................................................................... 8
   4.2 RSA-PFDH ....................................................................................................... 8
   4.3 RSA-PFDH\(^\oplus\) ............................................................................................. 9
   4.4 RSASSA-PKCS-v1.5 ......................................................................................... 9
   4.5 DSA ............................................................................................................... 10
5 Conclusion ............................................................................................................... 10
A Simulation Method in the CP-CT-ROM ................................................................ 12
B Security Proof for Signature Schemes ................................................................. 15
   B.1 Proof of Theorem 1 ......................................................................................... 15
   B.2 Proof of Theorem 2 ......................................................................................... 16
   B.3 Proof of Theorem 3 ......................................................................................... 17
   B.4 Proof of Theorem 4 ......................................................................................... 17
   B.5 Proof of Theorem 5 ......................................................................................... 17