Optimal platform sales mode in live streaming commerce supply chains

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Accepted: 24 June 2022
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Abstract
This paper investigates three common sales modes with live streaming commerce, including e-commerce platform mode, transferring mode and live streaming platform mode (abbreviated as E, T and L, respectively). Using game-theoretical method, we study how participants choose sales modes with consumer return. The findings show that for the seller and platforms, each mode may be the best, depending on basic net sales volume and channel rate. However, for the streamer, mode T will never be the best. On the other hand, product quality in mode L is always higher than that in mode T, and higher than that in mode E when live streaming platform’s basic net sales volume is high. Furthermore, we show that hybrid mode may generate higher profits for members, except for the live streaming platform; and we also study the impacts of transferring loss, gift-giving function and streamer’s dual-purpose on results through extensions, examining the robustness of the model, and deriving additional managerial insights.

Keywords Live streaming commerce · Mode selection · Product quality · Consumer return

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Published online: 29 July 2022
1 Introduction

Live streaming commerce has developed vigorously in recent years. In live streaming selling, people with public influences recommend the commodities, which can be carried out in an e-commerce platform or a live streaming platform respectively, or in both platforms simultaneously. Many platforms and sellers participate in live streaming commerce in China. With an average annual growth rate of more than 200% from 2016 to 2019, users in live streaming commerce are up to 524 million in 2020 and up to 635 million in 2021, and the number of multi-channel network institutions transfers to 28 thousand. Driven and supported by government policies, China’s live streaming commerce has developed rapidly and played an important role in revitalizing the economy since the COVID-19 epidemic. Moreover, many overseas platforms begin to test the waters. Walmart tried an hour-long live streaming selling in TikTok in 2020. Since then, Amazon, Shopee and AliExpress have also launched the live streaming function. Live streaming selling therefore becomes one of the vital strategies in e-commerce.

Due to the variety of platforms employed in live streaming and trading, participants are faced with the choice of different modes in live streaming commerce. There are three common modes in practice. The first mode is e-commerce platform mode (mode E) represented by Taobao and Shopee, with both live streaming and trading being finished in e-commerce platform. However, several content platforms (we call them live streaming platforms below) like TikTok and Instagram also launch live streaming, resulting in two more modes. One is transferring mode (mode T), in which the live streaming platform implements live streaming while trading is conducted in e-commerce platform. For example, audiences in KuaiShou live streaming room will transfer to Taobao through links, and TikTok also launches the minivans function in Indonesian to allow users to transfer to Shopee. The other is live streaming platform mode (mode L), in which both processes are carried out in the live streaming platform without transferring. For instance, TikTok set up “DouYin-Store” for trading function in 2020, and Facebook also launches “Facebook-Store”, offering a free live streaming and trading platform. Furthermore, each platform has

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1 Iresearch: Report on The Ecology of China’s live streaming commerce in 2020. http://www.100ec.cn/detail--6563340.html, 2020–7-13.
2 "Live streaming commerce" is speeding up. https://baijiahao.baidu.com/s?id=1688891932802092204&wfr=spider&for=pc, 2021–1-15.
3 Walmart is testing the waters with TikTok. https://baijiahao.baidu.com/s?id=1686394219710712803&wfr=spider&for=pc, 2020–12-18.
4 Cross-border e-commerce’s new trend: Live streaming commerce is changing the industry ecology. https://www.sohu.com/a/366749120_120057063, 2020–1-14.
5 TikTok is now available to Shopify merchants in UK and is set to expand to 15 countries. https://baijiahao.baidu.com/s?id=1692619870730131514&wfr=spider&for=pc, 2021–02-25.
6 Five live streaming commerce platforms: Only KuaiShou can transfer to other platforms, Wechat small program has low threshold. https://www.sohu.com/a/426336789_161795, 2020–10-21.
7 Facebook Shops are available for free. https://baijiahao.baidu.com/s?id=1667220918330806210&wfr=spider&for=pc, 2020–05-20.
different characteristics. According to research on China’s live streaming commerce industry in 2020, although with lower daily active users (DAU), e-commerce platforms have higher conversion rate due to user’s high purchasing motivation. Conversely, the live streaming platforms possess high DAU but a relatively low conversion rate because most users are not shopping-oriented. Based on the above conditions, TikTok in China only supports mode L while KuaiShou supports both modes T and L simultaneously. This motivates us to examine key factors influencing their mode decisions.

However, sometimes the product quality cannot be guaranteed in live streaming selling, resulting in the return of unmatched products. This is mainly attributed to the fact that many medium or top streamers have high bargaining powers to ask for “Lowest Price All-time”, which extremely squeezes sellers’ profit margins. According to the data posted by State Administration for Market Regulation in China, the complaints about live streaming account for 60% of all complaints received by the 12315 platform at the first three quarters in 2020. The qualified rate of spot check is merely 77.3% for products sold in live streaming in Pinduoduo and other four e-commerce providers in 2021. In addition, according to the report on China’s live streaming commerce industry in 2020, the average return rate of live streaming is about 30–50%, and for some live streaming rooms it is even as high as 70%, which is much higher than the 10–15% return rate of traditional e-commerce. Therefore, product quality and consumer return are significant factors that cannot be ignored when studying live streaming commerce, and this paper will investigate the product quality decision and the influence of consumer return on the decision-making.

With the above observations, several main questions arise in this research. (1) What are decisions of price, sales effort and product quality in different modes? (2) How do the sales modes affect product quality and net sales volume (NSV)? (3) For each participant, which mode is the best? To answer these questions, this paper considers a live streaming commerce supply chain consisting of a seller, a streamer, an e-commerce platform and/or a live streaming platform, which provides three mainstream modes (modes E, T and L) with the consideration of consumer return. The optimal decisions and mode selections are obtained. Besides, the impacts of the loss in transferring process, gift-giving function and the streamer’s dual-purpose attribute on results are further studied in extensions, followed by the research about a hybrid mode.

Our study differs from some existing research related to live streaming commerce. Many scholars focused on live streaming to find its incentive effect on consumers’ purchasing behaviors, mainly including the real-time reply function, emotional

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8 36 Institutes of Krypton: Research report on China’s live streaming commerce industry in 2020. https://36kr.com/p/986332005917833. 2020–12-02.
9 The problems in live streaming commerce are identified by the Consumers association. https://baijiahao.baidu.com/s?id=1684104917901952861&wfr=spider&for=pc. 2020–11-23.
10 Unqualified rate 22.7%! Shanghai spot-checks products in jingdong, Pinduoduo, Tmall, Xiaohongshu. https://sghexport.shobserver.com/html/baijiahao/2020/11/11/295337.html. 2021–11-11.
commitment, consumer engagement, and language style [7, 19, 31]. A few studies considered this problem from the perspective of supply chain management, in which they analyzed decisions of platform’s revenue sharing ratio and recommending fee, streamer’s advertisement, and seller’s price and random rewards [10, 15, 24, 40]. However, they mainly research the problem in the same live streaming mode. Different from the existing research, we provide the insights about the impacts of different modes on product quality and profits from the perspective of supply chain with the consideration of consumer return.

The main conclusions are obtained below. (1) As for profits, it is the best for the streamer to choose mode E when e-commerce platform’s basic NSV is low, or moderate but with low trading commission rate; otherwise, mode L is the best. For the seller, mode E (mode T) can be the best when NSV is high (low); otherwise, mode L can be the best. For both platforms, each mode can be optimal. (2) As for product quality and NSV, mode L always generates higher results than mode T, and they are also higher than mode E when live streaming platform’s basic NSV is high. (3) We find that hybrid mode may generate higher profits for members than each single mode, except for profit of the live streaming platform, while hybrid mode cannot have the highest product quality. Moreover, we also find the transferring loss, the gift-giving function and the streamer’s dual-purpose attribute will affect the choices of participants.

The rest of this paper is structured as follows. Section 2 reviews the related literature. Section 3 presents the formulations with three different modes. The comparisons and sensitivity analysis are carried out in Sect. 4. Section 5 makes several further extensions and discussions. Section 6 concludes this work. All the proofs can be found in the Appendix.

2 Literature review

This research is closely related to the following three streams: e-commerce platform supply chain, live streaming commerce, and product quality and consumer return.

2.1 E-commerce platform supply chain

E-commerce platform has drawn more and more public attention with the development of online shopping. Research on sales mode selection is an important topic. Abhishek et al. [1] compare two common sales modes, reselling mode and agent mode, and they find the agent mode is more efficient than the reselling mode with lower sales price. However, Tian et al. [33] find the competition among upstream sellers will reduce this likelihood. Zhang and Zhang [39] consider the mode choice of the online retailer when it seeks to make inroads into the offline market and has an information sharing strategy with the seller. Qin et al. [26] research different combinations of sales model and logistics service model, finding the seller’s preference is consistent with the enhancement of logistics service level. Qin et al. [27] also study the new phenomenon of sharing logistics services between e-retailers and streamers.
Besides, bundling strategy is another common topic in this area. Lin et al. [15] consider the competitive market of dual e-commerce platforms, and further study the influences of base users and competitive issues on price and bundling strategy. Guo et al. [5] also consider the bundling strategy with an agent sales mode.

Some research on e-commerce platforms is mainly about reselling and agent mode selection, without the consideration of live streaming. However, with the development of live streaming commerce, many e-commerce platforms develop the live streaming function or cooperate with other live streaming platforms. As a result, the existing research cannot be adopted into the live streaming environment. Our research fills the gap by researching new sales mode with the participation of live streaming platforms, in which both trading and live streaming functions of e-commerce platforms will be considered.

2.2 Live streaming commerce

With the popularity of live streaming commerce, related studies have emerged, and most researchers focus on the motivation and intention of consumers’ live streaming shopping behaviors. For example, Kim and Kim [12] pay attention to the core business model of this new e-commerce, use data from major commerce companies, and measure the effect of consumer-generated attributes such as “likes” in Facebook on sales of social sharing. Capitalizing on the real data from Alibaba, Tan et al. [31] research the effects of live-chatting on online shopping, and how could it be regulated by the sales and feedback scores. Hu and Chaudhry [7] build a comprehensive concept framework combining with relationship bonds, emotional commitment and consumer engagement according to the data from Taobao. Luo et al. [19] focus on the sales level of live streaming to find out how the language styles affect consumers’ purchasing behaviors on different types of products. Wongkitrungrueng et al. [36] analyze the data of Facebook from the perspective of streamers. Besides, Wu et al. [37] figure out the mechanism of consumer gift-giving when they can watch live streaming for free via the online survey. Lu et al. [17] add audiences externally to observe the relationship between audience size and tips, finding out a positive correlation between audience size and average tips.

Sequentially, studies about live streaming commerce supply chain management appeared. On the one hand, many scholars focus on different functions of live streaming platforms. For example, Tang and Huang [32] consider the free gift-giving behaviors of audiences, in which the gift revenue will be allocated between the platform and content producers. Li et al. [13] firstly discuss two different gift-giving behaviors, free-gift and paid-gift. Popescu and Crama [25] focus on the advertising and scheduling problems in the live streaming process. On the other hand, the live streaming is combined with e-commerce function. Feng and Wang [3] conclude different modes of live streaming and supply chains, as well as the functions of each member. Liu and Liu [16] explore how to set up the proportion of revenue sharing and recommendation fees for platforms, and how to make reasonable investment in the recommendation mechanism for streamers. Jiang and Cai [11] build a price decision model to research the impacts of consumer impulsive consumption...
and satisfaction on it, which shows its influence is non-monotonous. Peng et al. [24] conduct a study of rural agricultural products to explore how random rewards in live streaming will affect farmers’ income. Zhu and Liu [40] explore the coordination issues in the logistics service supply chain in live streaming e-commerce industry, discussing the influences of cost-sharing and the coordination contract on the level of effort of the logistics service.

Therefore, our research differs from the above-mentioned studies in three key aspects. Firstly, many studies only focus on the management of e-commerce platform or live streaming platform respectively, while we consider a supply chain with a streamer, a seller and both platforms simultaneously, in which the trading functions of live streaming platforms are considered and the sales mode selection is studied. Secondly, we explore the impacts of product quality and consumer return on decisions. It is vital for members to consider consumer return in live streaming with the increasing return rate nowadays, while the existing studies related to live streaming commerce with quality and return issues are still rare. Finally, some realistic factors such as the gift-giving function, the transferring loss between two platforms and the streamer’s dual-purpose attribute are considered in our study.

2.3 Product quality and consumer return

Product quality management is a long-term concern. Many scholars have carried out research on how to make reasonable product quality and price decisions. Seifbarghy et al. [28] analyze the quality and price decisions of a two-stage supply chain, under centralized and decentralized situation severally. Jiang and Yang [10] study the same decisions when the consumers can obtain the quality information through online reviews. Chen et al. [2] consider the quality decision in online, offline and dual-channel situations, finding the performance of supply chain could be promoted through adding a new channel. Jabarzare and Rasti-Barzoki [9] discuss the impacts of packaging on product quality in a dual-channel supply chain. Besides, many scholars consider the quality issue in the closed-loop supply chain. Maiti and Giri [20] analyze the quality decisions with different leaders in a closed-loop supply chain, in which the retailer is responsible to recycle. Guan and Chen [4] further research the interactive relationships between the information accesses and quality disclosure strategies of manufacturers.

Additionally, many researchers focus on the consumer return. For many scholars, consumer return policies are a topic of interest, and they usually compare different policies such as money-back guarantees (MBGs) and no-refund policies for retailers or manufacturers [8, 34]. Several scholars have also examined how return policies apply to omnichannel situations [21, 22]. The relationship between return quantity and product quality is also considered by many researchers. For example, Taleizadeh et al. [30] consider the situation where the quantity of returned commodities is affected by the quality in online shopping, and they research a joint strategy of price, quality and return policy. Yoo [38] and Li et al. [14] also carry out the similar studies aiming at this situation.
Therefore, based on existing research, we further discuss the trade-off of product quality and return quantity in the background of live streaming commerce, in which the product quality is harder to ensure and the return rate is higher than the traditional online shopping.

3 Model setup

Three mainstream sales modes in live streaming commerce are considered. (1) mode E: both live streaming and trading processes are carried out in e-commerce platform. (2) mode T: the streamer promotes products in live streaming platform while trading process will be realized in e-commerce platform. (3) mode L: both live streaming and trading processes are finished in live streaming platform. Therefore, the supply chain structures of three modes are exhibited in Fig. 1.

We summarize parameters and decision variables in Table 1. The subscript \( s = (E, T, L) \) indicates the type of modes (mode E, mode T, mode L) and the subscript \( i = (s, a, ep, lp, c) \) indicates different members (seller, streamer, e-commerce platform, live streaming platform and supply chain). The superscript \( ^* \) denotes the optimal value.

In order to make the model close to actual situations and facilitate the calculation, the following assumptions are proposed.

(1) The basic DAU of live streaming platform is higher than e-commerce platform, while the conversion rate is lower, that is, \( a_1 < a_2 \) and \( t_1 > t_2 \) exist.

Different platforms have different user portraits. Users in e-commerce platforms have strong shopping tendency, but their DAU are lower than live streaming platforms. However, the consumer’s shopping impulse in live streaming platform is weak and therefore the conversion rate is relatively low. For example, the DAU of TikTok is up to 600 million, while the DAU of Taobao is only 237 million.\(^{11}\) However, according to the data of QuestMobile, the average conversion rate of TikTok is only 8.1%, while it is over 65% in Taobao.\(^{12}\)

(2) The revenue sharing rates and live streaming service rates are the same for two platforms, while the trading commission rate in e-commerce platforms is higher, that is, \( \beta_2 < \beta_1 \).

The live streaming service rates in different platforms and the revenue sharing rates are usually determined by the market. However, after most sellers having settled into e-commerce platforms, live streaming platforms need to attract them through lower trading commission rates for building in-house e-commerce. For example, TikTok announces that new sellers entering into “DouYin-Store” enjoy the one-month trading commission rate with only 1%.\(^{13}\) Facebook even does not

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\(^{11}\) Pinduoduo surpassed Taobao in daily active users in just 5 years. [https://baijiahao.baidu.com/s?id=1693650015144214842&wfr=spider&for=pc](https://baijiahao.baidu.com/s?id=1693650015144214842&wfr=spider&for=pc). 2021–03-08.

\(^{12}\) QuestMobile: 2020 China mobile Internet annual report. [http://app.myzaker.com/news/article.php?pk=6018f10f15ec007e9199bfc2](http://app.myzaker.com/news/article.php?pk=6018f10f15ec007e9199bfc2). 2021–2–02.

\(^{13}\) How does “DouYin-Store” charge? [http://www.2gou.com/17775.html](http://www.2gou.com/17775.html). 2021–7–13.
charge any fees,\(^{14}\) while Taobao’s trading commission rate is generally around 3%-10%, and Tmall takes a percentage of no less than 5%.

(3) Both the quality improvement cost and the cost about sales effort are modeled as quadratic functions.

The quality improvement cost is regarded as a one-time investment, where the seller may enhance product quality by investing in designing quality, such as for a better assembly line, a better production machine, the updated computers, or a better supplier. Moreover, we model the quality improvement cost as a quadratic function, which explains improving the quality has an increasing incremental cost at high levels and then yields a diminishing return on the quality expenditure. This assumption is widely adopted in some research \([2, 9, 20]\). The investment cost for improving sales effort is also the same. For example, the quadratic function of advertisement investment is adopted by Lu and Navas \([17]\).

(4) The demand is related to the price and the level of sales effort, and the amount of returns is related to the product quality observed by consumers.

The demand has a linear negative correlation with product price and a linear positive correlation with sales effort level, and therefore \(Q = at - bp + re\). Besides, the return quantity is negatively correlated with product quality observed by consumers when they receive products, in which \(R = z - \varphi q\) exists. Li et al. \([14]\) and Taleizadeh et al. \([30]\) also come up with the similar assumptions in their studies.

(5) There is a value limitation of price coefficient of market demand:

\[
b > \max \left( \frac{2(1-a-\beta_1)\varphi^2}{k_2}, \frac{(1-\theta)ar^2}{k_1}, \frac{2(1-a-\beta_1)\varphi^2k_1+(1-\theta)ar^2k_2}{2k_1k_2} \right).
\]

This constraint ensures the non-negativity of decisions and demands. It can be also explained in reality, in which the streamer considers to promote sales effort only when consumers are relatively sensitive to price, otherwise, he can substantially increase the price to generate a higher profit without losing too many consumers, which is not true in reality.

Then, we illustrate the decision sequence in the supply chain. Recently, with the advent of professional streamers and multi-channel network institutions which aim to nurture streamers technically, streamers with substantial number of fans, high stickiness and strong cash ability have more and more bargaining power.\(^{15}\) They represent their fans to ask for quotation from sellers, making it difficult for small and medium-sized sellers to dominate the sales price. Therefore, we consider the streamer as the supply chain leader, deciding the sales effort and sales price firstly, and then the seller determines the product quality on the basis of given sales price.

\(^{14}\) Five live streaming commerce platforms: Only KuaiShou can transfer to other platforms, Wechat small program has low threshold. \https://www.sohu.com/a/426336789_161795. 2020–10-21.

\(^{15}\) The ROI of live streaming commerce is less than 1:2. \https://baijiahao.baidu.com/s?id=1687773702568945003&wfr=spider&for=pc. 2021–01-02.
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This is common for the customized products and products with short shelf-life in live streaming selling. For example, Yudonglai Food Company arranges the production of hotpot condiment immediately after receiving the orders via live streaming.\(^{16}\) Moreover, some liquor brands promote the customized liquor via live streaming, which results the sales before production process.\(^ {17}\)

Next, we will illustrate the profits of members in live streaming commerce supply chain among different modes respectively.

### 3.1 E-commerce platform mode (mode E)

In this situation, the live streaming and trading processes are all carried out in e-commerce platform. Therefore, the seller will give commissions (with a revenue sharing rate \(\alpha\)) to the streamer and pay the sales service fee (with a trading commission rate \(\beta_1\)) to the e-commerce platform based on NSV.\(^ {18}\) What worthy noting here is, the platform always knows the revenue sharing rate \(\alpha\) no matter how does the seller connect with the streamer. For example, if the seller wants to implement live streaming through Taobao, it must upload the revenue sharing rate in a platform called V mission which is also owned by Alibaba. Cooperation on live streaming skipping the platform is not allowed, otherwise, the streamer’s account will be shut down.\(^ {19}\) Therefore, the platform can monitor the revenue sharing rate, and the streamer needs to pay the live streaming service fee (with a live streaming service rate \(\theta\)) to the e-commerce platform based on its revenue through live streaming selling.\(^ {20}\) The profit functions of the seller, the streamer and the e-commerce platform in this mode are as follows:

\(^{16}\) “Chongqing Baby” was the big winner in live streaming commerce. http://guoqing.china.com.cn/2020-06/01/content_76113156.html. 2020–06-01.

\(^{17}\) Customized liquor in live streaming commerce. https://www.sohu.com/a/495293769_121016089. 2021–10-15.

\(^{18}\) 36 institutes of krypton: Research report on China’s live streaming commerce Industry in 2020. https://36kr.com/p/986332005917833. 2020-12-02.

\(^{19}\) How does Taobao charge? https://www.majia.com/article/482476. 2021–03-25.

\(^{20}\) How to calculate the streamer’s commission in live streaming? https://www.sohu.com/a/443243745_100163809. 2021-01-08.
Table 1  Notations for parameters and decision variables

| Model Parameters | Decision variables |
|------------------|--------------------|
| $a_1$ | Basic DAU of the e-commerce platform |
| $a_2$ | Basic DAU of the live streaming platform |
| $a_3$ | The common DAU of two platforms |
| $t_1$ | Conversion rate of DAU in e-commerce platform |
| $t_2$ | Conversion rate of DAU in live streaming platform |
| $\alpha$ | The revenue sharing rate (given by the seller to the streamer) |
| $\theta$ | The live streaming service rate (charged by platforms from the streamer) |
| $\epsilon$ | The channel rate (charged by the live streaming platform for external sales link) |
| $\beta_1$ | The e-commerce platform’s trading commission rate (paid by the seller for sales management) |
| $\beta_2$ | The live streaming platform’s trading commission rate (paid by the seller for sales management) |
| $\tau$ | The gift service rate (charged by the live streaming platform for gift-giving revenues) |
| $k_1$ | The unit cost of sales effort for the streamer |
| $k_2$ | The unit cost of quality improvement for the seller |
| $b$ | Price coefficient of the market demand |
| $r$ | Sales effort coefficient of the market demand |
| $\varphi$ | Quality level coefficient of the returned product quantity |
| $\delta$ | The retention rate of consumers in the transferring process |
| $\mu$ | The attention degree of the streamer to consumer surplus |
| $g$ | The gift-giving revenue for unit sales effort |
| $z$ | Basic consumer return quantity |
| $\sigma_1$ | The proportion of common DAU who are attracted in mode T |
| $\sigma_2$ | The proportion return products in mode E |
| $Q_s$ | Product sales volume under mode s |
| $R_s$ | Consumer return quantity under mode s |
| $\Delta Q_s$ | NSV of product under mode s |
| $\pi_{is}$ | The profit of $i$ in mode $s$ |
| $CS_s$ | The consumer surplus in mode $s$ |
| $SW_s$ | The social welfare in mode $s$ |
| $e$ | The sales effort decision of the streamer |
| $p$ | The unit sales price decided by the streamer |
| $q$ | The product quality level decision of the seller |

$$
\pi_{sE} = p(Q_E - R_E) (1 - \alpha + \beta_1) - \frac{1}{2}k_2q^2, \\
\pi_{aE} = p(Q_E - R_E) \alpha(1 - \theta) - \frac{1}{2}k_1e^2, \\
\pi_{epE} = p(Q_E - R_E) \beta_1 + p(Q_E - R_E) \alpha \theta.
$$
It should be noted that to prevent the fake sales data, the fees charged by platforms or the streamer are all based on the NSV, which means the returned product sales are excluded. For the seller, the first term is the revenue generated from selling products, and the second term is the cost for improving product quality. For the streamer, the first term is the sharing revenue generated from helping sell products for the seller, and the second term is the cost for enhancing the sales effort. For the e-commerce platform, the first term is the trading commission generated from the seller, and the other is the live streaming service revenue from the streamer.

### 3.2 Transferring mode (mode T)

In this mode, the streamer chooses to promote products through the live streaming platform with a higher DAU, while consumers need transfer to the e-commerce platform for purchasing. Therefore, the seller still gives commissions to the streamer and pays the sales service fee to e-commerce platform based on NSV. However, part of the commissions given to the streamer will be distributed to the live streaming platform as the permission fee (with a channel rate $\varepsilon$) for selling external products, which is also based on NSV. Therefore, the real revenue sharing rate gained by the streamer is $\alpha - \varepsilon$, and similar to mode E, the streamer still needs to pay the live streaming service fee based on its actual sharing revenue. For example, the live streaming platform in China named Kuaishou will charge sellers from Pinduoduo with a 6% channel rate, and the streamer also needs to pay Kuaishou with a 10% live streaming service rate based on its sharing revenue. Therefore, the profit functions of the seller, the streamer, the live streaming platform, and the e-commerce platform in this mode are given below:

$$\pi_{sT} = p(Q_T - R_T)(1 - \alpha - \beta_1) - \frac{1}{2}k_2q^2,$$

$$\pi_{aT} = p(Q_T - R_T)(\alpha - \varepsilon)(1 - \theta) - \frac{1}{2}k_1\varepsilon^2,$$

$$\pi_{lpT} = p(Q_T - R_T)\varepsilon + p(Q_T - R_T)(\alpha - \varepsilon)\theta,$$

$$\pi_{epT} = p(Q_T - R_T)\beta_1.$$  

Compared with the profit functions in mode E, the profit structures of the seller and the streamer remain unchanged. However, for the e-commerce platform, its revenue only comes from the sales service fees paid by the seller. For the live streaming platform’s profit function, the first term is the channel fees generated from the streamer, and the second term is the live streaming service revenue from the streamer.

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21 Taobao Union commission settlement rules. http://www.shihuizhu.com/news/newsDetails?id=875. 2020–10-10.

22 How does each platform charge for live streaming selling? https://www.zhihu.com/question/463102955. 2021–06-06.

23 What is the charging standard in KuaiShou? https://www.maijia.com/article/491060. 2021–07-09.

24 Five live streaming commerce platforms: Only KuaiShou can transfer to other platforms, Wechat small program has low threshold. https://www.sohu.com/a/426336789_161795. 2020–10-21.
3.3 Live streaming platform mode (mode L)

The live streaming and trading processes are all conducted in the live streaming platform. However, the live streaming platform usually has a lower trading commission rate. Besides, different from mode T, the streamer would not pay the channel fee because the products are sold through the live streaming platform itself. Therefore, the profit functions of the seller, the streamer and the live streaming platform are listed below:

\[
\pi_{sL} = p(Q_L - R_L)(1 - \alpha - \beta_2) - \frac{1}{2}k_2q^2,
\]

\[
\pi_{aL} = p(Q_L - R_L)\alpha(1 - \theta) - \frac{1}{2}k_1e^2,
\]

\[
\pi_{lpL} = p(Q_L - R_L)\beta_2 + p(Q_L - R_L)\alpha\theta.
\]

Here, the profit structures of the seller and the streamer also remain unchanged, while revenue of the live streaming platform includes both the sales service fee from the seller and the live streaming service fee paid by the streamer.

3.4 Decisions and performance

Solving the three models by backward induction method, we derive the optimal solutions, NSV, profits and welfares, as shown in Table 2 to Table 4.

Table 2 gives optimal decisions and NSV in three sales modes. The sales effort, price, product quality and NSV will all decrease with live streaming service rate, price coefficient and basic return quantity, while they will increase with the sales effort coefficient. Next, we substitute these optimal decisions into profit functions respectively to get optimal profits.

Table 3 exhibits optimal profits of the seller, the streamer, and two platforms in three modes. Profits of the seller and the streamer in three modes will also decrease with live streaming service rate and basic return quantity, and all the profits will increase with sales effort coefficient.

Next, Table 4 summarizes the supply chain profit and consumer surplus. Referring to Hafezalkotob [6], consumer surplus indicates the difference between the highest price consumers are willing to pay and the price they actually pay. Therefore, the consumer surplus in mode E is expressed as follows, and those in the other two modes are similar.

\[
CS_E = \int_p^{\frac{a_1t_1 - bp + re}{b}} [a_1t_1 - bp + re - (z - \varphi q)]dp = \frac{(a_1t_1 - bp + re)[a_1t_1 - bp + re - 2(z - \varphi q)]}{2b}.
\]

Similar to Sheu [29], we consider that social welfare is the sum of supply chain’s profit and consumer surplus, that is \(SW = CS + \pi_c\). Due to the complexity of results, social welfare will be omitted from the analysis here and will be analyzed through numerical examples.
To clearly investigate the impacts of key parameters on product quality, NSV and profits, we derive the first or second derivative with respect to revenue sharing rate, channel rate, various service rates and quality coefficient, respectively, which are shown in Corollary 1.

**Corollary 1** The influence of revenue sharing rate, channel rate, various rates and consumers’ coefficient on product quality, NSV and profits are shown as follows:

(1) \( \frac{\partial q^*}{\partial \theta} < 0, \frac{\partial q^*}{\partial \alpha} < 0, \frac{\partial q^*}{\partial \epsilon} < 0, \frac{\partial q^*}{\partial \beta_1} < 0; \)

(2) \( \frac{\partial \Delta Q^*}{\partial \theta} < 0, \frac{\partial \Delta Q^*}{\partial \alpha} > 0, \frac{\partial \Delta Q^*}{\partial \epsilon} < 0, \frac{\partial \Delta Q^*}{\partial \beta_1} < 0; \)

(3) \( \frac{\partial \pi_a^*}{\partial \theta} < 0, \frac{\partial \pi_a^*}{\partial \alpha} > 0, \frac{\partial \pi_a^*}{\partial \epsilon} < 0, \frac{\partial \pi_a^*}{\partial \beta_1} < 0; \)

(4) \( \frac{\partial^2 q^*}{\partial \theta^2} < 0, \frac{\partial^2 \pi_a^*}{\partial \alpha^2} < 0, \frac{\partial^2 \pi_a^*}{\partial \epsilon^2} < 0, \frac{\partial^2 \pi_a^*}{\partial \beta_1^2} < 0; \)

(5) \( \frac{\partial^2 q^*}{\partial \phi^2} > 0, \frac{\partial^2 \pi_a^*}{\partial \phi^2} > 0, \frac{\partial^2 \pi_s^*}{\partial \phi^2} < 0, \frac{\partial^2 \pi_p^*}{\partial \phi^2} < 0. \)

According to Corollary 1(1) and (2), product quality and NSV will decrease with live streaming service rate (\( \theta \)), trading commission rate (\( \beta_1 \)), and channel rate (\( \epsilon \)). The quality level will also decline with the increase of revenue sharing rate (\( \alpha \)), while the NSV will arise. Because higher revenue sharing rate shrinks profit margin of the seller but motivates the streamer to improve sales effort, which will finally increase NSV. From Corollary 1(4), profits of the seller and both platforms first increase and then decrease with their charge rate. Corollary 1(5) shows the supply chain’s profits have the similar relationships with all the rates. It implies the fact that higher revenue sharing rate, service rates and channel rate are not always better, which might bring negative effect on profits on the contrary.

Besides, coefficient about consumer return quantity also affects the results according to Corollary 1(6). Product quality, NSV and members’ profits (except the seller) will all increase with quality coefficient (\( \phi \)). Because high coefficient about quality motivates the seller to improve the quality, causing the decrease of return quantity and increase of profits. However, from perspective of the seller’s and supply chain’s profit, consumer’s moderate attention to product quality leads to the highest profit, while too much or too little attention will lower that. This can be attributed to the fact that excessive attention will improve quality but will increase sales price and the seller’s quality cost simultaneously, causing a large loss of price-oriented consumers. However, too little attention will demotivate the quality improvement, resulting in a substantial number of consumer return.

### 4 Model comparisons

In this section, the optimal solutions will be compared in three sales modes, and the best choice will be chosen. Moreover, results in some specific cases are provided.
Table 2  Optimal decisions and NSV under three modes

| Decisions | Mode E | Mode T | Mode L |
|-----------|--------|--------|--------|
| $e^*$     | $(1-\theta_k) \tau (a_1t_1-t_2)$ | $(1-\theta_k) \tau (a_1t_1-t_2)$ | $(1-\theta_k) \tau (a_1t_1-t_2)$ |
| $p^*$     | $2bh_k = (1-\theta_k^2) k_2 a - 2(1-a - \beta_l) \psi k_1$ | $2bh_k = (1-\theta_k^2) k_2 a - 2(1-a - \beta_l) \psi k_1$ | $2bh_k = (1-\theta_k^2) k_2 a - 2(1-a - \beta_l) \psi k_1$ |
| $q^*$     | $2bh_k = (1-\theta_k^2) k_2 a - 2(1-a - \beta_l) \psi k_1$ | $2bh_k = (1-\theta_k^2) k_2 a - 2(1-a - \beta_l) \psi k_1$ | $2bh_k = (1-\theta_k^2) k_2 a - 2(1-a - \beta_l) \psi k_1$ |
| $\Delta Q^*$ | $2bh_k = (1-\theta_k^2) k_2 a - 2(1-a - \beta_l) \psi k_1$ | $2bh_k = (1-\theta_k^2) k_2 a - 2(1-a - \beta_l) \psi k_1$ | $2bh_k = (1-\theta_k^2) k_2 a - 2(1-a - \beta_l) \psi k_1$ |

Table 3  Optimal profits of members under three modes

| Profits | Mode E | Mode T | Mode L |
|---------|--------|--------|--------|
| $\pi^*$ | $\frac{\epsilon^2}{2} k_2 (1-a - \beta_l) \kappa_1 (t_1 - t_2)^2 [2bh_k - (3-1-a - \beta_l) \psi k_1]^2$ | $\frac{\epsilon^2}{2} k_2 (1-a - \beta_l) \kappa_1 (t_1 - t_2)^2 [2bh_k - (3-1-a - \beta_l) \psi k_1]^2$ | $\frac{\epsilon^2}{2} k_2 (1-a - \beta_l) \kappa_1 (t_1 - t_2)^2 [2bh_k - (3-1-a - \beta_l) \psi k_1]^2$ |
| $\pi^a$ | $(1-\theta_k^2) k_2 (a_1t_1 - t_2)^2 [2bh_k - (3-1-a - \beta_l) \psi k_1]^2$ | $(1-\theta_k^2) k_2 (a_1t_1 - t_2)^2 [2bh_k - (3-1-a - \beta_l) \psi k_1]^2$ | $(1-\theta_k^2) k_2 (a_1t_1 - t_2)^2 [2bh_k - (3-1-a - \beta_l) \psi k_1]^2$ |
| $\pi^e$ | $\frac{\epsilon^2}{2} k_2 (a_1t_1 - t_2)^2 [2bh_k - (3-1-a - \beta_l) \psi k_1]^2$ | $\frac{\epsilon^2}{2} k_2 (a_1t_1 - t_2)^2 [2bh_k - (3-1-a - \beta_l) \psi k_1]^2$ | $\frac{\epsilon^2}{2} k_2 (a_1t_1 - t_2)^2 [2bh_k - (3-1-a - \beta_l) \psi k_1]^2$ |
| $\pi^p$ | $-\frac{\epsilon^2}{2} k_2 (a_1t_1 - t_2)^2 [2bh_k - (3-1-a - \beta_l) \psi k_1]^2$ | $-\frac{\epsilon^2}{2} k_2 (a_1t_1 - t_2)^2 [2bh_k - (3-1-a - \beta_l) \psi k_1]^2$ | $-\frac{\epsilon^2}{2} k_2 (a_1t_1 - t_2)^2 [2bh_k - (3-1-a - \beta_l) \psi k_1]^2$ |

4.1 Comparisons

Next, we conduct comparisons of decisions, NSV, profits and consumer surplus among three modes. For the sake of simplicity, we define the difference between basic sales volume and basic return quantity as basic NSV. Specifically, $\Delta_1 = a_1t_1 - z$ represents e-commerce platform’s basic NSV and $\Delta_2 = a_2t_2 - z$ means that in live streaming platform.

Theorem 1 The relationships of decisions in three modes are as follows:

(1) For the sales effort, $e^* > e^* > e^* > e^*$ if $0 < \beta_2 < \beta_1$ when $\Delta_1 \geq \Delta_2$; otherwise, $e^* > e^* > e^*$ if $0 < \beta_2 < \beta_1$ when $\Delta_1 \geq \Delta_2$.

(2) For the price $p$, $p^* > p^* > p^*$ if $0 < \epsilon < \epsilon v_1$, $e^* > e^* > e^*$ if $\epsilon v_1 \leq \epsilon$. 

\[ \text{(3) For the quantity } q^* \text{, } q^* = q^* = q^* \text{ if } \Delta_1 < \Delta_2. \]

\[ \text{(4) For the profit } \pi^* \text{, } \pi^* > \pi^* > \pi^* \text{ if } \Delta_1 \geq \Delta_2. \]

\[ \text{(5) For the consumer surplus } CS^* \text{, } CS^* > CS^* > CS^* \text{ if } \Delta_1 \geq \Delta_2. \]
| Mode  | Supply chain’s profit                                                                 | Consumer surplus                                                                 |
|-------|--------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
| Mode E | $\frac{k_1k_2(\theta_1-\theta_2)^2[2bk_1k_2-(1-\theta_2)^2k_2a^2-q^2k_1(1-a-a_1)3-a-\beta_1]}{2[2bk_1k_2-(1-\theta_2)^2k_2a-2(1-a-a_1)3-a-\beta_1]^2}$ | $\left\{ \frac{2z[2(1-a-a_1)\theta_2^2k_1 +(1-\theta)k_2^2r^2]}{2bk_1k_2(a_1t_1-3z)} + 2bk_1k_2(\alpha_1t_2-3z) \right\}$ |
| Mode T | $\frac{k_1k_2(\theta_1-\theta_2)^2[2bk_1k_2-(1-\theta_2)^2k_2a^2-q^2k_1(1-a-a_1)3-a-\beta_1]}{2[2bk_1k_2-(1-\theta_2)^2k_2a-2(1-a-a_1)3-a-\beta_1]^2}$ | $\left\{ \frac{2z[2(1-a-a_1)\theta_2^2k_1 +(1-\theta)k_2^2r^2]}{2bk_1k_2(a_1t_2-3z)} + 2bk_1k_2(\alpha_2t_2-3z) \right\}$ |
| Mode L | $\frac{k_1k_2(\theta_1-\theta_2)^2[2bk_1k_2-(1-\theta_2)^2k_2a^2-q^2k_1(1-a-a_1)3-a-\beta_1]}{2[2bk_1k_2-(1-\theta_2)^2k_2a-2(1-a-a_1)3-a-\beta_1]^2}$ | $\left\{ \frac{2z[2(1-a-a_1)\theta_2^2k_1 +(1-\theta)k_2^2r^2]}{2bk_1k_2(a_1t_2-3z)} + 2bk_1k_2(\alpha_2t_2-3z) \right\}$ |
(2) For the sales price, \(p_{L}^{*} > p_{T}^{*} > p_{E}^{*}\) when \(\Delta_{1} \geq D_{v_{1}}\Delta_{2}\)
\[
\begin{cases}
p_{L}^{*} > p_{E}^{*} \geq p_{T}^{*}, & \text{if } 0 < \beta_{2} < \beta_{1}
p_{L}^{*} \geq p_{E}^{*} > p_{T}^{*}, & \text{if } \beta_{1} \leq \beta_{2} < \beta_{1}\end{cases}
\]
when \(\Delta_{2} \leq \Delta_{1} < D_{v_{1}}\Delta_{2}\)
\[
\begin{cases}
p_{L}^{*} > p_{E}^{*} > p_{T}^{*}, & \text{if } 0 \leq \epsilon < \epsilon_{v_{2}}\text{ when } D_{v_{2}}\Delta_{2} \leq \Delta_{1} < \Delta_{2}\end{cases}
\]
and \(p_{L}^{*} > p_{E}^{*} > p_{T}^{*}\) when \(\Delta_{1} < D_{v_{2}}\Delta_{2}\).

(3) For the product quality, \(q_{L}^{*} \geq q_{E}^{*} \geq q_{T}^{*}\) when \(\Delta_{1} \geq D_{v_{3}}\Delta_{2}\);
\[
\begin{cases}
q_{L}^{*} > q_{E}^{*} \geq q_{T}^{*}, & \text{if } 0 < \beta_{2} < \beta_{T}^{\text{v}}\text{ when } \Delta_{2} \leq \Delta_{1} < D_{v_{3}}\Delta_{2}\text{;}
q_{E}^{*} \geq q_{L}^{*} > q_{T}^{*}, & \text{if } \beta_{T}^{\text{v}} \leq \beta_{2} < \beta_{1}\end{cases}
\]
when \(D_{v_{2}}\Delta_{2} \leq \Delta_{1} < \Delta_{2};\) and \(q_{L}^{*} > q_{E}^{*} > q_{T}^{*}\) when \(\Delta_{1} < D_{v_{2}}\Delta_{2}\).

Theorem 1 shows the relationships of decisions in different modes, which are mainly affected by basic NSV, channel rate, and live streaming service rate. According to Theorem 1(1), if live streaming platform’s basic NSV is low, mode E has the highest sales effort when its trading commission rate is high; otherwise, sales effort in mode L is the highest. If its basic NSV is high, mode L has the highest sales effort when channel rate is low, or else mode E has the highest one. As for product price and quality, mode E has the highest results when e-commerce platform’s basic NSV is high; when it is low, price and product quality in mode L are the highest; when it is in a medium range, the relationships will be influenced by channel rate and trading commission rate. The changes of product quality are consistent with sales price for that higher price means higher profit margin for the seller, which contributes to the improvement of product quality.

Therefore, it can be concluded from Theorem 1(2) and Theorem 1(3), for consumers, mode T can never be the best for product quality and mode L cannot be the best for price. They usually can purchase commodities with better quality in the high basic NSV platform, while with lower price in the platform with relatively low NSV. The reason is that in low basic NSV platform, the streamer needs to attract consumers at low price, and in high basic NSV platform, the seller has more capital to enhance product quality, which are consistent with traditional wisdom and practical operations. For the live streaming platform, mode L with in-house e-commerce is better for improving quality than mode T. Therefore, TikTok announces that displaying external sales links from other platforms like Taobao (mode T) is not allowed anymore, and only the links from internal “DouYin-Store” (mode L) are permitted. Moreover, comparing quality in modes L and E, we find only when basic NSV is \((0 < \beta_{2} < \beta_{E})\), will the live streaming platform in mode L provides higher quality products than e-commerce platform in mode E. This explains why TikTok charges low service fees and even offers services freely in the start-up period for building reputation.

**Theorem 2** For NSV, \(\Delta Q_{E}^{*} \geq \Delta Q_{L}^{*} > \Delta Q_{T}^{*}\) when \(\Delta_{1} \geq D_{v_{4}}\Delta_{2}\);
\[
\begin{cases}
\Delta Q_{E}^{*} > \Delta Q_{L}^{*} \geq \Delta Q_{T}^{*}, & \text{if } 0 < \beta_{2} < \beta_{E}^{\text{v}}\text{ when } D_{v_{2}} \leq \Delta_{1} < D_{v_{4}}\Delta_{2};
\Delta Q_{E}^{*} \geq \Delta Q_{L}^{*} > \Delta Q_{T}^{*}, & \text{if } 0 < \beta_{2} < \beta_{E}^{\text{v}}\text{ when } D_{v_{2}} \leq \Delta_{1} < D_{v_{4}}\Delta_{2};
\Delta Q_{L}^{*} > \Delta Q_{E}^{*} \geq \Delta Q_{T}^{*}, & \text{if } \epsilon_{v_{2}} \leq \epsilon < \alpha\end{cases}
\]
when \(D_{v_{2}}\Delta_{2} \leq \Delta_{1} < \Delta_{2};\) and \(\Delta Q_{L}^{*} > \Delta Q_{E}^{*} > \Delta Q_{T}^{*}\) when \(\Delta_{1} < D_{v_{2}}\Delta_{2}\).
Theorem 2 shows the relationships of NSV in three sales modes. Similar to product quality, mode T cannot have the highest NSV. When e-commerce platform’s basic NSV is high, mode E has the highest NSV; when it is low, mode L is the best. If it is in a middle level, the relationship will depend on channel rate and live streaming platform’s trading commission rate.

According to Theorems 1 and 2, when live streaming platform’s basic NSV is large ($\Delta_1 < D_{v2}\Delta_2$), although mode L has the highest sales price, it can also generate the highest NSV due to the best product quality. It indicates that for generating high NSV in live streaming commerce, product quality can contribute much even with high price. In reality, many hot products are usually of great quality, even if the price is not the lowest in the category. Therefore, it is short-sighted for streamers to blindly pursue “the lowest price”, and they should raise the price appropriately to ensure a reasonable profit. Thus, it is short-sighted for the e-commerce platform to reduce the lowest price to the lowest in the category.

Theorem 3 shows the relationships of profits among three modes. For the streamer, mode T cannot be the best choice. Mode E has the highest profit when e-commerce platform’s basic NSV is high, or it is moderate and with high live streaming platform’s trading commission rate; otherwise, mode L is the best mode. Because the sales price and NSV in mode E are the highest when e-commerce platform’s basic NSV is high, or is medium but live streaming platform’s trading commission rate
is high. However, when e-commerce platform’s basic NSV is low, or is medium but live streaming platform’s trading commission rate is also low, those in mode L are the highest. Mode T cannot have the highest NSV and price according to Theorem 1(3) and Theorem 2.

However, for the seller and the supply chain, three modes are all possible to be optimal, depending on the channel rate and basic NSV. Mode E is the best for the seller when e-commerce platform’s NSV is high; while mode T is the best when its NSV is low and channel rate is also low; otherwise, mode L can be the best. The reasons are similar to those for the streamer’s profit. When e-commerce platform’s basic NSV is high, the NSV and price in mode E are the highest, causing the highest seller’s profit in mode E. However, when its basic NSV is low, NSV and price in mode T is the highest if the channel rate is low, and also the profit, while those in mode L can be the highest if the channel rate is high.

As for the e-commerce platform, mode T is more profitable only when live streaming platform’s basic NSV is high and channel rate is low; otherwise, mode E is better. Finally, the live streaming platform will have higher profit in mode L than mode T only when its trading commission rate is relatively high and channel rate is low. This can explain that live streaming platforms like TikTok rise trading commission rate to a normal level one month later after the seller’s entry, which guarantees its long-term profit for setting up in-house e-commerce.

**Theorem 4** For consumer surplus, 1) $CS_E^* \geq CS_T^*$ when $\Delta_1 \geq \Delta_2$, and $CS_E^* \leq CS_T^*$ if $0 < \varepsilon < \varepsilon''$ when $\Delta_1 < \Delta_2$; 2) $CS_E^* \geq CS_L^*$ if $\Delta_1 \geq \Delta_1'$, otherwise $CS_E^* < CS_L^*$. 3) $CS_T^* < CS_L^*$ when $\varepsilon > \varepsilon''$, and $CS_T^* \geq CS_L^*$ if $\Delta_2 \geq \Delta_2'$ when $\varepsilon \leq \varepsilon''$.

Theorem 4 compares consumer surplus among three modes. All of the three modes may have the highest consumer surplus. Specifically, mode E can generate the highest consumer surplus when e-commerce platform’s basic NSV is high. With relatively low basic NSV, mode L has higher consumer surplus than mode E, and when simultaneously with low channel rate, that in mode T is also higher than mode E. Moreover, consumer surplus in mode L is always higher than mode T if channel rate is high. However, if channel rate is low, mode T has higher consumer surplus when live streaming platform’s basic NSV is high, otherwise, that in mode L is higher.

Combining Theorems 3 and 4, we intuitively exhibit the comparisons of supply chain’s profit and consumer surplus through numerical examples, and further obtain comparative results of social welfare, which are shown in Fig. 2.

From Fig. 2, we find the comparisons of social welfare resemble those of supply chain’s profit and consumer surplus, in which three modes all possibly have the highest results. This is consistent with the comparative results of supply chain’s profit in Theorem 3(5), in which the mode (E or T/L) with a higher basic NSV platform for live streaming will be the optimal, while for mode T and mode L, the relationships depend on channel rate. Combined with similar comparative results of consumer surplus.
surplus in Theorem 4, we obtain the comparative results of social welfare. Specifically, the social welfare in mode E is the highest when e-commerce platform’s NSV is relatively high, while mode T/L can be the highest on the contrary. For mode T and mode L, the former has the highest results only when its basic NSV is high and channel rate is low; otherwise, the latter is the highest.

Then, we will briefly demonstrate how the return quantity coefficient influences decision makers’ choice of mode through numerical examples, including the seller and the streamer. This shows that the consumer return affects the choice of live streaming commerce modes to some extent.

Figures 3 and 4 show the impacts of return quantity coefficient on the seller’s and streamer’s mode selection respectively. For the seller, we prove that there exists optimal return quantity coefficient for the profit as Corollary 1(6) shows. It is most likely that mode E will be chosen depending on the NSV of the e-commerce platform. However, the coefficient \( \varphi \) plays an important role in determining the choice between modes T and L. Specifically, the seller obtains higher profit in mode L with relatively low \( \varphi \). However, with the continuous increase of \( \varphi \), the seller’s profit in mode L will drop faster than in mode T, and the seller will switch to mode T from mode T gradually. For the streamer, the impacts of return quantity coefficient on mode selection are marginal. Profits in three modes are all rising with the increase of \( \varphi \), but mode T is always better than mode L and its increasing speed is also higher. The choice between modes E and T still largely depends on the basic NSV of the two platforms.

### 4.2 Specific cases

In the following, the comparative results will be analyzed in some specific market situations, where some findings are more intuitive.

**Corollary 2** When e-commerce platform and live streaming platform have the same basic NSV(\( \Delta_1 = \Delta_2 \)), we have \( e^*_L \geq e^*_T \geq e^*_E \), \( q^*_L \geq q^*_E \geq q^*_T \), \( p^*_L \geq p^*_E \geq p^*_T \), \( \Delta Q^*_L > \Delta Q^*_E \geq \Delta Q^*_T \), \( \pi^*_aL \geq \pi^*_aE \geq \pi^*_aT \), \( \pi^*_epE \geq \pi^*_epT \), while the seller’s optimal mode selection is still related to live streaming platform’s trading commission rate.

Corollary 2 shows the results when two platforms have same basic NSV. In this situation, mode L has highest product quality, NSV and the streamer’s profit. For price-oriented consumers, mode T will be the best because of the lowest price. For the e-commerce platform, mode E is better than mode T. However, in this condition, mode T will never be the best choice for the seller. Mode L is the most profitable when live streaming platform’s trading commission rate is low; otherwise, mode E is the best mode.

**Corollary 3** When trading commission rates in two platforms are the same(\( \beta_1 = \beta_2 \)). If \( \Delta_1 \geq \Delta_2 \) is true, then we have \( e^*_L \geq e^*_T > e^*_E \), \( p^*_L \geq p^*_E \geq p^*_T \), \( q^*_L \geq q^*_E > q^*_T \), \( \Delta Q^*_L \geq \Delta Q^*_E > \Delta Q^*_T \), \( \pi^*_aE \geq \pi^*_aL > \pi^*_aT \), \( \pi^*_stE \geq \pi^*_stL \geq \pi^*_stT \) and \( \pi^*_epE \geq \pi^*_epT \), while results are irrelevant to live streaming platform’s trading commission rate when \( \Delta_1 < \Delta_2 \).
Corollary 3 gives the comparative results when trading commission rates in two platforms are the same, which also exists in reality. For example, TikTok increases its trading commission rate to a normal standard 30 days after sellers settled in, in which condition the difference between two platforms’ trading commission rates is negligible. Different from basic models, as long as the e-commerce platform has a higher basic NSV than the live streaming platform, mode E will have the highest
product quality, NSV, and profits of the streamer and the seller. It implies that the ranges where mode E has the highest results are enlarged. Because the increase of live streaming platform’s trading commission rate reduces its advantage. However, when the e-commerce platform has a lower basic NSV, results are unchanged, in which mode L has the highest results while the relationships of the other two modes rely on channel rate.

Therefore, for the live streaming platform, when its conversion rate and basic NSV are low in initial period ($\Delta_1 \geq \Delta_2$), a low trading commission rate is better for its development. Otherwise, most streamers and sellers will choose mode E. However, when its basic NSV has developed to a certain level ($\Delta_1 < \Delta_2$), streamers and sellers will still choose mode L even if its trading commission rate is as high as e-commerce platforms. It is consistent with the fact that TikTok implements low trading commission rate at the beginning but then increases it.

**Corollary 4** When the live streaming platform allows selling from external platform without charging channel fee ($\epsilon = 0$), results are irrelevant to channel rate when $\Delta_1 \geq \Delta_2$; while if $\Delta_1 < \Delta_2$ exists, we have $e_L^s > e_T^s > e_E^s, p_L^s > p_T^s > p_E^s, q_L^s > q_T^s > q_E^s, \Delta Q_L^s > \Delta Q_T^s > \Delta Q_E^s$ and $\pi_{ae}^s > \pi_{at}^s > \pi_{aT}^s$.

Corollary 4 presents the results when the live streaming platform does not charge channel fees. For instance, the live streaming platform in China named Kuaishou will charge sellers from Pinduoduo with a 5% channel rate, while it is free for sellers in Taobao. When e-commerce platform’s basic NSV is lower than live streaming platform, mode L has the highest product quality, NSV and the streamer’s profit. However, when it is higher, results remain the same with the general situations, that is, both mode E and mode L are possible to be the highest for the above-mentioned dimensions, depending on live streaming platform’s trading commission rate. As for profit of the seller, all three modes may still be the best choice.

Therefore, even though there is no channel fee, mode T will still not have the highest product quality, NSV and the streamer’s profit. However, different from the general situations, mode T could generate better results than mode E when e-commerce platform’s basic NSV is relatively low, which can be attributed to the abandonment of channel fee.

**Corollary 5** When the live streaming platform gives up channel fee and adopts same trading commission rate as the e-commerce platform ($\epsilon = 0$ and $\beta_1 = \beta_2$), if $\Delta_1 \geq \Delta_2$ is true, we have $p_L^s \geq p_L^s = p_T^s, q_L^s \geq q_L^s = q_T^s, \Delta Q_L^s \geq \Delta Q_L^s = \Delta Q_T^s, \pi_{ae}^s \geq \pi_{al}^s = \pi_{at}^s = \pi_{ae}^s$ and $\pi_{aT}^s > \pi_{aE}^s$; while if there is $\Delta_1 < \Delta_2$, we have $e_L^s = e_T^s > e_E^s, p_L^s = p_T^s > p_E^s, q_L^s = q_T^s > q_E^s, \Delta Q_L^s = \Delta Q_T^s > \Delta Q_E^s, \pi_{al}^s = \pi_{at}^s \geq \pi_{aT}^s, \pi_{sl}^s = \pi_{st}^s > \pi_{sp}^s$ and $\pi_{lpT}^s > \pi_{lpE}^s$.

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25 Five live streaming commerce platforms: Only KuaiShou can transfer to other platforms, Wechat small program has low threshold. https://www.sohu.com/a/426336789_161795. 2020–10–21.
Corollary 5 indicates the results when channel rate is zero and two platforms carry out same trading commission rate. In this situation, differences among three modes mainly depend on the basic NSV of two platforms. When e-commerce platform has higher basic NSV, mode E will generate the highest product quality, NSV and profits of the seller and the streamer, although the product would also be more expensive. Otherwise, these results will be generated in modes T and L for the higher basic NSV of live streaming platform. As for the live streaming platform, mode L is the best choice. Because in mode T the sales service is provided by the e-commerce platform, thus the live streaming platform cannot obtain trading commission.

5 Extensions

In this section, we extend the basic models in more application scenarios. Specifically, we consider the following situations: loss of consumers in transferring in mode T; live streaming platform having the gift-giving functions, and the dual-purpose streamer. Our goal here is to investigate the applicability of main results and how these factors will influence the results.

5.1 Loss of consumers in transferring

In mode T, consumers will transfer to the e-commerce platform when they click sales links in live streaming platform. However, the increased shopping links may lead to the loss of consumers, especially for live streaming selling with impulsive consumption. There is a waiting time in transferring process, and the transferring may even fail, thus impatient consumers will give up buying. However, platforms integrating e-commerce and live streaming functions can avoid this problem to a large extent. Therefore, the loss of transferring process in mode T (transferring loss) is considered below, while the other two modes do not have the loss.

We introduce a parameter \( \delta \in [0, 1] \) to represent the retention rate of consumers in transferring process, and higher \( \delta \) indicates a lower degree of loss. Therefore, the real market demand in mode T is \( Q_T^a = a_2t_2\delta - bp + re \) (the superscript \( a \) indicates this extension model). Solving the extended models, we derive optimal results in this situation and show them in the Appendix. Through comparisons among different modes, Theorem 5 is obtained as follows.

**Theorem 5** The results in basic models have a strong robustness although with a loss in transferring process. The relationships between mode E and mode L are not affected, while there are fewer situations where mode T has higher results than the other two modes respectively.

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26 The healthy development of live streaming commerce needs cross-department supervision. https://baijiahao.baidu.com/s?id=1688002441423969861&wfr=spider&for=pc. 2021–01-05.

27 Why does not Pinduoduo have a shopping cart? http://www.woshipm.com/pd/3916042.html. 2020–05-26.
Theorem 5 shows the influence of transferring loss. In general, the ranges where mode T has higher results than the other two modes respectively will shrink. For example, compared with Theorem 2(3), the upper threshold of channel rate where product quality in mode T is higher than mode E decreases, which means the channel rate should be controlled with a lower value to keep higher quality in mode T. This can be attributed to the negative impacts of transferring loss in mode T, generating a lower real conversion rate in this mode. Therefore, the channel rate should be maintained in a lower level to attract the streamer and the seller. Different from basic models, the profit relationships of the seller in mode T and mode L are still affected by retention rate. When retention rate is high and channel rate is low, the seller’s profit in mode T will be higher than mode L; otherwise, mode L is always better than mode T.

Therefore, the advent of loss in transferring process motivates the streamer and the seller to turn to modes E and L away from mode T, and it also motivates the live streaming platform to choose mode L for building in-house e-commerce.

5.2 The gift-giving functions of live streaming platform

Before the popularity of live streaming selling, the main revenue sources for streamers are live streaming traffic and gifts from fans [14, 32], and these functions are still retained by many live streaming platforms. For example, live streaming rooms in TikTok are designed with gift-giving function, in which viewers need to purchase the “sound waves” before rewarding streamers. This is called paid gift-giving [14], and the platform will charge technical service fees from streamers. For instance, 30% to 70% of the streamer’s gift-giving revenues will be paid to TikTok. Compared with the gift-giving function in live streaming platforms, gifts in e-commerce platforms can be exchanged for free with experience scores, named as free gift-giving [14]. It has a relatively low degree of stimulation for streamers, and therefore we omit the effect of it in mode E.

In order to describe the gift-giving revenues, we identify a parameter \( g \) to represent the revenue for unit sales effort and the superscript \( b \) to indicate this extension model. The sales effort refers to the comprehensive effort made by streamers to promote products or build their own IP, in which higher level of sales effort often generates more gift-giving revenues. Also, the gift service rate \( \tau \) indicates technical service rate charged by the live streaming platform for gift-giving. Without consideration of e-commerce platform’s gift-giving function, the profit functions in mode E are unchanged. The profit functions of the seller and the e-commerce platform are also not affected in modes T and L, while these of the streamer and the live streaming platform will be changed as follows.

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28 How to calculate streamer’s live streaming commission in DouYin? https://www.zbku.net/zixun/201909141521.html. 2019-09-14.
According to the backward induction method, optimal results are obtained and are also showed in the Appendix. Compared with basic modes, the changes of optimal results are concluded in Theorem 6.

**Theorem 6** When the gift-giving function is considered, the changes of optimal results in modes $T$ and $L$ are as follows:

1) In mode $T$, we have: $e^*_T < e^{b*}_T$, $p^*_T < p^{b*}_T$, $q^*_T < q^{b*}_T$, $\Delta Q^*_T < \Delta Q^{b*}_T$, $\pi^*_sT < \pi^{b*}_sT$, $\pi^*_aT < \pi^{b*}_aT$, $\pi^*_lpT < \pi^{b*}_lpT$, $\pi^*_cT < \pi^{b*}_cT$, $CS^*_T < CS^{b*}_T$, $SW^*_T < SW^{b*}_T$.

2) In mode $L$, we have: $e^*_L < e^{b*}_L$, $p^*_L < p^{b*}_L$, $q^*_L < q^{b*}_L$, $\Delta Q^*_L < \Delta Q^{b*}_L$, $\pi^*_sL < \pi^{b*}_sL$, $\pi^*_aL < \pi^{b*}_aL$, $\pi^*_lpL < \pi^{b*}_lpL$, $\pi^*_cL < \pi^{b*}_cL$, $CS^*_L < CS^{b*}_L$, $SW^*_L < SW^{b*}_L$.

Theorem 6 shows the influence of gift-giving function on optimal results. Compared with results in basic models, the sales effort, product quality and price in modes $T$ and $L$ will all increase when considering gift-giving, meanwhile the NSV will also increase due to the decrease of return quantity. Moreover, profits of all the supply chain members will increase, as will consumer surplus and social welfare. This indicates that the implementation of gift-giving function is beneficial generally, although consumers need to pay a higher price for products.

Next, we compare the results among three modes within gift-giving function, the proofs of which can be found in the Appendix. The comparative results are concluded in Theorem 7.

**Theorem 7** When the gift-giving function is considered, the comparative results in basic models still have a strong robustness, while there are fewer situations where mode $E$ has higher results than the other two modes.

Theorem 7 illustrates the impacts of gift-giving function on comparative results among three modes. From aspects of optimal decisions and NSV, there are more situations where the sales effort, price and NSV of modes $T$ and $L$ are higher than mode $E$ respectively, and more situations where the product quality in mode $T$ is higher than mode $E$. This is because the gift-giving function brings extra incomes for the streamer and the live streaming platform, and the streamer is willing to improve sales effort, resulting in high sales price but also high product quality. However, it is more complicated to analyze the changes of product quality comparisons between modes $L$ and $E$, and the NSV comparisons between modes $T$ and $L$. Taking quality comparisons as an example, compared with Theorem 1, in order to encourage the seller to provide high quality products in mode $L$ than mode $E$, the
live streaming platform is required to have lower trading commission rate and basic NSV. There are also more situations where profits of the streamer, the seller and the e-commerce platform in modes T and L are respectively higher than mode E, compared with Theorem 3.

In general, the implementation of gift-giving function is beneficial to product quality, NSV and profits, especially for the live streaming platform, which will attract more streamers and sellers to carry out the live streaming through it. Therefore, many live streaming platforms also keep gift-giving function in reality.

5.3 The dual-purpose streamer

For streamers, the purchasing experiences of fans are of vital importance. A good shopping experience can strengthen their trust in streamers, thus improving the stickiness of fans. Therefore, for the streamers with vision and social responsibility, they will focus on consumer surplus besides profits, that is, they are dual-purpose streamers [34].

Therefore, we introduce a parameter \( \mu \in [0, 1] \) to indicate the streamer’s attention to consumer surplus, in which \( \mu = 0 \) represents a pure profit-driven streamer while \( \mu = 1 \) means a pure surplus-driven streamer [23]. Therefore, profit functions of the dual-purpose streamer in three modes are as follows, while those of the other members are unchanged, in which the superscript \( c \) indicates this extension model.

\[
\begin{align*}
\pi_{aE}^c &= p(Q_E^c - R_E^c)(1 - \theta) - \frac{1}{2}k_1e^2 + \mu \frac{Q_E^c(Q_E^c - 2R_E^c)}{2b}, \\
\pi_{aT}^c &= p(Q_T^c - R_T^c)(1 - \theta) - \frac{1}{2}k_1e^2 + \mu \frac{Q_T^c(Q_T^c - 2R_T^c)}{2b}, \\
\pi_{aL}^c &= p(Q_L^c - R_L^c)(1 - \theta) - \frac{1}{2}k_1e^2 + \mu \frac{Q_L^c(Q_L^c - 2R_L^c)}{2b}.
\end{align*}
\]

The optimal results can also refer to the Appendix. In view of the complexity of results, the comparisons will be analyzed by numerical experiments. Compared with basic models, the changes of results are presented in Observation 1.

**Observation 1** The sales effort level, NSV, profits of the streamer and the supply chain, consumer surplus and social welfare will all increase when the streamer is dual-purpose, while the price, product quality, and profits of two platforms will decrease. However, the seller’s profit will increase first but then decline with the streamer’s attention to consumer surplus.

Observation 1 shows the changes of results with a dual-purpose streamer. The streamer will enhance sales effort level and reduce price to incentive demand and consumer surplus. However, the product quality will drop due to a low price, while profit of the streamer will keep increasing. For the seller, streamer’s proper attention to consumer surplus is beneficial due to the increasing of NSV. However, excessive attention leads to a low sales price, causing decrease of product quality, which will
in turn demonstrate a high return rate and low profit. Even when the seller’s profit decreases, profit of the supply chain and social welfare still increase.

Therefore, similar to the results of Panda [23], it’s important for members to decide what extent they pay attention to consumer surplus. This is also applicable in the background of live streaming commerce with consumer return, in which the streamer’s proper attention to consumer surplus can improve profits, while too much attention leads to a poor product quality and bad reputation.

Then, we compared the results among three modes, which are concluded in Observation 2.

**Observation 2** *The comparisons between modes E and L with a dual-purpose streamer are similar to those in basic models, while there are obvious changes between mode T and the other two modes, especially when the streamer pays a high attention to consumer surplus.*

Observation 2 gives the comparative results of different modes with a dual-purpose streamer. The streamer’s attention to consumer surplus has similar influence on three modes, but it has significant influence on mode T. Different from basic models, when the streamer pays a high attention to consumer surplus, model T will have the highest sales effort, although this mode is still not optimal for product quality. As for NSV, even if the streamer only pays a little attention, it will greatly stimulate NSV in mode T and make it higher than that in other modes. From the perspective of profits, there are more situations for the seller having highest profit in mode T, and this mode can also be the best for the streamer with a high attention to consumer surplus. However, for two platforms, there are less situations where mode T is the best with the increasing attention. Finally, more situations will be generated for the supply chain gaining the highest profit in mode T.

Therefore, *the streamer’s dual-purpose attribute has significant effects on mode T*. It makes mode T possibly become an optimal choice for the streamer and the seller who are NSV-oriented or profit-oriented, and therefore motivates them to transfer to mode T, while it makes both profit-oriented platforms choose the other modes.

**6 The hybrid mode selection**

In basic models, we consider three mainstream sales modes respectively, in which the streamer only chooses either e-commerce platform or live streaming platform to advertise products. However, it is also common for them to implement live streaming through both platforms. In other words, they can choose hybrid mode. Combination of modes E and T is the most popular hybrid mode. For example, besides implementing live streaming selling in Taobao, many flagship stores such as Septwolves, Be & Cheery and Nike also employ live streaming in Kuaishou platform.29

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29 Kameila live streaming commerce, more popular than double 11. [https://www.sohu.com/a/282193781_428942. 2018–12-16]
Therefore, we will study a hybrid mode combining modes E and T in this subsection, in which live streaming selling is employed in both the e-commerce platform and the live streaming platform, finding out when should they choose the hybrid mode and when to carry out single mode.

The supply chain structure of this hybrid mode (with the subscript \(H\)) is shown in Fig. 5, and several details should be noticed. Firstly, two platforms have partial common DAU in reality, and we therefore use the parameter \(a_3\) to represent common DAU. Next, for common DAU, we define \(\sigma_1\) to indicate the proportion attracted in mode T. Therefore, the sales volume in mode E is \(Q^E_H = (a_1 - \sigma_1 a_3)t_1 - bp + re\) and in mode T is \(Q^T_H = [a_2 - (1 - \sigma_1)a_3]t_2 - bp + re\). Besides, for returned products, we use \(\sigma_2\) to represent the proportion of them in mode E. The total return quantity is \(R = z - \varphi q\), thus the return quantities of modes E and T are \(R^E_H = \sigma_2(z - \varphi q)\) and \(R^T_H = (1 - \sigma_2)(z - \varphi q)\) respectively.

We assume in e-commerce platform and the live streaming platform, the streamer’s sales effort and sales price are the same. Therefore, the profit functions of supply chain members are as follows:

\[
\begin{align*}
\pi_{sH} &= p(Q^E_H + Q^T_H - R)(1 - \alpha - \beta_1) - \frac{1}{2}k_2q^2, \\
\pi_{aH} &= p(Q^E_H - R^E_H)(1 - \theta) + p(Q^T_H - R^T_H)(\alpha - \epsilon)(1 - \theta) - \frac{1}{2}k_1e^2, \\
\pi_{lpH} &= p(Q^T_H - R^T_H)e + p(Q^T_H - R^T_H)(\alpha - \epsilon)\theta, \\
\pi_{epH} &= p(Q^E_H + Q^T_H - R^H)\beta_1 + p(Q^E_H - R^E_H)\alpha\theta.
\end{align*}
\]

We compare the product quality, NSV and profits in hybrid mode with three single modes through numerical experiments in Fig. 6, and the comparative results are concluded in Observation 3.

**Observation 3** *The hybrid mode cannot have the highest product quality, although it may generate higher quality than mode T or mode L. However, for NSV and profits of the seller, the streamer and the e-commerce platform, the hybrid mode may be the best, while it is not the best for the live streaming platform’s profit.*

Figure 6 and Observation 3 indicate the choices between the hybrid mode and single modes. As for product quality, as shown in Fig. 6 (a), the hybrid mode always causes lower quality than mode E, although it may bring higher quality than mode T or mode L. Therefore, hybrid mode cannot have the highest product quality. Because quality in mode E is always better. However, the hybrid mode can generate the highest NSV when common DAU is not too high (see Fig. 6 (b)).

From the perspective of profits, as shown in Fig. 6 (c) to Fig. 6 (f), the hybrid mode is not always better than each single mode. For the seller, when common DAU is low, implementing hybrid mode is beneficial with high live streaming platform’s conversion rate. When common DAU is high, hybrid mode generates the highest profit if its conversion rate is moderate; otherwise, mode E (mode T) is the best if the conversion rate is low (high).

For the streamer and the e-commerce platform, the hybrid mode performs best only when with low common DAU and moderate live streaming platform’s conversion rate; otherwise, single mode performs better. The reason is high common DAU will decrease the
advantages of hybrid mode obviously. However, when with common DAU and two platform has quite different basic NSV, the seller and the streamer will live stream on the platform with high basic NSV, by which they can achieve a high conversion for common DAU. Finally, the hybrid mode cannot be the most beneficial to live streaming platform’s profit. In hybrid mode, part of transactions will be completed in live streaming of the e-commerce platform. As a result, the transactions created by live streaming platform will be reduced, resulting in its lower profits.

Therefore, the hybrid mode does not always perform better than single modes. It is not the best for quality-oriented participants and profit-oriented live streaming platform. However, it is beneficial to the profit-oriented seller, the streamer and the e-commerce platform if common DAU is relatively low and conversion rate is moderate; otherwise, single mode performs better.

7 Conclusions and further research

7.1 Main conclusions

This paper considers a supply chain including the seller, the streamer, the e-commerce platform and/or the live streaming platform, and compares three typical sales modes. Then we discuss the results in more specific market situations, and further extend basic models into more scenarios with considerations of transferring loss, gift-giving function, and the dual-purpose streamer. We also explore the hybrid mode (combining modes E and T). The main conclusions are as follows.

From the perspective of profits, for the streamer, mode E is the best when e-commerce platform’s basic NSV is low, or moderate but with low trading commission rate; otherwise, mode L is the best. For the seller, mode E (T) is the best when e-commerce platform’s basic NSV is high (low and with low channel rate); otherwise, mode L can be the best. For both platforms, each mode may be optimal. Moreover, the hybrid mode may generate higher profits than single mode for members, except for live streaming platform. Besides, profits of the seller and both platforms will improve first but then decline with related rates (live streaming service rate, trading commission rate, and channel rate). Finally, consumer’s appropriate attention to product quality makes the supply chain achieve the highest profit.
From the perspective of product quality, mode L always generates higher quality than mode T, and also than mode E when live streaming platform’s basic NSV is high. Therefore, the live streaming platform can have better product quality in mode L with in-house e-commerce than mode T, and only with a high conversion rate and a low trading commission rate, can it have higher quality products than mode E. However, the hybrid mode will damage product quality compared with mode E. Moreover, we find that for generating high NSV in live streaming commerce, product quality can contribute much even with high price.

Besides, results in specific cases indicate that when two platforms have same basic NSV, mode L has the highest streamer’s profit; and the e-commerce platform will choose mode E; while the seller only chooses between modes E and L. With the same trading commission rate, mode E is the best for profits of the streamer and the seller if e-commerce platform’s basic NSV is high. Besides, if no channel rate is charged, mode L will generate the highest streamer’s profit when e-commerce platform’s basic NSV is low. Otherwise, results in these specific cases remain unchanged compared with basic models. Finally, the impacts of transferring loss, gift-giving function and dual-purpose streamer are discussed in extension models. Results show the findings of basic models have strong robustness in most cases.

7.2 Managerial insights

Based on observations in live streaming commerce, this study provides some insights about decision makings and mode selections for different participants. The following
questions are answered. 1) How could the seller realize a higher quality decision and the streamer realize a higher sales effort? 2) For the seller and the streamer, which platform is better for opening the online store or implementing live streaming? For platforms, is it necessary to implement the cross-platform coordination? Here are a few implementation actions being taken to improve surpluses for each party.

First, this study provides some suggestions for the seller’s product quality decision and the streamer’s sales effort decision. On the one hand, for sellers emphasizing product quality, cross-platform live streaming commerce is not recommended for them, in which they cannot pursue the best quality. While in other two modes, sellers need to pay extra attention to the basic NSV and commission rate when deciding product quality. Moreover, sellers should not blindly pursue the lowest price which damages product quality. On the other hand, we give similar suggestions about sales effort decisions for streamers. For example, cross-platform live streaming commerce is not recommended for streamers who pursue high level of sales effort, and when deciding the sales effort, focusing on the basic NSV and commission rate is also necessary for the streamers deciding the sales effort.

Next, we make some suggestions about mode selections for different participants. 1) For the seller, if the e-commerce platform has a high basic NSV, it is suggested to open the online stores in e-commerce platform and implement live streaming through it. However, if the e-commerce platform has a low basic NSV and sets a low channel rate, exerting live streaming through live streaming platform is more profitable. Under other conditions, exerting live streaming and opening stores in live streaming platform are suggested. 2) For the profit seeking streamer, cross-platform mode is always not recommended. When e-commerce platform’s basic NSV is high, or medium but with a high live streaming platform’s trading commission rate, he should cooperate with the seller in e-commerce platform and exert live streaming in the same platform. However, implementing live streaming in live streaming platform for the seller in that platform is suggested when e-commerce platform’s basic NSV is low, or medium but with a low live streaming platform’s trading commission rate. 3) For e-commerce platform, allowing the external consumers to transfer to its platform from live streaming platform is helpful when e-commerce platform’s basic NSV is low, or high but the channel rate is low. Otherwise, live stream by himself is better. For live streaming platform, only when his trading commission rate is high and the channel rate is low, can he obtain higher surplus by setting up the internal e-commerce function.

7.3 Future research

The research in this paper also has some limitations. For example, we do not consider the situation that the seller is a leader. Sometimes, sellers have more bargaining power with fierce competition among multi-channel network institutions and streamers. Some sellers with famous brands have a strong voice to decide the sales price obviously. Besides, sellers can carry out live streaming by themselves instead of cooperating with multi-channel network institutions, which will be more and more common in reality. Moreover, the seller and the streamer could propose a contract to coordinate the product quality and sales effort. These are directions for future research.
Appendix 1

Proofs of Tables 2–4

The streamer is the leader of the Stackelberg game in models, in which the response function of the seller is solved firstly through backward induction method. The second-order partial derivative of the seller’s profit function with respect to quality is always negative, therefore, it is a concave function with respect to \( q \) and there exists a unique optimal quality. By making the first-order derivative profit function of the seller with respect to \( q \) be zero, the response function of the seller could be obtained. Then, we substitute it into the streamer’s profit function to get the second-order partial derivatives with respect to \( e \) and \( p \) which are all negative, in which they demonstrate the existence of optimal \( e \) and \( p \). The optimal \( e \) and \( p \) can be obtained through making the first-order derivative profit function of the streamer with respect to \( e \) and \( p \) be zero simultaneously. The solving processes in other modes are similar and will not be described again here.

By substituting the optimal decisions into the corresponding functions, the optimal net sales volume, profits, consumer surplus and social welfare can be obtained after simplification.

Proof of Corollary 1

The first-order partial derivatives of the optimal decisions, net sales volume and profit of the streamer with respect to several parameters in three modes are solved, in which these results have same tendency in three modes. Therefore, we take the results in e-commerce platform mode as an example to exhibit the results, and the first-order partial derivative with respect to \( \varepsilon \) only exists in transferring mode.

\[
\frac{\partial q^*_E}{\partial \theta} = \frac{-\varphi k_1 k_2 \Delta_1 [2bk_1 - r^2 (1 - \beta_1 - \theta + \beta_1 \theta)]}{[2bk_1 k_2 - 2k_1 \varphi^2 (1 - \alpha - \beta_1) - (1 - \theta)ak_2 r^2]^2} < 0,
\]

\[
\frac{\partial q^*_E}{\partial \alpha} = \frac{-\alpha \varphi k_1 k_2 r^2 \Delta_1 (1 - \alpha - \beta_1)}{[2bk_1 k_2 - 2k_1 \varphi^2 (1 - \alpha - \beta_1) - (1 - \theta)ak_2 r^2]^2} < 0,
\]

\[
\frac{\partial q^*_E}{\partial \varepsilon} = \frac{-\varphi k_1 k_2 r^2 \Delta_2 (1 - \theta)(1 - \alpha - \beta_1)}{[2bk_1 k_2 - 2k_1 \varphi^2 (1 - \alpha - \beta_1) - (1 - \theta)ak_2 r^2]^2} < 0,
\]

\[
\frac{\partial q^*_E}{\partial \beta_1} = \frac{-\varphi k_1 k_2 \Delta_1 [2bk_1 - \alpha r^2 (1 - \theta)]}{[2bk_1 k_2 - 2k_1 \varphi^2 (1 - \alpha - \beta_1) - (1 - \theta)ak_2 r^2]^2} < 0,
\]

\[
\frac{\partial q^*_E}{\partial \varphi} = \frac{k_1 \Delta_1 (1 - \alpha - \beta_1)[2bk_1 k_2 + 2k_1 \varphi^2 (1 - \alpha - \beta_1) - (1 - \theta)ak_2 r^2]}{[2bk_1 k_2 - 2k_1 \varphi^2 (1 - \alpha - \beta_1) - (1 - \theta)ak_2 r^2]^2} > 0;
\]
\[
\frac{\partial^2 \pi^*_{epE}}{\partial \theta^2} = \frac{-ak_2r^2}{(2bk_1k_2 - 2k_1\varphi^2)(1 - \alpha - \beta_1) - (1 - \theta)ak_2r^2)^3} < 0,
\]
\[
\frac{\partial^2 \pi^*_{epE}}{\partial \alpha^2} = \frac{-4bk_1\varphi^2 + (1 - \theta)r^2[bk_2 - 3(1 - \beta_1)]\varphi}{[2bk_1k_2 - 2k_1\varphi^2(1 - \alpha - \beta_1) - (1 - \theta)ak_2r^2]^3} < 0,
\]
\[
\frac{\partial^2 \pi^*_{epE}}{\partial \varphi^2} = \frac{-6\varphi k_3(1 - \alpha - \beta_1)}{[2bk_1k_2 - 2k_1\varphi^2(1 - \alpha - \beta_1) - (1 - \theta)ak_2r^2]^3} < 0,
\]
\[
\frac{\partial^2 \pi^*_{lpE}}{\partial \varphi^2} = \frac{-k_2r^2(1 - \theta)}{[2bk_1k_2 - 2k_1\varphi^2(1 - \alpha - \beta_1) - (1 - \theta)ak_2r^2]^3} < 0,
\]
\[
\frac{\partial^2 \pi^*_{lpE}}{\partial \beta_1^2} = \frac{-2k_2\varphi^4([\alpha - \epsilon]\theta + \epsilon)\Delta_{\beta_1}^2k_1^3}{[2bk_1k_2 - 2k_1\varphi^2(1 - \alpha - \beta_1) - (1 - \theta)(\alpha - \epsilon)k_2r^2]^3} < 0;
\]

Next, we also take the results in e-commerce platform mode as an example to exhibit the second-order partial derivatives of profits of the seller, e-commerce platform, live streaming platform and supply chain with respect to various rates, and the partial derivatives with respect to \( \epsilon \) also only exist in transferring mode.
\[
\frac{\partial^2 \pi^*_{cE}}{\partial^2 \theta} = -2[bk_2 - \varphi^2(1 - \alpha - \beta_1)] < 0,
\]
\[
\frac{\partial^2 \pi^*_{cE}}{\partial^2 \alpha} = \frac{-2bk_2^2(1 - \theta)^2 - \varphi^2(bk_1 - (1 - \theta)r^2[2 - 1.5\beta_1 - (1 - \beta_1)\theta]) - k_1\varphi^4}{[2bk_1 - 2bk_2\varphi^2(1 - \alpha - \beta_1) - (1 - \theta)ak_2r^2]^3} < 0,
\]
\[
\frac{\partial^2 \pi^*_{cT}}{\partial^2 \alpha} = \frac{-2(1 - \theta)[bk_2 - \varphi^2(1 - \alpha - \beta_1)]}{[2bk_1 - 2bk_2\varphi^2(1 - \alpha - \beta_1) - (1 - \theta)ak_2r^2]^3} < 0,
\]
\[
\frac{\partial^2 \pi^*_{cT}}{\partial^2 \epsilon} = \frac{-2bk_1k_2 - 2\varphi^2k_1 + ak_2r^2(1 - \theta)}{[2bk_1 - 2bk_2\varphi^2(1 - \alpha - \beta_1) - (1 - \theta)ak_2r^2]^3} < 0,
\]
\[
\frac{\partial^2 \pi^*_{cE}}{\partial^2 \varphi} = \frac{-2k_1(1 - \alpha - \beta_1)(3 - \alpha - \beta_1)}{[2bk_1 - 2bk_2\varphi^2(1 - \alpha - \beta_1) - (1 - \theta)ak_2r^2]^3} < 0.
\]

**Proof of Theorem 1**

Firstly, the sales effort levels among different modes are compared respectively. For mode E and mode T, if \( e^*_E - e^*_T \geq 0 \), there then is \( \epsilon \geq \frac{A_1(C - Aar - B)(\Delta_1 - \Delta_2)}{A_2ar\Delta_1 + A_3(C - Aar - B)} = \epsilon_{v1} \), in which it is always true when \( \Delta_2 - \Delta_1 \leq 0 \), otherwise, when \( \Delta_2 - \Delta_1 > 0 \), \( e^*_E \geq e^*_T \) exists if \( \epsilon \geq \epsilon_{v1} \), while \( e^*_E < e^*_T \) if \( \epsilon < \epsilon_{v1} \). For mode E and mode L, the denominator of \( e^*_L \) is smaller than \( e^*_E \); therefore, there is \( e^*_L \geq e^*_E \) if \( \Delta_2 \geq \Delta_1 \); when \( \Delta_2 \leq \Delta_1 \), \( e^*_E - e^*_L \geq 0 \) is true if \( \beta_2 \geq \frac{R(C - Aar - B) - \Delta_1(C - Aar - B)}{2\varphi^2k_1\Delta_1} = \beta' \), otherwise, there is \( e^*_E < e^*_L \).

For mode T and mode L, \( e^*_L \) has a lower denominator but a higher numerator than \( e^*_T \), therefore, there must be \( e^*_L > e^*_T \).

Next, we consider comparing three modes together based on the above results. It is found that the comparative results mostly rely on the basic NSV, revenue sharing rate and sales service rate. Therefore, we summarize the results according to the basic NSV in the following: \( \begin{cases} 
\frac{e^*_E > e^*_L}{e^*_E > e^*_T} \text{ if } 0 < \beta_2 < \beta' \\
\frac{e^*_E > e^*_L}{e^*_E > e^*_T} \text{ if } \beta' < \beta \end{cases} \) exists when \( \Delta_1 \geq \Delta_2 \), while there is \( \begin{cases} 
\frac{e^*_L > e^*_T}{e^*_E > e^*_L} > e^*_E \text{ if } 0 < \epsilon < \epsilon_{v1} \text{ when } \Delta_1 < \Delta_2. 
\end{cases} \)

The prices among different modes are compared respectively. For mode E and mode T, there is \( p^*_E - p^*_T \geq 0 \) when \( \epsilon \geq \frac{(C - Aar - B)(\Delta_1 - \Delta_2)}{A_2r\Delta_1} = \epsilon_{v2} \). Therefore, \( p^*_E \geq p^*_T \) always exists when \( \Delta_1 \geq \Delta_2 \), while when \( \Delta_1 < \Delta_2 \), there is \( p^*_E \geq p^*_L \) if \( \epsilon \geq \epsilon_{v2} \) and \( p^*_E < p^*_L \) if \( \epsilon < \epsilon_{v2} \). For mode E and mode L, there is \( \beta_2 \geq \beta' \) if \( p^*_E - p^*_L \geq 0 \). Therefore, \( \beta_2 \) just has to be greater than a non-negative number if \( \Delta_2 \leq \frac{C - Aar - B}{C - Aar - B} \Delta_1 \), in which \( p^*_E \geq p^*_L \) always exists; if \( \Delta_2 > \frac{C - Aar - B}{C - Aar - B} \Delta_1 \), there is \( p^*_E \geq p^*_L \) when \( \beta_2 \geq \beta' \), otherwise, there is \( p^*_E < p^*_L \). For mode T and mode L, \( p^*_L \) has a lower denominator but a higher numerator than \( p^*_T \), therefore, there must be \( p^*_L > p^*_E \).
Next, we also consider comparing three modes together based on the above results. It should be noted that some thresholds are meaningless with a given $\Delta_1$. For example, there is $\beta'' - \beta_1 = (\Delta_2 - \Delta_1)/(C - Aar - B)/2^q k_1/\Delta_1 > 0$ when $\Delta_1 < \Delta_2$, which is contradictory to the assumption, therefore, $p^*_T > p^*_E$ is always true when $\Delta_1 < \Delta_2$. Besides, there is $\alpha_{v1} = (C - B)/(C - Aar - B)/\Delta_2 > 0$ when $\Delta_1 < (C - B)/\Delta_2$, which is also contradictory to the assumption, therefore, $p^*_E < p^*_T$ is always true. Putting the above results together, we have $p^*_E \geq p^*_L > p^*_T$ when $\Delta_1 \geq (C - Aar - B)/\Delta_2$.

The quality decisions among different modes are compared respectively. For mode $E$ and mode $L$, when $\Delta_2 > \Delta_1$ exists, there is $q^*_E - q^*_L \geq 0$ if $\alpha \geq (C - Aar - B)/(\Delta_2 - \Delta_1)$, otherwise, there is $q^*_E < q^*_L$. For mode $E$ and mode $T$, when $\beta_2 \geq (\Delta_2(1 - a)/(\Delta_2 - \Delta_1) \times (C - Aar - B)/B\Delta_1 + \Delta_2/(C - Aar - B)) = \beta''$ if $q^*_L - q^*_E \geq 0$ exists, therefore, there is $q^*_E \geq q^*_L$.

Next, we also consider comparing three modes together based on the above results. Similar to the comparative results of price decisions, we have $q^*_E \geq q^*_L > q^*_T$ when $\Delta_1 \geq (C - Aar - B)/(\Delta_2 - \Delta_1)$.

Proof of Theorem 2

The NSV among different modes are compared respectively. For mode $E$ and mode $T$, there is $\alpha \geq (C - Aar - B)/(\Delta_2 - \Delta_1)$ if $\Delta Q^*_E - \Delta Q^*_T \geq 0$ is true. Therefore, $\Delta Q^*_E \geq \Delta Q^*_T$ always exists when $\Delta_1 \geq \Delta_2$, while if $\Delta_1 < \Delta_2$, there is $\Delta Q^*_E \geq \Delta Q^*_T$ when $\alpha \geq (C - Aar - B)/(\Delta_2 - \Delta_1)$, otherwise, there is $\Delta Q^*_E < \Delta Q^*_T$.

For mode $E$ and mode $L$, when $\Delta Q^*_E - \Delta Q^*_L \geq 0$ exists, there is: $\Delta Q^*_L - \Delta L \geq (C - Aar - B)/\Delta_2 \geq (C - B)/\Delta_1 - \Delta_1(C - B)\beta_2 \geq \Delta_2(C - B)/\Delta_1 - \Delta_1(C - B)$.

Therefore, when $(C - B)/(\Delta_2 - \Delta_1) \geq 0$, that is, $\Delta_2 \leq (C - Aar - B)/(\Delta_2 - \Delta_1)$, there is $\beta_2 \geq (\Delta_2(C - B)/\Delta_1 - \Delta_1(C - B))/2^q k_1/\Delta_1 = \beta'''$, when
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Next, we integrate the comparative results among three modes together. When \( \Delta_1 < \Delta_2 \), \( \Delta \) and \( \beta_1 = \frac{(\Delta_1^3 - \Delta_2^3)}{2\Delta_1^2 \Delta_2} \) \( (C \cdot \text{Ar} - B) > 0 \) exists and it is contradictory to the assumption. Therefore, \( \pi^*_aL > \pi^*_aE \) is always true in this condition. At the same time, \( \epsilon v_4 - \alpha = -\frac{(C \cdot \text{Ar} - B) \Delta_3^2 + \Delta_1^2 A^2 r^2 \Delta_2^2}{A^2 r^2 + (C \cdot \text{Ar} - B) \Delta_1^2} < 0 \) exists, which indicates that \( \epsilon v_4 \) is always true and meaningful. Based on the above analysis, we have \( \pi^*_aL \geq \pi^*_aE > \pi^*_aT \) when \( \Delta_1 \geq \sqrt{\frac{C \cdot \text{Ar} - B}{C \cdot \text{Ar} - B_2} \Delta_2} \); if \( \Delta_2 \leq \Delta_1 < \sqrt{\frac{C \cdot \text{Ar} - B}{C \cdot \text{Ar} - B_2} \Delta_2} \) exists, there is

\[
\begin{cases} 
\pi^*_aL > \pi^*_aE \geq \pi^*_aT, & \text{if } 0 < \beta_2 < \beta_5^* \ 
\pi^*_aE \geq \pi^*_aL > \pi^*_aT, & \text{if } \beta_5^* \leq \beta_2 < \beta_1
\end{cases}
\]

when \( \Delta_1 < \Delta_2 \).

The comparison processes of seller’s profit among different modes are similar to the above process. Firstly, \( \epsilon \geq \epsilon v_2 \) can be obtained when \( \pi^*_sE \geq \pi^*_sT \). Therefore, \( \pi^*_sE \geq \pi^*_sT \) is always true when \( \Delta_2 \leq \Delta_1 \); if \( \Delta_2 > \Delta_1 \), there is still \( \pi^*_sE \geq \pi^*_sT \) if \( \epsilon \geq \epsilon v_2 \) while \( \pi^*_sE < \pi^*_sT \) if \( \epsilon < \epsilon v_2 \). For mode E and mode L, there is

\[
\Delta_1 \geq \frac{1}{\sqrt{h_1(C \cdot \text{Ar} - B_1) - \sqrt{h_2(C \cdot \text{Ar} - B)}}} = \epsilon v_5, \quad \text{otherwise, } \pi^*_sT < \pi^*_sL \text{ is true.}
\]

Finally, for mode T and mode L, there is \( \pi^*_sT \geq \pi^*_sL \) when \( \epsilon \leq \frac{\sqrt{h_1(C \cdot \text{Ar} - B_1) - \sqrt{h_2(C \cdot \text{Ar} - B)}}}{\sqrt{h_1(C \cdot \text{Ar} - B)}} = \epsilon v_2 \), otherwise, \( \pi^*_sT < \pi^*_sL \text{ is true. Due to the complexity of segmentations and thresholds, we will not integrate the comparative results together; it is the same for the comparative results of supply chain profit and consumer surplus.}

For the e-commerce platform, we only need to compare the profits in mode E and mode T. There is \( \epsilon \geq \frac{(C \cdot \text{Ar} - B)}{A r \Delta_1} \left( \frac{\beta_1}{a_0 + \beta_1} - \Delta_1 \right) = \epsilon v_6 \) if \( \pi^*_eE \geq \pi^*_eT \). Therefore, when \( \Delta_1 \geq \Delta_2 \sqrt{\frac{\beta_1}{a_0 + \beta_1}} \) exists, there is \( \pi^*_eE \geq \pi^*_eT \); otherwise, \( \pi^*_eE < \pi^*_eT \) is true.

For the live streaming platform, we just need to compare the profits in mode T and mode E. When \( \epsilon \geq \frac{(0a + \beta_2) [b^2k_2 \cdot A^2 (1 - \beta_1)] [(C \cdot \text{Ar} - B_1) A - \theta b k_2 \cdot A^2 (1 - \alpha \beta_1)] [(C \cdot \text{Ar} - B)]}{(1 - \theta) [b^2k_2 \cdot A^2 (1 - \alpha \beta_1)] [(C \cdot \text{Ar} - B)]} = \epsilon v_7 \) is true, \( \pi^*_lT \geq \pi^*_lE \); otherwise, \( \pi^*_lE < \pi^*_lT \).

For the profit of supply chain, we have \( \pi^*_cE \geq \pi^*_cT \) when \( \Delta_1 \geq \sqrt{\frac{C \cdot A (1 - \theta)(\alpha - \epsilon)^2 - m}{C \cdot A (1 - \theta) a^2 - m} \frac{C \cdot \text{Ar} - B}{C \cdot A (\alpha - \epsilon) r - B} \Delta_2} \); otherwise, \( \pi^*_cE < \pi^*_cT \) is true. For the comparative results between mode E and mode L, also the results between mode T and mode L, they are similar to the above comparison, in which the corresponding value
ranges for \( \pi^*_{cE} \geq \pi^*_{cL} \) and \( \pi^*_{cT} \geq \pi^*_{cL} \) are \( \Delta_1 \geq \frac{C-A(1-\theta)ra^2-m_1}{C-A(1-\theta)ra^2-m} \frac{C-Aar-B}{C-Aar-B_1} \Delta_2 \) and
\[
\frac{C-Aar-B}{C-A(\alpha-\varepsilon)\gamma-r-B} \sqrt{\frac{C-A(1-\theta)ra^2-m_1}{C-A(1-\theta)ra^2-m}} \geq 1.
\]

**Proof of Theorem 4**

If \( CS_E^* \geq CS_T^* \) is true, we have:

\[
\begin{align*}
&\{ A^2 [a_1 t_1 (C - 2B) + z(C - 2Aar)] [2z(Aar + B) + C(a_1 t_1 - 3z)] + 4A^2 z^2 r^2 (C - B - Aar)^2 \} \varepsilon^2 \\
&+ 2A r (C - B - Aar) [a_1 t_1 (C - 2B) + z(C - 2Aar)] [2z(Aar + B) + C(a_1 t_1 - 3z)] + (C - B - Aar) z \\
&\geq [2B(a_2 t_2 + z) + 4z (C + Aar)] \varepsilon + (C - B - Aar)^2 [a_1 t_1 (C - 2B) + z(C - 2Aar)] [2z(Aar + B) + C(a_1 t_1 - 3z)] - \\
&(a_2 t_2 (C - 2B) + z(C - 2Aar)) [2z(Aar + B) + C(a_2 t_2 - 3z)] \geq 0.
\end{align*}
\]

The left side of this inequality is a quadratic function with respect to \( \varepsilon \), in which both the quadratic and first terms are proved to be always positive while the constant term is uncertain. Therefore, the constant term is non-negative when \( \Delta_1 \geq \Delta_2 \) while it is negative when \( \Delta_1 < \Delta_2 \). On top of everything, if \( \Delta_1 \geq \Delta_2 \) is true, then \( CS_E^* - CS_T^* \geq 0 \) is always true if \( \varepsilon \geq 0 \); however, if \( \Delta_1 < \Delta_2 \), there exists a threshold \( \varepsilon^{v'} \), in which \( CS_E^* - CS_T^* \leq 0 \) is true if \( 0 < \varepsilon < \varepsilon^{v'} \) while there is \( CS_E^* \geq CS_T^* \) when \( \varepsilon \geq \varepsilon^{v'} \).

If \( CS_E^* \geq CS_L^* \) is true, we have:

\[
\begin{align*}
\Delta_1^2 &+ [4B z (C - B - Aar)(C - B_1 - Aar^2)] \Delta_1 \\
&(C - B - Aar)^2 (4z^2 (C - B_1 - Aar^2) + [a_2 t_2 (C - 2B_1) + z(C - 2Aar)] [2z(Aar + B_1) + a_2 t_2 C]) \geq 0.
\end{align*}
\]

The left side of this inequality is a quadratic function with respect to \( \Delta_1 \), in which both the quadratic and first terms are proved to be always positive while the constant term is negative. Therefore, there exists a threshold \( \varepsilon^{v'} \), in which \( CS_E^* - CS_L^* \leq 0 \) is true when \( 0 < \varepsilon < \varepsilon^{v'} \) while there is \( CS_E^* \geq CS_L^* \) if \( \Delta_1 \geq \Delta_1^{v'} \).

If \( CS_T^* \geq CS_L^* \) is true, we have:

\[
\begin{align*}
&\{ [(C - 2B)(C - B_1 - Aar)^2 C] [C - \{ C - B - A(\alpha - \varepsilon)r \}^2 (C - 2B_1 C)] \} \Delta_2 \\
&+ 4z [z (C - Aar)(B_1 - B) + ArB_1 \varepsilon] (C - B_1 - Aar) [C - B - A(\alpha - \varepsilon)r] \geq 0.
\end{align*}
\]

The left side of this inequality is a linear function with respect to \( \Delta_2 \). If the coefficient is non-negative, that is, when \( \varepsilon \leq \frac{(C-Aar-B_1)}{Ar} \sqrt{\frac{C-2B}{C-2B_1}} \frac{C-Aar-B}{C-Aar-B_1} \) exists, there

is \( CS_T^* \geq CS_L^* \) if \( \Delta_2 \geq \frac{4z [(C-Aar)(B_1-B)+ArB_1 \varepsilon] (C-B_1-Aar) (C-B-A(\alpha-\varepsilon)r)}{[[C-2B](C-B_1-Aar)^2 C] [C-B-A(\alpha-\varepsilon)r]^2 (C-2B_1 C)]} \) otherwise, \( CS_T^* < CS_L^* \) is true. However, if the coefficient is negative, that is, when \( \varepsilon > \frac{(C-Aar-B_1)}{Ar} \sqrt{\frac{C-2B}{C-2B_1}} \frac{C-Aar-B}{C-Aar-B_1} \) exists, it is impossible to have

\[
\begin{align*}
\Delta_2 &< \frac{4z [(C-Aar)(B_1-B)+ArB_1 \varepsilon] (C-B_1-Aar) (C-B-A(\alpha-\varepsilon)r)}{[[C-2B](C-B_1-Aar)^2 C] [C-B-A(\alpha-\varepsilon)r]^2 (C-2B_1 C)]} < 0, \text{ therefore, we have} \\
\Delta_2 &> \frac{4z [(C-Aar)(B_1-B)+ArB_1 \varepsilon] (C-B_1-Aar) (C-B-A(\alpha-\varepsilon)r)}{[[C-2B](C-B_1-Aar)^2 C] [C-B-A(\alpha-\varepsilon)r]^2 (C-2B_1 C)]} \text{ and } CS_T^* < CS_L^* \text{ is always true.}
\end{align*}
\]
Proofs of Corollaries 2–5

Corollaries 2–5 are all specific cases in the basic model. Therefore, we can get the corresponding results by matching specific market conditions to the results of the general cases, in which the processes are similar to the basic model and we will not repeat it here.

Appendix 2

Proof of Theorem 5

To obtain the impact of jumping loss on mode selection, we need to get the optimal decisions, NSV and profits firstly through backward induction method. In this situation, only mode T will be influenced by jumping loss. Therefore, we get the optimal results as follows, in which the superscript \(v\) indicates extension 5.1:

\[
\begin{align*}
\epsilon_v^T &= \frac{(1-\theta)k_2(a-\epsilon)(a_2t_2-\delta)-2(1-a-\beta_1)\psi^2k_1}{2bk_1k_2-(1-\theta)r^2k_2(a-\epsilon)-2(1-a-\beta_1)\psi^2k_1}, \\
\phi_v^T &= \frac{\theta k_2(1-a-\beta_1)(a_2t_2-\delta)-2(1-a-\beta_1)\psi^2k_1}{2bk_1k_2-(1-\theta)r^2k_2(a-\epsilon)-2(1-a-\beta_1)\psi^2k_1}, \\
\sigma_v^T &= \frac{|a_2t_2(2bk_1k_2-(1-\theta)r^2k_2(a-\epsilon)-2(1-a-\beta_1)\psi^2k_1)|}{2bk_1k_2-(1-\theta)r^2k_2(a-\epsilon)-2(1-a-\beta_1)\psi^2k_1}, \\
\Delta_v^T &= \frac{|a_2t_2(2bk_1k_2-(1-\theta)r^2k_2(a-\epsilon)-2(1-a-\beta_1)\psi^2k_1)|}{2bk_1k_2-(1-\theta)r^2k_2(a-\epsilon)-2(1-a-\beta_1)\psi^2k_1}.
\end{align*}
\]

After getting the optimal results of mode T in this situation, we need to compare the results among these modes. However, we only need to compare the results between mode T and mode E, also between mode T and mode L, because the relationships between mode E and mode L are not affected. The processes are the same and we omit the details here, only give the results below, in which \(\Delta_3 = a_2t_2\delta - \zeta\).

To exhibit the comparative results clearly, we summarize the thresholds in extension 5.1 as follows. We have

\[
\begin{align*}
\epsilon_v^a &= \frac{Aa(C-Ara-B)(\Delta_3 - \Delta_1)}{A^2Ar\Delta_1 + A\Delta_2(C-Ara-B)}, \\
\epsilon_v^a &= \frac{\Delta_1(C-B)(C-Aar-B) - \Delta_1(C-B)(C-Aar-B)}{\Delta_2(C-B)Ar}, \\
\epsilon_v^a &= \frac{\Delta_1 \sqrt{\Delta_1(C-Aar-B) + \Delta_2(C-Aar-B)}}{\Delta_2 \sqrt{\Delta_1 Ar}}, \\
\epsilon_v^a &= \left(\sqrt{\frac{1}{\theta a + \beta_1}} \Delta_3 - \Delta_1\right) \frac{(C-B)(C-Aar-B)}{Ar\Delta_1}, \\
\epsilon_v^a &= \left(\frac{2}{C-B-Aar} \Delta_1 \left\{a_2t_1(C-B) + C \left[\frac{a_2t_1(C-B) + C}{2(Aar+B+C) + 2C(a_2t_1-3z)} + 8A^2r^2z^2(C-B-Aar)\right]\right\}\right) \frac{1}{Ar\Delta_1}.
\end{align*}
\]
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$$e^{av} = \frac{(C-B-Aar)\left\{ [2z(Aar+B)+C(a_1+3z)]\left[a_1t_1(C-2B)+z(C-2Aar)\right]+2z(C-B-Aar)\left[\beta(\Delta_1+2z)+2z(C+Aar)\right] \right\}}{A[r(a_1t_1(C-2B)+z(C-2Aar))\left[2z(Aar+B)+C(a_1+3z)\right]+8A^2r^2z^2(C-B-Aar)]} \ .$$

The proofs of comparative results are similar to those of theorems in Sect. 4.2, then we omit them and only illustrate the results (Theorem 5a-5d) as follows.

**Theorem 5a** With the jumping loss in mode T, the relationships of optimal decisions among three modes are as follows:

1. **When** $\Delta_1 \geq \Delta_3$, there is $e^{a*}_E \geq e^{a*}_T$, and when $\Delta_1 < \Delta_3$, there is
   
   $$e^{a*}_E < e^{a*}_T \ , \ \text{if} \ 0 < \epsilon < e^{a}_v \ ; \ \text{when} \ \Delta_1 \leq \Delta_2 \ , \ \text{there is} \ e^{a*}_E \leq e^{a*}_L \ , \ \text{when} \ \Delta_1 > \Delta_2$$
   
   then we omit them and only illustrate the results (Theorem 5a-5d) as follows.

2. **When** $\Delta_1 \geq \Delta_3$, there is $p^{a*}_E \geq p^{a*}_T$, when $\Delta_1 < \Delta_3$, there is
   
   $$p^{a*}_E < p^{a*}_T \ , \ \text{if} \ 0 < \epsilon < e^{a}_v \ ; \ \text{when} \ \Delta_1 \geq \frac{C-Aar-B}{C-Aar-B_2} \Delta_2 \ , \ \text{there is} \ p^{a*}_E \geq p^{a*}_L \ , \ \text{when} \ \Delta_1 < \frac{C-Aar-B}{C-Aar-B_2} \Delta_2 \ , \ \text{there is}$$

3. **When** $\Delta_1 \geq \Delta_3$, there is $q^{a*}_E \geq q^{a*}_T$, when $\Delta_1 < \Delta_3$, there is

   $$q^{a*}_E \leq q^{a*}_T \ , \ \text{if} \ 0 < \epsilon < e^{a}_v \ ; \ \text{when} \ \Delta_1 \geq \frac{(C-Aar-B)(1-a)}{(C-Aar-B_2)(1-a-\beta_i)} \Delta_2 \ , \ \text{there is} \ q^{a*}_E \geq q^{a*}_L \ , \ \text{when} \ \Delta_1 < \frac{(C-Aar-B)(1-a)}{(C-Aar-B_2)(1-a-\beta_i)} \Delta_2 \ , \ \text{there is}$$

**Theorem 5b** With the jumping loss in mode T, the relationships of optimal NSV among three modes are as follows:

1. **When** $\Delta_1 \geq \Delta_3$, there is $\Delta Q^{a*}_E \geq \Delta Q^{a*}_T$, when $\Delta_1 < \Delta_3$, there is

   $$\Delta Q^{a*}_E \leq \Delta Q^{a*}_T \ , \ \text{if} \ 0 < \epsilon < e^{a}_v \ ; \ \text{when} \ \Delta_1 \geq \frac{(C-Aar-B)(1-a)}{(C-Aar-B_2)(1-a-\beta_i)} \Delta_2 \ , \ \text{there is} \ \Delta Q^{a*}_E \geq \Delta Q^{a*}_L \ , \ \text{when} \ \Delta_1 < \frac{(C-Aar-B)(1-a)}{(C-Aar-B_2)(1-a-\beta_i)} \Delta_2 \ , \ \text{there is}$$

2. **When** $\Delta_1 \geq \frac{C-Aar-B}{C-B} \Delta_2 \leq \frac{C-Aar-B}{C-Aar-B_2}(C-B) \Delta_2$, there is $\Delta Q^{a*}_E \geq \Delta Q^{a*}_L \ , \ \text{if} \ 0 < \beta_2 < \beta''' \ \text{when} \ \Delta_1 < \frac{C-Aar-B}{C-B} \Delta_2$, there is $\Delta Q^{a*}_E \leq \Delta Q^{a*}_L \ , \ \text{if} \ \beta''' \leq \beta_2 < \beta_1$.

3. **Theorem 5c** With the jumping loss in mode T, the relationships of optimal profits among three modes are as follows:
(1) When $\Delta_1 \geq \Delta_3$, there is $\pi_{st}^{as} \geq \pi_{st}^{al'}$, when $\Delta_1 < \Delta_3$, there is
\[
\begin{cases}
\pi_{st}^{as} < \pi_{st}^{al'}, & \text{if } 0 < \epsilon < \epsilon_{v4}^a; \\
\pi_{st}^{as} \geq \pi_{st}^{al'}, & \text{if } \epsilon_{v4}^a \leq \epsilon < \alpha.
\end{cases}
\]
when $\Delta_1 \geq \sqrt{\frac{C-Aar-B}{C-Aar-B_2}} \Delta_2$, there is $\pi_{st}^{as} \geq \pi_{st}^{al'}$; when
$\Delta_1 < \sqrt{\frac{C-Aar-B}{C-Aar-B_2}} \Delta_2$, there is
\[
\begin{cases}
\pi_{st}^{as} < \pi_{st}^{al'}, & \text{if } 0 < \beta_2 < \beta_{v5}^1; \\
\pi_{st}^{as} \geq \pi_{st}^{al'}, & \text{if } \beta_{v5}^1 \leq \beta_2 < \beta_1 \text{; } \pi_{st}^{as} \geq \pi_{st}^{al'}.
\end{cases}
\]

(2) When $\Delta_1 \geq \Delta_3$, there is $\pi_{st}^{as} \geq \pi_{st}^{as}$, when $\Delta_1 < \Delta_3$, there is
\[
\begin{cases}
\pi_{st}^{as} < \pi_{st}^{as}, & \text{if } 0 < \epsilon < \epsilon_{v2}^a; \\
\pi_{st}^{as} \geq \pi_{st}^{as}, & \text{if } \epsilon_{v2}^a \leq \epsilon < \alpha.
\end{cases}
\]
when $\Delta_1 \geq \sqrt{\frac{h_2}{h_1}} \frac{C-Aar-B}{C-Aar-B_1} \Delta_2$, there is $\pi_{st}^{as} \geq \pi_{st}^{sl'}$; when
$\Delta_1 < \sqrt{\frac{h_2}{h_1}} \frac{C-Aar-B}{C-Aar-B_1} \Delta_2$, there is
\[
\pi_{st}^{as} \leq \pi_{st}^{sl'}, \text{ when } \Delta_2 \geq \frac{(C-Aar-B_1) \sqrt{h_1}}{(C-Aar-B) \sqrt{h_2}} \Delta_3 \text{ there is }
\]
\[
\begin{cases}
\pi_{st}^{as} \leq \pi_{st}^{sl'}, & \text{if } 0 < \epsilon \leq \epsilon_{v5}^a; \\
\pi_{st}^{as} < \pi_{st}^{sl'}, & \text{if } \epsilon_{v5}^a < \epsilon < \alpha.
\end{cases}
\]

(3) When $\Delta_1 \geq \frac{a\theta}{\alpha_0 + \beta_1} \Delta_3$, there is $\pi_{ep}^{as} \geq \pi_{ep}^{al'}$, when $\Delta_1 < \frac{a\theta}{\alpha_0 + \beta_1} \Delta_3$, there is
\[
\begin{cases}
\pi_{ep}^{as} < \pi_{ep}^{al'}, & \text{if } 0 < \epsilon < \epsilon_{v6}^a; \\
\pi_{ep}^{as} \geq \pi_{ep}^{al'}, & \text{if } \epsilon_{v6}^a \leq \epsilon < \alpha.
\end{cases}
\]

(4) When $\beta_2 \leq \frac{a\theta((C-B)(C-Aar-B\Delta_1^2 - (C-B_1)(C-Aar-B)\Delta_1^2) \Delta_2^2)}{D_1^2(C-B_1)(C-Aar-B)\Delta_2^2}$, there is $\pi_{lp}^{as} \geq \pi_{lp}^{as}$
when
\[
\beta_2 > \frac{a\theta((C-B)(C-Aar-B\Delta_1^2 - (C-B_1)(C-Aar-B)\Delta_1^2) \Delta_2^2)}{D_1^2(C-B_1)(C-Aar-B)\Delta_2^2}, \text{ there is }
\]
\[
\begin{cases}
\pi_{lp}^{as} < \pi_{lp}^{as}, & \text{if } 0 < \epsilon < \epsilon_{v7}^a; \\
\pi_{lp}^{as} \geq \pi_{lp}^{as}, & \text{if } \epsilon_{v7}^a \leq \epsilon < \alpha.
\end{cases}
\]

(5) When $\Delta_1 \geq \frac{C-Aar-B}{C-Aar-B_1} \sqrt{\frac{C-Aar-B}{C-Aar-B_1}} \Delta_3$, there is $\pi c_{ET}^{as} \geq \pi c_{ET}^{as}$
otherwise, $\pi c_{ET}^{as} < \pi c_{ET}^{as}$; when $\Delta_1 \geq \frac{(C-Aar-B_1)(C-Aar-B)\Delta_2^2}{C-Aar-B_1} \Delta_3$, there is $\pi c_{ET}^{as} \geq \pi c_{ET}^{as}$
otherwise, $\pi c_{ET}^{as} < \pi c_{ET}^{as}$ when
\[
\frac{C-Aar-B_1 + 2\epsilon_{v7}^a \beta_1}{C-Aar-B_1 + 2\epsilon_{v7}^a \beta_1 + \Delta_3} \geq \frac{(C-Aar-B_1) \Delta_3}{C-Aar-B_1} \Delta_3, \text{ there is }
\]
\[
\begin{cases}
\pi c_{ET}^{as} \geq \pi c_{ET}^{as}, & \text{if } \epsilon \leq \epsilon_{v8}^a; \\
\pi c_{ET}^{as} < \pi c_{ET}^{as}, & \text{if } \epsilon > \epsilon_{v8}^a.
\end{cases}
\]

Theorem 5d With the jumping loss in mode $T$, the relationships of optimal consumer surplus among three modes is as follows:

(1) When $\Delta_1 \geq \Delta_3$, there is $CS_{ET}^{as} \geq CS_{ET}^{as}$, when $\Delta_1 < \Delta_3$, there is
\[
\begin{cases}
CS_{ET}^{as} < CS_{ET}^{as}, & \text{if } 0 < \epsilon < \epsilon_{v9}^a; \\
CS_{ET}^{as} \geq CS_{ET}^{as}, & \text{if } \epsilon_{v9}^a \leq \epsilon < \alpha.
\end{cases}
\]

(2) When $\Delta_1 \geq \Delta_4$, there is $CS_{ET}^{as} \geq CS_{ET}^{as}$, when $\Delta_1 < \Delta_4$, there is $CS_{ET}^{as} < CS_{ET}^{as}$;

(3) When $\epsilon > \epsilon_{v9}^a$, there is
\[
\begin{cases}
CS_{ET}^{as} \geq CS_{ET}^{as}, & \text{if } \Delta_2 \leq \Delta_2^2; \\
CS_{ET}^{as} < CS_{ET}^{as}, & \text{if } \Delta_2 > \Delta_2^2 \text{ when } \epsilon \leq \epsilon_{v9}^a, \text{ there is }
\]

Therefore, the overall comparative results are consistent with the basic model, while the thresholds are changed, in which $\epsilon_{v1}^a < \epsilon_{v2}^a < \epsilon_{v3}^a < \epsilon_{v4}^a$, $\epsilon_{v5}^a < \epsilon_{v6}^a$, $\epsilon_{v7}^a > \epsilon_{v7}^a$ and $\epsilon_{v8}^a < \epsilon_{v8}^a$ are always true.
Proof of Theorem 6

In order to obtain the influence of the gift-giving function on the optimal decisions, NSV and profits of different modes in the basic model, we need to obtain the optimal results according to the backward induction method. Since only mode T and mode L are affected by the gift-giving function of the live broadcast platform, the results obtained in these two modes are as follows, in which the superscript $b$ indicates extension $b$.

\[
el^*_T = \frac{1 - \theta}{2b_k}(a_2 t_2 - z) + g(1 - \tau)[bk_2 - \varphi^2(1 - \alpha - \beta_1)],
\]

\[
el^*_L = \frac{1 - \theta}{2b_k}(a_2 t_2 - z) + g(1 - \tau)[bk_2 - \varphi^2(1 - \alpha - \beta_2)],
\]

\[
p^*_T = \frac{k_1k_2(a_2 t_2 - z) + g r_k}{2b_kk_2 - (1 - \theta)r^2k_2(\alpha - \epsilon) - 2(1 - \alpha - \beta_1)\varphi^2k_1},
\]

\[
p^*_L = \frac{k_1k_2(a_2 t_2 - z) + g r_k}{2b_kk_2 - (1 - \theta)r^2k_2(\alpha - \epsilon) - 2(1 - \alpha - \beta_2)\varphi^2k_1},
\]

\[
\Delta Q^*_T = \frac{[k_1(a_2 t_2 - z) + g(1 - \tau)]bk_2 - (1 - \alpha - \beta_1)\varphi^2}{2b_kk_2 - (1 - \theta)r^2k_2(\alpha - \epsilon) - 2(1 - \alpha - \beta_1)\varphi^2k_1},
\]

\[
\Delta Q^*_L = \frac{[k_1(a_2 t_2 - z) + g(1 - \tau)]bk_2 - (1 - \alpha - \beta_2)\varphi^2}{2b_kk_2 - (1 - \theta)r^2k_2(\alpha - \epsilon) - 2(1 - \alpha - \beta_2)\varphi^2k_1},
\]

\[
x^*_T = \frac{(1 - \theta)}{2b_kk_2 - (1 - \theta)r^2k_2(\alpha - \epsilon) - 2(1 - \alpha - \beta_1)\varphi^2k_1},
\]

\[
x^*_L = \frac{(1 - \theta)}{2b_kk_2 - (1 - \theta)r^2k_2(\alpha - \epsilon) - 2(1 - \alpha - \beta_2)\varphi^2k_1},
\]

\[
\nu^*_T = \frac{k_2(1 - \alpha - \beta_1)[(a_2 t_2 - z)k_1 + g r(1 - \tau)]^2[2bk_2 - 3(1 - \alpha - \beta_1)\varphi^2]}{2[2bk_1k_2 - (1 - \theta)r^2k_2(\alpha - \epsilon) - 2(1 - \alpha - \beta_1)\varphi^2k_1]^2},
\]

\[
\nu^*_L = \frac{k_2(1 - \alpha - \beta_2)[(a_2 t_2 - z)k_1 + g r(1 - \tau)]^2[2bk_2 - 3(1 - \alpha - \beta_2)\varphi^2]}{2[2bk_1k_2 - (1 - \theta)r^2k_2(\alpha - \epsilon) - 2(1 - \alpha - \beta_2)\varphi^2k_1]^2},
\]

\[
\nu^*_c T = \frac{k_2(1 - \alpha - \beta_1)[(a_2 t_2 - z)k_1 + g r(1 - \tau)]^2[2bk_2 - 3(1 - \alpha - \beta_1)\varphi^2]}{2[2bk_1k_2 - (1 - \theta)r^2k_2(\alpha - \epsilon) - 2(1 - \alpha - \beta_1)\varphi^2k_1]^2}.
\]

However, in mode T and mode L, the expressions of profits in live streaming platform and supply chain, and consumer surplus are complex, and we therefore omit them here but compare them through numerical analysis.

The differences of these optimal results between basic model and the model with gift-giving function in mode T and mode L are listed below:
Proof of Theorem 7

After getting the optimal results, we need to compare the decisions, NSV and profits among different modes firstly as the same method in basic model. We only list the complicated part of the calculation process and omit the rest here.

(1) If \(e_T^{bs} - e_T^{bs} \geq 0\) exists, there is:

\[
\frac{2g(1 - \tau)[bk_2 - (1 - \alpha - \beta_1)\varphi^2]}{2bk_1k_2 - (1 - \theta)r^2k_2(\alpha - \epsilon) - 2(1 - \alpha - \beta_1)\varphi^2k_1} > 0,
\]

\[
\frac{\varphi g r(1 - \tau)(1 - \alpha - \beta_1)}{2bk_1k_2 - (1 - \theta)r^2k_2(\alpha - \epsilon) - 2(1 - \alpha - \beta_1)\varphi^2k_1} > 0,
\]

\[
\frac{(1 - \tau)grk_2}{2bk_1k_2 - (1 - \theta)r^2k_2(\alpha - \epsilon) - 2(1 - \alpha - \beta_1)\varphi^2k_1} > 0,
\]

\[
\frac{2g(1 - \tau)[bk_2 - (1 - \alpha - \beta_2)\varphi^2]}{2bk_1k_2 - (1 - \theta)r^2k_2(\alpha - \epsilon) - 2(1 - \alpha - \beta_2)\varphi^2k_1} > 0,
\]

\[
\frac{\varphi g r(1 - \tau)(1 - \alpha - \beta_2)}{2bk_1k_2 - (1 - \theta)r^2k_2(\alpha - \epsilon) - 2(1 - \alpha - \beta_2)\varphi^2k_1} > 0,
\]

\[
\frac{(1 - \tau)grk_2}{2bk_1k_2 - (1 - \theta)r^2k_2(\alpha - \epsilon) - 2(1 - \alpha - \beta_2)\varphi^2k_1} > 0,
\]

\[
\frac{(1 - \tau)[bk_2 - 2(1 - \alpha - \beta_1)\varphi^2]}{2bk_1k_2 - (1 - \theta)r^2k_2(\alpha - \epsilon) - 2(1 - \alpha - \beta_1)\varphi^2k_1} > 0,
\]

\[
\frac{(1 - \tau)[bk_2 - 2(1 - \alpha - \beta_2)\varphi^2]}{2bk_1k_2 - (1 - \theta)r^2k_2(\alpha - \epsilon) - 2(1 - \alpha - \beta_2)\varphi^2k_1} > 0,
\]

\[
\frac{gk_2r(1 - \alpha - \beta_1)(1 - \tau)[2\Delta_2k_1 + (1 - \tau)gr][2bk_2 - 3(1 - \alpha - \beta_1)\varphi^2]}{2[2bk_1k_2 - (1 - \theta)r^2k_2(\alpha - \epsilon) - 2(1 - \alpha - \beta_1)\varphi^2k_1]^2} > 0,
\]

\[
\frac{gk_2r(1 - \alpha - \beta_2)(1 - \tau)[2\Delta_2k_1 + (1 - \tau)gr][2bk_2 - 3(1 - \alpha - \beta_2)\varphi^2]}{2[2bk_1k_2 - (1 - \theta)r^2k_2(\alpha - \epsilon) - 2(1 - \alpha - \beta_2)\varphi^2k_1]^2} > 0,
\]

\[
\frac{\Delta Q_T^{bs} - \Delta Q_T^{bs}}{2bk_1k_2 - (1 - \theta)r^2k_2(\alpha - \epsilon) - 2(1 - \alpha - \beta_1)\varphi^2k_1} > 0,
\]

\[
\frac{\Delta Q_L^{bs} - \Delta Q_L^{bs}}{2bk_1k_2 - (1 - \theta)r^2k_2(\alpha - \epsilon) - 2(1 - \alpha - \beta_2)\varphi^2k_1} > 0,
\]

\[
\frac{\Delta Q_T^{bs} - \Delta Q_T^{bs}}{2bk_1k_2 - (1 - \theta)r^2k_2(\alpha - \epsilon) - 2(1 - \alpha - \beta_1)\varphi^2k_1} > 0,
\]

\[
\frac{\Delta Q_L^{bs} - \Delta Q_L^{bs}}{2bk_1k_2 - (1 - \theta)r^2k_2(\alpha - \epsilon) - 2(1 - \alpha - \beta_2)\varphi^2k_1} > 0,
\]

\[
\frac{\Delta Q_T^{bs} - \Delta Q_T^{bs}}{2bk_1k_2 - (1 - \theta)r^2k_2(\alpha - \epsilon) - 2(1 - \alpha - \beta_1)\varphi^2k_1} > 0,
\]

\[
\frac{\Delta Q_L^{bs} - \Delta Q_L^{bs}}{2bk_1k_2 - (1 - \theta)r^2k_2(\alpha - \epsilon) - 2(1 - \alpha - \beta_2)\varphi^2k_1} > 0,
\]

\[
\Delta Q_T^{bs} - \Delta Q_T^{bs} > 0 \quad \text{(Because } \Delta Q_T^{bs} - \Delta Q_T^{bs} > 0 \text{ and}
\]

\[
\Delta Q_L^{bs} - \Delta Q_L^{bs} > 0 \text{, because profits of each member in the model with gift-giving function are higher than these in basic model),}
\]

\[
\Delta Q_T^{bs} - \Delta Q_T^{bs} > 0 \text{, because profits of each member in the model with gift-giving function are higher than these in basic model).}
\]

\[
\text{Proof of Theorem 7}
\]

After getting the optimal results, we need to compare the decisions, NSV and profits among different modes firstly as the same method in basic model. We only list the complicated part of the calculation process and omit the rest here.

(1) If \(e_T^{bs} - e_T^{bs} \geq 0\) exists, there is:
$2\varphi^2[Aak_1\Delta_1 - 2g(C - Aar - B)(1 - \tau)]\beta_2 \geq (C - Aar - B)H_2$
\[+ 2Aa\varphi^2k_1\beta_1\Delta_2 - Aa(C - Aar - B_2)(\Delta_1 - \Delta_2).\]

Therefore, when the coefficient of $\beta_2$ is non-negative, which means when
$g \leq \frac{Aak_1\Delta_1}{2(1-\tau)(C-Aar-B)}$, there is $e_{b_2}^{b_2} \geq e_{L}^{b_2}$ if
$\beta_2 \geq \frac{2a\varphi^2k_1\beta_1\Delta_2 - Aa(C-Aar-B_2)(\Delta_1 - \Delta_2)}{2a\varphi^2k_1\Delta_2 - 2g(C-Aar-B)(1-\tau)} = \beta_{b_1}'$ is true, however, there is
$e_{b_2}^{b_2} < e_{L}^{b_2}$ if $\beta_2 < \beta_{b_1}'$; when the coefficient of $\beta_1$ is negative, that is, $g > \frac{Aak_1\Delta_1}{2(1-\tau)(C-Aar-B)}$, the results are quite similar.

(2) If $e_{b_2}^{b_2} < e_{L}^{b_2} \geq 0$ is true, there is:
$- 2g(1 - \tau)\{Aar\varphi^2(\beta_1 - \beta_2) + A \epsilon [bk_2 - \varphi^2(1 - \alpha - \beta_2)]\} \geq 2A\Delta_2\{[C - A(\alpha - \epsilon)r - B]a - (C - Aar - B_1)(\alpha - \epsilon)\}.$

Therefore, the coefficient of $g$ on the left-hand side of the inequality is negative. When
$g \leq \frac{Aa_1(C - A(a - \epsilon)r - B_1)\varphi^2(\beta_1 - \beta_2) + A \epsilon [bk_2 - \varphi^2(1 - \alpha - \beta_2)]}{2(1-\tau)(Aar - B)}$ exists, there is $e_{b_2}^{b_2} \geq e_{L}^{b_2}$. However, it is not true because $g$ is non-negative. Therefore, there is still $e_{b_2}^{b_2} < e_{L}^{b_2}$.

(3) If $q_{b_2}^{b_2} - q_{L}^{b_2} \geq 0$ is true, there is:
$- gr(1 - \tau)\{(1 - \alpha - \beta_2)[C - A(\alpha - \epsilon)r - B] - (1 - \alpha - \beta_2)(C - Aar - B_1)\} \geq k_1\Delta_2\{[C - A(\alpha - \epsilon)r - B](1 - \alpha - \beta_2) - (C - Aar - B_1)(1 - \alpha - \beta_1)\}.$

Therefore, the coefficient of $g$ on the left-hand side of the inequality is negative. When
$g \leq \frac{k_1\Delta_2(C - A(a - \epsilon)r - B_1)\varphi^2(\beta_1 - \beta_2) + A \epsilon [bk_2 - \varphi^2(1 - \alpha - \beta_2)]}{r(1-\tau)(Aar - B_1)}$ exists, there is $q_{b_2}^{b_2} \geq q_{L}^{b_2}$. However, it is not true because $g$ is non-negative. Therefore, there is still $q_{b_2}^{b_2} < q_{L}^{b_2}$.

(4) If $\Delta Q_{E}^{b_2} - \Delta Q_{L}^{b_2} \geq 0$ is true, there is:
$\{2a\varphi^2k_1[(C - B)\Delta_1 - \Delta_2(C - Aar - B)] - 2gr\varphi^2(1 - \tau)(C - Aar - B)\} \beta_2 \geq \Delta_2(C - B_2)(C - Aar - B) - \Delta_1(C - B)(C - Aar - B_2) + 2gr\varphi^2(1 - \tau)(C - Aar - B)\} \beta_2 \geq \Delta_2(C - B_2)(C - Aar - B) - \Delta_1(C - B)(C - Aar - B_2) + 2gr\varphi^2(1 - \tau)\}$
\[\{(C - B)(C - Aar - B)\}[bk_2 - \varphi^2(1 - \alpha)]\].

Therefore, when
$\frac{C - B}{C - Aar - B} \Delta_1 - \frac{gr(1 - \tau)}{k_1}$ exists, there is
$\beta_2 \geq \frac{\Delta_1(C - B_2)(C - Aar - B) - \Delta_2(C - B)(C - Aar - B_2) + 2gr\varphi^2(1 - \tau)(C - Aar - B)[bk_2 - \varphi^2(1 - \alpha)]}{2a\varphi^2k_1(C - B_2)(C - Aar - B) - 2gr\varphi^2(1 - \tau)(C - Aar - B)} = \beta_{b_2}'''$. If
$\Delta_2 > \frac{C - B(C - Aar - B_2)}{(C - B_2)(C - Aar - B)} \Delta_1 - \frac{2gr\varphi^2(1 - \tau)[bk_2 - \varphi^2(1 - \alpha)]}{C_2}$, then the numerator is always positive, otherwise, $\beta_{b_2}'''$ is negative and there is
$\frac{C - B}{C - Aar - B} \Delta_1 - \frac{gr(1 - \tau)}{k_1} > \frac{(C - B)(C - Aar - B_2)}{(C - B_2)(C - Aar - B)} \Delta_1 - \frac{2gr\varphi^2(1 - \tau)[bk_2 - \varphi^2(1 - \alpha)]}{C_2}$. Therefore, when
$0 < \Delta_2 \leq \frac{(C - B)(C - Aar - B_2)}{(C - B_2)(C - Aar - B)} \Delta_1 - \frac{2gr\varphi^2(1 - \tau)[bk_2 - \varphi^2(1 - \alpha)]}{C_2}$ is true, $\beta_{b_2}'''$ is negative.
and there is always \( \Delta Q^*_E \geq \Delta Q^*_L \); while when
\[
\frac{(C-B)2gr(1-\tau|\Delta_2|_{\alpha\theta})}{C-B} \leq \frac{(C-B)\alpha\theta}{C-B} \Delta_1 - \frac{\Delta_1^*}{\alpha\theta} < \frac{(C-B)\alpha\theta}{C-B} \Delta_1 - \frac{\Delta_1^*}{\alpha\theta} \text{ exists, there is}
\Delta Q^*_E \geq \Delta Q^*_L \text{ if } \beta_2 \geq \beta_2^* \text{ and } \Delta Q^*_E < \Delta Q^*_L \text{ if } \beta_2 < \beta_2^*. \]
Finally, when
\[
\Delta_2 > \frac{(C-B)\alpha\theta}{C-B} \Delta_1 - \frac{\Delta_1^*}{\alpha\theta} \text{ is true, } \beta_2^* \text{ is negative, and there exists } \Delta Q^*_E < \Delta Q^*_L.
\]

Also, we summarize the thresholds in extension 5.2 to exhibit the comparative results clearly. We have
\[
\varepsilon^b_{v1} = \frac{\Delta_1 (C-Aar-B) + H_1(C-Aar-B)}{A^2 a\theta \delta_1 + A^2 (C-Aar-B)}
\]
\[
\varepsilon^b_{v2} = \frac{(C-Aar-B)\alpha\theta}{C-B} \Delta_1 + \frac{(C-Aar-B)2gr(1-\tau|k_2|_{\alpha\theta}}{C-B} \Delta_1 - \frac{\Delta_1^*}{\alpha\theta} \text{ exists, there is}
\Delta Q^*_E \geq \Delta Q^*_L \text{ if } \beta_2 \geq \beta_2^* \text{ and } \Delta Q^*_E < \Delta Q^*_L \text{ if } \beta_2 < \beta_2^*. \]
Finally, when
\[
\Delta_2 > \frac{(C-B)\alpha\theta}{C-B} \Delta_1 - \frac{\Delta_1^*}{\alpha\theta} \text{ is true, } \beta_2^* \text{ is negative, and there exists } \Delta Q^*_E < \Delta Q^*_L.
\]

Next, we will exhibit the comparative results in Theorem 7a-c.

**Theorem 7a** With the gift-giving function in live streaming platform, the relationships of decisions among three modes are listed below:

1. When \( \Delta_1 \geq \Delta_2 + \frac{H_1}{\alpha\theta} \), there is \( \varepsilon^b_{E} \geq \varepsilon^b_{T} \), when \( \Delta_1 \leq \Delta_2 + \frac{H_1}{\alpha\theta} \), there is
\[
\begin{align*}
\varepsilon^b_{E} &< \varepsilon^b_{T} \text{ if } 0 < \varepsilon < \varepsilon^b_{v1}, \\
\varepsilon^b_{E} &\geq \varepsilon^b_{T} \text{ if } \varepsilon^b_{v1} \leq \varepsilon < \alpha^*.
\end{align*}
\]

2. When \( \Delta_1 \geq \Delta_2 + \frac{gr(1-\tau)}{\alpha\theta} \), there is \( \varepsilon^b_{E} \geq \varepsilon^b_{T} \), when \( \Delta_1 \leq \Delta_2 + \frac{gr(1-\tau)}{\alpha\theta} \), there is
\[
\begin{align*}
\varepsilon^b_{E} &< \varepsilon^b_{T} \text{ if } 0 < \beta_2 < \beta_2^*, \text{ when } g > \frac{\alpha\theta\delta_1}{(C-Aar-B)\Delta_2}, \\
\varepsilon^b_{E} &\geq \varepsilon^b_{T} \text{ if } \beta_2 \leq \beta_2^*.
\end{align*}
\]

3. When \( \Delta_1 \geq \Delta_2 + \frac{gr(1-\tau)}{\alpha\theta} \), there is \( \varepsilon^b_{E} \geq \varepsilon^b_{T} \), when \( \Delta_1 \leq \Delta_2 + \frac{gr(1-\tau)}{\alpha\theta} \), there is
\[
\begin{align*}
\varepsilon^b_{E} &< \varepsilon^b_{T} \text{ if } 0 < \beta_2 < \beta_2^*, \text{ when } g > \frac{\alpha\theta\delta_1}{(C-Aar-B)\Delta_2}, \\
\varepsilon^b_{E} &\geq \varepsilon^b_{T} \text{ if } \beta_2 \leq \beta_2^*.
\end{align*}
\]
(3) When $\Delta_1 \geq \Delta_2 + \frac{gr(1-r)}{k_1}$, there is $q_E^{b_2} \geq q_T^{b_2}$, when $\Delta_1 < \Delta_2 + \frac{gr(1-r)}{k_1}$, there is

\[
\begin{cases}
q_E^{b_2} < q_T^{b_2}, & \text{if } 0 < \varepsilon \leq \varepsilon_{b_2}^{v}\varepsilon; \\
q_E^{b_2} \geq q_T^{b_2}, & \text{if } \varepsilon_{b_2}^{v} < \varepsilon \leq \alpha; \\
q_E^{b_2} \geq q_L^{b_2}, & \text{when } \Delta_1 < \frac{(C-Aar-B)(1-a-B)}{(C-Aar-B)(1-a-B)} \Delta_2 + \frac{gr(1-r)}{k_1}, \text{ there is}
\end{cases}
\]

Theorem 7b With the gift-giving function in live streaming platform, the relationships of NSV among three modes are listed below:

(1) When $\Delta_1 \geq \Delta_2 + \frac{2gr(1-r)[bk_2-a^2(1-a-B)]}{C-B}$, there is $\Delta Q_E^{b_2} \geq \Delta Q_T^{b_2}$, when $\Delta_1 < \Delta_2 + \frac{2gr(1-r)[bk_2-a^2(1-a-B)]}{C-B}$, there is

\[
\begin{cases}
\Delta Q_E^{b_2} < \Delta Q_T^{b_2}, & \text{if } 0 < \varepsilon < \varepsilon_{b_2}^v; \\
\Delta Q_E^{b_2} \geq \Delta Q_T^{b_2}, & \text{if } \varepsilon_{b_2}^v \leq \varepsilon \leq \alpha; \\
\end{cases}
\]

(2) When $\Delta_1 > \frac{(C-Aar-B)[C-B]}{(C-Aar-B)[C-B]} \Delta_2 + \frac{gr(1-r)}{k_1}$, there is $\Delta Q_E^{b_2} \geq \Delta Q_T^{b_2}$, when $\Delta_1 < \frac{(C-Aar-B)[C-B]}{(C-Aar-B)[C-B]} \Delta_2 + \frac{gr(1-r)}{k_1}$, there is

\[
\begin{cases}
\Delta Q_E^{b_2} < \Delta Q_L^{b_2}, & \text{if } 0 < \varepsilon < \varepsilon_{b_2}^v; \\
\Delta Q_E^{b_2} \geq \Delta Q_L^{b_2}, & \text{if } \varepsilon_{b_2}^v \leq \varepsilon \leq \alpha; \\
\end{cases}
\]

(3) When $0 < \varepsilon < \varepsilon_{b_2}^v$, there is $\Delta Q_T^{b_2} < \Delta Q_L^{b_2}$, when $\beta_2 > \beta_{b_2}^{v}$.

\[
\begin{cases}
\Delta Q_T^{b_2} \geq \Delta Q_L^{b_2}, & \text{if } 0 \leq \varepsilon \leq \varepsilon_{b_2}^v; \\
\Delta Q_T^{b_2} < \Delta Q_L^{b_2}, & \text{if } \varepsilon_{b_2}^v < \varepsilon \leq \alpha; \\
\end{cases}
\]

Theorem 7c With the gift-giving function in live streaming platform, the relationships of profits among three modes are listed below:

(1) When $\varepsilon \geq \varepsilon_{b}^{v}$, there is $\pi_{aT}^{b_2} \geq \pi_{aT}^{b_2}$, when $\varepsilon < \varepsilon_{b}^{v}$, there is $\pi_{aT}^{b_2} < \pi_{aT}^{b_2}$, when $\beta_2 \geq \beta_{b_2}^{v}$, there is $\pi_{aE}^{b_2} \geq \pi_{aE}^{b_2}$, when $\beta_2 < \beta_{b_2}^{v}$, there is $\pi_{aE}^{b_2} < \pi_{aE}^{b_2}$.

(2) When $\varepsilon \geq \varepsilon_{b_2}^{v}$, there is $\pi_{sE}^{b_2} \geq \pi_{sT}^{b_2}$, when $\varepsilon < \varepsilon_{b_2}^{v}$, there is $\pi_{sE}^{b_2} < \pi_{sT}^{b_2}$, when $\Delta_1 \geq \sqrt{\frac{h_2}{h_1} C-Aar-B \Delta_2 + \frac{gr(1-r)}{k_1}}$, there is $\pi_{sE}^{b_2} \geq \pi_{sL}^{b_2}$, when $\Delta_1 < \sqrt{\frac{h_2}{h_1} C-Aar-B \Delta_2 + \frac{gr(1-r)}{k_1}}$, there is $\pi_{sE}^{b_2} < \pi_{sL}^{b_2}$.
thresholds are changed compared with those in basic model, in which $\varepsilon_{v,1}^b > \varepsilon_{v,2}^b > \varepsilon_{v,3}^b > \varepsilon_{v,6}, \beta_{b_1}^v > \beta_{b_2}^v, \beta_{b_3}^v > \beta_{b_4}^v, \beta_{b_5}^v > \beta_{b_6}^v, \beta_{b_7}^v > \beta_{b_8}^v$ and $\beta_{b_9}^v > \beta_{b_{10}}^v$ are always true.

### Evidence of Observation 1

In order to obtain the influence of the streamer’s dual-purpose on the optimal decisions, NSV and profits of different modes in the basic model; we need to obtain the optimal results according to the backward induction method. The decision results obtained are as follows, while the expressions of profits and NSV are too complicated to exhibit, in which the superscript $c$ indicates extension $c$.

\[
\begin{align*}
\epsilon^c_{x} & = -b^2 \alpha^2 k^2 (1 - \theta)^2 \Delta_r + \beta^2 k \varepsilon^c (1 - \theta)^2 + 2b^2 k \varepsilon^c (1 - \theta) + \beta^2 \varepsilon^c (1 - \theta) - \beta^2 \varepsilon^c (1 - \theta - \beta_1) - \alpha_{i} \varepsilon^c (1 - \theta - \beta_1) + \beta^2 \varepsilon^c (1 - \theta - \beta_1), \\
\epsilon^c_{z} & = -b^2 k^2 (1 - \theta)(a - \epsilon)^2 \Delta_r + k \varepsilon^c (1 - \theta)(a - \epsilon)^2 + \beta^2 \varepsilon^c (1 - \theta)(a - \epsilon)^2 + \beta^2 \varepsilon^c (1 - \theta)(a - \epsilon)^2,
\end{align*}
\]

Due to the complexity of expressions, we compare the results with dual-purpose through numerical figures, in which we compare the decisions, NSV, profits and welfare respectively. See Fig. 7.
Evidence of Observation 2

Due to the complexity of expressions, Observation 2 is also obtained through numerical examples, in which we compare the decisions, NSV, profits and welfare respectively.

We compare the results with dual-purpose with these in basic model through numerical figures, in which each row from left to right respectively indicates the comparative results in basic model, with a certain $\mu$ and with a various $\mu$.

See Fig. 8
Fig. 8 The relationships of results in three modes with dual-purpose streamer $b = 2, a_1 = 3000, a_2 = 6000, t_1 = 0.4, t_2 = 0.2, r = 1, z = 150, \varphi = 15, \alpha = 0.5, \varepsilon = 0.2, \beta_1 = 0.1, \beta_2 = 0.03, \theta = 0.03, k_1 = 80, k_2 = 100$
Fig. 8 (continued)
Evidence of Observation 3

According to the backward induction method, we can obtain the optimal decisions of members with the decision sequential. The optimal decisions are listed below, with which we can obtain the optimal NSV and profits of members respectively.

(i) The supply chain’s profit

(j) The consumer surplus

(k) The social welfare
The expressions of optimal NSV and profits are complicated and therefore we omit them here.

**Specific expressions and thresholds**

**Specific expressions**

\[
\Delta_1 = a_1 t_1 - z, \quad \Delta_2 = a_2 t_2 - z, \quad \Delta_3 = a_2 t_2 - z.
\]

\[
\Delta = \left\{ C^4 - 2(3B + B_1) C^3 + \{4B_1^2 + 12BB_1 + 4[B + Ar(a - \varepsilon)]\} C^2 - 8B_1 (B + B_1) [B + Ar(a - \varepsilon)] C + 4B^2 [B + Ar(a - \varepsilon)^2]\right\} z^2 + 4(C - 2B_1) t_1 \left\{ -0.5C^2 + 2BC - B[B + Ar(a - \varepsilon)] \right\} C a_2 z + (C - 2B)(C - 2B_1) t_2^2 C^2 a_2^2.
\]

\[
\Delta_1'' = \frac{\sqrt{\Delta - 4Bz(C - B_1 - Aar)}^2 (C - B - Aar)}{2C(C - 2B)(C - B_1 - Aar)^2},
\]

\[
\Delta_2'' = \frac{(C - Aar - B_1) \left\{ 2ABz(a - \varepsilon) + 2Bz(B - C) + \sqrt{\Delta} \right\}}{(2B_1 - C)(C + Aar - A\varepsilon + B - C)},
\]

\[
A = (1 - \theta)k_2 r, \quad B = 2(1 - a - \beta_1) \varphi^2 k_1, \quad B_1 = 2(1 - a - \beta_2) \varphi^2 k_1, \quad B_2 = 2(1 - a) \varphi^2 k_1, \quad C = 2bk_2 k_2.
\]

\[
h_1 = (1 - a - \beta_1) [2bk_2 - 3(1 - a - \beta_1) \varphi^2], \quad h_2 = (1 - a - \beta_2) [2bk_2 - 3(1 - a - \beta_2) \varphi^2],
\]

\[
m = \varphi^2 k_1 (3 - a - \beta_1), \quad m_1 = \varphi^2 k_1 (3 - a - \beta_2), \quad m_2 = \varphi^2 (1 - a - \beta_1), \quad m_2 = \varphi^2 (1 - a - \beta_2),
\]

\[
H_1 = 2g(1 - r)[bk_2 - \varphi^2 (1 - a - \beta_1)], \quad H_2 = 2g(1 - r)[bk_2 - \varphi^2 (1 - a)].
\]

\[
T = (C - 2B)(C - B_1 - Aar)^2 - (C - 2B_1)[C - B - A(a - \varepsilon)r]^2.
\]
\[ D_{11} = \frac{(C - Aar - B)(C - Aar - B_1)}{C - B} - \frac{(C - Aar - B)(1 - a)}{C - Aar - B_2}, \quad D_{12} = \frac{(C - Aar - B)(1 - a)}{C - Aar - B_2} \]

\[ D_{13} = \frac{(C - Aar - B)(1 - a)}{C - Aar - B_2}, \quad D_{14} = \frac{(C - Aar - B)(C - B)}{C - Aar - B_2} \]

\[ D_{25} = \frac{\sqrt{k_2}}{h_1} \frac{C - Aar - B}{C - Aar - B_1}, \quad D_{26} = \frac{\sqrt{\frac{\beta_1}{\alpha \theta + \beta_1}}}{h_1} \frac{C - Aar - B}{C - B}, \quad D_{27} = \frac{\sqrt{\frac{\beta_1}{\alpha \theta + \beta_1}}}{h_1} \frac{C - Aar - B}{C - B} \]

\[ D_{28} = \frac{a \theta (C - Aar - B)^2 - (C - B_1)(C - Aar - B_1)(C - Aar - B_2)^2}{C - B} \]

\[ D_{29} = \frac{C - Aar - B}{C - B} \sqrt{\frac{C - A(1 - \theta)(a - \epsilon)^2 r - m}{C - A(1 - \theta)a^2 r - m}}, \quad D_{30} = \frac{C - Aar - B}{C - Aar - B_1} \sqrt{\frac{C - A(1 - \theta)a^2 r - m}{C - A(1 - a - \epsilon)r - B}^2} \]

\[ D_{31} = \frac{C - Aar - B_2 + 2\varphi^2 k_1 \beta_2}{C - Aar - B_2 + 2\varphi^2 k_2 \beta_1 + Aar} \sqrt{\frac{C - A(1 - \theta)(a - \epsilon)^2 r - m}{C - A(1 - \theta)a^2 r - m}} \]

\[ D_{32} = \frac{4z_1[(C - Aar)(B_1 + 1) + Aar \epsilon (C - B_1 - Aar)](C - B - A(a - \epsilon)v)}{T} \]

**Thresholds in basic model**

\[ \epsilon_{i1} = \frac{Aar(C - Aar - B)(1 - \Delta_1)}{\Delta_1} \quad \epsilon_{i2} = \frac{(C - Aar - B)(\Delta_1 - 1)}{\Delta_1} \quad \epsilon_{i3} = \frac{(C - B)(C - Aar - B_1) - (C - B_1)(C - Aar - B)}{Ar(C - B_1)} \quad \epsilon_{i4} = \frac{a(C - Aar - B)(\Delta_1^2 - 1)}{Aar \Delta_1^2 + (C - Aar - B)\Delta_1^2} \]

\[ \epsilon_{i5} = \frac{\sqrt{k_1(C - Aar - B_1)} - \sqrt{k_2(C - Aar - B)}\sqrt{\Delta_1}}{\sqrt{k_2 Ar}} \quad \epsilon_{i6} = \frac{(\theta \alpha + \beta_2)(C - B_1)(C - Aar - B_1)}{Ar \Delta_1^2} \quad \epsilon_{i7} = \frac{(\theta \alpha + \beta_2)(C - B_1)(C - Aar - B_1)}{(\theta \alpha + \beta_2)(C - Aar - B_1)}^2 \]

\[ \epsilon'' = \frac{(C - B - Aar)\Delta_1 \left\{ a_1 A ar(2B - C) + \left[ C^2 - 4BC + 2B(Aar + B) \right] \right\}}{Ar \left[ a_1 A ar(2B) + z(C - 2Aar) \right][z(Aar + B) + C(a_1 - 3z)] + 8\varphi^2 z_1^2(C - B - Aar)^2} \]

\[ \epsilon''' = \frac{C - B_1 - Aar}{Ar} \sqrt{\frac{(C - 2B)}{(C - 2B_1)} - \frac{C - B}{C - B_1}}, \quad \beta'' = \frac{\Delta_2(C - Aar - B) - \Delta_1(C - Aar - B_1)}{2\varphi^2 k_1 \Delta_1} \]

\[ \beta''' = \frac{\Delta_2(C - Aar - B) - \Delta_1(1 - \alpha - \beta)(C - Aar - B_2)}{B \Delta_1 + \Delta_2(C - Aar - B)} \]

\[ \beta'''' = \frac{\Delta_2(C - B_2)(C - Aar - B) - \Delta_1(C - B)(C - Aar - B_2)}{2\varphi^2 k_1 \left[ (C - B)\Delta_1 - \Delta_2(C - Aar - B) \right]} \]

\[ \beta''''' = \frac{(C - B_2)(C - Aar - B) - (C - B)(C - Aar - B_2)}{2\varphi^2 k_1 Aar} \]

\[ \beta'''''' = \frac{\Delta_2^2(C - Aar - B) - \Delta_1(C - Aar - B_2)}{2\varphi^2 k_1 \Delta_1^2} \]
Acknowledgements This work was supported by the National Natural Science Foundation of China (No.72071081), Guangdong Basic and Applied Basic Research Foundation (No.2019A1515010792).

Declarations

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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