Numerical study of single bubble mobility in triangular and deltoid microchannels using the boundary element method

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Abstract. The study of bubbly liquid dynamics in microchannels of unconventional shapes is of great importance for different fields of science and industry. This work investigates the dynamics of the incompressible single bubbles in the slow periodic flow of viscous liquid in a triangular channel with a variable pressure gradient. The numerical approach used in this research is based on the boundary element method (BEM). This method is widely used for solving three-dimensional problems and problems in areas with complex geometry. The influence of the bubble’s initial position relative to the channel centerline on the bubble deformation, the relative velocity of the bubble, and its center of mass displacement in the channel are considered.

1. Introduction

The research of bubble flow in microchannels is of interest due to their frequent application in microfluidics, microanalysis, oil and gas industry, and manufacturing of composite materials. The interest in the study of the multiphase flow in porous media is due to the peculiarities of the size and geometry of the channel structure, which has a significant effect on the occurring processes. At present, various micro-models for representing a porous medium are being considered. The most frequently used models are simple capillary models with a porous medium viewed as a network of microchannels of complex shapes.

The study of the fluid flow features in such channels of complex geometry is important, both from the point of view of practical application and for deriving the theoretical principles of such processes, since these effects can significantly influence the nature of the motion of bubbles in a flow. The formation of bubbles during impregnation of reinforcement with a viscous liquid during the production of composite materials significantly affects the mechanical properties of the final sample. Thus, the study of methods of influencing the mobility of bubbles in the space between filaments is an important task.

In physical and mathematical modeling of the structure of such systems, the main task is to choose the most adequate representation of the pore space and develop its mathematical description, which takes into account their structural parameters as accurate as possible. Therefore, adequate numerical modeling of such processes based on effective methods and algorithms is extremely important.

At present, there are not many experimental works studying the features of bubbly fluid flow at the microlevel in microchannels of various shapes in great detail. Experimentally obtained characteristics may include bubble size and shape, bubble formation and growth rate, contact angle, liquid film thickness around bubbles, three-dimensional reconstruction of bubble shape, mixing
behavior, bubble velocity, and fluid velocity field. It is necessary to understand the relevant flow characteristics in the channels for the simulation of gas-liquid two-phase microfluidic devices. Simulation and analysis of microscale flow have to provide important insights into mechanisms and access to variables that cannot be directly measured in experiments.

In addition to taking into account the geometric features of the pore space, it is necessary to study the effect of external fields on multiphase flows. The application of pulsation and vibration is gaining more popularity due to higher efficiency [1] and easy implementation. In addition, pulsating flows, according to the data, show better mixing and improved mass and heat exchange [2, 3]. The study of pulsatile flows is particularly relevant at small scales because they are potentially usable in microdevices. Moreover, fundamental research examines the effects of pulsation on basic quantities, such as total pressure drop, lacking at the microscale. One of the methods for obtaining pulsating flows in microchannels is the application of a variable pressure gradient. In their papers [4], the authors have published analytical solutions of Navier-Stokes equations for time-dependent and variable flows for planar, circular, and annular flows.

Nowadays, it is extremely important to identify the features of the motion of bubbles at microscales in unconventional channel forms in a wide range of physical parameters. The main purpose of this work is the numerical study of the features of the effect of the channel wall curvature and the use of a pulsating flow on the mobility of single bubbles in microchannels.

2. Problem formulation and numerical approach

The dynamics of incompressible bubbles (index 2) in the slow periodic flow of viscous fluid (index 1) in a channel with variable pressure drop set at the channel’s inlet is considered. Since the slow flow is studied, the viscosity forces are much more significant than the inertia forces. It allows neglecting inertial terms in the calculations completely. All processes occur under isothermal conditions without taking into account the intermolecular Van der Waals forces. It is assumed that the dynamic viscosity can be neglected in contrast to the corresponding parameters of liquid. In this case, we consider models of an ideal gas and viscous liquid, the motion of which is described by the Stokes equations:

$$\nabla p + \mu \nabla^2 \mathbf{u} = 0, \quad \nabla \cdot \mathbf{u} = 0, \quad i = 1, 2$$

where $\mathbf{u}$ is the velocity, $p$ is the pressure, which includes a hydrostatic component, and $\mu$ is the dynamic viscosity.

The problem is solved under the following boundary conditions. For the flow in the channel, the no-slip condition on the sidewall is set. It is supplemented with periodicity conditions for the inlet and outlet sections of the channel. The velocities at the interphase boundary are equal, and the normal stress vector difference is set.

The bubble moves according to the kinematic condition

$$\frac{d\mathbf{x}}{dt} = \mathbf{u}(\mathbf{x}), \quad \mathbf{x} \in S,$$

where $\mathbf{u}(\mathbf{x})$ is the interface velocity, $\mathbf{x}$ is the radius-vector of the point, and $S$ is the interface.

For microfluidics, flows at low Reynolds numbers are typical; for them the Navier-Stokes equations can be simplified to the Stokes equations. The specificity of the Stokes equations is that they are linear and can be solved by the boundary element method (BEM). The solution of the presented problem is obtained using the three-dimensional BEM. This numerical approach is very effective in studying problems in areas with complex geometry since all calculations are related only to the boundary of considered particles and boundaries. In this paper, cases with small computational grids are considered and standard direct methods are used to solve the system of linear algebraic equations. A more detailed mathematical formulation of the problem and numerical implementation are described in [5].
3. Results and Discussion

To study the effect of curvature of the channel wall, in addition to the triangular channel cross-section, we considered a channel with a deltoid-shaped cross-section. We have chosen such a non-trivial case, since this shape corresponds to the space between densely packed cylindrical fibers in the reinforcing structure of composite materials. A qualitative triangulation of the channels with a deltoid and triangular cross-section (Fig. 1) are developed. The vertices of the deltoid and triangle coincide; the ratio of the channel length $L$ to the radius $R$ of the elements forming the porous system is the same for the channels and $L/R = 2$.

![Figure 1. Triangulation of the deltoid-shape ($N_\Delta = 10328, \ L/R = 2$) and triangular ($N_\Delta = 4620, \ L/R = 2$) cross-section channel.](image)

First of all, the steady flow of viscous fluid in the considered cross-sections channels at a given constant pressure drop $\Delta p$ over the channel fragment $L$ was studied. We calculated components of the velocity field and visualized the flow pattern along the channel fragment length (Fig. 2).

![Figure 2. Results of calculations of $U_x$ component of the velocity field in the $z = 0$ plane for two types of channel cross sections (deltoid-shape from the left and triangle from the right).](image)

The figure shows that the flow in this section is not symmetric and the maximum velocity is closer to the wide part of the channels. This distribution feature will influence the dynamics of dispersed inclusions in such channels.

Calculations were carried out to study the dynamics of incompressible bubbles in the channels under the action of the source of harmonic oscillations - the pressure drop that is set at the channel’s inlet. The effect of the initial position of the bubble relative to the channel centerline on the bubble deformation and the velocity of the center of mass relative to the average flow velocity in the channel...
was considered. The calculations were conducted at the following dimensional parameters: 
\[ \rho_1 = 1.1 \times 10^3 \text{ kg/m}^3, \; \gamma = 0.05 \text{ N/m}, \; \mu_1 = 0.3 \text{ Pa s}, \; \text{and} \; \mu_2 = 0 \text{ Pa s}, \] where \( \rho \) is the density, and \( \gamma \) is the surface tension. The radius of the individual fibers with the flow in between was \( R = 2.3 \times 10^{-3} \text{ m} \).

The pressure drop at pulsating flow in the channel was set as
\[ \Delta p = \Delta p + \sin(\omega \cdot t), \] with \( \omega = 2\pi \), and over the channel fragment \( L = 1.1 \times 10^{-1} \text{ m} \). All results are presented in dimensionless form and for dimensionless time \( t = t_{\text{dim}} = \gamma t_{\text{dim}} / (\mu a) \), where \( t_{\text{dim}} \) is the dimensional time, and \( a \) is the bubble radius that is equal to \( a = 0.07R \). The surface of the bubbles was discretized by a computational grid with \( N_\Delta = 1280 \) triangular elements. Initially, a spherical bubble was placed at an equal distance from the centerline of the channel \( \Delta h = 0.1R \) in three directions from the center in the \( yz \) plane. Thus, the initial coordinates of the mass center are \( y = -0.0481R; \; z = 0.0833R; \; y = -0.0481R, \; z = -0.0833R; \) and \( y = 0.1R, \; z = 0 \).

Fig. 3 shows the results of the test calculations.

**Figure 3.** Initial position of the bubble (a) of radius \( a = 0.07R \) with mass center at points (from top to bottom) \( y = -0.0481R; \; z = 0.0833R; \; y = -0.0481R, \; z = -0.0833R; \; y = 0.1R, \; z = 0 \), and its dynamics at various time from left to the right: \( t = 0, \; t = 2, \; t = 4 \) in deltoid (b) and triangular (c) channels.
One can see that the bubbles are deformed significantly in the deltoid-shape channel. These deformations are consistent with the curvature of the channel side wall due to the differences in cross-section shape and the asymmetric distribution of the gradient along the channel length (Fig.2). The bubbles in the triangular channel have moved further in the same time pass than bubbles in the deltoid channel.

To estimate the bubble mobility the velocity of center of mass of each bubble is calculated. Fig. 4 shows changes in time of the velocity of bubbles relative to the average channel velocity. As seen from the graphs, the bubble velocity increases (by 14% in deltoid channel, and by 10% in triangular channel) on average in all cases considering time intervals.

![Figure 4](image1.png)

**Figure 4.** The relative velocity of the mass center of bubbles.

The change in time of the distance of the center of mass of the bubble to the axis of the channel is considered (Fig. 5). One can see that there is a shift of the bubbles mass center toward the center of the channel (by 5% in deltoid channel, and by 3% in triangular channel on average for all cases) where the flow velocity is higher. Therefore, the relative velocities of these bubbles increase (Fig. 4).

![Figure 5](image2.png)

**Figure 5.** Variation of the distance of the mass center of bubbles to the channel centerline.
Figure 6. Variation of the position and relative velocity of the mass center of bubble.

Fig.6 shows changes in the relative velocity and position of the center of mass of one of the bubbles for constant and pulsating pressure gradients. It is seen, that the bubble in the deltoid channel with a pulsating flow has a higher velocity than with a flow with a constant pressure drop, although the displacement relative to this period is the same.

Conclusions
The features of the single incompressible bubble dynamics in microchannels with a cross-section in the form of deltoid and triangle under a variable pressure drop have been investigated. The features of the velocity field distribution in different sections inside the channels have been studied. It is shown that the flow along the channel axis is not symmetric, and this influences the dynamics of dispersed inclusions in such channels. The curvature of channel sidewalls and the initial position of the bubble relative to the channel centerline affect the bubble shape and velocity. The bubbles deform more significantly and move slowly in a deltoid-shaped channel due to the less distance to the wall. The relative velocities of the bubbles in the triangular channel are slightly higher than those in the deltoid channel because of the curved channel sidewalls. The bubbles in the deltoid channel are located closer to the walls where the fluid flow velocity is minimal. However, over time the bubbles in the channels move towards the center of the channel where the velocity is maximum, so their velocity increases. The observed displacement of the bubbles from the channel wall toward the center is due to the change of the velocity field gradient on the considered bubble sizes. For three cases of the location of the bubbles, the time variation of their relative velocity and distance from the channel axis has been considered under the impact of variable pressure drop. The graphs demonstrate that a bubble in a deltoid channel with a pulsating flow has a higher velocity but the same displacement relative to the axis than in a flow with a constant pressure drop. Thus, using a pulsating flow in the considered regime provides for an insignificant increase in the particle velocity. The revealed patterns in the characteristics of the dynamics of dispersed particles prove the need for more careful consideration of the features of the wall geometry of microchannels in porous materials.

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