Deterministic source of a train of indistinguishable single-photon pulses with single-atom-cavity system

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We present a mechanism to produce indistinguishable single-photon pulses on demand from an optical cavity. The sequences of two laser pulses generate, at the two Raman transitions of a four-level atom, the same cavity-mode photons without repumping of the atom between photon generations. Photons are emitted from the cavity with near-unit efficiency in well-defined temporal modes of identical shapes controlled by the laser fields. The second order correlation function reveals the single-photon nature of the proposed source. A realistic setup for the experimental implementation is presented.

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Deterministic sources of high-quality single-photon (SP) states are of great importance for quantum information processing [1]. A basic requirement for many quantum optics applications, including quantum computing with linear optics [2, 3], quantum cryptography [4] and entanglement swapping [5], is to have single photon pulses with well-defined identical shapes, frequency and polarization, as these schemes based on photon-interference effects are very sensitive to the parameters of SP pulses and their repetition rate. A good source has to ensure a pure SP state without mixture from both the multi-photon and zero-photon states, as well as to prevent the entanglement between the photons which degrade the purity of the SP state. Since the individual photons are usually emitted during the spontaneous decay of atomic systems, the SP sources must be immune to the environmental effects that induce the dephasing of atomic transitions. Most of the schemes proposed earlier to produce single photons on demand from solid state single emitters [6], organic molecules [7, 8], and quantum dots [9, 10] are confronted with this difficulty. Besides, they do not offer a high efficiency because of the isotropic nature of fluorescence that prevents to collect the photons, not to mention the spectral dephasing and inhomogeneity of solid-state emitters. Deterministic sources of single photons are realized also in cold atomic ensembles with feedback circuit [11, 12]. But these schemes are not suitable to generate SP train with an arbitrary repetition rate because of strong temporary bounds caused by the feedback and write-read processes.

At present, all the requirements mentioned above can be achieved together with a Δ-type atom trapped in high-finesse optical cavities [13, 14], where the single photons are generated via vacuum-stimulated Raman scattering of a classical laser field into a cavity mode. These systems not only provide a strong interaction between a photon and an atom, but also support very high collection efficiency due to the fact that the photons leave the cavity through one mirror with a transmissivity incomparably larger than that of the opposite one. By carefully adjusting the parameters of the laser pulse one can also easily control the waveform of output single photons. However, the main disadvantage of these schemes is the necessity to use a repumping field to transfer the population of the atom to its initial state after the generation of a cavity photon and only then to generate the next one. In this Letter we propose a scheme featuring a double Raman atomic configuration, which is able to deterministically generate a stream of identical SP pulses without using the repumping field, while maintaining the high generation efficiency, as well as providing simpler control of the output photon waveforms. It is interesting to note that a repumping field is not required also in a similar scheme for generating a sequence of single photons of alternating polarization [17].

Our scheme, illustrated in Fig. 1, involves a four-level atom trapped in a one-mode high-finesse optical cavity. The two ground states 1 and 2 and two upper states 3 and 4 of the atom are Zeeman sublevels (Fig. 1b), which are split by a magnetic field acting perpendicular to the cavity axis. The atom is initially prepared in one of the ground states, for instance, in state 1 of magnetic quantum number \( m = 0 \), and interacts in turns with a sequence of two pumping fields as shown in Fig. 1a. At first, a coherent \( \sigma^+ \) polarized field of Rabi frequency \( \Omega_1 \) applied between ground state 1 and excited state 4 transfers the atom to ground state 2 while creating a cavity-mode Stokes-photon. Then, after a programmable delay time \( \tau_d \), the \( \sigma^- \) polarized pump pulse \( \Omega_2 \) generates the anti-Stokes photon at the \( 3 \rightarrow 1 \) transition and transfers the atom back to ground state 1. The Stokes and anti-Stokes photons have identical frequencies, so that the cavity is coherently coupled to the atom on both transitions \( 4 \rightarrow 2 \) and \( 3 \rightarrow 1 \) with the rates \( g_1 \) and \( g_2 \), respec-
tively, resulting in the generation of linearly polarized cavity-photons in both cases. The laser fields are tuned to the two-photon resonance, while the one-photon detunings are very large compared to the Rabi frequencies and the cavity damping rate \( k \): \( \Delta_i \gg g_i, k, \Omega_i, i = 1, 2 \).

This condition makes the system robust against the spontaneous loss from upper levels and dephasing effects induced by other excited states. More importantly, in the off-resonant case the Raman process with effective atom-photon coupling \( G_i = g_i \Omega_i / \Delta_i, i = 1, 2 \), can be made much slower than the cavity field decay: \( G_i \ll k \). This ensures that a generated photon leaves the cavity long before the next cavity-photon is emitted and, hence, no entanglement between the photons will be created, if we also take into account that the coherence between atomic ground states is always zero. Therefore, we can construct identical wavepackets for outgoing photons independently from each other, as they are entirely determined by the temporal shape of the corresponding pump pulse. A remarkable feature of our scheme is that, despite the smallness of pulse. A remarkable feature of our scheme is that, determined by the temporal shape of the corresponding pump

We first solve the equations for the number of cavity photons and for their flux giving the SP detection probability. We carry out numerical calculations for realistic atomic systems in two cases. The first one, shown in Fig. 1, is \(^{85}\text{Rb}\) atom with the ground states \( 5S_{1/2}(F = 2, m_F = 0), 5S_{1/2}(F = 2, m_F = +1) \) and excited states \( 5P_{1/2}(F' = 1, m_{F'} = 0), 5P_{1/2}(F' = 1, m_{F'} = +1) \) as state 1.2 and 3.4 in our scheme, respectively. The central drawback of this scheme is that the spontaneous decay of state \( 5P_{1/2}(F' = 1, m_{F'} = +1) \) into the ground state \( 5S_{1/2}(F = 1) \) (level not shown) constitutes a loss channel that moves the system outside the considered level configuration. However, we show that even in this case the atom can generate about 80 identical SP pulses before falling into the ground \( F = 1 \) state. To restore the generation a repumping field must be applied to transfer the atom into the initial state. For continuous generation of SP, we consider a second case employing cycling transitions of the \( D_2 \) line in the \(^{85}\text{Rb}\) atom with \( 5S_{1/2}(F = 2) \) and \( 5P_{3/2}(F' = 3) \) as the ground and excited states. The main limitation in this case is the atom lifetime in cavities, which amounts to at most one minute \(^{10}\). In both cases, to analyze the dynamics of a coupled atom-cavity system under realistic conditions, we need to determine how other Zeeman sublevels, not shown in Fig. 1, impact the generation process. It is easy to see, however, that it does not matter what Zeeman sublevel of the ground state is initially populated. Indeed, in any case the atom emits cavity photons many times before jumping into a new pair of ground Zeeman sublevels via spontaneous decay of upper states; the generation of cavity photon from this new pair of Zeeman sublevels occurs in the above described manner, provided that \( \Delta_i \gg \Delta_B \), where \( \Delta_B = g_L \mu_B B \) is Zeeman splitting of atomic levels in the magnetic field \( B \) with \( g_L \) the Landé factor and \( \mu_B \) Bohr magneton.

The laser fields propagating perpendicular to the cavity axis are given by

\[
E_j(t) = E_j f_{j/2}^{1/2}(t) \exp(-i \omega_j t), \quad j = 1, 2.
\]  

where \( f_1(t) = \sum_{i=1}^{N} f_i(t) \) and \( f_2(t) = \sum_{i=1}^{N} f_i(t - \tau_d) \) represent the sum of \( N \) well-separated temporal modes with profiles \( f_i(t) \) and \( f_i(t - \tau_d) \) for the \( l \)th mode in the pump series 1 and 2, respectively. \( E_j \) is the peak amplitude of the field \( j \).

In far off-resonant case, we can adiabatically eliminate the upper atomic states 3 and 4 and write the effective Hamiltonian as

\[
H = \hbar (G_1 f_1^{1/2}(t) \sigma_{21} + G_2 f_2^{1/2}(t) \sigma_{12}) a^\dagger + h.c.
\]  

with \( \sigma_{ij} \) and \( a(a^\dagger) \) the atomic and cavity mode operators, respectively. The peak Rabi frequencies of the laser fields are given by \( \Omega_1 = \mu_{41} E_1 / \hbar, \Omega_2 = \mu_{32} E_2 / \hbar \) with \( \mu_{ij} \) the dipole matrix element of the \( i \rightarrow j \) transition.

For \( \Omega_i \) and \( g_i \) of the same order, the Stark shifts of the atomic ground levels, of the form \( \Omega_i^2 f_i(t) / \Delta_i \), and \( g_i^2 / \Delta_i \), are negligibly small with respect to the cavity linewidth \( k \) in the bad cavity limit \( G_i \ll k \) as considered here.

The system evolution is described by the master equation for the whole density matrix \( \rho \) for the atom and cavity mode \(^{18}\)

\[
\frac{d\rho}{dt} = -i \frac{\hbar}{\mathcal{E}} [H, \rho] + \frac{d\rho}{dt} \text{rel}
\]  

where the second term in the right hand side (rhs) accounts for all relaxations in the system. With the use of the Lindblad operator \( L[\hat{O}] \rho = \hat{O} \rho \hat{O}^\dagger - \rho \hat{O}^\dagger \hat{O} + \Gamma_1(t) L[\sigma_{21}] \rho \) it is written in the form

\[
\frac{d\rho}{dt} \text{rel} = k L[a] \rho + \Gamma_1(t) L[\sigma_{21}] \rho
\]
The first term in the rhs of this equation represents the cavity output coupling, while the second and third terms describe the optical pumping to ground states 2 and 1 from the states 1 and 2, respectively, and give rise to noise of corresponding rates

\[ \Gamma_1(t) = \frac{\Omega^2}{\Delta} f_1(t) \gamma_{42}, \quad \Gamma_2(t) = \frac{\Omega^2}{\Delta} f_2(t) \gamma_{31} \]  

The last term of Eq.(4) introduces the losses of atomic population due to the decay of upper atomic states 3 and 4 into states outside of the system with rates \( \gamma_{3,\text{out}} \) and \( \gamma_{4,\text{out}} \), respectively:

\begin{align*}
\Gamma_{1,\text{out}}(t) &= \frac{\Omega^2}{\Delta} f_1(t) \gamma_{4,\text{out}} = \Gamma_{1,\text{out}} f_1(t), \quad (6a) \\
\Gamma_{2,\text{out}}(t) &= \frac{\Omega^2}{\Delta} f_2(t) \gamma_{3,\text{out}} = \Gamma_{2,\text{out}} f_2(t). \quad (6b)
\end{align*}

Spontaneous emission channels corresponding to (i) the cycling transition returning the atom back to the starting state and to (ii) the atomic transfer to other ground sublevels with \( \Delta m_F = \pm 2 \), that lead to emitted photons not in the cavity mode, should be in principle included. However, in contrast to the decays of Eqs. (5) and (6), these channels neither change the population of the system, nor lead to any noises, therefore, they can be ignored.

Our aim is to find the flux of the output photons

\[ \frac{dn_{\text{out}}(t)}{dt} = \langle a_{\text{out}}^\dagger(t) a_{\text{out}}(t) \rangle \]  

which describes the sequence of outgoing SP wavepackets. Here \( n_{\text{out}}(t) \) is the mean photon number of the output field from the cavity, \( a_{\text{out}}(t) \), which is connected with the input \( a_{\text{in}}(t) \) and intracavity \( a(t) \) fields by the input-output formulation \[18\]

\[ a_{\text{out}}(t) + a_{\text{in}}(t) = \sqrt{k} a(t) \]  

and satisfies the commutation relation \[ a_{\text{out}}(t), a_{\text{out}}^\dagger(t') \rangle = \delta(t - t'). \] With the Hamiltonian (1), the Heisenberg-Langevin equation for \( a(t) \) is given by \[18\]

\[ \dot{a} = -i G_1 f_1^{1/2}(t) \sigma_{21} - i G_2 f_2^{1/2}(t) \sigma_{12} - (k/2) a + \sqrt{k} a_{\text{in}}(t) \]  

In the bad-cavity limit \( k \gg G_{1,2} \), we adiabatically eliminate the cavity mode \( a(t) \) yielding

\[ a = -\frac{2i}{k} [G_1 f_1^{1/2}(t) \sigma_{21} + G_2 f_2^{1/2}(t) \sigma_{12}] + \frac{2}{\sqrt{k}} a_{\text{in}}(t) \] 

Upon substituting this solution into the Hamiltonian, for the case of a vacuum input \( \langle a_{\text{in}}^\dagger(t) a_{\text{in}}(t) \rangle = 0 \), we eventually obtain from Eq.(3) the following equations for the atomic variables \( i, j = 1, 2; j \neq i \)

\begin{align*}
\langle \dot{\sigma}_{ij}(t) \rangle &= -[\alpha_i(t) + \Gamma_i(t) + \Gamma_{\text{out}}(t)] \langle \sigma_{ij}(t) \rangle \\
&+ [\alpha_j(t) + \Gamma_j(t)] \langle \sigma_{jj}(t) \rangle \\
\langle \dot{\sigma}_{21}(t) \rangle &= -\sum_{i=1}^2 [\alpha_i(t) + \Gamma_i(t)] \langle \sigma_{21}(t) \rangle \\
\end{align*}

which are subjected to initial conditions \( \langle \sigma_{11}(0) \rangle = 1, \langle \sigma_{22}(0) \rangle = 0 \). Here \( \alpha_i(t) = 4G_i^2 f_i(t)/k = \alpha_i f_i(t), i = 1, 2. \)

Thus, in the bad-cavity limit the problem is reduced to the solution of the dynamical equations for the atom. Equations (11) are easily solved analytically. However, the final solutions are lengthy and will be given here only graphically. We first discuss some properties of Eqs. (11). It is seen that state 1 (and similarly state 2) is populated in two ways: (i) via cavity photon generation with the rate \( \alpha_1(2) \) and (ii) by optical pumping of rate \( \Gamma_1(2) \), that gives for the signal-to-noise ratio \( R_{sn} = 4\gamma_1^2 f_1(t)/(k \gamma_{42}(31)) \), which must be quite large: \( R_{sn} \gg 1 \). The second observation is that the overall population of the atom after a total of \( n \) pulses of two laser sequences for \( \alpha_i T \gg 1 \) decreases as

\[ \langle \sigma_{11}(t) + \sigma_{22}(t) \rangle = (1 - n \Gamma_{\text{out}}/\alpha) \]  

Here it is assumed that \( \Gamma_{\text{out}} = \Gamma_{\text{out}} = \Gamma_{\text{out}}, \alpha_1 = \alpha_2 = \alpha \). Thus, the population leakage is negligibly small until \( n \alpha_i/\Gamma_{\text{out}} < 1 \). Further, the ground state coherence is always zero: \( \sigma_{21}(t) = 0 \), as expected due to the spontaneous nature of Raman transitions. Finally, from the explicit expression of the flux calculated from Eq.(7) for one pump pulse and considering \( \Gamma_{\text{out}} = 0 \)

\[ \frac{dn_{\text{out}}(t)}{dt} = \alpha_1(t) e^{-\int_{-\infty}^t \alpha_i(t') dt'} \]  

\[ \text{FIG. 2: (Color online) Flux (a), total number of output field photons (b) and population of atomic ground states 1 (solid) and 2 (dashed)(c) as a function of time (in units of inverse cavity decay } k^{-1}) \text{, in the case, when 4 pulses of each laser beam are applied with delay time } \tau_d = 3 \mu s. \text{ For the rest of parameters see the text.} \]
we conclude that the waveform of the emitted single-photon is simply related to the shape of the pump pulse and, thereby, is much easier controlled as compared to schemes proposed so far in literature. From this equation one finds \( n_{out}(t) = 1 - \exp\left(- \int_{-\infty}^{t} \alpha_1(t')dt'\right) \) showing that our system is able to produce photons with near-unit efficiency, if \( \alpha_1(t)T \gg 1 \).

In Fig. 2 the calculated flux and total number of output photons, as well as the populations of atomic ground states are shown for the case, when each laser sequence contains four Gaussian-shaped subpulses with duration \( T = 1 \mu s \). We present the results of cavity photon generation on the D1 line transition \( 5S_{1/2}(F = 2) \rightarrow 5P_{3/2}(F' = 1) \) in the \(^{85}\text{Rb}\) atom obtained with the following parameters: \( \Omega_{1,2} = 2\pi \times 10 \text{MHz} = 0.1\Delta(\Delta_1 \simeq \Delta_2 = \Delta) \), and \( (g_{1,2},k,\gamma_{sp})/2\pi = (10,3,6) \text{MHz} \), \( \gamma_{sp} \) being the total spontaneous decay rate of the upper states. A magnetic field of 20G produces the Zeeman splitting \( \Delta B/2\pi = 14 \text{MHz} \). These parameters are within experimental reach and ensure the fulfillment of all necessary conditions indicated above. The results demonstrate two important features of the scheme. The photons are generated deterministically at the leading edge of each pump pulse with identical duration \( T_{cav} \sim T/2 \) and time-symmetric wavepackets. The efficiency of one photon generation by each pump pulse is close to 100% (see Fig. 2b). As one can see in Fig. 2(a,c), the peak values of the generated SP pulses and of the atomic populations display a slight decrease in time caused by population losses through the channel \( 5P_{3/2}(F' = 1) \rightarrow 5S_{1/2}(F = 1) \). Just because of this fact the total number of emitted photons does not reach its maximum value 8 (Fig. 2b, solid line). Nevertheless, from Eq. (12) it follows that about \( n_{out} \sim \alpha_1/\Gamma_{out} \approx R_{sn} \approx 70 \) cavity photons are generated before the losses become significant. For comparison, \( n_{out} \) is shown also for the lossless case of D2 line cycling transition \( 5S_{1/2}(F = 2) \rightarrow 5P_{3/2}(F' = 3) \) (Fig. 2b, dashed line).

The probability of a joint detection of two photons produced in the train is given by the intensity correlation function

\[
G^{(2)}(t,\tau) = \langle a_{out}^\dagger(t)a_{out}^\dagger(t+\tau)a_{out}(t+\tau)a_{out}(t)\rangle \tag{14}
\]

where \( \tau \) is the time delay between the two photon detections. By applying the quantum regression theorem \( \text{[15,16]} \) and using the input-output relation Eq.(7), the second-order temporal correlation function \( G^{(2)}(t,\tau) \) is reduced to

\[
G^{(2)}(t,\tau) = k[\alpha_1(t+\tau)Z_1(t,\tau) + \alpha_2(t+\tau)Z_2(t,\tau)] \tag{15}
\]

where \( Z_i(t,\tau) = \langle a^\dagger(t)s_{ii}(t+\tau)a(t)\rangle, i = 1,2 \), as a function of \( \tau \), obey equations similar to Eqs.(10,11) with initial values \( Z_1(t,0) = \alpha_2(t)s_{22}(t))/k \) and \( Z_2(t,0) = \alpha_1(t)s_{11}(t))/k \). Since we are interested in the total probability of a joint detection as a function of the time delay \( \tau \), we have to integrate Eq.(16) over \( t \). The results of numerical calculations for \( G^{(2)}(\tau) = \int_{-\infty}^{\infty} G^{(2)}(t,\tau)dt \) are shown in Fig. 3. The temporal structure of \( G^{(2)}(\tau) \) reveals the characteristics of a pulsed source of light: the absence of a peak at delay time \( \tau = 0 \) is evidence of the single-photon nature of the source, and the individual peaks are separated by the pump pulses’ delay. The decrease in the peak amplitude of the probability of joint detection for increasing delay time results from having a finite train of emitted photons.

In conclusion, we have proposed a robust and realistic source of indistinguishable single-photons with identical frequency and polarization generated on demand in a well-defined spatio-temporal mode from a coupled double-Raman atom-cavity system. The high efficiency and simplicity of the scheme, free from such complications as repumping process and environmental dephasing, makes the generation of many SP identical pulses feasible. The removal of the repumping field is a principal task, because its usage strongly restricts the process: the repetition rate of emitting photons is limited by the acting time of the repumping field. Unlike this, our mechanism allows to freely change the repetition rate of the single-photon pulses up to zero, because even in this case non-entangled single-photons are generated. One of the most important applications of this property is to generate Fock states with a programmable number of photons. Moreover, in the good-cavity limit our scheme can serve as an one-atom laser with a controllable statistics of generated photons that provides the quantitative study of the quantum-to-classical transition in our system raised with gradual change of the parameters. These questions will be addressed in the future publications.

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![FIG. 3: Intensity correlation integrated over the single-photon train as a function of time delay \( \tau \) between the two photon detections. The parameters are the same as in Fig. 2](image-url)
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