Quantum fluctuations of space-time

Michael Maziashvili

Department of Physics, Tbilisi State University, 3 Chavchavadze Ave., Tbilisi 0128, Georgia
Institute of High Energy Physics and Informatization, 9 University Str., Tbilisi 0186, Georgia

Using a gedanken experiment providing presumably a minimal inaccuracy, the uncertainty contributions to the space-time measurement are precisely evaluated for clock and mirror respectively. The resulting expression of minimal uncertainty for the space(time) interval indicates the presence of minimal Planck scale observable length(time). The synthesis of quantum mechanics and general relativity predicts the UV and IR scales for Lorentz invariance violation. The influence of background radiation on the space-time measurement is estimated. Based on the minimal length uncertainty relation which takes into account the wavelength of a quantum used for distance measurement we evaluate the cumulative factor responsible for the magnification of the space-time fluctuation induced phase incoherence of a light propagating over a large distance. We notice that in view of the interferometric observations the quantum fluctuations of space-time in the braneworld model are enormously increased if the fundamental scale is taken much below the Planck one. Present approach to the uncertainty in distance measurement leads to new insight about the bounds of computation. The impact of the space-time fluctuations on the black hole physics is briefly emphasized.

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The possible detection of the space-time quantum fluctuations through the cumulative uncertainty in phase and wave vector direction of the light propagating over a large distance was elaborated by several authors in the last few years [1,2]. The smallness of the Planck length, \(l_p \sim 10^{-33}\) cm, characterizing the space-time foam physics takes the effect of space-time quantum fluctuations out of the range of our ordinary experiences, but the space-time-foam effects can cumulatively lead to a complete loss of phase information if the emitted radiation propagated a sufficiently large distance. Since the phase coherence of light from a distant point source is a necessary condition for the presence of diffraction patterns when the source is viewed through a telescope, such observations offer sensitive and uncontroversial test for the scenarios describing the quantum fluctuations of space-time. A question of paramount importance is to define the correct cumulative factor. In papers [3,4] it is argued that the magnification effect of the Planck scale fluctuations during the light traveling over a long distance is overestimated in [1,2] because of improper definition of the cumulative factor. First let us try to clarify this point.

That the space-time undergoes quantum fluctuations can be simply demonstrated by analyzing a gedanken experiment for space-time measurement [3]. Due to importance of a numerical factor characterizing amplitude of the space-time fluctuations we shall try to be as precise as possible in analyzing of gedanken experiment. Our measuring device is composed of a spherical clock localized in the region \(\delta x = 2r_c\) and having a mass \(m_c\), which also serves as a light emitter and receiver, and a spherical mirror of radius \(l/2\) and mass \(m\) surrounding the clock, \((r_c\) denotes the radius of the clock\), Fig.1. We are measuring a distance \(l\) by sending a light signal to the mirror under assumption that at the moment of light emission the centers of clock and mirror coincide. However, quantum uncertainties in the positions of the clock and mirror introduce an inaccuracy \(\delta l\) during the measurement. (Throughout this paper we set \(\hbar = c = k_B = 1\).)

Assuming the minimal uncertainty the spread in velocity of the clock can be found as \(\delta v_c = \delta p_c/m_c = 1/2m_c\delta x\) and analogously for the mirror as \(\delta v_m = 1/2(m + m_c)l\). After the time \(t = l\) elapsed by light to travel along the closed path clock-mirror-clock the total uncertainty during the measurement takes the form

\[
\delta l = \delta x + \frac{t}{2m_c\delta x} + \frac{t}{2(m + m_c)l}
\]

which after minimization with respect to \(\delta x\) gives

\[
\delta x = \sqrt{\frac{l}{2m_c}}, \quad \Rightarrow \quad \delta l_{\text{min}} = \sqrt{\frac{2l}{m_c} + \frac{1}{2(m + m_c)}}. \quad (1)
\]
This uncertainty diminishes with increasing masses of the mirror and clock. But the masses are limited by the requirement the device not to collapse into the black hole. In other words the size of the clock(mirror) should be greater than twice its gravitational radius. Following this discussion and using the expression of gravitational radius

\[ r_p = (Al_p^{2+n} m)^{(1-n)/(2+n)}, \quad A = \frac{8\Gamma^{(2+n)}}{(2+n)\pi^{(3+n)/2}}, \]

where \( n \) denotes the number of extra dimensions, one simply gets

\[ \delta l_{\text{min}} = 2^{\frac{3+2n}{2+n}} A^{\frac{1}{2+n}} (l_p^{2+n})^{\frac{1}{2+n}} + 2^n A l_p^{2+n} l^{-(1+n)}. \]  

(2)

From this equation it follows immediately that there exists the minimal observable length of the order of \( \sim l_p \). From Eq.\( (2) \) one readily gets the minimal uncertainty in time measurement, \( \delta t_{\text{min}} \), by replacing \( l_p \) and \( l \) with \( t_p \) and \( t \) respectively. The latter term in Eq.\( (2) \) becomes negligible in comparison with the first one if \( l \gg l_p \). In what follows we assume the condition \( l \gg l_p \) that results in the following expression for the minimal length uncertainty

\[ \delta l_{\text{min}} \approx 2^{\frac{3+2n}{2+n}} A^{\frac{1}{2+n}} (l_p^{2+n})^{\frac{1}{2+n}}. \]

(3)

Loosely speaking, in Eq.\( (2) \) one can consider the first and second terms as the uncertainties contributed to the measurement by the clock and mirror respectively. Due to maximal symmetry of the measuring device considered here one can hope it provides minimal uncertainty in length measurement.

The above discussion to be self-consistent let us notice that due to Eq.\( (3) \), the wavelength of photon used for a measurement may be known with the accuracy \( \delta \lambda \sim (l_p^{2+n})^{1/(3+n)} \) that results in additional uncertainty

\[ \delta l_{\text{min}} \sim (l_p^{2+n})^{1/(3+n)} + \frac{l}{\lambda} \delta \lambda \sim (l_p^{2+n})^{1/(3+n)} + \frac{2^{2+n} l}{\lambda^{2+n}}. \]

(4)

(For the sake of simplicity we assume that the wave length of the photon is not affected by the gravitational field of the clock). For \( \lambda \) can not be greater than \( l \) in order to measure this distance the latter term in Eq.\( (1) \) can be minimized by taking \( \lambda \sim l \). In this case the latter term becomes comparable to the first one and therefore justifies the Eq.\( (3) \). In the case when \( \lambda^{2+n} \ll (l_p^{2+n})^{1/(3+n)} \) the second term in Eq.\( (1) \) dominates and correspondingly the minimal uncertainty and metric fluctuations take the form

\[ \delta l_{\text{min}} \sim \frac{l_p}{\lambda}^{2+n} l, \quad \delta g_{\mu\nu} \sim \left( \frac{l_p}{\lambda} \right)^{2+n}. \]

(5)

In general the minimal length uncertainty has the form

\[ \delta l_{\text{min}} \sim l_p^{1-\alpha}, \]

(6)

where the parameter \( \alpha \) specifies different scenarios: \( \alpha = 2/3 \), \( \alpha = 1/2 \), \( \alpha = 1 \) and, as it shown in \( [3] \), for ADD braneworld model \( [4] \) with the size of extra dimensions much exceeding the fundamental Planck scale, \( \sim (\text{TeV})^{-1} \), \( \alpha = (2+n)/(3+n) \). From Eq.\( (6) \) one simply gets that the minimal length uncertainty and metric perturbations when one uses for measurement the quantum with wavelength \( \lambda \) take the form

\[ \delta l_{\text{min}} \sim \left( \frac{l_p}{\lambda} \right)^{\alpha} l, \quad \delta g_{\mu\nu} \sim \left( \frac{l_p}{\lambda} \right)^{\alpha}. \]

(7)

So that the space-time perturbation during the measurement when one uses the quantum with a wavelength \( \lambda \) to probe the space-time region with a linear size \( l \) is the greater the shorter the wavelength of the quantum used for measurement is. Therefore, for the amplification factor one gets

\[ \frac{\delta l}{\delta \lambda} \sim \frac{l}{\lambda}, \]

(8)

which is in complete agreement to what is used in \( [1, 2] \). So the key point missed in \( [3, 4] \) is that the dependence of space-time perturbations on the wavelength of quantum used for the measurement is not taken into account.

As it is discussed in \( [1, 2] \) due to amplification factor given by Eq.\( (8) \) the distant compact radiation sources should not produce the normal interference patterns that are often observed. Hence, if the experimental results analyzed in \( [1, 2] \) can be taken to be trustworthy then we must face the challenge of finding a new, self-consistent, formulation of the fundamental laws that agrees with experiment for the derivation of space-time quantum fluctuations is based on accepted principles of quantum mechanics and general relativity. Let us stress that the brane induced quantum gravitational fluctuations evaluated in [8] are unacceptably enhanced by taking the fundamental scale much below the Planck one. In view of the papers \( [1, 2] \) one simply finds that in the case of braneworld model with \( \sim \text{TeV} \) fundamental scale the theory fails at least by 10 orders of magnitude. Even if one takes the amplification factor \( (l/\lambda)^{1-\alpha} \), \( [3, 4] \), which seems to be not correct as it is discussed above, one finds that the braneworld model with \( \sim \text{TeV} \) fundamental scale fails by order of magnitude if only phase decoherence of the light detected by the telescope is taken into account and by 7 orders of magnitude if the fluctuations in direction of the wave vector is also taken into account.

In detecting the space-time foam one has to take into account the side factors as well that can overlap this effect. One of such factors can be the space-time perturbations caused by the background radiation. Since we are using a bit modified measuring device let us first revise the corresponding result obtained in [8]. We restrict ourselves to the case \( n = 0 \). Let us insert our measuring device into the background radiation with a temperature
Because of the background temperature the device acquires a mean velocity $\sim \sqrt{T/(m+m_0)}$. Assuming the gravitational radius of the device is not changed significantly due to background radiation and repeating the above discussion the expression of minimal uncertainty takes the form

$$\delta l_{\text{min}} \sim l_p^{2/3} l^{1/3} + l_p \sqrt{T}.$$ 

One sees that the present measuring device reduces significantly the influence of CMB on the length uncertainty relative to that one considered in §2. Namely, for $T = 2.7K$ the latter term in this equation becomes appreciable for distances $l^{1/6} \gtrsim 10^{11} \text{cm}^{1/6}$ much exceeding the present size of the observable universe. In general, when the photon with the wavelength $\lambda$ is used for the measurement one gets

$$\delta l_{\text{min}} \sim \frac{l_p^{2/3}}{\chi^{2/3}} + l_p \sqrt{\frac{T}{\lambda}}.$$ 

Correspondingly the effect of the CMB becomes negligible if $\lambda^{1/6} \ll l_p^{2/3} T^{1/2}$. In experiments considered in §2.3.2.4.1 $\lambda \sim \mu\text{m}$ and therefore this condition is satisfied with a great accuracy.

Now let us turn to the issue of computation for which the Eq. 7 can provide new insight. As it was shown in §1.1.1 the results of Salecker and Wigner §2 provide quite strong constraints on the ultimate capability of the computer. This idea was further developed in §1.1.1. Let us consider a simple computational system as a cube $l^3$ filled with the bits $l_b^3$ inside of which photon is bouncing representing therefore a bit-operation process. Using the Heisenberg uncertainty relation one simply concludes that the lifetime of the computer, i.e., the time during which it will be confined to the region $l^3$ is given by

$$T \sim l^2 M,$$

where $M$ is the mass of the computer. The computer not to collapse $l$ should be greater than its gravitational radius $\sim l_p^2 M$ resulting therefore in the notion of maximal lifetime and mass of computer with a given size $l^3$

$$M_{\text{max}} \sim \frac{l}{l_p^2}, \quad T_{\text{max}} \sim \frac{l^3}{l_p^2}.$$ 

In view of the existence of minimal insurmountable uncertainty in distance measurement the size of the bit cannot be smaller than $\delta l_{\text{min}}$. Simply speaking we need to write an information in the bit, read and move it to another bit. So that inside the region $l^3$ we need to operate with the quanta having the wavelength comparable to the size of a bit. Correspondingly from Eq. 7 one can evaluate the size of bit

$$\lambda \sim \left( \frac{l_p}{\chi} \right)^{\alpha} l, \quad \Rightarrow \quad l_b \sim \left( \frac{l_p}{l_{\text{min}}} \right)^{1/(1+\alpha)}, \quad (9)$$

the minimal time for bit-operation and the maximal number of bits

$$t \sim \left( \frac{l_p}{l_{\text{min}}} \right)^{1/(1+\alpha)} , \quad N \sim \left( \frac{l}{l_p} \right)^{3\alpha/(1+\alpha)}. \quad (10)$$

In this case for maximal number of operations one gets

$$N \frac{T_{\text{max}}}{t} \sim \left( \frac{l}{l_p} \right)^{(2+6\alpha)/(1+\alpha)}.$$ 

So this is the maximal number of operations the computer can accomplish in a given region with linear size $l$. In the above considered partition of $l^3$ into the bits $l_b^3$ the energy per bit is

$$E_b = \frac{M_{\text{max}}}{N} \sim \frac{1}{l_{\text{min}}^{3/4}} \frac{1}{l_p^{2+6\alpha}}.$$ 

Correspondingly from time-energy uncertainty relation one can say that the minimum time for bit-operation should be $E_b^{-1}$ which is less than $l_b$ used above and leads to the Margolus-Levitin §13 limit for maximum number of operations the computer can perform in region $l^3$ given by $\sim T_{\text{max}} M_{\text{max}}$. So in general there are two time scales for the bit, $l_b$ and $E_b^{-1}$. In order to arrive at the Margolus-Levitin limit unambiguously these two scales should coincide. Assuming this condition, $l_b = E_b^{-1}$, one gets

$$\frac{M}{E_b} = N, \quad l_b = \frac{l}{N^{1/3}}, \quad \Rightarrow \quad E_b = \frac{M^{1/4}}{l^{3/4}},$$

which by taking into account $l_b \sim (E_b l_p)^{\alpha} l$ implies

$$M \lesssim \frac{l_p^{2\alpha-1}}{l^{1+\alpha}}.$$ 

So, for $\alpha < 1$ one sees that this upper bound on the mass is less than $M_{\text{max}} \sim l/l_p^2$ and therefore the Margolus-Levitin limit is not attainable in this case for computer having the mass $M_{\text{max}}$. The case $\alpha = 1$ is exception allowing the Margolus-Levitin limit for maximum number of operations no matter what the mass of computer is.

It is of interest to analyze the minimal length uncertainty expression from the standpoint of Doubly-special relativity suggesting the Planck length as a second relativistic invariant besides the speed of light. This argument can be motivated by the Generalized Uncertainty Principle (GUP) ensuring the presence of minimal particle localization region with linear size of the order of Planck length §14, which in turn can also be derived by combining the uncertainty relation with gravitation §15. (The GUP derived via the quantum corrected gravitational potential §16 as well as for the ADD braneworld model §17 exhibits the linear size for the minimal localization region of the quantum to be of the order of $l_{\text{min}}$.)
Planck length as well). From the Eq. (3) one sees that if $l_p$ is invariant of the theory and $l$ transforms with respect to the special relativity then the Lorentz invariance is violated at the scale $\delta l_{\text{min}}$ and vice versa. By taking $l \sim \text{TeV}^{-1} \sim 10^{-16}\text{cm}$, which can be considered as a minimal length scale for which the Lorentz invariance is an experimental fact, one finds $\delta l_{\text{min}} \sim 10^{-16(1+\alpha)}$. So one can say that the transition from special relativity to the doubly-special regime takes place beneath the scale $\lesssim 10^{-16(1+\alpha)}\text{cm}$. When $\delta l_{\text{min}}$ approaches $\sim 10^{-16}\text{cm}$, the length scale $l \sim 10^{(33\alpha-16)/(1-\alpha)}\text{cm}$ determines the upper bound for the special relativity. From this expression one sees that for $\alpha = 1/2$ the upper bound for Lorentz invariance becomes unacceptable small. For $\alpha = 2/3$ the upper bound for Lorentz invariance is about $\sim \text{pc}$. This result is interesting in that it indicates every Lorentz invariant theory including the Maxwell electrodynamics should contain this characteristic scale beyond which the modification of the theory takes place. Correspondingly, the consistent treatment of the phase incoherence accumulation for electromagnetic wave traveling the distance much exceeding the length scale $\sim \text{pc}$ requires the modified theory to be known and used respectively.

Finally, following the paper, let us briefly emphasize the impact of space-time fluctuations on the black hole physics. The presence of unavoidable uncertainty in length measurement provided by all the space-time foamy models listed above results in fluctuations of the black hole thermodynamics simply because the gravitational radius can not be known exactly. Because of these fluctuations one can no longer argue that the information about the initial state of the body that collapsed to form the black hole is lost during the black hole evaporation. But it is principally impossible to keep track of these fluctuations that remains open the question about the unitarity in evolution of the initial state during the black hole formation and subsequent evaporation. The quantum corrections to the black hole entropy obtained in various scenarios are indiscernible because of these fluctuations. Fluctuations near the Planck scale become of the order of thermodynamic quantities themselves and therefore destroy the thermodynamical picture of the black hole. So that one should be cautious about the drawing sweeping conclusions based on the black hole thermodynamics near the Planck scale. For more details see 8.

To summarize, on the bases of gedanken experiment depicted in Fig.1 we have precisely evaluated the minimal uncertainty in space-time measurement. The expression of minimal uncertainty obtained in this way immediately results in minimal observable length(time) of the order of Planck length(time). We have specified the dependence of minimal length uncertainty on the wavelength of the quantum used for the measurement. On the bases of this relation the cumulative factor describing the phase and wave-vector uncertainties for the wave traveling over a large distance as well as bounds on the computation are evaluated. On the one hand the synthesis of uncertainty relation with gravitation leads to the GUP which shows that there is a minimal particle localization limit of the order of Planck length and therefore motivates the Planck length as a second invariant of the theory together with the speed of light. On the other hand combining together the uncertainty relations and general relativity one gets the minimal unavoidable uncertainty in space-time measurements which by taking into account the Planck length as an invariant exhibits the Lorentz invariance region from $\sim 10^{-26}\text{cm}$ to $\sim 10^{18}\text{cm}$. So one sees that one of the most striking results coming from synthesis of quantum mechanics and general relativity may be the prediction of Lorentz violation scales. Correspondingly, one can not make some sweeping conclusions on the bases of papers about the discrepancy between theory and experiment as further progress has likely to be achieved to provide new insights into the fundamental theories.

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* Electronic address: maziashvili@hepi.edu.ge

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size of clock is greater than $L$ one gets the Eq. \( \text{Eq.} \) with $n = 0$, i.e., the pure four-dimensional result. By the way one has to keep in mind the difference between the four and higher dimensional Planck scales.

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