MUTATION INVARIANCE FOR THE ZEROTH COEFFICIENTS OF THE COLORED HOMFLY POLYNOMIAL

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Abstract. We show that the zeroth coefficient of the cables of the HOMFLY polynomial (colored HOMFLY polynomials) does not distinguish mutants. This makes a sharp contrast with the total HOMFLY polynomial whose 3-cables can distinguish mutants.

1. Introduction

Let $P_K(a, z)$ be the HOMFLY polynomial of an oriented knot or link $K$ in $S^3$ defined by the skein relation

$$a^{-1}P_{\gamma^a}(a, z) + aP_{\gamma}(a, z) = zP_{\gamma}(a, z), \quad P_{\text{Unknot}}(a, z) = 1.$$ 

It is known that $P_K(a, z)$ is written in the form

$$P_K(a, z) = (az)^{-r_K+1}\sum_{i \geq 0} \gamma_i^K(a)z^{2i}$$

where $r_K$ denotes the number of the components of $K$. We call the polynomial $\gamma_i^K(a)$ the $i$-th coefficient (HOMFLY) polynomial of $K$.

One of an advantage of using the coefficient polynomials is that their skein formulae are simpler [Ka]. For example, the zeroth coefficient polynomial $\gamma_0^K(a)$ satisfies the skein formula

$$a^{-2}\gamma_0(a) + \gamma_0(a, z) = \begin{cases} \gamma_0(a) & \text{if } \delta = 0, \\ 0 & \text{otherwise.} \end{cases}$$

Here $\delta = \frac{1}{2}(r_{\gamma^a} - r_{\gamma}) + 1 \in \{0, 1\}$. Consequently, for each $i$ the $i$-th coefficient polynomial $\gamma_i^K(a)$ is computed in polynomial time with respect to the number of crossings [Pr]. This makes a sharp contrast with the total HOMFLY polynomial whose computation requires exponential time with respect to the number of crossings.

In this paper we show that the cables of the zeroth coefficient polynomial, the zeroth coefficient of the cables of the HOMFLY polynomial (so called the colored HOMFLY polynomial), do not distinguish mutants. For a coprime integers $p$ and $q$, we denote by $K_{p,q}$ the $(p, q)$-cable of the knot $K$.

Theorem 1. If two knots $K$ and $K'$ are mutant, then $\gamma_0^{K_{p,q}}(a) = \gamma_0^{K'_{p,q}}(a)$.

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Here two knots $K$ and $K'$ are called mutant if $K'$ is obtained from $K$ by mutation: we take a ball $B$ whose boundary intersects with $K$ at four points to get a tangle $Q$, then we cut $K$ along $Q$ and glue it back by rotating $Q$ (see Figure 1 (i)) to get another knot $K'$. If necessary, we change the orientation of $Q$ so that the $K'$ admits a coherent orientation.

![Mutation](image)

**Figure 1.** (i) Mutation (ii) Example of mutation: the Conway knot and the Kinoshita-Terasaka knot

It is often difficult to distinguish mutants since many familiar knot invariants, including the HOMFLY polynomial, take the same value for mutants. However, it may happen that the cables of a knot invariant $v$ can distinguish mutants, even if $v$ itself cannot. Namely, for mutants $K$ and $K'$, it may happen that $v(K) = v(K')$ but $v(K_{p,q}) \neq v(K'_{p,q})$. For example, the HOMFLY polynomial does not distinguish mutants, whereas its 3-cable can distinguish mutants, such as, the Conway knot and the Kinoshita-Terasaka knot [MC]. It may also happen that the cables of a knot invariant still fail to distinguish mutants – the cables of the Jones polynomial, the colored Jones polynomials, cannot distinguish mutants.

As shown in [Ta1], the cables of the zeroth coefficient polynomial are not determined by the HOMFLY polynomial so the mutation invariance is not obvious from the mutation invariance of the HOMFLY polynomial. The mutation invariance for the cables of the zeroth coefficient polynomial was studied [Ta2], where it was shown that 3-cables of the zeroth coefficient polynomial do not distinguish mutants by skein theoretic argument.

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**2. The zeroth coefficient polynomial in the context of the Kontsevich invariant**

In this section we discuss the zeroth coefficient polynomial in a point of view of quantum invariants. For basics of quantum invariants, we refer to [Oh]. We also refer to [LM1] for the relation between the HOMFLY polynomial and the Kontsevich invariant.
2.1. **Chord diagram and intersection graph.** A **chord** over the circle $S^1$ is a pair of distinct points $\{x, y\} \subset S^1$. We call the points $x$ and $y$ the **legs** of the chord. A **chord diagram** is a correction of mutually distinct $n$ chords $\{\{x_i, y_i\}\}_{i=1,\ldots,n}$ over $S^1$. The **degree** of the chord diagram $D$ is the number of chords, and we denote by $[D]$ the set of chords of $D$. As usual, we regard homeomorphic chord diagram as the same and express a chord diagram by drawing a diagram consisting of $S^1$ and chords connecting their legs.

The **space of chord diagram** $\mathcal{C} = \mathcal{C}(S^1)$ is a graded $\mathbb{C}$-vector space generated by chord diagrams, modulo 4T (four-term) relation in Figure 2. By taking the connected sum $\#$ as a multiplication, $\mathcal{C}$ is a graded commutative algebra. In the following, we will actually use the completion of $\mathcal{C}$ with respect to degrees, which we denote by the same symbol $\mathcal{C}$ by abuse of notation.

![Figure 2. 4T relation](image)

The **intersection graph** of a chord diagram $D$ is a graph $\Gamma(D)$ whose vertices is $[D]$, the set of chords of $D$. Two vertices $v$ and $w$ are connected by an edge if and only if two chords $v$ and $w$ intersect. Here we say that chords $v = \{a, b\}$ and $w = \{c, d\}$ intersect if the points $a$ and $b$ belong to the different components of $S^1 \setminus \{c, d\}$.

For a non-negative integer $n$, we say that $D$ is $n$-**colorable** if its intersection graph $\Gamma(D)$ is $n$-colorable. Namely, there is a map called a (vertex) coloring $c : \{\text{vertices of } \Gamma(D)\} = [D] \to \{1, \ldots, n\}$ such that $c(v) \neq c(w)$ if $v$ and $w$ are connected by an edge.

2.2. **Information carried by the coefficient polynomials.** In the following, we will always regard a knot $K$ as a framed knot with 0-framing.

Let $V_N$ be the standard $N$-dimensional representation of the lie algebra $\mathfrak{sl}_N$. Let $Q_K^{(\mathfrak{sl}_N, V_N)}(q)$ be the quantum $(\mathfrak{sl}_N, V_N)$ invariant of a knot $K$, which is related to the HOMFLY polynomial by

\[
Q_K^{(\mathfrak{sl}_N, V_N)}(q) = \frac{q^{\frac{N}{2}} - q^{-\frac{N}{2}}}{q^{\frac{1}{2}} - q^{-\frac{1}{2}}} P_K(q^{\frac{N}{2}}, q^{\frac{1}{2}} - q^{-\frac{1}{2}}).
\]

The (framed) Kontsevich invariant $Z(K)$ is an invariant of framed knots that takes value in $\mathcal{C}$. There is a map $W_N = W_{(\mathfrak{sl}_N, V_N)} : \mathcal{C} \to \mathbb{C}[[h]]$, called the **weight system** associated with $(\mathfrak{sl}_N, V_N)$ and we have

\[
W_N(Z(K)) = Q_K^{(\mathfrak{sl}_N, V_N)}(e^h).
\]

Combining (2.1) and (2.2), we get the following (cf. [LM1, Theorem 2.3.1])

\[
W_N(Z(K)) = \frac{e^{\frac{NH}{2}} - e^{-\frac{NH}{2}}}{e^{\frac{1}{2}} - e^{-\frac{1}{2}}} P_K(e^{\frac{NH}{2}}, e^{\frac{1}{2}} - e^{-\frac{1}{2}}).
\]
We expand the right hand side of (2.3) in a power series of $N$ and $h$ as

$$e^{\frac{Nh}{2}} - e^{-\frac{Nh}{2}} P_K(e^{\frac{Nh}{2}}, e^{\frac{h}{2}} - e^{-\frac{h}{2}}) = \sum_{i,j} c_{i,j}(K) h^i N^j. \quad (2.4)$$

We also expand the $i$-th coefficient polynomial in a power series of $(Nh)$ by putting $a = e^{\frac{Nh}{2}}$, as

$$\gamma_i^j(e^{\frac{Nh}{2}}) = \sum_{j=0}^{\infty} d_{i,j}(K)(Nh)^j. \quad (2.5)$$

Then we get an expansion

$$P_K(e^{\frac{Nh}{2}}, e^{\frac{h}{2}} - e^{-\frac{h}{2}}) = \sum_{i=0}^{\infty} \gamma_i^j(e^{\frac{Nh}{2}})(e^{\frac{h}{2}} - e^{-\frac{h}{2}})^{2i}$$

$$= \sum_{i=0}^{\infty} \left( \sum_{j=0}^{\infty} d_{i,j}(N^3 h^j) \right) \left( h^{2i} + (\deg h > 2i \text{ parts}) \right)$$

Since $\frac{e^{\frac{Nh}{2}} - e^{-\frac{Nh}{2}}}{e^{\frac{h}{2}} - e^{-\frac{h}{2}}} = N(1 + \text{power series on } N, h)$, by comparing (2.4) and (2.5) we get the following.

**Proposition 1.**

$$c_{i,j}(K) = \begin{cases} 0 & i \leq j \\ d_{i,j}(K) & i = j + 1 \end{cases}$$

**Remark 2.** The same argument shows that the $i$-th coefficient polynomial $\gamma_i^j$ is determined by $i$-diagonal part $\{c_{j+1, 1}(K), c_{j+2, 1}(K), \ldots, c_{j+1+2i, 1}(K)\}_{j=0, \ldots}$ of the HOMFLY polynomial.

**Remark 3.** One may notice that the situation is similar to the Melvin-Morton-Rozansky conjecture proven in [BG]. We expand the colored Jones polynomials, quantum $\mathfrak{sl}_2$ invariant with respect to $N$-dimensional irreducible representation, as a power series of $h$ and $N$. Then the coefficients vanish under the diagonal part, and the diagonal part is equivalent to the Alexander-Conway polynomial. See [BG] for details.

A *state* of a chord diagram $D$ is a map $\sigma : [D] \to \{+1, -1\}$. For a state $\sigma$, we associate an oriented surface $S_\sigma$ as follows. We start from the disk that bounds the outermost circle $S^1$ of the chord diagram. For a chord $c$ of $D$, if $\sigma(c) = +1$ we attach a 1-handle along $c$, and if $\sigma(c) = -1$ we do nothing (see Figure 3).

Let $s(\sigma)$ be the number of boundary components of the resulting surface $S_\sigma$ and let $n(\sigma)$ be the number of chords such that $\sigma(c) = -1$. Then $W_N(D)$ is given by the following formula [Ba, LM1, CDM].

$$W_N(D) = \sum_{\sigma : [D] \to \{+1, -1\}} (-1)^{n(\sigma)} N^{s(\sigma) - n(\sigma)} h^{\deg D} \quad (2.6)$$
Here \( \sigma \) runs over all the states of \( D \).

Let \( \sigma_+ \) be the state that assigns +1 for all the chords. We define the genus \( g(D) \) of \( D \) as the genus of the surface \( S_{\sigma_+} \). Then \( s(\sigma_+) = \deg(D) - 2g(D) + 1 \). By (2.6), for every state \( \sigma \), we have an inequality \( N_s(\sigma) - n(\sigma) \leq N_s(\sigma_+) - n(\sigma_+) = N_s(\sigma_+) \), and the equality occurs only if \( \sigma = \sigma_+ \). Therefore \( W_N(D) \) is of the form

\[
W_N(D) = N^{\deg D + 1 - 2g(D)} h^{\deg D} + \sum_{1 \leq j \leq \deg(D)} x_j N^{\deg D + 1 - 2g(D) - j} h^{\deg D}
\]

where \( x_j \in \mathbb{Z} \). This implies that for \( i \geq 0 \), \( W_N(D) \) contains a monomial \( N^{j+2i} h^j \) only if \( g(D) \leq i \). This observation and Proposition 1 identifies the information of the Kontsevich invariant carried by the zeroth coefficient polynomial.

**Theorem 4.** The zeroth coefficient polynomial \( \gamma_0^0(a) \) is determined by the genus 0 chord diagram part of the Kontsevich invariant. Namely, let \( C_0 \) be the subspace of \( C \) that is spanned by the chord diagram of genus 0, and \( p : C \to C_0 \) be the projection map. Then for knots \( K \) and \( K' \), if \( p(Z(K)) = p(Z(K')) \) then \( \gamma_0^0(a) = \gamma_0^0(a) \).

**Remark 5.** The same argument, together with Remark 2, shows that the \( j \)-th coefficient polynomial \( \gamma_j^0(a) \) is determined by the genus \( \leq j \) chord diagram part of the Kontsevich invariant.

### 3. PROOF OF MUTATION INVARIANCE

#### 3.1. Cables of the Kontsevich invariant

The effect of the cabling operation for the Kontsevich invariant is described as follows [BLT][LM2]. For a pair of integers \( (p, q) \), let \( l = l_{p,q} \) be a simple closed curve on a torus \( T = S^1 \times S^1 \) that represents \( p[S^1 \times \{\ast\}] + q[\{\ast\} \times S^1] \in H_1(T; \mathbb{Z}) \). Let \( \pi = S^1 \times S^1 \to S^1 \) be the projection map to the first factor. The restriction of \( \pi \) to \( l \) gives a \( p \)-fold covering map \( \pi = \pi_{p,q} : l = \widetilde{S^1} \to S^1 \). In the following, to distinguish the base circle \( S^1 \) and its \( p \)-fold covering, we denote the covering circle by \( \widetilde{S^1} \).

For a chord diagram \( D \) on \( S^1 \), we say that a chord diagram \( \widetilde{D} \) on \( \widetilde{S^1} \) is a lift of \( D \) if \( \pi(\widetilde{D}) = D \). For \( D = \{ \{x_1, y_1\}, \ldots, \{x_n, y_n\} \} \), \( \widetilde{D} \) is a lift of \( D \) if and only if \( \widetilde{D} = \{ \{\widetilde{x}_1, \widetilde{y}_1\}, \ldots, \{\widetilde{x}_n, \widetilde{y}_n\} \} \) where \( \widetilde{x}_i \in \pi^{-1}(x_i) \) and \( \widetilde{y}_i \in \pi^{-1}(y_i) \) for each \( i \).

We take a base point \( * \in S^1 \) so that * is different from all the legs of chords of \( D \). Then \( \pi^{-1}(S^1 \setminus *) \) is a union of disjoint intervals \( L_1, \ldots, L_p \). For \( x \in S^1 \setminus * \), we denote the lift of the point \( x \) which lies on \( L_i \) by \( x_i \).

We call a map \( s : \{ \text{Set of legs of chords of } D \} \to \{1, \ldots, p\} \) a leg \( p \)-coloring. There is a one-to-one correspondence between the set of all the leg \( p \)-colorings and
the set of all the lifts $\bar{D}$ of $D$: for a leg $p$-coloring $s$, we assign the lift $D_s$ so that the chord $\{x, y\}$ are lifted to the chord $\{x_s(x), y_s(y)\}$. See Figure 4.

![Figure 4. Lift of chord diagrams and leg $p$-colorings](image)

We define the map $\psi^{(p,q)} : (C(S^1) =) C \rightarrow C(= \mathcal{C}(\mathcal{S}^1))$ by

$$\psi^{(p,q)}(D) = \sum_s \bar{D}_s$$

where $s$ runs all the leg $p$-coloring. Using the map $\psi^{(p,q)}$, the cabling formula of Kontsevich invariant is written as follows [BLT, Theorem 1, Remark 3.4]:

$$Z(K(p,q)) = \left[ \psi^{(p,q)} \left( Z(K) \# \exp\left( \frac{q}{2p} \bigoplus \right) \right) \right] \# \exp\left( -\frac{q}{2} \bigoplus \right)$$

Here $\exp(D) = 1 + D + \frac{1}{2} D \# D + \cdots = \sum_{n=0}^{\infty} \frac{1}{n!} D \# D \cdots \# D$.

If $D$ is $p$-colorable, then there exists a 1-colorable lift $\bar{D}$ of $D$: For a $p$-coloring of $D$, we take a leg $p$-coloring $s$ of $D$ given by $s(x_i) = s(y_i) = c$ if the chord $\{x_i, y_i\}$ is colored by $c$. Then its corresponding lift $\bar{D}_s$ is 1-colorable. It is interesting to ask whether the converse is true or not:

**Question 1. Is a lift $\bar{D}$ of $D$ admits a 1-coloring, if and only if $D$ is $p$-colorable?**

Since $g(D) = 0$ if and only if $D$ is 1-colorable, it is the 1-colorable chord diagrams that contributes to the zeroth coefficient polynomial.

An affirmative answer to Question 1 implies that the zeroth coefficient polynomial of the $(p,q)$-cable is determined by the $i$-colorable chord diagram part of the Kontsevich invariant, for $i \leq p$.

3.2. 1-colorable lifts and mutation invariance. We say that a mutation is of *type A* if it preserves the boundary of strands pointwise (see Figure 5 (i)). Other two types of mutations are achieved by suitable compositions of mutation of type A (see Figure 5 (ii)).

A *share* $S(I,J)$ in a chord diagram $D$ is a union of disjoint intervals $I$ and $J$ in $S^1$ and chords of $D$ both of whose legs lie on $I \cup J$. A *mutation* of the chord diagram $D$ along a share $S$ is a chord diagram obtained by rotating or flyping (or both) a share $S$ (see Figure 6).

For mutants $K$ and $K'$, the Kontsevich invariant $Z(K')$ is obtained from $Z(K)$ by applying the mutation of each chord diagrams in $Z(K)$ along an appropriate share that corresponds to the mutation tangle $Q$ (see [CDM] for precise statement). Therefore we have the following.
Figure 5. (i) mutation of type A. (ii) other types of mutations are achieved by mutation of type A

Figure 6. Share and mutation of chord diagrams
Proposition 2. Let $V$ be a knot invariant that is governed by the Kontsevich invariant. That is, there is a map $W_V: C \rightarrow C$ such that $V(K) = W_V(Z(K))$. Assume that if $D$ and $D'$ are related by mutation of chord diagrams, $W_V(D) = W_V(D')$. Then $V$ does not distinguish mutants.

Actually, two chord diagram have the same intersection graph if and only if they are related by mutations and $V$ does not distinguish mutant if and only if $W_V$ only depends on the intersection graph $G$.

We say that a mutation of chord diagram is of type A if it flips the orientation of $I \cup J$ but it does not swap the position of $I$ and $J$. This corresponds to the type A mutation of a knot.

Let $D^\tau$ be the chord diagram that is obtained from $D$ by mutation of type A along a share $S = S(I \cup J)$. The following is the crucial observation.

Proposition 3. The number of 1-colorable lift of $\psi^{(p|q)}(D)$ and that of $\psi^{(p|q)}(D^\tau)$ is equal.

Proof. Let $\tilde{D}$ be a 1-colorable lift of $\psi^{(p|q)}(D)$. We construct a 1-colorable lift of $\psi^{(p|q)}(D^\tau)$ in the following way.

Let $X, Y$ be the connected components of $S^1 \setminus (I \cup J)$. We take a basepoint $* \in S^1$ as one of the boundary of $I$ so that four intervals $I, X, J, Y$ appear in this order along $S^1$. We denote the $i$-th lift of the interval $L \in \{I, J, X, Y\}$ by $L_i$.

We draw the chord diagram $\tilde{D}$ in the disk $D^2$ bounded by $S^1$. Since $\tilde{D}$ is 1-colorable, no two chords intersect each other. Let $[\tilde{D}]_{XY}$ be the set of chords of $\tilde{D}$ which are lifts of the chords in the share $S(X, Y)$ of $D$. For $A, B \in \{I_1, \ldots, I_p, J_1, \ldots, J_p\}$ we define $A \sim B$ if $A$ and $B$ lies in the same connected component of $D^2 \setminus [\tilde{D}]_{XY}$. By this equivalence relation, we decompose the set $\{I_1, \ldots, I_p, J_1, \ldots, J_p\}$ into the disjoint union of the equivalence classes as $\{I_1, \ldots, I_p, J_1, \ldots, J_p\} = T_1 \sqcup T_2 \sqcup \cdots \sqcup T_k$.

For each equivalence class $T_i$, we take a 2(\# $T_i$)-gon $R_i \subset D^2 \setminus [\tilde{D}]_{XY}$ obtained by connecting the boundary points of the intervals $I_s, J_t \in T_i$. Then we cut the disk $D^2$ along $R_i$ and glue it back by flying the region $R_i$ (see Figure 7), so that the orientation of each interval $I_s, J_t \in T_i$ are reversed, and that the interval which is a lift of $I$ (resp. $J$) is glued back to the lift of $I$ (resp. $J$) (see Figure 8).

By applying this operation for each equivalence class $T_i$, we get a 1-colorable chord diagram $\tilde{D}^\tau$ which is a lift of $\psi^{(p|q)}(D^\tau)$. This provides a bijection between the set of 1-colorable lift of $\psi^{(p|q)}(D)$ and that of $\psi^{(p|q)}(D^\tau)$.

$\square$

Proof of Theorem 4. Let $K$ and $K'$ be mutants. With no loss of generality we may assume that $K$ and $K'$ are related by mutation of type A. Then by Proposition 3 for each chord diagram $D$, both $D$ and its corresponding mutant $D^\tau$ produces the same number of 1-colorable chord diagrams in $\psi^{(p|q)}(D)$. This shows that 1-colorable chord diagram part of $\psi^{(p|q)}(Z(K))$ and $\psi^{(p|q)}(Z(K'))$ are the same. By (3.1), this shows that the 1-colorable chord diagram part of $Z(K_{p,q})$ and $Z(K'_{p,q})$ are the same. Since a chord diagram is 1-colorable if and only if its genus is zero, by Theorem 4 the zeroth coefficient polynomial of $K_{p,q}$ and $K'_{p,q}$ must be the same. $\square$

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Figure 7. A construction of 1-1 correspondence between 1-colorable lifts: We flip each region $R_i$ so that it glued back by reversing the orientation of each lifts of $I$ and $J$.

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