Thermal radiation from an expanding viscous medium

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The effects of viscosity on the space time evolution of QGP produced in nuclear collisions at RHIC energies have been studied. The entropy generated due to the viscous motion of the fluid has been taken into account in constraining the initial temperature by the final multiplicity (measured at the freeze-out point). The viscous effects on the photon spectra has been introduced consistently through the evolution dynamics and phase space factors of the particles participating in the production process. We notice a stronger effect on the photon spectra originating from QGP than hadronic matter. A detectable shift is observed in the space-time integrated \( p_T \) distribution of photons due to dissipative effects.

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I. INTRODUCTION

Nuclear collisions at Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) energies are aimed at creating a thermalized state of quarks and gluons called quark gluon plasma (QGP). The weakly interacting picture of the QGP stems from the perception of asymptotic freedom of QCD at high temperatures and densities. However, the experimental data from RHIC [1], especially the measured elliptic flow of hadrons indicate that the matter produced in Au+Au collisions exhibit properties which are more like a strongly interacting liquid than a weakly interacting gas. The magnitude of the transport coefficients can be used to understand the strength of the interaction within the QGP. Therefore, the study of the transport properties of QGP and hot hadrons is of paramount importance in characterizing the matter formed in heavy ion collisions (HIC) at relativistic energies. For example, the shear viscosity or the internal friction of the fluid symbolizes the ability to transfer momentum over a distance of about a mean free path. Therefore, in a system where the constituents interact strongly the transfer of momentum is performed easily - resulting in lower values of \( \eta/s \). Consequently such a system may be characterized by a small value of \( \eta/s \) where \( s \) is the entropy density. On the other hand, for a weakly interacting system the momentum transfer between the constituents become strenuous which gives rise to large \( \eta \). The importance of viscosity also lies in the fact that it damps out the variation in the velocity and makes the fluid flow laminar. A very small viscosity (large Reynolds number) may make the flow turbulent. A lower bound on the value of \( \eta/s \) has been found using AdS/CFT [2] (see also [3]).

Collisions between nuclei at ultra-relativistic energies produce charged particles - either in the hadronic or in the partonic state, depending on the collision energy. Interactions among these charged particles produce real photons. Because of their nature of interaction, the mean free path of photons in the medium (hadronic or partonic) is large compared to the size of the system formed in HIC. Therefore, photons emanating from such a system brings out the information of the source point very efficiently [4–6] (see [7–9] for review) and hence electromagnetic probes (photons and lepton pairs) may play crucial role in extracting the transport coefficients.

The effects of viscosity on the photon spectra resulting from HIC enter through two main factors: (i) the modification of the phase space factor due to the deviation of the system from equilibrium and (ii) the space time evolution of the matter governed by dissipative hydrodynamics. One more important issue deserves to be mentioned here. Normally, the initial temperature (\( T_i \)) and the thermalization time (\( \tau_i \)) are constrained by the measured hadron multiplicity (\( dN/dy \)). This approach is valid for a system where there is no viscous loss and the time reversal symmetry is valid. However, for a viscous system the entropy at the freeze-out point (which is proportional to the multiplicity) contains the initially produced entropy as well as the entropy produced during the space time evolution due to non-zero shear and bulk viscosity. Therefore, the amount of entropy generated during the evolution has to be subtracted from the total entropy at the freeze-out point and the remaining part which is produced initially should be used to estimate the initial temperature. Therefore, for a given \( dN/dy \) (which is associated with the freeze-out point) and \( \tau_i \) the magnitude of \( T_i \) will be lower in case of viscous dynamics compared to ideal flow.

Effects of viscosity on the transverse momentum distribution of photons was earlier considered in [11,12] and recently the interest in this field is renewed [13-14]. Beyond a certain threshold in collision energy the system is expected to be formed in QGP phase which will inevitably make a transition
to the hadronic matter later. The measured spectra contain contributions from both QGP and hadronic phases. Therefore, it becomes imperative to estimate the photon emission with viscous effects from QGP as well as hadrons and identify a kinematic window where photons from QGP dominate. While in some of the earlier works \[12, 14\] contributions from hadrons were ignored, in others \[10, 11\] the effects of dissipation on the phase space factors were omitted. In the present work we study the effects of viscosity on the thermal photon spectra originating from QGP and hadronic matter and argue that photons can be used as a very useful tool to estimate \(\eta/s\) and hence characterize the matter.

The paper is planned as follows. In the next section the kinetic theory formalism for evaluating the photon emission rate in the independent particle approximation \[15\] is discussed. In section III we discuss the viscous effects on the phase space distributions of the partons or hadrons participating in the photon production processes. Hydrodynamical evolution with viscous effects has been mentioned in section IV. Section V is devoted to results and section VI is dedicated to the summary and discussion.

II. PRODUCTION OF THERMAL PHOTONS

The transverse momentum \((p_T)\) distribution of photons from a reaction of the type: \(1 + 2 \rightarrow 3 + \gamma\) taking place in a thermal bath at a temperature, \(T\) is given by \[15\]:

\[
E d^3p \cdot \frac{N}{2(2\pi)^3} \int \frac{d^3p_1}{2E_1} \frac{d^3p_2}{2E_2} \frac{d^3p_3}{2E_3} f_1 f_2 (1 \pm f_3) \delta^{(4)}(p_1 + p_2 - p_3 - p) \left| \mathcal{M}^2 \right| \quad (1)
\]

where \(R\) is the rate of photon production per unit four-volume, \(N\) is the over all degeneracy for the reaction under consideration, \(p_i, E_i\) and \(f_i(E_i)\) are the three momentum, energy and thermal phase space factor of the particle \(i\) (either parton or hadron). \(\left| \mathcal{M}^2 \right|\) is the square of the invariant amplitude for the process under consideration. After some straightforward algebra Eq. (1) can be simplified to (see Appendix A):

\[
\frac{dR}{d^2p_T dy} = \frac{N}{16(2\pi)^3} \int p_{1T} d p_{1T} d p_{2T} d \phi_1 d y_1 d y_2 f_1 f_2 (1 \pm f_3) \frac{\left| \mathcal{M}^2 \right|}{p_{1T} \sin(\phi_1 - \phi_2) + p_{2T} \sin(\phi_2 - \phi_1)} \quad (2)
\]

The collision of nuclei at RHIC and LHC energies is expected to produce QGP. This state of matter, once created with high internal pressure will undergo rapid expansion and consequently will cool down to the temperature, \(T_c\) for QGP to hadron transition. In a first order phase transition scenario the system remains in a mixed phase of QGP and hadrons at \(T_c\) until a time where the entire QGP converts to hadrons. The thermal equilibrium may also be maintained in the hot hadronic phase till the freeze-out point (achieved at a temperature, \(T_F\)) where the mean free path of the hadrons is too large for collisions to take place.

The measured photon spectra \(dN/d^2p_T dy\) is the yield obtained after performing the space time integration over the entire evolution history - from the initial state to the freeze-out point. Therefore, Eq. (2) needs to be integrated over the four volume to connect the theoretical results with experiments:

\[
\frac{dN}{d^2p_T dy} \bigg|_{y=0} = \sum_{i=Q,M,H} \int d^4x \left[ \frac{dR}{d^2p_T dy} \bigg|_{y=0} \right] i \quad (3)
\]

where \(i \equiv Q, M, H\) represents QGP, mixed (coexisting phase of QGP and hadrons) and hadronic phases respectively. The effects of viscosity enter the photon spectra through the space time evolution governed by the dissipative hydrodynamics and the phase space factor, \(f_i’s\) in Eq. (2).

A. Thermal photons from QGP

The contribution from QGP to the spectrum of thermal photons due to annihilation \((q\bar{q} \rightarrow g\gamma)\) and Compton \((q\bar{q}) q \rightarrow q(\bar{q})\gamma)\) processes has been calculated in \[16, 17\] using hard thermal loop (HTL) approximation \[18\]. Later, it was shown that photons from the processes \[19\]: \(gg \rightarrow g\gamma\), \(q\bar{q} \rightarrow q\gamma\), \(q\bar{q} \rightarrow q\gamma\) and \(q\bar{q} \rightarrow g\gamma\) contribute in the same order \(O(\alpha_\gamma)\) as the Compton and annihilation processes. The complete calculation of emission rate from QGP to order \(\alpha_\gamma\) has been performed by resumming ladder diagrams in the effective theory \[20\]. However, in the present work we consider only the Compton and annihilation processes for photon production. We expect that the shift in the photon spectra from the ideal to the viscous scenario will not alter drastically with the replacement of the Compton + annihilation rates by the rate obtained in Ref. \[20\].

B. Thermal photons from hadronic matter

A set of hadronic reactions with all possible isospin combinations have been considered for the production of photons \[21, 24\] from hadronic matter. The relevant reactions and decays for photon production are: (i) \(\pi\pi \rightarrow \rho\gamma\), (ii) \(\pi\rho \rightarrow \pi\gamma\) (with \(\pi, \rho, \omega, \phi\) and \(a_1\) in the intermediate state \[23\]), (iii) \(\pi\pi \rightarrow \eta\gamma\) and (iv) \(\pi\eta \rightarrow \pi\gamma, \rho \rightarrow \pi\pi\gamma\)
and \( \omega \to \pi \gamma \). The corresponding vertices’s are obtained from various phenomenological Lagrangians described in detail in Ref. [21]. The effect of hadronic dipole form factors has been taken into account in the present work as in [24].

III. VISCOSOUS CORRECTION TO THE DISTRIBUTION FUNCTION

We assume that the system is slightly away from equilibrium which relaxes back to equilibrium through dissipative processes. Here we briefly recall the main considerations leading to the commonly used form for the first viscous correction, \( \delta f \) to the phase space factor, \( f \) defined as follows [25]:

\[
f(p) = f_0(1 + \delta f) = f_0 \left( 1 + \frac{p^\alpha p^\beta}{2T^2} \left[ C(\nabla_\alpha u_\beta) + A\Delta_{\alpha\beta} \nabla \cdot u \right] \right) \tag{4}\]

where \( f_0 \) is the equilibrium distribution function, \( \langle \nabla_\alpha u_\beta \rangle = \nabla_\alpha u_\beta + \nabla_\beta u_\alpha - \frac{4}{3} \Delta_{\alpha\beta} \nabla \gamma u^\gamma \), \( \Delta_{\alpha\beta} = g_{\alpha\beta} - u_\alpha u_\beta \), \( \nabla_\alpha = (g_{\alpha\beta} - u_\alpha u_\beta) \partial^\beta \), \( u_\mu \) being the four-velocity of the fluid. The coefficients \( C \) and \( A \) can be determined in the following way. Substituting \( f \) in the expression for stress-energy tensor \( T^{\mu\nu} \) we get,

\[
T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3} \varepsilon p^\mu p^\nu f_0(1 + \delta f)
\]

\[
= T_0^{\mu\nu} + \Delta T^{\mu\nu} \tag{5}
\]

where \( T_0^{\mu\nu} = (\epsilon + P) u^\mu u^\nu - g^{\mu\nu} P \) is the energy momentum tensor for ideal fluid. From general considerations [26], the dissipative part can be written as

\[
\Delta T^{\mu\nu} = \eta \langle \nabla^\mu u^\nu \rangle + \zeta \Delta^{\mu\nu} \nabla \cdot u \tag{6}
\]

Equating the part containing \( \delta f \) from [4] with [6], \( C \) and \( A \) can be expressed in terms of the coefficients of shear (\( \eta \)) and bulk (\( \zeta \)) viscosity respectively in terms of which the phase space distribution for the system can be written as:

\[
f = f_0 \left( 1 + \frac{\eta/s}{2T^2} p^\alpha p^\beta (\nabla_\alpha u_\beta) - \frac{\zeta/s}{3T^2} p^\alpha p^\beta \Delta_{\alpha\beta} \nabla \cdot u \right) \tag{7}
\]

For a boost invariant expansion in (1+1) dimension this can be simplified to get,

\[
f = f_0[1 + \delta f_\eta - \delta f_\zeta] \tag{8}
\]

where

\[
\delta f_\eta = \frac{\eta/s}{3T^2} (p_1^2 - 2p_z^2)
\]

and

\[
\delta f_\zeta = \frac{\zeta/s}{5T^3} (p_T^2 + p_z^2)
\]

where \( p_T^\prime = m_T \sin(y - \eta) \) is the z-component of the momentum in the fluid co-moving frame. The phase space distribution with viscous correction [5] thus enters the production rate of photon through Eq. [9].

IV. EXPANSION DYNAMICS

As mentioned before the \( p_T \) distribution of thermal photons is obtained by integrating the emission rate over the evolution history of the expanding fluid. Relativistic viscous hydrodynamics can be used as a tool for the space-time dynamics of the fluid.

![FIG. 1: Variation of temperature with proper time for different phases for various values of the shear viscosities. Inset shows the effect of viscosity on the cooling of the QGP phase (in an amplified scale) for different values of \( \eta/s \).](image)
the energy density varies with time as the conversion mixed phase the temperature remains constant but degeneracy for the QGP (hadronic) phase. In the Eq. 9 and solving for $\zeta$ when the conversion of QGP to hadronic matter is

$$(15)$$

Similarly the time variation of temperature in the hadronic phase is given by:

$$T = T_c \left( \frac{\tau_H}{\tau} \right)^{1/3} + \frac{2}{3T_H} \left( \frac{d}{s} \right)_H \left[ \left( \frac{\tau_H}{\tau} \right)^{1/3} - \frac{\tau_H}{\tau} \right]$$

In a realistic scenario the value of $\eta/s$ may be different for QGP [24–33] and hadronic phases [34–37]. However, in the present work we take the same value of $\eta/s$ both for QGP and hadronic matter.

V. RESULTS

In case of an ideal fluid, the conservation of entropy implies that the rapidity density $dN/dy$ is a constant of motion for the isentropic expansion [27]. In such circumstances, the experimentally observed (final) multiplicity, $dN/dy$ may be related to a combination of the initial temperature $T_i$ and the initial time $\tau_i$ as $T_i^3 \tau_i$. Assuming an appropriate value of $\tau_i$ (taken to be $\sim 0.6$ fm/c in the present case), one can estimate $T_i$.

For dissipative systems, such an estimate is obviously inapplicable. Generation of entropy during the evolution invalidates the role of $dN/dy$ as a constant of motion. Moreover, the irreversibility arising out of dissipative effects implies that estimation of the initial temperature from the final rapidity density is no longer a trivial task. We can, nevertheless, relate the experimental $dN/dy$ to the freeze-out temperature, $T_f$ and the freeze-out time, $\tau_f$ by the relation,

$$\frac{dN}{dy} = \pi R_A^2 4\pi \kappa$$

where $R_A$ is the radius of the colliding nuclei (we consider $AA$ collision for simplicity) and $\kappa$ is a constant $\sim 3.6$ for massless bosons.

To estimate the initial temperature for the dissipative fluid we follow the following algorithm. We treat $T_i$ as a parameter; for each $T_i$, we let the system evolve forward in time under the condition of dissipative fluid dynamics (Eq. 9) till a given freeze-out temperature $T_f$ is reached. Thus $\tau_f$ is determined. We then compute $dN/dy$ at this instant of time from Eq. 15 and compare it with the experimental $dN/dy$. The value of $T_i$ for which the calculated $dN/dy$ matches the experimental number is taken to be the value of the initial temperature. Once $T_i$ is determined the evolution of the system from the initial to the freeze-out stage is determined by the Eqs. 10, 12 and 13.

In Fig. 1 we display the variation of temperature with proper time. It is clear from the results shown in the inset (Fig. 1) that initial temperature for system which evolves with non-zero viscous effects is lower compared to the ideal case for a fixed $dN/dy$. 

![Image](image_url)
Because for a non-viscous isentropic evolution scenario the multiplicity (measured at the freeze-out point) is fixed by the initial entropy. However, for a viscous evolution scenario the generation of entropy due to dissipative effects contributes to the multiplicity. Therefore, for a given multiplicity (which is proportional to the entropy) at the freeze-out point one requires lower initial entropy, hence initial temperature will be lower. It is also seen (Fig. 1) that the cooling of the system is slower for viscous dynamics because of the extra heat generated during the evolution.

A. Photon spectra

In this section we present the shift in the $p_T$ distribution of the photons due to viscous effects. The integrand in Eq. 8 is a Lorentz scalar, consequently the Lorentz transformation of the integrand from the laboratory to the co-moving frame of the fluid can be effected by just transforming the argument, i.e. the energy of the photon ($E = p_T \cosh(y)$) in the laboratory frame should be replaced by $u_{\mu} p^\mu$ in the co-moving frame of the fluid, where $p^\mu$ is the four momentum of the photon.

The results presented here are obtained with vanishing bulk viscosity. The effects of viscosity enters into the photon spectra through the phase space factor as well as through the space time evolution. We would like to examine these two effects separately. For convenience we define two scenarios: (i) where the effects of viscosity on the phase space factor is included ($\delta f_\eta$ is non-zero in Eq. 8), but the viscous effects on the evolution are neglected ($\eta = 0$ in Eq. 9) and scenario (ii) where the effects of $\eta \neq 0$ are taken into account in the phase space factors as well as in the evolution dynamics. The space time integrated photon yield originating from the QGP in scenario (i) is displayed in Fig. 2. Note that the value of the initial temperatures for the results displayed in Fig. 2 are same (for all $\eta/s$) because the viscous effects on the evolution is ignored in scenario (i). The viscous effects on the $p_T$ distribution of the photons is distinctly visible. The higher values of $\eta/s$ makes the spectra flatter through the $p_T$ dependence of the correction, $\delta f_\eta$. Next we assess the effects of viscosity on photon spectra for scenario (ii). In Fig. 3 we depict the photon spectra for various values of $\eta/s$. In this scenario the value of $T_i$ is lower for higher $\eta/s$ for reasons described above. As a result the enhancement in the photon production due to change in phase space factor, $\delta f_\eta$ is partially compensated by the reduction in $T_i$ for non-zero $\eta$, which is clearly seen in the results displayed in Figs. 2 and 3.

In Figs. 4 and 5 we exhibit results for the hadronic phase for scenarios (i) and (ii) respectively. The effects of dissipation on the $p_T$ distribution of photons from hadronic phase is qualitatively similar to the QGP phase though the effect is stronger in the QGP phase than in the hadronic phase. It is expected that the observed shift in the photon spectra due to viscous effects may be detected in future high precision experiments. Finally in Figs. 6 and 7 we plot the $p_T$ spectra of photons for the entire life time of the thermal system i.e. the photon yield is obtained by summing up contributions from QGP, mixed and hadronic phases for different values of $\eta/s$. The effect of viscosity for the scenario (i) is stronger than (ii).
VI. SUMMARY AND DISCUSSIONS

We have studied the effects of viscosity on the evolving QGP produced in nuclear collisions at RHIC energies. The generation of entropy due to dissipation on the final (experimentally measured) multiplicity has been taken into account. The initial temperature has been constrained by the multiplicity (entropy) at the freeze-out point. The viscous effects on the photon spectra has been introduced consistently through the evolution dynamics and phase space factors of the particles participating in the production processes. It has been noticed that the effects of viscosity on the photons from QGP is stronger than those originating from hadrons. The QGP expected to be formed in Pb+Pb collisions at LHC energy will have longer life time and larger volume than that of RHIC i.e. the evolution dynamics at LHC will be dominated by QGP phase. Therefore, the chance of estimating the value of $\eta/s$ from photon at LHC is brighter. In view of the fact that the future experiments are progressing toward precision measurement the shift in the $p_T$ distribution of photons due to dissipative effects at LHC and RHIC may be detectable.

Before closing this section two comments are in order here. First, as mentioned before for the photon production rate from QGP we have used the Compton and annihilation processes. We have checked that the contribution from these two processes is down by a factor of 3-4 compared to the production rate obtained from the complete calculation of order
Taking these higher order processes into consideration in the present scenario involves a reevaluation of the photon production rates with thermal distribution factors containing viscous corrections. Secondly, we have confined only to the longitudinal flow of the matter in the present work ignoring the transverse kick (blue shift) received by the photons from radial flow [38]. However, both these factors will affect the photon spectra from ideal as well as dissipative scenarios in a similar fashion. Therefore, we expect the shift in the transverse momentum spectra of thermal photons in the presence of dissipative effects which is the main focus of the present work, will be detectable even when a more rigorous photon production rate along with transverse expansion is employed [39].

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Appendix A: Phase Space

In this appendix we derive Eq. 2 from 1. The photon production rate from the process, 1 + 2 → 3 + γ is given by,

\[ E \frac{dR}{dp} = \frac{1}{2(2\pi)^2} \int \int \int \int \frac{d^4p_1}{2E_1} \frac{d^4p_2}{2E_2} \frac{d^4p_3}{2E_1} \]

\[ f_1(E_1)f_2(E_2)[1 \pm f_3(E_3)|M|^2\delta(p_1 + p_2 - p_3 - p)|17] \]

Performing the \(d^4p_3\) integration using the delta function and using \(d^3p/E = p_T dp_T dy d\phi\) we get,

\[ E \frac{dR}{dp} \left[ \frac{1}{16(2\pi)^2} \int \int \int \int p_1 p_2 p_3 \phi_1 \phi_2 \int \int \int \right] \]

\[ f_1(E_1)f_2(E_2)[1 \pm f_3(E_3)|M|^2\delta(E_1 + E_2 - E_3 - E) \] (18)

where \(\phi_1\) and \(\phi_2\) are the angles made by the transverse momenta of first and second particles with the transverse momentum of the emitted photon. The momentum conservation along the \(z\)-direction: \(p_{3z} = p_{1z} + p_{2z} - p_z\) can be written in terms of rapidity as:

\[ m_{3T} \sinh y_3 = m_{1T} \sinh y_1 + m_{2T} \sinh y_2 - p_T \sinh y \] (19)

Now the energy, \(E_3\) can be written as:

\[ E_3 = m_{3T} \cosh y_3 = \sqrt{m_{3T}^2 + m_{3T}^2 \sinh^2 y_3} \] (20)

Substituting Eq. [19] in Eq. [20] we get,

\[ E_3 = \sqrt{[(m_{1T} \sinh y_1 + m_{2T} \sinh y_2 - p_T \sinh y)^2 + m_{3T}^2]} \] (21)

Considering the energy conservation \((E_3 = E_1 + E_2 - E)\) and writing the energies in terms of rapidity \((E_i = m_i \cosh y_i)\) we get,

\[ E_3 = m_{1T} \cosh y_1 + m_{2T} \cosh y_2 - p_T \cosh y \] (22)

Equating Eqs. [21] and [22] we have,

\[ m_{3T} = [m_{1T}^2 + m_{2T}^2 + p_T^2 + 2m_{1T}m_{2T} \cosh(y_1 - y_2) - 2m_{1T}p_T \cosh(y_1 - y) - 2m_{2T}p_T \cosh(y_2 - y)] \] (23)

However, we also have,

\[ m_{3T} = \left(\frac{p_{1T}^2 + p_{2T}^2}{2}\right)^{\frac{1}{2}} \]

\[ = \left[(p_{1T} + p_{2T} - p_T)^2 + m_3^2\right]^{\frac{1}{2}} \]

\[ = \left[p_{1T}^2 + p_{2T}^2 + p_T^2 + 2p_{1T}p_{2T} \cos(\phi_{12}) - 2p_{1T}p_T \cos(\phi_1) - 2p_{2T}p_T \cos(\phi_2) + m_3^2\right]^{\frac{1}{2}} \] (24)

where,

\[ \cos(\phi_{12}) = \cos(\phi_1) \cos(\phi_2) + \sin(\phi_1) \sin(\phi_2) \] (25)

Equating Eq. [23] with Eq. [24] leads to the expression,

\[ \left[(p_{1T} \cos \phi_1 - p_T) \cos \phi_2 + p_{1T} \sin \phi_1 \sin \phi_2 = \frac{1}{2p_T} \left[m_{1T}^2 + m_{2T}^2 - m_3^2 + 2m_{1T}m_{2T} \cosh(y_1 - y_2) - 2m_{1T}p_T \cosh(y_1 - y) - 2m_{2T}p_T \cosh(y_2 - y) + 2p_{1T}p_T \cos(\phi_1) \right] \] (26)

Solving Eq. [26] for \(\phi_2\) one gets,

\[ \phi_2^0 = \tan^{-1}\left(\frac{p_{1T} \sin \phi_1}{p_{1T} \cos \phi_1 - p_T}\right) - \cos^{-1}\left(\frac{H}{2R_{pT}}\right) \] (27)

where,

\[ R = \sqrt{p_{1T}^2 + p_T^2 - 2p_{1T}p_T \cos \phi_1} \] (28)

and,

\[ H = \left(m_{1T}^2 + m_{2T}^2 - m_3^2 \right) + 2m_{1T}m_{2T} \cosh(y_1 - y_2) - 2m_{1T}p_T \cosh(y_1 - y) - 2m_{2T}p_T \cosh(y_2 - y) + 2p_{1T}p_T \cos(\phi_1) \] (29)

Now we express the argument of the delta function in Eq. [18] as function of \(\phi_2\) as

\[ f(\phi_2) = \frac{E_1 + E_2 - E - E_3}{m_{1T} \cosh y_1 + m_{2T} \cosh y_2 - p_T \cosh y - \left|m_{3T}^2 + (m_{1T} \sinh y_1 + m_{2T} \sinh y_2 - p_T \sinh y)\right|^2 \] (30)
and performing the $\phi_2$ integration in Eq. 18 we get,

$$ E \frac{dR}{dp} = \frac{1}{16} \frac{N}{(2\pi)^6} \int_0^\infty p_{1T} dp_{1T} \int_0^\infty dp_{2T} \int_{-\infty}^\infty dy_1 \int_{-\infty}^\infty dy_2 \int_0^{2\pi} d\phi_1 f_1(E_1) f_2(E_2) \left[ 1 \pm f_3(E_3) \right] $$

\[ \frac{|M|^2}{|p_{1T} \sin(\phi_1 - \phi_2) + p_{T} \sin \phi_2|} \]  (31)

with the constraint

$$ |H| \leq 1 \] (32)

originating from $|\cos(\phi)| \leq 1$.

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