Grand unified theory constrained supersymmetry and neutrinoless double beta decay

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(October 12, 2018)

Abstract

We analyze the contributions to the neutrinoless double β decay ($0\nu\beta\beta$-decay) coming from the Grand Unified Theory (GUT) constrained Minimal Supersymmetric Standard Model (MSSM) with trilinear R–parity breaking. We discuss the importance of two-nucleon and pion-exchange realizations of the quark-level $0\nu\beta\beta$-decay transitions. In this context, the questions of reliability of the calculated relevant nuclear matrix elements within the Renormalized Quasiparticle Random Phase Approximation (pn-RQRPA) for several medium and heavy open-shell nuclei are addressed. The importance of gluino and neutralino contributions to $0\nu\beta\beta$-decay is also analyzed. We review the present experiments and deduce limits on the trilinear R-parity breaking parameter $\lambda'_{111}$ from the non-observability of $0\nu\beta\beta$-decay for different GUT constrained SUSY scenarios. In addition, a detailed study of limits on the MSSM parameter space coming from the $B \rightarrow X_s \gamma$ processes by using the recent CLEO and OPAL results is performed. Some studies in respect to the future $0\nu\beta\beta$-decay project GENIUS are also presented.

12.60.Jv,11.30.Er,23.40.Bw
I. INTRODUCTION

The neutrinoless double beta decay \((0\nu\beta\beta-\text{decay})\) is forbidden in the Standard Model (SM) since it violates lepton number by two units \((\Delta L = 2)\). Therefore this decay is a sensitive probe for different aspects of physics beyond SM. (For recent reviews see e.g. [1,2]). Many generalization of the SM admit the violation of laws of the SM to a small extent. In this context, non–observability of the \(0\nu\beta\beta\)-decay is used to constrain different extensions of the SM like those with the left–right symmetry [3,4], leptoquarks [5], R–parity violating supersymmetric (\(R_p\)/SUSY) models [6–12] and composite neutrinos [17,18].

The most widely discussed case in literature has been the upper bound on the light effective electron neutrino Majorana mass \(\langle m_{\nu} \rangle\) deduced from the experimental lower limit on the half-life of \(0\nu\beta\beta\)-decay [12]. Currently, the most restrictive limit on \(\langle m_{\nu} \rangle\) is found from \(0\nu\beta\beta\)-decay in \(^{76}\text{Ge}\) by the Heidelberg–Moscow collaboration [19]: \(\langle m_{\nu} \rangle \leq 0.4 - 1.3\) eV [1]. The uncertainty of this parameter is due to the ambiguity of \(0\nu\beta\beta\)-decay nuclear matrix elements. It is expected that the future double beta decay experiment GENIUS [20] based on 1 ton of enriched \(^{76}\text{Ge}\) would reach the sensitivity for \(\langle m_{\nu} \rangle (0.01 - 0.001\) eV).

Besides the simplest and the best known mechanism of lepton number violation based on the mixing of massive Majorana neutrinos advocated by different variants of the Grand Unified Theories (GUT) the R-parity violation proposed in the context of minimal supersymmetry extensions of the SM (MSSM) is becoming the most popular scenario for lepton number violation (see e.g. reviews [14,15]). The R-parity is a multiplicative quantum number defined as \(R = (-1)^{3B+L+2S}\) with B, L and S being the baryon, the lepton and the spin quantum numbers of the particle, respectively. Thus, all the SM particles have \(R=+1\), while their superpartners have \(R=-1\). We note that the R-parity conservation, which guarantee the baryon and lepton number conservation, is not required by gauge invariance or supersymmetry and might be broken explicitly or spontaneously at the Planck scale [16].

The R–parity can be broken by involving the bilinear and trilinear terms in superpotential of the MSSM. The bilinear terms generate non–zero vacuum expectation value for the sneutrino field, which leads to the lepton number violating interactions based on the neutralino–neutrino and chargino–electron mixing [12]. The trilinear terms represent the interactions, which violate directly the lepton number and lepton flavor [8–12]. Both ways influence the low–energy phenomenology and therefore using some exotic processes like the \(0\nu\beta\beta\)-decay one can impose limits on the parameters connected with the new physics.

Supersymmetric models with R-parity non-conservation (\(R_p/\text{MSSM}\)) have been extensively discussed in the literature (see e.g. [21,22]) and were also used for the study of R-parity violating trilinear term contribution to the \(0\nu\beta\beta\)-decay by Mohapatra [1] and Vergados [4]. The first calculations were concentrated only on the conventional two-nucleon mode of \(0\nu\beta\beta\)-decay, which assumes direct interactions between quarks of two-decaying neutrons [1]. A detail study of this mechanism was carried out in Ref. [3]. By using viable phenomenological assumptions about some of the fundamental parameters of the \(R_p/\text{MSSM}\) (e.g. the ansatz of universal sparticle masses and that the lightest neutralino is bino-like) it was found that the limit on the R-parity violating first generation Yukawa coupling \(\lambda_{111}'\) derived from the observed absence of the \(0\nu\beta\beta\)-decay is more stringent than the corresponding limit expected
from the forthcoming accelerator experiment at HERA. The authors end up with the conclusion that the gluino exchange R-parity violating mechanisms of the $0\nu\beta\beta$-decay dominates over the neutralino ones [8].

Another scenario for reduction of a number of supersymmetric parameters associated with the limit on $\lambda'_{111}$ has been outlined in Ref. [9]. The authors implemented relations among the weak scale values of all parameters entering the superpotential and the soft SUSY breaking Lagrangian and their values at the GUT scale. The obtained results showed the importance of the neutralino exchange mechanism. Similar studies have been performed also within gauge mediated SUSY breaking model [23,24]. We note that the GUT’s constrained SUSY scenarios have advantages of fewer degrees of freedom and usually predict more stringent limits on $\lambda'_{111}$ [23,24].

Recently, the dominance of the pion-exchange R-parity violating $0\nu\beta\beta$-decay mechanism (based on the double-pion exchange between the decaying neutrons) over the conventional two-nucleon one was proven for $^{76}$Ge isotope [10]. We note that the attention to the pion-exchange $0\nu\beta\beta$-decay mechanism was paid out first by Pontecorvo [25] and that this mechanism has been found less important in the presence of light Majorana neutrinos and more important, if heavy Majorana neutrinos are considered [26]. There are two advantages, which favor pion-exchange R-parity violating mode over the two-nucleon mode. First, the effective radius of the two-nucleon R-parity violating interaction is small because of exchange of heavy SUSY particles and therefore this mode is suppressed by the nucleon-nucleon repulsion at short distances. On the other side the pion-exchange mode leads to a long-range nuclear interaction, which is significantly less sensitive to short–hand correlations effects, as is mediated by light particles. Secondly, the enhancement of the pion-exchange mode has an origin in the bosonization of the $\pi^- \rightarrow \pi^+ + 2e^-$ vertex and is associated with the pseudoscalar hadronic current structure of the effective R-parity violating $0\nu\beta\beta$-decay Lagrangian on quark level [11,12]. The new pion-exchange mechanism of $0\nu\beta\beta$-decay offers more stringent limits on $\lambda'_{111}$ [10]. The one- and two-pion exchange realization of this mode have been discussed for experimentally interesting nuclear systems in Ref. [12]. Both gluino and neutralino exchange mechanisms have been found of comparable importance within the phenomenological scenario of this mode.

The procedure leading to the final constraint on $\lambda'_{111}$ consists of two parts: In the first step, the relevant SUSY parameters are determined within a proper SUSY scenario. Next, the reliable evaluation of $0\nu\beta\beta$-decay nuclear matrix elements has to be performed. Since the importance of the particle-particle residual interaction of the nuclear Hamiltonian for the description of open shell nuclear system was discovered [27], the proton-neutron Quasi-particle Random Phase Approximation (pn-QRPA) has been widely chosen in the calculations of the nuclear double beta decay transitions [12,28,35]. However, the extreme sensitivity of the calculated nuclear matrix elements as well as the collapse of the QRPA solution in the physically acceptable region of particle-particle strength address some questions about the predictive power of the obtained results [36,37]. The renormalized pn-QRPA (pn-RQRPA) without [38] and with proton-neutron pairing [39], which take into account the Pauli exclusion principle, do not collapse and offer more reliable results in respect to the QRPA [38,37]. This method has been used also in the recent nuclear structure studies of the $0\nu\beta\beta$-decay matrix elements [1,10,12,36]. Some other extensions of RQRPA can be also addressed in this context [13].
The goal of the present paper is to perform a comprehensive analysis of the R-parity violating trilinear term contribution to $0\nu\beta\beta$-decay within the GUT constrained MSSM. We shall focus our attention on a detailed pn-RQRPA study of the relative importance of the pion-exchange and two-nucleon modes of this process for experimentally interesting $0\nu\beta\beta$-decay nuclei. Our aim is also to discuss the present as well as possible future (imposed by GENIUS experiment) limit on the trilinear breaking SUSY parameter $\lambda'_{111}$ and identify this limit with the gluino or the neutralino exchange mechanisms. Moreover, all the calculations are performed within MSSM constrained by different processes, among which the most interesting are limits coming from $B \to X_s\gamma$ processes obtained by using the recent CLEO and OPAL results.

The paper is arranged as follows. In the next section (section II) we introduce the basic elements of the MSSM with the explicit R-parity breaking. Here we also discuss the main formulae relevant for the trilinear R-parity breaking contribution to $0\nu\beta\beta$-decay and the realistic pn–RQRPA nuclear structure method, which will be used for evaluation of nuclear matrix elements of interest. In section III we calculate the $0\nu\beta\beta$-decay matrix elements within the pn–RQRPA for experimentally interesting isotopes and analyze their uncertainties in respect to details of the nuclear model. In addition, we discuss the parameters of the MSSM and the importance of the gluino and the neutralino $0\nu\beta\beta$-decay mechanisms. The experimental constraints on the $0\nu\beta\beta$-decay are then used to constrain the first generation of lepton number violating Yukawa coupling constant $\lambda_{111}$ of supersymmetric particles. We close with a short summary and conclusions (section V).

II. THEORY

A. R-parity violating minimal supersymmetric standard model

The MSSM is based on the same gauge group as the SM. It is particle content required to implement supersymmetry in a consistent way is the minimal one. It is described by the superpotential which in the case of R-parity non-conservation contains both the R-parity conserving and breaking parts

$$W = W_0 + W_R,$$

where

$$W_0 = h_{ij}^{u} \hat{Q}_i \hat{H}_u \hat{d}_j^c + h_{ij}^{d} \hat{Q}_i \hat{H}_d \hat{d}_j^c + h_{ij}^{e} \hat{L}_i \hat{H}_d \hat{e}_j^c + \mu \hat{H}_d \hat{H}_u. \quad (2)$$

and

$$W_R = \lambda_{ijk} \hat{L}_i \hat{L}_j \hat{d}_k^c + \lambda'_{ijk} \hat{L}_i \hat{Q}_j \hat{d}_k^c + \lambda''_{ijk} \hat{Q}_i \hat{d}_j^c \hat{d}_k^c + \mu_j \hat{L}_j \hat{H}_u. \quad (3)$$

Here $\hat{Q}, \hat{L}$ denote the quark and lepton SU(2) doublet superfields, $\hat{u}^c, \hat{d}^c, \hat{e}^c$ the corresponding SU(2) singlets and $\hat{H}_u, \hat{H}_d$ the Higgs superfields whose scalar components give mass to up- and down-type quarks and leptons. In the R–parity breaking part we set $\lambda_{ijk} = \lambda'_{ijk} = 0$ to avoid the unsuppressed proton decay.

Since supersymmetry in the low-energy world is broken, the Lagrangian of the theory is supplemented with the "soft" supersymmetry breaking terms: They are given by:
\begin{align}
- \mathcal{L}_{\text{soft}} &= \left( A_{(i)j}^{B} h_{ij}^{B} Q_{i} H_{u} u_{j}^{c} + A_{(i)j}^{B} h_{ij}^{D} \bar{Q}_{i} H_{d} d_{j}^{c} + A_{(i)j}^{E} h_{ij}^{E} \bar{L}_{i} H_{d} c_{j}^{c} + \text{H.c.} \right) \\
&+ B\mu (H_{u} H_{u} + \text{H.c.}) + m_{H_{u}}^{2} |H_{u}|^{2} + m_{H_{d}}^{2} |H_{d}|^{2} \\
&+ \left( \frac{1}{2} M_{1} \bar{\psi}_{B} \psi_{B} + \frac{1}{2} M_{2} \bar{\psi}_{W} \psi_{W} + \frac{1}{2} m_{3} \bar{\psi}_{g} \psi_{g} + \text{H.c.} \right) \tag{4}
\end{align}

and

\begin{align}
- \mathcal{L}_{\text{soft.R}} &= \tilde{\lambda}_{ijk} \tilde{L}_{i} \tilde{L}_{j} \tilde{u}_{k}^{c} + \tilde{\lambda}'_{ijk} \tilde{L}_{i} \tilde{Q}_{j} \tilde{d}_{k}^{c} + \tilde{\lambda}''_{ijk} \tilde{u}_{i}^{c} \tilde{d}_{j}^{c} \tilde{d}_{k}^{c} + \tilde{\mu}_{2j}^{2} \tilde{L}_{j} \tilde{H}_{u} + \tilde{\mu}_{1j} \tilde{L}_{j} \tilde{H}_{d} \tag{5}
\end{align}

Here, the fields with tilde denote the scalar partners of the quark and lepton fields, while the \( \psi_i \) are the spin-\( \frac{1}{2} \) partners of gauge bosons.

In the present paper we concentrate on the trilinear terms R-parity breaking superpotential only, leaving complete treatment of bilinear terms for future studies.

Since our main goal is the R-parity MSSM description of the \( 0\nu\beta\beta \) decay, we have to derive the effective Lagrangian for that process from the superpotential. We will show below that this Lagrangian depends on many free supersymmetric parameters, like masses or coupling constants. In order to reduce the number of these parameters, we apply conditions motivated by supergravity theories unification for soft mass parameters at the GUT scale (see e.g. [10, 12]). We shortly present our method of finding the low-energy spectrum of the MSSM below.

We begin our procedure with running the gauge and Yukawa coupling constants up to the point where they unify (GUT scale). The initial conditions for the gauge couplings are set up by the values of \( \alpha_{em}(m_{Z}) = 1/127.89 \) and \( \alpha_s = 0.118 \). The values of Yukawa couplings at \( m_{Z} \) are connected with the standard model quark and lepton mass matrices in the usual way:

\begin{align}
M_{U} &= \frac{v \sin \beta}{\sqrt{2}} S^{U_{R}} Y_{U}^{T} S^{U_{L}}, \\
M_{D} &= \frac{v \cos \beta}{\sqrt{2}} S^{D_{R}} Y_{D}^{T} S^{D_{L}}, \\
M_{E} &= \frac{v \cos \beta}{\sqrt{2}} S^{E_{R}} Y_{E}^{T} S^{E_{L}} \tag{6}
\end{align}

Here, the unitary matrices \( S \) are connected with mixing among the MSSM fields of the matter sector induced by the electroweak symmetry breaking and \( K = S^{U_{L}} S^{D_{L}} \) is the Kobayashi-Maskawa matrix. In the matter sector primed mass eigenstates are related to their unprimed gauge eigenstate counterparts.

\begin{align}
u' &= S^{U_{L}} \nu, \\
qu' &= S^{U_{R}} c \nu^{T},
\end{align}

\begin{align}
d' &= S^{D_{L}} d, \\
e' &= S^{D_{R}} c e^{T},
\end{align}

\begin{align}
u' &= S^{N_{L}} \nu. \tag{7}
\end{align}

In above formulas, the following convention for the MSSM up an down-type Higgs vacuum expectation values are used:
\begin{align}
\langle H_d^0 \rangle = \frac{v_d}{\sqrt{2}}, \quad \langle H_u^0 \rangle = \frac{v_u}{\sqrt{2}}, \quad v = \sqrt{v_u^2 + v_d^2} = 246 \text{ GeV}, 
\end{align}

During the running procedure, the renormalization group equations (RGE’s) change between the weak and GUT scales at each particles mass threshold due to decoupling of states at the scales below their masses. At first running, we initially set this threshold at \( M_{SU3Y} = 1\text{ TeV} \), using the SM 2-loop RG equations below, and the MSSM RGE’s above that scale. When the gauge and Yukawa couplings get the GUT scale, we set all the soft scalar masses equal to the common scalar mass \( m_0 \), the soft trilinear couplings \( A \) to common \( A_0 \) and take gaugino masses to \( m_{1/2} \). In the next step one runs everything down back to the \( m_Z \) scale. We want to stress at this point that we do not use the full set of RGE’s appropriate for the R–parity broken MSSM \[43\]. We estimate that an influence of the R–parity breaking on other quantities running is marginal due to the smallness of \( \lambda \)’s. So, we limit our attention to RGE’s for MSSM with R–parity conserved \[44\].

It is well known that running of \( m^2_{H_u} \) is dominated by negative contribution from the top Yukawa coupling, which drives this parameter to a negative value at some scale causing dynamical electroweak symmetry breaking (EWSB). Thus the RGE’s improved supersymmetric potential naturally breaks \( SU(2) \times U(1)_Y \) to \( U(1)_{em} \), which additionally allows to express some GUT-scale free parameters in terms of low-energy ones. The tree-level Higgs potential has the form

\begin{align}
V_0 &= m^2_1 |H_d^0|^2 + m^2_2 |H_u^0|^2 + m^2_3 (H_d^0 H_u^0 + \text{H.c.}) + \frac{g^2_1 + g^2_2}{8} (|H_d^0|^2 - |H_u^0|^2)^2,
\end{align}

where \( m^2_1 \equiv m^2_{H_d} + \mu^2, m^2_3 \equiv B \mu \), and the phases of the fields are chosen such that \( m^2_3 < 0 \). Due to \( Q \) scale dependence of the soft Higgs parameters \( V_0 \) also strongly depends on \( Q \). Thus minimization of \( V_0 \) can produce very different vacuum expectation values \( v_u, v_d \) depending on \( Q \). That’s why it is necessary to minimize the full one-loop Higgs effective potential. The procedure leads to a set of two equations.

\begin{align}
|\mu|^2 + \frac{m^2_Z}{2} &= \frac{(m^2_{H_d} + \Sigma_d) - (m^2_{H_u} + \Sigma_u) \tan^2 \beta}{\tan^2 \beta - 1}, \\
\sin 2\beta &= \frac{-2 \mu B}{(m^2_{H_u} + \Sigma_u) + (m^2_{H_d} + \Sigma_d) + 2|\mu|^2}.
\end{align}

\( \Sigma_u, \Sigma_d \) are given e.g. in Ref. \[43\]. In order to minimize the stop contribution to the finite corrections, we take the minimization scale \( Q_{min} \) equal to the square root of mean square of stop masses.

The EWSB causes mixing among many particles. In particular, four gauginos mix to produce the so-called neutralinos with the mass matrix \( M_x \), which in the \( \psi = (\tilde{B}, \tilde{W}_3, \tilde{H}_1, \tilde{H}_2) \) basis has the form:

\begin{align}
M_x &= \begin{pmatrix}
M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W \\
0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W \\
-m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu \\
m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu & 0
\end{pmatrix},
\end{align}

with \( c_w = \cos(\theta_W), s_w = \sin(\theta_W), s_\beta = \sin(\beta), c_\beta = \cos(\beta) \) and \( M_1, M_2 \) being \( U(1) \) and \( SU(2) \) gaugino masses. At this point it is worth mentioning that the physical gluino mass is
related to the renormalized $\overline{D}R$ running mass $m_3$. For very heavy quarks this dependence can read [16]

$$m_3^{\text{pole}} \simeq m_3 \left[ 1 + \frac{\alpha_3}{4\pi} \left( 15 + 12I(r) \right) \right], \quad (13)$$

where $m_3$ and $\alpha_3$ are taken at $m_3$ and the loop function $I(r) = \frac{1}{2} \ln r + \frac{1}{2}(r-1)^2 \ln(1-r^{-1}) + \frac{1}{2} - 1$ for $r \geq 1$ with $r = m_3^2/m_3^2$.

Mixing in the gaugino sector effects with four physical neutralinos $\chi_i$.

$$\chi_i = \sum_{j=1}^{4} N_{ij} \psi_j, \quad (i = 1, 2, 3, 4). \quad (14)$$

The matrix $N$, diagonalizing the matrix $M_\chi$ is real and orthogonal. Thus, with real $N_{ij}$, neutralino masses are either positive or negative. If necessary, a negative mass can always be made positive by a redefinition of the relevant mixing coefficients $N_{ij} \rightarrow iN_{ij}$.

Similar mixing appears in the slepton and squark sector. The SUSY analog of mixing (11) in the SM sector is described as follows:

$$\tilde{u}' = \Gamma^U \left( \begin{array}{c} S_{UL} \tilde{u} + M^2_U \\ -\mu_M \cot \beta S_{UR} \tilde{u}^* \end{array} \right),$$

$$\tilde{d}' = \Gamma^D \left( \begin{array}{c} S_{DL} \tilde{d} \\ -\mu_M \tan \beta \end{array} \right),$$

$$\tilde{e}' = \Gamma^E \left( \begin{array}{c} S_{EL} \tilde{e} + M^2_E \\ -\mu_M \tan \beta \end{array} \right),$$

$$\tilde{\nu}' = \Gamma^N S_{EL} \tilde{\nu}, \quad (15)$$

The $6 \times 6$ squared mass matrices for the squarks and sleptons have a much more complicated form and involve parameters from both the supersymmetry breaking and conserving Lagrangians. They can be written in the following form:

$$M^2_{\tilde{u}} = \Gamma^U \left( \begin{array}{cc} S_{UL}^2 m_Q^2 + M_U^2 + \frac{m_3^2}{6} (3 - 4 \sin^2 \theta_W) \cos 2\beta \\ -\mu_M \cot \beta S_{UR} \tilde{u}^* S_{UL} \end{array} \right) \Gamma^U, \quad (16)$$

$$M^2_{\tilde{d}} = \Gamma^D \left( \begin{array}{cc} S_{DL}^2 m_Q^2 + M_D^2 + \frac{m_3^2}{6} (3 - 2 \sin^2 \theta_W) \cos 2\beta \\ -\mu_M \tan \beta S_{DR} \tilde{d}^* S_{DL} \end{array} \right) \Gamma^D, \quad (17)$$

$$M^2_{\tilde{e}} = \Gamma^E \left( \begin{array}{cc} S_{EL}^2 m_Q^2 + M_E^2 + \frac{m_3^2}{2} (1 - 2 \sin^2 \theta_W) \cos 2\beta \\ -\mu_M \tan \beta S_{ER} \tilde{e}^* S_{EL} \end{array} \right) \Gamma^E, \quad (18)$$
When the physical masses of scalar particles and tree level values of $\mu$ and $B\mu$ are found, one can set up thresholds for RGE’s of the gauge and Yukawa couplings and repeat the procedure by running everything up to the GUT scale. Setting the unification conditions for soft parameters again, it is possible to run everything back to the $m_Z$ scale. At that point we repeat calculation of mass eigenstates, run everything up to $Q_{\text{min}}$ and minimize the one-loop corrected Higgs potential mentioned above. Repeating such a procedure leads to fixed values of $\mu$ and $B\mu$. At the end, we check for conditions necessary for existence of a stable scalar potential minimum:

$$(\mu B)^2 > \left(|\mu|^2 + m_{H_u}^2 \right) \left(|\mu|^2 + m_{H_d}^2 \right),$$

$$2B\mu < 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2.$$ (19)

**B. Constraints**

Having obtained the low energy spectrum of the model, we are at a good point to test our scenario using constraints due to rare Flavor Changing Neutral Currents processes. It is well known that FCNC processes may serve as a strong constraint on the supersymmetric scenarios. Strong experimental suppression of FCNC transition puts upper bounds on various entries of the sfermion mass matrices at low energy. Many analyses of such constraints were performed for the SUGRA MSSM [17]. Some of them are also available for the Gauge Mediated Supersymmetry Breaking (GMSB) [18]. Without going into details of calculations, we summarize the basic ingredients of the applied procedure.

In our considerations we concentrate on limits on the low-energy spectrum coming from the $B \rightarrow X_s \gamma$ decay. This process is described by an effective Hamiltonian of the form [17,50]:

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} K^*_{ts} K_{tb} \sum_{i=1}^{8} C_i(\mu) P_i(\mu),$$ (20)

where $K$ is the Kobayashi-Maskawa matrix, $P_i$ are the relevant operators, and $C_i(\mu)$ are their Wilson coefficients. The coefficients $C_7$ and $C_8$ relevant in the subsequent analysis get contributions from both the SM and the MSSM interactions. The leading order and next-to-leading order SM contributions are discussed in [19]. For the MSSM case, only leading order contributions to those coefficients are available [17,50].

The constraints on the low-energy spectrum of our model are connected with present constraints on the $R_7$ parameter which measures the extra (MSSM) contributions to the $B \rightarrow X_s \gamma$ decay. Its definition reads:

$$R_7 \equiv 1 + \frac{C_7^{(0)\text{extra}}(m_W)}{C_7^{(0)\text{SM}}(m_W)}.$$ (21)

\footnote{NLO QCD corrections for SUSY scenario with charginos and one of the stops lighter than other sparticles can be found in [51].}
where index (0) stands for the leading order (LO) Wilson coefficients, and superscript extra for the SUSY (namely charged Higgs, chargino, neutralino and gluino) contributions.

Limits on the allowed values of $R_7$ are extracted from the present experimental limits on the branching ratio $\text{BR}(B \rightarrow X_s \gamma)$ recently measured by CLEO collaboration [21]: $\text{BR}(B \rightarrow X_s \gamma) = (2.50 \pm 0.47_{\text{stat}} \pm 0.39_{\text{syst}}) \times 10^{-4}$. The same process measured at ALEPH collaboration results with $\text{BR}(B \rightarrow X_s \gamma) = (3.11 \pm 0.80_{\text{stat}} \pm 0.72_{\text{syst}}) \times 10^{-4}$ [22]. Using the theoretical dependence of the theoretical expectations of $\text{BR}(B \rightarrow X_s \gamma)$ on $R_7$, comparison with the experimental data [50] and taking into account both theoretical and experimental errors, we end up with the following estimation of the $R_7$ range:

$$-6.6 < R_7 < -4.4 \text{ or } 0.0 < R_7 < 1.3.$$ (22)

In order to find numerical values of $R_7$ given in (21), we use the expressions for $C_7^{(0)\text{MSSM}}$ and $C_7^{(0)\text{SM}}$ listed in e.g. [47]. Then we exclude points which do not lie in the allowed region (22). In addition to $B \rightarrow X_s \gamma$ constraints, the free parameter space is limited by the conditions of proper electroweak symmetry breaking (19) and the condition of positive $(\text{mass})^2$ of mass eigenstates. In the last section we present the resulting constraints in tan $\beta - m_0$ and tan $\beta - m_{1/2}$ planes for $\mu$ of different signs. Moreover, the detailed analysis of $B \rightarrow X_s \gamma$ constraints is shown.

C. R-parity violating neutrinoless double beta decay

In this Subsection the main formulae relevant for R-parity violating $0\nu\beta\beta$-decay are presented. A detailed derivation of the effective $R_p$ quark-level Lagrangian for the $0\nu\beta\beta$-decay and its hadronization additionaly to derivation of the corresponding half-life time of this process can be found elsewhere Refs. [8–11].

The trilinear R-parity violating lepton part of the interaction Lagrangian of the MSSM takes the form [8]:

$$L_{\lambda_{111}} = -\lambda_{111}^{(0)}[\bar{u}_L, \bar{d}_L] \left( \begin{array}{c} e_R^c \\ -\nu_R^c \end{array} \right) \tilde{u}_R + (\bar{e}_L, \bar{\nu}_L) d_R \left( \begin{array}{c} \tilde{u}_L^* \\ -\tilde{d}_L^* \end{array} \right) + (\bar{u}_L, \bar{d}_L) d_R \left( \begin{array}{c} \tilde{e}_L^* \\ -\tilde{\nu}_L^* \end{array} \right) + \text{H.c.}. \right]$$ (23)

Together with the R-parity conserving MSSM Lagrangian describing interactions among gluinos, neutralinos, fermions and sfermions [53] after integration out of heavy degrees of freedom and carrying a Fierz transformation, one can obtain finally the $R_p$ SUSY induced quark-lepton interaction for $0\nu\beta\beta$-decay.

$$L_{\Delta e}^{\text{eff} = 2} = \frac{G_F^2}{2m_p} \bar{e}(1 + \gamma_5)e^e \left[ \eta_{PS} J_{PS} J_{PS} - \frac{1}{4} \eta_T J_T^{\mu\nu} J_{T\mu\nu} \right].$$ (24)

The color singlet hadronic currents in Eq. (24) are $J_{PS} = \bar{u}\sigma^{\mu\nu}d + \bar{u}\sigma^{\mu\nu}d$, $J_T^{\mu\nu} = \bar{u}\sigma^{\mu\nu}(1 + \gamma_5)d_a$, where $a$ is a color index and $\sigma^{\mu\nu} = (i/2)[\gamma^{\mu}, \gamma^{\nu}]$. The effective lepton-number violating parameters $\eta_{PS}$ and $\eta_T$ in Eq. (24) accumulate fundamental parameters of the $R_p$ MSSM.
\[\eta_{PS} = \eta_{\chi\bar{e}} + \eta_{\chi\bar{f}} + \eta_{\chi} + \eta_\tilde{g} + 7\eta'_\tilde{g}, \quad (25)\]
\[\eta_T = \eta_{\chi} - \eta_{\chi\bar{f}} + \eta_\tilde{g} - \eta'_\tilde{g}, \quad (26)\]

with

\[
\begin{align*}
\eta_\tilde{g} &= \frac{\pi\alpha_s}{6} \frac{\lambda_{11}^2}{G_F^2 m^2_\tilde{g}} \frac{m_p}{m_\tilde{g}} \left[ 1 + \left( \frac{m_{\tilde{d}_R}}{m_{\tilde{u}_L}} \right)^4 \right], \\
\eta_\chi &= \frac{\pi\alpha_2}{2} \frac{\lambda_{11}^2}{G_F^2 m^2_{\tilde{d}_R}} \sum_{i=1}^4 \frac{m_{\tilde{d}_R}}{m_{\chi_i}} \left[ \epsilon_{R_i}(d) + \epsilon_{L_i}(u) \left( \frac{m_{\tilde{d}_R}}{m_{\tilde{u}_L}} \right)^4 \right], \\
\eta_{\chi\bar{e}} &= 2\pi\alpha_2 \frac{\lambda_{11}^2}{G_F^2 m^2_{\tilde{d}_R}} \sum_{i=1}^4 \frac{m_{\tilde{d}_R}}{m_{\chi_i}} \epsilon_{L_i}(e) \frac{m_p}{m_{\tilde{u}_L}} ^2 , \\
\eta_{\chi\bar{f}} &= \frac{\pi\alpha_2}{12} \frac{\lambda_{11}^2}{G_F^2 m^2_{\tilde{d}_R}} \frac{m_p}{m_\tilde{g}} \left( \frac{m_{\tilde{d}_R}}{m_{\tilde{u}_L}} \right)^2 , \\
\eta'_{\tilde{g}} &= \frac{\pi\alpha_2}{2} \frac{\lambda_{11}^2}{G_F^2 m^2_{\tilde{d}_R}} \frac{m_p}{m_\tilde{g}} \left( \frac{m_{\tilde{d}_R}}{m_{\tilde{u}_L}} \right)^2 \sum_{i=1}^4 \frac{m_p}{m_{\chi_i}} \left[ \epsilon_{R_i}(d) \epsilon_{L_i}(e) + \epsilon_{L_i}(u) \epsilon_{R_i}(d) \left( \frac{m_{\tilde{d}_R}}{m_{\tilde{u}_L}} \right)^2 + \epsilon_{L_i}(u) \epsilon_{L_i}(e) \left( \frac{m_{\tilde{d}_R}}{m_{\tilde{u}_L}} \right)^2 \right]. \tag{27}
\end{align*}
\]

In Eq. \((27)\) \(G_F\) is the Fermi constant, \(m_p\) is the proton mass, \(\alpha_2 = g_2^2/(4\pi)\) and \(\alpha_s = g_s^2/(4\pi)\) are SU(2)_L and SU(3)_c gauge coupling constants respectively. \(m_{\tilde{u}_L}, m_{\tilde{d}_R}, m_{\tilde{g}}\) and \(m_{\chi_i}\) are masses of the u-squark, d-squark, gluino and neutralinos. We note that the matrix \(N_{ij}\) (see Eq. \((14)\)) rotates the 4 \(\times\) 4 neutralino mass matrix in Eq. \((12)\) to obtain the diagonal form \(\text{Diag}[m_\chi]\). We used the neutralino couplings in the form proposed in Ref. \([53]\):

\[
\begin{align*}
\epsilon_{L_i}(\phi) &= -T_3(\phi)N_{i2} + \tan\theta_W[T_3(\phi) - Q(\phi)]N_{i1}, \\
\epsilon_{R_i}(\phi) &= Q(\phi)\tan\theta_W N_{i1}. \tag{28}
\end{align*}
\]

The effective \(\Delta L_e = 2\) Lagrangian \((24)\) contains contributions from both gluino (SUSY parameters \(\eta_\tilde{g}\) and \(\eta'_\tilde{g}\)) and neutralino (SUSY parameters \(\eta_\chi, \eta_{\chi\bar{f}}, \eta_{\chi\bar{e}}\)) exchanges. The relevant Feynman diagrams associated with gluino \(\tilde{g}\) and neutralino \(\chi\) contributions to the \(0\nu\beta\beta\)-decay have been discussed in Refs. \([1,8]\).

The \(\bar{R}_p\)MSSM model gives the underlying transition of a down-quarks to an up-quarks (\(dd \rightarrow uu + 2e^-\)) only, and results in transformation of neutron into a proton. Till now three possibilities of hadronization have been considered. The most natural way is to incorporate the quarks in the nucleons which is the well-known two-nucleon mode \([1,8]\). But the intermediate SUSY partners are very heavy particles. Therefore in the two-nucleon mode the two decaying neutrons must come very close to each other, which is suppressed by the nucleon repulsion. Another possibility is to incorporate quarks undergoing the \(\bar{R}_p\)SUSY transition not in nucleons but in virtual pions \([1,10,12]\). If only one of the initial quark is placed in an intermediate pion, we end up with the one pion-exchange mode \([12]\). If all quarks are placed in two intermediate pions, one obtains the two-pion exchange mode, which dominates for the \(0\nu\beta\beta\)-decay of \(^{76}Ge\) \([10,12]\). The possibilities of incorporating quarks in heavier mesons have
not been considered so far. However, this way of hadronization is expected to be suppressed
due to heavier masses of exchange particles.

The half-life for the neutrinoless double beta decay regarding the above three possibilities
of hadronization of the quarks can be written in the form \[1,10,12\]

\[
[T_{1/2}^{0\nu}(0^+ \to 0^+)]^{-1} = G_{01} \left| \eta_T \mathcal{M}_N^{2N} + \left( \eta_{PS} - \eta_T \right) \mathcal{M}_f^{2N} + \frac{3}{8} \left( \eta_T + \frac{5}{8} \eta_{PS} \right) \mathcal{M}_{\pi N}^2 \right|^2.
\]

(29)

where

\[
\mathcal{M}_N^{2N} = c_A \left[ \alpha_{V,A}^{(0)} \mathcal{M}_{FN} + \alpha_{A}^{(0)} \mathcal{M}_{GTN} + \alpha_{V,A}^{(1)} \mathcal{M}_{F'} + \alpha_{A}^{(1)} \mathcal{M}_{GT'} + \alpha_{T}^{(0)} \mathcal{M}_{T'} \right],
\]

(30)

\[
\mathcal{M}_f^{2N} = c_A \left[ \alpha_{V,f}^{(0)} \mathcal{M}_{FN} + \alpha_{A,f}^{(0)} \mathcal{M}_{GTN} + \alpha_{V,A}^{(1)} \mathcal{M}_{F'} + \alpha_{A,A}^{(1)} \mathcal{M}_{GT'} + \alpha_{T,A}^{(1)} \mathcal{M}_{T'} \right],
\]

(31)

\[
\mathcal{M}_{\pi N}^2 = c_A \left[ \frac{4}{3} \alpha^{1\pi} \left( M_{GT-1\pi} + M_{T-1\pi} \right) + \alpha^{2\pi} \left( M_{GT-2\pi} + M_{T-2\pi} \right) \right] \]

(32)

with \(c_A = m_A^2/(m_pm_e)\). Here \(G_{01}\) is the standard phase space factor (see Ref. \[3,13\]) and \(m_A = 850\) MeV is the nucleon form factor cut-off (for all nucleon form factors the same cut-off is considered). The nucleon structure coefficients \(\alpha\)'s entering the
nuclear matrix elements of the two-nucleon mode calculated within the non-relativistic quark
model (NR) and the bag model (BM) are given in Table 3 of Ref. \[1\]. The structure coefficient
of the one-pion \(\alpha^{1\pi}\) and two-pion mode \(\alpha^{2\pi}\) are \[10,12\]: \(\alpha^{1\pi} = -0.044\) and \(\alpha^{2\pi} = 0.20\). The partial nuclear matrix elements of the \(R_p\) SUSY mechanism for the \(0\nu\beta\beta\)-decay appearing
in Eqs. \[30]-\[32\] are:

\[
\mathcal{M}_I = \langle 0^+_f | \sum_{i \neq j} \tau_{i}^+ \tau_{j}^+ \frac{R_0}{r_{ij}} F_I(x) \ | 0^+_i \rangle,
\]

\[
\mathcal{M}_J = \langle 0^+_f | \sum_{i \neq j} \tau_{i}^+ \tau_{j}^+ \frac{R_0}{r_{ij}} F_J(x) \sigma_i \cdot \sigma_j \ | 0^+_i \rangle,
\]

\[
\mathcal{M}_K = \langle 0^+_f | \sum_{i \neq j} \tau_{i}^+ \tau_{j}^+ \frac{R_0}{r_{ij}} F_K(x) S_{ij} \ | 0^+_i \rangle,
\]

(33)

where \(I = FN, F', J = GTN, GT', GT-k\pi\) and \(K = T', T-k\pi (k = 1, 2)\). The structure
functions \(F_{I,J,K}\) can be expressed as:

\[
F_{FN}(x) = F_{GTN}(x) = \frac{x_A}{48} (3 + 3x_A + x_A^2) e^{-x_A},
\]

\[
F_{F'}(x) = F_{GT'}(x) = \frac{x_A}{48} (3 + 3x_A - x_A^2) e^{-x_A},
\]

\[
F_T(x) = \frac{x_A^2}{48} e^{-x_A},
\]

\[
F_{GT-1\pi}(x_\pi) = e^{-x_\pi},
\]

\[
F_{T-1\pi}(x_\pi) = (3 + 3x_\pi + x_\pi^2) e^{-x_\pi},
\]

\[
F_{GT-2\pi}(x_\pi) = (x_\pi - 2) e^{-x_\pi},
\]

\[
F_{T-2\pi}(x_\pi) = (x_\pi + 1) e^{-x_\pi}.
\]

(34)
and we used the notations:  
\[ \hat{r}_{ij} = \frac{\mathbf{r}_i - \mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|}, \quad \mathbf{r}_{ij} = |\mathbf{r}_i - \mathbf{r}_j|, \quad x_A = m_A r_{ij}, \quad x_\pi = m_\pi r_{ij} \]
and  
\[ S_{ij} = 3 \sigma_i \cdot \hat{r}_{ij} \sigma_j \cdot \hat{r}_{ij} - \sigma_i \cdot \sigma_j. \]
Here \( \mathbf{r}_i \) is the coordinate of the i-th nucleon and \( R = r_0 A^{1/3} \) stands for the mean nuclear radius with \( r_0 = 1.1 \text{ fm} \).

In order to deduce constraints on the lepton number violating parameters \( \eta_{PS} \) and \( \eta_T \) from the non-observability of \( 0\nu\beta\beta \)-decay it is necessary to evaluate the nuclear matrix elements of two-nucleon \( (\mathcal{M}_q^{2N}, \mathcal{M}_f^{2N}) \) and pion-exchange \( (\mathcal{M}_\pi^{2N}) \) modes within an appropriate nuclear model.

III. NUCLEAR MODEL

We calculate the nuclear matrix elements within the proton-neutron renormalized Quasi-particle Random Phase Approximation (pn-RQRPA) \[38,39\], which is an extension of the pn-QRPA \[27,28\] by incorporating the Pauli exclusion principle for the fermion pairs. The advantage of the pn-RQRPA over the usual pn-QRPA is no collapsing RQRPA solution as one increases the strength of the particle-particle interaction within its physical values. Thus the results obtained by the pn-RQRPA method are more reliable, but the pn-RQRPA method requires coupled non-linear equation solutions, instead of the usual eigenvalue problem in the pn-QRPA formalism.

The pn-RQRPA method is suitable to deal with the nuclear structure aspects of beta transitions of open shell systems and allows to perform calculations in realistic large model spaces, which are unaccessible for the shell model calculations.

The pn-RQRPA formalism consists of two main steps: (i) The Bogoliubov transformation smears out the nuclear Fermi surface over a relatively large number of orbitals and (ii) the equation of motion in the quasiparticle basis determines then the excited states.

If only proton–proton and neutron–neutron pairing is considered the particle \((c^\tau_{\tau_m^\tau}, \tau = p, n)\) and quasiparticle \((a^\tau_{\tau_m^\tau}, \tau = p, n)\) creation and annihilation operators for the spherical shell model states are related to each other by the Bogoliubov-Valatin transformation:

\[
\begin{pmatrix}
  c^\tau_{\tau_m^\tau} \\
  \tilde{c}^\tau_{\tau_m^\tau}
\end{pmatrix} =
\begin{pmatrix}
  u_\tau & -v_\tau \\
  v_\tau & u_\tau
\end{pmatrix}
\begin{pmatrix}
  a^\tau_{\tau_m^\tau} \\
  \tilde{a}^\tau_{\tau_m^\tau}
\end{pmatrix},
\]  
(35)

where the tilde indicates the time-reversal operation \( \tilde{a}_{\tau_m^\tau} = (-1)^{j_r-m_r} a_{\tau_m^\tau}. \) The occupation amplitudes \( u \) and \( v \) and the single quasiparticle energies \( E_{\tau} \) are obtained by solving the BCS equation. Then one gets a nuclear Hamiltonian in quasiparticle representation

\[
H = \sum_{\tau_m^\tau} E_{\tau} a^\tau_{\tau_m^\tau} a^\tau_{\tau_m^\tau} + H_{22} + H_{40} + H_{04} + H_{31} + H_{13},
\]  
(36)

where \( H_{ij} \) is the normal ordered part of the residual interaction with \( i \) creation and \( j \) annihilation operators (see e.g. Ref. \[54\]).

In the framework of the pn-RQRPA the \( m^{th} \) excited state with the angular momentum \( J \) and the projection \( M \) is created by a phonon-operator \( Q^{m\dagger}_{JM^*} \)

\[
|m, JM\rangle = Q^{m\dagger}_{JM^*}|0_{\mathrm{RPA}}^+\rangle \quad \text{with} \quad Q^{m\dagger}_{JM^*}|0_{\mathrm{RPA}}^+\rangle = 0.
\]  
(37)

Here \( |0_{\mathrm{RPA}}^+\rangle \) is the ground state of the initial or the final nucleus and the phonon-operator \( Q^{m\dagger}_{JM^*} \) is defined by the ansatz:
\[ Q_{JM}^{m} = \sum_{pn} X_{(pn,Jm)}^{m} A^\dagger (pn, JM) + Y_{(pn,Jm)}^{m} \tilde{A}(pn, JM). \] (38)

\[ A^\dagger (pn, JM) (A(pn, JM)) \text{ is the two quasi-particle creation (annihilation) operator coupled} \]

to the good angular momentum \( J \) with projection \( M \), namely

\[ A^\dagger (pn, JM) = \sum_{mp,mn} C_{jm,p_{mn}}^{JM} a^\dagger_{pm} a_{mn}. \] (39)

In the pn-RQRPA the commutator of two-bifermion operators is replaced by its expectation value in the correlated QRPA ground state \(|0_{QRPA}^+\rangle \) (renormalized quasiboson approximation). Therefore we have

\[
\langle 0_{RPA}^+ | [A(pn, JM), A^\dagger (p'\ell', JM)] | 0_{RPA}^+ \rangle = \delta_{pp^\prime} \delta_{nn^\prime} \times \\
\{ 1 - \frac{1}{j_l} < 0_{RPA}^+ | [a^\dagger_n \tilde{a}_n]_{00} | 0_{RPA}^+ > - \frac{1}{j_k} < 0_{RPA}^+ | [a^\dagger_{n^\prime} \tilde{a}_{n^\prime}]_{00} | 0_{RPA}^+ > \},
\]

\[ =: D_{pn,Jm}. \] (40)

with \( j_p = \sqrt{2j_p + 1} \). Replacing \(|0_{RPA}^+\rangle \) in Eq. (40) by the uncorrelated BCS ground state leads to the usual quasiboson approximation (\( D_{pn} = 1 \)), which violates the Pauli exclusion principle by neglecting the terms coming from the commutation rules of the quasi-particles. From Eqs. (37) one can derive the RQRPA equation

\[
D^{-1/2} \begin{pmatrix} A & B \\ -B & -A \end{pmatrix} D^{-1/2} \begin{pmatrix} X^m \\ Y^m \end{pmatrix} = D^{-1/2} \begin{pmatrix} X^m \\ Y^m \end{pmatrix}. \] (41)

The matrices \( \overline{A} \) and \( \overline{B} \) are given as follows:

\[
\overline{A}_{pn,p'n'}^{Jm} = D_{pn,Jm}^{-1/2} \langle 0_{RPA}^+ | [A(pn, JM), [H, A^\dagger (p'\ell', JM)] | 0_{RPA}^+ \rangle D_{p'n',Jm}^{-1/2} \\
= E_p + E_n \delta_{pp'} \delta_{nn'} - 2[G(pn, p'n'; J)(u_p u_n u_{p'} u_{n'} + u_p u_n u_{p'} u_{n'}) - \\
F(pn, p'n'; J)(u_p v_n u_{p'} v_{n'} + v_p u_n u_{p'} u_{n'}) \] D_{pn,Jm}^{1/2} D_{p'n',Jm}^{1/2}, \] (42)

\[
\overline{B}_{pn,p'n'}^{Jm} = D_{pn,Jm}^{-1/2} \langle 0_{RPA}^+ | [A(pn, JM), [H, \tilde{A}(p'\ell', JM)] | 0_{RPA}^+ \rangle D_{p'n',Jm}^{-1/2} \\
= 2D_{pn,Jm}^{1/2} D_{p'n',Jm}^{1/2} [G(pn, p'n'; J)(u_p u_n v_{p'} v_{n'} + v_p u_n u_{p'} u_{n'}) - \\
F(pn, p'n'; J)(u_p v_n v_{p'} v_{n'} + v_p u_n u_{p'} u_{n'}) \]. \] (43)

Here, \( G(pn, p'n', J) \) and \( F(pn, p'n', J) \) are the particle-particle and particle-hole interaction matrix elements, respectively [54]. The coefficients \( D_{pn,Jm} \) are determined by solving numerically the system of equations [24]:

\[
D_{pn,Jm} = 1 - \frac{1}{2\sqrt{2} + 1} \sum_{j_n,j_m,j_{n'}} D_{pn',Jm',j_n'j_m'} J^2 |Y_{(pn',Jm')}^m|^2 \]

\[- \frac{1}{2\sqrt{2} + 1} \sum_{j_n,j_m,j_{n'}} D_{p'n',Jm',j_n'j_m'} J^2 |Y_{(p'n',Jm')}^m|^2. \] (44)
The selfconsistent scheme of the calculation of the forward- (backward-) going free variational amplitudes \( X^m (Y^{m_i}) \), the excited energies \( \Omega^m_{m_f} \) related to the ground state and the coefficients \( D_{pn,J^m} \) is a double iterative problem which requires the solution of coupled Eqs. (11) and (14).

Numerical treatment of the matrix elements (30)-(33) within the \( pn \)-RQRPA needs transformation of them to relative coordinates. After some tedious algebra one can obtain

\[
M_{\text{type}}^{I} = \sum_{j^{p,n,p',n',J}} (-)^{j_n+j_{p'}+J+j} (2J+1) \left\{ \begin{array}{ccc} j_p & j_n & J \\ j_{p'} & j_{n'} & J \end{array} \right\} \times
\]

\[
\langle p(1), p'(2); J | f(r_{12}) \tau_1^+ \tau_2^+ O^{I}_{\text{type}}(12) f(r_{12}) | n(1), n'(2); J \rangle \times
\]

\[
\langle 0^+_f | [c_p^+ c_{n'}]_f | J^m m_f \rangle > < J^m m_f | J^m m_i \rangle < [c_p^+ c_{n}]_f | 0^+_f > . \tag{45}
\]

In Eq. (15) \( O^{I}_{\text{type}}(12) \) represents the coordinate and spin dependent part of the nuclear two body transition operator for the \( \nu/\beta \beta \)-decay and can be expressed in the form:

\[
O^{I}_{\text{type}}(12) = H^{I}_{\text{type-F}}(r_{12}) + H^{I}_{\text{type-GT}}(r_{12}) \sigma_{12} + H^{I}_{\text{type-T}}(r_{12}) S_{12}. \tag{46}
\]

The short-range correlations between the two interacting protons (\( p(1) \) and \( p'(2) \)) and neutrons (\( n(1) \) and \( n'(2) \)) are simulated by using the correlation function \( f(r_{12}) \) in the non-antisymmetrized two-body matrix element (Eq. (14)). We adopt its following form:

\[
f(r_{12}) = 1 - e^{-\alpha r^2}(1 - br^2_{12}) \quad \text{with} \quad \alpha = 1.1 \text{ fm}^2 \quad \text{and} \quad b = 0.68 \text{ fm}^2. \tag{47}
\]

The one-body transition densities entering Eq. (15) are given as follows:

\[
\frac{< J^m m_i | [c_p^+ c_{n}]_f | 0^+_f >}{\sqrt{2J+1}} = (u_p^{(i)} v_n^{(i)} X^{m_i}_{(pn,J^m)} + v_p^{(i)} u_n^{(i)} Y^{m_i}_{(pn,J^m)}) \sqrt{D^{(i)}_{pn,J^m}}, \tag{48}
\]

\[
\frac{< 0^+_f | [c_p^+ c_{n}]_f | J^m m_f >}{\sqrt{2J+1}} = (v_p^{(f)} u_n^{(f)} X^{m_f}_{(pn,J^m)} + u_p^{(f)} v_n^{(f)} Y^{m_f}_{(pn,J^m)}) \sqrt{D^{(f)}_{pn,J^m}}. \tag{49}
\]

The index \( i \) (f) indicates that the quasiparticles and the excited states of the nucleus are defined with respect to the initial (final) nuclear ground state \( |0_i^+ \rangle \) (\( |0_f^+ \rangle \)). As the two sets of intermediate nuclear states generated from the initial and final ground states are not identical we use the overlap factor

\[
< J^+_{m_f} | J^+_{m_i} > \approx \sum_{pn} (X^{m_i}_{(pn,J^m)} X^{m_f}_{(pn,J^m)} - Y^{m_i}_{(pn,J^m)} Y^{m_f}_{(pn,J^m)}) \times
\]

\[
(v_p^{(i)} u_p^{(f)} + v_p^{(i)} v_p^{(f)}) (u_n^{(i)} u_n^{(f)} + v_n^{(i)} v_n^{(f)}) \tag{50}
\]

in definition (45) of the nuclear matrix elements.
IV. RESULTS AND DISCUSSION

A. Calculation of the nuclear matrix elements

The pn-RQRPA method has been applied for the calculation of the SUSY $0\nu\beta\beta$-decay nuclear matrix elements of the $A = 48, 76, 100, 116, 128, 130, 136$ and $150$ nuclear systems. The following single particle model spaces have been considered in these cases:

(i) For $^{48}Ca \rightarrow ^{48}Ti$ decay the nuclear model comprises 13 levels: $0s_{1/2}, 0p_{1/2}, 0p_{3/2}, 1s_{1/2}, 0d_{3/2}, 0d_{5/2}, 1p_{1/2}, 1p_{3/2}, 0f_{5/2}, 0f_{7/2}, 2s_{1/2}, 0g_{7/2}, 0g_{9/2}$.

(ii) For $^{76}Ge \rightarrow ^{76}Se, ^{82}Se \rightarrow ^{82}Kr$ decays the model space comprises 12 levels: $1s_{1/2}, 0d_{3/2}, 0d_{5/2}, 1p_{1/2}, 1p_{3/2}, 0f_{5/2}, 0f_{7/2}, 2s_{1/2}, 1d_{3/2}, 1d_{5/2}, 0g_{7/2}, 0g_{9/2}$.

(iii) For $^{96}Zr \rightarrow ^{96}Mo, ^{100}Mo \rightarrow ^{100}Ru$ and $^{116}Cd \rightarrow ^{116}Sn$ decays the model space comprises 16 levels: $1s_{1/2}, 0d_{3/2}, 0d_{5/2}, 1p_{1/2}, 1p_{3/2}, 0f_{5/2}, 0f_{7/2}, 2s_{1/2}, 1d_{3/2}, 1d_{5/2}, 0g_{7/2}, 0g_{9/2}, 1f_{5/2}, 1f_{7/2}, 0h_{9/2}, 0h_{11/2}$.

(iv) For $^{128}Te \rightarrow ^{128}Xe, ^{130}Te \rightarrow ^{130}Xe$ and $^{136}Xe \rightarrow ^{136}Ru$ decays the model space comprises 16 levels: $1s_{1/2}, 0d_{3/2}, 0d_{5/2}, 1p_{1/2}, 1p_{3/2}, 0f_{5/2}, 0f_{7/2}, 2s_{1/2}, 1d_{3/2}, 1d_{5/2}, 0g_{7/2}, 0g_{9/2}, 2p_{1/2}, 2p_{3/2}, 1f_{5/2}, 1f_{7/2}, 0h_{9/2}, 0h_{11/2}$.

(v) For $^{150}Nd \rightarrow ^{150}Sm$ decays the model space comprises 20 levels: $1s_{1/2}, 0d_{3/2}, 0d_{5/2}, 1p_{1/2}, 1p_{3/2}, 0f_{5/2}, 0f_{7/2}, 2s_{1/2}, 1d_{3/2}, 1d_{5/2}, 0g_{7/2}, 0g_{9/2}, 2p_{1/2}, 2p_{3/2}, 1f_{5/2}, 1f_{7/2}, 0h_{9/2}, 0h_{11/2}, 0i_{11/2}, 0i_{13/2}$.

The single particle energies have been calculated with a Coulomb-corrected Wood-Saxon potential. For the two-body interaction we used the nuclear G-matrix calculated from the Bonn one-boson exchange potential. The single quasiparticle energies and occupation amplitudes have been found by solving the BCS equations for protons and neutrons. Since our model spaces are finite the proton-proton and neutron-neutron pairing interactions have been renormalized according to Ref. [5]. In the calculation of the pn-RQRPA equation we also renormalized the particle–particle and particle–hole channels of the G-matrix interaction by introducing two parameters $g_{pp}$ and $g_{ph}$, i.e. $G(pn,p'n',J) \rightarrow g_{pp}G(pn,p'n',J)$ and $F(pn,p'n',J) \rightarrow g_{ph}F(pn,p'n',J)$. The value adopted from our previous calculations [10, 12, 36, 39] is $g_{ph} = 0.8$ and we discuss the stability of the nuclear matrix elements in respect to the $g_{pp}$ inside the expected physical range $0.8 \leq g_{pp} \leq 1.2$.

By the method outlined above we obtained the particular nuclear matrix elements of the two-nucleon mode $\mathcal{M}_{GT,N}, \mathcal{M}_{FN}, \mathcal{M}_{GT'}, \mathcal{M}_{F'}, \mathcal{M}_{T'}$ and of the pion-exchange mode $\mathcal{M}_{GT-k\pi}, \mathcal{M}_{T-k\pi}$ for all above mentioned nuclei. Their values are listed in Tab. 1 for $g_{pp} = 1.0$. It is worthwhile noticing that the two-nucleon matrix elements are considerably smaller in comparison with the pion-exchange ones. From that table one can see that the smallest nuclear matrix elements are associated with $A=48$ and $136$ systems. We suppose that it is connected with the fact of the closed shell in $^{48}Ca$ and $^{136}Xe$. Sharp Fermi level usually offers weaker transitions due to the Pauli blocking effect. The largest nuclear elements are associated with $A=150$ and $100$ systems. The difference between the Gamow-Teller matrix elements of $A=48$ and $A=150$ systems is about an order of magnitude. We also stress that the one-pion exchange mode is disfavored in respect to the two-pion exchange mode not only because of a considerably smaller value of the structure coefficient ($\alpha^{1\pi} = -0.044 \ll \alpha^{2\pi} = 0.20$) but also due to the partial mutual cancellation of $\mathcal{M}_{GT-1\pi}$ and $\mathcal{M}_{T-1\pi}$ in Eq. (32) for all studied nuclear systems. By multiplying the particular nuclear matrix elements
with corresponding structure coefficient one obtain the full two-nucleon $\mathcal{M}^{2N}_q$, $\mathcal{M}^{2N}_{f}$ and pion-exchange $\mathcal{M}^{\pi N}$ matrix elements [see Eqs. (30)-(32)]. The $\mathcal{M}^{2N}_q$ and $\mathcal{M}^{2N}_{f}$ have been calculated by using both the NR and the BM structure coefficients. By glancing at Tab. [4] we can see that the values of $\mathcal{M}^{2N}_q$ are considerably larger comparing to $\mathcal{M}^{2N}_{f}$ and less sensitive to the chosen type of structure coefficients. Also the pion-exchange matrix elements $\mathcal{M}^{\pi N}$ dominate by a factor 5 - 7 over $\mathcal{M}^{2N}_q$ element for all studied nuclear systems.

The sensitivity of the nuclear matrix elements $\mathcal{M}^{2N}_q$, $\mathcal{M}^{2N}_{f}$ and $\mathcal{M}^{\pi N}$ to the details of the nuclear model and to the effect of short-range correlations are presented in Table [1]. The advantage of the pn-RQRPA in respect to the QRPA method is that there is no collapse of the pn-RQRPA solution within the physically acceptable region of the particle-particle interaction parameter $g_{pp}$ (0.8 $\leq g_{pp} \leq 1.2$). We can see that the two-nucleon matrix element $\mathcal{M}^{2N}_q$ is rather small and unstable in respect to the changes of $g_{pp}$ within the discussed interval. Moreover, its value crosses zero for most nuclear systems. It means that if the deduced limits on $\lambda_{111}$ associated with this nuclear matrix element only, the predictive power of the result would be rather small. The matrix elements $\mathcal{M}^{2N}_q$ and $\mathcal{M}^{\pi N}$ are suppressed by the repulsion of the nucleon-nucleon interaction at short distances by the factor of about 5 and 3, respectively. Thus the large value of $\mathcal{M}^{\pi N}$ in comparison with $\mathcal{M}^{2N}_q$ is predominantly due to the effect of the bosonization of $\pi^- \rightarrow \pi^+ + 2e^-$ vertex. We note that variations of the values of $\mathcal{M}^{2N}_q$ and $\mathcal{M}^{\pi N}$ do not exceed 20 % - 30 % in respect to the their average values within the allowed $g_{pp}$ interval. Fig. [1] presents the dependence of the two-nucleon mode ($\mathcal{M}^{2N}_q$ and $\mathcal{M}^{2N}_{f}$) and the pion-exchange mode ($\mathcal{M}^{\pi N}$) calculated with and without short range correlations on the parameter $g_{pp}$ for $A=76$ system. One can see the large suppression of the matrix elements due to the short range correlations effect as well as rather stable behavior of the results on $g_{pp}$. The inclusion of ground state correlations beyond the QRPA within the pn-RQRPA method stabilizes the behavior of the studied matrix elements as a function of $g_{pp}$ and increases their predictive power, which is consistent with other studies [36].

### B. Constraints on R-parity violation

Here we shall analyze the constraints on R-parity violation using the current experimental lower half-life limits of the $0\nu\beta\beta$-decay for the nuclei listed in Table [11]. A subject of our interest is the dependence of the limits of three commonly used in literature quantities $\lambda'_{111}$, $\lambda''_{111}/[(m_{\tilde{q}}/100 \text{ GeV})^2(m_{\tilde{g}}/100 \text{ GeV})^{1/2}]$ and $\lambda''_{111}/[(m_{\tilde{e}}/100 \text{ GeV})^2(m_{\tilde{\chi}_0}/100 \text{ GeV})^{1/2}]$ on the GUT scale scenarios of the MSSM. Moreover, we compare contributions to final limits coming from two-nucleon and pion–exchange modes and also study the relevance of the gluino and the neutralino exchange mechanisms.

In the first step, we pay our attention to the constraints on free parameters of MSSM, particularly such SUSY parameters as $\tan \beta = \frac{\tilde{\mu}}{\tilde{\nu}_1}$, $m_0$, $m_{1/2}$, $A_0$ and $\text{sign}(\mu)$. The allowed space for these parameters is determined by some natural restrictions. We use the condition that the mass eigenstates of SUSY particles cannot be imaginary and consider the dynamical electroweak symmetry breaking conditions (Eq. (19)). Further, we require that the $R_7$ parameter associated with flavor changing neutral current processes fulfills relation (22). Using all the above assumptions, we have determined the excluded region for the parameters
tan β and m₀. It is shown in Figs. 2(a) and 2(b) where corresponding symbols indicate three main sources of constraints. The caption EWSB indicates the points excluded by improper electroweak symmetry breaking, b → s + γ the points eliminated by FCNC constraints and v.e.v. – the points, for which some of the mass eigenstates become imaginary. One can observe that this region depends strongly on sgn(µ): for µ being positive, the allowed region of tan β and m₀ is considerably larger. We find e.g. that for µ > 0 and small values of tan β (≤ 3) there is no restriction on m₀ while for µ < 0 there exists a lower bound on m₀ of about 240 GeV.

Further we have found that the constraints coming from the B → Xsγ decay are also especially sensitive to sgn(µ). In Figs. 3(a)–(b) the dependence of the R₇ parameter on m₀ and m₁/₂ is shown, for both positive and negative values of µ. We note that the SM contribution to R₇ is fixed for all the MSSM points at -0.188. The main SUSY contributions to this parameter come from charged Higgses and charginos and are shown on Figs. 3(c)–(d) for negative µ. One can see instantly the large negatives contribution coming from charged Higgses which is responsible for exclusion of the corresponding low values of m₀ and m₁/₂.

We proceed with the determination of limits on λ′₁₁₁, λ′₁₁₁/((m_q/100 GeV)²(m₃/100 GeV)²/(1/2)) and λ′₁₁₁/((mₐ/100 GeV)²(m₅₋₁₀₀ GeV)²/(1/2)) from the non observation of double beta decay as a function of SUSY free parameters. For this purpose, we use the 0νββ-decay half-life time (29) and the calculated nuclear matrix elements. We have found a very weak dependence of the quantities under discussion on the tan β and A₀ SUSY parameters (tan β influences mainly the Higgs (and neutralino) sector, while A₀ determines a very weak sfermion mixing). Therefore we present dependence on m₁/₂ and m₀ for positive µ only. The results are drawn in Figs. 4(a)–(c) for all nuclei we are interested in. One can see that the currently strongest bound on these quantities is deduced from the A=76 system followed by A=128. The limit on the trilinear R_p breaking SUSY parameter λ′₁₁₁ becomes less stringent with the increasing value of m₀ (Fig. 4(a)). Similar behavior is found for λ′₁₁₁/((m_q/100 GeV)²(m₃/100 GeV)²/(1/2)) also (Fig. 4(b)). On the contrary, the value of λ′₁₁₁/((mₐ/100 GeV)²(m₅₋₁₀₀ GeV)²/(1/2)) decreases with the increase of m₀. The reason for such a behaviour is connected with the fact that the mass of selectron is growing faster than the mass of squark as the value of m₀ is increased. In addition, the increase of its second power compensates for the modest increase of λ′₁₁₁ in respect to m₀. This can be seen explicitly from the dependence of the of squark, selectron, gluino and the lightest neutralino masses on m₀ as drawn in Fig. 5. We note that gluino and neutralino are gauginos and therefore are insensitive to the common scalar mass at the GUT scale m₀.

In Tab. 111 we present the limits on R_p coupling constants λ′₁₁₁ from the current lower bounds on the half-life time of the 0νββ-decay isotopes experimentally most promising. We consider two different scales for m₀ and m₁/₂: 100 GeV and 1 TeV. By glancing at Tab. 111 one finds out that the strongest limit on λ′₁₁₁ comes from the ⁷⁶Ge isotope and is 5.25 × 10⁻⁴ and 1.8 × 10⁻¹ for the considered values of m₀ and m₁/₂, respectively.

The experimental constraints on the half-life of the 0νββ-decay are expected to be more stringent in future. It would require the recalculation of the corresponding limits on λ′₁₁₁. For the experimentalists convenience and in order to make our results more general we introduce the GUT constrained SUSY sensitivity parameter c²YMSM(m₀, m₁/₂) of a given isotope Y, presented in Tab. 111. This parameter incorporates the elements of both R_p SUSY and nuclear structure theory and is related with the limit on λ′₁₁₁ as follows:
\[ \lambda'_{111}(m_0, m_{1/2}) \leq \xi_{Y}^{MSSM}(m_0, m_{1/2}) \times \left( \frac{10^{24} y}{T_{0\nu\beta\beta-ex}} \right)^{-1/4}. \] (51)

In this way the reader is provided with an easy algorithm of predicting desired limits with changing the experimental data.

A detailed study of the GUT constrained SUSY scenario for the $0\nu\beta\beta$-decay of $^{76}\text{Ge}$ offers the most stringent limit on R-parity breaking. At present this process is also the most perspective for the experimental detection in respect to the planned future experiment GENIUS [20]. In Figs. 5(a) and 5(b) we show the most stringent restriction on $\lambda'_{111}$ and $\lambda'_{111}/\left((\tilde{m}_e/100\text{ GeV})^2(\tilde{m}_\chi^0/100\text{ GeV})^{1/2}\right)$ as a function of both $m_0$ and $m_{1/2}$. In order to be more concrete we show the present bounds on $\lambda'_{111}$ together with those expected from the GENIUS $0\nu\beta\beta$-decay experiment for a given set of $m_0$ and $m_{1/2}$ parameters in Tab. IV. One can see that the GENIUS experiment is expected to improve the limits on $\lambda'_{111}$ by about a factor of 5 in respect to the current ones. Tab. IV contains also the calculated values of the corresponding masses of squark, selectron, gluino and lightest neutralino as well as the sensitivity parameter $\xi_{Y}^{76\text{Ge}}$ within the GUT constrained SUSY scenario.

We also discuss the importance of different $0\nu\beta\beta$-decay mechanisms on hadron and quark levels. The theoretical expression for half-life of $R_p 0\nu\beta\beta$-decay in Eq. (29) comprises the contributions from the two-nucleon mode and pion-exchange mode, which incorporate a different combination of $R_p$ SUSY parameters $\eta_T$ and $\eta_{PS}$. In the previous subsection we have shown that the pion-exchange nuclear matrix elements dominate over the two-nucleon ones. However, it is not the necessary condition for the dominance of pion-exchange mode. In order to clarify this point we calculate the limit on $\lambda'_{111}$ by considering only one of these mechanisms at one time. The dependence of $\lambda'_{111}$ on $m_0$ is drawn in Fig. 7 for $m_{1/2} = 100$ GeV and 500 GeV. We see that the pion-exchange mode offers a considerably stronger limit on $\lambda'_{111}$ than the two-nucleon mode, which can be safely neglected. We note that the peak appearing in the curves presenting the two-nucleon mode is a consequence of interference between neutralino and gluino contributions to final amplitude. The curve representing the limit on $\lambda'_{111}$ from the pion-exchange mode is free of such instabilities.

It remains to find out which of the neutralino and gluino $R_p$ exchange mechanisms is the most important one for the $0\nu\beta\beta$-decay process. In Fig. 8 we show the parameter $\lambda'_{111}$ as a function of $m_0$ for $m_{1/2}$ equals to 100 GeV and 500 GeV by considering only one of the above $R_p$ mechanisms. For $m_{1/2} = 100$ GeV and $m_0$ larger than about 200 GeV the gluino mechanism is the dominant one. On the other hand, for $m_{1/2} = 500$ GeV the neutralino exchange mechanism becomes prevalent within the whole considered interval of $m_0$. One concludes that none of these mechanisms is the most important in general. The answer to this problem depends on the details of the GUT constrained SUSY scenario and in particular on the values of $m_0$ and $m_{1/2}$ parameters.

V. CONCLUSIONS

In conclusions, we have presented a detailed analysis of the R-parity violating trilinear terms contribution to the $0\nu\beta\beta$-decay within the GUT constrained MSSM scenario. Both nuclear and particle physics aspects of this process have been discussed to some extent.
We calculated relevant nuclear matrix elements within the realistic pn-RQRPA method. A comparison of nuclear matrix elements belonging to the two-nucleon mode and the pion-exchange mode has been performed for $A = 48, 76, 82, 96, 100, 116, 128, 130, 136$ and $150$ nuclear systems. We have found that the $\pi$ mechanisms are larger by factor 5-7 mostly due to the strong $R_p$ $\pi^- \to \pi^+ + 2e^-$ transition. We suppose it is the explanation why the pion-exchange mode $0\nu\beta\beta$-decay is not favored for the mechanisms based on exchange of Majorana neutrinos having a different Lorentz structure and predicting weaker $R_p$ $\pi^- \to \pi^+ + 2e^-$ transition. Our studies show that $R_p$ pion-exchange mode nuclear matrix elements are less suppressed by the short-range correlation effects and are more stable in respect to the details of nuclear Hamiltonian than the two-nucleon matrix elements.

In the MSSM part of calculations we limited the space of the free SUSY parameters using necessary conditions for proper electroweak symmetry breaking and inducing other limits based on current FCNC (Flavor Changing Neutral Currents) experiments. The exclusion plots for the SUSY parameters are presented and the sensitivity of the excluded region to the sign of $\mu$ parameter is manifested. A detailed study of the $R_\tau$ parameter crucial for analysis of the FCNC $B \to X_s \gamma$ decay processes is also displayed and analyzed. We also present masses of squark, selectron, gluino and lightest neutralino in the GUT constrained MSSM, the knowledge of which is required for the analysis of the $R_p$ parameters.

Using the experimental lower bounds on the $0\nu\beta\beta$-decay half-life we then deduced current constraints on the $R_p$-MSSM parameters for different nuclei. The most restrictive constraints on the $R_p$ Yukawa coupling constant $\lambda'_{111}$ are found from the $^{76}\text{Ge}$ $0\nu\beta\beta$-decay experiment performed by the Heidelberg-Moscow Collaboration [19]. For the common SUSY masses $m_0$ and $m_{1/2}$ at the scale of 100 GeV and 1 TeV $\lambda'_{111}$ they are:

$$\lambda'_{111} \leq 5.3 \times 10^{-4} \text{ present}, \quad 1.1 \times 10^{-4} \text{ (GENIUS) for } m_0 = m_{1/2} = 100 \text{ GeV}, \quad (52)$$
$$\lambda'_{111} \leq 1.8 \times 10^{-1} \text{ present}, \quad 3.7 \times 10^{-2} \text{ (GENIUS) for } m_0 = m_{1/2} = 1 \text{ TeV}. \quad (53)$$

The present limit on $\lambda'_{111}$ can be improved by a factor of 5 if the future $0\nu\beta\beta$-decay experiment GENIUS is carried out and no signal about the $0\nu\beta\beta$-decay is detected.

Our studies have shown that the above limits are associated with the pion-exchange mode of the $0\nu\beta\beta$-decay, which is the dominant mechanism for this process at the hadron level. Therefore, bearing in mind the above nuclear structure analysis we argue that the obtained limits do depend very insignificantly on the nuclear physics uncertainties.

We have also dealt with the question which of the gluino and neutralino $0\nu\beta\beta$-decay mechanisms is more important. We have shown that there is no unique answer to this problem and the dominance of any of them is bound with a different choice of the SUSY parameters $m_0$ and $m_{1/2}$.

Finally, we conclude that the $0\nu\beta\beta$-decay imposes very restrictive limits on the important R-parity violating SUSY parameters also within the GUT constrained MSSM scenario. The obtained constraints on the trilinear $R_p$ SUSY parameter $\lambda'_{111}$ are able to compete with those derived from near future accelerator experiments.

ACKNOWLEDGMENTS

The authors acknowledge valuable discussions with S. G. Kovalenko on the R-parity violating MSSM. FS thanks the Department of Theoretical Physics at Maria Curie–Sklodowska
University for its hospitality. This work was supported in part by the State Committee for
Scientific Researches (Poland) Grant No. 2P03B00516 and by Grant Agency (Czech Republic) Contract No. 202/98/1216.
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TABLE I. Nuclear matrix elements for the R-parity violating SUSY mode of the $0\nu\beta\beta$-decay for the experimentally most interesting isotopes calculated within the renormalized pn-QRPA with $g_{pp} = 1.0$. $\mathcal{M} \times 10^n$ implies that the matrix element should be divided by $10^n$ to get the current numerical value. BM (NR) denotes that the nucleon structure coefficients of the bag model (non-relativistic quark model) have been considered.

$(\beta\beta)_0 e - decay : 0^+_{g.s.} \rightarrow 0^+_{g.s.}$ transition

| M. E.       | $^{48}Ca$ | $^{76}Ge$ | $^{82}Se$ | $^{96}Zr$ | $^{100}Mo$ | $^{116}Cd$ | $^{128}Te$ | $^{130}Te$ | $^{136}Xe$ | $^{150}Nd$ |
|-------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| $\mathcal{M}_{\text{GT}^--N} \times 10^2$ | 1.45  | 7.05  | 6.52  | 4.54  | 8.14  | 4.91  | 7.37  | 6.64  | 3.87  | 11.0  |
| $\mathcal{M}_{\text{F}^-N} \times 10^2$ | -0.58 | -2.48 | -2.28 | -1.67 | -2.94 | -1.78 | -2.68 | -2.43 | -1.44 | -4.09 |
| $\mathcal{M}_{\text{GT}^+} \times 10^2$ | -0.21 | -1.04 | -0.99 | -0.64 | -1.17 | -0.74 | -1.05 | -0.94 | -0.54 | -1.54 |
| $\mathcal{M}_{\text{F}^+} \times 10^3$ | 0.92  | 3.76  | 3.53  | 2.46  | 4.41  | 2.79  | 4.00  | 3.61  | 2.14  | 6.05  |
| $\mathcal{M}_{\text{T}^+} \times 10^3$ | -1.30 | -2.38 | -1.98 | -2.65 | -4.14 | -1.88 | -3.67 | -3.46 | -1.92 | -6.34 |
| $\mathcal{M}^2_{\tilde{q}^N}$ (BM) | -19.6 | -95.9 | -87.7 | -63.7 | -113. | -65.4 | -102. | -92.0 | -53.1 | -154. |
| $\mathcal{M}^2_{\tilde{f}^N}$ (BM) | -0.32 | 4.71  | 5.01  | 1.02  | 3.15  | 3.10  | 2.78  | 2.19  | 1.29  | 2.55  |
| $\mathcal{M}^2_{\tilde{q}^N}$ (NR) | -27.4 | -129. | -118. | -85.1 | -151. | -89.3 | -137. | -123. | -71.9 | -207. |
| $\mathcal{M}^2_{\tilde{f}^N}$ (NR) | -1.06 | 2.22  | 2.80  | -0.89 | -0.07 | 1.28  | -0.17 | -0.52 | -0.32 | -2.13 |
| $\mathcal{M}_{\text{GT}^-1\pi}$ | 0.25  | 1.30  | 1.23  | 0.77  | 1.43  | 0.92  | 1.25  | 1.10  | 0.61  | 1.85  |
| $\mathcal{M}_{T^-1\pi}$ | -0.53 | -1.02 | -0.87 | -1.11 | -1.73 | -0.78 | -1.57 | -1.48 | -0.84 | -2.70 |
| $\mathcal{M}_{\text{GT}^-2\pi}$ | -0.26 | -1.34 | -1.26 | -0.85 | -1.52 | -0.94 | -1.40 | -1.26 | -0.74 | -2.07 |
| $\mathcal{M}_{T^-2\pi}$ | -0.31 | -0.65 | -0.57 | -0.67 | -1.05 | -0.47 | -0.99 | -0.93 | -0.54 | -1.68 |
| $\mathcal{M}^2_{\pi N}$ | -147. | -625. | -583. | -428. | -750. | -436. | -692. | -627. | -367. | -1054. |
TABLE II. Sensitivity of nuclear matrix elements $M_{\bar{q}}^{2N}$, $M_f^{2N}$ and $M_{\pi N}$ for the $A=48, 76, 82, 96, 100, 116, 128, 130, 136$ and 150 nuclear systems calculated within the renormalized pn-QRPA to of nucleon short–range correlations (s.r.c) and to the factor $g_{pp}$, renormalizing the particle-particle interaction strength. In the calculations of nuclear matrix elements $M_{\bar{q}}^{2N}$ and $M_f^{2N}$ within two–nucleon mode the nucleon structure coefficients of the non–relativistic quark model was adopted.

| nucleus | $g_{pp}$ | $M_{\bar{q}}^{2N}$ | $M_f^{2N}$ | $M_{\pi N}$ |
|---------|---------|------------------|----------------|----------------|
|         | no s.r.c. | with s.r.c. | no s.r.c. | with s.r.c. | no s.r.c. | with s.r.c. |
| $^{48}Ca$ | 0.80     | -154. | -31.2 | -22.1 | -0.85 | -440. | -164. |
|         | 1.00     | -138. | -27.4 | -20.3 | -1.06 | -392. | -147. |
|         | 1.20     | -119. | -23.0 | -18.1 | -1.27 | -335. | -126. |
| $^{76}Ge$ | 0.80     | -711. | -143. | -90.8 | 3.67 | -2020. | -686. |
|         | 1.00     | -646. | -128. | -80.5 | 2.22 | -1831. | -625. |
|         | 1.20     | -581. | -113. | -78.2 | 0.81 | -1645. | -564. |
| $^{82}Se$ | 0.80     | -638. | -131. | -79.6 | 4.13 | -1832. | -638. |
|         | 1.00     | -581. | -119. | -74.2 | 2.80 | -1667. | -583. |
|         | 1.20     | -525. | -105. | -68.8 | 1.52 | -1503. | -529. |
| $^{82}Zr$ | 0.80     | -490. | -97.0 | -66.3 | 0.25 | -1383. | -479. |
|         | 1.00     | -437. | -85.1 | -61.1 | -0.89 | -1228. | -428. |
|         | 1.20     | -358. | -73.4 | -55.9 | 1.85 | -1077. | -376. |
| $^{100}Mo$ | 0.80     | -850. | -170. | -112. | 1.86 | -2408. | -832. |
|         | 1.00     | -764. | -151. | -104. | -0.07 | -2155. | -750. |
|         | 1.20     | -678. | -131. | -95.5 | -1.80 | -1908. | -668. |
| $^{116}Cd$ | 0.80     | -480. | -98.2 | -61.5 | 2.05 | -1365. | -474. |
|         | 1.00     | -440. | -89.3 | -57.5 | 1.28 | -1250. | -436. |
|         | 1.20     | -401. | -80.3 | -53.5 | 0.55 | -1135. | -398. |
| $^{128}Te$ | 0.80     | -778. | -155. | -103. | 1.81 | -2212. | -767. |
|         | 1.00     | -696. | -137. | -95.1 | -0.17 | -1976. | -692. |
|         | 1.20     | -615. | -118. | -87.5 | -2.08 | -1743. | -615. |
| $^{130}Te$ | 0.80     | -706. | -141. | -93.6 | 1.37 | -2008. | -697. |
|         | 1.00     | -631. | -123. | -86.7 | -0.52 | -1790. | -627. |
|         | 1.20     | -556. | -106. | -79.9 | -2.36 | -1573. | -556. |
| $^{136}Xe$ | 0.80     | -418. | -83.5 | -55.0 | 1.04 | -1191. | -413. |
|         | 1.00     | -369. | -71.9 | -50.8 | -0.32 | -1048. | -367. |
|         | 1.20     | -318. | -59.7 | -46.6 | -1.74 | -901. | -317. |
TABLE III. Upper limits on the lepton number non-conserving parameter $\lambda'_{111}$ deduced from the experimental lower limits of the $0\nu\beta\beta$-decay half-life time $T_{1/2}^{0\nu\beta\beta-exp(Y)}$ for the nuclei studied in this work. The MSSM SUSY parameters $m_0$ and $m_{1/2}$ are limited to two cases: 100 GeV and 1 TeV. According to Eq. (51) $\xi_Y^{MSSM}(m_0, m_{1/2})$ denotes the sensitivity of a given nucleus Y to the $\lambda'_{111}$ parameter. $G_{01}$ is the integrated kinematical factor for $0^+\rightarrow 0^+_g$. transition.

| nucleus | $G_{01} \times 10^{15}$ | $T_{1/2}^{0\nu\beta\beta-exp(Y)}$ [y] | $m_0 = m_{1/2} = 100$ GeV | $m_0 = m_{1/2} = 1$ TeV |
|---------|----------------|------------------|----------------|----------------|
| $^{48}$Ca | 803. | $9.5 \times 10^{21}$ | $5.90 \times 10^{-4}$ | $1.9 \times 10^{-3}$ |
| $^{76}$Ge | 7.93 | $1.1 \times 10^{25}$ | $9.56 \times 10^{-4}$ | $5.2 \times 10^{-4}$ |
| $^{82}$Se | 35.2 | $2.7 \times 10^{24}$ | $6.84 \times 10^{-4}$ | $1.7 \times 10^{-3}$ |
| $^{82}$Zr | 73.6 | $3.9 \times 10^{19}$ | $6.27 \times 10^{-4}$ | $7.9 \times 10^{-3}$ |
| $^{100}$Mo | 57.3 | $5.2 \times 10^{22}$ | $5.26 \times 10^{-4}$ | $1.1 \times 10^{-3}$ |
| $^{116}$Cd | 62.3 | $2.9 \times 10^{22}$ | $6.83 \times 10^{-4}$ | $1.6 \times 10^{-3}$ |
| $^{128}$Te | 2.21 | $7.7 \times 10^{24}$ | $1.23 \times 10^{-3}$ | $7.4 \times 10^{-4}$ |
| $^{130}$Te | 55.4 | $8.2 \times 10^{21}$ | $5.79 \times 10^{-4}$ | $1.9 \times 10^{-3}$ |
| $^{136}$Xe | 59.1 | $4.2 \times 10^{23}$ | $7.44 \times 10^{-4}$ | $9.2 \times 10^{-4}$ |
| $^{150}$Nd | 269. | $1.2 \times 10^{21}$ | $2.95 \times 10^{-4}$ | $1.6 \times 10^{-3}$ |
TABLE IV. The supersymmetric parameters with corresponding limits on $\lambda'_{111}$ derived from the best presently available experimental limit on half-life of $^{76}\text{Ge}$ $0\nu\beta\beta$-decay: $T_{1/2}^{\exp}(^{76}\text{Ge}) > 1.1 \times 10^{25}$ y [19]. Consequences of the expected half-life limit to be reached in the GENIUS experiment ($T_{1/2}^{\text{GENIUS}}(^{76}\text{Ge}) > 6 \times 10^{27}$ [18]) are also shown. The $m_0$, $m_{1/2}$ tan($\beta$), $A_0$ and $\text{sgn}(\mu)$ are the MSSM parameters. $m_{\tilde{q}}$, $m_{\tilde{e}}$, $m_{\tilde{\gamma}}$ and $m_{\chi_0^1}$ are the masses of squark, selectron, gluino and lightest neutralino, respectively. The $\xi_{^{76}\text{Ge}}^{MSSM}$ parameter is defined by Eq. (51).

$$\tan(\beta) = 3, A_0 = 500\text{ GeV, } \text{sgn}(\mu) = +1$$

| $m_0$ [GeV] | $m_{1/2}$ [GeV] | $m_{\tilde{q}}$ [GeV] | $m_{\tilde{e}}$ [GeV] | $m_{\tilde{\gamma}}$ [GeV] | $m_{\chi_0^1}$ [GeV] | $\xi_{^{76}\text{Ge}}^{MSSM}$ | $\lambda_{111}$ present [GeV] | $\lambda'_{111}$ GENIUS [GeV] |
|-------------|-----------------|------------------------|------------------------|------------------------|------------------------|------------------------|-----------------------------|-----------------------------|
| 100         | 100             | 250.9                  | 107.2                  | 261.9                  | 25.8                   | 9.56                   | 5.25 $\times 10^{-4}$       | 1.09 $\times 10^{-4}$       |
| 100         | 500             | 1136.0                 | 217.7                  | 1290.0                 | 208.0                  | 1.04                   | 5.73 $\times 10^{-3}$       | 1.19 $\times 10^{-3}$       |
| 100         | 1000            | 2281.0                 | 399.5                  | 2595.0                 | 420.2                  | 4.99                   | 2.74 $\times 10^{-2}$       | 5.67 $\times 10^{-3}$       |
| 500         | 100             | 548.1                  | 501.5                  | 298.7                  | 36.8                   | 1.15                   | 6.31 $\times 10^{-3}$       | 1.31 $\times 10^{-3}$       |
| 500         | 500             | 1237.0                 | 536.1                  | 1290.0                 | 208.3                  | 5.76                   | 3.16 $\times 10^{-2}$       | 6.54 $\times 10^{-3}$       |
| 500         | 1000            | 2333.0                 | 632.2                  | 2595.0                 | 420.2                  | 1.23                   | 6.76 $\times 10^{-2}$       | 1.40 $\times 10^{-2}$       |
| 1000        | 100             | 1025.0                 | 1001.0                 | 317.7                  | 39.8                   | 4.25                   | 2.33 $\times 10^{-2}$       | 4.83 $\times 10^{-3}$       |
| 1000        | 500             | 1511.0                 | 1019.0                 | 1407.0                 | 208.9                  | 1.57                   | 8.62 $\times 10^{-2}$       | 1.78 $\times 10^{-2}$       |
| 1000        | 1000            | 2490.0                 | 1072.0                 | 2596.0                 | 420.3                  | 3.27                   | 1.80 $\times 10^{-1}$       | 3.72 $\times 10^{-2}$       |
FIGURES

FIG. 1. The nuclear matrix elements of the two-nucleon mode ($M_{2N}^{\tilde{q}}$, $M_{2N}^{\tilde{g}}$) and pion-exchange mode ($M_{\pi N}^\pi$) of the $R_p$ SUSY contribution to the $0\nu\beta\beta$-decay of $^{76}\text{Ge}$ calculated within the pn-RQRPA. They are functions of the particle-particle interaction strength $g_{pp}$ with (a) and without (b) the two-nucleon short-range correlations.

FIG. 2. The constraints imposed by the $B \to X_s\gamma$ process, dynamical electroweak symmetry breaking condition (ESWB) and vacuum expectation values condition on the $\tan\beta$ and $m_0$ SUSY parameters. The excluded region is indicated by the corresponding symbols. We assume $m_{1/2} = 200$ GeV and $A_0 = 500$ GeV. Two cases of $\mu$ negative (a) and $\mu$ positive (b) are shown. The allowed space of parameters is considerably larger for positive $\mu$.

FIG. 3. The $R_7$ parameter determining the MSSM contributions to the flavor changing neutral current $B \to X_s\gamma$ decays is drawn against $m_0$ and $m_{1/2}$. The coefficient was calculated for $\tan\beta = 3$, $A_0 = 500$ and $\mu$ either positive (a) or negative (b). The charged Higgs (c) and chargino (d) contributions to $R_7$ for negative $\mu$ are also presented.

FIG. 4. The limits on (a) $\lambda'_{111}$ and (b) $\frac{\lambda'_{111}}{(m_{\tilde{g}}/100\text{GeV})^2(m_{\tilde{q}}/100\text{GeV})^{1/2}}$ deduced from the experimental lower bound on the half-life of the $0\nu\beta\beta$-decay of different nuclei are plotted as a function of $m_0$. Other free parameters are fixed as follows: $A_0 = 500$ GeV, $m_{1/2} = 500$ GeV, $\tan\beta = 3$ and $\mu > 0$ (see text for details). The needed nuclear matrix elements have been calculated within the pn-RQRPA.

FIG. 5. The masses of supersymmetric particles: $m_{\tilde{q}}$ (squark), $m_{\tilde{g}}$ (gluino), $m_{\tilde{s}}$ (selectron) and $m_{\chi}$ (the lightest neutralino) as a function of $m_0$ within the GUT constrained minimal supersymmetric standard model with R-parity breaking. Other free parameters are fixed: $m_{1/2} = 100$ GeV, $A_0 = 500$ GeV, $\tan\beta = 3$ and $\mu > 0$.

FIG. 6. The deduced limits on (a) $\lambda'_{111}$ and (b) $\frac{\lambda'_{111}}{(m_{\tilde{g}}/100\text{GeV})^2(m_{\tilde{e}}/100\text{GeV})^{1/2}}$ from the experimental lower bound on the half-life of the $0\nu\beta\beta$-decay in $^{76}\text{Ge}$ plotted as a function of $m_0$ and $m_{1/2}$. Other parameters are fixed as in the previous figures.

FIG. 7. The deduced from the lower bound on the half-life of the $0\nu\beta\beta$-decay in $^{76}\text{Ge}$ limit on $\lambda'_{111}$ plotted as a function of $m_0$. Two-nucleon mode (2n.-mode) and pion-exchange mode ($\pi$-mode) are considered separately. The parameters $m_{1/2}$ have been chosen to be 100 GeV or 500 GeV. Other parameters are fixed as in the previous figures. The dominance of the pion-exchange mode is easy to be noticed.
FIG. 8. The limit on $\lambda_{111}$ deduced from the experimental lower bound on the half-life of the $0\nu\beta\beta$-decay of $^{76}$Ge against $m_0$. The gluino (g-mech.) and neutralino (χ-mech.) mechanisms in the R-parity violating scenarios are considered separately for two values of $m_{1/2}$ (100 GeV and 500 GeV). Other parameters are the same as in Fig. 5. One can observe that for $m_{1/2} = 100$ GeV and a small value of $m_0$ up to 200 GeV the neutralino contribution dominates (gives stronger constraints), while for larger values of $m_0$ gluino mechanism becomes more important.
(a) $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$

with s.r.c.

(b) $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$

without s.r.c.
\( m_{1/2} = 200 \text{ GeV}, A_0 = 500 \text{ GeV}, \mu < 0 \)

\( m_{1/2} = 200 \text{ GeV}, A_0 = 500 \text{ GeV}, \mu > 0 \)
$\tan \beta = 3, A_0 = 500 \text{ GeV}, \mu > 0$  

Charged Higgs contribution

$\tan \beta = 3, A_0 = 500 \text{ GeV}, \mu < 0$  

Chargino contribution
2n-mode, $m_{1/2} = 100$ GeV
2n-mode, $m_{1/2} = 500$ GeV
π-mode, $m_{1/2} = 100$ GeV
π-mode, $m_{1/2} = 500$ GeV
$\lambda_{111}'$

$m_0$ [GeV]

g-mech., $m_{1/2} = 100$ Gev

g-mech., $m_{1/2} = 500$ Gev

$\chi$-mech., $m_{1/2} = 100$ Gev

$\chi$-mech., $m_{1/2} = 500$ Gev