Laser diodes with modulated optical injection: towards a simple signal processing unit?

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Abstract
The idea of using the dynamical behaviour of a semiconductor laser to perform a certain processing operation of an input signal has been around for quite a long time. While the unidirectional optical injection scheme seems well suited to such a target—with the injection serving as an optical carrier for the input signal—the impact of a modulation of the injection beam still requires thorough investigation. Here, we study the case of an optically injected laser with a simple single-tone modulation term added to the injection signal. We analyse the impact of amplitude modulation on the laser dynamics, and particularly focus on the evolution within the injection locking range. We highlight clear passband behaviour corresponding to the laser resonance at its relaxation oscillation frequency, and characterize its features for various parameter changes. Next, we report dramatic differences between amplitude and phase modulation as the latter quickly leads to a loss of the injection locking and to the emergence of chaotic dynamics in place from the passband response identified in the case of amplitude modulation. At last, we discuss the suitability of using laser diodes for signal filtering, as was recently proposed by others, and identify the main remaining issues that need to be overcome.

1. Introduction

Lasers and, in particular, semiconductor lasers are well known for having rich and complex dynamical behaviour intrinsically linked to exciting physical features and with the potential to lead to cutting-edge applications [1–3]. While one of the main practical interests at first was to understand how to avoid instabilities—which, by the way, still remain a significant issue nowadays [4–6]—the field has largely outgrown this initial problem to tackle a wide variety of topics: from application of optical chaos—chaos communication, random number generation or random key distribution [3]—to all-optical processing, high-frequency signal generation, RF over fiber or LIDAR systems [7–10]. These topics of research are also indirectly supported by the impressive growth of both the fields of microwave photonics [7, 11] and photonic integration [12]. On the one hand, the success of the former opens important opportunities for the field of laser dynamics as its successful development creates a fruitful and suitable context for harnessing the targeted behaviour and applications [7, 9, 13]. On the other hand, the latter makes it finally possible to overcome notorious stability issues and to drastically increase the complexity of a system without unreasonably increasing its cost and bulkiness [6, 14–16]. Simultaneously, generic foundry platforms and multi-project wafer runs also offer the exciting chance to make custom lasers and systems without the need to master all the steps of photonic integrated circuit growth [6, 17, 18].

A signal processing scheme necessarily implies that an input signal will modulate a given parameter of the system: to use laser diode dynamics it was therefore proposed to modulate the current of the laser itself or the injected signal via the master laser current or using an external modulator [7, 9]. Yet, only little attention was given to the impact of such modulation on the laser dynamics [9, 19, 20]. Of course, for systems targeting the generation of specific signals, the input is well controlled and there is thus no need to investigate the impact of an arbitrary input. On the other hand, when considering filtering applications—as in [21–24]—the fact that the
laser diode is a dynamically active device becomes a key issue. In this context, the work described in [22–24] is particularly interesting: the authors propose schemes based on injection-locked lasers to filter an input signal. Initially, such schemes relied on Fabry–Perot resonators, which were then upgraded with a semiconductor optical amplifier inside the cavity to improve its selectivity through the increase of the cavity Q-factor [25, 26]. The principle is therefore straightforward: due to the Fabry–Perot effect, only the resonating wavelengths are transmitted or reflected. Conceptually, Fabry–Perot semiconductor optical amplifiers are akin to Fabry–Perot laser diodes operated below threshold, thus, further increasing the gain inside the cavity naturally leads to laser emission. From then on, the system becomes active, and the wavelength selection occurs via the injection locking mechanism: if the input wavelength is close enough to that of the laser, then the laser is locked onto this wavelength, while other wavelengths will simply not be picked up. This works for potentially all longitudinal modes of the Fabry–Perot laser [5], but the use of single-mode distributed Bragg reflectors or distributed feedback lasers has been proposed to limit this periodicity [22–24]. Although using a laser in place of an amplified resonant cavity seems to be a convincing progress, one should also take into account that optical injection is known for triggering a wide variety of complex dynamical behaviour, including oscillations, excitability and chaos when the laser is not locked [27]. Depending on the use case, this issue can probably be somehow avoided or reduced. But if one wants to filter a carrier wave modulated by an input signal, considering the sensitivity of the laser diodes, this could also be expected to trigger complex dynamical behaviour—even within the locking range—which would, as a result, dramatically interfere with the filtering process.

Since optical injection has already been well studied, the key question here becomes: what happens when the injection beam is modulated? And it appears that this question did not attract much attention; which does not mean that modulated optical injection has never been used. In fact, because optical injection alone can trigger chaotic behaviour, the injection of a more complex signal was quickly identified as a possible solution to make the chaotic behaviour more complex and to improve its dynamical features for chaos-based applications. While direct modulation of the injection has been considered [28], the use of cascaded lasers—and thus the injection of a chaotic signal—proved to be an attractive and promising solution [29–31]. Alternatively, the slave laser can also be locked on the sidebands induced by the modulation of the optical injection, which forces the laser to generate so-called period-1 (or P1) oscillations [32]. This technique has been shown to be an excellent means to amplify [33], convert [34] or generate various high-frequency signals [35, 36]. Nevertheless, the behaviour of a semiconductor laser subject to optical injection with a simple harmonic modulation has, to the best of our knowledge, not been thoroughly investigated so far, even if very recent works showed that this problem also caught the interest of other groups [37, 38]. Yet, the overall impact of the modulation on the laser dynamics remains mostly unknown and is still to be clarified.

In this contribution, we focus on this simpler but also more fundamental topic. We first consider a single-tone sine modulation amplitude of the optical injection, and explore its impact on the laser dynamics. We particularly focus on the evolution within the locking range. We highlight a resonant response for a modulation frequency close to the relaxation oscillation (RO) frequency of the laser, which leads to an effect that is somehow similar to a frequency filter. We then discuss the evolution of the resonance bandwidth and peak frequency when varying the modulation strength and injection parameters. Last, we quickly investigate the effect of a square-wave amplitude modulation and of a sine modulation of the phase of the injected beam which we compare with previous results. Finally, we discuss the results from a filtering viewpoint and highlight the remaining key issues that still need to be overcome.

2. Numerical laser model including modulated optical injection

We start from a widely used model of an optically injected quantum well laser. The coupling is unidirectional from the master to the slave laser, thus only the slave laser is modelled and the injected signal is ideal. The laser rate equations thus read as follows:

\[
\frac{dE}{dt} = \frac{1}{2} (1 + i\alpha)nE + \kappa e^{i\Delta t}
\]

\[
\frac{dn}{dt} = -2\Gamma n - (1 + 2Bn)(|E|^2 - 1)
\]

with \(E\) and \(n\) being the electric field and carrier density of the slave laser, and \(\alpha\) the linewidth enhancement factor. As detailed in [27], the model is rescaled in time with respect to the RO frequency \(\omega_0\) which is therefore also the case of all the other time scales involved: \(\Gamma\) which gives the RO damping and \(B\) the carrier lifetime. Naturally, the optical injection term \(\kappa e^{i\Delta t}\) is also impacted, as the detuning \(\Delta\) between the slave and master laser, along with the injection rate \(\kappa\) are both rescaled and thus dimensionless. No spontaneous emission noise is considered in the model at this stage. Unless stated otherwise, we will use the same parameter values as in other contributions [27, 39]: \(B = 0.015, \alpha = 2\) and \(\Gamma = 0.035\). \(B\) and \(\Gamma\) are dimensionless due to the rescaling with respect to the RO
frequency. In addition, it is important to note that, due to the normalization detailed in [27], the injection current of the laser does not appear directly but is hidden in the normalized expression of the different parameters. Incidentally, this means that changing the value of the current is not straightforward as many parameters need to be rescaled. The parameter values considered here correspond to an injection current of 1.5 times the current at threshold and will be kept fixed in this manuscript. During our investigations, however, we have tested, on several different occasions, the fact that changing the laser injection current does not significantly alter the reported dynamical behaviour. Yet, because we did not perform a detailed investigation of this matter, we cannot fully confirm this point which would still require further investigations.

For the optical injection, we will consider a detuning in the range of $\Delta \in [-5, 5]$ and an injection rate in $\kappa \in [0, 1.5]$. In practice, the RO frequency of a semiconductor laser is typically in the range of 0.1 to 10 GHz. Thus, considering an RO frequency of 1 GHz provides a realistic estimation of the frequency scaling that is also straightforward to apply as the normalized time unit directly corresponds to nanoseconds. While several different modulation formats could be considered, we decided to first focus on a simple single-tone analog sine modulation. In this contribution, unless stated otherwise, we focus on a dual-sideband amplitude modulation for which the injection term becomes $\kappa(1 + A \sin(\omega_m t)) e^{i\Delta t}$, with $A$ being the amplitude of the modulation (with $0 < A < 1$), and $\omega_m$ the modulation frequency. Strictly speaking, an additional phase term inside the sine function could be added, but since we observed no impact of this term on our preliminary simulations, we have decided to neglect it for now. As the other time-related parameters, the modulation frequency is of course scaled to the RO frequency. Alternatively, we will briefly consider the case of a square-wave amplitude modulation—then the sine wave being simply replaced by a square function—and a phase modulation for which the injection terms become $\kappa e^{i(\Delta t + \Delta \phi)}$ with $0 < \Delta \phi < \pi$.

We simulate the time-evolution of our system using direct numerical integration of the rate equations after a phase-amplitude decomposition of the field equation. To analyse the dynamical behaviour when varying parameters, we extract the extrema of the generated time series for the output power $P = |E|^2$. This, however, suppresses the time/frequency information contained in the time series. For a representation in a 2D space, we only keep the number of extrema—i.e. we discard the value of each extrema—which provides a very synthetic overview of the dynamical complexity of the system, but which requires to be complemented by a temporal or spectral analysis of the dynamics. Mappings of the first and second largest Lyapunov exponents have been proposed with the advantage of providing extra input on the laser stability [39]. However, the use of the total number of extrema appeared, in our case, to be sufficient and more straightforward to implement. Yet, this approach requires us to set a minimal threshold separating two nearby extrema to avoid quantification and discretization errors. Hence, depending on this setting, small oscillations might remain undetected if the peak-to-peak amplitude is below this threshold, while numerical errors might be classified as distinct extrema. Within the framework of our study, these errors are only visible in the extrema maps in regions where a low number of extrema is recorded: e.g. P1 oscillations being misclassified as a steady state or P2 oscillations. The impact on the interpretation of the results is extremely limited, but, for the sake of accuracy, we will comment on these errors whenever required.

3. Overview of the dynamical effect of a modulated optical injection

The injection pulling/locking phenomenon is well known and well understood not only in lasers, but also in any coupled oscillator systems, even though the unidirectional coupling configuration might be more common in lasers. In short, if the injected signal is strong enough and its frequency is sufficiently close to the natural oscillation frequency of the slave oscillators, then injection locking can occur: in this case, the slave laser frequency is 'locked' to the frequency of the injected signal, i.e. the frequency of the master laser. In the injection rate versus detuning plane, the locking region appears as a smooth teeth-looking region, shown in white in figure 1(a). Around it, however, one can immediately see that, as already mentioned, optical injection induces a wide variety of dynamical behaviour [27].

3.1. Case of a low-amplitude low-frequency modulation

Introducing a modulation of the injected signal represents a significant qualitative change and, therefore, major dynamical changes in these instabilities can be expected. But, on the other hand, we could naively expect that the locking would be maintained, at least for a low modulation amplitude and/or frequency: with the output power of the slave laser showing oscillations similar to the amplitude modulation of the injected signal while its emission frequency is locked onto the frequency of the injected signal. In figure 1(b), we show the same extrema map as in panel (a) with a modulation amplitude of $A = 0.1$ and a modulation frequency $\omega_m = 0.1$, which could both be reasonably considered as relatively low values. And, indeed, we observe that the locking region has been almost completely turned from white to pink, i.e. two extrema are now recorded in the intensity time series,
meaning that the slave laser output power exhibits P1 oscillations, as shown in figure 1(d). From the time series, we can immediately confirm that the oscillation frequency indeed matches the modulation frequency of the optical injection. In the rest of this work we will refer to this regime as the ‘locked period-1’ or ‘locked P1’ dynamics. We can observe that for a weak injection $\kappa < 0.2$, the locking region remained white, e.g. only one extremum is detected. This misclassification can be easily attributed to the small amplitude of the output power oscillations, confirmed by the vertical scale of figure 1(d) showing a case that has been properly classified.

In the upper part of the locking range—corresponding to the lowest negative detuning values—we observe the emergence of a regular pulsed dynamic, see figure 1(c), that, at first glance, seems to be consistent with the known excitable behaviour of the laser in this region [27]. In the other regions where the laser is unlocked, we observe almost everywhere a large increase in the number of recorded extrema. From the time series, we observe that the laser dynamics are, typically, not significantly impacted—i.e. they are rather similar to the dynamics observed with a steady optical injection—but with the appearance of an additional modulation envelope at a frequency consistent with $\omega_{in}$, as displayed in figure 1(e).

While substantial dynamical changes might potentially be identified for certain sets of parameters, this first numerical insight tends to indicate that a low-amplitude, low-frequency modulation of the optical injection signal would mostly have a mild and predictable impact on the slave laser.

3.2. Evolution of the detuning-injection rate map for stronger/faster modulation

We now try to analyse the effect of a faster and/or stronger modulation of the injection signal based on the extrema maps obtained for $A$ equal to 0.1, 0.3 and 0.5, and for $\omega_{in}$ being set to 0.1, 0.5, 1, 2.5 and 5, which are shown in figure 2. Not surprisingly, the picture becomes more complex as both parameters have, as could be expected, a decisive impact on the laser dynamics.

Looking at the top row—i.e. panels (a1), (b1) and (c1)—we observe that increasing the amplitude of the modulation with a fixed frequency of $\omega_{in}$ = 0.1 (as in figure 1) leads to a shrinking of the locked period-1 oscillations. As the modulation amplitude increases, we see that the regular pulsed behaviour is triggered for lower detuning values. The modulation frequency is relatively low and, as a basic approximation, we could say that the system is simply moving along a horizontal line in the $(\kappa, \Delta)$ plane corresponding to the variation of $\kappa$ imposed by the modulation. When this path reaches the excitable region, which lies at lower injection rates, just above the locking range in figure 1(a), a pulse is triggered. As a result, increasing the modulation amplitude necessarily shrinks the P1 oscillation region. However, when the modulation frequency $\omega_{in}$ is increased, this interpretation quickly appears to become invalid, as shown by lower rows of figure 2.

Increasing the modulation frequency $\omega_{in}$ while keeping a low-amplitude modulation, i.e. the left column from (a1) to (a5), seems to have a relatively low impact on the locking region. A hook-shaped region with P1 dynamics appears around $\Delta = 0$ and $\kappa \approx 0.35$ in figure 2(a3) for $\omega_{in} = 1$, i.e. when the modulation frequency is identical to the RO frequency of the laser. In figure 2(a4) with $\omega_{in} = 2.5$, we see that this hook-shaped region is pushed towards a larger injection rate around $\kappa = 0.7$. In the same panel, we see the emergence of the so-called sideband locking regions corresponding to a detuning $\Delta = \pm \omega_{in}$ in which the slave laser is locked on the modulation sideband of the modulated injection signal. The emergence of clear P1 dynamics in this range is therefore perfectly in line with previous studies as this regime has been identified as a promising solution for high-frequency signal generation [33–36]. Going one step further to $\omega_{in} = 5$ in panel (a5), we remark that the features of the extrema map obtained when no modulation of the injection signal is present are resuracing,

![Figure 1](image.png)
especially for lower values of the injection rate. The modulation is five times faster than the RO frequency of the slave laser, and it looks like the influence of the modulation is already fading out. This is even better confirmed by the fact that, for lower injection rates $\kappa < 0.3$, increasing the modulation amplitude appears to have a quite limited effect: see panels (b.5) and (c.5) compared to other rows.

Alternatively, when the modulation frequency is of the same order of magnitude as the RO frequency of the laser, several qualitative changes can be observed. For $\omega_m = 0.5$, increasing the modulation amplitude—as shown in figures 2(b.2) and (c.2)—leads to a complete destabilization of the locked P1 dynamics. While new extrema are recorded, the period of the oscillation remains the same. Hence, this is not the result of a period-doubling bifurcation. However, we can observe a folding of the system’s trajectory in the phase space, which leads to the emergence of these additional extrema. When the modulation amplitude is further increased, this effect is relatively quickly suppressed: only a small region for large values of the injection rate remains unstable for $\omega_m = 1$, see (b.3) and (c.3).

Finally, we see that the most complex features are displayed in panels (b.3), (b.4), (c.3) and (c.4), that are for a relatively large modulation amplitude $A \geq 0.3$ and a modulation frequency equal or slightly above the RO frequency of the laser $\omega_m \geq 1$. While the detailed analysis of these features is way out of the scope of this paper, we can nonetheless remark that the hook-shaped regions—mentioned earlier—are significantly bigger for the larger modulation amplitude, like the sideband locking regions which are also significantly wider. At last, we observe in (c.3) two straight horizontal regions around a detuning of $\Delta \approx 2$ and $\Delta \approx 1.5$, where the system seems to exhibit some re-stabilization which does not correspond to any of the mechanisms mentioned.

**Figure 2.** Extrema maps in the detuning versus injection rate plane ($\Delta, \kappa$) for three different modulation amplitudes $A = 0.1$ (a), $A = 0.3$ (b) and $A = 0.5$ (c) from the left column to the right, and five different modulation frequencies $\omega_m = 0.1$ (1), $\omega_m = 0.5$ (2), $\omega_m = 1$ (3), $\omega_m = 2.5$ (4) and $\omega_m = 5$ (5) from the top to the bottom row. The color code is identical to that in figure 1.
previously. Further investigations might be required to clarify the laser behaviour observed for this range of parameters.

3.3. Case analysis of specific injection configurations

As highlighted in the previous sub-sections, the dynamical behaviour of the laser is rich and strongly dependent on the injection settings. To better analyse the response of the laser with respect to a modulation of the optical injection signal, we will investigate, in more detail, four distinct cases distributed across the locking range, i.e. four points in the detuning versus injection rate ($\Delta$, $\kappa$) maps:

- **Point A**: $\Delta = -0.35$, $\kappa = 0.35$
- **Point B**: $\Delta = -0.5$, $\kappa = 0.7$
- **Point C**: $\Delta = -0.5$, $\kappa = 1$
- **Point D**: $\Delta = -1$, $\kappa = 1$

For each case, we have first computed their extrema maps in the modulation amplitude versus modulation frequency ($A$, $\omega_m$) plane, which are displayed in the top row of figure 3. These are then complemented by a detailed bifurcation diagram and the corresponding spectral evolution obtained for $A = 0.5$ and an increasing modulation frequency. While being, obviously, only valid for a single injection setting, these maps and diagrams naturally provide a much finer overview of the laser behaviour when varying the amplitude and/or the frequency of the modulation. Thus, we can immediately see that the situation is once again fairly complex as multiple instability regions can be identified. Of course, for low-amplitude modulations the laser exhibits stable P1 dynamics, but, for medium/high amplitudes, we can still find two frequency windows for which the laser mostly shows apparently stable locked P1 dynamics. The same color code as in previous sections is used, and these regions are therefore displayed in pink. Isolated blue dots inside pink regions—i.e. detection of a few but more than two extrema, especially observed around $\omega_m \approx 1.8$ in figure 3(a.1) and $\omega_m > 3$ in (c.1)—correspond to small numerical instabilities due to, e.g. unusually long transient behaviour or small processing errors at the extrema detection stage (especially for high modulation frequencies). While they appear to be very visible due to

![Figure 3](https://example.com/figure3.png)

**Figure 3.** (X.1) Extrema maps in the modulation amplitude versus modulation frequency ($A$, $\omega_m$) plane, (X.2) bifurcation diagrams showing the extrema of the simulated intensity time series for $A = 0.5$—shown by the horizontal dashed line in the extrema maps—and (X.3) the corresponding evolution of the RF spectra obtained by computing the fast Fourier transform (FFT) of the intensity time series. All three items are shown for each of the four configuration points mentioned in the text, namely points A (a), B (b), C (c) and D (d). Again, the color code for the extrema maps is identical to that in figure 1.
the color code used, we can easily confirm via bifurcation diagrams that they have absolutely no impact on the analysis and interpretation of the data and can therefore be neglected.

Looking now at the bifurcation diagram computed for \( A = 0.5 \) for each point, we observe a clear resonance of the laser with the modulated injection for frequencies roughly in the range of \( 1 < \omega_m < 2 \). In all four cases highlighted in figure 3, the bifurcation diagrams exhibit relatively large and clean P1 oscillations in this range. The corresponding spectra confirm that the oscillation frequency is a match with the frequency of the injection signal modulation. On the lower modulation frequency side, this region of stable P1 oscillations is typically delimited by more complex dynamics—except for a low-amplitude modulation. Despite the emergence of additional extrema, only the modulation frequency and its harmonics appear in the spectral data. In figure 3(a.3), a resonance of the 2nd (3rd) harmonic frequencies can be seen around \( \omega_m \approx 0.5 (0.33) \), respectively; that is, when the harmonic frequency matches the slave laser RO frequency.

On the other hand, the laser also shows stable P1 oscillations for larger modulation frequencies, typ. \( \omega_m > 3 \). This region is separated from the previous one by two period-doubling bifurcations creating a clear bubble in the bifurcation diagrams in figures 3(a.2) and (b.2). While the same transition appears only for larger amplitudes in the case of point C, it is not observed for point D for which only a large region of stable P1 oscillations is observed for \( \omega_m > 1 \). In any case, even though the P1 oscillations are stable for large modulation frequencies, it is also obvious that their amplitudes are significantly lower than for \( \omega_m \approx 1 \) and keep decreasing as the modulation frequency is increased.

4. Characterization of the passband filtering induced by the laser response

The relatively strong resonance observed for \( \omega_m \approx 1 \) appears to be the dominant feature when considering the slave laser as a potential signal processing unit. Of course, this should not conceal the emergence of additional extrema or complex dynamics—these would clearly be detrimental and should be avoided—but the large variations of the oscillation amplitude due to the system’s resonance can certainly be expected to have a more important impact. As a result, we have decided to characterize this effect in more detail, analysing it as a passband filter applied to the modulated injection signal.

4.1. Influence of the modulation amplitude

We focus here on two main figures of merit: 1/ the peak amplitude frequency, i.e. the modulation frequency for which the stable P1 oscillations have the largest amplitude, and 2/ the 3 dB bandwidth, i.e. the size of the modulation frequency range within which the amplitude of the P1 oscillations is always larger that half the maximal oscillation amplitude (obtained at the peak amplitude frequency). From the bifurcation diagrams in figure 3, we can already see that the shape of the oscillation peak is not symmetrical. Thus, the peak amplitude frequency will not be at the center of the 3 dB passband. Nevertheless, these two figures of merit provide a valuable quantification of the filtering effect and remain suitable at this stage of the investigation.

In figure 4, we show the evolution of the peak amplitude frequency and the 3 dB bandwidth for an increasing modulation amplitude at each of the four points A, B, C and D identified in the previous section. The modulation amplitude is tuned from \( A = 0.02 \) to 0.5. Beyond this value the instabilities for low modulation frequencies exhibit oscillation amplitudes that are sufficiently large enough to prevent a straightforward estimation of the 3 dB bandwidth. We therefore decided to leave this range out of our current investigation.

First, we can observe that both the peak frequency and the bandwidth are strongly dependent on the injection configuration. Indeed, the four configurations that are considered here are distributed across the locking range and all lead to different results. Although the evolution for increasing modulation amplitudes is

Figure 4. Evolution of peak amplitude frequency (a) and the 3 dB bandwidth (b) for an increasing modulation amplitude for each of the four points A, B, C and D, shown as a full blue, dashed red, dotted yellow and dash-dotted purple line, respectively.
not strictly identical, the respective performances measured for each configuration remain identical across the considered amplitude range: the lowest bandwidth and lowest peak amplitude frequency are obtained for point A. Point B appears in the second rank for both figures of merit, which suggests that the tip of the tooth-shaped locking range corresponds to the most selective configurations. On the other side of the spectrum, we see that points C and D are significantly less selective. At the same time, we observe that the peak amplitude frequency for D is actually lower than that of C. This is the only inversion of ranks that we noticed between panels (a) and (b) in figure 4.

Next, in terms of variability, it appears that the peak amplitude frequency is more stable than the 3 dB bandwidth. Across all possible variations considered in figure 4, the peak amplitude frequency always remains between 1 and 1.4—which already represents a 40% variation—while the 3 dB bandwidth goes from 0.1 to 1.2. All the time scales of our model are rescaled with respect to the RO frequency of the slave laser. Since the latter is typically in the 0.1 to 10 GHz range, it is obvious that the selectivity and variability of the filtering effect will strongly depend on the specifications of the laser itself. Nevertheless, our simulation results tend to indicate that, for an RO frequency around 1 GHz, a passband between 200 MHz and 1.2 GHz could be expected.

4.2. Impact of the varying injection parameters

If we now look at the evolution of our two figures of merit when changing the injection configuration, i.e. when changing the detuning and the injection rate, we obtain the graphs displayed in figure 5. Of course, this is merely an example of the evolution that can be expected: the non-linearity and complexity of the system prevents a generalization of the reported behaviour to the whole locking range. Nevertheless, the reported trends seem to fit well with observations of other cases.

Increasing the detuning (in absolute value), as in figure 5(a), leads to a clear decrease in the peak amplitude frequency combined with a rather significant increase in the 3 dB bandwidth. In a sense, increasing the detuning has a similar effect to increasing the modulation amplitude. While the evolution of the peak amplitude frequency seems quite linear, the bandwidth exhibits an inflexion point around $\Delta \approx -1$ separating two fairly linear sections. We have observed that this inflexion point appears for different injection or laser parameters. While it seems to be less pronounced for smaller modulation amplitudes, it also appears to be significantly sharper for larger injection rates. Of course, the fact that such inflexion occurs for a detuning roughly equal to the RO frequency of the laser suggests that a certain interaction or coupling might be at stake. Yet, we were not able to identify the origin of this change in slope at this point. On the other hand, increasing the injection rate mostly leads to an increase of both the peak amplitude frequency and the bandwidth. For $\kappa < 0.4$, however, we observe a small decrease in the bandwidth before reaching a minimum at $\kappa \approx 0.45$. This effect might be linked to the fact that the system is reaching the boundary of the locking range on the low detuning side. This area might be slightly more stable for medium modulation frequencies as suggested by figure 2(b.2), where the system appears to be stable close to the boundary unlike in the rest of the locking range.

5. Influence of the modulation scheme on the laser dynamics

In this last section, we investigate the dynamical effect of two different modulation schemes. On one hand, we turn our initial sine-wave modulation into a square-wave signal. Being closer to a typical digital signal, this allows us to have quick pick into the potential of our system for data communication applications. On the other
hand, we also investigate the impact of a phase modulation as opposed to the amplitude modulation used in previous sections.

5.1. Square wave modulation
For a very low modulation frequency, the response of the laser for each edge of the square wave will be almost independent. In short, the laser will only see a single step function as a perturbation. In case the upper and lower parts of the square wave are both in the injection locking region, the perturbation simply triggers ROs which then slowly vanish as the laser settles on the new solution. This is what we see in figure 6 (a). Besides the fact that the response of the laser to the upward and downward edge is different—the transient appears to be much shorter when the injection rate is increased—we also observe another critical feature: the response of the laser is consistent, i.e. over one square-wave period the laser behaves exactly the same way. As a result, when looking at the extrema of the time series, we can identify a pattern which remains fixed as long as the period of the square-wave function is long enough to let the laser relax. This feature can be spotted in all diagrams shown in the first row of figure 7, and especially in (d.1). Yet, because of these oscillations, looking at the number of extrema becomes a quite unreliable way of quantifying the stability/instability of the system. Indeed, a small number of extrema can no longer be interpreted as a relatively stable response. In addition, as shown in figure 6(a), the output power variations induced by the modulation appear to be much smaller than the amplitude of the ROs. In short, to perform the same analysis that we did for the sine-wave modulation we need to refine our processing techniques. Although this goes out of the scope of the paper, simulating the system evolution for some particular cases already provides an important insight.

Figure 6. The simulated time series of the slave laser output power for a square-wave modulation of the injection beam for a frequency of $\omega_m = 0.01$ (a), $1.07$ (b), which is the frequency for which the largest oscillation amplitude is recorded, and $3$ (c). All three cases have been simulated for point C, i.e. $\kappa = 1$ and $\Delta = -0.5$. The reader should pay attention to the fact that the vertical scale used for panel (b) is different to that used for panels (a) and (c).

Figure 7. Bifurcation diagrams of the intensity time series for $A = 0.5$, and corresponding evolution of the RF spectra for a square-wave amplitude modulation of the injection signal for points A, B, C and D, shown in (a.1–a.2), (b.1–b.2), (c.1–c.2) and (d.1–d.2), respectively.
case modulation with output power, see phase-amplitude coupling in the laser cavity, the slave laser output exhibits a phase modulation with a steady so far. Considering again a low-frequency low-amplitude modulation, our simulations show that, despite the Finally, we consider the case of a phase modulation as opposed to the amplitude modulation that we focused on

5.2. Phase modulation

is inside the passband that we identify more high-frequency components.

periodic evolution closer to a sawtooth signal as the frequency is increased, as shown in figure 7. In between, for 0.4 − \( \pi \) to take into account the change in frequency induced by the locking. In addition, we limit the phase variations to the \([-\pi, \pi]\) range for better visualization of the evolution of the system’s dynamical behaviour. In some cases, however, it appears that the phase evolution is unbounded, indicating that the slave laser frequency is no longer locked to the frequency of the injected signal. As a result, the limitation in a 2\( \pi \) range induced discontinuities which are particularly detrimental to the FFT computation, as can be seen in panels (a.4), (b.4) and (c.4). Indeed, the modulation frequencies leading to an unbounded phase evolution can be directly spotted by an almost complete dark red line. Nevertheless, at this stage of our investigation, these cases can simply be disregarded as relevant information is available in other plots.

In short, this preview focusing on square-wave amplitude modulation tends to confirm previous analysis of the impact of the resonance at the RO frequency and resulting filtering effects. When the fundamental frequency is inside the passband that we identified earlier, the other spectral components are effectively filtered out. Yet, at the same time, any input at a lower frequency can be expected to trigger a complex nonlinear response.

5.2. Phase modulation

Finally, we consider the case of a phase modulation as opposed to the amplitude modulation that we focused on so far. Considering again a low-frequency low-amplitude modulation, our simulations show that, despite the phase-amplitude coupling in the laser cavity, the slave laser output exhibits a phase modulation with a steady output power, see figures 8(a.1) and (a.2). Yet, we also observe that the laser gets quickly destabilized, which naturally leads not only to phase fluctuations but also output power fluctuations, as shown in panels (b.1) and (b.2) of figure 8.

To go further, we again consider the four optical injection configurations already discussed and identified as points A, B, C and D. For each case, we simulate the system evolution for a fixed modulation amplitude of \( A = \pi/2 \) and modulation frequency between 0 < \( \omega_m \) < 5. Then, for each setting, we compute the bifurcation diagram and FFT spectrum of the output power time series and of the phase time series. The results are displayed in figure 9. Because all points are in the locking range, we do not consider the phase term \( \phi \) directly, but show \( \phi - \Delta \tau \) to take into account the change in frequency induced by the locking. In addition, we limit the phase variations to the \([-\pi, \pi]\) range for better visualization of the evolution of the system’s dynamical behaviour. In some cases, however, it appears that the phase evolution is unbounded, indicating that the slave laser frequency is no longer locked to the frequency of the injected signal. As a result, the limitation in a 2\( \pi \) range induced discontinuities which are particularly detrimental to the FFT computation, as can be seen in panels (a.4), (b.4) and (c.4). Indeed, the modulation frequencies leading to an unbounded phase evolution can be directly spotted by an almost complete dark red line. Nevertheless, at this stage of our investigation, these cases can simply be disregarded as relevant information is available in other plots.

Figure 8. The simulated time series of the output power (X.1) and phase (X.2) of the slave laser in a phase modulation of the injection beam for a frequency of \( \omega_m \) = 0.01 (a), 1.1 (b) and 3 (c). All three cases have been simulated for point B, i.e. \( \kappa = 0.7 \) and \( \Delta = -0.5 \). The reader should pay attention to the fact that the vertical scale used for panel (b.1) is different to that used for panels (a.1) and (c.1).
The most striking feature, with regards to the results obtained for amplitude modulation, is that only low-frequency modulation signals \( \omega_m \leq 0.2 \) or \( \leq 0.4 \) depending on the configuration) are going through the slave laser rather untouched: the peak-to-peak amplitude of the phase oscillations is similar to that of the input signal \( A = \pi/2 \) which corresponds to a peak-to-peak amplitude is \( \pi \) and the output power remains almost constant. At high-frequency, the phase modulation is significantly damped while notable oscillations can be seen for the output power: see figures 8(c.1) and (c.2). In between, instead of a clean passband around \( \omega_m \approx 1 \), we observe various complex behaviour for both the output power and the phase terms, as displayed in panels (b.1) and (b.2) of figure 8. In all cases, we also see in the FFT spectra the presence of a contribution at \( \omega_m/2 \), which was only observed for particular ranges of parameters for amplitude modulation and which suggests that a period-doubling bifurcation is playing a significant role in the unstable behaviour of the laser.

As a result, it therefore seems that the response of the laser to phase modulated optical injection is somewhat opposed to the one observed for amplitude modulation. While we see a response similar to a passband filter around the RO frequency when considering amplitude modulation, phase modulation leads to loss of locking and complex phase and amplitude fluctuations. Even if we can naturally expect a similar variation in behaviour to that for amplitude modulation, this opposition is a rather clear indication that a thorough and independent analysis of the phase modulation case will have to be performed.

6. Discussion and perspectives

Not surprisingly, our investigation shows once again that semiconductor lasers can have very rich dynamical behaviour. While optical injection was already a well-known trigger for complex dynamics, adding a modulating term to the mix does not naturally lead to a simplification of the overall picture. Nevertheless, if we come back to our initial focus of using an optically injected laser as a filter, our numerical study already provides some important insights.

First, even though this can be rather intuitive for people working in the field of laser dynamics, it is important to remark that there is an essential dependence of the laser dynamics to the selected type of modulation. Amplitude and phase modulation leads to dramatically different responses. Thus, the use of other types—such
as frequency, single-sideband, quadratic amplitude, amplitude or phase shift-keying modulation, and many others—will naturally have to be thoroughly tested and inspected. This is without even mentioning that going from a pure sine-wave to a square-wave modulation also showed a non-negligible impact. Of course, this can be seen as a major drawback in terms of application, but the wide variety of possible dynamics can also be seen as an exciting opportunity if we can find out how to successfully harness them.

Second, in the case of amplitude modulation, we have highlighted a clear passband filtering effect for frequencies around the RO frequency of the slave laser. It appears that such input can easily resonate inside the laser cavity and experience a significant amplification. Outside of this resonating range, the fluctuations of the output power are damped and have a much smaller amplitude. Taking a square-wave signal—with a much more complex spectral content than the initial sine wave—as an input confirms this analysis and the induced filtering effect. In addition, we have shown that the bandwidth of the passband can be tuned by changing the optical configuration. However, it appears that the selectivity would also depend on the modulation amplitude and, thus, on the input signal strength.

Third, even when the injection configuration lies within the locking range, instabilities can be triggered by modulations of the injected signal. In the case of amplitude modulation, these instabilities arise mostly for low-frequency modulations and have a fairly limited amplitude; smaller in fact than the oscillations obtained at the peak amplitude frequency. For phase modulation, however, we report severe instabilities leading to large frequency modulations and have a fairly limited amplitude; smaller in fact than the oscillations obtained at the RO frequency of the laser. From an application viewpoint, it is obviously essential to set up either a good mitigation strategy to avoid these instabilities and/or minimize their impact, or harness the resulting dynamics.

In a nutshell, it seems rather fair, at this stage, to say that optically injected semiconductor lasers might be unsuitable for filtering applications, especially as a general purpose filter. Yet, we believe that the variety of dynamical responses triggered by the modulation of the optical injection signal combined with the compactness and possible monolithic integration of this scheme on a photonic integrated circuit could be valuable for niche applications. Dual-optical injection in dual-wavelength lasers [49] with only one injection signal being modulated could certainly be advantageous exploited for generation of complex or chaotic high-frequency signals. Generation of high-frequency pulsed signals could also be considered using the excitable behaviour of injected semiconductor lasers. Nevertheless, in the end, the relevance of this scheme will necessarily have to be preceded by an even more thorough numerical and experimental exploration of the system’s behaviour.

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