Heavy-hadron molecules from light-meson-exchange saturation

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In the effective field theory framework the interaction between two heavy hadrons can be decomposed into a long- and a short-range piece. The long-range piece corresponds to the one-pion-exchange potential and is relatively well-known. The short-range piece is given by a series of contact-range interactions with unknown couplings, which substitute the less well-known short-range dynamics. While the general structure of the short-range potential between heavy hadrons is heavily constrained from heavy-quark symmetry, the couplings are still free parameters. Here we argue that the relative strength and the sign of these couplings can be estimated from the hypothesis that they are saturated by the exchange of light mesons, in particular the vector mesons \( \rho \) and \( \omega \), i.e. from resonance saturation. However, we propose a novel saturation procedure that effectively removes form-factor artifacts. From this we can determine in which spin and isospin configurations the low-energy constants are most attractive for specific two-heavy-hadron systems. In general the molecular states with lower isospins and higher spins will be more attractive and thus more probable candidates to form heavy-hadron molecules. This pattern is compatible with the interpretation of the \( X(3872) \) and \( P_c(4312/4440/4457) \) as molecular states, but it is not applicable to states with maximum isospin like the \( Z_c(3900/4020) \).

Heavy-hadron molecules might very well be the most popular type of exotic hadron [1,5]. The probable reason is their conceptual simplicity, which is only matched by the challenge of making concrete predictions in the molecular picture. Despite just being non-relativistic bound states of two heavy hadrons, the theoretical toolbox behind hadronic molecules has grown into a bewildering hodgepodge which is often diﬃcult to disentangle, to say the least. This is in contrast with the much more coherent descriptions offered by the quark model [4,5] or the theory behind quarkonium [6,10].

Yet the molecular picture has a few remarkable successes under its sleeves. They include the prediction of the \( X(3872) \) by Törnqvist [11], later detected by the Belle collaboration [12], and the prediction of three hidden-charm pentaquarks [13–19] (\( \Sigma, \bar{D}, \Sigma, \bar{D}^* \) molecules), which might very well correspond with the \( P_c(4312), P_c(4440) \) and \( P_c(4457) \) pentaquarks recently detected by the LHCb [20].

Regarding the \( X(3872) \), the most compelling evidence that it is molecular is its closeness to the \( D^*\bar{D}^* \) threshold [21–23] but its isospin-breaking decays [24] which are naturally reproduced in the molecular picture [25,27]. For the LHCb pentaquarks, though the molecular explanation is gaining traction [28,42], there are a few competing hypothesis about their nature [44,46].

Despite the numerous candidates and the intense theoretical interest, the qualitative and quantitative properties of the molecular spectrum are poorly understood. The present manuscript attempts to address this limitation by proposing a potential pattern in the spectrum of two-heavy-hadron bound states: for configurations without maximum isospin, the states with higher (light-quark) spin are expected to be lighter (i.e. more bound). This is the opposite pattern as with compact hadrons, for which mass usually increases with spin.

This pattern might explain why besides the \( X(3872) \) no other \( D^*\bar{D}^* \) molecule has been observed, as they should not be expected to be bound (with the exception of the \( 2^+ \) \( D^*\bar{D}^* \) configuration [47,48]) modulo other effects that could unbind it [49,50]. If applied to the light sector, it also explains why in the two-nucleon system the deuteron binds while the singlet state does not, or why the \( d^*(2380) \) is a \( \Delta \bar{\Delta} \) bound state [51] its spin should be \( J = 3 \). It also states that if the \( P_c(4440) \) and \( P_c(4457) \) are \( \Sigma, \bar{D}^* \) bound states, their expected quantum numbers are \( \frac{3}{2}^- \) and \( \frac{1}{2}^- \) respectively. This prediction, which agrees with a few theoretical analyses [40,43,53], will be put to the test by the eventual experimental determination of the quantum numbers of the pentaquarks.

This pattern is deduced from matching a contact-range description of the interaction between two heavy hadrons with a phenomenological description in terms of the potential generated by the exchange of light mesons. That is, we are considering the saturation of the low-energy constants by light-meson exchange (as in [54]). We will illustrate this idea with the one-pion-exchange (OPE) potential, which for two spin-\( \frac{1}{2} \), isospin-\( \frac{1}{2} \) hadrons reads

\[
V(q) = -\frac{g^2}{2f^2} \tau \left( \sigma_1 \cdot q \sigma_2 \cdot q \right) \frac{1}{q^2 + m^2} + \frac{1}{2} \left[ \sigma_1 \cdot q \sigma_2 q^2 \right] \frac{1}{q^2 + m^2} + \frac{1}{2} \left[ \sigma_1 \cdot q \sigma_2 q^2 \right] \frac{1}{q^2 + m^2},
\]

(1)

with \( g \) the axial coupling, \( f \sim 130 \text{ MeV} \) the pion decay constant, \( q \) the exchanged momentum and \( q = |q|, m_\pi \) the pion mass, \( \sigma_i \left( \tau_i \right) \) the Pauli matrices for hadron \( i = 1, 2 \) in spin (isospin) space, and \( \tau = \tau_1 \cdot \tau_2 \) an isospin factor. In the second line the potential has been decomposed into a spin-spin and a tensor piece. We will ignore the tensor piece, as it requires SD-wave mixing. We will consider the effect of OPE on the...
saturation of the couplings of the lowest-order contact-range potential, which is purely S-wave.

The idea behind saturation is to map the OPE potential into an effective potential of the type

\[ V_C(q) = C_0(\mu) + C_1(\mu) \bar{\sigma}_1 \cdot \bar{\sigma}_2, \]  

which requires a regulator (not explicitly written here), with \( \mu \) being a regularization scale (i.e., a cutoff), which we will choose around the mass of the exchanged light meson (\( \mu \sim m_\pi \) in this case) for saturation to work. If we expand the spin-spin piece of Eq. (1) in powers of \( q \),

\[ V(q) = -\frac{g^2}{6f^2} \tau \bar{\sigma}_1 \cdot \bar{\sigma}_2 + \ldots, \]  

then, by matching this expansion with the effective potential \( V_C \), we will deduce that OPE should not saturate the couplings:

\[ C_0^{\text{OPE}}(\mu \sim m_\pi) \sim 0, \quad C_1^{\text{OPE}}(\mu \sim m_\pi) \sim 0. \]  

However, this conclusion is premature. If we rewrite the \( q^2 \)-dependence as

\[ \frac{q^2}{q^2 + m_\pi^2} = 1 - \frac{m_\pi^2}{q^2 + m_\pi^2}, \]  

then the first contribution in the right-hand side is actually a Dirac delta. Owing to the finite size of the pions, this Dirac delta will acquire a finite size \( \sim 1/M \), with \( M \) the physical cut-off of the theory (probably a bit above 1 GeV). This does not necessarily coincide with the scale \( \mu \) we use for the effective interaction. In general saturation works best for \( \mu \sim m \) for the mass of the light meson, while for the exchange of a light meson to have physical meaning we need \( m < M \). From this, the saturation scale verifies \( \mu < m \), implying that in practice we can simply ignore contributions with a range shorter than \( 1/\mu (\sim 1/m) \), including the aforementioned delta. Thus for saturation purposes we will simply make the substitution

\[ \frac{q^2}{q^2 + m_\pi^2} \rightarrow -\frac{m_\pi^2}{q^2 + m_\pi^2}, \]  

in the exchange potential. This substitution rule leads to the saturated couplings

\[ C_0^{\text{OPE}}(\mu \sim m_\pi) \sim 0, \quad C_1^{\text{OPE}}(\mu \sim m_\pi) \sim -\frac{g^2}{6f^2} \tau. \]  

Finally, we can compare how well does the saturated contact-range interaction versus the potential from which it is derived. This is done in Figure 1 where we check that it works relatively well for the scattering length \( a_0 \) depending on the strength of the potential (the details of the regularization are explained in the caption). Particularly saturation correctly reproduces the existence of a bound state, which is signaled by a change of sign in the scattering length.

This idea can be extended to the exchange of other light mesons besides the pion, in particular the sigma, the rho and the omega. Actually in the case of the sigma there is no difference with the standard procedure [54], because its interaction does not generate a short-range delta. Regarding the rho and omega mesons, they will give rise to a central, a spin-spin and a tensor piece, where the difference now is that the spin-spin piece will contribute to the low-energy constants.

The interesting thing is how this applies to heavy-hadron molecules. Instead of using the standard superfield formalism, we will write the interaction between two heavy hadrons in the light-quark formalism described in Ref. [55]. The number of independent contact-range couplings depends on the ways to combine the light spins \( S_L1 \) and \( S_L2 \) of the two heavy hadrons 1 and 2: \( S_L1 \otimes S_L2 = |S_L1 - S_L2| \otimes \cdots \otimes (S_L1 + S_L2) \). This means, for instance, that in the \( DD \) and \( \Sigma \bar{D} \) families of molecules there are two independent couplings, in the \( \Sigma \Sigma \) family three independent couplings and in the \( D \bar{D} \) family four couplings. In addition, if the two heavy hadrons have different light spin, there is the possibility of additional couplings for operators involving the exchange of light spin (the \( \Lambda \Sigma \bar{c}_c \) system being an example). From this the S-wave contact-range interaction of two heavy hadrons can be written as

\[ V_C = C_0 + C_1 S_{L1} \cdot S_{L2} + C_2 \hat{Q}_{L,ij} \hat{Q}_{L,ij} + \ldots, \]  

that is, a series of the products of irreducible tensors built from the light-spin operators \( \hat{S}_{L1} \) and \( \hat{S}_{L2} \). The operator \( \hat{S}_{L} = S_L/[S_L^2] \) is a normalized spin operator, while the operator \( \hat{Q}_{L,ij} \) is the spin-2 product

\[ Q_{L,ij} = \frac{1}{2} [S_{L,i}S_{L,j} + S_{L,j}S_{L,i}] - \frac{S_L^2}{3} \delta_{ij}, \]  

which is later normalized as \( \hat{Q}_{L,ij} = Q_{L,ij}/|Q_{L,33}| \). Analogously we can define higher-spin products of \( S_{L1} \) and \( S_{L2} \).
To determine how to saturate the couplings $C_j$ of the effective potential, we will split it in two contributions coming from the scalar- and vector-meson potentials: $C_j = C_j^S + C_j^V$. We begin by writing the Lagrangians. For the interaction of a scalar meson with the light-quark degrees of freedom, the Lagrangian reads

$$L^S = g_\sigma q_L^\dagger \sigma q_L,$$

where $g_\sigma$ is a coupling constant, $\sigma$ is the scalar meson field and $q_L$ is a non-relativistic field for the light quarks within a heavy hadron. With this Lagrangian we end up with the potential

$$V_\sigma = -\frac{g_\sigma^2}{q^2 + m_\sigma^2},$$

for which saturation reads

$$C_{0j}^S \sim -\frac{g_\sigma^2}{m_\sigma^2}, \quad C_{Pj}^S \sim 0.$$  \hspace{1cm} (12)

For the vector mesons the Lagrangian can be written as the multipole expansion

$$L^V = L_{E0}^V + L_{M1}^V + L_{E2}^V + \ldots$$

$$= g_V q_L^\dagger V_0 q_L + \frac{f_V}{2M} q_L^\dagger \varepsilon_{ijk} S_{Lj} \partial_i V_k q_L + \frac{h_V}{2M^2} q_L^\dagger \hat{Q}_{Lij} \partial_i \partial_j V_0 q_L + \ldots,$$  \hspace{1cm} (13)

where we have explicitly written the electric charge, magnetic dipole and electric quadrupolar terms and with the dots indicating higher-order multipole terms. In this Lagrangian, $g_V$, $f_V$ and $h_V$ are coupling constants, $V_\mu = (V_0, V_i)$ is the vector meson field and $M$ is the typical mass scale for the size of the vector mesons. The number of terms depends on the spin of the light-quark degrees of freedom, where for $S_L = 0$ (e.g. $\Lambda_c$) there is only the electric term, for $S_L = \frac{1}{2}$ (e.g. $D, D^\ast$) there is also the magnetic dipole term, for $S_L = 1$ ($\Sigma_c, \Sigma_c^\ast$) we add the electric quadrupole term, and so on. For this Lagrangian it is easy to derive the one-boson-exchange potential [56] for a particular two-heavy-hadron system, where the contributions read

$$V_{E0} = + \frac{g_V^2}{q^2 + m_V^2},$$  \hspace{1cm} (14)

$$V_{M1} = - \frac{f_V^2}{4M^2} \frac{(S_{L1} \times \hat{q}) \cdot (S_{L2} \times \hat{q})}{q^2 + m_V^2},$$

$$= - \frac{2}{3} \frac{f_V^2}{4M^2} \frac{S_{L1} \cdot S_{L2}}{q^2 + m_V^2} + \ldots, \hspace{1cm} (15)$$

$$V_{E2} = + \frac{h_V^2}{4M^2} \frac{\hat{Q}_{L1ij} \hat{q} \hat{q}_i}{q^2 + m_V^2},$$

$$= + \frac{h_V^2}{36M^4} \frac{\hat{Q}_{L1ij} \hat{Q}_{L2ij}}{q^2 + m_V^2} + \ldots, \hspace{1cm} (16)$$

where for the M1 and E2 terms we isolate the S-wave piece in the second line. If we remove the Dirac-delta terms, we can deduce the saturation condition for vector-meson exchange. But first we have to distinguish between the $\omega$ and $\rho$ meson contributions. The most obvious difference is that the $\rho$ contribution contains an isospin factor that we have not explicitly written. Owing to the negative G-parity of the $\omega$, its contribution changes sign depending on whether we are dealing with the hadron-hadron or hadron-antihadron cases. Regarding the couplings, SU(3)-flavor symmetry and the OZI rule imply that the $\rho$ and $\omega$ couplings are identical for heavy hadrons in the 3 or 6 representation (which include all the cases considered here). After removing the Dirac-delta terms, we get the saturation conditions

$$C_0^V (\mu - m_V) \sim -\frac{g_V^2}{m_V^2} (\zeta + \hat{T}_1 \cdot \hat{T}_2),$$  \hspace{1cm} (17)

$$C_1^V (\mu - m_V) \sim \frac{f_V^2}{6M^2} (\zeta + \hat{T}_1 \cdot \hat{T}_2),$$  \hspace{1cm} (18)

$$C_2^V (\mu - m_V) \sim \frac{h_V^2 m_V^2}{36M^4} (\zeta + \hat{T}_1 \cdot \hat{T}_2),$$  \hspace{1cm} (19)

where $\zeta = \pm 1$ gives the contribution from the omega and $\hat{T}_i = T_i/T_i$ is the normalized isospin operator. The saturation condition generates $C_j$ couplings with consistent signs. From this we can see that for the isoscalar hadron-antihadron system the saturated couplings are always attractive:

$$C_j^V (I < I_1 + I_2) < 0.$$  \hspace{1cm} (20)

This does not imply that the potential is always attractive, because that will depend on the linear combination of $C_j$’s that conform the contact-range potential in a given channel. Yet, if we notice that the $C_j$’s follow a multipole expansion, the natural expectation is that terms involving higher multipoles will be smaller:

$$|C_j^V| < |C_j^V| \text{ for } J’ > J.$$  \hspace{1cm} (21)

This expectation is indeed confirmed by the LHCb pentaquark trio, provided they are molecular, as attested by a few theoretical works [37, 38, 40, 43].

To illustrate this idea we consider a few examples: 1) the $DD$ and $D\bar{D}$ family of molecules, 2) the $D\Sigma_c$ and $D\bar{\Sigma}_c$ family, and 3) the $\Sigma_c \Sigma_c$ and $\Sigma_c \Sigma_c^\ast$ one. We have summarized the form of the contact-range potential for these three cases in Table I. For the first case, which includes the X(3872), it is more convenient to define the contact-range potential in terms of the Pauli matrices (instead of the spin matrices)

$$V_c^{(1)} = C_0^V + S_{L1} \cdot S_{L2} C_1^V,$$  \hspace{1cm} (22)

for which vector saturation gives

$$C_0^V \sim \frac{g_V^2}{m_V^2} (\zeta + \tau_1 \cdot \tau_2),$$  \hspace{1cm} (23)

plus the analogous expression for $C_j^V$. From this it is clear that the $I = 0$ isoscalar configurations are guaranteed to be attractive. For the isovector configurations the $\rho$ and $\omega$ contributions cancel out: for the $C_0$ coupling there is still the scalar-meson contribution, which will result in attraction, while for
the C\textsubscript{1} coupling the sign will depend on how the SU(3)-flavor symmetry is broken. Alternatively, the exchange of the a\textsubscript{1} meson [57] would imply C\textsubscript{1}(I = 1) > 0 for the Z\textsubscript{c}(3900) and Z\textsubscript{c}(4020) resonances, which is compatible with their quantum numbers (J\textsuperscript{PC} = 1\textsuperscript{−+}). Thus it might be possible that the I = I\textsubscript{1} + I\textsubscript{2} configurations revert to the naive expectation of higher (light-quark) spin states having higher masses.

For the second case, the \( D\Sigma \) and \( D\Sigma \) family of molecules (which include the LHCb pentaquark trio), we define the contact-range potential as

\[ V_{C}^{(2)} = D_{0} + \vec{S}\cdot D_{1} , \]

where \( \vec{S}\cdot D_{1} \) refers to the spin-1 angular momentum matrices. Saturation in this case gives

\[ D_{0}^{V} \sim \frac{g_{V}g_{V}^{\prime}}{m_{V}}(\zeta + \tau_{1}\cdot T_{2}) , \]

plus the analogous expression for \( D_{1} \), with \( g_{V}^{\prime} \) the vector-meson coupling for the \( \Sigma \) and \( \Sigma^{\prime} \) baryons and \( T_{2} \) their isospin operators. This expression indicates that the isospin-\( \frac{1}{2} \) configurations are attractive for both the \( D\Sigma \) and \( D\Sigma \) cases. A second conclusion is that in the \( D\Sigma \) system the \( J^{P} = \frac{3}{2}^{−} \) configuration is expected to be more attractive than the \( J^{P} = \frac{1}{2}^{−} \) one, which implies that the quantum numbers of the \( P_{c}(4440) \) and \( P_{c}(4457) \) pentaquarks should be \( J^{P} = \frac{1}{2}^{−} \) and \( \frac{1}{2}^{−} \), respectively. A third conclusion is that the doubly charmed \( D^{\ast}\Sigma \) family of molecules are expected to be more tightly bound than the hidden-charm pentaquarks, owing to the different sign of \( \omega \) contribution [58].

Finally, if we apply it to the \( \Sigma \Sigma \) and \( \Sigma \Sigma \) family of molecules, the contact-range potential reads

\[ V_{C}^{(3)} = E_{0} + E_{1}\vec{S}_{L1}\cdot \vec{S}_{L2} + E_{2}\vec{Q}_{L1i}\vec{Q}_{L2ij} . \]

The vector-meson saturation of the couplings yields

\[ E_{0} \sim \frac{g_{V}^{2}}{m_{V}}(\zeta + \tau_{1}\cdot T_{2}) , \]

plus the analogous expressions for \( E_{1} \) and \( E_{2} \). From this the isoscalar and isovector \( 2^{−−} \) and \( 3^{−−} \) heavy baryonia are expected to be the most attractive.

To summarize, we propose a description of heavy-hadron molecules in terms of contact-range potentials that depend on a few couplings. The couplings are determined from saturation by scalar- and vector-meson exchange, where we propose a novel saturation procedure that takes into account the physical scale at which saturation is actually happening. The outcome is that it is possible to know the sign and relative strength of the two-heavy-hadron interaction, from which we can deduce a few qualitative properties of the heavy-molecular spectrum. The most interesting pattern is that for heavy molecular states with maximal isospin, we expect the configurations with higher light-quark spin to be more bound (or, equivalently, lighter if we refer to the mass of the states). This pattern is exactly the opposite of the one that is observed in standard compact hadrons, where mass usually increases with spin. The pattern is compatible with the quantum numbers of the \( X(3872) \) in the molecular picture and with the experimental absence of molecular partners of the \( X(3872) \) with smaller light-quark spin. The pattern also extends to the light sector, with the deuteron (neutron-proton, \( I(J) = 0(1) \)) and the recently observed \( d^{\prime}(2380) \) (\( \Lambda\Delta \), \( I(J) = 0(3) \)) being two illustrative examples. Yet the real test of the present idea will be the eventual experimental measurement of the quantum numbers of the \( P_{c}(4440) \) and \( P_{c}(4457) \) pentaquarks. If they are \( D^{\ast}\Sigma \) molecules, the saturation hypothesis suggests that the \( J = \frac{1}{2} \) state should be the most bound of the two, i.e. the spin of the \( P_{c}(4440) \) should be \( \frac{1}{2} \).

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**TABLE I.** Structure of the contact-range potential for the \( D\Omega \), \( D\Sigma \), \( D\Sigma \), and \( \Sigma \Sigma \) family of molecules. For configurations in which the isospin \( I \) of the molecule is not maximal, \( I < (I_{1} + I_{2}) \), all the couplings appearing in this table are expected to be negative in sign (i.e., attractive). If we take into account that the previous couplings are expected to be smaller as the multipole moment increases, then we arrive at the labels “Most”, “Yes”, “Likely” to characterize whether a particular molecular configuration is attractive.

| Molecule | \( J^{P} \) | \( V \) | Attractive? |
|----------|------------|--------------|-------------|
| \( D\Omega \) | 0\textsuperscript{++} | \( C_{0} \) | Yes |
| \( D\Sigma \) | 1\textsuperscript{++} | \( C_{0} + C_{1} \) | Most |
| \( D\Sigma \) | 1\textsuperscript{−−} | \( C_{0} - C_{1} \) | Likely |
| \( D\Sigma \) | 0\textsuperscript{−−} | \( C_{0} - 2C_{1} \) | Likely |
| \( D\Sigma \) | 1\textsuperscript{−−} | \( C_{0} - C_{1} \) | Likely |
| \( D\Sigma \) | 2\textsuperscript{++} | \( C_{0} + C_{1} \) | Most |
| \( \Sigma\Sigma \) | 0\textsuperscript{−} | \( E_{0} - \frac{2}{3}E_{1} \) | Likely |
| \( \Sigma\Sigma \) | 1\textsuperscript{−} | \( E_{0} + \frac{2}{3}E_{1} \) | Yes |
| \( \Sigma\Sigma \) | 1\textsuperscript{++} | \( E_{0} - \frac{2}{3}E_{1} + \frac{2}{3}E_{2} \) | Likely |
| \( \Sigma\Sigma \) | 1\textsuperscript{−+} | \( E_{0} + \frac{2}{3}E_{1} - \frac{2}{3}E_{2} \) | Likely |
| \( \Sigma\Sigma \) | 2\textsuperscript{−+} | \( E_{0} + \frac{2}{3}E_{1} + \frac{2}{3}E_{2} \) | Most |
| \( \Sigma\Sigma \) | 3\textsuperscript{−+} | \( E_{0} + \frac{2}{3}E_{1} + \frac{2}{3}E_{2} \) | Most |
| \( \Sigma\Sigma \) | 0\textsuperscript{−−} | \( E_{0} - \frac{13}{15}E_{1} + \frac{13}{15}E_{2} \) | Likely |
| \( \Sigma\Sigma \) | 1\textsuperscript{−−} | \( E_{0} - \frac{1}{3}E_{1} + \frac{2}{3}E_{2} \) | Likely |
| \( \Sigma\Sigma \) | 2\textsuperscript{−−} | \( E_{0} - \frac{1}{3}E_{1} + \frac{2}{3}E_{2} \) | Likely |
| \( \Sigma\Sigma \) | 3\textsuperscript{−−} | \( E_{0} + \frac{1}{3}E_{1} + \frac{2}{3}E_{2} \) | Most |
