Numerical simulation of gas-water two-phase seepage flow based on level set method

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Abstract. The mechanism of gas-water two-phase percolation was previously characterized based on percolation experiments. However, the changes in the gas-water interface and the volume fraction of the two phases in the process of gas-water percolation in porous media cannot be accurately described by experiments. This paper uses core casting thin sections and AutoCAD software to establish a digital core. Based on the idea of level set, a two-dimensional gas-water seepage model in microscopic pores is established. The finite element numerical simulation software is used to solve the model to describe the gas. The changes of the water two-phase interface are visually displayed. At the same time, Darcy’s law is used to compare the relative permeability curve obtained from the gas-water two-phase seepage experiment with the relative permeability curve obtained from the gas-water two-phase seepage finite element simulation. The results show that the gas-water two-phase seepage experiment and the gas-water two-phase seepage finite element numerical simulation have basically the same characterization of the seepage ability of porous media. There is obvious fingering phenomenon in the process of gas flooding, and the gas preferentially passes through the large pore throat. The smaller the pore throat, the greater the velocity of gas passing through the pore throat. The gas-water two-phase interface change described by the numerical simulation of the gas-water two-phase flow based on the digital core is more reliable.

Keywords: level set method, gas-water two-phase flow, numerical simulation, N-S equation, digital core.

1. Introduction
The two-phase seepage mechanism in porous media is of great significance for improving oil and gas reservoir recovery. Researches on microscopic two-phase seepage in rock are mostly based on physical simulation experiments and numerical simulation methods. Gas-water two-phase seepage physical model experiments mainly include conventional core displacement experiments [1-3], glass etching physical model experiments [4-5], core slice microscopic displacement experiments [6-8], etc., but these experiments simulate the cost is high, the operation is more cumbersome and difficult, the resolution of the equipment used in the displacement process is often low, and the microscopic displacement process and seepage characteristics of the pore scale cannot be clearly observed, and it is difficult to meet the needs of fine research on oil and gas development. For the numerical simulation of gas-water...
two-phase seepage flow, there are currently two major categories: network simulation and Boltzmann method [9-12]. The pore structure in the traditional model has been replaced by columnar throats and interconnected spherical pores. Although the pore network model can calculate the relative permeability and residual oil saturation very well, the pore throat structure is too ideal to truly show the phase interface changes and phase volume fraction changes during the two-phase flow. The Boltzmann method can qualitatively simulate the two-phase seepage of rock pores, but it cannot describe the characteristics of phase interface changes in the pores.

The level set method proposed by Osher and Sethian has been relatively mature in the field of tracking the free-phase interface during gas-liquid two-phase flow. Many scholars at home and abroad use this method to study the changes of phase characteristics in the process of two-phase seepage [13]. Bird et al. (2014) used Avizo to convert X-ray-derived tomographic images of rock samples into STL files and import them into Comsol to simulate single-phase fluid flow in the pore space [14]. Gao Yajun et al. (2016) change characteristics of the critical throat diameter ratio of pore-scale two-phase channeling based on the Levelset method [15]. Wu Feng et al. (2019) compared and verified the dynamic seepage characteristics of microscopic gas-water two-phase fluid characterized by microscopic visual glass etching displacement experiment and finite element numerical simulation [16]. More and more scholars use finite element numerical simulation methods to characterize the flow characteristics of single-phase or multi-phase fluids in porous media.

Although some microscopic mechanisms of fluid percolation in porous media can be studied qualitatively or semi-quantitatively through experiments, the theoretical quantitative description lacks necessary means and scientific and reasonable methods. Therefore, studying the seepage problem of porous media at the pore scale and establishing a set of theoretical models that can accurately describe the gas-water two-phase interface and the volume fraction change of the two-phase have very important theoretical significance for deep understanding of the gas-water two-phase seepage mechanism. Therefore, in this paper, combined with gas-water two-phase displacement experiments, using core casting thin slices, the digital core is described. Based on the NS equation, the laminar level set method is introduced to establish a gas-water two-phase flow numerical simulation model, using finite element numerical simulation software The model was solved, the change law of the gas-water two-phase mobile phase interface and the volume fraction of the phase at the pore scale was studied in detail, and the relative permeability curve of the digital core was solved by Darcy's law.

2. Introduction

2.1. Gas-water two-phase seepage experiment

Figure 1 is a diagram of an experimental device for unsteady gas-water two-phase displacement commonly used in laboratories. Use this device to conduct gas-water two-phase displacement experiments on a sandstone with good physical properties in a certain oil reservoir. The steps are as follows:

① After drying the sampled core, measure the basic physical properties of the sample length L, diameter d, porosity φ, permeability K, and mass m.
② After the core is saturated with formation water, measure the wet weight of the rock sample;
③ Put the core into the core holder and fix the experimental instrument;
④ Use the constant pressure displacement pump to displace the sample, and record the core after the pressure stabilizes The pressure at the inlet and outlet of the holder and the water flow rate are used to calculate the effective permeability of the water phase;
⑤ Adjust the outlet gas and water measurement units to keep the confining pressure constant, adjust the inlet pressure, and use humidified nitrogen to perform constant After the pressure is stabilized, record the current inlet and outlet pressures before and after the core holder, record the cumulative value of the liquid output, the value of the gas flowmeter, and accurately record the liquid production, gas production, and Displacement gas flow rate. When the gas drives the water to the bound water state, record the
displacement gas flow rate and the port pressure before and after the core holder in the bound water state.

⑥ Use the following formula to calculate the relative permeability and water saturation of unsteady gas and water:

\[ f_w(S_g) = \frac{\Delta V_w(t)}{\Delta V(t)} \]  
\[ K_{rw} = f_w(S_g) \frac{1}{1/[\bar{V}_t^i - \bar{V}_g^i]} \]  
\[ K_{rg} = K_{rw} \frac{\mu_g}{\mu_w} \left( 1 - \frac{f_w(S_g)}{f_w(S_g)} \right) \]  
\[ I = \frac{Q(t)}{Q_w} \]  
\[ S_g = \bar{V}_w^i(t) - \bar{V}(t) f_w(S_g) \]  
\[ S_w = 1 - S_g \]  

Among them, \( f_w \) is the water content; \( \bar{V}(t) \) is the dimensionless cumulative liquid production; \( \bar{V}_w(t) \) is the dimensionless cumulative water production; \( \bar{V}_g^i(t) \) is the dimensionless cumulative liquid production at time \( i \); \( \bar{V}_{w-1}(t) \) is \( i-1 \) Dimensionless cumulative liquid production at all times; \( K_{rg} \) is the relative permeability of the gas phase; \( K_{rw} \) is the relative permeability of the water phase; \( Q_w \) is the initial water output from the outlet end, mL/s; \( Q(t) \) is the output liquid volume, mL/s; \( S_g \) is the gas saturation of the outlet end surface; \( S_w \) is the water saturation of the outlet end surface.

Figure 1. Diagram of experimental device for gas-water two-phase seepage flow
2.2. Numerical simulation method of gas-water two-phase seepage flow

The numerical simulation method of gas-water two-phase seepage uses Navier-Stokes equation combined with level set method to construct a mathematical model of gas-water two-phase seepage.

The N-S equation is used to describe the equation of motion for the conservation of momentum in a viscous incompressible fluid:

\[ \rho \left( \frac{\partial \bar{u}}{\partial t} + \bar{u} \cdot \nabla \bar{u} \right) = -\nabla p + \nabla \left[ \mu \left( \nabla \bar{u} + \left( \nabla \bar{u} \right)^T \right) - \frac{2}{3} \left( \nabla \cdot \bar{u} \right) I \right] + F \]  

(7)

Where, \( \rho \) is the fluid density, \( p \) is the velocity vector, \( \bar{u} \) is the velocity component at time \( t \), \( \mu \) is the dynamic viscosity, and \( F \) is the external force per unit volume flow.

Use the level set function \( \phi \) to define the respective volume fractions of the gas-water two-phase flow. When it is water, \( \phi = 0 \), when it is gas, \( \phi = 100\% \), and use the reinitialized level set function \( \phi \) value to track the two-phase flow interface to study Dynamic characteristics of gas-water two-phase flow interface. The level set method is a numerical technique commonly used in interface tracking and shape modeling. Its basic idea is to abstract the fluid flow interface into a high one-dimensional space, usually represented by the zero level set of the level set function. Through continuous iterative evolution of the interface level set equation until it stabilizes and the evolution process ends, the shape of the flow interface is obtained. In the gas-water two-phase flow simulation process, the evolutionary iteration process can describe the dynamic change characteristics of the gas-water two-phase interface shape. The level set equation used to describe the change of the phase interface of the two-phase fluid can be written as:

\[ \frac{\partial \phi}{\partial t} + \bar{u} \cdot \nabla \phi = \gamma \nabla \cdot \left( \varepsilon_{ls} \nabla \phi - \phi (1 - \phi) \frac{\nabla \phi}{|\nabla \phi|} \right) \]  

(9)

Among them, the expression \( \frac{\partial \phi}{\partial t} + \bar{u} \cdot \nabla \phi \) on the right side of the equation is used to describe the movement of the phase interface.

In the formula, \( \phi \) is the level set variable, generally the value is \( 0 \leq \phi \leq 1 \), \( \phi = 0 \) means phase 1, and \( \phi = 1 \) means phase 2. The isosurface with a value of 0.5 is usually taken as the phase interface.

\( \varepsilon_{ls} \) is the interface thickness control parameter, generally \( h_{\text{max}} / 2 \), \( h_{\text{max}} \) is the maximum grid size, unit m.

\( \gamma \) is the level set function reinitializes the parameters, the unit m/s, generally the maximum velocity of fluid flow.

Fluid property control equation:

\[ \rho = \rho_1 + \left( \rho_2 - \rho_1 \right) \phi \]  

(10)

\[ \mu = \mu_1 + \left( \mu_2 - \mu_1 \right) \phi \]  

(11)

Where,

\( \rho_1 \) — Density of fluid 1, kg/m³; \( \rho_2 \) — Density of fluid 2, kg/m³;

\( \mu_1 \) — Viscosity of fluid 1, mPa·s; \( \mu_2 \) — Viscosity of fluid 2, mPa·s.

3. Model establishment and solution

3.1. Geometric model construction

A plane simulation model in 2D space is established, and the establishment of the micro-simulation geometric model is based on the micro-pore structure of the tight gas reservoir. First, analyze the pore
distribution characteristics according to the SEM image obtained under the microscope (as shown in Figure 2); then, use CAD drawing software, combined with geological understanding, to draw the pore distribution structure diagram of different lithofacies, and build a simulation. The base map of the geometric model is shown in Figure 3. Finally, the image drawn by CAD is imported into the finite element simulation software, and the image is scaled, cropped, and subtracted according to the actual size according to the scale, and the microscopic simulation geometric model is obtained, as shown in Figure 4. The geometric model constructed is mainly composed of two parts. One part is a connected pore bellow space, which serves as a fluid flow channel, and the other part is rock minerals, composed of multiple independent rock minerals. There is no fluid flow and is closed Unit body.

**Figure 2.** Microscopic scanning image of binary pores

**Figure 3.** Boundary processing by AutoCAD

**Figure 4.** Schematic diagram of the flow area model

Based on the topological structure vector diagram, the finite element calculation geometric structure can be established, and the geometric area is 114200μm². A boundary layer is added on the surface of solid particles to achieve better mesh accuracy (as shown in Figure 6). The geometric structure is divided into 50272 triangular meshes and 10323 quadrilateral meshes (boundary layer) by 825 points and 3723 edges.
For the gas-water two-phase flow model, the following assumptions are made:

1. The pores are filled with water under the initial conditions, the right side is displaced by pure gas, and the fluid flows out from the left;
2. The right side of the model is the inlet, and the boundary meets the requirements. The pressure boundary condition complies with the Dirichlet boundary condition in fluid mechanics, and the fluid reflux is forbidden, and there is no tangential velocity;
3. The left side of the model is the outlet, which also conforms to the Dirichlet boundary condition, which is the constant pressure boundary, allowing the fluid to be Gas and water are mixed, allowing tangential velocity;
4. The upper and lower sides of the model are closed boundaries and no fluid passes through.

3.2. Model boundary setting
After the geometric model is constructed, the setting of various domain conditions and boundary conditions in the physical field and the setting of the solving algorithm are also the key to the simulation of pore fluid micro-seepage, which can restrict the flow calculation process.

During the simulation of pore fluid microscopic seepage, the model makes the following assumptions: the pores are filled with oil phase under the initial conditions, and the pure water phase is displaced by the right side, and flows out from and only from the left side. According to the experimental parameter setting conditions, the model boundary conditions are set as the following boundary conditions (as shown in Figure 7): the upper and lower sides of the model are closed boundaries and no fluid passes through; the right is the fluid inlet, which satisfies the constant pressure boundary condition and conforms to the fluid Dirichlet boundary conditions in mechanics, and forbid fluid backflow, no tangential velocity; the left is the outlet, which also meets the Dirichlet boundary conditions, satisfies the constant pressure boundary, allows the fluid to be mixed with oil and water, and allows tangential velocity.
3.3. Model solution
The pore fluid micro-seepage simulation model is solved by using the COSMOSOLIntel Mathematical Core Function Library (MKL). The COSMOSOLIntel Mathematical Core Function Library (MKL) is a set of highly optimized and thread-safe mathematical routines and functions for high-performance engineering, science and finance. application. The cluster version of Intel MKL includes ScalAPACK and distributed memory fast Fourier transform, and provides linear algebra (BLAS, LAPACK, and SparseSolver), fast Fourier transform, vector math (VectorMath) and random number generator support. In COMSOL, you can use the PARDISO direct sparse matrix solver provided by MKL to solve large sparse linear equations. The solver is authorized by the University of Basel. It is easy to use, thread safe, and high-performance Memory-efficient software library. The specific solution model selection is shown in the figure below:

![Figure 8. Pore fluid micro-seepage simulation solver settings](image)

3.4. Calculation of relative permeability
According to the definition of porosity: porosity refers to the ratio of the sum of the volume of all pore spaces in a rock sample to the volume of the rock sample, expressed as a percentage, and the calculation formula for digital core porosity:

$$\varphi = \frac{V_p}{V_r} \times 100\% = \frac{V_p}{L \times H} \times 100\%$$  (12)

Among them, $V_p$ —the pore volume of the rock, which can be obtained by integrating the pore space in the model; $V_r$ —the total volume of the rock; $L$ —the length of the model; $H$ —the height of the model.

Permeability refers to the ability of a rock to allow fluid to pass under a certain pressure difference. It is generally calculated by Darcy's law:

$$k = \frac{Q \mu L}{A \Delta p}$$  (13)

Where, $Q$ —outlet flow; $\mu$ —dynamic viscosity; $L$ —pressure drop length; $\Delta p$ —pressure drop; $A$ —outlet area

Since the digital core model in this paper is a two-dimensional plane model, the calculation of the permeability of the model can be simplified as:
Where, \( u_D \) - Darcy velocity, the average Darcy velocity can be calculated by calculating the average velocity of the exit interface.

According to the above definition and calculation, the relative permeability of a phase at time \( t \) is equal to the ratio of the effective permeability at that time to the absolute permeability of the gas phase:

\[
K_{rg}(t) = \frac{K_g(t)}{K}
\]

\[
K_{rw}(t) = \frac{K_w(t)}{K}
\]

Where, \( K_{rg}(t) \) is the relative permeability of the gas phase; \( K_g(t) \) is the effective permeability at time \( t \) in the gas phase; \( K_{rw}(t) \) is the relative permeability of the water phase; \( K_w(t) \) is the effective permeability of the water phase; \( K \) is the absolute permeability of the gas phase.

According to the definition of water saturation, the volume fraction of the water phase at time \( t \) is the water saturation at that time:

\[
S_w(t) = \frac{V_w(t)}{V_p}
\]

Where, \( S_w(t) \) is the water saturation at time \( t \); \( V_w(t) \) is the volume of the water phase at time \( t \); \( V_p \) is the pore volume.

4. Results and discussion

4.1. Experimental results

The gas-water permeability curve of the rock sample measured at room temperature and pressure is shown in Figure 9. The obtained core irreducible water saturation and its corresponding gas relative permeability, isotonic point water saturation and isotonic point relative permeability are shown in Table 1.

![Figure 9. Experimental relative permeability curve](image-url)
Table 1. Gas drive water phase permeability test results

| Test Conditions | Irreducible water saturation | Relative permeability of gas phase under irreducible water saturation | Isotonic point water saturation | Isotonic permeability |
|-----------------|------------------------------|---------------------------------------------------------------------|-------------------------------|-----------------------|
| Temperature and pressure | 14.6% | 0.69 | 58% | 12.5mD |

4.2. Numerical simulation results
In order to facilitate the comparison with the gas-water two-phase seepage experiment, we keep the inlet pressure and outlet pressure of the numerical simulation model as well as the relevant water parameters consistent with the experiment, and calculate the volume fraction distribution of the rock sample at different times when the gas drives the water by finite element software. As shown in Figure 9.

![Figure 10](image)

Figure 10. The volume fraction distribution diagram of the rock sample at different times

It can be seen from Figure 9 that at t=10.4ms, the gas first enters the microscopic pore structure along the five inlets, and the water is displaced along the five throats, and the gas gradually enters the pores; at t=22.9ms, the upper part of the gas in the large pore throat displaces the water at a faster speed. The gas below the gas passes through the small pore throat, so the gas efficiency is slow; when t=29.1ms, the water in the upper large pore throat has been completely displaced. The water in the lower pore throat has not been completely replaced; at t=39.5ms, most of the water in the throat has been completely replaced, and only part of the pore throat has not been completely replaced. The simulation results show that during the entire gas-water displacement process, the gas has obvious fingering phenomenon, and the gas mainly displaces the water phase along the dominant channel.

According to Darcy's law, the absolute permeability of the digital core in the pure gas phase flow state can be calculated to be 82mD. At the same time, use the water and gas permeability calculation formulas at different times in Section 1.2 to obtain the effective permeability of the water and gas phases under different water phase volume fractions (that is, under different water saturations), as shown in Table 2.
### Table 2. Effective permeability of water phase and gas phase at different times

| Time (ms) | Water phase volume fraction (%) | Effective permeability of water phase (mD) | Effective gas permeability (mD) |
|----------|---------------------------------|------------------------------------------|---------------------------------|
| $t = 3.6$| 5.3                             | 0.23                                     | 64.11                           |
| $t = 8.2$| 11.5                            | 1.82                                     | 45.96                           |
| $t = 10.4$| 19.1                           | 3.65                                     | 33.28                           |
| $t = 15.2$| 35.4                           | 6.13                                     | 16.57                           |
| $t = 19.1$| 50.1                           | 8.91                                     | 9.02                            |
| $t = 22.9$| 55.8                           | 10.66                                    | 6.56                            |
| $t = 24.8$| 64.2                           | 13.94                                    | 3.44                            |
| $t = 29.1$| 78.5                           | 22.20                                    | 0.79                            |
| $t = 33.5$| 84.6                           | 27.19                                    | 0.37                            |
| $t = 39.5$| 92.4                           | 32.67                                    | 0.00                            |

4.3. Discuss

Calculate the relative permeability of the gas and water phases obtained by the numerical simulation, and compare the relative permeability curve obtained by the experiment with the relative permeability curve obtained by the numerical simulation, as shown in Figure 10.

![Figure 11. Comparison of relative permeability curves of gas and water phases](image)

It can be seen from the comparison chart of relative permeability curves that, since the digital core is an idealized porous medium, the original water saturation calculated by the numerical simulation is much smaller than the actual one. However, the end-point error of the relative permeability of the gas phase calculated by the digital simulation method and the relative permeability of the gas phase measured by the experimental method is only 10.5%, the error of the relative permeability of the water phase is only 5.1%, and the error of the isotonic point is only 4%. The relative permeability curve obtained by the experimental method is very close to the relative permeability curve obtained by the numerical simulation method, which shows the reliability of the numerical simulation method.

5. Conclusion

(1) Based on the unsteady state method, the relative permeability curve of a sandstone with good physical properties was measured;
(2) Based on the NS equation, the laminar flow level set method was introduced, and the gas-water two-phase flow numerical simulation model was established and used Finite element software solves the model;

(3) The gas-water two-phase numerical simulation results show that there is an obvious fingering phenomenon in the process of gas flooding. Gas preferentially passes through large pore throats. The smaller the pore throat, the gas passes through the pores. The greater the speed of the throat;

(4) The relative permeability curve of the digital core was calculated using Darcy’s law and compared with the results of the gas-water two-phase seepage experiment. The results show that the gas-water two-phase flow based on the level set method The digital simulation of phase percolation is more reliable for the characterization of gas-water two-phase percolation.

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