A simple approach to identify the proper relative permeability model

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Abstract. Relative permeability is one of the crucial input data for any reservoir simulation study. Experimentally, relative permeability tests can be executed by either steady state or unsteady-state flow procedures. Steady-state procedures have the advantage of easy calculations and the drawback of tedious long processes. Unsteady-state procedures take less time but need complex calculations. To shorten calculations of unsteady-state procedures, various models have been developed based on measuring permeability and saturation at initial and final stages of the flow process. This paper presents a simple approach to assess unsteady-state relative-permeability models for simulating a certain fluid displacement process. The Approach includes comparing the experimental average water saturation at the moment of breakthrough (Swavg)Bth with modelling (Swavg)Bth that can be estimated from the fractional-flow curve. The presented approach aims to identify the proper relative permeability model before conducting full reservoir simulation studies. This will improve the reservoir simulation process and shorten the history matching. The approach has been applied on six experiments of displacing brine by nitrogen gas in core samples of Berea sandstone. The results shown that two of the displacing experiments can be simulated with MBC model, one experiment can be simulated by Corey’s model, and one experiment can be simulated by Kam and Rossen model with error percent less than 1%. Modelling of the other two experiments gave error percent greater than 1% for MBC, Corey, Pirson, and Kam and Rossen models.

1. Introduction

The relative permeability concept is a useful approach to describe flow of one phase in existence of other phases. All rocks which represent hydrocarbon reservoirs contain more than one fluid flowing phase. Thus, relative permeability data are necessary for reservoir simulation studies. Relative permeability data should be obtained by experiments that best model the type of displacement that is thought to dominate reservoir flow performance [1]. Experimentally, relative permeability tests can be executed by either steady state or unsteady-state flow procedures [2-4]. Steady-state procedures have the advantage of easy calculations and the drawback of tedious long processes. Unsteady-state procedures take less time but need complex calculations [5-9]. To shorten calculations of unsteady-state procedures, various models have been developed based on measuring permeability and saturation at initial and final stages. The first paragraph after a heading is not indented (Bodytext style). Other paragraphs are indented (BodytextIndented style). of the flow process [10-16]. Artificial neural networks [17] and resistivity data [18] have been also used to predict relative permeability relationships. A correlation for relative permeability of complex behavior fluids such as gas-condensate fluids has been presented by Bang et al. [19]. Honarpour et al. [20] presented various approaches of laboratory
measurements and correlations for relative permeability. The familiar model used in the petroleum industry is the modified Brooks and Corey (MBC) model \[16, 21\]:

\[
K_{r1} = K_{r1}^* \left( \frac{S_1 - S_{1r}}{1 - S_{1r} - S_{2r}} \right)^{n_1}
\]

\[
K_{r2} = K_{r2}^* \left( \frac{1 - S_1 - S_{2r}}{1 - S_{1r} - S_{2r}} \right)^{n_2}
\]

where \(K_{r1}^*\) and \(K_{r2}^*\) are end-point relative permeabilities for phases 1 and 2, respectively. \(S_{1r}\) and \(S_{2r}\) are residual saturations for phases 1 and 2, respectively. \(n_1\) and \(n_2\) are exponents to be calculated from experimental data. Behrenbruch and Goda \[16\] indicated that values of \(n_1\) and \(n_2\) are a measure of the degree of heterogeneity of the core plugs, rather than wettability. This paper presents an approach to assess relative permeability models of unsteady-state methods versus experimental data. Four different relative-permeability models have been assessed to simulate displacing of brine by nitrogen gas in six sandstone cores with different absolute permeability.

2. Core-flooding experiments
Six core samples of Berea sandstone were used in the present core-flooding experiments. First, porosities of these core samples were measured by a gas porosity-meter. Second, the core samples were fully saturated with brine of 2.4 wt% of NaCl. Third, absolute permeability to brine of the core samples were measured using Darcy’s law. Forth, the brine was displaced from the core samples by injecting nitrogen gas with rate of 22.66 cm³/min (measured at room conditions) under confining pressure of 28 bar and atmospheric pressure at the outlet of the core sample. Pressure drop across the core sample, water recovery, and average water saturation in the core sample were recorded as functions of time. Gas relative permeability \(K_{rgwc}\) at the irreducible water saturation \(S_{wc}\) was calculated from Darcy’s law of gases. Properties of the core samples and results of the core-flooding experiments are shown in Table 1.

| TABLE 1. Core properties and results of core-flooding experiments. |
|-----------------|---------------|----------------|----------------|
| Core | Diameter (cm) | Length (cm) | Porosity % | K (μm²) | Swc | K_{rgwc} |
| A   | 3.832         | 3.925        | 13.487      | 0.00193 | 0.619 | 0.43     |
| B   | 3.795         | 3.894        | 16.71       | 0.03296 | 0.558 | 0.266    |
| C   | 3.795         | 3.895        | 17.818      | 0.00971 | 0.522 | 0.483    |
| D   | 3.808         | 2.592        | 20.559      | 0.02296 | 0.588 | 0.312    |
| E   | 3.799         | 7.718        | 16.599      | 0.06711 | 0.483 | 0.276    |
| F   | 3.794         | 7.627        | 18.023      | 0.10904 | 0.665 | 0.188    |

3. Relative permeability ratio
To assessed relative-permeability models it needs finding the relative permeability ratio of the displacement fluid to the displaced fluid for each model. Gas (nitrogen gas) is the displacement fluid and water (brine) is the displaced fluid in the present experiment. Four models have been assessed in the present paper; MBC model, Corey’s model \[10, 15\], Pirson’s model \[13, 15\], and Kam and Rossen model \[22\]. The relative permeability ratios of these models are as follows:

3.1. MBC model
The gas/water relative-permeability ratio can be obtained from dividing equation (2) by equation (1) considering that water is phase 1 and gas is phase 2. Then, \(K_{r1} = K_{rw}\), \(K_{r2} = K_{rg}\), \(S_1 = S_w\), \(S_{1r} = S_{wc}\), and...
S_{2r} = S_{gc}. Assuming n_1 = n_2 = 2 (as an approximate value for gas-water system), the result will be as follows:

\[ \frac{K_{rg}}{K_{rw}} = \left( \frac{K_{rgc}}{K_{rwgc}} \right) \left( \frac{1-S_{w}-S_{gc}}{S_{w}-S_{wc}} \right)^2 \]

(3)

In the present experiments, the core samples were saturated with brine before the displacement process, so that it can be practically considered S_{gc} = 0 and K_{rwgc} = 1.

3.2. Corey’s model
Corey proposed a simple mathematical expression for generating relative-permeability data of a gas-oil system [10, 15]. After replacing oil by water in Corey’s model, the gas/water relative permeability ratio will be:

\[ \frac{K_{rg}}{K_{rw}} = \frac{(S_g^*)^3 (2-S_g^*)}{(1-S_g^*)^4} \]

(4)

where \( S_g^* = \frac{S_g}{1-S_{wc}} \), and \( S_g \) is gas saturation.

3.3. Pirson’s model
Pirson derived from petrophysical considerations generalized relationships for determining the wetting and nonwetting phase relative permeability [13, 15]. The gas/water relative permeability ratio from Pirson’s model is:

\[ \frac{K_{rg}}{K_{rw}} = \frac{(1-S_{w}^*)[1-(S_{w}^*)^{0.25} (S_{w})^{0.5}]^{0.5}}{(S_{w}^*)^{0.5} (S_{w})^{3}} \]

(5)

where \( S_{w}^* = \frac{S_{w}-S_{wc}}{1-S_{wc}} \)

3.4. Kam and Rossen model
Kam and Rossen [22] estimated relative-permeability functions for gas and liquid phases by curve-fitting data for unconsolidated sand packs. The gas/water relative permeability ratio from Kam and Rossen model is:

\[ \frac{K_{rg}}{K_{rw}} = \left( \frac{1-S_{w}-S_{gc}}{1-S_{wc}-S_{gc}} \right)^{2.2868} \]

\[ \left( \frac{1-S_{w}-S_{gc}}{1-S_{wc}-S_{gc}} \right)^{1.9575} \]

(6)

4. Identifying the proper model
The present approach to assessed relative permeability models, consists of four steps:

4.1. Construct fractional-flow curve
The fractional-flow curve can be constructed as a function of water saturation from the following equation [15]:

\[ f_w = \frac{1}{1 + \frac{K_{rg}}{K_{rw}} \frac{\mu_w}{\mu_g}} \]

(7)
where \( f_w \) is water fractional-flow, \( \mu_w \) and \( \mu_g \) are water and gas viscosities (mPa.sec), respectively. At room conditions \( \mu_w = 1.1361 \) mPa.sec and \( \mu_g = 0.0176 \) mPa.sec. The gas/water relative-permeability ratio which is a function of water saturation, depends on the model to be assessed. Validity of equation (7) is restricted by validity of following assumptions of the fractional-flow theory: (1) the flow is incompressible, steady-state, and in one dimension, (2) effects of capillary pressure gradient and gravity are negligible \[15, 23\].

4.2. Estimate modelling \((S_{wavg})_{Bth}\) from Fractional-Flow Curve
The average water saturation behind the injected gas front at the breakthrough moment \((S_{wavg})_{Bth}\) can be estimated from the fractional-flow curve. Welge \[24\] showed that \((S_{wavg})_{Bth}\) can be determined graphically by drawing a tangent to the fractional flow curve, starting from the initial point \((S_w = 1)\) to intersect the line of \( f_w = 0 \). A schematic plot is shown in Fig. 1.

4.3. Determine experimental \((S_{wavg})_{Bth}\)
Average water saturation \((S_{wavg})\) can be experimentally determined as a function of time from material balance calculations. In the present study, the displaced water flow rate has been plotted versus \( S_{wavg} \). Then the experimental \((S_{wavg})_{Bth}\) has been determined at the moment at which the displaced water flow rate starts to decrease. Details are shown in the section of results and discussion.

4.4. Match experimental \((S_{wavg})_{Bth}\) with modelling \((S_{wavg})_{Bth}\)
The present approach uses matching of modelling \((S_{wavg})_{Bth}\) with experimental \((S_{wavg})_{Bth}\) as an index to assess validity of the relative-permeability models.

5. Results and discussion

5.1. Relative permeability ratio
Relative permeability ratio for each of the four models were calculated as a function of water saturation by substituting values of \( S_{wc} \) and \( K_{rgwc} \) (shown in Table 1) into equations (3), (4), (5), and (6). Fig. 2 shows relative permeability ratios of the four models for core sample A.
Figure 2. Gas/water relative-permeability ratio of different models for core A.

Fig. 2 shows that curves of gas/water relative-permeability ratio of MBC, Pirson, and Kam and Rossen models are close together at range of water saturation 0.7 to 0.9. Corey’s model gives a higher relative-permeability ratio than other models at water saturation less than 0.8, while Pirson's model gives a higher relative-permeability ratio at water saturations greater than 0.9. These differences in the gas/water relative permeability ratio among the models, resulting in diverse fractional-flow curve for the same displacement process in the same core sample as will be shown in the next subsection.

5.2. Estimation of modelling (Sw avg)Bth from fractional-flow curve

Fractional-flow curves have been constructed for each model in all core samples, using equation (7) with relative permeability ratio of each model. Then, Welge’s technique was used to estimate (Sw avg)Bth of each model. Fig. 3 shows fractional flow curves of the four models with Welge’s technique implemented for core sample A. It is observed that for the same displacement process in core sample A, the difference in relative permeability model results in different (Sw avg)Bth value and it will outcome in different estimated breakthrough time and reservoir performance. Thus identifying the most accurate relative permeability model to represent the displacement process is a condition to predict the most accurate reservoir performance.

Figure 3. Fractional-flow curves of relative-permeability models and Welge’s technique for core A.
5.3. Determination of Experimental ($S_{\text{avg}}^{\text{Bth}}$)

To determine the experimental ($S_{\text{avg}}^{\text{Bth}}$), water flow rate at the core outlet was calculated and plotted as a function of the average water saturation ($S_{\text{avg}}$). The experimental ($S_{\text{avg}}^{\text{Bth}}$) has been determined at the moment at which the displaced water flow rate starts to decrease. Fig. 4 shows determination of experimental ($S_{\text{avg}}^{\text{Bth}}$) for core sample A.

Figure 4. Determination of experimental ($S_{\text{avg}}^{\text{Bth}}$) in core sample A.

In Fig. 5 it is observed that the water flow rate initially increases since the water saturation at the outlet still at its initial value ($S_w = 1$). Then, the water flow rate starts decreasing at the moment of gas breakthrough, as a result of decreasing in water saturation at the core outlet. This decrease of water flow rate continues with decrease of water saturation till reaching flow rate of zero at the irreducible water saturation. The experimental ($S_{\text{avg}}^{\text{Bth}}$) can be determined more accurately if there is a gas flow meter or separator attached to the outlet of the flow line in the core-flooding equipment.

5.4. Matching experimental ($S_{\text{avg}}^{\text{Bth}}$) with modelling ($S_{\text{avg}}^{\text{Bth}}$)

Comparison of the experimental ($S_{\text{avg}}^{\text{Bth}}$) with the modelling ($S_{\text{avg}}^{\text{Bth}}$) of the four models has been made for each core sample. Fig. 5 shows matching the experimental ($S_{\text{avg}}^{\text{Bth}}$) with the modelling ($S_{\text{avg}}^{\text{Bth}}$) for core sample A. It is shown in Fig. 5 that Cory’s model gives the best match. The comparison process was mathematically expressed by percent of the absolute error as shown in Table 2. Cells with the shaded color in Table 2 indicate the proper model for each core sample. The proper model has been considered as that one with the lowest error percent and less than 1%. It has been found that none of the four models can be used to simulate the displacing process in core samples B and C. The absolute error percent of the four models are greater than 1% in these two core samples. This can be attributed to the non-matching of assumptions, data and theories that used in developing the four models to the displacement processes in cores C and D. In such cases, it needs to try different values for the exponents $n_1$ and $n_2$ of MBC model or assessing other models.

Figure 5. Matching the experimental ($S_{\text{avg}}^{\text{Bth}}$) with the modelling ($S_{\text{avg}}^{\text{Bth}}$) for core sample A.
Table 2. Comparison of the experimental ($S_{avg}$)_{Bth} with modelling ($S_{avg}$)_{Bth}.

| Core Sample | Experimental ($S_{avg}$)_{Bth} | MBC | Kam and Rossen | Pirson | Corey | Proper Model |
|-------------|-------------------------------|-----|---------------|--------|-------|--------------|
|             | Modelling ($S_{avg}$)_{Bth} | Abs | Modelling ($S_{avg}$)_{Bth} | Abs | Modelling ($S_{avg}$)_{Bth} | Abs | Modelling ($S_{avg}$)_{Bth} | Abs | Modelling ($S_{avg}$)_{Bth} | Abs | Proper Model |
| A           | 0.892                         | 0.88 | 1.345         | 0.904 | 1.345 | 0.945 | 5.942 | 0.894 | 2.224 | None |
| B           | 0.765                         | 0.83 | 8.497         | 0.888 | 16.08 | 0.937 | 22.48 | 0.873 | 13.725 | None |
| C           | 0.797                         | 0.856 | 7.403        | 0.879 | 10.29 | 0.932 | 16.94 | 0.867 | 7.905 | None |
| D           | 0.852                         | 0.855 | 0.352        | 0.895 | 5.047 | 0.94  | 10.33 | 0.881 | 3.286 | MBC |
| E           | 0.862                         | 0.805 | 6.613        | 0.868 | 0.696 | 0.93  | 7.899 | 0.851 | 1.392 | Kam and Rossen |
| F           | 0.852                         | 0.855 | 0.352        | 0.916 | 7.512 | 0.951 | 11.62 | 0.951 | 5.634 | MBC |

* $\text{Abs Err} \% = \left( \frac{\text{Modelling} (S_{avg})_{Bth} - \text{Experimental} (S_{avg})_{Bth}}{\text{Experimental} (S_{avg})_{Bth}} \right) \times 100$

6. Conclusions

1. A simple approach has been presented to assess different relative-permeability models to simulate certain displacement processes.
2. The approach included match of the experimental ($S_{avg}$)_{Bth} with the modelling ($S_{avg}$)_{Bth} that can be estimated from fractional-flow curve.
3. The present approach has been implemented to assess four relative permeability models to simulate six processes of displacing brine by nitrogen gas in different core samples.
4. Results shown that the displacement processes in two core samples (Samples D and F) can be simulated by MBC model, one core sample (Sample A) can be simulated by Corey model, and one core sample (Sample E) can be simulated by Kam and Rossen model.
5. The four relative permeability models gave error percent greater than 1% for the core samples B and C. Thus, it is not recommended to simulate the displacement processes in these two core samples by any of these four models.

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