Stability Analysis for Inclined Channel by an Angle 30° with The Presence of Magnetic Field

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ABSTRACT

In this research, we study the stability of a system of partial differential equations which represents fluid flow in an inclined channel and under the influence of a magnetic field perpendicular to the plane of the channel and the presence of radiation coefficients and when the channel has an inclination angle $\phi = 30$.

Keywords: Inclination Channel, Magnetic field, and stability.

1. Introduction

The flow and heat transfer of electrically conducting fluid in channels and circular pipes under the effect of a transverse magnetic field occurs in magnetohydrodynamic (MHD) generators, pumps and flow meters and have applications in nuclear reactors, filtration, geothermal systems and others.

In 1969, Torrance and Rockett studied the numerical solution of natural convection in enclosures with localized heating from below greeping flow to the onset of laminar instability and obtained solution of Gratshof number from $10^4$ to $10^5$, which agreements with the experimentally observed laminar flows. In 1977, Petry and Busse studied the convection in a rotating cylindrical annulus in the presence of a magnetic field.

In 1990, Daniess and Ong used A two-dimensional Galerkin formulation of the three dimensional oberbeck – Boussinesq equations to determine the onset of convection in an infinite rigid horizontal channel uniformly heated from bellow .In 1996, Busse and Clever studied the problem of thermal convection in a fluid layer heated from below and solved it numerically in the case when a strong vertical magnetic field permeates the layer, the stability of two dimensional convection rolls was studied for different values of the Hartmann number within the region (200–400). In 1997, Hartmann and Busse studied the convection in a rotating cylindrical annulus, they described many features to a good approximation by a system of three coupled amplitude equations.

In 2000, Almakrmi had studied Ekman boundary layer flow, he had studied its steady state solution and stability analysis. At the same year Aljoubory, studied the effect of buoyancy forces on the stability of a fluid flow between two plates one of them
is thermally insulated. In 2001, Malashetty, Umavathi and Kumar studied the problem of fully developed free convection of two fluid magnetohydrodynamic (MHD) flow in an inclined channel. In 2009, Singha studied an analytical solution to the problem of magnetohydrodynamic (MHD) free convection flow of an electrically conducting fluid between two heated parallel plates in the presence of an induced magnetic field. In 2011, Akbar, Hayat, Nadeem and Hendi, studied the effect of slip and heat transfer on the peristaltic flow of a third order fluid in an inclined asymmetric channel.

In this paper, the stability of this system raising from the heat transfer in an inclined channel with the presence of magnetic field and radiation has been investigated, the inclination angle was considered as $\phi = 30$ degree, it's found that the parameters $F_s$, $k$ and $w$ have a significant effect on the stability of the system.

2. The Model and Governing Equations:

Consider a fully-developed, steady laminar flow of fluid in the inclined channel at angle $(\phi = 30)$ degree, the distance between the walls of the channel is $2h$ apart. Choosing the coordinate system such that the $x-axis$ in the direction of the flow, $y-axis$ is measured perpendicular to the plane of the channel, whilst the $z-axis$ is in the direction mutually orthogonal to the other two axes, the boundaries of the inclined channel are taken to be perfectly conducting electrodes infinitely far apart so that any dependence upon the $z-coordinate$ vanishes.

In the model under consideration, the electric density has a single component $J_z$, the magnetic field has a component $B_z$ induced along the inclined channel in the direction of the flow, $B_z$ is zero and the component parallel to the $y-axis$ decomposed into two components $B_0 \sin \phi$ in the $x$ direction and $B_0 \cos \phi$ at $y$ direction. The electric field has a single component $E_0$ which is parallel to the $z-axis$, as illustrated in the figure below:

![Figure 1: The Inclined Channel at Angle $\phi = 30$](image)

Where the velocities $u$, $v$ and $w$ are zero at the edges and $T$ is temperature, $T_i$ represents the temperature of the lower plate and $T_2$ represents the temperature of the upper plate.

The governing equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$$  \hspace{1cm} \ldots(2.1)
\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -1 \frac{\partial P}{\rho_1 \partial x} + \nu \nabla^2 u - \frac{\sigma B^2}{\rho_1} v \sin \varphi - \rho g \sin \varphi. \] \quad (2.2)

\[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -1 \frac{\partial P}{\rho_1 \partial y} + \nu \nabla^2 v + \frac{\sigma B^2}{\rho_1} v \cos \varphi - \rho g \cos \varphi. \] \quad (2.3)

\[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho_1 C_p} \nabla^2 T - \frac{1}{\rho_1 C_p} \nabla q + \ldots + \frac{1}{\rho_1 C_p \sigma \mu_0} \left[ \frac{\partial B}{\partial y} \sin \varphi - \frac{\partial B}{\partial y} \cos \varphi \right]^2 \] \quad (2.4)

Where, \( \rho = \rho_1 \left[ 1 - \beta(T - T_i) \right] \)

with boundary conditions:

1. \( u = v = 0 \) at \( y = \pm h \)
2. \( T = T_i \), \( \frac{\partial T}{\partial y} = 0 \) at \( y = -h \)
3. \( T = T_i \), \( \frac{\partial T}{\partial y} = 0 \) at \( y = h \)

3. Non-dimensional Form:[11]

We define the following non-dimensional variables as follows:

\[ t = \frac{ht^*}{u_0}, \quad \tilde{B} = B_0 \mu_0 \sigma u_0 h b, \quad T = T_i \theta, \quad \tilde{q} = T_i^4 \sigma \tilde{Q} \]

\[ \tilde{q} = \frac{q}{T_i^4 \sigma}, \quad x = hx^*, \quad y = hy^*, \quad P = P^* \rho_1 u_0^2 \]

\[ u = u_0 u^*, \quad v = u_0 v^* , \]

After substituting (3.1) in equations (2.1), (2.2), (2.3) and (2.4), we get

\[ \frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0. \] \quad (3.2)

\[ \frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -1 \frac{\partial P^*}{\rho^* \partial x} + \frac{1}{\rho^*} \left( \frac{\partial^2 u^*}{\partial x^*^2} + \frac{\partial^2 u^*}{\partial y^*^2} \right) \frac{M^2}{Re} v^* \sin \varphi - \ldots - (E - H \theta) \sin \varphi. \] \quad (3.3)

\[ \frac{\partial v^*}{\partial t^*} + u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = -1 \frac{\partial P^*}{\rho^* \partial y} + \frac{1}{\rho^*} \left( \frac{\partial^2 v^*}{\partial x^*^2} + \frac{\partial^2 v^*}{\partial y^*^2} \right) + \frac{M^2}{Re} v^* \cos \varphi - \ldots - (E - H \theta) \cos \varphi \] \quad (3.4)

\[ \frac{\partial \theta}{\partial t^*} + u^* \frac{\partial \theta}{\partial x^*} + v^* \frac{\partial \theta}{\partial y^*} = \frac{\gamma}{\Pr \Re} \left[ \frac{\partial^2 \theta}{\partial x^*^2} + \frac{\partial^2 \theta}{\partial y^*^2} \right] - \frac{\gamma \text{Ec}}{B_0} \left[ \frac{\partial Q^*}{\partial x^*} + \frac{\partial Q^*}{\partial y^*} \right] + \ldots \]

\[ \ldots + \frac{M^2 \gamma \text{Ec}}{\Re} \left[ \frac{\partial b^*}{\partial y^*} \sin \varphi - \frac{\partial b^*}{\partial y^*} \cos \varphi \right]^2. \] \quad (3.5)

And since \( \text{div} \tilde{Q} = 16W \theta - 12W \) [11] and by multiplying equation (3.5) by \( \frac{\Pr \Re}{\gamma} \), we get

\[ \Pr \left[ \frac{\partial \theta}{\partial t^*} + u^* \frac{\partial \theta}{\partial x^*} + v^* \frac{\partial \theta}{\partial y^*} \right] = \left[ \frac{\partial^2 \theta}{\partial x^*^2} + \frac{\partial^2 \theta}{\partial y^*^2} \right] - 16F s \text{NW} \theta + 12F s \text{NW} + \ldots \]
\[ ... + M^2 F_s \left[ \frac{\partial b}{\partial y} \sin \varphi - \frac{\partial b}{\partial y^*} \cos \varphi \right]^2. \quad (3.6) \]

Where

\[ Re = \frac{h \rho u_b}{\mu} \] is the Reynolds number.

\[ Pr = \frac{\mu C_p}{k} \] is the Prandtl number.

\[ \gamma = \frac{C_v}{C_p} \] is the specific heat ratio.

\[ W = \alpha \rho h \] is Bouger number.

\[ Ec = \frac{u_0^2}{C_p T_1} \] is the Eckert number.

\[ Fs = Pr \cdot Ec \] is the new factor.

\[ M = B_0 \rho \left( \frac{\sigma}{\mu} \right)^{1/2} \] is Hartmann number.

\[ N = \frac{Re}{B_0}, \quad L = \frac{NB_0}{\gamma} \]

4. Stability Analysis:

We define the functions of velocities, pressure, Maxwell and the temperature as the following convenient form:

\[ u^* = u_1(x, y) + u_2(x, y, t) \]

\[ v^* = v_1(x, y) + v_2(x, y, t) \]

\[ p^* = p_1(x, y) + p_2(x, y, t) \]

\[ \theta = \theta_1(x, y) + \theta_2(x, y, t) \]

\[ b_x = b_x + b_y \]

by substituting equation (4.1) into equations (3.2), (3.3), (3.4) and (3.6), we get

\[ \frac{\partial (u_1 + u_2)}{\partial x^*} + \frac{\partial (v_1 + v_2)}{\partial y^*} = 0. \quad (4.2) \]

\[ \frac{\partial (u_1 + u_2)}{\partial t} + (u_1 + u_2) \frac{\partial (u_1 + u_2)}{\partial x^*} + (v_1 + v_2) \frac{\partial (u_1 + u_2)}{\partial y^*} = - \frac{\partial (p_1 + p_2)}{\partial x^*} + ... \]

\[ ... + \frac{1}{Re} \left[ \frac{\partial^2 (u_1 + u_2)}{\partial x^{*2}} + \frac{\partial^2 (u_1 + u_2)}{\partial y^{*2}} \right] - M^2 \frac{\partial^2 (v_1 + v_2) \sin \varphi - E \sin \varphi + H(\theta_1 + \theta_2) \sin \varphi}{\partial y^*} = 0. \quad (4.3) \]

\[ \frac{\partial (v_1 + v_2)}{\partial t} + (u_1 + u_2) \frac{\partial (v_1 + v_2)}{\partial x^*} + (v_1 + v_2) \frac{\partial (v_1 + v_2)}{\partial y^*} = - \frac{\partial (p_1 + p_2)}{\partial y^*} + ... \]

\[ ... + \frac{1}{Re} \left[ \frac{\partial^2 (v_1 + v_2)}{\partial x^{*2}} + \frac{\partial^2 (v_1 + v_2)}{\partial y^{*2}} \right] + M^2 \frac{\partial^2 (v_1 + v_2) \cos \varphi - E \cos \varphi + H(\theta_1 + \theta_2) \cos \varphi}{\partial y^*} = 0. \quad (4.4) \]

\[ \Pr L \left[ \frac{\partial (\theta_1 + \theta_2)}{\partial t} + (u_1 + u_2) \frac{\partial (\theta_1 + \theta_2)}{\partial x^*} + (v_1 + v_2) \frac{\partial (\theta_1 + \theta_2)}{\partial y^*} \right] = \frac{\partial^2 (\theta_1 + \theta_2)}{\partial x^{*2}} + ... \]
\[ \begin{align*}
\ldots & + \frac{\partial^2 (\theta_1 + \theta_2)}{\partial y^2} - 16 F s N W (\theta_1 + \theta_2) + 12 F s N W + \ldots \\
\ldots & + M^2 F s \left[ \frac{\partial(b_x + b_x)}{\partial y^2} \right]^2 (1 - 2 \sin \varphi \cos \varphi) \ldots (4.5) \\
\text{And therefore} \\
\text{Pr} \left[ \frac{\partial(\theta_1 + \theta_2)}{\partial t^*} + (u_1 + u_2) \frac{\partial(\theta_1 + \theta_2)}{\partial x^*} + (v_1 + v_2) \frac{\partial(\theta_1 + \theta_2)}{\partial y^*} \right] = \frac{\partial^2 (\theta_1 + \theta_2)}{\partial x^2} + \ldots \\
\ldots & + \frac{\partial^2 (\theta_1 + \theta_2)}{\partial y^2} - 16 F s N W (\theta_1 + \theta_2) + 12 F s N W + M^2 F s \left[ \frac{\partial(b_x + b_x)}{\partial y^2} \right]^2 (1 - 2 \sin \varphi \cos \varphi). \ldots (4.6) \\
\text{So} \\
\frac{\partial u_1}{\partial x^*} + \frac{\partial u_2}{\partial x^*} + \frac{\partial v_1}{\partial y^*} + \frac{\partial v_2}{\partial y^*} = 0. \ldots (4.7) \\
\frac{\partial u_1}{\partial t^*} + \frac{\partial u_2}{\partial t^*} + u_1 \frac{\partial u_1}{\partial x^*} + u_2 \frac{\partial u_2}{\partial x^*} + u_1 \frac{\partial v_1}{\partial x^*} + v_1 \frac{\partial v_1}{\partial y^*} + \ldots \\
\ldots + v_2 \frac{\partial v_2}{\partial y^*} = - \frac{\partial p_1}{\partial x^*} - \frac{\partial p_2}{\partial y^*} + \frac{1}{Re} \left[ \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_2}{\partial y^2} + \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_2}{\partial y^2} \right] + \ldots \\
\ldots + M^2 v_1 \sin \varphi - M^2 \frac{v_1}{Re} \sin \varphi - E \sin \varphi - H \theta_1 \sin \varphi - H \theta_2 \sin \varphi \ldots (4.8) \\
\frac{\partial v_1}{\partial t^*} + \frac{\partial v_2}{\partial t^*} + u_1 \frac{\partial v_1}{\partial x^*} + u_2 \frac{\partial v_2}{\partial x^*} + u_1 \frac{\partial v_2}{\partial y^*} + v_1 \frac{\partial v_1}{\partial y^*} + \ldots \\
\ldots + v_2 \frac{\partial v_2}{\partial y^*} = - \frac{\partial p_1}{\partial y^*} - \frac{\partial p_2}{\partial y^*} + \frac{1}{Re} \left[ \frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_2}{\partial y^2} + \frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_2}{\partial y^2} \right] + \ldots \\
\ldots + M^2 v_1 \cos \varphi + M^2 \frac{v_1}{Re} \cos \varphi - E \cos \varphi + H \theta_1 \cos \varphi + H \theta_2 \cos \varphi. \ldots (4.9) \\
\text{Pr} \left[ \frac{\partial \theta_1}{\partial t^*} + \frac{\partial \theta_2}{\partial t^*} + u_1 \frac{\partial \theta_1}{\partial x^*} + u_2 \frac{\partial \theta_1}{\partial x^*} + u_1 \frac{\partial \theta_2}{\partial x^*} + u_2 \frac{\partial \theta_2}{\partial x^*} \right] + \ldots \\
\ldots + \text{Pr} \left[ \frac{\partial \theta_2}{\partial t^*} + \frac{\partial \theta_2}{\partial t^*} + v_1 \frac{\partial \theta_2}{\partial y^*} + v_2 \frac{\partial \theta_1}{\partial y^*} + v_2 \frac{\partial \theta_2}{\partial y^*} \right] = \frac{\partial^2 \theta_1}{\partial x^2} + \frac{\partial^2 \theta_2}{\partial y^2} + \ldots \\
\ldots + \frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_2}{\partial y^2} - 16 F s N W \theta_1 - 16 F s N W \theta_2 + 12 F s N W + \ldots \\
\ldots + M^2 F s \left[ \frac{\partial b_x}{\partial y^*} \right]^2 + 2 \left[ \frac{\partial b_x}{\partial y^*} \right] \left[ \frac{\partial b_y}{\partial y^*} \right] + \left[ \frac{\partial b_y}{\partial y^*} \right]^2 (1 - 2 \sin \varphi \cos \varphi). \ldots (4.10) \\
\end{align*} \]

5. Unsteady State Equations:

This section is devoted to investigate the stability of the disturbance by using an approximated analytical method. From the differential equations (4.7), (4.8), (4.9) and (4.10) we get

\[ \frac{\partial u_2}{\partial x^*} + \frac{\partial v_2}{\partial y^*} = 0. \ldots (5.1) \]
\[
\begin{align*}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + u_2 \frac{\partial u}{\partial x} + u_2 \frac{\partial u}{\partial x} + v_1 \frac{\partial u}{\partial y} + v_2 \frac{\partial u}{\partial y} + v_2 \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \ldots \\
= \frac{1}{Re} \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] - \frac{M^2}{Re} v_2 \sin \phi + \Pi \cos \phi. \quad \ldots (5.2)
\end{align*}
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + u_2 \frac{\partial v}{\partial x} + u_2 \frac{\partial v}{\partial x} + v_1 \frac{\partial v}{\partial y} + v_2 \frac{\partial v}{\partial y} + v_2 \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \ldots \\
= \frac{1}{Re} \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] + \frac{M^2}{Re} v_2 \cos \phi + \Pi \sin \phi. \quad \ldots (5.3)
\]

\[
Pr \left[ \frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + u_2 \frac{\partial \theta}{\partial x} + u_2 \frac{\partial \theta}{\partial x} + v_1 \frac{\partial \theta}{\partial y} + v_2 \frac{\partial \theta}{\partial y} + v_2 \frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} - \ldots \ldots
\right]
\]

\[
16F^2 \Pi \theta^2 + M^2 F \left[ \frac{\partial^2 b}{\partial y^2} + \frac{\partial^2 b}{\partial y^2} \right] + \left( \frac{\partial^2 b}{\partial y^2} \right)^2 \right) \left(1 - \sin \phi \cos \phi \right). \quad \ldots (5.4)
\]

With boundary conditions and from (4.1) , we have:

\[
\begin{align*}
1 - u_2 = v_2 = 0 , \quad \theta_2 = 0 , \quad \frac{\partial \theta}{\partial y} = 0 & \quad \text{at} \quad y^* = -1 \\
2 - u_2 = v_2 = 0 , \quad \theta_2 = 0 , \quad \frac{\partial \theta}{\partial y^*} = 0 & \quad \text{at} \quad y^{**} = 1
\end{align*}
\]

6. **Disturbance in x and y Directions**:

To solve the linearized system (or to analyze the stability) and because the coefficient in the differential equations are independent of \( t \) and \( x \) we attempt to find the solution of the form [5]

\[
\begin{align*}
u_2 &= u(y)e^{i \alpha x}e^{a t} \\
v_2 &= v(y)e^{i \alpha x}e^{a t} \\
p_2 &= p(y)e^{i \alpha x}e^{a t} \\
\theta_2 &= \theta(y)e^{i \alpha x}e^{a t} \\
b_{x*} &= b_x(y)e^{i \alpha x}e^{a t}
\end{align*}
\]

Where \( k \) is wave number in the direction \( x \), and \( \alpha \) is the complex number which has the form \( \alpha = \alpha_i + i \alpha_2 \), \( \alpha_1 \), \( \alpha_2 \in R \) is the speed number. When \( \alpha_i > 0 \), the system is unstable while when \( \alpha_i < 0 \), the system is stable. Also \( u(y), v(y), p(y), \theta(y), b_x(y) \) are the amplitude functions.

After substitute (6.1) in the equations (5.1), (5.2), (5.3) and (5.4), we get:

\[
(iku + v')e^{ikx}e^{a t} = 0
\]

\[
(a u + u_i(iku) + \frac{\partial u}{\partial x^*}u + v_i u + \frac{\partial u}{\partial y^*} v + ikp - \frac{1}{Re} (-k^2u + u'') - \frac{M^2}{Re} v \sin \phi + \ldots
\]

\[
+ H \theta \sin \phi e^{ikx}e^{a t} = 0.
\]

\[
(au + u_i(ikv) + \frac{\partial v}{\partial x}u + v_i v' + \frac{\partial v}{\partial y} v + p' - \frac{1}{Re} (-k^2v + v'') - \frac{M^2}{Re} v \cos \Phi + \ldots
\]
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\[
\Pr L((a + u_1(ik\theta) + \frac{\partial}{\partial x^*} u + \frac{\partial}{\partial y^*} v) - k^2 \theta - \theta'' + 16F_{SW} \theta - ... - H\theta \cos \Phi) e^{ix} e^{at} = 0
\]

\[
... - M^2 F_s \left[ 2(\frac{\partial b_x}{\partial y^*})(b^*_1) + (b^*_1)^2 \right] (1 - 2 \sin \varphi \cos \varphi) e^{ix} e^{at} = 0
\]

Since $e^{ix} e^{at} \neq 0$, then

\[
iku + v' = 0
\]

\[
u + u_1(iku) + \frac{\partial u_1}{\partial x^*} u + \frac{\partial u_1}{\partial y^*} v + ikp + \frac{1}{Re} (-k^2 u + u'') + v_i u' -......
\]

\[
...... - \frac{M^2}{Re} \sin \varphi + H\theta \sin \varphi = 0
\]

\[
v + u_1(ikv) + \frac{\partial v_1}{\partial x^*} u + v_i v' + \frac{\partial v_1}{\partial y^*} v + p' - \frac{1}{Re} (-k^2 v + v'') -......
\]

\[
...... - \frac{M^2}{Re} \cos \varphi - H\theta \cos \varphi
\]

\[
a \Pr L((a + u_1(ik\theta) + \frac{\partial}{\partial x^*} u + v_i \theta' + \frac{\partial}{\partial y^*} v) - k^2 \theta - \theta'' + 16F_{SW} - ... - M^2 F_s \left[ 2(\frac{\partial b_x}{\partial y^*})(b^*_1) + (b^*_1)^2 \right] (1 - 2 \sin \varphi \cos \varphi) = 0
\]

And after linearization equations of (6.2) we get

\[
iku + v' = 0
\]

\[
(a + \frac{K^2}{Re})u + ikp - \frac{1}{Re} u'' - \frac{M^2}{Re} v \sin \varphi - H\theta \sin \varphi = 0
\]

\[
(a + \frac{K^2}{Re} - \frac{M^2}{Re} \cos \varphi) v + p' - \frac{1}{Re} v'' - H\theta \cos \varphi = 0
\]

\[
a \Pr L + k^2 + 16F_{SW} \theta - \theta'' = 0
\]

From (6.3), let $\phi = 30^\circ$, we have

\[
u = h
\]

\[
h' = (ik Re) p + (a Re + k^2) u + (\frac{M^2}{2} Re) v - (\frac{1}{2} H Re) \theta
\]

\[
p' = -(\frac{ik}{Re}) h + (\sqrt{3} - H) \theta - (a + \frac{k^2}{Re} - \sqrt{3} \frac{M^2}{2} Re)
\]

\[
v' = -(ik) u
\]

\[
\theta' = s
\]

\[
s' = (a \Pr L + k^2 + 16F_{SW}) \theta
\]

Where $X(y) = [u(y), h(y), p(y), v(y), \theta(y), s(y)]^T$, and
\[
\frac{dX}{dy} = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 \\
(a \Re + k^2) & 0 & ik \Re & - \frac{M^2}{2} & - \frac{H \Re}{2} \\
0 & -ik \Re & 0 & -(a + \frac{k^2}{\Re} - \frac{\sqrt{3} M^2}{2 \Re}) & \frac{\sqrt{3}}{2} H \\
-ik \Re & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
u \\
h \\
p \\
v \\
\theta \\
s
\end{bmatrix}
\]

And by using \(|\Omega - \lambda I| = 0\), where
\[
\Omega = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
A & 0 & B & W & G & 0 & 0 \\
0 & C & 0 & J & T & 0 & 0 \\
E & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & F & 0 & 0
\end{bmatrix}
\]

\[
A = (a \Re + k^2) , \quad B = (ik \Re) , \quad E = -(ik) , \quad J = (a + \frac{k^2}{\Re} - \frac{\sqrt{3} M^2}{2 \Re}) , \quad C = -(\frac{ik}{\Re}) ,
\]

Where
\[
F = (a \Pr L + k^2 + 16FsNW) , \quad G = -\frac{H \Re}{2} , \quad W = -\frac{M^2}{2} , \quad T = \frac{\sqrt{3}}{2} H
\]

we get
\[
\begin{vmatrix}
-\lambda & 1 & 0 & 0 & 0 & 0 & 0 \\
A & -\lambda & B & W & G & 0 & 0 \\
0 & C & -\lambda & J & T & 0 & 0 \\
E & 0 & 0 & -\lambda & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\lambda & 1 & 0 \\
0 & 0 & 0 & 0 & F & -\lambda & 0
\end{vmatrix} = 0
\]

or
\[
f(\lambda) = \lambda^6 - (F + A + BC)\lambda^4 - EW\lambda^3 + (BCF + AF - BEJ)\lambda^2 + EFW\lambda + EJBF = 0 \quad \ldots(6.5)
\]

Now, we solve equation (6.5) numerically by using (Maple 11) [7] to find the roots of this equation.

7. Conclusions

In this paper, we study the stability of the system under the effect of an inclination to the horizontal channel with the presence of magnetic field and radiation, when the system has inclined by angle \(\phi = 30^\circ\), it's noticed that the wave number \(k\) has a negative effect on the velocity function and depends it towards unstable and this is clear from figure (3). From figure (2), the increase of the factor \(Fs\) causes the increase to unstable probability of the system. Finally it is found that the increasing of Bouger number has an effect on the system towards stable, it's noticed from figure (4).
Stability Analysis for Inclined Channel by an Angle $30^\circ$ with The Presence of Magnetic Field

**Figure (2):** Effect of The Factor $F_s = 0.4, 0.04, 0.004$ at Angle $30^\circ$

**Figure (3):** Effect of The Wave Number $k = 2, 4, 8$ at Angle $30^\circ$

**Figure (4):** Effect of The Bouger Number $W = 0.1, 0.5, 0.9$ at Angle $30^\circ$
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