Model of the electro-weak, gravitational and strong interactions in the O-theory

V. Yu. Dorofeev
Friedmann Laboratory for Theoretical Physics

Abstract

Based on the matrix representation of octonion algebra, supplied with specific multiplication rule, the model of electroweak and gravitational interactions is built up. While electroweak interaction in this model is induced by charged W-bosons, other two forces appear to have slightly more complicated nature. Gravitational interaction coincides in the model with dipole interaction of a pair of charged bosons. The dipole consists of a charged vector bosons pair from the major octonion algebra fields. When the charged dipole pair interacts with the neutral bosons pair from the major octonion algebra fields, the charged bosons pair misses its mass. The drop in mass leads to appearance of far-ranging forces of gravitational interaction. Finally, strong interaction appears in the model as internal gravitational solution of 'black whole' type with the peculiar 'gravitational' constant. The solution is a product of interaction of major vector fields pair with charged W-bosons pair. It is inferred from the model that the state space is ten-dimensional. The space is built as a module of the matrix representation of octonion algebra over the particles field (O-module). Similarly to the Standard Weinberg-Salam theory, the particle mass here appears as the product of interaction of massless spinor fields and Higgs field from O-module representation.

Introduction

The idea of incorporation of octonion algebra into total field framework is a long-life story [1]. However, the early attempts were mostly concerned in the new Cayley octaves algebra introduction to the physics theory necessity rather than in seeking approach for unified field theory construction. Nowadays octonions are included in the emerging M-theory framework of unified interactions. Nevertheless the group approach to M-theory construction has a sensitive disadvantage: Cayley octaves algebras lose non-associativity of their lagrangian.

In the current work principally different approach to unified field theory construction is proposed. This approach implies the algebraic introduction of new...
interactions on the special algebra. Such approach has a number of advantages, though along with certain disadvantages. The main disadvantage is a necessity of new quantization schemes introduction. That becomes inevitable because the lagrangian loses its important group properties when considered on the newly introduced algebra. Therefore the problem of probabilistic interpretation of particles and the positivity of its energy arises. However after some reasoning that fact does not face so negative. As it follows from calculations, new particles should have higher mass therefore their influence within the modern energy scale could be neglected without losing any of group properties of $U(1) \times SU(2)$ symmetries of electro-weak interaction. In case of high energies non-associative nature of the lagrangian provides us with one specific opportunity: the order of multipliers we could choose according to our own preferences. That opportunity allows us to exclude undesirable summands. Thus in the proposed fields interaction model new quantization as well as renormalization methods, perhaps, could be found.

In [2] the author proposed generalization of the lagrangian from Weinberg-Salam theory to non-associative algebra and later in [3] proposed the scheme of gravitational interaction introduction. The main milestones of conducted research would be point out in the first paragraphs of the paper. After that strong interaction introduction scheme as a model of strong gravity, but in the framework of the approach developed here, and as a solution inside of “black whole” in the Schwarzschild metric, is proposed. New gravitation constant of strong gravitation is defined by the mass of charged $W$-bosons from weak interaction theory.

1 Dirac equation in the pseudo-riemannian space

Let $M$ be some pseudo-riemannian manifold and the coordinates $x(p) = (x^0, x^1, x^2, x^3) = x$ may be introduced in any point $p \in M$, metric is

$$ds^2 = g_{\mu \nu} dx^\mu dx^\nu$$

and the connectness is

$$\Gamma^\mu_{\nu \lambda} = \frac{1}{2} g^{\mu \kappa} (g_{\nu \kappa, \lambda} + g_{\nu \lambda, \kappa} - g_{\lambda \nu, \kappa})$$

Then the Riemannian tensor we define by

$$R^\tau_{\mu \nu \lambda} = \Gamma^\tau_{\mu \lambda, \nu} - \Gamma^\tau_{\mu \nu, \lambda} + \Gamma^\tau_{\sigma \nu} \Gamma^\sigma_{\mu \lambda} - \Gamma^\tau_{\sigma \lambda} \Gamma^\sigma_{\mu \nu}.$$  

Quadratic form (1) may be reduced to the diagonal form in any neighborhood of the point

$$ds^2 = H_0^2 dx^{(0)2} - H_1^2 dx^{(1)2} - H_2^2 dx^{(2)2} - H_3^2 dx^{(3)2}$$
We consider the metric of the physical space-time as metric in Minkowski space $M_4$ in the normal form

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 = \eta_{ab} dx^a dx^b \quad (5)$$

(Here we make distinction between the Greek indexes and Latin indexes: the Greek indexes are used with regard to pseudo-riemannian manifold and the Latin indexes are used with regard to the $M_4$.)

Build the tangent space in every point of the pseudo-riemannian space $M$. Assume the tangent space is the Minkowsky space $M_4$ with metric (5). Let

$$H_0 dx^{(0)} = c dt, H_1 dx^{(1)} = dx, H_2 dx^{(2)} = dy, H_3 dx^{(3)} = dz \quad (6)$$

Introduce the tetrads $h^a_\mu$ in the same points of the pseudo-riemannian space $M$ to connect the metric of pseudo-riemannian space to the metric of the $M_4$:

$$h^b_\mu h^\mu_a = \delta^b_a, \quad h^{(a)}_\mu h^\mu_a = g^{\mu\nu} \quad (7)$$

Let $h^a_\mu$ be tetrads [6]:

$$A^a = A^\mu h^a_\mu. \quad \text{Then}

$$\delta A^\mu = \delta (A^\mu h^a_\mu) = \delta A^\mu h^a_\mu + A^a \delta h^a_\mu = \delta A^a h^a_\mu + A^a h^a_{\nu\mu} \delta x^\nu = \Gamma^\mu_{\nu\lambda} A^\nu \delta x^\lambda \quad (8)$$

As $h^b_\mu h^\mu_a + h^b_{\mu\nu} h^\mu_a = 0$ so

$$\delta A^b = \gamma^b_{\lambda c} A^\lambda \delta x^c, \quad \gamma^b_{\lambda c} = h^b_{\mu\nu} h^\mu_a h^\nu_c \quad (9)$$

where $\gamma^b_{\lambda c}$ are Ricci coefficients.

In [7] free Dirac equation

$$\left( i \gamma^a \partial_a - m \right) e(x) = 0 \quad (10)$$

in the tangential space $M_4$ of the pseudo-riemannien space have the following form

$$\left( i h^a_\mu \gamma^a (\partial_\mu - i \Phi^1_\mu - \Gamma_\mu) - m \right) e(x) = 0 \quad (11)$$

where

$$\Gamma_\mu = h^a_\mu \Gamma_\mu = -\frac{1}{2} \gamma_{abc} \sigma^{bc}, \quad \sigma^{bc} = \frac{1}{4} \left[ \gamma^a, \gamma^b \right], \quad a, b = 0, 1, 2, 3 \quad (12)$$

and $\Phi^1_\mu$ is arbitrary real function.

Dirac matrixes comply with the following multiplication rule on matrixes $\sigma^{ab}$:

$$\gamma^a \sigma^{bc} = \frac{1}{4} \gamma^a \left[ \gamma^b, \gamma^c \right] = \frac{1}{2} \eta^{ab} \gamma^c - \frac{1}{2} \eta^{ac} \gamma^b - i \varepsilon^{abc \delta} \gamma^\delta \quad (13)$$

Then

$$-\gamma^a \Gamma_a = \frac{1}{4} h^\mu_a h^\nu_b h^c_{(c)\nu\mu} \left( \frac{1}{2} \eta^{ab} \gamma^c - \frac{1}{2} \eta^{ac} \gamma^b - i \varepsilon^{abc \delta} \gamma^\delta \right) \quad (14)$$
Assume, the metric (1) has form (4), then for different $a,b,c$ we get

$$
\Gamma^\lambda_{\mu\nu} h_c^{\lambda} h_b^{\mu} h_e^{\nu} = \frac{1}{2} g^{\mu\kappa} (g_{\nu\kappa,\lambda} + g_{\nu\kappa,\lambda} - g_{\lambda\nu,\kappa}) h_c^{\lambda} h_b^{\mu} h_e^{\nu} = 0
$$

(15)

so

$$
-\gamma^a \Gamma_a = \frac{1}{4} h_a^{\mu} h^{\nu(a)} h_{(c)\nu\mu} \gamma^c - \frac{1}{4} h_a^{\mu} h^\nu h_{(a)\mu\nu} \gamma^b = \frac{1}{4} h_a^{\mu} \gamma^c + \frac{1}{4} h^{\mu(a)} h_{b\mu} h_{(a)\nu} \gamma^b = \frac{1}{2} h_a^{\mu} \gamma^c
$$

(16)

and Dirac equation has the form [8]

$$
(i\gamma^a h^\mu (\partial_{\mu} - i\Phi^1_{\mu} + \Phi^2_{\mu}) - m)\psi = 0
$$

(17)

where

$$
\Phi^2_{\mu} = \frac{1}{2} \partial_{\mu} \left( \ln \frac{\sqrt{-g}}{H_{\mu}} \right)
$$

(18)

2  $\mathcal{O}$ - space

The doubling of quaternion algebra leads, in particular, to octonion algebra $\mathcal{O}$ [9], which is a linear space over the field of real numbers $\mathbb{R}$, for any $\mathbf{o}$ from octonion algebra $\mathcal{O}$. Octonion algebra cannot be represented by matrixes with traditional multiplication rule, but the special multiplication rule can be introduced, which allows such representation [5]

$$
\mathcal{O} = \{ \forall \mathbf{o} = \sum_{A=0}^{7} \alpha^A \hat{\Sigma}^A = \alpha^A \hat{\Sigma}^A, \quad \alpha^A \in \mathbb{R} \}
$$

(19)

where $\hat{\Sigma}^K$ are matrix and $(I, J, K = 1, 2, \ldots, 7)$:

$$
\hat{\Sigma}^I \cdot \hat{\Sigma}^J = -\delta^{IJ} + \varepsilon^{IJK} \hat{\Sigma}^K,
$$

(20)

where completely antisymmetric symbol $\varepsilon^{IJK}$ is not null if only

$$
\varepsilon^{123} = \varepsilon^{145} = \varepsilon^{176} = \varepsilon^{246} = \varepsilon^{257} = \varepsilon^{347} = \varepsilon^{365} = 1
$$

and $\hat{\Sigma}^0 = 1$ (matrice $2 \times 2$).

It is easy to ensure, that the multiplication rule leads to non-associative algebra, i.e.

$$
\{ \hat{\Sigma}^A, \hat{\Sigma}^B, \hat{\Sigma}^C \} = (\hat{\Sigma}^A \hat{\Sigma}^B) \hat{\Sigma}^C - \hat{\Sigma}^A (\hat{\Sigma}^B \hat{\Sigma}^C) = 2 \varepsilon^{ABCD} \hat{\Sigma}^D
$$

where $\varepsilon^{IJKL}$ is completely antisymmetric symbol, which is equal to one for the following expressions:

$$
\varepsilon^{1247} = \varepsilon^{1265} = \varepsilon^{2345} = \varepsilon^{2376} = \varepsilon^{3146} = \varepsilon^{3157} = \varepsilon^{4567} = 1
$$

(21)
Let $\Sigma^K = i\tilde{\Sigma}^K$, $K = 1, 2, \ldots, 7$ and

$$O = \{\forall o = a^A \Sigma^A, \quad a^A \in \mathbb{C}, A = 0, 1, \ldots, 7\} \quad (22)$$

where $(I = 1, 2, 3)\

$$\Sigma^0 = \begin{pmatrix} \sigma^0 & 0 \\ 0 & \sigma^0 \end{pmatrix}, \quad \Sigma^I = \begin{pmatrix} 0 & -i\sigma^I \\ i\sigma^I & 0 \end{pmatrix}$$

$$\Sigma^4 = \begin{pmatrix} -\sigma^0 & 0 \\ 0 & \sigma^0 \end{pmatrix}, \quad \Sigma^{4+I} = \begin{pmatrix} 0 & -\sigma^I \\ -\sigma^I & 0 \end{pmatrix} \quad (23)$$

Here $\sigma^0$ is a unit matrix and $\sigma^I, I = 1, 2, 3$ is a Pauli matrix:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (24)$$

Let’s define the specific multiplication rule for the abstract matrixes in the following way

$$o \ast o' = \begin{pmatrix} \lambda I & A \\ B & \xi I \end{pmatrix} \ast \begin{pmatrix} \lambda' I & A' \\ B' & \xi' I \end{pmatrix} = \begin{pmatrix} (\lambda\lambda' + \frac{1}{2}tr(AB'))I & \lambda A' + \xi' A + \frac{i}{2}[B, B'] \\ \lambda' B + \xi B' - \frac{i}{2}[A, A'] & (\xi\xi' + \frac{1}{2}tr(BA'))I \end{pmatrix} \quad (25)$$

The matrices $A, A', B, B$ of size $(2 \times 2)$ were introduced, together with unit matrix $I$ of the same size and complex numbers $\lambda, \xi, \lambda', \xi'$.

Multiplication rule (25) is defined for a wider matrix class than octonion algebra representation matrixes $\Sigma^a, a = 0, 1, 2, \ldots, 7$ introduced earlier. Two more matrixes could be introduced

$$f^8 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad f^9 = \begin{pmatrix} 0 & iI \\ -iI & 0 \end{pmatrix} \quad (26)$$

That maintains our specific multiplication rule (25).

So the set of matrixes

$$f^a = \Sigma^a, \quad a = 0, 1, \ldots, 7, \quad f^8, \quad f^9 \quad (27)$$

form a basis of linear space over the field of complex numbers, which is further called the extended octonion space, and is denoted for $O$.

That defines the space $O$ as non-associative $O$-module, where $O$ is alternative ring.

By means of multiplication (25) introduce the convolution on the $O$:

$$\langle O_1, O_2 \rangle = \frac{1}{2} \text{tr}(O_1 \ast O_2) = \frac{1}{2} \text{tr}(\begin{pmatrix} \lambda_1^I I & B_1^+ \\ A_1^- & \xi_1^I I \end{pmatrix} \ast \begin{pmatrix} \lambda_2^I I & A_2 \\ B_2^- & \xi_2^I I \end{pmatrix}) = \lambda_1^I \lambda_2 + \xi_1^I \xi_2 + \frac{1}{2} \text{tr}(B_1^+ B_2) + \frac{1}{2} \text{tr}(A_1^- A_2) \quad (28)$$

where $O \times O$ in $\mathbb{C}$. 

5
3 Methods of relieving of octonions from non-associativity

Probability model of non-associativity relieving.

\[ A \ast B \ast C \ast D = p_1((A \ast B) \ast C) \ast D + A \ast (B \ast C) \ast D \]  
(29)

\[ p_1 + p_2 = 1 \]

Here the frequencies \( p_1 \) and \( p_2 \) are introduced. These values depend on the number of the brackets permutations, which lead to the equal result in (29). For example, in (29) it is natural to admit \( c_{1b}^a = p_1 = 3/4 \) and \( c_{2b}^a = 1/4 \) because three different types of brackets permutations lead to the equal result in the first member of the right side of the equation (29) whereas only one type in the second member of the right side of (29).

The definition given in (29) is also applicable for greater number of elements.

Minimal model of non-associativity relieving.

\[ A \ast B \ast C \ast D = \min\{(A \ast B) \ast (C \ast D), A \ast (B \ast C) \ast D\} \]  
(30)

The maximal or minimax model is to be defined similarly.

Obviously, in associative case the same result is obtained to that of the models defined above.

4 Lagrangian of the O-theory

Let

\[ A_{ib}(x) = A_{ib}^A(x) \Sigma^A, \quad \Sigma^A \in \mathcal{O}_R, A = 0, 1, 2, \ldots, 7 \]  
(31)

\[ \mathcal{O}_R = \{\forall o_{ib} = \alpha^A \Sigma^A, \quad \alpha^A \in \mathbb{R}, A = 0, 1, 2, \ldots, 7\} \]

where \( A_{ib}^A(x) \in \mathbb{R}, x \in M_4, b = 0, 1, 2, 3 \).

In [2] the generalization of Weinberg-Salam lagrangian to non-associative algebra is proposed as follows (\( A, B = 0, 1, \ldots, 7 \)):

\[ \mathcal{L}_c = \mathcal{L}_f + (\partial_a \Psi \ast \frac{i}{2} q^A A^a A^b \Psi \ast \Sigma^A) \ast (\partial^a \Psi \ast \frac{i}{2} q^B A^a(B) \Sigma^B \ast \Psi) \]

\[ + \frac{i}{2} \mathcal{L} \ast \gamma_a(\overline{\partial}^a + \frac{i}{2} c_A q^A A^{a(A)} \Sigma^A) \ast L - \frac{i}{2} \mathcal{L} \ast \gamma_a(\overline{\partial}^a - \frac{i}{2} c_A q^A A^{a(A)} \Sigma^A) \ast L \]

\[ + \frac{i}{2} R \gamma_a(\overline{\partial}^a R + iq^0 a^a R) - \frac{i}{2} \mathcal{L} \ast (\overline{\partial}^a R - iq^0 a^a R) \]

\[ - \frac{i}{2} \mathcal{L} \ast \Psi \ast R - \frac{i}{2} \mathcal{L} \ast \Psi \ast L + m^2||\Psi||^2 - \frac{j}{4}||\Psi||^4 \]  
(32)

Here \( \Sigma^A \in \mathcal{O}, \quad \Psi, \Psi \in \mathcal{O} \).

The lagrangian of the free fields \( \mathcal{L}_f \) is \( (I, J, K, L = 1, \ldots, 7) \)

\[ \mathcal{L}_f = -\frac{1}{4} F_{ab}^0 F_{ab}^{0(0)} - \frac{1}{16} \text{tr}(\hat{F}_{ab} \ast \hat{F}_{ab}) = -\frac{1}{4} F_{ab}^0 F_{ab}^{0(0)} - \frac{1}{4} F_{ab}^0 F_{ab}^{0(K)} \]
\[ \Lambda^a = \Psi_f^\dagger \left( \frac{q^{(K)} m^2}{2f} A^a K A^{a(K)} + \frac{q^{(K)} A^a A^{a(g)}}{2f} A^a + \frac{q^{(K)} m^2}{2f} B a B^a - \frac{q^{(K)} m^2}{2f} A^3 B^a \\
+ \frac{\sigma^1 e_L}{2} \eta_L \gamma_a B^a e_L + \frac{\sigma^2 e_L}{2} \eta_L \gamma_a B^a e_L - \frac{\sigma^3 e_L}{2} \eta_L \gamma_a A^3 \nu_L \\
- \frac{q}{2} \eta_L \gamma_a e_L (A^0 - i A^2) - \frac{q}{2} \eta_L \gamma_a e_L (A^0 + i A^2) \\
+ \frac{i}{2} (\eta_L \gamma_a \partial^a e_L - \partial^a \eta_L \gamma_a e_L + \frac{4}{2} \eta_L \gamma_a \partial^a \nu_L - \partial^a \eta_L \gamma_a \nu_L) + \frac{m^4}{f} \\
+ \frac{i}{2} (\eta_R \gamma_a \partial^a e_R - \partial^a \eta_R \gamma_a e_R) + q^{(K)} \eta_R \gamma_a B^a e_R - \frac{\sqrt{2} \eta_m}{\sqrt{f}} (\eta_L e_R + \eta_R e_L) \\
- q^4 A^{a(4)} (\kappa_1 \eta_L \gamma_a e_L - \kappa_2 \eta_R \gamma_a e_L) - \frac{3}{2} q^6 A^{a(6)} \eta_L \gamma a e_L \\
- \frac{5}{4} q^6 A^{a(6)} + i q^5 A^{a(5)} \eta L \gamma a e_L - \frac{5}{4} q^6 A^{a(6)} - i q^5 A^{a(5)} \eta L \gamma a e_L \right) \]

(39)
Where the following notation is used $ie^{ij} A^{ij} q^i q^j = \delta^{ij}, \kappa_1, \kappa_2 \approx 10$.

So, the final lagrangian contains non-associative elements along with associative.

Consider non-associative summands from lagrangian $\mathcal{L}_o$. First of all, it is the quadratic member by the fields $A_k^\mu$

$$\frac{q^{ij} m^2}{2f} A_u^i A_u^{i(j)}$$

Apply probability model of non-associativity relieving to this member. Assume for non-zero components $c^{ij}_1 = c^{ji}_1 = 3/4, c^{ij}_2 = c^{ji}_2 = 1/4$. Hence due to the symmetry of the expression $q^i q^j A_{\mu}^i A_{\mu}^{i(j)}$ with respect to $i,j$ (there is no summing up by them) and the anti-symmetry of multiplier $A_{ij}^k$ (again with respect to $i,j$) this member equals zero.

In addition there is a non-associative term (34) in the lagrangian of free fields. Relieving from non-associativity for this member will lead to important physical properties of the lagrangian of the O-theory, therefore the discussion of methods of relieving it from non-associativity postpone to the appropriate section.

Assume the senior field $A_{a}^{K}, K = 4, 5, 6, 7$ equal zero, we come to the lagrangian of the Weinberg-Salam, the Standard Theory of weak interactions (ST) [10] in the appropriate gauge. Strictly speaking, the problem formulation concludes in choosing such generalization to $\mathcal{O}$-module that in the particular case of the minor fields we would get exactly the lagrangian of the ST.

5 Octonion lagrangian research

1. Current fields of $A_{k}^\mu, k = 0, 1, 2, 3$, as it is in the ST, indicate the presence of the vector bosons $Z_0, W$ and $W^*$, which appeared to be massive after their quadratic part of lagrangian and vacuum, induced by $\Psi_0$, research. This fact is expected, since the general principle of deducing the O-theory lagrangian concludes in obtaining such lagrangian $\mathcal{L}_o$ that, after excluding of the major fields $A_{k}^\mu, k = 4, 5, 6, 7$, we get the Lagrangian of the ST.

2. By analogy to the ST, the current containing part

$$q^4 A_{\mu}^{(4)} (\kappa_2 \bar{\psi}_L \gamma_\mu c_L - \kappa_1 \bar{\psi}_L \gamma_\mu \nu_L)$$

of the lagrangian (39) $\mathcal{L}_o$ gives the opportunity to introduce a neutral vector field $C_\mu = A_{4}^\mu$. Also, the form of quadratic terms of field $A_{4}^\mu$ allows us to introduce the lagrangian of a massive vector field $C_\mu$

$$\mathcal{L}_C = -\frac{1}{4}(\partial_\mu C_\nu - \partial_\nu C_\mu)(\partial^\mu C^\nu - \partial^\nu C^\mu) + \frac{m_{C}^2}{2} C_\mu C^\mu, \quad m_{C}^2 = \frac{(q^{(4)})^2 m^2}{f}$$

3. The field $A_{7}^\mu$ is special, because there are no currents. Research of the quadratic terms $A_{\mu}^{(4)}$ gives foundation to introduce the massive vector field $E_\mu =
$A^7_\mu$ and the lagrangian

$$L_E = -\frac{1}{4}(\partial_\mu E_\nu - \partial_\nu E_\mu)(\partial^\mu E^\nu - \partial^\nu E^\mu) + \frac{m^2_E}{2}E_\mu E^\mu, \quad m^2_E = \frac{(q^7)^2 m^2}{f} \quad (43)$$

4. The current containing part

$$\frac{5}{4}(q^6 A^{\mu(6)} + iq^5 A^{\mu(5)})\gamma_\mu e_L + \frac{5}{4}(q^6 A^{\mu(6)} - iq^5 A^{\mu(5)})\overline{\epsilon}_L \gamma_\mu \nu_L \quad (44)$$

gives foundation to introduce the charged vector field

$$D_\mu = \frac{1}{2q_D}(q^6 A^{\mu(6)} - iq^5 A^{\mu(5)}) \quad (45)$$

and the quadratic terms of the lagrangian $L_\omega$ allows to introduce a massive vector field $D_\mu$ with lagrangian (where $m^2_D = 2m^2 q^2_D/f$)

$$L_D = -\frac{1}{2}(\partial_\mu D^*_\nu - \partial_\nu D^*_\mu)(\partial^\mu D^\nu - \partial^\nu D^\mu) + m^2_D D^*_\mu D^\mu \quad (46)$$

However, that must be assumed $q^{(5)2} = q^{(6)2} = q^2 = q^2_D$, therefore $m_5 = m_6 = m_D = m_D^*$. But not everything is so smooth! There is another interesting term in the lagrangian $L_\omega$

$$-\frac{3}{2}q^6 A^{\mu(6)}\overline{\epsilon}_L \gamma_\mu e_L \quad (47)$$

In a way, this term is also responsible for the current. Indeed

$$-\frac{3}{2}q^6 A^{\mu(6)}\overline{\epsilon}_L \gamma_\mu e_L = -\frac{3}{4}(q_D \cdot D^*_\mu + q_D D_\mu)\overline{\epsilon}_L \gamma_\mu e_L \quad (48)$$

but this kind of the current violates the invariance of the lagrangian under the global transformation of the charge for vector fields: if

$$D_\mu \rightarrow e^{iQ} D_\mu, \quad e_L \rightarrow e^{iQ} e_L \quad \ldots \quad (49)$$

then

$$(q_D \cdot D^*_\mu + q_D D_\mu)\overline{\epsilon}_L e_L \rightarrow (e^{-iQ} q_D \cdot D^*_\mu + e^{iQ} q_D D_\mu) e^{-iQ} \overline{\epsilon}_L e^{iQ} e_L$$

$$\neq (q_D \cdot D^*_\mu + q_D D_\mu)\overline{\epsilon}_L e_L \quad (50)$$

Loss of global charge invariance is due to the appearance of a massive vector boson, with respect to a non-zero vacuum value $\Psi_0$. However, the unusual form of the current containing term (48) requires further study.
6 Solutions for vector bosons in the O-theory

We write the Euler-Lagrange equations for vector fields $A_{\mu}^k$, $k = 5, 6$ lagrangian $\mathfrak{L}_\alpha$. Let

$$F_{\nu\mu}^5 = A_{\nu,\mu}^5 - A_{\mu,\nu}^5, \quad F_{\nu\mu}^6 = A_{\nu,\mu}^6 - A_{\mu,\nu}^6$$

then we get

$$F_{\nu\mu}^{\mu(5)} + m_5^2 A_{\nu}^5 - \frac{1}{4} f^{IK\delta} q^{K\delta} q^{IJ} A^{\nu(I)} A_{\nu}^J A_{\mu}^J = 0 \quad (51)$$

$$F_{\nu\mu}^{\mu(6)} + m_6^2 A_{\nu}^6 - \frac{1}{4} f^{K\delta P} q^{K\delta} q^{J\delta} A^{\nu(J)} A_{\nu}^J A_{\mu}^J = 0 \quad (52)$$

where $m_i = m_D$, $i = 5, 6$. (In the equations (51 52) there is non-abelian part of the lagrangian fields $\mathfrak{L}_f$, which is there omitted.)

Part of the lagrangian, reflecting its non-associative nature, remained in $f^{IJKL}$. Non-zero values of $f^{IJKL}$ in the $\mathfrak{L}_\alpha$ are equal to $\pm 1$ only for some fields permutations (20). Choose $f^{IJKL} = -1$ according to minimax model. If the only non-zero $f^{4675} = -1$ in (51 – 52) we get

$$F_{\nu\mu}^{\mu(5)} + m_5^2 A_{\nu}^5 - q^{47} q^{56} A_{\mu}^4 A_{\nu}^6 A_{\mu}^7 = 0 \quad (53)$$

$$F_{\nu\mu}^{\mu(6)} + m_6^2 A_{\nu}^6 - q^{47} q^{56} A_{\mu}^4 A_{\nu}^6 A_{\mu}^7 = 0 \quad (54)$$

The convolution product $A_{\mu}^4 A_{\mu}^{\mu(7)}$ is a scalar and we shall believe it to be a constant on the small region $\Omega_\ell$. We also assume that all values in (53 – 54) are chosen so that

$$m_5^2 = q^{47} q^{56} A_{\mu}^{\mu(4)} A_{\mu}^7$$

in particular,

$$A_{\mu}^{\mu(4)} A_{\mu}^7 = \frac{m_5^2}{q^{47} q^{56}} \quad (55)$$

However, then there is a massless solution of the equations (53 – 54) for the vector-potentials $A_{\nu}^5$, $A_{\nu}^6$:

$$A_{\nu}^5 = A_{\nu}^6 \quad (56)$$

of the form (recall (55))

$$F_{\nu\mu}^{\mu(5)} = 0, \quad F_{\nu\mu}^{\mu(6)} = \partial_\nu D_\mu - \partial_\mu D_\nu$$

Thus the $D$-boson is massless. As a solution of the equation (57) in the spherical coordinate system it takes

$$A_{\mu}^5 = A_{\mu}^6 = (0, f(r), 0, 0) \quad (58)$$

or

$$A_{\mu}^5 = A_{\mu}^6 = (f(t), 0, 0, 0) \quad (59)$$

where $f(r)$ and $f(t)$ are arbitrary functions of the radius vector $r$ and time $t$, respectively.
On the other side

\[ E_{\mu}^\nu(C) + m_{C}^2 C^\nu = q^4 7 q^5 A_{\mu}^5 A^{\mu(6)} E^\nu \]

\[ E_{\mu}^\nu(E) + m_{E}^2 E^\nu = q^4 7 q^5 A_{\mu}^5 A^{\mu(6)} C^\nu \]  

(60)

Until now, the fields were considered solely as classic. When calculating the convolution \( C_\mu E^\mu \) it is not sufficient. Let the convolution \( C_\mu E^\mu \) be an average

\[ C_\mu E^\mu = \langle |\hat{C}_\mu \hat{E}^\mu| \rangle \neq 0 \]  

(61)

where

\[ \hat{C}_\mu = l^6(\hat{c}_s e^{-ikx} + \hat{c}_s e^{ikx}), \quad \hat{E}^\mu = r^6(\hat{c}_s e^{-ikx} + \hat{c}_s e^{ikx}), \quad k^2 = m^2 \]  

(62)

also we introduced creation and annihilation operators of Bose-particles and assumed the same mass for particles \( C \) and \( E \). Since the particles \( C, E \) are different and the operator \( \hat{c}, \hat{c} \) are commutative, then the right part of the (61) by the vacuum state is equal to zero. At the same time the (61) by the bound state is not equal to zero. For example, assume \( |1, 1 > = \hat{c}^+_+ > 0 \) then

\[ < 0)(\hat{c} + \hat{c}^+)(\hat{c}^+_+ + \hat{c})\hat{c}^+_+ |0 > = < 0)|\hat{c}^+_+ \hat{c}^+_+ |0 > \neq 0 \]

\[ < 0)|\hat{c}\hat{c}(\hat{c}^+_+ + \hat{c})|0 > = < 0)|\hat{c}^+_+ \hat{c}^+_+ |0 > \neq 0 \]  

(63)

Since the left side of (60) is zero (recall \( \partial_\mu C^\nu = \partial_\mu E^\nu = 0 \) [14], then \( A_{\mu}^5 A^{\mu(6)} \) should be required to be small.

### 7 The method of geometerization in the O-theory

As it was mentioned, the lagrangian \( \Lambda_o \) includes the current vector, which is non-invariant under the transformation of charge (50). The reason is that the currents

\[ q_D D_\alpha(y) \gamma^\alpha e_L(y) \quad q_D D_\alpha(y) \gamma^\alpha e_L(y) \]  

(64)

are included in the lagrangian as a sum. (Here \( y^a, a = 0, 1, 2, 3 \) are the variables of the flat Minkowsky space.) Moreover, by substituting this Lagrangian in the action we find that basically these terms are defined in distinct points of \( U \)

\[ S' = \frac{i}{2} \int_U (\gamma^\alpha (\partial_\alpha + \frac{3i}{2} q_D D_\alpha) e - (\partial^\alpha - \frac{3i}{2} q_D D_\alpha e))d^4y \]

\[ = \frac{i}{2} \int_U (\gamma^\alpha (\partial_\alpha + \frac{3i}{2} q_D D_\alpha) ed^4y - \frac{i}{2} \int_U (\partial_\alpha - \frac{3i}{2} q_D D_\alpha) \gamma^\alpha ed^4y \]  

(65)

(The differences between the left and right particles were neglected in the paper, despite the fact that basically the considered space was Riemannian-Cartan space. Detailed calculations show the matrix \( \gamma^3 \) leads to Riemannian-Cartan space [11], however, on the earth surface the effect is insignificant and unobservable in the modern empirical works ([12]-[13])).
On the other side in the initial lagrangian we had \( A_\mu^6(y) \). Therefore pair \( D \) and \( D^* \), also in action, should be defined at one point. To solve this problem, perform the procedure of geometrization.

Let's limit by the case of “sufficiently good connected \( \Omega \)” and “sufficiently well-defined \( \mathcal{L}_\omega \).

Consider a sufficiently large region of Minkowsky space \( \mathcal{U} = T \times \mathbb{R}^3 \) where \( T \) and \( \mathbb{R}^3 \) are compacts. Split the domain \( \mathcal{U} \) in an arbitrary way to \( n \) different small compact domains \( \mathcal{U}_i \subset \mathcal{U} \) with boundary \( \partial \mathcal{U}_i \) and with inner region \( U_i = \mathcal{U}_i \setminus \partial \mathcal{U}_i \)

\[
\bigcup_i \mathcal{U}_i = \mathcal{U}, \quad U_i \cap U_j = \emptyset, \quad i, j = 1, \ldots, n. \tag{66}
\]

In the region \( U_i \) arbitrary choose the points \( A_i \) and introduce local coordinates \( \xi \):

\[
y_i = y(A_i) + \xi_i \tag{67}
\]

Then the interval between any two points from any neighborhood \( U_i \) has form

\[
ds^2 = (dy^0)^2 - (dy^1)^2 - (dy^2)^2 - (dy^3)^2 \tag{68}
\]

Introduce manifold \( \overline{M} \) and split it to \( n \) regions \( \overline{M}_i \)

\[
\bigcup_i \overline{M}_i = \overline{M}, \quad M_i \cap M_j = \emptyset, \quad i, j = 1, \ldots, n. \tag{69}
\]

Define homeomorphisms for the domains \( M_i, \Omega_i \cup U_i \)

\[
f_i : M_i \rightarrow \Omega_i \subset \mathbb{R}^4,
\]

\[
g_i : \Omega_i \rightarrow U_i, \quad p_i = f_i^{-1} \circ g_i^{-1}(A_i) \tag{70}
\]

Denote \( x = x(p) = (x_0, \ldots, x_3), p \in M, x_\mu(p) \) the coordinates of the M, which are induced by \( f_i \). Assume \( g_i, f_i \) and coordinate systems in \( U_i \) and in \( \Omega_i \) satisfy

\[
y_\mu - y_\mu(p_i) = H_{\mu}^\nu(p_i)(x_\nu - x_\nu(p_i)) \tag{71}
\]

where \( H_{\mu}^\nu \) is a diagonal matrix.

Then the quadratic form of the interval with respect to \( p_i \) from the manifold \( M_i \) has the form:

\[
ds^2 = H_0^0(dx^0)^2 - H_1^1(dx^1)^2 - H_2^2(dx^2)^2 - H_3^3(dx^3)^2 \tag{72}
\]

Write the action of the O-theory lagrangian in the form of the Riemannian integral

\[
S_\omega = \int_U \mathcal{L}_\omega d^4y = \lim_{n \rightarrow \infty} \sum_{i=1}^n \mathcal{L}_\omega(A_i) \Delta U_i, \quad \Delta U_i = \Delta y_0 \Delta y_1 \Delta y_2 \Delta y_3 \tag{73}
\]
or

\[ S_\alpha = \lim_{n \to \infty} \sum_{i=1}^{n} \Omega_{\alpha}(p_i) \sqrt{-g(p_i)} \Delta^4 x, \quad \sqrt{-g(p_i)} = H_0 H_1 H_2 H_3 \] (74)

Rewrite the spinor part of the action

\[ \Delta S'_i = \left( \frac{i}{2} \bar{\psi} \gamma^\mu (H^{-1}_\mu \partial^\mu + \frac{3i}{4} q^6 A^{\mu(6)} - iq^5 A^{\mu(5)}) e ight. \\
- (H^{-1}_\mu \partial^\mu - \frac{3i}{4} q^6 A^{\mu(6)} + iq^5 A^{\mu(5)}) \bar{\psi} \gamma_\alpha e) \\
- \frac{5}{8} (q^6 A^{\mu(6)} + iq^5 A^{\mu(5)}) \bar{\psi} \gamma e - \frac{5}{8} (q^6 A^{\mu(6)} - iq^5 A^{\mu(5)}) \bar{\psi} \gamma_\mu e) \sqrt{-g(p_i)} \Delta^4 x \] (75)

Using the definition of vector fields \( D \) and \( D^* \) from (45) we come to the following expression for (75)

\[ \Delta S'_i = \left( \frac{i}{2} \bar{\psi} \gamma^\mu (\partial^\mu + \frac{3i}{4} qD D_\mu) e - (\partial^\mu - \frac{3i}{4} qD^* D^*_\mu) \bar{\psi} \gamma e) \\
- \frac{5}{4} qD^* D^*_\mu \bar{\psi} \gamma e - \frac{5}{4} qD D_\mu \bar{\psi} \gamma_\mu e) \Delta \Omega_i, \quad \Delta \Omega_i = \sqrt{-g(p_i)} \Delta^4 x \] (76)

From now on we neglect the decay of \( D \)-bosons into leptons, and therefore excluded the second string in (76) from further consideration.

In a small neighborhood \( U_i \) of the point of Minkowsky space we consider the derivative of the spinor Dirac particle as a covariant derivative in cur ved space, written in the tetrad formalism in the metric (4), for which tetrads \( H_\mu^a(p_i) \) are chosen in such way that

\[ \Phi_2^\mu(p_i) = \frac{3}{4} q^5 A^{\mu(5)}(p_i) \] (77)

where \( \Phi_2^\mu \) is same as (18).

In the new coordinates the action (76) has the following form

\[ \Delta S'_i = \left( \frac{i}{2} \bar{\psi} \gamma^\mu (\nabla^\mu + \frac{3i}{4} qD A^\mu_6) e - (\nabla^\mu - \frac{3i}{4} qD^* A^6_\mu) \bar{\psi} \gamma e) \right) \Delta \Omega_i \] (78)

which is equal to

\[ \Delta S'_i = \left( \frac{i}{2} \bar{\psi} \gamma^\mu \nabla^\mu \gamma e - \nabla^\mu \bar{\psi} \gamma^\mu \nabla^\mu e + \frac{3i}{4} (qD^* A^6_\mu e + qD A^6_\mu) \bar{\psi} \gamma^\mu e) \right) \Delta \Omega_i \] (79)

or

\[ \Delta S'_i = \left( \frac{i}{2} \bar{\psi} \gamma^\mu \nabla^\mu \gamma e - \nabla^\mu \bar{\psi} \gamma^\mu \nabla^\mu e + \frac{3i}{4} (qD^* A^6_\mu e + qD A^6_\mu) \bar{\psi} \gamma^\mu e) \right) \Delta \Omega_i \] (80)

Since \( A^6_\mu \) is given in the only point \( p_i \) and the signs of the interaction of particles \( D, D^* \) and electron are opposite, then

\[ \Delta S'_i = \frac{i}{2} (\bar{\psi} \gamma^\mu \nabla^\mu e) \Delta \Omega_i \] (81)
Assuming that the integrand is extendable to the whole $M$ we obtain

$$S' = \int_\Omega \frac{i}{2}(\overline{\psi}\gamma^\mu \nabla_\mu \psi - \nabla_\mu \overline{\psi} \gamma^\mu \psi) \sqrt{-g} dx^4 \quad (82)$$

The written out scheme coincides to the representation of the Riemannian integral for an ordinary problem of the GRG, so the above assumptions are feasible, because they are complied for some problems. For example, such scheme is suitable for the calculation of Tolman [15] and Friedmann (88) solutions.

In addition it is necessary to take in mind the GRG provides the link between matter and geometry through Einstein equations. It turns out, the equations connecting matter and geometry exist in the Lagrangian of the O-theory. Indeed, the lagrangian of the O-theory includes an unusual member

$$L'_f = \frac{1}{4} f^{IJKL} q^{IJ} q^{KL} (A^I_a A^J_b - A^I_b A^J_a)(A^{a(K)} A^{b(L)} - A^{b(K)} A^{a(L)})$$

$$= \int f^{IJKL} A^I_a A^J_b A^{a(K)} A^{b(L)} \quad (83)$$

which sign is not defined before specifying the method of non-associativity relieving.

Small regions $\Omega_i$ differ in the following way: there are regions of $\Omega_i$, which imply the existence of a virtual pair of $D + D^\ast$-bosons, and the regions where there is no such pair. In the areas where there is a virtual pair of $D + D^\ast$ it will be assumed a virtual pair of $C \cdot E$-bosons also exists (the contribution of the minor fields $A^B_{\mu}, B = 0, 1, 2, 3$ will be neglected since their mass compared to that of major field $A^B_{\mu}, B = 4, 5, 6, 7$ is negligible quantity).

Outside of the region of the implied virtual pair of $D + D^\ast$-bosons the gravitational vacuum is not formed, hence the average value of a virtual pair $C + E$ in such region is zero.

Action of the gravitational field in GRG has the following form [6]:

$$S_g = -\frac{1}{\kappa} \int_\Omega R \sqrt{-g} d\Omega = -\frac{1}{\kappa} \int_\Omega G \sqrt{-g} d\Omega - \frac{1}{\kappa} \int_\Omega \partial_\lambda (\sqrt{-g} w^\lambda) d\Omega \quad (84)$$

where $w^\lambda$ is vector [6] and

$$L_g = -\frac{1}{\kappa} G = -\frac{1}{\kappa} g^{\mu \nu} (\Gamma^\lambda_{\sigma \nu} \Gamma^\sigma_{\mu \lambda} - \Gamma^\lambda_{\sigma \lambda} \Gamma^\sigma_{\mu \nu}) \quad (85)$$

is lagrangian of the gravitational field.

We will not make distinction between $G$ and $R$ in the integrand because the divergence terms could omitted.

From here on, we assume (the validity of this assumption is verified in the cases of the Schwarzchild and Friedmann metric)

$$-\frac{1}{\kappa} \int_\Omega G \sqrt{-g} d\Omega = f^{4567} m_D^2 \int_\Omega A^5_\mu A^{67(0)} \sqrt{-g} d\Omega \quad (86)$$

(here $f^{4567} = 0, \pm 1$) and we come to Einstein equations

$$R_{\mu \nu} = \kappa T_{\mu \nu} \quad (87)$$

where $\kappa = m_D^2$. 

14
8 Friedmann solution

In this section it is shown in the flat Friedmannian space there is a self-consistent solution of octonion Lagrangian.

Consider homogeneous and isotropic Universe

\[ ds^2 = dx(0)^2 - a^2(t)(dx(1)^2 + dx(2)^2 + dx(3)^2) = a^2(\eta)(d\eta^2 - dl^2), \]  

with conformal time \( dt = a(\eta)d\eta \) and \( (\alpha, \beta = 1, 2, 3) \)

\[ g_{00} = a^2(\eta), \quad g_{\alpha\beta} = a^2(\eta)\eta_{\alpha\beta} \]  

Use (2) to find all the non-zero components of Christoffel symbol:

\[ \Gamma^0_{00} = a', \quad \Gamma^0_{\alpha\beta} = -\frac{a'}{a^3}g_{\alpha\beta}, \quad \Gamma^\alpha_{0\beta} = \frac{a'}{a}\delta^\alpha_\beta \]  

and estimate the value

\[ G = g^{\mu\nu}(\Gamma^\lambda_{\mu\nu}\Gamma^\kappa_{\lambda\kappa} - \Gamma^\lambda_{\mu\kappa}\Gamma^\kappa_{\nu\lambda}) = \frac{6a'^2}{a^4}, \]  

where the stroke means the derivative with respect to conformal time.

Write the Dirac equation in the Friedmann metric (88)

\[ (\gamma^\mu (H^\mu))^{-1}(\partial_\mu - i\Phi_j + \frac{1}{2}\partial_\mu \left( \ln \sqrt{-g} \right) - m)\psi = 0 \]  

From (18) and (92) we get

\[ q_D A^5_\mu = (2da/(a^2d\eta), \vec{0}) \]  

which is inline with (59).

Assume

\[ f^{4567} q^{56} q^{47} A^{(4)\mu} A^{(7)}_{\mu} = -\frac{3q_D^2}{2\kappa} \]  

then Lagrangian member (34), which is non-associative by fields, could be rewritten, using (77), as

\[ f^{4567} q^{56} q^{47} A^{(5)\mu} A^{(6)} A^{(7)}_{\mu} A^{(7)}_{\nu} = \frac{q^{56} q^{47} q_D^2}{q_D} f^{4567} A^{(4)\mu} A^{(7)}_{\mu} \left( \frac{2da}{a^2d\eta} \right)^2 \]

\[ = -\frac{6}{\kappa} \left( \frac{da}{a^2d\eta} \right)^2 = -\frac{6}{\kappa} \frac{a'^2}{a^4} = -\frac{1}{\kappa} G = \mathcal{L}_g \]  

(calculated based on the minimax non-associativity relieving method) and determines the gravitational field lagrangian.

Consider the right side of (60) equations of \( C \) and \( E \) bosons.

\[ F_{\mu\nu}^{(C)} + m_4^2 E = q^{56} q^{47} \frac{4a'^2}{q_D^2 a^4} E_{\nu} \]
\[ F_{\mu\nu}^{(E)} + m_E^2 E_{\nu} = q^5 q^{47} \frac{4a^2}{q_D a^4} C_{\nu} \]  

(96)

If we assume that the mass of \( C \) and \( E \)-bosons is large and the rate of Hubble recession is small, then, at the certain choice of constant \( 4q^5 q^{47}/q_D^2 \), the right side of (96) could indeed be regarded as negligibly small.

9 Schwarzchild solution

Consider the space where there is a massive spherically symmetrical object. Let this object be the source of octonion field. On great distances from the object the electro-weak interaction could be neglected therefore the object should be the source of solely the major octonion fields. Due to symmetry of the problem we can take on the great distances all the above fields are produced by vector-potential \( A_k^r = A_k^r(r) \), \( k = 4, 5, 6, 7 \).

Let, e.g. the electron, be moving in the space. The space where the electron is moving is by definition the Minkowsky space. In spherically symmetrical coordinates the metrics in the space forms

\[ ds^2 = dt^2 - dr^2 - r^2 (\sin^2 \theta d\varphi^2 + d\theta^2) \]

(97)

Let’s limit the consideration by left spinors. Rewrite the motion equation in octonion field of massive source, assumed it does not interact with the fields \( A_k^r \)

\[ (i\gamma^0 \partial_0 + i\gamma^r (\partial_r - \hat{\Sigma} \cdot \hat{\Sigma} - 3i q^6 A_6^r - 3q^5 A_5^r) + m) \psi = 0 \]  

(98)

with denotation

\[ \gamma^r = \gamma^1 \sin \theta \cos \varphi + \gamma^2 \sin \theta \sin \varphi + \gamma^3 \cos \theta, \quad \hat{\Sigma} = \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix} \]

where \( \hat{L} = \vec{r} \times \vec{p} \) is the angular moment operator [16].

In accordance with the general scheme of geotmetrization of the O-theory we get

\[ \frac{1}{2} \partial_r \left( \ln \sqrt{-g/H^r} \right) = -\frac{3}{4} q^5 A_5^r \]  

(99)

Introduce the general form of stationary spherically symmetric metric, obtained as the product of geotmetrization of the O-theory

\[ ds^2 = H_0^2(r) dt^2 - H_1^2(r) dr^2 - r^2 (\sin^2 \theta d\varphi^2 + d\theta^2) \]

(100)

therefore

\[ -\frac{3}{4} q^5 A_5^r = \frac{1}{2} \partial_r \left( \ln \frac{\sqrt{-g}}{H^r} \right) = \frac{H_0}{2H_0} = A_5^r = (0, A_5^r, 0, 0) \]  

(101)

For a weak field it is well-known [6]

\[ H_0^2 = g_{00} = 1 - r_s/r = f^2 \]  

(102)
where \( r_g \) is gravitational radius.

So assume \( H_0 \) to be known and find \( H_1(r) \). In the first approximation, with great \( r \), assume \( H_1 = 1 + C/r^n, n > 1 \).

\[
H_1 = 1 + C/r^n = 1 + \beta(r), \quad n > 1
\]  

(103)

Find gravitational field lagrangian (76) in metrics (100)

\[
G_g = g^\mu\nu (\Gamma^\lambda_\mu_\nu - \Gamma^\lambda_\mu_\nu \Gamma^\rho_\lambda_\nu - \Gamma^\lambda_\mu_\rho \Gamma^\nu_\lambda_\rho) = \frac{2}{r^2 H_1^2} + \frac{4H_{0,r}}{rH_1^2 H_0} - \frac{H_{0,r} H_{1,r}}{H_1^3 H_0}
\]  

(104)

Since the gravitational field (76) is \( 2/r^2 \) for the flat metric (97) then the lagrangian of the gravitational field in the E-theory, as it follows from (77) and (101), is

\[
G_g = G_{pl} + C_0 q^{(5)2} A^\mu (5)_\mu = \frac{2}{r^2} + C_0 \frac{4H_{0,r}}{9H_0^2}
\]  

(105)

Given \( \alpha(r), \beta(r) \) are infinitesimal, we see that matching (104) and (105) can only be achieved provided that

\[
H_0 H_1 = 1, \quad g_{11} = (1 - r_g/r)^{-1}
\]  

(106)

Finally the spherically-symmetrical Schwarzchild metric is obtained

\[
ds^2 = (1 - \frac{r_g}{r})dt^2 - \frac{dr^2}{1 - \frac{r_g}{r}} - r^2 (\sin^2 \theta d\varphi^2 + d\theta^2)
\]  

(107)

10 Geometrization of weak interaction in the O-theory

Choose a state \( \Psi \) for \( u \) and \( d \)-quarks (the solution found by analogy to the lepton sector, but with the other hypercharge operator \( Y \))

\[
\Psi = \sum_{i=0}^{9} \alpha_i u_i + \sum_{i=0}^{9} \beta_i d_i, \quad Y_{a,d} = \left( \begin{array}{c} -\frac{1}{3} \\ 0 \\ \frac{2}{3} \end{array} \right)
\]  

(108)

here \( \alpha_i \) and \( \beta_i \) are coefficients where \( \Psi \) corresponds to the lagrangian of the weak interactions.

Similarly to the case of weak interactions we write the total lagrangian of the O-theory, neglecting the distinction between left and right particles (\( A, B = 0, 1, \ldots, 7 \)):

\[
\mathcal{L}_\alpha = \mathcal{L}_f + (\partial_a \bar{\Psi}_\varphi \gamma^a A^a_\mu \Psi_\varphi * \Sigma^a) * (\partial^a \Psi_\varphi + \frac{i}{2} q^B A^a_\mu B^a \Sigma^B * \Psi_\varphi)
\]

\[
+ \frac{i}{2} \bar{\Psi}_\varphi \gamma_a (\partial^a + \frac{i}{2} c A^a_\mu A^{(a)A} \Sigma^a) * \Psi - \frac{i}{2} \bar{\Psi}_\varphi \gamma_a (\partial^a - \frac{i}{2} c A^a_\mu A^{(a)A} \Sigma^a) * \Psi
\]
\[
-\tilde{h} \Psi^* \psi^* \bar{\Psi} - \tilde{h} \Psi^* \psi^* \bar{\Psi} + m^2 |\Psi| \psi| - \frac{f}{4} |\Psi| \psi|^4 \quad (109)
\]

Let \( \Psi = \Psi \psi_0 \) from (38). Write some members of the lagrangian

\[
\mathcal{L}_\alpha = \frac{q(5)^2 m^2}{2f} A_a^5 A^{a(5)} + \frac{q(6)^2 m^2}{2f} A_a^6 A^{a(6)} + q^6 A^a(6) (\kappa_3 \bar{\psi} \gamma_\alpha u + \kappa_4 \bar{\psi} \gamma_\alpha d)
\]

\[
+ \kappa_5 (q^6 A^a(6) + iq^5 A^a(5)) \bar{\psi} \gamma_\alpha d + \kappa_6 (q^6 A^a(6) - iq^5 A^a(5)) \bar{\psi} \gamma_\alpha u
\]

\[
+ f^{1256} q^{12} q^{56} A_a^1 A^{a(2)} A_b^5 A^{b(6)}
\quad (110)
\]

Here the group constant of weak interactions \( q^{12} = g \) could be substituted. Thus the massive charged vector bosons \( D \) and \( D^* \) induced gravity, but with another constant

\[
\kappa_S = m_s^2 = g q^{56} A_a^1 A^{a(2)} = g q^{56} m_W^2
\quad (111)
\]

If \( m_W = 100 \text{ Gev}, m_s = 1 \text{ Gev}, g = 10^{-2}, \) then \( q^{56} = 10^{-2}! \)

It appears the group constant \( SU(2) \times U(1) \) and \( q^{56} \) possibly are equal! Or they are, at least, close.

The similar formula holds for the gravity

\[
\kappa_g = m_D^2 = g q^{56} A_a^4 A^{a(7)} = 10^{-2} q^{47} m_C m_E
\quad (112)
\]

Assume the mass of major vector fields are equal to the Planck mass (this assumption is reasonable, since it is assumed that they form a coupled state), we obtain \( q^{47} = 10^2 \).

Then the solution for a stable nucleus could be regarded as a solution inside of “black hole” in the Schwarzschild metric. The invisibility of free quarks is explained as a solution inside of “black hole”, that “has no hair” therefore it is impossible to extract any information except the information of its mass, charge and its angular momentum.

In fact, the previous arguments are not completely accurate. They refer to the case where “black hole” is not charged and has no rotational degrees of freedom. That is applicable to the lower states of mesons. Proton cannot be regarded as Schwarzschild metric external solution, because it is a charged particle. So the proton description should use the Reissner-Nordstrom solution [18]. That metric is a metric of a charged “black hole” with the charge of \( Q \) and the gravitational mass of \( M \), but with the other “strong gravitational” constant of \( \kappa_S \):

\[
ds^2 = (1 - \frac{2M}{r} + \frac{Q^2}{r^2}) dt^2 - \frac{dr^2}{1 - \frac{2M}{r} + \frac{Q^2}{r^2}} - r^2 (\sin^2 \theta d\varphi^2 + d\theta^2) \quad (113)
\]

(to be completely precise we could have used the Kerr-Newman geometry, if we had taken into account the spin and rotational degrees of freedom).

Pay attention to an interesting feature of this solution, which takes place in atoms: an increase in charge with no increase in weight leads to the impossibility of the existence of “black hole”. However complementing the nucleus with
neutrons increases the mass of the nucleus and leads to increase in its gravitational radius, and as a consequence in its stability. The relation between charge and mass of stable nucleus is

\[ Q < M = \frac{r_g}{2} \]  \hspace{1cm} (114)

However we should take into account the electrical repulsion of protons inside the nucleus. If the nucleus consists of protons only then the force of electrical repulsion increases and damages the nucleus. That fact is known from nuclear physics, but the model of the O-theory is a theoretical foundation.

11 Strong interactions in the O-theory

The research above shows the problem of quarks not flying-off nucleus could be solved in the framework of the O-theory as weak interaction manifestation. At the same time the geometrical nature of the effect, and therefore taking into account lagrangian non-associativity, leads to additional difficulties during the model quantization. On the other side, given the nucleus has high energy (the energy of the same order or higher than that of charged W-boson mass) the weak interaction current values and the geometrical structure and therefore the non-associative members in the theory apparently could be neglected. So, under the listed assumptions, the field part of lagrangian appears to be associative. Assume we neglect Higgs sector and limit our consideration by basis solution \( \Psi_0 \), then the O-theory lagrangian generally becomes associative. Therefore on the octonion algebra the convolution (32) appears to be associative scalar product of type (28). So we can build up special octonion symmetry group \( G_2 \). Once \( SU(3) \in G_2 \), the colour quark group \( SU_c(3) \) could be introduced to the model using standard method, i.e. fitting coefficients \( \alpha_k, \beta_k \) in (108). In this paper the described issue is not considered.

12 Conclusion

General framework for unified theory construction was proposed in the paper. As it had been predicted, finally we obtained the expression for current with coefficients, which gave us certain freedom (e.g. we were free to variate constant \( \kappa_3 \) inside certain limits etc.). That new current expression was caused by a number of new members appeared while generalization procedure of ST lagrangian. Basically, those members consist of fields \( A^A, A = 5, 6, 7, 8 \) and those ones appeared from multiplets on spinor fields of \( \mathcal{O} \) state space construction method. Should a general research be a start point? A physical nature of something could be a partial representation of that something and whether there is no interesting solution of a general problem detected then there is no interest in the general theory itself. For that reason a matter of principle for the author was the mathematical principles existence detection, which principles prove proposed unified theory physical foundation. Detailed symmetry of the
theory and quantization procedure research, from author’s point of view, should be conditioned by theory approximative schemes and could be a subject of further articles.

During gravitation build up process there was a remarkable feature of the developing formalism. Instead of introducing the space-time metric to the theory as a corollary of the variational principle for gravitational field action in terms of $R$, it was introduced as a corollary of geometrization method. Basically, the general relativity equations as a solution on the extremals also seems to be possible in the framework of the developed formalism. Such supposition is justified because the metrics is defined by vector-potentials $A^J_\mu$, the equations for which are Euler-Lagrange equations.

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