Signatures of primordial helicity in the CMBR
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We have studied ways in which helical primordial magnetic fields could be constrained by measurements of the cosmic microwave background radiation (CMBR). If there were helical flows in the primordial plasma at the time of recombination, they would produce parity violating temperature-polarization correlations ($C_T^B$ and $C_E^B$). However, the magnitude of helical flows induced by helical magnetic fields is unobservably small. We discuss an alternate scheme for extracting the helicity of a stochastically homogeneous and isotropic primordial magnetic field using Faraday rotation measure maps of the CMBR and the power spectrum of B-type polarization ($C_{BB}$).

1. Introduction

The improving quality of the CMBR measurements has made it possible to test different models of Early Universe physics. As more refined observations are made, a larger array of theoretical ideas will be put to the test and more details of the history of the universe will emerge.

A possibility that has already received some attention is that large-scale Parity (P) violation may be observed via the CMBR \cite{1,2}. In Ref. \cite{1} it was shown that a coherent magnetic field would induce non-zero P-violating correlations in the CMBR through Faraday rotation, while in Ref. \cite{2} the P violation was due to the dynamics of a pseudoscalar field. In Ref. \cite{3} we have examined the consequence of yet another possible source of P-violation (and also CP-violation), namely large-scale primordial helical magnetic fields\cite{4}. Helical fields could be expected from cosmic events such as electroweak (EW) baryogenesis \cite{4,5}. The reason for these expectations is that within the EW model production of baryons is accompanied by a change in the Chern-Simons number (CS), which can be interpreted as the net helicity in the $SU(2)_L \times U(1)_Y$ gauge fields. Changes in the CS are achieved via the production and decay of non-perturbative field configurations, such as sphalerons and linked loops of electroweak strings \cite{6}. Because these configurations produce magnetic fields, it is possible that the helicity in the non-Abelian fields associated with the CS could be inherited by magnetic fields present immediately after the EW phase transition. It has also been argued in Ref. \cite{6} that such helical magnetic fields could remain frozen in the primordial plasma and, provided sufficiently effective inverse cascading, develop cosmologically interesting strengths at the time of recombination.

Our approach in this work was to assume existence of a stochastic homogeneous and isotropic helical magnetic field at recombination and try to answer the following three questions, treated as independent from each other: 1) Could helical magnetic fields produce helical flows (kinetic helicity) in the primordial plasma? 2) Can one constrain primordial kinetic helicity using the CMBR? 3) Can one detect helical magnetic fields using the CMBR? We provide answers in the following sections.

2. Kinetic helicity from magnetic helicity?

We are interested in the effects of a statistically homogeneous and isotropic magnetic field, with possibly non-vanishing helicity. If we denote the Fourier amplitudes of the magnetic field by $b(k)$,
then
\begin{equation}
\langle b_i(k) b_j(k') \rangle = (2\pi)^3 \delta^{(3)}(k + k') \times [\delta_{ij} - \hat{k}_i \hat{k}_j] S(k) + \varepsilon_{ijk} \hat{k}_i A(k)].
\end{equation}

Here \( S(k) \) denotes the symmetric part and \( A(k) \) the antisymmetric part of the correlator. One can check that \( \langle \mathbf{B}(x) \cdot \lbrack \nabla \times \mathbf{B}(x) \rbrack \rangle \) only depends on \( A(k) \) and not on \( S(k) \). Therefore \( A(k) \) represents the helical component of the magnetic field and \( S(k) \) the non-helical component.

In the tight-coupling approximation it is the Lorentz force that drives flows in neutral plasma. An evaluation shows that the Lorentz force depends on \( S(k) \) but has no dependence on \( A(k) \). Therefore the velocity flow at last scattering is unaffected by the helical component of the magnetic field. In reality the coupling of photons to electrons is much stronger than that to protons and so the plasma at recombination is better treated as composed of two fluids: the electron-photon fluid and the proton fluid. A calculation, presented in full in Ref. [3] and closely following that of Harrison [10], shows that the electron-photon fluid will gain an angular velocity \( \omega_e = (en_e)^{-1} \nabla^2 \mathbf{B} \), where \( e \) and \( n_e \) are the electron charge and the number density. If we estimate \( |\nabla^2 \mathbf{B}| \sim B/L^2 \), where \( L \) is the coherence scale of the field, we find \( |\mathbf{v}| \sim |L \omega_e| \sim 10^{-18} (B_0/10^{-9} \text{G}) (1 \text{ kpc}/L_0) \) where \( B_0 \) and \( L_0 \) are the magnetic field strength and coherence scale at the present epoch. Compared to the velocities induced by gravitational perturbations (\( \sim 10^{-5} \)) the velocities induced by helical fields are insignificant.

3. Signatures of kinetic helicity

The CMBR anisotropies sourced by velocity flows are predominantly due to the Doppler effect. Observations of CMBR are usually presented in the form of spectral functions \( C_l^{XY} \), where \( X \) and \( Y \) stand for \( T \) (brightness temperature), \( E \) or \( B \) (so-called \( E \)- or \( B \)-type polarization) [3]. The correlators \( C_l^{EB} \) and \( C_l^{EB} \) are parity-odd, while all other correlators \( C_l^{XY} \) are parity-even. The presence of parity-violating (helical) flows will produce nonzero \( C_l^{EB} \) and \( C_l^{EB} \).

In Ref. [3] we have shown that \( C_l^{EB} \) and \( C_l^{EB} \) can be written as integral expressions depending on the Fourier transform of the average kinetic helicity \( \langle \mathbf{v} \cdot (\nabla \times \mathbf{v}) \rangle \). We have assumed a power law \( k \)-dependence for the relevant power spectra with a spectral index \( n \), and introduced a characteristic scale \( k_{\ast} \) and a characteristic strength \( v_0 \) of the helical flow. We then evaluated \( C_l^{EB} \) and \( C_l^{EB} \) for several different values of \( n \) and \( k_{\ast} \) to see if observations could, in principle, constrain \( v_0 \). We found that for \( n > -3 \) the bound is set by the cut-off provided by \( k_{\ast} \). Only for smaller \( n \) it is possible to constrain \( v_0 \).

Causality does, in general, constrain the value of the spectral index: \( n \geq 2 \). Thus, our analysis suggests that the CMBR will not be able to constrain primordial kinetic helicity unless helical flows were correlated on superhorizon scales.

4. A strategy to detect magnetic helicity

From previous sections one concludes that only the non-helical component of the magnetic field can have a signature in the Doppler contribution to the CMBR. If we could find another observable that is sensitive to both the non-helical and the helical components, we could combine observations and extract the helical component of the magnetic field. Such an observable is the Faraday rotation of linearly polarized sources due to light propagation through a magnetized plasma. The CMBR is expected to be linearly polarized and so any intervening magnetic fields will rotate the polarization vector by an angle

\begin{equation}
\theta = \frac{3}{2\pi e} \lambda_0^2 \int \tau(x) \cdot \mathbf{B} \cdot dl
\end{equation}

where \( \tau(x) \equiv n_e \sigma_T a \) is the differential optical depth along the line of sight, \( \lambda_0 \) is the observed wavelength of the radiation and \( \mathbf{B} \equiv \mathbf{B}_0 a^2 \) is the “comoving” magnetic field.

Faraday rotation depends on the free electron density, which becomes negligible towards the end of recombination. Therefore, the bulk of the rotation is produced during a relatively brief period of time when the electron density is sufficiently low for polarization to be produced and yet sufficiently high for the Faraday rotation to occur. The average Faraday rotation (in radians) between Thomson scatterings due to a tangled
magnetic field was calculated in Ref. \[11\] to be
\[ B \approx 0.08 \left( \frac{B_0}{10^{19} \text{G}} \right) \left( \frac{\nu_0}{30 \text{GHz}} \right)^2 \], where \( B_0 \) is the current amplitude of the field and \( \nu_0 \) is the radiation frequency observed today. The amplitude of the CMB polarization fluctuations is expected to be an order of magnitude lower than that of the temperature fluctuations. As discussed in Ref. \[12\], detecting a Faraday rotation of order 1° will require a measurement which is superior in sensitivity by another factor of 10^2. Such accuracy is at the limit of current experimental proposals but there is a hope that it will eventually be accomplished.

A polarization map of the CMBR at several wavelengths will (in principle) make it possible to obtain a wavelength independent “rotation measure” (RM). The expression for RM is that of eq. (2) divided by \( \lambda_0^2 \). One could then use a “rotation measure map” to find correlations of the RM:

\[ R R' \equiv \langle \text{RM(} \hat{n} \text{)RM(} \hat{n}' \text{)} \rangle, \]

where \( \hat{n} \) and \( \hat{n}' \) are two directions on the sky. Using Eq. (1) we find

\[ R R' = \left( \frac{3}{2 \pi e} \right)^2 \int \frac{d^3 k}{(2 \pi)^3} \left[ \alpha S(k) + \beta A(k) \right], \]

where \( \alpha \) and \( \beta \) are calculable functions given explicitly in Ref. \[3\]. A crucial feature of \( R R' \) is that it depends on both the helical and non-helical spectral functions \( S(k) \) and \( A(k) \).

The polarization spectra due to Doppler effect from plasma flows induced by tangled magnetic fields have already been calculated by Seshadri and Subramanian \[13\]. They computed the correlator \( C_{1}^{BB} \), which only depends on the non-helical spectral function \( S(k) \). This is because, as discussed in the previous section, \( A(k) \) is the force-free component of the magnetic field and does not induce any velocity in the last scattering surface. Hence, if we could use \( C_{1}^{BB} \) to obtain \( S(k) \) – which would only be possible assuming some functional form (such as a power law) for \( S(k) \) since \( S(k) \) occurs within some integrals – we could insert the result in the expression for \( R R' \) given in Eq. (2). This will isolate \( A(k) \) in Eq. (1) and, with some assumptions about the functional form of \( A(k) \), the cosmic magnetic helicity can be evaluated.

5. Summary

If there is kinetic helicity at last scattering, it would imprint a signature in the cross-correlators \( C_{1}^{TB} \) and \( C_{1}^{EB} \). Kinetic helicity can be induced by helical magnetic fields but the effect is too small to be significant since the helical component of magnetic fields is force-free. Instead we have proposed another strategy for detecting the helicity of primordial magnetic fields using polarization and rotation measure maps of the CMBR.

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