Accurate Terrain Estimation for Humanoid Robot Based on Disturbance State Observer

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ABSTRACT This study proposes a novel reactive terrain estimation system for humanoid robots from the perspective of a state observer. The design process is as follows: the underfoot force disturbance is converted into an underfoot position disturbance by using an admittance system, the disturbance state observation is designed with a closed-loop observation method, and finally, the observation state is switched by using a gait planning-based state machine. This study combines a one-step-ahead prediction technique with the algebraic operation of error dynamics, and the designed observer is called a synchronized error predictive observer. The observation error dynamics are analyzed by using the robustness theory to prove that the proposed method can reduce the ultimate bounded range of the observation error and the error in terrain estimation. This study has been validated through simulation and experiment using the UBTECH–Tsinghua WALKER-1 Prototype, which can accurately estimate the terrain height difference and orientation within 0.02 m-height of ground obstacles. The designed observer can effectively improve the accuracy and further reduce the instability that the gait control system may have to withstand.

INDEX TERMS Humanoid robots, prediction methods, terrain estimation, terrain factors.

I. INTRODUCTION

UBTECH and Tsinghua University jointly developed WALKER-1, a humanoid robot for home services, and its joint actuator selects a position control scheme. Humanoid robots can be regarded as floating-based robotic arms with a bipedal walking design, suitable for various human living environments, thus making home services the highest-demand application scenario. For the robot to be stable enough to serve in home scenarios, the gait stability problem of walking on uneven terrain needs to be overcome. The control strategies for robots walking on uneven terrain can be divided into disturbance rejection-based balance control and terrain estimation-based adaptive locomotion. In the first control strategy, the underfoot force disturbance from the uneven terrain and the contact force disturbance from other parts of the body are described as the same form of disturbance, then designs the control system so that the disturbance can converge. Thus, the robot can maintain its balance. In the second control strategy, a reactive sensor is used to estimate the underfoot terrain model, or an anticipatory sensor is used to estimate the upcoming terrain model. Then performs reflexive control or modifies gait planning trajectory based on the estimated state.

Balance control includes underfoot force and other contact force disturbances, enabling the robot to overcome uneven terrain while also providing a push recovery function. Adaptive locomotion, on the other hand, is a control strategy specifically designed for the robot to adapt to uneven terrain and has a faster transient response than balance control. Therefore, the best strategy for robots walking on uneven terrain is to use adaptive locomotion first to eliminate most of the landing effects on the robot and then use the balance control to reject the rest of the disturbance. Still, the premise is that the robot must have an accurate terrain estimation.

Since the terrain estimation can be classified as reactive-based or anticipatory-based depending on the type of sensor, where the anticipatory terrain estimation inevitably involves perceptual errors, the robot should first have stable and fast reactive terrain estimation. In the following, we review the reactive terrain estimation approaches in adaptive locomotion control for each research institution.
A. STATE OF THE ART

Depending on the robot’s hardware structure, terrain estimation can be based on general or special hardware. General hardware refers to structures with humanoid robot-related features, such as series-driven joints, thigh and shin parts, flat feet, and so on. The special hardware is the structure relative to the general hardware. For example, Yamaguchi et al. proposed a foot structure with four spikes [1], [2], Sugahara et al. designed a parallel driven joint robot [3], [4], Hashimoto et al. proposed a foot structure with a cam-type locking mechanism [5]–[7], Kang et al. designed a foot structure with optical distance sensor unit [8], and Sygulla et al. designed a LOLA robot with tactile sensors on the foot structure [9], [10]. Although the customized terrain estimation can be developed through unique foot structures and sensors, most robots still use general hardware and the little applicability of special hardware. Therefore, the scope of this paper is focused on the general hardware-based terrain estimation.

Among the general hardware-based terrain estimation, two main approaches can be classified as follows. The first one uses spline interpolation to modify the motion planning of the swing leg after sensing the ground obstacle, such as the landing time control designed by Huang et al. [11], the online trajectory generator designed by Park et al. for KHR-3 [12], and the short cycle walking pattern generation method proposed by Nishiwaki and Kagami [13]–[15]. The second one uses the compliance control of swing legs to complete terrain estimation, such as the virtual shock absorber designed by Kim et al. [16], the damping control method used by Yi et al. [17], and the leg length and foot posture control based on PD controller by Joe and Oh [18].

The underfoot terrain model described by the above terrain estimation is usually represented as the terrain height difference between two feet and the terrain orientations of each foot [19], [20], as shown in Fig. 1. In the double support phase, the height difference of two feet and the posture of the foot surface needs to be described together, while in the single support phase, only the orientation of the supporting foot surface needs to be used to describe it.

B. MOTIVATION

The difficulty for position-controlled humanoid robots encountering irregular terrain is that the terrain model is unknown in advance when using reactive terrain estimation. The reactive terrain estimation will uniformly regard uphill, downhill, and pit as all positional disturbances from the ground and cannot distinguish between them. The terrain estimation is only possible when the swing foot touches the ground. Therefore, the compliant control or terrain estimation must be moderate, neither reacting too fast to cause the swing foot to bounce when it touches the ground nor responding too slowly to receive too much landing impact.

For the implementation of terrain estimation approaches, recent research has focused on the compliance control of swing legs. The design of virtual spring damping for swing legs by controlling the extension and contraction of the leg, or the rotation of foot posture, to obtain terrain estimation or directly absorb the landing impact. However, there is no research to solve the terrain estimation problem from state observer design. From the perspective of state observer design, the virtual spring damping method is just a kind of open-loop observer, which has many drawbacks in practical engineering applications. For example, disturbances or inaccurate initial state estimation could lead to deviations between the estimated state and actual values, and the observation error may become larger and larger with time. Hence, the contributions of this study are as follows.

C. CONTRIBUTION

This study designs an accurate terrain estimation for a humanoid robot walking on uneven terrain, and its contributions are the following three points.

1) A novel disturbance state observer is designed to achieve reactive terrain estimation. Compared with open-loop observation methods such as virtual shock absorber, damping control, and PD control, the disturbance state observer is a closed-loop observation method, so the characteristics of the estimator can be improved from the design method.

2) For the design of the disturbance state observer, the one-step-ahead prediction technique and the algebraic
operation of the error dynamics are combined, so the designed observer is called a synchronized error predictive observer (SE-PO). Then, the robustness theory analysis proves that the proposed method can reduce the ultimate bounded range of the observation error and make the terrain estimation error smaller.

3) This study compares the performance of different terrain estimation methods in the Webot simulation and validates the proposed synchronized error predictive observer on a WALKER-1 prototype. The designed observer can effectively improve the accuracy of state-of-the-art techniques, providing a more accurate terrain estimation for adaptive locomotion, and further reducing the instability in the gait control system.

D. ORGANIZATION

Section II introduces the terrain estimation system, including the disturbance state observation, the gait planning-based state machine, the terrain estimation system framework, and the design of a synchronized error predictive observer. Figure 2 is the general framework of this study, which is suitable for the general hardware structure of humanoid robots. Section III is a theoretical analysis of robustness, divided into two parts: derivation of robustness index for observers and stability analysis of the observers. Section IV shows the simulation studies in the Webot environment to verify the effectiveness of the synchronized error predictive observer and compare it with the state-of-the-art methods such as virtual shock absorber, damping control, and PD control. Section V shows experimental results to validate the designed terrain estimation system on the WALKER-1 prototype using ground obstacles of 1cm and 2cm-height. Finally, Section VI is conclusions and future work.

II. TERRAIN ESTIMATION SYSTEM

The terrain estimation is addressed as a state observer design problem to enable a humanoid robot to perform faster and more accurate terrain estimation for uneven terrain. The goal is to design a disturbance state observer with a faster transient response to reduce estimated errors and achieve more accurate terrain estimation. The underfoot terrain model is the terrain height difference and the terrain orientation, both of which are position descriptions, and it is chosen to convert the underfoot force disturbance into an underfoot position disturbance using admittance systems.

Admittance control is a widely used technique in robot-environment interaction. First proposed by Hogan as a position-based impedance control concept [21]–[23], for which Ott, Keemink et al. have done detailed reviews [24], [25], and has been applied in many studies to control humanoid robot foot-environment contact as well. In this study, the admittance system is used as a reference model for disturbance state observation. In addition, the robot legs are modeled as an impedance system to design a synchronized error predictive observer. Since the design principle of terrain height difference is the same as disturbance state observation in terrain orientation, the following is an example of terrain height difference estimation.

A. DISTURBANCE STATE OBSERVATION

The reference model of the disturbance state observation was designed as an admittance system located at the feet, and the purpose of which was to transform the underfoot force disturbance into the observation states $\Delta H_l$ and $\Delta H_r$ of the underfoot position disturbance. The left and right legs of the robot were symmetrically designed; therefore, the following equations and pictures do not distinguish between the left and right legs and are expressed as the observation state $\Delta H_l$.

The admittance system model was developed: let the center of mass (CoM) of the ankle be denoted by $m$, and the masses of the thigh and shin parts be neglected. Fig. 3(a) depicts the admittance system. Where, $Z^R$ is the reference position of the foot center; $\dot{Z}^A_l$ is the output state of the admittance system. $F^U_{l/z}$ is the input force of the admittance system; $K_{adm}$ and $C_{adm}$ are the virtual spring damping, whose parameter configurations are related to the landing speed of the swing leg planning and must be obtained after parameter tuning.

When a Kalman filter is added, a phase delay occurs due to the noisy nature of the force sensor data, which can be considered as a first-order inertial system $(1 + Ts)^{-1}$ after an equivalent approximation. To consider the effect of this phase delay, the input force $F^U_{l/z}$ of the admittance system is designed for phase-lead compensation, such that

![Figure 2. Overall framework. (The light-gray and light-red blocks represent the joint space and the operational space respectively.)](image-url)
the dynamics equations of the admittance system can be presented as

\[
\begin{align*}
\dot{z}^\text{Adm} + C_{\text{Adm}} \dddot{z}^\text{Adm} + K_{\text{Adm}} \ddot{z}^\text{Adm} &= \hat{F}^\text{U}_z \\
\dot{\hat{F}}^\text{U}_z &= K_p \hat{F}^\text{M}_z + K_v \hat{F}^\text{M}_z
\end{align*}
\]

(1) (2)

where, \( \hat{F}^\text{M}_z \) is the measured data of the force sensor; \( \hat{F}^\text{M}_z \) is obtained by numerical differentiation; the ratio of \( K_p \) and \( K_v \) is 1 to \( T \); and \( T \) is the delay time caused by the Kalman filter.

The disturbance observation of the admittance system was an open-loop observation. To improve the accuracy and the dynamic response of the terrain estimation system, we chose to design a predictive observation. The measured state is predicted as a feedback state. A dynamic relationship exists between the input position of the impedance system (i.e., the observed state that needs to be reconstructed). Thus, the impedance system dynamics is given as follows:

\[
\begin{align*}
m\ddot{z}^\text{Imp} &= \dot{\hat{F}}^\text{Imp} - mg \\
\dot{\hat{F}}^\text{Imp} &= K_{\text{Imp}} (\dddot{z}^\text{Imp} - \Delta \dot{H}_t) + C_{\text{Imp}} \dddot{z}^\text{Imp}
\end{align*}
\]

(3) (4)

After establishing the disturbance state observation model of the terrain estimation system, it is necessary to distinguish between the underfoot reference force and the underfoot force disturbance to make the correct terrain estimation. The underfoot force disturbance is defined as the difference between the measurement and reference forces. A gait planning-based state machine is designed to simplify the reference force determination. The principle is to determine the foot state when the reference force is 0 according to gait planning. During this time, the measured force of the foot is the underfoot force disturbance (Fig. 4).

The robot is in a standby state before it starts walking and after it finishes walking. When starting to walk, the robot first enters the double support phase (DSP) of standing in place, where the swing foot and support foot observers are set to the previous state value. When the gait planning enters from the DSP to the single support phase (SSP), the swing leg is raised in the air; the swing leg observer is set to open, and the support leg observer is set to hold in case of early contact with the ground. When the gait planning returns from the SSP to the DSP, it requires the force sensor to determine if the swing foot has touched the ground (TG). If it has, the gait planning returns to the DSP; otherwise, it will enter the extended SSP and set both the swing foot and support foot observers to open until the swing foot touches the ground and return to the DSP. From the admittance and impedance systems and state machines introduced above, the terrain estimation system in the overall framework (Fig. 2) can be expanded, as shown in Fig. 5.

**B. SYNCHRONIZED ERROR PREDICTIVE OBSERVER**

This study combines the one-step-ahead prediction technique [26] with the algebraic operation of error dynamics [27], [28], so the designed observer is called a synchronized error predictive observer, and it’s working principle is briefly described as follows. First, the reference dynamics of the disturbance state observation is obtained using the admittance system. Next, the actual dynamics of the disturbance state observation is obtained using the impedance system. To asymptotically stabilize the observation error dynamics between the reference and actual dynamics, the feedforward observer is designed such that the predicted disturbance dynamics at frame \( k + 1 \) is eliminated in advance at frame \( k \), and the feedback observer is designed to control the convergence rate of the observation errors, such that the observation state \( \Delta \dot{H}_t \) of the underfoot position disturbance can be calculated. The state-space representation of the admittance system is given by (1) and (2) as follows:

\[
\begin{align*}
\begin{bmatrix}
\dddot{z}^\text{Adm}\left(k+1\right) \\
\dddot{z}^\text{Adm}\left(k+1\right)
\end{bmatrix} = \\
A_{\text{Adm}} \begin{bmatrix}
\dot{z}^\text{Adm}\left(k\right) \\
\dot{z}^\text{Adm}\left(k\right)
\end{bmatrix} + B_{\text{Adm}} \begin{bmatrix}
\dddot{z}^\text{Adm}\left(k\right) \\
\dddot{z}^\text{Adm}\left(k\right)
\end{bmatrix} + \begin{bmatrix}
\hat{F}^\text{U}_z\left(k\right) \\
\hat{F}^\text{U}_z\left(k\right)
\end{bmatrix}
\end{align*}
\]

(5)
In addition, the state space representation of the impedance system is given by (3) and (4) as

\[
\begin{bmatrix}
\dot{Z}_f^M(k+1) \\
\dot{Z}_g^M(k+1)
\end{bmatrix} = \begin{bmatrix}
A_{\text{imp11}} & A_{\text{imp12}} \\
A_{\text{imp21}} & A_{\text{imp22}}
\end{bmatrix}
\begin{bmatrix}
Z_f^M(k) \\
Z_g^M(k)
\end{bmatrix}
+ \begin{bmatrix}
B_{\text{imp11}} \\
B_{\text{imp21}}
\end{bmatrix} \Delta \hat{H}_f(k) - \begin{bmatrix}
B_{\text{g11}} \\
B_{\text{g21}}
\end{bmatrix} \sigma
\] (6)

The observation error state $\Delta \hat{H}_f(k)$ can be obtained after subtracting the output state $Z_f^M$ of the impedance system from the output state $\hat{Z}_f^M$ of the admittance system, where frames $k$ and $k+1$, $\hat{Z}_f^M(k)$, and $\hat{Z}_f^M(k+1)$ are defined as follows:

\[
\hat{Z}_f^M(k) = Z_f^M(k) - \hat{Z}_f^M(k)
\] (7)

\[
\hat{Z}_f^M(k+1) = Z_f^M(k+1) - \hat{Z}_f^M(k+1)
\] (8)

Substituting (5) and (6) into (8) yields the following expression for the observation error dynamics.

\[
\dot{Z}_f^M(k+1) = A_{\text{adm}} \hat{Z}_f^M(k) + B_{\text{adm}} \hat{\hat{F}}_{1z}(k) + B_{\text{g}} \sigma
- A_{\text{imp}} \hat{Z}_f^M(k) - B_{\text{imp}} \Delta \hat{H}_f(k)
\] (9)

After substituting (7) into (9), the observation error can be expressed as follows into two different forms through a mathematical algebraic operation:

\[
\dot{Z}_f^M(k+1) = A_{\text{adm}} \hat{Z}_f^M(k) + (A_{\text{adm}} - A_{\text{imp}}) \hat{Z}_f^M(k)
+ B_{\text{adm}} \hat{\hat{F}}_{1z}(k) + B_{\text{g}} \sigma - B_{\text{imp}} \Delta \hat{H}_f(k)
\] (10)

\[
\dot{Z}_f^M(k+1) = A_{\text{imp}} \hat{Z}_f^M(k) + (A_{\text{adm}} - A_{\text{imp}}) \hat{Z}_f^M(k)
+ B_{\text{adm}} \hat{\hat{F}}_{1z}(k) + B_{\text{g}} \sigma - B_{\text{imp}} \Delta \hat{H}_f(k)
\] (11)

Equations (10) and (11) are called type I and II synchronized errors, respectively. The difference between them is that the observation error system matrices $A_{\text{adm}}$ and $A_{\text{imp}}$ and the system states $\hat{Z}_f^M(k)$ and $\hat{Z}_g^M(k)$ are different. The convergence rate of the type I synchronized error can be changed by selecting the admittance system parameters; therefore, for the range of the admittance system parameters to be chosen arbitrarily without restriction, the type II synchronized error is represented by (11) in this study.

The observer design is divided into two parts as follows: disturbance cancellation and output regulation. In (11), $(A_{\text{adm}} - A_{\text{imp}}) \hat{Z}_f^M(k), B_{\text{adm}} \hat{\hat{F}}_{1z}(k)$, and $B_{\text{g}} \sigma$ are all predicted disturbance terms in frame $k+1$. The feedforward observer is used to eliminate these disturbances in frame $k$. The rest of the output regulation can be made by using the feedback observer to guarantee the closed-loop system stability. Accordingly, the synchronized error predictive observer is designed as follows:

\[
\Delta \hat{H}_f(k) = K_k \Delta \hat{Z}_f^M(k) + K_A \hat{Z}_f^M(k) + K_F \hat{\hat{F}}_{1z}(k) + K_g \sigma
\] (12)

where, $K_k$ is the feedback observation gain matrix, and $K_A$, $K_F$, and $K_g$ are the feedforward observation gain matrices calculated as follows:

\[
K_A = B_{\text{imp}}^{-1} (A_{\text{adm}} - A_{\text{imp}})
\] (13)

\[
K_F = B_{\text{imp}}^{-1} B_{\text{adm}}
\] (14)

\[
K_g = B_{\text{imp}}^{-1} B_{\text{g}}
\] (15)

The observation gain matrices of (13), (14), and (15) can be directly calculated if the parameters of the admittance and impedance systems are known. However, the observation error is not entirely eliminated in the disturbance cancellation process because $B_{\text{imp}}^{-1}$ is computed via the pseudo-inverse matrix; thus, after substituting (12) into (11), the observation error dynamics is reduced to

\[
\hat{Z}_f^M(k+1) = (A_{\text{imp}} - B_{\text{imp}} K_k) \hat{Z}_f^M(k) + \sigma
\] (16)
where, \( w(k) \) is the remaining error state of the disturbance
cancellation, which can be regarded as the disturbance signal
of the observer. Finally, the pole placement method can be
used to design the feedback observation gain matrix \( K_e \).

III. THEORETICAL ANALYSIS OF ROBUSTNESS

The performance of the synchronized error predictive
observer and other state-of-the-art methods for terrain esti-
lation will be related to the variation of uneven terrain and
the selection of parameters. To compare the different methods
mathematically, the Lyapunov second method for the linear
time-invariant discrete-time system performs a theoretical
analysis of robustness. First, deriving the robustness index
of the observer from the observation error dynamics, then
the stability analysis of the observer is performed to find the
mathematical expression of the ultimate bounded range of
the observation error. A virtual shock absorber is compared
with a synchronized error predictive observer to prove that the
proposed method in this study is more robust and can obtain
more accurate terrain estimation.

A. DERIVATION OF ROBUSTNESS INDEX FOR OBSERVERS

Consider the dynamics of the observation error using a syn-
chronized error predictive observer as in (16), and use the
Lyapunov second method to consider the energy variation of
the system such that the Lyapunov function is

\[
V \left[ \hat{Z}_e^2(k) \right] = \left( \hat{Z}_e^2(k) \right)^T P \hat{Z}_e^2(k)
\]

where \( P = P^T > 0 \). The first-order difference of the
Lyapunov function to time can be obtained as follows

\[
\Delta V \left[ \hat{Z}_e^2(k) \right] = V \left[ \hat{Z}_e^2(k+1) \right] - V \left[ \hat{Z}_e^2(k) \right]
\]

The Lyapunov difference function is obtained by substituting
the observation error dynamics (16) into (18) as

\[
\Delta V \left[ \hat{Z}_e^2(k) \right] = \left( \hat{Z}_e^2(k) \right)^T \left( A_{imp}^T + K_e^T B_{imp}^T \right) P \left( A_{imp} + B_{imp} K_e \right) \hat{Z}_e^2(k)
+ \hat{Z}_e^2(k)^T P \hat{Z}_e^2(k) + \hat{Z}_e^2(k)^T A_{imp}^T P w(k) + w(k)^T P w(k)
\]

The two lemmas are introduced to organize (19) further to
find the upper bound of the Lyapunov difference function.

Lemma 1: Let \( R \) be any positive definite matrix, for any
matrix (or vector) \( X \) and \( Y \) with appropriate dimensions,
the following inequality holds:

\[
X^T Y + Y^T X \leq X^T R^{-1} X + Y^T R Y
\]

Lemma 2 (Rayleigh Theorem): Let \( A \in \mathbb{C}^{n \times n} \) be a Hermitian
matrix with eigenvalues \( \lambda_{\min} = \lambda_1 \leq \ldots \leq \lambda_n = \lambda_{\max} \). For all vectors \( x \) that is not zero, the following inequality holds:

\[
\lambda_{\min} \leq \frac{x^H A x}{x^H x} \leq \lambda_{\max}
\]

Using Lemma 1, let \( X = PA_{imp} \hat{Z}_e^2(k) \), \( Y = w(k) \), \( R = \rho^2 I \)
be substituted into (20) to obtain the inequality as

\[
\left( \hat{Z}_e^2(k) \right)^T A_{imp}^T P w(k) + w(k)^T \left( P A_{imp} \hat{Z}_e^2(k) \right)
\leq \frac{1}{\rho^2} \left( \hat{Z}_e^2(k) \right)^T A_{imp}^T P P A_{imp} \hat{Z}_e^2(k) + \rho^2 w(k)^T w(k)
\]

In addition, let \( X = PB_{imp} K_e \hat{Z}_e^2(k) \), \( Y = w(k) \), \( R = \rho^2 I \)
be substituted into (20) to obtain the inequality as follows.

\[
\left( \hat{Z}_e^2(k) \right)^T K_e^T B_{imp}^T P w(k) + w(k)^T \left( PB_{imp} K_e \hat{Z}_e^2(k) \right)
\leq \frac{1}{\rho^2} \left( \hat{Z}_e^2(k) \right)^T K_e^T B_{imp}^T P P A_{imp} \hat{Z}_e^2(k) + \rho^2 w(k)^T w(k)
\]

Using Lemma 2, let \( X = w(k) \), \( A = P \), and \( c_P \) be the
maximum eigenvalues of the \( P \) matrix, and substitute into
(21) to obtain the inequality as

\[
w(k)^T P w(k) \leq c_P \rho^2 w(k)^T w(k)
\]

A new upper bound is obtained by substituting inequalities
(22), (23), and (24) back into (19).

\[
\Delta V \left[ \hat{Z}_e^2(k) \right] = V \left[ \hat{Z}_e^2(k+1) \right] - V \left[ \hat{Z}_e^2(k) \right]
\]

\[
\leq \left( \hat{Z}_e^2(k) \right)^T \left( \left( \frac{A_{imp}^T + K_e^T B_{imp}^T}{\rho^2} \right) P \left( A_{imp} + B_{imp} K_e \right) \right) \hat{Z}_e^2(k)
+ \left( 2 \rho^2 + c_P \right) w(k)^T w(k)
\]

If there exists a positive definite matrix \( P \) that can satisfy
the following Riccati-like inequality

\[
\left( \frac{A_{imp}^T + K_e^T B_{imp}^T}{\rho^2} P \left( A_{imp} + B_{imp} K_e \right) \right) + \left( \frac{K_e^T B_{imp}^T P B_{imp} K_e}{\rho^2} + Q \right) < 0
\]

where \( Q \) is a given, arbitrarily selectable weighting matrix, let
\( Q = I \) be the identity matrix for computational convenience.
A new upper bound for the Lyapunov difference function can
be obtained again by substituting (26) into (25)

\[
\Delta V \left[ \hat{Z}_e^2(k) \right] < - \left( \hat{Z}_e^2(k) \right)^T \hat{Z}_e^2(k) + \left( 2 \rho^2 + c_P \right) w(k)^T w(k)
\]

Summing the inequality (27) for the difference function,
from \( k = 0 \) to \( k = t_f - 1 \), we get

\[
V \left[ \hat{Z}_e^2(t_f) \right] - V \left[ \hat{Z}_e^2(0) \right] = \sum_{k=0}^{t_f-1} \left( \hat{Z}_e^2(k) \right)^T \hat{Z}_e^2(k) + \left( 2 \rho^2 + c_P \right) \sum_{k=0}^{t_f-1} w(k)^T w(k)
\]
The initial and terminal conditions for the observation error energy are

\[ V\left[ \hat{Z}_f^e(0) \right] = \left( \hat{Z}_f^e(0) \right)^T P \hat{Z}_f^e(0) = 0 \]  
(29)

\[ V\left[ \hat{Z}_f^e(T_f) \right] \geq 0 \]  
(30)

Finally, the robustness index of the observer is obtained by substituting (29) and (30) into (28)

\[ \sum_{k=0}^{T_f-1} \left( \left( \hat{Z}_f^e(k) \right)^T \hat{Z}_f^e(k) \right) \leq 2 \rho^2 + c_p \]  
(31)

The left-hand side of this inequality is the ratio of the energy of the error and disturbance states, which allows evaluating the ability of the system to withstand the external disturbances. The robustness index of the observer, $2 \rho^2 + c_p$, is a value between 0 and 1 and represents the degree of influence of the disturbance state on the system.

### B. STABILITY ANALYSIS OF THE OBSERVERS

From the simulation study in Section IV and the experimental results in Section V, the observation errors of the proposed synchronized error predictive observer and the state-of-the-art methods were all can converge. Only the observation errors may be different. The observation system must be Lyapunov stable, i.e., uniformly ultimate bounded (UUB). Therefore, the mathematical expression for the ultimate bounded range of the observation error can be obtained analytically from the derivation of the robustness index of the observer.

Since the disturbance state $w(k)$ in (31) is an unknown and time-varying signal, in order to be able to quantify the analysis, the supremum of the disturbance state energy is assumed to be $w_{bd}$ and the relationship between $w(k)$ and $w_{bd}$ is expressed in 2-norm as follows.

\[ \|w(k)\| \leq w_{bd} \]  
(32)

An upper bound for the Lyapunov difference function is obtained by substituting (32) into (27)

\[ \Delta V\left[ \hat{Z}_f^e(k) \right] < - \left( \hat{Z}_f^e(k) \right)^T \hat{Z}_f^e(k) + \left( 2 \rho^2 + c_p \right) w_{bd}^2 \]  
(33)

Because the system is UUB stable, its Lyapunov’s difference function must be negative, as follows

\[ \left( \hat{Z}_f^e(k) \right)^T \hat{Z}_f^e(k) + \left( 2 \rho^2 + c_p \right) w_{bd}^2 < 0 \]  
(34)

Then the ultimate bounded range of the observation error can be obtained as

\[ \left\| \hat{Z}_f^e(k) \right\| = \sqrt{2 \rho^2 + c_p} w_{bd} \]  
(35)

Next, the virtual shock absorber is used as an example to compare the ultimate bounded range with the synchronized error predictive observer. The corresponding mathematical expressions for the observer formula, the observation error dynamics, and the disturbance state are as follows. Since the output of the virtual shock absorber is the output using the admittance control, its observer formula is

\[ \Delta \hat{H}_1(k) = \hat{Z}_f^A(k) \]  
(36)

Substituting (36) into the observation error dynamics (11), it can be organized as follows.

\[ \hat{Z}_f^e(k+1) = A_{\text{imp}} \hat{Z}_f^e(k) + (A_{\text{adm}} - A_{\text{imp}} - B_{\text{imp}}) \hat{Z}_f^A(k) + B_{\text{adm}} \hat{F}_{UZ}(k) + B_g g + w(k) \]  
(37)

and further simplified to express as

\[ \hat{Z}_f^e(k+1) = A_{\text{imp}} \hat{Z}_f^e(k) + w_1(k) \]  
(38)

where the disturbance state $w_1(k)$ is

\[ w_1(k) = (A_{\text{adm}} - A_{\text{imp}} - B_{\text{imp}}) \hat{Z}_f^A(k) + B_{\text{adm}} \hat{F}_{UZ}(k) + B_g g + w(k) \]  
(39)

Also, consider the synchronized error predictive observer as (12), and represent its observation error dynamics as

\[ \hat{Z}_f^e(k+1) = (A_{\text{imp}} + B_{\text{imp}} \text{K}_c) \hat{Z}_f^e(k) + w_2(k) \]  
(40)

where the disturbance state $w_2(k)$ is

\[ w_2(k) = w(k) \]  
(41)

From the above comparison, the virtual shock absorber, which directly uses the output of the admittance control as the observation state, is substituted into the observation error dynamics to obtain the disturbance state $w_1(k)$. The synchronized error predictive observer, on the other hand, eliminates the disturbance term of the ground reaction force in the observation error dynamics, which yields the disturbance state $w_2(k)$. Again, the supremum of the disturbance state energy is assumed to be $w_{bd1}$, $w_{bd2}$ respectively, denoted as

\[ \|w_1(k)\| \leq w_{bd1} \]  
(42)

\[ \|w_2(k)\| \leq w_{bd2} \]  
(43)

and from (39) and (41), it follows that

\[ w_{bd2} < w_{bd1} \]  
(44)

According to (35), (42), (43) and (44), the ultimate bounded range of the observation error of the virtual shock absorber is more related to the magnitude of the ground reaction force disturbance, which makes the robot’s observation error affected by obstacles of different heights in the process of stepping on the ground obstacle. On the other hand, the synchronized error predictive observer maximizes eliminating the disturbance term of ground reaction force by one-step-ahead prediction in the error dynamics, which means the ultimate bounded range of observation error can be maximally unaffected by the change of ground reaction force. Therefore, it mathematically proves that the synchronized error predictive observer is more robust and can obtain more accurate terrain estimation.
IV. SIMULATION STUDIES

The terrain estimation system is first validated in a Webot simulation environment. To demonstrate that the proposed synchronized error predictive observer is a successful terrain estimation method, the virtual shock absorber, damping control, PD control, etc., of the state-of-the-art methods, are compared in the simulation. The parameters used were first determined on the experimental prototype and then used directly in the simulation for analysis and comparison, so the parameter values are shown in Table 1.

The simulation test steps are as follows. Three different heights of test boards, such as 0.01m, 0.015m, and 0.02m, are prepared and placed under the feet as ground obstacles of unknown height during in situ stepping. During the test, the observation state $\Delta \hat{H}_f$ of the underfoot position disturbance is used as the control input of the swing leg length to modify the desired output heights $z_{Dl}^f$ and $z_{Dr}^f$, as below:

$$z_{Dl}^f(k) = z_{Rl}^f(k) + \Delta \hat{H}_f(k)$$  \hspace{1cm} (45)
$$z_{Dr}^f(k) = z_{Rr}^f(k) + \Delta \hat{H}_r(k)$$  \hspace{1cm} (46)

where $z_{Rl}^f$, $z_{Rr}^f$, $z_{Rr}^f$ are the reference positions of the leg lengths in the gait planning. If the disturbance state observation is accurate, the desired output height of the swing leg will be the same as the ground undulation height, enabling the robot to contract the leg length to adapt to different ground heights during in situ stepping.

The simulation test scheme is divided into two parts. The first part is to independently verify the accuracy of the terrain height difference state and compare the performance of different observers through the full contact between the feet and the ground obstacles during in situ stepping. In the second part, the two best-performing observers are selected for comparison. The terrain height difference and orientation states are verified simultaneously through the partial contact between the feet and the ground obstacles during in situ stepping. Then the accuracy of the observer is evaluated by checking the balanced state of the robot.

First, since there is little difference in the simulation video screenshots using different observers, the first part of all simulation test procedures is represented in Fig. 6. Figures 7(a)–(d) show the simulation data using virtual shock absorber, damping control, PD control and synchronized error predictive observer in order, where the light-yellow bar,
FIGURE 8. Statistical analysis: compare the accuracy of different terrain estimation methods. (a) Observation means and standard deviations. (b) Relative errors and relative standard deviations.

light-orange bar and light-red bar represent the ground obstacle height of 0.01m, 0.015m and 0.02m used, respectively, which is convenient to compare with the observation data.

The above simulation process is repeated for ten sets of tests to obtain more informative statistics. The observation means and standard deviations of several terrain estimation methods were obtained under three different obstacle heights tests such as 0.01m, 0.015m, and 0.02m, as shown in Fig. 8(a). Then, the relative error values and standard deviations of several terrain estimation methods were obtained with different obstacle heights as the standard in Fig. 8(b). From the data analysis and error analysis in Fig. 8, the accuracy of the virtual shock absorber, damping control, and PD control methods are easily affected by the different heights of ground obstacles. In contrast, the synchronized error predictive observer has better robustness, and their relative errors all remain below 20%.

The second part of the simulation test is shown in Fig. 9, where the virtual shock absorber and the synchronized error predictive observer are selected for comparison. A ground obstacle of 0.015m-height is used for both tests, such that the landing foot made partial contact with the ground obstacle at the same position during in situ stepping. In this part, besides the terrain height observer, the terrain orientation observer was also used to make the foot posture adapt to the terrain.

As seen in the simulation screenshots of the control group in Fig. 9(a), the virtual shock absorber is susceptible to different heights of ground obstacles, which, together with the observation error in the terrain orientation, causes the robot to lose its balance and fall during in situ stepping. The screenshots of the experimental group in Fig. 9(b), on the other hand, show that the proposed synchronized error predictive observer does have better robustness and enables the robot to maintain its balance under the same conditions.
V. EXPERIMENTAL RESULTS

The experimental platform used in this study is the UBTECH-Tsinghua WALKER-1 prototype, as shown in Fig. 10. WALKER-1 has a total of 22 degrees of freedom (DoF), including 6 DoF for the left and right legs, 1 DoF for the waist, 3 DoF for the left and right arms, and 3 DoF for the head, and the overall robot is a position control scheme. For the sensor part, magnetic encoders are used for each joint, a 9-axis inertial measurement unit at the waist, and 6-axis force/torque sensors for each left and right ankle. Since the test method in this study is to make the robot perform in situ stepping, no waist, arm, or head movements are involved, so these joints are fixed during the experimental test.

In the terrain estimation system, the parameter values of the admittance and impedance systems were obtained through experimental measurements, estimation, and parameter tuning, as shown in Table 1. Only the stiffness and damping of the admittance system are slightly different from those used in the simulation studies, while the other parameters are the same as those used in the simulation studies.

The experimental test procedure is the same as that in the simulation studies. Flat ground is chosen as the default test environment, and ground obstacles of different heights are designed and placed randomly under the robot’s feet during the experiment. If the robot can adapt to the randomly placed blocks and step stably, the terrain estimation system passes the experimental validation. Since the performance of the control and experimental groups have been compared in the simulation studies, only the use of the proposed synchronized error predictive observer is validated in the experiments.

The experimental test scheme is divided into two parts, as in the simulation studies. The first part is to verify the accuracy of the terrain height difference state independently, using a 0.01m high yellow board and a 0.02m high red board to test the full contact between the landing foot and the ground obstacle, as shown in the snapshots in Fig. 11. In the experimental data of $\Delta H_l$ and $\Delta H_r$ in Fig. 12, the light-yellow bar and light-red bar represent the robot stepping on the 0.01m high yellow board and the 0.02m high red board.

From the experimental data, the observation states are not the same for both test boards every time, and the average error is 0.0012m, which is due to the error caused by the joint flexibility at the moment. Still, it does not affect the performance of the robot in situ stepping in the experiment.

Furthermore, in Fig. 12, there are many observations of underfoot position disturbance other than the light-red area and the light-yellow area because when the robot enters the single support phase from the double support phase, the swing leg just leaves the ground. The value of the force sensor is being reset to zero at this time, which is why the observer considers the underfoot force disturbance to enter the system. Still, this observation error does not affect the robot’s performance in situ stepping in the experiment either.

In the second part, to verify the estimation accuracy of both the terrain height difference and the terrain orientation states, a 0.01m high yellow rod and a 0.02m high red rod
were used to test the partial contact between the landing foot and the ground obstacle, as shown in the experimental snapshots in Fig. 13. The robot can maintain its balance in situ stepping without using other balance control strategies in the experimental results.

After the above experimental validation, the disturbance state observation designed in Section II-A can be sufficiently demonstrated. The synchronized error predictive observer in Section II-B can make the observation system response keep up with the stepping frequency of the robot under the gait cycle of 1.2s. Thus, the terrain estimation can reflect the observation states such as ground height difference and ground orientation of the uneven terrain during walking.

VI. CONCLUSION AND FUTURE WORKS

This study addresses the problem of reactive terrain estimation from the perspective of state observer design. The one-step prediction technique and the algebraic operation of error dynamics are combined to propose a synchronized error predictive observer to design a more accurate terrain estimation for humanoid robots in adaptive locomotion than the state-of-the-art methods.

The robustness index is derived from the observation error dynamics by using the robustness theory analysis. Then, the ultimate bounded range of the observation error to mathematically prove that the synchronized error predictive observer proposed in this study is more robust and can obtain more accurate terrain estimation.

In the simulation studies, the state-of-the-art methods are compared with the synchronized error predictive observer to verify that the novel terrain estimation can be maximally unaffected to the different heights of ground obstacles. Finally, the prototype experiments achieved full and partial contact tests between in situ stepping and ground obstacles without other balance control. The experimental results were consistent with the simulation of attaining accurate terrain estimation based on the disturbance state observer.

In future work, this more accurate terrain estimation can be used to design adaptive locomotion and balance control further to reduce the instability that the original gait control system needs to withstand to improve the balance performance of the robot during walking on uneven terrain.

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