Mach's Principle in the Acoustic World

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Abstract

The aim of this paper is to investigate the coupled oscillations of multiple bubbles within a cluster. The interaction between a bubble with the rest of the bubbles within a cluster yields an extra mass. For identical uniformly distributed bubbles, the induced extra mass is approximately equal to the virtual mass of the bubble under conditions similar to those imposed by Sciama according to the Mach's Principle.

Keyword: the acoustic world, bubbles cluster, Mach's Principle

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1. Introduction

Previous study of phenomena induced by acoustic waves in a fluid revealed an analogy between the acoustic world and the electromagnetic world \cite{1 and the cited works}. Acoustic waves transport energy and mass \cite{2} and generate wave packets and bubbles \cite{3}. Consequently, this yields a rise of the fluid’s inhomogeneities and, subsequently, a shift of the refractive index of the fluid. The effect of fluid non-homogeneity in the presence of a wave packet and a bubble is the deviation of an acoustic plane wave \cite{1, 4, 5}. The oscillating bubbles interact through the secondary Bjerknes forces \cite{6}. These forces are analogous to the electrostatic forces for phase difference dependent components \cite{7} and are analogous to gravitational forces for components independent of phase difference \cite{8}. The secondary Bjerknes forces are responsible for generating the clusters of bubbles \cite{9}.

As we will show in the following, some interesting effects can occur when the bubbles within a cluster oscillate in a coupled manner \cite{10-12}. One of the consequences is a growth in virtual mass of the oscillating bubble when \( N \) identical bubbles oscillate. This additional mass depends on the parameters of the bubble and of the liquid (bubble radius \( R_0 \) and the volume density of the fluid \( \rho \) ) and on the parameters of the cluster (cluster radius \( R_c \) and density of bubbles \( n_c \) or the average distance between the bubbles \( d_c = n_c^{-1/3} \)).

When the number of the bubbles in the cluster increases, this additional mass becomes significantly larger than that of the virtual mass of the bubble performing a radial oscillation. This occurrence is analogue to the dependence a particle’s mass to the parameters of the Electromagnetic World (i.e. the radius of the universe \( R_U \) and the particle density \( n_U \) or the average distance between particles \( d_U = n_U^{-1/3} \)) according to the Mach's Principle \cite{14, 15}.

In the second section of this paper we will derive the additional mass, mentioned above, when the bubbles within a cluster perform coupled oscillation.

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In the third section we will point out that the additional mass can induce certain effects to electrostatic and gravitational types of acoustic interactions.

Finally, some conclusions are listed in the last section.

2. Dynamics of coupled bubbles within a cluster

2.1. The issue of the interaction of N bubbles

The dynamics of interacting bubbles within a cluster can be described with the extended Rayleigh-Plesset equation (see e.g. [10-12]):

\[
R_i \ddot{R}_i + \frac{3}{2} \left(1 - \frac{\dot{R}_i}{3u}\right) \dot{R}_i^2 = \frac{1}{\rho} \left[ p_i^\infty + \frac{2\sigma}{R_i} - p_i \right] \left( \frac{R_{ui}}{R_i} \right)^3 - \frac{2\sigma}{R_i} + p_i - \frac{4\mu \dot{R}_i}{R_i} - \left( p_i^\infty - p_i^\infty \sin \omega t \right) - \sum_{j \neq i} \frac{R_j^2 \ddot{R}_j + 2R_j \dot{R}_j^2}{r_{ij}}
\]

where

\[
p_i = \rho \sum_{j \neq i} \frac{R_j^2 \ddot{R}_j + 2R_j \dot{R}_j^2}{r_{ij}} \equiv \rho \sum_{j \neq i} \frac{R_j^2 \ddot{R}_j}{r_{ij}}
\]

the the pressure resulting from the radiation scattered and absorbed by the other bubbles in the clusters, \( r_{ij} \) are the lengths between the centers of the bubbles \( i \) and \( j \), \( p_i \) is the inner pressure of the bubble, \( p_i^\infty = p_i^\infty \) is the pressure of the liquid away from the bubbles, \( \sigma \) is the surface tension coefficient of the liquid, \( \mu \) is the dynamic viscosity coefficient, \( R_{ui} \) is the equilibrium radius of the \( i \) bubble.

When oscillations have small amplitude \( R_i = R_{ui} \left[ 1 + x(t) \right] = R_{ui} \left( 1 + x \right) \), therefore

\[
\dot{R}_i = R_{ui} \dot{x}, \quad \ddot{R}_i = R_{ui} \ddot{x}.
\]

When omitting the pressure \( p_i \) in the equation (1), i.e. the linearized case, then the equation has the form

\[
\ddot{x} + 2\beta_v \dot{x} + \omega_{0i}^2 x = -\frac{A}{\rho R_{ui}^2} \cos \omega t,
\]

where natural angular frequency is

\[
\omega_{0i}^2 = \frac{1}{3\gamma} \left( \frac{p_i^\infty + 2\frac{\sigma}{R_i}}{\rho R_{ui}^3} \right) - 2\frac{\sigma}{\rho R_{ui}} + \frac{\omega_R^4 R_{ui}^2}{u^2 (1 + \omega_R^2 R_{ui}^2/u^2)} \right)^{1/2} \approx \frac{1}{R_{ui}} \left( \frac{p_{eq}^{(i)}}{\rho} \right)^{1/2},
\]

the whole damping is

\[
\beta_v = \beta_{visc} + \beta_{thir} + \beta_{acir},
\]

where radial viscous component \( \beta_{visc} \), radial thermal component \( \beta_{thir} \) and radial acoustic component \( \beta_{acir} \) are

\[
\beta_v = 2 \frac{\mu}{\rho R_{ui}^2}, \quad \beta_{thir} = 2 \frac{\mu_i}{\rho R_{ui}^2}, \quad \beta_{acir} = \frac{\omega_R^4 R_{ui}^2}{2u (1 + \omega_R^2 R_{ui}^2/u^2)} \approx \frac{\omega_R^4 R_{ui}^2}{2u}
\]
When replacing Eq. (3) in Eq. (2), this yield
\[ p_i = \rho \sum_{j=1}^{N} \frac{R_{ij}^3}{r_{ij}} \left[ \frac{(1 + x_j)^2 \ddot{x}_j + 2(1 + x_j) \dot{x}_j}{r_{ij}} \right] \]
\[ \approx \rho \sum_{j=1}^{N} \frac{R_{ij}^3}{r_{ij}} \frac{(1 + x_j)^2 \ddot{x}_j}{r_{ij}} \]
\[ \approx \rho \sum_{j=1}^{N} \frac{R_{ij}^3 \ddot{x}_j}{r_{ij}} \]  
\[ (8) \]

Assuming that bubbles are identical \( R_{01} = R_{02} = \ldots = R_{0i} = R_0 \), then Eq. (8) becomes
\[ p_i = \rho R_0^3 \sum_{j=1}^{N} \frac{\ddot{x}_j}{r_{ij}} \]  
\[ (9) \]

Equation (4) to which we add the pressure of the scattered radiation (9) becomes a system of equations [10, 11]
\[ \ddot{x}_i + 2\beta_i \dot{x}_i + \omega_{0i}^2 x_i = -\frac{A}{\rho R_0^2} \cos \omega t - \frac{1}{R_0^2} \sum_{j=1}^{N} \frac{R_{ij}^3 \ddot{x}_j}{r_{ij}}. \]
\[ (10) \]

For identical bubbles, the systems of equations (10) becomes
\[ \ddot{x}_i + 2\beta_i \dot{x}_i + \omega_{0i}^2 x_i = -\frac{A}{\rho R_0^2} \cos \omega t - \left( \sum_{j=1}^{N} \frac{R_{ij}^3}{r_{ij}} \right) \ddot{x}_i. \]
\[ (11a) \]

or
\[ \left( 1 + \sum_{j=1}^{N} \frac{R_{0}}{r_{ij}} \right) \ddot{x}_i + 2\beta_i \dot{x}_i + \omega_{0i}^2 x_i = -\frac{A}{\rho R_0^2} \cos \omega t. \]
\[ (11b) \]

When comparing Eqs. (4) and (11b) it is observed that the radiation pressure scattered by the other cluster bubbles induces an additional inertial term \( N_c = \sum_{j=1}^{N} \left( R_0 / r_{ij} \right) > 0 \), i.e. the inertial mass of each oscillating bubble increases from \( m = 4\pi R_0^3 \rho \) at
\[ m_N = 4\pi R_0^3 \rho \left( 1 + \sum_{j=1}^{N} \frac{R_0}{r_{ij}} \right) = m + mN_c, \quad N_c = \sum_{j=1}^{N} \frac{R_0}{r_{ij}}. \]
\[ (12) \]

Thus, the radial oscillation’s inertia for each bubble is also determined by the interaction with other bubbles
\[ m_N = m + mN_c = m + m_c \approx m, \quad m_c = mN_c, \quad N_c >> 1. \]
\[ (13) \]

We note that induced mass, \( mN_c \), is proportional to the virtual mass of the bubble. It follows that it is not possible to induce an extra mass if the virtual mass of the bubble is zero. This condition is also found for the phenomenon of induction of mass in the Electromagnetic World [16].

The increase in the mechanical inertia of each bubble within the cluster leads to changing the magnitudes of the other parameters of the bubble (see section 2.2!).

### 2.2. Oscillations of the bubbles within a cluster

In order to deduce the oscillations’ parameters of identical bubbles within a cluster, we proceed to divide Eq. (11b) against the coefficient of acceleration \( \ddot{x}_i \) in order to obtain the equation of the forced oscillator
\[ \ddot{x}_i + 2 \frac{\beta_i}{1 + \sum_{j=1}^{N} \frac{R_{0}}{r_{ij}}} \dot{x}_i + \frac{\omega_{0i}^2}{1 + \sum_{j=1}^{N} \frac{R_{0}}{r_{ij}}} x_i = -\frac{A}{\rho R_0^2} \cos \omega t \]
\[ (14) \]
or

\[ \ddot{x}_N + 2\beta_N \dot{x}_N + \omega_{Nv}^2 x_N = -\frac{A_N}{\rho R_0^2} \cos \omega t, \quad (15) \]

where we used the following parameters, according also to (12),

\[ \omega_{Nv}^2 = \frac{\omega_{0v}^2}{1 + N_c} < \omega_0^2, \quad \beta_N = \frac{\beta_l}{1 + \sum_{j \neq i} R_{0j}} = \frac{\beta_l}{1 + N_c} < \beta, \quad A_N = \frac{A}{1 + N_c} < A. \quad (16) \]

One can see from the above relations that, according to the assumption made in section 2.1, the increase of the virtual mass induces a decrease in its own natural angular frequency \( \omega_{Nv} < \omega_0 \), a decrease of the damping coefficient \( \beta_N < \beta \), as well as a decrease in the amplitude of oscillations \( A_N < A \) [13].

Assuming the solution of Eq. (15) is of the form \( x_N = a_N \cos(\omega t + \varphi_N) \), it results, according to [6]

\[ a_N = \frac{A_N}{\rho R_0^2 \left[ (\omega^2 - \omega_{0v}^2)^2 + 4\beta_N^2 \omega^2 \right]^{3/2}}, \quad \varphi_N = \arctan \frac{2\beta_N \omega}{(\omega^2 - \omega_{0v}^2)}. \quad (17) \]

Replacing Eq. (16) in Eq. (17) yields

\[ a_N = \frac{A}{\rho R_0^2 \left[ (\omega^2 (1 + N_c) - \omega_{0v}^2)^2 + 4\beta_l^2 \omega^2 \right]^{3/2}}, \quad \varphi_N = \arctan \frac{2\beta \omega}{(\omega^2 (1 + N_c) - \omega_{0v}^2)}. \quad (18a) \]

or

\[ a_N = \frac{A}{\rho R_0^2 \left[ (\omega^2 (1 + N_c) - \omega_{0v}^2)^2 + 4\beta_l^2 \omega^2 \right]^{3/2}}, \quad \varphi_N = \arctan \frac{2\beta \omega}{\omega^2 (1 + N_c) - \omega_{0v}^2}. \quad (18b) \]

At the resonance of velocity, \( \omega = \omega_{0hNv} = \omega_{0v}/\sqrt{1 + N_c} < \omega_{0v} \), the amplitude and the phase (18b) become

\[ a_{Nv} = \frac{A}{2\rho R_0^2 \beta_{hvr} \omega_{0v}^2}, \quad \varphi_{Nv} = \arctan \frac{\pi}{2}, \quad (19) \]

where \( \beta_{hvr} = \beta_r (\omega = \omega_{0Nv}) \) and

\[ \beta_{hvr} = \beta_{hr} + \beta_{accr} = \frac{2\mu}{\rho R_0^2} + \frac{2\mu_h (\omega = \omega_{0Nv})}{\rho R_0^2} + \frac{\omega_{0Nv} R_0}{2u}. \quad (20) \]

Special case \( \beta_{accr} \gg \beta_{hr} + \beta_{accr} \) yields \( \beta_{hvr} = \beta_{accr} = \omega_{0Nv} R_0/(2u) \), therefore the amplitude displayed in Eq. (19) becomes

\[ a_{Nvacc} = \frac{Au}{\rho R_0^2 \omega_{0Nv}^2} = \frac{Au (1 + N_c)^{3/2}}{\rho R_0^2 \omega_{0v}^2} = \frac{Au (1 + N_c)^{3/2}}{p_{eff}^{3/2}} = \frac{A (1 + N_c)^{3/2}}{p_{eff} \left( \frac{\rho u^2}{p_{eff}} \right)^{3/2}} \quad (21a) \]

or, for \( N_c >> 1 \),
When resonance occurs, one can see from Eq. (21b) that amplitude $a_{\text{N,ac}}$ depends on the parameters of the bubbles and of the cluster through $N_c$.

### 2.3. Cluster with uniform distribution of bubbles

We assume a spherical cluster with radius $R_c >> R_0$, having a large number of bubbles, $N >> 1$, 

$$N = \frac{4\pi R_c^3}{3} n_c,$$  

with uniformly distributed bubbles and the number density $n_c \approx 1/d_c^3$ (here $d_c$ is the average distance between the bubbles); hence 

$$N = \frac{4\pi R_c^3}{3d_c^3}.$$  

The sum $N_c = \sum_{j=1}^{N} (R_c/r_j)$ expressed in Eq. (12) can be approached through integral calculation 

$$N_c \approx R_0 \int_{d_c}^{R_c} \frac{n_c dV}{r} = R_0 \int_{d_c}^{R_c} 4\pi n_c r dr = 2\pi R_0 n_c \left( R_c^2 - d_c^2 \right) = \frac{2\pi R_0 R_c^2}{d_c^3} - \frac{2\pi R_0}{d_c^3} \approx \frac{2\pi R_0 R_c^2}{d_c^3}.$$  

In the following, we will assume that the test bubble is placed in the center of the cluster and it interacts with $dN = n_c dV$ bubbles contained in a spherical layer having thickness $dr$ and surface $4\pi r^2$.

For $R_c > d_c > R_0$, $N_c$ is much less than $N$ 

$$N_c \approx \frac{2\pi R_0}{d_c^3} R_c^2 - \frac{2\pi R_0}{d_c^3} \approx \frac{2\pi R_0 R_c^2}{d_c^3} \approx N \frac{3R_0}{2R_c} < N.$$  

When considering the distance from the bubble to the center of the cluster, we notice a force acting on each bubble, which is directed towards the center of the cluster (Eq.11 from [12]). This force is similar to the electrostatic force acting on a uniformly distributed charge or to a gravitational force acting on a uniformly distributed mass at a distance less than that of the radius of the distributed charge or mass ($r_0 < r_{\text{dist}} = R_c$). This issue will be a topic for a further paper.

### 3. Acoustic interactions tacking place in a cluster

#### 3.1. Interaction of electrostatic type

According to [6], the acoustic force for two identical bubbles, $R_{01} = R_{02} = R_0$, is 

$$F_a(r,\varphi) \approx -\frac{2\pi \rho \omega^2 R_0^3}{r^2} a^2, \cos \varphi = 1.$$  

Replacing Eq. (18) in Eq. (26), with $a = a_N$, yields
When resonance occurs, \( \omega^2 = \omega_{e,N}^2 \equiv \frac{\omega_0^2}{N_c} \), then
\[
F_{BN} (r) \equiv \frac{-2 \pi \omega^2 R_0^2}{\rho r^2} \cdot \frac{A^2}{\left[\left(\omega^2 (1 + N_c) - \omega_0^2\right)^2 + 4 \beta_c^2 \omega^2\right]} \approx \frac{-2 \pi \omega^2 A^2 R_0^2}{\rho r^2} \cdot \frac{\left[\left(\omega^2 N_c - \omega_0^2\right)^2 + 4 \beta_c^2 \omega^2\right]}{N_c >> 1}.
\] (27)

Adopting the natural angular frequency from Eq. (5) then Eq. (28) changes into
\[
F_{BN} (r) \equiv \frac{-2 \pi A^2 R_0^2}{2 \rho r^2 \beta_c^2 (a_{bh})} = \frac{-2 \pi A^2 R_0^2}{2 \rho r^2 \left[\frac{2 \mu}{\rho R_0^2} + \frac{2 \mu (\omega = \omega_{h,N}) + \omega_{e,N}^2 R_0}{2 \mu}\right]} \approx \frac{-2 \pi A^2 u^2}{\rho r^2 \omega_{e,N}^2}.
\] (28)

Adopting the natural angular frequency from Eq. (5) then Eq. (28) changes into
\[
F_{BN} (r) \equiv \frac{-2 \pi A^2 \rho u^2 R_0^2 N_c^2}{r^2 p_{eff}^2} = N_c^2 F_{Br} (r) >> F_{Br} (r),
\] (29)
i.e. a force much larger than the one acting between two bubbles not found in a cluster.

3.2. Interaction of gravitational type

According to Eq. (5) from [8] the gravitational force acting onto two identical bubbles which
are placed outside of a cluster of bubbles is
\[
F_{Bav} (r) = -\frac{2 \pi \rho \omega^2 R_0^2}{r^2} a^4, \cos \varphi = 1.
\] (30)

Instead of the above case, when the bubbles are placed within the cluster, i.e. when replacing
Eq. (18) in Eq. (30), with \( a = a_N \), the force becomes
\[
F_{Bav} (r) = \frac{-2 \pi A^2 A^4}{r^2 \rho^3 R_0^3} \left[\left(\omega^2 (1 + N_c) - \omega_0^2\right)^2 + 4 \left(\frac{\omega^2 R_0}{2 \mu} + \frac{2 \mu (\omega = \omega_{h,N})}{\rho R_0^2} + \beta_c^2\right)^2 \omega^2\right],
\] (31)
yielding the expression of the gravitational type force between two bubbles found within a
cluster.

When resonance occurs, \( \omega^2 = \omega_{e,N}^2 = \frac{\omega_0^2}{N_c + 1} \), the acoustic force of gravitational type
between two bubbles within a cluster is
\[
F_{Bav, r} (r) = \frac{-2 \pi A^4}{8 r^2 \rho^3 R_0^3} \left(\frac{\omega_{e,N}^2 R_0}{2 \mu} + \frac{2 \mu (\omega_{e,N})}{\rho R_0^2} + \beta_c^2\right)^4 \omega_{e,N}^2.
\] (32)

For \( \beta_{e,N} = \omega_{e,N}^2 R_0 / (2 \mu) >> \beta_{e,h} (\omega_{e,N}) \), the expression of the attractive force becomes
\[
F_{Bav, r} (r) = \frac{-2 \pi A^4}{r^2 \rho^3 R_0^3} \omega_{e,N}^2 \left(1 + \frac{4 \mu}{\rho \omega_{e,N}^2 R_0} + \frac{2 \mu (\omega_{e,N})}{\omega_{e,N}^2 R_0}\right)^4.
\] (33)

One can use the natural angular frequency from (16) to express the above attractive force. In
order to achieve this, we will consider the corrected form which is in Eq. (13) from paper [8]
\[
F_{\text{BNat}}(r) = \frac{-2\pi \gamma R_0^4 \rho^2 u^4 N_c^5}{r^2 p_{\text{eff}}^5 \left( 1 + \frac{4\gamma u \mu N_c}{p_{\text{eff}} R_0} + \frac{2\gamma p u R_0 N_c \beta_{\text{in}}(\omega_{\text{BNat}})}{p_{\text{eff}}} \right)} \approx \frac{-2\pi \gamma R_0^4 \rho^2 u^4 N_c^5}{r^2 p_{\text{eff}}^5 \left( 1 - \frac{4\gamma u \mu N_c}{p_{\text{eff}} R_0} - \frac{2\gamma p u R_0 N_c \beta_{\text{in}}(\omega_{\text{BNat}})}{p_{\text{eff}}} \right)}.
\]
(34)

According to Eq. (14a) from the same paper [8] one can approximate the force mentioned above for \( \beta_{\text{in}}(\omega_{\text{BNat}}) \approx (\gamma - 1) p_{\text{eff}} / 30 N_c \gamma^2 \rho \chi \) as
\[
F_{\text{BNat}}(r) \approx \frac{-2\pi \gamma^2 R_0^4 \rho^2 u^4 N_c^5}{r^2 p_{\text{eff}}^5} \left( 1 + \frac{2\gamma^2 3u^2 \mu^2 N_c^2}{p_{\text{eff}}^2 R_0^2} + \frac{2\gamma^3 \gamma^2 \rho^2 u^2 R_0^2 N_c^2 \beta_{\text{in}}^3}{p_{\text{eff}}^2 R_0^2} + \frac{2\gamma^3 \gamma^2 \rho^2 u^2 p \mu N_c^2 \beta_{\text{in}}^3}{p_{\text{eff}}^2 R_0^2} + \frac{2\gamma^4 \rho^2 u^4 \mu N_c^4 \beta_{\text{in}}^4}{p_{\text{eff}}^2 R_0^2} \right)
\]
(35)

In this latter form, the components proportional to \( p_{\text{eff}} \), for \( p_{\text{eff}} \approx 3\gamma \rho \), are
\[
F_{\text{BNat}} \left( R_0^6, r \right) \approx \frac{-2\pi N_c^3 R_0^4 A^4 \rho^2 u^4}{r^2 3^3 p_{\text{eff}}^5} \left( \frac{2(\gamma - 1)^7 u^2 R_0^2}{5^3 3^3 \gamma^6 \chi^2} + \frac{2(\gamma - 1)^7 N_c u^2 \mu R_0^2}{3^3 5^3 \gamma^6 \chi^2} \right)
\]
(36)

The first term in Eq. (36)
\[
F_{\text{BNat}} \left( R_0^6, r \right) \approx \frac{-m^2}{r^2} \left[ \frac{(\gamma - 1)^7 N_c^3 A^4 u^6}{2^3 3^5 5^2 \gamma^2 \rho_0^2 \chi^2} \right] = N_c^3 F_{\text{BNat}} \left( R_0^6, r \right),
\]
(37)
is much larger than the first term of the force acting between two bubbles out of the cluster.

3.3. The Mach’s Principle in Acoustic World

The analysis of the mechanical inertia induction phenomenon by coupling the oscillations of the bubbles in a cluster (section 2.2) leads to the hypothesis that this phenomenon is
The additional mass term, according to Eqs. (12) and (13) is a term according to the Mach’s Principle [14, 15]: mechanical inertia, as a property of a body, is also determined by the interaction with the other bodies in the Universe. In this case, the interaction between the bubbles that oscillate in phase is of electroacoustic nature. This interaction propagates with the velocity of acoustic waves such that \( R_e = u \tau_e \).

For a continuous distribution of bubbles into a cluster, according to Eq. (25), the rate of inertia increase is

\[
N_e = N \frac{3R_0}{2R_e} = \frac{NR_0}{R_e}, \quad R_e >> R_0.
\]

This relationship, for \( N_e \approx 1 \), is analogous to the relationship obtained by Sciama (Eq.7 from [14]) for gravitational interaction of particles in Universe (where the gravitational radius of the protons is \( R_g = Gm_p/c^2 \))

\[
Gm_\nu m_c \tau^2 = \frac{Gm_\nu}{c^2} \left( \frac{N_e}{R_\nu} \right) \left( \frac{R_\nu}{R_e} \right)^2 = \frac{N_\nu R_e}{R_\nu} \approx 1, \quad R_\nu = c \tau.
\]

For this condition, both the electroacoustic force (Eq. (29)) and the gravitoacoustic force (Eq. (37)) between the two cluster bubbles are identical to the electroacoustic force and the gravitoacoustic force for decoupled bubbles.

Therefore the relation \( N_e \approx 1 \) implies a relation between the radius of the cluster and the radius of the bubbles, \( R_e = NR_0 \) through the number of the bubbles.

For the case of a cluster with \( N \) identical bubbles, with \( N >> 1 \) and \( R_e << NR_0 \), we found out in the above study, that the induced mass \( m_c = mN_e \) is much larger than \( m \) since \( N_e \approx N(R_0/R_e) >> 1 \). We call \( N_e \) the acoustic Mach’s number .

### 4. Conclusions

The study of the coupling oscillations of the bubbles, contained within a cluster of bubbles, reveals the effect of increase the inertia of any bubble from the cluster. This increase is due to the scattered acoustic radiation by the other bubbles of the cluster.

For a large number of identical bubbles uniformly distributed, the induced mass has the size of the virtual mass of a bubble when \( N_e \approx 1 \). In this special case we’ve obtained a similar relation to that of the Sciama, for gravitational interaction.

The complete analogy may be accomplished when the cluster is neutral from an electroacoustically standpoint. The cluster contains \( N \) identical bubbles which oscillate in phase and \( N \) bubbles which oscillate in opposite phase, all uniformly distributed. In this case, the interactions between bubbles, placed at distances larger than the average distance between bubbles \( r > d_e \), are dipolar from an electroacoustic standpoint. At great distances, \( r >> d_e \), the dominant interaction is the gravitoacoustic interaction.

Another phenomenon that affects the interaction of bubbles within a cluster is the attenuation of wave intensity by scattering and absorption. The attenuation coefficient of the acoustic intensity is \( \mu_e = n_e \sigma_e \) within a cluster with uniformly distributed bubbles, where \( n_e \) is the number density of the bubbles and \( \sigma_e \) is the extinction cross section. \( D_e = 1/\mu_e \) is the
attenuation length. Some consequences of the mentioned effects will be studied in a further paper.

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