Measuring Cosmological Parameters with the SDSS QSO Spatial Power Spectrum Analysis to Test the Cosmological Principle

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ABSTRACT

In this paper we emphasize the importance of the Sloan Digital Sky Survey (SDSS) QSO clustering statistics as a unique probe of the Universe. Because the complete SDSS QSO sample covers a quarter of the observable universe, cosmological parameters estimated from the clustering statistics have an implication as a test of the cosmological principle, by comparing with those from local galaxies and other cosmological observations. Using an analytic approach to the power spectrum for the QSO sample, we assess the accuracy with which the cosmological parameters can be determined. Arguments based on the Fisher matrix approach demonstrate that the SDSS QSO sample might have a potential to provide useful constraints on the density parameters as well as the cosmic equation of state.

Keywords cosmology – large-scale structures of Universe: quasars

1. Introduction

The cosmological principle, the assumption of the homogeneity and isotropy of the Universe on large scales, is one of the most fundamental in the framework of the cosmology (e.g., Peebles 1993). As Lahav (2001,2002) reviewed, observations of the cosmic microwave background (CMB), the cosmic X-ray background, radio sources, and the Lyman-α forest strongly support the cosmological principle, though it is difficult to prove the principle definitely. Actually inhomogeneous cosmological models, which challenge the cosmological principle, have been proposed (Kantowski & Thomas 1998, Kantowski 2001, Barrett & Clarkson 2000, Celeria 2000). For example, it is claimed that a large local void universe might be viable, which is compatible with the observations of type Ia supernovae, the cosmic microwave isotropy, and the distribution of the local galaxies (Tomita 2000, 2001).

We emphasize the importance of the QSO clustering statistics as a unique and independent probe of the Universe. Especially it provides a chance to test the cosmological principle by

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comparing the cosmological parameters measured from the QSO clustering analysis with those from local galaxies and other cosmological observations. Recently the two degree field (2dF) QSO redshift (2QZ) Survey reported that a simply biased mass distribution explains the QSO spatial clustering and that the cosmological parameters can be measured (Croom et al. 2001, Hoyle et al. 2002a; 2002b, Outram et al. 2001). Their preliminary results with the 10k catalogue favor the cold dark matter (CDM) model with a cosmological constant, though the constraint is not very tight (see also Yamamoto 2002; Hereafter Paper I).

On the other hand, the Sloan Digital Sky Survey (SDSS) will aim to compile a homogeneous catalogue of \(10^5\) QSOs as well as \(10^6\) galaxies. The SDSS stores the 5 bands CCD imaging of \(10^4\) deg\(^2\) in the North Galactic Cap and of three \(2.5 \times 90\) deg\(^2\) strips in the South Galactic Cap. The QSO spectroscopic survey of the North Galactic Cap will detect \(10^5\) QSOs in the range of the redshift \(0 \lesssim z \lesssim 5\) with the limiting magnitude \(m = 19 \sim 20\). A QSO catalogue with the SDSS early data release has been reported (Schneider et al. 2002). Because the SDSS QSO sample will cover a quarter of the observable Universe, the cosmological parameters measured with the clustering statistics provide a unique implication for the cosmological principle. Therefore it is important to estimate the accuracy with which the QSO clustering statistics can measure the cosmological parameters.

Several authors have proposed possible methods (e.g., Ballinger, Peacock, & Heavens 1996, Matsubara & Suto 1996, Popowski et al. 1998), which originate from the Alcock-Paczynski’s test (1979). Very recently, Calvao et al. (2002) have claimed that a significant constraint on the density parameters as well as the cosmic equation of state of the dark energy, which is a quite important issue in physics and cosmology (see e.g., Caldwell, Dave & Steinhardt 1998), can be obtained from the Alcock-Paczynski test with the 2dF QSO sample. Concerning the SDSS QSO sample, however, a systematic assessment on the capability of measuring the cosmological parameters have not been well investigated, as far as we know. The reason might come from the fact that physical processes of the QSO formation and time-evolution have not been completely understood. On the other hand, the number count of the SDSS QSO sample in the early data release has been demonstrated by Schneider et al. (2002), which we use in our investigation.

Motivated by these situations, we revisit the QSO clustering statistics and assess the accuracy with which the cosmological parameters can be measured by the SDSS QSO sample. For this purpose we adopt the Fisher-matrix approach, which has been often used to assess the accuracy of measurement of the cosmological parameters from large surveys such as CMB anisotropy observations, galaxy redshift survey, and supernova data sets. The Fisher matrix approach bases the method on mathematical theorems, which enables us to clearly interpret results. The method adopted in the present paper is a simple extension of an approximate Fisher matrix approach using the power spectrum, which was originally formulated for galaxy samples (Tegmark et al.

\(^2\)Almost this work was completed, a similar paper by Matsubara and Szalay (2002) has been announced. Some differences are mentioned in the last section.
1998; Tegmark 1997). We use the power spectrum analysis extended so as to incorporate the various observational effects, the light-cone effect, the linear redshift distortion and the geometric distortion, which is useful for QSO samples (Paper I, Suto et al. 2000, Yamamoto & Suto 1999, Yamamoto & Nishioka 2001). This paper is organized as follows: In section 2, we briefly review formulas to evaluate the Fisher matrix element. In section 3, the best statistical errors of the cosmological parameters are shown assuming the complete SDSS QSO sample, which demonstrates the usefulness of the power spectrum analysis of the sample. In section 4, the problem of degeneracy in the parameter space is briefly discussed. Section 5 is devoted to conclusions. Throughout this paper we use the unit in which the light velocity equals 1.

2. Method

The Fisher matrix approach provides a useful technique to estimate errors in measuring cosmological parameters from a given data set (see e.g., Tegmark, Taylor & Heavens 1997, Jungman et al. 1996;1997). Tegmark et al. (1998) developed a useful approximate method to estimate the Fisher matrix element using the power spectrum of large surveys of galaxies. The method used in the present paper is a simple extension of their work. In an analysis of the QSO clustering, the additional observational effects, the light-cone effect and the geometric distortion effect, must be taken into account. The power spectrum analysis incorporating these effects can be developed by extending the work by Feldman, Kaiser & Peacock (1994; hereafter FKP). We start from briefly reviewing this power spectrum analysis useful for the QSO samples.

First we assume that the number density field of sources $n(s)$ is constructed from a data set, where $s$ denotes the three dimensional position of the sources. Here we assume that a distance-redshift relation $s(=|s|) = s(z)$ was adopted to plot the map (see below for the explicit definition $s(z)$). Following FKP, we define the density fluctuation field by

$$F(s) = \frac{n(s) - \alpha n_s(s)}{\left[ \int ds \tilde{n}(s)^2 \right]^{1/2}},$$  \hspace{1cm} (1)

where $\alpha$ is a constant, $n_s(s)$ is a synthetic catalogue, $\tilde{n}(s)$ is the expected mean number density. The optimum weight factor introduced in FKP can be fixed as a constant because $\tilde{n}P(k)$ is less than unity for the QSO sample. This means that the power spectrum analysis considered in the present paper does not require information on the evolution of the power spectrum as a function of the redshift. The power spectrum is constructed by the square of the Fourier coefficient of the fluctuation field defined by

$$\mathcal{F}(k) = \int ds F(s)e^{ik \cdot s}.$$ \hspace{1cm} (2)

Then the estimator of the power spectrum (before averaging over a shell of the radius $k(=|k|)$ in the Fourier space) is defined by subtracting the shot noise contribution $P_{\text{shot}}$ from $|\mathcal{F}(k)|^2$

$$P(k) = |\mathcal{F}(k)|^2 - P_{\text{shot}}.$$ \hspace{1cm} (3)
Using the two-point functions of \( n(s) \) and \( n_s(s) \) (see equation (2.1.5) in FKP), we can compute the expectation value of \( \langle |\mathcal{F}(k)|^2 \rangle \), which yields

\[
P_{\text{shot}} = (1 + \alpha) \frac{\int dz (dN/dz)}{\int dz W(z)}
\]

and

\[
\langle P(k) \rangle = \frac{\int dz W(z) P^a(k, z)}{\int dz W(z)}
\]

where we defined \( W(z) = (dN/dz)^2 (dz/ds)/s^2 \) by using the observable quantity \( dN/dz \), the number of sources per unit redshift and unit solid angle, instead of \( \bar{n}(s) \), which are related by

\[
dN = dss^2 \bar{n}(s),
\]

\( P^a(k, z) \) is the local power spectrum defined on a constant time surface of the redshift \( z \). In deriving (5), we adopted the distant observer approximation, \( |s_1 - s_2| \ll |s_1 + s_2| \). Under this approximation, the correlation function is related with \( P^a(k, z) \) as

\[
\xi(s_1, s_2) = (2\pi)^{-3} \int dk P^a(k, z(|s_1 + s_2|/2)) e^{i k (s_1 - s_2)}.
\]

In a similar way, the variance of the power spectrum (covariance matrix) can be approximately estimated

\[
\langle \Delta P(k) \Delta P(k') \rangle = \frac{2(2\pi)^3}{\kappa} \delta(k - k'),
\]

where \( \Delta P(k) = P(k) - \langle P(k) \rangle \), and

\[
\kappa = \Delta \Omega \int dss^2 \bar{n}(s)^2 = \Delta \Omega \int dz W(z)
\]

with the solid angle of a survey area \( \Delta \Omega \). Equation (5) is the power spectrum including the light-cone effect, which is incorporated by averaging the local power spectrum \( P^a(k, z) \) over the redshift with the weight factor \( W(z) \). The usefulness of the power spectrum (5) is discussed by comparing with results of the 2QZ group (Paper I).

In general, the Fisher matrix element is defined by

\[
F_{ij} = \left\langle -\frac{\partial^2 \ln L}{\partial \theta_i \partial \theta_j} \right\rangle,
\]

where \( \theta_i \) denotes the parameters and \( L \) is the probability distribution function of a data, given the model parameters. For simplicity, we adopt the approximation of the Gaussian probability distribution function for \( \Delta P(k) \)

\[
L \propto \exp \left[ -\frac{1}{2} \int dk \int dk' \Delta P(k) C(k, k')^{-1} \Delta P(k') \right],
\]

where the matrix \( C(k, k')^{-1} \) is the inverse of the covariance matrix \( \langle \Delta P(k) \Delta P(k') \rangle \). Following the work by Tegmark (1997), the Fisher matrix element reduces to

\[
F_{ij} = \frac{\kappa}{4\pi^2} \sum_{l=0, 2, \ldots} (2l + 1) \int_{k_{\text{min}}}^{k_{\text{max}}} dkk^2 \frac{\partial \langle P_l(k) \rangle}{\partial \theta_i} \frac{\partial \langle P_l(k) \rangle}{\partial \theta_j},
\]
where

\[
\langle P_l(k) \rangle = \int_0^1 d\mu L_l(\mu) \langle P(k) \rangle,
\]

and \( L_l(\mu) \) is the Legendre polynomial of the \( l \)-th order, and \( \mu \) denotes the directional cosine between the line-of-sight direction and the wave number vector. Note that \( \langle P(k) \rangle \) is a function of \( k \) and \( \mu \) under the distant observer approximation. This allows \( P^a(k, z) \) in equation (5) to be written \( P^a(k, \mu, z) \), which we model as (Ballinger et al. 1996)

\[
P^a(k, \mu, s) = \frac{1}{c_\parallel c_\perp} \times P_{QSO}(q_\parallel, q_\perp, \mu, c_\parallel, q_\perp, \sqrt{1 - \mu^2}/c_\perp, z),
\]

where \( P_{QSO}(q_\parallel, q_\perp, z) \) is the QSO power spectrum, \( q_\parallel \) and \( q_\perp \) are the wave number components parallel and perpendicular to the line-of-sight direction associated with the comoving coordinate of the real universe, \( c_\parallel \) and \( c_\perp \) are defined by \( c_\parallel = dr(z)/ds(z) \) and \( c_\perp = f_K(r(z))/s(z) \), respectively, with the comoving distance \( r(z) \) and the angular diameter distance \( f_K(r(z)) \) of the real universe. The explicit expression of \( r(z) \) is presented by equation (15). We model the QSO power spectrum of the spatial distribution incorporating the linear distortion (Kaiser 1987, see also Yamamoto Nishioka & Suto 1999), as follows:

\[
P_{QSO}(q_\parallel, q_\perp, z) = \left(1 + \frac{f(z)}{b(z)} \frac{q_\parallel^2}{q^2}\right)^2 b(z)^2 P_{\text{mass}}(q, z),
\]

where \( f(z) = d\ln D(z)/d\ln a(z) \) with the linear growth rate \( D(z) \) and the scale factor \( a(z) \), \( q^2 = q_\parallel^2 + q_\perp^2 \), \( b(z) \) is the scale independent bias factor, and \( P_{\text{mass}}(q, z) \) is the CDM mass power spectrum. The term in proportion to \( f(z) \) describes the linear distortion due to the peculiar velocity on large scales. We work within the linear theory of density perturbations because we consider the large scale QSO clustering statistics. In the present paper, we assume the Harrison-Zeldovich initial mass power spectrum and adopt the fitting formula of the cold dark matter transfer function by Eisenstein and Hu (1998), which is robust even when the baryon fraction is large.

Equation (11) is an extension of the previous result by Tegmark (1997). In the limit that the linear distortion, the geometric distortion, and the light-cone effect are switched off, equation (11) reproduces the result by Tegmark (1997). In the investigation in the present paper, the light-cone effect and the geometric distortion effect are particularly important. For a sample of a narrow range of the redshift, the light-cone effect can be negligible, however, it should be properly taken into account in the QSO spatial power spectrum analysis of the 2QZ sample and the SDSS sample, which consist of sources in the wide range of the redshift. In general, the geometric distortion becomes substantial for high-redshift samples. On the other hand, the linear distortion effect is rather minor in our investigation. It is well recognized that the linear distortion effect
is substantial and problematic in the Alcock & Paczynski’s geometric test (e.g., Ballinger et al. 1996). It is the fact that the anisotropic component of the power spectrum is sensitive to the linear distortion effect even at the high redshift. However, as we show in the next section, the dominant contribution constraining the cosmological parameters comes from the isotropic part of the power spectrum. The linear distortion effect only increases the overall amplitude of the isotropic part of the power spectrum in the scale independent bias model. This is the reason why the linear distortion is the minor effect in our investigation. This situation is in contrast to case of low redshift galaxy samples, in which the light-cone effect and the geometric distortion are negligible, while the linear distortion is rather important.

3. Results

In general, the comoving distance is written,

\[
r(z) = \int_0^z \frac{dz'}{H_0[\Omega_m(1+z')^3 + \Omega_K(1+z')^2 + \Omega_x(1+z')^3(1+w_Q)]^{1/2}},
\]

where the Hubble parameter is \(H_0 = 100 \text{ hkm/s/Mpc}\), \(\Omega_m\), \(\Omega_K\), and \(\Omega_x\) are the cosmological parameters of the densities and curvature, two of which are independent by virtue of the relation \(\Omega_m + \Omega_K + \Omega_x = 1\), and we introduced the constant cosmic equation of state parameter, \(w_Q\). In the present work, we focus on the four independent parameters \(\Omega_m\), \(\Omega_b\), \(\Omega_K\) and \(w_Q\). Here \(\Omega_m\) is the sum of the CDM and the baryon components. Following an analysis by the 2QZ group (Hoyle et al. 2002a), we choose \(s(z)\) to be the comoving distance with \(\Omega_m = 0.3\), \(\Omega_K = 0\), and \(w_Q = -1\):

\[
s(z) = \int_0^z \frac{dz'}{H_0[0.3(1+z')^3 + 0.7]^{1/2}}.
\]

Our results does not significantly depend on this choice of \(s(z)\). For \(dN/dz\) in the present paper, we adopt the recent result reported by Schneider et al. (2002) with the first SDSS QSO catalog. The QSO clustering bias in the SDSS sample has not been well investigated at present. However, Croom et al. reported on the time-evolution of the QSO bias by analyzing the 10k catalog of the 2QZ Survey (2001). According to them, the clustering amplitude does not show significant time evolution. For modeling the bias, following (Calvao et al. 2002), we first adopt the form

\[
b(z) = 1 + (b_0 - 1)D(z)^{-1.7},
\]

and determined the constant \(b_0(=1.2)\) so that the power spectrum best matches the 2QZ result (Hoyle et al. 2002a). For the linear growth rate of the model with \(w_Q \neq -1\), we adopted the fitting formula developed by Wang and Steinhardt (1998). Thus we vary the linear growth rate depending on cosmology. However, our result is not significantly altered by this modelling because the linear distortion effect is not substantial as described in the previous section.

Figure 1 shows the best statistical errors, \(\Delta \theta_i = 1/F_{ii}^{1/2}\), some of which are normalized by the target parameters (the parameters supposed to be true), \(\Omega_m = 0.28\), \(\Omega_K = 0\), \(\Omega_b = 0.04\) and
$w_Q = -1$, as functions of $k_{\text{max}}$ in equation (11). We fixed $k_{\text{min}} = 0.005 \, h \text{Mpc}^{-1}$ and assumed $\pi$ steradian of the survey area. The solid and dotted curves in each panel are the $l = 0$ and $l = 2$ components of $F_{ij}$, respectively. The dashed curve corresponds to the case that both components are summed up. Contribution of the higher modes $l \geq 2$ to the Fisher matrix element is small. Thus the contribution constraining the cosmological parameters is dominated by the component $l = 0$. The component $l = 2$ corresponds to the constraint from the anisotropic power spectrum $\langle P_2(k) \rangle$, while $l = 0$ corresponds to the constraint from the isotropic power spectrum $\langle P_0(k) \rangle$. The geometric distortion effect causes anisotropies of the power spectrum, and the Alcock-Paczynski’s geometric test utilizes this fact. The geometric distortion effect also alters the shape of the power spectrum $\langle P_0(k) \rangle$ due to the scaling of the wave number, equation (13). Therefore the constraint on the cosmological parameters in the present paper mostly relies on the latter effect that the shape of the isotropic power spectrum changes due to the geometric distortion.

Figure 2 shows the best statistical errors, $\Delta \theta_i = 1/F_{ii}^{1/2}$, as function of $k_{\text{min}}$ with fixed $k_{\text{max}} = 1 \, h \text{Mpc}^{-1}$. The target parameters and the meaning of the curves are same as those of Figure 1. These two figures indicate that the Fourier modes of $k \lesssim 0.01 h \text{Mpc}^{-1}$ and $k \gtrsim 0.1 h \text{Mpc}^{-1}$ are not significantly relevant to the integration for the Fisher matrix elements. Thus the Fourier modes of $0.01 h \text{Mpc}^{-1} \lesssim k \lesssim 0.1 h \text{Mpc}^{-1}$ dominate over the constraint on the cosmological parameters. This is a contrast to the case of the galaxy sample (Tegmark 1997). This difference traces back to the shot-noise dominance in the QSO sample. This result also suggests the validity of our assumption that nonlinear effect of density perturbations is small because the nonlinearity is influential on scales $k \gtrsim 0.1 h \text{Mpc}^{-1}$.

In the present paper, we have not considered the effect of the window function depending on the shape of a survey volume. The effect of the window function on the power spectrum for the 2QZ sample was investigated in the literature (Hoyle et al. 2002a). According to their result, the 2QZ power spectrum might be affected for $k \lesssim 0.02 h \text{Mpc}^{-1}$. Though a definite investigation using a mock sample is needed, for the case of the SDSS QSO sample, it might be expected that the effect of the window function appears on the larger length scales (smaller $k$) than that of the 2QZ power spectrum. From figures 1 and 2, as long as the window function effect in the SDSS QSO sample appears on the scale $k \lesssim 0.01 h \text{Mpc}^{-1}$, the estimation of the errors is not altered significantly.

We note that our result might depend on the modelling of the bias. This is because the Fisher matrix element is in proportion to the amplitude of the power spectrum of the QSO distribution. Figure 3 shows the same as Figure 1, but with adopting the bias model $b(z) = b_0 D(z)^{-1}$, in which the amplitude of the QSO clustering is almost constant as a function of the redshift $z$. Here we set $b_0 = 0.8$ so as to best match the 2QZ power spectrum (see also Paper I). The best statistical errors are worse than those of Figure 1, though the difference is not large. The reason of the smallness of the difference is explained as follows: We normalized the power spectrum so as to match the 2QZ power spectrum, in which the 2dF QSO sample is distributed in the range of redshift $0.3 \leq z \leq 2.3$. The difference is the contribution from the higher redshift $z \gtrsim 2$. The
bias model (17) has larger amplitude at the higher redshift, which yields larger amplitudes of the power spectrum and the Fisher matrix elements.

In the above investigations, we considered the simple bias models, whose amplitude we determined so that the power spectrum best matches the 2QZ result. This method might estimate the errors smaller. Then we also investigated the best statistical errors marginalized over parameters of the bias. We here consider the bias model parameterized by $b_0$ and $\nu$, as follows:

$$b(z) = 1 + \frac{(b_0 - 1)}{D(z)^\nu},$$

then marginalize the probability function with respect to $b_0$ and $\nu$. Explicitly, we considered the space of the parameters ($\Omega_m$, $b_0$, $\nu$) or ($w_Q$, $b_0$, $\nu$), and consider the Fisher matrix element marginalized by integrating the probability function with respect to $b_0$ and $\nu$. This process actually reduces the accuracy with which the cosmological parameters can be determined. For example, in the case with the target parameters $b_0 = 1.2$ and $\nu = 1.7$, we obtained the relative error $\Delta \Omega_m / \Omega_m \simeq 0.4$ and $\Delta w_Q \simeq 0.7$, where we assumed the same target parameters for the cosmological models as those of Figure 1. Similarly, we have $\Delta \Omega_m / \Omega_m \simeq 0.35$ and $\Delta w_Q \simeq 0.5$ for the target parameters $b_0 = 1.5$ and $\nu = 1$. We also have $\Delta \Omega_m / \Omega_m \simeq 0.25$ and $\Delta w_Q \simeq 0.35$ for $b_0 = 1.5$ and $\nu = 1.7$.

4. Degeneracy

In general the feasibility of determining the cosmological parameters is restricted by degeneracies in the parameter space. Actually many possible parameters can appear in generic cosmological model. Here we demonstrate some aspects of the degeneracies in the cosmological parameters. In Figure 4 we plot two dimensional confidence contours, where we assumed the same QSO sample and the bias model as those of Figure 1, $\Omega_m = 0.28$, $\Omega_\Lambda = 0.72$, $\Omega_b = 0.04$, $w_Q = -1$, and the Harrison-Zeldovich spectrum $n = 1$. Each panel shows the confidence contours on the two parameters, i.e., $\Omega_m$ and the other parameter labeled in each panel, with assuming the rest parameters to be fixed. Thus this figure shows the degeneracy between $\Omega_m$ and other parameters. From this figure it is clear that the uncertainty about $\Omega_b$ is problematic when determining $\Omega_m$. It also shows that the degeneracy between $\Omega_m$ and $n$, the index of the initial power spectrum, is relatively large. Thus the degeneracy between these parameters will be problematic in terms of determining all the parameters simultaneously. Nevertheless a useful constraint will be obtained from the QSO sample by combining it with constraints from Big Bang Nucleosynthesis, CMB anisotropies, Ia supernova, weak gravitational lensing and other observations. Then we can emphasize that the QSO sample provide a chance of a worthy complimentary test for the cosmological models.
5. Discussions and Conclusions

In this paper we have assessed the feasibility of measuring the cosmological parameters by means of the power spectrum analysis of the SDSS QSO sample, with an emphasis on a unique observational probe of the Universe. If tight constraints are provided, the result will have implications for testing the cosmological principle and inhomogeneous cosmological models. The best statistical errors are estimated using the Fisher matrix approach, which is a simple extension of the previous works for galaxy samples. For example, the best statistical errors are at 10% level for $\Omega_m$ and $\Omega_K$, a few times 10% for $w_Q$ and $\Omega_b$. These errors can increase to several $\times 10\%$, depending on the bias amplitude, when marginalizing over the parameters of the bias. Thus our result shows that the SDSS QSO sample might have a useful potential as a test of cosmological model. But, this does not mean that we have shown the capability of the Alcock-Paczynski test using the anisotropies of the power spectrum, because the constraint is almost from the isotropic component of the power spectrum. The cosmic degeneracy limits the capability of the method in determining all parameters with the best statistical accuracy simultaneously. However, the point is the uniqueness of the method as a probe, and the result would be useful for more robust understanding of the Universe.

Finally we mention comparisons with other works related with the present paper. Ballinger et al. (1996) used a method similar to the present paper, to predict a constraint from the 2dF QSO clustering statistics. Outram et al. (2001) have applied their method to the 2QZ sample. The 1-$\sigma$ error of $\Omega_m$ by Outram et al. (2001) is consistent with our result obtained by applying our method to the 2QZ sample. Very recently, Matsubara and Szalay (2002) have also reported a similar investigation based on an eigenmode analysis, while our method relies on the power spectrum analysis. Concerning the QSO sample, as we have discussed, the number density in proportion to $dN/dz$ and the modelling of the bias $b(z)$ can be sensitive to the final results. These modellings and the choice of the cosmological redshift space $s(z)$ are also different.

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Fig. 1.— The best statistical errors $\Delta \theta_i (= 1/F_{ii}^{1/2})$ as function of $k_{\text{max}}$, some of which are normalized by the target parameters $\Omega_m = 0.28$, $\Omega_K = 0$, $\Omega_b = 0.04$ and $w_Q = -1$. The solid and the dotted curve in each panel corresponds to $l = 0$ and $l = 2$ component, respectively. The dashed curve is the sum of the both. We adopted the solid angle of the survey area, $\Delta \Omega = \pi$ steradian.
Fig. 2.— The best statistical errors $\Delta \theta_i(= 1/F_{ii}^{1/2})$ as function of $k_{\text{min}}$. The target parameters and the meanings of the curves are same as those of Figure 1. Here we fixed $k_{\text{max}} = 1 \text{hMpc}^{-1}$.
Fig. 3.— Same as Fig. 1 but with the bias model of the form $b(z) = b_0 D(z)^{-1}$, where $b_0 (= 0.8)$ is used.
Fig. 4.— Confidence regions. We have chosen the target parameters as $\Omega_m = 0.28$, $\Omega_\Lambda = 0.72$, $\Omega_b = 0.04$, $w_Q = -1$ and $n = 1$. In the panels, we used the superscript ‘tr’ to denote the target parameters explicitly. Each panel shows the confidence contours on the two parameters, $\Omega_m$ and each of the other parameters, which is labeled in the panel, assuming that the rest parameters are fixed. The curves are the contours of the 68%, 95% and 99.7% confidence contours. Axes are normalized by the target parameter.