On the polarization and depolarization of the electromagnetic waves

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Abstract. We discuss a general description of the polarization of monochromatic electromagnetic waves that proves useful when the customary description in terms of Stokes parameters does not apply. We also show how this description can be exploited to study the depolarization of linearly polarized waves in the interior of porous model cosmic dust grains. The results that we discuss may affect our understanding of several problems that are relevant for astrobiology.

1. Introduction

The most important consequence of the Maxwell equations is probably the wave propagation of the electromagnetic energy. According to the Poynting theorem, that states the conservation of energy for a combined system of particles and fields [1], the direction of propagation of the electromagnetic energy is given by the Poynting vector, to which, according to its definition

\[ S = \frac{c}{4\pi} (E \times H), \]

both the electric and the magnetic fields are orthogonal. In this sense we could say that the electromagnetic waves are \textit{transverse}. However, it is seldom stressed that the transversality of the waves does not imply that, at any point, the plane swept, e.g., by the electric field as a function of time be orthogonal to the direction of \( S \). In fact, we deal very often with (homogeneous) plane waves and with spherical waves for both of which the plane swept by \( E \) is orthogonal to \( S \). Actually, the direction of \( S \) for a plane wave does not change either from point to point or as a function of time. For spherical waves the direction of \( S \) is always along the radius vector from the source and the plane swept by \( E \) is orthogonal to its direction. As a result, the state of polarization for these kinds of waves is fully described by the Stokes parameters [2], a set of four quantities with dimension of energy flux for whose measurement actual procedures are well established. From a mathematical point of view, the field of these waves is and remains, at any point and at any time, orthogonal to the unchanging direction of propagation and the Stokes parameters are built using the components of the field. As this is not the case for waves of general form a different description of their state of polarization is in order.
The purpose of this paper is just to point out one of these modes of description and to show how it can be applied to the study of the depolarization of linearly polarized waves within the cavities of model porous cosmic dust grains.

2. Field equations

We consider electromagnetic waves propagating in a homogeneous, isotropic, nonmagnetic medium and assume, without loss of generality, that all the fields depend on time through the factor $\exp(-i\omega t)$. Since all the fields should be real, we write, e.g.,

$$E(r, t) = \text{Re}[E(r) \exp(-i\omega t)],$$

although, in practice, we develop the theory by using the complex fields and only at the end of the algebraic manipulations we take the real part of the results.

For monochromatic fields the Maxwell equations take on the form

$$\nabla \times E = i\kappa_v B,$$
$$\nabla \times B = -i\kappa_v n^2 E,$$
$$\nabla \cdot (n^2 E) = 0,$$
$$\nabla \cdot B = 0,$$

where we define the propagation constant in vacuo and the refractive index as

$$\kappa_v = \frac{\omega}{c}, \quad n^2 = \mu \left( \varepsilon + i \frac{4\pi\sigma}{\omega} \right).$$

The usual algebraic manipulations led to the conclusion that both $E$ and $B$ are solution of the (decoupled) Helmholtz equations,

$$(\nabla^2 + \kappa_v^2 n^2)E = 0, \quad (\nabla^2 + \kappa_v^2 n^2)B = 0,$$

that satisfy the appropriate boundary conditions. Note that the decoupling of the equations (3) is a consequence of the assumed homogeneity of the medium.

At this stage, since we deal with monochromatic waves only, it is convenient to introduce the complex Poynting vector

$$\tilde{S} = \frac{c}{8\pi}[E \times H^*],$$

whose real part $S = \text{Re}(\tilde{S})$ gives the direction of propagation of the electromagnetic energy associated to a monochromatic wave. Note that, although $\tilde{S}$ comes from a time average [1], it is still dependent on $r$ so that both its magnitude and its direction may change from point to point.

3. Polarization

The time-independent part of the fields $E$ and $B$ is, in general, complex. Therefore, according to Born and Wolf [3], we define a pair of real vectors $p(r)$ and $q(r)$ such that

$$E(r) = p(r) + iq(r),$$

and, as a result

$$E(r, t) = \text{Re} \left[ (p(r) + iq(r)) \exp(-i\omega t) \right].$$
At any given point, say \( r_0 \), the vectors \( p \) and \( q \) define a plane on which \( E \) varies as a function of time. We first remark that, since \( E(r_0, t) \) is periodic, the tip of \( E \) should describe a closed curve on this plane. In order to show that this curve is an ellipse, let us define a pair of real vectors \( a \) and \( b \) such that

\[
p + iq = (a + ib) \exp(i\gamma),
\]

where \( \gamma \) is a real phase, that can be chosen so that \( a \) and \( b \) are mutually orthogonal. In fact, since

\[
\begin{align*}
a &= p \cos \gamma + q \sin \gamma, \\
b &= -p \sin \gamma + q \cos \gamma,
\end{align*}
\]

the requirement of mutual orthogonality of \( a \) and \( b \),

\[
(p \cos \gamma + q \sin \gamma) \cdot (-p \sin \gamma + q \cos \gamma) = 0,
\]

yields

\[
\tan 2\gamma = \frac{2p \cdot q}{p^2 - q^2}.
\]

As a result we can write

\[
E(r_0, t) = \text{Re}[(a(r_0) + ib(r_0)) \exp(-i(\omega t - \gamma))]
\]

and, by choosing a rectangular system of axes with origin at \( r_0 \) and the \( x \) and \( y \) axes along \( a \) and \( b \), respectively, we have

\[
\begin{align*}
E_x(r_0, t) &= a \cos(\omega t - \gamma), \\
E_y(r_0, t) &= b \sin(\omega t - \gamma),
\end{align*}
\]

i.e.

\[
\frac{E_x^2}{a^2} + \frac{E_y^2}{b^2} = 1.
\]

This shows that any monochromatic electromagnetic wave is, in general, elliptically polarized. Of course, according to the physical conditions, the ellipse may degenerate into a circle or into a straight line. In the latter case the area of the polarization ellipse,

\[
A = \pi ab,
\]

vanishes. Now, since \( E \), and of course also its complex conjugate \( E^* \), lie in the plane of the polarization ellipse, the real vector

\[
V = iE \times E^* = i(a + ib) \times (a - ib) = 2a \times b
\]

ind individuates the normal to this plane [4]. According to definition (5), the electric field rotates counterclockwise with respect to \( V \). In turn, the magnitude of \( V \), in view of the orthogonality of \( a \) and \( b \), is

\[
|V| = 2ab = \frac{2}{\pi} A
\]

and, therefore, vanishes for linear polarization. When \( V \neq 0 \), however, in general

\[
V \times S \neq 0,
\]
i.e. \( \mathbf{V} \) is not parallel to the direction of propagation of the wave and thus the plane of the polarization ellipse is not orthogonal to the direction of propagation, according to our statement in Sect. 1. Anyway, an important information is the sense of rotation of the field with respect to the direction of \( \mathbf{S} \). It is reasonable to assume that this information is given by the sign of the quantity

\[
V_s = \mathbf{V} \cdot \mathbf{S}/|\mathbf{S}|
\]

i.e. by the projection of \( \mathbf{V} \) in the direction of \( \mathbf{S} \). Of course, \( \mathbf{V} \) and \( \mathbf{S} \) can never become mutually orthogonal. Actually, it is an easy matter to see that \( V_s = 0 \) only when \( \mathbf{V} = 0 \), i.e. for linear polarization.

4. Spectral density tensor

The consideration in Section 3 should have convinced the reader that for an electromagnetic monochromatic wave of general form the state of polarization cannot be described by the components of the field that are orthogonal to the direction of propagation. In fact, as the vectors \( \mathbf{a} \) and \( \mathbf{b} \) depend on \( \mathbf{r} \), the state of polarization changes from point to point: only in the case of a homogeneous plane wave, the direction of the Poynting vector is constant from point to point. Then, for homogeneous plane waves, we can define a propagation vector (of the same constant direction of \( \mathbf{S} \)), to which the field is orthogonal and the customary Stokes parameters give a full description of the state of polarization. The Stokes parameters make sense also in the case of a spherical wave, because, as we remarked in Section 1, the direction of propagation is along the radius vector from the source, and the field is constantly orthogonal to this direction [5]. Although, plane and spherical waves are the most commonly dealt with, the considerations in Section 3 are not of a pure academic interest. In fact, the field scattered by an irregularly shaped particle in the near zone has just a general form and thus requires a general approach to the description of its polarization.

It is well known that all the information on the state of the field is given by the coherency dyad

\[
\rho = \mathbf{E}(\mathbf{r}) \otimes \mathbf{E}^*(\mathbf{r}),
\]

where \( \otimes \) denotes dyadic product. The representation of \( \rho \) in rectangular coordinates is known as the spectral density tensor

\[
S_{d3} = \begin{pmatrix}
E_x E_x^* & E_x E_y^* & E_x E_z^* \\
E_y E_x^* & E_y E_y^* & E_y E_z^* \\
E_z E_x^* & E_z E_y^* & E_z E_z^*
\end{pmatrix}.
\]

Actually, the elements of \( S_{d3} \) encompass full information on the state of polarization of the field. For instance, let us consider the pseudovector associated to \( S_{d3} \), whose components are

\[
F_i = \sum_{jk} \varepsilon_{ijk} (S_{d3})_{jk},
\]

where \( \varepsilon_{ijk} \) is the totally antisymmetric Ricci tensor. It is an easy matter to see that \( \mathbf{F} = \mathbf{E} \times \mathbf{E}^* = -i\mathbf{V} \).

In the case of a homogeneous plane wave one can take advantage of the transformation properties of \( S_{d3} \) to rotate the rectangular axes so that, e.g., the \( z' \) axis is parallel to the direction of propagation. As a consequence only the components of the field along the \( x' \) and \( y' \) axes are nonvanishing and we can recover the customary description of the polarization in terms of the Stokes parameters. Analogously, from the alternative representation of \( \rho \) in spherical coordinates, we can recover the Stokes parameters for a spherical wave.
5. Stokes parameters

Although the Stokes parameters are well known, it may be interesting to resume the technique by which they are introduced in the framework of the spectral density tensor. In fact, the same technique can be used to introduce generalized parameters that are appropriate for the case of waves of general form.

Let us consider a homogeneous plane wave whose spectral density tensor has been transformed to a frame of reference with the $z'$ axis in the direction of propagation. Denoting with a prime the rotated quantities, the tensor $S_{d3}'$ has only vanishing elements in the third row and third column. It is then convenient to rewrite

$$S_{d2}' = \begin{pmatrix} E_x'E_x'^* & E_y'E_y'^* \\ E_x'E_y'^* & E_y'E_y'^* \end{pmatrix},$$

so that the cartesian representation of $S_{d3}'$ appears as an actually two dimensional matrix. Now, it is well known that the most general $2 \times 2$ matrix can be expanded in terms of the two-dimensional unit matrix $1_2$ and of the three Pauli spin matrices $\sigma_i$ ($i = 1, 2, 3$): these matrices are, indeed, the generators of the Special Unitary Group SU(2). The expansion leads to the result

$$S_{d2}' = \frac{1}{2} \begin{pmatrix} I + Q & U - iV \\ U + iV & I - Q \end{pmatrix},$$

where

$$I = E_x'E_x'^* + E_y'E_y'^*,$$

$$Q = E_x'E_y'^* - E_y'E_x'^*,$$

$$U = E_x'E_y'^* + E_y'E_x'^*,$$

$$V = i(E_x'E_x'^* - E_y'E_y'^*),$$

are the spectral density Stokes parameters [1, 3]. The preceding definitions can be put in a more familiar form by remarking that, although the $z'$ axis should lie along the direction of propagation, the direction of the $x'$ and $y'$ axes, except for the mutual orthogonality, is arbitrary. Therefore by choosing the $z'x'$ plane as a plane of reference one gets, using the notation of van de Hulst [7]

$$I = E_l'E_l'^* + E_r'E_r'^*,$$

$$Q = E_l'E_r'^* - E_r'E_l'^*,$$

$$U = E_l'E_r'^* + E_r'E_l'^*,$$

$$V = i(E_l'E_l'^* - E_r'E_r'^*),$$

Since the determinant of $S_{d2}'$, Eq. (6), vanishes, the relation holds

$$Q^2 + U^2 + V^2 = I^2,$$

i.e., a purely monochromatic plane wave is fully polarized. Nevertheless, the plane waves that occur in practice, even when monochromatic to a high degree, are always wavepackets that include many frequencies in a narrow band centered on the nominal frequency $\omega_0$. In this respect, let us stress that $E$ depends on frequency so that also the spectral density Stokes parameters are frequency dependent. In fact, a wavepacket is characterized by a spectral tensor of the form

$$S(\omega_0, \Delta\omega) = \int_{\omega_0 - \Delta\omega/2}^{\omega_0 + \Delta\omega/2} S_{d2}'(\omega) \, d\omega.$$
The corresponding Stokes parameters $I$, $Q$, $U$, $V$, instead of equation (7), satisfy the relation

$$I^2 \geq Q^2 + U^2 + V^2,$$

so that one can define the degree of polarization of the packet as

$$p^2 = \frac{Q^2 + U^2 + V^2}{I^2} \leq 1.$$

Let us now go to see what is the analogous of the above quantities for a wave of general form. The most general $3 \times 3$ matrix can be expanded in terms of the unit matrix $\mathbf{1}_3$ and of the eight Gell-Mann matrices $\lambda_i$ ($i = 1, \ldots, 8$) that are the generators of the group SU(3) \cite{8}. Then, the expansion of the spectral density tensor reads

$$S_{d3} = \frac{1}{3} \Lambda_0 \mathbf{1}_3 + \frac{1}{2} \sum_{i=1}^{8} \Lambda_i \lambda_i = \begin{pmatrix}
\frac{1}{3} \Lambda_0 + \frac{i}{2} \Lambda_3 + \frac{1}{2\sqrt{3}} \Lambda_8 & \frac{1}{2} \Lambda_1 - i \frac{1}{2} \Lambda_2 & \frac{1}{2} \Lambda_4 - i \frac{1}{2} \Lambda_5 \\
\frac{1}{2} \Lambda_1 + i \frac{1}{2} \Lambda_2 & \frac{1}{2} \Lambda_0 - \frac{1}{2} \Lambda_3 + \frac{1}{2\sqrt{3}} \Lambda_8 & \frac{1}{2} \Lambda_6 - i \frac{1}{2} \Lambda_7 \\
\frac{1}{2} \Lambda_4 + i \frac{1}{2} \Lambda_5 & \frac{1}{2} \Lambda_6 + i \frac{1}{2} \Lambda_7 & \frac{1}{3} \Lambda_0 - \frac{1}{\sqrt{3}} \Lambda_8
\end{pmatrix}.$$

The expansion coefficients $\Lambda_i$, ($i = 0, \ldots, 8$) can be taken as the analogous of the spectral density Stokes parameters for a monochromatic wave of general form. In fact, for the case of a plane wave that propagates along the $z$ axis, $S_{d3}$ reduces to the form (6). Furthermore, the trace of both $S_{d3}$ and $S_{d2}$, give the intensity of the wave. Therefore, one can assume that the degree of polarization of a quasimonochromatic wavepacket of general form is given by

$$p^2 = \frac{1}{3} \sum_{i=1}^{8} \Lambda_i^2 \Lambda_0^2,$$

as suggested originally by Samson \cite{9} and by Barakat \cite{10}. The explicit expression of the parameters $\Lambda_i$ as a function of the cartesian component of the field is

$$\Lambda_0 = E_x E_x^* + E_y E_y^* + E_z E_z^*,$$
$$\Lambda_1 = E_x E_y^* + E_y E_x^*,$$
$$\Lambda_2 = i (E_x E_y^* - E_y E_x^*),$$
$$\Lambda_3 = E_x E_x^* - E_y E_y^*,$$
$$\Lambda_4 = E_z E_x^* + E_x E_z^*,$$
$$\Lambda_5 = i (E_x E_z^* - E_z E_x^*),$$
$$\Lambda_6 = E_z E_y^* + E_y E_z^*,$$
$$\Lambda_7 = i (E_y E_z^* - E_z E_y^*),$$
$$\Lambda_8 = \frac{1}{\sqrt{3}} (E_x E_x^* + E_y E_y^* - 2E_z E_z^*).$$

As an example of the information that can be gained from the knowledge of the generalized Stokes parameters, consider that the vector $\mathbf{V}$, that gives information on the polarization ellipse, turns out to be

$$\mathbf{V} = (\Lambda_7 \hat{e}_z - \Lambda_5 \hat{e}_y + \Lambda_2 \hat{e}_x).$$

Furthermore, Setälä et al. \cite{11} exploited the generalized Stokes parameters, and in particular Eq. (8), to describe the degree of polarization of the light emitted by a heated surface in the near zone.
6. Field within a sphere containing cavities

We stated in Section 4 that our considerations on the polarization apply to the field scattered in the near zone by an irregularly-shaped particle. Nevertheless, the formalism developed so far may be useful also to describe the polarization of the field in the interior of a particle. In the following Sections we focus on a homogeneous sphere containing eccentric spherical cavities, with the purpose of showing that when such a sphere is illuminated by a linearly polarized wave, the field within the cavities undergo a depolarization: More precisely, the field within the cavities acquires the features of an elliptically polarized wave.

The motivation of our choice is far from trivial. In fact, spheres containing cavities are considered to be good models for simulating the properties of porous cosmic dust grains. The presence in the cavities of circularly polarized ultraviolet light is relevant for understanding the photolytic processes that may give rise to biological homochirality. The results that we are going to discuss may thus affect our ideas on the photochemistry of racemic mixtures of chiral prebiotic molecules that led to a slight unbalance of the two symmetric conformations. This unbalance is considered as a prerequisite to life as we know it [23].

6.1. Calculation of the field

We consider a homogeneous sphere of refractive index \( n_0 \) and radius \( \rho_0 \) containing several spherical inclusions (that may be empty), numbered by an index \( \alpha \), of radius \( \rho_\alpha \) and refractive index \( n_\alpha \). Neither the radii nor the refractive indexes of the inclusions need to be equal to each other. The theory for the calculation of the field scattered by such a model scatterer has been published elsewhere [12, 13, 14], but the technique for calculating the field within the inclusions and for describing its state of polarization has never been discussed before. The geometry is sketched in Fig. 1 where only one inclusion is shown for the sake of clarity. We partition the space into three regions: the external region that is filled by a homogeneous nonabsorptive medium of refractive index \( n \), typically the vacuum; the interstitial region that is filled by the material of the host sphere; the region within the inclusions whose centres lie at \( \mathbf{R}_\alpha \). By assuming that all the fields depend on time through the factor \( \exp(-i\omega t) \), that is omitted throughout, we can define the propagation constants for the three regions described above as

\[
\begin{align*}
k &= nk_v, \quad k_0 = n_0 k_v, \quad k_\alpha = n_\alpha k_v,
\end{align*}
\]

where \( k_v = \omega/c \) is the propagation constant in vacuo. The incident field is the linearly polarized plane wave

\[
E_{\eta} = E_0 \hat{u}_\eta \exp(i \mathbf{k}_I \cdot \mathbf{r}),
\]

where \( \mathbf{k}_I = k\mathbf{\hat{k}}_I \) is the propagation vector, \( \hat{u}_\eta \) is the (unit) polarization vector whose index \( \eta = 1, 2 \) states whether the electric field is parallel (\( \eta = 1 \)) or perpendicular (\( \eta = 2 \)) to a fixed plane of reference through \( \mathbf{\hat{k}}_I \). The incident plane wave field and the field within the inclusions are expanded in a series of the vector multipole fields [15, 14]

\[
\begin{align*}
J^{(1)}_{lm}(\mathbf{r}, K) &= j_l(Kr)X_{lm}(\mathbf{\hat{r}}), \\
J^{(2)}_{lm}(\mathbf{r}, K) &= \frac{1}{K} \nabla \times J^{(1)}_{lm}(\mathbf{r}, K),
\end{align*}
\]

that are regular at the origin, where \( K \) is the propagation constant appropriate to the region of interest, \( j_l(Kr) \) is a spherical Bessel function and \( X_{lm} \) are vector spherical harmonics [1]; in turn, the superscripts 1 and 2 are the values of a parity index \( p \) that distinguishes the magnetic multipoles (\( p = 1 \)) from the electric ones (\( p = 2 \)). The expansion of the scattered field and of the field in the interstitial region requires to use also the multipole fields \( \mathbf{H}^{(p)}_{lm}(\mathbf{r}, K) \) that are identical to the \( \mathbf{J} \) multipole fields except for the substitution of the Hankel function of the first
kind \( h_l(Kr) \) in place of the Bessel functions \( j_l(Kr) \). The \( H \) fields satisfy the radiation condition at infinity.

The field in the external region is the superposition of the incident plane wave and of the field scattered by the whole object and has the expansion

\[
E_{\text{Ext}} = E_0 \sum_{p \text{l} m} \left[ J_{l m}(r,k) W_{l m}^{(p)}(r,k) A_{l m}^{(p)} + H_{l m}(r,k) A_{l m}^{(p)} \right],
\]

where the amplitudes of the scattered multipole fields \( A_{l m}^{(p)} \) bear the index \( \eta \) to recall the polarization of the incident field. The multipole amplitudes of the latter, \( W_{l m}^{(p)} \), encompass all the information on the direction of incidence and on the polarization, and are defined as

\[
W_{l m}^{(p)} = 4\pi i^{p+l+1} \hat{u}_\eta \cdot Z_{l m}(\hat{k}_1),
\]

where the asterisk denotes complex conjugation, and

\[
Z_{l m}^{(1)}(\hat{k}) = X_{l m}(\hat{k}), \quad Z_{l m}^{(2)}(\hat{k}) = X_{l m}(\hat{k}) \times \hat{k}
\]

are transverse harmonics [16]. In the interstitial region, the field is expanded as

\[
E_{\text{Ext}} = E_0 \sum_{p \text{l} m} \left[ \sum_{\alpha} H_{l m}^{(p)}(r,\eta_\alpha, k_\alpha) P_{\eta_\alpha l m}^{(p)} + J_{l m}(r, k_\alpha) P_{\eta_\alpha l m}^{(p)} \right],
\]

whereas the expansion of the field within the \( \alpha \)-th inclusion is

\[
E_{\text{Ext}} = \sum_{p \text{l} m} J_{l m}^{(p)}(r,\alpha, k_\alpha) C_{\eta l m}^{(p)}.
\]

In the preceding equations \( r_\alpha = r - R_\alpha \). Even the amplitudes \( P_{\eta_\alpha l m}^{(p)}, P_{\eta_\alpha l m}^{(p)} \) and \( C_{\eta l m}^{(p)} \) bear the index \( \eta \) to recall the polarization of the incident field. All the unknown amplitudes are

**Figure 1.** Geometry of a sphere containing inclusions. Only one inclusion is shown for the sake of clarity.
determined by the customary boundary conditions across the surface of the host sphere and across the surface of each one of the inclusions. In this respect we note that, since the expansions above contain multipole fields with different origins, using the appropriate addition theorem [14, 17] is necessary to express all the multipole fields with respect to the same origin. In particular, imposition of the boundary conditions across the surface of the host sphere yields a system of linear non-homogeneous equations for the amplitudes of the interstitial field. We write this system in matrix form as

$$\begin{pmatrix} (R)^{-1} + H & J_{-0} \\ R_W J_{0-} & (R_0)^{-1} \end{pmatrix} \begin{pmatrix} P \\ P_0 \end{pmatrix} = \begin{pmatrix} 0 \\ W \end{pmatrix}.$$ 

All the elements of the submatrices that appear in the preceding equation are explicitly defined in refs. [13] and [14]. Once the preceding system has been solved for the amplitudes $P_{\eta \alpha lm}$ and $P_{\eta \alpha 0lm}$, the boundary conditions across the surface of the inclusions yield the relation among these amplitudes and the amplitudes $C^{(p)}_{\eta \alpha lm}$ of the multipole fields within the $\alpha$-th inclusion.

7. Results and discussion
When the theory of the preceding section is applied to cosmic dust grains modeled as homogeneous spheres containing spherical empty cavities, we meet several aspects of the polarization of the field that are worth of a detailed study. Here we chose to restrict our analysis to the dependence of the sense of circular polarization on the geometry and size of the grains. To this end, we assume that the host sphere contains a single cavity, and defer to a further paper the study of spheres containing more than one cavity [18].

Most of the results that we are going to discuss refer to a host sphere of radius $\rho_0 = 100$ nm, because dust grains of this size are the most effective in extinguishing the starlight. Since we are interested in assessing if and to what extent the depolarization depends on the choice of the material, the interstitial region has been assumed to be filled either of astronomical silicates [19] or of amorphous carbon [20] or of ice [21]. We also considered a mixing composed of 30% silicates, 30% amorphous carbon and 40% ice: the dielectric function of this mixture has been

![Figure 2. Dielectric functions that we used in our calculations.](image-url)
Figure 3. $V_s$ at point F for polarization along the $x$ axis simulated using the Bruggeman mixing rule [22]. In this respect, we stress that the Bruggeman mixing rule has been applied to the interstitial material only, whereas the included cavity has been dealt with as a separate entity with refractive index 1. The wavelength dependence of the dielectric functions that we assumed for the various interstitial materials is reported in Fig. 2 in the range $0.1 \leq \lambda \leq 1 \mu m$. Actually, we found that for $\lambda > 1 \mu m$ no depolarization occurs, at least for model particles in the range of size that is appropriate to cosmic dust grains. The incident field is assumed to propagate along the $z$ axis, and both polarizations along the $x$- and the $y$-axis have been considered.

We start by calculating $V_s$ at points F and B of Fig. 1 for the case in which the host sphere contains a single empty cavity with radius $\rho_1 = 73.8$ nm, so that its volume is 40% of the volume of the host sphere. Actually, we considered several positions of the cavity within the host sphere but the result that we present in the following figures refer to the configuration in which the surface of the cavity is tangent to the surface of the host sphere with its center on the radius whose polar angles are $\vartheta_c = 60^\circ$ and $\varphi_c = 30^\circ$. In fact, we found that, at least as concerns the behavior of $V_s$ at points F and B, this is a configuration in which the depolarization effect is well visible. In this respect we stress that, as expected on simmetry considerations, when the center of the cavity lies on z axis $V_s = 0$ both at F and at B.

Figures 3 and 4 show $V_s$ calculated at point F, for polarization of the incident wave along the $x$ and the $y$ axis respectively, when the interstitial material is silicate, amorphous carbon, ice or the Bruggeman mixture. The results at point B are quite similar and are therefore not reported. The depolarization shows little dependence on the choice of the refractive index, the maximum value of $V_s$ occurring in the range $0.1-0.4 \mu m$ for any choice of the material. This is not surprising because a look to Fig. 2 shows that both the real and the imaginary part of the dielectric functions of these materials are rather similar in the wavelength range of interest. In view of the preceding remarks, since in this paper we want to discuss the general features of the depolarization effect, hereafter we report the results only for the case in which the interstitial
material is the Bruggeman mixture. However, the results reported in Figs. 3 and 4 show that
the depolarization is rather small for $\lambda = 0.5 \mu m$ and no depolarization occurs for $\lambda > 1.0 \mu m$.
Let us also stress that the change of the polarization of the incident field inverts almost exactly
the sign of $V_s$, i.e. the sense of the polarization. As changing the polarization is equivalent to a
rotation of the grain around the z axis, we can state that the sense of polarization appears to
depend strongly on $\varphi_c$.

We call the attention of the reader on the fact that for whatever choice of the interstitial
material and for whatever polarization of the incident field, the sense of rotation given by $V_s$
changes its sign at two wavelengths in the range between 0.15 and 0.30 $\mu m$. One of the aims of
this paper is just to focus on the parameters on which the wavelengths of these changes depend.
To this end we calculated $V_s$ for host spheres of radius from 100 nm to 300 nm containing a cavity
whose volume is 40\% of the volume of the host sphere. According to our statement above, the
interstitial material is the Bruggeman mixture. The results at point F are reported in Fig. 5 for
host spheres of radius $\rho_0 = 100$, 150, 200 and 250 nm. It is quite apparent that the wavelength at
which the change of the sense of polarization occurs depends on the radius of the grain although
the volume of the cavity is constantly 40\% of the total volume. The evolution of the changes of
the sense of polarization as a function of the radius of the host sphere, up to $\rho_0 = 300$ nm, are
reported in Fig. 6 The linear dependence of the position of the zeros of $V_s$ on the radius of the
host sphere should be noticed. On the contrary, the behavior is far from a linear one when we
consider the zeros of $V_s$ for host spheres of 100 nm containing a single cavity whose volume goes
from 0.05\% to 50\% of the volume of the host sphere. This is quite evident from the results
reported in Fig. 7. In any case, the results presented so far strongly suggest that the change
of the sense of polarization is fine-tuned by the size of the host sphere as well as by the ratio
of the volume of the host sphere and of the included cavity. Preliminary results for the case
of spheres containing several cavities confirm this suggestion, although the multiple scattering
processes among the cavities and between the cavities and the surface of the host sphere make

Figure 4. $V_s$ at point F for polarization along the y axis
Figure 5. $V_a$ for host spheres of variable size. The volume of the included cavity is, in all instances, 40% of the volume of the host sphere.

Figure 6. Evolution of the first and second change of the sense of polarization as a function of the radius of the host sphere. The included cavity has, in all instances, a volume of 40% of the volume of the host sphere.
more complex the interpretation of the results. Work on this subject is presently in progress.

8. Conclusions
A fundamental feature of the electromagnetic waves is their vector nature which, in turn, implies the need to take into account their state of polarization. For plane waves as well as for spherical waves the description of the state of polarization is effectively given by the Stokes parameters, the practical procedures for whose measurement is well established [3]. The definition of the Stokes parameters, however, lies on the assumption that there exist a well defined and constant direction of propagation of the waves to which the field is constantly orthogonal. Since plane and spherical waves are the most often met both in the theory and in the laboratory practice, one may be led to believe that the Stokes parameters give the most general description of the state of polarization.

In recent years, the researchers dealt with several cases in which waves of general form are involved. In the preceding sections we cited, as representative examples, the field scattered by an irregularly shaped particle in the near zone and the field in the vicinity of a heated surface [11]. In these cases the polarization ellipse has a changing orientation with respect to the direction of propagation that is individuated by the real part of the complex Poynting vector. As a result, one needs a more general description of the state of polarization, that, in turn, can be achieved through a suitable generalization of the Stokes parameters [6]. In Section 5 we stressed the relation between the generalized Stokes parameters and the vector \( \mathbf{V} \) that individuates the normal to the polarization ellipse and whose projection on the Poynting vector \( \mathbf{V}_S \) gives the sense of rotation of the field with respect to the direction of propagation of the wave.

We exploited \( \mathbf{V}_S \) to show that the field within a cavity embedded into a homogeneous sphere undergo a depolarization that gives to the field the features of an elliptically polarized wave. This finding may be important for our understanding of the origins of life. The discovery of amino acids with a significant enantiomeric excess in carbonaceous meteorites [24] offers an attractive alternative to a purely terrestrial origin of life on Earth [25, 26]. This enantiomeric excess...
is, indeed, believed to be generated by selective photolysis by ultraviolet circularly polarized light. However, starlight is either unpolarized or linearly polarized by dichroic extinction by the interstellar medium and, up today, no source of ultraviolet circularly polarized light has been found. According to our calculations, the existence of such external sources may be non necessary. Figures 3 and 4 show that the sense of rotation of the field, on which the formation of biomolecular homochirality depends, changes at least twice at wavelengths that, for host spheres of size appropriate to the cosmic dust grains, occur in the ultraviolet. Thus, the precise determination of wavelength at which these changes occur granted a detailed investigation. In fact, both the aminoacids and the sugars present strong circular dichroic bands just at frequencies in this range [28]. Now, one of the difficulties met by the assumption that biomolecular homochirality is induced by a source of circularly polarized light external to the grains, is just the fact that cutoff mechanisms must be invoked to ensure that the Kuhn-Condon sum rule is satisfied [27]. Our findings show that this is not necessary, as the wavelengths at which the sense of rotation changes are fine-tuned by the geometry of the grains.

Ultimately, although several problems still remain, we are confident that a wise exploitation of the formalism described in this paper may further improve our understanding of the origins of life.

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