New features of the maximal abelian projection  

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After fixing the Maximal Abelian gauge in $SU(2)$ lattice gauge theory we decompose the nonabelian gauge field into the so called monopole field and the modified nonabelian field with monopoles removed. We then calculate respective static potentials and find that the potential due to the modified nonabelian field is nonconfining while, as is well known, the monopole field potential is linear. Furthermore, we show that the sum of these potentials approximates the nonabelian static potential with 5% or higher precision at all distances considered. We conclude that at large distances the monopole field potential describes the classical energy of the hadronic string while the modified nonabelian field potential describes the string fluctuations. Similar decomposition was observed to work for the adjoint static potential. A check was also made of the center projection in the direct center gauge. Two static potentials, determined by projected $Z_2$ and by modified nonabelian field without $Z_2$ component were calculated. It was found that their sum is a substantially worse approximation of the $SU(2)$ static potential than that found in the monopole case. It is further demonstrated that similar decomposition can be made for the flux tube action/energy density.

1. Introduction 

It is well known \cite{1,2,3,4} that after Abelian projection in the Maximal Abelian gauge (MAG) \cite{5,6} the abelian string tension, calculated from the Abelian static potential, is very close to the nonabelian string tension and corresponding Coulomb term coefficient is about 1/3 of that in the nonabelian static potential. The former observation as many others supports the Abelian dominance (for review see e.g. \cite{7}). It was further discovered \cite{8,9,3} that the monopole static potential also has a string tension close to the nonabelian one and small Coulomb term coefficient. These observations are in agreement with conjecture that monopole degrees of freedom are responsible for confinement \cite{10}. 

The role of the other, nonrelated to monopoles, degrees of freedom was investigated to a lesser extent so far. We believe that studying the properties of the observables constructed out of these nonmonopole degrees of freedom will help to understand the properties of the infrared effective action. 

It is then interesting to see what kind of static potential one gets if one switches off the monopole contribution to the gauge field, i.e. if only off-diagonal gluons and the so called photon part of the abelian gluon field are left interacting with static quarks. 

Previously computations of this kind were made in \cite{11,12}, where it was shown that the topological charge, chiral condensate and effects of chiral symmetry breaking in quenched light hadron spectrum disappear after removal of the monopole contribution from the relevant operators. Similar computations were made within the scope of the $Z_2$ projection studies \cite{13}. It was shown that the modified gauge field with projected center vortices (P-vortices) removed pro-
roduces Wilson loops without area law, i.e. lacking confinement property. In fact we want to do a similar thing with monopoles. But we go one step further – we consider the sum of the static potentials \( V_{\text{mod}}(R) \) and \( V_{\text{mon}}(R) \), obtained from the Wilson loops of the modified gauge field (with monopole contribution removed) and from the Wilson loops of the monopole gauge field alone, respectively. We discover that \( V_{\text{mod}}(R) \) can be well fitted by pure Coulomb term and the sum \( V_{\text{mod}}(R) + V_{\text{mon}}(R) \) is a good approximation of the nonabelian static potential \( V(R) \) at all distances considered in our measurements. Furthermore, we find that a similar approximate decomposition holds for the adjoint static potential. We also check the decomposition of the static potential induced by the center projection. The approximate decomposition for the static potential suggests the possibility of such decomposition for the flux tube action/energy density. We also check this possibility.

The paper is organized as follows. In the next section we introduce the necessary definitions and describe details of our computations. In section 3 results for the various static potentials are presented. Section 4 is devoted to the action/energy density results. Finally, we conclude in section 5.

2. Definitions and simulation details

We study SU(2) lattice gauge theory with Wilson action. The abelian projection is made after fixing to the MAG. The abelian projection means coset decomposition of the nonabelian lattice gauge field \( U(s, \mu) \) into abelian field \( u(s, \mu) \) and coset field \( C(s, \mu) \):

\[
U(s, \mu) = C(s, \mu) u(s, \mu). \tag{1}
\]

The Abelian gauge field can be decomposed into a monopole (singular) part \( u_{\text{mon}}(s, \mu) \) and photon (regular) part \( u_{\text{ph}}(s, \mu) \):

\[
u(s, \mu) = u_{\text{mon}}(s, \mu) u_{\text{ph}}(s, \mu) \tag{2}
\]
or for the corresponding angles:

\[
\theta(s, \mu) = \theta_{\text{mon}}(s, \mu) + \theta_{\text{ph}}(s, \mu), \tag{3}
\]

where \( \theta(s, \mu), \theta_{\text{mon,ph}}(s, \mu) \in (-\pi, \pi) \) are defined by relations

\[
u(s, \mu) = e^{i\theta(s, \mu) \sigma_3}, \tag{4}
\]

\[
u_{\text{mon,ph}}(s, \mu) = e^{i\theta_{\text{mon,ph}}(s, \mu) \sigma_3}. \tag{5}
\]

\( \theta_{\text{mon}}(s, \mu) \) satisfies the equation [3]:

\[
\partial_\mu \partial_{\mu} \theta_{\text{mon}}(s, \mu) - \partial_\mu \partial_{\mu} \theta_{\text{mon}}(s, \mu) = 2\pi \partial_\mu m(s, \nu \mu), \tag{6}
\]

where \( \partial_\mu (\partial_{\mu} \mu) \) are lattice forward (backward) derivatives. The Dirac plaquette variable \( m(s, \mu \nu) \in \mathbb{Z} \) is determined by decomposition of the abelian plaquette angle \( \theta(s, \mu \nu) \equiv \partial_\mu \theta(s, \mu) - \partial_{\mu} \theta(s, \mu) \) into regular and singular parts:

\[
\theta(s, \mu \nu) = \overline{\theta}(s, \mu \nu) + 2\pi m(s, \nu \mu), \tag{7}
\]

\( \overline{\theta}(s, \mu \nu) \in (-\pi, \pi] \)

Equation (6) has solution

\[
\theta_{\text{mon}}(s, \mu) = -2\pi \sum_{s' \in Z} D(s - s') \partial_\mu m(s', \nu \mu), \tag{8}
\]

where \( D(s) \) is the lattice inverse Laplacian. This solution satisfies the Landau gauge condition \( \partial_\mu \theta_{\text{mon}}(s, \mu) = 0 \). The monopole gauge field \( \theta_{\text{mon}}(s, \mu) \) defined in eq. (3) reproduces more than 95% of monopoles on our lattices and it reproduces all monopoles in the infinite volume limit. This explains its name. We calculate the usual Wilson loops:

\[
W(C) = \frac{1}{2} \text{Tr} \left( \prod_{l \in C} U(l) \right), \tag{9}
\]

as well as monopole Wilson loops:

\[
W_{\text{mon}}(C) = \frac{1}{2} \text{Tr} \left( \prod_{l \in C} u_{\text{mon}}(l) \right), \tag{10}
\]

and the nonabelian Wilson loops with monopole contribution removed:

\[
W_{\text{mod}}(C) = \frac{1}{2} \text{Tr} \left( \prod_{l \in C} \tilde{U}(l) \right), \tag{11}
\]

where the modified nonabelian gauge field is defined as

\[
\tilde{U}(s, \mu) = C(s, \mu) u_{\text{ph}}(s, \mu). \tag{12}
\]

Note that \( u_{\text{ph}}(s, \mu) \) is abelian projection of \( \tilde{U}(s, \mu) \) and it has very few monopoles in the finite volume and no monopoles in the infinite volume.
Fixing MAG leaves unbroken $U(1)$ gauge symmetry. The general form of this transformation is
\[ \theta'(s, \mu) = \theta(s, \mu) + \partial_\mu \omega(s) + 2\pi n(s, \mu), \]
where $\theta'(s, \mu), \omega(s) \in (-\pi, \pi]$, $n(s, \mu) = 0, \pm 1$. Respectively, the Dirac plaquette variable changes as
\[ m'(s, \nu, \mu) = m(s, \nu, \mu) + \partial_\nu n(s, \mu) - \partial_\mu n(s, \nu). \]
Thus the monopole field eq. (3) depends on the choice of the $U(1)$ gauge:
\[ \theta_{mon}(s, \mu) = \theta_{mon}(s, \mu) + 2\pi n(s, \mu) - \delta(s, \mu), \]
where
\[ \delta(s, \mu) = 2\pi \sum_{s'} D(s - s') \partial_{s'\nu} n(s', \nu) \]
\[ - \frac{1}{N_{sites}} \sum_s n(s, \mu). \]
The monopole Wilson loop $W_{mon}(C)$ is invariant under the change of the monopole field due to the fact that $\sum_{l \in C} \delta(l) = 0$. This is not true for $W_{mod}(C)$ because of its nonabelian character. Indeed, the transformation for $\tilde{U}(s, \mu)$ is
\[ \tilde{U}'(s, \mu) = \Omega(s) \tilde{U}(s, \mu) \Omega(s + \mu) \Delta(s, \mu), \]
where
\[ \Omega(s) = \text{diag}\{e^{(-i\omega(s))}, e^{(i\omega(s))}\}, \]
\[ \Delta(s, \mu) = \text{diag}\{e^{-i\delta(s, \mu)}, e^{i\delta(s, \mu)}\}. \]
This transformation is not a gauge transformation and thus the Wilson loop $W_{mod}$ depends on the choice of the gauge used in the definition of the monopole field eq. (3). This problem is solved by using in the Dirac plaquettes $m(s, \mu)$ in some particular gauge. We choose the Landau gauge defined by the gauge condition:
\[ \max_{\omega} \sum_{s, \mu} \cos(\theta'(s, \mu)). \]
Another possible gauge condition would be the minimization of the number of the Dirac plaquettes:
\[ \min_{n_{\mu}} \sum_{s, \mu, \nu} (m'(s, \mu, \nu))^2. \]
Up to Gribov copies both conditions fix configuration of Dirac plaquettes $m(s, \mu)$ completely and thus $\theta_{mon}(s, \mu)$. In general, results for $W_{mod}$ can be different for different gauge conditions which fix the Dirac plaquette configuration. But for two gauge conditions introduced above we may hope that results are similar because fixing $U(1)$ Landau gauge also strongly reduces the number of Dirac plaquettes and thus we expect that for given lattice configuration number of Dirac plaquettes in these two gauges are close to each other.

We calculated $R \times T$ rectangular Wilson loops $W(R, T)$, $W_{mon}(R, T)$ and $W_{mod}(R, T)$. To extract the nonabelian static potential $V(R)$ link integration and smearing have been employed. Smearing was used in computations of the modified field potential $V_{mod}(R)$. Computations were done at $\beta = 2.5$ on $24^4$ lattices using 100 statistically independent configurations. To fix MAG 10 randomly generated gauge copies fixed by simulating annealing algorithm were used.

3. Static potential decomposition

In Fig. (left) we show the monopole $V_{mon}(R)$ and the modified field $V_{mod}(R)$ potentials. We find that $V_{mon}(R)$ is linear at large distances and has small curvature at small distances, as was observed many times before. Our result for $V_{mod}(R)$ is the first result for this potential. It can be seen from Fig. (left) that this potential is of the Coulombic type. Indeed it can be very well fitted by $V_{mod} - \alpha_{mod}/R$ with $\alpha_{mod} = 0.274(9)$. The fitting curve is shown in Fig. (left). Thus, removing the monopole contribution from the Wilson loop operator leaves us with Wilson loop which has no area law behavior, i.e. confinement property is lost. This result is similar to that obtained in after removing P-vortices.

In Fig. (left) we also compare the sum $V_{mon}(R) + V_{mod}(R)$ with the nonabelian potential $V(R)$. One can see that the nonabelian static potential is well approximated by this sum, i.e.
\[ V(R) \approx V_{mon}(R) + V_{mod}(R). \]
we find that

$$\frac{\left| \delta V(R) \right|}{V(R)} < 0.05,$$

where $$\delta V(R) = V(R) - (V_{\text{mon}}(R) + V_{\text{mod}}(R))$$. This observation can be formulated in the following way: the potential for the static sources, interacting with the nonabelian gauge field $$U(s, \mu)$$ can be approximated by the sum of the potential for the sources, interacting only with the monopole field $$u_{\text{mon}}(s, \mu)$$, and potential for the sources, interacting only with the modified field $$\tilde{U}(s, \mu)$$. All calculated potentials were fitted by the usual linear plus Coulomb functions. Results for the fit parameters are presented in Table 1.

In Fig. 1(right) we compare $$V_{\text{mod}}(R)$$ calculated with and without additional U(1) gauge fixing defined by eq. (21). One can see that without U(1) gauge fixing $$V_{\text{mod}}(R)$$ is substantially lower than it is after U(1) gauge fixing. Fitting with constant plus Coulomb fitting function we found the Coulomb coefficient two times smaller. Thus the approximate decomposition of the potential (23) does not hold without this gauge fixing.

The approximate relation (23) implies that for large $$T$$:

$$\langle W(R, T) \rangle \approx \kappa \langle W_{\text{mon}}(R, T) \rangle \langle W_{\text{mod}}(R, T) \rangle$$ (25)

Indeed we found that for $$R, T \geq 3$$ our data for unsmeared $$\langle W(R, T) \rangle$$ can be fitted by the right hand side of eq. (25) (with unsmeared $$W_{\text{mon}}$$ and $$W_{\text{mod}}$$) with $$\kappa = 0.886(9)$$.

Next we want to address the adjoint potential decomposition. We calculated nonabelian adjoint Wilson loop $$W_{\text{adj}}(R, T)$$, charge two monopole Wilson loop

$$W_{\text{mon},2}(R, T) = W_{\text{mon}}^2(R, T)$$

(26)

and adjoint Wilson loop $$W_{\text{mod,adj}}(R, T)$$ for the modified nonabelian field $$\tilde{U}(s, \mu)$$. From these Wilson loops the respective potentials $$V_{\text{adj}}(R)$$, $$V_{\text{mon},2}(R)$$ and $$V_{\text{mod,adj}}(R)$$ were extracted. In Fig. 2(right) we compare the sum $$V_{\text{mon},2}(R) + V_{\text{mod,adj}}(R)$$ with the adjoint potential $$V_{\text{adj}}(R)$$. As in the fundamental case we see approximate decomposition:

$$V_{\text{adj}}(R) \approx V_{\text{mon},2}(R) + V_{\text{mod,adj}}(R).$$

(27)
Table 1
Parameters of the potentials obtained by fits with function $V_0 - \alpha/R + \sigma R$.

| $aV(R)$          | $\sigma a^2$ | $\alpha$ | $aV_0$  |
|------------------|--------------|----------|---------|
| $aV_{mon}(R) + aV_{mod}(R)$ | 0.0310(3)   | 0.264(6) | 0.577(4) |
| $aV_{mod}(R)$    | 0.0311(3)    | -0.018(4)| -0.006(3)|
| $aV_{Z^2}(R) + aV_{mod,Z^2}(R)$ | 0.0224(15)  | 0.334(20)| 0.668(15)|
| $aV_{Z^2}(R)$    | 0.0249(6)    | -0.024(7)| -0.003(4)|
| $aV_{mod,Z^2}(R)$ | -0.029(6)   | 0.383(10)| 0.686(6) |

The potential $V_{mod,adj}(R)$ looks purely Coulombic. A Coulomb term coefficient $\alpha_{mod,adj} = 0.69(4)$ was found by fitting the data at $R \geq 2$. $V_{mon,2}(R)$ is linear with small curvature as was observed before in [3]. Our result [27] supports the conjecture [17,3] that the abelian charge two potential should be considered as the abelian projection for the adjoint potential.

As we noted above, in Ref. [13] it was shown that in the central gauge after removal of P-vortices the Wilson loop loses confinement property. In the MAG we found that removal of monopoles also leaves the Wilson loop without confinement property. Moreover, we found that the approximate decomposition [28] holds. It is then interesting to check similar decomposition for the central gauge. We made computations in the Direct Central (DC) gauge at $\beta = 2.5$ using half of the set of configurations which were used for computations in the MAG. Results are presented in Fig. 2(right). From comparison of Fig. 1(left) and Fig. 2(right) and from comparison of the respective fitting parameters, presented in Table 1 one can see that approximate decomposition works substantially better in MAG. This is not unexpected for large distances, since it is known that in the center gauges the P-vortex string tension is smaller than the monopole string tension in MAG.

Note that decomposition similar to eq. [24] is impossible for central projection since the charge two central projected Wilson loop is identity.

4. Flux tube profile

In this section we present our results for the flux tube action/energy density decomposition.

Taking derivative with respect to $\beta$ on the left and right hand sides of eq. (25) and dividing by respective expressions we obtain (for $T \rightarrow \infty$) \[^2\]:

$$\frac{\langle SW(R, T) \rangle}{W(R, T)} - \langle S \rangle \approx \left( \frac{\langle SW_{mon}(R, T) \rangle}{W_{mon}(R, T)} - \langle S \rangle \right) \times$$

$$\left( \frac{\langle SW_{mod}(R, T) \rangle}{W_{mod}(R, T)} - \langle S \rangle \right),$$

(28)

where $S$ is the nonabelian action averaged over the lattice. It is then interesting to check whether such approximate decomposition is valid also locally, i.e. whether the following approximate relation holds:

$$\langle F^2(s, \mu, \nu) \rangle_W \approx$$

$$\langle F^2_{mon}(s, \mu, \nu) \rangle_W + \langle F^2_{mod}(s, \mu, \nu) \rangle_W,$$

(29)

where

$$\langle F^2_{f}(s, \mu, \nu) \rangle_W \equiv \frac{\langle W_{f} F^2(s, \mu, \nu) \rangle}{\langle W_{f} \rangle} - \langle F^2(s, \mu, \nu) \rangle,$$

the lattice field strength squared is defined as

$$\frac{1}{2} F^2(s, \mu, \nu) = 1 - \frac{1}{2} \text{Tr} U(s, \mu, \nu),$$

(30)

$F^2_{mon}(s, \mu, \nu)$ and $F^2_{mod}(s, \mu, \nu)$ are defined analogously with respective plaquettes instead of

\[^2\]In this derivation we take into account that the Faddeev-Popov determinant has no explicit dependence on $\beta$. 

In Fig. 3 and Fig. 4 we show various longitudinal action and energy densities defined as follows:

\[
A_L(r, z) = \frac{1}{2} \langle F^2(\vec{s}, 0, 3) \rangle_W + \langle F^2(\vec{s}, 1, 2) \rangle_W,
\]

\[
E_L(r, z) = \frac{1}{2} \left( \langle F^2(\vec{s}, 0, 3) \rangle_W - \langle F^2(\vec{s}, 1, 2) \rangle_W \right),
\]

where \( r \) is the distance from the quark-antiquark axes, \( z \) coordinate along this axes. The distance between sources is \( R/a = 8 \). One can see from these figures that relation \( (23) \) indeed holds with rather good precision. Similar results were obtained for the transverse action and energy densities. In the computations of the action/energy density Wilson loops with \( T/a = 4 \) were used for the nonabelian and modified cases and with \( T/a = 6 \) for the monopole case.

5. Conclusions and discussion

Using abelian projection after fixing MAG we shown that the static potential \( V(R) \) can be approximately decomposed into two components, see eq. (23). Our preliminary results at smaller lattice spacing (\( \beta = 2.6 \)) indicate that in the continuum limit the agreement between left and right hand sides in (23) improves. One term in (23), \( V_{\text{mon}}(R) \), is due to the monopole gauge field contribution, another one, \( V_{\text{mod}}(R) \), is due to the contribution of the modified nonabelian gauge field \( \tilde{U}(s, \mu) \) with monopoles removed. Comparison of the parameters of these potentials, given in Table 1, with the effective string model predictions suggests the following interpretation: at large distances \( V_{\text{mon}}(R) \) is the classical energy of the flux tube and \( V_{\text{mod}}(R) \) is the flux tube fluctuations energy. This conclusion is supported by our results for the adjoint static potential, see Fig. 2(left).

It was checked whether a similar decomposition of \( V(R) \) holds for the center projection after fixing to DC gauge. We calculated projected \( Z_2 \) potential \( V_{Z_2}(R) \) and modified nonabelian field potential \( V_{\text{mod}, Z_2}(R) \). The latter was previously introduced and measured in [13]. We found that decomposition in this case holds with substantially
Figure 3. The longitudinal action densities: nonabelian (squares), monopole (diamonds), and sum of the monopole and modified (circles). (left): as functions of the coordinate $z$ along the $QQ$ axes; (right): as functions of the distance $r_\perp$ from this axis.

Figure 4. Same as in Fig. 3 but for the longitudinal energy densities.
less precision than in the monopole case. This can be seen from comparison of Figs. 1(left) and 2(right), and from Table 1. Both the string tension and the Coulomb coefficient obtained from the fits of the sum $V_{Z_2}(R) + V_{mod,Z_2}(R)$ are in worse agreement with respective parameters of the nonabelian potential than those obtained from the fits of the sum $V_{mod}(R) + V_{mod}(R)$.

The decompositions eq. (23) and eq. (27) are similar to decomposition in the compact $U(1)$ model [13] into the photon and monopole components of the static potential. In that model the action can be respectively decomposed into the monopole and photon parts without interaction term. Contrary to the compact $U(1)$ theory such term is unavoidable in the infrared effective action of the $SU(2)$ gluodynamics since the charge two component of the off-diagonal field $C(s,\mu)$ provides the string breaking of the charge two monopole potential $V_{mon,2}$ [19]. The relations eq. (23) and eq. (27) are hinting that this interaction term should be weak.

Our results for the action and energy densities in the quark-antiquark system, presented in Fig. 3 and Fig. 4 show that the approximate decomposition holds also for these quantities.

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