Scaffolding as a strategy to help student difficulties in proving group problems

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Abstract. This study aims to describe the use of scaffolding as a teaching strategy in helping students difficulties in proving group problems. To achieve that goal, learning is done with scaffolding strategies on group topics. Then the problem proving test is given on a group topic, then the interview is based on the student's proof answers. For this reason, this research is qualitative descriptive research. The subjects were students of Uniorw Tuban Mathematics education who were taking Algebra Structure Courses. Scaffolding strategy design refers to the mapping of proof indicators, and the level of scaffolding developed by [1] explaining, reviewing and restructuring, and developing conceptual thinking, then associated with the scaffolding strategy developed by [2], including offering explanations, inviting student participation, verifying and clarifying student understanding, modeling desired behavior, and inviting students to contribute instructions. To explore the ability to prove, interview guidelines refer to indicators of construction proof. The results showed that the ability of students in the initial steps of proof, proof of flow, using related concepts, the argument of proof, and the language of proof had a very good increase. So the ability of students to prove after using a strategy to pursue scaffolding has increased. The implication of scaffolding as a strategy becomes an alternative teaching choice in abstract algebra subjects.

1. Introduction
Groups are one of the topics in abstract algebra or algebra structure. Prominent characteristics of abstract algebra courses are topics that are loaded with definitions and theorems. Therefore, students are required to understand each definition and theorem they learn and can organize concepts in proving theorems or in problem-solving. According to [3], proving is "complex mathematical activities with logical, conceptual, social and problem-solving dimensions." Rav [4] said that proving is the way mathematicians demonstrate the use of mathematics to solve problems and show the truth of solving proposed problems. According to [5] proof is defined as a valid argument or mathematical claim that becomes an explicit reference to the "key" truth that is accepted for use in solving mathematical problems.

Proof has a very important role for mathematicians and mathematics education. According to [4] the problem of proof not only serves to show a mathematical statement that is true or false but has a broader meaning, for example, “verification, confirmation, systematization, inquiry, communication, and exploration.” Proof according to [6] is a daily practice of mathematicians. For mathematics teachers, teaching proof involves reasoning, trust, and communication to help make learning more
meaningful. Proof can be used to show that students understand and prove mathematics is not just learning to follow certain procedures. According to Reid in [7] explains that teaching students to write mathematical proofs correctly will develop students' deductive reasoning well. Even according to [8] proof as an explanatory aid and is the best help to explain what must be valued the highest.

Proof has a very important role in mathematics and mathematics education, but in class students still have difficulty in proving. [9] explain that the process of proof is undeniably complicated, involves various assumptions identifying student competencies, isolating the properties and structures given, and organizing logical arguments, each of which is not easy. The same thing was expressed by [6] explain that the complex nature of proof makes proof as a valuable tool in learning mathematics, difficulties arise when teaching and applying proof in the classroom. The results of the study [9–14], shows that the ability of students in proof in all classes has a poor understanding and has difficulty building evidence.

Referring to the description above, it is necessary to find a solution to overcome the difficulties of students in proving problems in abstract algebra. According to [15] student difficulties are categorized into two causes. Cause First, students do not understand a good conception in a mathematical proof, so do not know about what constitutes a mathematical proof. As a result, students cannot make valid proof, because they are not sure what proof is valid. The second reason, from students' difficulties, is that they may not understand theorems or concepts so that they are systematically applied incorrectly. The idea of a solution to help student difficulties in proving, according to [16] is through scaffolding. The reason for scaffolding is expressed by [17] that through the application of scaffolding can improve and develop students' understanding in mathematical proof.

Vygotsky defines scaffolding learning as the role of the teacher and others in supporting student development and provides a support structure to reach the next stage or level [18]. Stone [19] said that scaffolding teaching is a mechanism to observe the process by which students are helped to influence their learning potential. An important aspect of scaffolding learning is that helping is temporary because scaffolding by others who are knowledgeable is increasingly diminishing. Therefore, the goal of using scaffolding teaching strategies is for students to become independent and independent learners in problem solvers [20]. In this paper, the authors emphasize scaffolding as a teaching strategy to help students who have difficulty in proving. [18] explained that scaffolding as a teaching strategy for his initial ideas from Lev Vygotsky's sociocultural theory and his concept of the zone of proximal development (ZPD). In scaffolding learning, other more knowledgeable people, provide scaffolding or support to facilitate the development of other students. Scaffolding facilitates students' ability to build on internalize new information and previous knowledge.

Based on the results of [16] studies on mapping the scaffolding process with proof indicators can be summarized in Table 1 as follows.

| Indikator Bukti [21] | Level scaffolding [1] |
|----------------------|-----------------------|
| The First Step of Proof | Explaining, Reviewing, and restructuring |
| Flow of proof | Explaining, Reviewing, and restructuring |
| Related Concepts | Explaining, Reviewing, and restructuring |
| Proof Argument | Reviewing, Developing conceptual thinking, and making connections |
| Key Expression | Reviewing, restructuring |
| Proof Language | Reviewing |

To be implemented in classroom learning, a strategy that supports the results of the mapping is needed. [2] identified five different scaffolding strategies that are exploited by teachers to help students gain conceptual understanding, covering the five types of scaffolding that are commonly
used: offering explanations; invite student participation; verify and clarify student understanding; modeling desired behavior and inviting students to contribute instructions.

Offering explanation: Explanation is an explicit statement that is adjusted to students' understanding of what is being learned, and why, when and how it is used. Invite student participation: Students are allowed to join the process. After the teacher gives illustrations of thoughts, feelings or certain actions needed to complete the task, students have the opportunity to provide input to the part that they understand and know. Verifying and clarifying student understanding: If the understanding that appears makes sense, the teacher verifies student responses; if not, the teacher offers clarification. Modeling desirable behavior: This is teaching behavior that shows how a person must feel, think, or act in certain situations. In the case of proof of modeling, a template framework must be done. Invite students to contribute instructions: students are encouraged to contribute to delivering suggestions on how to complete assignments [2].

Based on the description above, the problem is how is the scaffolding strategy to help students who have difficulty in proving the problem? This trial is limited to group topics in abstract algebra subjects. The design of scaffolding as a strategy is constructed from the results of mapping the mapping indicators with the scaffolding level in Table 1.

2. Research Method
This study aims to describe scaffolding as a strategy to help students difficulty in proving group problems. To obtain a description of scaffolding as a strategy a modification of the results of the scaffolding process mapping and proof indicators [16] was carried out with a scaffolding strategy from [2]. Then do a trial on an abstract algebra class specifically the topic of the Group, on Unirow Tuban mathematics education students. At the end of the trial, there were 3 group problem verification tests. Furthermore, to explore the ability of students in proving the group's problems conducted exploration with proof-based interviews to the research subjects. For this reason, this type of research is qualitative exploration. The research subjects were students from the 2017 mathematics education class, Unirow Tuban, and for the subjects interviewed were 6 students (SI, DI, AA, NA, IS, and NP).

The main instrument in this research is the researcher himself, while the supporting instruments include: the test of proof of group problems and interview guidelines. The group problem proof test covers 3 (three) problems, namely: a) group problem with the set of conditions for membership and standard form operations; b) group problems with sets of ordinary numbers, but non-standard form operations; and c) group problems with the set containing conditions for membership and nonstandard form operations. Interview guidelines refer to indicators of evidence of construction, including the initial steps of proof, the flow of proof, related concepts, arguments, and language of proof. To determine the validity of the instrument, including the validity of the content, constructs, and language both for the proof test and interview guidelines validated by 5 (five) validators. Analysis of the results of the validation shows that the validity of the content, construct, and language of the instrument is valid.

To explore the construction of evidence can be determined by six categories, namely the first step of the proof, proof of flow, related concepts, proof of argument, key expressions, and language of proof [21]. The answer sheets from the group problem verification task and the results of the interviews were analyzed based on evidence construction indicators. Analysis of research data using the model of Milles and Huberman [22] namely data reduction, data presentation, and concluding.

3. Results and Discussion
Scaffolding design as a strategy is focused on improving the ability to prove problems. Indicators of proof of construction refer to six categories, namely the initial step of the proof, proof of flow, related concepts, proof of argument, key expressions, and language of proof [21]. But in this study focuses on five categories, namely: the first step of the proof, proof of flow, related concepts, the argument of proof, and language of proof.
3.1. The first step proof

Mapping the initial steps of proof with the scaffolding level of the results of [16] studies is explaining, reviewing, and restructuring. Activities that can be implemented in explaining include: showing and telling. Reviewing activities, including seeing, touching and verbalizing, prompting and probing, parallel modeling, and explaining and correcting. While restructuring activities include: meaningful context, simplifying problems, repeating students' conversations [1]. Scaffolding strategy to help students in proving difficulties through “offering explanations; invite student participation; verify and clarify student understanding; modeling desired behavior, and inviting students to contribute instructions” [2].

What are the implications? For group proofing include a) closed nature, b) associative nature, c) identity element, and d) inverse. These four characteristics have different definitions, so the initial steps of the proof are certainly different too. The implementation of scaffolding as a strategy in the initial stages of proof, the learning activities can be illustrated as follows:

3.1.1 Explaining. Explain the problem that will be proven, the goal is that students can understand the problem, the strategy: a) offering an explanation to students, maybe they have already faced a similar problem; b) invite student participation in understanding the problem; c) Verify and clarify student understanding; d) give modeling of the initial steps of proof, for example when proving closed nature. The definition of closed nature is "for each x, y member G, then xy member G". The initial modeling step of proof that can be given is "Take x, y of any G member". Another example when proving the nature of existence is identity, the definition is "there are e members of G such that for every x member G applies ex = xe = x". Modeling the initial steps of proof that can be given is "Suppose G has an identity element, for example, e member G" then find e member G such that ex = xe = x applies to any x member G. The keyword is when the definition "for each ...", Then the initial step of the proof is" take any ... ". Likewise, if the definition "exists ...", then the initial step of the proof is "suppose ..."). Then what if the definition contains "for every ..., there is ", such as "for every x member G, there is x\(^{-1}\) member G, so that it applies xx\(^{-1}\) = x\(^{-1}\)x = e", then step beginning of proof "take any x member ..., suppose x has an inverse, for example, x\(^{-1}\) ...", until finally, it gets x\(^{-1}\) that depends on x.

3.1.2 Reviewing. Conduct a review of the work of students when making the initial steps of proof, with the activity of seeing, touching and verbalizing the student's work. Perform prompting and probing to make students aware of mistakes, and help students understand. Give another modeling that is more specific to help students understand, and end by providing clarity and truth. The strategy is as follows: a) invite student participation to examine/discuss the work of other students; b) provide opportunities for students to verify and clarify student understanding that is being proven; and c) provide opportunities for students to contribute instructions to the work of other students.

3.1.3 Restructuring. Perform restructuring of student work when carrying out the initial steps of proof with activities providing meaningful context, and simplifying problems. Steps were taken: a) invite students to participate in restructuring the initial steps of proof; b) jointly verify the initial steps of proof and clarify students' understanding in the restructuring; and c) inviting students to contribute instructions in restructuring the initial steps of the proof.

3.2 Proof Flow

In the mapping proof flowchart indicator with scaffolding level results of [16] studies are the same as the initial steps of proof, namely explaining, reviewing, and restructuring. Activities that can be implemented are also almost similar to the initial steps of the proof. Scaffolding strategies to help student difficulties in the flow of proof can be through explaining; invite student participation; verify
and clarify student understanding; modeling desired behavior, and inviting students to contribute instructions [2]. Scaffolding implementation as a strategy in the proof flow indicator, learning activities can be illustrated as follows:

3.2.1 Explaining. Explain the problem that will be proven, the goal is that students can understand the problem, the strategy is: a) offer students the next step of proof that will be done; b) provide opportunities for students to participate in determining the next proof step; c) verify and clarify the student's idea of the next proof step; and d) if necessary provide systematic modeling of evidence (proof lines), for example when proving closed nature. After the student writes "Take x, y member G", what is the next step? So that the purpose of the proof is clearer, write what will be proven, for example, "Will be shown: xy member G". Furthermore, if the set G contains terms of membership, then describe x and y according to the terms of membership of G, for example, "If x members are G, then x = ..." (write according to the terms of membership of G). If the set G does not contain membership requirements, then x and y do not need to be parsed. Likewise, if the definition includes "for every ..., there is ...", such as "for every x member G, there is x\(^{-1}\) member G, so that it applies xx\(^{-1}\) = x\(^{-1}\)x = e", then flow of evidence can be given modeling as follows: a) Take any x ...; b) will be indicated: there are x\(^{-1}\) members of G, such that xx\(^{-1}\) = x\(^{-1}\)x = e\(^{\circ}\); c) describe the x-membership if G contains membership conditions; d) suppose x has an inverse, for example ...; and e) pay attention ... (perform algebraic operations to arrive at what will be shown)

3.2.2 Reviewing. Conduct a review of the work of students when conducting the flow of proof, with the activity of seeing, touching and verbalizing the student's work. Perform prompting and probing to make students aware of mistakes, and help students understand. Give another modeling that is more specific to help students understand, and end by providing clarity and truth. The strategy is as follows: a) invite student participation to examine/discuss the work of other students; b) provide opportunities for students to verify and clarify student understanding that is being proven; and c) provide opportunities for students to contribute instructions to the work of other students.

3.2.3 Restructuring. Restructuring the work of students when carrying out the flow of proof with activities providing meaningful context, and simplifying problems. Steps were taken: a) invite students to participate in restructuring the flow of proof; b) jointly verify the flow of proof and clarify the understanding of students in the restructuring; and c) inviting students to contribute instructions in restructuring the flow of evidence.

3.3 Related Concepts
The concept indicators connected to mapping with the scaffolding level of the results of [16] studies are the same as the initial steps of proof and the flow of proof, namely explaining, reviewing, and restructuring. The learning activities can be illustrated as follows:

3.3.1 Explaining. Explain the relation of the problem that will be proven with the previous concept, the goal of students being able to utilize prior knowledge, the strategy is: a) offering students the concepts of either the definition, theorem, or proven problems related to the problem to be proven; b) provide opportunities for students to participate in determining related concepts; c) verify and clarify the students' ideas about the proposed concepts; and d) inviting students to contribute about the concepts that are related and support the problem to be proven.

3.3.2 Reviewing. Conduct a review of students' ideas about concepts related to the problem that will be proven, by seeing, touching and verbalizing activities on student work. Perform prompting and probing to awaken student misconceptions, and assist students in understanding concepts, and end by providing clarity and truth. The strategy is as follows: a) inviting student participation to examine/discuss related concepts; b) provide opportunities for students to verify and clarify students'
understanding of concepts that will be used in proving; and c) provide opportunities for students to contribute related conceptual instructions.

3.3.3 **Restructuring.** Do a restructuring of the relationship between the concept and the problem that will be proven by activities providing meaningful context, and simplifying the problem. Steps were taken: a) invite students to participate in restructuring concepts by providing meaningful contexts and simplifying problems; b) jointly verify related concepts and clarify students' understanding of concepts in the restructuring; and c) inviting students to contribute instructions in restructuring the interrelation of concepts.

3.4. **Proof Argument**
Mapping the proof argument with the scaffolding level of the results of [16] studies, namely reviewing, developing conceptual thinking, and making connections. The reviewing activity emphasizes interpreting student actions, modeling, prompting and probing, and clarity and correctness. Learning activities by reviewing the arguments of student evidence with the problem to be proven. The strategy is as follows: a) invite student participation to examine the proof arguments used by students; b) provide the opportunity for students to verify and clarify the student proof arguments used in proving, through interpreting students, and prompting and probing; c) provide opportunities for students to contribute input on the argument of proof; d) provide a model of the commonly used proof argument, the premise and conclusion must be clear and logical, for example, "if .... then ...", "... if and only if ...", "because ..., then ... "; e) through prompting and probing students are motivated to develop conceptual thinking based on existing substantiation arguments; and f) motivate students so that the proof argument can be connected to the problem to be proven.

3.5 **Proof Language**
Mapping the language of evidence with the scaffolding level of the results of [16] studies, namely reviewing. The reviewing activity emphasizes interpreting student actions, modeling, prompting and probing, and clarity and correctness [1]. Learning activities by conducting a review of the language of proof of students with the problem to be proven, with the activity of interpreting the actions of student work. Perform prompting and probing to awaken student misconceptions, and assist students in making proof languages, and end by providing clarity and truth. The strategy is as follows: a) invite student participation to examine the language of proof used by students; b) provide opportunities for students to verify and clarify the language of student evidence used in proving; c) provide opportunities for students to contribute input on the language of evidence; and d) provide a model of proof language that is commonly used, for example, "if ..., then ....", "... if and only if ...", "because ..., then ...".

As an illustration of the three quiz questions, one of the problems with the type of set containing conditions for membership and non-standard operations is as follows.

For example: $G = \{(a, b) \mid a, b \in \mathbb{Q}\}$. Define the operation of the set $G$ as follows: $(a, b) \oplus (c, d) = (a + p \cdot a, bq), \forall (a, b), (p, q) \in G$. Investigate whether the set $G$ of the operation $\oplus$ is a group? Show!

Here the author presents some student answers, after following the learning with scaffolding strategy.
Look at Figure 1. The proof of SI for closed nature is very good. After following the learning with the scaffolding strategy shows that SI in the initial steps of proof, proof of flow, related concepts, an argument of proof, and the language of proof show that SI is very good. However, several subjects were found to be still wrong, including a) error in the initial steps of proof, when the set G contains membership requirements, For example: written “take any x, y ∈ Q” should be a member of G, because the set Q is a condition of membership; b) errors in the flow of proof, the case is the same when the set contains terms of membership, sometimes does not write what must be indicated, so the purpose is not clear, and does not spell out the terms of membership so that subsequent operations are wrong; c) errors in the use of related concepts, students are less able to connect between definitions or problems that have been proven, and can be used to prove the problem at hand; and d) the error in the argument of the proof lies in the formal relationship and the conclusion or cause and effect are sometimes not logical.

Scaffolding as a strategy to overcome student difficulties in proving group problems the results are very positive. This finding is in line with [23]'s suggestion, that the most common scaffolding strategy and one of the most appropriate strategies is to invite student participation to use in step-by-step work. It is natural for a teacher and students to use their test performance to be used as a scaffolding strategy that expresses students' thoughts and why they think that way. In the context of mathematics learning and students 'understanding of mathematical proof, [24] states that "students' capacity to engage in discussions about mathematical proof can be provided through scaffolding" through effective teaching practices. Scaffolding provides an opportunity for students to develop their independence, feelings, and self-confidence in working mathematically [25].

Then we look at the AA answer sheet in proof of the inverse nature in Figure 2.
The AA answer in Figure 2. shows that AA is not careful enough to pay attention to membership requirements, consequently, the conclusions are wrong. In the first step of the proof, proof of flow, and argument, and the language of proof is good. However, the ability to set membership requirements is poor or inaccurate, so the conclusions are wrong. Some students are not careful, because they do not pay attention to the requirements of the membership of the set. However, through the scaffolding strategy in the next exercise, they began to get used to working carefully. According to [19] there are several reasons for the success of the scaffolding process: 1) the scaffolding process for students can motivate problem-solving, 2) the scaffolding method will improve students' social relations, 3) the scaffolding process will increase student confidence in solving mathematical problems, and 4) this educational method can explain the mistakes and misunderstandings of students in the problem-solving process. Meanwhile, according to [26], scaffolding affects students both cognitively and emotionally, which impacts not only the learner's skills and knowledge but also the motivation and self-confidence of students when approaching a task.

**4. Conclusion**

The proof is important in mathematics education and mathematics. However, in the field of evidence, it is difficult for students and teachers to teach it. Alternative solutions that can be done is the modified scaffolding strategy by combining the scaffolding level with the construction indicators in the proof. The ability of students to prove group problems after using a scaffolding strategy has improved. The initial step of the proof, the ability of students to show very well in the initial step of the proof. Constraints are found when the given set contains membership requirements, especially on taking the set element. The flow of proof, the ability of students in the flow of proof also shows very well, the flow of proof that is shown is more systematic. Constraints are found when the given set contains membership requirements, especially the breakdown of membership according to specified conditions. As a result, the operation carried out became wrong. Another obstacle is the simplification that will be shown, so the evidentiary objective tends not to be achieved. Related concepts, students' abilities show
quite well in using related concepts. The disadvantage is that connection skills need to be trained, linking proven definitions, theorems or problems that can be used to prove the problem at hand. The argument of proof, the ability of students shows quite well in the argument of proof. The disadvantage is that proof of argument needs to be practiced, between premises and conclusions sometimes illogical. This is due to initial capabilities, connection related concepts are still lacking. Language of proof, the ability of students to show quite well in the language of proof. The disadvantage is that the language of proof needs to be trained, the language of proof especially cause and effect is sometimes illogical. This is due to weaknesses in the ability of the argument.

The implication of scaffolding as a strategy is to be a choice for lecturers who teach abstract algebra courses to help students in proving difficulties in algebra, especially groups. This research can be developed for other materials, for example, subgroups, normal subgroups, group homomorphism, or ring, so that it can strengthen the conclusion that scaffolding as an abstract algebra teaching strategy to improve the ability to prove abstract algebra problems.

5. References

[1] Anghileri J 2006 Scaffolding practices that enhance mathematics learning J. Math. Teach. Educ. 9 33–52
[2] Roehler L R and Cantlon D J 1997 Scaffolding: A powerful tool in social constructivist classrooms Scaffolding student Learn. Instr. approaches issues 6–42
[3] Weber K 2005 Problem-solving, proving, and learning: The relationship between problem-solving processes and learning opportunities in the activity of proof construction J. Math. Behav. 24 351–60
[4] Hanna G 2001 Gila Hanna Proof, Explanation and Exploration: an Overview 5–23
[5] Stylianides G J 2009 Reasoning-and-proving in school mathematics textbooks Math. Think. Learn. 11 258–88
[6] Imamoglu Y and Togrol A Y 2015 Proof Construction and Evaluation Practices of Prospective Mathematics Educators. 3 130–44
[7] Cyr S 2013 Development of beginning skills in proving and proof-writing by elementary school students [Online] Available www. cerme7. univ. rzeszow. pl/WG/1 Cerme7_WG1_Cyr. pdf.(January 31, 2013)
[8] Hanna G 2000 Proof, explanation and exploration: An overview Educ. Stud. Math. 44 5–23
[9] Healy L and Hoyles C 2000 A study of proof conceptions in algebra J. Res. Math. Educ. 31 396–428
[10] Knuth E J 2002 Teachers’ conceptions of proof in the context of secondary school mathematics J. Math. Teach. Educ. 5 61–88
[11] Miyazaki M 2000 Levels of proof in lower secondary school mathematics Educ. Stud. Math. 41 47–68
[12] Morris A K 2002 Mathematical reasoning: Adults’ ability to make the inductive-deductive distinction Cogn. Instr. 20 79–118
[13] Stylianides G J and Stylianides A J 2009 ABILITY TO CONSTRUCT PROOFS AND EVALUATE ONE’S OWN CONSTRUCTIONS1 Math. Educ. 166
[14] Weber K 2010 Mathematics majors’ perceptions of conviction, validity, and proof Math. Think. Learn. 12 306–36
[15] Weber K 2001 Student difficulty in constructing proofs: The need for strategic knowledge Educ. Stud. Math. 48 101–19
[16] Warli, Cintamulya I and Rahayu P 2020 Scaffolding process based on students diagnostic difficulties in proving group problems by using mathematics mapping J. Phys. Conf. Ser. 1422
[17] Miyazaki M, Fujita T and Jones K 2015 Flow-chart proofs with open problems as scaffolds for learning about geometrical proofs ZDM 47 1211–24
[18] Van Der Stuyf R R 2002 Scaffolding as a Teaching Strategy I . Scaffolding as a Teaching Strategy – Definition and Description Strategy
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[19] Amiripour P, Amir-Mofidi S and Shahvarani A 2012 Scaffolding as effective method for mathematical learning Indian J. Sci. Technol. 5 3328–31
[20] Hartman H 2002 Scaffolding & cooperative learning Hum. Learn. Instr. 23–69
[21] Isnarto I, Wahyudin W, Suryadi D and Dahlan J A 2014 Students’ proof ability: Exploratory studies of abstract algebra course Int. J. Educ. Res. 2 215–28
[22] Milles B M, Huberman A M and Saldana J 2014 Qualitative Data Analysis. Edisi Ketiga
[23] Bikmaz F H, Çelebi Ö, Ata A, Özer E, Soyak Ö and Reçber H 2010 Scaffolding Strategies Applied by Student Teachers to Teach Mathematics Educ. Res. Assoc. Int. J. Res. Teach. Educ. Int. J. Res. Teach. Educ. 1 25–36
[24] Blanton M L, Stylianou D A and David M M 2003 The nature of scaffolding in undergraduate students’ transition to mathematical proof Int. Gr. Psychol. Math. Educ. 113–20
[25] Williams L 2008 Tiering and scaffolding: Two strategies for providing access to important mathematics Teach. Child. Math. 14 324–30
[26] Dennen V P 2004 Cognitive apprenticeship in educational practice: Research on scaffolding, modeling, mentoring, and coaching as instructional strategies Handb. Res. Educ. Commun. Technol. 2 813–28