Thermo-electric figure of merit of quasi-two-dimensional Bi$_2$Te$_3$ nano-structures

M P Singh
Department of Physics, Govt. P. G. College Charkhari (Mahoba), Department of Higher Education, Uttar Pradesh India
E-mail: singhmps74@rediffmail.com

Keywords: quantum well, quantum size effect, figure of merit

Abstract
Quasi-two-dimensional nano-structures are characterized by radical changes in electron and phonon properties. Electron behavior in Q-2D nano-structures is modified by the presence of multiple sub-bands. For well widths smaller than 50 Å simplified model calculations based on the assumption of electrons occupying the lowest sub-band appears to be quite reasonable. For larger width others, parabolic sub-bands need to be considered. The present paper investigates the thermoelectric figure-of-merit of bismuth telluride which is five layered (Te-Bi-Te-Bi-Te) crystal with hexagonal symmetry with six equivalent valleys, in the range 40–300 Å and observe that for 100 Å well width three sub-band model and for 200 Å well width six sub-band model are good approximation at room temperature. Further it is observed that optimum carrier density for 200 Å well width is $3.2 \times 10^{24}$ m$^{-3}$ and maximum of ZT shifts towards lower concentration; and discusses the acceptability of single band approximation.

Introduction
Semiconductor thermoelectric materials are extensively used in power generation and heat pumps. The performance of thermoelectric devices is quantified by dimensionless figure-of-merit $ZT$, is defined as

$$ZT = \frac{\alpha^2 \sigma}{\lambda} T$$

where $\alpha$, $\sigma$, and $\lambda$ refer to the Seebeck coefficient, electrical conductivity, and thermal conductivity of the material, respectively [1–5]. Thermoelectric behavior of bulk material [6–9] is usually understood in terms of electronic and phonon properties, which have bearing on transport coefficients such as electrical and thermal conductivity and the Seebeck coefficient. Both electron and phonon properties show significant changes when the sample size is the order of deBroglie wavelength of the carrier and quantum effects become increasingly evident [10–15]. Search for efficient thermoelectric materials have led to intense theoretical and experimental study of conventional as well as engineered materials. Recent decades have witnessed keen interest on low-dimensional systems [16–21]. Several physical properties such as electron and phonon density of states undergo radical changes, which are quite unlike that in the bulk. For quasi two-dimensional system density of state at any particular energy is the sum over all parabolic sub-band below that point which is step like function and written as

$$\rho_{2D}(E) = \sum_{n=1}^{m} \left( \frac{m^*}{m} \Theta(E - E_n) \right)$$

where $\Theta$ is unit step function, $m^*$ is effective mass, $m$ is number of parabolic sub-band below particular energy $E_n$. The total number of carrier in Quasi-two-dimensional system may be written as carrier concentration per unit area

$$n_{2D} = \sum_{n=1}^{m} \frac{m^* k_B T}{\pi h^2} \int_{E_n}^{E_{max}} \left( \frac{1}{e^{(E - E_p)/k_B T} + 1} \right) d \left( \frac{E - E_p}{k_B T} \right) \Theta(E - E_n)$$

This content is licensed under Creative Commons Attribution 3.0
where $E_F$ quasi fermi energy for Q2D parabolic particular sub-band \[22\]. Moreover, particle interactions may also change significantly such as electron-alloy-disorder, electron-acoustic phonon and all this has an influence on thermoelectric properties in a complex manner \[23–25\].

In Q2-D system restriction on electronic motion along one-direction results is a typical sub-bands structure. Majority of theoretical work has used the simplified model (quantum size limit SQL) in which electron occupy only lowest sub-band. Except at very small width (\(<50\ \text{Å}\) SQL can be only a crude approximation. However, to obtain an accurate estimate of the influence of sub-band structure, one has to go beyond the confines of SQL. This paper deals with the effect of inclusion of multiple sub-bands on transport properties and thermoelectric figure-of-merit of bismuth telluride.

**Outline of theoretical model**

Let us consider Quasi-two-dimensional quantum well structure assuming parabolic multiple energy sub-band and current flow in x-direction and quantum confinement in z-direction. The electron dispersion relation is given by \[26–31\]

$$E = \frac{\hbar^2 k_x^2}{2m_x} + \frac{\hbar^2 k_y^2}{2m_y} + E_n$$

where $m_x$, $m_y$, $m_z$ are the effective mass along x-direction, y-direction and z-direction, respectively. $k_x$, $k_y$, $k_z$ are components of the wave vector. This dispersion relation shows free motion of electron in x-y plane and bound states in z-direction. Details of theoretical formulation for obtaining transport coefficient of a Q-2D structures have been presented in earlier work \[32\]. The electrical conductivity, electronic thermal conductivity, Lorentz factor and Seebeck coefficient can be expressed as

$$\sigma = \frac{1}{2\pi a} \frac{2k_B T}{h^2} \left[ \frac{m_y}{m_x} \right] e^2 \tau_6 A_6$$

where $m_x$, $m_y$, $m_z$ are effective mass along x-direction, y-direction and confined z-direction, $a$ is the well width, $k_x$, $k_y$ are components of the wave vector. This dispersion relation shows free motion of electron in x-y plane and bound states in z-direction. Details of theoretical formulation for obtaining transport coefficient of a Q-2D structures have been presented in earlier work \[32\]. The electrical conductivity, electronic thermal conductivity, Lorentz factor and Seebeck coefficient can be expressed as

$$\lambda_e = \left( \frac{k_B}{e} \right)^2 \sigma T$$

$$L_f = \left[ A_1 + 2A_2 + A_3 \right] \frac{A_6}{A_6} - \left[ \frac{A_4 + A_5}{A_6} \right]^2$$

$$\alpha = \frac{k_B}{e} \left[ \frac{E_F}{k_B T} \right] - \frac{A_4 + A_5}{A_6}$$

where coefficients $A_i (i = 1–6)$ expressed as

$$A_1 = \sum_{n=1}^{m} \int_{0}^{\infty} x^3 \left( -\frac{\partial f_n}{\partial x} \right) dx$$

$$A_2 = \sum_{n=1}^{m} \int_{0}^{\infty} x^2 \left( -\frac{\partial f_n}{\partial x} \right) dx$$

$$A_3 = \sum_{n=1}^{m} \int_{0}^{\infty} x \left( -\frac{\partial f_n}{\partial x} \right) dx$$

$$A_4 = \sum_{n=1}^{m} \int_{0}^{\infty} x \left( -\frac{\partial f_n}{\partial x} \right) dx$$

$$A_5 = \sum_{n=1}^{m} \int_{0}^{\infty} x \left( -\frac{\partial f_n}{\partial x} \right) dx$$

$$A_6 = \sum_{n=1}^{m} \int_{0}^{\infty} x \left( -\frac{\partial f_n}{\partial x} \right) dx$$

where $m$ is effective number of quantized conduction parabolic sub-bands below particular energy $E_{mc}$. Fermi distribution function $f_n = \frac{1}{\exp^{\xi} - 1}$, reduced carrier energy $x = \frac{E - E_n}{k_BT}$ and $\xi = \frac{E_n}{k_BT}$. Reduced Fermi energy $\xi = \frac{E_n}{k_BT}$ and reduced $\eta_n = \frac{E_n}{k_BT}$. One can be write for quasi-two-dimensional system $\tau_{ac} = \frac{\tau_6}{\tau_0}$. For a quasi-two-dimensional system acoustic phonon scattering relaxation time is given by $\frac{1}{\tau_{ac}} = \frac{3\pi^2 m \xi^2}{\rho \partial^2 \omega}$, $\Sigma$ is
deformation potential, and \( \rho \) is material density \([33, 34]\). From the expression of relaxation time for Q2D one may observe that relaxation time is independent of energy and depends upon the width of the layer.

**Thermoelectric figure-of-merit**

Q-2D structures with their somewhat unusual transport behavior in special situations, provide useful thermoelement materials. The primary aim of this paper is to exploit these properties to obtain a high thermoelectric performance exemplified the figure-of-merit \( ZT \) defined in equation (1). The thermal conductivity \( \lambda \) is expressed as a sum of electronic and lattice contributions

\[
\lambda = \lambda_e + \lambda_l
\]

(13)

On account of equations (3), (4) and (5) one can write electronic contribution to thermal conductivity

\[
\lambda_e = \left( \frac{k_b}{e} \right)^2 \frac{1}{2\pi a} \frac{2k_b T^2}{h^2} \sqrt{m_e e} \frac{v_B}{\tau_0} \left[ \frac{A_1 + 2A_2 + A_3}{A_6} - \left( \frac{A_4 + A_5}{A_6} \right)^2 \right] A_6
\]

since confinement direction Z-direction and current flow along X-direction one can defined mobility \( \mu_x = \frac{e\omega}{m_x} \)

The thermal conductivity \( \lambda \) is given as

\[
\lambda = \left( \frac{k_b}{e} \right)^2 \frac{1}{2\pi a} \frac{2k_b T^2}{h^2} \sqrt{m_e m_x e} \mu_x \left[ \frac{A_1 + 2A_2 + A_3}{A_6} - \left( \frac{A_4 + A_5}{A_6} \right)^2 \right] A_6 + \lambda_l
\]

(15)

The figure-of-merit of a quasi-two-dimensional structure with the help of the equations (1), (3), (6) and (15) can be expressed as

\[
ZT = \frac{\left[ \frac{\tau_x}{\tau_0} - \frac{A_1 + A_2}{A_6} \right]^2 A_6}{\left[ \frac{A_1 + 2A_2 + A_3}{A_6} - \left( \frac{A_4 + A_5}{A_6} \right)^2 \right] A_6 + \frac{1}{B}}
\]

(16)

where the expression \( B \) is sensitive upon materials properties of quantum well and \( B \) is given by as \([26–29]\)

\[
B = \frac{k_b^2 T^2 (m_e m_x)^{\frac{1}{2}} \mu_x}{\pi h^2 e \lambda_l a}
\]

(17)

The figure-of-merit may be optimized with respect to variations of \( B \) which depends upon intrinsic properties of the material. \( B \) is inversely proportional to width of layer ‘\( a \)’ and proportional to mobility of the carriers along x-direction. The large values of carrier mobility and small width may enhance the figure-of-merit. The figure-of-merit also depends upon doping of the material. In quasi two-dimensional structures due to quantization of conduction sub-bands doping of the material depends itself on layer width. Thus, for optimization of the figure-of-merit multiple parabolic sub-band model must be taken into account. The bismuth telluride has six equivalent valleys along slightly different orientations on the Brillouin zone. For simple numerical estimation of \( ZT \) it may be assumed that all six valleys have same orientation. Thus, expression for \( B \) equation (17) should be modified and is given by

\[
B = \frac{N_e k_b^2 T^2 (m_e m_x)^{\frac{1}{2}} \mu_x}{\pi h^2 e \lambda_l a}
\]

(18)

where \( N_e \) is number of equivalent valleys.

The lattice thermal conductivity can be modeled within the framework of the Boltzmann equation approach under the relaxation time approximation. One can express lattice thermal conductivity as

\[
\lambda_l = \frac{1}{3} \sum_j \int d\omega \; S_j(\omega) v_j \tau'
\]

(19)

where \( d\omega \; S_j(\omega) \) is the contribution to the specific heat form modes of polarization \( j \) at frequency \( \omega \), \( v_j \) is phonon group velocity, and \( \tau' \) is relaxation time depends upon temperature \([23]\). Reduction of \( \lambda_l \) is one of the primary concerns in any thermoelectric performance enhancement effort. In two-dimensional structures of interest to us the lattice thermal conductivity can be effectively reduced through boundary scattering and
through the alteration of phonon spectrum in low-dimensional structures [35–39]. The phonon transport in bulk materials is almost unaffected by the sample size as the phonon mean free path is already orders of magnitude lower than the sample size due phonon-phonon umklapp scattering. The sample size becomes effective when it is almost same or lower than the mean free path due to other processes. In this situation quantum effects become significant with radical changes in electronic and phonon properties. This leads to further reduction of the thermal conductivity due to changes in group velocity, density of states and scattering mechanism. The modification of the phonon group velocities and dispersion due to spatial confinement leads to a significant increase of the phonon relaxation rates, and as a result, there is a significant drop in the lattice thermal conductivity. These results have been used in the estimation of the thermoelectric figure-of-merit in bismuth telluride quantum wells [38, 39].

Results and discussions

The theoretical model described here has been applied to polycrystalline bismuth telluride. Various relevant parameters used in calculations are shown in table 1. In figure 1 we presented the power factor $\alpha^2\sigma (W/mK^2)$ plotted against with well width $a(m)$ ranging from 40 Å to 300 Å at 300 K for reduced Fermi Energy $\xi = 0.5$ with multiple parabolic sub-band for $m = 1$, $m = 2$ and $m = 3$. One can observe that quantum well width below 45 Å size quantum limit (SQL) are good approximation for power factor at 300 K for reduced Fermi Energy $\xi = 0.5$, however above this other parabolic sub-band are too important. The optimum power factor for SQL is 21.29 $\left(W/cm^2K\right)$ at the quantum well width 55 Å, when we have considered two parabolic sub-band then optimum power factor is 24.83 $\left(W/cm^2K\right)$ at the quantum well width 90 Å and when we have considered three parabolic sub-band then optimum power factor is 27.05 $\left(W/cm^2K\right)$ at the quantum well width 115 Å. That is with increase in number of sub-bands optimum power factor increased and maxima shifted towards larger quantum well width. The lattice contribution and electronic contributions to thermal conductivity of quantum wells depends upon well width. In figure 2 we have shown that variation of lattice thermal conductivity $\lambda_L (W/mK)$ with well width $a(m)$ at 300 K and variations of electronic thermal conductivity $\lambda_e (W/mK)$ with well width $a(m)$ at 300 K for reduced Fermi Energy $\xi = 0.5$ with multiples parabolic sub-bands for $m = 1$, $m = 2$ and $m = 3$. One can observe that for SQL optimum contribution of electronic thermal conductivity is 0.1796(W/mK) at 85 Å for 300 K and reduced Fermi Energy $\xi = 0.5$; for two parabolic sub-bands optimum contribution of electronic thermal conductivity is 0.254(W/mK) at 100 Å and for three parabolic sub-bands optimum contribution of electronic thermal conductivity is 0.2961(W/mK) at 120 Å. The lattice thermal conductivity decreases with decrease in quantum well width due to increase of phonon relaxation rate and effective mass of carriers. These results have been used in the estimation of the thermoelectric figure-of-merit in bismuth telluride quantum wells [38, 39].
Table 1. Physical parameter of Bi\textsubscript{2}Te\textsubscript{3} used in the calculations [28].

| $m_e/m_0$ | $m_t/m_0$ | $m_s/m_0$ | $N_s$ | $\mu_s$ | $a_0$ | $c_0$ | $v$ | $M(\text{Bi})$ | $M(\text{Te})$ |
|----------|----------|----------|------|--------|------|------|----|---------|---------|
| 0.02     | 0.08     | 0.32     | 6    | 1200 cm\textsuperscript{2} V s\textsuperscript{-1} | 4.3 Å | 30.3 Å | 3.0 × 10\textsuperscript{7} m s\textsuperscript{-1} | 3.47 × 10\textsuperscript{-25} kg | 2.12 × 10\textsuperscript{-25} kg |
conductivity is reduced with a reduction in well width and electronic thermal conductivity show an increase. The reduction in lattice thermal conductivity is more drastic as compared to the rise in the electronic part. However, as reduction of quantum well width electronic contribution of thermal conductivity attains optimum and then decreases. For three parabolic sub-bands both lattice and electronic contribution of thermal conductivity have equal contribution from below 95 Å, and value of each contribution is $0.2816 \text{ W m K}^{-1}$ at 95 Å; for two sub-bands from below 85 Å and value of each contribution is $0.2493 \text{ W m K}^{-1}$ at 85 Å; and for SQL from below 65 Å and value of each contribution is $0.1631 \text{ W m K}^{-1}$ at 65 Å. Total thermal conductivity decreases as the phonon contribution is large compared to the electronic contribution. Figure 3 displays variation of dimensionless figure-of-merit $ZT$ with reduced Fermi Energy $x$ at $T=300 \text{ K}$ for multiple parabolic sub-bands $m = 1, m = 2$ and $m = 3$. (a) Well width $a = 50 \text{ Å}$ (b) well width $a = 100 \text{ Å}$. In the figure 4 we display variation of figure-of-merit $ZT$ with reduced Fermi Energy $\xi$ and with carrier concentration $n_e (m^{-3})$ for well width $a = 200 \text{ Å}$ at $T = 300 \text{ K}$ for multiple parabolic sub-bands $m = 1, m = 2, m = 3, m = 4, m = 5$ and $m = 6$. Figure 5 shows variation of dimensionless figure-of-merit $ZT$ with well width $a(m)$ at 300 K for reduced Fermi Energy $\xi = 0.5$ with multiple parabolic sub-bands for $m = 1, m = 2$ and $m = 3$. At a well width ‘a’ of 50 Å one band model ($m = 1$ i.e. size quantum limit, SQL) presents approximately correct result. After consideration of up to two or three bands there is a minor change registered as compared to one band model. The maximum figure of merit $(ZT)$ approaches a value of 3.0 at the 50 Å at 300 K for reduced Fermi Energy $(\xi = 0.5)$. For 100 Å and 200 Å of well width one band model does not provide a correct representation. As well width increases more quantized conduction sub-bands are available for transportation of electron. Figures 3(a) & (b) and 4(a) & (b) display such behavior. The validity of SQL is quite obvious for low well widths. One band model results are valid for well width 90 Å [26]. However, we observe that with increase in well-width more and more sub-bands drop below the Fermi level and contribute to transport; and as well-width increases sub-bands come to nearer to each other and carrier have sufficient thermal energy to hop from one sub-band to other sub-band. From figures 3(b) & 4 one can observe that for 100 Å well width three sub-band model and for 200 Å well width six sub-band model are good approximation at room temperature. Further it is observed that optimum carrier density for 200 Å well width is $3.2 \times 10^{24} \text{ m}^{-3}$ and maximum of $ZT$ shifts towards lower concentration. In each sub-band carriers may acquire different velocities. Below 100 Å one observes a rapid rise in $ZT$. The curves for various sub-bands start coming close below about 60 Å indicating the validity of SQL. Above this SQL is no longer a valid approximation. Figure 6 displays percentage change in $ZT$ over the single
band model SQL at 300 K for reduced Fermi Energy $\xi = 0.5$ and is defined as $PCZT = \frac{(ZT)_{m_1} - (ZT)_{m_2}}{(ZT)_{m_3}} \times 100$ and $PCZT = \frac{(ZT)_{m_1} - (ZT)_{m_2}}{(ZT)_{m_3}} \times 100$. Deviation from SQL appears to be more pronounced for larger well widths. For three sub-bands the percentage change in ZT goes from about 35 to 58 as width increases from 105 to 200 Å.

In conclusions, the main results of Bi$_2$Te$_3$ quantum well in which thermal gradient present along x-axes and confinement along z-axes (1) as number of parabolic quantized sub-bands increases optimum power factor increased and maxima shifted towards larger quantum well width. (2) The lattice thermal conductivity is reduced with a reduction in well width. (3) The electronic thermal conductivity shows slow increase and attains a

Figure 3. Variation of figure-of-merit ZT with reduced Fermi Energy $\xi$ at $T = 300$ K for multiple parabolic sub-bands $m = 1, m = 2$ and $m = 3$. (a) Well width $a = 50$ Å (b) well width $a = 100$ Å.
maximum and then decreases as quantum well width reduced. (4) At a well width ‘a’ of 50 Å one band model (size quantum limit, SQL); for 100 Å well width three sub-band model and for 200 Å well width six sub-band model are presents approximately correct result at room temperature. (5) The optimum carrier density for 200 Å well width is \(3.2 \times 10^{24} \text{ m}^{-3}\). (6) The curves for various sub-bands start coming close below about 60 Å indicating the validity of SQL. (7) The maximum figure of merit \((ZT)\) approaches a value of 3.0 at the 50 Å at 300 K for reduced Fermi Energy \((\xi = 0.5)\). These properties of quantum wells can be utilized to optimized thermoelectric efficiency.
Acknowledgments

MPS is thankful to Dr Ambesh Dixit Assistant Professor IIT Jodhpur India, the Principal Dr H N Das Government Post Graduate College Charkhari Mahoba, Assistant Director Dr Sanjay Kumar Singh Department of Higher Education Allahabad Uttar Pradesh and Dr C M Bhandari Ex-Professor Department of Physics Allahabad University Allahabad for their valuable discussion and support.

Figure 5. Variation of figure-of-merit $ZT$ with well width $a(m)$ at 300 K for reduced Fermi Energy $\xi = 0.5$ with multiple parabolic sub-bands for $m = 1$, $m = 2$ and $m = 3$.

Figure 6. Variation of Percentage change in figure-of-merit $ZT$ with well width $a(m)$ at 300 K for reduced Fermi Energy $\xi = 0.5$ with respect to size quantum limits (SQL) for parabolic sub-bands $m = 2$ and $m = 3$. 
ORCID iDs

M P Singh @ https://orcid.org/0000-0002-4164-9856

References

[1] Ioffe A F 1957 Semiconductor Thermoelements and Thermoelectric Cooling (London: Infosearch)
[2] Drabble J R and Goldsmid H J 1961 Thermal Conduction in Semiconductors (London: Pergamon Press Inc.) ch 4, p 105
[3] Bhandari C M and Rowe D M 1983 Modern Thermoelectrics (Virginia: Reston publishing company, INC.)
[4] Bhandari C M and Rowe D M 1988 Thermal Conduction in Semiconductor (New Delhi: Wiley Eastern Limited) Ch-3, 4
[5] Nolas G S, Sharp I W and Goldsmid H J 2001 Thermoelectrics–Basic Principles and New Material Development (Heidelberg: Springer Series in Materials Science-45)
[6] Simon R 1962 J. Appl. Phys. 33 1830
[7] Bhandari C M and Rowe D M 1985 J. Phys. D: Appl. Phys. 18 873
[8] Bhandari C M 1994 CRC Handbook of Thermoelectrics ed D M Rowe (Boca Raton, FL: CRC Press) ch-4, 5, 6
[9] Singh M P and Bhandari C M 2004 Pramanac. Journal of Physics 62 1369
[10] Beenakker C W J and Houten H V 1991 Quantum transport in semiconductor nanostructures Solid State Physics: Advanced in Research and Application ed H Ehrenreich and D Turnbull (SanDiego USA: Academic Press Inc.) vol-44
[11] Dresselhau M S 2001 Solid State Physics Part-1 Transport Properties of Solids 6.732 Fall, chap-9, pp-139
[12] Rosas A R, Riera R, Marin I L and Compo G 2002 Handbook of Thin Film Materials ed H S Nalwa vol-5 (San Diego USA: Academic) ch-6
[13] Johnson E A Electron in quantum semiconductor structures Low-Dimensional Semiconductor Structures ed K Barnham and D Vvedensky (Cambridge: Cambridge University Press) ch-2, 4, 6
[14] Singh M P and Bhandari C M 2003 Solid St. Comm. 127 649
[15] Singh M P and Bhandari C M 2003 Indian Journal of Pure and Applied Physics 41 950–3
[16] Mahan G D 1998 Good thermoelectrics Solid State Physics ed H Ehrenreich and F Saepen (San Diego: Academic Press) vol-51
[17] Ibragimov G V 2002 J. Phys: Condens. Matter 14 8245–8152
[18] Dresselhau M S et al 2001 Quantum wells and quantum wires for potentials thermoelectric application Recent Trends in Thermoelectric Materials, Research III, Semiconductor and Semimetals (New York: Academic) ch T M Trritt vol-71
[19] Bejinari I and Kantser V 2008 Phys. Rev. B 78 115322
[20] Yeigel O C and Srivastava G P 2014 Journal of Philosophical Magazine, Part B: Condensed Matter Physics 94
[21] Poudel B et al 2008 Science 320 634–8
[22] Mao J, Liu Z and Ren Z 2016 NPJ Quantum Materials 2016 16028
[23] Harrison P 2005 Quantum Well, Wires and Dots (Padstow, UK: TJ International) Ch. 2 page 24–9
[24] Klemens P G 1958 Thermal conductivity and lattice vibrational mode Solid State Phys ed F Seitz and D Turnbull (New York: Academic) vol 7, page 1–94
[25] Ridley B K 1982 J. Phys. C. Solid State Physics 15 5899–917
[26] Ibragimov G B and Bhandari C M 2002 J. Phys.: Condens. Matter 14 8143–52
[27] Hicks L D and Dresselhau M S 1993 Phys. Rev. B 47 12727
[28] Hicks L D and Dresselhau M S 1993 Phys. Rev. B 47 16631
[29] Sofo J O and Mahan G D 1994 Appl. Phys. Lett. 65 2690
[30] Dresselhau M S, Dresselhau G, Sun X, Zange Z, Cronin S B and Koga T 1999 Conference Proceedings Physics of Solid State 41 679
[31] Rogacheva E I et al 2002 Appl. Phys. Lett. 80 2690
[32] Rogacheva E I et al 2003 Nanotechnology 14 53
[33] Singh M P and Bhandari C M 2005 Solid St. Comm. 133 29
[34] Ridley B K 1981 Quantum Process in Semiconductors (New York: Oxford university press)
[35] Ridley B K 1997 Electron Phonon in Semiconductor Multilayers (USA: Cambridge university Press) Chap. 2, 11
[36] Khitun A, Wang K L and Chen G 2000 Nanotechnology 11 327
[37] Venkatasubramanian R, Siivola E, Colpitts T and O’Quinn B 2001 Nature 413 597
[38] Balandin A and Wang K L 1998 J. Appl Phys. 84 6149
[39] Balandin A and Wang K L 1998 Phys. Rev. B 58 1544