Cylindrical neutrosophic single-valued number and its application in networking problem, multi-criterion group decision-making problem and graph theory

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Abstract: In this study, the authors envisage the neutrosophic number from various distinct rational perspectives and viewpoints to give it a look of a conundrum. They focused and analysed neutrosophic fuzzy numbers when indeterminacy and falsity functions are dependent on each other, which serves an indispensable role for the uncertainty concept. Additionally, the idea of cylindrical neutrosophic single-valued number is focused here, when the indeterminacy and falsity functions are dependent to each other using an influx of different logical and innovative graphical representation. They also developed the score and accuracy function for this particular cylindrical neutrosophic single-valued number and analysed some real-life problems like networking critical path model problem and minimal spanning tree problem of operation research field when the numbers are in cylindrical neutrosophic ambiance. They also introduced a multi-criterion group decision-making problem in this cylindrical neutrosophic domain. This noble thought will help us to solve a plethora of daily life problems in the neutrosophic arena.

1 Introduction

The uncertainty theory plays an influential role in handling various real-life models in the field of science and engineering. In the current era, the multi-criteria decision making (MCDM) process has been attracted much attention to several researchers as it can nicely handle many real-life challenging problems in many frontline areas like a financial investment, recruitment policies, clinical diagnosis of disease, design of the complex circuit, etc. It is not an over-stated fact that the fuzzy set theory plays a very crucial role in decision-making problems, especially when decision-makers work in an uncertain environment. The theories of uncertainty have geared up dramatically after the introduction of the fuzzy set by Zadeh [1] and intuitionistic fuzzy set by Atanassov [2]. Avarasov presents the legendarine idea of an intuitionistic fuzzy set where he introduced the concept of membership function as well as a non-membership function of belongingness. Liu and Yuan [3] ignited the concept of a triangular intuitionistic fuzzy set, which is the congenial mixture of triangular fuzzy set and intuitionistic fuzzy set. Ye [4] introduces the idea of trapezoidal intuitionistic fuzzy set and successfully applied it in multi-criteria decision-making problems. Smarandache [5] manifests the idea of a neutrosophic set. Neutrosophic set considers the truth membership function, the indeterminacy membership function, and the falsity membership function simultaneously, which are more constructive and as well as applicable rather than the general fuzzy and intuitionistic fuzzy set concepts (see Fig. 1).

Neutrosophic number mainly deals with undetermined, insufficient and inconsistent data. With the advancement of research day-by-day, we see that a single-valued neutrosophic set concept is introduced, and basically, it is the extension part of neutrosophic set Wang et al. [6]. Ye [7] formulates the concept of simplified neutrosophic sets, and Peng et al. [8, 9] developed some ideas on new operations and aggregation operators. Recently, researchers have given their attention on neutrosophic set they developed a different kind of extensions of the neutrosophic set, such as different forms of triangular neutrosophic sets [10], bipolar neutrosophic sets [11–14] and multi-valued neutrosophic sets [15].

Recently, many researchers and scientists started their work in neutrosophic domain and they introduced some ideas which can be applied in the field of engineering problem. There are still different approaches that can be made in defining a cylindrical neutrosophic single-valued number when the indeterminacy and falsity functions are dependent.

Needless to say that the aspects of neutrosophic sets are very much pertinent to dealt our real-life problems in a more meaningful way. Smarandache [5] defines that the sum of the belongingness function, non-belongingness function, and indeterminacy function is less than or equal to 2 whenever two peripherals are dependent. In contrast, the third peripheral one is independent of the other two. We focused on the cylindrical neutrosophic single-valued number, which is a generalised version of the dependence neutrosophic number developed by Broumi et al. [16]. Here, we consider the sum of the squares of two dependent membership functions less than or equals to 1 and third membership function equals to 1 (considering only the 1st quadrant, since all the membership functions belong to [0, 1]) which is geometrically denotes a 1/4 the portion of a cylinder having a radius of 1 unit.

In this paper, we have introduced the concept of cylindrical neutrosophic number in the neutrosophic domain and analysed it in detail, which is previously coined by Smarandache [17]. All general neutrosophic numbers will satisfy the cylindrical neutrosophic single-valued number. Additionally, we also manifested an important score and accuracy function in the cylindrical neutrosophic arena and apply it in various problems like critical path problem, minimal spanning tree problem and multi-criteria group decision-making problem. Also, comparison analysis is done with the established methods in each case and sensitivity analysis is performed in MCGDM technique. Some important articles [17–23] are also published in this field recently.
1.1 Structure of the paper

In this paper, Section 1 contains the introduction part, Section 2 includes some preliminaries of the proposed work, Section 3 represents the score and accuracy function in the cylindrical environment, Section 4 encompass networking problem and its comparisons, Section 5 filled with MCGDM problem in the cylindrical arena, Section 6 contains minimal spanning tree problem, its comparisons, finally, Section 7 covers the conclusion of the paper.

2 Mathematical preliminaries

Definition 1: (Neutrosophic set): A set \( \tilde{S} \) in the universal discourse \( X \), symbolically denoted by \( x \), is called a neutrosophic set \( [5] \) if \( \tilde{S} = \{x; T(S) x \}, I(S) x \}, F(S) x \} x \in X \} \), where \( T(S) x \rightarrow [0, 1] \) is said to be the true membership function, which has the degree of belongingness, \( I(S) x \rightarrow [0, 1] \) is said to be the indeterminacy membership, having a degree of uncertainty, and \( F(S) x \rightarrow [0, 1] \) is said to be the incorrect membership, which has the degree of non-belongingness of the decision-maker.

\[ T(S) x \}, I(S) x \}, F(S) x \} \rightarrow [0, 1] \]

Definition 2: (Single-valued neutrosophic set): A neutrosophic set \( \tilde{S} \) in Definition 1, also known as single-valued neutrosophic Set (signeuS) if \( x \) is a single-valued independent variable.

\[ \tilde{S} = \{x; T(S) x \}, I(S) x \}, F(S) x \} x \in X \} \]

If there exist three points \( a_0, b_0 \) and \( c_0 \), for which \( T(S) a_0 \} = 1, I(S) b_0 \} = 1 \) and \( F(S) c_0 \} = 1 \), then the signeuS is called neut-normal.

Definition 3: [Single-valued neutrosophic number]: Single-valued neutrosophic number \( \beta \) is defined as \( \tilde{\beta} = \{(p^i, q^i, r^i, s^i); \alpha, \beta, \gamma \in [0, 1] \} \) Here \( T(S) x \rightarrow [0, \alpha] \), the indeterminacy membership function \( I(S) x \rightarrow [\beta, 1] \) and the fallacious membership function \( F(S) x \rightarrow [\gamma, 1] \} \) are defined as follows:

\[ T(S) = \begin{cases} \frac{p^i}{\alpha} & \frac{1}{q^i} \leq x \leq q^i \\ \frac{r^i}{\gamma} & \frac{1}{s^i} \leq x \leq s^i \\ 0 & \text{otherwise} \end{cases} \]

\[ I(S) = \begin{cases} \frac{r^i}{\beta} & \frac{1}{s^i} \leq x \leq s^i \\ 1 & \text{otherwise} \end{cases} \]

\[ F(S) = \begin{cases} \frac{p^i}{\gamma} & \frac{1}{q^i} \leq x \leq q^i \\ \frac{r^i}{\beta} & \frac{1}{s^i} \leq x \leq s^i \\ 1 & \text{otherwise} \end{cases} \]

Definition 4: (Neutrosophic set: when indeterminacy and falsity functions are dependent \( [5] \)): A set \( \tilde{S} \) in the universal discourse \( X \), symbolically denoted by \( x \), is called a neutrosophic set if \( \tilde{S} = \{x; T(S) x \}, I(S) x \}, F(S) x \} x \in X \} \), where
It is to be noted that, $T_{\text{neu}}(x), I_{\text{neu}}(x)$ and $F_{\text{neu}}(x)$ exhibits the following relation:

$$0 \leq T_{\text{neu}}(x) + I_{\text{neu}}(x) + F_{\text{neu}}(x) \leq 2$$

**Definition 5:** (Spherical neutrosophic single-valued number): A number $s_{\text{ns}} = \{T_{\text{neu}}(x), I_{\text{neu}}(x), F_{\text{neu}}(x)\}$ is called a spherical neutrosophic single-valued number [15] if it satisfies the following relation:

$$0 \leq \left( T_{\text{neu}}(x) \right)^2 + \left( I_{\text{neu}}(x) \right)^2 + \left( F_{\text{neu}}(x) \right)^2 \leq 3,$$

where $T_{\text{neu}}(x): X \rightarrow [0, \sqrt{3}], I_{\text{neu}}(x): X \rightarrow [0, \sqrt{3}], F_{\text{neu}}(x): X \rightarrow [0, \sqrt{3}]$.

**Definition 6:** (Cylindrical neutrosophic single-valued set): A set $C_{\text{neuS}}$ in the universal discourse $X$, symbolically denoted by $x$, is called a cylindrical neutrosophic set [17] if

$$C_{\text{neuS}} = \{x; T_{\text{Cneu}}(x), I_{\text{Cneu}}(x), F_{\text{Cneu}}(x) \mid x \in X\},$$

where $T_{\text{Cneu}}(x): X \rightarrow [0, 1]$ is the correct membership function, $I_{\text{Cneu}}(x): X \rightarrow [0, 1]$ is the indeterminacy membership function and $F_{\text{Cneu}}(x): X \rightarrow [0, 1]$ is an untrue membership function.

It is to be noted that, $T_{\text{Cneu}}(x), I_{\text{Cneu}}(x)$ and $F_{\text{Cneu}}(x)$ exhibits the following relation:

$$\left( T_{\text{Cneu}}(x) \right)^2 + \left( I_{\text{Cneu}}(x) \right)^2 \leq 1,$$

For convenience, $C_{\text{neu}} = \{T_{\text{neu}}(x), I_{\text{neu}}(x), F_{\text{neu}}(x)\}$ is defined as a cylindrical neutrosophic single-valued number (CnNFN), generally denoted as, $C_{\text{neu}} = (T_{\text{Cneu}}, I_{\text{Cneu}}, F_{\text{Cneu}})$. A cylindrical neutrosophic single-valued set is represented graphically in Fig. 2.

**Fig. 2 Geometrical representation of a cylindrical neutrosophic single-valued set**

### 3 Score and accuracy functions

Score function and accuracy function play an essential role in uncertainty theory during the ranking of fuzzy numbers. The need for score and accuracy function is to compare or convert a fuzzy number into a crisp number. Score function and accuracy function of a cylindrical neutrosophic single-valued number are entirely depending on the value of truth membership indicator degree, falsity membership indicator degree, and indeterminacy membership indicator degree. Here we propose a score function as follows:

For any CnNFN, $C_{\text{neu}} = (T_{\text{Cneu}}, I_{\text{Cneu}}, F_{\text{Cneu}})$ it is to be noted that

- Beneficiary degree of truth function = $2(T_{\text{Cneu}})^2$.
- Non-beneficiary degree of falsity function = $(F_{\text{Cneu}})^2$.
- And the indeterminacy degree of indeterminacy function = $(I_{\text{Cneu}})^2$.

Thus, we defined the score function as $SC_{\text{Cneu}} = \frac{2(T_{\text{Cneu}})^2 - (I_{\text{Cneu}})^2 - (F_{\text{Cneu}})^2}{2}$, where $SC_{\text{Cneu}} \in [-1, 1]$ and the accuracy function is defined as

$$AC_{\text{Cneu}} = \frac{2(T_{\text{Cneu}})^2 + (I_{\text{Cneu}})^2 + (F_{\text{Cneu}})^2}{2},$$

where $AC_{\text{Cneu}} \in [0, 2]$.

Now we conclude that

- If $C_{\text{neu}} = (1, 0, 0)$ then, $SC_{\text{Cneu}} = 1$ and $AC_{\text{Cneu}} = 1$.
- If $C_{\text{neu}} = (0, 1, 1)$ then, $SC_{\text{Cneu}} = 1$ and $AC_{\text{Cneu}} = 1$.
- If $C_{\text{neu}} = (0, 0, 0)$ then, $SC_{\text{Cneu}} = 0$ and $AC_{\text{Cneu}} = 0$.

#### 3.1 Relationship between any two cylindrical neutrosophic single-valued numbers

Let us consider any two cylindrical neutrosophic single-valued numbers defined as follows:

$$C_{\text{neu}} = (T_{\text{Cneu}}, I_{\text{Cneu}}, F_{\text{Cneu}})$$

if

1. $SC_{\text{Cneu}} > SC_{\text{Cneu}'}$, then $C_{\text{neu}} > C_{\text{neu}'}$.
2. $SC_{\text{Cneu}} < SC_{\text{Cneu}'}$, then $C_{\text{neu}} < C_{\text{neu}'}$.
3. $SC_{\text{Cneu}} = SC_{\text{Cneu}'}$, then
   - (i) $AC_{\text{Cneu}} > AC_{\text{Cneu}'}$, then $C_{\text{neu}} > C_{\text{neu}'}$.
   - (ii) $AC_{\text{Cneu}} < AC_{\text{Cneu}'}$, then $C_{\text{neu}} < C_{\text{neu}'}$.
   - (iii) $AC_{\text{Cneu}} = AC_{\text{Cneu}'}$, then $C_{\text{neu}} \sim C_{\text{neu}'}$. 

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4 Critical path problem in cylindrical neutrosophic single-valued number environment

Project management is anxious with nominating, planning, execution and regulation of projects to communicate or crossed collaborator’s need or demand from the project. There are two different approaches to project management: (i) critical path method (CPM) and (ii) program evaluation and review technique (PERT). CPM indicates the longest path of a project duration network. The estimated value of activity span time maybe differs under specific environments, and it might not be presented accurately due to the occurrence of the error by way of measuring or due to instruments, etc. The neutrosophic set theory is much suitable to model uncertainty, which is related to parameters like activity duration time, resource reachable in CPM. Especially, in this paper, we take one networking problem related to the operational research field and we focused and analysed this engineering problem in the cylindrical neutrosophic domain. CPM is a project management procedure for the ongoing designing that represents the critical and non-critical functions with the success of stopping time frame issues. The critical path is always reflected among the schedule, project phases throughout the lifecycle. A few established works in critical and shortest path methods are recently developed in the neutrosophic arena [24–28].

A few important terms associated with CPM are defined as follows:

*Activity: It is any part of a project having an exact start and end and it might use some sort of methods involving time, some materials, effort, appliances, etc.*

*Event or node: The starting and ending points of activities symbolised by circles are referred to as nodes or events.*

*Critical path: Critical path is the longest path in the networking problem.*

The steps in the algorithm of a CPM are as follows:

- It identifies the particular tasks and milestones.
- The exact order of the activities is also determined by CPM.
- A network diagram can be constructed.
- For each completion of the work, the estimated time required can be calculated.
- The critical path is thus decided.

*Forward pass:* At the time of the forward movement, for the first occurrence when it starts with a time of zero, the calculations begin from left to right till the final event occurs. Let’s assume $ES_i$ denotes the earliest time of having the event $i$ for any sort of activity $(i, j)$, then $S_j = ES_i + t_j$. If there exists greater than one activity for an event, then the earliest starting time for that particular event is calculated as $ES_j = \max\{ES_i + t_j\}$ for all such activities emerging from node $i$ and intruding to node $j$.

*Backward pass:* At the time of backward movement, it starts with the final node and the calculation starts from right to left till the starting event. Let’s consider $LF_i$ depicts the newest completed time of the event $i$, for any sort of activity $(i, j)$, then the value of $F_j$ is calculated as $F_j = LF_i - t_j$. If there exists more than one activity for an event, then the current completed time for that particular event is calculated as $LF_j = \min\{LF_i - t_j\}$ for all sorts of activities that emerge from node $j$ and enters to node $i$.

In our proposed model, we consider all the activities are cylindrical neutrosophic single-valued number and to compare two distinct $SpNFN$ we utilise the concept of the score function

$$SC_{C_{new}} = \frac{2(T_{C_{new}})^2 - (t_{C_{new}})^2 - (F_{C_{new}})^2}{2}$$ (1)

to estimate the minimum and maximum values. If the score values are equal, then two $SpNFN$ are estimated by using the accuracy function defined as

$$AC_{C_{new}} = \frac{(T_{C_{new}})^2 + (t_{C_{new}})^2 + (F_{C_{new}})^2}{2}$$ (2)

4.1 Flowchart for the problem

Flowchart for the problem is shown in Fig. 3.

A CPM problem is considered with numerical data, which is described in Table 1.

Here we want to find the following things:

- Draw the project network.
- Compute the earliest and latest finish time.
- Find the critical path and total project duration.

The above problem has been solved by the following steps:

*Step 1: Network diagram (see Fig. 4).*

**Fig. 3 Flowchart for the problem**

| Nodes | Description | Predecessors | Activity |
|-------|-------------|--------------|----------|
| A     | selection of different vehicles | — | $a = [0.75; 0.5; 0.5]$ |
| B     | selection of different destination | — | $b = [0.5; 0.45; 0.35]$ |
| C     | selection of driver | A | $e = [0.6; 0.5; 0.5]$ |
| D     | final plan and blueprint | B | $n = [0.55; 0.55; 0.45]$ |
| E     | route-map creation | A | $O = [0.65; 0.4; 0.5]$ |
| F     | fuel consumption for different places | — | $d = [0.8; 0.6; 0.6]$ |
| G     | instruction and training | C | $h = [0.7; 0.55; 0.45]$ |
| H     | initial startup time | D | $m = [0.5; 0.5; 0.4]$ |
| I     | vehicle testing timing | A | $c = [0.4; 0.2; 0.5]$ |
| J     | run the system | E, G, H | $p = [0.45; 0.4; 0.4]$ |
| K     | final destination | F, I, J | $g = [0.6; 0.4; 0.6]$ |

| $f$ | $[0.7; 0.55; 0.45]$ |
| $i$ | $[0.7; 0.45; 0.35]$ |
| $f$ | $[0.8; 0.65; 0.65]$ |
Step 2: Now, we consider the start time level as 0. To compute the earliest start time left to right for each activity, we believe the score function (1). When multi-connection is merged into a particular node, we just take the score function (1) to evaluate the maximum value for the earliest start time. If the core values are the same in many cases, then we compute the accuracy function (2) to calculate the maximum value for the earliest start time. Similarly, for the latest finish time, the process is the same from right to left. Still, in the case of multi-connection, we consider the score function (1) and just take the minimum value to evaluate the latest finish time. If the core values are the same in many cases, then we compute the accuracy function (2) to calculate the latest finish time. Then we observe the particular nodes where both the earliest start time and latest finish time are the same and construct the critical path.

Step 3: After computation, the earliest start time and latest finish time, we have a network, as shown in Fig. 5.

So, expected project duration – 0.96 Days ≈ 1 Day (approx) with the critical path $A \rightarrow C \rightarrow G \rightarrow J \rightarrow K$.

4.2 Comparison table

We compared our proposed work with previously established works formulated by the authors [29–32] to compute the minimum optimal cost of the above problem.

The comparison table is given in Table 2.

5 Multi-criterion group decision-making method in a cylindrical neutrosophic single-valued set environment

Nowadays, researchers of our universe are very much interested in doing multi-criterion decision-making problems. In the case of this problem, we consider a finite number of alternatives as well as a finite number of attributes with different types of weight function for a different number of decision-makers. It is not a very easy task to evaluate the attribute value in terms of crisp numbers due to the presence of impreciseness. The information of the attribute values is of cylindrical number in nature. The goal of this method is to find out a comparison between the alternatives and attributes maintaining the weight of the decision-makers such that we can easily find out the best alternatives and the worst ones. Already many of the researchers invented some ideas on MCDM [29, 33–47] problem. Still, in this new cylindrical neutrosophic domain, we consider a multi-criterion group decision-making problem, and we focused and analysed this problem using our developed score and accuracy functions. We proposed an algorithm to solve this MCGDM problem and utilised mathematical operators for simplification and, lastly, use the score value for the ranking section. Additionally, we compared our results with the established methods done before and tally the ranking order in different situations. Also, sensitivity analysis can be performed by changing different kinds of attribute weights and observe the ranking order in distinct cases.

Fig. 4 Network diagram

Fig. 5 Earliest start time and the latest finish time
Table 2 Comparison table

| Approach            | Results                                                                 |
|---------------------|-------------------------------------------------------------------------|
| our proposal        | \( A \rightarrow C \rightarrow G \rightarrow J \rightarrow K \)     |
| Deli et al. [29]    | Utilising crispification skill observes that negative components are arises, thus it cannot be determinable. |
| Chakraborty et al.  | Utilising crispification skill observes that negative components are arises, thus it cannot be determinable. |
| Wang [31]           | Utilising crispification skill observes that negative components are arises, thus it cannot be determinable. |
| Biswas et al. [32]  | Utilising crispification skill observes that negative components are arises, thus it cannot be determinable. |

5.1 Illustration of the MCGDM problem

We consider the problem as follows:

Let \( P = \{ P_1, P_2, P_3, \ldots, P_m \} \) is the distinct alternative set and \( Q = \{ Q_1, Q_2, Q_3, \ldots, Q_n \} \) is the distinct attribute set, respectively. Let \( W_e = \{ W_{e1}, W_{e2}, W_{e3}, \ldots, W_{en} \} \) be the weight set associated with the attributes \( Q \) where each \( W_e \geq 0 \) and also satisfies the relation \( \sum_{i=1}^{n} W_{ei} = 1 \). We also consider the set of decision-makers \( D = \{ D_1, D_2, D_3, \ldots, D_k \} \) associated with alternatives whose weight vector is defined as \( \theta = (\theta_1, \theta_2, \theta_3, \ldots, \theta_k) \), where each \( \theta_i \geq 0 \) and also satisfies the relation \( \sum_{i=1}^{k} \theta_i = 1 \), this weight vector will be selected according to the decision-makers quality of judgment, knowledge, thinking power etc.

5.2 Algorithm for solving the above MCGDM problem

Step 1: Creation of decision matrices. First, we create the decision matrices for each decision maker’s choice associated with alternatives versus attribute functions. We consider the member of the matrices in the cylindrical neutrosophic environment, so all \( a_{ij} \) are the member of the cylindrical neutrosophic set. The associated matrix is defined as follows:

\[
X^P = \begin{pmatrix}
\cdot & Q_1 & Q_2 & Q_3 & \cdots & Q_n \\
P_1 & a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\
P_2 & a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\
P_3 & \ddots & \ddots & \ddots & \ddots \\
P_m & a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn}
\end{pmatrix}
\]  

(3)

Step 2: Creation of a weighted single decision matrix. To obtain a single group decision matrix we all use the operation \( \bar{a}_{ij} = \bar{\sum_{i=1}^{n} W_{ei}X^P} \) for each individual decision matrix \( X^P \). Thus, we get the new matrix as

\[
X = \begin{pmatrix}
\cdot & Q_1 & Q_2 & Q_3 & \cdots & Q_n \\
P_1 & a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\
P_2 & a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\
P_3 & \ddots & \ddots & \ddots & \ddots \\
P_m & a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn}
\end{pmatrix}
\]  

(4)

Step 3: Creation of a weighted priority matrix using the weight vector. To obtain a single column decision matrix, we all use the operation \( \bar{a}_{ij} = \bar{\sum_{i=1}^{n} \theta_i \bar{a}_{ij}} \), \( p = 1, 2, \ldots, m \) for each individual column, and hence we get the decision matrix as

\[
X = \begin{pmatrix}
\cdot & \bar{Q}_1 \\
\bar{P}_1 & a_{11}^w \\
\bar{P}_2 & a_{21}^w \\
\ddots & \ddots \\
\bar{P}_m & a_{m1}^w
\end{pmatrix}
\]  

(5)

Step 4: Ranking. Now, we consider the score and accuracy function and convert the matrix (5) into a crisp form such that we could evaluate the best alternative corresponding to the best attributes. We consider the score values according to the increasing order and choose the best fit result. The highest value will give us the best result and the lowest one will give the worst one.

5.3 Illustrative example

We consider a problem to choose the best option to purchase a mobile. We know that lots of mobile companies are available in the market and they have different components with different qualities and facilities. So, it is a multi-criterion decision-making problem with different types of decision-makers. We choose the problem as follows:

\( P_1 = \) Mobile company 1, \( P_2 = \) Mobile company 2, \( P_3 = \) Mobile company 3 are the alternatives.
\( Q_1 = \) Camera quality, \( Q_2 = \) Proccesor quality, \( Q_3 = \) Service facility are the attributes.

We consider there are three decision-makers \( D_1 = \) young aged, \( D_2 = \) Middle aged, \( D_3 = \) Old aged having weight function \( D = \{ 0.30, 0.40, 0.30 \} \) and also we consider the weight vector associated with the attribute function \( \theta = [0.37, 0.33, 0.30] \).

Step 1: Creation of decision matrices using a numerical value: We consider the matrices according to each decision maker’s choice related to attributes and attribute functions. All the members of the matrices are of cylindrical neutrosophic nature. So, the decision matrices are

\[
M^P = \begin{pmatrix}
\cdot & \bar{Q}_1 & \bar{Q}_2 & \bar{Q}_3 \\
\bar{P}_1 & 0.6, 0.3, 0.4 & 0.55, 0.35, 0.25 & 0.5, 0.4, 0.3 \\
\bar{P}_2 & 0.65, 0.45, 0.35 & 0.62, 0.4, 0.4 & 0.7, 0.4, 0.3 \\
\bar{P}_3 & 0.52, 0.4, 0.3 & 0.6, 0.4, 0.4 & 0.45, 0.35, 0.25
\end{pmatrix}
\]

Table 3 Attribute weight versus ranking result

| Attribute weight | Decision matrix | Ranking       |
|------------------|-----------------|---------------|
| \( 0.37,0.33,0.30 \) | \( 0.1875 \)    | \( P_2 > P_3 > P_1 \) |
| \( 0.37,0.33,0.30 \) | \( 0.2366 \)    | \( P_2 > P_3 > P_1 \) |
| \( 0.37,0.33,0.30 \) | \( 0.1909 \)    | \( P_2 > P_3 > P_1 \) |
| \( 0.25,0.30,0.45 \) | \( 0.1796 \)    | \( P_2 > P_3 > P_1 \) |
| \( 0.25,0.30,0.45 \) | \( 0.2336 \)    | \( P_2 > P_3 > P_1 \) |
| \( 0.25,0.30,0.45 \) | \( 0.2063 \)    | \( P_2 > P_3 > P_1 \) |
| \( 0.25,0.50,0.25 \) | \( 0.1951 \)    | \( P_2 > P_3 > P_1 \) |
| \( 0.25,0.50,0.25 \) | \( 0.2184 \)    | \( P_2 > P_3 > P_1 \) |
| \( 0.25,0.50,0.25 \) | \( 0.1859 \)    | \( P_2 > P_3 > P_1 \) |
| \( 0.33,0.33,0.34 \) | \( 0.1877 \)    | \( P_2 > P_3 > P_1 \) |
| \( 0.33,0.33,0.34 \) | \( 0.2384 \)    | \( P_2 > P_3 > P_1 \) |
| \( 0.33,0.33,0.34 \) | \( 0.1935 \)    | \( P_2 > P_3 > P_1 \) |
| \( 0.27,0.45,0.28 \) | \( 0.1905 \)    | \( P_2 > P_3 > P_1 \) |
| \( 0.27,0.45,0.28 \) | \( 0.2255 \)    | \( P_2 > P_3 > P_1 \) |
| \( 0.27,0.45,0.28 \) | \( 0.1898 \)    | \( P_2 > P_3 > P_1 \) |
for decision maker $D_1$

$$M^2 = \begin{pmatrix}
    - & Q_1 & Q_2 & Q_3 \\
    P_1 & (0.7, 0.4, 0.5) & (0.65, 0.5, 0.4) & (0.5, 0.4, 0.3) \\
    P_2 & (0.7, 0.4, 0.35) & (0.6, 0.5, 0.4) & (0.5, 0.4, 0.3) \\
    P_3 & (0.55, 0.4, 0.35) & (0.4, 0.2, 0.1) & (0.7, 0.4, 0.1) \\
\end{pmatrix}$$

For decision maker $D_2$

$$M^3 = \begin{pmatrix}
    - & Q_1 & Q_2 & Q_3 \\
    P_1 & (0.6, 0.4, 0.3) & (0.5, 0.4, 0.35) & (0.55, 0.45, 0.35) \\
    P_2 & (0.65, 0.4, 0.35) & (0.7, 0.5, 0.4) & (0.5, 0.3, 0.2) \\
    P_3 & (0.4, 0.25, 0.15) & (0.45, 0.2, 0.1) & (0.45, 0.25, 0.25) \\
\end{pmatrix}$$

For decision maker $D_3$

Step 2: Creation of weighted single decision matrix

$$M = \begin{pmatrix}
    - & Q_1 & Q_2 & Q_3 \\
    P_1 & (0.64, 0.37, 0.41) & (0.58, 0.43, 0.34) & (0.52, 0.42, 0.32) \\
    P_2 & (0.67, 0.42, 0.35) & (0.64, 0.47, 0.4) & (0.56, 0.37, 0.27) \\
    P_3 & (0.5, 0.36, 0.28) & (0.48, 0.26, 0.19) & (0.55, 0.26, 0.19) \\
\end{pmatrix}$$

Step 3: Creation of a weighted priority matrix using the weight vector

$$M = \begin{pmatrix}
    (0.58, 0.41, 0.36) \\
    (0.63, 0.38, 0.42) \\
    (0.51, 0.30, 0.22) \\
\end{pmatrix}$$

Step 4: Ranking: Now, we consider the score and accuracy defined in Section 3 and converts the cylindrical neutrosophic single-valued numbers into crisp one; thus we get the final ideal decision matrix as

$$M = \begin{pmatrix}
    0.1875 \\
    0.2365 \\
    0.1909 \\
\end{pmatrix}$$

Now, we arrange the numbers in ascending order to get $0.1875 < 0.1909 < 0.2365$. Thus ranking of the priority alternatives will be as $P_2 > P_3 > P_1$.
5.3 Results and sensitivity analysis

The main idea of sensitivity analysis is to exchange weights of the attribute values keeping the rest of the terms are fixed. Here a sensitivity analysis is done to understand how the attribute weights of each criterion affecting the relative matrix and their ranking. The result of sensitivity analysis is shown in Table 3 and Figs. 6 and 7 show the associated weights of different attribute functions and the priority of alternatives, respectively.

Table 5 Necessary data for minimal spanning tree problem

| Connection | Weights          | Score value |
|------------|------------------|-------------|
| i          | (0.4; 0.4; 0.35) | 0.019       |
| d          | (0.55; 0.45; 0.6) | 0.021       |
| j          | (0.59; 0.6; 0.41) | 0.042       |
| g          | (0.6; 0.6; 0.5)   | 0.055       |
| f          | (0.6; 0.5; 0.5)   | 0.055       |
| c          | (0.65; 0.6; 0.5)  | 0.110       |
| k          | (0.65; 0.6; 0.5)  | 0.118       |
| e          | (0.65; 0.45; 0.55) | 0.17        |
| a          | (0.7; 0.5; 0.5)   | 0.24        |
| f          | (0.75; 0.6; 0.5)  | 0.258       |
| b          | (0.8; 0.6; 0.6)   | 0.28        |
| h          | (0.85; 0.55; 0.65) | 0.36        |

5.4 Comparison of our work with established work

We compared our work with other previous work described by Garg [41], Deli et al. [29] and Aslam et al. [42]. It is observed that the best alternatives in each case are \( P_2 \).

The comparison is shown in Table 4.

6 Minimal spanning tree problem in the cylindrical neutrosophic environment

Minimal spanning tree is a very important and interesting topic graph theory domain Prim’s and Kruskal’s algorithms are often using to solve a minimal spanning tree problem [7]. A procedure was established for getting the value of the minimum spanning tree of
a single-valued neutrosophic graph. Based on the similarity measure, Mandal and Basu [48] put forward an approach in a neutrosophic environment that searches the minimal spanning tree problems by considering the inconsistency, incompleteness and indeterminacy of the information. The minimal spanning tree problem in the bipolar neutrosophic environment was analysed by Mullai et al. [49]. Further, researchers developed [30, 50–55] lots of work in the neutrosophic domain. Now, one of the key questions will arise if the edges are in the cylindrical neutrosophic domain, then what is the process to solve this kind of problem? The main purpose of this section is to find out a minimal spanning tree from the graph where all the edges exhibit cylindrical neutrosophic single-valued number.

6.1 General idea

Spanning tree: In a connected, weighted graph $G$, where $T$ is a subgraph of $G$ and tree $T$ contains all the vertices of $G$ with the minimum possible number of edges, the tree $T$ is called a spanning tree.

Minimal spanning tree: Let us consider a graph $G$, which is a weighted graph. A spanning tree with the minimum weight in $G$ is referred to as a minimum spanning tree.

We consider a graph in a cylindrical neutrosophic environment. Now, we proposed an algorithm to find out the minimal spanning tree from a graph, whose weights are cylindrical neutrosophic type in nature.

In our proposed algorithm

- List all edges (except self-loop) of the graph.
- To find out the minimum spanning tree, we utilise the score function (1) concept to compare all the edges which contain minimum weight and then list all the edges of the graph in order of non-decreasing weight.
- Select any edge having the smallest weight and if more than one edge having smallest weight then uses accuracy function (2) and take the smallest one.
- Out of all remaining edges, select an edge having the smallest one, which does not create a circuit with the edges which already included.
- Continue this process until all the vertices were included.

6.2 Illustrative example

Obtain a minimal spanning tree of the following graph (see Fig. 8).

Where 1, 2, …, 8 are the vertices of the graph and $a$, $b$, $c$, …, $k$ are the edges of the graph as follows: $a = [(0.7; 0.5; 0.5)]$, $b = [(0.8; 0.6; 0.6)]$, $c = [(0.6; 0.5; 0.5)]$, $d = [(0.55; 0.45; 0.55)]$, $e = [(0.65; 0.45; 0.55)]$, $f = [(0.75; 0.6; 0.5)]$, $g = [(0.6; 0.6; 0.5)]$, $h = [(0.85; 0.55; 0.65)]$, $i = [(0.4; 0.4; 0.35)]$, $j = [(0.55; 0.6; 0.4)]$, $k = [(0.65; 0.6; 0.5)]$.

We need to find out the minimal spanning tree.

6.3 Solution procedure

Step 1: List all the edges

$a = [(0.7; 0.5; 0.5)]$, $b = [(0.8; 0.6; 0.6)]$, $c = [(0.6; 0.5; 0.5)]$, $d = [(0.55; 0.45; 0.6)]$, $e = [(0.65; 0.45; 0.55)]$, $f = [(0.75; 0.6; 0.5)]$, $g = [(0.6; 0.6; 0.5)]$, $h = [(0.85; 0.55; 0.65)]$, $i = [(0.4; 0.4; 0.35)]$, $j = [(0.55; 0.6; 0.4)]$, $k = [(0.65; 0.6; 0.5)]$.

Step 2: Using our proposed score function (1) list all the edges in order of non-decreasing weight (see Table 5).

Step 3: Minimal spanning tree is shown in Fig. 9.

| Approach          | Results                          |
|-------------------|---------------------------------|
| our proposal      | 0.47 units.                     |
| Deli et al. [29]  | Utilising crispification skill observes that negative components are arises, thus it cannot be determinable. |
| Wang [31]         | Utilising crispification skill observes that negative components are arises, thus it cannot be determinable. |
| Biswas et al. [32]| Utilising crispification skill observes that negative components are arises, thus it cannot be determinable. |
| Brouni et al. [16]| Utilising crispification skill observes that negative components are arises, thus it cannot be determinable. |

Here this is the minimal spanning tree given in Fig. 9 having weight 0.47 units.

6.4 Comparison table

We compared our proposed work with previously established works formulated by the authors [29–32] to compute the minimum optimal cost of the above problem. The comparison table is given in Table 6.

Table 6 Comparison table

| Approach          | Results                          |
|-------------------|---------------------------------|
| our proposal      | 0.47 units.                     |
| Deli et al. [29]  | Utilising crispification skill observes that negative components are arises, thus it cannot be determinable. |
| Wang [31]         | Utilising crispification skill observes that negative components are arises, thus it cannot be determinable. |
| Biswas et al. [32]| Utilising crispification skill observes that negative components are arises, thus it cannot be determinable. |
| Brouni et al. [16]| Utilising crispification skill observes that negative components are arises, thus it cannot be determinable. |

7 Conclusion

In this particular paper, we establish the idea of cylindrical neutrosophic single-valued numbers from different outlook and aspects for analysing its true character. We are furnished with the score and accuracy function concept used in converting a neutrosophic number into a crisp number finding its significance for decision-making problems. We solve a networking problem in operational research domain using the cylindrical neutrosophic single-valued set environment and also done a minimal spanning tree problem giving a far ambidextrous result.

Consequently, we have inspected the following outcomes as:

- The cylindrical neutrosophic single-valued number can be the better choice of decision-makers rather than neutrosophic numbers when falsity and indeterminacy function are dependent.
- The concept of a cylindrical neutrosophic single-valued number is better than a general neutrosophic number.
- The numbers can be taken for different real-life application model.

As a final conclusion, we can say that we applied for the cylindrical neutrosophic single-valued number in different problems like critical path method, MCDM and spanning tree problem, which can be easily and successfully applied to other problems related with science and engineering field. This noble thought will help us to solve a plethora of daily life problems in the uncertainty arena. Further, researchers can apply this conception of cylindrical neutrosophic numbers in various fields like a technology-based problem, diagnoses problem, mathematical modelling, pattern recognition problem, cloud computing, image sensing problem, mobile computing problem etc. The future study can be continued by forming different types of cylindrical neutrosophic numbers with various applications. There is still a massive amount of work in this field; hence much spectacular study can be explored with cylindrical neutrosophic parameters.

8 References

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