TECHNICAL NOTE

Application of rotation rate sensors in an experiment of stiffness ‘reconstruction’

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Received 7 February 2013, in final form 8 April 2013
Published 13 June 2013
Online at stacks.iop.org/SMS/22/077001

Abstract

Preliminary results of an application of rotation rate sensors in dynamic identification of a vibrating beam are reported. Three rotational sensors, measuring the response of a cantilever beam to kinematic harmonic excitations are applied in the reconstruction of its stiffness. A successful reconstruction of 15% stiffness drops is demonstrated. With the development of angular sensors and the decrease in their cost one can expect further progress of this new area of vibration-based damage detection.

(Some figures may appear in colour only in the online journal)

1. Introduction

The sensors measuring rotational vibrations are attracting more and more attention from researchers because of their huge potential for application in various fields of engineering activity, such as aerospace control, automotive, defense, or even consumer electronics. For more information about their development, see e.g. [1–3]. Recently these sensors have found particular applications in the new, emerging field of rotational seismology [4, 5], which adds three new dimensions to the conventional, translation ground motions (rotations about $x$, $y$, $z$ axes).

The rotation sensors, which are also called angular sensors or gyro sensors, can measure the angle rate (rotational velocity) in the range of hundreds of radians per second or rotational accelerations with an accuracy $0.01^\circ \text{s}^{-2}$ or better. Details can be found on the respective web sites of the producers (e.g. Systron Donner, ATA, Jewell etc).

As the accuracy of angular sensors improves and prices decrease there is also a possibility to apply them in structural health monitoring (SHM). Their application in SHM can be beneficial because rotations of beams in bending are more sensitive to stiffness variations than their respective transversal motions. So far, these intuitive expectations are supported only by numerical simulations, which indicate numerous potential advantages from the application of rotational measurements in SHM [6, 7]. Recently the role of rotations in structural response proved to be yet more important as they appeared as a side effect of strong seismic waves propagating in buildings [8].

The purpose of this paper is to report preliminary results of the application of the Systron Donner HZ1-100-100 rotation rate sensor in stiffness reconstructions of a simple beam model.

2. Stiffness reconstruction of frame structures using harmonic vibrations

The damage in SHM is usually modeled as isolated ‘cuts’ of structural elements, representing local losses of stiffness (see e.g. [9–11]). However in some areas of SHM, particularly for typical, civil engineering structures (e.g. reinforced concrete frames [12–14], buildings modeled as shear beams [8, 15] or masonry structures) it is not possible to localize damage in the form of such simple ‘cuts’, with localized stiffness losses. Instead, more difficult tasks have to be
undertaken, which aim at formal ‘reconstructions’ of spatially distributed stiffness variations. The solutions of respective inverse problems require the application of sophisticated optimization algorithms and substantial computer power. For this reason, research efforts in this branch of SHM concentrate on the development of more and more efficient numerical algorithms [16]. To improve damage detection or reconstruction, harmonic excitations of structures are often applied (e.g. [17–19]).

Consider the matrix equation of motion of an undamped, discrete dynamic system under harmonic excitations, with angular frequency \( \omega_p \) [rad s\(^{-1}\)]:

\[
\mathbf{M} \ddot{\mathbf{d}} + \mathbf{K}^d \mathbf{d} = \mathbf{P}_0 \mathbf{e}^{\text{int} \omega_p t}.
\] (1)

Its solution takes the familiar form:

\[
(\mathbf{K}^d - \omega_p^2 \mathbf{M}) \mathbf{u} = \mathbf{P}_0,
\] (2)

where \( \mathbf{K}^d, \mathbf{M}, \mathbf{d}, \mathbf{\ddot{d}}, \mathbf{u}, \mathbf{P}_0 \) are the stiffness and inertia matrices, vectors of response displacements and accelerations as well as respective amplitudes of harmonic response displacements and excitation forces. The superscript \( d \) indicates that the stiffness matrix refers to the damaged state. It should be noted that the lack of damping effects in equation (1) does not influence the computation results of ‘reconstructions’ in structural health monitoring procedures if the excitation frequency \( p \) is far enough from the resonance zones.

Using the finite element method (FEM), the global stiffness matrix of the damaged structure \( \mathbf{K}^d \) can be written in a form, in which damage contributions to particular finite elements are described by vectors of multipliers \( \alpha \) measuring the loss of its stiffness \((0 \leq \alpha_i \leq 1)\):

\[
\mathbf{K}^d = \sum_{i=1}^{n} \mathbf{K}_{ii}^d = \sum_{i=1}^{n} \alpha_i \mathbf{K}_{ii}^\text{ue}.
\] (3)

Substituting equation (3) into equation (2) one arrives at the following equation

\[
\left( \sum_{i=1}^{n} \alpha_i \mathbf{K}_{ii}^\text{ue} - \omega_p^2 \mathbf{M} \right) \mathbf{u} = \mathbf{P}_0,
\] (4)

which can be used to compute displacement amplitudes for certain excitations and a particular FEM structural model. In order to quantify the difference between actual (measured) \( \mathbf{u}_m \) and reconstructed (computed) amplitudes \( \mathbf{u}_c \) the following functional \( J(\alpha) \) is defined:

\[
J(\alpha) = \sum_{j=1}^{n_d} \left( \frac{u_j^c(\alpha) - u_j^m}{u_j^m} \right)^2
\] (5)

where \( n_d \) refers to the number of measured amplitudes under harmonic excitations. Minimization of equation (5) leads to the vectors of stiffness multipliers \( \alpha \) describing the actual damage state of the modeled structure. Since harmonic excitations and measurements of the monitored structure can be repeated many times, even a single sensor and single harmonic exciter can be used sequentially during such experimental stiffness ‘reconstructions’. By including rotational measurements the respective number of measured and computed responses \( n_d \) may increase or the measurements can be limited to include only rotational degrees of freedom.

Effective minimization of equation (5) requires application of sophisticated optimization algorithms. For this purpose genetic algorithms and local Levenberg–Marquardt minimization are applied in this paper (see e.g. [19]).

To assess the effectiveness of ‘reconstruction’, one can either measure the maximum difference among all the analyzed differences of reconstructions \( \text{ME} = \text{maximum error} \), or the so-called weighted, average error \( (\text{WAE}) \) may be defined:

\[
\text{WAE} = \sqrt{\frac{1}{n_c} \sum_{i=1}^{n_c} \left( \frac{\alpha_i^c - \alpha_i^d}{\alpha_i^d} \right)^2},
\] (6)

for which the summation takes place over all \( n_c \) ‘reconstructed’ finite elements.

Minimization of equation (5) was successfully applied in the computer simulations of stiffness reconstructions using translation degrees of freedom only [16, 17] as well as the rotational degrees of freedom [7]. The question remained: to what extent rotational measurements can effectively be applied in actual experimental ‘reconstructions’?

3. Description of the experiment

Before the rotational sensors are applied to measure the large reinforced concrete frames met in civil engineering it will be beneficial to gather some experience on a smaller scale. Such small, laboratory models should however represent a similar level of axis rotations as expected from the full-scale frames. After an analysis of what was available, it was decided to carry out the experiments on small span (750 mm), cantilever beams made of Plexi with cross-section \( b = 80 \text{ mm} \) and \( h = 14 \text{ mm} \) in the ‘intact’ state (figures 1 and 2) and reduced stiffness representing the ‘damaged’ state (figure 2). It should be noted that such small beams are much easier to excite in laboratory conditions by using kinematic excitations, than to actually exert the actuator motion on them. For this reason a heavy,
Figure 2. Plan view of the analyzed Plexi beam: (a) ‘intact’ beam and (b) ‘damaged’ beam—with stiffness drops (all dimensions are in mm).

Figure 3. Dynamic model of a cantilever beam with three sub-sections and six degrees of freedom, under vertical, kinematic, harmonic excitations with amplitude $A_0$ and frequency $f_p$ [Hz].

Figure 4. Close view of the rotation rate sensor Horizon HZI-100-100 and nuts glued to Plexi beam to compensate for mass loss due to stiffness reduction.

long span, simply supported steel beam was prepared to act as a support for the analyzed beams. The steel beam was excited in the vertical direction by an actuator controlled by the HBM Instron system, while the analyzed, Plexi beam was clamped at the mid-span of the steel beam (figure 1).

This solution made it possible to excite kinematic vibrations of the Plexi beam. The vibrating beam model is shown schematically in figure 3. In addition to three translational coordinates $d_1$, $d_2$ and $d_3$, three additional rotational coordinates $d_4$, $d_5$ and $d_6$ are shown.

To model the loss of stiffness (‘damage’), selected cross sections are reduced. A 15% stiffness drop was modeled for the beam reported in this paper. Since actual stiffness losses often do not lead to mass reductions (e.g. in case of reinforced concrete structures [12]) the loss of mass from the beam cross-section reductions needed to be compensated by attaching additional masses to the beam. This was done by gluing nuts on both sides of the beam in specific, appropriate proportions (see figure 4).

The harmonic motion of the actuator was controlled by the Instron system, while data acquisition was done by a multi-channel system (Hottinger MCG Plus). To measure translational vibrations along the $d_1$, $d_2$ and $d_3$ axes, three miniature accelerometers PCB 333B52 were applied. The angular motions, described by coordinates $d_4$, $d_5$ and $d_6$ were measured using three rotation rate sensors (Systron Donner HZ1-100-100 $\pm 100^\circ$ s$^{-1}$ range, sensitivity: 0.06$^\circ$ s$^{-1}$). The kinematic motion of the main beam was also independently monitored by a portable, MEMS acceleration Sequoia device attached to a laptop through a USB port.

Two models of the 750 mm long Plexi beam were measured and analyzed:

- the first, ‘intact’ one, with a constant cross-section $b = 80$ mm and $h = 14$ mm (see upper part of figure 2) and
- the second, ‘damaged’ one, with decreased cross sections (see lower part of figure 2), leading to 15% drops of stiffness.

The material data of Plexi was density $\rho = 1318.7$ kg m$^{-3}$ and Young modulus $E = 4.51$ GPa. The fundamental, natural frequency of the tested cantilever beam (with all the sensors attached) was 6.90 Hz, while for the ‘damaged’ one (with compensated mass loses) it was 6.23 Hz. This was important to know before the experiment started, as the ‘reconstruction’ method described in sections 2 and 3, by definition, avoids vibrations close to resonances [17–19].

4. Analysis of simulation and experimental results

4.1. Simulation results

Computer simulations were carried out by analyzing the effectiveness of stiffness reconstructions in three cases of reconstructions:
Figure 5. Plots presenting the rotational frequency response (rotation rate) for the ‘intact’ (a) and ‘damaged’ (b) beams (degrees of freedom 4–6 from figure 3).

(1) using only amplitudes of acceleration translations (degrees of freedom 1, 2, 3 from figure 3) and the amplitude of excitations \( A_0 \),

(2) using combined translation accelerations and rotation rates (degrees of freedom 1–6 from figure 3) as well as the amplitude of excitations \( A_0 \),

(3) using only amplitudes of rotation rates (degrees of freedom 4–6 from figure 3) and the amplitude of excitations \( A_0 \).

Various amplitude levels and excitation frequencies were analyzed. The optimization software (see e.g. [7, 19]) easily reconstructed stiffness and mass drops in this case. For example for an excitation frequency \( f_p = 4.5 \) Hz \( (\omega_p = 28.3 \text{ rad s}^{-1}) \) and a beam tip amplitude of displacements equal to 1.5 mm, the WAE errors (equation (6)) and maximum (ME) errors for the above three cases were:

- (1) including only amplitudes of acceleration translations WAE = 0.067%, ME = 0.178%,
- (2) including combined 3 translations and 3 rotations WAE = 0.033%, ME = 0.114%,
- (3) for the three rotations only: WAE = 0.016%, ME = 0.041%.

The next stage of simulations was carried out by adding measurement noise to the simulated signals (for details see: [7, 19]). Up to about 2% noise, it was still possible to obtain acceptable results of reconstructions using the computer simulations.

Figure 6. Results of the experimental ‘reconstruction’ for the Plexi cantilever beam under harmonic kinematic excitations at a frequency \( f_p = 4.5 \) Hz.

4.2. Experimental results

The tests were carried out by measuring harmonic vibrations of the Plexi beams for various excitation frequencies, out of the resonance zones, at a low excitation level.

In the analyzed example these were excitations leading to a tip displacement amplitude of about 1.2–1.5 mm. In figure 5 the resulting amplitudes of rotation rates for coordinates 6, 5, 4 (figure 3) are shown for both an ‘intact’ beam as well as the ‘damaged’ beam, vibrating at frequencies of 4.0, 4.5 and 5.0 Hz. In table 1, detailed results of measurements of the amplitudes from all the sensors are given for vibration frequencies of 4.5 Hz.

Similarly, as in the simulations of section 4.1, three numerical optimization models were applied:

(a) with three translations (1, 2, 3 in figure 3),
(b) six combined translations/rotations (1–6 in figure 3) and
(c) rotations only (4, 5, 6 in figure 3).

In this preliminary stage reported here, only selected combinations of excitation frequencies and amplitude levels were analyzed. So far the model (c), including only rotations, allowed successful reconstructions. Example results for an excitation frequency \( f_p = 4.5 \) Hz and amplitude of the beam tip response equal to about 1.2 mm are shown in figure 6. The errors of the reconstruction were WAE = 2.48% and ME = 3.87%.

5. Conclusion

Earlier numerical analyses [6] indicated the potential advantages of measuring the rotational degrees of freedom of vibrating beams and frames in structural health monitoring. Successful numerical reconstructions of stiffness losses for up to 32 reconstructed zones were possible thanks to the application of Genetic Algorithms and Levenberg–Marquardt local minimization procedures [7]. These numerical reconstructions were still possible with simulated noise reaching a level of 2% with respect to the measured signal.
Table 1. Key values of amplitudes of vibrations describing the excitation and response of tested Plexi beams under harmonic excitations with a frequency $f_p = 4.5$ Hz.

| Description                                      | Degree of freedom (figure 3) | 'Intact' beam | 'Damaged' beam | Unit |
|--------------------------------------------------|-------------------------------|---------------|----------------|------|
| Amplitude of (kinematic) excitation accelerations | 0.383                         | 0.383         | m s$^{-2}$     |      |
| Amplitude of (kinematic) excitation displacements| 0.480                         | 0.480         | mm             |      |
| Amplitude of tip transversal accelerations       | 3                             | 0.945         | m s$^{-2}$     |      |
| Amplitude of tip transversal displacements       | 3                             | 1.182         | mm             |      |
| Amplitude of rotational velocity (rotation rate) of the tip of the beam| 6 | 1.612 | 2.512 $^\circ$ s$^{-1}$ | |
| Amplitude of rotation of the tip of the beam     | 6                             | 0.057         | $^\circ$ deg   |      |
| Amplitude of transversal accelerations           | 2                             | 0.646         | m s$^{-2}$     |      |
| Amplitude of rotational velocity (rotation rate) | 5                             | 1.526         | $^\circ$ s$^{-1}$ | |
| Amplitude of transversal accelerations           | 1                             | 0.462         | m s$^{-2}$     |      |
| Amplitude of rotational velocity (rotation rate) | 4                             | 1.047         | $^\circ$ s$^{-1}$ | |

The purpose of this paper is to report the first, successful experimental reconstructions of cantilever beam stiffness drops using three, rotation rate sensors. Compared to the results reported in [7] the size of the minimization problem was small enough not to cause any problems with the optimization convergence of numerical simulations. When it comes, however, to actual, experimental reconstructions, obtaining the solution is not so easy. Among the three applied minimization approaches, using three translations, six combined translations/rotations and three rotations, so far only the last approach appeared successful at an acceptable level of accuracy. It is particularly interesting to note here that when minimizing functions of many variables in SHM the smaller models may sometimes offer better results over those with an information excess.

Nevertheless, experiments will continue for various types of beam models and sensor configurations. The analyses of stiffness losses with mass reductions, further development of minimization algorithms and the application of other methods of nondestructive damage detection are planned to be carried out by ourselves and will be reported soon. Progress in angular sensors’ range and resolution makes this new branch of vibration-based damage reconstruction more and more promising.

Acknowledgments

This research was supported in part by the Polish NSF Grant N506 289037 ‘Simulation and experimental investigations of rotation measurements in dynamic identification of bar structures’. The Authors wish to thank Dr Bronislaw Jędraszak (Opole University of Technology) and Dariusz Knapek (EC Test Systems) for their assistance in carrying out the experiments.

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