Apparent CPT Violation in Neutrino Oscillation from Dark Non-Standard Interactions

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A natural realization of CPT violation in neutrino oscillation can arise due to the coupling to a light scalar or vector dark matter (DM). The dark non-standard interaction (NSI) is associated with the $\gamma_0$ matrix in neutrino's effective propagator and hence corrects the neutrino Hamiltonian as dark matter potential, in the same way as the ordinary matter effect. The effect is, however, inversely proportional to the neutrino energy and hence appears as a correction to the neutrino mass squared. Due to a sign difference in the corrections for neutrino and anti-neutrino modes, the neutrino oscillation receives CPT violation from the dark NSI. Seeing difference in the neutrino and anti-neutrino mass squared differences not necessarily leads to the conclusion of CPT symmetry breaking in the fundamental Lagrangian but can indicate light DM and its coupling with neutrinos.

Introduction – The CPT theorem is one of a few robust predictions of the relativistic local quantum field theories (QFT) [1, 2]. As long as a theory satisfies three conditions: 1) Lorentz invariance, 2) hermiticity of the Hamiltonian, and 3) locality, it is invariant under the combined CPT transformation. The CPT violation then unavoidably indicates violation of at least one of the three conditions. Note that these three conditions are quite fundamental and measuring CPT violation is a direct probe of the underlying structure of the Nature.

A phenomenological consequence of the CPT symmetry is that a particle and its anti-particle must have exactly the same mass and lifetime. Measuring the difference in the particle and anti-particle masses and lifetimes is then a direct probe of the CPT symmetry. This applies to the neutral Kaon and neutrino systems. Although the constraint from the neutral Kaon system seems quite stringent, $|m(K^0) - m(K^-)|/m_K < 6 \times 10^{-18}$ [3], a more natural parametrization is in terms of the mass squared. First, the parameter that appears in the Lagrangian is $m_K^2$ rather than $m_K$. Even for the neutrino system, although the fermion mass appears as $m_\nu$ in the Lagrangian, it is the mass squared terms in the Hamiltonian that control the oscillation pattern. Using the mass squared parametrization, the Kaon constraint $|m^2(K^0) - m^2(K^-)| < 0.25 \text{eV}^2$, reads much weaker and the neutrino system actually gives better constraint [4].

Neutrinos are more fundamental particles than the neutral Kaons and hence are probably better probes of the fundamental CPT symmetry [5]. Currently neutrino oscillation provides the most stringent bound, $|\Delta m^2_{21} - \Delta m^2_{31}| < 5.9 \times 10^{-5} \text{eV}^2$ and $|\Delta m^2_{31} - \Delta m^2_{32}| < 1.1 \times 10^{-3} \text{eV}^2$ [6]. The future DUNE experiment can further push the limit to $|\Delta m^2_{31} - \Delta m^2_{32}| < 8.1 \times 10^{-5} \text{eV}^2$ [5]. In addition to causing difference in the oscillation patterns for neutrinos and antineutrinos, the presence of CPT violation has many other phenomenological consequences, such as neutrino-to-antineutrino transitions [7] and baryogenesis [8].

Possible violation of the CPT theorem can arise from Lorentz violation [9, 10], non-locality [11], non-commutative geometry [12], or Ether potential [13]. In this letter we provide a natural realization of CPT violation as environmental dark NSI. Without introducing CPT symmetry breaking at the Lagrangian level, a splitting in the neutrino and anti-neutrino masses can arise when neutrinos travel through the DM medium. The Lorentz and consequently CPT invariances are violated by the environmental DM medium. Combining different types of neutrino oscillation experiments can help us to identify this CPT violation.

The Dark NSI – Neutrino oscillation can happen if neutrino masses are non-degenerate and the mixing from flavor to mass eigenstates [14, 15] is nontrivial. In vacuum, the neutrino oscillation is totally determined by the neutrino mass matrix. However, the oscillation pattern can receive environmental effect if neutrinos propagate through matter [16, 17]. From the forward scattering with matter particles, either electron or nuclei, neutrino propagator can receive corrections [18, 19]. Even without mass term, neutrino oscillation can happen in matter [16].

If DM is a fundamental particle, our universe is immersed in a sea of DM particles. According to the astrophysical constraints, the local DM energy density is $\rho_\chi \approx 0.47 \text{GeV/cm}^3$ [20] and its number density is inversely proportional to its mass $n_\chi = \rho_\chi/m_\chi$. With small enough mass, there would be a plenty of DM particles surrounding us. Due to the Pauli exclusion principle, the light DM ($\lesssim 100 \text{ eV}$) can only be bosons, either scalar or vector particles. In this letter, we first focus on the scalar case while the conclusion can also apply to the vector one. If the scalar DM particle has interaction with neutrinos, the relevant Lagrangian is

$$-\mathcal{L} = \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{2} M_{\alpha\beta} \bar{\nu}_\alpha \nu_\beta + y_{\alpha\beta} \phi \bar{\nu}_\alpha \nu_\beta + h.c., \quad (1)$$
with a Yukawa coupling between the light DM $\phi (\equiv \chi)$ and neutrinos.

When neutrinos propagate through the DM medium, both neutrino and DM particles are present as real particles. The forward scattering in Fig. 1 is then described by the scattering matrix element
\[
\langle \nu_\alpha (p_\nu) \phi (p_\phi) | T e^i \frac{d^4 \tau}{\mathcal{L}} | \nu_\beta (p_\nu) \phi (p_\phi) \rangle,
\]
with neutrino and DM being the external particles. By definition, the forward scattering has zero momentum exchange among the external particles. Consequently, the initial- and final-state neutrinos (DM particles) have exactly the same momentum $p_\nu (p_\phi)$. The resulting correction to the two-point function can generally decompose as $\delta \Gamma \equiv \delta \Gamma^\nu \gamma_\mu + \delta M$ and the neutrino (anti-neutrino) Hamiltonian expands into
\[
H \approx \frac{(M + \delta M)(M + \delta M)^\dagger}{2E_\nu} \mp \delta \Gamma^0.
\]
While the mass potential $\delta \Gamma^0$ receives an opposite sign, the mass term correction $\delta M$ is the same for both neutrino and antineutrino modes. The effective Hamiltonian (3) can generally apply for various matter effects that neutrino can experience [18, 21]. Considering the fact that the DM particles around the Earth are non-relativistic, we just need to keep the dominant time component, $p_\phi \approx m_\phi \gamma_0$. The leading-order correction is
\[
\delta \Gamma_{\alpha \beta} \approx \sum_j y_{\alpha j} y^*_{\beta j} \frac{\rho_\chi}{m_\phi^2} \frac{\gamma_0}{E_\nu}.
\]
An interesting feature is that Eq. (4) has energy dependence, rather than the energy-independent SM matter potential [18] or the mass term correction from the scalar NSI [21]. This leads to significantly different phenomenological consequences. Since the first term in (3) is also inversely proportional to the neutrino energy $E_\nu$, the correction from (4) then appears as correction to the mass squared term instead,
\[
H = \frac{M^2}{2E_\nu} \mp \frac{1}{E_\nu} \sum_j y_{\alpha j} y^*_{\beta j} \frac{\rho_\chi}{m_\phi^2} \frac{\gamma_0}{2E_\nu} \equiv \frac{M^2 \pm \delta M^2}{2E_\nu},
\]
where $\delta M^2_{\alpha \beta} \equiv \mp \frac{2\alpha}{m_\phi^2} \sum_j y_{\alpha j} y^*_{\beta j}$. From its coupling with DM, neutrinos receive an opposite mass squared correction from that of anti-neutrinos. This is essentially an apparent violation of the CPT symmetry due to the environmental effect.

At first sight, it may seem strange why a chirality-flipping Yukawa coupling in (1) can lead to chirality-conserving correction (4). Although it is true that Yukawa coupling does flip chirality, two Yukawa vertices in Fig. 1 can flip the neutrino chirality twice and conserve the neutrino chirality. In addition, the non-zero momentum flow in the neutrino propagator of (1) provides $1/E_\nu$ dependence and promotes the $\gamma_0$ term to correction of the neutrino mass squared term.

The earlier studies [22] focused on the fuzzy DM scenario which is equivalent to replacing the scalar DM field in (1) by $\phi \rightarrow \sqrt{2}\rho_\chi \cos(m_\phi \tau)/m_\phi$ with time variation. Nevertheless, this effect is essentially correction to the neutrino mass $\delta M$ rather than the mass squared term, $\delta M^2$. As already indicated in (3), the correction to the neutrino mass term has no sign difference between neutrino and anti-neutrino. For a complex scalar, $\phi = |\phi| e^{i\alpha_\phi}$, the $\delta M^2$ correction is time independent and then the time-dependent $\delta M$ term in (3) can be safely ignored if we only consider the time-averaged data.

Note that being fuzzy DM is not necessary for sizable dark NSI effect on neutrino oscillation. With proportionally larger Yukawa coupling and mass, the light DM can have large enough dark NSI as the fuzzy one. For example, the effect scales as $y_\phi/m_\phi$ in the condensation case. It is definitely possible to relax the mass and Yukawa coupling range while maintaining the size of dark NSI. The forward scattering contribution is actually of the same order as the condensation one. While the former scales as $y^2_\phi \rho_\chi/m_\phi^2$ and contributes to $\delta M^2$, the later scales as $y_\phi \sqrt{\rho_\chi}/m_\phi$ and contributes to $\delta M$.

In addition, [23] studied the matter effect from both fermion and scalar fields. Their study is for totally different environment, in supernova or the host plasma of the Early Universe. With a $f\bar{f} \nu_L \phi$ term, the neutrino can receive matter potential from both $f$ and $\phi$ backgrounds that are present in supernova or the Early Universe. The fermion $f$ can be either a DM fermion or sterile neutrino. For both cases, the matter effect is always recognized as potential, rather than correction to the neutrino mass squared term.

**Phenomenological Consequences** – To get a better sense of the dark NSI, we parametrize the correction to $MM^\dagger$ in general as
\[
\delta M^2 \equiv \Delta m^2_a \begin{pmatrix} \eta_{ee} & \eta_{e\mu} & \eta_{e\tau} \\ \eta_{e\mu}^* & \eta_{\mu\mu} & \eta_{\mu\tau} \\ \eta_{e\tau}^* & \eta_{\mu\tau}^* & \eta_{\tau\tau} \end{pmatrix},
\]
where the atmospheric mass squared difference $\Delta m^2_a \approx 2.7 \times 10^{-3} eV^2$ is the larger one of the two characteristic scales in $MM^\dagger$ while dimensionless parameters $\eta_{\alpha\beta}$ parameterize the size of the dark NSI in the unit of $\Delta m^2_a$. All simulations are done with NuPro [24].
we show the dark NSI effect on the reactor

Sizable effect appears with $\delta M^2_{\alpha\beta} \sim \Delta m^2_{ij}$, or equivalently, $m_\phi/y_{\alpha\beta} \sim \sqrt{2\rho_\chi/\Delta m^2_{ij}} \approx (0.043 \sim 0.25) \text{ eV}$ for $\rho_\chi = 0.3 \text{ GeV/cm}^3$, $\Delta m^2_{21} = 7.55 \times 10^{-5} \text{ eV}^2$ and $\Delta m^2_{31} = 2.50 \times 10^{-3} \text{ eV}^2$ [25]. The previous studies [22] have used the time variation of experimental data to constrain the CPT conserving part $\delta M \propto y_\phi$ in (3) to roughly $10 \sim 15\%$ uncertainty of the mass scale. Since the CPT violating correction $\delta M^2 \sim \mathcal{O}(\delta M)^2$, we can expect percentage level of CPT violation and can be even as large as $\eta_{\alpha\beta} \sim \mathcal{O}(0.1)$ for $2 \sim 3\sigma$ confidence level. Note that only the ratio $m_\phi/y$ matters and the dark matter mass can span a large range, $(10^{-22} \sim 10^{-5}) \text{ eV}$, as long as the coupling scales proportionally within the perturbative range.

Being a correction to the mass squared term, the dark NSI effect is energy independent according to (5). Even at low energy, the dark NSI effect can be significant, for example, in the solar and reactor neutrino oscillations. Most importantly, the neutrino and anti-neutrino modes have the opposite signs which provide an extra way of identification from the scalar NSI [21].

In Fig. 2 we show the dark NSI effect on the reactor neutrino oscillations. The effect at the Daya Bay experiment [26] is quite moderate since its oscillation is modulated by the larger mass squared difference $\Delta m^2_{31}$. With $\eta_{\alpha\alpha} = 0.01$, the dark NSI contributes only $1\%$ of $\Delta m^2_{31}$ which is just around the Daya Bay precision. However, the effect is significant at the medium-baseline JUNO experiment [27]. The lower-frequency oscillation modulated by the smaller $\Delta m^2_{21}$ is just $3\%$ of $\Delta m^2_{31}$ and is comparable to the dark NSI. The JUNO experiment can significantly improve the probe of the dark NSI.

For the 1-2 mixing sector, the KamLAND reactor anti-neutrino measurement [28] has mismatched contour from the solar neutrino measurements at SNO [29], Borexino [30], and SK [31] for both the mass square difference $\Delta m^2_{21} \equiv \Delta m^2_{21}$ and the solar mixing angle $\theta_1 \equiv \theta_{12}$ [3, 25]. While KamLAND gives $\Delta m^2_{21} = 7.54^{+0.19}_{-0.18} \times 10^{-5} \text{ eV}^2$ and $\sin^2 \theta_{12} = 0.316^{+0.034}_{-0.026}$, the solar data prefers $\Delta m^2_{21} = 4.82^{+1.29}_{-0.60} \times 10^{-5} \text{ eV}^2$ and $\sin^2 \theta_{12} = 0.310^{+0.014}_{-0.014}$ [31]. It is possible for the dark NSI to reconcile these two datasets. Fitting the Borexino 2017 [30] and the SK [31] data sets, the $\delta \chi^2$ curves in Fig. 3 for the $\eta_{\mu}$ and $\eta_{\tau}$ elements clearly shows an extra minimum which is even lower than the minimum with vanishing dark NSI. The coupling of bosonic DM with neutrino provides a natural realization of the CPT violation to explain the long-standing discrepancy. Between the two local minima, there is a high peak around $\eta_{\mu} \approx -0.01$.

We use the 2-$\nu$ formalism

$$M^2_{2\nu,2\bar{\nu}} \equiv \Delta m^2_{31} \left[ s^2_\alpha s^2_\beta c^2_s + c^2_\alpha s^2_\beta - c^2_\alpha c^2_\beta \right] \pm c_{\nu,\bar{\nu}} \Delta m^2_{\alpha\beta} \left[ \eta_{\alpha\alpha} \eta_{\mu\mu} - \eta_{\alpha\beta} \eta_{\mu\mu} \right],$$

where $c_{\nu,\bar{\nu}} = 1$, to quantitatively understand these re-
sults. Diagonalizing $M^{2}_{
u}$ and $M^{2}_{D}$ gives two sets of $(\Delta m^{2}_{\text{eff}})$ and $(\theta_{\text{eff}})$ to account for the different measured values from the reactor anti-neutrino and solar neutrino experiments.

Since the experimentally measured variables are those effective ones of the $M^{2}_{\nu}$ and $M^{2}_{D}$ for the neutrino and anti-neutrino modes, respectively, it is more convenient to use the subtraction trick [21]. In other words, we first reconstruct $M^{2}_{\nu}$ with the measured or effective variables. Correspondingly, $c_{\nu}=2$ and $c_{\nu}=0$. As $c_{\nu}s_{\nu} \approx \frac{2}{3}$ and $\Delta m^{2}_{\nu}/\Delta m^{2}_{\text{eff}} \approx 30$, the off-diagonal elements of $M^{2}_{\nu}$ vanishes with $\eta_{\nu\mu} \approx -0.01$, leading to unrealistic $(\theta_{\text{eff}}) = 0$ and hence the high peak in the $\delta \chi^{2}$ curve.

The dark NSI has sizable effect at both low and high energy regions, crossing the black SI curve in the intermediate region. The $\delta \chi^{2}$ curve has a global minimum at $\eta_{\nu\mu} = -\eta_{\nu\tau} \approx -0.16$ which approaches the smaller $\Delta m^{2}_{\text{eff}}$ solution. At 2$\sigma$ level, $\epsilon_{e\mu}$ and $\epsilon_{e\tau}$ can be as large as $\pm 0.03$ which can further relax to $\pm 0.1$ at 3$\sigma$ confidence level. Better measurement of the solar neutrino fluxes at the SNO+ [32] and Jinping neutrino [33] experiments can help to identify the dark NSI.

With $\eta_{\alpha\beta} = 3\%$, the dark NSI effect on the CP measurement with accelerator neutrinos is already significant, see Fig. 4. While most of the $\eta_{\alpha\beta}$ elements deforms the biprobability contour around the SI one, the deviation by $\eta_{\nu\mu}$ can significantly change the picture. This is understandable since the CP measurement is mainly about the $\mu \rightarrow e$ transitions and hence is sensitive to any modification in the $e\mu$ element. Unfortunately, the $\eta_{\nu\mu}$ element at percentage level is not well constrained by the solar neutrino oscillation, see Fig. 3. Other complementary searches are necessary to guarantee the CP sensitivity at the accelerator type experiments against the dark NSI.

The atmospheric neutrino oscillation might provide such a complementary channel. As shown in Fig. 5, the $\eta_{\nu\mu}$ element can significantly modify the oscillation behaviors, especially around the MSW resonances which is the most important region to the neutrino mass hierarchy measurement with atmospheric neutrinos. With large event rate at PINGU [37] and ORCA [38], or the ability of INO [39] in distinguishing neutrino from antineutrino, good sensitivity on $\eta_{\nu\mu}$ can be expected with prior knowledge on the neutrino mass hierarchy.

**Conclusion** – The CPT violation can appear without breaking the CPT symmetry in the fundamental Lagrangian. Instead, it can arise as environmental effect and act as a manifestation of the coupling between neutrino and light DM. A new channel of probing the light DM appears in the neutrino oscillation. In addition to affecting the low-energy reactor and solar neutrino oscillations, the dark NSI can phenomenologically fake the genuine Dirac CP phase in the accelerator experiments. To guarantee the CP sensitivity, a synergy among various types of neutrino experiments is necessary.
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Supplementary Material

When neutrinos propagate through dark matter, both neutrino and dark matter particles are present as real particles. The forward scattering is then described by scattering matrix element

\[
\langle \nu_\alpha(p_\nu) \phi(p_\phi) \rvert T e^{i \int d^4x \mathcal{L}} \nu_\beta(p_\nu) \phi(p_\phi) \rangle = \langle \nu_\alpha(p_\nu) \phi(p_\phi) \rangle.
\]  

(S1)

By definition, forward scattering has no momentum exchange among particles. Consequently, the initial- and final-state neutrinos (dark matter particles) have exactly the same momentum \( p_\nu \) (\( p_\phi \)). A direct consequence is that the two scalar dark matter fields \( \phi \) share exactly the same wave function. The scalar dark matter particles can be replaced as \( \phi \rightarrow \sqrt{2p_\phi/m_\phi} \) times a creation or annihilation operator for the initial- or final-state states, \( \langle \phi \rvert \rightarrow (\sqrt{2p_\phi/m_\phi} ) a_\phi(0) \) or \( \langle \phi \rvert \rightarrow (\sqrt{2p_\phi/m_\phi} ) (a_\phi) \), respectively. To eliminate \( a_\phi^\dagger \) and \( a_\phi \), the leading contribution is the one shown in Fig. 1 with two \( \phi \nu \) vertices,

\[
i \delta S_{\beta\alpha} = \frac{1}{2} \langle \nu_\beta \phi \rvert T(\nu_\nu \phi \nu \nu)(\nu_\nu \phi \nu \nu) \rangle \langle \nu_\nu \phi \nu \nu \rangle.
\]  

(S2)

for the transition \( \nu_\alpha \rightarrow \nu_\beta \). With second quantization, the neutrino field and state are defined as \( \nu = a_\nu + b_\nu \) and \( \nu = a_\nu^\dagger (0) \). Consequently, contraction can only happens between \( \nu \) and \( \nu_\nu \) as well as between \( \bar{\nu} \) and \( \nu_\nu \). Since there is no difference between the two vertices in (S2), we can use the contraction of neutrino operators to fix the order of these two vertices, \( \nu_\nu \) contracts with the first and \( \nu \) with the second, as shown in (S3). Then there are two different ways of contracting the DM field \( \phi \) and its external state \( \phi \),

\[
i \delta S_{\beta\alpha} = \frac{1}{2} \langle \phi \nu \nu \rvert T(\nu_\nu \phi \nu \nu)(\nu_\nu \phi \nu \nu) \rangle \langle \nu_\nu \phi \nu \nu \rangle.
\]  

(S3)

The remaining one neutrino and one anti-neutrino fields would contract to become a neutrino propagator, as depicted in Fig. 1. The sample procedure can be repeated for anti-neutrino \( \bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta \) transition,

\[
i \delta S_{\beta\alpha} = \frac{1}{2} \langle \bar{\nu}_\beta \phi \rvert T(\nu_\nu \phi \nu \nu)(\nu_\nu \phi \nu \nu) \rangle \langle \nu_\nu \phi \nu \nu \rangle.
\]  

(S4)

In the transition matrix \( \delta S_{\beta\alpha} \) for neutrino propagation, the contracted neutrino operators are already next to each other but for the anti-neutrino one \( \delta S_{\beta\alpha} \), the neutrino operators need odd number of permutations to put paired ones together. This leads to a minus sign difference between the neutrino and anti-neutrino cases

\[
\delta S_{\beta\alpha} = \bar{\nu}_\beta \bar{\nu}_\alpha \nu_\alpha, \quad \delta S_{\beta\alpha} = -\bar{\nu}_\alpha \nu_\beta \bar{\nu}_\beta.
\]  

(S5)

Adding these corrections to the neutrino kinetic terms,

\[
\bar{u}_\beta(p_\nu - M + \delta \Gamma) \beta_\alpha u_\alpha a^\dagger a = \bar{v}_\alpha(-p_\nu - M + \delta \Gamma) \alpha_\beta v_\beta b^\dagger b = 0.
\]  

(S6)

From (S5) to (S6), the sign associated with \( \delta \Gamma \) is compensated by the permutation of neutrino operators while a sign difference now appears in the momentum part. Or equivalently, the effective propagator is the summation of all diagrams,

\[
\pm \bar{p}_\nu - M \sum_{n=0}^\infty \left( \frac{i \delta \Gamma}{\pm \bar{p}_\nu - M} \right)^n = \frac{i}{\pm \bar{p}_\nu - M + \delta \Gamma},
\]  

(S7)

for neutrino and anti-neutrino, respectively.

If we generally decompose the two-point function as \( \delta \Gamma \equiv \delta \Gamma^\mu_\mu + \delta M \), the neutrino (anti-neutrino) Hamiltonian expands as

\[
H \approx \frac{(\delta M + \delta M)(\delta M + \delta M)}{2E_\nu} \mp \delta \Gamma_0.
\]  

(S8)

While the matter potential \( \delta \Gamma_0 \) receives an opposite sign, the mass term correction is the same for the neutrino and antineutrino modes. The formalism (S6) and (S8) is quite general for various matter effects that neutrino can experience [18, 21]. Note that the neutrino (anti-neutrino) oscillation is described by \( H (H^T) \), respectively, due to the different flavor assignments in (S5) and (S6).

The concrete form of the two-point functions \( i \delta \Gamma_\alpha_\beta \) can be written down according to the Feynman diagrams in Fig. 1,

\[
\delta \Gamma_\alpha_\beta = \rho_\phi(v_\phi) \sum_j y_{\alpha j} y_{\beta j}^\dagger \left[ \pm (\bar{p}_\nu + p_\phi) - m_\nu \right] + \frac{i}{\pm (\bar{p}_\nu - p_\phi) - m_\nu},
\]  

(S9)

where \( p_\phi^2 = m_\phi^2 \) for on-shell neutrinos. First, let us move the \( \gamma \) matrices to the numerator

\[
\delta \Gamma_\alpha_\beta = \frac{i \rho_\phi(v_\phi) \sum_j y_{\alpha j} y_{\beta j}^\dagger}{m_\phi^2} \left[ \pm (\bar{p}_\nu + p_\phi) + m_\nu \right] + \frac{\pm (\bar{p}_\nu - p_\phi) + m_\nu}{m_\phi^2 - 2p_\nu \cdot p_\phi},
\]  

(S10)

whether the denominators have been simplified as \( (p_\nu \pm p_\phi)^2 - m_\nu^2 = p_\phi^2 \pm 2p_\nu \cdot p_\phi \) for on-shell neutrinos. Since the momentum of the non-relativistic light DM is much smaller than the neutrino momentum, \( p_\phi \sim m_\phi (1, \bar{v}_\phi) \ll p_\nu \), the denominators are dominated by \( \pm 2p_\nu \cdot p_\phi \approx \pm 2m_\phi E_\nu \). Then the common term \( \pm \bar{p}_\nu + m_\nu \) in the two numerators cancel with each other, leaving only the \( \bar{p}_\phi \) term. Considering the fact that dark matter particles around Earth are quite non-relativistic nowadays, we just need to keep the dominating time component,
\( \dot{p}_0 \approx m_0 \gamma_0 \). In addition, from neutrino to anti-neutrino, the momentum in the propagator receive a minus sign to account for the opposite fermion flow, leading to the overall sign in (S5). Keeping only the leading order, we can get

\[
\delta \Gamma_{\alpha\beta} \approx \sum_j y_{\alpha j} y_{\beta j}^* \frac{\rho_\chi}{m_\phi^2 E_\nu} \gamma_0, \tag{S11}
\]

with the total density \( \rho_\chi \) from averaging over the DM velocity distribution, \( \int \rho_\phi(\vec{v}) d\vec{v} = \rho_\chi \).

An interesting feature is that (S11) has energy dependence, rather than the energy-independent SM matter potential [18] or mass term correction from scalar NSI [21]. This leads to significantly different phenomenological consequences. Since the first term in (S8) is also inversely proportional to neutrino energy, the correction from (S11) then appears as correction to the mass squared term,

\[
H = \frac{M^2}{2E_\nu} + \frac{1}{E_\nu} \sum_j y_{\alpha j} y_{\beta j}^* \frac{\rho_\chi}{m_\phi^2} = \frac{M^2 + \delta M^2}{2E_\nu}, \tag{S12}
\]

where \( \delta M^2_{\alpha\beta} = \pm \frac{2\alpha}{m_\phi^2} \sum_j y_{\alpha j} y_{\beta j}^* \). If neutrino travels inside the ordinary matter, there is an extra contribution from the matter potential induced by the SM charged currents. Due to the presence of light DM, neutrinos receive opposite mass squared correction than anti-neutrinos. This is essentially a manifest violation of CPT symmetry due to environmental effect.

At first sight, it may seem strange why a helicity-flipping Yukawa coupling in (1) can lead to helicity-conserving correction (S11). Although it is true that Yukawa coupling does flip helicity, two Yukawa vertices in Fig. 1 can flip the neutrino helicity twice and conserve the neutrino helicity. In addition, the non-zero momentum flow in the neutrino propagator of (1) provides \( 1/E_\nu \) dependence and promotes the \( \gamma_0 \) term to correction of the neutrino mass squared term.

For vector DM particle \( V \), it can couple with neutrino current

\[
- \mathcal{L} \ni \frac{1}{2} m_V^2 V_\mu V^\mu + \frac{1}{2} M_{\alpha\beta} \bar{\nu}_\alpha \nu_\beta + g_{\alpha\beta} \bar{\nu}_\alpha \gamma^\mu \nu_\beta. \tag{S13}
\]

Following the same procedure of sandwiching action \( S \) with external fields and contracting particle creation versus annihilation operators in pair, we can derive the effective two-point function

\[
\delta \Gamma_{\alpha\beta} = -g_{\alpha j} g_{\beta j} \frac{\rho_\nu (\nu \nu)}{m_V^2} \epsilon_\alpha (p_{\nu}) \epsilon_\beta^* (p_{\nu}) \times \left[ \gamma^\alpha \frac{p_{\nu}^\mu + p_{\nu}^\mu + m_\nu}{m_V^2 + 2p_{\nu} \cdot p_{\nu}} \gamma^\beta + \gamma^\beta \frac{p_{\nu}^\mu - p_{\nu}^\mu + m_\nu}{m_V^2 - 2p_{\nu} \cdot p_{\nu}} \gamma^\alpha \right]. \tag{S14}
\]

Since the incoming and outgoing dark matter particles share the same momentum, \( p_{\nu} = p_{\nu}^\prime \), the two polarization vectors are actually the same, \( \epsilon (p_{\nu}^\prime) = \epsilon (p_{\nu}) \equiv \epsilon \). In addition, it is possible to choose convention to make the polarization vectors real. Then the indices \( \alpha \) and \( \beta \) in (S14) can interchange with each other and consequently we can first factorize out the two \( \gamma \) matrices on the side,

\[
\delta \Gamma_{\alpha\beta} = -g_{\alpha j} g_{\beta j} \frac{\rho_\nu (\nu \nu)}{m_V^2} \epsilon_\alpha (p_{\nu}) \epsilon_\beta (p_{\nu}) \times \gamma^\alpha \left[ \frac{p_{\nu}^\mu + p_{\nu}^\mu + m_\nu}{m_V^2 + 2p_{\nu} \cdot p_{\nu}} + \frac{p_{\nu}^\mu - p_{\nu}^\mu + m_\nu}{m_V^2 - 2p_{\nu} \cdot p_{\nu}} \right] \gamma^\beta. \tag{S15}
\]

Then, we can use the same argument as the scalar case to eliminate the \( \dot{p}_\nu + m_\nu \) terms in the numerator and \( \dot{p}_\nu \approx m_\nu \gamma_0 \).

With non-relativistic dark matter, the three polarization vectors can be chosen as the three spatial unit vector along \( x, y, \) and \( z \) axes, \( \epsilon_i = (0, e_i) \), respectively. For \( m_V \ll E_\nu \), we only need to consider the \( \dot{p}_\nu \) term. Since DM is non-relativistic, its contribution is dominated by \( \dot{p}_\nu \approx m_\nu \gamma_0 \). The two identical polarization vectors \( \epsilon_\alpha \) and \( \epsilon_\beta \) can symmetrize their indices, \( \epsilon_\alpha \epsilon_\beta = \epsilon_\beta \epsilon_\alpha \). This significantly simplifies the \( \gamma \) matrices, \( \epsilon_\alpha \epsilon_\beta \gamma^\alpha \gamma^\beta = 2(\epsilon \cdot \gamma) \epsilon^0 + \epsilon^0 \). Then the effective potential reduces to a form close to fermion propagator with at most linear combination of \( \gamma \) matrices. Since the polarization vectors are orthogonal and have only spatial components, the \( 2(\epsilon \cdot \gamma) \epsilon^0 \) term vanishes at the leading order. The two-point function then simplifies to

\[
\delta \Gamma_{\alpha\beta} \approx \sum_j g_{\alpha j} g_{\beta j} \frac{\rho_\nu}{m_V^2 E_\nu} \gamma_0. \tag{S16}
\]

Consequently, the correction from vector dark matter to neutrino oscillation takes the same form as the scalar case (S12) with \( \delta M^2_{\alpha\beta} = \pm 2 \sum_j g_{\alpha j} g_{\beta j} \frac{\rho_\nu}{m_V^2} \), which is similar as the scalar case with the Yukawa couplings \( y \) replaced by the gauge couplings \( g \).