Revealing Reliable Signatures  
by Learning Top-Rank Pairs

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Abstract. Signature verification, as a crucial practical documentation analysis task, has been continuously studied by researchers in machine learning and pattern recognition fields. In specific scenarios like confirming financial documents and legal instruments, ensuring the absolute reliability of signatures is of top priority. In this work, we proposed a new method to learn “top-rank pairs” for writer-independent offline signature verification tasks. By this scheme, it is possible to maximize the number of absolutely reliable signatures. More precisely, our method to learn top-rank pairs aims at pushing positive samples beyond negative samples, after pairing each of them with a genuine reference signature. In the experiment, BHSig-B and BHSig-H datasets are used for evaluation, on which the proposed model achieves overwhelming better pos@top (the ratio of absolute top positive samples to all of the positive samples) while showing encouraging performance on both Area Under the Curve (AUC) and accuracy.

Keywords: writer-independent signature verification · top-rank learning · absolute top

1 Introduction

As one of the most important topics in document processing systems, signature verification has become an indispensable issue in modern society \cite{12}. Precisely, it plays important role in enhancing security and privacy in various fields, such as finance, medical, forensic agreements, etc. As innumerable significant documents are signed almost every moment throughout the world, automatically examining the genuineness of the signed signatures has become a crucial subject. Since misjudgment is hardly allowed especially in serious and formal situations like in forensic usages, obtaining “highly reliable” signatures is of great importance.

Fig.\textsuperscript{1} illustrates two scenarios of signature verification. In the writer-dependent scenario (a), it is possible to prepare the verifiers specialized for individuals. In

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In this paper, we propose a new method to learn top-rank pairs for highly-reliable writer-independent signature verification. The proposed method is inspired by top-rank learning [3,4,5]. Top-rank learning is one of the ranking tasks but is different from the standard ranking task. Fig. 2 shows the difference of top-rank learning from standard learning to rank. The objective of the standard ranking task (a) is to determine the ranking function that evaluates positive samples much higher than negative samples as possible. This objective is equivalent to maximize AUC. In contrast, top-rank learning (b) has a different objective to maximize absolute top positives, which are highly-reliable positive samples in the sense that no negative sample has a higher rank than them. In (c), the very beginning of the ROC of top-rank learning is a vertical part; this means that there are several positive samples that have no negative samples ranked higher than them. Consequently, Top-rank learning can derive absolute top positives.
as highly reliable positive samples. The ratio of absolute top positives over all positives is called “$\text{pos@top}$.”

The proposed method to learn top-rank pairs accepts a paired feature vector for utilizing the promising property of top-rank learning for writer-independent signature verification. As shown in Fig. 1 (b), the writer-independent scenario is based on the pairwise evaluation between a query and a genuine reference. To integrate this pairwise evaluation into top-rank learning, we first concatenate $q$ and $g$ into a single paired feature vector $x = q \oplus g$, where $q$ and $g$ denote the feature vector of a query and a genuine reference, respectively. The paired vector $x$ is treated as positive when $q$ is written by the genuine writer and denoted as $x^+$; similarly, when $q$ is written by a forgery, $x$ is negative and denoted as $x^-$. We train the ranking function $r$ with the positive paired vectors $\{x^+\}$ and the negative paired vectors $\{x^-\}$, by top-rank learning. As the result, we could have highly reliable positives as absolute top positives; more specifically, a paired signature in the absolute top positives is “a more genuine pair” than the most genuine-like negative pair (i.e., the most-skilled forgery), called top-ranked negative, therefore highly reliable. Although being an absolute top positive is much harder than just being ranked higher, we could make a highly reliable verification by using the absolute top positives and the trained ranking function $r$.

To prove the reliability of the proposed method in terms of pos@top, we conduct writer-independent offline signature verification experiments with two publicly-available datasets: BHSig-B and BHSig-H. We use SigNet [1] as not only the extractor of deep feature vectors ($q$ and $g$) from individual signatures but also a comparative method. SigNet is the current state-of-the-art model of offline signature verification. The experimental results prove that the proposed method outperforms SigNet not only pos@top but also other conventional evaluation metrics, such as accuracy, AUC, FAR, and FRR.

Our contributions are arranged as follows:

- We propose a novel method to learn top-rank pairs. This method is the first application of top-rank learning to conduct a writer-independent signature verification task to the best knowledge of the authors, notwithstanding that the concepts of top-rank learning and absolute top positives are particularly appropriate to highly reliable signature verification tasks.
- Experiments on two signature datasets have been done to evaluate the effect of the proposed method, including the comparison with the SigNet. Especially, the fact that the proposed method achieves higher pos@top proves that the trained ranking function gives a more reliable score that guarantees “absolutely genuine” signatures.

## 2 Related Work

The signature verification task has attracted great attention from researchers since it has been proposed [6,7]. Generally, signature verification is divided into online [3] as well as offline [9,10] fashions. Online signatures offer pressure and stroke order information that is favorable to time series analysis methods [11]...
like Dynamic Time Warping (DTW) [12]. On the other hand, offline signature verification should be carried out only by making full use of image feature information [13]. As a result, acquiring efficacious features from offline signatures [7,14,15] has become a highly anticipated challenge.

In recent years, CNNs have been widely used in signature verification tasks thanks to their excellent representation learning ability [16,17]. Among CNN-based models, Siamese network [18,19] is one of the common choices when it comes to signature verification tasks. Specifically, a Siamese network is composed of two identically structured CNNs with shared weights, particularly powerful for similarity learning, which is a preferable learning objective in signature verification. For example, Dey et al. proposed a Siamese network-based model that optimizes the distances between two inputted signatures, shows outstanding performance on several famous signature datasets [1]. Wei et al. also employed the Siamese network, and by utilizing inverse gray information with multiple attention modules, their work showed encouraging results as well [20]. However, none of the those approaches target revealing highly reliable genuine signatures.

To acquire the highly reliable genuine signatures, learning to rank [21,22] is a more reasonable approach than learning to classify. This is because learning to rank allows us to rank the signatures in order of genuineness. Bipartite ranking [23] is one of the most standard learning-to-rank approach. The goal of the bipartite ranking is to find a scoring function that gives a higher value to positive samples than negative samples. This goal corresponds to the maximization of AUC (Area under the ROC curve), and thus bipartite ranking has been used in various domains [24,25,26].

As a special form of bipartite ranking, the top-rank learning strategy [3,4,5] possesses characteristics that are more suitable for absolute top positive hunting. In contrast to the standard bipartite ranking, top-rank learning aims at maximizing the absolute top positives, that is, maximizing the number of positive samples ranked higher than any negative sample. Therefore, top-rank learning is suitable for some tough tasks that require possibly highly reliable (e.g., medical diagnosis [27]). To the best of our knowledge, this is the first application of top-rank learning to the signature verification task.

TopRank CNN [27] is a representation learning version of the conventional top-rank learning scheme, which combines the favorable characteristics of both CNN and the top-rank learning scheme. To be more specific, when encountered with entangled features in which positive is chaotically tied with negative, conventional top-rank learning methods without representation learning capability like TopPush [3] can hardly achieve a high pos@top. That is to say, the representation learning ability of CNN structure makes the TopRank CNN a more powerful top-rank learning method. Moreover, for avoiding the easily-happen over-fitting phenomenon, TopRank CNN considerately attached the max operation with the p-norm relaxation.

Despite the superiority of ranking schemes, studies that apply ranking strategies on signature verification tasks are still in great demand. Chhabra et al. in [28] proposed a Deep Triplet Ranking CNN, aiming at ranking the input sig-
natures in genuine-like order by minimizing the distance between genuine and anchors. In the same year, Zheng et al. proposed to utilize RankSVM for writer-dependent classification, to ensure the generalization performance on imbalanced data [2]. However, even if they care about ranking results to some extent, no existing studies have been dedicated to the absolute top genuine signatures yet. To address this issue, this work is mainly designed to focus on obtaining the absolute top genuine signatures, implemented by learning top-rank pairs for writer-independent offline signature verification tasks.

3 Learning Top-Rank Pairs

Fig. 3 shows the overview of the proposed method to learn top-rank pairs for writer-independent offline signature verification. The proposed method consists of two steps: a representation learning step and a top-rank learning step. Fig. 3(a) shows the former step and (b) and (c) show the latter step.

3.1 Feature representation of paired samples

As shown in Fig. 1(b), each input is a pair of a genuine reference sample $g$ and a query sample $q$ for writer-independent signature verification. Then the
paired samples \((g, q)\) are fed to some function to evaluate their discrepancy. If the evaluation result shows a large discrepancy, the query is supposed to be a forgery; otherwise, the query is genuine.

Now we concatenate the two \(d\)-dimensional feature vectors \((g, q)\) into a \(2d\)-dimensional single vector as shown in Fig. 3(a). Although the concatenation doubles the feature dimensionality, it allows us to treat the paired samples in a simple way. Specifically, we consider a (Genuine \(g\), Genuine \(q\))-pair as a positive sample with the feature vector \(x^+ = g \oplus q^+\) and a (Genuine \(g\), Forgery \(q^−\))-pair as a negative sample with \(x^− = g \oplus q^−\). If we have \(m\) (Genuine, Genuine)-pairs and \(n\) (Genuine, Forgery)-pairs, we have two sets \(\Omega^+ = \{x^+_i | i = 1, \ldots, m\}\) and \(\Omega^- = \{x^-_j | j = 1, \ldots, n\}\).

Under this representation, the writer-independent signature verification task is simply formulated as a problem to have a function \(r(x)\) that gives a large value for \(x^+_i\) and a small value for \(x^-_j\). Ideally, we want to have \(r(x)\) that satisfies \(r(x^+_i) > r(x^-_j)\) for arbitrary \(x^+_i\) and \(x^-_j\). In this case, we have a constant threshold \(\theta\) that satisfies \(\max_{1 \leq j \leq n} r(x^-_j) < \theta < \min_i r(x^+_i)\). If \(r(x) > \theta\), \(x\) is simply determined as a (Genuine, Genuine)-pair. However, in reality, we do not have the ideal \(r\) in advance; therefore we need to optimize (i.e., train) \(r\) so that it becomes closer to the ideal case under some criterion. In 3.2, pos@top is used as the criterion so that trained \(r\) gives more absolute tops.

As indicated in Fig. 3(a), each signature is initially represented as a \(d\)-dimensional vector \(g\) (or \(q\)) by SigNet \([1]\), which is still a state-of-the-art signature verification model realized by metric learning with the contrastive loss. Although it is possible to use another model to have the initial feature vector, we use SigNet throughout this paper. The details of SigNet will be described in 3.4.

### 3.2 Optimization to learn top-rank pairs

We then use a top-rank learning model for optimizing the ranking function \(r(x)\). As noted in Section 1, top-rank learning aims to maximize pos@top, which is formulated as:

\[
pos@top = \frac{1}{m} \sum_{i=1}^{m} I \left( r(x^+_i) > \max_{1 \leq j \leq n} r(x^-_j) \right),
\]

where \(I(z)\) is the indicator function. pos@top evaluates the number of positive samples with a higher value than any negative samples. The positive samples that satisfy the condition in Eq. 1 are called absolute top positives or just simply absolute tops. Absolute tops are very “reliable” positive samples because they are more positive than the top-ranked negative, that is, the “hardest” negative \(\max_{1 \leq j \leq n} r(x^-_j)\).

Among various optimization criteria, pos@top has promising properties for the writer-independent signature verification task. Maximization of pos@top is equivalent to the maximization of absolute tops — this means we can have reliable positive samples to the utmost extent. In a very strict signature verification task, the query sample \(q\) is verified as genuine only when the concatenated vector
$x = g \oplus q$ becomes one of the absolute tops. Therefore, having more absolute tops by maximizing pos@top will give more chance that the query sample is completely trusted as genuine.

Note that we call $r(x)$ as a “ranking” function, instead of just a scoring function. In Eq. (1), the value of the function $r$ is used just for the comparison of samples. This suggests that the value of $r$ has no absolute meaning. In fact, if we have a maximum pos@top by a certain $r(x)$, $\phi(r(x))$ also achieves the same pos@top, where $\phi$ is a monotonically-increasing function. Consequently, the ranking function $r$ specifies the order (i.e., the rank) among samples.

We will optimize $r$ to maximize $x^+$ in pos@top. Top-Rank Learning is the existing problem to maximize pos@top for a training set whose samples are individual (i.e., unpaired) vectors. In contrast, our problem to learn top-rank pairs is a new ranking problem for the paired samples, and applicable to various ranking problems where the relative relations between two vectors are important.

As noted in 3.3 and shown in Fig. 3 (b), we train $r$ along with a deep neural network (DNN) to have a reasonable feature space to have more pos@top. However, there are some risks when we maximize Eq. (1) directly using a DNN, if it has a high representation flexibility. The most realistic case is that the DNN overfits some outliers or noise. For example, if a negative outlier is distributed over the positive training samples, achieving the perfect pos@top is not a reasonable goal.

To avoid such risks, we employ the $p$-norm relaxation technique [29,27]. More specifically, we convert the maximization of pos@top into the minimization of the following loss:

$$L_{\text{TopRank}} (\Omega^+, \Omega^-) = \frac{1}{m} \sum_{i=1}^{m} \left( \sum_{j=1}^{n} (l(r(x_i^+)) - r(x_j^-))^p \right)^\frac{1}{p},$$

where $l(z) = \log(1+e^{-z})$ is a surrogate loss. Fig. 3 (c) illustrates $L_{\text{TopRank}}$. When $p = \infty$, Eq. (2) is reduced to $L_{\text{TopRank}} = \frac{1}{m} \sum_{i=1}^{m} \max_{1 \leq j \leq n} l(r(x_i^+)) - r(x_j^-))$, which is equivalent to the original pos@top loss of Eq. (1). If we set $p$ at a large value (e.g., 32), the Eq. (2) approaches the original loss of pos@top. In [27], it is noted that it is better not to select a too large $p$, because it has a risk of over-fitting and the overflow error in the implementation.

3.3 Learning top-rank pairs with their representation

In order to have a final feature representation to have a more pos@top, we apply a DNN to convert $x$ non-linearly during the training process of $r$. Fig. 3 (b) shows the process with the DNN. Each of the paired feature vectors in a minibatch is fed to a DNN. In the DNN, the vectors are converted to another feature space.

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Theoretically, learning top-rank pairs can be extended to handle vectors obtained by concatenating three or more individual vectors. With this extension, our method can rank the mutual relationship among multiple vectors.
and then their ranking score $r$ is calculated. The parameters of DNN are trained by the loss function $L_{\text{TopRank}}$.

3.4 Initial features by SigNet

As noted in 3.1, we need to have initial vectors ($g$ and $q$) for individual signatures by an arbitrary signature image representation method. SigNet [1] is the current state-of-the-art signature verification method and achieved high performance in standard accuracy measures. As shown in Fig. 3 (a), SigNet is based on metric learning with a contrastive loss and takes a pair of a reference signature and a query signature as its input images. For all pairs of reference and query signatures, SigNet is optimized to decrease the distance between (Genuine, Genuine)-pairs and increase the distance between (Genuine, Forgery)-pairs. The trained network can convert a reference image into $g$ and a query into $q$. Then a positive sample $x^+_i$ or a negative sample $x^-_j$ is obtained by the concatenation $g \oplus q$, as described in 3.4.

Theoretically, we can conduct the end-to-end training of SigNet in Fig. 3 (a) and DNN in (b). In this paper, however, we fix the SigNet model after its independent training with the contrastive loss. This is simply to make the comparison between the proposed method and SigNet as fair as possible. (In other words, we want to purely observe the effect of pos@top maximization and thus minimize the extra effect of further representation learning in SigNet.)

One might misunderstand that the metric learning result by SigNet and the ranking result by top-rank learning in the proposed method are almost identical;
however, as shown in Fig. 4, they are very different. As we emphasized so far, the proposed method aims to have more pos@top; this means we have a clear boundary between the absolute tops and the others. In contrast, SigNet has no such function. Consequently, SigNet might have a risk that a forgery has a very small distance with a genuine. Finally, this forgery will be wrongly considered as one of the reliable positives, which are determined by applying a threshold $\lambda$ to the distance by SigNet.

4 Experiments

In this section, we demonstrate the effectiveness of the proposed method on signature verification tasks. Specifically, we consider a comparative experiment with SigNet which is known as the outstanding method for the signature verification tasks.

4.1 Datasets

In this work, the BHSig260 offline signature dataset is used for the experiments, which consists of two subsets where one set of signatures are signed in Bengali (named BHSig-B) and the other in Hindi (named BHSig-H). The BHSig-B dataset includes 100 writers in total, each of them possesses 24 genuine signatures and 30 skillfully forged signatures. On the other hand, BHSig-H dataset contains 160 writers, each of them own genuine and forged signatures same as BHSig-B. In the experiments, both of the datasets are divided into training, validation, and test set to the ratio of 8:1:1.

Following the writer-independent setting, we evaluate the verification performance using the pair of signatures. That is, the task is to verify that the given pair is (Genuine, Genuine) or (Genuine, Forgery). We prepare a total of 276 (Genuine, Genuine)-pairs for each writer and a total of 720 (Genuine, Forgery)-pairs for each writer.

4.2 Experimental Settings

**Setting of the SigNet** SigNet is also based on a Siamese network architecture, whose optimization objective is similarity measurement. In the experiment, we followed the training setting noted in [1], except for the modifications of the data division.

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† Available at [http://www.gpds.ulpge.es/download](http://www.gpds.ulpge.es/download)
‡ CEDAR dataset that is also used in [1] was not used in this work because it has achieved 100% accuracy in the test set. Besides, GPDS 300 and GPDS Synthetic Signature Corpus datasets are restricted from obtaining
Table 1: The comparison between the proposed method and SigNet on BHSig-B and BHSig-H datasets.

| Dataset | Approaches | pos@top (↑) | Accuracy (↑) | AUC (↑) | FAR (↓) | FRR (↓) |
|---------|------------|-------------|--------------|---------|---------|---------|
| BHSig-B | proposed   | 0.283       | 0.806        | 0.889   | 0.222   | 0.222   |
|         | SigNet     | 0.000       | 0.756        | 0.847   | 0.246   | 0.247   |
| BHSig-H | proposed   | 0.114       | 0.836        | 0.908   | 0.179   | 0.178   |
|         | SigNet     | 0.000       | 0.817        | 0.891   | 0.192   | 0.192   |

Setting of the proposed method As introduced in Section 3.4, we used the extracted features from the trained SigNet (124G Floating point operations (FLOPs)) as the initial features. For the learning top-rank pairs with their representation, we used a simple architecture, 4 fully-connected layers (2048, 1024, 512 and 128 nodes respectively) with ReLU function (1G FLOPs). The hyperparameter $p$ of the loss function $2$ is chosen from $\{2, 4, 8, 16, 32\}$ based on the validation pos@top. As a result, we obtained $p=4$ for BHSig-B and $p=16$ for BHSig-H. The following results and the visualization are obtained by using these hyper-parameters.

4.3 Evaluation metrics

In the experiment, pos@top, accuracy, AUC, False Rejection Rate (FRR), and False Acceptance Rate (FAR) are used to comprehensively evaluate the proposed method and SigNet.

- pos@top: The ratio of the absolute top (Genuine, Genuine) signature pairs to the number of all of the (Genuine, Genuine) signature pairs (see also Eq. (1)).
- Accuracy: The maximum result of the average between True Positive Rate (TPR) and True Negative Rate (TNR), following the definition in [1].
- AUC: Area under the ROC curve.
- FAR: The ratio of the number of falsely accepted (Genuine, Forgery) signature pairs divided by the number of all (Genuine, Forgery) signature pairs.
- FRR: The ratio of the number of falsely rejected (Genuine, Genuine) signature pairs divided by the number of all (Genuine, Genuine) signature pairs.

4.4 Quantitative and qualitative evaluations

The quantitative evaluations of the proposed method and SigNet on two datasets are shown in Table 1. Remarkably, the proposed method achieved an overwhelming better performance on pos@top, while pos@top of SigNet is 0 for both datasets. This proves that the proposed method can reveal absolute top positive signature pairs (i.e., highly reliable signature pairs). Furthermore, the proposed method also outperformed the comparison method on all other evaluation criteria, accuracy and AUC, lower FAR, and FRR.
The ROC curves of the proposed method and SigNet on two datasets are shown in Fig. 5, respectively, each followed by a corresponding zoom view of the beginning part of ROC curves. As a more intuitive demonstration of Table 1, other than the larger AUC, it is extremely obvious that the proposed method achieved higher pos@top, which is the True Positive Rate (TPR) for \( x = 0 \).

Figs. 6 (a) and (b) show the distributions of features from the proposed method mapped by Principal Component Analysis (PCA) on two datasets, colored by ranking scores (normalized within the range \([0,1]\)). From the visualizations, we can easily notice the absolute top positive sample pairs distinguished by the top-ranked negative.

Following the feature distributions, Figs. 6 (c) and (d) give a more intuitive representation of the ranking order for the learning top-rank pairs. These graphs include information of (1) where did the top-ranked negative appear and (2) how many (Genuine, Genuine) and (Genuine, Forgery)-pairs are scored to the same rank. It could be observed that the first negative appeared after a portion of positive from these two graphs. On the other hand, Figs. 6 (e) and (f) show the ranking conditions of SigNet. Since the first negative pair shows on the top of the ranking, this causes 0 pos@top for both datasets.

As shown in Fig. 7, the absolute top (Genuine, Genuine)-pairs in (a) and (b) show great similarity to their counterparts. Especially, the consistency in their strokes and preference of oblique could be easily noticed even with the naked eye. On the other hand, both the non-absolute top (Genuine, Genuine)-pairs in (c) and (d) and (Genuine, Forgery)-pairs in (e) and (f) show less similarity to their corresponding signatures, no matter whether they are written by the same writer or not. Notwithstanding that they are all assigned with low ranking scores by learning top-rank pairs, such resemblance between two different classes could easily incur misclassification in conventional methods. Thus, our results shed light on the validity of the claimed effectiveness of the proposed method to maximize the pos@top.
Fig. 6: (a) and (b) PCA visualizations of feature distribution for the proposed method. The top-ranked negative and the absolute top positives are highlighted. (c)-(f) Distributions of the ranking scores as histograms. The horizontal and vertical axes represent the ranking score and #samples, respectively.
5 Conclusion

As a critical application especially for formal scenarios like forensic handwriting analysis, signature verification played an important role since it has been proposed. In this work, we proposed a writer-independent signature verification model for learning top-rank pairs. What is novel and interesting for this model is that the optimization objective of top-rank learning is to maximize the pos@top, to say the highly reliable signature pairs in this case. This optimization goal has fulfilled the requirement of the intuitive need of signature verification tasks to acquire reliable genuine signatures, not only to naively classify positive from negative. Through two experiments on data set BHSig-B and BHSig-H, the effectiveness of pos@top maximization has been proved compared with a metric learning-based network, the SigNet. Besides, the performance of the proposed model on the AUC, accuracy, and other common evaluation criteria frequently used in signature verification shown encouraging results as well.

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