Foundations of decomposition for manufacturing geometrical products

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Abstract
Geometrical tolerancing was developed to improve the weakness of previous tolerance systems to handle imperfect form and ambiguous references and was primarily developed for assembly of components. Although there have been recent modifications to increase tolerance zone’s utility, it is still basically a go/no-go system with components being in or out of tolerance. The use of tolerance zones is causing real industrial problems in the specification of high valued precision products. For example: in healthcare, the specification of the geometrical shape of the cup in total hip-joint replacements by a simple tolerance zone is allowing some cups to fail, by dislocating out of position, prematurely. A new design of non-spherical head is beginning to appear and the market requires improved specification. Further mathematical decomposition for the specification of the tolerancing zone is required to distinguish between good and failing functional geometries [1, 2].

The connection with filtration is explored. In particular the definitions for ‘primary mapping’ contained in ISO 16610-1 [3] is developed to include the foundations of decomposition in terms of structures called ‘complete lattices’. One very useful property of complete lattices is the existence of a smallest subset of lattice elements (called lattice generators) that can reconstruct the whole of the lattice using just joint (or alternatively meet) operations. These lattice generators constitute a basis (or frame) of the lattice and form a measurement scale for the nesting index of the associated decomposition/filter.

This definition of decomposition, given here, is universal and goes beyond just decomposition of geometrical products. Examples will be given to illustrate the utility of the definition to other aspects of smart manufacturing, including: formal concept analysis [4] for smart information systems.

1. Introduction

The history of the specification of manufactured geometrical products includes many basic decomposition operations (breaking down a component into a combination of simpler objects) for further analysis. This includes: partitioning, filtration, and association operations etc [5], and also decomposing a cylinder into straightness of the axis and the generatrix, plus the roundness. Surface texture has more sophisticated decompositions starting in 1933 with the Abbott Firestone curve (material ratio curve) [6], primary for looking at running-in/wear of a surface. The first B46 1 Standard issued March 1940 [7] contained the decomposition of a profile into roughness and waviness but had no primary connection to the manufacturing process. The primary connection to the signature from traditional machine tools came a little later in 1944 [8] with Roughness being due to the cutting action, Waviness due to vibrations, chatter, heat treatment or warping strains, and Form errors being due to flexure of the work in the machine (tool), or lack of straightness in the ways.

In the past seventy years, although the analysis of the decompositions has become more sophisticated, the primal decompositions have remained virtually the same. Manufacturing technology has made great progress, in particular freeform and structured surfaces where decomposition into simple geometrical shapes (planes, spheres, cylinders, prisms, revolutes, etc.) is not appropriate. The use of tolerance zones is causing
Figure 1. Additive manufactured component: roughness, waviness and lay is not an appropriate decomposition.

Figure 2. (a) and (b) are POSETS while (c) is not since the two left hand dots have two connections.

Figure 3. Examples of Dedekind–MacNeille completion: with the POSET at the bottom and the complete lattice at the top.
real industrial problems in the specification of high valued precision products. For example: In healthcare, the specification of the geometrical shape of the cup in total hip-joint replacements by a simple tolerance zone is allowing some cups to fail, by dislocating out of position, prematurely. A new design of non-spherical head is beginning to appear and the market requires improved specification. Further mathematical decomposition for the specification of the tolerance zone is required to distinguish between good and failing functional geometries [1, 2]. Further the decomposition into roughness, waviness, and lay, is not appropriate for modern processes such as additive manufacturing, see figure 1. This paper develops the foundations of decomposition to enable new appropriate decompositions to be developed for 21st century manufacturing for geometrical products.

2. Decomposition foundations

In this paper we will demonstrate the very close connection between decomposition and a special kind of filter based on ‘primary mappings’ contained in ISO 16610-1 [3]. Primary mappings are idempotent that is to say applying the same primary mapping twice is equivalent to only applying it once. The set of nesting indices for primary mappings is ordered but not necessarily totally ordered, some pairs of nesting indices may not be able to be compared for order. Thus the nesting indices of primary mappings form a Partially Ordered SET or POSET see figure 2.

The following theorem is very important for the connection between POSETs (primary mappings nesting indices) and complete lattices [9].

![Figure 4. Examples of lattices of elementary scales. (red dots) (a) Ordinal lattice (b) interordinal lattice (c) biordinal lattice and (d) nominal lattice.](image-url)

![Table 1. Attributes of the planets +Pluto.](table-url)
2.1. Dedekind–MacNeille completion

Every POSET can be order embedded (i.e. preserve the order structure of the POSET) in the unique smallest complete lattice that contains the given POSET, see figure 3.

A Lattice is very simple mathematical structures consisting of a POSET in which any two elements, a & b say, have two binary operations defined on them called ‘meet’ (infimum of a & b) and ‘join’ (supremum of a & b) [9].

A complete lattice is a lattice in which the join and the meet exits for every subset of that lattice. Hence a finite complete lattice will have a greatest element called the top and a lowest element called the bottom, see figure 3.

One very useful property of complete lattices is the existence of a smallest subset of lattice elements (called lattice generators) that can reconstruct the whole of the lattice using just joint (or alternatively meet) operations. These lattice generators constitute a basis (or frame) of the lattice and form a measurement scale for the nesting index of the associated decomposition/filter. Thus every individual decomposition corresponds to an associated element in the lattice which can be labelled by the subset of the lattice generators that can reconstruct that particular element (see figure 4).

This definition of decomposition is universal and goes beyond just decomposition of geometrical surfaces. An example will be given in the next section to illustrate the utility of the definition to other aspects of smart manufacturing, namely ‘formal concept analysis’ [4, 9] for smart information systems.

3. Illustrative example

Autonomous Manufacturing will require access to the huge manufacturing knowledge-base (National and International Standards, Materials data-sheets, etc) amassed by Humans. The structure of knowledge needs to be determined and encapsulated into smart data-bases that are machine readable.

Lattice Theory, through formal concept analysis [4, 9], offers a natural setting in which to discuss and analyse hierarchies of concepts. It provides a first step in the creation of a hierarchical structure of knowledge for smart data-bases by means of a hierarchical decomposition of concepts via a scale.

Unfortunately surface texture examples are either very simplistic (Roughness, Waviness, and Lay), or too large for a paper (formal concept analysis for ISO standards, etc.). There has been success with formal concept analysis with ISO standards with the recognition of a missing GPS operator ‘reconstruction’ which has now been added to ISO 17450-1 2011 clause 3.4.1.7. [5].

The following example illustrates how formal concept analysis can be used to decompose a set of entities with associated attributes that is not trivial nor too large for illustration.

Table 1 gives some attributes of the planets + Pluto in our Solar System.

Formal concept analysis is able to take this table of attributes and turn it into a complete lattice, figure 5 (for some reason in the formal concept analysis literature lattices are used that are upside down to those in the rest of the paper and in the lattice literature).

The resulting lattice has the attributes as the generators with s-s, near and no, running down the right hand side and yes, far, m-size running down the left hand side and large just off the left hand side. The analysis has shown that several of the concepts have not been fully decomposed (Uranus-Neptune, Jupiter-Saturn, Earth-Mars, and Mercury Venus) and that alternative attributes are necessary to produce a full decomposition. Even though, formal concept analysis has produced an hierarchical decomposition of the concepts, according to the specified attributes, ready to be used for further analysis in smart data-bases.

4. Conclusions and further work

Within the paper the concept of Decomposition and its Mathematical foundations have been introduced:
Decompositions can be embedded into a Complete lattice; Decomposition Scales are the Lattice generators.

It has been demonstrated that the mathematical foundations are universal and can be used for:

- Decomposition of geometry;
- Decomposition of the structure of knowledge.

Complete Lattices can provide the foundations for a mathematical framework for AI and data analytics to empirically decompose structures with no a priori assumptions.

Contained within the paper is a vision of one possible future pathway for the specification and verification of geometrical products, particularly surface texture, through decomposition.

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