Multiple Tactical Missiles Cooperative Attack With Formation-Containment Tracking Requirement Along the Planned Trajectory

XINGGUANG XU¹², CHANGRONG CHEN², ZHANG REN¹, AND SHUSHENG LI²

¹School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, China
²Beijing Institute of Mechanical and Electrical Engineering, Beijing 100074, China
Corresponding author: Xingguang Xu (xuxingguang@buaa.edu.cn)

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ABSTRACT This paper studies the cooperative guidance and control problems with formation-containment tracking requirement under directed topologies for multiple tactical missiles. The states of the formation-leaders can not only keep a parallel triangle formation but also make tracks for the state trajectory generated by the tracking-leader, while the states of followers converge to the convex hull formed by those of the formation-leaders. Firstly, an integrated cooperative guidance and control framework is proposed, and combat missions are distinguished in line with tactical missiles rewarded different functions in practical applications. Then, a terminal slide mode guidance law with impact angle is presented in three-dimensional space, which ensures attack missile to attack the target in finite time. Sufficient conditions for multiple tactical missiles to achieve formation-containment tracking are derived. On the basis of Lyapunov theory, it is shown that the expected formation-containment tracking can be realized by reconnaissance and decoy missiles in the presence of the tracking-leaders’ unknown control input. In addition, in the case of unknown drag force, the six degrees of freedom missile controller is designed by combining genetic algorithm and disturbance observer to dynamically adjust the rudder deflection and thrust, ensuring stable tracking of overload command during cooperative attack. Finally, numerical simulations validate the feasibility and effectiveness of the proposed results.

INDEX TERMS Cooperative guidance and control framework, formation-containment tracking, terminal slide mode guidance law, genetic algorithm, disturbance observer.

I. INTRODUCTION

Recently salvo attack of tactical missiles has been regarded as an effective approach to penetrate anti-air defense systems. Researches on cooperative guidance and control under the background of multiple missiles combat application have been hot and attractive issues. To achieve salvo attack, we can perform cooperative guidance and control of tactical missiles through two different strategies, i. e., individual homing and cooperative homing. For individual homing, a suitable common impact time is pre-programmed manually beforehand, and then the open-loop or closed-loop guidance and control laws are adopted for each tactical missile individually to achieve the designated impact time independently. At present, the scheme used in this kind of guidance law design can be divided into two main categories. One is based on the traditional proportional guidance law. In [1], a feedback term of time-to-go error was added to the guidance law, and an impact-time-control guidance (ITCG) law was proposed. [2] designed a proportional navigation with a variable coefficient, which was based on the relationship between the remaining flight time and the effective navigation ratio. Another category is based on the modern control theory. In addition to impact time, the coordinated impact angle needs to be considered in order to make the missile warhead obtain better killing effects. [3] linearized the model under the small heading error assumption, the impact time and impact angle can be controlled simultaneously using the guidance law based on the optimal control theory. In [4], the guidance law based on the LQR theory was presented to
attack the uniform motion target with expected impact angle and minimum miss-distance.

Unlike individual homing, a hierarchical framework free from predesigned impact time is built for cooperative homing via online data links and communication topology. In [5], missiles were classified into many groups, where each group belonged to the centralized leader-follower framework, and the leaders of different groups communicated with each other through the nearest-neighbor topology. Finite-Time Cooperative Guidance (FTCG) laws considering the saturation constraint on field-of-view (FOV) were firstly addressed to accommodate the communication topology in a single group, and then an improved sequential approach was proposed to FTCG-FOV to accommodate the communication topology between groups. [6] investigated the uncertainty of communication composed of stochastic network and additive noise, analysed the time-to-go error of each missile in the communication network, and set a new impact time through the data link using the mean square error.

Although the aforementioned cooperative homing to realize salvo attack has been widely investigated, combat missions are not distinguished according to various characteristics of tactical missiles in practical applications, and the formation-containment tracking issues are not considered. To improve the penetration effectiveness under the condition of system-of-systems combat, combat scenarios are proposed as follows: (i) Attack missile with warhead should penetrate the enemy’s anti-air defense systems to destroy a target either moving at a relatively low speed or non-maneuvering, and more space on the missile can be taken up to load explosives. (ii) Reconnaissance missiles mounting detecting equipment are required to realize an expected formation pattern while tracking the designated trajectory generated by attack missile simultaneously, which can promote the detection precision in the way of detecting the target cooperatively. (iii) Decoy missiles will converge to the convex hull spanned by reconnaissance missiles synchronously, and interfere the enemy defenses by the loaded interference power amplifiers. In addition, the safety of attack missile can be guaranteed due to the fact that reconnaissance and decoy missiles will disperse the fire power of the enemy. Therefore, the proposed methods outperform the conventional multi-missile combat mode. As far as the author knows, there are few literatures on the formation-containment tracking for multiple tactical missiles. With the rapid development of consensus theory (see [7]–[18] and references therein), more studies related with consensus strategies are extended to solve the formation or containment problems of the multi-agent system. [7] presented formation control problems for the multi-agent system utilizing consensus-based approaches, where a given geometry was approached by the states or outputs of multiple agents, and provided proofs that some traditional formation methods such as virtual structure, behavior-based ones, and leader-follower can be converted into the consensus based ones. Time-varying formation protocols for the multi-agent system with general linear dynamics and switching directed interaction topologies were dealt with in [8]. Besides achieving the time-varying formation, the entire multi-agent system can also make tracks for the desired trajectory through formation tracking protocols in [9], [10]. In [11], containment control problems were investigated for general linear multi-agent systems with time-varying delays, where the states or outputs of the followers were required to converge to the convex hull spanned by those of leaders. Recently, due to the increasing researches on formation control and containment control, the formation-containment problem has been frequently considered, which requires that the states or outputs of leaders achieve desired formation while the states or outputs of followers converge to the convex hull formed by those of leaders. [12], [13] addressed formation-containment analysis and design problems for time-delayed high-order linear time-invariant multi-agent systems with directed interaction topologies or heterogeneous architecture. Formation-containment control problems for multi-UAV systems with directed topologies were presented in [14], and a formation-containment platform with multiple UAVs was demonstrated.

To the best of our knowledge, even though each of the four aspects listed above including formation, containment, formation-tracking and formation-containment has been taken into account in the existing literatures, formation-containment tracking problems considering all of the four aspects remain to be further investigated.

In this paper, formation-containment tracking problems for multiple tactical missiles subjected to directed topologies are proposed. Firstly, an integrated cooperative guidance and control framework is presented, in which the guidance layer, coordination layer, and control layer are assigned their own engineering problems by guidance engagement geometry, kinematic dynamics and actuator control loop, respectively. At the theoretical level of formation-containment tracking, tactical missiles rewarded different functions act as the tracking-leader, formation-leader and follower. Then a terminal slide mode variable structure guidance law with impact angle constraint is designed for the tracking-leader, which guarantees attack missile hit the target under the proposed guidance law in finite time. Moreover, formation-containment tracking protocols are derived utilizing neighboring information of different agents. It is proved that formation-leaders can maintain a time-varying formation with target enclosing achieved, meanwhile the followers are supposed to converge to the convex hull specified by the formation-leaders. Genetic algorithm and disturbance observer (DOB) are employed to design the control systems for salvo attack of different missiles, which allow all six degrees of freedom (DOF) of the missile to be controlled using four control input through thrust and rudder deflection. Finally, a formation-containment tracking anti-ship scenario is introduced, where 6 tactical missiles are considered to perform a cooperative attack against a stationary target. The numerical simulation results are addressed to illustrate the performance of the obtained results.
Compared with the relevant results on formation-containment tracking control, the main contributions of the current paper are threefold. Firstly, an integrated cooperative guidance and control framework is proposed. Multiple tactical missiles can not only realize cooperative attack autonomously but also constitute a predefined time-varying formation-containment geometry while tracking the trajectory of attack missile. In [2], [5], [6], [19], [20], [23], only time or space cooperative guidance problems were considered and there existed no formation or containment process. Secondly, the tracking-leader is free from certain input. Attack missile adopts the sliding mode control method to construct the homing guidance law, which acts as an unknown control input for the tracking-leader on the formation-containment tracking control level. In [10], the formation-leader performed a formation tracking task without the tracking-leader control input. The protocols to cope with cases of a tracking-leader with naught or available input cannot be extended to the non-cooperative tracking-leader case. Thirdly, the dynamic characteristics of the tactical missile in control loop are taken into account in parallel to the consideration of formation-containment tracking in guidance loop. Genetic algorithm can quickly adjust the parameters of rudder/axial overload control loop, which enables the tactical missile to track the specific overload command required to follow the desired cooperative trajectory smoothly and quickly. A DOB in axial overload control loop is provided with no prior knowledge of the drag force. In [7]–[9], [11]–[13], [15]–[17], the dynamics for each agent were restricted to swarm systems without concrete objects, and the practical application of formation or containment approaches for missiles were not investigated. In [20], [21], [23], the missiles were assumed to be mass points. Because the dynamic characteristics will affect the reaction speed of the tactical missile to the cooperative guidance command dramatically, the results for aircrafts free from the dynamic characteristics cannot be directly applied to salvo attack considering missiles’ dynamic characteristics and the cooperative guidance requirement simultaneously. Only 3 DOF cooperative problems were considered in [22]. Although 6 DOF cooperative problems were investigated in [24], [25], explicit knowledge of the drag coefficient was required in [24], while controller involving turbojet engine thrust was not considered in [25] with respect to axial overload control. It should be noted that [26] only focused on the mid-course guidance of cooperative attack of multiple missiles formation, and cooperative engagement process involving dynamic characteristics was ignored.

The remainder of this paper is organized as follows. Section II introduces some useful results and basic concepts and gives the problem formulation of the formation-containment tracking cooperative attack for multiple tactical missiles. In Section III, the integrated cooperative guidance and control framework is proposed, and the approach to implement the guidance law during the engagement, design the formation-containment tracking protocols and conduct the control of each missile through the rudder/axial overload control loop are presented. Numerical simulations are discussed in detail in Section IV. At last, Section V concludes this paper.

II. PRELIMINARIES AND PROBLEM DESCRIPTION

A. BASIC CONCEPTS ON GRAPH THEORY

A directed graph with \( M + N + 1 \) nodes is described by \( G = (V, E, W) \), where \( V = \{v_0, v_1, \ldots, v_{M+N}\} \) is the set of nodes, \( E \subseteq \{(v_i, v_j) : v_i, v_j \in V; i \neq j\} \) denotes the set of edges, and \( W = [w_{ij}] \in \mathbb{R}^{(M+N+1) \times (M+N+1)} \) represents the adjacency matrix with nonnegative weights \( w_{ij} \). An edge of graph \( G \) is defined as \( e_{ij} = (v_i, v_j) \). The weight \( w_{ij} > 0 \) if and only if \( e_{ij} \in E \), and \( w_{ii} = 0 \), \( i = 0, 1, \ldots, M + N \). Denote \( N_l = \{v_j \in V : (v_j, v_l) \in E\} \) by the set of neighbors of node \( v_l \). A sequence of ordered edges \((v_{i_1}, v_{i_2}, \ldots, v_{i_{g+1}})(k = 1, 2, \ldots, g + 1)\) is said to be a directed path from \( v_{i_1} \) to \( v_{i_g} \). Let \( \text{deg}_{in}(v_l) = \sum_{v_j \in V} w_{lj} \), denote the in-degree of node \( v_l \), then the in-degree matrix of \( G \) is described by \( D = \text{diag}(\text{deg}_{in}(v_1), \ldots, \text{deg}_{in}(v_{M+N})) \). The Laplacian matrix of the graph \( G \) is defined as \( L = D - W \).

The multi-agent system is made up of 1 tracking-leader, \( N \) formation-leaders and \( M \) followers. If an agent has no neighbors, it is called a tracking-leader. The formation-leader can only receive information from other formation-leaders or the tracking-leader. The neighbors of a follower cannot be anything but a cluster of other followers or formation-leaders. For notational brevity, denote by \( E = \{1, 2, \ldots, N\} \) and \( F = \{1 + N, 2 + N, \ldots, M + N\} \). In this paper, consider a multiple tactical missile system with \( M + N + 1 \) missiles, and each is denoted as an agent within the multi-agent system.

B. GUIDANCE AND CONTROL GEOMETRY

The three-dimensional (3D) homing guidance geometry is depicted in Fig.1, where \( OXYZ, OX_1Y_1Z_1 \) are the inertial coordinate system and the line-of-sight (LOS) coordinate system, respectively. \( R \) is the range-to-go between the missile and the target along the LOS direction, \( q_\delta \) represents the LOS angle in the pitching direction, \( q_\beta \) is the LOS angle in the yawing direction. One obtains

\[
\frac{d}{dt}r = \frac{\delta}{dt}r + \omega \times r
\]  

(1)
The ballistic coordinate system is given by

\[ r = \begin{bmatrix} R & 0 & 0 \end{bmatrix}^T \]

where \( r \) is the relative position vector of missile and target. \( \frac{\delta}{\delta t}, \frac{\delta}{\delta t} \) denote taking time derivative of \( r \) in OXYZ and \( OX_4Y_4Z_4 \), respectively. \( \omega = \begin{bmatrix} 0 & R\dot{q}_e & -R\dot{q}_e \cos q_e \end{bmatrix}^T \) represents the angular velocity in \( OX_4Y_4Z_4 \). Thus, time derivative of Eq. (1) can be expressed as

\[
\frac{d^2}{dt^2} r = \frac{\delta^2}{\delta t^2} r + \frac{\delta}{\delta t} (\omega \times r) + \omega \times (\omega \times r) \tag{2}
\]

The coordinate transformation from the LOS coordinate system to the inertial system can be described as follows

\[
\mathcal{L}_1 = \begin{bmatrix} \cos q_e & \sin q_e & 0 \\ -\sin q_e & \cos q_e & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \beta & 0 & -\sin \beta \\ 0 & 1 & 0 \\ \sin \beta & 0 & \cos \beta \end{bmatrix} = \begin{bmatrix} \cos q_e \cos \beta & \sin q_e \cos \beta & \sin q_e \sin \beta \\ -\sin q_e \cos \beta & \cos q_e \cos \beta & -\sin q_e \sin \beta \\ \sin \beta & 0 & \cos \beta \end{bmatrix} \tag{3}
\]

The control system design geometry is given in Fig. 2, where \( OX_2Y_2Z_2 \) is the ballistic coordinate system. The flight-path angle and speed bank angle are described by \( \theta \) and \( \psi_v \). Let state variables be \( x = \begin{bmatrix} x_1, x_2 \end{bmatrix}^T, \quad x_1 = \begin{bmatrix} q_e - q_{ed} \end{bmatrix}, \quad x_2 = \begin{bmatrix} q_e - q_{bd} \end{bmatrix}, \quad x_3 = \begin{bmatrix} \dot{q}_e \end{bmatrix} \), \( x_4 = \begin{bmatrix} \dot{q}_e \end{bmatrix} \), where \( q_{ed} \) and \( q_{bd} \) are the expected LOS angle within 3D space. Therefore, the equations of missile-target engagement take the form

\[
\begin{align*}
\dot{x}_1 &= x_3 \\
\dot{x}_2 &= x_4 \\
\dot{x}_3 &= \frac{-2\dot{R}}{R} x_2 - x_2^2 \sin x_1 \cos x_1 + \frac{1}{\cos x_1} \frac{a_{M\beta}}{R} - \frac{1}{\cos x_1} \frac{a_{\beta\beta}}{R} \\
\dot{x}_4 &= -\frac{2\dot{R}}{R} x_4 + 2x_3 x_4 \tan x_1 + \frac{1}{\cos x_1} \frac{a_{M\beta}}{R} - \frac{1}{\cos x_1} \frac{a_{\beta\beta}}{R} 
\end{align*} \tag{4}
\]

In this paper, taking the target as stationary or shifting at a relatively low speed, then let \( a_{\beta\beta} = a_{\beta\beta} = 0 \), and further define

\[
\begin{align*}
\zeta_1 &= [x_1, x_2]^T, \quad \zeta_2 = [x_3, x_4]^T, \quad x = \left[ \begin{bmatrix} \zeta_1 \end{bmatrix}^T, \begin{bmatrix} \zeta_2 \end{bmatrix}^T \right]^T, \\
\mathcal{R}(x) &= \left[ \begin{bmatrix} \frac{-2\dot{R}}{R} x_2 - x_2^2 \sin x_1 \cos x_1 - \frac{2\dot{R}}{R} x_4 + 2x_3 x_4 \tan x_1 \end{bmatrix} \right]^T, \\
\mathcal{N}(x) &= \text{diag} \left\{ \frac{1}{R} \frac{1}{\cos x_1} \right\} \mathcal{A} = \left[ \begin{bmatrix} \frac{a_{M\beta}}{a_{\beta\beta}} \end{bmatrix}^T \right]
\end{align*} \tag{7}
\]

Eq. (7) is rewritten as

\[
\begin{align*}
\dot{\zeta}_1 &= \zeta_2 \\
\dot{\zeta}_2 &= \mathcal{R}(x) + \mathcal{N}(x) \mathcal{A}
\end{align*} \tag{8}
\]

Coordinate transformations Eqs. (3) and (4) are further adopted to transfer the control input from the LOS coordinate system to the ballistic coordinate system, and assuming the overload in \( X_2 \)-direction for the formation-leader is steered to be zero, one obtains that

\[
\begin{align*}
\dot{\zeta}_1 &= \zeta_2 \\
\dot{\zeta}_2 &= \mathcal{R}(x) + \mathcal{H}(x) \mathcal{D}
\end{align*} \tag{9}
\]

where \( \mathcal{D} = [n_{\zeta_1}, n_{\zeta_2}]^T \) stands for overload command along \( Y_2 \)-direction and \( Z_2 \)-direction in \( OX_2Y_2Z_2 \) and (10), as shown at the bottom of the next page.

1) DYNAMIC MODELING

Based on the relationship between missile motion state and overload, one can obtain

\[
\begin{align*}
\dot{v} &= g(n_v - \sin \theta) \\
\dot{\theta} &= g(n_v - \cos \theta) / v \\
\psi_v &= -gn_{\zeta_1} \cos \theta / v
\end{align*} \tag{11}
\]
where \([n_x, n_y, n_z]^T\) is overload of missile in the ballistic coordinate system. The speed, flight-path angle and speed bank angle are described by \(\psi, \theta\) and \(\psi_v\). \(g\) is gravity acceleration. According to dynamical equations, overload can be written as

\[
\begin{align*}
0 &= \frac{1}{mg} \left( T \cos \alpha \cos \beta - X \right) \\
n_x &= \frac{1}{mg} \left( T \sin \alpha \cos \gamma_v + \cos \alpha \sin \beta \sin \gamma_v \right) + Y \cos \gamma_v - Z \sin \gamma_v \\
n_y &= \frac{1}{mg} \left( T \sin \alpha \sin \gamma_v - \cos \alpha \sin \beta \cos \gamma_v \right) + Y \sin \gamma_v + Z \cos \gamma_v
\end{align*}
\]

where \(T\) is engine thrust. Drag force \(X\), lift force \(Y\), lateral force \(Z\) are the components of aerodynamic force. \(\alpha, \beta, \gamma_v\) represent attack angle, sideslip angle and velocity slope angle. We can get these three angles based on the following geometric equation

\[
\begin{align*}
\sin \beta &= \cos \theta \left[ \cos \gamma \sin (\psi - \psi_v) + \sin \phi \sin \gamma \cos (\psi - \psi_v) \right]
\sin \alpha &= \cos \theta \left[ \sin \phi \cos \gamma \cos (\psi - \psi_v) - \sin \gamma \sin (\psi - \psi_v) \right]
\sin \gamma_v &= \cos \alpha \sin \beta \sin \psi - \sin \alpha \sin \beta \cos \gamma \cos \phi + \cos \beta \sin \gamma \cos \phi / \cos \theta
\end{align*}
\]

where \(\gamma\) is the roll angle, \(\phi\) is the pitch angle, \(\psi\) is the yaw angle. According to the transformation relationship between inertial coordinate system and body coordinate system \([27]\), one can obtain that

\[
\begin{pmatrix}
\dot{\psi} \\
\dot{\phi} \\
\dot{\gamma}
\end{pmatrix} =
\begin{pmatrix}
0 & \sin \gamma & \cos \gamma \\
0 & \frac{\cos \gamma}{\sin \phi} & -\sin \phi / \sin \gamma \\
1 & -\tan \phi \cos \gamma & \tan \phi \sin \gamma
\end{pmatrix}
\begin{pmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{pmatrix}
\]

The angular velocities are described in the body fixed frame attached to the center of mass, and the rotational motion equations are written as

\[
\begin{pmatrix}
\dot{\omega}_x \\
\dot{\omega}_y \\
\dot{\omega}_z
\end{pmatrix} = - \begin{pmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{pmatrix} \times J
\begin{pmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{pmatrix} + \begin{pmatrix}
M_x \\
M_y \\
M_z
\end{pmatrix}
\]

where \(m\) is mass of missile, \(J \in \mathbb{R}^{3 \times 3}\) is the moment of inertia, \([M_x, M_y, M_z]^T\) is the force and aerodynamic torque applied to the missile. Ignoring the effect of damping torque, the approximations of the forces and moments are given by

\[
\begin{align*}
X &= c_x qS \\
Y &= \left( c_d^\alpha \alpha + c_d^\delta \delta \right) qS \\
Z &= \left( c_z^\alpha \alpha + c_z^\beta \beta \right) qS
\end{align*}
\]

\[
\mathcal{H}(x) = \frac{\mathcal{N}(x)}{g}
\]

\[
\begin{pmatrix}
\sin q_x \sin \theta \cos (q_\beta - \psi_v) + \cos q_x \cos \theta \sin q_x \sin (q_\beta - \psi_v) \\
\sin \theta \sin (\psi_v - q_\beta) \\
\cos (q_\beta - \psi_v)
\end{pmatrix}
\]

where \(q = 1/2 \rho v^2\) is dynamic pressure, and \(\rho\) stands for air density. \(S, L, \delta_x, \delta_y, \delta_z\) denote reference area, reference length, aileron angular deflection, rudder angular deflection and elevator angular deflection. \(c_x, c_y, c_z, \delta_x, \delta_y, \delta_z\) are the aerodynamic coefficients, and \(m_x, m_y, m_z, h_x, h_y, h_z\) are the aerodynamic moment coefficients.

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In this study, a 3D many-to-one engagement scenario with formation-containment tracking requirement is considered, and the following assumptions are adopted.

**Assumption 1:** The nonzero overload $n_i(t)$ is bounded and there exists a maximum magnitude of the overload $n_m$ such that $\|n_i(t)\| \leq n_m$ due to the finite actuation capability for physical actuators during flight.

**Assumption 2:** Considering that long-term motion characteristics are mainly determined by long period mode in flight, short period process and the effect of rotation in the control process is neglected in the guidance realization.

**Assumption 3:** The directed graph among the tracking-leader and the formation-leaders in $G$ contain a spanning tree, in which the tracking-leader is the root node. There is at least one directed path between each follower and some formation-leader.

Under Assumption 3, the Laplacian matrix $L \in \mathbb{R}^{(N+M+1) \times (N+M+1)}$ associated with the directed graph $G$ has the following form

$$
L = \begin{bmatrix}
0 & 0 & 0 \\
L_{21} & L_{22} & 0 \\
0 & L_{32} & L_{33}
\end{bmatrix}
$$

where $L_{21} \in \mathbb{R}^{N \times 1}$, $L_{22} \in \mathbb{R}^{N \times N}$, $L_{32} \in \mathbb{R}^{M \times N}$ and $L_{33} \in \mathbb{R}^{M \times M}$.

Let $\dot{h}_i(t) = [v_i(t), \nu_i(t)]^T$, $A_1 = [1, 0]^T$, and $B = [0, 1]^T$. Assume that $A = A_1B^T$, and $(A, B)$ is stabilizable.

Therefore Eq. (17) and (18) can be rewritten as

$$
\dot{h}_i(t) = A \dot{h}_i(t) + Bu_i(t)
$$

**Definition 1:** The formation-leader $i$ is said to achieve the expected time-varying formation tracking specified by $h(t)$ if

$$
\lim_{t \to \infty} (\dot{h}_i(t) - h_i(t) - \dot{h}_0(t)) = 0, \quad i \in E
$$

where $h_i(t) \in \mathbb{R}^3 (i = 1, 2, \cdots, N)$ characterize the desired time-varying formation configuration, and $h_i(t) = [h_{ix}(t), h_{iy}(t), h_{iz}(t)]^T$ denotes piecewise continuously differentiable formation vector for the formation-leader $i$.

**Definition 2:** The follower $j$ is said to achieve containment if

$$
\lim_{t \to \infty} \left(\dot{h}_j(t) - \sum_{k=1}^{N} \lambda_{jk} \dot{h}_k(t)\right) = 0, \quad j \in F
$$

where $\lambda_{jk} (j \in F; k \in E)$ represent nonnegative constants satisfying $\sum_{k=1}^{N} \lambda_{jk} = 1$ for the follower $j$.

**Definition 3:** The multiple tactical missile system Eq. (21) is said to achieve formation-containment tracking if Eq. (22) and (23) hold for all the formation-leaders and followers, respectively.

Not that all the results hereafter can be easily extended to the higher dimensional case with $n \geq 2$ using the Kronecker product. In the three-dimensional space, $n$ is set to be 3.

**Lemma 1 [28]:** Consider the following nonlinear system

$$
\dot{\xi} = \Upsilon(\xi)
$$

where $\Upsilon(0) = 0$, $\xi \in \mathbb{R}^n$, $U_0 \in \mathbb{R}^n$ is defined as an open-loop field at the origin of the system. Given a positive definite function $V(\xi)$, where $\xi \in U_0$, if there exist real numbers $k > 0$, $\alpha \in (0, 1)$ satisfying

$$
\dot{V}(\xi) + kV(\xi)^\alpha \leq 0
$$

then the origin of the system is a finite-time stable equilibrium, and the pause time $t_r$ depends on the initial value $\xi(0) = \xi_0$, its upper bound is

$$
\frac{V(\xi_0)^{1-\alpha}}{k(1-\alpha)}
$$

**Lemma 2 [16]:** Under Assumption 3, there exists a diagonal matrix $D_E = diag[de_1, de_2, \cdots, de_N]$ with $de_i > 0 (i = 1, 2, \cdots, N)$ such that $F_E = D_EL_1 + L_1^T D_E > 0$, and there is a diagonal matrix $D_F = diag[df_1, df_2, \cdots, df_M]$ with $df_j > 0 (j = 1, 2, \cdots, M)$. One such $D_E$ and $D_F$ can be calculated by $[de_1, de_2, \cdots, de_N]^T = (L_2^T)^{-1}1_N$ and $[df_1, df_2, \cdots, df_M]^T = (L_3^T)^{-1}1_M$.

The formation-containment tracking issues for multiple tactical missiles considered in this current paper are mainly focused as follows: (1) how to construct a hierarchical framework for cooperative attack with formation-containment tracking requirement; (2) how to implement a terminal guidance with a desired terminal impact angle in finite time against stationary target within 3D space; (3) under what conditions formation-containment tracking can be accomplished with the tracking-leader of uncertain control input; (4) how to develop the formation-containment tracking control protocol and 6-DOF missile control system associated with nonlinear uncertainties.

**III. MAIN RESULTS**

**A. INTEGRATED COOPERATIVE GUIDANCE AND CONTROL FRAMEWORK**

For the proposed integrated cooperative guidance and control framework, different tactical missiles are firstly divided into attack missiles, reconnaissance missiles and decoy missiles according to their respective combat missions (see Table 1). Without loss of generality, a schematic diagram of cooperative attack with formation-containment tracking requirement is shown in Fig.3, in which multiple tactical missiles communicate with each other through data links, and achieve cooperative guidance and control along the attack direction using the nearest neighbors’ information.

Instead of completing the combat mission independently by high-performance multi-purpose missiles or platforms, the capabilities are scattered to multiple missiles, which jointly form a combat system to complete the mission. The battlefield orientation of the formation-containment tracking framework is that the attack missile converges to a given impact angle in the course of engagement, and strikes the heavily defended targets in finite time to guarantee better lethal performances of the warheads. Reconnaissance missiles with detection equipment are expected to generate specific formation configurations, which expands the
TABLE 1. Combat missions division among different missiles.

| Classification | Roles            | Guidance mode                                      | Mount devices | Combat mission            |
|----------------|------------------|---------------------------------------------------|---------------|---------------------------|
| Attack missile | Tracking-leader  | Terminal slide mode variable structure guidance law with impact angle | Warhead       | Killing a target          |
| Reconnaissance missile | Formation-leader   | Formation-tracking | Seeker       | Acquiring information about the state of target motion |
| Decoy missile  | Follower         | Interference power amplifier                      | Interfering the anti-air defense missile system |

FIGURE 3. Skeleton of cooperative attack with formation-containment tracking requirement.

cooperative search radius of the target. Then the detected target location is broadcast to attack missile that will adjust its flight online to realize fixed-point attack. Decoy missiles suppress and disperse the fire power of the enemy in the confrontation area. Consequently, the missile group will survive the threat and penetrate the enemy’s anti-air defense systems successfully, forming an overwhelming advantage against the surrounding combat opponents.

Then the flowchart of the proposed cooperative framework listed in Fig.4 is implemented through a step-by-step layer such as the guidance layer, coordination layer and control layer. Missile and target guidance engagement geometry, kinematic dynamics and actuator control loop are applied to depict different layers.

Remark 1: Overloads in diverse directions are coordinated variables in the proposed framework, which link different layers together. In the course of attack missile engagement, the overload design in Section III.B has been transformed from the LOS coordinate system (OX₄Y₄Z₄) to the ballistic coordinate system (OX₂Y₂Z₂) in Section II.C during the engagement process of attack missile. The cooperative scheme in Section III.C is implemented in the inertial coordinate system (OXYZ). Then the ballistic coordinate system is introduced into the three-loop missile autopilot and thrust adjustment design procedure in Section III.D. Accordingly, coordinate transformations from the ballistic coordinate system to the inertial system, and on the contrary, from the inertial system to the ballistic coordinate system are implemented once, respectively. For the conciseness and clarity, the subscript of different coordinate systems will be omitted throughout this paper.

B. TERMINAL SLIDE MODE VARIABLE STRUCTURE GUIDANCE LAW WITH IMPACT ANGLE

Considering the 3D guidance system (9), define the sliding mode surface as follows

$$s = [s_1, s_2]^T$$  \hspace{1cm} (27)

where $s_1 = x_1 + \frac{1}{2}x_3^{p_1/q_1}$, $s_2 = x_2 + \frac{1}{2}x_4^{p_2/q_2}$ is the longitudinal and lateral motion sliding mode surface, respectively. $\beta_1 > 0$ and $\beta_2 > 0$ are any positive constants. Positive odd numbers $p_1 > 0$, $q_1 > 0$, $p_2 > 0$ and $q_2 > 0$ are chosen to satisfy $1 < p_1/q_1 < 2$. Design the terminal guidance law as

$$D = -\mathcal{H}(x)^{-1} \left[ \mathcal{R}(x) + \Omega(x) + \eta \text{sgn}(s) \right]$$ \hspace{1cm} (28)

where

$$\Omega(x) = \begin{bmatrix} \beta_1 \frac{q_1}{p_1} x_3^{2-p_1/q_1} & \beta_2 \frac{q_2}{p_2} x_4^{2-p_2/q_2} \end{bmatrix}^T,$$

$$\eta = \text{diag}(\eta_1, \eta_2), \text{sgn}(s) = \begin{bmatrix} \text{sgn}(s_1) & \text{sgn}(s_2) \end{bmatrix}^T.$$

The stability analysis of the guidance loop is shown in Appendix.

Then we can get the following theorem.

Theorem 1: In the process of terminal guidance for attack missile, the guidance law (28) can ensure that the 3D guidance system (9) converges to the sliding mode surface $s = 0$ in finite time and the system converges to zero on the sliding mode surface in finite time.

Remark 2: The guidance law based on sliding mode control method is applied widely in the existing literatures, and the LOS angle and angle rate is usually chosen as the sliding mode surface (see [29-36]). Theoretically speaking, the smaller the parameters $\beta_i (i = 1, 2)$ and $q_i/p_i (i = 1, 2)$ are, the shorter the time of system (9) converges to the sliding mode surface $s = 0$, which indicates a better control performance. However, when the state of system has reached the sliding mode surface $s = 0$, the smaller the parameters $\beta_i (i = 1, 2)$ are, the longer the time of system (9) converges to the equilibrium point, which may result in an unacceptable performance or even destabilize the system. Therefore,
\[ (I_N \otimes BK_1) \]
\[ \times \left( \sum_{j=0}^{N} \omega_{ij} \left( (\delta_i(t) - h_i(t)) - (\delta_j(t) - h_j(t)) \right) \right) \]
\[ + \sigma g(I_M \otimes BK_1) f \]
\[ \times \left( \sum_{j=0}^{N} \omega_{ij} \left( (\delta_i(t) - h_i(t)) - (\delta_j(t) - h_j(t)) \right) \right) \]
\[ + (I_N \otimes A) \delta_E \]
\[ \hat{\vartheta}_F(t) = (I_M \otimes BK_2) \]
\[ \times \left( \sum_{j=1+N}^{M+N} \omega_{ij} \left( (\delta_i(t) - h_i(t)) - (\delta_j(t) - h_j(t)) \right) \right) \]
\[ + \sigma g(I_M \otimes BK_2) f \]
\[ \times \left( \sum_{j=1+N}^{M+N} \omega_{ij} \left( (\delta_i(t) - h_i(t)) - (\delta_j(t) - h_j(t)) \right) \right) \]
\[ + (I_M \otimes A) \hat{\delta}_F \]

\[ \text{Remark 3:} \] Attack missile maintains an axial overload of zero through the tangential balance between thrust and drag force (further discussed in Section III.D), which is beneficial to ensure the warhead security. Convergence to the desired impact angle within a finite time is important in most practical guidance applications [29], which is conductive to the killing effects of the warheads in time-varying modern warfare.

**C. FORMATION-CONTAINMENT TRACKING ANALYSIS AND DESIGN**

In this section, velocity, positions and acceleration vectors take the form defined in the inertial coordinate system. Consider the following time-varying formation-containment tracking protocol

\[ u_i(t) = K_1 \sum_{j=0}^{N} \omega_{ij} \left( (\delta_i(t) - h_i(t)) - (\delta_j(t) - h_j(t)) \right) \]
\[ + \sum_{j=0}^{N} \tau g f \left( \sum_{j=0}^{N} \omega_{ij} \left( (\delta_i(t) - h_i(t)) - (\delta_j(t) - h_j(t)) \right) \right), \quad i \in E \]
\[ u_i(t) = K_2 \sum_{j=1+N}^{M+N} \omega_{ij} \left( \delta_i(t) - \delta_j(t) \right) \]
\[ + \sigma g f \left( \sum_{j=1+N}^{M+N} \omega_{ij} \left( \delta_i(t) - \delta_j(t) \right) \right), \quad i \in F \]

where \( u_i(t) = [u_i(t)_{(x)}, u_i(t)_{(y)}, u_i(t)_{(z)}]^T \) is the acceleration along axial, vertical and lateral directions. \( K_p = \left[ k_{p1}, k_{p2} \right] (p = 1, 2) \), are constant gain matrices. \( \tau \) and \( \sigma \) denote positive constants, and \( f(s(t)), s(t) \in \mathbb{R}^n \) is a nonlinear function to be ascertained later.

The following theorem provides sufficient conditions for system (21) with the proposed protocols to accomplish formation-containment tracking.

Let \( \hat{\delta}_E = \left[ \hat{\delta}_1^T(t), \hat{\delta}_2^T(t), \ldots, \hat{\delta}_N^T(t) \right]^T \) and \( \hat{\delta}_F = \left[ \hat{\delta}_1^{1+N}(t), \hat{\delta}_2^{1+N}(t), \ldots, \hat{\delta}_M^{1+N}(t) \right]^T \). Under protocols (29) and (30), multi-agent system (21) can be transformed as follows.

\[ \hat{\vartheta}_F(t) = (I_M \otimes BK_2) \]
\[ \times \left( \sum_{j=1+N}^{M+N} \omega_{ij} \left( (\delta_i(t) - h_i(t)) - (\delta_j(t) - h_j(t)) \right) \right) \]
\[ + \sigma g(I_M \otimes BK_2) f \]
\[ \times \left( \sum_{j=1+N}^{M+N} \omega_{ij} \left( (\delta_i(t) - h_i(t)) - (\delta_j(t) - h_j(t)) \right) \right) \]
\[ + (I_M \otimes A) \hat{\delta}_F \]

\[ \text{Theorem 2:} \] The multiple tactical missile system under protocols (29) and (30) accomplish formation-containment tracking if the following conditions hold simultaneously.

(i) For all \( i \in E, h_{i_k}(t) = h_{i_k}(t), \) that is,
\[ h_{i_k}(t) = A h_{i_k}(t) \]
\[ \text{(33)} \]

(ii) Let \( K_1 = \rho_1 B^T P, \) and \( K_2 = \rho_2 B^T P \). Choose
\[ \rho_1 \geq \frac{\lambda_{\max}(D_1)}{\lambda_{\min}(\Phi^T)}, \rho_2 \geq \frac{\lambda_{\max}(G_1)}{\lambda_{\min}(E^T)} \]
\[ \text{(34)} \]

\( P \) is the positive definite solution to the following algebraic Riccati equation
\[ A^T P + PA - PBB^T P + I_n = 0 \]
\[ \text{(35)} \]

(iii) Consider the following nonlinear function \( f(s(t)) \)
\[ f(s(t)) = \begin{cases} \frac{s(t)}{||s(t)||}, & ||s(t)|| \neq 0 \\ 0, & ||s(t)|| = 0 \end{cases} \]
\[ \text{(36)} \]

where \( s(t) \in \mathbb{R}^m \) represents
\[ B^T P \left( \sum_{j=0}^{N} \omega_{ij} \left( (\delta_i(t) - h_i(t)) - (\delta_j(t) - h_j(t)) \right) \right) \]
\[ \text{or} \]
\[ B^T P \left( \sum_{j=1}^{N+M} \omega_{ij} \left( \delta_i(t) - \delta_j(t) \right) \right) \]
\[ \text{(37)} \]

(iv) Specify \( \tau \) and \( \sigma \) in advance, and the values are relatively large with
\[ \tau \geq n_m, \quad \sigma \geq \tau \]
\[ \text{(38)} \]

Proof: Let \( \hat{\vartheta}_E(t) = \left[ \hat{\delta}_1^T(t), \hat{\delta}_2^T(t), \ldots, \hat{\delta}_N^T(t) \right]^T \), where \( \hat{\delta}_i(t) = \delta_i(t) - h_i(t) - \delta_0(t) \).

Define \( \Gamma(t) \) and \( \Xi(t) \) as
\[ \Gamma(t) = (L_1 \otimes I_n) \hat{\delta}_E \]
\[ \Xi(t) = (L_3 \otimes I_n) \hat{\delta}_F(t) + (L_2 \otimes I_n) \hat{\delta}_E(t) \]
\[ \text{(39)} \]

From Eq. (38) and (39), one has
\[ \Gamma(t) \]
\[ = (I_N \otimes A) \Gamma(t) + (L_{11} \otimes BK_1) \Gamma(t) + \tau (L_{11} \otimes B) G(\Gamma(t)) \]
\[ - (L_{11} I_n \otimes Bu(t)) + (L_{11} \otimes A) h(t) - (L_{11} \otimes I_n) h(t) \]
\[ \text{(40)} \]
\( \tilde{z}(t) = (I_M \otimes A)z(t) + (L_3 \otimes BK_2)z(t) \\
+ \sigma g(L_1 \otimes B)G(z(t)) + \tau g(L_2 \otimes B)F(\Gamma(t)) \\
+(L_2 \otimes BK_1)\Gamma(t) \)  

(41)

where

\[ \Gamma(t) = [\Gamma_1^T(t), \Gamma_2^T(t), \ldots, \Gamma_N^T(t)], \]

\[ z(t) = [z_{1,N}^T(t), z_{2,N}^T(t), \ldots, z_{M+N}^T(t)], \]

\[ F(\Gamma(t)) = [f^T(\Gamma_1(t)), f^T(\Gamma_2(t)), \ldots, f^T(\Gamma_N(t))], \]

\[ F(z(t)) = [f^T(z_{1,N}(t)), f^T(z_{2,N}(t)), \ldots, f^T(z_{M+N}(t))]. \]

The Formation-Leaders Can Achieve Formation Tracking: Construct the Lyapunov candidate function as follows:

\[ V_E(t) = \Gamma^T(t)(DE \otimes P)\Gamma(t) \]  

(42)

where \( DE \) is a positive diagonal matrix described in Lemma 2.

Taking the derivative of \( V_E(t) \) along the trajectory of Eq. (40) obtains

\[ \dot{V}_E(t) = \Gamma^T(t)(DE \otimes (PA + A^TP))\Gamma(t) \\
- \rho_1 \Gamma^T(t)(\Phi_E \otimes PBB^T)\Gamma(t) \\
- 2\tau g \Gamma^T(t)(DE_{L11} \otimes PB)F(\Gamma(t)) \\
\times 2 - \Gamma^T(t)(DE_{L11} \otimes PB)(1_N \otimes u_0(t)) \\
+ 2\Gamma^T(t)(DE_{L11} \otimes P)\dot{h}(t) \]  

(43)

where \( \Phi_E = DE_{L11} + L_{11}^TDE > 0 \) and \( \dot{h}(t) = (I_N \otimes A)\dot{h}(t) \) - \( \dot{h}(t) \).

Based on Eq. (36), one has

\[ \Gamma_1^T(t)PBf(\Gamma_1(t)) = \left\|B^TP\Gamma_1(t)\right\|, \quad i \in E \]  

(44)

\[ \Gamma_i^T(t)PBf(\Gamma_1(t)) \leq \left\|B^TP\Gamma_1(t)\right\| \quad \|f(\Gamma_i(t))\| \leq \left\|B^TP\Gamma_1(t)\right\|, \quad i \neq j \]  

(45)

Thus, one can further has

\[ -2\tau g \Gamma^T(t)(DE_{L11} \otimes PB)F(\Gamma(t)) \]

\[ = -2\tau g \sum_{i=1}^{M+N} d_{i} \Gamma_1^T(t)PB \sum_{j=1}^{N} \omega_{ij}(f(\Gamma_j(t)) - f(\Gamma_j(t))) \]

\[ = -2\tau g \sum_{i=1}^{M+N} d_{i} \omega_{i0} \Gamma_1^T(t)PBf(\Gamma_1(t)) \]

\[ \leq -2\tau g \sum_{i=1}^{M+N} d_{i} \omega_{i0} \left\|B^TP\Gamma_1(t)\right\| \]  

(46)

Under Assumption 1, one can obtain

\[ -2\Gamma^T(t)(DE_{L11} \otimes PB)(1_N \otimes u_0(t)) \]

\[ = -2 \sum_{i=1}^{N} d_{i} \omega_{i0} \Gamma_1^T(t)PBu_0(t) \]

\[ \leq 2 \sum_{i=1}^{N} d_{i} \omega_{i0} \left\|B^TP\gamma_i(t)\right\| \quad \|u_0(t)\| \]

\[ \leq 2\eta_m \sum_{i=1}^{N} d_{i} \omega_{i0} \left\|B^TP\gamma_i(t)\right\| \]  

(47)

From Eq. (46) and (47), it holds that

\[ \dot{V}_E(t) \]

\[ \leq \Gamma^T(t)(DE \otimes (PA + A^TP)\Gamma(t) \\
- \rho_1 \Gamma^T(t)(\Phi_E \otimes PBB^T)\Gamma(t) + 2(\tau - \eta_m) \times \sum_{i=1}^{N} d_{i} \omega_{i0} \left\|B^TP\gamma_i(t)\right\| + 2\Gamma^T(t)(DE_{L11} \otimes P)\dot{h}(t) \]  

(48)

Substituting Eq. (34) and (37) into (48) leads to

\[ \dot{V}_E(t) \leq \Gamma^T(t)(DE \otimes (PA + A^TP - PBB^T)P)\Gamma(t) \]

\[ + 2\Gamma^T(t)(DE_{L11} \otimes P)\dot{h}(t) \]  

(49)

Since Eq. (33) and (35) hold, it follows from (49) that

\[ \dot{V}_E(t) \leq -\Gamma^T(t)(DE \otimes I_n)\Gamma(t) \]

(50)

Therefore, it can be concluded that \( \Gamma(t) \rightarrow 0 \) as \( t \rightarrow \infty \), that is, Eq. (22) is required. Furthermore,

The Followers Can Achieve Containment:

Construct the Lyapunov candidate function as follows:

\[ V_F(t) = \Xi^T(t)(DF \otimes P)\Xi(t) \]  

(51)

where \( DF \) is a positive diagonal matrix specified in Lemma 2. The time derivative of \( V_F(t) \) along the trajectories of Eq. (41) satisfies

\[ \dot{V}_F(t) = \Xi^T(t)(DF \otimes (PA + A^TP))\Xi(t) \\
- \rho_2 \Xi^T(t)(DF_{L3} + L_{11}^TDF \otimes PBB^T)\Xi(t) \\
- 2\tau g \Xi^T(t)(DF_{L2} \otimes PB)F(\Xi(t)) \\
+ 2\tau g \Xi^T(t)(DF_{L2} \otimes PB)F(\Gamma(t)) \\
+ 2\Xi^T(t)(DF_{L2} \otimes PBK_1)\Gamma(t) \]  

(52)

One gets from Eq. (44) and (45) that

\[ 2\tau g \Xi^T(t)(DF_{L3} \otimes PB)F(\Xi(t)) \]

\[ \leq 2\tau g \sum_{i=1+M+N}^{M+N} df_i \left\|B^TP\Xi_i(t)\right\| \sum_{j=1}^{N} \omega_{ij} \]  

(53)

Recalling that \( \|f(\Gamma_i(t))\| \leq 1 \), therefore

\[ 2\tau g \Xi^T(t)(DF_{L2} \otimes PB)F(\Gamma(t)) \]

\[ = 2\tau g \sum_{i=1+M+N}^{M+N} df_i \Xi_i^T(t)PB \sum_{j=1}^{N} \omega_{ij}(f(\Gamma_j(t)) \]

\[ \leq 2\tau g \sum_{i=1+M+N}^{M+N} df_i \left\|B^TP\Xi_i(t)\right\| \sum_{j=1}^{N} \omega_{ij} \]  

(54)
Moreover,

\[
2\Xi^T(t)(D_F L_2 \otimes PBK_1)\Gamma(t) \\
= 2\Xi^T(t)(\sqrt{D_F} \otimes I_n)(\sqrt{D_F} L_2 \otimes PBK_1)\Gamma(t) \\
\leq \frac{1}{2} \Xi^T(t)(D_F \otimes I_n)\Xi(t) \\
+ 2\left\| (\sqrt{D_F} L_2 \otimes PBK_1)\Gamma(t) \right\|^2
\]

(55)

Noting that Eq. (34) holds, from Eq. (53), (54), and (55), it can be obtained that

\[
\dot{V}_T(t) \\
= \Xi^T(t)(D_F \otimes (PA + A^TP - PBB^TP))\Xi(t) \\
+ \frac{1}{2} \Xi^T(t)(D_F \otimes I_n)\Xi(t) \\
+ 2\left\| (\sqrt{D_F} L_2 \otimes PBK_1)\Gamma(t) \right\|^2 \\
+ 2(\tau - \sigma)g \sum_{i=1}^{M+N} d_{ij} \left\| B^TP\Xi_i(t) \right\|_2 \sum_{j=1}^{N} \omega_{ij} \\
\leq \Xi^T(t)(D_F \otimes (PA + A^TP - PBB^TP))\Xi(t) \\
+ \frac{1}{2} \Xi^T(t)(D_F \otimes I_n)\Xi(t) \\
+ 2\left\| (\sqrt{D_F} L_2 \otimes PBK_1)\Gamma(t) \right\|^2 \\
- \frac{1}{2} \Xi^T(t)(D_F \otimes I_n)\Xi(t) \\
\leq 2\left\| (\sqrt{D_F} L_2 \otimes PBK_1)\Gamma(t) \right\|^2 - \frac{1}{2\lambda_{\text{max}}(P)} V_T(t)
\]

(56)

Under the condition that \( \lim_{t \to \infty} \Gamma(t) = 0 \), then it can be concluded that \( \Xi(t) \to 0 \) as \( t \to \infty \). In other words, definition (23) is accomplished. The conclusion of Theorem 2 can be drawn. \( \square \)

**Remark 4:** The guidance law implemented by the tracking-leader devised in Section III.B acts as an unknown input to formation-leaders. Note that the second term of Eq. (29) based on the sliding mode control provides an effective approach to make up for the tracking-leader’s unknown input. To avoid undesirable chattering of the closed-loop system, the term \( s(t)/\|s(t)\| \) in Eq. (36) can be replaced by \( s(t)/\|s(t)\| + \omega \), where \( \omega \) is a positive constant. Due to the unknown control input of the leader, the existing methods in [9, 10] are no longer applicable for swarm systems in the absence of explicit knowledge of the formation-leaders’ control input. Unlike the containment case in [11], there exist external inputs for each follower, thus the containment methods cannot be used directly in the formation-containment tracking case in this paper. The second term of Eq. (29) is utilized to suppress the external inputs from the formation-leaders.

**Remark 5:** From the above analysis, reconnaissance missile outweighs attack missile on the maneuver capability, while decoy missile is more mobile than reconnaissance missile. In the overall design stage, the performance parameters should be designated consistent with the overload requirements of different types of missiles. More powerful maneuver capability can be guaranteed through relatively larger pneumatic actuators, lighter weight or engine with more specific impulse.

### D. CONTROL SYSTEMS DESIGN FOR SALVO ATTACK OF TACTICAL MISSILES

Given the expected overload \( n_{xc}, n_{wc}, n_{zc} \) in Section III.B and III.C, the rudder and axial overload controllers based on genetic algorithm and DOB combined with the transition process are proposed to drive the plant to the three direction reference overloads. The change of coordinate from the inertial system to the ballistic coordinate system is adopted through Eq. (4) in this section. Furthermore, to facilitate control systems design, the missile control model is decomposed into four channels, i.e., yawing channel, pitching channel, rolling channel and velocity channel.

1) **DESIGN OF RUDDER CONTROL LOOP**

**Channel 1 Controller Design for Yawing Channel:**

After the decomposition, we first propose the conventional two-loop overload autopilot with proportion-integral (PI) correction, which is arranged transition process in yawing channel. The following transition process during the transition period \( T_0 \) is constructed with respect to \( n_{zc} \) devised in the preceding section.

\[
\text{trns} (T_0, t) = \begin{cases} 
\frac{1}{2} \left\{ 1 + \sin \left[ \pi \left( \frac{t}{T_0} - \frac{1}{2} \right) \right] \right\}, & t \leq T_0 \\
1, & t > T_0
\end{cases}
\]

(57)

The block diagram of the controller for yawing channel is shown in Fig. 5. The controller in yawing channel is designed as

\[
\delta_y = K_{p1} (\tilde{n}_{zc} - n_z) + K_{i1} \int (\tilde{n}_{zc} - n_z) \, dt + K_{d1} \omega_y
\]

(58)

Deflection angle of rudder \( \delta_y \) in Eq. (58) can control the actual \( \beta \) to track the required sideslip angle and finally steer \( n_z \) to \( \tilde{n}_{zc} \), where \( \tilde{n}_{zc} \) is the output of a transition process in the form of \( \tilde{n}_{zc} = n_{zc} \text{trns} (T_0, t) \) and the command for the controller to keep up with. \( K_{p1}, K_{i1} \) and \( K_{d1} \) are controller parameters.
Note that the missile may be statically unstable for a sizable flight envelope, which makes the controller design domain narrow and difficult to find a workable solution. To cope with it, genetic algorithm is employed to find and optimize the optimal control parameters in this paper. The parameters selecting problem can be formulated in the following form:

\[
\begin{align*}
\text{find } &: K_{p1}, K_{i1}, K_{d1} \\
\text{min } &: J_{yaw} \\
\text{s.t. } &: \text{Eq.(58)}
\end{align*}
\]

where \( J_{yaw} \) denotes performance index, which can be calculated by

\[
\begin{align*}
\min J_{yaw} &= \begin{cases} \\
\int_{0}^{\infty} \left[ w_{yaw1} |e_{yaw}(t)| + w_{yaw2} |\delta_y(t)| \right] \text{dt} & e_{yaw}(t) \leq 0 \\
\int_{0}^{\infty} \left[ w_{yaw1} |e_{yaw}(t)| + w_{yaw2} |\delta_y(t)| \right] \text{dt} + w_{yaw3} |e_{yaw}(t)| & e_{yaw}(t) > 0
\end{cases}
\end{align*}
\]

(60)

where \( e_{yaw}(t) = \bar{n}_{zc} - n_{c} \) is the tracking error of the lateral overload. To ensure the transition process of the lateral overload fast and smoothly without overshoot, penalty terms for tracking error \( e_{yaw} \) and deflection angle \( \delta_y \) are introduced. The weight coefficients for the performance index \( J_{yaw} \) are selected as \( w_{yaw1}, w_{yaw2} \) and \( w_{yaw3} \).

Remark 6: The penalty terms for tracking error and deflection angle have the merits of the optimal flight control law such as good tracking, energy conserving. The last term of the second equation of Eq. (60) is utilized to alleviate the control effort when the tracking overshoot situation appears. Based on the above analysis, it is indicated that solving the genetic algorithm problem (59) with performance index (60) yields that each missile has good autopilot dynamic response capability, ensuring the control actuation system to track the guidance command without delay.

Channel 2 Controller Design for Pitching Channel:

Similar to the yawing channel case, the controller in pitching channel is derived as

\[
\delta_z = K_{p2} (\bar{n}_{zc} - n_{z}) + K_{i2} \int (\bar{n}_{zc} - n_{z}) \text{dt} + K_{d2} \omega_z
\]

(61)

Deflection angle of elevator \( \delta_z \) in Eq. (61) can control the actual \( \alpha \) to track the required attack angle and finally steer \( n_y \) to \( \bar{n}_{yc} \), where \( \bar{n}_{yc} \) is the output of a transition process in the form of \( \bar{n}_{yc} = n_{yc}\text{trm}(T_0, t) \) and the command for the controller to track. \( K_{p2}, K_{i2} \) and \( K_{d2} \) are controller parameters to be determined appropriately. Fig.6 depicts the block diagram of the controller for pitching channel.

The following performance index on the base of genetic algorithm is also given as

\[
\begin{align*}
\min J_{pitch} &= \begin{cases} \\
\int_{0}^{\infty} \left[ w_{pitch1} |e_{pitch}(t)| + w_{pitch2} |\delta_z(t)| \right] \text{dt} & e_{pitch}(t) \leq 0 \\
\int_{0}^{\infty} \left[ w_{pitch1} |e_{pitch}(t)| + w_{pitch2} |\delta_z(t)| \right] \text{dt} + w_{pitch3} |e_{pitch}(t)| & e_{pitch}(t) > 0
\end{cases}
\end{align*}
\]

(62)

where \( e_{pitch}(t) = \bar{n}_{yc} - n_{y} \) is the tracking error of the vertical overload. \( w_{pitch1}, w_{pitch2} \) and \( w_{pitch3} \) are the weight coefficients for the performance index \( J_{pitch} \).

Channel 3 Controller Design For Rolling Channel:

For Skid-to-Turn (STT) missile, its rolling channel is generally held to be stable, hence, the command of the roll angle \( \gamma_c \) is set to be zero. Design the control law as follows

\[
\delta_x = K_{p3} (\gamma_c - \gamma) + K_{d3} \omega_x
\]

(63)

where actual and expected roll angel is \( \gamma, \gamma_c \) = 0, respectively, \( \delta_x \) is the deflection angle of aileron. Selecting parameters \( K_{p3}, K_{d3} \) is obtained via the proposed approach. The block diagram of the controller for rolling channel is demonstrated by Fig.7. Design the following performance index for rolling channel

\[
\begin{align*}
\min J_{roll} &= \begin{cases} \\
\int_{0}^{\infty} \left[ w_{roll1} |e_{roll}(t)| + w_{roll2} |\delta_x(t)| \right] \text{dt} & e_{roll}(t) \leq 0 \\
\int_{0}^{\infty} \left[ w_{roll1} |e_{roll}(t)| + w_{roll2} |\delta_x(t)| \right] \text{dt} + w_{roll3} |e_{roll}(t)| & e_{roll}(t) > 0
\end{cases}
\end{align*}
\]

(64)

where \( e_{roll}(t) = \gamma_c - \gamma \), \( w_{roll1}, w_{roll2} \) and \( w_{roll3} \) are the weight coefficients.
2) DESIGN OF AXIAL OVERLOAD CONTROL LOOP

Channel 4 Controller Design for Velocity Channel:

Assumption 4: The fuel equivalence ratio $\Phi$ affects the thrust $T$ through an approximated linear relationship as shown in Eq. (65). $T$ remains to be zero when $\Phi$ is set to be zero. After $\Phi$ is increased to 1, $T$ changes to the maximum.

\[
T = T_m \cdot \Phi
\]

where $T_m$ is the maximum thrust of the powered missile.

In the light of assumption 4, the fuel equivalence ratio is controlled by the PI controller, which is designed as

\[
\Phi = K_p (T_c - T) + K_i \int (T_c - T) \, dt
\]

where $K_p$ and $K_i$ are control parameters to be chosen, $T_c$ is given reference thrust coincident with the above-mentioned expected axial overload (19), (28), (29) and (30). And it is worth mentioning that the axial overload for attack missile is set to be zero. Fig. 8 presents the block diagram of the controller for velocity channel.

Considering the fact that the expected axial overload $n_{xc}$ is generated from the expressions of $\mathcal{D}$ in Eq. (28) and $u(t)$ in Eq. (29) and (30), it follows from the first equation of Eq. (12) that

\[
T_c = \frac{m n_{xc} + X}{\cos \alpha \cos \beta}
\]

Obviously, if we obtain the exact information of the expected axial overload $n_{xc}$, drag force $X$ and the current flight attitude, the reference thrust $T_c$ can be derived accordingly. However, due to modeling uncertainties and unpredictable disturbances in practical applications, drag force cannot be estimated accurately. For the uncertain drag force, we regard drag force as the lumped disturbance with $d_s = -\frac{X}{m}$, and there exists a positive constant $\Delta d'$ such that $|d_s| \leq \Delta d'$. Therefore, the velocity dynamics with external disturbance $d_s$ can be formulated as

\[
\dot{V} = \frac{T \cos \alpha \cos \beta}{m} - g \sin \theta + d_s
\]

Next, a DOB is proposed to estimate the external disturbance in the following theorem.

**Theorem 3:** For the dynamics in velocity channel described by Eq. (68), the following DOB is constructed to estimate $d_s$ as

\[
\begin{align*}
e_v &= V - z_{11} \\
z_{11} &= \frac{T \cos \alpha \cos \beta}{m} + z_{12} - g \sin \theta + \frac{\alpha_1}{\varepsilon} e_v \\
z_{12} &= \frac{\alpha_2}{\varepsilon^2} e_v
\end{align*}
\]

where $e_v$ is the observation error of the velocity $V$. $z_{11}$ and $z_{12}$ are the observation value of $V$ and $d_s$.

The DOB for $d_s$ can guarantee that $z_{11} \to V(t)$ and $z_{12} \to d_s(t)$ as $t \to \infty$ if this following case holds. $\alpha_1$, $\alpha_2$ are positive constants and the characteristic polynomial $\lambda^2 + \alpha_1 \lambda + \alpha_2$ meets Hurwitz conditions.

**Proof:** Define the observing error as $E = [e_1, e_2]^T$, where $e_1 = \frac{V - z_{11}}{\varepsilon}$, $e_2 = d_s - z_{12}$. We can get form Eq. (69) that

\[
\begin{align*}
e_1 &= \dot{V} - z_{11} \\
e_2 &= e_1 \dot{d}_s - z_{12}
\end{align*}
\]

And the observing error equation can be written as

\[
e \dot{E} = A_e E + \varepsilon B_e \dot{d}_s
\]

Define the Lyapunov function as $V_0 = e^T P_e E > 0$. Thus, there is

\[
\begin{align*}
\dot{V}_0 &= e^T P_e \dot{E} + \varepsilon e^T P_e \dot{d}_s \\
\dot{V}_0 &= (A_e E + \varepsilon B_e \dot{d}_s)^T P_e E + e^T P_e (A_e E + \varepsilon B_e \dot{d}_s) \\
\dot{V}_0 &= e^T (A_e^T P_e E + \varepsilon (B_e^T P_e E + e^T P_e A_e E) + e^T P_e B_e \dot{d}_s) \\
\dot{V}_0 &\leq \varepsilon e^T P_e E + 2 e \|P_e B_e\| \|E\| \|\dot{d}_s\| \\
\dot{V}_0 &\leq -\lambda_{min} (Q_e) \|E\|^2 + 2 \varepsilon \Delta d' \|P_e B_e\| \|E\| \\
\dot{V}_0 &\leq \frac{\alpha_1 \lambda_1}{\lambda_2} \lambda_2 \|E\|^2 + 2 \varepsilon \Delta d' \|P_e B_e\| \|E\|
\end{align*}
\]

where $\lambda_{min} (Q_e)$ is the minimum nonzero eigenvalue of matrix $Q_e$. The positive constant $\varepsilon$ is sufficiently small such that $\varepsilon < \frac{\lambda_{min} (Q_e)}{2 \|P_e B_e\| \|E\|^2}$, then it holds that $\dot{V}_0 < 0$, which means the estimation error for velocity channel based on DOB is asymptotically stable. This completes the proof of Theorem 3.

After obtaining the estimation of $d_s$ from DOB, replace $d_s$ with $z_{12}$ and then Eq. (67) is given by

\[
T_c = \frac{m n_{xc} - m z_{12}}{\cos \alpha \cos \beta}
\]

To comprehensively demonstrate the above-mentioned analysis, the overall control block diagram for velocity channel is
shown in Fig.9. The final designed controller is composed by three equations, i.e., Eq. (65), (66), and (74).

Remark 7: The turbojet engine thrust modeling is simplified as a linear model, and many details of the mathematical model are ignored. To deal with it, the effect of engine dynamics modeling error can be incorporated into the expression of $d_{\alpha}$. Although acceleration tracking controller design for multiple missiles is presented in [24], [25], the proposed cooperative controller is not applicable in that the explicit knowledge of the drag coefficient and the consideration of turbojet engine thrust were absent.

IV. NUMERICAL SIMULATIONS

In this section, the formation-containment tracking guidance and control of multiple tactical missiles to attack a stationary warship for berthing refueling in three dimensions is investigated to illustrate the effectiveness of the proposed strategies. The multiple tactical missile system is required to perform a salvo attack, where attack missile is designed to break through the interception of the anti-air defense system and strike the target with terminal impact angle constraint to achieve maximum lethality, and constitutes the formation-containment tracking motion modes together with reconnaissance and decoy missiles.

A. ASCENARIO

Assume that there is a STT missile group with 1 attack missile, 3 reconnaissance missiles and 2 decoy missiles, which are set as the tracking-leader (labeled by 0), the formation-leaders (labeled by 1-3) and the followers (labeled by 4,5) from the theoretical level, respectively. The simulations are conducted in the MATLAB2017b on a personal computer with the simulation step 0.002s and miss distance 10m. The missile model parameters are mass $m = 735$kg, moment of inertia $J = \text{diag}(25, 350, 350)$kg·m², reference area $S = 1m^2$, reference length $c = 1m$, and maximum thrust $T_m = 8kN$. Refer to http://dx.doi.org/10.21227/h3k0-b213 for the detailed data associated with the aerodynamic coefficients, and local aerodynamics can be obtained through linear interpolation. The initial conditions of 6 missiles and the warship are displayed in Table 2.

The directed communication graph of 6 missiles with 0-1 weights is demonstrated in Fig.10.

The attack mission of missile 0 is to attack in two different phases. In the first phase, during $t \in [0s, 100s)$, missile 0 is at a trimmed cruise conditions: $v = 270m/s$, and $h = 4000m$. In the second phase, during $t \in [100s, 117.5s)$,

$$
\text{missile0 attack } (-29000, 4000, -1750) \quad (270, 0, 0)
$$

From the expression of $h_E(t)$, one can obtain that the states of missiles 1-3 are expected to locate at the three vertexes of a regular triangle formation respectively and rotate at the angular speed of 0.5 rad/s while tracking the motion trajectory of missile 0. Moreover, missiles 4, 5 will move into the interior of the desired regular triangle formed by the state trajectories of missiles 1-3.

B. BSIMULATION RESULTS

In the simulation, the guidance law parameters are designed to be $\beta_1 = \beta_2 = 0.065$, $p_1 = p_2 = 5$, $q_1 = q_2 = 3$, $\eta_1 = \eta_2 = 0.008$. Note that sign function $\text{sgn}(s)$ may cause undesirable chattering, saturation function $\text{sat}(s)$ is employed to eliminate the chattering. The effectiveness of the proposed approach in Section III.B can be illustrated in Fig.11. As is shown in Fig.11 (b), it can be concluded that missile 0 can attack the target precisely and the maximum miss distance is 0.1197m. Fig.11 (c) shows that the LOS angle $q_{i}$ in vertical plane and $q_{\beta}$ in horizontal plane converge to about $-80^o$.
and $-60^\circ$ in about 12 and 8 seconds, respectively, showing that the impact angle criterion is satisfied with negligible tracking errors. From the aforementioned simulation results, one sees that the high precision strike requirement with impact angles can be achieved by attack missile in finite time under the proposed guidance law.

$$P$$ can be solved from Eq. (35) as

$$P = \begin{bmatrix} 1.7321 & 1 \\ 1 & 1.7321 \end{bmatrix}.$$ 

Based on Theorem 2, the control parameters in Eq. (34), (37) can be chosen as $\rho_1 = 1.5$, $\rho_2 = 0.8$, $\tau = 6$, and $\eta = 8$.

Fig. 12 depicts the state trajectory snapshots of the 6 missiles at different time instants, where the tactical missiles from 0 to 5 are marked by black pentagram, green asterisk, blue asterisk, glaucous asterisk, red circle, and purple circle respectively, and the convex hull spanned by the states of missiles 1-3 is denoted by blue solid lines. The evolution of the formation-containment tracking geometric configuration can be clearly seen from Fig. 12 (a-d).

According to the aerodynamic feature of the missile, the following aerodynamic coefficients around the trim condition are obtained using the linear interpolated lookup table.

$$\begin{align*}
    c_{\delta^y} &= 0.01937 \\
    m_{c_{\delta^y}} &= -7.97 \times 10^{-3}, \\
    c_{\delta^z} &= -0.01937 \\
    m_{c_{\delta^z}} &= -7.97 \times 10^{-3}, \\
    c_{\alpha^z} &= 0.2341 \\
    m_{c_{\alpha^z}} &= 0.01995, \\
    c_{\beta^z} &= -0.2581 \\
    m_{c_{\beta^z}} &= 0.01995, \\
    m_{\delta^z} &= -0.01067
\end{align*}$$

To optimize the control parameters, genetic algorithm listed in Section III.D is applied with roughly given initial search values. Taking yawing channel for missile 0 as an example, let $w_{\text{yaw}1} = 0.05$, $w_{\text{yaw}2} = 1$, $w_{\text{yaw}3} = 1$, and the search scopes for parameters in yawing controller (57)
are set to be $K_{p1} \in [0, 0.15], K_{i1} \in [0, 0.2]$ and $K_{d1} \in [0, 2]$. By setting the numbers of iterations and individuals, the optimal control parameters in yawing channel are converged asymptotically via the genetic algorithm such that $K_{p1} = 0.1215, K_{i1} = 0.0531, K_{d1} = 1.6100$. Similar to the yawing channel, it can also be calculated that the optimal control parameters in pitching controller (61) and rolling controller (63) are $K_{p2} = 0.1172, K_{i2} = 0.0209, K_{d2} = 1.5306, K_{p3} = 0.1109, K_{d3} = 0.0239$. As exhibited in Fig.13 (a-c), after the control parameters adjustment, performance indexes in yawing, pitching and rolling channels reach and stay on the minimum conditions, respectively. Fig.13 (d-f) reveal that satisfactory system responses can be guaranteed within the solved controller parameter set. From Fig.14, it can be deduced that the actual overload in Y-direction and Z-direction can track respective command ideally due to the proposed controllers (58), (61), (63). Additionally, vertical and lateral overload command can be tracked through reasonable rudder deflection angles. Fig.15 demonstrates the estimation of the uncertain drag force of missile 0 by DOB (69). It can be concluded from Fig.15 (a) that the DOB can estimate the uncertain drag force with high precision, which is used to adjust the throttle setting of the turbojet engine through the controller (66). Thus, the overload of attack missile in X direction keeps zero according to the tangential balance between thrust and drag force, as displayed in Fig.15 (b). Fig.15 (c) depicts that the amplitude and transition of the throttle setting are quite reasonable and stable despite the uncertainty of the drag force estimation. The trajectory of the position for the 6 missiles within 117.55s is shown in Fig.16. Fig.17 displays that the curves of the formation and containment errors approach to zero over time for each missile. The oscillations of formation tracking errors and containment errors at the moment $t = 100s$ are induced by the switching from cruise phase to homing guidance. The effectiveness of the proposed DOB, the good tracking of the three direction reference overloads and the smooth transitions of flight parameters are exhibited.
the proposed DOB could estimate the uncertain drag force with high precision, and the desired reference overload can be effectively kept up with under this cooperative scheme through reasonable actuator actions. Therefore, the multiple tactical missile system can realize cooperative flight with the designated formation-containment tracking geometry.

V. CONCLUSIONS

This paper investigated cooperative guidance and control problems with formation-containment tracking requirement for multiple tactical missiles. An integrated cooperative guidance and control framework was proposed, and combat missions were distinguished in accordance with diverse characteristics of tactical missiles in practical applications. A terminal slide mode guidance law with impact angle was introduced in 3D space, which guaranteed attack missile to attack the target in finite time. Sufficient conditions for multiple tactical missiles to achieve formation-containment tracking were developed. It was proved that formation-leaders can keep certain time-varying formation tracking, and the followers would converge to the convex hull specified by the formation-leaders. In the way of genetic algorithm and disturbance observer combined with the transition process, all 6 DOF of missiles were successfully tracked during their cooperative attack utilizing four control input through thrust and deflection in the presence of uncertain drag force. A formation-containment tracking anti-ship scenario with 6 tactical missiles was introduced, and numerical simulations were presented to confirm the feasibility and effectiveness of the theoretical results of this paper. An interesting direction for follow-up research work is to extend cooperative attack approaches with formation-containment tracking requirement to the condition in which the target is not stationary and the interaction topology can be directed and switching.

APPENDIX

Here gives the stability analysis of guidance loop for attack missile.

(1) Under the condition that $s_i \neq 0$.

According to Eq. (27), one has

$$
\dot{s} = \left[ \begin{array}{c}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4
\end{array} \right] = \left[ \begin{array}{c}
1 \\
1 \\
\beta_1 q_1 p_1 x_3^{1-p_1/q_1-1} \\
\beta_2 q_2 x_4^{1-p_2/q_2-1}
\end{array} \right]
$$

$$
\times \left[ \begin{array}{c}
\beta_1 q_1 x_3^{2-p_1/q_1} \\
\beta_2 q_2 x_4^{2-p_2/q_2} \\
1 \\
0
\end{array} \right] + \left[ \begin{array}{c}
\dot{x}_3 \\
\dot{x}_4
\end{array} \right]
$$

$$
\hat{s} = W(x) [\mathbf{Q}(x) + \dot{s}_2]
$$
where
\[
W(x) = \begin{bmatrix} \frac{1}{\beta_1} q_1 x_1^3 & 0 \\ 0 & \frac{1}{\beta_2} q_2 x_2^4 \end{bmatrix}.
\]

According to Eq.(9), one can obtain
\[
\dot{s} = W(x) [\Omega(x) + R(x) + H(x)D] = -W(x) \eta \text{sgn}(s).
\] (A.2)

Define the candidate Lyapunov function as \( V_1 = s^T \dot{s} \). Taking the derivative of \( V_1 \) yields
\[
\dot{V}_1 = 2s^T \dot{s} = -2s^T W(x) \eta \text{sgn}(s)
\leq -\left( \frac{2}{\beta_1} q_1 x_1^3 \eta_1 + \frac{2}{\beta_2} q_2 x_2^4 \eta_2 \right)
\leq -\alpha(x_3, x_4) V_1^{0.5}
\] (A.3)

Then Eq. (A.3) can be rewritten as
\[
\dot{V}_1 + \alpha(x_3, x_4) V_1^{0.5} \leq 0
\] (A.4)

where
\[
\alpha(x_3, x_4) = \min \left( \frac{2}{\beta_1} q_1 x_1^3 \eta_1, \frac{2}{\beta_2} q_2 x_2^4 \eta_2 \right).
\]

Furthermore, the following two cases are discussed

Case 1: \( \alpha(x_3, x_4) \neq 0 \). Eq. (A.4) clearly satisfies the form of Lemma 1.

Case 2: \( \alpha(x_3, x_4) = 0 \). For the sake of simplicity in description, let \( \chi_1 = x_3, \chi_2 = x_4 \), then \( \alpha(x_1, x_2) = \chi(x_1, x_2) \).

(i) If \( \chi_1 > 0 \), it follows from Eq. (9) and (28) that
\[
\begin{align*}
\dot{\chi}_1 &= -\eta_1 \text{sgn}(s) \\
\dot{\chi}_2 &= -\eta_2 \text{sgn}(s)
\end{align*}
\] (A.5)

If \( s_j > 0 \), it can be inferred that \( \dot{\chi}_j = -\eta_j \). Else if \( s_j < 0 \), there is \( \dot{\chi}_j = \eta_j \). It can be inferred that \( \chi_1 = 0 \) is not an attractor of the system. Thus, there exists two arbitrarily small positive numbers \( \delta_i (i = 1, 2) \) such that when \( |\chi_i| < \delta_i \), for \( s_j > 0 \), the system state is transferred from \( \chi_i = \delta_i \) to \( \chi_i = -\delta_i \) in finite time, for \( s_j < 0 \), the system state is transferred from \( \chi_i = -\delta_i \) to \( \chi_i = \delta_i \) in finite time. When \( |\chi_1| > \delta_1 \) and \( |\chi_2| > \delta_2 \), due to the fact that \( p_1 > 0, q_1 > 0, p_2 > 0 \) and \( q_2 > 0 \) are positive odd numbers with \( 1 < p_i / q_i < 2 \), there is \( \alpha(x_1, x_2) \geq \alpha(\delta_1, \delta_2) > 0 \). Then Eq. (A.4) satisfies the form of Lemma 1, the system (9) can converge to the sliding mode surface \( s = 0 \) in finite time.

(ii) \( \chi_1 = 0, \chi_2 \neq 0 \) (i + j = 3)

Owing to the discussion in (i), then \( V_1 = s^T \dot{s} = s^2 \), and \( \dot{V}_1 = -2 s^T \dot{s} / \eta V_1^{0.5} \), it satisfies the form of Lemma 1. The system (9) can converge to the sliding mode surface \( s = 0 \) in finite time.

(2) Under the condition that \( s_j = 0 \), one gets that
\[
\begin{align*}
x_1 + \frac{1}{\beta_1} x_3^{p_1/q_1} &= 0 \\
x_2 + \frac{1}{\beta_2} x_4^{p_2/q_2} &= 0
\end{align*}
\] (A.6)

Construct the candidate Lyapunov function as \( V_2 = x_1^2 + x_2^2 \), on differentiating \( V_2 \) and substituting Eq. (A.6), thus
\[
\begin{align*}
\dot{V}_2 &= 2x_1 \dot{x}_1 + 2x_2 \dot{x}_2 \\
&= 2x_1 (-\beta_1 x_1^{p_1/q_1-1}) + 2x_2 (-\beta_2 x_2^{p_2/q_2-1}) \\
&= -2\beta_1 x_1^{p_1/q_1-1} - 2\beta_2 x_2^{p_2/q_2-1} \\
&
\leq -2\beta_1 x_1^{p_1/q_1-1} - 2\beta_2 x_2^{p_2/q_2-1} \\
&
\leq -2\beta V_2^{0.5} \\
&
\leq -2\beta V_2^{0.5}
\end{align*}
\]

where
\[
\beta = \min \left( \beta_1^{p_1/q_1}, \beta_2^{p_2/q_2} \right),
\]
\[
\gamma = \min \left( \frac{q_1/p_1 + 1}{2}, \frac{q_2/p_2 + 1}{2} \right).
\]

We can get
\[
\dot{V}_2 + 2\beta V_2^{0.5} \leq 0
\] (A.7)

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XINGGUANG XU was born in 1988. He received the B.S. degree from Northwestern Polytechnical University, in 2011, started graduate study at Harbin Institute of Technology, and the M.S. degree from the third institute of China Aerospace Science and Industry Corporation, in 2014. He is currently pursuing the Ph.D. degree in navigation, guidance, and control with Beihang University (BUAA). He is also an Engineer with the Beijing Institute of Mechanical and Electrical Engineering. His research interests include aircraft design, fault-tolerant control, and cooperative control of multiagent systems.

CHANGRONG CHEN was born in 1994. He received the B.S. degree from the Harbin Institute of Technology, in 2017, and the M.S. degree from the third institute of China Aerospace Science and Industry Corporation, in 2020. His research interests include aircraft control system designs, adaptive control, and cooperative control of multiagent systems.

ZHANG REN received the B.S. degree in aircraft guidance, the M.S. degree in navigation, and the Ph.D. degree in simulation from Northwestern Polytechnical University, in 1982, 1985, and 1994, respectively. He held the Visiting Professor position at the University of California at Riverside and Louisiana State University, USA, respectively. He is currently a Professor with the School of Automation Science and Electronic Engineering, Beihang University. His research interests include aircraft guidance, navigation and control, fault detection, isolation and recovery, and cooperative control of multiagent systems.

SHUSHENG LI was born in 1962. He is currently a Professor and a Ph.D. Supervisor with the third institute of China Aerospace Science and Industry Corporation. He is also a National Outstanding Contribution Expert in the field of tracks and controls and granted a special allowance from the State Council. His research interests include aircraft design, fault diagnosis, tracks and controls, and design for testability.