RetroRenting: An Online Policy for Service Caching at the Edge

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ABSTRACT

The rapid proliferation of shared edge computing platforms has enabled application service providers to deploy a wide variety of services with stringent latency and high bandwidth requirements. A key advantage of these platforms is that they provide pay-as-you-go flexibility by charging clients in proportion to their resource usage through short-term contracts. This affords the client significant cost-saving opportunities, by dynamically deciding when to host (cache) its service on the platform, depending on the changing intensity of requests.

A natural caching policy for our setting is the Time-To-Live (TTL) policy. We show that TTL performs poorly both in the adversarial arrival setting, i.e., in terms of the competitive ratio, and for i.i.d. stochastic arrivals with low arrival rates, irrespective of the value of the TTL timer.

We propose an online caching policy called RetroRenting (RR) and show that in the class of deterministic online policies, RR is order-optimal with respect to the competitive ratio. In addition, we provide performance guarantees for RR for i.i.d. stochastic arrival processes and prove that it compares well with the optimal online policy. Further, we conduct simulations using both synthetic and real world traces to compare the performance of RR and its variants with the optimal offline and online policies. The simulations show that the performance of RR is near optimal for all settings considered. Our results illustrate the universality of RR.

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1 INTRODUCTION

Widespread adoption of smartphones and other handheld devices over the last decade has been accompanied with the development of a wide variety of mobile applications providing a plethora of services. These applications often rely on cloud computing platforms [10] to enable delivery of high-quality performance any-where, anywhere to resource-constrained mobile devices. However, the last few years have seen the emergence of applications based on machine learning, computer vision, augmented/virtual reality (AR/VR) etc. which are pushing the limits of what cloud computing platforms can reliably support in terms of the required latency and bandwidth. This is largely due to the significant distance between the end user and the cloud server, which has led the academia and the industry to propose a new paradigm called edge computing [19] whose basic tenet is to bring storage and computing infrastructure closer to the end users. This can help enable applications with ultra-small network latency and/or very high bandwidth requirements, which cannot be reliably supported by the backhaul connection. As a concrete example, consider a user in a wildlife sanctuary, capturing the scene around her live on a mobile device, which relays the image/video to an edge server. Using its much higher computational and storage capabilities, an application on the edge server can continually detect species of plants, animals, birds and relay this information back to the end user device where it can be overlaid onto the live stream to provide a much richer viewing experience. Broader applications of edge computing include industrial robotics/drone automation, AR/VR-based infotainment and gaming, autonomous driving and the Internet of Things (IoT). While there are now several industry offerings of dedicated edge computing platforms, e.g., Amazon Web Services [1] and Microsoft Azure [2], there have also been proposals to augment cellular base stations [8] and WiFi access points [24] so that they can act as edge servers.

Edge computing platforms enable an application provider to ‘cache’ its service at servers close to the end users. In this paper, we say that a service is cached at an edge server, if all the data and code needed to run the service has been downloaded from a remote/back-end server (possibly in the cloud) and cached on the edge server. Thus the edge server can handle service requests on its own without requiring to communicate with the back-end server. Edge servers are often limited in computational capability as compared to cloud servers [22], and hence there might be a limit on the number of parallel requests they can serve for the cached service. An application provider can avail this ability to cache on the edge server in return of a cost which is in proportion to the amount of resources used and/or the duration of rental. Since computing platforms usually provide pay-as-you-go flexibility [16], the client can dynamically decide when to cache or evict the service at the edge, depending on the varying number of arriving service requests. The application provider needs to design an efficient service caching policy which can help minimize the overall cost of deploying the service.

Most of the literature on caching policies has focused on the related content caching problem [5], which deals with the problem of delivering content (for example video or music) to end users by deploying storage caches close to the end user. There are several key differences between the content caching and the service caching problem. In the former, if a content is not currently cached and a request arrives for it (cache miss), the content has to be fetched...
We show that TTL performs poorly both in terms of the competitive ratio, which is popular in the content caching literature. Under the TTL policy, each cache miss triggers a download of the service to the edge server. We consider two classes of request arrival processes: (i) adversarial arrivals: the request sequence is arbitrary and the performance of any online policy is measured by its competitive ratio, which provides a worst-case guarantee on its performance for any request arrival sequence in comparison to the optimal off-line policy, which has knowledge of the entire arrival sequence a-priori. (ii) i.i.d. stochastic arrivals: requests are generated according to an i.i.d. stochastic process and we compare the expected cost of a proposed policy with that of the optimal online policy.

1.1 Our Contributions
A natural policy for our setup is the Time-To-Live (TTL) policy [7], which is popular in the content caching literature. Under the TTL policy, each cache miss triggers a download of the service to the edge server where it is then retained for some fixed amount of time. We show that TTL performs poorly both in terms of the competitive ratio for arbitrary arrivals and for i.i.d. stochastic arrivals with low arrival rates, irrespective of the value of the TTL timer. Given the limitations of TTL, we propose an online caching policy called RetroRenting (RR) which uses the history of request arrivals to decide when to cache or evict the service at the edge server. Under the adversarial request setting, we show that RR is order-optimal with respect to the competitive ratio in the class of deterministic online policies. In addition, we also provide performance guarantees for RR under i.i.d. arrivals and prove that it compares very well with the optimal offline policy in this setting. In addition to our analytical results, we conduct simulations using both synthetic and real world traces to compare the performance of RR and its variants with the optimal offline and online policies. Our simulations show that the performance of RR is near optimal for all settings considered. These results combined illustrate the universality of RR.

1.2 Related Work
Mobile applications have increasingly become more and more demanding in terms of their bandwidth and latency requirements. This, along with the advent of new time-critical applications such as the Internet of Things (IoT), AR/VR and autonomous driving has necessitated the migration of a part of the storage and computing capabilities from remote servers to the edge of the network. See [14, 15, 18] for a survey of various edge computing architectures and proposed applications. The emergence of such edge computing platforms [14, 15, 18] has been accompanied with various academic works which model and analyse the performance of such systems. We briefly discuss some of the relevant works in the literature.

One approach towards designing efficient edge computing systems is to formulate the design problem as a large one-shot static optimization problem which aims to minimize the cost of operating the edge computing platform [4, 8, 17, 22, 27]. [17] considers such a problem in a heterogeneous setting where different edge nodes have different storage or computation capabilities and various services have different requirements. The goal is to find the optimal service placement scheme subject to the various constraints. The authors show that the problem is in general NP-hard and propose constant factor approximation algorithms. A similar problem is considered in [4] which looks at the setting where an edge server is assisting a mobile unit in executing a collection of computation tasks. The question of which services to cache at the edge and which computation tasks to offload are formulated as a mixed integer non-linear optimization problem and the authors design a reduced-complexity alternative minimization based iterative algorithm for solving the problem. Similar problems have also been considered in [8, 22, 27]. Our work differs from this line of work in that we are interested in designing online algorithms which adapt their service placement decisions over time depending on the varying number of requests.

One approach to modeling time-varying requests is to use a stochastic model as done in [9, 23, 26] which assumes that requests follow a Poisson process and then attempts to minimize the computation latency in the system by optimizing the service caching and task offloading decisions. [9] considers a setting where the underlying distribution for the request process is apriori unknown and uses the framework of Contextual Combinatorial Multiarmed Bandits to learn the demand patterns over time and make appropriate caching decisions. Finally, [23] considers a Markovian model for user mobility and uses a Markov Decision Process (MDP) framework to decide when and which services to migrate between different edge servers as the users move around. Our work differs from these works in that in addition to stochastic request models, we also focus on the case of arbitrary request arrival processes and provide ‘worst-case’ guarantees on the performance of our proposed schemes instead of ‘average’ performance guarantees. This can be vital in scenarios where the arrival patterns change frequently over time, making it difficult to predict demand or model it well as a stochastic process.

The work closest to ours is [28] which considers an edge server with limited memory $K$ and an arbitrary request process for a catalogue of services. This work studies the design of service caching policies which minimize the cost incurred by the edge server for deploying the various services. The authors propose an online algorithm called ReD/LeD and prove that the competitive ratio of the proposed scheme is at most $10K$. Unlike [28], we study the problem from the perspective of an application provider and design cost-efficient service caching policies which dynamically decide when to cache or evict the service at the edge.
Finally, as mentioned before, the problem of service caching does resemble the content caching problem but with some key differences. Content caching has a rich history, see for example [3, 5, 6, 20, 21, 25]. A popular class of online caching policies is the Time-To-Live (TTL) policy [7], which downloads a content to the cache upon a cache miss and then retains it there for a certain fixed amount of time. In this work, we consider a variant of the TTL policy for service caching and demonstrate that it performs poorly in several cases.

2 SYSTEM SETUP

2.1 Network Model

We study a system consisting of a back-end server and an edge server in proximity to the end-user. The back-end server always stores the service. The edge server has sufficient storage to cache the service. This storage space can be rented by paying a renting cost.

The computation power at the edge server is limited. When a user makes a request, it can be served by the edge server for free if the service is cached on the edge server and the edge server has sufficient computation power to process it, or by the back-end server at a non-zero cost. The back-end server can serve all the requests that are routed to it.

2.2 Arrival Process

We consider a time-slotted system and consider both adversarial and stochastic request arrivals. In the adversarial setting, we make no assumptions on the request sequence. In the stochastic setting, we make the following assumption.

Assumption 1. (i.i.d. stochastic arrivals) The number of requests arriving in a time-slot is independent and identically distributed across time-slots. More specifically, let \( X_t \) be the number of requests arriving in time-slot \( t \). Then, for all \( t \),

\[
\mathbb{P}(X_t = x) = p_x \text{ for } x = 0, 1, 2, \ldots
\]

2.3 Sequence of Events in a Time-slot

The following sequence of events occurs in each time-slot. We first have request arrivals. If the service is cached on the edge server, requests are served locally subject to the constraints on the computation power of the edge server, else requests are forwarded to the back-end server. The system then makes a caching decision (fetch/evict/no change).

2.4 Cost Model and Constraints

Our cost model builds on the model proposed in [28] and extends it to the setting where cache space can be rented in a dynamic manner by paying a renting cost. For a given caching policy \( \mathcal{P} \), the total cost incurred in time-slot \( t \), denoted by \( C_t^\mathcal{P} \), is the sum of the following three costs.

- **Service cost** \( C_{S,t}^\mathcal{P} \): Each request forwarded to the back-end server is served at the cost of one unit.

- **Fetch cost** \( C_{F,t}^\mathcal{P} \): On each fetch of the service from the back-end server to cache on the edge-server, a fetch cost of \( M(>1) \) units is incurred.

- **Rent cost** \( C_{R,t}^\mathcal{P} \): A renting cost of \( c(\geq 0) \) units is incurred to cache the service on the edge server for a time-slot.

Since the edge server has limited computation power, the number of requests that can be served by the edge server in a time-slot is limited to \( \kappa \in \mathbb{Z}^+ \), where \( \mathbb{Z}^+ \) is the set of all positive integers. Let \( r_t \) be an indicator of the event that the service is cached on the edge server during time-slot \( t \). It follows that

\[
C_{t}^\mathcal{P} = C_{S,t}^\mathcal{P} + C_{F,t}^\mathcal{P} + C_{R,t}^\mathcal{P}
\]

where,

\[
C_{S,t}^\mathcal{P} = \begin{cases} X_t - \min\{X_t, \kappa\} & \text{if } r_t = 1 \\ X_t & \text{otherwise} \end{cases}
\]

\[
C_{F,t}^\mathcal{P} = \begin{cases} M & \text{if } r_t = 0 \text{ and } r_{t+1} = 1 \\ 0 & \text{otherwise} \end{cases}
\]

\[
C_{R,t}^\mathcal{P} = \begin{cases} c & \text{if } r_t = 1 \\ 0 & \text{otherwise} \end{cases}
\]

Remark 1. We limit our discussion to the case where \( c \in [0, \kappa) \) because, for \( c \geq \kappa \), it is optimal to forward all requests to the back-end server, irrespective of the value of \( M \) and the arrival sequence.

2.5 Algorithmic Challenge

The algorithmic challenge is to design a policy which decides when to cache the service on the edge server. Caching policies can be divided into the following two classes.

Definition 1. (Types of Caching Policies)

- **Offline Policies**: A policy in this class knows the entire request arrival sequence a-priori.

- **Online Policies**: A policy in this class does not have knowledge of future arrivals.

We design an online policy which makes caching decisions based on the request arrivals thus far and the various costs and constraints, i.e., the rent cost \( c \), the fetch cost \( M \), and the edge service constraint \( \kappa \).

2.6 Metric and Goal

The optimal offline and online policies serve as benchmarks to evaluate the performance of the proposed policy. We use different cost metrics for the adversarial and stochastic request arrival settings.

2.6.1 Adversarial arrivals. For the adversarial setting, we compare the performance of a policy \( \mathcal{P} \) with the performance of the optimal offline policy (OPT-OFF). The goal is to design a policy \( \mathcal{P} \) which minimizes the competitive ratio \( \rho^\mathcal{P} \) defined as

\[
\rho^\mathcal{P} = \sup_{a \in \mathcal{A}} \frac{C(a)}{\text{OPT-OFF}(a)},
\]

where \( \mathcal{A} \) is the set of all possible finite request arrival sequences, \( C(a) \), \( \text{OPT-OFF}(a) \) are the overall costs of service for the request arrival sequence \( a \) under online policy \( \mathcal{P} \) and the optimal offline policy respectively.

2.6.2 i.i.d. stochastic arrivals. For i.i.d. stochastic arrivals (Assumption 1), we compare the performance of a policy \( \mathcal{P} \) with the performance of the optimal online policy (OPT-ON). The goal is to minimize \( \sigma^\mathcal{P} \), defined as the ratio of the expected cost incurred by
policy $P$ in $T$ time-slots to that of the optimal online policy in the same time interval. Formally,

$$\sigma^P(T) = \frac{\mathbb{E}\left[\sum_{t=1}^{T} c^P_t\right]}{\mathbb{E}\left[\sum_{t=1}^{T} c^{OPT-ON}_t\right]}.$$  \hspace{1cm} (3)

where $c^P_t$ is as defined in (1).

3 MAIN RESULTS AND DISCUSSION

In this section, we state and discuss our main results. We provide outlines of the proofs in Section 5 and the details of the proofs are discussed in Section 6, Appendix C, and Appendix D.

3.1 Our Policy: RetroRenting (RR)

A caching policy determines when to fetch and cache the service and when to evict the service from the cache. The RR policy makes these decisions in each time-slot by evaluating if it made the right decision in hindsight. We first provide an overview of the RR policy.

To fetch: Let the service not be cached at the beginning of time-slot $t$ and $t_{evict} < t$ be the time when the service was most recently fetched by RR. The RR policy searches for a time-slot $\tau$ such that $t_{evict} < \tau < t$, and the total cost incurred is lower if the service is fetched in time-slot $\tau - 1$ and cached during time-slots $\tau$ to $t$ than if the service is not cached during time-slots $\tau$ to $t$. If there exists such a time $\tau$, the RR policy fetches the service in time-slot $t$.

To evict: Let the service be in the cache at the beginning of time-slot $t$ and $t_{fetch} < t$ be the time when the service was most recently fetched by RR. The RR policy searches for a time-slot $\tau$ such that $t_{evict} < \tau < t$, and the total cost incurred is lower if the service is not fetched during time-slots $\tau$ to $t$ and fetched in time-slot $t$ than if the service is cached during time-slots $\tau$ to $t$. If there exists such a time $\tau$, the RR policy evicts the service in time-slot $t$.

Refer to Algorithm 1 for a formal definition of the RR policy. The notation used in Algorithm 1 is summarized in Table 1.

| Symbol | Description |
|--------|-------------|
| $t$ | Time index |
| $M$ | Fetch cost |
| $c$ | Rent cost per time-slot |
| $\kappa$ | Maximum number of requests that can be served by the edge server in a time-slot |
| $x_t$ | Number of requests arriving in time-slot $t$ |
| $r_t$ | Indicator variable; 1 if the service is cached in time-slot $t$ and 0 otherwise |
| $(x_t - \kappa)^+$ | $\max\{x_t - \kappa, 0\}$ |

Table 1: Notation used in Algorithms 1 and 3

Remark 2. Note that in time-slot $t$, the computation and storage complexities of the RR policy scale as $O(t)$ (if either $t_{fetch} = 0$ or $t_{evict} = 0$). This is indeed a limitation of the RR policy since, in the worst case, the computational and storage complexities increase linearly with time.

To overcome this limitation, we propose an efficient variant of the RR policy called the RR$_u$ policy. The only difference between the RR and RR$_u$ policies is that, at time $t$, the RR$_u$ policy considers the arrival sequence in the previous at most $u$ time-slots to make its caching decision whereas the RR policy can potentially look at the entire arrival sequence from $t = 0$ to make its caching decision for each time-slot (refer to lines 7 and 15 in Algorithm 1). Therefore, under RR$_u$, the range for $\tau$ in lines 7 and 15 in Algorithm 1 are replaced with $\max\{t_{evict}, t - u\} < \tau < t$ and $\max\{t_{fetch}, t - u\} < \tau < t$ respectively. Refer to Algorithm 3 in Appendix A for the complete definition of RR$_u$.

Remark 3. Note that the computational and storage complexity of the RR$_u$ policy is $O(u)$ and does not scale with time as was the case for the RR policy. Since $x_t > 0$, for all $t$, for the conditions in lines 7 and 15 of RR$_u$ to be satisfied, $u > \frac{M}{\kappa - \kappa + 1}$ and $u > \frac{M}{\kappa}$ respectively. Therefore, we impose the condition that $u > \max\{\frac{M}{\kappa - \kappa + 1}, \frac{M}{\kappa}\}$.

3.2 Performance guarantees for RR and RR$_u$

3.2.1 Adversarial arrivals. Our first theorem characterizes the performance of RR in the adversarial arrivals setting.
Theorem 2. Let $\rho^{RR}$ be the competitive ratio of RR as defined in (2). Then,

$$\rho^{RR} \leq \left( 5 + \frac{\kappa}{M} - \frac{4c}{\kappa} \right).$$

Since this result holds for all finite request arrival sequences, Theorem 2 provides a worst-case guarantee on the performance of the RR policy as compared to that of the optimal offline policy. Recall that unlike the RR policy, the optimal offline policy knows the entire arrival sequence a-priori.

The competitive ratio of RR improves as the fetch cost ($M$) and rent cost ($c$) increase, however, it increases linearly with $\kappa$. Our next result shows that the competitive ratio of any deterministic online policy increases linearly with $\kappa$.

Theorem 3. Let $\cal P$ be any deterministic online policy and let $\rho^{p}$ be the competitive ratio of this policy as defined in (2). Then,

$$\rho^{p} \geq \begin{cases} 1 + \frac{\kappa}{c + M} & \text{if } \kappa \geq \frac{c(M + M)}{M} \\ \frac{\kappa}{c} & \text{otherwise.} \end{cases}$$

From Theorems 2 and 3, we conclude that the RR policy is order optimal with respect to the edge server computation constraint ($\kappa$) for the setting considered. This is one of the key results of this work. While Theorem 2 gives a worst-case guarantee on the performance of the RR policy, in our subsequent analytical and simulation results, we observe that for the request sequences considered, the performance of the RR policy is significantly closer to that of the offline optimal policy than the bound in Theorem 2 suggests.

3.2.2 Stochastic arrivals. Next we characterize the performance of the RR and RR$_{u}$ policies for i.i.d. stochastic arrivals (Assumption 1, Section 2). Recall that, under Assumption 1, in each time-slot, the number of request arrivals is $x$ with probability $p_{x}$ for $x = 0, 1, \cdots$.

Our next lemma characterizes the difference between the expected cost incurred in a time-slot by our policies and the optimal online policy.

Lemma 4. Let $\Delta_{t}^{p} = \mathbb{E}[C_{t}^{p} - C_{t}^{OPT\text{-}CON}]$, $\mu = \mathbb{E}[\min\{X_{t}, \kappa\}]$,

$$f(\kappa, \lambda, M, \mu, c) = (M + \mu) \left( \frac{\lambda M}{\mu - c} \exp\left( -\frac{2(\mu - c)^{2}M}{\kappa^{2}} \right) \right) + \exp\left( -\frac{2(\lambda - 1)^{2}M(\mu - c)}{\lambda \kappa^{2}} \right), \text{ and}$$

$$g(\kappa, \lambda, M, \mu, c) = (c + M) \left( \frac{\lambda M}{c - \mu} \exp\left( -\frac{2(c - \mu)^{2}M}{\kappa^{2}} \right) \right) + \exp\left( -\frac{2(\lambda - 1)^{2}(c - \mu)M}{\lambda \kappa^{2}} \right).$$

Then, under Assumption 1,

- Case $\mu > c$:

$$\Delta_{t}^{RR} \leq \min_{\lambda: \lambda > 1 \text{ and } t > \frac{\lambda M}{\mu - c}} f(\kappa, \lambda, M, \mu, c),$$

$$\Delta_{t}^{RR}_{u} \leq \min_{\lambda: \lambda < \frac{\mu - c}{\mu - \mu} \text{ and } t > \frac{\lambda M}{\mu - c}} g(\kappa, \lambda, M, \mu, c).$$

- Case $\mu < c$:

$$\Delta_{t}^{RR} \leq \min_{\lambda: \lambda > 1 \text{ and } t > \frac{\lambda M}{\mu - c}} f(\kappa, \lambda, M, \mu, c),$$

$$\Delta_{t}^{RR}_{u} \leq \min_{\lambda: \lambda < \frac{\mu - c}{\mu - \mu} \text{ and } t > \frac{\lambda M}{\mu - c}} g(\kappa, \lambda, M, \mu, c).$$

We thus conclude that for $t$ large enough, the difference between the cost incurred by RR/RR$_{u}$ and the optimal online policy in time-slot $t$, decays exponentially with $M$ and $|\mu - c|$. Theorem 5. Let $\nu = \mathbb{E}[X_{t}]$ and $\mu = \mathbb{E}[\min\{X_{t}, \kappa\}]$. Recall the definition of $\sigma^{p}$ given in (3).

- Case $\mu > c$: For the function $f$ defined in Lemma 4,

$$\sigma^{RR}(T) \leq \min_{\lambda > 1} \left( 1 - \frac{\lambda M}{\mu - c} \frac{M + \gamma + \nu + 1}{T} \right) + \frac{\lambda M}{\mu - c} \frac{M + \gamma + \nu + 1}{T}.$$ 

- Case $\mu < c$: For the function $g$ defined in Lemma 4,

$$\sigma^{RR}(T) \leq \min_{\lambda < \frac{\mu - c}{\mu - \mu}} \left( 1 - \frac{\lambda M}{\mu - c} \frac{M + \gamma + \nu + 1}{T} \right) + \frac{\lambda M}{\mu - c} \frac{M + \gamma + \nu + 1}{T}.$$ 

Remark 4. The bounds obtained in Lemma 4 and Theorem 5 hold for all i.i.d. stochastic arrival processes. We note that the bounds worsen as $\kappa$ increases. This is a consequence of using Hoeffding’s inequality to bound the probability of certain events. It is important to note that significantly tighter bounds can be obtained for specific i.i.d. processes by using the Chernoff bound instead of Hoeffding’s inequality. In the next section, via simulations, we show that the performance of RR and its variants does not worsen as $\kappa$ increases. We thus conclude that the deterioration of the performance guarantees with increase in $\kappa$ is a consequence of the analytical tools used and not fundamental to RR and its variants.
We use Theorem 5 to conclude that for $T$ large enough, the bound on the ratio of the expected cost incurred by RR/RR$_u$ in $T$ time-slots to that of the optimal online policy (OPT-ON) in the same time interval decays as $M$ increases.

Often, policies designed with the objective of minimizing the competitive ratio tend to perform poorly on average in typical stochastic settings. Similarly, policies designed for specific stochastic arrival processes can have poor competitive ratios if they perform poorly for certain 'corner case' arrival sequences. The performance guarantees for RR obtained in this section show that RR performs well in both the adversarial and the i.i.d. stochastic setting. This is a noteworthy feature of the RR policy.

3.3 Performance of TTL
In this section, we focus on the TTL policy which is widely used and studied in the classical caching literature. TTL serves as a benchmark to compare the performance of RR and RR$_u$.

The TTL policy fetches and caches the service whenever there is a miss, i.e., the service is requested but is not cached on the edge server. There is a timer associated with the fetch, which is set to a fixed value ($L$) right after the service is fetched. If the service is not requested before the timer expires, the service is evicted from the cache. If a request arrives while the service is cached, the timer is reset to its initial value of $L$. Refer to Algorithm 2 for a formal definition.

**Algorithm 2: TTL Policy**

1. Input: Request arrival sequence $x_t$, $t > 0$, TTL value $L$
2. Output: Caching Strategy $r_t$, $t > 0$
3. Initialize: Caching variable $r_1 = 0$ and timer $t = 0$
4. For each time-slot $t$
   - if $x_t = 0$ then
     - if timer = 0 then
       - $r_{t+1} = 0$
     - else
       - $r_{t+1} = 1$, timer = timer + 1
   - else
     - $r_{t+1} = 1$, timer = $L$
5. end

Our next result provides a lower bound on the competitive ratio of the TTL policy.

**Theorem 6.** Let $p^{TTL}$ be the competitive ratio of the TTL policy with TTL value $L$ as defined in (2). Then,

$$p^{TTL} \geq \begin{cases} 1 + \frac{M}{L + c} & \text{if } 1 \leq \kappa < M + c, \\ \frac{\kappa + Lc + M}{c + \min\{Lc, M\}} & \text{otherwise.} \end{cases}$$

The key takeaway from Theorem 6 is that unlike the RR policy, the performance of the TTL policy deteriorates as the fetch cost ($M$) increases. This is a consequence of the fact that the TTL policy fetches and caches the service on a miss irrespective of the value of $M$, whereas for high values of $M$, RR and the optimal offline policy might choose not to fetch the service at all. Note that the performance of both RR and TTL deteriorates with increase in $\kappa$.

Next, we characterize the performance of TTL for i.i.d. stochastic arrivals.

**Lemma 7.** Let $\Delta^{TTL}_t = \mathbb{E}[C^{TTL}_t - C^{OPT-ON}_t]$ and $\mu = \mathbb{E}[\min\{X_t, \kappa\}]$. Recall that $p_0 = \mathbb{P}(X_t = 0)$. Then, under Assumption 1, if $\mu < c$,

$$\Delta^{TTL}_t = p_0 \min\{t-1, L\} (1 - p_0) M + \left(1 - p_0 \min\{t-1, L\}\right) (c - \mu).$$

For $t > 1$, $\min\{t - 1, L\} \geq 1$, and therefore,

$$\Delta^{TTL}_t \geq \min\{(1 - p_0)(p_0 M + c - \mu), c - \mu\}. \quad (4)$$

We thus conclude that for low arrivals rates, the difference between the cost incurred by TTL and the optimal online policy in time-slot $t$, increases with $M$ and $c - \mu$. This illustrates the limitations of TTL for the i.i.d. stochastic setting with low arrival rates. The sub-optimality of TTL is a consequence of the fact that the TTL policy fetches and caches the service on a miss irrespective of the value of $M$ and $\mu$, whereas for high values of $M$ and low values of $\mu$, RR and the optimal online policy choose not to fetch the service at all.

**Theorem 8.** Let $\nu = \mathbb{E}[X_t]$ and $\mu = \mathbb{E}[\min\{X_t, \kappa\}]$. Recall the definition of $\sigma^p_t$ given in (3). If $\mu < c$,

$$\sigma^{TTL}(T) \geq \left(1 - \frac{1}{T}\right) \min\left(\frac{(1 - p_0)(p_0 M + c - \mu), c - \mu}{\nu}\right).$$

From Theorem 8, we conclude that for low request arrival rates ($\mu < c$), the performance of TTL deteriorates with increases in $M$. Contrary to this, the performance of RR and RR$_u$ approaches the performance of the optimal online policy as $M$ increases (Theorem 5). We thus conclude that for low arrival rates and high fetch cost, TTL is sub-optimal.

TTL policies perform well in content caching, where on a miss, the requested content is fetched from the back-end server by all policies including TTL. However, as discussed above, on a miss in our service caching setting, there are two options: (a) request forwarding which forwards the request to the back-end server for service; and (b) service fetch which fetches all the data and code needed for running the service from the back-end server and caches it on the edge server. The cost for these two actions is different. By definition, TTL always takes the second option, whereas, for low request arrival rates and high fetch cost, RR, its variants and the optimal online and offline policies use the first option. This explains the poor performance of TTL for service caching.

4 SIMULATION RESULTS
In this section, we compare the performance of various caching policies via simulations. The simulation parameters for each set of simulation are provided in the figure caption.

4.1 Stochastic Arrivals
For the first set of simulations, we focus on arrival processes satisfying Assumption 1. We compare the performance of RR, RR$_u$, the optimal offline policy and the optimal online policy. Each datapoint in the plots is averaged over 10000 requests.
This can be explained as follows. If \( \mu > c \) in this case, therefore, \( \mu \) in the condition the RR policy checks to fetch and cache the service to the back-end server. However, for small values of \( \mu - c \), the condition the RR policy checks to fetch and cache the service (Step 8 in Algorithm 1) is not very unlikely. This leads to multiple fetch–store–evict cycles and therefore a higher cost than the optimal offline policy. As \( M \) and/or \( \mu - c \) increase, this event becomes less probable. The case when \( \mu > c \) can be argued along similar lines.

### 4.1.1 i.i.d. Bernoulli Arrivals

In Figures 1-3, we consider Bernoulli arrivals with parameter \( p \), i.e., \( X_t = 1 \) with probability \( p \) and \( X_t = 0 \) otherwise. Recall that \( \kappa \geq 1 \) and \( \mu = E[\min\{X_t, \kappa\}] \). Since \( X_t \leq 1 \) in this case, therefore, \( \mu = p \). We fix \( \kappa = 1 \) and define \( \gamma = \max\{\frac{M}{\pi - \pi^*}, \frac{M}{\kappa}\} \). We compare the performance of RR, RR\(_{OL}\), RR\(_{OLY}\), RR\(_{OLY}\), RR\(_{OLY}\) with the optimal offline and online policies.

The gap between RR\(_{OL}\) and RR decreases with increase in \( \mu \). The performance of the RR policy is quite close to that of the optimal online policy for all parameter values considered. The performance gap between the optimal offline policy and the RR policy is very small compared to the bound on competitive ratio obtained in Theorem 2. We see that the gap between the performance of the RR and optimal online policy increases as \( M \) and/or \( |\mu - c| \) decrease. This can be explained as follows. If \( \mu < c \), the optimal online policy does not fetch/store the service and forwards all the requests to the back-end server. However, for small values of \( \mu - c \) and \( M \), the condition the RR policy checks to fetch and cache the service (Step 8 in Algorithm 1) is not very unlikely. This leads to multiple fetch–store–evict cycles and therefore a higher cost than the optimal online policy. As \( M \) and/or \( \mu - c \) increase, this event becomes less probable. The case when \( \mu > c \) can be argued along similar lines.

### 4.1.2 i.i.d. Poisson Arrivals

In Figure 4, we consider the case where the arrival process is Poisson with parameter \( \lambda \). We vary \( \kappa \) and define \( \gamma = \max\{\frac{M}{\pi - \pi^*}, \frac{M}{\kappa}\} \). We see that the performance of all policies improves with increase in \( \kappa \). The performance of RR is very close that of the optimal offline and the optimal online policies. As before, the gap between RR\(_u\) and RR decreases with increase in \( u \).

### 4.2 Trace-driven Simulations

For the next set of simulations, we use trace-data obtained from a Google Cluster [12]. We use a time-slot duration small enough to ensure that there is at most one request in a time-slot. This trace-data has requests for four types of jobs/services identified as "Job 0", "Job 1", "Job 2", and "Job 3". In this section, we present results for "Job 2" (Figures 5 and 6). Results for the other jobs are qualitatively similar and are available in Appendix B.

For this set of simulations, we fix \( \kappa = 1 \) and \( \gamma = \max\{\frac{M}{\pi - \pi^*}, \frac{M}{\kappa}\} \). We compare the performance of RR and its variants with the optimal offline policy. We note that the performance gap between
We divide time into frames such that Frame $i$ when OPT-OFF downloads the service for the $i$th time. We refer to the time interval before the beginning of the first frame as Frame 0. Note that by definition, in all frames, except maybe the last frame, there is exactly one eviction by OPT-OFF.

We use the properties of RR and OPT-OFF to show that each frame in which OPT-OFF evicts the service has the following structure (Figure 7):
- RR fetches and evicts the service exactly once each.
- RR does not cache the service at the beginning of the frame.

We divide Frame $i$ into four sub-frames defined as follows.
- $i.a$: OPT-OFF does not cache the service while RR does.
- $i.b$: OPT-OFF and RR both cache the service.
- $i.c$: OPT-OFF and RR both don’t cache the service.
- $i.d$: OPT-OFF does not cache the service while RR does.

We use the two previous steps to show that the cumulative difference in service and rent costs incurred by RR and OPT-OFF in a frame is upper bounded by $2M + \kappa$. Since the fetch cost under RR and OPT-OFF in a frame is equal, we have that the total cost incurred by RR and OPT-OFF in a frame differs by at most $4M + \kappa$.

We show that once fetched, OPT-OFF caches the service for at least $\frac{M}{\omega}$ time-slots (Lemma 9). We thus conclude that the total cost incurred by OPT-OFF in a frame is lower bounded by $M + \frac{M}{\omega} + \kappa$. We use this to upper bound the ratio of the cost incurred by RR and cost incurred by OPT-OFF in the frame.

The cost incurred by RR and OPT-OFF in Frame 0 is equal. We then focus on the last frame. If OPT-OFF does evict the service in the last frame, the analysis is identical to that of the previous frame. Else, we upper bound the ratio of the cost incurred by RR and cost incurred by OPT-OFF in the frame.

The final result then follows from stitching together the results obtained for individual frames.
5.2 Proof Outline for Theorem 3
We divide the class of deterministic online policies into two sub-classes. Any policy in the first sub-class caches the service during the first time-slot. All other polices are in the second sub-class.

For each policy in either sub-class, we construct a specific arrival sequence and compute the ratio of the cost incurred by the deterministic online policy and an alternative policy. By definition, this ratio serves as a lower bound on the competitive ratio of the deterministic online policy.

5.3 Proof Outline for Theorem 5
We first characterize a lower bound on the cost per time-slot incurred by any online policy (Lemma 17).

Next, we focus on the case where \( \mu > c \). We upper bound the probability of the service not being in the cache during time-slot \( t \) under RR and RR\(_{u}\) using Hoeffding’s inequality [13]. The result then follows by the fact that conditioned on the service being in the cache in time-slot \( t \), the expected total cost incurred by RR and RR\(_{u}\) is at most \( c + \mathbb{E}[X_t - \min(X_t, \kappa)] = c + \mathbb{E}[X_t] - \mu \) and is upper bounded by \( M + c + \mathbb{E}[X_t] \) otherwise. We then consider the case where \( \mu < c \). We upper bound the probability of the service being in the cache or being fetched in time-slot \( t \) under RR and RR\(_{u}\) using Hoeffding’s inequality [13]. The result then follows by the fact that conditioned on the service not being in the cache and not being fetched in time-slot \( t \), the expected total cost incurred by RR is at most \( \mathbb{E}[X_t] \) and is upper bounded by \( M + c + \mathbb{E}[X_t] \) otherwise.

5.4 Proof Outline for Theorem 6
Depending on the value of the system parameters \( (M, c, \kappa) \), we construct specific arrival sequences and compute the ratio of the cost incurred by TTL and OPT-OFF for these sequences. By definition, this ratio serves as a lower bound on the competitive ratio of TTL in each case.

5.5 Proof Outline for Theorem 8
Under the TTL policy with TTL value \( L \), the service is in the cache during a given time-slot if and only if it is requested at least once in the \( L \) previous time-slots. We compute the probability of the event \( G \) defined as the event that the service is requested at least once in the \( L \) previous time-slots. We compute the conditional expected cost incurred by the TTL policy in time-slot conditioned on \( G \) and \( G^c \) to obtain the result.

6 PROOFS
In this section, we discuss the proofs of Theorems 2 and 5. Due to space limitations, proofs of Theorems 3, 6 and 8 are relegated to Appendix D. The notation used in this section is given in Table 2.

6.1 Proof of Theorem 2
We use the following lemmas to prove Theorem 2. Due to space limitations, the proofs of Lemma 9 – 15 are given in Appendix C.

The first lemma provides a lower bound on the duration for which OPT-OFF caches the service once it is fetched.

| Symbol     | Description                                      |
|------------|--------------------------------------------------|
| \( t \)   | Time index                                      |
| \( M \)   | Fetch cost                                      |
| \( c \)   | Rent cost per time-slot                         |
| \( x_t \) | Number of requests arriving in time-slot \( t \) |
| \( \Sigma_t \) | \( \min\{X_t, \kappa\} \)                    |
| \( \delta_t \) | \( X_t - \Sigma_t \)                          |
| \( r^i(t) \) | Indicator variable; 1 if the service is cached by \( i \) \text{OPT-OFF} during time-slot \( t \) and 0 otherwise |
| \( \eta \) | A caching policy                                |
| \( C_0(n, m) \) | Total cost incurred by the offline optimal policy in the interval \([n, m]\) |
| \( C_{\text{TTL}}(n, m) \) | Total cost incurred by the offline optimal policy in the interval \([n, m]\) |
| Frame \( i \) | The interval between the \( i^{\text{th}} \) and the \((i+1)^{\text{th}}\) fetch by the offline optimal policy |
| \( C_{\text{OPT-OFF}}(i) \) | Total cost incurred by the offline optimal policy in Frame \( i \) |
| \( C_{\text{RR}}(i) \) | Total cost incurred by RR in Frame \( i \) |

Table 2: Notation

Lemma 9. Once OPT-OFF fetches the service, it is cached for at least \( \frac{M}{c-k} \) slots.

The next lemma gives an upper bound on the number of requests that can be served by the edge server (subject to its computation power constraints) in a time-interval such that RR does not cache the service during the time-interval and fetches it in the last time-slot of the time-interval.

Lemma 10. Let \( r^{\text{RR}}(n-1) = 1, r^{\text{RR}}(t) = 0 \) for \( n \leq t \leq m \) and \( r^{\text{RR}}(m+1) = 1 \). Then \( \sum_{i=n}^{m} X_i < (m-n)c + 2M + \kappa \).

Consider the event where both RR and OPT-OFF have cached the service in a particular time-slot. The next lemma states that given this, OPT-OFF evicts the service before RR.

Lemma 11. If \( r^{\text{RR}}(n) = 1, r^{*}(t) = 1 \) for \( n \leq t \leq m \), and \( r^{*}(m + 1) = 0 \). Then, \( r^{\text{RR}}(t) = 1 \) for \( n + 1 \leq t \leq m + 1 \).

Consider the case where both RR and OPT-OFF have cached the service in a particular time-slot. From the previous lemma, we know that, OPT-OFF evicts the service before RR. The next lemma gives a lower bound on the number of requests that can be served by the edge server (subject to its computation power constraints) in the interval which starts when OPT-OFF evicts the service from the cache and ends when RR evicts the service from the cache.

Lemma 12. Let \( r^{*}(n-1) = 1, r^{*}(n) = 0, r^{\text{RR}}(t) = 1 \) for \( n-1 \leq t \leq m \) and \( r^{\text{RR}}(m+1) = 0 \). Then \( \sum_{i=n}^{m} X_i \geq (m-n)c - 2M \).

Our next result states that RR does not fetch the service in the interval between an eviction and the subsequent fetch by OPT-OFF.
Lemma 13. If \( r^*(n-1) = 1 \), \( r^*(t) = 0 \) for \( n \leq t \leq m \), and \( r^*(m+1) = 1 \), then RR does not fetch the service in time-slots \( n, n+1, \ldots, m+1 \).

The next lemma states that in the interval between a fetch and the subsequent eviction by OPT-OFF, RR caches the service for at least one time-slot.

Lemma 14. If \( r^*(n-1) = 0 \), \( r^*(t) = 1 \) for \( n \leq t \leq m \) and \( r^*(m+1) = 0 \), then, for some \( n < t \leq m \), \( r^{RR}(t) = 1 \).

If both RR and OPT-OFF cache the service in a particular time-slot, from Lemma 11, we know that OPT-OFF evicts the service before RR. The next lemma states that RR evicts the service before the next time OPT-OFF fetches it.

Lemma 15. If \( r^*(n-1) = 1 \), \( r^*(t) = 0 \) for \( n \leq t \leq m \), \( r^*(m+1) = 1 \), and \( r^{RR}(n-1) = 1 \), then, RR evicts the service by the end of time-slot \( m \) and \( r^{RR}(m+1) = 0 \).

To compare the costs incurred by RR and OPT-OFF we divide time into frames \([1, t_1-1], [t_1, t_2-1], [t_2, t_3-1], \ldots\), where \( t_i-1 \) is the time-slot in which OPT-OFF fetches the service for the \( i \)-th time for \( i \in \{1, 2, \ldots\} \). Our next result characterizes the sequence of events that occur in any such frame.

Lemma 16. Consider the interval \([t_i, t_{i+1}-1]\) such that OPT-OFF fetches the service at the end of time-slot \( t_i-1 \) and fetches it again at the end of time-slot \( t_{i+1}-1 \). By definition, there exists \( \tau \in [t_i, t_{i+1}-2] \) such that OPT-OFF evicts the service in time-slot \( \tau \). RR fetches and evicts the service exactly once each in \([t_1, t_2-1]\). The fetch by RR is in time-slot \( t_f^{RR} \) such that \( t_1 \leq t_f^{RR} \leq \tau \) and the eviction by RR is in time-slot \( t_e^{RR} \) such that \( \tau < t_e^{RR} < t_2 \) (Figure 8).

\[
\begin{aligned}
&t_i-1 \quad t_f^{RR} \quad \tau \quad t_e^{RR} \quad t_{i+1}-1 \\
\text{Frame } i &
\end{aligned}
\]

Figure 8: Illustration of Lemma 16 showing download and eviction by OPT-OFF and RR in the \( i \)-th frame. Downward arrows represent fetches, upward arrows indicate evictions. Black and red arrows correspond to the OPT-OFF and RR policy respectively. The two bars below the timeline indicate the state of the cache under OPT-OFF and RR. The solid black and solid red portions represent the intervals during with OPT-OFF and RR cache the service respectively.

Proof. Without loss of generality, we prove the result for \( i = 1 \). Since \( r^*(t_1-1) = 0 \), \( r^*(t_1) = 1 \) for \( t_1 \leq t \leq \tau \) and \( r^*(\tau) = 0 \) then by Lemma 14, \( r^{RR}(t_1) = 1 \) for some \( t_1 < t_f^{RR} \leq \tau \). In addition, by Lemma 15, \( r^{RR}(t_1) = 0 \). Therefore, RR fetches the service at least once in the interval \([t_1, t_2-1]\).

By Lemma 11, if \( t_f^{RR} < \tau \), since both RR and OPT-OFF cache the service during time-slot \( t_f^{RR} + 1 \), OPT-OFF evicts the service before RR, therefore, once fetched, RR does not evict the service before time-slot \( \tau + 1 \), i.e., \( r^{RR}(\tau) = 1 \) for \( t_f^{RR} + 1 \leq \tau \leq \tau + 1 \).

Since \( r^*(\tau) = 1 \), \( r^*(t_1) = 0 \) for \( \tau + 1 \leq t \leq t_2 - 1 \) and \( r^*(t_2) = 1 \), then by Lemma 15, RR evicts the service in time-slot \( t_e^{RR} \) such that \( \tau < t_e^{RR} = t_2 - 1 \). In addition, once evicted at \( t_e^{RR} \leq t_2 - 1 \), RR does not fetch it again in the before time-slot \( t_2 \) by Lemma 13.

This completes the proof.

Proof of Theorem 2. As mentioned above, to compare the costs incurred by RR and OPT-OFF we divide times into frames \([1, t_1-1], [t_1, t_2-1], [t_2, t_3-1], \ldots\), where \( t_i-1 \) is the time-slot in which OPT-OFF downloads the service for the \( i \)-th time for \( i \in \{1, 2, \ldots\} \).

For convenience, we account for the fetch cost incurred by OPT-OFF in time-slot \( t_i \) in the cost incurred by OPT-OFF in Frame \( i \). Given this, the cost under RR and OPT-OFF is the same for \([1, t_1-1]\) (Frame 0) since both policies don’t cache the service in this period.

Note that if the total number of fetches made by OPT-OFF is \( k < \infty \), there are exactly \( k + 1 \) frames (including Frame 0). The \( (k + 1) \)-th frame either has no eviction by OPT-OFF or OPT-OFF evicts and then never fetches the service.

We now focus on Frame \( i \) such that \( 0 < i < k \), where \( k \) is the total number of fetches made by OPT-OFF.

Without loss of generality, we focus on Frame 1. Recall the definitions of \( \tau, t_f^{RR}, \text{ and } t_e^{RR} \) from Lemma 16, also seen in Figure 8. By Lemma 16, we have that RR fetches and evicts the service exactly once each in \([t_1, t_2-1]\) such that the fetch by RR is in time-slot \( t_f^{RR} \) such that \( t_1 \leq t_f^{RR} \leq \tau \) and the eviction by RR is in time-slot \( t_e^{RR} \) such that \( \tau < t_e^{RR} < t_2 \).

Both OPT-OFF and RR makes one fetch in the frame. Hence the difference in the fetch costs is zero. We now focus on the service and rent cost incurred by the two policies.

Let \( \tau_1 = t_f^{RR} - t_1 \). By Lemma 10, the number of requests that can be served by the edge server in \([t_1, t_f^{RR}]\) is at most \( 2M + \tau_1 c + \kappa \). Since RR does not cache the service in \([t_1, t_f^{RR}]\), the rent cost incurred in \([t_1, t_f^{RR}]\) by RR is zero and the service cost incurred in \([t_1, t_f^{RR}]\) by RR is at most \( 2M + \tau_1 c + \kappa + \sum_{l=t_1}^{t_f^{RR}} \delta_l \). OPT-OFF rents storage in \([t_1, t_f^{RR}]\) at cost \( cr_1 \) and incurs a service cost of \( \sum_{l=t_1}^{t_f^{RR}} \delta_l \) in \([t_1, t_f^{RR}]\).

Hence difference in the service and rent cost incurred by RR and OPT-OFF in \([t_1, t_f^{RR}]\) is at most \( 2M + \kappa \).

The service and rent cost incurred by OPT-OFF and RR in \([t_f^{RR} + 1, \tau] \) are equal.

Let \( \tau_2 = t_e^{RR} - \tau \). By Lemma 12, the number of requests that can be served by the edge server in \([\tau + 1, t_e^{RR}]\) is at least \( \tau_2 c - 2M \). The service cost incurred by OPT-OFF in \([\tau + 1, t_e^{RR}]\) is at least \( \tau_2 c - 2M + 1 \).
This completes the characterization for Frame 1 to k − 1.

We now focus on Frame k, which is the last frame. There are two possible cases, one where OPT-OFF evicts the service in Frame k, in which case the analysis for Frame k is identical to that of Frame 1, and the other when OPT-OFF does not evict the service in Frame k. We now focus on the latter.

Given that OPT-OFF downloads the service in time-slot $t_k - 1$, there exists $m > t_k$ such that $\sum_{l=t_k}^{m} x_l \geq M + (m - t_k + 1)c$. By Step 8 in Algorithm 1, RR downloads the service at the end of time-slot m. Let $t_k = m - t_k$. By Lemma 10, the number of requests that can be served by the edge server during these $t_k$ time-slots is at most $2M + t_k c + \kappa$. Since RR does not cache the service during these $t_k$ time-slots, the rent cost incurred by RR is zero and the service cost incurred by RR is at most $2M + t_k c + \kappa + \sum_{l=t_k}^{t_k + t_k - 1} \delta_l$. OPT-OFF rents storage during these $t_k$ time-slots at cost $c t_k$ and the service cost incurred by OPT-OFF is $\sum_{l=t_k}^{t_k + t_k - 1} \delta_l$. There is no difference between the cost of RR and OPT-OFF after the first $t_k$ slots in Frame k. It follows that

$$C^{RR}(k) - C^{OPT-OFF}(k) \leq 2M + \kappa.$$ (6)

From (5) and (6),

$$c^{RR}(k) \leq \left(3 - \frac{2c}{\kappa}\right) C^{OPT-OFF}(k) + \kappa \leq \left(5 + \frac{\kappa}{M} - \frac{4c}{\kappa}\right) C^{OPT-OFF}(k).$$

Stitching together the results obtained for all frames, the result follows.

## 6.2 Proof of Theorem 5

**Lemma 17.** Let $X_t$ be the number of requests arriving in time-slot $t$, $\nu = \mathbb{E}[X_t]$, $\nu = \min\{X_t, \kappa\}$ and $\mu = \mathbb{E}[X_t]$. Under Assumption 1, let $\mathbb{E}[C^{OPT-ON}_t]$ be the cost per time-slot incurred by the OPT-ON policy. Then,

$$\mathbb{E}[C^{OPT-ON}_t] \geq \min\{c + \nu - \mu, \nu\}.$$

**Proof.** If the service is in the cache in time-slot $t$, the expected cost incurred is $c + \mathbb{E}[X_t - \min\{X(t), \kappa\}] = c + \nu - \mu$.

If the service is not cached at the edge server, the expected cost incurred is at least $\nu$. This proves the result.

**Lemma 18.** Let $X_t$ be the number of requests arriving in time-slot $t$, $X_t = \min\{X_t, \kappa\}$ and $\mu = \mathbb{E}[X_t]$. Then, for $(c - \mu)\tau + M > 0$,

$$\mathbb{P}\left(\sum_{t=\tau-\tau+1}^{\tau} X_t \geq \tau c + M\right) \leq \exp\left(-\frac{2((c - \mu)\tau + M)^2}{\tau^2}\right),$$

and for $(\mu - c)\tau + M > 0$,

$$\mathbb{P}\left(\sum_{t=\tau-\tau+1}^{\tau} X_t < \tau c\right) \leq \exp\left(-\frac{2((\mu - c)\tau + M)^2}{\tau^2}\right).$$

**Proof.** Follows by Hoeffding’s inequality [13].

**Proof of Lemma 4.** We first consider the case when $\mu > c$. We define the following events: $E_{t_1, t_2} : \sum_{l=t_1}^{t_2} X_l < (t_2 - t_1)c - M, E_r = \bigcup_{t=t_1}^{t} E_{t, r}, E_r : \sum_{l=\tau-\tau+1}^{\tau-1} X_l \geq \frac{\lambda M}{\mu - c} + M$. By Lemma 18, it follows that

$$\mathbb{P}(E_{t_1, t_2}) \leq \exp\left(-\frac{2(\mu - c)^2(t_2 - t_1 + 1)}{\kappa^2}\right),$$

and therefore,

$$\mathbb{P}(E_r) \leq \sum_{t_1=1}^{\tau - \frac{\lambda M}{\mu - c}} \exp\left(-\frac{2(\mu - c)^2(\tau - t_1 + 1)}{\kappa^2}\right) \leq \exp\left(-\frac{2(\mu - c)^2}{\kappa^2}\right) \cdot \frac{1}{1 - \exp\left(-\frac{2(\mu - c)^2}{\kappa^2}\right)}.$$

(7)

Using (7) and the union bound,

$$\mathbb{P}(E) \leq \exp\left(-\frac{2(\mu - c)^2}{\kappa^2}\right) \cdot \left[\frac{\lambda M}{\mu - c} + M\right] \cdot \frac{1}{1 - \exp\left(-\frac{2(\mu - c)^2}{\kappa^2}\right)}.$$

(8)

By Lemma 18,

$$\mathbb{P}(F^c) \leq \exp\left(-\frac{2(\mu - c)^2}{\kappa^2}\right) \cdot \left[\frac{\lambda M}{\mu - c} + M\right] \cdot \frac{1}{1 - \exp\left(-\frac{2(\mu - c)^2}{\kappa^2}\right)}.$$

(9)
By (8) and (9),
\[
\mathbb{P}(E^c \land F) \geq 1 - \left[ \frac{\lambda M}{c - \mu} \right] \exp\left(-\frac{2(\mu - c)p M}{\kappa^2}\right) \left[ 1 - \exp\left(-\frac{2(\mu - c)\tau}{\kappa}\right) \right].
\]
(10)

Consider the event \( G = E^c \land F \) and the following three cases.

Case 1: The service is in the cache during time-slot \( t - \left[ \frac{\lambda M}{c - \mu} \right] + 1 \) to \( t - 1 \). It follows that in this case, the service is in the cache during time-slot \( t \).

Case 2: The service is not in the cache during time-slot \( t - \left[ \frac{\lambda M}{c - \mu} \right] + 1 \) to \( t - 2 \). In this case, the service is not fetched in time-slots \( t - 1, \tau, \tau - 1 \). Conditioned on \( F \), by the properties of the RR and RR\( u \) policies, condition in Step 8 in Algorithms 1 and 3 is satisfied for \( \tau \leq t - \left[ \frac{\lambda M}{c - \mu} \right] \). It follows that in this case, the service is not in the cache during time-slot \( t \).

Case 3: The service is not in the cache during time-slot \( t - \left[ \frac{\lambda M}{c - \mu} \right] \) and is not fetched in time-slots \( t - \left[ \frac{\lambda M}{c - \mu} \right] + 1 \) to \( t - 2 \). In this case, the service is not fetched in time-slots \( t - 1, \tau - 1 \). Conditioned on \( F \), by the properties of the RR and RR\( u \) policies, condition in Step 8 in Algorithms 1 and 3 is satisfied for \( \tau \leq t - \left[ \frac{\lambda M}{c - \mu} \right] \). It follows that in this case, the service is not in the cache during time-slot \( t \).

We thus conclude that conditioned on \( G = E^c \land F \), the service is in the cache during time-slot \( t \). We now compute the expected cost incurred by the RR policy. By definition,
\[
\mathbb{E}[c_t^{RR}] = \mathbb{E}[c_t^{RR}| G] \mathbb{P}(G) + \mathbb{E}[c_t^{RR}| G^c] \mathbb{P}(G^c).
\]
Note that, \( \mathbb{E}[c_t^{RR}| G] = c + v - \mu + (M + \mu) \mathbb{P}(G^c) \leq M + c + v \). Therefore,
\[
\mathbb{E}[c_t^{RR}] = c + v - \mu + (M + \mu) \mathbb{P}(G^c)
\]
\[
\leq c + v - \mu + (M + \mu) \times \left( \left[ \frac{\lambda M}{c - \mu} \right] \exp\left(-\frac{2(\mu - c)p M}{\kappa^2}\right) \left[ 1 - \exp\left(-\frac{2(\mu - c)\tau}{\kappa}\right) \right] \right)
\]
+ \exp\left(-\frac{2(\lambda - 1)^2 M(c - \mu)}{\kappa^2}\right). 
\]
(11)

For the RR policy, we optimize over \( \lambda > 1 \) to get the tightest possible bound. By Lemma 17 and (11), we have the result for RR. For the RR\( u \) policy, we optimize over \( 1 < \lambda < \left[ \frac{(\mu - c)\mu}{M} \right] \) to get the tightest possible bound. By Lemma 17 and (11), we have the result for RR\( u \). For RR\( u \), the upper limit on \( \lambda \) comes due to the fact that RR\( u \) goes back at most \( u \) time-slots to make caching/eviction decisions (refer to event \( F \) above).

Next, we consider the case when \( \mu < c \). We define the following events \( F_{t_1, t_2} : \sum_{i=1}^{t_2} \mathbb{X}_i \geq (t_2 - t_1)c + M, F_t = \bigcup_{t_1=1}^{t-1} F_{t_1, t_2}, E : \sum_{i=t-1}^{t-1} \mathbb{X}_i \geq \frac{\lambda M}{c - \mu} \).

By Lemma 18, it follows that
\[
\mathbb{P}(F_{t_1, t_2}) \leq \exp\left(-\frac{2(c - \mu)^2(t_2 - t_1 + 1)}{\kappa^2}\right),
\]
and therefore,
\[
\mathbb{P}(F_t) \leq \sum_{i=t_1=1}^{t-1} \exp\left(-\frac{2(c - \mu)^2(\tau - t_1 + 1)}{\kappa^2}\right)
\]
\[
\leq \exp\left(-\frac{2(c - \mu)^2\tau}{\kappa^2}\right).
\]
(12)

Using (12) and the union bound, \( \mathbb{P}(F_t) \) and \( \mathbb{P}(F_{t-1}) \) are upper bounded by
\[
\left[ \frac{\lambda M}{c - \mu} \right] \exp\left(-\frac{2(c - \mu)^2\tau}{\kappa^2}\right).
\]
(13)

By Lemma 18,
\[
\mathbb{P}(F_t^c \land F_{t-1}^c \land E) \geq 1 - \left[ \frac{\lambda M}{c - \mu} \right] \exp\left(-\frac{2(c - \mu)^2\tau}{\kappa^2}\right).
\]
(14)

Consider the event \( G = F_t^c \land F_{t-1}^c \land E \) and the following three cases.

Case 1: The service is not in the cache during time-slot \( t - \left[ \frac{\lambda M}{c - \mu} \right] + 1 \) to \( t - 1 \). Conditioned on \( F_t^c \), by the properties of the RR and RR\( u \) policies, the service is not fetched in time-slots \( t - \left[ \frac{\lambda M}{c - \mu} \right] \) to \( t - 1 \). It follows that in this case, the service is not in the cache during time-slot \( t \).

Case 2: The service is in the cache during time-slot \( t - \left[ \frac{\lambda M}{c - \mu} \right] \) and is not fetched in time-slot \( \tau \) such that \( t - \left[ \frac{\lambda M}{c - \mu} \right] + 1 \leq \tau \leq t - 2 \). Conditioned on \( E \), by the properties of the RR and RR\( u \) policies, condition in Step 8 in Algorithms 1 and 3 is satisfied for \( \tau \leq t - \left[ \frac{\lambda M}{c - \mu} \right] \). It follows that in this case, the service is in the cache during time-slot \( t \).

Case 3: The service is in the cache during time-slot \( t - \left[ \frac{\lambda M}{c - \mu} \right] \) and is not fetched in time-slot \( \tau \) such that \( t - \left[ \frac{\lambda M}{c - \mu} \right] + 1 \leq \tau \leq t - 1 \). It follows that in this case, the service is in the cache during time-slot \( t \).

We thus conclude that conditioned on \( G = F_t^c \land F_{t-1}^c \land E \), the service is not in the cache during time-slot \( t \). In addition, conditioned on \( F_t^c \), the
service is not fetched in time-slot $t$. We now compute the expected cost incurred by the RR policy. By definition,
\[
\mathbb{E}[c_{t}^{RR}] = \mathbb{E}[c_{t}^{RR}|G] \mathbb{P}(G) + \mathbb{E}[c_{t}^{RR}|G^c] \times \mathbb{P}(G^c).
\]
Note that, $\mathbb{E}[c_{t}^{RR}|G] = v$, $\mathbb{E}[c_{t}^{RR}|G^c] \leq c + v + M$. Therefore,
\[
\mathbb{E}[c_{t}^{RR}] = v + (c + M)\mathbb{P}(G^c)
\leq v + (c + M) \times \left( \frac{\lambda M}{c - \mu} \exp \left( -\frac{2(c - \mu)^2}{\lambda^2} \right) \right)
+ \exp \left( -\frac{2(\lambda - 1)^2}{\lambda k^2} \right).
\]
(16)

For the RR policy, we optimize over $\lambda > 1$ to get the tightest possible bound. By Lemma 17 and (16), we have the result for RR. For the RR$_u$ policy, we optimize over $1 < \lambda < (\mu - c\muM)/M$ to get the tightest possible bound. By Lemma 17 and (16), we have the result for RR$_u$. For RR$_u$, the upper limit on $\lambda$ comes due to the fact that RR$_u$ goes back at most $u$ time-slots to make caching/eviction decisions (refer to event $E$ above).

Proof of Theorem 5. Follows by Lemma 17, (11), (16), the definition of $\sigma^p$ in (3), and the fact that the cost incurred in a time-slot by any policy is upper bounded by $M + c + v$.

For the RR policy, we optimize over $\lambda > 1$ to get the tightest possible bound. For RR$_u$ policy, we optimize over $1 < \lambda < (\mu - c\muM)/M$ to get the tightest possible bound. For RR$_u$, the upper limit on $\lambda$ comes due to the fact that RR$_u$ goes back at most $u$ time-slots to make caching/eviction decisions.

7 CONCLUSIONS

In this work, we focus on designing online strategies for service caching on edge computing platforms. We show that the widely used and studied TTL policies do not perform well in this setting. This is because, on a miss, TTL fetches the data and code needed for the request arrival processes. Via analysis for adversarial and i.i.d. stochastic arrivals, we show that the RR policy is optimal and that its variants perform well for a wide array of request arrival processes.

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APPENDIX A

We provide a formal definition of the RR_u policy.

**Algorithm 3: RetroRenting u (RR_u)**

1. **Input**: Fetch cost \( M \) units, rent cost \( c \) units per time-slot, integer \( u > 0 \), request arrival sequence: \( x_t, t > 0 \)
2. **Output**: Caching strategy \( r_t, t > 0 \)
3. **Initialize**: Caching variable \( r_1 = t_{\text{fetch}} = t_{\text{evict}} = 0 \)
4. **for each** time-slot \( t \) **do**
   5. \( r_{t+1} = r_t \)
   6. **if** \( r_t = 0 \) **then**
      7. **for** max \( \{ t_{\text{evict}}, t - u \} < \tau < t \) **do**
         8. **if** \( \sum_{l=\tau}^t x_l \geq (t - \tau + 1)c + M + \sum_{l=\tau}^t (x_l - \kappa)^+ \), then
            9. \( r_{t+1} = 1, t_{\text{fetch}} = t \)
            10. **break**
      11. **end**
   12. **if** \( r_t = 1 \) **then**
      13. **for** max \( \{ t_{\text{fetch}}, t - u \} < \tau < t \) **do**
          14. **if** \( \sum_{l=\tau}^t x_l + M < (t - \tau + 1)c + \sum_{l=\tau}^t (x_l - \kappa)^+ \), then
              15. \( r_{t+1} = 0, t_{\text{evict}} = t \)
              16. **break**
      17. **end**
   18. **end**

**APPENDIX B**

We present simulation results for trace-data obtained from a Google Cluster [12]. We use a time-slot duration small enough to ensure that there is at most one request in a time-slot. This trace-data has requests for four types of jobs/services identified as "Job 0", "Job 1", "Job 2", and "Job 3". In this section, we present results for "Job 0" (Figures 9, 12), "Job 1" (Figures 10, 13), and "Job 3" (Figures 11, 14).

**APPENDIX C**

In this section, we discuss the proofs of Lemmas 9 – 15. The next three lemmas are used to prove Lemmas 9 – 15.

**Lemma 19.** If \( r^n (n - 1) = 0, r^n (t) = 1 \) for \( n \leq t \leq m \) and \( r^n (m + 1) = 0 \), then, \( \sum_{l=n}^m \delta_l \geq M + (m - n + 1)c \).

**Proof.** The cost incurred by OPT-OFF in \( n \leq t \leq m \) is \( M + (m - n + 1)c \sum_{l=n}^m \delta_l \). We prove Lemma 19 by contradiction. Let us assume

\[
\sum_{l=n}^m \delta_l < M + (m - n + 1)c
\]

that \( \sum_{l=n}^m \delta_l < M + (m - n + 1)c \). We construct another policy \( \eta \) which behaves same as OPT-OFF except that \( r_\eta (t) = 0 \) for \( n \leq t \leq m \). The
that $C^\eta(n,m) - C^{OPT-OFF}(n,m) = \sum_{l=n}^{m} x_l - M - (m-n+1)c$, which is negative by our assumption. This contradicts the definition of the OPT-OFF policy, thus proving the result.

The next lemma shows that if the number of requests that can be served by the edge server in a time-interval exceeds a certain value (which is a function of the length of that time-interval) and the service is not cached at the beginning of this time-interval, then OPT-OFF fetches the service at least once in the time-interval.

**Lemma 20.** If $r^*(n-1) = 0$, and $\sum_{l=n}^{m} x_l \geq M + (m-n+1)c$, then OPT-OFF fetches the service at least once in the interval from time-slots $n$ to $m$.

**Proof of Lemma 20.** We prove Lemma 20 by contradiction. We construct another policy $\eta$ which behaves same as OPT-OFF except that $\eta_l(t) = 1$ for $n \leq t \leq m$. The total cost incurred by $\eta$ in $n \leq t \leq m$ is $C^\eta(n,m) = M + (m-n+1)c + \sum_{l=n}^{m} \delta_l$. It follows that $C^\eta(n,m) - C^{OPT-OFF}(n,m) = M + (m-n+1)c - \sum_{l=n}^{m} x_l$, which is negative. Hence there exists at least one policy $\eta$ which performs better than OPT-OFF. This contradicts the definition of the OPT-OFF policy, thus proving the result.

Our next lemma characterizes a necessary condition for RR to fetch the service.

**Lemma 21.** If $r^{RR}(n) = 0$ and $r^{RR}(n+1) = 1$, then, by the definition of the RR policy, $\exists r < n$ such that $\sum_{l=n-\tau+1}^{n} x_l \geq r\kappa + M$. Let $\tau_{\min} = \min \left\{ \tau \mid \sum_{l=n-\tau+1}^{n} x_l \geq r\kappa + M \right\}$. Then, $\tau_{\min} \geq \frac{M}{\kappa c}$.

**Proof.** Since $\sum_{l=n-\tau+1}^{n} x_l \leq \tau\kappa$, $\tau_{\min} \geq \tau_{\min}c + M$, and the result follows.

**Proof of Lemma 9.** Suppose OPT-OFF fetches the service at the end of the $(n-1)^{th}$ time-slot and evicts it at the end of time-slot $m > n$. From Lemma 19, $\sum_{l=n}^{m} x_l \geq M + (m-n+1)c$. Since $\sum_{l=n}^{m} x_l \leq (m-n+1)c, (m-n+1)c \geq M + (m-n+1)c$, i.e., $(m-n+1) \geq \frac{M}{\kappa c}$. This proves the result.

**Proof of Lemma 10.** Given $r^{RR}(m) = 0$ and $r^{RR}(m+1) = 1$, we have that $\exists r > 0$ and $m - \tau + 1 \geq n$ such that $\sum_{l=m-\tau+1}^{m} x_l \geq r\kappa + M$ and $\sum_{l=m-\tau+2}^{m} x_l \leq (\tau-1)\kappa + M$. In addition, since $r^{RR}(m-\tau+1) = 0$, we have that $\sum_{l=n}^{m-\tau} x_l < (m-\tau-n+1)c + M$. 

...
By definition,
\[
\sum_{l=n}^{m} x_l = (\sum_{l=n}^{m-\tau} x_l) + \sum_{l=m-\tau+1}^{m} x_l < (m - \tau - n + 1)c + M + \kappa + (\tau - 1)c + M < (m - n)c + 2M + \kappa.
\]
A consequence of this result is that, for \( n' \) such that \( n \leq n' \leq m - \tau \),
\[
\sum_{l=n'}^{m} x_l < (m - n')c + 2M + \kappa.
\]
\[\Box\]

Proof of Lemma 11. We prove this by contradiction. Let \( \exists \exists m < m \) such that \( r^{RR}(m + 1) = 0 \). Then, from Algorithm 1, there exists an integer \( \tau > 0 \) such that \( \sum_{l=m-\tau+1}^{m} x_l < \tau c - M \).

The cost incurred by OPT-OFF in the interval \( m - \tau + 1 \) to \( m \) is \( \tau c + \sum_{l=m-\tau+1}^{m} \delta_j \).

Consider an alternative policy \( \eta \) for which \( r_{\eta}(t) = 0 \) for \( m - \tau + 1 \leq t \leq m \), \( r_{\eta}(m + 1) = 1 \), and \( r_{\eta}(t) = r(t) \) otherwise. It follows that
\[
C^{\eta} - C^{\text{OPT-OFF}} = \sum_{l=m-\tau+1}^{m} x_l + M - \tau c \quad \text{which is negative by our assumption. This contradicts the definition of the OPT-OFF policy, thus proving the result.}
\]
\[\Box\]

Proof of Lemma 12. Given \( r^{RR}(m) = 1 \) and \( r^{RR}(m + 1) = 0 \), we have that \( \exists \exists \tau > 0 \) such that \( \sum_{l=m-\tau+1}^{m} x_l < \tau c - M \) and \( \sum_{l=m-\tau+1}^{m} x_l \geq (\tau - 1)c - M \). In addition, since \( r^{RR}(m + 1) = 1 \), we have that \( \sum_{l=n}^{m} x_l \geq (m - \tau - n + 1)c - M \).

By definition,
\[
\sum_{l=n}^{m} x_l = (\sum_{l=n}^{m-\tau} x_l) + \sum_{l=m-\tau+1}^{m} x_l \geq (m - \tau - n + 1)c - M + 0 + (\tau - 1)c - M = (m - n)c - 2M.
\]
Thus proving the result. A consequence of this result is that, for \( n' \) such that \( n \leq n' \leq m - \tau \), \( \sum_{l=n'}^{m} x_l < (m - n')c + 2M \).
\[\Box\]

Proof of Lemma 13. We prove this by contradiction. Let RR fetch the service in time-slot \( t \) where \( n \leq t \leq m - 1 \).

Then from Algorithm 1, there exists an integer \( \tau > 0 \) such that \( t - \tau \geq n \) and \( \sum_{l=t-\tau+1}^{t} x_l \geq \tau c + M \). If this condition is true, by Lemma 20, OPT-OFF would have fetched the service at least once in the interval \( t - \tau + 1 \) and \( t \) for all \( n \leq t \leq m - 1 \). Hence RR does not fetch the service between \( n \) and \( m - 1 \). \[\Box\]

Proof of Lemma 14. We prove this by contradiction. Let \( t^{RR}(t) = 0 \) for all \( n \leq t \leq m \). Then by the definition of the RR policy, \( \sum_{l=t-\tau+1}^{t} x_l < \tau c + M \) for any \( \tau > 0 \) and \( \tau \leq t - n + 1 \). If we choose \( t = m \) then \( \sum_{l=t}^{m} x_l < (m - n + 1)c + M \), which is false from Lemma 19. This contradicts our assumption. \[\Box\]

Proof of Lemma 15. We prove this by contradiction. Assume that RR does not evict the service in any time slot \( t \) for all \( n \leq t \leq m \). Then from the definition of the RR policy, \( \sum_{l=t-\tau+1}^{t} x_l + M \geq \tau c \) for all \( \tau > 0 \) such that \( 0 < \tau \leq t - n + 1 \). As a result, at \( t = m \), \( \sum_{l=m}^{m} x_l + M \geq (m - n + 1)c \). Given this, it follows that OPT-OFF will not evict the service at the end of time-slot \( n - 1 \). This contradicts our assumption. By Lemma 13, RR does not fetch the service in the interval between an eviction and the subsequent fetch by OPT-OFF. Therefore, \( r^{RR}(m + 1) = 0 \). \[\Box\]

APPENDIX D
Proof of Theorem 3

Proof. Let \( P \) be a given deterministic online policy and \( C^P(a) \) be the cost incurred by this policy for the request sequence \( a \).

We first focus on the case where \( P \) does not cache the service during the first time-slot. We define \( t^{(1)}_{\text{fetch}} \geq 1 \) as the first time the policy \( P \) fetches the service when there are \( K \) request arrivals each in the first \( t^{(1)}_{\text{fetch}} \) time-slots. Since \( P \) is an online deterministic policy, the value of \( t^{(1)}_{\text{fetch}} \) can be computed a-priori.

Consider the arrival process \( a \) with \( K \) request arrivals each in the first \( t^{(1)}_{\text{fetch}} \) time-slots and no arrivals thereafter. It follows that \( C^P(a) \geq K^{(1)}_{\text{fetch}} + M + c \).

Consider an alternative policy \( \text{ALT} \) which caches the service in time-slots 1 to \( t^{(1)}_{\text{fetch}} \) and does not cache it thereafter. It follows that \( C^{\text{ALT}}(a) = ct^{(1)}_{\text{fetch}} + M \). By definition, \( \rho^P \geq \frac{K^{(1)}_{\text{fetch}} + M + c}{ct^{(1)}_{\text{fetch}} + M} \).

Therefore,
\[
\rho^P \geq \begin{cases} 1 + \frac{\kappa}{c + M} & \text{if } \kappa \geq \frac{c(c + M)}{M} \\ \frac{\kappa}{c} & \text{otherwise.} \end{cases}
\]

Next, we focus on the case where \( P \) caches the service during the first time-slot. We define \( t^{(1)}_{\text{evict}} \geq 1 \) as the first time the policy \( P \) evicts the service when there are no request arrivals each in the first \( t^{(1)}_{\text{evict}} \) time-slots. Since \( P \) is an online deterministic policy, the value of \( t^{(1)}_{\text{evict}} \) can be computed a-priori.

Consider the arrival process \( a \) with no request arrivals each in the first \( t^{(1)}_{\text{evict}} \) time-slots and \( \kappa \) arrivals in time-slot \( t^{(1)}_{\text{evict}} + 1 \). It follows that \( C^P(a) \geq ct^{(1)}_{\text{evict}} + M + \kappa \). Consider an alternative policy \( \text{ALT} \) which does not cache the service in time-slots 1 to \( t^{(1)}_{\text{evict}} \).
we construct a request sequence where a request arrives in the first time-slot and no requests arrive thereafter. OPT-OFF does not fetch the service and the total cost of service per request for this request sequence under OPT-OFF is one unit. Let $\rho^{OPT-OFF}$ be the cost of service per request for this request sequence under TTL. TTL fetches the service on a request arrival and stores it on local edge server for $L$ time slots. Thus the cost of service incurred by TTL per request is $C_{TTL} = 1 + M + Lc$. Therefore, $\rho^{TTL} = 1 + M + Lc$.

Next, we consider the case where $\kappa \geq M + c$ and $Lc \leq M$. For this setting we construct a request sequence where $\kappa$ requests arrive in the first time-slot and no requests arrive thereafter. In this case, $C_{TTL} = u(\kappa + Lc + M)$. Consider an alternative caching policy which fetches the service before the first time-slot and caches it for one time-slot. The total cost of service for this policy is $M + c$. It follows that

$$\rho^{TTL} \geq \frac{\kappa + M + Lc}{M + c}.$$  

Next, we consider the case where $\kappa \geq M + c$ and $Lc \leq M$. For this setting we construct a request sequence where $\kappa$ requests arrive in time-slots $1, L + 2, 2L + 3, \ldots, (u - 1)L + u$ and no requests arrive in the remaining time-slots. In this case, $C_{TTL} = u(\kappa + Lc + M)$. Consider an alternative caching policy which fetches the service before the first time-slot and caches it till time-slot $(u - 1)L + u$. The total cost of service for this policy is $M + ((u - 1)L + u)c$. It follows that

$$\rho^{TTL} \geq \sup_{u \geq 1} \frac{u(\kappa + Lc + M)}{M + ((u - 1)L + u)c} = \frac{\kappa + Lc + M}{Lc + c}.$$  

The result follows from the three cases. □

Proof of Theorem 8

Proof of Lemma 7. We compute the expectation of the cost incurred by the TTL policy in time-slot $t$. Let $G$ be the event that there are no arrivals in time-slots $\max\{t - L, 1\}$ to $t - 1$. Under Assumption 1, $\mathbb{P}(G) = p_{0}^{\min\{L, t - 1\}}$. For the TTL policy, conditioned on $G$, the service is not cached at the beginning of time-slot $t$ and all requests received in time-slot $t$ are forwarded to the back-end server. In addition, the service is fetched if at least one request is received in time-slot $t$. It follows that $\mathbb{E}[C_{TTL}^{TTL}] = (1 - p_{0}) + v$. If the service is cached in time-slot $t$, the TTL policy pays a rent cost of $c$ and up to $\kappa$ results are served at the edge. The remaining requests are forwarded to the back-end server. It follows that

$$\mathbb{E}[C_{TTL}^{TTL}|G^c] = c + \mathbb{E}[X_{t} - \min\{X_{t}, \kappa\}] = c + v - \mu.$$  

Note that

$$\mathbb{E}[C_{TTL}^{TTL}] = \mathbb{E}[C_{TTL}^{TTL}|G] \mathbb{P}(G) + \mathbb{E}[C_{TTL}^{TTL}|G^c] \mathbb{P}(G^c)$$

$= (1 - p_{0}) + v) p_{0}^{\min\{L, t - 1\}}$

$$+ (c + v - \mu)(1 - p_{0}^{\min\{L, t - 1\}}).$$

Moreover, $\mathbb{E}[C_{OPT-OFF}] \leq v$. It follows that $\Delta^{TTL} \geq (1 - p_{0}) p_{0}^{\min\{L, t - 1\}} + (c - \mu)(1 - p_{0}^{\min\{L, t - 1\}})$, thus proving the result. □