Constraints on absolute neutrino Majorana mass from current data

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We present new constraints on the neutrino Majorana masses from the current data of neutrinoless double beta decay and neutrino flavour mixing. With the latest results of $0\nu\beta\beta$ progresses from various isotopes, including the recent calculations of the nuclear matrix elements, we find that the strongest constraint of the effective Majorana neutrino mass is from the $^{136}\text{Xe}$ data of the EXO-200 and KamLAND-Zen collaborations. Further more, by combining the $0\nu\beta\beta$ experimental data with the neutrino mixing parameters from new analyses, we get the mass upper limits of neutrino mass eigenstates and flavour eigenstates and suggest several relations among these neutrino masses.

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Introduction — The experiments with solar, atmospheric, reactor and accelerator neutrinos in recent decades [1–9] have provided compelling evidences for the neutrino oscillation. In the three-generation neutrino framework, the neutrino oscillation is caused by the nonzero neutrino masses due to the mixing among the mass eigenstates in the flavour eigenstates. There have been a number of new measurements on the neutrino mass splitting recently [10–13]. However, the absolute mass scale of neutrinos is still unclear yet. It is also not clarified whether neutrinos are Dirac or Majorana fermions.

Fortunately, the neutrinoless double beta decay ($0\nu\beta\beta$) process provides us some extra information on neutrino properties. The double beta decay of eleven nuclei, whose $Q$-values are larger than 2 MeV such as $^{76}\text{Ge}$, $^{82}\text{Se}$, $^{100}\text{Mo}$, $^{130}\text{Te}$ and $^{136}\text{Xe}$, have been studied up to date [14–22]. For almost all the eleven isotopes, the two-neutrino double beta decay ($2\nu\beta\beta$) processes

$$(A, Z) \rightarrow (A, Z + 2) + 2e^- + 2\bar{\nu}_e,$$

are observed. The $0\nu\beta\beta$ process

$$(A, Z) \rightarrow (A, Z + 2) + 2e^-,$$  \hspace{1cm} (2)

if occurs, can serve as signals of new physics such as the total lepton number violation and the Majorana nature of neutrinos. The only observation of decay events was reported in the year of 2001 by the Heidelberg-Moscow experiment [14, 15], which claimed the half-life time $T_{0\nu}^{1/2} = 2.23^{+0.44}_{-0.31} \times 10^{25}$ yr on $^{76}\text{Ge}$ at 68\% CL. But the data from the GERmanium Detector Array (GERDA) experiment [21] in 2013 indicate that the half-life time of $^{76}\text{Ge}$ should be no lower than $2.5 \times 10^{25}$ yr (90\% CL), with the Heidelberg-Moscow result unconfirmed. Recently, the Enriched Xenon Observatory (EXO-200) collaboration [22] and the Kamioka Liquid Scintillator Anti-Neutrino Detector-Zero Neutrino Double Beta (KamLAND-Zen) collaboration [19] released their new results from which we can obtain the strongest constraint on neutrino Majorana mass.

We can only obtain the effective Majorana neutrino mass from the half-life time of $0\nu\beta\beta$ process. In order to get constraints of each mass eigenstates and flavour eigenstates, it is necessary to combine the data of double beta decay experiments and oscillation experiments, since the oscillation provides us two mass square differences, as well as the mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ and the CP phase $\delta$ in the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix [23, 24].

In this paper we provide new constraints on absolute neutrino Majorana mass from analysis of latest data of $0\nu\beta\beta$ processes, combined with also new results of neutrino mixing parameters. In Sec. II, we obtain the upper limits of the effective Majorana neutrino mass. Then we calculate the mass constraints of different eigenstates by using the CP phase as a variable in Sec. III and Sec. IV. In Sec. V, we present the conclusion and some discussions on some possible relations among neutrino masses.

Effective Majorana neutrino mass — The half-life time of $0\nu\beta\beta$ process is determined by three kinds of contributions, i.e., the particle mass factor, the phase space integral factor $G_{0\nu}$ and the nuclear matrix element (NME) $M_{0\nu}$. Both the latter two vary with different isotopes. If only considering the contribution of the light Majorana neutrino, the half-life time of a given isotope $(A, Z)$ is written as [25, 26]

$$[T_{0\nu}(0^+ \rightarrow 0^+)]^{-1} = \left(\frac{m_{\beta\beta}}{m_e}\right)^2 G_{01}(Q_{\beta\beta}, Z)|M_{0\nu}(A)|^2,$$  \hspace{1cm} (3)

where $m_e$ is the rest mass of electron. Here we introduce the effective Majorana neutrino mass

$$m_{\beta\beta}^0 = \sum_j U_{e j}^2 m_j,$$  \hspace{1cm} (4)
to represent the contribution of the Majorana neutrino during the decay process. $U_{ij}$ ($l = e, \mu, \tau$, $j = 1, 2, 3$) are the elements of product matrix of the PMNS matrix $V$ and a diagonal matrix which contains two additional Majorana CP phases $\alpha_2$ and $\alpha_3$.

\[
V(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = \begin{pmatrix}
 c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
 -c_{12}s_{23}s_{13}e^{i\delta} - s_{12}c_{23} & -s_{12}s_{23}s_{13}e^{i\delta} + c_{12}c_{23} & s_{23}c_{13} \\
 -c_{12}s_{23}s_{13}e^{i\delta} + s_{12}s_{23} & -s_{12}s_{23}s_{13}e^{i\delta} - c_{12}c_{23} & c_{23}c_{13}
\end{pmatrix},
\]

(5)

\[
U = V \cdot \begin{pmatrix}
 1 & 0 & 0 \\
 0 & e^{i\frac{\theta_{23}}{2}} & 0 \\
 0 & 0 & e^{i\frac{\theta_{13}}{2}}
\end{pmatrix},
\]

(6)

where $s_{ij}$ and $c_{ij}$ denote $\sin \theta_{ij}$ and $\cos \theta_{ij}$. The flavour mixing of the three neutrino generations is written as

\[
\nu_l(x) = \sum_j U_{lj} \nu_j(x),
\]

(7)

where the subscripts $l = e, \mu, \tau$ denote the flavour eigenstates and $j = 1, 2, 3$ represent the mass eigenstates.

### TABLE I: The lower limits of half-life time $T^{0\nu}_{1/2}$ of several isotopes observed by recent experiments [15–22].

| isotope | experiment       | $T^{0\nu}_{1/2}$ [10^{24}yr] |
|---------|-----------------|-------------------------------|
| 76Ge    | Heidelberg-Moscow | 9.6                           |
|         | GERDA           | 25                            |
| 82Se    | NEMO-3          | 0.32                          |
| 100Mo   | NEMO-3          | 1.0                           |
| 130Te   | Cuoricino       | 4.1                           |
| 136Xe   | KamLAND-Zen 2012 | 5.7                           |
|         | KamLAND-Zen 2013 | 19                            |
|         | KamLAND-Zen 2014 | 26                            |
|         | EXO-200         | 11                            |

### TABLE II: The phase space integral factors [26–28] and the nuclear matrix elements from some approximation methods [29–33] for these five isotopes.

| isotope | 76Ge | 82Se | 100Mo | 130Te | 136Xe |
|---------|------|------|-------|-------|-------|
| $G_{01}$ [10^{-14} yr^{-1}] | 0.63 | 2.70 | 4.40  | 4.10  | 4.30  |
| ISM(U)  | 2.81 | 2.64 | -     | 2.65  | 2.19  |
| EDF(U)  | 4.60 | 4.22 | 5.08  | 5.13  | 4.20  |
| IBM-2   | 5.42 | 4.37 | 3.73  | 4.03  | 3.33  |
| QRPA-A  | 5.16 | 4.66 | 5.42  | 3.90  | 2.18  |
| SPQRPA-B | 5.82 | 5.66 | 5.15  | 4.70  | 3.36  |

For each of the five isotopes mentioned above in Table I, the phase space integral factor is determined and

### TABLE III: The upper limits of effective Majorana neutrino mass from recent experiments, with $m_{\beta\beta,1}^0$ and $m_{\beta\beta,2}^0$, which are related to the NMEs, denoting the minimal and maximal values of the mass upper limits.

| isotope      | experiment       | $m_{\beta\beta,1}^0$ [eV] | $m_{\beta\beta,2}^0$ [eV] |
|--------------|-----------------|-----------------|-----------------|
| 76Ge         | Heidelberg-Moscow | 0.357            | 0.739            |
|              | GERDA           | 0.221            | 0.458            |
| 82Se         | NEMO-3          | 0.971            | 2.082            |
| 100Mo        | NEMO-3          | 0.473            | 0.923            |
| 130Te        | Cuoricino       | 0.243            | 0.470            |
| 136Xe        | KamLAND-Zen 2012 | 0.246            | 0.471            |
|              | KamLAND-Zen 2013 | 0.135            | 0.258            |
|              | KamLAND-Zen 2014 | 0.115            | 0.221            |
|              | EXO-200         | 0.177            | 0.339            |

### TABLE IV: The global fit of neutrino mixing parameters [36], with NH and IH denoting the normal and inverted hierarchies of the mass eigenstates. The second square difference of mass is defined as $\Delta m^2 = m_1^2 - (m_2^2 + m_3^2)/2$ in normal hierarchy, and $-\Delta m^2$ in inverted hierarchy.

| parameter   | best fit±1σ | 3σ range |
|-------------|-------------|----------|
| $\Delta m^2_{12}$ (10^{-5} eV^2) | 7.54±0.26 | 6.99 → 8.18 |
| $\Delta m^2_{13}$ (10^{-3} eV^2) (NH) | 2.43±0.06 | 2.19 → 2.62 |
| $\Delta m^2_{23}$ (10^{-2} eV^2) (IH) | 2.42±0.07 | 2.17 → 2.61 |
| $\sin^2 \theta_{12}$ (NH or IH) | 0.307±0.016 | 2.59 → 3.59 |
| $\sin^2 \theta_{23}$ (NH) | 0.386±0.024 | 0.331 → 0.637 |
| $\sin^2 \theta_{23}$ (IH) | 0.392±0.020 | 0.335 → 0.663 |
| $\sin^2 \theta_{13}$ (NH) | 0.0241±0.0025 | 0.0169 → 0.0313 |
| $\sin^2 \theta_{13}$ (IH) | 0.0244±0.0023 | 0.0171 → 0.0313 |

has been calculated [26–28]. However, the NMEs are more complicated and we can only obtain them by approximation methods. There are several common approaches, e.g., the interacting shell model (ISM) [29], energy density functional (EDF) method [30], interacting boson model (IBM) [31], quasiparticle random phase approximation (QRPA) [32] and self-consistent renormalized quasiparticle random phase approximation (SR-PRA) [33]. The NMEs from some of these methods
together with the phase space factors are listed in Table II.

According to Eq. (3), we can obtain the expression of the effective Majorana neutrino mass with the half-life time as known quantity,

\[ m_{\beta\beta}^{0v} = \frac{m_e}{M_{0v}\sqrt{G_{01}^{2}\Gamma_{1/2}}} \]  

(8)

The results of the constrains on \( m_{\beta\beta}^{0v} \) from different experiments are showed in Table III. Comparing with the half-life time data in Table I, we find that the EXO-200 and KamLAND-Zen experiments provide the strongest constraint on the effective Majorana neutrino mass

\[ m_{\beta\beta}^{0v} < m_{\beta\beta,\text{min}} = 0.115 \text{ eV}, \]  

(9)

based on the \(^{136}\text{Xe}\) data and EDF approximation approach. It is necessary to notice that though the GERDA group reports a strong lower limit of the \( 0\nu\beta\beta \) decay half-life time, it does not provide the strongest constraint on Majorana neutrino mass [34], since the phase space integral factor of \(^{136}\text{Xe}\) is much larger than that of \(^{76}\text{Ge}\).

**Mass eigenstates** — Now we already obtain an upper limit of \( m_{\beta\beta}^{0v} \). According to Eq. (4), if considering the mixing parameters we get to the constraints of three mass eigenstates of Majorana neutrino. Here we use the global fitting results [35, 36] of mixing parameters listed in Table IV as the inputs. The latest T2K result [37] suggests the negative maximal violation of CP phase \( \delta = -90^\circ \) or \( 270^\circ \), which agrees with the prediction of the maximal CP violation (\( \delta = \pm 90^\circ \)) from the \( \mu-\tau \) interchange symmetry [38]. In our calculation, we take the CP phase \( \delta \) as a parameter too. Since there is little information about the Majorana CP phases, we hypothesize that \( \alpha_{21} = \alpha_{31} = 0 \). Thus, we have only three mass eigenvalue as unknowns in the three equations below,

\[ m_{\beta\beta}^{0v} = |(c_{12}c_{13})^2 m_1 + (s_{12}c_{13})^2 m_2 + s_{13}^2 e^{-2i\delta} m_3|, \]  

(10)

\[ m_2^2 - m_1^2 = \Delta m^2_{21}, \]  

(11)

\[ m_3^2 - m_1^2 = \Delta m^2 + \frac{1}{2} \Delta m_{21} \text{ (NH)}, \]  

(12)

\[ m_1^2 - m_3^2 = \Delta m^2 - \frac{1}{2} \Delta m_{21} \text{ (IH)}. \]  

(13)

Then we draw the curves that show how the upper limits of neutrino mass eigenvalues \( m_1, m_2, m_3 \) change with the CP phase \( \delta \), as seen from Fig. 1.

In the figure we can find that these three functions \( m_i(\delta) \) (\( i = 1, 2, 3 \)) are all trigonometric functions with the same phase. The period of the \( m_i(\delta) \) is not \( 2\pi \) but \( \pi \). The \( \pi \) period is resulted from the fact that the \( \delta \) appears as \( 2\delta \) form in Eq. (9). If \( \delta \) satisfies the maximal CP violation assumption, namely \( \delta = \pm 90^\circ \), the sign of \( \delta \) has no influence for constraining the neutrino Majorana mass in this way, for the range of \( \pm 90^\circ \) is just the right period of one \( \pi \).

The first two mass eigenvalues \( m_1 \) and \( m_2 \) are very close to each other. The third one has a larger difference with respect to them, but the distinction is still within 10 meV. The error bars in the figure reflect the \( \pm 1\sigma \) range influences of the mixing parameters. This error range almost covers the difference between \( m_1 \) and \( m_2 \). This indicates that the three mass eigenstates \( \nu_i \) (\( i = 1, 2, 3 \)) are degenerate: \( \nu_1 \) and \( \nu_2 \) are strongly degenerate, and \( \nu_3 \) is a little bit weaker. That helps us to understand why the mixing angle between \( \nu_1 \) and \( \nu_2 \) is large. On the other hand, since the level of error is related to \( m_{\beta\beta}^{0v} \), it is necessary to improve the observation on \( 0\nu\beta\beta \) process, for getting stronger mass upper limits or even exactly events to enrich the knowledge of the neutrino mass eigenvalues.

**Flavour eigenstates** — In the three-generation neutrino framework, there are three mass eigenstates that evolve with the time and three flavour eigenstates that take part in the weak interaction. Equipped with the upper limits of each mass eigenvalue from the effective Majorana neutrino mass, many calculations become fea-
sible. It is the flavour eigenstate mass constraints that interest us most since they have the direct correlation to the dynamic processes. We introduce the mass operator \( m \) that satisfies the relation

\[
m_j = \langle \nu_j | m | \nu_j \rangle,
\]

where \( j = 1, 2, 3 \) for the mass eigenstates. From Eq. (7), the masses of flavour eigenstates, i.e., \( m_l = \langle \nu_l | m | \nu_l \rangle \) \((l = e, \mu, \tau)\), are written as

\[
m_e = |c_{12}c_{13}|^2m_1 + |s_{12}c_{13}|^2m_2 + |s_{13}e^{-i\delta}|^2m_3, \quad (15)
m_\mu = | - c_{12}s_{23}s_{13}e^{i\delta} - s_{12}c_{23}|^2m_1 \\
+ | s_{12}s_{23}s_{13}e^{i\delta} + c_{12}c_{13}|^2m_2 + |s_{23}c_{13}|^2m_3, \quad (16)
m_\tau = | - c_{12}c_{23}s_{13}e^{i\delta} + s_{12}s_{23}|^2m_1 \\
+ | - s_{12}c_{23}s_{13}e^{i\delta} - c_{12}s_{23}|^2m_2 + |c_{23}c_{13}|^2m_3. \quad (17)
\]

From 0\( \nu\beta\beta \) processes we obtain the upper limit for the summed neutrino mass

\[
\sum_l m_l = \sum_j m_j \leq 0.37 \text{ eV (NH)} \text{ or } 0.35 \text{ eV (IH)}, \quad (24)
\]

which is comparable with the bound 0.23 eV obtained from cosmological observations [39].

**Conclusion** — In conclusion, by analysing the latest results of 0\( \nu\beta\beta \) processes from various isotopes, we give a new constraint of the effective Majorana neutrino mass in \(^{136}\text{Xe} \) with the EXO-200 and KamLAND-Zen data [19, 22]. The strongest mass upper limit of \( m_{\beta\beta}^{0\nu} \) ranges from 0.115 to 0.339 eV depending on different choices of the nuclear matrix elements. Further more, combining with the global fitting results of neutrino mixing parameters, we calculate the upper mass limits of three mass eigenstates \( \nu_1, \nu_2, \nu_3 \) and three flavour eigenstates \( \nu_e, \nu_\mu \) and \( \nu_\tau \). The mass eigenvalue upper limits are very close to each other, and are periodic with period \( \pi \). There are some hierarchical and invariant relations.
The studies on the nature of neutrinos are beneficial to explore new physics. The $0\nu\beta\beta$ decay process is important to explore whether the neutrinos are Majorana fermions or Dirac fermions, and it is also very useful to provide the limits of their absolute mass scales, as reflected from our analysis. Such kind of experiments can provide more strong constraints on the effective Majorana neutrino mass in the future, and they can also clarify possible relations among mass eigenstates and flavour eigenstates of neutrinos.

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