RR Lyrae pulsational temperature scales: On the consistency between different empirical relations

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ABSTRACT
In this paper, by assuming as correct equilibrium temperatures of RRab Lyrae variables those defined from Carney, Storm & Jones (1992, CSJ), we show that temperatures derived from (B-V) colour (mean colour over the pulsational cycle calculated on the magnitude scale) by Bessel, Castelli & Plez 1998 (BCP) transformations, are consistent with CSJ equilibrium temperatures within a probable error of \( \delta \log T_e = \pm 0.003 \). As a consequence it is shown that the pulsational temperature scale, temperature-period-blue amplitude (\( T_{\text{eff}} = f(P, A_B) \)), provided by De Santis (1996, DS96) by (B-V) colour of about 70 stars of Lub’s sample, is a suitable relation, reddening and metallicity free, to calculate equilibrium temperatures for RRab variables. This relation is independent from variable mass and luminosity within a large range of period-shift from the mean period-amplitude relation valid for the Lub’s sample of variables. On the contrary, it is also shown that a temperature-amplitude-metallicity relation is strictly dependent from the period-amplitude relation of the sample used for calibrating it: we prove this means that it is dependent from both mass and luminosity variations of variables.

Key words: stars: evolution – stars: interiors – globular clusters: general

1 INTRODUCTION.
In a recent paper, Carretta, Gratton & Clementini (2000, CGC) noticed that the temperature - period - amplitude relation of CSJ may hide some degeneracy due to some correlation existing between some of these parameters, and then may be inadequate to estimate the luminosity level of RR Lyrae variables in globular clusters by using the \( \log P - \log T_e \) plane. On this point, a first concern was pointed out by Catelan (1998, hereinafter C98), who suggested that an empirical temperature - amplitude - metallicity relation would be a better approach, with respect to a calibration including the period, "since period-shift caused by luminosity variations could be misinterpreted as being due to temperature variations". Besides, it is worth to note that De Santis & Cassisi (1999, hereinafter DC99), by adopting the temperature - period - blue amplitude relation of DS96 for determining the luminosity level of horizontal branch stellar structures, did not find evidence for a luminosity dependence of this pulsational temperature scale (see their Figure 12). Since period and amplitude can be determined with great accuracy and independently of both distance and reddening of a variable, the availability of a reliable relation connecting pulsational properties with the equilibrium temperature of RR Lyrae variables would be of outstanding relevance. Therefore, we have investigated the properties of both temperature - period - amplitude and temperature - amplitude - metallicity relations. The plan of this paper is the following: in Section 2, after showing that temperatures derived from mean (B-V) colour (calculated over the pulsational cycle on the magnitude scale) by using the BCP transformations reproduce equilibrium temperatures within a probable error (corresponding to 50% probability to get the right value within the error bar) of \( \delta \log T_e = \pm 0.003 \), we verify that residuals from both temperature-period-amplitude and temperature-amplitude-metallicity pulsational scales are not correlated with period; in Section 3 we show that temperature-amplitude-metallicity relation is a function of the period-shift from the mean period-amplitude relation valid for the Lub’s sample used for calibrating it; in Section 4 it is shown that the DS96 temperature-period-amplitude relation is not dependent on the period-shift from the mean period-amplitude relation valid for the Lub’s sample. This occurrence allows us to infer that temperature-period-amplitude relation does not depend on both mass and luminosity variations, at least within given ranges of period and period-shift. In Section 5 some relevant points about the DS96 pulsational temperature scale are remarked. In Section 6 some applications of temperature-period-amplitude relation to highly evolved variables are discussed. Finally, in Section 7 the conclusions of the present analysis are summarized.
2 FIELD RR LyRAE VARIABLES: THE CARNEY, STORM & JONES’S SAMPLE.

By using equilibrium temperature obtained through a BW procedure applied to a sample of field RRab Lyrae of CSJ, C98 has calibrated a temperature-amplitude-metallicity relation. As residuals do not show any period dependence (see below), it is possible to conclude that "the period term is not significant in this pulsational temperature relation."

On the other hand, DS96 has shown that a fine agreement exists between his temperature-period-amplitude relation:

\[ \log T_e(P, A_B) = -0.1094 \log P + 0.0134 A_B + 3.770 \]

and the equilibrium temperatures of CSJ sample. In fact it is: \[ \frac{\partial \log T_e}{\partial \log P} = 0.99 \pm 0.09 \] with a probable error \[ p.e. = \pm 0.003 \] and a correlation coefficient \[ r = 0.94 \]. In addition, also in this case, residuals are not correlated with the pulsational period (see below) and this means that "the period term must be significant". In the following we propose to find an explanation to this apparent contrast. To this aim we define:

- \( T_{eq} \): equilibrium temperature of CSJ variables;
- \( T_{eq}(P, A_B) \): pulsational temperature as defined in DS96;
- \( T_{eq}(BCP) \): temperature obtained from the \((B-V)_{0}\) color of CSJ sample by using the BCP transformations, where \((B-V)_{0}\) is the dereddened mean color over the pulsational cycle magnitude weighted;
- \( T_{C98} \): pulsational temperature defined by C98.

First of all, we show in Figure 1 the fine agreement between the \( \log T_{eq}(BCP) \) and DS96 pulsational temperature scale applied to the same sample of variables.

As it is shown in figure 1, these temperature scales are in a one-to-one correspondence, with a \( p.e. = \pm 0.003 \) and \( r = 0.96 \). Thus, since the correlation coefficient is of the order of 1, by the transitive property, it is possible to confirm that also \( \log T_{eq}(BCP) \), calculated from the dereddened \((B-V)_{0}\) color, provides the equilibrium temperature of RRab variables. Besides, by defining residuals between \( \log T_{eq}(BCP) \) and \( \log T_{eq} \) as: \( \delta \log T_{eq}(BCP) = \log T_{eq}(BCP) - \log T_{eq} \), it is possible to show in Figure 2a that residuals between these temperatures scales must be considered as purely accidental.

Now, we define residuals between \( \log T_{eq} \) of CSJ and pulsational temperature scale of DS96 as: \( \delta \log T_{eq} = \log T_{eq} - \log T_{eq}(P, A_B) \). In Figure 2b it is shown that no systematic trend is present neither with respect to period nor with respect to blue amplitude (big filled circles correspond to the two highly evolved stars DX Del and SS Leo). Finally, we define the difference between \( \log T_{eq} \) of CSJ and \( T_{C98} \), obtained by using the Catelan’s formula, as \( \delta \log T_{C98} = \log T_{eq} - \log T_{C98} \). Also in this case no systematic trend is present in Figure 2c with respect to both period and blue amplitude. However, it is worth to note that the highly evolved star SS Leo is in clear disagreement with the Catelan’s pulsational scale by about \( \delta \log T_{e} = 0.02 \).

Thus we have an evidence that both temperature-period-amplitude (DS96) and temperature-amplitude-metallicity (C98) relations do not show a period dependence of residuals. This apparent contrast may be explained only by supposing a dependence between the blue amplitude and the period. This concern may be verified easily. In fact, from the CSJ sample we have the following period-blue amplitude-metallicity mean relation:

\[ \log P = -0.13(\pm 0.03) A_B - 0.10(\pm 0.01) [Fe/H] + -0.220(\pm 0.027) \]

Figure 1. Comparison between \( \log T_e(BCP) \) and \( \log T_e(P, A_B) \) temperature scales. The one-to-one correspondence is shown. (see Section 2 for more details)

with \( r = 0.96 \), where the metallicity term denotes the dependence of the period-amplitude relation from both the appropriate mean mass and mean luminosity for variables of a given metallicity. Then, if we substitute \( \delta \log T_{eq} \) in the eq. (1) given by DS96, we obtain the following temperature-amplitude-metallicity:

\[ \log T_e = 0.027 A_B + 0.011 [Fe/H] + 3.795 \]

This temperature scale is in a good one-to-one correspondence with the equilibrium temperature scale, with a \( p.e. = \pm 0.004 \) and a scale coefficient \( \frac{\partial \log T_e}{\partial \log P} = 1.00 \pm 0.13 \), while residuals are not correlated with the period of variables, being \( \frac{\partial \log T_e}{\partial A_B} = -0.007 \pm 0.016 \).

Before closing this Section, it should be noted that the agreement between different temperature scales \( T_{eq} \), \( T_{eq}(BCP) \) and \( T_{eq}(P, A_B) \) is only a purely empirical result and, in principle, it might depend on the adopted sample of variables. However, in Section 4 we will show that \( T_{eq}(P, A_B) \) relation is largely independent from the sample of variables used to calibrate it. In addition, it is possible to verify that the variables of CSJ sample are within the range of validity of this relation (for this range see Section 4). Thus we are confident on the reliability of our conclusions.

3 FUNDAMENTAL RR LyRAE VARIABLES IN GLOBULAR CLUSTERS.

In the previous Section we have tested the period dependence of \( \log T_e - \log P - A_B \) and \( \log T_e - A_B - [Fe/H] \) relations by using the field RRab variables of CSJ sample. We have shown that both temperature scales are in a good agreement with the equilibrium temperatures values reported in Table 4 of CSJ, without any systematic period dependence of residuals. In addition, we have shown that the \( \log T_e - A_B - [Fe/H] \) relation must be considered as the resultant of both the \( \log T_e = f(P, A_B) \) and the \( \log P = g(A_B, [Fe/H]) \) relations.

Now we will repeat these tests for the RRab variables in the GCs M68 and IC 4499, by adopting \( T_{eq}(BCP) \) as static temperature on the basis of results obtained in the previous Section. For M68, we adopt the photometric data by

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We define \( \delta \log T_e(P, A_B) = \log T_e(BCP) - \log T_e(P, A_B) \) and \( \delta \log T_e(C98) = \log T_e(BCP) - \log T_e(C98) \). In addition, we introduce the quantity: \( \Delta \log P(A_B) = \log P(\text{observed}) - \log P(\text{CSJ}) \), which represents the period shift, at fixed amplitude, from the period-amplitude relation of CSJ sample (eq. 2).

The observed \( \log P - A_B \) relation valid for M68 variables has a slope \( \frac{d \log P}{dA_B} = -0.09 \pm 0.04 \), that is different than the one obtained for the CSJ sample. In Figure 3a it is shown that residuals \( \delta \log T_e(P, A_B) \) are independent from \( \Delta \log P(A_B) \). The same procedure is applied to residuals of C98 temperature scale (Figure 3b). It is clearly shown that \( \frac{d \log T_e(C98)}{d \Delta \log P(A_B)} = -0.15 \pm 0.06 \).

The same effect is observed in IC4499 where the period-amplitude relation is also remarkably different from CSJ sample, that is: \( \frac{d \log P}{dA_B} = -0.09 \pm 0.02 \). Also in this case residuals \( \delta \log T_e(P, A_B) \) results independent from \( \Delta \log P(A_B) \) (Figure 3a) whilst the dependence of \( \delta \log T_e(C98) \) from \( \Delta \log P(A_B) \) is evident. In fact we have: \( \frac{d \log T_e(C98)}{d \Delta \log P(A_B)} = -0.18 \pm 0.04 \), as it is shown in Figure 3b. It is evident that, if the observed period is greater than that predicted from the period-amplitude-metallicity relation valid for CSJ sample, the temperature provided by the Catelan’s formula is overestimated.

Thus we have the evidence that the \( \log T_e(C98) \) is a function of the period-shift from the period-amplitude relation valid for CSJ sample. On the basis of these evidences we can confirm that the \( \log T_e(A_B, [Fe/H]) \) relation is only the particular case of the \( \log T_e(P, A_B) \) relation when a fixed period-amplitude-metallicity relation is adopted. The consequence is that a temperature-amplitude-metallicity relation is strictly valid only for the sample of variables used to calibrate it, or for samples of variables with the same period-amplitude relation.

### 4 ON THE RANGE OF VALIDITY OF TEMPERATURE-PERIOD-AMPLITUDE RELATION.

In the previous Section we have shown that the validity of \( \log T_e - A_B - [Fe/H] \) relation is strictly correlated to the proper period - amplitude relation of variables sample. Now we will test the extent of the validity of the \( \log T_e(P, A_B) \) relation. To this aim it is worth to remember that DS96 relation was calibrated on 67 field RR Lyrae of ab type from the Lub’s (1979) sample (see Castellani & De Santis 1994, hereinafter CD94, and references therein). The mean period-amplitude-metallicity valid for this sample of variables is

\[
\log P = -0.12 A_B - 0.12 [Fe/H] - 0.275
\]

thus we define \( \Delta \log P(A_B) = \log P(\text{observed}) - \log P(4) \), the proper period-shift for each variable (P(4) is the period provided by equation 4). By defining \( \delta \log T_e(P, A_B) = \log T_e(BCP) - \log T_e(P, A_B) \), in Figures 4a and 4b it is shown that \( \delta \log T_e(P, A_B) \) is not correlated with both period and period shift of these variables (big filled circles refer to two RRab of Lub sample, FW Lup and AN Ser, whose periods are within the same range of the other 67 field RRab Lyrae, omitted when DS 96 relation was calibrated).
Thus, for a sample of variables in a globular cluster, the period-amplitude relation may be considered as a mean relation valid for the mean mass and the mean luminosity appropriate for that cluster. Then, for each variable, the period-shift shows the effect due to mass and/or luminosity difference from those mean values.

In the case of field variables, the stars in the sample cover a range in metallicity and, therefore, the appropriate mean relation is metallicity-dependent. However, the metallicity term in the mean relation does not denote a physical dependence on the metal content of these stars, but it mimics the true dependence of the variable mass and luminosity on the metallicity. This concern may be easily verified by using the CSJ star sample. In fact, by adopting log $T_{eq}$ and log $P$ from their Table 4, and using the van Albada & Baker (1971, hereinafter vA&b) equation we can calculate the light-to-mass ratio ($A$ parameter) for the stars in the sample:

$$A = \log L - 0.81 \log M = 1.19 \log P + 4.14 \log T_{eq} - 13.687$$

Then, we have the following calibration:

$$\log P = -0.13(\pm0.02)A_B + 1.08(\pm0.09)A - 2.057(\pm0.021)6$$

with $r = 0.97$, whilst metallicity dependence is dispensed (see Figures 5a and 5b). This confirms that metallicity term hides the physical role of the light-to-mass ratio term in the period-amplitude relation of CSJ sample (eq.2).

Thus, also in this case the period-shift of a variable from eq.(2) is due to mass and/or luminosity difference from those valid for the mean relation 2).

So, by assuming the mean period-amplitude-metallicity relation, we fix the zero point in the period - amplitude plane from which we measure differences in mass and/or luminosity of variables by period-shift method. Thus, the independence of the log $T_{eq}(P, A_B)$ from the period-amplitude relation means that this pulsational temperature scale is not affected by mass and luminosity effects (at least, within the ranges of period and period-shift above - mentioned ).

5 SOME RELEVANT POINTS ABOUT THE DS96 PULSATIONAL TEMPERATURE SCALE.

Before concluding this paper, it is interesting to note that in our analysis no argument was necessary to infer the reliability of the zero-point of the DS96 pulsational temperature scale. Nevertheless, since its accuracy is required to derive the zero-point of the absolute magnitude scale for RR Lyrae variables and, then, to settle the short versus long distance scale issue, it is necessary to define the limit of validity of the adopted value. The temperature-period-blue amplitude relation calibrated from Lub’s data by CD94 was obtained by using temperatures derived from $(B-V)_0$ by Sandage (1990a), corrected for a residual reddening effect by using the color-effective temperature relation provided by Butler et al. (1978). Previously, Caputo & De Santis (1992) had obtained, from the same sample, a $(B-V)_0 = A_B - [Fe/H]$ relation by using the reddening and metallicity estimations given by Lub (1979). DS96 found that the equilibrium temperature scale, as defined in the CSJ paper by means of Stefan’s law, is the same function of both blue amplitude and period as given by CD94 (the dependence on the metallicity being negligible), provided that the zero-point is smaller by
δ log Te = 0.011. Thus, in that paper the zero-point was corrected consequently. Finally, in DC99 this pulsational temperature scale was tested in the \((B-V)_0 - A_B\) plane by adopting four independent sets of transformations (see their Figures 2, 3 and 4); Buser & Kurucz (1978, hereinafter BK78), Kurucz (1992, hereinafter K92), BCP - improperly quoted as Castelli, Gratton & Kurucz (1997a,b) by DC99, and Yale semiempirical transformations (Green 1988, hereinafter Y88). The check was performed by using the observational data for RRab Lyrae variables in three globular clusters: M3, M15 and M68. The following metallicities were used \([Fe/H]_M3 = -1.3\) and \([Fe/H]_{M15,M68} = -2\) (it is worth to note that an error in the metallicity of 0.1 dex gives a difference in the calculated temperature by transformations less than 0.001 in log \(T_e\)). A good agreement was found with BK78, BCP and Y88 transformations. Thus we conclude that the use of DS96 pulsational temperature scale corresponds to the combined use of one of the transformations above cited and the Lub’s reddening scale, with the evident advantage that DS96 relation is “reddening and metallicity free” (in the sense that it does not require the knowledge of metallicity and reddening when it is applied to variables of a given cluster). Moreover, periods and amplitudes are determined with a greater accuracy than colours. It is worth noticing that, in the case of a significant error in the zero-points of the reddening scale of Lub or of the above mentioned transformations, it would be necessary to change the zero-point of the DS96 pulsational temperature scale.

### 6 THE CASE OF HIGHLY EVOLVED VARIABLES.

In Section 4 we have defined the period and the period - shift (from Lub’s period - amplitude relation) and the ranges of validity of the log \(T_e(P, A_B)\) relation; now we test our scenario in the following three cases of highly evolved stars:

a) The CSJ sample of field RRab Lyrae includes two highly evolved stars: DX Del and SS Leo. BW results for both these stars show an high luminosity with respect to the luminosity predicted by the Mbol - [Fe/H] relation of CSJ92. Their pulsational period and period - shift are:

DX Del: \(log P = -0.325\) and \(\Delta log P(A_B) = +0.04\)

SS Leo: \(log P = -0.203\) and \(\Delta log P(A_B) = +0.07\)

Both period and period-shift of these stars are in the range of validity of DS96 log \(T_e(P, A_B)\) relation and their residuals \(\Delta \log T_e(P, A_B)\) are within the statistical fluctuations (see Figure 2b);

b) the variables I-42 and I-100 in M3 are, notoriously, two highly evolved stars. Their period and period-shift are respectively:

I-42: \(log P = -0.038\) and \(\Delta log P(A_B) = +0.10\)

I-100: \(log P = -0.001\) and \(\Delta log P(A_B) = +0.14\)

In this case periods are out of the range of validity of the log \(T_e(P, A_B)\) relation and their residuals \(\Delta \log T_e(P, A_B)\) show a systematic trend with respect to the general behaviour of M3 RRab variables in the \(\delta \log T_e(P, A_B) - \Delta \log P(A_B)\) plane (Figure 4c, the observational data are from Sandage 1990b);

c) finally, we consider the case of V9 in 47 Tucanae. In this case we have \(log P = -0.133\) and \(\Delta log P(A_B) = +0.22\) and these parameters are, marginally, within the ranges of validity of the log \(T_e(P, A_B)\) relation. Storm et al 1994 (S94), by a BW analysis, found for this variable the following parameters: \(\log Teq = 3.833\), \(M_V = 0.32\), \(M_{bol} = 0.32\) (case A : their preferred case). On this basis, by using the vA&B equation, one derives the following value for the mass: \(M = 0.46M_\odot\). This pulsational mass is too low, running against a reasonable agreement with the theoretical evolutionary scenario. In fact, for the metallicity of 47 Tuc \([Fe/H] = -0.7\) RRab masses vary in a range of few hundredths around 0.6\(M_\odot\). However, we can propose a different consistent scenario. In the work by DS96, by using the pulsational temperature scale log \(T_e(P, A_B)\), the following pulsational relations for A parameter and \(M_V\) have been derived:

\[A = \log L - 0.81 \log M = 0.737 \log P + 0.055 A_B + 1.930\]

\[M_V = 1.842 \log P - 0.137 A_B + 0.31\]

Observed pulsational parameters are \(P = -0.1326\) and \(A_B = 1.35\). Thus we obtain:

\(M_V = 0.37\), \(\log T_e(P, A_B) = 3.803\), \(log L = 1.75\) (by assuming \(M_{bol} = 4.75\) ) and \(A = 1.907\). The resulting mass is \(M = 0.64M_\odot\), a value consistent, within the statistical accuracy, with the evolutionary predictions. Thus in this scenario, V9 luminosity is consistent with BW result, but its BW temperature is unacceptable. However, as period and period-shift of V9 are only marginally within the range of validity of log \(T_e(P, A_B)\) relation, this interpretation is actually only matter of opinion.

### 7 SUMMARY

Now, we summarize the conclusions of the present analysis. In Section 2 we have shown that \(T_e(BCP)\), calculated from the dereddened (B-V) colour by BCP transformations, provides equilibrium temperature scale of RRab variables. In Section 3, by assuming \(T_e(BCP)\) as static temperature, we have shown that Catelan’s pulsational temperature scale is a function of variables period-shift from the period - amplitude relation valid for the CSJ variables sample. The conclusion was that a temperature-amplitude-metallicity relation is strictly valid, only for the sample of variables used to calibrate it, or for samples of variables with the same period-amplitude relation. In Section 4 we have shown that temperature-period-amplitude relation of DS96 is independent from the period-amplitude relation proper of variables sample. The consequence is that this pulsational temperature scale is not affected by mass and luminosity effects, within the following period and period-shift (from Lub’s period-amplitude relation) ranges: \(-0.45 < \log P < -0.13\) and \(-0.10 < \Delta \log P(A_B) < +0.20\).
Acknowledgements: Many thanks are due to Dr. Mar- cio Catelan for his interesting comments and constructive criticism during the preparation of this text. Likewise, the author is grateful to Dr. Bruce W. Carney for suggestions and comfortable discussion after the writing of this paper. Special thanks go to Dr. Santi Cassisi and Dr. Anna Piersi- moni for many helpful suggestions and for the help provided in the preparation of an early draft of this paper. The author is also grateful to Dr. Martin Stift for his encouraging comments after the reading of this text. Finally, we wish to thank the referee, Dr. Raffaele Gratton, for his pertinent comments and constructive remarks and suggestions during the refereeing process.

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a) $\log T_{\text{BCP}} - \log T_{\text{eq}}$

b) $\log T_{\text{eq}} - \log T_e\ (P, A_B)$

c) $\log T_{\text{eq}} - \log T_e\ (C98)$
b) M68

IC 4499

a) M68

IC 4499

ΔlogP(A_B)
