Collective excitation of trapped degenerate Fermi gases

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We show that the slow driving of a focused laser beam through the cloud of trapped cold fermions allows for the collective excitation in the system. The method, proposed originally by us for bosons, seems to be quite feasible experimentally — it requires only a proper change in time of the potential in atomic traps, as realized in laboratories already.

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The experimental realization of the Bose-Einstein condensate (BEC) \[ \text{[1]} \] in a trapped Bose gas has triggered considerable interest in the field of cold degenerate atomic gases \[ \text{[2,3]} \]. The relatively weak interaction between atoms allows for precise experimental manipulation of an atomic gas. In particular collective excitations of the BEC such as vortices and solitons \[ \text{[4–7]} \] (in the mean field language – see also \[ \text{[8]} \]) have been studied in details both theoretically and experimentally. Nowadays, it is also possible to cool down Fermi gases to a regime where effects of the quantum statistics become noticeable \[ \text{[9–11]} \]. Therefore, it would be interesting to contrast their collective behaviors \[ \text{[12–14]} \] with those of the BEC.

Very recently we have proposed a new simple scheme that allows for creation of both solitons and vortices in a BEC using an appropriate time dependent modification (by an additional tightly focused laser beam) of the trapping potential \[ \text{[15,16]} \]. The aim of the present work is to discuss possible collective excitations of trapped fermions and to show that the very same scheme as for bosons may be utilized successfully to create excitation in a cold Fermi gas. A numerical attempt to create such excitations using the phase imprinting method \[ \text{[17]} \], has been recently made \[ \text{[18,19]} \]. It is not clear, however, which states are prohibited for spin-polarized identical fermions due to the Pauli exclusion principle. Consequently, it is natural to consider first a noninteracting particle model as the atoms may interact only through vanishingly small \( p \)-wave collisions. Our excitation method originally proposed for noninteracting bosons remains valid even for quite strong interaction between particles. We expect thus that consideration of the noninteracting Fermi system as a first approximation is perfectly legitimate.

It is well known that BCS transition may occur in the quantum cold gas of fermions at a very low temperature \( T_c \) \[ \text{[18,19]} \]. While in the noninteracting particle model this effect is absent, we shall show that creation of collective excitations may occur both for \( T = 0 \) (neglecting BCS pairing) and for temperatures that are greater than \( T_c \) but sufficiently low that the effects of quantum statistics are noticeable.

The paper is organized as follows. Firstly we specify what we consider as a collective excitation in the cold fermionic gas on a one-dimensional (1D) example. Secondly, we present the basic idea of the excitation scheme still for 1D noninteracting particle model at zero temperature limit. Finally we move to the 3D model for an experimentally realistic temperature indicating that signatures of the collective excitations can be observed in a laboratory.

A gas of bosons being in a product state of ground, single particle (e.g. mean field) states is a standard approximate description of the BEC. It is natural then to consider as a collective excitation a situation in which all particles are simultaneously transferred to some excited state of the mean field effective potential. This results in the so called “dark soliton” \[ \text{[20]} \]. Since all particles are both initially and finally in the same state, the mean field description of the soliton creation requires an efficient mechanism for transfer the population from the ground to the first excited state.

The situation is quite different for Fermi system for which each state can be at most single occupied (we assume a spin-polarized identical fermions as realized experimentally). Consider a 1D Fermi gas at \( T = 0 \) prepared in an ideal collective ground state of the harmonic trap. For \( N \) identical fermions this is equivalent to assuming that all oscillator eigenstates, \( \psi_n(x) \), from \( n=0 \) (single particle ground state) to \( n=N-1 \) are occupied with other levels being necessarily empty. The wave function of the \( N \) fermion system may be represented then by the Slater determinant

\[
\Psi(x_1, x_2, \ldots, x_N) = \frac{1}{\sqrt{N!}} \begin{vmatrix} \psi_0(x_1) & \psi_0(x_2) & \cdots & \psi_0(x_N) \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{N-1}(x_1) & \psi_{N-1}(x_2) & \cdots & \psi_{N-1}(x_N) \end{vmatrix}.
\]

The corresponding single particle reduced probability density reads

\[
\varrho(x) = \int dx_2 \ldots dx_N |\Psi(x, x_2, \ldots, x_N)|^2 = \frac{1}{N} \sum_{i=0}^{N-1} |\psi_i(x)|^2.
\]
which (employing Christoffel-Darboux formula \[2\]) may be reduced to
\[
\rho(x) = \frac{\exp(-x^2/2)}{\sqrt{\pi N!}2^{N-1}} [N!H_{N-1}(x) - (N-1)H_N(x)H_{N-2}(x)], \tag{2}
\]
where \(H_N(x)\) denote standard Hermite polynomials. This density is shown in Fig. 1a.

![FIG. 1. Single particle reduced probability density for \(N = 300\) fermions in the 1D model, at temperature \(T = 0\), before [panel (a)] -ground state] and after [panel (b) -collectively excited state] the sweeping of the perturbation. For \(x\) we use the harmonic oscillator unit of length, i.e. \(\sqrt{\hbar/m\omega}\).

Now since each fermion occupies a different state, there is some ambiguity in defining a collective excitation. We define it by requiring that each fermion undergoes a single excitation. Since they are indistinguishable, the final wave function is then the Slater determinant involving the oscillator functions from \(n = 1\) to \(n = N\), the corresponding single particle density is shown in Fig. 1b. This is a most complete analogy to the collective excitation of bosons where all \(N\) particles gain a single excitation too. The difference is that bosons enjoy the same initial (and final) state while the fermions need to be excited from necessarily different states. Importantly, all bosons pass from an even to an odd state in an ideal case, leading to the creation of a “dark soliton”, with a node in the center. The collective excitation of a fermionic gas leaves the ground state empty that results in a dip in the single particle probability density (Fig. 1b). In the 1D model the relative depth of the dip scales with number of fermions as \(1/\sqrt{N}\).

How to realize such a final “collective” state? Surprisingly we show below that a patient way of exciting fermions one by one, is quite simple and feasible. Moreover, it turns out that in a practical implementation it seems to be an almost trivial extension of the scheme we proposed for an efficient excitation of a Bose gas!

As in the original suggestion for the bosonic case \[13\] we propose to modify the harmonic trapping potential by an additional tightly focused laser beam which we sweep through the trap. Provided that the frequency of this laser beam is sufficiently detuned from an internal atomic transition (still being close to the resonance so that two atomic levels can be considered only) the upper level may be eliminated adiabatically from the analysis (see e.g. \[21\]). Then the effective potential for the atomic motion, in the 1D model, reads
\[
V(x) = \frac{x^2}{2} + U_0(x_0) \exp\left(-\frac{(x-x_0)^2}{2\sigma^2}\right), \tag{3}
\]
where \(U_0(x_0)\) is proportional to the laser intensity, \(x_0 = x_0(t)\) is a time-dependent position of the center of the laser beam, while \(\sigma\) is directly related to the cross-section of the (gaussian) laser beam. We assume the trapping harmonic oscillator units, i.e. \(\omega t\) for time and \(\hbar/m\omega\) for length, where \(m\) stands for an atomic mass.

Assume we prepare the fermionic sample in the state \([\Psi]\) in a pure harmonic trap. Next we turn on the laser, initially focused at the left edge of the trap \((x_0(0) \ll 0)\) and move the focus across the trap towards the center realizing the potential \([\mathcal{P}]\). For all numerical simulations presented below we take \(\sigma = 0.2\) and
\[
U_0(x_0) = (a - bx_0) \arctan(x_0), \tag{4}
\]
with \(a, b > 0\). In the bosonic case we took \(U_0(x_0)\) proportional to \(\arctan(x_0)\) \[14\]. In the present case we have to add an additional prefactor ensuring that the perturbation is not negligible in comparison with the harmonic part (which is large far from the center of the trap). We have chosen the simplest linear form of such a prefactor taking \(a = 18, b = 0.5\) in the numerical examples, the results are not very sensitive to precise values of \(a\) and \(b\). During the excitation process \(x_0\) changes linearly with time according to \(x_0(t) = x_0(0) + 0.02t\).

To understand the effect produced by sweeping of the laser induced potential across the trap it is sufficient to consider the change of single particle energy levels as a function of the center of the laser beam \(x_0\). The corresponding level dynamics is depicted in Fig. 2. Observe that the levels undergo a series of orderly arranged very narrow avoided crossings (not only between low lying levels but also between the highest ones). Assume \(x_0\) is changed sufficiently slowly so as to follow the energy levels adiabatically except in the immediate vicinities of avoided crossings. Here, since the avoided crossings are extremely narrow we assume that they are passed diabatically. Under these premises the excitation scenario works as follows. We start with the laser beam situated at negative value of \(x_0\) (with large \(|x_0|\)) and increase \(x_0\) up to \(x_0 = 0\) where the sweeping process ends (when the center of the laser beam reaches the center of the harmonic trap its intensity is reduced to zero). With increasing \(x_0\) we pass diabatically the consecutive avoided crossings encountered on the way. The important avoided
crossings occur when one of the levels is occupied. Assuming that the highest occupied level is \( N - 1 \), the diabatic passage via the avoided crossing of the \( N \) with the \( N - 1 \) level populates the \( N \) level leaving practically empty the \( N - 1 \) state (the Landau-Zener effect). The next crossing, occurring at slightly larger \( x_0 \) value populates now the empty \( N - 1 \) level from \( N - 2 \), another one populates \( N - 2 \) at the expense of \( N - 3 \) and so on. The last avoided crossing assumed to be passed diabatically (all \( N \) that, provided all \( N \) populates \( n = 1 \) state and leaves empty \( n = 0 \). It is clear that, provided all \( N \) avoided crossings are passed diabatically (all \( N \) particles are successively excited), we realize the desired, described above final Slater determinant in-

![Figure 2](image.png)

**FIG. 2.** Energy levels of the potential (3) as a function of \( x_0 \). Panel (a) shows the behavior of low lying energy levels while in the panel (b) the energy levels around \( n = 180 \) are presented. Note a series of very narrow avoided crossings between the neighboring energy levels (for clarity consecutive levels are drawn using solid and dashed lines). The inset in panel (a) shows a vastly enlarged avoided crossing between the ground and the first excited state around \( x_0 \approx -6.2 \). In the figure we use the harmonic oscillator units, i.e. \( \hbar \omega \) for energy and \( \sqrt{\hbar/m\omega} \) for length.

We have shown [15,16] that a similar laser sweeping scheme works very efficiently in the bosonic case. There, however, a single avoided crossing (between the ground and the first excited state) had to be passed diabatically by \( N \) bosons. In the present situation we have to cross diabatically \( N \) different avoided crossings.

Apart from the experimental approach which is obviously beyond the scope of the present letter, the other way to test the proposed scheme is the numerical integration of the time-dependent Schrödinger equation. Since we deal with \( N \) noninteracting particles the problem of integration of \( N \) dimensional Schrödinger equation reduces to the problem of integration of \( N \) independent single particle equations for time evolution of the wave functions \( \psi \), see [4]. Such an approach, with the parameters assumed above [i.e. \( \sigma, U_0(x_0) \) and \( x_0(t) \)] yields to transfer of particle from \( n \) level to \( n + 1 \) level with the probability \( p \) higher than 99.95 \%, as we have checked for \( n \leq 180 \). This indicates that the proposed mechanism of excitation is extremely efficient and to the end of this paper we will show analytical results assuming \( p = 1 \).

So far we have analyzed the system at the zero temperature limit. The lowest attainable temperature of a trapped Fermi gas in the recent experiments corresponded to 0.2\( T_F \) [10,11], where \( T_F \) is the Fermi temperature. To consider the finite temperature effect on the excitation process we switch to the 3D model of noninteracting fermions. We assume a cigar-shaped harmonic trap with an additional local gaussian well created by the laser beam

\[
V(x,y,z) = \frac{x^2}{2} + \frac{\omega^2}{2\omega} (y^2 + z^2) + U_0(x_0) \exp \left( -\frac{(x-x_0)^2}{2\sigma^2} \right),
\]

where we have chosen \( \omega/\omega = 10 \) as an example. The other parameters of the potential are exactly the same as in the 1D case. The laser beam is directed along \( z \) axis. Its crosssection in the \( xy \) plane is assumed to be tightly focused in the \( x \) direction having at the same time large waist along \( y \). In effect the gaussian character in the potential is realized in the \( x \) direction only, while in the \( y \) direction the laser intensity remains constant on the size of the Fermi cloud. Changing the trapping potential by sweeping of the laser beam allows for excitations in the \( x \) degree of freedom of the system. The excitation in this degree is exactly the same as in the 1D case considered previously because the 3D model of noninteracting particle is separable.

Assume we have \( N \) fermions in the harmonic trap distributed among the energy levels according to the Fermi-Dirac statistics with \( T > 0 \). The corresponding single particle probability density (integrated over \( y \) and \( z \) coordinates) reads

\[
\rho(x) = \frac{1}{N} \sum_{n_x} g_{n_x} |\psi_{n_x}(x)|^2,
\]

where we have shown [15,16]
where the sum runs over occupied $\psi_{n_x}$ states only. The degeneracy factor $g_{n_x}$ takes into account that several fermions may share the same $\psi_{n_x}$ state with different other quantum numbers (basically $\sum n_x g_{n_x} = N$). Next we perform the potential sweeping that realizes excitation in the $x$ degree of freedom, i.e. each occupied $\psi_{n_x}$ state goes to $\psi_{n_x+1}$ with a probability practically equal to unity. In Fig. 3 we show the results of the excitation process for $N = 40000$ fermions in the temperature $T = 0.3T_F$. For example that corresponds to $T = 0.2 \mu K$ for $\omega = 2\pi \cdot 50$ Hz being within the range of current experiments [9–11]. The results presented in Fig. 3 were obtained by averaging over 100 realizations of $N$ fermions in the temperature $T = 0.3T_F$. A successful excitation results in a central dip clearly visible in the figure. Note also that in the present case we can employ much more particles than in the 1D model and the dip in the probability density possesses still a considerable depth. That is due to the simple fact that before the potential sweeping the $\psi_{n_x}$ state with $n_x = 0$ contributes many $g_0$ times to the density $\rho(x)$. Consequently, after the excitation, the lack of the $n_x = 0$ state is much more significant than in the 1D case where such a state could be occupied once only.

As a difficulty, however, one might realize that the width of the dip is of the order of a few percent of the whole distribution of atoms in a harmonic trap. This might preclude its direct observation. To overcome this difficulty one may, for the detection purposes, supplement the proposed scheme by the “free fall expansion”. Assume that after the sweep by the laser beam and the corresponding creation of the collective excitation we turn off the trap completely. The group of fermions will fall in the gravitational field, the cloud will expand simultaneously. It is important to realize that the corresponding single particle density also expands roughly preserving its shape (and in particular the central dip structure). The single particle density after the free expansion lasting $2\pi / \omega$ (i.e. $20$ ms for $\omega = 2\pi \cdot 50$ Hz) is shown in the inset of Fig. 3. Observe that the width of the dip can quickly reach the size comparable to the initial size of the whole cloud.

To summarize, we have presented a simple method that allows for creation of collective excitation in a trapped Fermi gas in analogy to a soliton-like state of a Bose-Einstein condensate. The results shown in this work are based on the noninteracting particle model. We believe, that in analogy to the bosonic case, an interaction between fermions (much weaker than in the bosonic case) will not affect the efficiency of the proposed scheme.

The method can be also applied to create collective excitation in the so-called Tonks gases (see, e.g., [23] and references therein). Indeed, impenetrable Bosons trapped in a quasi 1D potential can be modeled by a noninteracting Fermi gas. Hence, the excitation scheme presented here becomes directly applicable to such a Bose system.

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FIG. 3. Single particle reduced probability density after the excitation process for $N = 40000$ fermions. The figure represents average over 100 samples corresponding to the Fermi-Dirac distribution with $T = 0.3T_F$. The inset shows the same density after a free expansion lasting one period of the harmonic trap $(2\pi / \omega)$. In the figure we use the harmonic oscillator unit of length, i.e. $\sqrt{\hbar / m \omega}$.

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