Rigorous Definition of Quantum Field Operators in Noncommutative Quantum Field Theory∗

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Abstract

The space, on which quantum field operators are given, is constructed in any theory, in which the usual product between test functions is substituted by the $\star$-product (the Moyal-type product). The important example of such a theory is noncommutative quantum field theory (NC QFT). This construction is the key point in the derivation of the Wightman reconstruction theorem.

Key words: Noncommutative quantum field theory, Axiomatic approach, Wightman reconstruction theorem.

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1 Introduction

Quantum field theory (QFT) as a mathematically consistent theory was formulated in the framework of the axiomatic approach in the works of Wightman, Jost, Bogoliubov, Haag and others ([1] - [4]).

Within the framework of this theory on the basis of most general principles such as Poincaré invariance, local commutativity and spectrality, a number of fundamental physical results, for example, the CPT-theorem and the spin-statistics theorem were proven [1] - [3].

Noncommutative quantum field theory (NC QFT) being one of the generalizations of standard QFT has been intensively developed during the past years (for reviews, see [5, 6]). The idea of such a generalization of QFT ascends to Heisenberg and was initially developed in Snyder’s work [7]. The present development in this direction is connected with the construction of noncommutative geometry [8] and new physical arguments in favor of such a generalization of QFT [9]. Essential interest in NC QFT is also due to the fact that in some cases it is a low-energy limit of string theory [10].

The simplest and at the same time the most studied version of noncommutative theory is based on the following Heisenberg-like commutation relations between coordinates:

$$[\hat{x}_\mu, \hat{x}_\nu] = i \theta_{\mu\nu}, \quad (1)$$

where $\theta_{\mu\nu}$ is a constant antisymmetric matrix.

The relation (1) breaks the Lorentz invariance of the theory, while the symmetry under the $SO(1, 1) \otimes SO(2)$ subgroup of the Lorentz group survives [11].

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NC QFT can be formulated also in commutative space by formally replacing the usual product of operators by the star (Moyal-type) product:

$$\varphi(x) \star \varphi(x) = \exp\left(\frac{i}{2} \theta_{\mu\nu} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial y^\nu}\right) \varphi(x)\varphi(y)|_{x=y}. \quad (2)$$

This product of operators can be extended to the corresponding product of operators in different points:

$$\varphi(x_1) \star \cdots \star \varphi(x_n) = \prod_{a<b \leq n} \exp\left(\frac{i}{2} \theta_{\mu\nu} \frac{\partial}{\partial x^\mu_a} \frac{\partial}{\partial x^\nu_b}\right) \varphi(x_1) \cdots \varphi(x_n). \quad (3)$$

Wightman approach in NC QFT was formulated in [12] and [13]. For scalar fields the CPT theorem and the spin-statistics theorem were proven in the case $\theta_{\mu\nu} = 0$. In [14] - [16] it was shown that the main classical axiomatic results are valid or have analog in NC QFT at least if time commutes with spatial variables, i.e. $\theta_{0\nu} = 0$.

In [12] it was proposed that Wightman functions in the noncommutative case can be written down in the standard form

$$W(x_1, \ldots, x_n) = \langle \Psi_0, \varphi(x_1) \cdots \varphi(x_n) \Psi_0 \rangle, \quad (4)$$

where $\Psi_0$ is the vacuum state. However, unlike the commutative case, these Wightman functions are only $SO(1,1) \otimes SO(2)$ invariant. In fact in [12] the CPT theorem has been proven in the commutative theory, where Lorentz invariance is broken up to $SO(1,1) \otimes SO(2)$ symmetry, as in the noncommutative theory it is necessary to use the $\star$-product at least in coinciding points.

In [13] it was proposed that in the noncommutative case the usual product of operators in the Wightman functions has to be replaced by the Moyal-type product (3) both in coinciding and different points. Such a product of operators reflects the natural physical assumption, that noncommutativity should change the product of operators not only in coinciding points, but also in different ones.

Actually it seems very natural to use in different points not the $\star$-product itself, but the following generalization of the $\star$-product in coinciding points:

$$\varphi(x) \star \varphi(x) \rightarrow \xi(x-y) \varphi(x) \star \varphi(y), \quad \xi(0) = 1, \quad (5)$$

where $\xi(x-y)$ is some function rapidly falling if $|x-y|^2 \gg \theta$. For example, $\xi(x-y)$ can be arbitrary continuous function, satisfying the inequality:

$$|\xi(x-y)| \leq C \exp\left(-\frac{|x-y|^2}{\theta}\right), \quad (6)$$

where $C$ is a some positive number,

$$\theta \equiv \max_{\mu,\nu} |\theta_{\mu\nu}|, \quad |x-y| \equiv \max_{i=0,1,2,3} |x_i - y_i|. \quad (7)$$

In [13] it was shown that in the derivation of axiomatic results, the concrete type of product of operators in various points is insignificant.

The Wightman functions can be generally written down as follows [13]:

$$W(x_1, \ldots, x_n) = \langle \Psi_0, \varphi(x_1) \tilde{\star} \cdots \tilde{\star} \varphi(x_n) \Psi_0 \rangle. \quad (7)$$

The meaning of $\tilde{\star}$ depends on the considered case. In particular,

$$\varphi(x)\tilde{\star}\varphi(y) = \varphi(x) \star \varphi(y), \quad (8)$$

$$\varphi(x)\tilde{\star}\varphi(y) = \xi(x-y) \varphi(x) \star \varphi(y), \quad (9)$$
\[ \varphi(x) \hat{\star} \varphi(y) = \varphi(x) \varphi(y), \quad x \neq y; \]
\[ \varphi(x) \hat{\star} \varphi(x) = \varphi(x) \star \varphi(x) \]  
(10)

It will be shown below that eqs. (10) have to be substituted by their regularization.

Let us stress that actually the field operator given at a point cannot be a well-defined operator \[3\]. Well-defined operator is a smoothed operator:

\[ \varphi_f \equiv \int \varphi(x) f(x) \, dx, \]  
(11)

where \( f(x) \) is a test function. Correspondingly Wightman functions are generalized functions in the space of test functions. In QFT the standard assumption is that all \( f(x) \) are test functions of tempered distributions. On the contrary, in the NC QFT the corresponding generalized functions can not be tempered distributions as the \( \star \)-product contains infinite number of derivatives. As is well known (see, for example, \[1\]) that there could be only a finite number of derivatives in any tempered distribution.

The formal expression \(7\) actually means that the scalar product of the vectors \( \Phi_k = \varphi_{f_k} \cdots \varphi_{f_1} \Psi_0 \) and \( \Psi_n = \varphi_{f_{k+1}} \cdots \varphi_{f_n} \Psi_0 \) is the following:

\[ \langle \Phi_k, \Psi_n \rangle = \int W(x_1, \ldots, x_n) f_1(x_1) \cdots f_k(x_k) \hat{\star} f_{k+1}(x_{k+1}) \cdots \hat{\star} f_n(x_n) \]
\[ d x_1 \cdots d x_n, \quad W(x_1, \ldots, x_n) = \langle \Psi_0, \varphi(x_1) \cdots \varphi(x_n) \Psi_0 \rangle. \]  
(12)

In paper \[18\] it was shown that the series

\[ f(x) \star f(y) = \exp \left( \frac{i}{2} \gamma_{\mu \nu} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial y^\nu} \right) f(x) f(y) \]  
(13)

converges if \( f(x) \in S^\beta, \quad \beta < 1/2, \quad S^\beta \) is a Gel’fand-Shilov space \[17\]. The similar result was obtained also in \[19\].

## 2 Rigorous Definition of Quantum Field Operators in NC QFT

Let us define rigorously quantum field operator \( \varphi_f \). To this end we construct a closed and nondegenerate space \( J \) such that operators \( \varphi_f \) be well defined on dense domain of \( J \).

As in the commutative case every vector of \( J \) can be approximated with arbitrary accuracy by the vectors of the type

\[ \varphi_{f_1} \cdots \varphi_{f_n} \Psi_0, \]

where \( \Psi_0 \) is a vacuum vector. In other words the vacuum vector \( \Psi_0 \) is cyclic.

The difference of noncommutative case from the commutative one is that the action of the operator \( \varphi_f \) is defined by the \( \star \)-product.

Construction of space \( J \) we begin with the introduction of set \( M \) of breaking sequences of the following kind

\[ g = \{ g_0, g_1, \ldots, g_k \}, \]  
(14)

where \( g_0 \in \mathbb{C}, \quad g_1 = g_1^1 (x_1), \quad x_1 \in \mathbb{R}^4, \quad g_i = g_i^j (x_1) \hat{\star} \cdots \hat{\star} g_i^j (x_i), \quad x_j \in \mathbb{R}^4, 1 \leq j \leq i, \) particularly \( \hat{\star} \) is determined by the formulae \(8\) - \(10\), \( k \) depends on \( g \). Addition and multiplication by complex numbers of the above mentioned sequences are defined component by component. It is obvious, that \( C \, g \in M, \) if \( g \in M \).

The every possible finite sums of the sequences belonging \( M \) form space \( J'_0 \) on which action of the operator \( \varphi_f, \quad f = f(x), \quad x \in \mathbb{R}^4 \) will be determined.
Certainly, to determine the $\ast$-product, functions $g_k$ should have sufficient smoothness. As stated above the $\ast$-product is well-defined, if $g_k$ belongs to one of Gel’fand-Shilov spaces $S^\beta$, $\beta < 1/2$ [17]. Moreover, $f \ast g_k \in S^\beta$ with the same $\beta$ [18]. It gives us the possibility to determine Wightman functions as generalized functions corresponding to the above mentioned Gel’fand-Shilov space.

The operator $\varphi_f$ is defined as follows

$$\varphi_f g = \{f g_0, f \ast g_1, \ldots, f \ast g_k\},$$

(15)

where $f \ast g_i = f(x) \ast g_i^1(x_1) \ast \cdots \ast g_i^i(x_i)$

We assume that

$$f \ast g_i = f(x) \ast g_i^1 + f(x) \ast h_i.$$  

(16)

If $\ast$ is defined by one of the equations (8) - (10), then it is evident that eq. (16) is fulfilled. In accordance with eq. (16) any vector of space $J'_0$ is a sum of the vectors belonging to set $M$, the operator $\varphi_f$ is determined on any vector of space $J'_0$ and $\varphi_f \Phi \in J'_0, \forall \Phi \in J'_0$.

The scalar product of vectors in $J'_0$ we define with the help of Wightman functions $W(x_1, \ldots, x_n) \equiv \langle \Psi_0, \varphi(x_1) \cdots \varphi(x_n) \Psi_0 \rangle$. We consider firstly a chain of vectors: vacuum vector $\Psi_0 = \{1, 0, \ldots, 0\}$, $\Phi_1 = \varphi_{f_1} \Psi_0$, $\ldots$, $\Phi_k = \varphi_{f_k} \cdots \varphi_{f_1} \Psi_0$, $f_i = f_i(x_i)$, $x_i \in \mathbb{R}^4$.

According to eq. (15), $\Phi_k = \{0, \ldots, f_k \ast \cdots \ast f_1, 0 \ldots 0\}$.

Similarly, $\Psi_n = \varphi_{f_k+1} \cdots \varphi_{f_1} \Psi_0 = \{0, \ldots, f_k+1 \ast \cdots \ast f_n, 0 \ldots 0\}$. It is obvious, that $J'_0$ is a span of the vectors of such a type.

In what follows we consider the case $\ast = \ast$. Let us point out that the definition of $\ast$ by formulas (9) - (11) leads to the final results same with this case.

The scalar product of vectors $\Phi_k$ and $\Psi_n$ is

$$\langle \Phi_k, \Psi_n \rangle = \langle \Psi_0, \varphi_{f_1} \cdots \varphi_{f_k+1} \cdots \varphi_{f_n} \Psi_0 \rangle =$$

$$\int W(x_1, \ldots, x_n) \bar{f}_1(x_1) \ast \cdots \ast \bar{f}_k(x_k) \ast f_{k+1}(x_{k+1}) \ast \cdots \ast f_n(x_n) d x_1 \ldots d x_n.$$  

(17)

The adjointed operator $\varphi_f^*$ is defined by the standard formula. If operator $\varphi_f$ is Hermitian then $\varphi_f^* = \varphi_f$. In this paper we consider only Hermitian (real) operators, but the construction can be easily extended to complex fields.

Let us show now that a condition

$$\langle \Phi_k, \Psi_n \rangle = \overline{\langle \Psi_n, \Phi_k \rangle}$$

(18)

is fulfilled, if (as well as in commutative case [11]),

$$W(x_1, \ldots, x_n) = \overline{W(x_n, \ldots, x_1)}.$$  

(19)

Really, in accordance with (17)

$$\langle \Psi_n, \Phi_k \rangle = \langle \Psi_0, \varphi_{f_1} \cdots \varphi_{f_{k+1}} \varphi_{f_k} \cdots \varphi_{f_1} \Psi_0 \rangle =$$

$$\int W(x_n, \ldots, x_1) \bar{f}_n(x_n) \ast \cdots \ast \bar{f}_{k+1}(x_{k+1}) \ast f_k(x_k) \ast \cdots \ast f_1(x_1) d x_1 \ldots d x_n.$$  

(20)

The required condition is satisfied, since owing to antisymmetry of $\theta^\mu

\bar{f}_n(x_n) \ast f_{n-1}(x_{n-1}) \ast \cdots \ast f_1(x_1) = f_1(x_1) \ast \cdots \ast f_n(x_n)$.  

According to the formula (17), the scalar product of any vectors $g \in M$ and $h \in M$ is

$$\langle g, h \rangle = \sum_{k=0}^{\infty} \sum_{m=0}^{\infty} \int d x_1 \ldots d x_k d y_1 \ldots d y_m.$$  

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proceed from the function defined by eq. (10) to its regularization: we have to put one in different points then given construction can also be fulfilled, only in the different points in the general case as well.

\[ J(x, y) = \langle x^+, y^+ \rangle + \langle x^-, y^- \rangle. \]  

(23)

It is easy to see that we can introduce in \( J \) the positive-definite scalar product \((\cdot, \cdot)\), namely

\[ (x, y) = \langle x^+, y^+ \rangle - \langle x^-, y^- \rangle. \]  

(24)

We can introduce norm using this product \( \|x\| = (x, x)^{1/2} \).

Evidently, \( \|x^+\| = (x^+, x^+)^{1/2}, \|x^-\| = (-\langle x^-, x^- \rangle)^{1/2} \).

The closure of \( J_0 : J_0 = J_0^+ \oplus J_0^- \) is a Krein space.

The space \( J_0 \), in turn, can contain isotropic subspace \( \tilde{J} \). Factor-space \( J = \tilde{J}_0/\tilde{J} \) evidently is a nondegenerate space.

Thus, we have constructed closed and nondegenerate space \( J \) such that operators \( \varphi_f \) are determined on dense domain \( J_0 \). Hence, the axiom of cyclicity of vacuum is fulfilled.

Let us construct in \( J \) the scalar product of any two vectors \( \Phi \) and \( \Psi \). It is obvious, that there exist sequences of vectors \( \Phi_n \in J_0 \) and \( \Psi_m \in J_0 \) such that

\[ < \Phi, \Psi > = \lim_{n, m \to \infty} < \Phi_n, \Psi_m >. \]  

(25)

We shall note, that condition \( \theta^{\mu \nu} = 0 \) was not used and thus given above construction is valid in the general case as well.

Let us stress that if the \( \ast \) product acts only in coinciding points and is substituted by usual one in different points then given construction can also be fulfilled, only in the different points we have to put \( \theta^{\mu \nu} = 0 \). But in this case the function \( f(x, y) = f(x)f(y), x \neq y, f(x, x) = f(x) \ast f(x) \) is not continuous in the points \( x = y \). In order to overcome this difficulties let us proceed from the function defined by eq. [10] to its regularization:

\[ f_\alpha(x, y) = \eta(x - y) \varphi(x) \ast \varphi(y), \]  

(26)

\[ \eta(x - y) = 1, \text{ if } |x - y|^2 < \alpha - \varepsilon, \eta(x - y) = 0, \text{ if } |x - y|^2 \geq \alpha, \]

As any vector of space \( J_0^\prime \) is a finite sum of the vectors belonging to the set \( M \), formula (21) defines the scalar product of any two vectors in \( J_0^\prime \).

As well as in commutative case, we need to pass from \( J_0^\prime \) to nondegenerate and closed space \( J \).

The space \( J_0^\prime \) can contain isotropic, i.e. orthogonal to \( J_0^\prime \) vectors which, as is known, form subspace \( [20] \). Designating isotropic space as \( \tilde{J}_0 \) and passing to factor-space \( J_0 = J_0^\prime/\tilde{J}_0 \), we obtain nondegenerate space, i.e. a space which does not contain isotropic vectors. Let us note, that if \( g \in J_0 \) then \( \varphi_f g \in \tilde{J}_0 \). For closure of space \( J_0 \) we assume, as well as in commutative case, that \( J_0 \) is a normalized space. If the metrics of \( J_0 \) is positive, norm \( \Phi \equiv \|\Phi\| \) can be defined by the formula \( \|\Phi\| = \langle \Phi, \Phi \rangle^{1/2} \). \( \tilde{J}_0 \) is a closure of \( J_0 \), this closure is carried out with the help of standard procedure - closure to the introduced norm. In gauge theories in case of covariant gauge it is necessary to introduce indefinite metric (see e.g. [21]). Let us recall that axiomatic formulation was extended on gauge theories, first of all, in papers of Morchio and Strocchi [22]. The standard space, which is considered in axiomatic theory of gauge fields, is a Krein space [20].

Let us recall that nondegenerate space admits fundamental decomposition if

\[ J_0 = J_0^+ \oplus J_0^-, \]  

(22)

where \( J_0^\pm \) is a space with the definite positive (negative) metric, \( J_0^+ \bot J_0^- \). In other words, if \( x \in J_0 \), then \( x = x^+ + x^- \), \( x^\pm \in J_0^\pm \), \( x^+ \bot x^- \), that is \( \langle x^+ , x^- \rangle = 0 \).

It is obvious that

\[ \langle x, y \rangle = \langle x^+, y^+ \rangle + \langle x^-, y^- \rangle. \]  

(23)

It is easy to see that we can introduce in \( J \) the positive-definite scalar product \((\cdot, \cdot)\), namely

\[ \langle x, y \rangle = \langle x^+, y^+ \rangle - \langle x^-, y^- \rangle. \]  

(24)

We can introduce norm using this product \( \|x\| = (x, x)^{1/2} \).

Evidently, \( \|x^+\| = (x^+, x^+)^{1/2}, \|x^-\| = (-\langle x^-, x^- \rangle)^{1/2} \).

The closure of \( J_0 : J_0 = J_0^+ \oplus J_0^- \) is a Krein space.

The space \( J_0 \), in turn, can contain isotropic subspace \( \tilde{J} \). Factor-space \( J = \tilde{J}_0/\tilde{J} \) evidently is a nondegenerate space.

Thus, we have constructed closed and nondegenerate space \( J \) such that operators \( \varphi_f \) are determined on dense domain \( J_0 \). Hence, the axiom of cyclicity of vacuum is fulfilled.

Let us construct in \( J \) the scalar product of any two vectors \( \Phi \) and \( \Psi \). It is obvious, that there exist sequences of vectors \( \Phi_n \in J_0 \) and \( \Psi_m \in J_0 \) such that

\[ < \Phi, \Psi > = \lim_{n, m \to \infty} < \Phi_n, \Psi_m >. \]  

(25)
α is arbitrary, ε can be taken arbitrary small without loss of continuity of η(x − y).

It is evident that if α ≫ θ, then expressions (19) and (20) practically coincide.

Now let us pass to the limit θμν = 0, that is proceed to the commutative case. In this case f(x) · f(y) → f(x) f(y) and we come to the construction of space J in the commutative case. Let us point out that the first step in the standard construction of this space [1], [2] is the introduction of sequences g determined by the formula (14), in which, however, gi ≡ gi(x1, . . . , xi) are smooth functions of variables xj ∈ R4. We shall note that in the commutative case, starting with J′ 0, we shall come to the same space J. Really, as space of functions of a type g1(x1) g2(x2) . . . gi(xi) is dense in space of functions gi(x1, . . . , xi) [1], [2], we can complete J′ 0 up to the space of the above mentioned sequences and then carry out the standard construction of space J.

Remark In fact we have obtained a very general construction, which is valid not only in the case of NC QFT, but also for any case, when usual product of the functions is substituted by the new one, if the following conditions are satisfied:

i Corresponding functions belong to the some space, such that Wightman functions are defined as generalized functions (functionals) over this space;

ii The scalar product in this space is defined by eq. (17), but condition (19) may be substituted by the new one;

iii There exists the passage to the standard multiplication.

3 Conclusions

We have rigorously constructed field operators in NC QFT. This construction is important for any rigorous treatment of the axiomatic approach to NC QFT via NC Wightman functions and the derivation of rigorous results such as CPT and spin-statistics theorems.

The carried out construction of the closed and nondegenerate space, such that operators ϕf are determined on its dense domain, corresponds to the theorem named as ”main” in [2] and opens a way to derivation of the reconstruction theorem in noncommutative field theory, that we are going to make.

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