Potential of Interaction between Non-Abelian Charges in the Yang-Mills-Higgs Model

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Abstract

The problem of two nonrelativistic chromoelectric and chromomagnetic charges in a Higgs vacuum is considered in classical field theory. An approximation of interaction potential is constructed on the basis of a numerical solution to the equations of gluostatics. The concept of a non-Abelian Abrikosov vortex is discussed. It is shown that the results of Ginzburg-Landau theory for the tension of a string between magnetic charges can be directly extended to the non-Abelian case.

Introduction

At present, the mechanism of dual superconductivity is one of the most appealing concepts that are invoked in attempts at explaining quark confinement [1]. It is assumed that the physical vacuum hinders the penetration of gluon fields to large distances from the sources of charge and compresses lines of force in the same manner as tube does. This pattern is well known in the Abelian theory of superconductivity for magnetic charges. The mean-field approximation as exemplified by Ginzburg-Landau theory is quite sufficient for obtaining a quantitative description of arising physical situation—and in particular, for evaluating the tension coefficient [2]. Since the microscopic theory of confinement has yet to be developed, the above furnishes sufficient motivation for performing a meanfield analysis of the problem of non-Abelian charges in a vacuum that possesses the properties of a gluon superconductor. In this formulation of the problem, Yang-Mills theory is supplemented with Higgs fields, and the response of a medium to the presence of non-Abelian charges (at the macroscopic level) is described by two phenomenological constants in a phenomenological Lagrangian. An attempt of this kind has recently been made in [3], where the approximation of gluostatics developed
earlier for the case of trivial vacuum [4]–[6] was used to describe nonrelativistic heavy particles carrying non-Abelian charges. It was shown that the equations of gluostatics with Higgs fields are consistent and that these equations describe Yang-Mills fields generated by the charges themselves and the response of the vacuum to the presence of these charges. Some general properties of solutions to these equations and the possible scenarios of charge confinement have also been discussed.

This article reports on some results obtained from a numerical analysis of the equations of gluostatics involving Higgs fields. The results of numerical calculations are used to construct a simple approximation for the potential interaction between non-Abelian charges. By means of a direct substitution, it is shown that there are vortex-type excitations in the Yang-Mills-Higgs system and that these excitations are identical to Abrikosov vortices. Owing to this, it turns out that the numerical results for the tension coefficient that were obtained in [2] for a funnel-shaped potential of interaction between magnetic charges in a superconductor directly apply to chromomagnetic charges in a nontrivial vacuum.

1 Chromoelectric charges

In the model in question, the Lagrangian density has the form

\[ L = \frac{1}{4} \tilde{G}^{\mu\nu} \tilde{G}_{\mu\nu} - j^\mu \tilde{A}_\mu - \frac{1}{2} D^\mu \tilde{\chi} D_\mu \tilde{\chi} - \frac{\lambda}{4} \{ \tilde{\chi}^2 - F^2 \}^2, \]

(1)

where \( \tilde{A}_\mu \) and \( \tilde{\chi} \) are the triplets of, respectively, Yang-Mills and Higgs fields (hereafter, the analysis is restricted to the case of the SU(2) group); \( j^\mu \) is the density of the external-source currents; \( \tilde{G}_{\mu\nu} = \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu + g \tilde{A}_\mu \times \tilde{A}_\nu \) is the strength tensor of the gluon field (the cross denotes vector product in isotopic space); and the covariant derivative is defined as \( D^\mu \tilde{\varphi} = \partial^\mu \tilde{\varphi} + g \tilde{A}^\mu \times \tilde{\varphi} \).

The equations for the Yang-Mills and Higgs fields are obviously given by

\[ D^\mu \tilde{G}_{\mu\nu} + g D_\nu \tilde{\chi} \times \tilde{\chi} = \tilde{j}_\nu, \]

\[ D^\mu D_\mu \tilde{\chi} - \lambda \{ \tilde{\chi}^2 - F^2 \} \tilde{\chi} = 0. \]

(2)

In the gluostatic approximation, we retain only the zeroth component of the 4-current. We have

\[ \tilde{j}^0 = \tilde{\rho} = \tilde{P}_1 \delta(x - x_1) + \tilde{P}_2 \delta(x - x_2), \]
where \( P_i, x_i, i = 1, 2 \) are the vector-charges and coordinates of the particles involved. Further, the vector-charges of the particles represent a convenient basis—\( \tilde{P}_1, \tilde{P}_2, \tilde{P}_3 = \tilde{P}_1 \times \tilde{P}_2 \)—in which we expand solutions to equations (2).

The zeroth component \( \tilde{A}_0 = \tilde{\varphi} \) of the gluon field then appears as a linear combination of the vector-charges of the particles,

\[
\tilde{\varphi}(x, t) = \varphi_1(x) \tilde{P}_1(t) + \varphi_2(x) \tilde{P}_2(t),
\]

while the vector field is proportional to the third component of the three basis vectors:

\[
\tilde{A}(x, t) = a(x) \tilde{P}_3(t).
\]

The potentials and the vector fields are functions of the spatial coordinate and particle coordinates: \( \varphi_i(x | x_1, x_2), a(x | x_1, x_2) \).

By virtue of the condition

\[
\partial^\mu \tilde{j}_\mu + g \tilde{A}^\mu \times \tilde{j}_\mu = 0,
\]

which ensures the consistency of equations (2), the basis generally proves to be rotating in isotopic space in the direction of the vector \( \tilde{\Omega} = \varphi_1^* \tilde{P}_1 + \varphi_2^* \tilde{P}_2 \), with frequency \( |\tilde{\Omega}| \), whereas the functions \( \varphi_1 = \varphi_1(x_2) \), and \( \varphi_2^* = \varphi_2(x_1) \) determine the values of the potentials and the points where the charges reside.

Nontrivial solutions to the system of equations (4) were studied in [3] for the Higgs field represented as a linear combination of the vector-charges of the particles

\[
\tilde{\chi}(x, t) = \chi_1(x) \tilde{P}_1(t) + \chi_2(x) \tilde{P}_2(t).
\]

Following factorization of the contributions of the resulting functions \( a, \varphi^T = ||\varphi_1, \varphi_2|| \), and \( \chi^T = ||\chi_1, \chi_2|| \), the system of equations of gluostatics assumes the form

\[
\begin{align*}
DD\Phi + g^2\{\Phi J_\chi\}C\chi &= \delta, \\
DD\chi + g^2\{\Phi J_\chi\}C\Phi &= \lambda\{\chi JC\chi - F^2\}\chi, \\
\nabla \times \nabla \times a &= gj_\varphi + gj_\chi = 0, \\
\mathbf{j}_\varphi &= \Phi JD\Phi, \quad \mathbf{j}_\chi = \chi JD\chi.
\end{align*}
\]

where the column \( \Phi \) is the difference of the columns \( \varphi \) and \( \varphi^* \), \( \varphi^T = ||\varphi_1, \varphi_2||; \Phi = \varphi - \varphi^* \),
$D_{kl} = \nabla \delta_{kl} + gaC_{kl}$ ($k,l = 1,2$) is the covariant derivative,
$\delta^T = \|\delta(x - x_1), \delta(x - x_2)\|$, and $C$ and $J$ are the $2 \times 2$ matrices

\[
C = \begin{pmatrix}
-(\tilde{P}_1\tilde{P}_2) & -(\tilde{P}_2\tilde{P}_2) \\
(\tilde{P}_1\tilde{P}_1) & (\tilde{P}_1\tilde{P}_2)
\end{pmatrix}, \quad
J = \begin{pmatrix}
0 & 1 \\
-1 & 0
\end{pmatrix}.
\]

The parentheses in the above expression denote scalar products of vector-charges in isotopic space, and $\delta$ is the delta function that describes the charge source of intensity equal to unit.

A more general formulation assumes the use of additional components that have nonzero projections on other basis vectors as well. In contrast to electrodynamics, non-Abelian theory involves an extra degree of freedom associated with the choice of the relative orientation of the Yang-Mills and Higgs fields in isotopic space. The resulting system of equations is rather cumbersome, but the regularities in the changes that its solutions suffer as the result of this generalization may be traced by considering, instead of (5), the limiting case in which the Higgs field have a nonzero projection only onto the third vector of the basis:

\[
\tilde{\chi}(x,t) = \chi(x)\tilde{P}_3(t).
\] (7)

Instead of the system of equations (6), we then have

\[
DD\Phi - g^2\chi^2\Phi = \delta,
\]
\[
\Delta\chi - g^2\Phi^2\chi = \lambda(\chi^2 - F^2)\chi,
\]
\[
\nabla \times \nabla \times a - g j_\varphi = 0, \quad j_\varphi = \Phi JD\Phi.
\] (8)

Let us supplement this system of equations with boundary conditions. The delta sources on the righthand sides of (6) and (8) are eliminated by isolating the Coulomb term in the solutions for the field $\varphi$ as $\varphi_c$

\[
\varphi' = \varphi_c + \varphi.
\]

Experience gained from the calculations and the analysis performed previously for the equations of gluostatics in the trivial vacuum reveal that, at large values of the coupling constant $g$, we must approximate pointlike source more accurately, taking into account induced charge density [7]. However, for the parameter values considered below, the above-type superposition involving the Coulomb solution and simple boundary conditions at the points
where the charges are located are quite sufficient for achieving a reasonably high accuracy of the calculations and a reasonably fast convergence of iterations. The quantities $\varphi$ were treated as parameters in one version of the calculations and were determined in solving numerically the problem with free boundary conditions of the second kind at the points where the charges are located $\frac{\partial \varphi_1}{\partial n} |_{\mathbf{x}=\mathbf{x}_2} = 0$, $\frac{\partial \varphi_2}{\partial n} |_{\mathbf{x}=\mathbf{x}_1} = 0$.

An advantageous feature of the rotating basis is that the boundary conditions at spatial infinity (when the calculation is performed in a large box) are specified in a very simple form: $\varphi|_G \to 0$, $a_{\rho,z}|_G \to 0$. In this case, the Higgs field must approach the vacuum value $\tilde{\chi}^2 = F^2$, whence it follows, in particular, that, if the charges are equal in magnitude ($|\tilde{P}_1| = |\tilde{P}_2| = P$), we have $\chi_1|_G = \chi_2|_G = \chi_{as} = F/(2 P \cos(\theta/2))$, where $\theta$ is the angle between the vector-charges in isotopic space (this solution is obviously singular at $\theta = \pi$). In the case specified by equation (5), the boundary condition at infinity has the form $\chi|_G = F$. The formulation of the problem admits the possibility of imposing additional first- and second-order boundary conditions on the field $\tilde{\chi}$ that are analogous to the above boundary conditions for the fields $\varphi$ at the points where the charges reside.

The potential of interaction between the two charges is determined by integrating over space the symmetrized field-energy density

$$T = \mathcal{E}_e + \mathcal{E}_m,$$

$$\mathcal{E}_e = \frac{1}{2} E_{\Phi} J C E_{\Phi} + \frac{g^2}{2} \tilde{P}_3^2 \{ \Phi J \chi \}^2 + \frac{1}{2} E_{\chi} J C E_{\chi} + \frac{\lambda}{4} \{ \tilde{\chi}^2 - F^2 \}^2, \quad (9)$$

$$\mathcal{E}_m = \frac{1}{2} \tilde{P}_3^2 H^2,$$

where $E_{\Phi} = D \Phi$, $E_{\chi} = D \chi$, and $H = \nabla \times a$.

Integration by parts makes it possible to recast the expression for the energy density $\mathcal{E}_e$ of the gluoelectric field into the form

$$\mathcal{E}_e = -\frac{1}{2} \varphi J C \delta - \frac{g^2}{2} \tilde{P}_3^2 \{ \varphi J \chi \} \{ \Phi J \chi \} - \frac{g^2}{2} \tilde{P}_3^2 \{ \varphi J E_{\Phi} \} a +$$

$$+ \frac{g^2}{2} \tilde{P}_3^2 \{ \Phi J \chi \}^2 + \frac{\lambda}{4} \{ F^4 - \tilde{\chi}^4 \}, \quad (10)$$

which is more convenient for numerical calculations.

Let us first consider the case in which the Higgs field is specified by equation (5) and analyze solutions to the system of equations (6). The energy
of the gluomagnetic field and the contributions to the energy that come from the five terms in (10) are presented in the table as functions of the distance between the charges [after integration over the space of energy densities with our numerical solution to the system of equations (6)]. For the parameters, we choose the following characteristic values: $g = 1, P = 1, \lambda = 1, F = 1, \theta = \pi - \pi/10$ (in connection with the choice of $\theta$, it should be noted that, in numerical calculations, the vector charges cannot be taken to be strictly parallel because $\chi_{as}$ is singular in this configuration). The calculations were performed for the free boundary conditions at the points where the charges are located. If the boundary conditions of the first order are used and if the parameter values are chosen as above, the minimum of energy can be achieved by varying $\phi^*$; as a result, we arrive at the same solution. It should also be noted that the singular contributions that describe self-interaction were eliminated in the same way as in electrodynamics.

From the table, we can see that the fourth and fifth terms represent the largest corrections to the first term, but they are of opposite signs and compensate each other. The second and third terms and the contribution of the gluomagnetic energy are negligibly small. Following a natural regularization of divergences, the first term is expressed in terms of $\phi^*$ and takes the form

$$V_{int} = -\frac{\phi_1^* + \phi_2^*}{2} (\tilde{P}_1 \tilde{P}_2).$$

Figure 1 displays the potential $V_{int}$ as a function of the distance $r$ between the charges for $F = 0, 1, 2, 3$ (in the table, $\lambda, g, P,$ and $\theta$ are specified as above). For the trivial condensate $F = 0$, we naturally arrive at the Coulomb interaction represented in Fig. 1 by points. As $F$ is increased, the curves are shifted below, this shift being equidistant at large $r$.

In discussing the data presented in the table, we have already noted that, owing to cancellation of the leading corrections, the interaction potential is determined primarily only by the values of the fields $\phi$ at the points where the charges reside. Thus, the detailed distribution of the vector and Higgs fields may prove not very important, and even a rough approximation to the numerical solution will yield a reasonable result for the interaction potential. Indeed, let us assume that the vector field is negligibly small in the system of equations (6), and let us approximate the higgs field in the entire space by its asymptotic value. The equations for the fields $\phi$ then become trivial,
and the required solution is found straightforwardly (the angle $\theta$ drops from final expressions). As a result, we obtain

$$\varphi_1 = -\frac{1 + e^{-gF|x-x_2|}}{8\pi |x-x_1|} - \frac{1 - e^{-gF|x-x_2|}}{8\pi |x-x_2|},$$

$$\varphi_2 = -\frac{1 - e^{-gF|x-x_1|}}{8\pi |x-x_1|} - \frac{1 + e^{-gF|x-x_2|}}{8\pi |x-x_2|}.$$ (12)

In the difference $\varphi_1 - \varphi_2$, the Coulomb components cancel out completely. If this were not the case, the fourth term in expression would guarantee a linear growth of the potential (this provides a convenient tool for checking the accuracy of our calculations)[3].

The required constants $\tilde{\varphi}$ are now given by

$$\tilde{\varphi}_1 = \tilde{\varphi}_2 = -\frac{1 + e^{-gF|x_1-x_2|}}{8\pi |x_1-x_2|} - \frac{gF}{8\pi}.$$ (13)

The interaction potential can then be approximated as

$$V_{\text{int}} = \left(\tilde{P}_1 \tilde{P}_2\right) \left\{\frac{1 + e^{-gF}r}{r} + gF\right\}.$$ (14)

The following comment is in order. Only for $\lambda \gg 1$ could we hope that the above crude approximation is reasonable. This is the limit in which the Higgs fields differ noticeable from their asymptotic values in the region of dimension $r \leq \lambda^{-1/2}$ and make a negligibly small contribution to the total energy integral. But owing to cancellation of the significant corrections, this approximation proves valid down to $\lambda \sim 1$, which is a surprising result.

Figure 2 shows the interaction potential as a function of the coupling constant $g$ at a fixed distance between the particles ($r = 5$). On the curve presented in this figure, we can find the $d$ value corresponding to the onset of a nonlinear regime (it is quite conceivable, however, that this restriction is peculiar to the computational algorithm that we used and has nothing to do with real physics).

It was indicated in [3] that the mode of solutions that is characterized by the suppression of the vector field $|a| = 0$, $j_\chi = j_\Phi$ must manifest itself with increasing $\lambda$. Figure 3 shows the energy of the gluomagnetic field as a
function of $\lambda$. We can see that the vector field in the system is suppressed as the superconducting properties of the medium are enhanced.

It only remains for us to consider the case described by the system of equations (8), which corresponds to the Higgs field proportional to the third basis vector. It can easy be seen that the charges are now coupled only by short-range forces because, in this field configuration, the gluoelectric fields are screened at large distances, the screening factor being $e^{-gF r}$. For the same reason, the generation of gluomagnetic fields is insignificant. As a result, the charges do not interact at large distances, so that, instead of (14) we approximately have

$$V_{\text{int}} = \frac{(\vec{P}_1 \vec{P}_2)}{4\pi} e^{-gF r} r.$$

From the above results, it follows that solutions of the type (3) are energetically favorable in the case of attraction between the particles [$(\vec{P}_1 \vec{P}_2) < 0$], whereas the configuration (7) is preferable in the case of repulsion. All these solutions are stable in classical field theory. Solutions are determined (“strenghtted”) by the boundary conditions, which specify the choice of the transverse component of the Higgs field lying in the plane spanned by the vector-charges of the particles. An effective potential that is expected to arise in quantum theory will contain the above solutions with a factor of continual integration.

A simple comparison of the potential (14) with the potential used in the model of heavy quarkonia (see [8]),

$$V(r) = -\frac{\alpha}{r} + \beta r + V_0,$$

where $\alpha = 0.27$, $\beta = 0.25$ GeV$^2$, and $V_0 = -0.76$ GeV ($r$ is measured in GeV$^{-1}$), leads to the following estimates of the constants:

$$g \cong 2.6, \quad F \cong 1.1, \quad \left(\frac{g^2}{8\pi} \cong 0.27, \quad \frac{g^3F}{8\pi} \cong 0.76\right).$$

At the same time, potential models yield a much lower value for the coupling constant $g$ [9]:

$$g(J/\psi) \cong 1.55, \quad g(\Upsilon) \cong 1.42, \quad \alpha_s(J/\psi) \cong 0.19, \quad \alpha_s(\Upsilon) \cong 0.16.$$
by the potential models of heavy quarkonia will be studied elsewhere in greater detail.

In this section, we have discussed the case in which the non-Abelian charges are placed in a strong gluonic superconductor. For the sake of completeness, we present an approximation for the case in which the vacuum reduces to a trivial one \( (\lambda \to 0, F \to 0) \). For not overly large coupling constants \( (g^2/(4\pi) < 1) \), the interaction potential that takes into account the contribution of the gluomagnetic field has the form

\[
V_{\text{int}} = \frac{(\tilde{P}_1 \tilde{P}_2) + \gamma \tilde{P}_3^2}{4\pi r}, \quad \gamma = \frac{g^2}{4\pi} \frac{6 - \pi^2/2}{16\pi}.
\]  

(16)

\section{Chromomagnetic charges}

It was noted in [3] that the system of equations (7), which corresponds to the approximation of gluostatics, can also be used to describe magnetic charges. We will illustrate this statement by means of an explicit substitution. In the absence of chromoelectric charges, we begin by eliminating singular delta-function sources from the system of equations and seek a particular solution of the form \( \Phi = 0 \) (in the case under study, the column \( \tilde{\phi} \) is considered as a free parameter).

The resulting system of equations,

\[
\begin{align*}
\mathbf{D} \mathbf{D} \chi & = \lambda \{ \chi J C \chi - F^2 \} \chi, \\
\nabla \times \nabla \times a + g j_\chi & = 0, \quad j_\chi = \chi J D \chi,
\end{align*}
\]  

(17)

describes the expulsion of a gluomagnetic field from a Higgs condensate just in the same way as this occurs in the Abelian theory of superconductivity. From the symmetry properties of the system of equations and from the fact that the eigenvalues of the matrix \( C \) are complex conjugate to each other \( (\mu_{1,2} = \mp i|\tilde{P}_3|) \), it follows that a Higgs doublet can be described by one complex-valued function. The equations of the theory are then equivalent to the Ginzburg-Landau system of equations. Explicitly, this can be demonstrated by means of the substitutions

\[
\chi_1 = \left\{ \frac{\psi}{1 - e^{-i\theta}} + \frac{i \psi^*}{1 + e^{i\theta}} \right\} \frac{F}{\sqrt{2 |\tilde{P}_3|}},
\]
\[ \chi^2 = \left\{ \frac{\psi}{1 - e^{i\theta}} - \frac{i \psi^*}{1 + e^{-i\theta}} \right\} \frac{F}{\sqrt{2 |P_3|}}. \]  

Instead of two real-valued functions, transformations (18) define a complex-valued function \( \psi \) (here, \( \psi^* \) is the complex conjugate of \( \psi \)). In addition, we introduce the scale transformations \( \nabla \rightarrow gF \nabla \) and \( a \rightarrow \frac{F}{|P_3|} a \), which reduce the equations for the function \( \psi \) and the vector field \( a \) to the canonical form

\[
(\nabla - ia)^2 \psi = \kappa \{|\psi|^2 - 1\} \psi,
\]

\[
\nabla \times \nabla \times a - \frac{i}{2} \{\psi(\nabla + ia) \psi^* - \psi^*(\nabla - ia) \psi\} = 0,
\]

where \( \kappa = \frac{\lambda}{g^2} \).

It is well known [10] that the system of equations (19) has solutions with a quantized magnetic flux. In the initial variables, these solutions correspond to Abrikosov vortices with the magnetic flux

\[
\oint a \, dl = Q_n = \frac{2\pi n}{g |P_3|}, \quad n = \pm 1, \pm 2, \ldots,
\]

where \( l \) is a closed contour that circumvents the vortex in the unperturbed condensate. Formally, the flux of the gluomagnetic field is then given by

\[
\oint \tilde{A} \, dl = \frac{2\pi n}{g} \frac{P_3}{|P_3|}.
\]

The gluoelectric field identically vanishes

\[
\tilde{E} = \tilde{P}_1 D \Phi_1 + \tilde{P}_2 D \Phi_2 \equiv 0,
\]

and so does the gauge-invariant \( \text{t} \) Hooft tensor of the field strength; that is, we have

\[
F_{\mu\nu} = \check{\chi}^a G_{\mu\nu}^a - \frac{1}{g} \check{\chi}^a (D_\mu \check{\chi} \times D_\nu \check{\chi})^a \equiv 0,
\]
where $\tilde{\chi} = \chi/|\chi|$. The energy density is concentrated near the vortex core and is given by

$$E = g^2 F^4 \mathcal{E}, \quad \mathcal{E} = \frac{1}{2}(\nabla \times \mathbf{a})^2 + \frac{1}{2}|(\nabla - i\mathbf{a})\psi|^2 + \frac{\kappa}{4}(|\psi|^2 - 1)^2. \quad (20)$$

At $\kappa = 1/\sqrt{2}$, the integral of energy is quantized and has the linear density $\int E_n \, ds = \pi F^2 n$. In the Abelian case, solutions with large $|n|$ are stable for $\kappa < 1/\sqrt{2}$ [11]. In the case being considered, the gluomagnetic field does not necessarily disturb a gluonic superconductor, because there is an additional degree of freedom associated with the relative orientation of the Yang-Mills and Higgs fields in isotopic space. Thus, solutions of the Abrikosov-vortex type are metastable, but they may lie comparatively far from the instability region. Solutions of this type have already been used in the theory of electroweak interaction and are referred to as W and Z strings [12].

Unfortunately, magnetic charges separated by large distances can be described only by solving numerically equations (19) with the input singular Dirac potential

$$A_D = \frac{g}{4\pi} \frac{e_{\varphi}}{\rho} \left\{ \frac{z - d}{[\rho^2 + (z - d)^2]^{1/2}} - \frac{z + d}{[\rho^2 + (z + d)^2]^{1/2}} \right\}, \quad (21)$$

where $\mathbf{e}_\varphi$ is a unit vector, $\rho$ and $z$ are the radial coordinates, and $\pm d$ are the points at the $z$ axis where the magnetic poles reside. The results of such numerical investigations are presented in [4], among other things, the tension coefficient for a funnel-like potential was calculated in these studies as a function of the parameters of the Ginzburg-Landau potential. Formulas (18), which establish relationship between the Abelian and non-Abelian models, can be used to extend these results to the case of chromomagnetic charges in a gluonic superconductor.

**Conclusion**

The problem of two nonrelativistic chromoelectric and chromomagnetic charges in a Higgs vacuum with the properties of a gluonic superconductor has been considered within classical field theory. It has been shown that there is a direct analogy with the Abelian theory of superconductivity, where magnetic charges are confined, while electric charges are not (by charge confinement,
we mean here a linear growth of the energy of particle interaction with distance). It has been found that Abelian modes do not exhaust all possible states of chromoelectric charges; for these, there exist states whose energy is less than the Coulomb (Yukawa) energy. It is conceivable that the inclusion of loop corrections may reconcile the results presented in this study with available data on the potential of heavy quarkonia, thereby ensuring confinement of charges.

A simple representation is given above for a certain class of solutions to the Yang-Mills-Higgs equations that describes non-Abelian Abrikosov vortices. The solutions obtained here can be used in constructing models of elementary particles, which can then be considered either as closed vortex lines in the bulk of a Higgs condensate or as droplets of a dual condensate with a quantized gluoelectric field frozen into them. In the latter case, the source and outflux of the gluoelectric field from the droplet surface can be interpreted as a quark-antiquark pair.

Considering the dual pattern, we arrive, for the dual fields (potentials) and charges, at the scenario in which the fates of chromoelectric and chromomagnetic charges are interchanged: the former are confined, while the latter are free.

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Gluoelectric and gluomagnetic contributions to the energy

| $r$  | $V_e^I$  | $V_e^{II}$ | $V_e^{III}$ | $V_e^{IV}$ | $V_e^{V}$ | $V_m$   |
|------|----------|------------|-------------|------------|-----------|--------|
| 0.1  | $-7.58 \cdot 10^{-1}$ | $1.0 \cdot 10^{-6}$ | $6.4 \cdot 10^{-9}$ | $4.97 \cdot 10^{-3}$ | $-4.90 \cdot 10^{-3}$ | $7.5 \cdot 10^{-4}$ |
| 0.6  | $-1.35 \cdot 10^{-1}$ | $-1.3 \cdot 10^{-6}$ | $2.7 \cdot 10^{-7}$ | $1.86 \cdot 10^{-2}$ | $-1.84 \cdot 10^{-2}$ | $1.3 \cdot 10^{-5}$ |
| 1    | $-8.94 \cdot 10^{-2}$ | $-1.9 \cdot 10^{-6}$ | $-2.7 \cdot 10^{-6}$ | $2.34 \cdot 10^{-2}$ | $-2.32 \cdot 10^{-2}$ | $2.6 \cdot 10^{-6}$ |
| 3    | $-5.08 \cdot 10^{-2}$ | $-2.3 \cdot 10^{-6}$ | $-4.2 \cdot 10^{-6}$ | $2.84 \cdot 10^{-2}$ | $-2.81 \cdot 10^{-2}$ | $3.7 \cdot 10^{-7}$ |