The systematic construction of free fermionic heterotic string gauge models

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Abstract. It has been shown that the string landscape consists of roughly $10^{500}$ string vacua. While the construction of all of these models is currently made infeasible by technological limits, systematic construction schemes can be employed to explore regions of the landscape. These surveys can provide insight into the theory that purely analytical analysis cannot. We discuss one such systematic survey scheme being developed and deployed at Baylor University as well as results of a recently launched survey of layer 2 gauge models.

1. Introduction
It is well known that the number of possible string derived models is on the order of $10^{500}$ [1,2]. Consequently, any efforts to explore this landscape of string vacua require the use of high-performance computing and a choice of construction method. Each construction method has access to different, overlapping regimes of the landscape; here we will focus on the weakly coupled free fermionic heterotic string (WCFFHS) construction formalism [3–7]. The WCFFHS formalism has produced some of the most phenomenologically viable models to date [8–42] and is ideal for computer construction. Random examinations of the landscape, using this formalism, have been performed in the past [43,44]; however, due to the many-to-one nature of this construction a random survey of the input parameters has many endemic problems that are non-trivial to address [45]. One way to deal with these problems is to systematically survey the valid input parameters.

Two software frameworks, currently under development at Baylor University, are being designed and used specifically for the purpose of performing such systematic surveys of the WCFFHS landscape. One such framework, the Gauge Framework, focuses on systematically building gauge models in four to ten large spacetime dimensions. These surveys serve multiple purposes including aiding in attempts at understanding and reducing the redundancies inherent to the construction method.

A detailed account of the WCFFHS formalism from the perspective of gauge model building is provided in [46]. The term “gauge model” and the definition of “uniqueness” used in this study were defined therein, and are thus omitted here for brevity. Presently we are extending the layer 1 survey previously considered to 6, 8, and 10 large spacetime dimensions. To that end we review how the input space was modified to produce such models, section 2. From there a few basic results from that survey are presented, section 3, focusing on the distribution of unique models over the allowed orders and dimensions as well as the occurrence of GUT groups. We conclude with a some preliminary results from a recently launched layer 2 survey, section 4.
2. Gauge Model Building
The Gauge Framework focuses on the construction of gauge models, as described in [46]. These models are in many ways the most simplistic models that one can build, and can be thought of as a basis from which more complex models can be built. This makes them interesting as a starting point for systematic surveys because they can be used to guide further searches. Here the WCFFHS construction method is reviewed with gauge models in mind.

Within the free fermionic framework two inputs are required, the set of basis vectors, \( A \), and the GSO projection coefficient matrix, \( k \). In order to systematically build these models, it is necessary to systematically build the input pairs, \((A, k)\), ensuring that all of the modular invariance constraints are met. Our primary deviation from [46] is in the subset of possible basis vector sets considered which is explained below. Beyond this the process follows as before; no changes have been made to the construction of states nor the possible GSO projection matrices, \( k \).

2.1. Basis Vectors
In this study, all basis vector sets, \( A \), are considered in \( D = 4, 6, \ldots, 10 \) large spacetime dimensions. For clarity, a basis vector set is defined as:

\[
A = \left\{ \vec{\alpha}_i \mid \vec{\alpha}_i \in \mathbb{Q}^{40-2D} \cap (-1, 1)^{40-2D} \right\},
\]

(1)

where \( i = 1, 2, \ldots, L \). For our purposes we will always take \( l \geq 3 \) and will refer to \( L = l - 2 \) as the layer. Each of these basis vectors represents the boundary conditions of complex worldsheet fermion degrees of freedom. By convention, \( \alpha^i_j \) with \( j = 1, \ldots, (14-D) \) are taken to represent the boundary conditions of the left-moving supersymmetric string and \( \alpha^i_j \) with \( j = 11, \ldots, (26-D) \) to represent the right-moving bosonic string boundary conditions. The order of each basis vector, \( N_i \), is defined to be the smallest integer such that

\[
N_i \alpha^i_j = 0 \pmod{2}.
\]

(2)

Of course, the possible basis vectors are constrained by modular invariance in such a way that

\[
N_i \alpha^2_i = \begin{cases} 
0 \pmod{8} & \text{if } N_i \text{ even} \\
0 \pmod{4} & \text{if } N_i \text{ odd}
\end{cases}
\]

(3)

and

\[
N_{ij} \vec{\alpha}_i \cdot \vec{\alpha}_j = 0 \pmod{4},
\]

(4)

where \( N_{ij} = \text{LCM}(N_i, N_j) \).

Since the basis vector sets are \( L = 1 \), there are three basis vectors, two of which will always be the same for every basis vector set generated:

- The first basis vector, denoted \( \mathds{1} \), is the all periodic basis vector: \((1^{14-D} \parallel 1^{26-D})\).
- The second basis vector is the SUSY generator, \( S \), which is \((1^4 0^{10-D} \parallel 0^{26-D})\).

Generating these basis vectors can introduce excessive redundancy thereby hindering systematic search algorithms. However, an algorithm for efficiently generating only the modular invariant basis vectors was developed and implemented in [47].

\footnote{We do not allow for chiral Ising models here. So all left- or right-moving real fermions can be paired to form left- or right-moving complex fermions.}
3. Layer 1 Results
Traditionally, the collection of string derived, low energy effective field theories (LEEFTs) is referred to as the landscape. However, because we are interested less in the field theories and more in the mapping from the WCFFHS input space to this landscape, only those LEEFTs that are mapped to by the particular input subspace defined above are considered. For these purposes it is sufficient to refer to the LEEFTs to which these inputs map as the “FFHS gauge landscape.”

Using the WCFFHS formalism, all unique, layer 1 gauge models from order 2 through 22 in 4, 6, 8 and 10 large spacetime dimensions have been constructed. The number of unique models generated as well as how they are distributed amongst the possible number of spacetime supersymmetries is given in Table 1.

| $D$ = 10 | $D$ = 8 | $D$ = 6 | $D$ = 4 |
| --- | --- | --- | --- |
| $\mathcal{N} = \mathcal{N}_{\text{max}}$ | 2 | 13 | 18 | 68 |
| $\mathcal{N} = 0$ | 6 | 50 | 73 | 509 |
| Groups in Both Flavors | 1 | 6 | 18 | 50 |
| Total Number of Models | 4,953,930 | 12,493,632 | 29,079,534 | 85,859,757 |

There is significant growth in both the total number of models and the number of unique models as the space is compactified. This comes as no surprise and results from an increase in the number of fermionic degrees of freedom, that is, computationally the input space has grown because the basis vectors are longer. However, something of particular interest is that the growth in the number of unique models is faster than that in the total number of models. To clarify, the overall efficiency, while still abysmal, improves as the space is compactified. Concretely, roughly $1.6 \times 10^{-4}$% of the models generated at $D = 10$ are unique while at $D = 4$ the number is a bit higher, $6.7 \times 10^{-4}$%. This is not necessarily something one might expect and could either be a general property of the formalism or may result from the mechanisms used to reduce redundancy. As of now, this is unclear and further work is to be done.

In addition to the way the unique models distribute across the number of large spacetime dimensions, we can look at the distribution across the order, Figure 1. In all of the dimensions considered, the production of new, unique, supersymmetric models falls off by order 22 while the production of $\mathcal{N} = 0$ models falls off by order 24. This results from the fewer constraints on the models and the fact that there are no odd order $\mathcal{N} = 0$ models. One might suspect that beyond order 24 no unique models will be found. To address this, a higher order survey is already underway, the results of which will be released in a future work. All together these results suggest that, while the input space is effectively infinite, the redundancy in the many-to-one mapping from input space to model space is such that a general systematic survey is reasonable. Further, these results conform to the conclusion that the landscape is in fact finite.

We go on to consider the GUT groups occurring in each of the dimensions in question, Table 2. Specific focus is given to the Flipped-$SU_5$, $E_6$, $SO_{10}$, $SU_3 \times SU_2 \times SU_2$, (PS), $SU_3 \times SU_2 \times SU_2$, (LRS), and $SU_3 \times SU_2 \times U_1$ gauge groups. As this study focuses primarily on the gauge content alone, we have chosen to consider models that have promising groups for unification but have
neglected the matter content. Strictly speaking, this means that the models constructed and presented here may not, and in all likelihood are not, true GUT models. However, it may be possible to construct models with favorable matter content without changing the gauge group by adding nontrivial left-movers to these gauge models.

Table 2: Occurrence of GUT Groups by Dimension - The values represent the percentage of unique models possessing at least one such group factor.

|        | $\mathcal{N} = \mathcal{N}_{\text{max}}$ |        | $\mathcal{N} = 0$ |
|--------|-----------------------------------------|--------|------------------|
|        | $D = 10$ | $D = 8$ | $D = 6$ | $D = 4$ | $D = 10$ | $D = 8$ | $D = 6$ | $D = 4$ |
| $\mathcal{F}SU_5$ | 0% | 0% | 0% | 5.9% | 0% | 0% | 5.5% | 20.9% |
| $E6$ | 0% | 7.6% | 11.1% | 8.8% | 0% | 8% | 11.0% | 9.6% |
| $SO_{10}$ | 0% | 0% | 11.1% | 13.2% | 0% | 8% | 13.7% | 13.9% |
| PS | 0% | 0% | 0% | 5.9% | 0% | 2% | 8.2% | 16.3% |
| LRS | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 8.7% |
| MSSM | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 14.7% |

One thing to note here is the occurrence of the LRS and MSSM in $D = 4$ alone. This may be specific to gauge models or it may suggest that those gauge groups are more easily produced in $D = 4$. This is a possibility due to the increased number of degrees of freedom. Surveys looking at generic models to determine whether this is, in fact, the case are being prepared and will likely launch after more of the gauge landscape has been explored.
4. Layer 2 Results

Of particular interest in this study are the preliminary results from the ongoing layer 2 survey. To date, the survey has built all models from order (2, 2) to order (2, 7) in $D = 4$. This amounts to approximately 149 million models, significantly more than was found in the $D = 4$, layer 1, order 2 to 22 survey. This is expected as adding additional layers increases the number of degrees of freedom exponentially. However, while one might expect this to increase the number of unique models generated, current results suggest that this is not the case. In fact, not a single unique model has been generated beyond those found in the previous layer 1, $D = 4$ survey. To some degree this redundancy is expected. For example, we know that the number of unique models generated at order (2, 3) is exactly the same as at order 6. However, this only accounts for layer 2 models whose orders share no non-unit divisors. Because very little work has been done to build models of high order, this redundancy has never been observed and could prove quite fundamental to understanding the WCFFHS mini-landscape.

5. Conclusion

While many gauge surveys are still underway, results are already being produced that are significant from a model building standpoint. We are finding that the free fermionic region of the landscape is smaller than was originally expected. Additionally we are seeing indicators that $D = 4$ is somewhat favorable to model production as compared to $D = 6$, 8 and 10; the increased efficiency of unique model generation as well as the increased occurrence of left-right symmetric and standard model gauge groups make $D = 4$ more productive. Further still, results were presented that suggest that increasing layer may not yield unique models beyond those which layer 1 can generated. Future work will primarily focus on computationally exploring the layer 2 region of the landscape as well as analytic considerations of the layer 2 results presented herein.

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