Quantum hair of black holes out of equilibrium

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Abstract

Classically, the black hole (BH) horizon is completely opaque, hiding any clues about the state and very existence of its interior. Quantum mechanically and in equilibrium, the situation is not much different: Hawking radiation will now be emitted, but it comes out at an extremely slow rate, is thermal to a high degree of accuracy and thus carries a minimal amount of information about the quantum state within the BH. Here, it is shown that the situation is significantly different when a quantum BH is out of equilibrium. We argue that the BH can then emit “supersized” Hawking radiation with a much larger amplitude than that emitted in equilibrium. The result is a new type of quantum hair that can reveal the state and composition of the BH interior to an external observer. Moreover, the frequency and amplitude of the new hair can be explained by the observer without invoking any new physical principles. The new hair decays at a parametrically slow rate in comparison to the Schwarzschild time scale and can be detected through the emission of gravitational waves (and possibly other types of waves); for example, during and after a BH-merger event. The current discussion is motivated by a previous analysis, in the context of a recently proposed polymer model for the BH interior, that implies emissions just like those described here. We expect, however, that the new hair is a model-independent property of quantum BHs.
1 Introduction

It was not too long ago when the classical picture of the interior of a black hole (BH) was more or less accepted. However, thanks to the so-called firewall argument [1, 2] (and as recently reviewed in [3]) along with its less-celebrated forerunners [4, 5, 6], a conflict between the classical description of a BH and the principles of quantum theory has been revealed. As such, a variety of non-classical models of the interior have emerged as leading candidates for a successor. (See, however, [7, 8].) But a consensus point of view on the correct description of the interior is still lacking, as evident from the ongoing and rather intense debate (see [9] for a summary). Some have suggested that the BH interior should, one way or another, be expelled from the accessible part of spacetime (e.g., [10]). We, on the other hand, have suggested that the BH interior is composed of highly excited, interacting, long, closed strings — essentially, a “ball of string” or a collapsed polymer [11]. Others have, however, proposed entirely different compositions for the interior (e.g., [12]).

The recent detections of gravitational waves (GWs) from BH mergers [13] have elevated what was an abstract academic debate about the laws of quantum gravity to a more tangible discussion about the expected signatures in the GW data of either a non-empty or an excised interior [14, 15]. Meaning that each new proposal about the nature of BHs will have to confront such data as it comes in from subsequent merger observations. Besides the resulting GWs, there could also be data from the emission of electromagnetic waves and neutrinos, although neither has been detected so far. Until now, the data has been completely consistent with the predictions of classical general relativity [16, 17] (see, however, [15, 18]), but it is too soon to reach any
We have recently analyzed some of the consequences of our proposed polymer description of the BH interior. In particular, it was argued in [19] (also see [20]) that GW observations could provide a means for distinguishing our model from that of a classical BH as well as from other candidate models. The idea is that the interior matter of a polymer BH, which can be effectively viewed as a fluid, will support pulsating modes in essentially the same way that a relativistic star does. These fluid modes would exist in addition to the standard spacetime modes of the exterior, and so their spectrum would then be added onto that of the ring-down or quasinormal modes (QNMs) of a perturbed BH. The polymer has an outer surface that behaves just like a BH horizon in the limit $h \to 0$ but is otherwise only partially opaque [21]. Models without such an effective horizon would likely have a spectrum that differs even more substantially from that of a classical BH [22].

The bottom line is that a fluid-like description of the BH interior gives rise to a new type of quantum hair, which is emitted with a parametrically lower frequency $\omega_I \sim v_I/R_S$ and a parametrically longer damping time $\tau_I \sim R_SC/v_I^2$ [19] in comparison to the QNMs of its classical counterpart. Here, $R_S$ is the Schwarzschild radius, $c$ is the speed of light and $v_I < c$ is the velocity of sound for a fluid mode from the $I^{th}$ class. For the polymer model in particular, the parametric difference is due to the introduction of a new scale, the string scale, and therefore a new dimensionless parameter $g_s = l_P/l_s$, the ratio of the Planck scale to the string scale. In this case, $v_I/c = g_s$ for what would be the most experimentally accessible class of modes.
Here, we would like to discuss how a coupling between internal fluid modes and emitted GWs (or other types of waves) can occur from the perspective of an observer on the outside. The external observer does have the prerogative of ignoring all knowledge about the interior but, then again, should be able to explain all phenomena in the framework of classical GR. Hawking has made this same point in [23]; to wit, ‘All data on a “hidden” surface compatible with the observer’s limited information are equally probable.’ From this perspective, the interior of the BH is an imaginary construction whose sole justification is to serve as a mental crutch to help explain the properties of the emitted radiation. After all, the BH horizon is supposed to prevent just such an emission as the interior is causally disconnected as far as this outside observer is concerned [3].

Since the picture does seem sensible enough from an internal point of view [19, 20], what needs to be shown is that an external observer will attribute the source of the additional (fluid) modes to perturbations of the exterior spacetime and not those of the BH interior. Establishing this to be true is the primary objective of the current paper, and we are indeed able to confirm that the two perspectives are consistent.

The key to resolving the conflicting viewpoints is the realization that this external perspective for the fluid modes is really no more or less paradoxical than that of Hawking radiation itself [24, 23]. In spite of some arguments that the Hawking effect can be linked to mechanisms like pair production, quantum tunneling and so on, one can only learn about the interior indirectly by observations on the outside. And so, as the above quotation correctly implies, any explanation of the Hawking process is just as viable as any
other as long as its predictions are consistent with what is known or could be known about BHs. Hence, there has to be a vantage point for which the Hawking modes originate in the exterior spacetime because, just like for any other form of matter, this radiation is not permitted to escape through the horizon.

What we will then argue is that, when viewed externally, the fluid modes are describing “supersized” Hawking emissions. This is because each such event represents a large-amplitude coherent state of photons, gravitons, etc. (akin to a electromagnetic or gravitational field) rather than a single boson. And, same as for the standard case, the supersized modes must appear to have originated in the exterior spacetime. For either choice, regular or supersized, we will assert that this exterior picture is consistent as long as the BH has, to some degree, deviated from its equilibrium state. The degree of deviation depends on the amount of energy that is injected into the specific mode and, therefore, also determines the amplitude of the respective emission.

We will also address the puzzling absence of (damped) relativistic modes in the interior, which was a central finding in [19]. The question of interest is whether this result is an artifact of our particular model or a physical consequence of a more general nature. We do find that this is indeed a general phenomenon, from both the internal and external perspectives.

Although relying on the results of a particular model, we expect that many of the ideas and conclusions should apply just as well to any “BH-like” object; which is meant as an exotic spacetime containing exotic matter that can exist inside of an ultra-compact object while somehow resisting gravitational collapse. This object should, simultaneously, exhibit all of the
standard properties of a BH when viewed from the outside. Note that this excludes models lacking a “horizon-like” outer surface such as gravastars and wormholes.

The above claims are substantiated and further discussed in Section 2, followed by a summary in Section 3. Throughout the paper, we ignore numerical factors of order one and fundamental constants are only made explicit when needed for clarity. For simplicity, three large spacelike dimensions and a non-rotating BH are assumed. When we refer to BHs (including the use of the subscript $BH$), the polymer model is implied unless stated otherwise. As the current results are often compared to those in [19], we follow this earlier treatment and assume that the fluid modes are scalars.

2 The external perspective

An external observer can, from her perspective, only see “stuff” which is on her side of the horizon. Whatever is supposed to be leaking out of the BH, whether it be conventional or “supersized” Hawking radiation, must have originated from outside the horizon as far as this observer is concerned. When the BH is close to its equilibrium state, this outside observer can use a “horizon-locking gauge” to describe the near-horizon geometry [25, 26]. In this gauge, the equilibrium position of the compact object’s outer surface (or effective horizon) stays at $r = R_S$ up to some high order in the relative strength of the perturbation.

To understand how an outside observer would interpret the supersized quantum radiation, it is necessary to know about departures from equilib-
Figure 1: Visualization of the deformed horizon. Scalar (left), dipole (center) and quadrupole (right) deformations are shown. The dashed, black circles depict the position of the unperturbed horizon at \( r = R_S \) and the solid, blue shapes depict its respective deformations.

To get a handle on this, let us first recall a classical analogue: a tidal deformation of the horizon of a slowly rotating BH due to an external perturbation. This was first discussed by Hartle \[27\] and later by O’Sullivan and Hughes \[28\] (see, in particular, Appendix B1 of \[28\] and also \[25, 26\]), who visualized this setup by embedding a deformed sphere in a three-dimensional, flat Euclidean space. The basic idea was to lock up the position of the sphere’s outer surface (as described above) and rather interpret its deformation as a perturbation in the associated Ricci curvature. But one can just as well choose a gauge for which the deformation is interpreted as the difference between the location of the outer surface and \( R_S \). This difference would, in our case, be the extent that the internal fluid is either protruding out of or sinking into the fiducial horizon.

Figure 1 can help one to visualize the gravitational coupling between
the deformed horizon and the external observer. For example, a static quadrupole deformation of the horizon (rightmost panel, Fig. 1) changes the sphere from its unperturbed shape by an amount that scales with the strength of the perturbation times the second Legendre polynomial for the polar angle \( P_2(\theta) \propto 3 \cos^2 \theta - 1 \). More generally, the position and shape of the deformed horizon can be expected to oscillate in time.

Perhaps contrary to expectations, the deformed surface can include both depressions and protrusions irrespective of the direction of the perturbing force, as illustrated in Fig. 1. Our interest is in places where the horizon is depressed inwards (equivalently, where the fluid protrudes outwards), as this implies that a portion of the interior has been momentarily “exposed” to the exterior spacetime. From an external point of view, such a depression of the horizon can only be explained by the BH absorbing a flux of negative energy, just as the emission of standard Hawking radiation is normally explained [29]. However, the negative flux is really just a story that an outside observer has to invent to reconcile energy conservation with the flux of positive energy emanating out from the BH.

In the classical case of tidal horizon deformations, an outgoing flux occurs only for the superradiant modes of a rotating BH. In the case of Hawking radiation emerging from a polymer BH, the outgoing flux and compensating negative flux are explained, internally, by a quantum effect that allows small loops of string to break off and detach from the string-filled interior. Super-sized Hawking radiation should be similar but, in this case, a large portion of string would be detached collectively in a short span of time.

Also from an internal perspective, the relative deformation \( \Delta L/L \) of the
horizon due to a particular restoring force (say the $I^{th}$ one) is the ratio

$$\frac{\Delta L}{L} \bigg|_I \sim \frac{(\Delta E)_I}{E} = \frac{p_I}{\rho} \approx \frac{v_I^2}{c^2},$$

(1)

where $v_I$ is again the sound velocity for the $I^{th}$ mode, $p_I$ is its pressure and $\rho \sim M_{BH}/R^3_S$ is the total energy density ($M_{BH}$ is the BH mass). Here, we have employed standard relations from thermodynamics, between stress and strain and between the pressure-to-energy-density ratio and sound velocity.

Equation (1) can be used to obtain an expression for the redshift at the outermost extent of the protruding fluid,

$$\sqrt{-g_{tt}} \bigg|_I = \sqrt{1 - \frac{R_S}{r_I}} = \sqrt{1 - \frac{R_S}{R_S \left(1 + \frac{\Delta L}{L} \bigg|_I\right)}} \approx \sqrt{\frac{\Delta L}{L} \bigg|_I},$$

(2)

that is,

$$\sqrt{-g_{tt}} \bigg|_I \approx \frac{v_I}{c}.$$

(3)

The above estimates will be used to determine the mode frequencies as measured by an observer in the exterior, which will tell us if her observations are consistent with those from an interior point of view.

For future reference, it should be noted that Eqs. (2) and (3) are not compatible with a relativistic speed of sound $v_I = c$ since, in this case, $\Delta L/L$ cannot be small. This is the first indication that the limiting case of $v_I = c$ is problematic.

### 2.1 Hawking radiation

To illustrate the procedure of determining the external frequencies, it is useful to start with the familiar case of standard Hawking radiation. Let us then

\footnote{Unlike in \cite{19}, we now use the energy $E$ in place of the free energy $F$, as these and their order-by-order corrections scale in parametrically the same way in the polymer model.}
begin with

$$\langle \Delta E \rangle_H \sim T_H = \frac{1}{R_S}$$

(4)

and

$$E = M_{BH},$$

(5)

from which it follows that

$$\frac{\langle \Delta E \rangle_H}{E} \sim \frac{1}{R_S M_{BH}} = \frac{1}{S_{BH}}$$

(6)

and then

$$\sqrt{-g_{tt}}|_H = \sqrt{\frac{1}{S_{BH}}} = \frac{l_P}{R_S},$$

(7)

where a subscript of $H$ indicates an associated property (e.g., $T_H$ is the Hawking temperature), $S_{BH}$ is the BH entropy and $l_P$ is the Planck length.

This is the redshift at the location of the protruding fluid and, therefore, the location of the source as far as an external observer is concerned. But what frequency would this observer assign to a Hawking mode at the same point? It is natural to ascribe a wavelength of $l_P$ to a near-horizon Hawking mode. The logic here follows that of 't Hooft’s “brick-wall” model [30]. Formally, one can start with $$\langle \Delta L \rangle_H = \frac{L}{E} \langle \Delta E \rangle_H = \frac{R_S}{S_{BH}} = \frac{l_P}{R_S},$$ which corresponds to a proper length of $l_P$. And so the observer assigns this mode with a frequency at the source of $$\omega^{(H)}_{\text{source}} = c/l_P.$$ The frequency at the location of the external observer is then found by redshifting its value at the source. This process yields the expected result,

$$\omega^{(H)}_{\text{ext}} = \omega^{(H)}_{\text{source}} \sqrt{-g_{tt}}|_H = T_H.$$  

(8)

This is not exactly groundbreaking physics, as the above argument runs along the same lines as that of the membrane paradigm [31]. The difference
here, though, is that we did not have to conjecture a location for the (so-called) stretched horizon before determining the redshift.

The polymer model realizes the same value for the Hawking temperature from an internal perspective [21]. A small loop of string which has broken off from one of the typically long loops in what is a bound state of interacting, highly excited, closed strings will have some probability of escape, and a calculation reveals that both the rate and energy of emission agree with $T_H$. In this way, consistency between the exterior and interior perspectives has been established, at least as far as it concerns the case of conventional Hawking radiation.

2.2 “Supersized” Hawking radiation

2.2.1 Frequency of emission

Let us next consider some non-relativistic fluid mode, beginning with its frequency as seen by an external observer. We already know that the redshift at the location of the protruding fluid (which is exterior to but still in the vicinity of $r = R_S$) is $v_I/c$, and so it becomes prudent to ask about the mode frequency at this same “source” location. An external observer, who is unaware of the fluid, would confuse these non-relativistic fluid modes with relativistic spacetime QNMs, for which the wavelength at the source would be approximately $R_S$. She would therefore assign them a frequency at the source of

$$\omega_{\text{source}}^{(I)} = \frac{c}{R_S}, \quad (9)$$
from which one can deduce that

\[ \omega_{\text{ext}}^{(I)} = \omega_{\text{source}}^{(I)} \sqrt{-g_{tt}} \big|_{I} = \frac{v_I}{R_S}, \tag{10} \]

again as expected (see the Introduction). The difference between the internal and external perspectives can then only be one of interpretation.

Since the frequencies redshift, one might wonder why the energies \((\Delta E)_I\) in Eq. (1) do not. In reality they do but, in both cases (standard and supersized), the values that were used for \((\Delta E)_I\) and \(E\) are already what would be measured by an asymptotic observer. We know this because the standard Hawking case can be used to calibrate all other cases. The two energies \((\Delta E)_I\) and \(E\) at the source would then be blueshifted from their asymptotic values in the same way, leaving their ratio undisturbed.

### 2.2.2 Coupling to the exterior and decay time

To complete our consistency check, the estimates from [19] for an emitted energy of \((\Delta E)_I = v_I^2 E_I\) and a damping time of \(\tau_I = R_S/v_I^2\) need to be similarly reproduced from an external perspective. Here, \(E_I\) is the amount of energy which has been injected into the \(I^{\text{th}}\) mode by the deforming force. (It was assumed in [19] that \(E_I \sim M_{BH}\).)

For an exterior observer, the supersized Hawking radiation is relativistic and has a frequency of \(\omega_I = v_I/R_S\). Therefore, she must conclude that the wavelength of the radiation at distances far away from the horizon is \(\lambda_I = R_S c/v_I\). The same conclusion can be arrived at by the fact that, like before, the wavelength near the source must be \(\lambda \sim R_S\), which then asymptotically redshifts to \(\lambda_I \sim R_S c/v_I\).
Now consider that the same observer attributes the source with a radial size of about $R_S$. She then just needs to know that the transmission cross-section for such long wavelength modes through a proportionally smaller surface of area $A$ is determined by the ratio $A/\lambda^2$, which translates into $R_S^2/\lambda^2 = v_I^2$ for the case at hand. We can conclude that the coupling or efficiency of emission goes as $v_I^2$, so that the energy in the emitted wave scales as

$$(\Delta E)_I \sim E_I v_I^2,$$

in agreement with the internal perspective [19]. This is based on the assumption that most of the mode energy is being emitted in the form of coherent waves rather than dissipating as heat.

Meanwhile, the damping time for any given mode $\tau_I$ is directly related to the corresponding relaxation time of the BH. The latter can be deduced with an inspection of

$$\frac{dE_I}{dt} \propto (\Delta E)_I \sim v_I^2 E_I,$$

which implies a relaxation time that scales with the inverse of $v_I^2$ and likewise for the damping time $\tau_I$. One is then led to the expected result of $\tau_I \sim R_S/v_I^2$, where the factor of $R_S$ follows simply for dimensional reasons and the knowledge that the Schwarzschild time is the only classically available time scale.

In summary, the supersized Hawking radiation oscillates with a frequency of $\omega_I = v_I/R_S$, carries away an energy of $(\Delta E)_I = E_I v_I^2$ (where it is expected that $E_I \sim M_{BH}$) and decays with a characteristic time of $\tau_I = R_S/v_I^2$. This can be compared to the standard Hawking emissions with a

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frequency of $\omega_H = 1/R_S$, an effective coupling of $v_H^2 = 1$ (cf. $\omega_I = v_I/R_S$) and an emitted energy of $1/R_S$, leading to a decay time of $\tau_H = R_S/v_H^2 = R_S$, again just as expected.

### 2.2.3 Coupling to gravitational waves

Also of interest is the strength of the coupling of the fluid modes to external GWs, as this along with $(\Delta E)_I$ is what determines the amplitude of the emitted GWs. The coupling strength can be determined using Einstein’s quadrupole formula

$$\langle h \rangle \sim \frac{1}{r} \ddot{Q}.$$  \hfill (13)

This means that, for an external observer,

$$\langle h \rangle_I \sim \ddot{Q}_I \sim (\Delta E)_I R_S^2 \omega_I^2 \sim (E_I v_I^2)(R_S \omega_I)^2 \sim E_I v_I^4,$$  \hfill (14)

where the factor of $R_S^2$ can be attributed to the quadrupole moment of the emitting object and a dot denotes a time-derivative. Also, the two factors of frequency are due to the pair of time derivatives, which further suppresses the amplitude of the emitted GWs. Equation (14) agrees with the internal version of the same calculation [19].

### 2.2.4 Absence of relativistic modes

From an internal perspective, the absence of relativistic fluid modes can be traced to the polymer being near its equilibrium state and an incompatibility between the two boundary conditions that any fluid mode is required to satisfy: vanishing at the center of the object and outgoing at its surface. Moreover, the leading correction to the (free) energy has to be parametrically
small, $\Delta E/E < 1$ (see Eq. (1) and the comment just before Subsection 2.1), from which it follows that $v^2/c^2 = \Delta E/E < 1$.

From an external perspective, it is rather the continuity of the emission at $v_I = 1$ which makes the emission of such waves impossible. This is because $v_I > 1$ is unphysical and therefore unacceptable, the amplitude of such faster-than-light waves has to vanish identically. The condition of continuity then implies that the amplitude of waves for which $v_I = 1$ must similarly vanish.

Let us explain the continuity argument in a more detailed way: An external relativistic mode could never have been redshifted, as this would imply that it had been sourced by a fluid mode whose sound velocity was faster than the speed of light. Now consider that, for a (would-be) relativistic fluid mode, $\omega = \alpha c/R_S$ and $\lambda = R_S/\alpha$, where $\alpha$ is some constant of order 1 which takes into account any neglected numerical factors. But if its wavelength is indeed $R_S/\alpha \simeq R_S$, this mode must have originated somewhere close to the horizon and must then have experienced a significant redshift. Conversely, to suffer no redshift, it would have to be produced far away from the horizon with a wavelength that is parametrically larger than $R_S$. Such a mode could not possibly be under the influence of the BH and so — even if it somehow defied the condition of continuity and did exist — an external observer would not consider it to be part of the BH’s QNM spectrum. This argument does not preclude the existence of the standard class of relativistic spacetime QNMs, as these are a consequence of waves in the exterior spacetime and not of fluid modes from inside the BH.
2.2.5 The potential barrier

Finally, we would now like to show that the gravitational potential barrier at about $\frac{3}{2}R_S$ does not affect in any significant way the emission of the supersized Hawking radiation; an assumption that was implicit in [19].

To understand this claim, let us consider a massless particle with a modest angular momentum; then the peak in the barrier goes as $1/R_S$ when expressed in units of energy (rather than units of energy squared as it normally appears). That the peak is of the same order as $T_H$ is what explains the famous grey-body factors affecting the emission of the standard Hawking radiation (e.g., [9]). On the other hand, the energy of a supersized emission is of order $M_{BH}v_I^2$ for scalar modes and $M_{BH}v_I^4$ for gravitons, as $E_I \sim M_{BH}$ can be expected. Meaning that the ratio of the radiated energy to the height of the barrier is $M_{BH}v_I^4/(1/R_S) = S_{BH}v_I^4 \gg 1$ for the fluid modes of interest (i.e., those for which the resulting GWs could be experimentally detected). A supersized emission, which is really a large coherent state of gravitons, will not be affected by barrier at all. This has become a classical problem in which the energy of the wave far exceeds that of the potential barrier.

3 Conclusion

It was shown that, if an ultra-compact object is non-empty but does have a surface that acts effectively as a BH horizon, interior modes can nevertheless couple to emitted GWs or, for that matter, other types of waves (such as electromagnetic waves and neutrinos). An external observer will view the interior modes as supersized Hawking emissions which originated close
to but outside the equilibrium position of the horizon. Moreover, we have shown that the same point of view applies just as well to standard Hawking radiation.

Although these conclusions rely on the intuition gained from studying the polymer model for the BH interior [19] (also [20]), we believe that they are not specific to the polymer model and would readily carry over to any ultra-compact object containing non-trivial fluid-like matter and having an outer surface that acts like a BH horizon to some level of approximation.

The resulting picture is suggestive of a new type of BH hair for which a parametrically long time of shedding is required. In fact, the existence of novel BH hair should be part and parcel for any BH-like object containing non-trivial matter and could yet be the key which unlocks the door to the secretive world behind the horizon. Such revelations could come about through the observation of GWs resulting from BH mergers, hopefully in the near future.

We have also addressed the absence of damped relativistic modes in the interior, even though these are ubiquitous in some of the analogous calculations for relativistic stars. There is, however, some evidence that the suppression of relativistic fluid modes is a more general phenomenon (e.g., [33, 34]). If so, our analysis could prove helpful in a broader range of studies.

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