Spin and charge ordering in high-$T_c$ cuprates has been reported in neutron-scattering experiments of La$_{1.475}$Nd$_{0.4}$Sr$_{0.125}$CuO$_4$ (Ref. 1) and La$_{1.875}$(Ba,Sr)$_{0.125}$CuO$_4$ (Ref. 2) at the hole concentration $x = 1/8$. The ordered phase is called the stripe phase, because the ordering is explained by a stripe structure that consists of one-dimensional (1D) charge domain walls with the filling of 1/2 hole per site (quarter filling) separating antiferromagnetic (AF) spin domains in antiphase. The presence of the quarter-filled charge river has been confirmed by angle-resolved photoemission spectroscopy experiments for La$_{1.28}$Nd$_{0.6}$Sr$_{0.12}$CuO$_4$. At around $x \sim 1/8$ the superconducting transition temperature $T_c$ is strongly suppressed, which is known as the 1/8 anomaly.

The stripe phase emerges below the temperature of the structural phase transition from the low-temperature orthorhombic (LTO) to tetragonal (LTT) phase. Transport properties exhibit anomalous behaviors in the stripe phase with LTT structure. For example, the Hall constant $R_H$ becomes strongly temperature dependent in the stripe phase and it eventually almost vanishes, $R_H \sim 0$ at temperature $T \sim 0.45$. The vanishing of $R_H$ is interpreted as a particular consequence of strong correlation in the stripe structure where the formation of the 1D charge stripe with an equal concentration of hole and electron carriers (due to the strong on-site Coulomb interaction) leads to the particle-hole symmetry and thus the vanishing of the off-diagonal conductivity $\sigma_{xy}$.

Below the LTO-LTT transition temperature, the thermoelectric power $Q$ also shows a sudden decrease similar to $R_H$ at around $x = 1/8$ and becomes negative at $x = 1/8$ in high-temperature tetragonal cuprates. In addition, the electronic specific heat coefficient $\gamma$ at $x = 1/8$ is very small at low temperatures. These thermodynamic and thermoelectric anomalies are likely to be related to the formation of static stripe. In the present paper, we investigate thermodynamic properties such as the chemical potential $\mu$ and the entropy $s$ in the stripe phase of cuprates, and clarify the origin of the suppression of $Q$ as well as $s$ observed in the experiments. We employ in this study the $t$-$J$ model with a stripe potential that stabilizes the charge stripe, and use the finite-temperature Lanczos method for small clusters to calculate the temperature dependence of the thermodynamic properties. We find that the entropy $s$ is suppressed in the stripe phase as a consequence of the formation of the 1D charge stripes accompanied by the development of AF spin domains. Such a stripe formation simultaneously leads to weak temperature dependence of $\mu$. The thermoelectric power obtained from the temperature dependence of $\mu$ becomes small and negative under the strong stripe potential. The suppression of $s$ and $Q$ is consistent with experimental data in the stripe phase of La$_{1.6-x}$Nd$_{0.4}$Sr$_x$CuO$_4$.

In order to study the thermodynamics of the stripe phase of cuprates, we use the finite-temperature Lanczos technique within the grand-canonical ensemble for the $t$-$J$ model with a stripe potential. The prototype $t$-$J$ model reads

\[ H_{tJ} = -t \sum_{\langle ij \rangle s} (\hat{c}_{j \sigma}^\dagger \hat{c}_{i \sigma} + \text{H.c.}) + J \sum_{\langle ij \rangle} \left( \mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i n_j \right), \]

where no double occupancy of sites is allowed. In numerical studies we use $J/t = 0.4$ to be in the strong-correlation regime relevant to cuprates. The stripes are stabilized by introducing an attractive hole potential $V$ along the stripe

\[ H_{st} = -V \sum_{i \in \text{stripe}} (1 - n_i), \]

which on one hand in a small system enhances the intrinsic tendency of the $t$-$J$ model towards the stripes structures and on the other hand represents the effects of
extrinsic features favorable to the formation of the stripes such as lattice anisotropy.\textsuperscript{15}

We consider here numerically two types of $t$-$J$ clusters with $N = N_x \times N_y$ sites: (i) $(N_x, N_y) = (4, 4)$ with periodic boundary conditions in both the $x$ and $y$ directions and (ii) $(N_x, N_y) = (4, 5)$ with a periodic boundary condition in the $x$ direction and an open boundary condition in the $y$ direction. The stripe potential Eq. 2 is introduced into a row along the $x$ direction of the $4 \times 4$ cluster and into a middle leg of the $4 \times 5$ cluster.

First, we examine the dependence of the entropy on the stripe potential $V$. The entropy density $s$ for the hole concentration $x$ and temperature $T$ is given by

\begin{equation}
  s = \frac{1}{N} \left[ k_B \ln \Omega + \frac{(H_{tJ} + H_{sd}) - \mu (1 - x) N}{T} \right],
\end{equation}

where $k_B$ is the Boltzmann factor, $\Omega$ is the partition function, $\langle \cdots \rangle$ is the averaging within the ground-canonical ensemble, and $\mu$ is the chemical potential calculated separately. Figure 1 shows $s$ as a function of $T$ for $x = 1/8$ in the $4 \times 4$ cluster and $x = 1/10$ in the $4 \times 5$ cluster (two holes in both the clusters) at various values of $V/t$ in the range of $0 \leq V/t \leq 3$. Note that at large $V/t$ the two holes are confined within the stripe\textsuperscript{14} effectively leading to quarter-filled 1D charge stripes. We observe that with increasing $V/t$ $s$ gradually decreases. The reduction of $s$ is stronger at low temperatures below roughly $T \sim J/(=0.4t)$ than at high temperatures around $T \sim t$. Since the decrease of $s$ at high temperatures becomes significant when $V > t$, the restriction of the space available for the hole motion due to the stripe potential must be the predominant source of the decrease. It appears plausible that for $T < J$ AF spin correlations develop inside the spin domain from which holes are expelled\textsuperscript{14}. The contribution to the entropy is then small both from the AF domains without holes (in analogy to the Heisenberg model where $s \propto T^2$) as well from 1D metallic stripes, unlike in the homogeneously doped $t$-$J$ model where $s$ remains anomalously large even at $T < J$.\textsuperscript{12} The suppression of $s$ in the stripe phase of La$_{1.6-x}$Nd$_{0.4}$Sr$_x$CuO$_4$ has been indeed observed experimentally via the small electronic specific heat coefficient $\gamma$ at $x = 1/8$ below $T \sim 30$ K\textsuperscript{14}.

Next, let us consider the thermoelectric power $Q$ in the stripe phase. Since the experimental data mentioned above are just an average from the contributions parallel and perpendicular to the stripe, it is necessary to take an average of $Q$ over the two directions to compare with the experiments. Within the linear response theory, $Q$ with the electrical and heat currents in the $\alpha$ direction can be expressed in terms of the current-current correlation function $C_{j\alpha J\alpha}(\omega)$ and the energy current-current
The correlation function $C_{J_{E_a,J_{a'}}}(\omega)$,

$$Q = \frac{1}{eT} \left[ \mu - \frac{C_{J_{E_a,J_{a'}}}(\omega \to 0)}{C_{J_{a,J_{a'}}}(\omega \to 0)} \right], \quad (4)$$

where $\mu$ is the chemical potential, and $e$ is the electric charge. Instead of a complete analysis of $C_{J_{E_a,J_{a'}}}(\omega)$, which is straightforward but significantly involved, we rely on an approximate relation which has been verified within the $t$-$J$ model in the regime of weak to moderate doping that $C_{J_{E_a,J_{a'}}}(\omega)$ is proportional to $C_{J_{a,J_{a'}}}(\omega)$. This leads to a simplified expression for the thermoelectric power

$$Q \sim \frac{1}{eT} [\mu(T) - \mu(T = 0)]. \quad (5)$$

Here we note that this approximation removes the difference of $Q$ along the parallel and perpendicular directions, since $\mu(T)$ has no anisotropy. As a result of this, we do not need to take the average. As shown below, the experimental features of $Q$ in the stripe phase is explained by using Eq. (5), indicating the validity of the approximation. More precise calculations of Eq. (4) remains to be done in future.

We examine $\mu$ before discussing $Q$. Figure 2 shows $\mu(T)$ for various values of $V/t$ in the two clusters with two holes. The chemical potential at $T = 0$ is calculated by using the following equation: $\mu(T = 0) = [E_0(1) - E_0(3)]/2$, where $E_0(N)$ is the ground state energy of the $N$-hole system. At $V/t = 0$, $\mu$ increases with increasing $T$, being consistent with the previous report that the slope is positive below $x \sim 0.15$. While the slopes at high temperatures near $T \sim t$ do not change so much when $V/t$ is turned on, those at low temperatures below $T \sim J$ become smaller and change the sign from positive to negative at large $V$ [$V/t = 3$ and $2$ in Figs. 2(a) and 2(b), respectively]. The appearance of a negative slope of $\mu$ again has to be related to the formation of the 1D charge river. To establish this, we show in Fig. 3 the temperature dependence of $\mu$ for a 16-site 1D $t$-$J$ ring with 8 holes (quarter filling at $x = 0.5$). We take an antiperiodic boundary condition in order to obtain the nondegenerate ground state. We find that the slope of $\mu$ for $J/t = 0.4$ is negative when $T/t \gtrsim 1.5$. The negative slope at around $T/t = 0.2$ is consistent with the data at large $V$ in Fig. 2. In Fig. 2, the negative slope region extends to lower temperatures when the data are extrapolated to the $T = 0$ values. This seems to be different from the temperature dependence of $\mu$ for $J/t = 0.4$ below $T/t \sim 0.15$ in Fig. 3. However, with reducing $J/t$, we find that the negative slope region extends to lower temperatures (see $\mu$ for $J/t = 0.1$), indicating qualitatively similar behavior with the data at large stripe potential. Therefore, it is as likely that the value of $J/t$ is effectively reduced inside the charge stripes. Actually, it may be possible that the coupling of spins in the stripes with those in the AF spin domains induces frustration effects that effectively reduce the exchange interaction inside the stripes. This possibility should be examined in a detail in the future.

According to Eq. (5), we evaluate $Q(T)$, which is shown in Fig. 4. At the temperatures shown in the figure, $Q$ decreases with increasing $V$. This means that the formation of the stripe phase reduces $Q$, being consistent...
with experimental data that $Q$ is strongly suppressed in the stripe phase of La-based cuprates. Fixing the value of $V$, $Q$ decreases with decreasing temperature, being again consistent with the experimental data below the LTO-LTT transition temperature. At large $V$, $Q$ becomes negative. As discussed above, this may be associated with the presence of the quarter-filled 1D system with strong correlations. Very interestingly, negative $Q$ is obtained in such a quarter-filled system when $T$ is much larger than $t$ but much smaller than the on-site Coulomb repulsion $U$. In this limit, $Q$ is given by $\mu / (e T)$, leading to $Q = -(k_B / e) \ln \left(2x / (1-x)\right) = -(k_B / e) \ln 2$ at $x = 0.5$. Since the negative slope of $\mu$ in the quarter-filled 1D $t$-$J$ model shown in Fig. 3 is smoothly connected to the value $-k_B \ln 2$ at $T \to \infty$, the negative $Q$ at large $V$ in Fig. 4 may be the realization of characteristics in the quarter-filled system with strong correlations. From Fig. 4 we can also conclude that in a broader $4 \times 5$ system $Q$ decreases faster with imposed potential $V/t$. This seems plausible since one expects that in a larger lattice, in particular with open boundary conditions, the $t$-$J$ model tends inherently towards stable stripe structures, hence generally a weaker $V/t$ is enough to stabilize stripes, although the values of $V/t$ taken here are still large probably due to small system sizes.

Based on the results above, we further discuss the physical meaning of the reduction of $Q$ in the stripe phase. The temperature derivative of $\mu$ is related to the density derivative of $s$ according to the Maxwell’s relation

$$\left(\frac{\partial \mu}{\partial T}\right)_x = \left(\frac{\partial s}{\partial x}\right)_T. $$

The small temperature dependence of $\mu$ in the presence of the stripe potential shown in Fig 2 implies small variation of the entropy at around $x = 1/8$. In fact, $s$, which is an increasing function with respect to $x$ at around $x = 1/8$ for $V = 0.25$ is suppressed near $x = 1/8$ with increasing $V$, leading to the small value of $(\partial s/\partial x)_T$. Therefore, the reduction of $Q$ induced by the small temperature dependence of $\mu$ is intimately related to the small variation of the entropy with respect to the hole concentration. This is reasonable because holes added to the stripe phase enter into the charge domain wall (stripe) not affecting AF spin domains and therefore induce only a small change of the entropy. We can thus conclude that the suppression of $Q$ below the LTO-LTT transition is a consequence of the reduction of the entropy accompanied by small concentration dependence.

In summary, we have examined the entropy, chemical potential, and thermoelectric power in the stripe phase of high-$T_c$ cuprates, by using the finite-temperature Lanczos technique for the $t$-$J$ model with a stripe potential. We find that the thermoelectric power becomes small when the stripe potential is introduced. This is understood as a consequence of the reduction of the entropy associated with the formation of the one-dimensional charge rivers characteristic of the stripe phase. The suppression of the thermoelectric power as well as the entropy is consistent with the experimental data in the stripe phase of La$_{1-x}$Nd$_x$CuO$_4$. From both the preceding and this works, we conclude that the anomalies of the Hall constant, thermoelectric power, and entropy in the stripe phase have all the origin in the formation of the quarter-filled one-dimensional charge stripes.

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