A Model for FGM/SMA Bilayer Beams Accounting for Asymmetric SMA Behavior

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Abstract. A new model is proposed for bilayer cantilever beams consisting of shape memory alloy (SMA) and functionally graded material (FGM) layers and subjected to loading at the tip. The model accounts for tensile-compressive SMA behavior through the use of an appropriate set of constitutive relations and considers the deformation of the beam within the frame of the Timoshenko beam theory. The derivation of the model proceeds by first determining the correct sequence in which different solid phase structures develop within the SMA layer as the load is applied, resulting in the formation of distinct solid phase regions. The boundaries separating these regions are then located and used in deriving appropriate moment and shear force equations throughout a complete loading-unloading cycle. The model is capable of tracking the deviation of the neutral surface with respect to the mid-plane and the distribution of martensite in the beam as the load varies. The nonlinear variation of stress and strain in a cross section of the beam is also considered. Results of beam deflection and neutral surface deviation are presented and validated against finite element simulations of a 3D model of a beam consisting of a Nitinol and CNT-reinforced layers.

1. Introduction
Since their discovery in the early 1980s [1], FGMs have been used in various fields such as aerospace, electronics, medical technology, energy conversion, etc. This rapid development was motivated by the distinctive performance of FGMs, which cannot usually be achieved using monolithic or conventionally associated materials. Examples of such properties include high fracture toughness, low in-plane transverse stresses, enhanced thermal barrier performance, wear resistance, etc. [2]. In contrast, SMAs are well known for their superelasticity and shape memory effect [3], which are made possible by the ability of these alloys to undergo the so-called martensitic transformation between phases of different crystallographic symmetries. Combining FGMs and SMAs has therefore allowed the creation of “smart” FGM composites, combining the advantages of both types of materials and possibly paving the way for unprecedented engineering applications.

FGM/SMA laminates have recently attracted the interest of several research groups. In this regard, Viet et al. [3] proposed an analytical model for FGM/SMA laminates subjected to tip load based on the ZM model for SMAs and Timoshenko beam theory. Liu et al. [4] presented an analytical model for functionally graded SMA composite beams subjected to bending based on the Euler-Bernoulli theory. Sepiani et al. [5] developed a theoretical model for a FGM/SMA laminate beam subjected to arbitrary thermal loading and boundary conditions, etc. [6,7]. A limitation of the existing literature is the common disregard of tensile-compressive asymmetry in the response of the SMA constituent and of the behavior of the beam during unloading. Both these limitations are addressed here. Moreover, the deformation of the beam is described within the frame of the Timoshenko beam theory and the influence of the neutral
axis deviation, as the load is varied, on the resulting moments and shear forces acting in the beam is fully considered.

2. Analytical model of the FGM/SMA laminated composite cantilever beam

The model is derived using the ZM constitutive theory to describe the behavior of the SMA component [8]. Multiple versions of the ZM model have been developed over the years, collectively accounting for most of the unusual SMA effects [8-17].

2.1. Beam structure and material properties of FGM

The beam structure with the SMA layer bonded onto the FGM is shown in Figure 1.

![Figure 1. Geometric configuration and boundary conditions of the SMA/FGM beam. (a) Longitudinal view; (b) cross section view.](image)

In the figure, \( w \) and \( h \) are the width and height of the composite, and \( h_f \) is the FGM thickness. \( h_s \) is the distance from the mid-plane to the interface between FGM and SMA layers computed as \( h_s = h/2 - h_f \). Young’s modulus, Poisson’s ratio and density of FGM vary continuously along the thickness according to the volume fraction of its constituents [18]. The Young’s modulus of the FGM, \( E_F \), is given by

\[
E_F = E_b + (E_i - E_b) \left( \frac{y + \Delta - h_f}{h/2 - h_f} \right)^n
\]

where, \( E_i \), \( E_b \) are Young’s moduli of the inclusion and matrix, and \( y \) the distance of the neutral axis to the mid-plane. The FGM is assumed to consist of an epoxy matrix with Young’s modulus \( E_b = 3 \text{ GPa} \) with Carbon nanotube inclusions (CNTs) of Young’s modulus \( E_i = 1000 \text{ GPa} \).

2.2. Analytical stress-strain relation for the SMA layer considering asymmetry

Based on the loading functions for forward and reverse phase transformations accounting for the tensile-compressive stress asymmetry in the extended 3D ZM model for SMAs [19], the following reduced form of the loading functions is obtained:

\[
F_{js} = \text{sign}(\alpha) \left[ \frac{E_{MA} + P_{MA}}{2} \sigma_{jx}^2 + \sigma_{jx} e_{0x} \right] + \left\{ a - b - \text{sign}(\alpha) \left[ G + (\alpha - \beta) \left( k_h \left( \frac{3}{4} e_{0x}^3 - c_h e_{0x}^3 \right) \right) \right] \right\} z
\]

\[
-\text{sign}(\alpha) \left[ \frac{\beta}{2} k_h \left( \frac{3}{4} e_{0x}^3 - c_h e_{0x}^3 \right) \right]^{2/3} + C(T) - a
\]

where

\[
\text{sign}(\alpha) = \begin{cases} 1 & \text{for loading} \\ -1 & \text{for unloading} \end{cases}
\]

In Eq. (2), \( E_{MA} \) and \( P_{MA} \) are material parameters related to Young’s moduli of austenite and martensite, \( e_0 \) is the maximum orientation strain. \( G \), \( a \), \( b \), \( \alpha \), \( \beta \) are material parameters that govern the size of the superelastic hysteresis loop and hardening during forward and reverse phase transformation, \( C(T) \) is a
temperature dependent energy term that governs the temperature dependence of the stress transformation thresholds of phase transformation, $k_h$ and $c_h$ are parameters that govern the extent of asymmetry between tension and compression.

The initial and final stresses for forward phase transformation, respectively denoted $\sigma_-^{MF}$ and $\sigma_+^{MF}$ can be found by substituting $z=0$ and $z=1$, respectively, into Eq. (1) to obtain

$$
\begin{align*}
\sigma_-^{MF} &= -\varepsilon_{0x} + \frac{\varepsilon_{0x}^2 + 2\left(\varepsilon_{0x} E_1 + P_{MA}\right)}{E_{MA} + P_{MA}} \left(1/2C(T) + a\right) \\
\sigma_+^{MF} &= -\varepsilon_{0x} + \frac{\varepsilon_{0x}^2 - 2\left(\varepsilon_{0x} E_1 + P_{MA}\right)}{E_{MA} + P_{MA}} \left(a - G - b - (\alpha - 3\beta)/2\Gamma + C(T) + a\right)
\end{align*}
$$

where $\Gamma = \left(k_h \left(\frac{3}{4}\varepsilon_{0x}^3 - c_h \varepsilon_{0x}^3 \frac{3}{3.8}\right)^{3/2}\right)$.

Following the procedure in [20-25], the stress in the tensile and compressive regions during loading ($j=1$) and unloading ($j=-1$) can be written in terms of strain as

$$
\sigma_{jx} = \Pi_{j1x}^\varepsilon + \Pi_{j2x}^\varepsilon \varepsilon_{jx} + \Pi_{j3x}^\varepsilon \varepsilon_{jx}^2 + \Pi_{j4x}^\varepsilon \varepsilon_{jx}^3 + \Pi_{j5x}^\varepsilon \varepsilon_{jx}^4 = \frac{\Pi_{j1x}^\varepsilon}{3}
$$

where the coefficients are constants obtained following the procedure described in [20] and given by the following expressions:

$$
\Pi_{j1x} = \frac{2\varepsilon_{0x} \left(E_{MA} + P_{MA} + \frac{1}{E_m} - \frac{1}{E_A}\right)}{E_{MA} + P_{MA} \left(1/E_m - 1/E_A\right)}
$$

$$
\Pi_{j2x} = \frac{2\varepsilon_{0x} \left[\varepsilon_{0x}^2 - \text{sign}(\varepsilon) \left(\alpha - b - (\alpha - \beta)\Gamma\right)\right] - G \left(\frac{\beta}{2} \Gamma + C(T) + \text{sign}(\varepsilon)\alpha\right) \left(\frac{1}{E_m} - \frac{1}{E_A}\right)}{E_{MA} + P_{MA} \left(1/E_m - 1/E_A\right)}
$$

$$
\Pi_{j3x} = \frac{2\varepsilon_{0x} \left[\varepsilon_{0x}^2 - \text{sign}(\varepsilon) \left(\alpha - b - (\alpha - \beta)\Gamma\right)\right] - G \left(\frac{\beta}{2} \Gamma + C(T) + \text{sign}(\varepsilon)\alpha\right)}{E_{MA} + P_{MA} \left(1/E_m - 1/E_A\right)}
$$

$$
\Pi_{j4x} = \frac{2\varepsilon_{0x} \left[\varepsilon_{0x}^2 - \text{sign}(\varepsilon) \left(\alpha - b - (\alpha - \beta)\Gamma\right)\right] - G \left(\frac{\beta}{2} \Gamma + C(T) + \text{sign}(\varepsilon)\alpha\right)}{E_{MA} + P_{MA} \left(1/E_m - 1/E_A\right)}
$$

2.3. Moment equation

By assuming perfect bonding between the FGM and SMA layers, the strain is continuous across the FGM-SMA interface [26, 27]. The Timoshenko theory for the beam then gives

$$
\begin{align*}
\varepsilon &= y d\theta / dx = yk \\
d\omega / dx &= \gamma - \theta
\end{align*}
$$

where $\omega$ is the vertical displacement of the mid-plane, $\theta$ is the rotation of the normal to the mid-plane, $k = d\theta/dx$ is the curvature, $\gamma$ is the transverse shear strain, and $y$ is the vertical coordinate with respect to the neutral plane. During loading, the SMA beam can experience three stages including stages 1, 2 and 3 according to the number of material phases appearing in the beam. The details of these stages can be found in [20]. In this work, the notations $F_{xj\pm}$, $M_{ij\pm}$, $Q_{ij\pm}$, $\Delta_{ij\pm}$ and $k_{ij\pm}$ as introduced to respectively indicate the force normal to the cross section, the bending moment, the shear force, the neutral axis deviation from the mid-plane in section $i$, and the curvature in section $i$. The width-normalized force, width-normalized moment and width-normalized shear force in section $i$ are then defined as $\bar{F}_{xj\pm} = \frac{F_{xj\pm}}{2k_{ij\pm}}$, $\bar{M}_{ij\pm} = \frac{M_{ij\pm}}{2k_{ij\pm}^2}$, and $\bar{Q}_{ij\pm} = \frac{Q_{ij\pm}}{2k_{ij\pm}}$, respectively.
4

\[ F_{jiz} / w, \bar{M}_{jiz} = M_{jiz} / w, \text{ and } \bar{Q}_{jiz} = Q_{jiz} / w. \]

A section, denoted \( s_i \), is defined as a longitudinal portion of the beam containing cross sections with similarly ordered arrangements of solid phase regions (Figure 2).

![Figure 2](image)

Figure 2. Two examples of possible beam structures in a longitudinal cross section view during loading. (a) Stage 2; (b) stage 3.

In Figure 2, austenite is in blue, martensite is in red, and regions of phase mixture are shown in yellow. The FGM material is in graded black.

Starting from an austenitic beam, which is not shown here, applying a monotonically increasing tip load results in increasing moment with highest magnitude at the lower left tip of the beam. Phase transformation therefore starts at that point and propagates to fill the yellow region in Figure 2(a). As the load increases further, austenite is fully transformed into martensite in the region highlighted in red in Figure 2(b).

The distance of the boundary between austenite and the mixed-phase region, and between the mixed-phase region and martensite to the neutral axis are given by

\[
\begin{align*}
\gamma_{2iz} &= \frac{\sigma_{2iz}^{Mf}}{k_{yi} E_A} = \frac{\rho_{1i}}{k_{yi}} \\
\gamma_{2iz} &= \frac{\sigma_{2iz}^{Mf} + \epsilon_{02} E_M}{k_{yi} E_M} = \frac{\rho_{2i}}{k_{yi}}
\end{align*}
\]

where \( \rho_{1i} = \sigma_{1i}^{Mf} / E_A \) and \( \rho_{2i} = \left( \sigma_{2i}^{Mf} + \epsilon_{02} E_M \right) / E_M \). The first equation in (6) are obtained by considering elastic behavior of austenite at the onset of forward martensite transformation, while the second equation is obtained considering the stress-strain relation in [20], specified to the case where the martensite volume fraction \( z \) is equal to 1, indicating complete forward phase transformation.

Due to the influence of neutral axis variation during forward phase transformation in the SMA layer, there is no unique beam phase structure. Thus, the geometric relation and axial force equilibrium are used to identify the correct beam phase structure by comparison among corresponding curvatures. For example, the curvature \( \Delta_{2.3i} \) of a boundary cross section between \( s_2 \) and \( s_3 \) in Fig.2a based on axial force equilibrium and geometric relation during is obtained by solving the system

\[
\begin{align*}
\Delta_{2.3i} &= h_i - \frac{\rho_{1i}}{k_{2i}} \\
\sum \bar{F}_{2.3i} &= \left( \Pi_{12k} - \Pi_{12k}^* \right) y + \Pi_{12k}^* \frac{k_{23h}^2 y^2}{2} + \Pi_{12k}^* \frac{k_{23h}^2 y^3}{3} + \Pi_{12k}^* \frac{k_{32h}^3 y^4}{4} + \Pi_{12k}^* \frac{k_{32h}^3 y^5}{5} \right) \left( \gamma_{2i} - \Delta_{2.3i} \right) \\
+ &k_{23h} \left[ E_0 \left( E_i - E_0 \right) \left( \psi_{12h} + \psi_{23h} \right) \right]^{n+1} \left[ \psi_{12h} (n+1) y - \psi_{23h} y \right] \left[ \psi_{12h} (n+1) (n+2) \right] \right]_{h_i - \Delta_{2.3i}} = 0
\end{align*}
\]
where constants $\psi_1 \pm$ and $\psi_2 \pm$ are $\psi_1 \pm = 1/(h/2 - h_s)$ and $\psi_2 \pm = (\Delta h \pm - h_s)/(h/2 - h_s)$. The first equation in (7) expresses that the relation between the neutral axis deviation $\Delta h_2$ and $y$ coordinate of the interface between boundaries 2 and 3. The second equation expresses that the sum of resultant axial forces, obtained by integration of the normal stress over a cross sectional area, is equal to zero. Once the correct structure is determined, the analytical relation between moment, curvature and neutral axis deviation, and between shear force and shear strain can be obtained for each beam section. Without loss of generality, we demonstrate the derivation procedure of such relation for section $s_4$ of case 4. Indeed, the equilibrium of forces and moments for particular cross section of $s_4$ is written as follows:

$$
\begin{align}
\overline{F}_{s4} &= \int_{-h/2-A_{sl}}^{h/2-A_{sl}} E_M \left( k_{sl} \right) \left( y - \epsilon_{sl} \right) dy + \int_{-h/2-A_{sl}}^{h/2-A_{sl}} \sigma_{sl} dy + \int_{-h/2-A_{sl}}^{h/2-A_{sl}} k_{sl} E_F y dy = 0 \\
\overline{M}_{s4} &= \int_{-h/2-A_{sl}}^{h/2-A_{sl}} E_M \left( k_{sl} \right) \left( y - \epsilon_{sl} \right) dy + \int_{-h/2-A_{sl}}^{h/2-A_{sl}} \sigma_{sl} dy + \int_{-h/2-A_{sl}}^{h/2-A_{sl}} k_{sl} E_F y^2 dy = 0
\end{align}
$$

(8)

where $k_{sl}$ is the curvature and $\Delta h_{sl}$ the deviation of the neutral axis through section $s_4$. Substituting the expressions of the stresses from Eq. (4), and the values of boundary distances in Eq. (6) into Eq. (8), the relation for section $s_4$ of case 4 is written as

$$
\begin{align}
\overline{F}_{s4} &= \int_{-h/2-A_{sl}}^{h/2-A_{sl}} E_M \left( k_{sl} \right) \left( y - \epsilon_{sl} \right) dy + \int_{-h/2-A_{sl}}^{h/2-A_{sl}} \sigma_{sl} dy + \int_{-h/2-A_{sl}}^{h/2-A_{sl}} k_{sl} E_F y dy = 0 \\
\overline{M}_{s4} &= \int_{-h/2-A_{sl}}^{h/2-A_{sl}} E_M \left( k_{sl} \right) \left( y - \epsilon_{sl} \right) dy + \int_{-h/2-A_{sl}}^{h/2-A_{sl}} \sigma_{sl} dy + \int_{-h/2-A_{sl}}^{h/2-A_{sl}} k_{sl} E_F y^2 dy = 0
\end{align}
$$

(9)

The formulation of the volume fraction $z$ in terms of the coordinate $y$ measured with respect to the neutral axis within the transformed region is considered to be linear following [20]:

$$
z_{s4} = a_{s4} \gamma + b_{s4}
$$

(10)

where $z$ is equal to 1 at the boundary between the mixed-phase and martensitic regions. The coefficients in Eq. (10) are $a_{s4} = (1 - z_{s4int})/(\rho_{2s}/k_{s4} - h_s + \Delta h_{s4})$ and $b_{s4} = 1 - a_{s4}\rho_{2s}/k_{s4}$.

Following [20], the shear force in section $s_4$ is obtained based on Eq. (10) as

$$
\begin{align}
\overline{Q}_{s4} &= K_G a_{s4} \left[ \ln \left( G_M a_{s4} (G_s - G_M) \gamma - \Delta h_{s4} \right) \right]^{\gamma_{s4} - \gamma_{0s}} \\
&+ K_G \left( \gamma_{s4} - \gamma_{0s} \right) \left( G_M \gamma_{s4}^2 \gamma_{0s} + G_s a_{s4} \gamma^2 \gamma_{0s} + G_M a_{s4} \gamma^2 \gamma_{0s} + G_M a_{s4} \gamma^2 \right) \left( \gamma_{s4} \gamma_{0s} \right)
\end{align}
$$

(11)

where the magnitude $\gamma_0$ of the transformation shear strain is $\gamma_0 = \sqrt{3} \epsilon_{sl}/2$; $K_s$ is the Timoshenko shear correction factor for rectangular sections $K_s = 5/6$; and $G_M$, $G_s$ and $G_t$ are the shear moduli of the FGM layer corresponding to Young’s moduli $E_F$, $E_b$ and $E_t$, respectively.

3. Finite Element Analysis and Validation of the Model

Finite element analysis of a 3D model of the beam is carried out using a (UMAT) that implements the ZM model in ABAQUS to simulate the behavior of the SMA layer. The dimensions and material properties are identical to those used in solving the analytical model. The bottom-up mesh tool in ABAQUS is used to mesh the FGM layer into 26 thin layers, each assigned a different set of material properties based on Eq. (1). The computation of the deflection of the beam using the analytical model
is carried out in two steps: a curvature function is determined from fitting discrete curvature data along the beam length using a trust region algorithm in MATLAB [32], the deflection is then found by double integration [20].

4. Results and Discussions

The input parameters of the beam used in the analytical method solution and 3D FEA are provided in Table 1.

| Material properties | Dimensions |
|---------------------|------------|
| $E_A$ 61500 MPa    | $a$ 6.8920 MPa | $\epsilon_h$ 1.299 |
| $E_M$ 24000 MPa    | $b$ 6.9091 MPa | $w$ 0.5 mm |
| $\nu$ 0.3          | $k_h$ 3.29    | $L$ 50 mm |
| $\nu$ 0.3          | $k_h$ 3.29    | $L$ 50 mm |
| $2750$ MPa         | $G$ 4.6556 MPa | $h$ 3 mm |
| $A_f^0$ 40 °C      | $G$ 4.6556 MPa | $h_f$ 1 mm |
| 0.4381 MPa/°C      | $k_T$ 2.492 MPa |
| $\epsilon_{0+}$ 4 % | $\epsilon_{0-}$ −2.5 % |

Figure 3. Position of the neutral axis along the length of beam for an upward load $P_{l+} = 13 \, N$, for $T=45 \, ^{\circ}C$ and $n=4$.

The position of the neutral axis along the length of beam for an upward load $P_{l+} = 13 \, N$, for $T=45 \, ^{\circ}C$ and $n=4$ calculated using FEA and the analytical model is shown in Figure 3.

Figure 4. Beam deflection under upward and downward loads $|P_{l\pm}| = 11 \, N$ for $n = 4$ and $T=45 \, ^{\circ}C$. 
A good agreement is obtained with a maximum percent difference between the two methods of approximately 4% in terms of axial deviation from the mid-plane at the clamped face. Figure 4 demonstrates the shape of the beam subjected to upward and downward loads of magnitude $P_{\pm}=11\,N$ for $n=4$ and $T=45\,^\circ C$ based on analytical model. The difference in the deflected shape is due to different distributions of solid phase regions in each case.

5. Conclusions

In this work, an analytical model was derived for a FGM/SMA bilayer cantilever beam subjected to tip load. The model considers asymmetric behavior of the SMA in tension and compression and accounts for the variation of the position of the neutral axis of the beam with the applied load through a complete loading cycle. The results in terms of beam deflection profile and neutral axis deviation from the mid-plane based on the analytical model are validated against 3D FEA. The results indicate that under the same tip load magnitude, the overall stiffness of the FGM/SMA beam for a given load magnitude depends on the direction of the applied load because of asymmetric geometry and SMA material response.

6. References

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