Tilt-induced anomalous thermal transport in normal/superconducting borophane junctions

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We study charge and heat transport in normal-metal/superconductor (NS) hybrid junction, based on a tilted anisotropic Dirac material. Using the extended Blonder-Tinkham-Klapwijk formalism, the conductance spectra of NS borophane, a two-dimensional Dirac semimetal with two tilted anisotropic Dirac cones in its dispersion, is investigated. Completely different from the usual normal-metal-superconductor junctions, in spite of the large mismatch in Fermi wavevectors of the normal-metal and superconductor sides of the borophane NS junction, the electron-hole conversion happens with unit probability at normal incidences. Furthermore, we demonstrate that in the heavily doped superconducting regime for heavily doped normal borophane, the electron-hole conversion happens with unit probability, approximately at any incident angle. The dependence of the Andreev with Fermi energy and bias voltage, enable us, selecting the retroconfiguration or specular configuration in types of Andreev reflection processes. We numerically establish an anomalous behavior of thermal conductance in borophane. The tilting of the Dirac cones gives rise to an anomalous behavior in the thermal conductance of the NS hybrid junction of borophane, such that the thermal conductance decreases by increasing the temperature. Our findings will have potential applications for transport and energy control in superconducting quantum interference devices and hybridized mesoscopic systems.

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I. INTRODUCTION

The Andreev reflection (AR) is a scattering process between a normal-metal/superconductor (N/S) hybrid junction, in which the electron-hole conversion occurs for the excitation energies of the incident electrons $\varepsilon$ being lower than the superconducting energy gap $\varepsilon < \Delta_S$ [1]. In this process, an electron with an energy $\varepsilon$ (relative to the Fermi energy $E_F$) and spin polarization $s$, upon hitting the N/S interface, is retro reflected as a hole with the same energy but opposite spin direction $-s$. This conversion process, results in a finite conductance in the N/S junction at the bias voltages below the superconducting energy gap $\varepsilon < \Delta_S$ [2].

As is known, in a conventional normal metal-superconductor (NS) junction, an incident electron from the normal side is reflected back as an electron or a positive charged hole (known as retro AR), retraces almost the path of the incident electron. This feature stems from the fact that both an incident electron and a reflected hole are in the conduction band.

Another novel AR phenomena, known as specular Andreev reflection has been discovered by Beenakker [3, 4], in graphene-based superconducting hybrid structures, in which the hole reflected along a specular path of the incident electron, is resulted from the sublattice pseudospin degree of freedom of electrons with a gap (non-superconducting) in its band dispersion [5, 6], which is absent in ordinary metal-superconductor interfaces. Perfect AR has been also proposed in the NS junction of topological insulator [7, 8].

The normal-superconducting interfaces have been a widely used proposal to realize exotic transport behaviors, especially in recent years when plenty of new materials with massless linear excitations or nontrivial topological properties are discovered and used to fabricate these interfaces [3, 4, 9–14].

Recent advances in fabrication technologies have made exploring two-dimensional materials possible for applications, which in turn has triggered a tremendous interest. Transport properties in conventional normal metal-superconductor heterojunction have been investigated extensively by many authors and many interesting results have been obtained.

The proximity effect in two-dimensional materials was studied early on in graphene and many striking properties of superconducting graphene have been clarified [3, 18–20].

In the case of specular Andreev reflection for undoped graphene ($E_F = 0$), a hole is generated in the valence band. In doped graphene ($E_F > 0$), the Andreev reflection can be normal or specular, depending on the energy of the incoming electron. In heavily doped graphene ($E_F > \Delta$), only normal Andreev reflection is present for subgap energies, since the distance from Fermi level to the valence band is too large for specular AR to occur. In an undoped graphene, Andreev reflection is an inter-band scattering at all excitation energies, which is not possible in regular metals, with no excitation gap separating the conduction and valence bands. In the regime $0 < E_F < \Delta$, depending on the incident electron energy $E$, the Andreev reflection has a form of either normal or specular.

The tunneling conductance [18, 20] and Josephson current [21] in graphene junctions are oscillatory functions.
of the width and height of the barrier at the interface.

It is known that, in conventional NS junctions, electric and thermal conductances reflect the magnitude or symmetry of the superconductor energy gap [1, 2]. Compared to the linear temperature dependence in a bulk superconductor, explained from the Wiedemann-Franz law, its thermal conductivity is exponentially suppressed at the low temperatures $k_B T \ll \Delta_0$ [22], because Andreev reflection completely blocks the subgap flow of energy and thus their thermal conductivity is often negligible. The story is considerably different in hybrid mesoscopic structures of NS because the heat flow through superconductor may become significant [23]. Applying an electric current in the normal metal-superconductor junction, is used for refrigeration of electrons in the normal metal which can be used for the realization of microcoolers [24–26], high-sensitive detectors and quantum devices [27, 28].

While charge and thermal conductance in normal/superconductor junction of the isotropic Dirac materials have been studied [3, 8, 16–21, 29–35], but to our knowledge, thermal Andreev reflection in a zero-gap semiconductors, with tilted anisotropic Dirac cones, have not yet been reported in the literature. Tilted Dirac have been predicted in a series of materials, including deformed graphene [36, 37], partially hydrogenated Dirac [38], the surface of topological crystalline insulators, such as the (001) surface of SnSe [39, 40], organic compound $\alpha$-(BEDT-TTF)$_2$I$_3$ [36, 41–43], 8-$Pmmn$ borophene [44–49].

Among various allotropes of boron, as carbons neighbor in the periodic table, graphene-like two-dimensional (2D) structure of boron known as borophene has attracted great attention due to its fascinating properties and promising applications in nanoelectronics [68–71].

However, theoretical calculations show that due to the imaginary frequencies in phononic dispersion of free-standing 2D borophene is unstable against long-wavelength periodic vibration [52, 72], needing a substrate to be stabilized. A feasible proposal to dynamically stable borophene is the chemical fictionalization using surface hydrogenation. First-principles calculations show that the hydrogenation of borophene, is a viable method to stabilize borophene in the vacuum without a substrate [52]. Compared with that of borophene, borophene, a monolayer of fully hydrogenated borophene has a remarkable Fermi velocity which is nearly four times higher than that of graphene. It displays a huge electrical and magnetic anisotropy [53–55], along with highly anisotropic mechanical properties [56]. In contrast to the buckled direction, that shows off a semiconductor behavior, along the valley direction, borophane shows a metallic trend with a linear current-voltage curve [73]. Lower crystal symmetry of borophane, in contrast to graphene, causes the asymmetric velocity parameters which results in the two tilted Dirac cones at $K_D$ and $-K_D$, in the effective low-energy Hamiltonian of borophane. The Bravais lattice constants of the conventional orthorhombic unit cell of the buckled crystal structures of borophene and borophane, are $a_x = 1.62 \text{ Å}$, $a_y = 2.85 \text{ Å}$ and $a_z = 1.92 \text{ Å}$ and $a_x = 2.81 \text{ Å}$, respectively [50–52] and contains 4 atoms per unit cell. Notice that the buckling height of $h = 0.96 \text{ Å}$ in borophane reduces to $h = 0.81 \text{ Å}$, upon hydrogen adsorption in borophane [55].

Due to the electrons in Fermi surface arising from the hybridized states of the $\sigma$ and $\pi$ bond, 2D boron structure may be a pure single-element intrinsic superconducting material with the highest $T_c$ (higher than the liquid hydrogen temperature) on conditions without high pressure and external strain which can be modified by strain and doping [74–76].

A first-principles study reveals that borophene is the first known materials with high-frequency plasmons in the visible spectrum [77]. Furthermore, in this borophene polymorph, the anisotropic plasmon mode remains undamped for higher energies along the mirror symmetry direction in which the anisotropic Friedel oscillation in borophene behaves like $r^{-3}$ in the large-$r$ limit [78].

Motivated by the great interest in search of tilted Dirac material, we wish to examine whether the tilt leads to qualitatively different physics in normal-metalsuperconductor hybrid junction of borophane. However, to our best knowledge, answers to these question is still lacking. In this paper, using the extended Blonder-Tinkham-Klapwijk formalism, we study the charge and heat transport in a NS hybrid contact based on the fully hydrogenated borophene (borophane) and in the 8-$Pmmn$ 2D boron Polymorph.

The rest of the paper is organized as follows. In Sec. II, the model Hamiltonian and lattice structure of borophane is introduced and then the method which is used to calculate the differential conductance of charge and heat transport in normal-metalsuperconductor hybrid junction of borophane is explained using the extended Blonder-Tinkham-Klapwijk formalism in Sec. III. In Sec. IV, we present and describe our numerical results. Finally, we conclude and summarize our main results in Sec. V.

II. MODEL HAMILTONIAN OF SUPERCONDUCTING BOROPHENE

In the following we consider a two dimensional normal/superconducting borophane junction, occupying the $xy$ plane while the superconducting region occupies $x > 0$ (region S) and the normal region extending $x < 0$ (region N). The proposed setup is schematically shown in Fig. II. Consider the BCS pairing in the S region, the electron and hole excitations are described by the Bogoliubov-de Gennes (BdG) equation which has the form [79]

$$
\begin{pmatrix}
H_0(k) - \mu(r) & \Delta_S(r) \\
\Delta_S^*(r) & \mu(r) - H_0^*(k)
\end{pmatrix}
\begin{pmatrix}
u \\
\bar{v}
\end{pmatrix} = \varepsilon
\begin{pmatrix}
u \\
\bar{v}
\end{pmatrix},
$$

in which $H_0(k)$ is the effective single-particle low-
energy Hamiltonian for borophane, for excitations near
the two Dirac points $K_D = (\pm 0.64, 0) \text{Å}^{-1}$ [78, 80, 81].

\[ H_0(k) = \hbar v_{0x} k_x \sigma_x + \hbar v_{0y} k_y \sigma_y + \hbar v_1 k_z \sigma_0 - U(r) \]  

(2)

Here, $\sigma_x, \sigma_y$ are the Pauli matrices for the pseudospin
representing the lattice degree of freedom while $\sigma_0$ is
the $2 \times 2$ identity matrix. The suggested values of the
direction-dependent velocities, in units of $(x10^5 \text{ m/s})$, are
specified as $v_{0x} = 19.58, v_{0y} = 6.32,$ and $v_t = -5.06.$

$\mu(r)$ and $\Delta_S(r)$ are the chemical potential and superconducting
order parameter, respectively, which can be
expressed as

\[ \Delta_S(r) = \begin{cases} 0 & \text{if } x < 0, \\ \Delta_0 & \text{if } x > 0 \end{cases} \]  

(3)

A superconducting electrode covers the region $x > 0,$
thus the electrostatic potential $U(r)$ is taken to be

\[ U(r) = \begin{cases} 0 & \text{if } x < 0, \\ -U_0 & \text{if } x > 0 \end{cases}. \]  

(4)

The energy dispersion for quasiparticles in the superconducting
region is written as

\[ E_S(k) = U(r) + v_t k_x + \tau \sqrt{\Delta_0^2 + (\mu_S + \sqrt{v_{0x}^2 k_x^2 + v_{0y}^2 k_y^2})^2}, \]  

(5)

in which $\tau = 1(-1),$ denotes the conduction (valence) band in borophane.

Two right-moving and two left-moving modes for a
fixed energy two electron and two hole modes in the normal borophane regions are written as

\[ E_{e(h)}^c(k) = \hbar v_{1e} k_x + \hbar \sqrt{v_{0x}^2 k_x^2 + v_{0y}^2 k_y^2} + \mu, \]  

(6)

\[ E_{e(h)}^v(k) = \hbar v_{1e} k_x - \hbar \sqrt{v_{0x}^2 k_x^2 + v_{0y}^2 k_y^2} + \mu, \]  

(7)

in which $c(v)$ denote the conduction (valence) band in borophane and $k = \sqrt{k_x^2 + k_y^2}.$ The anisotropic and tilted
Dirac crossing along the $\Gamma-Y$ direction in the rectangular
Brillouin zone of borophane is obtained from the band dispersion Eqs.6 and 7, as

The eigenfunctions of the Dirac-Bogoliubov quasiparticles of energy $\varepsilon$ are given by

\[ \psi^S_+ = e^{ik_z x} e^{iqy} \begin{pmatrix} a_+ \\ b_+ \\ c_+ \\ 1 \end{pmatrix}, \]  

(8)

\[ \psi^S_- = e^{-ik_z x} e^{iqy} \begin{pmatrix} a_- \\ b_- \\ c_- \\ 1 \end{pmatrix}, \]  

(9)

from the $\varepsilon^+$ and $\varepsilon^-$ branches of the spectrum, respectively,
which describing right-moving electron- and left-moving holelike quasiparticles that either decay exponentially as $x \to \infty$ (for subgap solutions when $\varepsilon < \Delta_0$) or
propagate along the $x$ direction (for supragap solutions when $\varepsilon > \Delta_0$).

\[ a_+ = \frac{\Delta_0(\Delta_0^2 + E_F^2 - w_+^2 + |Z_+|^2)}{2\Delta_0 E_F^2 Z_+}, \]  

(10)

\[ b_+ = \frac{\Delta_0^2(E_F^2 - w_+^2 + (E_F^2 + w_+^2)(E_F^2 - w_+^2) - |Z_+|^2)}{2\Delta_0 E_F^2 Z_+}, \]  

\[ c_+ = \frac{Z_+(-\Delta_0^2 + (E_F^2 - w_+^2) - |Z_+|^2))}{2\Delta_0 (E_F^2 - w_+^2) + (E_F^2 + w_+^2)((E_F^2 - w_+^2) - |Z_+|^2)}, \]  

where $Z_\pm = v_{0x} k_{e(h)}^\ell + iv_{0y} q$ and $w_\pm = u_0 + v_t k_{e(h)}^\ell - \varepsilon,$
with the longitudinal wave vector $k_{e(h)}^\ell$ for electronlike
(holelike) quasiparticles in S region, respectively, that are
the solutions of the energy-momentum relation, which
can be obtained by solving the following equation

\[ a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0 = 0 \]  

(11)

in which
calculate the expectation value of carrier velocity using all states carry the same amount of quasiparticle current and states of the form

$$a_0 = (\Delta_0^2 - \varepsilon^2 + E_F^2 + 2\varepsilon u_0 - u_0^2)^2 + 2(\Delta_0^2 - E_F^2 - (\varepsilon - u_0)^2)v_{0y}q^2 + v_{0y}q^4,$$
$$a_1 = 4(\varepsilon - u_0)v_1(\Delta_0^2 - E_F^2 - (\varepsilon - u_0)^2)^2,$$
$$a_2 = -2(E_F^2v_{0x}^2 + u_0^2v_{0y}^2 + E_F^2v_{0y}^2 - 3u_0v_{0x}^2 + \varepsilon^2(v_{0x}^2 - 3v_{0y}^2) - 2\varepsilon u_0(v_{0x}^2 - 3v_{0y}^2) + \Delta_0^2(-v_{0x}^2 + v_{0y}^2) + v_{0y}^2(-v_{0x}^2 + v_{0y}^2)q^2),$$
$$a_3 = 4(\varepsilon - u_0)(v_{0x} - v_1)v_1(v_{0x} + v_1),$$
$$a_4 = (v_{0x}^2 - v_{0y}^2)^2,$$

(12)

Inside N region, the solutions of BdG equation are two states of the form

$$\psi_{\pm} = \frac{1}{\sqrt{N_e}} e^{\pm i k_x x} e^{i q y} \begin{pmatrix} \tau e^{\pm i \beta_k^\pm} \\ 1 \\ 0 \\ 0 \end{pmatrix},$$

(13)

for the valence band electrons and

$$\psi_{\pm} = \frac{1}{\sqrt{N_h}} e^{\pm i \alpha k_x x} e^{i q y} \begin{pmatrix} 0 \\ \tau e^{\pm i \beta_k^\pm} \\ 1 \end{pmatrix},$$

(14)

$$\beta_k^{\pm} = \tan^{-1} \left[ \frac{v_{0y}q}{v_{0x}k_{x}^{(h)}} \right]$$

with \( \alpha = 1 \) for retroreflection (conduction band holes) and \( \alpha = -1 \) for specular reflection (valence band holes). It should be noted that, due to the translational invariance in the y direction, the corresponding component of momentum, \( q \) is conserved. The prefactors \( 1/\sqrt{N_e}, 1/\sqrt{N_h} \) ensure that all states carry the same amount of quasiparticle current. In order to obtain these normalizing factors, we calculate the expectation value of carrier velocity using the formula \( j_x = \psi^* \frac{\partial \psi}{\partial k_x} \psi = v_x \). Furthermore, the \( x \) and \( y \) components of the velocity operator is defined as \( v = \frac{1}{h} \nabla_k E(k) \) and can be derived as

$$v_{x,\tau} = v_t + \tau \frac{v_{0y}^2 k_{x}^{(h)}}{\sqrt{v_{0x}^2 k_{x}^{(h)} - v_{0y}^2 q^2}},$$

(15)

$$v_{y,\tau} = \tau \frac{v_{0y}q}{\sqrt{v_{0x}^2 k_{x}^{(h)} - v_{0y}^2 q^2}},$$

(16)

### III. Model and Theory

Let us consider an incident electron from the normal side of the junction with energy \( \varepsilon \) and transverse momenta \( q \). Taking into account both Andreev and normal reflection processes, the wave functions in the normal and superconducting regions, can be written as

$$\psi_N = \psi_N^e + r_e \psi_N^h - r_A \psi_N^h,$$

(17)

$$\psi_S = t_e \psi_S^e + t_h \psi_S^h,$$

(18)

Here, \( \psi_N^{e,h} \) and \( \psi_S^{e,h} \) are the solutions of BdG equation for the quasiparticles inside N and S regions, respectively. \( r_e \) and \( r_A \) denote the reflection coefficients of normal and Andreev reflections, respectively. The superscripts "e" and "h" denote the electronlike and holelike excitations, respectively.

The boundary conditions for the wave functions and current conservation in the \( x \) direction, at the interface can be written as

$$\psi_N \big|_{x=0} = \psi_S \big|_{x=0},$$

$$v_{x,N} \psi_N \big|_{x=0} = v_{x,S} \psi_S \big|_{x=0}$$

(19)

which leads to the analytical expressions for the reflection coefficients \( r_e \) and \( r_A \) as:

$$r_e = \frac{-a_+ c_+ + a_- c_+ + b_+ c_+ e^{i \beta_k^h} - b_- c_+ e^{i \beta_k^h} + e^{i \alpha \beta_k^h} (a_- - a_+ + (-b_- + b_+) e^{i \beta_k^h}) \alpha}{a_+ c_- - a_- c_+ + b_+ c_- e^{i \beta_k^h} + b_- c_- e^{i \beta_k^h} + e^{i \alpha \beta_k^h} (-a_- + a_+ + (b_- - b_+) e^{i \beta_k^h}) \alpha},$$

$$r_h = \frac{a_+ c_+ + a_- c_+ + b_+ c_+ e^{i \beta_k^h} - b_- c_+ e^{i \beta_k^h} + e^{i \alpha \beta_k^h} (a_- - a_+ + (-b_- + b_+) e^{i \beta_k^h}) \alpha}{(c_- - c_+)(e^{i \beta_k^h} - e^{i \beta_k^h})}$$

(20)

As we know, an incident electron with a subgap energy \( \epsilon < \Delta_0 \) from the conduction-band, can be either nor-
FIG. 2. Schematic illustration of the reflection and transmission processes in the NS junction of borophane. The filled and open circles denote the electron and a hole, respectively. When the excitation energy becomes smaller than the superconducting order parameter in the normal region (\(\varepsilon < \Delta_0\)) there are no propagating modes in the superconducting side, so that electrons injected from the normal region are reflected back either as an electron or a hole. However, there are two scattering processes for holes: if the excitation energy \(\varepsilon\) becomes smaller (larger) than the normal region Fermi energy, \(\varepsilon < E_F\) (\(\varepsilon > E_F\)), electron and hole originate from the same band (retroreflection-left panel) or from different bands (specular reflection-middle panel) of borophane dispersion, respectively. When Andreev reflection takes place, the transmitted Cooper pair is located at the Fermi level of the superconductor. In S region (right panel), the blue arrow represents an evanescent mode (when \(\varepsilon < \Delta_0\)), while the black ones denotes the propagating modes (when \(\varepsilon > \Delta_0\)).

normally reflected as an electron or Andreev reflected as a hole in the same band.

A. Andreev conductance

In this section, we investigate properties of the differential tunneling conductance of the borophane NS junction using the well-known Blonder-Tinkham-Klapwijk (BTK) formula [2, 82]

\[
G(\varepsilon) = G_0 \sum_{\pm K_0} \sum_{q} \left[1 - |r(\varepsilon, q)|^2 + \frac{|p_h| \cos \phi_A \cos \phi}{|p_e| \cos \phi} |r_A(\varepsilon, q)|^2 \right],
\]

(21)

where \(G_0 = \frac{\pi^2}{\hbar} N(\varepsilon)\) is the ballistic conductance of a monolayer borophane N transverse modes [83, 84], and \(P_e(h)\) is the wavevector of the electron (hole) quasi-particles, inside the N region.

The density of states \(N(\varepsilon)\), can be obtained by solving the following equation

\[
N(\varepsilon) = \frac{1}{(2\pi)^2} \int_0^\infty k' dk' \delta(\varepsilon - \varepsilon_{k'})
\]

(22)

Performing this integral over energy, one finds the following expansion for the density of states:

\[
N(\varepsilon) = \frac{k^\ast(\varepsilon, \phi)}{v_t \cos(\phi) + \sqrt{v_{0x}^2 \cos^2(\phi) + v_{0y}^2 \sin^2(\phi)}}
\]

(23)

where \(\phi = \tan^{-1}(k_y/k_x)\) and the wave vector \(k^\ast(\varepsilon, \phi)\) is given by

\[
k^\ast(\varepsilon, \phi) = \frac{-2v_t \cos(\phi) + \tau \sqrt{2} \sqrt{v_x^2 + v_y^2 + (v_x^2 - v_y^2) \cos(2\phi)}}{v_x^2 + v_y^2 - v_t^2 + (v_x^2 - v_y^2 - v_t^2) \cos(2\phi)},
\]

(24)

B. Heat transport by normal/superconducting borophane junctions

In order to investigate the thermal transport properties of the proposed N/S structure, assuming a temperature gradient \(\Delta T\) through the junction, using relation \(\kappa = \lim_{\Delta T \to 0} J_Q/\Delta T\), with \(J_Q\) the heat current density, we present the behavior of the thermal conductance given by [87, 88]

\[
\kappa = \kappa_0 \int_0^\infty d\varepsilon \int_{-\pi/2}^{\pi/2} d\phi \cos \phi \left[1 - |r(\varepsilon, \phi)|^2 - \frac{|p_h| \cos \phi_A \cos \phi}{|p_e| \cos \phi} |r_A(\varepsilon, \phi)|^2 \right] \frac{\varepsilon^2}{(k_B T)^2 \cosh^2(\frac{\varepsilon}{2T})},
\]

(25)

in which \(\kappa_0 = \frac{k_B W}{8\pi^2 \hbar}\) is a constant parameter corresponding to the N/N metallic conductance [85] the thermal
conductance at zero temperature. We replace the zero-temperature superconducting order parameter $\Delta_S$ in Eq. (1) with the temperature-dependent one, $\Delta_S(T) = 1.76 k_B T_C \tanh(1.74 \sqrt{T_C / T - 1})$. Here $T_C$ is the critical temperature of the superconductor.

IV. NUMERICAL RESULTS

In this section, based on Eqs. 21 and 25, we present our numerical results for the AR process of the N/S hybrid structure of borophane in the physical regime. Since attaining the regime $E_F \ll \Delta_0$, in experiments may be difficult, so it is of importance to consider the regime of comparable $E_F$ and $\Delta_0$, in which retro-reflection changes to specular Andreev reflection. We set $\Delta_0 = 0.01eV$ for zero temperature order parameter in all our results presented in this section, with $E_F$ and $E_F'$, the Fermi energies in the normal and superconducting regions, respectively. Let us now consider the regime where the Fermi surfaces of the normal metal and the superconductor is aligned $U(r) = 0$.

To calculate the probability of the electron-hole conversion for subgap energies ($\varepsilon < \Delta_0$), we show the behavior of the probability of normal and Andreev processes for an incident electron with a subgap energy $\varepsilon / \Delta_0 = 0.01$, in terms of the angle of incidence, in Fig. 20 for several values of the normal region chemical potential $E_F$. Panel (a) is for $E_F' / \Delta_0 = 10$ and panel (b) is for $E_F' / \Delta_0 = 10^3$.

Since the transmission into the superconductor region is forbidden for subgap energies ($\varepsilon < \Delta_0$), one obviously verifies that $|r_A|^2 + |r_A'|^2 = 1$. The electron-hole conversion at normal incidence ($\phi = 0$), happens with unit probability ($|r_A|^2 = 1$), regardless of the amount of the Fermi energy of the normal borophane. This is completely different from usual normal-metal-superconductor contact, in which Andreev reflection is suppressed at any angle of incidence, if the Fermi wave lengths at the two sides of the interface are very different.

More interestingly, in the heavily doped superconducting regime ($E_F' / \Delta_0 \gg 1$), for heavily doped normal region ($E_F / \Delta_0 \gg 1$), only normal Andreev reflection is present for any incident angle.

As the experimentally measurable Andreev conductance contains both the conductance due to Andreev reflection and electron tunneling, we present the behavior of the normalized total Andreev conductance of the N/S structure $G / G_0$ as a function of the bias voltage $\varepsilon / \Delta_0$ in Fig. 4, for different values of normal borophane Fermi energy $E_F / \Delta_0$.

As seen in this figure, the conductance displays two limiting behaviors for $E_F \gg \Delta_0$ or $E_F \ll \Delta_0$. At $E_F = 0$ a sharp coherence peak is appeared in the differential conductance. For the case of $E_F \ll \Delta_0$, behavior of the differential conductance is similar to the normal metal-insulator-superconductor (NIS) junction of graphene [18]. As usual for an NS junction [86], the conductance has a singularity at $\varepsilon = \Delta_0$. As bias voltage increases toward $\varepsilon / \Delta_0 = 1$, the conductance becomes larger toward $G = 2G_0$ then with increasing the bias voltage, the conductance becomes lower toward a saturation conductance. Interestingly, a zero-bias conductance peak appears in the conductance spectra. This two peak structure is just similar to the case of normal metal-insulator d-wave superconductor junction [89].

In Fig. 5, we briefly explore the behavior of the differential conductance of a NS junction of borophane, in the heavily doped superconducting regime ($E_F' / \Delta_0 \gg 1$), where $E_F' / \Delta_0 = 1000$. In contrast to the above considered limit (low doping of the superconducting region), the conductance peak related to the bias voltage of $\varepsilon / \Delta_0 = 1$ becomes smooth as $E_F$ increases. It has been shown that, for subgap energies ($\varepsilon / \Delta_0 < 1$), in the regime $E_F' = E_F$, the standard situation of perfect Andreev reflection is recovered, with a sharp drop at the gap edge, corresponding to the onset of quasiparticle transmittance into the superconductor side. Exactly the same as unconventional anisotropic d-wave superconductor-graphene junctions, the subgap conductance is always close to $2G_0$, but becomes more constant with increasing $E_F$.

A similar qualitative behavior for the differential conductance occurs when $E_F' = E_F$. As we can see in

![Fig. 3. Normal and Andreev reflection probabilities versus the angle of incidence for $\varepsilon / \Delta_0 = 0.01$, for several values of the normal region chemical potential ($E_F / \Delta_0$). (a) for $E_F' / \Delta_0 = 10$ and (b) for $E_F' / \Delta_0 = 10^3$.](image)
Andreev conductance saturated to a constant value $G = G_0$, where for $E_F/\Delta_0 > 500$, the differential conductance is approximately independent of the Fermi energy of the normal borophane.

On the other hand, study of the dependence of the differential Andreev conductance $G/G_0$ on the Fermi energy of the normal region ($E_F/\Delta_0$), is crucial for experimental accessibility of our proposals. We give the results for $E_F/\Delta_0 = 100$, in the two regimes $\varepsilon/\Delta_0 > 1$ and $\varepsilon/\Delta_0 < 1$ in the Figs. 7 and 8, respectively.

For $\varepsilon/\Delta_0 > 1$, the conductance related to the nearly zero Fermi energy ($E_F \to 0$), for each value of the bias voltage $\varepsilon$, reached to value $G = G_0$. In the high bias

Fig. 6, a two peak structure appears for the case where $E_F/\Delta_0 = 1$. In the limit of very large $E_F/\Delta_0$, the standard situation of perfect Andreev reflection is recovered.

The same as Figs. 4 and 5, but now for when $E_F/\Delta_0 = 10^3$. For $\varepsilon/\Delta_0 < 1$, the conductance related to the nearly zero Fermi energy ($E_F \to 0$), increases with increasing the bias voltage $\varepsilon$, reached to value $G = 4G_0$ for $\varepsilon/\Delta_0 = 1$. In the limit of $E_F/\Delta_0 = 1$, however, the conductance related to each bias voltage is identical.

For $\varepsilon/\Delta_0 > 1$, behavior of the differential conductance is entirely different. With increasing the bias voltage, the Andreev conductance saturated to a constant value $G = G_0$, where for $E_F/\Delta_0 > 500$, the differential conductance is approximately independent of the Fermi energy of the

FIG. 4. Normalized Differential conductance (in units of the ballistic value $G_0 = 4Ne^2/h$) of the borophane N/S junction, for several values of the normal region chemical potential $E_F/\Delta_0$ for $E_F/\Delta_0 = 10$.

FIG. 5. Normalized Differential conductance (in units of the ballistic value $G_0 = 4Ne^2/h$) of the borophane N/S junction, for several values of the normal region chemical potential $E_F/\Delta_0$ for $E_F/\Delta_0 = 10^3$.

FIG. 6. Same as Figs. 4 and 5, but now we have fixed $E_F = E_F$ for all curves.

FIG. 7. Dependence of the normalized differential conductance (in units of the ballistic value $G_0 = 4Ne^2/h$) of the borophane N/S junction, on the chemical potential $E_F/\Delta_0$, for $E_F/\Delta_0 = 100$. 

$\varepsilon/\Delta_0 = 4$ $\varepsilon/\Delta_0 = 0$ $\varepsilon/\Delta_0 = 0.01$ $\varepsilon/\Delta_0 = 0.1$ $\varepsilon/\Delta_0 = 1$ $\varepsilon/\Delta_0 = 10$ $\varepsilon/\Delta_0 = 100$ $\varepsilon/\Delta_0 = 1000$ $G/G_0$ $E_F/\Delta_0 = 50$ $E_F/\Delta_0 = 100$ $E_F/\Delta_0 = 200$ $E_F/\Delta_0 = 300$ $E_F/\Delta_0 = 400$ $E_F/\Delta_0 = 800$ $E_F/\Delta_0 = 1000$
voltage regime ($\varepsilon/\Delta_0 \gg 1$), Andreev conductance is independent of the normal region Fermi energy.

For $\varepsilon/\Delta_0 < 1$, the Andreev conductance related to the zero Fermi energy ($E_F \rightarrow 0$), has an increasing behavior with increasing the bias voltage $\varepsilon$, and reaches a maxium value ($G(E_F = 0) = 4G_0$) at zero Fermi energy.

In the limit of $E'_F = E_F$, however, the conductance related to each bias voltage reaches a constant value ($G(E'_F = E_F) = 2G_0$). Turning on the normal region Fermi energy, away from the zero Fermi energy, the Andreev conductance suddenly drop to a lower constant value for $\varepsilon/\Delta_0 = 1$. It is seen that in a certain Fermi energy, with increasing the bias voltage $\varepsilon/\Delta_0$, the conductance increases such that saturates toward a constant value $G = 2G_0$.

We now proceed to investigate how the thermal conductance spectra of a N/S borophane change with temperature. Figure 9 shows the normalized thermal conductance as a function of the temperature $T/T_C$ for various $E_F$ with $E'_F/\Delta_0 = 1$. We obtain an exponential dependence of the thermal conductance on temperature. Contrary to the graphene NS junctions [3, 4], thermal conductance decreases by increasing the temperature. The anomalous Nernst and thermal Hall effects in a semimetal, with two linearized-tilted Dirac cones, has already been demonstrated [91, 92]. At a certain temperature, the thermal conductance decreases with increasing the Fermi energy $E_F/\Delta_0$.

Figure 10 shows normalized thermal conductance as a function of $T/T_C$ for various $E_F$ for heavily doped superconducting regime ($E'_F \gg \Delta_0$, where $E'_F/\Delta_0 = 100$). A similar qualitative behavior is seen for the thermal conductance, when $E_F/\Delta_0 = 100$, as seen in Fig. 10. As can be seen, as doping of the superconducting region increases, the thermal conductance falls off at a higher rate.

V. CONCLUSION

We study charge and heat transport in normal-metal/superconductor (NS) hybrid junction, based on
a tilted anisotropic Dirac material. Using the extended Blonder-Tinkham-Klapwijk formalism, the conductance spectra of NS borophane, a two-dimensional Dirac semimetal with two tilted anisotropic Dirac cones in its dispersion, is investigated. Completely different from the usual normal-metal-superconductor junctions, in spite of the large mismatch in Fermi wavevectors of the normal-metal and superconductor sides of the borophane NS junction, the electron-hole conversion happens with unit probability at normal incidences. Furthermore, we demonstrate that in the heavily doped superconducting regime for heavily doped normal borophane, the electron-hole conversion happens with unit probability, approximately at any incident angle. Interestingly, a zero-bias conductance peaks appears in the conductance spectra. This two peak structure is just similar to the case of normal metal-insulator d-wave superconductor junction.

Exactly the same as unconventional anisotropic d-wave superconductor-graphene junctions, the subgap conductance is always close to $2G_0$, but becomes more constant with increasing $E_F$. The dependence of the Andreev with Fermi energy and bias voltage, enable us, selecting the retroconfiguration or specular configuration in types of Andreev reflection processes. We numerically establish an anomalous behaviour of thermal conductance in borophane. The tilting of the Dirac cones gives rise to an anomalous behaviour in the thermal conductance of the NS hybrid junction of borophane, such that the thermal conductance decreases by increasing the temperature. Our findings will have potential applications for transport and energy control in superconducting quantum interference devices and hybridized mesoscopic systems.

VI. ACKNOWLEDGMENTS

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