$D_{sJ}(2317)$ meson production at RHIC

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Production of $D_{sJ}(2317)$ mesons in relativistic heavy ion collisions at RHIC is studied. Using the quark coalescence model, we first determine the initial number of $D_{sJ}(2317)$ mesons produced during hadronization of created quark-gluon plasma. The predicted $D_{sJ}(2317)$ abundance depends sensitively on the quark structure of the $D_{sJ}(2317)$ meson. An order-of-magnitude larger yield is obtained for a conventional two-quark than for an exotic four-quark $D_{sJ}(2317)$ meson. To include the hadronic effect on the $D_{sJ}(2317)$ meson yield, we have evaluated the absorption cross sections of the $D_{sJ}(2317)$ meson by pion, rho, anti-kaon, and vector anti-kaon in a phenomenological hadronic model. Taking into consideration the absorption and production of $D_{sJ}(2317)$ mesons during the hadronic stage of heavy ion collisions, we find that the final yield of $D_{sJ}(2317)$ mesons remains sensitive to its initial number produced from the quark-gluon plasma, providing thus the possibility of studying the quark structure of the $D_{sJ}(2317)$ meson and its production mechanism in relativistic heavy ion collisions.

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I. INTRODUCTION

A narrow $D_{sJ}(2317)$ meson was recently observed by the BABAR Collaboration [1] in the inclusive $D^{*+}_s\pi^0$ invariant mass distribution from $e^+e^-$ annihilation and confirmed by the Belle Collaboration in $B$ meson decay [2]. This meson has the natural spin-parity $J^P = 0^+$ and a mass below what was obtained from the QCD sum rule approach [3] and quark model calculations [4] for a normal two-quark state $c\bar{s}$. The $D_{sJ}(2317)$ meson has thus been considered as a possible candidate for the exotic four-quark states that were studied in the bag model [5, 6], QCD sum rules [7] and the non-relativistic potential model [8]. It is also possible that the $D_{sJ}(2317)$ meson is simply a $DK$ molecule or atom. Determination of the $D_{sJ}(2317)$ meson width is limited by experimental resolutions to a value of less than 4.6 MeV/c$^2$ [2]. The small width of $D_{sJ}(2317)$ meson is not surprising as its mass is below the threshold of $DK$ system and can only decay into the kinematically allowed but isospin violated channel of $D\pi$ state. Theoretically, the decay width of $D_{sJ} \rightarrow D_s\pi$ has been studied using the QCD sum rules, and its value varies from a few keV [9] to a few tens keV [10] depending on the assumed flavor state of four quarks or two quarks, respectively. A more phenomenological approach based on the $^3P_0$ model [11] also gives a narrow width of about a few tens keV for a normal two-quark $D_{sJ}(2317)$ meson.

Studying the mechanism for $D_{sJ}(2317)$ meson production in nuclear reactions is useful for understanding its quark structure. In Ref. [12], a coupled-channel quark model has been used to study the production of a two-quark $D_{sJ}(2317)$ meson and its radial excitations in hadronic reactions. Production of $D_{sJ}(2317)$ in relativistic heavy ion collisions has also been studied [13]. It was found that for a four-quark $D_{sJ}(2317)$ meson, a much large yield is obtained if one takes into account the diquark-diquark interactions in the produced quark-gluon plasma. Since the $D_{sJ}(2317)$ meson is not expected to survive in the quark-gluon plasma, it is more likely to be produced at hadronization of the quark-gluon plasma either statistically or via quark coalescence. Its final abundance in a heavy ion collision depends, however, also on its absorption and production probability in the subsequent hadronic matter.

In the present paper, we study $D_{sJ}(2317)$ meson production in central heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) in a kinetic model that starts from the final stage of the quark-gluon plasma, goes through a mixed phase of quark-gluon and hadronic matters, and finally undergoes the hadronic expansion. The production of $D_{sJ}(2317)$ mesons from the quark-gluon plasma is modeled by the constituent quark coalescence model, which has been shown to describe reasonably not only the particle yields and their ratios [14] but also their transverse momentum spectra and anisotropic flows [15, 16, 17]. The predicted number of $D_{sJ}(2317)$ mesons is found to depend on its quark structure, with the two-quark state giving an order-of-magnitude larger value than the four-quark state. The $D_{sJ}(2317)$ meson can be absorbed and also produced in subsequent hadronic matter via the reactions $\pi D_{sJ} \rightarrow K^* D(K^*)$, $\rho D_{sJ} \rightarrow KD$, $KD_{sJ} \rightarrow \rho D(\pi D^*)$, and $K^* D_{sJ} \rightarrow \pi D$. The cross sections for these reactions are evaluated in a phenomenological hadronic model with coupling constants and form factors involving the $D_{sJ}(2317)$ meson determined from the QCD sum rules. Taking into account the hadronic absorptions, the predicted number of $D_{sJ}(2317)$ mesons produced in central heavy ion collisions is shown to be of the order of 1000 per event.

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effect in heavy ion collisions via a kinetic approach, we find that the final number of $D_{sJ}(2317)$ mesons remains sensitive to its initial number produced from the quark-gluon plasma. Studying $D_{sJ}(2317)$ meson production in heavy ion collisions thus provides the possibility to study both its production mechanism and quark structure.

This paper is organized as follows. In Section II, the dynamics of heavy ion collisions at RHIC is described. Production of $D_{sJ}(2317)$ meson from the initial quark-gluon plasma via the quark coalescence model is discussed in Section III. The absorption cross sections, is described in Appendix B. Values of other parameters of the fire-cylinder are determined from fitting the measured transverse energy $\varepsilon_{\text{trans}} \approx 788$ GeV as well as the extracted freeze out temperature $T_F = 125$ MeV and transverse flow velocity $\approx 0.65c$ of midrapidity hadrons in central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. Assuming that the fire-cylinder expands isentropically, the fraction of hadronic matter during the mixed phase is found to increase approximately linearly, and the time dependence of the temperature of the fire-cylinder obtained in Ref.18 can be parameterized as

$$T(\tau) = T_C - (T_H - T_F) \left( \frac{\tau_H - \tau}{\tau_F - \tau_H} \right)^{0.8},$$

where $T_H$ is the temperature of the hadronic matter at the end of the mixed phase and is thus the same as the critical temperature $T_C$ for the quark-gluon plasma to hadronic matter transition. As in Ref.18, we take $T_H = T_C = 175$ MeV. The freeze out temperature $T_F = 125$ MeV then leads to a freeze out time $\tau_F \approx 17.3$ fm/c.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{(Color online) Time evolution of the abundance of pions, kaons, anti-kaons, and nucleons including contributions from decays of resonances (a) and the ratio of anti-baryon to baryon abundances (b) of mid-rapidity particles in central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV.}
\end{figure}

For normal hadrons such as pions, kaons, anti-kaons, and nucleons, they are taken to be in chemical equilibrium with baryon chemical potential $\mu_B = 30$ MeV, charge chemical potential $\mu_Q = 0$ MeV, and strangeness chemical potential $\mu_S = 10$ MeV. The nonzero strange chemical potential is needed to account for observed $K^+ / K^-$ ratio in medium. Ne-
glecting the time dependence of the chemical potentials, which have been shown to vary weakly with the temperature of an isentropically expanding matter in heavy ion collisions at RHIC \cite{23}, time evolution of the abundance of pions, kaons, anti-kaons, and nucleons has been shown in Fig. 1 of Ref. \cite{13}. Including also contributions from decays of resonances, time evolution of the abundance of these hadrons is shown in Fig. 1(a). It is seen that the total numbers of pions, kaons and anti-kaons do not change much during the hadronic stage while the nucleon number decreases significantly as temperature drops. Their final numbers at freeze out are 926, 133, 113, and 47, respectively, and are comparable to those measured in experiments. In Fig 1(b), the time evolution of the ratio of anti-baryons to baryons is shown, and it changes from an initial value of 0.75 to a final value of 0.66, which is also comparable to the measured value of about 0.7.

III. D_{sJ}(2317) MESON PRODUCTION FROM THE QUARK-GLUON PLASMA

A. The coalescence model

In the coalescence model, the production of D_{sJ}(2317) mesons that are produced from the quark-gluon plasma is given by the product of a statistical factor g_{D_{sJ}} which denotes the probability of finding these quarks in any one of these color-spin-isospin basis states, i.e., \( g_{D_{sJ}} = 1/3^2 \times 1/2^2 = 1/36 \) or \( 1/3^4 \times 1/2^4 = 1/1296 \) for a two-quark or four-quark \( D_{sJ}(2317) \) meson, respectively.

The \( \sigma \) in Eq. (3) denotes an element of a space-like hypersurface \( \sigma_C \) at hadronization \( \Delta t \). In terms of the proper time \( \tau = (t^2 - z^2)^{1/2} \), the longitudinal momentum-energy rapidity \( \eta = (1/2) \ln (p_L/p_T) \) and space-time rapidity \( \eta = (1/2) \ln (p_L/p_T) \), the polar coordinates \( r \) and \( \phi \) in the transverse plane, the covariant volume element can be written as \( p \cdot \sigma = \tau m r \cosh(y - \eta) r dr d\phi d\eta \).

For the phase-space distribution functions of quarks in the fire-cylinder, they are taken to be the same as in the description of the heavy ion collision dynamics, i.e., they are uniformly distributed in the transverse plane and their momentum distributions are relativistic Boltzmannian in the transverse direction but uniform in rapidity along the longitudinal direction. Also, the Bjorken correlation of equal spatial \( \eta \) and momentum \( y \) rapidities is imposed, which is consistent with the small difference \( |y - \eta| \leq 0.5 \) seen in the transport model \cite{22}. Explicitly, the quark momentum distribution per unit rapidity at \( T_C \) is

\[
\begin{align*}
\frac{d^2f_q}{d^3p} &= \frac{g_\delta(\eta - y)}{(2\pi)^3} \frac{\gamma (m_T - p_T \cdot \beta - \mu_q)}{T_C} \exp \left( -\frac{\gamma (m_T - p_T \cdot \beta - \mu_q)}{T_C} \right),
\end{align*}
\]

where \( g_\delta = 6 \) is the color-spin degeneracy of a quark, \( \beta = (r/R) v_C \) is the radial-dependent transverse flow velocity, \( \gamma = (1 - \beta^2)^{-1/2} \), and the quark chemical potential \( \mu_q = \mu_b + \mu_s \). The abundances of quarks at the end of the QGP phase in central Au+Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV described by the expanding fire-cylinder model of previous section are \( N_u = N_s \approx 245 \), and \( N_b \approx 149 \) at \( \tau_C \), if we take into account the effect of gluons by converting them into quarks according to the quark flavor composition in the quark-gluon plasma as in Ref. \cite{17}. For charm quarks, we assume that they are in thermal equilibrium in the quark-gluon plasma, which is supported by the large elliptic flow of electrons from charm meson decays that are observed in experiments \cite{27,28} and transport models \cite{29,30}. Their number is, however, at present uncertain as it depends on whether charm quarks can be produced from the quark-gluon plasma. If we assume that the latter contribution is unimportant, then the number of charm quarks \( N_c \) produced in heavy ion collisions is simply given by the product of the charm quark number \( N_{NN}^{NN} \) produced from an initial hard nucleon-nucleon scattering and the total number of binary collisions (about 960) in central Au+Au collisions. Using \( N_{NN}^{NN} \) from the PYTHIA program, we obtain \( N_c \approx 1.5 \). The value of \( N_c \) increases to about 3 and 7 if we use the \( N_{NN}^{NN} \) from the PHENIX \cite{31} and...
the STAR experiment \[^{32}\text{STAR}\] respectively. In the following study, we will use \(N_c = 3\) for the calculation and discuss the sensitivity of our results to the change in the value for \(N_c\). For the charm quark mass, it is taken to be \(m_c = 1.5\) GeV.

For the Wigner distribution function of the \(D_{sJ}(2317)\) meson, instead of using a function of Lorentz invariant four-dimensional relative coordinates and momenta such as in Refs. \[^{12}\text{12}\] \[^{32}\text{32}\], we take it to be a function of three-dimensional relative coordinates and momenta for simplicity, based on the harmonic oscillator wave functions. Specifically, it is obtained from assuming that the wave functions of the quarks are those of a harmonic oscillator with an oscillator frequency \(\omega\). For a two-quark \(D_{sJ}(2317)\) meson with \(J^P = 0^+\), its \(c\bar{s}\) quarks are in the relative \(p\)-wave and its Wigner distribution function is thus \[^{33}\text{33}\]

\[
J^W_{D_{sJ}}(x;p) = \frac{(16y^2/3 - 8 + 16\sigma^2k^2)}{\exp\left(-\frac{y^2}{\sigma^2} - \frac{\sigma^2k^2}{2}\right)}.
\]

In the above, we have used the usual definitions \(y = x_1 - x_2\) and \(k = (m_s p_1 - m_c p_2)/(m_c + m_s)\) for the relative coordinate and momentum of the two quarks, respectively. For the width parameter \(\sigma\), it can be related to the oscillator frequency \(\omega\) by \(\sigma = 1/(\mu \omega)^{1/2}\) with the reduced mass \(\mu\) given by \(\mu = m_c m_s/(m_c + m_s)\).

If the \(D_{sJ}(2317)\) meson is a four-quark meson, its four quarks are all in relative \(s\)-waves, which then leads to the Wigner distribution function

\[
J^W_{D_{sJ}}(x;p) = \frac{(16y^2/3 - 8 + 16\sigma^2k^2)}{\exp\left(-\frac{y^2}{\sigma^2} - \frac{\sigma^2k^2}{2}\right)}.
\]

where the relative coordinates \(y_i\) and momenta \(k_i\) are related to the quark coordinates \(x_i\) and momenta \(p_i\) by the Jacobian transformations:

\[
\begin{align*}
y_1 &= \frac{x_1 - x_2}{\sqrt{2}}, \\
y_2 &= \sqrt{\frac{3}{2}} \left(\frac{m_c}{m_c + m_s} x_1 + \frac{m_s}{m_c + m_s} x_2 - x_3\right), \\
y_3 &= \sqrt{\frac{3}{4}} \left(\frac{m_c}{m_c + m_s + m_q} x_1 + \frac{m_s}{m_c + m_s + m_q} x_2 + \frac{m_q}{m_c + m_s + m_q} x_3 - x_4\right),
\end{align*}
\]

and

\[
\begin{align*}
k_1 &= \sqrt{\frac{m_c p_1 - m_c p_2}{m_c + m_s}}, \\
k_2 &= \sqrt{\frac{3 m_q (p_1 + p_2) - (m_c + m_s) p_3}{m_c + m_s + m_q}}, \\
k_3 &= \sqrt{\frac{3 m_q (p_1 + p_2 + p_3) - (m_c + m_s + m_q) p_4}{m_c + m_s + 2m_q}},
\end{align*}
\]

It can be shown that the product of the Jacobians for the coordinate and momentum transformations is equal to one.

The width parameter \(\sigma\) for the \(i\)-th relative coordinate in a four-quark \(D_{sJ}(2317)\) meson is again given by \(\sigma_i = 1/(\mu_i \omega)^{1/2}\) with the reduced masses

\[
\begin{align*}
\mu_1 &= \frac{2m_c m_s}{m_c + m_s}, \\
\mu_2 &= \frac{3 m_q (m_c + m_s)}{2 m_c + m_s + m_q}, \\
\mu_3 &= \frac{4 m_q (m_c + m_s + m_q)}{3 m_c + m_s + 2m_q}.
\end{align*}
\]

We note that the reduced mass \(\mu_1\) of the \(c\bar{s}\) quark pair in a four-quark \(D_{sJ}(2317)\) meson is a factor of two larger than that in a two-quark \(D_{sJ}(2317)\) meson due to differences in the definitions of the relative coordinate and momentum.

### B. Number of \(D_{sJ}(2317)\) mesons produced from the quark-gluon plasma

To evaluate the number of \(D_{sJ}(2317)\) mesons produced from the quark-gluon plasma requires information on the oscillator frequency \(\omega\) through the width parameter \(\sigma\) in the \(D_{sJ}(2317)\) meson Wigner distribution function, which is related to the size of \(D_{sJ}(2317)\). Since the latter is not known empirically, we choose the value of the oscillator frequency to fit instead the root-mean-square charge radius of the \(s\)-wave charmed \(D_s^+(c\bar{s})\) meson. Taking its Wigner distribution function similar to Eq. \((\text{8})\) but with only one relative coordinate and momentum, we obtain the following mean-squared charge radius for the \(D_s^+(c\bar{s})\) meson:

\[
\langle r_{D_s^+}^2 \rangle_{ch} = \frac{2}{3} \langle (x_1 - Y)^2 \rangle + \frac{1}{3} \langle (x_2 - Y)^2 \rangle = \frac{m_c^2 + 2m_s^2}{3 (m_c + m_s)} \langle y^2 \rangle = \frac{m_c^2 + 2m_s^2}{2 (m_c + m_s)^2} \langle y^2 \rangle.
\]

In the above, \(Y = (m_c x_1 + m_s x_2)/(m_c + m_s)\) is the center-of-mass coordinate of \(c\bar{s}\) quarks, and we have used the relation \(\langle y^2 \rangle = (3/2)\sigma^2\) between the mean-square width and the parameter for two quarks in the relative \(s\)-wave as in the \(D_s^+(c\bar{s})\) meson. Using the value \(\langle r_{D_s^+}^2 \rangle_{ch} \approx 0.124\) fm\(^2\) determined from the light-front quark model \[^{34}\text{34}\], we find that \(\sigma \approx 0.60\) fm and \(\hbar \omega \approx 300\) MeV.

For a two-quark \(D_{sJ}(2317)\) meson, whose quarks are in the relative \(p\)-wave, the relation between the mean-square distance of the two quarks and the width parameter is \(\langle y^2 \rangle = (5/2)\sigma^2\). Using above determined width parameter \(\sigma\), we obtain the following root-mean-square radius for a two-quark \(D_{sJ}(2317)\) meson:

\[
\langle r_{D_{sJ}}^2 \rangle_{two}^{1/2} = \frac{1}{\sqrt{2}} \frac{(m_c + m_s)^{1/2}}{m_c + m_s} \langle y^2 \rangle^{1/2}
\]
For a four-quark $D_{sJ}(2317)$ meson, its three size parameters are $\sigma_1 = 1/(\mu_1 \omega)^{1/2} \approx 0.42 \text{ fm}$, $\sigma_2 = 1/(\mu_2 \omega)^{1/2} \approx 0.58 \text{ fm}$, and $\sigma_3 = 1/(\mu_3 \omega)^{1/2} \approx 0.6 \text{ fm}$. The resulting root-mean-square radius of a four-quark $D_{sJ}(2317)$ meson is then

$$\langle r^2(D_{sJ}) \rangle_{\text{four}}^{1/2} = \left[ \frac{3}{4} \left( \frac{m_c + m_s}{m_c + m_s} \right)^2 \sigma_1^2 + \frac{9}{16} \left( \frac{m_c + m_s}{m_c + m_s} \right)^2 \sigma_2^2 + \frac{1}{2} \left( \frac{m_c + m_s + m_q}{m_c + m_s + 2m_q} \right)^2 \sigma_3^2 \right]^{1/2} \approx 0.62 \text{ fm},$$

which is somewhat larger than that of a two-quark $D_{sJ}(2317)$ meson.

The coalescence integral in Eq.(3) can be evaluated analytically if we expand the hyperbolic functions to first order, neglect the transverse flow, and use non-relativistic momentum distributions for quarks. The first approximation is valid for $|y| \leq 0.5$ considered in present study. Although the transverse flow affects strongly the transverse momentum spectrum of produced $D_{sJ}(2317)$ mesons, it only has a small effect on its number. As shown in Appendix A, these approximations lead to the following numbers of produced $D_{sJ}(2317)$ mesons from quark coalescence: $\sim 1.9 \times 10^{-2}$ for a two-quark $D_{sJ}(2317)$ meson and $\sim 1.1 \times 10^{-3}$ for a four-quark $D_{sJ}(2317)$ meson. These numbers are about a factor of two larger than those obtained from numerically evaluating the coalescence integral using the Monte Carlo method of Ref.15, which gives about $9.8 \times 10^{-3}$ and $4.2 \times 10^{-4}$ per unit rapidity for the two-quark and four-quark $D_{sJ}(2317)$ mesons, respectively, largely due to the use of the relativistic quark distribution functions.

Since equilibrium thermal models have been successfully employed in describing the experimental data for the yields and ratios of many hadrons in heavy ion collisions at RHIC 33, 34, it is of interest to compare the predicted number of $D_{sJ}(2317)$ mesons from the coalescence model with that from the statistical model. In terms of the charm fugacity $\gamma_C$ and the strangeness chemical potential $\mu_S$, this model gives the following number of produced $D_{sJ}(2317)$ mesons at hadronization:

$$N^\text{stat}_{D_{sJ}} = \gamma_C \int_{s_h} \frac{p^4 d^4 p_{\text{d}}}{(2\pi)^4} E f_{D_{sJ}}(x,p) \approx \frac{V_H \gamma_C e^{\mu_S/T_H}}{(2\pi)^4} \int dm_T m_T^2 e^{-\frac{m_T^2}{2\mu^2}} \times I_0 \left( \frac{\gamma_H \beta_H \mu_T}{T_C} \right) \approx 5.2 \times 10^{-2},$$

where $f_{D_{sJ}}(x,p)$ is the thermal distribution function of $D_{sJ}(2317)$ mesons, given by an expression similar to Eq.(I) for quarks and $I_0$ is the modified Bessel function. In obtaining the numerical value in the last line of above equation, we have used $V_H \approx 1.908 \text{ fm}^3$, $T_H = 175 \text{ MeV}$, $\beta_H = 0.3 \text{ c}$, $\mu_S = 10 \text{ MeV}$, and the charm fugacity $\gamma_C \approx 8.4$. The latter ensures that the numbers of charmed hadrons produced statistically at hadronization is same as the number of charm quarks $N_c$ in the quark-gluon plasma. Specifically, we have $N_D \approx 1.1$, $N_{D^*} \approx 1.5$, $N_{D_s} \approx 0.31$, and $N_{\Lambda_c} \approx 0.11$, giving a total of about 3 charmed hadrons. We note that the number of $D_{sJ}(2317)$ mesons produced in the statistical model is independent of its quark structure, contrary to that in the coalescence model in which the yield for a two-quark $D_{sJ}(2317)$ meson is about a factor of twenty larger than that for a four-quark one.

### IV. HADRONIC EFFECTS ON THE DsJ(2317) MESON

#### A. $D_{sJ}(2317)$ meson absorption cross sections by hadrons

FIG. 2: Born diagrams for $D_{sJ}(2317)$ absorption by $\pi$, $\rho$, $\bar{K}$, and $K^*$ mesons.

The abundance of $D_{sJ}(2317)$ mesons can change during the expansion of the hadronic matter as a result of absorption by pions, rho mesons, anti-kaons, and vector anti-kaons. Neglecting reactions with a $D_s$ meson in the final states, which are suppressed as a result of the presence of the isospin violated vertex $D_{sJ}D_s\pi$, we have the following reactions:

$$\pi D_{sJ} \rightarrow K D^*(K^*D), \quad \rho D_{sJ} \rightarrow K D, \quad \bar{K} D_{sJ} \rightarrow \rho D(\pi D^*), \quad K^* D_{sJ} \rightarrow \pi D,$$

as shown in Fig.2 for the lowest-order Born diagrams. The cross sections for these reactions can be evaluated using the interaction Lagrangians

$$\mathcal{L}_{\rho KK} = ig_{\rho KK}(\bar{K} \bar{\rho} \rho K - \bar{\rho} \rho \bar{K} K) \cdot \vec{\rho},$$
\[
\mathcal{L}_{\rho_{DD}} = -ig_{\rho_{DD}}(\bar{D}\slashed{\tau}\bar{\rho}_5D - \bar{\rho}_5\slashed{D}D) \cdot \bar{\rho}^5,
\]
\[
\mathcal{L}_{K^*\pi} = ig_{K^*\pi}K^*_\mu \slashed{\tau} \cdot (K\partial^\mu \bar{\pi} - \partial^\mu K \bar{\pi}) + \text{H.c.,}
\]
\[
\mathcal{L}_{D^*\pi} = -ig_{D^*\pi}D^*_\mu \slashed{\tau} \cdot (D\partial^\mu \bar{\pi} - \partial^\mu D \bar{\pi}) + \text{H.c.,}
\]
\[
\mathcal{L}_{D_{sJ}DK} = g_{D_{sJ}DK} D_{sJ} \bar{D} K D_{sJ}.
\]

In the above, \( \slashed{\tau} \) are Pauli matrices for isospin, and \( \slashed{\rho} \) denote the pion and rho meson isospin triplet, respectively; and \( K = (K^+, K^0)^T \) and \( K^* = (K^{*+}, K^{*0})^T \) denote the pseudoscalar and vector strange meson isospin doublet, respectively. The isospin doublet pseudoscalar \( D \) and vector \( D^* \) mesons are defined in a similar way.

For coupling constants, we use \( g_{\rho_{DD}} = 2.52 \) from the vector dominance model (VDM) \(^{32,33}\), \( g_{K^*\pi} = 3.25 \) \(^{39}\), and \( g_{D^*\pi} = 6.3 \) \(^{40}\) from the decay widths of \( K^* \) and \( D^* \), respectively, and \( g_{pKK} = 3.25 \) from the \( SU(3) \) symmetry \(^{41}\). For the coupling constant \( g_{KDD_{sJ}} \), it has been studied in the QCD sum rules, and its value depends strongly on the quark structure of \( D_{sJ}(2317) \) meson. The predicted values are \( g_{D_{sJ}DK} = 9.2 \text{ GeV} \) if the \( D_{sJ}(2317) \) meson is a two-quark state \(^{12}\) and \( g_{D_{sJ}DK} = 3.15 \text{ GeV} \) if it is a four-quark state \(^{43}\).

The amplitudes for the reactions shown in Fig.2 are given by

\[
\mathcal{M}_1 = -\tau_{ij}^a g_{D_{sJ}DK} K g_{K^*\pi} \frac{1}{t - m_K^2} (2p_2 - p_4) \mu^i \epsilon_2^a,
\]
\[
\mathcal{M}_2 = \tau_{ij}^a g_{D_{sJ}DK} g_{D^*\pi} \frac{2p_2 - p_4}{t - m_D^2} \mu^i \epsilon_2^a,
\]
\[
\mathcal{M}_{3a} = \tau_{ij}^a g_{D_{sJ}DK} g_{KK} \frac{2p_2 - p_4}{t - m_K^2} \mu^i \epsilon_2^a,
\]
\[
\mathcal{M}_{3b} = \tau_{ij}^a g_{D_{sJ}DK} g_{DD} \frac{1}{u - m_D^2} (2p_2 - 2p_3) \mu^i \epsilon_2^a,
\]
\[
\mathcal{M}_4 = \mathcal{M}_1(p_2 \leftrightarrow -p_4),
\]
\[
\mathcal{M}_5 = \mathcal{M}_2(p_2 \leftrightarrow -p_3; p_3 \leftrightarrow p_4),
\]
\[
\mathcal{M}_6 = \mathcal{M}_3(p_2 \leftrightarrow -p_4).
\]

In the above, the matrix element \( \tau_{ij}^a \) takes into account the isospin states of the particles in a reaction, with \( a \) denoting those of isospin triplet \( \pi \) and \( \rho \) mesons, and \( i \) and \( j \) those of isospin doublet \( K, K^*, D, \) and \( D^* \) mesons. The momenta \( p_1 \) and \( p_2 \) are those of initial state particles while \( p_3 \) and \( p_4 \) are those of final state particles on the left and right side of a diagram. The usual Mandelstam variables are given by \( s = (p_1 + p_2)^2 \), \( t = (p_1 - p_3)^2 \), and \( u = (p_1 - p_4)^2 \).

To obtain the full amplitudes, one needs in principle to carry out a coupled-channel calculation in order to avoid violation of unitarity. Such an approach is, however, beyond the scope of present study. To prevent the artificial growth of the tree-level amplitudes with the energy, we introduce instead form factors at interaction vertices, which are taken to have the form \(^{44}\)

\[
F(q^2) = \frac{\Lambda^2}{\Lambda^2 + q^2},
\]

where \( q^2 \), taken in the center of mass, is the squared three momentum transfer for \( t \) and \( u \) channels, or the squared three momentum of either the incoming or outgoing particles for \( s \) channel. For the cutoff parameter \( \Lambda \), we use \( \Lambda = 1.3 \text{ GeV} \) for vertices involving an off-shell \( K \) meson and \( \Lambda = 3.7 \text{ GeV} \) for those involving an off-shell \( D \) meson. These values are determined from the QCD sum-rule calculations given in Appendix B for the \( D_{sJ}DK \) three-point functions. Although the calculations are only for a two-quark \( D_{sJ}(2317) \) meson, we use them also for a four-quark \( D_{sJ}(2317) \) meson as well as for other vertices in the diagrams in Fig.2. We expect this to be a reasonable assumption as a study of the \( X(3872) \) meson in the QCD sum rules has indicated that both the form and the cut-off of its form factor are not significantly different between a two-quark and a four-quark \( X(3872) \) meson \(^{45}\).

The isospin- and spin-averaged cross section is then given by

\[
\sigma_n = \frac{1}{64\pi s N_f N_S} \frac{p_f}{p_i} \int d\Omega \overline{|M_n|^2} F^4,
\]

where \( \overline{|M_n|^2} \) denotes the squared amplitude obtained from summing over the isospins and spins of both initial and final particles, with \( F \) denoting the appropriate form factors at interaction vertices. The factors \( N_f = (2I_1 + 1)(2I_2 + 1) \) and \( N_S = (2S_1 + 1)(2S_2 + 1) \) in the denominator are due to averaging over the isospins \( I_1 \) and \( I_2 \) as well as the spins \( S_1 \) and \( S_2 \) of initial particles.

The three-momenta in the center of mass of initial and final particles are denoted by \( p_i \) and \( p_f \), respectively.

![FIG. 3: (color online) Cross sections for the absorption of a four-quark \( D_{sJ}(2317) \) meson by \( \pi, \rho, K, \) and \( K^* \) mesons via reaction \( \pi D_{sJ} \rightarrow K D^*(K^*D), \rho D_{sJ} \rightarrow K D, K D_{sJ} \rightarrow \rho D(K D^*), \) and \( K^* D_{sJ} \rightarrow \rho D \).](image-url)
energy $s_0^{1/2}$ of a reaction for the scenario that it is a four-quark state. Aside near the threshold of a reaction, where the cross section can be very large or small depending on whether the reaction is exothermic or endothermic, most cross sections are less than 1 mb except the reaction $\pi D_{sJ} \rightarrow K^* D$, which has a peak value of about 2 mb. If the $D_{sJ}(2317)$ meson is a two-quark state, its absorption cross sections are about a factor of nine larger than corresponding ones shown in Fig. 3 as the coupling constant $g_{D_{sJ}DK}$ is about a factor of three larger for a two-quark $D_{sJ}(2317)$ meson than for a four-quark one.

The $D_{sJ}(2317)$ meson can also be produced in the hadronic matter by the inverse reactions $KD^*(DK^*) \rightarrow \pi D_{sJ}$, $KD \rightarrow \rho D_{sJ}$, $\rho D(\pi D^*) \rightarrow K D_{sJ}$, and $\pi D \rightarrow K^* D_{sJ}$, with cross sections related to those of absorption reactions via the detailed balance relations.

B. Thermally averaged $D_{sJ}(2317)$ meson absorption cross sections

In the kinetic model to be used in the next section for studying $D_{sJ}(2317)$ meson absorption and production in hadronic matter, the thermally averaged cross sections are needed. In terms of the thermal distribution functions $f_i(p)$ of $D_{sJ}(2317)$ mesons and other hadrons, the thermally averaged cross section $\sigma_{ab \rightarrow cd}$ for the reaction $ab \rightarrow cd$ is given by [40]

$$\langle \sigma_{ab \rightarrow cd} \rangle = \frac{\int d^3p_a d^3p_b f_a(p_a) f_b(p_b) \sigma_{ab \rightarrow cd} v_{ab}}{\int d^3p_a d^3p_b f_a(p_a) f_b(p_b)}$$

$$= \left[4\alpha_a^2 K_1(\alpha_a) \alpha_b^2 K_1(\alpha_b)\right]^{-1}$$

$$\times \int_0^\infty dz [z^2 - (\alpha_a + \alpha_b)^2][z^2 - (\alpha_a - \alpha_b)^2]$$

$$\times K_1(z) \sigma(s = z^2 T^2),$$

(19)

with $\alpha_i = m_i/T$, $z_0 = \max(\alpha_a + \alpha_b, \alpha_c + \alpha_d)$, $K_1$ being the modified Bessel function, and $v_{ab}$ denoting the relative velocity of initial two interacting particles $a$ and $b$, i.e.,

$$v_{ab} = \sqrt{(p_a \cdot p_b)^2 - m_a^2 m_b^2} / E_a E_b.$$

(20)

The thermally averaged absorption cross sections of the $D_{sJ}(2317)$ meson by $\pi$, $\rho$, $K$, and $K^*$ as functions of the temperature of the hadronic matter are shown in Fig. 4 for the case that $D_{sJ}(2317)$ is a four-quark meson. It is seen that the thermally averaged cross section of the dominant reaction $\pi D_{sJ} \rightarrow K^* D$ has values less than 0.6 mb. This value is again about a factor of nine larger if the $D_{sJ}(2317)$ is a two-quark meson.

V. TIME EVOLUTION OF THE $D_{sJ}(2317)$ MESON ABUNDANCE IN HADRONIC MATTER

A. Rate equation for $D_{sJ}(2317)$ meson production in heavy ion collisions

In terms of thermally averaged cross sections and the densities of $\pi$, $\rho$, $K$, and $K^*$ mesons, the time evolution of the $D_{sJ}(2317)$ meson abundance in the hadronic matter is determined by the kinetic equation

$$\frac{dN_{D_{sJ}}(t)}{dt} = R_{QGP}(t) + \sum_{a,b,c} (\sigma_{D_{sJ} \rightarrow ab} v_{D_{sJ}}) n_a(0)(t)$$

$$\times \left[N_{D_{sJ}}(t) \frac{n_c(0)}{n_c(0)} - N_{D_{sJ}}(t)\right],$$

(21)

where $n_a(t)$ and $n_c(0)$ are, respectively, the equilibrium densities of light meson type $a$ and charmed meson type $c$ in the hadronic matter at proper time $t$ when its temperature is $T$ according to Eq. (1), and $N_{D_{sJ}}(t)$ is the equilibrium number of $D_{sJ}(2317)$ mesons given by Eq. (14) using the temperature and flow velocity at proper time $t$. Since hadronization of the quark-gluon plasma takes a finite time of $T_H - T_C \approx 2.5 \text{ fm/c}$, $D_{sJ}(2317)$ mesons are produced from the quark-gluon plasma in the mixed phase, with a rate proportional to the volume of the quark-gluon plasma. This is included in Eq. (21) through the term $R_{QGP}(t)$. Since the fraction of the quark-gluon plasma during the mixed phase decreases almost linearly...
with the proper time, we can approximately write

$$R_{QGP}(\tau) = \begin{cases} N_{D_{s,J}}^{(0)} / (\tau_H - \tau_C), & \tau_C < \tau < \tau_H; \\ 0, & \text{Otherwise.} \end{cases}$$  \tag{22}$$

In the above, $N_{D_{s,J}}^{(0)}$ is the total number of $D_{s,J}(2317)$ mesons produced from the quark-gluon plasma. In the following calculations, it is obtained either from the coalescence model by evaluating Eq. (13) numerically using the Monte-Carlo method or from the statistical model using Eq. (13). In writing Eq. (21), we have assumed that the total number of charmed hadrons is conserved during the evolution of the hadronic matter as charms are not likely to be produced and destroyed in the hadronic matter because of their small production and annihilation cross sections [47, 48, 49].

$$\text{FIG. 5: Time evolution of the } D_{s,J}(2317) \text{ meson abundance in central Au+Au collisions at } \sqrt{s_{NN}} = 200 \text{ GeV for different initial numbers of } D_{s,J}(2317) \text{ mesons produced from the quark-gluon plasma.}$$

B. $D_{s,J}$ meson yield in relativistic heavy ion collisions

In Fig. 5, the abundance of $D_{s,J}(2317)$ mesons in central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV is shown as a function of the proper time of the fire-cylinder. Since the initial number of $D_{s,J}(2317)$ mesons produced from the quark-gluon plasma via quark coalescence is below the equilibrium number for both the two-quark and four-quark $D_{s,J}(2317)$ mesons, its number increases during the hadronic evolution as shown by the dashed and solid lines, respectively. The final number of $D_{s,J}(2317)$ mesons is about $3.0 \times 10^{-2}$ if the $D_{s,J}(2317)$ meson is a two-quark state and is about $6.0 \times 10^{-3}$ if it is a four-quark meson. Although the ratio ($\sim 5$) between the final numbers for the two and four-quark $D_{s,J}(2317)$ mesons is smaller than that ($\sim 20$) for initially produced $D_{s,J}(2317)$ mesons, it is still appreciable. This result differs significantly from the predictions of the statistical model. In this case, the $D_{s,J}(2317)$ meson number decreases slightly to $3.8 \times 10^{-2}$ during the hadronic evolution if it is a two-quark meson as shown by the dash-dotted line, and it remains essentially unchanged during hadronic evolution if it is a four-quark meson as shown by the dotted line. Since the final yield of $D_{s,J}(2317)$ mesons in the coalescence model is much smaller for a four-quark state than for a two-quark state and also that from the statistical model, studying $D_{s,J}(2317)$ meson production in relativistic heavy ion collisions thus provides the possibility of understanding not only its production mechanism but also its quark structure.

Above results are obtained by assuming that there are three charm quarks in the quark-gluon plasma, based on the number of charm quarks measured by the PHENIX collaboration in $p+p$ collisions. If this number is increased by a factor of two, which is closer to that expected from the STAR experiment on $d+Au$ collisions, the final $D_{s,J}(2317)$ meson numbers in both the coalescence and statistical models are increased by about a similar factor. A similar reduction factor is seen in the final $D_{s,J}(2317)$ meson numbers in heavy ion collisions if the total charm quark number is reduced by a factor of two as given by the PYTHIA program for $p+p$ collisions.

VI. SUMMARY

Using the quark coalescence model, we have predicted the yield of $D_{s,J}(2317)$ mesons in central Au+Au collisions at RHIC. Contrary to the prediction of the statistical model, the initial number of $D_{s,J}(2317)$ meson produced at the end of the quark-gluon stage of heavy ion collisions depends sensitively on whether it is a two-quark or a four-quark meson, with the former giving an order of magnitude larger number than the latter. To take into account the effects of absorption and production during subsequent hadronic evolution, we have used a hadronic model to evaluate the cross sections for the absorption of the $D_{s,J}(2317)$ meson by pion, rho meson, anti-kaon, and vector anti-kaon in the tree-level Born approximation. With empirical masses and coupling constants as well as form factors from the QCD sum rules, we have found that all these cross sections are small, except the reaction $\pi D_{s,J} \rightarrow K^*D$, which has a peak cross section of about 2 mb, if the $D_{s,J}(2317)$ is a four-quark meson, but they are about ten times larger if it is a conventional two-quark meson. Including these reactions in a kinetic model based on a schematic hydrodynamic description of relativistic heavy ion collisions, we have studied the time evolution of the abundance of $D_{s,J}(2317)$ mesons in these collisions. Our results show that the large difference in the initial numbers given by the quark coalescence model for the two-quark and four-quark $D_{s,J}(2317)$ mesons re-
mains appreciable at freeze out. On the other hand, the $D_{sJ}(2317)$ number determined from the statistical model is essentially unchanged if the $D_{sJ}(2317)$ is a four-quark meson and only change slightly if it is a two-quark meson. Studying $D_{sJ}(2317)$ production at RHIC and also at the forthcoming LHC thus offers the possibility to understand its quark structure and production mechanism. To achieve this goal requires, however, accurate information on the number of charm quarks produced during initial hard nucleon-nucleon collisions, as increasing or decreasing the initial quark number by a factor affects the final $D_{sJ}(2317)$ meson number by a similar factor. Also, it is important to know if charm quarks can be produced from the quark-gluon plasma as this would affect the initial $D_{sJ}(2317)$ mesons produced from the quark-gluon plasma as well.

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APPENDIX A: APPROXIMATE EVALUATION OF THE COALESCENCE INTEGRAL

For a hypersurface of constant proper time and a distribution with Bjorken correlation between $y$ and $\eta$, we can expand the hyperbolic function in the coalescence integral to first order in $y$ and $\eta$ if we consider $D_{sJ}(2317)$ meson production at midrapidity with $|y| < 0.5$. In this case, the invariant phase-space factor in Eq. (A1) can be approximated by

$$p_i \cdot d\sigma_i \frac{d^3p_i}{E_i} \approx d^3x_i d^3p_i.$$  \hspace{1cm} (A1)

Neglecting the transverse flow and treating quarks non-relativistically, we can then use the relation

$$\sum_{i=1}^{n} \frac{p_i^2}{2m_iT} = \frac{K^2}{2MT} + \sum_{i=1}^{n-1} \frac{k_i^2}{2\mu_iT},$$  \hspace{1cm} (A2)

where $M = \sum_{i=1}^{n} m_i$, to express the quark Boltzmann momentum distribution functions in terms of the total $K$ and relative $k_i$ momenta. Using also the total and relative $y_i$ coordinates of the quarks, we obtain the following expression for the number of $D_{sJ}(2317)$ mesons produced from quark coalescence:

$$N_{D_{sJ}} = g_{D_{sJ}} \prod_{j=1}^{n} \rho_j \frac{1}{2} \int \frac{d^3y_i \ d^3k_i}{f_{D_{sJ}}(y_i,k_i)} \bar{f}_{D_{sJ}}(k_i).$$  \hspace{1cm} (A3)

In the above, we have made use of the fact the Wigner function of $D_{sJ}(2317)$ meson is factorable in both the relative coordinates and momenta of its constituent quarks. Because of $-0.5 \leq y \leq \eta \leq 0.5$, the momentum space integral in Eq. (A3) reduces to a two-dimensional one, as the momentum integral in the $z$-direction gives simply one.

If the $D_{sJ}(2317)$ meson is a p-wave two-quark meson, evaluating the integrals in Eq. (A3) with its Wigner function given by Eq. (5) leads to the following number of $D_{sJ}(2317)$ mesons produced from quark coalescence:

$$N_{D_{sJ}}^{\text{two}} \approx \frac{1}{27} N_c N_f \frac{(4\pi)^{3/2} \mu T \sigma^5}{V_C (1 + 2\mu_i T \sigma^2)^2} \approx 1.9 \times 10^{-2},$$  \hspace{1cm} (A4)

For a four-quark $D_{sJ}(2317)$ meson with its Wigner function given by Eq. (6), the number of $D_{sJ}(2317)$ mesons produced from quark coalescence is

$$N_{D_{sJ}}^{\text{four}} \approx \frac{1}{1296} N_c N_f (N_u N_u + N_d N_d) \times \prod_{i=1}^{3} \frac{(4\pi \sigma^2)^{3/2}}{V_C (1 + 2\mu_i T \sigma^2)} \approx 1.1 \times 10^{-3}. \hspace{1cm} (A5)$$

APPENDIX B: THE $D_{sJ}DK$ FORM FACTOR

In this appendix, we compute the $D_{sJ}DK$ form factor using the QCD sum rules [50, 51]. In this approach, the short-range perturbative QCD is extended by the Wilson operator product expansion (OPE) of the correlators, which results in a series in powers of the squared momentum with Wilson coefficients. The convergence at low momentum is improved by using a Borel transform. The expansion involves universal quark and gluon condensates. Equating the quark-based calculation of a given correlator to the same correlator that is calculated using hadronic degrees of freedom via a dispersion relation then provides the sum rules from which a hadronic quantity can be estimated.

We shall use the three-point function to evaluate the $D_{sJ}DK$ form factor by following the procedure suggested in Ref. [52] and further extended in [53]. This means that we shall calculate the correlators for an off-shell $D$ meson and then for an off-shell $K$ meson, requiring that the corresponding extrapolations to the respective poles lead to the same unique coupling constant.

The three-point function associated with a $D_{sJ}DK$ vertex with an off-shell $D$ meson is given by

$$\Gamma_{\mu}^{(D)}(p,p') = \int d^4 x \ d^4 y \langle 0| T\{j_{\bar{D}_{sJ}}(x)D(y)j_{D_{sJ}}(0)\}|0\rangle.$$
states contribution to the matrix element in Eq. (B1):

\[ \Gamma_\mu(p, p') = \frac{m_{D_{0j}} m^2_{\pi}}{m_c} F_K f_D f_{D_{0j}} \left( \frac{m^2_{\pi}}{m_c} - m^2_D \right) \left( p'^2 - m^2_D \right) \times \]
\[ \times \frac{g^2_{D_{0j}DK}}{(q^2 - m^2_D)} p'_\mu + \text{higher resonances} . \]  

In deriving Eq. (B2), we have made use of

\[ \langle D_{0j}(p) K(p') D(q) \rangle = \frac{g^2_{D_{0j}DK}}{(q^2 - m^2_D)} q^2, \]  

where \( q = p' - p \), and the decay constants \( F_K \) and \( f_D \) and \( f_{D_{0j}} \) are defined by the matrix elements

\[ \langle 0 | j_5_{\mu} | K(p') \rangle = i \gamma_\mu F_K, \]  

\[ \langle 0 | j_D | D(q) \rangle = \frac{m^2_D f_D}{m_c}, \]  

and

\[ \langle 0 | j_{D_{0j}} | D_{0j}(p) \rangle = m_{D_{0j}} f_{D_{0j}}. \] 

The contribution of higher resonances and continuum in Eq. (B2) will be taken into account as usual in the standard form of Ref. [54], through the continuum thresholds \( s_0 \) and \( u_0 \) for the \( D_{0j} \) and \( K \) mesons, respectively.

The QCD side, or theoretical side, of the vertex function is evaluated by performing Wilson’s operator product expansion of the operator in Eq. (B1). Expressing \( \Gamma_\mu \) in terms of the invariant amplitudes,

\[ \Gamma_\mu(p, p') = F_1(p^2, p'^2, q^2) p_\mu + F_2(p^2, p'^2, q^2) p'_\mu, \]  

we can write a double dispersion relation for each one of the invariant amplitudes, \( F_1 \), over the virtualities \( p^2 \) and \( p'^2 \) holding \( Q^2 = -q^2 \) fixed:

\[ F_1^{(p^2, p'^2, Q^2)} = -\frac{1}{4\pi^2} \int_{m_c}^{\infty} ds \int_0^{\infty} du \frac{\rho_I(s, u, Q^2)}{(s - p^2)(u - p'^2)}, \]  

where \( \rho_I(s, u, Q^2) \) equals the double discontinuity of the amplitude \( F_I(p^2, p'^2, Q^2) \) on the cuts \( m_c^2 \leq s \leq \infty \) and \( 0 \leq u \leq \infty \), which can be evaluated using Cutkosky’s rules. Finally, in order to suppress the condensates of higher dimension and at the same time reduce the influence of higher resonances, we perform a double Borel transform in both variables \( p^2 = -p'^2 \rightarrow M^2 \) and \( p'^2 = -p^2 \rightarrow M'^2 \). Equating the two representations described above, we obtain the following sum rule in the structure \( p'_\mu \):

\[ \frac{m_{D_{0j}} m^2_{\pi}}{m_c} F_K f_D f_{D_{0j}} g^2_{D_{0j}DK}(Q^2) e^{-m^2_{D_{0j}}/M^2} e^{-m^2_K/M'^2} \]
\[ = (Q^2 + m^2_D) \left[ m_c < \bar{s}s > e^{-m^2_{D_{0j}}/M^2} \right. \]
\[ \left. - \frac{1}{4\pi^2} \int_{m_c}^{\infty} ds \int_{u_{\text{max}}}^{s_0} du \exp(-s/M^2) \exp(-u/M'^2) \right. \]
\[ \times f(s, t, u) \theta(u_0 - u) \],  

where \( t = -Q^2 \) and

\[ f(s, t, u) = \frac{3}{2|\lambda(s, u, t)|^{1/2}} \left( m_c^2 + 2m_c m_s - s + \right. \]
\[ + ((2m_c^2 + 2m_c m_s - s - t + u)(m_c^2(s - t + u) + \]
\[ s(t + u - s)[\lambda(s, u, t)]^{-1}}, \]  

with \( \lambda(s, u, t) = s^2 + u^2 + t^2 - 2su - 2st - 2tu \), and \( u_{\text{max}} = s + t - m^2_D - st/m_c^2 \).

We use the same parameters as in Ref. [42]: \( m_s = 0.15 \text{GeV}, m_c = 1.26 \text{GeV}, F_K = 0.16 \text{GeV}, f_D = 1.865 \text{GeV}, m_K = 0.948 \text{GeV}, m_{D_{0j}} = 2.317 \text{GeV}, \) \( f_{D_{0j}} = 0.23 \text{GeV}, f_{D_{0j}} = 0.225 \text{GeV}, \) \( \langle \bar{s}s \rangle = 0.8 \langle m_q \rangle \), with \( \langle m_q \rangle = -(0.245)^3 \text{GeV}^3 \). For the continuum thresholds, we take \( s_0 = (6.3 \pm 0.1) \text{GeV}^2 \) and \( u_0 = (m_K + \Delta u)^2 \) with \( \Delta u = 0.5 \text{GeV} \).

We also use the same Borel window as in Ref. [42]: \( 10 \text{GeV}^2 \leq M^2 \leq 20 \text{GeV}^2 \) and work at a fixed ratio \( M'^2/M^2 = 0.64/m^2_{D_{0j}} \). We find a good Borel stability in this region of the Borel mass. Fixing \( M^2 = 15 \text{GeV}^2 \), we show in Fig. 6 by the filled circles the momentum dependence of \( g^2_{D_{0j}DK}(Q^2) \).

Since the present approach can not be used at small values of \( Q^2 \), extracting the \( g_{D_{0j}DK} \) coupling from the form factor requires extrapolation of the curve to the mass of the off-shell meson \( D \). In order to do this, we fit the QCD sum-rule results with an analytical expression. We have obtained a reasonable fit using a monopole form:

\[ g^2_{D_{0j}DK}(Q^2) = \frac{92.4}{Q^2 + 14.1}, \]  

where the numbers are in units of \( \text{GeV}^2 \). This fit is also shown in Fig. 6 by the solid line. From Eq. (B11), we get \( g_{D_{0j}DK} = g^2_{D_{0j}DK}(Q^2 = -m^2_{D_{0j}}) = 8.7 \).

To check the consistency of above fit, we also evaluate the form factor at the same vertex, but for an off-shell kaon. In this case, we have to evaluate the three-point function

\[ \Gamma_{\mu}^{(K)}(p, p') = \int d^4x d^4y \langle 0 | \{ j_5(x) j_{5\mu}(y) j_{5j_{D_{0j}}}(0) \} | 0 \rangle \times \]
\[ \times e^{ip'x} e^{i(p-p')y} \].  

(B12)
Proceeding in a similar way, we obtain the following sum rule:

\[
\frac{m_{D_s}m_D^2}{m_c^2}F_K^2f_Df_{D_s,j}g_{D_{s,J}DK}^{(K)}(Q^2)e^{-m_{D_s,j}^2/M^2}e^{-m_D^2/M'2}
\]

\[
= \frac{Q^2 + m_D^2}{4\pi^2} \int_{u_{\text{min}}}^{s_0} ds \int_{u_{\text{min}}}^{u_0} du e^{-s/M^2} e^{-u/M'2}
\]

\[
\times g(s,t,u),
\]

where \( u_{\text{min}} = m_c^2 - \frac{m_D^2}{s-m_D^2} \) and

\[
g(s,t,u) = \frac{3}{\alpha(s,u,t)}\sqrt{s}m_c^2(s-t+3u)
\]

Using now \( u_0 = (m_D + \Delta_u)^2 \) with \( \Delta_u = 0.5 \text{GeV} \) and \( M^2 = m_D^2/\Delta_{M,D}^2 \), we find that the results are also rather stable as a function of the Borel mass. Fixing \( M^2 = 15 \text{ GeV}^2 \), we show in Fig. 6 by the squares the QCD sum-rule results for \( g_{D_{s,J}DK}^{(K)}(Q^2) \). A good fit of these results can be obtained using an exponential form:

\[
g_{D_{s,J}DK}^{(K)}(Q^2) = 7.98 e^{-Q^2/1.75},
\]

where \( 1.75 \) is in units of \( \text{GeV}^2 \), as shown in Fig. 6 by the dashed line. From Eq. (B15) we get \( g_{D_{s,J}DK}^{(K)}(Q^2 = -m_D^2) \) above and the result obtained in Ref. [42] for this coupling constant: \( g_{D_{s,J}DK} = 8.9 \pm 0.9 \).

Considering the uncertainties in the continuum thresholds, and the difference in the values of the coupling extracted when the \( D \) meson or the kaon is off-shell, our result for the \( D_{s,J}DK \) coupling constant is thus \( g_{D_{s,J}DK} = 8.9 \pm 0.9 \).

From the parameterizations in Eqs. (B11) and (B15), we can also get information about the cut-off \( (\Lambda) \) in the form factors. We see that the cut-off is much bigger when the \( D \) meson is off-shell (\( \Lambda \approx 3.7 \text{GeV} \)) than when the kaon is off-shell (\( \Lambda \approx 1.3 \text{GeV} \)), in agreement with the results obtained in refs. [52, 53].

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