Effects of construction history on the stress distribution under a sand pile

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We report experiments on cohesionless granular piles to determine the effect of construction history on the static stress distribution. The stresses beneath the piles are monitored using a very sensitive capacitive technique. The piles are formed either by release of granular material from a relatively small output (localized source), or from a large diameter sieve (homogeneous rain). The stress profiles resulting from localized source inputs have a clear stress dip near the center of the pile while the results from an homogeneous rain show no stress dip. We also show that the stress profiles scale simply with the pile height. Experiments on wedges-shaped piles show the same effects but to a lesser degree.

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Granular systems have captured much recent interest because of their rich phenomenology, and important applications. Static arrays show inhomogeneous spatial stress profiles called stress chains, where forces are carried primarily by a small fraction of the total number of grains. Recent numerical simulations and experiments have shown that the structure and the nature of these chains plays a critical role in the dynamics and statics of dense granular systems even in the absence of strong disorder of the granular packings (see Fig. 1). Necessarily, the presence of these chains must be reflected in the continuum constitutive relations which are needed to close the governing equations and thereby, solve even the simplest boundary value problems in granular statics.

The stress profile under a static pile of granular material provides a useful method for probing the effects of stress chains and the history of their formation. The literature contains many experiments examining stress profiles under static piles of granular material. Although there are a number of such studies, they are not in mutual agreement. In addition, a number of competing constitutive models have been invoked to explain the experimental observations.

The present experiments have been carried out with the aim of resolving the experimental conflict by determining as carefully as possible the relation between the preparation of a heap and the stress profile at its base. It is important to clarify the effect of construction technique on the stress structure for the following reasons:

1. To help understand the wide variation in past data.
2. To test some of the theories which depend explicitly on construction history.

Of possible pile geometries, conical, and wedge-shaped heaps have been the most frequently studied. Many of the experiments on conical piles have indicated, contrary to simple intuition, that there is a dip in the pressure profile beneath the center. The existence of a dip in the stress profile for wedge-shaped piles is an open question.

There are important technical considerations in determining whether there is a stress dip. The most important of these is the fact that even modest deformations either of the surface supporting the pile or of the force detector may lead to erroneous measurements. In addition, if the pile is formed by dropping material onto the heap from a considerable height, as opposed to gentler deposition methods, it is likely that residual stresses become frozen into the heap. In such a case, or for a very heavy load, there is likely to be a characteristic length associated with the deformation of the pile under its own weight.

There are only a few reported experiments addressing the influence of the construction technique or filling rate on the stress. (However, two of us have probed the effect of the granular packing history on the mean pressure at the bottom of a silo.) Regarding sand-piles in particular, we are aware of only one set of experiments that considered the effect of construction technique on stress profiles, namely the work of Lee and Herrington for wedge-shaped piles of sand. These authors constructed piles using three different methods and found that the different construction techniques yielded results that were identical within the resolution of their instruments; no dip was recorded.

In the present experiments, we explore the effects of construction procedures on the pressure profiles using two different methods to build both conical and wedge-shaped heaps. We use detectors with very high resolution and very small deflection. We also build these piles on rigid base plates. In the case of a conical pile, we explicitly investigate the scaling of this profile with total mass for piles with an height ratio up to three times that of the smallest pile, corresponding to a mass ratio of ~ 30.

Several details of these experiments are important. We used sand of diameter 1.2 mm ± 0.4 mm and angle of repose 32°. The base plate on which we constructed most of these piles was a 15.0 mm thick duralumin support which was adequate to prevent deflection under the weight of the pile. Some additional experiments were carried out using a 1.3 mm steel base. These experiments used a fixed funnel height, and they are discussed below. For a typical sand pile of $H = 8 cm$ height, we estimated the maximal sagging of the bottom plate to be $w_m = 6.5 \mu m$. Therefore $w_m/H = 10^{-5}$, a value that was smaller by $\sim 10^{-3}$ than the relative deflection for which sagging of the base might create a significant perturbation. A single capacitive normal stress (i.e. pressure) sensor of diameter of 11.3 mm (9 grain diameters) was placed flush with the surface of the base plate. We then determined the normal stress at various locations along the radial axis of the conical piles or along the short edge of the wedge-shaped piles by repeated construction of heaps with the same mass of sand. The resolution of the measuring device was 0.25% of the typical maximum stress for an 8 cm pile, which corresponded to a vertical deflection of the sensor of $\sim 1.3 \mu m$. We tested the consistency of the measurements with different membrane thicknesses, and we found...
consistent results within experimental resolution. Here, we present data obtained with only one of these membranes which had a thickness $t = 100 \mu m$. The sensor was calibrated against the hydrostatic pressure of a water column. However, the response of the sensor to known weights of granular material was consistently somewhat smaller, by a factor of $\sim 0.9$, than for water. We emphasize that this reduction was constant throughout the measurements. In particular, using a calibration based on granular mass, we consistently found that the integrated weight of the pile was correct.

We constructed both types of heaps by two qualitatively different procedures. The first of these used a funnel and we refer to it as the ‘localized source’ procedure; the second used a sieve, and we refer to it as the ‘raining procedure’. The following paragraphs give details on each method. Fig. 2 and Fig. 3 show photographs of these two configurations.

The localized-source procedure: We formed the pile using a funnel with an outlet that was much smaller than the final pile diameter. The funnel lifted steadily and slowly so that the outlet was always slightly above the apex during the heap formation. This approach, as opposed to a fixed funnel height, avoided effects from the deposition of particles with large kinetic energies that varied with the distance between the apex of the heap and the bottom of the funnel. For conical piles, the sand emptied from a conical funnel of outlet diameter 11.7 mm ($\approx 10$ grains) onto the duralumin plate; the latter had a diameter larger than that of the final heap. For wedge-shaped piles the sand emptied from a wedge-shaped funnel with an outlet that was 11.7 mm in the short direction and 20 cm in the long direction. The dimensions of the supporting surface and the bottom of the wedge-shaped pile were 20 cm X 26 cm. Boundaries consisting of two Plexiglas walls 2.0 cm thick and taller than the peak of the pile preserved the wedge-shape as the pile formed; the remaining two sides, parallel to the long direction of the wedge, were open. The sensor was placed halfway between the supporting walls, and at various distances from the centerline of the heap. During the experiments we measured the volume of the known mass of granular material forming the pile to determine its average volume fraction, $\rho$.

The raining procedure: The second construction method was designed to build up a pile in which the stress chains, and hence the principal stress directions were more nearly in the vertical direction. The containers from which the sand was poured had cross-sectional dimensions slightly larger than the platform on which the heap formed. The bottom of these containers were wire meshes with 0.40 cm diameter holes. To form the heaps, the containers were filled while resting on the platform; they were then raised slowly above the platform, allowing a steady rain of sand onto the heap. Excess sand, at angles greater than the angle of repose, was allowed to avalanche off the platform. For this procedure, the base platform had the same size as the bottom of the final pile. The final mass of sand and the pile volume was measured at the end of the procedure. For conical piles we used a cylindrical container and a supporting platform of diameter 26 cm (236 grain diameters). For the wedge-shaped piles, we used a rectangular box with dimensions 20 cm X 26 cm; the platform was identical to the one used in the localized-source procedure.

Pressure profiles and photographs of the final conical and wedge-shaped piles are shown in Figs. 2 and center of the heap is scaled by $R$, where $R$ is the pile radius for conical heaps, and the distance from the center axis for wedge-shaped heaps. The pressure is scaled by the hydrostatic pressure, $\rho gH$. The bars represent the standard deviation of several independent runs, not experimental error, which is about 0.25%.

The entire weight of each pile was integrated by curve-fitting the profile and integrating over the base area. This calculation is then compared with the known weight of the pile. For conical piles and localized-source wedge-shaped piles the error between both measurements is about 1.5% or less. For the raining procedure applied to wedge-shaped piles, we observe a discrepancy as large as 8%. This relatively large “missing mass” for the wedge-shaped pile may be caused by a screening effects of the walls which support some of the weight.

Data for the conical piles created by the localized-source method show a clear pressure minimum at $r/R = 0$. A maximum in the stress of $\sim 0.6 \rho gH$, occurs at a position $r/R \approx 0.3$ which agrees reasonably well with previous conical pile data. The dimensionless stress

![FIG. 2. Normal stress profiles beneath conical piles of granular materials of height $H$.](image)
at \( r/R = 0 \), 0.3\( \rho gH \), is \( \sim 50\% \) lower than the maximum stress. Experiments performed with a fixed height funnel show a larger pressure difference between the maximum at \( r/R = 0 \) and the value at \( r/R = 0 \). This suggests that the particles pack differently with different deposition energies. The pressure is clearly largest near or at \( r = 0 \) in the case of the raining procedure with maximum value 0.6\( \rho gH \).

We have investigated the issue of heap size by considering conical piles built with the localized source procedure. Specifically, we obtained data for for heap heights spanning 4.5 cm to 14.0 cm by simply stopping the filling process at various stages to obtain stress data. This variation by \( \sim 3 \) in the maximum height of the piles corresponds to a variation of \( \sim 30 \) in the mass, and hence the peak stress. The resulting data are displayed in Fig. 4. While there is some scatter in the results, the normalized profiles collapse surprisingly well. The peak occurs consistently at \( r/R = 0.3 \), and the stress at \( r = 0 \) is consistently \( \sim 50\% \) of the peak stress. This finding disagrees with earlier studies by Jotaki et al. [14], who also examine conical piles formed by pouring from funnels. These authors found that that the larger piles had deeper dips in the stress at the center. The difference between this data and ours is that Jotaki et al use a fixed funnel height for a given heap height. Larger piles were formed by setting the funnel progressively higher. Material dropped from the larger heights had more energy than for smaller heights. The height dependence observed by Jotaki et al may be explained by a density difference in the packings, with a corresponding height dependence of the scaled stresses. In experiments where we fixed the funnel height at a height \( z > H \) above the base, we found that the stress dips were deeper than for the experiments where we gradually raised the funnel.

A dip does not occur in the profiles of the heaps created by the raining method. Rather, there is a peak pressure of about 0.6 at \( r/R = 0 \), and a steady drop in the pressure moving out towards the edge of the pile.

For the wedge-shaped piles we find results qualitatively identical to conical piles. For the raining procedure, the stress profile shows no indication of a central dip. However, for the localized-source case there is a clear minimum at \( r/R = 0 \). The value of this dip is notably smaller than for the analogous conical heap, i.e. only 15\% lower than the maximum stress, rather than 50\% lower. The pressure at the center is about 0.65\( \rho gH \). The maximum in the stress occurs at \( r/R \approx 0.25 \) with a value of about 0.75\( \rho gH \). While the dip is smaller than the conical pile case, there is a definite variation in the shapes of the profiles which indicates a difference in the stress structure caused by the deposition process.

An important question concerns the dependence of the stress profile on the heap size. Earlier experiments [14,15] suggested that the size and relative position of the stress maximum may vary with the size of the pile. Alternatively, Radjai [24] has suggested that the relative size of the funnel opening to the size of the heap may be important.

To conclude, we have shown that the construction history affects the pressure distribution at the bottom of a sand pile on a rigid base. These experiments were conducted for conical and wedge shaped piles. We observed the existence of a pressure dip at the center of a sand pile if the filling procedure corresponded to a localized-source. We found that the pressure profile scaled linearly

![FIG. 3. Normal stress profiles beneath wedge-shaped piles of granular material of height \( H \). The piles are made by different construction techniques illustrated by the accompanying photographs (see text). The distance from the center of the profiles is normalized by the pile radius \( R \) and stresses are normalized by \( \rho gH \).](image1)

![FIG. 4. Normal stress profiles for different pile heights \( H \) in localized source experiments for a conical pile. The distance from the center of the profiles is normalized by the pile radius \( R \) and stresses are normalized by \( \rho gH \).](image2)
with the pile height, within the experimental scatter. It seems likely that the progressive formation of the pile by successive small avalanches leads to the occurrence of a pressure dip. In the case of a more uniformly vertical filling via a raining procedure, the dip disappears. A localized-source procedure with a fixed pouring height tends to produce a height dependent stress profile (with a dip). We have shown that the dip in these experiments cannot be caused by a deformation of the base. If small deflections of the base (order $10^{-5}$) were an issue then, that effect should appear in both the localized-source and raining procedures, and would also prevent the collapse of the data for different heap heights.

A heuristic explanation of the mechanism producing the dip is that the flow of particles during the localized-source procedure forms stress chains oriented preferentially in the direction of the slope (c.f. Fig 1.). These chains form arches which shield the center from some of the weight, thereby forming the dip. These effects agree qualitatively with the explanations of Witmmer et al. Explanations for the magnitude and of the dip its variation with geometry and are still lacking. We will present additional details and a more extensive comparison to theory elsewhere.

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