Effect of hexagonal synthetic aperture on the optical system with focal error

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Abstract. We introduce a new approach by using hexagonal synthetic aperture in an optical system and obtaining a high-resolution image. This paper illustrates two cases of an optical system with a hexagonal synthetic aperture where the study was included, First, study the effect of a hexagonal synthetic aperture on a diffraction-bound optical system, Second, study the effect of hexagonal synthetic aperture on an optical system with a focal error of 25λ and 50λ. The results showed a significant improvement in the central intensity, a clear decrease in the secondary peaks, and a decrease in the width of the central intensity curve, which gives a high-resolution image as a result of improving the signal to noise ratio.

Introduction

More than 80 years ago, the first synthetic aperture system was implemented in a Michaelson interferometer [1]. The use of aperture in any optical system improves the quality of the image through that system [2-5] and this is what prompted researchers to study different shapes of those apertures, such as circular, square, rectangular, ellipse ... and others [6-10]. But with the advancements in the field of optics, it has been found that the effect on the central density is greater when using synthetic aperture having the same shape as a single aperture [11-16].

The radiation of a point object passage through a particular optical system distorts image in the image plane and the reason for this is due to both diffraction and aberration [17], what should be recognized is that no matter how much aberration is reduced, diffraction is still a feature of waves. Diffraction and aberration cause the image to be distributed similar to a Gaussian distribution as it extends to a diffuse spot and does not appear as a dot when the source is point [18]. There are several functions such as the spread function (point, linear, and edge) that we rely on to evaluate wave front
efficiency, And no matter how smart data processing methods are developed, it is not possible to produce an image that matches the dimensions of the body completely because the visual system is affected by several factors, Including the shape of the body, the shape of the aperture, the type of lighting used, the wavelength and the types of aberration, all these factors directly affect and play an important role in the distribution of intensity and its spread in the plane of the image [19].

The point spread function is one of the most important functions through which the performance of the optical system can be evaluated and because it is affected by both the diffraction caused by the aperture, the amount and type of aberrations, so the effect of the focal error on the point spread function will be studied using the synthetic hexagonal apertures [20].

**Deriving the point spread function for hexagonal aperture**

By using Fourier transform to pupil function, the complex amplitude at the point \((u, v)\) in plane of image

\[
f(x, y) = \tau(x, y)e^{i(kw(x, y))}
\]

where: \(\tau(x, y)\) pupil transparency or transmission function, \(e^{i(kw(x, y))}\) is aberration function of wave front. \(W(x, y)\) is focal error aberration and \((x, y)\) is exit pupil coordinate. The equation of focal error given by

\[
W(x, y) = w_{20}(x^2 + y^2)
\]

The integral formula for point spread function

\[
F(u, v) = \frac{\int \int f(x, y) dx dy}{n. f \int \int f(x, y)e^{i\pi(ux+vy)} dx dy}
\]

\(F(u, v)\) is complex amplitude and \((n. f = \frac{36N^2}{17\pi^2})\) is Normalizing factor of synthetic aperture (which is calculated by using diffraction limit system), where \(N\) is aperture number .

From Figure 1 we found \(x = \hat{x} + x_j\) and \(y = \hat{y} + y_j\)

![Multiple –Aperture Configuration](image)

**Figure 1: Multiple –Aperture Configuration**

let’s assume \((m = 2\pi v)\) and \((z = 2\pi u)\) where \(m\) and \(z\) represent the image axies ,

\[
F(z, m) = \frac{36N^2}{17\pi^2}\int \int f(\hat{x}, \hat{y})e^{i(z\hat{x}+m\hat{y})} d\hat{x} d\hat{y} \sum_{j=1}^{N} e^{i(zx_j+my_j)}
\]
Since we are dealing with a hexagonal aperture and upon entering the aberrations factor from Eq. 1 and Eq. 2 into Eq. 5 we get

$$F(z, m) = \frac{36N^2}{17\pi^2} \int \tau(x, y)e^{ikw_0(x^2+y^2)} e^{i(x\hat{x}+m\hat{y})} \, dx \, dy \sum_{j=1}^{N} e^{i(x_j+m_j)}$$

(6)

The aberration factor is taken in terms of wavelength when a wave number equal to $\left(\frac{2\pi}{\lambda}\right)$

$$F(z, m) = \frac{36N^2}{17\pi^2} \int \tau(x, y)e^{i2\pi[w_0(x^2+y^2)+(x\hat{x}+m\hat{y})]} \, dx \, dy \sum_{j=1}^{N} e^{i(x_j+m_j)}$$

(7)

By substituting the integral boundary in order to include the area of compound aperture as shown in Figure 2, we get

$$F(z, m) = \frac{36N^2}{17\pi^2} \left[ \int_{\frac{2\pi}{N}}^{\frac{2\pi}{N}} \int_{\frac{2\pi}{N}}^{\frac{2\pi}{N}} \tau(x, y) e^{i2\pi w_0(x^2+y^2)} e^{i2\pi(x\hat{x}+m\hat{y})} \, d\hat{x} \, d\hat{y} \sum_{j=1}^{N} e^{i(x_j+m_j)} \right]$$

(8)

By using the relationship

$$e^{i\theta} = \cos \theta + i \sin \theta$$

(9)

The Eq. 8 can be written as follows
The reason for this is the similarity in the intensity distribution of the two axes, therefore, we keep one axes.

\[
\int_{\frac{a}{N}}^{\frac{a}{N}} \int_{0}^{\frac{2a}{N}} \left( \sum_{j=1}^{N} \cos(\theta x_j + \phi y_j) + i \sin(\theta x_j + \phi y_j) \right) + \int_{\frac{a}{N}}^{\frac{a}{N}} \int_{0}^{\frac{2a}{N}} \left( \sum_{j=1}^{N} \cos(\theta x_j + \phi y_j) + i \sin(\theta x_j + \phi y_j) \right)
\]

Multiply the function \( F(z, m) \) by its complex conjugate, then we obtain the point spread function (PSF) when the incident light is incoherent.

\[
G(z) = |F(z)|^2
\]
\[ G(z) = \frac{36 N^2}{17 \pi^2} \left[ \int \frac{x}{\sqrt{N}} \int \frac{\partial}{\partial x} \cos 2\pi[w_20(x^2 + y^2) + (z\dot{x})] + i \sin 2\pi[w_20(x^2 + y^2) + (z\dot{x})] \right] d\dot{x} d\dot{y} \left[ \int \frac{x}{\sqrt{N}} \int \frac{\partial}{\partial x} \cos 2\pi[w_20(x^2 + y^2) + (z\dot{x})] + i \sin 2\pi[w_20(x^2 + y^2) + (z\dot{x})] \right] d\dot{x} d\dot{y} \]
FIGURE 3. The point spread function for single hexagonal aperture for diffraction limit system and focal errors 0.25λ and 0.5λ.

FIGURE 4. The point spread function for hexagonal synthetic apertures for diffraction limit system when apertures number (N=1,2,3,4,5 and 6).
FIGURE 5. The point spread function for hexagonal synthetic apertures for optical system with focal error $0.25\lambda$ when apertures number ($N=1, 2, 3, 4, 5$ and $6$).

FIGURE 6. The point spread function for hexagonal synthetic apertures for optical system with focal error $0.5\lambda$ when apertures number ($N=1, 2, 3, 4, 5$ and $6$).
Discussion and analysis

The main purpose of using synthetic apertures is to reduce the effect of both aberration and diffraction that cause distortion of the image produced by a particular optical system. Figure 3 shows the effect of the focal error of an optical system that includes a single hexagonal aperture where the central intensity is observed to decrease from 1 in the state of diffraction limit system to 0.822 when the focal error 0.25λ and further decreased and reached 0.477 when the focal error 0.5λ. From this it is clear that an increase in the amount of aberration in the optical system leads to a decrease in the power resolution that system and thus losses in the central intensity, which affects the quality of the image.

When the system is aberration-free (diffraction limit system), we find that the central intensity is constant for all apertures and equals 1, which is the largest magnitude of the central intensity. Note Figure 4. However, we observe an increase in the number of secondary peaks when using the hexagonal synthetic aperture, due to the increase in the amount of diffraction caused by the increased edges of the synthetic aperture compared to the single aperture.

In Figure 5, we note that the optical system was affected by an aberration (focal error) equal to 25λ in addition to being affected by the diffraction associated with any wave. We note that the central intensity that decreased due to aberration, increases when the number of apertures increases, noting that the width of the central intensity curve and the secondary peaks are not affected, as shown by Figures 4 and 5 and Increasing the amount of focal error to 0.5λ Figure 6, the central intensity increases from 0.477 for an optical system with a single hexagonal aperture until it approaches the maximum intensity value with the number of apertures increases.

From the above, it becomes evident that there has been a modification in the value of the maximum intensity in the plane of the image, so we get the greatest intensity in the center. The reason for this is that the structural aperture has a wide field of view in comparison with the single aperture.

Conclusions
1- Focal error affects the central intensity and leads to distortion in the image, and the hexagonal synthetic apertures increases the intensity and improves the visual system.
2- odd apertures (3 and 5) give better results than even apertures (2,4,6) in terms of width of the central intensity curve and the height of the secondary peaks, which increases the optical system power resolution.
3- The synthetic aperture increases the number of secondary peaks

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