Study on implicit finite-element method for steel-arch support

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Abstract. The steel-arch structure in soft-rock tunnels is very small and densely arranged. Too many elements would exist if the solid element is adopted for its simulation, and applying these elements in actual engineering practice is difficult. Therefore, an implicit finite-element method (FEM) to simulate the support effect of steel arches was proposed based on the principle of stiffness superposition. First, an equivalent mechanical model of a steel arch was established based on the principle of stiffness equivalence and deformation-coordination hypothesis. Then, a new concept called auxiliary element was presented. According to this concept, an implicit finite-element model of the equivalent element of the steel arch was developed using the reconstruction technology of elements. The equivalent-stiffness equation of the implicit equivalent element was derived based on the interpolation theory and element-equilibrium equation of FEM, and the equivalent stiffness of the implicit equivalent element was superposed on the stiffness matrix of the auxiliary element to simulate the support effect of the steel arch. Finally, a numerical case was presented to verify the rationality of this method and correctness of the programming. Establishing a solid model of the steel arch using this method was not necessary. Thus, the modeling work was greatly simplified, and adjusting the support scheme was convenient, which provided a new numerical simulation method for the support design of steel arch in soft-rock tunnels.

1. Introduction

Many water diversion and water-transfer projects are under construction or being planned in China to solve the problem of uneven distribution of water resources. The diversion tunnels in these projects are dozens or even hundreds of kilometers long, such as the under construction Yunnan Central Water Diversion Project whose diversion tunnel is 611.99-km long. Long-distance water-conveyance tunnels often inevitably pass through weak interlayers, fault fracture zones, and other unfavorable geological conditions, and steel-arch structures are often used as initial support to minimize the deformation of the surrounding rock and ensure safety of the tunnel, which are the main bearing structures in soft-rock tunnels[1]. Therefore, studying the mechanism of steel-arch supporting structures in soft-rock tunnels and proposing a reasonable and efficient numerical simulation method for the steel-arch structures are very important.

To date, many scholars have conducted a series of studies on steel-arch supporting structures using on-site monitoring, analytical calculation, and numerical simulation. In terms of on-site monitoring and analytical calculation, Wang X Y[2], Wen J Z[3], and Moreira N[4] derived the internal-force analytical formula of the initial support using the foundation curved-beam theory and back-analysis method based on on-site monitoring data. Qu H F[5] studied the adaptability of steel arch and steel grid using on-site monitoring and analytical calculation. Cao X P[6] performed a comparative analysis of the different support schemes for high-ground-stress soft-rock tunnels using field tests. Hu S M[7] obtained the safety
factor of an initial tunnel support using the convergence-constraint method. Li S C et al.\cite{8} established a mechanical-analysis model of a steel arch using the elastic thin-shell theory, improved the surrounding rock-support characteristic curve, and analyzed the mechanical characteristics of the initial support. The on-site monitoring method is very costly and uneconomical; thus, its promotion and application is disadvantageous. In contrast, the finite-element (FE) numerical simulation is widely used owing to its economy and convenience. At present, numerical simulations are mainly divided into two categories. One is the use of the principle of equivalent compressive stiffness on the equivalent elastic modulus of a steel arch instead of concrete for FE calculation\cite{9}, i.e., the elastic-modulus equivalent method. By applying the equivalent elastic-modulus method, Hu S M et al.\cite{10} conducted a comprehensive analysis of the force and deformation of the initial support of a loess tunnel. Zhou J et al.\cite{11} investigated the step-by-step support timing of deeply buried tunnels. Li X F et al.\cite{12} analyzed the influence of the initial-support parameters on the stability of the surrounding rock in a tunnel by combining the field-monitoring data. The other category is the use of beam element to simulate a steel-arch structure and calculate the stress and strain of the steel arch\cite{13}. Wang K Z et al.\cite{14} derived an analytical formula for the unit-support force of a steel arch and applied it to a numerical simulation to simulate the supporting effect of a steel arch. The results of the abovementioned studies provide an important reference for the steel-arch support design of soft-rock tunnels. However, the calculation of the equivalent method is too rough and inconsistent with the real situation, and the three-dimensional modeling that adopts the beam element in the simulation is complicated.

Therefore, the present study developed an equivalent mechanical model of a steel arch based on the principle of stiffness equivalence and deformation-coordination hypothesis. Then, an element-reconstruction technology was proposed to generate an implicit FE model of the steel arch, which avoided the establishment of a large number of solid steel-arch models. Third, by applying the principle of FE stiffness superposition, the stiffness of the equivalent element was superimposed into the proposed auxiliary element-stiffness matrix to simulate the supporting effect of the steel arch. An implicit FE method (FEM) for simulating the supporting effect of the steel arch was developed, which greatly simplified the modeling work in the early stage and facilitated adjustment of a later support plan. Finally, a numerical example was provided to verify the rationality of the proposed implicit FE simulation method for a steel arch and the correctness of the program.

2. Equivalent mechanical model of the steel-arch supporting structure

2.1. Sectional equivalence of a steel-arch supporting structure

The cross-sectional form of a steel-arch structure is complex, such as the commonly used “H-shaped” and “I-shaped” steel. To simplify the calculation, the complex section of the steel arch is considered as equivalent to a rectangular section based on the principle of stiffness equivalence, and the equivalent parameters are calculated in this study. By considering the “I-shaped steel” as an example, the force distribution diagram of the steel-arch structure is shown in Figure 1.

![Figure 1. Force schematic of a steel arch.](image)

The external load (axial force, shear force, and bending moment) is borne by the steel arch itself and material A in the trough. (When concrete is not sprayed, material A in the trough is air. After concrete is sprayed, the material in the trough is concrete.) Figure 1 shows that the shear force, axial force, and
bending moment borne by the steel arch itself and material A are $Q_1$, $N_1$, and $M_1$ and $Q_2$, $N_2$, and $M_2$, respectively. According to Literature\cite{8}, we obtain the following relationships:

$$
K_{eq} = K_1 + K_2
$$
$$
D_{eq} = D_1 + D_2
$$

where $K_{eq}$ and $D_{eq}$ are the equivalent bending and equivalent compressive stiffness, respectively. $K_1$ and $K_2$ are the bending stiffness of the steel arch and material A, respectively. $D_1$ and $D_2$ are the compressive stiffness of the steel arch and material A, respectively. Figure 2 shows that according to the theory of material mechanics, we have

$$
K_1 = E_1 I_x, \quad D_1 = E_1 A_1
$$
$$
K_2 = E_2 \left( \frac{bh^3}{12} - I_x \right), \quad D_2 = E_2 (bh - A_1)
$$
$$
K_{eq} = E_{eq} \left( \frac{bh^3}{12} \right), \quad D_{eq} = E_{eq} h_{eq}
$$

where $E_1$, $E_2$, and $E_{eq}$ are the elastic modulus of the steel arch, elastic modulus of material A, and equivalent elastic modulus, respectively. $I_x$ and $A_1$ are the section moment of inertia and section area along the axis of the steel arch, respectively, and $h_{eq}$ is the equivalent height.

![Figure 2. Schematic of the section equivalence of the steel arch.](image)

Combining Equations (1) and (2) yields the equations for the equivalent height and equivalent elastic modulus as follows:

$$
h_{eq} = \left( \frac{12(K_1 + K_2)}{D_1 + D_2} \right)^{1/2}
$$
$$
E_{eq} = \frac{D_1 + D_2}{bh_{eq}}.
$$

Finally, according to the weight of the cross-sectional area, the equivalent Poisson’s ratio is calculated as follows:

$$
\mu_{eq} = \frac{A_1 \mu_1 + (bh - A_1) \mu_2}{bh}
$$

where $\mu_{eq}$, $\mu_1$, and $\mu_2$ are the equivalent Poisson’s ratio, Poisson’s ratio of the steel arch, and Poisson’s ratio of material A, respectively.
2.2. Constitutive relationship of the steel-arch supporting structure

By using the abovementioned equivalent-rigidity method, the complex section of a steel arch can be simplified into a rectangular section. Furthermore, the equivalent hexahedral element of the steel arch (hereinafter referred to as the equivalent element) can be used to simulate the supporting effect of the steel arch relative to the elastic constitutive relationship. The stress–strain relationship of the equivalent element is expressed as follows:

\[ \{\sigma\} = [D_e]\{\varepsilon\} \tag{6} \]

where \( \{\sigma\} \) is the stress component of the equivalent element, \([D_e]\) is the elastic matrix, and \(\{\varepsilon\} \) is the strain component of the equivalent element.

Simultaneously, a new concept called stress back-calculation coefficient is proposed to inversely calculate the stress of the steel arch. The concept is based on the following basic assumptions. (1) The steel arch and material A share the load. (2) The deformations of the steel arch and material A coordinate with each other without a relative slippage. (3) The stress back-calculation coefficients of the six stress components of the steel arch are the same.

On the basis of the abovementioned assumptions, we consider the axial deformation as an example to derive the expression of the stress back-calculation coefficient. Because the deformations of the steel arch and material A coordinate with each other without a relative slippage, we have

\[ \frac{\sigma_{n1}}{E_1} = \frac{\sigma_{n2}}{E_2} \tag{7} \]

where \( \sigma_{n1} \) and \( \sigma_{n2} \) are the axial stress of the steel arch and material A, respectively. Because the axial force of the equivalent element is borne by the steel arch and material A, we have

\[ \sigma_{bh\text{sq}} = \sigma_{n1}A_e + \sigma_{n2}(bh - A_e) \tag{8} \]

where \( \sigma_n \) is the axial stress of the equivalent element.

Solving simultaneous Equations (7) and (8) yields the axial stress of the steel arch as follows:

\[ \sigma_{n1} = \frac{E_1bh_{\text{sq}}}{A_e E_1 + (bh - A_e)E_2} \cdot \sigma_n = \lambda \cdot \sigma_n \tag{9} \]

where \( \lambda \) is the stress back-calculation coefficient of the steel arch. Then, the stress components of the steel arch can be obtained by

\[ \{\sigma_t\} = \lambda \cdot \{\sigma\} \tag{10} \]

where \( \{\sigma_t\} \) and \( \{\sigma\} \) are the stress components of the steel arch and equivalent element, respectively. Combining Equations (6) and (10) yields the constitutive relationship of the steel arch.

\[ \{\sigma_t\} = \lambda \cdot [D_e]\{\varepsilon\} \tag{11} \]

3. Implicit FEM of a steel-arch support

In actual engineering practice, the steel-arch support structures are densely arranged. If solid-element modeling is used, the number of model grids is large, which greatly increases the modeling workload and the difficulty in future calculation and processing. In addition, adjusting the support scheme is not convenient. Therefore, an implicit FEM is proposed in this paper to simulate the supporting effect of the steel arch. This method does not require establishment of a solid model of a steel arch. Instead, by constructing an auxiliary element, the stiffness of the equivalent element is superimposed on the stiffness matrix of the auxiliary element to simulate the supporting effect of the steel arch. Simultaneously, an implicit equivalent-element reconstruction program is developed to facilitate the superposition of the stiffness of the equivalent element and visual display of the stress and deformation of the steel arch.
3.1. Reconstruction technology of the implicit equivalent element

We first describe the auxiliary element. Auxiliary element refers to a layer of thin elements whose thickness is the same as that of the equivalent element. It is arranged at the tunnel excavation surface. Its role is to serve as a carrier for the superposition of the equivalent-element stiffness. We need to note that by assigning different material parameters for the auxiliary element, different support schemes can be simulated. When concrete is not sprayed, the auxiliary element can be considered as an air element with a very small parameter to simulate the independent supporting effect of the steel arch. After concrete is sprayed, the auxiliary element is set as a concrete element, which can simulate the combined supporting effect of the steel arch and concrete.

Obviously, the prerequisite for superposition of the equivalent-element stiffness is to determine the node coordinate information of all equivalent elements in the FE model. Therefore, according to the topological relationship between the plane and auxiliary hexahedral element, a pre-processing program for the implicit FE simulation of steel arches in soft-rock tunnels is developed using the Python programming language, i.e., the implicit equivalent-element reconstruction program.

Figure 3 shows that we first build a layer of auxiliary elements according to the equivalent height calculated by Equation (3). Then, according to selected steel-arch support spacing \( \alpha \), \( n = \lceil l / \alpha \rceil \) cutting planes can be constructed in the range of the auxiliary elements \( y = (0, l) \). Finally, according to the topological relationship between the cutting surface and auxiliary element, all line information can be obtained by cutting. Then, distance \( b \) is adjusted along the \( y \) direction to reconstruct all equivalent elements.

![Figure 3. Reconstruction of the implicit equivalent element.](image)

The reconstruction technology of the implicit equivalent element not only provides a precondition for stiffness superposition of all equivalent elements but also paves the way for the visual display of the stress and deformation of all steel arches in the later stage.

3.2. Principle of stiffness superposition of implicit equivalent element

Figure 4 shows that the equivalent element described in Section 2.1 is implicit in the auxiliary element. By using the implicit equivalent-element reconstruction program with the auxiliary elements, as presented in Section 3.1, the geometric information of all implicit equivalent elements can be obtained. The superposition of the stiffness of the equivalent element is described as follows.

We assume that the node displacements of the auxiliary and equivalent elements are \( \{ \delta \} \) and \( \{ \delta' \} \), respectively, i.e.,

\[
\{ \delta \} = \{ u_1, v_1, w_1, \cdots, u_b, v_b, w_b \} \\
\{ \delta' \} = \{ u', v', w', \cdots, u'_b, v'_b, w'_b \}
\]  

(12)

where \( u, v, \) and \( w \) represent the displacement of a node in the \( x, y, \) and \( z \) directions, respectively. From the principle of shape-function interpolation, we obtain

\[
\{ \delta' \} = [N]\{ \delta \}
\]  

(13)
where \([N]\) is the shape-function matrix, which can be expressed as follows:

\[
[N] = \begin{bmatrix}
N_1^1 & 0 & 0 & N_2^1 & 0 & 0 & \cdots \\
0 & N_1^1 & 0 & 0 & N_2^1 & 0 & \cdots \\
0 & 0 & N_1^1 & 0 & 0 & N_2^1 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots \\
N_1^{24} & 0 & 0 & N_2^{24} & 0 & 0 & \cdots \\
0 & N_1^{24} & 0 & 0 & N_2^{24} & 0 & \cdots \\
0 & 0 & N_1^{24} & 0 & 0 & N_2^{24} & \cdots \\
\end{bmatrix}
\] (14)

where \(N_j^i\) is the value of the shape function of the \(j\)th node of the auxiliary element at the \(i\)th node of the equivalent element.

\[\begin{array}{c}
\text{The auxiliary element} \\
\text{The equivalent element}
\end{array}\]

Figure 4. Schematic of the stiffness superposition of the implicit equivalent element.

We assume that the node loads of the auxiliary and equivalent elements are \(\{F\}\) and \(\{f\}\), respectively; therefore,

\[
\{F\} = \{F_{x1}, F_{y1}, F_{z1}, \ldots, F_{x8}, F_{y8}, F_{z8}\}
\]

\[
\{f\} = \{f_{x1}, f_{y1}, f_{z1}, \ldots, f_{x8}, f_{y8}, f_{z8}\}
\] (15)

According to the principle of static equivalent of FE, we have

\[
\{F\} = [N]^T \{f\}.
\] (16)

The FE equilibrium equations of the auxiliary and equivalent elements are expressed as follows:

\[
[K_{eq}][\delta] = \{f\}
\]

\[
[\Delta K_{au}][\delta] = \{F\}
\] (17)

where \([K_{eq}]\) and \([\Delta K_{au}]\) are the stiffness matrix of the equivalent element and that superposed into the auxiliary element, respectively. Combining Equations (13), (16), and (17) yields \([\Delta K_{au}]\) as

\[
[\Delta K_{au}] = [N]^T [K_{eq}] [N].
\] (18)

3.3. Iterative-calculation method of implicit FE for a steel-arch support

In this study, the elastoplastic-damage constitutive model is used to simulate the mechanical properties of the surrounding rock, and the auxiliary and equivalent elements are regarded as elastic media to simulate the supporting effect of the steel arch. The incremental variable damage-stiffness method proposed by Yu Y T and Xiao M\(^{[15]}\) is used for the iterative solution.
In each iterative-calculation process, displacement vector of the auxiliary element \{\delta\} can be obtained. According to the principle of shape-function interpolation, displacement vector of the equivalent element \{\delta^i\} can be obtained using Equation (13), and stress component of the equivalent element \{\sigma\} can be calculated as follows:

$$\{\sigma\} = [D_i][B][\delta^i]$$  \hspace{1cm} (19)

where \([D_i]\) and \([B]\) are the elastic and strain matrices of the equivalent element. Then, stress components of the steel arch \{\sigma_j\} can be obtained using Equation (10).

In the iterative-calculation process, when the maximum stress of the steel arch exceeds the yield strength of steel, the stiffness matrix provided by the equivalent element to the auxiliary element, i.e., \([\Delta K_i]\), needs to be set to \([\theta]\). At the same time, the stress increment that exceeds the yield surface, namely, \{\Delta \sigma_j\} = \left(\frac{\sigma_{\text{max}}}{\sigma_y} - 1\right) \cdot \{\sigma_j\} , is transformed into node loads and applied to the auxiliary-element nodes in the reverse direction using Equation (20). Then, the next step of the iterative calculation is performed.

$$\{\Delta F\} = [N]^T \cdot \iint_{\Omega} [B]^T \{\Delta \sigma_j\} dx dy dz$$  \hspace{1cm} (20)

4. Case analysis
We consider the excavation and support of a circular tunnel in a class-V surrounding-rock geological condition as an example. Then, a numerical model for implicit simulation of the steel arch is established, and the implicit equivalent element proposed in this paper is used to simulate the supporting effect of the steel arch. We compare the results before and after the support to illustrate the rationality of the implicit FE simulation method of the steel arch and the validity of the program.

4.1. Establishment of the implicit numerical model
The selected steel is Q235 whose yield strength is 235 MPa, \(E_i = 200 \text{ GPa}\), and \(\mu_i = 0.3\). The basic parameters of the steel-arch structure are listed in Table 1.

| Type  | \(b\) | \(h\) | \(A_1\) | \(I_1\) | Layout spacing |
|-------|------|------|------|------|-------------|
| I20a  | 100  | 200  | 35.578 | 2370  | 0.4         |

This study only simulates the single-support function of the steel-arch structure. The mechanical parameters of material A and the material in the auxiliary element are the same as those of the air element. In other words, the elastic modulus is \(1.0 \times 10^6 \text{ GPa}\), and the Poisson’s ratio is 0.41. The mechanical parameters of the class-V surrounding rock are listed in Table 2.

| Material | Elastic modulus | Poisson ratio | Cohesion | Inner friction angle | Tensile strength | Compressive strength |
|----------|-----------------|---------------|----------|---------------------|-----------------|---------------------|
| Rock     | 0.80            | 0.35          | 0.10     | 16.70               | 0.08            | 12.00               |

According to the parameters of the steel arch listed in Table 1, the equivalent mechanical parameters can be obtained using the Equations (3), (4), and (5) presented in Section 2.1, i.e., \(h_{eq} = 0.28\), \(E_{eq} = 23.44 \text{ GPa}\), and \(\mu_{eq} = 0.39\). According to these equivalent parameters, the numerical model of the implicit simulation for the steel-arch support is shown in Figure 5.
Figure 5. Numerical model of the implicit simulation of the steel-arch support.

The calculation ranges are 154.23 m in the x direction, 20.00 m in the y direction, and 145.92 m in the z direction. The excavation diameter of the tunnel is 4.60 m, and the burial depth is 85.00 m. Because the width of the steel arch is only 0.1 m and the layout spacing is 0.4 m, when the explicit solid element is used to simulate the steel arch, the size of the model element in the y direction needs to be controlled, which can inevitably lead to a large number of grids in the explicit model. However, by applying the implicit simulation method proposed in this paper, the number of grids in the implicit numerical model is only 19,300, which greatly reduces the number of grids.

4.2. Analysis of the calculation results

The self-weight stress field is used as the initial condition for the calculation, and the lateral pressure coefficient is \( \mu / (1 - \mu) = 0.538 \). The boundary conditions are as follows: normal constraint in the x and y directions, full constraint in the z-axis negative direction, and no constraint in the z-axis positive direction.

Two calculation conditions are considered, namely, (1) before and (2) after the support. In the calculation of the steel-arch support, we consider the surrounding-rock excavation-load release coefficient of 0.8 in the calculation without loss of generality. A typical section (\( y = 10.0 \) m) is selected to perform a series of comparative analyses according to the following four aspects: damage zone, rock deformation, steel-arch deformation, and steel-arch stress.

4.2.1. Comparative analysis of the damage zone of the surrounding rock.

Figures 6 and 7 show the distribution map of the damage zone under conditions (1) and (2), respectively. We can observe that the damage to the sidewalls of the cavern is more serious, the development depth of the plastic zone in the sidewalls is significantly larger than that at the top and bottom of the cavern, and a partially cracking zone appears. The distribution trend of the damage zone under the two conditions is completely consistent, which shows the validity and accuracy of the implicit simulation calculation.

Figure 6. Distribution of the damage zone under condition (1).

Figure 7. Distribution of the damage zone under condition (2).
The list in Table 3 shows that although the distribution trend of the damage under the two conditions is consistent, the magnitude of the damage indexes under condition (2) is obviously smaller than that under condition (1). This result indicates that the implicit simulation method of the steel arch proposed in this paper can rationally simulate the supporting effect of the steel arch.

| Condition | Plastic Zone | Cracking Zone | Total damage zone | Plastic dissipation energy |
|-----------|--------------|---------------|-------------------|---------------------------|
|           | m³           | m³            | m³                | t·m                       |
| (1)       | 8981.6       | 261.8         | 9243.8            | 738.6                     |
| (2)       | 7163.6       | 74.1          | 7237.7            | 328.6                     |

4.2.2. Comparative analysis of the rock deformation.

By taking into account the symmetry of the model and boundary conditions, half of the surrounding rock is taken into consideration. We consider points A and B in Figure 5 as the starting and end points, respectively. Then, the change trend of the resultant surrounding-rock displacement along the circumference of the cavern is shown in Figure 8.

![Figure 8](image)

**Figure 8.** Comparison of the resultant displacement under the two conditions.

We can observe that the deformation of the surrounding rock shows a trend in which the top and bottom are larger and the middle part of the sidewalls is smaller. The deformation trends of the surrounding rock under the two conditions are the same. However, the magnitude of the deformation under condition (2) is obviously smaller than that under condition (1), which not only shows that the steel arch can obviously minimize the deformation of the surrounding rock but also proves the rationality of the implicit simulation method for the steel arch and the correctness of the programming.

4.2.3. Analysis of the deformation and stress of the steel arch.

The deformation pattern of the steel arch along the circumference of the cavern is shown in Figure 9, which shows that the deformation pattern of the steel arch is basically the same as that of the surrounding rock, whereas the deformation value is significantly smaller than that of the surrounding rock. The main reasons for the difference are that the elastic load of the surrounding rock is instantaneously released after excavation of the cavern and the surrounding rock has already undergone a large deformation before the steel arch can support it.

The nephogram of the first principal stress of the steel arch is shown in Figure 10, which shows that the stress in the steel arch is relatively large at the sidewalls, top, and bottom of the cavern and relatively small in the remaining area. The maximum stress is $-182.00$ MPa, which is still within the yield-strength range of steel, indicating that the steel arch has a certain safety reserve.
5. Conclusions

- The sectional equivalence of a steel-arch structure was investigated using the principle of stiffness equivalence, and the equivalent-element method proposed in this paper was applied to simulate the supporting effect of the steel arch. Then, the constitutive relationship of the steel arch was derived based on the hypothesis of deformation coordination. Finally, the equivalent mechanical model of the steel-arch model was established.

- An element-reconstruction method of the implicit equivalent element was proposed, and the program was implemented using Python. The formula for stiffness equivalence of the implicit equivalent element was derived based on the principle of shape-function interpolation and equilibrium equation of FEM. Furthermore, the iterative-calculation method of the implicit FEM for the steel-arch support was provided. Finally, an implicit simulation method of the steel-arch support was developed.

- A numerical example was used to verify the rationality of the implicit FEM for the steel-arch support and the correctness of the program.

- The implicit FEM of the steel-arch support proposed in this paper greatly simplified the modeling work and facilitated the adjustment of the support scheme, which provided a new numerical simulation method for the design of the initial support for soft-rock tunnels.

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