Meson decay in an independent quark model

Hakan ÇİFTÇİ and Hüseyin KORU
Gazi University, Faculty of Art and Science, 06500 Ankara, TURKEY
March 25, 2022

Abstract
Leptonic decay widths and leptonic decay constants of light vector mesons and weak leptonic decay widths and weak decay constants of light and heavy pseudoscalar mesons have been studied in a field-theoretic framework based on the independent quark model with a scalar-vector power-law potential. The results are in very good agreement with the experimental data.

PACS numbers: 12.39.Ki, 12.39.Pn, 13.20.-v

1 Introduction

Though quantum chromodynamics is considered to be the underlying theory of strong interaction between quarks and gluons at the structural level of hadrons, many low-energy phenomena such as spectroscopy, static electromagnetic properties, can not be explained by first-principles application of QCD. Therefore one needs to resort to phenomenological models. For example, a potential model with an equally mixed scalar-vector harmonic potential\(^1\) of independent quarks in a relativistic Dirac framework has been used to study several low-energy phenomena in the baryonic sector such as octet baryon masses\(^2\), magnetic moments\(^3\), weak electric form factors\(^4\), nucleon electromagnetic form factors and charge radii\(^5\). In addition to the harmonic potential, with power-law potential such as \(r^\nu (\nu > 0)\) heavy-heavy and light-heavy quarkonium states have been studied and the results are in good agreement with experimental results\(^6\). This model has also been successful in explaining pion mass,
its decay constant\textsuperscript{7} and the radiative decay\textsuperscript{8} of ordinary light and heavy mesons. Because of this wide range of applicability of the model to both baryons and mesons, it has proved to be a rather simple and successful alternative to the cloudy bag model\textsuperscript{9}. The purpose of this work is to extend its applicability to the study of the leptonic decay of vector mesons such as $\rho$, $\omega$, $\phi$ and weak leptonic decay of light and heavy pseudoscalar mesons for an equally mixed scalar-vector power-law potential such as $Ar^\nu$. The leptonic decay width of heavier vector mesons in the charm and bottom quark sector have been studied in the nonrelativistic approach through the Van Royen-Weisskopf formula with radiative corrections\textsuperscript{10}. But the same approach is not suitable for ordinary vector mesons in the light flavor sector, where the constituent quark dynamics is more relativistic. On the other hand, for weak leptonic decays, while many nonrelativistic quark model calculations\textsuperscript{11} suggest that $f_K > f_D > f_B$ ($f_M$ is the weak leptonic decay constant), some of the models based on QCD sum rules\textsuperscript{12} and lattice calculations\textsuperscript{13} predict more or less a constant $f_M$ between $K$ and $B$ mesons. Capstick and Godfrey\textsuperscript{14} calculate the hadronic matrix elements for the relativitized quark model expression for $f_M$. They find $f_{B_c} > f_{D_s} > f_K > f_D > f_{B_s} > f_B > f_\pi$, but the calculated value of the ratio $f_K/f_\pi \approx 1.75$ is much higher than the experimental value of 1.22. The leptonic decay widths and decay constants $f_V$ of the light vector mesons have been calculated by using an equally mixed scalar vector harmonic potential\textsuperscript{15}. Also, weak leptonic decay constants, $f_M$, of pseudoscalar mesons have been calculated using the same potential and are found to satisfy $f_{B_c} > f_{D_s} > f_D > f_K > f_{B_s} > f_B > f_\pi$\textsuperscript{16}. Potential used in Ref. 15 and 16 is harmonic. Since the potential between quarks is weaker than the harmonic potential, the potential used in this study is $Ar^{0.2} + V_0$ which gives very good results for heavy-heavy and light-heavy quarkonium states and radiative decay of ordinary light and heavy mesons\textsuperscript{6,8}. Both in Ref. 15,16 and in this study, the potential does not include Coulomb term because Coulomb-like vector interaction are believed to have less prominent role for the light mesons.
2 Potential model

The quark-confining interaction in a hadron, which is believed to be generated by the nonperturbative multigluon mechanism, is not possible to calculate theoretically from first-principles within QCD. Therefore, from a phenomenological point of view, the present model assumes that the quark and antiquark in a hadron core are independently confined by an average flavor-independent potential \( V(r) \) of the form

\[
V(r) = \frac{1}{2}(1 + \beta)(Ar^\nu + V_0), \quad A > 0 \quad \text{and} \quad \nu > 0. \tag{1}
\]

For this potential, Dirac equation can be written as

\[
[\vec{\alpha} \cdot \vec{p} + \beta m + V(r)] \Psi(r) = E \Psi(r) \tag{2}
\]

where \( \vec{\alpha} \) and \( \beta \) are Dirac matrixes. Eq. (2) has two solutions with positive and negative energy given respectively in the forms

\[
\psi_\Lambda(r) = \left[ \frac{i g_\Lambda(r)/r}{\vec{\sigma} \cdot \hat{f}_\Lambda(r)/r} \right] U_\Lambda(\hat{r}) \tag{3}
\]

\[
\phi_\Lambda(r) = \left[ \frac{\vec{\sigma} \cdot \hat{r} f_\Lambda(r)/r}{g_\Lambda(r)/r} \right] \tilde{U}(\hat{r}) \tag{4}
\]

where \( \Lambda = (nljm) \) represents the set of Dirac quantum numbers specifying the eigenmodes. The spin angular parts \( U_\Lambda(\hat{r}) \) and \( \tilde{U}_\Lambda(\hat{r}) \) are described as

\[
U_{ljm}(\hat{r}) = \sum_{m_1, m_s} \langle lm_1 m_s | jm \rangle Y_{l_1}^{m_1}(\hat{r}) \chi_{1/2}^{m_s}, \tag{5}
\]

\[
\tilde{U}_{ljm}(\hat{r}) = (-1)^{j+m-l} U_{lj-m}(\hat{r}) \tag{6}
\]

Substituting Eq. (3) or Eq. (4) into Eq. (2), one obtains (for \( n = 0, l = 0 \))

\[
\left[ \frac{-1}{2} \frac{d^2}{dr^2} + \lambda_q A r^\nu \right] g_\Lambda(r) = \lambda_q (E - m - V_0) g_\Lambda(r), \tag{7}
\]

\[
f_\Lambda(r) = \frac{1}{\lambda_q} \left( \frac{1}{dr} - \frac{1}{r} \right) g_\Lambda(r), \tag{8}
\]

where \( \lambda_q = E + m \), \( E \) is energy of confined quark and \( m \) is quark mass. Using the substitution \( \rho = (\lambda_q A)^{\frac{1}{\nu+2}} r \) for convenience, Eq. (7) reduces to the form

\[
\left[ \frac{-1}{2} \frac{d^2}{d\rho^2} + \rho^\nu \right] g(\rho) = \epsilon g(\rho), \tag{9}
\]
where

$$\epsilon = \left[ \frac{\lambda'^2}{A^2} \right]^{\nu+2} (E - m - V_0),$$

(10)

and \( g(\rho) \) is chosen as

$$g(\rho) \approx \rho \exp \left( - (x\rho)^d \right),$$

(11)

where \( x \) and \( d \) are variation parameters and they are obtained by minimizing \( \epsilon \). Hence they are solutions of

$$\frac{\partial \epsilon}{\partial x} = 0,$$

(12)

$$\frac{\partial \epsilon}{\partial d} = 0.$$

(13)

Using Eq. (9), Eq. (11) and Eq. (12), \( \epsilon \) is found to be

$$\epsilon = \left( \frac{\nu + 2}{\nu} \right) ax^2$$

(14)

where \( x = \left( \frac{b\nu}{2\gamma} \right)^{\frac{1}{\nu+2}}, \ a = \left( \frac{d+1}{8} \right) 2^{\frac{d}{\nu}} \frac{\Gamma\left(\frac{d}{\nu}\right)}{\Gamma\left(\frac{3}{\nu}\right)} \), \( b = 2^{\frac{d}{\nu}} \frac{\Gamma\left(\frac{d+1}{\nu}\right)}{\Gamma\left(\frac{3}{\nu}\right)} \). When Eq. (13) is calculated numerically, \( d \) is obtained as 1.502 and using this value \( \epsilon \) is found as 1.3268 which is very close to the results of the WKB method and 1/N-expansion.

Thus, the lowest eigenmodes corresponding to the positive and negative energies have the respective explicit forms

$$\psi_{\Lambda(1s_1/2)}(r) = \frac{1}{\sqrt{4\pi}} \left[ \frac{ig(r)/r}{\vec{\sigma} \cdot \hat{r} f(r)/r} \right] \chi_m,$$

(15)

$$\phi_{\Lambda(1s_1/2)}(r) = \frac{1}{\sqrt{4\pi}} \left[ \frac{\vec{\sigma} \cdot \hat{r} f(r)/r}{-i g(r)/r} \right] \bar{\chi}_m,$$

(16)

where the two component spinors \( \chi_m \) and \( \bar{\chi}_m \) denote \( \chi_\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \chi_\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \) and \( \bar{\chi}_\uparrow = \begin{pmatrix} 0 \\ -i \end{pmatrix}, \bar{\chi}_\downarrow = \begin{pmatrix} i \\ 0 \end{pmatrix} \). Using Eq. (8) and Eq. (5), the radial parts in the upper and lower component solutions corresponding to a quark flavor \( q \) are

$$g_q(r) = \frac{N_q}{r_{0q}} \left( \frac{r}{r_{0q}} \right)^d \exp \left( - \left( \frac{x}{r_{0q}} \right)^d r^d \right),$$

(17)

$$f_q(r) = -\frac{N_q d}{\lambda_q r_{0q}} \left( \frac{x}{r_{0q}} \right)^d r^d \exp \left( - \left( \frac{x}{r_{0q}} \right)^d r^d \right),$$

(18)

where, \( N_q \) is normalization constant obtained from the equation

$$N_q^2 \int_0^{\infty} \left( f_q^2(r) + g_q^2(r) \right) dr = 1.$$
3 Quark-antiquark momentum distribution

Knowing the quark-antiquark eigenmodes in the ground-state of meson, it is possible to obtain their corresponding momentum distribution amplitude. If $G_q(p, \lambda, \lambda')$ is the amplitude for finding a bound quark of flavor $q$ in its eigenmode $\Phi^{(+)}_{q\lambda}(r)$ in a state of definite momentum $p$ and spin projection $\lambda'$, then it is given by\textsuperscript{15,17}

$$\Phi^{(+)}_{q\lambda}(r) = \sum_{\lambda'} \int d^3pG_q(p, \lambda, \lambda') \sqrt{\frac{m}{E_p}} U_q(p, \lambda') \exp(i\vec{p} \cdot \vec{r}). \quad (20)$$

Where $U_q(p, \lambda')$ is the usual free Dirac spinors. Eq. (20) can be easily inverted to yield

$$G_q(p, \lambda, \lambda') = \sqrt{\frac{m}{E_p}} U^\dagger_q(p, \lambda') \int d^3r \Phi^{(+)}_{q\lambda}(r) \exp(-i\vec{p} \cdot \vec{r}). \quad (21)$$

Thus, it is found as

$$G_q(p, \lambda, \lambda') = G_q(p) \delta_{\lambda \lambda'}, \quad (22)$$

where

$$G_q(p) = \frac{i\pi N_q}{\sqrt{2dr_0q}} \beta^q \frac{1}{\lambda_q} (E_p + E_q) \sqrt{\frac{m + E_p}{E_p}} H(d, z), \quad (23)$$

where, $E_p = \sqrt{p^2 + m^2}$, $E_q$ is the solution of Eq. (10), $\beta = \left(\frac{q}{r_0q}\right)^d$, $z = \frac{mE_p}{2\sqrt{2}}$ and

$$H(d, z) = \frac{d^2}{2^{d+2}z^2} \int_0^\infty y^{\frac{d+1}{2}} \exp\left(-\left(\frac{y}{2}\right)^d\right) J_{\frac{3}{2}}(yz) dy. \quad (24)$$

4 Leptonic decay widths

Now following Margolis and Mendel\textsuperscript{17} one can represent the ground state of a neutral vector meson such as $(\rho, \omega, \phi)$ with a particular spin projection $S_V$ and zero momentum as

$$|V(0), S_V\rangle = \frac{\sqrt{3}}{\sqrt{N(0)}} \sum_{q,\lambda_1,\lambda_2} \int d^3pG_q(p)C_{\lambda_1,\lambda_2}^{SV} \langle V| b^\dagger_q(p, \lambda_1) \tilde{b}^\dagger_q(-p, \lambda_2) |0\rangle. \quad (25)$$

Here, $b^\dagger_q(p, \lambda_1)$ and $\tilde{b}^\dagger_q(-p, \lambda_2)$ operating on the vacuum state are quark and antiquark creation operators, respectively. The summation with the flavor coefficient $\zeta_q^V$ and the spin configuration coefficient $C_{\lambda_1,\lambda_2}^{SV}$ represents the appropriate SU(6) spin-flavor structure of the particular vector meson $V$ with its spin projection $S_V$ and zero
momentum. The factor $\sqrt{3}$ is due to the effective color singlet configuration of the meson. $N(0)$ represents the overall normalization, which is given by

$$N(0) = \frac{1}{(2\pi)^3} \int_0^\infty d^3p |G_q(p)|^2.$$  
(26)

Assuming that the main contribution to the leptonic decay process of neutral vector mesons such as $\rho$, $\omega$, $\phi$ comes from single virtual photon creation from the annihilation of the bound quark-antiquark pair inside of the meson, $S$-matrix element in configuration space can be written as

$$S_{fi} = \langle e^-(k_1, \delta_1)e^+(k_2, \delta_2)| - ie^2 \int d^4x_1 d^4x_2 (\bar{\psi}_e(x_2)\gamma^\mu\psi_e(x_2)D_{\mu\nu}(x_2 - x_1)$$

$$\times \sum_q e_q\bar{\psi}_q(x_1)\gamma^\nu\psi_q(x_1)) |V, S_V \rangle$$  
(27)

Where, $D_{\mu\nu}(x_2 - x_1)$ is the photon propagator, $\psi_e(x)$ and $\psi_q(x)$ are the free lepton and quark fields, respectively. After some standard calculations (details of the calculation can be found in Ref. 15), one obtains

$$\Gamma(V \to e^+e^-) = \frac{4\pi}{3} \alpha_{em}^2 M_V f_V^2.$$  
(28)

Where, $f_V$ is known as the leptonic decay constant and in this model it can be written as

$$f_V^2 = \frac{2\langle e_q \rangle_V^2 I_V^2}{3\pi^2 M_V^3 J_V}.$$  
(29)

Here,

$$\langle e_q \rangle_{\rho,\omega,\phi} = \left(\frac{1}{\sqrt{2}}, \frac{1}{3\sqrt{2}}, \frac{1}{3}\right);$$

$$I_V = \int_0^\infty dpd^2 \left(2 + \frac{m}{E_p}\right) G_q(p),$$

$$J_V = \int_0^\infty dpd^2 |G_q(p)|^2.$$  
(30)

$I_V$ and $J_V$ values can be calculated numerically by computer.

5 Weak leptonic decay widths

Here, the weak leptonic decay of charged pseudoscalar mesons such as $\pi^\pm$, $K^\pm$, $D^\pm$, $D_s^\pm$, $B^\pm$ and $B_c^\pm$ are considered. Assuming that the main contribution to the
weak leptonic decay processes comes from the single virtual boson creation from the
annihilation of the bound quark-antiquark pair inside the pseudoscalar meson \( M \),
the \( S \)-matrix element in configuration space is written as\(^{16}\)

\[
S_{fi} = \langle l(k_1, \delta_1) \bar{\nu}(k_2, \delta_2) \left| \frac{-iG_F}{\sqrt{2}} \right| \int d^4x \overline{\psi}_l(x) \gamma^\mu \left(1 - \gamma^5\right) \psi_l(x) \times \sum_{q_m,q_n} \nu_{q_m q_n} \psi_{q_m}(x) \gamma_\mu \left(1 - \gamma^5\right) \psi_{q_n}(x) |M(0)\rangle.
\]

Where, \( G_F \) is the Fermi coupling constant, \( \nu_{q_m q_n} \) are the CKM matrix elements and
\(|M(0)\rangle\) is given by\(^{16}\)

\[
|M(0)\rangle = \sqrt{\frac{3}{N(0)}} \sum C_{q_1 q_2}^M(\lambda_1, \lambda_2) \times \int d^3p \left[G_{q_1}(p)G_{q_2}^*(\bar{p})\right]^{\frac{1}{2}} b_{q_1}^\dagger(p, \lambda_1) \bar{b}_{q_2}^\dagger(-p, \lambda_2) |0\rangle,
\]

where, \( C_{q_1 q_2}^M(\lambda_1, \lambda_2) \) stands for the appropriate SU(6) spin-flavor coefficients for the
pseudoscalar meson \( M \). \( N(0) \) represents the overall normalization, which is given
by

\[
N(0) = \frac{1}{(2\pi)^3} \int_0^\infty d^3p \left(G_{q_1}(p)G_{q_2}^*(\bar{p})\right).
\]

After some standard calculations, which can be found in Ref. 16, one obtains

\[
\Gamma(M \rightarrow l\bar{\nu}l) = \frac{G_F^2}{8\pi} |\nu_{q_1 q_2}|^2 M_p m_l^2 \left(1 - \frac{m_l^2}{M_p^2}\right)^2 f_M^2,
\]

where \( f_M \) is the weak decay constant, having the form

\[
f_M^2 = \frac{3 I_M^2}{2\pi^2 M_p J_M}.
\]

Here \( M_p \) is the mass of pseudoscalar meson, \( m_l \) is the mass of lepton and \( I_M \) and
\( J_M \) are found as

\[
I_M = \int_0^\infty dp p^2 A(p) \left[G_{q_1}(p)G_{q_2}^*(\bar{p})\right]^{\frac{1}{2}},
\]

\[
J_M = \int_0^\infty dp p^2 \left[G_{q_1}(p)G_{q_2}^*(\bar{p})\right],
\]

and

\[
A(p) = \frac{(E_{p_1} + m_{q_1})(E_{p_2} + m_{q_2}) - p^2}{[E_{p_1} E_{p_2} (E_{p_1} + m_{q_1})(E_{p_2} + m_{q_2})]^2},
\]

where \( E_{p_i} = \sqrt{p^2 + m_{q_i}^2} \).
6 Results

The calculations involve the potential parameters of the model \((A, V_0, \nu)\) and the quark masses \((m_u = m_d, m_s, m_c, m_b)\). The potential parameters are chosen to be \(A = 0.68\) GeV, \(V_0 = -0.3961\) GeV, \(\nu = 0.2\).

The light-quark masses \(m_u = m_d\) and \(m_s\) are obtained from \(\omega\) and \(\phi\) mesons as \(m_u = m_d = 0.078\) GeV, \(m_s = 0.3\) GeV, and from \(D^\pm\) and \(B^\pm\) \(m_c = 1.3\) GeV, \(m_b = 4.81\) GeV.

When these parameters are used the masses of \(D, D_s, B, B_s, B_c, w\) and \(\phi\) can be calculated to be almost the same with their experimental values. Also, with the same parameters, radiative decay widths of light and heavy mesons have been calculated and the obtained results are close to their experimental values. Using this potential parameters and quark masses given above, the calculated results are shown in Table I, Table II and Table III.

7 Conclusion

In this paper, the independent particle model approach has been used to investigate leptonic decay of light vector mesons and weak leptonic decay of light and heavy pseudoscalar mesons. The Dirac equation in a power-law potential has been solved with variation technique using special type wave function. Leptonic decay widths and decay constant \(f_V\) and weak leptonic decay widths and decay constant \(f_M\) have been calculated and are compared with the results of other theoretical investigations and experiments. The results say to us that the potential used in this study can be considered as interaction potential for quarks. Because of the model structure it is assumed that quarks do not interact with each other. However, there are spin-spin and other hyperfine interactions between quarks. Adding these interactions, good results, especially in the calculation of meson mass spectra, could be obtained. Such interaction terms can be found in Ref. 24,25.
Acknowledgments

We thank T.M. Aliev and H. Akcay for useful discussions.

References

1. N.Barik, B.K.Dash and M.Das, Phys.Rev. D31 (1985) 1652,
   P.Leal Ferrera, Lett. Nuovo cimento 20 (1977) 157,
   P.Leal Ferrera and N.Zagury, ibid 20 (1977) 511
2. N.Barik and B.K.Dash, Phys. Rev. D33 (1986) 1925
3. N.Barik and B.K.Dash, Phys. Rev. D34 (1986) 2803
4. N.Barik, B.K.Dash and M.Das, Phys. Rev. D32 (1985) 1725
5. N.Barik and B.K.Dash, Phys. Rev. D34 (1986) 2052
6. H.Akcay and H.Ciftci, J. Phys. G22 (1996) 455
7. N.Barik, B.K.Dash and P.C.Dash, Pramana J. Phys. 29 (1987) 543
8. N.Barik, P.C.Dash and A.R.Panda, Phys. Rev. D46 (1992) 3856,
   N.Barik, P.C.Dash, Phys.Rev. D49 (1994) 299, H.Ciftci (unpublished)
9. A.W.Thomas, Adv. Nucl. Phys. 13 (1983) 1
10. R.Van Royen and V.F.Weisskopf, Nuovo cimento A50 (1967) 617
11. S.N.Sinha, Phys. Lett. B178 (1986) 110,
    G.Godfrey, Phys. Rev. D33 (1986) 1391
12. L.J.Reinders, Phys. Rev. D38 (1988) 947,
    S.Narison, Phys. Lett. B198 (1987) 104,
    C.A.Dominguez and N.Power, Phys. Lett. B197 (1987) 423
13. M.B.Govale, et al., Phys. Lett. B206 (1988) 113,
    R.M.Wokshyn et al. Phys. Rev. D39 (1989) 978
14. S.Capstick and S.Godfrey, Phys. Rev. D41 (1990) 2856
15. N.Barik, P.C.Dash and A.R.Panda, Phys. Rev. D47 (1993) 1001
16. N.Barik and P.C.Dash, *Phys. Rev.* **D47** (1993) 2788

17. B.Margolis and R.R.Mandal, *Phys. Rev.* **D28** (1983) 468,
    C.Hayne and N.Isgnur, *Phys.Rev.* **D25** (1982) 1944

18. H.Kraseman, *Phys.Lett.* **B96** (1980) 397

19. E.Glowich, *Phys.Lett.* **B91** (1980) 271

20. M.Claudsun, *Harward Report No.* **91** (1981) (unpublished)

21. C.Bernard, *et al., Phys. Rev.* **D38** (1980) 3540

22. H.W.Hamber, *Phys. Rev.* **D39** (1989) 896

23. C.Caso *et al., The European Physical Journal* **C3** (1998) 1

24. Ho-Meoyng Choi and Chueng-Ryong Ji, *Phys. Lett.* **B460** (1999) 461

25. Ho-Meoyng Choi and Chueng-Ryong Ji, *Phys. Rev.* **D59** (1999) 074015
Table I. Leptonic decay widths and the decay constant $f_V$ in keV in comparison with the results of other researchers and the experiment.

| Meson | $\Gamma (V \to e^+e^-)$ | Experiment$^{23}$ | Ref. 15 | Ref. 17 |
|-------|-------------------------|-------------------|--------|--------|
| $\rho$ | 6.37                    | 6.77±0.32         | 6.26(8.1) | 7.8 |
| $\omega$ | 0.684            | 0.6±0.02          | 0.67(0.87) | 0.84 |
| $\phi$ | 1.46                    | 1.37±0.05         | 1.58(1.84) | 1.69 |

| Meson | $f_V$ | Experiment$^{23}$ | Ref. 15 | Ref. 17 |
|-------|-------|-------------------|--------|--------|
| $\rho$ | 0.193 | 0.2±0.04          | 0.19(0.22) | 0.21 |
| $\omega$ | 0.0624 | 0.06±0.01         | 0.06(0.07) | 0.07 |
| $\phi$ | 0.0813 | 0.08±0.01         | 0.08(0.09) | 0.07 |
Table II. Decay constants of pseudoscalar mesons in MeV in comparison with the results of other model and the experiment. Experimental values are taken from Ref. 16.

| Model         | $f_\pi$    | $f_K$    | $f_D$    | $f_{D^*}$ | $f_B$    | $f_{B_s}$ | $f_{B_c}$ |
|---------------|------------|----------|----------|-----------|----------|-----------|-----------|
| Expt.\textsuperscript{16,24} | 131.73±0.15 | 160.6±1.3 | <219     | 137-304   | -        | -         | -         |
| This work     | 131.4      | 150.2    | 181.1    | 235.9     | 208.3    | 257.3     | 377.2     |
| Ref. 16       | 138        | 157      | 161      | 205       | 122      | 154       | 221       |
| Ref. 14       | 100        | 153      | 149      | 160       | 96       | 111       | 141       |
| Ref. 20       | -          | -        | 172      | 196       | 149      | 170       | 255       |
| Ref. 18       | 139        | 176      | 150      | 210       | 125      | 175       | 425       |
| Ref. 19       | 178        | 182      | 148      | 166       | 98       | -         | -         |
| Ref. 21       | -          | -        | 174±53   | 234±72    | 105±34   | 155±75    | -         |
| Ref. 22       | 141±21     | 155±21   | 282±28   | -         | 183±28   | -         | -         |
Table III. Partial decay widths $\Gamma (M \to l\bar{l})$ in MeV and the branching ratio $B (M \to l\bar{l})$ of pseudoscalar mesons in comparison with the experiment. ($B (M \to l\bar{l}) = \tau_M \frac{\Gamma (M \to l\bar{l})}{\tau_M}$. $\tau_M$ is the mean lifetime of meson $M$.)

| Process          | $\Gamma (M \to l\bar{l})$ | $B (M \to l\bar{l})$ | Expt. 23 $B (M \to l\bar{l})$ |
|------------------|---------------------------|----------------------|-------------------------------|
| $\pi^\pm \to \mu^\pm \nu_\mu$ | $2.523 \times 10^{-14}$ | 0.994                | 0.999877±0.0000004            |
| $\pi^\pm \to e^\pm \nu_e$     | $3.238 \times 10^{-18}$ | 1.276 $10^{-4}$      | (1.23±0.004) $10^{-4}$       |
| $K^\pm \to \mu^\pm \nu_\mu$  | $3.000 \times 10^{-14}$ | 0.563                | 0.6351±0.0018                |
| $K^\pm \to e^\pm \nu_e$      | $7.724 \times 10^{-19}$ | 1.45 $10^{-5}$       | (1.55±0.07) $10^{-5}$        |
| $D^\pm \to \mu^\pm \nu_\mu$  | $1.795 \times 10^{-13}$ | 2.87 $10^{-4}$       | <7.2 $10^{-4}$               |
| $D^+_s \to \mu^\pm \nu_\mu$  | $6.240 \times 10^{-12}$ | 4.41 $10^{-3}$       | $4^{+2.2}_{-2.0} \times 10^{-3}$ |
| $B^\pm \to \mu^\pm \nu_\mu$  | $1.693 \times 10^{-16}$ | 4.15 $10^{-7}$       | <2.1 $10^{-5}$               |
| $B^+_c \to \mu^\pm \nu_\mu$  | $8.960 \times 10^{-14}$ | -                    | -                            |
| $D^\pm \to \tau^\pm \nu_\tau$ | $4.720 \times 10^{-13}$ | 7.540 $10^{-4}$      | -                            |
| $D^+_s \to \tau^\pm \nu_\tau$ | $6.090 \times 10^{-11}$ | 4.3 $10^{-2}$        | (7±4) $10^{-2}$              |
| $B^\pm \to \tau^\pm \nu_\tau$ | $3.770 \times 10^{-14}$ | 9.25 $10^{-5}$       | <0.57 $10^{-3}$              |
| $B^+_c \to \tau^\pm \nu_\tau$ | $2.140 \times 10^{-11}$ | -                    | -                            |