We discuss the junction conditions in the context of the Randall-Sundrum model with the Gauss-Bonnet interaction. We consider the $Z_2$ symmetric model where the brane is embedded in an $AdS_5$ bulk, as well as a model without $Z_2$ symmetry in which the brane (in this case called by tradition “shell”) separates two metrically different $AdS_5$ regions. We show that the Israel junction conditions across the membrane (that is either a brane or a shell) have to be modified if more general equations than Einstein’s, including higher curvature terms, hold in the bulk, as is likely to be the case in a low energy limit of string theory. We find that the membrane can then no longer be treated in the thin wall approximation. We derive the junction conditions for the Einstein-Gauss-Bonnet theory including second order curvature terms and show that the microphysics of Gauss-Bonnet thick membranes may, in some instances, be simply hidden in a renormalization of Einstein’s constant.

I Introduction

Motivated by recent developments in high energy physics [1,2,3,4] there is at present a considerable increase of activity in the domain of cosmology with extra dimensions. In these models gravity is assumed to act in a $n$-dimensional “bulk” while the standard model interactions are confined to a 4-dimensional slice (“brane” worldsheet) of this multi-dimensional spacetime. Randall and Sundrum have recently proposed two models in which all the matter is confined to a 4-dimensional brane worldsheet embedded in a 5-dimensional anti-de Sitter ($AdS_5$) spacetime with imposed $Z_2$ metric symmetry, $w \to -w$ ($w$ denotes the fifth dimension and the metric is expressed in the Gaussian normal coordinates) [5].

What is usually done in General Relativity, when, e.g., studying the gravitational collapse of spherical bodies is to join two metrically different solutions of the Einstein field equations. Integration of the Einstein equations across the surface separating the two regions leads to the Israel junction conditions ([6], see also Appendix A for a review) relating the surface stress-energy tensor to the discontinuity of the extrinsic curvature across the surface. The sign of the extrinsic curvature, and thus the form of the junction conditions, depend on the definitions of normal vectors in the neighbourhood of the surface. However, once the directions of the normal vectors on each side of the surface are fixed, e.g., pointing in a defined positive sense, the formalism becomes unambiguous and yields in general a nonzero stress-energy tensor for the surface (massive “shell”).

On the other hand matter of $Z_2$ symmetric $AdS_5$ branes, which connect two metrically identical solutions of the Einstein equations, arises formally by a flip of
the normal vectors at one or the other side of the boundary surface while preserving
the form of Israel junction conditions. This flip of the normals across the brane
describes the connection, via the Israel junction conditions, of a bulk region and its
mirror image. Were no formal flip of the normal vectors performed, i.e. dropping
out of the $Z_2$ symmetry, one would join in a topologically trivial way two com-
plementary parts of the $AdS_5$ spacetime across a non-massive boundary surface.
Hence, the $Z_2$ symmetry dropped out, a massive shell must separate two $AdS_5$
regions with different cosmological constants [7].

If these brane or shell models are to be the low energy limit of string theory,
it is likely that the field equations include higher curvature terms. In particular,
the lowest non-linear curvature terms derive from the Gauss-Bonnet Lagrangian,
which, in five dimensions, is the only non-linear term in the curvature which yields
second order field equations (see e.g. [8] and references therein). The Randall-
Sundrum model with the Gauss-Bonnet correction has recently been considered in ref. [9-11]. However, only in [7] are the peculiarities of the junction conditions
carefully examined.

In this paper based on [7] we derive the junction conditions across the membrane
(either brane or shell) taking into account the Gauss-Bonnet correction and con-
clude that in this case the thin wall approximation fails. When second (and higher)
curvature terms are taken into account the junction conditions formally include
products of distributions as well as products of distributions and non-infinitely
differentiable functions: they are therefore not well defined in the distributional
sense. The complete evolution of the Einstein-Gauss-Bonnet universe must be
studied more carefully taking into account the internal structure of the membrane.
Nevertheless, the microphysics of the thick membrane may, at late times, be hidden
in a renormalization of Einstein’s constant.

II Einstein-Gauss-Bonnet membranes

The membrane cosmological models with the $AdS_5$ bulk are solutions of the Einstein
equations $G_{AB} + \Lambda g_{AB} = 0$ everywhere except on the membrane $\Sigma$. In brane
cosmology the cosmological constant $\Lambda$ is the same on each side of $\Sigma$, in shell
cosmology it jumps from $\Lambda_+$ to $\Lambda_-$. Now, in order to take into account the Gauss-
Bonnet correction, we shall consider the gravitational action

$$S_g = \int d^5x \sqrt{-g}(-2\Lambda + R + \alpha L_2)$$

(1)

with

$$L_2 = R_{ABCD}R^{ABCD} - 4R_{AB}R^{AB} + R^2$$

(2)

where $\alpha$ is a coupling constant, and where $R_{ABCD}$, $R_{AB}$ and $R$ are the Riemann
tensor, the Ricci tensor and the scalar curvature of the five dimensional metric $g_{AB}$
with determinant $g$. The corresponding field equations, outside the membrane, are

$$\Lambda g_{AB} + G_{AB} + \alpha H_{AB} = 0$$

(3)
with
\[ H_{AB} \equiv 2R_{ALMN}R_B^{LMN} - 4R_{AMBN}R_B^{MN} - 4R_{AM}R_B^M + 2RR_{AB} - \frac{1}{2}g_{AB}L_2. \] (4)

Contrarily to Einstein’s equations, the equations (3) possess, for \( \alpha \neq 0 \) and a given value of the cosmological constant \( \Lambda \), two (anti) de Sitter solutions
\[ R_{ABCD} = L_\pm (g_{AC}g_{BD} - g_{AD}g_{BC}) \]
with
\[ L_\pm = \frac{1}{4\alpha} \left(-1 \pm \sqrt{1 + 4\alpha \Lambda/3}\right). \] (5)

In brane cosmology we shall choose one or the other solution everywhere in the bulk. In shell cosmology, we shall choose the solution \( L_+ \) on one side of the shell and the solution \( L_- \) on the other side. (Shell cosmologies are therefore more satisfactory in Einstein-Gauss-Bonnet theory as one does not have to impose different cosmological constants on each side of the shell.)

In order to get the stress-energy tensor on the membrane, one proceeds along Israel’s line. We first choose a Gaussian coordinate system \( (w, x^\mu) \) such that the metric reads
\[ ds^2 = dw^2 + \gamma_{\mu\nu}dx^\mu dx^\nu = dw^2 - n^2(\tau, w)d\tau^2 + S^2(\tau, w)[d\chi^2 + f_k^2(\chi)(d\theta^2 + \sin^2\theta d\phi^2)], \] (6)
where \( w = 0 \) is the equation of the membrane \( \Sigma \) and functions \( n(\tau, w), S(\tau, w) \) being given in ref. [12]. In this coordinate system the extrinsic curvature of the surfaces \( w = \text{constant} \) is simply given by
\[ K_{\mu\nu} = -\frac{1}{2}\frac{\partial \gamma_{\mu\nu}}{\partial w}. \] (7)
It jumps across the membrane from \( K_{\mu\nu}^+ \) to \( K_{\mu\nu}^- \) (with \( K_{\mu\nu}^+ = -K_{\mu\nu}^- \) in the case of branes) and this discontinuity can be described in terms of the Heaviside distribution.

Expressing now the Riemann tensor (3) in terms of \( K_{\mu\nu} \) and the four dimensional Riemann tensor of the metric \( \gamma_{\mu\nu} \) we then obtain from (3), everywhere outside the membrane (see Appendices A and B for the ‘4+1’ decompositions of \( G_{\mu\nu}^A \) and \( H_{\mu\nu}^A \))
\[ \Lambda\delta_{\mu\nu} + G_{\mu\nu} + \alpha H_{\mu\nu} = (1 + 4\alpha L) \left( \frac{\partial K^\nu}{\partial w} - \delta^\nu_\mu \frac{\partial K}{\partial w} \right) + \ldots \] (0) (8)
where \( K \equiv \gamma^{\alpha\beta}K_{\alpha\beta} \), where \( L = L_+ \) or \( L_- \), and where the dots stand for terms containing at most first order \( w \)-derivatives of \( \gamma_{\mu\nu} \).

In Einstein’s theory, \( \alpha = 0 \), and (8), in the vicinity of \( \Sigma \), is well defined in a distributional sense: \( \partial K^\mu/\partial w \) can be expressed in terms of the Dirac distribution and the integration of (8) across the membrane gives Israel’s junction conditions, that is the stress-energy tensor on the membrane in terms of the jump in the extrinsic curvature, eq. (28).

When \( \alpha \neq 0 \) on the other hand, (8) is not well defined in a distributional sense, as \( L \) cannot be considered as an infinitely \( w \)-differentiable function. Indeed, in shell cosmology, \( L \) jumps from \( L_+ \) to \( L_- \) across \( \Sigma \), and in brane cosmology,
\( L = L_+ = L_- \) is continuous across \( \Sigma \), but, because of the reflexion symmetry, has a discontinuous \( w \)-derivative. This mathematical obstruction simply means that, in Einstein-Gauss-Bonnet theory, membranes cannot be treated in the thin wall approximation: the jumps in the extrinsic curvature and in \( L \) or its derivative have to be described in detail within specific microphysical models.

When the thickness of the membrane is taken into account, the distributions \( \partial K^\nu_{\mu} / \partial w \) and \( L \) are replaced by rapidly varying but \( C^\infty \) functions. Supposing that the metric keeps the form (6), we can define from the \( \tau-\tau \) component of (8) the sharply peaked function (cf. (36) in Appendix B)

\[
\kappa \rho \equiv 3(1 + 4\alpha L) \frac{\partial K^\chi_{\chi}}{\partial w} \quad (9)
\]

(The \( \chi-\chi \) component of (8) is redundant thanks to the conservation equation, that is the Bach-Lanczos identity (34).) In the vicinity and inside the membrane \( K^\chi_{\chi} \) can be written as

\[
K^\chi_{\chi} = \frac{1}{2} \tilde{K}^\chi_{\chi} + \frac{1}{2} \hat{K}^\chi_{\chi} f(\tau, w) \quad (10)
\]

where \( \tilde{K}^\chi_{\chi} \equiv K^{\chi+}_{\chi+} + K^{\chi-}_{\chi-} \) and where the function \( f(\tau, w) \), which varies rapidly from \(-1\) to \(+1\) across \( \Sigma \), encapsulates its microphysics. Similarly, in the case of brane cosmology, we can write

\[
L = \tilde{L} g_b(\tau, w) \quad (11)
\]

where \( \tilde{L} = L_+ \) or \( L_- \) and where \( g_b(\tau, w) \) is some even function of \( w \) which varies rapidly from \(+1\) to \(+1\) across the brane. In shell cosmology on the other hand

\[
L = \frac{1}{2} L + \frac{1}{2} \hat{L} g_s(\tau, w) \quad (12)
\]

where \( g_s(\tau, w) \) varies rapidly from \(-1\) to \(+1\) and \( \hat{L} = -\frac{1}{2\alpha} \), \( \hat{L} = \frac{1}{2\alpha} \sqrt{1 + 4\alpha \Lambda} \).

Integrating (9) across \( \Sigma \) we therefore get the energy density of the membrane as

\[
\kappa \rho \equiv \int_{-\eta}^{+\eta} dw \kappa \rho = 3 \tilde{K}^\chi_{\chi} \left[ 1 + 2\alpha \hat{L} \int_{-\eta}^{+\eta} dw g_b \frac{\partial f}{\partial w} \right] \quad (13)
\]

in the case of branes, and,

\[
\kappa \rho \equiv \int_{-\eta}^{+\eta} dw \kappa \rho = \frac{3}{2} \sqrt{1 + \frac{4\alpha \Lambda}{3}} \hat{K}^\chi_{\chi} \int_{-\eta}^{+\eta} dw g_s \frac{\partial f}{\partial w} \quad (14)
\]

in the case of shells (the fact that one does not recover the results of Einstein’s theory when \( \alpha = 0 \) is not surprising as \( L_+ \) is divergent in that case). Now, if \( f \) or \( g_b/s \) depend on \( \tau \), the integrals in (13)-(14) are some functions of \( \tau \). But, if \( f \) and \( g_b/s \) do not depend on time, which is probably to be expected whenever the brane has reached a stationary state, that is at late times, then the integrals in (13)-(14) are just numbers. In this case then, the microphysics of the membrane is simply hidden in a renormalization of the Einstein constant \( \kappa \).
The decomposition of $H^A_B$ is given in full generality in Appendix B, eq. (35). Higher curvature terms typically induce terms with higher powers in extrinsic curvature, or products of components of the Riemann and extrinsic curvature tensors. Since these terms are obviously no well defined in the distributional sense, the thin wall formalism is no longer applicable beyond the linear order in curvature tensors. The behaviour of the membranes in the context of such theories needs to be studied more carefully, taking into account the microphysics of the thick membrane, as is done in e.g. [16]. Nevertheless, at late times, the microphysics of an Einstein-Gauss-Bonnet membrane can be hidden in a renormalization of Einstein’s constant.

Appendix

A Junction conditions for non-null surfaces in General Relativity

This appendix summarizes the junction conditions in the theory of Einstein (Lanczos [13], Darmois [14], Misner and Sharp [15], Israel [6]). Suppose we are given a 4-dimensional hypersurface ($\Sigma$) in a 5-dimensional spacetime (metric $g_{AB}$) which can be imagined as the element of a family of surfaces. The normal vectors $n^A$ to this family of surfaces are not null: $n_A n^A \equiv \epsilon = \pm 1$. They are all oriented in a positive direction defined in the bulk. Let the surface be either spacelike ($\epsilon = -1$) or timelike ($\epsilon = +1$). As an aid in deriving junction conditions we introduce Gaussian normal coordinates in the neighbourhood of $\Sigma$. The metric $g_{AB}$ has the form

$$ds^2 = \epsilon dw^2 + \gamma_{\mu\nu} dx^\mu dx^\nu,$$

and the extrinsic curvature of the surfaces $w = constant$ is

$$K_{\mu\nu} = -\frac{1}{2} \frac{\partial \gamma_{\mu\nu}}{\partial w}. \quad (16)$$

The curvature tensor of the metric $g_{AB}$ can be expressed in terms of the intrinsic curvature of 4-dimensional hypersurface ($\gamma_{\mu\nu}$) and of its extrinsic curvature; one gets the so-called Gauss-Codazzi equations. In the special case of Gaussian normal coordinates the equations simplify to

$$R_{w\mu w\nu} = \frac{\partial K_{\mu\nu}}{\partial w} + K_{\rho\nu} K^\rho_{\mu}, \quad (17)$$

$$R_{w\mu \rho\nu} = \nabla_\mu K_{\rho\nu} - \nabla_\rho K_{\mu\nu}, \quad (18)$$

$$R_{\lambda\mu\nu\rho} = \frac{4}{4} R_{\lambda\mu\nu\rho} + \epsilon [K_{\mu\nu} K_{\lambda\rho} - K_{\mu\rho} K_{\lambda\nu}], \quad (19)$$

where $\nabla_\rho$ is the covariant derivative with respect to the 4-dimensional metric $\gamma_{\mu\nu}$. From (17)-(19) we obtain the decomposition of the Ricci tensor ($R_{AB} = g^{CD} R_{CABD}$) and of the scalar curvature ($R = g^{AB} R_{AB}$) as:

$$R_{ww} = \gamma^{\mu\nu} \frac{\partial K_{\mu\nu}}{\partial w} + Tr(K^2), \quad (20)$$

$$R_{w\mu} = \nabla_\mu K - \nabla_\nu K^\nu_{\mu}, \quad (21)$$

$$R_{\mu\nu} = \frac{4}{4} R_{\mu\nu} + \epsilon \left[ \frac{\partial K_{\mu\nu}}{\partial w} + 2 K^\rho_{\mu} K_{\rho\nu} - K K_{\mu\nu} \right], \quad (22)$$

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\[ R = 4R + \epsilon \left[ 2\gamma_{\mu\nu} \frac{\partial K_{\mu\nu}}{\partial w} + 3Tr(K^2) - K^2 \right], \quad (23) \]

where we defined \( K \equiv K_{\mu\mu} \) and \( Tr(K^2) \equiv K_{\mu\nu}K_{\mu\nu} \).

In terms of the intrinsic and extrinsic curvature of the 4-dimensional hypersurfaces \( w = \text{constant} \), the Einstein tensor \( (G^A_B = R^A_B - (1/2)\delta^A_B R) \) and the field equations have components

\[
G^w_w = -\frac{1}{2} 4R + \frac{1}{2} \epsilon \left[ K^2 - Tr(K^2) \right] = \kappa T^w_w, \quad (24)
\]

\[
G^\mu_w = \epsilon \left[ \nabla_\mu K - \nabla_\nu K^\nu_\mu \right] = \kappa T^\mu_w, \quad (25)
\]

\[
G^\nu_w = 4G^\mu_\nu + \epsilon \left[ \frac{\partial K^\mu_\nu}{\partial w} - \delta^\mu_\nu \frac{\partial K}{\partial w} \right] + \epsilon \left[ -KK_\mu^\nu + \frac{1}{2} \delta_\mu^\nu Tr(K^2) + \frac{1}{2} \delta_\mu^\nu K^2 \right] = \kappa T^\nu_w. \quad (26)
\]

If the stress-energy tensor \( T^A_B \) contains a 'delta-function contribution' at \( \Sigma \), the integral of \( T^A_B \) with respect to the proper distance \( w \) measured perpendicularly through \( \Sigma \),

\[
T^A_B \equiv \lim_{\eta \to 0} \left[ \int_{-\eta}^\eta T^A_B dw \right], \quad (27)
\]

is non-zero and represents the surface stress-energy tensor. In this case the extrinsic curvature must be a distribution of 'Heaviside type' at \( \Sigma \). Integral (27) applied on equations (24)-(26) yields the junction conditions relating the stress-energy tensor of \( \Sigma \) to the discontinuity of the extrinsic curvature at \( \Sigma \). In the passage to the limit \( \eta \to 0 \) only the terms \( \sim (\partial K^\mu_\nu/\partial w) \) contribute to yield

\[
\kappa T^w_w = 0,
\]

\[
\kappa T^\mu_w = 0,
\]

\[
\kappa T^\nu_w = \epsilon (\dot{K}^\mu_\nu - \delta^\mu_\nu \dot{K}), \quad \text{where} \quad \dot{K}^\mu_\nu \equiv K^\mu_\nu(0+) - K^\mu_\nu(0-). \quad (28)
\]

It is useful to denote \( K^+_{\mu\nu} \equiv K_{\mu\nu}(0+) \), \( K^-_{\mu\nu} \equiv K_{\mu\nu}(0-) \).

As for the intrinsic geometry of \( \Sigma \), it must be continuous across \( \Sigma \); this is the second junction condition completing equations (28). If there are no 'delta singularities' contained in \( T^A_B \), the bulk is sliced by massless 'boundary surfaces'.

### B Junction conditions for the theory of Einstein-Gauss-Bonnet

The theory of Einstein-Gauss-Bonnet is based on the following Lagrangian

\[
\mathcal{L} = \sqrt{-g} \left[ -2\Lambda + R + \alpha L_2 \right], \quad (29)
\]

where \( g \) is the determinant of the 5-dimensional metric \( g_{AB} \) and \( \alpha \) is a constant of the dimension of \([\text{length}]^2\). \( L_2 \) is the Gauss-Bonnet Lagrangian which reads

\[
L_2 = R_{ABCD}R^{ABCD} - 4R_{AB}R^{AB} + R^2. \quad (30)
\]

*We analyse the particular case of a 5-dimensional spacetime sliced by 4-dimensional hypersurfaces.*
The Euler variation of $\mathcal{L}$ gives the following field equations:

$$\Lambda g_{AB} + G_{AB} + \alpha H_{AB} = 0.$$  \hspace{1cm} (31)

$G_{AB}$ is the Einstein tensor and $H_{AB}$ is its analogue stemmed from the Gauss-Bonnet part of the Lagrangian, $L_2$.

$$G_{AB} \equiv R_{AB} - \frac{1}{2} g_{AB} R,$$  \hspace{1cm} (32)

$$H_{AB} \equiv 2 \left[ R_{ABCD} R^{CD} - R_{AB} R^{CD} + R R_{AB} \right] - \frac{1}{2} g_{AB} L_2.$$  \hspace{1cm} (33)

$G^A_B$ and $H^A_B$ satisfy the Bianchi and Bach-Lanczos identities respectively

$$\nabla_A G^A_B = 0, \quad \nabla_A H^A_B = 0.$$  \hspace{1cm} (34)

In order to derive the junction condition in the theory of Einstein-Gauss-Bonnet we need to express $H_{AB}$ in terms of the intrinsic curvature of hypersurfaces $w = \text{constant}$ and their extrinsic curvatures. We adopt the notation used in Appendix A in which the ’4+1’ decomposition of the Einstein tensor $G_{AB}$ is shown (expressions (24)-(26)).

Inserting the decomposition of the curvature tensors (17)-(23) into (33) one finds the following results:

$$H^A_B \text{ does not contain terms } \sim \left( \frac{\partial K^\mu_\nu}{\partial w} \right)^2 \text{ as one would expect since } H^A_B \text{ contains terms } \sim (R_{ABCD})^2. \text{ Further, there are no terms linear in } (\frac{\partial K^\mu_\nu}{\partial w}) \text{ in } H^w_w \text{ and } H^w_\mu \text{ so that these junction conditions correspond to those in the theory of Einstein: } T^w_w = T^w_\mu = 0. \text{ The components } H^\mu_\nu \text{ are}

$$

$$H^\mu_\nu = \left\{ \frac{\partial K^\mu_\nu}{\partial w} \right\} (2T \tau (K^2) - 2K^2) + \left\{ \frac{\partial K^\lambda_\nu}{\partial w} \right\} \left( 4KK^\lambda_\nu - 4K^\lambda_\nu K^\beta_\lambda \right) + \left\{ \frac{\partial K^\lambda_\mu}{\partial w} \right\} \left( 4KK^\lambda_\mu - 4K^\lambda_\mu K^\beta_\lambda \right) + \left\{ \frac{\partial K^\alpha_\beta}{\partial w} \right\} (4K^\mu_\nu K^\alpha_\beta - 4K^\alpha_\mu K^\beta_\nu) + \left\{ \frac{\partial K^\mu_\alpha}{\partial w} \right\} \left( 4KK^\alpha_\beta - 4K^\alpha_\beta K^\mu_\nu \right) + \left\{ \frac{\partial K^\alpha_\beta}{\partial w} \right\} \left( 4K^\mu_\nu K^\alpha_\beta - 4K^\alpha_\mu K^\beta_\nu \right) + \epsilon \left( -4R^\alpha_\mu R^\nu_\alpha \frac{\partial K^\alpha_\beta}{\partial w} - 4R^\alpha_\nu \frac{\partial K^\nu_\alpha}{\partial w} \right) + \epsilon \left( -4R^\alpha_\mu \frac{\partial K^\alpha_\beta}{\partial w} + 4R^\alpha_\nu R^\nu_\alpha \frac{\partial K^\alpha_\beta}{\partial w} - 2\delta^\alpha_\nu R^\alpha_\nu \frac{\partial K^\mu_\mu}{\partial w} \right) + \ldots,$$

$$\text{(35)}$$

where ’\ldots’ includes terms of zeroth order in $(\partial K^\mu_\nu/\partial w)$ which disappear in the passage to the limit $\eta \rightarrow 0$ of the integration (27).

In the case the metric has the form

$$ds^2 = dw^2 - n^2(\tau, w)d\tau^2 + S^2(\tau, w)[d\chi^2 + f^2_\chi(\chi)(d\theta^2 + \sin^2 \theta d\phi^2)]$$

In the case the metric has the form

$$ds^2 = dw^2 - n^2(\tau, w)d\tau^2 + S^2(\tau, w)[d\chi^2 + f^2_\chi(\chi)(d\theta^2 + \sin^2 \theta d\phi^2)]$$
we have $\epsilon = +1$ and

$$H^\tau = -12L(\tau, w) \frac{\partial K\chi}{\partial w} + \ldots \quad \text{with} \quad L(\tau, w) \equiv -\left(\frac{K\chi}{S}\right)^2 + \frac{\dot{S}^2 + kn^2}{n^2 S^2}$$

(36)

and where $K\chi = -S'/S$, a prime denotes $\partial/\partial w$. Outside the membrane, spacetime is anti-de Sitter, $n$ and $S$ are given in ref. [12], and $L(\tau, w) \to L_{\pm}$ (as an explicit calculation shows). In the vicinity and inside the membrane the function $L(\tau, w)$ is either continuous with discontinuous $w$-derivative (case of branes) or discontinuous (case of shells) and can be modelled by the expressions (11)-(12) in the text.

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