Diquark-diquark-antiquark model for pentaquarks with hidden charm: current status and problems

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Abstract

The $J/\psi p$ signals at 4380 MeV and 4450 MeV which were seen by the LHCb collaboration in the decay $\Lambda^0_{cb} \rightarrow K^- J/\psi p$ are discussed following the hypothesis of the existence of diquark-diquark-antiquark composite states. The discussed problems concern:
(i) estimation of masses and mass splittings for low-lying states,
(ii) determination of decay modes in the quark recombination scheme,
(iii) unitarity and analyticity constrains for pentaquark propagators,
(iv) hadronic deuteron-like components in the diquark-diquark-antiquark multiplet.

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1 Introduction

Data for the $\Lambda^0_{cb} \rightarrow p J/\psi K^-$ decay provide a definite argument for the existence of a pentaquark, a baryon system in $p J/\psi$ spectra which has the following quark content: $P_{cuud} = \bar{c}(cuud)$. In terms of the antiquark-diquark-diquark states it can be presented as a three-body system:

$$P = a \left( \bar{c}^\alpha \cdot D_{cq1}^\beta \cdot D_{q2q3}^\gamma \epsilon_{\alpha\beta\gamma} \right) + b \left( \bar{c}^\alpha \cdot D_{q2q3}^\beta \cdot D_{cq1}^\gamma \epsilon_{\alpha\beta\gamma} \right),$$

$$D_{cq1}^\beta = c^{\beta_{\gamma \gamma'}} q_{1\gamma_{\gamma'}} e^{\beta_{\gamma\gamma'}} e^{\beta_{\gamma'\gamma''}} \epsilon_{\gamma\gamma''}, \quad D_{q2q3}^\gamma = q_{2\gamma_{\gamma'}} q_{3\gamma'} e^{\gamma_{\gamma'}} e^{\gamma_{\gamma''}} (1)$$

where $\alpha, \beta, \gamma$ refer to color indices. Let us stress that both terms in (1) have the same weight in color space. The right-hand side diquarks are forming baryons (it is conventional condition). The second term in Eq. (1) gives a zero result for the states under consideration. Taking into account coordinate wavefunction lead us to nonvanishing second term in Eq. (1), however such states are radial excited ones and they are out of our consideration (see Appendix A).
A diquark is a color triplet member, similar to a quark, and the right-hand side of Eq. (1) presents a three-body system with a color structure similar to that in low-lying baryons. It was supposed in Refs. 2-4 that it results in the formation of a diquark-diquark-antiquark system. Following this idea it is possible to perform a classification of such baryon states and give estimations of their masses. Estimation of the charmed diquark masses is given in Ref. 5 where diquark-antidiquark states with hidden charm are studied.

The notion of the diquark was introduced by Gell-Mann.6 Diquarks were discussed for baryon states during a long time, see the pioneering papers 7-11 and recent studies 12-17. The systematization of baryons in terms of the quark-diquark states is presented in Refs. 18, 19.

Pentaquarks built of light-light and light-heavy diquarks present a natural extension of multiquark schemes studied in the last decade for mesons.20-26

The LHCb collaboration observed two resonance states in the $J/\psi p$ -spectrum1 the broad state $P(4380)$ and the narrow one $P(4450)$. These resonances generated a wide discussion.27-43

We will concentrate on discussing the narrow state because there are different versions of interpretation for a broad state (for example see Ref. 44).

We suppose that narrow pentaquark states are formed by decays which are not $s$-wave recombination ones.45 Such a narrow state is $P(4450)$ with the width $\sim 40 \pm 20$ MeV. In the present paper we calculate the recombination transitions for multiplet ($\bar{c}cuud$). We see that the $P(4450)$ state does not decay in $s$-wave recombination mode. (The $s$-wave transition $P(4450) \rightarrow J/\psi + N(J = \frac{3}{2})$ is impossible because of the large mass of $N(J = \frac{3}{2})$ state). There is not such a problem for all other multiplet members. The narrow $P(4450)$ recombines in $d$-wave $P(4450) \rightarrow J/\psi + p$, so the $d$-wave transition gives a comparatively small value of width.

2 Pentaquarks as composite states of light diquark ($qq'$), charmed diquark ($qc$) and antiquark ($\bar{c}$)

In Ref. 4 it was supposed that the genuine antiquark-diquark-diquark state in the LHCb experiment is the narrow $P_\pi^-(4450)$ while the broad $P_\pi^+(4380)$ is the result of rescatterings of final state hadrons.

2.1 Color-spin-isospin structure of the pentaquark

Generally, in color space the wave function of the antiquark-diquark-diquark pentaquark is represented by Eq. (1). As it was mentioned above, in the case under consideration we have the first term of Eq. (1) only. It is written as follows:

$$\bar{c}^\alpha \cdot D_{c_{q_1}}^\beta \cdot D_{q_2q_3}^\gamma \epsilon_{\alpha\beta\gamma} = \bar{c}^\alpha (c_{q_1})^\beta (q_2q_3)^\gamma \epsilon_{\alpha\beta\gamma}$$

$$= -(\bar{c}^\alpha c_{q_1})(q_{1\gamma} q_{2\gamma'} q_{3\gamma''} \epsilon_{\gamma\gamma'}) + (\bar{c}^\alpha q_{1\alpha})(c_{\gamma} q_{2\gamma'} q_{3\gamma''} \epsilon_{\gamma\gamma''})$$

$$= -(\bar{c}c)(q_1 q_2 q_3) + (\bar{c}q_1)(c q_2 q_3)$$
where the color-flavor wave function of pentaquark reads:

$$P_{cquud} = -\sqrt{\frac{1}{3}} \vec{c} \cdot (cu) \cdot (ud) + \sqrt{\frac{2}{3}} \vec{c} \cdot (cd) \cdot (uu).$$ \hspace{1cm} (3)$$

We discuss a scheme in which the exotic baryon states are formed by standard QCD-motivated interactions (gluonic exchanges, confinement forces) but in addition with diquarks as constituents.

We work with four diquarks, two scalars $S_{cq}^{1I_s, J^J_z}$ and two axial-vectors $A_{cq}^{1I_s, J^J_z}$:

$$S_{cq}^{1/2, 0^+}(I = 1/2, J = 0), \quad A_{cq}^{1/2, 1^+}(I = 1/2, J = 1),$$

$$S_{cq}^{00, 0^+}(I = 0, J = 0), \quad A_{cq}^{00, 1^+}(I = 1, J = 1),$$

where $I$ and $J$ refer to isospin and spins of the diquarks. In terms of these diquarks the color-flavor wave function of pentaquark reads:

$$P_{c\cdot cq\cdot q''} = \epsilon^{\alpha} \cdot \epsilon_{\beta \gamma} \begin{vmatrix} S_{cq}^\beta \\ A_{cq}^\gamma \end{vmatrix} \begin{vmatrix} S_{q''}^\beta \\ A_{q''}^\gamma \end{vmatrix}$$ \hspace{1cm} (5)$$

We have six diquark-diquark states:

$$P_{c\cdot cq\cdot q''} = \epsilon^{\alpha} \cdot \begin{vmatrix} (S_{cq}S_{q''})^{\alpha}(1/2; 0^+) \\ (S_{cq}A_{q''})^{\alpha}(1/2; 1^+) \\ (A_{cq}S_{q''})^{\alpha}(1/2; 1^+) \\ (A_{cq}A_{q''})^{\alpha}(1/2; 0^+) \end{vmatrix}$$ \hspace{1cm} (6)$$

with the isospin and spin-parity numbers of the diquark-diquark pair: $I = \frac{1}{2}, \frac{3}{2}$ and $J^P = 0^+, 1^+, 2^+$.

So, the low-lying $s$-wave pentaquark multiplet $P(I, J^P)$ reads:

$$P_{c\cdot (cq)\cdot (q'q'')} = \begin{vmatrix} P(1/2, 1^-) \\ P(1/2, 1^-), P(1/2, 3^-), P(3/2, 1^-), P(3/2, 3^-) \\ P(1/2, 1^-), P(1/2, 3^-) \end{vmatrix}$$ \hspace{1cm} (7)$$

2.2 Masses of the $s$-wave pentaquarks

It was understood already relatively long ago that the mass splitting of hadrons can be well described in the framework of the quark model by the short-ranged spin-spin interactions of the constituents.\textsuperscript{10,18} For mesons and baryons the mass formulae discussed by Glashow read\textsuperscript{38}

$$M_M = \sum_{j=1,2} m_{q(j)} + a \frac{\vec{s}_1 \vec{s}_2}{m_{q(1)} m_{q(2)}},$$

$$M_B = \sum_{j=1,2,3} m_{q(j)} + b \sum_{j>\ell} \frac{\vec{s}_j \vec{s}_\ell}{m_{q(j)} m_{q(\ell)}},$$

$$3$$
where \( \vec{s}_j \) and \( m_{q(j)} \) refer to spins and masses of the constituents. Mass splitting parameters in Eq. (8), \( a \) and \( b \), are characterized by the size of the colour-magnetic interaction and the size of the discussed hadron, the short-range interaction is supposed in Ref. [10]. For the 36-plet mesons (\( q\bar{q} \)) and 56-plet baryons (\( qqq \)) formulae of Eq. (8) work well.

The only exception is the pion, its calculated mass is \( \sim 350 \text{ MeV} \) that point out the existence of additional forces in the pseudoscalar channel (possibly, istanton-induced forces).

It seems natural to apply modified formulae (8) to pentaquark systems. For multiquark states from Ref. [1] we have:

\[
M_{q_1q_2-q_3c} = 3m_{D(q_1q_2)} + 4\Delta \left( \vec{\mu}_{D(q_1q_2)} \vec{\mu}_{D(q_3c)} + \vec{\mu}_{D(q_1q_2)} \vec{\mu}_{c} + \vec{\mu}_{c} \vec{\mu}_{D(q_3c)} \right) \\
3m_{D(q_1q_2)} + m_{D(q_3c)} + m_{\bar{c}}
\]

where \( \vec{\mu}_{D} \) and \( \vec{\mu}_{c} \) are color-magnetic moments of diquarks and \( c \)-quark, \( \Delta \) is the parameter of spin splitting. Diquarks are considered as composite systems of quarks analogous to light nuclei. The magnetic moments are written as sums of quark magnetic moments:

\[
\vec{\mu}_{D(q_1q_2)} = \frac{\vec{s}_{q_1}}{m_{q_1}} + \frac{\vec{s}_{q_2}}{m_{q_2}}, \quad \vec{\mu}_{D(q_3c)} = \frac{\vec{s}_{q_3}}{m_{q_3}} + \frac{\vec{s}_{c}}{m_{c}} \simeq \frac{\vec{s}_{q_3}}{m_{q_3}},
\]

\[
\vec{\mu}_c = \frac{\vec{s}_c}{m_c} \simeq 0.
\]

In our estimations we take into account that \( m_c >> m_q \).

Estimation of diquark masses is the most problematic issue in the study of diquarks (see for example Ref. [10]). Basing on Refs. [18], we estimate the masses of scalar \( S (J^P = 0^+) \) and axial \( A (J^P = 1^+) \) diquarks as follows (in MeV units):

\[
m_q = 330, \quad m_c = 1450, \\
m_{S(q_1q_2)} = 700, \quad m_{S(q_3c)} = 2000, \\
m_{A(q_1q_2)} = 800, \quad m_{A(q_3c)} = 2100.
\]

Correspondingly we write a set of the low-lying pentaquark states:

\[
\begin{align*}
I = 1/2 & \quad I = 3/2 \\
P_{\frac{1}{2}+}^{(c\bar{c})} & (4450), P_{\frac{1}{2}+}^{(c\bar{c})} (4450), P_{\frac{1}{2}+}^{(c\bar{c})} (4450), \\
P_{\frac{1}{2}+}^{(s\bar{s})} & (4300), P_{\frac{1}{2}+}^{(s\bar{s})} (4300), P_{\frac{1}{2}+}^{(s\bar{s})} (4300), \\
P_{\frac{3}{2}+}^{(c\bar{c})} & (4150), P_{\frac{3}{2}+}^{(c\bar{c})} (4150), P_{\frac{3}{2}+}^{(c\bar{c})} (4150), \\
P_{\frac{3}{2}+}^{(s\bar{s})} & (4000), P_{\frac{3}{2}+}^{(s\bar{s})} (4000), P_{\frac{3}{2}+}^{(s\bar{s})} (4000),
\end{align*}
\]

Masses are given in MeV units, an uncertainty in the determination of masses is of the order of \( \pm 150 \text{ MeV} \). It should be noted that the process \( P \rightarrow \text{meson} + \text{baryon} \rightarrow P \) can shift mass values in Eq. (12). Such a rescattering was taken into account for tetraquark states in Ref. [51]. Following Ref. [51] we estimate this shift to be of the order of 100 MeV.
3 Spin-isospin structure of pentaquarks

3.1 Pentaquark $P_{c\bar{c}u\bar{u}u}^{1\frac{1}{2}+\frac{3}{2}+\frac{3}{2}+}(4450)$

In the antiquark-diquark-diquark scheme the isospin structure of the $P_{c\bar{c}u\bar{u}u}^{1\frac{1}{2}+\frac{3}{2}+\frac{3}{2}+}(4450)$ with spin and isospin projections can be written as follows:

$$P_{c\bar{c}u\bar{u}u}^{1\frac{1}{2}+\frac{3}{2}+\frac{3}{2}+}(4450) = -\frac{1}{\sqrt{3}} \vec{c} \cdot \vec{u}, \quad A_{c\bar{c}ud}^{1\frac{1}{2}+\frac{3}{2}+\frac{3}{2}+}, \quad A_{ud}^{10,11}, \quad A_{cd}^{1\frac{1}{2}+\frac{3}{2}+\frac{3}{2}+}, \quad A_{uu}^{11,11}$$  \hspace{1cm} (13)

The wave functions for axial diquarks read:

$$A_{c\bar{c}ud}^{1\frac{1}{2}+\frac{3}{2}+\frac{3}{2}+} = c^+u^+, \quad A_{ud}^{10,11} = \frac{1}{\sqrt{2}}(d^+u^+ + u^+d^+),$$

$$A_{cd}^{1\frac{1}{2}+\frac{3}{2}+\frac{3}{2}+} = c^+d^+, \quad A_{uu}^{11,11} = u^+u^+. \hspace{1cm} (14)$$

Taking into account color structure, we can rewrite expression (13) as follows:

$$P_{c\bar{c}u\bar{u}u}^{1\frac{1}{2}+\frac{3}{2}+\frac{3}{2}+}(4450) = -\frac{1}{\sqrt{3}} \vec{c} \cdot \vec{u}, \quad A_{c\bar{c}ud}^{1\frac{1}{2}+\frac{3}{2}+\frac{3}{2}+}, \quad A_{ud}^{10,11}, \quad A_{cd}^{1\frac{1}{2}+\frac{3}{2}+\frac{3}{2}+}, \quad A_{uu}^{11,11} =$$

$$= -\frac{1}{\sqrt{3}} (\vec{c} \cdot \vec{u} A_{c\bar{c}ud}^{1\frac{1}{2}+\frac{3}{2}+\frac{3}{2}+} + \vec{c} \cdot \vec{u} A_{ud}^{10,11} + \vec{c} \cdot \vec{u} A_{cd}^{1\frac{1}{2}+\frac{3}{2}+\frac{3}{2}+}) +$$

$$+ \frac{2}{3} \frac{1}{\sqrt{3}} (\vec{c} \cdot \vec{u} A_{uu}^{11,11} + \vec{c} \cdot \vec{u} A_{uu}^{11,11}). \hspace{1cm} (15)$$

The white quark-antiquark and three-quark states should be projected on states of real mesons and baryons. We need mesons:

$$J/\psi^+ = c^+c^+, \quad \bar{D}^{*0\uparrow} = c^+u^+, \quad D^{*-\uparrow} = c^+d^+, \hspace{1cm} (16)$$

and baryons:

$$N^{1\frac{1}{2}+\frac{3}{2}+\frac{3}{2}+} = \sqrt{\frac{2}{3}} d^+ A_{uu}^{11,11} - \sqrt{\frac{1}{3}} u^+ A_{ud}^{10,11},$$

$$\Sigma_+^{++}(10,\frac{3}{2} \frac{3}{2} \frac{3}{2}) = c^+ A_{ud}^{10,11},$$

$$\Sigma_+^{++}(10,\frac{3}{2} \frac{3}{2} \frac{3}{2}) = c^+ A_{uu}^{11,11}. \hspace{1cm} (17)$$

Therefore we write for the pentaquark wave function:

$$P_{c\bar{c}u\bar{u}u}^{1\frac{1}{2}+\frac{3}{2}+\frac{3}{2}+}(4450) = -J/\psi^+ N^{1\frac{1}{2}+\frac{3}{2}+\frac{3}{2}+} - \sqrt{\frac{1}{3}} \bar{D}^{*0\uparrow} \Sigma_+^{++}(10,\frac{3}{2} \frac{3}{2} \frac{3}{2}) + \sqrt{\frac{2}{3}} D^{*-\uparrow} \Sigma_+^{++}(11,\frac{3}{2} \frac{3}{2} \frac{3}{2}). \hspace{1cm} (18)$$

The state $P_{c\bar{c}u\bar{u}u}^{1\frac{1}{2}+\frac{3}{2}+\frac{3}{2}+}(4520 \pm 150)$ is a good candidate to be a state which was observed by LHCb (5/2 + (4540 ± 4)) with a width of $\Gamma = 39 \pm 24$ MeV.

Recombination to $\bar{D}^{*0\uparrow} \Sigma_+^{++}$ and $D^{*-\uparrow} \Sigma_+^{++}$ channels is strongly suppressed because their thresholds (4460 and 4464 MeV correspondingly) are larger than the pentaquark mass. So the leading recombination channel is $P \rightarrow J/\psi + p$ which is possible in d-wave only. s-wave transition $P \rightarrow J/\psi + N(J = \frac{3}{2})$ has the threshold 4817 MeV. That explains the relatively small width of $P(4450)$. 

5
3.1.1 Propagator

Resonance propagator:

\[
\frac{1}{m^2 - s + G^2 \cos^2 \gamma |k_{J/\psi N^*}| - iG^2 \sin^2 \gamma k_{J/\psi p}^5 - iG^2 k_{D^* \Sigma^*}}
\]

\[= \frac{1}{m_{P(1/2^-)}^2 - s - iG^2_{J/\psi p} k_{J/\psi p}^5 - iG^2 k_{D^* \Sigma^*}}
\]

Here:

\[m_{P(1/2^-)}^2 = m^2 + G^2 \cos^2 \gamma |k_{J/\psi N^*}|, \quad G_{J/\psi p}^2 = G^2 \sin^2 \gamma \]

Thresholds under discussion:

\[M_{J/\psi} + M_{N^*} = (3100 + 1720) MeV = 4820 MeV, \]
\[M_{D^*} + M_{\Sigma^*} = (2010 + 2520) MeV = 4520 MeV \]
\[M_{J/\psi} + M_p = (3100 + 940) MeV = 4040 MeV \]

Hadron momenta in the pentaquark region \(\sqrt{s} \sim 4450 \text{ MeV}\):

\[|k_{J/\psi N^*}| = \sqrt{\left[-s + (M_{J/\psi} + M_{N^*})^2\right] \left[s - (M_{J/\psi} - M_{N^*})^2\right]} \quad (s < (M_{D^*} + M_{\Sigma^*})^2) \]
\[k_{D^* \Sigma^*} = i \sqrt{\left[-s + (M_{D^*} + M_{\Sigma^*})^2\right] \left[s - (M_{D^*} - M_{\Sigma^*})^2\right]} \quad s < (M_{D^*} + M_{\Sigma^*})^2 \]
\[k_{D^* \Sigma^*} = \sqrt{\left[s - (M_{D^*} + M_{\Sigma^*})^2\right] \left[s - (M_{D^*} - M_{\Sigma^*})^2\right]} \quad s > (M_{D^*} + M_{\Sigma^*})^2 \]
\[k_{J/\psi p} = \sqrt{\left[s - (M_{J/\psi} + M_p)^2\right] \left[s - (M_{J/\psi} - M_p)^2\right]} \quad s > (M_{J/\psi} + M_p)^2 \]

Amplitude of production of pentaquark with background contribution reads:

\[A = A_{\text{background}} + m_{P(1/2^-)}^2 - s - iG^2_{J/\psi p} k_{J/\psi p}^5 - iG^2 k_{D^* \Sigma^*} \]

\[= A_{\text{background}} \left[1 + \frac{\mu_{\text{prod}}^2}{m_{P(1/2^-)}^2 - s - iG^2_{J/\psi p} k_{J/\psi p}^5 - iG^2 k_{D^* \Sigma^*}} \right] \]

At complex \(\mu_{\text{prod}}^2\) and \(m_{P(1/2^-)} \sim 4450 \text{ MeV}\) we have resonance-cusp interplay:

\[\sigma \sim |A|^2 \]

with

\[H_{\text{prod}}^2 = |\mu_{\text{prod}}^2| e^{i\Phi_{\text{prod}}} \quad \left|m_{P(1/2^-)}^2 \right| \lesssim m_{P(1/2^-)}^2 \]
\[\left[G_{J/\psi p}^2 k_{J/\psi p}^5 \right]_{\sqrt{s} \sim 4500} \lesssim 50 \text{ MeV}, \quad \left[G_{D^* \Sigma^*}^2 k_{D^* \Sigma^*} \right]_{\sqrt{s} \sim 4500} \lesssim 50 \text{ MeV} \]
3.2 Pentaquark $P_{\bar{c}S_{cu}, S_{ud}}$

Combination $P = \bar{c}S_{cu} \cdot S_{ud}$ contains scalar diquarks $S_{cu}^{1/2,00} = [c^+u^+ - c^+u^+]1/2$ and $S_{ud}^{00,00} = [u^+d^+ - u^+d^+ - d^+u^+ + d^+u^+]1/2$:

$$P_{\bar{c}1(cuud)}^{1/2,1/2} = \bar{c}^+ S_{cu}^{1/2,00} S_{ud}^{10,00}. \quad (26)$$

Using (2) expression (26) can be represented as follows:

$$P_{\bar{c}1(cuud)}^{1/2,1/2} = \bar{c}^+ [c^+u^+ - c^+u^+] \frac{1}{\sqrt{2}} S_{ud} = \frac{1}{\sqrt{2}} \left\{ -\bar{c}^+ c \cdot u S_{ud} + \bar{c}^+ u \cdot c S_{ud} + \bar{c}^+ c \cdot \bar{c} S_{ud} + S_{ud} \right\}. \quad (27)$$

Now we need expressions for the meson parts:

$$\bar{c}^+ c^+ = J/\psi^\dagger, \quad \bar{c}^+ c^+ = \frac{1}{\sqrt{2}} (J/\psi(0) - \eta_c),$$

$$\bar{c}^+ u^+ = \bar{D}^*0^\dagger, \quad \bar{c}^+ u^+ = \frac{1}{\sqrt{2}} (\bar{D}^*0(0) - \bar{D}^0), \quad (28)$$

and for baryon parts ($\Lambda_c^+(cud)$ and $p(uud)$):

$$\Lambda_c^{+(00,\frac{1}{2}+)} = \bar{c}^+ S_{ud}^{00,00} , \quad (29)$$

$$N_{\frac{3}{2}+\frac{3}{2}+}^{1/2,1/2} = \frac{1}{\sqrt{2}} \left[ \frac{2}{3} d^A_{uu}^{11,11} - \sqrt{2} d^A_{uu}^{11,10} + \frac{1}{3} u^A_{ud}^{10,11} + \frac{1}{3} \right] \frac{1}{\sqrt{2}} u^A_{ud}^{00,00} ,$$

$$N_{\frac{3}{2}+\frac{3}{2}+}^{1/2,1/2} = \frac{1}{\sqrt{2}} \left[ \frac{2}{3} d^A_{uu}^{11,11} - \sqrt{2} d^A_{uu}^{11,10} + \frac{1}{3} u^A_{ud}^{10,11} + \frac{1}{3} \right] \frac{1}{\sqrt{2}} u^A_{ud}^{00,00} ,$$

where $N_{\frac{3}{2}+\frac{3}{2}+}^{1/2,1/2}$ is some radial excitation of proton $N_{\frac{3}{2}+\frac{3}{2}+}^{1/2,1/2}$. Taking into account expressions (28) and (29) we can write:

$$P_{\bar{c}1(cuud)}^{1/2,1/2} = \frac{1}{\sqrt{2}} \left\{ J/\psi^\dagger (p^+ + p') \right\} \frac{1}{\sqrt{2}} \left\{ \bar{D}^*0^\dagger (\bar{D}^0 - \bar{D}^0) \Lambda_c^{++} + \frac{1}{\sqrt{2}} (J/\psi(0) - \eta_c) (p^+ + p') \right\} . \quad (30)$$

Combination $P = -\bar{c}S_{ud} \cdot S_{cu}$ (see Eq. (1)) gives zero result without taking into account the coordinate wave function.

3.3 Pentaquark $P_{\bar{c}(Acu \cdot A_{ud})j=0}^{1/2,1/2}$ ($\sim 4100$)

Isospin and spin structure of pentaquark state $P_{\bar{c}(Acu \cdot A_{ud})j=0}^{1/2,1/2}$ is:

$$P_{\bar{c}A_{cu}, A_{ud}}^{1/2,1/2} = -\frac{1}{\sqrt{3}} \bar{c}^+ \left[ A_{cu}^{11,11} A_{ud}^{10,11} A_{ud}^{10,11} + A_{cu}^{10,11} A_{ud}^{10,11} + A_{cu}^{11,11} A_{ud}^{10,11} \right] .$$
In the case of diquark-diquark spin

In terms of mesons and baryons it can be represented in form:

\[ 3.4 \text{ Pentaquark} \]

and baryons:

\[
\Sigma_c^{+(10, \frac{1}{2})} = \sqrt{\frac{2}{3}} c^\dagger A_{ud}^{10,11} - \sqrt{\frac{1}{3}} c^\dagger A_{ud}^{10,10},
\]

\[
\Sigma_c^{++(11, \frac{1}{2})} = \sqrt{\frac{2}{3}} c^\dagger A_{uu}^{11,11} - \sqrt{\frac{1}{3}} c^\dagger A_{uu}^{11,10},
\]

we can write for the state \( P_{c(A_{cu}.A_{ud})}^{\frac{1}{2} \frac{1}{2} \frac{1}{2}} \):

\[
P_{c(A_{cu}.A_{ud})}^{\frac{1}{2} \frac{1}{2} \frac{1}{2}} = \frac{1}{2} J/\psi^0(-N^{\frac{1}{2} \frac{1}{2} \frac{1}{2}} + N^{\frac{1}{2} \frac{1}{2} \frac{1}{2}}) +  \]

\[
+ \frac{1}{2\sqrt{2}}(J/\psi^0)\eta_c(-N^{\frac{1}{2} \frac{1}{2} \frac{1}{2}} + N^{\frac{1}{2} \frac{1}{2} \frac{1}{2}}) - \]

\[
- \frac{1}{\sqrt{6}}D^{0\theta}\Sigma_c^{+(10, \frac{1}{2})} - \frac{1}{2\sqrt{3}}(D^{0\theta} - D^0)\Sigma_c^{+(10, \frac{1}{2})} + \]

\[
+ \frac{1}{\sqrt{3}}D^{*-\theta}\Sigma_c^{++(11, \frac{1}{2})} + \frac{1}{\sqrt{6}}(D^{*-\theta} - D^-)\Sigma_c^{++(11, \frac{1}{2})}. \tag{34}
\]

\[ \text{3.4 Pentaquark} P_{c(A_{cu}.A_{ud})}^{\frac{1}{2} \frac{1}{2} \frac{1}{2}} (\sim 4240) \]

In the case of diquark-diquark spin \( j_{AA} = 1 \), pentaquark state \( P_{c(A_{cu}.A_{ud})}^{\frac{1}{2} \frac{1}{2} \frac{1}{2}} \) can be written as follows (upper indecies in right part of the expression represent spin and its projection):

\[
P_{c(A_{cu}.A_{ud})}^{\frac{1}{2} \frac{1}{2} \frac{1}{2}} = -\frac{1}{\sqrt{3}}[\bar{c}(A_{cu}A_{ud})]^{1J_z} \frac{1}{2} \frac{1}{2} + \sqrt{\frac{2}{3}}[\bar{c}(A_{cu}A_{ud})]^{1J_z} \frac{1}{2} \frac{1}{2}. \tag{35}
\]

In terms of mesons and baryons it can be represented in form:

\[
P_{c(A_{cu}.A_{ud})}^{\frac{1}{2} \frac{1}{2} \frac{1}{2}} = J/\psi^0 \left( -\frac{1}{3\sqrt{2}}p^\dagger + \frac{1}{3\sqrt{2}}p^\dagger + \frac{1}{3\sqrt{2}}N^{\frac{1}{2} \frac{1}{2} \frac{1}{2}} \right) + \frac{1}{\sqrt{6}}J/\psi^0 N^{\frac{1}{2} \frac{1}{2} \frac{3}{2}} + \]

\[
+ J/\psi^0 \left( \frac{1}{6}p^\dagger - \frac{1}{6}p^\dagger - \frac{1}{3}N^{\frac{1}{2} \frac{3}{2} \frac{1}{2}} \right) + \eta_c \left( \frac{1}{2}p^\dagger - \frac{1}{2}p^\dagger \right) + \]

\[
+ D^{0\theta} \left( \frac{1}{3\sqrt{6}}\Sigma_c^{10, \frac{1}{2}} - \frac{1}{3\sqrt{6}}\Sigma_c^{10, \frac{3}{2}} + \frac{1}{\sqrt{6}}D^{0\theta}\Sigma_c^{+10, \frac{1}{2}} + \]

\[
\right. \]
As previously we use axial-vector diquarks (14) and write:

\[ + D^{*-(0)} \left( \frac{1}{3\sqrt{3}} \Sigma_{c}^{++11, \frac{1}{2}} + \frac{2\sqrt{2}}{3\sqrt{3}} \Sigma_{c}^{++11, \frac{3}{2}} \right) + D^{-} \left( \frac{1}{3\sqrt{3}} \Sigma_{c}^{++11, \frac{3}{2}} + \frac{\sqrt{2}}{3\sqrt{3}} \Sigma_{c}^{++11, \frac{1}{2}} \right) + \]

\[ + \bar{D}^{0\uparrow} \left( \frac{1}{3\sqrt{6}} \Sigma_{c}^{+10, \frac{1}{2}} - \frac{1}{3\sqrt{3}} \Sigma_{c}^{+10, \frac{3}{2}} \right) + \frac{1}{3\sqrt{2}} \bar{D}^{0\downarrow} \Sigma_{c}^{+10, \frac{3}{2}} = \]

\[ - D^{*-(}) \left( \frac{-\sqrt{2}}{3\sqrt{3}} \Sigma_{c}^{++11, \frac{1}{2}} + \frac{1}{3\sqrt{3}} \Sigma_{c}^{++11, \frac{3}{2}} \right) - \frac{1}{3} D^{*-(} \Sigma_{c}^{++11, \frac{3}{2}}. \quad (36) \]

3.5 Pentaquark \( P_{c(cu)u}^{3J_2} \) near 4450 MeV

It is the isotopic counterpart of the observed state \( P_{c(cu)u}^{1J_2} \) (4450). We consider here the case with \( I_z = \frac{1}{2} \), namely, the state with the same quark content as \( P_{c(cu)u}^{1J_2} \) (4450). The \( P_{c(cu)u}^{3J_2} \) can be seen in the decay \( \Sigma_{0}^{0} \to K^- J/\psi N \pi \), in the spectrum \( J/\psi \Delta^+(1240) \to J/\psi N \pi \).

In the quark-diquark model the spin-isospin wave function of the \( P_{c(cu)u}^{3J_2} \) with spin-isospin projections \( I_z = \frac{1}{2} \), \( J_z = \frac{5}{2} \) is written as:

\[ P_{c(cu)u}^{3\frac{1}{2}, \frac{5}{2}} = \sqrt{2} \left( \frac{2}{3} \right) c^+ A_{11}^{10,11} A_{ud}^{10,11} + \frac{1}{\sqrt{3}} c^+ A_{11}^{10,11} A_{uu}^{11,11} \quad (37) \]

As previously we use axial-vector diquarks (14) and write:

\[ P_{c(cu)u}^{3\frac{1}{2}, \frac{5}{2}} = \sqrt{2} \left( \frac{2}{3} \right) c^+ u^d A_{11}^{10,11} + \frac{1}{\sqrt{3}} c^+ A_{ud}^{10,11} \]

\[ + \frac{1}{\sqrt{3}} \left( -c^+ d^u A_{uu}^{11,11} + c^+ A_{uu}^{11,11} \right). \quad (38) \]

The white quark-antiquark and three-quark states should be projected on states of real mesons (16) and baryons (17), (59). Therefore we write for the pentaquark spin-flavor wave function:

\[ P_{c(cu)u}^{3\frac{1}{2}, \frac{5}{2}} = -J/\psi \Delta^{+(\frac{1}{2}, \frac{3}{2})} + \sqrt{3} D^{0\uparrow} \Sigma_{c}^{+(10, \frac{3}{2})} + \frac{1}{3} D^{*-(} \Sigma_{c}^{+(11, \frac{3}{2})}. \quad (39) \]

Let us emphasise, that states under consideration are s-wave states, we consider them as dominant ones.

3.6 Pentaquark \( P_{c(cu)u}^{3\frac{1}{2}, \frac{3}{2}} A(udj_{1}) \)

Pentaquark state \( P_{c(cu)u}^{3\frac{1}{2}, \frac{3}{2}} A(udj_{1}) \) for the case \( j_{AA} = 1 \) can be written as follows:

\[ P_{c(cu)u}^{3\frac{1}{2}, \frac{3}{2}} A(udj_{1}) = -\frac{1}{\sqrt{3}} \left( \frac{1}{3} c^+ A_{11}^{10,11} A_{ud}^{10,11} + \frac{1}{\sqrt{3}} A_{ud}^{11,11} A_{uu}^{11,11} \right) = \]

\[ = \left( \frac{2}{3} \right) c^+ \left( \frac{1}{3} A_{11}^{10,11} A_{ud}^{10,11} - A_{ud}^{11,11} A_{uu}^{11,11} \right) + \left( \frac{2}{3} \right) c^+ \left( \frac{1}{3} A_{11}^{10,11} A_{ud}^{10,11} - A_{ud}^{11,11} A_{uu}^{11,11} \right). \quad (40) \]
Using formulae from \[3\] we can rewrite this expression in terms of mesons and baryons:

\[
P_{\bar{c}A(cu)A(ud)}^{1/2, 3/2, 3/2} = \frac{1}{\sqrt{3}} J/\Psi^{\uparrow} p^\dagger - \frac{1}{\sqrt{3}} J/\Psi^{\downarrow} p^\dagger - \frac{1}{2\sqrt{3}} J/\Psi^{\uparrow} N^{2/3, 2/3} + \\
+ \frac{1}{2\sqrt{2}} J/\Psi^{(0)} N^{1/2, 3/2} + \frac{1}{2\sqrt{2}} \eta_c N^{1/2, 3/2} + \frac{\sqrt{2}}{3} D^{*0} \Sigma_c^{+10, 4/3} - \frac{1}{6} D^{*0} \Sigma_c^{+10, 2/3} - \\
- \frac{2}{3} D^{*0} \Sigma_c^{-11, 2/3} + \frac{1}{3\sqrt{2}} D^{*+0} \Sigma_c^{+11, 2/3} + \frac{1}{2\sqrt{6}} D^{0(0)} \Sigma_c^{+10, 4/3} - \\
- \frac{1}{2\sqrt{6}} D^{*0} \Sigma_c^{+10, 2/3} - \frac{1}{2\sqrt{3}} D^{*-0} \Sigma_c^{+11, 2/3} + \frac{1}{2\sqrt{3}} D^{-} \Sigma_c^{-11, 2/3}. \tag{41}
\]

### 3.7 Pentaquark \(P_{\bar{c}A(cu)A(ud)}^{1/2, 3/2, 3/2}\)

Pentaquark state \(P_{\bar{c}A(cu)A(ud)}^{1/2, 3/2, 3/2}\) for the case \(j_{AA} = 2\) can be written as follows:

\[
P_{\bar{c}A(cu)A(ud)}^{1/2, 3/2, 3/2} = \frac{-\sqrt{5}}{2\sqrt{2}} \eta_c N^{1/2, 3/2} - \frac{3\sqrt{2}}{2\sqrt{6}} J/\Psi^{(0)} N^{1/2, 3/2} + \frac{\sqrt{3}}{2\sqrt{5}} J/\Psi^{0} N^{1/2, 3/2} + \\
+ \frac{1}{\sqrt{2}} \bar{D}^{*0} \Sigma_c^{+10, 4/3} - \frac{\sqrt{6}}{2\sqrt{5}} \bar{D}^{*0(0)} \Sigma_c^{+10, 4/3} - \frac{\sqrt{5}}{2\sqrt{6}} \bar{D}^{0} \Sigma_c^{+10, 4/3} - \\
- \frac{\sqrt{2}}{2\sqrt{5}} D^{*+0} \Sigma_c^{+11, 2/3} + \frac{\sqrt{5}}{2\sqrt{3}} D^{-} \Sigma_c^{-11, 2/3} + \frac{3}{2\sqrt{5}} D^{-*0} \Sigma_c^{+11, 2/3}. \tag{43}
\]

### 3.8 Pentaquark \(P_{\bar{c}S(cu)A(ud)}^{1/2, 3/2, 3/2}\)

Using diquarks this state can be represented as follows:

\[
P_{\bar{c}S(cu)A(ud)}^{1/2, 3/2, 3/2} = \frac{-1}{\sqrt{3}} \left[ \bar{c} \cdot S_{\bar{c}u}^{1/2, 0} A_{ud}^{10, 1J_z} \right]^{1/2} + \frac{2}{3} \left[ \bar{c} \cdot S_{\bar{c}u}^{-1/2, 0} A_{uu}^{11, 1J_z} \right]^{1/2} = \\
= \frac{-1}{\sqrt{3}} \left[ \frac{2}{3} \bar{c} \cdot S_{\bar{c}u}^{1/2, 0} A_{ud}^{10, 11} - \frac{1}{\sqrt{3}} \bar{c} \cdot S_{\bar{c}u}^{-1/2, 0} A_{ud}^{10, 10} \right] + \\
+ \sqrt{3} \left[ \frac{2}{3} \bar{c} \cdot S_{\bar{c}u}^{1/2, 0} A_{uu}^{11, 11} - \frac{1}{\sqrt{3}} \bar{c} \cdot S_{\bar{c}u}^{-1/2, 0} A_{uu}^{11, 10} \right]. \tag{44}
\]

Using formulae from \[3\] we can rewrite this expression in terms of mesons and baryons:

\[
P_{\bar{c}S(cu)A(ud)}^{1/2, 3/2, 3/2} = \frac{-\sqrt{2}}{3} J/\Psi^{(0)} N^{1/2, 3/2} - \frac{\sqrt{2}}{12} J/\Psi^{(0)} p^\dagger + \frac{\sqrt{2}}{12} J/\Psi^{(0)} p^\dagger + \frac{\sqrt{2}}{4} \eta_c p^\dagger + \\
+ \frac{\sqrt{3}}{3} J/\Psi^{0} N^{1/2, 3/2} + \frac{1}{3} J/\Psi^{0} N^{1/2, 3/2} - \frac{1}{6} J/\Psi^{0} p^\dagger - \frac{1}{6} J/\Psi^{0} p^\dagger - \frac{\sqrt{2}}{4} \eta_c p^\dagger. \tag{45}
\]
Using meson and baryon states we write:

\[-\frac{\sqrt{6}}{18}D^{0\psi}\Sigma_c^{+10,\frac{1}{2}} + \frac{1}{3}D^{0\psi}\Sigma_c^{+10,\frac{3}{2}} + \frac{\sqrt{3}}{6}D^{0\psi}\Sigma_c^{+10,\frac{5}{2}} - \frac{\sqrt{3}}{9}D^{0\psi}\Sigma_c^{+10,\frac{1}{2}} + \frac{\sqrt{6}}{9}D^{*0(0)}\Sigma_c^{+10,\frac{3}{2}} + \frac{\sqrt{3}}{2}D^{*0(0)}\Sigma_c^{+10,\frac{5}{2}} - \frac{\sqrt{3}}{9}D^{*0(0)}\Sigma_c^{+11,\frac{1}{2}} - \frac{\sqrt{6}}{6}D^{-}\Sigma_c^{+11,\frac{3}{2}} - \frac{\sqrt{6}}{18}D^{*-(0)}\Sigma_c^{++11,\frac{3}{2}} + \frac{\sqrt{6}}{9}D^{*-(0)}\Sigma_c^{++11,\frac{5}{2}} - \frac{2\sqrt{3}}{9}D^{*-(0)}\Sigma_c^{++11,\frac{7}{2}} + \frac{\sqrt{2}}{3}D^{*-(0)}\Sigma_c^{++11,\frac{9}{2}}.\]

### 3.9 Pentaquark $P^{\frac{1}{2}+\frac{1}{2}+\frac{1}{2}}_{\bar{cA}(cu)S(ud)}$

Here we use axial-vector diquarks $A_\Lambda^{\frac{1}{2}+\frac{1}{2}} = c^\dagger u^\dagger$, $A_{\Sigma_c}^{\frac{1}{2}+\frac{1}{2}} = [c^\dagger u^\dagger + c^\dagger d^\dagger]^{\frac{1}{2}}$ and scalar diquark $S_{ud}^{00,00} = \frac{1}{2}[u^\dagger d^\dagger - u^\dagger d^\dagger - d^\dagger u^\dagger + d^\dagger u^\dagger]$: \[P_{\bar{cA}(cu)S(ud)}^{\frac{1}{2}+\frac{1}{2}+\frac{1}{2}} = \frac{\sqrt{2}}{3}A_{cu}^{\frac{1}{2}+\frac{1}{2}}S_{ud}^{00,00} - \frac{1}{\sqrt{3}}\bar{c}^\dagger A_{\Sigma_c}^{\frac{1}{2}+\frac{1}{2}}S_{ud}^{00,00} = \frac{\sqrt{2}}{3}A_{cu}^{\frac{1}{2}+\frac{1}{2}}S_{ud}^{00,00} - \frac{1}{\sqrt{3}}\bar{c}^\dagger A_{\Sigma_c}^{\frac{1}{2}+\frac{1}{2}}S_{ud}^{00,00} = \frac{1}{\sqrt{6}}[-\bar{c}^\dagger c^\dagger \cdot u^\dagger S_{ud}^{00,00} + c^\dagger u^\dagger \cdot c^\dagger S_{ud}^{00,00}] = \frac{1}{\sqrt{6}}[-\bar{c}^\dagger c^\dagger \cdot u^\dagger S_{ud}^{00,00} + c^\dagger u^\dagger \cdot c^\dagger S_{ud}^{00,00}]. \] (46)

Using meson and baryon states we write:

\[
P_{\bar{cA}(cu)S(ud)}^{\frac{1}{2}+\frac{1}{2}+\frac{1}{2}} = -\frac{1}{\sqrt{6}}(J/\psi^{(0)} + \eta_c)(p^\dagger + p') + \frac{1}{\sqrt{3}}(D^{*0(0)} + D^0)\Lambda_c^{++} + \frac{1}{\sqrt{6}}(J/\psi^{(0)} - \eta_c)(p^\dagger + p') - \frac{1}{\sqrt{6}}(D^{*0(0)} - D^0)\Lambda_c^{++} + \frac{1}{\sqrt{6}}(J/\psi^{(0)} - \eta_c)(p^\dagger + p') - \frac{1}{\sqrt{6}}(D^{*0(0)} - D^0)\Lambda_c^{++}. \] (47)

### 4 Conclusion

We considered the recombination process for pentaquark transition to the meson-baryon channel. With the fixed set of quarks we have two transitions in color space. Both color channels have the same weight. However, different meson-baryon channels have different weights due to different phase space and different thresholds. We analyzed in detail all possible transitions for the system $\bar{c}cuud$. We suppose that the $s$-wave recombination leads to large decay widths. We see that the $J = \frac{5}{2}$ channel only can not decay in the $s$-wave mode; $d$-wave recombination is possible and naturally leads to dumping of width. There is no such dumping of width for other members of pentaquark multiplet with $J = \frac{1}{2}, \frac{3}{2}$. So we can expect that $P^{\frac{3}{2}}, P^{\frac{3}{2}}$ states are wide and corresponding poles are drowned in complex energy plane. So it is natural that $P(4450)$ is seen experimentally as a narrow peak only.\[1\]
Acknowledgments

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A Vanishing of the states $\bar{c}(q_2q_3)(cq_1)$

This state reads:

$$-\bar{c}^\alpha \cdot D_\alpha^\beta D_\gamma^\gamma (\epsilon_{\alpha\beta\gamma}) = (\bar{c}^\alpha q_2 \alpha)(q_3 \beta \beta', q_1 \beta', \epsilon^{\beta\beta' \beta''}) - (\bar{c}^\alpha q_3 \alpha)(q_2 \beta \beta', q_1 \beta', \epsilon^{\beta\beta' \beta''})$$

$$= (\bar{c} q_3)(q_3 q_1) - (\bar{c} q_3)(q_2 q_1) = 0.$$

The zero results due to flavor-spin symmetry of the light diquark: $D_{q_2q_3} = D_{q_3q_2}$. Let us consider as an example combination $P = \bar{c}A_{u^d} \cdot A_{c^u}$:

$$\bar{c}^\uparrow \cdot A_{u^d} \cdot A_{c^u} = \bar{c}^\uparrow \cdot \frac{1}{\sqrt{2}}(u^d \cdot d^\uparrow) \cdot c^\uparrow u^\uparrow$$

$$= \frac{1}{\sqrt{2}} (\bar{c}^\uparrow \cdot u^d \cdot c^\uparrow u^\uparrow + \bar{c}^\uparrow \cdot d^\uparrow \cdot c^\uparrow u^\uparrow)$$

$$= \frac{1}{\sqrt{2}} (-\bar{c}^\uparrow u^\uparrow \cdot d^\uparrow c^\uparrow u^\uparrow + \bar{c}^\uparrow d^\uparrow \cdot u^\uparrow c^\uparrow u^\uparrow - \bar{c}^\uparrow d^\uparrow \cdot u^\uparrow c^\uparrow u^\uparrow + \bar{c}^\uparrow u^\uparrow \cdot d^\uparrow c^\uparrow u^\uparrow) = 0.$$

B Baryons in terms of diquarks

B.1 Proton: $N^+(uud), I = \frac{1}{2}$

Proton $N^{\frac{3}{2}+\frac{3}{2}}_j$:

$$p^\uparrow = \frac{1}{3\sqrt{2}} u^\uparrow A^{10,10}_{ud} - \frac{1}{3} u^\uparrow A^{10,11}_{ud} - \frac{1}{3} d^\uparrow A^{11,10}_{uu} + \frac{\sqrt{2}}{3} d^\uparrow A^{11,11}_{uu} + \frac{1}{\sqrt{2}} u^\uparrow S^{00,00}_{ud},$$

$$p^\downarrow = -\frac{1}{3\sqrt{2}} u^\uparrow A^{10,10}_{ud} + \frac{1}{3} u^\uparrow A^{10,11}_{ud} + \frac{1}{3} d^\uparrow A^{11,10}_{uu} - \frac{\sqrt{2}}{3} d^\uparrow A^{11,11}_{uu} - \frac{1}{\sqrt{2}} u^\uparrow S^{00,00}_{ud}.$$  \hspace{1cm} (49)

Proton $N^{\frac{1}{2}+\frac{1}{2}}_j$:

$$p^\uparrow = -\frac{1}{3\sqrt{2}} u^\uparrow A^{10,10}_{ud} + \frac{1}{3} u^\uparrow A^{10,11}_{ud} + \frac{1}{3} d^\uparrow A^{11,10}_{uu} - \frac{\sqrt{2}}{3} d^\uparrow A^{11,11}_{uu} + \frac{1}{\sqrt{2}} u^\uparrow S^{00,00}_{ud},$$

$$p^\downarrow = \frac{1}{3\sqrt{2}} u^\uparrow A^{10,10}_{ud} - \frac{1}{3} u^\uparrow A^{10,11}_{ud} - \frac{1}{3} d^\uparrow A^{11,10}_{uu} + \frac{\sqrt{2}}{3} d^\uparrow A^{11,11}_{uu} - \frac{1}{\sqrt{2}} u^\uparrow S^{00,00}_{ud}.$$  \hspace{1cm} (51)

Nucleon $N^{\frac{3}{2}+\frac{3}{2}}_j$:

$$N^{\frac{3}{2}+\frac{3}{2}} = -\frac{1}{3} u^\uparrow A^{10,11}_{ud} + \frac{2}{3} d^\uparrow A^{11,11}_{uu},$$

$$\text{ (53)}$$
\[ N^{\frac{3}{2}, \frac{3}{2}} = -\frac{1}{3} u^\dagger A_{ud}^{(10,11)} - \frac{\sqrt{2}}{3} u^\uparrow A_{ud}^{(10,10)} + \frac{\sqrt{2}}{3} d^\dagger A_{uu}^{(11,11)} + \frac{2}{3} d^\uparrow A_{uu}^{(11,10)}, \]
\[ N^{\frac{3}{2}, -\frac{3}{2}} = -\frac{1}{3} u^\dagger A_{ud}^{(10,1-1)} - \frac{\sqrt{2}}{3} u^\uparrow A_{ud}^{(10,10)} + \frac{\sqrt{2}}{3} d^\dagger A_{uu}^{(11,1-1)} + \frac{2}{3} d^\uparrow A_{uu}^{(11,10)}, \]
\[ N^{\frac{3}{2}, -\frac{3}{2}} = -\sqrt{\frac{1}{3}} u^\dagger A_{ud}^{(10,1-1)} + \sqrt{\frac{2}{3}} d^\uparrow A_{uu}^{(11,10)}. \]

**B.2 Delta:** \( \Delta^+(uuud), I = \frac{3}{2} \)

Delta \( \Delta^+(\frac{3}{2}, \frac{3}{2}) \):
\[ \Delta^+(\frac{3}{2}, \frac{3}{2}) = -\frac{1}{3} d^\dagger A_{uu}^{11,10} + \frac{\sqrt{2}}{3} d^\uparrow A_{uu}^{11,11} + \frac{2}{3} u^\uparrow A_{ud}^{10,11} - \frac{\sqrt{2}}{3} u^\dagger A_{ud}^{10,10}, \]
\[ \Delta^+(\frac{3}{2}, -\frac{3}{2}) = \frac{1}{3} d^\dagger A_{uu}^{11,10} - \frac{\sqrt{2}}{3} d^\uparrow A_{uu}^{11,1-1} - \frac{2}{3} u^\uparrow A_{ud}^{10,1-1} + \frac{\sqrt{2}}{3} u^\dagger A_{ud}^{10,10}. \]

Delta \( \Delta^+(\frac{3}{2}, \frac{3}{2}) \):
\[ \Delta^+(\frac{3}{2}, \frac{3}{2}) = \sqrt{\frac{1}{3}} d^\dagger A_{uu}^{11,11} + \sqrt{\frac{2}{3}} u^\dagger A_{ud}^{10,11}, \]
\[ \Delta^+(\frac{3}{2}, -\frac{3}{2}) = \frac{\sqrt{2}}{3} d^\dagger A_{uu}^{11,10} + \frac{1}{3} d^\uparrow A_{uu}^{11,11} + \frac{\sqrt{2}}{3} u^\uparrow A_{ud}^{10,11} + \frac{2}{3} u^\dagger A_{ud}^{10,10}, \]
\[ \Delta^+(\frac{3}{2}, -\frac{3}{2}) = \sqrt{\frac{1}{3}} d^\dagger A_{uu}^{11,1-1} + \sqrt{\frac{2}{3}} u^\dagger A_{ud}^{10,1-1}. \]

**B.3 Lambda:** \( \Lambda_c^+(cud), I = 0 \)

Lambda \( \Lambda_c^+(00, \frac{1}{2}) \):
\[ \Lambda_c^+(00, \frac{1}{2}) = c^\dagger S_{ud}^{00,00}, \]
\[ \Lambda_c^+(00, -\frac{1}{2}) = c^\dagger S_{ud}^{00,00}. \]

**B.4 Sigma:** \( \Sigma_c^+(cud), I = 1 \)

Sigma \( \Sigma_c^+(10, \frac{1}{2}) \):
\[ \Sigma_c^+(10, \frac{1}{2}) = \sqrt{\frac{2}{3}} c^\dagger A_{ud}^{10,11} - \sqrt{\frac{1}{3}} c^\uparrow A_{ud}^{10,10}, \]
\[ \Sigma_c^+(10, -\frac{1}{2}) = -\sqrt{\frac{2}{3}} c^\dagger A_{ud}^{10,1-1} + \sqrt{\frac{1}{3}} c^\uparrow A_{ud}^{10,10}. \]
Sigma $\Sigma_c^{+(10, \frac{3}{2} J_z)}$:

\[
\Sigma_c^{+(10, \frac{3}{2} \frac{3}{2})} = c^\dagger A_{ud}^{10,11},
\]
\[
\Sigma_c^{+(10, \frac{3}{2} \frac{1}{2})} = \sqrt{\frac{1}{3}} c^\dagger A_{ud}^{10,11} + \sqrt{\frac{2}{3}} c^\dagger A_{ud}^{10,10},
\]
\[
\Sigma_c^{+(10, \frac{3}{2} - \frac{1}{2})} = \sqrt{\frac{1}{3}} c^\dagger A_{ud}^{10,11-1} + \sqrt{\frac{2}{3}} c^\dagger A_{ud}^{10,10},
\]
\[
\Sigma_c^{+(10, \frac{3}{2} - \frac{3}{2})} = c^\dagger A_{ud}^{10,11-1}.
\]

**B.5 Sigma: $\Sigma_c^{++}(cuu), I = 1$**

Sigma $\Sigma_c^{++(11, \frac{3}{2} J_z)}$:

\[
\Sigma_c^{++(11, \frac{3}{2} \frac{3}{2})} = \sqrt{\frac{2}{3}} c^\dagger A_{uu}^{11,11} - \sqrt{\frac{1}{3}} c^\dagger A_{uu}^{11,10},
\]
\[
\Sigma_c^{++(11, \frac{3}{2} \frac{3}{2})} = -\sqrt{\frac{2}{3}} c^\dagger A_{uu}^{11,11-1} + \sqrt{\frac{1}{3}} c^\dagger A_{uu}^{11,10}.
\]

Sigma $\Sigma_c^{++(11, \frac{1}{2} J_z)}$:

\[
\Sigma_c^{++(11, \frac{1}{2} \frac{1}{2})} = c^\dagger A_{uu}^{11,11},
\]
\[
\Sigma_c^{++(11, \frac{1}{2} \frac{3}{2})} = \sqrt{\frac{1}{3}} c^\dagger A_{uu}^{11,11} + \sqrt{\frac{2}{3}} c^\dagger A_{uu}^{11,10},
\]
\[
\Sigma_c^{++(11, \frac{1}{2} \frac{1}{2})} = \sqrt{\frac{1}{3}} c^\dagger A_{uu}^{11,11-1} + \sqrt{\frac{2}{3}} c^\dagger A_{uu}^{11,10},
\]
\[
\Sigma_c^{++(11, \frac{1}{2} \frac{3}{2})} = c^\dagger A_{uu}^{11,11-1}.
\]

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