Dependence of the polytropic index of plasma on magnetic field

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Abstract

Determination of the polytropic index of plasma in a magnetic field provides description of plasma physics ranging from laboratory plasma in the Earth to black hole in the Universe. Accordingly, a lot of efforts have been devoted to revealing the dependence of polytropic index of plasma on magnetic field. A recent experiment performed in a magnetic nozzle reported the dependency of the polytropic index on the magnetic field strength in that the polytropic index changes from $5/3$ to unity with decreasing the magnetic field strength. In this letter, we show that the polytropic index of plasma does not depend on magnetic field if the radial electric field is sustained. The polytropic index is measured to be close to 2, higher than the previously reported value, regardless of the change in the strength and configuration of magnetic field.

In terms of plasma modeling in a magnetic field, the equation of state is indispensable and is used to simplify the equations of magnetohydrodynamics. Polytropic index is a key parameter in the equation of state and describes the expansion and compression process of the plasma transferring heat between the system and its surrounding [1]. Accordingly, precise observation of the polytropic index is crucial to understand the macroscopic plasma physics in the Universe [2–9] and Earth [10–17].

In astrophysics or laboratory-generated magnetized plasma, non-adiabatic behavior of the electrons has been attracting great interest and remains a challenge to identify [2–17]. The analysis of the causes of the low polytropic index observed in the solar wind has mainly focused on the consequence of thermal process such as an addition of the heat flux via the dissipation of the turbulence, setting the subject of heat flux as halo and the strahl [3, 8, 9]. Unlike the thermodynamic property of space plasma, the polytropic index of electrons in laboratory magnetic nozzle is causally related to the ambipolar potential [13–15]. Recent intensive studies on electron thermodynamics have shown various polytropic indexes between 1 and $5/3$ [11–15], and different polytropic index for each research group is explained by the energy-dependent behavior of electrons in the electric and magnetic fields.

The research has been further deepened, and two important studies are drawing interests [16, 17]. It has been found that by removing the axial electric field in direction parallel to divergent magnetic field, electrons can undergo adiabatic expansion that only interacts with divergent magnetic fields, showing polytropic index closer to $5/3$ [16]. In the most recent study [17], the dependence on electron thermodynamics and magnetic field strength is revisited, and a new perspective has been presented that the adiabatic expansion is possible only when cross-field transport of electrons is restricted via radial electric field. It was demonstrated that the strength of radial electric field is proportional to the magnetic field strength (current of nozzle coil), and eventually the polytropic index becomes dependent on the magnetic field strength. Then, a fundamental question arises. If all the electric field elements are maintained regardless of magnetic field change, will the polytropic index be dependent on the magnetic field?

In this letter, we investigate the dependence of electron polytropic index and magnetic field by removing the variation of the electric field in all magnetic field configurations. The S-nozzle device and magnetic
field structure used in this study consists of two main regions; a filament plasma source; a grounded expansion chamber (figure 1(a)). The two different regions (the source and expansion chamber) are separated by the nozzle field coil to form convergent–divergent magnetic field configuration. The plasma source consists of the thorium filaments biased negatively by a power supply (ps2; \(-100\) V) to accelerate the thermionic electrons at the axial position of \(-0.34\) m. The filament is heated by a current source (ps1; 71–77 A). Then, beam-electrons are generated by the acceleration of the thermionic electrons with energy equivalent to the voltage difference between the filament and sheath boundary. These beam-electrons are responsible for the initial ionization of the discharge via collisions with neutral species. Eventually, the electron energy distributions undergo energy spread in the distribution function through the heating and energy loss [18–21]. Based on the results of previous study that the adiabatic expansion of electrons can occur by the limitation of cross-field diffusion via formation of the radial electric field [17], we devise a design that can generate the radial electric field even at low magnetic field strength. We apply the characteristics of the electron beam produced plasma [22] that the axial confinement of energetic electrons can create a radial plasma potential gradient. Accordingly, the length of the filament is minimized (3 cm).

Using a mass flow controller with a flow-rate of 3.5 sccm, argon gas is injected through a gas feeding port in the source region. The base pressure is maintained under \(10^{-6}\) Torr, and operating pressure is fixed at 0.38 mTorr. For the assumption of collisionless sheath around the collecting probe, the Larmor radius of the electrons should be considered for design of a probe (i.e., probe tip radius, 0.15 mm; minimum electron Larmor radius, 0.50 mm). The plasma is collisionless at the relevant length scale (electron-neutral collisional mean free path of 1.3 m). The electron energy probability functions (\(eepf\)) are obtained by calculating the second derivative of the measured \(I–V\) characteristic with the assumption of isotropic plasmas [23]. Each \(eepf\) is averaged over 110 shots to minimize error, and the error of plasma potential measurement is less than 3%. Accordingly, the error bar of plasma parameters is much smaller than the symbol size.
Collisions with nearby field-aligned electrons, a constant field structure, the medium strength condition (70 A) is assigned as low times at three nozzle coil currents (low, 30 A; medium, 70 A; high, 115 A) (figure 1(b)). To adjust the nozzle field coil and guiding field coil. The magnetic field strength at the nozzle throat has a difference of up to 3.8 nozzle field magnet, and the magnetic field structure is changed by adjusting both the currents of the nozzle sustained in all experimental sets. Axial variations of properties of the plasma, namely, electron density, axial electric field is not present (figures 3(b) and 4(b)), so it can be assumed that the electric field is non-local electron kinetics [24, 25]. In this experiment, the ambipolar electric field is kept constant, so the non-Maxwellian distribution was found before in DC and RF discharges and explained in the context of the deepf source and expanding regions. Radial variation of plasma potential

\[ \psi \] are predicted to be the result of the difference in the strength and structure of the magnetic field in the source and expanding regions. Radial variation of plasma potential \( V_p \) of two experimental sets is examined (figures 3(a) and 4(a)). Compared to previous studies [17], a radial electric field can be generated even at a relatively low \( B_z \), and the change in the electric field is negligible in all magnetic field configurations. The axial electric field is not present (figures 3(b) and 4(b)), so it can be assumed that the electric field is sustained in all experimental sets. Axial variations of properties of the plasma, namely, electron density \( n_e \), and effective electron temperature \( T_{\text{eff}} \) with different condition of \( B_z \) and \( L_B \) are shown in figures 3(c) and (d) and figures 4(c) and (d), respectively. As \( B_z \) increases, only an increase in the absolute value of the electron density is observed [figure 3(c)], showing less pronounced variation compared with previous studies [17]. In the previous studies, rapid spatial changes in \( n_e \) were observed at low nozzle current conditions, and it was explained by frequent cross-field diffusion due to weakened radial electric field. Such an effect is not observed in our results [26] because the radial electric field is sustained in all experiment conditions.

In the experiments to change \( L_B \), variation of \( n_e \) and \( T_{\text{eff}} \), in each condition is weak at the position closer to the nozzle throat due to identical configuration of the magnetic field at the source region and the nozzle throat (figures 4(c) and (d)). The spatial variation of \( T_{\text{eff}} \) is distinctive, and the higher \( L_B \), the slower the axial change in \( n_e \) and \( T_{\text{eff}} \) [26].

For an electron gas in the collisionless condition, where only interactions between particles are perfectly elastic collisions, the evolution of the electron gas along magnetic field line can be modeled with a polytropic law

\[ T_{\text{eff}} = C_e (\psi) n_e^{\gamma_e - 1}, \quad (1) \]

where \( \gamma_e \) is the polytropic index, respectively. Since the electron gas is strongly magnetized in the absence of collisions with nearby field-aligned electrons, a constant \( C_e (\psi) = T_{\text{eff,0}} / (n_{e,0})^{\gamma_e - 1} \) where magnitudes with
sub index 0 can be evaluated at the nozzle throat (0.1 m), can be defined by the origin condition of each magnetic field flux surface \( \psi \) with the assumption of the axis symmetry. Thereby, the polytropic law of the electron gas in a magnetic nozzle system can be written as

\[
\frac{T_{\text{eff}}}{T_{\text{eff},0}} = \left( \frac{n_e}{n_{e,0}} \right)^{\gamma_e - 1}.
\]

(2)

From a kinetics perspective, the polytropic equation of state is evaluated from

\[
\log \int_0^\infty \varepsilon^{3/2} f(\varepsilon) \, d\varepsilon = \gamma_e \log \int_0^\infty \varepsilon^{1/2} f(\varepsilon) \, d\varepsilon + \log \left[ \frac{3}{2} \left( \frac{T_{\text{eff},0}}{n_{e,0}} \right)^{\gamma_e - 1} \right],
\]

(3)

where \( \varepsilon \) is the electron kinetic energy and \( f(\varepsilon) \) is the eepf [13]. In all experiments of changing magnetic field strength and structure, the dynamics of the electrons follows a polytropic law with index \( \gamma_e = 2.10 \pm 0.04(3\sigma) \) considering the propagation of uncertainty (figures 5(a)–(e)). The measured \( \gamma_e \) differs from the adiabatic value found in previous experiments [17]. In the previous experiment, continuous strengthening of the radial electric field was observed as the magnetic field strength increases. Accordingly, the relationship between \( \gamma_e \) and \( B_z \) in the previous experiment does not show asymptotic value, showing \( \gamma_e \) increasing steadily. In contrast, we observe a constant \( \gamma_e \) regardless of the changes in magnetic field. The measured \( \gamma_e \) is rather closer to 2 higher than that of previous studies [11–17], suggesting the following points. Previous studies on thermodynamics of electrons only emphasized that isothermally behaving trapped electrons in the presence of an axial electric field can induce \( \gamma_e \) close to 1 [13–17]. However, the change in the degree of freedom of electrons due to the presence of a radial electric field that confines ions is overlooked. Unlike previous studies that set the degree of freedom to 3 (\( \gamma_e = 5/3 \) for adiabatic), this study verifies that the degree of freedom of electrons can be reduced to 2 (\( \gamma_e = 2 \) for adiabatic) by a radial electric field (reduced crossed transport of electrons via ion confinement). After all,
when using the polytropic equation in performance scaling and modeling of magnetic nozzle, it implies that a clear setting of the degree of freedom of the system is essential.

In this paper, we clearly demonstrate the dependency of polytropic index with magnetic field. This study shows that the variation of magnetic field strength and structure does not affect the polytropic index of electrons in a system where the electric field is sustained. Eventually, it is worth emphasizing that the variation of the radial and axial electric fields contributes to the difference in the polytropic index observed in each research group via confinement by magnetic moment well or violation of frozen-in condition due to cross-field transport [10–17]. That is, it is indicated that even with the identical condition of magnetic field (strength and structure) and axial electric field, the electron thermodynamics can be close to isothermal if the radial electric field contributes to increase the cross-field transport of electrons. This suggests that the radial distribution of plasma parameters inside the plasma source can be a major factor in magnetic nozzle efficiency. It is also meaningful that we provide theoretical and modeling group a new polytropic index for magnetic nozzle device. Finally, it is emphasized that the newly revealed thermodynamic property of electrons must be reconsidered in the modeling of the magnetic nozzle efficiency such as detachment and ion acceleration.

In addition to the fact that the polytropic index is closer to 2 regardless of the magnetic field structure and strength, it is meaningful that the equilibrated state is rather bi-Maxwellian distribution (or Kappa distribution) in the far-field region where the magnetic field strength is weak and the cooling is completed. When two thermalized groups in the eepfs observed in the far-field region are fitted with the Maxwellian distribution, we can estimate that the electron group with kinetic energy of about 6 eV or more has a lower cooling rate [27]. This observation in the laboratory plasma may give a clue for the origin for the non-Maxwellian distributions that frequently observed in space [28, 29] and require further research in more depth.
Figure 5. Dependency of $\gamma_\varepsilon$ on $B_z$; (a) low, (b) medium, and (c) high $B_z$. The medium $B_z$ condition is assigned as low $L_B$. Then (d) and (e) are results of medium and high $L_B$ structure, respectively. The polytropic index determined by log–log relationship between $p_\varepsilon$ and $n_\varepsilon$ averaged over $eepf/s$ by the upper and lower limit curves from the $3\sigma$ rule drawn as solid lines.

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Data availability

The data that support the findings of this study are available within this article.

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