Difference of Photometric Properties between Regular and Nonregular Miras in the Magellanic Clouds

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Abstract

Mira variables are asymptotic giant-branch pulsating stars with long pulsation periods and large amplitudes in optical bands. By applying the random forest algorithm to the I-band light curves for the Miras in the Magellanic Clouds, we have classified these Miras into regular Miras and nonregular Miras. Nonregular Miras exhibit a long-term variation in addition to their primary pulsation periods. Our results confirm that the period–luminosity relation for maximum light has a small dispersion, but only occurs on the regular oxygen-rich Miras, which we recommend to be applied in future distance-scale work. We have also collected multiband photometry for these Miras to perform a spectral energy distribution fitting with stellar and dust components, showing that a significant fraction of dust is present around the nonregular Miras. According to our results, we believe that the periodic long-term variations seen in the nonregular Miras might be due to the presence of dust.

Unified Astronomy Thesaurus concepts: Mira variable stars (1066); Periodic variable stars (1213); Pulsating variable stars (1307); Long period variable stars (935)

Supporting material: machine-readable tables

1. Introduction

Mira variables (hereafter Mira) are low- and intermediate-mass pulsating stars on the asymptotic giant branch (AGB) that express large periodic variation in the optical and near-infrared (NIR) bands (Whitelock 2012), with pulsation periods spanning from ~100 to ~1500 days and amplitude variations of ΔI > 0.8 mag and ΔV > 2.5 mag. They are also very cool red giants with effective temperatures around 3000 K and radii of a few hundred Solar radii (Trabucchi et al. 2021). Miras can be divided into oxygen-rich (O-rich) and carbon-rich (C-rich; for example, see Cioni et al. 2001; Riebel et al. 2010) similar to other AGB stars.

At the beginning of the twentieth century, the correlation between the pulsation periods and apparent magnitudes existed for the classical Cepheids (Leavitt & Pickering 1912) known as the period–luminosity (PL) relation or Leavitt Law and can be used as distance indicators. The first PL relation for Miras was found using NIR observations, as presented in Glass & Evans (1981). Until now, several papers that have also attempted to devise the PL relations for Miras can be found in the literature (see, e.g., Feast 1984; Feast et al. 1989; Whitelock et al. 2008; Yuan et al. 2017; Bhardwaj et al. 2019). Furthermore, long-period variables (LPVs), including Miras in the Large Magellanic Cloud (LMC), were found to exhibit several sequences of the PL relation (Wood et al. 1999; Soszyński et al. 2007) in the optical and NIR bands, at which Miras occupied sequence C on these PL relations. Some Miras and LPVs also exhibit a significant long-term trend extending several thousand days without apparent periodic variations (Whitelock et al. 1997). The amplitude of these long-term trends was smaller at longer wavelengths (Whitelock et al. 2003).

The goal of our work is to investigate the light curves of Miras in the LMC and Small Magellanic Cloud (SMC), collected from the third phase of the Optical Gravitational Lensing Experiment (OGLE-III; Udalski et al. 2008) and supplemented with photometric data available from SIMBAD archive. We first classified the LMC and SMC Miras into regular and nonregular Miras in Section 2. The light curves of regular Miras can be represented as a simple sinusoidal function without additional variations. In contrast, the light curves of nonregular Miras are superpositions of a sinusoidal function and a long-term variation. In this study, we assume the long-term variation is periodic. The main results of our analysis will be presented in Sections 3 and 4 for the regular and nonregular Miras, respectively. We then performed a multiband analysis on these Miras in Section 5, followed by the conclusions given in Section 6.

2. Light-curve Classification

We retrieved the OGLE-III I-band light curves for 1663 and 352 Miras in the LMC and SMC, respectively, classified by the OGLE team (Soszyński et al. 2009, 2011). These photometric data cover the time span between 3000 to 4500 days. Visual inspections of a small subset of these light curves revealed that some Miras are regular Miras with single periods, while other Miras exhibit long-term variation or multiperiodic behaviors as mentioned in the Introduction (see Figure 1 for representative examples). To classify the Mira light curves into regular and nonregular Miras, we employed the powerful machine-learning (ML) techniques that are becoming popular and widely used in astronomy, especially for classification purposes (Raschka 2015). We selected the random forest (RF) algorithm as our ML classifier due to its high efficiency, simple use, and high modifiability, which has been applied to a variety of data sets to classify astronomical sources (see, e.g., Breiman 2001; Pattnaik et al. 2021).
The RF algorithm required users to select “features” to perform the classification. In this work, we selected 12 light-curve features generated from the Python package FATS (Feature Analysis for Time Series; Nun et al. 2015, 2017). These light-curve features are the Amplitude, Beyond1Std, CAR_sigma, CAR_tau, Mean, LinearTrend, PercentAmplitude, PeriodLS, Std, MaxSlope, Q31, and Psi_CS. Definitions of these features can be found in the FATS document\(^1\) and will not be repeated here. After initial visual inspections on a subset of the light curves, we selected 100 and 150 regular and nonregular Miras that appeared significantly distinct to be our training data to ensure we have at least 100 representative light curves for both types of Miras as our training sets. Then we used the RF classification subroutine available from the Scikit-learn (Pedregosa et al. 2011) package to perform the light-curve classification.

The RF algorithm was comprised of a large number of decision trees. Each decision tree can classify a selected target

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\(^1\) [http://isadoranun.github.io/tsfeat/FeaturesDocumentation.html](http://isadoranun.github.io/tsfeat/FeaturesDocumentation.html)
with the given input features. An example of such a decision tree is shown in Figure 2. The RF algorithm will perform a majority voting based on all decision trees to avoid errors caused by using just a single decision tree. Our RF has 5000 decision trees in our study, and the minimum number of decision leaves was set to 50. Based on our training data, the ranks of the feature importance are displayed in Figure 3, at which the three most important features are the Amplitude, Beyond1std, and CAR sigma. If some of the features were removed during the training process, we found that the accuracy of the classification would decrease. Therefore, we kept all 12 features in our RF classification, achieving an...
accuracy of 93.2%. The resulting receiver-operating-characteristic (ROC) curve is presented in Figure 4, showing that the RF algorithm can perform reasonably well for our classification purpose. We have also used 70% of the training data to be the test data to check the performance of the RF classification. The result is presented in Figure 5, showing that the true–false ratios (TFR) for the regular and the nonregular Miras are 91% and 94.5%, respectively. As both values are greater than 90%, we believe the RF algorithm classification can be applied to classify the Miras into regular and nonregular Miras based on their light-curve features.

Based on our RF classification, there were 694 (642 in the LMC and 52 in the SMC) regular Miras and 1321 (1021 in the LMC and 300 in the SMC) nonregular Miras in our samples. A digital table that included the numerical values for all of the features for our data is available on Zenodo: doi:10.5281/zenodo.6233291.

3. Regular Miras

Based on the results found in Section 2, the RF algorithm classified 642 (232 C-rich Miras and 410 O-rich Mira) and 52 (38 C-rich and 14 O-rich) regular Miras in the LMC and SMC, respectively. The light curves of these regular Miras shared similar patterns with sinusoidal variations; a few examples are illustrated in the middle panels of the subfigures in Figure 6. Therefore, we used the Lomb–Scargle (LS) periodogram, implemented in astropy (Astropy Collaboration et al. 2013; Astropy Collaboration, et al. 2018), to calculate their periods. One common pattern for the light curves of regular Miras was that their magnitudes corresponding to the maximum light were very stable, while at minimum light, they displayed a large fluctuation.

We first divided the light curve for the regular Miras into individual pulsation cycles based on the computed LS period. For each pulsation cycle, if the number of data points was more than 15, the data set was fitted with a sinusoidal curve and then the curve was used to obtain the maximum, mean, and minimum magnitudes. Finally, the averages and standard deviations of the maximum, mean, and minimum magnitudes were calculated by combining the cycles’ values. The upper and lower panels in each subfigure of Figure 6 present the stable magnitudes at maximum light and unstable magnitudes at minimum light, respectively. Figure 7 presents the correlation between the various standard deviation (σ). According to Figure 7, σ mean was correlated with σ min; in contrast, σ max is in general smaller than σ mean. Therefore, we can see that the values for ⟨m max⟩ are more stable than ⟨m mean⟩.

Interestingly, the magnitudes at minimum light for some regular Miras were found to exhibit variations with time. Some variations seem to be periodic, so we tried to find the possible periods. We performed a period search using trial periods in three ranges: those smaller than 3000 days, between 3000 and 10,000 days, and larger than 10,000 days. After the best periods (P min) were found, a simple sinusoidal function was used to fit the magnitudes at minimum light (dashed curves in Figure 6) and a standard deviation of residuals value was calculated. If the standard deviation of the residuals value is smaller than 0.04, then the magnitudes at minimum light were considered to be periodic. We split the behavior of magnitudes at minimum light into three different classes. The first class was for the regular Miras with magnitudes at minimum light that did not show regular periodicity, the second class for those
Figure 6. Examples of light curves for regular Miras in the LMC (subfigures on the left) and SMC (subfigures on the right). The middle panel presents the $I$-band light curve taken from the OGLE-III in each subfigure. The top and bottom panels show the determined magnitudes at maximum and minimum light, respectively, for each pulsation cycle (whenever available). The top two subfigures are examples of light curves with minimal light magnitudes that do not exhibit any periodicity. The middle two subfigures are the light curves in which the magnitude at minimum light can be fitted with a periodic sinusoidal function (shown as dashed curves) with periods $P_{\text{min}}$ of around 1000 days. The bottom two subfigures are similar to the subfigures in the middle, except $P_{\text{min}}$ was found to be much longer than 3000 days.
exhibiting periodicity in the range of 300–3000 days, and third class for those much longer than 3000 days. The numbers of regular Miras in the first, second, and third classes were 197, 363, and 134, respectively. We suggested that if $P_{\text{min}}$ is longer than 3000 days, the magnitudes at minimum light may not be truly periodic, but with some long-term trends. In contrast, if $P_{\text{min}}$ was around or greater than 1500 days, then there will be pairs of maximum and minimum points in the observed time span, so we believed that the period should be real. In Table 1, we present the average magnitudes and the corresponding dispersion for the available pulsation cycles. The determined $P_{\text{min}}$ values, whenever available, were also given in Table 1.

Table 1

| MIRA_ID         | Spectral type | Period (day) | $<m_{\text{max}}>$ (mag.) | $\sigma_{\text{max}}$ (mag.) | $<m_{\text{mean}}>$ (mag.) | $\sigma_{\text{mean}}$ (mag.) | $<m_{\text{min}}>$ (mag.) | $\sigma_{\text{min}}$ (mag.) | $P_{\text{min}}$ (day) | $E(V-I)$ (mag.) |
|-----------------|---------------|--------------|---------------------------|-------------------------------|-----------------------------|-------------------------------|---------------------------|----------------------------|----------------|----------------|
| OGLE-LMC-LPV-00082 | O-rich        | 451.5        | 13.85                     | 0.14                          | 14.43                       | 0.36                          | 14.65                     | 0.07                       | 0              | 0.189          |
| OGLE-LMC-LPV-00115 | C-rich        | 176.1        | 14.57                     | 0.06                          | 14.99                       | 0.17                          | 15.56                     | 0.28                       | 0              | 0.219          |
| OGLE-LMC-LPV-00355 | O-rich        | 154.6        | 13.79                     | 0.04                          | 14.23                       | 0.10                          | 14.71                     | 0.14                       | 1099.9        | 0.147          |
| OGLE-LMC-LPV-00743 | O-rich        | 216.1        | 13.60                     | 0.08                          | 14.11                       | 0.16                          | 14.83                     | 0.18                       | 2400.7        | 0.222          |
| OGLE-LMC-LPV-00881 | O-rich        | 120.2        | 13.84                     | 0.02                          | 14.25                       | 0.06                          | 14.67                     | 0.16                       | 3515.4        | 0.169          |

Note. The table is published in its entirety in machine-readable format. A portion is shown here for guidance regarding its form and content. (This table is available in its entirety in machine-readable form.)

Figure 7. Correlations of the standard deviations $\sigma$ for the maximum, mean, and minimum magnitudes. The left panels are for $\sigma_{\text{mean}}$ vs. $\sigma_{\text{max}}$, and the right panels show $\sigma_{\text{mean}}$ vs. $\sigma_{\text{min}}$, separated for the C-rich (top panels) and O-rich (bottom panels) regular Miras. The dashed lines indicate a 1:1 correlation.

2 We emphasize that we are using the light curves from OGLE-III. Confirming or disproving the very long periods of $P_{\text{min}}$ has to wait for the OGLE-IV light curves, which were not publicly available when this paper was written.

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maximum light was found to be smaller than its counterpart at mean light. Following Bhardwaj et al. (2019), we applied a two-slope model and a quadratic model to fit the PL relations of both the O-rich and C-rich regular Miras in our sample. The adopted two-slope model is:

\[ m = a + b_1 \log P - \log P_b, \]  

(1)

where the slope \( b_1 \) is different for the short-period \((P < 300 \text{ days})\) and long-period \((P > 300 \text{ days})\) Miras. The quadratic model is:

\[ m = a + b_1 \log P - \log P_b + b_2 (\log P - \log P_b)^2, \]  

(2)

where the break period is adopted as \( P_b = 300 \text{ days} \) (Bhardwaj et al. 2019). Extinction corrections were done using the optical

![Figure 8. PL relations for regular Miras in the LMC. The green dashed lines and the blue solid lines are the fitted PL relations using the two-slope model at maximum (black circles) and mean (red triangles) light, respectively.](image)

### Table 2

Parameters of the Fitted PL Relations for Regular Miras

|                  | Two-slope Model | Quadratic Model |
|------------------|-----------------|-----------------|
|                  | \( a \) | \( b_1 \) | \( b_2 \) | \( \sigma \) | \( a \) | \( b_1 \) | \( b_2 \) | \( \sigma \) |
| **LMC only**     |               |                |                |               |                |                |                |               |
| Max              | 14.75 ± 0.56  | -0.44 ± 0.23   | -4.95 ± 0.88   | 0.312         | Max           | 13.70 ± 0.02  | -1.60 ± 0.19  | -6.76 ± 1.04  | 0.307         |
| Mean             | 16.08 ± 0.57  | -0.77 ± 0.23   | -2.66 ± 0.67   | 0.284         | Mean          | 14.19 ± 0.02  | -1.26 ± 0.18  | -3.38 ± 0.96  | 0.285         |
| **C-rich (\( N = 225 \))** |               |                |                |               |                |                |                |               |
| Max              | 18.21 ± 0.33  | -2.00 ± 0.14   | -4.01 ± 0.29   | 0.327         | Max           | 13.23 ± 0.02  | -2.70 ± 0.09  | -0.97 ± 0.34  | 0.323         |
| Mean             | 16.04 ± 0.39  | -0.78 ± 0.17   | -2.63 ± 0.42   | 0.401         | Mean          | 14.01 ± 0.02  | -1.45 ± 0.11  | -0.97 ± 0.43  | 0.403         |
| **O-rich (\( N = 406 \))** |               |                |                |               |                |                |                |               |
| Max              | 16.17 ± 0.26  | -1.08 ± 0.11   | -5.10 ± 0.51   | 0.334         | Max           | 13.46 ± 0.01  | -2.31 ± 0.09  | -1.67 ± 0.35  | 0.371         |
| Mean             | 15.68 ± 0.27  | -0.61 ± 0.11   | -2.56 ± 0.52   | 0.372         | Mean          | 14.10 ± 0.01  | -1.33 ± 0.09  | -1.67 ± 0.36  | 0.372         |
| **All (\( N = 631 \))** |               |                |                |               |                |                |                |               |
| Max              | -4.61 ± 0.89  | -0.26 ± 0.37   | -3.25 ± 0.38   | 0.337         | Max           | -5.27 ± 0.02  | -1.73 ± 0.20  | -7.53 ± 1.09  | 0.326         |
| Mean             | -3.60 ± 0.85  | -0.45 ± 0.35   | -2.31 ± 0.30   | 0.293         | Mean          | -4.75 ± 0.02  | -1.38 ± 0.18  | -3.66 ± 0.97  | 0.289         |
| **C-rich (\( N = 240 \))** |               |                |                |               |                |                |                |               |
| Max              | -1.18 ± 0.35  | -1.81 ± 0.15   | -3.80 ± 0.23   | 0.325         | Max           | -5.78 ± 0.02  | -2.75 ± 0.09  | -1.01 ± 0.35  | 0.329         |
| Mean             | -3.02 ± 0.41  | -0.74 ± 0.18   | -2.44 ± 0.31   | 0.401         | Mean          | -4.96 ± 0.02  | -1.52 ± 0.11  | -1.01 ± 0.43  | 0.405         |
| **O-rich (\( N = 419 \))** |               |                |                |               |                |                |                |               |
| Max              | -2.80 ± 0.31  | -1.08 ± 0.13   | -3.92 ± 0.22   | 0.370         | Max           | -5.52 ± 0.01  | -2.35 ± 0.09  | -1.86 ± 0.36  | 0.387         |
| Mean             | -3.60 ± 0.33  | -0.48 ± 0.14   | -2.53 ± 0.21   | 0.370         | Mean          | -4.86 ± 0.01  | -1.40 ± 0.09  | -1.86 ± 0.36  | 0.375         |
| **All (\( N = 659 \))** |               |                |                |               |                |                |                |               |

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reddening maps of the LMC and SMC (Skowron et al. 2021; the values of $E(V-I)$ are listed in the last column of Table 1) by adopting an averaged value of $R_V = 3.41$ and $R_V = 2.74$ (Cardelli et al. 1989) for the LMC and SMC, respectively.

We fitted these two models of the PL relations using maximum and mean light for regular Miras in two sample sets: one with LMC samples and another with samples combining LMC and SMC. We adopted a distance of 49.59 kpc (Pietrzyński et al. 2019) and 62.44 kpc (Graczyk et al. 2020) for the LMC and SMC, respectively, when combining the samples of regular Miras in the Magellanic Clouds. The parameters of the fitted PL relations are summarized in Table 2, while Figures 8 and 9 present the PL relations for the regular Miras in the LMC only, fitted with the two-slope and quadratic models, respectively. Similarly, the PL relations for the combined sample for the LMC and SMC were also fitted to increase the number of data points in the sample. The results are shown in Figures 10 and 11.

It can be seen from Table 2 that the reduction in the PL dispersion at maximum light only occurs for O-rich regular Miras, both in the sample from LMC only and the sample that combines the LMC and SMC. In Ita & Matsunaga (2011), C-rich Miras did not exhibit a linear relation in the $I$-band with their primary pulsation period because they had different levels of circumstellar extinction. In contrast, we found that C-rich regular Miras display mild linear trends in their PL relation. Furthermore, for C-rich regular Miras, the dispersion of the PL relation at the maximum light was larger than the dispersion at the mean light using both regression models. Hence, PL relations based on the samples combining both O-rich and C-rich regular Miras were mostly influenced by the C-rich Miras. As a result, the PL relation dispersions at maximum and mean light were comparable to each other when the samples of O-rich and C-rich regular Miras were combined (see Table 2). Finally, we pointed out that the PL relation for the C-rich regular Miras at maximum light displays a steep and “up-turn” slope, as demonstrated in Figure 8, for $P_b \sim 350$ days. This is because the C-rich Miras with period longer than 350 days have a larger amplitude;
hence the magnitudes at maximum light display larger variation than their shorter-period counterparts.

4. Nonregular Miras

For the nonregular Miras classified in Section 2, we fitted their OGLE-III $I$-band light curves with two components:

$$I(t) = I_0 + L^1(t) + L^2(t),$$

$$= I_0 + \sum_{i=1}^{3} A_i \cos \left( 2\pi \frac{t}{P_i} + \phi_i \right) + \sum_{i=1}^{3} B_i \cos \left( 2\pi \frac{t}{P_i} + \phi_i \right).$$

In the above equations, $P_s$ represents the short-term or primary pulsation period, and $P_l$ is the long-term period. We applied the Nelder–Mead algorithm to search for the best-fit $P_s$–$P_l$ pairs by evaluating the $\chi^2$ values from the fitted light curves. We restricted the range of $P_s$ to be between 100 and 1000 days (using the periods found from the LS periodogram, $P_{LS}$ as initial guesses) and $P_l$ to be between 1000 and 5000 days. To speed up the calculations, a pair of $P_s$–$P_l$ was set as a starting point; then about eight pairs around the starting point were selected to fit the function. The pair with the smallest $\chi^2$ value served as the starting point for the next iteration. This procedure was repeated until a local minimum was reached, and the best fitting values of $P_s$ and $P_l$ were determined, as presented in Table 3. We picked 10 nonregular Miras to perform a Monte Carlo simulation by simulating 100 light curves for each of them using the observed errors. These simulated light curves were then run through the same Nelder–Mead algorithm. Based on our simulations, we estimated the $P_l$ error to be about 100 days. The distribution of the $P_l/P_s$ ratio was shown in the left panel of Figure 12, which peaks around 5 with a long tail toward a ratio of $\sim 20$ for the C-rich nonregular Miras. Furthermore, no correlation was found between the $P_l/P_s$ ratio and the pulsation period $P_s$, as shown in the right panel of Figure 12, implying $P_l$ was independent of the pulsational properties of Miras. Figure 13 presents several examples of the nonregular Miras, together with the best-fit $I(t)$ to their $I$-band light curves based on the determined $P_s$–$P_l$ pairs.

Figure 11. Same as in Figure 9, but for the combined samples of regular Miras in the LMC and SMC.

Figure 12. Left Panel: histogram of $P_l/P_s$ ratio for nonregular Miras in the sample. Right Panel: $P_l/P_s$ ratios as a function of $P_s$. 

Figure 13. Examples of the nonregular Miras, together with the best fit to their $I$-band light curves.
It is clear that magnitudes at both the maximum and minimum light are unstable, thus showing that nonregular Miras are unsuitable for deriving PL relations. This is further confirmed in Figure 14, which displays nonregular Miras having no clear trend in the PL relations. We have also removed the long-term variations, \( L(t) \), in Equation (3), and only kept the \( I_0 + L'(t) \) for the nonregular Miras light curves.

An example shown in Figure 15 suggests that the resulting light curves were similar to those of regular Miras. Using the same methodology described in Section 3, we derived the PL relations at both maximum and mean light after removing \( L(t) \). The resulting PL relations, as presented in Figure 16, do not exhibit a clear correlation similar to that of regular Miras. We also found that the scatter was larger at maximum light than in

**Table 3**

| MIRA_ID          | Spectral Type | \( P_s \) (days) | \( P_l \) (days) | \( P_{OGLE} \) (days) |
|------------------|---------------|------------------|------------------|-----------------------|
| OGLE-LMC-LPV-00055 | C-rich        | 288.98           | 6089.38          | 290.9                 |
| OGLE-LMC-LPV-00094 | C-rich        | 331.56           | 2089.78          | 332.3                 |
| OGLE-LMC-LPV-00144 | C-rich        | 370.50           | 2727.32          | 364.2                 |
| OGLE-LMC-LPV-00225 | C-rich        | 494.30           | 5974.18          | 504.2                 |

*Note.* The table is published in its entirety in machine-readable format. A portion is shown here for guidance regarding its form and content.

(This table is available in its entirety in machine-readable form.)

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Figure 13. Examples of light curves for nonregular Miras. The black curves are the best-fit light curves based on the determined short- and long-term periods, as described in the text. The dashed curves are the long-term variation \( L(t) \) given by Equation (3).

Figure 14. Examples of light curves for nonregular Miras. The black curves are the best-fit light curves based on the determined short- and long-term periods, as described in the text. The dashed curves are the long-term variation \( L(t) \) given by Equation (3).
mean light for these PL relations. However, when comparing the left panel of Figures 14 to 16, the O-rich nonregular Mira, after removing the $L(t)$ term, were located closer to the quadratic PL relations based on the regular Miras. In contrast, the C-rich nonregular Miras did not correlate between the pulsation periods and magnitudes at neither mean nor maximum light. Based on previous studies, a C-rich Mira that presented a long-term trend on its light curve may be due to the presence of circumstellar reddening (Feast et al. 1984), similar to some AGB stars that had a feature of a thick dust shell (Whitelock et al. 2003). We speculated dust might play an important role for C-rich nonregular Miras.

5. Spectral Energy Distribution Fittings

For our sample of Magellanic Cloud Miras, we have also collected their multiband photometry from the SIMBAD database, whenever available, to perform the spectral energy

![Figure 14. PL relations for nonregular Miras in the LMC, using either the $P_s$ (left panel), the $P_l$ (middle panel), or the $P_l/P_s$ ratios (right panel) as independent variables.]

![Figure 15. Examples of light curves for nonregular Miras after removing the long-term variation $L(t)$.]
distribution (SED) fittings (at which the same SIMBAD database and methodology was applied in Lee et al. 2021). This multiband photometry was mostly single-epoch random-phase measurements from various catalogs (see Table 4). Nevertheless, we fitted our SED only to the maximum values for a given filter. These included 196/143 and 28/56 regular/nonregular Miras in the LMC and SMC. We pointed out that, in the case of nonregular Miras, the multiband photometry from the SIMBAD database can only be found for the C-rich Miras to perform the SED fitting. Extinctions for each filter, $A_\lambda$, were corrected using the optical extinction map from Skowron et al. (2021) together with the reddening law adopted from Cardelli et al. (1989). Our model for the SED fitting consists of two blackbody radiation functions to represent the star and the dust components. These two components were modeled as a projected 2D “disk” from two spheres with different radii. When performing the SED fitting, we restricted the ranges of the blackbody temperature between 2000 K and 4000 K for the stellar component and between 10 K to 1800 K for the dust component. Same as in Section 3, we adopted a distance of 49.59 kpc and 62.44 kpc, for the LMC and SMC, respectively, when converting the (extinction-corrected) multiband observed

![Figure 16. PL relations at maximum light (left panel) and at mean light (right panel) for nonregular Miras after removing the long-term variations. The curves are the fitted quadratic models as given in Table 2 and not the fittings to the O-rich nonregular Miras.](image)

### Table 4

List of Broadband Photometric Data Catalogs used in SED Fitting

| Catalog Name                        | Filter        | References          |
|-------------------------------------|---------------|---------------------|
| NOMAD-1 Catalog                     | Ks, H, J      | Zacharias et al. (2004) |
| The Guide Star Catalog Version 2.3.2 | i, F, J       | Lasker et al. (2008)  |
| The PPMXL Catalog                   | Ks, H, J      | Roeser et al. (2010) |
| XPM Catalog                         | Ks, H, J      | Fedorov et al. (2011) |
| SPM 4.0 Catalog                     | Ks, H, J, V, B| Girard et al. (2011) |
| UCAC4 Catalog                       | Ks, H, J      | Zacharias et al. (2012) |
| AP0P                                | Ks, H, J      | Qi et al. (2015)    |
| UCAC5 Catalogue                     | G             | Zacharias et al. (2017) |
| 2MASS All-Sky Catalog of Point Sources | K, H, J     | Cutri et al. (2003) |
| Gaia DR1                            | G             | Gaia Collaboration et al. (2016) |
| Gaia DR2                            | Grp, G, Gbp   | Gaia Collaboration et al. (2018) |
| Gaia EDR3                           | Grp, G, Gbp   | Gaia Collaboration (2020) |
| WISE All-sky Data Release           | Ks, H, W1, W2, W3, W4 | Cutri et al. (2012) |
| AllWISE Data Release                | Ks, H, W1, W2, W3, W4 | Cutri et al. (2021) |
| SkyMapper Southern Sky Survey. DR1.1 | g, r, i, z    | Wolf et al. (2018) |
| The CatWISE2020 catalog             | W1, W2        | Marocco et al. (2021) |
| The band-merged unWISE Catalog      | W1, W2        | Schlafly et al. (2019) |
| ASAS-SN catalog                     | Ks, H, J, G, V, W1, W2, W3, W4 | Jayasinghe et al. (2018) |
| TESS Input Catalog v8.0             | Ks H, J, V, B, W1, W2, W3, W4, | Stassun et al. (2019) |
| ATLAS all-sky stellar ref. catalog, ATLAS-REFCAT2 | H, J, G      | Toney et al. (2018) |
| The HSOY catalog                    | H, J, G       | Altmann et al. (2017) |
| VEXAS DR2 catalogs                  | Ks, J, Y, W1, W2, W3, W4 | Khramtsoy et al. (2021) |
Figure 17. Examples of SED fittings for regular Miras (left column) and nonregular Miras (right column). The data points in blue crosses are the available photometric data taken from the SIMBAD database. The red dashed and blue dotted curves are the best-fit blackbody radiation function for representing the star and the dust components, respectively. The sum of these two components is shown as the black solid curves. In the left column, the black dashed curves are the resulting SED fit using the DESK model (see text for details).
magnitudes to the fluxes. Comparisons of the fitted SED for both Miras showed that the peak of the fitted blackbody curve for the dust component in nonregular Miras was much higher than that for the stellar component, as presented in Figure 17 for six examples. This suggested that nonregular Miras had more dust than regular Miras. Figure 18 shows the composite fitted SEDs at infrared for regular and nonregular Miras.

We have also used a Python package, the Dusty Evolved Star Kit (DESK; Goldman 2020), to fit the SED and compare it to our result. We used the Oss-Orich-bb model and Zubko-Crich-bb model available from DESK, and the temperatures scale for the stellar component was set in between 2600 K and 3400 K, while for the dust component, the temperature range is 600 K–1200 K. In the case of O-rich regular Miras, the results from the DESK fitting were consistent with our simple blackbody SED fittings. In contrast, DESK predicted a large far-infrared flux excess for the C-rich regular Miras, implying a larger dust abundance than our blackbody SED fittings. In the case of nonregular Miras, neither the C-rich nor the O-rich model in DESK can be fitted well to the observed SED. Therefore, we retain a two-component blackbody model when fitting the SED to the nonregular Miras.

Based on the results, the majority of the fluxes for the regular Miras were from the stellar components. In contrast, the dust components for the nonregular Miras contributed most of the overall fluxes. Due to the different levels of infrared excess, dust-rich and dust-poor Miras will have different distributions on the NIR color–color diagram. In Figure 19, NIR colors in the JHK band for several samples of Miras were compared, including the JHK-band photometry from the Two Micron All Sky Survey (2MASS; Cutri et al. 2003) and the Large Magellanic Cloud Near-Infrared Synoptic Survey (LMCNISS; Yuan et al. 2017) for Miras in the LMC. In general, nonregular Miras tend to have larger values of NIR colors (black points in Figure 19) than regular Miras (red points in Figure 19), even though there were some nonregular Miras having smaller JHK-band colors and located within the region occupied by regular Miras (and vice versa). This behavior was similar to the independent samples of dust-rich and dust-poor Miras presented in the literature.

Figure 18. Composite of the fitted SED between the I-band and 10 micron. The left for the regular Miras and the right for nonregular Miras. It is clear that the peaks of the SED for regular Miras are around $\sim 1.26$ micron (or log $\lambda \sim 0.1$), but for nonregular Miras the peaks are around $\sim 1.6$ to $\sim 6.3$ micron (or log $\lambda \sim 0.2$ to $\sim 0.8$).

Figure 19. JHK-band color–color diagram for various samples of Miras. The blue and green points are the dust-poor and dust-rich Miras presented in Whitelock et al. (2000) and Whitelock & Feast (2000), respectively. The red and black crosses are the regular and nonregular Miras in the LMC, respectively, based on the single-epoch 2MASS data. Similarly, the red and black triangles are the regular and nonregular Miras in the LMC, respectively, based on the multiepoch LMCNISS data.
The general dust-poor and dust-rich nature of regular and nonregular Miras, respectively, can also be seen from their distinct behaviors in the color–magnitude diagram (CMD). Figure 20 displays typical examples of the OGLE-III $V$, $I$-band light curves (top panels), the $(V-I)$ color curves (middle panels, constructed using the $V$- and $I$-band data points that were separated within a day), and the corresponding CMD (bottom panels) at various pulsation phases throughout the pulsation cycles for a regular (left panel) and a nonregular (right panel) LMC Miras. Distributions of the $I$-band magnitudes and $(V-I)$ color from all of the available data points on the $VI$-band light curves for the regular and nonregular LMC Miras are displayed in Figure 21. Although there are some overlapping regions, the regular and nonregular Miras occupied different regions on the CMD. The $I$-band magnitudes for regular Miras were confined between $\sim 11$ and $\sim 17$ mag but showed a large scatter extended to $\sim 22$ mag (due to large variation in their light curves) in the case of the nonregular Miras.

As demonstrated in Figure 20, the loci on CMD for the regular Miras followed the expectation of a pulsating star: the
color became bluer when the star was brighter (due to the Stefan–Boltzmann law). In contrast, the loci of the nonregular Miras on the CMD showed the opposite trend: the overall color became bluer when the star was fainter. We notice this trend only occurs during or around the minimum of the long-term trend (that could cover a few pulsation cycles, as shown in the right panels of Figure 20 with data points after the observing time of ∼3200 days). For pulsation cycles near the maximum of the long-term trend, the behavior of the loci on CMD follows the normal pulsation (the two left-most data points in the middle-right panel of Figure 20, with observing time at ∼3000 days and \((V-I)\) colors around 3.2–3.3). The phenomenon of colors becoming bluer when the stars are fainter, especially at or near the flux minimum, is known as the "blueing effect". This effect can be seen in some young stars with circumstellar dust (the UXor-type stars; see Bibo & The 1990; Grady et al. 1995; Herbst & Shevchenko 1999; Huang et al. 2019) due to scattering from the obscured dust. In the case of nearby Miras and semiregular variables, Ireland et al. (2004) showed that, based on interferometric observations, scattering by dust in the inner circumstellar shell is important, and in general, this could make the color index to be blue. To our knowledge, there is little information in the literature about the photometric variation of optical light in such dust-obscured Miras. We believe that this “blueing effect” is vital in the study of dust-obscured Miras. Again, general trends of redder/bluer colors when the regular/nonregular Miras become fainter can be seen from the composite CMD, as shown in Figure 21.

Some by-products obtained from the two-component blackbody SED fitting are the radii of the stellar and dust components. In Table 5, we summarized the fitted temperatures (in Kelvin) and radius (in Solar radii) for both components in our sample of regular Miras. The left panel of Figure 22 presents the corresponding period–radius (PR) relation for the regular Miras, at which the regular Miras followed a well-defined PR relation. In the left panel of Figure 22, we included a "death line", as defined in Trabucchi et al. (2021), to represent the maximum pulsation periods for a given stellar radius based on a series of theoretical model calculations. The regular Miras are located below this line. In the right panel of Figure 22, we presented a similar PR relation for the dust components, wherein no correlation was found between the pulsation periods of regular Miras and the radii of the dust components. In Figure 23, we present the PR relation for the C-rich nonregular Miras that is similar to Figure 22, where \(P_s\) and \(P_d\) are adopted from Table 3. There seem to exist some trends between \(R_d\) and \(P_s\), for \(\log R_d\) between 2.5 and 4.0, in the case of the nonregular Miras, but in general, no obvious correlations were found between the radii and periods, neither for the stellar nor for the dust component based on our SED fitting results.

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**Figure 22.** PR relation for regular Miras, separated for the stellar component (left panel) and the dust component (right panel), based on the results of our SEDs fitting (see Table 5). The dashed line on the left panel is the “death line” adopted from Trabucchi et al. (2021).

**Figure 23.** PR relation for nonregular Miras, separated for the stellar component with \(P_s\) (left panel), the dust component with \(P_d\) (middle panel), and the dust component with \(P_l\) (right panel), based on the results of our SEDs fitting (see Table 5). The dashed line on the left panel is the “death line” adopted from Trabucchi et al. (2021).
Table 5

Results of the SEDs Fitting for the Regular and Nonregular Miras

| MIRA_ID          | Spectral type | Type | Ts (K) | Rs (R_\odot) | Td (K) | Rd (R_\odot) |
|------------------|---------------|------|--------|--------------|--------|--------------|
| OGLE-LMC-LPV-00082 | O-rich        | R    | 3147.43 | 147.67       | 91.73  | 19965.99     |
| OGLE-LMC-LPV-00355 | O-rich        | R    | 3160.09 | 135.89       | 214.57 | 2704.37      |
| OGLE-LMC-LPV-00743 | O-rich        | R    | 3014.52 | 213.43       | 15.92  | 19962.04     |
| ...              | ...           | ...  | ...    | ...          | ...    | ...          |

Note. The table is published in its entirety in machine-readable format. A portion is shown here for guidance regarding its form and content.

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6. Summary

In this work, we classified the Magellanic Cloud Miras into 694 regular Miras and 1321 nonregular Miras based on their OGLE-III J-band light curves through machine learning. For regular Miras, we found that, at maximum light, their magnitude is relatively stable, while at minimum light, a large fraction exhibit periodic variations. The dispersion of the PL relation at maximum light is smaller for O-rich, while for C-rich is larger than their corresponding mean light counterparts.

For the nonregular Miras, we simultaneously determined the periods corresponding to the short- and long-term variations, and no clear correlation was seen between these periods and the apparent magnitudes even after removing the long-term variations in their light curves. Our results also suggest the (periodic) long-term variations are not directly associated with pulsations because after removing the long-term trends, the O-rich nonregular Miras were located close to the PL relation fitted from the regular Miras (Figure 16).

Using the available multiband photometry, we performed SED fitting of our sample of Miras with two blackbody radiation functions for the stellar and dust components. Results based on our SED fitting, together with the evidence from the CMD and the JHK-band color-color diagram, showed that a large abundance of dust could be found in the nonregular Miras. In contrast, regular Miras did not exhibit evidence for the presence of dust. This suggested that a large fraction of dust found in the nonregular Miras could be responsible for their long-term variations shown in the light curves.

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Software: astropy (Astropy Collaboration et al. 2013; Astropy Collaboration, et al. 2018), FATS (Nun et al. 2015, 2017), Scikit-learn (Pedregosa et al. 2011), DESK (Goldman 2020).

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