Perturbative fragmentation of vector colored particle into bound states with a heavy antiquark

V.V.Kiselev[1], A.E.Kovalsky[2]

Russian State Research Center "Institute for High Energy Physics", Protvino, Moscow Region, 142284, Russia.

Abstract

The fragmentation function of vector particle into possible bound S-wave states with a heavy antiquark is calculated in the leading order of perturbative QCD for the high energy processes at large transverse momenta with the different behaviour of anomalous chromo-magnetic moment. One-loop equations are derived for the evolution of fragmentation function moments, which is caused by the emission of hard gluons by the vector particle. The integrated probabilities of fragmentation are given. The distribution of bound state over the transverse momentum with respect to the axis of fragmentation is calculated in the scaling limit.

1 Introduction

An interesting problem concerning properties of interaction beyond the Standard Model is to study production of hadrons containing leptoquarks [1] which are scalar and vector color-triplet particles appearing in Grand Unification Theories, provided that their total width is much less than the QCD-confinement scale, $\Gamma_{LQ} \ll \Lambda_{QCD}$. Recently the production of $(qLQ)$–baryons in the case of scalar leptoquark was discussed [2].

In this work we study the high energy production of heavy leptoquarkonium containing a vector leptoquark. The results can be used to calculate the fragmentation of vector local diquarks into baryons (a similar approach was applied to the production of $\Omega_{ccc}$ in [3]). For the sake of convenience the local color-triplet vector field will be referred to as the leptoquark in this paper.

New problem arising in this case is a choice of the lagrangian for the vector leptoquark interaction with gluons. Indeed, to the lagrangian of a free vector field

$$-1/2H_{\mu\nu}\bar{H}^{\mu\nu},$$

where $H_{\mu\nu} = \partial_\mu U_\nu - \partial_\nu U_\mu$, $U_\mu$ is the vector complex field with derivatives substituted by covariant ones, we can add the gauge invariant term proportional to

$$S^{\alpha\beta}_{\mu\nu}G^{\mu\nu}U_\beta\bar{U}_\alpha,$$

where $S^{\alpha\beta}_{\mu\nu} = 1/2(\delta^{\alpha\beta}\delta_{\mu\nu} - \delta^{\alpha\mu}\delta_{\nu\beta})$ is the tensor of spin, $G^{\mu\nu}$ is the gluon field strength tensor. It leads to the appearance of a parameter in the gluon–leptoquark vertex (the so-called anomalous chromomagnetic moment, see Section 2). In this work we discuss the production of a 1/2-spin bound state containing the heavy vector particle at various values of this parameter.

[1] E-mail: kiselev@mx.ihep.su
[2] Moscow Institute of Physics and Technology, Dolgoprudny, Moscow region.
At high transverse momenta, the dominant production mechanism for the heavy leptoquarkonium bound states is the leptoquark fragmentation, which can be calculated in perturbative QCD after the isolation of soft binding factor extracted from the non-relativistic potential models. The corresponding fragmentation function is universal for any high energy process for the direct production of leptoquarkonia.

In the leading $\alpha_s$-order, the fragmentation function has a scaling form, which is the initial condition for the perturbative QCD evolution caused by the emission of hard gluons by the leptoquark before the hadronization. The corresponding splitting function differs from that for the heavy quark because of the spin structure of gluon coupling to the leptoquark.

In this work the scaling fragmentation function is calculated in Section 2 for two different cases of the anomalous chromomagnetic moment behaviour. The limit of infinitely heavy leptoquark, $m_{LQ} \to \infty$, is obtained from the full QCD consideration for the fragmentation. The distribution of bound state over the transverse momentum with respect to the axis of fragmentation is calculated in the scaling limit in Section 3. The splitting kernel of the DGLAP-evolution is derived in Section 4, where the one-loop equations of renormalization group for the moments of fragmentation function are obtained and solved. These equations are universal, since they do not depend on whether the leptoquark will bound or free at low virtualities, where the perturbative evolution stops. The integrated probabilities of leptoquark fragmentation into the heavy leptoquarkonia are evaluated in Section 5. The results are summarized in Conclusion.

2 Fragmentation function in leading order

The contribution of fragmentation into direct production of heavy leptoquarkonium has the form

$$d\sigma[l_H(p)] = \int_0^1 dz \, d\hat{\sigma}[LQ(p/z), \mu] \, D_{LQ\to l_H}(z, \mu),$$

where $d\sigma$ is the differential cross-section for the production of leptoquarkonium with the 4-momentum $p$, $d\hat{\sigma}$ is that of the hard production of leptoquark with the scaled momentum $p/z$, and $D$ is interpreted as the fragmentation function depending on the fraction of momentum carried out by the bound state. The value of $\mu$ determines the factorization scale. In accordance with the general DGLAP-evolution, the $\mu$-dependent fragmentation function satisfies the equation

$$\frac{\partial D_{LQ\to l_H}(z, \mu)}{\partial \ln \mu} = \int_z^1 \frac{dy}{y} P_{LQ\to LQ}(z/y, \mu) \, D_{LQ\to l_H}(y, \mu),$$

where $P$ is the kernel caused by emission of hard gluons by the leptoquark before the production of heavy quark pair. Therefore, the initial form of fragmentation function is determined by the diagram shown in Fig.1, and, hence, the corresponding initial factorization scale is equal to $\mu = 2m_Q$. Furthermore, this function can be calculated as an expansion in $\alpha_s(2m_Q)$. The leading order contribution is evaluated in this Section.

Consider the fragmentation diagram in the system, where the momentum of initial leptoquark has the form $q = (q_0, 0, 0, q_3)$ and the leptoquarkonium one is $p$, so that

$$q^2 = s, \quad p^2 = M^2.$$
Figure 1: The diagram of leptoquark fragmentation into the heavy leptoquarkonium.

In the static approximation for the bound state of leptoquark and heavy quark, the quark mass is expressed as $m_Q = rM$, and the leptoquark mass equals $m = (1 - r)M = \bar{r}M$. The gluon–vector leptoquark vertex has the form

$$T_{\alpha\mu\nu}^{VVg} = -ig_s t^a [g_{\mu\nu}(q + \bar{r}p)_\alpha - g_{\mu\alpha}((1 + \bar{r})\bar{r}p - \bar{r}q)_\nu - g_{\nu\alpha}((1 + \bar{r})q - \bar{r}p)_\mu - g_{\nu\alpha}((1 + \bar{r})q - \bar{r}p)_\mu],$$

(2)

where $\bar{r}$ is the anomalous chromomagnetic moment, $t^a$ is the QCD group generator. The sum over the vector leptoquark polarizations with the momentum $q$ ($q^2 = s$) depends on the choice of the gauge of free field lagrangian (for example, the Stueckelberg gauge), but the fragmentation function is a physical quantity and has not to depend on the gauge parameter changing the contribution of longitudinal components of the vector field. So, the sum over polarization can be taken in the form

$$P(q)_{\mu\nu} = -g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{s}.$$ 

The matrix element of the fragmentation into the baryon with the spin of 1/2 has the form

$$\mathcal{M} = -\frac{2\sqrt{2\pi\alpha_s}}{9\sqrt{M^3}} \frac{R(0)}{r\bar{r}(s - m^2)^2} P(q)_{\rho\delta} P(\bar{r}p)_{\mu\eta} T_{\alpha\mu\nu}^{VVg} \rho_{\alpha\beta} \bar{q}\gamma^\delta(\hat{p} - M)\gamma^\eta\gamma^5 l_H \mathcal{M}_0^\delta,$$

(3)

where the sum over the gluon polarization is written down in the axial gauge with $n = (1, 0, 0, -1)$

$$\rho_{\mu\nu}(k) = -g_{\mu\nu} + \frac{k_{\mu}n_{\nu} + k_{\nu}n_{\mu}}{k \cdot n},$$

with $k = q - (1 - r)p$. The spinors $l_H$ and $\bar{q}$ correspond to the leptoquarkonium and heavy quark associated to the fragmentation. $\mathcal{M}_0$ denotes the matrix element for the hard production
of leptoquark at high energy, \( R(0) \) is the radial wave-function at the origin. The matrix element squared and summed over the helicities of particles in the final state has the form

\[
|\bar{M}|^2 = W_{\mu\nu} M_{\mu 0}^\mu M_{\nu 0}^\nu
\]

In the limit of high energies \( q \cdot n \to \infty \), \( W_{\mu\nu} \) behaves like

\[
W_{\mu\nu} = -g_{\mu\nu} W + R_{\mu\nu},
\]

where \( R_{\mu\nu} \) can depend on the gauge parameters and leads to scalar formfactor terms which are small in comparison with \( W \) in the limit of \( q \cdot n \to \infty \). Define

\[
z = \frac{p \cdot n}{q \cdot n}.
\]

The fragmentation function is determined by the expression [7]

\[
D(z) = \frac{1}{16\pi^2} \int ds \theta\left( s - \frac{M^2}{z} - \frac{m_{LQ}^2}{1-z} \right) W,
\]

where \( W \) is defined in (4). The integral in the expression for the fragmentation function diverges logarithmically at a constant value of anamalous chromomagnetic moment if \( \alpha \) does not equal \(-1\).

In this work we consider two sets for the behaviour of anamalous chromomagnetic moment. Set I is defined by \( \alpha = -1 \). Here we observe that the obtained fragmentation function coincides with that for the scalar leptoquark [3] except the factor of \( 1/3 \)

\[
D(z) = \frac{8\alpha_s^2}{243\pi} \frac{|R(0)|^2 \, z^2 (1-z)^2}{M^3 r^2 r^2} \left[ \frac{1}{1-r z} \right]^6 \cdot \left\{ 3 + 3r^2 - (6 - 10r + 2r^2 + 2r^3) z + \left( 3 - 10r + 14r^2 - 10r^3 + 3r^4 \right) z^2 \right\},
\]

which tends to

\[
\tilde{D}(y) = \frac{8\alpha_s^2}{243\pi} \frac{|R(0)|^2 \, (y - 1)^2}{m_{LQ}^3} \frac{y^6}{r} \left\{ 8 + 4y + 3y^2 \right\},
\]

at \( r \to 0 \) and \( y = (1 - (1-r)z)/(rz) \). The limit of \( \tilde{D}(y) \) is in agreement with the general consideration of \( 1/m \)-expansion for the fragmentation function [8], where

\[
\tilde{D}(y) = \frac{1}{r} a(y) + b(y).
\]

The dependence on \( y \) in \( a(y) \) is the same as for the fragmentation of the heavy quark into the quarkonium [7].

Set II: \( \alpha \) behaves like \(-1 + AM^2/(s - m_{LQ}^2)\). The obtained fragmentation function is

\[
D(z) = \frac{8\alpha_s^2}{243\pi} \frac{|R(0)|^2 \, z^2 (1-z)^2}{16M^4 r^2 r^2} \left[ \frac{1}{1-r z} \right]^6 \cdot \left\{ 16(3 + 3r^2 - 6z + 10rz - 2r^2z - 2r^3 z + 3z^2 - 10z^2 r + 14z^2 r^2 - 10z^2 r^3 + 3z^2 r^4) + A(3A + 24r - 6A - 2rzA - 32rz - 16r^2z + 3z^2 A + 8z^2 A + 3z^2 r^2 A - 32z^2 r^2 + 24z^2 r^3) \right\}.
\]

\(^3\)In paper [2] an arithmetic error was made, which slightly affects upon the final result at small \( r \).
Figure 2: The fragmentation function of leptoquark into the heavy leptoquarkonium, the $N$-factor is determined by $N = \frac{8\alpha_s^2}{243\pi} \frac{|R(0)|^2}{m^2_A r^2(1-r)z}$, the fragmentation function for Set I is shown by the dashed line, the fragmentation function for Set II ($A = 3$) is given by the solid line at $r = 0.02$.

It tends to

$$
\tilde{D}(y) = \frac{8\alpha_s^2}{243\pi} \frac{|R(0)|^2}{16y^6} \frac{(y-1)^2}{m^2_Q} \frac{16(8 + 4y + 3y^2) + A(8A - 8yA + 3y^2A - 64 + 16y)}{r},
$$

at $r \rightarrow 0$ and $y = (1 - (1 - r)z)/(rz)$. The perturbative functions in the leading $\alpha_s$-order are shown in Fig. 4 at $r = 0.02$. We see hard distributions, which become softer with the evolution (see ref. [2]).

3 Transverse momentum of leptoquarkonium

In the system with an infinite momentum of the fragmentating leptoquark its invariant mass is expressed by the fraction of longitudinal leptoquark momentum $z$ and transverse momentum with respect to the fragmentation axis $p_T$ (see Fig. [4]) as

$$
s = m^2 + \frac{M^2}{z(1-z)}[(1-(1-r)z)^2 + t^2],
$$

where $t = p_T/M$. The calculation of diagram in Fig. [4] gives the double distribution

$$
\frac{d^2P}{ds \, dz} = D(z, s),
$$
where the Set I function \(D\) has the form

\[
D(z, s) = \frac{256\alpha_s^2}{81\pi} \frac{|R(0)|^2}{r^2 r^2} \frac{M^3}{[1 - r z]^2(s - m^2)^4}
\]

\[
\left\{ r r^2 + r(1 + r - z(1 + 4r - r^2)) \frac{s - m^2}{M^2} - z(1 - z) \left( \frac{s - m^2}{M^2} \right)^2 \right\},
\]

For Set II, we find

\[
D(z, s) = \frac{8\alpha_s^2}{81\pi} \frac{|R(0)|^2}{r^2 r^2} \frac{M^3}{[1 - r z]^2(s - m^2)^4}
\]

\[
\left\{ 8r(A - 4 + 4r)^2(1 - z + rz)^2 + \\
+ 2(-A - 4 - 4r + zA + 4z - rzA + 16rz - 4r^2z) \\
(1 - z + rz)(A - 4 + 4r) \frac{s - m^2}{M^2} - 32(1 - z)z \left( \frac{s - m^2}{M^2} \right)^2 \right\}.
\]

The distribution of leptoquarkonium over the transverse momentum can be obtained by the integration over \(z\)

\[
D(t) = \int_0^1 dz \, D(z, s) \frac{2M^2t}{z(1 - z)}.
\]

For Set I we have

\[
D(t) = \frac{64\alpha_s^2}{81\pi} \frac{|R(0)|^2}{3(1 - r)^5M^3} \frac{1}{t^6}
\]

\[
\left\{ t(30r^3 - 30r^4 - 61t^2r + 45r^2t^2 + 33r^3t^2 - \\
-17r^4t^2 + 3t^4 - 9rt^4 + 15r^2t^4 - 9r^3t^4) - \\
(30r^4 - 99r^2t^2 - 54r^3t^2 + 27r^4t^2 + 9t^4 + 18rt^4 - 6r^2t^4 + \\
+18r^4t^4 + 3r^6t^4 + 3t^6 - 9rt^6 + 9r^2t^6)\arctg\left( \frac{1 - r}{r + t^2} \right) + \\
24(2r^3t + rt^3 + r^2t^3) \ln\left( \frac{r^2(1 + t^2)}{r^2 + t^2} \right) \right\}.
\]

The distribution for Set II is given in Appendix. The typical form of distribution over the transverse momentum is shown in Fig. 3.

4 Hard gluon emission

The one-loop contribution can be calculated in the way described in the previous Sections. This term does not depend on the part of leptoquark-gluon vertex with the anomalous chromomagnetic moment, therefore the splitting kernel coincides with that for the scalar leptoquark. It equals

\[
P_{LQ \to LQ}(x, \mu) = \frac{4\alpha_s(\mu)}{3\pi} \left[ \frac{2x}{1 - x} \right]_+,
\]
where the "plus" denotes the ordinary action: \[ \int_0^1 dx f_+(x) \cdot g(x) = \int_0^1 dx f(x) \cdot [g(x) - g(1)] \]. The scalar leptoquark splitting function can be compared with that of the heavy quark

\[
P_{Q\rightarrow Q}(x, \mu) = \frac{4\alpha_s(\mu)}{3\pi} \left[ \frac{1 + x^2}{1 - x} \right]_+,\]

which has the same normalization factor at \( x \rightarrow 1 \). Furthermore, multiplying the evolution equation by \( z^n \) and integrating over \( z \), one can get from (1) the \( \mu \)-dependence of moments \( a(n) \) up to the one-loop accuracy of renormalization group

\[
\frac{\partial a(n)}{\partial \ln \mu} = -\frac{8\alpha_s(\mu)}{3\pi} \left[ \frac{1}{2} + \ldots + \frac{1}{n+1} \right] a(n), \quad n \geq 1. \tag{13}
\]

At \( n = 0 \) the right hand side of (13) equals zero, which means that the integrated probability of leptoquark fragmentation into the heavy leptoquarkonium is not changed during the evolution, and it is determined by the initial fragmentation function calculated perturbatively in Section 2.

The solution of equation (13) has the form

\[
a_{(n)}(\mu) = a_{(n)}(\mu_0) \left[ \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right]^{16}{\frac{\beta_0}{35}} \left[ \frac{1}{2} + \ldots + \frac{1}{n+1} \right], \tag{14}
\]

where one has used the one-loop expression for the QCD coupling constant

\[
\alpha_s(\mu) = \frac{2\pi}{\beta_0 \ln(\mu/\Lambda_{QCD})},
\]

where \( \beta_0 = 11 - 2n_f/3 \), with \( n_f \) being the number of quark flavors with \( m_q < \mu < m_{LQ} \). Relation (14) is universal one, since it is independent of whether the leptoquark is free or bound at the virtualities less than \( \mu_0 \). We can use the evolution for the fragmentation into the heavy leptoquarkonium. The leptoquark can lose about 20% of its momentum before the hadronization [2].
5 Integrated probabilities of fragmentation

As has been mentioned above, the evolution conserves the integrated probability of fragmentation which can be calculated explicitly

\[
\int dz \, D(z) = \frac{8\alpha_s^2}{81\pi} \frac{|R(0)|^2}{16m_Q^2} w(r). \tag{15}
\]

For Set I, we have

\[
w(r) = \frac{16[(8 + 15r - 60r^2 + 100r^3 - 60r^4 - 3r^5) + 30r(1 - r + r^2 + r^3) \ln r]}{15(1 - r)^7}. \tag{16}
\]

For Set II, we find \((A = 3)\)

\[
w(r) = \left\{ (143 + 701r - 1882r^2 + 3250r^3 - 3245r^4 + 2017r^5 - 936r^6 - 48r^7) + \\
30r(25 - 21r + 43r^2 + r^3 + 8r^4 + 16r^5) \ln r \right\} \frac{1}{15(1 - r)^9}. \tag{17}
\]

The \(w(r)\) functions are shown in Fig. 4 at low \(r\).

6 Conclusion

In this work the dominant mechanism for the production of bound states of spin 1/2 of a local color-triplet vector field with a heavy antiquark is considered for high energy processes at large transverse momenta, where the fragmentation contributes as the leading term. We investigate two sets of the anomalous chromomagnetic moment behaviour. Set I is defined by \(\bar{\alpha} = -1\) (the expression for the fragmentation function diverges logarithmically at a constant value of anomalous chromomagnetic moment if \(\bar{\alpha}\) is not equal to \(-1\)). Here we observe that the obtained fragmentation function coincides with that for the scalar leptoquark up to the factor of 1/3

\[
D(z) = \frac{8\alpha_s^2}{243\pi} \frac{|R(0)|^2}{M^3r^2\bar{r}^2} \frac{z^2(1 - z)^2}{[1 - \bar{r}z]^6} \cdot \left\{ 3 + 3r^2 - (6 - 10r + 2r^2 + 2r^3)z + \\
+ (3 - 10r + 14r^2 - 10r^3 + 3r^4)z^2 \right\}, \tag{18}
\]

where \(r\) is the ratio of heavy quark mass to the mass of the bound state. In the infinitely heavy leptoquark limit, \(D(z)\) has the form, which agrees with what expected from general consideration of \(1/m\)-expansion for the fragmentation function. Set II is defined by \(\bar{\alpha} = -1 + AM^2/(s - m_{LQ}^2)\). The fragmentation function for Set II differs from that for Set I:

\[
D(z) = \frac{8\alpha_s^2}{243\pi} \frac{|R(0)|^2}{16M^3r^2\bar{r}^2} \frac{z^2(1 - z)^2}{[1 - \bar{r}z]^6} \cdot \\
\left\{ 16(3 + 3r^2 - 6z + 10rz - 2r^2z - 2r^3z + \\
+ 3z^2 - 10z^2r + 14z^2r^2 - 10z^2r^3 + 3z^2r^4) + \\
+ A(3A + 24r - 6zA - 2rzA - 32rz - 16r^2z + 3z^2A + \\
+ 2z^2rA + 8z^2r + 3z^2r^2A - 32z^2r^2 + 24z^2r^3) \right\}. \tag{19}
\]
Figure 4: The w-functions for the leptoquark fragmentation into the heavy leptoquarkonium versus the fraction $r = m_Q/M$. The curves correspond to Sets as in Fig. 3.

The distribution of bound state over the transverse momentum with respect to the axis of fragmentation is calculated in the scaling limit. The corresponding distribution functions are given by the expression (11) for Set I and the expression from Appendix. The hard gluon corrections caused by the splitting of vector leptoquark are taken into account so that the evolution kernel has the form

$$P_{LQ \rightarrow LQ}(x, \mu) = \frac{4\alpha_s(\mu)}{3\pi} \left[ \frac{2x}{1-x} \right]_+,$$

which results in the corresponding one-loop equation for the moments of fragmentation functions (see eqs.(13), (14)). The numerical estimates show that the probabilities of fragmentation into bound states with c- and b-quarks of heavy vector leptoquark with the mass about 400 GeV are of the order of $10^{-3-4}$. This suppression makes the experimental observation of such states rather difficult. However we can use the perturbative expressions for the model of fragmentations into doubly heavy baryons, where the integrated probabilities are of the order of $10^{-1-2}$. The results will be used to investigate the fragmentation of doubly heavy vector diquarks into baryons elsewhere.
This work was in part supported by the Russian Foundation for Basic Research, grants 99-02-16558 and 96-15-96575.

7 Appendix

The expression for the distribution over the transverse momentum for Set II ($\alpha = -1 + \frac{3M^2}{s-m_{LQ}^2}$) is given by

$$D(t) = \frac{8a_s^2}{81\pi} \frac{|R(0)|^2}{3(1-r)^7M^3} \frac{1}{t^6} \left\{ 30r^3t - 270r^4t + 720r^5t - 480r^6t - 727t^3r + 2441r^2t^3 - 
-2041r^3t^3 - 694r^4t^3 + 664r^5t^3 - 272r^6t^3 + 129t^5 - 285t^5r + 
+477r^2t^5 - 489r^3t^5 + 312r^4t^5 - 144r^5t^5 -(30r^4 - 240r^5 + 480r^6 + 
+t^2(-909r^2 + 2412r^3 - 45r^4 - 1080r^5 + 432r^6) + 
+t^4(171 + 288r - 492r^2 + 696r^3 - 165r^4 + 264r^5 + 48r^6) + 
+t^6(129 - 156r + 321r^2 - 168r^3 + 144r^4))\arctg\left(\frac{(1-r)t}{r+t^2}\right) + 
+192tr(1-r)(-2t^2(1-r^2) + (1-4r)r^2)\ln\left(\frac{t^2(1+r^2)}{r^2+t^2}\right) \right\}. \quad (21)$$

References

[1] W.Buchmüller, R.Rückl, D.Wyler, Phys. Lett. B191, 442 (1987).
[2] V.V.Kiselev, Phys. Rev. D58, 054008 (1998); V.V.Kiselev, Phys. Atom. Nucl. 62, 300 (1999) [Yad. Fiz. 62, 335 (1999)].
[3] V.A.Saleev, hep-ph/9906515 (1999).
[4] E.Braaten, S.Fleming, T.C.Yuan, Ann. Rev. Nucl. Part. Sci. 46, 197 (1997).
[5] A.Martin, Phys. Lett. 93B, 338 (1980).
[6] W.Buchmüller, C.H.H.Tye, Phys. Rev. D24, 132 (1981).
[7] E.Braaten, T.C.Yuan, Phys. Rev. Lett. 71, 1673 (1993); E.Braaten, K.Cheung, T.C.Yuan, Phys. Rev. D48, 4230 (1993); E.Braaten, K.Cheung, T.C.Yuan, Phys. Rev. D48, 5049 (1993).
[8] R.L.Jaffe, L.Randall, Nucl. Phys. B415, 79 (1994).