Marginally Deformed Rolling Tachyon around the Tachyon Vacuum in Open String Field Theory

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Abstract

We investigate the string field theory around the tachyon vacuum. A pure gauge form of the solution is constructed at the tachyon vacuum. For a special choice of the gauge function for the pure gauge form, marginal deformation from the tachyon vacuum is allowed due to the nontrivial roles of Schnabl’s analytic vacuum solution. We obtain an exact rolling tachyon solution which describes the late time behaviors of D-brane decay.


1 Introduction

There are two well-known vacua in open bosonic string field theory (OSFT) [1]: the unstable (perturbative) vacuum and the tachyon (nonperturbative) vacuum. M. Schnabl obtained the analytic solution for the tachyon vacuum [2]. After Schnabl’s work, there has been remarkable progress in understanding OSFT [3]-[32]. Especially, many works have been devoted to the construction of analytic solutions in bosonic string [14, 15, 18, 19, 22, 25, 30] and superstring [16, 17, 22, 23, 28] field theories, which correspond to the exactly marginal deformations of the boundary conformal field theory (BCFT) [33]. For the earlier works on marginal deformations in OSFT, see Refs. [34, 35, 36, 37, 38, 39, 40].

One of the most interesting examples of marginal deformations is the time dependent solution, referred to as rolling tachyon, in open string theory. Much of the interest in the rolling tachyon solution, however, have been concentrated on the deformations from the unstable vacuum, such as the recently developed marginal deformations in OSFT. Since the known marginal deformations are exact but perturbative solutions in the perturbation parameter \(\lambda\) and the closed forms are not known up to now, the knowledge of the marginally deformed rolling tachyon in OSFT is restricted to the physics around the unstable vacuum, and several puzzles still exist.

The rolling tachyon solutions [41, 42] in BCFT, the boundary string field theory (BSFT), and the low energy effective field theories describe the tachyon matter interpreted as closed string radiations from the D-brane decay. During the decay process of D-brane, which is encoded in the dynamics of the tachyon field, the pressure of the system approaches zero monotonically from a negative value, maintaining a constant energy density, and the tachyon field grows monotonically and approaches the tachyon vacuum, which is located at infinity of the tachyon field.

In OSFT, however, the different behaviors of rolling tachyon appear in the level truncated field theory [13, 14, 45]. In the (0,0)-level truncation of \(L_0\)-eigenstate expansion, the rolling tachyon solution overshoots the tachyon vacuum and oscillates with ever-growing amplitude. Moreover, the pressure of the system has similar oscillating behaviors. The qualitatively similar behaviors also appear in the \(p\)-adic string theory also [43]. These unexpected results (that the tachyon field does not roll from the unstable vacuum to the tachyon vacuum) were also confirmed in the higher level [45, 46]. Even in the exact marginal deformation for rolling tachyon, these oscillating behaviors seem to appear [14, 15], although they are difficult to confirm since the coefficients in the series expansion for the tachyon solution can be obtained numerically, except for several ones, and are restricted to few coefficients.

The aforementioned is a puzzle in the time dependent behaviors of OSFT. How, then, can the puzzling behaviors of rolling tachyon in OSFT be reconciled with the well-known behaviors in BCFT, BSFT, and other low energy effective theories? Several trials were conducted to find the answer to this question, where the time dependent gauge transformation [45, 20], other kinds of time dependent solution [47] and the properties of the partition function in a two-dimensional sigma model [21] were used.

In this work, we try to solve the puzzle of the rolling tachyon solution in OSFT.
This is done by considering the rolling tachyon marginal deformation from the tachyon vacuum. We investigate the string field theory around the tachyon vacuum background, which is explicitly known by Schnabl [2]. The action for the string field around the tachyon vacuum has the same form as the action around the unstable vacuum when BRST operator $Q_B$ at the unstable vacuum is replaced by BRST operator $\tilde{Q}$ at the tachyon vacuum. We construct an analytic solution perturbatively in a parameter $\lambda$ at the tachyon vacuum, using the remarkable properties of wedge states with operator insertions on the worldsheet boundary [48]. Since we insert a matter operator on the boundary in the profile of string field $\tilde{\Psi}$ obtained in section 3, it seems that $\tilde{\Psi}$ is not well-defined at the tachyon vacuum which has no worldsheet boundary. However, it does not cause any problem since $\tilde{\Psi} = 0$ represents the tachyon vacuum by construction.

In section 2, it is shown, by explicitly solving equation of motion and appropriate gauge transformations, that the vanishing cohomology of $\tilde{Q}$ at the tachyon vacuum is directly connected to the pure gauge forms for the well-known perturbative type of solution in OSFT. We argue the validity of the pure gauge solutions for some cases corresponding to large gauge transformation.

In section 3, we consider the marginal deformations around the tachyon vacuum. Through a special choice of ghost number zero string state $\phi$, we construct the marginally deformed solutions. We apply the marginally deformed solution to the rolling tachyon vertex operator $e^{-X_0}$, which describes the late time behaviors of D-brane decay. We obtain the explicit rolling tachyon profile around the tachyon vacuum. Finally, the conclusions that are arrived at in this work are presented in section 4.

## 2 Pure Gauge Solution around the Tachyon Vacuum

Let us first briefly review the bosonic OSFT around the tachyon vacuum. The OSFT around the tachyon vacuum solution is described by the action

$$\tilde{S}[\tilde{\Phi}] = -\frac{1}{2}\langle \tilde{\Phi}, \tilde{Q}\tilde{\Phi} \rangle - \frac{1}{3}\langle \tilde{\Phi}, \tilde{\Phi}^*\tilde{\Phi} \rangle,$$

where the open string coupling constant $g_o = 1$ is set for simplicity, $\langle \cdot, \cdot \rangle$ is the BPZ inner product, `$*$' denotes Witten’s star product, and $\tilde{Q}$ is the new BRST operator at the tachyon vacuum. The BRST operator $\tilde{Q}$ acts on string field $\chi$ of ghost number $gh(\chi)$ through

$$\tilde{Q}\chi = Q_B\chi + \Psi^*\chi - (-1)^{gh(\chi)}\chi^*\Psi,$$

where $Q_B$ is the BRST operator at the unstable vacuum, and $\Psi$ represents Schnabl’s analytic vacuum solution in $B_0\Psi = 0$ gauge [2],

$$\Psi \equiv \lim_{N \to \infty} \left[ \sum_{n=0}^{N} \psi_n' - \psi_N \right]$$
with

\[ \psi_0 = \frac{2}{\pi} c_1 |0\rangle, \]

\[ \psi_n = \frac{2}{\pi} c_1 |0\rangle \star |n\rangle \star B_1^c c_1 |0\rangle, \quad (n \geq 1), \]

\[ \psi'_0 \equiv \frac{d\psi_0}{dn} |_{n=0} = Q_B(B_1^c c_1 |0\rangle) = K_1^c c_1 |0\rangle + B_1^c c_0 c_1 |0\rangle, \]

\[ \psi'_n \equiv \frac{d\psi_n}{dn} = c_1 |0\rangle \star K_1^c |n\rangle \star B_1^c c_1 |0\rangle, \quad (n \geq 1). \quad (4) \]

Here \( B_1^c \) and \( K_1^c \) are defined on the upper half plane (UHP) as

\[ B_1^c = \int_{C_L} \frac{d\xi}{2\pi i}(1 + \xi^2)b(\xi), \]

\[ K_1^c = \int_{C_L} \frac{d\xi}{2\pi i}(1 + \xi^2)T(\xi), \quad (5) \]

where contour \( C_L \) runs counterclockwise along the unit circle with \( \text{Re} \xi < 0 \). Action (1) is invariant under the gauge transformation

\[ \delta \tilde{\Phi} = \tilde{Q} \tilde{\Phi} + \tilde{\Phi} \star \tilde{\Lambda} - \tilde{\Lambda} \star \tilde{\Phi} \quad (6) \]

for a ghost number zero state \( \tilde{\Lambda} \) and satisfies the equation of motion,

\[ \tilde{Q} \tilde{\Phi} + \tilde{\Phi} \star \tilde{\Phi} = 0. \quad (7) \]

In solving the equation of motion (7), we use a perturbative method in some parameter \( \lambda \), which is a well-known method in OSFT. For example, the recently developed marginally deformed solutions in OSFT used this perturbative method [14, 15]. The solution of Eq. (7) has the form,

\[ \tilde{\Psi} = \sum_{n=1}^{\infty} \lambda^n \tilde{\phi}_n \quad (8) \]

with \( \tilde{\phi}_n \) satisfying the relations

\[ \tilde{Q} \tilde{\phi}_1 = 0, \quad (9) \]

\[ \tilde{Q} \tilde{\phi}_n = -\sum_{k=1}^{n-1} \tilde{\phi}_k \star \tilde{\phi}_{n-k}, \quad (n \geq 2). \quad (10) \]

Solution (8) satisfies Eq. (7) at each order of \( \lambda \).

In this section, by explicitly solving equations (9) and (10), it is shown that all perturbative solutions like (8) at the tachyon vacuum have pure gauge like forms.\(^1\) \(^2\)

\(^1\)We use the conventions of Ref. [3].

\(^2\)The ordinary piece \( \sum_{n=0}^{\infty} \lambda^{n+1} \psi_n \) of Schnabl’s vacuum solution (9), which starts from \( Q_B \)-exact state \( \psi_0 = Q_B(B_1^c c_1 |0\rangle) \), can be considered as a perturbative solution and is a pure gauge form. The solution becomes nontrivial, however, at the special value \( \lambda = 1 \). For the earlier studies of the pure gauge forms in OSFT, see Refs. [36, 37, 38, 40].
to the gauge transformations. The vanishing cohomology at the tachyon vacuum is
directly connected to the form of the solution. By introducing the homotopy operator

\[ A = -\frac{\pi}{2} \int_1^2 dr B^2 |r) , \]  

which satisfies \( \tilde{Q}A = I \) with identity string state \( I \) of star product algebra, Ellwood
and Schnabl proved that all \( \tilde{Q} \)-closed states are \( \tilde{Q} \)-exact at the tachyon vacuum \([6]\).
From this fact it can be seen that the only form of solution which satisfies Eq. (9) is
a \( \tilde{Q} \)-exact state

\[ \tilde{\phi}_1 = \tilde{Q} \phi = Q_B \phi + \Psi * \phi - \phi * \Psi , \] 

where \( \phi \) is a ghost number zero state. Unlike other backgrounds, in the tachyon
vacuum this is a unique form of solution in the \( \lambda \)-order of solution \([8]\).

From now on, we determine \( \tilde{\phi}_n \), \( (n \geq 2) \), in a given order of \( \lambda \) from the Eq. (10).
When \( n = 2 \), the Eq. (10) is given by

\[ \tilde{Q} \tilde{\phi}_2 = -\tilde{\phi}_1 * \tilde{\phi}_1 = -\tilde{Q} \phi * \tilde{Q} \phi = \tilde{Q} \left( (\tilde{Q} \phi) * \phi \right) , \] 

Then the following solution of the Eq. (13) is obtained:

\[ \tilde{\phi}_2 = (\tilde{Q} \phi) * \phi + \tilde{Q} \chi_2 , \] 

where \( \chi_2 \) is a ghost number zero string state. Up to \( \lambda^2 \)-order, perturbative solution
\([8]\) is given by

\[ \tilde{\Psi} = \lambda \tilde{Q} \phi + \lambda^2 \tilde{\phi}_2 . \] 

Inserting the Eq. (15) into the action (11), we obtain

\[ S = \lambda^4 \left[ -\frac{1}{2} \langle \tilde{\phi}_2 , \tilde{Q} \tilde{\phi}_2 \rangle - \langle \tilde{Q} \phi , \tilde{Q} \phi * \tilde{\phi}_2 \rangle \right] + O(\lambda^5) . \] 

The terms of the \( \lambda^4 \)-order in the action (16) are completely determined by the truncated solution (15). The higher order of \( \lambda \) in the action (16), however, is not fixed by \( \tilde{\Psi} \) given in Eq. (15). As such, the valid action in this order of \( \lambda \) is the \( \lambda^4 \)-term in Eq. (16). Then it can be easily found that the term \( \tilde{Q} \chi_2 \) in Eq. (14) is a gauge
degree in the action (16) up to \( \lambda^4 \)-order. Through the simplest gauge choice \( \tilde{Q} \chi_2 = 0 \) in (15), the following is obtained:

\[ \tilde{\phi}_2 = (\tilde{Q} \phi) * \phi . \] 

Similarly, when \( n = 3 \) in Eq. (10), we have an equation

\[ \tilde{Q} \tilde{\phi}_3 = -\tilde{\phi}_1 * \tilde{\phi}_2 - \tilde{\phi}_2 * \tilde{\phi}_1 = \tilde{Q} \left( (\tilde{Q} \phi)^2 \right) , \]
where we use the notation

\[ \phi^n \equiv \underbrace{\phi \ast \phi \ast \ldots \ast \phi}_n. \]

From the Eq. (18), we obtain

\[ \tilde{\phi}_3 = (\tilde{Q}\phi) \ast \phi^2 + \tilde{Q}\chi_3, \tag{19} \]

where \( \chi_3 \) is also an arbitrary ghost number zero string state. Then the perturbative solution (8) up to the \( \lambda^3 \)-order is given by

\[ \tilde{\Psi} = \lambda\tilde{Q}\phi + \lambda^2(\tilde{Q}\phi) \ast \phi + \lambda^3\tilde{\phi}_3. \tag{20} \]

Using the same procedure as in the case of \( \tilde{\phi}_2 \), we can fix the gauge by choosing \( \tilde{Q}\chi_3 = 0 \) from the action in the \( \lambda^5 \)-order. Then the gauge fixed \( \tilde{\phi}_3 \) will be given by

\[ \tilde{\phi}_3 = (\tilde{Q}\phi) \ast \phi^2. \tag{21} \]

By repeating the aforementioned procedures, the gauge fixed \( \tilde{\phi}_n \) can be obtained from \( \lambda^{n+2} \)-term in OSFT action (1):

\[ \tilde{\phi}_n = (\tilde{Q}\phi) \ast \phi^{n-1}. \tag{22} \]

As a result, perturbative solution (8) is represented as a pure gauge solution

\[ \tilde{\Psi} = \sum_{n=1}^{\infty} \lambda^n(\tilde{Q}\phi) \ast \phi^{n-1} = \tilde{Q}\phi \ast \frac{\lambda}{1 - \lambda\phi} = e^{-\tilde{\Lambda}} \ast (\tilde{Q}e^{\tilde{\Lambda}}), \tag{23} \]

where the ghost number zero string state \( \tilde{\Lambda} \) is given by

\[ \tilde{\Lambda} = -\ln(1 - \lambda\phi) = \sum_{n=1}^{\infty} \frac{\lambda^n}{n} \phi^n. \]

For any pure gauge form of solution \( \tilde{\Psi} = \tilde{U}\tilde{Q}\tilde{U}^{-1} \) with a ghost number 1 string state \( \tilde{U} \), we can find a perturbative type solution by setting \( \lambda\phi = \mathcal{I} - \tilde{U} \) at the tachyon vacuum. A similar pure gauge form in terms of BRST operator \( Q_B \) for Schnabl's vacuum solution was found by Okawa [3].

Since the action (1) is invariant under the infinitesimal (small) gauge transformation (6), it is also invariant under the gauge function \( e^{\tilde{\Lambda}} \) in Eq. (24) which is \(^3\)By using \( \tilde{Q}A = \mathcal{I} \) at the tachyon vacuum, the following pure gauge solution can also be constructed:

\[ \tilde{\Psi} = \sum_{n=1}^{\infty} \lambda^n(\tilde{Q}\phi) \ast (A \ast \tilde{Q}\phi)^{n-1}. \tag{23} \]

This solution can be constructed only at the tachyon vacuum. The solution (24) can also be reduced to the simplest solution (24) by gauge fixing it from the relation \( A \ast \tilde{Q}\phi = \phi - \tilde{Q}(A\phi) \).
generated by small gauge transformations. In this case, where $e^{\tilde{\Lambda}}$ can be deformed to the identity string state $I$, the pure gauge solution (24) has no physical meaning. In some cases, however, $e^{\tilde{\Lambda}}$ state cannot be deformed continuously into the identity string state at the tachyon vacuum. Then the action is not invariant for the pure gauge solution (24) (i.e., $\tilde{S}[e^{-\Lambda}\tilde{Q}e^{\Lambda}] \neq 0$). In the gauge theory, this type of gauge transformation is called large gauge transformation. In this case, the pure gauge solution has a nontrivial physical meaning. In the subsection 3.2, an explicit form of $\phi$, which corresponds to the large gauge transformation, will be introduced.

### 3 Marginal Deformations

The string state $\tilde{\phi}_n$ in Eq. (22) is a wedge state with operator insertions on the worldsheet boundary. If string state $\phi$ in the construction of solution (24), which is made from some operator insertion into the SL(2,R) vacuum $|0\rangle$, is well-defined, $\tilde{\phi}_n$ will not cause any divergence in the calculations of BPZ inner products since the separations among the boundary insertions do not go to zero by construction. As we discussed in the previous section, the only constraint for $\phi$ is its ghost number (i.e., $gh(\phi) = 0$). Since the solution herein, however, around the tachyon vacuum, is the pure gauge form, choosing the physically acceptable matter operator will become nontrivial.

An important and well-known solution in OSFT is the marginally deformed solution [14, 15]. As an application of the construction, we consider matter operators which give exactly marginal deformations in BCFT [33]. These operators are of particular interest in the study of tachyon condensation and the cohomology class of BRST operator $Q_B$ at the unstable vacuum.

We can also construct a pure gauge solution around the unstable vacuum by replacing the BRST operator $\tilde{Q}$ with $Q_B$ in the Eq. (24), apart form the issue of the gauge invariance of pure gauge form. However, we cannot obtain the marginal deformation by the pure gauge solution around the unstable vacuum, except for the cases in Refs. [18, 23] which have special prescriptions. The reason is following: The first term in the pure gauge solution for the BRST operator $Q_B$ is

$$\lambda Q_B \phi,$$

where $\phi$ is a ghost number zero string state and include an exactly marginal operator $V$. To extract the contribution to the marginal deformation $c_1 V(0)|0\rangle$, we can use a test state $c_0 c_1 \tilde{V}(0)|0\rangle$ with a primary operator $\tilde{V}$. But the contribution of $\lambda Q_B \phi$ to $c_1 V(0)|0\rangle$ vanishes always since

$$\langle c_0 c_1 \tilde{V}, Q_B \phi \rangle = -\langle Q_B(c_0 c_1 \tilde{V}), \phi \rangle = 0.$$

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4In the construction of marginal deformation for photon around the unstable vacuum, the pure gauge solution was used by Fuchs et al. [18, 23]. In the paper the non-normalizable states were used and the counterterms were added to obtain some meaningful results which are independent of the non-normalizable states.
Therefore, we cannot obtain the marginally deformed solution from the pure gauge solution around the unstable vacuum without special prescriptions.

3.1 Marginally deformed operators

Let us consider an exactly marginal operator called $V$ with conformal dimension one in the construction of ghost number zero string state $\phi$ in Eq. (24). After some investigations of the construction of $\phi$, we found that the most plausible and simple choice for $\phi$ is 5

$$\phi = V(0)B_1^Lc_1|0\rangle,$$

where $B_1^L$ was defined in Eq. (5). Due to the properties of $B_1^L$, the solution is significantly simplified. The other ghost number zero states, for instance, $\phi = V(0)|0\rangle$ or $\phi = V(0)B_1^Lc_0|0\rangle$, can also be considered. These cases, however, do not give marginal deformations since the term $V(0)c_1|0\rangle$, which corresponds to the marginal deformation in the $\lambda$-order in the resulting solution $\tilde{\Psi}$ in Eq. (24), vanishes. In these reasons, we suggest a special choice for $\phi$ given in Eq. (25), and we examined this fact for the rolling tachyon marginal operator $V = e^{\pm X^q}$, which has a nontrivial physical meaning, as will be seen in the next subsection. For the general marginal operators, further investigation is needed.

In what follows in this section, we restrict our interest to the case of $\phi$ given in Eq. (25). As was discussed about our pure gauge solution in section 2, we have to choose the matter operator $V$ in the gauge function,

$$e^\lambda = \frac{1}{1 - \lambda \phi} = \frac{1}{1 - \lambda V(0)B_1^Lc_1|0\rangle},$$

which gives nontrivial physical meanings.

Under the assumption that a matter operator $V$ makes the solution $\tilde{\phi}_n$ nontrivial, we write down the explicit form of $\tilde{\phi}_n$ in Eq. (22) as

$$\tilde{\phi}_n = \left( V(0)c_0|0\rangle + \partial V(0)c_1|0\rangle + V(0)c_1K_1^L|0\rangle \right) \ast J^{n-2} \ast V(0)B_1^Lc_1|0\rangle$$

$$+ \Psi \ast J^{n-1} \ast V(0)B_1^Lc_1|0\rangle$$

$$- V(0)B_1^Lc_1|0\rangle \ast \Psi \ast J^{n-2} \ast V(0)B_1^Lc_1|0\rangle,$$

(27)

where $J \equiv V(0)|0\rangle$, and the following relations are used:

$$Q_B\phi = V(0)c_0|0\rangle + \partial V(0)c_1|0\rangle + V(0)c_1K_1^L|0\rangle,$$

$$\phi^n = J^{n-1} \ast V(0)B_1^Lc_1|0\rangle.$$  

(28)

In obtaining the expression $\phi^n$ in Eq. (28), we used the relations,

$$B_1^L \phi_1 \ast \phi_2 = B_1 \phi_1 \ast \phi_2 + (-1)^{g(\phi_1)} \phi_1 \ast B_1^L \phi_2,$$

$$\{B_1, c_1\} = 1, \quad B_1|0\rangle = 0, \quad (B_1^L)^2 = 0.$$ 

5For the special choice of $\phi = V(0)B_1^Lc_1|0\rangle$, the pure gauge solution $\tilde{\phi}_n$ is the same as the solution since $A \ast \tilde{\phi} = A \ast Q_B \phi = A \ast \Psi \ast \phi - A \ast \phi \ast \Psi = \phi$, where we used the facts, $A \ast \phi = 0$, $Q_BA = I - |0\rangle$, and $A \ast \Psi = B_1^Lc_1|0\rangle$. 

8
Recently, marginally deformed exact solutions around the unstable vacuum are obtained in bosonic string \cite{14,15,18,19,22,25,30} and superstring \cite{16,17,22,23,28} field theories. If the operator $V$ has a regular OPE with itself, the solutions are well-defined. When the OPE of $V$, however, is singular, divergencies arise in the solutions, and one needs to add counterterms to regularize it at each order of $\lambda$ \cite{15} or renormalize the operator $V$ \cite{25,28}. In the construction of $\tilde{\phi}_n$ in Eq. (27), there is no divergence even if the case for the operator $V$ with singular OPE would be considered, as explained earlier.

### 3.2 Rolling tachyon: Late time behaviors of D-brane decay

As a very important application of the solution \cite{24}, we consider the rolling tachyon vertex operator, $V(y) = e^{\pm \sqrt{\alpha'} X^0(y)}$, where $y$ is the boundary coordinate on UHP. In $\alpha' = 1$ units, $V$ is the dimension one primary operator. Since we are considering the string field theory around the tachyon vacuum, we choose

$$V(y) = e^{-X^0(y)}$$

for definiteness. In physical point of view, the deformation of tachyon field $e^{-X^0(0)c_1|0\rangle}$ in the $\lambda$-order represents the facts that the system will reach to the tachyon vacuum in the far future.

By inserting rolling tachyon vertex operator (29) into the Eq. (25), we obtain

$$\phi = e^{-X^0(0)} B^L_1 c_1|0\rangle.$$  

(30)

Then the gauge function $e^{\hat{\Lambda}}$ is given by

$$e^{\hat{\Lambda}} = \frac{1}{1 - \lambda e^{-X^0(0)} B^L_1 c_1|0\rangle}.$$  

(31)

The above gauge function is an example that generates the so-called large gauge transformation corresponding to nontrivial physics. The reason for this is the following: One cannot deform the gauge function $e^{\hat{\Lambda}}$ in Eq. (31) to identity string state $I$ since we can always rescale the gauge parameter $\lambda$ by the time $x^0$ (zero mode of $X^0$) translation.

From the expression for $\tilde{\phi}_n$ given in Eq. (27) we obtain the string field solution corresponding to rolling tachyon marginal deformation,

$$\tilde{\Psi} = \sum_{n=1}^{\infty} \lambda^n \tilde{\phi}_n = \sum_{n=1}^{\infty} \lambda^n (A_n + B_n + C_n)$$

$$= \sum_{n=1}^{\infty} \lambda^n \left( \beta_n e^{-nX^0(0)c_1|0\rangle} + \cdots \right),$$

(32)

where $\beta_n$ is the coefficient of the tachyon profile and

$$A_n = (Q_B\phi) \ast \phi^{n-1}$$
\[
\begin{align*}
B_n &= \Psi \ast \phi^n \\
C_n &= -\phi \ast \Psi \ast \phi^{n-1} \\
&= -e^{-X^0(0)}B^L_1c_1|0\rangle 
\end{align*}
\]

with \( J = e^{-X^0(0)}|0\rangle \). In the third step of the Eq. (32), the tachyon component was separated, and \( \cdots \) indicates the higher level fields.

By using a test string state
\[
\chi_n = e^{nX^0(0)}c_0c_1|0\rangle
\]
the coefficient \( \beta_n \) in Eq. (32) can be extracted by
\[
\beta_n = \langle \chi_n, \tilde{\phi}_n \rangle,
\]
where we omit volume factor \((2\pi)^D\delta^D(0)\) for the \( D \) spacetime dimensions arising from the BPZ inner products \( \langle \cdot, \cdot \rangle \) for simplicity. For the conventions of the matter and ghost correlation functions, see Appendix. Then the exact expression for \( \beta_n \) is given by
\[
\beta_n = \beta_n^A + \beta_n^B + \beta_n^C,
\]
where
\[
\beta_n^A \equiv \langle \chi_n, A_n \rangle = \frac{1}{2} \left( \frac{2}{\pi} \right)^{n^2+n} \frac{\partial}{\partial x} \left[ a^{n^2+n-1} \left\{ \left( x - \frac{1}{2a} \sin(2ax) \right) \sin^2 a + \left( 1 - \frac{1}{2a} \sin(2a) \right) \sin^2(ax) \right\} \times \prod_{j=2}^{n} \sin^2\left( a(x-j) \right) \prod_{2 \leq k \leq m} \sin^2\left( a(k-m) \right) \right]_{x=1} \\
+ \left( \frac{2}{\pi} \right)^{n^2+n} \frac{\partial}{\partial \theta} \left[ b^{n^2+n-1} \sin^2 b \left( 1 - \frac{1}{2b} \sin(2b) \right) \times \prod_{j=0}^{n-2} \sin^2\left( b(m+j-1) \right) \prod_{0 \leq k \leq l} \sin^2\left( b(k-l) \right) \right]_{m=2},
\]
\[
\beta_n^B \equiv \langle \chi_n, B_n \rangle = \left( \frac{2}{\pi} \right)^{n^2+n} \sum_{m=0}^{C} \frac{\partial}{\partial m} \left[ c^{n^2+n-1} \sin^2 c \left( 1 - \frac{1}{2c} \sin(2c) \right) \prod_{2 \leq j \leq k} \sin^2\left( c(j-k) \right) \prod_{i=2}^{n+1} \sin^2\left( c(m+i) \right) \right],
\]
\[
\beta_n^C \equiv \langle \chi_n, C_n \rangle = -2 \left( \frac{2}{\pi} \right)^{n^2+n} \sum_{m=0}^{C} \frac{\partial}{\partial m} \left[ c^{n^2+n-1} \cos c \sin^2 c \left( \cos c - \frac{1}{c} \sin c \cos(2c) \right) \times \prod_{j=3}^{n+1} \sin^2\left( c(m+j-1) \right) \prod_{3 \leq k \leq l} \sin^2\left( c(k-l) \right) \right]_{m=3}^{n+1} \sin^2 c \prod_{i=3}^{n+1} \sin^2\left( c(m+i) \right)
\]

(37)
Figure 1: Graph of $\bar{T}(x^0)$.

|       | $n=1$    | $n=2$    | $n=3$    | $n=4$    | $n=5$    | $n=6$    |
|-------|----------|----------|----------|----------|----------|----------|
| $\bar{\beta}_A$ | 0.0      | 3.9161   | 0.096432 | 9.0328$\times 10^{-5}$ | 2.4981$\times 10^{-9}$ | 1.4941$\times 10^{-15}$ |
| $\bar{\beta}_B$ | 8.7380   | 0.35979  | 0.00080278 | 5.9455$\times 10^{-8}$ | 9.7343$\times 10^{-14}$ | 2.5567$\times 10^{-21}$ |
| $\bar{\beta}_C$ | -7.7380  | -3.9711  | -0.050663 | -1.9498$\times 10^{-5}$ | -1.6748$\times 10^{-10}$ | -2.2996$\times 10^{-17}$ |
| $\bar{\beta}$   | 1.0      | 0.30483  | 0.046572 | 7.0889$\times 10^{-5}$ | 2.3307$\times 10^{-9}$ | 1.4711$\times 10^{-15}$ |

Table 1: Several few coefficients for $\bar{\beta}_A$, $\bar{\beta}_B$, $\bar{\beta}_C$, and $\bar{\beta}$.

with

$$a = \frac{\pi}{n+1}, \quad b = \frac{\pi}{m+n-1}, \quad c = \frac{\pi}{m+n+2}.$$  

In the calculations of $\beta_B$ and $\beta_C$, we have to use the Schnabl’s vacuum solution $\Psi$ which is composed of the ordinary piece $\sum_{n=0}^{\infty} \psi_n'$ and the phantom piece $-\psi_\infty$. The ordinary piece has nontrivial contributions to $\beta_n$ and makes a convergent series. However, the phantom piece has no contribution to $\beta_n$ since the coefficients of $\psi_N$ in $L_0$-level truncation (level zero in this case) go to zero as $O(N^{-3})$ for a large $N$.

The detailed calculations for BPZ-inner products of $\beta_A$, $\beta_B$, and $\beta_C$ are given in Appendix.

The numerical results for the first few $\beta_n$ are

$$
\begin{align*}
\beta_1 &= -0.042740, \quad \beta_2 = -0.013018, \quad \beta_3 = -0.0019905, \\
\beta_4 &= -3.0298 \times 10^{-6}, \quad \beta_5 = -9.9612 \times 10^{-11}, \quad \beta_6 = -6.2872 \times 10^{-17}. \quad (38)
\end{align*}
$$

We can easily obtain the higher coefficients of $\beta_n$ by adjusting the number of significant digits to increase numerical precision in the computer program. In the convention
herein, the tachyon potential is unbounded from below at $T \to +\infty$. As such, the value of the tachyon field at the unstable vacuum is greater than that at the tachyon vacuum. Since the deformation at the true vacuum is being considered, the rolling tachyon deformation, which describes the decay of the unstable D-brane from the unstable vacuum to the tachyon vacuum, is positive, i.e., $T > 0$. As we have shown in Eq. (38), all coefficients $\beta_n$ are negative. Therefore, the physical solution for the tachyon profile in Eq. (39) corresponds to $\lambda < 0$. After rescaling by translation in the time direction, we can set $\lambda = -1$ in Eq. (32).

If we normalize $\beta_n$ by using $\beta_1$ for convenience, the resulting tachyon profile in Eq. (32) is given by

$$\bar{T}(X^0) \equiv \frac{T(X^0)}{|\beta_1|} = e^{-X^0} + \sum_{n=2}^{\infty} (-1)^{n+1} \beta_n e^{-nX^0} = e^{-X^0} - 0.3048 e^{-2X^0} + 0.04657 e^{-3X^0} - 7.089 \times 10^{-5} e^{-4X^0} + 2.331 \times 10^{-9} e^{-5X^0} - 1.471 \times 10^{-15} e^{-6X^0} + \cdots, \quad (39)$$

where $\bar{\beta}_n \equiv \beta_n/\beta_1$. The behaviors of the rescaled tachyon field $\bar{T}(x^0)$ are plotted in Fig.1. To obtain physical intuitions for the roles of BRST operator $Q_B$ and the vacuum solution $\Psi$ in the tachyon profile, we summarize the first few normalized coefficients, $\bar{\beta}_A^A \equiv \beta_A^A/\beta_1, \bar{\beta}_B^B \equiv \beta_B^B/\beta_1$, and $\bar{\beta}_C^C \equiv \beta_C^C/\beta_1$ in Table 1. As can be seen in Table 1, in the late time behaviors of the tachyon profile (encoded in $\bar{\beta}_1$) the roles of tachyon vacuum solution $\Psi$ are crucial in the rolling tachyon deformation, and there is no contribution from $\bar{\beta}_1^A$ which comes from the BRST charge $Q_B$. From the next order of tachyon coefficients $\beta_n, (n \geq 2)$, the contributions of tachyon vacuum solution (encoded in $\bar{\beta}_n^B$ and $\bar{\beta}_n^C$) rapidly decrease and those of $Q_B$-term become dominant.

Tachyon field $T(x^0)$ decreases monotonically and approaches zero at $x^0 \to \infty$, as can be seen in Fig.1. The system approaches the tachyon vacuum without oscillating behaviors at a late time of the D-brane decay. A similar behavior of the tachyon profile at a late time was reported in Ref. [20] by neglecting the contributions of matter correlators. However, our result suggests that in the far past, the tachyon profile had wild oscillating behaviors. Therefore, it seems that our solution connects the wild oscillations to the tachyon vacuum. Since the tachyon field, however, is not gauge invariant, it is difficult to identify the oscillating behaviors in the far past in this paper as those of the marginally deformed rolling tachyon around the unstable vacuum, which were obtained in Refs. [14, 15].

4 Conclusion

In this work, we investigated the analytic solutions around the tachyon vacuum in OSFT. Based on the fact that all $\bar{Q}$-closed states at the tachyon vacuum are $\bar{Q}$-exact
due to the existence of homotopy operator $A$ [6], we construct a pure gauge solution from the perturbative type of solution (8). This situation at the tachyon vacuum in OSFT is reminiscent of that at the spatial infinity in gauge theory, such as the four-dimensional Euclidean Yang-Mills theory with an instanton solution. Like the gauge theory, the pure gauge solutions, which correspond to the large gauge transformations, are physically nontrivial.

As an application of the construction herein of a pure gauge solution around the tachyon vacuum, we considered the marginally deformed rolling tachyon vertex operator $V = e^{-X^0}$, in which the system stays at the tachyon vacuum in the far future. After some investigations, we found that a special choice of the ghost number zero string state $\phi$ in Eq. (30) and corresponding gauge function $e^{\tilde{\Lambda}}$ given in Eq. (31) make the pure gauge solution nontrivial for the rolling tachyon vertex operator. In this special choice, the pure gauge solution corresponds to the large gauge transformation. Then the gauge function $e^{\tilde{\Lambda}}$ cannot be deformed to the identity string state $I$ since gauge parameter $\lambda$ represents a translation along the time direction. We found that under the choice of $\phi$ in Eq. (30), we could make the marginal deformation due to the nontrivial roles of the analytic tachyon vacuum solution $\Psi$ in Eq. (3). We explicitly obtained the tachyon profile, which is believed to describe the behaviors of D-brane decay at late times. The coefficients $\beta_n$ represent exponentially decreasing behaviors, which are similar to those of tachyon coefficients at the unstable vacuum [14, 15]. According to the results of this study, the tachyon profile decrease monotonically and approaches the tachyon vacuum asymptotically at a late time of the D-brane decay. These behaviors of the tachyon profile around the tachyon vacuum were also obtained in Ref. [20] by neglecting the roles of matter correlators. Our results, however, show that there were wild oscillations in the far past. Therefore, our solution seems to connect the wild oscillations of tachyon profile to the tachyon vacuum.

By using the remarkable property of OSFT, we can relate the marginally deformed rolling tachyon around the unstable vacuum and our solution around the tachyon vacuum. If $\tilde{\Psi}_u$ is the rolling tachyon solution [14] satisfying $Q_B \tilde{\Psi}_u + \tilde{\Psi}_u \ast \tilde{\Psi}_u = 0$ around the unstable vacuum, the string field $(-\Psi + \tilde{\Psi}_u)$ is the solution around the tachyon vacuum since it satisfies the equation of motion,

$$
\tilde{Q}(-\Psi + \tilde{\Psi}_u) + (-\Psi + \tilde{\Psi}_u) \ast (-\Psi + \tilde{\Psi}_u) = 0,
$$

where we used the fact that $Q_B \Psi + \Psi \ast \Psi = 0$. Though $(-\Psi + \tilde{\Psi}_u)$ and our solution $\tilde{\Psi}$ satisfy the same equation of motion and use the same rolling tachyon vertex operator, the two solutions are gauge inequivalent since the action values for the two are different due to the presence of Schnabl’s solution in the former case. Here we used the fact that the BPZ inner products including $\tilde{\Psi}_u$ or $\tilde{\Psi}$ in the calculation of the actions are trivially zero from the momentum conservation in the products. Therefore to find the relation between $\tilde{\Psi}_u$ and $\tilde{\Psi}$, we have to obtain the late time behaviors of $\tilde{\Psi}_u$ around the tachyon vacuum. However, it was not known yet.

If the two rolling tachyon marginal solutions around the unstable vacuum and the solution presented herein describe the same physical situation, although further
investigation in this direction is needed, the result of this study suggests the following possibilities. The first possibility is that the puzzling oscillating behaviors can be eliminated through the resummation of the series form of the marginal solutions, if such is possible. Actually, this possibility is not very promising since the oscillating behaviors were examined by using various methods in literatures [43, 44, 45, 46]. Even in the analytic solution [14, 15], the oscillating behaviors were almost confirmed from the behaviors of the coefficients of the tachyon profile. The results presented in this paper also support the oscillating behaviors before the system approaches the tachyon vacuum. The other possibility is that by using a time dependent gauge transformation, the rolling tachyon solution, which connects the unstable vacuum to the tachyon vacuum without the oscillating behaviors, as suggested in literatures [15, 20], can be obtained. Although it is difficult to give concluding remarks based on the results of this study, we think that our results can, hopefully, shed some light on the puzzle of rolling tachyon solution (tachyon matter problem) in OSFT, since we fixed the behaviors of the tachyon profile at the late time of D-brane decay. Since the tachyon field alone, however, is not gauge invariant, the physical meanings of the tachyon profile, which were obtained through marginal deformation at the unstable vacuum, and our result at the tachyon vacuum are not clear. Further investigation in this direction is needed.

As we have seen in Refs. [18, 23] or in Schnabl’s vacuum solution⁶, pure gauge solutions are useful in obtaining physically nontrivial solutions. Of course, to obtain meaningful results, gauge degree must be avoided. In this sense, the construction of pure gauge solution at the tachyon vacuum herein can be used to obtain meaningful solutions, for instance, marginal solutions or soliton solutions, through various methods. Extension of our method to supersymmetric string field theory is also an interesting subject.

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A Conventions and Calculations for $\beta^A_n$, $\beta^B_n$, and $\beta^C_n$

We calculate the correlators, which come from BPZ-inner products, in semi-infinite cylinder(SIC) (frequently called as sliver frame) with coordinate $z$. The coordinate $z$

⁶For the breakdown of gauge symmetry in the ordinary piece when $\lambda = 1$, see the section 5 of Ref. [6].
has the relation
\[ z = f(\xi) = \frac{2}{\pi} \tan^{-1} \xi \]  
(40)
with coordinate $\xi$ in UHP. We basically follow the notations used in literatures [3, 10, 15].

Using appropriate mappings $f_i(\xi)$ from UHP to SIC, we can obtain the following relation between BPZ-inner product and correlation function in conformal field theory for general type of BPZ-inner product,
\[
\langle \phi_0, \phi_1 \ast \phi_2 \ast \cdots \phi_n \rangle = \langle f_0 \circ \phi_0(0) f_1 \circ \phi(0) \cdots f_n \circ \phi_n(0) \rangle_{\mathcal{W}_1},
\]  
(41)
where $\langle \cdots \rangle_{\mathcal{W}_n}$ denotes a correlation function on SIC with $\mathcal{W}_n$ called as wedge state surface with circumference,
\[
-\frac{1}{2} (1 + n) \leq \text{Re}(z) \leq \frac{1}{2} (1 + n),
\]  
(42)
$\phi_i$ denotes a generic state in the Fock space and $\phi_i(0)$ represents the corresponding operator having the relation $|\phi_i\rangle = \phi_i(0)|0\rangle$, and
\[
f_j(\xi) = \frac{2j}{n+1} + \frac{4}{(n+1)\pi} \tan^{-1} \xi, \quad (0 \leq j \leq n).
\]  
(43)
Then under the assumption that $\phi_i(0)$ is a primary operator with conformal dimension $h_i$ for simplicity, we obtain
\[
\langle \phi_0, \phi_1 \ast \phi_2 \ast \cdots \phi_n \rangle = \left( \frac{4}{(n+1)\pi} \right)^n \left( \sum_{i=0}^{n} h_i \right) \langle \phi_0(0) \phi_1 \left( \frac{2}{n+1} \right) \cdots \phi_j \left( \frac{2j}{n+1} \right) \cdots \phi_n \left( \frac{2n}{n+1} \right) \rangle_{\mathcal{W}_1}
\]
\[
= \left( \frac{2}{\pi} \right)^n \sum_{i=0}^{n} h_i \langle \phi_0(0) \phi_1(1) \cdots \phi_n(n) \rangle_{\mathcal{W}_n},
\]  
(44)
where in the second step we rescaled the coordinate, $z \to \frac{n+1}{2} z$.

### A.1 Ghost and matter correlators

We use the following convention for the ghost correlator on UHP,
\[
\langle c(\xi_1)c(\xi_2)c(\xi_3) \rangle_{UHP} = (\xi_1 - \xi_2)(\xi_1 - \xi_3)(\xi_2 - \xi_3).
\]  
(45)
From the conformal transformation from UHP to SIC, we obtain the ghost correlator on the wedge state surface $\mathcal{W}_\alpha$,
\[
\langle c(z_1)c(z_2)c(z_3) \rangle_{\mathcal{W}_\alpha, g} = \left( \frac{1 + \alpha}{\pi} \right)^3 \sin s_{12} \sin s_{13} \sin s_{23}
\]  
(46)
with definition $s_{ij} \equiv \frac{\pi(z_i - z_j)}{1 + \alpha}$, where $\langle \cdot \rangle_{W_{\alpha}, g}$ denotes the ghost correlator. Using the relations $\{B_0 + B_0^\star, c(z)\} = z$ and $\{B, c(z)\} = 1$, we obtain a very useful formula in the calculations of ghost correlators,

$$\langle B c(z_1) c(z_2) c(z_3) c(z_4) \rangle_{W_{\alpha}, g} = \frac{(1 + \alpha)^2}{\pi^3} \left[ -z_1 \sin s_{23} \sin s_{24} \sin s_{34} + z_2 \sin s_{13} \sin s_{14} \sin s_{34} - z_3 \sin s_{12} \sin s_{14} \sin s_{24} + z_4 \sin s_{12} \sin s_{13} \sin s_{23} \right], \quad (47)$$

where $B_0$ is the zero mode of the $b$ ghost in the $z$ coordinate, $B_0^\star$ is its BPZ conjugate, and

$$B = \int \frac{dz}{2\pi i} b(z) = \frac{\pi}{2} f \circ B_L^0. \quad (48)$$

When $B$ is located between two operators at $t_1$ and $t_2$ with $\frac{1}{2} < t_1 < t_2$, the contour of the integral can be taken to be $-V_\alpha^+ + \alpha$ with $2t_1 - 1 < \alpha < 2t_2 - 1$. Here the oriented straight lines $V_\alpha^\pm$ in SIC is defined as

$$V_\alpha^\pm = \left\{ z \mid \Re(z) = \pm \frac{1}{2}(1 + \alpha) \right\},$$

orientation : $\pm \frac{1}{2}(1 + \alpha) - i\infty \longrightarrow \pm \frac{1}{2}(1 + \alpha) + i\infty. \quad (49)$

On the other hand, for the matter correlators, we use the two point function

$$\langle X^\mu(\xi) X^\nu(\xi') \rangle_{UHP} = -2\eta^{\mu\nu} \ln |\xi - \xi'|.$$

Then the general $n$-point correlator on UHP is given by

$$\langle e^{ik_1 \cdot X(z_1)} e^{ik_2 \cdot X(z_2)} \ldots e^{ik_n \cdot X(z_n)} \rangle_{UHP} = (2\pi)^D \delta^D(k_1 + k_2 + \ldots + k_n) \prod_{1 \leq i < j}^{n} |\xi_i - \xi_j|^{2k_i k_j} \quad (50)$$

Using the conformal transformation from UHP to SIC, we obtain the matter correlator on SIC,

$$\langle e^{ik_1 \cdot X(z_1)} e^{ik_2 \cdot X(z_2)} \ldots e^{ik_n \cdot X(z_n)} \rangle_{W_{\alpha}, m} = (2\pi)^D \delta^D(k_1 + k_2 + \ldots + k_n) \prod_{1 \leq i < j}^{n} \left| \sin \left( \frac{\pi(z_i - z_j)}{1 + \alpha} \right) \right|^{2k_i k_j}, \quad (51)$$

where $\langle \cdot \rangle_{W_{\alpha}, m}$ denotes the matter correlator on the wedge state surface $W_{\alpha}$. 

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A.2 Calculation of $\beta_n^A$

Using the formulas (44), (46), (47), and (51) defined in this Appendix, we can calculate the tachyon coefficient $\beta_n$ in Eq. (33). Firstly we calculate $\beta_n^A$ which comes from the contribution of $Q_B$,

$$\beta_n^A = \langle \chi_n, A_n \rangle,$$  \hfill (52)

where

$$\chi_n = e^{nX^0(0)}c_0c_1|0\rangle,$$

$$A_n = \left[ \partial \left( e^{-X^0(0)}c(0) \right) |0\rangle + e^{-X^0(0)}c_1K_1^L|0\rangle \right] \ast J^{n-2} \ast e^{-X^0(0)}B_1^Lc_1|0\rangle.$$  \hfill (53)

From the formula (44) we obtain

$$\beta_n^A = \left( \frac{2}{\pi} \right)^{n^2+n-1} \left[ \frac{\partial}{\partial x} \left\{ \langle \partial c(0)c(0)c(x)Bc(n) \rangle \rangle_{W_n, g} \right. \right.$$

$$\times \left( e^{nX^0(0)}e^{-X^0(2)}e^{-X^0(3)} \cdots e^{-X^0(n)} \right)_{W_n, m} \right]_{x=1}$$

$$+ \frac{\partial}{\partial m} \left\{ \langle \partial c(0)c(0)c(1)Bc(m+n-2) \rangle \rangle_{W_{m+n-2}, g} \right. \right.$$

$$\times \left( e^{nX^0(1)}e^{-X^0(m)}e^{-X^0(m+1)} \cdots e^{-X^0(m+n-2)} \right)_{W_{m+n-2}, m} \right\}_{m=2},$$  \hfill (54)

where we used the Eq. (48) and the facts,

$$f \circ c_0|0\rangle = \partial c(0)|0\rangle, \quad f \circ c_1|0\rangle = \left( \frac{\pi}{2} \right) c(0)|0\rangle, \quad K_1^L|0\rangle = \left( \frac{2}{\pi} \right) \frac{\partial}{\partial m} |m\rangle |m=2\rangle.$$

Using the translation symmetry on SIC, we can rewrite the ghost correlators in Eq. (54) as

$$\langle \partial c(0)c(0)c(x)Bc(n) \rangle \rangle_{W_n, g} = \langle Bc(-1)\partial c(0)c(0)c(x) \rangle \rangle_{W_n, g},$$

$$\langle \partial c(0)c(0)c(1)Bc(m+n-2) \rangle \rangle_{W_{m+n-2}, g} = \langle Bc(-1)\partial c(0)c(0)c(1) \rangle \rangle_{W_{m+n-2}, g}. $$  \hfill (55)

Then applying formulas (47), (51), and (55) to the Eq. (54), we obtain the explicit expression of $\beta_n^A$ in Eq. (33).

A.3 Calculation of $\beta_n^B$

In the calculation of $\beta_n^B$ we use the tachyon vacuum solution $\Psi$ given in Eq. (3). Then $B_n$ in Eq. (33) is rewritten as

$$B_n = \Psi \ast \phi^n = \Psi \ast J^{n-1} \ast e^{-X^0(0)}B_1^Lc_1|0\rangle$$

$$= \sum_{m=0}^{\infty} \frac{\partial}{\partial m} \left[ \frac{2}{\pi} c_1|0\rangle \ast |m+1\rangle \ast J^{n-1} \ast e^{-X^0(0)}B_1^Lc_1|0\rangle \right]$$

$$- \lim_{N \to \infty} \left[ \frac{2}{\pi} c_1|0\rangle \ast |N+1\rangle \ast J^{n-1} \ast e^{-X^0(0)}B_1^Lc_1|0\rangle \right]. $$  \hfill (56)
The tachyon vacuum solution is composed of the ordinary piece \( \sum_{n=0}^{\infty} \psi_n' \) and the phantom piece \( -\psi_{\infty} \). However, the contribution of the phantom piece which corresponds to the last term in Eq. (56) to \( \beta^n_B \) vanishes since

\[
\langle \chi_n, \psi_N * \phi^n \rangle \sim O \left( \frac{1}{N^3} \right)
\]

for large \( N \). By neglecting the phantom piece in the calculation of \( \beta^n_B \), we obtain

\[
\beta^n_B = \left( \frac{2}{\pi} \right)^{n^2+n-1} \sum_{m=0}^{\infty} \frac{\partial}{\partial m} \left[ \langle \partial c(0)c(1)Bc(m+n+1) \rangle_{w_{m+n+1},g} \right. \\
\left. \times \left( e^{nX^0(0)}e^{-X^0(1)}e^{-X^0(m+1)}e^{-X^0(m+2)}e^{-X^0(m+3)} \right)_{w_{m+n+1},g} \right] .
\]

(57)

Similarly to the case of \( \beta^n_A \), by using the translation symmetry on SIC and the ghost and matter correlators (47) and (51) we can obtain the expression \( \beta^n_B \) in Eq. (37).

A.4 Calculation of \( \beta^n_C \)

Similarly to the case of \( \beta^n_B \) in the previous subsection, by neglecting the phantom piece in the calculation of \( \beta^n_C \) we obtain

\[
\beta^n_C = - \left( \frac{2}{\pi} \right)^{n^2+n-1} \sum_{m=0}^{\infty} \frac{\partial}{\partial m} \left[ \langle \partial c(0)c(1)Bc(m+n+1) \rangle_{w_{m+n+1},g} \right. \\
\left. \times \left( e^{nX^0(0)}e^{-X^0(1)}e^{-X^0(m+1)}e^{-X^0(m+2)}e^{-X^0(m+3)} \right)_{w_{m+n+1},g} \right] .
\]

(58)

In the calculation of the ghost correlator in Eq. (58), we use the relations \{\( B \), \( c(z) \)\} = 1, \( B^2 = 0 \), and the translation symmetry on SIC and obtain

\[
\langle \partial c(0)c(1)Bc(m+n+1) \rangle_{w_{m+n+1},g} = \langle Bc(-1)\partial c(0)c(1) \rangle_{w_{m+n+1},g} - \langle Bc(-1)\partial c(0)c(1) \rangle_{w_{m+n+1},g} .
\]

(59)

Using the relations (47), (51), and (59), we obtain the explicit expression of \( \beta^n_C \) in Eq. (37).

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