Using Reconfigurable Intelligent Surfaces for UE Positioning in mmWave MIMO Systems

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Abstract

A reconfigurable intelligent surface (RIS) consists of massive meta elements, which results in a reflection path between a base station (BS) and user equipment (UE). In wireless localization, this reflection path aids in positioning accuracy, especially when the line-of-sight (LOS) path is subject to severe blockage and fading. We develop a RIS-aided positioning framework to locate a UE in environments where the LOS path may or may not be available. We first estimate the RIS-aided channel parameters from the received signals at the UE. To reduce algorithmic complexity, we propose a linear combination of the estimated UE positions from the direct and reflection paths, which is shown to be approximately the maximum likelihood estimator under the large-sample regime when the estimates from different paths are independent. We optimize the RIS phase shifts to improve the positioning accuracy, and extend the proposed approach to the case with multiple BSs and UEs. We derive the Cramér–Rao bound (CRB) and demonstrate numerically that our proposed method approaches the CRB.

I. INTRODUCTION

The reconfigurable intelligent surface (RIS) has been proposed as an aid to wireless communication systems. A RIS consists of many low-cost meta elements [1], [2], through which the performance of existing wireless communication systems can be improved without significant additional hardware cost. Different from a relay, a RIS passively reflects the received signal, changing only its phase shift before transmission to a user equipment (UE) [3]–[5]. With the help of a RIS, a reflected transmission path...
can be established if the direct transmission path is blocked, which makes the RIS potentially useful for urban or indoor communications [6]. Compared to traditional wireless communication with transmit beamforming, the phase shifts of a RIS can be configured to achieve passive beamforming [7]–[9] for RIS-aided systems. With properly designed passive beamforming, many works in the literature have shown that the RIS can improve various system performance metrics, such as spectral efficiency [7], [9], received signal to noise ratio (SNR) [10], [11] and bit error rate [12]. These works suggest that RIS can play an important role in future wireless communication systems.

In this paper, we investigate the use of RIS in user equipment (UE) positioning. Localization or positioning is an important task in wireless communications [13], [14]. In 5G systems, positioning of the UE has diverse applications, including industrial use cases, smart mobility, and location-based services. The use of 5G millimeter wave (mmWave) has the potential to provide better positioning accuracy compared to the Global Positioning System (GPS) [15]. As such, the 3GPP Release 16 [16] has incorporated standards for location management in the 5G NR framework.

In [17], a positioning method based on received signal strength (RSS) of mmWaves was presented. By using sufficient measurements and multiple access points (APs), the position of a UE can be estimated by the trilateration technique, which utilizes the ranges estimated through measuring the received signal power. In [18], by using one AP, a hybrid RSS and angle of arrival (AoA) positioning scheme is developed to provide estimates of both the distance and the orientation of the target. In [19], the authors train a noise-free RSS model and then use it to position UEs with noisy RSS. However, for the RSS-based methods, the positioning accuracy is determined by the signal strength model, and the signal strength is often corrupted by small-scale fading, which is challenging to estimate, especially when the measurement time of the signal is insufficient.

Apart from the RSS-based methods, some works [20]–[22] obtain the position of a UE in mmWave systems by estimating the time of arrival (ToA) and AoA or angle of departure (AoD). Therefore, the positioning task can be treated as a channel estimation problem. Because of the limited scattering of paths in mmWave communication, compressed sensing (CS) methods can be employed to reduce the measurement time and computational complexity [23], [24]. Specifically, in [20], the maximum likelihood estimate (MLE) for time of delay and AoD is discussed under the line-of-sight (LOS) scenario with the base station (BS) being equipped with massive antennas and the UE having one antenna. In [25], the authors estimate the position and rotation angle of the UE using a single BS, and the proposed method can
be applied in a non-LOS (NLOS) environment. In [21], the received signal measurements are structured as a tensor, based on which the channel parameters such as ToAs, AoAs and AoDs are extracted. In [22], a tensor-based channel estimation method for positioning and mapping was proposed for diffuse multipaths.

Since a RIS creates a reflection path between a BS and UE, the UE can utilize the measurements from this reflection path as additional information for positioning. Some works have shown that the positioning accuracy improves with the size of the RIS. The Cramér-Rao lower bound (CRB) of the positioning accuracy is analyzed in [26]–[30]. However, few existing literature have developed practical positioning algorithms for a RIS-aided system. Indoor positioning using the RSS is investigated by [31], [32], which estimates the position of a UE using the probability distribution of the RSS. In [33], the authors consider channel estimation and geometric mapping for positioning under the twin-RIS scenario.

In this paper, we develop a novel positioning and inference framework for RIS-aided systems using channel estimation techniques. Our approach is not limited to using the RSS measurements. Different from the existing works [31], [32], we formulate our problem under the general case where there may be more than one RIS. In contrast with existing RIS-aided channel estimation methods [34]–[36] that estimate the cascaded channel by assuming that the direct channel is estimated in advance, we estimate the channel parameters such as ToAs, AoAs and AoDs of the direct and reflection paths jointly. In addition, different from the geometric mapping in [33], our proposed inference model considers the estimation accuracy of the channel parameters, which yields a UE position estimation error close to the theoretical CRB.

The main contributions of this paper are summarized as follows:

• We consider the down-link MIMO-OFDM setup in this work. Direct estimation of the UE position from the received signals is computationally expensive as it involves a nonlinear and non-convex optimization. Therefore, we propose a two-step positioning framework. In the first step, we estimate the channel parameters of the direct and reflection paths. In the second step, we obtain an estimate of the UE position from the channel parameters of each path. We derive the CRB of the UE position estimate under our positioning framework.

• To infer the UE position from the different estimates corresponding to the direct and reflection paths, we perform a linear combination of these estimates. The linear combination weights depend only on the covariance of the UE position estimates. We show when the estimates from different paths
are independent, the proposed linear combination is approximately the MLE of the UE position in the large-sample regime.

- To optimize the positioning framework, we propose an approach for designing the RIS phase shifts. Specifically, the phase shift design problem is to maximize the expectation of the reflection path gain, which can be then solved using singular value decomposition.

One challenge is to distinguish the direct and reflection paths. In this work, different from the existing works where the path with the smallest delay is assigned as the direct path, we distinguish the direct and the reflection paths by ranking a path quantity related to its power level. This method is more robust if the SNR is low. Our proposed RIS-aided positioning framework is also readily extended to the multi-UE and multi-BS scenarios.

The rest of this paper is organized as follows. In Section II, the signal and channel model, and our system assumptions are introduced. In Section III, we derive the CRB of the UE positioning error under the signal and channel model. The proposed RIS-aided channel parameter estimation approach is discussed in Section IV. In Section V, we propose the fusion method to infer the UE position from the estimated channel parameters. In Section VI, we propose the method to optimize the RIS phase shifts and discuss the extension of our positioning framework to the multi-UE and multi-BS scenarios. We present numerical results in Section VII. Finally, we conclude in Section VIII.

**Notations:** A bold lower case letter $a$ is a vector and a bold capital letter $A$ represents a matrix. $A^\top$, $A^H$, $A^{-1}$, $\text{tr}(A)$, $|A|$, $\|A\|_F$ and $\|a\|_2$ are, respectively, the transpose, Hermitian, inverse, trace, determinant, Frobenius norm of $A$, and the 2-norm of $a$. $[A]_{i,:}$, $[A]_{:,i}$, $[A]_{i,j}$, and $[a]_i$ are, respectively, the $i$th column, $i$th row, $i$th row and $j$th column entry of $A$, and the $i$th entry of vector $a$. The operation $\text{vec}(A)$ stacks the columns of $A$ to form a column vector. $\text{Col}(A)$ is the column space of matrix $A$. We use $\text{diag}(a)$ to represent a diagonal matrix with the vector $a$ on the main diagonal. The circular symmetric complex Gaussian distribution with mean $\mu$ and variance $\sigma^2$ is given by $\mathcal{CN}(\mu, \sigma^2)$. We use $\otimes$ to denote the Kronecker product.

**II. SYSTEM MODEL**

In this section, we present our system model and assumptions. We first discuss the channel model, which includes the BS-RIS links, RIS-UE links and BS-UE link. We then present the received signal model at the UE.

December 2, 2021 DRAFT
Fig. 1. Positioning of a UE with the aid of a RIS.

A. Channel Model

We assume that the BS has a uniform rectangular array (URA) with \( N \) antennas. There are \( Q \) RISs and each is equipped with a URA of \( M \) elements. The UE has \( D \) antennas. In this work, we assume that the position of every RIS is known by the BS and UE. Without loss of generality, we adopt a coordinate system with the BS at its origin and the URA of the BS in \( y-z \) plane (see Fig. 1 for an illustration). Each RIS’ URA is assumed to be contained in a \( x-z \) plane perpendicular to the \( y-z \) plane of the BS URA.

We also assume that the UE’s antennas are contained in a horizontal plane parallel to the BS URA, but with a possibly different orientation. Let \( \overline{M}_R \in \mathbb{R}^{3 \times 3} \) be the rotation matrix associated with the UE, given by

\[
\overline{M}_R = -R_3(\alpha_3)R_2(\alpha_2)R_1(\alpha_1)
\]

\[
= - \begin{bmatrix}
\cos \alpha_3 & \sin \alpha_3 & 0 \\
-\sin \alpha_3 & \cos \alpha_3 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \alpha_2 & \sin \alpha_2 \\
0 & -\sin \alpha_2 & \cos \alpha_2
\end{bmatrix}
\begin{bmatrix}
\cos \alpha_1 & \sin \alpha_1 & 0 \\
-\sin \alpha_1 & \cos \alpha_1 & 0 \\
0 & 0 & 1
\end{bmatrix},
\]

where \( \alpha_1, \alpha_2, \alpha_3 \) are the Euler angles with respect to (w.r.t.) the UE. For convenience, we define \( M_R = [\overline{M}_R]_{2:3, \cdot} \). In this work, we assume that \( \overline{M}_R \) is known a priori by the UE.

We suppose that the communication system uses OFDM with \( K \) subcarriers. For the \( k \)th subcarrier, the channel from the BS to the \( q \)th RIS is denoted as \( G_{k,q} \in \mathbb{C}^{M \times N} \), the channel from the UE to the \( q \)th RIS is \( H_{r,k,q} \in \mathbb{C}^{D \times M} \), and the channel from the BS to the UE is \( H_{d,k} \in \mathbb{C}^{D \times M} \).
1) BS-RIS links: In this work, we model the BS-RIS channel as a mmWave channel. We assume that each RIS is placed at a sufficient height (e.g., on a tall building) so that there is a LOS path between the BS and the RIS.

From the OFDM assumption, the $k$th subcarrier of the $q$th BS-RIS channel is [37], [38]

$$
G_{k,q} = h_{R,i,q} e^{-i2\pi f \frac{4}{N} \tau_{i,q}} a_R(f_{R,i,q}, v_{R,i,q}) a_B^H(g_{B,i,q}, v_{B,i,q}),
$$

where $i = \sqrt{-1}$, $h_{R,i,q} = \alpha_{R,i,q} \sqrt{\beta_{R,i,q} MN}$ with $\beta_{R,i,q}$ being the large scale path gain and $\alpha_{R,i,q}$ being a complex-valued channel coefficient. $W$ is the transmission bandwidth, and $\tau_{i,q}$ is the propagation delay of the signal from the BS to the $q$th RIS. In particular, $a_R(f_{R,i,q}, v_{R,i,q}) \in \mathbb{C}^{N \times 1}$ and $a_B(g_{B,i,q}, v_{B,i,q}) \in \mathbb{C}^{N \times 1}$ are, respectively, the URA response vectors of the RIS and BS, where

$$
f_{R,i,q} = \sin \theta_{R,i,q} \cos \phi_{R,i,q}, \quad v_{R,i,q} = \cos \theta_{R,i,q},
$$

$$
g_{B,i,q} = \sin \theta_{B,i,q} \sin \phi_{B,i,q}, \quad v_{B,i,q} = \cos \theta_{B,i,q},
$$

with the $\theta_{R,i,q}$ (or $\theta_{B,i,q}$) and $\phi_{R,i,q}$ (or $\phi_{B,i,q}$) being the elevation and azimuth AoAs (or AoDs) associated the BS-RIS link, respectively. To be more precise, the URA response vectors $a_R(f, g)$ and $a_B(f, g)$ in (2) are given by

$$
a_R(f, v) = a_R(f) \otimes a_R(v),
$$

$$
a_B(g, v) = a_B(g) \otimes a_B(v),
$$

where $a_R(f) = \frac{1}{M^{1/2}} [1, \exp(i\pi f), \ldots, \exp(i\pi f(M^{1/2} - 1))]$ and $a_B(g) = \frac{1}{N^{1/2}} [1, \exp(i\pi g), \ldots, \exp(i\pi g(N^{1/2} - 1))]$.

2) RIS-UE link: For the channel between the $q$th RIS and the UE, we again assume that a LOS path exists between the RIS and the UE. The $k$th subcarrier channel of the RIS-UE link is given by

$$
H_{r,k,q} = h_{R,i,q} e^{-i2\pi f \frac{4}{N} \tau_{i,q}} a_U(g_{U,i,q}, v_{U,i,q}) a_R^H(f_{R,i,q}, v_{R,i,q}),
$$

where $h_{R,i,q} = \alpha_{R,i,q} \sqrt{\beta_{R,i,q} MD}$ with $\beta_{R,i,q}$ being the large scale path gain and $\alpha_{R,i,q}$ being complex-valued channel coefficient, and $\tau_{i,q}$ is the delay. The URA response vector $a_R(f_{R,i,q}, v_{R,i,q})$ is given in (5) and $a_U(g_{U,i,q}, v_{U,i,q}) \in \mathbb{C}^{D \times 1}$ is the URA response vector of the UE, where

$$
f_{R,i,q} = \sin \theta_{R,i,q} \sin \phi_{R,i,q}, \quad v_{R,i,q} = \cos \theta_{R,i,q},
$$
\[
\begin{bmatrix}
g_{U_r,q} \\
v_{U_r,q}
\end{bmatrix} = \mathbf{M}_R \begin{bmatrix}
\sin \theta_{R_z,q} \cos \phi_{R_z,q} \\
\sin \theta_{R_z,q} \sin \phi_{R_z,q} \\
\cos \theta_{R_z,q}
\end{bmatrix},
\]  
(9)

with \(\theta_{R_z,q}\) and \(\phi_{R_z,q}\) being the elevation and azimuth AoDs associated with the RIS-UE link. Abusing terminology, we refer to \((f_{R_z,q}, v_{R_z,q})\) as the AoD of the \(q\)th RIS, and \((g_{U_r,q}, v_{U_r,q})\) as the AoA of the UE on the reflection path.

The URA response vector of the UE is

\[
a_U(g, v) = a_{\tilde{U}}(g) \otimes a_{\tilde{U}}(v),
\]  
(10)

where \(a_{\tilde{U}}(g) = \frac{1}{D_1} \left[ 1, \exp(i\pi g), \ldots, \exp(i\pi g(D_1^2 - 1)) \right] \in \mathbb{C}^{D_1^2 \times 1}\). For example, the response due to the \(k\)th subcarrier on the RIS-UE link is given by \(a_U(g_{U_r,q}, v_{U_r,q})\).

3) BS-UE link: We model the BS-UE link channel using the Rician fading model, given by

\[
H_{d,k} = \mathbf{\bar{H}}_{d,k} + \sqrt{1/(1 + K_d)} \mathbf{Z}_{d,k},
\]  
(11)

where \(K_d\) is the Rician factor, \(\mathbf{\bar{H}}_{d,k}\) is the deterministic component or the LOS path, and \(\mathbf{Z}_{d,k}\) denotes the small-scale fading whose entries are independent and identically distributed (i.i.d.) according to \(\mathcal{CN}(0, \beta_d)\) with \(\beta_d\) being the large scale path gain. The expression of \(\mathbf{\bar{H}}_{d,k}\) is given by

\[
\mathbf{\bar{H}}_{d,k} = h_d e^{-i2\pi \frac{w}{K_d} r_d} a_U(g_{U_d}, v_{U_d}) a_B(g_{B_d}, v_{B_d}),
\]  
(12)

where we let \(h_d = \sqrt{\frac{K_d}{1 + K_d}} \beta_d N D \alpha_d\) with \(\alpha_d\) being complex-valued channel coefficient, and the URA response vectors of the UE and the BS, \(a_U(g_{U_d}, v_{U_d})\) and \(a_B(g_{B_d}, v_{B_d})\) are defined in (10) and (6), respectively. We have

\[
g_{B_d} = \sin \theta_{B_d} \sin \phi_{B_d}, \quad v_{B_d} = \cos \theta_{B_d},
\]  
(13)

\[
\begin{bmatrix}
g_{U_d} \\
v_{U_d}
\end{bmatrix} = \mathbf{M}_R \begin{bmatrix}
\sin \theta_{B_d} \cos \phi_{B_d} \\
\sin \theta_{B_d} \sin \phi_{B_d} \\
\cos \theta_{B_d}
\end{bmatrix},
\]  
(14)

where \(\theta_{B_d}\) and \(\phi_{B_d}\) are the elevation and azimuth AoDs associated the BS-UE link. Abusing terminology, we refer to \((g_{B_d}, v_{B_d})\) as the AoD of the BS, and \((g_{U_d}, v_{U_d})\) as the AoA of the UE on the LOS path.
In summary, using the channel models of the BS-RIS link in (2), the RIS-UE link in (7), and the BS-UE link in (11), the effective channel between the BS and UE on the $k$th subcarrier can be written as

$$H_k = H_{d,k} + \sum_{q=1}^{Q} H_{r,k,q} \Theta_q G_{k,q} + \tilde{Z}_{d,k}$$

(15)

where $\Theta_q = \text{diag}(\theta_q)$ with $\theta_q = [e^{i\theta_1^{(q)}}, \ldots, e^{i\theta_M^{(q)}}]$ denoting the phase shift of the $q$th RIS, and $h_{r,q} = h_{R_1,q} h_{R_2,q} a_B^H(f_{R_2,q}, v_{R_2,q}) \Theta_q a_R(f_{R_1,q}, v_{R_1,q})$. For convenience, we denote $H_k = H_{d,k} + \sum_{q=1}^{Q} H_{r,k,q} \Theta_q G_{k,q}$ in (15).

Here, we define the channel parameters as

$$\eta = [\eta_d^\top, \eta_{r,1}^\top, \ldots, \eta_{r,Q}^\top]^\top \in \mathbb{R}^{(7+5Q)},$$

(16)

where $\eta_d = [\text{Re}\{h_d\}, \text{Im}\{h_d\}, \tau_d, g_{U_d}, v_{U_d}, g_{B_d}, v_{B_d}]^\top$, $\eta_{r,q} = [\text{Re}\{h_{r,q}\}, \text{Im}\{h_{r,q}\}, \tau_{r,q}, g_{U_{r,q}}, v_{U_{r,q}}, g_{B_{r,q}}, v_{B_{r,q}}]^\top$.

We denote the position of the UE as $p_U = [x_{U}, y_{U}, z_{U}]^\top$, and the position of the $q$th RIS as $p_{R,q} = [x_{R,q}, y_{R,q}, z_{R,q}]^\top$. To relate the channel parameters $\eta$ to the UE position, let

$$\xi = [p_U^\top, \text{Re}\{h_d\}, \text{Im}\{h_d\}, \text{Re}\{h_{r,1}\}, \text{Im}\{h_{r,1}\}, \ldots, \text{Re}\{h_{r,Q}\}, \text{Im}\{h_{r,Q}\}]^\top.$$ 

(17)

Then, we can define a function $F(\xi) = \eta$ from the relations of (3), (4), (8), (9), (13), (14), and the following equalities:

$$\tau_d = \|p_U\|_2/c, \quad \tau_{r,q} = \|p_U - p_{R,q}\|_2/c$$

$$\theta_{B,d} = \arccos\frac{z_{U}}{\|p_U\|_2}, \quad \phi_{B,d} = \arctan 2(y_{U}, x_{U})$$

$$\theta_{R_1,q} = \arccos\frac{z_{U} - z_{R,q}}{\|p_U - p_{R,q}\|_2},$$

$$\phi_{R_2,q} = \arctan 2(y_{U} - y_{R,q}, x_{U} - x_{R,q}).$$

(18)
B. Received Signal at the UE

Suppose that the UE receives signals over $T$ time slots. From the channel model (15), the received signal at the UE at each time $t = 1, \ldots, T$ on the $k$th subcarrier is given by

$$r_k(t) = H_k x(t) + n_k(t), \quad (19)$$

where $x_k(t) \in \mathbb{C}^{N \times 1}$ is the transmitted signal from the BS at time $t$, and $n_k(t) \in \mathbb{C}^{D \times 1}$ is a noise vector with entries i.i.d. according to the complex Gaussian distribution $\mathcal{CN}(0, \sigma^2)$ and independent across time. Let $R_k = [r_k(1), \ldots, r_k(T)] \in \mathbb{C}^{D \times T}$, $X = [x(1), \ldots, x(T)] \in \mathbb{C}^{D \times T}$, and $N_k = [n_k(1), \ldots, n_k(T)] \in \mathbb{C}^{D \times T}$. We assume that the transmitted signals are orthogonal, i.e., $XX^H = T/DI$, where $I$ is the identity matrix. Moreover, the transmit power is assumed to be unit, i.e., $\|x(t)\|_2^2 = 1$, for $t = 1, \ldots, T$. The compact form of the received signal in (19) is given by

$$R_k = H_k X + N_k. \quad (20)$$

Right multiplying (20) by $D/TX^H$, we have

$$(D/T)R_kX^H = H_k + (D/T)N_kX^H. \quad (21)$$

The entries in $(D/T)N_kX^H$ are i.i.d. Gaussian $\mathcal{CN}(0, \sigma^2D/T)$ random variables. Here, we define $\tilde{R}_k = (D/T)R_kX^H$ and recalling the definition of $H_k$ in (15), we obtain

$$\tilde{R}_k = \tilde{H}_k + \tilde{Z}_{d,k} + (D/T)N_kX^H = \tilde{H}_k + \tilde{N}_k, \quad (22)$$

where we denote $\tilde{N}_k = \tilde{Z}_{d,k} + (D/T)N_kX^H$, and its entries follows $\mathcal{CN}(0, \tilde{\sigma}^2)$ with $\tilde{\sigma}^2 = \frac{D}{T}\sigma^2 + \frac{\beta_d}{1 + K_d}$. Our objective is to infer the position of the UE by using the observations $\{\tilde{R}_k\}_{k=1}^K$ in (22). Because directly estimating the UE position from (22) is challenging, we first estimate the channel parameters, from which the UE position is then inferred.

III. CRB FOR UE POSITION ESTIMATION

In this section, we derive the CRB for the UE position estimation based on the observations in (22). We will compare the performance of the proposed method against this bound in the numerical results in Section VII.
A. FIM of the channel parameters $\eta$

Recall that our observations are $\tilde{R}_k = \tilde{H}_k + \tilde{N}_k$ in (22). We perform two steps to obtain the Fisher information matrix (FIM). In the first step, we compute the FIM w.r.t. $\eta$ of (16). For any unbiased estimator $\hat{\eta}$, we have

$$\mathbb{E}[(\hat{\eta} - \eta)(\hat{\eta} - \eta)^T] \succeq \left( \sum_{k=0}^{K-1} F_{\eta}^{(k)} \right)^{-1},$$

(23)

where $F_{\eta}^{(k)} \in \mathbb{R}^{(7+5Q) \times (7+5Q)}$ is the FIM of $\eta$ based on the observations from the $k$th subcarrier. Accordingly, the FIM of $\eta$ based on the observations from all the $K$ subcarriers is

$$F_{\eta} = \sum_{k=0}^{K-1} F_{\eta}^{(k)}. $$

(24)

Because the noise in (22) is Gaussian, we have the following

$$\ln f(\tilde{R}_k | \eta) = -\frac{1}{\hat{\sigma}^2} \text{tr}((\tilde{R}_k - \tilde{H}_k)(\tilde{R}_k - \tilde{H}_k)^H) + C,$$

(25)

where $C$ is a normalization constant. The $(i, j)$-th element of $F_{\eta}^{(k)}$ is then given by

$$[I_{\eta}^{(k)}]_{i,j} = -\mathbb{E} \left[ \frac{\partial^2 \ln f(\tilde{R}_k | \eta)}{\partial \eta_i \partial \eta_j} \right].$$

After simplifications, we have

$$[I_{\eta}^{(k)}]_{i,j} = \frac{2}{\hat{\sigma}^2} \left\{ \text{tr} \left( \frac{\partial \tilde{H}_k^H}{\partial \eta_i} \frac{\partial \tilde{H}_k}{\partial \eta_j} \right) \right\}. $$

(26)

Appendix A provides detailed derivation for terms in the FIM.

B. FIM for the UE position parameters $\xi$

To derive the FIM for the UE position parameters $\xi$, we use the relation $F(\cdot)$ in (18). The Jacobian matrix $J \in \mathbb{R}^{(7+5Q) \times (5+2Q)}$ of $F$ is given in Appendix B. The FIM for $\xi$ is then given by

$$F_{\xi} = J^T \sum_{k=1}^{K} F_{\eta}^{(k)} J \in \mathbb{R}^{(5+2Q) \times (5+2Q)}. $$

(27)
Accordingly, a lower bound for the MSE of the UE position is as follows:

$$\text{MSE}(p_U) \geq \text{tr} \left( |F^{-1}_\xi|_{1:3,1:3} \right).$$

(28)

**Proposition 1.** Let $$\xi_d = [p_U^T, \text{Re}\{h_d\}, \text{Im}\{h_d\}]$$ and $$\xi_{r,q} = [p_U^T, \text{Re}\{h_{r,q}\}, \text{Im}\{h_{r,q}\}]$$. When we only utilize parameters associated with the direct path for the UE positioning task, the error covariance matrix

$$C_{p_U}^{(d)} = \mathbb{E}[(\hat{p}_U^{(d)} - p_U)(\hat{p}_U^{(d)} - p_U)^T]$$

satisfies the following bound,

$$C_{p_U}^{(d)} \succeq \left[ \left( J_d^T \bar{C}_{\eta_d} J_d \right)^{-1} \right]_{1:3,1:3},$$

(30)

where $$\bar{C}_{\xi_d} = (J_d^T \bar{C}_{\eta_d}^{-1} J_d)^{-1}$$ with $$\bar{C}_{\eta_d} = [F^{-1}]_{1:7,1:7}, J_d = \frac{\partial \eta_d}{\partial \xi_d} \in \mathbb{R}^{7 \times 5}$$, and $$F_{\eta_d} \in \mathbb{C}^{7 \times 7}$$ is the FIM of $$\eta_d$$.

When we only utilize parameters associated with the qth RIS path for the UE positioning task, the error covariance matrix

$$C_{p_U}^{(r,q)} = \mathbb{E}[(\hat{p}_U^{(r,q)} - p_U)(\hat{p}_U^{(r,q)} - p_U)^T]$$

satisfies the following bound,

$$C_{p_U}^{(r,q)} \succeq \left[ \left( J_{r,q}^T \bar{C}_{\eta_{r,q}} J_{r,q} \right)^{-1} \right]_{1:3,1:3},$$

(32)

where $$\bar{C}_{\xi_{r,q}} = (J_{r,q}^T \bar{C}_{\eta_{r,q}}^{-1} J_{r,q})^{-1}$$ with $$\bar{C}_{\eta_{r,q}} = [F^{-1}]_{5q+3:7+5q,5q+3:7+5q}, J_{r,q} = \frac{\partial \eta_{r,q}}{\partial \xi_{r,q}} \in \mathbb{R}^{5 \times 5}$$, and $$F_{\eta_{r,q}} \in \mathbb{C}^{5 \times 5}$$ is the FIM of $$\eta_{r,q}$$.

**Proof.** See Appendix C.

\[ \Box \]

**IV. Estimation of Channel Parameters**

In this section, we formulate optimization problems to estimate the AoDs from the BS $$(g_{B_d}, v_{B_d})$$, and propagation delays $$\tau_d$$ and $$\{\tau_{r_{2,q}}\}_{q=1}^Q$$ along the LOS path from the BS to the UE and the reflection paths from each RIS to the UE, respectively. We also estimate the AoAs at the UE $$(g_{U_d}, v_{U_d})$$ and $$(g_{U_{r,q}}, v_{U_{r,q}})$$ along the LOS path and reflection paths, respectively.
Because the noise \( \tilde{N} \) in (22) is Gaussian, the MLE of \( \eta \) of (16) is given by the following:

\[
\min_{\eta} \sum_{k=0}^{K-1} \| \tilde{R}_k - H_k \|_F^2. \tag{33}
\]

However, directly solving the above problem is challenging because it is nonlinear and nonconvex in \( \eta \). However, we note that the rank of \( H_k \) in (22) is \( Q + 1 \). We can leverage this low-rank property to estimate the channel parameters.

### A. Estimation of AoD \((g_{B_d}, v_{B_d})\)

The AoD \((g_{B_d}, v_{B_d})\) is for the BS-UE link given in (13). We discuss the estimation of \( g_{B_d} \). The estimation of \( v_{B_d} \) is done similarly. We reshape \( \{ \tilde{R}_k \}_{k=0}^{K-1} \) over the dimensions of \( a_{\tilde{B}}(g_{B_d}) \) and \( \{ a_{\tilde{B}}(g_{B_r,q}) \}_{q=1}^{Q} \) as

\[
R_B = [a_{\tilde{B}}(g_{B_d}), a_{\tilde{B}}(g_{B_r,1}), \ldots, a_{\tilde{B}}(g_{B_r,Q})] Q_B + N_B \in \mathbb{C}^{\sqrt{N} \times \sqrt{NDK}}. \tag{34}
\]

Since \( g_{B_r,q} \) for all \( q \) in (4) is known a priori as we assume that the position of the \( q \)th RIS is known, we only need to estimate \( g_{B,d} \) from (34) by solving

\[
\min_{f_{g_{B_d}}} \| R_B - A_B Q_B \|_F^2
\]
subject to \( Q_B = (A_B^H A_B)^{-1} A_B^H R_B. \) \tag{35}

where \( A_B = [A_{B_r}, a_{\tilde{B}}(g_{B_d})] \) with \( A_{B_r} = [a_{\tilde{B}}(g_{B_r,1}), \ldots, a_{\tilde{B}}(g_{B_r,Q})] \). We assume \( g_{B_r,q} \) is distinct for each \( q \), which can be achieved by carefully deploying the RISs. Thus \( A_B^H A_B \) is invertible and we have the following result.

**Lemma 1.** The problem in (35) is equivalent to

\[
\max_{g_{B_d}} \left\| \tilde{a}_{\tilde{B}}^H(g_{B_d}) R_B \right\|_2^2, \tag{36}
\]

where \( \tilde{a}_{\tilde{B}}(g_{B_d}) = \frac{a_{\tilde{B}}(g_{B_d}) - P_r a_{\tilde{B}}(g_{B_d})}{\| a_{\tilde{B}}(g_{B_d}) - P_r a_{\tilde{B}}(g_{B_d}) \|_2} \) and \( P_r = A_{B_r} (A_{B_r}^H A_{B_r})^{-1} A_{B_r}^H \).

**Proof.** Note that \( P_r a_{\tilde{B}}(g_{B_d}) \) is the orthogonal projection onto the column space of \( A_{B_r} \), and \( \tilde{a}_{\tilde{B}}(g_{B_d}) \) is the residual vector of projection with normalization. Therefore, \([A_{B_r}, a_{\tilde{B}}(g_{B_d})]\) spans the same subspace as \([A_{B_r}, \tilde{a}_{\tilde{B}}(g_{B_d})]\). For convenience, we define \( \tilde{A}_{B_r} \) as the Gram–Schmidt orthogonalization of columns
in $A_B$. We have

$$\text{Col}(\{A_B, a_B(g_{B_d})\}) = \text{Col}(\{\tilde{A}_B, \tilde{a}_B(g_{B_d})\})$$

$$= \text{Col}(\{\tilde{A}_B, \tilde{a}_B(g_{B_d})\}),$$

By defining $\tilde{A}_B = [\tilde{A}_B, \tilde{a}_B(g_{B_d})]$, one can check that $\tilde{A}_B^H \tilde{A}_B = I$. Therefore, from the equivalence in subspaces, the residual of $R_B$ w.r.t. $\text{Col}(A)$ is same as $\text{Col}(\tilde{A})$. The objective function in (35) is then given by

$$\|R_B - \tilde{A}_B \tilde{Q}_B\|_F^2$$

$$= \text{tr}(R_B^H R_B - R_B^H \tilde{A}_B \tilde{Q}_B - \tilde{Q}_B^H \tilde{A}_B^H R_B + \tilde{Q}_B^H \tilde{A}_B^H \tilde{A}_B \tilde{Q}_B)$$

$$= \text{tr}(R_B^H R_B - R_B^H \tilde{A}_B \tilde{Q}_B)$$

$$= \text{tr}(R_B^H R_B - R_B^H \tilde{A}_B \tilde{A}_B^H R_B),$$

(37)

where $\tilde{Q}_B = (\tilde{A}_B^H \tilde{A}_B)^{-1} \tilde{A}_B^H R_B$, and the last inequality comes from $\tilde{A}_B^H \tilde{A}_B = I$. Therefore, we have

$$\arg \min_{g_{B_d}} \text{tr}(R_B^H R_B - R_B^H \tilde{A}_B \tilde{A}_B^H R_B)$$

$$= \arg \max_{g_{B_d}} \|\tilde{A}_B^H R_B\|_F^2$$

$$= \arg \max_{g_{B_d}} \|\tilde{a}_B(g_{B_d}) R_B\|_2^2,$$

which is exactly the problem provided in (36). This concludes the proof. \qed

The variable of optimization $g_{B_d}$ in problem (36) is scalar and various standard optimization techniques can be applied to find the optimal solution. Suppose $\hat{g}_{B_d}$ is the optimal solution found. Let

$$\hat{Q}_B = \arg \min_{Q_B} \|R_B - [A_B, a_B(g_{B_d})] Q_B\|_F^2,$$

(38)

where $\hat{Q}_B = [[\hat{Q}_B]_{r,:}, \cdots, [\hat{Q}_B]_{Q,:}, [\hat{Q}_B]_{d,:}]$. Note that the values of $\|\hat{Q}_B\|_2^2$ and $\|\hat{Q}_B\|_2^2$ are related to the energy of the direct and reflection paths. Therefore, we can sort the paths according to the values of $\|\hat{Q}_B\|_2^2$ and $\|\hat{Q}_B\|_2^2$. We save the estimated order of the path energies as $s_t \in \mathbb{R}^{Q+1}$. This path order is utilized to distinguish the direct and reflection paths in the following subsections.
B. Estimation of $\tau_d$ and $\{\tau_{r,q}\}_{q=1}^Q$

We define

$$a_C(\tau) = [1, e^{-i2\pi \frac{w}{K} \tau}, \ldots, e^{-i2\pi \frac{(K-1)w}{K} \tau}]$$

(39)

and reshape $\{\tilde{R}_k\}_{k=0}^{K-1}$ over the dimensions of $a_C(\tau_d)$ and $\{a_C(\tau_{r,q} + \tau_{r,q})\}_{q=1}^Q$ to obtain

$$R_D = [a_C(\tau_d), a_C(\tau_{r,1} + \tau_{r,1}), \ldots, a_C(\tau_{r,1} + \tau_{r,1})]_{Q} + N_D \in \mathbb{C}^{K \times DN},$$

(40)

where $Q_D \in \mathbb{C}^{2 \times DN}$ and $N_D \in \mathbb{C}^{K \times DN}$. We use the multiple signal classification (MUSIC) method to estimate delays from the observations in (40).

Note that the column space of $R_D$ is spanned by $a_C(\tau_d)$ and $\{a_C(\tau_{r,q} + \tau_{r,q})\}_{q=1}^Q$. Letting $A_D = [a_C(\tau_d), a_C(\tau_{r,1} + \tau_{r,1}), \ldots, a_C(\tau_{r,1} + \tau_{r,1})]$, the covariance of (40) is

$$C_D = A_D Q_D Q_D^H A_D^H + \sigma^2 I.$$

(41)

Intuitively, when the noise level is low, the covariance matrix $C_D$ in (41) can be approximated by the covariance of the signal part, i.e., $A_D Q_D Q_D^H A_D^H$. This is the underlying methodology of MUSIC. The covariance matrix in (41) can be estimated by using the sample correlation matrix $\hat{C}_D = R_D R_D^H$. Let $[w_1, w_2, \ldots, w_{ND}]$ be the eigenvectors of $\hat{C}_D$, where $w_i$ corresponds to the $i$th largest eigenvalue. Then, letting $W^c_H = [w_{Q+2}, w_{Q+3}, \ldots, w_{ND}]$, the estimation of the delays $\tau_d, \{\tau_{r,q}\}_{q=1}^Q$ is achieved by the following:

$$\text{find } Q + 1 \text{ peaks: } 1/\|a_C^H(\tau) W^c_H \|_2^2 \text{ with } \tau \leq K/W.$$

(42)

Suppose the estimated delays are $\{\hat{\tau}_i\}_{i=1}^{Q+1}$. A heuristic way to distinguish the delay for the direct path is to use the minimum delay estimated. However, this approach may result in errors when the SNR is low, as our simulation in Section VII shows. Therefore, we use (38) instead to assign the delays for the direct and reflection paths. Specifically, after estimating $\{\hat{\tau}_i\}_{i=1}^{Q+1}$, denoting $\hat{A}_D = [a_C(\hat{\tau}_1), \ldots, a_C(\hat{\tau}_{Q+1})]$, we find

$$Q_D = \arg \min_{Q_D} \|R_D - \hat{A}_D Q_D\|_F^2 = (\hat{A}_D^H \hat{A}_D)^{-1} \hat{A}_D^H R_D.$$

Then, we use the path ordering $s_t$ from the sorting of (38) and $\{||Q_D||_2^2\}_{i=1}^{Q+1}$ to assign the estimated
delays to path indices. Let the matched estimated delays be \( \{ \hat{\tau}_d, \hat{\tau}_{r_2, 1}, \ldots, \hat{\tau}_{r_2, Q} \} \).

C. Estimation of AoAs \( (g_{U_r, q}, v_{U_r, q}) \) and \( (g_{U_d}, v_{U_d}) \)

We present only the method to estimate \( g_{U_r, q} \) and \( g_{U_d} \). The same approach can be applied to the estimation of \( v_{U_r, q} \) and \( v_{U_d} \). We reshape \( \{ \tilde{R}_k \}_{k=0}^{K-1} \) over the dimension of \( a_\tilde{U}(g_{U_d}) \) and \( \{ a_\tilde{U}(g_{U_r, q}) \}_{q=1}^{Q} \) as \( R_U \in \mathbb{C}^{2 \times \sqrt{D_0}N_K} \),

\[
R_U = [a_\tilde{U}(g_{U_d}), a_\tilde{U}(g_{U_r, 1}), \ldots, a_\tilde{U}(g_{U_r, Q})] Q_U + \tilde{N}_U, \tag{43}
\]

where \( Q_U \in \mathbb{C}^{2 \times \sqrt{D_0}N_K} \) and \( \tilde{N}_U \in \mathbb{C}^{\sqrt{D_0}N_K \times D_0} \). Note that the signal part of the column space of \( \tilde{R}_U \) is spanned by \( A_U = [a_\tilde{U}(g_{U_r, 1}), \ldots, a_\tilde{U}(g_{U_r, Q}), a_\tilde{U}(g_{U_d})] \). Similar to Section IV-B, we utilize the MUSIC method and (38) to obtain the estimation of \( \{ \hat{g}_{U_r, q} \}_{q=1}^{Q} \) and \( \hat{g}_{U_d} \) as \( \{ \hat{g}_{U_r, q} \}_{q=1}^{Q} \) and \( \hat{g}_{U_d} \). Using the same technique, we can obtain the estimated \( \{ \hat{v}_{U_r, q} \}_{q=1}^{Q} \) and \( \hat{v}_{U_d} \) as \( \{ \hat{v}_{U_r, q} \}_{q=1}^{Q} \) and \( \hat{v}_{U_d} \), respectively.

D. Estimation of \( h_d \) and \( \{ h_r \}_{q=1}^{Q} \)

Using the estimates \( (\hat{\tau}_d, \hat{\tau}_{r_2, q}, \hat{v}_{U_d}, \hat{v}_{B_d}, \hat{v}_{B_d}) \), and \( \{ \hat{\tau}_{r_2, q}, \hat{\tau}_{r_2, q}, \hat{v}_{U_r, q}, \hat{v}_{U_r, q}, \hat{v}_{U_r, q} \}_{q=1}^{Q} \), we solve the following problem to estimate \( h_d \) and \( h_r \):

\[
\hat{h} = \arg \min_h \sum_{k=0}^{K-1} \| R_k - \tilde{H}_k \|_F^2
\]

subject to \( (\tau_d, g_{U_d}, v_{U_d}, g_{B_d}, v_{B_d}) = (\hat{\tau}_d, \hat{g}_{U_d}, \hat{v}_{U_d}, \hat{g}_{B_d}, \hat{v}_{B_d}) \)

\[
(\tau_{r_2, q}, g_{U_r, q}, v_{U_r, q}) = (\hat{\tau}_{r_2, q}, \hat{g}_{U_r, q}, \hat{v}_{U_r, q}), \tag{44}
\]

where \( \hat{h} \) is the estimate of \( h = [h_d, h_{r, 1}, \ldots, h_{r, Q}]^T \). From the formulation in (22), we can vectorize the matrix as

\[
\mathbf{r}_s = \text{vec}([\tilde{R}_0, \ldots, \tilde{R}_{K-1}]), \quad \hat{\mathbf{r}}_d = \text{vec}([\tilde{H}_{d, 0}, \ldots, \tilde{H}_{d, K-1}]), \quad \hat{\mathbf{r}}_{r, q} = \text{vec}([H_{r, 0, q} G_{0, q}, \ldots, H_{r, K-1, q} G_{K-1, q}]).
\]

Then, the formulated problem in (44) can be expressed as

\[
\hat{h} = \arg \min_h \| \mathbf{r}_s - \tilde{\mathbf{R}} \mathbf{h} \|_2^2,
\]

where \( \tilde{\mathbf{R}} = [\hat{\mathbf{r}}_d, \hat{\mathbf{r}}_{r, 1}, \ldots, \hat{\mathbf{r}}_{r, Q}] \). The solution is \( \mathbf{h} = (\tilde{\mathbf{R}}^\ast \tilde{\mathbf{R}})^{-1} \tilde{\mathbf{R}}^\ast \mathbf{r}_s \).
V. UE POSITION ESTIMATION

In this section, we present a fusion method to infer the position of the UE from the estimated channel parameters.

A. Fusion via Linear Combination

Recall that the error covariance matrices of $\hat{p}^{(d)}_U$ and $\{\hat{p}^{(r,q)}_U\}_{q=1}^Q$ are given by (29) and (31), respectively. The following lemma presents the proposed fusion method based on the error covariance matrices.

**Lemma 2.** Assume that the estimations of $\hat{p}^{(d)}_U$ and $\{\hat{p}^{(r,q)}_U\}_{q=1}^Q$ are based on independent measurements, and the covariance matrices are in (29) and (31). Then, the optimal linear combination of $\hat{p}^{(d)}_U$ and $\{\hat{p}^{(r,q)}_U\}_{q=1}^Q$ is given by

$$\hat{p}_U = C_{p_U} ((C_{p_U}^{(d)})^{-1} \hat{p}_U^{(d)} + \sum_{q=1}^Q (C_{p_U}^{(r,q)})^{-1} \hat{p}_U^{(r,q)}),$$

where $C_{p_U} = ((C_{p_U}^{(d)})^{-1} + \sum_{q=1}^Q (C_{p_U}^{(r,q)})^{-1})^{-1}$.

**Proof.** To obtain a linear combination of $\hat{p}^{(d)}_U$ and $\{\hat{p}^{(r,q)}_U\}_{q=1}^Q$ as the estimate of the UE position, we let the expression of the estimation of UE position be

$$\hat{p}_U = A_d \hat{p}_U^{(d)} + \sum_{q=1}^Q B_q \hat{p}_U^{(r,q)},$$

where $A_d \in \mathbb{C}^{3\times3}$ and $B_q \in \mathbb{C}^{3\times3}, \forall q$. In order to obtain an unbiased estimator, it must have $A_d + \sum_{q=1}^Q B_q = I$. To minimize the MSE of $\hat{p}_U$, we need to solve the following problem:

$$\min_{A_d, \{B_q\}_{q=1}^Q} \text{tr}(\mathbb{E}[(\hat{p}_U - p_U)(\hat{p}_U - p_U)^T])$$

subject to $A_d + \sum_{q=1}^Q B_q = I$. (47)

Substituting the expression of (46) and taking first order derivative of the objective function in (47) give

$$A_d = C_{p_U} (C_{p_U}^{(d)})^{-1}, \quad B_q = C_{p_U} (C_{p_U}^{(r,q)})^{-1}.$$  

This concludes the proof. \qed
Remark 1. Since we assume independence among the UE estimations from these paths, according to Proposition 1, we have the following bounds

\[
C^{(d)}_{p_U} \succeq \left[ (J^T d F_{\hat{\eta}, d} J_d)^{-1} \right]_{1:3, 1:3} = \tilde{C}^{(d)}_{p_U}, \tag{48}
\]

\[
C^{(r,q)}_{p_U} \succeq \left[ (J^T r,q F_{\hat{\eta}, r,q} J_{r,q})^{-1} \right]_{1:3, 1:3} = \tilde{C}^{(r,q)}_{p_U}, \tag{49}
\]

where we denote the bounds as \(\tilde{C}^{(d)}_{p_U}\) and \(\tilde{C}^{(r,q)}_{p_U}\). We define \(\tilde{C}^{(r,q)}_{p_U} = ((\tilde{C}^{(d)}_{p_U})^{-1} + \sum_{q=1}^{Q} (\tilde{C}^{(r,q)}_{p_U})^{-1})^{-1}\). Therefore, when the exact error covariances in (45) are not available, we can employ the lower bounds in (48) and (49).

\[
\hat{p}_U = \tilde{C}^{(d)}_{p_U} \left( (\tilde{C}^{(d)}_{p_U})^{-1} \hat{p}^{(d)}_U + \sum_{q=1}^{Q} (\tilde{C}^{(r,q)}_{p_U})^{-1} \hat{p}^{(r,q)}_U \right). \tag{50}
\]

B. Asymptotic MLE

We now show that the proposed linear combination in (50) is approximately the MLE in the asymptotic regime of large sample size. We first introduce the extended invariance principle (EXIP), which is asymptotically equivalent to the MLE. Then we show that the proposed linear combination is approximately the optimal solution of EXIP.

Theorem 1 (EXIP theorem [39]). Suppose the loss function for estimating the parameters \(\xi\) is given by \(L(y; \xi)\), where \(y \in \mathbb{R}^{Z \times 1}\) are observations. Suppose there exists a function \(\eta = F(\xi)\) with loss function \(L(y; \eta) = L(y; F(\xi)) = L(y; \xi)\). The estimation of \(\xi\) and \(\eta\) are given by

\[
\hat{\xi} = \arg \min \limits_\xi L(y; \xi), \quad \hat{\eta} = \arg \min \limits_\eta L(y; \eta).
\]

If \(\lim_{N \to \infty} \hat{\eta} = \lim_{N \to \infty} F(\hat{\xi})\), then

\[
\hat{\xi} = \arg \min \limits_\xi [\hat{\eta} - F(\xi)]^T W [\hat{\eta} - F(\xi)]
\]

is asymptotically equivalent to \(\hat{\xi}\) as \(Z \to \infty\), where \(W\) is

\[
W = \mathbb{E} \left[ \frac{\partial^2 L(y; \eta)}{\partial \eta \eta^T} \right]_{\eta=\hat{\eta}}.
\]

In (18), the UE position parameters are related to the channel parameters \(\eta\) via a function \(F(\cdot)\). We can apply the EXIP approach to obtain the UE position estimate from the channel parameter estimates.
Specifically, from the estimator $\hat{\eta}$ in Section IV, applying Theorem 1, we can solve the following weighted least squares problem

$$
\hat{\xi} = \arg \min_{\xi} [\eta - F(\xi)]^\top W [\hat{\eta} - F(\xi)],
$$

(52)

where the weight matrix $W$ is given by

$$
W = \mathbb{E} \left[ \frac{\partial^2 L(\{ \tilde{R}_k \}_{k=1}^K; \eta)}{\partial \eta \eta^\top} \right]_{\eta = \hat{\eta}} = F_{\hat{\eta}},
$$

and the loss function $L(\{ \tilde{R}_k \}_{k=1}^K; \eta) = \sum_{k=1}^K \| \tilde{R}_k - \tilde{H}_k \|_F^2$ (see (33)). Note that the inference model in [33], [40] with geometric mapping is equivalent to letting $W = I$ in (52). In particular, the gradient-based method can be utilized to find the optimum in (52), which is, however, sensitive to the initialization. In what follows, we will show that (50) is the approximate solution of the problem (52).

Let $F_d$ and $F_{r,q}$, $q = 1, \ldots, Q$, be functions such that $\eta_d = F_d(\xi_d)$ and $\eta_{r,q} = F_{r,q}(\xi_{r,q})$. The objective function in (52) is approximate to the following,

$$
(\hat{\eta} - F(\xi))^\top W (\hat{\eta} - F(\xi))
\approx
\begin{bmatrix}
    F_d(\hat{\xi}_d) - F_d(\xi_d) \\
    F_{r,1}(\hat{\xi}_{r,1}) - F_{r,1}(\xi_{r,1}) \\
    \vdots \\
    F_{r,1}(\hat{\xi}_{r,Q}) - F_{r,Q}(\xi_{r,Q})
\end{bmatrix}^\top
\begin{bmatrix}
    J_d(\hat{\xi}_d - \xi_d) \\
    J_{r,1}(\hat{\xi}_{r,1} - \xi_{r,1}) \\
    \vdots \\
    J_{r,1}(\hat{\xi}_{r,Q} - \xi_{r,Q})
\end{bmatrix}
\begin{bmatrix}
    F_d(\hat{\xi}_d) - F_d(\xi_d) \\
    F_{r,1}(\hat{\xi}_{r,1}) - F_{r,1}(\xi_{r,1}) \\
    \vdots \\
    F_{r,1}(\hat{\xi}_{r,Q}) - F_{r,Q}(\xi_{r,Q})
\end{bmatrix},
$$

(53)

where $\hat{\xi}_d$ is inferred from $\hat{\eta}_d$, and $\hat{\xi}_{r,q}$ is inferred from $\hat{\eta}_{r,q}$. Further details are in Section V-C. The first approximation is from that $F_d(\hat{\xi}_d) \approx \hat{\eta}_d$ and $F_{r,q}(\hat{\xi}_{r,q}) \approx \hat{\eta}_{r,q}$. The second approximation in (53) holds
from the Taylor series expansion. Letting the first order derivative of (53) be zero gives

$$\hat{\xi} = (J^\top F^\eta J)^{-1} J^\top F^\eta \left[ \begin{array}{c} J_d \hat{\xi}_d \\ J_{r,1} \hat{\xi}_{r,1} \\ \vdots \\ J_{r,Q} \hat{\xi}_{r,Q} \end{array} \right].$$  

(54)

Therefore, the solution in (54) is the approximate solution of (52), which is asymptotically MLE in the large-sample regime.

The following proposition shows that the optimal linear combination in (50) is equivalent to the solution in (54) when the paths are independent.

**Proposition 2.** Suppose the paths are independent, in other words, $F^\eta$ has the following form

$$F^\eta = \left[ \begin{array}{cccc} F_{\eta,d} & 0 & \cdots & 0 \\ 0 & F_{\eta,r,1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & F_{\eta,r,Q} \end{array} \right].$$  

(55)

Then, the solution in (54) is equivalent to (50), that is $[\hat{\xi}]_{1:3} = \tilde{C}_{p_U} ((\tilde{C}_{p_U}^{(d)})^{-1} \tilde{p}_U^{(d)} + \sum_{q=1}^{Q} (\tilde{C}_{p_U}^{(r,q)})^{-1} \tilde{p}_U^{(r,q)})$.

**Proof.** See Appendix D. □

In summary, we have shown that the optimal linear combination in (50) is approximate to the optimal solution of EXIP method when the paths are independent. Therefore, it is approximately equivalent to the MLE in large-sample region.

**C. Estimation of $\tilde{p}^{(d)}_U$ and $\{\tilde{p}^{(r,q)}_U\}_{q=1}^{Q}$**

1) **Inferring $\tilde{p}^{(d)}_U$ from $\tilde{\gamma}_d$**: We first focus on the direct path, and discuss how to obtain the refined channel parameters associated with the direct path. Define $f = [g_{U_d}, v_{U_d}, g_{B_d}, v_{B_d}]^\top$, and $z = [f_{B,d}, g_{B_d}, v_{B_d}]^\top$ with $f_{B,d} = \sin \theta_{B_d} \cos \phi_{B_d}$. From (13) and (14), we have $f = [M_R; 0, I]z = \tilde{A}z$. However, the estimation result $\hat{f} = [\hat{g}_{U_d}, \hat{v}_{U_d}, \hat{g}_{B_d}, \hat{v}_{B_d}]^\top$ may not satisfy the above relation due to corruption of noise. We employ the weighted least squares method by solving

$$\min_z (\hat{f} - \tilde{A}z)^\top \tilde{C}_f^{-1} (\hat{f} - \tilde{A}z), \text{ subject to } \|z\|_2^2 = 1,$$  

(56)
where $C_\hat{f} = [F^{-1}_q]_{2,3;2,5}$. If we ignore the constraint, the solution is given by

$$\hat{z} = (\hat{A}^\top C_\hat{f}^{-1} \hat{A})^{-1} \hat{A}^\top C_\hat{f}^{-1} \hat{f}.$$ 

Then we project this solution to the feasible region of the problem in (56). The estimation of $(\hat{d}, \hat{\theta}_{B,d}, \hat{\phi}_{B,d})$ is given by

$$\hat{d} = c \tau_d, \hat{\theta}_{B,d} = \arccos \hat{\nu}_{B,d}, \hat{\phi}_{B,d} = \arcsin(\hat{g}_{B,d}/\sin \hat{\theta}_{B,d}).$$

The estimated UE position $\mathbf{p}^{(d)}_{U}$ from the direct path is then given by

$$\begin{align*}
\dot{z}_{U_d} &= \hat{d} \cos \hat{\theta}_{B_d} = \hat{d} \hat{v}_{B,d} \\
\dot{x}_{U_d} &= \hat{d} \sin \hat{\theta}_{B_d} \cos \hat{\phi}_{B_d} = \hat{d} \sqrt{1 - \hat{g}^2_{B,d} - \hat{v}^2_{B,d}} \\
\dot{y}_{U_d} &= \hat{d} \sin \hat{\theta}_{B_d} \sin \hat{\phi}_{B_d} = \hat{d} \hat{g}_{B,d}.
\end{align*}$$

2) Inferring $\mathbf{p}^{(r,q)}_{U}$ from $\hat{\gamma}_{r,q}$: For the $q$th reflection path, we define $g_{R_2,q} = \sin \theta_{R_2,q} \sin \phi_{R_2,q}$ and $f_{U,r,q} = -\sqrt{1 - g_{U,r,q}^2 - \nu_{U,r,q}^2}$. Based on the relations in (1) and (9), we have

$$\begin{bmatrix}
   f_{R_2,q} \\
   g_{R_2,q} \\
   v_{R_2,q}
\end{bmatrix} =
\begin{bmatrix}
   \sin \theta_{R_2,q} \cos \phi_{R_2,q} \\
   \sin \theta_{R_2,q} \sin \phi_{R_2,q} \\
   \cos \theta_{R_2,q}
\end{bmatrix}
= M_{R}^{-1}
\begin{bmatrix}
   f_{U,r,q} \\
   g_{U,r,q} \\
   v_{U,r,q}
\end{bmatrix}.$$  

Therefore, $[\hat{f}_{R_2,q}, \hat{g}_{R_2,q}, \hat{v}_{R_2,q}] = M_{R}^{-1}[f_{U,r,q}, \hat{g}_{U,r,q}, \hat{v}_{U,r,q}]$. The estimation of $\{d_{2,q}, \hat{\theta}_{R_2,q}, \hat{\phi}_{R_2,q}\}_{q=1}^Q$ is given by

$$\hat{d}_{2,q} = c \tau_{r_2,q}, \hat{\theta}_{R_2,q} = \arccos \hat{\nu}_{R_2,q}, \hat{\phi}_{R_2,q} = \arctan(\hat{g}_{R_2,q}/\hat{f}_{R_2,q}).$$

Then, the UE position can be estimated as

$$\begin{align*}
\dot{z}_{U_r,q} - z_{R,q} &= \hat{d}_{2,q} \cos \hat{\theta}_{R_2,q} = \hat{d}_{2,q} \hat{v}_{R_2,q}, \\
\dot{x}_{U_r,q} - x_{R,q} &= \hat{d}_{2,q} \sin \hat{\theta}_{R_2,q} \cos \hat{\phi}_{R_2,q} = \hat{d}_{2,q} \hat{f}_{R_2,q}, \\
\dot{y}_{U_r,q} - y_{R,q} &= \hat{d}_{2,q} \sin \hat{\theta}_{R_2,q} \sin \hat{\phi}_{R_2,q} = \hat{d}_{2,q} \hat{g}_{R_2,q}.
\end{align*}$$

VI. DISCUSSIONS

In this section, we propose methods to optimize the phase shifts of a RIS for the purpose of positioning a UE. We also discuss the extension of our proposed framework to the multi-BS and multi-UE scenarios.
A. Design of RIS Phase Shifts

We consider the RIS-aided positioning scenario in Fig. 2, where the RIS aims to serve multiple UEs. Specifically, the phase shifts of a RIS are designed to serve the UEs with elevation angles in the range $[\theta_l, \theta_u]$ and azimuth angles in $[\phi_l, \phi_u]$. For example, the LOS between the BS and UEs within this region of interest may be blocked with high probability. The phase shifts of the RIS are designed to aid these UEs.

Recall that the gain of the reflection path is proportional to $|a_{R_1}^H(f_{R_2}, q_{R_2}) \Theta_q a_{R_2}(f_{R_1}, q_{R_1})|$. Since the quantities $f_{R_1, q}$ and $v_{R_1, q}$ are unknown a priori, we make an unbiased design $\Theta_q$ based on the served UEs in the following.

For convenience, we combine $\Theta_q a_{R_1}(f_{R_1, q}, v_{R_1, q})$ as one variable $\tilde{\theta}_q \in \mathbb{C}^{M \times 1}$. If the UEs are uniformly distributed in elevation range $[\theta_l, \theta_u]$ and azimuth range $[\phi_l, \phi_u]$, then we consider the following optimization problem:

$$
\max_{\tilde{\theta}_q} \mathbb{E} \left[ |(a_{R_1}(f) \otimes a_{R_2}(v))^H \tilde{\theta}_q |^2 \right]
$$

subject to $f = \sin \theta \cos \phi, \ v = \cos \theta$

$$
\theta \sim U[\theta_{q,l}, \theta_{q,u}], \ \phi \sim U[\phi_{q,l}, \phi_{q,u}]. \quad (58)
$$

Since directly solving (58) is challenging, we define a matrix $D_A \in \mathbb{C}^{M \times Z}$ with its column having the form of $a_{R_1}(f) \otimes a_{R_2}(v)$, where $(f, v)$ is chosen in a discretized range. Therefore, we have the following
approximation

\[ \mathbb{E} \left[ \left| (a_R(f) \otimes a_R(v))^H \hat{\theta}_q \right|^2 \right] \approx \frac{1}{Z} \| D_A^H \hat{\theta}_q \|^2_2. \] (59)

Then, we can reformulate the problem in (58) as

\[ \hat{\theta}_q = \operatorname{arg \ max}_{\theta_q} \| D_A^H \theta_q \|^2_2. \] (60)

If there is no constraint for \( \hat{\theta}_q \), the solution is the dominant left singular vector of \( D_A \). In order to satisfy the constraint imposed on \( \hat{\theta}_q \), we let \( \hat{\theta}_q \) be the complex angle of the dominant left singular vector of \( A \).

The design of phase shifts of \( q \)th RIS \( \Theta_q \) is then given by \( \text{diag}(\theta_q) \) with

\[ \angle \theta_q = -\angle a_R(f_{R_1,q}, v_{R_1,q}) + \angle \hat{\theta}_q. \] (61)

B. Extension to Multiple UEs

Assume there are \( L \) users. Let the true position of UE \( l \) be \( p_{U,l} = p_{U,1} + \Delta_l \) with \( \Delta_l \) being the relative position w.r.t. the reference UE 1 with position \( p_{U,1} \). We assume that the relative positions of the UEs are known through inter-UE measurements and message exchanges [41]–[43]. Assume each UE \( l \) first estimates its position independently as \( \hat{p}_{U,l} \) with error covariance \( C_{p_{U,l}} \). To leverage on inherent correlations among the UEs, we employ the linear combination of estimates as described in Section V-A,

\[ \hat{p}_{U,1} = \sum_{l=1}^{L} A_l (\hat{p}_{U,l} - \Delta_l), \hat{p}_{U,l} = \hat{p}_{U,1} + \Delta_l, \] (62)

where \( A_l \in \mathbb{C}^{3\times3} \) is the combining matrix. By using the similar statements as Lemma 2, we can minimize the MSE of the estimates, i.e., \( \sum_{l=1}^{L} \text{tr}(\mathbb{E}[(\hat{p}_{U,l} - p_{U,l})(\hat{p}_{U,l} - p_{U,l})^H]) \), and obtain the expression of \( A_l \) in (62) as follows,

\[ A_l = \left( \sum_{l=1}^{L} C_{p_{U,l}}^{-1} \right)^{-1} C_{p_{U,l}}^{-1}. \]

Therefore, the estimated positions of UEs are given by

\[ \hat{p}_{U,1} = \left( \sum_{l=1}^{L} C_{p_{U,l}}^{-1} \right)^{-1} \sum_{l=1}^{L} C_{p_{U,l}}^{-1} (\hat{p}_{U,l} - \Delta_l), \] (63)

\[ \hat{p}_{U,l} = \hat{p}_{U,1} + \Delta_l. \]
After the fusion, the resulting error covariance of each UE is given by \((\sum_{l=1}^{L} C_{p,U,l}^{-1})^{-1}, \forall l\). This result can be utilized for the case where there are multiple BSs, which we discuss in the following subsection. In particular, when the error covariance \(C_{p,U,l}\) in (63) is not available, we can employ the lower bound as an alternative, which can still achieve near optimal performance as we analyzed in Section V-A and Section V-B.

C. Extension to Multiple BSs

We now consider the case where there are \(P\) BSs. The received signal of the \(k\)th subcarrier at the UE is given by

\[
\mathbf{R}_k = \sum_{i=1}^{P} \mathbf{H}_{i,k} \mathbf{X}_i + \mathbf{N}_k, \tag{64}
\]

where \(\mathbf{X}_i \in \mathbb{C}^{D \times T}\). As in the case of a single BS, we assume \(\mathbf{X}_i \mathbf{X}_i^H = T/D \mathbf{I}\). Here, we further assume that \(\mathbf{X}_i \mathbf{X}_j^H = 0, \forall i \neq j\). Thus, right multiplying both sides of (64) with \(D/T \mathbf{X}_i^H\) yields

\[
\frac{D}{T} \mathbf{R}_k \mathbf{X}_i^H = \left( \sum_{i=1}^{P} \frac{D}{T} \mathbf{H}_{i,k} \mathbf{X}_i \right) \mathbf{X}_i^H + \frac{D}{T} \mathbf{N}_k \mathbf{X}_i^H
\]

\[
= \mathbf{H}_{i,k} + \frac{D}{T} \mathbf{N}_k \mathbf{X}_i^H. \tag{65}
\]

From the signal transmitted by the \(i\)th BS, we can estimate the UE position by using the proposed method for a single BS, i.e., \(\hat{\mathbf{p}}^{(i)}_U\). Let the error covariance be \(C_{p,U}^{(i)}\). Then, the estimate from different BSs can be fused by linear combination as

\[
\hat{\mathbf{p}}_U = \sum_{i=1}^{P} \mathbf{B}_i \hat{\mathbf{p}}^{(i)}_U, \tag{66}
\]

where \(\mathbf{B}_i \in \mathbb{C}^{3 \times 3}\) is the combining matrix. Similarly, by minimizing the MSE of \(\hat{\mathbf{p}}_U\) in (66), i.e.,

\[
\text{tr}(\mathbb{E}[\hat{\mathbf{p}}_U - \mathbf{p}_U](\hat{\mathbf{p}}_U - \mathbf{p}_U)^H)],
\]

the final estimate from different BSs is given by

\[
\hat{\mathbf{p}}_U = \left( \sum_{i=1}^{P} C_{p,U}^{(i)} \right)^{-1} \sum_{i=1}^{P} C_{p,U}^{(i)} \hat{\mathbf{p}}^{(i)}_U.
\]

VII. NUMERICAL RESULTS

In this section, we evaluate the proposed RIS-aided positioning method. We verify the UE positioning accuracy achieved by comparing to the CRB under varying the noise level. We also verify the channel parameter estimation accuracy. Numerical experiments are also conducted to provide insights into the
TABLE I
SIMULATION PARAMETERS

| Parameter                     | Value                  |
|-------------------------------|------------------------|
| Number of BS antennas        | $N = 100$              |
| Number of UE antennas        | $D = 64$               |
| RIS size                     | $M = 400$              |
| Transmission bandwidth       | $W = 100$MHz           |
| Carrier frequency            | $f_c = 30$GHz          |
| Number of OFDM subcarriers   | $K = 32$               |
| Rician factor                | $K_d = 100$            |
| Number of time slots         | $T = 6 \times 10^5$   |

impact of the direct path loss exponent on the UE positioning accuracy. Finally, we present the simulation results for the multi-UE and multi-BS scenarios.

In the simulations, we utilize the root mean-square error, $\text{RMSE} = \sqrt{\mathbb{E}[\| \hat{\mathbf{p}}_U - \mathbf{p}_U \|^2]}$, to measure the positioning accuracy, where $\hat{\mathbf{p}}_U$ is the estimated UE position. Throughout our experiments, we use the parameter settings in Table I.

A. UE Positioning Accuracy

![Graph](image)

Fig. 3. UE position estimation RMSE versus the noise level.

In this simulation, we evaluate the UE positioning accuracy of the proposed RIS-aided positioning method with a single BS and UE. All position coordinates are measured in meters with the BS at the origin as illustrated in Fig. 1. The UE position is $\mathbf{p}_U = [50, 10, 20]$, where $z_U = 20$ means the UE height...
of 20m from the BS. One RIS is at position $p_{R,1} = [30, -5, 2]$, where $z_{R,1} = 2$ means the RIS height of 2m from the BS. The path loss exponent for the direct path is $L_d = 4.5$, and the path loss exponent for reflection path is $L_r = 2$. We evaluate the positioning accuracy following methods:

- The proposed positioning method that distinguishes the direct path based on the path energy, labeled as “Direct+reflection paths, proposed method”.
- The proposed positioning method that distinguishes the direct path based on the estimated delay, labeled as “Direct+reflection paths, delay-based”.
- The positioning method with EXIP [39], labeled as “Direct+reflection paths, EXIP”.
- The positioning method with geometric mapping [33], [40], labeled as “Direct+reflection paths, GM”.
- The positioning method that utilizes only the direct path, labeled as “Direct path only”.

We observe from Fig. 3 that our proposed method outperforms the benchmark approaches, with RMSE close to the CRB when the SNR is high. The result also verifies that distinguishing the paths based on the path energy provides better performance compared to the delay-based approach. We also observe that fusing the estimates from the direct and reflection paths achieves a better accuracy than using only the direct path, which validates the effectiveness of the RIS.

B. Channel Parameter Estimation Accuracy

In Figs. 4 and 5, we evaluate the RMSE of the channel parameters, i.e., $(\tau_d, \theta_{B_d}, \phi_{B_d})$ and $(\tau_{R_2,1}, \theta_{R_2,1}, \phi_{R_2,1})$, by using proposed method. The CRBs of the estimators are also plotted as the benchmark. The simulation settings are the same as those in Fig. 3. As we expect in Figs. 4 and 5, the RMSEs of the estimated parameters are all close to their CRBs, which validates the effectiveness of the proposed method.

C. Direct Path Loss Exponent

In Fig. 6, we evaluate the positioning accuracy of the proposed method under different path loss exponents for the direct path. The simulation settings are the same as those in Fig. 3 except that $L_d = \{3, 3.1, \ldots, 5\}$ and $1/\sigma^2 = 95$dB. The path loss exponent captures the blockage severity of the direct path with larger path loss exponent meaning more severe blocking. We observe from Fig. 6 that when the direct path is not severely blocked ($L_d = 3$), the fusion result has similar performance as the “direct path only” case. As the path loss exponent increases, the positioning error using only the direct path increases,
while the proposed method still produces an accurate result. This verifies the proposed positioning method can adapt to different fading scenarios of the direct path.

### D. Multi-UE and Multi-BS Scenarios

In Fig. 7, we compare the positioning accuracy achieved by using multiple BSs and multiple UEs with the scenario of single BS and single UE. The positions of the two BSs are at $[0, 0, 0]$ and $[0, 10, 0]$. 

![Fig. 4. Channel parameter estimation RMSE of the direct path versus the noise level.](image)

![Fig. 5. Channel parameter estimation RMSE of the reflection path versus the noise level.](image)
The positions of the two UEs are at $[50, 10, 20]$ and $[52, 10, 20]$. The position of the RIS is at $[30, -5, 2]$. The path loss exponents are $L_d = 4.5$ and $L_r = 2$. In Fig. 7, by using the techniques in Section VI, the proposed positioning method in this scenario also achieves performance close to the theoretical bound. From Fig. 7, when more than one BS and UE can cooperate and exchange information, the positioning accuracy can be further improved.
VIII. Conclusions

In this paper, we have developed a RIS-aided positioning framework. The framework consists of first estimating the RIS-aided channel parameters from received signals, and then using these estimates to infer the UE position. Through an optimal linear combination of estimates from the direct and reflection paths, the proposed fusion method is shown via the EXIP framework to approximate the MLE asymptotically when the estimates are independent and the number of samples is large. The advantage of our approach is computational tractability, making it amendable to real-time implementation, as compared to direct estimation of the UE position from the received signals. Moreover, the proposed RIS-aided positioning method can be readily extended to the multi-BS and multi-user scenario. Through simulation studies, we demonstrated the positioning accuracy of the proposed method, which shows that it is close to the CRB and can adapt to different channel fading scenarios.

Appendix A

CRB Derivation

Taking the derivatives of $\mathbf{H}_k$ w.r.t. the parameters of the direct path, we have

$$\frac{\partial \mathbf{H}_k}{\partial \tau_d} = -i2\pi \frac{kW}{K} \mathbf{H}_{d,k},$$

$$\frac{\partial \mathbf{H}_k}{\partial \text{Re}\{h_d\}} = \frac{\mathbf{H}_{d,k}}{h_d}, \quad \frac{\partial \mathbf{H}_k}{\partial \text{Im}\{h_d\}} = i \frac{\mathbf{H}_{d,k}}{h_d},$$

$$\frac{\partial \mathbf{H}_k}{\partial g_{U_d}} = h_{d,k} a_U(g_{U_d}, v_{U_d}) a_B^H(g_{B_d}, v_{B_d}),$$

$$\frac{\partial \mathbf{H}_k}{\partial v_{U_d}} = h_{d,k} a_U(g_{U_d}, v_{U_d}) a_B^H(g_{B_d}, v_{B_d}),$$

$$\frac{\partial \mathbf{H}_k}{\partial g_{B_d}} = h_{d,k} a_U(g_{U_d}, v_{U_d}) a_B^H(g_{B_d}, v_{B_d}),$$

and the derivative w.r.t. the parameters of the reflection path, we obtain

$$\frac{\partial \mathbf{H}_k}{\partial \tau_{r,q}} = -i2\pi \frac{kW}{K} \mathbf{H}_{r,k,q},$$

$$\frac{\partial \mathbf{H}_k}{\partial \text{Re}\{h_{r,q}\}} = \frac{\mathbf{H}_{r,k,q}}{h_{r,q}}, \quad \frac{\partial \mathbf{H}_k}{\partial \text{Im}\{h_{r,q}\}} = i \frac{\mathbf{H}_{r,k,q}}{h_{r,q}},$$

$$\frac{\partial \mathbf{H}_k}{\partial g_{U_{r,q}}} = h_k a_U(g_{U_{r,q}}, v_{U_{r,q}}) a_B^H(g_{B_{r,q}}, v_{B_{r,q}}),$$

$$\frac{\partial \mathbf{H}_k}{\partial v_{U_{r,q}}} = h_k a_U(g_{U_{r,q}}, v_{U_{r,q}}) a_B^H(g_{B_{r,q}}, v_{B_{r,q}}).$$
We also have
\[
\frac{\partial a_U(g_U,v_U)}{\partial g_U} = (i\pi a_G(g_U) \circ E_U) \otimes a_C(v_U),
\]
\[
\frac{\partial a_U(g_U,v_U)}{\partial v_U} = a_G(g_U) \otimes (i\pi a_G(v_U) \circ E_U),
\]
\[
\frac{\partial a_B(g_B,v_B)}{\partial g_B} = (i\pi a_B(g_B) \circ E_B) \otimes a_B(v_B),
\]
\[
\frac{\partial a_B(g_B,v_B)}{\partial v_B} = a_B(g_B) \otimes (i\pi a_B(v_B) \circ E_B),
\]
\[
\frac{\partial a_R(f_R,v_R)}{\partial f_R} = (i\pi a_R(f_R) \circ E_R) \otimes a_R(v_R),
\]
\[
\frac{\partial a_R(f_R,v_R)}{\partial v_R} = a_R(f_R) \otimes (i\pi a_R(v_R) \circ E_R),
\]

where \( E_U = [0, 1, \ldots, D^{1/4}]^T \), \( E_B = [0, 1, \ldots, N^{1/4}]^T \), and \( E_R = [0, 1, \ldots, M^{1/4}]^T \).

**APPENDIX B**

**JACOBIAN MATRIX DERIVATION**

In this appendix, we derive the Jacobian matrix \( J \in \mathbb{R}^{(7+5Q) \times (5+2Q)} \) used in (27). Recall that \( \eta \) in (16) and \( \xi \) in (17), we write \( J \) in the following form:
\[
J = \begin{bmatrix}
\tilde{J}_d^T & \tilde{J}_{r,1}^T & \cdots & \tilde{J}_{r,Q}^T
\end{bmatrix}^T,
\]

where \( \tilde{J}_d = \frac{\partial \eta}{\partial \xi} \in \mathbb{R}^{7 \times (5+2Q)} \) and \( \tilde{J}_{r,q} = \frac{\partial \eta}{\partial \xi} \in \mathbb{R}^{5 \times (5+2Q)} \). We first derive the Jacobian matrix of the direct path \( \tilde{J}_d \), whose entries are can be obtained through
\[
\frac{\partial r_d}{\partial p_U} = -\frac{p_U^T}{c\|p_U\|^2}, \quad \frac{\partial \text{Re}\{h_d\}}{\partial \text{Re}\{h_d\}} = 1, \quad \frac{\partial \text{Im}\{h_d\}}{\partial \text{Im}\{h_d\}} = 1,
\]
\[
\frac{\partial s_d}{\partial p_U} = \frac{\partial s_d}{\partial \theta_B} \frac{\partial \theta_B}{\partial p_U} + \frac{\partial s_d}{\partial \phi_B} \frac{\partial \phi_B}{\partial p_U},
\]

with \( s_d \) denoting any entry in \( \eta_d \). We have
\[
\frac{\partial \theta_B}{\partial p_U} = \frac{1}{\|p_U\|^3} \left( x_U z_U, y_U z_U, -x_U^2 - y_U^2 \right),
\]
\[
\frac{\partial \phi_B}{\partial p_U} = \left[ -\frac{y_U}{x_U^2 + y_U^2}, \frac{x_U}{x_U^2 + y_U^2}, 0 \right],
\]
\[
\frac{\partial g_B}{\partial \theta_B} = \cos \theta_B \sin \phi_B, \quad \frac{\partial g_B}{\partial \phi_B} = \sin \theta_B \cos \phi_B
\]
\[
\frac{\partial v_B}{\partial \theta_B} = -\sin \theta_B, \quad \frac{\partial v_B}{\partial \phi_B} = 0,
\]
When we utilize only the BS-UE link for UE positioning, the FIM is given by

\[ \frac{\partial g_{U_d}}{\partial \theta_{B_d}} = [M_R]_{1,1} \cos \theta_{B_d} \cos \phi_{B_d} + [M_R]_{1,2} \cos \theta_{B_d} \sin \phi_{B_d} - [M_R]_{1,3} \sin \theta_{B_d}, \]

\[ \frac{\partial g_{U_d}}{\partial \phi_{B_d}} = -[M_R]_{1,1} \sin \phi_{B_d} \sin \theta_{B_d} + [M_R]_{1,2} \cos \phi_{B_d} \sin \theta_{B_d}, \]

\[ \frac{\partial v_{U_d}}{\partial \theta_{B_d}} = [M_R]_{2,1} \cos \theta_{B_d} \cos \phi_{B_d} + [M_R]_{2,2} \cos \theta_{B_d} \sin \phi_{B_d} - [M_R]_{2,3} \sin \theta_{B_d}, \]

\[ \frac{\partial v_{U_d}}{\partial \phi_{B_d}} = -[M_R]_{2,1} \sin \phi_{B_d} \sin \theta_{B_d} + [M_R]_{2,2} \cos \phi_{B_d} \sin \theta_{B_d}. \]

We next derive the Jacobian matrix of the reflection path \( \bar{J}_{r,q} \), whose entries are can be obtained through

\[ \frac{\partial \tau_{r,q}}{\partial \bar{P}_U} = \frac{\bar{P}_U^T - \bar{P}_{R,q}^T}{\| \bar{P}_U - \bar{P}_{R,q} \|^2}, \quad \frac{\partial \text{Re}\{h_{r,q}\}}{\partial \text{Re}\{h_{r,q}\}} = 1, \quad \frac{\partial \text{Im}\{h_{r,q}\}}{\partial \text{Im}\{h_{r,q}\}} = 1 \]

\[ \frac{\partial s_r}{\partial \bar{P}_U} = \frac{\partial s_r}{\partial \theta_{R,q}} \frac{\partial \bar{P}_U^T}{\partial \phi_{R,q} + \frac{\partial \phi_{R,q}}{\partial \bar{P}_U^T}}, \]

with \( s_r \) denoting any entry in \( \eta_r \). Here, we denote \( \bar{P}_{U,q} = P_U - P_{R,q} = [\tilde{x}_{U,q}, \tilde{y}_{U,q}, \tilde{z}_{U,q}]^T \). Therefore,

\[ \frac{\partial \theta_{R,q}}{\partial \bar{P}_U} = \frac{[\tilde{x}_{U,q}, \tilde{y}_{U,q}, \tilde{z}_{U,q}] - [\tilde{x}_{U,q}, \tilde{y}_{U,q}, \tilde{z}_{U,q}]}{\| \bar{P}_{U,q} \|^2 \left( 1 - \frac{\| \bar{P}_{U,q} \|^2}{2} \right)^{\frac{3}{2}}}, \]

\[ \frac{\partial \phi_{R,q}}{\partial \bar{P}_U} = \begin{bmatrix} -\frac{\tilde{x}_{U,q}}{\tilde{x}_{U,q}^2 + \tilde{y}_{U,q}^2} & -\frac{\tilde{x}_{U,q}}{\tilde{x}_{U,q}^2 + \tilde{y}_{U,q}^2} & 0 \end{bmatrix}, \]

\[ \frac{\partial g_{U,q}}{\partial \theta_{R,q}} = [M_R]_{1,1} \cos \theta_{R,q} \cos \phi_{R,q} + [M_R]_{1,2} \cos \theta_{R,q} \sin \phi_{R,q} - [M_R]_{1,3} \sin \theta_{R,q}, \]

\[ \frac{\partial g_{U,q}}{\partial \phi_{R,q}} = -[M_R]_{1,1} \sin \theta_{R,q} \sin \phi_{R,q} + [M_R]_{1,2} \cos \phi_{R,q} \sin \theta_{R,q}, \]

\[ \frac{\partial v_{U,q}}{\partial \theta_{R,q}} = [M_R]_{2,1} \cos \theta_{R,q} \cos \phi_{R,q} + [M_R]_{2,2} \cos \theta_{R,q} \sin \phi_{R,q} - [M_R]_{2,3} \sin \theta_{R,q}, \]

\[ \frac{\partial v_{U,q}}{\partial \phi_{R,q}} = -[M_R]_{2,1} \sin \theta_{R,q} \sin \phi_{R,q} + [M_R]_{2,2} \cos \phi_{R,q} \sin \theta_{R,q}. \]

**APPENDIX C**

**PROOF OF PROPOSITION 1**

When we utilize only the BS-UE link for UE positioning, the FIM is given by

\[ \mathbf{C}_{\xi_d} = (J_d^T \mathbf{C}_{\xi_d}^{-1} J_d)^{-1}, \]

(67)
where $\overline{C}_{\eta_d} = \left[F_{\eta}^{-1}\right]_{1:7,1:7}$, and $J_d = \frac{\partial \eta_d}{\partial \xi_d} \in \mathbb{R}^{7 \times 5}$. Thus, the error covariance matrix satisfies the following:

$$C_{p_U}^{(d)} \succeq [\overline{C}_{\xi_d}]_{1:3,1:3}.$$  \hspace{1cm} (68)

Since $F_{\eta_d}^{-1} \succeq \left[F_{\eta}^{-1}\right]_{1:7,1:7}$, the following equation holds,

$$\overline{C}_{\xi_d} = \left( J_d^T \left( \left[F_{\eta}^{-1}\right]_{1:7,1:7} \right)^{-1} J_d \right)^{-1} \succeq \left( J_d^T F_{\eta_d} J_d \right)^{-1}. \hspace{1cm} (69)$$

Therefore, combining (68) and (69), we have

$$C_{p_U}^{(d)} \succeq [\overline{C}_{\xi_d}]_{1:3,1:3} \succeq \left( J_d^T F_{\eta_d} J_d \right)^{-1} \hspace{1cm} (30).$$

This concludes the proof in (30).

Similarly, when only the $q$th RIS link is utilized for UE positioning, we can also obtain

$$C_{p_U}^{(r,q)} \succeq [\overline{C}_{\xi_{r,q}}]_{1:3,1:3} \succeq \left( J_{r,q}^T F_{\eta_{r,q}} J_{r,q} \right)^{-1} \hspace{1cm} (32).$$

This concludes the proof for (32).

**APPENDIX D**

**PROOF OF PROPOSITION 2**

It suffices to show that

$$\left( J_d^T F_{\eta} J \right)^{-1} J_d \hat{\xi}_d = \overline{C}_{p_U} (\overline{C}_{p_U}^{(d)})^{-1} \hat{p}_{p_U}^{(d)}. \hspace{1cm} (70)$$

The components for reflection paths can be proved similarly. Thus, it is sufficient to prove the following two equalities:

$$\left( J_d^T F_{\eta_d} J \right)^{-1} \left[ J_d \hat{\xi}_d \right]_{1:3,;} = \overline{C}_{p_U} (\overline{C}_{p_U}^{(d)})^{-1} \hspace{1cm} (70)$$

$$\left( J_d^T F_{\eta_d} J \right)^{-1} \left[ J_d \hat{\xi}_d \right]_{1:7,;} = \overline{C}_{p_U} (\overline{C}_{p_U}^{(d)})^{-1} \hspace{1cm} (71)$$

Here, we write

$$F_{\eta_d} = \begin{bmatrix} F_{\eta_d}^{(h)} & F_{\eta_d}^{(h,p)} \\ F_{\eta_d}^{(p,h)} & F_{\eta_d}^{(p)} \end{bmatrix} \quad \text{and} \quad F_{\eta_{r,q}} = \begin{bmatrix} F_{\eta_{r,q}}^{(h)} & F_{\eta_{r,q}}^{(h,p)} \\ F_{\eta_{r,q}}^{(p,h)} & F_{\eta_{r,q}}^{(p)} \end{bmatrix}, \hspace{1cm} (72)$$

December 2, 2021  DRAFT
where \( \mathbf{F}_{\eta_d}^{(h)} \in \mathbb{C}^{2 \times 2} \), \( \mathbf{F}_{\eta_d}^{(p)} \in \mathbb{C}^{5 \times 5} \), \( \mathbf{F}_{\eta_r}^{(h)} \in \mathbb{C}^{2 \times 2} \), \( \mathbf{F}_{\eta_r}^{(p)} \in \mathbb{C}^{3 \times 3} \), and the remaining matrices have matching dimensions. Then, for the first term in the product on left-hand side (L.H.S.) of (70), we can calculate

\[
J^T \hat{\mathbf{F}}_\eta J = \begin{bmatrix} \mathbf{Z}_{1,1} & \mathbf{Z}_{1,2} \\ \mathbf{Z}_{2,1} & \mathbf{Z}_{2,2} \end{bmatrix},
\]  

(73)

where we denote

\[
\mathbf{Z}_{1,1} = (J_d^{(p)})^T \mathbf{F}_{\eta_d}^{(p)} J_d^{(p)} + \sum_{q=1}^{Q} (J_r^{(p)})^T \mathbf{F}_{\eta_r}^{(p)} J_r^{(p)} ,
\]

\[
\mathbf{Z}_{1,2} = \begin{bmatrix} (J_d^{(p)})^T \mathbf{F}_{\eta_d}^{(p)} & (J_r^{(p)})^T \mathbf{I}_{\eta_r} & \cdots & (J_r^{(p)})^T \mathbf{I}_{\eta_r,Q} \end{bmatrix} ,
\]

\[
\mathbf{Z}_{2,1} = \mathbf{Z}_{1,2}^T ,
\]

\[
\mathbf{Z}_{2,2} = \begin{bmatrix} \mathbf{F}_{\eta_d}^{(h)} & 0 & \cdots & 0 \\ 0 & \mathbf{I}_{\eta_r,1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{I}_{\eta_r,Q} \end{bmatrix} .
\]

From (73), we have

\[
[(J^T \hat{\mathbf{F}}_\eta J)^{-1}]_{1:3,1:3} = (\mathbf{Z}_{1,1} - \mathbf{Z}_{1,2} \mathbf{Z}_{2,2}^{-1} \mathbf{Z}_{2,1})^{-1} \begin{bmatrix} \mathbf{I} & -\mathbf{Z}_{1,2} \mathbf{Z}_{2,2}^{-1} \end{bmatrix}.
\]  

(74)

From the definitions of \( \tilde{\mathbf{C}}_{p_d}^{(d)} \) in (48) and \( \tilde{\mathbf{C}}_{p_u}^{(r,q)} \) in (49), we can check that \([ (J^T \hat{\mathbf{F}}_\eta J)^{-1}]_{1:3,1:3} = \tilde{\mathbf{C}}_{p_u} \). Therefore, (74) can be rewritten as

\[
[(J^T \hat{\mathbf{F}}_\eta J)^{-1}]_{1:3,1:3} = \tilde{\mathbf{C}}_{p_u} \begin{bmatrix} \mathbf{I} & -\mathbf{Z}_{1,2} \mathbf{Z}_{2,2}^{-1} \end{bmatrix} .
\]  

(75)

For the second and third terms in the product on the L.H.S. of (70), we have

\[
[J^T \hat{\mathbf{F}}_\eta J]_{1:7} J_d = \begin{bmatrix} \begin{bmatrix} (J_d^{(p)})^T \mathbf{F}_{\eta_d}^{(p)} J_d^{(p)} & (J_d^{(p)})^T \mathbf{F}_{\eta_d}^{(p,h)} \\ \mathbf{F}_{\eta_d}^{(h,p)} J_d^{(p)} & \mathbf{F}_{\eta_d}^{(h)} \end{bmatrix} \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix} .
\]  

(76)
Combining (75) and (76) gives

\[
\left[ (J^\top \bar{F}_\eta J)^{-1} \right]_{1:3, :} [J^\top \bar{F}_\eta J]_{1:7, :} \bar{J}_d = \bar{C}_{p_d} [ (J_d^{(p)} \bar{F}_\eta^{(p)} J_d^{(p)}) - (J_d^{(p)} \bar{F}_\eta^{(h)} (F^{(h,p)}_\eta J_d^{(p)})^{-1} F^{(h,p)}_d J_d^{(p)}) 0].
\]

We can verify that

\[
(J_d^{(p)} \bar{F}_\eta^{(p)} J_d^{(p)}) - (J_d^{(p)} \bar{F}_\eta^{(h)} (F^{(h,p)}_\eta J_d^{(p)})^{-1} F^{(h,p)}_d J_d^{(p)}) = (\bar{C}_{p_d})^{-1}.
\]

Thus, we have proved (70) and (71). This concludes the proof.

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