Reply to “Comment on ‘Minimal size of a barchan dune’”

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We reply to the preceding comment by Andreotti and Claudin [Phys. Rev. E 76, 063301 (2007)] on our paper [Phys. Rev. E 75, 011301 (2007)]. We show that the equations of the dune model used in our calculations are self-consistent and effectively lead to a dependence of the minimal dune size on the wind speed through the saturation length. Furthermore, we show that Meridiani Planum ripples are probably not a good reference to estimate the grain size of Martian dune sands: the soil in the ripple troughs at the landing site is covered with nonerodible elements (“blueberries”), which increase the minimal threshold for saltation by a factor of 2. We conclude that, in the absence of large fragments as the ones found at the landing site, basaltic grains of diameter \( d = 500 \pm 100 \ \mu \text{m} \) that compose the large, typical dark Martian dunes [K. S. Edgett and P. R. Christensen, J. Geophys. Res. 96, 22765 (1991)] probably saltate during the strongest storms on Mars. We also show that the wind friction speed \( u_f \approx 3.0 \text{ m/s} \) that we found from the calculations of Martian dunes is within the values of maximum wind speeds that occur during Martian storms a few times a decade [R. E. Arvidson et al., Science 222, 463 (1983); H. J. Moore, J. Geophys. Res. 90, 163 (1985); R. Sullivan et al., Nature (London) 436, 58 (2005); D. J. Jerolmack et al., J. Geophys. Res. 111, E12S02 (2006)]. In this manner, the dune model predicts that Martian dunes can be formed under present Martian conditions, with no need to assume other conditions of wind and atmosphere that could have prevailed in the past.

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In the preceding comment [1], Andreotti and Claudin [Phys. Rev. E 76, 063301 (2007)] claim to find inconsistencies in the dune model which has been used by Parteli et al. [2] in the study of the minimal size of barchan dunes. This model, which consists of a coupled set of equations for the wind profile over the topography, the sand flux, and the evolution of the topography with time, has been originally presented in Refs. [3,4], later improved in Refs. [5,6], and repeatedly tested through successful quantitative comparison with real wind tunnel data and with real dunes measured in the field [3,6–8].

In Ref. [2], Parteli et al. studied, with the dune model, the role of the wind strength and interdune flux for the shape and the size of the minimal dune, and used the results to obtain the wind velocity on Mars from the minimal size of Martian dunes.

The first criticism of Andreotti and Claudin [1] is that the dune model is not self-consistent. They state that the saturation length of the sand flux, which determines the minimal dune size, should not decrease with the wind velocity because the relaxation rate is limited by the grain inertia. Next, Andreotti and Claudin [1] find that the grain size of the ripples at Meridiani Planum landing site on Mars is \( d = 87 \pm 25 \ \mu \text{m} \), which is much smaller than the grain size \( d = 500 \pm 100 \ \mu \text{m} \) of the larger, dark Martian dunes, as obtained from thermal inertia data [9] and used in the calculations of Parteli et al. [2].

Andreotti and Claudin, then, propose an alternative explanation for the dependence of the minimal size on the wind speed: the effect of slopes.

The comments of Andreotti and Claudin [1] are constructive and the issues addressed by these authors deserve to be discussed in depth. We organize the present reply paper following the same structure of the preceding comment [1]: Sec. I, regarding the modeling of the flux saturation length and the self-consistency of the dune model; Sec. II, concerning the grain size of Martian dune sand and the reliability of the value of Martian wind velocity obtained by Parteli et al. [2]; and Sec. III, concerning the effect of slopes on the minimal dune size.

I. SAND TRANSPORT MODEL

The first criticism of Andreotti and Claudin [1] refers to an apparent inconsistency in our sand transport model. They say that, since the grain inertia is not included in the evolution of the sand flux, the saturation length determined by the ejection process can be smaller than the length needed for the grains to reach their asymptotic trajectory.

Indeed, in the current model for sand transport we assume that the characteristic length for the relaxation of the mean grain velocity in the saltation layer is much smaller than the flux relaxation length determined by grain ejection. This can lead to a discrepancy with the full model for wind shear velocities \( u_s \) far from the threshold \( u_w \). In the following we calculate a modified saturation length that takes into account both processes and show that the saturation length \( l_s \) is determined by the ejection process for the typical range of shear velocities found on Earth, i.e., \( u_s < 3 u_w \approx 0.7 \text{ m/s} \).

Notice that all previous sand dunes simulation results performed with the current sand transport model are included within both ranges [2–6].

Following the original approach of Sauermann et al. [3] the saltation belt is modeled as a granular fluid layer characterized by a vertically averaged mean velocity \( \vec{u} \) and grain density \( \rho \). Both magnitudes obey the mass and momentum conservation equations averaged over the \( z \) axis. The mass conservation over the saltation layer reads [3]
\[ \frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{u} = \rho \left( 1 - \frac{\rho}{\rho_s} \right) \left( 1 - \frac{\rho}{\rho_s} \right), \]  

where the right-hand term accounts for the interchange of particles between the saltation layer and the surface mainly due to the ejected grains by the splash. This term describes the relaxation toward saturation \( \rho_s \) of the grain density \( \rho \) in the saltation layer. Here, the saturation density, defined as the maximum amount of grains carried by the wind with a given shear velocity, is given by \( \rho_s = \frac{\rho_{th}}{2} \tau_{th} (U_s^2 - 1) \) and the characteristic saturation time \( T_s(u) = 2au/\left[ \gamma g (U_s^2 - 1) \right] \), where \( U_s = u_e/u_{th} \) is the relative wind shear velocity, \( \tau_{th} = \rho_s U_{th}^2 \) is the threshold shear stress, \( g \) is the gravity, \( \gamma \) and \( \alpha \) are model parameters, and \( \rho_f \) is the fluid density [3,6].

Furthermore, the model assumes that the saltation layer over a flat surface is only subjected to a mean wind drag force and a friction force. The latter accounts for the momentum lost during the inelastic grain collisions with the bed. The momentum conservation over a flat bed is given by [3]

\[ \frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = \frac{g}{u_{\text{fall}}} [\vec{v}(\rho) - \vec{u}] - \frac{g}{2\alpha} \vec{u} - \frac{g}{2\alpha} \vec{u}, \]

where \( u_{\text{fall}} \) is the grain settling velocity. The first right-hand term represents a Newtonian drag force exerted by the wind with an effective velocity \( \vec{v} \), while the second gives the bed friction.

### A. Linear analysis

For stationary one-dimensional (1D) profiles, the coupled system (1 and 2) has the equilibrium solution \( \left( \rho_s, u_e \right) \), which corresponds to the saturated state. Introducing the linear perturbations around the homogeneous solution \( \rho(x) = \rho_s [1 + \bar{\rho}(x)] \) and \( u(x) = u_e [1 + \bar{u}(x)] \), and selecting \( v(\rho) \) in the momentum equation, the linearized system becomes

\[ \frac{\partial \bar{\rho}}{\partial x} = -\frac{\bar{\rho}}{l_\rho} + \frac{\bar{u} \bar{v}}{l_v} \frac{\partial \bar{v}}{\partial x} = -\frac{\bar{u}}{l_v}, \]

where the characteristic relaxation lengths for the mean density and velocity of the saltation layer are, respectively,

\[ l_\rho = u_e T_s(u_e) = \frac{2au_e^2}{\gamma g (U_s^2 - 1)}, \]

\[ l_v = \frac{\sqrt{2\alpha}}{2g} u_{\text{fall}} u_e. \]

Here, the mean saturated grain velocity is given by \( u_e = u_{ed} [a + b (U_s - 1)] \), where the constants \( a \) and \( b \) depend on the grain diameter and the grain to fluid density ratio [3,6]. The relaxation length \( l_\rho \) accounts for the relaxation due to the ejection process, while \( l_v \) includes the grain inertia, given by the falling velocity \( u_{\text{fall}} \) in the momentum balance. From the linear system Eq. (3) the largest relaxation length toward saturation, defined as the saturation length \( l_s \), is given by

\[ l_s = \frac{2l_\rho}{(1 + l_\rho/l_v) - [1 - l_\rho/l_v]} = \max(l_\rho, l_v). \]

**Figure 1.** Dependence of the density, velocity, and overall relaxation lengths \( l_\rho \), \( l_v \), and \( l_s \), respectively, with the wind shear ratio \( \bar{U} \) for Earth conditions.

The soil of the Meridiani Planum landing site is covered with hematite spherules and fragments reaching millimeters in size. These hematite particles or “blueberries” are much larger and denser than the typical basaltic sand of Martian dunes. The landing site is in fact a field of coarse-grained ripples, whose interiors consist of fine basaltic sand in the

**II. THE SIZE AND DENSITY OF GRAINS ON MARS**

In the comment, Andreotti and Claudin [1] propose that the grains that constitute the sand of Martian dunes have diameter \( d = 87 \pm 25 \mu m \). This value has been obtained by the same authors in a previous work [10], in which they analyzed recent photographs of Martian ripples taken by the rovers at Meridiani Planum. However, the value of grain diameter obtained by Andreotti and Claudin [1] from the analysis of the Meridiani Planum ripples is much smaller than the grain size of the typical large intracratere dunes as obtained from thermal inertia data, i.e., \( d = 500 \pm 100 \mu m \) [9]. In fact, grain size determinations made from Mars orbit are far from being unambiguous [11] since detailed knowledge of grain size distribution, grain shapes, and other variables are required for accurate measurements of particle size.

In this manner, there is no doubt that the work of Claudin and Andreotti [10] is of relevance since the measurements of grain sizes performed by these authors are based on images of unprecedented resolution. However, care must be taken before generalizing their results of grain sizes obtained from the Meridiani Planum ripples to the typical large dark dunes on Mars.

**A. Threshold for saltation at Meridiani Planum**

The soil of the Meridiani Planum landing site is covered with hematite spherules and fragments reaching millimeters in size. These hematite particles or “blueberries” are much larger and denser than the typical basaltic sand of Martian dunes. The landing site is in fact a field of coarse-grained ripples, whose interiors consist of fine basaltic sand in the

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{u} = \rho \left( 1 - \frac{\rho}{\rho_s} \right) \left( 1 - \frac{\rho}{\rho_s} \right). \]

\[ \frac{\partial \bar{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = \frac{g}{u_{\text{fall}}} [\vec{v}(\rho) - \vec{u}] - \frac{g}{2\alpha} \vec{u} - \frac{g}{2\alpha} \vec{u} = -\frac{\bar{u}}{l_v}. \]
range of 50−125 μm, but which are armored with coarse grains at their crests [12,13]. Hematite particles with a median diameter of about 1.0 mm cover more than 75% of the crest area of all ripples. On the other hand, the coarse-grain coverage in ripple troughs is of almost 50%, whereas interripple areas are composed mostly of intact spherules having diameter of several millimeters, with median 3.0 mm. In comparison, “granule” ripple troughs on Earth have an insignificant coverage of large fragments, the coarse particles remaining almost entirely on the ripple crests [13–15].

The role of the blueberries for the transport of sand at Meridiani Planum ripple troughs is dramatic. It is well known that the presence of large particulates shielding a sand bed increases the minimal wind velocity \( u_{rt} \) for entrainment of the finer grains into saltation [16,17]. Gillette and Stockton [16] found experimentally that the minimal threshold \( u_{rt} \) of erodible grains with diameter \( d = 107 \) μm increased by a factor of \( k = 2.5 \) in the presence of nonerodible grains with diameters about \( D = 2.0 \)−4.0 mm having spatial coverage of 45%. In fact, the geometrical properties of these experiments are very similar to the ones of Meridiani Planum ripples. Indeed, a value of \( k = 2.0 \) was found later by Nickling and McKenna Neuman [17] from experiments with larger particles, where \( d = 270 \) μm and \( D = 18 \) mm.

On the basis of the results from the experiments mentioned in the last paragraph, it was possible to explain the formation of the Meridiani Planum ripples. As demonstrated in recent publications [12,13], there are strong evidences that the minimal wind velocity required to mobilize the sand grains at Meridiani Planum ripple troughs has been effectively increased by a factor \( k \) of about 2.0−2.5, as observed in experiments with sand bed shielded by nonerodible roughness mentioned above.

In the absence of nonerodible large fragments, the minimal wind velocity required to entrain sand grains into saltation can be calculated with Eq. (A1) [18]. We follow the idea of Jerolmack et al. [13] and calculate the modified threshold for saltation at Meridiani Planum \( ku_{rt} \) taking the average value \( k = 2.25 \). The result is shown by the dashed curve in Fig. 2. In this figure, the full, straight line represents the maximum allowed wind friction speed during the gusts of dust storm at Meridiani Planum: \( u_s = 3.5 \) m/s. This value of wind speed, which is probably achieved once in intervals of years [12], is estimated to be an upper bound because larger wind speeds would result in saltation of the hematite spherules, which evidently did not occur during formation of the ripples. As explained previously, the winds that formed the ripples at Meridiani Planum landing site have friction speed \( u_s \) in the range 2.5−3.5 m/s, the lower bound corresponding to the minimal threshold for creeping motion of the hematite grains [13].

Although the estimation of the modified threshold for saltation (dashed line of Fig. 2) is very crude, it suggests that the wind strength that sculpted the soils of Meridiani Planum was just sufficient to entrain the grains of smallest saltation threshold values, as recognized in Ref. [10]. The dashed area of Fig. 2 corresponds to the range of grain sizes that are entrained by the wind into saltation at Meridiani Planum \( 69 \leq d \leq 168 \) μm, assuming \( u_{rt} \) is about 2.25 times the value calculated with Eq. (A1). The minimum for saltation occurs in fact at about 100 μm, which is well within the range of grain sizes of the sand found in the interior of coarse-grained ripples, on the matrix bed in the ripple troughs, and within small pits and craters at Meridiani Planum, which apparently serve as particle traps [12].

For illustration, the value of \( u_{rt} \) obtained with Eq. (A1) for the grain diameter of Martian dunes \( d = 500 \) μm, is shown by the filled circle in Fig. 2. The empty circle shows the modified threshold \( 2.25u_{rt} \) for \( d = 500 \) μm. We see that saltation of such coarse grains at the Meridiani Planum landing site would require a wind of \( u_s = 5.0 \) m/s, which is much larger than the maximum value, 3.5 m/s. However, it is clear from Fig. 2 that basaltic grains much larger than those of the landing site can be entrained by a wind of strength \( u_s = 3.5 \) m/s, in places where sand is not shielded by nonerodible elements.

In conclusion, the threshold for saltation transport at the Meridiani Planum landing site is modified due to the presence of nonerodible hematite fragments on the soil. Thus, provided other factors as sand induration [19] are not affecting the local threshold for saltation, it is very plausible that the grains of Martian dunes, which have diameter \( d = 500±100 \) μm [9], are effectively entrained by formative winds of strength 2.5−\( u_s = 3.5 \) m/s under present Martian conditions, since the threshold for entrainment of such coarse grains is exceeded at such values of \( u_s \).

### B. The wind velocity that forms dunes on Mars

The main criticism in the comment by Andreotti and Claudin regarding the results on Martian dunes is that the
dune model predicts that “very strong” winds [1] are required to form the Martian dunes. We recall the value of Martian wind shear velocity obtained in Parteli et al. from the minimal dune size [2]: \( u_s = 3.0 \) m/s. However, values of \( u_s \) about 3.0 m/s are within maximum values of shear velocity on Mars, and occur only during the strongest dust storms [20]. Sand transport on Mars is, thus, expected to consist of short duration events (a few minutes) a few times a decade [12,21], and does not occur under typical Martian wind velocities, which are between 0.3 and 0.7 m/s [22]. We conclude that the value \( u_s = 3.0 \) m/s found in Parteli et al. [2] from the shape of Martian sand dunes is consistent with real values of wind velocities expected to occur during sand transport on Mars.

Furthermore, as shown in Ref. [23], Martian dunes of different shapes and sizes and at different locations on Mars can be explained without necessity to assume that they were formed “in the past under very strong winds” as stated in Ref. [1]. The calculations using the model presented in Parteli et al. [2] show that the wind velocity on Mars in fact does not exceed 3.0 m/s.

### III. Linear Stability Analysis: Relation Between the Unstable Dune Wavelength and the Saturation Length

In their comment, Andreotti and Claudin [1] also proposed a mechanism to understand the apparent scaling of the minimal dune size with the inverse of the wind shear stress, in addition to the scaling of the saturation length which arises from the derivation of the dune model (Sec. I).

Following the work of Rasmussen et al. [24], Andreotti and Claudin include the dependence of the threshold shear stress \( u_{th}^2 \) on the local slope \( \alpha = \partial_h \), into the linear stability analysis of the equations for the dune evolution. Originally the scaling of the saturated flux with the local slope is not only reduced to the threshold shear stress and can be written as [24]

\[
q_s = \chi \left[ u_s^2 - u_{th}^2 \right] \frac{(1 + \partial_h)}{(1 + \partial_h \tan \theta)} \left( 1 + \partial_h / \tan \theta \right),
\]

where \( \theta \) is the repose angle and \( u_s \) is the wind shear velocity.

With this slope dependence the linear perturbation of the normalized saturated flux in the Fourier space can be written as

\[
\hat{q}_s = \frac{1}{U_s - 1} \left( U_s^2 (A + iB) - \frac{i}{2 \tan \theta} (U_s^2 + 1) \right) \hat{h},
\]

where \( U_s = u_s / u_{th} \) and \( \hat{h} \) is the surface Fourier transform. From the linear stability analysis the surface perturbation \( \hat{h} \) is unstable [25] and the most unstable wavelength \( \lambda \) previously found by Andreotti and Claudin [1] is slightly corrected to

\[
\lambda = \frac{6 \pi A U_s^2}{U_s^2 (2 \tan \theta B - 1) - 1} I_s(U_s),
\]

where the dependence with the wind shear velocity is determined by the term \( 2 \tan \theta B - 1 \). For a repose angle \( \theta = 34^\circ \) and a typical value \( B = 1.5 \) this dependence is strong. However, \( B \) is not constant and decreases with the ratio \( \lambda / z_0 \), where \( z_0 \) is the aerodynamic roughness. This dependence of the wind-surface coupling parameter \( B \) with the most unstable wavelength cannot be ignored and leads to a new mechanism in the dune size selection. Summarizing, in agreement with Ref. [1], by taking into account the slope effect in the shear velocity threshold, the contribution of the wind speed to the dune size selection becomes a complex issue that needs further study.

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### Appendix: Threshold Wind Velocity for Saltation

Iversen and White [18] proposed the following equation to calculate the threshold \( u_{th} \) for direct entrainment of grains into saltation

\[
A = \frac{0.129 \left( 1 + 6.0 \times 10^{-7} / \rho_{grain} d_{2.5} 0.5 \right)}{(1.928 \ Re_{st}^{0.092} - 1)^{0.5}}
\]

for \( 0.03 < \text{Re}_{st} < 10 \) and

\[
A = 0.129 (1 + 6.0 \times 10^{-7} / \rho_{grain} d_{2.5} 0.5)
\times \left( 1 - 0.0858 \exp(-0.0617 / (\text{Re}_{st} - 10)) \right)
\]

for \( \text{Re}_{st} \geq 10 \), where \( \nu \) is the kinematic viscosity, \( \text{Re}_{st} \) is the friction Reynolds number \( \text{Re}_{st} = u_{th} d / \nu \), and the constant \( 6.0 \times 10^{-7} \) has units of kg m\(^{-0.5}\) s\(^{-2}\), while all other numbers are dimensionless. The solid curve in Fig. 2 shows \( u_{th} \) as function of the grain diameter calculated with Eq. (A1) using \( \rho_{grain} = 3200 \) kg/m\(^3\), \( \rho_{fluid} = 0.02 \) kg/m\(^3\), and \( \nu = 6.35 \times 10^{-4} \) m\(^2\)/s [13].
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