Statistics of biased tracers in variance-suppressed simulations

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Simulations

DM Models

Taken from Lovell et. al. (2012)

Galaxy Formation

Taken from Pillepich et. al. (2019)

Non-linear structure formation

Taken from Angulo et. al. (2020)

Taken from Zennaro et. al. (2021)

Taken from Pellejero-Ibañez et. al. (2021)
Simulations are noisy due to having finite volume.

Variance Suppression Methods

Angulo & Pontzen (2016), Chartier et. al (2020), ...

Taken from Schneider et. al. (2016)
Fixing

\[ P(|\delta(k)|, \theta_k) = \frac{|\delta|}{\pi L^3 P} e^{-|\delta|^2/L^3 P} . \]

Fix amplitudes of the initial modes to:

\[ |\delta_L(k)| = \sqrt{P(k)} \quad \theta(k) \in [0, 2\pi] \]

\[ \delta_L(k) = \sqrt{P(k)}e^{i\theta(k)} \]

\[ \delta(k)\delta(-k) \approx \sqrt{P(k)}e^{i\theta(k)} \sqrt{P(k)}e^{-i\theta(k)} = P(k) \]

Taken from Villaescusa-Navarro et al. (2018)
Fixing and Pairing

Pairing

\[ \delta_A(k) = \sqrt{P(k)} e^{i\theta(k)} \]

\[ \delta_B(k) = \sqrt{P(k)} e^{i(\theta(k)+\pi)} = -\delta_A(k) \]

Simulation A \( z = 0 \)  A-IC \( z = 99 \)  B-IC \( z = 99 \)  Simulation B \( z = 0 \)

Taken from Pontzen et. al. (2016)
Can F&P help make this more precise?

\[ P_{gg}(k) = \sum_{i,j \in \{\delta, \delta^2, s^2, \nabla^2 \delta\}} b_i b_j P_{ij}(k) + \frac{1}{\bar{n}} \]
COLA Simulations

L-PICOLA: Implementation of the COLA algorithm (Howlett et. al. 2015).

This is a PM N-body code that works in a frame comoving with LPT trajectories. Much faster with good accuracy.

- Ensemble of 200 Gaussian simulations
- Ensemble of 200 F&P simulations
Testing for Biases
Variance Reduction
Hybrid Lagrangian Bias Model

- Are these spectra biased in F&F simulations?
- By how much are the variances reduced?
- What is the remaining level of noise (model error)?

Compute $P_{ij}$ in perturbation-theory

Numerically

Analytically
Quantitative Calculation

Using the approximate expressions for the power-spectra in the F&P regime we can compute their variances:

\[ \sigma_{ij}^2 = \left\langle (P_{ij}^{F&P})^2 \right\rangle - \left\langle P_{ij}^{F&P} \right\rangle^2 \]
We can forecast the simulation sizes necessary for the analysis of Euclid.

Using the coevolution relations determined in Zennaro et. al (2021)

\[
\sigma^2_E = \frac{P^2_{\text{EUCLID}}}{N_k}
\]

| $V_E [\text{Gpc}/h]^3$ | $\bar{n}_{\text{ELG}} [h/\text{Mpc}]^{-3}$ | $z$ | $b_1^k$ |
|------------------------|-------------------|-----|---------|
| 33.05                  | $1.83 \times 10^{-3}$ | 0.85 – 0.95 | 0.38 |

Using the largest value of $b_2$ allowed by the relations determined in Zennaro et. al (2021)
Conclusions

Upshots

- Fixing & Pairing enables unbiased and precise measurements of basis spectra
- Characterization and clear qualitative explanation of variance reduction in basis spectra
- Theoretical calculations able to predict the remaining variances and forecast model error
- Developed and tested method to reduce variance in terms unaffected by F&P

Take-home Message

- Fixing & Pairing greatly eases the task of using N-Body simulations to build models for galaxy clustering, and does so without introducing biases;
- Robust way to know a priori the effect of F&P in n-point functions, and to compute their remaining variances

For the Future

- Make the optimization procedure more robust
- Use it to identify the properties responsible for this variance cancellation

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