From weak-scale observables to leptogenesis

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Abstract

Thermal leptogenesis is an attractive mechanism for generating the baryon asymmetry of the Universe. However, in supersymmetric models, the parameter space is severely restricted by the gravitino bound on the reheat temperature $T_{RH}$. For hierarchical light neutrino masses, it is shown that thermal leptogenesis can work when $T_{RH} \sim 10^9$ GeV. The low-energy observable consequences of this scenario are $\text{BR}(\tau \rightarrow \ell \gamma) \sim 10^{-8} - 10^{-9}$. For higher $T_{RH}$, thermal leptogenesis works in a larger area of parameter space, whose observable consequences are more ambiguous. A parametrisation of the seesaw in terms of weak-scale inputs is used, so the results are independent of the texture chosen for the GUT-scale Yukawa matrices.

1 Introduction

Leptogenesis [1] is an appealing mechanism for producing the baryon asymmetry of the Universe [2]. In the seesaw model [3], heavy singlet (“right-handed”) neutrinos $\nu_R$ decay out-of-equilibrium, producing a net lepton asymmetry, which is reprocessed by Standard Model (SM) $B+L$ violating processes [4] into a baryon asymmetry. A natural and cosmology-independent way to produce the $\nu_R$ is by scattering in the thermal plasma. This scenario is referred to as “thermal leptogenesis”. However, the lightest $\nu_R$ can be produced only if their mass $M_1$ is less than the reheat temperature $T_{RH}$ of the plasma after inflation. In addition, the asymmetry is proportional to $M_1$ [5], so there is a lower bound on $M_1$ to get a large enough asymmetry. This implies $10^8$ GeV $< M_1 < T_{RH}$.

The seesaw is an attractive minimal extension of the SM that generates the observed small $\nu$ masses. Three right-handed neutrinos, with large majorana masses $M_i$, are added to the Standard Model, along with a Yukawa matrix for the neutrinos. It is desirable to supersymmetrise the seesaw, to address the hierarchy between the weak scale and the $M_i$. In the SUSY seesaw, $T_{RH}$ must be low enough to avoid over-producing gravitinos [6], the canonical bound for gravity mediated SUSY breaking is $T_{RH} \lesssim 10^9$ GeV. The aim of this paper is to identify the parameter space where thermal leptogenesis can work, taking $M_1 \sim T_{RH} \sim 10^9$ GeV.

The SUSY seesaw has more low-energy consequences than the non-SUSY version. It induces lepton flavour violating (LFV) entries in the slepton mass matrix, which can lead to radiative lepton decays [8], such as $\mu \rightarrow e\gamma$, at experimentally accessible rates. Eighteen parameters are required to define the neutrino and sneutrino mass matrices (in the charged lepton mass eigenstate basis), which is the same number as there are high scale inputs for the seesaw model [9]. It can be shown that the SUSY seesaw can be parametrised with the sneutrino and light neutrino mass matrices [10], in a texture model independent way. That is, the high-scale physical inputs of the SUSY seesaw—the $\nu_R$ masses $M_i$ and Yukawa coupling
Y_L—can be “reconstructed” from the neutrino and sneutrino mass matrices\(^1\). The baryon asymmetry can therefore be expressed as a function of weak scale observables. In this paper we identify the ranges of experimentally measurable quantities which are consistent with thermal leptogenesis. This phenomenological analysis differs from previous work \([11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22]\) by making minimal assumptions about the high scale theory: we assume the SUSY seesaw and universal soft masses at the GUT scale. The usual approach is to assume a GUT-scale texture that generates the desired neutrino mass matrix, and discuss leptogenesis—the aim here is to input the slepton mass matrix instead of a texture. This analysis should be consistent with all GUTs and texture choices covered by these assumptions.

Section 2 includes notation, and a review of leptogenesis and our parametrisation of the seesaw model. Section 3 presents approximate analytic expressions for the quantities on which leptogenesis depends. The low energy signatures of the parameter space where thermal leptogenesis works are discussed in section 4. CP violation is briefly discussed in section 5. The results are discussed and summarised in section 6.

2 Review

The observed deficits in muon neutrinos from the atmosphere \(^{23}\) and in electron neutrinos from the sun \(^{24, 25, 26}\) can be fit with small neutrino mass differences. The recent KamLAND observation of a $\bar{\nu}_e$ deficit from reactors confirms the neutrino mass explanation of the solar neutrino puzzle \(^{27}\). The small $\Delta m^2$ are consistent with three patterns of neutrino mass: hierarchical ($\Delta m_{atm}^2 = m_3^2$, $\Delta m_{sol}^2 = m_2^2$), degenerate ($m_3 \approx m_2 \approx m_1 \gg \Delta m_{atm}^2$) and quasi-Dirac ($m_3^2 \approx m_2^2 \approx \Delta m_{atm}^2$, $\Delta m_{sol}^2 = m_3^2 - m_2^2$). The leptogenesis scenario considered in this paper, where the $\nu_R$ are produced by scattering in the plasma, does not work for degenerate $m_i$ \(^{[5]}\) (see also \(^{[28]}\) for a detailed discussion). The quasi-Dirac spectrum could be interesting, although it is possibly disfavoured by supernova data \(^{29}\). We assume the $m_i$ are hierarchical, so the neutrino masses are much smaller than the charged lepton and quark masses. These small masses can be naturally understood in the seesaw model.

In subsection 2.1, the seesaw is reviewed from the top-down; introducing new physics at a high scale $M_X$, and seeing its low energy implications. This approach has been followed by many model builders who construct a natural or symmetry-motivated structure of the high-scale mass and Yukawa matrices, and then study its low energy consequences. See e.g. \(^{30}\) for early works that produce neutrino mass matrices with small mixing angles, and \(^{31}\) for more complete up-to-date references. Lepton flavour violation due to the SUSY seesaw, which could be observed in $\ell_j \to \ell_i \gamma$ \(^{8, 32}\) or in slepton production and decay at colliders \(^{33}\) has also been extensively studied from a top down approach (see e.g citations of \(^{8, 32}\)). Recent studies (for instance \(^{34}\)) have considered the branching ratios for $\ell_j \to \ell_i \gamma$ in models that induce the two observed large mixing angles among the light leptons.\(^2\).

Subsection 2.3 is a “top-down” review of leptogenesis, which is the obvious approach \([11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22]\) since the asymmetry is generated at high scales. See \([11]\) for examples and models. The translation between this approach and our bottom-up phenomenological analysis is not obvious, so it is difficult to relate our work to these papers. A phenomenological analysis of leptogenesis in non-SUSY (so no LFV) SO(10) models was discussed in \([16, 22]\), with particular attention to the low-energy CP violation. A Yukawa-matrix independent analysis has also been done in the case where there are only two right-handed neutrinos \(^{17}\).

\(^1\)This “reconstruction” would require universal soft masses at the GUT scale, and improbable experimental accuracy at the weak scale, so is in practice impossible.

\(^2\)see also \(^{35}\) for a more phenomenological discussion of $\mu \to e\gamma$
2.1 Notation and Numbers

We consider the supersymmetric see-saw for two reasons: first, supersymmetry stabilizes the Higgs mass against the quadratic divergences that appear due to heavy particles (e.g., the right-handed neutrinos). Secondly, the slepton masses enter our bottom-up parametrization of the see-saw.

The lepton part of the superpotential reads

\[ W_{\text{lep}} = e_R^c T Y_e L \cdot H_d + \nu_R^c T Y_\nu L \cdot H_u - \frac{1}{2} \nu_R^c T M \nu_R^c, \]

where \( L_i \) and \( e_R (i = e, \mu, \tau) \) are the left-handed lepton doublet and the right-handed charged-lepton singlet, respectively, and \( H_d (H_u) \) is the hypercharge \(-1/2 ( +1/2)\) Higgs doublet. \( Y_e \) and \( Y_\nu \) are the Yukawa couplings that give masses to the charged leptons and generate the neutrino Dirac mass, and \( M \) is a \( 3 \times 3 \) Majorana mass matrix. This is the minimal see-saw; additional terms are possible, for instance in SO(10) models a small triplet \( v_{3\nu} (T) \) is probable \[30\], leading to a \( \nu_L (T) \nu_L \) mass term.

We work in the left-handed basis where the charged lepton mass matrix is diagonal, and in a basis of right-handed neutrinos where \( M \) is diagonal

\[ D_M \equiv \text{diag}(M_1, M_2, M_3), \]

with \( M_i \geq 0 \), and \( M_1 < M_2 < M_3 \). In this basis, the neutrino Yukawa matrix must be non-diagonal, but can always be diagonalized by two unitary transformations:

\[ Y_\nu = V_R^T D_Y V_L, \]

where \( D_Y \equiv \text{diag}(y_1, y_2, y_3) \) and \( y_1 \ll y_2 \ll y_3 \). Later in the paper, we will assume that \( D_Y \) is hierarchical, with a steeper hierarchy than is in the light neutrino mass matrix: \((y_1/y_2)^2 \ll m_1/m_2\).

It is natural to assume that the overall scale of \( M \) is much larger than the electroweak scale or any soft mass. Therefore, at low energies the right-handed neutrinos are decoupled and the corresponding effective Lagrangian contains a Majorana mass term for the left-handed neutrinos: \( \delta \mathcal{L}_{\text{lep}} = -\nu_R^c T m_\nu \nu_R^c + \text{h.c.} \), with

\[ m_\nu = m_D^T \cdot M^{-1} \cdot m_D = Y_\nu^T \cdot M^{-1} \cdot Y_\nu \langle H_u^0 \rangle^2. \]

We define the Higgs vev, \( \langle H_u^0 \rangle^2 = v_u^2 = v^2 \sin^2 \beta \), where \( v = 174 \text{ GeV} \). In the basis where the charged-lepton Yukawa matrix, \( Y_e \) and the gauge interactions are diagonal, the \( [m_\nu] \) matrix can be diagonalized by the MNS \[35\] matrix \( U \) according to

\[ U^T [m_\nu] U = \text{diag}(m_1, m_2, m_3) \equiv D_m, \]

where \( U \) is a unitary matrix that relates flavour to mass eigenstates

\[ \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \]

and the \( m_i \) can be chosen real and positive, and ordered such that \( m_1 < m_2 < m_3 \). Assuming hierarchical left-handed \( \nu \) masses, we take \( m_2^2 = \Delta m^2_{\text{atm}} = 2.7 \times 10^{-3} \text{eV}^2 \) \[34\] and \( m_3^2 = \Delta m^2_{\text{solar}} = 7.0 \times 10^{-5} \text{eV}^2 \) \[40\]. This corresponds to \( m_3 = 5.2 \times 10^{-2} \text{eV} (3.9 - 6.3 \times 10^{-2} \text{eV} \text{ at 90\% C.L.}), \) and \( m_2 = 8.2 \times 10^{-3} \text{eV} (7 - 15 \times 10^{-3} \text{eV at 3 \sigma}) \). \( m_1 \) is unknown, usually unimportant, and we take it to be \( m_2/10 \). As we
shall see, the baryon asymmetry is weakly dependent on $\tan \beta$ in the parametrisation we use, so we set $\sin \beta = 1$.

$U$ can be written as

$$U = V \cdot \text{diag}(e^{-i\phi/2}, e^{-i\phi'/2}, 1),$$

where $\phi$ and $\phi'$ are CP violating phases, and $V$ has the form of the CKM matrix

$$V = \begin{pmatrix}
    c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\
    -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}c_{12}e^{i\delta} & s_{23}s_{13} \\
    s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}c_{12}e^{i\delta} & c_{23}s_{13}
\end{pmatrix}. \tag{8}$$

The numerical values of the angles are $0.28 \leq \tan^2 \theta_{sol} \leq 0.91$ (3$\sigma$), with best fit point $\tan^2 \theta_{sol} = 0.44$ [10], so $\theta_{sol} = 41$. We take $\theta_{atm} = \pi/4$. The CHOOZ angle $\theta_{13}$ is experimentally constrained $\sin \theta_{13} \leq 0.2$ [11]. Considerable effort and thought has gone into designing experiments sensitive to smaller values of $\theta_{13}$. J-PARC hopes to reach $O(0.05)$ [12], and a neutrino factory could detect $\theta_{13}$ as small as $0.02 \rightarrow 0.001$ [12,13].

We assume a simple gravity-mediated SUSY breaking scenario, with universal soft masses at the scale $M_X$. The sneutrino mass matrix (in the charged lepton mass eigenstate basis) can be written in the leading log approximation as

$$(m_{\tilde{e}}^2)_{ij} \sim \text{ (diagonal part)} - \frac{3m_0^2}{8\pi^2}[Y^\dagger_{\nu}]_{ik}[Y_{\nu}]_{kj} \ln \frac{M_X}{M_k}, \tag{9}$$

where “diagonal-part” includes the tree level soft mass matrix, the radiative corrections from gauge and charged lepton Yukawa interactions, and the mass contributions from F- and D-terms.

The branching ratio for $\ell_j \rightarrow \ell_i \gamma$ can be estimated

$$\frac{BR(\ell_j \rightarrow \ell_i \gamma)}{BR(\ell_j \rightarrow \ell_i \nu_{\ell_j})} \sim \frac{\alpha^3}{G_F m_{\tilde{L}}^2} |y_{\tilde{L}}^*_{kj} V_{LL_i} V_{LL_j}^*|^2 \frac{2\tan^2 \beta \simeq 10^{-7}}{y_{\tilde{L}_i}} \left(\frac{m_{\nu_{\ell_j}}}{100 \text{GeV}}\right)^4 \left(\frac{\tan \beta}{2}\right)^2 \tag{10}$$

where $C \sim O(0.001 \div 0.01)$, and $V_{LL}$ diagonalises the second term of eqn (6). More accurate formulae for the branching ratios can be found in [32]. To further simplify these estimates, it would be convenient to assume that $V_{LL} = V_L$. That is, the lepton asymmetry will be a function of the angles of $V_L$, and it would be simplest to estimate $\ell_j \rightarrow \ell_i \gamma$ using the angles of $V_L$ for those of $V_{\tilde{L}}$. This a reasonable approximation when $\theta_{Li} \gg \frac{\Delta}{\theta_{Rij}} (i < j)$, where $\theta_{Rij} (\theta_{Lij})$ is an angle of $V_R (V_L)$. For hierarchical Yukawa eigenvalues, this is likely to be true, even if an angle $\theta_R$ in $V_R$ is large, because the usual texture estimate for $\theta_{Li}$ would be $\sqrt{y_i/y_j}$. We assume this condition is verified, so the principle contribution to $[m_{\tilde{e}}^2]_{ij} \propto y_{\tilde{L}_i}^2 V_{LL_i} V_{LL_j}$.

Table 2.1 lists the current upper limits on the $\mu \rightarrow e\gamma$, $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ branching ratios, and the corresponding bounds on $V_{Li}$ that can be estimated from eqn (10) [44]. It also contains the hoped for sensitivity of some anticipated rare decay searches. Colliders could also be sensitive to flavour violating slepton masses [45].

Leptogenesis will depend on angles of $V_L$. In the remainder of the paper, we will claim that “leptogenesis predicts an observable $BR(\ell_j \rightarrow \ell_i \gamma)$”, if the $V_L$ elements required exceed the last column of table 2.1 (last three rows), with $y_3 = 1$. It is clear that the branching ratios can be decreased, for fixed $V_L$, by decreasing $y_3$ and adjusting weak scale SUSY parameters. However, if SUSY is discovered, these masses and mixing angles could in principle be measured at colliders, and some information about the
magnitude of $y_3$ could be available through the renormalisation group equations [45]. This assumption of universal soft masses at the scale $M_X$ will not be crucial for our conclusions. Additional contributions to the off-diagonal soft masses are unlikely to cancel the ones we discuss, so the lower bounds we set on LFV branching ratios, from requiring leptogenesis to work, should remain.

$$BR(\mu \to e\gamma) < 1.2 \times 10^{-11}$$  
(PSI)  
$$y_3^2 V_{L32}^* V_{L31} < .006$$

$$BR(\tau \to e\gamma) < 2.7 \times 10^{-6}$$  
(CLEO)  
$$y_3^2 V_{L33}^* V_{L31} \approx 12$$

$$BR(\tau \to \mu\gamma) < 1.1 \times 10^{-6}$$  
(CLEO)  
$$y_3^2 V_{L33}^* V_{L32} < 9$$

$$BR(\mu \to e\gamma) \sim 10^{-14\pm15}$$  
(PSI/ufact)  
$$y_3^2 V_{L32}^* V_{L31} \sim 3 \times 10^{-4}$$

$$BR(\tau \to e\gamma) \sim 10^{-9}$$  
(BABAR/BELLE)  
$$y_3^2 V_{L33}^* V_{L31} \sim 0.2$$

$$BR(\tau \to \mu\gamma) \sim 10^{-9}$$  
(BABAR/BELLE/LHC)  
$$y_3^2 V_{L33}^* V_{L32} \sim 0.2$$

Table 1: Current limits [46] and hoped for sensitivities [47] of some experiments. The numerical bounds in the right column are multiplied by $(m_{SUSY}/100 \text{ GeV})^2 \left(\frac{2}{\tan \beta}\right)$. If $\tan \beta$ is large, these rare decays are sensitive to smaller angles in $V_L$ [48].

Various CP violating phases in the neutrino and slepton mass matrices could be measured in upcoming experiments. However, the experimental sensitivity to the phases depends on the magnitude of unmeasured real parameters. Anticipated $0\nu\beta\beta$ experiments may be sensitive to a maximal phase $\phi'$, for the neutrino mass spectrum we consider. The minimum value of the angle $\delta$ that could be measured at a $\nu$ factory depends on $\Delta m_{32}^2, \Delta m_{21}^2$, and $\theta_{13}$ (see e.g. [43]), so there is no foreseeable clear upper bound. The imaginary part of the product of off-diagonal slepton masses $\Im \{m_{\tilde{\nu}}^{ij}\} = J (m_{\tilde{\nu}_2} - m_{\tilde{\nu}_1}) (m_{\tilde{\nu}_3} - m_{\tilde{\nu}_1}) (m_{\tilde{\nu}_1} - m_{\tilde{\nu}_3})$ could be measured in slepton flavour oscillations down to $J = 10^{-3}$ [10]. $J$ depends on the magnitude of the $m_{\tilde{\nu}}^{ij}$ as well as their phases, so all the phases in the (s)lepton sector can be of order 1.

2.2 Review of the parametrisation

It was shown in [10] that the seesaw can be parametrised from the bottom-up, using the neutrino and sneutrino mass matrices. See [50] for applications. A similar phenomenological parametrisation of the non-SUSY seesaw [41] could be used, if the scale $M$ of right-handed masses was low enough to measure dimension six operators $\propto 1/M^2$. We briefly review [10] here.

It is in principle possible to extract the matrix

$$P \equiv Y_{\nu}^\dagger Y_{\nu} = V_L^\dagger D_Y^2 V_L$$ (11)

from its contribution to the renormalisation group running of the slepton mass matrix. This relies critically on having universal soft masses at the GUT scale, and on very precise measurements of sneutrino masses and decays. It is therefore unrealistic [10]. However, since SUSY has not yet been discovered, $D_Y$ and $V_L$ can be used as inputs in a “bottom-up” parametrisation of the seesaw.
The aim is to determine $\mathbf{Y}_{\nu}$ and $\mathcal{M}$ from $\left[m_{\nu}\right]$ and $P$. $V_L$ and $D_Y$ can be determined from $P$, and used to strip the Yukawas off $\left[m_{\nu}\right]$:  
\[
D_Y^{-1} V_L^* \frac{\left[m_{\nu}\right]}{v_u^2} V_L D_Y^{-1} = V_R^* D_M^{-1} V_L^* = \mathcal{M}^{-1},
\]  
(12)
where the left hand side of this equation is known ($\left[m_{\nu}\right]$ is one of the inputs, and $V_L$ and $D_Y$ were obtained from eq. (11)). Therefore, $V_R$ and $D_M$ can also be determined. This shows that, working in the basis where the charged lepton Yukawa coupling, $\mathbf{Y}_e$, the right-handed Majorana mass matrix, $\mathcal{M}$, and the gauge interactions are all diagonal, it is possible to determine uniquely the heavy Majorana mass matrix, $\mathcal{M}$, and the neutrino Yukawa coupling, $\mathbf{Y}_\nu = V_R^* D_Y V_L$, starting from $\left[m_{\nu}\right]$ and $\mathbf{Y}_e^T \mathbf{Y}_\nu$.

2.3 Leptogenesis, and the upper bound

The see-saw mechanism provides a natural framework to generate the baryon asymmetry of the Universe, defined as $\eta_B = (n_B - n_{\bar{B}})/s$, where $s$ is the entropy density. As was shown by Sakharov, generating a baryon asymmetry requires baryon number violation, C and CP violation, and a deviation from thermal equilibrium. These three conditions are fulfilled in the out-of-equilibrium decay of the right-handed neutrinos and sneutrinos in the early Universe. In the remainder of this paper, “right-handed neutrinos”, and the shorthand notation $\nu_R$, refer to both right-handed neutrinos and right-handed sneutrinos.

In gravity mediated SUSY breaking scenarios, these is an upper bound from gravitino production on the reheat temperature $T_{RH}$ of the Universe after inflation. The gravitino has a mass $m_{3/2} \sim m_{SUSY}$ and only gravitational interactions with SM particles, so it is very weakly coupled, and long-lived. If a significant number of them decay at or after Big Bang Nucleosynthesis, they could disrupt the predicted abundances of light elements. Gravitinos can be created by various mechanisms in the early Universe, such as scattering in the thermal plasma, or direct coupling to the inflaton (preheating). The latter is effective, but avoidable. The number density of gravitinos produced in scattering increases with the plasma temperature, so the bound on $n_{3/2}$ sets an upper bound on the reheat temperature of the Universe after inflation of

$$T_{RH} \lesssim 10^9 \rightarrow 10^{12}\text{GeV}$$

(13)
(corresponding to $m_{3/2} \sim 100\text{ GeV} \rightarrow 10\text{ TeV}$). This bound assumes that the gravitino decays; there are models where the gravitino is the LSP, which allow $T_{RH} \lesssim 10^{11}\text{ GeV}$.

Let us briefly review the mechanism of generation of the BAU through leptogenesis. At the end of inflation, a certain number density of right-handed neutrinos, $n_{\nu_R}$, is somehow produced. If these right-handed neutrinos $\nu_R$ decay out of equilibrium, a lepton asymmetry can be created. The subsequent ratio of the lepton excess to the entropy density $s$ is given by

$$\eta_L = \frac{n_\ell - n_{\bar{\ell}}}{s} = \sum_i \frac{n_{\nu_R}}{s} \epsilon_i \tilde{d}_i.$$  

(14)

The CP-violating parameter $\epsilon_i$ is determined by the particle physics model that gives the masses and couplings of the $\nu_R$. The value of $n_{\nu_R}/s$ depends on the mechanism to generate the right-handed neutrinos. We assume the $\nu_R$ are generated by Yukawa scattering in the thermal plasma, in which case $n_{\nu_R}/s \lesssim n_{eq}/s \approx 2/g_*, n_{eq}$ is the equilibrium number density of massless particles, and $g_* \simeq 300$ is the number of propagating states in the supersymmetric plasma. This also implies an

$^3$in our conventions, $n_{\nu_R} = (n_{\nu_R} + n_{\bar{\nu}_R})/2$. The .2 is an approximation to $g_* n_{eq}/s = \zeta(3)135/(8\pi^4)$.
upper bound on the $\nu_R$ mass: $M_1 \lesssim T_{RH}$. Finally, $\tilde{d}_1$ is the fraction of the produced asymmetry that survives after $\nu_R$ decay. To ensure $\tilde{d}_1 \sim 1$, lepton number violating interactions (decays, inverse decays and scatterings) must be out of equilibrium when the right-handed neutrinos decay. In the case of the lightest right-handed neutrino $\nu_{R_1}$, this corresponds approximately to

$$K = \frac{\Gamma_{D_1}}{2H|_{T=\tilde{M}_1}} < 1$$

(15)

where $H$ is the Hubble parameter at the temperature $T$, and $\Gamma_{D_1}$ the $\nu_{R_1}$ decay rate. There are two competing requirements on the $\nu_R$ parameters—the couplings must be large enough to produce a thermal distribution, but small enough that the $\nu_R$ decay out of equilibrium. Thermal leptogenesis has been carefully studied in [15]. The numerical results of [15] suggest that $n_{\nu_R} < n_{\nu_R}^e$: either $n_{\nu_R}$ does not attain its equilibrium number density, or lepton number violating interactions wash out a significant fraction of the asymmetry as it is produced. Defining an effective light neutrino “mass”

$$\frac{\tilde{m}_1}{v^2_u} = \frac{8\pi}{\sqrt{\frac{\Gamma_{D_1}}{\tilde{M}_1} M_1}} (\hat{Y}_\nu Y_\nu^\dagger)_{11}$$

(16)

$n_{\nu_R} \tilde{d}_1/s \gtrsim 10^{-4}$ is realised for $[58] 5 \times 10^{-5} \text{ eV} \lesssim \tilde{m}_1 \lesssim 10^{-2} \text{ eV}$. The precise numerical bound on $\tilde{m}_1$ depends on $M_1$, and can be found in [15].

For $\tilde{m}_1 > 10^{-4} \text{ eV}$ and $M_1 \sim 10^9 \text{ GeV}$, the dilution factor $d_1$ can be approximated [58, 59]

$$\frac{n_{\nu_R} \tilde{d}_1}{s} \equiv d_1 \sim \frac{1}{6g_*} \frac{1}{\sqrt{K^2 + 1}}$$

(17)

with $K \simeq 910\tilde{m}_1/eV$ from eqn (16). This is a slight modification of the approximation, to ensure that it falls between the $M_1 = 10^8 \text{ GeV}$ and $10^{10} \text{ GeV}$ lines of [11], in the relevant range $0.001 \text{ eV} \lesssim \tilde{m}_1 \lesssim 0.1 \text{ eV}$. The exact numerical factor is important, because it is difficult to get a large enough asymmetry. Multiplying $d_1$ by a factor of a few significantly increases the parameter space where thermal leptogenesis can work. The approximation (17) neglects the decrease in $d_1$ at $\tilde{m}_1 \lesssim 10^{-4} \text{ eV}$, which is due to underproduction of $\nu_{R_1}$ in scattering. This is reasonable, because $\tilde{m}_1 \gtrsim m_1$ [5], and we take $m_1 = m_2/10$.

The last step is the transformation of the lepton asymmetry into a baryon asymmetry by non-perturbative B+L violating (sphaleron) processes [4], giving

$$\eta_B = \frac{C}{\eta_{B-L}} = (3 - 9) \times 10^{-11},$$

(18)

where $C = 8/23$ in the Minimal Supersymmetric Standard Model. Big Bang Nucleosynthesis constrains $\eta_B$ to lie in the range of eqn (18). In a flat Universe, the CMB determines $\eta_B \simeq (0.75 - 1.0) \times 10^{-10}$ [58]. The wider BBN range is used in this paper, because it is difficult to generate a large enough $\eta_B$.

The CP asymmetry can be approximated as

$$\epsilon_1 \simeq -\frac{3}{8\pi} \frac{1}{|Y_\nu Y_\nu^\dagger|_{11}} \sum_j \text{Im} \left\{ |Y_{\nu^*} Y_{\nu^*}^\dagger|_{1j}^2 \left( \frac{M_1}{M_2} \right) \right\}$$

(19)

$$= -\frac{3}{8\pi} \frac{M_1}{|Y_\nu Y_\nu^\dagger|_{11}} \text{Im} \left\{ |Y_{\nu^*} m_{\nu^*}^*|_{11} \right\}.$$  

(20)

\(^4\text{See [20] for a detailed analysis of thermal leptogenesis at higher temperatures, including the effects of } \nu_R^2 \text{ and } \nu_R^3\)
if the lepton asymmetry is generated in the decay of the lightest right-handed neutrino, and if the masses of the right-handed neutrinos are hierarchical. Were the asymmetry produced in the decay of $\nu_{R2}$ or $\nu_{R3}$, it would depend on a different combination of couplings.

It is straightforward to show that if $\epsilon_1$ is written
\[
|\epsilon_1| = \frac{3}{8\pi v_u^2} M_1 m_3 \delta_{HMY}
\]
then eqn (20) implies the upper bound $\delta_{HMY} \leq 1$. The numerical results of [21, 60] agree with this constraint. Using eqns (14) and (18), this can be transformed into a lower bound on $M_1$:
\[
M_1 \gtrsim \frac{\eta_B}{C} \left[ \frac{n_{\nu_R} + n_{\bar{\nu}_R}}{s} \frac{3}{8\pi} \frac{m_3}{v_u^2} d_1 \right]^{-1} = 10^9 \left( \frac{\eta_B}{3 \times 10^{-11}} \right) \left( \frac{0.05 \text{eV}}{m_3} \right) \left( \frac{4 \times 10^{-4}}{d_1} \right) \text{GeV}.
\]

Setting $m_3$ to its 90% C.L. upper bound 0.063 eV, and $d_1$ to its maximum value $n_{\nu_R}/s \simeq 45/(2\pi^4 g_*)$, implies $M_1 > 3 \times 10^8$ GeV.

This lower bound on $M_1$ comes very close to the gravitino bound eqn (13) on the reheat temperature. Thermal production of the $\nu_R$ requires $M_1 < \sim T_{RH}$ so either $\epsilon$ is close to its upper bound, or $M_1, T_{RH} > 10^9$ GeV, or thermal leptogenesis does not generate the observed baryon asymmetry. We explore the first option, and somewhat the second. The third possibility, non-thermal $\nu_R$ production, has been discussed by many authors (see e.g. references of [61]).

3 Analytic approximations for $\delta_{HMY}$, $M_1$, $\tilde{m}_1$

At least three inputs are required to parametrise thermal leptogenesis [18, 11, 28]. A possible choice would be the mass $M_1$ and decay rate $\propto \tilde{m}_1$ of the $\nu_R$, and the CP asymmetry $\epsilon_1$. However, $\epsilon \propto M_1$, so we use $M_1$, $\tilde{m}_1$ and $\delta_{HMY}$ (introduced by Hamaguchi, Murayama and Yanagida), where $\delta_{HMY}$ measures how close $\epsilon$ comes to saturating its upper bound. Note however, that $\delta_{HMY}$ is not a CP phase.

This section contains simple analytic approximations indicating the dependence of leptogenesis parameters on measurable quantities, such as neutrino masses and rare LFV decays. We used this approximation, with attention to the phases, in [37]. A similar, somewhat simplified version was introduced in [16].

The inputs for the analytic approximation are:
\[
V_L, D_Y, U, \ [m_\nu]
\]
Two of the angles of $U$ are known, and the CHOOZ angle is bounded above. The eigenvalues $y_i$ of the neutrino Yukawa matrix are unknown, and realistically cannot be determined from the sneutrino mass matrix. It seems reasonable to assume a hierarchy for the $\{y_i\}$, since we measure hierarchical Yukawas for the quarks and charged leptons. The $y_i$ remain as variables in the equations; we will discover that only the smallest eigenvalue $y_1$ is relevant, and can be “traded” for the mass $M_1$ of the lightest $\nu_R$, which is tightly constrained. $V_L$ contains three unknown angles, related to the lepton flavour violating decays $\ell_j \rightarrow \ell_i \gamma$. There are three phases in both $U$ and $V_L$, all are unknown, and assumed to be chosen to maximise the baryon asymmetry.

The lightest eigenvalue and corresponding eigenvector of $\mathcal{M}$ are estimated in the first Appendix, which also contains some simple (but illuminating) 3-d plots of leptogenesis parameters. The mass of

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5If the hierarchy in $Y_\nu$ is similar to that of the quarks and charged leptons, then a hierarchy in the $M_i$ is natural.
the lightest $\nu_R$ is
\[ |M_1| \approx \frac{y_1^2 v_u^2}{|W_{1j}^2 m_j|}, \tag{24} \]
where the matrix $W = V_L U$ is the rotation from the basis where the $\nu_L$ masses are diagonal to the basis where the neutrino Yukawa matrix $Y_\nu = Y^\dagger \nu$ is diagonal. There are three limiting values for $M_1$, corresponding to $M_1 \approx y_1^2 v_u^2/m_1$: $M_1 \rightarrow y_1^2 v_u^2/m_1$ when $W_{1,3} \rightarrow 1$, $M_1 \rightarrow y_1^2 v_u^2/m_1$ when $W_{1,3}, W_{1,2} \rightarrow 0$, and $M_1 \rightarrow y_1^2 v_u^2/m_2$ when $W_{1,3} < m_2/m_3$, $W_{1,2} \rightarrow 1$. This is easy to see in figure 4.

$M_1$ is the only quantity relevant for leptogenesis which depends on $y_1$. The latter is effectively unmeasurable; it is constrained by theoretical expectations, and by the requirement that the analytic approximation be self-consistent. Theoretically, the eigenvalues of $Y_\nu$ are expected to be hierarchical, and of order the quark or lepton Yukawas, so figure 4 is plotted with $y_1 \approx 10^{-4}$. The approximations of this section are consistent, provided that the dropped $O(y_1^2, y_2^2/y_1^2)$ terms are smaller than the $O(m_1/m_3)$ terms which are kept. This is the case for $y_1 \sim 10^{-4}$. Since $y_1$ is unmeasurable and only weakly constrained, it can be adjusted, as function of $m_i$ and $W_{1,1}$, to obtain a value for $M_1$ where leptogenesis could work. In fact, since $M_1 \propto y_1^2$ is tightly constrained, the requirement $M_1 \approx 10^9$ GeV “determines” $y_1$.

The eigenvector (50) can be used to evaluate the $\nu_{R1}$ decay rate: eqn. (10) becomes
\[ \tilde{\nu}_1 \approx \frac{\sum_k |W_{1k}^2 m_k^2|}{\sqrt{\sum_n |W_{1n}^2 m_n|^2}} \tag{25} \]
$\tilde{\nu}_1$ has various limits: $\tilde{\nu}_1 \rightarrow m_3$ for $W_{1,3}$ large, $\tilde{\nu}_1 \rightarrow m_2$ for $W_{1,2}$ large and $W_{1,3} < m_2/m_3$, , and $\tilde{\nu}_1 \rightarrow m_1$ when $W \rightarrow 1$. This is easy to see from the RHS of figure 4. In the $W \rightarrow 1$ limit, washout is minimised, because the dilution factor $d_1 \propto 1/m_1$.

To saturate the upper bound (21), $\delta_{HMY}$ needs to approach 1. Evaluating eq. (20) with the eigenvector (50), gives
\[ \delta_{HMY} = \frac{\text{Im} \left\{ \sum_{k,m} W_{1k}^2 m_3^3 W_{1n}^2 m_n \right\}}{m_3 |\sum_n W_{1n}^2 m_n| (\sum_j |W_{1j}^2 m_j^2|)} \approx \frac{|W_{11} W_{12}^2 m_1 m_2^2 + |W_{11} W_{13}^2 m_1 m_3^2 | + |W_{12} W_{13}^2 m_2 m_3|}{m_3 (\sum_n |W_{1n}^2 m_n|) (\sum_j |W_{1j}^2 m_j^2|)} \tag{26} \]
This paper is about the relation between real low energy observables (such as $BR(\mu \rightarrow e \gamma)$) and the baryon asymmetry, so scant attention will be paid to the phases in $U$ and $V_L$. For most of parameter space, the phases can be chosen such that $\delta_{HMY}$ is larger than the second expression in eqn (20).

This second expression is plotted on the LHS in figure 5. $\delta_{HMY}$ can approach 1 if the numerator (upstairs) is dominated by $m_2^3 m_3$ or by $m_3^2 m_1$. This is because of the $m_3$ in the denominator. If the $m_3^2 m_1$ dominates upstairs, then $\delta_{HMY}$ will approach 1 when $W_{12}^2 m_j \approx W_{13}^2 m_1$ and $|W_{11}^2 m_n| \approx |W_{13}^2 m_3|$, or equivalently, when
\[ \frac{m_1^2}{m_3^2} < W_{13}^2 < \frac{m_1}{m_3} \ and \ \frac{m_1}{m_2} < \frac{m_3}{m_3} \ \tag{27} \]
This corresponds to the highest ridge in $\delta_{HMY}$ in figure 5. Notice that the position of the peak depends sensitively on the lightest neutrino mass $m_1$.\footnote{everywhere but when the three terms upstairs have equal magnitude}
If $m_3^2m_2$ dominates upstairs, then $\delta_{HMY} \to 1$ when
\[
W_{12}^2 \frac{m_2^2}{m_3^2} < W_{13}^2 < W_{12}^2 \frac{m_2^2}{m_3^2} \text{ and } W_{12}^2 < W_{12}^2 \frac{m_2^2}{m_3^2}.
\]
(28)

This corresponds to the shoulder at slightly large $W_{13}$, which is cut by $W_{12} \sim 1$ in the LH plot of figure 5.

Finally, for $W_{13}$ very small, $\delta \to m_2/m_3 \sim 0.1$ along the ridge at $W_{12} \sim 0.1$. This corresponds to the $m_3^2m_1$ term dominating upstairs, and arises when
\[
W_{11}^2 \frac{m_1^2}{m_2^2} < W_{12}^2 \frac{m_1^2}{m_2^2} \text{ and } W_{12}^2 < W_{12}^2 \frac{m_1^2}{m_3^2}.
\]
(29)

Although $\delta_{HMY}$ does not reach its maximum value for these parameters, the washout is small, so the baryon asymmetry generated is only slightly too small. As we will see in figure 3, it is large enough if $M_1, T_{RH} \sim 10^{10}$ GeV are allowed.

4 When does thermal leptogenesis work?

The baryon asymmetry can be approximated as
\[
\eta_B \simeq \frac{8d_1}{23} \frac{3}{8\pi v_u^2} M_1 m_3 \delta_{HMY}
\]
by combining eqns (14), (18), and (21). This is plotted in figure 1, which suggests that $\eta_B$ can be large enough.

The issue is whether a large enough asymmetry can be generated, so the observational upper limit on $\eta_B$ is unimportant. Also, the asymmetry calculated here is the upper bound corresponding to maximal CP violation, so it can be reduced by taking smaller phases. We use the one-σ observational lower bound on $\eta_B$ from nucleosynthesis: $\eta_B \gtrsim 3 \times 10^{-11}$. To obtain a large enough baryon asymmetry by thermal leptogenesis, the parameters $M_1, \delta_{HMY}$, and $d_1$ must occupy narrow ranges. The washout effects are minimised when the $\nu_R$ decay rate is small, which corresponds to $W \to 1$. In this case, $n_{\nu_R}d_1/s = d_1 < 10^{-3}$ which implies the lower bound $\epsilon \gtrsim 10^{-7}$. (If $\epsilon \gtrsim 10^{-6}$ can be obtained, then $d_1 \sim 10^{-4}$ is large enough.) Eqn (21) implies a lower bound on $M_1$ to get $\epsilon_1$ large enough. In addition, $M_1 \lesssim T_{RH}$ is required for thermal production; the canonical SUSY gravitino bound is $T_{RH} \lesssim 10^9$ GeV, so $5\times 10^{15}$GeV $\lesssim M_1 \lesssim T_{RH}$. Since $\epsilon \simeq 10^{-7}$ is required, for $M_1 \lesssim 10^9$ GeV one must have $\delta_{HMY} \to 1$. The parameter space of choice can be summarised as
\[
\text{few } \times 10^8 \text{GeV } \lesssim M_1 \lesssim \text{few } \times 10^9 \text{GeV}
\]
(31)
\[
d_1 \to \frac{45}{2\pi^4 g_*}, g_* = 230
\]
As can be seen from figures 4 — 5, it is difficult to simultaneously satisfy these conditions. $M_1$ and $d_1$ increase as $W_{13}, W_{12} \to 0$, but $\delta$ decreases.

The analytic approximations of the previous section show that the baryon asymmetry depends on $U$ and the first row of $V_L$ (via $W_{ij}$), on the light neutrino masses $m_i$, on the lightest neutrino Yukawa $y_1$, and on phases. These real parameters are known, or could be experimentally constrained in the next
Figure 1: 3-d plot of $\eta_B = 8d_1\epsilon/23$, for central neutrino mass values, $m_1 = m_2/10$ and $M_1 = 10^9$ GeV.

On the left, $\eta_B$ is plotted as a function of $\omega_{12} \simeq \log|W_{12}|$ and $\omega_{13} \simeq \log|W_{13}|$. On the right, $\eta_B$ is plotted as a function of $\omega_{13}$ and $\chi_{12}$, defined such that $W_{12} = \cos\theta_{W_{13}}\sin(\theta_{sol} - 10^{\chi_{12}}\pi/2)$. The RHS measure on parameter space is more sensible, see the discussion after eqn(34).

20 years—with the exception of $m_1$, $y_1$, and $V_{L12}$. For the purposes of this paper, $V_{L12}$ is included with the measurable angles, and $y_1$ is determined as a function of $m_1$, by requiring that $M_1$ be in the range where leptogenesis could work. The baryon asymmetry then becomes independent of $y_1$. Some subtle dependence on $m_1$ remains: the area and location of the high ridge in figure 1 depend on $m_1$, but the baryon asymmetry and low energy footprints do not. This is discussed in the Appendix about $m_1$.

Notice also that it could be expected to have a similar hierarchy in the neutrino Yukawas as in the other fermions, in which case $y_1 \sim h_u$, $h_e$ or $h_d$. This gives

$$M_1 \sim \left(\frac{y_1}{h_u}\right)^2 \left(\frac{m_2}{W_{1j}^2 m_j}\right) 3 \times 10^6 \text{ GeV}$$

(32)

where $W_{1j}^2 m_j$ is usually of order $m_2$. If $y_1 \simeq h_u$, then $\eta_B$ is too small over most of parameter space. This was found in some models by [15]. The baryon asymmetry can be large enough, for $y_1 \simeq h_u$, in the small area of parameter space where $W_{1j}^2 m_j \simeq m_1 \lesssim m_2/100$. This is in the $m_1$ Appendix too.

The baryon asymmetry depends weakly on $\tan\beta$, when $M_1$ is taken as an input, and $d_1$ is approximated as $\propto 1/\tilde{m}_1 \propto \sin^2\beta$. The $m_i$ are experimentally measured, and therefore independent of $\sin^2\beta$, so it is clear from eqn (30) that the $\sin\beta$ dependence arises entirely from $\tilde{m}_1$. If instead $M_1 = (y_1^2 v_2^2)/|W_{1j}^2 m_j|$, then $\eta_B \propto \sin^4\beta$. In both cases, larger $\sin\beta$ is marginally favoured.

The parameters $W_{12}$ and $W_{13}$ are convenient, because they summarise the unknown mixing angles and phases. The physically relevant quantities for leptogenesis ($M_1, \epsilon, ...$) can be plotted as a function of the two real unknowns $|W_{12}|$ and $|W_{13}|$. However, the $W_{1j}$ are not observable—the matrix $W$ is related
to the more physical matrices $V_L$ and $U$ by $W = V_L U$. Recall that $V_L$ rotates from the basis where the neutrino Yukawa matrix $Y_
u$ is diagonal to the basis where $Y_e$ is diagonal, and $U$ rotates from the basis where $[m_e]$ is diagonal to the basis where $Y_e$ is diagonal. $W$ can be written

\[
W_{13} = V_{L11} \sin \theta_{13} e^{-i \delta} + V_{L12}/\sqrt{2} + V_{L13}/\sqrt{2} \\
W_{12} = V_{L11} \sin \theta_{sol} + V_{L12}(\cos \theta_{sol} - \sin \theta_{sol} \sin \theta_{13} e^{i \delta})/\sqrt{2} \\
- V_{L13}(\cos \theta_{sol} + \sin \theta_{sol} \sin \theta_{13} e^{i \delta})/\sqrt{2}
\]

(33)

(34)

where $\theta_{12} = \theta_{sol}$ and $\theta_{23} = \pi/4$ in the MNS matrix.

From a model building perspective \[31\], there are two natural limits for $W$. The most popular is for the large leptonic mixing angles to come from the seesaw structure of the light neutrino mass matrix. In this case, $V_L \sim 1$ can easily arise, so $W \sim U$. This is similar to the quark sector, where the CKM matrix (the analogue of $V_L$) has small angles. An example of this is texture models where the large atmospheric mixing angle is due to a $\nu_R$ mass eigenstate having approximately equal Yukawa couplings to $\nu_\mu$ and $\nu_\tau$ \[62\]. Alternatively, the electron Yukawa matrix $Y_e$ could be “the odd man out”; it could have large off-diagonal elements in a basis where the neutrino mass matrix $[m_e]$ and $Y_e$ are simultaneously almost diagonal \[63\]. In this case $V_L \sim U^\dagger$ and $W \sim 1$. These two cases are discussed in the following two subsections. Figure 1 suggests that thermal leptogenesis can work for $W \sim 1$. As we shall see, a large enough asymmetry is also possible in the $V_L \rightarrow 1$ limit, if a slightly larger $T_{R,H}$ is allowed.

The plots are functions of $\log W_{13}$ and $\log W_{12}$, rather than, e.g. $W_{12}$ and $W_{13}$. It is sensible to use logarithmic measure on unknown physical parameters\[7\] because it is equally probable for mixing angles to have any order of magnitude between $e.g.10^{-3}$ and 1. However, $W_{12}$ are not physical parameters, so this reasoning does not apply to them. Specifically, values of $W_{12} \approx \sin \theta_{sol}$ arise in the presumably small area of parameter space where $V_L \approx U^\dagger$. This reasoning does apply to the CHOOZ angle and the unknown angles of $V_L$, but we prefer to plot $\eta_B$ as a function of two unknowns, rather than four. So a more appropriate measure on $W_{12}$ might be logarithmic in the difference away from $U_{12} \approx \sin \theta_{sol}$. Therefore, on the RHS of figure 1 is plotted the same function as on the LHS, but as a function of $\omega_{13} \approx \log(W_{13})$, and $\chi_{12}$, the latter defined such that

\[
W_{12} = \sin(\theta_{sol} - 10^{\chi_{12}} \pi/2).
\]

(35)

$\chi_{12} \approx \log(W_{12} - U_{12})$ is an approximation to the log of the unknown angles of $V_L$ (see eqn 33). So the RHS plot tells us the same information as its twin on the left: the asymmetry is largest if a large angle in $V_L$ cancels the large solar angle in the MNS matrix $U$.

A final technical comment: $W \sim 1$ and $V_L \sim 1$ mean the 12 and 13 matrix elements are small $\lesssim .1$. $W \sim 1$ means that $W$ maximises $\eta_B$, so $W_{12} \lesssim .1$ and $.01 \lesssim W_{13} \lesssim .1$. $V_L \sim 1$ includes both the possibilities that the angles of $V_L$ are smaller, or larger, than the CHOOZ angle.

Section 4.1 studies the phenomenological consequences of sitting in the region where thermal leptogenesis works easily, which corresponds approximately to $W_{12} \lesssim .1$, $.01 \lesssim W_{13} \lesssim .1$. Then in section 4.2 some of the parameters which are fixed in figure 1 are varied, so a large enough baryon asymmetry can be generated for $V_L = 1$. The parameter space between these two limits is discussed in section 4.3.

4.1 $W \sim 1$

The parameter space where $\eta_B$ is largest in figure 1 corresponds roughly to

\[
.01 \lesssim W_{13} \lesssim .1 \quad W_{12} \lesssim .1
\]

(36)

\[7\]this choice of measure is neither unique nor universally agreed on
This can be understood from the analytic approximation \(30\). We fix \(M_1 \simeq 10^9\) GeV, so \(\eta_B \propto d_1 \delta_{HMY}\). The factor \(d_1\) is largest when the \(\nu_R\) decay rate \(\Gamma \propto \tilde{m}_1\) is smallest, so more of the asymmetry survives when \(W \to 1\) (see the expression \(24\)). \(\delta_{HMY}\) parametrises how close \(\epsilon\) can come to its upper bound \(24\). For \(m_1 \sim m_2/10\), the values of \(W_{12}, W_{13}\) where \(\delta_{HMY}\) is maximised (eqn \(24\)) correspond to eqn \(36\). \(\eta_B\) is maximal at smaller \(W_{13}\) than \(\delta_{HMY}\), as can be seen by comparing figures 1 and 4. This is due to the lepton number washout encoded in \(d_1\), which is faster at larger \(W_{13}\).

To obtain \(W_{12}\) and \(W_{13}\) in this region, \(V_L\) must have the form

\[
V_L = R_{23}[U_{\odot}]^\dagger \tag{37}
\]

where \(R_{23}\) is an unspecified complex rotation in the 23 plane (written in the form of eqn (8) with \(\theta_{12} = \theta_{13} = 0\), \(S = \sin \theta_{23}, C = \cos \theta_{23}\), and taking a 23 phase \(\alpha\)), and \([U_{\odot}]\) is a matrix whose angles are roughly those of the MNS matrix \(\pm 1\). The unknown \(R_{23}\) appears because leptogenesis only depends on the first row of \(W\).

It is interesting to study the implications for \(\ell_j \to \ell_i\gamma\) of eqn \(67\). Taking \([U_{\odot}] = U\)

\[
V_{L31} = Se^{i(\alpha + \phi'/2)}c_{13}s_{12} + Cs_{13}e^{i\delta} \\
\simeq S e^{i(\alpha + \phi'/2)}s_{sol} + Cs_{13}e^{i\delta} \\
V_{L32} = Se^{i(\alpha + \phi'/2)}(c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta}) + Cs_{23}13 \\
\simeq S e^{i(\alpha + \phi'/2)}c_{sol}/\sqrt{2} + C/\sqrt{2}
\]

For generic values of \(S\), this implies \(V_{L32} \sim 1\), so an experimentally accessible \(\tau \to \mu\gamma\) branching ratio. If, on the other hand, \(S\) is tuned to make \(V_{L32} \to 0\), then \(BR(\tau \to \mu\gamma)\) would be unobservable. However, in this case \(V_{L31} \sim -\sin \theta_{sol}/\sqrt{1 + \cos^2 \theta_{sol}} \simeq 1/\sqrt{3}\), so \(\tau \to e\gamma\) should be observable. One can conclude that if leptogenesis takes place in the \(W \sim 1\) peak of figure 1, then one or both of \(\tau \to \mu\gamma\) and \(\tau \to e\gamma\) should have a branching ratio \(\gtrsim 10^{-9}\) (according to the leading log approximation of the introduction). Similarly, \(BR(\mu \to e\gamma)\) should be \(\gtrsim 10^{-14}\) if \(S\) or \(s_{13} \gtrsim 10^{-3}\).

### 4.2 \(V_L = 1\)

It is barely possible to get a large enough baryon asymmetry when the angles in \(V_L\) are small, although this is not evident from figure 1. In the limit \(V_L \to 1\), the matrix \(W \to U\), so \(W_{12} \simeq \sin \theta_{sol}\) and \(W_{13} \simeq \sin \theta_{13}\). In figure 1 \(\eta_B\) is at least a factor of 6 to small at \(\log W_{12} \sim -0.5\) (equivalently, \(\chi_{12}\) small), but there is a bump at \(W_{13} \sim 1\). In this section, \(\eta_B\) at \(V_L = 1\) is increased by varying \(m_3, m_2\) and \(M_1\); \(V_L\) close to the identity is discussed in the following subsection.

If \(V_L = 1\), eqns \(24\) and \(26\) give

\[
M_1 = \frac{y_{12}^2 v_0^2}{m_2 s_{12}} \\
\tilde{m}_1 = m_2
\]

To maximise the asymmetry, \(M_1\) is taken to be \(f \times 10^9\) GeV, where \(f\) is a few (this determines \(y_{12}^2 = M_1 m_2 s_{12}^2/v_0^2 \approx 7.2 f \times 10^{-8}\) ). In figure 10 of 11, the lepton asymmetry is plotted as a function of \(\tilde{m}_1\) for various values of \(M_1\) and \(\epsilon_1 = 10^{-6}\). This plot shows that for \(\epsilon \simeq 10^{-6}\), a large enough asymmetry can be generated when \(\tilde{m}_1 \simeq m_2\). Note that the washout effects are correctly included in this plot of 11, so this result does not depend on the analytic approximation of eqn 17. Also, the asymmetry increases as \(\tilde{m}_1\) decreases, so small values of the solar mass are preferred.
The upper bound of eqn (21) implies that for $\epsilon = 10^{-6}$, $f_{m3}\delta_{HMY} = 0.25\text{eV}$. From the experimentally allowed range of $m_3$ given after eqn (6), we see that $M_1 \sim 3 \times 10^9\text{ GeV}$ is required, assuming $\delta \sim 1$ is also possible. For $V_L = 1$, 

$$
\delta_{HMY} \simeq \frac{s_{13}^2 m_3^3 s_{12}^2 m_2 \sin(\phi' - 2\delta)}{m_3 (s_{12}^2 m_2^2 + s_{13}^2 m_3^2) s_{12}^2 m_2}
$$

which approaches 1 when $s_{13} \simeq s_{12} m_2 / m_3$. In the RH plot of figure 1, $V_L = 1$ corresponds to $W_{13} = \sin \theta_{13}$ and $\chi_{12} \to -\infty$. In figure 2 $\delta_{HMY}$ is plotted as a function of $\theta_{13}$ on the LHS; the analytic approximation to $\eta_B$ (eqn (30)) is plotted on the RHS. So thermal leptogenesis “works” at $V_L = 1$, for $M_1 \sim 6 \times 10^9\text{ GeV}$.

Phrased another way: for arbitrarily small $\ell_j \to \ell_i \gamma$ branching ratios, requiring thermal leptogenesis to work predicts the CHOOZ angle $\theta_{13}$. If $m_3$ is taken at its 90\%C.L. upper bound, and $M_1 \sim 6 \times 10^9\text{ GeV}$, then $\eta_B \sim 3 \times 10^{-11}$ can be obtained. This requires a CHOOZ angle of $\theta_{13} \sim 4 \times 10^{-2}$, and phases which satisfy $2\delta - \phi' = \pi/2$.

Figure 2: On the LHS $\delta_{HMY}$, and on the RHS the analytic approximation to $\eta_B$, as a function of $S_{13} = \log[\sin \theta_{13}]$. $\delta_{HMY} \sim 1$ is required to get a large enough asymmetry, see the discussion in section 4.2. The remaining parameters are $\tan^2 \theta_{sol} = 0.44$, $m_2 = 7 \times 10^{-3}\text{ eV}$, $m_3 = 6.3 \times 10^{-2}\text{ eV}$, and for the $\eta_B$ plot, $V_L = 1$ and $M_1 = 4 \times 10^9\text{ GeV}$.
4.3 $V_L$ from 1 to $U^\dagger$

It is clear from the RHS of figure 11 that if a large enough asymmetry can be generated at $V_L = 1$ ($\chi_{12} \sim -3$), then enough baryons can be generated along the ridge leading to the peak, and also along the “other” ridge at $W_{12} \sim m_1/m_2$. To sit on this second ridge requires $W \sim 1$, so has the same experimental signatures as discussed in section 4.1. This section is about the $W_{13} \sim m_2/m_3$ ridge stretching from $V_L \sim 1$ to the $W \sim 1$ peak.

Starting from the small $\chi_{12}$, flat section of the ridge and moving towards the peak corresponds to allowing small matrix elements $V_{L12}, V_{L13} \ll 1$. In this limit,

$$W_{13} \approx \sin\theta_{13} + V_{L12}/\sqrt{2} + V_{L13}/\sqrt{2}$$

and $W_{13} \sim 0.04$ is required to get a large enough asymmetry. Unfortunately, $W_{13} \sim 0.04$ determines a sum of three unknowns: $\theta_{13} \gtrsim 0.4$ could be observed, $V_{L13} \sim 0.04$ induces a potentially observable $\tau \to e\gamma$ signal, but $V_{L12}$ has no observable consequences. For $V_L \sim 1$, the $V_{L12}$ contribution to $m^2_0$ (eqn 64) is suppressed by $y_{22}^2 \sim 10^{-4}$.

Figure 13 is a contour plot in $\omega_{13}$ and $\chi_{12}$ space of the approximation (60) to $\eta_B$. The contours enclose the area when $\eta_B > 2 \times 10^{-11}$, for $M_1 = f \times 10^6$ GeV, central values of $m_3$ and $m_2$, $m_1 = m_2/10$, and are labelled by $f$. Allowing $f > 1$ significantly increases the available parameter space. This corresponds to increasing $M_1$ (which should be $\lesssim T_{RH}$), or increasing $m_3$, which is constrained by atmospheric neutrino oscillations, or for $\chi_{12} \lesssim 0.5$, to decreasing $m_2$, which is constrained by solar neutrino experiments and KamLAND. Perhaps the most palatable way to increase $f$ is to allow $T_{RH} \sim 10^{10}$ GeV. The value of $\eta_B$ chosen, $\eta_B = 2 \times 10^{-11}$, is minimal. To obtain the CMB favoured $\eta_B \approx 9 \times 10^{-11}$, would require values of $f$ that were four times larger.

5 CP violation

In a previous paper[37], we discussed the relation between the leptonic phases that could be measured at low energy, and the CP violation required for leptogenesis. We assumed that $\epsilon$ was large enough, and studied the relative importance of the neutrino factory phase $\delta$ and the double beta decay phase $\phi'$ for leptogenesis. If the right-handed neutrinos $\nu_{R1}$ are produced non-thermally, getting $\epsilon$ large enough may not be a significant constraint (see e.g. [61] for a discussion and references). However, we have seen that it is a challenge when the $\nu_{R1}$ are produced thermally. So in this section, we briefly discuss the relative importance of low-energy phases for thermal leptogenesis—imposing the constraint that $\epsilon$ is large enough.

It is well known that there is no linear relation between the “leptogenesis phase” and $\delta$ or $\phi'$[62]. That is, the lepton asymmetry can be non-zero when $\delta = \phi' = 0$, and it can be zero when $\delta, \phi' \neq 0$. To overcome this, we introduced a statistical notion of “overlap” between the leptogenesis phase and the low energy phases of our parametrisation. The overlap $O_{\delta}$ aimed to quantify the relative importance of the phase $\delta$ for leptogenesis, assuming that all the low-energy phases were $O(1)$. In [37], we considered the cases where $W_{13}^2 W_{12}^2 m_3^2 m_2$, or $W_{13}^2 W_{11}^2 m_3^2 m_1$, is the most important term upstairs in $\delta_{HMY}$. That is, we consider $V_L = 1$, $V_L \sim 1$ and the case of large $V_L$ angles that do not exactly cancel those in $U$. This occurs over most of the parameter space where $\epsilon$ could be large enough. However, $\epsilon$ is largest in the small area of parameter space where $W \sim 1$ and $W_{13}^2 W_{11}^2 m_3^2 m_1$ dominates upstairs in $\delta_{HMY}$. So let us now consider which low-energy phases are important for leptogenesis in this case.

Writing the phases explicitly gives

$$\epsilon \propto \Im\{W_{11}^2 W_{13}^2\} = \Im\{e^{i\phi} [V_{L11} c_{13} c_{12} + |V_{L12}| e^{i2\phi_{12}} (-s_{23} c_{12} s_{13} e^{i2\phi})]$$

15
\[ w_{13} \]

\[ X_{12} \]

Figure 3: Contour plot of \( \eta_B \), as a function of \( \omega_{13} \simeq \log[ W_{13} ] \) and \( \chi_{12} \simeq \log[ V_{L12} + V_{L13} ] \). The contours enclose the area when \( \eta_B > 2 \times 10^{-11} \), for \( M_1 = f \times 10^9 \) GeV, central values of \( m_3 \) and \( m_2 \), and \( m_1 = m_2/10 \). In the direction of increasing area, the lines correspond to \( f = 1, 3, 6 \) and 9.

\[
\begin{align*}
+ |V_{L13}|^2 & \equiv \left( s_{23}s_{12} - c_{23}c_{12}s_{13} e^{i \delta} \right)^2 \times \left[ |V_{L11}|^2 s_{13} e^{-i \delta} + |V_{L12}|^2 e^{i \phi} s_{23}c_{13} \right] \\
+ |V_{L13}|^2 & \equiv \left( s_{23}e^{i \phi} - c_{23}c_{13} \right)^2
\end{align*}
\]

(39)

where \( \phi_{1j} \) is the phase of \( V_{1j} \). \( V_{1j} \sim U_{j1}^* e^{i \omega_{1j}} \), because \( W \sim \text{diag}\{ e^{i \omega_1}, e^{i \omega_2}, 1 \} \). The “neutrinoless double beta decay phase” \( \phi' \) is irrelevant for leptogenesis, because it only enters into \( W_{12} \). The phase \( \phi \) of \( m_1 \) will always be important, because \( W_{11} \propto e^{-i \phi/2} \), so \( \epsilon \) will be a sum of terms \( \propto \sin(m_1 \phi + ...) \).

Both the phases \( \phi_{12} \) and \( \phi_{13} \) of \( V_{L12} \) and \( V_{L13} \) are likely to have significant overlap with the leptogenesis phase, because \( V_L \approx U^\dagger \) so the \( |V_{L1j}|^2 \) are large. The “neutrino factory phase” \( \delta \) always multiplies the CHOOZ angle, which suppresses its contribution to \( \epsilon \).

The three weak-scale phases which have significant “overlap” with the leptogenesis phase, in the area of parameter space near \( W \sim 1 \), are therefore \( \phi_{12}, \phi_{13} \) and \( \phi \). This is unfortunate, because although

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\(^8\)This constraint on \( W \) is what we usually refer to as \( W \sim 1 \). Since \( V_{11} \) is real, \( \omega_1 \simeq -\phi/2 \).
there is some hope of measuring \( \phi' \) and \( \delta \), there is no foreseeable experiment to determine any of these three.

6 Discussion and summary

This paper has discussed leptogenesis in a minimal model of the SUSY seesaw, with gravity mediated SUSY breaking and universal soft masses at a high scale. It uses a parametrisation of the model in terms of

\[
D_{m_\nu}, U, D_Y, V_L
\]

where \( D_{m_\nu} \) is the diagonal light majorana neutrino mass matrix (assumed hierarchical), \( U \) is the MNS matrix, \( D_Y \) is the diagonal neutrino Yukawa matrix, and \( V_L \) diagonalises \( Y_\nu Y_\nu^T \). The notation is briefly defined in table 6, giving the equation numbers of more detailed definitions. The angles of the unitary matrix \( V_L \) can be related, in SUSY models, to the rates for \( \ell_j \to \ell_i \gamma \) because \( Y_\nu^T Y_\nu \) contributes to the renormalisation group equations for the slepton masses. This allows the right handed neutrino masses and Yukawa couplings to be expressed as a function of quantities which could be measured, in principle or in practise, at the weak scale. This parametrisation is briefly reviewed in section 2.2.

| \( Y_\nu = V_R^T D_Y V_L \), \( y_i \) | neutrino Yukawa, eigenvalues | 4 |
|-----------------------------|---------------------------------|---|
| \( \mathcal{M}, \bar{\mathcal{M}}_1 \) | \( \nu_R \) mass matrix, lightest eigenvalue | 11, 24 |
| \( V_L \) | \( V_L Y_\nu Y_\nu^T V_L = D_Y^2 \) | 5 |
| \( \theta_{ij}, \phi, \phi', \delta \) | phases of \( V_L \) | section 11 |
| \( m_\nu \), \( m_i \) | light neutrino mass matrix | 5 |
| \( U \) | MNS matrix | 6 |
| \( W \) | \( V_L U \) | 56 |
| \( \omega_{1j} \) | \( \approx \log |W_{1j}| \) | 51 |
| \( \chi_{12} \) | \( \approx \log (U_{12} - W_{12}) \) | 35 |
| \( \eta_L \) | lepton asymmetry | 41 |
| \( \epsilon \) | CP asymmetry | 20, 52 |
| \( \delta_{HMY} \) | \( \epsilon / \epsilon_{\text{max}} \) | 21, 20 |
| \( m_1 \) | \( \propto \nu_R \) decay rate | 10, 20 |
| \( d_1, d_1 \) | dilution factor of lepton asymmetry | 17, 14 |
| \( \eta_B \) | baryon asymmetry | 18, 30 |

Table 2: Table of notation, with a brief description and the equation number of a more complete definition.

The baryon asymmetry produced in leptogenesis depends on the number density of right-handed neutrinos which decay, on \( \epsilon \equiv \) the average lepton asymmetry produced per decay, and on the survival probability of the asymmetry in the thermal plasma after it is produced. We consider the “thermal leptogenesis” scenario, in which the right-handed neutrinos are produced by scattering interactions in the plasma. Non-thermal production mechanisms are also possible, perhaps even probable, but depend on additional parameters from the sector which produces the right-handed neutrinos. Both the thermally produced \( \nu_R \) number density, and the survival probability of the lepton asymmetry in the plasma after it is produced, can be computed from the reheat temperature of the plasma after inflation \( T_{RH} \), and from the seesaw parameters. These processes have been carefully studied in [18, 11].
A convenient analytic approximation to the numerical results of \[18\] is used in this paper. A single function \(d_1\) (see equation \[17\]) is defined as the number density of \(\nu_R\times\) the survival probability of the lepton asymmetry once it is produced. So the baryon to entropy ratio today is \(\eta_B \simeq 8d_1\epsilon/23\). \(^9\)

The \(CP\) asymmetry produced in the decay of the lightest \(\nu_R\) is bounded above (for hierarchical \(M_i\) and \(m_j\)):

\[
\epsilon < \frac{3M_1m_3}{8\pi v_u^2}
\]

(41)

where \(M_1\) is the mass of the \(\nu_{R1}\) and \(m_3 = \sqrt{\Delta m_{32}^2}\). Since \(d_1\) cannot exceed \(45/(2\pi^4y_\lambda)\) for thermally produced \(\nu_{R1}\), obtaining \(\eta_B > 3 \times 10^{-11}\) requires \(M_1 > 3 \times 10^8\) GeV.

In section \[4\] approximate analytic formulae for the lightest \(\nu_R\) mass \(M_1\), for the \(CP\) asymmetry \(\epsilon\) and for the baryon asymmetry \(\eta_B\), are given in terms of our weak-scale parameters. These approximations are valid for hierarchical \(M_i\) and neutrino Yukawa eigenvalues \(y_i\). The baryon asymmetry can be written as a function

\[
\eta_B(m_2, m_3, \theta_{23}, \theta_{12}; \theta_{13}, V_{L12}, V_{L13}, y_1, m_1, \text{phases})
\]

(42)

where “known” low-energy parameters precede the semi-colon. We concentrate on the dependence of \(\eta_B\) on real parameters, assuming that the phases can be chosen to maximise the asymmetry.

It is interesting that the baryon asymmetry only depends on 5 of the 8 unknown real parameters in eqn \[41\]. Two of these, \(\theta_{13}\) and \(V_{L13}\), are possibly measurable; the constraints that thermal leptogenesis imposes on them will be discussed later for different areas of parameter space. On the other hand, there are no foreseen experiments that could determine \(y_1, m_1\), and \(V_{L12}\). \(V_{L12}\) is included in the discussion with \(\theta_{13}\) and \(V_{L13}\), because these three unknowns can be exchanged for the 12 and 23 elements of \(W = V_LU\). This simplifies expressions and is convenient for plotting. The dependence of \(\eta_B\) on \(m_1\) is subtle, comparatively unimportant, and discussed in an Appendix. The lightest right-handed neutrino mass \(M_1\), and therefore the baryon asymmetry, is proportional to \(y_1^2\). \(y_1^2\) is exchanged for \(M_1\).

This is a peculiar exchange— why do we want to use a GUT-scale mass as input in our weak-scale parametrisation? The off-diagonal elements of \(V_L\), and the \(y_i\), are related to lepton flavour violating off-diagonal slepton mass matrix entries (to which processes like \(\ell_j \rightarrow \ell_i\gamma\) are sensitive), and to slepton mass differences. The smallest neutrino Yukawa \(y_1\) makes negligible contributions to both these effects.

However, \(M_1 > 3 \times 10^8\) GeV is required for thermal leptogenesis to have any hope of working, and if SUSY is discovered, the sparticle spectrum could give some indication of the gravitino mass, and therefore the allowed reheat temperature \(T_{RH} > M_1\). So we “determine” \(y_1\) by requiring that thermal leptogenesis \(\text{could produce a large enough asymmetry: } 3 \times 10^8\) GeV < \(M_1 < T_{RH}\). Then we study the requirements on the remaining parameters such that the asymmetry \(\epsilon\) is large enough. These additional requirements may have observable consequences.

The analytic formulae of section \[4\] are simple and compact, but nonetheless difficult to visualise. The asymmetry depends on the first row of the matrix \(W\), so for qualitative understanding, we show 3-dimensional figures of leptogenesis parameters as a function of \(\omega_{13} \sim \log W_{13}\), and \(\omega_{12} \sim \log W_{12}\). Logarithmic measure is reasonable for unmeasured but observable matrix elements—which the \(W_{ij}\) are not. For small angles in \(V_L\) (see section \[4\] for a general discussion), the \(W_{ij}\) can be related to the more physical matrix elements \(V_{L1k}\) and \(U_{ij}\): \(W_{13} \sim \theta_{13} + V_{L12} + V_{L13}\), \(W_{12} \sim \sin\theta_{sol} + V_{L12} + V_{L13}\). To present the area of parameter space where leptogenesis works with a more physical measure, we therefore plot the baryon asymmetry as a function of \(\omega_{13}\) and \(\chi_{12} \sim \log[\sqrt{V_{L12} + V_{L13}}]\) in figures \[1\] and \[3\].

We define thermal leptogenesis to “work” if it can produce \(\eta_B \gtrsim 3 \times 10^{-11}\), as required by Big Bang Nucleosynthesis. For \(M_1 \simeq 10^9\) GeV (consistent with the canonical gravitino bound \(T_{RH} \sim 10^9\)

\(^9\)where the 8/23 arises in the transformation of the lepton asymmetry into a baryon asymmetry by the electroweak \(B + L\) violating processes.
What parameter values must be observed, if thermal leptogenesis works in an MSUGRA model?

Suppose first that \( f \approx 1 \), which corresponds to \( M_1 \sim 10^9 \)GeV for central values of the light neutrino masses. Thermal leptogenesis works in the area of figure 3 at \( \omega_{13} \sim -2 \) and \( \chi_{12} \sim -0.5 \). This small area of parameter space is discussed in section 4.4, and occurs if \( W = V_L U \sim 1 \). The phenomenological consequences of this area of parameter space are unambiguous: the branching ratio of \( \tau \to \mu \gamma \), or \( \tau \to e \gamma \), should be large. More concretely, at least one of \( V_{L32} \) or \( V_{L31} \) is \( O(1/\sqrt{2}) \), so according to the estimates of table 2, \( BR(\tau \to e\gamma) \gtrsim 10^{-8} \).

From a theoretical model building perspective, this area of parameter space corresponds to the neutrino Yukawa and light mass matrices \( Y_\nu \) and \([m_\nu]\) being almost simultaneously diagonalisable. The large leptonic mixing angles arise in the rotation from this basis to the one where the charged lepton Yukawa matrix \( Y_e \) is diagonal.

The baryon asymmetry is largest at this point for two reasons. The \( \nu_R \) decay rate (eqn 10) is small, so lepton number violation is slow after the asymmetry is produced, and more of the asymmetry survives. Secondly, the asymmetry produced is almost maximal; it comes within a factor of \( O(1) \) of the upper bound eqn 11. This is discussed after eqn 26.

The area of this parameter space, where \( \eta_{13} \) is largest, depends on the smallest neutrino mass \( m_1 \). The plots are made with \( m_1 = m_2/10 \); the area shrinks as \( m_1 \) decreases. This peak in \( \eta_{13} \) only exists for \( m_1 \neq 0 \). It is interesting that the CP asymmetry and the low-energy footprints of this area of parameter space are independent of \( m_1 \). However, the number density of \( \nu_R \) (and therefore the asymmetry) decreases for \( m_1 \lesssim 10^{-5} \) eV, and our approximation fails. See the Appendix for a discussion.

In brief, if a sparticle spectrum consistent with gravity mediated SUSY breaking is measured, with a gravitino mass of \( m_{3/2} \sim 100 \) GeV \( (T_{RH} \sim 10^9 \) GeV), then \( \tau \to \mu \gamma \) or \( \tau \to e \gamma \) must be observable for thermal leptogenesis to work.

Now consider the enlarged parameter space allowed by \( M_1/(10^9 \) GeV) \( \equiv f > 1 \) in figure 3: thermal leptogenesis works for \( W_{13} \sim m_2/m_3 \) and pretty much all values of \( W_{12} \). This sets one constraint on the three "physical" matrix elements \( \sin \theta_{13}, V_{L13} \) and \( V_{L12} \). As discussed in sections 4.2 and 4.3, it can be satisfied if any one of the angles is \( O(m_2/m_3) \). These possibilities have different weak-scale implications.

If \( V_L \) has small angles like the CKM matrix, \( V_{L13}, V_{L12} \ll 0.1 \), then leptogenesis requires that the CHOOZ angle \( \theta_{13} \approx 0.04 \), which is close to its current experimental bound. This implies that the baryon asymmetry is determined by parameters which can be measured in the neutrino sector\(^{10} \); \( m_3, m_2, \theta_{13} \) and the phases \( \delta \) and \( \phi \).

If the CHOOZ angle \( \theta_{13} \ll 0.1 \), then it is still possible to sit on the \( W_{13} \sim 0.04 \) ridge, by having \( V_{L13}, \) or \( V_{L12} \sim 0.04 \). The former angle is related to \( \tau \to e \gamma \), and could perhaps be measured in this process. Unfortunately, \( V_{L12} \) appears in the RGEs multiplying \( y_2^2 \), the middle Yukawa eigenvalue, so has no observable consequences in the slepton mass matrix. So thermal leptogenesis can "work" when \( \theta_{13} \) and \( BR(\ell_j \to \ell_i \gamma) \) are unobservably small.

The matrix elements \( V_{L13}, \) and \( V_{L12} \) are small along most of the \( W_{13} \) ridge currently under discussion. Many texture models occupy this area of parameters space, where the CKM-like matrix \( V_L \) (between the bases where \( Y_e \) and \( Y_\nu \) are diagonal) has small angles. The large angles of the MNS matrix then arise from the majorana structure of \( Y^T M^{-1} Y \).

\(^{10}\) Caveat: \( \eta_B \propto M_1 m_3/m_2 \) in this case, and we "determine” \( M_1 \approx 6 \times 10^9 \) GeV by requiring \( \eta_B \) large enough. If \( m_3 \) (\( m_2 \)) is larger (smaller) than the current best-fit values, \( \eta_B \) increases.
The baryon asymmetry is larger along the ridge than in the rest of parameter space, because the washout is moderate—$m \sim m_2$—and because the CP asymmetry $\epsilon$ approaches its upper bound. Notice that the baryon asymmetry on this ridge is independent of $m_1$—the solution remains as $m_1 \to 0$.

As discussed after eqn (26), there are two limits in which $\epsilon$ is maximal: the ridge where $W_{13} \sim m_2/m_3$, and the previously discussed peak. There is an orthogonal ridge in figure 3, at $\chi_{12} \sim -0.5$ ($W_{12} \sim 0.1$), where $\delta_{HY} \lesssim m_2/m_3$, but washout is minimised. It has the same observable footprints as the peak.

Until now we have considered if thermal leptogenesis works, then what should we see at low energy? Allowing $T_{RH} \sim 10^{10}$ GeV, it seems just about all phenomenology is consistent with thermal leptogenesis: large $\tau \to \mu \gamma$, $\theta_{13} \sim 0.04$, observable $\tau \to e \gamma$, nothing observable at all...So now consider the inverse question: are there weak-scale observations that can rule out thermal leptogenesis? The previous discussion is vague because SUSY has not been discovered. Clearly thermal leptogenesis does not work if $W_{13}$ is too big or too small. Since all the terms which contribute to $W_{13}$ cannot be measured, no experimental lower bound can be set. However, one could tell that $W_{13}$ is too large, for instance if large $V_{L13}$ ($\tau \to e \gamma$) is measured. The analysis of this paper relies crucially on the assumption that the $\nu_R$ are produced thermally. A larger number density of the lightest $\nu_R$, $n_{\nu_R}/s$, could be produced non-thermally, so a large enough baryon asymmetry could be produced with a smaller $\epsilon$. This would enlarge the available parameter space. Furthermore, if the $\nu_R$ are produced non-thermally, they could be $\nu_R^2$ or $\nu_R^3$, making the formulae for $\epsilon$ inapplicable.

In summary, we study the baryon asymmetry resulting from the decay of the lightest right-handed neutrino $\nu_{R1}$, assuming the $\nu_R$s are produced thermally. We present compact analytic approximations for the quantities relevant to thermal leptogenesis, in terms of the light neutrino masses, the MNS matrix, the smallest eigenvalue of the neutrino Yukawa matrix $Y_\nu$, and the matrix $V_L$ which diagonalises $Y_\nu$ on its SU(2) doublet indices. In the MSUGRA scenario, we can trade these parameters for the neutrino and sneutrino mass matrices ($m_\nu$ and $m_{\tilde{\nu}}^2$), or more usefully, for $m_\nu$, for the branching ratios of lepton flavour violating decays $\ell_j \to \ell_i \gamma$, and for the lightest right-handed neutrino mass $M_1 \lesssim T_{RH}$. We find a small area of parameter space where a large enough baryon asymmetry is generated for $T_{RH} \sim 10^9$ GeV. It corresponds to large off-diagonal elements in $m_{\tilde{\nu}}^2$, and therefore observable $\tau \to e \gamma$. For $T_{RH} \sim 10^{10}$ GeV, leptogenesis can also work for smaller off-diagonal elements in $m_{\tilde{\nu}}^2$.

### Acknowledgements

Thanks to Oxford where we started this, and to Valencia, for a warm and sunny welcome when it was being completed. I am grateful to Marco Peloso for encouragement and asking the right questions, and to Michael Plümiacher for many discussions, comments and for careful reading of the manuscript. I particularly thank Alejandro Ibarra for innumerable productive discussions and important contributions.

### Note added

After this work was completed, related analyses appeared.

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11If there are additional sources of flavour violation in the slepton masses, (e.g.) non-universal soft masses) this does not work.
7 Appendix: the approximation and plots

In this Appendix, the lightest eigenvalue and corresponding eigenvector of $\mathcal{M}$ are estimated, using an approximation borrowed from diagonalising neutrino mass matrices in R-parity violating theories.

It is first convenient to scale some powers of the smallest Yukawa out of the hermitian matrix $\mathcal{M}^{-1}\mathcal{M}^{-1}$:

$$v_i^4 \mathcal{M}^{-1} \mathcal{M}^{-1} = D_Y^{-1} V_L [m_\nu]^T D_Y^{-2} V_L^* [m_\nu] V_L^* D_Y^{-1} = \frac{\Lambda}{y_1^2}. \quad (43)$$

This can be written more compactly as

$$\frac{\Lambda}{y_1^2} = D_Y^{-1} \Delta \mathcal{D}^{-1}, \quad (44)$$

by defining

$$\Delta = V_L^* [m_\nu] V_L^T = V_L^* D m_\nu U^T V_L^T \equiv W^* D m_\nu W^T, \quad (45)$$

where the matrix $W = V_L U$ is the rotation from the basis where the $\nu_L$ masses are diagonal to the basis where the neutrino Yukawa matrix $Y_\nu^T Y_\nu$ is diagonal.

The matrix $\Lambda$ can be written

$$[\Lambda]_{ij} = (\bar{\lambda}_i) \cdot (\bar{\lambda}_j^*) = \sum_k (\lambda_i)_k (\lambda_j^*)_k, \quad (46)$$

where

$$\bar{\lambda}_i = \frac{y_1}{y_i} \left( \frac{\Delta_{ii}}{y_1 \Delta_{2i} / y_2} \right) \frac{\Delta_{3i}}{y_1 \Delta_{3i} / y_3}. \quad (47)$$

If the hierarchy in the $y_i$ is steeper than in the $m_j$, and/or that the angles in $W$ are large, then

$$|\bar{\lambda}_1|^2 \gg |\bar{\lambda}_2|^2, |\bar{\lambda}_3|^2 \quad (48)$$

so the largest eigenvalue of $\Lambda \ (= v_i^4 y_1^4 / |\mathcal{M}_1|^2)$ is

$$|\mathcal{M}_1| \approx \frac{y_1^2 v_\alpha^2}{\sqrt{|\lambda_1|^2}} \approx \frac{y_1^2 v_\alpha^2}{|\Delta_{11}|} \approx \frac{v_\alpha^2}{|W_{ij}^2 m_j|}, \quad (49)$$

with associated eigenvector (normalised $\bar{\lambda}_1$):

$$\bar{\lambda}_1 \approx \left( \frac{\Delta_{11}^*}{y_1 \Delta_{21}^*} \right) \times \frac{1}{\Delta_{11}^*} \quad (50)$$

In figure $4$, $M_1$ is plotted as a function of $\omega_{12} \approx \log W_{12}$ and $\omega_{13} \approx \log W_{13}$, for $y_1 = 10^{-4}$, and using the central values of $m_i$ listed after eqn (6). The precise definition is

$$W_{12} = \cos \theta_{W12} \sin \theta_{W12}, \quad W_{13} = \sin \theta_{W13}$$

with $\theta_{W1j} = 10^{\omega_{1j}} \pi / 2. \quad (51)$

This Appendix contains many three dimensional plots of functions that will enter into the equation for the baryon asymmetry. The aim of these figures is to give a qualitative impression; quantitatively clearer
contour plots of the baryon asymmetry are in the body of the paper. In figure 4 there are three limiting values for $M_1$, corresponding to $M_1 \simeq y_1^2 v_2^2/m_1$: $M_1 \to y_1^2 v_2^2/m_3$ when $W_{13} \to 1$, $M_1 \to y_1^2 v_2^2/m_1$ when $W_{13}, W_{12} \to 0$, and $M_1 \to y_1^2 v_2^2/m_2$ when $W_{13} < m_2/m_3, W_{12} \to 1$.

The $\nu_R$ decay rate can be evaluated with the eigenvector (50), which gives eqn (25). $\tilde{m}_1$ has three limits—$m_1, m_2, m_3$—depending on the values of $W_{1j}$. The logarithm of $\tilde{m}_1/m_3$ is plotted on the RHS of figure 4. $\tilde{m}_1$ must be in the range given after eq. (16), which implies $\log(\tilde{m}_1/m_3) \lesssim -1.4$.

Finally, the CP asymmetry $\epsilon$, eqn. (20), can be evaluated with the eigenvector (50) to obtain

$$\epsilon \simeq -\frac{3\Delta_{11}^2}{8\pi|\Delta Y^2\Lambda|_{11}} \text{Im} \left\{ \frac{[\Delta Y^2 \Delta Y^T]_{11}}{|\Delta Y^2 \Delta Y^T \Lambda^2|_{11}} \right\} = \frac{3\Delta_{11}^2}{8\pi \sum_j |W_{1j}|^2 m_{\nu_j}^2} \text{Im} \left\{ \frac{\sum_k W_{ik}^2 m_{\nu_k}^3}{\sum_n W_{in}^2 m_{\nu_n}} \right\},$$  (52)

where terms of order $y_1/y_2$ and $y_1/y_3$ have been dropped. $\delta_{HMY} \propto \epsilon_1/M_1$ is given in eqn (26), and plotted on the LHS of figure 5. $\epsilon_1$ is plotted on the RHS; it peaks on the ridge of eqn (27) because this is where the larger values of $M_1$ and $\delta_{HMY}$ overlap.

The results in the remainder of the paper are based on the analytic approximations of this section. How reliable are these equations? The eqn (44) for $\Lambda = y_1^2 [M_i M^i]$ is exact, but the formula we use for $\epsilon_1$ assumes hierarchical $M_i$, so it is consistent to assume this in solving for the eigenvalues and eigenvectors of $\Lambda$. The approximation is that the first column (or row) of $y_1^2 D_Y^{-1} \cdot \Delta \cdot D_Y^{-1}$ is the lightest eigenvalue, multiplying its eigenvector. It breaks down if the elements of the second or third row/column become of order $\Delta_{11}$, as one can see by writing the eigenvector in a basis rotated by a small angle from the eigenbasis. $y_1 \Delta_{12}/y_2, y_1 \Delta_{13}/y_3 \simeq \Delta_{11}$ could occur if
1. the $M_i$ were of similar magnitude, rather than hierarchical. This is “unlikely”, because the hierarchy in $D_Y$ is much steeper than in $[m_\nu]$. 

2. $m_1$ too small—if $m_1/m_2, m_1/m_3 < y_2^2$, then the terms being kept are smaller than the neglected ones. This is discussed in an appendix.

8 Appendix: $m_1 \ll m_2/10$

In this paper, we assumed a hierarchical spectrum for the light neutrino masses: $\Delta m^2_{\text{atm}} = m_3^2 - m_2^2, \Delta m^2_{\text{sol}} = m_3^2$, so the smallest neutrino mass $m_1$ is unlikely to be measured with anticipated data. However, it enters our formulae for the baryon asymmetry, as does the smallest Yukawa $y_1$. In the body of the paper, we fixed $m_1 = m_2/10$, and determined $y_1$ as a function of $M_1$ and our weak scale parameters, by requiring $M_1$ to be in the range allowed by leptogenesis. In this Appendix, we discuss the dependence of our results on $m_1$. For most values of $W_{12}$ and $W_{13}$, $m_1$ is irrelevant because $|W_{13}|^2 m_1 \ll |W_{12}|^2 m_2, |W_{13}|^2 m_3$. However, $m_1$ cannot be dropped from our analytic expressions, for $W$ close to the identity. This is the area of parameter space where $\eta_B$ is maximal; the remainder of the Appendix is restricted to this area of parameter space. We are interested in how $\eta_B$ scales with $m_1$, and in whether our analytic approximation is still valid.

The maximum value of $\epsilon$, eqn 41, is independent of $m_1$, if $M_1$ is independent of $m_1$. We have fixed $M_1 \simeq T_{RH}$, which determines $y_1^2$ as a function of $W_{1n}^2 m_n \sim m_1$. So varying $m_1$ allows $y_1$ to vary:

Figure 5: On the LHS (RHS), $\delta_{HMY} (\epsilon)$ as a function of $\omega_{12} \simeq \log [W_{12}]$ and $\omega_{13} \equiv \log [W_{13}]$. On the RHS, $M_1$ is taken as a function of $y_1 = 10^{-4}$ and other inputs. Both plots are for central values of the neutrino masses.
$m_1 \sim m_2/100$ would allow leptogenesis to work for $y_1 \sim h_u$, which could be theoretically attractive.

As discussed after eqn (24), $\epsilon$ approaches its upper bound (equivalently $\delta_{HMY} \sim 1$) on the peak in figure 4 where $m_3^2/m_2^2 \sim W_{13}^2$ and $W_{12}^2 < m_1^2/m_2^2$. As $m_1$ decreases, the area in $W_{12}, W_{13}$ space where $\delta_{HMY} \sim 1$ decreases, but the maximum value is unchanged. The numerical values of $W_{12}$ and $W_{13}$ where the maximum is reached will also decrease, making this parameter space increasingly “fine-tuned” ($W$ very close to the identity is unlikely to be stable under renormalisation group running).

The $\nu_R$ decay rate must have values in the range given after eqn (14), to ensure $\eta_B$ as large as possible. We first concern ourselves with the upper bound: $\hat{m}_1 < 3 \times 10^{-3}$ eV. If $W_{1m}^2 m_{13}^2 \sim W_{23}^2 m_{13}^2$ and $W_{1j}^2 m_j \sim W_{11}^2 m_1$, as required to maximise $\delta_{HMY}$, then from eqn (24), $\hat{m}_1 \sim W_{13}^2 m_{13}^2/m_1$. To maximise $\eta_B \sim \delta_{HMY}/\hat{m}_1$, requires $W_{13}^2 \sim m_3^2/m_2^2$, so that the decay rate is slow enough, but $\delta_{HMY}$ is still $O(1)$. So as $m_1$ decreases from $m_2/10$ to $10^{-3}m_2$, the area of the peak on the RHS of figure 4 will shrink, but the height is unchanged.

For smaller values of $m_1$, the asymmetry will decrease. This is because $\hat{m}_1$ is small, so $\nu_R$ production in the plasma is inefficient (see [11]).

It is straightforward to check that our analytic approximation holds, in the shrinking area of parameter space where $W_{13} \sim m_1/m_3$ and $W_{12} < m_1/m_2$, provided that $\frac{y_3^2}{m_1} \ll \frac{y_2^2}{m_2}$. This is the condition that $\frac{y_e^2 v^2_u}{m_1}$ is the lightest $\nu_R$ mass. So the analytic approximation fails as $m_1$ approaches $\frac{y_3^2}{y_2^2} m_2$.

Finally, the low energy prediction of the peak are independent of $m_1$, because they follow from requiring that $W \sim 1$. As $m_1$ decreases, $W$ must approach the identity more and more closely, so $V_L$ becomes more precisely $U^\dagger$. But whether $V_L \sim U^\dagger$, or $V_L = U^\dagger$, the expectation remains that $\tau \rightarrow \mu \gamma$ or $\tau \rightarrow e \gamma$ should be observable.

So in summary, the magnitude of the baryon asymmetry on the peak of figure 4 is independent of $m_1$ for $10^{-3}m_2 < m_1 < m_2/10$. As $m_1$ decreases, the location of the peak shifts to smaller $W_{13}$, and its area will shrink. We cannot say anything for $m_1 < 10^{-3}m_2$: our analytic formulae indicate that $\eta_B$ will decrease, but the approximation they are based on is unreliable.

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