High-precision measurement of atmospheric mass-squared splitting with T2K and NOvA

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ABSTRACT: A precise measurement of the atmospheric mass-squared splitting $|\Delta m_{\mu\mu}^2|$ is crucial to establish the three-flavor paradigm and to constrain the neutrino mass models. In addition, a precise value of $|\Delta m_{\mu\mu}^2|$ will significantly enhance the hierarchy reach of future medium-baseline reactor experiments like JUNO and RENO-50. In this work, we explore the precision in $|\Delta m_{\mu\mu}^2|$ that will be available after the full runs of T2K and NOvA. We find that the combined data will be able to improve the precision in $|\Delta m_{\mu\mu}^2|$ to sub-percent level for maximal 2-3 mixing. Depending on the true value of $\sin^2 \theta_{23}$ in the currently-allowed 3$\sigma$ range, the precision in $|\Delta m_{\mu\mu}^2|$ will vary from 0.87% to 1.24%. We further demonstrate that this is a robust measurement as it remains almost unaffected by the present uncertainties in $\theta_{13}$, $\delta_{\text{CP}}$, the choice of mass hierarchy, and the systematic errors.

KEYWORDS: T2K, NOvA, Precision, $\Delta m_{\mu\mu}^2$, Medium-baseline reactor experiments
1 Introduction

Recent discovery of a moderately large value of $\theta_{13}$ [1–4] has provided an edge for the present generation long-baseline superbeam experiments to explore the remaining fundamental unknowns like neutrino mass hierarchy (MH), octant of $\theta_{23}$ and the leptonic CP-violation. T2K [5, 6] and NO$\nu$A [7–10] are the two current generation experiments that have potential to shed light on these remaining unknowns using the $\theta_{13}$ driven $\nu_\mu/\bar{\nu}_\mu \rightarrow \nu_e/\bar{\nu}_e$ appearance channel [11–18]. Another important consequence of the large value of $\theta_{13}$ is that it has enabled the medium-baseline reactor oscillation (MBRO) experiments like JUNO [19] and RENO-50 [20] to resolve MH [21–29]. While it is important for T2K and NO$\nu$A to address these pressing issues, it has been pointed out in [25, 27, 28, 30, 31] that the sensitivity of MBRO experiments to MH can be significantly improved by a high-precision measurement of $|\Delta m^2_{\mu\mu}|$. T2K and NO$\nu$A can do this measurement via the $\nu_\mu/\bar{\nu}_\mu \rightarrow \nu_\mu/\bar{\nu}_\mu$ disappearance channel,

\begin{equation}
P(\nu_\mu/\bar{\nu}_\mu \rightarrow \nu_\mu/\bar{\nu}_\mu) = 1 - \sin^2 2\theta_{\mu\mu} \sin^2 \left( \frac{\Delta m^2_{\mu\mu} L}{4E} \right).
\end{equation}
Here $|\Delta m_{\mu\mu}^2|$ and $\theta_{\mu\mu}$ are the effective two-flavor atmospheric mass-squared splitting and mixing angle, measured in muon neutrino disappearance oscillation experiments $[30,31]^1$, 

$$\begin{align}
\Delta m_{\mu\mu}^2 &= \Delta m_{31}^2 - \Delta m_{21}^2 (\cos^2 \theta_{12} - \cos \delta_{\text{CP}} \sin \theta_{13} \sin 2\theta_{12} \tan \theta_{23}), \\
\sin^2 2\theta_{\mu\mu} &= 4 \cos^2 \theta_{13} \sin^2 \theta_{23} (1 - \cos^2 \theta_{13} \sin^2 \theta_{23}).
\end{align}$$

On the one hand, precision in $|\Delta m_{\mu\mu}^2|$ can mitigate the challenge in the absolute energy scale uncertainty in MBRO experiments, thus enhancing their sensitivity to MH. On the other hand, comparison of the effective $|\Delta m_{\mu\mu}^2|$ from muon-flavor oscillation experiments and the corresponding effective $|\Delta m_{\mu\mu}^2|$ from electron-flavor oscillation experiments can provide additional MH information $[30–33]$. Recently, it has been demonstrated that a precision of 1% on $|\Delta m_{\mu\mu}^2|$ can improve JUNO’s sensitivity to MH from $\Delta \chi^2 = 10$ to $\Delta \chi^2 = 18$ in a six-years run $[27]$. Besides addressing the need of MBRO experiments, a precise $|\Delta m_{\mu\mu}^2|$ measurement, along with a precision measurement of $|\Delta m_{\mu\mu}^2|$, is a crucial step towards validating the 3-flavor oscillation model $[31,33]$. An accurate $|\Delta m_{\mu\mu}^2|$ measurement will also severely constrain the neutrino mass models $[34]$ and itself a key input for neutrinoless double beta decay searches $[35]$. 

Currently the most precise information on $|\Delta m_{\mu\mu}^2|$ comes from the MINOS experiment. A two-flavor analysis based on its complete run gives $|\Delta m_{\mu\mu}^2| = 2.41^{+0.09}_{-0.10} \times 10^{-3}$ eV$^2$ $[36]$, which corresponds to a relative 1σ precision of $\sigma(\Delta m_{\mu\mu}^2) = 3.94\%^2$. The latest disappearance analysis from T2K experiment based on its 3.86% of the total exposure, i.e. $3.01 \times 10^{20}$ protons on target (p.o.t), gives $|\Delta m_{32}^2| = 2.44^{+0.17}_{-0.15} \times 10^{-3}$ eV$^2$ $[37]^3$. The current T2K precision is only $\sigma(\Delta m_{32}^2) = 6.56\%$. In this paper, we explore whether it is plausible to reach the 1% precision with the combined data from T2K and NOνA. These two experiments will gather copious statistics from the muon disappearance channel, enabling a high-precision measurement of $\Delta m_{\mu\mu}^2$. In Sec. 2, we briefly mention the key experimental features of T2K and NOνA and provide the simulation details adapted in this work. In Sec. 3, we discuss the precision in $|\Delta m_{\mu\mu}^2|$ achievable by these two experiments and its dependence on various factors. Finally, we give our concluding remarks in Sec. 4.

2 Experimental specifications and simulation details

2.1 The T2K experiment

The Tokai to Kamioka (T2K) experiment is the first experiment to observe the three flavor effects in neutrino oscillations and its main objective is to measure $\theta_{13}$ by observing $\nu_\mu/\bar{\nu}_\mu \rightarrow \nu_e/\bar{\nu}_e$ oscillations. Neutrinos are produced in the J-PARC accelerator facility in Tokai and are directed towards the 22.5 kton water Čerenkov Super-K detector placed in Kamioka, 295 km away at a 2.5° off-axis angle $[5]$. For muon charged-current quasi-elastic (CCQE) events, the energy resolution is $\sigma_E(\text{GeV}) = 0.075\sqrt{E/\text{GeV}} + 0.05$. The $\nu_\mu$
beam peaks sharply at 0.6 GeV, which is very close to the 1st oscillation maximum of $P_{\mu e}$.
The flux falls off rapidly, such that, there is hardly any at energies greater than 1 GeV.
The experiment plans to run with a proton beam power of 750 kW with proton energy of 30 GeV for 5 years in $\nu$ mode only. This corresponds to a total exposure of $8 \times 10^{21}$ protons on target (p.o.t). The neutrino flux is monitored by the near detectors, located 280 m away from the point of neutrino production. The background information and other details are taken from references [12, 38].

2.2 The NO$\nu$A experiment

The NO$\nu$A (NuMI$^4$ Off-axis $\nu_e$ Appearance) experiment [9, 10, 39] uses FermiLab’s NuMI $\nu_{\mu}/\bar{\nu}_{\mu}$ beamline and is scheduled to start taking data from late 2013. A 14 kton Totally Active Scintillator Detector (TASD) will be placed in Ash River, Minnesota which is 810 km away at an off-axis angle of 14 mrad (0.8°). This off-axis narrow-width beam peaks at 2 GeV. A 0.3 kton near detector will be located at the FermiLab site to monitor the un-oscillated neutrino or anti-neutrino flux. It aims to determine the unknowns such as MH, leptonic CP-violation, $\theta_{13}$ and the octant of $\theta_{23}$ by the measurement of $\nu_{\mu}/\bar{\nu}_{\mu} \rightarrow \nu_e/\bar{\nu}_e$ oscillations. For the CCQE muon events, the energy resolution is $\sigma_E(\text{GeV}) = 0.06 \sqrt{E}/\text{GeV}$. The experiment is scheduled to run for 3 years in $\nu$ mode followed by 3 years in $\bar{\nu}$ mode with a NuMI beam power of 0.7 MW and 120 GeV proton energy, corresponding to $6 \times 10^{20}$ p.o.t per year.

2.3 Simulation details

We use GLoBES [40, 41] to carry out all the simulations in this work. The true values of neutrino oscillation parameters have been taken to be: $\Delta m^2_{21} = 7.5 \times 10^{-5}$ eV$^2$, $\sin^2 \theta_{12} = 0.3$ [42, 43], $|\Delta m^2_{\mu\mu}| = 2.41 \times 10^{-3}$ eV$^2$ [36, 44], and $\sin^2 2\theta_{13} = 0.089$ [1, 4, 45, 46]. $\Delta m^2_{31}$ is calculated based on $\Delta m^2_{\mu\mu}$ and other values using Eq. 1.2 assuming different true MH and $\delta_{CP}$. The value of $\Delta m^2_{31}$ is calculated separately for normal hierarchy (NH where $m_3 > m_2 > m_1$) and for inverted hierarchy (IH where $m_2 > m_1 > m_3$) using this equation where $\Delta m^2_{\mu\mu}$ is taken to be +ve for NH and -ve for IH. We have taken into account the present 3$\sigma$ uncertainty of $\sin^2 \theta_{23}$ in the range 0.36 to 0.66 [42, 43] both in simulated data and in fit. Note that, we perform a full three-flavor analysis in obtaining the results. We find that the true value of $\delta_{CP}$ has little impact to the precision of $|\Delta m^2_{\mu\mu}|$. Therefore, in this work, $\delta_{CP}(\text{true}) = 0$ has been assumed for all the results. The experimental features of T2K and re-optimized NO$\nu$A are the same as considered in reference [13]. We consider the nominal set of systematics i.e. normalization error of 2.5% and 10% on signal and background respectively for both the experiments. We also consider the tilt error$^5$ on signal and backgrounds to incorporate the energy-scale uncertainty. In this work, we consider 0.01% tilt error for NO$\nu$A and 0.1% tilt error for T2K, for both signal and backgrounds. The impact of different assumptions on systematics has been studied further in Sec. 3.4.

Fig. 1 shows the survival event spectra (CCQE muon events) for T2K and NO$\nu$A for three different choices of $|\Delta m^2_{\mu\mu}|$. These three different choices correspond to the best-

$^4$Neutrinos at the Main Injector.

$^5$Here “tilt” describes a linear distortion of the event spectrum.
fit and the 3σ upper and lower bounds. The total events corresponding to $|\Delta m_{\mu\mu}^2| = 2.1 \times 10^{-3}$ eV$^2$, $2.41 \times 10^{-3}$ eV$^2$, and $2.7 \times 10^{-3}$ eV$^2$ are 230, 153, and 114 respectively, with a three-years $\nu$ run in NO$\nu$A. The corresponding numbers for T2K are 369, 300, and 318 with a five-years $\nu$ run. The upper left (right) panel shows the event spectrum for the experiment T2K (NO$\nu$A). The ratio of oscillated to un-oscillated event spectrum are given in the lower panels. Fig. 1 shows that the first oscillation minima are clearly seen in both experiments due to their excellent energy resolution for CCQE muon events. This enables them to perform an accurate measurement of $|\Delta m_{\mu\mu}^2|$.

![Figure 1: Reconstructed event spectrum for the experiments T2K (left panels) and NO$\nu$A (right panels) for the three different values of $|\Delta m_{\mu\mu}^2|$ corresponding to the present best-fit and upper and lower 3σ limits. Only CCQE $\nu_\mu$ survival events have been considered. The top panels show the event spectra while the bottom panels show the ratio of oscillated over un-oscillated events as a function of the reconstructed energy. We have assumed NH, $\sin^2 2\theta_{23} = 0.5$, $\sin^2 2\theta_{13} = 0.089$, and $\delta_{CP} = 0$.](image)

The precision of $|\Delta m_{\mu\mu}^2|$ is calculated using the conventional least chi-squared method. To calculate the $\Delta \chi^2$, the observed number of events are simulated using a particular choice of the true parameters. These are then contrasted with the events generated using another test set of oscillation parameters. This procedure is repeated for all the test values of oscillation parameters in their respective allowed intervals. We marginalize over test $\sin^2 2\theta_{13}$ in its 2σ range, over test $\delta_{CP} \in [-180^\circ, 180^\circ]$ and over test $\sin^2 \theta_{23}$ in the 3σ range. We impose a Gaussian prior in $\sin^2 2\theta_{13}$ with 5% uncertainty [47]. The solar parameters are kept fixed; and so is the Earth matter density. GLoBES performs a binned-spectral analysis using a Poissonian definition of the $\Delta \chi^2$. The relative 1σ precision of $|\Delta m_{\mu\mu}^2|$ is
defined as
\[ \sigma(\Delta m_{\mu\mu}^2) = \frac{(\Delta m_{\mu\mu}^2)^{+3\sigma} - (\Delta m_{\mu\mu}^2)^{-3\sigma}}{6} \times \frac{100}{2.41 \times 10^{-3} \text{eV}^2} \% , \] (2.1)
where \(2.41 \times 10^{-3} \text{eV}^2\) is the present best-fit of \(\vert \Delta m_{\mu\mu}^2 \vert\). \((\Delta m_{\mu\mu}^2)^{+3\sigma}\) and \((\Delta m_{\mu\mu}^2)^{-3\sigma}\) are the two values of \(\vert \Delta m_{\mu\mu}^2 \vert\) at which \(\Delta \chi^2 = 9\); with \((\Delta m_{\mu\mu}^2)^{+3\sigma}\) being the larger of the two.

3 Study of the \(\vert \Delta m_{\mu\mu}^2 \vert\) precision

In the following subsections, we study the effect of various important issues like contribution from appearance channel, the effect of uncertainty in \(\sin^2 \theta_{23}\) and the effect of difference systematic uncertainties, on the precision of \(\sigma(\vert \Delta m_{\mu\mu}^2 \vert)\). Finally, we show how the precision of \(\sigma(\vert \Delta m_{\mu\mu}^2 \vert)\) is going to improve with increasing statistics from these two experiments.

3.1 Effect of appearance and disappearance data

Fig. 2 shows the \(\Delta \chi^2\) vs. test \(\vert \Delta m_{\mu\mu}^2 \vert\) for the NO\(\nu\)A experiment, assuming NH (IH) to be the true hierarchy in the left (right) panel, \(\sin^2 \theta_{23}(\text{true}) = 0.5\) and \(\vert \Delta m_{\mu\mu}^2 \vert(\text{true}) = 2.41 \times 10^{-3} \text{eV}^2\). All test parameters have been marginalized over, except the solar parameters as we explained earlier. It can be seen from Fig. 2 that the precision is dominated by the disappearance data. The combined data of disappearance and appearance channels improves the precision by 0.04\%, compared to disappearance alone. The contribution of appearance channel to the determination of \(\vert \Delta m_{\mu\mu}^2 \vert\) is very small. For completeness, we still include both appearance and disappearance data in this work.

![Figure 2: \(\Delta \chi^2\) vs. test \(\vert \Delta m_{\mu\mu}^2 \vert\) for NO\(\nu\)A. Left (Right) panel corresponds to NH (IH) being the true hierarchy. The relative contribution to the sensitivity from disappearance and appearance channels is shown. Here we take \(\vert \Delta m_{\mu\mu}^2 \vert = 2.41 \times 10^{-3} \text{eV}^2\), \(\sin^2 \theta_{23}(\text{true}) = 0.5\), \(\sin^2 2\theta_{13}(\text{true}) = 0.089\), and \(\delta_{\text{CP}}(\text{true}) = 0\).](image-url)
3.2 Precision of $|\Delta m_{\mu\mu}^2|$ with T2K and NOνA

In Fig. 3, we compare the precision of the two experiments T2K and NOνA in measuring $|\Delta m_{\mu\mu}^2|$. The left (right) panel corresponds to NH (IH) being the true hierarchy. As before, we assume $\sin^2 \theta_{23}^{(\text{true})} = 0.5$. We find that, after full runs, NOνA will give $\sigma(|\Delta m_{\mu\mu}^2|) = 1.45\%$, while T2K will give a more precise measurement $\sigma(|\Delta m_{\mu\mu}^2|) = 1.16\%$. The reason is that T2K has more statistics.

![Figure 3: $\Delta \chi^2$ vs. test $|\Delta m_{\mu\mu}^2|$ for T2K and NOνA alone and the combined data. Left (Right) panel corresponds to NH (IH) being the true hierarchy. Here $|\Delta m_{\mu\mu}^2|^{\text{(true)}} = 2.41 \times 10^{-3} \text{ eV}^2$, $\sin^2 \theta_{23}^{(\text{true})} = 0.5$, $\sin^2 2\theta_{13}^{(\text{true})} = 0.089$, and $\delta_{CP}^{(\text{true})} = 0$.](image)

We next explore the potential of combined data. A precision of $\sigma(|\Delta m_{\mu\mu}^2|) = 0.87\%$ can be obtained after the full runs of these two experiments. Thus, if the 2-3 mixing is maximal, then a less than 1% accurate determination of $|\Delta m_{\mu\mu}^2|$ can be achieved.

It can also be seen from the $\Delta \chi^2$ vs. test $|\Delta m_{\mu\mu}^2|$ plots that the precision of $|\Delta m_{\mu\mu}^2|$ is essentially independent of the hierarchy. Thus for simplicity, from here onwards, we show results only for NH assumed to be the true hierarchy.

3.3 Impact of 2-3 mixing angle on $|\Delta m_{\mu\mu}^2|$ precision

Recent MINOS results hint at a non-maximal $\sin^2 2\theta_{23}$ [36]. Global analysis [42, 43] suggests two degenerate values of $\theta_{23}$, one in the lower octant and the other in the higher octant. The leading term in the muon disappearance probability is dependent on $\sin^2 \theta_{23}$ as shown in Eq. 1.3. Thus, this parameter is expected to affect the precision in the measurement of $|\Delta m_{\mu\mu}^2|$ directly. In this section, we study the dependence of $|\Delta m_{\mu\mu}^2|$ precision on the true value of $\sin^2 \theta_{23}$.

Fig. 4 shows the effect of $\sin^2 \theta_{23}$ on the determination of $|\Delta m_{\mu\mu}^2|$. The left panel shows the $3\sigma$ allowed regions in the $|\Delta m_{\mu\mu}^2|(\text{test}) - \sin^2 \theta_{23}(\text{true})$ plane for the experiments T2K, NOνA, and combined. The right panel depicts the corresponding relative $1\sigma$ precision on
It can be seen that the best precision can be achieved for the maximal 2-3 mixing case and it deteriorates as the mixing deviates from maximal. With the combined data of T2K and NOνA, a $\sigma(|\Delta m^2_{\mu\mu}|) = 0.87\%$ is achievable for $\sin^2\theta_{23}(\text{true}) = 0.5$. For the most conservative choice of $\sin^2\theta_{23}(\text{true}) = 0.36$ or 0.66 at the 3$\sigma$ allowed limits, the precision deteriorates to $\sigma(|\Delta m^2_{\mu\mu}|) = 1.24\%$. The results are more or less symmetrical around the maximal mixing.

![Figure 4](image_url)

**Figure 4**: Left panel shows the 3$\sigma$ allowed regions in the $|\Delta m^2_{\mu\mu}|(\text{test}) - \sin^2\theta_{23}(\text{true})$ plane for the experiments T2K, NOνA, and combined. Right panel depicts the corresponding relative 1$\sigma$ precision on $|\Delta m^2_{\mu\mu}|$. Here true hierarchy is NH, $|\Delta m^2_{\mu\mu}|(\text{true}) = 2.41 \times 10^{-3} \text{ eV}^2$, $\sin^2\theta_{13}(\text{true}) = 0.089$, and $\delta_{CP}(\text{true}) = 0$.

### 3.4 Effect of systematic uncertainties

Here we study in detail the impact of the systematic uncertainties on the measurement of $\Delta m^2_{\mu\mu}$. For this purpose, we consider three different sets of assumptions on systematics. The default choice of systematics has been already mentioned in Sec. 2.3. In the second set of systematic errors, we increase the normalization error to 10% and 20% for both signal and background, for both experiments, while keeping the tilt errors same as before. In the third set, we further increase the tilt error as well, to 10% for both signal and backgrounds, for both the experiments. The possible effect of these three set of systematics on $\Delta m^2_{\mu\mu}$ precision is shown in table 1. It can be seen that systematics play a minor role in the measurement of $|\Delta m^2_{\mu\mu}|$. 

| $\sin^2\theta_{23}(\text{true})$ | $|\Delta m^2_{\mu\mu}|(\text{true})$ | $\sigma(|\Delta m^2_{\mu\mu}|)$ |
|-----------------------------|-------------------------------|--------------------------|
| 0.36                        | $2.41 \times 10^{-3}$ eV$^2$ | 0.87\%                   |
| 0.41                        | $2.41 \times 10^{-3}$ eV$^2$ | 0.87\%                   |
| 0.46                        | $2.41 \times 10^{-3}$ eV$^2$ | 0.87\%                   |
| 0.51                        | $2.41 \times 10^{-3}$ eV$^2$ | 0.87\%                   |
| 0.56                        | $2.41 \times 10^{-3}$ eV$^2$ | 0.87\%                   |
| 0.61                        | $2.41 \times 10^{-3}$ eV$^2$ | 0.87\%                   |
| 0.66                        | $2.41 \times 10^{-3}$ eV$^2$ | 0.87\%                   |
Table 1: Effect of systematic uncertainties on the relative 1\(\sigma\) precision of |\(\Delta m^2_{\mu\mu}\)|.

|        | NO\(\nu\)A | T2K     |
|--------|------------|---------|
| Signal norm. err : Signal tilt err | 2.5% : 0.01% | 2.5% : 0.1% |
| Bkg. norm. err : Bkg. tilt err    | 10% : 0.01%  | 10% : 0.1%   | 10% : 10% |
| Bkg. norm. err : Bkg. tilt err    | 10% : 0.1%   | 10% : 0.1%   | 10% : 10% |

Relative precision \(\sigma(|\Delta m^2_{\mu\mu}|)\)

- 0.87%
- 0.94%
- 0.95%

3.5 Evolution of |\(\Delta m^2_{\mu\mu}\)| precision with statistics

![Figure 5](image)

In Fig. 5, we study the improvement in the precision of |\(\Delta m^2_{\mu\mu}\)| as the statistics increases for T2K, NO\(\nu\)A, and adding their data. The x-axis shows the fraction of the total statistics for these experiments. For NO\(\nu\)A, we assume equal run time in neutrino and anti-neutrino modes at any given fractional statistics. Left and right panels present the results for the most optimistic (0.5) and the most pessimistic (0.36) values of \(\sin^2 \theta_{23}(\text{true})\) respectively. For T2K, the precision improves from 3.5% to 1.16% as their statistics increases from 10% to 100% for the maximal mixing case. But, when we combine the data from T2K and NO\(\nu\)A with equal fractional statistics, the precision improves from 2.9% to 0.87%. A precision of \(\sigma(|\Delta m^2_{\mu\mu}|) = 1\%\) can be achieved if 80% of the total data from the two experiments is available for maximal mixing. For the pessimistic case (right panel), the precision on |\(\Delta m^2_{\mu\mu}\)| improves from 3.5% to 1.24% as the combined statistics increases from 10% to 100%.

We have checked that a precision of 0.75% is achievable with the combined data from T2K and NO\(\nu\)A if their energy resolution can be improved by a factor of 2 assuming...
maximal 2-3 mixing. We also would like to point out that with the present energy resolution, the precision can be improved by simply increasing their statistics. A precision of 0.61% can be obtained if the statistics of these two experiments are doubled. It clearly suggests that this measurement is still statistically dominated for the present run-plans of T2K and NOνA.

4 Summary and Conclusions

High-precision measurement of $|\Delta m_{\mu\mu}^2|$ is crucial in validating the 3-flavor neutrino oscillation model. It also serves as a key input to the neutrino mass models and to the neutrinoless double beta decay searches. In addition, a sub-percent measurement of $|\Delta m_{\mu\mu}^2|$ is mandatory for the MBRO experiments to obtain a reasonably good sensitivity to neutrino MH. In the foreseeable future, presently running T2K and upcoming NOνA experiments can provide a more accurate measurement of $|\Delta m_{\mu\mu}^2|$ beyond the current MINOS precision. In this paper, we have studied in detail the expected precision in $|\Delta m_{\mu\mu}^2|$ that can be achieved after the complete runs of T2K and NOνA experiments.

| True $\sin^2 \theta_{23}$ | T2K (5ν) | NOνA (3ν + 3¯ν) | T2K + NOνA |
|--------------------------|----------|-----------------|------------|
| 0.36                     | 1.53%    | 2.33%           | 1.24% (2.41$^{+0.09}_{-0.09}$) |
| 0.50                     | 1.16%    | 1.45%           | 0.87% (2.41$^{+0.07}_{-0.06}$) |
| 0.66                     | 1.53%    | 2.26%           | 1.24% (2.41$^{+0.09}_{-0.09}$) |

Table 2: Relative 1σ precision on $|\Delta m_{\mu\mu}^2|$ considering different true values of $\sin^2 \theta_{23}$. Results are shown for T2K, NOνA, and their combined data. In the last column, inside the parentheses, we also give the 3σ allowed ranges of test $|\Delta m_{\mu\mu}^2| (\times 10^{-3} \text{ eV}^2)$ around its best-fit.

It can be seen from Table 2 that T2K (NOνA) can measure $|\Delta m_{\mu\mu}^2|$ with a relative 1σ precision of 1.45% (1.16%) assuming maximal 2-3 mixing. Combining the data from these two experiments, a sub-percent precision is achievable. It clearly demonstrates the possible synergy between these two experiments with different energy spectra and baselines. We have also studied the dependency of this measurement on the true value of $\sin^2 \theta_{23}$. The precision in $|\Delta m_{\mu\mu}^2|$ can vary in the range of 0.87% to 1.24% depending on the true value of $\sin^2 \theta_{23}$ in its currently-allowed 3σ region. As expected, for maximal 2-3 mixing, we have the best measurement of 0.87% (see Table 2). Any analysis assuming the full runs of these two long-baseline experiments can now assume a 1σ prior of $\sim 1\%$ on $|\Delta m_{\mu\mu}^2|$. In the last column, inside the parentheses, we also present the 3σ allowed ranges of test $|\Delta m_{\mu\mu}^2| (\times 10^{-3} \text{ eV}^2)$ around its best-fit. This is a very robust measurement in the sense that it is quite immune to the present uncertainties in $\sin^2 2\theta_{13}$, $\delta_{CP}$, choice of hierarchy, and the systematic errors. This high-precision measurement of $|\Delta m_{\mu\mu}^2|$ by the current generation experiments T2K and NOνA will certainly provide a boost for the physics reach of MBRO experiments in addressing the neutrino mass hierarchy.
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A Impact of 1-3 mixing angle on $|\Delta m_{\mu\mu}^2|$ precision

The $\nu_\mu \rightarrow \nu_\mu$ survival probability is independent of $\theta_{13}$ to the first order. Therefore, the precision measurement of $|\Delta m_{\mu\mu}^2|$ should not be affected much by this parameter. This is indeed the case as shown in Fig. 6 where we have presented the precisions of $|\Delta m_{\mu\mu}^2|$ for the best-fit as well as the $3\sigma$ upper and lower limits of true $\sin^2 2\theta_{13}$. The $|\Delta m_{\mu\mu}^2|$ precision achieved by the experiment NO$\nu$A is not affected by the uncertainty in $\sin^2 2\theta_{13}$. The same is true for T2K.

![Figure 6: $\Delta\chi^2$ vs. $|\Delta m_{\mu\mu}^2|$ (test) for NO$\nu$A. Left (right) panel corresponds to true NH (IH). The effect of $\sin^2 2\theta_{13}$ on the precision of $|\Delta m_{\mu\mu}^2|$ has been shown for three different true $\sin^2 2\theta_{13}$ values: the current best-fit and the 3$\sigma$ upper and lower limits. Here $|\Delta m_{\mu\mu}^2|$ (true) = $2.41 \times 10^{-3}$ eV$^2$, $\sin^2 2\theta_{23}\text{(true)} = 0.5$, and $\delta_{CP}\text{(true)} = 0.$](image)

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