Partons and QCD in high-energy polarized pp collisions

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We present and use a technique for implementing in a fast way, and without any approximations, higher-order calculations of partonic cross sections into global analyses of parton distribution functions. The approach, which is set up in Mellin-moment space, is particularly suited for analyses of future data from polarized proton-proton collisions at RHIC, but not limited to this case. We also briefly discuss the effects of soft-gluon resummations on spin asymmetries.

1. Introduction

Current and future dedicated high-energy spin experiments are expected to vastly broaden our understanding of the nucleon spin structure. Processes in polarized lepton-nucleon and proton-proton scattering will be studied \[\text{(1)}\] that will give access to, in particular, the spin-dependent gluon distribution $\Delta g$ and the flavor-SU(2) breaking in the polarized nucleon sea, $\Delta \bar{u} - \Delta \bar{d}$. Having available at some point in the near future spin data on various different reactions, one needs to tackle the question of how to determine such polarized parton densities from the measurements. This is not at all a new problem: in the unpolarized case, several groups perform such “global analyses” of the plethora of data available there \[\text{(2)}\]. The strategy is in principle clear: an ansatz for the parton distributions at some initial scale $\mu_0$, given in terms of appropriate functional forms with a set of free parameters, is evolved to a scale $\mu_F$ relevant for a certain data point for a certain cross section. Then the parton densities at scale $\mu_F$ are used to compute the theoretical prediction for the cross section, and a $\chi^2$ value is assigned that represents the quality of the comparison to the experimental point. This is done for all data points to be included in the analysis, and subsequently the parameters in the ansatz for the parton distribution functions are varied, until eventually a minimum in $\chi^2$ is reached.

In practice, this approach is difficult if the partonic scattering is treated beyond the lowest order of perturbation theory (PT). The numerical evaluation of the hadronic cross section at higher orders is usually a rather time-consuming procedure as it often requires several tedious numerical integrations, not only for the convolutions with the parton densities, but also for the phase space integrations in the partonic cross section. The fitting procedure outlined above, on the other hand, usually requires thousands of computations of the cross section for any given data point, and so the computing time required for a fit easily becomes excessive even on modern workstations.

In the unpolarized case, a way to get around this problem is based on the fact that the parton densities are already known here rather accurately \[\text{(3)}\]. As a consequence, the theory answer for a certain cross section is expected to change in a very predictable way when going from the lowest-order Born level to the first-order approximation. It is then possible to pre-calculate a set of correction factors $K_i$ (i running over the data points), and to simply multiply them in each step of the fitting procedure to the lowest-order approximation for the cross section, the latter being
usually much faster to evaluate than that involving higher order terms. In the polarized case, however, it is in general not at all clear whether such a strategy will work. Here, the parton densities are known with much less accuracy so far. It is therefore not possible to pre-calculate higher-order correction factors that one would be able to keep fixed throughout the fit, while using “fast” lowest-order expressions for the partonic cross sections. For instance, the spin-dependent parton distributions, as well as the polarized partonic cross sections, may have zeros in the kinematical regions of interest, near which the predictions at lowest order and the next order will show marked differences.

Our goal is to find a way of implementing efficiently, and without approximations, higher-order expressions for any hadronic cross section into the fitting procedure. As was recently shown [6,7], this can be achieved in a very simple and straightforward way by going to Mellin space. A related technique was first discussed in [8].

In the following we will outline our method and apply it to an example, the production of prompt photons at high transverse momentum in polarized proton-proton collisions at RHIC. The sensitivity of this reaction to the gluon distribution via the LO Compton subprocess is the main reason why this process will be the flagship measurement of gluon resummation on spin asymmetries. As a first case study for future global analyses we also carry out a “toy” analysis of polarized deep-inelastic scattering (DIS) and prompt photon data projected for RHIC.

2. Hadronic Cross Sections and the Mellin Moment Technique

A general spin-dependent cross section in longitudinally polarized pp collisions, differential in a certain observable $O$ and integrated over experimental bins in other kinematical variables $T$, can be written as

$$\frac{d\Delta \sigma^H}{dO} = \frac{1}{2} \left[ \frac{d\sigma^H}{dO} (++ - \frac{d\sigma^H}{dO} (+-)) \right]$$

$$= \sum_{a,b} \int_{x_{a0}^{x_{a1}}} dT \int_{x_{b0}^{x_{b1}}} dx_a \int_{x_{c0}^{x_{c1}}} dx_b \times \Delta f_a(x_a, \mu_F) \Delta f_b(x_b, \mu_F) \times \frac{d\Delta \hat{\sigma}_H}{dO dT} (x_a P_A, x_b P_B, P_H, T, \mu_R, \mu_F),$$

where the arguments $(++)$ and $(+-)$ in the first line refer to the helicities of the incoming protons. The $\Delta f_i$ are the spin-dependent parton distributions, defined as

$$\Delta f_i(x, \mu_F) \equiv f_i^+(x, \mu_F) - f_i^-(x, \mu_F),$$

where $f_i^+(f_i^-)$ denotes the number density of a parton-type $f_i$ with helicity ‘$+$’ (‘$-$’) in a proton with positive helicity, carrying the fraction $x$ of the proton’s momentum. Note that for some observables, a fragmentation function may be additionally present in Eq. (1). The scale $\mu_F$ is the factorization scale for initial state collinear singularities and reflects the certain amount of arbitrariness in the separation of short-distance and long-distance physics embodied in Eq. (1). The other scale, $\mu_R$, in Eq. (1) is the renormalization scale, introduced in the procedure of renormalizing the strong coupling constant. Finally, the sum in Eq. (1) is over all contributing partonic channels $a + b \rightarrow H + X$, with $d\Delta \hat{\sigma}_H$ the associated spin-dependent partonic cross section, defined in complete analogy with the first line of Eq. (1). They are perturbative and hence are expanded in powers of $\alpha_s$.

The Mellin moments of the polarized parton distribution functions are defined as

$$\Delta f_i^m(\mu) = \int_0^1 dx \ x^{-1} \Delta f_i(x, \mu).$$

The parton distributions in Bjorken-$x$ space are recovered from the moments by an inverse Mellin transform:

$$\Delta f_i(x, \mu) = \frac{1}{2\pi i} \int_{C_n} dn \ x^{-n} \Delta f_i^m(\mu),$$

where $C_n$ denotes a contour in the complex $n$ plane that has an imaginary part ranging from $-\infty$ to $\infty$ and that intersects the real axis to the right of the rightmost poles of the $\Delta f_i^m(\mu)$. The crucial, but simple, step in applying moment techniques to Eq. (1) is to express the
$\Delta f_i(x, \mu_F)$ by their Mellin inverses in Eq. [4][8]. One subsequently interchanges integrations and arrives at
\[
\frac{d\Delta \hat{H}}{dO} = \sum_{a,b} \int_{C_n} \frac{dn}{2\pi i} \int_{C_m} \frac{dm}{2\pi i} \Delta f^n_a \Delta f^m_b \times \int_{x_{a,n}^{i.r.}}^{1} dx_a \int_{x_{b,m}^{i.r.}}^{1} dx_b \ x_a^{-n} x_b^{-m} \times \frac{d\Delta \hat{H}}{dOdT} (x_A, x_B, P_T, T, \mu_R, \mu_F) \\
= \sum_{a,b} \int_{C_n} \int_{C_m} dm \Delta f^n_a (\mu_F) \Delta f^m_b (\mu_F) \times \Delta \hat{H}_{ab}(n, m, O, \mu_R, \mu_F) . \tag{5}
\]
One can now pre-calculate the quantities $\Delta \hat{H}_{ab}(n, m, O, \mu_R, \mu_F)$, which do not depend at all on the parton distribution functions, prior to the fit for a specific set of the two Mellin variables $n$ and $m$, for each contributing subprocess and in each experimental bin. Effectively, one has to compute the cross sections with complex "dummy" parton distribution functions $x_a^{-n} x_b^{-m}$. All the tedious and time-consuming integrations are already dealt with in the calculation of the $\Delta \hat{H}_{ab}(n, m, O, \mu_R, \mu_F)$.

The double inverse Mellin transformation which finally links the parton distributions with the pre-calculated $\Delta \hat{H}_{ab}(n, m, O, \mu_R, \mu_F)$ of course still needs to be performed in each step of the fitting procedure. However, the integrations over $n$ and $m$ in Eq. (5) are extremely fast to perform by choosing the values for $n, m$ in $\Delta \hat{H}_{ab}(n, m, O, \mu_R, \mu_F)$ on the contours $C_n, C_m$ simply as the supports for a Gaussian integration. The point here is that the integrand in $n$ and $m$ falls off very rapidly as $|n|$ and $|m|$ increase along the contour, for two reasons: first, each parton distribution function is expected to fall off at least as a power $(1-x)^3$ at large $x$, which in momentum space converts into a fall-off of $\sim 1/n^4$ or higher. Secondly, we may choose contours in momentum space that are bent by an angle $\alpha - \pi/2$ with respect to the vertical direction; a possible choice is shown in Fig. 1. Then, for large $|n|$ and $|m|$, $n$ and $m$ will acquire large negative real parts, so that $(x_a)^{-n}$ and $(x_b)^{-m}$ decrease exponentially along the respective contours. This helps for the numerical convergence of the calculation of the $\Delta \hat{H}_{ab}(n, m, O, \mu_R, \mu_F)$ and also gives them a rapid fall-off at large arguments.

3. Example: Prompt Photons at RHIC

In Ref. [8] we have presented two practical applications of the Mellin technique: the semi-inclusive production of hadrons in polarized DIS, and the production of prompt photons in polarized $pp$ collisions at RHIC. Here we will briefly recall our findings for the latter process. To be specific, we consider the transverse momentum ($p_T$) distribution of the prompt photon, integrated over a certain experimental bin in its pseudorapidity $\eta$. Thus, we have "$O \equiv p_T$" and "$T \equiv \eta$" in Eq. (4).

For prompt photon production, the lowest-order partonic reactions are $q + \bar{q} \rightarrow \gamma + g$ and $q + g \rightarrow \gamma + q$, the latter channel being sensitive to the polarized gluon distribution. The next-to-leading order (NLO) corrections are also available [9] and will be used in our case study.

We use $\sqrt{s} = 200$ GeV and consider five values of $p_T$ which will be experimentally accessible at RHIC, $p_T = [12.5, 17.5, 22.5, 27.5, 32.5]$ GeV. We average over $|\eta| < 0.35$ in pseudorapidity and impose an isolation cut on the photon [8], for which
we choose the criterion proposed in [4] with parameters $R = 0.4$, $\epsilon = 1$. A positive feature of this isolation criterion is the absence of a fragmentation contribution to prompt photon production. We choose the renormalization and factorization scales $\mu_R = \mu_F = p_T$.

Our first goal here is to show that the method based on Eq. (3) actually works and correctly reproduces the result obtained within the direct, but “slow”, calculation via Eq. (1). Also, we need to establish an optimal size of the grids that yields excellent accuracy but is still calculable in, say, a few hours of CPU time on a standard workstation. Fig. 2 compares the results based on Eq. (5), referred to as “Mellin technique”, to those of Eq. (1), for various sizes of the grid in $n, m$. Here we have used the polarized parton densities of [4] (“standard” set).

For a more detailed comparison, we split up the contributions to the NLO prompt photon cross section into three parts, associated with the reactions $q + \bar{q} \to \gamma + X$ and $q + g \to \gamma + X$ that are already present at Born level, and all other processes that arise only at NLO. One notices that in each case already a grid size of $64 \times 64$ values yields excellent accuracy. Even a $56 \times 56$ grid is acceptable apart from a minor deviation occurring for $q\bar{q}$ scattering in the vicinity of a zero in the partonic cross section.

The crucial asset of the Mellin method is the speed at which one can calculate the full hadronic cross section, once the grids $\Delta \tilde{\sigma}_{ab}^\gamma(n, m, p_T, \mu_R, \mu_F)$ have been pre-calculated. For the $64 \times 64$ grid, we found that 1000 evaluations of the full NLO prompt photon cross section take only about 10-15 seconds on a standard workstation. Note that this number includes the evolution (in moment space) of the parton distributions from their input scale to the scale $p_T$ relevant to this case. Clearly, an implementation into a full parton density fitting procedure is now readily possible.

To give an example, we finally perform a “toy” global analysis of the available data on polarized deep-inelastic scattering [8] and of fictitious data on prompt photon production at RHIC [2], which we project by simply calculating $A_{LL}^\gamma$ to NLO using the sets of polarized and unpolarized parton distributions of [11] and [4], respectively. For an estimate of the anticipated 1σ errors on the “data” for $A_{LL}^\gamma$, we use the numbers reported in [2]. We subsequently apply a random Gaussian shift of the pseudo-data, allowing them to vary within 1σ. The “data”, as well as the underlying theoretical calculation of $A_{LL}^\gamma$ based on the spin-dependent parton densities of [4] (solid line), are shown in Fig. 3(a).

Next, we perform a large number of fits to the full, DIS plus projected prompt photon, “data set”. We simultaneously fit all polarized parton densities, choosing the distributions of [11] as the input for the $\Delta q$, $\Delta \bar{q}$, but using randomly chosen values for the parameters in the ansatz for the polarized gluon distribution at the input scale $\mu_0$. Regarding the details of the evolution, we stay within the setup of [11], but we choose a more flexible ansatz for $\Delta g$:

$$\Delta g(x, \mu_0) = N x^\alpha (1 - x)^\beta (1 + \gamma x) g(x, \mu_0),$$

which also allows for a zero. $g(x, \mu_0)$ is the unpolarized gluon density [8] at the input scale of [11]. Ideally, thanks to the strong sensitivity of the prompt photon reaction to $\Delta g$, the gluon density in each fit should return close to the function we assumed when calculating the fictitious prompt photon “data”, in the region of $x$ probed by the data. Indeed, as shown in Fig. 3(b), this happens. The shaded band illustrates the deviations of the gluon densities obtained from the global
fits to the “reference $\Delta g$” used in generating the pseudo-data. It should be stressed that

As is expected, the gluon density is rather tightly constrained in the $x$-region dominantly probed by the prompt photon data. This is true in particular at $x \approx 0.15$, as a result of the most precise “data point” for $A_{LL}^\gamma$ at $p_T = 12.5$ GeV. Fig. 3(b) shows also two extreme gluon densities with first moments $\Delta g^1(\mu_0) = \pm 0.8$ (dotted lines), which are both in perfect agreement with all presently available DIS data. It should be noted that future measurements of $A_{LL}^\gamma$ at RHIC at $\sqrt{S} = 500$ GeV and for similar $p_T$ values would further constrain $\Delta g$ in the $x$-region between 0.05 and 0.1. Although our analysis still contains a certain bias by choosing only the framework of [11] for the fits as well as by our choice of what $\chi^2$ values are still tolerable, it clearly outlines the potential and importance of upcoming measurements of $A_{LL}^\gamma$ at RHIC for determining $\Delta g$.

4. Spin asymmetries and resummation

In the unpolarized case, a pattern of disagreement between theoretical predictions and experimental data for prompt photon production has been observed in recent years [12], not globally curable by ‘fine-tuning’ the gluon density [13]. The main problems reside in the fixed-target region, where NLO theory dramatically underpredicts some data sets. At collider energies, as relevant to RHIC, there is less reason for concern.

In view of this, various improvements of the theoretical framework have been developed. One of them resorts to applying ‘threshold’ resummation to the prompt photon cross section [14], which organizes to all orders in $\alpha_s$ large logarithmic corrections to partonic hard scattering associated with emission of soft gluons. As the partonic c.m.s. energy $\hat{s}$ approaches its minimum value at $\hat{s} = 4p_T^2$, corresponding to ‘partonic threshold’ when the initial partons have just enough energy to produce the photon and the recoiling jet, the phase space available for gluon bremsstrahlung vanishes, resulting in corrections to the partonic cross section $d\hat{\sigma}/dp_T$ as large as $\alpha_s^k \ln^{2k}(1 - 4p_T^2/\hat{s}) \hat{\sigma}^\text{Born}$ at $k$-th order in $p_T$. Threshold resummation [14] organizes this singular behavior of $d\hat{\sigma}/dp_T$ to all orders. It is again carried out in Mellin-$n$ moment space, where the
logarithms have the form $\alpha_k^s \ln^{2k}(n)^{2\text{Born}}(n)$. Its application is particularly interesting in the fixed-target regime, since here the highest $x_T$ are attained in the data and the discrepancy between data and theory is largest.

In phenomenological applications [16,17] of threshold resummation, one finds a significant, albeit not sufficient, enhancement of the theory prediction in the fixed-target regime at large values of $p_T/\sqrt{s}$, accompanied by a dramatic reduction of scale dependence [16,17]. Thanks to the universal structure of soft-gluon emission, it is straightforward to apply threshold resummation to the polarized cross section. Fig. 4 shows the resulting effects on the spin asymmetry $A_{LL}$, for a 'toy' example that assumes a fictitious polarized set-up of the E706 experiment. Details are as in [16]. Even though Fig. 4 does not directly refer to the case of RHIC, it is good news that resummation effects cancel to a large extent in $A_{LL}$ for our present example.

A similar cancellation of higher-order effects associated with soft-gluon emission is found for the parity-violating longitudinal single-spin asymmetry for jet production in $p\bar{p}$ collisions, which has been proposed [18,2] as a candidate indicator for physics beyond the Standard Model and is also measurable at RHIC.

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