Ambiguous Persuasion: An Ex-Ante Formulation*

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Abstract

Consider a persuasion game where both the sender and receiver are ambiguity averse with maxmin expected utility (MEU) preferences and the sender can choose to design an ambiguous information structure. This paper studies the game with an ex-ante formulation: The sender first commits to a (possibly ambiguous) information structure and then the receiver best responds by choosing an ex-ante message-contingent action plan. Under this formulation, I show it is never strictly beneficial for the sender to use an ambiguous information structure as opposed to a standard (unambiguous) information structure. This result is shown to be robust to the receiver having non-MEU Uncertainty Averse preferences but not to the sender having non-MEU preferences.

JEL: C72, D81, D83

Keywords: Bayesian persuasion, ambiguity aversion, maxmin expected utility, uncertainty averse preferences, dynamic consistency

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1 Introduction

Strategic information provision, or more specifically persuasion, is valuable for an informed sender in influencing the action taken by a receiver (Bergemann and Morris, 2019; Kamenica, 2019). The sender is able to influence the receiver’s decision by providing information that shapes the uncertainty the receiver is facing. The literature has focused primarily on Bayesian persuasion in which the sender is only allowed to expose the receiver to uncertainty that can be fully described by a probability measure. For decisions under uncertainty, however, ambiguity aversion is recognized as an important phenomenon when facing uncertainty that is not describable by a probability measure, and it can influence behaviors (see a survey by Gilboa and Marinacci (2013)). This raises the question in the context of persuasion, first addressed by Beauchêne, Li, and Li (2019) (BLL henceforth): If the receiver is known to be ambiguity averse, can the sender benefit from strategically introducing ambiguity into their communication to better influence the receiver’s decisions?

Indeed, BLL answer this question by studying a persuasion game under the assumption that both the sender and receiver are ambiguity averse with preferences represented by the Maxmin Expected Utility (MEU) model (Gilboa and Schmeidler, 1989). In addition, the sender can choose to commit to an ambiguous information structure, an ambiguous experiment, which determines the closed convex set of probability measures over states and messages shared by both players. The receiver observes the sender’s choice of experiment and a realized message, then chooses an action according to their updated preferences. Among other results, they demonstrate that the sender can strictly benefit from choosing an ambiguous experiment and provide methods to characterize its extent.1

This paper analyzes the persuasion game under the same assumptions as in BLL but with an ex-ante formulation: After the receiver observes the sender’s choice of experiment but before any message realizes, they choose a complete message-contingent action plan (this is what I will refer to as the ex-ante stage for the receiver). The main result of this paper, Theorem 1, shows that under the ex-ante formulation of the persuasion game, the sender can never strictly gain from introducing ambiguity into their communication. In other words, a seemingly small difference in formulations of the game leads to a dramatic change in perspective on the value of ambiguous communication to the sender.

These contrasting conclusions stem from the well-known non-equivalence between ex-ante and interim decisions by an MEU decision maker when they update their pref-

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1See Cheng (2021) for an alternative characterization using concavification.
ferences with any fully consequentialist updating rule including the one assumed in BLL (Hanany and Klibanoff, 2007, 2009; Siniscalchi, 2009). Formally speaking, such receiver is dynamically inconsistent as their interim optimal actions are different from the ones specified by their ex-ante optimal action plan.\(^2\) While BLL’s persuasion game models the receiver as choosing their interim optimal action, the ex-ante formulation models the receiver as choosing actions according to their ex-ante optimal plan.

Theorem 2 extends Theorem 1 by showing that the assumption the receiver’s preferences are MEU is inessential as long as the receiver continues to be uncertainty averse and to share the same set of measures as the sender. The assumption that the receiver is uncertainty averse (an axiom introduced by Schmeidler (1989)) is much weaker than MEU. Cerreia-Vioglio, Maccheroni, Marinacci and Montrucchio (2011) characterize and provide a representation theorem for such preferences and show that they include most of the models of ambiguity averse preferences appearing in the literature, e.g., MEU, ambiguity-averse smooth ambiguity preferences (Klibanoff, Marinacci and Mukerji, 2005) and variational preferences (Maccheroni, Marinacci and Rustichini, 2006).

One direction in which Theorem 1 is not robust, is regarding the assumption the sender’s preferences are MEU. Cheng, Klibanoff, Mukerji and Renou (2023) show that a less ambiguity averse sender may have strict gains from ambiguous communication even when other assumptions of Theorem 1 are maintained and explore when and how the sender can benefit from ambiguity. In the present paper, I provide an example of such gains for non-MEU senders in Section 3.

The following example provides some intuition of why the sender benefits in BLL’s persuasion game but no longer so in the ex-ante formulation:

**Example 1.** Suppose there are two equally likely states of the world: \(\{0, 1\}\). A representative voter (the receiver) is choosing between two policies: \{Status Quo, New\}. The Status Quo is considered the safer action as its outcomes are known, while the New policy may be better or worse depending on the states. Formally, let the voter’s payoff under each action and state be given by the following payoff matrix

\[
\begin{array}{cc}
0 & 1 \\
\hline
\text{Status Quo} & 0 & 0 \\
\text{New} & 1 & -0.5 \\
\end{array}
\]

\(^2\)See Section 5.1 for a more detailed discussion.
A politician (the sender) who benefits from the Status Quo strictly prefers the voter to choose it in all states. They can design and commit to an experiment (e.g. a policy experiment) to influence the voter’s choice.

Without any additional information, the receiver strictly prefers New to Status Quo given the states are equally likely. Under Bayesian persuasion, the best the sender can achieve is to induce the receiver to choose Status Quo three-quarters of the time.\(^3\)

When both players have MEU preferences and the receiver chooses their interim optimal action, BLL show that the sender can induce the receiver to always choose Status Quo with an ambiguous experiment. Specifically, the sender can commit to revealing the state perfectly but be ambiguous about which message reveals which state. After the receiver observes any message, if they take both possibilities into account, then their updated belief is fully ambiguous about the state.\(^4\) As a result, taking the Status Quo becomes the interim optimal action according to the MEU criterion with such an updated belief.

The receiver’s interim optimal actions correspond to the message-contingent action plan that takes the action Status Quo after every message, which gives the receiver an ex-ante payoff of zero. The plan of taking the action New after every message, however, gives the receiver a strictly positive ex-ante payoff under the same ambiguous experiment.\(^5\) As a result, the plan of always taking the status quo cannot be induced by this ambiguous experiment once the receiver maximizes their ex-ante payoff. In other words, the sender’s gain in BLL’s persuasion game is achieved by inducing message-contingent action plans that are not inducible in the ex-ante formulation.

The remainder of this paper is organized as follows. Section 1.1 reviews the related literature. Section 2 introduces the ambiguous persuasion game. Section 3 gives the main result when both players are MEU decision-makers. Section 4 generalizes the main result to Uncertainty Averse preferences. Section 5 provides some discussions and concludes.

\(^3\)This is obtained by committing to a probabilistic information structure, a statistical experiment with two messages such that the receiver’s posterior given these two messages are 0 and 2/3 in terms of the probability of state 1. The probabilities of sending these two messages are 1/4 and 3/4 respectively.

\(^4\)This is true when the receiver applies the full Bayesian updating as in BLL.

\(^5\)This can be seen by noticing that the receiver’s ex-ante payoff from this plan under any state-revealing statistical experiment is the same and positive.
1.1 Related Literature

This paper contributes to the literature on Bayesian persuasion initiated by Kamenica and Gentzkow (2011), and more specifically, the line of research combining persuasion and ambiguity aversion (Beauchêne, Li and Li, 2019; Hedlund, Kauffeldt and Lammert, 2020; Nikzad, 2021; Cheng, 2022; Kosterina, 2022). While this paper shows that an MEU sender cannot benefit from ambiguous persuasion in the ex-ante formulation, in a companion paper, Cheng, Klibanoff, Mukerji and Renou (2023) characterize when and how a non-MEU sender can benefit from ambiguous persuasion. The ex-ante formulation adopted by the present paper, where the receiver chooses a message-contingent plan has also been used in other contexts, for example, the study of belief polarization (Baliga, Hanany and Klibanoff, 2013), information order (Li and Zhou, 2016, 2020), and incomplete information games (Hanany, Klibanoff and Mukerji, 2020).

The issue of dynamic consistency in ambiguous persuasion is also discussed in BLL and Pahlke (2022). But they take different views than the present paper. They consider dynamic consistency through the lens of rectangularity (Epstein and Schneider, 2003), which is a restriction imposed on the ambiguous experiment, that, if satisfied, ensures the MEU receiver is dynamically consistent under full Bayesian updating. However, not all ambiguous experiments satisfy such a constraint, for example, those used in Examples 1 and 2 are not rectangular. BLL show that if the sender is constrained to only rectangular ambiguous experiments, then there is no gain for the sender. The nature of their result is fundamentally different from the main result of this paper, as the sender here is not constrained when choosing experiments.

Pahlke (2022) introduces rectangularity to ambiguous experiments by introducing correlations between the realization of states and which statistical experiment is used to generate the message. By doing so, the receiver’s interim optimal actions will remain optimal according to the ex-ante belief with correlation, as it effectively enlarges the set of priors to its rectangular hull. As a result, in her paper, the sender’s optimal payoff from ambiguous persuasion is the same as in BLL’s characterization. In contrast, this paper studies the receiver’s decision based on her original ex-ante belief without such correlation.

There is a connection of this paper with papers on Bayesian persuasion when the players’ priors are heterogeneous (Alonso and Câmara, 2016; Laclau and Renou, 2017; Galperti, 2019). Given an ambiguous experiment, because the sender and receiver’s utility functions are different, the minimizer of their ex-ante payoffs may turn out to be different thus as if they have different priors. However, notice that this heterogeneity arises endoge-
nously, while all the listed papers consider exogenously heterogeneous priors. Therefore, the results and findings of this paper cannot be obtained as implications of those papers.

Evidently, Shishkin and Ortoleva (2023) conducted a lab experiment to study the subjects’ behaviors under an ambiguous experiment exactly the same as the one used in Example 1. They show that a majority of the subjects indeed choose their ex-ante optimal action. Their finding suggests the importance and relevance of analyzing the ambiguous persuasion game under the ex-ante formulation.

2 Ex-Ante Formulation of the Persuasion Game

I consider a persuasion game between a sender and a receiver under the same assumptions as in BLL but with the ex-ante formulation. Let $\Omega$ be a finite set states of the world.$^6$ For any finite set $X$, let $\Delta(X)$ denote the set of all probability distributions on $X$ endowed with the topology of weak convergence. The sender and receiver have a common prior $p \in \Delta(\Omega)$ with full support. There is a finite set $A$ of feasible actions the receiver can choose from. If the receiver chooses $a \in A$, the payoff to the sender and receiver are given by $u_s(a, \omega)$ and $u_r(a, \omega)$, respectively when the state is $\omega$.

Let $M$ denote a finite set of messages (with $|M| \geq \min\{|\Omega|, |A|\}$). In a Bayesian persuasion problem, the sender chooses and commits to a statistical experiment $\pi$, which is a mapping from $\Omega$ to $\Delta(M)$. Let $\pi(m|\omega)$ denote the probability of sending message $m$ in state $\omega$ under the experiment $\pi$. Notice that a statistical experiment $\sigma$ induces a joint prior $p_{\pi} \in \Delta(\Omega \times M)$ given by

$$p_{\pi} \in \Delta(\Omega \times M) = \pi(m|\omega)p(\omega).$$

Under the ex-ante formulation, the receiver observes the sender’s commitment to a statistical experiment and chooses a message-contingent action plan. Let a function $f : M \rightarrow \Delta(A)$ denote a generic contingent plan such that $f(m)(a)$ is the probability of taking action $a \in A$ after message $m \in M$ realizes. Let $\mathcal{F}$ denote the set of all contingent plans endowed with the topology of pointwise convergence. Given a statistical experiment

$^6$I study a finite environment for the ease of exposition and to be consistent with our working paper, Cheng, Klibanoff, Mukerji and Renou (2023). See Appendix A for the generalization to infinite spaces.
\[ U_r(\pi, f) = \sum_{m, a, \omega} f(m)(a)u_r(a, \omega)p_\pi(\omega \times m). \]

Similarly, the sender’s payoff is given by
\[ U_s(\pi, f) = \sum_{m, a, \omega} f(m)(a)u_s(a, \omega)p_\pi(\omega \times m). \]

Formally, the timing of the (Bayesian) persuasion game is given as follows:
1. Sender chooses and commits to a statistical experiment.
2. Receiver observes the sender’s commitment and chooses a contingent plan.
3. Nature draws the state, message, and action according to the prior and players’ strategies and payoff realizes.

The sender’s Bayesian persuasion program is defined by:
\[
\max_{\pi, f} U_s(\pi, f), \quad \text{subject to } f \in \arg\max_{f \in F} U_r(\pi, f).
\]

In an ambiguous persuasion game, in addition to statistical experiments, the sender can also choose and commit to an ambiguous experiment. As defined in BLL, an ambiguous experiment \( \Pi \) is a closed and convex set of statistical experiments. Moreover, neither the sender nor the receiver knows which statistical experiment is going to be used to generate the messages. As a result, after the sender commits to an ambiguous experiment and the receiver chooses a contingent plan, both players need to evaluate their payoffs in the presence of ambiguity.\(^7\)

As each statistical experiment induces a joint prior \( p_\pi \), an ambiguous experiment \( \Pi \) induces a set of joint priors denoted by \( C_{\Pi} \). Formally,
\[
C_{\Pi} := \{ p_\pi \in \Delta(\Omega \times M) : \pi \in \Pi \},
\]

\(^7\)It is important that the sender has no control over which statistical experiment is used. Otherwise, the receiver may be able to infer which experiment will be used from the sender’s preference. To achieve this, the sender can either delegate the choice of the statistical experiment to an unknown and payoff-irrelevant third party or let it depend on an exogenously ambiguous event, say the draw from an Ellsberg urn.
which is a closed and convex subset of $\Delta(\Omega \times M)$. Throughout this paper, both the sender and receiver are assumed to take the set $C_{\Pi}$ as the set of joint priors they deem relevant for evaluating their payoffs.

Both players are assumed to be ambiguity averse. For now, assume both players’ preferences are represented by the Maxmin Expected Utility (MEU) model with $C_{\Pi}$ being the set of possible distributions. Then given an ambiguous experiment $\Pi$, the receiver’s ex-ante payoff from choosing the contingent plan $f$ is given by

$$U_{r}^{MEU}(\Pi, f) = \min_{\pi \in \Pi} U_{r}(\pi, f) = \min_{p_{\pi} \in C_{\Pi}} \sum_{m,a,\omega} f(m)u_{r}(a, \omega)p_{\pi}(\omega \times m).$$

Similarly, the sender’s ex-ante payoff from the ambiguous experiment $\Pi$ when the receiver chooses the contingent plan $f$ is given by

$$U_{s}^{MEU}(\Pi, f) = \min_{\pi \in \Pi} U_{s}(\pi, f) = \min_{p_{\pi} \in C_{\Pi}} \sum_{m,a,\omega} f(m)u_{s}(a, \omega)p_{\pi}(\omega \times m).$$

The timing of the ambiguous persuasion game is analogously the same as the Bayesian persuasion game except that the sender can choose and commit to an ambiguous experiment and if this is the case, Nature also needs to draw a statistical experiment ambiguously.

The MEU sender’s ambiguous persuasion program when facing an MEU receiver is defined by

$$\max_{\Pi, f} U_{s}^{MEU}(\Pi, f),$$

subject to $f \in \arg \max_{f \in F} U_{r}^{MEU}(\Pi, f)$.

Note that this program is in general different from program (3) in BLL as

$$f \in \arg \max_{f \in F} U_{r}^{MEU}(\Pi, f)$$

does not imply $f(m)$ remains optimal when the receiver applies full Bayesian updating to update the prior conditioning on the message $m$. 


3 No Gains from Ambiguous Persuasion

The main result of this paper shows that, under the ex-ante formulation of the persuasion game, there is no benefit to the sender from strategically introducing ambiguity into their communication.

**Theorem 1.** When both players are MEU decision-makers, the sender’s maximum payoff from the ambiguous persuasion program coincides with the maximum payoff from the Bayesian persuasion program.

The proof of Theorem 1 relies on the use of the Minimax theorem. Specifically, it implies that for any contingent plan $f^*$ that is a best response to $\Pi$, there must exist $\pi^* \in \Pi$ such that $f^*$ is also a best response to $\pi^*$. As a result, the sender is weakly better off by committing to $\pi^*$ than committing to $\Pi$ as they induce the same contingent plan by the receiver and the sender is an MEU decision-maker.

**Proof of Theorem 1.** Suppose the sender commits to some ambiguous experiment $\Pi$. Then $f$ can be induced by $\Pi$ if and only if

$$f \in \arg \max_{f \in F} \min_{p \in C_{\Pi}} \sum_{m,a,\omega} f(m)(a)u_r(a,\omega)p_\pi(\omega \times m),$$

i.e., $f$ is the solution to the maxmin program. Notice the objective function for the maxmin program is linear and thus continuous in $f$ and $p_\pi$. Moreover, $C_{\Pi}$ is a closed and convex subset of a compact set $\Delta(\Omega \times M)$, thus also compact. $F$ is a finite Cartesian product of the convex and compact set $\Delta(A)$, thus is also convex and compact under the product topology. Therefore, all the conditions for Sion’s minimax theorem (Sion, 1958) hold, we have

$$\max_{f \in F} \min_{p \in C_{\Pi}} \sum_{m,a,\omega} f(m)(a)u_r(a,\omega)p_\pi(\omega \times m) = \min_{p \in C_{\Pi}} \max_{f \in F} \sum_{m,a,\omega} f(m)(a)u_r(a,\omega)p_\pi(\omega \times m),$$

i.e., a saddle value for the program exists. Compactness of $F$ and $C_{\Pi}$ further guarantees the existence of saddle points (see Theorem 8.1 in Aubin (2002), for example). Therefore, any solution $f^*$ must be part of a saddle point, i.e., there exists some $\pi^* \in \Pi$ such that $f^*$ is also a best response to $\pi^*$.
When the sender commits to this ambiguous experiment $\Pi$ and induces $f^*$, one has

$$U_s^{MEU}(\Pi, f) = \min_{p_\pi \in C_{\Pi}} \sum_{m, a, \omega} f^*(m)(a)u_s(a, \omega)p_\pi(\omega \times m)$$

$$\leq \sum_{m, a, \omega} f^*(m)(a)u_s(a, \omega)p_{\pi^*}(\omega \times m) = U_s(\pi^*, f^*),$$

i.e., is weakly worse than committing to the statistical experiment $\pi^*$ that also induces $f^*$. Therefore, the sender’s payoff from any ambiguous experiment is always weakly dominated by a statistical experiment, which implies the optimum of the two programs must coincide.

**Remark 1.** Theorem 1 conveys a very strong message: When both players are MEU decision makers and ambiguity is introduced without inducing dynamically inconsistent choices, then there will be no benefit to the sender. In other words, all the benefits from ambiguous persuasion characterized in BLL with the interim formulation come from exploiting the receiver’s dynamic inconsistency under ambiguity.

**Remark 2.** As revealed by the proof, this impossibility result crucially relies on two forces: First, any contingent plan that is the best response by the receiver to some ambiguous experiment must also be the best response to some possible statistical experiment. Second, under MEU, the sender evaluates the receiver’s contingent plan against the worst possible statistical experiment, which makes them weakly worse off than just using the statistical experiment (in the set) that induces the same plan.

Notice the first force does not rely on the MEU assumption as long as the Minimax theorem holds for the receiver’s program. However, MEU turns out to be the crucial assumption for the second force. Whenever the sender deviates from MEU, the same argument will no longer be guaranteed to hold: When $\pi^*$ in the proof is indeed the worst case for the sender but the sender is not MEU, he may get a strictly higher payoff than the worst case. Then using $\Pi$ will be strictly better than using $\pi^*$ for the sender. To see this more concretely, consider the following example:

**Example 2.** Suppose there are two equally likely states of the world: $\{0, 1\}$. The receiver has three feasible actions: $\{a, b, c\}$. The sender’s payoff depends only on the action: $u_s(c) = 2$, $u_s(b) = 1$, and $u_s(a) = 0$. The receiver’s payoff is given by the following payoff matrix:
There are two possible messages \( \{m_1, m_2\} \) and let \( f(m_1) f(m_2) \) denote a contingent plan \( f \).

The optimal statistical experiment is given by \( \pi^* \), which generates posteriors (denoted by the probability of state 1) \( 1/4 \) and \( 3/4 \) with equal probabilities. Given this experiment, the receiver takes action \( b \) and \( c \) at the two posteriors respectively. The sender’s ex-ante payoff is

\[
U_s(\pi^*, bc) = \frac{1}{2} u_s(b) + \frac{1}{2} u_s(c) = \frac{3}{2}
\]

Consider a statistical experiment \( \pi_\epsilon \) which generates posteriors \( (1/4 - \epsilon) \) and \( 3/4 \) for some \( \epsilon > 0 \). Because of Bayes plausibility, the first posterior is generated with a probability of \( 1/(2 + 4\epsilon) \), strictly less than \( 1/2 \). Hence, if fixing the receiver’s contingent plan, the sender’s ex-ante payoff will be strictly higher under \( \pi_\epsilon \) compared with \( \pi^* \):

\[
U_s(\pi_\epsilon, bc) = \frac{1}{2 + 4\epsilon} u_s(b) + \frac{1 + 4\epsilon}{2 + 4\epsilon} u_s(c) = \frac{3 + 8\epsilon}{2 + 4\epsilon} > \frac{3}{2} = U_s(\pi^*, bc).
\]

Let the ambiguous experiment \( \Pi \) be the closed and convex hull of \( \{\pi_\epsilon, \pi^*\} \) and notice that

\[ bc \in \arg \max_{f \in F} U^\MEU_f (\Pi, f), \]

since \( \pi^* \) is the minimizer of \( U_r(\pi, bc) \) among all experiments in \( \Pi \). Thus, \( bc \) can be induced when the sender commits to \( \Pi \).

On the other hand, \( \pi^* \) turns out to be also the minimizer of \( U_s(\pi, bc) \) among all experiments in \( \Pi \), that is,

\[ U^\MEU_s (\Pi, bc) = U_s(\pi^*, bc). \]

This makes sure that the sender cannot gain from ambiguous persuasion when she is an MEU decision-maker. However, whenever the sender is not MEU, then \( U_s(\Pi, bc) > U^\MEU_s (\Pi, bc) \), the sender clearly gets a strictly higher payoff than the optimal Bayesian persuasion. This is true when, for example, the sender has smooth ambiguity preferences or \( \alpha \)-MEU preferences.
In the next section, I provide a generalization of Theorem 1 by allowing the receiver to have more general ambiguity averse preferences.

4 Uncertainty Averse Receiver

The most general family of preferences that displays ambiguity aversion is the Uncertainty Averse preferences, defined and axiomatized in Cerreia-Vioglio, Maccheroni, Marinacci and Montrucchio (2011). According to the Uncertainty Averse representation, the receiver’s ex-ante payoff from choosing a plan $f$ when the sender commits to an ambiguous experiment $\Pi$ is given by

$$U^{UAP}_r(\Pi, f) = \min_{p \in \Delta(\Omega \times M)} G_\Pi \left( \sum_{m,a,\omega} f(m)(a)u_r(a, \omega)p(\omega \times m), p \right)$$

for some function $G_\Pi : T \times \Delta(\Omega \times M) \to (-\infty, +\infty]$ where

$$T = [\min_{a,\omega} u_r(a, \omega), \max_{a,\omega} u_r(a, \omega)]$$

and satisfies:

(i) $G_\Pi(\cdot, \cdot)$ is lower semi-continuous and quasi-convex.

(ii) $G_\Pi(\cdot, p)$ is increasing for all $p \in \Delta(\Omega \times M)$.

(iii) $\min_{p \in \Delta(\Omega \times M)} G_\Pi(t, p) = t$ for all $t \in T$.

(iv) $G_\Pi(\cdot, p)$ is extended-valued continuous on $T$ for all $p \in \Delta(\Omega \times M)$.

(v) $C_\Pi$ is the closed and convex hull of

$$\text{dom}_{\Delta} G_\Pi := \bigcup_{t \in T} \{ p \in \Delta(\Omega \times M) : G_\Pi(t, p) < \infty \}.$$ 

The first four are standard conditions for a representation of Uncertainty Averse preferences. (v) is the additional requirement that the receiver still views $C_\Pi$ as the set of relevant joint priors according to her Uncertainty Averse preferences. Notice that if the receiver has MEU preferences (as a special case of Uncertainty Averse preferences), this requirement simply reduces to the previous requirement on the receiver’s set of joint priors. 

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8See Cerreia-Vioglio, Maccheroni, Marinacci and Montrucchio (2011) for the required axioms and discussions.
On the other hand, as discussed before, the sender remains an MEU decision maker as it turns out to be crucial for the impossibility result. The MEU sender’s ambiguous persuasion program when facing an Uncertainty Averse receiver is defined by

$$\max_{\Pi, f} U^{MEU}_s(\Pi, f),$$

subject to \( f \in \arg\max_{f \in F} U^{UAP}_r(\Pi, f). \)

The following theorem shows that the impossibility result still holds in this more general environment.

**Theorem 2.** When the sender is an MEU decision maker and the receiver is an Uncertainty Averse decision maker, the sender’s maximum payoff from the ambiguous persuasion program coincides with the maximum payoff from the Bayesian persuasion program.

The proof of Theorem 2 is almost the same as Theorem 1 except for a bit more work to show conditions for the minimax theorem holds.

**Proof of Theorem 2.** Suppose the sender commits to some ambiguous experiment \( \Pi \). Then \( f \) can be induced by \( \Pi \) if and only if

$$f \in \arg\max_{f \in F} \min_{p \in C_{\Pi}} \tilde{G}_{\Pi}(f, p) := \arg\max_{f \in F} \min_{p \in C_{\Pi}} G_{\Pi} \left( \sum_{m, a, \omega} f(m)(a) u_r(a, \omega)p(\omega \times m), p \right).$$

Both \( F \) and \( C_{\Pi} \), as argued in the proof of Theorem 1, are compact convex subsets of linear topological spaces. For each \( p \in C_{\Pi}, \tilde{G}_{\Pi} (\cdot, p) \) is continuous following from continuity of \( G_{\Pi} (\cdot, p) \) and linearity of \( \sum_{m, a, \omega} f(m)(a) u_r(a, \omega)p(\omega \times m) \). \( \tilde{G}_{\Pi} (\cdot, p) \) is quasi-concave following from monotonicity of \( G_{\Pi} (\cdot, p) \) and linearity of \( \sum_{m, a, \omega} f(m)(a) u_r(a, \omega)p(\omega \times m) \). For each \( f \in F, \tilde{G}_{\Pi}(f, \cdot) \) is lower semi-continuous and quasi-convex following from condition (i).

Thus all the conditions for Sion’s minimax theorem hold and thus imply that

$$\max_{f \in F} \min_{p \in C_{\Pi}} G_{\Pi} \left( \sum_{m, a, \omega} f(m)(a) u_r(a, \omega)p(\omega \times m), p \right) = \min_{p \in C_{\Pi}} \max_{f \in F} G_{\Pi} \left( \sum_{m, a, \omega} f(m)(a) u_r(a, \omega)p(\omega \times m), p \right).$$
Again, it further implies that any solution \( f^* \) must be part of a saddle point, i.e., there exists some \( p^* \in C_\Pi \) such that

\[
f^* \in \max_{f \in F} G_\Pi(f, p^*) = \arg \max_{f \in F} G_\Pi \left( \sum_{m,a,\omega} f(m)(a)u_r(a, \omega)p^*(\omega \times m), p^* \right).
\]

Then monotonicity of \( G_\Pi(\cdot, p) \) implies that

\[
G(f^*, p^*) \geq G(f, p^*) \iff \sum_{m,a,\omega} f^*(m)(a)u_r(a, \omega)p^*(\omega \times m) \geq \sum_{m,a,\omega} f(m)(a)u_r(a, \omega)p^*(\omega \times m),
\]

i.e., \( f^* \) is also optimal for the receiver against \( p^* \) with \( p^\pi = p^* \). From this point on, exactly the same argument as in the proof of Theorem 1 applies.

\[\square\]

5 Discussions and Conclusions

5.1 Interpretations of the Ex-Ante Formulation

This paper studies ambiguous persuasion under an ex-ante formulation and finds the results are very different from BLL’s interim formulation. In Bayesian persuasion, there is no such difference as the receiver maximizes expected utility and applies Bayesian updating, thus is always dynamically consistent. This is so more fundamentally because the receiver’s preference in this case satisfies Savage’s sure-thing principle (Savage, 1972; Ghirardato, 2002). Ambiguity-averse preferences, however, necessarily violate the sure-thing principle and as a result, dynamic consistency cannot be guaranteed in general. Without digging into the behavioral interpretations, the easiest interpretation of the ex-ante formulation is that the receiver has to make a contingent plan and cannot change it thereafter. This is the case when, for example, the sender offers a contingent contract (specifying states and actions) and the receiver is willing to follow and sign at the ex-ante stage.

One behavioral interpretation is when the receiver is sophisticated enough to anticipate that they will be dynamically inconsistent under ambiguity and want to maximize their ex-ante payoff. Then the receiver would be willing to “commit” to their ex-ante optimal plan. A behavioral characterization of preferences that value such commitments in decision-making is given by Siniscalchi (2011). It is important to note that this “commit-
“Commitment” is only credible for the receiver to tie their own hand. However, this commitment is not credible for the receiver to force the sender to reveal more information. A possible way to implement this type of commitment is that the receiver may choose to set up an algorithm for taking actual actions.

Another behavioral interpretation is when the receiver updates their ambiguous belief with dynamic consistency as a criterion. That is, the receiver chooses an updating rule that makes sure their ex-ante optimal action remains interim optimal after updating. A behavioral characterization of a family of such updating rules is given by Hanany and Klibanoff (2007, 2009). However, it is known that dynamic consistency in updating can be obtained only when the updating rule is partially consequentialist. An updating rule is (fully) consequentialist if (i) the updated belief treats the counterfactual events as null events; (ii) the updating does not depend on which decision problem is considered, i.e., the available and optimal alternatives. The dynamically consistent updating rules satisfy (i) but necessarily violate (ii), thus is partially consequentialist. If the receiver chooses such an updating rule, then there will be no difference between the interim and ex-ante formulation, and both agree with the results obtained in this paper.

5.2 Concluding Remarks

Facing an ambiguity-averse receiver in a persuasion game, the sender might hope to exploit such aversion by introducing ambiguity into their communication. The present paper highlights that whether the sender is able to do so crucially depends on how the receiver best responds under ambiguity. Under the ex-ante formulation where the receiver best responds by choosing a message-contingent action plan, this paper shows that an MEU sender cannot benefit when facing any ambiguity (uncertainty) averse receiver. However, the sender will be able to benefit when she is ambiguity averse but not as extreme as MEU. The characterization of such a benefit is in the companion paper, Cheng, Klibanoff, Mukerji and Renou (2023).

Appendix A A More General Environment

In this appendix, I show that the conclusions in Theorem 1 and 2 remain true when the environment is not necessarily finite. Notice the only argument one needs to show is that conditions for a minimax theorem hold.
Let $\Omega$ and $A$ be compact subsets of Polish spaces, i.e., complete separable metric spaces. For any Polish space $X$, let $\Delta(X)$ denote the set of all Borel probability measures on $X$ endowed with the topology of weak convergence. The sender and receiver have a common prior $p \in \Delta(\Omega)$. Let $u_s$ and $u_r$ be Borel measurable continuous functions from $\Omega \times A$ to $\mathbb{R}$. Thus, if the receiver chooses $a \in A$, the utilities of the sender and receiver are given by $u_s(a, \omega)$ and $u_r(a, \omega)$ when the state is $\omega$.

Let $M$ be a finite set of messages. I adopt the distributional strategy approach (Milgrom and Weber, 1985) to define a statistical experiment by the Borel probability measure it induces over $\Omega \times M$, i.e., $\pi \in \Delta(\Omega \times M)$ such that $\pi(\omega \times M) = p(\omega)$ for all $\omega \in \Omega$. The latter constraint is just a form of Bayes Plausibility requirement. Let $\Delta_p(\Omega \times M)$ denote the set of all statistical experiments, it is a compact set following from the compactness of $\Omega \times M$ (Aliprantis and Border, 2006). An ambiguous experiment $\Pi$ is then defined as a closed convex subset of $\Delta_p(\Omega \times M)$.

The receiver chooses a message-contingent action plan, formally defined as a function $f : M \to \Delta(A)$. Let $\mathcal{F} = \Delta(A)^M$ denote the set of all contingent plans endowed with the product topology. Thus, under the product topology, it remains a compact convex linear topological space. Notice all the conditions for Sion’s minimax theorem are satisfied in this more general environment, thus the same conclusions hold.

**Theorem 3.** Conclusions of Theorem 1 and Theorem 2 are true in this more general environment.

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9When $M$ is infinite, compactness of the receiver’s set of strategies cannot be guaranteed. Nonetheless, there exist (more specialized) minimax theorems that require weaker conditions so can lead to the same conclusion, see McLinden (1984) for example.
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