Octave-spanning frequency combs have been successfully demonstrated in Kerr nonlinear microresonators. These microcombs rely on both engineered dispersion, to enable generation of frequency components across the octave, and on engineered coupling, to efficiently extract the generated light into an access waveguide while maintaining a close to critically coupled pump. The latter is challenging, as the spatial overlap between the access waveguide and the ring modes decays with frequency. This leads to strong coupling variation across the octave, with poor extraction at short wavelengths. Here, we investigate how a waveguide wrapped around a portion of the resonator, in a pulley scheme, can improve the extraction of octave-spanning microcombs, in particular at short wavelengths. We use the coupled-mode theory to predict the performance of the pulley couplers and demonstrate good agreement with experimental measurements. Using an optimal pulley coupling design, we demonstrate a 20 dB improvement in extraction at short wavelengths compared to straight waveguide coupling. © 2019 Optical Society of America

Kerr solitons generated in nonlinear microresonators [1] are promising for many applications in telecommunications [2], range measurement [3], and optical frequency metrology [4]. However, for frequency metrology in particular, many applications require octave-spanning bandwidth for full stabilization through the f-2f technique [5]. Suitable engineering of the resonator dispersion profile for octave bandwidth [6, 7], or even super-octave bandwidth [8], has been widely reported, and octave-spanning soliton frequency combs have been demonstrated [9, 10], along with f-2f stabilization [4, 11]. Such stabilization requires sufficient power at the frequencies of interest. This ultimately depends not only on the generated intracavity field and ability to take advantage of effects like targeted dispersive wave (DW) emission [7, 12, 13], but also on the extraction of the intracavity field, usually through evanescent coupling to an inplane waveguide (or waveguides) for microring resonators. Efficient extraction over an octave of bandwidth is particularly nontrivial due to the wavelength dependence of both the phase-matching and the spatial-mode overlap between the resonator and waveguide modes.

In this Letter, we characterize an approach to overcome this challenge, particularly at short wavelengths, based on a pulley configuration in which a portion of the access waveguide is wrapped around the microring [14]. Though utilized in our recent octave-spanning microcomb works [4, 9, 11], this approach was not studied in detail. Here, we present a basic coupled-mode theory (CMT) formalism to design the pulley to improve resonator-waveguide coupling, thus comb extraction at short wavelengths, while maintaining desirable coupling in the pump and long wavelength bands. One consequence of pulley coupling is the introduction of narrow spectral windows in which essentially no coupling occurs, due to a complete phase mismatch between the ring and waveguide modes. Consequently, it is important to control the spectral position of these windows in which no coupling occurs, which we refer to as antiphase-matched frequencies, so that they are separated from the regions of interest, namely the pump and DW frequencies. Experimentally, we validate both the control of the pulley antiphase-matched frequencies and the enhancement of short wavelength extraction, by ≈20 dB relative to conventional point coupling using straight waveguides.

A number of computational approaches have been used to model coupling between ring resonators and waveguides [14–16]. Here, we model resonator-waveguide coupling in an integrated planar geometry by considering only the region over which their fields interact [Fig. 1(a)], with microring outer radius...
The effects of the coupling coefficient, $\kappa_{r\rightarrow wg}$, the phase-mismatch $\Delta \beta$, and the mode overlap $\Gamma$ can be combined into a single quantity describing the coupling strength known as the coupling quality factor $Q_c$, defined as

$$Q_c = \frac{\omega R_{wg}^R}{c_0 |\kappa_{r\rightarrow wg}|^2},$$

where $R_{wg}^R$ is the group index of the ring resonator. It is convenient to compare $Q_c$ with the intrinsic quality factor $Q_i$ and to define the extraction efficiency as $\eta = (1 + Q_i/Q_c)^{-1}$.

The basic challenge that we address is conceptually illustrated in Fig. 1 and is easy to explain using this CMT framework. Straight waveguide coupling to a ring resonator involves a limited interaction length over which the waveguide and resonator modes spatially overlap, leading to close to a point-like coupling region, particularly for small-diameter rings. At long wavelengths (low frequencies) and for a carefully chosen gap size $G$, the overlap $\Gamma$ can be appreciable enough that a short interaction length is adequate, yielding $Q_c$ comparable to $Q_i$ (i.e., critical coupling). However, as seen in Figs. 1(a) and 1(b), as the wavelength decreases (frequency increases), each mode is more confined, leading to a reduction in $\Gamma$, and $Q_c$ increases exponentially with frequency. This results in poor coupling at short wavelengths, with $Q_c$ orders of magnitude higher than $Q_i$ [Fig. 1(b)]. This is problematic for octave-spanning combs, as illustrated in Fig. 1(c). Here, the spectrum of the intracavity field is simulated by solving the Lugiato–Lefever equation with the open-source pylLE package [18], for a geometry appropriate for supporting octave-span operation. Though the dispersion has been engineered to support nearly harmonic dispersive waves at 280 and 155 THz, the $>$100x difference in $Q_c$ will lead to very different out-coupled powers (given a $Q_i \approx 3 \times 10^6$ that is not expected to significantly vary with wavelength), as seen in the plot of $\eta$ in Fig. 1(c). This will be a major impediment to direct self-referencing.

To overcome this issue, it is possible to increase the interaction length between the waveguide and the ring by wrapping the former around the latter, resulting in a pulley coupling design [14] shown schematically in Fig. 2(c). We note that the overlap coefficient $\Gamma$ in Eq. (2) is independent of the position along the optical path $L$, and the accumulated phase has to be accounted for across the pulley length $L_c$, the length for which the gap is constant between the ring and the waveguide. Thus Eq. (1) becomes

$$\kappa_{r\rightarrow wg} = \Gamma(\omega)L_c e^{i \phi} dL = \Gamma L_c \text{sinc}(L_c \sqrt{(\Delta \beta/2)^2 + \Gamma^2}),$$

with $\text{sinc}(x) = \sin(x)/x$. Hence, Eq. (3) can be rewritten as

$$Q_c = \frac{\omega R_{wg}^R}{c_0} \frac{2\pi R}{\Gamma L_c \text{sinc}(L_c \sqrt{(\Delta \beta/2)^2 + \Gamma^2})^2}.$$  

The above accounts only for the region where the resonator-waveguide gap is constant, and not where the waveguide bends toward and away from the ring, as seen in Fig. 2(c). To account for this, we evaluate Eq. (1) in these regions, resulting in a coupling coefficient $K_i$. The ratio $K_i/\Gamma_{\text{pulley}}(\omega)$ gives the effective length of the curved portion [Fig. 2(c)]. Hence, one can then introduce an effective pulley length $L_{\text{eff}}(\omega) = L_c + 2K_i(\omega)/\Gamma_{\text{pulley}}(\omega)$ that replaces the pulley length in Eq. (4).
and are not observed for a straight waveguide due to the limited interaction length (over which the gap is continuously varying). To investigate further, we calculate $Q_c$, using parameters that correspond to the experimental system studied, that is, 780 nm thick silicon nitride (Si$_3$N$_4$) microrings that are symmetrically clad in silica (SiO$_2$), with $R = 23.3$ μm. We pick $RW = 1600$ nm (resulting in the simulated frequency comb shown in [Fig. 1(b)], along with $W = 550$ nm and $G = 700$ nm. As shown in Fig. 2, the antiphase-matching condition results in sharp peaks in $Q_c$ for frequencies that vary with $L_c$. At these frequencies, regardless of the overlap between the ring and the waveguide modes, no transfer of energy occurs. The behavior of $Q_c$ on either side of the resonances is important for octave-spanning comb applications. On the blue side (short wavelengths), the overall increase in interaction length results in smaller $Q_c$ (improved coupling) than in the straight waveguide case. On the red side (longer wavelengths), the difference in $\Delta n_{\text{eff}}$, accumulated over $L_c$, keeps $Q_c$ larger than in the straight waveguide case, where the rings are generally overcoupled. The net result is a reduced wavelength dependence in $Q_c$ (outside the antiphase-matched window) than for a straight waveguide. We also note that when the pulley is sufficiently long, higher orders of antiphase-matching can be satisfied (i.e., $k > 1$), leading to multiple resonances in $Q_c$.

To validate the CMT modeling, we first verify the pulley resonance behavior through linear transmission measurements of devices designed to show a $Q_c$ resonance within the 182 THz to 207 THz tuning range of our laser source. We keep the pulley parameters fixed, namely gap $G = 800$ nm, waveguide width $W = 750$ nm, and pulley coupling length $L_c = 40$ μm, while the ring width $RW$ is varied. This results in variation of the effective index of the microring $n_{\text{eff}}^\text{ring}$, leading to a modification of the antiphase-matching condition and hence the spectral position of the corresponding frequencies. By fitting ~25 resonances of the first-order TE mode family that appear within the laser scan range, we extract the spectral dependence of $Q_c$ [Fig. 3(a)] for each $RW$, taking into account internal losses, coupling, and backscattering [19].

Simulations of $Q_c$ through CMT match both the values and the trend of the experimental $Q_c$, including the divergence at

Interestingly, Eqs. (4) and (5) suggest that resonances in the coupling will happen according to

$$\Delta n_{\text{eff}}^\text{wg} = 2 \frac{c_0}{\alpha} \sqrt{(k\pi/L_c)^2 - \Gamma^2}; \quad k \in \mathbb{N}. \quad (6)$$

Physically, these resonances correspond to locations where the access waveguide and the ring waveguide are antiphase-matched

Fig. 2. (a) Difference in effective index between the ring and waveguide mode $\Delta n_{\text{eff}}^\text{wg}$ (black) and the antiphase-matching condition [right-hand side of Eq. (6)] for $L_c = [15, 17, 26]$ μm (blue, red, green). $L_c = 26$ μm supports an antiphase condition for both the first and second order (solid and dashed green lines). (b) $Q_c$ for the corresponding pulley lengths. (c) Cartoon depicting the behavior of the electric field for the three frequencies shown as dashed lines in (b), corresponding to the DW and pump frequencies, for $L_c = 15$ μm.
resonance. Moreover, one can reproduce the variation in the $Q_c$ antiphase-matching spectral position with $RW$ due to the change of effective index. The simulations also show the difference of sensitivity in the dimension between the ring and the waveguide. As the waveguide is narrower with a mode less confined than the ring, a small variation of its width results in a significant change in its effective index. Hence, by only changing the waveguide width by 10 nm, the antiphase-matching frequency shifts by about 5 THz. To achieve the same shift, one needs to modify the $RW$ by 50 nm. This gives the ability to tune the position of the pulley antiphase-matched frequency while keeping the microring resonator at a fixed geometrical dimension that is likely already dispersion-optimized.

Outside the tunable laser range, it is possible to extract the position of the pulley antiphase-matched frequency by measuring the spectra of modulation instability (MI) combs generated through strong pumping ($P_{pmp} = 200$ mW) of the resonators at $\approx 1550$ nm, on the blue-detuned side of a cavity resonance. The spectral components in these MI combs are not phase-locked, and the overall comb acts as a quasi-continuous-wave, spectrally broadband source. Thus, the pulley coupling can be studied using these states over a spectral range as broad as the comb bandwidth.

To confirm this, we measure the MI comb spectra of the devices characterized linearly [Fig. 3(b)]. We observe that the position of the antiphase-matched frequency obtained through linear characterization of the device (by extracting $Q_c$) and through measuring the position of the dip in the MI comb is consistent. The latter method also agrees with the CMT simulations, and the antiphase-matched frequency is within the fabrication uncertainty.

We now compare a pulley coupling design optimized for extraction of an octave-spanning microcomb, namely, with a coupling antiphase-matched frequency in between the pump and the short wavelength DW, against the straight waveguide coupling for the same ring parameters. We first characterized $Q_c$ (through linear transmission measurements) in both the pump band and near the short wavelength DW, around 193 and 280 THz, respectively [Fig. 4(a)] using two continuous tunable lasers centered around 1550 and 1050 nm. We were unable to measure any resonance of the first-order TE mode in the 280 THz range for the straight waveguide, as expected from simulations where $Q_c$ is orders of magnitude higher than the expected $Q_c \approx 3 \times 10^6$ (as measured in other bands). In contrast, the pulley devices, for both $L_e = 15 \mu m$ and $L_e = 17 \mu m$, exhibit a difference in $Q_c$ of only 1 order of magnitude between the two bands and show good agreement with the values predicted by the CMT. Finally, from MI comb spectra [Fig. 4(b)], the advantage of using the pulley coupling approach for extraction is apparent. Pumping both the straight waveguide and the $L_e = 15 \mu m$ pulley devices such that the long DW and overall comb shape are the same, the pulley coupling shows a clear advantage in extracting the short DW with a $>20$ dB increase in power obtained. This enhancement of short DW extraction has recently been applied in studies of octave-spanning soliton microcombs [4,9,11,20].

In conclusion, we have presented a CMT formalism to design pulley couplers to help with the extraction of octave-spanning spectra from chip-integrated, microring-based frequency combs. We use the CMT to elucidate the roles of the phase mismatch and spatial overlap in the wavelength-dependent coupling spectrum. Finally, we show that using such pulley coupling increases by $\approx 20$ dB the extraction of the short wavelength part of an octave-spanning frequency comb compared to the same resonator with a straight waveguide coupling.

**Funding.** Defense Advanced Research Projects Agency (ACES, DODOS); National Institute of Standards and Technology (NIST-on-a-chip); UMD NIST PML (70NANB10H193).

**REFERENCES**

1. T. J. Kippenberg, A. L. Gaeta, M. Lipson, and M. L. Gorodetsky, *Science* **361**, eaan8083 (2018).
2. J. Wu, X. Xu, T. G. Nguyen, S. T. Chu, B. E. Little, R. Morandotti, A. Mitchell, and D. J. Moss, *IEEE J. Sel. Top. Quantum Electron.* **24**, 1 (2018).
3. M.-G. Suh and K. J. Vahala, *Science* **359**, 884 (2018).
4. D. T. Spencer, R. Holzwarth, and T. W. Hänsch, *Nature* **557**, 81 (2018).
5. T. Udem, R. Holzwarth, and T. W. Hänsch, *Nature* **416**, 233 (2002).
6. Y. Okawachi, M. R. E. Lamont, K. Luke, D. O. Carvalho, M. Yu, M. Lipson, and A. L. Gaeta, *Opt. Lett.* **39**, 3535 (2014).
7. Q. Li, T. C. Bries, D. A. Westley, J. R. Stone, B. R. Ilc, S. A. Diddams, S. B. Papp, and K. Srinivasan, in *Frontiers in Optics* (OSA, 2015), paper FW6C–5.
8. G. Moille, Q. Li, S. Kim, D. Westly, and K. Srinivasan, *Opt. Lett.* **43**, 2772 (2018).
9. Q. Li, T. C. Bries, D. A. Westley, T. E. Drake, J. R. Stone, B. R. Ilc, S. A. Diddams, S. B. Papp, and K. Srinivasan, *Optica* **4**, 193 (2017).
10. M. H. P. Pfeiffer, C. Herkommer, J. Liu, H. Guo, M. Karpov, E. Lucas, M. Zervas, and T. J. Kippenberg, *Optica* **4**, 684 (2017).
11. T. C. Bries, J. R. Stone, T. E. Drake, D. T. Spencer, C. Fredrick, Q. Li, D. Westly, B. R. Ilc, K. Srinivasan, S. A. Diddams, and S. B. Papp, *Opt. Lett.* **43**, 2933 (2018).
12. V. Brasc, M. Geiselmann, M. H. P. Pfeiffer, and T. J. Kippenberg, *Opt. Express* **24**, 29312 (2016).
13. X. Yi, Q.-F. Yang, X. Zhang, K. Y. Yang, X. Li, and K. Vahala, *Nat. Commun.* **8**, 14869 (2017).
14. E. S. Hosseini, S. Yegnanarayanan, A. H. Atabaki, M. Soltani, and A. Adibi, *Opt. Express* **18**, 2127 (2010).
15. M. Chin and S. Ho, *J. Lightwave Technol.* **16**, 1433 (1998).
16. M. Bahadori, M. Nikdast, S. Rumley, L. Y. Dai, N. Janosik, T. Van Vaerenbergh, A. Gazman, Q. Cheng, R. Polster, and K. Bergman, *J. Lightwave Technol.* **36**, 2767 (2018).
17. A. Yariv and P. Yeh, *Photonics: Optical Electronics in Modern Communications* (Oxford University, 2007).
18. G. Moille, Q. Li, X. Lu, and K. Srinivasan, *J. Res. Natl. Inst. Stand. Technol.* **124**, 124012 (2019).
19. M. Borselli, T. Johnson, and O. Painter, *Opt. Express* **13**, 1515 (2005).
20. S.-P. Yu, T. C. Bries, G. T. Moille, X. Lu, S. A. Diddams, K. Srinivasan, and S. B. Papp, *Phys. Rev. Appl.* **11**, 044017 (2019).