Generalized Grad-Shafranov equation for gravitational Hall-MHD equilibria

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Abstract

The consistent theoretical description of gravitational Hall-MHD (G-Hall-MHD) equilibria is of fundamental importance for understanding the phenomenology of accretion disks (AD) around compact objects (black holes, neutron stars, etc.). The very existence of these equilibria is actually suggested by observations, which show evidence of quiescent, and essentially non-relativistic, AD plasmas close to compact stars, thus indicating that accretion disks may be characterized by slowly varying EM and fluid fields. These (EM) fields, in particular the electric field, may locally be extremely intense, so that AD plasmas are likely to be locally non-neutral and therefore characterized by the presence of Hall currents. This suggests therefore that such equilibria should be described in the framework of the Hall-MHD theory. Extending previous approaches, holding for non-rotating plasmas or based on specialized single-species model equilibria which ignore the effect of space-time curvature, the purpose of this work is the formulation of a generalized Grad-Shafranov (GGS) equation suitable for the investigation of G-Hall-MHD equilibria in AD’s where non-relativistic plasmas are present. For this purpose the equilibria are assumed to be generated by a strong axisymmetric stellar magnetic field and by the gravitating plasma characterizing the AD.

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I. INTRODUCTION AND MOTIVATIONS

This work is part of a research investigation of accretion disks arising around compact objects (black holes, neutron stars, etc.). In this first paper we formulate a generalized Grad-Shafranov equation which determines the self-generated equilibrium magnetic field, via a quiescent dynamo effect.

The occurrence of dynamo effects of this type is essentially ubiquitous in character both in laboratory and space plasmas. In particular, it is well known that the phenomenon may exist even in plasmas which are at fluid (or kinetic) equilibrium, or sufficiently close to it, in the sense that the fluid fields (and, respectively, the kinetic distribution function) characterizing the plasma are stationary (or quasi-stationary). This corresponds to the appearance of a self-generated magnetic field in quiescent plasmas, i.e., in the absence of significant turbulence phenomena. The same may happen also in accretion disk (AD) plasmas. In particular, the self-generated magnetic field may become comparable in magnitude, or even larger, than that may be produced by the central object. This circumstance, it should be stressed, might occur even in the case of low density AD plasmas. In this paper we intend to pose the problem of the theoretical description of self-consistent gravitational plasma equilibria appropriate for modelling AD plasmas (in this context, what is meant by an "equilibrium" is stationary-flow solution). Their investigation is of fundamental importance for understanding the phenomenology of accretion disks. The very existence of these equilibria is actually suggested by observations, which show evidence of quiescent and essentially non-relativistic, AD plasmas close to compact stars, thus indicating that accretion disks may be characterized by slowly varying EM and fluid fields. Gravitational Hall-MHD equilibria (G-Hall-MHD) represent an important aspect of the phenomenology accretion disk (AD) plasmas. This is true especially for those AD which occur near compact objects (black holes, neutron stars, etc.) where the presence of strong EM fields is expected. These fields, and particularly the electric field, may locally be extremely intense, so that AD plasmas are likely to be locally non-neutral and therefore characterized by the presence of Hall currents. In this paper we analyze the combined role of the diamagnetic effects arising in a rotating plasma, Hall currents produced by local deviations from charge neutrality, and the influence of space-time curvature produced by the central object. It is important to stress, however, that two classes of equilibria can in principle be distinguished, which are respectively fluid
and also kinetic. In the first case only the fluid fields are required to be stationary, while in the second also the kinetic distribution function must be stationary. In addition, for kinetic equilibria, additional prescriptions can in principle be added. Therefore, the two problems, although evidently related, should be treated separately. The purpose of this work is the investigation of the first type of equilibrium. In particular, here we intend to address the problem of the formulation of a generalized Grad-Shafranov (GGS) equation suitable for the investigation of G-Hall-MHD equilibria in AD’s. The investigation of kinetic equilibria is instead left to the accompanying paper [8]. Based on a two-fluid description of the plasma, in which relativistic corrections due to space-time curvature are taken into account only by introducing a suitable Pseudo-Newtonian potential, the theory is applicable to the investigation of axisymmetric equilibria which occur in the central regions of the AD where the plasma particles are taken to still be non relativistic in the sense that their rotational velocities are much smaller than $c$. In the same region the plasma flow velocity and current density of each plasma species are determined self-consistently by taking into account the relevant diamagnetic currents, including the Hall current and the gravitational drift produced by space-time curvature. The theory permits the explicit numerical determination of the equilibrium magnetic field. The approach appears to be relevant for the investigation of a variety of possible plasma equilibria in AD’s. Despite earlier investigations, carried out by several authors, including for example Chandrasekhar (1956, [1]) and Morozov and Solov’ev (1963, [2]) and more recently by Krasheninnikov and Catto [3], Throumoulopoulos and Tasso (2001) [4], McClements and A. Thyagaraya (2001, [5]) and by Ghanbari and Abbassi (2004, [6]), a consistent theoretical description of these equilibria - a challenge for astrophysicists and mathematical physicists alike - has not yet been obtained. We refer in particular both to the approximate solutions and to the nature of the physical models adopted so far. For example, in some of previous works, factorized approximate solutions for the dipolar magnetic field were used [3,4], while in many cases the AD model is based on a single-fluid ideal-MHD description, assuming quasi-neutral rotating or non-rotating plasmas (see for example [3]) which are assumed to be subject to the action of a purely classical Newtonian potential for the central object. Other treatments [6], which investigate the problem of self-gravitating accretion disks, adopt a self-similar solution method while ignoring completely relativistic effects and the possible presence of self-generated EM fields. In a subsequent development McClements and Thyagaraja [5] tackled the problem of constructing the relevant set of fluid
equations, in particular a generalized Grad-Shafranov equation for the magnetic field, to be applied to numerical simulations of gravitational AD plasmas. Their approach, while still considering only a purely non-relativistic quasi-neutral plasma, represents an interesting development. In fact, adopting a more realistic two-fluid model based on ideal-MHD equations, it allows - in principle - the numerical investigation of gravitational plasma equilibria with arbitrary flows and subject to the action of an arbitrary self-consistent (axisymmetric) magnetic field. Nevertheless several aspects of the theory deserve further investigation. A first basic issue is related to the very formulation of the theoretical model appropriate for AD plasmas. In fact, AD’s close to compact objects can be characterized by extremely intense EM (both magnetic and electric) fields as well by gravitational fields. It follows that AD plasmas are likely to be locally non-neutral and therefore Hall currents may be present. This suggests therefore that such equilibria should be described in the framework of the Hall-MHD theory. These equilibria are here denoted as gravitational Hall-MHD (G-Hall-MHD) equilibria. In addition, for the description of equilibria occurring close to compact stars, the effect of space-time curvature is expected to become significant. Another problem to be further clarified, however, is related to the determination of the self-generated toroidal magnetic field arising in AD plasmas. In fact at equilibrium, in the absence of radial flows and of an externally-generated toroidal magnetic field, the self-consistent magnetic field is expected to be purely poloidal. Its precise relationship with the poloidal magnetic field in the plasma as well with the relevant diamagnetic currents driven by plasma inhomogeneities and flows has yet to be determined. Finally, an interesting theoretical issue is related to the possible existence of bifurcated plasma equilibria. These might correspond to AD’s characterized - in some suitable sense - by the presence of ”high” or ”low” magnetic fields respectively. Besides the existence of the bifurcation effect, an interesting closely related question is obviously its possible role in the determination of the observed phenomenology of AD’s occurring close to compact objects. In this paper, the focus of the investigation is on the combined role of the diamagnetic effects produced by the rotating plasma, and in particular the influence of Hall current and of space-time curvature, for the possible generation of *quiescent dynamo effects* in these equilibria. This permits a detailed analysis of the formation and the structure of these equilibria occurring in the central regions of the AD where non-relativistic plasmas are actually expected. Extending previous approaches, (in particular Krashennikov and Catto, 1999 [3] and Throumoulopoulos and Tasso, 2001 [4]
and McClements and Thyagaraja, 2001 [5]), the main goal of this work is the formulation of a generalized Grad-Shafranov (GGS) equation suitable for the investigation of G-Hall-MHD equilibria in AD’s where non-relativistic plasmas are present (Cremaschini et al., 2008 [7]). For this purpose the equilibria are assumed to be generated by a strong axisymmetric stellar magnetic field and by the gravitating plasma characterizing the AD. Basic features of the theoretical model adopted include: the assumptions of finite plasma rotation, two-species fluid fields, divergence-free electric current density and (primarily) toroidal plasma current. As a consequence, an equilibrium equation is obtained from Ampere’s law for the poloidal magnetic flux function. An approximate solution method for the GGS equation is presented which allows one to construct systematically approximate analytical solutions near the equatorial plane of the AD. The approach may be relevant for the investigation of a variety of possible G-Hall-MHD equilibria in AD’s.

II. CONSTRUCTION OF GRAVITATIONAL-HALL-MHD EQUILIBRIA

Let us now assume that the AD plasma is formed by two species of charged particles: one species of ions and one of electrons. The plasma is considered stationary, ”collisionless” (or, in a proper sense, weakly collisional) and axisymmetric, i.e., independent of the azimuth $\varphi$ when referred to a set of cylindrical coordinates ($R, \varphi, z$) for which the axis $R = 0$ is a principal axis. In such a case a complete description of the plasma can be obtained in terms of the species-dependent number density ($n_s$), flow velocity ($V_s$) and tensor-pressure ($\Pi_s$), together with the EM fields $\{\mathbf{E} = -\nabla \phi, \mathbf{B}\}$, both externally produced and/or self-generated, and the effective gravitational potential produced by the central object and by the plasma itself, in which the plasma is immersed. In particular, denoting by $\mathbf{b}$ the local unit vector of the magnetic field $\mathbf{B}$, ignoring electron inertia contributions and the effect of binary collisions, the species momentum balance equations read respectively for ions and electrons

$$m_i n_i \mathbf{V}_i \cdot \nabla \mathbf{V}_i + \nabla \cdot \Pi_i + m_i n_i \nabla R U^{eff} - q_i n_i \mathbf{E} - \frac{q_i}{c} n_i \mathbf{V}_i \times \mathbf{B} = 0,$$

$$\nabla \cdot \Pi_e - q_e n_e \mathbf{E} - \frac{q_e}{c} n_e \mathbf{V}_e \times \mathbf{B} = 0.$$  

It follows that the plasma current density takes the form $\mathbf{J} = \mathbf{J}_\parallel \mathbf{b} + \mathbf{J}_\perp H + \mathbf{J}_\perp D$, where $\mathbf{J}_\parallel$ is the parallel current density to be defined so as to fulfill identically the isochoricity condition
\( \nabla \cdot \mathbf{J} = 0 \). Here \( \mathbf{J}_{\perp H} = \frac{\rho \mathbf{E} \times \mathbf{b}}{B} \) and \( \mathbf{J}_{\perp D} = \frac{\rho \mathbf{E}^{(d)} \times \mathbf{b}}{B} \) denote respectively the so-called Hall and diamagnetic current densities, while \( \rho \) and \( \mathbf{E}^{(d)} \) are the local charge density of the plasma and the diamagnetic electric field \( \mathbf{E}^{(d)} \) at equilibrium, which is defined so that

\[
\rho \mathbf{E}^{(d)} \equiv -m_i n_i \mathbf{V}_i \cdot \nabla \mathbf{V}_i - \nabla \cdot \mathbf{\Pi} - m_i n_i \nabla \mathbf{R} U_{\text{eff}}.
\]  

Here \( U_{\text{eff}} \) denotes a suitable effective (pseudo-Newtonian) gravitational potential taking into account relativistic corrections of the Newtonian potential \cite{7}. Moreover, we assume that for a strongly magnetized plasma, the tensor pressure \( \mathbf{\Pi} = \sum_{s=e,i} \mathbf{\Pi}_{ss} \) is diagonal, i.e., \( \mathbf{\Pi}_{ss} \) is of the form

\[
\mathbf{\Pi}_{ss} = P_{ss} \mathbf{b} b + P_s (\mathbf{e}_b - \mathbf{b} b),
\]

where \( P_{ss} \) and \( P_s \) denote respectively the scalar parallel and perpendicular pressures for each species defined with respect to the magnetic field direction (\( b \)). In the following they are assumed to be suitably prescribed scalar fields, for example, to be considered as specified by means of an appropriate fluid or kinetic closure condition or directly determined via experimental observations or from numerical experiments. Then

\[
\nabla \cdot \mathbf{\Pi} = \nabla P_s + \mathbf{b} \mathbf{b} \cdot \nabla \left( \frac{P_{ss} - P_s}{B} \right) + (P_{ss} - P_s) \mathbf{b} \cdot \nabla \mathbf{b},
\]

where invoking again Ampere’s law and neglecting finite-\( \beta \) effects, one obtains \( \mathbf{b} \cdot \nabla \mathbf{b} \cong \nabla \ln B \cdot (\mathbf{e}_b - \mathbf{b} b) \). In the following we shall ignore the effects due to the gravitational self-field of the plasma and require

\[
P_{ss} = P_s + \delta_1 P_s,
\]

\[
\mathbf{V}_s (\mathbf{r}, t) = V_{Ts} \mathbf{e}_\varphi + \delta_2 V_R \mathbf{e}_R + \delta_2 V_z \mathbf{e}_z,
\]

\[
\mathbf{B} = \mathbf{B}_p + \varepsilon \mathbf{B}_T \equiv \nabla \psi \times \nabla \varphi + \delta_3 I(\psi) \mathbf{e}_\varphi,
\]

where

\[
\delta_1, \delta_2, \delta_3 \ll 1,
\]

where \( \delta_i \) \( (i = 1, 2, 3) \) are dimensionless parameters. Here the notation is standard. In particular \( \mathbf{e}_\varphi = R \nabla \varphi \), \( \mathbf{e}_R \) and \( \mathbf{e}_z \) are the toroidal, radial and \( z \)-direction unit vectors, \( \Omega_s \) is the toroidal angular velocity of each species and \( \psi \) is a suitable poloidal flux function. Moreover, \( V_{Ts} = \Omega_s R \), \( V_R \) and \( V_z \) are the toroidal, radial and \( z \)-direction flow velocities, while \( \mathbf{B}_p \) and \( \delta_3 \mathbf{B}_T \) denote the poloidal and toroidal magnetic fields.
III. G-HALL-MHD EQUILIBRIA WITH A PURELY POLoidal MAGNETIC FIELD

Let us now investigate in particular the case in which the radial and z-direction flow velocities and the toroidal magnetic field are negligible while the fluid pressure of each species is isotropic. In this case the gravitational-Hall-MHD (G-Hall-MHD) formulation is simply achieved by invoking the stationary Maxwell equations for the equilibrium EM fields and the momentum equations for \( \mathbf{V}_s \) for each species. Let us determine explicitly the toroidal angular rotation velocity \( \Omega_s \) for each species. In this case, invoking the ordering \( \delta_i \) and neglecting corrections proportional to \( \delta_i \) (for \( i = 1, 2, 3 \)), the \( \nabla R \) component of the momentum equation for ions gives the bifurcated solutions:

\[
\Omega_i^\pm = \frac{1}{2m_i} \left\{ -q_i \frac{(\nabla \psi \cdot \nabla R)}{cR} \pm \sqrt{\left( q_i \frac{(\nabla \psi \cdot \nabla R)}{cR} \right)^2 + 4 \frac{m_i}{n_i} \left( \frac{1}{n_i} \nabla_R P_i + q_i \nabla_R \phi + m_i \nabla_R U_{\text{eff}} \right)^2} \right\},
\]

where \( q_i \) is the ion electric charge (while in the following \( q_e \) denotes the electron charge). For electrons, neglecting electron-inertia effects and considering again the \( \nabla R \) component of the momentum equation, one obtains

\[
\Omega_e \approx c \frac{\partial \phi}{\partial \psi} + c \frac{1}{q_e n_e} \frac{\partial P_e}{\partial \psi}. \tag{9}
\]

Finally, in the same approximation, the \( \nabla z \) components of the same equations gives respectively two constraint equations for the (isotropic) scalar pressure of each species \( P_s = n_s T_s \), i.e., respectively for ions and electrons

\[
\nabla_z P_i + n_i q_i \left( -\nabla_z \phi + \frac{R \Omega_i}{cR} (\nabla \psi \cdot \nabla z) \right) + n_i m_i \nabla_z U_{\text{eff}} = 0. \tag{10}
\]

\[
\nabla_z P_e + n_e q_e \left( -\nabla_z \phi + \frac{R \Omega_e}{cR} (\nabla \psi \cdot \nabla z) \right) = 0. \tag{11}
\]

These results show at once that in the present case:

- The fluid (G-Hall-MHD) equilibria admit generally two distinct solutions \( \Omega_i^\pm \) for the ion angular velocity. Both of them are in principle admissible. Nevertheless, one can see that in the limit in which all the terms become small and negligible except the gravitational one, one recovers the customary Keplerian (or pseudo-Keplerian) solution. Nevertheless, in general AD plasmas, both signs are admissible and they
correspond to the two possible rotational directions of the ion species (clockwise or counterclockwise, according to the unit vector which defines the reference system). As a basic implication, in the two cases the parallel and the diamagnetic currents may differ significantly. In such a case the bifurcation is expected to give rise to quite disparate physical conditions for the AD, which correspond respectively to the occurrence of "strong" and "weak" poloidal magnetic fields in the AD plasma.

The existence domain of the velocity solution is determined by requiring the strictly positivity of the argument of the square root:

$$\left( q_i \left( \frac{\nabla \psi \cdot \nabla R}{cR} \right) \right)^2 + 4 \frac{m_i}{R} \left[ \frac{1}{n_i} \nabla _R P_i + q_i \nabla _R \phi + m_i \nabla _R U_{eff} \right] > 0 \quad (12)$$

This domain is determined by the magnetic field, the pressure, the density, the electrostatic and gravitational profiles. Whenever the above condition is violated, the ion velocity becomes imaginary. As this is a non-physical solution, we say that in the domains where this happens the equilibrium does not exist.

- The treatment of the gravitational field produced by the central massive object is handled by the introduction of a pseudo-Newtonian potential $G_{eff}$ [7]. This allows us to take into account relativistic corrections carried by the local space-time curvature.

- The density and pressure profiles for ions and electrons ($n_i, n_e$ and $P_i, P_e$) remain in principle arbitrary, being subject only to the species constraint equations [10]. In particular, if the density profiles are prescribed together with the pressure of each species on the equatorial plane, [i.e., $P_s(R, z = 0)$], Eq.(10) determines uniquely, for each species $s$, the vertical profile (i.e., the $z$–dependence) of the its partial pressure $P_s$.

- In the central region of the AD where $\Omega_e$ is sufficiently large [see Eq.(9)] the electrostatic field, in particular its component $E_\psi = -\nabla \psi \cdot \nabla \phi / |\nabla \psi|$ which drives the toroidal rotation of the electrons, is expected to become very strong.

Let us now investigate the relevant Maxwell equations: the Poisson equation for the electrostatic field $\mathbf{E}$ and Ampere’s law for the magnetic field. These give respectively for the electrostatic potential $\phi$ and the poloidal flux function the pde’s

$$\frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial \phi}{\partial R} \right) + \frac{\partial^2 \phi}{\partial z^2} = -4\pi \left( |q_i|n_i - |e|n_e \right), \quad (13)$$
\[
\frac{1}{R} \frac{\partial^2 \psi}{\partial z^2} \frac{1}{R} \frac{\partial^2 \psi}{\partial R^2} = -\frac{4\pi R}{c} (|q_i| n_i \Omega_i - |e| n_e \Omega_e),
\]
(14)
to be identified respectively with the Poisson equation and the generalized Grad-Shafranov equation.

IV. CONDITIONS OF VALIDITY AND CLOSURE CONDITIONS

We now make a few comments on the theory developed here. The first one concerns the assumption of a weakly collisional plasma for the AD, whereby the various (plasma) species are allowed to have different velocities. This requires that the mean free path for all of the species must be assumed to be much larger than the characteristic macroscopic scale length \( L \) of the relevant fluid fields \( \phi_1, ..., \phi_N \). In particular, \( L \) can be identified locally with \( L = \inf_{i=1,N} (L_i) \), where \( L_i^{-1} = |\nabla \ln \phi_i| \) is the scale length associated with the gradient of the fluid field \( \phi_i \). This assumption has the important consequence that the current density (and hence the self-consistent magnetic field which it generates) can become very strong even in a low density plasma. A second aspect concerns the physical conditions in which Hall-current effects are expected to play a significant role for G-Hall-MHD equilibria. In this regard it is important to point out again that Eq. (9) implies that in the central regions of the AD, where \( \Omega_e \) is sufficiently large, the electrostatic field can be expected to become very strong. Hence, local deviations from quasi-neutrality, should be expected in these equilibria. This means that the Debye length \( \lambda_{Ds} = 1/\sqrt{4\pi \sum_{s} Z_s^2 e^2 n_s / T_s} \) of each species may become locally comparable to the macroscopic characteristic scale length \( L \) of the plasma equilibrium. The third aspect concerns the peculiar feature discovered here for G-Hall-MHD equilibria, related to the existence of bifurcated solutions for the ion toroidal angular velocity. It is obvious that, for a prescribed magnetic field (produced by the central object), the direction of ion flow velocity in the outer regions of the AD uniquely determines the sign of the ion toroidal angular frequency also in the central region. As a consequence, it depends on the direction of injection for the charged particles at the external boundary of the disk, where the plasma flow is expected to be almost Keplerian. With the specification of this boundary condition the equilibrium solution therefore becomes unique.

Another issue is related to the closure conditions for the Hall-MHD equations. To resolve the indeterminacy in the density and pressure of each species the equilibrium magnetic and
electric fields have been determined under the assumption of suitably prescribing the pressure of each species on the equatorial plane $P_s(R, z = 0)$. We can motivate the necessity and the use itself of the closure relations by the following arguments:

1) this is a standard approach in plasma physics. For example, the same technique is used while studying plasma equilibria for laboratory plasmas;

2) generally, the set of fluid equations for plasmas may not be closed even at equilibrium, at least in the case of locally non-Maxwellian kinetic equilibria.

In general, the closure problem may be solved by adopting one of the following alternative approaches:

1) taking the density or temperature profiles for each species from experimental data. This would require observational data to be reduced and compared with theoretical models.

On the other hand, the theoretical model also suggests possible interesting developments related specifically to the adoption of:

2) phenomenological fluid closure conditions (from numerical experiments);

3) kinetic closure conditions, in which the undetermined fluid fields are prescribed via kinetic theory.

Finally - as indicated above - a completely different viewpoint lies in the investigation of kinetic equilibria. This involves the requirement that the G-Hall-MHD equilibrium corresponds actually to an underlying kinetic equilibrium, to be suitably prescribed. This point and related issues relevant for the fluid/kinetic evaluation of G-Hall-MHD equilibria will be discussed in greater detail in the accompanying paper [8].

V. CONCLUSIONS

In this paper the relevant equations which describe G-Hall-MHD equilibria have been investigated. From the above analysis we summarize the following main aspects:

1) the relevant equations describing fluid equilibria of this type have been obtained under the assumption of axial symmetry and a collisionless plasma;

2) the effect produced by equilibrium diamagnetic currents generated by the plasma, including non-neutrality and the Hall current, have been taken into account;

3) the effect of differential rotation produced by the non rigid toroidal rotation of the AD has been taken into account by including the toroidal angular frequency of each species;
4) allowance has been made for the inclusion of relativistic effects by taking into account the space-time curvature generated by the central compact object;

5) no assumption of local quasi-neutrality has been invoked.

The present theory can be generalized in several ways, to include - in particular - the kinetic treatment of G-Hall-MHD equilibria (to be discussed in the accompanying paper [8]), species-dependent flow velocities having both toroidal and radial flow velocities, as well as the additional presence of a toroidal magnetic field produced by the plasma flows, of an anisotropic pressure tensor and of self-gravitating AD’s. Based on the present theory, G-Hall-MHD equilibria can be investigated utilizing a perturbative solution method based on a power series expansion near to the equatorial plane \((z = 0)\) \([7]\). This permits an analysis of the combined role of Hall, diamagnetic and relativistic curvature effects, which are all expected to play a significant role in the central regions of AD’s.

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Notice

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[1] S. Chandrasekhar, ApJ 124, 232 (1956).
[2] A. I. Morozov and L. S. Solov'ev, Sov. Phys.- Dokl. 8, 243 (1963).
[3] S. I. Krashennikov and P. J. Catto, Phys. Lett. A 260, 502 (1999).
[4] G. N. Throumoulopoulos, H. Tasso, Geophys. Astroph. Fluid Dynamics 94, 249 (2001).
[5] K. G. McClements and A. Thyagaraya, Mon. Not. Roy. Astr. Soc., 323(3), 733 (2001).
[6] J. Ghanbari and S. Abbassi, Mon. Not. Roy. Astr. Soc., 350(4), 1437 (2004).
[7] C. Cremaschini, A. Beklemishev, J. C. Miller and M. Tessarotto, *Gravitational MHD equilibria in the presence of differential rotation*, contributed paper to be presented at 7th PAMIR International Conference on Fundamental and Applied MHD (Presqu’île de Giens, Toulon, France, September 8 - 12, 2008).
[8] C. Cremaschini, A. Beklemishev, J. C. Miller and M. Tessarotto, *Axisymmetric gravitational MHD equilibria in the presence of plasma rotation*, contributed paper to be presented at RGD26 (Kyoto, Japan, July 21-25, 2008).