Decays of the $\hat{\rho}(1^{-+})$ Exotic Hybrid and $\eta-\eta'$ Mixing

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Abstract

QCD sum-rules are used to calculate the $\hat{\rho}(1^{-+}) \to \pi\eta, \pi\eta'$ decay widths of the exotic hybrid in two different $\eta-\eta'$ mixing schemes. In the conventional flavour octet-singlet mixing scheme, the decay widths are both found to be small, while in the recently-proposed quark mixing scheme, the decay width $\Gamma_{\hat{\rho} \to \eta\pi} \approx 250\text{ MeV}$ is large compared with the decay width $\Gamma_{\hat{\rho} \to \eta'\pi} \approx 20\text{ MeV}$. These results provide some insight into $\eta-\eta'$ mixing and hybrid decay features.

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1 Introduction

In recent years, progress has been made in the search for exotic mesons such as glueballs, hybrids and multi-quark states. Investigation of these states will not only extend our understanding of hadronic physics beyond the standard quark model, but will also provide insight into non-perturbative aspects of QCD.

Independent observations of an exotic isovector $J^{PC} = 1^{-+}$ state are of particular interest because of its special quantum numbers and decay modes identifying it as a hybrid meson. The $\hat{\rho}(1405)$ with mass $1392^{+25}_{-22}$ MeV, width $333 \pm 50\text{ MeV}$ decaying into $\eta\pi$ was observed by both E852 and Crystal Barrel in very different production processes [1, 2]. The $\hat{\rho}(1600)$ with mass $1593 \pm 8\text{ MeV}$, width $168 \pm 20\text{ MeV}$, decaying into $\rho\pi$, was also observed by E852 [3]. Though the underlying structure of this state is not yet understood, a new important decay mode $\hat{\rho} \to \eta'\pi$ has been recently reported, where the exotic resonance has a mass $1597 \pm 10\text{ MeV}$ and a width $340 \pm 40 \pm 50\text{ MeV}$ [3].

Spectra and decay characteristics of hybrids have been studied with a variety of theoretical methods such as the MIT bag model, flux tube model, potential model, quark-gluon constituent model, QCD sum rules and lattice gauge theory, but the exploration is neither complete nor definitive [4]. The exotic $1^{-+}$ hybrid is generally predicted to be the lowest lying hybrid state, but predictions of its decay rate vary widely in these analyses motivating further investigation.

Theoretical studies of hybrid decays using three point function sum-rules have identified $\hat{\rho}(1^{-+}) \to \rho\pi$ as the dominant decay mode compared to $\hat{\rho}(1^{-+}) \to \pi\eta, \eta'\pi$ [5, 6]. However, the complications associated with $\eta-\eta'$ mixing (which can potentially include a gluonic component) is a possible source of the discrepancy between these theoretical investigations and experimental observations which find comparable widths in the $\eta\pi$ and $\rho\pi$ channels.

The mixing of $\eta$ and $\eta'$ has been the subject of many investigations [7]. In the traditional mixing scheme, the $\eta, \eta'$ states are regarded as superpositions of the flavor octet and the flavor singlet with a single mixing angle which can be determined from physical processes. However, recent investigations show that this picture may

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not be correct [8] and a new orthogonal quark-content basis has been proposed [9]. Decay constants and the mixing angle have been extracted in this new scheme. Further insight into \( \eta - \eta' \) mixing can thus be gained from calculation of \( 1^- \) hybrid decays to \( \eta \pi, \eta' \pi \) in the two schemes, and comparing with the existing experiments in these channels.

Issues of \( \eta - \eta' \) mixing are reviewed in Section 2, including the interpolating currents needed in the sum-rule method. The QCD sum-rules for the decays \( \hat{\rho} \rightarrow \pi \eta', \pi \eta \) are developed in Section 3, and their analysis is given in Section 4. The final Section is reserved for conclusions.

## 2 \( \eta, \eta' \) mixing

Conventionally, \( \eta, \eta' \) states are considered as superpositions of an \( SU(3) \) flavor octet \( \eta_8 \) and a flavor singlet \( \eta_0 \) with one mixing angle \( \theta \) as

\[
|\eta\rangle = |\eta_8\rangle \cos \theta - |\eta_0\rangle \sin \theta, \quad |\eta'\rangle = |\eta_8\rangle \sin \theta + |\eta_0\rangle \cos \theta.
\]

(1)

The decay constants are defined by

\[
\langle 0|J_{5\mu}^i|p(p)\rangle = i f_p^i p_\mu \quad (i = 8, 0; \quad P = \eta, \eta'),
\]

(2)

where \( J_{5\mu}^8 \) denotes the \( SU(3)_F \) octet and \( J_{5\mu}^0 \) the \( SU(3)_F \) singlet axial-vector current. It is typically assumed that the decay constants follow the pattern of state mixing:

\[
\begin{align*}
\langle \eta \rangle = f_8 \cos \theta, \\
\langle \eta' \rangle &= f_0 \sin \theta,
\end{align*}
\]

(3)

with the decay constants \( f_8, f_0 \) are defined by couplings of \( \eta_8 \) and \( \eta_0 \) to the divergence of the relevant axial vector currents:

\[
\begin{align*}
\partial_\mu J_{5\mu}^8 &= \frac{2}{\sqrt{6}} (m_u \bar{u} i \gamma_5 u + m_d \bar{d} i \gamma_5 d - 2m_s \bar{s} i \gamma_5 s), \\
\partial_\mu J_{5\mu}^0 &= \frac{2}{\sqrt{3}} (m_u \bar{u} i \gamma_5 u + m_d \bar{d} i \gamma_5 d + m_s \bar{s} i \gamma_5 s) + \frac{1}{\sqrt{3}} \frac{3 \alpha_s}{4 \pi} G_{F}^{A} \tilde{G}_{F}^{A}.
\end{align*}
\]

(4)

where \( G_{F}^{A} \) is the gluonic field strength tensor and its dual is \( \tilde{G}_{F}^{A} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} G_{F}^{A} \). Through analyses of physical processes involving \( \eta, \eta' \), the mixing angle is respectively found to be \( \theta \simeq -10^\circ \) or \( \theta \simeq -23^\circ \) according to the quadratic and the linear Gell-Mann-Okubo mass formula.

However, recent investigations indicate that the three parameters \( f_8, f_0, \theta \) representation of the decay constants defined in [8] do not seem to follow the same mixing pattern as the physical states [8]. Motivated by the different influences on the flavor structure of the breaking of \( SU(3)_F \) and the state mixing of vector and tensor mesons, a new \( \eta - \eta' \) mixing scheme is proposed [8]. In this scheme, the two orthogonal basis states \( \eta_\phi \) and \( \eta_s \) are related to the physical states by the transformation

\[
\begin{pmatrix}
\eta \\
\eta'
\end{pmatrix} = U(\phi) \begin{pmatrix}
\eta_\phi \\
\eta_s
\end{pmatrix},
\]

(5)

where \( U \) is a unitary matrix defined by

\[
U(\phi) = \begin{pmatrix}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi
\end{pmatrix},
\]

(6)

and ideal mixing corresponds to the case \( \phi = 0 \).

In the same way, the decay constants are defined as

\[
\langle 0|J_{5\mu}^i|p(p)\rangle = if_p^i p_\mu \quad (i = q, s; \quad P = \eta, \eta'),
\]

(7)
where \( j_{5\mu}^q \) denotes the axial vector currents with quark content \( i = q, s \). Their explicit form are
\[
\begin{align*}
  j_{5\mu}^q &= \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d) \quad (8) \\
  j_{5\mu}^{5} &= \bar{s}\gamma_\mu\gamma_5 s.
\end{align*}
\]

with divergences
\[
\begin{align*}
  \partial_\mu j_{5\mu}^q &= \frac{1}{\sqrt{2}}[2m_u\bar{u}i\gamma_5 u + 2m_d\bar{d}i\gamma_5 d + \frac{\alpha_s}{2\pi}G\tilde{G}], \\
  \partial_\mu j_{5\mu}^{8} &= 2m_s\bar{s}i\gamma_5 s + \frac{\alpha_s}{4\pi}G\tilde{G},
\end{align*}
\]

where \( G\tilde{G} \equiv G_{\mu\nu}^A G_A^{\mu\nu} \). Through these divergences, the decay constants can also be written as
\[
\langle 0|\partial_\mu j_{5\mu}^{i}|p\rangle = M_i^p f_i^p .
\]

Phenomenological analysis in this scheme reveals that the decay constants follow the pattern of the state mixing corresponding to a mixing angle \( \phi = 39.3^\circ \pm 1^\circ \) [9].

### 3 Sum-Rules for exotic hybrid decays to \( \eta\pi, \eta'\pi \)

Following the sum-rule methods of [3][9], the three point function used for the sum rule corresponding to \( \bar{q}(1^{-+}) \rightarrow \pi\eta', \pi\eta \) is
\[
\Gamma_\mu(p, q) = i \int d^4x d^4y e^{i(qx+py)} \langle 0|T j_\pi(x)\partial_\nu j_{5\nu}(y)O_\mu(0)|0 \rangle = T_s(p + q) + T_A(p - q) \mu
\]

where
\[
\begin{align*}
  j_\pi(x) &= (m_u + m_d) : \bar{u}(x)i\gamma_5 d(x) : \\
  O_\mu(x) &= g : \bar{u}\gamma_\mu T^a d G_\mu^a(x) :.
\end{align*}
\]

In the flavor octet-singlet mixing scheme, the interpolating currents \( \partial_\nu j_{8\nu}^5 \) for these processes are \( \partial_\nu j_{8\nu}^0 \) and \( \partial_\nu j_{8\nu}^8 \), while in the new quark \( \eta - \eta' \) mixing scheme, the appropriate currents are \( \partial_\nu j_{8\nu}^0 \) and \( \partial_\nu j_{8\nu}^8 \).

At leading order in perturbation theory and to lowest order in the non-strange quark masses, the quark components of the currents can be ignored in the correlation function compared to the gluonic component. Thus the gluonic component of the currents provides the dominant QCD contribution. Since this gluonic contribution to the correlation function is proportional to \( (p - q)_\mu \), we focus on the anti-symmetric factor \( T_A(p, q) \). Using the results of [3] leads to the following theoretical expressions for \( T_A^i \) at the symmetric Euclidean point \( -p^2 = -q^2 = -(p + q)^2 = Q^2 \gg \Lambda^2 \)
\[
\begin{align*}
  T_A^8 &= 0, \\
  T_A^0 &= \frac{\sqrt{3}\alpha_s}{3} \left\{ \frac{3\alpha_s}{2\pi^2} \langle \bar{u}u + \bar{d}d \rangle \ln \frac{Q^2}{\nu^2} + \frac{1}{4\pi Q^2} g(\bar{u}G_u + \bar{d}G_d) + \frac{13}{18Q^4} (\bar{u}u + \bar{d}d) \langle \alpha_s G^2 \rangle \right\}, \\
  T_A^q &= \frac{\sqrt{2}\alpha_s}{3} \left\{ \frac{3\alpha_s}{2\pi^2} \langle \bar{u}u + \bar{d}d \rangle \ln \frac{Q^2}{\nu^2} + \frac{1}{4\pi Q^2} g(\bar{u}G_u + \bar{d}G_d) + \frac{13}{18Q^4} (\bar{u}u + \bar{d}d) \langle \alpha_s G^2 \rangle \right\}, \\
  T_A^s &= \frac{\alpha_s}{3} \left\{ \frac{3\alpha_s}{2\pi^2} \langle \bar{u}u + \bar{d}d \rangle \ln \frac{Q^2}{\nu^2} + \frac{1}{4\pi Q^2} g(\bar{u}G_u + \bar{d}G_d) + \frac{13}{18Q^4} (\bar{u}u + \bar{d}d) \langle \alpha_s G^2 \rangle \right\},
\end{align*}
\]

where \( T_A^i (i = 0, 8, q, s) \) are results corresponding to different currents in [11]. Note that since \( \partial_\nu j_{8\nu}^8 \) has no gluon anomaly term, the leading-order perturbative contribution to \( T_A^8 \) is suppressed by non-strange quark mass factors and can consistently be set to zero within our approximation. A fascinating aspect of this three-point function is the suppression of perturbative corrections compared with nonperturbative effects.
To get the phenomenological representation of the three point function, we define

$$\langle 0| j_\tau(0) |\pi \rangle = f_\pi m_\pi^2, \quad \langle 0| O_{\mu}(0) |\rho \rangle = \sqrt{2} f_\rho M_\rho^3 \varepsilon_{\mu},$$

(17)

where $\varepsilon_{\mu}$ is the polarization vector of the hybrid. From the discussions in Section 2, the interpolating current $\partial_\nu f_{\nu\mu}$ couples to both the $\eta$ and $\eta'$ states, indicating that the saturation with one channel in the three point function is insufficient. Thus $\eta-\eta'$ mixing effects require saturation of the three point function with two channels: $\hat{\rho} \rightarrow \eta \pi$ and $\hat{\rho} \rightarrow \eta' \pi$. For simplicity, the contributions of single pole terms and high excited states are omitted in our results, leading to the phenomenological result

$$T^A_\lambda(Q^2) = g_{\hat{\rho}\eta\pi} f_\pi m_\pi^2 \frac{f_\eta M_\eta^2}{Q^2 + m_\pi^2} \frac{\sqrt{2} f_\rho M_\rho^3}{Q^2 + M_\rho^2} + g_{\hat{\rho}\eta'\pi} f_\pi m_{\eta'}^2 \frac{f_{\eta'} M_{\eta'}^2}{Q^2 + m_{\eta'}^2}$$

(18)

where $g_{\hat{\rho}\eta\pi}$, $g_{\hat{\rho}\eta'\pi}$ are the strong couplings and $i = 0$, $8$, $q$, $s$.

Equating the theoretical side with the phenomenological side and taking a Borel transformation of $Q^2 T^A_\lambda$, we obtain

$$g_{\hat{\rho}\eta\pi} \frac{f_\eta M_\eta^2}{f_\pi} \frac{M_\eta^2}{M_\rho^2 - M_\eta^2} \left( e^{-M_\eta^2 \tau} - e^{-M_\rho^2 \tau} \right) + g_{\hat{\rho}\eta'\pi} \frac{f_\eta M_\eta^2}{f_\pi} \frac{M_{\eta'}^2}{M_\rho^2 - M_{\eta'}^2} \left( e^{-M_{\eta'}^2 \tau} - e^{-M_\rho^2 \tau} \right) = 0,$$

(19)

$$= \frac{\sqrt{6} \tilde{\alpha}_s(\tau)}{8 \tau^2} \left\{ \frac{\tilde{\alpha}_s}{\pi^2} + \frac{13}{27} \langle \alpha_s G^2 \rangle \tau^2 \right\},$$

(20)

$$g_{\hat{\rho}\eta\pi} \frac{f_\eta M_\eta^2}{f_\pi} \frac{M_\eta^2}{M_\rho^2 - M_{\eta}^2} \left( e^{-M_{\eta}^2 \tau} - e^{-M_\rho^2 \tau} \right) + g_{\hat{\rho}\eta'\pi} \frac{f_\eta M_\eta^2}{f_\pi} \frac{M_{\eta'}^2}{M_\rho^2 - M_{\eta'}^2} \left( e^{-M_{\eta'}^2 \tau} - e^{-M_\rho^2 \tau} \right)$$

$$= \frac{\tilde{\alpha}_s(\tau)}{4 \tau^2} \left\{ \frac{\tilde{\alpha}_s}{\pi^2} + \frac{13}{27} \langle \alpha_s G^2 \rangle \tau^2 \right\},$$

(21)

$$g_{\hat{\rho}\eta\pi} \frac{f_\eta M_\eta^2}{f_\pi} \frac{M_\eta^2}{M_\rho^2 - M_{\eta}^2} \left( e^{-M_{\eta}^2 \tau} - e^{-M_\rho^2 \tau} \right) + g_{\hat{\rho}\eta'\pi} \frac{f_\eta M_\eta^2}{f_\pi} \frac{M_{\eta'}^2}{M_\rho^2 - M_{\eta'}^2} \left( e^{-M_{\eta'}^2 \tau} - e^{-M_\rho^2 \tau} \right)$$

$$= \frac{\sqrt{2} \tilde{\alpha}_s(\tau)}{8 \tau^2} \left\{ \frac{\tilde{\alpha}_s}{\pi^2} + \frac{13}{27} \langle \alpha_s G^2 \rangle \tau^2 \right\},$$

(22)

where $\tau$ is the Borel variable and the PCAC relation

$$m_\rho^2 f_\rho^2 = -(m_u + m_d) \langle \bar{u}u + \bar{d}d \rangle$$

(23)

has been used. The notation $g^{os}$ is a reminder that the strong couplings are referenced to the octet-singlet mixing scheme, while $g^{gm}$ indicates the quark mixing scheme. The above sets of equations can be solved for the strong couplings which then lead to the decay widths through

$$\Gamma_{\hat{\rho} \rightarrow \pi \pi'} \simeq |g_{\hat{\rho} \pi \pi'}|^2 \frac{M_{\rho}}{48\pi} \left( 1 - \frac{M_{\rho}^2}{M_{\pi}^2} \right)^3.$$

(24)

Results of this analysis will be presented in the next Section.

## 4 Decay widths for $\hat{\rho} \rightarrow \pi \eta'$, $\pi \eta$

For the analysis of the sum-rules, we use QCD and hybrid meson parameters consistent with

$$\langle \alpha_s G^2 \rangle = 0.07 GeV^4, \quad \Lambda^{(3)} = 350 \text{ MeV}$$

(25)

$$M_{\rho} = 1.6 \text{ GeV}, \quad f_\rho = 30 \text{ MeV}.$$  

(26)
Combined with the ref. [4] values for the $\eta$ and $\eta'$ decay constants in the two mixing schemes

$$f_\eta^0 = 0.144 \text{ GeV}, \quad f_{\eta'}^0 = 0.125 \text{ GeV}, \quad f_{\eta}^s = 0.13 \text{ GeV}, \quad f_{\eta'}^s = 0.12 \text{ GeV}, \quad f_{\eta}^8 = 0.190 \text{ GeV}. \quad (27)$$

$$f_{\eta'}^0 = 0.178 \text{ GeV}, \quad f_{\eta'}^s = 0.011 \text{ GeV}, \quad f_{\eta'}^8 = 0.012 \text{ GeV}, \quad f_{\eta}^8 = 0.190 \text{ GeV}. \quad (28)$$

Other parameters occurring in the sum-rule are completely standard: $f_\rho = 132 \text{ MeV}$, $M_\eta = 0.548 \text{ GeV}$, $M_{\eta'} = 0.958 \text{ GeV}$. Via (13)–(22), these parameters completely determine the strong couplings and hence the decay widths in the two mixing scenarios. A difficulty associated with this procedure is the large uncertainty associated with $f_\rho$ ($25 \text{ MeV} < f_\rho < 50 \text{ MeV}$) [10], and the lack of freedom to consider smaller values of the hybrid mass $M_{\rho'}$ that would be consistent with the $\rho'(1405)$. However, since $f_\rho$ is a common scale on the phenomenological side of the sum-rules, ratios of couplings can be obtained independent of $f_\rho$ with the freedom to vary $M_{\rho'}$. We will also ignore any uncertainties associated with the remaining input parameters since the symmetric kinematic point and the simple phenomenological model used in the sum-rule introduces uncertainties which are not easily modelled. Thus our results should be considered as determining the characteristic scales of the decay widths for the processes $\rho \rightarrow \eta \pi$, $\rho \rightarrow \eta' \pi$. In particular, the quark mixing scheme predicts a large value for $\Gamma_{\rho \rightarrow \eta \pi}$ that is comparable to $\Gamma_{\rho \rightarrow \rho \pi}$ and is consistent with experimental observations.
5 Conclusions

In this paper, the sum-rules describing $1^{-+}$ exotic hybrid decays $\hat{\rho} \to \eta\pi$, $\eta'\pi$ have been examined in two different $\eta$-$\eta'$ mixing schemes.

In the singlet-octet scheme, we reproduce the previous sum-rule results $[5, 6]$ which find $\Gamma_{\hat{\rho} \to \eta\pi} \sim \Gamma_{\hat{\rho} \to \eta'\pi} \ll \Gamma_{\hat{\rho} \to \rho\pi}$ which differs from the experimentally observed decay pattern $\Gamma_{\hat{\rho} \to \eta\pi} \sim \Gamma_{\hat{\rho} \to \eta'\pi} \sim \Gamma_{\hat{\rho} \to \rho\pi}$ $[1, 2, 3]$. By contrast, the quark mixing scheme results in a significant enhancement of the $\eta\pi$ channel, leading to $\Gamma_{\hat{\rho} \to \eta\pi} \approx 250$ MeV comparable to the experimental observations. However, we find a much smaller width in the $\eta'\pi$ channel; $\Gamma_{\hat{\rho} \to \eta'\pi} \approx 20$ MeV and in general $\Gamma_{\hat{\rho} \to \eta\pi}/\Gamma_{\hat{\rho} \to \eta'\pi} \sim 15$ contrary to the experimental observation $[3]$.

Theoretical refinements extending our analysis beyond the symmetric momentum point and employing a more elaborate phenomenological model could be considered, but it seems unlikely that such refinements would generate a sufficiently large value of $\Gamma_{\hat{\rho} \to \eta'\pi}$ to accommodate the observed value. However, there exists evidence for a gluonic component in the $\eta$-$\eta'$ system $[12]$ which would have an impact on our analysis, and could enhance the width of the $\hat{\rho} \to \eta'\pi$ channel.

Further theoretical work and experimental confirmation of $\Gamma_{\hat{\rho} \to \eta'\pi}$ are necessary to reach a definitive conclusion, but the quark mixing scheme result $\Gamma_{\hat{\rho} \to \eta\pi} \approx 250$ MeV comparable to the experimental value provides support for the quark $\eta$-$\eta'$ mixing scheme rather than the singlet-octet approach.

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Figure 1: Strong couplings as a function of the Borel parameter $\tau$ in the octet-singlet $\eta-\eta'$ mixing scheme. The solid curve represents $g_{\rho\eta\pi}$ and the dashed curve represents $g_{\rho\eta'\pi}$. The extremum of the curves provides the sum-rule prediction of the couplings.
Figure 2: The strong coupling ratio $|g^{\alpha\beta}_{\rho\pi\pi} / g^{\alpha\beta}_{\rho'\pi\pi}|$ as a function of the hybrid mass $M_\rho$ in the octet-singlet $\eta-\eta'$ mixing scheme.
Figure 3: Strong couplings as a function of the Borel parameter $\tau$ in the quark scheme for $\eta-\eta'$ mixing. The solid curve represents $g^{\eta\eta}_{\rho\rho}$ and the dashed curve represents $g^{\eta\eta'}_{\rho\rho}$. The extremum of the curves provides the sum-rule prediction of the couplings.
Figure 4: The strong coupling ratio $\left| \frac{g_{\rho\eta\pi}^{qm}}{g_{\rho\eta'\pi}^{qm}} \right|$ as a function of the hybrid mass $M_\rho$ in the quark scheme for $\eta-\eta'$ mixing.