Cost Function Learning in Memorized Social Networks With Cognitive Behavioral Asymmetry

Yanbing Mao©, Jinning Li©, Naira Hovakimyan©, Fellow, IEEE,
Tarek Abdelzaher©, Fellow, IEEE, and Christian Lebiere©

Abstract—This article investigates the cost function learning in social information networks, wherein human memory and cognitive bias are explicitly taken into account. We first propose a model for social information-diffusion dynamics, with a focus on the systematic modeling of asymmetric cognitive bias represented by confirmation bias and novelty bias. Building on the dynamics model, we then propose the M³IRL—a memorized model and maximum-entropy-based inverse reinforcement learning—for learning cost functions. Compared with the existing model-free IRLs, the characteristics of M³IRL are significantly different here: no dependence on the Markov decision process principle, the need for only a single finite-time trajectory sample, and bounded decision variables. Finally, the effectiveness of the proposed social information-diffusion model and the M³IRL algorithm is validated by the online social media data.

Index Terms—Asymmetric confirmation bias, asymmetric novelty bias, cost function learning, human memory, inverse reinforcement learning (IRL), social information-diffusion dynamics.

I. INTRODUCTION

With the holistic combination of artificial intelligence, communication, and information technology, social media has pushed our society into far-reaching and globally connected communities, wherein people can express a wide variety of unfiltered opinions, sentiments, and emotions, regarding, e.g., health, illness, and health services [1]. The consequential challenges that we are facing are the widespread low-quality information (e.g., misinformation and disinformation) from malicious information sources with inauthentic behavior [2], [3], [4]. To fight malicious information sources, the game-theoretic framework of competitive information diffusion was initially formulated in [5], [6], [7], [8], [9], and [10], where the malicious information sources disperse the low-quality information while the defenders spread the truthful information to counter the influence of low-quality information on public. The game-theoretic formulations rely on a common assumption that the game players know each other’s cost function for decision-making. To pave the way to fight against malicious information sources in a game-theoretic adversarial setting, we, thus, investigate the problem: how to learn the implicit cost functions of game players?

Cost function learning is a much more challenging and deeper problem, which has deep roots in the inverse optimal control (IOC) [11] and then later studied in the context of inverse reinforcement learning (IRL) [12], [13], [14], [15], [16]. However, IRLs in social information networks with humans in the loop are not explored yet. The practical challenges hindering the exploration include the following.

1) Human Memory: Recent investigation on social (mis)information spread from the perspective of cognitive architecture reveals the significant influence of human memory in the information consumption [17], [18], which contradicts the common assumption of the Markov decision process (MDP) in both IOC and IRL [12], [13], [14], [15], [16], [19], [20], [21].

2) Dynamic External Stimulus: The dynamic external stimulus pushes the cost functions of decision-making to be dynamic accordingly. For example, before the outbreak of the COVID-19 crisis in the U.S., the news agency Fox News dispersed the misinformation that COVID-19 is a hoax, which is driven by the event of U.S. president impeachment, and lately changed the tune due to the possible lawsuit [22]. In dynamic scenarios, the most recent trajectory of evolving opinions can yield a much more accurate inference of cost functions than a large number of trajectory samples collected under different external stimuli.

The seminal model-based IRL with local optimality [13] provides a potential building block to address the challenges induced by human memory and dynamic external stimulus. Specifically, the model-based IRL needs only a single finite-time trajectory and removes the assumption that the expert demonstrations are globally probabilistically optimal. Inspired by the model-based IRL, we propose the M³IRL: a memorized model and maximum-entropy-based IRL with local optimality.

The proposed M³IRL applying in social information-diffusion networks is model-based. The inference accuracy
of cost functions via M3IRL, thus, relies on the trustworthiness of the social information-diffusion model. Therefore, the challenge moving forward is what is a social information-diffusion aggregate model that can well describe how humans consume and propagate information as well as how beliefs evolve? Recently, with the wide use of social media [23], in conjunction with automated news generation with the help of artificial intelligence technologies [24], [25], the dynamics of social information-diffusion has gained vital importance in studying misinformation spread and political polarization. Meanwhile, it has been revealed that cognitive bias, especially confirmation bias and novelty bias, plays a key role in the misinformation spread and polarization evolution. In particular, it is well understood that the confirmation bias helps create “echo chambers” within online social networks [26], [27], in which misinformation and polarization thrive [28], [29]. Recently, Xu et al. [23] and Abdelzaher et al. [30] revealed the significant influence of consumer preferences for outlying content (due to novelty bias) on the opinion polarization in the modern era of information overload. Hence, the challenge pertaining to modeling the information diffusion dynamics is how to capture human cognitive bias in information consumption and spread?

The Hegselmann–Krause (HK) model [31] has been recognized for the capability of capturing confirmation bias [32], through imposing bounded confidence on opinion distances. The HK model involves a discontinuity in the influence impact, i.e., an individual completely ignores the opinions that are “too far” from hers. The discontinuity, however, renders the steady-state analysis difficult. As a remedy, the tractable continuous (state-dependent) social-influence models were proposed in [6], [33], [34], [35], and [36] to capture the confirmation bias. We note that, in most of the social problems, e.g., president election and product rating, humans hold an asymmetric cognitive bias (i.e., the same opinion distance can result in different influence weights) in their opinion evolutions. However, the HK model with symmetric confidence boundary [31] and the continuous models [6], [33], [34], [35], [36] can only capture the symmetric confirmation bias (i.e., the same opinion distance results in same influence weights), and the HK model with asymmetric confidence boundaries [31] can only partially capture the asymmetric confirmation bias. In addition, the existing information diffusion models do not consider capturing the novelty bias yet [6], [31], [33], [34], [35], [36], [37], [38]. To obtain a fairly accurate model of social information for the proposed M3IRL, we provide systematic modeling guidance of asymmetric cognitive bias, represented by confirmation bias and novelty bias.

The contributions of this article are summarized as follows.

1) We propose a model of social information-diffusion dynamics, which explicitly takes subconscious bias, human memory, confirmation bias, and novelty bias into account. Meanwhile, we provide systematic modeling guidance for capturing asymmetric confirmation bias and asymmetric novelty bias.

2) Building on the proposed model of social information diffusion, we propose the M3IRL for learning the cost functions of target individuals in social networks with humans in the loop.

3) Given a library of basis functions that constitute the cost functions in social information networks, we validate the effectiveness of proposed M3IRL using the online social media data.

II. PRELIMINARIES

A. Notation

The social system is composed of $n$ individuals. The interaction among individuals is modeled by a digraph $G = (V, E)$, where $V = \{v_1, \ldots, v_n\}$ is the set of vertices representing the individuals, and $E \subseteq V \times V$ is the set of edges of the digraph $G$ representing the influence structure. Other notations that are used throughout this article are included in Table I.

To end this section, we introduce the definitions of interested cognitive bias in particular.

*Definition 1*: The confirmation bias is generally referred to cognitive behavior where a person gives larger weight to evidence that confirms his belief and undervalues evidence that could disprove it [39].

*Definition 2*: The novelty bias refers to humans’ preferences for outlying content [30].

*Definition 3*: The subconscious bias refers to an individual’s innate opinion, which is based on inherent personal characteristics, e.g., socioeconomic conditions where the individual grew up and/or lives in [37].

B. Social Information-Diffusion Dynamics

We consider the following model of social information diffusion (adopted from our prior model [6], [40], [41]), which will be used to derive the model-based cost function learning algorithm:

\[ x_i(k + 1) = a_i(x_i(k), \tau_i)x_i + \sum_{j \in V} c_{ij}(x_i(k), \tau_i)x_j(k), \quad i \in V \]  \hspace{1cm} (1)

where we clarify the notations and variables.\n
1) $x_i(k) \in [-1, 1]$ is individual $v_i$’s evolving opinion at time $k$; $s_i \in [-1, 1]$ is her subconscious bias.

2) $c_{ij}(x_i(k), \tau_i)$ represents the influence weight of individual $v_j$ on $v_i$, and

\[ c_{ij}(x_i(k), \tau_i) = \begin{cases} > 0, & \text{if } (v_i, v_j) \in E \\ = 0, & \text{otherwise} \end{cases} \]  \hspace{1cm} (2)
where $\tau_i$ denotes individual $i$’s memory horizon. For example, at current time $k$, individual $v_i$ has a memory of a topic information over the time $k-1, k-2, \ldots, k-\tau_i$.

3) The state-dependent influence weight $c_{ij}(x, k, \tau_i)$ is proposed to capture $v_i$’s cognitive bias induced by the conjunctive confirmation bias and novelty bias

$$c_{ij}(x, k, \tau_i) = \gamma(x_i(x, k, \tau_i), x_j(k)) + \gamma(x_i(x, k, \tau_i), x_j(k))$$

(3)

where $\gamma(x_i(x, k, \tau_i), x_j(k)) \geq 0$ is proposed to capture the novelty bias and $\gamma(x_i(x, k, \tau_i), x_j(k)) \geq 0$ describes the confirmation bias.

4) $\xi_i(x, k, \tau_i)$ in (3) denotes individual $v_i$’s sensed expectation from her memory of surrounding opinions over the memory horizon $[k-\tau_i, k-\tau_i+1, \ldots, k]$. The surroundings in real life can include individual’s neighbors and the information sources that she follows. $\xi_i(x, k, \tau_i)$ is defined as

$$\xi_i(x, k, \tau_i) \equiv \sum_{l=k-\tau_i}^{k} m_i(t) \cdot \sum |V| c_{ij}(x, t, \tau_i) x_j(t) + \sum_{l=k-\tau_i}^{k} \sum |V| m_i(v) \cdot c_{ip}(x, v, \tau_i)$$

(4)

The time-varying function $m_i(t)$ in (4) is proposed to indicate the influence of memory horizon on the sensed expectation, which satisfies

$$0 \leq m_i(t) \leq m_i(t + 1) \ \forall t \in \mathbb{N}.$$ (5)

5) $\xi_i(x, k, \tau_i)$ in (3) denotes individual $v_i$’s sensed expectation from her own memory

$$\xi_i(x, k, \tau_i) \equiv \sum_{l=k-\tau_i}^{k} m_i(t) \cdot x_i(t) + \sum_{l=k-\tau_i}^{k} m_i(v)$$

(6)

6) $a_i(x, k, \tau_i) \geq 0$ is the “resistance parameter” of individual $v_i$ on her subconscious bias. To guarantee $x_i(k) \in [-1, 1]$ for $\forall k \in \mathbb{N}$ and $\forall i \in \mathbb{V}$, it is determined in such a way that

$$a_i(x, k, \tau_i) + \sum_{j \in \mathbb{V}} c_{ij}(x, k, \tau_i) = 1 \ \forall i \in \mathbb{V}.$$ (7)

Remark 1: The imposed condition (5) indicates the decaying influence of memory horizon on the individuals’ real-time sensed exceptions (4) and (6). $m_i(t)$ can be a function of decaying activation, e.g., base-level activation, proposed in the cognitive architecture [17], [18].

C. Problem Formulation

It is well understood and has been demonstrated that confirmation bias helps create “echo chambers” within online social networks [26], [27], in which misinformation and polarization thrive [28], [29]. Meanwhile, Xu et al. [23] and Abdelzaher et al. [30] recently revealed the significant influence of novelty bias on the opinion polarization. In addition, more than 40 years of studies in cognitive and social psychology have revealed that the asymmetry effect/bias (i.e., the distance from X to Y may be estimated differently from Y to X) is a universal phenomenon, ranging from psychological similarity estimations [42] to social perception [43]. From the perspective of modeling the social information-diffusion dynamics, how to systematically capture the realistic asymmetric cognitive bias is not explored yet. The first problem that we will address is pertaining to the model, whose solution will constitute the base of model-based cost function learning.

Problem 1: What is the systematic modeling guidance for the social information-diffusion dynamics that can capture the asymmetric confirmation bias and the asymmetric novelty bias?

Generally, an individual has an implicit individual/joint cost function for (e.g., political-gain-driven, profit-driven, and curiosity-driven) decision-making. Moreover, the game-theoretic formulations for fighting against malicious information sources rely on an assumption that the players know each other’s cost function for decision-making [6], [7]. Learning the cost functions is, thus, indispensable for the feasibility of game-theoretic defense strategies. Meanwhile, we note that the dynamic external stimulus pushes the cost functions of decision-makers to be dynamic as well [22]. In the dynamic scenarios, the most recent trajectory rather than a larger number of trajectory samples collected under different stimuli is more desired for a more accurate inference of cost function. Building on the answer to Problem 1, the accurate cost function learning constitutes the second problem.

Problem 2: Given the model of social information-diffusion dynamics (1) that well captures asymmetric confirmation bias and asymmetric novelty bias, how to leverage the most recent trajectory to learn the cost functions of target individuals?

With the solutions to Problems 1 and 2, the proposed framework of cost function learning is presented in Fig. 1, where the red and blue nodes denote the observed target individuals. Specifically, we first collect a finite-time trajectory of evolving opinions and actions. The trajectory sample is then used to fit the proposed model of social information diffusion. We next input the fit model, the trajectory sample, and the library of basis functions to the M3IRL for comput-
This behavior is formally described by

\[
\text{Part 2}
\]

Alex and Bob have the same opinion distance from George’s, and they three serve as the combination coefficients associated with basis functions.

III. PROBLEM 1: ASYMMETRIC COGNITIVE BIAS

We first use the confirmation bias as an example to describe the asymmetry bias phenomenon in opinion evolution, and the expected behavior that the influence weight \( c(x, k, t_i) \), should capture. We then extend the modeling mechanism to the novelty bias. The cognitive behavior due to confirmation bias and novelty bias that the model shall capture is described in Fig. 2. For the sake of simplifying the presentation, we refer \( x_a, x_b, \) and \( x_g \) (dropping out \( \text{and} \), and \( \text{respectively.} \) We suppose that the topic being discussed is “COVID-19 Is a Hoax.” The hierarchy representations of \( x_i(t) \) are illustrated by Fig. 3(i), where 1 and –1 correspond to “completely opposing” and “completely supporting” the claim, respectively.

A. Behavior Due to Asymmetric Confirmation Bias

We describe George’s five behavior in reality if he holds confirmation bias toward the opinions of his neighbors Alex and Bob. The five corresponding behavioral scenarios are shown in Fig. 3(ii-1)–(ii-5), respectively.

Fig. 3(iii-1) (Neutral Opinion With the Same Distance): The \( i \neq 0 \) indicates that George neither supports nor opposes the claim. In this scenario, George should treat the opinions of Alex and Bob equally, as long as they have the same distance from his opinion. This behavior is formally described by

\[
\text{Part 2}
\]

are in the same domain of opposing the claim. However, Bob is more hesitating in his opinion and more likely to leave the current domain in his next opinion evolution process, while George and Alex are more stubborn. In this scenario, George should also favor Alex’s opinion more, which is described by

\[
\text{Part 2}
\]

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\[
\text{Part 2}
\]
Alex are in the same domain of opposing the claim, their opposing degrees have large distance, i.e., Alex is much more stubborn, while George is much more hesitating and 2) Bob is in the other domain, but he likes George and is very hesitating. In this scenario, George should favor Bob’s opinion more. This expected behavior is described by

$$
\zeta(x_g, x_a, x_b) > \zeta(x_g, x_a, x_b),
$$

if $|x_g - x_b| = \zeta(x_g, x_a, x_b)$ \[n\] and $x_g \cdot x_b < 0$ and $x_g \cdot x_a > 0$. \( \zeta \) \( \zeta \)

Remark 2: Taking $\zeta > 0$ as an example and considering $0 < \zeta(x_g, x_a, x_b) < 1$, the condition (12) implies that $\zeta(x_g, x_b) > \zeta(x_g, x_a)$. This means that George puts a larger influence weight on $x_b$ than $x_a$ when the ratio of their opinion differences is larger than the threshold, i.e., $(|x_g - x_a|/|x_g - x_b|) = (1/\zeta(x_g, x_a, x_b)) > 1$.

B. Definition: Asymmetric Cognitive Bias

Based on the practical asymmetry behavior described in Section III-A, we first present the formal definition of capturing asymmetric confirmation bias and then extend the definition to the asymmetric novelty bias.

Definition 4: The influence weight $\zeta(x, k, \tau_i, x_j(k))$ in (3) is said to capture the asymmetric confirmation bias if it satisfies (8)–(12) simultaneously.

Lamberson and Soroka [44] revealed that negative information—compared with positive information—is more “outlying” since it is far away from expectations. Inspired by the discovery, an individual’s sensed novelty/outlying/surprising degree of information is measured in terms of sensed expectation from her surroundings in memory. For example, if George holds novelty bias, he will prefer news/opinions that have larger distance with her own surrounding expectation $\tau_g(x, k, \tau_i)$. According to the same logic that describes the expected behavior (8)–(12) due to asymmetric confirmation bias, the expected asymmetry behaviors due to asymmetric novelty bias are formally described by

$$
\tau_g(x, \tau_i, x_a) = \tau_g(x, x_b), \quad \text{if } x_a = -x_b \quad \text{and} \quad x_g = 0
$$

$$
\tau_g(x, \tau_i, x_a) < \tau_g(x, x_b), \quad \text{if } |\tau_g - x_a| = |\tau_g - x_b| \quad \text{and} \quad x_g \cdot x_a < 0 \geq \tau_g \cdot x_b
$$

$$
\tau_g(x, \tau_i, x_a) < \tau_g(x, x_b), \quad \text{if } |\tau_g - x_a| = |\tau_g - x_b| \quad \text{and} \quad \tau_g \cdot x_a < x_g \cdot x_b \geq 0
$$

$$
\tau_g(x, \tau_i, x_a) < \tau_g(x, x_b), \quad \text{if } |\tau_g - x_a| = |\tau_g - x_b| \quad \text{and} \quad x_g \cdot x_a < 0 \geq \tau_g \cdot x_b
$$

$$
\tau_g(x, \tau_i, x_a) < \tau_g(x, x_b), \quad \text{if } |\tau_g - x_a| = |\tau_g - x_b| \quad \text{and} \quad x_g \cdot x_a \geq 0
$$

$$
\tau_g(x, \tau_i, x_a) < \tau_g(x, x_b), \quad \text{if } |\tau_g - x_a| = |\tau_g - x_b| \quad \text{and} \quad \tau_g \cdot x_a < x_g \cdot x_b \geq 0
$$

$$
\tau_g(x, \tau_i, x_a) = \tau_g(x, x_b), \quad \text{if } |\tau_g - x_a| = |\tau_g - x_b| \quad \text{and} \quad \tau_g \cdot x_a > 0.
$$

The definition of capturing asymmetric novelty bias is then formally presented as follows.

Definition 5: The influence weight $\tau(x, k, \tau_i, x_j(k))$ in (3) is said to capture the asymmetric novelty bias if it satisfies (13)–(17) simultaneously.

C. Relevant Work: Confirmation Bias Models

We now examine the existing models in capturing asymmetric confirmation bias.

1) Hegselmann–Krause Model: The seminal HK model [31], i.e.,

$$
x_i(k + 1) = \frac{1}{|N_i(k)|} \sum_{j \in N_i(k)} x_j(k), \quad i \in \mathbb{V}
$$

has been well recognized for capturing bias to some extent [23], [28], [32]. Depending on the upper confidence level $\tau_i$ and the lower confidence level $\xi_i$ in (18b), the HK model (18) can partially capture the asymmetric confirmation bias. Concretely, if $\tau_i = \xi_i$, the model (18) can only capture the symmetric confirmation bias. For example, given $\tau_i = \xi_i = 0.4$ and $x_i = 0.2$, if $x_a = 0.7$ and $x_b = -0.3$, according to the logic (18b), individual $v_i$ will abandon both the opinions $x_a = 0.7$ and $x_b = -0.3$ completely; if $x_a = 0.45$ and $x_b = -0.05$, according to (18b), individual $v_i$ will prefer the opinions $x_a = 0.45$ and $x_b = -0.05$ equally. It is obvious that the model in the scenario of $\tau_i = \xi_i$ describes the symmetric behavior.

2) Continuous State-Dependent Influence: To investigate polarization and homogeneity, the DeGroot model with continuous state-dependent influence is proposed in [33], [34].

$$
x_i(k + 1) = \frac{\sum_{j \in \mathbb{V}} c(x_i(k), x_j(k))x_j(k)}{\sum_{j \in \mathbb{V}} c(x_i(k), x_j(k))}, \quad i \in \mathbb{V}
$$

The model can only capture symmetric confirmation bias. For example, given $x_a = -0.3$, $x_b = 0.5$, and $x_j = 0.1$, according to (19b), individual $v_i$ exhibits symmetric behavior:

$$
c(x_i, x_a) = c(x_i, x_b) \quad \text{since } |x_i - x_a|^2 = |x_j - x_b|^2 = 0.16.
$$

D. Asymmetric Cognitive Bias Modeling

We present the systemic guidance of modeling asymmetric confirmation and novelty bias. In this article, we construct $c(x_g, x_a)$ and $\tau_g(x_g, x_a)$ to have the following general forms:

$$
\zeta (x_g, x_a) = \frac{1}{g_g} \left[ \tilde{f}_g (x_a) - \tilde{f}_g (x_g) \right] \geq 0
$$

$$
\tau_g (x_g, x_a) = \frac{1}{g_g} \tilde{f}_g (x_g) - \tilde{f}_g (x_a) \geq 0.
$$

We first present the sufficient and necessary conditions of the model (20) to satisfy (8)–(12) in the following theorem, whose proof appears in Appendix A.
Theorem 1: The influence weight \( c(x_a, x_a) \) given in (20) satisfies (8)–(12) if and only if
\[
\frac{g}{g} \left( \frac{\partial}{\partial} + \frac{\partial}{\partial} x_a \right) \text{ is strictly decreasing w.r.t. the distance} | \frac{\partial}{\partial} x_a |
\]
\[
\frac{g}{g} \left( \frac{\partial}{\partial} x_a \right) > \frac{\partial}{\partial} x_a + \frac{\partial}{\partial} x_a, \text{ if } |x_a - x_a| = |x_b - x_a|, x_a > x_b \text{ and } x_a > 0
\]
\[
\frac{g}{g} \left( \frac{\partial}{\partial} x_a \right) < \frac{\partial}{\partial} x_a + \frac{\partial}{\partial} x_a, \text{ if } |x_a - x_a| = |x_b - x_a|, x_a < x_b \text{ and } x_a < 0
\]
\[
\frac{g}{g} (0) = \frac{\partial}{\partial} x_a + \frac{\partial}{\partial}(-x_a).
\]

To address Problem 2, we propose the M3IRL: a memorized IR model. To address Problem 2, we propose the M3IRL: a memorized IR model.

IV. PROBLEM 2: M3IRL ALGORITHM

To address Problem 2, we propose the M3IRL: a memorized IR model. To address Problem 2, we propose the M3IRL: a memorized IR model.

Algorithm 1 M3IRL Algorithm

Input: Trajectory sample (34), basis function library:

1. Fit the social information-diffusion model (1) using the trajectory sample (34);
2. Compute the gradient and Hessian according to Eq. (47) and Eq. (48), respectively;
3. Compute the approximate of conditional probability distribution according to Eq. (46);
4. Compute the preference parameter vectors \( \alpha \) and \( \theta \) according to Eq. (51) using the library of basis functions Eq. (52)–(55).

Remark 3: The trajectory sample (34) denoted by \( T(u) \) is due to the implication that the evolving opinions \( x \) depend on the actions of observed targets.

The action space of observed targets is defined as

\[
\mathcal{U}[l] \triangleq [-1, 1]^{[l]} \triangleq [-1, 1] \times [-1, 1] \times \cdots \times [-1, 1]
\]

such that \( u(k) \in \mathcal{U}[l] \).

In the framework of cost function learning, we assume that each target’s cost function denoted by \( r_i(x, u) \) consists of \( p \) basis functions

\[
\begin{align*}
\bar{r}_i(x, u) &= \sum_{q=1}^{p} \beta_{iq} c_q(x, u), \quad \text{with} \quad \sum_{q=1}^{p} |\beta_{iq}| = 1 \quad (38a) \\
\bar{c}_q(x, u) &\triangleq \sum_{j=k}^{k+1-1} \bar{c}_q(\bar{x}(j), u(j)), \quad q = 1, 2, \ldots, p. \quad (38b)
\end{align*}
\]

The targets usually do not have identical importance on the public opinion evolution, which is due to, for example, different numbers of their followers. Motivated by this, we impose importance weights on each target’s cost function, which can be leveraged to transform multiple cost functions into a single one, i.e.,

\[
r_i(u) = \sum_{i \in \mathcal{V}} a_i r_i(x, u), \quad \text{with} \quad a_i \geq 0 \quad \text{and} \quad \sum_{i \in \mathcal{V}} a_i = 1. \quad (39)
\]
Considering that and (38), we define a set of parameter vectors
\[
\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_{|\mathcal{T}|} - 1, \alpha_{|\mathcal{T}|}]^T \in \mathbb{R}^{|\mathcal{T}|} \tag{40}
\]
\[
\theta = [\theta_1, \theta_2, \ldots, \theta_{|\mathcal{P}| - 1}, \theta_{|\mathcal{P}|}]^T \in \mathbb{R}^p \tag{41}
\]
\[
\Delta = [\Delta_1; \Delta_2; \ldots; \Delta_{|\mathcal{T}| - 1}] \in \mathbb{R}^{|\mathcal{T}|}. \tag{42}
\]

We present the approximation of log-likelihood of joint action policy (44) via local optimization in the following theorem, whose proof appears in Appendix B.

**Theorem 3:** Under Assumption 1-2), the log-likelihood of the conditional probability distribution (44), i.e., \( \log p(u|\theta, \alpha, \tilde{x}(k)) \), is approximated as
\[
\tilde{L}(\theta, \alpha) = -\frac{1}{2} u^T \mathbf{H} u + u^T \mathbf{h} + \sum_{i=1}^{|\mathcal{T}|} \log \left( \frac{|\mathbf{h} - \mathbf{H}u_i|}{e^{-\mathbf{h}^T \mathbf{H}u_i} - e^{-|\mathbf{h}|}} \right). \tag{46}
\]

**Remark 5:** Following the same proof path, the approximation of the log-likelihood of conditional probability distribution was first presented in [13] as
\[
\tilde{L}(\theta, \alpha) = -\frac{1}{2} u^T \mathbf{H} u + \frac{1}{2} \log |\mathbf{H}| - \frac{1}{2} \log 2\pi
\]

which is derived under an implicit assumption that the decision variables are unbounded such that the Gaussian integral, i.e., \( \int_{-\infty}^{\infty} e^{-u^T \mathbf{u}} du = \sqrt{\pi} \), can be leveraged. However, the approximation via Gaussian integral cannot be applied to the social problem as studied in this article since the range of decision variables is constrained to the bounded set \([-1, 1]\).

**C. Computation of \( \mathbf{H} \) and \( \mathbf{h} \)**

Moving forward is the computation of \( \mathbf{H} \) and \( \mathbf{h} \) for solving the log-likelihood approximation (46). Under Assumption 1-1), we obtain from (39) and (45) that
\[
\mathbf{h} = \sum_{i=1}^{|\mathcal{T}|} \alpha_i \left( \frac{\partial r_i(x, u)}{\partial u} + \frac{\partial x}{\partial u} \left( \frac{\partial r_i(x, u)}{\partial x} \right) \right) \tag{47}
\]
\[
\mathbf{H} = \sum_{i=1}^{|\mathcal{T}|} \alpha_i \left( \frac{\partial^2 r_i(x, u)}{\partial u^2} + \left( \frac{\partial x}{\partial u} \right)^T \frac{\partial^2 r_i(x, u)}{\partial x \partial u} \frac{\partial x}{\partial u} \right) + \left( \frac{\partial x}{\partial u} \right)^T \frac{\partial^2 r_i(x, u)}{\partial x^2} \frac{\partial x}{\partial u} \tag{48}
\]

where
\[
\frac{\partial x}{\partial u} \triangleq \left[ \begin{array}{c} \frac{\partial x}{\partial u} \\ \frac{\partial x}{\partial u} \\ \vdots \\ \frac{\partial x}{\partial u} \end{array} \right], \quad \frac{\partial^2 r_i(x, u)}{\partial x \partial u} \triangleq \left[ \begin{array}{ccc} \frac{\partial^2 r_i(x, u)}{\partial x \partial u} & \cdots & \frac{\partial^2 r_i(x, u)}{\partial x \partial u} \\ \frac{\partial^2 r_i(x, u)}{\partial x \partial u} & \cdots & \frac{\partial^2 r_i(x, u)}{\partial x \partial u} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 r_i(x, u)}{\partial x \partial u} & \cdots & \frac{\partial^2 r_i(x, u)}{\partial x \partial u} \end{array} \right]. \tag{49}
\]
Remark 6: Under Assumption 1-1), the Hessian matrix $H$ in [13] is derived as

$$H = \sum_{i=1}^{[T]} \alpha_i \left( \frac{\partial^2 r_i(x, u)}{\partial u^2} + \left( \frac{\partial x}{\partial u} \right)^T \frac{\partial^2 r_i(x, u)}{\partial x^2} \frac{\partial x}{\partial u} \right)$$

which is due to the assumption of MDP imposed on the system model.

With $H$ and $h$ at hand, the preference parameter vectors $\alpha$ and $\theta$ can be obtained by solving the following nonlinear constraint optimization problem:

$$\begin{align*}
(\theta^*, \alpha^*) &= \arg \max_{\theta, \alpha} \{ \tilde{L}(\theta, \alpha) \} \\
\text{s.t.} \quad &\sum_{q=1}^{p} |\theta_q| = 1 \\
&\sum_{i \in T} \alpha_i = 1, \alpha_i \geq 0 \quad \forall i \in T
\end{align*}$$

(51a)

(51b)

where $\tilde{L}(\theta, \alpha)$ is given in (46).

D. Basis Functions

In this section, as an example, we present a library of basis functions that can cover targets’ cost functions. We then leverage the proposed M3IRL algorithm to infer the associated preference parameter vectors $\theta$ and $\alpha$ of targets’ cost functions. The considered basis functions are

$$c_1(x, u) = \sum_{t=k}^{k+1} \sum_{j=1}^{[T]} (1 - \tilde{x}_j(t))^2 \quad (52)$$

$$c_2(x, u) = \sum_{t=k}^{k+1} \sum_{j=1}^{[T]} (1 + \tilde{x}_j(t))^2 \quad (53)$$

$$c_3(x, u) = \sum_{t=k}^{k+1} \sum_{j=1}^{[T]} \tilde{x}_j^2(t), \quad (54)$$

$$c_q-|[\Xi] + 1(x, u) = \sum_{t=k+1}^{k} (u_q(t) - u_q(t-1))^2, \quad q \in \mathbb{T}. \quad (55)$$

Remark 7: In minimization, the basis functions (52)-(54) indicate the objectives of steering the public opinions to +1, -1, and 0, respectively. The basis functions (55) imply a behavioral motivation of stubbornness. The implicit motivation representations for decision-making can be inferred from the conjunctive $\theta$ and $\alpha$.

Equations (46) and (51) indicate that the inference of preference parameters needs the computations of $h$ and $H$. Furthermore, the relations (47) and (48) imply that the computations of $h$ and $H$ rely on the computations of $(\partial r_j(x, u)/\partial x)$, $(\partial r_j(x, u)/\partial u)$, $(\partial^2 r_j(x, u)/\partial x^2)$, $(\partial^2 r_j(x, u)/\partial u^2)$, and $(\partial^2 r_j(x, u)/\partial x \partial u)$, which are carried out in [45, Appendix D].

V. EMPIRICAL VALIDATION

We collected nine Twitter users’ tweets from January 2021 to September 2021 to validate the effectiveness of the proposed information-diffusion model and the M3IRL algorithm. The network structure of nine users is shown in Fig. 4, where nodes 8 and 9 are identified as information sources. It follows from Fig. 4 that $\mathbb{H} = \{1, 2, \ldots, 7\}$ and $\mathbb{T} = \{8, 9\}$.

A. Data Processing

The collected tweets are centered on the topic of “COVID-19 vaccine,” and the tweets are sampled biweekly. The tweets are encoded as numerical values in the range $[-1, 1]$, via the BERT Twitter sentiment analysis, which is trained using the Sentiment140 dataset with 1.6 million tweets. The collected tweets with encoded values are available at Twitter_Vaccine_data.xlsx: https://github.com/ymao578/Social-Data. The encoded $x \in [-1, 0]$ denotes the opinion of opposing the COVID-19 vaccine, $x = 0$ is the neutral opinion, and $x \in (0, 1]$ represents the opinion of supporting the COVID-19 vaccine. A few examples of encoded tweets are given as follows.

1) “Scientists” and “doctors” at The Lancet and Nature have blood on their evil hands $\Rightarrow -1$.

2) #COVID19 #CDCExperts is this true? Claim falling Ill or death after receiving COVID vaccine predictable and actually a good thing $\Rightarrow -0.08$.

3) You think that I am against vaccination, I don’t. I don’t either endorse it nor against it. I don’t like the way of handling the data. $\Rightarrow 0$.

4) I am pro (some) vaccines, but they need to be tested for seven to ten years. They need to go through the proper testing. I want long-term data on vaccine outcomes. mRNA technology is synthetic how can that be good for your body? $\Rightarrow 0.216666667$.

B. Model Fitting

The considered decaying-influence model of memory is a simplified version of the base level function in the ACT-R declarative memory model [17], [18]

$$m_i(v) = \log(v^{-d_i} + 1), \quad v \in \{1, 2, \ldots, \tau_i\}, \quad d_i > 0 \quad (56)$$

1BERT Twitter Sentiment Analysis: https://github.com/OthSay/bert-tweets-analysis

2Sentiment140 dataset: https://www.kaggle.com/kazanova/sentiment140
where \( \tau_i \) is individual \( n_i \)'s memory horizon and \( d_i \) is the fitting parameter from real data.

To simplify the model fitting, we ignore humans’ innate opinions in the model, and let \( X_1 = X_2 = \cdots = X_7 = \emptyset \) and \( d_1 = d_2 = \cdots = d_7 = d \). According to Theorems 1 and 2, the social influence models that aim to capture the asymmetric confirmation bias and novelty bias are chosen as shown in the equation at the bottom of the page, where \( a_1 > 0 \) is the fitting parameter. Under the settings, according to the dynamics (1) and the relation (3), the model (32), without consideration of innate opinions, is rewritten as

\[
x_i(k+1) = \sum_{j \in \mathbb{N}} c_{ij}(x_i, k, \tau_i) x_j(k) + \sum_{j \in \mathbb{N}} c_{ij}(x_i, k, \tau_i) u_j(k)
\]

where \( i \in \mathbb{N} \). To fit the model from real data, the considered loss function is

\[
e = \frac{1}{2} \sum_{i=1}^{18} \| x_i(k) - \hat{x}(k) \|^2,
\]

where \( \hat{x}(k) \) denotes the real data of opinion at time \( k \) (given \( \hat{x}(1) = x(1) \)).

We let \( \tau_i = 2, \forall i \in \mathbb{N} \), which means that the fit model assumes that all the humans’ memory horizons are 1 month due to the biweekly sampling rate. The fit model parameters are summarized as follows.

1) \( \mathbb{T} = -1 \) and \( d = 6.01 \).
2) \( a_1 = 1.8, a_2 = 2.2, a_3 = 1.4, a_4 = 2.2, a_5 = 0.2, a_6 = 1 \), and \( a_7 = 2.2 \).

C. Cost Functions Learning via M^3IRL

We use the most recent three data (sampling over 1.5 months; mid-August to end-September) to learn the cost functions. With the encoded data and fit model parameters, we obtain from [45, Appendix D] that

\[
\begin{align*}
\mathbf{u}^\top \mathbf{h} &= 0.3662(a_1 \theta_{11} + a_2 \theta_{21}) - 0.4609(a_1 \theta_{12} + a_2 \theta_{22}) \\
&\quad - 0.0473(a_1 \theta_{13} + a_2 \theta_{23}) - 0.4166(a_1 \theta_{14} + a_2 \theta_{24}) \\
\mathbf{u}^\top \mathbf{H} \mathbf{u} &= 0.027(a_1 \theta_{11} + a_1 \theta_{21} + a_2 \theta_{13} + a_2 \theta_{22}) \\
&\quad + a_2 \theta_{23}) + 0.7351(a_1 \theta_{14} + a_2 \theta_{24}) \\
\mathcal{C}(\mathbf{h}, \mathbf{H}) &= \frac{4}{\sum_{i=1}^4} \log \left( \frac{\mathbf{w}_i}{\mathbf{e}^{\mathbf{m}_i} - \mathbf{e}^{-\mathbf{m}_i}} \right)
\end{align*}
\]

where

\[
\begin{align*}
\mathbf{w}_1 &= -0.8512(a_1 \theta_{11} + a_2 \theta_{21}) + 1.7902(a_1 \theta_{12} + a_2 \theta_{22}) \\
&\quad - 0.0305(a_1 \theta_{13} + a_2 \theta_{23}) + 1.3636(a_1 \theta_{14} + a_2 \theta_{24}) \\
\mathbf{w}_2 &= -0.5547(a_1 \theta_{11} + a_2 \theta_{21}) + 0.9536(a_1 \theta_{12} + a_2 \theta_{22}) \\
&\quad + 0.1994(a_1 \theta_{13} + a_2 \theta_{23}) - 1.3636(a_1 \theta_{14} + a_2 \theta_{24}) \\
\mathbf{w}_3 &= -0.1860(a_1 \theta_{11} + a_2 \theta_{21}) + 0.2281(a_1 \theta_{12} + a_2 \theta_{22}) \\
&\quad + 0.0211(a_1 \theta_{13} + a_2 \theta_{23}) - 1.3636(a_1 \theta_{14} + a_2 \theta_{24}) \\
\mathbf{w}_4 &= -0.5622(a_1 \theta_{11} + a_2 \theta_{21}) + 0.6720(a_1 \theta_{12} + a_2 \theta_{22})
\end{align*}
\]

+ 0.0549(a_1 \theta_{13} + a_2 \theta_{23}) - 1.3636(a_1 \theta_{14} + a_2 \theta_{24}).

The preference coefficients are obtained through solving (51) via the constraint optimization toolbox “fmincon” of MATLAB

\[
a_8 = 0.2040, \quad a_9 = 0.7960 \quad \text{(57a)}
\]

\[
[\theta_{81}, \theta_{82}, \theta_{83}, \theta_{84}] = [-0.0852, 0, -0.1799, 0.735] \quad \text{(57b)}
\]

\[
[\theta_{91}, \theta_{92}, \theta_{93}, \theta_{94}] = [0.7815, 0, -0.2185]. \quad \text{(57c)}
\]

With the basis functions (52)–(55), the learned cost functions of two information sources are

\[
\begin{align*}
r_8(\mathbf{x}, \mathbf{u}) &= -0.0852 \sum_{t=0}^{k+1} \sum_{j=1}^{d} (1 - x_j(t))^2 \\
&\quad + 0.735 \sum_{t=0}^{k+1} \sum_{j=1}^{d} (1 + x_j(t))^2 \\
&\quad - 0.1799 \sum_{t=0}^{k+1} (u_8(t) - u_9(t))^2 \quad \text{(58)}
\end{align*}
\]

\[
\begin{align*}
r_9(\mathbf{x}, \mathbf{u}) &= 0.7851 \sum_{t=0}^{k+1} \sum_{j=1}^{d} (1 - x_j(t))^2 \\
&\quad - 0.2185 \sum_{t=0}^{k+1} (u_8(t) - u_9(t))^2. \quad \text{(59)}
\end{align*}
\]

Through observing the learned coefficient \( a_8 \) and \( a_9 \) and cost functions (58) and (59), we infer the following.

1) The information source IS8 aims at manipulating the opinions of the public to be against the COVID-19
vaccine (i.e., $x(k) \in [-1, 0]^2$), which is indicated by the first two terms in the right-hand side of (58). This inference can be demonstrated by IS8’s evolving opinions in Fig. 5.

2) The information source IS9 aims at leading the opinions of the public to support the COVID-19 vaccine (i.e., $x(k) \in (0, 1]^2$), which is indicated by the first term in the right-hand side of (59). This inference can be demonstrated by IS9’s evolving opinions in Fig. 5.

3) Both information sources IS8 and IS9 prefer to spread outlying opinions or dislike spreading persistent opinions, which is implied by the third and second terms in the right-hand side of (58) and (59), respectively. The inference can be partially demonstrated by the sharp jumping opinions in the trajectories of IS8 and IS9 in Fig. 5.

4) $a_0 = 0.7960 > a_8 = 0.2040$ implies that the information source IS9 has a bigger influence than the information source IS8 on the evolving opinions of the observed social community. The inference can be partially demonstrated by the observation of Fig. 5 in conjunction with Fig. 6 that more evolving opinions are in the supporting range $(0, 1)$ than the opposing range $[-1, 0]$, and IS9 supports COVID-19 vaccine.

**D. Comparision**

We use the most recent four action data (sampling over two months: early-August to end-September) to learn the cost functions. Following the same step to obtain (57), we have

$$a_8 = 0.2086, \quad a_9 = 0.7914$$

$$[\theta_{b1}, \theta_{b2}, \theta_{b3}, \theta_{b4}] = [-0.0806, 0.0001, -0.1853, 0.7340]$$

$$[\theta_{h1}, \theta_{h2}, \theta_{h3}, \theta_{h4}] = [0.7843, 0, 0, -0.2156].$$

With the basis functions (52)-(55), the learned cost functions of two information sources are

$$r_8(x, u) = -0.0806 \sum_{i=0}^{k-t-1} \sum_{j=1}^{k+t-1} (1 - x_j(t))^2 + 0.734 \sum_{i=0}^{k-t-1} \sum_{j=1}^{k+t-1} (1 + x_j(t))^2$$

$$+ 0.0001 \sum_{i=0}^{k-t-1} (u_q(t) - u_q(t - 1))^2 - 0.1853 \sum_{i=k+1}^{k+t-1} (u_q(t) - u_q(t))^2$$

$$r_9(x, u) = 0.7843 \sum_{i=0}^{k-t-1} \sum_{j=1}^{k+t-1} (1 - x_j(t))^2 - 0.2156 \sum_{i=k+1}^{k+t-1} (u_q(t) - u_q(t))^2$$

which, in conjunction with (58) and (59), indicates that the learned cost functions vary with the length of the finite-time action trajectory. This also means that the learned cost functions via our proposed M3 are dynamic, which can capture the influence of dynamic external stimulus on the decision-making of information sources.

**VI. CONCLUSION**

In this article, we have proposed the social information-diffusion model that explicitly takes human memory, asymmetric confirmation bias, and asymmetric novelty bias into account. Based on the proposed model, we have proposed the M3IRL algorithm to learn the cost functions of target individuals. Real data validations suggest the effectiveness of the derived M3IRL algorithm and the proposed public opinion evolution model.

As a part of future research, we will investigate the generalization of the cost function learning framework for large-scale social networks with the incorporation of communication detection and classification.

**APPENDIX A**

**PROOF OF THEOREM 1**

**A. Sufficient Condition**

Without loss of generality, we let $x_a > 0$. With the consideration of (23), the condition (26) is equivalent to

$$|f_g(x_a) - f_g(0)| = f_g(x_a) - f_g(0) = f_g(0) - f_g(-x_a)$$

$$= |f_g(-x_a) - f_g(0)|$$

which, in conjunction with (22) and (23), leads to the behavior (8).

We now consider the condition

$$|x_g - x_a| = |x_g - x_a| \quad \text{and} \quad x_g x_g > x_b x_g > 0$$

which is the union of conditions in (24) and (25). If $x_g > 0$, the relationship in (60) implies that $x_a - x_g = x_g - x_b > 0$ and $x_a > x_b$, which follows from (23) and (24) that imply $|f_g(x_a) - f_g(x_b)| = f_g(x_a) - f_g(x_b) < |f_g(x_b) - f_g(x_a)| = f_g(x_b) - f_g(x_a)$. We then can obtain from (22) and (20) that

$$\mathcal{L}(x_g, x_a) > \mathcal{L}(x_g, x_b), \quad \text{if} \quad |x_b - x_g| = |x_g - x_a|, x_a > x_b$$

and $x_g > 0$. (61)
If $x_g < 0$, with the consideration of (25), following the same steps to derive (61), we have

$$\mathcal{L}(x_g, x_a) > \mathcal{L}(x_g, x_b), \text{ if } |x_b - x_g| = |x_a - x_g|, \ x_a < x_b \text{ and } x_g < 0.$$  

(62)

We note that the union of (61) and (62) is equivalent to

$$\mathcal{L}(x_g, x_a) > \mathcal{L}(x_g, x_b), \text{ if } |x_b - x_g| = |x_a - x_g| \text{ and } x_a > x_b.$$  

(63)

Meanwhile, it is straightforward to observe from (9) and (10) that their union is also equivalent to (63). We thus conclude that the conjunctive conditions (22)–(25) lead to the behavior (9) and (10).

Without loss of generality, we let $x_g \geq x_a \geq 0$. It follows from (23) that $|f(x_0) - f(x_0)| = |f(x_0) - f(x_0)|$. If $x_a$ decreases to $x_b$ and $x_a \geq 0$, we then have $|f(x_0) - f(x_0)| < |f(x_0) - f(x_0)|$ and $|x_a - x_a| < |x_b - x_b|$. Considering (22) and (20), we have $c(x_a, x_a) > c(x_a, x_b)$. If $x_a$ increases to $x_b$ such that $x_a - x_a > x_b - x_a > 0$, we obtain from (23) that $|f(x_0) - f(x_0)| < |f(x_0) - f(x_0)|$ and $|x_a - x_a| < |x_b - x_b|$. Considering (22) and (20), we then have $c(x_a, x_a) > c(x_a, x_a)$. We, thus, conclude that the conjunctive conditions (22) and (23) imply

$$\mathcal{L}(x_a, x_a) > \mathcal{L}(x_a, x_a), \text{ if } |x_a - x_a| < |x_a - x_a|, \ x_a \geq 0 \text{ and } x_b \geq 0.$$  

(64)

In the case of $0 > x_a \geq x_a$, following the same steps to derive (64), we have

$$\mathcal{L}(x_a, x_a) > \mathcal{L}(x_a, x_a), \text{ if } |x_a - x_a| < |x_a - x_a|, \ x_a \leq 0 \text{ and } x_b \leq 0.$$  

(65)

The results (64) and (65) indicate that the conjunctive conditions (22) and (23) result in the behavior (11).

Let us consider the condition

$$x_g \cdot x_b < 0 \text{ and } x_g \cdot x_a > 0.$$  

(66)

If $x_g > 0$, the condition (66) implies that $x_b < 0$ and $x_a > 0$. Without loss of generality, we let $x_g < x_a - x_g$. Then, in the light of (61) and (64), we, respectively, obtain

$$\mathcal{L}(x_g, x_b) < \mathcal{L}(x_g, x_a), \text{ if } |x_b - x_b| = |x_a - x_a|.$$  

(67)

$$\mathcal{L}(x_g, 0) > \mathcal{L}(x_g, x_a), \text{ if } 0 < x_g < x_a - x_a.$$  

(68)

which are due to the facts: $x_a > x_b$, $x_g > 0$, $|0 - x_a| > |x_b - x_b|$, and $x_a > 0$. Considering that $x_b > 0$, the inequalities (67) and (68) indicate that there exists a $x_b < 0$ such that $|x_a - x_a| > |x_b - x_b|$ and $\mathcal{L}(x_a, x_a) > \mathcal{L}(x_g, x_a)$. We, thus, can summarize that there exists an $x_b$ such that

$$\mathcal{L}(x_g, x_a) > \mathcal{L}(x_g, x_a), \text{ if } |x_b - x_b| < |x_a - x_a|, \ x_b > 0.$$  

(69)

Also, considering (66), if $x_a < 0$, according to the same logic used to derive (69), we can conclude that there exists an $x_b$ such that

$$\mathcal{L}(x_g, x_b) > \mathcal{L}(x_g, x_a), \text{ if } |x_a - x_a| < |x_b - x_a|, \ x_a < 0.$$  

(70)

Let us denote $\mathcal{L}(x_g, x_a, x_b) = \left(\frac{|x_a - x_a|}{|x_g - x_a|}\right) < 1$, by which it is straightforward to verify that the union of (69) and (70) is equivalent to (12). We, here can conclude that the conjunctive conditions (22)–(25) result in the behavior (12).

1) Necessary Condition: The necessary condition is proven via contradiction, i.e., assuming that one of the conditions (22)–(25) does not hold and then proving that the influence model $\mathcal{L}(x, x_j)$ cannot capture the behavior (9)–(12) simultaneously.

We assume that (22) does not hold, i.e., $\mathcal{L}(f(x_a) - f(x_a))$ is nondecreasing w.r.t. the distance $|f(x_a) - f(x_a)|$. We let $x_g = x_a = x_a$ and we have $|f(x_a) - f(x_a)| = f(x_a) - f(x_a)$. If $x_a$ decreases to $x_a$, we then have $|f(x_a) - f(x_a)| < |f(x_a) - f(x_a)|$. As a consequence, we obtain from (20) that

$$\mathcal{L}(x_a, x_a) \leq \mathcal{L}(x_a, x_a)$$  

(71)

which contradicts with (11). We now consider the case that $\mathcal{L}(f(x_a) - f(x_a))$ is nondecreasing w.r.t. $|f(x_a) - f(x_a)|$ and $f(c)$ is nonincreasing w.r.t. $c$. Let us set $0 < x_b < x_a < x_a$. We, thus, have $|x_a - x_a| < |x_a - x_a|$, $|f(x_a) - f(x_a)| > |f(x_a) - f(x_a)|$, and (71). Following the same analysis method, we can conclude that, if the conditions (22)–(25) do not hold, we have the contradicting behavior (71) with (11).

APPENDIX B

PROOF OF THEOREM 3

Considering (45), the second-order Taylor expansion of $r(\hat{u})$ around $u$

$$r(\hat{u}) \approx r(u) + (\hat{u} - u) u^T \frac{\partial^2 u}{\partial u^2} (\hat{u} - u) + \frac{1}{2} (\hat{u} - u) u^T H(u - u) (\hat{u} - u).$$

(72)

We can now obtain

$$\int_{u \in U} e^{-r(u) + (\hat{u} - u) u^T \frac{\partial u}{\partial u} (\hat{u} - u) + \frac{1}{2} (\hat{u} - u) u^T H(u - u) (\hat{u} - u)} d\tilde{u}$$

$$= e^{-r(u) + (\hat{u} - u) u^T H(u - u) (\hat{u} - u) + \frac{1}{2} (\hat{u} - u) u^T H(u - u) (\hat{u} - u)}$$

$$\approx e^{-r(u) + (\hat{u} - u) u^T H(u - u) (\hat{u} - u) + \frac{1}{2} (\hat{u} - u) u^T H(u - u) (\hat{u} - u)}$$

$$= e^{-r(u) + (\hat{u} - u) u^T H(u - u) (\hat{u} - u) + \frac{1}{2} (\hat{u} - u) u^T H(u - u) (\hat{u} - u)}$$

$$= e^{-r(u) + (\hat{u} - u) u^T H(u - u) (\hat{u} - u) + \frac{1}{2} (\hat{u} - u) u^T H(u - u) (\hat{u} - u)}$$

$$= e^{-r(u) + (\hat{u} - u) u^T H(u - u) (\hat{u} - u) + \frac{1}{2} (\hat{u} - u) u^T H(u - u) (\hat{u} - u)}$$

$$= e^{-r(u) + (\hat{u} - u) u^T H(u - u) (\hat{u} - u) + \frac{1}{2} (\hat{u} - u) u^T H(u - u) (\hat{u} - u)}$$

(73)

$$= e^{-r(u) + (\hat{u} - u) u^T H(u - u) (\hat{u} - u) + \frac{1}{2} (\hat{u} - u) u^T H(u - u) (\hat{u} - u)}$$

(74)

where (73) from its previous step is obtained via considering Assumption 1-2.)
Substituting (72) with (74) into (44) yields
\[
p(u(\theta, \alpha, \tilde{x}(k)) \approx e^{\theta \cdot \left(\sum_{h \in \mathcal{U}} e^{\theta \cdot \left(u - H(u) - \alpha \cdot \tilde{u}\right) - f_{\theta,\alpha}(h - \tilde{H}_u)}\right)^{-1}} = e^{-\tilde{H}_u + \alpha \cdot \tilde{u}} \prod_{i=1}^{[\mathcal{T}]} e^{\theta \cdot \left(h - \tilde{H}_u\right)} - e^{-\left(h - \tilde{H}_u\right)}
\]
by which we obtain \(\tilde{p}(\theta, \alpha) \approx \log p(u(\theta, \alpha, \tilde{x}(k)))\), where \(\tilde{p}(\theta, \alpha)\) is given in (46).

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Yanhing Mao received the Ph.D. degree in electrical and computer engineering from the State University of New York at Binghamton, Binghamton, NY, USA, in 2019. From 2019 to 2022, he was a Post-Doctoral Researcher with the Department of Computer Science and the Department of Mechanical Engineering, University of Illinois at Urbana-Champaign, Urbana, IL, USA. He is currently an Assistant Professor with the Engineering Technology Division, Wayne State University, Detroit, MI, USA. His current research focuses on physics artificial intelligence (AI), self-driving vehicles, social cybersecurity, and social information dynamics and control.

Jinning Li received the B.S. degree from Shanghai Jiao Tong University, Shanghai, China, in 2019. He is currently pursuing the Ph.D. degree in computer science with the University of Illinois at Urbana-Champaign (UIUC), Urbana, IL, USA. Before joining UIUC, he was a Machine Learning Engineer with Pony.ai Inc., Fremont, CA, USA. His current research focuses on data mining, graph mining, natural language processing, and social networks.

Naira Hovakimyan (Fellow, IEEE) received the Ph.D. degree in physics and mathematics with a major in optimal control and differential games from the Institute of Applied Mathematics, Russian Academy of Sciences, Moscow, Russia, in 1992. She is currently a W. Grafton and Lillian B. Wilkins Professor of mechanical science and engineering with the University of Illinois at Urbana-Champaign, Urbana, IL, USA. She is a Co-Founder and a Chief Scientist with IntelinAir, Indianapolis, IN, USA. Her research interests are in control and optimization, autonomous systems, machine learning, neural networks, game theory, and their applications in aerospace, robotics, mechanical, agricultural, electrical, petroleum, biomedical engineering, and elderly care. Dr. Hovakimyan is a fellow and a Life Member of the American Institute of Aeronautics and Astronautics.

Tarek Abdelzaher (Fellow, IEEE) received the Ph.D. degree in computer science from the University of Michigan, Ann Arbor, MI, USA, in 1999. He is currently a Sohaib and Sara Abbasi Professor of computer science and a Willett Faculty Scholar with the University of Illinois at Urbana-Champaign, Urbana, IL, USA, with over 300 refereed publications in real-time computing, distributed systems, sensor networks, and IoT. Dr. Abdelzaher is a fellow of ACM. He received the IEEE Outstanding Technical Achievement and Leadership Award in Real-time Systems in 2012, the Xerox Research Award in 2011, and several best paper awards. He has served as the Editor-in-Chief of journal Real-Time Systems for 20 years, an Associate Editor of IEEE TMC, IEEE TPDS, ACM ToSN, ACM TIIoT, and ACM ToIT, among others, and the chair of multiple top conferences in his field.

Christian Lebiere received the B.S. degree in computer science from the University of Liège, Liège, Belgium, in 1986, and the Ph.D. degree from the School of Computer Science, Carnegie Mellon University, Pittsburgh, PA, USA, in 1990. During his graduate career, he studied connectionist models and algorithms and was a Co-Developer of the cascade-correlation neural network learning algorithm that has been used in hundreds of practical applications and scientific papers. He is currently a Research Faculty directing the FMS Cognitive Modeling Group at the Psychology Department, Carnegie Mellon University. Since 1991, he has been working on the development of the ACT-R hybrid cognitive architecture and was coauthor with John R. Anderson of the 1998 book The Atomic Components of Thought. His main research interests are cognitive architectures and their applications to psychology, artificial intelligence, human–computer interaction, decision-making, intelligent agents, cognitive robotics, and neuromorphic engineering.