HYDRODYNAMICS OF ACCRETION ONTO BLACK HOLES

M.-G. Park\textsuperscript{1,2} and J. P. Ostriker\textsuperscript{1}

\textsuperscript{1} Princeton University Observatory, Princeton, NJ 08544, USA
\textsuperscript{2} Kyungpook National University, Department of Astronomy and Atmospheric Sciences, Taegu 702-701, KOREA

ABSTRACT

Spherical and axisymmetric accretion onto black holes is discussed. Physical processes in various families of solutions are explained and their characteristics are summarized. Recently discovered solutions of axisymmetric flow provide us with various radiation efficiency and spectrum, which may successfully model diverse accretion systems. Possible role of preheating is also speculated. The various families of solutions can be plotted as trajectories on the plane of \((l, e)\) [or \((l, \dot{m})\)] where \(l\) is the luminosity in units of the Eddington luminosity, \(\dot{m}\) is the similarly defined mass accretion rate and \(e\) is the efficiency defined by \(e \equiv L/\dot{M}c^2 = l/\dot{m}\).

We discuss the domains on these planes where solutions are known or expected to be unstable to either spherical or non-spherical perturbations. Preliminary analysis indicates that a preheating instability will occur along the polar direction of the advection dominated flow for \(e \gtrsim 10^{-2}\).

I. INTRODUCTION

The generic accretion flow can be broadly classified as (quasi) spherical or axisymmetric one. The angular momentum of the flow and its transfer processes will determine whether the flow is spherical or not. True spherically symmetric flow is possible only when the angular momentum is rigorously zero. However, when the angular momentum is small enough and there exists a process that can effectively remove or transport the angular momentum, then the flow maintains a roughly spherical shape (Loeb and Laor 1992; Narayan and Yi 1995a). If the angular momentum is significant, the flow becomes rotationally supported and is flattened to disk.

Until recently, most works on spherical accretion flow have focused on zero angular momentum flow and those on axisymmetric flow on thin flat disk. Purely spherical flow is physically well defined and has high degrees of symmetry, which make it possible to treat hydrodynamics, radiative transfer and various gas-radiation microphysics rather accurately. Yet it might be too ideal for real astrophysical situations. The disk flow produces more radiation for a given mass accretion and the solution is well understood in some parameter ranges except the angular momentum transport process remains uncertain. However, the true solution of the disk flow requires solving the two-dimensional hydrodynamic and radiation transport equations, a quite daunting challenge even with today’s enormous computing power.

In this review, we use the dimensionless luminosity \(l \equiv L/L_E\) and the dimensionless mass accretion rate \(\dot{m} \equiv \dot{M}/\dot{M}_E\) where \(L\) is the luminosity of the accretion flow, \(L_E\) the Eddington luminosity, \(\dot{M}\) the mass accretion rate and \(\dot{M}_E\) the Eddington mass accretion rate, \(\dot{M}_E \equiv L_E/c^2\). In some literature, the Eddington accretion rate divided by the radiation efficiency \(e \equiv L/(\dot{M}c^2) = l/\dot{m}\) is used as the basic unit. In accretion onto a black hole, \(e\) can be arbitrary unlike in accretion onto a neutron star.

II. SPHERICAL FLOW

The first study of spherical accretion onto compact objects dates back more than forty years (Bondi 1952). In this classic work, hydrodynamics of polytropic flow is studied within the Newtonian dynamics, and it is found that either a settling or transonic solution is mathematically possible for the gas accreting onto
compact objects. Especially, the accretion rate is highest for the transonic solution. Relativistic version of the same problem was solved by Michel (1972) twenty years later, after the discovery of celestial X-ray sources. He showed that accretion onto the black hole should be transonic.

However, the real accretion necessarily involves radiative processes, which make the flow non-adiabatic and, at the same time, more diverse and interesting. In general, compression of the infalling gas raises the temperature of the accreting gas and then radiation is produced by the radiative cooling of heated gas, which in turn interacts with the other part of the flow via scattering and absorption, thereby heats or cools the upstream gas. So understanding the accretion flow involves simultaneously solving for the gas and radiation field for given outer boundary conditions. This is particularly important for accretion onto black holes because the lack of the hard surface in a black hole makes the radiation efficiency unknown a priori. In accretion onto neutron star, the surface potential of the star determines how much luminosity is generated for a given accretion rate, regardless of the details in radiative and hydrodynamic processes. In accretion onto black hole, the radiation efficiency is determined by the state of the gas at all radii. If the gas is kept at low temperature, most of the gravitational potential energy would be lost to the hole in the form of gas kinetic energy. If the gas is kept at high temperature by the radiative heating or other processes, the luminosity is increased and less kinetic energy is lost to the hole.

Spherical accretion onto a black hole is generally specified by the mass accretion rate and by the luminosity and the spectrum of the radiation field. As long as two body processes are important, the solutions become scale free and are applicable to black holes of arbitrary mass (Chang and Ostriker 1985). These scale-free solutions then depend primarily on the dimensionless parameters $\lambda$ and $\dot{m}$ or $\lambda$ and $e$ (Ostriker et al. 1976). However, if we require the self-consistency between the gas and the radiation field, only certain combinations of the accretion rate and the luminosity (plus spectrum) are allowed, that is a given type of solution is described by a line on the $(\lambda, \dot{m})$ plane. Now we describe these solutions one by one in the order of increasing $\dot{m}$.

Fig. 1. Spherical and axisymmetric accretion flow solutions in luminosity vs mass accretion rate plane.
Fig. 2. Spherical and axisymmetric accretion flow solutions in luminosity vs radiation efficiency plane.

2.1 Solutions with $\dot{m} \ll 1$

Since the electron scattering optical depth from infinity down to the horizon is roughly given by

$$\tau_{es} = \dot{m}(r/r_s)^{-1/2}$$

where $r_s$ is the Schwarzschild radius, $\dot{m} \ll 1$ flow is optically thin to scattering, and whatever photons produced escape without difficulty. Any kind of interaction between photons and gas particles is minimal. Gas is freely falling so $v \propto (r/r_s)^{-1/2}$ and its temperature has the virial value, $T \propto (r/r_s)^{-1}$, until electrons become relativistic (Shapiro 1973). In Newtonian approximation, $\gamma = 5/3$ accreting flow has a constant Mach number, because the sound speed is increasing as fast as the infall velocity (Bondi 1952).

But in the relativistic regime, the sound speed or the electron temperature is increasing less rapidly and the flow becomes supersonic with increasing Mach number for $T \gtrsim 10^8$ K (Shapiro 1973, Park 1990a). The temperature can reach as high as few $10^{10}$ K, producing a hard relativistic bremsstrahlung spectrum. Yet the luminosity is quite small due to the very low density. Any radiative heating, especially Compton heating, for this family of solutions can be ignored because of the low luminosity. Typical solutions are marked as large empty circles in $\dot{m} < 1$ region of Figures 1 and 2 and their radiation efficiency $e$ is less than $10^{-8}$ (Park 1990a).

2.2 Low-Temperature Solutions with $\dot{m} \geq 0.1$

As $\dot{m}$ approaches 0.1, bremsstrahlung and atomic cooling becomes more efficient and gas cools down $\sim 10^4$ K, the equilibrium temperature of the surrounding gas. Depending on the mass of the hole, the flow can form an effectively optically thick\(^1\) core with blackbody radiation field inside. This regime represents the transition from adiabatic to non-adiabatic flow (Park 1990a; Nobili et al. 1991).

\[^1\] $\tau^* \equiv (3\tau_{es}\tau_{abs})^{1/2} \gg 1$ where $\tau_{abs}$ is the absorption optical depth.
As the gas density at the boundary increases further, so does the mass accretion rate, and the flow becomes effectively optically thick out to a larger radius. The flow is maintained at a low temperature. These solutions are quite similar to those in a stellar interior except that the flow is infalling by the gravity and $PdV$ work is the source of the luminosity (Flammang 1982, 1984; Soffel 1982; Blondin 1986; Nobili et al. 1991). Typical solutions are shown in Figures 1 and 2 as large triangles (Nobili et al. 1991).

An important and interesting radiation transport process called radiation trapping happens inside certain radius where $v > c/(\tau_{es} + \tau_{abs})$ is satisfied. When the diffusion speed of photons is slower than the flow’s bulk velocity, most of the photons are carried inward with the flow (Begelman 1978). The correct treatment of this process requires relativistic radiative transport equations, at least up to the order $(v/c)^1$ (Flammang 1982, 1984; Park 1990a; Nobili et al. 1991; Park 1993). In relativistic flow, especially in an optically thick one, the flux seen by the observer comoving with the flow and the flux seen by the stationary observer (relative to the coordinate) should be carefully distinguished. The momentum transferred to the gas from the radiation is closely linked to the former, a comoving-frame flux, and the luminosity seen by the observer at infinity to the latter, a fixed-frame flux. (Mihalas and Mihalas 1984; Park 1993). For an accretion flow, the comoving flux is always larger than the fixed frame flux and should be less than the Eddington flux for steady inflow (Park and Miller 1991). This radiation trapping and the low temperature of the flow make this family of solutions very inefficient radiators with $e \sim 10^{-7}$.

### 2.3 High-Temperature Solutions with $\dot{m} \gtrsim 0.1$

All this would change if gas can be preheated by outcoming hard radiation produced at smaller radii. The existence of higher temperature, higher luminosity accretion solution due to this preheating was first proved by Wandel et al. (1984) in simplified treatment. A more accurate and relativistic treatment show that these higher luminosity and higher efficiency steady-state solutions exist only for $3 \lesssim \dot{m} \lesssim 100$ (Park and Ostriker 1989; Park 1990a,b; Nobili et al. 1991). So there exist two different families of solutions for given $\dot{m}$, low temperature and high-temperature one (Fig. 1).

The gas temperature can reach up to $10^9 \sim 10^{10}$ K; the flow is optically thick to scattering, and the radiation trapping occurs near the hole. However, it is optically thin to absorption, $\tau^* \ll 1$. Bremsstrahlung photons produced in the inner region are upscattered by hot electrons around, which subsequently heat cool electrons in the outer part by Comptonization. These solutions are plotted in Figures 1 and 2 as large empty circles (Park 1990a; Nobili et al. 1991). These solutions are much more luminous, $l \simeq 10^{-4} - 10^{-2}$, and produce much harder photons than the low-temperature solutions of the same mass accretion rate. Still, they are much less efficient radiators, $e \sim 10^{-4}$, than the thin disk.

There are ways to increase the radiation efficiencies. The dissipational heating of turbulent motion and the magnetic field reconnection (Mészáros 1975; Maraschi et al. 1982) and self-consistently sustained high electron-positron pair production (Park and Ostriker 1989) are two examples. Solutions of the former type are shown as small squares (Maraschi et al. 1982) with $e$ as high as $\sim 0.1$ and the latter type as small filled circles (Park and Ostriker 1989) in Figures 1 and 2. They constitute yet another families of solutions for given $\dot{m}$. Though these solutions are quite attractive because of the high efficiency and high temperature, they are quite likely to be subject to the preheating instability described below. The reason that these solutions are found in steady-state calculation at all is that either preheating is ignored (Mészáros 1975; Maraschi et al. 1982) or only the inner part of the flow is considered (Park and Ostriker 1989). It is possible to maintain the steady flow under significant preheating by having a shock (Chang and Ostriker 1984). However, no self-consistent solutions with a steady shock is yet constructed, although there is some indication that such solution might exist around $\dot{m} \sim 1$ or $\dot{m} \sim 100$ (Park 1990b; Nobili et al. 1991).

### 2.5 Preheating

When radiation and gas interact, both momentum and energy are transferred from one to the other. The luminosity of a given spherical accretion flow has a well known upper limit above which photons would give the gas particles too much momentum to infall steadily. When gas is fully ionised and opacity is dominated by the Thomson scattering, this limit is the well-known Eddington luminosity.

Ostriker et al. (1976) found another limit based on the energy transfer. They found that if the gas is preheated too much around the sonic point, it gets too hot to accrete in steady-state. When the gas temperature at the sonic point is $10^8$ K and the Compton temperature of the preheating radiation is $10^8$ K,
the disruption of the accretion can occur at \( l < 0.01 \). This is why Park (1990a,b) and Nobili et al. (1991) could not find high-temperature self-consistent solutions for \( \dot{m} \lesssim 3 \) and \( \dot{m} \gtrsim 100 \): in the former parameter region, the photon energy of the preheating radiation is too high and in the latter, the luminosity is too high.

This disruption of flow by preheating happens in a square region labeled I in Figures 1 and 2. The exact location of the boundary depends on various conditions of the gas and the radiation field as well as the shape of the cooling curve. Under certain conditions, the region could be much smaller than the one shown in Figures 1 and 2 (Ostriker et al. 1976; Cowie et al. 1978; Bisnovatyi-Kogan and Blinnikov 1980; Stellingwerf and Buff 1982; Stellingwerf 1982; Krolik and London 1983).

The time-dependent behavior of the preheated flow was investigated by Cowie et al. (1978) with analytic analysis and hydrodynamic simulations (see also Grindlay 1978). They found two distinct type of time dependent behaviors both inside and near the boundaries of region I. High luminosity, low efficiency flow (region II in Figs. 1 and 2) develops preheating inside the sonic point, which produces recurrent flaring in short time intervals, \( \sim 100(M/M_\odot) \) sec, with its average luminosity similar to the steady-state solutions. On the other hand, in higher efficiency, lower luminosity flow (region III in Figs. 1 and 2) preheating outside the sonic point induces longer time scale, \( \sim 10^9(M/M_\odot) \) sec, changes in accretion rate and luminosity. Even the solution outside the preheating regime appears to have short time variability due to the preheating (Zampieri et al. 1996).

The fact that high luminosity, high efficiency steady-state solutions would suffer the preheating instability implies the time-dependent nature of these solutions. As the luminosity and efficiency of the accretion increase, we expect more variability in the flow and in the emitted radiation in general.

III. AXISYMMETRIC ACCRETION

3.1 Thin Disk

Until recently most works on non-spherical accretion have focused on the thin disk accretion (see Pringle 1981, Treves et al. 1988, Chkrabarti 1996 for reviews). Since the pioneering work by Shakura and Sunyaev (1973), Pringle and Rees (1972), and Novikov and Thorne (1973), thin disk models with so-called \( \alpha \) viscosity have been successfully applied to many astronomical objects including various X-ray sources (see Frank et al. 1985 for reviews).

Thin disk accretion model has merits of being simple: the equations become one-dimensional (in radius) and all physical interaction can be described by local quantities. Gas is rotating with Keplerian angular momentum, which is transported radially by viscous stress. The gravitational potential energy is locally converted to heat by this viscous process. The gas cools by radiating in the vertical direction, thereby not affecting any other part of the flow. The radiation efficiency depends only on the location of the inner disk boundary and is generally \( \sim 0.1 \). So any thin accretion disk model lies on the dotted line \( e = 0.1 \), denoted as TD, in Figures 1 and 2. However, its application cannot be extended to high luminosity systems due to the instability at a high mass accretion rate (Lightman and Eardley 1974; Shakura and Sunyaev 1976; Pringle 1976; Piran 1978) nor to hard X-ray sources due to the low temperature of the disk. Although Shapiro et al. (1976) discovered another family of thin disk solutions that have very high electron temperature, they are also found to be thermally unstable (Pringle 1976; Piran 1978).

3.2 Slim Disk

If the dimensionless mass accretion rate \( \dot{m} \) approaches \( e^{-1} \), the vertical height of the disk becomes comparable to the radius and the disk is not thin any more. In this regime, the radial motion of the gas becomes important and the angular momentum of the gas is below the Keplerian value unlike in thin disk. Abramowicz et al. (1988) improved over the thin disk approach by incorporating the effect of pressure in the radial motion to find new type of solutions in high \( \dot{m} \) regime. They are stable against the viscous and thermal instabilities. Viscously dissipated energy can now be advected into the hole along with the gas in addition to being radiated away through the surface of the disk. They named these solutions ‘slim disk’ because the disk is not thin, yet not so thick that vertically integrated equations are valid. This work shows that disk accretion too can have efficiency other than \( \sim 0.1 \). These solutions are represented in Figures 1 and 2 as the dot-dashed curve (SD) at the end of \( e = 0.1 \) line (Szuszkiewicz et al. 1996).
3.3 Self-Similar Flow

To get the slim disk solutions, Abramowicz et al. (1988) had to explicitly integrate the hydrodynamic equations that have a critical point. However, recently Narayan and Yi (1994) found that the equations admit self-similar solutions if the advective cooling is always a constant fraction of the viscous dissipation. These solutions are very simple and at the same time very restrictive. The density, velocity, angular velocity, and the total pressure are simple power laws in radius: \( \rho \propto r^{-3/2}, \ v \propto r^{-1/2}, \ \Omega \propto r^{-3/2}, \ \text{and} \ P \propto r^{-5/2}. \) Unfortunately the temperature of these solutions is always close to the virial value and the vertically integrated equations may not be valid.

They addressed this question in the next work (Narayan and Yi 1995a). By assuming the self-similar form of physical quantities in radius, the axisymmetric two-dimensional hydrodynamic equations are reduced into one-dimensional equations in polar angle only, and easily solved. The solutions show a radial velocity of a few percent of free-fall value at the equatorial plane and zero radial velocity on the pole. So the gas is preferentially accreted along the equatorial plane. However, the flow is always subsonic because the pressure is close to the virial value. Since only the transonic accretion is allowed, this makes the solution inapplicable close to the hole\(^2\).

One important result of this work is that physical quantities calculated from the vertically integrated equations generally agree with the polar angle averaged quantities. This justifies using vertically integrated one-dimensional equations, i.e., slim disk equations, even to the two-dimensional accretion flow. But rigorously, this convenience applies only to the self similar solutions.

Due to the simplicity of the solutions, many diverse microphysics can be incorporated into these solutions without much difficulty (Narayan and Yi 1995b). The typical solutions with low \( \dot{m} \) have efficiency much smaller than the standard thin disk (dashed line in Figs. 1 and 2). These advection-dominated accretion models with low \( \dot{m} \) successfully explain various low luminosity sources which have been hard to model with highly efficient thin disk, like Sgr A\(^*\) (Narayan et al. 1995), NGC 4258 (Lasota et al. 1996), and X-ray transients (Narayan et al. 1996a; Chen and Taam 1996). Typical solutions are denoted as NY in Figures 1 and 2 (Narayan and Yi 1995b). All these models have most of the gravitational potential energy of the accreted matter advected into the hole without ever being converted to the radiation. So these flows are quite similar to the almost adiabatic, high temperature, low \( \dot{m} \) spherical solutions of Shapiro (1973). In a way it should be because the above self-similar forms of various physical quantities are exactly those in the non-relativistic adiabatic spherical accretion flow. In fact, optically thin two-temperature spherical accretion with a magnetic field also produces a spectrum that roughly agrees with that of Sgr A\(^*\) (Melia 1992, 1994).

Another interesting feature of these models is that because the total pressure is a constant fraction of the virial value, the flow has to be two-temperature and/or some magnetic field should exist, otherwise the radiation spectrum will be simply that of the free-fall, virial temperature plasmas.

3.4 Unified Description

All these seemingly different solutions are actually just different families of solutions for the axisymmetric accretion flows with angular momentum. In each family, appropriate assumptions are made to ease the difficulty of solving the relevant equations. Specific name is attached to each family to describe the physical characteristics of the flow, e.g., thin disk, slim disk, and advection dominated flow. Although slim disk solutions and self-similar flow solutions are the results of different approach—the former from the one-dimensional approach and the latter from the self-similarity in two-dimensional flow—their final results agree in general, albeit not in detail, and the description of these axisymmetric flows in consistent manner has become finally possible (Chen et al. 1995).

A very revealing way of presenting these solutions is to look at the relation between the mass accretion rate \( \dot{m} \) and the surface mass density \( \Sigma \) at a given radius. Different families of solutions appear as specific curves in \( \dot{m} \) vs \( \Sigma \) plane (Fig. 3). \(^3\) Since the shape of the curves depends on the mass of the hole and the value of the viscosity parameter \( \alpha \), we describe a specific case where \( M = 10 \text{M}_\odot \) and \( \alpha = 0.01 \) (Chen et al. 1995). Solutions can be broadly separated into (vertically) effectively optically thin and thick flows. The left dotted

\(^2\) However, recent global solutions of slim disk equations show that self-similar solutions describe the flow reasonably well at intermediate radii (Narayan et al. 1996b; Chen et al. 1996).

\(^3\) The solutions are for \( \alpha = 0.01, \ M = 10 \text{M}_\odot, \ \text{and} \ r = 20GM/c^2 \). The surface mass density \( \Sigma \) is in g/cm\(^3\).
curve (SLE,NY) in Figure 3 represents the former and the right S shape solid curve (GTD,RTD,SD) the latter. The former exists only for $\dot{m} \ll 1$ and the latter for any value of $\dot{m}$.

The lower (GTD) and middle (RTD) branch of S curve are the classic gas pressure dominated and radiation pressure dominated thin disk solutions of Shakura and Sunyaev (1973). In both, radiative cooling dominates over the advective one, and the lower branch is thermally and viscously stable while the middle branch is unstable to both type of instability. The upper (SD) branch of S curve is the original slim disk solution in which the advective cooling dominates and stabilizes the radiation pressure dominated flow (Abramowicz et al. 1988). This solution is not geometrically thin. The disk scale height can be comparable to the cylindrical radius, especially when $\dot{m} > e^{-1}$. The temperature of these S curve solution is not high, $\lesssim 10^8$ K, and the radiation spectrum will be the superposition of modified blackbodies at different temperatures. However, they are very efficient radiators, $e \sim 0.1$.

Fig. 3. Various axisymmetric accretion solutions in accretion rate vs surface mass density plane.

The right (SLE) branch of the effectively optically thin solutions represent the two-temperature hot disk solutions of Shapiro et al. (1976), which are viscously stable yet thermally unstable (Pringle 1976; Piran 1978) and radiative cooling dominated. The left (NY) branch is the advection dominated flow of Narayan and Yi (1994) and Abramowicz et al. (1995), in which most of the viscously generated heat is advected into the hole stabilizing the flow. The gas temperature is very high, $\sim 10^9$ K, and the radiation spectrum will be that of Comptonized synchrotron and bremsstrahlung. The radiation efficiency of the right branch is always $\sim 0.1$ and that of the left branch has roughly $l \propto \dot{m}^2$ and $e \propto \dot{m}$ as in optically thin spherical accretion (Narayan and Yi 1995b).

As in spherical accretion, there can be other interesting family of solutions if we consider the pair production (Kunsunose and Mineshige 1992 and references therein) or various type of shocks (Chakrabarti 1996 and references therein), which we will not discuss further.

Preheating and Outflows

One interesting aspect of axisymmetric accretion unexplored so far is the effect of preheating. In thin disk flow, any part of the disk is free from the radiation produced in the other parts of the disk either due to the geometry (vertical direction) or due to the high absorption optical depth (radial direction). But in true two-dimensional flow, hard radiation generated at the inner part of the flow should go through the outer part. This radiative heating will be more complex than in a spherical flow. For example, the self-similar flow of Narayan and Yi (1995a) has zero radial velocity along the pole, around which viscous heating, radiative cooling, and advective cooling are very small compared to those in the equatorial region. So this part of the flow can be very easily heated by Comptonization to a high temperature, possibly higher than the virial
value. It is quite plausible that this could develop some interesting phenomena, like relaxational oscillations seen in spherical flow (Cowie et al. 1976) or simply outflow. Two-dimensional flow with steady accretion onto the equatorial plane and time-dependent accretion or outflow in the polar direction will be quite useful in explaining some high energy sources. We know that time-dependence and outflow are the norm rather than the exception in these sources. Thus the potential instability of the self-similar solutions to preheating in the polar region should be seen as a virtue of the solution. A preliminary analysis indicates that a preheating instability will first occur along the polar direction for $\epsilon \gtrsim 10^{-2}$. Two-dimensional numerical hydro simulations show that accretion and outflow can coexist (Chakrabarti and Molteni 1993; Molteni et al. 1994; Ryu et al. 1995; Molteni et al. 1996). Preheating might be yet another ingredient in the outflow and the time variabilities in accreting flows.

IV. CONCLUSION

Rapid and exciting recent developments in the theory of accretion, especially on the viscous axisymmetric flow, are the significant steps toward understanding various astronomical sources that are believed to be powered by the accretion onto black holes. Although the theory is far from complete and we may have to wait many years to understand the real three-dimensional hydrodynamics and radiative transfer, it already is rather successful in explaining most X-ray sources. In addition, the incomplete part of our understanding could be the key to the remaining mysteries since the most attractive current quasi spherical solutions are likely to be unstable to the formation of (unsteady) jets.

ACKNOWLEDGMENTS

We thank Xingming Chen and Insu Yi for useful discussions, especially Xingming Chen for kindly providing the data for Figure 3. This work is partly supported by the Professor Dispatch Program of Korea Research Foundation and NSF grant AST 94-24416.

REFERENCES

Abramowicz, M. A., X. Chen, S. Kato, J.-P. Lasota, and O. Regev, Astrophys. J., 438, L37 (1995).
Abramowicz, M. A., B. Czerny, J. P. Lasota, and E. Szuszkiewicz, Astrophys. J., 332, 646 (1988).
Begelman, M. C., Mon. Not. R. Ast. Soc., 184, 53 (1978).
Bisnovatyi-Kogan, G. S., and S. I. Blinnikov, Mon. Not. R. Ast. Soc., 191, 711 (1980).
Blondin, J. M., Astrophys. J., 308, 755 (1986).
Bondi, H., Mon. Not. R. Ast. Soc., 112, 195 (1952).
Chang, K. M., and J. P. Ostriker, Astrophys. J., 288, 428 (1985).
Chakrabarti, S. K., Phys. Rep., 266, 229 (1996).
Chakrabarti, S. K., and D. Molteni, Astrophys. J., 417, 671 (1993).
Chen, X., M. A. Abramowicz, and J.-P. Lasota, preprint, astro-ph/9607020 (1996).
Chen, X., M. A. Abramowicz, and J.-P. Lasota, R. Narayan, and I. Yi, Astrophys. J., 443, L61 (1995).
Chen, X., and R.E. Taam, Astrophys. J. in press (1996).
Cowie, L. L., J. P. Ostriker, and A. A. Stark, Astrophys. J., 226, 1041 (1978).
Flammang, R. A., Mon. Not. R. Ast. Soc., 199, 833 (1982).
Flammang, R. A., Mon. Not. R. Ast. Soc., 206, 589 (1984).
Frank, J., A. R. King, and D. J. Raine, Accretion Power in Astrophysics, Cambridge U. Press, Cambridge (1985).
Grindlay, J. E., Astrophys. J., 221, 234 (1978).
Krolik, J. H., and R. A. London, Astrophys. J., 167, 18 (1983).
Kusunose, M., and S. Mineshige, Astrophys. J., 440, 100 (1995).
Lasota, J.-P., M. A. Abramowicz, X. Chen, J. Krolik, R. Narayan, and I. Yi, Astrophys. J., 462, 142, (1996).
Lightman, A. P., and D. M. Eardley, Astrophys. J., 187, L1 (1974).
Loeb, A., and A. Laor, Astrophys. J., 384, 115 (1992).
Maraschi, L., R. Roasio, and A. Treves, Astrophys. J., 253, 312 (1982).
Melia, F., Astrophys. J., 387, L25 (1992).
Melia, F., Astrophys. J., 426, 577 (1994).
Mészáros, P., Astr. Astrophys., 44, 59 (1975).
Michel, F. C., Astrophys. Space Sci., 112, 195 (1972).
Mihalas, D., and B. W. Mihalas, Foundations of Radiation Hydrodynamics, Oxford University Press, Oxford
Molteni, D., G. Lanzafame, and S. K. Chakrabarti, *Astrophys. J.*, **425**, 161 (1994).
Molteni, D., D. Ryu, and S. K. Chakrabarti, *preprint*, astro-ph/9605116 (1996).
Narayan, R., S. Kato, and F. Honma, *preprint*, astro-ph/9607019 (1996b).
Narayan, R., J. E. McClintock, and I. Yi, *Astrophys. J.*, **457**, 821 (1996a).
Narayan, R., and I. Yi, *Astrophys. J.*, **428**, L13 (1994).
Narayan, R., and I. Yi, *Astrophys. J.*, **444**, 231 (1995a).
Narayan, R., and I. Yi, *Astrophys. J.*, **452**, 710 (1995b).
Narayan, R., I. Yi, and R. Mahadavan, *Nature*, **374**, 623 (1995).
Novikov, I. D., and K. S. Thorne, in *Black Holes*, ed. C. DeWitt and B. DeWitt, Gordon and Breach, New York (1973).
Ostriker, J. P., R. McCray, R. Weaver, and A. Yahil, *Astrophys. J.*, **208**, L61 (1976).
Park, M.-G., *Astrophys. J.*, **354**, 64 (1990a).
Park, M.-G., *Astrophys. J.*, **354**, 83 (1990b).
Park, M.-G., *Astr. Astrophys.*, **274**, 642 (1993).
Park, M.-G., and G. S. Miller, *Astrophys. J.*, **371**, 708 (1991).
Park, M.-G., and J. P. Ostriker, *Astrophys. J.*, **347**, 679 (1989).
Piran, T., *Astrophys. J.*, **221**, 652 (1978).
Pringle, J. E., *Mon. Not. R. Ast. Soc.*, **177**, 65 (1976).
Pringle, J. E., *Ann. Rev. Astron. Astrophys.*, **19**, 137 (1981).
Pringle, J. E., and M. J. Rees, *Astr. Astrophys.*, **21**, 1 (1972).
Ryu, D., G. L. Brown, J. P. Ostriker, and A. Loeb, *Astrophys. J.*, **452**, 364 (1995).
Shakura, N. I., and R. A. Sunyaev, *Astr. Astrophys.*, **24**, 337 (1973).
Shakura, N. I., and R. A. Sunyaev, *Mon. Not. R. Ast. Soc.*, **175**, 613 (1976).
Shapiro, S. L., *Astrophys. J.*, **180**, 531 (1973).
Shapiro, S. L., A. P. Lightman, and D. N. Eardley, *Astrophys. J.*, **204**, 187 (1976).
Soffel, M. H., *Astr. Astrophys.*, **116**, 111 (1982).
Stellingwerf, R. F., *Astrophys. J.*, **260**, 768 (1982).
Stellingwerf, R. F., and J. Buff, *Astrophys. J.*, **260**, 755 (1982).
Szuszkiewicz, E., M. A. Malkan, and M. A. Abramowicz, *Astrophys. J.*, **458**, 474 (1996).
Treves, A., L. Maraschi, and M. Abramowicz, *Pub. Ast. Soc. Pac.*, **100**, 427 (1988).
Wandel, A., A. Yahil, and M. Milgrom, *Astrophys. J.*, **282**, 53 (1984).
Zampieri, L., J. C. Miller, and R. Turolla, *preprint*, astro-ph/9607030 (1996).
Fig. 2
Fig. 3