Effects of water-cement ratio and notches to the flexural strength of concrete

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Abstract. It is often assumed that flexural strength of concrete has less significance impact on overall concrete strength. However, from fracture mechanics point-of-view tensile is an element the mechanics always look into due to cracking does associate with tension. In the research, fracture is translated into physical laboratory experiment by introducing notches. Physical laboratory works on concrete beams with three-point bend test configuration under static load and calculating outputs from laboratory with numerical equations. Three-point bend test method is conducted because from the testing, tensile strength or also recognised as flexural strength of concrete for each water-cement ratio could be attain. Thus, the aim of this article is to reveal and discuss the pattern of flexural strength of concrete on different water-cement ratio. The testing follows conventional fracture three-point bend test on concrete but with revised version by testing notched concrete beams. Normal three-point bend tests were run on concrete beams with different notch sizes; 30 mm, 15 mm, and 5 mm respectively. There were three water-cement ratio decided in concrete mix; 0.3, 0.4, and 0.5. Thus, the trend of flexural strength of concrete follows the trend of water-cement ratio. Flexural strength increases when water-cement ratio increases up to water-cement ratio 0.5.

Keywords - flexural strength; water-cement ratio; concrete

1. Introduction

Flexural strength or bending stress are also recognised as Modulus of Rupture\cite{1}. For concrete or brittle material, these parameters represent tensile strength of a concrete. Flexural strength could be obtained using two types of testing in concrete which are three-point bending and four-point bending test\cite{2, 3}. The setting of these test have the similar two supporting rollers below the concrete beam and the distance between the rollers at each ends is measured as “span”. The importance of flexural strength could be clearly noticed by fatigue and fracture. Fatigue and fracture can be interpreted through a material’s endurance.

1.1 Three-Point Bending Test and Four-Point Bending Test

The differences are three-point has only one loading force at the middle of the concrete beam specimen transmitted through a roller, and four-point has two loading forces transmitted through two rollers with certain diameter. Three-point bend test emphasizes on its maximum stress in the middle
and decreases the residual stress at other points. On the other hand, four-point produces uniform stress between the loading rollers [4, 5].

2. Concrete mix design

If the materials involved in concrete mix are measured and controlled carefully, then the actual result relating strength and water-cement ratio of a concrete mix will definitely have attained and acceptable [6, 7, 8]. Accordingly, the research tightly controls every material related in casting concrete. In the research, in concrete mix there were four components were properly controlled for all three water-cement ratio in casting – fine aggregate, course aggregate, cement, and water.

Standard percentage of fine aggregate passing 600 µm is given approximately within 39% to 46% [9, 10, 11]. Thus, obeying the standards, four sets of fine aggregate used to mix in concrete were sieved to check the compliance with the standards and uniform. The ideal condition of either fine or course aggregate to cast concrete is surface-saturated-dry (SSD) [12, 13]. Generally, the moisture in fine aggregate shall be within 0.2% to 2.5% [12, 14]. The research managed to comply with the allowable moisture as mentioned in literatures and standard above where average moisture are less than 2%. Moisture of fine aggregate were measured using soil moisture meter. The instrument used were accurate and reliable under any soil condition.

![Figure 1](image1.jpg)

**Figure 1.** Few places within fine aggregate store were checked as to conform within the range using soil moisture meter.

The course aggregate were standardized to averagely 10 mm in diameter as in Figure 2. Cement used specifically Type I (CEM I) ordinary Portland cement. The slump for every concrete mix were checked corresponding to BS EN 12350-2 and all the slumps are in the acceptable range [15, 16]. Water-cement ratio 0.3, 0.4, and 0.5 for the concrete mix slump is 25 mm, 47 mm, and 57 mm correspondingly as in Figure 3. About 24 hours after casting, the concrete specimens were cured for 28 days until testing. The temperature of the water in curing tank were monitored and did not allowed to exceed 27°C as highlighted in ASTM C31 and ACI 308 [17, 18].

![Figure 2](image2.jpg)

**Figure 2.** 10-mm course aggregate.
3. Laboratory testing and configuration

3.1 Three-Point Bending Test
The defaulted configuration of laboratory experiment is three-point bend test as shown Figure 1. Three-point bend test is chosen as to emphasize on the ultimate tensile/flexural strength of concrete. Three-point bend test accentuates on its maximum stress in the middle and eliminates residual stress at elsewhere in the concrete beam specimen [5]. Thus, three-point test for concrete consists of three components: top-loading roller, bottom-support roller, and concrete beam specimen itself.

Basically, mechanisms that involve directly with physical experimental works and the calculation afterwards follows references made by RILEM and previous work from Hu’s researches which eventually proved-derivation from ASTM E399-90. Bottom-support roller and top-loading roller are both 30-mm in diameter based on RILEM TC 162-TDF and RILEM TC 89-FMT as shown in figure below [19, 20]:

![Figure 4. 30-mm diameter of top and bottom roller.](image)

3.2 Size of specimen
Corresponding to RILEM, the magnitude of span-to-depth ratio (S/W) of the concrete beam should be designed more than 2.5 [21]. Since the techniques in conducting the experiment inspired by Xiao Zhi Hu which he followed ASTM, henceforth ratio of S/W for concrete beam specimen were made 8 – meaning that the span, S is 800 mm and the depth of the beam is 100 mm [22]. Next, referring to
RILEM, the notch depths should be approximately 1/3 from the beam’s depth and the notch width is less than 5 mm [23]. Overall, in deciding the size of beam, few standards were referred which finally made the beam 1065 mm in length, 110 mm in width, and 100 mm in depth [21][22][24]. As mentioned earlier, the difference between these beams are the water-cement ratios and notch sizes as shown in Figure 5, Figure 6, and Figure 7 below.

**Figure 5.** Dimensions for concrete beam specimen with 5-mm (5.9 mm) notch.

**Figure 6.** Dimensions for concrete beam specimen with 15-mm (14 mm) notch.

**Figure 7.** Dimensions for concrete beam specimen with 30-mm (31.68 mm) notch.

### 3.3 Test procedure

One of the important factors in running three-point bend test is deciding the proper loading rate as inappropriate loading rate will cause inaccurate results. Thus, based on thorough literature reviews and experiences conducting the test previously, 1 MPa per minute is considered as reasonable and suitable loading rate which produces more steady results [23, 25, 26]. Secondly, in order to obtain stable data outputs, the top roller which act as loading source made to touch the specimen a bit – just about 0.1 MPa which it was suggested by ASTM C78 [26].
3.4 Formulas and calculations

The methodology commences by conducting three-point bend test on concrete beams. There are few variables that has to control constant such as (i) cement type, (ii) concrete constituents and its mixtures, (iii) the dimensions of the beam like width, height, length and span of the concrete beam, (iv) average grain size $G$ and possibly maximum grain size $d_{max}$. Of all those, notch depths, $a_0$ are the manipulative variables – as the main graph-plotting is peak load or maximum load on concrete beams, $P_{\text{max}}$ against different notch depths [27].

The linear equation governing $P_{\text{max}} - A_e$ involves few beam dimensional factors as shown below:

$$P_{\text{max}} = f_t \cdot A_e$$

(1)

where $P_{\text{max}}$ is simple peak load exerted on beam in three-point bend test, $f_t$ is tensile strength of the concrete beam, and $A_e$ is equivalent area. Thus, $A_e$ is the manipulative variables (x-axis), $P_{\text{max}}$ acts as responding variable (y-axis), and the tensile strength $f_t$ is the gradient of the graph. Equivalent area includes the entire dimensional factor of beam and given as below:

$$A_e = \frac{(W - a_0)(W - a_0 + 3G)}{1.5\left(\frac{S}{B}\right)^2 \left(1 + \frac{a_e}{3G}\right)^{1.5}}$$

(2)

where $W$ is the depth of specimen, initial crack depth $a_0$, $G$ is the average grain size, $S$ is the span where the distance between supports touches the beam, $a_e$ is the equivalent crack, and $B$ is beam’s width. Equivalent crack $a_e$ in Equation 3 is calculated using initial notch depth $a_0$, and a geometrical factor $Y(\infty)$ consisting of ratio $a_0$ and $W$, as follows:

$$a_e = \left[\frac{(1-\alpha)^2 \cdot Y(\alpha)}{1.12}\right]^{2} \cdot a_0$$

(3)

where $Y(\infty) = 1.106 - 1.552\alpha + 7.71\alpha^2 - 13.53\alpha^3 + 14.23\alpha^4$ suitable for the research specimen’s span-depth ratio (S/W) of 8; and $\alpha = a_0 / W$. Equivalent crack $a_e$ is considered as true structural parameter, since it is completely regulated by the specimen size and geometry.

4. Results and discussion

Typically, lower water-cement ratio in concrete mix results higher compressive strength of concrete corresponds to [28]. The trend for tensile or flexural strength in some literatures or standards are proportional to the compressive strength of concrete according to classes of concrete [29]. However, the trend for flexural strength in this research seems to be the other way round. But then again, it is wise to bear in mind there were few differences in this research which justify the contradiction to those mentioned – the technique the concrete specimens were prepared, size of specimen, moisture and curing condition of the concrete specimen are several factors that influence the variation in flexural test [30, 31].
4.1 Flexural strength against water-cement ratio variation

Figure 8. Flexural Strength of plain concrete beam specimens for different water-cement ratios and notch depths.

Figure 8 shows plain concrete beam which is without notch attain the highest strength and followed by the smallest notch depth of 5.9 mm, then the intermediate notch depth 14 mm and finally the concrete beams largest notch depth 31.68 mm have the least strength for water-cement ratio of 0.3, 0.4, and 0.5 respectively. For 31.68 mm notched beam, the increase between water-cement ratio 0.3 and 0.4, and slight decrement between water-cement ratio 0.4 and 0.5 are about 8.13% and 9.55% respectively. For 14 mm notched beam, the increment between water-cement ratios is roughly 2%. On the other hand, the flexural strength develops from 3.02% to 5.67% between the water-cement ratios for 5.9 mm notched beam.

The trend of escalating flexural strength with respect to the water-cement increment is agreed by few literatures [32, 33]. It is important to understand that there were many factors that cause the contradictory as mentioned in earlier paragraph.

Figure 9. A section of cement paste made up of Ordinary Portland Cement (OPC) [34].

Note: c: unhydrated cement; C-S-H: calcium silicate hydrate; CH: calcium hydroxide; p: pore; Circle ‘A’ contains C-S-H, CH and few elements that is not visible.
Based on Figure 9 and Figure 10 from the corresponding literatures, it could be seen that highest amount of C-S-H produces highest strength as in Figure 10(a) of cement paste used in concrete mix with water-cement ratio 0.3, and least strength for less C-S-H in concrete mix with 0.5 water-cement ratio as in Figure 10(c) [35]. Literally, the influence of C-S-H in concrete might be express in term of its compressive strength [35]. When the water-cement ratio in concrete mix is low such as 0.3, the pores in the mix could be reduce and enhance C-S-H density which eventually makes the concrete stronger [36]. Based on the comparison in Figure 10(a), Figure 10(b) and Figure 10(c), the darker dots in the image were the pores – and it can be seen that Figure 10(a) which represent water-cement ratio 0.3 has the least dark dots (pores) and lighter image (which some of it represents un-hydrated cement and C-S-H).

Table 1 shows compressive strength on three concrete mixes with different water-cement ratio respectively. As per explanation from literature above, the compressive strength decreases accordingly with respect to increment of water-cement ratio.

| Specimen Number | Water-Cement ratio (w/c) | Compressive Strength (MPa) |
|-----------------|--------------------------|---------------------------|
|                 | 0.3                      | 0.4                       | 0.5                       |
| 1               | 77.131                   | 67.167                    | 44.345                    |
| 2               | 75.814                   | 64.096                    | 48.269                    |
| 3               | 73.022                   | 58.503                    | 42.814                    |
| Average         | 75.322                   | 63.255                    | 45.143                    |

However, it does not mean that concrete mix with water-cement ratio 0.5 is unworkable. As it is noticeable in Figure 10(c), the cement paste has minimal light spots. The light spots (around six large light spots) are clearly visible in Figure 10(a) and Figure 10(b), they were un-hydrated cement – meaning that the cement paste did not fully hydrate as to compare to Figure 10(c) which represent concrete mix with more moisture, water-cement 0.5.

The reason the compressive strength of different water-cement ratio was enlightened because it may compare and provide understanding of flexural behaviour on concrete. Exceptionally, in discussing on flexural in concrete, there are more additional aspects that have to be taken into consideration. In testing flexural in concrete through three-point bend test, when the loading is exerted on the specimen, top part of the beam will experience compression and bottom part will be under tension [37, 38]. Thus, it is not a literal process of failure.

It is beneficial to recall, lower water-cement ratio in concrete mix will result higher compressive strength but decreases its tensile or flexural strength [28, 33]. Let’s focus at the centre of the beam when the load is applied. For water-cement ratio 0.3, the compressive strength which is the...
top part of the beam is more dominant – meaning that higher compressive strength and the bottom part which experience tension is weaker. Inversely, for concrete with water-cement ratio 0.5, the bottom part which is tensile strength is more governing and top part which under compression is less dominant. Thus, it can be seen the trend is parabolic-shape where water-cement ratio 0.4 is the peak and water-cement 0.3 and 0.5 are lower like the curve for 31.68-mm notched beam in Figure 8. In fact, generally, overall trend is not as far as parabolic-alike curve but the optimum water-cement ratio varies for the plain and different notch depths.

For concrete mix with water-cement ratio 0.3, it has higher compressive strength, higher brittleness, but lower flexural strength compared to the concrete mix with water-cement ratio 0.4 and 0.5 [39]. For this research, concrete mix with water-cement ratio of 0.4 displays decency. It is because for the top (compression) and bottom (tension) part of the beam for water-cement ratio 0.4, both are equally mediocre – it makes the beam more stable. Hence, it is such a way supporting each other which eventually results higher overall strength.

5. Conclusion
Generally, flexural strength decreases when water-cement ratio increases accordingly and flexural strength is reverse of its compressive strength for concrete mix with water-cement ratio of 0.3, 0.4, and 0.5. In term of notch size, larger notch depth concrete beam results lower flexural strength. Thus, water-cement ratio and size of notch have significant effects on its flexural strength.

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