Investigation of \textit{cscs} tetraquark in the chiral quark model

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Inspired by the recent observation of exotic resonances $X(4140)$, $X(4274)$, $X(4350)$, $X(4500)$ and $X(4700)$ reported by several experiment collaborations, we investigated the four-quark system $cs\bar{c}\bar{s}$ with quantum numbers $J^{PC} = 1^{++}$ and $0^{++}$ in the framework of the chiral quark model. Two configurations, diquark-antidiquark and meson-meson, with all possible color structures are considered. The results show that no molecular state can be formed, but the resonance may exist if the color structure of meson-meson configuration is $8 \otimes 8$. In the present calculation, the $X(4274)$ can be assigned as the $cs\bar{c}\bar{s}$ tetraquark states with $J^{PC} = 1^{++}$, but the energy of $X(4140)$ is too low to be regarded as the tetraquark state. $X(4350)$ can be a good candidate of compact tetraquark state with $J^{PC} = 0^{++}$. When the radial excitation is taken into account, the $X(4700)$ can be explained as the 2S radial excited tetraquark state with $J^{PC} = 0^{++}$. As for $X(4500)$, there is no matching state in our calculation.

I. INTRODUCTION

Recently, several exotic resonances were observed in the invariant mass distribution of $J/\psi\phi$. In 2009, the CDF Collaboration found the $X(4140)$ with mass $M = 4143.0 \pm 2.9 \pm 1.2$ MeV and width $\Gamma = 11.7^{+6.7}_{-4.7} \pm 3.7$ MeV in $B^+ \rightarrow J/\psi\phi K^+$ decay [1]. In 2010, a narrow resonance $X(4350)$ with mass $M = 4350.6^{+4.6}_{-5.1} \pm 0.7$ MeV and width $\Gamma = 13^{+18}_{-9} \pm 4$ MeV was reported by Belle Collaboration in $\gamma\gamma \rightarrow J/\psi\phi$ process and the possible spin parity is $J^{PC} = 0^{++}$ or $2^{++}$ [2]. A few years later, the exotic resonance $X(4140)$ was observed by some other collaborations including LHCb, D0, CMS, and BABAR [3–6]. In 2011, another resonance $X(4274)$ with mass $M = 4274.4 \pm 1.9$ MeV and width $\Gamma = 32.3 \pm 7.6$ MeV was observed by the CDF Collaboration in $B^+ \rightarrow J/\psi\phi K^+$ decay with $3.1\sigma$ significance [7]. In 2016, the LHCb Collaboration performed the first full amplitude analysis of the $B^+ \rightarrow J/\psi\phi K^+$ process and the existence of the $X(4140)$ and $X(4274)$ was confirmed. Their quantum numbers are fixed to be $J^{PC} = 1^{++}$ [8]. At the same time, the Collaboration observed another two resonances, $X(4500)$ and $X(4700)$ with $J^{PC} = 0^{++}$. Their masses and decay widths have been determined as [9]

\[
\begin{align*}
M_{X(4500)} &= (4506 \pm 11^{+12}_{-15}) \text{ MeV}, \quad (1) \\
\Gamma_{X(4500)} &= (92 \pm 21^{+21}_{-20}) \text{ MeV}, \quad (2) \\
M_{X(4700)} &= (4704 \pm 10^{+14}_{-24}) \text{ MeV}, \quad (3) \\
\Gamma_{X(4500)} &= (120 \pm 31^{+42}_{-53}) \text{ MeV}. \quad (4)
\end{align*}
\]

With the discovery of these exotic resonances, many theoretical work has been performed, such as approaches based on quark models [10–14], QCD sum rules [15], etc. In the framework of relativized quark model, the $X(4140)$ can be regarded as the $cs\bar{c}\bar{s}$ tetraquark ground state, and the $X(4700)$ can be explained as the 2S excited tetraquark state [10]. Based on the simple color-magnetic interaction model, possible ground $cs\bar{c}\bar{s}$ tetraquark states in the diquark-antidiquark configuration have been investigated, and the interpretation of $X(4500)$ and $X(4700)$ needs orbital (radial or angular) excitation [11]. Deng et al. investigated the hidden charmed states in the framework of the color flux-tube model, and they found the energy of the first radial excited state ($cs\bar{c}\bar{s}$) with $2^0 D_0$ is in full accord with that of the state $X(4700)$ [12]. In a simple quark model with chromomagnetic interaction, Stancu suggested that the $X(4140)$ could possibly be the strange partner of $X(3872)$ in a tetraquark interpretation [13]. Ortega et al. claimed that the $X(4140)$ resonance appears as a cusp in the $J/\psi\phi$ channel due to the near coincidence of the $D_{s}^{+} D_{s}^{*+}$ and $J/\psi\phi$ mass thresholds when the nonrelativistic constituent quark model was employed [14]. According to the QCD sum rules, Chen et al. pointed out that the $X(4500)$ and $X(4700)$ may be interpreted as the $D$-wave $cs\bar{c}\bar{s}$ tetraquark states with opposite color structures [15]. It should be emphasized that most of these explanations do not agree with each other, and many of these investigations neglected the spin-orbital interaction when the orbital excitation are taken into account.

To see whether these exotic resonances can be described by $cs\bar{c}\bar{s}$ tetraquark systems with $J^{PC} = 0^{++}, 1^{++}$, we do a high precision four-body calculation based on the framework of the chiral quark model, which described the hadron spectra and hadron-hadron interaction well [16, 17]. The high precision few-body method, Gaussian expansion method (GEM) [18], is employed for this purpose. Two configurations, $(qq)(q\bar{q})$ (meson-meson) and $(qq)(\bar{q}\bar{q})$ (diquark-antidiquark) are considered. All the color constructions for each configuration are taken into account. For meson-meson configuration, the color structures are $1 \otimes 1$ and $8 \otimes 8$, and for diquark-antidiquark configuration, $3 \otimes 3$ and $6 \otimes 6$. To explain the two higher exotic resonances $X(4500)$ and $X(4700)$, the orbital excitation with the inclusion of spin-orbital interaction are invoked. To expose the structures of the states, the distances between two quarks (antiquarks) for given states are calculated.

This paper is organized as follows. In Sec. II, the chi-

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The chiral quark model has achieved great success when describing hadron spectra and hadron-hadron interactions [17]. The specific introduction of the chiral quark model can be found in Ref. [16]. The Hamiltonian of csśś tetraquark systems, which is shown below, includes the mass, the kinetic energy and different kinds of interactions. These interactions include the confinement $V^C$, one-gluon-exchange $V^G$ and Goldstone bosons exchanges $V^X(\chi = \pi, \kappa, \eta)$, only $\eta$ exchange plays a role between $s$ and $\bar{s}$. Scalar meson exchange $V^\sigma$ is not included in these interactions because it is expected to exist between $u(\bar{u})$ and $d(\bar{d})$ only. Due to the existence of orbital excitation, the spin-orbit coupling terms are also taken into consideration.

$$H = \sum_{i=1}^{4} \left( \frac{p_i^2}{2m_i} - T_{cm} \right) + \sum_{i<j}^{4} \left( V_{ij}^C + V_{ij}^G + \sum_{\chi=\pi, \kappa, \eta} V_{ij}^X + V_{ij}^{CL,LS} + V_{ij}^{GL,LS} \right), \quad (5)$$

$$V_{ij}^C = -a_c \alpha_i^c \cdot \alpha_j^c \left[ \frac{1}{r_{ij}} - \frac{2\pi}{3m_i m_j} \sigma_i \cdot \sigma_j \delta(r_{ij}) \right], \quad (6)$$

$$V_{ij}^G = \frac{\alpha_s}{4\pi} \sum_{\lambda} \frac{L_0}{\lambda} \cdot \sigma_i \cdot \sigma_j \left[ Y(m_{l_\lambda} r_{ij}) - \frac{\alpha_s^3}{m_{l_\lambda}^3} Y(\lambda_{l_\lambda} r_{ij}) \right] \sum_{\alpha_\lambda} \sum_{\chi=\pi, \kappa, \eta} \lambda_i^\alpha \cdot \lambda_j^\alpha \delta(r_{ij}), \quad (7)$$

$$V_{ij}^\pi = \frac{g_{\pi}^2}{4\pi} \frac{m_{\pi}}{m_{l_\pi}} \frac{L_0}{\lambda} \cdot \sigma_i \cdot \sigma_j \left[ Y(m_{l_\pi} r_{ij}) - \frac{\alpha_s^3}{m_{l_\pi}^3} Y(\lambda_{l_\pi} r_{ij}) \right] \sum_{\alpha_\lambda} \sum_{\chi=\pi, \kappa, \eta} \lambda_i^\alpha \cdot \lambda_j^\alpha \delta(s_{ij}), \quad (8)$$

$$V_{ij}^\eta = \frac{g_{\eta}^2}{4\pi} \frac{m_{\eta}}{m_{l_\eta}} \frac{L_0}{\lambda} \cdot \sigma_i \cdot \sigma_j \left[ Y(m_{l_\eta} r_{ij}) - \frac{\alpha_s^3}{m_{l_\eta}^3} Y(\lambda_{l_\eta} r_{ij}) \right] \sum_{\alpha_\lambda} \sum_{\chi=\pi, \kappa, \eta} \lambda_i^\alpha \cdot \lambda_j^\alpha \delta(s_{ij}), \quad (9)$$

Where $T_{cm}$ is the kinetic energy of the center of mass motion; $Y(x)$ is the standard Yukawa functions; the $\sigma$ and $\lambda$ represent Pauli and Gell-Mann matrices, respectively; and the strong coupling constant of one-gluon exchange is $\alpha_s$, its running property is given as:

$$\alpha_s(\mu_{ij}) = \frac{\alpha_0}{\ln \left( (\mu_{ij}^2 + \mu_0^2) / \Lambda_0^2 \right)}, \quad (11)$$

where $\mu_{ij}$ represents the reduced mass of two interacting particles.

For the diquark-antidiquark configuration, the sub-clusters $qq$ and $\bar{q}\bar{q}$ can be treated as compound bosons $Q$ and $\bar{Q}$ with no internal orbital excitation. If the relative orbital angular excitations between the two clusters is $L$, the four-body spin-orbit interactions can be simply expressed as follows [19, 20]:

$$V_{QQ}^{CL,LS} = -a_c \alpha_i^Q \cdot \alpha_j^Q \left[ \frac{1}{4M_Q M_{\bar{Q}}} \right] L \cdot S, \quad (12)$$

$$V_{QQ}^{GL,LS} = -a_s \alpha_i^Q \cdot \alpha_j^Q \left[ \frac{3}{8M_Q M_{\bar{Q}}} \right] \frac{X^3}{X^3} L \cdot S, \quad (13)$$

where the $M_Q(M_{\bar{Q}})$ is the total mass of sub-cluster; $X$ is the distance between the two clusters, and $S$ is the total spin of the tetraquark state. This simplification can be generalized to color-octet meson sub-clusters in meson-meson configuration.

In this paper, the model parameters of chiral quark model are directly taken from our previous work [21], which is shown in Table I. These parameters are obtained by fitting the meson spectrum. Some of the calculated meson spectral have been given in Table II.

Next, we will introduce the wave functions for csśś tetraquark. A quark (antiquark) has four degrees of freedom, including orbit, flavor, spin, and color. For each degree of freedom, first we construct the wave function for each sub-cluster, then coupling the wave functions of two sub-clusters to get the wave functions for the final tetraquark systems.

For spatial part, the total spatial wave functions of tetraquark systems can be obtained by coupling three relative orbital motion wave functions:

$$\psi_{LM_L} = \left[ [\psi_{L_1}(r_{12}) \psi_{L_2}(r_{34})]_{L_12} \psi_{L_3}(R) \right]_{LM_L}, \quad (14)$$

where $\psi_{L_1}(r_{12})$ and $\psi_{L_3}(r_{34})$ are the relative orbital motions between two particles in each sub-cluster with angular momentum $l_1$ and $l_2$, respectively; and $\psi_{L_3}(R)$ is the
TABLE I: Parameters of chiral quark model.

| Parameters   | Values | Parameters   | Values |
|--------------|--------|--------------|--------|
| $m_u$(MeV)   | 313    | $\Delta_2 = \Delta_6 (fm^{-1})$ | 4.2    |
| $m_d$(MeV)   | 313    | $\Delta_2 = \Delta_6 (fm^{-1})$ | 5.2    |
| $m_s$(MeV)   | 536    | $g_{2s}(/4\pi)$ | 0.54   |
| $m_c$(MeV)   | 1728   | $\theta_0(^{\circ})$ | -15    |
| $m_b$(MeV)   | 5112   | $a_0$(MeV) | 101    |
| $a_s$(fm$^{-1}$) | 0.70  | $\Delta$(MeV) | -78.3  |
| $a_l$(fm$^{-1}$) | 3.42  | $\alpha_0$ | 3.67   |
| $a_o$(fm$^{-1}$) | 2.77  | $\Delta_0$(fm$^{-1}$) | 0.033  |
| $b_o$(fm$^{-1}$) | 2.51  | $\mu_0$(MeV) | 36.976 |

relative orbital wave function between two sub-clusters with angular momentum $L_r$. In the present calculation, we do not consider the orbital excitation in each sub-cluster, so set $l_1 = l_2 = 0$ and $L_r = L$, which is the total orbital angular momentum of tetraquark systems. In GEM, three relative orbital motion wave functions are all expanded by Gaussian functions \[18\]:

$$\psi_{lm}(r) = \sum_{n=1}^{n_{\text{max}}} c_{nl} \phi_{nlm}^G(r)$$ \quad (15)

$$\phi_{nlm}^G(r) = N_{nl} r^l e^{-\nu_n r^2} Y_{lm}(\hat{r})$$ \quad (16)

$$N_{nl} = \left(\frac{2l+1}{\pi^{3/2}}\right)^{1/2} \left(\frac{\Gamma(l+\nu_n)}{\Gamma(l+1)}\right)^{1/2} \frac{1}{r_{\text{min}}} \nu_n$$ \quad (17)

where $N_{nl}$ are normalization constants, and the expansion coefficients $c_{nl}$ are obtained by solving the Schrödinger equation. The Gaussian size parameters are set according to the following geometric progression:

$$\nu_n = \frac{1}{r_n^2}, r_n = r_{\text{min}} a^{n-1}, a = \left(\frac{r_{\text{max}}}{r_{\text{min}}}ight)^{1/n_{\text{max}}}$$ \quad (18)

where the $n_{\text{max}}$ is the number of Gaussian functions, and $a$ is the ratio coefficient. After parameter optimization through continuous calculation, we found the calculation results begin to converge, when $n_{\text{max}} = 7$.

For spin part, since the spin of each quark (antiquark) is 1/2, the spin of each sub-cluster can only be 0 or 1. After coupling the spins of the two sub-clusters, the total spin of tetraquark may be 0, 1, and 2. The total spin $S$ of tetraquark system is obtained from the coupling: $S_1 \otimes S_2 \rightarrow S$, where $S_1$ and $S_2$ represent the spins of two sub-clusters. We use $\chi_i$($i = 1 \sim 6$) to denote the total spin wave functions of tetraquark systems. All of the six possible spin channels are given as:

$$\chi_1: 0 \otimes 0 \rightarrow 0 \chi_2: 1 \otimes 1 \rightarrow 0$$

$$\chi_3: 0 \otimes 1 \rightarrow 1 \chi_4: 1 \otimes 0 \rightarrow 1 \chi_5: 1 \otimes 1 \rightarrow 1$$

$$\chi_6: 1 \otimes 1 \rightarrow 2$$ \quad (19)

TABLE II: Meson spectra (unit: MeV).

| mesons $E$ | PDG [22] $E$ | mesons $E$ | PDG [22] $E$ |
|------------|--------------|------------|--------------|
| $\pi$      | 140.1        | 139.6      | 1953.4      | 1968.3      |
| $\rho$     | 774.4        | 775.3      | 2080.2      | 2112.2      |
| $\omega$   | 708.2        | 782.7      | 1015.8      | 1019.5      |
| $K$        | 496.4        | 493.7      | 824.0       | 957.8       |
| $K^*$      | 918.4        | 891.8      | 2983.6      | 2983.9      |
| $D$        | 1875.4       | 1869.7     | 3096.4      | 3096.9      |
| $D^*$      | 1986.3       | 2010.3     | \[20\]

For flavor part, the isospins of all quarks (antiquarks) are all zero, so there is no need to consider the coupling of the isospin. We use $\varphi_j (j = 1 \sim 3)$ to represent the total flavor wave functions of tetraquark systems and they can be written as:

$$\varphi_1 = (cc)(ss), \quad \varphi_2 = (cs)(sc),$$

$$\varphi_3 = (cs)(\bar{c}s)$$ \quad (20)

where $\varphi_1$ and $\varphi_2$ are for meson-meson configuration and $\varphi_3$ is for diquark-antidiquark configuration.

For color part, there are four different color wave functions of tetraquark systems, which are denoted by $\omega_k$ ($k = 1 \sim 4$). $\omega_1$ and $\omega_2$ stand for the color singlet-singlet $1\otimes1$ and octet-octet $8\otimes8$ in meson-meson configuration, respectively. The remaining two color wave functions $\omega_3$ and $\omega_4$ stand for the color antitriplet-triplet $3\otimes3$ and sextet-antisextet $6\otimes6$ in diquark-antidiquark configuration, respectively. The specific construction process and forms of these color wave functions can be found in Ref. [21].

Finally, the total wave functions for the tetraquark systems can be given as:

$$\Psi_{ijk}^{LJ\lambda j} = A_{\lambda j} \psi_{i} \chi_{j} \varphi_{k},$$

$$(i = 1 \sim 6, \ j = 1 \sim 3, \ k = 1 \sim 4),$$

where $J$ is the total angular momentum and $M_J$ is the 3rd component of the total angular momentum. Due to the two quarks (antiquarks) in $cs\bar{c}s$ tetraquark systems are all non-identical particles, the the antisymmetrization operator $A = 1$.

The eigenenergies of the tetraquark systems can be obtained by solving the following Schrödinger equation:

$$H \Psi_{ijk}^{LJ\lambda j} = E_{ijk}^{LJ\lambda j} \Psi_{ijk}^{LJ\lambda j},$$ \quad (22)

where the Hamiltonian $H$ and wave functions $\Psi_{ijk}^{LJ\lambda j}$ have been given in Eq.(5) and Eq.(21), respectively.

III. NUMERICAL RESULTS AND DISCUSSIONS

The $J^{PC}$ of these exotic resonances observed recently in $B^+ \rightarrow J/\psi K^+$ decay are fixed to $1^{++}$ and $0^{++}$. In
TABLE III: Energy of S-wave \(cs\bar{s}\) tetraquark states with \(J^{PC} = 1^{++}\) (unit: MeV).

| Channel         | \(E_{1S}\)  | \(E_{c1}\)  | \(E'_{c1}\) | \(E_{th}^{th}\) | \(E_{exp}^{th}\) |
|-----------------|-------------|-------------|-------------|----------------|-----------------|
| \(\psi S\chi_1^1\phi_1\omega_1\) | 4112.7      | 4112.7      | 4116.9      | 4112.2         | 4116.4(J/\psi\phi) |
| \(\psi S\chi_1^1\phi_1\omega_2\) | 4305.2      | 4305.2      | 4309.4      |                |                 |
| \(\psi S\chi_2^2\phi_2\omega_1\) | 4033.8      | 4033.7      | 4080.7      | 4033.5         | 4080.5(D_0\,D_0^*) |
| \(\psi S\chi_{p}^3\phi_3\omega_2\) | 4370.6      | 4370.8      | 4417.8      |                |                 |
| \(\psi S\chi_{p}^3\phi_3\omega_3\) | 4343.4      | 4332.7      | -           |                |                 |
| \(\psi S\chi_{p}^3\phi_3\omega_4\) | 4361.5      | 4374.3      | -           |                |                 |

TABLE IV: Energy of S-wave \(cs\bar{s}\) tetraquark states with \(J^{PC} = 0^{++}\) (unit: MeV).

| Channel         | \(E_{1S}\)  | \(E_{c1}\)  | \(E'_{c1}\) | \(E_{th}^{th}\) | \(E_{exp}^{th}\) |
|-----------------|-------------|-------------|-------------|----------------|-----------------|
| \(\psi S\chi_1^1\phi_1\omega_1\) | 3810.7      | 3810.6      | 3942.0      | 3941.7(\eta_c\eta') |                  |
| \(\psi S\chi_1^1\phi_1\omega_2\) | 4359.1      | 4360.2      | 4491.6      |                | 4712.7          | 4844.1         |
| \(\psi S\chi_2^2\phi_2\omega_1\) | 4114.0      | 4114.0      | 4118.2      | 4112.2         | 4116.4(J/\psi\phi) | 4601.2         | 4605.4         |
| \(\psi S\chi_2^3\phi_3\omega_1\) | 4273.4      | 4273.4      | 4277.6      |                | 3936.6(D_0\,D_0) | 4910.6         | 4940.4         |
| \(\psi S\chi_3^4\phi_4\omega_2\) | 3908.0      | 3908.0      | 3837.8      | 3906.8         |                | 4224.4(D_0\,D_0^*) | 4719.4         |
| \(\psi S\chi_3^4\phi_4\omega_3\) | 4376.1      | 4376.3      | 4406.1      |                | 4655.4          | 4719.4         |
| \(\psi S\chi_3^4\phi_4\omega_4\) | 4161.6      | 4161.6      | 4225.6      | 4224.4         |                | 4643.5          | 4641.7         |

our current calculation, only the states with these two sets of quantum numbers are considered. Due to the parities of these exotic resonances is positive, the total orbital angular momentum \(L\) must be even, so does \(L_r\), according to Eq.(14). Meanwhile, these exotic resonances all have the definite positive \(C\) parity, we need to combine the spin and flavor wave functions to construct the eigenstates of the charge conjugate operator. We use \((qq)^{S_1}(qq)^{\bar{S}_2})^S\) and \((qq)^{S_1}(qq)^{\bar{S}_2})^\bar{S}\) to represent the combination of spin and flavor wave functions. According to the Ref. [23], all of these eigenstates with positive \(C\) parity are given as:

\[
\begin{align*}
\chi_{5}\phi_1 &= [(cc)^1(s\bar{s})^1]^1, \\
\chi_{p}\phi_2 &= \frac{1}{\sqrt{2}}([(cc)^0(s\bar{s})^1]^1 + [(cc)^1(s\bar{s})^0]^1), \\
\chi_{p}\phi_3 &= \frac{1}{\sqrt{2}}([(cc)^0(s\bar{s})^0]^1 + [(cc)^1(s\bar{s})^0]^1), \\
\chi_{1}\phi_1 &= [(cc)^0(s\bar{s})^0]^0, \\
\chi_{2}\phi_2 &= [(cc)^1(s\bar{s})^0]^0, \\
\chi_{2}\phi_3 &= [(cc)^0(s\bar{s})^0]^0, \\
\chi_{6}\phi_1 &= [(cc)^0(s\bar{s})^1]^2, \\
\chi_{6}\phi_2 &= [(cc)^1(s\bar{s})^1]^2, \\
\chi_{6}\phi_3 &= [(cc)^0(s\bar{s})^0]^1,
\end{align*}
\]

where the \(\chi_p\) in Eq.(24) and Eq.(25) is \(\frac{1}{\sqrt{2}}(\chi_3 + \chi_4)\).

Generally, we only consider the \(S\)-wave states for the low-lying state, i.e., \(L_r = 0\) for states with \(J^{PC} = 1^{++}\). However, for the high-lying states, \(X(4500)\) and \(X(4700)\), we will take into account the \(D\)-wave states, i.e., \(L_r = 2\) for some states with \(J^{PC} = 0^{++}\).

First, we calculated the energies of the \(S\)-wave \(cs\bar{s}\) states with \(J^{PC} = 1^{++}\) and \(0^{++}\). The results are given in Table III and Table IV, respectively. In the tables, \(E_{1S}\) (\(E_{2S}\)) denotes the energy of the first (second) \(S\)-wave state with single channel calculation, and \(E_{c1}\) (\(E_{c2}\)) represents the energy through coupling of two different color structures. \(E_{th}^{th}\) (\(E_{exp}^{th}\)) represents the theoretical (experimental) two-body threshold. Because the theoretical calculation cannot reproduced the experimental data exactly for meson spectrum, so we make a correction, \(E' = E - E_{th}^{th} + E_{exp}^{th}\) for the meson-meson configuration to minimize the theoretical uncertainty. \(E_{c1}'\) (\(E_{c2}'\)) represents the corrected energy of \(E_{c1}\) (\(E_{c2}\)). For diquark-antidiquark configuration, the correction is not applied because no asymptotic physical state in this case.

From Table III and Table IV, we can see that the energies of color singlet-singlet \((1 \otimes 1)\) structure are all a bit high than the corresponding theoretical thresholds. The adiabatic energy of the system in this case is shown in Fig. 1 (the adiabatic energy is obtained by setting the number of Gaussians for the relative motion between two sub-clusters to 1). When we gradually increase the di-
distance between the two sub-clusters, the energy slowly tend to theoretical threshold. This phenomenon suggests that the two mesons tend to stay away and no bound states can be formed when the color structure is $1 \otimes 1$. The reason for this phenomenon is that the Goldstone bosons exchange between the two sub-clusters is too weak to bind the two mesons together. When the color structure is octet-octet ($8 \otimes 8$), the energies are generally higher than the corresponding color singlet-singlet ($1 \otimes 1$) structure. According to Fig.1(b), the two colorful sub-clusters cannot fall apart or get too close. That’s because the existence of confinement $V_C$ hinders the too far separation of the two colorful clusters. This phenomenon suggests that the resonances with compact tetraquark structure may exist in our present calculation when the color configuration of the meson-meson configuration is octet-octet ($8 \otimes 8$). Due to the small overlap between the two color structures, $1 \otimes 1$ and $8 \otimes 8$, the coupling between two color structures is small, which makes the resonance possible. When the configuration is diquark-antidiquark, the two color structures antitriplet-triplet ($3 \otimes \bar{3}$) and sextet-antisextet ($6 \otimes \bar{6}$) have the similar energies and the coupling between them is rather strong. According to Fig.1(c) and Fig.1(d), whether $3 \otimes 3$ or $6 \otimes \bar{6}$, the two sub-clusters are all colorful and they cannot stay too far away from each other, and short-range repulsion prevent the two sub-clusters from getting too close. So the resonances with compact tetraquark structure may also exist in diquark-antidiquark configuration.

For the states with $J^{PC} = 1^{++}$ (Table III), the energies of $1S$ ground state are all between 4300 MeV and 4420 MeV except the state with color structure $1 \otimes 1$. The corrected energy $E_{c_1}'$ of hidden color($8 \otimes 8$) channel is around 4309.4 MeV and the lowest energy of diquark-antidiquark configuration is about 4332 MeV. Both energies are not far from the mass of $X(4274)$. The mix of two configurations may reduced the energy a little, the lowest energy of the state with $J^{PC} = 1^{++}$ is expected to approach the experimental value of the $X(4274)$. So the $X(4274)$ can be a candidate of compact tetraquark state in our present calculation. As for $X(4140)$, the energy is too low and there is no matching state for it in our calculations. For the states with $J^{PC} = 0^{++}$ (Table IV), the energies of $1S$ ground state are all between 4277 MeV and 4492 MeV except the state with color structure $1 \otimes 1$. It is worth mentioning that the energy
explained as the 2nd resonance recently found in experiment by LHCb Collaboration. The other exotic state of $X(4700)$ cannot be explained by tetraquark structure. For the $D$-wave excitations, the energies of meson-meson configuration with color structure $1 \otimes 1$ are still very close to the corresponding theoretical threshold. That’s because the too long distance between the two clusters makes the effect of the $D$-wave excitation negligible. When the color structure is $8 \otimes 8$ or the configuration is diquark-antidiquark, the energies of $D$-wave excitation are all too high to be explained as the $X(4700)$ and $X(4500)$. So it’s not appropriate to use angular excitation to explain these exotic resonances in our present calculation.

In order to analyze the possible structure of the $cs\bar{s}\bar{s}$ tetraquark, we calculated the distances between two $q(\bar{q})$ for $(cs)(\bar{s}\bar{s})$ and $(cs)(\bar{c}s)$ ground states with $J^{PC} = 0^{++}$ (see Table VI). The distances between any two $q(\bar{q})$ are very close and now the structure is a compact tetraquark structure. After coupling the two color structures for meson-meson configuration, the change is small, because of the weak coupling between these two color structures. For the diquark-antidiquark configuration in Table VII, whatever the color structure, the distance between any two $q(\bar{q})$ is small and now the structure is also very compact. After coupling the two color structures for diquark-antidiquark configuration, the change is obvious. Strong coupling between these two color structures is the cause of this phenomenon. In addition, the relative kinetic energy between two light $q(\bar{q})$ is greater than that of two heavy $q(\bar{q})$. That’s why the distance between two light $q(\bar{q})$ is bigger than that of two heavy $q(\bar{q})$ in our calculations.

### Table V: Energy of $D$-wave $cs\bar{s}\bar{s}$ tetraquark states with $J^{PC} = 0^{++}$ (unit: MeV)

| Channel | $E_{1D}$ | $E_{c1}$ | $E_{th,exp}$ | $E_{th}^{exp}$ |
|---------|---------|---------|-------------|--------------|
| $[\psi D_{J}\chi]^{0}_{J} \psi_{2\omega_{1}}$ | 4116.6 | 4116.4 | 4120.6 | 4112.2 |
| $[\psi D_{J}\chi]^{0}_{J} \psi_{2\omega_{2}}$ | 5173.8 | 5174.0 | 4227.2 | 4160.4 |
| $[\psi D_{J}\chi]^{0}_{J} \psi_{2\omega_{3}}$ | 4853.8 | 4847.6 | - | - |
| $[\psi D_{J}\chi]^{0}_{J} \psi_{2\omega_{4}}$ | 5173.8 | 5177.6 | - | - |

**IV. SUMMARY AND OUTLOOK**

In this work, we investigated the $cs\bar{s}\bar{s}$ tetraquark states in the framework of chiral quark model and try to explain those exotic resonances recently observed in the invariant mass distribution of $J/\psi\phi$. Two configurations, $(q\bar{q})(\bar{q}\bar{q})$ and $(q\bar{q})(\bar{q}\bar{q})$, with all possible spin and color structures
are taken into consideration. We found that the \((q\bar{q})(q\bar{q})\) configuration can not form the bound states when color structure is \(1 \otimes 1\) because the Goldstone bosons exchange is too weak to bound the two mesons. If the color structure of \((q\bar{q})(q\bar{q})\) is \(8 \otimes 8\) or the configuration is \((qq)(\bar{q}\bar{q})\), the resonances can be formed. In our calculation, the \(X(4274)\) and \(X(4350)\) in experiment can be regarded as the ground state of \(cs\bar{c}\bar{s}\) compact tetraquark states. The \(X(4700)\) can be explained to be the \(2S\) radial excitation but the \(X(4500)\) has no match in present calculation. Due to the too low energy of \(X(4140)\), it is impossible to use tetraquark state to explain this exotic resonances in our work. The current work doesn’t consider the coupling between the configurations \((q\bar{q})(q\bar{q})\) and \((qq)(\bar{q}\bar{q})\) and the coupling of \(S\)-wave and \(D\)-wave is also ignored. These legacy situations will be considered in the future work.

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