Overview of pion-nucleus interaction at low energies

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Predictions of the existence of well-defined deeply bound pionic atom states for heavy nuclei and the eventual observation of such states by the \((d,^3He)\) reaction have revived interest in the pion-nucleus interaction at threshold and in its relation to the corresponding pion-nucleon interaction. Explanation of the ‘anomalous’ \(s\)-wave repulsion in terms of partial restoration of chiral symmetry and/or in terms of energy-dependence effects have been tested in global fits to pionic atom data and in a recent dedicated elastic scattering experiment. The role of neutron density distributions in this context is discussed in detail for the first time.

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I. MOTIVATION AND BACKGROUND

The motivation for presenting this overview is the revival in recent years of interest in the pion-nucleus interaction at threshold and in particular in its relation to the corresponding pion-nucleon interaction. For low energy pions one may relate the two interactions by the Ericson Ericson (EE) potential, which is inserted into the Klein-Gordon (KG) equation to calculate shifts and widths of pionic atom levels. Traditionally, experiments on pionic atoms involved the measurement of X-rays from atomic transitions of \(\pi^-\) which terminate when the nuclear absorption of pions becomes dominant. It was believed that in any case the deepest atomic levels for heavy nuclei will be too broad to be well-defined. Friedman and Soff predicted in 1985 that such states, owing to the repulsive \(s\)-wave \(\pi N\) interaction at threshold, are sufficiently narrow so as to be well defined, and three years later Toki and Yamazaki discussed ways to populate such states. The first observation of these states was by Yamazaki et al. Figure illustrates how nuclear absorption becomes dominant when the angular momentum \(l\) goes down and how its initial increase is greatly reduced due to the repulsion of the wavefunction out of the nucleus, which is the mechanism responsible for making the level widths sufficiently small to be observed.

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FIG. 1: Radiation (dashed) and total widths of pionic atom levels in Pb.

FIG. 2: $\chi^2$ vs. the neutron-excess radius parameter $\gamma$ for different shapes of the neutron density (lower part); derived values of the isovector parameter $b_1$ (upper).
Interest in the pion-nucleus interaction at low energies has been focused on the s-wave part of the optical potential 

\[ q(r) = -4\pi(1 + \frac{\mu}{M})\{\tilde{b}_0(r)[\rho_n(r) + \rho_p(r)] + b_1[\rho_n(r) - \rho_p(r)]\} - 4\pi(1 + \frac{\mu}{2M})4B_0\rho_n(r)\rho_p(r), \]

(1)

where \( M \) is the mass of the nucleon and \( \rho_n \) and \( \rho_p \) are the neutron and proton density distributions normalized to the number of neutrons \( N \) and number of protons \( Z \), respectively. The function \( \tilde{b}_0(r) \) is given in terms of the local Fermi momentum \( k_F(r) \) corresponding to the isoscalar nucleon density distribution:

\[ \tilde{b}_0(r) = b_0 - \frac{3}{2\pi}(b_0^2 + 2b_1^2)k_F(r). \]

(2)

This term shows enhanced repulsion compared to the free pion-nucleon interaction, resulting mostly from the in-medium isovector s-wave \( \pi N \) amplitude \( b_1 \) which plays a dominant role due to the nearly vanishing of the corresponding isoscalar amplitude \( b_0 \). Some extra repulsion comes also from the empirical dispersive component of the two-nucleon absorption term \( B_0 \). Recent interest in this topic included attempts to explain the enhancement of the s-wave repulsion in terms of chiral-motivated density dependence of the pion decay constant [6] or by energy-dependent effects [7, 8].

In Section II we discuss the role of the neutron density distributions which, together with the proton distributions, form an essential ingredient of the EE potential [1, 2]. In Section III we present results from global fits to pionic atom data, with emphasis placed on dependence on the neutron distribution used in the analysis. Recent extensions to the scattering regime are presented in Section IV. Section V is a summary.

II. THE ROLE OF NEUTRON DENSITIES

With nine parameters in the EE pion-nucleus optical potential [2] the only way of gaining significant information from fits to pionic atom data is to perform large scale fits to as wide a data base as possible. The proton density distributions \( \rho_p \) are known quite well from electron scattering and muonic X-ray experiments, and can be obtained from the nuclear charge distributions by numerical unfolding of the finite size of the proton. In contrast, the neutron densities \( \rho_n \) are not known to sufficient accuracy and we have therefore performed fits while scanning over these densities. This procedure is based on the expectation that for a large data set over the whole of the periodic table some local variations will cancel out and that an average behavior may be established.

Experience showed [8] that the feature of neutron density distributions which is most relevant in determining strong interaction effects in pionic atoms is the radial extent, as represented for example by \( r_n \), the neutron density rms...
radius. Other features such as the detailed shape of the distribution have only minor effect. For that reason we chose the rms radius as the prime parameter in the present study and adopted the two-parameter Fermi distribution both for the proton and for the neutron density distributions as follows:

\[ \rho_{n,p}(r) = \frac{\rho_{0n,0p}}{1 + \exp((r - R_{n,p})/a_{n,p})} . \] (3)

The use of single-particle densities is not expected to be more appropriate when there are many nuclei far removed from closed shells and when in any case parameters of \( \rho_n \) are being varied. In order to allow for possible differences in the shape of the neutron distribution, the ‘skin’ and the ‘halo’ forms of Ref. 9 were used, as well as an average of the two. In this way we have used three shapes of the neutron distribution for each value of its rms radius all along the periodic table. The rms radius \( r_n \) for the various nuclei was parameterised using the following expression for the neutron-proton differences

\[ r_n - r_p = \gamma \frac{N - Z}{A} + \delta , \] (4)

and scanning over the values of the parameter \( \gamma \). This approach has been made in analyses of antiprotonic atoms data [9, 10].

III. RESULTS FOR PIONIC ATOMS

Figure 2 shows values of \( \chi^2 \) for 100 data points in the lower part and the derived values of the s-wave parameter \( b_1 \) in the upper part, as functions of the neutron radius parameter \( \gamma \). It is seen that the best fit is obtained for the ‘skin’ shape and that the \( b_1 \) parameter is then in good agreement with its free-space value. We note, however, that this best fit is obtained for a value of \( \gamma \) which leads to unacceptably large neutron rms radii in heavy nuclei [11]. Attempting to introduce finite-range folding into the potential, as was successfully done with antiprotons [10], the fits deteriorate monotonically with increasing range. However, when finite range is introduced separately into the s-wave and the p-wave parts of the potential, it is found that a finite range with an rms radius of 0.9\( \pm \)0.1 fm applied only to the p-wave part, leads to the best fit and Fig. 2 shows that it is obtained for \( \gamma \) close to 1.0 fm, which is acceptable when comparing with other sources of information on neutron radii. The discrepancy between the value obtained for \( b_1 \) and its free-space value is clearly observed. It is also found that \( \text{Re}B_0/\text{Im}B_0 = -2 \), which is unacceptable [2]. Both results represent the well known ‘anomaly’, or enhanced repulsion in nuclei.

In the chiral-approach of Weise [6] the in-medium \( b_1 \) is related to possible partial restoration of chiral symmetry in dense matter, as follows. Since \( b_1 \) in free-space is well approximated in lowest chiral-expansion order by the Tomozawa-
Weinberg expression $b_1 = -\mu_{\pi N}/8\pi f_\pi^2 = -0.08 \, m_\pi^{-1}$, then it can be argued that $b_1$ will be modified in pionic atoms if the pion decay constant $f_\pi$ is modified in the medium. The square of this decay constant is given in leading order, as a linear function of the nuclear density,

$$f_\pi^2 = f_\pi^2 - \frac{\sigma}{m_\pi^2} \rho$$  \hspace{1cm} (5)

with $\sigma$ the pion-nucleon sigma term. This leads to a density-dependent isovector amplitude such that $b_1$ becomes

$$b_1(\rho) = \frac{b_1(0)}{1 - 2.3 \rho}$$  \hspace{1cm} (6)

for $\sigma=50$ MeV and with $\rho$ in units of fm$^{-3}$. With this ansatz the best fit is obtained for $\gamma$ close to 1.0 fm and with $b_1$ almost in agreement with the free value. When the energy dependence of the $b_0$ parameter is also included [7, 8], Fig. 4 shows a perfect agreement between the derived $b_1$ and its free value, for acceptable neutron rms radii. For this case the dispersive term Re$B_0$ (not shown) turns out to be consistent with zero, in contrast to the unacceptably large repulsive values obtained for the conventional potential. It is worth mentioning that although the values of $r_n$ obtained here agree with the values found from analyses of antiprotonic atoms [9, 10], the latter favor the ‘halo’ shape for the neutron density distribution. This could be the result of using the over-simplified Fermi distribution and the

FIG. 3: Same as Fig. 2 but for finite range in the $p$-wave term.
The fact that whereas pionic atoms are sensitive to densities around 50% of the central value, antiprotonic atoms sample regions where the density is only 5% of that of nuclear matter.

Finally, it should be noted that the ‘deeply bound’ pionic atom states fit precisely into the picture emerging from global fits to conventional pionic atom data in the sense that predictions made with potentials obtained from fits to the latter are in full agreement with experimental results for the former. That is a natural consequence of the repulsion of the atomic wavefunction out of the nucleus which prevent really deep penetration of the deeply-bound atomic wavefunctions.

IV. ELASTIC SCATTERING AT 21.5 MEV

With the picture emerging from global analyses of pionic atoms it is interesting to extend the study of the $s$-wave part of the pion-nucleus potential into the scattering regime, where below 30 MeV the pions penetrate well into nuclei. In the scattering scenario, unlike in the atomic case, one can study both charge states of the pion, thus increasing sensitivities to isovector effects and to the energy dependence of the isoscalar amplitude due to the Coulomb interaction. Note that both were found to play a role in the atomic case. For that reason precision measurements of
elastic scattering of 21.5 MeV $\pi^+$ and $\pi^-$ by several nuclei were performed very recently at PSI [12, 13] and analyzed in terms of the same effects as in pionic atoms. The experiment was dedicated to studying the elastic scattering of both pion charge states and special emphasis was placed on the absolute normalization of the cross sections, which was based on the parallel measurements of Coulomb scattering of muons. The potentials tested were the conventional (C) one, the chiral motivated potential of Weise [6] (W), the energy-dependent potential [7, 8] (E) and a potential with both effects included (EW).

Table I summarizes the results, showing that clearly the better fits to the data require that at least one of these effects is included, and that the derived $b_1$ agrees with the free $\pi N$ interaction only when the chiral-motivated density dependence is included.

V. SUMMARY

Global analyses of strong interaction effects in pionic atoms, fitting parameters to the EE potential across the periodic table, consistently led to ‘anomalous’ repulsion in the $s$-wave part of the potential when compared to the free $\pi N$ interaction at threshold. This is particularly clear when the dependence on rms radii of neutron density distributions is considered. Introducing into the $s$-wave part of the potential a chiral-motivated dependence on density of the isovector interaction and the energy dependence of the isoscalar interaction fully explain the enhanced repulsion and the best fit is then obtained with neutron densities that are in agreement with other sources of information. Dedicated experiment at 21.5 MeV measuring the elastic scattering of $\pi^\pm$ by several targets show that the pion-nucleus potential changes smoothly from threshold into the scattering regime. Enhanced repulsion is observed also at 21.5 MeV, and is accounted for by the same mechanisms as for pionic atoms. However, in contrast with pionic atoms, the quality of fits to the data clearly require the inclusion of at least the chiral-motivated density dependence.

**TABLE I: Values of $b_1$ from fits to elastic scattering of 21.5 MeV $\pi^\pm$ by Si, Ca, Ni and Zr. $b_1 = -0.088 \pm 0.001(m_\pi^{-1})$ for the free $\pi N$ interaction.**

| model | C     | W     | E     | EW    |
|-------|-------|-------|-------|-------|
| $b_1 (m_\pi^{-1})$ | $-0.114 \pm 0.006$ | $-0.081 \pm 0.005$ | $-0.119 \pm 0.006$ | $-0.083 \pm 0.005$ |
| $\chi^2$ for 72 points | 134 | 88 | 80 | 88 |
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