Coherence revival under the Unruh effect and its metrological advantage

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In this paper, we investigate the quantum coherence extraction between two accelerating Unruh-DeWitt detectors, coupling to a scalar field in (3 + 1)-dimensional Minkowski spacetime. We find that quantum coherence as a nonclassical correlation can be generated through the Markovian evolution of the detector system, just like quantum entanglement. However, with growing Unruh temperature, in contrast to monotonously degrading entanglement, we find that quantum coherence exhibits a striking revival phenomenon. For certain detectors’ initial state choices, the coherence measure will reduce to zero at first and then grow to an asymptotic value. We verify such coherence revival by inspecting its metrological advantage on the quantum Fisher information (QFI) enhancement. Since the maximal QFI bounds the accuracy of quantum parameter estimation, we conclude that the extracted coherence can be utilized as a physical resource in quantum metrology.

Keywords: Open quantum system, Unruh effect, Quantum coherence, Quantum Fisher information

I. INTRODUCTION

Nonclassical correlations have long been regarded as a key physical resource. A notable example is quantum entanglement, which plays a central role in many areas of fundamental physics [1, 2], such as quantum information theory, black hole physics, and cosmology. Other types of quantum correlations beyond entanglement, such as nonlocality [3] and quantum discord [4], were also exploited. However, there are more general notions on quantum resources characterizing the nonclassicality that makes a system inherently quantum. In recent years, intensive works have been done on quantum coherence, which encapsulates the defining features of quantum theory in arbitrary dimensions, from the superposition principle of single qubit to quantum correlations among multipartite. Quantum coherence constitutes a powerful resource [5–7], which was wildly applied in quantum metrology [8], biological systems [9] and thermodynamics [10]. Moreover, on a fundamental level, it has been shown [11, 12] that coherence and entanglement are intimately related as they are interchangeable and can be unified into a single resource (see also [13, 14] for a review).

On the other hand, it was realized that a designed physical system can extract the intrinsic nonclassical correlations of quantum vacuum. For instance, two initially uncorrelated Unruh-DeWitt (UDW) detectors in Minkowski spacetime can generate nonlocal entanglement under relativistic acceleration motion [15, 16], and be witnessed by various measures, such as uncertainty relation [17, 18] and Bell-type inequalities [19, 20]. In this context, the residual entanglement of the final equilibrium is a result of competition between the Unruh decoherence and entanglement generation from the Markovian evolution of the detectors that interacts with the background field. In other words, even with the entanglement generation, the Unruh effect solely plays the role of an environment decoherence [21].

Beyond entanglement among multipartite quantum systems, it is intriguing to exploit the possibility of extracting other measures on the quantumness of quantum vacuum. Earlier study [22] has observed that the extraction of quantum coherence is possible even for single UDW detector. For certain initial energy of the background field and the duration of the interaction, a detector moving with uniform acceleration can extract a larger amount of coherence from field states compared to a detector at rest. In other words, the rate of coherence loss may sometimes become slower for a moving detector than for a detector at rest. Similarly, for a spatially extended UDW detector, coherence harvesting and swelling were also observed [23]. Such extracted coherence was shown to be catalytic, meaning the same amount of coherence can be repeatedly extracted.

In this paper, we concern with the quantum coherence extraction for a system of multi-UDW detectors via a coupling to a massless scalar field in 3 + 1 Minkowski spacetime. In particular, we exploit the quantum coherence of a combined system with two UDW detectors, both under a uniform acceleration. Motivated by the entanglement generation for multi-UDW detectors [15], we anticipate that quantum coherence can also be generated after the system approaches its equilibrium, even with two detectors initially in an incoherent state. Starting from general initial states, however, we are interested in the necessary conditions that detectors should satisfy to ensure the coherence extraction, i.e., the residual coherence at equilibrium is larger than the initial quantum coherence of detectors.

While quantum fluctuation of the background field plays a role of an environment, we could treat two-detectors as an open quantum system [24], whose reduced density matrix evolves via a Lindblad-type master equation derived by tracing out the environmental degrees of
freedom. For the detectors with uniform acceleration $a$, this formalism may lead us to a naive expectation that just as illustrated for entanglement in various scenarios [25, 26], Unruh effect or environmental decoherence should also result in a monotonous decay of quantum interference in two-detectors system. However, we find that quantum coherence indeed exhibits a striking revival phenomenon that for a certain choice of detectors’ initial state, coherence measure [5] will reduce to zero at first then grow up and eventually approach to an asymptotic value, while Unruh temperature $T_U = a/2\pi$ keep growing during the process.

As an important physical resource, we expect the above coherence revival to be verified in some practical quantum process by providing enhanced performance. In metrological tasks, nonclassicality rather than entanglement is necessary to achieve quantum advantages [27, 28]. As a particular nonclassicality which is more general than entanglement, quantum coherence is intimately related to the quantum Fisher information (QFI) [29, 30], a key quantity bounding the ultimate precision of the estimation cost of any state is determined by its QFI [31]. As an important physical resource, we expect the above coherence revival to be verified in some practical quantum process by providing enhanced performance. In metrological tasks, nonclassicality rather than entanglement is necessary to achieve quantum advantages [27, 28].

Modeling each detector as a two-level atom, the total Hamiltonian of the combined system of detectors and background field is

$$H = \frac{\omega}{2} \Sigma_3 + H_\Phi + \mu H_1,$$

where $\omega$ is the energy level spacing of the atom and $\Sigma_3$ is one of symmetrized bipartite operators $\Sigma_i \equiv \sigma_i^{(A)} \otimes \mathbf{1}^{(B)} + \mathbf{1}^{(A)} \otimes \sigma_i^{(B)}$, defined by Pauli matrices $\{\sigma_i^{(\alpha)} \mid i = 1, 2, 3\}$ with superscript $\{\alpha = A, B\}$ labeling distinct atoms. $H_\Phi$ is the Hamiltonian of free massless scalar fields $\Phi(t, x)$ satisfying standard Klein-Gordon equation. The interaction Hamiltonian can be written in a dipole form between atoms and fluctuating field bath [33], say $H_I = (\sigma_2^{(A)} \otimes \mathbf{1}^{(B)}) \Phi(t, x_A) + (\mathbf{1}^{(A)} \otimes \sigma_2^{(B)}) \Phi(t, x_B)$, where $x_A$ and $x_B$ label the space positions of two atoms at same time $t$.

Assuming a weak coupling between two-atom system and environment ($\mu \ll 1$), such that the initial state of combined system is approximated as $\rho_{\text{tot}}(0) = \rho_{AB}(0) \otimes |0\rangle \langle 0|$, where $\rho_{AB}(0)$ is the initial state of the detectors and $|0\rangle$ is the field vacuum. Obviously, the dynamics of $\rho_{\text{tot}}$ should be governed by a unitary evolution via von Neumann equation $\dot{\rho}_{\text{tot}}(\tau) = -i[H, \rho_{\text{tot}}(\tau)]$, where $\tau$ is the proper time of the atom. Whenever the typical time scale of the environment is much smaller than that of the detectors [24], we can further assume the detectors undergoing a Markovian evolution2. The reduced dynamics of the detectors can be obtained by integrating over the background field degrees from the $\rho_{\text{tot}}(\tau)$, driven by a quantum dynamical semigroup of completely positive map. Eventually, the open system dynamics should be governed by a Lindblad-type master equation [34, 35]

$$\frac{\partial \rho_{AB}(\tau)}{\partial \tau} = -i[H_{\text{eff}}, \rho_{AB}(\tau)] + \mathcal{L} [\rho_{AB}(\tau)],$$

where

$$\mathcal{L} [\rho] = \sum_{i,j=1,2,3}^{A,B} \sum_{\alpha, \beta = A, B} \frac{C^{(\alpha \beta)}_{ij}}{2} \left[ 2\sigma_j^{(\beta)} \rho_{AB} \sigma_i^{(\alpha)} - \{\sigma_i^{(\alpha)} \sigma_j^{(\beta)} \}, \rho_{AB} \right],$$

representing a dissipative evolution attributed to the interaction between the detectors and the external field. The Kossakowski matrix $C^{(\alpha \beta)}_{ij}$ can be explicitly resolved. After introducing the Wightman function of scalar field $G^{(\alpha \beta)}(x, x') = \langle 0 | \Phi(t, x^{(\alpha)}) \Phi (t', x^{(\beta)}) | 0 \rangle$, its Fourier transform is

$$G^{(\alpha \beta)}(\lambda) = \int_{-\infty}^{\infty} d\tau \ e^{i\lambda \tau} G^{(\alpha \beta)}(\tau),$$

The reliability of Markovian approximation is a delicate issue. However, for weak coupling, possible non-Markovian correction may be robust only at early-time evolution [32]. Since only the asymptotic equilibrium state of the detector is concerned, we can safely neglect the difference between Markovian and non-Markovian solutions.

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1 For instance, in the context of quantum thermodynamics, coherence cost of any state is determined by its QFI [31].
where the superscript $\alpha, \beta = \{A, B\}$ labeling distinct atoms. For two-atom system, one can easily find that $G^{(AA)} = G^{(BB)}$ and $G^{(AB)} = G^{(BA)}$, which lead to $G^{(AA)} = G^{(BB)} \equiv G_0$ and $G^{(AB)} = G^{(BA)}$.

The master equation (2) enables us to describe the asymptotic equilibrium states of detectors at large times, which are determined by the competition between environment dissipation and quantum correlations generated through the Markovian evolution of detectors [15]. For two-atom system, the initial interatomic separation $L = \|x^{(A)} - x^{(B)}\|$ is a control parameter of correlation generation. Thus, the Kossakowski matrices now become distance-dependent since in general $G^{(AB)} = G^{(BA)} \equiv G(\omega, L) = G_0(\omega)f(\omega, L)$ for two separated atoms [33], where $f(\omega, L)$ is an even function of frequency $\omega$. One would not be surprised that the correlation generation between atoms would be more effective for smaller distance $L$, and becomes impossible for an infinitely large separation. In fact, it was shown [25] that there always exists a proper $L$, below which the generated correlations can persist under environment dissipation. Therefore, we can concisely fix a small interatomic separation and only be concerned about the influence of environment decoherence on the equilibrium states of detectors. In such a situation, all the Kossakowski matrices become equal $C_{ij}^{AA} = C_{ij}^{BB} = C_{ij}^{AB} = C_{ij}^{BA} \equiv C_{ij}$ [36], where the coefficients $C_{ij}$ are determined by a decomposition

$$C_{ij} = \frac{\gamma_+ + \delta_{ij} - \frac{1}{2}\gamma_-}{2} - \frac{\gamma_-}{2} \epsilon_{ijk} \delta_{3k} + \gamma_0 \delta_{3i} \delta_{3j},$$

where

$$\gamma_\pm = \mathcal{G}(\omega) \pm \mathcal{G}(-\omega), \quad \gamma_0 = \mathcal{G}(0) - \gamma_+ / 2.$$  \hspace{1cm} (6)

Moreover, the interaction with the external scalar field would induce a Lamb shift contribution for the detector effective Hamiltonian $\mathcal{H}_{\text{eff}} = \frac{1}{2} \omega \sigma_3$, in terms of a renormalized frequency $\tilde{\omega} = \omega + i\gamma \mathcal{K}(\omega) \equiv \mathcal{K}(\tau + i\beta)$, where $\beta = 1/\tau_U = 2\pi/\alpha$ is recognized. Translating it into frequency space, one has

$$\mathcal{G}(\lambda) = e^{i\beta \omega} \mathcal{G}(-\lambda).$$  \hspace{1cm} (7)

Using translation invariance $\langle 0 | \Phi(x(0)) \Phi(x(\tau)) | 0 \rangle = \langle 0 | \Phi(0) | 0 \rangle$, after some algebras, we find (6) can be resolved as

$$\gamma_+ = \int_{-\infty}^{\infty} d\tau e^{i\lambda \tau} \langle 0 | \Phi(\tau) \Phi(0) | 0 \rangle = (1 + e^{-i\gamma \omega}) \mathcal{G}(\omega),$$
$$\gamma_- = \int_{-\infty}^{\infty} d\tau e^{i\lambda \tau} \langle 0 | [\Phi(\tau), \Phi(0)] | 0 \rangle = (1 - e^{-i\gamma \omega}) \mathcal{G}(\omega),$$

holding true for generic interacting fields. For later use, we also introduce the ratio

$$\gamma = \gamma_- / \gamma_+ = \frac{1 - e^{-i\gamma \omega}}{1 + e^{-i\gamma \omega}} = \tanh(\beta \omega / 2),$$

which depends solely on the Unruh temperature $T_U$ due to the frequency KMS condition (7).

By expressing the reduced density matrix of the two-system as

$$\rho_{AB}(\tau) = \frac{1}{4} \left[ 1^A \otimes 1^B + \sum_{i=1}^{3} \rho_i \Sigma_i + \sum_{i,j=1}^{3} \rho_{ij} \sigma_i^A \otimes \sigma_j^B \right],$$

and inserting it back into (2), one can derive [17] the reduced density matrix of two-UDW detectors at equilibrium in an X-type structure

$$\rho_{AB}(\omega, \beta, \Delta_0) = \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & C & D & 0 \\ 0 & D & C & 0 \\ 0 & 0 & 0 & B \end{pmatrix},$$

where

$$A = \frac{(3 + \Delta_0)(\gamma - 1)^2}{4(3 + \gamma)^2}, \quad B = \frac{(3 + \Delta_0)(\gamma + 1)^2}{4(3 + \gamma)^2},$$
$$C = \frac{3 - \Delta_0 - (\Delta_0 + 1)\gamma^2}{4(3 + \gamma)^2}, \quad D = \frac{\Delta_0 - \gamma^2}{2(3 + \gamma)^2}.$$  \hspace{1cm} (12)

We observe that the final equilibrium state of the two-detectors system now depends on the ratio $\gamma$ characterizing the thermal nature of the Unruh effect, as well as the choices of initial state encoded in $\Delta_0 = \sum_i \text{Tr}[\rho_{AB}(0)\sigma_i^A \otimes \sigma_j^B]$, a dimensionless constant of motion satisfying $-3 \leq \Delta_0 \leq 1$ to keep $\rho_{AB}(0)$ positive.

### III. QUANTUM COHERENCE AND ITS REVIVAL

#### III.1. Quantum coherence monotone

We first review some elementary concepts concerning coherence measures. The notion of coherence admitted in this paper is identified as the one in [5], where the following set of axioms should be satisfied to specify a reasonable measure of quantum coherence.

For a given fixed basis $\{ |i\}$, the set of incoherent states $\mathcal{I}$ is the set of quantum states with diagonal density matrices with respect to this basis. Incoherent completely positive and trace-preserving maps (ICPTP) are maps that map every incoherent state to another incoherent state. Given this, we say that $\mathcal{C}$ is a proper measure of quantum coherence if it satisfies the following properties:

- (C1). $\mathcal{C}(\rho) \geq 0$ for any quantum state $\rho$ and equality holds iff $\rho \in \mathcal{I}$.
- (C2a). The measure is non-increasing under a ICPTP map $\Phi$, i.e., $\mathcal{C}(\rho) \geq \mathcal{C}(\Phi(\rho))$.
- (C2b). Monotonicity for average coherence under selective outcomes of ICPTP: $\mathcal{C}(\rho) \geq \sum_\alpha \rho_\alpha \mathcal{C}(\rho_\alpha)$, where $\rho_\alpha = K_{\alpha} \rho K_{\alpha}^\dagger / \text{Tr}[K_{\alpha} \rho K_{\alpha}^\dagger]$ for all $K_{\alpha}$ with $\sum_\alpha K_{\alpha}^\dagger K_{\alpha} = 1$ and $K_{\alpha} K_{\beta} \subseteq \mathcal{I}$.
- (C3). Convexity, i.e. $\mathcal{C}(\rho + (1-\lambda)\sigma) \geq \mathcal{C}(\rho + (1-\lambda)\sigma)$, for any density matrix $\rho$ and $\sigma$ with $0 \leq \lambda \leq 1$. 


A general distance-based coherence quantifier can be found [13], which satisfies all the conditions mentioned above, regarding the minimal distance between the target state and a given incoherent state. Using quantum relative entropy

\[ S(\rho||\sigma) = \text{Tr}[\rho \log \rho] - \text{Tr}[\rho \log \sigma], \]

one coherence monotone is \(\min_{\sigma \in \mathcal{Z}} S(\rho||\sigma)\) which can be recasted into [5]

\[ C_{R.E.}(\rho) = S(\rho_{\text{diag}}) - S(\rho), \]

where \(S(\rho) = -\text{Tr}(\rho \log \rho)\) is the von Neumann entropy for the state \(\rho = \sum_{ij} \rho_{ij} |i\rangle \langle j|\), and \(\rho_{\text{diag}} = \sum_{i} \rho_{ii} |i\rangle \langle i|\) is derived from a dephasing operation on a density matrix.

In the next section, we will investigate the above coherence monotone for a two-UDW detector system. We should remark that we do not use another well-known \(l_1\)-norm monotone as authors of [22, 23] did, because monotone (14) is a more general measure even for the infinite-dimensional system to avoid potential divergent [37]. Although, in the present case, these two monotones are equivalent, we remain using (14) for possible extension in future work.

### III.2. Coherence revival

In our context, by diagonalizing the density matrix (11), we obtain the related eigenvalues

\[
\lambda_1 = \frac{(1 - \gamma)^2(3 + \Delta_0)}{4(3 + \gamma^2)}, \quad \lambda_2 = \frac{(1 + \gamma)^2(3 + \Delta_0)}{4(3 + \gamma^2)},
\]

\[
\lambda_3 = \frac{(1 - \gamma^2)(3 + \Delta_0)}{3 + \gamma^2}, \quad \lambda_4 = \frac{1 - \Delta_0}{4},
\]

and eigenvectors

\[
|\lambda_1\rangle = |00\rangle, \quad |\lambda_2\rangle = |11\rangle,
\]

\[
|\lambda_3\rangle = \frac{|10\rangle - |01\rangle}{\sqrt{2}}, \quad |\lambda_4\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}.
\]

We straightforwardly calculate the relative entropy as

\[ C_{R.E.}(\omega, T_U, \Delta_0) = -2C \log C + \lambda_3 \log \lambda_3 + \lambda_4 \log \lambda_4, \]

which depends on the energy spacing of atom \(\omega\), the initial state choice of detectors encoded in \(\Delta_0\), as well as Unruh temperature \(T_U\) proportional to the detectors' acceleration.

Since \(\omega\) and \(T_U\) are combined in \(\gamma\) (9), for simplicity, we specify \(\omega = 1\) and depict the related quantum coherence in Fig.1, as a function of initial state choice \(\Delta_0\) and Unruh temperature \(T_U\). Since the Unruh effect, in general, is recognized as an environment decoherence [38], we may naively expect that for arbitrary fixed initial state, quantum coherence \(C_{R.E.}\) would be a monotonously decreasing function of Unruh temperature \(T_U\). However, we observe directly from Fig.1 that the quantum coherence is a non-monotonous function of \(T_U\) corresponding to a class of initial states. In particular, there is a revival of coherence, i.e., as Unruh temperature \(T_U\) grows, the quantum coherence degrades to zero at first and then increases to a nonvanishing asymptotic value \(C_{\text{asymp}}\) obtained from (17) by taking the infinite acceleration limit \(\gamma \to 0\).

In the upper line of Fig.2, we plot the quantum coherence as a function of initial state choice \(\Delta_0\) and Unruh temperature \(T_U\), for different values of the detector's energy spacing. By numerical analysis, we find that above coherence revival occurs when the initial state satisfying \(\Delta_0 \in (0, 1]\) bounded by the white solid curves in Fig.2(a)-2(c). We also note that the revival of coherence is suppressed for the larger energy spacing of a UDW detector.

A striking difference between quantum coherence and entanglement generated under the Unruh effect [15] needs to be highlighted. It has been shown that the survival of entanglement for the equilibrium state of two accelerating detectors results from competition between the Unruh decoherence and entanglement enhancement from Markovian dynamics. In other words, during the process of entanglement generation, the Unruh effect solely plays a role in decoherence; therefore, as the Unruh temperature grows, the survived entanglement degrades. To see this, we directly calculate the negativity for the final equilibrium state of two accelerating UDW detectors, a measure of distillable entanglement. It is defined by

\[ \mathcal{E}_{\text{neg}}(\rho) = \frac{1}{2} \sum_i (|\lambda_i| - \lambda_i) = -\sum_{\lambda_i < 0} \lambda_i, \]

where \(\lambda_i\) are the negative eigenvalues of partial transposed density matrix. The value of negativity ranges from 0 for a separable state, to 0.5 for a maximally entangled state.
Form (11), we can straightforwardly obtain [17]

$$\mathcal{E}_{\text{neg}} = \max \left\{ \frac{\sqrt{(\Delta_0 - \gamma^2)^2 + \gamma^2 (3 + \Delta_0)^2}}{2 (3 + \gamma^2)} - \frac{\lambda_1 + \lambda_2}{2}, 0 \right\},$$

(18)

which reaches maximum 0.5 at $\Delta_0 = -3$. In the lower line of Fig.2, we plot the negativity for the final equilibrium state of the two-detector system for $\omega = 1, 3, 5$. For varying initial state preparation, the negativity (18) is vanishing at $\Delta_0 = \frac{-\gamma^2 - 3}{3 - \gamma}$, presented by the red solid curve in Fig.2(d)-2(f). It is obvious that entanglement degrades monotonously with growing Unruh temperature, i.e., no revival of entanglement can happen for any initial state preparation. This clearly demonstrates the distinction between entanglement and coherence [11].

### III.3. A concrete example

We illustrate our result in a concrete example. Choosing the initial state of two-detector in a product form

$$\rho_{\text{initial}} = \rho_A(0) \otimes \rho_B(0).$$

(19)

The state of each detector can be written in Bloch form

$$\rho_A(0) = \frac{1}{2} (1 + n \cdot \sigma), \quad \rho_B(0) = \frac{1}{2} (1 + m \cdot \sigma),$$

(20)

where $n$ and $m$ are two unit Bloch vectors. Without loss of generality, taking $n = (0, 0, 1)$ and $m = (0, \sin \theta, \cos \theta)$, we have $\Delta_0 = n \cdot m = \cos \theta$ where $\theta \in [0, \pi]$ is angle between two vectors giving $\Delta_0 \in [-1, 1]$.

The quantum coherence of the initial state (19) is

$$C_{\text{R.E.}}(0) = H_{\text{binary}} \left( \frac{\cos \theta + 1}{2} \right),$$

(21)

where $H_{\text{binary}}(x) \equiv -x \log x - (1-x) \log(1-x)$ is binary entropy of variable $x$.

After the Markovian evolution, the final equilibrium state of two detectors possesses quantum coherence (17). Defining the change between the final state coherence and those of the detector initially as

$$\delta C_{\text{R.E.}} = C_{\text{R.E.}}(\omega, T_U, \theta) - C_{\text{R.E.}}(0).$$

(22)

Once $\delta C_{\text{R.E.}} > 0$ (i.e., more coherence for detectors’ final state than it was had initially), the quantum coherence has been generated through the interaction between the detectors’ system and the background field. By designed operation protocol, one may expect such coherence increment to be properly extracted.

Numerically, we depict $\delta C_{\text{R.E.}}$ in Fig.3. Starting from an incoherent initial state with $\theta = 0, \pi$, one has $C_{\text{R.E.}}(0) = 0$ but a nonvanishing $\delta C_{\text{R.E.}} > 0$ for two-detector at final equilibrium, which indicates a generation of quantum coherence through its evolution. For general initial states, we know that a coherence revival can occur with choice $\theta \in [0, \pi/2]$ (i.e., $\Delta_0 \in (0, 1]$) for sufficiently large Unruh temperature. However, by numerically evaluating, we recognize only regions bounded by the red solid lines in Fig.3 can have $\delta C_{\text{R.E.}} > 0$, referring to a coherence extraction for sufficiently large $T_U$, i.e., the coherence of two-detector’s final state is enhanced compared to its initial state. On the other hand, the region in Fig.3 without coherence extraction indicates that the generated coherence via Markovian evolution of the detectors cannot counteract the effect of Unruh decoherence.

### IV. METROLOGICAL ADVANTAGE

In general, quantum coherence is recognized as a physical resource that can be utilized to improve the performance of various quantum information tasks. One particular example is the close relation between non-classical coherence and quantum metrological tasks, where the quantumness encoded in coherence can be used to enhance the accuracy of estimation of extremely sensitive parameters, e.g., Unruh temperature, spacetime curvature in relativistic context [39–44]. This section aims to examine the metrological advantage of coherence revival found before. In particular, we will evaluate the QFI for the two-UDW detector, with Unruh temperature as a parameter chosen to be optimally estimated. We will show that the revival coherence corresponds to the enhanced QFI and, therefore, may provide a significant quantum advantage in metrological tasks.

In any quantum metrological task, QFI gives a lower bound to the mean-square error in the parameter estimation via the Cramér–Rao inequality $\text{Var}(\hat{\lambda}) \geq [\mathcal{N} \mathcal{F}_X]^{-1}$, where $\mathcal{N}$ is the number of repeated measurements. In terms of the symmetric logarithmic derivative (SLD) operator $L_X$, which satisfies $\partial_X \rho = \frac{i}{2} \{\rho, L_X\}$, the QFI is defined as $\mathcal{F}_X = \text{Tr}[\rho(X)L_X^2]$. For a diagonalized density matrix like $\rho = \sum_i \lambda_i |\psi_i\rangle \langle \psi_i|$, the related QFI can be further written as [45, 46]

$$\mathcal{F}_X = \sum_{i=\pm} \sum_{i\neq j=\pm} \frac{(\partial_X \lambda_i)^2}{\lambda_i} + \sum_{i\neq j=\pm} \frac{2(\lambda_i - \lambda_j)^2}{\lambda_i + \lambda_j} |\langle \psi_i | \partial_X \psi_j \rangle|^2,$$

(23)

where the summations run over all eigenvalues satisfying $\lambda_i \neq 0$ and $\lambda_i + \lambda_j \neq 0$.

We choose to estimate Unruh temperature $T_U$. The related QFI can be straightforwardly calculated from (23) by substituting the eigenvalues (15) of the diagonalized density matrix of the detectors system. According to different preparations of initial state encoded in $\Delta_0 \in [-3, 1]$, we come to two classes of QFI:

(i) $\Delta_0 = -3$, only one nonvanishing eigenvalue $\lambda_4 = 1$, which gives $\mathcal{F}_T = 0$;

(ii) $\Delta_0 \in (-3, 1]$, all four eigenvalues (15) are nonvan-
FIG. 2. (Upper) The quantum coherence of the final equilibrium state of two-detectors system as a function of initial state preparation encoded by $\Delta_0$ and Unruh temperature $T_U$. We demonstrate the coherence monotone $C_R.E.$ for fixed energy spacing of detector with $\omega = 1, 3, 5$. In each case, coherence revival can exist for the initial states with $\Delta_0 \in (0, 1]$. (Lower) Entanglement measured by negativity is plotted as a function of initial state preparation encoded by $\Delta_0$ and Unruh temperature $T_U$. For any fixed initial states, no revival of entanglement with respect to Unruh temperature can occur.

FIG. 3. The quantum coherence extraction for two UDW detectors (fixed $\omega = 1$). For initial states preparation with $\theta \in [0, \pi/2]$, coherence revival occurs for sufficiently large Unruh temperature. Further, for initial states with $\theta$ chosen inside the regions bounded by the red solid lines, detectors can possess more quantum coherence at equilibrium than their initial state, which means a coherence extraction can happen.

Since $\partial_T \lambda_4 = 0$, we have the QFI

$$F_T = \sum_{i=1,2,3} \frac{(\partial_T \lambda_i)^2}{\lambda_i} = \frac{\gamma^6 - 9\gamma^4 - \gamma^2 + 9}{\gamma^6 + 9\gamma^4 + 27\gamma^2 + 27} \cdot \omega^2(3 + \Delta_0).$$

With specific energy spacing of detector $\omega = 1, 3, 5$, we depict the QFI in Fig.4 as a function of Unruh temperature and initial state choice $\Delta_0$. The first thing that steps out is that similar to quantum coherence rather than entanglement, with fixed initial state choice, the QFI of the two-UDW detector is not a monotonous function of Unruh temperature. We note that a local peak exists as Unruh temperature grows, which means the largest precision of estimation on $T_U$ can be achieved at relatively low acceleration. Such non-monotonicity of the QFI with respect to Unruh temperature was also found in the single-detector case recently [47].

Comparing Fig.4 and the upper line of Fig.2, we find further similarity between quantum coherence and the QFI. It is obvious that the maximal value of the QFI peak can be achieved for initial state preparation with $\Delta_0 = 1$, i.e., as approaching $\Delta_0 = 1$, the increasing of QFI corresponds to enhanced quantum coherence revival. For larger $\omega$, the coherence revival is heavily suppressed, and correspondingly, we find that the maximal value of the QFI peak also decreases conspicuously.

In general, the quantumness of a physical system can be utilized as a physical resource in various quantum pro-
cesses. Although both quantum coherence and entanglement can serve as a resource, due to the complementary non-monotonous behaviors shared by coherence and QFI under growing $T_U$, we suggest that it is the coherence rather than the entanglement dominates the QFI’s local peak and the maximal value. In other words, with coherence revival, one can significantly improve the precision in estimating Unruh temperature, thus endowing quantum coherence a metrological advantage.

V. SUMMARY AND DISCUSSIONS

In this paper, we explore the quantum coherence generation between two accelerating Unruh-DeWitt detectors. We find coherence can be generated through the Markovian detector evolution, just like entanglement dynamics in the same scenario. While the monotonous generation of quantum entanglement is well-known, we find that for certain choices of detectors’ initial state, under growing Unruh temperature, coherence measure may have non-monotonous behavior. We verify such coherence revival by inspecting its advantage in enhancing the QFI of Unruh temperature estimation. As the maximal QFI bounds the accuracy of quantum measurement, we conclude that the extracted coherence may be utilized as a physical resource in quantum metrology.

It seems curious at first glance that coherence monotone (14) and quantum entanglement have distinctive behaviors under Unruh decoherence, as many works have indicated [13] that both of them are physical resources that are interchangeable with each other. However, we note that, unlike entanglement, coherence is a basis-dependent concept. If we adopt the viewpoint that a preferred basis of coherence only emerges by the environment (e.g., the energy conservation makes the energy eigenbases naturally selected in the paper as preferred bases), with varying Unruh temperature, we would better cautiously refer to the related background states as the different environment. In this context, the coherence revival could be a demonstration of so-called “einselction” [48] in a relativistic framework.

On the other hand, the intimate relation between entanglement dynamics and coherence generation of multi-UDW detectors may be reexamined from a thermodynamic perspective. In particular, for an open quantum system, its evolution irreversibility can be characterized by the entropy production arising from non-equilibrium quantum processes ascribed to system correlations. It has been recently shown [49] that the entropy production of an open quantum system can further be interpreted as an interplay between population dynamics and coherence dynamics. Extending this analysis to the context of Unruh decoherence may shed new light on our understanding of the quantum nature of the Unruh effect. We will exploit these interesting topics in the future.

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