Improved GM-SVR combined prediction model of pavement skid resistance condition based on finite data

Xing Chen\textsuperscript{1a}, JinLiang Xu\textsuperscript{1b*}, RiShuang Sun\textsuperscript{2c}

\textsuperscript{1}School of Highway, Chang'an University, Xi'an, Shaanxi, 710064, China
\textsuperscript{2}Shandong Provincial Communications Planning and Design Institute Group Co., Ltd, Jinan, Shandong, 250031, China
\textsuperscript{a}370012719@qq.com, b*xujinliang@chd.edu.cn, c*handsomepb@139.com

Abstract—The pavement skid resistance condition of expressway has an important impact on driving safety. With the increase of service life, the pavement skid resistance condition decreases year by year, and an accurate prediction model is of great significance to improve the level of traffic safety. Firstly, the improved GM (1, 1) model and support vector machine regression model are established, and then the two are combined by entropy weight method to obtain the GM-SVR prediction model. In this paper, the skidding resistance index (SRI) of a certain section of Lezi expressway for limited years (2016, 2018 and 2020) is used as the basic data to predict the SRI values in 2021 and 2022. In order to verify the accuracy of the model, the pavement condition index (PCI) of the same road section in 2013, 2015 and 2017 are used as the basic data to predict the PCI values in 2018 and 2019; Taking MAE, MSE and MAPE as test indicators, the predicted values and measured values in 2018 and 2019 are compared and analyzed to test the prediction accuracy.

1. Introduction

According to statistics, there were 247,646 traffic accidents in China in 2019, including 215,009 motor vehicle traffic accidents\textsuperscript{[1]}, accounting for 86.8% of the total accidents. Serious traffic safety problems have been widely concerned by the whole society. The decline of pavement skid resistance condition of expressway is likely to cause vehicle slip and long braking distance, which threatens driving safety and has attracted the attention of scholars at home and abroad. In view of this phenomenon, scholars have carried out research on pavement durability and preventive maintenance, among which the research on pavement performance prediction model is the focus of scholars. Scientific and reasonable prediction model is helpful for relevant departments to master the change law of pavement skid resistance condition, and take measures in advance for sections with serious decline of pavement skid resistance condition to prevent accidents. Therefore, the establishment of pavement skid resistance condition prediction model is of great significance to the formulation of dynamic control measures, reduce the occurrence of traffic accidents and improve the level of road traffic safety.

At present, the prediction models of pavement performance at home and abroad mainly include deterministic model, probabilistic model, bionic model and grey prediction model. Deterministic models, such as the PPI equation of asphalt pavement performance decay proposed by Sun Lijun\textsuperscript{[2]} mainly use the initial performance index of asphalt pavement and road service life. It is very simple and convenient to use, but it can not make rational use of dynamic data and can only be used to predict short-term data. Probabilistic model can express the fuzziness of pavement performance degradation under the action of
various factors. Markov prediction model is a commonly used probabilistic model. Dong Shi\cite{3} classified the pavement structure and traffic grade, calculated the Markov state transition matrix of pavement decay according to the principle of panel data, and predicted the pavement performance. The most widely used bionic model is neural network model. Zhang Kaixing\cite{4} applied BP neural network algorithm to pavement performance prediction in Guangdong Province and proposed a comprehensive performance evaluation model considering pavement structure strength index; Bo Xu\cite{5} predicted the temperature of asphalt pavement in cold areas through the improved BP neural network model; Angela J. Haddad\cite{6} constructed rutting prediction model by extracting data from long-term pavement neural database and using deep neural network learning.

Although neural network has the ability of ultra-high speed and fault tolerance, the calculation is cumbersome, and a large amount of data is needed for model training. At present, the maintenance cycle of expressway in China is short, and the pavement needs to be maintained in three or four years, so it is difficult to obtain the data of pavement skid resistance condition in long-term natural state. Therefore, the research on prediction method under limited pavement data has higher practical value. Grey prediction model is widely used in pavement performance prediction because of its advantages of less data, high prediction accuracy and simple calculation method. Wang\cite{7} applied the model to pavement flatness prediction; Wang Zhixiang\cite{8} distributed the weights of each measured sub index based on the hierarchical variable weight method, and established the grey GM (1, 1) prediction model of asphalt pavement technical condition index; Tang\cite{9} et al used the GM (1, 1) model to predict the monthly attenuation model of pavement condition index and pavement quality index, which has high prediction accuracy. Yang Guohua\cite{10} used variable weight to construct the background value, introduced the fitting residual to construct the polynomial into the model, and established the improved GM (1, 1) model; Yu Xiaohe\cite{11} improved the dependence of the grey model on the initial value and predicted the pavement performance of asphalt pavement.

In order to overcome the shortcomings of single model, many scholars have studied the combined prediction model. Wang XuanCang\cite{12} established a combined prediction model for the performance of asphalt pavement in combination with grey correlation analysis and support vector machine, which has higher accuracy than the traditional grey model; Li Hailian\cite{13} established a prediction model based on IFA-SVM combined with support vector machine theory and improved firefly algorithm, and predicted multiple indices of pavement performance; Wang Hainian\cite{14} combined American PME model, grey prediction model and mechanical empirical method to predict rutting; Le Tianhan\cite{15} predicted Subgrade Frost Heave based on non isochronous GM (1, 1) prediction model and support vector machine for residual correction. Yi Zhang predicted wind power generation based on the combined model of PSO-SVR and grey prediction\cite{16}. The above research results show that compared with a single prediction model, the combined model has better applicability and accuracy.

Considering the existing pavement performance prediction models at home and abroad, this paper first establishes the improved gray model GM (1, 1) and support vector machine regression model, and then calculates the weight of the two by entropy weight method to obtain the GM-SVR prediction model. Then, taking the skidding resistance index (SRI) of a certain section of Lezi expressway for limited years (2016, 2018 and 2020) as the basic data, the SRI values in 2021 and 2022 are predicted by GM-SVR model. In order to verify the accuracy of the model, the pavement condition index (PCI) of the same road section in 2013, 2015 and 2017 is used as the basic data to predict the PCI values in 2018 and 2019, and MAE, MSE and MAPE are used as test indicators to test the prediction accuracy of the score analysis between the predicted values and the measured values in 2018 and 2019. The combined model combines the applicability of the two methods for small samples\cite{17}, and realizes the high-precision prediction of pavement skid resistance condition with limited data.

2. Basic principle of model

2.1 Grey prediction model GM (1, 1)

Grey prediction is a theory for predicting systems containing uncertain factors\cite{18}. The grey prediction
model identifies the difference of development trend between system factors, generates and processes the original data to find the law of system change, generates a strong regular data sequence, and then establishes the corresponding differential equation model to predict the future development trend of things.

This grey prediction model is like GM (1, 1), GM (1, 1 / sin)\(^{[19]}\), but the most commonly used is GM (1, 1) model. Its basic idea is to accumulate the original data to generate a new sequence so as to weaken the randomness of the original data. Then the differential equation model of the generated and changed sequence is established. The establishment of grey model mainly includes the following steps:

### 2.1.1 Level ratio test and data transformation processing

Assuming that the original data sequence is \( X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\} \), the level ratio test shall be conducted on the original data sequence before modeling so as to judge the feasibility of modeling. The calculation method of inspection parameter \( \sigma(k) \) is shown in (1). If \( \sigma(k) \) meets \( \sigma(k) \in (e^{n+1}, e^{n+1}) \), the original data can be used directly; Otherwise, the original data shall be translated.

\[
\sigma(k) = \frac{x^{(0)}(k-1)}{x^{(0)}(k)}, k = 1, 2, 3, \ldots, n
\] (1)

When the level ratio test meets the requirements, the original data is accumulated once to generate (1-AGO) sequence:

\[
X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\} \tag{2}
\]

\[
x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 1, \ldots, n
\] (3)

### 2.1.2 Establishment of Grey Differential Equation and Solving Parameters

The expression form of the GM (1, 1) model is:

\[
x^{(0)}(k) + az^{(1)}(k) = b \tag{4}
\]

The whitening equation of this model is:

\[
\frac{dx^{(i)}(t)}{dt} + ax^{(i)}(t) = b \tag{5}
\]

In (4), \( a \) is the development coefficient, which determines the fitting ability of the algorithm; \( b \) is the endogenous control grey number, which determines the prediction ability of the algorithm\(^{[20]}\); \( z^{(1)}(k) \) is each element in the generated background value sequence, representing the approximate trapezoidal area surrounded by \( x^{(1)}(k) \) and \( x^{(1)}(k-1) \), the \( z^{(1)}(k) \) of traditional GM (1, 1) is obtained according to (6):

\[
z^{(1)}(k) = \frac{1}{2} (x^{(1)}(k-1) + x^{(1)}(k)), k = 2, 3, \ldots, n
\] (6)

Since the background value is the key factor affecting the accuracy of the grey model, when (6) is used for calculation, large errors will occur at the time of large fluctuation in the time series interval. Since the cumulative sequence approximately satisfies the exponential function relationship, the following optimization formula can be used for calculation\(^{[15]}\):

\[
Z^{1}(t_i) = \frac{X^{1}(t_i) - X^{1}(t_{i-1})}{\ln X^{1}(t_i) - \ln X^{1}(t_{i-1})} \tag{7}
\]

The values of \( a \) and \( b \) obtained by the least square method are:
\[ \hat{a} = [a, b]^T = (B^T B)^{-1} B^T Y \] (8)

where:

\[
B = \begin{pmatrix}
-z^{(1)}(2) & 1 \\
-z^{(1)}(3) & 1 \\
\vdots & \vdots \\
-z^{(1)}(n) & 1
\end{pmatrix},
Y = \begin{pmatrix}
x^{(0)}(2) \\
x^{(0)}(3) \\
\vdots \\
x^{(0)}(n)
\end{pmatrix}
\]

By substituting \(a\) and \(b\) into (5) to solve the differential equation and reducing the results, the final predicted time response equation (9) can be obtained as follows:

\[ \hat{x}(k+1) = (x^{(0)}(1) - \frac{b}{a})(1-e^a)e^{-ak} \] (9)

### 2.1.3 Model accuracy test

There are three commonly used model test methods: residual test, correlation test and a posteriori test. Because the posterior error test method can directly judge the advantages and disadvantages of data fitting without comparing with other data, this paper selects the posterior error test method to test the accuracy of the model[21]. The test method evaluates the accuracy of the model through the two indices variance ratio \(C\) and small residual probability \(P\). The test reference table is shown in Table 1. The expressions of \(C\) and \(P\) are shown in (10) and (11).

\[ C = \frac{S_2}{S_1} \] (10)

\[ p = P\{\bar{\Delta}(0) - \bar{\Delta}(0) < 0.6745S_1\} \] (11)

Where \(S_1\) is the standard deviation of the original sequence, \(S_2\) is the standard deviation of the residual, \(\Delta(0)(i)\) is the fitted residual, and \(\bar{\Delta}(0)\) is the mean value of the residual. The expressions are as follows:

\[ S_1 = \sqrt{\frac{\sum_{i=1}^{n}[x^{(0)}(i) - \bar{x}^{(0)}]^2}{n-1}} \] (12)

\[ S_2 = \frac{\sqrt{\sum_{i=1}^{n}[\Delta(0)(i) - \bar{\Delta}(0)]^2}}{n-1} \] (13)

\[ \Delta(0)(i) = x^{(0)}(i) - \hat{x}(0)(i) \] (14)

\[ \bar{\Delta}(0) = \frac{1}{n} \sum_{i=1}^{n} \Delta(0)(i) \] (15)

| Variance ratio \(C\) | Small residual probability \(P\) | Model accuracy |
|----------------------|-----------------------------|---------------|
| < 0.35               | > 0.95                      | Excellent     |
| < 0.50               | > 0.80                      | Qualified     |
| < 0.65               | > 0.70                      | Barely Qualified |
| > 0.65               | < 0.70                      | Unqualified   |
2.2 Support vector machine regression
Support vector machine regression (SVR) is a regression model derived from support vector machine. SVM \(^{[22]}\) is a binary classification model, which can solve the problems of small samples, nonlinearity, high dimension and so on. The basic idea of SVM classification model is to solve the separation hyperplane which can correctly divide the training data set and has the largest geometric interval; the SVR regression model is to find a regression hyperplane and make all the data of a set closest to the set of the plane, so as to achieve the purpose of regression prediction. The basic idea of SVR is as follows:

Set the original dataset as: 
\[
(x_1, y_1), (x_2, y_2), \ldots, (x_m, y_m), x \in \mathbb{R}^n, y \in \mathbb{R},
\]
and the equation of the hyperplane is:
\[
f(x) = \omega^T x + b \quad (16)
\]
If the "distance" to the sample point farthest from the hyperplane is the smallest, then:
\[
\left\{ \begin{array}{l}
\min \frac{1}{2} \| \omega \|^2 \\
\text{s.t. } |y_i - (\omega^T x_i + b)| \leq \varepsilon, \forall i
\end{array} \right.
\]
(17)
By introducing kernel function and Lagrangian variation, the final regression function is:
\[
f(x) = \omega^* \cdot x + b = \sum_{i=1}^{m} (a_i - a_i^*) K(x_i \cdot x) + b \quad (18)
\]
where \(\omega\) and \(b\) are hyperplane coefficients, \(\varepsilon\) is any positive number, \(a_i\) and \(a_i^*\) are sample support vectors, \(K(x_i \cdot x)\) is inner product kernel function, which is used to map nonlinear samples to high-dimensional space. In order to achieve better fitting accuracy, radial basis function kernel function is selected in this model\(^{[23]}\).

Different from the general linear regression, the data in the interval band in the SVR model does not calculate the loss if and only if the absolute value of the gap between \(f(x)\) and \(y\) is greater than \(\varepsilon\); Besides, SVR optimizes the model by maximizing the width of the spacer and minimizing the total loss\(^{[24]}\).

2.3 Entropy weight combination method
In the combined prediction model, the weight coefficients of different models are very important for the accuracy of the final prediction results. In information theory system, information and entropy are a pair of relative concepts. Information is the measure of order in the system, and entropy is the measure of disorder. Entropy weight rule is a method to weight an index by applying the corresponding entropy meaning. If the entropy value of an index is small, the greater the amount of information it contains, the higher the weight; The smaller the entropy, the smaller the weight.

In the establishment of the model in this paper, if the error series of the prediction results obtained by a prediction model is larger, the model will be given a smaller weight, and vice versa.

3. Establishment of combined prediction model
Firstly, this paper takes the original data as the input of the improved GM (1, 1) model and SVR model, and obtains two single prediction models through MATLAB. The relative fitting errors of the two models are calculated respectively, the error proportion and entropy value are calculated according to the idea of entropy weight method, and the weight coefficients of the last two models are calculated. Finally, the final prediction model can be obtained by combining and adding their weight coefficients. The specific modeling process is shown in Figure 1.
The establishment of the combined model mainly relies on the powerful matrix calculation ability of MATLAB and the SVM software package libsvm developed and designed by Professor Lin Zhiren of Taiwan University. The software package can be used to solve the problems of epsilon SVR and nu SVR. In the process of SVR operation, the key step is to adjust the relevant parameters (mainly penalty parameter C and kernel function parameter g). Generally, the idea of cross validation is used to select the optimal parameters\cite{25}. Because k-fold cross validation (k-cv) in CV verification can effectively avoid the occurrence of over learning and under learning, this paper selects k-cv method to calculate the optimal C and G. That is, the original data is divided into k groups, each subset data is used as a verification set, and the rest of the k-1 subset data is used as a training set. In this way, K models are obtained, and the average of the classification accuracy of the final verification sets of the K models is used as the performance index under the k-cv.

4. Model validation

4.1 Basic data

In order to predict the skid resistance performance of expressway pavement, the pavement skidding resistance index (SRI) of a section in the downward direction of Leshan Zigong Expressway in Sichuan Province in 2016, 2018 and 2020 is selected as the research sample to predict the SRI values in 2021 and 2022.

In order to verify the accuracy of the model, this paper selects the pavement condition index (PCI) of the corresponding road section to test the model. Take the PCI values of this section in 2013, 2015 and 2017 as input, predict the PCI values in 2018 and 2019, compare them with the measured values, and analyze the accuracy of the model. The measured values of PCI and SRI are shown in Table 2 and Table 3.

| Table 2 The value of SRI |
|--------------------------|
| Year  | 2016 | 2018 | 2020 |
|-------|------|------|------|
| SRI   | 95.88| 95.74| 86.68|

| Table 3 The value of PCI |
|--------------------------|
| Year  | 2013 | 2015 | 2017 | 2018 | 2019 |
|-------|------|------|------|------|------|
| PCI   | 98.40| 95.35| 93.30| 89.62| 86.39|

4.2 Model prediction

Firstly, the level ratio test of SRI and PCI data is carried out according to formula (1). The results are: $\sigma_{SRI} = (1.0015,1.1045)$ and $\sigma_{PCI} = (1.0319,1.0219)$, both of them meet $\sigma(k) \in (0.6065,1.6487)$, so GM (1, 1) modeling can be carried out.
The improved GM (1, 1) and SVR calculation models are established in MATLAB, and then the entropy weight combination is realized through the code to calculate the weight of each method. Then, the PCI value and SRI value are used as model inputs for prediction, and the prediction results are Table 4 and Table 7 respectively. Meanwhile, in order to test the prediction results of the model, this paper selects the traditional GM (1, 1) model and the GM (1, 1) model which uses SVR to modify the residual error to compare with the GM-SVR combination model in this paper. The fitting results of each model are output in MATLAB. The prediction results of each model are shown in Table 4.

| Year | Measured Value | Predictive Value |
|------|----------------|------------------|
|      | Traditional GM (1, 1) | Residual improvement GM (1, 1) | GM-SVR |
| 2013 | 98.40           | 98.464           | 98.197  |
| 2015 | 95.35           | 95.234           | 95.299  | 95.478  |
| 2017 | 93.30           | 93.235           | 93.300  | 93.463  |
| 2018 | 89.62           | 87.485           | 87.550  | 89.590  |
| 2019 | 86.39           | 85.649           | 85.714  | 88.051  |

Table 5 GM (1, 1) accuracy test results

| p   | C       | Model Accuracy |
|-----|---------|----------------|
| SRI | 1       | 0.0546 Excellent |
| PCI | 1       | 0.0226 Excellent |

4.3 Result analysis

4.3.1 Grey model accuracy test
Since the establishment of this model depends on the traditional GM (1, 1), it is necessary to test the accuracy of the model. It can be seen from Table 5 that under the input of SRI and PCI data, the P and C values of GM (1, 1) model meet P > 0.95 and C < 0.35, so the accuracy of grey prediction model is excellent.

4.3.2 Analysis of prediction results
To verify the accuracy of the model, this paper uses the PCI value of the same road section to evaluate the prediction accuracy of the model, and makes a comparative analysis with other prediction models.

Table 7 SRI prediction results of GM-SVR model

| Year | Measured Value | Predictive Value |
|------|----------------|------------------|
| 2016 | 95.88          | 95.917           |
| 2018 | 95.74          | 95.007           |
| 2020 | 86.68          | 86.963           |
| 2021 | 74.458         | 74.458           |
| 2022 | 71.497         | 71.497           |
model is more consistent, while the other two models deviate greatly, which shows that the stability of GM-SVR model is higher than that of other models over time.

In order to measure the effect of model prediction, this paper selects mean square error (MSE), mean absolute percentage error (MAPE) and mean absolute error (MAE) to evaluate each model\textsuperscript{[26]}. The calculated values of various indicators are shown in Table 4. By comparing the indices of each model, it is obvious that the evaluation indices of traditional GM (1, 1) are greater than the residual modified GM (1, 1) model and GM-SVR combined model because it is a single prediction model; The residual correction GM model is the second. The indices of the combined GM-SVR model based on entropy weight method are the smallest, which shows that the combined model established in this paper has better prediction accuracy than the traditional prediction model.

The GM-SVR model tested above has smaller prediction error and higher accuracy. Therefore, this model is used to predict the SRI of this section in 2021 and 2022. The results are shown in Table 7, and the prediction of pavement skid resistance condition with limited data.

5. Conclusion

Based on the results and discussions presented above, the conclusions are obtained as below:

(1) This paper combines the accuracy of grey theory for short-term prediction of small sample data and the applicability of SVR to small sample and nonlinear data processing, and uses entropy weight method to combine them to establish the prediction model of pavement skid resistance condition. Mean square error (MSE), mean absolute percentage error (MAPE) and mean absolute error (MAE) are used to evaluate the fitting results, and finally a prediction model with fitting accuracy meeting the requirements is obtained.

(2) GM-SVR prediction model can predict the pavement data of limited years, which is in line with the actual characteristics of road maintenance in China. It can provide a guarantee for highway driving safety and has high practical value.

(3) GM-SVR model excavates the internal laws of pavement performance data from a mathematical point of view, and the prediction results are reliable. This paper only takes the skidding resistance index and pavement condition index as examples. This model can also be used to predict the changes of other pavement indices, and has certain universality.

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