Twist Three Distribution $f_\perp(x, k^\perp)$ in Light-front Hamiltonian Approach

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Abstract

We calculate the twist three distribution $f_\perp(x, k^\perp)$ contributing to Cahn effect in unpolarized semi-inclusive deep inelastic scattering. We use light-front Hamiltonian technique and take the state to be a dressed quark at one loop in perturbation theory. The 'genuine twist three' contribution comes from the quark-gluon interaction part in the operator and is explicitly calculated. $f_\perp(x, k^\perp)$ is compared with $f_1(x, k^\perp)$.
I. INTRODUCTION

Transverse momentum dependent parton distributions (TMDs) [1] have gained a lot of interest recently. In collinear hard scattering processes, for example, in deep inelastic scattering (DIS) the large virtuality $Q^2$ of the hard probe (virtual photon) introduces a longitudinal direction. A plane perpendicular to that is the transverse plane. Ordinary parton distributions (pdfs) measured in inclusive processes like DIS do not give any information on the transverse momentum distributions of quarks and gluons. TMDs can be measured in processes when sufficient transverse momentum is measured in the final state; for example in semi-inclusive deep inelastic scattering (SIDIS) where a hadron with transverse momentum $P^\perp_h$ is measured or in Drell-Yan process where the transverse momentum of the virtual photon is measured. Factorization for some processes involving TMDs have been proven at twist two at one loop order and argued to hold for all orders [2, 3]. The TMDs involve an operator structure which is bilocal both in the light cone as well as in the transverse direction; and a path ordered exponential of the line integral of the gauge field (gauge link) is necessary for color gauge invariance. In light front gauge, the link in the light cone direction becomes unity, but contribution comes from the part of the link at light-cone infinity involving the transverse component of the gauge field. It has been found that this is process dependent in general [4] and may contribute at leading order in $1/Q$ [5]. However for fragmentation process the shape of the Wilson line has no effect on observables [6]. In SIDIS and Drell-Yan the TMDs are simply connected by a reversal of sign [7]. In more complicated processes like hadron production in hadron-hadron collisions, although the standard universality of TMDs does not hold, predictivity is not lost [8]. However, very recently in [9] it has been shown that such ‘generalized’ factorization does not hold for all hadroproduction processes.

There are 32 quark TMDs including twist two, three and four. The higher twist or subleading in $1/Q$ TMDs contain one or more ‘bad’ light cone component of the quark field, and the operator involves quark-gluon interaction term. The subleading twist TMDs are important as they contribute in several single spin as well as azimuthal asymmetries in the kinematical range of present experiments. Experimental data on several of these asymmetries are now available [10]. Interpretation of these subleading twist asymmetries are more challenging as they involve
several higher twist distribution and fragmentation functions. Here model calculations of these functions play an important role. Twist three TMDs are related to twist two TMDs and ‘genuine twist three part’ through equation of motion relations \([11]\). In certain models some other relations between the TMDs exist based on Lorentz invariance. The Lorentz invariance relations do not hold in QCD due to the presence of the gauge link \([12]\) whereas the equation of motion relations still hold \([11]\).

Among the twist three TMDs there are few model calculations of \(f_{\perp}(x, k^\perp)\). This is time-reversal even and plays an important role in \(\cos \phi_h\) asymmetry in unpolarized SIDIS, the so-called Cahn effect \([13]\). The unpolarized SIDIS cross section depends on the azimuthal angle \(\phi_h\) between the lepton plane and the hadron production plane, and on the transverse momentum of the detected hadron. The \(\phi_h\) dependence of the SIDIS cross section has been experimentally detected by the EMC collaboration \([14]\). If one neglects the explicit quark gluon interaction terms in the distribution and fragmentation functions then this \(\cos \phi_h\) dependence of the cross section is given in terms of the unpolarized distribution and fragmentation functions at \(1/Q\) level. This effect has been investigated in a parton model approach by introducing a phenomenologically motivated intrinsic \(k^\perp\) dependence \([15]\). \(f_{\perp}(x, k^\perp)\) has been calculated in a simple spectator model in \([16]\). A bag model result of \(f_{\perp}(x, k^\perp)\) has been given in \([17]\).

In this work we calculate \(f_{\perp}(x, k^\perp)\) in light front Hamiltonian approach. Instead of using the Feynman diagrams, we expand the state in Fock space in terms of multi-parton light-front wavefunctions. The partons are on-mass shell interacting objects having non-vanishing transverse momenta and thus they can be called field theoretic partons. The advantage is that these wave functions are Lorentz boost invariant \([18]\), so we can truncate the Fock space expansion to a few particle sector in a boost invariant way. We take the state to be a dressed quark at one loop in QCD. The two particle light-front wave functions (LFWFs) can be calculated analytically for a quark at one loop using the light-front Hamiltonian. Using the constraint equation in light-front gauge, the bad component of the fermion field, \(\psi(\gamma)\), is eliminated. The operator has a mass dependent part, a \(k^\perp\) dependent part and a quark-gluon interaction part. The distribution can be expressed in terms of overlaps of LFWFs. In addition to diagonal overlaps there are particle number changing off-diagonal overlaps. Twist three distributions \(g_T(x), e(x)\) and \(h_L(x)\) have been investigated in this approach before \([19,21]\). In the next section, we present
II. TWIST THREE DISTRIBUTION $f_{\perp}(x, k^\perp)$

The transverse momentum dependent distribution $f_{\perp}(x, k^\perp)$ is defined as

$$\frac{k^i}{P^+}f_{\perp}(x, k^\perp) = \int \frac{dy^-d^2y^\perp}{4(2\pi)^3} e^{iP^+y^-x} e^{-ik^+y^+} \langle P \mid \bar{\psi}(0)U(0, y)\gamma^i\psi(y^-, y^\perp) \mid P \rangle \mid_{y^+ = 0}. \quad (1)$$

$U(0, y)$ is the path ordered exponential (link) required for color gauge invariance. For transverse momentum dependent distributions, the bilocality in the operator is both in the longitudinal as well as in the transverse direction. In the light cone gauge, $A^+ = 0$, the gauge link in the longitudinal or light-cone direction becomes unity, but contribution will come from the transverse gauge link at light cone infinity which can not be set to unity in this gauge. It has been found recently that this part of the gauge link gives important contribution even at twist two level, in particular in the case of time-reversal odd observables [4]. However, in the following, we neglect the contribution from the transverse gauge link.

We take $i = 1$. We have, using the light-front projection operators $\Lambda^\pm = \frac{1}{2}\gamma^0\gamma^\pm$;

$$\bar{\psi}(0)\gamma_i\psi(y^-, y^\perp) = \psi^{(-)\dag}(0)\alpha^\perp\cdot\partial\perp + \psi^{(+)\dag}(0)\alpha^\perp\cdot\partial\perp \psi^{(-)}(y^-, y^\perp). \quad (2)$$

In light-front gauge, $A^+ = 0$, the 'bad' component, $\psi^{(-)}$ is constrained, and the equation of constraint is given by [22]

$$\psi^{(-)}(y) = \frac{1}{i\partial^+}(i\alpha^\perp\cdot\partial^\perp + g\alpha^\perp\cdot A^\perp + \beta m)\psi^{(+)}(y); \quad (3)$$

where the operator $\frac{1}{i\partial^+}$ is defined as [22]

$$\frac{1}{\partial^+}f(x^-) = \frac{1}{4}\int_{-\infty}^{\infty} dy^- \epsilon(x^- - y^-) f(y^-). \quad (4)$$

The antisymmetric step function is given by

$$\epsilon(x^-) = -\frac{i}{\pi} \mathcal{P} \int \frac{d\omega}{\omega} e^{i\omega x^-}; \quad (5)$$

$\mathcal{P}$ denotes the principal value. Using the equation of constraint the field $\psi^{(-)}$ can be removed.

The operator has three parts:

$$O_{k^\perp} = \psi^{(+)\dag}(0) \left[ (\alpha^\perp\cdot\partial^\perp) (\frac{1}{\partial^+}) \alpha^1 + \alpha^1 (\frac{1}{\partial^+}) (\alpha^\perp\cdot\partial^\perp) \right] \psi^{(+)}(y); \quad (6)$$
\[ O_g = g \psi^{(+\dagger)}(0) \left[ (\alpha^\perp \cdot A^\perp)(\frac{1}{-i\partial^+})\alpha^1 + \alpha^1(\frac{1}{i\partial^+})(\alpha^\perp \cdot A^\perp) \right] \psi^{(+)}(y); \] (7)

\[ O_m = m \psi^{(+\dagger)}(0)\gamma^1 \left[ \left( \frac{1}{-i\partial^+} \right) - \left( \frac{1}{i\partial^+} \right) \right] \psi^{(+)}(y). \] (8)

For the dynamical field \( \psi^{(+)} \) we use two component formalism \[22\] | \( P, \sigma \rangle \) is a proton state of momentum \( P \) and helicity \( \sigma \). The state can be expanded in Fock space in terms of multi-parton LFWFs. Instead of the proton we take the state to be a dressed quark. Fock space expansion of such a state can be written as :

\[
| P, \sigma \rangle = \phi_1 b^\dagger(P, \sigma | 0) \]

\[
+ \sum \frac{dk_1^+d^2k_1^\perp}{\sqrt{2(2\pi)^3k_1^+}} \frac{dk_2^+d^2k_2^\perp}{\sqrt{2(2\pi)^3k_2^+}} \sqrt{2(2\pi)^3P^+}\delta^3(P-k_1-k_2) \phi_2(P, \sigma | k_1, \sigma_1; k_2, \lambda_2) b^\dagger(k_1, \sigma_1)a^\dagger(k_2, \lambda_2 | 0). \] (9)

Here \( a^\dagger \) and \( b^\dagger \) are bare gluon and quark creation operators respectively and \( \phi_1 \) and \( \phi_2 \) are the multiparton wave functions. We introduce Jacobi momenta \( x_i, q_i^\perp \) such that \( \sum_i x_i = 1 \) and \( \sum_i q_i^\perp = 0 \). They are defined as

\[ x_i = \frac{k_i^+}{P^+}, \quad q_i^\perp = k_i^\perp - x_i P^\perp. \] (10)

Also, we introduce the wave functions,

\[ \psi_1 = \phi_1, \quad \psi_2(x_i, q_i^\perp) = \sqrt{P^+}\phi_2(k_i^+ , k_i^\perp); \] (11)

which are independent of the total transverse momentum \( P^\perp \) of the state and are boost invariant. The two particle wave function depends on the helicities of the quark and gluon. Using the eigenvalue equation for the light-cone Hamiltonian, this can be written as \[23\],

\[
\psi_{2\sigma_1, \lambda}(x, q^\perp) = \frac{x(1-x)}{(q^\perp)^2 + m^2(1-x)^2} \frac{1}{\sqrt{(1-x)}} \frac{g}{\sqrt{2(2\pi)^3}} T^a \chi^a_{\alpha_1} \left[ -2 \frac{q^\perp}{1-x} - \frac{\bar{\sigma}^\perp \cdot q^\perp}{x} \bar{\sigma}^\perp \right]
+ im\bar{\sigma}^\perp \frac{(1-x)}{x} \chi^a \epsilon^*_\lambda \psi_1. \] (12)

\( m \) is the bare mass of the quark, \( \bar{\sigma}_1 = \sigma_2, \bar{\sigma}_2 = -\sigma_1 \). \( \psi_1 \) actually gives the normalization of the state \[23\]:

\[ | \psi_1 |^2 = 1 - \frac{\alpha_s}{2\pi} C_f \log \frac{Q^2}{\mu^2} \int_\epsilon^{1-\epsilon} dx \frac{1+x^2}{1-x}, \] (13)
to order $\alpha_s$. Here $\epsilon$ is a small cutoff on $x$. We have taken the cutoff on the transverse momenta to be $Q$, the large scale of the process. In the above expression, we have neglected subleading finite pieces. $\mu$ is a small scale such that $(q^\perp)^2 > \mu^2 >> m^2$.

Contribution from $O_m$ is zero. $O_{k^\perp}$ has contribution from single particle sector as well as two particle sector of the state and $O_g$ will get contribution from an overlap of a single particle and a two-particle light-front wave functions. The contributions from $O_{k^\perp}$ are:

$$
\int \frac{dy^- d^2 y^\perp}{4(2\pi)^3} e^{2P^+ y^-} e^{-i k^\perp y^\perp} \langle P \mid O_{k^\perp} \mid P \rangle = \delta(1 - x) \frac{P^1}{P^+} \delta^2(k^\perp - P^\perp) \langle \psi_1 \rangle^2
$$

$$
+ \int d^2 q^\perp q^\perp + xP^1 \pi \frac{P^1}{P^+} \delta^2(k^\perp - q^\perp - xP^\perp)
$$

$$
= \frac{P^1}{P^+} \delta(1 - x) \delta^2(k^\perp - P^\perp) \langle \psi_1 \rangle^2
$$

$$
+ \frac{\alpha_s}{2\pi^2} C_f \int d^2 q^\perp \delta^2(k^\perp - q^\perp - xP^\perp) \frac{[\(q^\perp)^2 + m^2(1 - x)^2]}{(q^\perp)^2 + m^2(1 - x)^2} xP^+ (14)
$$

Here we have summed over the helicity of the state. The Fock space expansion of the interaction part of the operator can be written as

$$
O_g^{(1)} = g \sum_{\text{spins}} \int (dk_1) \int (dk_2) \int [dk_3] \chi_{\lambda_1}^\dagger \left( \epsilon_{\lambda_3}^1 i \sigma_3 \epsilon_{\lambda_3}^2 \right) \chi_{\lambda_2} e^{-\frac{i}{2} k_2^\dagger y^- + i k_2^\perp y^\perp}
$$

$$
\left[ \frac{1}{k_1^+ - k_3^+} b_{\lambda_1}^\dagger(k_1) b_{\lambda_2}(k_2) a_{\lambda_3}(k_3) + \frac{1}{k_1^+ + k_3^+} b_{\lambda_1}^\dagger(k_1) b_{\lambda_2}(k_2) a_{\lambda_3}^\dagger(k_3) \right]
$$

$$
(15)
$$

$$
O_g^{(2)} = g \sum_{\text{spins}} \int (dk_1) \int (dk_2) \int [dk_3] \chi_{\lambda_1}^\dagger \left( \epsilon_{\lambda_3}^1 i \sigma_3 \epsilon_{\lambda_3}^2 \right) \chi_{\lambda_2}
$$

$$
e^{-\frac{i}{2} k_2^\dagger y^- + i k_2^\perp y^\perp} \left[ \frac{1}{k_2^+ + k_3^+} b_{\lambda_1}^\dagger(k_1) b_{\lambda_2}(k_2) a_{\lambda_3}(k_3) e^{-\frac{i}{2} k_3^\perp y^- + i k_3^\perp y^\perp} + \frac{1}{k_2^+ - k_3^+} b_{\lambda_1}^\dagger(k_1) b_{\lambda_3}(k_3) a_{\lambda_3}^\dagger(k_3) e^{i \frac{x}{2} k_3^\perp y^- - i k_3^\perp y^\perp} \right].
$$

$$
(16)
$$

Here we have used the notations $(dk) = \frac{dk^+ d^2 k^\perp}{2(2\pi)^3 \sqrt{k^+}}$ and $[dk] = \frac{dk^+ d^2 k^\perp}{2(2\pi)^3 \sqrt{k^+}}$. As we stated above, the interaction part of the operator gives $\psi_1^1 \psi_2^2$ and $\psi_2^1 \psi_1^2$ type terms. Contribution from $O_g$ is given by:

$$
\int \frac{dy^- d^2 y^\perp}{4(2\pi)^3} e^{2P^+ y^-} e^{-i k^\perp y^\perp} \langle P \mid O_g \mid P \rangle = -\frac{\alpha_s}{2\pi^2} C_f \int d^2 q^\perp \frac{q^1}{P^+} \frac{1}{(x(1 - x))^2} \delta^2(k^\perp - q^\perp - xP^\perp). \tag{17}
$$
Here we have used explicit form of $\psi_{2,s_1,\lambda}(x,q^\perp)$.

In the frame where $P^\perp = 0$, one has

$$xf_\perp(x,k^\perp) = \frac{\alpha_s}{2\pi^2}\left(\frac{(k^\perp)^2(1+x^2)/(1-x) + m^2(1-x)^3}{[(k^\perp)^2 + m^2(1-x)^2]^2}\right)
= \frac{1}{(1-x)[(k^\perp)^2 + m^2(1-x)^2]} \cdot \frac{1}{(1-x)[(k^\perp)^2 + m^2(1-x)^2]^2} = \alpha_s \frac{[(k^\perp)^2x^2 + m^2(1-x)^2(1-x)]}{(1-x)[(k^\perp)^2 + m^2(1-x)^2]^2}. \quad (18)$$

The twist two unpolarized distribution $f_1(x,k^\perp)$ can be calculated using the definition

$$f_1(x,k^\perp) = \int \frac{dy^\perp d^2y^\perp}{4(2\pi)^3} e^{ip^\perp y^\perp} e^{-ik^\perp y^\perp} \langle P | \bar{\psi}(0)U(0,y)\gamma^+\psi(y^\perp,y^\perp) | P \rangle |_{y^\perp = 0}. \quad (19)$$

The operator neglecting the gauge link is of the form $2\psi^{(+)}(0)\psi^{(+)}(y^\perp,y^\perp)$. For a dressed quark state, one gets

$$f_1(x,k^\perp) = \delta(1-x)\delta^2(k^\perp - P^\perp)|\psi_1|^2 + \frac{\alpha_s}{2\pi^2}C_f \int d^2q^\perp \delta^2(k^\perp - q^\perp - xP^\perp) \frac{[(q^\perp)^2(1+x^2) + m^2(1-x)^3]}{[(q^\perp)^2 + m^2(1-x)^2]^2}. \quad (20)$$

In the frame $P^\perp = 0$ one gets

$$f_1(x,k^\perp) = \frac{\alpha_s}{2\pi^2}C_f \frac{[(k^\perp)^2(1+x^2) + m^2(1-x)^3]}{[(k^\perp)^2 + m^2(1-x)^2]^2}; \quad (21)$$

neglecting the single particle contribution at $x = 1$ and $k^\perp = 0$. The above result agrees with [24]. Note that in order to get the correct behaviour at $x = 1$ one has to include the single particle contribution and the normalization of the state Eq. (13). Comparing we see the equation of motion relation

$$xf_\perp = x\tilde{f}_\perp + f_1 \quad (22)$$

is satisfied, with $\tilde{f}_\perp$ is the genuine twist three quark-gluon interaction part which in our calculation, comes from $O_g$.

In the TMDs we did not use the large $k^\perp$ approximation. However in the limit of large $k^\perp$, the twist three distribution has $\frac{1}{(k^\perp)^2}$ behaviour as shown in [25].
FIG. 1: (Color online) Plots of (a) $f_1(x, k^\perp)$ and (b) $xf_\perp(x, k^\perp)$ vs $x$ and $k = |k^\perp|$. We have taken $m = 0.3$ GeV. $k^\perp$ is in GeV.

In Fig. 1 we have plotted $f_1(x, k^\perp)$ and $xf_\perp(x, k^\perp)$ as functions of $x$ and $k^\perp$. Substantial difference is observed in relatively lower $k^\perp$ region, in fact $xf_\perp$ also becomes negative. This is due to the quark-gluon interaction contribution to the twist three distribution, and unlike the bag model $[17]$. We took $m = 0.3$ GeV. We have divided both plots by $\frac{\alpha_s}{2\pi^2}$. One has to be careful not to compare the numerical results of the dressed quark calculations with experimental data. However, the qualitative behaviour is interesting as unlike phenomenological models, the genuine twist three part comes from explicit calculation of the quark gluon interaction term.

In the integrated distribution, there is an integration over $k^\perp$. The operator is bilocal only in minus direction. As a result, the gauge link is only in the light-cone direction and becomes unity in the light cone gauge. The operator can still be separated into three parts, $O_m$, $O_{k^\perp}$ and $O_g$ using the equation of constraint for $\psi^{(-)}$. $O_m$, as before gives zero contribution. Contribution of $O_g$ is zero after $k^\perp$ integration, due to rotational symmetry. The entire contribution comes from $O_{k^\perp}$:

$$
\int \frac{dy^-}{8\pi} e^{\frac{P^- + y^-}{2}} \langle P \mid O_{k^\perp} \mid P \rangle = \frac{P^1}{P^+} \left[ \delta(1 - x) + \frac{\alpha_s}{2\pi} \log \frac{Q^2}{\mu^2} C_f \int dx \frac{1 + x^2}{1 - x} \right].
$$

(23)

rhs is the twist two unpolarized distribution function $f_1(x, Q^2)$. This is expected as when integrated distributions are concerned, the transverse component of the bilocal current given by Eq. (2) with bilocality only in the minus direction, has the same parton interpretation as the plus component. Note that we get nonzero result only when $P^\perp$ is nonzero $[23]$. 

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III. DISCUSSION

In this paper, we calculate the twist three distribution $f_{\perp}(x,k_{\perp})$ in light-front Hamiltonian approach. This distribution is known to play an important role in the observed Cahn effect in unpolarized SIDIS. Instead of a proton state we take the state to be a dressed quark at one loop in QCD. The advantage is that the higher Fock space component (two particle) LFWF can be calculated analytically. These play an important role in the higher twist distributions. The partons, that is, the quarks and gluons have non-zero transverse momenta and they interact. The transverse momentum dependence of the two-particle LFWF is obtained by solving the eigenvalue equation of the light-front Hamiltonian. At $O(\alpha_s)$ this calculation is exact. However, we neglect the contribution from the gauge link at light-cone infinity. The operator has three parts, an intrinsic transverse momentum dependent term, a mass term and a 'genuine twist three' quark-gluon interaction term. Contribution from each of these terms are calculated using overlaps of LFWFs. The equation of motion relation connecting $f_{\perp}(x,k_{\perp})$ to the twist two unpolarized distribution $f_{1}(x,k_{\perp})$ and a quark-gluon interaction part is shown to hold. $xf_{\perp}(x,k_{\perp})$ differs substantially in qualitative behaviour from $f_{1}(x,k_{\perp})$ in low $k_{\perp}$ region. The last part vanishes when integrated over $k_{\perp}$ and one gets the same information as in $f_{1}(x,Q^2)$.

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