Symmetries of hadrons after unbreaking the chiral symmetry

L.Ya. Glozman, C.B. Lang, Mario Schröck

Universität Graz

Graz, October 3, 2012

[PRD 86 (2012) 014507, arXiv:1205.4887]
Outline

Motivation and introduction

Quark propagator

Mesons

Baryons

Summary
Key questions to QCD

- How is the hadron mass generated in the light quark sector?
- How important is chiral symmetry breaking for the hadron mass?
- Are confinement and chiral symmetry breaking directly interrelated?
- Is there parity doubling and does chiral symmetry get effectively restored in high-lying hadrons?
- Is there some other symmetry?
The Banks–Casher relation

The lowest eigenmodes of the Dirac operator are related to the quark condensate of the vacuum:

\[ \langle \bar{\psi} \psi \rangle = -\pi \rho(0) \]

- \( \rho(0) \): density of the lowest quasi-zero eigenmodes of the Dirac operator
- Here the sequence of limits is important: \( V \to \infty \) then \( m_q \to 0 \)
“Unbreaking” chiral symmetry

- Our goal is to construct hadron correlators out of *reduced* quark propagators which exclude a variable number of the lowest Dirac eigenmodes (see also, e.g., [DeGrand, PRD 69 (2004)]).

- we use the Hermitian Dirac operator $D_5 \equiv \gamma_5 D$ (real eigenvalues)

- split the quark propagator $S \equiv D^{-1}$ into a low mode (lm) part and a *reduced* (red) part

\[
S = \sum_{i \leq k} \frac{1}{\mu_i} |v_i\rangle \langle v_i| \gamma_5 + \sum_{i > k} \frac{1}{\mu_i} |v_i\rangle \langle v_i| \gamma_5 = S_{\text{lm}(k)} + S_{\text{red}(k)}
\]
“Unbreaking” chiral symmetry

- Our goal is to construct hadron correlators out of reduced quark propagators which exclude a variable number of the lowest Dirac eigenmodes (see also, e.g., [DeGrand, PRD 69 (2004)]).

- We use the Hermitian Dirac operator $D_5 \equiv \gamma_5 D$ (real eigenvalues).

- Split the quark propagator $S \equiv D^{-1}$ into a low mode (lm) part and a reduced (red) part

\[
S = \sum_{i \leq k} \mu_i^{-1} |v_i\rangle \langle v_i| \gamma_5 + \sum_{i > k} \mu_i^{-1} |v_i\rangle \langle v_i| \gamma_5
\]

\[
= S_{\text{lm}}(k) + S_{\text{red}}(k)
\]

- In this work we investigate the reduced (red) part of the propagator

\[
S_{\text{red}}(k) = S - S_{\text{lm}}(k)
\]
The setup

- 161 configurations [Gattringer et al., PRD 79 (2009)]
- size $16^3 \times 32$
- two degenerate flavors of light fermions, $m_\pi = 322(5)$ MeV
- lattice spacing $a = 0.1440(12)$ fm
- Chirally Improved (CI) Dirac operator [Gattringer, PRD 63 (2001)] (approximate solution of the Ginsparg-Wilson equation)
- three different kinds of quark sources: Jacobi smeared narrow (0.27 fm) and wide (0.55 fm) sources and a $P$ wave like derivative source → serves a large operator basis for the variational method.
The lattice quark propagator

The tree-level quark propagator is

$$S_0(p) = \frac{1}{ip + m_0}$$
The lattice quark propagator

The tree-level quark propagator is

\[ S_0(p) = \frac{1}{i\not{p} + m_0} \]

\[ S_0(p) \rightarrow S_{\text{bare}}(a; p) = Z_2(\mu; a)S(\mu; p) \]
The lattice quark propagator

The tree-level quark propagator is

$$S_0(p) = \frac{1}{i\not{p} + m_0}$$

$$S_0(p) \rightarrow S_{\text{bare}}(a; p) = Z_2(\mu; a) S(\mu; p)$$

the renormalized quark propagator

$$S(\mu; p) = \frac{1}{i\not{p} A(\mu; p^2) + B(\mu; p^2)} = \frac{Z(\mu; p^2)}{i\not{p} + M(p^2)}.$$

We calculate $S_{\text{bare}}(a; p)$ in Landau gauge on the lattice and therefrom extract

- the renormalization function $Z(\mu; p^2)$
- the renormalization point independent mass function $M(p^2)$
The quark propagator under eigenmode reduction

The dynamically generated mass decreases with the truncation level → restoration of the chiral symmetry.

[M. Schröck, PLB 711 (2012), arXiv:1112.5107]
The quark propagator under eigenmode reduction

- the dynamically generated mass decreases with the truncation level → restoration of the chiral symmetry.

[M. Schröck, PLB 711 (2012), arXiv:1112.5107]
The quark propagator under eigenmode reduction

- the dynamically generated mass decreases with the truncation level → restoration of the chiral symmetry.

[M. Schröck, PLB 711 (2012), arXiv:1112.5107]
The quark propagator under eigenmode reduction

- the dynamically generated mass decreases with the truncation level → restoration of the chiral symmetry.

[M. Schröck, PLB 711 (2012), arXiv:1112.5107]
The quark propagator under eigenmode reduction

- the dynamically generated mass decreases with the truncation level → restoration of the chiral symmetry.

[M. Schröck, PLB 711 (2012), arXiv:1112.5107]
Motivation and introduction

Quark propagator

Mesons

Baryons

Summary

The quark propagator under eigenmode reduction

- the dynamically generated mass decreases with the truncation level
  → restoration of the chiral symmetry.

[M. Schröck, PLB 711 (2012), arXiv:1112.5107]
The quark propagator under eigenmode reduction

- the dynamically generated mass decreases with the truncation level → restoration of the chiral symmetry.

[M. Schröck, PLB 711 (2012), arXiv:1112.5107]
The quark propagator under eigenmode reduction

- the dynamically generated mass decreases with the truncation level → restoration of the chiral symmetry.

[M. Schröck, PLB 711 (2012), arXiv:1112.5107]
The quark propagator under eigenmode reduction

- the dynamically generated mass decreases with the truncation level → restoration of the chiral symmetry.

[M. Schröck, PLB 711 (2012), arXiv:1112.5107]
Reminder: chiral symmetry and its breaking

When neglecting the two lightest quark masses, the QCD Lagrangian becomes invariant under the symmetry group

$$SU(2)_L \times SU(2)_R \times U(1)_A$$

- axial vector part of the $SU(2)_L \times SU(2)_R$ symmetry is broken spontaneously in the vacuum
- vector part is (approximately) preserved
- $U(1)_A$ axial symmetry is not only broken spontaneously but also explicitly (axial anomaly)
Mesons

We explore the following isovector mesons which, in a chirally symmetric world, would be related via the following symmetries

\[
\begin{array}{c|c}
SU(2)_L \times SU(2)_R \text{ (axial)} & U(1)_A \\
\rho \leftrightarrow a_1 & \rho \leftrightarrow b_1
\end{array}
\]

- can we restore the chiral symmetry and if, what happens to confinement?
- how does the mass of the light mesons change?
- what happens to the \(U(1)_A\) axial symmetry?
Meson masses vs. Dirac eigenmode reduction level

- heavy $\rho$ meson: mass not due to dynamical chiral symmetry breaking
Meson masses vs. Dirac eigenmode reduction level

- heavy $\rho$ meson: mass not due to dynamical chiral symmetry breaking
- degeneracy of $\rho$ and $a_1$: restoration of the $SU(2)_L \times SU(2)_R$ chiral symmetry
Meson masses vs. Dirac eigenmode reduction level

- heavy $\rho$ meson: mass not due to dynamical chiral symmetry breaking
- degeneracy of $\rho$ and $a_1$: restoration of the $SU(2)_L \times SU(2)_R$ chiral symmetry
- degeneracy of $\rho$ and $\rho'$: hint to a higher symmetry which includes $SU(2)_L \times SU(2)_R$ as a subgroup
Meson masses vs. Dirac eigenmode reduction level

- heavy $\rho$ meson: mass not due to dynamical chiral symmetry breaking
- degeneracy of $\rho$ and $a_1$: restoration of the $SU(2)_L \times SU(2)_R$ chiral symmetry
- degeneracy of $\rho$ and $\rho'$: hint to a higher symmetry which includes $SU(2)_L \times SU(2)_R$ as a subgroup
- nondegeneracy of $\rho$ and $b_1$: $U(1)_A$ remains broken, still existence of confined states
Do we really still observe exponentially decaying states?

- the noise in the correlators (l.h.s.) decreases under Dirac low-mode truncation
- as a consequence the effective mass plots (r.h.s.) become more stable than in full QCD!
Motivation and introduction

Quark propagator

Mesons

Baryons

Summary

$a_1(J^{PC} = 1^{++})$

Do we really still observe exponentially decaying states?

- the noise in the correlators (l.h.s.) decreases under Dirac low-mode truncation
- as a consequence the effective mass plots (r.h.s.) become more stable than in full QCD!
**b_1 (J^{PC} = 1^{+-})**

Do we really still observe exponentially decaying states?

- the noise in the correlators (l.h.s.) decreases under Dirac low-mode truncation
- as a consequence the effective mass plots (r.h.s.) become more stable than in full QCD!
Baryons

The $\Delta - N$ splitting is usually attributed to the hyperfine spin-spin interaction between valence quarks. The realistic candidates for this interaction are:

- the spin-spin color-magnetic interaction
- the flavor-spin interaction related to the spontaneous chiral symmetry breaking

What happens to the $\Delta - N$ splitting after restoration of the chiral symmetry?
Do the masses of the nucleon and the $N(1535)$ meet?
Baryon masses vs. Dirac eigenmode reduction level

- heavy $N(+)$: mass not due to dynamical chiral symmetry breaking

M. Schröck

Symmetries of hadrons after unbreaking the chiral symmetry
Baryon masses vs. Dirac eigenmode reduction level

- heavy $N(+)$: mass not due to dynamical chiral symmetry breaking
- parity doubling of $N(+) \text{ and } N(-)$
Motivation and introduction
Quark propagator
Mesons
Baryons
Summary

Baryon masses vs. Dirac eigenmode reduction level

- heavy \( N(+) \): mass not due to dynamical chiral symmetry breaking
- parity doubling of \( N(+) \) and \( N(-) \)
- degeneracy of two \( N(+) \) and \( N(-) \) states: hint to a higher symmetry which includes \( SU(2)_L \times SU(2)_R \) as a subgroup

M. Schröck

Symmetries of hadrons after unbreaking the chiral symmetry
**Motivation and introduction**

**Quark propagator**

**Mesons**

**Baryons**

**Summary**

---

**Baryon masses vs. Dirac eigenmode reduction level**

![Graph showing baryon masses vs. Dirac eigenmode reduction level](image)

- **heavy $N(\,\pm\,)$**: mass not due to dynamical chiral symmetry breaking
- **parity doubling of $N(\,\pm\,)$ and $N(\mp\,)$**
- **degeneracy of two $N(\,\pm\,)$ and $N(\mp\,)$ states**: hint to a higher symmetry which includes $SU(2)_L \times SU(2)_R$ as a subgroup
- **distinguished excited states of $\Delta(\,\pm\,)$**: confinement persists
- **$\Delta-N$ splitting reduces to $\approx 50\%$**

---

M. Schröck

Symmetries of hadrons after unbreaking the chiral symmetry
\[ N(J^P = 1/2^+) \]

Motivation and introduction

Quark propagator

Mesons

Baryons

Summary

\[ N(J^P = 1/2^+) \]

all states: correlators, red(20),

\[ \begin{array}{cccccc}
0 & 2 & 4 & 6 & 8 & 10 & 12 \\
10^{-7} & 10^{-6} & 10^{-5} & 10^{-4} & 10^{-3} & 10^{-2} & 10^{-1}
\end{array} \]

lowest state(s): eff. masses, red(20),

\[ \begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
0.5 & 1 & 1.5 & 2
\end{array} \]

state 0

state 1

state 2

state 3

state 4

state 5

\[ N(J^P = 1/2^+) \]

all states: correlators, red(64),

\[ \begin{array}{cccccc}
0 & 2 & 4 & 6 & 8 & 10 & 12 \\
10^{-7} & 10^{-6} & 10^{-5} & 10^{-4} & 10^{-3} & 10^{-2} & 10^{-1}
\end{array} \]

lowest state(s): eff. masses, red(64),

\[ \begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
0.5 & 1 & 1.5 & 2
\end{array} \]

state 0

state 1

state 2

state 3

state 4

state 5

M. Schröck

Symmetries of hadrons after unbreaking the chiral symmetry
**Motivation and introduction**

**Quark propagator**

**Mesons**

**Baryons**

**Summary**

\[ N(J^P = 1/2^-) \]

*all states: correlators, red(12),

*state 0

*state 1

*state 2

*state 3

*state 4

*state 5

\[ N(J^P = 1/2^-) \]

*lowest state(s): eff. masses, red(12),

*state 0

*state 1

\[ N(J^P = 1/2^-) \]

*all states: correlators, red(64),

*state 0

*state 1

*state 2

*state 3

*state 4

*state 5

\[ N(J^P = 1/2^-) \]

*lowest state(s): eff. masses, red(64),

*state 0

*state 1
\(J^P = 3/2^+\)

**Motivation and introduction**

**Quark propagator**

**Mesons**

**Baryons**

**Summary**

\[ \Delta \left( J^P = \frac{3}{2}^+ \right) \]

- All states: correlators, red(16)
- Lowest state(s): eff. masses, red(16)

![Graphs showing correlators and effective masses for states 0, 1, and 2 for J^P = 3/2^+](image)

- All states: correlators, red(128)
- Lowest state(s): eff. masses, red(128)

![Graphs showing correlators and effective masses for states 0, 1, and 2 for J^P = 3/2^+](image)
**Motivation and introduction**

**Quark propagator**

**Mesons**

**Baryons**

**Summary**

\[ \Delta (J^P = 3/2^-) \]

- **all states: correlators, red(16)**
- **state 0**
- **state 1**
- **state 2**

- **lowest state(s): eff. masses, red(16)**
- **state 0**
- **state 1**

- **all states: correlators, red(128)**
- **state 0**
- **state 1**
- **state 2**

- **lowest state(s): eff. masses, red(128)**
- **state 0**
- **state 1**
Summary

- low lying eigenvalues of the Dirac operator are associated with chiral symmetry breaking
- we have computed hadron propagators while removing increasingly more of the low lying eigenmodes of the Dirac operator
- the confinement properties remain intact, i.e., we still observe clear bound states for all of the studied hadrons
- the mass values of the vector meson chiral partners $a_1$ and $\rho$ approach each other: restoration of $SU(2)_L \times SU(2)_R$
- no degeneracy between $\rho$ and $b_1$: $U(1)_A$ axial anomaly untouched
- the nucleon and the $N(1535)$ become degenerate
- the spin-spin color-magnetic interaction and the flavor-spin interaction are of equal importance for the $\Delta - N$ splitting
Low-mode contribution of $D$ and $D_5$ to the $\pi$ and $\rho$ correlators
Low-mode contribution of $D$ and $D_5$ to the $\pi$ and $\rho$ correlators

\begin{align*}
C_\pi(t) &= 1 \times 10^8 \\
C_\rho(t) &= 1 \times 10^7
\end{align*}
### ρ interpolators

| #_ρ | interpolator(s) |
|-----|-----------------|
| 1   | \( \bar{a}_n \gamma_k b_n \) |
| 8   | \( \bar{a}_w \gamma_k \gamma_t b_w \) |
| 12  | \( \bar{a}_{\partial_k} b_w - \bar{a}_w b_{\partial_k} \) |
| 17  | \( \bar{a}_{\partial_i} \gamma_k b_{\partial_i} \) |
| 22  | \( \bar{a}_{\partial_k} \epsilon_{ijk} \gamma_j \gamma_5 b_w - \bar{a}_w \epsilon_{ijk} \gamma_j \gamma_5 b_{\partial_k} \) |

Interpolators for the ρ-meson, \( J^{PC} = 1^{--} \). The first column shows the number, the second shows the explicit form of the interpolator. cf. [Engel et al., PRD 82 (2010), arXiv:1005.1748]
$a_1$ interpolators

| $a_1$ | interpolator(s) |
|-------|-----------------|
| 1     | $\bar{a}_n \gamma_k \gamma_5 b_n$ |
| 2     | $\bar{a}_n \gamma_k \gamma_5 b_w + \bar{a}_w \gamma_k \gamma_5 b_n$ |
| 4     | $\bar{a}_w \gamma_k \gamma_5 b_w$ |

$a_1$-meson, $J^{PC} = 1^{++}$, cf. [Engel et al., PRD 82 (2010), arXiv:1005.1748]
### $b_1$ interpolators

| $\# b_1$ | interpolator(s) |
|-----------|-----------------|
| 6         | $\bar{a}_\partial_k \gamma_5 b_n - \bar{a}_n \gamma_5 b_{\partial k}$ |
| 8         | $\bar{a}_\partial_k \gamma_5 b_w - \bar{a}_w \gamma_5 b_{\partial k}$ |

$b_1$-meson, $J^{PC} = 1^{+-}$, cf. [Engel et al., PRD 82 (2010), arXiv:1005.1748]
**N interpolators**

- $N(i) = \varepsilon_{abc} \Gamma_1^{(i)} u_a (u_b^T \Gamma_2^{(i)} d_c - d_b^T \Gamma_2^{(i)} u_c)$
- $N(+): 1, 2, 4, 14, 15, 18$
- $N(-): 1, 7, 8, 9$

| $\chi^{(i)}$ | $\Gamma^{(i)}_1$ | $\Gamma^{(i)}_2$ | smearing | #N |
|------------|-----------------|-----------------|----------|----|
| $\chi^{(1)}$ | 1 | $C \gamma_5$ | $(nn)n$ | 1 |
| | | | $(nn)w$ | 2 |
| | | | $(nw)n$ | 3 |
| | | | $(nw)w$ | 4 |
| | | | $(ww)n$ | 5 |
| | | | $(ww)w$ | 6 |
| $\chi^{(2)}$ | $\gamma_5$ | $C$ | $(nn)n$ | 7 |
| | | | $(nn)w$ | 8 |
| | | | $(nw)n$ | 9 |
| | | | $(nw)w$ | 10 |
| | | | $(ww)n$ | 11 |
| | | | $(ww)w$ | 12 |
| $\chi^{(3)}$ | $i$ | $C \gamma_t \gamma_5$ | $(nn)n$ | 13 |
| | | | $(nn)w$ | 14 |
| | | | $(nw)n$ | 15 |
| | | | $(nw)w$ | 16 |
| | | | $(ww)n$ | 17 |
| | | | $(ww)w$ | 18 |

cf. [Engel et al., PRD 82 (2010), arXiv:1005.1748]
Δ interpolators

- $\epsilon_{abc} \ u_a \left( u_b^T \ C \gamma_k \ u_c \right)$
- $\Delta^+): 1, 2, 3$
- $\Delta^-(−): 1, 2, 3$

| smearing | #$\Delta$ |
|----------|-----------|
| $(nn)n$  | 1         |
| $(nn)w$  | 2         |
| $(nw)n$  | 3         |
| $(nw)w$  | 4         |
| $(ww)n$  | 5         |
| $(ww)w$  | 6         |

cf. [Engel et al., PRD 82 (2010), arXiv:1005.1748]