Constraining the Dark Matter Vacuum Energy Interaction Using the EDGES 21 cm Absorption Signal

Yuting Wang1 and Gong-Bo Zhao1,2,3

1 National Astronomy Observatories, Chinese Academy of Science, Beijing, 100101, People’s Republic of China; ytwang@nao.cas.cn
2 University of Chinese Academy of Sciences, Beijing, 100049, People’s Republic of China; gbzhao@nao.cas.cn
3 Institute of Cosmology and Gravitation, University of Portsmouth, Portsmouth, PO1 3FX, UK

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Abstract

The recent measurement of the global 21 cm absorption signal reported by the Experiment to Detect the Global Epoch of Reionization Signature (EDGES) Collaboration is in tension with the prediction of the ΛCDM model at a 3.8σ significance level. In this work, we report that this tension can be released by introducing an interaction between dark matter and vacuum energy. We perform a model parameter estimation using a combined data set including EDGES and other recent cosmological observations, and find that the EDGES measurement can marginally improve the constraint on parameters that quantify the interacting vacuum, and that the combined data set favors the ΛCDM at a 68% confidence level. This proof-of-concept study demonstrates the potential power of future 21 cm experiments to constrain the interacting dark energy models.

Key words: cosmology: observations – dark energy

1. Introduction

Recently, the Experiment to Detect the Global Epoch of Reionization Signature (EDGES) Collaboration reported an excess 21 cm absorption signal at the effective redshift z ~ 17 (Bowman et al. 2018). The amplitude of this observed signal is $T_{21} = -500^{+200}_{-100}$ mK, where the error, including potential systematic uncertainties, is at the 99% confidence level (CL; Bowman et al. 2018). Surprisingly, this measurement is in tension with the theoretical prediction in the standard ΛCDM cosmology at about a 3.8σ significance level, namely, the measured $T_{21}$ almost doubles the ΛCDM prediction, which is $T_{21} = -209$ mK (Barkana 2018).

Much attention from the astrophysics community has been attracted to this discovery, and various interpretations have been proposed to explain the discrepancy. As $T_{21} \propto [1 - T_{\text{CMB}}(z)/T_S(z)]/H(z)$, where $T_{21}$ is the measured intensity of the 21 cm radiation relative to the cosmic microwave background (CMB) temperature $T_{\text{CMB}}(z)$, $T_S(z)$ is the spin temperature of the hydrogen gas, and $H(z)$ is the Hubble parameter, there are in principle three ways (and their combinations) to make $T_{21}$ more negative to be compatible with the EDGES measurement: (A) reduce the spin temperature $T_S$ by introducing new cooling mechanisms, e.g., the dark matter-baryon scattering (Barkana 2018; Berlin et al. 2018; Fialkov et al. 2018; Muñoz & Loeb 2018), (B) raise $T_{\text{CMB}}$ by additional radio background (Ewall-Wice et al. 2018; Feng & Holder 2018; Fraser et al. 2018), or (C) reduce the Hubble parameter (Costa et al. 2018; Hill & Baxter 2018).

In this paper, we propose to release the tension by reducing the Hubble parameter through the interaction between dark matter and dark energy. Specifically, we consider the interacting vacuum energy model4 proposed in Wands et al. (2012), and perform a parameter estimation for this model using a joint data set including EDGES and other kinds of recent cosmological measurements.

This paper is organized as follows. In the next section, we present a brief description of the 21 cm absorption observable. Then we introduce the interacting vacuum energy model in Section 3, before showing the cosmological constraint on the interacting vacuum energy model using observations in Section 4. The last section is devoted to a conclusion and discussions.

2. The 21 cm Absorption Signal

At the Rayleigh–Jeans limit, the brightness temperature of the observed radiation field is (Field 1958; Furlanetto et al. 2006),

$$T_b(z, \nu) = T_{\text{CMB}}(z)e^{-\tau_\nu} + T_S(z)(1 - e^{-\tau_\nu}),$$

where $z$ and $\nu$ denote redshift and frequency, respectively, and $\tau_\nu$ is the optical depth of the inter-galactic medium at frequency $\nu$. $T_S(z)$ is the spin temperature, and $T_{\text{CMB}}$ is the CMB temperature which evolves with redshift as $T_{\text{CMB}}(z) = 2.725(1+z)$ K.

The 21 cm signal is caused by the hyperfine splitting of neutral hydrogen atoms. The transition from the triplet state to the singlet state corresponds to the emission of photons of wavelength at 21 cm, whose frequency is $\nu_0 = 1420.4$ MHz. The intensity of the 21 cm radiation relative to the CMB temperature is thus

$$T_{21}(z) \approx \frac{T_S(z) - T_{\text{CMB}}(z)}{1 + z} \tau_{\nu_0}(z),$$

$$\tau_{\nu_0}(z) = \frac{2e^2/n_A n_H}{16k_B^2/\nu_0 T_S(z)H(z)},$$

where $c$ is the speed of light, $h$ the reduced Planck constant, and $k_B$ the Boltzmann constant. $A_{10} = 2.85 \times 10^{-15}$ s$^{-1}$ is the emission coefficient of the spontaneous transition from the triplet state to the singlet state. $n_H$ is the number density of neutral hydrogen, and $H(z)$ is the Hubble parameter as a

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4 The idea of decaying vacuum energy is a recurring concept to explain the accelerating expansion of the universe (Bertolami 1986; Freese et al. 1987; Chen & Wu 1990; Carvalho et al. 1992; Berman 1991; Pavon 1991; Al-Rawaf & Taha 1996; Shapiro & Sola 2002; Sola 2011).
function of redshift. In ΛCDM, H(z) can be approximated as 
\[ H_0 \sqrt{\Omega_M(1+z)^3} \] at \( z \gg 1 \), such that the optical depth in 
Equation (3) can be rewritten as (Furlanetto et al. 2006)

\[
\tau_{\alpha}^{\Lambda CDM} \approx 8.6 \times 10^{-3} \chi_M \left( \frac{T_{\text{CMB}}(z)}{T_b(z)} \right) \left( \frac{\Omega_b h^2}{0.02} \right) \cdot \left( \frac{0.15}{\Omega_m h^2} \right) \left( \frac{1+z}{10} \right)^{1/2},
\]

(4)

where \( \chi_M \) is the neutral hydrogen fraction, and \( \Omega_b h^2 \) and \( \Omega_m h^2 \)
are the physical baryon and matter densities, respectively.

3. The Interacting Vacuum Energy Model

With the presence of the interacting vacuum energy, the continuity equations for the interacting vacuum V and dark matter \( \rho_{dm} \) are

\[
\dot{V} = Q, \\
\dot{\rho}_{dm} + 3H\rho_{dm} = -Q,
\]

(5)

where \( Q \) is the interaction between the vacuum energy and dark matter. Vacuum energy has a non-varying equation of state, i.e., \( w \equiv -1 \), but a time-evolving energy density due to the interacting term \( Q \), which is different from the interacting dark energy models discussed in Costa et al. (2018), where the equation of state of dark energy is a constant \( w \), but \( w \approx -1 \). In this work, we consider an interaction of the form

\[
Q = 3\alpha H \frac{\rho_{dm} V}{\rho_{dm} + V}.
\]

(6)

Note that \( \alpha \) can be a function of time in general. In this work, we parameterize the time-dependence as

\[
\alpha(a) = \alpha_0 + \alpha_\alpha (1 - a).
\]

(7)

As shown, \( \alpha \) approaches \( \alpha_0 \) and \( \alpha_0 + \alpha_\alpha \) in limits of \( a = 1 \)
and \( a = 0 \), respectively, and interpolates linearly in between. In this model, \( \alpha_\alpha = -\alpha_0 / \alpha_\alpha \), thus it can be used as an indicator of the dynamics of the vacuum. The effective equation of state for vacuum energy is

\[
w_{V}^{\text{eff}} = -1 - \alpha(a) - \frac{\rho_{dm}}{\rho_{dm} + V}.
\]

(8)

This parametrization of \( \alpha(a) \) can realize a quintom-like effective dark energy with the equation of state crossing \(-1 \), as shown in the upper left and the lower right regions of Figure 1. In contrast, a constant interaction parameter, i.e., \( \alpha_\alpha \) as discussed in Wang et al. (2013, 2014), can only yield a quintessence-like dark energy with \( w_{V}^{\text{eff}} > -1 \) for a negative \( \alpha_0 \) or a phantom-like dark energy with \( w_{V}^{\text{eff}} < -1 \) for a positive \( \alpha_0 \).

The Friedmann equation reads as

\[
H^2 = \frac{8\pi G}{3} \left[ \rho_b + \rho_\gamma + \rho_{dm} + V \right],
\]

(9)

where baryons and radiation follow the standard conservation equations. The expansion history of the universe can be solved by combining Equations (5), (6), (7), and (9). Apparently, any non-zero \( \alpha \) yields a modification of expansion history compared with that in the ΛCDM model (Wang et al. 2013). As the 21 cm temperature \( T_{21} \) depends on \( \tau_{s,0} \), which further depends on the expansion rate \( H \) through Equation (3), the interacting vacuum model can leave imprints on the 21 cm observables.

At the perturbation level, we consider an energy flow that is parallel to the 4-velocity of dark matter, i.e., \( Q_{dm}^\mu = -Q_{dm} \delta_{\mu}^{\nu} \). In this case dark matter particles follow geodesics as in ΛCDM, but the continuity equation gets modified, namely, the velocity perturbation for dark matter is not affected by the interaction and obeys the standard equation (Wang et al. 2013, 2014)

\[
\delta_{dm} = 0.
\]

(10)

Thus we will evolve the perturbation equations in a synchronous gauge that is comoving with the dark matter.\(^5\)

Meanwhile, the dark matter density contrast \( \delta_{dm} \) evolves in the dark matter-comoving frame (Wang et al. 2013, 2014),

\[
\delta_{dm} = -\frac{\delta}{2} + \frac{Q}{\rho_{dm}} \delta_{dm}.
\]

(11)

where \( \delta \) is the scalar mode of metric perturbations in the synchronous gauge. In this gauge, the vacuum energy is spatially homogeneous, i.e., \( \delta V = 0 \).

4. Observational Constraints

We use a modified version of Code for Anisotropies in the Microwave Background (CAMB; Lewis et al. 2000)\(^6\) to compute the theoretical prediction of \( T_{21}(z) \) using Equations (2)–(9), given a set of cosmological parameters,

\[
P \equiv \{ \omega_b, \omega_\gamma, \Theta_\gamma, \tau, n_s, A_s, \alpha_0, \alpha_\alpha \},
\]

(12)

where \( \omega_b \) and \( \omega_\gamma \) are the physical baryon and cold dark matter densities, respectively; \( \Theta_\gamma \) is 100 times the ratio of the sound horizon to the angular diameter distance at decoupling; \( \tau \) is the reionization optical depth; \( n_s \) and \( A_s \) are the spectral index and the amplitude of the primordial power spectrum, respectively.

\(^5\) If the initial value of \( \delta_{dm} \) is set to zero, it would remain zero at all times.

\(^6\) Available at https://camb.info.
and \( \alpha_0 \) and \( \alpha_a \) parameterize the strength of the interacting vacuum in the form of Equation (7).

It is assumed that the spin temperature \( T_s(z) \) fully couples to the gas temperature \( T_G(z) \) at redshifts of \( z \approx 15 \sim 20 \), as indicated by the observed 21 cm signal from EDGES, and as discussed in recent paper (Xiao et al. 2018), and we compute the evolution of \( T_G(z) \) using RECFAST (Seager et al. 1999, 2000; Wong et al. 2008; Scott & Moss 2009).\(^7\) The evolution equation of the gas temperature \( T_G(z) \) is given in Seager et al. (1999) and in Scott & Moss (2009), i.e.,

\[
\frac{dT_G(z)}{dz} = \frac{T_G(z) - T_{\text{CMB}}(z)}{H(z)(1 + z)} + \frac{2T_G(z)}{1 + z},
\]

(13)

where \( t_c(z) \) is the Compton-heating timescale, i.e.,

\[
t_c(z) = \frac{3m_e c}{8\sigma_T a_R T_{\text{CMB}}^4(z)} \left[ \frac{1 + f_{\text{He}}(z) + x_e(z)}{x_e(z)} \right]
\]

(14)

Here \( m_e \) is the electron mass, \( c \) is the speed of light, \( \sigma_T \) is the Thomson scattering cross section, \( a_R \) is the radiation constant, \( f_{\text{He}}(z) \) is the fractional abundance of helium by number, and \( x_e(z) \) is the free electron fraction normalized to the total hydrogen number density. The decoupling time between gas and CMB is at \( H \approx 1/t_c(z) \).

As shown in Equation (13), the evolution of the gas temperature also depends on \( H(z) \). In order to figure out the effect of \( H(z) \) on the EDGES signal, we show the cosmic expansion rate \( H(z) \) and Compton-heating rate \( 1/t_c(z) \) in the \( \Lambda \)CDM and the interacting vacuum energy model with fixed parameters, i.e., \( \alpha_0 = -0.5 \) and \( \alpha_a = 0 \) in the upper panel of Figure 2. It is seen that the interacting vacuum model has a smaller \( H(z) \), but the decoupling time between gas and CMB at which \( H \approx 1/t_c(z) \) in the interacting vacuum energy model has very little change, compared with that in \( \Lambda \)CDM. In the lower panel of Figure 2, we show the CMB temperature and gas temperatures in the \( \Lambda \)CDM and interacting vacuum energy model with fixed parameters, i.e., \( \alpha_0 = -0.5 \) and \( \alpha_a = 0 \). Both models have very close gas temperatures. Therefore, according to Equation (2) we can see that a reduced value of \( H(z) \) in

\(^7\) Available at http://www.astro.ubc.ca/people/scott/recfast.html.

\(^8\) Available at https://cosmologist.info/cosmomc/.

![Figure 2](https://www.astro.ubc.ca/people/scott/recfast.html)

**Figure 2.** Upper panel: the cosmic expansion rate and Compton-heating rate in the \( \Lambda \)CDM and the interacting vacuum energy model with fixed parameters, i.e., \( \alpha_0 = -0.5 \) and \( \alpha_a = 0 \). Lower panel: CMB temperature and gas temperatures in the \( \Lambda \)CDM and the interacting vacuum energy model with fixed parameters, i.e., \( \alpha_0 = -0.5 \) and \( \alpha_a = 0 \).

![Figure 3](https://www.astro.ubc.ca/people/scott/recfast.html)

**Figure 3.** Illustration of the intensity of the 21 cm signal relative to the CMB temperature, \( T_{21}(K) \), for various values of \( \alpha_0 \) and \( \alpha_a \). The color bar indicates the values of \( \alpha_a \). The black solid curve corresponds to the case in which \( \alpha \) does not evolve with time (\( \alpha_a = 0 \)). The intersect between the black solid curve and the vertical gray dashed line, marked by a yellow star, denotes the \( \Lambda \)CDM model. The hatched region illustrates the observed 21 cm signal from EDGES at a 99% CL.

Equation (3) would be the main contribution to an increase of the amplitude of the 21 cm signal.

In Figure 3, we show \( T_{21} \), as defined in Equation (2), for various values of \( \alpha_0 \) and \( \alpha_a \), with other cosmological parameters fixed at values consistent with a Planck 2015 cosmology (Planck Collaboration et al. 2016). As shown, the \( \Lambda \)CDM model (\( \alpha_0 = \alpha_a = 0 \)), denoted by the star, is in tension with the EDGES measurement at a 99% CL illustrated by the hatched region. However, interacting vacuum models can in principle release the tension, namely, \( T_{21} \) can be pushed into the hatched region by a large range of the \( \alpha_0 \) and \( \alpha_a \) parameters.

We then perform a Monte Carlo Markov Chain global fit for the parameters in Equation (12) using a modified version of CosmoMC\(^7\) (Lewis & Bridle 2002) with a combined data set including,

1. the angular power spectra of temperature and polarization measurements of CMB from the Planck mission (Planck Collaboration et al. 2016);
2. the Joint Light-curve Analysis sample of supernovae (SNe) measurements (Betoule et al. 2014);
3. the Baryonic Acoustic Oscillations (BAO) distance measurements from 6dFGS (Beutler et al. 2011), Sloan Digital Sky Survey (SDSS) DR7 Main Galaxy Sample (Ross et al. 2015), Lyα forest of BOSS DR11 quasars (Font-Ribera et al. 2014; Delubac et al. 2015), and BOSS DR12 with tomographic information (Wang et al. 2017; Zhao et al. 2017); joint BAO and Redshift Space Distortions (RSD) measurements from WiggleZ (Blake et al. 2012) and from eBOSS DR14 (Zhao et al. 2019); and RSD measurements from 6dFGS (Beutler et al. 2012), 2dFGRS (Percival et al. 2004), SDSS LRG (Samushia et al. 2012), and VIPERS (de la Torre et al. 2013);
4. and the local \( H_0 \) measurement using the Cepheids, i.e., \( H_0 = 73.24 \pm 1.74 \text{ km s}^{-1} \text{ Mpc}^{-1} \) (Riess et al. 2016).

The results are summarized in Table 1 and in Figures 4 and 5. To quantify the constraint on \( \alpha_0 \) and \( \alpha_a \) from the EDEGS measurement, we show the 68%, 95%, and 99% CL contours of
Figure 4. Contour plots for the parameters \( \alpha_0, \alpha_a \) derived from different data combinations including EDGES alone (shaded regions and solid curves in the left corner; the solid curves from left to right denote the 68%, 95%, and 99% CL contours, respectively), CMB + SNe + BAO + RSD + \( H_0 \) (blue dashed), and CMB + SNe + BAO + RSD + \( H_0 \) + EDGES (solid green). The yellow star marks the \( \Lambda \)CDM model. The yellow dashed curves denote the 68%, 95%, and 99% CL contours of \( \alpha_0 \) and \( \alpha_a \) simultaneously, we only vary \( \alpha_0 \) and \( \alpha_a \), namely, the error bars of \( \alpha_0 \) and \( \alpha_a \) are tightened by 4% and 0.2%, respectively, and the figure of merit, which is the determinant of the inverse covariance matrix for \( \alpha_0 \) and \( \alpha_a \), gets improved by 10%, as shown in Table 1. The \( \Lambda \)CDM model is compatible with data within a 68% CL in both cases. This is expected as the comparatively low precision of the EDGES measurement makes it difficult to compete with the remaining combined data sets.

To see the evolution history of \( \alpha \) allowed by current observations, we reconstruct \( \alpha(a) \) using the constraint that we derived with the functional form assumed in the first place, and show the result in Figure 5. As expected, adding the EDGES data barely changes the reconstruction, and a sweet spot, the epoch at which the error of \( \alpha \) gets minimized, shows up at \( a \sim 0.5 (z \sim 1) \) in both cases, and at this epoch, the best-fit value of \( \alpha \) changes sign, i.e., energy transfers from dark matter to vacuum energy at early times (\( z \gtrsim 1 \)) and vice versa at late times (\( z \lesssim 1 \)).

5. Conclusion

The recent measurement of the 21 cm brightness temperature performed by the EDGES team has attracted wide attention, partially due to the fact that the observed signal is far below what is expected in a \( \Lambda \)CDM model. Interpretations have been proposed, and most of which focus on the nature of dark matter.

In this work, we perform a proof-of-concept study of the potential power of 21 cm measurements to constrain the possible interaction between dark matter and dark energy. We find that EDGES alone can yield a non-trivial constraint on \( \alpha_0 \) and \( \alpha_a \), parameters quantifying the interaction (with all other parameters fixed), and an interaction vacuum model is able to explain the measured 21 cm brightness temperature.

Given the large uncertainty in the current EDGES measurement, it marginally improves the constraint on \( \alpha_0 \) and \( \alpha_a \), on top of a compilation of recent measurements of SNe, CMB, BAO, and RSD, and the \( \Lambda \)CDM model agrees with the combined data sets within a 68% CL. An improved test will benefit from more realistic systematic uncertainties in the future. Additionally, future 21 cm measurements, such as the square kilometer array,\(^{10}\) will provide a much more precise measurement on \( \alpha \), which offers a new probe to shed light on the nature of dark energy and dark matter.

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\(^{9}\) As EDEGS alone cannot constrain all of the parameters in Equation (12) simultaneously, we only vary \( \alpha_0 \) and \( \alpha_a \) in this case while the other parameters are fixed to the values derived from the Planck 2015 measurement.

\(^{10}\) More information is available at https://www.skatelescope.org/.
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ORCID iDs
Yuting Wang  @ https://orcid.org/0000-0001-7756-8479  
Gong-Bo Zhao  @ https://orcid.org/0000-0003-4726-7614

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