Large Majorana Mass
from the Anomalous $U(1)$ Gauge Symmetry

Naoyuki Haba\textsuperscript{1}

\textit{Department of Physics, Nagoya University}
\textit{Nagoya, JAPAN 464-01}

Abstract

We show a new and simple model which induces large Majorana masses of right-handed neutrinos. It is based on $U(1)_X$ anomalous gauge symmetry, which is cancelled by the Green-Schwarz mechanism. These Majorana masses can solve the solar neutrino problem by the vacuum oscillation mechanism. The superpotential of this model is scale invariant. The field contents are simple extensions of the MSSM, which have four standard gauge singlet fields, two extra vector-like fields, and right-handed neutrinos in addition to the MSSM.

\textsuperscript{1}E-mail: haba@eken.phys.nagoya-u.ac.jp
1 Introduction

The SUSY theory which guarantees the smallness of scalar masses is the most attractive candidate beyond the standard model. It is likely that the SUSY theory is an effective theory of string. In this paper, we investigate the possibility that such a theory can accommodate the solar neutrino problem. The solar neutrino problem suggests the existence of Majorana neutrinos with masses of $O(10^{10} \sim 10^{12})$ GeV for the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism, or $O(10^{12} \sim 10^{14})$ GeV for the vacuum oscillation (VO) mechanism[1][2][3]. Then how can we understand these mass scales within the framework of the string inspired SUSY theory? We take here the string inspired model which has no scales except for the string scale and soft SUSY breaking terms. In this framework, it is difficult to build the model which can derive large Majorana masses. Although some models have succeeded to produce such an intermediate scale by using non-renormalizable interactions[4], we show here another simple model in which the superpotential have no dimensionful parameters. The superpotential is scale invariant, that is, renormalizable and composed of only cubic interactions. We know that the cubic coupling of the superpotential is calculable in some string theory. So we can search for a string compactification that might lead to this type of models. In the model proposed here, the anomalous $U(1)_X$ gauge symmetry is crucial in generating suitable Majorana masses. The $U(1)_X$ gauge anomaly is cancelled by the Green-Schwarz mechanism[5]. This model can naturally solve the solar neutrino problem by the VO mechanism.

We introduce four singlet fields, two vector-like extra generations, and right-handed neutrinos in addition to usual spectrum of the MSSM. These additional fields play following roles:

1. Four singlet fields are introduced to make the superpotential scale invariant
and generate large Majorana masses. By the suitable $U(1)_X$ charge assignment, we will show that only one of them acquire desired vacuum expectation value (VEV) and other three singlet fields get weak (SUSY breaking) scale VEVs. The former generates the Majorana masses and the latter give the so-called $\mu$ term and masses of extra generations.

2. Two vector-like extra generations are needed to satisfy the condition of the Green-Schwarz mechanism. Unless there are fields which have color and/or EM charges, we have no solution for the Green-Schwarz mechanism. The existence of extra vector-like generations is the general feature in string theories, where the anomaly cancellation of $SU(3)_c \times SU(2)_L \times U(1)_Y$ is automatic.

3. Right-handed neutrinos are added to solve the solar neutrino problem by the see-saw mechanism.

Section 2 is devoted to the explanation of the anomalous $U(1)_X$ gauge symmetry. In section 3, we study the new model. Section 4 gives summary and discussion.

2 Anomalous $U(1)_X$ Symmetry

If there is the anomalous $U(1)_X$ symmetry, a Fayet-Iliopoulos term is induced from the string loop effects\[6\]. In this case, $D$-term becomes

$$D = \frac{g_S^2 M_S^2}{192 \pi^2} \text{Tr} \ q + \sum_i \epsilon_i^{-1} q_i |S_i|^2,$$

where $g_S$ and $q_i$ are the string coupling, and the charge of the anomalous $U(1)_X$ gauge symmetry, respectively. $S_i$ is the scalar component of the $i$th superfield. $M_S$ is the string scale\[7\] which is given by

$$M_S \simeq 5.27 \times g_S \times 10^{17} \text{ GeV}.$$
\(\varepsilon_i\) is the small parameter which can be derived from moduli parameters as shown in the next section.

The scalar potential with soft breaking mass parameters becomes

\[
V = \frac{1}{2} D^2 + \sum_i m_i^2 |S_i|^2. \tag{3}
\]

Here we neglect \(F\) terms and other soft breaking terms. We will see that it is justified for the model shown in the next section. The stationary condition for \(S_i\) is

\[
\frac{\partial V}{\partial S_i} = D q_i S_i^* + m_i^2 S_i^* = 0. \tag{4}
\]

If \(S_i^*\) gets the non-zero VEV, we obtain the relation

\[
D = -\frac{m_i^2}{q_i}. \tag{5}
\]

Note that this requires

\[
m_i^2 \propto q_i. \tag{6}
\]

It is clear that Eq.(5) cannot be satisfied for an arbitrary \(U(1)_X\) charge. Then, we can conclude that the only \(i\)th field denoted by \(\phi\) has the large VEV. The value of VEV becomes

\[
\langle \phi \rangle = \left[ \left( -\frac{g_S^2}{192\pi^2} M_S^2 \text{Tr} q - \frac{m_i^2}{q_\phi} \right) \varepsilon_\phi \right] \approx \left( -\frac{g_S^2}{192\pi^2} \frac{\text{Tr} q}{q_\phi} \varepsilon_\phi \right) M_S \tag{7}
\]

from Eq.(5). In the next section, we will show that this VEV yields Majorana masses of right-handed neutrinos.

3 A Simple Model

Now we consider a simple model which has large Majorana masses of right-handed neutrinos. This model is renormalizable in contrast to the models proposed in Ref.[8].
The superpotential $W$ has only cubic interactions. It assures us of using Eq.(3) for the scalar potential because we assume that squarks and sleptons do not obtain VEVs.

\[ W = y_{ia} L_i H_1 E_c^a + y_{ij}^n L_i H_2 N_j^c + y_{ij}^d Q_i H_1 D_c^j + y_{ia}^u Q_i H_2 U_c^a \\
+ f_{ij}^{(1)} \phi N_i^c N_j^c + f^{(2)} X H_1 H_2 + f_a^{(3)} Y \overline{U}^c U_a^c + f_a^{(4)} Z \overline{E} E_a^c, \]

where $Q_i, L_i, D_i, N_i,$ and $H_{1,2}$ are quark doublets, lepton doublets, right-handed down-sector quarks, right-handed neutral leptons, and Higgs doublets, respectively. $X, Y, Z,$ and $\phi$ are standard gauge singlet fields. The indices $i, j,$ and $a$ denote numbers of generations, which satisfy $i, j = 1 \sim 3$ and $a = 1 \sim 4$. There exist the fourth generation and mirror fields denoted by $\overline{U}^c$ and $\overline{E}^c$ for right-handed up-sector quarks $U^c$ and right-handed charged leptons $E^c$, respectively. By introducing these extra vector-like fields, we can get useful models which have rich CP structures[9]. As we said before, we assume that only fields $X, Y, Z, H_{1,2},$ and $\phi$ get VEVs in order not to break color and EM charge, and in order to keep SUSY unbroken down to the weak scale. Then the $f^{(2)}$ coupling only contribute to the stationary condition in Eq.(4) for the $F$ term. This $F$ term effect can not let the $j$th field $(i \neq j)$ satisfy Eq.(3) without unnatural fine-tuning. So we can say that if the field has a distinct $U(1)_X$ charge, only $\langle \phi \rangle$ is large and $\langle X \rangle, \langle Y \rangle, \langle Z \rangle$, and $\langle H_{1,2} \rangle$ are of order weak (SUSY breaking) scale. The suitable $\mu$ term is effectively derived from $f^{(2)} \langle X \rangle[10]$.

The mixed anomaly coefficients for $U(1)_X$ and $SU(3)_c \times SU(2)_L \times U(1)_Y$ are given by

\[ C_1 = \frac{1}{6} (3[q_{H_1} + q_{H_2}] + 3[q_Q + 8q_{U^c} + 2q_{D^c} + 3q_L + 6q_{E^c}] \\
+ 8q_{U^c} + 8q_{D^c} + 6q_{E^c} + 6q_{E^c}), \]

\[ C_2 = \frac{1}{2} (q_{H_1} + q_{H_2} + 3[3q_Q + q_L]). \]
\[ C_3 = \frac{1}{2} (3[q_Q + q_{Ue} + q_{Dc}] + q_{Ue} + q_{Ue}) , \quad (11) \]

\[ C_{grav} = \frac{1}{24} \text{Tr} \, q. \quad (12) \]

Here, \( C_1, C_2, \) and \( C_3 \) are the coefficients of the mixed \( U(1)_X \times U(1)_Y^2, U(1)_X \times SU(2)_L^2, \) and \( U(1)_X \times SU(3)_c^2 \) anomalies, respectively. \( C_{grav} \) is the gravitational anomaly mixed with \( U(1)_X \). The Green-Schwarz mechanism for anomaly cancellation requires

\[ \frac{C_1}{k_1} = \frac{C_2}{k_2} = \frac{C_3}{k_3} = \frac{C_{grav}}{k_{grav}}. \quad (13) \]

Where, \( k_i \) denotes the Kac-Moody level, which satisfies \( k_1 = 5/3, k_2 = k_3 = k_{grav} = 1 \). More general case is argued in Ref.\[11\]. The \( U(1)_X^2 \times U(1)_Y \) anomaly denoted by \( C_{XXY} \) cannot be cancelled by the Green-Schwarz mechanism. Then the equation

\[ C_{XXY} = -q_{H_1}^2 + q_{H_2}^2 + 3[q_Q^2 - 2q_{Ue}^2 + q_{Dc}^2 - q_L^2 + q_{Ee}^2] \]
\[ -2q_{Ue}^2 + 2q_{Ue}^2 + q_{Ee}^2 - q_{Ee}^2 \]
\[ = 0 \quad (14) \]

must be satisfied to the anomaly. As for \( C_{XXX} \), there is a possibility that other unknown particles may contribute. So we donot consider it.

There are fourteen independent \( U(1)_X \) charge parameters. However, we want to consider only cubic interactions for \( W \) as Eq.(8), these charge parameters are reduced to six independent parameters by using the eight relations as

\[ q_{\phi} + 2q_{Nc} = 0, \]
\[ q_L + q_{H_1} + q_{Ee} = 0, \]
\[ q_L + q_{H_2} + q_{Nc} = 0, \]
\[ q_Q + q_{H_1} + q_{Dc} = 0, \]
\[ q_Q + q_{H_2} + q_{Ue} = 0, \quad (15) \]
\[ q_X + q_{H_1} + q_{H_2} = 0, \]
\[ q_Y + q_{U^c} + q_{U^{\tau}} = 0, \]
\[ q_Z + q_{E^c} + q_{E^{\tau}} = 0. \]

Now we take \( q_{H_1}, q_{H_2}, q_Q, q_{U^{\tau}}, q_{E^{\tau}}, \) and \( q_{N^c} \) for six independent parameters. Imposing these constraints, three equations of the Green-Schwarz mechanism for the anomaly cancellation Eq.(13) becomes

\begin{align*}
4q_{H_1} + q_{H_2} + 10q_{Q} - 3q_{N^c} - q_{U^c} & = 0, \quad (16) \\
12q_{H_1} - 6q_{H_2} + 30q_{Q} - 15q_{N^c} - 3q_{U^c} - 6q_{E^{\tau}} & = 0, \quad (17) \\
25q_{H_1} + 35q_{H_2} + 10q_{Q} - 2q_{N^c} - 10q_{U^c} & = 0. \quad (18)
\end{align*}

The equation (14) becomes to be

\[ C_{XXY} = -q_{H_1}^2 + q_{H_2}^2 + 3q_{Q}^2 - 8(q_{Q}^2 + q_{H_2}^2) + 3(q_{Q} + q_{H_1})^2 \\
-3(q_{N^c} + q_{H_2})^2 + 4(-q_{H_1} + q_{H_2} + q_{N^c})^2 + 2q_{U^c}^2 - q_{E^{\tau}}^2 \\
= 0. \quad (19)\]

From Eqs.(17) and (18), \( f_{ij}^{(1)} \) terms induce supersymmetric Majorana mass terms. The value of Majorana mass \( M_M \) becomes

\[ M_M \simeq \sqrt{-\frac{g_S^2}{8\pi^2} \frac{C_{grav}}{q_\phi} \varepsilon_\phi} \, M_S \simeq \sqrt{-\frac{1}{50\pi} \frac{C_{grav}}{q_\phi} \varepsilon_\phi} \, M_S \simeq \sqrt{\frac{1}{1200\pi} \alpha \varepsilon_\phi} \, M_S, \quad (20)\]

where,

\[ \alpha \equiv -\frac{24 C_{grav}}{q_\phi}. \quad (21)\]

We use \( g_S^2/4\pi \simeq g_{GUT}^2/4\pi \simeq 1/25 \) in Eq.(20). In addition to the above four equations (16)∼(19), we get one more equation from the positivity of \( \alpha \), that is

\[ 13q_{H_1} + 11q_{H_2} + 2q_{Q} + 2q_{N^c}(1 + \alpha) - 2q_{U^c} = 0, \quad \alpha > 0. \quad (22)\]
Now we get the system of six unknown parameters and five equations. We can easily solve these five homogeneous equations. The result is that all $U(1)_X$ charges can be parameterized by $\alpha$. The ratio of the charges of $H_{1,2}$ becomes

$$\frac{q_{H_2}}{q_{H_1}} = \frac{-2375\alpha^2 + 7900\alpha + 5832}{2125\alpha^2 + 7600\alpha + 5832}, \quad (23)$$

and other charges are obtained from Eqs.(15) \sim (18). In fact, there is another solution of Eq.(23), which is $q_{H_2}/q_{H_1} = -1$. However, we cannot get a large Majorana mass because $q_\phi = 0$ in this case.

Now we are in a place to discuss about values of $\alpha$ and $\varepsilon_\phi$. As for $\alpha$, the massless condition induces $\alpha \geq O(10^{-2})$ in the $Z_N$ orbifold model\cite{12}. On the other hand, $\varepsilon_\phi$ might be written in terms of the moduli field $T$ and the modular weight $n$ of $\phi$ as

$$\varepsilon_\phi \simeq \frac{1}{(T + T^*)^n}. \quad (24)$$

Here $T$ can be the order $O(10)$\footnote{It is favorable for the string gauge unification\cite{13,14}.} and the modular weight $n$ can be positive in the twisted sector of the orbifold model\footnote{Since $\phi$ has the $U(1)_X$ charge, $n$ cannot be large integer in practice. However $n = 1$ is really possible for the specific orbifold model as in Ref.\cite{14}.}. Then some orbifold models might really derive $\alpha \cdot \varepsilon_\phi \simeq 10^{-2}$.

In this case, the Majorana mass becomes $6.1 \times 10^{14}$ GeV from Eq.(20). This is suitable for the VO mechanism solution of the solar neutrino problem. However it is impossible to obtain the MSW solution which demands $\alpha \cdot \varepsilon_\phi \simeq 10^{-6}$ in ordinary compactifications.

## 4 Summary and Discussion

We have studied a new and simple model which can induce large Majorana masses of the right-handed neutrinos, which can solve the solar neutrino problem by the

\footnote{We thank Professor Y. Kawamura for the discussion about $\alpha$ and $\varepsilon_\phi$.}
VO mechanism. There is no scale in the original superpotential. The mass scale of Majorana masses is generated dynamically. It is the result of anomalous $U(1)_X$ gauge symmetry, which are cancelled by the Green-Schwarz mechanism. By the assignment of $U(1)_X$ charges, we can obtain suitable Majorana masses for the solar neutrino problem. Do the situation change if we change the number of singlet fields and/or vector-like generations? In the case of decreasing the number of fields, we have no solutions of the Green-Schwarz mechanism. It is easily shown by constructing the model which contain only $D^c(U^c, E^c, N^c, Q, L)$, $D^c(U^c, E^c, N^c, Q, L)$, and one singlet in addition to NMSSM with cubic terms. We have also no suitable solution in the next three cases;

1. NMSSM +$(D^c, D^c) + (E^c, E^c)$+ two singlets + right-handed neutrinos.

2. NMSSM +$(D^c, D^c) + (N^c, N^c)$+ two singlets + right-handed neutrinos.

3. NMSSM +$(U^c, U^c) + (N^c, N^c)$+ two singlets + right-handed neutrinos.

As for the case of increasing the number of fields, the solution of the Green-Schwarz mechanism may become the simple integer charge of $U(1)_X$. It is worth searching whether this $U(1)_X$ charge assignment is derived from orbifold models or not.

**Acknowledgements**

I would like to thank Professor Y. Kawamura for useful discussions and comments. I am also grateful to Professor A. I. Sanda and Professor T. Matsuoka for useful comments and careful reading of the manuscript.
References

[1] M. Gell-Mann, P. Ramond and S. Slansky, in: Supergravity, eds. D. Freedman et al. (North-Holland, Amsterdam, 1979);
    T. Yanagida, KEK lectures, eds. O. Sawada et al. (1980) p.912; Phys. Rev. D23 196 (1981);
    R. Mohapatra and S. Senjanovic, Phys. Rev. Lett. 44 912 (1980).

[2] S. Bludman, D. Kennedy, and P. Langacker, Nucl. Phys. B374, 373 (1992);
    L. Krauss, E. Gates, and M. White, Phys. Lett. B299, 94 (1993);
    P. I. Krastev and S. T. Petcov, Phys. Lett. 299, 99 (1993)
    S. Bludman et al., Phys. Rev. D47, 2220 (1993).

[3] K. S. Hirata et al., Phys. Lett. B280, 146 (1992).

[4] M. Masip, Phys. Rev. D46, 3610 (1992);
    N. Haba, C. Hattori, M. Matsuda, T. Matsuoka and D. Mochinaga, Phys. Lett. B337, 63 (1994);
    N. Haba, C. Hattori, M. Matsuda, T. Matsuoka and D. Mochinaga, Prog. Theor. Phys. 92, 153 (1994).

[5] M. Green and J. Schwarz, Phys. Lett. 149, 117 (1984).

[6] M. Dine, N. Seiberg and E. Witten, Nucl. Phys. B289, 589 (1987);
    J. J. Attick, L. Dixon and A. Sen, Nucl. Phys. B292, 109 (1987);
    M. Dine, I. Ichinose and N. Seiberg, Nucl. Phys. B293, 253 (1987).

[7] V. S. Kaplunovsky, Nucl. Phys. B307, 145 (1988);
    V. S. Kaplunovsky, Nucl. Phys. B382, 436 (1992).
[8] L. Ibáñez and G. G. Ross, Phys. Lett. **B332**, 100, (1994);  
P. Binetruy and P. Ramond, Phys. Lett. **B350**, 49 (1995);  
hep-ph/9508322, M. M. Robinson and J. Ziabicki;  
hep-ph/9511317, T. Gherghetta, G. Jungman and E. Poppitz;  
T. Banks, Y. Grossman, E. Nardi and Y. Nir, Phys. Rev. **D52**, 5319 (1995);  
E. J. Chun, Phys. Lett. **B367**, 226 (1996);  
[12] Y. Katsuki, Y. Kawamura, T. Kobayashi, N. Ohtsubo and K. Tanioka, Prog. Theor. Phys. **82**, 171 (1989);  
Y. Katsuki, Y. Kawamura, T. Kobayashi, N. Ohtsubo, Y. Ono and K. Tanioka,  
Nucl. Phys. **B341**, 611 (1990).  
[11] L. E. Ibáñez, Phys. Lett. **B303**, 55 (1993).  
[14] L. E. Ibáñez, D. Lüst, and G. G. Ross, Phys. Lett. **B272**, 251 (1991);  
L. E. Ibáñez and D. Lüst, Nucl. Phys. **B382**, 305 (1992).