Abstract—Compressor maps are one of the main elements describing the behaviour of centrifugal compressors. Although the compressor map is often provided by the manufacturer, there may be changes during the lifetime of the compressor due to refurbishments or wear. Since the compressor maps are often used in real-time optimization problems, there is a need for simple approximation methods. This paper focuses on approximation of physical models using Chebyshev polynomials instead of third order polynomials which are unable to capture some aspects of the compressor behaviour. Chebyshev polynomials capture the characteristics better than third order polynomials. They provide a flexible tool for compressor map approximation and analysis.

I. INTRODUCTION

Centrifugal compressors are widely used in gas transport networks. One of the main elements necessary for the analysis and control is a compressor map, sometimes called compressor performance map [1]. It shows the relationships between the flowrate, pressure ratio and efficiency of a compressor [2]. Even though the terms ‘compressor map’ and ‘compressor performance map’ are used interchangeably, it is worth noting that in certain works ‘compressor map’ is used to describe the relationship only between the flowrate and the pressure ratio, whereas ‘compressor performance map’ is focused on the efficiency [3].

Although the compressor map is provided by the manufacturer, there may be changes during the lifetime of the compressor due to refurbishments or wear. Compressors have a long life and may be in operation for 30 years or more. The operating conditions, such as gas composition or inlet pressure and temperature are not constant and their influence on a compressor map cannot be neglected [4], [5]. Nonetheless, in most cases the gas composition is assumed constant and additional parameters are introduced in order to compensate for the pressure and temperature changes, i.e. corrected mass flow and corrected speed [4]. This work focuses on the invariant compressor map.

The authors of [6] show how the compressor map may be modelled from first principles with knowledge of some key parameters such as internal dimensions of the impeller (e.g. tip diameter) and blade angle in the diffuser. The map may also be reconstructed from operating data. A typical use for a compressor map is within a real-time optimization framework [3] where there is a need to be able to perform calculations on the map in real-time. For this purpose, a look-up table or other simple approximation is of great value. This paper considers the use of Chebyshev polynomials for approximation of the map. A meta-modelling approach is presented where the polynomials are fitted to data generated from a first principles model.

A compressor map is a graphical depiction of the pressure versus mass flow rate characteristic of the compressor over a range of compressor speeds. Calculating the characteristics of a compressor using physical modelling is based on the assumption that the compressor system can be modelled as an isentropic process in series with an isobaric process including entropy increase [6]. This approach takes into account various losses inside the compressor (see [7], [6], [8]). However, physical modelling requires detailed knowledge about the geometry of the compression unit which is not always available. A data driven approach overcomes this difficulty and to create a model of the process using polynomial approximation. This approach is mainly based on classical polynomials proposed by [9] in which they were applied for analysis of transient states in a compressor and further exploited in [10], who developed a piecewise continuous polynomial approximation. The compressor map is approximated with second order polynomial for negative mass flow and quadratic or third order polynomial otherwise. In [11], the authors investigated spline approximation combined with physical modelling. However, most authors use a single third order (cubic) polynomial [6]. In [12] the approximation using third order polynomials was successfully used for controller design for a compressor. However, this approach is unable to capture some aspects of the compressor behaviour and to preserve the shape of the characteristics. The main contribution of this work is a new method of approximation of compressor maps based on Chebyshev polynomials.

The paper is organised as follows. The first section introduces the compressor map calculation derived from first principles. The next two parts present approximation methods using third order and Chebyshev polynomials. Then the efficacy of both methods is described as the number of measurements varies. The paper ends with discussion of depicted results and suggestions for future work.

II. COMPRESSOR MAP

A compressor map captures the relationship between the pressure rise inside the compressor as a function of
of the mass flow \( m \) and compressor rotational velocity \( N \). In this article, the compressor described in [8] was used. The compressor map for this unit can be found in Figure 1 and is described with the equation (1)

\[
\psi(m, N) = \begin{cases} 
c_a m^2 + \psi_+(0, N), & m \leq 0 
\psi_+(m, N), & m > 0
\end{cases}
\]

where the pressure rise for positive mass flow in a compressor is modelled as

\[
\psi_+(m, N) = \frac{p_2}{p_{01}} = \left(1 + \eta_2(m, N)\frac{\Delta h_{0c,ideal}}{T_{01}c_p}\right)^{\frac{\gamma-1}{\gamma}}
\]

where \( p_2 \) - pressure downstream of a compressor, \( p_{01} \) - inlet pressure, \( T_{01} \) - inlet temperature, \( c_p \) - specific heat capacity, \( \kappa \) - specific heat ratio. The parameter \( c_a \) defines the slope for the negative mass flow rate side of the characteristics and is estimated from experimental data [13].

The isentropic efficiency of a compressor is defined as

\[
\eta_i(m, N) = \frac{\Delta h_{0c,ideal}}{\Delta h_{0c,ideal} + \Delta h_{loss}}
\]

where \( \Delta h_{0c,ideal} \) - total specific enthalpy delivered to the gas and \( \Delta h_{loss} = \Delta h_{ci} + \Delta h_{bf} + \Delta h_{di} + \Delta h_{ii} \) - sum of friction (subscript \( f \)) and incidence (second subscript \( i \)) losses in the impeller (first subscript \( i \)) and diffuser (subscript \( d \)).

Gravdahl in [8] includes also efficiency loss

\[
\Delta \eta_{loss} = \Delta \eta_c + \Delta \eta_{bf} + \Delta \eta_v + \Delta \eta_d
\]

where: \( \Delta \eta_c \) - clearance loss, \( \Delta \eta_{bf} \) - backflow loss, \( \Delta \eta_v \) - volute loss, and \( \Delta \eta_d \) - diffusion loss.

Finally, \( \eta_i(m, N) \) denotes overall efficiency of the compressor as function of mass flow \( m \) and rotational velocity \( N \)

\[
\eta_i(m, N) = \frac{\Delta h_{0c,ideal}}{\Delta h_{0c,ideal} + \Delta h_{loss}}
\]

detailed description and derivation of (2) and (1) and can be found in [8].

The full characteristics of the compressor calculated according to [8] are in Figure 1. As can be seen, this approach gives a realistic depiction of a compressor map.

III. COMPRESSOR MAP APPROXIMATION

Since the model in form (1) requires detailed knowledge of the geometry of the compressor, it might not be possible to use it. The data-driven modelling based on polynomial approximation is another approach which can be used for drawing a compressor map. Table I presents the comparison between physical modelling and polynomial approximation. Although a physical model is more accurate and gives better insight to the behaviour of the compressor, it requires a detailed knowledge about the parameters of the compressor unit. Polynomial approximation methods overcome this requirement at the expense of accuracy.

The numerical data for this section was taken from the physical model calculated in [8] and described in Section II. Since the data for approximation come from a continuous physical model, it is assumed that the measurements can be done for \( N \) ranging from 25000 rpm to 55000 rpm, and for any mass flow from the interval \([-0.2, 0.8]\). Two datasets of measurements were considered for each rotational velocity:

- Five measurements. According to [6], five measurements distributed across the range of mass flow rates should be enough to capture the behaviour of a compressor.
- Twenty measurements. For the flat curves which occur for \( N \in \{20000, 25000\} \) (Fig. 1), five measurements do not preserve the desired shape.

The results were compared using two indicators:

- Approximation error \( e_k \), where where \( k \) is an index denoting the types of approximating polynomials, third order and Chebyshev, respectively. The formula used for calculation is

\[
e_k = \sum_{N_i} \sum_{w_i} (\Psi_k(m_i, N_i) - \Psi(m_i, N_i))^2
\]

where \( (m_i, N_i) \) are the points where measurements \( \Psi(m_i, N_i) \) were taken from physical model, \( \Psi_k \) is

![Fig. 1: Identified compressor map. The red line indicates the maximal value for each rotational velocity. The rotational velocity \( N \) is in revolutions per minute [rpm]](image_url)

| TABLE I: Comparison between physical modelling and polynomial approximation |
|---------------------------------|--------------------------|
|                                | Physical modelling        | Polynomial approximations |
| **Advantages**                  |                          |                          |
| Gave better insight to the     | Does not require          | Less accurate             |
| behaviour of the compressor    | compressor maps/additional|                         |
|                                | experiments               |                         |
| **Disadvantages**               | Requires detailed knowledge of the compressor or additional identification problem |                         |
|                                | Accuracy depends on      |                         |
|                                | identification results    |                         |
|                                | The characteristics is    |                         |
|                                | not differentiable for    |                         |
|                                | \( m = 0 \)              |                         |
the calculated characteristics, \( i = 1, \ldots, 5 \) or \( i = 1, \ldots, 20 \), and \( j = 1, \ldots, 5 \).

- Shape indicator defined as linear interpolation between maximum value of the characteristics calculated for positive mass flow for each rotational velocity. The purpose of the shape indicator is to give a visual indication of how well the approximations match the compressor map in Fig.1. This pressure versus mass flow lines all have a peak for some non-zero mass flow rate, and the shape indicator joins the peaks. The shape indicator for a good approximation should broadly match the dashed red line which is shown in Fig. 1.

It is worth noting that the shape indicator is close to the theoretical surge line defined as function of rotational speed. Nevertheless, the practical surge line might not coincide with theoretical results.

### A. Third order polynomial approximation

This section examines the construction of compressor characteristics by means of third order polynomials using the method recommended in [6], in order to provide a baseline comparison with the Chebyshev polynomial approximation. The algorithm is as follows:

1. For each rotational velocity \( N \) find at least four values of pressure.
2. Find an approximation in form of third order polynomial as function of mass flow.
3. Take the coefficients of the approximating polynomials for a given power of mass and find an approximation as function of rotational speed.

The last step ensures that the characteristics are continuous with respect not only to mass flow, but also to the rotational speed \( N \). The resulting characteristics have the form

\[
\psi(m, N) = c_0(N) + c_1(N)m + c_2(N)m^2 + c_3(N)m^3 \quad (7)
\]

where

\[
c_i(N) = c_{i0} + c_{i1}N + c_{i2}N^2 + c_{i3}N^3 \quad (8)
\]

The parameters \( c_{ij} \), \( i, j = 0, 1, 2, 3 \), were found through least squares optimization.

1) **Approximation of five measurements:** According to [6], it is sufficient to take five data points for each rotational velocity \( N \) in order to obtain an accurate approximation. They are depicted in Figure 2b.

The polynomials (7) are (Fig. 2a):

\[
\begin{align*}
\psi(m, 25000) &= -0.9387m^3 + 0.9521m^2 - 0.4151m + 1.2469 \\
\psi(m, 30000) &= -1.0056m^3 + 0.8813m^2 - 0.3785m + 1.3647 \\
\psi(m, 40000) &= -1.4351m^3 + 1.0042m^2 - 0.2994m + 1.6877 \\
\psi(m, 50000) &= -1.8751m^3 + 1.1624m^2 - 0.1988m + 2.1803 \\
\psi(m, 55000) &= -2.1880m^3 + 1.2837m^2 - 0.1102m + 2.5083
\end{align*}
\quad (9)
\]

The resulting compressor map can be found in Figure 2b. The third order polynomials did not preserve the shape of the curves for most rotational velocities \( N \). The red dotted line in Fig. 2b is the shape indicator which gives a visual indicator of the maximum values of pressure calculated for each positive mass flow rate. The target for the shape indicator is the red dashed line as shown in Fig 1. However, the shape indicator in Figure 2b shows that the fitted curves fail to capture several of the maxima in the pressure versus flow curves that are expected for positive mass flow rates.

2) **Approximation of 20 measurements:** The second approach used 20 points in order to find the approximation. They are depicted in Figure 3b.

The resulting compressor map can be found in Figure 3b. Even though the data points used for approximation approximated the shape of expected curves, the shape indicator line in Fig. 3b shows that the approximation did not preserve it. The curves were flattened by the approximation and the maxima for most of the rotational velocities are shifted to \( m = 0 \).
The parameters $m$ and $n$ denote the numbers of data points available. There are two assumptions about $(x_i, y_j)$ which must be taken into account. The formula (11) assumes that $(x_i, y_j) \in [-1,1]^2$ and $x_i = \cos \left( \frac{i\pi}{n} \right), y_j = \cos \left( \frac{j\pi}{m} \right)$. Therefore, the measurement must be taken according to these two formulas. Moreover, it is necessary to provide a linear scaling for mass flow and rotational velocity:

$$x_i = 2m_i - 0.6,$$
$$y_j = \frac{1}{15000}N_j - \frac{8}{3}.$$

The parameters $r$ and $n$ used for approximation depend on the number of measurements and are gathered in Table II.

**TABLE II: Parameters $r$ and $n$ used for simulation**

| $r$ | $n$ | 5 | 20 measurements |
|-----|-----|---|-----------------|
| 5   | 5   | 5 | 20              |

1) **Approximation of five measurements:** The first approach used five data points. The resulting compressor map is shown in Figure 4.

![Approximation](image)

(a) Approximation as a function of mass flow. The circles denote the points where measurements were taken, the dots denote approximation. From the top: $N = 55000$ rpm, $N = 50000$ rpm, $N = 40000$ rpm, $N = 30000$ rpm, $N = 25000$ rpm

(b) Approximation with shape indicator. The rotational velocity $N$ is in revolutions per minute [rpm]

Fig. 3: Third order polynomials with 20 data points

The polynomials (7) are (Fig. 3a):

$$\psi(m, 25000) = -0.553m^3 + 0.5431m^2 - 0.3529m + 1.2650$$
$$\psi(m, 30000) = -0.693m^3 + 0.5535m^2 - 0.3351m + 1.3810$$
$$\psi(m, 40000) = -1.0436m^3 + 0.6048m^2 - 0.2498m + 1.7065$$
$$\psi(m, 50000) = -1.4252m^3 + 0.6661m^2 + 0.1603m + 2.1929$$
$$\psi(m, 55000) = -1.7957m^3 + 0.8926m^2 - 0.0729m + 2.5253$$

(10)

**B. Chebyshev polynomials approximation**

This section compares the approach using third order polynomials illustrated above with an approximation using Chebyshev polynomials. It was based on ‘chebfun’ package, an open-source Matlab toolbox developed by researchers from Oxford University [14]. It was primarily designed for univariate functions analysis and then extended to two variables. The two-dimensional version is described by formula (11) from [14], [15]

$$f(x, y) \approx p_M(x, y) = \sum_{i=0}^{r-1} \sum_{j=0}^{n-1} a_{ij}(M)T_i(y)T_j(x)$$

(11)

The parameters $n$ and $r$ denote the numbers of measurements for $x$ and $y$, $a_{ij}(M)$ - coefficients of the approximation as functions of the parameter $M$. The functions $T_k(x)$ are Chebyshev polynomials given by the trigonometric formula

$$T_k(x) = \cos(k \cos^{-1}(x))$$

(12)

The algorithm proposed in [15] adjusts the values of $a_{ij}(M)$ so that predicted values of $p_M(x, y)$, $i = 0, \ldots, r - 1$, $j = 1, \ldots, n - 1$ are close in a least squares sense to the measured values $f(x, y)$. The parameter $M$ is called the rank of approximation and describes the number of terms used to calculate the coefficients $a_{ij}(M)$ on the right hand side of (11) [15]. $M$ can take values between 1 and $\min\{r, n\}$.

Applied to modelling of a compressor map, the variables $x_i$ and $y_j$ will be mass flow $m_i$ and rotational velocity $N_j$ which can be measured. The values $f(x_i, y_j)$ denote in this case the pressure ratio. The parameters $r$ and $n$ are the numbers of data points available. There are two assumptions about $(x_i, y_j)$ which must be taken into account. The formula (11) assumes that $(x_i, y_j) \in [-1,1]^2$ and $x_i = \cos \left( \frac{i\pi}{n} \right), y_j = \cos \left( \frac{j\pi}{m} \right)$. Therefore, the measurement must be taken according to these two formulas. Moreover, it is necessary to provide a linear scaling for mass flow and rotational velocity:

$$x_i = 2m_i - 0.6,$$
$$y_j = \frac{1}{15000}N_j - \frac{8}{3}.$$

The parameters $r$ and $n$ used for approximation depend on the number of measurements and are gathered in Table II.

**TABLE II: Parameters $r$ and $n$ used for simulation**

| $r$ | $n$ | 5 | 20 measurements |
|-----|-----|---|-----------------|
| 5   | 5   | 5 | 20              |

The coefficients of the approximation $a_{ij}$ form a $5 \times 5$ matrix (transposed in order to enable easier comparison with...
The shape indicator can be divided into two parts. For small rotational velocities, \( N < 30000 \) rpm, the curves are flat and do not resemble the characteristics in Fig. 1. However, for \( N > 30000 \) rpm, the approximation preserved the shape of the curves and the shape indicator looks similar to the reference curve in Fig. 1.

2) **Approximation of 20 measurements**: The resulting compressor map for 20 data points is shown in Figure 5.

![Fig. 5: Chebyshev approximation with shape indicator - 20 data points](image)

The coefficients of the approximation \( a_{ij} \) form a \( 5 \times 20 \) matrix (transposed due to space requirements):

\[
\begin{bmatrix}
1.71769 & 0.59641 & 0.0967 & 0.02637 & 0.00607 \\
-0.15752 & -0.01929 & -0.00226 & -0.00065 & 0.00061 \\
-0.03545 & -0.04042 & -0.00836 & -0.00225 & -0.00125 \\
-0.03254 & -0.01877 & -0.00323 & -0.00087 & -0.00036 \\
0.03178 & 0.00813 & 0.00125 & 0.00034 & 0.00002 \\
-0.01124 & -0.00137 & 0.00035 & 0.00009 & 0.00013 \\
-0.001149 & -0.00254 & -0.00046 & -0.00012 & -0.00007 \\
0.00423 & 0.00142 & 0.00023 & 0.00006 & 0.00001 \\
-0.00289 & 0 & 0.00002 & 0.00001 & 0.00002 \\
0.00049 & -0.00037 & -0.00007 & -0.00002 & -0.00002 \\
0.00069 & 0.00054 & 0.00009 & 0.00003 & 0.00001 \\
-0.00116 & -0.0007 & -0.00012 & -0.00003 & -0.00001 \\
0.00034 & 0.0001 & 0.00002 & 0 & 0 \\
0.00006 & 0.00056 & 0.00001 & 0.00003 & 0.00002 \\
-0.00069 & -0.00021 & -0.00003 & -0.00001 & 0 \\
0.00059 & 0.00018 & 0.00003 & 0.00001 & 0 \\
0.00045 & 0.00027 & 0.00005 & 0.00001 & 0.00001 \\
0.00004 & 0.00019 & 0.00004 & 0.00001 & 0.00001 \\
0.00031 & 0.00021 & 0.00004 & 0.00001 & 0 \\
-0.00026 & -0.00021 & -0.00004 & -0.00001 & 0
\)

(13)

\[
a_{ij}^T = \begin{bmatrix}
0.0342 \\
-0.0262 \\
0.0661
\end{bmatrix}
\]

The shape indicator in Fig. 5 shows that Chebyshev polynomials preserved the shape of the characteristics for the whole range of rotational velocities. In comparison to the reference characteristics in Fig. 1, it is shifted to the left, but, nevertheless, it suggests that approximation using Chebyshev polynomials is more accurate than using third order polynomials.

**IV. DISCUSSION**

The Table III presents the approximation error for the third order and Chebyshev polynomials.

|                  | Third order polynomials | Chebyshev polynomials |
|------------------|-------------------------|-----------------------|
| **Five points**  | 0.0262                  | 0.0084                |
| **20 points**    | 0.0861                  | 0.0542                |

The Chebyshev polynomials give smaller errors, especially in case of five data points approximation where there is a difference of one order of magnitude. The Figures 2b and 5 confirm that they give more accurate results.

The Table IV presents the approximation error for the third order and Chebyshev polynomials.

**TABLE IV: Approximation error**

|                  | Third order polynomials | Chebyshev polynomials |
|------------------|-------------------------|-----------------------|
| **Five points**  | 0.0262                  | 0.0084                |
| **20 points**    | 0.0861                  | 0.0542                |

An analogous comparison is done for Chebyshev approximation with matrices given by (13) and (14). The analysis of the matrices and the approximation error suggests that this approximation method exhibits properties similar to the third order polynomials approximation. Nonetheless, Figures 4 and 5 show that the Chebyshev approximation is more sensitive to the number of measurements. Even though the approximation error is larger for 20 measurements, the shape indicator for 20 measurements is closer to the baseline curve than the indicator from Fig. 4.

The computational complexity of both methods is also worth considering. The third order polynomial approximation consists in principle of two separate least-square polynomial curve fitting algorithms: the first one with respect to mass flow, the second one with respect to rotational speed. The complexity of each step is given by \( O(C^2K) \) where \( C \) - number of parameters, \( K \) - number of measurements [16]. In this case, \( C = 4 \) since third order polynomials are used in both algorithms, \( K = 5 \) for five measurements and \( K = 20 \) for 20 measurements.

The Chebyshev polynomial approximation has the complexity of form \( O(M(r \log r + n \log n)) \) [15]. Here \( M = 2 \) - internal parameter of chebfun package, and \( r \) and \( n \) are given in Table II. Table IV presents the comparison of both methods.
There are significant differences between the two approaches. The approximation using third order polynomials is more computationally expensive than Chebyshev polynomials for both five and 20 measurements. The number of operations required to approximate 20 measurements using third order polynomials combined with insufficient shape preservation suggests that increasing the number of measurements is not necessary in this method; five data points are sufficient, as suggested in [6]. Chebyshev polynomials based approximation, even though sensitive to the number of measurements, is less computationally expensive.

### TABLE IV: Computational complexity of two approaches

|                | Third order polynomials | Chebyshev polynomials |
|----------------|-------------------------|-----------------------|
| **Five points** | \(O(4^2 \cdot 5) + O(4^2 \cdot 5) \approx 2O(80)\) | \(O(2(5 \log 5 + 5 \log 5)) \approx O(32)\) |
| **20 points**  | \(O(4^2 \cdot 20) + O(4^2 \cdot 5) \approx O(320) + O(80)\) | \(O(2(10 \log 20 + 10 \log 20)) \approx O(136)\) |

(a) Third order polynomials approximation

(b) Chebyshev polynomials approximation

Both methods can be used directly in Matlab software; the chebfun package required for approximation with Chebyshev polynomials is available on the authors’ website [14]. Table V gathers the advantages and disadvantages of both approaches.

### TABLE V: Comparison of polynomial approximations

| Advantages | Third order polynomials | Chebyshev polynomials |
|------------|-------------------------|-----------------------|
| Direct implementation | Preserve the shape of the curves for negative mass flow | Preserve the shape of the curves for positive and negative mass flow |
| Not sensitive to number of measurements | Accurate for small number of measurements | More flexible |

| Disadvantages | Third order polynomials | Chebyshev polynomials |
|---------------|-------------------------|-----------------------|
| Do not preserve the shape of the curves for positive mass flow | Require chebfun package | Require initial data analysis (scaling) |
| Similar results for five and 20 points | Sensitive to number of measurements | |
| Computational expensive | | |

### V. CONCLUSION

The compressor map is an important element in the analysis of a compression system. Often provided by the manufacturer of the compressor, it can change during the lifetime of the system. Since real-time optimization is one of the main applications of a compressor map, there is a need to provide a simple method which will perform the calculations in real-time. One of the solutions is to use third order polynomial approximation approach. However, this method does not preserve some of the features of the compressor map. This paper applies the polynomial approximation based on Chebyshev polynomials to reconstruct the map created with first principles approach.

The approximation using Chebyshev polynomials captures all the important aspects of compressor behaviour. Moreover, the comparison shows that Chebyshev polynomials allow the map to be reconstructed accurately from fewer points than third order polynomials. This can be useful when using experimental data since there may only be a few operating points available for the calculations. As suggested by one of the reviewers, the method will be also useful when dealing with compressor instability, as well as with the optimization of the working point of the compressor itself because it improves the accuracy.

Nevertheless, the validation of approximation using Chebyshev polynomials on experimental data is a matter for future research; it will also take into account the efficiency maps. Further works include also its application in dynamic modelling of a compressor train.

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