Light fermion mass generation in dynamical symmetry breaking

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Abstract

We reconsider the question of mass generation for fermions coupled to a set of gauge bosons (in particular, the electroweak gauge bosons) when the latter get their masses through the Goldstone bosons originating in a simple (i.e. not extended) technicolor sector. The fermion global chiral symmetries are broken by including four-fermion interactions. We find that the system can be nonperturbatively unstable under fermion mass fluctuations driving the formation of an effective coupling between the technigoldstone bosons and the ordinary fermions. Minimization of an effective action for the corresponding composite operators describes then dynamical generation of light fermion masses $\sim M \exp(-k/g^2)$, where $M$ is some cutoff mass.

\footnote{Research supported by NSF grant NSF-PHY 9819686}
1 Introduction

Electroweak dynamical symmetry breaking (DSB) provides a natural and attractive mechanism for generating the $W$ and $Z$ masses. It has proven much more difficult, however, to satisfactorily account for the quark and lepton masses within such a framework. Extended technicolor, walking technicolor, top condensation, and top-color assisted technicolor are among the various proposals that have been investigated. (For reviews and references, see [1].)

In this paper we reconsider the question of fermion mass generation in a framework employing only simple (i.e. not extended) technicolor. Specifically, we investigate the following question. Consider the system consisting of a simple technicolor sector, electroweak interactions, and the ordinary quarks and leptons. In addition, postulate four-fermi interactions among the ordinary fermions that explicitly break their global chiral symmetries. Thus, restricting to one fermion family only, introduce the interactions [2]:

$$\mathcal{L}_{4f} = \frac{4\pi^2}{N_c \Lambda^2} \left[ G_1 (\bar{q}_i^L q_j^R)(\bar{q}_k^R q_l^L) + G_3 (\bar{q}^i_L q^j_R) \tau^3_{jk}(\bar{q}^k_R q^l_L) + G_2 (\bar{q}^i_L q^j_R) \epsilon_{ik} \epsilon_{jl}(\bar{q}^k_R q^l_L) + \text{h.c.} \right]$$

(1)

where sum over color indices is understood, $i, j, k, l$ are isospin indices, and $\Lambda$ some UV cutoff. [1] reduces the weak symmetry group to just $SU(2)_L \times U(1)_Y$. Recall that in the standard Higgs model this is a function fulfilled by the Yukawa couplings. Note that for sufficiently large values of the $G_1$ coupling, this four-fermion interaction can induce dynamic chiral symmetry breaking and mass generation (NJL model), as in fact is assumed in top condensation schemes [1]. Here we will always assume that four-fermion couplings in (1) are below their critical value for inducing any mass generation effects solely by themselves.

The electroweak gauge bosons are assumed to acquire mass through the Goldstone bosons associated with CSB in the technicolor sector. We then ask whether in this system mass generation for the ordinary fermions can also occur.

Now since, with only the interactions specified above present, the ordinary fermions can communicate with the techniquark sector only through the electroweak gauge bosons, and gauge interaction vertices preserve chirality, it is clear that this cannot happen to any finite order in the couplings. The question is whether it can happen nonperturbatively. The conventional answer to this would appear to be negative: the weak interactions are ‘too weak’ to produce such dynamical mass generation. This, however, is actually a spurious argument. The weakness of the weak interactions could be no more relevant here than it is in the generation of the $W$ and $Z$ masses. It is the strong technicolor interactions that produce the necessary Goldstone bosons, and the relevant question is whether an effective Yukawa coupling can form nonperturbatively.

Indeed, the symmetry of the three terms is $SU(2)_L \times SU(2)_R \times U(1)_Y \times U(1)_A$, $SU(2)_L \times U(1)_Y \times U(1)_V \times U(1)_A$, and $SU(2)_L \times SU(2)_R \times U(1)_V$, respectively.
between these Godstone bosons and the quarks. What determines this, in physical terms, is whether the coupled system is unstable under mass fluctuations. If it is, then even very weak interactions can drive the instability (even though the instability cannot be seen to any finite order in perturbation theory).

The same question arises more generally when the standard electroweak gauge bosons are replaced by some other set of gauge bosons at a different, perhaps much higher, mass scale. In fact it is in this form that the question would more likely be pertinent to mass generation for all three fermion generations, as well as more general use of DSB in the construction of models. In the following we consider only the case of the standard electroweak interactions with one fermion family, and the simplest QCD-like technicolor sector; but the same analysis applies in the more general context. This analysis leads to a definition of an effective action, and hence, by minimization, to a set of self-consistent equations for dynamically generated effective Yukawa vertices. Unfortunately, consideration of these equations in general entails very considerable computational complexity, but the physical context is transparent. The trivial (perturbative) solution is always a solution, but one finds that, depending on the model, a nontrivial solution may also exist. From the structure of the self-consistent equations, it follows that a nontrivial solution, when it exists, generically describes dynamical generation of fermion masses $\sim M \exp(-k/g^2)$, where $k$ depends on the couplings $G_i$ in (1), and $M$ is some (cutoff) mass. In the case of the electroweak interactions treated below this gives very small fermion masses $\sim m_W \exp(-k/g^2)$.

2 Preliminaries

We consider the minimal technicolor theory with technicolor gauge group $G_{TC}$, and two flavors of fundamental representation massless techniquarks $Q = (U, D)$. (It is important for our purposes to work within a model that allows a certain amount of computation. $G_{TC} = SU(2)$ may in fact be the only experimentally still viable QCD-like simple technicolor model.) The global chiral group is then $SU(2)_L \times SU(2)_R$, and is spontaneously broken by the strong technicolor interaction to diagonal $SU(2)$ resulting in mass $M$ for the technifermions, and a triplet of Goldstone bosons (the technipions) $\phi^a$. The corresponding broken chiral generators are given by $\gamma^5 \tau^i$, $i = 1, 2, 3$, where $\tau^i$ are the Pauli matrices. The coupling of the Nambu-Goldstone bosons to the technifermions is, for momenta below a cutoff $\Lambda \sim M$, adequately represented in a simple NJL model (for review and references, see e.g. [3]), or an equivalent linear sigma model effective description, giving

$$\phi^a \left\langle Q \right\rangle = \gamma^5 \tau^a G P(p + k, p)$$

(2)
with (dimensionless) wave-function $P(p + k, p) = 1 + O(k/M)$. For momenta above $\Lambda$, $P(p, k)$ decays rapidly to zero – the detailed UV behavior will in fact be irrelevant for physics at scales well below $M$. The effective coupling $G$ is related to the techniupion decay constant $F$ by the GT relation $M = FG$. The triplet of pions is accompanied by a massive bound state, the sigma or real higgs scalar, of mass $2M$.

For application to standard electroweak theory, the techniquarks form an electroweak left-handed doublet $Q_L = (U_L D_L)$, and right-handed singlets $U_R, D_R$. Anomalies are cancelled by assigning hypercharge 0 to $Q_L$, and $\pm 1/2$ to $U_R, D_R$, respectively. As it is well-known, the electroweak gauge bosons then acquire mass through the pole in their polarization tensor (Schwinger mechanism) generated by the coupling of the Goldstone bosons (figure 1). The resulting mass matrix reproduces the familiar electroweak gauge boson mass matrix with $m_W = gF/2$.

It should be noted that the pole contribution represented in figure 1 gives only the $k^\mu k^\nu$ part of the polarization tensor $\Pi_{ab}^{\mu\nu}(k) = (k^2 g_{\mu\nu} - k^\mu k^\nu)\Pi_{ab}(k^2)$. It is not easy to identify directly in terms of diagrammatic contributions, such as in figure 1, the accompanying $g_{\mu\nu}$-part which must be there because of gauge invariance. This is typical of DSB computations where one often must rely on general, in principle non-perturbative, constraints to trace the symmetry breaking effects. In the present case, the transverse polarization tensor is related to the 3-gauge boson proper vertex by a Ward identity, which in the zero momentum transfer limit becomes

$$
limit_{q, k \to 0} q^\mu \Gamma_{abc}^{\mu\nu\lambda}(q, k, -k - q) = -i(k^2 g^{\epsilon\lambda} - k^\epsilon k^\lambda)[(T^a - B^a), \Pi(k^2)]_{bc}.
$$

Here $T^a$ are the adjoint generators of $SU(2) \times U(1)$. (Coupling constants are included in the definition of generators.) The matrix $B^a$, also proportional to $T^a$, involves the FP ghosts and is actually irrelevant for the development below. The pole contribution $\Pi_{ab}(k^2) \to \mu_{ab}^2/k^2$ then satisfies

$$
limit_{q, k \to 0} q^\mu \Gamma_{abc}^{\mu\nu\lambda}(q, k, -k - q) = (g^{\epsilon\lambda} - k^\epsilon k^\lambda)[T^a, \mu^2]_{bc}.
$$

A Goldstone pole with nonvanishing residue in the 3-gauge boson proper vertex then implies a nonvanishing symmetry-violating mass matrix such that the commutator $[T^a, \mu^2]$ on the r.h.s. does not vanish. The occurrence of such poles follows from the existence of nonvanishing effective $\phi^\pm W^\mp V$ couplings, $(V = \gamma, Z)$, as is easily verified.

Figure 1: Schwinger mechanism
by explicit computation using (2) and the GT relation. The resulting residues on the l.h.s. of (4) precisely match the commutator on the r.h.s. computed with the mass matrix surmised from figure 1.

3 How the fermions can get mass

Consider now the coupling of the ordinary quarks and leptons. It suffices to consider a singlet doublet \( q = (u, d) \). The gauge boson-quark-quark proper vertex is related to the inverse quark propagator \( iS^{-1}(p) = /p - \Sigma(p) \) by the non-Abelian version of the original QED WI. In the zero momentum transfer limit one has

\[
\lim_{q \to 0} q_{\mu} \Gamma_{ij}^{\mu a}(p + q, p) = -i \Sigma_{il}(p) [t_{ij}^a - B_{ij}^a(p)] + i \gamma^0 [t_{il}^a - B_{il}^a(p)] \gamma^0 \Sigma_{lj}(p). \tag{5}
\]

The \( t^a \)'s denote the generators in the fermion representation. Again, the quantity \( B_{ij}^a \) involves the FP ghosts and need not be given explicitly here as it does not enter the argument in the following.

If the l.h.s. does not vanish, i.e. if the gauge boson-quark-quark proper vertex \( \Gamma_{ij}^{\mu a} \) acquires a pole, (5) shows that the quark self-energy \( \Sigma(p) \) must possess a symmetry violating part resulting in a (dynamically generated) nonvanishing mass matrix. Now \( \Gamma_{ij}^{\mu a} \) will acquire a pole if an effective vertex

\[
\lim_{q \to 0} q_{\mu} \Gamma_{ij}^{\mu a}(p + q, p) = -i \Sigma_{il}(p) [t_{ij}^a - B_{ij}^a(p)] + i \gamma^0 [t_{il}^a - B_{il}^a(p)] \gamma^0 \Sigma_{lj}(p). \tag{5}
\]

linking the Golstone bosons to the quarks can be dynamically generated. Since communication with the techniquark sector occurs only through the exchange of gauge bosons, it is clear that, starting with massless bare quarks, this cannot happen to any finite perturbative order. But nonvanishing contributions to (5) can arise in the presence of mass fluctuations, i.e. massive quark propagators, and the question is whether a consistent nonperturbative solution fixing a nonzero mass can exist.

To lowest order in the electroweak couplings, such nonvanishing contributions are shown in figure 2. The computation of graphs in figure 2 and in what follows is done as follows. Propagators for internal gauge boson lines are in Landau gauge, and, correspondingly, all external lines are transverse except for those external lines taken with longitudinal polarization as part of the statement of a WI (incoming line of momentum \( q \) in the above cases). As always in dynamical symmetry breaking computations, the use of the Landau gauge is practically mandatory. It automatically ensures that the Goldstone pole remains massless, and it contributes only at a vertex to which the particular longitudinal leg(s) specified in the WI is (are) attached - the transverse
Landau propagator automatically eliminating graphs with longitudinal Goldstone pole contribution in all other vertices. All self-energies are assumed resummed giving dressed propagators.

Connecting the φ’s in figure 2 to incoming W⁺ (W⁻) (cp. figure 1), one obtains, for quark momentum p → 0, a contribution to the vertex \( \Gamma_{\mu a}^{ij} \) given by:

\[
\mp ig \frac{q^\mu}{q^2 + i\epsilon} 3e^2 \left[ J(M, m_W, m_V, m_u) q_1 m_u \frac{1}{2} (1 \mp \gamma^5) + J(M, m_W, m_V, m_d) q_2 m_d \frac{1}{2} (1 \pm \gamma^5) \right] \frac{\tau^\pm}{2} .
\]  

Upper (lower) signs refer to incoming W⁺(W⁻), and

\[
J(M, m_W, 0, m) = \frac{1}{16\pi^2} \left[ \ln \left( \frac{m_W^2}{m^2} \right) + O(g^2, \frac{m_W^2}{M^2}, m \ln m) \right] \]  

\[
J(M, m_W, m_Z, m) = \frac{1}{16\pi^2} \left[ \left( \frac{m_Z^2}{m^2} \right) + O(g^2, \frac{m_W^2}{M^2}, m \ln m) \right] .
\]  

In (7) \( m_V = 0 \) or \( m_Z \) for \( V = \gamma \) or \( Z \), resp.; for \( V = Z \) one must also replace \( e^2 \rightarrow e^2 \tan^2 \theta_W \). Also, \( q_u \) \( q_d \) denote the elm charges of the quark or lepton doublet \((u, d)\). In obtaining (7) we made use of the GT relation, and approximated the fermion self-energy by

\[
\Sigma(k) \approx \Sigma(0) = m_q = \text{diag}(m_u, m_d) .
\]  

Consider then (3) at \( p = 0 \). Inserting (4) in the l.h.s. of (3), one sees that, as expected, the WI is (trivially) satisfied by the perturbative solution \( m = 0 \). There is, however, also the possibility of a nonperturbative solution \( m \neq 0 \) if

\[
3e^2 \sum_{V=\gamma,Z} J(M, m_W, m_V, m_u) q_u = 1 \]  

\[
3e^2 \sum_{V=\gamma,Z} J(M, m_W, m_V, m_u) q_d = -1 .
\]  

Then (11), (12) give

\[
m_q = m_W \exp \left\{ - \frac{8\pi^2}{e^2} \frac{1}{3|q|} + \frac{1}{2} \left( \frac{m_W^2}{m_Z^2} \right) \tan^2 \theta_W \right\} \left( 1 + O(g^2, \frac{m_W^2}{M^2}) \right) , \quad q = u, d
\]  

Figure 2: Contribution to (6) in presence of mass fluctuations.
with \( q_u > 0, \ q_d < 0 \).

It should be noted that the loop involving the gauge bosons is UV convergent and receives little contribution from momenta well above \( m_W \). In particular, in the case of photon exchange practically the entire contribution to the integral, the logarithmic term in (8), comes from the IR regime well below \( m_W \). In fact, the integral becomes singular in the limit \( m_q \to 0 \). This justifies the approximation (10). Thus, (3) is solved by a dynamical fermion \( \Sigma(k) \) taken to be a slowing varying function representing a soft mass \( \sim m_q \) given by (13) for momenta well below \( M \), and falling off rapidly in the UV region \( \sim M \). One is essentially approximating \( \Sigma(k) \) by a step function. In a more refined approximation the falloff is fixed by the correction terms in (8), (9).

(Analogous remarks apply of course also to the dynamically generated \( m_W, m_Z \), and techniquark masses.)

The possible existence of such a nonperturbative solution to the WI would signify that the \( m_q = 0 \) solution is unstable under any nonzero mass fluctuation. This situation occurs in a wide range of mass generation phenomena, including chiral symmetry breaking in QCD. We noted that there is no smooth \( m \to 0 \) limit in the conditions (11)-(12). Also, the nonperturbative form \( \sim \exp(-\text{const.}/e^2) \) for the resulting mass ratio is characteristic of mass generation driven by gauge interactions as opposed to quadratically divergent scalar or four-Fermi interactions.

By the same token, however, multiple gauge boson exchanges beyond the lowest two gauge boson exchange of figure 2 cannot, in general, be ignored. This is because an \( n+1 \) gauge boson exchange contribution to (3), for example, may in general contain \( \left( \ln(m_W^2/m^2) \right)^n \) terms which, for \( m \) of the form (13), are comparable to the lowest order two-rung exchange of figure 2. Thus the higher loop contributions cannot be ignored, and the solution (13) cannot be trusted.

This is in fact illustrated by turning to the remaining WI’s (4), i.e. those for the generators \( t^3, t^0 \). The latter, corresponding to incoming photon, is trivially satisfied: due to the nonchiral photon coupling, contributions to both sides of (5) (in particular pole contributions such as those of arising from figure 2), vanish identically. In the case of incoming \( Z \) connecting to \( \phi^0 \), however, it is not hard to check that there is no nonvanishing contribution to the proper \( \phi^0 \bar{u}u \) and \( \phi^0 \bar{d}d \) vertices analogous to figure 2, i.e. involving the exchange of just two gauge bosons. (This in fact reflects the structure of the electroweak gauge boson mass matrix.) As can be explicitly verified, nonvanishing such contributions to (4) begin with the exchange of three gauge bosons.

Though (13) cannot be trusted, the computation leading to it does show how fermion mass generation mediated by the generation of an effective coupling (3) between technipions and quarks is in principle possible and consistent with gauge invariance. The problem is how to estimate the effective vertex (3). Contributions to the blob on the r.h.s. in (3) involve not only direct exchange of gauge bosons, but also technistrong interaction effects through, e.g., further Golstone boson exchanges. Thus a nonvanishing contribution of the type of figure 2 gives rise to further processes of the type:
These, assuming a solution for nonzero mass of the form (13), can indeed give a larger contribution than other direct gauge boson exchange processes, and in fact further generate an infinite set of such exchanges by iteration. As always in DSB one cannot expect to be able to express a nonperturbatively generated effective vertex by any finite set of particular contributions. Rather, one has to determine the effective vertex, in this case (6), through self-consistent equations which implicitly incorporate infinite sets of graphs without double-counting.

4 Self-consistent equations for dynamical mass generation

The only systematic way of obtaining such equations is through the construction of the effective action for composite operators \[ \phi q \bar{q} \] here applied to \( \phi q \bar{q} \). This is done as follows. One introduces a source \( v_{ij}(x, \bar{y}, y) \) coupled to \( \phi(x) \bar{q}(y) q_j(y) \) in the functional integral for techniquarks, electroweak gauge bosons and quarks to obtain the generating functional \( iW[v] = \ln Z[v] \). (Note that \( \phi \) is itself a composite field, but we assume that the techniquark sector is adequately described by the effective linear sigma model description, cp. (3).) The effective action \( \Gamma \) for the 3-point vertex

\[
g'_{ij} = \frac{\delta W[v]}{\delta v^a_{ij}} = \Delta^{ab} S_{ik} \gamma^b_{kl} S_{lj} \quad (14)
\]

is then defined by

\[
\Gamma[\gamma] = W[v] - v \cdot g, \quad \frac{\delta \Gamma}{\delta g} = -v. \quad (15)
\]

Here \( S, \Delta \) denote the full quark and Goldstone propagators, and hence \( \gamma \) represents the proper vertex (3). The effective action \( \Gamma[\gamma] \) can be expanded in the form:

\[
\Gamma[\gamma] = \Gamma_0 - \frac{1}{2} \gamma^b_{ik} S_{jl} \Delta_{ba} S_{ki} \gamma^a_{lj} + \Gamma_3[\gamma], \quad (16)
\]

where

\[
\Gamma_3 = \left\{ \text{sum of all only trivially 3PR vacuum graphs} \right\}. \quad (17)
\]
(Obvious condensed notation with summation-integration over repeated generalized indices is used.) An only trivially 3PR (3-particle-reducible) graph is a 2PI (2-particle irreducible) graph that can be cut into two parts by cutting three lines if and only if one and only one of the two parts consists of a single 3-point vertex. Otherwise the graph is 3PI. All graphs are computed with dressed propagators.

The $\gamma^a_{ij}$ vertices are determined by the minimization of the effective action (13):

$$\frac{\delta \Gamma[\gamma]}{\delta \gamma} = 0.$$  \hspace{1em} (18)

Imposing (18) gives, to lowest order in the skeleton loop expansion (17), the following coupled set of self-consistent equations:

$$\begin{align*}
\begin{array}{c}
\includegraphics{eq19a} \\
= \quad \includegraphics{eq19b} \\
+ \quad \includegraphics{eq19c} \\
+ \quad \includegraphics{eq19d} \\
+ \quad \includegraphics{eq19e} \\
= \quad \includegraphics{eq19f} \\
+ \quad \includegraphics{eq19g} \\
+ \quad \includegraphics{eq19h} \\
+ \quad \includegraphics{eq19i}
\end{array}
\end{align*}$$

(19)

There is also the corresponding set of equations with the $\phi^\pm \bar{u} d$, $\phi^- \bar{d} u$ vertices on the l.h.s. In (19)-(20) oriented wavy lines represent $W^\pm$'s and oriented dashed lines represent $\phi^\pm$'s, with positive charge flow into a vertex in the direction of the arrow. Unoriented wavy and dashed lines represent $\gamma$, $Z$, and $\phi^0$, respectively. For brevity, each graph depicted in (19)-(20) is understood to stand for a set of graphs that, in addition to the graph in question, also includes all ‘crossed’ graphs that may be formed by permutations of its vertices. The small filled blobs represent the techniquark-loop-induced vertices between $W^\pm$, $Z$, $\gamma$ and technipions, e.g

$$\begin{align*}
\begin{array}{c}
\includegraphics{eq21a} \\
\equiv \quad \includegraphics{eq21b} \\
= -ig^2 s_W \frac{1}{2} g^\mu\nu + O(k^2/M^2),
\end{array}
\end{align*}$$

(21)
(with $s_W \equiv \sin \theta_W$, $k$ of the order of the external momenta). The last graph on the r.h.s. in (19) and (20) is the contribution from the four-quark interactions (square vertices).

The Goldstone boson is massless (Landau gauge). The self energy in the dressed quark propagator in (19)-(20), on the other hand, is related by the WI (5) to the vertex (6). Indeed, making use of the GT relation, (5) is seen to be equivalent to the following relation between the self energy, in the approximation (10), and the effective $\gamma$ vertices:

$$
\begin{align*}
\phi^+ & \quad d, u \\
\phi^- & \quad u, d \\
\phi^0 & \quad d, u
\end{align*}
$$

$$
= \pm \frac{1}{\sqrt{2}} g \left[ \frac{m_u}{m_W} \frac{1}{2} (1 \mp \gamma^5) - \frac{m_d}{m_W} \frac{1}{2} (1 \pm \gamma^5) \right]. \quad (22)
$$

$$
\frac{1}{2} g \frac{m_q}{m_W} \tau^3 \gamma^5
$$

(23)

(In (22) upper (lower) signs refer to incoming $d$ ($u$)).

(19)-(20) are to be solved together with (22), (23). Note that these equations are indeed what one would basically expect to get in a self-consistent Hartree-Fock approximation for the vertices (22)-(23). The effective action (17), however, provides in principle a systematic approximation scheme. $m_u = m_d = 0$ is seen to be, trivially, always a solution; we are looking for a nontrivial nonperturbative $m_q \neq 0$ solution.

Note that, when terms suppressed by inverse powers of $M$ are neglected, the techniquarkloop-induced vertices are essentially those of the standard model (cp. second equality in (21)). Even so, the system (19)-(20) appears rather formidable due to the remaining 2-loop structures and the large number of graphs involved.

What makes computations feasible is the fact that in the limit of vanishing external momenta all 2-loop graphs are in principle explicitly computable in terms of Spence (di-log) functions [5]. By a variety of tricks, every 2-loop integral with numerator momentum tensor structures can be reduced to a series of 2-loop scalar field theory integrals, which in turn can be related to standard types involving four propagators. These procedures generally generate large number of terms. The scalar integrals can then be explicitly integrated in terms of dilogs, which can finally be expanded in ratios of masses. In the case at hand the use of Landau gauge gives numerator tensor structures with up to six powers of momenta making computation of graphs very laborious and lengthy. The structure of the results, however, is rather easily stated.

In computing the r.h.s. of (19)-(20) we assume that $m_q = m_d$, $m_u$ are much smaller than all other mass scales, and are interested in exhibiting the expected large double and single pure $\ln(m_q/m_W)$ terms. Fortunately, not all graphs in (19)-(20) provide such large logs. Examination of the expansion in mass ratios of all possible dilogs resulting from the various graphs shows that only graphs containing a photon line give such logs.
It should be pointed out that this is not really a consequence of the masslessness of the photon, but of the (near) degeneracy of the $Z$, $W^+$, and $W^-$ masses. We also take the cutoff in (I) to be of the order of the technicolor CSB cutoff.

We have computed explicitly planar graphs (taking the techniquark loop structure into account) on the r.h.s of (19)-(20). Though this is a lengthy computation, the structure of the resulting two conditions for $m_d$, $m_u$ is simple. Taking $G_2$ in (I) to be small compared to $G_1$, $G_3$, one finds:

\[
\left(\frac{g^2}{16\pi^2}\right)^2 \left[ s_w q_q \left( \frac{9}{8} \ln^2 \left( m_W^2 / m_q^2 \right) + \left( c_1 + c'_1 \ln \left( \frac{M^2}{m_q^2} \right) \ln \left( \frac{m_W^2}{m_q^2} \right) \right) \right] \right.
\]

\[+ O\left( \ln^2 \left( \frac{M^2}{m_W^2} \right), 1, \frac{m_W}{M^2}, \frac{m_q}{m_W}, \ln \left( \frac{m_{u,d}}{m_W} \right) \right) \] \[+ \left( G_1 \pm G_3 \right) \left[ 1 + O\left( \frac{m_q^2}{M^2} \ln \left( \frac{M^2}{m_q^2} \right) \right) \right] = \pm 1
\]

for $m_q = m_u$, $q_q = q_u$ and upper sign, and for $m_q = m_d$, $q_q = q_d$ and lower sign. $c_1$, $c'_1$ are numerical constants of order unity, and only the order of all other resulting terms is indicated. (24) then gives:

\[
m_q \sim m_W \exp \left\{ - \frac{8\pi^2}{g^2} \left( \frac{1 - C_q}{s_W^2 |q_q| c_2} \right)^{1/2} \right\}, \quad q = u, d
\]

where $c_2 = 9/8$, and $C_u = (G_1 + G_3)$, $C_d = G_1 - G_3$.

The masses obtained in (25) are naturally tiny compared to $m_W$. We have not examined the possibility of solutions to (19)-(20) with $m_q$ not much smaller than all other mass scales.

A complete derivation of (25) would include also computation of the nonplanar contributions in (19)-(20) which has not yet been completed. These will change the numerical value of the coefficient $c_2$, but should not change the qualitative behavior.

### 5 Discussion

The exponential form of the solution (25) is due to the presence of the gauge interactions which are responsible for the pure log terms in (24). The role of the four-fermion interactions (I), however, should now be noted. They contribute the factors $(1 - C_q)$ in the exponent. Variations in the values of the couplings in (I), therefore, may produce very large variations in the dynamically generated masses. More importantly, these couplings may be adjusted to stabilize (25) against higher corrections from the expansion (17), i.e. render all higher powers of logs in higher loop contributions smaller.

It is useful to contrast this to mass generation driven only by four-fermion interactions, as in top condensation models. In that case one considers only the last diagram

\[\text{A well-known example of this arises in the minimization of the Coleman-Weinberg potential.}\]
on the r.h.s. of (19)-(20). Correspondingly one has to balance the \((m_q^2/M^2) \ln(M^2/m_q^2)\) piece of the term proportional to the \(G_i\)'s on the l.h.s. of (24) against a constant. A solution then requires extreme fine-tuning of the couplings \(G_i\) (above a critical (strong) value) in order to obtain a small mass \(m_q\) relative to the cutoff \(M\). In the present case, we need not take the four-fermion couplings in (1) to be above critical to drive mass generation, and \(m_q^2 \ln m_q^2\) terms are unimportant as they can produce only tiny corrections to the leading behavior (25). The constant piece contributions in (24) coming from (1) can then have the important effects pointed out above without any excessive fine-tuning.

We explored the possibility of the above nonperturbative mechanism for fermion mass generation within the familiar context of the standard minimal electroweak model. The mechanism, however, is quite general, and may be realized in a wider context and at different scales. Not all contributions to all the quark and lepton masses need of course arise at a single scale. A natural application for the proposed mechanism is in models where the electroweak gauge bosons are replaced by a system of gauge bosons with some nondegenerate masses (cp. remarks preceding (24)) of a much higher, perhaps unification, scale. The analog of (24) then produces very large natural hierarchies. One such simple model will be treated elsewhere.

I would like to thank Z. Bern and A. Grant for discussions.

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4This is again the usual fine-tuning due to quadratic divergences with a fundamental Higgs in a different guise.