Rectification of electronic heat current by a hybrid thermal diode

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Designing an efficient thermal diode

As discussed in the main text, an efficient thermal diode can be realized by means of a $N_L$INISIN$_R$ chain, in which the N electrode is coupled to the phonon bath. This can be efficiently realized through a thermalizing normal metal probe $P_N$ [1]. As shown in Fig. 1a, the value of the resistance $R_N$ has a strong influence on the behavior of the rectification efficiency $\mathcal{R}$ vs. the bias temperature $T_{\text{bias}}$: not only it modifies the maximum value of $\mathcal{R}$ ($\mathcal{R}_{\text{max}}$), but it also moves the maximum point. This is strictly related to the diode working principle: at low temperatures the S electrode works as a thermal bottleneck in both the forward and reverse configuration. Under equal $T_{\text{bias}}$, we obtain $\mathcal{R} \gg 1$ if the N part of the diode is efficiently thermalized to the bath temperature $T_{\text{bath}}$, and $R_N$ plays an important role in this process. As a matter of fact, if $P_N$ is too transparent, $T_N$ and $T_S$ in the forward configuration become too close to $T_{\text{fur}}$, on the contrary, if $P_N$ is too opaque, in the reverse configuration $T_N$ gets significantly higher than $T_{\text{rev}}$. This delicate trade-off leads to a non-monotonic behavior of $\mathcal{R}_{\text{max}}$ vs. $R_N$, as shown in Fig. 1b.

The role of $R_T$, on the other side, turns out not to be so crucial to optimize the efficiency of the heat diode. Increasing or decreasing $R_T$ leads, respectively, to the enhancement or reduction of the thermal gradients at the diode’s output in both the configurations. This yields a non-monotonic behavior of $\mathcal{R}_{\text{max}}$ vs. $R_T$ that does not differ significantly from the result we obtained for our device ($\mathcal{R}_{\text{max}}$ shows a relative variation of $\sim 10\%$ for $R_T$ spanning from $1 \, \text{k}\Omega$ to $300 \, \text{k}\Omega$).

In order to design an efficient thermal diode, the ingredient that has to be also considered is $J_{e-\text{ph},N}$, i.e., the electron-phonon relaxation in the N electrodes. In our experiment we exploited Al$_{0.98}$Mn$_{0.02}$ because of its favorable oxidation properties and of the reduced electron-phonon coupling at low temperatures. As a matter of fact, this material follows a $T^6$ power law with $\Sigma_{\text{AlMn}} = 4.5 \times 10^9 \, \text{WK}^{-6}\text{m}^{-3}$ [2, 3]. Another material that is typically used to fabricate N electrodes is copper (Cu), which is characterized by a $T^5$ dependence and $\Sigma_{\text{Cu}} = 3 \times 10^9 \, \text{WK}^{-5}\text{m}^{-3}$ [4, 5].

In Fig. 1b we compare the maximum rectification efficiency obtained at $T_{\text{bath}} = 50 \, \text{mK}$ as a function of $R_N$ for two different choices of the N material. We limit our analysis to the same range of $T_{\text{bias}}$ we explored experimentally. The solid lines correspond to a diode made of Al$_{0.98}$Mn$_{0.02}$ and Al, identical to the one we measured, whereas the dashed lines stand for
a device where all the N parts are made of Cu. In the latter case, the $T^5$ dependence of the electron-phonon coupling has dramatic consequences on the efficiency of the device. This stems from the stronger energy relaxation that affects all the N electrodes, thereby levelling temperature gradients across the whole device.

Finally, we consider the role of the electron-phonon coupling for an increasing $T_{\text{bath}}$. To this end, we define the optimal rectification ratio $R_{\text{opt}}$ as the one corresponding to the maximum value between $R$ and $1/R$. In Fig. 1c we plot $R_{\text{opt}}$ vs. $T_{\text{bath}}$ for the same cases analyzed in Fig. 1b. As explained in the main text, the optimal efficiency of the Al$_{0.98}$Mn$_{0.02}$ diode changes direction ($R_{\text{opt}} < 1$) [6, 7] at $T_{\text{bath}} = 300$ mK, indicating a strong reduction in the effectiveness of $P_N$. This switching temperature results from the competition between the energy release through the probe ($\propto T^2$) and the electron-phonon coupling ($\propto T^6$) affecting all the N electrodes of the structure. In the case of Cu, the switching point occurs at lower $T_{\text{bath}}$, since the electron-phonon coupling in this material is stronger at low temperatures ($\propto T^5$).

**Electrical vs. thermal behavior of the device**

The electrical behavior of the device is symmetric whereas the thermal one is not because of the subtle difference between the electric and heat current. Let us focus first on the NIS junction at the core of the thermal diode. Under thermal bias, the heat current ($J$) flowing between the S electrode at temperature $T_S$ and the N electrode at temperature $T_N$ connected by a tunnel barrier with normal-state resistance $R_\Omega$ reads [4]:

$$J(T_S, T_N) = \frac{2}{e^2 R_\Omega} \int_0^\infty dEEN(E, T_S)[f(E, T_S) - f(E, T_N)],$$

where $f(E, T) = [1 + \exp(E/k_B T)]^{-1}$ is the Fermi-Dirac distribution function and $N(E, T) = |R[E + i\Gamma/\sqrt{(E + i\Gamma)^2 - \Delta^2(T)}]|$ is the smeared (by non-zero $\Gamma$) normalized Bardeen-Cooper-Schrieffer density of states in the superconductor [8]. On the other side, the electric current ($I$) flowing through the same junction under voltage bias $V$ can be written as follows [4]:

$$I(T_S, T_N, V) = \frac{1}{2e R_\Omega} \int_{-\infty}^\infty dEEN(E, T_S)[f(E - eV, T_N) - f(E + eV, T_N)].$$

The peculiar dependence on temperature of the heat current $J$ makes it non-symmetric (i.e., without definite parity) under thermal bias reversal, i.e., $|J(T_S, T_N)| \neq |J(T_N, T_S)|$, which is at the origin of the thermal rectifying character of the NIS junction. By contrast, as it can
be seen from the expression of the electric current, upon voltage bias reversal one obtains $|I(T_S, T_N, V)| = |I(T_S, T_N, -V)|$, which yields a symmetrical differential conductance. The electrical response of the whole device is the results of the series connection of additional NIN and NIS tunnel junctions, which all possess symmetric behavior from the electrical point of view.

If we focus into the whole diode, thermal asymmetry arises also as consequence of the asymmetric coupling to the phonon bath. This fact, again, does not affect the electric current flowing through the device. Indeed, the electric measurements shown in the main text were obtained through the series connection of two heater wires and at a uniform structure temperature equal to $T_{\text{bath}}$, while letting the rest of wires electrically open. As a consequence, no electric current was flowing through the P$_N$ probe in this case.
Supplementary Material References

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FIG. 1. **Role of the coupling strength to the phonon bath.** (a) Rectification efficiency $\mathcal{R}$ vs. $T_{\text{bias}}$ calculated for different values of the probe resistance $R_N$ at $T_{\text{bath}} = 50 \text{ mK}$. The solid line is the result corresponding to our device with $R_N = 21.5 \text{ k}\Omega$, as obtained from the theoretical fit of the experimental data. (b) Maximum rectification efficiency ($\mathcal{R}_{\text{max}}$) vs. $R_N$ calculated for two possible choices of the N material in the device at $T_{\text{bath}} = 50 \text{ mK}$. (c) Optimal rectification efficiency $\mathcal{R}_{\text{opt}}$ vs. $T_{\text{bath}}$ for the same cases analyzed in panel B. Lines are guides to the eye. The curves have been calculated by setting $R_N$ equal to the values obtained from the experimental fitting at each $T_{\text{bath}}$. Shadowed region indicates the regime where $\mathcal{R} < 1$. 
$T_{\text{bath}} = 50 \text{ mK}$

(a) $R_N (\text{k}\Omega)$ vs $T_{\text{bias}}$ (K)

(b) $R_{\text{max}}$ vs $R_N (\text{k}\Omega)$

(c) $R_{\text{opt}}$ vs $T_{\text{bath}}$ (K)

Al$_{0.98}$Mn$_{0.02}$

Cu

$\max$

$\text{opt}$