COMPACT INVERSE CATEGORIES

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An inverse category is a category that comes with a contravariant involution \( \dagger \) that acts as the identity on objects and satisfies \( f = ff\dagger f \) and \( ff\dagger gg\dagger = gg\dagger ff\dagger \) on morphisms [2]. The one-object case, of inverse monoids, has been well-studied [4]. In particular, abelian inverse monoids obey a structure theorem [3]: any abelian inverse monoid is a semilattice of abelian groups. In the many-object case, any inverse category gives rise to a semilattice-shaped family of groupoids in a similar way, but not in a functorial way, and it is generally impossible to recover the inverse category from this family without a degree of commutativity.

From the perspective of computer science, inverse categories provide semantics for typed reversible programs. To model recursion, it would be desirable to have additional compact closed structure. We show that compact inverse categories generalise abelian inverse monoids to multiple objects, and extend the structure theorem: any compact inverse category is a semilattice of compact inverse groupoids. The latter are also known as coherent 2-groups or crossed modules, and have several characterisations [1]. This structure theorem crucially uses features inherent in compact categories such as traces and scalars.

Based on joint work with Robin Cockett.

References

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