Spin-dependent Seebeck effect in asymmetric four-terminal systems with Rashba spin-orbit coupling

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Abstract – We propose a new type of spin-dependent Seebeck effect (SDSE) emerging from the Rashba spin-orbit coupling in asymmetric four-terminal electron systems. This system generates spin currents or spin voltages along the longitudinal direction parallel to the temperature gradient in the absence of magnetic fields. The remarkable result arises from the breaking of the reflection symmetry along the transverse direction. In the meantime, the SDSE along the transverse direction, the so-called the spin Nernst effect, with spin currents or spin voltages perpendicular to the temperature gradient, can be simultaneously realized in our system. We further find that it is possible to use the temperature differences between four leads to tune the spin-dependent Seebeck coefficients.

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Spin caloritronics, the study of the relationship between spin current and heat flow in magnetic materials, has attracted much attention in recent years [1–10], in particular after the discovery of the spin Seebeck effect (SSE) in metallic ferromagnets [1]. Analogous to the conventional Seebeck effect generating electric voltages, the SSE refers to the generation of spin currents (voltages) in the presence of a temperature gradient [11]. In general, the SSE can be divided into two categories [12]: i) the longitudinal SSE in which spin currents are generated parallel to the temperature gradient; ii) the transverse SSE, also called the spin Nernst effect, in which spin currents are generated perpendicular to the temperature gradient. The transverse SSE was first observed in metallic ferromagnetic films [1,13], ferromagnetic semiconductors [4], and magnetic insulators [3] by using the inverse spin-Hall effect. Later on, the longitudinal SSE has also been confirmed experimentally in magnetic insulators [14,15]. All of these experiments on the SSE have been performed for magnetic materials, in which the spin current is carried by magnons.

In non-magnetic materials, the spin current is carried by conduction-electrons rather than by magnons or magnetic solitons. The generation of spin-polarized electron current driven by a temperature gradient is called spin-dependent Seebeck effect (SDSE) [11]. It has been reported that a very large magnetic field is needed to realize the SDSE experimentally [16]. Theoretically, Liu et al. [17] and Cheng et al. [18] have investigated the SDSE in non-magnetic materials, two-dimensional (2D) mesoscopic electron systems, in the presence of both spin-orbit coupling (SOC) and magnetic fields. In the absence of magnetic fields, the transverse SDSE, analogous to the spin-Hall effect, has been studied [19]. Both the intrinsic transverse SDSE due to the SOC [20,21] and the extrinsic transverse SDSE due to the spin-dependent skew scatterings [22] have been theoretically investigated. However, there exists no theoretical treatment on the occurrence of the transverse SDSE.
of the longitudinal SDSE in the absence of magnetic fields.

In this letter, we propose a system realizing the longitudinal SDSE in the absence of magnetic fields and suggest a practical scheme for implementing such a device composed of 2D mesoscopic asymmetric four terminals with the Rashba SOC. Several developments in the study of the spin-Hall effect and Nernst effect [23–26] though for symmetric systems, make such systems a practical possibility.

Figure 1 shows the schematic diagram of a square region of width \( L \) in the \( x-y \) plane connected with four ideal leads also of width \( L \). \( T_\mu \) is the temperature at lead \( \mu \), where \( \mu = 1, 2, 3, 4 \). The temperatures at leads 1 and 3 are \( T_1 = T + \gamma \Delta T \) and \( T_3 = T - (1 - \gamma) \Delta T \) with \( 0 \leq \gamma \leq 1 \), respectively. The temperatures at leads 2 and 4 are \( T \). We break the reflection symmetry along the transverse direction by introducing an isosceles-triangle region as shown in fig. 1. Such asymmetric samples have been fabricated by Matthews et al. [26] in studying the thermal rectification effect. The spin-coherence length of typical 2D electron gas, say, GaAs/AlGaAs, can be estimated of the order of 10 \( \mu \)m at low temperatures. This is due to the fact that the spin-decoherence time is around 100 ps and the Fermi velocity is between \( 10^5 \) m/s and \( 10^6 \) m/s [27]. Since the size of our system is 250 nm, much smaller than the spin-decoherence length, the spin relaxation and spin-decoherence processes become irrelevant.

The electric current \( I_{\mu\sigma} \) into lead \( \mu \) can be calculated from the Landauer-Büttiker formula [28,29]

\[
I_{\mu\sigma} = \sum_{\nu\sigma'} L_{\mu\sigma,\nu\sigma'}^{(0)} (V_{\nu\sigma'} - V_{\nu\sigma}) + \sum_{\nu\sigma'} L_{\mu\sigma,\nu\sigma'}^{(1)} (T_{\mu\nu} - T_{\nu\mu}). \tag{1}
\]

Here we define the spin polarization \( \sigma (\sigma = \uparrow, \downarrow) \) in the \( z \)-direction. \( L_{\mu\sigma,\nu\sigma'}^{(n)} \) is the transport coefficient with \( n = 0, 1 \), in which \( T_{\mu\nu} \) is the energy-dependent transmission coefficient from lead \( \nu \) with spin \( \sigma' \) (noted as \( \{\nu\sigma'\} \)) to lead \( \mu \) with spin \( \sigma \) (noted as \( \{\mu\sigma\} \)). \( V_{\nu\sigma'} \) (\( V_{\nu\sigma} \)) is the spin-dependent potential related bias of \( \{\mu\sigma\} \) (\( \{\nu\sigma'\} \)), and \( f_0(E) = 1/[\exp((E - E_F)/(k_B T)) + 1] \) [21,28]. We employ the tight-binding Hamiltonian [30] to calculate the transmission coefficient

\[
H = \sum_{i\sigma} c_{i\sigma}^\dagger c_{i\sigma} - t \sum_{\langle i,j \rangle \sigma} c_{i\sigma}^\dagger c_{j\sigma} + i S \sum_{i} \left( c_{i\uparrow}^\dagger c_{i\downarrow} + c_{i\downarrow}^\dagger c_{i\uparrow} \right) + H.C., \tag{2}
\]

where \( c_{i\sigma}^\dagger \) and \( c_{i\sigma} \) are the creation and annihilation operators of electrons on site \( i \) with spin \( \sigma \). The on-site energy is \( \varepsilon_i = 4t + V(i) \), where \( V(i) \rightarrow \infty \) when \( i \) is inside the triangle region and \( V(i) = 0 \) in the rest of the region. The parameter \( t = \hbar^2/2m^*a^2 \) represents the hopping energy with the effective mass \( m^* \) between the nearest-neighboring sites (\( i, j \)) of the lattice constant \( a \). The parameter \( tS = V_{\text{SOC}}/2a \) is expressed by the Rashba SOC constant \( V_{\text{SOC}} \). \( \delta_x \) and \( \delta_y \) are the unit vectors along the \( x \) - and \( y \)-directions. The transmission coefficient \( T_{\mu\sigma,\nu\sigma'}(E) = T[i\Sigma_{\mu\sigma} G^{R}\Sigma_{\nu\sigma'} G^{A}] \) can be calculated by the Green’s function method [29,31] where \( G^{R}(G^{A}) = (G^{R}(G^{A}))^{\dagger} \) of the retarded (advanced) Green's function. Here \( \Sigma_{\mu\sigma} = i[\Sigma_{\mu\sigma}^{R} - \Sigma_{\mu\sigma}^{A}] \) with \( \Sigma_{\mu\sigma}^{R} \) being the self-energy, and \( \Sigma_{\mu\sigma}^{A} = [i\Sigma_{\mu\sigma}^{A}]^{\dagger} \). In the low-temperature limit \( T \ll E_F/k_B \), the \( E_F \)-dependent transport coefficient can be expressed as [32]

\[
I^{(1)}_{\mu\sigma,\nu\sigma'} = \frac{\pi^2 k_B^2 T}{3h} \frac{dT_{\mu\sigma,\nu\sigma'}(E)}{dE} \bigg|_{E=E_F}. \tag{3}
\]

The spin-dependent Seebeck coefficient (SDSC) for closed-boundary (CB) conditions by setting the voltage as \( V_{\mu\sigma} = 0 \) for all \( \{\mu\sigma\} \) and that for open-boundary (OB) conditions by setting the spin current as \( I_{\mu\sigma} = 0 \) for all \( \{\mu\sigma\} \) are written as

\[
S_{\mu}^{l} = \frac{\hbar}{2e} \frac{I_{\mu\uparrow} - I_{\mu\downarrow}}{\Delta T}, \quad S_{\mu}^{V} = \frac{V_{\mu\uparrow} - V_{\mu\downarrow}}{2\Delta T}, \tag{4}
\]

where \( (I_{\mu\uparrow} - I_{\mu\downarrow})/2\hbar \) is the spin current and \( V_{\mu\uparrow} - V_{\mu\downarrow} \) represents the spin accumulation in lead \( \mu \). \( S_{\mu=1,3}^{l} \left( S_{\mu=1,3}^{V} \right) \) describe the longitudinal SDSCs under CB (OB) conditions. Similarly, \( S_{\mu=2,4}^{l} \left( S_{\mu=2,4}^{V} \right) \) describe the transverse SDSCs under CB (OB) conditions (see fig. 1). The substitution of the spin current \( I_{\mu\sigma} \) of eq. (1) into eq. (4) yields the analytical expressions of \( S_{\mu=1,3}^{l} \left( S_{\mu=1,3}^{V} \right) \) with the definition of \( \Delta_{\mu\sigma} = [I_{\mu\uparrow,\mu\uparrow}^{(1)} + L_{\mu\sigma,\mu\sigma'}^{(1)}] - [L_{\mu\sigma,\mu\sigma'}^{(1)} + I_{\mu\sigma,\mu\sigma'}^{(1)}] \) of the form

\[
S_{\mu=1,3}^{l} = \frac{\hbar}{2e} [\Delta_{12} + \Delta_{14}] \gamma + \Delta_{13}], \tag{5}
\]

\[
S_{\mu=1,3}^{V} = \frac{\hbar}{2e} [(\Delta_{32} + \Delta_{34})(1 - \gamma) + \Delta_{31}], \tag{5}
\]

while the analytical expressions of \( S_{\mu=2,4}^{l} \) become

\[
S_{\mu=2,4}^{l} = \frac{\hbar}{2e} [-\gamma \Delta_{\mu1} + (1 - \gamma) \Delta_{\mu3}]. \tag{6}
\]
Equations (5) and (6) provide two features on the SDSC: i) at low temperature, both the longitudinal and the transverse SDSCs show linear temperature dependence. This is because \( L^{(1)}_{\sigma\sigma',\nu\nu'} \) and \( \Delta_{\mu} \) are proportional to \( T \) from eq. (3); ii) the sharp energy-dependent transmission coefficient is because \( L^{(1)}_{\sigma\sigma',\nu\nu'} \) is proportional to the slope on \( E \) of \( T_{\sigma\sigma',\nu\nu'}(E) \) as seen from eq. (3).

In our numerical calculations, the width of the square region \((L \times L)\) is taken to be \( L = 20a = 250 \text{ nm} \) with \( a = 12.5 \text{ nm} \). The side length of the isosceles triangle is 35 nm and the bottom width is 50 nm. We choose \( t = 5 \text{ meV} \) and \( T = 0.58 \text{ K} (= 0.011/\text{KB}) \) which ensures that the electron transport is within the ballistic regime. The Rashba SOC constant \( V_{\text{SOC}} \) is chosen to be \( 2.5 \times 10^{-11} \text{ eV} \cdot \text{m} \), for exemplifying InSb- and InAs-based 2D electron systems [33] unless specified.

We first give the relations among \( \Delta_{\mu} \) to make the arguments on the SDSE transparent when the system has the \( D_2 \) symmetry without triangle region. The time-reversal symmetry yields the equality \( L^{(1)}_{\sigma\sigma',\nu\nu'} = L^{(1)}_{\sigma\sigma',\nu\nu'} \) which further yields \( \Delta_{\mu\nu} = -\Delta_{\mu\nu} \), where \( \sigma \) means spin reversing [34]. We summarize other relations in footnote 1. From footnote 1, we find that \( \Delta_{12} = -\Delta_{32} = \Delta_{34} = -\Delta_{14} \) and \( \Delta_{13} = \Delta_{31} = 0 \). Substituting these into eq. (5), the SDSC yields \( S^L_1 = S^L_4 = 0 \). These results indicate that there is definitely no longitudinal SDSE in the symmetric systems, but the transverse SDSE occurs as in the case of ref. [21] (see fig. 1). It is straightforward to obtain \( S^L_2 = -S^L_3 = \Delta_{23}h/(2e) \). In an asymmetric system, we find \( \Delta_{12} = -\Delta_{32} \neq -\Delta_{14} = \Delta_{34} \) and \( \Delta_{13} = \Delta_{31} = 0 \), but \( \Delta_{12} \neq \Delta_{13} \) in eq. (5). Therefore, both the longitudinal SDSE and the transverse SDSE can be simultaneously realized in our asymmetric system. This feature originates from the symmetry breaking from \( D_2 \) to \( C_3 \) with the etched triangle region. The physical implications for the above can be obtained as follows: i) \( L^{(1)}_{\mu\sigma,\nu(\mu+1)\sigma} = L^{(1)}_{\mu\sigma,\nu(\mu+1)\sigma} \) in footnote 1 does not hold when \( \mu = 1, 2, 3, 4 \); ii) \( L^{(1)}_{\mu\sigma,\nu(\mu+1)\sigma} = L^{(1)}_{\mu\sigma,\nu(\mu+2)\sigma} \) and \( L^{(1)}_{\mu\sigma,\nu(\mu+1)\sigma} = L^{(1)}_{\mu\sigma,\nu(\mu+1)\sigma} \) do not hold when \( \mu = 2, 4 \); iii) \( L^{(1)}_{\mu\sigma,\nu(\mu+2)\sigma} = L^{(1)}_{\mu\sigma,\nu(\mu+2)\sigma} \) in footnote 1 does not hold when \( \mu = 1, 3 \).

Figures 2(a) and (b) show the longitudinal SDSCs \((S^L_{\mu=1,3})\) and the transverse SDSCs \((S^T_{\mu=2,4})\) as a function of \( E_F \) in both the symmetric and the asymmetric systems taking \( \gamma = 0 \). The CB conditions are employed in calculations. Figure 2(a) for the asymmetric system realizes that \( S^L_{\mu=1,3} \), oscillate strongly with varying \( E_F \). Both the sign and the amplitude of \( S^L_{\mu=1,3} \), oscillate drastically since the transmission coefficients depend highly on \( E_F \). In contrast, there exists no longitudinal SDSE in symmetric system. Figure 2(b) shows that \( S^T_{\mu=2,4} \) oscillate as well with \( E_F \) in both the symmetric and the asymmetric systems. The inequality \( S^L_2 \neq -S^L_3 \) holds for the asymmetric system, differently from the case \( S^L_2 = -S^L_3 \) for the symmetric system. This indicates

\[ S^L_2 = -S^L_3 = \Delta_{23}h/(2e) \]
that, in the symmetric system, the spin current through lead 2 is exactly the same as the spin current through lead 4. There is no spin current in lead 1 and lead 3, which results in the absence of longitudinal SDSC. In the asymmetric system, the spin current through lead 2 is not equal to the spin current through lead 4. A portion of spin current flows into the longitudinal direction yielding non-zero longitudinal SDSC as shown in fig. 1.

Figures 2(c) and (d) present the longitudinal SDSCs \( S_{\mu=1,3}^V \) and the transverse SDSCs \( S_{\mu=2,4}^V \) in both the symmetric and the asymmetric systems with the OB conditions. Unlike the CB conditions, \( S_{\mu=1,3}^V \) cannot be solved analytically. The numerically calculated \( S_{\mu=1,3}^V \) in fig. 3(c) vary rapidly with \( E_F \) in the asymmetric system. For CB conditions, there is no longitudinal SDSE in the symmetric system. Figure 2(d) shows that \( S_{\mu=2,4}^V \) oscillate rapidly with \( E_F \) in both the symmetric and the asymmetric systems. The situation of \( S_{\mu=2,4}^V \) is realized only in the case of the symmetric system, indicating that the spin accumulation in lead 2 is exactly the same as that in lead 4 with the reverse sign, and there is no spin accumulation in lead 1 and lead 3. In the asymmetric system, the electron spins accumulate in all four leads.

From the results in figs. 2(a)–(d), we find that spin current or spin accumulation along both the longitudinal and the transverse directions can be generated by applying a temperature gradient when the symmetry along the transverse direction is broken.

Figure 3(a) shows the calculated results for the longitudinal SDSC vs. the Rashba SOC constant \( V_{SOC} \) under CB conditions in both symmetric and asymmetric systems for \( \gamma = 0 \) and \( E_F = 0.5 \text{ meV} \). In the asymmetric system, we find that \( S_{\mu=1,3}^I \) vanish for \( V_{SOC} = 0 \) because of no spin-splitting without the SOC. \( S_{\mu=1,3}^I \) vary with increasing \( V_{SOC} \) and the amplitudes become larger due to stronger spin-splitting. For some values of \( V_{SOC} \) such as \( 8 \times 10^{-11} \text{ eV} \cdot \text{m} \), not only the amplitude but also the sign of \( S_{\mu=1,3}^I \) are different from \( S_{\mu=1}^I \). In the symmetric system, there is no longitudinal SDSE independent of \( V_{SOC} \).

Finally, we consider the effect of the temperature parameter \( \gamma \) on the longitudinal SDSC. As shown in fig. 1, \( T_1 - T_3 = \Delta T \), \( T_1 - T_2 = T_1 - T_4 = \gamma \Delta T \), and \( T_2 - T_3 = T_4 - T_3 = (1 - \gamma) \Delta T \). Figure 3(b) shows the longitudinal SDSC vs. \( \gamma \) for the CB conditions in both symmetric and asymmetric systems when \( V_{SOC} = 2.5 \times 10^{-11} \text{ eV} \cdot \text{m} \) and \( E_F = 0.45 \text{ meV} \). In the asymmetric system, \( S_{\mu=1,3}^I \) vary linearly with \( \gamma \). This linear dependence comes from eq. (5). The absolute value of SDSC in lead 1 is larger (smaller) than that in lead 3 for \( \gamma < 0.5 (\gamma > 0.5) \). When \( \gamma = 0.5 \), \( S_{\mu=1}^I \) and \( S_{\mu=3}^I \) become the same: \( S_{\mu=1}^I = S_{\mu=3}^I = h/(4e)[\Delta_{12} + \Delta_{13} + 2\Delta_{14}] \). It is easy to understand that similar results \( S_{\mu=1}^I = S_{\mu=3}^I \), though not presented in this letter, can be found for the OB conditions when \( \gamma = 0.5 \). We point out that one can choose \( \gamma = 0 (\gamma = 1) \) to obtain a large SDSC in lead 1 (lead 3). Such feature enables us to enhance the longitudinal SDSC. In the symmetric system, there is no longitudinal SDSE no matter how large \( \gamma \) is.

To summarize, we have proposed a new type of the SDSE emerging from the Rashba SOC in asymmetric four-terminal electron systems, which exhibits both the longitudinal and the transverse SDSE in the absence of magnetic fields. Our calculations have revealed that the longitudinal SDSE can be realized by the breaking of the reflection symmetry along the transverse direction. This is due to the fact that the spin flow along the transverse direction induced by the temperature gradient splits into the longitudinal direction by the triangle region. Furthermore, our calculations show that the Fermi energy, the Rashba SOC constant, and the temperature parameter \( \gamma \) affect severely the SDSC. The mechanism of our results is due neither to skew scattering nor to the side jump. Basically, the spin current from lead 2 to lead 4 always exists regardless of the existence of the triangle. The transverse SDSE due to the intrinsic spin-Hall effect becomes relevant in the presence of spin-orbit coupling [20,21]. Moreover, the triangle geometry breaks the symmetry of the spin-dependent current flow from lead 2 to lead 4 by splitting into two currents \( I_{1\uparrow} - I_{1\downarrow} \) and \( I_{3\uparrow} - I_{3\downarrow} \) into lead 1 and lead 3. Therefore, the triangle is a splitter of the spin current. Our findings should be realized in asymmetric four-terminal mesoscopic electron systems, for examples, such as InSb- or InAs-based 2D systems with the Rashba SOC.
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