Charmless Hardronic Decays $B_u \to VV$: Angular Distributions, Direct CP Violation and Determination of the Unitary Triangle

B. Tseng$^a$ and Cheng-Wei Chiang$^b$

$^a$ Institute of Physics, Academia Sinica
Taipei, Taiwan 115, Republic of China

$^b$ Department of Physics, Carnegie Mellon University
Pittsburgh, PA 15213, USA

Abstract

Two-body charmless nonleptonic decays of $B_u \to VV$ are studied within the generalized factorization approach using a recent calculation of the effective Wilson coefficients $c_{i}^{\text{eff}}$, which are not only renormalization-scale and -scheme independent but also gauge invariant and infrared finite. After making a universal ansatz for the nonfactorizable contributions, we parametrize these effects in terms of $N_{c}^{\text{eff}}(LL)$ and $N_{c}^{\text{eff}}(LR)$, the effective numbers of colors arising from $(V-A)(V-A)$ and $(V-A)(V+A)$ four-quark operators, respectively. Three different schemes for these contributions are considered: (i) the naive factorization, (ii) the large-$N_{c}$ improved factorization, and (iii) our preferred choice: $(N_{c}^{\text{eff}}(LL), N_{c}^{\text{eff}}(RR)) = (2, 5)$. We present the full angular distribution of all charmless $B_u \to VV$ decays in both transversity and helicity frames. Direct CP violation in these normalized angular correlation coefficients is not negligible in $B_u^- \to K^{*-}\rho^0, K^{*-}\omega$, and direct CP violation in the partial rate difference for $B_u^- \to K^{*-}\omega, K^{*-}\rho$ and $\rho^-\omega$ can be as large as 45%, 25%, $-10\%$, respectively. Due to the sizable QCD penguin contributions in $\rho^-\omega$, the determination of the unitary triangle $\alpha$ via this decay mode is more promising than via $\rho^-\rho^0$. It is also encouraging to determine the unitary triangle $\gamma$ through $B_u^- \to K^{*-}\rho$ because of $N_{c}$-insensitivity and the not-so-small tree contribution. The impacts of a negative $\rho$ on the branching ratios and CP violation are studied. We also comment on the theoretical uncertainties and their possible impacts.

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$^1$E-mail: btseng@phys.sinica.edu.tw
$^2$E-mail: chengwei@andrew.cmu.edu
In past years we have witnessed remarkable progress in the study of exclusive charmless $B$ decays. Experimentally, CLEO [1, 2] has discovered many new two-body decay modes

$$B \to \eta' K^\pm, \eta K^0, \pi^\pm K^0, \pi^\pm K^{\mp}, \pi^0 K^\pm, \rho^0 \pi^\pm, \rho^\mp \pi^\pm, \omega K^\pm,$$

and found a possible evidence for $B \to \phi K^*$. Moreover, CLEO has provided new improved upper limits for many other decay modes. While all the measured channels are penguin dominated, the most recently measured $\rho^0 \pi^-$ and $\rho^\mp \pi^\pm$ modes are dominated by the tree diagrams. In the meantime, updates and new results of many $B \to PV$ decays with $P = \eta, \eta', \pi, K$ and $V = \omega, \phi, \rho, K^*$ as well as $B \to PP$ decays will be available soon. With the $B$ factories Babar and Belle starting to collect data, many exciting and harvest years in the arena of $B$ physics and $CP$ violation are expected to come.

An earlier systematic study of exclusive nonleptonic two-body decays of $B$ mesons was made in [3]. Since then, many significant improvements and developments have been achieved over past years [4]. For example, a next-to-leading order effective Hamiltonian [3, 6, 7] for current-current operators and QCD as well as electroweak penguin operators [5] becomes available. The renormalization scheme and scale problems with the factorization approach for matrix elements can be circumvented by employing scale- and scheme-independent effective Wilson coefficients. Heavy-to-light form factors have been computed using QCD sum rules, lattice QCD and potential models. Besides, the gauge and infrared regulator dependence problem of the effective Wilson coefficients has also been resolved in [9]. Finally, a theoretical framework, namely the generalized factorization approach [10], has been shown to be useful for the understanding of the experimental data.

In our previous studies [12, 13, 14], we have completed all branching ratios of $B_{u,d,s} \to PP, VP$ and $VV$. It is known that there is rich physics in the $VV$ decay modes [14, 15]: in addition to the average quantity such as the branching ratio, there are more observables in the $VV$ modes shown in the angular distribution, from which we can get more information on the dynamics. Aside from the partial rate difference, there are also more $CP$-violating observables in the angular distributions of $VV$ decay modes which can provide more contents with possible fingerprints of new physics beyond the standard model. The advantage of having a large number of observables in $VV$ modes also results in some useful strategies of determining the unitary triangles [17]. While the contribution of the electromagnetical penguin (EWP), which does show effects on the $\rho^- \rho^0$ decay mode [18], is neglected in some early studies [16], we include it in our calculations. Meanwhile, all these earlier studies [16, 13] suffer from the gauge and infrared regulator dependence problem of the effective Wilson coefficients. In this letter, we present an updated analysis based on the gauge invariant effective Wilson coefficients. The helicity and transversity amplitudes appearing in the angular distributions of all $B_u \to VV$ and their $CP$ violating observables are calculated. Topics about determination of the possible $CP$-conserving final state interaction (FSI) phases and/or possible $CP$-violating phases from new physics, a revival possibility of negative $\rho$ and its impact on $CP$ violation and the determination of the unitary triangle are discussed in brief.

Let us begin with a brief description of the theoretical framework. The relevant effective $\Delta B = 1$ weak Hamiltonian is

$$\mathcal{H}_{\text{eff}}(\Delta B = 1) = \frac{G_F}{\sqrt{2}} \left[ V_{ub} V_{uq}^* (c_1 O_1^{u} + c_2 O_2^{u}) + V_{cb} V_{cq}^* (c_1 O_1^{c} + c_2 O_2^{c}) - V_{tb} V_{tq}^* \sum_{i=3}^{10} c_i O_i \right] + \text{h.c.},$$

where $q = d, s$, and

$$O_1^{u} = (\bar{u}b)_{V-A} (\bar{q}u)_{V-A}, \quad O_1^{c} = (\bar{c}b)_{V-A} (\bar{q}c)_{V-A}.$$
\[ O_2^\mu = (\bar{q}b)_{V-A}(\bar{u}u)_{V-A}, \quad O_2^c = (\bar{q}b)_{V-A}(\bar{c}c)_{V-A}, \]

\[ O_{3(5)} = (\bar{q}b)_{V-A} \sum_{q'} (\bar{q}'q')_{V-A(V+A)}, \quad O_{4(6)} = (\bar{q}_\alpha b_\beta)_{V-A} \sum_q (\bar{q}_\beta q'_\alpha)_{V-A(V+A)}, \]

\[ O_{7(9)} = \frac{3}{2}(\bar{q}b)_{V-A} \sum_{q'} e_{q'}(\bar{q}'q')_{V+A(V-A)}, \quad O_{8(10)} = \frac{3}{2}(\bar{q}_\alpha b_\beta)_{V-A} \sum_q e_q(\bar{q}_\beta q'_\alpha)_{V+A(V-A)}, \]

with \((\bar{q}_1 q_2)_{V_{\pm A}} \equiv \bar{q}_1 \gamma_{\mu}(1 \pm \gamma_5)q_2\). In Eq. (2), \(O_{3-6}\) are QCD penguin operators and \(O_{7-10}\) are electroweak penguin operators, and \(c_i(\mu)\) are Wilson coefficients which have been evaluated to the next-to-leading order (NLO) \([3, 8]\). One important feature of the NLO calculation is the renormalization-scheme dependence of the Wilson coefficients (for a review, see \([7]\)). In order to ensure the \(\mu\) and renormalization scheme independence for the physical amplitude, the matrix elements, which are evaluated under the factorization hypothesis, have to be computed in the same renormalization scheme and renormalized at the same scale as \(c_i(\mu)\). However, as emphasized in \([10]\), the matrix element \((O)_{\text{fact}}\) is scale independent under the factorization approach and hence it cannot be identified with \((O(\mu))\). Incorporating QCD and electroweak corrections to the four-quark operators, we can redefine \(c_i(\mu)(O_i(\mu)) = c_i^{\text{eff}}(O_i)_{\text{tree}},\) so that \(c_i^{\text{eff}}\) are renormalization scheme and scale independent. Then the factorization approximation is applied to the hadronic matrix elements of the operator \(O\) at tree level. Recently, the controversy on gauge dependence and infrared singularity associated with the effective Wilson coefficients, criticized in \([11]\), is resolved in \([8]\). Gauge invariance of the decay amplitude is maintained under radiative corrections by assuming on-shell external quarks. (For a more detailed discussion, see \([4, 13]\)). In this letter, we will utilize these recently-calculated gauge-invariant Wilson coefficients and thus our results do not suffer from these gauge dependence and infrared singularity controversies. The numerical values for \(c_i^{\text{eff}}\) are shown in the last column of Table I of \([13]\), where \(\mu = m_b(m_b), \Lambda_{\text{MS}}^{(5)} = 225 \text{ MeV}, m_t = 170 \text{ GeV}\) and \(k^2 = m_b^2/2\) are used. From which we can see that the Wilson coefficients for \(b \to s\) and \(\bar{b} \to \bar{s}\) are almost the same and those for \(b \to d\) and \(\bar{b} \to \bar{d}\) are slightly different.

In the naive factorization approach, only the factorizable contributions are considered. However, as indicated by the \(B \to D P (V)\), contributions from the nonfactorizable amplitudes, which cannot be calculated in this naive factorization approach, are important for the understanding of the data \([20, 21]\). The spirit of the generalized factorization approach is to incorporate these nonfactorizable contributions in a phenomenological way: we parametrize these contributions, determine them from a few decay modes and then make predictions for the other modes. For the \(B (D) \to PP, PV\) decays \((P: \text{pseudoscalar meson, } V: \text{vector meson})\), there is only one single form factor (or Lorentz scalar) involved in the decay amplitude. Thus, the effects of nonfactorization can be lumped into the effective parameters \(a_i^{\text{eff}}\) \([11, 14]\):

\[ a_{2i}^{\text{eff}} = c_{2i}^{\text{eff}} + c_{2i-1}^{\text{eff}} \left( \frac{1}{N_c} + \chi_{2i} \right), \quad a_{2i-1}^{\text{eff}} = c_{2i-1}^{\text{eff}} + c_{2i}^{\text{eff}} \left( \frac{1}{N_c} + \chi_{2i-1} \right), \]

where \(c_{2i, 2i-1}^{\text{eff}}\) are the Wilson coefficients of the 4-quark operators, and nonfactorizable contributions are characterized by the parameters \(\chi_{2i}\) and \(\chi_{2i-1}\). We can parametrize the nonfactorizable contributions by defining an effective number of colors \(N_c^{\text{eff}}\), called \(1/\xi\) in \([23]\), as \(1/N_c^{\text{eff}} = (1/N_c) + \chi\). Thus the nonfactorizable effects are effectively incorporated in the factorization approach, that is the generalized factorization framework. However, the general amplitude of \(B(D) \to VV\) decay consists of three independent Lorentz scalars, corresponding to the \(S-, P-\) and \(D-\)wave amplitudes. Consequently, it is in general not possible to define effective \(a_i\) unless nonfactorizable terms contribute in equal weight to all covariant amplitudes. In this letter, we, following \([13, 16, 18, 22]\),...
make a further universal assumption for the nonfactorizable contributions to the different invariant amplitudes, i.e. \( \chi_{A1} = \chi_{A2} = \chi_{A3} = \chi_V \), so that all the nonfactorizable contributions for the different covariant amplitudes are equally weighted.

Different factorization approaches used in the literature can be classified by the effective number of colors \( N_c^{\text{eff}} \). The so-called “naive” factorization discards all the nonfactorizable contributions and takes \( 1/N_c^{\text{eff}} = 1/N_c = 1/3 \), whereas the “large-\( N_c \) improved” factorization \[^{[24]}\] drops out all the subleading \( 1/N_c \) terms and takes \( 1/N_c^{\text{eff}} = 0 \). In this paper, in addition to predictions from these two “homogeneous” nonfactorizable pictures, which assume that \( (N_c^{\text{eff}})_1 \approx (N_c^{\text{eff}})_2 \approx \cdots \approx (N_c^{\text{eff}})_10 \), we also present results from the “heterogeneous” one, which considers the possibility of \( N_c^{\text{eff}}(LR) \neq N_c^{\text{eff}}(LL) \). The consideration of the “homogeneous” nonfactorizable contributions, which is commonly used in the literature, has its advantage of simplicity. However, as argued in \[^{[12]}\] due to the different Dirac structure of the Fierz transformation, nonfactorizable effects in the matrix elements of \((V - A)(V + A)\) operators are a priori different from that of \((V - A)(V - A)\) operators, i.e. \( \chi(LR) \neq \chi(LL) \). Since \( 1/N_c^{\text{eff}} = 1/N_c + \chi \), theoretically it is expected that

\[
\begin{align*}
N_c^{\text{eff}}(LL) &\equiv \left( N_c^{\text{eff}} \right)_1 \approx \left( N_c^{\text{eff}} \right)_2 \approx \left( N_c^{\text{eff}} \right)_3 \approx \left( N_c^{\text{eff}} \right)_4 \approx \left( N_c^{\text{eff}} \right)_9 \approx \left( N_c^{\text{eff}} \right)_{10}, \\
N_c^{\text{eff}}(LR) &\equiv \left( N_c^{\text{eff}} \right)_{5} \approx \left( N_c^{\text{eff}} \right)_{6} \approx \left( N_c^{\text{eff}} \right)_{7} \approx \left( N_c^{\text{eff}} \right)_{8}. \quad (5)
\end{align*}
\]

We can thus make predictions based on different schemes for these nonfactorizable contributions as done in \[^{[13]}\]. The main goal of this studies is to make predictions as much as possible with effective one set indicated by the limited experimental data. This “minimal-fitting and global-predictions” will make theoretical analysis simple and powerful. In this short letter, we will use three different sets: (i) the naive factorization, (ii) the large-\( N_c \) improved factorization, and (iii) our the preferred choice \((N_c(LL), N_c(LR)) \approx (2, 5)\). The first two schemes are used as a reference and the third set is based on our analysis of the recent experimental data from CLEO (readers are referred to \[^{[13]}\]).

Let’s briefly discuss the input parameters: four Wolfenstein parameters characterizing the Cabibbo-Kobayashi-Maskawa (CKM) matrix are used with \( \lambda = 0.2205 \) and \( A = 0.815 \). As for the parameters \( \rho \) and \( \eta \), different updated analyses \[^{[23, 24, 25]}\] have been performed. In these fits, it is clear that \( \sqrt{\rho^2 + \eta^2} = 0.41 \) is slightly larger than the previous analysis. For our purposes in the present paper we will employ the values \( \rho = 0.175 \) and \( \eta = 0.370 \). Though not be completely ruled out, a negative \( \rho \) is disfavored by these global analyses. However, it is found that a negative \( \rho \) is preferable by the CLEO data \[^{[28, 13]}\]. The impact of a negative \( \rho \) is discussed with a simple sign-flip of \( \rho \). Under the factorization hypothesis, the decay amplitudes are expressed as the products of decay constants and form factors. We follow the standard parameterizations for decay constants and form factors \[^{[23]}\]. For values of the decay constants, we take \( f_\pi = 132 \text{ MeV} \), \( f_K = 160 \text{ MeV} \), \( f_\rho = 210 \text{ MeV} \), \( f_{K^*} = 221 \text{ MeV} \), \( f_\omega = 195 \text{ MeV} \) and \( f_\phi = 237 \text{ MeV} \). Concerning the heavy-to-light mesonic form factors, we will use the BSW results evaluated in the relativistic quark model \[^{[23]}\] with the proper \( q^2 \)-dependence adopted from the heavy-quark symmetry.

3. To set up our notation, we will briefly discuss the angular distributions of \( B \to VV \) and CP violating observables. The most general covariant amplitude for a \( B \) meson decaying into a pair of vector mesons has the form \[^{[14, 19]}\]:

\[
A(B(p) \to V_1(k)V_2(q)) = \epsilon_{V_1}^{\mu}\epsilon_{V_2}^{\nu} \left( a g_{\mu\nu} + \frac{b}{m_{V_1}m_{V_2}} p_{\mu} p_{\nu} + i \frac{c}{m_{V_1}m_{V_2}} \epsilon_{\mu\alpha\beta} k^{\alpha} q^{\beta} \right), \quad (6)
\]
where, $\epsilon_{V_1}$, $\epsilon_{V_2}$ and $m_{V_1}$, $m_{V_2}$ represent the polarization vectors and masses of the vector mesons $V_1$ and $V_2$, respectively. These invariant amplitudes $a$, $b$, and $c$ have the advantage of being directly related to the decay constants and form factors under the generalized factorization approach.

However, it is customary to express the angular distributions of $B \to VV$, with each vector meson subsequently decaying into two particles, in terms of the helicity amplitudes, for which we use the notation: $H_\lambda = \langle V_1(\lambda) V_2(\lambda) | H_{wkl} | B \rangle$ for $\lambda = 0, \pm 1$. The relations between the helicity and invariant amplitudes are $H_0 = -a x - b (x^2 - 1)$, and $H_\pm = a \pm \sqrt{x^2 - 1} c$. In general, the explicit form of the angular distribution depends upon the spin of the decay products of the two decaying vector mesons. To be specific, we will take for the purpose of demonstration the angular distribution of the decays $B \to V_1(\to P_1 P'_1) V_2(\to P_2 P'_2)$, where $P_1^{(i)}$ and $P_2^{(j)}$ denote pseudoscalar mesons. An example is $B^- \to K^{*-} \rho^0 \to (K\pi^-) (\pi^+ \pi^-)$. The normalized angular distribution for this type of decay is:

$$\frac{1}{\Gamma} \frac{d^3 \Gamma}{d \cos \theta_1 d \cos \theta_2 d \phi} = \frac{9}{8\pi} \left( \frac{\Gamma_T}{4 \Gamma} \sin^2 \theta_1 \sin^2 \theta_2 + \frac{\Gamma_L}{\Gamma} \cos^2 \theta_1 \cos^2 \theta_2 \right. $$

$$+ \frac{1}{4} \sin 2\theta_1 \sin 2\theta_2 [\alpha_1 \cos \phi - \beta_1 \sin \phi] $$

$$+ \frac{1}{2} \sin^2 \theta_1 \sin^2 \theta_2 [\alpha_2 \cos 2\phi - \beta_2 \sin 2\phi] \right),$$

where

$$\Gamma_T = \frac{|H_{+1}|^2 + |H_{-1}|^2}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2}, \quad \Gamma_L = \frac{|H_0|^2}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2},$$

$$\alpha_1 = \frac{\text{Re} (H_{+1}H_0^* + H_{-1}H_0^*)}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2}, \quad \beta_1 = \frac{\text{Im} (H_{+1}H_0^* - H_{-1}H_0^*)}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2},$$

$$\alpha_2 = \frac{\text{Re} (H_{+1}^*H_0)}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2}, \quad \beta_2 = \frac{\text{Im} (H_{+1}^*H_0)}{|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2}.$$  

Here $\theta_1 (\theta_2)$ is the angle between the $P_1 (P_2)$ three-momentum vector in the $V_1 (V_2)$ rest frame and the $V_1 (V_2)$ three-momentum vector defined in the $B$ rest frame, and $\phi$ is the angle between the normals to the planes defined by $P_1P'_1$ and $P_2P'_2$, in the $B$ rest frame.

To take the advantage of more easily extracting the CP-odd and -even components, the angular distribution is often written in the linear polarization basis, which is defined, according to the notation in [9], in the following form of the decay amplitude:

$$(B_q(t) \to V_1V_2) = \frac{A_0(t)}{x} \epsilon_{V_1}^{*T} \epsilon_{V_2}^{*L} - A_\parallel(t) \epsilon_{V_1}^{*T} \cdot \epsilon_{V_2}^{*L} \sqrt{2} - i A_\perp(t) \epsilon_{V_1}^{*T} \times \epsilon_{V_2}^{*L} \cdot \hat{p}_{V_2} \sqrt{2},$$  

where $x \equiv p_{V_1} \cdot p_{V_2} / (m_{V_1} m_{V_2})$ and $\hat{p}_{V_2}$ is the unit vector along the direction of motion of $V_2$ in the rest frame of $V_1$. The transversity amplitudes $A_\parallel$, $A_0$ and $A_\perp$ are related to the helicity ones by $A_0 = H_0$, $A_\parallel = \frac{1}{\sqrt{2}} (H_{+1} + H_{-1})$, and $A_\perp = \frac{1}{\sqrt{2}} (H_{+1} - H_{-1})$. The normalized differential decay rate in terms of transversity amplitudes is then given by

$$\frac{1}{\Gamma} \frac{d^3 \Gamma}{d \cos \psi d \cos \theta d \phi} = \frac{9}{8\pi} \left( \frac{\Gamma_L}{\Gamma} \cos^2 \psi \sin^2 \theta \cos^2 \varphi + \frac{\Gamma_\parallel}{2 \Gamma} \sin^2 \psi \sin^2 \theta \sin^2 \varphi \right.$$  

$$+ \frac{\Gamma_\parallel}{2 \Gamma} \sin^2 \psi \cos^2 \theta - \frac{\zeta}{2 \sqrt{2}} \sin 2\psi \sin 2\theta \sin 2\varphi$$

$$- \frac{\xi_1}{2} \sin^2 \psi \sin 2\theta \sin \varphi + \frac{\xi_2}{2 \sqrt{2}} \sin 2\psi \sin 2\theta \cos \varphi \right).$$
\[ \Gamma_T = \frac{|A_0|^2 + |A_1|^2}{|A_0|^2 + |A_1|^2 + |A_\perp|^2}, \quad \Gamma_{||} = \frac{|A_1|^2}{|A_0|^2 + |A_1|^2 + |A_\perp|^2}, \]
\[ \xi_1 = \frac{\text{Im}(A_1 A_1^*)}{|A_0|^2 + |A_1|^2 + |A_\perp|^2}, \quad \xi_2 = \frac{\text{Im}(A_1 A_1^*)}{|A_0|^2 + |A_1|^2 + |A_\perp|^2}, \]

and we take the rest frame of \( V_1 \), \( V_2 \) moves in the x direction, and the z axis is perpendicular to the decay plane of \( V_2 \to P_2 P_2' \) and we assume that \( p_\rho(P_2) \) is nonnegative. \((\theta, \phi)\) is the angular coordinates of \( P_1 \) and \( \psi \) is that of \( P_2 \), both in the rest frame of \( V_1 \).

By measuring the six coefficients in the angular distribution of \( B \to VV \) and their corresponding conjugate processes, we can construct rich CP violating observables, in addition to the usual partial rate difference \( \Delta = T_1 - T_2 \). Since it is easy to extract the CP information from the measurements in the transversity basis, we will only concentrate on this basis. With \( \eta, \xi_1, \xi_2 \) for \( B_u^- \) decays, and similarly \( \bar{\tau}, \bar{\xi}_1, \bar{\xi}_2 \) for \( B_u^+ \) decays, the CP violating observables for the transversity amplitudes can be constructed as: \( T_1 = \xi_1 + \bar{\xi}_1, T_2 = \zeta - \bar{\zeta} \), and \( T_3 = \xi_2 + \bar{\xi}_2 \) for the processes with the same branching ratios in both \( B_u^- \) decays and their conjugate processes. For the processes with different branching ratios, we could use the same definition for the unnormalized distributions.

**4.** In this letter, we calculate all the angular distributions and direct CP violation in the helicity \( \| \) and transversity bases. Our results \( \| \) are shown in Table I, from which we find: (1) all the charmless \( B_u \to VV \) decay modes are dominated by the longitudinal polarized state and the P-wave amplitudes in these decay modes are small, thus all the charmless \( B_u \) decays are dominated by the CP-even components and have only small angular correlation asymmetries \((i.e.\ CP\ violation\ in\ the\ angular\ correlation\ coefficients)\) associated with the imaginary terms, and (2) the imaginary terms appearing in the angular correlation are all small, especially they vanish for the \( \rho^- \rho^0, K^{*0} \rho^- \), \( K^{*+} \phi, K^{*-} K^{*0} \) and \( \rho^- \phi \) modes, and the normalized angular correlation coefficients in those decay modes are the same for the \( B_u \) decay modes and their conjugate modes. The later feature is a general phenomenon of all the processes involving only one factorized amplitude \( X^{(B^-VV)} \) (defined in Eqs. (1)), for example the \( K^{*0} \rho^-, K^{*-} \phi, K^{*-} K^{*0} \) and \( \rho^- \phi \) modes in charmless \( B_u \) decays. It results from the fact that the standard CP-violating weak phase and the CP-conserving perturbative strong phase are all factored out into a common factor in the processes with only one \( X^{(B^-VV)} \). Thus all the angular correlation coefficients, which appear in the distributions in a bilinear from \( A_f A_\gamma^* \), are all real and their differences only show up in the partial rate difference .

One origin of this general feature comes form the universality ansatz for the nonfactorizable contributions. A measurement of non-negligible imaginary terms for these decay modes does indicate a possible deviation of the universality ansatz and/or a nontrivial phase among different amplitudes.

The factorized amplitude of \( B^- \to \rho^- \rho^0 \) is
\[ A(B^- \to \rho^- \rho^0) = \frac{G_F}{\sqrt{2}} \left[ V_{ub} V_{ud}^*(a_1 + a_2) - V_{td} V_{ub}^* \frac{3}{2} (a_7 + a_9 + a_{10}) \right] X^{(B^- \rho^0, \rho^-)} \]

\(^3\)To save the space, we shall only show the results in the tranversity basis.

\(^4\)The relevant formulas of decay amplitudes can be found in [13]
where the factorized term $X^{(BV_1,V_2)}$ has the expression:

$$X^{(BV_1,V_2)} \equiv \langle V_2||\bar{q}_2q_3\rangle_{\nu-A}|0\rangle\langle V_1||\bar{q}_1b\rangle_{\nu-A} = -if_{V_2}m_2\left[\left(\varepsilon_1^* \cdot \varepsilon_2^*\right)(m_B + m_1)A_1^{BV_1}(m_2^2) - \left(\varepsilon_1^* \cdot p_B\right)(\varepsilon_2^* \cdot p_B)\frac{2A_2^{BV_1}(m_2^2)}{(m_B + m_1)} + i\epsilon_{\mu\nu\alpha\beta}\varepsilon_2^\mu\varepsilon_1^\nu p_\alpha p_\beta \frac{2V^{BV_1}(m_2^2)}{(m_B + m_1)}\right]. \quad (13)$$

Since the CKM matrix for the tree and penguin contributions are comparable, this decay is dominated by the largest $a_1$ and hence it is $N_c$-stable. Due to isospin symmetry, the QCD penguin does not contribute to this decay mode. The electroweak penguin (EWP), though making little contributions, cannot be neglected when discussing the CP asymmetry: without the EWP contributions, the partial rate asymmetry will be zero. The small $\Delta$ reflects the small contributions from the EWP and thus the actual EWP contributions can be determined by the measurement of the partial rate asymmetry.

It is instructive to compare $B^- \to \rho^-\omega$ with $B^- \to \rho^-\rho$. Although the average branching ratios for these two decay modes are almost the same, the physics involved are quite different. The factorized amplitude for $B^- \to \rho^-\omega$ is

$$A(B^- \to \omega\rho^-) = V_{ub}V_{ud}^*\left\{a_1X^{(BV_1,\omega^-)} + a_2X^{(BV_2,\omega^-)}\right\} - V_{tb}V_{td}^*\left\{(a_4 + a_{10})X^{(BV_1,\rho^-)} + (2a_3 + a_4 + 2a_5 + \frac{1}{2}(a_7 + a_9 - a_{10})X^{(BV_1,\omega^-)}\right\}. \quad (14)$$

Since the CKM factors in the tree and penguin parts are comparable, this decay mode is still dominated by the tree diagram with the largest $a_1$. Unlike the case of $B^- \to \rho^-\rho$, the QCD penguin does make sizable contributions to $B^- \to \rho^-\omega$. The QCD penguin contributions, which are smeared out in the average quantities of the branching ratio, do show their impacts on the angular correlation coefficients. With sizable QCD penguin contributions, direct CP violation in this decay mode can be as large as $-5\%-10\%$. While the partial rate asymmetry depends less upon the factorization scheme, the angular correlation asymmetries are highly sensitive to $N_c$. Pursuant to sizable QCD penguin contributions, we wish to emphasize that a determination of the unitary triangle $\alpha$ via this decay mode is more promising than via $\rho^-\rho$.

With the replacement of $\rho^-$ by $K^{*-}$, the QCD penguin becomes the dominant mechanism in $B^- \to K^{*-}\omega(\rho)$ due to the CKM factors involved. The comparable tree and penguin contributions result in a significant partial rate asymmetry, however their destructive interference makes the branching ratio smaller. The general amplitude for $B^- \to K^{*-}\omega$ is

$$A(B^- \to \omega K^{*-}) = V_{ub}V_{us}^*\left\{a_1X^{(BV_1,K^{*-})} + a_2X^{(BV_2,K^{*-})}\right\} - V_{tb}V_{ts}^*\left\{(a_4 + a_{10})X^{(BV_1,K^{*-})} + (2a_3 + 2a_5 + \frac{1}{2}(a_7 + a_9))X^{(BV_2,K^{*-})}\right\}. \quad (15)$$

While the average branching ratios do not show significant changes in these three schemes, the predicted CP-violating observables and imaginary angular correlation asymmetries are dramatically different. The naive factorization and our preferred factorization predict a positive partial rate difference, but the large-$N_c$ improved factorization predicts a negative and smaller one. Direct CP violation in the partial rate asymmetry of this decay mode in our preferred factorization scheme (and also in the naive factorization scheme) can be larger than 40 %. It is also found that the angular correlation asymmetries, especially $T_3$, in this decay mode and also in the $B^- \to \rho^0 K^{*-}$ decay mode are not negligible.
The amplitude for \( B^- \to \rho^0 K^{*-} \) is
\[
A(B^- \to \rho^0 K^{*-}) = V_{ub} V_{us}^* \left\{ a_1 X(B\rho^0,K^{*-}) + a_2 X(BK^{*-}\rho^0) \right\} \\
- V_{tb} V_{ts}^* \left\{ (a_4 + a_{10}) X(B\rho^0,K^{*-}) + \frac{3}{2}(a_7 + a_9) X(BK^{*-}\rho^0) \right\}.
\]
(16)
The pattern of \( Br(B^- \to \rho^0 K^{*-}) > Br(B^- \to \omega K^{*-}) \), which is independent of the factorization scheme, comes from the behaviour of the QCD penguin: a destructive interference among the \( a_4 \) and \( a_3 + a_5 \) terms in \( \omega K^{*-} \) makes its branching ratio smaller than that of \( \rho K^{*-} \), where the latter does not suffer from the destructive interference. Since there is only one dominant QCD penguin contribution in this \( N_c \)-stable \( \rho K^{*-} \) mode, the interferences between the tree and penguin contributions have the same behaviours within three factorization schemes, and the associated partial rate asymmetries predicted thus have a definite sign no matter which factorization scheme is used. Due to \( N_c \)-insensitivity and the existence of not-so-small tree contributions, a determination of the unitary triangle \( \gamma \) through this decay mode is encouraging.

All the analysis and conclusions mentioned above are based on a positive \( \rho \), which is favored by the global fitting. However, it has been shown that a negative \( \rho \) does show an improvement between the theory and experiments. The impact of a negative \( \rho \) on \( B_u \to V V \) is studied with a simple sign-flip of \( \rho \) and is discussed for the following four different classes. The first class is the \( b \to s \) purely-penguin modes, consisting of \( K^{*-0} \rho^- \) and \( K^{*-} \phi \), where the CKM factors involved in these decay modes are not sensitive to \( \rho \). Thus the branching ratios for \( B_u^\pm \) decays are not sensitive to the sign of \( \rho \) and almost the same. The partial rate asymmetry is then very small and not so interesting. The second class is the \( b \to d \) purely-penguin modes, consisting of \( K^{*-}K^{*0} \) and \( \rho^- \phi \), where the involved CKM factor is \( V_{tb} V_{ts}^* \) which is very sensitive to the sign of \( \rho \). A sign-flip for the \( \rho \) from a positive one to a negative one can enhance the branching ratio by a factor of two. Because of this enhancement in the branching ratio, the partial rate difference in these two decay modes is suppressed. While \( K^{*-}K^{*0} \) mode is \( N_c \)-stable, \( \rho^- \phi \) mode is highly sensitive to the factorization scheme we used, and the associated CP-violating observables too. While the two classes discussed so far are purely-penguin processes which are simple to analyze, the next two classes having both tree and penguin contributions are more subtle. For the third class, which involves \( b \to s \) transition and consists of \( K^{*-}\rho \) and \( K^{*-}\omega \), the penguin contribution plays the dominant role with a sizable tree contribution. The destructive (constructive) interference between tree and penguin contributions makes the branching ratio smaller (larger) and partial rate asymmetry larger (smaller) with positive (negative) \( \rho \), respectively. The decays in the last class with \( \rho^-\rho \) and \( \rho^-\omega \) have comparable CKM factors in the tree and penguin parts and thus are dominated by the largest \( a_1 \) term. Because of this and the absence of the QCD penguin contribution, \( \rho^-\rho \) has nearly the same branching ratio as its conjugate process and thus the associated partial rate difference is small and insensitive to the factorization schemes and the sign of \( \rho \). It is very interesting that the impact of negative \( \rho \) on the \( \rho^-\omega \) mode shows an opposite behaviour to the third class: one gets a smaller (larger) branching ratio and larger (smaller) partial rate difference with a negative (positive) \( \rho \), respectively.

5. In this letter, we revisit two-body charmless nonleptonic decays of \( B \to V V \) by employing the generalized factorization approach in which the effective Wilson coefficients \( c_{i}^\text{eff} \) are renormalization-scale and -scheme independent while factorization is applied to the tree-level hadronic matrix elements. Contrary to previous studies, our \( c_{i}^\text{eff} \) do not suffer from gauge and infrared problems. Following the standard approach, we make a further universal assumption for the nonfactorizable contributions and thus these nonfactorizable effects can then be parametrized in terms of \( N_{c}^\text{eff}(LL) \) and \( N_{c}^\text{eff}(LR) \), the effective numbers of colors arising from \((V - A)(V - A)\) and \((V - A)(V + A)\).
four-quark operators, respectively. The full angular distributions of charmless decays are calculated in terms of not only the helicity basis but also the transversity basis. In addition to the partial rate difference, we also calculate other CP violating observables. Results from three different schemes for the nonfactorizable contribution, the naive factorization, large-$N_c$ improved factorization and our preferred choice-the optimized hetero-factorization are presented.

Our main results are the following:

- The longitudinal polarization dominates over other polarization states (about 90%) for all charmless VV decay modes with the beautiful pattern: $A_0 \gg A_\| > A_\perp$, i.e. the P-wave (CP-odd component) amplitudes are small. Thus all the charmless $B_u \to VV$ decays are governed by the CP-even components and the imaginary angular distribution coefficients are small.

- For those processes involving only one factorized amplitude $X^{(B^-V,V)}$, such as $\overline{K}^0 \rho^-$, $K^+ \phi$, $K^0 K^0$, $\rho^- \rho^0$ and $\rho^- \phi$ modes, the imaginary terms appearing in the angular distribution are all zero. Thus a measurement of these imaginary terms will show whether there is a deviation from the universal ansatz for the nonfactorizable contributions, a possible final state phase, or even a nontrivial phase from the new physics.

- Though having the largest branching ratios, $B_u^\pm \to \rho^\pm \rho$, which are dominated by the tree diagrams, have a small partial rate difference. Thus the most exciting decay mode from the viewpoint of a large branching ratio and also a large partial rate difference is $B_u^- \to \rho^- \omega$, which has a sizable penguin contribution. Due to this sizable penguin contribution, we would like to emphasize that a possible determination of the unitary triangle $\alpha$ is more promising in $B_u^- \to \rho^- \omega$ than in $B_u^- \to \rho^- \rho$.

- Since the penguin contributions play a major role in $B_u^- \to K^* \rho (\omega)$, large $N_c$-improved factorization predicts a large cancellation between the tree $a_1$ and $a_2$ terms and thus a smaller partial rate asymmetry. For $B_u^- \to K^* \rho$, which is not sensitive to the information of nonfactorization, the CP-violating observables have the same sign in these three factorization schemes. Because of the $N_c$-insensitivity and the not-so-small tree contributions, a determination of the unitary triangle $\gamma$ through this decay mode is encouraging. Though having the largest partial rate asymmetry, $B^- \to K^* \omega$ suffers from the theoretical uncertainty in the nonfactorizable contents. The quantum interference among the tree and penguin contributions is changed and thus the relevant sign of the partial rate asymmetry is also changed when we use the large-$N_c$ improved factorization instead of the naive factorization. Direct CP violation in the partial rate asymmetry of the $B^- \to K^* \omega$ mode in our preferred factorization scheme (and also in the naive factorization scheme) can be larger than 40%. The angular correlation asymmetries predicted for $B^- \to \rho^0 (\omega) K^*$ are not negligible.

- To show the parametric correlation with $\rho$, we also make some predictions based on a negative $\rho$. A negative $\rho$ will enhance the penguin contributions with $V_{td}$ which is proportional to $(1 - \rho - i\eta)$ and change the interference among the tree and penguin contributions especially for those involved $V_{ub}$ as $(\rho - i\eta)$. A negative $\rho$ has little impact on the $b \to s$ purely-penguin modes, but can enhance the branching ratios and thus reduce the associated partial rate asymmetries of $b \to d$ purely-penguin modes by a factor of two with a sign-flip done in this analysis. For the decay modes with both tree and penguin contributions, there are two totally different behaviours: the branching ratios are enhanced (reduced) and the partial
rate differences are reduced (enhanced) for the $b \to s$ ($b \to d$) transition, except for the $\rho^-\rho$ mode which is governed by the tree contribution and has nearly the same branching ratio as its conjugate process. The partial rate difference for $\rho^-\rho$ is small and insensitive to the factorization schemes and the sign of $\rho$.

Finally, we discuss some uncertainties in our calculation and their possible impacts.

- In this letter, we have neglected the $W$-annihilation (WA) and the space-like penguin (SP) contributions. The WA and SP do not appear in the $B^-_u \to \rho^-\phi$ mode (likewise, the $W$-exchange contribution and SP also disappear in $B_d \to \rho(\omega)\phi$), thus these processes do not suffer from the uncertainties due to nonspectator contributions. The impact of WA on the purely-penguin modes may be quite significant. For $B^-_u \to K^{*-}\phi$, the CKM-suppressed WA with the largest $a_1$ may have a large effect under the condition that a large cancellation among the QCD penguin contributions happens. Likewise for $B^-_u \to K^{*-}K^{*0}$, WA with the largest $a_1$ may also have a large influence on this decay mode because of the comparable CKM factors for the penguin and WA.

- The perturbative strong phase is included in our calculation, while soft final-state interaction (FSI) phases are not considered in this paper. The soft FSI phases do have large impacts on the angular correlation coefficients, especially for classes involving one $X^{(B,V,V)}$ where the imaginary terms are all vanishing. However, a measurement of non-vanishing imaginary terms for this class of decay modes is not necessarily claimed to be a direct confirmation of the FSI phase in $B$ decays since they can be generated by the CP-violating phases induced from new physics.

- Our results for CP observables are evaluated at $k^2 = m_b^2/2$. It is known that CP violation is sensitive to the $k^2$ we used. Besides, the physics of $B_u \to VV$ is sensitive to the form factors we used as shown in the recent paper [13]. These topics will be discussed in a separate publication.

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References

[1] For a review of CLEO measurements on charmless $B$ decays, see K. Lingel, T. Skwarnicki, and J.G. Smith, Annu. Rev. Nucl. Part. Sci. 48, 253 (1998).

[2] Y. S. Gao and F. Würthwein, [hep-ex/9904008]
[3] L.L. Chau, H.Y. Cheng, W.K. Sze, H. Yao, and B. Tseng, *Phys. Rev.* **D43**, 2176 (1991); **D58**, (E)019902 (1998); **D45**, 3143 (1992).

[4] I. Halperin and A. Zhitnitsky, *Phys. Rev.* **D56**, 7247 (1997); E.V. Shuryak and A. Zhitnitsky, *Phys. Rev.* **D57**, 2001 (1998); A.L. Kagan and A.A. Petrov, [hep-ph/9707354](http://arxiv.org/abs/hep-ph/9707354); N.G. Deshpande, B. Dutta, and S. Oh, *Phys. Rev.* **D57**, 5723 (1998); [hep-ph/9712445](http://arxiv.org/abs/hep-ph/9712445); A. Datta, X.G. He, and S. Pakvasa, *Phys. Lett.* **B419**, 369 (1998); A. Ali, J. Chay, C. Greub, and P. Ko, *Phys. Lett.* **B424**, 161 (1998); A. Ali and C. Greub, *Phys. Rev.* **D57**, 2996 (1998); M.R. Ahmady, E. Kou, and A. Sugamoto, *Phys. Rev.* **D58**, 014015 (1998); D.S. Du, C.S. Kim, and Y.D. Yang, *Phys. Rev.* **B419**, 369 (1998). D.S. Du, Y. D. Yang, G. Zhu, *Phys. Rev.* **D59** 014007 (1999).

[5] A.J. Buras, M. Jamin, M.E. Lautenbacher, and P.H. Weisz, *Nucl. Phys.* **B370**, 69 (1992); A.J. Buras, M. Jamin, and M.E. Lautenbacher, *Nucl. Phys.* **B408**, 209 (1993).

[6] M. Ciuchini, E. Franco, G. Martinelli, L. Reina, and L. Silvestrini, *Z. Phys.* **C68**, 255 (1995).

[7] G. Buchalla, A.J. Buras, and M.E. Lautenbacher, *Rev. Mod. Phys.* **68**, 1125 (1996); A.J. Buras, [hep-ph/980647](http://arxiv.org/abs/hep-ph/980647).

[8] R. Fleischer, *Phys. Lett.* **B332**, 419 (1994); N.G. Deshpande and X.G. He, *Phys. Lett.* **B336**, 471 (1994); N.G. Deshpande, X.G. He, and J. Trampetic, *Phys. Lett.* **B345**, 847 (1994); G. Kramer, W.F. Palmer and H. Simma, *Phys. Rev.* **D52**, 6411 (1995); D.S. Du and M.Z. Yang, *Phys. Lett.* **B358**, 123 (1995); D.S. Du and L. Guo, *Z. Phys.* **C75**, 9 (1997).

[9] H.Y. Cheng, H.n. Li, and K.C. Yang, [hep-ph/9902239](http://arxiv.org/abs/hep-ph/9902239) (1999).

[10] H.Y. Cheng, *Int. J. Mod. Phys.* **A4**, 495 (1989); *Phys. Lett.* **B335**, 428 (1994); *Phys. Lett.* **B395**, 345 (1997).

[11] A.N. Kamal and A.B. Santra, *Phys. Rev.* **D51**, 1415 (1995); *Z. Phys.* **C72**, 91 (1996); A.N. Kamal, A.B. Santra, and R.C. Verma, *Phys. Rev.* **D53**, 2506 (1996); A.N. Kamal, A.B. Santra, and F. Ghoroddoussi, Nuovo Cim. **111A**, 165 (1998); F.M. Al-Shamali and A.N. Kamal, Eur. Phys. J. **C4**, 669 (1998); *Phys. Rev.* **D59**, 054020 (1999).

[12] H.Y. Cheng and B. Tseng, *Phys. Lett.* **B415**, 263 (1997); *Phys. Rev.* **D58**, 094005 (1998).

[13] Y.H. Chen, H.Y. Cheng, B. Tseng, and K.C. Yang, [hep-ph/9903453](http://arxiv.org/abs/hep-ph/9903453).

[14] B. Tseng, *Phys. Lett.* **B446**, 125 (1999); Y.H. Chen, H.Y. Cheng, and B. Tseng, *Phys. Rev.* **D59**, 074003 (1999).

[15] G. Valencia, *Phys. Rev.* **D39**, 3339 (1989).

[16] G. Kramer and W.F. Palmer, *Phys. Rev.* **D45**, 193 (1992); *Phys. Lett.* **B279**, 181 (1992); *Phys. Rev.* **D46**, 2969, 3197 (1992); G. Kramer, T. Mannel and W.F. Palmer, *Z. Phys.* **C55**, 497 (1992); G. Kramer, W.F. Palmer and H. Simma, *Nucl. Phys.* **B428**, 77 (1994); *Z. Phys. C66*, 429 (1995).

[17] D. Atwood and A. Soni, *Phys. Rev.* **D59**, 013007 (1999).

[18] A.N. Kamal and C.W. Luo, *Phys. Lett.* **B388**, 633 (1996).
[19] A.J. Buras and L. Silvertrini, hep-ph/9806278.

[20] M. Neubert and B. Stech, in Heavy Flavours, edited by A.J. Buras and M. Lindner, 2nd ed. (World Scientific, Singapore, 1998).

[21] H.Y. Cheng and B. Tseng, Phys. Rev. D51, 6295 (1995).

[22] A. Ali, G. Kramer, and C.D. Lü, Phys. Rev. D58, 094009 (1998); D59, 014005 (1998).

[23] M. Wirbel, B. Stech, and M. Bauer, Z. Phys. C29, 637 (1985).

[24] A.J. Buras, J.-M. Gérard, and R. Rückl, Nucl. Phys. B268, 16 (1986).

[25] F. Parodi, P. Roudeau, and A. Stocchi, hep-ex/9903063 and references therein.

[26] S. Mele, hep-ph/9810333.

[27] A. Ali and D. London, hep-ph/9903535.

[28] N.G. Despande, X.G. He, W.S. Hou, and S. Pakvasa, Phys. Rev. Lett. 82, 2240 (1999); X.G. He, W.S. Hou, and K.C. Yang, hep-ph/9902256.

[29] J.L. Rosner, Phys. Rev. D42, 3732 (1991); A. S. Dighe, I. Dunietz, H. J. Lipkin and J. L. Rosner, Phys. Lett. B369, 144 (1996).
Table A.1  Branching ratios, partial rate asymmetries, angular correlation coefficients in the transversity basis for various $B_u^- \rightarrow VV$ processes. Results are obtained using the BSW form factors in the absence of soft FSI phases. The Wolfenstein parameters $(\rho, \eta) = (0.175, 0.370)$ are used in different factorization schemes (FS's): (a) the naive factorization scheme, (b) the large-$N_c$ improved factorization scheme, and (c) our preferred factorization scheme: $((N_c(LL), N_c(LR)) = (2, 5))$. The impact of a negative $\rho$ is studied in (d) our preferred factorization scheme: $(N_c(LL), N_c(LR)) = (2, 5)$ with $(\rho, \eta) = (-0.175, 0.370)$. Values in the parentheses correspond to the conjugate processes.

| Process       | FS  | BR $\times 10^{-6}$ | $a_{CP}$ [%] | $\Gamma_\perp$ [%] | $\Gamma_\parallel$ [%] | $\zeta$ [%] | $\xi_1$ $\times 10^{-3}$ | $\xi_2$ $\times 10^{-3}$ |
|---------------|-----|---------------------|---------------|-------------------|------------------------|------------|------------------------|------------------------|
| $K^*\phi^-\rho^0$ | (a) | 4.80                | 0.21          | 84.2              | 9.6                    | 6.2        | -28.5                  | 0                      | 0                      |
|               |     | (4.78)              |               | (84.2)            | (9.6)                  | (6.2)      | (-28.5)                | (0)                    | (0)                    |
|               | (b) | 0.290               | 0.69          | 84.2              | 9.6                    | 6.2        | -28.5                  | 0                      | 0                      |
|               |     | (0.286)             |               | (84.2)            | (9.6)                  | (6.2)      | (-28.5)                | (0)                    | (0)                    |
|               | (c) | 3.92                | 0.13          | 84.2              | 9.6                    | 6.2        | -28.5                  | 0                      | 0                      |
|               |     | (3.91)              |               | (84.2)            | (9.6)                  | (6.2)      | (-28.5)                | (0)                    | (0)                    |
|               | (d) | 3.79                | 0.13          | 84.2              | 9.6                    | 6.2        | -28.5                  | 0                      | 0                      |
|               |     | (3.78)              |               | (84.2)            | (9.6)                  | (6.2)      | (-28.5)                | (0)                    | (0)                    |
| $K^*\rho^0$   | (a) | 5.15                | 0.05          | 87.6              | 7.1                    | 5.3        | -24.9                  | 0                      | 0                      |
|               |     | (5.145)             |               | (87.6)            | (7.1)                  | (5.3)      | (-24.9)                | (0)                    | (0)                    |
|               | (b) | 7.82                | 0.06          | 87.6              | 7.1                    | 5.3        | -24.9                  | 0                      | 0                      |
|               |     | (7.81)              |               | (87.6)            | (7.1)                  | (5.3)      | (-24.9)                | (0)                    | (0)                    |
|               | (c) | 4.03                | 0.12          | 87.6              | 7.1                    | 5.3        | -24.9                  | 0                      | 0                      |
|               |     | (4.02)              |               | (87.6)            | (7.1)                  | (5.3)      | (-24.9)                | (0)                    | (0)                    |
|               | (d) | 3.892               | 0.01          | 87.6              | 7.1                    | 5.3        | -24.9                  | 0                      | 0                      |
|               |     | (3.891)             |               | (87.6)            | (7.1)                  | (5.3)      | (-24.9)                | (0)                    | (0)                    |
| $K^*\rho^0$   | (a) | 4.78                | 20.40         | 88.3              | 6.8                    | 4.9        | -24.5                  | -0.06                  | 5.45                    |
|               |     | (3.16)              |               | (88.6)            | (6.7)                  | (4.8)      | (-24.3)                | (0.25)                 | (-2.21)                |
|               | (b) | 5.76                | 9.48          | 88.3              | 6.8                    | 4.9        | -24.5                  | -1.08                  | 9.76                    |
|               |     | (4.76)              |               | (88.7)            | (6.6)                  | (4.7)      | (-24.2)                | (0.62)                 | (-5.63)                |
|               | (c) | 4.43                | 24.43         | 88.3              | 6.8                    | 4.9        | -24.5                  | -0.30                  | 3.00                    |
|               |     | (2.69)              |               | (88.5)            | (6.7)                  | (4.8)      | (-24.3)                | (-0.03)                | (0.30)                  |
|               | (d) | 7.68                | 12.69         | 88.2              | 6.8                    | 5.0        | -24.5                  | -0.22                  | 1.94                    |
|               |     | (5.95)              |               | (88.3)            | (6.8)                  | (4.9)      | (-24.5)                | (-0.04)                | (0.39)                  |
| $K^*\omega$   | (a) | 2.56                | 42.62         | 87.4              | 7.2                    | 5.4        | -25.1                  | 0.39                   | -3.38                   |
|               |     | (1.03)              |               | (87.2)            | (7.3)                  | (5.5)      | (-25.3)                | (-0.24)                | (2.09)                  |
|               | (b) | 1.62                | -9.70         | 91.2              | 5.5                    | 3.3        | -22.1                  | 5.41                   | -46.6                   |
|               |     | (1.99)              |               | (91.5)            | (5.4)                  | (3.1)      | (-22.1)                | (-2.62)                | (22.6)                  |
|               | (c) | 2.76                | 46.03         | 87.7              | 7.1                    | 5.2        | -24.9                  | 0.59                   | -5.06                   |
|               |     | (1.02)              |               | (87.3)            | (7.3)                  | (5.4)      | (-25.2)                | (-0.66)                | (5.65)                  |
|               | (d) | 4.93                | 21.13         | 87.7              | 7.1                    | 5.2        | -24.9                  | 0.30                   | -2.56                   |
|               |     | (3.21)              |               | (87.6)            | (7.1)                  | (5.3)      | (-25.0)                | (-0.27)                | (2.33)                  |
| Process | FS | BR $\times 10^{-6}$ | $a_{CP}$ [%] | $\Gamma_\perp$ [%] | $\Gamma_\parallel$ [%] | $\frac{\Gamma_{1}}{\Gamma_{2}}$ [%] | $\xi$ [%] | $\xi_1$ [$\times 10^{-3}$] | $\xi_2$ [$\times 10^{-3}$] |
|--------|----|---------------------|--------------|------------------|------------------|------------------|-------|-----------------|-----------------|
| $K^*-K^{0}$ | (a) | 0.26 | -4.97 | 87.5 | 7.6 | 4.9 | -25.8 | 0 | 0 |
| | | (0.29) | — | (87.5) | (7.6) | (4.9) | (25.8) | (0) | (0) |
| | (b) | 0.39 | -4.37 | 87.5 | 7.6 | 4.9 | -25.8 | 0 | 0 |
| | | (0.43) | — | (87.5) | (7.6) | (4.9) | (25.8) | (0) | (0) |
| | (c) | 0.20 | -4.74 | 87.5 | 7.6 | 4.9 | -25.8 | 0 | 0 |
| | | (0.22) | — | (87.5) | (7.6) | (4.9) | (25.8) | (0) | (0) |
| | (d) | 0.36 | -2.69 | 87.5 | 7.6 | 4.9 | -25.8 | 0 | 0 |
| | | (0.38) | — | (87.5) | (7.6) | (4.9) | (25.8) | (0) | (0) |
| $\rho^-\phi$ | (a) | 0.0136 | 0.37 | 84.3 | 9.0 | 6.7 | -27.5 | 0 | 0 |
| | | (0.0135) | — | (84.3) | (9.0) | (6.7) | (27.5) | (0) | (0) |
| | (b) | 0.27 | -3.03 | 84.3 | 9.0 | 6.7 | -27.5 | 0 | 0 |
| | | (0.29) | — | (84.3) | (9.0) | (6.7) | (27.5) | (0) | (0) |
| | (c) | 0.0096 | -1.11 | 84.3 | 9.0 | 6.7 | -27.5 | 0 | 0 |
| | | (0.0094) | — | (84.3) | (9.0) | (6.7) | (27.5) | (0) | (0) |
| | (d) | 0.0175 | -0.57 | 84.3 | 9.0 | 6.7 | -27.5 | 0 | 0 |
| | | (0.0173) | — | (84.3) | (9.0) | (6.7) | (27.5) | (0) | (0) |
| $\rho^-\rho^0$ | (a) | 13.53 | -0.40 | 90.5 | 5.4 | 4.1 | -22.1 | 0 | 0 |
| | | (13.64) | — | (90.5) | (5.4) | (4.1) | (22.1) | (0) | (0) |
| | (b) | 7.61 | -0.46 | 90.5 | 5.4 | 4.1 | -22.1 | 0 | 0 |
| | | (7.68) | — | (90.5) | (5.4) | (4.1) | (22.1) | (0) | (0) |
| | (c) | 17.13 | -0.38 | 90.5 | 5.4 | 4.1 | -22.1 | 0 | 0 |
| | | (17.26) | — | (90.5) | (5.4) | (4.1) | (22.1) | (0) | (0) |
| | (d) | 16.17 | -0.37 | 90.5 | 5.4 | 4.1 | -22.1 | 0 | 0 |
| | | (16.29) | — | (90.5) | (5.4) | (4.1) | (22.1) | (0) | (0) |
| $\rho^-\omega$ | (a) | 12.75 | -9.51 | 90.5 | 5.4 | 4.1 | -22.2 | -0.014 | 0.23 |
| | | (15.43) | — | (90.5) | (5.4) | (4.1) | (22.2) | (0.012) | (-0.19) |
| | (b) | 7.98 | -5.17 | 90.5 | 5.4 | 4.1 | -22.2 | 0.0040 | -0.066 |
| | | (8.85) | — | (90.5) | (5.4) | (4.1) | (22.2) | (0.007) | (-0.10) |
| | (c) | 15.69 | -8.43 | 90.5 | 5.4 | 4.1 | -22.2 | -0.008 | 0.12 |
| | | (18.58) | — | (90.5) | (5.4) | (4.1) | (22.2) | (0.007) | (-0.10) |
| | (d) | 12.03 | -10.72 | 90.5 | 5.4 | 4.1 | -22.2 | -0.006 | 0.10 |
| | | (14.92) | — | (90.5) | (5.4) | (4.1) | (22.2) | (0.011) | (-0.18) |