FORM FACTOR DESCRIPTION OF THE NON-COLLINEAR
COMPTON SCATTERING TENSOR

Wei Lu

Center de Physique Théorique*, Ecole Polytechnique
91128 Palaiseau Cedex, France

(July 1997)

Abstract

We present a parameterization of the non-collinear (virtual) Compton scattering tensor in terms of form factors, in which the Lorentz tensor associated with each form factor possesses manifest electromagnetic gauge invariance. The main finding is that in a well-defined form factor expansion of the scattering tensor, the form factors are either symmetric or antisymmetric under the exchange of two Mandelstam variables, $s$ and $u$. Our decomposition can be used to organize complicated higher-order and higher-twist contributions in the study of the virtual Compton scattering off the proton. Such procedures are illustrated by use of the virtual Compton scattering off the lepton. In passing, we note the general symmetry constraints on Ji’s off-forward parton distributions and Radyushkin’s double distributions.

*Unité propre 14 du Centre National de la Recherche Scientifique.
I. INTRODUCTION

Recently, there is much revived interest in the virtual Compton scattering (VCS). By VCS, people usually mean the scattering of a virtual photon into a real photon off a proton target

\[ \gamma^*(q) + N(P, S) \rightarrow \gamma^*(q') + N(P', S') \, . \]

As usual, three Mandelstam variables are defined for this process: 

\[ s \equiv (q + P)^2, \quad t \equiv (q - q')^2, \quad u \equiv (P - q')^2 \, . \]

Due to the momentum conservation \( P + q = P' + q' \), there is the following constraint:

\[ s + t + u = q^2 + q'^2 + 2m^2, \tag{1} \]

where \( m \) is the proton mass.

The object of study is the following scattering tensor

\[ T^{\mu\nu}(q, P, S; q', P', S') = i \int d^4\xi e^{iq'\cdot\xi} \langle P', S'|T[J^\mu(\xi)J^\nu(0)]|P, S\rangle \, , \tag{2} \]

where \( J \) is the quark electromagnetic current in the proton and \( T \) stands for the time-ordering of the operators. At present, much of interest is focused on the deeply VCS (DVCS), which is a very special kinematic region of the generic VCS. It has been claimed that the dominant mechanism in the DVCS is the VCS off a massless quark \([1]\). Correspondingly, two different approaches to the DVCS tensor have been developed: the Feynman diagram expansion \([1,2]\) and operator product expansion (OPE) \([6,4]\).

A careful reader might be aware of such a fact: At the leading twist expansion of the DVCS tensor, both in the Feynman diagram expansion and in OPE approach, the resultant expressions do not possess manifest electromagnetic gauge invariance. The purpose of this paper is to remedy the case by presenting a full form factor parameterization of the non-collinear Compton scattering tensor. With the help of our decomposition of the scattering tensor, one can safely ignore the higher-twist terms at leading-twist expansion and recover the electromagnetic gauge invariance by brute force. Hopefully, our form factor description can be used to organize complicated calculations as one goes beyond leading twist and/or leading order.

We confess that we are not the very first to attempt to develop a form factor parameterization of the VCS tensor. As early as in 1960s, Berg and Lindner \([7]\) ever reported a parameterization of the VCS tensor in terms of form factors. The virtue, also an implicit assumption, in their decomposition is that the scattering tensor can be put into a form of direct products of the Lorentz tensors and Dirac bilinears, i.e., the Lorentz index of the VCS tensor is not carried by the gamma matrices. In fact, all the leading twist expansions of the DVCS tensor so far assume such a factorized form. A drawback of the Berg-Lindner decomposition is that they employed a lot of momentum combinations which have no specific crossing and time reversal transformation properties. As a consequence, the form factors they defined possess no specific symmetry properties under crossing and time reversal transformations. Moreover, their decomposition lacks a term associated with the Lorentz structure \( \epsilon^{\mu\nu\alpha\beta} q_\alpha q'_\beta \), which has been shown by recent researches to be a carrier of leading-twist contributions.

It should be stressed that there is no unique decomposition of the Compton tensor. A few years ago, Guichon, Liu and Thomas \([8]\) worked out a general decomposition of the
VCS tensor, which contains no explicit proton spinors. Their decomposition is nice for the discussion of the generalized proton polarizabilities, as has done in Ref. \[8\]. Recently, the decomposition of the VCS tensor of such type has been refined by Drechsel et al. \[9\] within more extensive contexts. However, a decomposition of the VCS tensor without explicit Dirac bilinear structures is of very limited use for the present Feynman diagram expansion and OPE analysis of the DVCS tensor.

Hence, it is desirable to reconstruct a parameterization of the Compton scattering tensor in terms of form factors with explicit Dirac structures, which constructs the subject of this paper. To make our arguments more transparent, we will first consider the Lorentz decomposition for the double VCS off a lepton, then transplant our results onto the proton case. [By double VCS we mean that both the initial- and final-state photons are virtual. Correspondingly, we will refer the usual VCS to as the single VCS in distinction.] The reason for adopting such a strategy is that in quantum electrodynamics, it is more convenient to discuss the chiral properties of the Dirac bilinears. At the later stage, we will reduce our results for the double VCS to the real Compton scattering (RCS) as well as the single VCS. Such a procedure will greatly facilitate the discussion of the symmetry properties of the single VCS form factors.

The decomposition of the Compton tensor is essentially subject to the symmetries that it observes, so we begin with a brief discussion of the symmetry properties of the Compton scattering. First, the current conservation requires that

\[ q_\mu T^{\mu\nu}(q, P, S; q', P', S') = q'_\nu T^{\mu\nu}(q, P, S; q', P', S') = 0 . \]  

Second, parity conservation tells us

\[ T^{\mu\nu}(q, P, S; q', P', S') = T^{\nu\mu}(\bar{q}, \bar{P}, -\bar{S}; \bar{q}', \bar{P}', -\bar{S}') , \]  

where \( \bar{q}^\mu \equiv q^\mu \), and so on. Third, time reversal invariance demands

\[ T^{\mu\nu}(q, P, S; q', P', S') = T^{\nu\mu}(\bar{q}', \bar{P}', S'; \bar{q}, \bar{P}, \bar{S}) . \]  

Fourthly, there is a crossing symmetry for the Compton scattering, namely,

\[ T^{\mu\nu}(q, P, S; q', P', S') = T^{\nu\mu}(-q', P, S; -q, P', S') . \]  

By combining (3) with (4), we have

\[ T^{\mu\nu}(q, P, S; q', P', S') = T^{\nu\mu}(q', P', S'; q, P, -S) . \]  

That is to say, the adjoint parity-time-reversal transformation amounts to \( \mu \leftrightarrow \nu, q \leftrightarrow q', P \leftrightarrow P', S \rightarrow -S' \) and \( S' \rightarrow -S \). Furthermore, combining (3) with (5) yields

\[ T^{\mu\nu}(q, P, S; q', P', S') = T^{\nu\mu}(-q, P', -S'; -q', P, -S) . \]  

In fact, the Compton scattering respects more symmetries than summarized above. For example, it is subject to the momentum and angular momentum conservations. In the case of collinear scattering, the angular momentum conservation exerts further constraints on the Compton scattering. To show this, we digress to the helicity amplitude description of the Compton scattering.
In the expansion of the Compton scattering amplitude, a fundamental question that must be answered in advance is that how many independent state vectors there are in a complete basis. This can be most naturally done by counting independent helicity amplitudes. Here we stress that each independent helicity amplitude corresponds to one observable independent form factor in the Compton scattering tensor, while there is no simple one-to-one correspondence between helicity amplitudes and form factors.

Let us consider the most general non-collinear double VCS off a massive lepton ($l$). Because the massive lepton and the virtual photon have 2 and 3 helicity states respectively, there are $2 \times 3 \times 2 \times 3 = 36$ helicity amplitudes. By parity conservation, only half of these helicity amplitudes are independent. In the non-collinear case, the other symmetries cannot further reduce the number of independent helicity amplitudes. Similarly, there are 12 and 8 independent helicity amplitudes for the non-collinear single VCS and RCS off the massive lepton.

In the collinear scattering limits, however, time reversal invariance and angular momentum conservation impose further constraints on the Compton scattering. We denote a generic Compton scattering helicity amplitude $A(\lambda_q, \lambda_l; \lambda_q', \lambda_l')$, where $\lambda$'s are the helicities of the corresponding particles. By time reversal invariance, there is

$$A(\lambda_q, \lambda_l; \lambda_q', \lambda_l') = A(\lambda_q', \lambda_l'; \lambda_q, \lambda_l). \quad (9)$$

To discuss the constraints from angular momentum conservation, we need to distinguish two collinear limits:

$$\lambda_q - \lambda_l = \pm (\lambda_q' - \lambda_l'). \quad (10)$$

where $\pm$ corresponds to the forward and backward collinear scattering, respectively. As a result, only a small fraction of the Compton helicity amplitudes survive in the collinear limits. We summarize those surviving (independent) helicity amplitudes in Table 1.

The above helicity amplitude analysis implies a thorny fact: As one approaches the collinear limits, there is significant degeneracy in the form factor parameterization of the Compton scattering tensor. Here we emphasize that one must avoid the over-degeneracy of the form factor description in the collinear cases as much as possible. As will be clarified, some form factor parameterizations of the Compton scattering tensor, albeit applicable in the non-collinear cases, might become ill-defined in the collinear scattering limits.

Now we investigate the general structure of the non-collinear Compton tensor. As stated before, the Berg-Lindner decomposition assumes the following form

$$T^{\mu\nu}(q, P, S; q', P', S') = \sum_i \bar{U}(P', S') \Gamma_i U(P, S) t_i^{\mu\nu} F_i, \quad (11)$$

where $\Gamma_i$s are gamma matrices (saturated with particle momenta if carrying Lorentz indices), $t_i^{\mu\nu}$'s Lorentz (pseudo)-tensors constructed from the relevant particle momenta (the metric tensor and the Levi-Civita tensor may be involved), and $F_i$s Lorentz invariant form-factor like objects. As a matter of fact, all of the recent research results about the DVCS tensor can be tailored into the form of Eq. (11).

Now we justify Eq. (11) for the non-collinear Compton scattering. In principle, one can assume a decomposition for the Compton tensor in which there is no explicit Dirac bilinears.
Then, the Lorentz indices of the scattering tensor can be carried by the metric tensor, the Levi-Civita tensor, the particle momenta and lepton spin four-vectors. We will not write down any such decompositions. Rather, we note that the lepton spin four-vector can carry the free Lorentz index now. The spin four-vector \( S \) of a lepton of momentum \( P \) is subject to \( S \cdot P = 0 \), so it can be expressed in terms of three \textit{non-collinear} particle momenta. For the non-collinear scattering, one can write down, say

\[
S^\mu = (S \cdot K_1) P^\mu + (S \cdot K_2) q^\mu + (S \cdot K_3) q'^\mu, \tag{12}
\]

where \( K_1, K_2 \) and \( K_3 \) are three momentum combinations whose expressions we do not need.

As a consequence, one can eliminate the lepton spin four-vectors from the building blocks that carry the free Lorentz indices and lump \( S \cdot K_j \) into the form factors. At this stage, if one displays the Dirac bilinears, the decomposition of the Compton scattering tensor assumes the structure of Eq. (11).

From the above justification, we see that some subtleties will arise as one goes to the collinear limits of the Compton scattering. That is, Eq. (11) is inapplicable to the discussion of the transverse proton spin dependence of the Compton scattering amplitude in the collinear limits. Fortunately, the collinear scattering are only very special kinematic limits of the VCS. So it is still desirable to develop a form factor parameterization of the non-collinear Compton scattering with the general structure of Eq. (11).

The symmetries impose further constraints on the decomposition of Eq. (11). For a generic VCS, its form factors depend on 4 independent kinematical variables. Though there is much interest in the small-\(|t|\) limit behavior of the single VCS off the proton, we insist in choosing \( s, u, q^2 \) and \( q'^2 \) (\( t \) being an auxiliary quantity) as four independent kinematical variables for the form factors. The reason for doing so is that the crossing transformation of the Compton scattering essentially relates its \( s \)- to \( u \)-channel contributions or vice versa. Under the crossing transformation, \( s \leftrightarrow u \) and \( q^2 \leftrightarrow q'^2 \). Further, under the time reversal (or the adjoint parity-time-reversal) transformation, there is \( q^2 \leftrightarrow q'^2 \). So, it is a natural choice for us to define the form factors in such a way that all of them possess specific symmetry properties under the crossing and time-reversal transformations. To this end, we demand that the Lorentz tensor and Dirac bilinear associated with each form factor are either symmetric or antisymmetric under the crossing and time-reversal transformations.

At this stage, we recognize that it is more useful to talk about the adjoint crossing-(parity)-time-reversal transformation properties of the single VCS form factors, for which people usually take into account the on-shell condition \( q'^2 = 0 \) of the final photon in practical calculations.

Now we set about the construction of the form factor description of the non-collinear Compton scattering tensor. As claimed earlier, we begin with the double VCS off a lepton. We first consider the case of a massless lepton. Then, its helicity and chirality coincide with each other. For the massless lepton, the chiral symmetry holds exactly, so the lepton helicity is conserved in the Compton scattering. As a consequence, there are only 9 independent helicity amplitudes and accordingly 9 complex form factors for the non-collinear double VCS off the massless lepton. All the spinor bilinears must be chiral-even, so only the vector and axial-vector Dirac structures, \( \gamma_\alpha \) and \( \gamma_\alpha \gamma_5 \), get into work. To saturate the Lorentz indices carried by the Dirac matrices, we choose \( q + q' \), \( P \) and \( P' \) as 3 independent momenta. Obviously, there are only two nontrivial, independent Dirac structures: \( \bar{U}(P', S')(\bar{q} + \bar{q}'\gamma_5)U(P, S) \).
and \( U(P', S')(\hat{q} + \hat{q}')\gamma_5 U(P, S) \).

Now we construct proper gauge-invariant Lorentz tensors to match \( U(P', S')(\hat{q} + \hat{q}')U(P, S) \). In doing so, we keep it in mind to render our choices of independent Lorentz tensors possess specific crossing and parity-time-reversal transformation properties. Using the metric tensor, we can write down \(-(q \cdot q')g^{\mu\nu} + q^\mu q'^\nu\). As index \( \mu \) is carried by particle momenta, we can write down only two independent momentum combinations because of the momentum conservation and gauge invariance. Similarly for index \( \nu \). So, we have 4 more independent tensors without invoking the metric tensor. Our choices are \( A^\mu B^\nu \), \( A_1^\mu B_1^\nu \), \( A_1^\nu B_1^\mu \), and \( A_1^\mu B_1^\nu \), where

\[
A^\mu = (q'^\mu - \frac{q \cdot q'}{P \cdot q} P^\mu) + (q^\mu - \frac{q \cdot q'}{P' \cdot q'} P'^\mu),
\]

\[
A_1^\mu = q^\mu - \frac{q^2}{q \cdot q'} q'^\mu,
\]

\[
B^\nu = (q'^\nu - \frac{q \cdot q'}{P' \cdot q'} P'^\nu) + (q^\nu - \frac{q \cdot q'}{P \cdot q'} P'^\nu),
\]

\[
B_1^\nu = q'^\nu - \frac{q^2}{q \cdot q'} q'^\nu.
\]

By construction, \( A^\mu B^\nu \), \( A_1^\mu B_1^\nu \), \( A_1^\nu B_1^\mu \), and \( A_1^\mu B_1^\nu \) have specific symmetry properties under the crossing and time-reversal transformations.

To match \( U(P', S')(\hat{q} + \hat{q}')\gamma_5 U(P, S) \), we need to invoke one Levi-Civita tensor. If the Levi-Civita tensor is demanded to carry two free Lorentz indices, we have \( \epsilon^{\mu\nu\alpha\beta} q_\alpha q'_\beta \) by gauge invariance. As one of the Lorentz indices is carried by the particle momentum, at our disposal are \( A^\mu D^\nu + B^\nu C^\mu \), \( A^\mu D^\nu - B^\nu C^\mu \), \( A_1^\mu D^\nu + B_1^\nu C^\mu \), \( A_1^\nu D^\mu - B_1^\mu C^\nu \), where

\[
C^\mu = \epsilon^{\mu\nu\alpha\beta} q_\alpha P_\beta P'_\gamma,
\]

\[
D^\nu = \epsilon^{\mu\nu\alpha\beta} q'_\alpha P_\beta P'_\gamma.
\]

Again, \( A^\mu D^\nu + B^\nu C^\mu \), \( A^\mu D^\nu - B^\nu C^\mu \), \( A_1^\mu D^\nu + B_1^\nu C^\mu \), \( A_1^\nu D^\mu - B_1^\mu C^\nu \) have specific symmetry properties under the crossing and time-reversal transformations.

By definition, \( A^\mu D^\nu \), \( A_1^\mu D^\nu \), \( B^\nu C^\mu \), and \( B_1^\nu C^\mu \) are independent of each other. On the other hand, the antisymmetric property of \( \epsilon^{\mu\nu\alpha\beta} q_\alpha q'_\beta \) tells us it may have 6 non-vanishing components. Due to the current conservation conditions, Eq. (3), only 4 of them are independent. Therefore, \( \epsilon^{\mu\nu\alpha\beta} q_\alpha q'_\beta \) can be expanded in terms of \( A^\mu D^\nu \), \( A_1^\mu D^\nu \), \( C^\mu B_1^\nu \), and \( C^\nu B_1^\mu \). In fact, one can directly construct the following identity:

\[
\epsilon^{\mu\nu\alpha\beta} q_\alpha q'_\beta = \frac{(P \cdot A_1 - P' \cdot A_1)A^\mu D^\nu - (P \cdot A - P' \cdot A)A_1^\mu D_1^\nu}{(P \cdot A)(P' \cdot A_1) - (P' \cdot A)(P \cdot A_1)}
+ \frac{-(P \cdot B_1 - P' \cdot B_1)C^\mu B_1^\nu + (P \cdot B - P' \cdot B)C^\nu B_1^\mu}{(P \cdot B)(P' \cdot B_1) - (P' \cdot B)(P \cdot B_1)}.
\]

Notice that this identity holds only for the non-forward Compton scattering. This is where the subtleties arise in choosing four Lorentz pseudo-tensors to match \( U(P', S')(\hat{q} + \hat{q}')\gamma_5 U(P, S) \). If one selects \( A^\mu D^\nu \), \( B^\nu C^\mu \), \( A_1^\mu D^\nu \), and \( C^\nu B_1^\mu \), all of them will drop out in the collinear limits. As we stressed, we need to avoid the over-degeneracy in the collinear limits.
as much as possible. On the other hand, all of recent studies indicate that the $\epsilon^{\mu\nu\alpha\beta}q_\alpha q'_\beta$ term incorporates the leading twist contributions. Hence, we choose $\epsilon^{\mu\nu\alpha\beta}q_\alpha q'_\beta$, $A^\mu D^\nu + B^\nu C^\mu$, $A_1^\mu D_\nu + C^\mu B_1^\nu$, and $A_1^\mu D_\nu - C^\mu B_1^\nu$ as 4 independent Lorentz pseudo-tensors to match with $U(P',S')(q + \gamma')\gamma_5 U(P,S)$. [In the Berg-Lindner decomposition, there is no $\epsilon^{\mu\nu\alpha\beta}q_\alpha q'_\beta$ term.] Notice that $\epsilon^{\mu\nu\alpha\beta}q_\alpha q'_\beta$ survives the collinear limits.

Thus, we have identified 9 independent structures for the non-collinear double VCS off the massive lepton.

Now we take into the lepton mass effects. Then, the helicity of a massive lepton is no longer in coincidence with its chirality. The chiral-even lepton state is roughly in the helicity-$+\frac{1}{2}$ state, with a helicity-$-\frac{1}{2}$ contamination of $O(m/Q)$, where $m$ and $Q$ are the lepton mass and a high energy interaction scale, respectively. The inclusion of the lepton mass effect will generate 9 more structures, which flip the lepton helicity. Writing down the Dirac bilinears is essentially an expansion according to the chiral structure. Although the expansion according to the lepton helicity is not coincident with that according to the chiral structure, the number of independent terms in any complementary expansion should be equal. Hence, the lepton mass will generate 9 more independent chiral-odd structures in the general decomposition.

In constructing independent chiral-odd Dirac bilinears, we have 1, $\gamma_5$ and $\sigma^{\alpha\beta}$ at our disposal. We first consider the pseudo-scalar Dirac structure $\bar{U}(P',S')\gamma_5 U(P,S)$. Remind that there should be at least two spin-dependent form factors in the collinear limit, because 4 independent helicity amplitudes survive in the collinear limits of double VCS. To avoid possible over-degeneracy in the collinear limits, we select again $\epsilon^{\mu\nu\alpha\beta}q_\alpha q'_\beta$, $A^\mu D^\nu + B^\nu C^\mu$, $A_1^\mu D_\nu + C^\mu B_1^\nu$, and $A_1^\mu D_\nu - C^\mu B_1^\nu$ as 4 independent Lorentz pseudo-tensors to match with $\bar{U}(P',S')\gamma_5 U(P,S)$.

Now we consider the tensor Dirac structures. By choosing $q + q'$, $P$ and $P'$ as 3 independent particle momenta, we have $\bar{U}(P',S')\sigma^{\alpha\beta}P_\alpha P'_\beta U(P,S)$, $\bar{U}(P',S')\sigma^{\alpha\beta}(q + q')_\alpha P'_\beta U(P,S)$, $\bar{U}(P',S')\sigma^{\alpha\beta}P_\alpha(q + q')_\beta U(P,S)$. By use of the Dirac equation, one can show that (1) $\bar{U}(P',S')\sigma^{\alpha\beta}P_\alpha P'_\beta U(P,S)$ is equivalent to the scalar Dirac structure; and 2) Both $\bar{U}(P',S')\sigma^{\alpha\beta}(q + q')_\alpha P'_\beta U(P,S)$ and $\bar{U}(P',S')\sigma^{\alpha\beta}P_\alpha(q + q')_\beta U(P,S)$ reduce to a combination of the the vector and scalar Dirac structures. We choose $\bar{U}(P',S')\sigma^{\alpha\beta}P_\alpha P'_\beta U(P,S)$ as an independent Dirac structure, which can be matched with $-(q \cdot q')g^{\mu\nu} + q^{\mu}q'^\nu$, $A^\mu B^\nu$, $A_1^\mu B'^\nu + A^\mu B_1'^\nu$, $A_1^\mu B'^\nu + A^\mu B_1'^\nu$, and $A_1^\mu B_1'^\nu$.

Thus, we have specified 9 independent chiral-odd structures for the non-collinear double VCS off the massive lepton.

Now we are in a position to make our suggestion about the Lorentz decomposition of the non-collinear double VCS off the massive lepton:

$$T^{\mu\nu} = \left(\begin{array}{c}
\frac{-(q \cdot q')g^{\mu\nu} + q^{\mu}q'^\nu}{su} U(P',S')(f_1(q + \gamma') + f_2 \frac{i\sigma^{\alpha\beta}P_\alpha P'_\beta}{2m}) U(P,S) \\
\frac{A^\mu B^\nu}{su} U(P',S')(f_3(q + \gamma') + f_4 \frac{i\sigma^{\alpha\beta}P_\alpha P'_\beta}{2m}) U(P,S) \\
\frac{A_1^\mu B'^\nu + A^\mu B_1'^\nu}{su} U(P',S')(f_5(q + \gamma') + f_6 \frac{i\sigma^{\alpha\beta}P_\alpha P'_\beta}{2m}) U(P,S) \\
\frac{A_1^\mu B'^\nu - A^\mu B_1'^\nu}{su} U(P',S')(f_7(q + \gamma') + f_8 \frac{i\sigma^{\alpha\beta}P_\alpha P'_\beta}{2m}) U(P,S)
\end{array}\right)$$
transition amplitude. Generally speaking, they are symmetric under the adjoint crossing-parity-time-reversal transformation. More concretely, they are symmetric under the adjoint crossing-parity-time-reversal transformation. However, the form factors can be employed to perform consistency check of theoretical calculations. We summarize the crossing and parity-time-reversal transformation properties of the form factors in Table 2. The various symmetry transformation properties of the form factors, just note that the form factors are functions of $s$, $u$, $q^2$ and $q'^2$. From our procedure to establish the above decomposition, the reader can convince himself that our decomposition is complementary in the non-collinear case. The $s$ and $u$ factors in Eq. (20) can be understood as the remnants of the $s$- and $u$-channel propagators.

The Compton scattering off the proton and that off the lepton observe the same symmetries, while the proton is a composite object. Therefore, Eq. (20) applies as well to the non-collinear double VCS off the proton. The soft physics in the proton, in relation to the chiral symmetry breaking, does not bring about extra problems in decomposing the VCS amplitude. From now on, we understand Eq. (20) to be the general decomposition of the double VCS tensor for the proton.

By construction, the form factors in Eq. (20) are either symmetric or antisymmetric under the crossing and time reversal transformations. The crossing symmetry properties of the form factors can be read off straightforwardly. To obtain the time-reversal transformation properties of the form factors, just note that the form factors are functions of $s$, $u$, $q^2$ and $q'^2$, irrelevant of the spin state of the proton. Then, one can put each of the protons in a specific helicity state to show how the form factors transform under the adjoint parity-time-reversal transformations. We summarize the crossing and parity-time-reversal transformation properties of the form factors in Table 2. The various symmetry transformation properties of the form factors can be employed to perform consistency check of theoretical calculations. Of more interest are the symmetry properties of the form factors under the adjoint crossing, parity and time-reversal transformations, where $s$ and $u$ exchange their roles. They are especially useful for the study of the single VCS, where the on-shell condition $q'^2 = 0$ of the final photon is usually implicit in the calculations.

Now we address the reduction of Eq. (20) to the cases of the single VCS. Regarding the single VCS, the on-shell condition of the final-state photon implies as well the Lorentz condition of the final-state photon. Therefore, all of its $B_i^{\gamma}$-related terms become unobservable. As a consequence, we are left with 12 independent, observable form factors $f_{1,2,3,4,5+7,6+8}$, and $g_{1,2,3,4,5+7,6+8}$. For convenience, we introduce the shorthand $f_{i\pm j} = f_i \pm f_j$. Similarly for the $g$-type form factors. Notice that $f_{5+7,6+8}$ and $g_{5+7,6+8}$ have no specific transformation properties under individual crossing and parity-time-reversal transformation. However, they are symmetric under the adjoint crossing-parity-time-reversal transformation. More concretely, they are symmetric under $s \leftrightarrow u$.

Notice that the single VCS tensor contains more information than the corresponding transition amplitude. Generally speaking, $f_{5-7,6-8,9,10}$ and $g_{5-7,6-8,9,10}$ do not vanish at $q'^2 = 0$ on their own. They do not make contributions to the single VCS amplitude simply

\begin{equation}
\begin{aligned}
&+ \frac{A^\mu B_i^{\gamma}}{su} \bar{U}(P', S') \left( f_9 (q + q') + f_{10} \frac{i \sigma^{\alpha \beta} P_\alpha P'_\beta}{2m} \right) U(P, S) \\
&+ \frac{i e^{\mu \nu \alpha} q_\alpha q_\nu}{su} \bar{U}(P', S') \left( g_1 (q + q') \gamma_5 + g_2 \frac{(P \cdot P') \gamma_5}{2m} \right) U(P, S) \\
&+ \frac{i (A_{\mu} D_{\nu} + C_{\mu} B_{\nu})}{(P \cdot P')_{su}} \bar{U}(P', S') \left( g_3 (q + q') \gamma_5 + g_4 \frac{(P \cdot P') \gamma_5}{2m} \right) U(P, S) \\
&+ \frac{i (A_{\mu} D_{\nu} - C_{\mu} B_{\nu})}{(P \cdot P')_{su}} \bar{U}(P', S') \left( g_5 (q + q') \gamma_5 + g_6 \frac{(P \cdot P') \gamma_5}{2m} \right) U(P, S),
\end{aligned}
\end{equation}

where $f_i$ and $g_i$ are dimensionless complex form factors, dependent on $s$, $u$, $q^2$ and $q'^2$. From our procedure to establish the above decomposition, the reader can convince himself that our decomposition is complementary in the non-collinear case. The $s$ and $u$ factors in Eq. (20) can be understood as the remnants of the $s$- and $u$-channel propagators.

The Compton scattering off the proton and that off the lepton observe the same symmetries, while the proton is a composite object. Therefore, Eq. (20) applies as well to the non-collinear double VCS off the proton. The soft physics in the proton, in relation to the chiral symmetry breaking, does not bring about extra problems in decomposing the VCS amplitude. From now on, we understand Eq. (20) to be the general decomposition of the double VCS tensor for the proton.

By construction, the form factors in Eq. (20) are either symmetric or antisymmetric under the crossing and time reversal transformations. The crossing symmetry properties of the form factors can be read off straightforwardly. To obtain the time-reversal transformation properties of the form factors, just note that the form factors are functions of $s$, $u$, $q^2$ and $q'^2$, irrelevant of the spin state of the proton. Then, one can put each of the protons in a specific helicity state to show how the form factors transform under the adjoint parity-time-reversal transformations. We summarize the crossing and parity-time-reversal transformation properties of the form factors in Table 2. The various symmetry transformation properties of the form factors can be employed to perform consistency check of theoretical calculations. Of more interest are the symmetry properties of the form factors under the adjoint crossing, parity and time-reversal transformations, where $s$ and $u$ exchange their roles. They are especially useful for the study of the single VCS, where the on-shell condition $q'^2 = 0$ of the final photon is usually implicit in the calculations.

Now we address the reduction of Eq. (20) to the cases of the single VCS. Regarding the single VCS, the on-shell condition of the final-state photon implies as well the Lorentz condition of the final-state photon. Therefore, all of its $B_i^{\gamma}$-related terms become unobservable. As a consequence, we are left with 12 independent, observable form factors $f_{1,2,3,4,5+7,6+8}$, and $g_{1,2,3,4,5+7,6+8}$. For convenience, we introduce the shorthand $f_{i\pm j} = f_i \pm f_j$. Similarly for the $g$-type form factors. Notice that $f_{5+7,6+8}$ and $g_{5+7,6+8}$ have no specific transformation properties under individual crossing and parity-time-reversal transformation. However, they are symmetric under the adjoint crossing-parity-time-reversal transformation. More concretely, they are symmetric under $s \leftrightarrow u$.

Notice that the single VCS tensor contains more information than the corresponding transition amplitude. Generally speaking, $f_{5-7,6-8,9,10}$ and $g_{5-7,6-8,9,10}$ do not vanish at $q'^2 = 0$ on their own. They do not make contributions to the single VCS amplitude simply
because the contraction of their associated Lorentz tensors with the polarization vector of
the final-state photon vanish. To obtain these form factors, one could extrapolate the data
from $q'^2 \neq 0$, which is beyond the scope of this work.

Another interesting reduction of Eq. (24) is to go to the non-collinear RCS. Imposing
the on-shell condition both on the initial and final-state photons, we are left with 8 observable,
independent form factors: $f_{1,2,3,4}$ and $g_{1,2,3,4}$. In other words, we can reach the non-collinear
RCS by simply dropping those terms constructed with $A_{\mu}^{\nu}$ and/or $B_{\nu}^{\nu}$. Our conclusion is
consistent with the independent helicity counting by Kroll, Schürmann and Guichon [14],
but disagrees with the claim made by Berg and Lindner that there are only 6 non-vanishing
form factors for the non-collinear RCS.

Here we remark that the Berg-Lindner claim was incorrect, because it was based on
an abuse of the crossing symmetry of the Compton scattering. It is literally true that
the single VCS form factors depend on only 3 independent kinematical variables if the on-
shell condition of the final photon is taken into account. As far as the crossing symmetry
properties are concerned, however, $q'^2$ must be taken as an independent kinematical
variable as the single VCS is discussed. In Ref. [1] it is assumed that the single VCS form factors
are three-argument functions, so its discussion about the crossing symmetry properties are
incorrect. In addition, the crossing symmetry for the proton was employed in Ref. [7] to
eliminate two form factors. We note that the fermion crossing symmetry, which is essentially
a charge conjugation symmetry, can be used to relate the Compton scattering off the proton
to that off the anti-proton, so it does not generate any constraints on the VCS form factors.

A straightforward application of Eq. (20) is to recover the manifest electromagnetic
gauge invariance in the leading twist expansion of the DVCS tensor [1,2,4]. Notice that in
these leading twist expansions, all of the involved nonperturbative matrix elements, such
as Ji’s off-forward parton distributions (OFPD) and Radyushkin’s double distributions, are
color gauge invariant by definition. In addition, all the leading twist contributions in these
expansions are associated with the Lorentz structure of types $g_{\mu\nu} + \cdots$ and $\epsilon_{\mu\nu\cdots}$. To recover
the electromagnetic gauge invariance, one can simply replace in Refs. [1,2,4] $g_{\mu\nu} + \cdots$ and
$\epsilon_{\mu\nu\cdots}$ by $g_{\mu\nu} - q'^{\mu} q^{\nu}/(q \cdot q')$ and $\epsilon_{\mu\nu\cdots} q_{\alpha} q'_{\beta}$ respectively, without need to look into non-leading
terms.

The usefulness of our decomposition of the non-collinear Compton scattering tensor is
more than above. It can be expected that the study of the VCS will inevitably go beyond
leading twist and leading order. In fact, some progress along this line has been witnessed
[11]. The basic use of Eq. (20) lies in helping theoreticians organize complicated calculations
in the study of higher-twist and higher-order contributions.

In the following, we illustrate such procedures by expanding the Born-level amplitude for
the double VCS off the massless lepton in terms of form factors. Since we are working with
the massless lepton, there are only 9 double VCS form factors. Namely, all the form factors
with an even subscript drop out in Eq. (20) now. From this heuristic example, we will verify
our analysis of the symmetry properties of the Compton form factors. In addition, we will
learn that there do exist some physical quantities in Nature that cannot be accessed directly
by experiments but can, in principle, be extracted by extrapolation.

To project out the form factors, we first multiply both sides of Eq. (20) with the complex
conjugate of $\bar{u}(p',s')(\mathbf{q} + \mathbf{q}')u(p,s)$, and perform the spin sum over the initial and final-state
leptons so as to eliminate the Dirac spinors. Here the $g$-type form factors drop out because
the lepton has been assumed to be massless. Then, we saturate the Lorentz indices of the resulting tensor equations in turn with the Lorentz tensors associated with each form factor. As a result, we obtain for the $f$-type form factors the algebraic equations of the following form

\[
\begin{pmatrix}
c_1 \\
c_3 \\
c_{5+7} \\
c_{5-7} \\
c_9
\end{pmatrix} = \begin{pmatrix}
a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\
a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\
a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\
a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\
a_{51} & a_{52} & a_{53} & a_{54} & a_{55}
\end{pmatrix} \begin{pmatrix}
f_1 \\
f_3 \\
f_{5+7} \\
f_{5-7} \\
f_9
\end{pmatrix} .
\]

(21)

Similarly, by employing $\bar{u}(p', s')(\not{q'} - \not{q})\gamma_5 u(p, s)$, we have for the $g$-type form factors,

\[
\begin{pmatrix}
d_1 \\
d_3 \\
d_{5+7} \\
d_{5-7}
\end{pmatrix} = \begin{pmatrix}
b_{11} & b_{12} & b_{13} & b_{14} \\
b_{21} & b_{22} & b_{23} & b_{24} \\
b_{31} & b_{32} & b_{33} & b_{34} \\
b_{41} & b_{42} & b_{43} & b_{44}
\end{pmatrix} \begin{pmatrix}
g_1 \\
g_3 \\
g_{5+7} \\
g_{5-7}
\end{pmatrix} ,
\]

(22)

To save space, we omit the concrete expressions for $c_i$, $d_i$, $a_{ij}$ and $b_{ij}$.

Straightforward algebra gives us

\[
f_1^{(0)}(s, u, q^2, q'^2) = \frac{s-u}{s+u} ,
\]

(23)

\[
f_3^{(0)}(s, u, q^2, q'^2) = \frac{(s-q^2)(u-q^2)(s-q'^2)(u-q'^2)}{(s^2-u^2)(su-q'^2q'^2)} ,
\]

(24)

\[
f_{5+7}^{(0)}(s, u, q^2, q'^2) = -\frac{(s-q^2)(u-q'^2)(s+u-q^2)}{(s-u)(su-q'^2q'^2)} ,
\]

(25)

\[
f_{5-7}^{(0)}(s, u, q^2, q'^2) = -\frac{(s-q^2)(u-q'^2)(s+u-q^2)}{(s-u)(su-q'^2q'^2)} ,
\]

(26)

\[
f_9^{(0)}(s, u, q^2, q'^2) = \frac{(s+u)[s^2+u^2+su-(q^2+q'^2)(s+u)+q^2q'^2]}{(s-u)(su-q'^2q'^2)} ,
\]

(27)

\[
g_1^{(0)}(s, u, q^2, q'^2) = \frac{(q^2-q'^2)(s+u)[s^2+u^2-(q^2+q'^2)(s+u)+2q^2q'^2]}{[2su-(q^2+q'^2)(s+u)+q^4+q'^4][s^2+u^2+2su-4q^2q'^2]} ,
\]

(28)

\[
g_3^{(0)}(s, u, q^2, q'^2) = \frac{(s-q^2)(u-q'^2)(s-q'^2)(u-q^2)(s+u-q^2)}{(s-u)(su-q'^2q'^2)[2su-(q^2+q'^2)(s+u)+q^4+q'^4]} ,
\]

(29)

\[
g_{5+7}^{(0)}(s, u, q^2, q'^2) = \left\{ (s+u)(s+u-q^2-q'^2)[s^2+u^2-(q^2+q'^2)(s+u)+q^2q'^2] \right. \\
\left. [-su(s+u)+q^2(s^2+u^2)+q^2q'^2(s+u)+2q^2q'^4)] \right. \\
\times \left\{ (s-u)(su-q'^2q'^2)[s^2+u^2+2su-4q^2q'^2] \\
\left. [2su-(q^2+q'^2)(s+u)+q^4+q'^4]\right)^{-1} ,
\]

(30)

\[
g_{5-7}^{(0)}(s, u, q^2, q'^2) = \left\{ (s+u)(s+u-q^2-q'^2)[s^2+u^2-(q^2+q'^2)(s+u)+q^2q'^2] \\
\left. [-su(s+u)+q^2(s^2+u^2)+q^2q'^2(s+u)+2q^4q'^2)] \right. \\
\times \left\{ (s-u)(su-q'^2q'^2)[s^2+u^2+2su-4q^2q'^2] \\
\left. [2su-(q^2+q'^2)(s+u)+q^4+q'^4]\right)^{-1} .
\]

(31)
where the superscript \((0)\) labels the lowest-order results. Obviously, Eqs. (23-31) are consistent with our general symmetry analysis summarized in Table 2.

Letting \(q'^2 = 0\) in Eqs. (23-31), we obtain the form factors for the single VCS off the massless lepton. Evidently, in the single VCS case, \(f_{5-7}, f_9\) and \(g_{5-7}\) do not vanish themselves. Moreover, all the form factors have specific symmetry properties under \(s \leftrightarrow u\), while \(f_{5-7}\) and \(g_{5-7}\) have no specific symmetry properties under \(q^2 \leftrightarrow q'^2\).

As we have pointed out, \(f_{5-7}, f_9\) and \(g_{5-7}\) do not come into action in the single VCS amplitude, due to the Lorentz conditions of the final photon. Such a fact informs us that they can be replaced by arbitrary numbers, in any complete, gauge-invariant expansions of the single VCS tensor for the massless lepton (quark). In another word, even \(f_{5-7}, f_9\) and \(g_{5-7}\) are included explicitly in the bases for expanding the single VCS tensor, they cannot be solved out uniquely. By explicit calculations, one can easily show that all \(a_{ij}\) and \(b_{ij}\) with index 4 and/or 5 vanish in the case of the single VCS. Hence, \(c_{5-7}, c_9\) and \(d_{5-7}\) must vanish, which is a manifestation of the electromagnetic gauge invariance. The above fact sounds trivial to the expansion of the Born amplitudes, but serves as a very useful consistency check in the practical calculations of loop corrections. Note that one can also construct projectors (with Lorentz tensor and Dirac bilinear structures) for all of the form factors that function in the single VCS.

Our form factor decomposition of the non-collinear Compton scattering tensor has further implications to the study of Ji’s OFPDs and Radyushkin’s double distributions. In the leading-twist Feynman diagram expansion of the DVCS off the proton, the underlying dynamics is believed to be the single VCS off the massless quark. A virtue of our decomposition, Eq. (21), is that the Lorenz tensors for single VCS tensor of the massless quark are the same as those in the DVCS tensor of the proton. Since the VCS off the quark and that off the proton are subject to the same symmetry constraints, one can naturally conclude that both the OFPDs and double distributions possess some symmetry properties. These symmetry properties will impose some constraints as one attempts to model the OFPDs and double distributions.

In Ji’s expansion [11] of the DVCS tensor, a light-like momentum \(p'\) in connection with the average of the initial- and final-state proton momenta is introduced. Then, the momenta of the initial- and final-state protons are approximated as \((1 + \xi)p\) and \((1 - \xi)p\), respectively, where \(\xi (0 < \xi < 1)\) is the analog of the Bjorken variable. Correspondingly, the momenta of the initial- and final-state partons participating in the hard single VCS are effectively taken as \((x + \xi)p\) and \((x - \xi)p\). One can easily show that two partonic Mandelstam variables are related to their hadronic counterparts via

\[
\hat{s} \equiv [q' + (x - \xi)p]^2 \simeq \frac{x - \xi}{1 - \xi} s, \quad (32)
\]

\[
\hat{u} \equiv [q' - (x + \xi)p]^2 \simeq \frac{x + \xi}{1 + \xi} u. \quad (33)
\]

A DVCS form factor of the proton can be roughly thought of as the convolution of the corresponding parton form factor with an appropriate OFPD. The quark form factor is either symmetric or antisymmetric under \(\hat{s} \leftrightarrow \hat{u}\), which amounts to \(s \leftrightarrow u\) and \(\xi \rightarrow -\xi\). Hence, the symmetry properties of the proton form factor demands that the OFPDs satisfy the following relations:
\[ H(x, \xi, \Delta^2) = H(x, -\xi, \Delta^2) , \]  
\[ E(x, \xi, \Delta^2) = E(x, -\xi, \Delta^2) , \]  
\[ \bar{H}(x, \xi, \Delta^2) = \bar{H}(x, -\xi, \Delta^2) , \]  
\[ \bar{E}(x, \xi, \Delta^2) = \bar{E}(x, -\xi, \Delta^2) , \]  
where \( \Delta^2 \equiv (P' - P)^2 \) is the Mandelstam variable \( t \). In fact, the above relations can be derived from the definitions of these OFPDs directly by time reversal invariance. Further, one can show \([12]\) that all of the OFPDs are real, with the help of Eqs. (34-37).

Now let us consider Radyushkin’s expansion \([2]\) of the DVCS tensor, in which the momenta of the initial- and final-state partons are approximated as \( xP + yr \) and \( xP - \bar{y}r \) respectively, with \( r \equiv P - P' \) and \( \bar{y} = 1 - y \). Here two partonic Mandelstam variables read

\[ s \equiv [q' + xp - (1 - y)r]^2 \simeq \bar{y}(s + u) - xu , \]  
\[ u \equiv [q' - xp - yr]^2 \simeq -y(s + u) - xu . \]

Obviously, \( s \leftrightarrow u \) implies \( y \rightarrow -\bar{y} \) and \( \bar{y} \rightarrow -y \). Now, the nonperturbative physics is incorporated by two double distributions \( F(x, y) \) and \( G(x, y) \). Consequently, \( F(x, y) \) and \( G(x, y) \) must be invariant under the transformation \( y \rightarrow -\bar{y} \) and \( \bar{y} \rightarrow -y \). Here we recall that \( F(x, y) \) is actually defined by the following leading-twist expansion of the proton matrix:

\[
\int \frac{d\lambda d\eta}{(2\pi)^2} e^{i\lambda(x+\xi y)-i\eta(x-\bar{y}z)} \langle P', S'| \bar{\psi}(\lambda n) \gamma^\alpha \psi(\eta n)|P, S \rangle = \bar{U}(P', S') \gamma^\alpha U(P, S)F(x, y) + \cdots ,
\]  
\[ (40) \]

where \( \zeta \equiv r \cdot n \) and \( n \) is a light-like vector with an inverse momentum dimension. Hence, we can effectively write down

\[ F(x, y) \equiv F(x; y, \bar{y}) , \]  
\[ (41) \]

That is to say, \( y \) and \( \bar{y} \) function in the double distributions as if they were two independent variables. To examine the symmetry properties of \( F(x, y) \), one can put each of the protons in a helicity eigenstate. Then, by use of time reversal invariance, one can quickly show

\[ F(x; y, \bar{y}) = F(x; -\bar{y} , -y) , \]  
\[ (42) \]

Similarly, there is

\[ G(x; y, \bar{y}) = G(x; -\bar{y} , -y) . \]  
\[ (43) \]

Equations \([42] [43]\) are a useful guide as one parameterizes \( F(x, y) \) and \( G(x, y) \).

In Ref. \([3]\), there was an observation that the double distributions are purely real in some toy models. In fact, this is generally true in QCD. To show this, just take the complex conjugate of Eq. \([10]\). There will be \( F^*(x; y, \bar{y}) = F(x; -\bar{y} , -y) \). Combining this with Eq. \([12]\), we know that \( F(x, y) \) is real. The proof that \( G(x, y) \) is real goes the same way.

In closing, we remark the limitations of our form factor description of the Compton scattering tensor. It is applicable to the non-collinear Compton scattering, both real and virtual. As going to the collinear limits, however, it becomes pathological. The case is the
worst as one attempts to discuss the transverse proton spin dependence of the Compton amplitude in the collinear limits. There is no remedy in our present scheme to parameterize the Compton scattering tensor in terms of form factors. In fact, the drawbacks of our decomposition are shared by all of the present Feynman diagram expansions and OPE analyses of the proton DVCS tensor. To develop a form factor description of the Compton scattering tensor suitable for taking the collinear limits, one can demand that the gamma matrices carry free Lorentz indices. At present, we have not seen any advantages in adopting such a scenario.

ACKNOWLEDGMENTS

The author thanks M. Anselmino, M. Diehl, M. Glück, T. Gousset, P.A.M. Guichon, Xiangdong Ji, Boqiang Ma, B. Pire, A. Radyushkin, J.P.Ralston and E. Reya for useful discussions and/or correspondence. In particular, he is grateful to M. Diehl and B. Pire for helping clarify the constraints of the crossing symmetry on the Compton scattering tensor.

Table 1. Surviving independent helicity amplitudes of the various Compton scattering in the collinear scattering limits.

| $\gamma^*(q) + N(P) \rightarrow \gamma^*(q') + N(P')$ | back | forward | $\gamma^*(q) + N(P) \rightarrow \gamma(q') + N(P')$ | back | forward | $\gamma(q) + N(P) \rightarrow \gamma(q') + N(P')$ | back | forward |
|---|---|---|---|---|---|---|---|---|
| $A(1, \frac{1}{2}; 1, \frac{1}{2})$ | $A(0, \frac{1}{2}; -1, -\frac{1}{2})$ | $A(0, \frac{1}{2}; -\frac{1}{2}, \frac{1}{2})$ | $A(1, \frac{1}{2}; -1, -\frac{1}{2})$ | $A(0, \frac{1}{2}; 1, \frac{1}{2})$ | $A(0, \frac{1}{2}; -\frac{1}{2}, \frac{1}{2})$ | $A(1, \frac{1}{2}; 1, \frac{1}{2})$ | $A(1, \frac{1}{2}; -1, -\frac{1}{2})$ |
| $A(0, \frac{1}{2}; 0, -\frac{1}{2})$ | $A(0, \frac{1}{2}; 1, -1)$ | $A(0, \frac{1}{2}; 1, \frac{1}{2})$ | $A(0, \frac{1}{2}; -1, -\frac{1}{2})$ | $A(0, \frac{1}{2}; 1, \frac{1}{2})$ | $A(0, \frac{1}{2}; 1, -1)$ |

Table 2. Crossing and parity-time-reversal transformation properties of the double VCS form factors. The plus and minus signs represent that the form factor is symmetric and antisymmetric, respectively.

| form factor | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ | $f_6$ | $f_7$ | $f_8$ | $f_9$ | $f_{10}$ | $g_1$ | $g_2$ | $g_3$ | $g_4$ | $g_5$ | $g_6$ | $g_7$ | $g_8$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $s \leftrightarrow u$ & $q^2 \leftrightarrow q'^2$ | $-$ | $+$ | $-$ | $-$ | $+$ | $-$ | $+$ | $-$ | $+$ | $-$ | $+$ | $-$ | $-$ | $-$ | $+$ | $-$ | $-$ | $+$ |
| $q^2 \leftrightarrow q'^2$ | $+$ | $+$ | $+$ | $+$ | $-$ | $-$ | $-$ | $+$ | $+$ | $+$ | $-$ | $-$ | $-$ | $-$ | $+$ | $-$ | $-$ | $-$ |
| $s \leftrightarrow u$ | $-$ | $-$ | $+$ | $-$ | $+$ | $-$ | $+$ | $+$ | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ | $-$ |
REFERENCES

[1] X. Ji, Phys. Rev. Lett. 78, 610 (1997); Phys. Rev. D53, 7114 (1997).
[2] A.V. Radyushkin, Phys. Lett. 380B, 417 (1996); Phys. Lett. 385B, 333 (1996); hep-ph/9704207 to appear in Phys. Rev. D.
[3] C. Hyde-Wright, Proceedings of the second ELFE workshop, Saint Malo, France, 1996, eds. N. d’Hose at al., to be published in Nucl. Phys. A (1997).
[4] Z. Chen, Columbia preprint CU-TP-835, hep-ph/9705279.
[5] M. Diehl, T. Gousset, B. Pire, and J. P. Ralston, hep-ph/9706344.
[6] K. Watanabe, Prog. Th. Phys. 67, 1834 (1982).
[7] R.A. Berg and C.N. Lindner, Nucl. Phys. 26, 259 (1961).
[8] P.A.M. Guichon, G.Q. Liu and A. W. Thomas, Nucl. Phys. A591, 606 (1995).
[9] D. Drechsel, G. Knöchlein, A. Yu. Korchin, A. Metz and S. Scherer, MKPH-T-97-11, nucl-th/9704064.
[10] P. Kroll, M. Schürmann and P.A.M. Guichon, Nucl. Phys. A598, 435 (1996).
[11] X. Ji and J. Osborne, hep-ph/9707254.
[12] X. Ji, private communication.