DEM simulation of internal erosion around a submerged defective pipe

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Abstract. Internal erosion caused by broken sewer pipes often leads to ground subsidence in urban area, which is a major risk to public safety and has caused substantial socioeconomic loss. In order to ensure the ground stabilization and the safety of buried pipelines, it is necessary to understand the process of internal erosion around a submerged defective pipe. In this paper, the Dynamic Fluid Mesh (DFM) is coupled with the three-dimensional discrete element method (DEM) to simulate internal erosion in gap-graded soils above a defective pipe. In this fluid-solid coupling scheme, the fluid mesh can be generated according to the soil skeleton formed by coarse particles and updated at regular intervals. Seepage forces are calculated and applied on solid particles in the DEM model. The approach accounts for permeability and porosity changes due to soil skeleton deformation and internal erosion. In this study, some gap-graded soils samples with different size ratio are established. A defective pipe is placed below the sample. After that, different hydraulic gradients are applied to the sample. Fine particles are washed away from the hole in the pipe. The results indicate that the erosion process can be divided into three stages according to changes in the erosion rate. In the initial stage, numerous fine particles are washed away, and the flow rate increases with the increase of eroded particles. Subsequently, the erosion rate decreases and the flow rate tends to reach a steady state. Finally, only a small proportion of particles fall down from the outlets and the erosion rate levels off to zero gradually. Parametric studies show that the increase of hydraulic gradient increase the eroded particle mass. The number of erosion particles from the bottom layer is much larger than those from the upper layers as more fine particles in the upper layers are locked.

1. Introduction
Internal erosion around a defective pipeline is a hazard for public safety. The investigation of erosion mechanism around a defective pipeline is significant for pipe design, inspection and maintenance [1]. In 2020, Chang et al. [2] reported a laboratory experimental test of internal erosion which was conducted by using gap-graded soils. Based on the experiment, we simulate the internal erosion around a defective pipeline with water coming in from both sides.
In this study, the Dynamic Fluid Mesh (DFM) method is proposed to investigate the internal erosion. DEM software PFC3D [3] is employed to create a particle assembly representing the gap-graded soils. The fluid flow scheme is implemented via through the Python programming feature in PFC3D. The flow rate varies due to the variation of porosity and permeability [4].

2. Dynamic Fluid Mesh (DFM) Method

A program has been developed that combines the DEM with the DFM (dynamic fluid mesh). The flow chart for the calculation procedure is shown in Figure 1. In this coupling model, Space tetrahedral shape fluid mesh is generated firstly in DEM software (Figure 2). Subsequently, porosity and permeability for each fluid element are calculated by. The Poisson’s equation (Eq. (1)) is solved to obtain the pressure gradient and calculate flow rate by Darcy law. The internal erosion is calculated. The fluid mesh, permeability and flow force are updated every 4,000 steps depending on the computer’s capabilities in the model. If the total calculated steps reach the set calculated time, the program terminates.

\[ \nabla \left( \frac{K}{\mu \varepsilon} \nabla p \right) = 0 \quad (1) \]

where, \( \nabla p \) is the pressure gradient, \( K \) is the permeability of the simple, \( \mu \) is the fluid dynamic viscosity, and \( \varepsilon \) is the porosity of the assemble.

The tetrahedron meshes are generated by using the python package named scipy, which triangulates points distributed in space and gives the center coordinate and volume of every tetrahedron. The process is called spatial delaunay triangularizations. According to the tetrahedron generation criterion, in the interior of circumsphere of the tetrahedron can not contain any other points [5]. In DEM, center points of particles and contact points between walls and balls can be used to generate tetrahedron meshes. In gap-graded soils model, considering the limitation of computing power, only larger particles center points and contacts on the boundaries are used as nodes of meshes generally (Figure 2). The meshes change with the variation of nodes position dynamically in the process of calculation at appropriate intervals.
Figure 2 Fluid mesh generated based on the skeleton of coarse particles in the DEM model of gap-graded soils; the yellow and blue balls represent the coarse particles and fine particles, respectively.

After the meshes are formed, we use K-DTree algorithm which is included in scipy package to search the $N$ closest centre points of meshes to each fine particle. The K-DTree algorithm is a data structure that divides the k-dimensional data space. It is mainly used for searching key data in multi-dimensional space [6]. The fine particles are assumed in the closest mesh firstly, and the sum of the volume of space tetrahedron formed by the particle and any three mesh nodes of the rest of meshes are calculated subsequently. Comparing the sum of the volume and volume of mesh, we can tell which mesh the fine particle belongs to. Finally, the flow rate and the drag force applied on particles can be obtained by traditional fluid-particle interaction theories [4].

3. Numerical Model Setup

A numerical experiment is simulated to study internal erosion of gap-graded soils by coupling DFM-DEM. The numerical modelling is carried out by utilizing discrete element code PFC3D [7] and the rolling resistance linear model provided in the software is chosen in the model. The rolling resistance contact model is based on the linear model with incorporating a torque acting on the contacting pieces to counteract rolling motion. The model could provide reasonable compensation to the shape effect when idealized spherical particles are used to mimic the irregularly shaped granular materials. More details can be found in Zhang et al. [8].

In this study, three numerical models are set up to run. The models are generated in a rectangular box of 0.12×0.125×0.05 m (W×H×D). The assembles are generated in a central rectangular box with an arch in the bottom (Figure 3). The number of initial particles is 18705, the number of coarse particles is 207, and fine particles is 18498. In the specimen, the diameter of coarse particles is on average 8 times larger than that of fine particles. The particle size distribution for the model is shown in Figure 4. The parameters of the sample are shown in Table 1. Fine particles account for 20% of the total weight in the specimen. The fluid enters from the both sides of the model and outflows from the top of the arch (Figure 3). The top of the arch is a hole with 0.03 m in diameter. The vertical pressure at the top is 10 kPa and all the other walls are fixed. The water pressures on the both sides are set as 0.1, 1.0 and 5.0 MPa.
Figure 3 (a) The schematic of internal erosion model; (b) sample and mesh, yellow balls represent coarse particles and blue balls represent small particles.

Table 1 Input parameters in the numerical model.

| Model parameters                          | Values         |
|-------------------------------------------|----------------|
| Normal stiffness-coarse particles         | 6.4e5 N/m      |
| Normal stiffness-fine particles           | 1.2e5 N/m      |
| Average radius-coarse particles           | 5.47 mm        |
| Average radius-fine particles             | 1.07 mm        |
| Normal-to-shear stiffness ratio           | 2.0            |
| Coefficient of friction                   | 0.6            |
| Rolling resistance coefficient            | 0.1            |
| Particle density                          | 2650 kg/m³     |
| Stiffness of lateral wall                 | 1.2e4 N/m      |
| Stiffness of end wall                     | 1.2e5 N/m      |
Figure 4 Particle size distribution for the numerical specimen of gap-graded soils.

The centroids of coarse particles and contact points of coarse particles-walls are used to generate the fluid meshes (Figure 3 (b)). The top confining pressure is 10 kPa. The rest of the walls are fixed. The initial pressure field and fluid velocity of the model with 0.1 kPa injection pressure in each grid are shown in Figure 5 (a) and (b), respectively.

Figure 5 The initial pressure field and fluid velocity of the model with 0.1 kPa in each grid.

4. Numerical Results

Three groups models with different injection pressure are simulated to study the relationship between injection pressure and mass of internal erosion. Based on the first case in Section 3 with 0.1 kPa injection pressure, two additional cases are run by using the 1 kPa, 5 kPa injection pressure and other settings of these three cases are kept the same. The results are shown in Figure 6 and Figure 7.

Figure 6 shows the process of internal erosion in defective pipe under 0.1 injection pressure and the evolution of erosion area. It can be seen from the figure that the erosion area is mainly located above the outlet.
According to Figure 7, the evolution of the internal erosion can be divided into three stages according to the variation of erosion rate, i.e. initial stage, deceleration stage and stabilization stage. In the initial stage, particles above the holes are washed down by the fluid and the effect of gravity. In this stage, large quantities of particles are carried away by gravity and water flow. In the deceleration stage, the erosion rate slows down gradually. Most of these particles produced at this stage are initially far away from the outlet. As these particles are removed, the erosion rate slows down further. In the stabilization stage, the erosion process tends to stop gradually. In this stage, a small proportion of particles are still produced. The fluid velocity in the mesh also remains steady at this stage. Although the eroded particle mass is different due to different particle sizes and particle shapes, the shape of erosion curve agrees well with the results of laboratory erosion experiments.

As the injection pressure increases, the mass of internal erosion increases. In the initial stage, the difference of eroded mass under different injection pressure increases with calculation step increasing. In the deceleration stage, the difference is widest and there is a slight reduction in the stop stage. Subsequently, the gap tends to be stable.

Based on internal erosion mass curve, the lost particles can also be classified into three categories, i.e. particles above the exit, particles away from the exit and particles blocked by baseplate and coarse particles. Particles above the exit are easy to loss due to the force applied by gravity and flow. In addition to this, these particles are less affected by friction and mainly washed away in the initial stage. Different from particles of the first kind, for particles of the second kind, friction has more effect than force applied by fluid. The erosion rate of these particles is slow, which means more calculation steps are needed and accidental factors such as collisions between particles may play a role. The third kind particles are mainly affected by obstruction of baseplate or coarse particles and these particles are difficult to be washed down. When the external conditions change, such as hydraulic gradient, confining pressure and so on, the content of particles in both parts will change. e.g. when hydraulic gradient is high, particles are less affected by friction and washed away quickly.
5. Conclusion
In this study, a new approach named dynamic fluid mesh method (DFM) that coupling DEM and fluid flow has been proposed for internal erosion analysis. The meshes can be updated at regular interval according to the variation of coarse particles skeleton. Moreover, the approach also accounts for permeability and porosity changes due to solid deformation and internal erosion. Based on the method, an internal erosion model has been presented by coupling DEM model with the fluid flow model to simulate the evolution of the internal erosion around submerged defective pipe in gap-graded soils. Some simulation results are summarized below.

The erosion area is located above the outlet and the area expands with the loss of fine particles. The process of internal erosion can be divided into three stages according to the variation of erosion rate. In the first stage, large quantities of particles are carried away by flow and the velocity of flow in the pores rises rapidly. The erosion rate slows down gradually in the second stage and tends to zero in the third stage. The velocity of water in the mesh also remains steady at the second and third stages. As the hydraulic gradient increases, the mass of internal erosion increases.

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