Heavy Quark Parton Distributions:
Mass-Dependent or Mass-Independent Evolution?\textsuperscript{1}

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Abstract. In a consistently formulated pQCD framework incorporating non-zero mass heavy quark partons, there is still the freedom to define parton distributions obeying either mass-independent or mass-dependent evolution equations, contrary to statements made in a recent paper by MRRS. With properly matched hard cross-sections, different choices merely correspond to different factorization schemes, and they yield the same physical cross-sections. We demonstrate this principle in a concrete order $\alpha_s$ calculation of the DIS charm structure function. We also examine the proper matching between parton definitions and subtractions in the hard cross-section near threshold where the calculation is particularly sensitive to mass effects of the heavy quark. The results obtained from the general-mass formalism are quite stable against different choices of scale and exhibit a smooth transition in the threshold region (using either mass-independent or mass-dependent evolution), in contrast to results of another recently proposed scheme.

Recent improved measurements of heavy quark production in both leptoproduction and hadroproduction demand better theoretical understanding of the underlying QCD physics. One must consider the changing role of the heavy quark (denoted by $H$) from threshold to the very high energy limit available in current and future experiments. Specifically, we address the question of the mass dependence in the evolution of the parton distribution functions (PDF’s), $f_{a/P}(x,\mu)$.

In the ACOT factorization scheme, [1,2] the heavy quark mass ($m_H$) is fully incorporated in the hard cross-section and threshold ($\mu = m_H$) matching conditions on $\alpha_s(\mu)$ and $f_{a/P}(x,\mu)$. This scheme has the desirable feature that the evolution of all partons, including the heavy quark, is controlled by

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the mass-independent $\overline{\text{MS}}$ kernels; \textit{i.e.}, $f_{a/P}(x, \mu)$ satisfy the well established evolution equations. This is a precise consequence of the renormalization scheme in the theory with mass, \textit{it is not an approximation}.\footnote{As the $\overline{\text{MS}}$ scheme yields mass-independent renormalization constants to all orders, the evolution kernels must be mass-independent. As $m_H \to 0$, the massive hard scattering $\sigma$’s reduce to precisely the zero-mass $\overline{\text{MS}}$ $\sigma$’s without any finite renormalization; in this sense, the ACOT scheme is uniquely the minimal massive extension of the $\overline{\text{MS}}$ scheme. [1]}

A general mass formalism can, in principle, be implemented with factorization prescriptions different from that of ACOT. In a consistently formulated theory, physical predictions from different schemes must remain the same up to higher-order corrections. In a recent paper, MRRS [3] proposed a procedure within the general mass approach involving \textit{mass-dependent splitting functions} for the evolution equations. They questioned the validity of the ACOT scheme because of its use of mass-independent evolution. It is important to clarify this question and determine what schemes are truly viable, since further development of heavy quark theory and phenomenology rely on the availability of dependable calculations.

\section{SCHEME INDEPENDENCE OF PHYSICAL CROSS SECTION}

We consider charm production in DIS in a general renormalization/factorization scheme $R$. The dominant contributions are:\footnote{We refer to the term in Eq. 1 as: $\sigma_{\text{TOT}} = \sigma_{\text{HE}} + (\sigma_{\text{HC}} - \sigma_{\text{SUB}})$, where $\sigma_{\text{TOT}}$ represents the total physical cross section, $\sigma_{\text{HE}}$ the heavy-flavor excitation term ($\gamma^* c \to c$), $\sigma_{\text{HC}}$ the heavy-flavor creation term ($\gamma^* g \to c\bar{c}$), and $\sigma_{\text{SUB}}$ the subtraction term.} \footnote{To focus on the heavy quark parton singularities (of the form $\ln(m_H/Q)$), throughout the paper it is understood that the collinear singularities associated with light partons are always subtracted from $\sigma$ according to the usual $\overline{\text{MS}}$ scheme.}

\begin{equation}
\sigma_{\text{phys}} = R f_{c/P} \otimes R \sigma_{c\gamma^* \to e} + R f_{g/P} \otimes R \sigma_{g\gamma^* \to c\bar{c}} + \mathcal{O}(\alpha_s^2) + O(\alpha_s^2) + \mathcal{O}(\alpha_s^2)
\end{equation}

$R\sigma^n$ denotes the \textit{hard cross section} in a given renormalization scheme $R$ at order $n$. $\sigma^n$ denotes the scheme independent \textit{partonic cross section}.$^4$ $R\tilde{\sigma}^n$ denotes the \textit{subtraction} terms which removes the potential heavy quark collinear singularities from $\sigma^n$. The physical cross section $\sigma_{\text{phys}}$ is scheme independent. The choice of subtraction $R\tilde{\sigma}^n$ defines the scheme, and hence the parton distributions $R f_{a/P}$ and their associated splitting functions $R P_{a \to b}$. In fact, to this order these terms are connected by the relation: $R\tilde{\sigma}_{g\gamma^* \to c\bar{c}}^{(1)} = R f_{c/g}(1) \otimes \sigma_{c\gamma^* \to c}^{(0)}$, where $R f_{c/g}(1)$ is the $\mathcal{O}(\alpha_s^1)$ perturbative distribution of $c$ in $g$. For the complete $\sigma_{\text{SUB}}$ we have:
FIGURE 1. $F_2$ vs. $\mu$ for DIS $c$-production. a) $F_2^{HE}$, $F_2^{SUB}$ and the difference $F_2^{HE} - F_2^{SUB}$. The solid curves are for the mass-independent evolution scheme, and the dashed curves are for the mass-dependent evolution scheme. b) $F_2^{TOT}$ and $F_2^{HE} - F_2^{SUB}$. The difference between the mass-independent evolution and mass-dependent evolution for $F_2^{TOT}$ is higher order and comparable or less than the $\mu$-variation.

\[
\sigma_{SUB} = R \frac{f_{g/P} \otimes R \tilde{\sigma}_{g\gamma^* \rightarrow c\bar{c}}^{(1)}}{2\pi} \int_{m_H^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \frac{R \tilde{P}_{g \rightarrow c} \otimes \sigma_{c\gamma^* \rightarrow c}^{(0)}}{2\pi} (2)
\]

Near threshold ($\mu \sim m_H$), we have

\[
\sigma_{HE} = R \frac{f_{c/P} \otimes R \tilde{\sigma}_{c\gamma^* \rightarrow c}^{(0)}}{2\pi} \int_{m_H^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \frac{R \tilde{P}_{g \rightarrow c} \otimes \sigma_{c\gamma^* \rightarrow c}^{(0)}}{2\pi} + \mathcal{O}(\alpha_s^2) (3)
\]

which reflects the fact that $\sigma_{SUB} \simeq \sigma_{HE}$, by construction. This yields $\sigma_{TOT} \simeq \sigma_{HC} + \mathcal{O}(\alpha_s^2)$ independent of the specific choice of scheme $R$, and therefore independent of the choice of $R \tilde{\sigma}_{g\gamma^* \rightarrow c\bar{c}}^{(1)}$ and $R \tilde{P}_{g \rightarrow c}^{(1)}$. This is the key mechanism that compensates the different effects of the mass-independent vs. mass-dependent evolution, and yields a $\sigma_{TOT}$ which is identical up to higher-order terms.

II MASS-DEPENDENT OR MASS-INDEPENDENT EVOLUTION?

Ref. [3] strongly advocated the use of mass-dependent evolution of the parton distributions, and questioned the correctness of using the familiar $\overline{\text{MS}}$ mass-independent evolution in ref. [1]. Is this criticism justified? No—provided the proper matching between the hard scattering cross sections and the parton distributions is observed. Although the ACOT scheme chose to use a mass-independent evolution for its simplicity and natural connection to the familiar zero-mass parton results, this choice is not an approximation—the physical predictions are insensitive to this choice. We explicitly demonstrate this by implementing a mass-dependent evolution scheme in the same formalism, and
FIGURE 2. $F_2$ vs. $Q^2$ for DIS $c$-production. The scale choice is: $\mu^2 = Q^2/4$. a) The general mass formalism with mass-dependent evolution. b) The MRRS scheme with mass-dependent evolution. This figure is taken directly from ref. [3], Fig.6b. The curves are re-labeled to correspond to our notation.

compare the new results with the existing mass-independent ACOT calculation. In the next section, we shall compare this new massive calculation with that of MRRS.

In Fig. 1 we display the separate contributions to $F_2$ for both mass-independent and mass-dependent evolution. The matching properties discussed earlier are best examined by comparing the (scheme-dependent) $F_2^{HE}$ and $F_2^{SUB}$ contributions of Fig.1a. We observe the following. i) Within each scheme, $F_2^{HE}$ and $F_2^{SUB}$ are well matched near threshold. ii) The matching of $F_2^{HE}$ and $F_2^{SUB}$ ensures that the scheme dependence of $F_2^{TOT}$ is properly of higher-order in $\alpha_s$.

The lesson is clear: the choice of a mass-independent $\overline{\text{MS}}$ or a mass-dependent (non-$\overline{\text{MS}}$) evolution is purely a choice of scheme, and becomes simply a matter of convenience—there is no physically new information gained from the mass-dependent evolution.

III $F_2^{TOT}$ SHOULD MATCH FIXED-ORDER RESULT NEAR THRESHOLD

A necessary consistency check of any proposed general mass scheme in the important intermediate energy region is that it yield the correct limits for threshold and asymptotic energies. Although this feature is, in principle, built into the formalism discussed in Eq. 2, it is essential to verify that the expected behavior actually appears in specific implementation of the scheme—in particular, near the threshold region where the required cancellation between the $F_2^{HE}$ and $F_2^{SUB}$ terms is delicate.

Fig.2 compares $F_2$ vs. the physical variable $Q$ using the same charm mass, scale choice, and format of an existing MRRS plot, (cf. Fig.6b, Ref. [3]). In Fig.2a, $F_2^{TOT}$ smoothly interpolates between $F_2^{HC}$ in the threshold region,
and $F_{2}^{HE}$ in the high energy region, demonstrating the desired matching.\footnote{The intuitive reason for these physical limits is identical to the original mass-independent scheme. [1] In the threshold region the “heavy quark” behaves more like an extrinsic heavy object, while for high energies the “heavy quark” behaves like an intrinsic parton.} In contrast, the corresponding curve in Fig.2b (taken from Ref. [3]) shows a sudden rise at threshold which indicates a lack of expected cancellation between $F_{2}^{HE}$ and $F_{2}^{SUB}$ just above threshold. Since both calculations use the same (MRRS) evolution kernel, this difference can only be due to the implementation of the subtraction procedure: Fig.2a uses the general mass formalism described above, [1,2]; Fig.2b uses the procedure described in Ref. [3].

### IV $F_{2}^{TOT}$ SHOULD BE STABLE UNDER SCHEME/SCALE CHOICE

Irrespective of smoothness of the theory calculation, a physical prediction must be stable against the choice of artificial theoretical parameters such as the renormalization and factorization scale $\mu$. Fig.3a shows our calculation (again, using the mass-dependent evolution) with a different choice of $\mu$ compared to Fig.2a. In Fig.3b we note that while $F_{2}^{HE}$ changed dramatically with choice of $\mu$, this change is compensated by $F_{2}^{SUB}$ such that $F_{2}^{TOT}$ remains stable.

The corresponding comparison is not available for the MRRS procedure. If the threshold behavior evident in Fig.2b persists for other choices of scale, the position of the sudden rise in cross-section will move with the choice of scale; that would be cause for concern.\footnote{For a scale choice of $\mu^2 = Q^2/k^2$, the charm quark evolution turns on at $Q^2 = k^2m_c^2$, where $k^2$ is an arbitrary parameter.}
V SUMMARY

In conclusion, we find that the presence or absence of heavy quark mass in the evolution kernels has no physical effect on cross sections—there is no physically new information gained from a mass-dependent evolution.

A mass-dependent evolution scheme has inherent difficulties. The definition of the mass-dependent splitting kernels introduces a kinematic ambiguity since it is impossible to have an on-shell massless particle split into two collinear massive on-shell particles (e.g., $g \rightarrow c\bar{c}$); this leads to a number of different definitions for the mass-dependent splitting kernels. [5] Since the PDF’s in the mass-dependent scheme are not $\overline{\text{MS}}$, the hard scattering cross sections $\hat{\sigma}$ must be converted from the $\overline{\text{MS}}$ scheme to this scheme. At NLO, this conversion, as well as the PDF evolution, requires two-loop splitting kernels ($\mathcal{P}^{(2)}$) computed in the mass-dependent scheme; these results do not yet exist.

The hallmark of any consistently formulated pQCD theory is: i) proper matching between $F_{2}^{HE}$ and $F_{2}^{SUB}$ at threshold, and ii) insensitivity of the physical $F_{2}^{TOT}$ to the arbitrary scheme/scale choice. The properly implemented general mass formalism satisfies these criteria. The ACOT scheme (with mass-independent $\overline{\text{MS}}$ evolution) has the added advantage that the known mass-independent $\overline{\text{MS}}$ kernels follow naturally from the renormalization scheme adopted.

REFERENCES

1. Aivazis,M., Collins,J.C., Olness,F., & Tung,W.K., Phys.Rev. D50, 3102 (1994); Aivazis,M., Olness,F., & Tung,W.K., Phys.Rev.Lett. 65, 2339 (1990).
2. Tung, W.K., “The Heavy Quark Parton Oxymoron”, hep-ph/9706480, contribution to DIS97 Workshop, Chicago.
3. Martin, A.D., Roberts, R.G., Ryskin, M.G., and Stirling, W.J., hep-ph/9612449.
4. Collins, J.C., & Tung, W.K., Nucl. Phys. B278, 934 (1986); Collins, J.C., CERN preprint in prep.; Collins, J.C., Soper, D.E., & Sterman, G., in Perturbative QCD. A.H. Mueller, ed., World Sci. Pub.,(1989); & references therein.
5. Gluck, M., Hoffmann, E., & Reya, E., Z. Phys. C13, 119 (1982); Eichten, E., Hinchliffe, I., Lane, K., and Quigg, C., Rev. Mod. Phys. 56, 579 (1984), add. ibid. 58, 1065 (1986); Kim, C.S., Kim, S.M., and Olsson, M.G., hep-ph/9307299.
6. Martin,A.D., Stirling,W.J., & Roberts,R.G., Int.J.Mod.Phys.A10, 2885 (1995).
7. Lai, H.L., et al., Phys. Rev. D55, 1280 (1997).