Incoherent magnetization dynamics in strain mediated switching of magnetostrictive nanomagnets

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Abstract

Micromagnetic studies of the magnetization change in magnetostrictive nanomagnets subjected to stress are performed for nanomagnets of different sizes. The interplay between demagnetization, exchange and stress anisotropy energies is used to explain the rich physics of size-dependent magnetization dynamics induced by modulating stress anisotropy in planar nanomagnets. These studies have important implications for strain mediated ultralow energy magnetization control in nanomagnets and its application in energy-efficient nanomagnetic computing devices.

Keywords: nanomagnetic computing, incoherent switching, magnetostrictive, magnetization dynamics

(Some figures may appear in colour only in the online journal)

1. Introduction

The non-volatility and unprecedented energy efficiency of nanomagnetic logic and memory devices have spurred investigations of different types of magnetization reversal schemes in nanomagnets. The typical methods employed to induce magnetization reversal include the use of electric current-generated magnetic field [1], spin transfer torque (STT) [2], current-driven domain wall motion [3], current-induced spin–orbit torque or spin Hall effect [4, 5], and strain generated by applying an electrical voltage to a multiferroic nanomagnet [6–11]. While the field and spin torque driven magnetization dynamics have been studied extensively, strain mediated magnetization dynamics is a relatively new concept which promises high energy efficiency in switching nanomagnetic devices [12, 13] but has not been studied as extensively as the others.

The physics underlying these different switching mechanisms have pronounced differences, particularly the manner in which the energy landscape of the nanomagnet (figure 1(a), (b)) is modified by the switching agent (figures 1(c)–(e)). An external magnetic field reshapes the potential energy landscape of the nanomagnet in such a way that the magnetization aligns itself along the direction of the applied field (figure 1(c)). However, unlike a magnetic field, STT is non-conservative in nature and hence cannot be modeled and explained within a potential energy framework (figure 1(d)). Spin-polarized current is utilized to transfer spin angular momenta to the resident spins in a nanomagnet, which reorients the magnetic moments in the nanomagnet resulting in magnetization reversal [14, 15]. A full 180° switching (or complete magnetic reversal) is achievable using any of these strategies. In contrast, the maximum possible rotation that can be induced in magnetostrictive nanomagnets by strain is 90° (figure 1(e)). There are ways around this;
precisely timed application and withdrawal of stress [16], dipole coupling with a neighboring nanomagnet (Figure 1(f)) [17], or sequential application of stress along different directions (with the aid of carefully designed electrode configuration) [18] can bring about a complete 180° rotation.

Many theoretical studies of magnetization dynamics use the macrospin approximation in which the magnetization rotation is considered to be a collective and coherent rotation of spins. This approach, however, lacks the framework to study the intricate details of the micromagnetic configuration during the switching process. Extensive studies have been conducted to examine the effects of incoherent magnetization rotation under magnetic fields [19–22] or STT [23–30]. However, incoherent magnetization rotation in shape anisotropic nanomagnets subjected to stress is yet to be rigorously addressed and analyzed. In this study, we have performed rigorous micromagnetic simulations to study the peculiarities of the incoherent magnetization dynamics induced by modulation of the stress anisotropy in a nanomagnet with applied strain.

2. Theory and micromagnetic modeling

In relatively small nanomagnets (∼50 nm lateral dimensions), the magnetization is spatially uniform owing to exchange interaction and all the spins rotate more or less in unison (coherently) when the magnetization evolves from one state to another. However, in nanomagnets having larger dimensions, the magnetization states are predominantly non-uniform, particularly during the switching process. These size effects are investigated in elliptical disk-shaped nanomagnets of various dimensions, specifically for stress induced magnetization dynamics. Uniaxial compression/tension is generated along the major axis (easy axis) of the ellipse to rotate the magnetization from the easy toward the in-plane hard axis (minor axis of the ellipse). During this magnetization rotation, the interplay between the exchange, magnetostatic and stress anisotropy energies determines whether the magnetization rotation is coherent or not, and the final magnetic configuration that is stabilized.

We performed micromagnetic simulations using the MuMax package [31]. Our geometry was discretized into 2 × 2 × 3 nm³ cells. Since MuMax does not have an inbuilt functionality to incorporate the effect of stress directly, we accomplish this through the use of uniaxial magneto-crystalline anisotropy, based on their equivalent contribution to the effective field.

In the MuMax framework [31], the magnetization dynamics is simulated using the Landau–Lifshitz–Gilbert equation:

\[
\frac{\partial \vec{m}}{\partial t} = \left( -\gamma \frac{\vec{m} \times \vec{H}_{\text{eff}}}{1 + \alpha^2} + \alpha \times (\vec{m} \times (\vec{m} \times \vec{H}_{\text{eff}})) \right),
\]

where \( m \) is the reduced magnetization (\( M / M_{\text{sat}} \)), \( M_{\text{sat}} \) is the saturation magnetization, \( \gamma \) is the gyromagnetic ratio and \( \alpha \) is the Gilbert damping coefficient. The quantity \( H_{\text{eff}} \) is the
Table 1. Material parameters (FeGa alloy at about 19–20 atomic percent gallium).

| Parameters                      | Values                  |
|---------------------------------|-------------------------|
| Exchange stiffness constant ($A_{K}$) | $1.6 \times 10^{-11}$ J m$^{-1}$ |
| Saturation magnetization ($M_s$)     | $1.32 \times 10^{6}$ A m$^{-1}$ |
| Gilbert damping constant ($\alpha$)  | 0.017                   |
| Magnetostrictive coefficient $(3/2)\lambda_s$ | $4 \times 10^{-4}$ |

The effective magnetic field which is given by

$$\vec{H}_{\text{eff}} = \vec{H}_{\text{demag}} + \vec{H}_{\text{exchange}} + \vec{H}_{\text{stress}}.$$  (2)

Here, $H_{\text{demag}}$ and $H_{\text{exchange}}$ are the demagnetization (or magnetostatic) field and the effective field due to exchange coupling evaluated in the MuMax framework in the manner described in [31].

We used the uniaxial anisotropy field to incorporate the effect of stress which is modeled in MuMax using the following effective field term

$$\vec{H}_{\text{anis}} = \frac{2K_{u1}}{\mu_0 M_{\text{sat}}} (\vec{u} \cdot \vec{m}) \vec{u} + \frac{4K_{u2}}{\mu_0 M_{\text{sat}}} (\vec{u} \cdot \vec{m})^2 \vec{u},$$  (3)

where, $K_{u1}$ and $K_{u2}$ are first and second order uniaxial anisotropy constants and $\vec{u}$ is the unit vector in the direction of the anisotropy. For $K_{u2} = 0$, equation (3) reduces to

$$\vec{H}_{\text{anis}} = \frac{2K_{u1}}{\mu_0 M_{\text{sat}}} (\vec{u} \cdot \vec{m}) \vec{u}.$$  (4)

The effective field due to an external uniaxial stress can be expressed as

$$\vec{H}_{\text{stress}} = \frac{3\lambda_s \sigma}{\mu_0 M_{\text{sat}}} (\vec{m} \cdot \vec{s}) \vec{s},$$  (5)

where, $(3/2)\lambda_s$ is the saturation magnetostriction, $\sigma$ is the external stress (Pa) and $\vec{s}$ is the unit vector in the direction of the applied stress. Comparing equations (4) and (5)

$$K_{u1} = \frac{3\lambda_s \sigma}{2}$$  (6)

and this equation was used to find the value of $K_{u1}$ to simulate the effect for a given uniaxial stress $\sigma$ applied in the same direction as the uniaxial anisotropy.

We chose iron–gallium to study the magnetization dynamics as it has moderate Gilbert damping constant ($\alpha$) and magnetostrictive coefficient $(3/2)\lambda_s$ which are between those of Ni (low $\alpha$, $\lambda_s$) and Terfenol-D (high $\alpha$, $\lambda_s$). Table 1 lists the material parameters used in the simulation [32, 33].

### 3. Results

#### 3.1. Pre-stress equilibrium states

For an ideal single domain elliptical nanomagnet, the in-plane hard and easy axes for the magnetization lie along the minor and major axis of the elliptical disk, respectively. The energy difference between the two states, i.e. when the magnetization is along the hard axis and when the magnetization is along the easy axis, is the in-plane energy barrier height, denoted by $\Delta E_1$ in table 2. While choosing the different nanomagnet dimensions to study the effect of stress, the aspect ratios (ratio of the major axis to the minor axis to the thickness) were held constant which, in turn, keeps the demagnetization factors identical over all chosen dimensions. In other words, all magnets that have been studied have approximately the same value of the critical stress ($\sigma_c$) under the macrospin assumption, where critical stress is defined as the stress required to make the stress anisotropy energy equal to the in-plane barrier height (i.e. $\sigma_c = \Delta E_1/(3/2)\lambda_s$). The barrier height and corresponding critical stress values calculated for the chosen nanomagnet dimensions are shown in table 2. Note that the slight differences in $\sigma_c$ originate from the use of identical discretization volumes for all nanomagnet geometries that results in a relatively better staircase approximation in the larger nanomagnets.

In order to observe the size effects, the initial magnetization was set along the major axis, following which the nanomagnet’s spins were allowed to relax. These relaxed or equilibrium states are shown in figure 2. The energy barrier per unit volume ($\Delta E_1/V$) between the states uniformly magnetized along the minor axis and those uniformly magnetized along the major axis are independent of the nanomagnet volume. However, the difference in energy per unit volume between the states uniformly magnetized along the major axis and the relaxed states ($\Delta E_{\text{relaxed}}/V$) increases with increasing volume. Thus, the energy difference per unit volume between the states uniformly magnetized along the hard axis and the ‘relaxed’ states (which is essentially $\Delta E_1^*/V = \Delta E_1/V + \Delta E_{\text{relaxed}}/V$)

![Figure 2. Equilibrium (relaxed) states of different sized nanomagnets showing spatial variation in the magnetization orientation](image-url)
First, the nanomagnet with the smallest dimensions (50 nm × 40 nm × 3 nm) is examined. Uniaxial stress was applied along the major axis and ramped up in discrete steps of 15 MPa. After each step, the magnetization was allowed to settle to the intermediate equilibrium state. Two intermediate stress points (the critical stress $\sigma^*_c$ and the stress which rotates the magnetization by exactly $90^\circ$) were also added to the set to provide a complete picture. Ideally, if the process was entirely coherent, then these two stress values would be the same, i.e. the critical stress would abruptly rotate the magnetization from the major to the minor axis. Any difference between these two values is a measure of the incoherency. As can be seen in figure 3, the magnetization rotates in a predominantly coherent manner under stress (figures 3(a) and (b)) and aligns along the minor axis ($\sim90^\circ$ rotation) at a stress of 63 MPa (figure 3(c)) while the critical stress shown in table 2 is 57.41 MPa. The small difference is due to the minor incoherency of the rotation process. Upon removal of this stress, the magnetization has an equal probability of either rotating back to its initial orientation or flipping by $180^\circ$. This magnetization rotation picture most closely matches the macrospin (coherent rotation) approximation.

In nanomagnets of intermediate dimensions (100 nm × 80 nm × 6 nm), the magnetization goes through a ‘C-state’ that is stable under stress application (figure 4). The ‘C-state’ is so-named because the spins curl into the shape of the letter ‘C’. Stress was varied in increments of 15 MPa from 0 MPa up to a maximum of 180 MPa. As stress increases, the magnetization component along the minor axis gradually increases, but there still exists a non-zero component of the magnetization along the major axis (figure 4(c)). In other words, even a stress as high as 180 MPa does not rotate or modify the magnetization orientation so much that the memory of the prior state is completely erased. Thus, upon removal of stress, the magnetization always returns to its initial state (figure 4(d)) because of the retained memory. Also, because the magnetization reverts to the original state upon withdrawal of stress, the magnetization change is ‘reversible’ and the state visited under stress is ‘volatile’.

Interestingly, in larger sized nanomagnets (200 nm × 160 nm × 12 nm), the magnetization begins to rotate at lower stress (10 MPa) (figure 5(a)), compared to the smaller and intermediate sized magnets, in which no significant magnetization rotation is observed until a stress of 45 MPa is applied. The magnetization attains a very stable vortex state with stress as low as 20 MPa (figure 5(b)). Further increase in stress results in a small increase in the magnetization vectors oriented along the minor axis ($\pm x$-axis) in figure 5(c). The net magnetization, however, remains zero. The vortex state is a stable state and the nanomagnet remains in this state even if stress is withdrawn after the vortex formation, resulting in ‘non-volatility’. Therefore, in larger nanomagnets, the magnetization change is ‘irreversible’ and the state visited under stress is ‘non-volatile’ in stark contrast to the case for intermediate sized nanomagnets.

Note that for all of the cases, we performed the simulations dynamically but we have only included snapshots at various times of this dynamic simulation.

### 3.3. Study of the energy profile

The magnetization switching behavior of nanomagnets of different sizes can be explained by studying the energies involved. For this purpose, exchange, demagnetization, stress anisotropy and the total energies were calculated and plotted against applied stress.
For the smallest magnet, the demagnetization and stress anisotropy energies with all magnetization pointing exactly along the minor axis are shown by two reference lines (dashed lines in figure 6). Exchange energy variation is almost negligible, which explains the coherency of the switching process. There is an increase in demagnetization energy when the magnetization rotates from the major toward the minor axis. The magnetization orientation remains unchanged until a stress of 45 MPa is applied, since the demagnetization energy cost of rotating away from the easy axis is more than the lowering of stress anisotropy energy achieved by such a rotation. Thereafter, the magnetization starts to rotate, because the rate of increase of the demagnetization energy with increasing stress is less than the rate at which stress anisotropy decreases due to stress induced rotation (shown in figure 6). Therefore, the magnetization rotates to a slightly lower energy state. At 63 MPa stress, the stress anisotropy energy completely erodes the barrier between the easy and hard axis. Consequently, the minor axis orientation becomes energetically favorable and the magnet completes its magnetization rotation to this new easy state.

For the intermediate and large sized magnets, demagnetization and exchange energies of the vortex state are illustrated through two reference lines (dashed lines in figures 7 and 8). The calculation of total energy shows that vortex state is the energy minimum. In order to reach the vortex state from the initial equilibrium state, the nanomagnets need to go through intermediate states of high exchange energy. Hence, there exists a barrier between the vortex and initial equilibrium state which originates from the change in exchange energy it has to undergo to reach this vortex state. Therefore, the reduction in stress anisotropy energy and demagnetization energy should exceed the increase in exchange coupling energy to form the vortex state.

In the intermediate sized magnet, in the range of stress applied (~180 MPa, shown earlier in figure 4) the magnet stabilizes itself in the C-shaped metastable state, although the vortex state is still the ultimate energy minimum state. Here the C-shape persists up to large stresses as the exchange coupling penalty incurred in forming a vortex state is very high at the these lateral dimensions. On the other hand, the barrier between the initial equilibrium state and the vortex state is small for larger sized magnets. Therefore, at a stress of only 20 MPa, the magnetization settles to a vortex configuration in the larger sized nanomagnets (figure 8). Demagnetization and exchange energies of the nanomagnet meet the corresponding reference lines once the vortex is formed. Further increase in stress results in an increase in the component of the magnetization vector along the minor axis. Thus, a small increase in the demagnetization energy can be observed.

4. Conclusions

We have shown that the interplay between the exchange, demagnetization and stress energy can lead to a rich variety of magnetic configurations in planar shaped anisotropic nanomagnets under stress. These have been analyzed without including the effect of thermal noise to give us a simple physical picture. We discuss the magnetization behavior in the presence of thermal noise in the accompanying online supplementary material (section A, part i). There is no qualitative change in the behavior in the presence of thermal noise for the two extreme cases: small nanomagnets with near coherent magnetization rotation and large nanomagnets where stress easily induces a stable vortex state. However, in the

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intermediate nanomagnet, in addition to the C-shape magnetization configuration formed in the absence of thermal noise, an intermediate configuration, where most of the magnetization points along the hard axis, also emerges when thermal noise is added.

Furthermore, the three cases discussed here capture the essential physics of the size dependent strain mediated switching process. We have explored five additional intermediate sizes with the same aspect ratios (online supplementary section A, part ii) and different aspect ratios at constant volume (section A, part iii) to show that in such cases the qualitative switching behavior does not differ from those discussed in the main paper.

This work can have implications for the use of nanomagnets switched with stress for low energy computing cases the qualitative switching behavior does not differ from the vortex state is entered into nanomagnets is perhaps the most viable switching strategy. If the field required to nudge the magnetization away from this vortex state is so high that it might negate any energy advantage of strain induced switching unless the energy cost of such a “reset” operation can be amortized over a large number of nanomagnets. Furthermore, these studies can shed light on some recent observations of irreversible magnetization change in large nanomagnets [34].

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