Nonpropagation of massive mode on $\text{AdS}_2$

in topologically massive gravity

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Abstract

Making use of Achucarro-Ortiz (AO) type of dimensional reduction, we study the topologically massive gravity with a negative cosmological constant on $\text{AdS}_2$ spacetimes. For a constant dilaton, this two-dimensional model also admits three $\text{AdS}_2$ vacuum solutions, which are related to two $\text{AdS}_3$ and warped $\text{AdS}_3$ backgrounds with an identification upon uplifting three dimensions. We carry out the perturbation analysis around these backgrounds to find what is a physically propagating field. However, it turns out that there is no propagating massive mode on $\text{AdS}_2$ background, in contrast to the Kaluza-Klein (KK) type of dimensional reduction. We note that two dimensionally reduced actions are different and thus, the non-equivalence of their on-shell amplitudes is obtained.

PACS numbers: 04.60.Kz, 04.70.Dy, 03.65.Sq, 03.65.-w.
Keywords: topologically massive gravity; perturbation; dimensional reduction.

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1 Introduction

The gravitational Chern-Simons terms with a coupling constant $K$ in 3D Einstein gravity produce a physically propagating massive graviton \[^{[1]}\]. This topologically massive gravity with a negative cosmological constant $\Lambda = -1/l^2$ (TMG$_{\Lambda}$) gives us the AdS$_3$ solution \[^{[2, 3]}\]. For the Newton’s constant $G_3 > 0$, the massive modes carry negative energy on the AdS$_3$ background. In this sense, the AdS$_3$ background may not be a stable vacuum for $K \neq 0$. The opposite case of $G_3 < 0$ may cure the problem, but induces a negative mass for the BTZ black hole. Another relevant issue is what is the number $N_c$ of physical degrees of freedom (DOF) for a massive mode for $K \neq 0$. Now one may arrive at a consensus with $N_c = 1$.

There is a possibility for avoiding negative energy when choosing the chiral point of $K = l$. At this point, the massive mode becomes a massless left-moving graviton, which carries no energy and may be considered as a pure gauge. However, this special point raises many questions \[^{[1]}\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\]. At the chiral point of $K = l$, a physical degree of freedom seems to be propagating when using the linearized analysis. Even though the canonical analysis has shown that the DOF is one ($N_c = 1$) at the chiral point \[^{[8, 10]}\ 14\], a single DOF is not yet confirmed to be $\psi^{\text{new}}$ which is regarded as an additional mode for the graviton \[^{[13]}\]. Actually, as was claimed in \[^{[14]}\], the transition to the chiral coupling does not make a critical change on the form of the Poisson bracket algebra. This may imply that a choice of $K = l$ is nothing special in the canonical analysis.

In general, it is not easy to handle the gravitational Chern-Simons terms even though they are invariant under diffeomorphism and Weyl rescaling of the metric \[^{[15, 16]}\]. Hence, one needs to seek another way to investigate the TMG$_{\Lambda}$. In this end, one may introduce conformal transformation to single out a conformal degree of freedom (a dilaton $\phi$) and then use the Kaluza-Klein (KK) ansatz to obtain an effective two-dimensional action of 2DTMG$_{\Lambda}$, which will be a gauge and coordinate invariant. Actually, this was performed by introducing the metric of $ds^2_{KK} = \phi^2 [g_{\mu\nu} dx^\mu dx^\nu + (d\theta + A_\mu dx^\mu)^2]$. Saboo and Sen \[^{[17]}\] have used the 2DTMG$_{\Lambda}$ to obtain the entropy of extremal BTZ black hole \[^{[18]}\] by applying the entropy function formalism. Furthermore, the authors in \[^{[19]}\] have used the entropy function approach to find three distinct vacuum solutions of the 2DTMG$_{\Lambda}$.

Recently, we have studied the topologically massive gravity with a negative cosmological constant on AdS$_2$ spacetimes by making use of the KK type of dimensional reduction \[^{[20]}\]. For a constant dilaton, this two-dimensional model of 2DTMG$_{\Lambda}$ admits three AdS$_2$ vacuum solutions, which are related to the AdS$_3$ with a positive/negative charge and warped AdS$_3$ backgrounds \[^{[21, 22, 23, 24, 25, 26]}\] with an identification upon uplifting three dimensions. We have carried out the perturbation analysis around these backgrounds to find what is a
physically propagating field on the $\text{AdS}_2$ background whose curvature is $\ddot{R} = -2/v$. It turns out that a perturbation mode $\delta F = (h - f/e)$ of $F$ as a dual scalar of the Maxwell field is a nonpropagating field in the absence of the Chern-Simons terms. However, it becomes a massive mode in the presence of the Chern-Simons terms, whose equation is given by $(\dddot{\nabla}^2 - m_\pm^2)\delta F = 0$ with $m_\pm^2 = \frac{2}{v} - \frac{1}{4v} \left(1 \pm \frac{1}{\sqrt{5}}\right) \left(5 \mp \frac{1}{\sqrt{5}}\right)$ for two $\text{AdS}_2$ solutions. This confirms clearly that the DOF is one ($N_c = 1$) for the topologically massive gravity [1].

In this work, we will consider the AO type of dimensional reduction (AOTMG$_\Lambda$) based on the metric of Eq. (7) introduced by Achucarro and Ortiz [29]. They had exactly recovered the correspondence between the BTZ black hole and $\text{AdS}_2$ black hole. In this approach, we note that the “linear dilaton” was mainly used for recovering the $\text{AdS}_2$ black hole [30, 31, 32, 33, 34, 35] and confirming the $\text{AdS}_2$/CFT$_1$ correspondence [36, 37, 38]. In the presence of the gravitational Chern-Simons terms [39], the authors in [40] have first considered the AOTMG$_\Lambda$ to find $\text{AdS}_2$ solutions.

For “constant dilation”, we will perform perturbation analysis for the AOTMG$_\Lambda$. This means that we are working in the near-horizon geometry $\text{AdS}_2 \times S^1$ of extremal BTZ black hole. A perturbation mode $\delta F = (h - f/e)$ is a gauge-artifact in the absence of the Chern-Simons terms. We find that $\delta F$ remains a massless, redundant mode even in the presence of the Chern-Simons terms, in contrast to the previous result for the KK reduction case [20].

2 TMG$_\Lambda$

Now, let us start with the action for topologically massive gravity with a negative cosmological constant (TMG$_\Lambda$) given by [1]

$$I_{\text{TMG}_\Lambda} = \frac{1}{16\pi G_3} \int d^3x \sqrt{-g} \left[R_3 - 2\Lambda + \frac{K}{2} \varepsilon^{lmn} \Gamma^p_l \left(\partial_m \Gamma^q_{np} + \frac{2}{3} \Gamma^q_{ml} \Gamma^r_{np}\right)\right],$$

(1)

where $\varepsilon^{lmn}$ is the tensor defined by $\varepsilon^{lmn}/\sqrt{-g}$ with $\varepsilon^{012} = 1$. We choose the Newton’s constant $G_3 > 0$. The Latin indices of $l, m, n, \cdots$ denote three dimensional tensors. The $K$-term is called the gravitational Chern-Simons terms. It is the third order derivative correction to the 3D Einstein gravity. Here we choose “+” sign to avoid negative graviton energy [5].

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1 On the other hand, we expect to have a massive mode with 2 DOF when adding the Pauli-Fierz action of $I_{PF} = m_{PF}^2 \int d^3x \sqrt{-g} \left[h^{mn} h_{mn} - (h^m_m)^2\right]$ [27]. However, we have to use a different method to deal with a massive field in the TMG$_\Lambda$, which is supposed to be a massive mode including a gravivector. The reason is clear because the bilinear Chern-Simons term of $I_{CS} = K \int d^3x \sqrt{-g} h^{mn} \epsilon_{mn}^{(1)}$ is basically different from $I_{PF}$, where $\epsilon_{mn}^{(1)}$ is the linearized Cotton tensor [2]. It was shown that $I_{CS}$ is a square-root of $I_{PF}$ [28].
Varying this action leads to the Einstein equation
\[ G_{\mu\nu} + KC_{\mu\nu} = 0, \tag{2} \]
where the Einstein tensor including the cosmological constant is given by
\[ G_{\mu\nu} = R_{\mu\nu
\frac{3}{2}} - \frac{1}{l^2} g_{\mu\nu}, \tag{3} \]
and the Cotton tensor is
\[ C_{\mu\nu} = \varepsilon_{\mu pq} \nabla_p (R_{\mu
\frac{3}{2}} - \frac{1}{4} g_{\mu q} R_{\nu}). \tag{4} \]

We note that the Cotton tensor \( C_{\mu\nu} \) vanishes for any solution to \( G_{\mu\nu} = 0 \) for Einstein gravity, so all solutions to general relativity are also solutions of the TMG_\Lambda. Hence, the BTZ black hole solution [18] appears for the \( K = 0 \) case. For the \( K \neq 0 \) case, the warped black hole solution for \( \nu^2 = \left( \frac{l}{3R} \right)^2 > 1 \), which is asymptotic to the warped AdS_3, was found as [23, 25, 26]
\[ ds^2_{wBH} = -\tilde{N}^2 dt^2 + \frac{l^4}{4R^2 \tilde{N}^2} dr^2 + l^2 \tilde{R}^2 \left( d\theta + \tilde{N}^\theta dt \right)^2, \tag{5} \]
where
\[ \tilde{R}^2(r) = \frac{r^4}{4} \left[ 3(\nu^2 - 1)r + (\nu^2 + 3)(r_+ + r_-) - 4\nu \sqrt{r_+ r_- (\nu^2 + 3)} \right], \]
\[ \tilde{N}^2(r) = \frac{l^2(\nu^2 + 3)(r - r_+)(r - r_-)}{4 R^2(r)}, \]
\[ \tilde{N}^\theta(r) = \frac{2\nu r - \sqrt{r_+ r_- (\nu^2 + 3)}}{2 \tilde{R}^2(r)}. \tag{6} \]

We note that this metric reduces to the BTZ metric in a rotating frame when choosing \( \nu^2 = 1 \).

### 3 AO type reduction of TMG_\Lambda

Let us perform AO type of dimensional reduction using the metric [29]
\[ ds^2_{AO} = g_{\mu\nu}(x) dx^\mu dx^\nu + \phi^2(x) \left[ d\theta + A_\mu(x) dx^\mu \right]^2 \tag{7} \]
without conformal transformation [20]. Here \( \theta \) is a coordinate that parameterizes an \( S \) with period \( 2\pi l \). Hence, its isometry is factorized as \( G \times U(1) \). After the \( \theta \)-integration, the action reduces to an interesting two-dimensional TMG_\Lambda action
\[ I_{\text{AO-TMG}_\Lambda} = \frac{l}{8G_3} \int d^2x \sqrt{-g} \phi \left( R + \frac{2}{l^2} - \frac{1}{4} \phi^2 F_{\mu\nu} F^{\mu\nu} \right) + \frac{Kl}{32G_3} \int d^2x \phi^2 \left( R e^{\mu\nu} F_{\mu\nu} + \phi^2 e^{\mu\nu} F_{\mu\rho} F^{\rho\sigma} F_{\sigma\nu} \right). \tag{8} \]
Here $R$ is the 2D Ricci scalar with $R_{\mu\nu} = R_{\mu\nu}/2$ and $\phi$ is the dilaton. Also $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\epsilon^{01} = 1$. The Greek indices of $\mu, \nu, \rho, \cdots$ represent two dimensional tensors. Hereafter we choose $G_3/l = 1/8$ for simplicity. We mention that this action was recently used to find AdS$_2$ solutions [40].

Introducing a dual scalar $F$ of the Maxwell field through

$$F = -\frac{\epsilon^{\mu\nu} F_{\mu\nu}}{2\sqrt{-g}},$$

(9)

equations of motion for $\phi$ and $A_\mu$ are given by

$$R + \frac{2}{l^2} + \frac{3}{2} \phi^2 F^2 - K\phi F(R + 2\phi^2 F^2) = 0,$$

(10)

$$\epsilon^{\mu\nu} \partial_\mu \left[ \phi^3 F - \frac{K}{2} \phi^2 (R + 3\phi^2 F^2) \right] = 0,$$

(11)

respectively. The equation of motion for $g_{\mu\nu}$ is described as follows:

$$g_{\mu\nu} \left( \nabla^2 \phi - \frac{1}{l^2} \phi + \frac{1}{4} \phi^3 F^2 \right) - \nabla_\mu \nabla_\nu \phi$$

$$- \frac{K}{2} \left[ g_{\mu\nu} \left( \frac{1}{2} R\phi^2 F + \phi^4 F^3 + \nabla^2 (\phi^2 F) \right) - \nabla_\mu \nabla_\nu (\phi^2 F) \right] = 0.$$

(12)

Moreover, the above equation can be further separated into the trace part

$$\nabla^2 \phi - \frac{2}{l^2} \phi + \frac{1}{2} \phi^3 F^2 - K \left[ \frac{1}{2} R\phi^2 F + \phi^4 F^3 + \frac{1}{2} \nabla^2 (\phi^2 F) \right] = 0,$$

(13)

and the traceless part

$$\frac{1}{2} g_{\mu\nu} \nabla^2 \phi - \nabla_\mu \nabla_\nu \phi - \frac{K}{2} \left[ \frac{1}{2} g_{\mu\nu} \nabla^2 (\phi^2 F) - \nabla_\mu \nabla_\nu (\phi^2 F) \right] = 0.$$

(14)

Note that the “traceless part” does not play no role in obtaining vacuum solutions. However, when carrying out the perturbation analysis in the next section, this equation imposes a nontrivial constraint on the propagation of physical modes. This contrasts to our previous KK type reduction case [20], where the “traceless part” did not put any additional constraint on the propagation degrees of freedom. (See the footnote 2.)

Now, we wish to find the AdS$_2$ background as a vacuum solution to equations of motion. In case of a constant dilaton, from Eqs. (10) and (13), we have the condition

$$(1 - \frac{3}{2} K\phi F) \left( \frac{4}{l^2} - \phi^2 F^2 \right) = 0,$$

(15)
which implies three different relations between $\phi$ and $F$

$$\phi_{\pm} = \pm \frac{2}{lF}, \quad \phi_w = \frac{2}{3KF}. \quad (16)$$

Note here that for $K = l/3$, $\phi_w$ reduces to $\phi_+$. Assuming the line element preserving $G = SL(2, R)$ isometry

$$ds^2_{AdS_2} = v \left(-r^2 dt^2 + \frac{dr^2}{r^2}\right), \quad (17)$$

we have the AdS$_2$-spacetimes, which satisfy

$$\bar{R} = \frac{2}{v}, \quad \bar{\phi} = u, \quad \bar{F} = e/v, \quad (18)$$

where $\bar{F}_{10}$ = $\partial_1 \bar{A}_0 - \partial_0 \bar{A}_1$ = $e$ with $\bar{A}_0 = er$ and $\bar{A}_1 = 0$. In order to find the whole solutions of AdS$_2$ type, we may use the entropy function formalism [17] because it provides an efficient way to obtain AdS$_2$ solution as well as entropy of extremal black hole [19, 20]. The entropy function is defined as

$$\mathcal{E}(u, v, e, q) = 2\pi \left[qe - \mathcal{F}(u, v, e)\right], \quad (19)$$

where $\mathcal{F}(u, v, e)$ is the Lagrangian density $\mathcal{L}_{\text{AOTMG}}$ evaluated when using Eq. (18),

$$\mathcal{F}(u, v, e) = -2u + \frac{2uv}{l^2} + \frac{u^3e^2}{2v} + K\frac{u^2e}{v} \left(1 - \frac{u^2e^2}{v}\right). \quad (20)$$

Here we have equations of motion upon the variation of $\mathcal{E}$ with respect to $u$, $v$, and $e$

$$-2 + \frac{2v}{l^2} + \frac{3u^2e^2}{2v} + K\frac{2ue}{v} \left(1 - \frac{u^2e^2}{v}\right) = 0, \quad (21)$$

$$\frac{2u}{l^2} - \frac{u^3e^2}{2v^2} - K\frac{u^2e}{v^2} \left(1 - \frac{u^2e^2}{v}\right) = 0, \quad (22)$$

$$q - \frac{u^3e}{v} - K\frac{u^2e}{v} \left(1 - 3\frac{u^2e^2}{2v}\right) = 0, \quad (23)$$

respectively. Associated with the constant dilaton solutions [16], we also obtain three kinds of AdS$_2$ solutions similar to the KK case as fellows.

(1) For $u = \frac{2v}{le} (\phi_+ = \frac{2}{lF})$, one has AdS$_2$-solution with a positive charge $q$

$$u = \sqrt{-\frac{ql^2}{2(l-K)}}, \quad v = \frac{l^2}{4}, \quad e = \sqrt{\frac{l-K}{2q}}, \quad (24)$$

(2) For $u = \frac{-2v}{le} (\phi_- = -\frac{2}{lF})$, one has AdS$_2$-solution with a negative charge $q$

$$u = -\sqrt{-\frac{ql^2}{2(l+K)}}, \quad v = \frac{l^2}{4}, \quad e = -\sqrt{\frac{l+K}{-2q}}, \quad (25)$$

6
(3) For \( u = \frac{6Kl^2}{l^2 + 27K^2} \) \( (\phi_w = \frac{2}{3KF}) \), one has warped AdS\(_2\)-solution with a positive charge \( q \)
\[
u = \sqrt{\frac{9Kql^2}{l^2 + 27K^2}}, \quad v = \frac{9K^2l^2}{l^2 + 27K^2}, \quad e = \sqrt{\frac{4Kl^2}{q(l^2 + 27K^2)}}.
\]

Then, the corresponding entropies of the extremal black holes are given by
\[
S_+ = \frac{2\pi}{e} (l - K) \sim 2\pi \sqrt{\frac{ql}{6} \times \frac{12(l - K)}{l}}, \quad l \geq K
\]
\[
S_- = \frac{2\pi}{e} (l + K) \sim 2\pi \sqrt{\frac{ql}{6} \times \frac{12(l + K)}{l}}, \quad l \geq -K
\]
\[
S_w = \frac{2\pi}{e} \frac{8Kl^2}{l^2 + 27K^2} \sim 2\pi \sqrt{\frac{ql}{6} \times \frac{96Kl}{l^2 + 27K^2}}, \quad K > 0.
\]

The last relations (\( \sim \)) may be confirmed by the Cardy formula if \( ql \) is the eigenvalue of \( L_0 \)-operator of dual CFT\(_2\). However, the AdS/CFT correspondence is not still specified, because we have two AdS\(_2\) solutions \([36, 37, 38, 40]\): one is AdS\(_2\) with constant dilaton and near-horizon chiral CFT\(_2\) (AdS\(_2\)/CFT\(_2\) correspondence, in this work), and the other is AdS\(_2\) with linear dilaton and asymptotic CFT\(_1\) (AdS\(_2\)/CFT\(_1\) correspondence). Moreover, we note the entropy relation of \( S_+ = \frac{4\pi l}{3e} = S_w \) for the \( K = l/3 \) case, which implies a close connection between solution (1) and (3).

We note that for (24), (25), and (26), their background metric (7) can be rewritten as the extremal (warped) black holes using Poincare coordinates \((t, r, z)\), respectively \[19\]
\[
ds_{AO}^2 = \bar{g}_{\mu\nu} dx^\mu dx^\nu + (\bar{\phi})^2 (d\theta + \bar{A}_\mu dx^\mu)^2 \rightarrow
\]
\[
ds_+^2 = \frac{l^2}{4} \left[ -r^2 dt^2 + \frac{dr^2}{r^2} + (dz + r dt)^2 \right],
\]
\[
ds_-^2 = \frac{l^2}{4} \left[ -r^2 dt^2 + \frac{dr^2}{r^2} + (dz - r dt)^2 \right],
\]
\[
ds_w^2 = \frac{9K^2l^2}{27K^2 + l^2} \left[ -r^2 dr^2 + \frac{dr^2}{r^2} + \frac{4l^2}{27K^2 + l^2} (dz + r dt)^2 \right]
\]
with \( \theta = ez \). This show clearly that the isometry of \( SL(2, R) \times U(1) \) also persists in the AO type of dimensional reduction, similar to the KK case.

### 4 Perturbation of AOTMG\(_\Lambda\) on AdS\(_2\)

Now, we consider the perturbation modes of the dilaton, graviton, and dual scalar of the Maxwell field around the AdS\(_2\) background as \[20\]
\[
\phi = \bar{\phi} + \varphi, \quad g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \quad F = \bar{F}(1 + \delta F),
\]

\[7\]
where the bar variables denote the AdS$_2$ background as $\bar{\phi} = u$, $\bar{g}_{\mu\nu} = v \text{ diag}(-r^2, r^{-2})$, and $\bar{F} = e/v$. This background corresponds definitely to the near-horizon geometry of the extremal BTZ (warped) black holes, factorized as AdS$_2 \times S^1$. Here we note that the perturbation of the Maxwell field is defined as
\[ F_{10} = \bar{F}_{10} + \delta F_{10}, \] (35)
where $\delta F_{10} = \partial_1 a_0 - \partial_0 a_1$ and $\delta F_{10} = -f$. On the other hand, the perturbation fields are chosen to be
\[ h_{\mu\nu} = -h\bar{g}_{\mu\nu}, \quad \delta F = \left( h - \frac{f}{e} \right). \] (36)

### 4.1 Perturbation with $K = 0$

First, let us briefly summarized the $K = 0$ case, which is actually equivalent to the KK reduction, because this provides a reference case. However, this case is equivalent to the KK reduction case even though their perturbed equations of (10), (11), (13), and (14) take slightly different forms as
\[ \delta R(h) + \frac{3u^2 e^2}{v^2} \left( \frac{1}{u} \phi + \delta F \right) = 0, \] (37)
\[ u^3 \left( \frac{3}{u} \phi + \delta F \right) = 0, \] (38)
\[ \nabla^2 \phi - \frac{2\phi}{l^2} + \frac{u^3 e^2}{v^2} \left( \frac{3}{2u} \phi + \delta F \right) = 0, \] (39)
\[ \left( \frac{1}{2} \bar{g}_{\mu\nu} \nabla^2 - \nabla_{\mu} \nabla_{\nu} \right) \phi = 0 \] (40)
with the linearized Ricci scalar $\delta R(h) = (\nabla^2 - 2/v)h$. It is known from the counting of DOF that all of these modes belong to pure gauge because there is no DOF for the graviton $h_{mn}$ propagating on the AdS$_3$ background in the 3D Einstein gravity. Therefore, it is necessary to show that all modes of $\phi, h$, and $f$ are non-propagating on the AdS$_2$ background. For this purpose, we compute the on-shell exchange amplitude by plugging external sources $T, J_\phi$, and $J_f$ into Eqs. (37), (38) and (39) without constraint. Under the source condition of $T = \frac{e^2}{u} J_f$, the on-shell exchange amplitude take a contact form
\[ \bar{A}^{K=0} = \frac{1}{2} \int d^2 p \left[ \frac{1}{u^3} J_f^2 \right]. \] (41)
As a result, the effective 2D gravity theory, which is correctly matched with the original 3D Einstein gravity, has no physically propagating modes.
4.2 Perturbation with $K \neq 0$

For the $K \neq 0$ case, perturbed equations of motion are complicated to be

\[
\delta R + \frac{3u^2e^2}{v^2} \left( \frac{\varphi}{u} + \delta F \right) - \frac{Kue}{v} \left[ \delta R + \left( \frac{6u^2e^2}{v^2} - \frac{2}{v} \right) \left( \frac{\varphi}{u} + \delta F \right) \right] = 0, \quad (42)
\]

\[
u^3 \left( \frac{3\varphi}{u} + \delta F \right) - \frac{Ku^2v}{2e} \left[ \delta R + \left( \frac{12u^2e^2}{v^2} - \frac{4}{v} \right) \varphi + \frac{6u^2e^2}{v^2} \delta F \right] = 0, \quad (43)
\]

\[
\bar{\nabla}^2 \varphi - \frac{2\varphi}{l^2} + \frac{u^3e^2}{v^2} \left( \frac{3\varphi}{2u} + \delta F \right)
- \frac{Ku^2e}{2v} \left[ \delta R + \left( \bar{\nabla}^2 - \frac{2}{v} \right) \varphi + \frac{2ue^2}{v^2} \varphi + \frac{6u^2e^2}{v^2} \left( \frac{\varphi}{u} + \delta F \right) \right] = 0, \quad (44)
\]

\[
\left( \frac{1}{2} \bar{g}_{\mu\nu} \bar{\nabla}^2 - \bar{\nabla}_\mu \bar{\nabla}_\nu \right) \left[ \varphi - \frac{Ku^2e}{2v} \left( \frac{2\varphi}{u} + \delta F \right) \right] = 0. \quad (45)
\]

Let us first consider the two AdS$_2$ solutions (24) and (25). Considering the relation $v = u^2e^2$, and inserting $\delta R$ in Eq. (43) into Eq. (42), we obtain a relation between $\delta F$ and $\varphi$ as

\[
\delta F = \alpha_\pm \left( \frac{\varphi}{u} \right), \quad (46)
\]

where $\alpha_\pm$ is determined to be

\[
\alpha_\pm = -\left( \frac{8K \mp 3l}{4K \mp l} \right). \quad (47)
\]

On the other hand, $\delta R$ in either Eq. (42) or Eq. (43) can be simplified to show the relation

\[
\delta R = \pm \frac{16e}{l^3} \left( \frac{8K \mp 3l}{4K \mp l} \right) \varphi. \quad (48)
\]

At this point, we check that for $K = 0$ case, $\alpha_\pm = -3$, which gives $\delta F = -3\varphi/u$. In this case, one finds that $\delta R = (6/uv)\varphi$. It again returns to Eq. (37), implying an unusual propagation of $(\bar{\nabla}^2 - 2/v)^2h = 0$. Considering Eq. (45) leads to the constraint on the mode relation

\[
\frac{\varphi}{u} = \frac{1}{2} \left[ \frac{Kue}{v - Kue} \right] \delta F. \quad (50)
\]

At this stage, it seems appropriate to comment on the case of the KK reduction [20]. In this case, the linearized equation of the “traceless part” is given by

\[
\left( \frac{1}{2} \bar{g}_{\mu\nu} \bar{\nabla}^2 - \bar{\nabla}_\mu \bar{\nabla}_\nu \right) \left[ \varphi - \frac{Ke}{2v} \delta F \right] = 0. \quad (49)
\]

In contrast to the AO type reduction, this equation does not impose any additional constraint because $\varphi = \frac{Ke}{2v} \delta F$ is the same mode relation appeared in Eq. (4.17) in Ref. [20].
With \( u = \pm \frac{2v}{l} \), the above constraint leads to a relation between \( \varphi \) and \( \delta F \) as

\[
\frac{\varphi}{u} = \left[ \frac{K}{\pm l - 2K} \right] \delta F, \tag{51}
\]

respectively.

It is now important to note that for compatibility, Eqs. (46) and (51) leads to the condition

\[
K = \pm \frac{l}{3} \tag{52}
\]

for the two AdS\( _2 \) solutions of \( u = \pm \frac{2v}{l} \), respectively. This compatibility condition could also be derived by eliminating \( \varphi \) in Eqs. (42) and (43) when using (51). We rewrite them as a matrix equation

\[
\begin{pmatrix}
\frac{Kl}{4} & \frac{(4K \mp l)(K \mp l)}{(2K \mp l)^2} \\
\frac{2K \mp l}{2K} & \frac{4(K \mp l)(K \mp l)}{(2K \mp l)^2}
\end{pmatrix}
\begin{pmatrix}
\delta R \\
\delta F
\end{pmatrix} =
\begin{pmatrix}
0 \\
0
\end{pmatrix} \rightarrow M_{\pm} \mathcal{H} = 0 \tag{53}
\]

with \( \mathcal{H} = (\delta R, \delta F) \). Then, \( \delta R \) and \( \delta F \) have nontrivial solutions iff the determinant of \( M_{\pm} \) is zero as

\[
\det M_{\pm} = 0 \rightarrow \pm \frac{(3K \mp l)(K \mp l)}{(2K \mp l)^2} = 0 \tag{54}
\]

which implies Eq. (52). Thus, the linearized equation of the traceless part puts on an important constraint such that

\[
\frac{\varphi}{u} = \delta F. \tag{55}
\]

Considering the compatibility condition of \( K = \pm l/3 \), we obtain the relation between the perturbed fields as

\[
\left( \nabla^2 - \frac{2}{v} \right) h = -\frac{2}{uv} \varphi = -\frac{2}{v} \delta F. \tag{56}
\]

On the other hand, the remaining equation (44) takes the form

\[
\left( 1 \mp \frac{3K}{l} \right) \left( \nabla^2 + \frac{2}{v} \right) \varphi = 0, \tag{57}
\]

which does not provide any useful information when imposing the condition of \( K = \pm l/3 \).

At this stage, we note that the previous approach of linearized equations without sources did not show clearly what kind of modes are really propagating on the AdS\( _2 \) background. Hence, we need to take into account the on-shell exchange amplitude with external sources. We note that in Appendix, we initially compute the on-shell exchange amplitude without the constraint. Then, we require the condition (52) on the on-shell exchange amplitude.
For the two AdS$_2$ solutions, the coupled equations in (53) take the form with external sources

\[
\left(\frac{K}{2K \mp l}, \frac{(4K \mp l)(K \mp l)}{4(8K \mp 2l)(K \mp l)^3}\right) \left(\begin{array}{c} \delta R \\ \delta F \end{array}\right) = \left(\begin{array}{c} \pm \frac{1}{u} J_f \\ \mp J_\varphi \end{array}\right) \tag{58}
\]

which allow $\delta R$ and $\delta F$ to express in terms of the sources as

\[
\delta R = \frac{32e^3(8K \mp 3l)J_f - (4K \mp l)J_\varphi}{(3K \mp l)l^5}, \tag{59}
\]

\[
\delta F = \frac{(2K \mp l)[K^5J_\varphi - 32e^3(2K \mp l)J_f]}{4l^3(3K \mp l)(K \mp l)}. \tag{60}
\]

On the other hand, using Eqs. (59) and (60), the remaining equation (78) (equivalently, (44) with $-T$) leads to the source condition

\[
T = \frac{e^2}{v^2} J_f. \tag{61}
\]

Finally, making use of (61), (59), (60), and (50), the Fourier-transformed on-shell amplitude

\[
\bar{A}_\pm^K = \frac{1}{2} \int d^2 p \left[ v_\varphi(p) J_\varphi + vh(p) \left( -T + \frac{e^2}{v^2} J_f \right) - \frac{e^2}{v} \delta F(p) J_f \right] \tag{62}
\]

reduces to

\[
\bar{A}_\pm^K = \frac{1}{u^5 l} \int d^2 p \frac{1}{\det M_\pm} \left[ (2K \mp l)J_f^2 - \frac{K^2u^6v^2}{2K \mp l}J_\varphi^2 \right], \tag{63}
\]

which contains contact terms without poles. Hence, this does not contribute to the interaction between the separate sources. This is also recovered from Eq. (54) with the source condition (61). However, we observe that Eq. (63) blows up because $\det M_\pm = 0$ when imposing the compatibility condition $K = \pm l/3$. Hence, the AO type reduction is not a proper way to obtain propagating modes on the near-horizon geometry of AdS$_2 \times S^2$ of extremal black holes.

Now, let us consider the perturbation around the warped AdS$_2$ solution (26). By making use of $u = 2v/3Ke$, (42) and (43) give

\[
\delta F = -\frac{2(27K^2 - 2l^2)}{27K^2 - 5l^2} \left( \frac{\varphi}{u} \right), \tag{64}
\]

while Eq. (45) implies

\[
\delta F = \frac{\varphi}{u}. \tag{65}
\]

Thus, equating (64) and (65) leads again to the compatibility condition of $K = \pm l/3$ even for the warped AdS$_2$ solution. Moreover, Eqs. (42) and (43) can be rewritten as

\[
\left(\begin{array}{cc} 1 & \frac{20}{9K^2} - \frac{12}{7l^2} \\ 1 & -\frac{16}{9K^2} + \frac{24}{7l^2} \end{array}\right) \left(\begin{array}{c} \delta R \\ \delta F \end{array}\right) = \left(\begin{array}{c} 0 \\ 0 \end{array}\right) \rightarrow M_\pm H = 0. \tag{66}
\]
From the condition of nontrivial solutions for $\delta R$ and $\delta F$ as

$$\det M_w = \frac{4(9K^2 - l^2)}{K^4l^2} = 0,$$

we again confirm the compatibility condition $K = \pm l/3$ for the warped AdS$_2$ solution. The perturbed fields are satisfied as

$$\left(\nabla^2 - \frac{2}{v}\right) h = -\frac{8}{l^2} \delta F = -\frac{8}{ul^2} \phi$$

while the remaining equation (67) does not give rise to any further information as before, upon using the compatibility condition.

On the other hand, for the warped AdS$_2$ solution, the coupled equations (66) take the form with the external sources

$$\left(\begin{array}{cc} 1 & \frac{20}{9K^2} - \frac{12}{l^2} \\ 1 & -\frac{16}{9K^2} + \frac{24}{l^2} \end{array}\right) \left(\begin{array}{c} \delta R \\ \delta F \end{array}\right) = \left(\begin{array}{c} \frac{(27K^2 + l^2)^3 e^3}{162K^5l^6} J_f \\ -3J_\phi \end{array}\right).$$

Now, $\delta R$ and $\delta F$ are rewritten in terms of the external sources as

$$\delta R = \frac{1}{(9K^2 - l^2)} \left(\frac{(27K^2 + l^2)^3(27K^2 - 2l^2)}{729K^5l^6} e^3 J_f - \frac{1}{3}(27K^2 - 5l^2)J_\phi\right),$$

$$\delta F = \frac{-1}{(9K^2 - l^2)} \left(\frac{(27K^2 + l^2)^3 e^3}{648K^3l^4(9K^2 - l^2)} J_f + \frac{3K^2l^2}{4(9K^2 - l^2)} J_\phi\right).$$

On the other hand, using (70) and (71), the remaining equation (78) (equivalently, (44) with $-T$) leads to the source condition (61).

Finally, using (70) and (71), one can obtain the Fourier-transformed on-shell amplitude for the warped AdS$_2$ case as

$$A^K = \frac{2}{K^2l^2} \int d^2p \frac{1}{\det M_w} \left[\frac{(27K^2 + l^2)^4 e^5}{273^6K^5l^6} J_f^2 - \frac{34K^5l^6}{2\epsilon(27K^2 + l^2)^2} J_\phi^2\right],$$

which is exactly the same with Eq. (85) when imposing the source condition.

5 Discussions

We have studied the topologically massive gravity with a negative cosmological constant on AdS$_2$ spacetimes by making use of the AO type of dimensional reduction. We have obtained that for a constant dilaton, the two-dimensional model of AOTMG$_3$ admits three AdS$_2$ solutions. As was shown in Eqs. (31), (32), and (33), these are related to AdS$_3$ with
positive/negative charge and warped AdS$_3$-solution with an identification upon uplifting three dimensions similar to the KK reduction case. However, it turns out that there is no propagating massive mode on AdS$_2$, in contrast to the KK case. This shows that the AOTMG$_\Lambda$ based on the AO type of the dimensional reduction is not appropriate for describing a massive propagation of the TMG$_\Lambda$ on AdS$_2$ spacetimes, even though it was successfully used to derive the entropies of extremal BTZ and warped black holes when applying the entropy function formalism.

How we could understand the disappearance of massive modes under the AO type of dimensional reduction? Is it related to the isometry? For topologically massive gravity without a negative cosmological constant, it was known that the exact theory has no nontrivial solutions that admits a hypersurface-orthogonal Killing vector [44]. This means that assuming too much isometry may eliminate all of propagating modes. In this work, the isometry of AdS$_3$, SL$(2,R) \times$ SL$(2,R)$ is broken to the isometry of AdS$_2$, SL$(2,R) \times U(1)$ when performing the AO reduction. At this stage, we note that two types of dimensional reduction provide the same isometry of SL$(2,R) \times U(1)$. Also these give the same field contents of $\varphi, \delta F$, and $h$. Therefore, it is reasonable to consider that the dimensional reduction does not eliminate all propagating modes.

An important thing to remark is not an isometry breaking, but the role of the dilaton. It seems that replacing $g_{\mu\nu}$ by $\phi^2 g_{\mu\nu}$ in Eq. (7) leads to the metric for the KK reduction. However, we emphasize that this replacement does not lead to the 2DTMG$_\Lambda$ action obtained by the KK reduction:

\[
\mathcal{I}_{2\text{DTMG}} = \frac{l}{8G_3} \int d^2 x \sqrt{-g}(\phi R + \frac{2}{\phi}(\nabla \phi)^2 + \frac{2}{l^2} \phi^3 - \phi^4 F_{\mu\nu} F^{\mu\nu}) + \frac{Kl}{32G_3} \int d^2 x \left( R \epsilon^{\mu\nu} F_{\mu\nu} + \epsilon^{\mu\nu} F_{\mu\rho} F^{\rho\sigma} F_{\sigma\nu} \right),
\]

(73)

where the kinetic term of $\phi$ appears in the Einstein action, while the dilaton never appears in the Chern-Simons terms. Actually, this operation leads to a different form of

\[
\mathcal{I}_{2\text{DTMG}} + \frac{Kl}{32G_3} \int d^2 x \frac{2}{\phi} \nabla_\mu \phi \nabla^\mu (\epsilon^{\mu\nu} F_{\mu\nu}).
\]

(74)

For $K = 0$, we have proven that two are equivalent [11] because two of $\mathcal{I}_{2\text{DTMG}}$ and $\mathcal{I}_{\text{AOTMG}}$ describe the same on-shell amplitude [11]. However, for $K \neq 0$, two are inequivalent because $\mathcal{I}_{2\text{DTMG}}$ “does not” contain the dilaton in the Chern-Simons terms, whereas $\mathcal{I}_{\text{AOTMG}}$ “does” contain the dilaton in the Chern-Simons terms. In the KK reduction case, the disappearance of the dilaton persists in the linearized perturbation theory. Hence, there is obviously no coupling between $\varphi$ and $\delta F$. This makes $\delta F$ massive for the $K \neq 0$ case. On the other hand,
as $I_{\text{AOTMG}}$ is shown in Eq. (8), the dilaton appears in the Chern-Simons terms. As a result, all perturbed modes of $\varphi, \delta F$ and $h$ become coupled to each other, eliminating a massive mode in contrast to the KK reduction case.

At this stage, we note that the constraint (50) (compatibility condition (52)) obtained from the traceless part is not compatible with the finiteness of on-shell amplitude for two AdS$_2$ solutions because it blows up as (63) shows. Also, as is shown in (72), the on-shell amplitude blows up for the warped solution. This implies that the 2D action $I_{\text{AOTMG}}$ is not suitable for describing the near-horizon geometry AdS$_2 \times S^1$ of extremal black holes.

In conclusion, the AO type of dimensional reduction with “a constant dilaton” is not a correct way to study the near-horizon geometry AdS$_2 \times S^1$ of extremal black hole from topologically massive gravity, while the KK type of dimensional reduction provides a promising scheme to investigate the near-horizon geometry of extremal black hole [20]. Importantly, we have observed that in the presence of the gravitational Chern-Simons terms (for $K \neq 0$), two actions are inequivalent and thus, the non-equivalence of their on-shell amplitudes is obtained.

Finally, we would like to mention that the AO type of dimensional reduction is useful for studying the AdS$_2$ black hole without the gravitational Chern-Simons terms [30].

Acknowledgement

The authors thank D. Grumiller for communication on physical degrees of freedom. Y.-W. Kim was supported by the Korea Research Foundation Grant funded by Korea Government (MOEHRD): KRF-2007-359-C00007. Two of us (Y. S. Myung and Y.-J. Park) were supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MEST) through the Center for Quantum Spacetime (CQUeST) of Sogang University with grant number 2005-0049409.

Appendix: On-shell exchange amplitudes with external sources ($K \neq 0$ case without constraint)

Let us start with the bilinear action with external sources

$$A^K = \int d^2 x \left[ \delta_2 L_{\text{AOTMG}}(h, \varphi, f) + \sqrt{-g} \left( \varphi J_\varphi + h_{\mu\nu} T^{\mu\nu} + \frac{c}{\ell^2} f J_f \right) \right]. \quad (75)$$
Then, the linearized equations with external sources are given by

$$
\delta R + \frac{3u^2e^2}{v^2} \left( \frac{\varphi}{u} + \delta F \right) - \frac{Kue}{v} \left[ \delta R + \left( \frac{6u^2e^2}{v^2} - \frac{2}{v} \right) \left( \frac{\varphi}{u} + \delta F \right) \right] = -J_\varphi, \quad (76)
$$

$$
u^3 \left( \frac{3\varphi}{u} + \delta F \right) - \frac{Ku^2v}{2e} \left[ \delta R + \left( \frac{12u^2e^2}{v^2} - \frac{4}{v} \right) \varphi + \frac{6u^2e^2}{v^2} \delta F \right] = -J_f, \quad (77)
$$

$$
\nabla^2 \varphi - \frac{2}{l^2} \varphi + \frac{u^3e^2}{v^2} \left( \frac{3\varphi}{2u} + \delta F \right) - \frac{Ku^2e}{2v} \left[ \delta R + \left( \nabla^2 - \frac{2}{v} \right) \left( \frac{2\varphi}{u} + \delta F \right) + \frac{2ue^2}{v^2} \varphi + \frac{6u^2e^2}{v^2} \left( \frac{\varphi}{u} + \delta F \right) \right] = -T. \quad (78)
$$

If the above sources are turned off, these are the same equations of (42), (43), and (44), respectively. Hence we could follow the same diagonalizing process in Sec. 4.2 with the sources. As was shown before, we first consider the AdS$_2$ solutions ($\phi_{\pm}$) with $u = \pm (2v/e)$. By solving Eq. (77) for $\delta R$ and inserting it into Eq. (76), we have $\delta F$ with the sources as

$$
\delta F = \alpha_{\pm} \frac{\varphi}{u} + \alpha_s, \quad (79)
$$

where $\alpha_{\pm}$ is given by Eq. (47) and $\alpha_s$

$$
\alpha_s = \mp \frac{Kl^3 J_\varphi + 8e^3 (1 \mp \frac{2K}{l}) J_f}{(K \mp l)(4K \mp l) l}. \quad (80)
$$

Now $\delta R$ can be read off from Eq. (77) as

$$
\delta R = \left( \nabla^2 - \frac{2}{v} \right) h = \beta_{\pm} \varphi + \beta_s, \quad (81)
$$

where $\beta_{\pm} = (16e/l^3)\alpha_{\pm}$ and $\beta_s$

$$
\beta_s = 32e^3 Kl^4 J_f \pm \frac{(6K \mp l) l [J_\varphi \mp \frac{32e^3 (2K \mp l)}{Kl^3} J_f]}{(K \mp l)(4K \mp l) l}. \quad (82)
$$

Finally, making use of (79) and (81), we obtain from Eq. (78) the dilaton equation expressed in terms of sources

$$
\left( \nabla^2 - \frac{2}{v} \right) \varphi = -\left( \frac{4K \mp l}{3K \mp l} \right) \left( T - \frac{e^2}{v^2} J_f \right) + \frac{1}{(K \mp l)(3K \mp l)} \left[ 4Ke^2 (2K \mp l) J_f \mp \frac{K^2l^3}{8e} J_\varphi \right]. \quad (83)
$$

After obtaining the Fourier-transformed fluctuations from (79), (81), and (83), and making a tedious calculation, we arrive at the Fourier-transformed on-shell amplitude induced by the
external sources as
\[
\tilde{A}_\pm^K = \frac{1}{2} \int d^2 p \left[ \frac{1}{(K \pm l)(3K \mp l)} \left( \pm \frac{32e^5(2K \mp l)^2}{l^5} J^2 \mp \frac{K^2l^5}{32e} J^2 \right) \right.
\]
\[
\pm \frac{16e^3(8K \mp l)}{l^3(3K \mp l)} \left( \frac{T}{\bar{p}} + \frac{3}{2} \right) J^2 \pm \frac{4e(8K \mp l)}{l(3K \mp l)} \left( \frac{T}{\bar{p}} + \frac{3}{2} \right)^2 J^2 \right] \]  
(84)

for two AdS$_2$ ($\phi_{\pm}$) solutions. We easily check that for the $K = 0$ case with the source condition $T = \frac{e^2}{\bar{p}} J_f$, the Fourier-transformed on-shell amplitude shows no pole.

For the warped AdS$_2$ solution with $u = 2v/3Ke$, by repeating the tedious whole procedure as the AdS$_2$ case, we finally arrive at the Fourier-transformed on-shell amplitude
\[
\tilde{A}_w^K = \frac{1}{2} \int d^2 p \left[ \frac{1}{9(K^2 - l^2)^2} \left( \frac{(27K^2 + l^2)^4e^5}{27 \cdot 6^3 K^5 l^6} J^2 - \frac{34K^5 l^6}{2e(27K^2 + l^2)^2} J^2 \right. \right.
\]
\[
\left. \left. + \frac{e(27K^2 + l^2)(27K^2 - 2l^2)}{9K l^2} \left( \frac{T}{\bar{p}} + \frac{3}{2} \right)^2 J^2 \right] . \right. \]  
(85)

We mention that under the source condition $T = \frac{e^2}{\bar{p}} J_f$, the Fourier-transformed on-shell amplitude also shows no pole. However, both $\tilde{A}_\pm^K$ and $\tilde{A}_w^K$ blow up in the limit of $K \to l/3$, showing a signal for the failure of the action $\mathcal{L}_{\text{AOTMG}_{\Lambda}}$, which is introduced in order to describe a massive propagating mode in the AdS$_2$ spacetimes.

References

[1] S. Deser, R. Jackiw, S. Templeton, Annals Phys. 140, 372 (1982), Erratum-ibid. 185, 406 (1988), Annals Phys. 281, 409 (2000).

[2] W. Li, W. Song, A. Strominger, J. High. Energy Phys. 0804, 082 (2008) [arXiv:0801.4566 [hep-th]].

[3] W. Li, W. Song, A. Strominger, [arXiv:0805.3101 [hep-th]].

[4] S. Carlip, S. Deser, A. Waldron, D. K. Wise, Class. Quant. Grav. 26, 075008 (2009) [arXiv:0803.3998 [hep-th]].

[5] D. Grumiller, N. Johansson, J. High. Energy Phys. 0807, 134 (2008), [arXiv:0805.2610 [hep-th]].
[6] G. Giribet, M. Kleban, M. Porrati, J. High. Energy Phys. **0810**, 045 (2008), arXiv:0807.4703 [hep-th].

[7] M. I. Park, J. High. Energy Phys. **0809**, 084 (2008), arXiv:0805.4328 [hep-th].

[8] D. Grumiller, R. Jackiw, N. Johansson, arXiv:0806.4185 [hep-th].

[9] S. Carlip, S. Deser, A. Waldron, D. K. Wise, Phys. Lett. B **666**, 272 (2008), arXiv:0807.0486 [hep-th].

[10] S. Carlip, J. High. Energy Phys. **0810**, 078 (2008), arXiv:0807.4152 [hep-th].

[11] A. Strominger, arXiv:0808.0506 [hep-th].

[12] I. Sachs, S. N. Solodukhin, J. High. Energy Phys. **0808**, 003 (2008), arXiv:0806.1788 [hep-th].

[13] Y. S. Myung, Phys. Lett. B **670**, 220 (2008), arXiv:0808.1942 [hep-th].

[14] M. Blagojevic, B. Cvetkovic, J. High. Energy Phys. **0905**, 073 (2009), arXiv:0812.4742 [gr-qc].

[15] G. Guralnik, A. Iorio, R. Jackiw, S. Y. Pi, Annals Phys. **308**, 222 (2003), arXiv:hep-th/0305117.

[16] D. Grumiller, W. Kummer, Annals Phys. **308**, 211 (2003), arXiv:hep-th/0306036.

[17] B. Sahoo, A. Sen, J. High. Energy Phys. **0607**, 008 (2006), arXiv:hep-th/0601228.

[18] M. Banados, C. Teitelboim, J. Zanelli, Phys. Rev. Lett. **69**, 1849 (1992), arXiv:hep-th/9204099.

[19] M. Alishahiha, R. Fareghbal, A. E. Mosaffa, J. High. Energy Phys. **0901**, 069 (2009), arXiv:0812.0453 [hep-th].

[20] Y. S. Myung, Y. W. Kim, Y. J. Park, J. High. Energy Phys. **0906**, 043 (2009), arXiv:0901.2141 [hep-th].

[21] K. A. Moussa, G. Clement, C. Leygnac, Class. Quant. Grav. **20**, L277 (2003), arXiv:gr-qc/0303042.

[22] A. Bouchareb, G. Clement, Class. Quant. Grav. **24**, 5581 (2007), arXiv:0706.0263 [gr-qc].
[23] D. Anninos, W. Li, M. Padi, W. Song, A. Strominger, J. High. Energy Phys. 0903, 130 (2009), arXiv:0807.3040 [hep-th].

[24] K. A. Moussa, G. Clement, H. Guennoune, C. Leygnac, Phys. Rev. D 78, 064065 (2008), arXiv:0807.4241 [gr-qc].

[25] G. Compere, S. Detournay, Class. Quant. Grav. 26, 012001 (2009), arXiv:0808.1911 [hep-th].

[26] J. J. Oh, W. Kim, J. High. Energy Phys. 0901, 067 (2009), arXiv:0811.2632 [hep-th].

[27] M. Fierz, W. Pauli, Proc. R. Soc. 173, 211 (1939).

[28] E. A. Bergshoeff, O. Hohm, P. K. Townsend, Phys. Rev. Lett. 102, 201301 (2009), arXiv:0901.1766 [hep-th].

[29] A. Achucarro, M. E. Ortiz, Phys. Rev. D 48, 3600 (1993), arXiv:hep-th/9304068.

[30] J. Navarro-Salas, P. Navarro, Nucl. Phys. B 579, 250 (2000), arXiv:hep-th/9910076.

[31] D. Louis-Martinez, G. Kunstatter, Phys. Rev. D 52, 3494 (1995), arXiv:gr-qc/9503016.

[32] D. Grumiller, R. McNees, J. High. Energy Phys. 0704, 074 (2007), arXiv:hep-th/0703230.

[33] Y. S. Myung, Y. W. Kim, Y. J. Park, Mod. Phys. Lett. A 23, 91 (2008), arXiv:0707.3314 [gr-qc].

[34] Y. S. Myung, Y. W. Kim, Y. J. Park, Phys. Rev. D 78, 044020 (2008), arXiv:0804.0301 [gr-qc].

[35] Y. S. Myung, Y. W. Kim, Y. J. Park, Phys. Rev. D 76, 104045 (2007), arXiv:0707.1933 [hep-th].

[36] T. Hartman, A. Strominger, J. High. Energy Phys. 0904, 026 (2009), arXiv:0803.3621 [hep-th].

[37] M. Cadoni, M. R. Setare, J. High. Energy Phys. 0807, 131 (2009), arXiv:0806.2754 [hep-th].

[38] M. Cadoni, M. Melis, P. Pani, arXiv:0812.3362 [hep-th].

[39] M. Adak, T. Dereli, Class. Quant. Grav. 21, 2275 (2004), arXiv:gr-qc/0403016.
[40] M. Alishahiha, F. Ardalan, J. High. Energy Phys. **0808**, 079 (2008), [arXiv:0805.1861](https://arxiv.org/abs/0805.1861) [hep-th].

[41] M. Cadoni, Phys. Lett. B **395**, 10 (1997), [arXiv:hep-th/9610201](https://arxiv.org/abs/hep-th/9610201).

[42] H. W. Lee, Y. S. Myung, J. Y. Kim, Phys. Rev. D **52**, 5806 (1995), [arXiv:hep-th/9510122](https://arxiv.org/abs/hep-th/9510122).

[43] D. Birmingham, S. Mokhtari, Phys. Rev. D **74**, 084026 (2006), [arXiv:hep-th/0609028](https://arxiv.org/abs/hep-th/0609028).

[44] A. N. Aliev, Y. Nutku, Class. Quant. Grav. **13**, L29 (1996), [arXiv:gr-qc/9812089](https://arxiv.org/abs/gr-qc/9812089).

[45] S. Randjbar-Daemi, A. Salam, J. A. Strathdee, Nucl. Phys. B **214**, 491 (1983).