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Invariant generalized almost complex structures on real flag manifolds. (English)

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Summary: We characterize those real flag manifolds that can be endowed with invariant generalized almost complex structures. We show that no $GM_2$-maximal real flag manifolds admit integrable invariant generalized almost complex structures. We give a concrete description of the generalized complex geometry on the maximal real flags of type $B_2$, $G_2$, $A_3$, and $D_l$ with $l \geq 5$, where we prove that the space of invariant generalized almost complex structures under invariant $B$-transformations is homotopy equivalent to a torus and we classify all invariant generalized almost Hermitian structures on them.

MSC:

53D18 Generalized geometries (à la Hitchin)
14M15 Grassmannians, Schubert varieties, flag manifolds
53C15 General geometric structures on manifolds (almost complex, almost product structures, etc.)
53C30 Differential geometry of homogeneous manifolds

Keywords:
generalized geometry; real flag manifold; integrability; B-transformation

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