Progressive suppression of spin relaxation in 2D channels of finite width

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We have investigated spatio-temporal kinetics of electron spin polarization in semiconductor narrow 2D strip and explored the ability to manipulate spin relaxation. Information about spin of the conduction electrons and mechanisms of spin rotation is incorporated into transport Monte Carlo simulation program. A model problem, involving linear-in-k splitting of the conduction band, responsible for the D’yakonov-Perel’ mechanism of spin relaxation in the zinc-blende semiconductors and heterostructures, is solved numerically to yield the decay of spin polarization of an ensemble of electrons in the 2D channel of finite width. For very wide channels, a conventional 2D value of spin relaxation is obtained. With decreasing channel width the relaxation time soars rapidly by orders of magnitude. Surprisingly, the cross-over point between 2D and quasi-1D behavior is found to be at tens of electron mean-free paths. Thus, classically wide channels can effectively suppress electron spin relaxation.

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I. INTRODUCTION

Spintronics, a nontrivial extension of the conventional electronics, adds functionality utilizing the carrier spin degree of freedom. Spin can potentially be used as a by far more capacious quantum information storage cell, be involved in the transfer of information, for elaborate schemes of information processing, both quantum and classical, and be integrated with electric-charge counterparts in combined designs. Electron or nuclear spin manipulation is believed to be the key component of the potential realizations of a quantum computer (for example, see Ref. [3]).

Several devices were proposed so far including spin hybrid tunneling structures with magnetic layers and spin-based memory. The most notable success was obtained with giant magnetoresistance (GMR) effect devices that rely on the variation of electron scattering in a multilayer stack of ferromagnetic films separated by nonmagnetic materials. Though the variation of the current through the structure is small, it is sufficient for detection, and the excellent sensitivity of the device to weak external magnetic fields (typically 1% change of resistance per oersted) opened the door for massive data storage applications. Spin coherence is a pivotal prerequisite for the operability of the prospective spintronic devices.

Concerning experimental realization of spintronic elements, most propositions rely on the injection of spin-polarized electrons from the ferromagnetic layer and suffer from the poor quality of the ferromagnet-semiconductor interfaces that produce a large number of surface states causing strong spin relaxation and reducing the polarization of the injected electrons to only a few percent. Since the injected electron crosses the metal-semiconductor interface twice in spin-valve designs, i.e., at the source and drain terminals of the device, the importance of the interface problem multiplies. Furthermore, spin relaxation in the active region of the device adds to the complications as well.

In our study, we consider the possibilities for suppressing spin relaxation of the conduction electrons and evaluate different approaches. It is extremely interesting to investigate what happens with the spin relaxation in a 2D electron gas as the long strip of finite width is formed using electrostatic squeezing split-gate technique or by another method. These systems can be used to connect active elements in integrated chip designs and can even become a part of the active device as we continue to reduce size of the element. Eventually we should reach the 1D limit exhibiting no spin relaxation. This transition to the 1D behavior was recently documented in the computer simulation of the Datta and Das spin transistor reported by Bournel et al. that inspired our research. We concentrate our efforts on the definition of the cross-over regions from the point of view of the spin relaxation time and the description of its behavior in the broad region around the cross-over points.

The rest of this paper is organized as follows: We start with a brief account of spin relaxation mechanisms in semiconductors in order to identify the most relevant. We discuss its transformation to lower-dimensional systems, as well as different possibilities to suppress destructive spin relaxation (Sec. II). The model of a narrow patterned 2D electron gas channel is described in Sec. III. Results of Monte Carlo simulation of spin relaxation in the channel are given in the next section (Sec. III), along with a comprehensive discussion of the relaxation regimes in terms of the channel width and a value of the spin splitting of electron subbands (Sec. III). A brief summary follows at the end.

II. MECHANISMS OF SPIN RELAXATION

There are several mechanisms that can cause spin relaxation of conduction electrons (see Ref. [4] for an up-to-date informal review):
i) The mechanism of D’yakonov and Perel’ (DP) takes into consideration that spin splitting of the conduction band in zinc-blende semiconductors at finite wave vectors is equivalent to an effective magnetic field that causes electron spin to precess. For electron experiencing random multiple scattering events the orientation of this effective field is random and that leads to the spin relaxation.

ii) Bir-Aronov-Pikus (BAP) processes involve a simultaneous flip of electron and hole spins due to electron-hole exchange coupling.

iii) Spin relaxation due to momentum relaxation is possible directly through spin-orbit coupling [Elliot-Yafet (EY) process].

iv) Spin relaxation can take place as a result of hyperfine interaction of electron spins with magnetic momenta of lattice nuclei, the hyperfine magnetic field being randomly changed due to the migration of electrons in the crystal.

BAP processes require a substantial hole concentration that is not available in the unipolar doped structures. EY processes are suppressed in 2D environments — target of our prime interest. Thus, we concentrate our efforts on the DP mechanism as the most relevant one for the case considered.

A. DP mechanism and change of the dimensionality

As a result of the relatively low zinc-blende crystal symmetry, the effective $2 \times 2$ electron Hamiltonian for the conduction electrons contains spin dependent terms that are cubic in the electron wave vector $k$

$$H = \eta' [\sigma_x k_y (k_z^2 - k_x^2) + \text{etc}].$$

(1)

The constant $\eta'$ reflects the strength of the spin splitting in the conduction band whose value is defined by the details of the semiconductor band structure, $\sigma_i$ ($i = x, y, z$) are the Pauli matrices, other terms in Eq. (1) should be obtained by the cyclic permutation of the indices.

As the system dimensionality is changed from the 3D to 2D by, for example, the extremely strong spacial confinement in the third direction — that can be achieved in the semiconductor heterostructures — modifications to the character of the spin relaxation occur as well. First of all, an average wave vector in the direction of the quantum confinement (axis $z$) is large, so the terms in the spin splitting involving $k_z^2$ will dominate. This results in the orientation of the effective magnetic field in the plane of the 2D electron gas ($x - y$ plane). Still the elementary rotations around random axes, all laying in one plane, do not commute with each other, so the electrons reaching the same final destination by different trajectories will have different spin orientations. For longer times, more and more distinguishable trajectories will become possible and this will lead to a progressive reduction of the averaged spin polarization of the electron ensemble. Since it is rather easy to design a structure with $|k_z| > k_F$, the splitting for a typical electron will be larger and the rate of spin relaxation will be enhanced. Similar behavior can be naively expected with the further reduction of the dimensionality to the 1D case. Let the axis $x$ be along wire in what follows. Here again the main terms will contain spacial-confinement multipliers $k_y^2$, $k_z^2$. The principal difference with the 2D case is that now all rotations are limited to a single axis direction and they commute with each other. Apart from the systematic rotation, spin polarization does not disappear with time. All particles, independently of the number and the sequence of the scattering events, that reach the same final point B will have the same spin orientation. This statement is relaxed if one allows intersubband scattering in the one-dimensional system. This type of scattering becomes progressively more important for wider and wider quantum wires with more and more subbands involved. Thus, we will recover, as one can predict, the 2D or 3D case in the limit of very thick quantum wires.

Actually, there are two types of terms that appear in the effective-mass Hamiltonian for the 2D electron gas: the bulk-asymmetry induced (Bychkov–Rashba) and structure-asymmetry-induced (Bychkov–Rashba). They are of the same functional form

$$H = \eta \sigma \cdot [k \times \hat{z}] \equiv \eta (\sigma_x k_y - \sigma_y k_x),$$

(2)

where $\hat{z}$ is a unit vector in the $z$ direction. We are not interested here in the origin of the spin splitting term in our model problem and simply assume its presence with a wave-vector dependence given by Eq. (2). (see Refs. 20,21 for the band-structure calculation of the linear-in-$k$ spin splitting in heterostructures simultaneously treating bulk- and structure-induced asymmetry.)

This form of the spin Hamiltonian is equivalent to the precession of the spin in the effective magnetic field

$$H = \frac{\hbar}{2} \sigma \cdot \Omega_{\text{eff}}, \text{ where } \Omega_{\text{eff}} \equiv \eta_{\text{DP}} \mathbf{v} \times \hat{z}. \quad (3)$$

Here the particle velocity $\mathbf{v} = \hbar \mathbf{k}/m^*$, and an obvious substitution $\eta \to \eta_{\text{DP}}$ is done for convenience. $\eta_{\text{DP}}$ is expressed in inversed length units. For a particle moving ballistically the distance $1/\eta_{\text{DP}}$ spin will rotate to the angle $\phi = 1$. We remind that the quantum-mechanical description of the evolution of the spin $1/2$ is equivalent to the consideration of the classic momentum $S$ with the equation of motion

$$\frac{dS}{dt} = \Omega_{\text{eff}} \times S.$$

The reciprocal effect of electron spin on the orbital motion through spin-orbit coupling can often be ignored due to the large electron kinetic energy in comparison to the typical spin splittings and strong change of the momentum in scattering events.
B. Possibilities to influence spin relaxation time

In addition to understanding of the reasons governing spin depolarization of carriers, we wish to consider and assess the possibilities to actively influence these destructive processes in order to improve parameters and gain new functionality of the future spintronic devices. Following are the potential approaches for manipulating spin relaxation times:

i) A rather simple observation follows directly from the essence of the regime of motional narrowing (small elementary spin rotations during ballistic electron flights). Since \( \tau_S^{-1} = \tau_p/(\Omega L)^2 \), reduction of momentum relaxation time, \( \tau_p \), leads to the suppression of spin relaxation. On the other hand, this will lead to increasing of broadenings as well as decoherence, and can worsen device parameters.

ii) Since the bulk-asymmetry- and structure-asymmetry-induced spin splittings are additive with the same \( k \)-dependence, it is possible to tune combined spin splittings in the conduction band to a desired value through manipulation of the external electric field.

iii) Additional spin splitting, which is independent of the electron wave vector will fix the precession axis. An evident possibility here is the Zeeman effect. The time of spin relaxation scales in the presence of the external magnetic field, \( B \), as

\[
\frac{1}{\tau_S(B)} = \frac{1}{\tau_S(0)} \left( 1 + \frac{\Omega L}{\langle r \rangle} \right)^2
\]

where \( h\Omega_L = g\mu_B B \) is a Zeeman splitting of electron spin sublevels. This equation suggests that for \( \Omega L \tau_p = 1 \) spin relaxation time will double.

iv) There is a possibility of controlling the spin relaxation of the conduction electrons by doping. The first realization was reported in \( \delta \)-doped heterostructures and a several-orders-of-magnitude longer spin memory has recently been observed in \( n \)-doped structures. \( \Omega \)

v) When the channel width, \( L \), is comparable to the magnitude of the electron mean free path, \( L_p \), from the classical point of view, the sequential alteration of one of the wave vector components should effectively reduce spin relaxation (reflective boundaries). Scattering on the boundaries (diffusive boundaries) will decrease \( \tau_p \) as well and can potentially influence spin relaxation. Quantum mechanically, the channel narrowing leads to the quantization of the electron transverse motion in the strip and absence of spin relaxation without intersubband scattering.

The first four possibilities are considered to some extent in the scientific literature or are just evident consequences of the relaxation mechanisms. The fifth deserves a more thorough analysis. To check the effect of the patterning of the 2D electron gas into a strip of some large width \( L \), we developed a simple Monte Carlo simulator encompassing random scattering of the particles in the channel, reflection from the boundaries and spin rotation during free flight due to spin splitting of the conduction band. Now we describe in more detail our working model.

III. MODEL

As a model system we consider a strip of the 2D electron gas. The third dimension (axis \( z \)) is quantizing and it is absolutely irrelevant to our consideration of the particle movement in the real space, since the intersubband gap is larger than all other energy scales involved.

i) We will assume that all particles have the same velocity \( |v| \).

ii) Scattering is considered to be elastic and isotropic in order to retain model simplicity; the former assumption preserves velocity modulus, the latter one eliminates any correlations between directions of the particle velocity before and after the scattering event.

iii) We neglect electron-electron interactions and consider all particles to be independent.

iv) An assumption that all scatterers are completely uncorrelated leads to an exponential distribution of times between any two consecutive scattering events, their average is called the momentum relaxation time, \( \tau_p \). This time corresponds to the mean-free path \( L_p = |v|\tau_p \).

v) The problem spin Hamiltonian is given in the form of Eq. (2) and influences only spin coordinate. We ignore the reciprocal effect of the spin on the motion in the real space.

vi) The width of the channel is large, several or even tens of mean-free paths so that it is permissible to consider classically the electron real-space movement.

vii) At first we consider only reflecting boundary conditions at the borders of the 2D strip. Reflecting channel boundaries preserve longitudinal component of the particle velocity and change sign of the normal component in collisions. Later, we will compare our results with diffusive boundaries.

A. Types of experiments

For simplicity, in all of our experiments particles will be injected into the system at some particular point \( A \) (input terminal) at time \( t = 0 \) with spin \( S \). As the particle experiences multiple scattering events a diffusive pattern of motion is formed with, for an isotropic system, a Gaussian distribution

\[
\Gamma(r) \sim \frac{1}{\langle r \rangle} \exp \left( -\frac{r^2}{\langle r \rangle^2} \right)
\]

that broadens as time increases: \( \langle r \rangle \sim L_p \sqrt{t/\tau_p} \).

Evidently, there are multiple possibilities for experimental setups. The definition of spin relaxation, obtained in these experiments will vary correspondingly. Now, we consider several important possibilities:

i) The most informative type of experiment would be
to obtain the average spin \( \langle S \rangle \) as a function of time \( t \) at each point B (output terminal). Results of all other experimental configurations can be derived by partial integration of this correlation function.

ii) At time \( t \) average \( \langle S \rangle \) is calculated for the whole ensemble independent of the real-space position of electrons. Optical experiments are considered likely to deliver information of this type, because of the limited possibilities to focus optical system and fundamental restrictions.

iii) Particles, reaching output terminal are removed from the system immediately, \( \langle S \rangle \) is measured as a function, i.e., of the interterminal distance. Individual particles can spend a substantial time in the system, depending on their trajectories. This type of experiment is the most probable variant in electric experiments where points A and B can be identified as real device gates. Made from ferromagnetic materials, gates can inject polarized electrons and sense the polarization of the drain flux, delivering information about the average spin of carriers.

Let us now show, that our result for spin relaxation does indeed depend on the definition of the experiment. As a simple example we consider a pure 1D case. From the point of view of the first and third experiments there is no spin relaxation. There exists a systematic rotation of the spin proportional to the distance from the interaction point A. Independent of the number of scattering events and individual trajectories, all particles reaching point B will have exactly the same spin orientation, but it would be different for different choices of point B. Thus, the transverse component of the spin \( S_y(x) = S^0_y \cos(\eta_{DP}x) \).

For the second realization we readily conclude that the average spin

\[
\langle S_y \rangle = \int dx \ S_y(x) \Gamma(x) \sim S^0_y \exp \left( -\frac{1}{4} \eta_{DP}^2 (x)^2 \right) \\
\sim S^0_y \exp \left( -\frac{1}{4} \eta_{DP}^2 L_p^2 t^2 \tau_p \right),
\]

that is an exponential decay of spin polarization in the spatially more and more broadening electron distribution.

Note that in the 2D case the same phenomenon of the systematic rotation of the electron spin takes place in addition to the (2D–3D specific) DP spin relaxation. This systematic rotation on the angle \( \phi = \eta_{DP} r \times \hat{z} \) for the particle real space transfer \( r \) is again independent on the details of individual trajectories.

IV. RESULTS OF SIMULATION

Fig. 4 shows the time dependence of the average spin polarization \( \Sigma \langle S_y \rangle \) in the channel. It is found that apart from the systematic rotation this dependence is essentially the same for all points inside channel that have a substantial electron occupation, so it is possible to calculate reliably a value of spin polarization. The calculation is performed for the D’yakonov-Perel’ parameter \( \eta_{DP} L_p = 0.05 \) and different channel widths. For this particular graph the trajectories of \( N = 5 \times 10^3 \) electrons are traced that gives a standard deviation in the definition of the spin polarization on the order of \( \sqrt{N} \sim 10^{-2} \) (throughout our investigation \( N = 10^3 - 10^5 \)).

A strong dependence of the polarization decay on the channel width is obtained. The decay is found to be approximately exponential, apart from the small initial interval. This is the region where the diffusive regime of electron motion and spin rotation is not yet established \( (\sqrt{t/\tau_p} \sim 1) \). It overlaps or is followed by the region where the typical trajectory of a randomly scattered particle does not reach channel boundaries yet. Here the relaxation should be essentially the same as in the unpatterned 2D gas.

This generally exponential temporal behavior of spin polarization justifies an introduction of the spin relaxation time, the dependence of this decay time on the channel width is presented in Fig. 4b. In case of sufficiently wide channels so defined relaxation time approaches the 2D limit of \( \tau_{S}^{2D} \), for narrow channels \( \tau_{S} \) scales as \( L^{-2} \). In Fig. 4c we summarize results of our simulation of channels with a fixed width and different values of D’yakonov-Perel’ term \( \eta_{DP} \). As the constant \( \eta_{DP} \) becomes larger and larger, the system behavior gradually goes out of the regime of motional narrowing since elementary rotations on the elementary free flights are not small anymore. This leads to a quick spin relaxation as reflected by the reduction and saturation of \( \tau_{S} \). On the other side of this dependence is a steep increase in the relaxation time as \( \eta_{DP} \) decreases. We have found an intermediate region, where \( \tau_{S}^{-1} \) scales as a second power of the DP constant, that is followed by a forth-power dependence for very small values of \( \eta_{DP} \).

Thus, in the well-established regime of motional narrowing and in the limit of sufficiently narrow channels, we propose an asymptotic formula \( \tau_{S} \sim \eta_{DP}^4 L^{-2} \).

To identify an effect of particular choice of the boundary conditions we have simulated a channel with diffusive boundaries. The particle that reaches the channel border is scattered back into the channel with equal probabilities for all directions of the new particle velocity. The results of the calculation repeat closely the case of the reflective boundaries up to the very narrow channel with widths of only several mean free paths. No systematic deviation of the spin relaxation time is observed for wider channels.

V. REGIMES OF THE DP RELAXATION IN A CHANNEL

Thus, we distinguish the following regimes of spin relaxation in channels of finite width as we vary both independent parameters \( \eta_{DP} \) and \( L \) of the problem in consideration:
FIG. 1. Spin relaxation in a channel. (a) Time dependence of the spin polarization, calculated for different channel widths. DP constant $\eta_{DP}$ is fixed to be 0.05. The trajectories of $5 \times 10^3$ particles are traced that defines a standard deviation $\sim 10^{-2}$ for the calculated averages. Close-to-exponential decays of the polarization permit to define spin relaxation time, $\tau_S$; (b) $\tau_S$ as a function of the channel width $L$; (c) Spin relaxation time in dependence on the DP spin splitting constant $\eta_{DP}$ at fixed channel width $L = 40L_p$.

FIG. 2. Different regimes of the spin relaxation on the plane $(\eta_{DP}, L)$ of model parameters: $\eta_{DP}L_p \gtrsim 1$ — elementary rotations during free flights are not small, $\tau_S \sim \tau_p$; $\eta_{DP}L_p < 1$, $\eta_{DP}L \gtrsim 1$ — 2D spin relaxation, $\tau_S^{2D} \sim \tau_p(\eta_{DP}L_p)^{-2}$; $\eta_{DP}L < 1$ — supression of spin relaxation, quasi-1D regime, $\tau_S \sim \tau_S^{2D}(\eta_{DP}L)^{-2} \sim \tau_p \eta_{DP}^{-4}L^{-2}$; $L \lesssim L_p$ — $L$ substitutes $L_p$, quantum mechanical quantization in the channel.

First of all, for very large spin splitting ($\eta_{DP}L_p \gtrsim 1$) we violate a general condition for the motional narrowing regime for the DP spin relaxation. Each elementary rotation is not small and the information about the spin polarization is lost already after the first random scattering event (see as well Fig. 2 as a guide). For this regime $\tau_S \sim \tau_p$ and is the shortest of all regimes.

When $\eta_{DP}$ is small ($\eta_{DP}L_p < 1$), we are back in the regime of motional narrowing and a well known equation $\tau_S^{2D} \sim \tau_p(\eta_{DP}L_p)^{-2}$ defines the time of spin relaxation in the 2D system. Now we narrow the strip of the 2D electron gas. The behavior is unchanged until $\eta_{DP}L \sim 1$. For smaller channel widths ($\eta_{DP}L < 1$) DP spin relaxation is suppresed very effectively, with $\tau_S \sim \tau_S^{2D}(\eta_{DP}L)^{-2}$. For $L < L_p$ the channel width $L$ acts as a new mean-free path in the system, substituting $L_p$ in equations. Actually, this region does not satisfy our assumption about classical motion of particles in real space; the transverse...
motion is quantized and this system should be considered as a quantum wire with multiple subbands. The crossover points define broad regions of mixed behavior that becomes more definitive as we move out of them.

We conducted analysis presented here for spin relaxation in the most convenient from our point of view units. Now, we will check that the chosen range of parameters is well within the reasonable limits present in the contemporary heterostructures. For the asymmetric GaAs/AlGaAs quantum well with the electron concentration of $10^{12} \text{ cm}^{-2}$ in the conduction band is on the order of $0.2 - 0.3 \text{ meV}$ at the Fermi energy (see recent paper by Wissinger et. al[22] and references therein). This 2D electron concentration corresponds to the $k_F \approx 3 \times 10^5 \text{ cm}^{-1}$. Samples with $L_p \gg 1/q = 3 \times 10^{-6} \text{ cm}$ are readily available in laboratories. Furthermore, this splitting corresponds to $\eta_{DP} = 6 - 9 \times 10^4 \text{ cm}^{-1}$. This set of parameters is on the border of the regimes of the motional narrowing and large elementary rotations. Reducing the electron concentration in the 2D electron gas and, thus, structure asymmetry due to the internal electric field will push parameters into well established regime of motional narrowing, suitable for the experimental observation of the described phenomena. Datta and Das[1] give an estimate of $\pi/\eta_{DP} = 7 \times 10^{-5} \text{ cm}$ for some particular InGaAs/InAlAs heterostructures.

**VI. SUMMARY**

In conclusion, we have investigated spin-dependent transport in semiconductor narrow 2D channels and explored the possibility of suppressing spin relaxation. Our approach is based on a Monte Carlo transport model and incorporates information on conduction band electron spins and spin rotation mechanisms. Specifically, an ensemble of electrons experiencing multiple scattering events is simulated numerically to analyze the decay of electron spin polarization in channels of finite width due to the D'yakonov-Perel' mechanism. We have been able to identify different regimes of the spin relaxation in the 2D channels of finite width and obtain dependencies of the spin relaxation time on the width $L$ and DP parameter $\eta_{DP}$. The most attractive for the future spintronic applications is a regime of the suppressed spin relaxation with the relaxation time, $\tau_S$, scaling as $L^{-2}$.

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