The current status of axion physics is presented. There still exists the axion window $10^9 \text{ GeV} \leq F_a \leq 10^{12} \text{ GeV}$. The recent CAST solar axion search experiment on the axion-photon-photon coupling strength has to be improved by a factor of 100 to reach down to the region of superstring axions. The calculable $\bar{\theta}$ and $m_u = 0$ cases for strong CP solutions, and axino cosmology in SUSY extension of axion are also commented.

Keywords: Axion, Axino, Strong CP Problem, Superstring Axion.

1. Introduction

Modern cosmology needs dark matter and dark energy in the universe: $\Omega_{\text{CDM}} \simeq 0.23, \Omega_\Lambda \simeq 0.73$. There are several particle physics candidates for CDM: LSP, axion, axino, gravitino, LKP and other hypothetical heavy particles with some kind of $Z_2$ symmetries.

The old electroweak scale axion is the pseudo-Goldstone boson arising from breaking the global Peccei-Quinn (PQ) symmetry. The very light axion is the invention from the need to solve the strong CP problem through PQ symmetry with electroweak singlet field(s). Superstring axions may be in this very light axion category. The existence of instanton solutions in nonabelian gauge theories needs $\theta$ vacuum, introducing a CP violating interaction. In the $\theta$ vacuum, the physically meaningful interaction is parametrized by

$$\tilde{\theta} = \frac{\theta}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} = \{F \tilde{F}\}; \quad \tilde{\theta} = \theta_{\text{QCD}} + \theta_{\text{weak}}$$

where $\theta_{\text{QCD}}$ is the value determined from high energy scale and $\theta_{\text{weak}} = \ldots$
Arg. Det. $M_q$ is the one contributed when the electroweak CP violation is introduced. Here $\bar{\theta}$ is the final value taking into account the electroweak CP violation. For QCD to become a correct theory, this CP violation by $\bar{\theta}$ must be sufficiently suppressed. A nonvanishing value $\bar{\theta}$ contributes to the neutron electric dipole moment $d_n$. From the experimental limit, we obtain the bound

$$|d_n| < 0.63 \times 10^{25} \text{ ecm} \rightarrow |ar{\theta}| < 10^{-9}.$$ 

Why is this so small? It is the strong CP problem. There are three types for the solution: (1) Calculable $\bar{\theta}$, (2) Massless up quark, and (3) Axion.

One may argue that there were no strong CP problem in the beginning. In particular in 5D extension, since the instanton solution is the one in 4D. I think this does not work or at best belongs to the calculable $\bar{\theta}$ type, because in the 4D effective theory one can always consider a 4D theory after integrating out the 5th coordinate. Let us briefly comment on two solutions first.

- **The Nelson-Barr type:** CP violation is introduced spontaneously. So, original Yukawa couplings are real. Spontaneous CP violation is introduced at high energy by introducing vectorlike heavy quarks so that they mix with light quarks. If the heavy vectorlike quarks are not introduced, the CP violation of light quarks originated by the high energy scale CP violation will be tiny due to the decoupling theorem. Not to be affected by the decoupling theorem and to guarantee a tree level Arg. Det. $M_q = 0$, specific forms for Yukawa couplings are assumed: SU(2)xU(1) breaking real VEVs appear only between $F - F$ Yukawas, and CP violating phases in the VEVs appear only in $F - R$ Yukawas, where $F$ are the SM fermions and $R$ are the heavy fermions. If heavy vectorlike fermions are integrated out, the effective Yukawa coupling structure of the low energy sector is of the Kobayashi-Maskawa form.

- **Massless up quark:** Suppose that we chiral-transform a quark,

$$q \rightarrow e^{i\gamma^5 \alpha} q.$$

It is equivalent to changing $\theta \rightarrow \theta - 2\alpha$. Thus, if it is allowed to have such a symmetry then strong CP problem is not present. The massless quark case belongs here. This solution was known from the very beginning of the strong CP problem but was not taken seriously because the up quark seemed to be massive. The problem is, “Is $m_u = 0$ allowed phenomenologically?” The famous
up/down quark mass ratio from chiral perturbation theory (cPT) calculation is $m_u/m_d = \frac{5}{9}$. But physics below 100 GeV is more involved. There is the determinental interaction of ’t Hooft, pictorially shown as Below the electroweak scale quarks obtain mass. Suppose, the up quark is massless. Then, there is no strong CP problem. But chiral perturbation theory can be done with instanton generated up quark mass from the above ’t Hooft interaction, $m_u = m_d m_s/\Lambda$, where $\Lambda$ is at the QCD scale. So it is the problem whether the instanton calculus really gives the desired magnitude, in which case $\theta$ is still unphysical. In the community, still there is a disagreement on this issue: Kaplan and Manohar (KM), and Choi belongs to the positive group, and Leutwyler (L) belongs to the negative group. CP even observables do not see $m_u$. From the figure, for example, we have $m_u, eff = m_d m_s/\Lambda$, $m_d, eff = m_d + m_u m_s/\Lambda \simeq m_u m_s/\Lambda$, $m_s, eff \simeq m_s$. But CP odd observables see $m_u$. Is $Z = m_u/m_d$ small? KM shows from the 2nd order cPT $Z \simeq 0.2$, and they could not rule out the $m_u = 0$ case. Explicitly, cPT has the $L_7$ parameter in the term $L_7(M^\dagger U - MU^\dagger)$, where $M = 3 \times 3$ mass matrix and $U = 3 \times 3$ matrix for meson fields. KM shows $m_u = 0 : cPT \Rightarrow \begin{cases} L_7 \sim +1.5 \times 10^{-4} \text{ or} \\ (2L_8 - L_5) \simeq (-1.2 \sim -2.5) \times 10^{-3} \end{cases}$ (1) where $L_8, L_5$ are another parameters in the cPT. On the other hand L attempted to compute $L_7$, using the QCD sum rule for the SU(3) singlet pseudoscalar $\eta'$ dominance (similarly to the vector meson dominance),

$L_7 \simeq L_{7,\eta'} \simeq (-2^{-4}) \times 10^{-4}$ (Gasser – Leutwyler coefficients) (2) with a notable sign difference from (1). If (2) were true, the case $m_u = 0$ is ruled out. But Choi argues that if $\eta'$ gets mass from instanton calculus, which is the modern wisdom on the U(1) problem
resolution, he can change the sign of (2) to
\[ L_7 \simeq (3 \sim 8) \times 10^{-4}. \] (3)

So, we can have the possibility of \( m_u = 0 \).

In recent years, lattice calculation has been performed toward this issue.\(^{12}\) In \( 16^3 \times 32 \) lattice calculation, they obtain: \( 2L_8 - L_5 \sim 10^{-4} \) and \( m_u/m_d = 0.484 \pm 0.027 \). If true, \( m_u = 0 \) is ruled out.

I consider that the problem on \( m_u = 0 \) is not completely settled yet, even though \( m_u \neq 0 \) seems to be the majority opinion of the community.

- These show that the axion solution is the most compelling solution which is discussed in the subsequent section.

2. Axion

The axion potential is of the form where the vacuum is shown as a bullet.

\[ \begin{array}{c}
\text{The vacuum stays there for a long time, and oscillates when the Hubble time}
\end{array} \]

\( (1/H) \) is larger than the oscillation period\((1/m_a): H < m_a \). This occurs when the temperature is about 1 GeV. Axion is directly related to \( \theta \). Its birth was from the PQ symmetry whose spontaneous breaking introduced a dynamical degree, a pseudo-Goldstone boson called axion. But “pseudo-Goldstone” nature is specific in axion in that axion is a pseudoscalar \( a \) without any potential except that arising from,

\[ \frac{1}{32\pi^2} \frac{a}{F_a} F \tilde{F} = \frac{a}{F_a} \{ F \tilde{F} \}. \] (4)

This kind of nonrenormalizable term can arise in several ways. The first important scale is \( F_a \), defining the strength of nonrenormalizable interaction. It can arise from higher dimensional fundamental interactions with the Planck scale \( F_a \),\(^{13,14}\) from composite models with the composite scale \( F_a \),\(^{15}\) from spontaneously broken renormalizable field theories. In the last case, the global symmetry must have the gluon anomaly and is called the PQ symmetry.\(^2\) If this PQ symmetry is spontaneously broken, there arises a pseudo-Goldstone boson coupling to the anomaly with the global symmetry breaking scale \( F_a \).

In QFT, a very light axion is embedded in the phase of a complex \( \text{SU}(2)_L \) singlet scalar field \( s \), (it may contain very tiny components \((\leq 10^{-7}) \) from
SU(2)_L doublet phases),
\[ s = \frac{V + \rho e^{ia/F_a}}{\sqrt{2}}, \quad a \equiv a + 2\pi N_{DW} F_a, \quad V = N_{DW} F_a. \quad (5) \]

So, \( F_a \) is in general smaller than \( \langle s \rangle \). The potential arising from the anomaly term after integrating out the gluon field is the axion potential. Three properties of the axion potential are known:

(i) It is periodic with \( 2\pi F_a \) periodicity,

(ii) The minima are at \( \langle a \rangle = 0, 2\pi F_a, 4\pi F_a, \cdots, 2,16 \).

(iii) A set of minima is identical, leaving to a few \( (N_{DW}) \) distinct vacua.\(^{17}\)

\[ \sim m\Lambda_{QCD}^3 \]

\[ \tilde{\theta} = 0 \]

Fig. 1. Vacua are at \( \tilde{\theta} = 2\pi \). The height of the axion potential is given by the instanton interaction and boson mixing.

The height of the axion potential is the scale \( \Lambda \) of the nonabelian gauge interaction and the boson mixing as shown in Fig. 1. We simply take this value as the QCD scale, but in fact it is \( m\Lambda_{QCD}^3 \) where \( m \) is the light quark mass.\(^{18}\) The dominant one \( \Lambda_{QCD}^3 \) corresponds to the \( \eta' \) potential. If there are quarks, the height is adjusted since as we have seen before a massless quark makes it flat. The \( u \) and \( d \) quark phenomenology gives

\[ V[a] = \frac{Z}{(1 + Z)^2} f_\pi^2 m_\pi^2 \left( 1 - \cos \frac{a}{F_a} \right). \quad (6) \]

The essence of the axion solution is that \( \langle a \rangle \) seeks \( \tilde{\theta} = 0 \) in the evolving universe whatever happened before. It is a cosmological solution\(^{20}\) as shown in Fig. 1. The weak CP violation makes the minimum of the potential shifted a little bit at \( \tilde{\theta} = O(10^{-17}) \). The axion mass is given by \( m_a \simeq (10^7 \text{GeV}/F_a) \) 0.6 eV.

There are several laboratory experiments, restricting the axion decay constants: (i) meson decays, \( J/\Psi \rightarrow a + \gamma, \Upsilon \rightarrow a\gamma, K^+ \rightarrow \pi^+ + a \), (ii) beam dump experiments, \( p(\text{or } e^-)N \rightarrow aX, a \rightarrow \gamma\gamma \) and \( e^+e^- \), (iii) and
nuclear deexcitation, \( N^* \rightarrow Na, a \rightarrow \gamma\gamma \) and \( e^+e^- \). Thus, we obtain the inner space bound \( F_a \geq 10^4 \) GeV from the laboratory experiments. So, from the beginning, it was known that the PQWW axion, arising from the electroweak scale, is ruled out.\(^{21}\) Thus, \( F_a \) has to be very large, having led to the so-called invisible axion. But, there is a possibility of detecting it,\(^{22}\) and hence it should be called a very light axion or sub-meV axion.

3. Axion window to outer space

But the stringent lower bounds on the axion decay constant comes from the outer space observations. Firstly, stellar evolutions, if axion existed, are affected by axion emissions and the successful standard energy loss mechanism due to weak interactions restricts the axion mass toward a smaller region, or the axion decay constant to a larger region. The stringent bound comes from the study of supernova evolution,\(^{23}\) especially from the SN1987A study to give \( F_a \geq 10^9 \) GeV.\(^{24}\) On the other hand, the very interesting upper bound on \( F_a \) is obtained from the axionic contribution to dark energy in universe.\(^{20}\)

3.1. Stars

The current supernova\(^{24}\) (globular cluster\(^{25}\)) limit on \( F_a \) is \( 10^9 \) GeV(\( 10^{10} \) GeV). It uses primarily the Primakoff process with the following coupling,\(^{3}\)

\[
\mathcal{L}_{a\gamma\gamma} = -c_{a\gamma\gamma} \frac{a}{F_a} \frac{e^2}{32\pi^2} F_{em} F_{em} \implies \mathbf{E} \cdot \mathbf{B} \text{ interaction} \quad (7)
\]

\[
c_{a\gamma\gamma} = \tilde{c}_{a\gamma\gamma} + 6 \sum_{i=\text{light}} \hat{\alpha}_i Q_{em,i}^2 \simeq \tilde{c}_{a\gamma\gamma} - 1.93, \quad Z = \frac{2}{9} \quad (8)
\]

\[
\tilde{c}_{a\gamma\gamma} = \text{determined from high energy physics} \quad (9)
\]

\[
\hat{\alpha}_u \simeq \frac{1}{1+Z}, \quad \hat{\alpha}_d \simeq \frac{Z}{1+Z} \quad (10)
\]

where the chiral symmetry breaking of \( u, d \) quarks are taken into account. The number 1.93 corresponds to \( Z = \frac{2}{9} \). Since the instanton contribution to light quark masses is present,\(^{9,10}\) we may take a band around 1.93.

In the hot plasma in stars, once produced, they most probably escape the core of the star and take out energy. This contributes to the energy loss mechanism of star and should not dominate the luminocity: (i) The Primakoff process: \( \gamma \rightarrow a \) (present in any model): \( g_{a\gamma\gamma} < 0.6 \times 10^{-10} \text{ GeV}^{-1} \) or \( F_a > 10^7 \) GeV, and \( 0.4 \text{ eV} < m_a < 200 \) keV ruled out because too heavy to produce,
(ii) Compton-like scattering: $\gamma e \rightarrow ae$ (DFSZ axion has $ae$ coupling).

$$g_{ae} < 2.5 \times 10^{-13}, \quad 0.01 \text{ eV} < m_a < 200 \text{ keV}, \quad \text{and}$$

(iii) SN1987A, $NN \rightarrow NNa \times 10^{-10} < g_{aN} < 3 \times 10^{-7} \Rightarrow F_a > 0.6 \times 10^9 \text{ GeV}.$

Stellar evolution uses the energy loss mechanism, with the aforementioned lower bound on $F_a$. But laboratory experiments can offer a more effective bound on $F_a$ than just the energy loss mechanism, as done by the CAST (CERN axion solar telescope) experiment of Fig. 2.\textsuperscript{26}

3.2. Universe

In the standard big bang cosmology (SBB), there is a severe domain wall problem.\textsuperscript{27} The SBB allows only the domain wall number $N_{DW} = 1.\textsuperscript{28}$ But the most interesting inflationary cosmology solves this domain wall problem at one stroke if the reheating temperature after inflation is lower than $F_a$. The inflationary cosmology seems to get support from the COBE and WMAP observations of density perturbations in the early universe, and NDW problem is not an issue since in SUSY models the reheating temperature is required to be smaller than $10^9$ GeV.$\textsuperscript{29}$ In axion cosmology,
the following items are important:

- The axion decay constant $F_a$,
- Axion couplings to $\gamma, e, p, n$,
- The domain wall number $N_{DW}$.

If a singlet scalar VEV $\langle s \rangle$ breaks the PQ symmetry, then $F_a = \langle s \rangle / N_{DW}$ defines the domain wall number. Reheating temperature after inflation is required to be below $F_a$ if $N_{DW} > 1$.

Axions are created at $T \simeq F_a$, but the universe does not change $\langle a \rangle$ until $H \simeq m_a(T = 1\text{GeV})$. Then, the classical field $\langle a \rangle$ starts to oscillate. From the harmonic oscillator type energy density $m_a^2 F_a^2$, we have $m_a \times$ number density $\Rightarrow$ CDM-like energy:

$$\rho_a(T_\gamma) = m_a(T_\gamma) n_a(T_\gamma) \simeq 2.5 \frac{F_a}{M_P} \frac{F_a m_a}{T_1} T_\gamma^3 \left( \frac{A(T_1)}{F_a} \right)^2 \tag{11}$$

where the oscillation start-up temperature $T_1$ is the strong interaction scale 1 GeV and $T_\gamma$ is the present temperature. If $F_a$ is large ($> 10^{12}$ GeV), then the axion energy density dominates the energy density of the universe. Since the energy density is proportional to the number density, it behaves like a CDM.

### 3.3. Axion window and search for cosmic axions

The above astro- and cosmological-bounds on $F_a$ are summarized as

$$10^9 \text{GeV} \leq F_a \leq 10^{12} \text{GeV} \tag{12}$$

If axions are the CDM component of the universe, then they can be detected. The feeble coupling can be compensated by a huge number of axions. The number density $\sim F_a^2$, and the cross section $\sim 1/F_a^2$, and there is a hope of detecting it. Sikivies cavity detector with dimension of tens of cm has been used to give coarse bounds on axion parameters in the axion mass of order $10^{-5}$ eV.

### 4. Axions from superstring

Superstring tells us definite things about global symmetries. If axion is present, it is better to be realized in superstring. They are the bosonic degrees in $B_{MN}$ (MI-axion is $B_{\mu\nu}$ and MD-axion is $B_{ij}$) and furthermore additional massless bosons from compactification are candidates. Superstring does not allow global symmetries. But there is an important exception to this claim: the shift symmetry of $H_{\mu\nu\rho}$, which gives the MI-axion.
It is the only allowed global symmetry. $B_{ij}$ are generally heavy, but it is a model dependent statement.

The superstring axion decay constants are expected near the string scale which is too large: $F_a > 10^{16}$ GeV. A key question in superstring models is “How can one obtain a low value of $F_a$?” An idea is the following:

In some compactifications, anomalous U(1) results, where U(1) gauge boson eats the MI-axion to become heavy. Earlier, this direction, even before discovering anomalous U(1) gauge boson, was pointed out by Barr. It became a consistent theory after discovering the anomalous U(1). Then, a global symmetry survives down the string scale. $F_a$ may be put in the axion window. It was stressed in several references.

However, this idea does not work necessarily, as will be commented later.

Somehow MD-axion(s) may not develop a large superpotential terms. But the problem here is the magnitude of the decay constant. MD-axion decay constants were tried to be lowered by localizing them at fixed points. It uses the flux compactification idea and it is possible to have a small $F_a$ compared to the string scale as in the RS model. One needs the so-called throat as schematically shown in Fig. 3.

![Fig. 3. A schematic view of the $S_2 \times S_2 \times S_1$ throat. At the tip, one elongated $S_2$ can shrink to a point shown as a star. The un-shrunk $S_2$ has the cycle $C_2$ where a MD-axion resides as a harmonic 2-form.](image)

**Axion mixing** Even if we lowered some $F_a$, we must consider hidden sector also. In this case, axion mixing must be considered. There is an important theorem.
Cross theorem on potential heights and decay constants: Suppose two axions \( a_1 \) with \( F_1 \) and \( a_2 \) with \( F_2 \) \( (F_1 \ll F_2) \) couples to two nonabelian groups whose scales having a hierarchy, \( \Lambda_1 \ll \Lambda_2 \). The higher potential \( \Lambda_2 \) couples to both axions. Then, the diagonalization process chooses the larger potential \( \Lambda_2 \) corresponds to the smaller decay constant \( F_1 \), and the smaller potential \( \Lambda_1 \) corresponds to the larger decay constant \( F_2 \).

So, just obtaining a small decay constant is not enough. Hidden sector may steal the smaller decay constant. It is likely that the QCD axion chooses the larger decay constant. Recently, the mixing effect has been stressed by I.-W. Kim et. al.\(^{19,40}\) And most probably, our axion will couple to the \( \mu \) term:

\[
H_u H_d f(S_1, S_2, \ldots)
\]

After all, the topologically attractive \( B_{MN} \) may not be the axion we want. Let us go back to earlier field theoretic very light axion. In string models, its effect toward phenomenology was not calculated before. Now we have an explicit model for MSSM,\(^{41}\) and we can see here whether the idea of approximate global symmetry is realized. It must be that at sufficiently higher orders the PQ symmetry is broken. In the \( Z_{12-I} \) model, we calculate the axion-photon-photon coupling\(^{40}\) whose result is shown in Fig. 2. But the decay constant is at the GUT scale. In this kind of calculation, there are so many Yukawa couplings to consider. For example, we encountered \( O(10^4) \) terms for \( d=7 \) superpotential terms and it is not a trivial task to find an approximate PQ symmetry direction.

In addition, we point out that the MI-axion with anomalous \( U(1) \) always has a large decay constant since most of the fields are charged under this anomalous \( U(1) \). Phenomenologically successful axion must need an approximate PQ symmetry.

An approximate PQ global symmetry with discrete symmetry in SUGRA was pointed out long time ago: given by Lazarides and Shafi\(^{42}\) for a discrete \( Z_3 \times Z_3 \). But this field theoretic method does not guarantee that string models realize this idea. In this sense, an explicit demonstration of an approximate PQ symmetry is vital for a string matter axion.

5. SUSY extension and axino

The SUSY extension always introduces gravitino. Gravitinos produced thermally after inflation decay very late in cosmic time scale \( (>10^3 \text{ s}) \).
and can dissociate the light nuclei by its decay products. Not to have too many gravitinos, the reheating temperature must be bounded, \( T_r \leq 10^9 \text{GeV} \) (old value), \( T_r \leq 10^7 \text{GeV} \) (new but model dependent value). Therefore, in SUSY theories we must consider the relatively small reheating temperature, and the domain wall problem does not matter in axion cosmology.

The SUSY extension with the strong CP solution via axion introduces its superpartner axino. Its cosmological significance is in that it can serve as keV range warm dark matter or GeV range cold dark matter. Let us comment on its CDM possibility.

For axino to be CDM, it must be stable or practically stable. Without the \( R \)-parity conservation, this can not happen. Thus, we require the practical \( R \)-parity conservation for the possibility of axino CDM. In addition, for axino to be LSP it must be lighter than the lightest neutralino whose mass is expected to be around 100 GeV. Thus, the estimation of the axino mass is of prime importance. The conclusion is that there is no theoretical upper bound on the axino mass and axino mass can be easily in the GeV range. Since axion is almost massless, one expects that its superpartners are massless in the first approximation. Its scalar partner, saxion, obtains the mass of order the soft terms after SUSY breaking. Its cosmological effect is relatively late decaying nature, adding more photons after its decay.

Regarding mass, saxion is like the other SM SUSY scalars.

However, the axino mass is intimately related to the SUSY breaking scenario and symmetries of the superpotential. The PQ symmetry allows the following superpotential:

\[
W = f Z (S_1 S_2 - F_a^2), \quad Z, S_1, S_2 : \text{singlets}
\]

There also exist SUSY breaking soft terms. Thus, the following potential is obtained

\[
V = |f|^2 (|S_1|^2 + |S_2|^2) |Z|^2 + (A_1 f S_1 S_2 Z - A_2 f F_a^2 Z + \text{h.c.})
\]

which determines the VEV of \( Z \). Since \( S_1 \) and \( S_2 \) are of order \( F_a \), \( \langle Z \rangle \) is of order the \( A \) term. Thus, the fermion partners have the mass matrix of the form

\[
\begin{pmatrix}
0 & m_{\tilde{a}} & f F_a \\
m_{\tilde{a}} & 0 & f F_a \\
f F_a & f F_a & 0
\end{pmatrix}, \quad m_{\tilde{a}} = f \langle Z \rangle
\]

The lightest eigenvalue is the axino mass. The others are of order \( F_a \). As shown above, the axino mass is basically a free parameter, and expected to
be smaller than the naive SUSY breaking scale due to the small coupling. Most probably, it is lighter than neutralino $\chi$. But its mass can be much smaller than the SUSY scale as shown from a superpotential of the form,\textsuperscript{48}

$$W' = fZ(S_1S_2 - X^2) + \frac{1}{3}(X - M)^3$$

where $X$ carries the vanishing PQ charge. This potential is much more complicated to analyze. We show $m_{\tilde{a}} = O(A - 2B + C) + O(m_{3/2}/F_{\tilde{a}})$.

For the standard pattern of soft terms, we have $B = A - m_{3/2}$ and $C = A - 2m_{3/2}$.\textsuperscript{49} Thus, the axino mass is of order keV. Even the tree level axino mass needs the knowledge on the full superpotential, we treat the axino as the LSP which is the most probable choice. Its mass is left as a free parameter. KeV axinos can be warm dark matter which is thermal relic. GeV axinos can be CDM. In this case, the reheating temperature must be low.\textsuperscript{45}

If gravitino is the next lightest LSP (NLSP), $m_{\tilde{a}} < m_{3/2} < m_{\chi}$, the gravitino problem can be resolved,\textsuperscript{50} since the thermally produced gravitinos would decay to axino and axion which do not affect BBN produced light elements.

Most probably, $\chi$ would be the NLSP, and the thermal production mechanism restricts the reheating temperature after inflation as summarized in Fig. 4. At high reheating temperature, thermal production contributes dominantly in the axino production. Even though the reheating temperature is below critical energy density line, there still exists the CDM possibility by the non-thermal production (NTP) axinos. Covi \textit{et. al}.\textsuperscript{45} shows

$$NTP: \quad \Omega_{\tilde{a}}h^2 = \frac{m_{\tilde{a}}}{m_{\chi}}\Omega_{\chi}h^2 \quad \text{for} \quad m_{\tilde{a}} < m_{\chi} < m_{3/2}$$

In Fig. 4, NTP axinos can be CDM for relatively low reheating temperature $< 10$ TeV, in the region

$$10 \text{ MeV} < m_{\tilde{a}} < m_{\chi}, \quad \text{NTP axino as CDM possibility.}$$

The shaded region corresponds to the MSSM models with $\Omega_{\chi}h^2 < 10^4$, but a small axino mass renders the possibility of axino closing the universe or just 30% of the energy density. If all SUSY mass parameters are below 1 TeV, then $\Omega_{\chi}h^2 < 100$ and sufficient axino energy density requires

$$m_{\tilde{a}} > 1 \text{ GeV}, \quad \begin{cases} \text{Low reheating is good in view of} \\ \text{the recent gravitino problem.} \\ \text{But not good with leptogenesis.} \end{cases}$$
Fig. 4. The solid line gives the upper bound from thermal production on the reheating temperature as a function of the axino mass. The dark region is the region where non-thermal production can give cosmologically interesting results ($\Omega_{\tilde{a}} h^2 \approx 1$).

6. Conclusion

I reviewed strong CP and axion. In particular,

- Solutions of the strong CP problem: Nelson-Barr, $m_u = 0$, axion. Axion $a$ is the most attractive and plausible solution.
- Axions can contribute to CDM. Maybe solar axions are easier to detect. Most exciting is, it confirms instanton physics by observation.
- Tried to present a superstring matter axion coupling for the first time. A QCD axion from superstring may be a window to string.
- With SUSY extension, $O(\text{GeV})$ axino can be CDM. It is difficult to detect this axino from the DM search, but possible to detect at LHC as missing energy.

7. Acknowledgments

This work is supported in part by the KRF ABRL Grant No. R14-2003-012-01001-0. J.E.K. is also supported in part by the KRF grants, No. R02-
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