Students thinking process type of non-routine problems based on mathematical ability in senior high school

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Abstract. Mathematics will not escape the problem solving from a simple level to a high level. Problems are given by the teacher affect the thinking process of students. Thinking processes occur when students are interested in collecting information based on initial thinking to identify problems, find solutions and succeed in solving them. There are two types of thinking processes that are predictive and functional types. The purpose of this research is to describe the thinking process of students in high school in problem-solving with the non-routine problem. This research uses the descriptive qualitative method. The research subjects consisted of grade XI students of Senior High School 3 which includes excellent schools in Boyolali, selected by purposive sampling. The results of this study showed that six students with details of two high-ability students, two medium-ability students, and two low-ability students. Moreover, high-ability students and medium-ability students used predictive thinking process type while low-ability students used the functional thinking process type. It can be concluded that students with predictive thinking process type of mathematical ability are better than the type of functional thinking process.

1. Introduction

Mathematics is the most important subject that is so instrumental in advancing the civilization of a nation and also to assist human beings in understanding and mastering other problems [1]. Mathematics as a tool/means that can be used to help humans in overcoming the problems that exist in life [2]. Mathematics is essential and becomes a primary subject that is useful in solving problems that exist in life. Problem is a situation in which the individual wants to do something but does not know what to do yet, and problem-solving is “to learn what to do in cases in which one does not know what to do” [3]. Moreover, problem-solving entails engaging in a task for which the solution process is not identified beforehand [4]. Mathematical problem solving has been the focus of school mathematics [5]. Mathematics problem solving is not a topic but a process underlie the whole mathematics programmes which contextually helped concepts and skills to be learned [6]. Large numbers of students have not acquired the basic skills they need in mathematics as a result, many students were reported to face difficulties in mathematics particularly in mathematics problem solving [7].
Solving student problems in mathematics is called mathematical ability. Mathematical abilities in Indonesia is still at the stage of receiving information and memorizing the problem set by the teacher. This happens because the problem given is still a matter of routine where students memorize step by step to solve problems. If students are given problems that are slightly different from the questions given by the student, they will find it difficult. They were reported to face difficulties in making the accurate perceptions and interpretations, memorizing and retrieving facts, giving concentrations and using their logic thinking [8].

The ability of students to solve math problems needs to be improved one of them by giving non-routine questions. Non-routine problems are this kind of problem differs from others in that they may not be solved through several operations [9]. Their solution requires a series of activities and abilities such as operating knowledge, organizing and classifying data, being able to see relations, finding the rules, and reaching generalities. Good problems as those "which occur in the environment where students are", "which forces students to develop strategies and practices them" and create environments for students to introduce new terms "[4]. It is a fact that non-routine problems are "good problems" and hold a significant place in teaching problem solving [10]. The provision of non-routine problems is one of the alternative teachers in improving students' ability to solve problems.

The development of students' abilities in solving problems is needed in solving mathematical problems. As an active and structural process in learning mathematics, students need to undertake control and agency over their learning and problem-solving activities [11]. Problem-solving in learning mathematics is important because in addition to developing students' ability to build new knowledge and reflect the problem-solving process [12]. The process of thinking on mathematical problem solving by students is seen in the problem-solving step [13]. One of the problems solving steps used is the steps of Bransford and Stein. The problem solving introduced is IDEAL problem solving, namely the problem-solving model that is able to improve thinking skills and improve skills in problem-solving processes [14]. According to Bransford and Stein, IDEAL problem solving is designed to help identify and understand different parts of problem-solving, each letter symbolizing an essential component in the problem-solving process. IDEAL stands for I-Identify problem, D-Define goal, E-Explore possible strategies, Anticipate outcomes and actions, L-Look back [14].

Problem-solving is an individual activity or effort for finding the right solution of a problem [15]. The problem-solving process is called the thinking process [16]. Problem-solving has a connection to the thinking process [12]. The process shows the sequence of events that must be passed. This means that problem solving is carried out by process sequence, such as an individual's problem in obtaining the right problem solving [12]. Every student has a different thinking process. Thinking is a process that consists of receiving information (from inside or outside the student), processing, storing and recalling that information from students' memories [17]. Thinking types can be divided into two, namely the type of predictive thinking and the type of functional thinking. The type of predictive thinking is a way of thinking with a tendency to see the relationship between two or more concepts in decision making, while the type of functional thinking is a way of thinking with more emphasis on seeing the link and how to implement decisions [17].

From research findings are students with high abilities which they use in solving non-routine problems are very high, the ability of students who they use in solving non-routine problems are above the mid-level and low ability that they use in solving non-routine problems are below the mid-level [13]. Studied with high-achievers and they reached the similar result that students infrequently executed the strategies [18]. On the other hand, performances of high-achieving students were higher than those of other achievement levels. Consequently, significant relationships were found between levels of strategies and achievement levels.

Based on the previous explanation, the researcher needs to conduct the research related to the thinking process type in a nonroutine problem. This research aims to describe the thinking process of students in high school in problem-solving with a non-routine problem.
2. Method
This research was qualitative research. According to Creswell in qualitative research, researchers identify a research problem based on trends in the field or need to explain why something happened [19]. The research was conducted in Senior High School 3 which includes excellent schools in Boyolali class XI in year 2017/2018. Purposive sampling did the selected subjects. According to Creswell the purpose of the selection of subjects with purposive sampling is that the researchers can choose individuals [19]. The selected subjects of six students with details of two students with high math skills, two students with moderate math skills and two students with low math skills.

Grouping students based on the results of the math ability test scores that have been given to all students of class XI natural science 4. In this study to determine the position of students in the ability level group using the deviation standard. This is done to simplify the grouping of each subject based on the tests that are possessed after obtaining results when testing a mathematical ability.

This instruments in this research are the non-routine problem in a circle. The non-routine problem given to the subjects.

The task of non-routine problem, Determine the limit of the value of h so that the \( R(h, -1) \) point is located outside the circle \( x^2 + y^2 - 4x + 2y - 4 = 0 \)

3. Results and discussion
Mathematics ability tests carried out in class XI natural science 4 in Senior High School 3 which includes excellent school Boyolali were 36 students. The test was held in February 2018, to determine the subject of the study. The problem in this test consists of 5 essay questions. Based on the test scores of students’ mathematical ability obtained data there are 11 high-ability students, 11 moderate-capable students, and 14 low-ability students. If made in percent form can be seen in table 1 below.

| Ability Category | Score  | In Percentage (%) |
|------------------|--------|--------------------|
| High             | 61 – 100 | 30.55%             |
| Medium           | 46 – 60  | 30.55%             |
| Low              | 0 – 45   | 38.90%             |
| **Summary**      |         | **100%**           |

Based on the results of the mathematical ability test to determine six research subjects, namely two subjects from high mathematical abilities, two subjects from average math skills and two subjects from low mathematical abilities. The determination of this subject is also based on the results of interviews with the teacher. Students selected as research subjects can be seen from table 2 below.

| Student Mathematics Ability Category | Subject Initials |
|--------------------------------------|------------------|
| High                                 | H1 & H2          |
| Medium                               | M1 & M2          |
| Low                                  | L1 & L2          |

Based on table 2 it was obtained that there are 6 research subjects who will be analyzed subject type thinking in solving mathematical problems with non-routine questions on circle material. Some of the students answers can be analyzed as follows:
3.1. Analysis of type thinking students high ability H1

Based on Figure 1, it showed that students could identify problems even if they do not write the given and asked the question. Students understand the concept of work by substituting point R to the equation of the circle. Students do a zero test to determine a positive or negative sign. Students describe a number line to determine the value boundaries h. Students can write conclusions boundaries h.

Based on Figure 2, it showed that students could identify problems even if they do not write the given and asked question questions. Students understand the concept of work by looking for a center point P to the equation of the circle then substituting to the formula looking for the radius. After that substitute the center point \((2, -1)\), point \((h, -1)\) in the equation outside the circle. Students look for the value \(h_1, 2\) by using the ABC formula to determine the values of \(h_1\) and \(h_2\). Students describe a number line to determine the value boundaries h. Students do a zero test to determine a positive or negative sign. Students can write conclusions about boundaries that are positive.

Figure 1 and figure 2 includes the type of predictive thinking because students are able to understand the problem. Students are able to know what concepts will be used. Students are able to connect several concepts in work so that students are able to determine the expected results.
3.2. Analysis of type thinking students medium ability M1 and M2

![Figure 3. Answer M1.](image)

Based on figure 3 it showed that students could identify problems even if they do not write the given and asked question questions. Students understand the concept of work by looking for a center point \( P \) to the equation of the circle then substituting to the formula looking for the radius. After that substitute the center point \((2, -1)\), point \((h, -1)\) in the equation outside the circle. Students factor in the value \( h \). Students describe a number line to determine the value boundaries \( h \). Students do a zero test to determine a positive or negative sign. An error occurred here, where students take negative signs which should be positive. So the students' answers are wrong in writing conclusions boundaries that are negative.

![Figure 4. Answer M2.](image)

Based on figure 4 it showed that students could identify problems even if they do not write the given and asked question questions. Students can identify problems even if they do not write the given and asked the question. Students can write a point formula outside the circle. Students use the concept of a point outside the circle and the equation of the circle on the problem. The sign should be more than (>) Because it is less precise, it does not draw zero lines and test lines.

Figure 3 and figure 4 includes the type of predictive thinking because students are able to understand the problem. Students are able to know what concepts will be used. Students are able to connect several
concepts in work. Even though students have not been able to determine the right answer, students have been able to connect the concepts in work.

3.3. Analysis of type thinking students low ability L₁ and L₂

![Figure 5. Answer L₁.](image)

Based on figure 5 it showed that students could identify problems even if they do not write the given and asked question questions. Students understand the concept of work by looking for a center point $P$ to the equation of the circle. However, students stop here. Students assume that 2 is the counterpart of the value $h$.

![Figure 6. Answer L₂.](image)

Based on figure 6 it showed that students could write down what is the given and asked questioned. Students do not understand the concepts used to complete. Students tend to want to complete without planning quickly. Students understand the concept of work by looking for a center point $P$ to the equation of the circle. However, students stop here. Students assume that 2 is the counterpart of the value $h$.

Figure 5 and figure 6 includes the type of functional thinking because students want to quickly carry out completion rather than plan for completion. Students do not know what concept concepts will be used so that students have not been able to relate the concept concepts that have been obtained previously.

Based on the results of the study, each student has a different way of solving mathematical problems in solving mathematical problems and problems encountered. This is possible because different abilities of each individual are different. This in accordance with Ormrod opinion states that problem solving is using (transferring) existing knowledge and skills to answer unanswered questions or difficult situations [20]. This means that to solve problems in a learning requires quite a lot of mastery of concepts well as a basis for students to solve a given problem [21]. Each student has a different ability in mastering mathematical concepts, so it has a great influence on the ability to solve mathematical problems.

Based on the results of the study it can be concluded that students with high mathematical abilities and students with moderate abilities are included in the type of predictive thinking process. Students with low abilities are included in the type of functional thinking process. The number of students included in the type of predictive thinking process is more than students who have the type of functional thinking. This is in accordance with the opinion Schwank which states that it is very difficult find subjects who...
think functionally [22]. However, furthermore Schwank stating that in Indonesia and China can be found student who think functional usualy women students [22].

4. Conclusion
Students' mathematical abilities are divided into three, namely high, medium and low. Each category of mathematical abilities will influence the students' thinking process of solving problems. Students with high math skills in solving non-routine problems can be resolved well. All stages of completion can be traversed. Students with high mathematical abilities and average math skills can be said to be students with predictive thinking types. This is because students can connect several concepts that exist. While the ability of low students was included in the type of functional thinking where students only use one concept without connecting with the concepts that have been studied.

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