NURBS interpolation for tangential point tracking grinding of eccentric shaft

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Abstract: The sine eccentric shaft is one of the most important parts for industrial robot RV reducer, and its machining quality has immediate effect on the overall device performance. The eccentric shaft X-C linkage grinding model is built based on the tangential point tracking machining principle, and the method of calculating the roundness error for eccentric circle grinding is proposed here. The research is performed on non-uniform rational B-spline (NURBS) interpolation for the coordinates’ densification of eccentric shaft X-C linkage grinding, and the algorithm process is put forward. The process of eccentric grinding is simulated and analysed by wheel inversion envelope method. The results show that the eccentric tangent point tracking model proposed is correct. Compared with the linear interpolation, NURBS interpolation can remarkably improve the accuracy of eccentric shaft tangent point tracking grinding.

1 Introduction

The mechanical transmission normally requires the conversion between rotational and reciprocal rectilinear movement, for which using the eccentric shaft structure is a common method. Eccentric shaft parts are extensively used in mechanical equipment such as diesel engine, petrol engine, reducer, compressor, and hydraulic pump. In recent years, industrial robots have been increasingly extensively used in industry-related fields; RV reducer, thanks to its small size, high rigidity, light weight, high precision, and transmission ratio etc., have become key components of industrial robot, and its cost accounts has exceeded one-third of overall robot [1]. RV reducer principally comprises such complex parts as eccentric shaft, cycloid, pin gear shell, and planet carrier, of which the machining precision immediately determines the performance of reducer [2]. As shown in Fig. 1, since eccentric shaft is a position regulating part of RV reducer, extremely rigorous requirements have been placed on the machining quality and precision of its eccentric circle section. In view of the fact that the roundness accuracy of eccentric circle for eccentric shaft of industrial robot RV reducer is typically required to be within ±5 μm, the machining efficiency and precision of eccentric shafts manufactured in the form of eccentric fixture by means of turning or grinding can no longer meet the requirements of industrial robot. It has turned into an important development tendency of non-circular shaft parts machining that the processing of the entire contour realised through X-C shaft linkage grinding based on the tangential point tracking rationale where X-axis drives the feed of grinding carriage while C-axis drives the rotation of work piece. The constant change of the position and direction of grinding point and the wheel radius leads to complicated calculation of part machining coordinates during tangential point tracking grinding. Since tangential point tracking grinding and its error compensation technology directly involve the core technology and commercial interests of CNC manufacturers, only a few literatures about tangential point tracking grinding model and process were reported [3], only Chen G et al. proposed special complex processing model for camshaft or crankshaft parts [4–6]. Many factors may affect the machining precision and surface quality of eccentric shaft, and numerical interpolation is one of the most important factors. Interpolation method determines the trajectory of NC machining points and the continuity of machine tool motion. In recent years, some simplified spline interpolation algorithms have been emerged in high-end CNC systems [7–9]. Featuring extremely smooth and continuous first-order derivative and second-order derivative, NURBS curve offers unified mathematical expression of circle, ellipse, parabola, hyperbola, and conic curve etc. NURBS curve expediently enables partial modification, and the change to any part of the curve will not affect other parts [10, 11]. All these properties constitute the huge advantage of NURBS interpolation in terms of the machining of parts with complex-curved surfaces [12]. Here, eccentric shaft eccentric part of the tangent point tracking processing is the object of study. In order to reduce the grinding error, the NURBS interpolation is applied to the eccentric tangent point tracking grinding point interpolation calculation process.

2 Building the tangential point tracking grinding model of eccentric shaft

2.1 Eccentric shaft tangent point tracking grinding principle

Fig. 2 shows the relationship diagram of eccentric shaft X-C linkage grinding. In the figure, O is the base circle centre of eccentric shaft; O₁ is the centre of eccentric circle; O₂ stands for grinding wheel centre; e is the eccentric distance; r represents eccentric circle radius; R is the grinding wheel radius. During grinding processing, the eccentric shaft rotates about base circle centre O at angular velocity ω. The position of grinding point ‘A’ varies with the rotation angle C. In order to ensure that the wheel and eccentric shaft are always tangent, X-axis wheel carriage needs to move the appropriate displacement Sₓ as the grinding point ‘A’ changes. When the eccentric shaft driven by C-axis motor turns one revolution, grinding point ‘A’ accomplish the movement of entire eccentric circle. Since this method requires the precise linkage between the X-axis and the C-axis to ensure that the
The contour equation of eccentric circle is dependent on the derivation $\rho$ could be converted into polar coordinate $(\rho, \varphi)$ by substituting (1) for (2).

$$\rho = \sqrt{x^2 + y^2}$$

$$\varphi = \begin{cases} 
\arctg(y/x); (x > 0, y \geq 0) \\
\pi + \arctg(y/x); (x < 0) \\
2\pi + \arctg(y/x); (x > 0, y \leq 0) \\
\pi/2; (x = 0, y > 0) \\
3\pi/2; (x = 0, y < 0) 
\end{cases}$$

As shown in Fig. 2, for X-C linkage grinding of eccentric circle contour, the grinding point ‘A’, grinding wheel centre ’O1’, and the centre of eccentric circle ’O2’ need to be kept on the same straight line at any angular displacement C; that is, grinding wheel is in normal contact with eccentric shaft contour, and is perpendicular to the tangent at grinding point. In Fig. 2, $\beta$ represents the angle between eccentric circle normal and polar radius; $\alpha_A$ is the angle between the parallel line of the normal of grinding point and X-axis. The computing equation for X-C grinding coordinates of eccentric circle obtained based on differential geometry principle and literature [13] is shown in (3).

$$S_x = \sqrt{(\rho \cos \beta + R)^2 + (\rho \sin \beta)^2}$$

$$C = \varphi + \beta - \alpha_A$$

$\alpha_A$ = arctg($\rho \sin \beta$)($\rho \cos \beta + R$)

2.3 Roundness error analysis for tangential point tracing grinding of eccentric shaft

In the case of X-C linkage grinding, eccentric shaft and grinding wheel are the relative movement to each other. From the perspective of inverse movement, the entire machining is substantially a kind of grinding wheel enveloping for eccentric circle contour when the eccentric shaft is considered stationary. For the convenient observation and calculation in Cartesian coordinate system, the eccentric circle grinding process is analyzed through inverse movement envelope of grinding wheel.

As shown in Fig. 3, grinding wheel inversion envelope method is applied to analysis by assuming the eccentric shaft remains stationary while the grinding wheel moves inversely around workpiece. When the actual coordinate of C-axis is $C_i$, the actual coordinate of X-axis is $S_{xi}$, at which point the grinding wheel centre is $O_2 (X_i, Y_i)$. As shown in (4), the coordinates of grinding wheel centre and eccentric shaft grinding coordinate is the relationship between polar coordinate system and Cartesian coordinate system.

$$\begin{cases} 
X_i = S_{xi} \cos C_i \\
Y_i = S_{xi} \sin C_i 
\end{cases}$$

Interpolation and other factors make it impossible for actual grinding coordinate ($S_{xi}$, $C_i$) to coincide with command trajectory, thereby resulting in roundness error of eccentric circle. The geometric relationship-based equation for calculation of eccentric circle roundness error $\varepsilon$ is given in (5). From the geometric relationship, the eccentric circle roundness error $\varepsilon$ can be calculated as shown in (5).

$$\varepsilon = \sqrt{(S_{xi} \cos C_i - e)^2 + (S_{xi} \sin C_i)^2} - R - r$$

3 NURBS interpolation algorithm for grinding eccentric shaft

As the changes of tangential points during tangential point tracing grinding brings about heavy programming load, the interval between two adjacent machining points is usually 1°, which is to say, every revolution of eccentric shaft tangential point tracing grinding involves 360 discrete machining points; the interpolation calculation between machining points is often performed with
CNC system. As grinding wheel must run with a high linear velocity during grinding operation so as to guarantee favourable machining quality and efficiency. For that purpose, large grinding wheel radius is required, while the eccentric shaft of RV reducer is small in size, that means $R \gg r$. From the view of wheel inverse movement envelope method mentioned in the previous section, these discrete machining points actually represent the centre of grinding wheel forward coordinates. Therefore, the interpolation trajectory between machining coordinate points may exert a significant effect on eccentric shaft machining precision.

To assure a smooth and continuous acceleration curve, it is essential to use third- or higher NURBS spline, but the higher of the NURBS spline, the larger the peak value of velocity and acceleration curve; moreover, the NURBS spline higher than third power goes against engineering practice since it requires a large amount of calculation. Hence, cubic NURBS spline is employed here to study the coordinate interpolation algorithm for X-C movement envelope method mentioned in the previous section, here to study the coordinate interpolation algorithm for X-C tangential point tracing grinding of eccentric shaft. Assume the coordinate point for one machining revolution of tangential point tracing X-C linkage grinding of eccentric shaft is $q_i(S_{xi}, C_i)$ ($i = 1, 2, \ldots, n$), and obtain the interpolation trajectory of a NURBS curve.

### 3.1 NURBS curve representation

NURBS curve involves three equivalent expressions, i.e. rational parameterisation, rational basis function, and homogeneous coordinate table. Cubic NURBS curve can be expressed as a piecewise rational fraction as shown in (6) [14]:

$$ p(u) = \sum_{i=0}^{n} o_i d_i N_i(u) $$

where vector point $d_i$ ($i = 1, 2, \ldots, n$) is control vertices that are connected sequentially into a control polygon. The $o_i$ ($i = 1, 2, \ldots, n$) is the weight factor: First and last weight factors are equal to one respectively.

$$ N_i(u) = \begin{cases} 1 & \text{if } u_i \leq u < u_{i+1} \\ \frac{u - u_i}{u_{i+1} - u_i} N_i(u) + \frac{u_{i+2} - u}{u_{i+2} - u_{i+1}} N_{i+1}(u) & \text{otherwise} \end{cases} $$

### 3.2 Knot vector parameterisation

There are four commonly used methods for data point parameterisation in NURBS: they are uniform parameterisation, accumulated chord length parameterisation, centripetal parameterisation, and welfare parameterisation, among which accumulated chord length parameterisation is generally employed; it is extensively used since it faithfully reflects the distribution of data points by chord length. This method overcomes problems resulting from uniform parameterisation performed in the event of uneven distribution of data points by chord length [15].

This method realises the correspondence from each node interval to the chord length between the two points of the corresponding curve. Let $L$ be the sum of chord length after the normalisation of forward difference vector. For cubic non-periodic NURBS spline curve, the multiplicity of nodes at both ends is normally set to 4; three end points are added at each side, and there would be $n + 6$ end points in total. In doing so, the end points of NURBS curve obtain end point properties of Bezier curve, that is, the first and last end points of curve coincide with that of control vertex, while the first and last end points of curve are tangent to first and last sides of control polygon. Re-order the nodes as shown in (8).

$$ u_i = u_n = u_0 = u_0 = 0 $$

$$ L = \sum_{i=1}^{n-1} |p_{i+1} - p_i| $$

$$ u_i = u_{i-1} + \frac{|p_{i+1} - p_{i-1}|}{L} (i = 5, \ldots, n + 2) $$

$$ u_{n+1} = u_{n+2} = u_{n+5} = u_{n+6} = 1 $$

### 3.3 Boundary conditions of NURBS curve

The boundary conditions of NURBS curve include tangent vector condition, parabola condition, and free end condition etc. Tangent vector condition is frequently used for two-dimensional curve. From a mechanical perspective, tangent vector condition is equivalent to the fixed support at end of beam and has a fixed tangential direction. Modular length is associated with the parameters employed and is not the invariant of curve. In the case of arc-length parameterisation, tangent vector has unit modular length, hence, as a parameterisation method that is similar to arc-length parameterisation method, accumulated chord length parameterisation normally offers unit tangent vector to enhance the fairness at the end of spline curve, as shown in (9).

$$ d_1 - d_0 = \frac{(u_1 - u_0)\omega_0}{3\omega_0} p_n^0 $$

$$ d_{n+2} - d_{n+1} = \frac{(u_{n+2} - u_{n+1})\omega_{n+1}}{3\omega_{n+2}} p_n^0 $$

As for X-C coordinates of eccentric circle grinding, the centre of eccentric circle at initial position of grinding is on the positive of X-axis; hence, $p_0^0 = p_n^0 = 0$.

### 3.4 Calculation of control vertex

In consideration of boundary conditions, two control vertices are normally added to completely describe the curve shape. After parameterisation, the control point (10) could be obtained through bringing the boundary condition into (6). Then, solving this equation can get the control vertices.

When $p_0 = p_n = 0$, $d_1 - d_0 = 0 \times e_0$, and $d_{n+2} - d_{n+1} = 0 \times e_{n+2}$, hence, boundary condition could be identified as $b_1 = -1$, $c_1 = -1$, $a_{n+2} = -1$, $b_{n+2} = 1$, $e_0 = 0$, $e_{n+2} = 0$; (10) has the form of chasing method, but chasing method is not applicable as some variables may be zero during the solution, so the equation should be solved by the big sparse matrix method [16].

$$\begin{bmatrix} b_0 e_0 \\ a_0 b_0 c_0 \end{bmatrix} = \begin{bmatrix} d_1 \\ e_1 \end{bmatrix} $$

$$\begin{bmatrix} a_1 b_1 c_1 \\ a_2 b_2 c_2 \\ \ddots \\ a_{n+1} b_{n+1} c_{n+1} \end{bmatrix} \begin{bmatrix} d_n \\ e_n \end{bmatrix} = \begin{bmatrix} d_{n+1} \\ e_{n+1} \end{bmatrix} $$

where
After the control point is determined, the point on cubic NURBS curve can be obtained by inserting points into the interval \([u_{i-1}, u_i])\), that is, accomplish the NURBS curve compacting. Equation (6) can be expressed in a segmented form, the expression of the \(i\)th segment of third-power non-uniform rational B-spline curve should be (11). According to this equation, the \(i\)th segment of curve is only associated with the neighbouring four segments of curve, thus exhibiting an excellent local property.

\[
p_i(u) = \frac{\sum_{j=0}^{n_i} \omega_j N_j(u)}{\sum_{j=0}^{n_i} \omega_j N_j(u)} \quad u \in [u_i, u_{i+1}] \tag{11}
\]

4 Simulation verification

The X-C machining coordinates of eccentric shaft grinding are calculated based on tangential point tracing grinding model, while the accuracy of model is demonstrated by grinding wheel inversion envelope method. NURBS interpolation algorithm is used to machining coordinates calculation which obtained by the eccentric tangent point tracking model, and comparison is made with linear interpolation to verify the effect of NURBS interpolation.

4.1 Simulation of eccentric shaft tangential point tracing grinding

Take the eccentric shaft of an RV reducer with model 1GNC00157 as an example, where the radius of eccentric circle \(r = 26.3\), and the eccentric distance \(e = 5.17\); assuming grinding wheel radius \(R = 100\) mm. The parameters \(\theta\) can be uniformly separated in the range of \(0-2\pi\), and assume the point of per machining revolution \(n = 360\). Bring these parameters into (1) and (2), can get 360 sets of eccentric circle profile coordinates; substitute the 360 sets of polar coordinates and grinding wheel radius and grinding wheel radius \(R\) into (3), can get the X-C linkage tangential point tracing grinding coordinates of this eccentric shaft, as shown in Fig. 4.

To verify the correctness of the tangent tracking model, using the wheel reversal envelope method to simulate the X-C machining grinding process of the eccentric shaft. Take the grinding coordinates into (4), can get the coordinates of grinding wheel centre. Drawing each grinding wheel contour, the envelope trajectory shown in Fig. 5. It can find out that the grinding wheel envelope trajectory coincides with eccentric circle profile, which demonstrates the tangential point tracing model algorithm is correct.

4.2 NURBS interpolation and error analysis for eccentric shaft grinding

Despite the CAM process can calculate the machining coordinates, the NC system still needs to adapt the interpolation algorithm to complete the entire contour processing; that is, complete the densification of the machining points. For eccentric shat tangent point grinding coordinates \((S_z, C)\), using different interpolation methods, the error and surface quality caused by the interpolation is also different. For complex curve contour processing, micro-segment linear interpolation fit machining is most employed. From the wheel reversal envelope method mentioned above, can find out that the eccentric shaft machining trajectory is substantially equivalent to the grinding wheel centre coordinates. Therefore, according to the principle of linear interpolation and calculation method and (5), can calculate the interpolation error; the result of linear interpolation error of eccentric shaft tangential point tracing machining is shown in Fig. 6.

As shown in Fig. 6, linear interpolation roundness error at linear interpolation command points of eccentric shaft is very small; the roundness error between two command points increases and then decreases; the overall range of maximum error resulting from linear interpolation is about 3 \(\mu m\). Compared with \(5\mu m\) eccentricity roundness accuracy ordered, it is clear that the error caused by linear interpolation is large.

Substituting the discrete grinding coordinates based on the tangential point tracing for eccentric shaft into (6)–(11) and assuming the weight factor \(\omega_i = 1\), can obtain about 3,600 NURBS interpolation points coordinates. The comparison performed by grinding wheel inverse envelope method between the results of NURBS interpolation and linear interpolation for tangential point tracing machining coordinates of eccentric shaft is shown in Fig. 7.

Furthermore, the NURBS interpolation trajectory error could be calculated by substituting the machining coordinates obtained by NURBS interpolation calculation into (5), which shown in Fig. 8. As shown in Fig. 8, the maximum roundness error for tangential point tracing machining of eccentric shaft caused by NURBS interpolation is <0.05 \(\mu m\). When compared with linear interpolation, the peak value of interpolation error decreases by 98.36%. Furthermore, from Fig. 7, can find out that NURBS...
Interpolation curve has unified representation of complex curve for its extremely smooth and continuous first-order derivative and second-order derivative; when compared with linear interpolation, NURBS interpolation perfectly preserves original nature of curve, thereby significantly reducing interpolation error, and the surface may also be more smooth.

5 Conclusion

Eccentric shaft grinding simulation and model error analysis demonstrate that the tangential point tracing grinding method proposed here is correct. The comparison between NURBS interpolation and linear interpolation indicates that NURBS interpolation can effectively reduce interpolation error and improve eccentric shaft machining precision. Despite its capability of reducing tangential point tracing error for eccentric shaft grinding, NURBS interpolation has some intrinsic defects such as the determination of weight factor and the excessively large amount of calculation for back calculation of control point etc. These defects restrict the application of NURBS interpolation in CNC system, and they need to be further studied.

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7 References

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Fig. 7 Comparison of eccentric shaft tangent tangential point tracing processing between NURBS and linear interpolation

Fig. 8 Roundness error caused by eccentricity tangent point tracking processing NURBS interpolation