The importance of the quadrupole of a binary system relies mainly in its connection to gravitational radiation via Einstein’s famous quadrupole formula [1]. This formula is beautifully confirmed by Taylor’s binary pulsar [2, 3], the indirect proof of the existence of gravitational waves, for which Hulse and Taylor have been awarded with the Nobel price in 1993. Direct detection of gravitational waves will (hopefully) be realized in the next few years by the numerous experiments operating today.

The question whether gravitational waves from binary systems can be detected via their effects on the propagation of photons has been addressed repeatedly in the past. In [4] to [8] the deflection angle and the time delay caused by a gravity wave passing through the path of the photon is analyzed from different points of view, and only in 1998 the solid conclusion to this problem was given by Kopeikin et al. [7]: these effects are too faint to be detectable today.

In [7, 8] the problem is faced in full generality, determining the time delay, $\Delta t$, and deflection angle, $\alpha$, caused by a localized source of gravitational radiation, $\mathbf{D}$, acting as a deflector of light (see Fig. 1). In [7, 8], a multipole expansion for the energy momentum tensor of the source is used. In [7] this expansion is truncated at the quadrupole and the resulting gravitational field is expressed in terms of the deflector mass, its angular momentum and quadrupole.

In this paper we study the effect of the lens quadrupole not on gravity wave production, but on the scalar potential which is responsible for lensing. Especially, we want to compute its contribution to microlensing. The quadrupole is the lowest multipole contributing to the emission of gravitational waves. The scalar potential, however, contains also the larger monopole and dipole contributions. Naively, one might expect that therefore the quadrupole is irrelevant, unobservable. We shall show here that this is not the case. For impact parameters close to the critical line, which are those with large amplification, the quadrupole contribution to the amplification is significantly enhanced and can become observable. Furthermore, mass and angular momentum are conserved quantities, while the quadrupole is in general time dependent. It will therefore introduce a time-dependence in the microlensing signal which we determine. A direct detection of such a modulation can give important information about the quadrupole of the lens and therefore on the gravitational radiation it emits. This can be very useful for the observation of gravitational radiation from the system. But even if the effect is not observed directly, because of the under sampling of data or because the period of the lens system is of the same order than the time-scale of microlensing event, it has to be taken into account as a possible source of error in static parameters estimations, especially for high amplification events. We conclude the paper with a rate estimation, taking into account only compact binaries of our galaxy as example. However, we observe that, more generally, such a modulation effect can be given by any object with a varying quadrupole. Our result gives a new tool to determine the quadrupole of a unknown system independent of its nature.

We stress that the modulation can be detected only if the period of the system is smaller than the time-scale of microlensing $i.e.$ for relatively compact binaries with periods of less than about 30 days.

II. THE AMPLIFICATION FACTOR

We work in the thin lens approximation, which means that we may project the lens mass distribution into a plane and we consider impact parameters $d = |\xi|$ much smaller than the distances $r$ and $r_0$ in Figure 1. Fur-
thermore, we assume the condition $\frac{\omega r^2}{a^2} \ll 1$ where $\omega$ is the frequency of the binary, so that retardation inside the lens plane can be neglected. We employ the center of mass system, i.e. the coordinate system where the center of mass of the binary is at rest at position $x = 0$. Up to the quadrupole, the gravitational lens potential is then given by \cite{9}

$$
\Psi(x, t^*) = \left[ M + \epsilon_{pqk} k_p S_q \partial_j + \frac{1}{2} I_{pq}^{TT}(t^*) \partial_p \partial_q \right] \ln d, \quad (1)
$$

where we have set $G = c = 1$ and $t^*$ denotes retarded time, $t^* = t - r$. Here $k$ is the unit vector pointing from the source to the observer, $M$ is the total mass of the system, $S$ is its angular momentum and $I_{pq}^{TT}$ is the transverse traceless quadrupole tensor, projected into the plane normal to $k$,

$$
S^q(t) = \frac{1}{2} \varpi^{qpr} \int d^3x \left( x^p T^{qr} (x, t) - x^r T^{0p} (x, t) \right) \ln d, \quad (2)
$$

$$
I_{pq} = \int d^3x \rho(x, t) (x_q x_p - \frac{1}{3} |x|^2 \delta_{qp}), \quad (3)
$$

$$
I_{pq}^{TT} = \left[ \delta_{ip} \delta_{jq} - \frac{1}{2} (\delta_{pq} + k_p k_q k_i k_j) \right] I_{ij}. \quad (4)
$$

$T^{\mu\nu}(x)$ denotes the energy momentum tensor of the source and $\rho(x, t) = T^{00}$ is the energy density. Since the background spacetime is Minkowski, spatial index positions are irrelevant. The time delay and the deflection angle can be expressed in terms of $\Psi$ as \cite{7}

$$
\Delta = -4\Psi + 2M \ln (4\pi r_0), \quad \alpha_i = 4 \partial_i \Psi. \quad (5)
$$

The amplification of a far away light source with $r_0 \gg r$, is now easily determined as $\mu = 1 - \frac{1}{2 \det(A)}$, where $A$ is the Jacobian of the lens map (see e.g. \cite{9}),

$$
A_{ij} = \delta_{ij} - 4r \partial_i \partial_j \Psi(x, t^*). \quad (6)
$$

Without loss of generality, we fix the orientation of the coordinate system so that the $x_1$ axis is aligned with the impact vector and the third axis is normal to the lens plane. Hence $\xi_1 = d, \xi_2 = 0$ and $k_1 = k_2 = 0, k_3 = 1$. From Eqs. \cite{10, 11} one then obtains the following expression for $\mu$,

$$
\mu^{-1} = \det(A) = 1 - \frac{16a^2}{d^2} \left[ \frac{M^2}{d^2} + 4\frac{S^2_k + S^2_{2k}}{d^2} + 4M \left( \frac{S^2_k}{d^2} + \frac{3(I_{11} + \frac{1}{2} I_{33})}{d^4} \right) + 24 \frac{S^2_k (I_{11} + \frac{1}{2} I_{33})}{d^6} \right]. \quad (7)
$$

We use that in our coordinate system $I_{ij}^{TT}$ is entirely determined by $I_{11}^{TT} = -I_{22}^{TT} = I_{11} + \frac{1}{2} I_{33}$ and $I_{12}^{TT} = I_{12}$. The quadrupole tensor $I_{ij}$ has to be evaluated at retarded time $t^* = t - r$. The largest term containing the quadrupole $\propto 12(I_{11} + \frac{1}{2} I_{33})/d^4$ is suppressed with respect to the monopole $\propto M^2/d^2$, by a factor $\frac{(a/d)^4}{d^2}$. Therefore, one might suggest that systems with large orbits have the strongest contribution from the quadrupole. This is true, but in this case the time variation may not be visible if the period of the system is larger than the duration of the event, $T = 2\pi a^{3/2}/\sqrt{M} \geq d/v$. Here $v$ is the source velocity (projected into the lens plane).

Furthermore, our expansion breaks down at $a \simeq d$ since higher multipoles can no longer be neglected and the microlensing event probes the full matter distribution of the lens. This more complex phenomenon has been studied extensively in the literature, see e.g. Refs. \cite{10} to \cite{21}. Here we restrict ourselves to $a/d < 0.3$, say. We are mainly interested in compact binaries which usually also generate a significant amount of gravitational waves.

The amplification is largest close to the critical line defined by $\det(A) = 0$. Neglecting the sub-dominant contributions this corresponds to the Einstein radius $r_E = 2\sqrt{Mr}/c$, the critical impact parameter. For large amplification, $\Delta = |d - d_c|/d_c \ll 1$, the effect of the quadrupole is enhanced by a factor $\Delta^{-1}$. To illustrate this, we consider a binary with angular momentum normal to the lens plane, and take into account only the dominant contribution from the quadrupole in Eq. \cite{7}, $\frac{12I_{ij}^{TT}}{d^4}$. We parameterize the quadrupole term by

$$
\frac{12(I_{11} + \frac{1}{2} I_{33})(t^*)}{d^3} = \gamma(t^*) \frac{M}{d} \epsilon^2. \quad (8)
$$

Here $\gamma(t^*)$ is a dimensionless function of order unity which contains the information about the time dependence of the quadrupole and hence, e.g. on the frequency of the gravitational wave emitted by the system. The amplification can then be approximated by

$$
\mu^{-1} \simeq 1 - \left( \frac{d}{a} \right)^4 [1 + \gamma \epsilon^2], \quad \epsilon = \frac{a}{d}. \quad (8)
$$
We want to consider the case \( d = d_c (1 + \Delta) \) with \( \Delta \ll 1 \). In this case we have to lowest order in the small parameters \( \epsilon \) and \( \Delta \)
\[
\mu \simeq \frac{1}{4\Delta - \gamma \epsilon^2} = \frac{\Delta^{-1}}{4 - \gamma \epsilon^2/\Delta}.
\] (9)

We conclude that the contribution from the quadrupole is significant if the ratio \( \epsilon^2/\Delta \simeq 4\mu \epsilon^2 \) is significant, say larger than a few percent.

As usual, our ray optical approach gives rise to a divergence of the amplification when the impact parameter approaches the critical value \( d_c = 2\sqrt{M} \cdot r \). The amplification grows indefinitely when \( d \) decreases toward the critical value. At distances smaller than \( d_c \) there are in principle multiple images, but they are too close together to be resolved by present optical telescopes. The divergences in the geometrical optics treatment is removed in the correct treatment using wave optics [9].

### III. EXAMPLES

In this section we present some examples for the modulation of the magnification by microlensing by compact binaries in our galaxy. It is worth to stress that such a modulation can be produced by any system having a varying quadrupole, and our result can be applied to more general cases than the ones discussed here. In Figs. 2 and 3 we choose for the BS two equal masses \( M_1 = M_2 = 1.4M_\odot \) in circular orbit in the lens plane so that the spin is aligned with the 3-axis, \( S_1 = S_2 = 0 \). We consider a background source moving with 100km/s relative to the lens D.

In Fig. 2 the amplification is plotted as function of time for a neutron star or white dwarf binary. A rotation period of \( T = 10^5 \) sec, corresponding to an orbital radius of \( a \simeq 4.5 \times 10^8 \) km is assumed. The binary is placed at distance \( r = 200 \) pc. The impact parameter is \( d = (1 + 10^{-3}) r_E \) yielding an amplification of about \( \mu \simeq 250 \). This is very close to the critical impact parameter \( d_c = r_E = 2.2 \times 10^8 \) km which gives infinite amplification (within a ray optical treatment). The quadrupole modulation amounts to 43% of the static contribution at maximum amplification. Our naive estimate gives a relative contribution \( 4\mu \epsilon^2 \simeq 0.4 \) from the quadrupole, which is in the right ballpark.

In the second example, plotted in Fig. 3 we take a deflector somewhat further away, \( r = 700 \) pc, and consider a binary with period, \( T = 2 \times 10^8 \) sec = 23 days. For this system, \( a \simeq 3.4 \times 10^7 \) km. The impact parameter of this case is \( d = 1.04 r_E \) yielding a maximal amplification of about 7. The quadrupole modulation amounts still to 11% at maximum. Here the modulation signal is less significant since \( \Delta = (d - d_c)/d_c \) is larger, and also the period of the system is significantly longer.

In order to gain a good qualitative intuition of this phenomenon one can visualize the lines of equal amplification for a lens system. For a point-mass they are circles around the deflector. If a non-vanishing quadrupole moment is present, the circle of divergence as well as the circles of equal amplification are deformed as shown in Figure 4. They are simply the solutions of the equation \( \mu^{-1} = 0 \) and \( \mu^{-1} = \text{constant} \) respectively, as a function of the angle between \( \xi \) and the direction of the vector relating the two stars of the binary. This vector rotates with the period of the system and with it the deformed circles. The observed modulation of the amplification
comes from the rotation of the non-circular curves of constant amplification in the lens plane. The deformation of the circles is quite faint but since the amplification becomes so large close to the critical line, the quadrupole is nevertheless observable. Furthermore, our quantitative examples show that quadrupole modulations of high importance can appear using quite reasonable parameters for the lensing system, and for this reason, even if the modulations cannot be detected directly, they have to be included in the error budget for microlensing events with high magnification.

We stress that the modulation can only be observed if the period of the binary is shorter than the time interval over which the amplification is significant.

It is interesting that observing such an event in the LISA range of frequencies, permits at the same time to determine the frequency and direction of the binary as source for gravitational waves. This will allow to detect the corresponding gravity wave signal out of the confusion noise in the LISA data [22].

The idea to detect gravity waves via microlensing has been studied in [23], and more recently in [24], but in these works the gravity wave source and the static lens are two different objects, the first being far enough from the second to make the quadrupole time dependence discussed here unimportant.

In this paper we consider the quadrupole variation of the deflector itself and we study its contribution to the scalar lens potential. The effect from the also emitted dynamical gravitational wave is much smaller than the one considered here in the frequency range we are interested in \((10^{-6} - 10^{-3}Hz)\), since it is proportional to the second time derivative of the quadrupole.

However, measuring the modulation of \(\mu\) provides access to information about the variation of \(I_{ij}\) itself. For example, it allows to predict the frequency and amplitude of the gravity waves emitted by the system.

As we have seen above, in order for the modulation to be measurable, the microlensing event has to reach rather high magnification.

We focus on a galactic bulge survey where the observed field is about \(8^\circ \times 8^\circ = (\Delta \varphi)^2\) centered on galactic coordinates \(l \sim 4^\circ, b \sim -6^\circ\), which contains about \(N_s = 5 \cdot 10^7\) bulge stars [23]. Using the binary population model of Tutukov–Yungelson [26] we can estimate the number of galactic black hole, neutron star and white dwarf binaries to be about \(3 \times 10^5\). Within the volume swept by the light rays coming from our sources, we expect to find about a fraction (see Fig. 5)

\[
x \sim \frac{r^3(\Delta \varphi)^2}{3D_{gal}R_{gal}^2 \pi} \approx 10^{-4}
\]

of these binaries, leading to \(N_b \approx 27000\). Here \(R_{gal} \approx 15kpc\) is the radius of the galactic disk and \(D_{gal} \approx 300pc\) is its thickness and \(r \sim D_{gal}/2\sin(6^\circ) = 1.4kpc\) is the apparent thickness of the galactic disc in the direction of the survey.

The cross section \(\sigma\) of the events for which the modulation is visible is about \(\sigma = 0.6d^2\). With this the fraction \(f\) of the observed field covered by a binary per unit time is given \(f = \sqrt{\sigma v}/A\), where \(v\) is the center of mass velocity of the source with respect to the binary and \(A = r^2(\Delta \varphi)^2\) is the area of the observed field. With \(r = 700pc\) and \(v = 100km/sec\) we obtain an event rate

\[
\eta = \frac{\sqrt{\sigma v}N_s N_b}{A} \sim 0.1 \text{ events/year}.
\]  

Note also, that we have taken into account only microlensing by compact binaries. Main sequence binaries which are sufficiently close, so that \(a/d < 0.3\), say may very well contribute a more substantial event rate. Furthermore, the modulations might be due to different objects with a varying quadrupole, e.g cosmic strings [27].
This event rate seems not out of reach of observations and it is already interesting to investigate whether such an event is not present in existing microlensing surveys, that is whether a varying quadrupole behavior may fit one of the exotic microlensing events detected so far.

Even if the modulation cannot be detected directly, the contributions from the quadrupole and the angular momentum are sufficiently important that they have to be included in the error budget for microlensing events with high magnification, $\mu \gtrsim 7$, if one wants to reach an accuracy in the predicted light curve of about 10%.

Let us finally estimate the contribution of the dipole. The dominant effect coming from the spin is the term $4MS/d^3$. Parameterizing $|S| \sim Ma^2\omega \sim Ma_o$, where $\omega$ denotes the frequency of the binary and $d_o \sim \omega a$ is the orbital velocity. Hence $v_o \simeq 10^{-3} \sqrt{M_\odot/3a} 10^7 \text{km}/a$. Its relative contribution is of the order of $M_\odot d^{-3}/(M_\odot d^{-2}) \simeq (a/d)\omega a = \omega a_o \ll 1$, where $v_o$ denotes the orbital velocity of the binary,

$$v_o \simeq \sqrt{\frac{M}{a}} \simeq 10^{-3} \sqrt{\frac{M}{3M_\odot}} \frac{10^7 \text{km}}{a}.$$ 

When $d$ approaches $d_c$ also this term is parametrically enhanced leading to a magnification

$$\mu S \simeq \frac{\Delta^{-1}}{4(1 + \epsilon v_o/\Delta)}.$$ 

(11)

This term is significant only for very compact and therefore very fast binaries, $a \sim 10^8 \text{km} - 10^9 \text{km}$, which then have to be sufficiently close so that $d_c$ is relatively small and $a/d$ is still significant for $d \simeq d_c$. More precisely one finds

$$\epsilon v_o = \sqrt{\frac{R_o a d_c}{2r d}}.$$ 

Hence we need $\frac{\mu S}{\Delta} \geq 10^{-2}/\mu^2$ for the spin amplification to amount to at least 10%. This looks quite unreasonable for amplifications which are not gigantic. This term is also more difficult to disentangle from the monopole since it is time independent like the latter.

V. CONCLUSION

In this paper we have derived and discussed a new effect which leads to a modulation of the light curve on microlensing events from compact time dependent source where the impact parameter $d$ is larger than the size of the source. A simple analysis of the amplification modulation of such an event allows to determine important parameters of the lens quadrupole such as its frequency and its amplitude. These can then be used to predict the gravitational radiation emitted from the system.

This effect is of particular importance in microlensing events by compact binaries. The relative contribution to the magnification from the quadrupole being of the order of $4\mu(a/d)^2$, the effect is most significant for high magnification. A rate estimation for galactic compact binaries shows that typical microlensing surveys towards the galactic bulge should detect about one such event every decade. But even in cases where it is not observed directly, the effect has to be included in the error budget for the microlensing light curve.

This work is supported by the Swiss National Science Foundation (FNS).

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