Can One Phase Induce All CP Violations Including Leptogenesis?

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Abstract

In the framework of a SUSY SO(10) model a phase is generated spontaneously for the $B - L$ breaking VEV. Fitting this phase to the observed CP violating $K, B$ decays all other CP breaking effects are uniquely predicted. In particular, the amount of Leptogenesis can be explicitly calculated and found to be in the right range and sign for the BAU.

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CP violation is directly observed only in the decays of the K and B mesons. The present experimental results [1] are consistent at the moment with the standard model (SM). I.e. CP breaking is induced by a phase in the Cabibbo, Kobayashi, Maskawa (CKM) mixing matrix of the quarks.

Extensions of the SM using right-handed (RH) neutrinos, that account for the neutrino oscillations, involve in general phases which allow for CP violation in the leptonic sector also. This CP breaking is difficult to observe but may be detected as soon as neutrino factories are available. The observation of neutrino-less double beta decays may be also an indication for Majorana phases in the neutrino sector [2].

Spontaneous generation of baryon asymmetry in the universe (BAU) needs CP violation [3]. It is clear now that it requires also extension of the SM, while baryon asymmetry in the universe (BAU) à la Fukugita and Yanagida [4] due to leptogenesis [5] is the most popular and promising theory for the BAU.

Where is the CP breaking coming from?
CP breaking can be induced via phases in the Yukawa coupling, in the interactions of the LH and RH gauge bosons and in the VEVs. Phases in the spontaneously generated VEVs lead naturally to violation of CP. This spontaneous breaking can also help to solve the strong CP problem [7] [8].
The spontaneous violation of CP was already suggested long ago by T. D. Lee [9]. In the framework of $SO(10)$ GUT spontaneous breaking was first discussed by Harvey, Reiss and Ramond [10]. Recently, Bento and Branco [11] added to the SM a heavy Higgs scalar with a $B - L$ violating VEV to generate spontaneous CP violation.

In general, the known CP violation in the hadronic sector is not related to the leptonic one. Even the CP breaking needed for leptogenesis is usually independent of that in the leptonic sector. Hence, CP violation in the leptonic sector is in general not predictable. Predictability can be gained only in terms of a specific model. There are quite a few models relating CP violation in the neutrino sector to leptogenesis [12] but no conventional SUSY GUT which connects the leptogenesis to the observed violation in the K and B decays is presently known.

I would like to suggest in this paper that the one and only origin for CP violation is a spontaneous breaking at high energies. A phase in the $B - L$ breaking VEV can induce all manifestations of CP violation. This phase can be fixed by the observed breaking in the K and B decays and the other CP violations are then predicted. In particular, we will show explicitly that within a SUSY $SO(10)$ model the amount and sign of leptogenesis are as is needed to have the right BAU.

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Let me first show how a phase can be spontaneously generated in the $SU(5)$ singlet component of a scalar $126$ representation of $SO(10)$. It was already pointed out by Harvey, Ramond and Reiss \cite{10} that there is a natural way to break CP spontaneously at high energies. This is due to the fact that $(126)^4$ is $SO(10)$ invariant. $\Phi_{126}$ is the Higgs representation used to break down $B-L$. Its $SU(5)$ singlet component gives also masses to the heavy RH neutrinos. The corresponding large VEV induces also small VEVs in the components of $\Phi_{126}$ that transform like $2_L$ under the SM \cite{13} which play a role in the light fermion mass matrices.

Assume that all the parameters in the $SO(10)$ invariant Lagrangian are real. If the three fermionic families are in $\Psi_{16}$'s, only $\Phi_{10}$, $\Phi_{126}$ and $\Phi_{120}$ can contribute to the mass terms:

$$16 \times 16 = (10 \oplus 126)_S \oplus (120)_{AS}$$  \hspace{1cm} (1)

Suppose we have chosen global symmetries that dictate a (super-)potential of the form\footnote{Note that $10$ is a real representation.} \cite{11}

$$V(\lambda_1, \lambda_2, \ldots) = V_0 + \ldots + \lambda_1 (\Phi_{10})^2_S + (\Phi_{126})^2_S + (\Phi_{126})^4_S + (\Phi_{120})^4_S$$  \hspace{1cm} (2)

and that those are the only phase dependent terms after the spontaneous breaking\footnote{For a detailed discussion of possible scalar potentials see Ref. \cite{10}. The $[(\Phi_{126})^2_S + (\Phi_{126})^4_S]$ part serves also to break the continuous global symmetries avoiding massless Nambu-Goldston bosons.}. If the $SU(5)$ singlet component of $\Phi_{126}$, $\Phi_{126}$ acquire a VEV as well as the right component of $\Phi_{10}$:

$$< \Phi_{10} > = \frac{v}{\sqrt{2}} \quad \quad < \Phi_{126} > = \frac{\Upsilon}{\sqrt{2}} e^{i\alpha}$$  \hspace{1cm} (3)

The phase dependent part of the potential can be then written as

$$V(v, \Upsilon, \alpha) = A \cos(2\alpha) + B \cos(4\alpha).$$  \hspace{1cm} (4)

For $B$ positive and $|A| > 4B$ the absolute minimum of the potential is obtained with

$$\alpha = \frac{1}{2} \arccos\left(\frac{A}{4B}\right).$$  \hspace{1cm} (5)

This spontaneous generation of a phase in the large VEV $\Upsilon$, will generate also phases in the induced small VEVs which give mass to the light fermions. Those will lead to CP violation in the quark and lepton sectors. The value of the spontaneously generated phase $\alpha$ depends on arbitrary parameters in
the Higgs potential. Its actual value can be however fixed by the requirement that the phases of the induced light VEVs will give the observed CP violation in the K,B decays. All other manifestations of CP violation will be then explicitly predicted. In particular the amount and sign of leptogenesis are then predicted in models where $M^{\nu}_{\text{Dirac}}$ is known.

Let me now explicitly calculate the amount of leptogenesis in a SUSY SO(10) model where a phase is generated spontaneously in the $B-L$ breaking VEV. The model was developed in a series of papers [14] [15]. It was originally aimed to find explicitly the mixing angles which are hidden in the SM, like RH rotations. Those allow to calculate explicitly e.g. the proton decay branching ratios as well as all mass matrices and in particular the Dirac neutrino mass matrix and the RH neutrino mass matrix which are needed for the calculation of the leptogenesis. We will use here the mass matrices given in Ref. [15]. This is a renormalizable SUSY SO(10) model i.e. $B-L$ is broken via $\Phi_{126} + \Phi_{126}$ while $\Phi_{126}$ gives mass to the RH neutrinos (without using non-renormalizable contributions). The origin of CP breaking in the model is a phase in the SU(5) singlet component of one $\Phi_{126}$. A global horizontal symmetry $U(1)_F$ dictates the asymmetric Fritzsch texture [10] for the fermionic mass matrices and the possible VEVs in the different Higgs representations. By fitting the free parameters to the observed masses and CKM matrix a set of non-linear equations is obtained. These equations have five solutions which obey all the restrictions, i.e. five sets of explicit mass matrices. The Dirac neutrino mass matrices have the texture:

$$M^{\nu}_{\text{Dirac}} = \begin{pmatrix} 0 & A & 0 \\ B & 0 & C \\ 0 & D & E \end{pmatrix}. \quad (6)$$

They are given explicitly in Appendix I.

The RH neutrino mass matrices have the following form in our model:

$$M_{\nu_R} = e^{i\alpha} \begin{pmatrix} 0 & a & 0 \\ a & 0 & 0 \\ 0 & 0 & -b \end{pmatrix} M_R. \quad (7)$$

Where the real $a, b > 0$. The corresponding eigenmasses are given in Table 1.

Table 1. The masses of the RH neutrinos for the five solutions in $10^{13}$ GeV.

| Solution $10^{13}$GeV | 1     | 2     | 3     | 4     | 5     |
|-----------------------|-------|-------|-------|-------|-------|
| $M_1 = M_2$           | 5.2   | 9.1   | 16    | 18    | 12    |
| $M_3$                 | $8 \times 5.2$ | $7 \times 9.1$ | $3 \times 16$ | $3 \times 18$ | $2 \times 12$ |
What is Leptogenesis?
Out of equilibrium CP violating decays of RH neutrinos, $N_i$, produce excess of the lepton number $\delta L \neq 0$. This will induce baryon asymmetry through $B+L$ conserving sphaleron processes [4] [5] [6].
The amount of CP violation in these decays is:

$$
\epsilon_i = \frac{\Gamma(N_i \rightarrow L_i + \Phi) - \Gamma(N_i^\dagger \rightarrow L_i^\dagger + \Phi^\dagger)}{\Gamma(N_i \rightarrow L_i + \Phi) + \Gamma(N_i^\dagger \rightarrow L_i^\dagger + \Phi^\dagger)}
$$

Knowing the details of CP violation in the leptonic sector as well as the RH mixing angles\(^3\), one is able to calculate explicitly the BAU via leptogenesis. This is the main test of the model.

Let us denote the Dirac neutrino mass matrix $M_{\nu}^{Dirac}$ in the basis where $M_{\nu}^R$ is real diagonal with positive eigenvalues: $M_D$. In this basis $\epsilon_i$ can be expressed as follows

$$
\epsilon_i = \frac{1}{8\pi v^2 (M_D^T M_D)_{ii}} \sum_j \text{Im}[(M_D^T M_D)^2_{ij}] f(M_j^2/M_i^2)
$$

where

$$
f(x) = \sqrt{x} \left[ \ln(1 + \frac{1}{x}) + \frac{2}{x - 1} \right]
$$

and $v = 174 \times \sin \beta$ GeV \(^4\).

$M_{\nu R}$ is given in eq.(7) and its eigenmasses in Table 1.

It is diagonalized by a matrix $U$

$$
U^T M_{\nu R} U = \text{diag}(M_1, M_2, M_3) = M_3 \text{ diag}(\frac{M_1}{M_3}, \frac{M_1}{M_3}, 1)
$$

$$
U = OP \quad \text{where} \quad P = e^{-(i/2)\alpha} \text{ diag}(i, 1, i)
$$

and

$$
O = \begin{pmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
0 & 0 & 1
\end{pmatrix}
$$

In this basis, in terms of eq. (6)

\(^3\)Note, that $M^\dagger M$ is diagonalized using the RH mixing matrix.

\(^4\)tan $\beta = 10$ is used in the model [15]
\[
M_D^\dagger M_D = \begin{pmatrix}
1/2(|A|^2 + |B|^2 + |D|^2) & i/2(|A|^2 - |B|^2 + |D|^2) & 1/\sqrt{2}(B^\dagger C - D^\dagger E) \\
-i/2(|A|^2 - |B|^2 + |D|^2) & 1/2(|A|^2 + |B|^2 + |D|^2) & i/\sqrt{2}(B^\dagger C + D^\dagger E) \\
1/\sqrt{2}(BC^\dagger - DE^\dagger) & -i/\sqrt{2}(BC^\dagger + DE^\dagger) & |C|^2 + |E|^2
\end{pmatrix}.
\]

This gives the following general results

\[\text{Im}((M_D^\dagger M_D)_{12}(M_D^\dagger M_D)_{12}) = \text{Im}((M_D^\dagger M_D)_{21}(M_D^\dagger M_D)_{21}) = 0 \quad (8)\]

\[(M_{D}^{\dagger} M_{D})_{11} = (M_{D}^{\dagger} M_{D})_{22}\]

Due to the degeneracy of \(N_1, N_2\), the decay of both contributes to \(\epsilon_i\). However, eq. (8) avoids the possible singularity in \(f(x)\). Hence,

\[\epsilon_L = \frac{1}{8\pi v^2 (M_D^\dagger M_D)_{11}} (\text{Im}[(M_D^\dagger M_D)_{22}^2] + \text{Im}[(M_D^\dagger M_D)_{23}^2]) f(M_3^2/M_1^2).\]

The BAU is given then (in the minimal supersymmetric SM) as

\[Y_B = -1/3 \frac{\epsilon_L}{g^*} d_{B-L}\]

where \(g^* = 228.75\) and \(d_{B-L}\) is the dilution factor due to inverse decay washout effects and lepton number violating scattering. It must be obtained by solving the corresponding Boltzmann equation. There are different approximate solutions in the literature. The frequently used approximate solution [17] is good only for

\[K = \frac{\tilde{m}_1 M_P}{1.7 \times 8\pi v^2 g^*} = \frac{\tilde{m}_1 (eV)}{1.08 \times 10^{-3} (eV)} > 1\]

where \(\tilde{m}_1 = \frac{(M_D^\dagger M_D)_{11}}{M_1}\). In our model however, \(K \approx 10^{-2}\).

Buchmüller, Di Bari and Plümer [6] studied recently in detail both cases \(K > 1\) and \(K < 1\). They found that for \(K < 1\) one must take into account thermal corrections due to the gauge bosons and the top quark. Hence, \(d_{B-L}\) depends on “initial conditions” and they found that for \(K \approx 10^{-2}\)

\[10^{-4} \geq d_{B-L} \leq 10^{-2}\]

Hirsch and King [18] give empirical approximate solutions for the case \(K \ll 1\). The solution corresponding to our model is

\[\log_{10}(d_{B-L}) = 0.8 \times \log_{10}(\tilde{m}_1 eV) + 1.7 + 0.05 \times \log_{10}(M_1/10^{10} GeV).\]

See Figure 9 in their paper where \(d_{B-L}\) is called \(\kappa_f\).
I will use this expression to have a definite prediction. The results for the five solutions are given in Table 2.

Table 2. The CP asymmetry \( \epsilon_L \), the dilution factor \( d_{B-L} \) and the Baryon asymmetry \( Y_B \) for the five solutions.

| Solution | \( \epsilon_L \)       | \( d_{B-L} \)       | \( Y_B \)         |
|----------|------------------------|---------------------|-------------------|
| 1        | \(-6.5 \times 10^{-7}\) | 0.0064              | \(6.1 \times 10^{-12}\) |
| 2        | \(-6.6 \times 10^{-5}\) | 0.0074              | \(7.1 \times 10^{-10}\) |
| 3        | \(-7.4 \times 10^{-5}\) | 0.0088              | \(9.5 \times 10^{-10}\) |
| 4        | \(-1.3 \times 10^{-6}\) | 0.009               | \(1.7 \times 10^{-11}\) |
| 5        | \(-5.6 \times 10^{-5}\) | 0.06                | \(4.9 \times 10^{-10}\) |

This must be compared with the experimental results:

- BOOMerANG and DASI \(^{19}\)
  \[0.4 \times 10^{-10} \leq Y_B \leq 1.0 \times 10^{-10}\]
- WMAP and Sloan Digital Sky Survey \(^{20}\)
  \[Y_B = (6.3 \pm 0.3) \times 10^{-10}\]

Hence:
- Solutions 1 and 3 are probably excluded. The other solutions are consistent with the experimental observation, especially if the uncertainty in \( d_{B-L} \) is taken into account.
- All solutions have the right sign. This is the main prediction of the model in view of the uncertainty in \( d_{B-L} \).

I must emphasize that there is no ambiguity in the prediction of the sign because of the following reasons:

a) The sign of \( M_1 \) must be positive because \( \epsilon_i \) is calculated in terms of \( M_D \) which is the neutrino Dirac mass matrix in the basis where the RH neutrino mass matrix (7) is diagonal, real and positive.

b) The parameters and especially the phases of \( M_{\nu}^{Dirac} \) (6) are fixed without ambiguity for each one of the above solutions, although one cannot write explicitly their dependence on \( \alpha \). As was mentioned before, the entries to the mass matrices are solutions of non-linear equations in which the induced components of \( \Phi_{126} \) (with the phase \( \alpha \)) are involved. The physical value of \( \alpha \) is then fixed by requiring that \( J_{\text{Jarlskog}} \sim 10^{-5} \) to be \( \alpha \sim 0.003 \).

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\(^6\)In a recent paper Frampton, Glashow and Yanagida in ref. \(^{12}\) presented a model where the sign of the BAU can be related to the CP violation in neutrino oscillation experiments. In our model both CP violation in the neutrino oscillation as well as the sign of the BAU are predicted in terms of CP violation in the quark sector.
To complete the predictions of the model let me use the complex lepton mixing matrix $U_{PMNS}$ of Ref. [15] (see Appendix II) to give the amount of CP violation in the neutrino oscillation

$$J_{\text{leptons}} = Im(U_{11}U_{22}^*U_{12}^*U_{21}^*)$$

and the value of $<m_{ee}>$

$$<m_{ee}> = \sum_{i=1}^{3} (U_{e1}^2)m_i$$

relevant for the neutrino-less double-beta decay $\beta\beta_0\nu$. See Table 3.

Table 3. The CP violation invariant for the leptonic sector $J_{\text{leptons}}$ and the effective neutrino mass for the neutrino-less double-beta decay for the five solutions.

| Solution | 1      | 2      | 3      | 4      | 5      |
|----------|--------|--------|--------|--------|--------|
| $J_{\text{leptons}}$ | 0.0092 | 0.00059 | 9.8$\times 10^{-6}$ | 7.8$\times 10^{-6}$ | 6.6$\times 10^{-6}$ |
| $<m_{ee}>$ | 0.0031 | 0.005  | 0.0068 | 0.0056 | 0.0029 |

CONCLUSIONS

I presented in the paper the following observations:

CP is naturally broken spontaneously at high energies in $SO(10)$ GUTs. A phase is generated in a VEV and not in the Yukawa couplings, as it is usually done. This can be used as the only origin CP violation.

In the framework of a SUSY $SO(10)$ model that uses this idea, fitting to the observed CP violation, as it is reflected in the CKM matrix, fixes uniquely the CP breaking in the leptonic sector without free parameters. An explicit calculation of leptogenesis in this model gives solutions consistent with the range and sign of the observed BAU$^8$.

Our model applies the conventional see-saw mechanism $^{22}$, it is possible however, to use a similar program for the type II see-saw $^{23}$ as well $^{24}$. The large value of the RH neutrino mass can be incompatible with the gravitino problem if SUSY is broken in the framework of mSUGRA. Possible

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$^7$m_1 in our solutions is of $O(10^{-3} eV)$.

$^8$“A common origin for all CP violations” was suggested recently also by Branco, Parada and Rebelo $^{21}$. They use a non-SUSY SM extended by adding scalar Higgs, leptons and exotic vector-like quarks. The complex phase is generated spontaneously in the VEV of the heavy singlet scalar meson. The connection with the low energy CP violation in the hadronic sector is obtained only via mixing with the exotic quarks. They give also no explicit value for the leptogenesis.
solutions are discussed in the literature. E.g. Ibe, Kitano, Murayama and Yanagida \cite{25} presented very recently a nice solution based on anomaly mediated SUSY breaking.

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APPENDIX I
The Dirac neutrino mass matrices for the five solutions (for $\tan \beta = 10$) in GeV.

| Solution | GeV | 1       | 2       | 3        | 4       | 5        |
|----------|-----|---------|---------|----------|---------|---------|
| $\text{Re}(M^\text{Dirac}_{\nu})_{12}$ | 17.486 | 26.953  | -41.320 | -41.320 | -28.274 |
| $\text{Im}(M^\text{Dirac}_{\nu})_{12}$ | 0.0394  | 0.0607  | 0.0929  | -0.0929 | -0.06356|
| $\text{Re}(M^\text{Dirac}_{\nu})_{21}$ | 17.654  | 27.120  | -41.218 | -41.218 | -28.172 |
| $\text{Im}(M^\text{Dirac}_{\nu})_{21}$ | 0.0394  | 0.0607  | 0.0929  | -0.0929 | -0.06356|
| $(M^\text{Dirac}_{\nu})_{23}$ | -113.425 | -142.425 | 116.073 | 82.073  | 102.073 |
| $(M^\text{Dirac}_{\nu})_{32}$ | -14.700  | 14.302  | 10.695  | 44.695  | 24.695  |
| $\text{Re}(M^\text{Dirac}_{\nu})_{33}$ | -127.913 | -176.670 | 146.103 | 146.103 | 78.715  |
| $\text{Im}(M^\text{Dirac}_{\nu})_{33}$ | -0.3152 | -0.4249 | 0.2788  | 0.2788  | 0.1271  |
APENDIX II

The leptonic mixing matrix for the different solutions.

| Solution | 1     | 2     | 3     | 4     | 5     |
|----------|-------|-------|-------|-------|-------|
| Re($U_{PMNS}$)_{11} | -0.8583 | 0.8136 | 0.7465 | 0.8579 | 0.8740 |
| Im($U_{PMNS}$)_{11}  | 0.000004 | 0.00034 | -0.000001 | -0.000001 | 0.000001 |
| Re($U_{PMNS}$)_{12}  | -0.5104 | -0.5778 | -0.6589 | -0.5059 | -0.4806 |
| Im($U_{PMNS}$)_{12}  | -0.000007 | 0.000007 | -0.00027 | -0.00021 | -0.0002 |
| Re($U_{PMNS}$)_{13}  | -0.0526 | -0.0644 | 0.0927 | 0.0897 | 0.0717 |
| Im($U_{PMNS}$)_{13}  | 0.000002 | 0.00026 | 0.00042 | 0.00004 | 0.00003 |
| Re($U_{PMNS}$)_{21}  | -0.3496 | -0.4869 | -0.4653 | -0.3754 | -0.2492 |
| Im($U_{PMNS}$)_{21}  | 0.00191 | 0.00190 | 0.00212 | 0.0017 | 0.00088 |
| Re($U_{PMNS}$)_{22}  | 0.6567 | -0.6168 | -0.6167 | -0.7364 | -0.5670 |
| Im($U_{PMNS}$)_{22}  | -0.0030 | 0.0029 | 0.00260 | 0.0031 | 0.00018 |
| Re($U_{PMNS}$)_{23}  | -0.6682 | -0.6185 | -0.6350 | -0.5628 | -0.7829 |
| Im($U_{PMNS}$)_{23}  | 0.0031 | 0.00285 | 0.0029 | 0.0026 | 0.0028 |
| Re($U_{PMNS}$)_{31}  | -0.3756 | -0.3176 | -0.4756 | -0.3508 | -0.4172 |
| Im($U_{PMNS}$)_{31}  | 0.00082 | 0.00085 | 0.00216 | 0.0009 | 0.0011 |
| Re($U_{PMNS}$)_{32}  | 0.5552 | -0.6168 | -0.4309 | -0.4492 | -0.6664 |
| Im($U_{PMNS}$)_{32}  | -0.00121 | 0.00127 | 0.0009 | 0.00097 | 0.0014 |
| Re($U_{PMNS}$)_{33}  | 0.7421 | 0.7832 | 0.7669 | 0.82168 | 0.6179 |
| Im($U_{PMNS}$)_{33}  | -0.00163 | -0.00204 | -0.0020 | -0.00213 | -0.0016 |