The non-forward BFKL amplitude and rapidity gap physics

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Abstract

We discuss the BFKL approach to processes with large momentum transferred through a rapidity gap. The Mueller and Tang scheme to the BFKL non-forward parton-parton elastic scattering amplitude at large $t$, is extended to include higher conformal spins. The new contributions are found to decrease with increasing energy, as follows from the gluon reggeisation phenomenon, and to vanish for asymptotically high energies. However, at moderate energies and high $|t|$, the higher conformal spins dominate the amplitude. We illustrate the effects by studying the production of two high $E_T$ jets separated by a rapidity gap at HERA energies. In a simplified framework, we find excellent agreement with the HERA photoproduction data once we incorporate the rapidity gap survival probability against soft rescattering effects. We emphasize that measurements of the analogous process in electroproduction may probe different summations over conformal spins.
1 Introduction

High energy hadronic scattering with large momentum transfer $|t| \gg \Lambda_{QCD}^2$ and rapidity $y \gg 1$ is an excellent testing ground for perturbative QCD. The most interesting is the case when colour is not exchanged in the interaction. Our understanding of such processes (hard colour singlet exchange) is based on the BFKL equation [1, 2, 3] which resums the gluonic ladder diagrams in the leading logarithmic approximation1. The best known processes in which the behaviour of the scattering amplitude may be tested are elastic vector meson production [4, 5, 6], diffractive, proton dissociative $\gamma p$ scattering [7] and events with gaps between jets in the appropriate kinematic regime [8, 9, 10, 11, 12, 13]. Two main approaches to determine non-forward BFKL amplitudes have been proposed.

The first approach relies on the conformal symmetry of the leading logarithmic BFKL equation, which permits an analytical solution of the problem [14, 3]. The applications follow the Mueller and Tang [9] subtraction scheme to obtain the elastic parton-parton scattering amplitude. It is valid for an asymptotically large rapidity gap. However, it is doubtful whether this asymptotic formula may be used for the currently available measurements. The main problem is, that the Mueller-Tang cross-section for gaps between jets is, even for $y \sim 5$, much smaller than the lowest order two-gluon exchange cross-section. Of course, one has to include the gluon reggeization factor which suppresses the infrared sensitive part of the two-gluon amplitude [8, 15], but still this contribution appears to be very important [12, 13] for the region of $y$ probed in the current experiments [16].

The other approach is based on numerical studies of the non-forward BFKL equation [4, 12, 13]. The main advantage of this method is that it does not require any restrictions imposed on $y$ and takes into account all the available details of the impact factors. An important ingredient of this framework is that it is possible to go beyond the leading logarithmic approximation by including some phenomenological modifications of the BFKL kernel which are expected to resum a major part of the higher order corrections, like the running of the coupling constant along the ladder and the imposition of the consistency constraint [17, 18, 7].

The main purpose of this paper is to generalize the analytical approach so that the BFKL parton scattering elastic amplitude can be used at lower values of $y$. We will demonstrate that in this case we have to include the higher conformal spin contributions and so we are able to trace the phenomenon of gluon reggeization in a representation given by the conformal

\footnote{Recently, also the next-to-leading corrections have become available [4].}
Figure 1: The Feynman diagram illustrating the mechanism of photo- and electroproduction of two high $E_T$ jets separated by a rapidity gap.

eigenfunctions of the BFKL kernel. Possible phenomenological effects of higher conformal spins in the forward BFKL amplitude have been discussed, for example, in [19, 20].

From the experimental point of view, the process of interest (with a large rapidity gap between two high $E_T$ jets) has been observed at the Tevatron [16] and now data are becoming available for the analogous process at HERA [21, 22]. For example, in Fig. 1 we show the production mechanism for the diffractive process at HERA. Originally, it was claimed [16] that the Tevatron observations strongly disagree with the BFKL approach. However a closer study shows that the contradiction disappears when we allow for the effects of (i) hadronisation, (ii) the survival probability of the rapidity gaps and (iii) asymmetric non-asymptotic BFKL contributions. Numerically, at these energies it was found at the partonic level [12, 13] that the elastic parton-parton amplitude may be well described by two reggeized gluon exchange. Indeed, as we will show in Section 2, the non-asymptotic (i.e. higher conformal spin) components may be summed to give reggeized two gluon exchange. We note that the conformal spin components contribute up to the value of the conformal spin $n \sim \sqrt{E_T/k_0}$, where the infrared physical cutoff $k_0$ is driven by the size of the incoming state.

In Section 3 we consider the corresponding process at HERA: that is two high $E_T$ jets separated by a rapidity gap as shown in Fig. 1. Here we have a second variable, the photon virtuality $Q^2$, which allows us to change the size of the incoming $q\bar{q}$ state. In this way one can study the effect of varying the number of conformal spin components. We have attempted to summarize the essential points of the analysis in a self-contained, and more physical, way when describing the application in Section 3.
2 Elastic parton-parton scattering amplitude

The quark-quark elastic cross-section at high energies reads

\[ \frac{d\sigma_{qq}}{dt}(y) = \frac{4\alpha_s^4}{81\pi} |A(y, t)|^2, \]  

(1)

where the amplitude \( A(y, t) \) is

\[ A(y, t) = \int d^2k \, d^2k' \, f^a(k, k'; y). \]  

(2)

2.1 Two gluon exchange

In the \( y \to 0 \) limit, the BFKL amplitude reduces to its lowest level approximation of the exchange of two elementary gluons, i.e.

\[ f^a(k, k'; 0) = \frac{\delta(k - k')}{k^2(q - k)^2}. \]  

(3)

The integral over the tranverse momenta in eq. (2) is infrared divergent in this limit. One may, however, introduce an infrared cutoff \( k_0^2 \) and modify the gluon propagators \( 1/k^2 \to 1/(k^2 + k_0^2) \). Such cutoff has a physical origin. It may be related either to hadron sizes or to the gluon propagation length in the QCD vacuum. With such a substitution, (2) becomes

\[ A(y = 0, t) \simeq \frac{2\pi}{q^2} \log(q^2/k_0^2) \]  

(4)

where \( q \) is the momentum transfer. For the production of a pair of high \( E_T \) jets, we have \( q \simeq E_T \). At not large rapidities and large \( |t| \), the most important effect of the BFKL evolution is that the exchanged gluons become reggeized. This accounts for a no-emission amplitude from a system of two gluons in the colour singlet state, where one of the gluons is much softer than the other. To a good approximation, it leads to an additional factor multiplying the gluon propagator close to the singular point

\[ \frac{1}{k^2 + k_0^2} \to \frac{1}{k^2 + k_0^2} \left( \frac{k^2}{q^2} \right)^z \]  

(5)

with \( z = 3\alpha_s y/(2\pi) \). Then for moderate \( y \)

\[ A(y, t) \simeq \frac{2\pi}{q^2} \int_0^{q^2} dk^2 \, \frac{1}{k^2 + k_0^2} \left( \frac{k^2}{q^2} \right)^z. \]  

(6)

For \( y \neq 0 \) the integration in (6) may be safely performed, and the limit \( k_0^2 \to 0 \) taken, to obtain

\[ A(y, t) = \frac{2\pi}{q^2} \frac{1}{z} \left[ 1 - \left( \frac{k_0^2}{q^2} \right)^z \right] \to \frac{2\pi}{z q^2}. \]  

(7)

We shall trace how the above expression arises from the summation over the conformal spins.
2.2 BFKL conformal components

The LO BFKL expression for $f^q(k, k'; y)$ is [14, 13]

$$f^q(k, k'; y) = \frac{1}{(2\pi)^6} \sum_{n=-\infty}^{\infty} \int d\nu \left\{ \frac{\nu^2 + n^2/4}{[p^2 + (n-1)^2/4][\nu^2 + (n+1)^2/4]} \times \exp[\omega_n(\nu)y] I^1_{n,\nu}(k, q) I^2_{n,\nu}^*(k', q) \right\} \tag{8}$$

where

$$\omega_n(\nu) = \frac{3\alpha_s}{\pi}[2\psi(1) - \psi(1/2 + |n|/2 + i\nu) - \psi(1/2 + |n|/2 - i\nu)] \tag{9}$$

are the eigenvalues of the BFKL kernel. The functions $I^s_{n,\nu}$ are constructed from the impact factors $\Phi^s(k, q)$ and the eigenfunctions of the BFKL kernel. To be precise

$$I^s_{n,\nu}(k, q) = \Phi^s(k, q) \int d^2\rho_1 d^2\rho_2 E_{n,\nu}(\rho_1, \rho_2) \exp(ik\rho_1 + i(q-k)\rho_2) \tag{10}$$

where the eigenfunctions take the form

$$E_{n,\nu}(\rho_1, \rho_2) = \left( \frac{\rho_1 - \rho_2}{\rho_1 \rho_2} \right)^h \left( \frac{(\rho_1 - \rho_2)^*}{\rho_1 \rho_2} \right)^{\tilde{h}} \tag{11}$$

Here we have used the complex representation of transverse vectors $k$ and $\rho$ (which reveals the conformal symmetry), that is $k = k_x + ik_y$ and $\rho = \rho_x + i\rho_y$. The powers $h = 1/2 + n/2 + i\nu$ and $\tilde{h} = 1/2 - n/2 + i\nu$ are the conformal weights.

For parton scattering, the impact factors $\Phi^s(k, q)$ are constant functions of $k$. In other words for a point-like parton we have $\rho_1 = \rho_2$, and hence the eigenfunctions (11), the impact factors (10) and the amplitude $f^s$ of (8), vanish identically. However one can not use the expansion (8) over the conformal eigenfunctions $E_{n,\nu}$ for coloured initial objects. In such a case we face an infrared divergency in the integrations over the $\rho_i$ which correspond to contributions proportional to $\delta(k)$ or $\delta(q - k)$. For a colourless object these divergent terms are absent, since they are multiplied by zero – the total colour charge. Thus one has to consider the parton spectators which compensate the colour charge of our active quark. Assuming that these spectators are located at rather large distances we may follow the Mueller-Tang prescription, as was shown in [15].

The generalisation of Mueller-Tang prescription for arbitrary conformal spins reads:

$$\left( \frac{\rho_1 - \rho_2}{\rho_1 \rho_2} \right)^h \left( \frac{\rho_1 - \rho_2}{\rho_1 \rho_2} \right)^* \tilde{h} \to \left( \frac{\rho_1 - \rho_2}{\rho_1 \rho_2} \right)^h \left( \frac{\rho_1 - \rho_2}{\rho_1 \rho_2} \right)^* \tilde{h} - \left( \frac{1}{\rho_2} \right)^h \left( \frac{1}{\rho_2} \right)^* \tilde{h} - \left( \frac{-1}{\rho_1} \right)^h \left( \frac{-1}{\rho_1} \right)^* \tilde{h} =$$
\[
\left( \frac{\rho_1 - \rho_2}{\rho_1 \rho_2} \right)^{\hat{h}} \left( \left( \frac{\rho_1 - \rho_2}{\rho_1 \rho_2} \right)^* \right)^{\hat{h}} \left( \frac{1}{|\rho_2|} \right)^{1+2i\nu} \left( \frac{\rho_2}{|\rho_2|} \right)^{-n} - \left( \frac{1}{|\rho_1|} \right)^{1+2i\nu} \left( -\frac{\rho_1}{|\rho_1|} \right)^{-n}.
\] (12)

Note the minus sign in the last term, which will result in the cancellation of contributions with odd \(n\). We set \(\Phi^s(q,k) = 1\) for \(s = 1, 2\) and substitute (12) into (11) to obtain

\[
I_{n,\nu}^s(k,q) = - (2\pi)^3 i^n \left[ \delta(q - k) + (-1)^n \delta(k) \right] \frac{1}{q} \left( \frac{q}{2} \right)^{2i\nu} \frac{\Gamma(1/2 + n/2 - i\nu)}{\Gamma(1/2 + n/2 + i\nu)}.
\] (13)

We insert the last result into eq. (8) and then integrate over \(k\) and \(k'\) to obtain the non-forward amplitude:

\[
A(y,t) = 4 \frac{q^2}{q^2} \sum_{m=-\infty}^{\infty} \int d\nu \left\{ \frac{\nu^2 + m^2}{\left[ \nu^2 + (m - 1/2)^2 \right] \left[ \nu^2 + (m + 1/2)^2 \right]} \exp[\omega_{2m}(\nu) y] \right\}.
\] (14)

The last formula represents the desired generalisation of the Mueller-Tang result for the quark-quark elastic scattering amplitude. Note, that only contributions from even conformal spins \(n = 2m\) are left in the sum.

### 2.3 Sum over conformal spins

The resulting amplitude has the following properties. For very large \(y\) all the components with \(|m| > 0\) get suppressed because in this case \(\omega_{2m}(\nu) < 0\). Then, indeed, it is enough to retain only the leading term with \(m = 0\) (i.e. \(n = 0\)), which gives rise to an increasing part of the amplitude with the famous LO BFKL intercept

\[
A(y,t) \sim y^{-3/2} \exp \left( \frac{12 \log 2 \alpha_s}{\pi} y \right).
\] (15)

However, for \(y \to 0\) the expression is divergent, as expected from an inspection of the two gluon exchange amplitude, due to high \(1/|m|\) asymptotics in the terms under the sum (8):

\[
\int d\nu \frac{\nu^2 + m^2}{\left[ \nu^2 + (m - 1/2)^2 \right] \left[ \nu^2 + (m + 1/2)^2 \right]} = \begin{cases} 
\pi & \text{for } m = 0, \\
\frac{\pi}{|m|} \frac{m^2 - 1/8}{m^2 - 1/4} & \text{for } m \neq 0.
\end{cases}
\] (16)

For \(y > 0\) the divergence disappears due to the presence of the \(\exp[\omega_{2m}(\nu) y]\) term. Namely, for large \(m\) one has

\[
\omega_{2m}(\nu) \leq \omega_{2m}(\nu = 0) = \frac{6 \alpha_s}{\pi} \left[ -\log(|m|) - \gamma_E \right] + O(1/m^2),
\] (17)
which bounds from above the suppression factor in the sum over $m$ to be $|m|^{-6\alpha_s/\pi}$. This changes the $1/|m|$ asymptotics of the summed terms and ensures the convergence of the infinite sum. Note, that for small $y$ (i.e. $z \to 0$) the limiting behaviour of the amplitude is

$$A(y, t) \sim \sum_{m=1}^{\infty} \frac{8\pi}{q^2} m^{-1-4z} = \frac{2\pi}{zq^2} + \text{regular terms}. \quad (18)$$

Thus we have reproduced the answer (6,7), which was obtained by accounting for the gluon reggeization when the infrared cutoff $k_0 \to 0$.

In Fig. 2 we illustrate the result of taking a finite number of terms in the sum over the conformal moments in (14). We first plot the sum of terms with all conformal spins $n$ between -4 and 4, and then show the sum for terms between -20 and 20. Only even values of conformal spins contribute.
2.4 Insight from comparison with naive estimates

At very high rapidity the amplitude is dominated by the $n = 0$ component, while at low rapidity it is mainly two gluon exchange $\mathcal{I}$. We may therefore try to approximate the full amplitude by the sum of these two contributions. To compare this approximation with amplitude summed over conformal spins, we have to understand how the infrared parameter $k_0$ reveals itself in the BFKL expression $\mathcal{B}$ which was written in terms of conformal eigenfunctions. It turns out that the impact factor $I_{n,\nu}$ goes to zero for $|n| > n_{\text{max}}$, where the value of $n_{\text{max}}$ is regulated by the ratio $q/k_0$.

Let us focus on the integral $\mathcal{I}$ defining the function $I_{n,\nu}(k, q)$ for configurations with, say, $k \ll q$, which dominate at low $y$. In this case the typical values of $\rho_1$ and $\rho_2$ in integral $\mathcal{I}$ are $|\rho_1| \gg |\rho_2|$. The scale for $|\rho_1|$ is set by the size $R$ of the initial colour multipole.

Then, one may rewrite

$$
\left( \frac{\rho_1 - \rho_2}{\rho_1 \rho_2} \right)^h \left( \frac{\rho_1 - \rho_2}{\rho_1 \rho_2} \right)^{\bar{h}} \left( \frac{1}{\rho_2} \right)^h \left( \frac{1}{\rho_2^*} \right)^{\bar{h}} \left( 1 - h \frac{\rho_2}{\rho_1} - \bar{h} \frac{\rho_2^*}{\rho_1^*} \right) \tag{19}
$$

where only the leading correction in $|\rho_1/\rho_2|$ to the result obtained in the limit $R \to \infty$, is retained. Thus, a rough estimate of the relative correction to $\mathcal{I}$ coming from the fact that the size $R$ is finite, from the term proportional to $h \rho_2/\rho_1$, is given by the ratio

$$
C_{n,\nu} = \frac{- \int d^2 \rho_2 \left( h \rho_2 / R \right) \exp(i q \rho_2) \rho_2^{-h} (\rho_2^*)^{-\bar{h}}}{\int d^2 \rho_2 \exp(i q \rho_2) \rho_2^{-h} (\rho_2^*)^{-\bar{h}}} \tag{20}
$$

The integrals in $\mathcal{I}$ are of the same form as those used for the derivation of $\mathcal{B}$. In particular, the result for the denominator is already known to be

$$
\frac{2\pi i^n}{q} \left( \frac{q}{2} \right)^{2i\nu} \frac{\Gamma(1/2 + n/2 - i\nu)}{\Gamma(1/2 + n/2 + i\nu)} \tag{21}
$$

and the integral in the numerator may be obtained from $\mathcal{I}$ by substitutions $i\nu \to i\nu - 1/2$ and $n \to n - 1$. Thus, it is straightforward to find that

$$
C_{n,\nu} = \frac{(n/2 + i\nu)^2 - 1/4}{i q R} \tag{22}
$$

It may be seen that the typical value of $|\nu|$ in integral $\mathcal{B}$ is $|\nu| \approx |n|/2$ for large $|n|$. Then $|(n/2 + i\nu)^2 - 1/4| \approx n^2/2$ and after including a similar result from the term in eq. $\mathcal{I}$ containing $\bar{h} \rho_2^*/\rho_1^*$, the estimate of the total correction is

$$
2|C_{n,\nu}| \approx \frac{n^2}{q R} \tag{23}
$$
The Mueller-Tang scheme breaks down when the correction factor becomes large $|C_{\nu\nu}| \gtrsim 1$ and $I^n_{\nu\nu}$ gets suppressed in relation to the r.h.s of (13). Noting that $k^2_0 \simeq 1/R^2$, we find from (23) that the sum over conformal spins should be extended to

$$n_{\text{max}} = \sqrt{\frac{q}{k_0}}.$$  \hspace{1cm} (24)

We may compare the result of the summation up to $n_{\text{max}}$ with naive estimates given by a simple addition of the $n = 0$ contribution and the amplitude given by the two reggeized gluon exchange (B). The dashed curves in Fig. 2 show results for three values of the ratio $q/k_0$, namely $4^2$, $20^2$ and $\infty$. The agreement between the naive estimates and the truncated sum over conformal spins can be understood as follows. At $y = 0$ the sum over $n$ in (8) takes form

$$A(y = 0, t) \simeq \frac{8\pi}{q^2} \left( \frac{1}{2} + \sum_{m=1}^{n_{\text{max}}/2} \frac{1}{m} \right) \simeq \frac{2\pi}{q^2} \log \left( \frac{q^2}{k_0^2} \right),$$  \hspace{1cm} (25)

where we have used (14) and (16). In this way we reproduce the leading behaviour of the amplitude given by (B). Indeed, as seen in Fig. 2, the naive sum of the $n = 0$ component and the two (reggeized) gluon contribution (B) for the two cases of $q/k_0 = 4^2$ and $20^2$ are rather close to the sum over the conformal spins $|n|$ up to 4 and 20 respectively.[3]

Note that we do not have any non-perturbative effects modifying the gluon perturbative propagators. However, the sensitivity to the infrared details decreases with increasing $y$, since these details influence more the contributions from higher conformal spins, which get suppressed with $y$. The suppression is faster for larger $|n|$. In view of the above discussion and the recent results of Ref. [12, 13], the extension to include higher conformal spins is necessary to understand the Tevatron data for hard colour singlet exchange.

3 Phenomenological consequences for gaps between jets

As we have just discussed, the above formalism is relevant to processes mediated by colour singlet exchange with high momentum transfer $q$. As noted, the classic example is the production of a pair of high $E_T$ jets separated by large rapidity gap $y$. We emphasize that the conventional (asymptotic) BFKL amplitude only dominates at high rapidities well above the

\footnote{Note that, in general, the naive sum slightly overestimates the true result as the two gluon contribution (B) already contains part of the $n = 0$ component.}
reach of present experiments. This is illustrated by the discrepancy between the $n = 0$ curve and the full result (given by the continuous curves on Fig. 2), in the region $3\alpha_s y / \pi < 1$ currently sampled experimentally. Instead, in this region it is necessary to sum all the contributions with conformal spins up to $n_{\text{max}} = \sqrt{q/k_0}$, where $k_0$ is a physical infrared cutoff. On the other hand, in this domain the amplitude is well described by two reggeized gluon exchange, as illustrated by the dotted curves in Fig. 2 (note, that the uppermost curve corresponds $q/k_0 \to \infty$ whereas the lower dotted curves correspond to the choices $\sqrt{q/k_0} = 4$ and 20). The physical meaning of gluon reggeization, that is of the factor $(k^2/q^2)^z$ in (26), is that it reflects the fact that the emissions of extra gluons are forbidden within the rapidity gap interval $y$, so that pure elastic parton-parton scattering occurs. This is equivalent to the normal Sudakov-like suppression. The latter suppression is the probability not to emit gluons with transverse momentum $p_t$ in the interval $(k_0, q)$, which takes the form $\exp(-n_g)$. The quantity $n_g$, the anticipated average number of emissions, is

$$n_g = \int_{k_0^2}^{q^2} \frac{dp_t^2}{p_t^2} \frac{3\alpha_s}{\pi} y.$$  

Thus, the non-forward lowest-order two-gluon exchange amplitude should be multiplied by the suppression factor

$$\exp(-n_g/2) = \left(\frac{k_0^2}{q^2}\right)^{3\alpha_s y/(2\pi)},$$  

as given in (26). Thus we see that in the BFKL amplitude the suppression is generated by the resummation of the virtual corrections.

The hard colour singlet exchange has been investigated experimentally at the Tevatron [16] by measuring events with gaps between jets. In [11] these data were compared with BFKL results in the standard Mueller-Tang approximation. It was found that the rising $E_T$ dependence of the gap fractions may be reproduced by the model only when a fixed value of coupling constant is used. The assumed lack of running of the coupling with increasing momentum transfer is, however, difficult to motivate. The experimental status of the gap fraction dependence on the jet separation in rapidity, $\Delta y$, is not so certain as the observed $E_T$ dependence. In particular, CDF results indicate a decreasing tendency at high $\Delta y$, contrary to D0 data. The error bars are still too large to claim inconsistency and both the experimental distributions agree with a flat $\Delta y$ dependence. This is, however, incompatible with the predictions based on the Mueller-Tang approximation, which give a steep rise of the gap fraction at large rapidity [11].

In the Tevatron kinematical conditions the bulk of data comes from $E_T \sim 20$ GeV and $\Delta y \sim 5$. Then, if our choice of the cutoff scale $k_0 \sim 1$ GeV is correct, the Mueller-Tang approximation gives about a half of the contribution to the scattering amplitude, thus about
Figure 3: Gap fraction for photoproduction of two high $E_T$ jets separated by a rapidity gap $\Delta \eta$. The experimental data [22] are obtained with the rapidity gap events defined as those with the maximal total transverse energy flow in between the high-$E_T$ jets to be $E_T^{\text{cut}} = 0.5$ GeV. The continuous and dashed curves show the partonic non-leading BFKL prediction, with and without the gap survival probability factor included respectively, and the dotted curve is the LO BFKL prediction without the gap survival probability factor taken into account.

25% of the cross-section. After setting $\alpha_s = 0.17$, as in [11], this may be seen in Fig. 2 by comparing the sum over conformal spins up to $n = 4$ with $n = 0$ component, at $3\alpha_s\gamma/\pi \sim 0.8$. Therefore the Mueller-Tang approximation should not be used to describe the available Tevatron data unless the cut-off $k_0$ is much larger than we expect.

Recently [12, 13] it has been demonstrated that when the gluon reggeization phenomenon is accounted for, no significant discrepancies between the data and BFKL results appear in either the $E_T$ or the $\Delta y$ distribution. This conclusion holds both for fixed and running couplings, provided they are consistently used in the amplitude at the scale set by the typical virtuality of each vertex.

The result obtained in Refs. [12, 13], from theoretical considerations based on the BFKL framework, was confronted with experimental data from the CDF and D0 collaborations [16].
Besides the hard parton-parton scattering amplitude for colour singlet exchange, other important effects, like hadronisation corrections and the gap survival probability were included using a complete Monte Carlo treatment. It was found \cite{12, 13} that the full BFKL prediction was in agreement with the Tevatron data, contrary to calculations based on the asymptotic Mueller-Tang approximation.

An interesting way of studying this effect in more detail is to observe events with a large rapidity gap between two high $E_T$ jets in diffractive deep inelastic scattering at HERA, as sketched in Fig. 1. Since one jet comes from photon dissociation, the infrared cutoff $k_0$ is controlled by the photon virtuality $Q^2$. Therefore, there is the possibility to vary $k_0$ while retaining the same kinematics of the hard parton-parton interaction. In this way, we avoid complications from hadronisation, variation of parton densities etc. Another advantage of electroproduction is that we have high survival probability, $S^2$, of the rapidity gap against soft rescatterings, that is $S^2 \approx 1$, contrary to the analogous jet production process at the Tevatron.

At present, however, data are available at HERA \cite{22} for the photoproduction of jets separated by a rapidity gap. These data are compared with the corresponding non-forward BFKL predictions in Fig. 3. The two uppermost curves are obtained using the solutions to the BFKL equation given in Ref. \cite{12, 13} and correspond to BFKL amplitudes with and without incorporating resummations of higher order effects. In the leading order BFKL calculation a fixed value of $\alpha_s = 0.17$ was used, whereas the running coupling was taken in the non-leading BFKL case, which explains why the LO BFKL curve lies so low. Nevertheless, for photoproduction the probability of soft rescattering, which produces secondaries in the rapidity gap, is not negligible. To calculate the resulting suppression factor $S^2$ we have used the formalism of Ref. \cite{23}. Recall that, there, the model was tuned to describe the available soft $pp$ and $p\bar{p}$ interactions throughout the CERN ISR–Tevatron energy range. Assuming Vector Meson Dominance, the rescattering in photoproduction may occur between a virtual vector meson and the proton. Data are not available for soft vector meson–proton scattering. However using the additive quark model, and the analogy between the light vector mesons $V = \rho, \omega, \ldots$ and the pion, we expect\footnote{Note that the ZEUS collaboration \cite{24} estimate that $\sigma_{tot}(\pi p) = 31 \pm 4$ mb, by observing the interference between the pions from $\rho$ meson and $\pi^+\pi^-$ background.}

$$\sigma_{tot}(Vp) \simeq 30 \text{ mb}$$

at the collision energies relevant to the HERA data, that is $W \approx 200$ GeV. With this cross
After taking this factor into account we obtain the final prediction shown by the lower continuous curve in Fig. 3. There is thus an excellent agreement between the data and the theory.

It is appropriate to comment on the approximations made to obtain this prediction for the photoproduction of jets separated by a rapidity gap at HERA. We have assumed that the effects of hadronization and of producing gaps in the conventional colour-octet exchange scattering are small. We expect the experimental cut $E_T^{\text{cut}}$ on soft secondaries to suppress these effects.

Moreover, although we believe our two-channel eikonal calculation of the survival factor $S^2$, using the framework of [23], is the best that can be done at present, we note that again it relies on soft phenomena. Clearly it is important to study all these effects in more detail within a Monte Carlo framework as in [12, 13]. This will, among other things, allow a study of the influence of the parameter $E_T^{\text{cut}}$ in the gap definition [25] on the gap fraction.

When the HERA luminosity increases, it will be particularly informative to observe electroproduction of high $E_T$ jets separated by large rapidity gaps. This will allow $Q^2$, as well as the jet $E_T$, to be varied, and hence conformal spin summations up to different $n_{\text{max}}$ to be probed. Moreover, here, there are no uncertainties connected with the survival factor $S^2$, since it is predicted to be 1 for this process. These data will therefore allow a test of QCD radiative effects, which are important ingredients in all non-forward diffractive phenomena.

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Inputting the cross section of (28), the model [23] predicts the elastic slope $B \simeq 11 \text{ GeV}^{-2}$, in agreement with ZEUS observations for $\gamma p \rightarrow pp$. 

\footnotesize

4Inputting the cross section of (28), the model [23] predicts the elastic slope $B \simeq 11 \text{ GeV}^{-2}$, in agreement with ZEUS observations for $\gamma p \rightarrow pp$. 

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