Search for $b \to u$ transitions in $B^{+} \to [K^{-}\pi^{+}\pi^{0}]DK^{0}$ decays

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We present a study of the decays $B^+ \rightarrow DK^+$ with $D$ mesons reconstructed in the $K^+\pi^-\pi^0$ or $K^-\pi^+\pi^0$ final states, where $D$ indicates a $D^0$ or a $\bar{D}^0$ meson. Using a sample of $474 \times 10^6$ $B\bar{B}$ pairs collected with the BABAR detector at the PEP-II asymmetric-energy $e^+e^-$ collider at SLAC.
I. INTRODUCTION

CP violation effects are described in the standard model of elementary particles with a single phase in the Cabibbo-Kobayashi-Maskawa quark mixing matrix $V_{ij}$ [1]. One of the unitarity conditions for this matrix can be interpreted as a triangle in the plane of Wolfenstein parameters [2], where one of the angles is $\gamma = \text{arg} (-V_{ub}^* V_{ud} / V_{cb}^* V_{cd})$. Various methods to determine $\gamma$ using $B^+ \to DK^+$ decays have been proposed [3–5]. In this paper, we consider the decay channel $B^+ \to DK^+$ with $D \to K^- \pi^\pm \pi^0$ [6] studied through the Atwood-Dunietz-Soni (ADS) method [4]. In this method, the final state under consideration can be reached through $b \to c$ and $b \to u$ processes as indicated in Fig. 1. We obtain the final error of this measurement, are uncorrelated.

following the ADS method, we search for $B^+ \to K^- \pi^\pm \pi^0 \ell^- \nu_\ell$ events, where the favored $B^+ \to D^{0} K^+$ decay, followed by the doubly-Cabibbo-suppressed $D^0 \to K^- \pi^\pm \pi^0$ decay, interferes with the suppressed $B^+ \to D^{0} K^+$ decay, followed by the Cabibbo-favored $D^0 \to K^- \pi^\pm \pi^0$ decay. These are called “opposite-sign” events because the two kaons in the final state have opposite charges. We also reconstruct a larger sample of “same-sign” events, which mainly arise from the favored $B^+ \to D^{0} K^+$ decays followed by the Cabibbo-favored $D^0 \to K^- \pi^\pm \pi^0$ decays. We define $f \equiv K^- \pi^\pm \pi^0$ and $f^\prime \equiv K^- \pi^\pm \pi^0$. We extract

$$R^+ = \frac{\Gamma(B^+ \to [f]D^0 K^+)}{\Gamma(B^+ \to [f^\prime]D^0 K^+)}.$$  

we measure the ratios $R^+ = \frac{\Gamma(B^+ \to [f]D^0 K^+)}{\Gamma(B^+ \to [f^\prime]D^0 K^+)}$. We obtain $R^+ = (5.17^{+12.10}_{-10.15}(\text{stat})^{+2.13}_{-2.29}(\text{sys})) \times 10^{-3}$ and $R^- = (12.22^{+10.16}_{-10.15}(\text{stat})^{+2.13}_{-2.29}(\text{sys})) \times 10^{-3}$, from which we extract the upper limits at 90% probability: $R^+ < 23 \times 10^{-3}$ and $R^- < 29 \times 10^{-3}$. Using these measurements, we obtain an upper limit for the ratio $r_B$ of the magnitudes of the $b \to u$ and $b \to c$ amplitudes $r_B < 0.13$ at 90% probability.

\[ R^+ = \frac{\Gamma(B^+ \to [f]D^0 K^+)}{\Gamma(B^+ \to [f^\prime]D^0 K^+)} \]

from the selected $B^+$ and $B^-$ samples, respectively.

While our previous analysis [7] used another set of observables:

$$R^+ = \frac{\Gamma(B^+ \to [f]D^0 K^+)}{\Gamma(B^+ \to [f^\prime]D^0 K^+)}.$$  

we prefer to use observables defined in Eqs. (1) and (2) since their statistical uncertainties, which dominate in the final error of this measurement, are uncorrelated.

The amplitude of the two-body $D$ decay can be written as

$$A(B^+ \to D^{0} K^+) = |A(B^+ \to D^{0} K^+)| r_B e^{i\delta_B},$$  

where $r_B = |A(B^+ \to D^{0} K^+)| / |A(B^+ \to D^{0} K^+)|$ is the ratio of the magnitudes of the $b \to u$ and $b \to c$ amplitudes, $\delta_B$ is the CP conserving strong phase, and $\gamma$ is the CP violating weak phase. For the three-body $D$ decay we use similarly defined variables:

$$r^2_D = \frac{\Gamma(D^0 \to f)}{\Gamma(D^0 \to f)}.$$  

\[ r^2_D = \frac{\int d\tilde{m}^2_{\text{DPS}}(\tilde{m})}{\int d\tilde{m}^2_{\text{DPS}}(\tilde{m})} \]
SEARCH FOR $b \to u$ TRANSITIONS IN 

$\delta_D = (47^{\pm14}_{\pm11})^a$, are used in the signal yield estimation and $r_B$ extraction. The ratio $r_D$ has been measured in different experiments and we take the average value $r_D^b = (2.2 \pm 0.1) \times 10^{-3}$ [9]. Its value is small compared to the present determination of $r_B$, which is taken to be $(0.106 \pm 0.016)$ [10]. According to Eqs. (8) and (9), this implies that the measurements of ratios $R^\pm$ are mainly sensitive to $r_B$. For the same reason, the sensitivity to $\gamma$ is reduced, and therefore the main aim of this analysis is to measure $R^\pi$, $R^\rho$, and $r_B$. The current high precision on $r_B$ is based on several earlier analyses by the BABAR [7,11–13], BELLE [14–16], and CDF [17] Collaborations.

This paper is an update of our previous analysis [7] based on $226 \times 10^6 B\bar{B}$ pairs and resulting in a measurement of $R = (13^{+12}_{-10}) \times 10^{-3}$, which was translated into the 95% confidence level limit $r_B < 0.19$.

The results presented in this paper are obtained with 431 fb$^{-1}$ of data collected at the $Y(4S)$ resonance with the BABAR detector at the PEP-II $e^+e^-$ collider at SLAC, corresponding to $474 \times 10^5 B\bar{B}$ pairs. An additional “off-resonance” data sample of 45 fb$^{-1}$, collected at a center-of-mass (CM) energy 40 MeV below the $Y(4S)$ resonance, is used to study backgrounds from “continuum” events, $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, o, or c$).

II. EVENT RECONSTRUCTION AND SELECTION

The BABAR detector is described in detail elsewhere [18]. Charged-particle tracking is performed by a five-layer silicon vertex tracker and a 40-layer drift chamber. In addition to providing precise position information for tracking, the silicon vertex tracker and drift chamber measure the specific ionization, which is used for identification of low-momentum charged particles. At higher momenta, pions and kaons are distinguished by Cherenkov radiation detected in a ring-imaging device. The positions and energizes of photons are measured with an electromagnetic calorimeter consisting of 6580 thallium-doped CsI crystals. These systems are mounted inside a 1.5 T solenoidal superconducting magnet. Muons are identified by the instrumented flux return, which is located outside the magnet.

The event selection is based on studies of off-resonance data and Monte Carlo (MC) simulations of continuum and $e^+e^- \rightarrow Y(4S) \rightarrow B\bar{B}$ events. The BABAR detector response is modeled with GEANT4 [19]. We also use EVTGEN [20] to model the kinematics of $B$ meson decays and JETSET [21] to model continuum backgrounds processes. All selection criteria are optimized by maximizing the $S/\sqrt{S+B}$ ratio, where $S$ and $B$ are the expected numbers of the opposite-sign signal and background events, respectively. In the optimization, we assume an opposite-sign branching fraction of $4 \times 10^{-6}$ [9].

The charged kaon and pion identification criteria are based on a likelihood technique. These criteria are typically 85% efficient, depending on the momentum and polar angle, with misidentification rates at the 2% level. The $\pi^0$ candidates are reconstructed from pairs of photon candidates with an invariant mass in the interval [119, 146] MeV/$c^2$ and with total energy greater than 200 MeV. Each photon should have energy greater than 70 MeV.

The neutral $D$ meson candidates are reconstructed from a charged kaon, a charged pion, and a neutral pion. The correlation between the tails in the distribution of the $K\pi\pi^0$ invariant mass, $m_D$, and the $\pi^0$ candidate mass, $m_{\pi\pi}$, is taken into account by requiring $|m_D - m_{\pi\pi}|$ to be within 24 MeV/$c^2$ of its nominal value [9], which is 1.5 times the experimental resolution.

The $B^+$ candidates are reconstructed by combining $D$ and $K^+$ candidates, and constraining them to originate from a common vertex. The probability distribution of the cosine of the $B$ polar angle with respect to the beam axis in the CM frame, $\cos\theta_B$, is expected to be proportional to $(1 - \cos^2\theta_B)$. We require $|\cos\theta_B| < 0.8$.

We measure two almost independent kinematic variables: the beam-energy substituted (ES) mass $m_{ES} = \sqrt{(s/2 + \tilde{p}_0 \cdot \tilde{p}_B)^2 - E_0^2 - p_B^2}$, and the energy difference $\Delta E = E_B - \sqrt{s}/2$, where $E$ and $\tilde{p}$ are the energy and momentum, the subscripts $B$ and $0$ refer to the candidate $B$ meson and $e^+e^-$ system, respectively, $\sqrt{s}$ is the center-of-mass energy, and $E_B$ is measured in the CM frame. For correctly reconstructed $B$ mesons, the distribution of $m_{ES}$ peaks at the $B$ mass, and the distribution of $\Delta E$ peaks at zero. The $B$ candidates are required to have $\Delta E$ in the range $[-23, 23]$ MeV ($\pm 1.3$ standard deviations). We consider only events with $m_{ES}$ in the range [5.20, 5.29] GeV/$c^2$.

In less than 2% of the events, multiple $B^+$ candidates are present, and in these cases we choose that with a reconstructed $D$ mass closest to the nominal mass value [9]. If more than one $B^+$ candidate share the same $D$ candidate, we select that with the smallest $|\Delta E|$. In the following, we refer to the selected candidate as $B_{sig}$. All charged and neutral reconstructed particles not associated with $B_{sig}$, but with the other $B$ decay in the event, $B_{other}$, are called the rest of the event.

III. BACKGROUND CHARACTERIZATION

After applying the selection criteria described above, the remaining background is composed of nonsignal $B\bar{B}$ events and continuum events. Continuum background events, in contrast to $B\bar{B}$ events, are characterized by a jetlike topology. This difference can be exploited to discriminate between the two categories of events by means of a Fisher discriminant $F$, which is a linear combination of six variables. The coefficients of the linear combination are chosen to maximize the separation between signal and continuum background so that $F$ peaks at 1 for signal and at $-1$ for continuum background. They are determined with samples of simulated signal and continuum events,
and validated using off-resonance data. In the Fisher discriminant, we use the absolute value of the cosine of the angle between $B_{\text{sig}}$ and $B_{\text{other}}$ thrust axes, where the thrust axis is defined as the direction maximizing the sum of the longitudinal momenta of all the particles. Other variables included in $F$ are the event shape moments $L_0 = \sum_i p_i$, and $L_2 = \sum_i p_i |\cos \theta_i|^2$, where the index $i$ runs over all tracks and energy deposits in the rest of the event; $p_i$ is the momentum; and $\theta_i$ is the angle with respect to the thrust axis of the $B_{\text{sig}}$. These three variables are calculated in the CM system. We also use the distance between the decay vertices of $B_{\text{sig}}$ and $D$, the distance of closest approach between $K$ meson tracks belonging to signal decay chain, and $|\Delta t|$, the absolute value of the proper time interval between the $B_{\text{sig}}$ and $B_{\text{other}}$ decays [22]. The latter is calculated using the measured separation along the beam direction between the decay points of $B_{\text{sig}}$ and $B_{\text{other}}$ and the Lorentz boost of the CM frame. The $B_{\text{other}}$ decay point is obtained from tracks that do not belong to the reconstructed $B_{\text{sig}}$, with constraints from the $B_{\text{sig}}$ momentum and the beam-spot location. We use $m_{\text{ES}}$ and $F$ to define two regions: the fit region, defined as $5.20 < m_{\text{ES}} < 5.29$ GeV/$c^2$ and $-5 < F < 5$, and the signal region, defined as $5.27 < m_{\text{ES}} < 5.29$ GeV/$c^2$ and $0 < F < 5$.

The $B\bar{B}$ background is divided into two components: nonpeaking (combinatorial) and peaking. The latter consists of $B$-meson decays that have a well-pronounced peak in the $m_{\text{ES}}$ signal region. One of the decay channels which can mimic opposite-sign signal events, is the $B^+ \to D\rho^+$ decay with $D \to K^+ K^-$ and $\rho^+ \to \pi^+ \pi^0$. In order to reduce this contribution, we veto events for which the invariant $K^+ K^-$ pair mass $m_{K^+ K^-}$ is $|m_{K^+ K^-} - M_{D(PDG)}| > 20$ MeV/$c^2$ (with the $D$ meson invariant mass, $M_{D(PDG)}$, taken to be 1864.83 MeV/$c^2$ [9]). Simulations indicate that the remaining background is negligible.

Another possible source of peaking $B\bar{B}$ background is the decay $B^+ \to D\pi^+$ with $D \to K^+ \pi^- \pi^0$, which can contribute to the signal region of the same-sign sample due to the misidentification of the $\pi^+$ as a $K^+$. The number of events is expected to be about 8% of the total same-sign signal sample (see Table I).

The charmless $B^+ \to K^+ K^- \pi^+ \pi^0$ decay can also contribute to the signal region. The branching fraction of this decay has not been measured. Therefore the size of this background is estimated from the sidebands of the reconstructed $D$ mass, $1.904 < M_D < 2.000$ GeV/$c^2$ or $1.700 < M_D < 1.824$ GeV/$c^2$. The result of the study is reported in Table I. In the final fit, we fix the yield of the same-sign $B\bar{B}$ peaking background to the sum of charmless and open-charm events. The opposite-sign background in the final event sample is assumed to be negligible.

The overall reconstruction efficiency for signal events is $(9.6 \pm 0.1)\%$ for opposite-sign signal events and $(9.5 \pm 0.1)\%$ for same-sign signal events. These numbers are equal within the uncertainty as expected. The composition of the final sample is shown in Table I.

### IV. FIT PROCEDURE AND RESULTS

To measure the ratios $R^+$ and $R^-$, we perform extended maximum-likelihood fits to the $m_{\text{ES}}$ and $F$ distributions, separately for the $B^+$ and $B^-$ data samples. We write the extended likelihood functions $L^{\pm}$ as

$$L^{\pm} = e^{-N} \frac{N^{N}}{N!} \cdot \prod_{j=1}^{N} f^{\pm}(x_j, \theta, N^\prime),$$

where

$$f^{\pm}(x, \theta, N^\prime) = \frac{1}{N^\prime} \bigg[ \frac{\sum_{i=1}^{N_{\text{sig}}} f_{\text{sig},\pm}(x_j, \theta_{\text{sig},\pm})}{1 + R^\pm} \bigg] + \frac{\sum_{i=1}^{N_{\text{bkg}}} f_{\text{bkg},\pm}(x_j, \theta_{\text{bkg},\pm})}{1 + R^\pm},$$

where $f_{\text{sig},\pm}(x_j, \theta_{\text{sig},\pm})$, $f_{\text{sig},\pm}(x_j, \theta_{\text{sig},\pm})$, and $f_{\text{bkg},\pm}(x_j, \theta_{\text{bkg},\pm})$ are the probability density functions (PDFs) of the hypotheses that the event is a same-sign signal, opposite-sign signal, or a background event ($B_i$ are the different background categories used in the fit), respectively. $N$ is the number of events in the selected sample, and $N^\prime$ is the expectation value for the total number of events. The symbol $\theta$ indicates the set of parameters to be fitted. $N_{\text{bkg}}^{\text{total}}$ is the total number of signal events, $R^\pm = N_{\text{bkg}}^{\text{total}} / N_{\text{sig}}$, for the decays of the $B^\pm$ meson, and $N_{\text{bkg}}^{\text{total}}$ is the total number of events of each background component. For the opposite-sign events, the background comes from continuum and $B\bar{B}$ events. The peaking $B\bar{B}$ background is introduced as a separate component in the fit to the same-sign sample. The fit is performed to the $B^+$ sample (consisting of 15 706 events) to determine $R^+$ and $R^-$.
to the $B^-$ sample (consisting of 15057 events) to determine $R^-$. The PDFs for $R^+$ and $R^-$ fits are identical. The $R_{ADS}$ ratio is fitted to the same likelihood ansatz, but to the combined $B^+$ and $B^-$ data sample.

Since the correlations among the variables are negligible, we write the PDFs as products of the one-dimensional distributions of $m_{ES}$ and $\mathcal{F}$. The absence of correlation between these distributions is checked using multidimensional distributions of $m_{ES}$. The PDF parameters are derived from data when possible. We write the PDFs as products of the one-dimensional distributions for the $B^+$ and $B^-$ samples, including the extracted number of signal and background events and their statistical errors.

The data are well described by the overall fit result (solid blue line) which is the sum of the signal, continuum, nonpeaking, and peaking $B\bar{B}$ backgrounds.

![Graph](image-url)

**FIG. 2** (color online). Distribution of (a,b) $m_{ES}$ (with $\mathcal{F} > 0.5$) and (c,d) $\mathcal{F}$ (with $m_{ES} > 5.27$ GeV/$c^2$) and the results of the maximum-likelihood fits for the combined $B^+$ and $B^-$ samples (extracting $R_{ADS}$), for (a,c) opposite-sign and (b,d) same-sign decays. The data are well described by the overall fit result (solid blue line) which is the sum of the signal, continuum, nonpeaking, and peaking $B\bar{B}$ backgrounds.

| Sample | $B^+$ | $B^-$ | $B^+$ and $B^-$ |
|--------|-------|-------|-----------------|
| $R$, $10^{-3}$ | $5^{+12}_{-10}$ | $12^{+12}_{-10}$ | $9.1^{+8.2}_{-7.6}$ |
| $N_{ES}^{B^+}$ | 1032 ± 41 | 946 ± 39 | 1981 ± 57 |
| $N_{ES}^{B^-}$ | 305 ± 52 | 120 ± 36 | 402 ± 65 |
| $N_{ES}^{B\bar{B}}$ | 315 ± 44 | 329 ± 44 | 644 ± 62 |
| $N_{ES}^{cont}$ | 10290 ± 111 | 10017 ± 105 | 20329 ± 154 |
| $N_{ES}^{cont}$ | 3660 ± 69 | 3539 ± 68 | 7203 ± 76 |

The fits to the $m_{ES}$ for $\mathcal{F} > 0.5$ and the $\mathcal{F}$ distribution with $m_{ES} > 5.27$ GeV/$c^2$ are shown in Fig. 2, for the
the contributions related to the uncertainties on the PDF parameters. To evaluate
these, we repeat the fit varying the PDF parameters for each fit species within their statistical errors, taking into account correlations among the parameters (labeled as “PDF error” in Table III).

To evaluate the uncertainties arising from peaking background contributions, we repeat the fit varying the peaking B\(B\) background contribution within its statistical uncertainties and the errors of branching fractions, \(B\), used to estimate the contribution. For the opposite-sign events, only the positive part of the probability distribution is used in the evaluation.

Differences between data and MC (labeled as “Simulation” in Table III) in the shape of the \(f\) distribution are studied for signal components using the data control samples of \(B^{+} \rightarrow D \pi^{+}\) with \(D \rightarrow K^{+} \pi^{-} \pi^{0}\). These parameters are expected to be slightly different between the \(B \rightarrow D \pi\) and \(B \rightarrow D K\) samples. We conservatively take the systematic uncertainty as the difference in the fit results from the nominal parameters set (using MC events) and the parameters set obtained using the \(B \rightarrow D \pi\) data sample.

The systematic uncertainty attributed to the cross feed between opposite-sign and same-sign events has been evaluated from the MC samples. The number of same-sign events passing the selection of the opposite-sign events is taken as a systematic uncertainty. The efficiencies for same-sign and opposite-sign events were verified to be the same within a precision of 3% [25]. We hence assign a systematic uncertainty on \(R^{\pm}\) based on variations due to changes in the efficiency ratio by \pm 3%.

The systematic uncertainties for the ratios \(R^{+}\), \(R^{-}\), and \(R_{\text{ADS}}\) are summarized in Table III. The overall systematic errors represent the sum in quadrature of the individual uncertainties.

| Source                  | \(R^{+}\) | \(R^{-}\) | \(R_{\text{ADS}}\) |
|-------------------------|-----------|-----------|-------------------|
| PDF error               | +1.1      | 1.1       | 1.0               |
| Same-sign peaking background | 0.2      | 0.5       | 0.2               |
| Opposite-sign peaking background | -0       | +0       | +0               |
| Simulation              | 0.6       | 0.6       | 0.7               |
| \(B\) errors            | 0.2       | 0.6       | 0.4               |
| Cross feed contribution | 0.1       | 0.4       | 0.3               |
| Efficiency ratio        | 0.1       | 0.1       | 0.3               |
| Combined uncertainty    | +1.2      | +1.6      | +1.4              |

The dark and light shaded zones represent the 68% and 90% probability regions, respectively.
are translated into a probability distribution for \( r_B \) using Eqs. (8) and (9) simultaneously. We assume the following prior probability distributions: for \( r_D \) a Gaussian with mean \( 4.7 \times 10^{-2} \) and standard deviation \( 3 \times 10^{-3} \) [9]; for \( k_D \) and \( \delta_D \), we use the likelihood obtained in Ref. [8], taking into account a 180 degree difference in the phase convention for \( \delta_D \); for \( \gamma \) and \( \delta_B \) we assume a uniform distribution between 0 and 360 degrees, while for \( r_B \) a uniform distribution in the range [0, 1] is used. We obtain the posterior probability distribution shown in Fig. 4. Since the measurements are not statistically significant, we integrate over the positive portion of that distribution and obtain the upper limit \( r_B < 0.13 \) at 90% probability, and the range

\[
 r_B \in [0.01, 0.11] \quad \text{at 68\% probability,} 
\]

and 0.078 as the most probable value.

**VII. SUMMARY**

We have presented a study of the decays \( B^+ \to D^0 K^\pm \) and \( B^\mp \to D^0 K^\pm \), in which the \( D^0 \) and \( \bar{D}^0 \) mesons decay to the \( K^\pm \pi^\pm \pi^0 \) final state using the ADS method. The analysis is performed using \( 474 \times 10^6 \) \( B\bar{B} \) pairs, the full \( BABAR \) data set. Previous results [7] are improved and superseded by improved event reconstruction algorithms and analysis strategies employed on a larger data sample.

The final results are

\[
 R^+ = (5^{+12}_{-10}(\text{stat})^{+4}_{-4}(\text{syst})) \times 10^{-3}, 
\]

\[
 R^- = (12^{+12}_{-10}(\text{stat})^{+2}_{-4}(\text{syst})) \times 10^{-3}, 
\]

\[
 R_{ADS} = (9.1^{+0.2}_{-0.3}(\text{stat})^{+1}_{-3}(\text{syst})) \times 10^{-3}, 
\]

from which we obtain 90\% probability limits

\[
 R^+ < 23 \times 10^{-3}, 
\]

\[
 R^- < 29 \times 10^{-3}, 
\]

\[
 R_{ADS} < 21 \times 10^{-3}. 
\]

From our measurements, we derive the limit

\[
 r_B < 0.13 \quad \text{at 90\% probability.} 
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