Nucleon EDM and rare decays of $\eta$ and $\eta'$ mesons

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The modern advanced experimental techniques realized in the experiments on electron’s electric dipole moment (EDM) are based on that idea and have the sensitivity in the experiments on electron’s electric dipole moment (EDM). The experimental limits for the corresponding branching ratios are quite soft. I relate these decay branching ratios to the value of the induced nucleon EDM and translate the experimental limits on the neutron EDM into much more stringent constraints on these decay rates: \( \frac{\Gamma_{\eta \to \pi\pi}}{\Gamma_{\eta}^{\text{full}}} \lesssim 3.5 \times 10^{-14} \) and \( \frac{\Gamma_{\eta' \to \pi\pi}}{\Gamma_{\eta'}^{\text{full}}} \lesssim 1.8 \times 10^{-17} \).

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The first proposal of experimental search of CP-violating effects in atoms was made almost 40 years ago [1]. The modern advanced experimental techniques realized in the experiments on electron’s electric dipole moment (EDM) are based on that idea and have the sensitivity of \( d_e \sim 10^{-26} \text{e cm} \) [2].

Apart from the electron EDM, experimental searches for the EDM of the neutron are on-going. Current experimental limit on the value of the electric dipole moment (EDM) of the neutron is \( d_n \lesssim 2.9 \times 10^{-26} \text{e cm} \) [3]. From theoretical point of view, a non-zero EDM could imply non-zero values for the QCD \( \theta \)-term as the latter can induce an EDM [4].

Recently, an experimental program has been under consideration at JLab Hall D, pursuing the goal of a higher precision determination of the rare decay rates for processes \( \eta(\eta') \to \pi^0 \gamma \gamma \) and \( \eta(\eta') \to \pi \pi \) within the GlueX experiment [5]. The latter decay channel explicitly violates the CP-conservation. It becomes clear if considering this decay in the rest frame of the initial \( \eta \) meson. Since it is spinless, the resulting pair of pions can only be produced in the S-wave, but in this way the intrinsic parities of the initial and final state are opposite, while the C-parity is not affected.

In this letter, I will address the limits on rare decays \( \eta(\eta') \to \pi \pi \). I will show that provided such decays do go at a non-zero rate, this would induce an effective CP-violating \( \eta N N \) coupling. Consequently, the photon-nucleon coupling will obtain a CP-violating contribution due to virtual \( \eta \)-loops. Since such a coupling, the electric dipole moment (EDM) of the nucleon is tightly constrained by experimental observations, this imposes a very stringent constraint on the \( \eta(\eta') \to \pi \pi \) decay branching ratios.

PDG quotes the following branching ratios for these decays [3]:

\[
\begin{align*}
\frac{\Gamma(\eta \to \pi \pi)}{\Gamma_{\eta}^{\text{full}}} &< \begin{cases} 3.3 \times 10^{-4}, & \pi^+ \pi^- \\ 4.3 \times 10^{-4}, & \pi^0 \pi^0 \end{cases} \\
\frac{\Gamma(\eta' \to \pi \pi)}{\Gamma_{\eta'}^{\text{full}}} &< 2\% 
\end{align*}
\]

The masses and full widths of the two mesons are quoted in PDG [3] as \( m_{\eta} = 547.75 \pm 0.12 \text{ MeV} \), \( \Gamma_{\eta}^{\text{full}} = 1.29 \pm 0.07 \text{ keV} \) and \( m_{\eta'} = 958.75 \pm 0.14 \text{ MeV} \), \( \Gamma_{\eta'}^{\text{full}} = 0.202 \pm 0.016 \text{ MeV} \), respectively.

![FIG. 1: $\eta(\eta') \to \pi \pi$ decay process. The solid square represents the CP-violating vertex.](image)

The only form of the effective Lagrangian that can lead to the process shown in Fig.1 has the form

\[
\mathcal{L} = f_{\eta \pi \pi} m_{\eta} \eta \pi_a \pi_b \delta_{ab},
\]

with \( f_{\eta \pi \pi} \) the corresponding coupling constant that is chosen to be dimensionless, \( m_{\eta} \) the mass of the \( \eta \)-meson, and the indices \( a, b \) refer to isospin. Similar form holds for \( \eta' \).

The corresponding decay widths can be calculated using the above Lagrangian:

\[
\Gamma_{\eta \to \pi \pi} = \sqrt{m_{\eta}^2 - 4m_{\eta}^2} |f_{\eta \pi \pi}|^2
\]

The experimental limits of Eq. (1) can be translated into bounds on the coupling constants,

\[
\begin{align*}
|f_{\eta \pi \pi}| &\lesssim 2.3 \times 10^{-4} \\
|f_{\eta' \pi \pi}| &\lesssim 1.5 \times 10^{-2}
\end{align*}
\]

In turn, a CP-violating coupling of \( \eta \)'s to two pions will induce an effective CP-violating \( \eta N N \) vertex

\[
\mathcal{L}_{\eta N N}^{CP} = \bar{g}_{\eta N N} \bar{N} N \eta
\]

with the corresponding coupling \( \bar{g}_{\eta N N} \). To estimate the value of this coupling, I will use heavy baryon ChPT formalism with pions, \( \eta \) and \( \rho \) mesons and the nucleon. I do not include the nucleon resonances and the \( \Delta \) into this calculation. These resonances may be included in...
the nucleon EDM, one can use effective field theory for
via virtual loops, as shown in Fig.3.
the nucleon, the
(b graph) vanishes due to the isospin dependence. The
contribution is non-zero, while the ted-pole contribution
contribute to this vertex. However, only the diagram a)
regularization with \( \Delta = \eta N N \) for
QCD
that I will take to be equal to \( \Lambda_{QCD} \)
are not very well known. The former can range
with \( \eta N N \)
and \( \eta \rho \) loops (diagrams b) are shown. Solid square rep-
the meson momentum. The couplings \( g_{\eta NN} \) and
\( g_{\eta \rho NN} \) are not very well known. The former can range
between 0.5 and 1.5 [9], depending on the model which
is used to calculate or extract it from the data. Further-
more, along with the PV coupling shown above, also the
PS coupling \( g_{\eta NN} \tilde{N} \gamma_5 N \) is allowed for the \( \eta \)'s unlike for
pions. While they are equivalent for on-shell nucleons,
inside the loop the use of the one or the other coupling
will in general lead to different results. In the calculation,
I will use the PV coupling and will assume that this PV
coupling for \( \eta \) and \( \eta' \) are of order 1 and are roughly equal.

Finally, the nucleon electromagnetic vertex with real
photons is given by
\[
\Gamma^\mu(q) = e \left[ e N \gamma^\mu + \kappa N i\sigma^{\mu\nu} \frac{q_\nu}{2 M_N} + \tilde{d}_N i\gamma_5 \sigma^{\mu\nu} \frac{q_\nu}{2 M_N} \right]
\]  
(9)
where the dimensionless \( \tilde{d}_N \) is the electric dipole
moment (EDM) of the nucleon measured in units of the
nuclear magneton \( \frac{e}{2 M_N} \), and the index \( N = p, n \) indicates
whether the nucleon is the proton or the neutron, respect-
ively. Since the only direct experimental constraint on
the value of the EDM is for the neutron, I will compute
the induced neutron EDM in which case the photon cou-
plies to the anomalous magnetic moment of the neutron.
The calculation is simplest to perform in HBChPT for-
mulation.
To calculate the contribution of the vector meson loops
shown in Fig. 3b, we need to define the effective La-
grangian for \( \gamma N (\gamma \rho) \) vertex, as well as the vector meson
coupling to the nucleon. These are given by [9]:
\[
L_{\gamma N} = \frac{e \lambda_V}{4 m_\rho} \epsilon_{\mu\nu\sigma\beta} F^{\mu\nu} V^{\sigma\beta} \eta,
\]
\[
\Gamma^\mu_{V NN} = \tilde{N} \left( g_\rho^{V} V^\mu + g_\pi^{V} i\sigma^{\mu\nu} \frac{q_\nu}{2 M_N} \right) \tau V N
\]  
(10)
with the electromagnetic field strength tensor \( F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \) and the vector meson field tensor \( V^{\mu\nu} =

FIG. 2: Induced CP-violating \( \eta(\eta')NN \) vertex. The solid
square represents the CP-violating \( \eta(\eta')\pi \pi \) vertex.

lowest order ChPT Lagrangian in the heavy baryon for-
malism is well known and I refer the reader to Ref. [8]
for the details. The calculation leads to the following
relation:
\[
\bar{g}_{\eta NN} = - f_{\eta \pi \pi} m_\eta \left( \frac{g_{\pi NN}}{2 M} \right)^2 \frac{m_\pi}{16 \pi} \left( \frac{2}{\pi} \left( \Delta + \ln \frac{\mu^2}{m_\pi^2} \right) + 1 \right)
\]  
(6)
where \( g_{\pi NN} = g_{\Delta NN} \) is the pion-nucleon coupling con-
stant, \( F_\pi \) being the pion decay constant. Similar re-
sult holds for \( \eta' \). The above loop calculation contains
a divergence that is regularized by means of dimensional
regularization with \( \Delta = \frac{1}{2} - \gamma_E + \ln(4\pi) \) the usual MS
scheme subtraction, and \( \mu \) is the renormalization scale
that I will take to be equal to \( \Lambda_{QCD} \). When perform-
ing the renormalization within the full underlying the-
ory, the divergent part should cancel. I will use the re-
maining finite part to estimate the limits on the effective
CP-violating \( \eta NN \) coupling implied by the experimental
limits on \( \eta(\eta') \) decays,
\[
|\tilde{g}_{\eta NN}| \sim f_{\eta \pi \pi} m_\eta \left( \frac{g_{\pi NN}}{2 M} \right)^2 \frac{m_\pi}{16 \pi} \lesssim 1.8 \times 10^{-5}
\]
\[
|\tilde{g}_{\eta' NN}| \sim f_{\eta' \pi \pi} m_{\eta'} \left( \frac{g_{\pi NN}}{2 M} \right)^2 \frac{m_\pi}{16 \pi} \lesssim 1.9 \times 10^{-3}
\]  
(7)

In the presence of the CP-violating coupling of \( \eta \)'s to
the nucleon, the \( \eta \)'s can contribute to the nucleon EDM
via virtual loops, as shown in Fig.2.
To estimate the contribution of such an interaction to
the nucleon EDM, one can use effective field theory for
\( \eta \)'s like for pions.
The usual, CP-conserving \( \eta NN \) coupling is given by
\[
L_{\eta NN} = \frac{g_{\eta NN}}{2 M} \tilde{N} \gamma_5 N \eta
\]  
(8)
with \( k \) the meson momentum. The couplings \( g_{\eta NN} \) and
\( g_{\eta' NN} \) are not very well known. The former can range
\[ \partial^\mu V^\alpha - \partial^\alpha V^\mu, \] respectively. The isospin factor \( \tau_V \) is 1 for the \( \omega \) and \( \tau_3 \) for \( \rho^0 \).

The \( V \gamma \gamma \) coupling constants values can be deduced from the \( V \to \eta \gamma \) decay widths,

\[ \Gamma_{V \to \eta \gamma} = \frac{\alpha_{em} (m_\eta^2 - m_\gamma^2)^3}{24 \, m_V^3 m_\eta^2} \lambda_V^2 \] (11)

and the \( V \eta' \gamma \) couplings from the \( \eta' \to V \gamma \) decay widths with

\[ \Gamma_{\eta' \to V \gamma} = \frac{\alpha_{em} (m_\eta^2 - m_\gamma^2)^3}{8 \, m_\eta^2} \lambda_V^2. \] (12)

The empiric values of the parameters introduced above are summarized in Table I:

| \( \eta \gamma \) (MeV) | \( g_\eta^V \) | \( g_\eta' / g_\eta^V \) | \( \lambda_V \) | \( \lambda_V' \) |
|-----------------|---------|-----------------|---------|---------|
| \( \rho \)      | 775.8   | 2.4             | 0.9     | 1.18    |
| \( \omega \)    | 782.6   | 16              | 0.25    | 0.43    |

TABLE I: Parameters for the vector mesons.

I take the values of the \( VNN \) couplings from [9], and obtain the \( V \eta \gamma \) couplings from the corresponding decay widths given in [9] as explained above.

With the use of these vertices, one obtains the estimate for the loop contribution shown in Fig. 3 a and b:

\[ d_n^{a, \eta} = \kappa_n \frac{g_{\eta N N} g_{\eta' N N} I(m_\eta^2)}{8 \pi^2} \] (13)

\[ d_n^{a, V} = \lambda_V \tau_V \frac{(g_\eta^V + g_\eta'^V) g_{\eta N N}}{8 \pi^2} \frac{m_\eta^2 I(m_\eta^2) - m_{\eta'}^2 I(m_{\eta'}^2)}{m_V^2 - m_{\eta'}^2}, \]

where I used the notation

\[ I(m^2) = \Delta + \ln \frac{M^2}{\mu^2} - \frac{m^2}{M^2} \ln \frac{m^2}{M^2} + 2 \]

\[ - \frac{\sqrt{m^2(4M^2 - m^2)}}{M^2} \left[ \arctan \frac{2M^2 - m^2}{m \sqrt{4M^2 - m^2}} + \frac{\sqrt{m^2(4M^2 - m^2)}}{4M^2 - m^2} \right] \] (14)

In the above equation, \( \mu \) stands for the regularization scale. In this case, the natural scale at which the divergencies cancel is at least of order of the mesonic masses in the loops, rather than \( \Lambda_{QCD} \) as for the pion loops. For the numeric estimates, I will set this scale to the nucleon mass. The \( \eta' \) contribution obtains by substituting the respective mass and couplings into Eq. (13).

We can now proceed with numeric estimates for the induced neutron EDM. For this, I will require that every individual contribution to the neutron EDM does not exceed the experimental limit. Evaluating the expressions for each contribution, the most stringent limits for both \( CP \)-violating \( \eta(\eta')NN \)-couplings come from \( \eta \), and \( V \eta' \) loops for the \( \eta' \). These constraints read

\[ |g_{\eta NN}| \lesssim 1.6 \times 10^{-10} \]

\[ |g_{\eta' NN}| \lesssim 5.8 \times 10^{-11}. \] (15)

Of course, if a full EFT calculation would be possible, these bounds might shift either way due to enhancement or partial cancellation of different contributions, both among the calculated meson loop effects alone and with the contributions of the nucleon resonances that are not included in this analysis. If these latter effects will come with the same relative sign, the resulting limit on \( g_{\eta NN} \) will become stronger, then the results of Eq. (15) will still hold. The only significant qualitative difference will arise if the effects that were not considered here would tend to cancel the loop contributions that I provided. However, such cancellations can only lead to significant changes in the estimates if they occur at 99\% or even more percent level, so that the bound on the \( CP \)-violatin in \( \eta \)-decay is loosened by several orders of magnitude. An example of such cancellation is observed in the case of the magnetic polarizability of the nucleon where the large diamagnetic contribution from the pion loops are cancelled by a large paramagnetic contribution due to \( N \to \Delta \) electromagnetic transition that comes with the opposite sign. As a result, the magnetic polarizability of the proton is about ten times smaller than its electric polarizability [10].

A precise cancellation of physically different contributions at a level of 1\% or even below that would indicate an existence of an unknown symmetry that prevents the mechanisms considered here from contributing to the EDM. The precision of our knowledge of the interactions of the \( \eta \)'s and vector mesons with the nucleons and nucleon resonances, both experimentally and theoretically, is far from the level that would allow to observe such a symmetry, if it is to exist at all.

Comparing now Eq. (15) to the bounds derived from the experimental limits on the \( CP \)-violating \( \eta \)'s decays, Eq. (4), we find a discrepancy of five orders of magnitude for \( |g_{\eta NN}| \) and eight orders of magnitude for \( |g_{\eta' NN}| \).

The recursive calculation leads to the EDM-induced constraint onto the \( \eta \pi \pi \) coupling constants:

\[ f_{\eta \pi \pi} \lesssim 2 \times 10^{-9} \]

\[ f_{\eta' \pi \pi} \lesssim 4.3 \times 10^{-10} \] (16)

Finally, recalling the relation between these couplings and the corresponding decay branching ratios obtained earlier in Eq. (3), it is possible to deduce the naturalness constraints onto these decay rates from the experimental limits on neutron EDM:

\[ \frac{\Gamma(\eta \to \pi \pi)}{\Gamma_{\eta \pi \pi}^{full}} \lesssim 3.5 \times 10^{-14}, \]

\[ \frac{\Gamma(\eta' \to \pi \pi)}{\Gamma_{\eta' \pi \pi}^{full}} \lesssim 1.8 \times 10^{-17}. \] (17)
These limits are more stringent than the current experimental limits by 10 orders of magnitude for $\eta$, and by 15 orders of magnitude for $\eta'$. 

In summary, I considered the $\eta(\eta') \rightarrow \pi\pi$ decay channels that explicitly violate parity and time-reversal conservation. I constructed a Lagrangian for such an interaction and derived the induced $CP$-violating coupling of $\eta$'s to the nucleon using heavy baryon ChPT. If this coupling is non-zero, it generates a contribution to the electric dipole moment of the nucleon through virtual $\eta$, $\eta\rho^0$ and $\eta\omega$ loops. The tight experimental constraints on the neutron EDM lead to the conclusion that the current experimental limits on branching ratios for the $\eta$s decaying to two pions are highly underconstrained. I provide the naturalness bounds on these branching ratios, $\Gamma_{\eta \rightarrow \pi \pi} \lesssim 3.5 \times 10^{-14}$ and $\Gamma_{\eta' \rightarrow \pi \pi} \lesssim 1.8 \times 10^{-17}$. These results indicate that a direct experimental search for the signal in these decay channels is not feasible in the near future.

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