Decomposition method for zonal resource allocation problems in telecommunication networks

I V Konnov¹ and A Yu Kashuba²

¹ Department of System Analysis and Information Technologies, Kazan Federal University, Kazan 420008, Russia
² LLC "AST Povolzhye", Kazan, 420029, Russia

E-mail: Igor.Konnov@kpfu.ru

Abstract. We consider problems of optimal resource allocation in telecommunication networks. We first give an optimization formulation for the case where the network manager aims to distribute some homogeneous resource (bandwidth) among users of one region with quadratic charge and fee functions and present simple and efficient solution methods. Next, we consider a more general problem for a provider of a wireless communication network divided into zones (clusters) with common capacity constraints. We obtain a convex quadratic optimization problem involving capacity and balance constraints. By using the dual Lagrangian method with respect to the capacity constraint, we suggest to reduce the initial problem to a single-dimensional optimization problem, but calculation of the cost function value leads to independent solution of zonal problems, which coincide with the above single region problem. Some results of computational experiments confirm the applicability of the new methods.

1. Introduction

Despite the existence of powerful processing and transmission devices, increasing demand of different telecommunication services and its variability lead to serious congestion effects and inefficient utilization of network resources. This situation forces one to replace the fixed allocation rules with more flexible mechanisms, which are based on proper mathematical models; see e.g. [1]–[3]. In particular, spectrum sharing is now one of the most critical issues in this field and various adaptive mechanisms have been suggested. Most papers are devoted to game-theoretic models and implementation of decentralized iterative methods for finding the Nash equilibrium points or their generalizations; see e.g. [4, 5]. At the same time, various optimization based mechanisms are also suggested; see e.g. [6, 7, 5, 3].

In this paper, we consider some problems of optimal allocation of a homogeneous resource in telecommunication networks such that the income received from users payments is maximized and the implementation costs of the network operator are minimized. We first present an optimization formulation for the case where the network manager aims to distribute some homogeneous resource (bandwidth) among users of one region with quadratic charge and fee functions. These convex quadratic optimization problems can be solved by simple and efficient solution methods. We describe some modifications for this special problem. Next, we consider a more general resource allocation problem for a provider of a wireless communication network divided into zones (clusters); which was formulated as a convex optimization problem in [8, 9]. Now, since the price functions are affine, we
obtain again a convex quadratic optimization problem having capacity and zonal balance constraints. Unlike [8, 9], we now suggest to apply the dual Lagrangian method with respect to only capacity constraint. Therefore, we replace the initial problem with a single-dimensional optimization problem, however, calculation of its cost function value requires independent solution of zonal problems. Each of these problems coincides with the above single region resource allocation problem and can be solved by the simple algorithms suggested. In such a way we develop a new dual decomposition approach for solution finding, whose implementation is simpler essentially in comparison with the methods from [8, 9]. We present results of computational experiments which confirm the applicability of the new method.

2. Simple resource allocation model

Let us consider a single telecommunication network with nodes (users). The general problem of a network manager is to find an optimal allocation of a limited homogeneous resource among the users in order to maximize the total payment received from the users and to minimize the total network implementation expenses. That is, \( x \) is an unknown quantity of the resource offered by the network, with the capacity bounds \( x \in [0, b] \), which yields the network expense (cost of implementation) \( u(x) \). Similarly, \( y_i \) is the unknown resource offered to user \( i \in I \) and \( \varphi_i(y_i) \) is the fee (incentive) value paid by node \( i \) with the capacity bounds \( y_i \in [0, \alpha_i] \), where \( I \) is the index set of users. The network manager problem is formulated as follows:

\[
\max_{(x, y) \in D} \sum_{i \in I} \varphi_i(y_i) - u(x),
\]

where \( y = (y_i)_{i \in I} \),

\[
D = \left\{ (x, y) \left| \sum_{i \in I} y_i = x, 0 \leq y_i \leq \alpha_i, i \in I, 0 \leq x \leq b \right. \right\}.
\]

Suppose that the set \( D \) is non-empty, functions \( u(x) \) and \( -\varphi_i(y_i) \) are convex and quadratic, i.e.,

\[
\varphi_i(y_i) = 0.5 \alpha_i y_i^2 + \beta_i y_i, \quad \alpha_i < 0, i \in I; u(x) = 0.5 x^2 + \delta x, \quad \gamma > 0.
\]

Then (1) is a convex quadratic optimization problem, which can also be treated as a two-side auction models with one trader where all the participants have affine price functions; see [10, 11]. For these problems there exist many rather efficient solution methods; see e.g. [12] and references therein. They are mostly based on duality theory.

Following this approach, write the Lagrange function of problem (1) with the negative sign:

\[
M(x, y, p) = u(x) - \sum_{i \in I} \varphi_i(y_i) - p \left( x - \sum_{i \in I} y_i \right)
\]

\[
= (u(x) - px) - \sum_{i \in I} (\varphi_i(y_i) - py_i)
\]

\[
= (0.5 x^2 + \delta x - px) - \sum_{i \in I} (0.5 \alpha_i y_i^2 + \beta_i y_i - py_i)
\]

In order to find a value of the dual cost function

\[
\theta(p) = \min_{x \in [0, b], y \in [0, \alpha]} M(x, y, p),
\]

where \( a = (\alpha_i)_{i \in I} \), we have to solve one-dimensional problems:

\[
\min_{0 \leq x \leq \delta_k} \rightarrow (0.5 x^2 + \delta x - px),
\]
and
\[
\min_{\alpha, \gamma, \varepsilon_i} \to (-0.5\alpha, \gamma_i^2 - \beta_i \gamma_i + p y_i),
\]
for \( i \in I \). Solutions of these problems denoted by \( x(p) \) and \( y_i(p), \ i \in I \), respectively, are defined uniquely. Set \( \tilde{x}(p) = (p - \delta) / \gamma \) and \( \tilde{y}_i(p) = (\beta_i - p) / \alpha_i \), then
\[
x(p) = \begin{cases} 0 & \text{if } p \leq \delta, \\ b & \text{if } p \geq \delta + \beta_i, \\ \tilde{x}(p) & \text{otherwise;}
\end{cases} \quad y_i(p) = \begin{cases} 0 & \text{if } p \geq \beta_i, \\ a_i & \text{if } p \leq \beta_i + \alpha_i a_i, \\ \tilde{y}_i(p) & \text{otherwise;}
\end{cases}
\]
for \( i \in I \);
(2)

It follows that the function \( \theta(p) \) is concave and differentiable with
\[
\theta'(p) = \sum_{i \in I} y_i(p) - x(p).
\]
Besides, the one-dimensional dual problem
\[
\max_p \theta(p)
\]
coincides with the simple equation
\[
\theta'(p) = 0,
\]
where \( \theta'(p) \) is non-increasing. If \( p^* \) is the solution of (3), then we can find the solution of the initial problem (1) from (2) by setting \( p = p^* \).

If we set \( p'' = \max_{i \in I} \beta_i \) and \( p' = \delta \), then the case \( p'' < p' \) gives immediately the zero solutions in accordance with (2). So we can consider only the non-trivial case where \( p' < p'' \). Then by (2) we must have \( \theta'(p') > 0 \) and \( \theta'(p'') < 0 \). These properties enable us to find a solution of (3) by the simple bisection algorithm, denoted as Algorithm (BS). Given an accuracy \( \varepsilon > 0 \) and the initial segment \([p', p'']\), we take \( \tilde{p} = 0.5(p' + p'') \), calculate \( \theta'(\tilde{p}) \). Then we set \( p' = \tilde{p} \) if \( \theta'(\tilde{p}) > 0 \) and \( p'' = \tilde{p} \) otherwise, until \( (p'' - p') < \varepsilon \).

We can also utilize various heuristic algorithms. For instance, we describe a simple Algorithm (SQ). Define \( I_s = \{ i \in I \mid \beta_i > p' \} \), set \( y_i = 0 \) for \( i \not\in I_s \) and re-arrange the indices in \( I_s \) to have the descending order for the values of \( \beta_i \). Then find two sequential indices \( i_l \) and \( i_{l+1} \) in \( I_s \) such that \( \Delta_i < 0 \) and \( \Delta_{i_{l+1}} > 0 \), where
\[
\Delta_i = \sum_{i=1}^{L} y_i (\beta_i) - x(\beta_i).
\]
Then find \( p^* \) such that \( \theta'(p^*) = 0 \) in the segment \([\beta_{i_l}, \beta_{i_{l+1}}]\).

3. Multi-zonal network problem

Let us consider a more general model where a telecommunication network is divided into several zones (clusters). The problem of a manager of the network is to find the optimal allocation of a limited homogeneous network resource among the zones in order to maximize the total profit containing the total income received from consumers’ fees and negative resource implementation costs; see [8, 9].

Let us use the following notation:
\[ n \text{ is the number of zones;} \]
\[ I_k \text{ is the index set of users (currently) located in zone } k \ (k = 1, ..., n); \]
\[ B \text{ is the total resource supply (the total bandwidth) for the system (network);} \]
\[ x_k \text{ is an unknown quantity of the resource allotted to zone } k \text{ with the upper bound } b_k \text{ and } \]
\[ f_k(x_k) \text{ is the cost of implementation of this quantity of the resource for zone } k \ (k = 1, ..., n); \]
\[ y_i \text{ is the resource amount received by user } i \text{ with the upper bound } a_i \text{ and } \phi(y_i) \text{ is the} \]
\[ \text{charge value paid by user } i \text{ for the resource value } y_i. \]

The network manager problem is the optimization problem involving capacity and balance constraints:

\[
\max \rightarrow \sum_{k=1}^{n} \left( \sum_{i \in I_k} \phi_i(y_i) - f_k(x_k) \right),
\]  
subject to

\[
\sum_{k=1}^{n} x_k \leq B; \tag{5}
\]
\[
\sum_{i \in I_k} y_i = x_k, \quad k = 1, ..., n; \tag{6}
\]
\[
0 \leq y_i \leq a_i, \quad i \in I_k, \quad 0 \leq x_k \leq b_k, \quad k = 1, ..., n. \tag{7}
\]

That is, (6) provides the balance for demand and supply in each zone, (7) involves capacity constraints for users and network supply values in each zone, and (5) gives the upper bound for the total resource supply.

In what follows we assume that there exists at least one feasible point satisfying conditions (5)–(7), all the functions \( f_k(x_k) \) and \( -\phi_i(y_i) \) are convex and quadratic, i.e.

\[
\phi_i(y_i) = 0.5\alpha_i y_i^2 + \beta_i y_i, \quad \alpha_i < 0, \quad i \in I_k, \]
\[
f_k(x_k) = 0.5\gamma_k x_k^2 + \delta_k x_k, \quad \gamma_k > 0; \quad k = 1, ..., n. \tag{8}
\]

This means that (4)–(8) is a convex quadratic optimization problem. However, due to large dimensionality and inexact data one can meet serious drawbacks in solving this problem with usual finite or penalty solution methods. In order to create an efficient method, we have to take into account its separability and apply certain decomposition approach. However, the standard duality approach using the Lagrangian function with respect to all the functional constraints leads to the multi-dimensional dual optimization problem. We will apply another approach, which was suggested in [13].

Let us define the Lagrange function of problem (4)–(7) as follows:

\[
L(x, y, \lambda) = \sum_{k=1}^{n} \left[ \sum_{i \in I_k} \phi_i(y_i) - f_k(x_k) \right] - \lambda \left( \sum_{k=1}^{n} x_k - B \right).
\]

We utilize the Lagrangian multiplier \( \lambda \) only for the total resource bound. We can now replace problem (4)–(7) with its dual:

\[
\min_{\lambda \geq 0} \rightarrow \psi(\lambda), \tag{9}
\]

where
\[ \psi(\lambda) = \max_{(x,y) \in W} L(x,y,\lambda) = \lambda B + \max_{(x,y) \in W} \sum_{i=1}^{n} \left[ \sum_{k \in I_i} \phi_i(y_i) - f_{k_i}(x_i) - \lambda x_i \right]. \]

\[ W = \left\{ (x,y) \mid \sum_{i \in I_k} y_i = x_i, \ 0 \leq y_i \leq a_i, i \in I_k, \ 0 \leq x_i \leq b_i, k = 1, \ldots, n \right\}. \]

By duality (see e.g. [14, 15]), problems (4)–(7) and (9) have the same optimal value. But solution of (9) can be found by one of well-known single-dimensional optimization algorithms; see e.g. [15]. In order to calculate the value of \( \psi(\lambda) \) we have to solve the inner problem:

\[ \text{max} \to \sum_{k=1}^{n} \left[ \sum_{i \in I_k} \phi_i(y_i) - f_{k_i}(x_i) - \lambda x_i \right], \]

subject to

\[ \sum_{i \in I_k} y_i = x_i, \ 0 \leq y_i \leq a_i, i \in I_k, \]

\[ 0 \leq x_i \leq b_i, k = 1, \ldots, n. \]

Obviously, this problem decomposes into \( n \) independent zonal optimization problems

\[ \text{max} \to \left[ \sum_{i \in I_k} \phi_i(y_i) - f_{k_i}(x_i) - \lambda x_i \right], \]

(10)

subject to

\[ \sum_{i \in I_k} y_i = x_i, \ 0 \leq y_i \leq a_i, i \in I_k, \]

\[ 0 \leq x_i \leq b_i ; \]

for \( k = 1, \ldots, n \). Each \( k \)-th independent zonal problem (10) clearly coincides with problem (1) where

\[ \phi_i(y_i) = 0.5x_i^2 + \beta_i y_i, i \in I_k, \]

\[ u(x) = f_{k_i}(x_i) + \lambda x_i = 0.5x_i^2 + (\delta_i + \lambda)x_i. \]

Therefore, we can find its solution by the algorithms of Section 2.

4. Numerical experiments
The methods were implemented in C++ with a PC with the following facilities: Intel(R) Core(TM) i7-4500, CPU 1.80 GHz, RAM 6 Gb.

The initial intervals for choosing the dual variable \( \lambda \) were taken as [0,1000]. Values of \( b_i \) were chosen by trigonometric functions in [1,11], values of \( a_i \) were chosen by trigonometric functions in [1,2]. Value \( B \) were taken equal 1000. The number of zones was varied from 5 to 105, the number of users was varied from 210 to 1010. Users were distributed in zones either uniformly or according to the normal distribution.

The coefficients of the functions \( f_{k_i}(x_i) \) and \( \phi_i(y_i) \) from (8) were taken as

\[ \gamma_k = 2(|\sin(2k + 2)| + 1), \delta_k = |\cos(k + 1)| + 3, \]

and
\[ \alpha_i = -6|\cos(2i + 1)| - 6, \beta_i = |\sin(i + 2)| - 1. \]

For all the methods of finding solution of problem (4)–(7) the accuracy of upper dual problem solution were varied from \(10^{-1}\) to \(10^{-4}\). The accuracy of lower level problem solution in Algorithm (BS), was fixed and equal to \(10^{-2}\). For each set of the parameters made 1000 tests. Let \(J\) denote the total number of users, \(T\) the total processor time in seconds. The results of computations are given in Tables 1–3.

**Table 1. Results of testing with \(J = 510, n = 70\).**

| \(\epsilon\) | \(T_{\epsilon}: \text{Algorithm (SQ)}\) | \(T_{\epsilon}: \text{Algorithm (BS)}\) |
|---|---|---|
| \(10^1\) | 0.0183 | 0.0018 |
| \(10^2\) | 0.0217 | 0.0017 |
| \(10^3\) | 0.0247 | 0.0020 |
| \(10^4\) | 0.0289 | 0.0022 |

**Table 2. Results of testing with \(n = 70, \epsilon = 10^{-2}\).**

| \(J\) | \(T_{\epsilon}: \text{Algorithm (SQ)}\) | \(T_{\epsilon}: \text{Algorithm (BS)}\) |
|---|---|---|
| 210 | 0.0043 | 0.0009 |
| 310 | 0.0088 | 0.0009 |
| 410 | 0.0142 | 0.0012 |
| 510 | 0.0217 | 0.0017 |
| 610 | 0.0303 | 0.0022 |
| 710 | 0.0397 | 0.0026 |
| 810 | 0.0515 | 0.0032 |
| 910 | 0.0639 | 0.0039 |
| 1010 | 0.0791 | 0.0048 |

**Table 3. Results of testing with \(J = 510, \epsilon = 10^{-2}\).**

| \(n\) | \(T_{\epsilon}: \text{Algorithm (SQ)}\) | \(T_{\epsilon}: \text{Algorithm (BS)}\) |
|---|---|---|
| 5 | 0.0221 | 0.0004 |
| 15 | 0.0217 | 0.0005 |
| 25 | 0.0218 | 0.0006 |
| 35 | 0.0217 | 0.0006 |
| 45 | 0.0214 | 0.0017 |
| 55 | 0.0220 | 0.0010 |
| 65 | 0.0215 | 0.0011 |
| 75 | 0.0223 | 0.0010 |
| 85 | 0.0216 | 0.0012 |
| 95 | 0.0220 | 0.0012 |
| 105 | 0.0219 | 0.0015 |

As we can see from the results in the tables, in all the cases the suggested methods were rather effective in finding a solution. Moreover, for the same accuracy, both the methods gave the same numbers of upper iterations, so that the main difference was in the processor time which showed that utilization of Algorithm (BS) for inner optimization problems give better performance.

5. Conclusions
We considered several problems of optimal resource allocation in telecommunication networks. We presented simple and efficient solution methods for the case where the network manager aims to
distribute some homogeneous resource (bandwidth) among users of one region with quadratic charge and fee functions. Next, we considered a more general problem for a provider of a wireless communication network divided into zones. By using the dual Lagrangian method with respect to the capacity constraint, we suggest to reduce the initial problem to a single-dimensional optimization problem, where calculation of the cost function value leads to independent solution of zonal problems, which coincide with the above single region problem. Some results of computational experiments confirm the applicability of the new methods.

Acknowledgments
In this work, the authors were supported by the RFBR grant, project No. 16-01-00109a. Also, the first author was supported by grant No. 297689 from Academy of Finland.

References
[1] Courcoubetis C and Weber R 2003 Pricing Communication Networks: Economics, Technology and Modelling (Chichester: John Wiley & Sons)
[2] Stańczak S, Wiczanowski M and Boche H 2006 Resource Allocation in Wireless Networks. Theory and Algorithms (Berlin: Springer)
[3] Wyglinski A M, Nekovee M and Hou Y T (eds.) 2010 Cognitive Radio Communications and Networks: Principles and Practice (Amsterdam: Elsevier)
[4] Leshem A and Zehavi E 2009 Game theory and the frequency selective interference channel: A practical and theoretic point of view IEEE Signal Process 26 pp 28–40
[5] Raoof O and Al-Raweshidy H 2010 Auction and game-based spectrum sharing in cognitive radio networks. in: Game Theory ed Q Huang (Rijeka: Sciy) ch 2 pp 13–40
[6] Huang J, Berry R A and Honig M L (2006) Auction-based spectrum sharing ACM/Springer Mobile Networks and Appl. 11 pp 405–418
[7] Koutsopoulos I and Iosifidis G 2010 Auction mechanisms for network resource allocation. in: Proc. of Workshop on Resource Allocation in Wireless Networks, WiOpt 2010 pp 554–563
[8] Konnov I V, Kashina O A and Laitinen E 2011 Optimisation problems for control of distributed resources Int. J. Model., Ident. and Contr. 14 pp 65–72
[9] Konnov I V, Kashina O A and Laitinen E 2012 Two-level decomposition method for resource allocation in telecommunication network Int. J. Inf. Wirel. Comm. 2 pp 150–155
[10] Konnov I V 2006 On modeling of auction type markets Issled. Inform. 10 pp 73–76, Available at SSRN: http://ssrn.com/abstract=2482282
[11] Konnov I V 2015 An alternative economic equilibrium model with different implementation mechanisms Adv. Model. Optim. 17 pp 245–265
[12] Patriksson M and Strömberg C 2015 Algorithms for the continuous nonlinear resource allocation problem: New implementations and numerical studies Eur. J. Oper. Res. 243 pp 703–722
[13] Konnov I V, Kashuba A Yu and Laitinen E 2015 A simple dual method for optimal allocation of total network resources Recent Advances in Mathematics. Proceedings of the International Conference "PMAMCM 2015" ed I J Rudas (Zakynthos) pp 19–21
[14] Polyak B T 1983 Introduction to Optimization (Moscow: Nauka) (Engl. transl. in Optimization Software, New York, 1987)
[15] Konnov I V 2013 Nonlinear Optimization and Variational Inequalities (Kazan: Kazan Univ. Press)

11th International Conference on "Mesh methods for boundary-value problems and applications" IOP Publishing IOP Conf. Series: Materials Science and Engineering 158 (2016) 012054 doi:10.1088/1757-899X/158/1/012054