A parametric permeability study for a simplified vertebra based on regular microstructures

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The following work focuses on studying the permeability and its dependency on geometric parameters on the macro-scale representing the underlying microstructure. In particular, regular microstructures are studied such that the computation of permeability is straightforward and the geometric parameters can be modified in a systematic manner. The permeabilities of the respective geometries are then derived by numerical upscaling of pore-scale velocities computed by using Abaqus’ Stokes-flow solver. Based on these results, the permeabilities were found to be strongly dependent on the specific surface area and the respective porosity. This indicates the potential to estimate the order of magnitude for the permeability of real-word applications using these two parameters. Furthermore, the tortuosity can be beneficial to obtain better permeability estimations.

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1 Introduction

Percutaneous vertebroplasty is a common medical procedure, in which bone cement is injected inside the porous trabecular structure of a weakened or fractured vertebral bone in order to restore its load-carrying capacity. In order to prevent cases of cement leakage outside the vertebra, it is desired to have a model that can simulate the filling behaviour of the bone-cement upon injection. The bone-cement flow inside the vertebra can be conveniently modelled and simulated using the Theory of Porous Media (TPM), cf. [1]. However, this requires the knowledge of the permeability of the vertebra. A direct determination of the permeability requires the exact geometric details of the internal microstructure, e.g. via micro-CT measurements. This, however, cannot be obtained in vivo. Hence, this study aims to investigate if the permeability can be estimated indirectly, by studying the influence of macro-scale geometric parameters on the permeability. This is done by simulating Stokes flow through regular microstructures to compute the permeability and varying the geometric parameters of the microstructure in order to observe the changes in the permeability. This is explained in the following section.

2 Method

Close-packed structures of uniform-sized spheres, e.g. of FCC-type, are used to carry out a parametric study with respect to three parameters: porosity \( n^F \), specific surface area \( S \), and tortuosity \( \tau \). The porosity depends on the packing structure of the spheres. The specific surface area is the ratio of the surface area of the solid part to its volume, and hence can be changed by varying the radius of the spheres. Permeabilities were computed for various values of \( n^F \) and \( S \) using the following procedure:

(i) Compute the flow velocity: For the sake of numerical efficiency, only a repeating substructure (unit cell) of the packed structure is modelled. A pressure gradient is applied in a prescribed flow direction. In the pore-scale model, the flow domain is discretised using finite elements. The Stokes-flow is solved using the Abaqus/CFD solver.

(ii) Upscale: Based on the volumetric flow, \( Q \), which is obtained as output from Abaqus/CFD, the equivalent seepage velocity, \( w_F \), can be determined by \( Q = n^F A \cdot w_F \). Hereby, \( n^F \) describes the porosity and \( A = A n \), where \( n \) is the normal vector in the direction of applied pressure gradient and \( A \) is the corresponding cross-sectional area.

(iii) Calculate the permeability: The well-known Darcy law is obtained from the TPM after combining the restrictions of the entropy inequality with constitutive assumptions in the balance of momentum for the fluid, cf. [2]. Assuming a creeping flow regime and neglecting gravity, the intrinsic permeability \( K^S \) can be obtained from Darcy’s law via

\[
K^S = \frac{\mu^{FR} A n}{\mu^{FR} \grad p}.
\]

Therein, \( \mu^{FR} \) is the dynamic viscosity of the fluid and \( p \) is the fluid pressure.

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The tortuosity is defined as the ratio between the length of the flow channels through a porous medium and the end-to-end length. In this case, the tortuosity is calculated by substituting the computed permeability value in the Kozeny-Carman equation, i.e., \[ K^S = \frac{(n_F)^3}{[2 \pi^2 (1 - n_F^2) S^2]} \], cf. [3].

### 3 Results

As one can see in Fig. 1, the permeability increases with porosity. This is expected since higher porosity typically means larger pores and wider channels for the fluid to flow through. Furthermore, there is an increase in orders of magnitude of the permeability with relatively smaller changes in the specific surface area. A low specific surface area implies larger particle sizes and, hence, wider gaps. As a result, the resistance to flow is reduced and the permeability increases.

![Fig. 1: Permeability \((K^S)\) versus porosity \((n_F^2)\) for different values of specific surface area \((S)\).](image)

The results are promising since they show that the order of magnitude of the permeability can potentially be estimated from the porosity and the specific surface area – provided that the pores are sufficiently interconnected. The possible range of tortuosity can be narrowed down to a small range if the type of porous medium is known. This can be used to obtain a better estimate of the permeability. Furthermore, the results are applicable to microstructures with any kind of internal shape. This is because in spite of using only sphere-packing structures in this study, the parameters used are of general nature and therefore, applicable to other geometries as well. These parameters are in no way exhaustive, although they serve as a good first step. For a more detailed parametric investigation in the future, further parameters need to be included, e.g. constrictivity.

### 4 Discussion

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