Transient Conjugate Free Convection from a Vertical Flat Plate in a Porous Medium Subjected to a Sudden Change in Surface Heat Flux

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Abstract

The paper presents a theoretical study using the Kármán-Pohlhausen method for describing the transient heat exchange between the boundary-layer free convection and a vertical flat plate embedded in a porous medium. The unsteady behavior is developed after the generation of an impulsive heat flux step at the right-hand side of the plate. Two cases are considered according to whether the plate has a finite thickness or no thickness. The time and space evolution of the interface temperature is evidenced.

Keywords: Transient convection, Porous media.

1. Introduction

The importance of heat transfer phenomena associated with free convection in porous media is well known. Interest in this phenomenon has been motivated by such diverse engineering problems as geothermal energy extraction, storage of nuclear waste material, ground-water flows, pollutant dispersion in aquifers and packed-bed reactors, to mention just a few applications [1].

Owing to its fundamental and practical importance, the conjugate coupling heat transfer between a free convection flow and a vertical flat plate of finite thickness embedded in a porous medium has received particular attention [2-7]. Various approaches were used to deal with the difficulties associated with the simultaneous solution of the flow and thermal boundary layers and the longitudinal and transversal heat conduction in the solid plate. Despite the existing results in the open literature, they do not yet provide a complete description of this important problem, which has a bearing on many practical applications, particularly those related to energy conservation in buildings [8].

The point we wish to take up here is that of the transient conjugate free convection due to a vertical flat plate embedded in a porous medium. At a given time the right-hand side of the plate is suddenly subjected to a uniform heat flux, while the left-hand side of the plate is thermally insulated [9]. The present study is conducted in two phases: with finite thickness or without thickness of the plate, respectively. Analytical and numerical solutions are presented for all possible values of time and space evolution of the interface temperature.

2. Plate with Thickness

Consider unsteady free convection flow due to a semi-infinite vertical flat plate of finite thickness $a$ adjacent to a semi-infinite fluid-saturated porous medium. Initially, the whole system is at a temperature $T_0$, but subsequently the left-hand side of the plate is suddenly raised to, and held at a uniform heat flux $q_u$. The physical model and coordinate system is shown in Fig. 1. Assuming that the porous medium is isotropic and homogeneous and that the fluid is incompressible, the boundary layer and the Boussinesq approximations are invoked to obtain the following equations.

The equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

The Darcy's Law

$$u = \frac{gK\beta}{\nu} (T_f - T_s)$$

The equation of energy in the fluid-porous medium

$$\sigma \frac{\partial T_f}{\partial t} + u \frac{\partial T_f}{\partial x} + v \frac{\partial T_f}{\partial y} = \alpha_s \frac{\partial^2 T_f}{\partial y^2}$$

and the equation of the heat transfer inside the solid plate

$$\frac{\partial T_s}{\partial t} = \frac{\partial^2 T_s}{\partial y^2}$$

where $(x, y)$ are the Cartesian coordinates along and normal to the plate, $(u, v)$ are the velocity components in the $(x, y)$ directions, $t$ is the time, $T_f$ and $T_s$ are the temperatures of the fluid-saturated porous medium and the solid plate, respectively, and $g$, $\beta$, $\nu$, $K$, $\alpha_s$, and $\alpha_s$ are the physical constants. Equations (1)-(4) are subject to the following initial and boundary conditions.

For the fluid-porous medium ($y \geq 0$)

$$u = v = 0, \quad T_f = T_f^0 \quad \text{at} \quad t = 0 \quad \text{or} \quad x = 0$$

$$u = 0, \quad \frac{\partial T_f}{\partial y} = 0 \quad \text{as} \quad y \to \infty$$

For the solid ($-a \leq y \leq 0$)

$$T_s = T_s^0 \quad \text{at} \quad t = 0 \quad \text{or} \quad x = 0$$

For the fluid-solid interface

$$T_f = T_s = T_f^0 \quad \text{on} \quad y = 0 \quad \text{and} \quad t > 0$$

$$q_u = -\beta \frac{\partial T_f}{\partial y} = k_f \frac{\partial T_f}{\partial y} - k_s \frac{\partial T_s}{\partial y} \quad \text{on} \quad y = 0 \quad \text{and} \quad t > 0$$

where $T_f$ is the interface temperature and $k_f$ and $k_s$ are the thermal conductivities of the fluid and solid, respectively.
Further, a second-order Kármán-Pohlhausen temperature profile [10-15] is assumed in the fluid-porous medium to have

\[ \frac{\partial}{\partial y} \theta = \frac{1}{2} \left( 1 - \frac{2 \theta}{\delta} \right) y^2 + \frac{1}{k} \left( 1 - \frac{2 \theta}{\delta} \right) y + \theta, \]

where \( \delta \) is the boundary-layer thickness and

\[ \theta_0 = \frac{k}{a} (T_0 - T_\infty), \quad \theta_1 = \frac{k}{a} (T_1 - T_\infty), \quad k = \frac{k}{k}. \]

To obtain the integral form of the governing equations for transient conjugate free convection in a vertical porous layer, equations (1)-(4) are integrated across the boundary layer to yield

\[ \frac{\Gamma}{3} \frac{\partial}{\partial x} (\rho \theta) + \frac{\partial}{\partial x} (\rho \theta y) = \frac{2 \theta}{\delta} \frac{\partial}{\partial x} \left( \theta + \frac{2 \theta}{3k} \right) = \frac{1}{k} \left( 1 - \frac{2 \theta}{\delta} \right) \]

subject to

\[ \delta = \theta = 0 \quad \text{at} \ t = 0 \ \text{or} \ x = 0, \]

where \( \Gamma = \frac{\sigma \alpha}{\alpha_f} \). They are hyperbolic sets of partial quasi-linear differential equations, which have two characteristic curves. By using the method of characteristics, the equations of direction of the characteristics are

\[ dx = 0 \quad \text{and} \quad \frac{\Gamma}{3} \left( \frac{4}{3} + k \delta \right) dx = \frac{\Gamma}{3} \left( 2 + k \delta \right) dt \]

so that the wave speed in the porous medium is

\[ \frac{9 \theta \alpha}{10}. \]

The same assumptions and restrictions are considered as in the previous section, but with a semi-infinite flat plate without thickness \( (a = 0) \). In this configuration, there are no conduction phenomena. It is noticed that equation has only one characteristic curve and the wave speed now is

\[ \frac{9 \theta \alpha}{10}. \]

The same behaviour of the temperature distribution \( \theta \) can also be seen in Fig. 3.

3. Plate without Thickness

The interface temperature distribution \( \theta \) is illustrated in Fig. 2 to show that although the value of \( \theta \) increases continuously with both in \( t \) and \( x \), its slope exhibits a discontinuity at whose value depends on \( t, \Gamma \), and \( k \). This discontinuity suggests a sudden change in the heat transfer characteristics that can be attributed to the presence of an essential singularity in the governing equations [16-23]. It is also noticed that \( \theta \) increases continuously with time and approaches for large time the corresponding steady state value.

![Fig. 2. Interface temperature for plate with thickness](image)

![Fig. 3. Interface temperature for plate without thickness](image)

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