Relay-Aided Secure Broadcasting for Visible Light Communications

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Abstract

A visible light communication broadcast channel is considered, in which a transmitter communicates with two legitimate receivers in the presence of an external eavesdropper. A number of trusted cooperative half-duplex relays are deployed to aid with securing the transmitted data. Transmitting nodes are equipped with single light fixtures, containing multiple light emitting diodes, and receiving nodes are equipped with single photo-detectors, rendering the considered setting as a single-input single-output system. Transmission is amplitude-constrained to maintain operation within the light emitting diodes’ dynamic range. Achievable secrecy rate regions are derived under such amplitude constraints for this multi-receiver wiretap channel, first for direct transmission without the relays, and then for multiple relaying schemes: cooperative jamming, decode-and-forward, and amplify-and-forward. Superposition coding with uniform signaling is used at the transmitter and the relays. Further, for each relaying scheme, secure beamforming vectors are carefully designed at the relay nodes in order to hurt the eavesdropper and/or benefit the legitimate receivers. Superiority of the proposed relaying schemes, with secure beamforming, is shown over direct transmission. It is also shown that the best relaying scheme depends on how far the eavesdropper is located from the transmitter and the relays, the number of relays, and their geometric layout.

I. INTRODUCTION

Visible light communications (VLC) technology is a promising candidate for future high-speed communication systems, offering solutions to spectrum congestion issues in conventional radio
frequency systems [1], [2]. The broadcast property in VLC, however, calls for careful design of secure communications to protect legitimate users from potential eavesdroppers, especially in public areas. Physical layer security is a powerful technique to deliver, provably, secure data, see, e.g., [3]. In this work, we design physical layer secure relaying schemes for a broadcast VLC channel with an external eavesdropper.

Recently, there have been several works on physical layer security aspects in VLC, see, e.g., [4]–[20]. The idea of employing an external friendly node that transmits jamming signals to degrade the eavesdropper channel is investigated in [4], [5], under amplitude constraints that are imposed such that the light emitting diodes (LEDs) within their dynamic range, with [4] focusing on uniform signaling and [5] focusing on truncated Gaussian signaling. Achievable secrecy rates for the multiple-input single-output (MISO) VLC channel are derived in [6], which are then used for transmit beamforming signal design for the MISO setting in [7]. References [8], [9] also derive achievable secrecy rates for the MISO VLC channel and design transmit beamforming signals, yet with focus on truncated generalized normal signaling, showing improvement over rates achieved by both uniform and truncated Gaussian signaling. Further improvements are later shown in [10] by using discrete signaling with finite number of mass points. Discrete signaling is also considered in [11], in which closed-form achievable secrecy rates for single-input single-output (SISO) VLC channels are derived. Reference [12] considers a multiple-input multiple-output (MIMO) VLC channel and derives achievable secrecy rates via designing transmit covariance matrices for uncorrelated symmetric logarithmic-concave input distributions. Secrecy outage probabilities are derived in [13]–[15] with multiple eavesdroppers, via tools from stochastic geometry and spatial point processes. Security aspects of hybrid radio frequency/VLC setups are considered in [16], [17]. A multiple-transmitter and multiple-eavesdropper scenario with one legitimate user is considered in [18], in which secrecy outage probabilities and ergodic secrecy rates with and without transmitters’ cooperation are derived. References [19], [20] are the most closely related to our work, in which broadcast VLC channels with confidential messages are considered and achievable secrecy sum rates are derived.

In this paper, we investigate the role of using extra luminary sources acting as trusted co-operative half-duplex relays in securing a two-user broadcast VLC channel from an external eavesdropper. The idea of relaying in VLC has been previously studied in a single-user setting,
with no external eavesdroppers, in [21]. In our setting, an amplitude constraint is imposed upon the transmitted signal for the LEDs to operate within their dynamic range. Under such amplitude constraint, we first derive an achievable secrecy rate region, without using the relays, based on superposition coding with uniform signaling at the source. We then invoke the relays, and derive achievable secrecy rate regions for several relaying schemes: *cooperative jamming*, *decode-and-forward*, and *amplify-and-forward*, in all of which an amplitude constraint also applies to the relays’ transmissions. For each relaying scheme, we design *secure beamforming* signals to maximize the achievable rates under the relays’ amplitude constraints. The design of the beamforming signals is based on formulating optimization problems that are inferred from the derived achievable secrecy rates. Results show the enhancement, in general, of the achievable secrecy rates using the relays, and that the best relaying scheme highly depends on the eavesdropper’s distance from the transmitter and the relays, and also on the number of relays and how they are geometrically laid out.

**II. System Model**

We consider an indoor VLC channel in which a transmitter (source) communicates with two legitimate receivers (users) in the presence of an external eavesdropper. The source is mounted on the ceiling, and is equipped with one light fixture that contains multiple LEDs modulated by the same current signal. The two users, and the eavesdropper, are assumed to lie geometrically on a two-dimensional plane close to the floor, and are each equipped with a single photo detector (PD).

The source’s LEDs are driven by a fixed, positive bias current that sets the illumination intensity. The data signal, \( x \in \mathbb{R} \), is superimposed on the bias current to modulate the instantaneous optical power emitted from the LEDs. The source employs superposition coding [22] to transmit two messages \( x_1 \) and \( x_2 \) to the first and the second user, respectively, by setting

\[
x = \alpha x_1 + (1 - \alpha)x_2
\]

for some \( \alpha \in [0, 1] \) that determines the priority of each user. In VLC, since the signal is modulated onto the intensity of the emitted light, it must satisfy peak amplitude constraints that are imposed by the dynamic range of typical LEDs to avoid clipping distortion. An amplitude
Fig. 1. An indoor VLC system model in which a source communicates with two legitimate users in the presence of an eavesdropper. A number of cooperative trusted relays assist with the source’s transmission.

Constraint, $A > 0$, is enforced as follows:

$$\alpha |x_1| + (1 - \alpha) |x_2| \leq A \quad \text{a.s.}$$ (2)

Let $h_1$, $h_2$, and $h_e$ denote the positive, real-valued channel gains between the source and the first user, second user, and eavesdropper, respectively. $h_1$ and $h_2$ are known at the source. Without loss of generality, let $h_1 > h_2$, and hence the first (strong) user decodes the second (weak) user’s message first then uses successive interference cancellation to decode its own message, while the weak user decodes its message by treating the strong user’s interfering signal as noise [22]. We denote by $y_1$, $y_2$, and $y_e$ the received signals, in the electric domain, at the strong user, the weak user, and the eavesdropper, respectively. These are

$$y_1 = h_1 x + n_1,$$ (3)

$$y_2 = h_2 x + n_2,$$ (4)

$$y_e = h_e x + n_e,$$ (5)

where $n_1$, $n_2$, and $n_e$ are i.i.d. $\sim \mathcal{N}(0, 1)$ noise terms.

A number of extra luminary sources acting as trusted cooperative half duplex relay nodes are available to aid with securing data from the eavesdropper. Such relay nodes can be, e.g., on the walls of the room in between the source and the users, or hanging from the ceiling in between

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1We choose to normalize the noise variances in this paper for simplicity of presentation, and take that effect on the signal-to-noise ratio (SNR) into the amplitude constraint’s value. That is, the SNR is now given by the square of the channel gain multiplied by the square of the amplitude constraint.
the source and the users, see Fig. 1. Let there be $K$ relays, and denote the channel gains from the source to the relays by the vector $h_r \triangleq [h_{r,1}, \ldots, h_{r,K}]$. Let $g_1$, $g_2$, and $g_e$ denote the $K$-length channel gain vectors from the relays to the strong user, the weak user, and the eavesdropper, respectively. The channels from the relays to the users and the eavesdropper are known at the relays. Similar to the source, we assume an amplitude constraint, $\bar{A} > 0$, applies to the each relay’s transmitted signal.

In the following sections, we derive achievable secrecy rates when the source and the relays transmit their data using uniform signaling schemes. We first compute the rates without using the relays, and then compare them to the rates achieved under various relaying strategies: cooperative jamming, decode-and-forward, and amplify-and-forward.

III. DIRECT TRANSMISSION

In this section, we derive an achievable secrecy rate region via direct transmission, i.e., without using the relay nodes. We state the result in the following theorem:

**Theorem 1** The following secrecy rate pair, for the strong and weak users, is achievable via direct transmission for a given $\alpha$:

$$
r_{1,s} = \left[\frac{1}{2} \log \left( \frac{1 + \frac{2h_1^2\alpha^2A^2}{\pi e}}{1 + \frac{h_e^2\alpha^2A^2}{3}} \right) \right]^{+},$$

$$
r_{2,s} = \left[\frac{1}{2} \log \left( \frac{1 + \frac{2h_2^2A^2}{\pi e}}{1 + \frac{h_e^2\alpha^2A^2}{3}} \right) \right]^{+},$$

where the second subscript $s$ is to denote secrecy rates, and $[\cdot]^+ \triangleq \max(\cdot, 0)$.

**Proof:** See Appendix A. ■

Observe that for $\alpha = 1$, we obtain $r_{2,s} = 0$ since $\frac{2}{\pi e} < \frac{1}{3}$, and $r_{1,s}$ coincides with the SISO achievable rate derived in [6], since the signal now is only directed toward one user (the strong user). The opposite holds for $\alpha = 0$ as well. It is also clear from (6) and (7) that the strong user’s achievable secrecy rate is positive if and only if (iff)

$$
\frac{2}{\pi e} h_1^2 > \frac{1}{3} h_e^2.
$$

2 All vectors in this paper are column vectors.
3 The log terms in this paper denote natural logarithms.
and that the weak user’s achievable secrecy rate is positive iff
\[
\left( \frac{2}{\pi e} - \frac{\alpha^2}{3} \right) h_2^2 + \left( \frac{2\alpha^2}{\pi e} - \frac{1}{3} \right) h_e^2 > \left( \frac{1}{9} - \frac{4}{\pi^2 e^2} \right) \alpha^2 h_2^2 h_e^2.
\] (9)

Thus, achieving positive secrecy rates depends on the relative channel conditions between the users and the eavesdropper. In the following sections, we enhance the achievable secrecy rates above by using cooperative trusted relays.

IV. COOPERATIVE JAMMING

In this section, we discuss the cooperative jamming scheme. In such, the relays cooperatively transmit a jamming signal \( Jz \), simultaneously with the source’s transmission, to confuse the eavesdropper. Here, \( J \in \mathbb{R}^K \) is a beamforming vector and \( z \) is a random variable that are both to be designed under the following constraints:

\[
|z| \leq \bar{A} \quad \text{a.s.,} \quad (10)
\]
\[
|J| \preceq 1_K, \quad (11)
\]
where \( 1_K \) is an all ones \( K \)-length vector, and the inequality \( \preceq \) is element-wise. The received signals at the legitimate users and the eavesdropper are now given by

\[
y_1 = h_1 x + g_1^T J z + n_1, \quad (12)
\]
\[
y_2 = h_2 x + g_2^T J z + n_2, \quad (13)
\]
\[
y_e = h_e x + g_e^T J z + n_e, \quad (14)
\]
where the superscript \( T \) denotes the transpose operation.

In order not to harm the legitimate users, the beamforming vector is designed such that

\[
g_1^T J = g_2^T J = 0, \quad (15)
\]
which is guaranteed if \( K \geq 3 \) relays, making the matrix \( G^T \triangleq [g_1 \ g_2]^T \) have a non-empty null space. Let us denote the beamforming vector satisfying (15) by \( J_o \). We now have the following result:
Theorem 2 The following secrecy rate pair, for the strong and weak users, is achievable via cooperative jamming for a given $\alpha$:

$$r_{1,s}^J = \left[ \frac{1}{2} \log \left( 1 + \frac{2h_e^2\alpha^2A^2}{\pi e} \right) - \frac{1}{2} \log \left( 1 + \frac{h_e^2\alpha^2A^2}{1 + \frac{2(g_e^TJ_o)^2A^2}{\pi e}} \right) \right]^+, \quad (16)$$

$$r_{2,s}^J = \left[ \frac{1}{2} \log \left( 1 + \frac{2h_e^2}{\pi e} \right) - \frac{1}{2} \log \left( 1 + \frac{h_e^2A^2}{1 + \frac{2(g_e^TJ_o)^2A^2}{\pi e}} \right) \right]^+, \quad (17)$$

where the superscript $J$ is to denote the cooperative jamming scheme.

Proof: See Appendix B. \hfill \square

We now proceed to find the optimal beamforming vector $J_o$ that maximally degrades the eavesdropper’s channel. In view of (16) and (17), by direct first derivative analysis, one can show that $r_{1,s}^J$ is increasing in $(g_e^TJ_o)^2$ iff

$$h_e^2\alpha^2A^2 > \frac{\pi e}{2} - 3 \approx 1.27, \quad (18)$$

and that $r_{2,s}^J$ is increasing in $(g_e^TJ_o)^2$ iff

$$h_e^2 (1 - \alpha^2) A^2 > \frac{\pi e}{2} - 3 \approx 1.27. \quad (19)$$

We note that, as a direct consequence of the data processing inequality [22], sending a jamming signal can only degrade the eavesdropper’s channel. It is clear, however, that the inequalities in (18) and (19) do not hold all the time, and hence sending a jamming signal might actually benefit the eavesdropper. This is justified though since we only derive lower bounds on the achievable secrecy rates, as opposed to exact computations. Whenever the secrecy rate (of either user) is increasing in $(g_e^TJ_o)^2$, we find the optimal beamforming vector $J_o^*$ by solving the following optimization problem:

$$\max_{J_o} (g_e^TJ_o)^2$$

$$\text{s.t. } G^TJ_o = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$|J_o| \leq 1_K. \quad (20)$$

To solve the above problem, we first introduce the following notation regarding the projection
matrix onto the null space of a matrix $A \in \mathbb{R}^{K \times K}$:

$$
P^\perp(A) \triangleq I_K - A (A^T A)^{-1} A^T,
$$

where $I_K$ denotes the $K \times K$ identity matrix. The optimal $J_o^*$ vector then should be of the form

$$
J_o^* = P^\perp(G) u_J
$$

for some vector $u_J \in \mathbb{R}^K$ to be designed. Hence, choosing $u_J = c_J P^\perp(G) g_e$ maximizes the objective function of problem (20) for some constant $c_J \in \mathbb{R}$. Finally, to satisfy the amplitude constraint, we choose the constant $c_J$ such that

$$
J_o^* = \frac{P^\perp(G) g_e}{\max_i (|P^\perp(G) g_e|)_i},
$$

where $(l)_i$ denotes the $i$th component of a vector $l$.

V. DECODE-AND-FORWARD

In this section, we discuss the decode-and-forward scheme. Communication occurs over two phases. In the first phase, the source broadcasts its messages to both the legitimate users and relays. In the second phase, the relays decode the received messages and forward them to the users. The eavesdropper overhears the transmission over the two phases.

The received signal at the relays in the first phase is

$$
y_r = h_rx + n_r,
$$

where $n_r \sim \mathcal{N}(0, I_K)$ represents the Gaussian noise in the source-relays channels. In the second phase, the $i$th relay decodes its received signal to find $x_1$ and $x_2$, re-encodes $x_1$ into $\tilde{x}_1$ and $x_2$ into $\tilde{x}_2$ using independent codewords, and then forwards them to the users using superposition coding after multiplying its transmitted signal by a constant $d_i \in \mathbb{R}$ to be designed. Effectively, the relays’ transmitted signal in the second phase is given by $dx_r$, with $d = [d_1, d_2, \ldots, d_K]$.

\footnote{Note that $P^\perp(\cdot)$ can be defined to operate on vectors as well, denoting a projection onto their orthogonal complements in the space.}
and \( x_r \) given by

\[
x_r = \alpha \tilde{x}_1 + (1 - \alpha) \tilde{x}_2.
\] (25)

That is, we assume the relays use the same \( \alpha \) fraction as the source. The following constraints hold at the relays:

\[
\alpha |\tilde{x}_1| + (1 - \alpha) |\tilde{x}_2| \leq \tilde{A} \quad \text{a.s.,} \tag{26}
\]

\[
|d| \leq 1_K. \tag{27}
\]

The received signals at the legitimate users and the eavesdropper in the second phase are given by

\[
y_r^1 = g_1^T d x_r + n_r^1, \tag{28}
\]

\[
y_r^2 = g_2^T d x_r + n_r^2, \tag{29}
\]

\[
y_e = g_e^T d x_r + n_e, \tag{30}
\]

where the superscript \( r \) is to denote signals received from the relays, and the noise terms \( n_r^1 \), \( n_r^2 \), and \( n_e \) are i.i.d. \( \sim \mathcal{N}(0, 1) \).

For the number of relays \( K \geq 2 \), we propose designing the beamforming vector \( d \) to satisfy the following:

\[
g_e^T d = 0 \tag{31}
\]

so that the eavesdropper does not receive any useful information in the second phase. We denote such beamforming signal by \( d_o \). If \( K \geq 3 \), then it will hold that both \( g_1^T d_o \) and \( g_2^T d_o \) are non-zero a.s. We now have the following theorem:

**Theorem 3** The following secrecy rate pair, for the strong and weak users, is achievable via decode-and-forward for a given \( \alpha \):

\[
 r_{1,s}^{DF} = \frac{1}{2} \left[ r_{1}^{DF} - \frac{1}{2} \log \left( 1 + \frac{h_2^2 \alpha^2 A^2}{3} \right) \right]^+, \tag{32}
\]

\[
 r_{2,s}^{DF} = \frac{1}{2} \left[ r_{2}^{DF} - \frac{1}{2} \log \left( \frac{1 + h_2^2 A^2}{1 + \frac{2h_2^2 \alpha^2 A^2}{\pi e}} \right) \right]^+, \tag{33}
\]
where the superscript $DF$ is to denote the decode-and-forward scheme, and $r_1^{DF}$ and $r_2^{DF}$ given by (34) and (35), respectively, at the top of this page.

**Proof:** See Appendix C. \[\Box\]

In view of (34) and (35), we see that $r_1^{DF}$ is increasing in $(g_1^T d_o)^2$, while direct first derivative analysis shows that $r_2^{DF}$ is increasing in $(g_1^T d_o)^2$ iff $\alpha \leq \sqrt{\frac{2/\pi e}{1/3}} \approx 0.838$, yet this condition can be ignored since $r_{s,2}^{DF}$ can only be positive if $\alpha \leq 0.838$. Therefore, we propose the following optimization problem to find the best beamforming vector:

$$
\max_{d_o} \alpha (g_1^T d_o)^2 + (1 - \alpha) (g_2^T d_o)^2 \\
\text{s.t.} \quad g_1^T d_o = 0 \\
|d_o| \leq 1_K.
$$

(36)

To satisfy the first constraint, the optimal $d_o^*$ should be of the form

$$
d_o^* = \mathcal{P}^\perp(g_e) u_d \triangleq F_d u_d
$$

(37)

for some vector $u_d \in \mathbb{R}^K$ to be designed, with $\mathcal{P}^\perp(\cdot)$ as defined in (21). To choose the best $u_d$, we rewrite the objective function of the above problem slightly differently as follows:

$$
\mathbf{u_d}^T F_d \left( \alpha g_1 g_1^T + (1 - \alpha) g_2 g_2^T \right) F_d \mathbf{u_d}.
$$

(38)

Therefore, the optimal $u_d$ is given by

$$
u_d = c_d \mathbf{v}_d,
$$

(39)
where \( c_d \in \mathbb{R} \) is a constant, and \( v_d \) is the leading eigenvector of the matrix

\[
F_d \left( \alpha g_1 g_1^T + (1 - \alpha) g_2 g_2^T \right) F_d,
\]

(40)
i.e., the eigenvector corresponding to the largest eigenvalue of the matrix. Finally, we choose \( c_d \) to satisfy the amplitude constraint as follows:

\[
u_d = \frac{v_d}{\max_i(|v_d|)}.
\]

(41)

VI. AMPLIFY-AND-FORWARD

In this section, we discuss the amplify-and-forward scheme. As in the decode-and-forward scheme, communication occurs over two phases. However, in the second phase, the \( i \)th relay merely re-sends its received signal from the first phase after multiplying (amplifying) it by a constant \( a_i \in \mathbb{R} \) to be designed. Effectively, the relays’ transmitted signal in the second phase is given by \( \text{diag}(y_r) \cdot a \), where \( \text{diag}(l) \) is the diagonalization of the vector \( l \), and the following amplitude constraint holds at the relays:

\[|\text{diag}(y_r) \cdot a| \leq 1_{K} \bar{A} \text{ a.s.} \]

(42)
The received signals at the legitimate users and the eavesdropper in the second phase are given by

\[
y_r^r = g_1^T \text{diag}(y_r) \cdot a + n_r^r, \]

(43)
\[
y_r^c = g_2^T \text{diag}(y_r) \cdot a + n_r^c, \]

(44)
\[
y_e^r = g_e^T \text{diag}(y_r) \cdot a + n_e^r. \]

(45)

As in the decode-and-forward scheme, for \( K \geq 2 \) relays, we propose designing the beamforming vector \( a \) to satisfy

\[g_e^T \text{diag}(h_r) \cdot a = 0 \]

(46)
so that the eavesdropper does not receive any useful information in the second phase. We denote such beamforming signal by \( a_o \). Further, for \( K \geq 3 \) relays, it holds that both \( g_1^T \text{diag}(h_r) \cdot a_o \) and \( g_2^T \text{diag}(h_r) \cdot a_o \) are non-zero a.s. We now have the following theorem:
Theorem 4 The following secrecy rate pair, for the strong and weak users, is achievable via amplify-and-forward for a given $\alpha$:

$$r_{AF,1}^{AF} = \frac{1}{2} \left[ \frac{1}{2} \log \left( 1 + \frac{2\kappa_2^2 \alpha^2 A^2}{\pi e} \right) - \frac{1}{2} \log \left( 1 + \frac{h_2^2 \alpha^2 A^2}{3} \right) \right]^+, \quad (47)$$

$$r_{AF,2}^{AF} = \frac{1}{2} \left[ \frac{1}{2} \log \left( 1 + \frac{2\kappa_2^2 \alpha^2 A^2}{1 + \kappa_2^2 \alpha^2 A^2} \right) - \frac{1}{2} \log \left( 1 + \frac{h_2^2 A^2}{3} \right) \right]^+, \quad (48)$$

where the superscript $AF$ is to denote the amplify-and-forward scheme, and

$$\kappa_j^2 \triangleq h_j^2 \frac{(g_j^T \text{diag}(h_r) a_o)^2}{1 + (g_j^T a_o)^2}, \quad j = 1, 2. \quad (49)$$

**Proof:** See Appendix D. \[\blacksquare\]

In view of (47) and (48), we see that $r_{AF,1}^{AF}$ is increasing in $\kappa_1^2$, while direct first derivative shows that $r_{AF,2}^{AF}$ is increasing in $\kappa_2^2$ iff $\alpha \leq \sqrt{2/\pi e} \approx 0.838$, yet again this condition can be ignored (as we did in the decode-and-forward case) since $r_{AF,2}^{AF}$ can only be positive if $\alpha \leq \sqrt{2/\pi e} \approx 0.838$. Therefore, we propose the following fractional optimization problem to find the best beamforming vector that maximizes $j$th user’s rate, $j = 1, 2$:

$$\max_{a_o} \frac{(g_j^T \text{diag}(h_r) a_o)^2}{1 + (g_j^T a_o)^2}$$

s.t. $g_e^T \text{diag}(h_r) a_o = 0$

$$|\text{diag}(y_r) a_o| \leq 1_{K\bar{A}}. \quad (50)$$

To solve the above fractional program, we introduce the following auxiliary problem:

$$p_j^{AF}(\lambda) \triangleq \max_{a_o} \left( g_j^T \text{diag}(h_r) a_o \right)^2 - \lambda \left( 1 + (g_j^T a_o)^2 \right)$$

s.t. $g_e^T \text{diag}(h_r) a_o = 0$

$$|\text{diag}(y_r) a_o| \leq 1_{K\bar{A}} \quad (51)$$

for some $\lambda \geq 0$. One can show the following: 1) $p_j^{AF}(\lambda)$ is decreasing in $\lambda$; and 2) the optimal solution of problem (50) is given by $\lambda^*$ that solves $p_j^{AF}(\lambda^*) = 0$ [23]. Hence, one can find an upper bound on $\lambda^*$ that makes $p_j^{AF}(\lambda) < 0$ and then proceed by, e.g., a bisection search, to find $\lambda^*$. Focusing on problem (51), we first note that, to satisfy the first constraint, the optimal $a_o$
should be of the form

\[ a_o = \mathcal{P}^\perp(\text{diag}(h_r)g_e)u_a \triangleq F_a u_a \]  

(52)

for some vector \( u_a \in \mathbb{R}^K \) to be designed. To choose the best \( u_a \), we rewrite the objective function as

\[ u_a^T F_a \left( \text{diag}(h_r)g_j^T \text{diag}(h_r) - \lambda g_j g_j^T \right) F_a u_a. \]  

(53)

Hence, the optimal \( u_a \) is given by

\[ u_a = c_a v_a, \]  

(54)

where \( c_a \in \mathbb{R} \) is a constant, and \( v_a \) is the leading eigenvector of the matrix

\[ F_a \left( \text{diag}(h_r)g_j^T \text{diag}(h_r) - \lambda g_j g_j^T \right) F_a. \]  

(55)

We choose \( c_a \) to satisfy the amplitude constraint as follows:

\[ u_a = \frac{v_a}{\max_i (|\text{diag}(y_r)v_a|)_i} A. \]  

(56)

Finally, let \( a_o^{(j)} \) be the solution of problem (50). We propose using the following beamforming vector:

\[ a_o^a = \alpha a_o^{(1)} + (1 - \alpha) a_o^{(2)}. \]  

(57)

VII. NUMERICAL EVALUATIONS

In this section, we validate our results via numerical evaluations. We consider a room of size 5 × 5 × 3 cubic meters. With the origin tuple \((0, 0, 0)\) denoting the center of the room’s floor, the source is located at \((0, 0, 3)\), the strong user at \((0.75, 0.75, 0.7)\), and the weak user at \((-1.25, 0.75, 0.7)\). We consider \( K = 5 \) relays located at the following positions: \((0.1, 0.1, 2)\), \((0.1, -0.1, 2)\), \((0, 0, 2)\), \((-0.1, 0.1, 2)\), and \((-0.1, -0.1, 2)\), see the plan view in Fig. 2. The channel gain between two nodes \( q_1 \) and \( q_2 \) is given by [24]

\[ \frac{A_{det}(m + 1)}{2\pi l_{q_1,q_2}^2} \left( \frac{|z_{q_1} - z_{q_2}|}{l_{q_1,q_2}} \right)^{m+1}. \]  

(58)
where $A_{det} = 10^{-4}$ squared meters is the PD’s physical area, $m = -\log(2)/\log(\cos(\phi_2/2))$ is the order of Lambertian emission, with $\phi_2 = 60^\circ$ denoting the LED semi-angle at half power, and $l_{q_1,q_2}$ denoting the distance between the two nodes. We set the amplitude constraints to $A = 10^7$ and $\bar{A} = 10^6$. We also ignore optimizing the term $\lambda$ in the amplify-and-forward scheme for simplicity, and set it to 1.

In Fig. 3, the achievable secrecy rate regions for the schemes proposed in this paper, along with that of the direct transmission scheme are shown, through basically taking the union over $0 \leq \alpha \leq 1$ of the achievable secrecy rate pairs, and then applying time sharing if needed to convexify the resulting regions. The solid lines in Fig. 3 are when the eavesdropper is located at $(0, 1.75, 0.7)$. We see in this case that all the proposed schemes perform strictly better than direct
transmission. The dashed lines in Fig. 3 are when the eavesdropper is located a bit further away from the source (and the relays) at $(0, 2, 0.7)$. We see in this case that larger secrecy rates are achievable for all schemes, yet both the decode-and-forward and amplify-and-forward schemes perform worse than direct transmission, since the channels from the relays to the eavesdropper are relatively worse. We also notice the slight improvement of the cooperative jamming scheme in this case over direct transmission. It is clear from this figure that the best relaying scheme depends on the distance to the eavesdropper, and that the proposed relaying schemes achieve higher secrecy rates than direct transmission when the eavesdropper is closer to the source.

In Fig. 4 we show the effect of the eavesdropper’s distance from the source on the secrecy sum rate for a fixed $\alpha = 0.8$. Such value of $\alpha$ is merely chosen as it better shows the varying performances of the relaying schemes compared to other values of $\alpha$, under the considered system parameters. We vary the eavesdropper’s location from $(0, 0.75, 0.7)$ to $(0, 2.25, 0.7)$, i.e., we only change its location’s second coordinate’s value. We observe from the figure that clearly the secrecy sum rate increases, for all schemes, as the eavesdropper’s distance from the source increases. We also note that at relatively close locations, the proposed relaying schemes achieve strictly positive rates, as opposed to the zero rate achieved via direct transmission. This again shows how useful the proposed relaying schemes become, compared to direct transmission, when the eavesdropper is relatively close to the source.
We elaborate on this latter note in Fig. 6 in which we fix the eavesdropper’s position at 
$(0, 1, 0.7)$, and vary the locations of the relays by basically varying the point around which their second coordinate is centered. Specifically, we let the relays be located at $(0.1, c_y + 0.1, 2)$, $(0.1, c_y - 0.1, 2)$, $(0, c_y, 2)$, $(-0.1, c_y + 0.1, 2)$, and $(-0.1, c_y - 0.1, 2)$ and vary the center point $c_y$ from 0 to 1.5, see the plan view in Fig. 5. We see from the figure that direct transmission achieves zero secrecy rates for all values of $c_y$, since the eavesdropper is relatively closer to the source than the legitimate users. On the other hand, all the proposed relaying schemes achieve strictly positive secrecy rates, with varying performances. We notice, in particular, that the relatively simple cooperative jamming scheme performs best when the relays are closest to the eavesdropper.

Finally, in Fig. 7 we explore the effect of a different aspect on the secrecy sum rate: the
Fig. 7. Effect of number of relays on the secrecy sum rates of the proposed schemes. The eavesdropper is located at (0, 1.25, 0.7). The relays are located along the corner and mid side points of a square of side length $2\ell$ meters, centered at (0, 0, 2). Solid lines are when $\ell = 0.1$, and dashed lines are when $\ell = 0.5$.

Fig. 8. Plan view of the geometric layout of the system, in which the number of relays is varying, as well as their relative distance from each other. Either the layout in green with $\ell = 0.1$, or that in brown with $\ell = 0.5$ is chosen to employ the varying number of relays.

We consider the situation in which the eavesdropper is located relatively close to the source at (0, 1.25, 0.7), and place a varying number of relays along the corners and sides of a square of side length $2\ell$ meters, centered at (0, 0, 2). Specifically, we locate one relay at the center of the square, at (0, 0, 2), and the remaining relays at either the corners: $(\ell, \ell, 2)$, $(-\ell, \ell, 2)$, $(\ell, -\ell, 2)$, and $(-\ell, -\ell, 2)$; or at the centers of the sides: $(\ell, 0, 2)$, $(0, \ell, 2)$, $(-\ell, 0, 2)$, and $(0, -\ell, 2)$. We vary the number of relays, $K$, from 3 to 9 relays, and plot the achievable secrecy sum rate in each case. The solid
lines in Fig. 7 are when $\ell = 0.1$ meters, while the dashed lines are when $\ell = 0.5$ meters, see the plan view in Fig. 8. We see from the figure that direct transmission achieves zero secrecy rates, since the eavesdropper is relatively closer to the source than the legitimate users, while all the proposed schemes achieve strictly positive secrecy sum rates, which is increasing with the number of relays. We also see varying performances of the proposed schemes when $\ell$ changes. In particular, when the relays are relatively close to each other with $\ell = 0.1$, decode-and-forward performs best, with cooperative jamming catching up with amplify-and-forward when $K \geq 5$. On the other hand, when the relays are relatively far away from each other with $\ell = 0.5$, cooperative jamming performs similar to amplify-and-forward when $K = 3, 4$ before outperforming it when $K \geq 5$, and outperforms decode-and-forward for $K \geq 7$. In this latter case of $\ell = 0.5$, the performance of decode-and-forward saturates after $K = 4$ and extra deployed relays do not add more benefit, which is mainly because they are relatively far away from the source. The results in Fig. 7 show that both the number of relays, and their geometrical layout are crucial system aspects that should be carefully designed to meet a desired system performance.

VIII. **Conclusion and Future Directions**

A VLC broadcast channel in which a transmitter communicates with two legitimate receivers in the presence of an external eavesdropper has been considered. Under an amplitude constraint, imposed to allow the LEDs to operate within their dynamic range, an achievable secrecy rate region has been derived, based on superposition coding with uniform signaling. Then, trusted cooperative half-duplex relay nodes have been introduced in order to assist with securing the data from the eavesdropper via multiple relaying schemes: cooperative jamming, decode-and-forward, and amplify-and-forward. Secure beamforming signals have been carefully designed at the relays to enhance the achievable secrecy rates. It has been shown that the best relaying scheme varies according to the distance from the transmitter (and the relays) to the eavesdropper, and also on the number of relays and their geometrical layout.

Extending the approaches in this paper to the case with multiple transmitting LED fixtures and/or multiple receiving PDs would be of interest as a future direction. In addition, one could also consider deriving achievable secrecy rate regions based on different distributions other than uniform, such as discrete and truncated generalized normal distributions, that are used in the literature. Another direction would be to consider the case in which the eavesdropper’s location
is not known at the transmitter, or known within some boundaries. In the former case, the goal would be deriving secrecy outage probabilities, while in the latter case, the goal could be deriving a worst case achievable secrecy rate region.

**Appendix**

A. Proof of Theorem 1

Given $\alpha$, the following secrecy rates, for the strong and weak users, are achievable for this multi-receiver wiretap channel [25]:

\[
\begin{align*}
  c_{1,s} &= \left[ I(x; y_1|x_2) - I(x; y_e|x_2) \right]^+, \\
  c_{2,s} &= \left[ I(x_2; y_2) - I(x_2; y_e) \right]^+,
\end{align*}
\]

(59)

(60)

where $I(\cdot; \cdot)$ denotes the mutual information measure [22]. Now let the transmitted symbols $x_1$ and $x_2$ represent two independent uniformly distributed random variables on $[-A, A]$. Clearly, this satisfies the amplitude constraint in (2). Let us now drop the superscript + for simplicity of presentation. We proceed by lower bounding $c_{1,s}$ as follows:

\[
\begin{align*}
  c_{1,s} &= I(x; h_1(\alpha x_1 + (1 - \alpha) x_2) + n_1|x_2) - I(x; h_e(\alpha x_1 + (1 - \alpha) x_2) + n_e|x_2) \\
  &= I(x_1; h_1\alpha x_1 + n_1) - I(x_1; h_e\alpha x_1 + n_e) \\
  &= \mathbb{H}(h_1\alpha x_1 + n_1) - \mathbb{H}(h_e\alpha x_1 + n_e) \\
  &\geq \frac{1}{2} \log \left( e^{2\mathbb{H}(h_1\alpha x_1)} + e^{2\mathbb{H}(n_1)} \right) - \frac{1}{2} \log \left( 2\pi e \left( h_e^2\alpha^2 A^2 + 1 \right) \right) \\
  &= \frac{1}{2} \log \left( h_1^2\alpha^2 4A^2 + 2\pi e \right) - \frac{1}{2} \log \left( 2\pi e \left( h_e^2\alpha^2 A^2 + 1 \right) \right) \\
  &= r_{1,s},
\end{align*}
\]

(61)

(62)

(63)

(64)

(65)

(66)

where $\mathbb{H}(\cdot)$ in (63) denotes the differential entropy measure [22], and (64) follows by lower bounding the first (positive) term in (63) by the entropy power inequality (EPI) [22] and upper bounding the second (negative) term in (63) by plugging in a Gaussian $x_1$, instead of uniform, with the same variance, since Gaussian maximizes differential entropy [22]. Next, we proceed similarly to lower bound $c_{2,s}$ as follows:

\[
\begin{align*}
  c_{2,s} &= I(x_2; h_2(\alpha x_1 + (1 - \alpha) x_2) + n_2) - I(x_2; h_e(\alpha x_1 + (1 - \alpha) x_2) + n_e)
\end{align*}
\]

(67)
\begin{align*}
    & \geq \alpha \ln (h_2 x_1 + n_2) + (1 - \alpha) \ln (h_2 x_2 + n_2) - \ln (h_2 x_1 + n_2) \\
    & - \ln (h_\epsilon (\alpha x_1 + (1 - \alpha) x_2) + n_e) + \ln (h_\epsilon \alpha x_1 + n_e) \\
    & \geq \alpha \ln (h_2 x_1 + n_2) + (1 - \alpha) \ln (h_2 x_2 + n_2) - \ln (h_2 x_1 + n_2) \\
    & - \ln (h_\epsilon (\alpha x_1 + (1 - \alpha) x_2) + n_e) + \ln (h_\epsilon \alpha x_1 + n_e) \\
    & \geq \frac{1}{2} \log \left( e^{2\ln(h_2 x_1)} + e^{2\ln(n_2)} \right) - \frac{1}{2} \log \left( 2\pi e \left( \frac{h_2^2 \alpha^2 A^2}{3} + 1 \right) \right) \\
    & - \frac{1}{2} \log \left( 2\pi e \left( \frac{h_\epsilon^2 \alpha^2 A^2}{3} + 1 \right) \right) + \frac{1}{2} \log \left( e^{2\ln(h_\epsilon \alpha x_1)} + e^{2\ln(n_\epsilon)} \right) \\
    & \geq \frac{1}{2} \log \left( e^{2\ln(h_2 x_1)} + e^{2\ln(n_2)} \right) - \frac{1}{2} \log \left( 2\pi e \left( \frac{h_2^2 \alpha^2 A^2}{3} + 1 \right) \right) \\
    & - \frac{1}{2} \log \left( 2\pi e \left( \frac{h_\epsilon^2 \alpha^2 A^2}{3} + 1 \right) \right) + \frac{1}{2} \log \left( e^{2\ln(h_\epsilon \alpha x_1)} + e^{2\ln(n_\epsilon)} \right) \\
    & = \ln \left( h_2 (\alpha x_1 + (1 - \alpha) x_2) + n_2 \right) - \ln \left( h_2 \alpha x_1 + n_2 \right) \\
    & - \ln \left( h_\epsilon (\alpha x_1 + (1 - \alpha) x_2) + n_e \right) + \ln \left( h_\epsilon \alpha x_1 + n_e \right) \\
    & \geq \alpha \ln (h_2 x_1 + n_2) + (1 - \alpha) \ln (h_2 x_2 + n_2) - \ln (h_2 x_1 + n_2) \\
    & - \ln (h_\epsilon (\alpha x_1 + (1 - \alpha) x_2) + n_e) + \ln (h_\epsilon \alpha x_1 + n_e) \\
    & \geq \frac{1}{2} \log \left( e^{2\ln(h_2 x_1)} + e^{2\ln(n_2)} \right) - \frac{1}{2} \log \left( 2\pi e \left( \frac{h_2^2 \alpha^2 A^2}{3} + 1 \right) \right) \\
    & - \frac{1}{2} \log \left( 2\pi e \left( \frac{h_\epsilon^2 \alpha^2 A^2}{3} + 1 \right) \right) + \frac{1}{2} \log \left( e^{2\ln(h_\epsilon \alpha x_1)} + e^{2\ln(n_\epsilon)} \right) \\
    & \geq \frac{1}{2} \log \left( e^{2\ln(h_2 x_1)} + e^{2\ln(n_2)} \right) - \frac{1}{2} \log \left( 2\pi e \left( \frac{h_2^2 \alpha^2 A^2}{3} + 1 \right) \right) \\
    & - \frac{1}{2} \log \left( 2\pi e \left( \frac{h_\epsilon^2 \alpha^2 A^2}{3} + 1 \right) \right) + \frac{1}{2} \log \left( e^{2\ln(h_\epsilon \alpha x_1)} + e^{2\ln(n_\epsilon)} \right) \\
    & = r_{2,s},
\end{align*}

where (69) follows by Jensen’s inequality (concavity of differential entropy) [22]; (70) follows by using EPI to lower bound the positive terms of (69) together with the fact that $h_2 x_1 + n_2$ and $h_2 x_2 + n_2$ have the same distribution, and plugging in a Gaussian $x_1$ and $x_2$, instead of uniform, with the same variances to upper bound the negative terms of (69); and (71) follows since $\alpha \leq 1$. This concludes the proof.

\subsection*{B. Proof of Theorem 2}

We first note that, different from direct transmission, over here we have another random variable $z$ involved in the calculations. To emphasize the difference, we denote the secrecy rates in (59) and (60) by $c_{1,s}^d$ and $c_{2,s}^d$, respectively. We now proceed with the same approach as that followed in the proof of Theorem 1. Specifically, we let $x_1$ and $x_2$ be two independent uniformly distributed random variables on $[-A, A]$, and let $z$ be uniformly distributed on $[-\bar{A}, \bar{A}]$, independently of $x_1$ and $x_2$. We then expand the mutual information terms constituting $c_{1,s}^d$ and $c_{2,s}^d$ in terms of differential entropy, lower bound positive terms by EPI (and Jensen’s inequality if need be), and upper bound negative terms by plugging in Gaussian random variables with the same variances, instead of uniform. Specific justifications of intermediate steps are as in
the proof of Theorem [11] and are thus omitted for brevity. We also drop the superscript \( + \) for convenience.

A lower bound on \( c_{1,s}^J \) is now given by

\[
c_{1,s}^J = \mathbb{I}(x_1; h_1 \alpha x_1 + n_1) - \mathbb{I}(x_1; h_e \alpha x_1 + g_e^T J_0 z + n_e) \tag{74}
\]

\[
\geq \frac{1}{2} \log \left( e^{2h(h_1 \alpha x_1)} + e^{2h(n_1)} \right) - \frac{1}{2} \log(2\pi e) \\
- \frac{1}{2} \log \left( 2\pi e \left( h_1^2 \alpha^2 A^2 \right) + \frac{1}{3} \left( g_e^T J_0 \right)^2 + 1 \right) + \frac{1}{2} \log \left( e^{2h(g_e^T J_0) + e^{2h(n_e)}} \right) \tag{75}
\]

Similarly, we lower bound \( c_{2,s}^J \) as follows:

\[
c_{2,s}^J = \mathbb{I}(x_2; h_2 (\alpha x_1 + (1 - \alpha) x_2) + n_2) - \mathbb{I}(x_2; h_e (\alpha x_1 + (1 - \alpha) x_2) + g_e^T J_0 z + n_e) \tag{76}
\]

\[
\geq \frac{1}{2} \log \left( e^{2h(h_2 x_1)} + e^{2h(n_2)} \right) - \frac{1}{2} \log \left( 2\pi e \left( h_2^2 \alpha^2 A^2 \right) + \frac{1}{3} \left( g_e^T J_0 \right)^2 + 1 \right) \\
- \frac{1}{2} \log \left( 2\pi e \left( h_2^2 \alpha^2 A^2 \right) + \frac{1}{3} \left( g_e^T J_0 \right)^2 + 1 \right) + \frac{1}{2} \log \left( e^{2h(g_e^T J_0) + e^{2h(n_e)}} \right) \tag{77}
\]

\[
\geq \frac{1}{2} \log \left( e^{2h(h_2 x_1)} + e^{2h(n_2)} \right) - \frac{1}{2} \log \left( 2\pi e \left( h_2^2 \alpha^2 A^2 \right) + 1 \right) \\
- \frac{1}{2} \log \left( 2\pi e \left( h_2^2 \alpha^2 A^2 \right) + \frac{1}{3} \left( g_e^T J_0 \right)^2 + 1 \right) + \frac{1}{2} \log \left( e^{2h(g_e^T J_0) + e^{2h(n_e)}} \right) \tag{78}
\]
\[
- \frac{1}{2} \log \left( 2\pi e \left( \frac{h_e^2 A_e^2}{3} + (g_e^T J_o)^2 \frac{A^2}{3} + 1 \right) \right) + \frac{1}{2} \log \left( h_s^2 e^2 A^2 + (g_e^T J_o)^2 \alpha^2 4 \bar{A}^2 + 2\pi e \right)
\]
\[= r_{2,s}^j. \]  

This concludes the proof.

**C. Proof of Theorem 3**

We let the relays employ the same decoding technique of the strong user: first decode the weak user’s message by treating the strong user’s interfering signal as noise, and then use successive interference cancellation to decode the strong user’s message. Using the decode-and-forward lower bound in [26, Theorem 16.2], the following secrecy rates are achievable:

\[
c_{1,s}^{DF} = \frac{1}{2} \left[ \min \left\{ \mathbb{I} \left( x_1; y_1, y_1^r \mid x_2, \bar{x}_2 \right), \min_i \mathbb{I} \left( x_1; y_{r,i} \mid x_2 \right) \right\} - \mathbb{I} \left( x; y_e \mid x_2 \right) \right]^+, \]
\[
c_{2,s}^{DF} = \frac{1}{2} \left[ \min \left\{ \mathbb{I} \left( x_2, \bar{x}_2; y_2, y_2^r \mid x_2 \right), \min_i \mathbb{I} \left( x_2; y_{r,i} \mid x_2 \right) \right\} - \mathbb{I} \left( x_2; y_e \mid x_2 \right) \right]^+,
\]

where the extra \( \frac{1}{2} \) term is due to sending the same information over two phases of equal durations. By the independence of \( x_j \) and \( \bar{x}_j \), \( j = 1, 2 \), we have

\[
\mathbb{I} \left( x_1, \bar{x}_1; y_1, y_1^r \mid x_2, \bar{x}_2 \right) = \mathbb{I} \left( x_1; h_1 \alpha x_1 + n_1 \right) + \mathbb{I} \left( \bar{x}_1; g_1^T d_o \alpha \bar{x}_1 + n_1^r \right),
\]
\[
\mathbb{I} \left( x_2, \bar{x}_2; y_2, y_2^r \mid x_2 \right) = \mathbb{I} \left( x_2; h_2 \alpha (x_1 + (1 - \alpha)x_2) + n_2 \right) + \mathbb{I} \left( \bar{x}_2; g_2^T d_o (\alpha \bar{x}_1 + (1 - \alpha)\bar{x}_2) + n_2^r \right).
\]

To derive the lower bounds on \( c_{1,s}^{DF} \) and \( c_{2,s}^{DF} \), we proceed as in the proof of Theorem 1 by lower bounding the positive terms above by EPI (and Jensen’s inequality if need be), and upper bounding the negative terms above by plugging in Gaussian random variables with the same variances instead of uniform. This directly gives \( r_{1,s}^{DF} \) and \( r_{2,s}^{DF} \). Specific details are merely the same as in the proof of Theorem 1 and are omitted for brevity.

**D. Proof of Theorem 4**

We note that the \( j \)th user, \( j = 1, 2 \), can view the system as the following \( 1 \times 2 \) SIMO system:

\[
\begin{bmatrix}
y_j \\
y_j^r
\end{bmatrix} = \begin{bmatrix} h_j \\
g_j^T \text{diag} \left( h_r \right) a_o \end{bmatrix} x + \begin{bmatrix} n_j \\
n_j^r
\end{bmatrix},
\]

(90)
where the noise term $\tilde{n}_j^r \triangleq \begin{bmatrix} g_j^T \text{diag}(n_r) a_o + n_j^r \end{bmatrix}$, which is $\sim \mathcal{N} \left(0, 1 + (g_j^T a_o)^2 \right)$. The $j$th user then applies the capacity achieving maximal ratio combining [27] to get the following sufficient statistic:

$$\tilde{y}_j \triangleq h_j y_j + \frac{g_j^T \text{diag}(h_r) a_o}{1 + (g_j^T a_o)^2} y_j^r$$

(91)

$$\triangleq h_j y_j + \frac{h_j, r}{\sigma_{j, r}} y_j^r.$$ (92)

Therefore, the following secrecy rates are now achievable:

$$c_{1,s}^{AF} = \frac{1}{2} \left[ \mathbb{I} (x; \tilde{y}_1 | x_2) - \mathbb{I} (x; y_e | x_2) \right]^+,$$ (93)

$$c_{2,s}^{AF} = \frac{1}{2} \left[ \mathbb{I} (x_2; \tilde{y}_2) - \mathbb{I} (x_2; y_e) \right]^+,$$ (94)

where the extra $\frac{1}{2}$ term is due to sending the same information over two phases of equal durations, as in the decode-and-forward scheme. We now proceed with lower bounding the positive mutual information terms in (93) and (94); the negative terms are handled exactly as in the proof of Theorem 11. For the strong user, we have

\[
\mathbb{I} (x; \tilde{y}_1 | x_2) = \ln \left( \left( h_1^2 + \frac{h_{1, r}^2}{\sigma_{1, r}^2} \right) \alpha x_1 + h_1 n_1 + h_{1, r} \tilde{n}_1^r \right) - \ln \left( h_1 n_1 + h_{1, r} \tilde{n}_1^r \right)
\]

(95)

\[
\geq \frac{1}{2} \log \left( e^{2h \left( \left( h_1 + \frac{h_{1, r}}{\sigma_{1, r}} \right) \alpha x_1 \right)} + e^{2h (h_1 n_1) + e^{2h (h_{1, r} \tilde{n}_1^r)}} \right) - \frac{1}{2} \log \left( 2 \pi e \left( h_1^2 + \frac{h_{1, r}^2}{\sigma_{1, r}^2} \right) \right)
\]

(96)

\[
= \frac{1}{2} \log \left( h_1^2 + \frac{h_{1, r}^2}{\sigma_{1, r}^2} \right)^2 \alpha^2 4 A^2 + (2 \pi e) \left( h_1^2 + \frac{h_{1, r}^2}{\sigma_{1, r}^2} \right) - \frac{1}{2} \log \left( 2 \pi e \left( h_1^2 + \frac{h_{1, r}^2}{\sigma_{1, r}^2} \right) \right)
\]

(97)

\[
= \frac{1}{2} \log \left( 1 + \frac{2 \kappa_1^2 \alpha^2 A^2}{\pi e} \right).
\]

Similarly, for the weak user, we have

\[
\mathbb{I} (x_2; \tilde{y}_2) = \ln \left( \left( h_2^2 + \frac{h_{2, r}^2}{\sigma_{2, r}^2} \right) \alpha x_1 + (1 - \alpha) x_2 + h_2 n_2 + \frac{h_{2, r} \tilde{n}_2^r}{\sigma_{2, r}^2} \right)
\]

(99)

\[
- \ln \left( h_2^2 + \frac{h_{2, r}^2}{\sigma_{2, r}^2} \right) \alpha x_1 + h_2 n_2 + \frac{h_{2, r} \tilde{n}_2^r}{\sigma_{2, r}^2} \right)
\]

\[
\geq \alpha \ln \left( \left( h_2^2 + \frac{h_{2, r}^2}{\sigma_{2, r}^2} \right) x_1 + h_2 n_2 + \frac{h_{2, r} \tilde{n}_2^r}{\sigma_{2, r}^2} \right) + (1 - \alpha) \ln \left( \left( h_2^2 + \frac{h_{2, r}^2}{\sigma_{2, r}^2} \right) x_2 + h_2 n_2 + \frac{h_{2, r} \tilde{n}_2^r}{\sigma_{2, r}^2} \right)
\]
\[ - \frac{1}{2} \log \left( \frac{2\pi e}{(2\pi e)^2 + (h_2^2 + h_2^2) r^2} \right) \]

(100)

\[ \geq \frac{1}{2} \log \left( e^{2h_2} \left( \frac{h_2^2 + h_2^2}{\sigma_{2,r}^2} \right)^x_1 + e^{2h_2 \nu_2} + e^{2h_2 \rho_2} \left( \frac{h_2^2 + h_2^2}{\sigma_{2,r}^2} \right)^x_3 \right) \]

(101)

\[ \frac{1}{2} \log \left( \frac{2\pi e}{(2\pi e)^2 + (h_2^2 + h_2^2) r^2} \right) \]

(102)

\[ = \frac{1}{2} \log \left( 1 + \frac{2\alpha_2^2 A^2}{\pi e} \left( \frac{1}{1 + \frac{2\alpha_2^2 A^2}{\pi e}} \right) \right). \]

(103)

This concludes the proof.

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