Review of Low Energy Neutrinos

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Abstract. Some issues regarding low energy neutrinos are reviewed. We focus on three aspects i) We show that by employing very low energy (a few keV) electron neutrinos, neutrino disappearance oscillations can be investigated by detecting recoiling electrons with low threshold spherical gaseous TPC’s. In such an experiment, which is sensitive to the small mixing angle $\theta_{13}$, the novel feature is that the oscillation length is so small that the full oscillation takes place inside the detector. Thus one can determine accurately all the oscillation parameters and, in particular, measure or set a good limit on $\theta_{13}$. ii) Low threshold gaseous TPC detectors can also be used in detecting nuclear recoils by exploiting the neutral current interaction. Thus these robust and stable detectors can be employed in supernova neutrino detection. iii) The lepton violating neutrinoless double decay is investigated focusing on how the absolute neutrino mass can be extracted from the data.

1. Introduction.
The discovery of neutrino oscillations can be considered as one of the greatest triumphs of modern physics. It began with atmospheric neutrino oscillations [1] interpreted as $\nu_\mu \rightarrow \nu_\tau$ oscillations, as well as $\nu_e$ disappearance in solar neutrinos [2]. These results have been recently confirmed by the KamLAND experiment [3], which exhibits evidence for reactor antineutrino disappearance. As a result of these experiments we have a pretty good idea of the neutrino mixing matrix and the two independent quantities $\Delta m^2$, e.g. $|m_3^2 - m_2^2|$ and $|m_3^2 - m_1^2|$. Fortunately these two $\Delta m^2$ values are vastly different [4], see Eq. (14) below. This means that the relevant $L/E$ parameters are very different. Thus for a given energy the experimental results can approximately be described as two generation oscillations. For an accurate description of the data, however, a three generation analysis is necessary.

In all of these analyses the oscillation length is much larger than the size of the detector. So one is able to see the effect, if the detector is placed in the right distance from the source. It is, however, possible to design an experiment with an oscillation length of the order of the size of the detector. This is achieved, if one considers a neutrino source with as low as practical average neutrino energy, such as a triton source with a maximum energy of 18.6 keV. Thus the average oscillation length is 6.5m, which is smaller than the radius of 10m of a spherical gaseous TPC detector [5]. Such low energy events can be detected by measuring recoiling electrons with a low threshold spherical TPC detector.

In a typical supernova an energy of about $10^{53}$ ergs is released in the form of neutrinos [6],[7]. These neutrinos are emitted within an interval of about 10 s after the explosion and they travel to Earth undistorted, except that, on their way to Earth, they may oscillate into other flavors. Thus for traditional detectors relying on the charged current interactions
precise event rate may depend critically on the specific properties of the neutrinos. The time integrated spectra in the case of charged current detectors, like the SNO experiment, depend on the neutrino oscillations [8]. Recently it has become feasible to detect neutrinos by exploiting the neutral current interaction [9] and measuring the recoiling nucleus. One employs gaseous TPC detectors with low threshold energies. A description of the NOSTOS project and details of the spherical TPC detector with sub-keV threshold are given in [5],[10]. The whole system looks stable, robust and easy to maintain. The neutral current interaction, through its vector component, can lead to coherence, i.e. an additive contribution of all neutrons in the nucleus (the vector contribution of the protons is tiny).

Finally, in spite of the great process been made in understanding neutrinos mainly via neutrino oscillations, some fundamental issues remain unsettled. First are the neutrinos Dirac or Majorana particles? What is the absolute scale of the neutrino masses? The first question can practically be answered only by neutrinoless double beta decay, see, e.g., earlier reports [11] and references therein. Furthermore neutrinoless double beta decay offers the best hope for answering the second question down to a few meV.

2. Coherent neutrino nucleus scattering

The standard neutral current left handed weak interaction can be cast in the form:

\[ \mathcal{L}_\text{II} = \frac{G_F}{\sqrt{2}} [\bar{\nu}_\alpha \gamma^\mu (1 - \gamma^5) \nu_\alpha] \left[ \bar{q} \gamma_\mu (g_V q) - g_A (q) \gamma^5 q \right] \] (1)

(diagonal in flavor space). At the nucleon level we get:

\[ \mathcal{L}_\text{II} = \frac{G_F}{\sqrt{2}} [\bar{\nu}_\alpha \gamma^\mu (1 - \gamma^5) \nu_\alpha] \left[ \bar{N} \gamma_\mu (g_V N) - g_A (N) \gamma^5 N \right] \] (2)

with

\[ g_V (p) = \frac{1}{2} - 2 \sin^2 \theta_W \simeq 0.04 \ , \ g_A (p) = 1.27 \frac{1}{2} ; \ g_V (n) = -\frac{1}{2} , \ g_A (n) = -\frac{1.27}{2} \] (3)

Beyond the standard level one has further interactions which need not be diagonal in flavor space [12]. We are not, however, going to discuss such issues here. The cross section for elastic neutrino nucleon scattering has extensively been studied [6],[13].

The energy of the recoiling particle can be written in dimensionless form as follows:

\[ y = \frac{2 \cos^2 \theta}{(1 + 1/x_\nu)^2 - \cos^2 \theta} \ , \ y = \frac{T_{\text{recoil}}}{m_{\text{recoil}}} , x_\nu = \frac{E_\nu}{m_{\text{recoil}}} \] (4)

The maximum energy occurs when \( \theta = 0 \), \( y_{\text{max}} = \frac{2}{(1 + 1/x_\nu)^2 - 1} \), in agreement with Eq. (2.5) of earlier work. [6]. One can invert Eq. 4 and get the neutrino energy associated with a given recoil energy and scattering angle. From the above expressions we see that the vector current contribution, which may lead to coherence, is negligible in the case of the protons. Thus the coherent contribution may come from the neutrons and is expected to be proportional to the square of the neutron number. The neutrino-nucleus coherent cross section takes the form:

\[ \left( \frac{d \sigma}{d T_A} \right)_{\text{weak}} = \frac{G_F^2 A m_N}{2\pi} \left( \frac{N^2}{4} \right) F_{\text{coh}} (A, T_A, E_\nu), \]

\[ F_{\text{coh}} (A, T_A, E_\nu) = F(T_A) \left[ \left( 1 + \frac{A - 1}{A} \frac{T_A}{E_\nu} \right) + \left( 1 - \frac{T_A}{E_\nu} \right)^2 \right] \]

\[ \left( 1 - \frac{A - 1}{A} \frac{1}{m_N E_\nu/T_A - 1} \right) - \frac{A m_N T_A}{E_\nu^2} \] (5)

where \( F(T_A) \) is the nuclear form factor.

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3. Supernova Neutrinos

The number of neutrino events for a given detector depends on the neutrino spectrum and the distance of the source. We will consider a typical case of a source which is about 10 kpc, i.e. \( D = 3.1 \times 10^{22} \) cm (of the order of the radius of the galaxy) with an energy output of \( 3 \times 10^{53} \) ergs with a duration of about 10 s. Furthermore we will assume for simplicity that each neutrino flavor is characterized by a Fermi-Dirac like distribution times its characteristic cross section and we will not consider here the more realistic distributions, which have recently become available [14]. This is adequate for our purposes. Thus:

\[
\frac{dN}{dE_\nu} = \sigma(E_\nu) \frac{E_\nu^2}{1 + e^{E_\nu/T}} = \frac{\Lambda}{J T} \frac{x^4}{1 + e^x}, \quad x = \frac{E_\nu}{T}
\]

(6)

with \( J = \frac{31 \pi^6}{252} \), \( \Lambda \) a constant and \( T \) the temperature of the emitted neutrino flavor. Each flavor is characterized by its own temperature as follows:

\[ T = 8 \text{ MeV for } \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau \text{ and } T = 5 (3.5) \text{ MeV for } \bar{\nu}_e (\nu_e) \]

The constant \( \Lambda \) is determined by the requirement that the distribution yields the total energy of each neutrino species.

\[
U_\nu = \frac{\Lambda T}{J} \int_0^\infty dx \frac{x^5}{1 + e^x} \Rightarrow \Lambda = \frac{U_\nu}{T}
\]

We will further assume that \( U_\nu = 0.5 \times 10^{53} \) ergs per neutrino flavor. Thus one finds:

\[ \Lambda = 0.89 \times 10^{58} (\nu_e), \ 0.63 \times 10^{58} (\bar{\nu}_e), 0.39 \times 10^{58} \text{ (all other flavors)} \]

The differential event rate (with respect to the recoil energy) is proportional to the quantity:

\[
\frac{dR}{dT_A} = \frac{\lambda(T)}{J} \int_0^\infty dx F_{coh}(A, T_A, xT) \frac{x^4}{1 + e^x}
\]

(7)

with \( \lambda(T) = (0.89, 0.63, 0.39) \) for \( \nu_e, \bar{\nu}_e \) and all other flavors respectively. This is shown in Figs. 1 and 2. Summing over all the neutrino species we can write [10]:

\[
\text{No of events} = C_\nu K(A, (T_A)_{th}) \frac{K(40, (T_A)_{th})}{K(40, (T_A)_{th})} Qu(A)
\]

(8)
with
\[
C_\nu = 153 \left( \frac{N}{22} \right)^2 \frac{U_\nu}{0.5 \times 10^{38} \text{ergs}} \left( \frac{10 \text{kpc}}{D} \right)^2 \frac{P}{10 \text{Atm}} \left( \frac{R}{4 \text{m}} \right)^{3} \frac{300}{T_0}
\]
(9)

\( K(A, (T_A)_\text{th}) \) is the rate at a given threshold energy divided by that at zero threshold. It depends on the threshold energy, the assumed quenching factor and the nuclear mass number. It is unity at \((T_A)_\text{th} = 0\). From the above equation we find that, ignoring quenching, the following expected number of events:

\[
1.25, 31.6, 153, 614, 1880 \text{ for He, Ne, Ar, Kr and Xe (10)}
\]

respectively. For other possible targets the rates can be found by the above formulas or interpolation.

The quantity \( Qu(A) \) is the quenching factor [15]-[16], assuming a threshold energy \((T_A)_\text{th} = 100 \text{eV}\). The parameter \( Qu(A) \) takes the values:

\[
0.49, 0.38, 0.35, 0.31, 0.29 \text{ for He, Ne, Ar, Kr and Xe (11)}
\]

respectively. The effect of quenching is larger in the case of heavy targets, since the average energy of the recoiling nucleus is smaller. The effect of quenching is exhibited in Figs 3 and 4 for the two interesting targets Ar and Xe. We should mention that it is of paramount importance to experimentally measure the quenching factor. The above estimates were based on the assumption of a pure gas. Such an effect will lead to an increase in the quenching factor and needs be measured.

Anyway the number of expected events including quenching and \( E_\text{th} = 0.1 \text{ keV} \) becomes:

\[
0.61, 12.0, 53.5, 190, 545 \text{ for He, Ne, Ar, Kr and Xe (12)}
\]

The inclusion of the form factor is important only in the case of Xe, in which case the above number of events becomes 415.
4. Neutrino oscillations
The neutrino mixing can be parametrized as follows:

\[
U = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & 0 & s_{13} e^{i\delta} \\
0 & 1 & 0 \\
-s_{13} e^{i\delta} & 0 & c_{13}
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
\tag{13}
\]

where \( s_{ij} = \sin \theta_{ij} \) and \( c_{ij} = \cos \theta_{ij} \).

The neutrino oscillation data can be summarized as follows [4]:

\[
\begin{pmatrix}
\Delta m^2_{31} (10^{-3} \text{eV}^2) \\
\Delta m^2_{21} (10^{-5} \text{eV}^2) \\
\sin^2 \theta_{12} \\
\sin^2 \theta_{23} \\
\sin^2 \theta_{13}
\end{pmatrix}
\begin{pmatrix}
1.3 \\
8.1 \\
0.3 \\
0.5 \\
0.00
\end{pmatrix}
\begin{pmatrix}
1.7 - 2.9 \\
7.3 - 8.7 \\
0.25 - 0.34 \\
0.38 - 0.64 \\
\leq 0.028
\end{pmatrix}
\begin{pmatrix}
1.4 - 33 \\
7.2 - 9.1 \\
0.23 - 0.38 \\
0.38 - 0.68 \\
\leq 0.047
\end{pmatrix}
\tag{14}
\]

In a three generation model the electron neutrino disappearance probability is given by:

\[
P(\nu_e \to \nu_e) = 1 - \cos^2 \theta_{12} \sin^2 2\theta_{12} \sin^2 \Delta_{21}
\]

\[
- \cos^2 \theta_{12} \sin^2 \theta_{13} \sin^2 \Delta_{32} - \sin^2 \theta_{12} \sin^2 2\theta_{13} \sin^2 \Delta_{32}
\tag{15}
\]

with

\[
\Delta_{21} = \frac{\Delta m^2_{31} L}{2E_\nu}, \quad \Delta_{32} = \frac{\Delta m^2_{31} L}{2E_\nu}
\tag{16}
\]

In the presence of neutrino mixing both oscillation lengths contribute to the electron neutrino disappearance. For \( |\Delta_{21}| \ll |\Delta_{32}| \) and \( \theta_{13} \ll 1 \) we get

\[
P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta_{12} \sin^2 \frac{\Delta m^2_{31} L}{2E_\nu} - \sin^2 \theta_{13} \sin \frac{\Delta m^2_{31} L}{2E_\nu}
\tag{17}
\]

\[
P(\nu_e \to \nu_e) \simeq 1 - \sin^2 2\theta_{12} \sin^2 \pi \frac{L}{50L_{32}} - \sin^2 2\theta_{13} \sin \pi \frac{L}{L_{32}}
\tag{18}
\]

with

\[
L_{32} = \frac{2E_\nu}{\pi \Delta^2_{32}} = \text{small oscillation length}, \quad L_{12} \approx 50L_{23} = \text{large oscillation length}
\]

These are shown in Figs 5-6.

5. Neutrino mass Limits from astrophysics and triton decay
The neutrino oscillation data alone cannot determine the absolute neutrino mass scale and the sign of \( \Delta m^2_{32} \). We thus distinguish the following cases

- Normal Hierarchy:

\[
\Delta m^2_{SUN} = m_2^2 - m_1^2, \quad \Delta m^2_{ATM} = m_3^2 - m_1^2
\]

\[
m_1, \quad m_2 = \sqrt{\Delta m^2_{SUN} + m_1^2}, \quad m_3 = \sqrt{\Delta m^2_{ATM} + m_1^2}
\]
Figure 5. The small oscillation length $\nu_e$ disappearance expected to be seen in a TPC low energy electron detector.

Figure 6. On top of the large oscillation length seen in reactor experiments we show the small oscillation length due to $\theta_{13}$.

- Inverted Hierarchy:
  \[
  \Delta m_{SUN}^2 = m_2^2 - m_1^2, \quad \Delta m_{ATM}^2 = m_2^2 - m_3^2
  \]
  \[
  m_3, m_2 = \sqrt{\Delta m_{ATM}^2 + m_3^2}, \quad m_1 = \sqrt{\Delta m_{ATM}^2 - \Delta m_{SUN}^2 + m_3^2}
  \]

- The degenerate scenario.
  \[
  m_1 = m_2 = m_3 >> \sqrt{|\Delta m_{23}^2|}
  \]

These problems can be tackled from other experiments as follows:

(i) The astrophysics limit as follows:
  - Normal Hierarchy:
    \[
    m_1 + \sqrt{\Delta m_{SUN}^2 + m_1^2} + \sqrt{\Delta m_{ATM}^2 + m_1^2} \leq m_{astro}
    \]
  - Inverted Hierarchy:
    \[
    m_3 + \sqrt{\Delta m_{ATM}^2 + m_3^2} + \sqrt{\Delta m_{ATM}^2 - \Delta m_{SUN}^2 + m_3^2} \leq m_{astro}
    \]

  Unfortunately at present this limit is not very stringent [17] $m_{astro} < 0.71eV$

(ii) The triton decay limit
  - Normal Hierarchy:
    \[
    c_{12}^2 c_{13}^2 m_1^2 + s_{12}^2 c_{13}^2 (\Delta m_{SUN}^2 + m_1^2) + s_{13}^2 (\Delta m_{ATM}^2 + m_1^2) \leq m_{decay}^2
    \]
  - Inverted Hierarchy:
    \[
    s_{13}^2 m_3^2 + s_{12}^2 c_{13}^2 (\Delta m_{ATM}^2 + m_3^2) + + c_{12}^2 c_{13}^2 (\Delta m_{ATM}^2 - \Delta m_{SUN}^2 + m_3^2) \leq m_{decay}^2
    \]

  This limit is also not very stringent [18] $m_{decay} < 2.2eV$

The only process which at present offers the best chance of reaching limits comparable to the neutrino oscillation data is neutrinoless double beta decay.
6. Neutrinoless Double Beta Decay.

Double beta decay of a nucleus $A(N,Z)$ occurs when the ordinary $\beta$ decay to the nucleus $A(N, Z \pm 1)$ is forbidden due to energy conservation or angular momentum mismatch, while the decay to one of the nuclei $A(N, Z \pm 2)$ is allowed. Ignoring the non exotic decays involving neutrinos, the following decays are possible:

$$N(A,Z) \rightarrow N(A,Z + 2) + e^- + e^- \quad (0\nu \beta\beta \text{-decay})$$

(the corresponding two neutrino decay has already been observed in many systems).

Furthermore, omitting the non exotic processes accompanied by neutrinos, the following processes are possible:

$$N(A,Z) \rightarrow N(A,Z - 2) + e^+ + e^+ \quad \text{(double positron emission)}$$

$$e^+_b + N(A,Z) \rightarrow N(A,Z - 2) + e^+ \quad \text{(electron positron conversion)}$$

$$e^-_b + e^+_b + N(A,Z) \rightarrow N(A,Z-2) + 2 \text{ X-rays (Double electron capture)}$$

We will limit our discussion on the first of these processes, which is the most important experimentally. We will adopt the most popular view and assume that the process proceeds via intermediate neutrinos. The relevant half life time is given by:

$$[T_{1/2}^{0\nu}]^{-1} = G_{01}^{0\nu} \frac{|< m\nu >|}{m_e} \Omega_\nu$$

where $G_{01}^{0\nu}$ is well understood kinematical factor, $\Omega_\nu$ is the nuclear matrix element and $< m\nu >$ is the average neutrino mass given by:

$$< m\nu > = c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 e^{i\alpha} m_2 + s_{13}^2 e^{i\beta} m_3$$

with $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$, $m_i$, $i = 1, 2, 3$ are the neutrino masses and $\alpha$ and $\beta$ the two relevant Majorana phases.

Once the nuclear matrix elements are known $< m\nu >$ can be extracted from the data. From this the lightest neutrino mass can be inferred, if neutrino oscillation data are incorporated. This analysis has already been done [19], [20] and we are not going to elaborate on it here. The main conclusions are: If the degenerate scenario holds double beta decay observation is within the goals of the current experiments. If the inverted hierarchy holds, there exists a lower bound on the value of $< m\nu >$ which is within the reach of the currently planned future experiments. If, however, the normal hierarchy scenario holds there is no such lower bound and the road towards measuring $< m\nu >$ may be very long. The above conclusions depend, however, on the assumption that the neutrino mass mechanism dominates in $0\nu \beta\beta$ decay.

There exist many other mechanisms which may contribute to double beta decay. The most prominent are intermediate heavy neutrinos and R-parity, and consequently lepton violating, supersymmetry [11].

We will extend our formalism to consider the case of right handed currents: The mixing matrix is a $6 \times 6$ and takes the form:

$$U = \left( \begin{array}{cc} \nu^0_L & \nu^0_{Lc} \\ \nu^0_{cL} & U^{11} \\ \nu^0_{c\mu} & U^{21} \\ \nu^0_{c\tau} & U^{22} \end{array} \right) \left( \begin{array}{c} \nu_L \\ N_L \end{array} \right)$$

where

$$\nu^0_L = (\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}) ; \quad \nu^0_{Lc} = (\nu^c_{eL}, \nu^c_{\mu L}, \nu^c_{\tau L}) \leftrightarrow \text{right handed neutrino}$$
\[ \nu_L = (\nu_{1L}, \nu_{2L}, \nu_{3L}) \text{ light}, \ N_L = (N_{1L}, N_{2L}, N_{3L}) \text{ heavy} \]

If the right handed neutrino does not exist:

\[ U = U^{11} = U^{\text{MNS}} \]

Quite generally the half-life takes the form [11]:

\[ [T_{1/2}^{0\nu}]^{-1} = G_{01}^{\nu} \left\{ |X_L|^2 + |X_R|^2 - \tilde{C}_1^L X_L X_R + \ldots \right\} \]

\[ + \tilde{C}_2 |\lambda| X_L \cos \psi_1 + \tilde{C}_3 |\eta| X_L \cos \psi_2 + \tilde{C}_4 |\lambda|^2 + \tilde{C}_5 |\eta|^2 \]

\[ + \tilde{C}_6 |\eta| \cos(\psi_1 - \psi_2) + \text{Re}(\tilde{C}_2 \lambda X_R + \tilde{C}_5 \eta X_R) \} , \]

(22)

Where the left handed contribution is:

\[ X_L = \eta_\nu \Omega_\nu + \eta_K^L \Omega_N + \eta_{\text{SUSY}} \Omega_{\text{SUSY}} \]

(23)

with \( \Omega_\nu \), \( \Omega_N \), \( \Omega_{\text{SUSY}} \) the nuclear matrix elements, associated with light neutrinos, heavy neutrinos and SUSY contributions respectively, while \( \eta_\nu \), \( \eta_K^L \), \( \eta_{\text{SUSY}} \) lepton violating parameters:

\[ \eta_\nu = \frac{m_\nu}{m_e} , \ \ < m_\nu > = \sum_k (U_{ek}^{(1)})^2 e^{i \alpha_k} m_k \eta_K^L = \sum_k (U_{ek}^{(12)})^2 e^{i \Phi_k} \frac{m_e}{M_e} \]

with \( \alpha_k \), \( \Phi_k \) Majorana phases and \( m_p \) (\( m_e \)) being the proton (electron) mass. We see now that we have two types of interference.

- The interference between the various left handed contributions.

It is thus possible that the light neutrino mass mechanism may be cancelled by the other contributions, so that the experiments go below the inverted hierarchy and still do not see the 0\( \nu \) double beta decay. This, however, cannot happen in all nuclear systems, since the nuclear matrix elements behave very differently. The light neutrino operator is long range, but the one resulting from heavy particle exchange is short ranged. To be more specific consider two left handed mechanisms, one light (\( \nu \)) and one heavy H. Then

\[ \frac{m_e/\Omega_\nu}{\sqrt{T_{1/2}(0\nu)G_{01}^{\nu}}} = < m_\nu > + \eta_H m_e \frac{\Omega_H}{\Omega_\nu} \]

(24)

or

\[ \eta_H = \frac{b - < m_\nu >}{m_e r} , \ \ r = \frac{\Omega_H}{\Omega_\nu} , \ \ b = \frac{m_e/\Omega_\nu}{\sqrt{T_{1/2}(0\nu)G_{01}^{\nu}}} \]

(25)

Now for two targets \( A_1 \) and \( A_2 \) we get

\[ < m_\nu > = \frac{b_1 r_2 - b_2 r_1}{r_2 - r_1} \]

More than two targets over determine the system and provide tests.

If only the light neutrino mechanism is operative

\[ \Rightarrow b_1 = b_2 = b \] we get the standard expression \( < m_\nu > \), which is independent of the target:

In an analogous fashion we can write:

\[ < m_\nu > = \left[ 1 + (\eta_K^L/\eta_\nu) (\Omega_N(A)/\Omega_\nu(A)) \right] \]

(26)

Assuming that the cancellation in the target \( A_0 \) is almost complete (given by \( c, c << 1 \)), we find

\[ < m_\nu > = < m_\nu > \left[ 1 - c \frac{\Omega_\nu(A_0)}{\Omega_\nu(A)} \frac{\Omega_N(A)}{\Omega_N(A_0)} \right] \]

(27)
Even though for the neutrino mass mechanism recent and more reliable nuclear matrix elements have appeared [21], we need the matrix elements for the other mechanisms within the same QRPA model. So we are going to use the set provided by the earlier paper [22] (without p-n pairing). The factor inside the square bracket is plotted as a function of A for $A_0 = ^{76}\text{Ge}$ and $A_0 = ^{136}\text{Xe}$ in Figs 7 and 8.

**Figure 7.** The apparent $< m_\nu >$ when there exists 100% (thick curve), 90% (fine curve) and 80% (dotted curve) cancellation in the target $A_0 = ^{76}\text{Ge}$.  

**Figure 8.** The same as in 7 in the case of $A_0 = ^{136}\text{Xe}$.  

It is clear that such a cancellation cannot occur in all nuclear systems.

- Interference between left and right handed lepton currents.

One can have two amplitudes independent of the neutrino mass if the chirality of the neutrinos is opposite. The associated lepton violating parameters are indicated by $\lambda$ and $\eta$ given by:

$$\eta = \epsilon \eta_{RL}, \quad \lambda = \kappa \eta_{RL} \quad (28)$$

$$\eta_{RL} = \sum_j^3 (U_{ej}^{(21)} U_{ej}^{(11)}) e^{i\alpha_j} \quad (29)$$

$$\kappa = m_L^2/m_R^2, \quad \epsilon = \tan\zeta. \quad (30)$$

$m_L, m_R =$ gauge boson masses $\zeta =$ the gauge boson mixing angle.

In the experimental limit on $^{76}\text{Ge}$ after using the nuclear matrix elements of [22] we obtain the constraint on the $< m_\nu, \lambda$ and $< m_\nu, \eta$ plane shown in Figs 9 and 10 respectively.

7. Conclusions

In this brief review we considered some aspects of neutrino physics, which in the frontiers of research after the discovery of neutrino oscillations.

We first considered neutrinos as probes to detect and study supernova explosions. We have seen that it is quite simple to detect typical supernova neutrinos in our galaxy, provided that such a supernova explosion takes place (one explosion every 30 years is estimated [23]). The idea is to employ a small size spherical TPC detector filled with a high pressure noble gas and measure nuclear recoils. An enhancement of the neutral current component is achieved via the coherent effect of all neutrons in the target. Thus employing, e.g., Xe at 10 Atm, with a feasible threshold energy of about 100 eV in the detection of the recoiling nuclei, one expects between 400 and 1900 events, depending on the quenching factor and the nuclear form factor. We believe that networks of such dedicated detectors, made out of simple, robust and cheap technology, can
be simply managed by an international scientific consortium and operated by students. This network comprises a system, which can be maintained for several decades (or even centuries). This is is a key point towards being able to observe few galactic supernova explosions.

We then examined processes which depend on the mixing and mass of the neutrinos. We have shown that there are many advantages in using very low energy neutrinos in the few keV and detecting them via measuring electron recoils using low threshold gaseous spherical TPC detectors. Then one can look for oscillations induced by the small mixing angle $\theta_{13}$ and the the small oscillation length, associated with the large $\Delta m^2_{23}$, of tens of meters. Then the full oscillation takes place inside the detector. With the realistic goal of measuring the distance up to 1% we hope to measure or put stringent limits on the small mixing angle $\theta_{13}$. At the same time such an experiment will put a limit on the neutrino magnetic moment at the level of $10^{-12}\mu_b$. It can also measure the Weinberg angle down to essentially zero momentum transfer.

We have also seen that neutrinoless double beta decay is the only process to decide whether neutrinos are Dirac or Majorana particles and the best process to settle the question of the absolute scale of neutrino mass. In order to be able to do so reliable nuclear matrix elements must be available. Furthermore, even though the neutrino mass mechanism is the most popular scenario in this post neutrino oscillation period, other mechanisms may contribute or even dominate this process. Interference between such mechanisms may invalidate any conclusions we draw about the neutrino mass scale. This indeed maybe a problem, if one analyzes the data of only one target. We have seen, however, that such ambiguities may be resolved, if one analyzes data from many targets or experiments with different signatures, e.g. tracking versus deposited energy.

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