Solitons of the Self-dual Chern-Simons Theory on a Cylinder

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Abstract

We study the self-dual Chern-Simons Higgs theory on an asymptotically flat cylinder. A topological multivortex solution is constructed and the fast decaying property of solutions is proved.
1 Introduction

In this paper, we prove the existence and the fast decaying property of topological Chern-Simons vortices on \((2 + 1)\) space \((R \times M, \ dt^2 - g_{ij}dx^i dx^j)\) whose spatial manifold \(M\) is a cylinder. The spatial metric \(g_{ij}\) on \(M\) is asymptotically flat, so that each end of the cylinder is close to \((R^2 - B_E(r_0), \delta_{ij})\), the outside of a large ball of radius \(r_0\) in Euclidean space \(R^2\). The metric on cylinder \(M\) is given in a general form. For an example, it includes the wormhole of Einstein-Rosen bridge, which connects two asymptotically flat universes. For the Higgs potential term, we take the model developed on \((2 + 1)\) space \((R \times R^2, dt^2 - dx_1^2 - dx_2^2)\) by Hong-Kim-Pac and Jackiew-Weinberg to study vortex solutions of the Abelian Higgs model carrying both electric and magnetic charges [1, 2].

In the context of recent physics topics including string theory and brane world scenario, asymptotic cylindrical geometry formed by topological defects attracts attention. When the smooth brane world of Randall-Sundrum type [3] is considered with extra-dimensions, the bulk topological defects form their asymptotic space which is a cylinder of finite neck [4]. In particular, a crucial role of angular momentum carrying self-dual extended objects is clear in making a stable (1/4)-supersymmetric tubular D2-brane [5]. Since the solitons of our interest are self-dual and spinning, which live in a cylindrical background spacetime, they can be viewed as one candidate of 0-brane counterparts and their worldvolume realization may become a sort of supertubes or brane worlds.

Over the past decade, many attention have been given to the above model [1, 2] on spatial manifold \(R^2\). For the Chern-Simons Higgs theory on flat \(R^2\) a topological multi-vortex solution and a nontopological multi-vortex solution exist [6]-[9]. Inspired by the work of Hoopt, periodic solutions on a torus or a sphere were studied [10]-[16]. Schiff [17] constructed a background metric model on spatial manifold \(R^2\) and studied the radial symmetric case. Generalizations to non-radial background metric models on the spatial manifold \(R^2\) were studied for nontopological solutions and topological solutions [18, 19]. Recently the topology of the configuration space of this model on \(R^2\) and a cylinder is analyzed [20] and an Abelian gauge field theory on complex line bundle over a compact surface is developed [21].
Though one of the final goals in this direction is to study the existence of self-gravitating multi-soliton configurations, present researches are limited to those with rotational symmetry due to complications [22, 23].

In the previous work [6], the distance function is used for the existence proof of solitons on \((2 + 1)\) space \(R \times R^2, dt^2 - dx_1^2 - dx_2^2\). Since the distance function is no longer smooth on cylinder \(M\), we construct an approximate solution using the Green function on small sets around the centers of vortices to show the existence. The Maximum principle is used to estimate the asymptote of solitons.

2 Chern-Simons Equation

In this section we review the Bogomolnyi bound of the Chern-Simons Higgs theory coupled to background gravity [17]. The static metric \(G_{\mu\nu}\) on \((2 + 1)\) space \(R \times M\) is given by

\[
ds^2 = dt^2 - g_{ij}(x^k)dx^i dx^j \quad (i, j, k, ... = 1, 2),
\]

where \(g_{ij}\) is a metric on a two-dimensional cylindrical spatial manifold \(M\). Throughout this paper, we assume that there exists a smooth compact subset \(K \subset M\) such that \((M - K, g_{ij})\) has two disjoint connected components \(C_1\) and \(C_2\), where \(C_1 = (R^2 - B_E(0, R_0), h_1\delta_{ij})\) and \(C_2 = (R^2 - B_E(0, R_0), h_2\delta_{ij})\). In the above, \(R^2 - B_E(0, R_0)\) denotes the complement of the Euclidean ball of radius \(R_0\) whose center at the origin in \(R^2\), and \(h_1\) and \(h_2\) are smooth positive functions on \(R^2 - B_E(R_0)\) satisfying \(\alpha^{-1} < h_1, h_2 < \alpha\) for some positive constant \(\alpha\).

The static scalar potential \(V(|\phi|)\) is taken to be

\[
V(|\phi|) = \frac{e^4}{\kappa^2} |\phi|^2 (|\phi|^2 - v^2)^2.
\]

The action is given by

\[
S = \int d^3 x \sqrt{G} \left[ \kappa \frac{\epsilon_{\mu\nu\rho}}{4 \sqrt{G}} F_{\mu\nu} A_\rho + G^{\mu\nu} D_\mu \phi D_\nu \phi - V(|\phi|) \right],
\]

where \(\phi = e^{i\Theta} |\phi|\) is a complex scalar field, \(A_\mu\) a U(1) gauge field, \(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\), \(D_\mu = \partial_\mu - ie A_\mu\), and \(\sqrt{G} = \sqrt{\det(G_{ij})}\) \((\sqrt{g} = \sqrt{\det(g_{ij})})\). All components are assumed to
be static. The symmetric energy-momentum tensor is
\[ T_{\mu\nu} = (D_\mu \phi D_\nu \phi + D_\nu \phi D_\mu \phi) - G_{\mu\nu} \left[ G^{\rho\sigma} D_\rho \phi D_\sigma \phi - V(\phi) \right]. \] (2.4)

Since a Riemann surface admits an isothermal coordinates on a sufficiently small neighborhood of each point, the metric can be written as \( g_{ij} = h(x_1, x_2) \delta_{ij} = \sqrt{g} \delta_{ij} \) on this neighborhood for a positive function \( h \). From now on, we use this coordinate system. The equation of motion with respect to \( A_0 \) is given by:
\[ A_0 = -\frac{k F_{12}}{2\sqrt{g}e^2 |\phi|^2}. \] (2.5)

Assume that \( \phi \) is integrable in the following Eq. (2.6),
\[ E = \int_M T_{00} \, dV_g \] (2.6)
\[ = \int_M d^2x \left| \frac{k F_{12}}{2\sqrt{h}e\phi} \mp \frac{\sqrt{h}e^2}{k} \phi(|\phi|^2 - v^2) \right|^2 + |(D_1 \pm iD_2)\phi|^2 \mp \int_M d^2xeF_{12} \]
where \( dV_g = \sqrt{g} \, d^2x \). The first-order Bogomolnyi equations are
\[ F_{12} = \pm \frac{2e^3}{\kappa^2} |\phi|^2 (|\phi|^2 - v^2) \sqrt{g}, \] (2.7)
\[ D_1\phi \pm iD_2\phi = 0. \] (2.8)

The second equation (2.8) expresses the spatial components of the gauge field \( A_i \) in terms of the scalar field, i.e., \( eA_i = -\partial_i \Theta \mp \epsilon_{ij} \partial_j \ln |\phi| \). Substituting it into the first Bogomolnyi equation (2.7) together with the conformal gauge, we have
\[ \partial^2 \ln |\phi| = \frac{2e^4}{\kappa^2} \sqrt{g}|\phi|^2 (|\phi|^2 - v^2) \mp \epsilon^{ij} \partial_i \partial_j \Theta, \] (2.9)
where Dirac-delta function like contribution of the scalar phase \( \Theta \) comes from multi-valued function. We define:
\[ \check{F}_{12} \equiv \frac{e F_{12}}{\sqrt{g}} = \pm \frac{\partial^2 \ln |\phi|^2}{2\sqrt{g}} \]
\[ = \pm \frac{2e^4 v^4}{\kappa^2} \left| \frac{\phi}{v} \right|^2 \left( \left| \frac{\phi}{v} \right|^2 - 1 \right). \] (2.10)

To make energy finite, \( |\phi| \) can have four possible asymptote, i.e. \( |\phi| \to v \), or zero at the infinity of \( C_1 \) or \( C_2 \). A solution of Eq. (2.9) is called a topological solution if \( |\phi| \to v \neq 0 \) at the infinity of \( C_1 \) and \( C_2 \). In this paper, we consider the topological solution only.
3 Existence and Behavior of a Solution

In this section, we show that a topological solution with arbitrary prescribed vortex centers can be constructed and the solution decays fast. The proof is different from the case of spatial manifold $\mathbb{R}^2$, where $\ln(x_1^2 + x_2^2)$ is used to construct a solution [3]. The main reason is that the distance function is not smooth on cylinder $M$. There are at least two points on $M$ where the distance function is not differentiable.

From now on, we denote that $M$ be a cylinder, $d(p, q)$ be the Riemannian distance between two points $p$ and $q$ on $(M, g)$ and $B(p, r) = \{q \in M \mid d(p, q) < r \}$. Using the isothermal coordinates system, we let $\Delta = \frac{1}{\sqrt{g}}(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}) (\Delta_0 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2})$, $|\nabla u|(\|u\|_E)$ and $\delta (\delta_E)$ be the Laplacian, the norm of the gradient and Dirac-delta function with respect to the metric $g_{ij}$ (Euclidean metric), respectively. We denote $\|w\|_p = (\int_M |\nabla w|^p dV_g)^{\frac{1}{p}}$ and $H^2_1$ be the Sobolev space, which is the completion of $C^\infty_c(M)$ with respect to the norm $\|w\| = (\int_M |\nabla w|^2 + w^2 dV_g)^{\frac{1}{2}}$.

Note that Eq. (2.10) is equivalent to Eq. (3.11) in the following Theorem 1 by taking $\frac{|\phi|}{e} = e^w$ and $k = 2e^2v^2$. We state the main Theorem.

Theorem 1. There exists a topological solution for the following self-dual Chern-Simons vortex equation on $(M, g)$

$$\Delta w = e^w(e^w - 1) + 4\pi \sum_{k=1}^n \delta_{p_k}(p)$$  \hspace{1cm} (3.11)

with the boundary condition

$$\lim_{d(p, p_0) \to \infty} w = 0,$$

for some fixed point $p_0 \in M$. Moreover, $w$ satisfies $-ae^{-b d(p, p_0)} \leq w(p) < 0$ and $|\nabla w|(p) < ae^{-b d(p, p_0)}$ at infinity for some positive constants $a$ and $b$.

Remark. The quantity $|\phi|^2 - 1$, $|D_1\phi|^2 + |D_2\phi|^2$, and $F_{12}^* = \frac{F_{12}}{\sqrt{g}}$ all decay exponentially fast. This fact easily comes from Theorem 1.

Proof. We divide the proof into two steps.

Step 1. Existence of a solution.
We construct a solution \( w \) with \( w \to -\infty \) around centers of vortices. It is easy to see that any \( H^2 \) solution \( w \) of Eq. (3.11) satisfies \( w \leq 0 \) on the outside of centers of vortices, by multiplying \( u \) on Eq. (3.11) and integrating over a large smooth subdomain of \( M \). Let \( \{p_1, \ldots, p_n\} \) be the arbitrary prescribed centers of vortices on \( M \), which may not be distinct. Take \( i_0 \) be the injective radius of \((M, g)\), which means that \( B(p, i_0) \) is diffeomorphic to an open ball in Euclidean space \((\mathbb{R}^2, \delta_{ij})\). Denote \( \epsilon_1 = \{1, i_0/2, d(p_i, p_j)/4 \mid p_i \neq p_j\} \) and \( L_0 = \max_{k=1,\cdots,n} \{4, 4d(p_1, p_k)\} \). We decompose \( M = \Omega_1 \cup \Omega_2 \cup \Omega_3 \), where

\[
\Omega_1 = \{ p \in M \mid \min_{k=1,\cdots,n} d(p, p_k) \leq \epsilon_1 \}, \\
\Omega_2 = M - B(p_1, L_0), \\
\Omega_3 = M - (\Omega_1 \cup \Omega_2). 
\]

By the existence of a Green function on a Riemannian manifold [24], there exists \( u_k \) on \( B(p_k, \epsilon_1) \) for \( k = 1, \cdots, n \), satisfying the following properties

\[
\Delta u_k(p) = 4\pi m_k \delta_{p_k}(p) \quad \text{when} \quad p \in B(p_k, \epsilon_1), 
\]

and

\[
u_k(p) = \ln \epsilon_1^{2m_k} \quad \text{when} \quad p \in \partial B(p_k, \epsilon_1),
\]

where \( m_k \) is the multiplicity at \( p_k \). We take \( \overline{\nu} \) be a smooth function on \( M \) with

\[
\overline{\nu}(p) = \begin{cases} 
    u_k & \text{if } x \in B(p_k, \epsilon_1) \text{ for } k = 1, \cdots, n, \\
    0 & \text{if } x \in \Omega_2,
\end{cases}
\]

and \( \ln \epsilon_1^{2n} \leq \overline{\nu}(p) \leq 0 \) for \( p \in \Omega_3 \). We define \( S(p) = e^{\overline{\nu}(p)} \). The metric on \( B(p_k, r) \) approaches to \( \delta_{ij} \) as \( r \to 0 \) in the normal coordinates and \( S(p) \to d(p, p_k)2m_k \) as \( p \to p_k \) [24]. Note that \( S(p) = 1 \) on \( p \in \Omega_2 \). Take \( w = \overline{\nu} + u \), then Eq. (3.11) becomes

\[
\Delta u = Se^u(Se^u - 1) + h,
\]

where \( h = -\Delta \overline{\nu} + 4\pi \sum_{k=1}^n \delta_{p_k}(p) \) is a smooth function whose support lies in a compact set \( \Omega_3 \). A critical point of functional \( E \) defined on \( H^2 \) is a solution of Eq. (3.15), where

\[
E(u) = \int_M |\nabla u|^2 + (Se^u - 1)^2 + 2hu \, dV_g,
\]

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Using the basic inequality \((e^t - 1)^2 \geq \frac{|t|^2}{(1+|t|)^2}\), we estimate the second term of Eq. (3.16)
\[
\int_M (Se^u - 1)^2 dV_g = \int_{M-B(p_1,L_0)} (e^u - 1)^2 dV_g + \int_{B(p_1,L_0)} (Se^u - 1)^2 dV_g \\
\geq \int_{M-B(p_1,L_0)} (e^u - 1)^2 dV_g \\
\geq \int_M \frac{|u|^2}{(1+|u|)^2} dV_g - c_1
\]
(3.17)
for \(u \in H^1_1\), where \(c_1\) is the volume of \(B(p_1, L_0)\).

Since \((M, g)\) is an asymptotically flat cylinder, the Sobolev Imbedding Theorem holds on \((M, g)\), i.e., there exists a positive constant \(c_2\) such that
\[
\int_M f^2 dV_g \leq c_2^4 \left( \int_M |\nabla f| dV_g \right)^2 \text{ for } f \in H^1_1(M)\] [24]. Following [6, 25], we estimate the last term in (3.16). Set \(f = u^2\), then
\[
\int_M u^4 dV_g \leq c_2 \left( \int_M |u \nabla u| dV_g \right)^2 \\
\leq c_2 \int_M u^2 dV_g \int_M |\nabla u|^2 dV_g,
\]
(3.18)
and
\[
\left( \int_M u^2 dV_g \right)^2 \leq \left[ \int_M \left( \frac{|u|}{1+|u|} \right) (1+|u|)|u| dV_g \right]^2 \\
\leq 2 \int_M \left( \frac{|u|}{1+|u|} \right)^2 dV_g \int_M u^2 + u^4 dV_g.
\]
(3.19)
From Eqs. (3.18) and (3.19),
\[
\int_M u^2 dV_g \leq 2 \int_M \left( \frac{|u|}{1+|u|} \right)^2 dV_g \left( 1 + c_2 \int_M |\nabla u|^2 dV_g \right).
\]
(3.20)
Using \(\sqrt{2ab} \leq |a| + |b|\),
\[
\left( \int_M u^2 dV_g \right)^{\frac{1}{2}} \leq \int_M \frac{|u|^2}{(1+|u|)^2} dV_g + c_2 \int_M |\nabla u|^2 dV_g + 1.
\]
(3.21)
We can bound the last integral in Eq. (3.16):
\[
2 |\int_M h u dV_g| \leq 2 \| h \|_4 \| u \|_4 \leq c_3 \| h \|_4 \left( \| u \|_2 \| \nabla u \|_2 \right)^{\frac{1}{2}}
\]
(3.22)
\[
\leq \epsilon \| u \|_2 + \frac{c_4}{\epsilon} \| \nabla u \|_2 + c_5
\]
(3.23)
\[
\leq \epsilon \left( \| u \|_2 + \| \nabla u \|_2^2 \right) + c_6
\]
(3.24)
\[
\leq \epsilon \left( \int_M \frac{|u|^2}{(1+|u|)^2} dV_g + (1+c_2) \| \nabla u \|_2^2 \right) + c_7,
\]
(3.25)
where $c_3, \ldots c_7$ and $\epsilon$ are some constants. In the above, the Hölder inequality, (3.18) and (3.21) are used in (3.22), (3.24), (3.25) and (3.26), respectively. There exists a constant $c_8$ such that

$$E(u) \geq \int_M |\nabla u|^2 dV_g + \int_M \frac{|u|^2}{(1+|u|)^2} dV_g - c_1$$

$$- \epsilon \left[ \int_M \frac{|u|^2}{(1+|u|)^2} dV_g + (1+c_2) \int_M |\nabla u|^2 dV_g \right] - c_7$$

$$\geq (1-\epsilon(1+c_2)) \int_M |\nabla u|^2 dV_g + (1-\epsilon) \int_M \frac{|u|^2}{(1+|u|)^2} dV_g - c_8.$$  \hspace{1cm} (3.27)

By taking small $\epsilon$ and Eq. (3.21),

$$E(u) \geq c_9 \left( \int_M |\nabla u|^2 + u^2 dV_g \right)^{1/2} - c_{10},$$  \hspace{1cm} (3.28)

for some positive constants $c_9$ and $c_{10}$. Therefore $E(u)$ is coercive on $H^1$ and $\inf_{u \in H^1} E(u)$ is finite. Moreover, $E(u)$ is weakly lower semi-continuous on $H^1$. We take a minimizing sequence $\{u_n\}$ for $\inf_{u \in H^1} E(u)$. Then $\{u_n\}$ is bounded on $H^1$, which has a subsequence $\{u_{n_k}\}$ converging to $u \in H^1$, a minimizer for $\inf_{u \in H^1} E(u)$. By the elliptic regularity, $u$ is smooth. Finally, $u$ satisfies Eq. (3.13). The existence of a solution for Eq. (3.11) is proved.

**Step 2. Behavior of a solution.**

In this part, we study the behavior of the solution of Eq. (3.11). Since the distance function is not smooth, we need to find a smooth function which can tell the behavior of the solution.

Let $p_0$ be a fixed point in $M$. Take large $R_1 > R_0$ so that $\{p_1, \ldots p_n\} \subset B(p_0, R_1)$ and $M - B(p_0, R_1) \subset M - B(p_0, R_0) \subset C_1 \cup C_2$, where $C_1 = (R^2 - B_E(0, R_0), h_1 \delta_{ij})$ and $C_2 = (R^2 - B_E(0, R_0), h_2 \delta_{ij})$ with $\alpha^{-1} < h_1, h_2 < \alpha$. Consider $p \in M - B(p_0, R_1)$. Then, either $p \in C_1$ or $C_2$. Using the definition of $C_1$ and $C_2$, regard $p$ as a point in $(R^2 - B_E(0, R_0), \delta_{ij}) \subset (R^2, \delta_{ij})$. Define $r_e(p)$ be the Euclidean distance from $p$ to the origin of $R^2$ and $f_1(p) = -ae^{-br_e(p)}$ for $p \in M - B(p_0, R_1)$. Note that $f_1(p)$ is differentiable and $\Delta f_1(p) = \frac{1}{\sqrt{g}} \Delta_0 f_1(p)$ for $p \in M - B(p_0, R_1)$ in the canonical coordinates system. For any given small $\epsilon > 0$, there exists a constant $\delta_1$ so that $e^t \geq 1 - \epsilon > 0$ and $1 - e^t \geq (\epsilon - 1)t > 0$ for $-\delta_1 \leq t \leq 0$. Take positive constant $b$ so that $b < (1 - \epsilon)/\sqrt{\alpha}$.

To estimate the lower bound of the solution, we use the Maximum principle (cf. [26]). Since $\Delta w \leq 0$ and $w \leq 0$, we have $-c||w||_{L^2(B(p,1))} \leq w(p) < 0$ at infinity for some positive
constant $e^{26}$. From $\int_M w^2dV_g < \infty$, $w$ decays to zero uniformly at infinity. Since $\alpha^{-1}\delta_{ij} < g_{ij} < \alpha\delta_{ij}$ on $M - B(p_0, R_1) \subset C_1 \cup C_2$, we can take sufficiently large $R_2 > R_1$ so that there exist positive constants $a$ and $\delta_2 (<< \delta_1)$ with $-\delta_1 \leq f_1(p) = -ae^{-br(x)} \leq -\delta_2$ for $p \in \partial B(p_0, R_2)$ and $w(p) > -\delta_2$ for $p \in M - B(p_0, R_2)$.

Using $\alpha^{-1}\delta_{ij} < g_{ij} < \alpha\delta_{ij}$ and $-\delta_1 < f_1(p) < 0$ for $p \in M - B(p_0, R_2)$,
\[
\Delta f_1 - e^{f_1}(e^{f_1} - 1) \geq \frac{1}{\sqrt{g}}\Delta_0 f_1 - (1-\varepsilon)^2 f_1 \\
\geq \frac{1}{\sqrt{g}}(-\frac{b}{r_\varepsilon} + b^2)f_1 - (1-\varepsilon)^2 f_1 \\
\geq \left(\alpha(b^2 - \frac{b}{r_\varepsilon}) - (1-\varepsilon)^2\right)f_1 \\
> 0.
\]

Take $F(x, t) = e^t(e^t - 1)$, then $\partial_t F(x, t) > 0$ for $t > -\delta_1 > -\ln 2$. Therefore $F(x, f_1) - F(x, w) = \lambda(x)(f_1 - w)$ for $\lambda(x) > 0$ and $-\delta_1 < f_1, w \leq 0$. From (3.29), we have the following estimate on $M - B(p, R_2)$,
\[
\Delta(f_1 - w)(p) \geq e^{f_1}(e^{f_1} - 1) - e^{w}(e^{w} - 1) \\
\geq \lambda(p)(f_1 - w).
\]

If $f_1 - w > 0$ on some domain $D \subset M - B(p, R_2)$, then $\Delta(f_1 - w) > 0$ on $D$. By the Maximum Principle, $f_1 - w$ can not have maximum inside of $D$. Since $f_1 = w$ on $\partial D$ and $f_1 > w$ on $D$, $D$ must be the empty set. Therefore, $f_1 = -ae^{-br(x)} \leq w$ on $M - B(p_0, R_2)$. Since the metric satisfies $\alpha^{-1}\delta_{ij} < g_{ij} < \alpha\delta_{ij}$ on $C_1$ and $C_2$, there exists constant $\mu \geq 1$ such that $\mu^{-1}d(p, p_0) < r(\varepsilon(p)) < \mu d(p, p_0)$ for $p \in M - B(p_0, R_2)$. Finally we conclude that $-ae^{-\frac{b}{\mu}d(p, p_0)} \leq w(p)$ on $M - B(p_0, R_2)$.

Next consider decay estimates for $|\nabla w|$. Taking partial derivative $\partial_{x_i}$ to Eq. (3.11), we have:
\[
\Delta w_i = e^{w}(2e^{w} - 1)w_i
\]
at infinity where $w_i = \partial_{x_i}w$. Since $e^{w} \rightarrow 1$, $\Delta w_i = \zeta(x)w_i$ where $\zeta(x)$ converges to one at infinity. Similar estimates, as was done for $w$, produce $|w_i(p)| < a'e^{-\mu'd(p, p_0)}$ for some constants $a', b'$, and $\mu'$. This completes the proof of Theorem 1.
4 Conclusion

For an asymptotically flat cylindrical spatial manifold $M$, we proved the existence and decaying property of a topological multi-vortex solution of the Chern-Simons Higgs theory in $(2 + 1)$ space $R \times M$, which have been previously studied on $(2 + 1)$ space $R \times R^2$. The related questions for other prescribed asymptotic solutions in $(2 + 1)$ space $R \times M$ with $|\phi| \to v$ or zero at infinity of $C_1$ or $C_2$ need future study.

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