An Extraction of $V_{cb}$ from the Semi-Leptonic $\bar{B} \to D^*$ Decay

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We present an extraction of $V_{cb}$ from a lattice calculation of the $\bar{B} \to D^* \ell \nu$ decay matrix elements. We obtain $|V_{cb}|\sqrt{\frac{\tau_{D^*}}{\tau_{D^{*0}}}} = 0.037^{+0.019}_{-0.017}$ from a single parameter fit to the new CLEO data.

1. Introduction

The $\bar{B} \to D^* \ell \nu$ decay proceeds via the spectator process whereby the bottom quark in the $B$ meson decays, through the weak interaction, to a charm quark forming a $D$ meson; the common light quark (u or d) takes no part in the interaction. Thus the current matrix element is directly proportional to the Cabibbo Kobayashi Maskawa ($CKM$) matrix element $V_{cb}$.

The vector and axial vector current matrix elements can be parametrised in terms of four form factors, $h_i$'s, which are independent of the meson masses:

$$\frac{\langle D^*(v',\epsilon)|V^\mu|\bar{B}(v)\rangle}{\sqrt{m_{D^*}m_B}} = ih_V(v \cdot v') \epsilon^{\mu\nu\lambda\sigma} \epsilon'_\nu v'_\lambda v_\sigma$$

$$\frac{\langle D^*(v',\epsilon)|A^\mu|\bar{B}(v)\rangle}{\sqrt{m_{D^*}m_B}} = (1 + v \cdot v')h_{A1}(v \cdot v')\epsilon^{\mu\nu\lambda\sigma} \epsilon'_\nu v'_\lambda v_\sigma$$

$$- \epsilon^\ast \cdot v\{h_{A2}(v \cdot v')v^\mu + h_{A3}(v \cdot v')v'^\mu\}, \tag{1}$$

where $v$ and $v'$ are the meson four-velocities. In the limit of infinite heavy quark mass these form factors, and those of the associated $\bar{B} \to Dl\bar{\nu}$ decay, are related through an exact spin-flavour symmetry to a single universal function $\xi(v \cdot v')$, the Isgur-Wise function \cite{Isw}.

2. Calculation Details

We use a $24^3 \times 48$ lattice at $\beta = 6.2$. This corresponds to an inverse lattice spacing $a^{-1} = 2.73(5) \text{ GeV}$ \cite{Hed}. This calculation is based on 60 $SU(3)$ gauge field configurations with quark propagators generated using an $O(a)$ improved fermion action \cite{Hed}.

We compute the three point correlators using the standard source method \cite{Hed} choosing $t = 24$ as the extension point. We then symmetrize about this point using Euclidean time reversal. Matrix elements are obtained by fitting to the form of the three point correlator at large Euclidean time with meson energy factors constrained to values from two point correlator fits. We use a local, $O(a)$ improved, weak current of the form $\bar{b}(1 - \gamma_5)\gamma_\mu c$ and rescale this to the continuum using the renormalization constants $Z_V$ and $Z_A$.

These preliminary results are for degenerate ‘bottom’ and ‘charm’ quarks, $\kappa_{Q'}$ and $\kappa_Q = 0.12900$, and for three values of the light quark mass, $\kappa_q = 0.14144, 0.14226$ and 0.14262. We give the $B$ meson lattice momentum $(0, 0, 0)$ and $(1, 0, 0)$ and inject up to $\sqrt{4}$ units at the current operator, the corresponding $v \cdot v'$ lie between 1.0 and 1.3. Statistical errors are calculated using a bootstrap procedure for 100 samples. Correlations between timeslices are taken into account.

3. Axial Vector Form Factor

In the infinite heavy quark limit the axial vector form factor, $h_{A1}$, is exactly the Isgur-Wise function, $\xi(v \cdot v')$. For finite heavy quark mass there exist two sources of symmetry breaking: radiative corrections, $R$, from renormalization of the heavy quark current by hard gluon exchange, and power corrections in the inverse heavy quark mass:

$$h_{A1} = (1 + R + O(\frac{1}{m_Q}) \cdots) \xi(v \cdot v'). \tag{2}$$

The radiative corrections are perturbative QCD corrections and hence can be evaluated analytically in a model independent way. We calcu-
late these using Neubert’s short distance expansion of heavy quark currents \cite{5}. The corrections proportional to inverse powers of the heavy quark mass are non-perturbative. They are model dependent. However, Luke’s theorem \cite{6} demands that $O(\frac{1}{m_Q})$ corrections to $h_{A_1}$ vanish at zero recoil leaving corrections of $O(\frac{1}{m_Q^2})$; see \cite{7}.

We can define the radiatively corrected axial vector form factor, $h_R^{A_1}$, as:

$$h_R^{A_1} = h_{A_1} (1 + R) = \xi (v \cdot v').$$  \hspace{1cm} (3)

We use the BSW \cite{8} parametrisation of the Isgur-Wise function:

$$\xi (v \cdot v') = \frac{2}{1 + v \cdot v'} \exp \{ (2\rho^2 - 1) \frac{1 - v \cdot v'}{1 + v \cdot v'} \}$$ \hspace{1cm} (4)

and perform a two parameter fit of the radiatively corrected axial vector form factor, $Z_A^{-1} h_R^{A_1}$, to the BSW type model:

$$Z_A^{-1} h_R^{A_1} = s \xi \rho,$$  \hspace{1cm} (5)

where $\rho^2$ is the slope parameter at zero recoil and $s = Z_A^{-1}$; using $\xi (1) = 1$ from heavy quark symmetry. Figure 1 shows a fit to our data for our heaviest light quark mass.

4. Axial Vector Current Renormalisation

The value of $Z_A$ obtained from the above fit will contain $O(\frac{1}{m_Q})$ corrections. Figure 2 shows a plot of $Z_A$ for a range of non-degenerate heavy quark kappa values; the ‘bottom’ quark at $\kappa_Q = 0.12900$ and 0.12100, four ‘charm’ quark masses and the light quark at $\kappa_q = 0.14144$. The consistency of values suggests that that $Z_A$ is at most only a weak function of the heavy quark masses. This is particularly important when studying ratios of form factors \cite{7}. $Z_A$ appears to have a value of around 1.1.

5. Extraction of $V_{cb}$

The differential decay rate for $\bar{B} \rightarrow D^* l \bar{\nu}$, $\Gamma$, can be expressed in the form:

$$f(\Gamma \bar{B} \rightarrow D^* l \bar{\nu}) = |V_{cb}| \xi (v \cdot v') = s \xi \rho_{lat}.$$ \hspace{1cm} (6)

We extract $V_{cb}$ from a one parameter fit, $s$, of the experimental data to our two parameter ‘BSW’ model constraining $\rho^2$ to $\rho^2_{lat}$, the value of $\rho^2$ obtained from our fit to $Z_A^{-1} h_R^{A_1}$.

Before extracting $V_{cb}$ we extrapolate the light quark mass to the chiral limit and obtain $\rho^2_{lat} = 1.1^{+5}_{-3}$. Figures 3 and 4 show $V_{cb}$ fits to the new CLEO \cite{9} and new ARGUS \cite{10} data respectively.

We obtain from the fit to the CLEO data:

$$|V_{cb}| \sqrt{\frac{\tau_B}{1.49 \text{ps}}} = 0.037^{+0.03}_{-0.03},$$ \hspace{1cm} (7)

where $\tau_B$ is the lifetime of the $B$ meson. The first set of errors are due to experimental uncertainties while the second are due to uncertainties in our lattice determination of $\rho^2$. The value of $V_{cb}$ obtained from a fit to the ARGUS data is in excellent agreement.
6. Vector Form Factor

Finally Figure 5 shows a plot of the radiatively corrected vector form factor, \( h_V^R \), for our heaviest light quark mass. \( h_V \) is not protected by Luke’s theorem and contains large \( O\left( \frac{1}{m_Q} \right) \) corrections. Correcting for \( Z_V \) from our study of the \( \bar{B} \to Dl\bar{\nu} \) decay matrix elements [1], we observe \( O\left( \frac{1}{m_Q} \right) \) corrections of 30 – 40 %. This is consistent with Neubert’s predictions [2].

7. Conclusions

We have determined \( V_{cb} \) from a lattice computation of the \( \bar{B} \to D^*l\bar{\nu} \) form factors. This value is in good agreement with other estimates; in particular with our value obtained, assuming heavy quark symmetry, from the \( \bar{B} \to Dl\bar{\nu} \) decay [3].