Stress state of underground opening support considering its interaction with rock mass

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Abstract. This study addresses stress state of support systems in underground mine openings when the boundary of an opening is incompletely relaxed from forces commensurable with the initial stress state of rock mass. The calculation algorithm is based on the stiffness matrix composed of mechanical characteristics of intact rock mass. The features of the stress state of underground mining opening support in various modes of deformation of the opening boundary are determined.

1. Introduction

Mining in difficult ground conditions and great depths under initial stresses comparable with ultimate strength of rocks impose exclusive standards on mine support systems meant to ensure safe operation within the entire mine life. Mechanical characteristics of mine support and technology of its installation are selected based on the accepted model of stress state of the support and enclosing rock mass [1–3].

An adequate description of the stress state of rock mass and mine support needs analyzing their joint deformation. This process can conditionally be divided into two stages. At the first stage, free displacement of the boundary of an underground opening takes place. At the second stage, the boundary of the opening gets in contact with the support elements and their joint deformation starts [1, 4, 5]. Accordingly, at the first stage, it is required to find free displacements of the underground opening boundary. These displacements are caused by external forces that arise upon partial relaxation of the boundary from initial stresses of rocks mass before mining. The rest forces affect both surrounding rock mass and support system, govern the stress state of the support and influence the further alteration of stresses in rocks. Thus, at the second stage, it is necessary to solve the problem on deformation of a continuum medium with different physical and mechanical processes, simulating materials of rock mass and support system.

Many design procedure account for the diversity of geotechnical conditions of support installation in underground openings with different cross sections. However, some problems in the theory of stress state of mine support systems remain yet unsettled. For example, the problem connected with modeling joint deformation of support system elements and surrounding rock mass [2, 6, 7].

One of potential formulations and the related algorithm of calculation are proposed in [5, 8]. The developed algorithm accurately accounts for features of joint deformation of underground opening boundary and support system, and adequately determines the resultant stresses. This paper demonstrates capabilities of the offered approach in terms of the problem on surrounding rock mass around an underground opening with support system at variable mechanical characteristics of the support and parameters of the contact interaction.
2. Algorithm of finding stress state of support system and surrounding rock mass

The algorithm of finding stress state of underground opening support and surrounding rock mass considers the process of deformation in two stages. The first stage is determining mechanical behavior of rock mass without opening. The second stage is finding additional stresses due to the opening. In the conventional approach of calculating additional stresses, it is required to outline the boundary of a newly created opening and to calculate stresses at this boundary. The implementation of the algorithm involves much difficulty connected with setting of boundaries of successively created underground and calculating nodal forces at these boundaries [9]. In the algorithm proposed in this paper, the computational domain, including subdomains for elements of the support system, is split into finite elements. The mechanical properties of the elements in the underground opening are assumed to be the same as before the opening formation, i.e., the computation domain models an intact rock mass. The condition to express formation of the underground opening is vanishing of all stresses in the elements in the opening. It is suggested to fulfill this condition using the iterative method of initial stresses [10, 11]. In the finite elements modeling the opening, the vector of the initial nodal forces is calculated:

\[ \{F\} = \int [B]^T \{\sigma^0\} dV. \]

Here, \([B]^T\) is the matrix connecting strains and displacements of the elements; \(V\) is the volume of a finite element; \(\{\sigma^0\}\) is the vector of initial stresses equal to the stresses in the related element at the first step of iteration.

For using the stiffness matrix constructed for rock mass without excavations in calculation of stresses in mine support made of material having different physical and mechanical properties than rock mass, an iteration algorithm has been built. Under elastic deformation of the support and rock mass, the strains \(\varepsilon_{ij}\) in the finite elements modeling the support and calculated using the stiffness matrix of the initial rock mass are assumed as the true strains. By the Hooke law, the possible stresses \(\sigma_{ij}^*\) in the finite elements with mechanical properties of the support are calculated:

\[ \sigma_{ij}^* = \lambda_i \varepsilon_{ij} \delta_{ij} + 2 \mu_i \varepsilon_{ij} \]

where \(\lambda_i, \mu_i\) are the Lamé constants of the support material; \(\delta_{ij}\) is the Kronecker delta; \(\varepsilon = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}\) is the volumetric strain.

The difference of the stresses \(\sigma_{ij}^*\) and \(\sigma_{ij}\), where \(\sigma_{ij}\) found using the unaltered stiffness matrix are assumed as the initial stresses is given by:

\[ \sigma_{ij}^0 = \sigma_{ij} - \sigma_{ij}^* \]

Then, the algorithm of initial stresses is applied. The vector of the initial nodal forces is added to the vector of the external loads. The new strain state of rock mass is determined. Assuming the obtained strains are valid, the procedure is repeated. The iterative process is continued until two successive approximations differ by a small preset value.

3. Stress state of rock mass and support at different mechanical properties of the support

The proposed algorithm for stress state of rocks mass in the vicinity of an underground opening with support with regard to mechanical characteristics of the support is used in the scenario of “instantaneous” installation of the support when all displacements of the underground opening boundary occur under condition of stiff contact with the support [8]. The algorithm assumes geometry of the opening, mechanical properties of the rock mass and support, as well as the initial stresses. It is set that the rock mass has Young’s modulus \(E = 50000\) MPa and Poisson’s ratio \(\nu = 0.25\). The volume density of rocks and support is 0.03 MN/m³. The initial stress state of rock mass is governed by the rock weight and, in accord with Dinnik’s hypothesis:

\[ \sigma_y^0 = -\phi H; \quad \sigma_z^0 = -\nu \phi H / (1 - \nu); \quad \tau_{xy}^0 = 0. \]

The underground is shaped as a semicircle with radius 7 m and bottom at a depth of 750 m. The thickness of the support is 0.5 m.
Figure 1 shows the contour line of the principal stresses $\sigma_1$ and $\sigma_2$ in case of the support material possessing $E=100000$ MPa and $\nu=0.2$. In this case, the highest compressive stresses in the computational domain are observed in the vicinity of the internal boundary of the support (Figure 1b). The highest tensile stresses appear in the floor of the underground opening (Figure 1a). A small concentration zone of tensile stresses, both in terms of level and effective area, forms in the arch of the support.

![Figure 1](image1.png)

**Figure 1.** Distribution of principal stresses (a) $\sigma_1$ and (b) $\sigma_2$ (MPa) in mine support and adjacent rock mass in the scenario of “instantaneous” installation of the support with Young’s modulus $E = 100000$ MPa.

The qualitative behavior of the stress distribution in the computation domain remains unaltered upon the change in the mechanical characteristics of the support material, although the stresses change their values. The calculated stresses for different Young’s moduli of the support are compiled in table 1. It follows from the data in table 1 that the value of Young’s modulus has little influence on the first principal stress $\sigma_1$. In the concentration zone of the second principal stress $\sigma_2$, its values grow with increasing Young’s modulus.

| Young’s modulus, MPa | $|\sigma_1^c|_{\text{max}}$, MPa | $|\sigma_1^t|_{\text{max}}$, MPa | $|\sigma_2^c|_{\text{min}}$, MPa | $|\sigma_2^t|_{\text{max}}$, MPa |
|----------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| 100000               | 27                            | 11                            | 2                             | 134                           |
| 150000               | 28                            | 11                            | 6                             | 161                           |
| 200000               | 28                            | 11                            | 8                             | 180                           |
| 250000               | 28                            | 12                            | 9                             | 195                           |
| 300000               | 27                            | 12                            | 9                             | 206                           |

![Figure 2](image2.png)

**Figure 2.** Distribution of principal stresses (a) $\sigma_1$ and (b) $\sigma_2$ (MPa) in mine support and adjacent rock mass in the scenario of “instantaneous” installation of the support with Young’s modulus $E = 300000$ MPa.
For better understanding of influence exerted by Yong’s modulus of the support material on stresses in adjacent rock mass, Figure 2 shows the contour line of the stresses $\sigma_1$ and $\sigma_2$ in case of $E = 300000$ MPa. Nearby the underroof part of the support, the uniaxial compression state is formed. The zone of compressive stresses covers mostly the floor rocks of the underground opening, while the highest compression is reached in the support (Figure 2b).

4. Stress state of rock mass and mine support in case of free deformation of the underground opening boundary

Let us assess free displacement of the underground opening boundary before it touches the mine support. The calculation uses the parameters assumed in Figure 2. First, we calculated free displacements of the boundary of the unsupported opening. Figure 2 presents the distribution of the second principal stresses $\sigma_2$ in installation of the support after the free displacement of the opening boundary makes 20% of the total free displacements.

![Figure 3. Principal stress $\sigma_2$ (MPa) in mine support and adjacent rock mass after 20% displacement of unsupported opening boundary (support has $E = 300000$ MPa).](image)

From the comparison of Figures 2 and 3, it is conclude that when the support installation takes place after the opening boundary has freely displaced by 20%, the maximum compressive stresses reduce considerably (2 times). The abutment pressure zone forms nearby the vertical boundaries of the underground opening as against its formation in the floor of the opening in case of instantaneous installation of the support (Figure 2b). The qualitative change in the stress state pattern in the abutment pressure zone takes place as the values of the free displacement of the underground opening boundary increase. Figure 4 shows the contour lines of $\sigma_2$ after free displacement of the opening boundary by 50% of its total free displacement.

![Figure 4. Principal stress $\sigma_2$ (MPa) in mine support and adjacent rock mass after 50% displacement of unsupported opening boundary (support has $E = 300000$ MPa).](image)
The patterns of $\sigma_2$ in Figures 2 and 3 most largely differ in terms of the stress redistribution in the abutment pressure zone. Two concentration zones of vertical stresses appear. One zone results from deformation of rock mass around the unsupported underground opening. The other zone ensues from joint deformation of the support and adjacent rock mass. Under initial conditions of Figure 4, the level of the compressive stresses in the concentration zones in the support is higher than in rock mass. In case of higher values of free displacement of the underground opening boundary, the maximum compressio will be observed in rock mass.

5. Conclusions
1. The increase in Young’s modulus of the support material leads to successive displacement of the abutment pressure zones (concentration of the compressive stresses $\sigma_2$) from their position along the vertical boundaries to the floor of the underground opening.
2. When the support is installed after the boundary of the unsupported opening has freely partway displaced, stresses redistribute in the zones of abutment pressure: two concentrations zones of vertical stresses appear. One zone forms in rock mass as a result of its deformation around the unsupported opening. The second zone arises as a consequence of joint deformation of the support and adjacent rock mass.

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