Gauge Physics of Finance: simple introduction

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Abstract

In this paper we will state the fundamental principles of the gauge approach to financial economics and demonstrate the ways of its application. In particular, modeling of real pricing processes will be considered for an example of S&P500 market index. Derivative pricing and portfolio theory are also briefly discussed.

1 Introduction: Why is it financial physics that has been tackled?

Experts say that people rest when they contemplate symmetrical figures. One of the ways to explain this phenomenon is that a symmetrical picture is easy to be grasped by the eye, which makes an impression that it is comprehensible and easy to control as it can be decomposed into simpler ideal figures that are well known and have been recognizable since ones childhood. This discourse can be applied to the cognition process in general: cognizing new things we decompose them into fundamental components (blocks) and establish links between them. The more fundamental components there are at our disposal, the more beautiful and comprehensive the real picture is.

During its long period of existence physics has accumulated a wide range of blocks that are the building blocks of an edifice of contemporary knowledge of the physical world. These blocks are special since they have been specially selected for describing sophisticated systems consisting of a great number of elements that interact with each other. That is why it is not no wonder that these blocks are made use of in other areas besides physics that are associated with response to the impact of many interrelated factors. Urban development, traffic jams, economic issues are only several examples of such applications.

Like any other branch of science designed to provide quantitative description, physics uses the language of mathematics. However, there is a distinctive difference between mathematics and physics. This difference lies in the way of setting a task. As we advance in tackling the subject of our studies – financial modeling – we will consider

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how financial mathematics is different from financial physics – a branch of science that is just emerging. To put it in a nutshell, financial mathematics deals with the question "How?". For instance, how to form an effective investment portfolio or how to assess and hedge derivatives? Financial physics deals with the question "Why?". Below the three most important "Whys" we are going to deal with further are listed:

1. Why do we observe certain statistical characteristics of real pricing charts?

2. Why do traders analyze pricing charts and use technical analysis that is discarded by all economists?

3. Why does the Black-Scholes equation not always describe real prices?

It may seem that a practitioner is more interested in "How?" rather than in "Why?". However, from history it is known that after the question Why? has been answered, there is an answer to the question How? as well. For example, in contemporary financial mathematics almost all answers to the question How? are derived from the answer to the question "Why?": because long-term opportunities to gain no-risk returns greater than the returns from a bank deposit should not exist, that is, because arbitrage opportunities should not exist in a market of rational agents. The latter statement leads to a hypothesis of an efficient market, a model of stochastic processes for price description, martingales and other tools of financial mathematics. This is the basis for the huge industry of quantitative analysis and financial economics. It is the no-arbitrage assumption that financial physics is going to attack, by accepting the existence of short-life arbitrage opportunities.

Let us look into the no-arbitrage condition closer. A great number of factors affect prices constantly, ranging from fundamental information release (which happens comparatively rarely) to the appearance at the market of an investor ready to make a timely deal which can change quotations. Price changes cause situations when some assets are more or less attractive than other assets with the same level of risk. The arbitrage (or no-risk returns) is only a particular example of the described above mis-pricing. It is clear that such a situation can not last for long, and rational investors (or almost rational) will smooth out the situation by buying profitable and selling unprofitable assets (let us put aside for now bubbles and crashes), after which the return on assets will again reflect their level of risk properly. In other words, the arbitrage opportunities will cease to exist. Thus, the no-arbitrage assumption may be considered as a statement of the instantaneous character of the smoothing described above and of the infinite speed of money flows. But is it really so? Provided that there are market imperfections and the money flows do not transfer instantly, to what extent can the no-arbitrage picture be justified? Can money flows be disregarded?

It is here that we tackle financial physics. Technically, a solution of any physical problem boils down to realization of what can be disregarded without distorting the picture and keeping the most essential details. On the other hand, all physics turns around the transfer of matter or energy, forces engendering these processes, non-equilibrium dynamics and equilibration process. Continuing this chain, it can be said that all financial physics consists of the answers to the questions about money flows.
disregarding whether they are described with the help of the gauge theory (which will be tackled later) or the theory of critical phenomena or the computer modeling of behavior of a great number of agents-investors. Summarize all said with the first formula in the article, financial physics can be defined as

\[
\text{Financial Physics} = \text{Financial Mathematics} + \text{Money flows} \quad (1)
\]

Further, we are going to demonstrate how the tools of modern theoretical physics, its images and objects, physical "blocks" are used to build a theory describing the short-term ("fast") dynamics of money flows and addressing three questions "Why?" of financial physics. We will see that there is a certain similarity between gauge theories of fundamental interactions and financial economics arising from general beautiful symmetries and mathematical (differential geometrical) structures. Such a similarity essentially limits the number of scenarios of mathematical description and leads to theories that have been studied intensively by physicists during the recent 50 years. In this approach, excess return plays the role of a force field (an analogue of an electromagnetic field), and money flows created by it play the role of charge currents. Uncertainty immanent to the financial market becomes an analogue of quantization inducing uncertainty into physical theories, while transaction costs turn into an effective charge mass resulting in inertia and limited time of the money flows reaction. These analogies as may seem futile at first glance come easily from the mathematical point of view. Studying price statistics in such a model we achieve the picture which is very close to a real one, and the first question Why? is successfully answered. The second "Why?" will be addressed when we have a closer look at how the system develops within a very short period of time when casual fluctuations have not yet washed out the trend. In this case, the movement of prices and money flows is determined by a system of equations resulting in indicators developed in technical analysis and used by traders. Adjustments to the Black-Scholes equation will appear naturally when we reformulate financial derivative pricing policy in terms of a new theory. Further, we will consider each of these situations and discuss them in more details.

It should be understood that financial physics like any other economic theory can not be taken for a panacea or Holly Grail. However, it is a step on the way to deeper understanding and more perfect theory. One should not expect to have the same number of quantitative coincidences as in Quantum Electrodynamics. It goes without saying that the laws regulating the microworld and galaxies are different from those regulating the peoples society, and they can not be interchanged mechanically. On the other hand, it is impractical to let our prejudices stop us making use of evident analogies and the experience acquired in physical sciences. Moreover, besides physical methods there is also physical methodology, in which one assumes that unless an experiment negates the theory it has the right to exist.

2 Analogy with Electrodynamics

We will start with a simple example. Let us assume that the spot exchange rate of a dollar to a pound is \( F(t) \) at time \( t \) while their respective interest rates are \( r_1 \) and
r_2. Assuming that all relevant information that can effect the exchange rates between 
t and t + dt is known and is reflected in these rates we ask how F(t) and F(t + dt) 
are interrelated. It is easy to find that this interrelation can be expressed by a simple 
equation:

\[ F(t)(1 + r_2) = F(t + dt)(1 + r_1) \]  

(2)

Let us suggest that for some reasons the right side of the equation is greater than 
the left one. This will immediately result in a response from "smart money" that will 
borrow pounds at time t, exchange them immediately into dollars, deposit the dollars 
until time t + dt at an interest rate r_1, after which exchange them back to pounds. 
The assumption that the right side of equation (2) is greater than the left one will 
ensure a no-risk profit from an arbitrage transaction described above. However, this 
condition can not last for long: since few sellers will choose to sell dollars at time t and 
few buyers will choose to buy dollars at time t + dt at initial prices, the exchange rates 
will change until equality (2) is redeemed. It is also easy to make certain that the left 
part of equation (2) can not stay greater than the right one for a long time. Equation 
(2) expresses the condition of absence of arbitrage. However, we are more interested 
in the restoration process rather than in equation (2) itself.

The process of restoration or, as we are going to refer to it, relaxation will take 
some time the length of which is determined by the market liquidity as well as market 
imperfections such as transaction costs and the bid-ask spread. The same factors will 
define the speed of the relaxation. For instance, the relaxation goes faster when the 
deviation from a balanced price is great, which attracts a large number of arbitrageurs 
despite transaction costs compared with cases when deviations are little and transaction 
costs make transactions profitable only for big arbitrageurs. The closer the market is 
to perfection, the less time relaxation takes and the higher its speed is.

Using this example we can identify two issues that will play a very important role 
in further studies.

2.1 Arbitrage and paths

Arbitrage or gaining returns from mispricing are always associated with the flow of 
assets along two different routes having a common beginning and end. In the previous 
example we have compared route (a1): pounds at the time t \rightarrow dollars at the time t 
\rightarrow dollars at the time t + dt \rightarrow pounds at the time t + dt with route (a2): pounds 
at the time t \rightarrow pounds at the time t + dt. Returns gained from these two routes of 
transactions are expressed by equation (2) assuming there is no arbitrage. Similarly, a 
pair of routes (b1) and (b2) can be introduced which begin and end in dollars.

Instead of two routes, a closed path of the assets flow can be studied which follows 
the first route from the starting point to the end, and the second route from the end 
to the starting point. In our case it will be a cyclic path (c): pounds at the time t 
\rightarrow dollars at the time t \rightarrow dollars at the time t + dt \rightarrow pounds at the time t + dt 
\rightarrow pounds at the time t. Having assigned to each segment of this path a respective 
exchange or interest factor (assuming that each segment that flows backwards has an 
inverse factor) and having multiplied these factors along the path and subtracted one
we will get an equation:

\[ R(c) = F^{-1}(t)(1 + r_2)^{-1}F(t + dt)(1 + r_1) - 1 \]  

which is equal to the discounted profit from an arbitrage transaction when "cash" follows route (a1), and debts follow route (a2). Further, we will use a term "excess return on the arbitrage operation" to define this value.

Besides a cyclic path (c) there is another cyclic path (-c) which is derived from (c) by changing the flow direction. This path can be also described by an equation:

\[ R(-c) = F(t)(1 + r_2)F^{-1}(t + dt)(1 + r_1)^{-1} - 1 \]

which is equal to the discounted return on an arbitrage operation when "cash" follows route (b1), and debts follow route (b2). Combining equations (3) and (4) one can obtain the following value:

\[ R = R(c) + R(-c) \]

defining an opportunity to carry out a (certain) profitable arbitrage operation. Quantity \( R \) is not negative and is equal to zero only if there is no arbitrage. In this case, equation (2) is equivalent to the equation \( R = 0 \). It is more convenient to use value \( R \) rather than equations (4) and (5) separately especially when we do not know which particular operation is profitable.

### 2.2 Charges and Forces

Let us go back to a mechanism of establishing balance described in a paragraph after formula (2). Speaking in general terms we can conclude that "cash" flows from under-valued assets in overpriced assets and "debts" flow backwards, so if "cash" flows like charged particle experiencing a force then "debts" behave like particles with opposite charge. What is more, flowing in such a way assets make this very force change diminishing its value. In physics this effect is referred to as screening. Thus, we may conclude that a financial system behaves in the same way as a system of charges in a force field which is created and changed by these charges. Applying physical blocks we can determine that a financial system "cash"-"debts"-arbitrage seems to look like classical electrodynamics though not in conventional three-dimensional space but in strange discrete space (time remained unchanged). In our example with dollars and pounds this new financial space consists of two points only – a point "dollars" and a point "pounds" – where assets "jump" from one point into another.

At first glance such an analogy with electrodynamics may seem purely superficial for economists as well as physicists. Frankly speaking, there are several ways of writing an equation describing migration and screening, electrodynamics being just one alternative. That is why without additional arguments favoring electrodynamics seems artificial and unjustified. Such additional arguments are provided in the form of general powerful symmetry which singles out electrodynamics from a great number of other competitive theories.

Let us study equations (3) and (4) in more detail. It is easy to find that they have certain remarkable properties: they do not change when currency units and their
respective exchange and interest rates are changed. For instance, we assume that between time $t$ and $t + dt$ it was decided to use pence instead of pounds. This decision will not affect the dollar’s interest rate and exchange rate at time $t$, but it will diminish the exchange rate 100 times at time $t + dt$ (when a pound will cost 100 new units – pence) and increase the interest factor 100 times – having deposited one pound one will get $100(1 + r_2)$ pence. Factor 100 will vanish and equations (3) and (4) will remain unchanged. We could have applied such consideration to simultaneous change of both dollars and pounds, which would not alter the result – equations (3) and (4) based on closed paths do not change. Going further and applying this approach to any traded (exchangeable) assets one can see that equations like (3), (4) and (5) remain unchanged when the assets units scale changes disregarding whether it is currency, bonds or stocks. In the first case the change in the scale would mean denomination, in other cases the change of the traded lot or merge and split.

Let us suggest now that we are trying to construct a theory that has the property of not changing when the units of measurements are arbitrary chosen, that is, it does not depend on the choice of the financial assets units: the currency nominal value, the lot size, etc. It is no doubt that the real world has this property, at least to certain extent agents do not start behaving in a different way only because they are dealing with 100 pence instead of pounds or if there are only 50 shares in the lot instead of 100. In building this theory one can only use such mathematical objects that remain unchanged when the units of measurement are changed, for instance, equations (3), (4) and (5). We can prove that in this case the simplest nontrivial theory will be an equivalent of electrodynamics! It is this very property that distinguishes electrodynamics from a great number of other competitive theories.

Building a model theory we did not start from searching for symmetry of this theory just by chance. We were aided by the methodology of physics. All contemporary physical theories are built based on the symmetry inherent in them. The general theory of relativity is built based on the assumption that the choice of the coordinates is not essential for the statement of the law of bodies movement (This looks rather like our freedom to choose the units in equations (3-5)). Quantum electrodynamics can be built on the assumption that the wave function phase can not be physically seen and its selection is arbitrary. Theories of weak and strong interactions also have fundamental symmetries of selection of physically equivalent objects. Physicists refer to such symmetries as gauge symmetries. The theory of the financial market described in this paper can be also called a gauge one - the Gauge Theory of Arbitrage (GTA) since the role of the force field in this theory is played by the excess return on arbitrage operation. From a mathematical point of view, this theory looks very much like electrodynamics with only one difference between them: instead of a local group of quantum phase rotation a local group of dilatations of financial assets units is used.

### 2.3 GTA Geometry (can be skipped when read for the first time)

Using a more formal approach one can assume that the inner reason for the similarity between all contemporary physical theories and the GTA lies in the fact that all of them
are descriptions of dynamics in geometrically complex spaces called by mathematicians fibre bundles. Such spaces consist of a base $B$ (for instance, Minkowski space or only two straight lines D and P) and fibres $F$ adhered to each point $B$. Such fibres collected in one point make a bundle, which accounts for a generic name of such spaces. Imagine that we are watching a particle moving in such a fibre bundle. The particle can move inside the fibre as well as between fibres so that its position can be described by two characters ($x$ and $y$): $x$ refers to the particle coordinate on the base $B$, while $y$ denotes the particle coordinate in the fibre $F(x)$ corresponding to point $x$. Putting aside the issue of coordinates on the base we can ask a question: ”What if coordinates in different fibres are not adjusted to each other?” In this case changing of coordinates does not say anything about the real change of particle position which is characterized by change of ”real” coordinates – exactly like the rate of return does not say anything until inflation is accounted and the real rate of return is calculated by the Fisher’s formula. Like in the example with inflation, we must subtract from the total change of the coordinate its superficial change which is associated with zero real change and which is determined by coordinates disagreement in different fibres. It is this superficial change that determines the rule for coordinates comparison in coordinate systems of different fibres or, as mathematicians put it, parallel transport.

Going back to the main topic of our discussion we can see that the abstract discourses described above look as if they were specially designed for the financial market. Assume we have two currencies (two points on the base) and want to compare four dollars and three pounds (numbers 3 and 4 in coordinate systems of fibres corresponding to points ”dollar” and ”pound” on the base). At first glance fours notes seem more attractive than three. However, anyone will prefer to have three pounds because at an exchange rate 1.67 dollars per one pound one can get five dollars for these three pounds, which is much better than to have only four. We can see that when assets transformed from four dollars into three pounds the real change was equal to +1 dollar instead of initial -1. In our case the real value of four dollars was 2.40 pounds, and the fictitious change was equal to 1.60 pounds. When compared 2.40 pounds differ from three pounds by 60 pence, which is equivalent to 1 dollar exactly. For a mathematician all these would mean that 2.40 pounds are equal to 4 dollars under parallel transport from one point on the base to another, and a covariant (real) difference of three pounds and four dollars is equal to 60 pence.

Net Present Value gives us another financial example of parallel transport. Assume that we can choose between 100 pounds now or 103 pounds a year later. At first glance one hundred and three (the same currency!) notes seem more attractive than one hundred. However, a reader is likely to choose one hundred because at an interest rate 5% one hundred pounds now will become 105 pounds in a year, which is definitely better than 103. Instead of counting all pounds a year later we can count them now. Then, we will be able to compare 100 pounds with the discounted by present value of 103 pounds, that is the Net present Value (NPV) equal to $98.10=103/(1+0.05)$. Again one can see that as the assets move in time from pounds at present to pounds a year later the real change is 2 pounds instead of initial +3, the parallel translation of 100 pounds amounting to 105 pounds and the covariant (real) difference being 2.

After we have defined parallel transport, we can address an issue of the difference
of results of parallel transport carried out by different routes on the base which have common beginning and end. However, from technical point of view it is more convenient to deal not with a pair of routes but with one cyclic path that covers one route moving forward and the second one moving backwards. The difference between the initial value of the transported amount and its value after the parallel transport along the closed path determines the curvature tensor of the fibre bundles, and, as it follows from formulas (3) and (4), directly results in excess rate of return on arbitrage operation and preconditions the assets movement. That is why comparison of the excess rate of return with the electromagnetic field intensity is not accidental both make particles move and both are equal to elements of the curvature tensor! In physics different results of parallel transport always mean the affect of forces on the transported particle, and the value of these forces is equal to elements of the curvature tensor. Introduction of the value R in formula (5) is not accidental as well it is directly associated with the square of the curvature tensor which is equal to the field energy both in our example as well as in electromagnetic field.

To sum up, we can say that when financiers buy and sell securities, exchange currency or calculate NPV, they make parallel transports in fibre bundles. What is more, they have been making that for more than 100 years without knowing about it. It is like Moliere’s famous hero who was very surprised when he learned that he spoke prose. The absence of arbitrage and free lunches looks very much like the condition of energy minimization, which is ubiquitous in classical physics.

2.4 Uncertainty and Quantization

The picture we have built is based on the assumption that all information capable to affect prices is known in advance, which means there is no uncertainty. The real financial world, as you understand, is far from this picture. Any financial operation is associated with a risk immanent to the financial world, that is why we have to generalize the theory in case of uncertain or random prices.

Describing random characters we are not able to predict their exact value such as the exchange rate, for example, in a month time. What we can do is only predict the probability with which we can expect a certain value of the exchange rate. It is difficult to define the very concept of probability in this case. According to the definition, probability is equal to the number of times a certain result is expected to be achieved under identical experimental conditions. This means that applying probabilistic description we expect that the experiment will be repeated in the exactly the same way many times. It is evident that this condition can not be met in the example with the exchange rate: there is only one April and only one May in 1998. Trying to repeat the experiment in May-June we will encounter different conditions on the market, and consequently, we will have to change the conditions of the experiment. Moreover, the result of the experiment in April-May may affect the outcome of the experiment in May-June. We will try to overcome these difficulties using conditional probabilities and short periods of time when it can be assumed that all outer factors remain unchanged.

Like random values, random paths are also described by probabilities, in this case
the probability of covering the whole path. Such a probability is referred to as the path weight. The weight fully defines a statistical model and the choice of the weight is a key step in building a theory. Here again we are facing the problem of choice. To make our choice we will use the following considerations: The weight must keep the symmetry of the theory built before stochasticity could enter it, that is the weight must be based on gauge-invariant quantities like (3), (4) and (5). In the situation when the world becomes less and less accidental, the weight must identify paths with minimum mispricing opportunities, that is the weight must identify the paths on which the quantity (5) has a minimum value. In the approximation where the speed of money flows is infinite and they do not effectively participate in the description, the theory, according to equation (1), must reproduce the results of financial mathematics. Traders represent a homogeneous ensemble. Such an assumption distinguishes this model from the previously proposed ones that represented the market as a combination of smart money and ”noise traders”. ”Smart money” know the real price and remove mispricing, while ”noise traders” behave unreasonably and represent a crowd. Remember that our target is to describe the short-term dynamics when all traders are professional participants of the market, and it would be odd to divide them into ”smart” and ”noise”. It is more realistic to assume that all traders behave in a different way in terms of individuality but in the same way in terms of statistics, and their understanding of the real price is formed according to a stochastic distribution. The latter must be gauge symmetric. Of course, a question how to model traders behavior still remains. In other words, it is unclear whether an assumption about return maximization is sufficient or one should introduce additional factors such as risk aversion, herd behavior, etc. This question will be addressed in the next paragraph.

It can be demonstrated that a simple theory built on the basis of these considerations will be as similar to quantum electrodynamics (in imaginary time) as a theory without stochasticity is similar to the classical electrodynamics.

3 Answering the Three Questions ”Why?”

After having described the general principles of construction of the theory we will consider its applications and see how this theory answers the three questions Why? of financial physics put in the first paragraph.

3.1 Statistical Characteristics of Real Price Charts

If one suggests that price changes are associated with new fundamental information, the chart of a return must look like the path of the particle movement affected by a random force. After some time, the return value may be defined with some probability, and the distribution function of this probability will be a Gaussian function, which means return will have normal distribution. However, it was noticed long ago that real profitability distribution functions do not look like Gaussian distribution: at the same average values and variation they have ”fat” tails and a higher and narrower peak. In addition, the peak height comes down eventually not according to a conventional distribution law
but according to the law \( t^{-a} \) where exponent \( a \) depends on the concrete security and is often approximately equal to 0.7. The latter is called by physicists an anomalous scaling behavior and is associated with a nontrivial internal dynamics. The description of this dynamics will be an answer to the first question "Why?".

The scaling behavior can be explained by the Fractional Market Hypothesis (FMH) (see Peters, 1995 for discussion and motivation) according to which, in the market there are investors with different investment time frames but the same general laws regulating investors behavior which do not depend on the time frame. Should the FMH be accepted, it is sufficient to describe the behavior of investors with short time frames to obtain the description of the system in general. It is here that the gauge theory comes on to the stage.

Fig. 1 and 2 show the results of a statistical study (Mantegna and Stanley, 1995) of the price distribution function of a portfolio of stocks belonging to 500 largest companies (squares) traded on the New York Stock Exchange (market index S&P500) and a model distribution function (a solid line) achieved from a simple model of the Gauge Theory of Arbitrage (Ilinski and Stepanenko, 1998). Fig. 1 shows the law of the distribution function height descent with time (the anomalous scaling law), while Fig. 2 demonstrates the function distribution at one minute. While the scaling behavior owes its appearance to the FMH, the distribution function type was defined due to statistical laws of gauge dynamics. It is a nearly ideal matching of the theory and the "experiment", which is similar to the accuracy of quantum electrodynamics. Fig. 2 also shows normal distribution (long dashes) and the popular Levi distribution (short dashes) having the right scaling behavior. Both of them are not as accurate in description as the model distribution.

3.2 Technical Analysis and the Efficient Market

A conventional simplified understanding of an efficient market is based on the assumption that all relevant information is already reflected in the price, and the price may change only due to some news. Since the news can not be forecast by virtue of its definition, the price changing is a random process which is not affected by the already known information, say price history. Thus, technical analysis using price history for forecasting future prices falls out of the law, and a trader instead of following a system approach in trading is resorted to toss a coin. It is unlikely that she or he will accept this suggestion.

The cause of this seeming contradiction lies in the unclear statement of the efficient market hypothesis. An accurate statement calls for rationality from investors and availability of a real pricing model and assumes that the real price may deviate of from the model only occasionally. The model price itself can depend on time and be defined by the available information, for instance, previous prices. Let us consider the following example. Assume that due to new information the return of certain securities has been increased compared to other securities with a similar risk. More profitable securities will be purchased until the return is leveled. The process of leveling takes time, and it should be accounted for in the construction of an ideal model. More details about equilibration can be taken from price history. If this washing out of the
mispricing takes finite amount of time, the price history analysis and the market forces behind it may become a key factor in building a model of the dynamics of future prices or, strictly speaking, of their average values. The latter brings us back to technical analysis representing a set of empirical (a physicist will say phenomenological) rules for prediction of the model prices and making respective investment decisions. Comparison of the characteristic time of return fluctuation and the time of relaxation determines the applicability of technical analysis. The assumption about immediate relaxation leads to the initial simplified definition of the efficient market.

All said above can be applied to the Gauge Theory of Arbitrage. In the previous paragraph it was demonstrated how a stochastic description appears in the gauge theory. The aim of this paragraph is to show how technical analysis emerges in the same framework. To a certain extent, technical analysis plays the role of classical mechanics on which the quantum theory is founded. Within relatively short time frames any quantum dynamics comes down to the classical one – if a quantum particle is released and its behavior is observed within a short time frame, although it will be impossible to define its exact location, the degree of uncertainty of its location will be negligible and its mean value will be the same as the result of a solution of the classical equations of motions (that is why such an approximate description is referred to as quasi-classical). Uncertainty grows with time, and a quasi-classical solution degrades and deforms. Stochastization takes place.

It can be demonstrated that in a quasi-classical approximation (for short time frames) a simple gauge model used in the previous paragraph to find the distribution function come down to a system of equations the solution to which corresponds to two known technical analysis indices the Negative Volume and the Positive Volume Indices. These indices describe the behavior of rational and irrational investors, respectively. The fact that both indices have appeared in one model is not a mere coincidence the statistical behavior of traders was defined as an irrational distribution of solutions around a rational average one, and that is why in quasi-classical equations both a rational and a noise components are found. It seems logical to refer to a market of such agents as a quasi-efficient market.

To sum up, technical analysis in the gauge theory corresponds to quasi-classical dynamics and describes dynamics within short time frames which are followed by stochastization resulting in a realistic statistical description. The question why technical analysis can be applied to different investment time frames can be answered by the Fractional Market Hypothesis.

3.3 Portfolio Theory and Derivative Pricing

A term "investment portfolio" is a key term in a contemporary financial theory. In assessing derivatives a portfolio replicating a derivative plays an important role and determines the cost of the derivative. In the Asset Pricing Theory an optimal portfolio defines the average asset growth as a function of its correlation with the key market parameters. In both cases prices are considered to be a random process, and the task comes down to elimination and averaging stochasticity and assessing securities under the condition of absence of arbitrage opportunities. The next step to be taken should be
setting up a portfolio theory which would, on the one hand, accept temporary arbitrage opportunities (they always exist in the market) and, on the other hand, provide for elimination of such opportunities and allow for accounting a no-random component of pricing movement and the technical analysis predictions. All these tasks can be addressed within the frames of the Gauge Theory of Arbitrage.

As it was demonstrated in previous paragraphs, the gauge approach provides for description of the market response to appearance of arbitrage opportunities and relaxation caused by it. It also allows to model the realistic price behavior and apply technical analysis tools having selected an adequate classical theory. One of the applications of this approach is finding adjustments to the Black-Scholes equation when virtual arbitrage opportunities and complicated pricing processes are present. To this end, two sorts of traders should be introduced into the theory speculators and arbitrageurs. The former are involved in modeling a realistic pricing movement, while the latter account for diminishing of arbitrage opportunities. Such classification does not seem artificial, and can be virtually seen in the market. In the frames of an infinitely fast response of investors, as one might have expected, the task can be limited by the solution of the Black-Scholes equation. However, within limited time of relaxation the gauge theory generates adjustments to the equation or, strictly speaking, makes it an integro-differential due to the memory effects. The same approach can be also applied for deriving adjustments to the equations of the Arbitrage Pricing Theory and, eventually, to the Capital Asset Pricing Model bearing in mind that in the latter case arbitrage opportunities are likely to appear not between a derivative and a replicating portfolio but between two different portfolios.

Finally, one more comment should be made. Besides the effects described above, a gauge approach has one more advantage it allows the inclusion of such market imperfections as bid-ask spread and transaction costs. To introduce them into the model one has just to insert additional factors into the matrix of model transitional probabilities.

4 Conclusion

The aim of this paper was to acquaint readers with a number of new concepts and tasks arising when analyzing the influence of money flows on the random pricing process. To a certain extent all scenarios described above were known earlier, and we have just made an attempt to give an idea of a quantitative description of the processes on which these scenarios are based. Technical issues were not addressed in order to make the principles of the theory clearer.

Although from the formal point of view the theory is similar to quantum electrodynamics, it is unlikely to be as fundamental as its prototype. It is rather a new language convenient for formulating the problems and searching for their solution. This language is technically advanced, and we hope that we have managed to persuade the readers, at least partly, that it is adequate and useful. Using the terms of this language one can introduce new elements relevant for individual problems (such as behavior peculiarities of investors or market restrictions), which will allow to extend the range of problems
that can be modeled.

In conclusion, we can give an example of an algorithm for building a pricing model of a concrete instrument within the framework of the gauge approach. The first step is to select a classical gauge-invariant theory for money flows which correlates with a certain tool of technical analysis applicable to this particular sort of securities. Then, following a formal prescription one should introduce a stochastic factor, which means to construct a quantum theory. The next step is to verify the validity of the stochastic description of the model on historical data. By this time all the parameters of the theory will have been determined, and the pricing model can be considered completed. After that, the tested model can be used by investors with different purposes, for instance, for arbitrage, speculation, hedging and risk management.

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FIG.1 Theoretical (solid line) and experimental (squares) probability of return to the origin (to get zero return) $P(0)$ as a function of time. The slope of the best-fit straight line is $-0.712 \pm 0.025$. The theoretical curve converges to the Brownian value 0.5 as time tends to one month.

FIG.2 Comparison of the $\Delta = 1$ min theoretical (solid line) and observed (squares) probability distribution of the return $P(r)$. The dashed line (long dashes) shows the gaussian distribution with the standard deviation $\sigma$ equal to the experimental value 0.0508. Values of the return are normalized to $\sigma$. The dashed line (short dashes) is the best fitted symmetrical Levy stable distribution.
