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Abstract

The production of $d'$ dibaryons in heavy ion collisions due to the elementary process $NN \rightarrow d'\pi$ is considered. The $NN \rightarrow d'\pi$ cross section is estimated using the vacuum $d'$ width $\Gamma_{d'} \approx 0.5$ MeV extracted from data on the double charge exchange reactions on nuclei. The $d'$ production rate per single collision of heavy ions is estimated at an incident beam energy of $1\, A\cdot GeV$ within the framework of the Quantum Molecular Dynamics transport model. We suggest to analyse the invariant mass spectrum of the $NN\pi$ system in order to search for an abundance of events with the invariant mass of the $d'$ dibaryon. The $d'$ peak is found to exceed the statistical fluctuations of the background at a $6\sigma$ level for $2 \times 10^5 \cdot A$ central collisions of heavy ions with the atomic number $A$.

keywords: heavy ion collisions, dibaryons

14.20.Pt, 21.65.+f
I. INTRODUCTION

In the last years the possibility of the existence and the properties of a narrow dibaryon \( d' (T = 0, J^p = 0^-) \) have been widely discussed both from the experimental and theoretical points of views [1] - [8]. So far, the narrow resonance structure is unambiguously established only in the double charge exchange (DCE) reactions on nuclei from \(^7\text{Li}\) to \(^{56}\text{Fe}\) [6]. The peaks in the forward DCE reaction cross sections at the incident kinetic energy of pions \( T_\pi \approx 50 \text{ MeV} \) are well described assuming that the dominant contribution of this process stems from a \( d' \) dibaryon with a mass \( M_{d'} = 2063 \text{ MeV} \) and a free decay width \( \Gamma_{d'} \approx 0.5 \text{ MeV} \). The \( d' \) mechanism is presently the only one which allows to explain the observed narrow structures. It is therefore important to find an evidence for a \( d' \) dibaryon in other reactions. At CELSIUS (Uppsala), the reaction \( pp \rightarrow pp\pi^+\pi^- \) was studied [8]. The analysis of the \( pp\pi^- \) invariant mass spectrum revealed a \( 4\sigma \) peak at \( M \approx M_{d'} \). The reported data are, however, still preliminary. Further plans to search for a \( d' \) dibaryon include proton-proton collisions and photo- and electro-production on deuterons.

The existence of narrow dibaryon resonances has important consequences for physics of dense nuclear matter. When the density is increased beyond a critical value, the formation of a Bose condensate of the dibaryons becomes energetically favourable. This phenomenon has been studied in Refs. [9] - [11].

In the present work, the possibility of the \( d' \) production in heavy ion collisions is investigated for incident beam energies around 1 \( A \cdot \text{GeV} \). Such a beam is, e.g., available at SIS at GSI (Darmstadt). We investigate the \( d' \) production rate and the \( pp\pi^- \) background within the framework of the Quantum Molecular Dynamics (QMD) transport approach [12] which is well established and describes successfully many observables of heavy ion reactions [13] - [16].

In the next Sect., the \( NN \rightarrow d'\pi \) cross section is calculated using as an input value the vacuum \( d' \) width \( \Gamma_{d'} \approx 0.5 \text{ MeV} \) extracted from the DCE reactions. We expect \( 2 \times 10^{-4} \) \( d' \) per single \( NN \) collision. In Sect. 3 we start with a qualitative discussion of the \( d' \) production
dynamics in heavy ion collisions. Using reasonable assumptions, we give an estimate for the $d'$ production rate and for the number of heavy ion collisions needed to separate the $d'$ peak from the background.

In Sect. 4, the $d'$ production in heavy ion collisions is studied with the use of the QMD simulations. The $d'$ production rate is calculated for central collisions of the ions $^{20}\text{Ne}$, $^{59}\text{Ni}$, $^{108}\text{Ag}$, and $^{197}\text{Au}$. The invariant mass distribution of the background $pp\pi^-$ events is analysed in Sect. 5. We found that it is possible to distinguish a $6\sigma d'$ peak from the analysis of at least $2 \times 10^5 \cdot A$ collisions. Corresponding experiments at the SIS have collected up to now about $10^8$ events. Thus the possibility for the search for the $d'$ dibaryon in heavy ion collisions is apparently a realistic aim.

II. CROSS SECTION FOR $D'$ PRODUCTION IN NUCLEON-NUCLEON COLLISIONS

The possible $NNd'\pi$ couplings are carefully analysed in Ref. [3]. Thus we start with the form of the $NNd'\pi$ vertex suggested in this work

$$T_{NNd'\pi} = g\gamma_1 C\gamma_5 \tau_i \psi_2 \pi_i d'.\tag{1}$$

Here, $C = \gamma_2 \gamma_0$ and $g$ is a form factor which depends on the momenta of the particles involved. The two nucleons in the vertex (II) are in the relative $^1S_0$ state, so they are affected by the long-distance attraction resulting in a virtual pole and by the short-distance repulsion. The final-state interaction of two nucleons substantially modifies the $d'$ decay rate. On the other hand, the initial-state interaction of the nucleons in the reaction $NN \rightarrow d'\pi$ modifies the cross section.

First we assume a constant form factor $g$ and neglect the interaction of the nucleons. Then we discuss in details the final- and initial-state interactions and the influence of the nontrivial momentum dependence of the form factor $g$.

The total width $\Gamma_{d'}^{(0)}$ of the $d'$ dibaryon for a constant $g$ with no final-state interactions can be calculated as
\[ \Gamma_{d'}^{(0)} = g^2 \frac{3}{64\pi^2} \sqrt{\mu M_{d'} T_{\pi, \text{max}}^2}. \]  

Here, \( T_{\pi, \text{max}} = ((M_{d'} - \mu)^2 - 4m^2)/(2M_{d'}) \approx 45 \text{ MeV} \) is the maximum pion kinetic energy in the \( d' \) decay, \( \mu, m, \) and \( M_{d'} \) are the pion, nucleon, and dibaryon masses, respectively. Due to the isotopic invariance, the partial decay widths are given by

\[ \Gamma_{d' \rightarrow pp\pi^-}^{(0)} = \Gamma_{d' \rightarrow pn\pi^0}^{(0)} = \frac{1}{3} \Gamma_{d'}^{(0)}. \]

The same relations hold true for the widths corrected to the final-state interactions.

The total cross section of the dibaryon production \( \sigma_{pp \rightarrow d'\pi^+}^{(0)} \) equals

\[ \sigma_{pp \rightarrow d'\pi^+}^{(0)} = g^2 \frac{1}{8\pi} \frac{k}{p} \]  

where \( k \) and \( p \) are the pion and nucleon momenta in the center-of-mass frame. In the different isotopic channels, the \( d' \) production cross sections are related by

\[ \sigma_{pp \rightarrow d'\pi^+}^{(0)} = \sigma_{nn \rightarrow d'\pi^-}^{(0)} = 2\sigma_{pn \rightarrow d'\pi^0}. \]

The same relations are valid for the cross sections corrected by the initial-state interactions.

In terms of the \( d' \) width, the \( pp \rightarrow d'\pi^+ \) cross section takes the form

\[ \sigma_{pp \rightarrow d'\pi^+}^{(0)} = \frac{8\pi}{3} \frac{1}{\sqrt{\mu M_{d'}}} \frac{1}{T_{\pi, \text{max}}^2} \frac{k}{p}. \]  

If the final-state interaction of the nucleons in the \( d' \) decay and the initial-state interaction of the nucleons in the \( d' \) production are taken into account, we obtain both, a corrected width \( \Gamma_{d'} \) and a corrected cross section \( \sigma_{pp \rightarrow d'\pi^+} \):

\[ \Gamma_{d'} = \Gamma_{d'}^{(0)} \eta_F \quad \text{and} \quad \sigma_{pp \rightarrow d'\pi^+} = \sigma_{pp \rightarrow d'\pi^+}^{(0)} \eta_I. \]  

The corrected cross section contains, as compared to Eq. (4), an additional factor \( \eta_I/\eta_F \)

\[ \sigma_{pp \rightarrow d'\pi^+} = \frac{8\pi}{3} \frac{1}{\sqrt{\mu M_{d'}}} \frac{1}{T_{\pi, \text{max}}^2} \frac{k}{p} \frac{\eta_I}{\eta_F}. \]  

In contrast to Ref. [3] where the factors \( \eta_I \) and \( \eta_F \) have been determined from different sources, here we use a method which allows to calculate both, \( \eta_I \) and \( \eta_F \), consistently. The
common scale of $\eta_I$ and $\eta_F$ remains in our method unfixed due to the unknown asymptotic behaviour of the nucleon scattering phase shifts at high energies. The common scale factor, however, drops out from the ratio $\eta_I/\eta_F$ entering into Eq. (5). The present method is based on the integral representation of the Jost function [17]

$$D_I(T) = \exp \left( - \frac{1}{\pi} \int_0^\infty \frac{\delta(T')}{{T}' - T} d{T}' \right).$$

(7)

Here, $T$ is the nucleon kinetic energy in the laboratory frame. The value of $T$ is connected to the invariant energy $s = (p_1 + p_2)^2$ by the relation $s = 4m^2 + 2mT$. In Eq. (7), $\delta(T)$ is the phase shift in the dominant $^1S_0$ partial wave state of the two nucleons in the $d'$ decay.

It is known from scattering theory that the influence of the final-state interaction on the outgoing particles can be taken into account by dividing the bare amplitude by the Jost function. The same prescription is valid for the initial-state interaction. The amplitudes with two external nucleon lines get therefore factors $1/D_I(T)$.

The enhancement factor $\eta_I$ entering Eq. (3) is simply

$$\eta_I(T) = \frac{1}{|D_I(T)|^2},$$

while the factor $\eta_F$ can be calculated by averaging the same function $\eta(T)$ over the phase space distribution of two nucleons originating from the decay $d' \rightarrow NN\pi$.

The phase $\delta(T)$ is experimentally known up to $T = 1000$ MeV [18] and can be extrapolated to infinity in a model dependent way. We use the experimental data in the interval $T = 0 \div 1000$ MeV and results of the model [19] for $T > 1000$ MeV. The uncertainties in the asymptotic behaviour of the phase do, however, not significantly influence the ratio $\eta_I/\eta_F$. The function $\beta(T) = \frac{1}{2} \ln (\eta(T)/\eta(0))$ is shown in Fig. 1 for the interval $T = 0 \div 1000$ MeV (solid curve). It is seen that it rapidly falls off from $\beta(0) = 0$ to a minimum value of $\beta(T_1) \approx -3.3$ at a kinetic energy of $T_1 \approx 600$ MeV. In the interval $T = 700 \div 1000$ MeV, $\beta(T)$ is a slowly varying function. For comparison, we plot in Fig. 1 the function $\beta(T)$ calculated in the effective-range approximation (see Appendix). The ratio $\eta_I/\eta_F$ in our approach becomes
Inserting $M_{d'} = 2063$ MeV and $\Gamma_{d'} = 0.5$ MeV, as suggested by the interpretation of the DCE reactions \[ , \] into Eq. (8) we obtain for the cross section

$$
\sigma_{pp \rightarrow d'\pi^+} = 300 \frac{k}{p} \mu b.
$$

(9)

In the reaction $NN \rightarrow d'\pi$, the threshold energy is $T_0 = 710$ MeV. At $T \approx 730$ MeV, and $k/p \approx 1/10$ one obtains $\sigma_{pp \rightarrow d'\pi} = 30 \mu b$. The cross section (8) should be compared to the experimental data on the $d'$ production in the $pp \rightarrow pp\pi^+\pi^-$ reaction \[ . \] At $T = 750$ MeV, where the total cross section of the $pp \rightarrow pp\pi^+\pi^-$ reaction is about $10 \div 20 \mu b$, the $d'$ contribution is estimated to be at a 7% level. It means that at this energy $\sigma_{pp \rightarrow d'\pi} \approx 15 \cdot 0.07 \cdot 3 \approx 3 \mu b$, i.e. the empirical value is about ten times smaller than the value given by Eq. (8). Hence, we are forced to conclude that the constant form factor $g$ does not allow to calculate the cross section $\sigma_{pp \rightarrow d'\pi}$ correctly from the width $\Gamma_{d'}$. Thus, we need to introduce a form factor to suppress the $NNd\pi$ vertex at high $NN$ energies. One possible choice of the form factor is the following

$$
g^2 = \frac{g'^2}{1 + \frac{T^2}{m_{FF}^2}}
$$

(10)

where $m_{FF}$ is a cut-off mass. The required suppression allows to determine the mass parameter $m_{FF} \approx 240$ MeV. Finally, we get for the cross section

$$
\sigma_{pp \rightarrow d'\pi} = \frac{300}{(1 + \frac{T^2}{m_{FF}^2})} \frac{k}{p} \mu b.
$$

(11)

Fig. 2 shows the cross section versus the nucleon kinetic energy from the $d'\pi$ threshold up to $T = 1000$ MeV.

It is seen from Eq. (7) that the enhancement factor $\eta(T)$ approaches unity with increasing $T$. In the potential framework, the final- and initial-state interaction effects disappear with increasing the energy. In our case $\eta_I/\eta_F \approx 1/5$ and $\eta_I$ is almost constant in the interval $T = 700 \div 1000$ MeV. One can thus assume that at $T = 700 \div 1000$ MeV initial-state interaction
effects are essentially switched off. The result \( \eta_I/\eta_F \approx 1/5 \) should be compared to the result of Ref. [3]: \( \eta_F \approx 5 \), \( \eta_I \approx 1/10 \), and \( \eta_I/\eta_F \approx 1/50 \). The order-of-magnitude difference is apparently related to the effect of a \( d' \) form factor implicitly included in calculations of the enhancement factor \( \eta_I \) performed in Ref. [3]. The effective-range approximation gives \( \eta_F \approx 3 \) and \( \eta_I \approx 1 \) (see Appendix).

III. NAIVE ESTIMATE FOR \( D' \) PRODUCTION IN HEAVY ION COLLISIONS

In the previous section the cross section \( pp \to d'\pi^+ \) has been estimated using thereby as an input value the vacuum \( d' \) width \( \Gamma_{d'} \approx 0.5 \) MeV. This cross section turned out to be rather small. For a kinetic energy of \( T = 1 \) GeV of the first nucleon in the rest frame of the second nucleon the \( d' \) production cross section reaches a value of \( \sigma_{pp \to d'\pi^+} \approx 10 \) \( \mu \)b (see Fig. 2). The total nucleon-nucleon cross section at the same energy is about \( \sigma_{NN}^{\text{tot}} \approx 40 \) mb. The ratio between the total \( d' \) production cross section averaged over the isospin states of the initial nucleons and the total nucleon-nucleon cross section is about

\[
\xi = \frac{3\sigma_{pp \to d'\pi^+}}{\sigma_{NN}^{\text{tot}}} \approx 2 \times 10^{-4}.
\]  

We thus expect \( \xi \approx 2 \times 10^{-4} \) \( d' \) dibaryons to be produced per single \( NN \) collision.

In heavy ion collisions the number of the elementary \( NN \) scatterings is already at intermediate energies relatively large and thus the \( d' \) production rate is enhanced. The mean free path \( \ell_f \approx 1 \) fm of a nucleon in a heavy nucleus is small as compared to the nuclear size. Therefore a nucleon from the colliding nucleus undergoes several scatterings with nucleons from the target nucleus, loses its energy and finally cannot produce a \( d' \) dibaryon (\( \sqrt{s} < M_{d'} \)) anymore. A naive estimate of the number of the \( d' \) dibaryons per a single \( AA \) collision yields

\[
n_{d'} = \xi A \approx 0.04 \left( \frac{\xi}{2 \times 10^{-4}} \right) \left( \frac{A}{200} \right).
\]

The \( d' \) production rate in symmetric and central heavy ion collisions is roughly enhanced by a factor \( A \) corresponding to the number of the projectile/target nucleons as compared to \( pp \) collisions. The background in heavy ion collisions is more complicated.
We wish to select $NN\pi$ events to test the invariant mass distribution with respect to the presence of a peak caused by the $d'$ dibaryon. For total number of the pions produced per single $AA$ collision at an energy of 1 $A\cdot$GeV, we assume $N_\pi \approx A/4$. This number is an upper limit for the pion production rate in the central heavy ion collisions, which coincides roughly with the prediction of the so-called Harris systematics [20] at that energy. However, recent experiments [21] indicate that pion production is suppressed in heavy systems with respect to the Harris systematics. The total number $N_t$ of those $NN\pi$ events which have to be analysed is then

$$N_t = 2A^2 N_\pi N_C \approx 4 \times 10^6 \left( \frac{A}{200} \right)^3 N_C,$$

with $N_C$ being the number of nucleus-nucleus collisions.

For the further discussion we denote with $n_F(p)$ and $n_B(k)$ the momentum space distribution functions of the outgoing nucleons and pions, respectively. The number density of $NN\pi$ triplets with respect to the $NN\pi$ invariant mass $M_t$ is given by

$$\frac{1}{N_C} \frac{dN_t}{dM_t^2} = \frac{g^2_{\text{nucl}} g_{\text{pion}}}{2!} \int \frac{dp_1}{(2\pi)^3} \frac{dp_2}{(2\pi)^3} \frac{dk}{(2\pi)^3} \delta \left( \left( p_1 + p_2 + k \right)^2 - M_t^2 \right) n_F(p_1) n_F(p_2) n_B(k)$$

(15)

where $g_{\text{nucl}} = 2 \times 2$, $g_{\text{pion}} = 3$ are statistical factors for nucleons and pions. Since we are interested in the values of $M_t$ close to the $NN\pi$ threshold, one can write

$$\frac{1}{N_C} \frac{dN_t}{dM_t} \approx \frac{g^2_{\text{nucl}} g_{\text{pion}}}{2!} \frac{1}{(2\pi)^6} \Phi_3(M_t) \int \frac{dP}{(2\pi)^3} \left( \frac{M_t}{\sqrt{M_t^2 + P^2}} \right) n_F^2 \left( \frac{m}{M_t} P \right) n_B \left( \frac{\mu}{M_t} P \right)$$

(16)

where

$$\Phi_3(M_t) = \int dp_1 dp_2 dk \delta^4(p_1 + p_2 + k - P) \approx \frac{\pi^3}{2} (\mu M_t)^{3/2} T_{\pi,\text{max}}^2(M_t)$$

(17)

is essentially the phase space of the $NN\pi$ system. Here, $P = (\sqrt{M_t^2 + P^2}, P)$, $m$ and $\mu$ are nucleon and pion masses, and $T_{\pi,\text{max}} = ((M_t - \mu)^2 - 4m^2)/(2M_t)$ is the maximum kinetic energy of the pions in the center-of-mass frame of the $NN\pi$ system for a given invariant mass $M_t$. 

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Near the $NN\pi$ threshold, the dependence of the density $dN_t/dM_t$ on the invariant mass $M_t$ is mainly governed by the factor $T_{\pi, \text{max}}^2(M_t)$. One can therefore write

$$
\frac{1}{N_t} \frac{dN_t}{dM_t} \approx \frac{T_{\pi, \text{max}}^2(M_d)}{\int_{2\, \text{GeV}}^{2.3\, \text{GeV}} T_{\pi, \text{max}}^2(M_t) dM_t} \approx 4 \times 10^{-4} \, \text{MeV}^{-1}.
$$

(18)

The density of the background events in vicinity of the $d'$ peak is estimated as

$$
\frac{1}{N_C} \frac{dN_t}{dM_t} \approx 2A^2 N_\pi \frac{dN_t}{N_t} \approx 1600 \left( \frac{A}{200} \right)^3.
$$

(19)

In the case of the $^{197}\text{Au} + ^{197}\text{Au}$ collisions, we thus obtain about 1600 background events per 1 MeV bin of spectrum of the invariant mass $M_t$.

The actual bin width $\Delta M_t$ which we are interested in is given by the maximum of the $d'$ width $\Gamma_d \approx 0.5$ MeV and the experimental resolution $(\Delta M_t)^\text{exp}$. The number of the background events in this interval,

$$
\frac{\Delta N_t}{N_C} = 2A^2 N_\pi \frac{dN_t}{N_t} \Delta M_t \approx 1600 \left( \frac{A}{200} \right)^3 \left( \frac{\Delta M_t}{1 \, \text{MeV}} \right),
$$

(20)

should be compared to the number of the $d'$ events given by Eq. (13).

Suppose we have $N_C$ collisions and we want to have a statistically significant peak at a $n_\sigma \sigma$ level with $n_\sigma \geq 6$. We should require

$$
n_\sigma \sqrt{N_C \Delta n_t} \leq N_C n_d',
$$

(21)

with $\Delta n_t = \Delta N_t/N_C$ being the number of the $NN\pi$ events per single $AA$ collision in the interval $\Delta M_t$. We get

$$
N_C \geq n_\sigma^2 \frac{\Delta n_t}{n_d'} \approx 4 \times 10^7 \left( \frac{n_\sigma}{6} \right)^2 \left( \frac{A}{200} \right) \left( \frac{2 \times 10^{-4}}{\xi} \right)^2 \left( \frac{\Delta M_t}{1 \, \text{MeV}} \right).
$$

(22)

The above estimates give a rough idea on the number of collisions needed to reveal the $d'$ peak. We see that light nuclei require smaller number of the collisions and therefore are better candidates for $d'$ searches. In order to obtain a more precise estimate and to check the $A$ dependence of the results, we perform in Sects. 4 and 5 Monte Carlo simulations for the $d'$ production in central heavy ion collisions within the QMD model. We analyse also additional cuts and show that they allow to reduce the value $N_C$ by a factor $1/2$. 

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IV. SIMULATION OF $D'$ PRODUCTION IN HEAVY ION COLLISIONS

The simulations were done within the framework of the Quantum Molecular Dynamics (QMD) model [12]. QMD is a semi classical transport model which accounts for relevant quantum aspects like the Fermi motion of the nucleons, stochastic scattering processes including Pauli blocking in the final states, the creation and reabsorption of resonances and the particle production. The time evolution of the individual nucleons is thereby governed by the classical equations of motion

$$\frac{\partial p_i}{\partial t} = -\frac{\partial H}{\partial q_i}, \quad \frac{\partial q_i}{\partial t} = \frac{\partial H}{\partial p_i},$$

(23)

with the classical $N$–body Hamiltonian $H$

$$H = \sum_i \sqrt{p_i^2 + M_i^2} + \frac{1}{2} \sum_{i \neq j} \left( U_{ij} + U_{ij}^{Yuk} + U_{ij}^{Coul} \right).$$

(24)

The Hamiltonian, Eq. (24), contains two-body potential interactions which are finally determined as classical expectation values from the local Skyrme forces $U_{ij}$ supplemented by a phenomenological momentum dependence,

$$U_{ij} = \alpha \left( \frac{\rho_{ij}}{\rho_0} \right) + \beta \left( \frac{\rho_{ij}}{\rho_0} \right)^\gamma + \delta \ln^2 \left( \epsilon |p_i - p_j|^2 + 1 \right) \frac{\rho_{ij}}{\rho_0},$$

(25)

and an effective Coulomb interaction $U_{ij}^{Coul}$,

$$U_{ij}^{Coul} = \left( \frac{Z}{A} \right)^2 \frac{e^2}{|q_i - q_j|} \text{erf} \left( \frac{|q_i - q_j|}{\sqrt{4L}} \right).$$

(26)

Here, $\rho_{ij}$ is a two-body interaction density defined as

$$\rho_{ij} = \frac{1}{(4\pi L)^{3/2}} e^{-(|q_i - q_j|)^2/4L}$$

(27)

and erf is the error function. The parameters $\alpha, \beta, \gamma, \delta, \epsilon$ of the Skyrme interaction, Eq. (25), are determined such as to reproduce the saturation density ($\rho_0 = 0.17 \text{ fm}^{-3}$), the binding energy ($E_B = -16 \text{ MeV}$) for normal nuclear matter at a fixed incompressibility $K_\infty$, as well as the correct momentum dependence of the real part of the nucleon-nucleus optical potential [13]. In the present calculations we use a soft equation of state with $K_\infty =$
200 MeV corresponding to the momentum-dependent interaction (SMD), Eq. (25), which is known to reproduce reasonably well the reaction dynamics in the considered energy range. The Yukawa-type potential $U_{ij}^{\text{Yuk}}$ in Eq. (24) serves to improve the surface properties and stability of the initialised nuclei.

Two-body collisions are implemented by Monte Carlo methods, the Pauli principle is thereby taken into account for the final states. Nucleons while propagating collide with each other stochastically provided that the distance between centroids of the two Gaussian wave packets is less than $d_{\text{min}} = \sqrt{\sigma_{\text{tot}}(\sqrt{s})/\pi}$. For the inelastic nucleon-nucleon channels, we include the relevant baryonic resonances, i.e. $\Delta(1232)$ and $N^*(1440)$. The pions originate from the respective resonance decays. In particular we include the resonance production and rescatterings in the inelastic $NN$ collisions: the one-pion decays of $\Delta$ and $N^*$, the two-pion decays of $N^*$, and the one-pion reabsorption processes. For details see Refs. [14,15].

Since the production cross section of the $d'$ dibaryons is small, these particles are treated perturbatively. The $d'$ dibaryons are produced at kinematically favourable conditions in the $pp$ and $nn$ collisions with a probability

$$P = \frac{\sigma_{pp\rightarrow d'\pi^+}}{\sigma_{NN}^{\text{tot}}} ,$$

while in the $pn$ collisions the $d'$ dibaryons are produced with a probability $P/2$. Here, $\sigma_{NN}^{\text{tot}}$ is the total (elastic + inelastic) $NN$ cross section and $\sigma_{pp\rightarrow d'\pi^+}$ is given by Eq. (11). The results of our simulations for the $d'$ production rate in 1000, 50, 25, and 100 central collisions of the ions $^{20}Ne$, $^{50}Ni$, $^{108}Ag$, and $^{197}Au$ are shown on Fig. 3 (a). The impact parameter $b$ is assumed to be less, respectively, than 0.7, 1.3, 2, and 3 fm (minimal bias). In these intervals, the probability for the $d'$ production is not sensitive to the values of $b$. The $d'$ production rate increases linearly with the atomic number $A$, and so our results can be presented in the form

$$n_{d'} \approx 10^{-4} \cdot A .$$ (28)
V. THE INVARIANT MASS DISTRIBUTION OF THE BACKGROUND $PP\pi^-$ EVENTS

The number of the $d'$ dibaryons produced per single collision is much smaller than the number of the $NN\pi$ background events in the vicinity of the $d'$ mass (according to Sect. 3). It is therefore reasonable to search for cuts which are able to enhance the $d'$ signal.

The first possible cut takes the fact into account that the $d'$ is an isosinglet state. Only $1/12$ of all $NN\pi$ events form an isosinglet state (we assume isotropy of the $NN\pi$ events in the isotopic space). The selection of isosinglet $NN\pi$ events could reduce the background by a factor of 12. Such a selection is, however, impossible. Let us select only $pp\pi^-$ events. The total reduction of the background is also $1/12$. Since the branching ratio of the $d' \rightarrow pp\pi^-$ decay is $1/3$, the number of the $d'$ events is reduced as well. The relative reduction of the number $N_C$ of collisions needed to reveal the $d'$ peak is, according to Eq. (22), $3^2/12 = 3/4$.

The second possible cut can be obtained by the selection of protons with an invariant mass $M_{pp} < 2m + 15$ MeV. Such a cut not much influences the $d'$ signal, since for the dominant part of the $d'$ decays ($\approx 2/3$) the invariant mass of the two protons $M_{pp}$ is less than $2m + 15$ MeV. This is just the effect of the final-state interaction of two protons in the $d' \rightarrow pp\pi^-$ decay. In Fig. 1, this effect is displayed in the fast increase of the function $\beta(T)$ at small nucleon kinetic energies $T$. The reduction of the background can be estimated as in Eq. (16) and gives

$$\frac{T^2_\pi(M_{pp} = 2m) - T^2_\pi(M_{pp} = 2m + 15 \text{ MeV})}{T^2_\pi(M_{pp} = 2m)} \approx \frac{1}{3}$$

(29)

where $T_\pi(M_{pp})$ is the kinetic energy of pions in the $NN\pi$ system for a given invariant mass $M_{pp}$ of the two nucleons. In particular, $T_\pi(M_{pp} = 2m) = T_{\pi,\text{max}} \approx 45$ MeV. The relative reduction of the value $N_C$ is $(1/3) \cdot (3/2)^2 = 3/4$.

The $d'$ dibaryon decay products are distributed isotropically in the nucleus-nucleus center-of-mass frame, while the nucleons in heavy ion collisions are produced with a higher probability in the forward and backward directions. The angular distribution of the outgoing
protons averaged over 100 $^{197}\text{Au} + ^{197}\text{Au}$ collisions is shown in Fig. 4. The cut in the polar angle of the beam can provide an additional reduction of the lower limit for the number of the collisions required to reveal the $d'$ peak. The background is reduced by a factor 2, while the number of the $d'$ events is reduced by a factor 3/4 (see below). The relative reduction of the number $N_C$ is small $(1/2) \cdot (4/3)^2 = 8/9$.

These three cuts when applied separately are not much efficient. Nevertheless, being combined they reduce the value $N_C$ by a factor $(3/4) \cdot (3/4) \cdot (8/9) = 1/2$.

The simulations of the $pp\pi^-$ background are made within the framework of the QMD model for 1000, 50, 25, and 100 central collisions of the ions $^{20}\text{Ne}$, $^{59}\text{Ni}$, $^{108}\text{Ag}$, and $^{197}\text{Au}$ at the beam energy 1000 MeV per nucleon. In order to exclude complex fragments from the analyses, we take only protons into account which have no neighbouring nucleons closer than 3 fm in the final stage of the reaction. Here we trace the evolution of the reaction up to 80 fm/c, a time where almost all resonances ($\Delta$ and $N^*(1440)$) have decayed into nucleons and pions.

The invariant mass distribution of the $pp\pi^-$ events in the interval $2.02 < M_t < 3$ GeV is shown in Fig. 5 (a) for one typical $^{197}\text{Au} + ^{197}\text{Au}$ collision. This distribution has a maximum at $M_t \approx 2.3$ GeV. The upper limit in the integral entering the denominator of Eq. (18) is therefore reasonable.

In Fig. 5 (b) we show the invariant mass distribution of the events in the interval $2.02 - 2.1$ GeV, averaged over 100 collisions $^{197}\text{Au} + ^{197}\text{Au}$. According to Eq. (14), the density of the $NN\pi$ events is proportional to the phase space of the $NN\pi$ system. Near the $NN\pi$ threshold, the phase space is proportional to square of the difference $M_t - M_{th}$ where $M_{th} = 2m + \mu$. We see that in this region the density of the $pp\pi^-$ events is fitted well by the curve $C(M_t - M_{th})^2$, in agreement with the expression (13). From the fit, we determine the coefficient $C = 2.6 \times 10^4$ GeV$^{-2}$. In vicinity of the $d'$ dibaryon mass, the average number of the background $pp\pi^-$ events equals $\Delta N_t/N_C = 55$. This number should be compared with the number in the right hand side of Eq. (20). Here we got 1600 $NN\pi$ events per 1 MeV or, equivalently, 1600/12 = 130 $pp\pi^-$ events per 1 MeV. The estimate of Sect. 3 is therefore
in reasonable agreement with the results of the QMD simulations.

In Fig. 5 (c) we show the distribution of the $pp\pi^-$ events for 100 collisions $^{197}Au + ^{197}Au$, grouped in 10 bins of a width of $\Delta M_t = 1$ MeV around the $d'$ dibaryon mass. The solid curve is the best linear fit to this distribution. This distribution is now further analysed in Fig. 6.

First, we construct the histogram (a): The number of the bins versus the deviation of the number of the $pp\pi^-$ events from the mean values determined by the solid straight line in Fig. 5 (c). In the vicinity of the $d'$ mass, the number of $NN\pi$ triplets is about $\Delta N_t \approx 5500$. From Fig. 6 (a) we deduce the RMS deviation $\sigma \approx 80$ in the number $\Delta N_t$. The Poisson law requires $\sigma \approx \sqrt{\Delta N_t} \approx 75$. The background fluctuations are therefore distributed in agreement with the Poisson statistics.

In Fig. 6 (b), the same results are shown for the cut $M_{pp} < 2m + 15$ MeV. The total number of the $pp\pi^-$ events is reduced approximately by a factor 3: We obtain $\Delta N_t \approx 2000$ triplets, in the excellent agreement with the estimate of Eq. (29). The RMS deviation $\sigma \approx 65$ is somewhat larger. One can expect that the $\sigma$ is reduced by a factor $\sqrt{3} = 1.7$, while we get only a reduction by a factor of $\approx 1.3$. This deviation can, however, be explained by statistical fluctuations due to the relatively small number of the bins (= 10).

The angular cut $|\cos \theta^{c.m.s.}_p| < 0.75$ reduces the background by a factor of two, while the number of the detected $d'$ events is reduced by a factor $3/4$. The results for the double cut $M_{pp} < 2m + 15$ MeV and $|\cos \theta^{c.m.s.}_p| < 0.75$ are shown in Fig. 6 (c). Now we get only $\Delta N_t \approx 1100$ triplets. The RMS deviation $\sigma \approx 30$ is again in good agreement with the Poisson law, i.e. $\sigma \approx \sqrt{\Delta N_t} \approx 33$.

To summarise, the selection of the $pp\pi^-$ events suppresses the background by a factor 12. The second cut over the invariant masses of the two protons suppresses the background by a factor 3. The third cut over the proton scattering angles suppresses the background by a factor 2. The number of the detected $d'$ events decreases by a factor $1/3$ in the first case, decreases by a factor $2/3$ in the second case, and decreases by a factor $3/4$ in the third case. The number $N_C$, which is the ratio between the background $\Delta n_t$ and square of the signal
n_{d'}^2 (see Eq. (22)), decreases by a factor 1/2.

The fluctuations of the $pp\pi^-$ background summed up over all ion collisions are described reasonably well by the Poisson statistics, and so Eq. (22), on which we base the estimate for the total number of collisions needed to reveal the $d'$ peak, are apparently justified.

At the same time, fluctuations of the $pp\pi^-$ background from one collision to another due to the variations of the impact parameter and due to variations of the coordinates and momenta of the nucleons inside of the colliding nuclei, are significant. These fluctuations are analysed in Fig. 7 with no cuts (a), with the cut $M_{pp} < 2m + 15$ MeV (b), and with the double cut $M_{pp} < 2m + 15$ MeV and $|\cos \theta_{c.m.s.}^{p}| < 0.75$ (c). The histograms (a) - (c) show the number of simulated collisions as a function of the number of all possible $pp\pi^-$ combinations per single collision in the bin at the $d'$ dibaryon mass $M_{d'} = 2.063$ GeV with width $\Delta M_t = 1$ MeV. The reduction of the background by a factor 3 on the histogram (b) and further by a factor 2 on the histogram (c) as compared to the histograms (a) and (b), respectively, is clearly seen, while the RMS deviations are 2-3 times greater than one could expect from the Poisson distribution. It means that the distinction between different collisions has essentially a dynamical origin. If all triplets from all collisions are mixed, the Poisson distribution starts to work, as we discussed above.

With all the cuts taken into account, the number of the $pp\pi^-$ events in a bin $\Delta M_t = 1$ MeV around the $d'$ mass per one collision equals $\Delta n_t = 11$ (see Fig. 7 (c)). This number should be compared with the number of the detected $d'$ dibaryons in the channel $pp\pi^-$ $n_{d'} \approx 0.02 \cdot (1/3) \cdot (2/3) \cdot (3/4) = 0.0033$. It means that the number of the $^{197}Au + ^{197}Au$ collisions needed to reveal the $d'$ peak at a 6$\sigma$ level is

$$N_C \geq n_{d'}^2 \frac{\Delta n_t}{n_{d'}^2} \approx 3.6 \times 10^7.$$  \hspace{1cm} (30)

The $A$ dependence of the number of the background events and of the ratio $\Delta n_t / n_{d'}^2$ are analysed in Figs. 3 (b) and (c). The results of the QMD simulations confirm essentially the naive estimates of Sect. 3: The value $\Delta n_t$ increases as $A^3$, while the ratio $\Delta n_t / n_{d'}^2$ increases linearly with $A$. In the general case, Eq. (30) reads
\[ N_C \geq 2 \times 10^5 \cdot A. \] (31)

The limit on the value \( N_C \) is inverse proportional to square of the \( d' \) production cross section and depends linearly on the interval \( \Delta M_t = \max(\Gamma_{d'}, (\Delta M_t)^{\text{exp}}) \) where \((\Delta M_t)^{\text{exp}}\) is the resolution of the detectors. The \( d' \) cross section in Eq. (11) is well defined provided that the signal seen at CELSIUS is really connected to the \( d' \) dibaryon. Otherwise the used value of \( \sigma_{pp \rightarrow d'\pi^+} \) gives the upper limit for the \( d' \) production cross section, for the number \( n_{d'} \) of the detected \( d' \) dibaryons, and the minimum value for the number \( N_C \).

VI. SUMMARY

We considered the production of the \( d' \) dibaryons in heavy ion collisions. Due to smallness of the cross section for the \( d' \) dibaryon production in the elementary \( NN \) collisions, the production of the \( d' \) dibaryons in heavy ion collisions was considered as a perturbation to the reaction dynamics. The number of the \( d' \) dibaryons was calculated using the QMD model for \( ^{20}Ne, ^{59}Ni, ^{108}Ag, \) and \( ^{197}Au \) central collisions at the energy 1000 MeV per nucleon. We obtained \( n_{d'} \approx 10^{-4} \cdot A \) dibaryons per single collision. This number was compared with the background in the invariant mass distribution of the \( pp\pi^- \) events. The comparison showed that at least \( 2 \times 10^5 \cdot A \) collisions are needed to make the \( d' \) signal statistically significant. Light nuclei are more promising candidates for the \( d' \) searches. If data on the \( pp\pi^- \) invariant mass distribution from earlier experiments are already available, the search for the \( d' \) peak can be made both with light and heavy nuclei. This is a realistic task to collect statistics sufficient to reveal the \( d' \) peak, and thus the search for the \( d' \) dibaryon seems to be a possibility which should be probed first by analysing the available heavy ion data accordingly.

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APPENDIX. THE ENHANCEMENT FACTOR $\eta_F$ FOR REACTION $D' \rightarrow NN\pi$ IN THE EFFECTIVE-RANGE APPROXIMATION

The Jost function in the effective range approximation has the form (see [17], Ch.9)

$$1/D(k) = \frac{k + i\alpha}{k + i\gamma}$$

where

$$\frac{1}{2}r_e(\alpha - \gamma) = 1, \quad \frac{1}{2}r_e\alpha\gamma = 1/a.$$  

Here, $a$ is the scattering length, $r_e$ the effective radius, and $k$ is the nucleon momentum in the center-of-mass frame of two nucleons. In our case, $a = 23.7$ fm, $r_e = 2.67$ fm, and

$$k = \sqrt{\frac{1}{2}M_d(T_{\pi,\text{max}} - T_\pi)}.$$  

In Fig. 1, the value $\beta(T) = \frac{1}{2}\ln|D(0)/D(T)|^2$ where $T = 2k^2/m$ is plotted (dashed curve).

The factor $\eta_F$ is obtained by averaging the value $|1/D(k)|^2$ over the phase space of the $NN\pi$ system

$$\eta_F = \frac{\int |1/D(k)|^2 \sqrt{T_\pi(T_{\pi,\text{max}} - T_\pi)} dT_\pi}{\int \sqrt{T_\pi(T_{\pi,\text{max}} - T_\pi)} dT_\pi}.$$  

This expression can be transformed as follows

$$\eta_F = \frac{4i}{\pi} \int_C k_0^2 z + \frac{\alpha^2}{k_0^2 z + \gamma^2} \sqrt{z(z - 1)} dz.$$  

The contour integral runs around the cut $[0, 1]$, $Re(z)$ decreases when $Im(z)$ is positive and increases when $Im(z)$ is negative. Here,

$$k_0 = \sqrt{\frac{1}{2}M_dT_{\pi,\text{max}}}.$$  

We move the contour $C$ to infinity and obtain

$$\eta_F = 1 + 4(\alpha^2 - \gamma^2)(\sqrt{k_0^2 + \gamma^2} - \gamma)^2/k_0^4.$$  

Substituting $\alpha \approx 2/r_e >> \gamma \approx 1/a$ and $k_0 = 210$ MeV $>> \gamma$, we get

$$\eta_F \approx 1 + 16/(r_e k_0)^2 \approx 3.$$  

For $T = 800$ MeV and $k = \sqrt{\frac{1}{2}mT} = 600$ MeV, $\eta_I = |1/D(k)|^2 \approx 1.$
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FIGURE CAPTIONS

Fig. 1: The function $\beta(T) = \frac{1}{2} \ln (\eta(T)/\eta(0))$ versus the laboratory kinetic energy $T$ of the incoming nucleon. The solid curve represents the calculation with the Jost function (7), while the dashed curve is due to the effective-range approximation described in Appendix.

Fig. 2: The cross section $\sigma_{pp \rightarrow d'\pi^+}$ versus the laboratory kinetic energy $T$ from the $d'\pi$ threshold up to 1000 MeV for the reaction $pp \rightarrow d'\pi^+$.

Fig. 3: (a) The $d'$ production rate $n_{d'}$ versus the atomic number $A$ of the colliding nuclei $^{20}$Ne, $^{59}$Ni, $^{108}$Ag, and $^{197}$Au at 1 A·GeV incident energy. The errors display variations of the value $n_{d'}$ due to variations of the impact parameter and initial coordinates and momenta of nucleons in the colliding nuclei. The solid straight line is the linear fit of the value $n_{d'}$. (b) The number $\Delta n_t$ of the $pp\pi^-$ triplets per 1 MeV of the $pp\pi^-$ invariant mass in vicinity of the $d'$ peak versus atomic number $A$ of the colliding nuclei. The errors give statistical fluctuations. The solid curve is the $A^3$ fit of the value $\Delta n_t$. (c) The ratio between the background $pp\pi^-$ events and square of the number of the $d'$ dibaryons versus atomic number $A$. The solid straight line is the linear fit of the ratio $\Delta n_t/n_{d'}^2$.

Fig. 4: The proton polar angular distribution in the center-of-mass system of the colliding nuclei in central ($b < 3$ fm) $^{197}$Au + $^{197}$Au collisions at 1 A·GeV incident energy.

Fig. 5: The distribution of the $pp\pi^-$ background events in central ($b < 3$ fm) $^{197}$Au + $^{197}$Au collisions at 1 A·GeV incident energy. (a) The $pp\pi^-$ background in the interval $2 \div 3$ GeV of the invariant mass $M_t$ for a single typical collision. (b) The $pp\pi^-$ background near threshold averaged over $N_C = 100$ collisions. According to Eq. (16), the distribution has a simple threshold behaviour: $\Delta N_t = C(M_t - M_{th})^2$. It is seen that the parabolic curve is an excellent fit to the distribution near the threshold. (c) The total
$pp\pi^-$ background at threshold (see scale of $M_t$) for $N_C = 100$ collisions. The straight solid line gives the best fit for these 10 bins. The deviations of the $\Delta N_t$ from the mean values indicated by the solid line are in good agreement with the Poisson distribution (see Figs. 6 and discussion in the text). The value $\Delta N_t = (dN_t/dM_t)\Delta M_t$ is the number of the $pp\pi^-$ events in the interval $\Delta M_t$. In the histograms (a), (b), and (c), $\Delta M_t = 10, 1, and 1 \text{ MeV}$, respectively.

Fig. 6: The numbers of the bins of Fig. 5 (c) with the fixed deviations from the mean values of the $\Delta N_t$, indicated by the solid line, with no cuts (a), with the cut over the two-proton invariant masses $M_{pp} < 2m + 15 \text{ MeV}$ (b), and with the double cut over the two-proton invariant masses $M_{pp} < 2m + 15 \text{ MeV}$ and the proton scattering angles $|\cos \theta_p^{c.m.s.}| < 0.75$ in the center-of-mass system of the colliding nuclei (c). The RMS deviations $\sigma = 80, 65, and 30$ should be compared with the mean values $\Delta N_t = 5500, 2000, \text{ and } 1100$ in the cases (a), (b), and (c), respectively (the mean values $\Delta N_t/N_C$ with $N_C = 100$ can be read off from the upper right corners of Figs. 7 (a) - (c)). The fluctuations of the background $\sigma \approx \sqrt{\Delta N_t}$ do not contradict to the Poisson statistics.

Fig. 7: The distribution of 100 simulations of $^{197}Au + ^{197}Au$ collisions as a function of the number of the $pp\pi^-$ combinations (triplets) per single collision in the region $M_d' \pm 0.5 \text{ MeV}$ with no cuts (a), with the cut over the two-proton invariant mass $M_{pp} < 2m + 15 \text{ MeV}$ (b), and with the double cut over the two-proton invariant mass $M_{pp} < 2m + 15 \text{ MeV}$ and the proton scattering angles $|\cos \theta_p^{c.m.s.}| < 0.75$ in the center-of-mass system of the colliding nuclei (c).
FIGURES

FIG. 1.
FIG. 4.
