UTILITY OF THE WEAK TEMPERATURE GRADIENT APPROXIMATION FOR EARTH-LIKE TIDALLY LOCKED EXOPLANETS

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ABSTRACT

Planets in M dwarf stars’ habitable zones are likely to be tidally locked with orbital periods of the order of tens of days. This means that the effects of rotation on atmospheric dynamics will be relatively weak, which requires small horizontal temperature gradients above the boundary layer of terrestrial atmospheres. An analytically solvable and dynamically consistent model for planetary climate with only three free parameters can be constructed by making the weak temperature gradient (WTG) approximation, which assumes temperatures are horizontally uniform aloft. The extreme numerical efficiency of a WTG model compared to a three-dimensional general circulation model (GCM) makes it an optimal tool for Monte Carlo fits to observables over parameter space. Additionally, such low-order models are critical for developing physical intuition and coupling atmospheric dynamics to models of other components of planetary climate. The objective of this paper is to determine whether a WTG model provides an adequate approximation of the effect of atmospheric dynamics on quantities likely to be observed over the next decade. To do this, we first tune a WTG model to GCM output for an Earth-like tidally locked planet with a dry, 1 bar atmosphere, then generate and compare the expected phase curves of both models. We find that differences between the two models would be extremely difficult to detect from phase curves using the James Webb Space Telescope. This result demonstrates the usefulness of the WTG approximation when used in conjunction with GCMs as part of a modeling hierarchy to understand the climate of remote planets.

Key words: planets and satellites: atmospheres – planets and satellites: detection

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1. INTRODUCTION

In recent years the number of known exoplanets has grown from only a handful of gas giants to over a thousand planets, including several roughly Earth-sized rocky planets. There are several ongoing and planned surveys searching nearby stars for planets including small, Earth-like habitable zone planets around M dwarf stars, such as MEarth (Berta et al. 2013), APACHE (Giacobbe et al. 2012), CARMENES (Quirrenbach et al. 2012), and NGTS (Wheatley et al. 2013). This is important because M dwarfs represent 75% of the stars in the galaxy and their planets are likely to be the most easily detectable Earth analogs. Planets near the habitable zone, with temperatures moderate enough to maintain liquid water, are of particular interest because water is essential for life on Earth.

It is likely that planets in the habitable zone of M dwarf stars will be in spin-orbit resonance, usually in the “tidally locked” configuration with one side always facing the star (Kasting et al. 1993). Climate modeling studies have shown that a tidally locked planet with an atmosphere only 10% of the mass of Earth’s atmosphere is capable of transporting enough heat to the cold nightside to prevent atmospheric collapse by condensation there (Joshi et al. 1997; Joshi 2003; Tarter et al. 2007; Scalo et al. 2007). Understanding the characteristics of tidally locked, rocky planet atmospheres is therefore of great interest. Observational techniques have been proposed to probe the climate and structure of such planets both by photometry (e.g., Seager & Deming 2009; Cowan et al. 2012a; Cowan et al. 2012b; Cowan et al. 2012c; Fujii & Kawahara 2012) and low resolution spectroscopy (e.g., Beichman 1998; Fridlund 2000; Selsis et al. 2011).

Tidally locked planets in the habitable zone of M dwarfs should tend to have a relatively long rotation period (typically tens of days) and therefore a weak Coriolis force. An analogous situation arises in the tropics of rapidly rotating planets, such as Earth, because the horizontal component of the Coriolis force vector is small there. When the Coriolis force is weak (Rossby Number, \( R_o \gtrsim 1 \)), advection balances the pressure gradient. For typical wind speeds this allows only a very weak pressure gradient and therefore temperature gradient (Charney 1963) and we may reasonably approximate horizontal atmospheric temperature gradients as zero (uniform temperature) above the boundary layer, the lower region of the atmosphere where frictional forces are important (Pierrehumbert 1995). This situation is referred to as the “weak temperature gradient” (WTG) approximation (Sobel et al. 2001), and it is brought about by gravity waves. The WTG approximation is therefore only valid if the timescale for gravity wave propagation around a planet is short compared to the radiative timescale, which excludes its application to extremely hot and/or thin atmospheres (Perez-Becker & Showman 2013; Showman et al. 2013). Three-dimensional general circulation model (GCM) simulations suggest that the WTG approximation is a useful guiding principle for tidally locked habitable zone planets (Merlis & Schneider 2010), which leads to greatly simplified models of the atmospheric dynamics of such planets (Pierrehumbert 2011) that can be coupled to models of other critical processes for planetary climate (Kite et al. 2011).

In addition to elucidating relevant physics, a WTG model can be run millions of times extremely quickly and could therefore be used in conjunction with observations to constrain atmospheric parameters. There are two main advantages to fitting a WTG model to data as opposed to a simple phenomenological model (Cowan & Agol 2008; Cowan et al. 2013) or energy balance model (Cowan & Agol 2011; Lewis et al. 2013). First, a
WTG model is based on a dynamically consistent framework so that behavior constrained by the phase curve is guaranteed to satisfy the relevant dynamical equations. In contrast, an energy balance model might, for example, diffuse heat in an unphysical way. Second, as we will explain in Section 2, a WTG model includes a solid surface and solves for the surface temperature, which would be useful for interpreting phase curves of Earth-like planets.

The goal of this paper is to determine whether a WTG model could be feasibly distinguished from a GCM using thermal phase curve photometry, the type of observations relevant to atmospheric dynamics likely to be available over the next decade. To do this we run a GCM at a variety of rotation rates, keeping other parameters constant, then tune a WTG model to the GCM output and compare the thermal phase curves that each would produce when measured by a remote observer. In practice, a WTG model would be tuned to observations and a GCM requires many unknown input parameters that would also need to be tuned to observations, so tuning the WTG model to the GCM is a reasonable methodology. We find that in the regime of dry Earth-like ≈1 bar atmospheres, the phase curve a WTG model produces would be nearly indistinguishable from that a GCM produces for a tidally locked habitable zone planet orbiting a nearby M dwarf star. This suggests that for many situations the WTG approximation is a sufficient description of atmospheric dynamics for such planets. GCMs would remain situations the WTG approximation is a sufficient description of orbiting a nearby M dwarf star. This suggests that for many WTG model produces would be nearly indistinguishable from regim of dry Earth-like to the GCM is a reasonable methodology. We find that in the practice, a WTG model would be tuned to observations and a GCM output and compare the thermal phase curves that keeping other parameters constant, then tune a WTG model to atmospheric dynamics likely to be available over the next phase curve photometry, the type of observations relevant to could be feasibly distinguished from a GCM using thermal

Surface energy balance yields

\[ (1 - \alpha)S[\theta] + e_\alpha \sigma T_s^4 = \sigma T_a^4 + a \cdot (T_a^\ast - T_a), \]

where \( S[\theta] = \{ F_\ast \sin \theta, \quad \forall \theta > 0, \quad \forall \theta < 0 \} \), \( F_\ast \) is the incident stellar flux at the substellar point, \( \alpha \) is the albedo, \( \sigma \) is the Stefan–Boltzmann constant, \( e_\alpha \) is the atmospheric longwave emissivity, and \( a \) is the surface-to-mid-troposphere exchange constant, which implicitly includes sensible heat flow by dry turbulent exchange and atmospheric convection. We have assumed that the atmosphere is transparent to solar radiation, but has some longwave opacity. We take the surface albedo to be a constant and the surface emissivity to be one. We set \( a \) to zero wherever \( T_a > T_s \), since, to a first approximation, stable stratification prevents sensible heat exchange and convection. Finally, the global mean energy balance at the TOA can be written as

\[ \int_{-90}^{90} (1 - \alpha)S[\theta] \cos \theta d\theta = \int_{-90}^{90} \left( (1 - e_\alpha)\sigma T_s^4 + e_\alpha \sigma T_a^4 \right) \cos \theta d\theta. \]

Equations (1) and (2) can be solved for the constant atmospheric temperature \( T_a \) and the surface temperature as a function of theta \((T_a(\theta))\) if \( \alpha \), \( e_\alpha \), and \( a \) are specified. We choose the albedo \((\alpha)\) of the WTG model so that it matches the global mean TOA albedo of the GCM. We then choose \( e_\alpha \) and \( a \) to minimize the least square distance between the WTG model and the GCM for the following parameters: the TOA upward longwave flux at the substellar point, the TOA upward longwave flux averaged over the nightside, and the surface temperature at the substellar point.

3. RESULTS

3.1. Comparison of WTG Model to GCM

Inspection of the GCM results indicates that the WTG approximation is reasonable. For example, when the rotation period is set to five days, the temperature at a given pressure level in the mid-troposphere (≈0.5 bar) varies by less than 10 K around the planet (not shown). As a result, the WTG model can be easily tuned to reproduce the broad features of the GCM (Figure 1), despite the fact that it has only three tunable parameters. The similarity between the longwave fluxes

Note that we assume here that \( F_\ast \) is known. If \( F_\ast \) is not known, then \( F_\ast(1 - \alpha) \) should be treated as a single unknown parameter so that the model still has three tunable parameters.
in the WTG model and the GCM when plotted as a function of the angle from the terminator is striking, and emphasizes the excellent performance of the WTG approximation.

It is useful to confirm that the WTG parameters after the fit are physically reasonable and comparable to those in the GCM. The albedo in the WTG model is set to exactly match the average TOA albedo of the GCM, so there is a direct correspondence between the models. The emissivity, $ea$, is a parameter meant to impart the gross behavior of longwave radiation into the WTG model. The fact that the net “greenhouse forcing,” or difference between surface and TOA longwave fluxes, is similar between the GCM and WTG model (Figure 1) confirms that the fit to $ea$ is reasonable. The final WTG parameter, $a$, the surface-to-mid-troposphere exchange constant, cannot be easily compared to a particular parameter in the GCM because it accounts for the combination surface turbulent exchange and atmospheric convection.

Consistent with Merlis & Schneider (2011), the rotation rate has remarkably little effect on TOA and surface upward longwave flux profiles in the GCM when they are plotted as a function of the angle from the terminator (Figure 2). This is likely due to the fact that even at high (Earth-like) rotation rates, the tropics, which dominate emission, still obey the WTG approximation. Others have observed a dynamical transition when the Rossby radius falls below the planetary radius at rotation periods less than a few days (Edson et al. 2011; Leconte et al. 2013; Yang et al. 2013) that leads to increased equatorial superrotation (positive zonal wind at the equator). We observe this as well, but find that even in this regime the longwave flux profiles do not deviate much from those at low rotation rates (Figure 2).

3.2. Difficulty of Observing Model Differences

Measurement of variations in the disk-integrated broadband thermal phase curve due to a planet’s hot dayside periodically facing toward and away from Earth is the main tool we have to gain insight into the atmospheric dynamics of tidally locked exoplanets. An example of a major success of this technique is its use on the hot Jupiter HD 189733b (Knutson et al. 2007; Knutson et al. 2009) to confirm the prediction (Showman & Guillot 2002) of equatorial superrotation and eastward advection of the hot spot that results from strong heating on the dayside and cooling on the nightside (Showman & Polvani 2011). Although the Earth-like planets under consideration would be far less luminous and emit further in the infrared, their phase curves would still likely be observable sufficiently close to Earth ($\lesssim$20 pc) with next generation telescopes (Selsis et al. 2011; Cowan et al. 2012b; Yang et al. 2013).

As a test of the WTG approximation, we compare the thermal phase curves that the GCM and WTG model would produce for a distant observer using the methodology of Yang et al. (2013), considering different observer inclination angles. We calculate the phase curve as the variation in total thermal flux that would be observed as the planet orbits the star assuming a stellar radius $R_*$ = 0.2 $R_\odot$, a stellar temperature $T_*$ = 3000 K, a planetary radius $R_p = R_\oplus$, and a planetary emission temperature $T_{ef} = 240$ K. We assume that the Mid-Infrared Instrument (MIRI) on the James Webb Space Telescope (JWST) will be the most advantageous instrument in the near-term, and assume that we could integrate the thermal flux from 10 to 28 $\mu$m using this instrument.

In order to calculate the expected precision, we scale the photometric precision of $4 \times 10^{-5}$ obtained by Knutson et al. (2012) using the Spitzer Space Telescope, which is about 50% larger than the photon noise. Phase curve measurements will be easiest to interpret for a transiting planet, so we assume a distance appropriate for the nearest transiting tidally locked habitable zone terrestrial planet. A conservative estimate of 0.04 habitable zone terrestrial planets per M dwarf yields an expected distance of 20 pc, which we will adopt as our fiducial value. We also consider distances of 5 pc and 10 pc.
Figure 3. Comparison of the predicted phase curves of the GCM run with a 10 day orbital period and the WTG model. The horizontal axis is the phase (360 × [time]/[Orbital Period]). The vertical axis is the deviation (in ppm) of the total (including both star and planet) infrared flux from its mean value. The error bar represents the approximate expected precision of a one-day integration using the MIRI instrument on JWST for an Earth-sized planet at a distance of 20 pc. Differences between the WTG model and the GCM would not be detectable at this distance.

(A color version of this figure is available in the online journal.)

Figure 4. Maximum difference between phase curves that the WTG model and the GCM would produce assuming an Earth-sized planet orbiting an M star with radius 0.2 \(R_\odot\). The difference is plotted in ppm and should be compared to an instrumental precision for one-day integrations using the MIRI instrument on the JWST of 129 ppm if the system is at a distance of 20 pc, 65 ppm at 10 pc, and 32 ppm at 5 pc.

(A color version of this figure is available in the online journal.)

because different estimates of planetary frequency (Dressing & Charbonneau 2013; Morton & Swift 2013) and habitable zone width (Kopparapu 2013; Yang et al. 2013) yield 0.5–1 habitable zone terrestrial planets per M dwarf. Finally, we assume an integration time of one day.

Assuming a transiting planet at 20 pc yields a precision of 129 ppm, at which the phase variations of a dry Earth-sized terrestrial planet with a 10 day orbital period should be observable at 7-\(\sigma\) precision (Figure 3). The maximum difference between the WTG model simulation and the GCM simulation would be entirely undetectable at this distance (Figure 3). Even if the planet were at 5 pc (32 ppm precision), the maximum difference between the WTG model and GCM would still probably be undetectable (2.5-\(\sigma\)). Given that all detection significances would increase linearly with planetary radius, the difference between the WTG model and the GCM would only start to be detectable for a large super-Earth at a distance of 5 pc (assuming that this difference does not scale strongly with planetary radius and that the atmosphere is not extremely thin).

When we vary the planetary rotation rate and inclination angle, we find that the maximum difference between the WTG model and the GCM would generally not be detectable (Figure 4). The most important exception is for relatively low rotation rates (long period orbits) in near transiting configuration. The difference between the WTG model and the GCM might be detectable for these planets, particularly if they are at distances of less than 10 pc. It is interesting that the WTG gradient approximation actually becomes less effective at lower rotation rates. This is due to the sharper profiles of infrared emission to space as a function of angle from the terminator at lower rotation rates (Figure 2), which are hard for the WTG model to fit.

4. DISCUSSION AND CONCLUSIONS

We find that a WTG model provides a very good approximation of the atmospheric dynamics of a tidally locked terrestrial planet orbiting an M dwarf. If we assume that the planet is (1) dry, i.e., that the effects of moisture and clouds on the infrared emission to space are small, and (2) the atmosphere is sufficiently thick, we find that a WTG model could not easily be distinguished observationally from a full three-dimensional GCM calculation of the atmospheric dynamics using the MIRI instrument on the JWST and assuming the planet orbits a nearby M dwarf star. Although we did tune the WTG model to the GCM, the fact that this is possible with only three WTG model parameters indicates that the WTG approximation is an
acceptable first approximation for understanding dry Earth-like M dwarf planets. It is reasonable to expect that many planets in or near the habitable zone of M dwarfs will be fairly dry due to reduced volatile delivery (Raymond et al. 2007) and nightside ice trapping (Menou 2013). If a measured thermal phase curve is consistent with clouds having a minor or no effect, then a Monte Carlo fit of the three-parameter WTG model to the observed thermal phase curve should provide a good description of the dynamical behavior of the atmosphere.

Of course, the habitable zone planets that will be of most interest are the ones that actually have liquid water at the surface on the dayside in at least some regions, and will therefore tend to have clouds. In this case, interpretation of the phase curve would require some understanding of the cloud behavior (Yang et al. 2013). Since we have shown that the WTG approximation is a good approximation of the atmospheric dynamics, it should be possible to construct a version of the WTG model incorporating cloud effects with only one or two extra parameters that could be fit to observed phase curves. In such cases it would be beneficial to also run a GCM to confirm that implied cloud behavior is reasonable, which demonstrates the value of using a hierarchy of climate models to understand the climate of exoplanets.

Aside from the direct implications of using WTG models to decipher the basics of atmospheric dynamics and planetary climate from thermal phase curves, it is important to emphasize that the fact that this works reasonably well justifies the use of the WTG approximation in theoretical studies, e.g., by Pierrehumbert (2011) and Kite et al. (2011). This is extremely beneficial because of the physical insight that the WTG approximation can provide into the problem of understanding the climate of tidally locked terrestrial planets.

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