We present a new type of probabilistic model which we call DISsimilarity COefficient Networks (DISCO Nets). DISCO Nets allow us to efficiently sample from a posterior distribution parametrised by a neural network. During training, DISCO Nets are learned by minimising the dissimilarity coefficient between the true distribution and the estimated distribution. This allows us to tailor the training to the loss related to the task at hand. We empirically show that (i) by modeling uncertainty on the output value, DISCO Nets outperform equivalent non-probabilistic predictive networks and (ii) DISCO Nets accurately model the uncertainty of the output, outperforming existing probabilistic models based on deep neural networks.

1 Introduction

We are interested in the class of problems that require the prediction of a structured output \( y \in \mathcal{Y} \) given an input \( x \in \mathcal{X} \). Complex applications often have large uncertainty on the correct value of \( y \). For example, consider the task of hand pose estimation from depth images, where one wants to accurately estimate the pose \( y \) of a hand given a depth image \( x \). The depth image often has some occlusions and missing depth values and this results in some uncertainty on the pose of the hand. It is, therefore, natural to use probabilistic models that are capable of representing this uncertainty. Often, the capacity of the model is restricted and cannot represent the true distribution perfectly. In this case, the choice of the learning objective influences final performance. Similar to Lacoste-Julien et al. [11], we argue that the learning objective should be tailored to the evaluation loss in order to obtain the best performance with respect to this loss. In details, we denote by \( \Delta_{\text{training}} \) the loss function employed during model training, and by \( \Delta_{\text{task}} \) the loss employed to evaluate the model’s performance.

We present a simple example to illustrate the point made above. We consider a training dataset \( D = \{x_n, n = 1..N\} \) sampled from a mixture of two bidimensional Gaussian. We train two probabilistic models, Model A and Model B, to capture the data probability distribution. Each model is able to represent a bidimensional Gaussian distribution with diagonal covariance parametrised by \( (\mu_1, \mu_2, \sigma_1, \sigma_2) \). Each model is trained by minimising its probabilistic objective function, defined by the training loss \( \Delta_{\text{training}} \). We refer the reader to the appendix for details on the objective function.

For \( x = (x_1, x_2), x' = (x'_1, x'_2) \in \mathbb{R}^2 \), the training loss of Model A emphasises the first dimension of the data, that is, \( \Delta_A(x - x') = (10 \times (x_1 - x'_1)^2 + 0.1 \times (x_2 - x'_2)^2)^{1/2} \). In the case of Model B this is the opposite, that is, \( \Delta_B(x - x') = (0.1 \times (x_1 - x'_1)^2 + 10 \times (x_2 - x'_2)^2)^{1/2} \). None of the models will be able to recover the true data distribution since they do not have the ability to represent a mixture of Gaussian. In other words, we cannot avoid model error, similarly to the real data scenario. Each model performs a grid search over the best parameters values for \( (\mu_1, \mu_2, \sigma_1, \sigma_2) \). Figure [1] shows the contours of the Mixture of Gaussian distribution of the data (in black), and the contour of the Gaussian fitted by each model (in red and green). Detailed setting of this example is available in the appendix.
As expected, the fitted Gaussian distributions differ according to $\Delta_{\text{training}}$ employed. Table 1 shows that the loss on the test set, evaluated with $\Delta_{\text{task}}$, is minimised if $\Delta_{\text{training}} = \Delta_{\text{task}}$. This simple example illustrates the advantage to being able to tailor the model’s training objective function to have $\Delta_{\text{training}} = \Delta_{\text{task}}$. This is in contrast to the commonly employed learning objectives we present in Section 2, that are agnostic of the evaluation loss.

In order to alleviate the aforementioned deficiency of the state-of-the-art, we introduce DISCO Nets, a new class of probabilistic model. DISCO Nets represent $P_T$, the true posterior distribution of the data, with a distribution $P_G$ parametrised by a neural network. We design a learning objective based on a dissimilarity coefficient between $P_T$ and $P_G$. The dissimilarity coefficient we employ was first introduced by Rao [21] and is defined for any non-negative symmetric loss function. Thus, any loss function can be incorporated in our setting, allowing the user to tailor DISCO Nets to his or her needs. Finally, contrarily to existing probabilistic models presented in Section 2, DISCO Nets do not require any specific architecture or training procedure, making them an efficient and easy-to-use class of model.

2 Related Work

Deep neural networks, and in particular, Convolutional Neural Networks (CNNs) are comprised of several convolutional layers, followed by one or more fully connected (dense) layers, interleaved by non-linear function(s) and (optionally) pooling. Recent probabilistic models use CNNs to represent non-linear functions of the data. We observe that such models separate into two types. The first type of model does not explicitly compute the probability distribution of interest. Rather, these models allow the user to sample from this distribution by feeding the CNN with some noise $z$. Among such models, Generative Adversarial Networks (GAN) presented in Goodfellow et al. [6] are very popular and have been used in several computer vision applications, for example in Denton et al. [1], Radford et al. [20], Springenberg [24] and Yan et al. [27]. A GAN model consists of two networks, simultaneously trained in an adversarial manner. A generative model, referred as the Generator $G$, is trained to replicate the data from noise, while an adversarial discriminative model, referred as the Discriminator $D$, is trained to identify whether a sample comes from the true data or from $G$. The GAN training objective is based on a minimax game between the two networks and approximately optimizes a Jensen-Shannon divergence. However, as mentioned in Goodfellow et al. [6] and Radford et al. [20], GAN models require very careful design of the networks’ architecture. Their training procedure is tedious and tends to oscillate. GAN models have been generalized to conditional GAN (cGAN) in Mirza and Osindero [15], where some additional input information can be fed to the Generator and the Discriminator. For example in Mirza and Osindero [15] a cGAN model generates tags corresponding to an image. Gauthier [4] applies cGAN to face generation. Reed et al. [22] propose to generate images of flowers with a cGAN model, where the conditional information is a word description of the flower to generate. While the application of cGAN is very promising, little quantitative evaluation has been done. Furthermore, cGAN models suffer from the same difficulties we mentioned for non-conditional GAN. Another line of work has developed towards the use of statistical hypothesis testing to learn probabilistic models. The works presented in Dziugaite et al. [2] and Li et al. [13] propose to train generative deep networks with an objective function based on the Maximum Mean Discrepancy (MMD) method. The MMD method, from Gretton et al. [7, 8], is a statistical hypothesis testing method to assess if two probabilistic

---

1At the time writing, we do not have access to the full paper of Reed et al. [22] and therefore cannot take advantage of this work in our experimental comparison.
distributions are similar. As mentioned in Dziugaite et al. [2], the MMD test can be seen as playing the role of an adversary.

The second type of model approximates intractable posterior distributions with use of variational inference. The Variational Auto-Encoders (VAE) presented in Kingma and Welling [9] is composed of a probabilistic encoder and a probabilistic decoder. The probabilistic encoder is fed with the input $x \in \mathcal{X}$ and produces a posterior distribution $P(z|x)$ over the possible values of noise $z$ that could have generated $x$. The probabilistic decoder learns to map the noise $z$ back to the data space $\mathcal{X}$. The training of VAE uses an objective function based on a Kullback-Leibler Divergence. VAE and GAN models have been combined in Makhzani et al. [14], where the authors propose to regularise autoencoders with an adversarial network. The adversarial network ensures that the posterior distribution $P(z|x)$ matches an arbitrary prior $P(z)$.

In hand pose estimation, imagine the user wants to obtain accurate positions of the thumb and the index finger but does not need accurate locations of the other fingers. The task loss $\Delta_{\text{task}}$ might be based on a weighted L2-norm between the predicted and the ground-truth poses, with high weights on the thumb and the index. Existing probabilistic models cannot be tailored to task-specific losses and we propose the DISsimilarity COefficient Networks (DISCO Nets) to alleviate this deficiency.

3 DISCO Nets

We begin the description of our model by specifying how it can be used to generate samples from the posterior distribution, and how the samples can in turn be employed to provide a pointwise estimate. In the subsequent subsection, we describe how to estimate the parameters of the model using a training data set.

3.1 Prediction

**Sampling.** A DISCO Net consists of several convolutional and dense layers (interleaved by non-linear function(s) and possibly pooling) and takes as input a pair $(x, z) \in \mathcal{X} \times \mathcal{Z}$, where $x$ is input data and $z$ is some random noise. Given one pair $(x, z)$, the DISCO Net produces a value for the output $y$. In the example of hand pose estimation, the input depth image $x$ is fed to the convolutional layers. The output of the last convolutional layer is flattened and concatenated with a noise sample $z$. The resulting vector is fed to several dense layers, and the last dense layer outputs a pose $y$. From a single depth image $x$, by using different noise samples, the DISCO Net produces different pose candidates for the depth image. This process is illustrated in Figure 2. Importantly, DISCO Nets are flexible in the choice of the architecture. For example, the noise could be concatenated at any stage of the network, including at the start.

![Figure 2: For a single depth image $x$, using 3 different noise samples $(z_1, z_2, z_3)$, DISCO Nets output 3 different candidate poses $(y_1, y_2, y_3)$ (shown superimposed on the depth image). The depth image is from the NYU Hand Pose Dataset of Tompson et al. [26], preprocessed as in Oberweger et al. [16]. Best viewed in color.](image)

We denote $P_G$ the distribution that is parametrised by the DISCO Net’s neural network. For a given input $x$, DISCO Nets provide the user with samples $y$ drawn from $P_G(y|x)$ without requiring the expensive computation of (often intractable) partition function. In the remainder of the paper we consider $x \in \mathbb{R}^{d_x}$, $y \in \mathbb{R}^{d_y}$ and $z \in \mathbb{R}^{d_z}$.

**Pointwise Prediction.** In order to obtain a single prediction $y$ for a given input $x$, DISCO Nets use the principle of Maximum Expected Utility (MEU), similarly to Premachandran et al. [19].
The prediction $y_{\Delta_{task}}$ maximises the expected utility, or rather minimises the expected task-specific loss $\Delta_{task}$, estimated using the sampled candidates. Formally, the prediction is made as follows:

$$y_{\Delta_{task}} = \arg\max_k EU(y_k) = \arg\min_k \sum_{k'=1}^{K} \Delta_{task}(y_k, y_{k'}')$$ (1)

where $(y_1, ..., y_K)$ are the candidate outputs sampled for the single input $x$. Details on the MEU method are in the appendix.

### 3.2 Learning DISCO Nets

**Objective Function.** We want DISCO Nets to accurately model, with $P_G(y|x)$, the true probability of the data $P_T(y|x)$. In other words, $P_G(y|x)$ should be as similar as possible to $P_T(y|x)$. This similarity should be evaluated with respect to the loss specific to the task at hand. Thus, given any non-negative symmetric loss function between two outputs $\Delta(y, y')$ with $(y, y') \in Y \times Y$, we employ a diversity coefficient that is the expected loss between two samples drawn randomly from the two distributions. Formally, the diversity coefficient is defined as:

$$\text{DIV}_{\Delta}(P_T, P_G) = \sum_{x \in X} \sum_{y \in Y} \sum_{y' \in Y} \Delta(y, y')P_T(y|x)P_G(y'|x)P_T(x)dydy' dx$$ (2)

Intuitively, we should minimise this diversity so that $P_G(y|x)$ is as similar as possible to $P_T(y|x)$. However there is uncertainty on the output $y$ to predict for a given $x$. In other words, $P_T(y|x)$ is diverse and $P_G(y|x)$ should be diverse as well. Thus we encourage $P_G(y|x)$ to provide sample outputs, for a given $x$, that are different from each other by minimising the following dissimilarity coefficient:

$$\text{DISC}_{\Delta}(P_T, P_G) = \text{DIV}_{\Delta}(P_T, P_G) - \gamma \text{DIV}_{\gamma\Delta}(P_T, P_G)$$ (3)\[\gamma \in [0, 1]

The dissimilarity $\text{DISC}_{\Delta}(P_T, P_G)$ is the difference between the diversity between $P_T$ and $P_G$, and an affine combination of the diversity of each distribution. These coefficients were introduced by Rao $[2]$ with $\gamma = \frac{1}{2}$ and used for latent variable models in Kumar et al. $[10]$. We do not need to consider the term $\text{DIV}_{\Delta}(P_T, P_T)$ as it is a constant in our problem, and thus DISCO Nets’ objective function is defined as follows:

$$F = \text{DIV}_{\Delta}(P_T, P_G) - \gamma \text{DIV}_{\gamma\Delta}(P_T, P_G)$$ (4)

When minimising $F$, the term $\gamma \text{DIV}_{\gamma\Delta}(P_T, P_G)$ encourages $P_G(y|x)$ to be diverse. The value of $\gamma$ balances between the two goals of $P_G(y|x)$, that are providing accurate outputs while being diverse.

**Optimisation.** Let us consider a training dataset composed of $N$ examples input-output pairs $D = \{(x_n, y_n), n = 1...N\}$. In order to train DISCO Nets, we need to compute the objective function $[4]$. We do not have access to the true probability distributions $P_T(y, x)$ and $P_T(x)$. To overcome this deficiency, we construct estimators of each diversity term $\text{DIV}_{\Delta}(P_T, P_G)$ and $\text{DIV}_{\gamma\Delta}(P_T, P_G)$. First, we take an empirical distribution of the data, that is, taking ground-truth pairs $(x_n, y_n)$. We then estimate each distribution $P_G(y|x_n)$ by sampling $K$ outputs from our model for each $x_n$. This gives us an unbiased estimate of each diversity term, defined as:

$$\hat{\text{DIV}}_{\Delta}(P_T, P_G) = \frac{1}{N} \sum_{n=1}^{N} \frac{1}{K} \sum_{k=1}^{K} \Delta(y_n, G(z_k, x_n; \theta))$$

$$\hat{\text{DIV}}_{\gamma\Delta}(P_T, P_G) = \frac{1}{N} \sum_{n=1}^{N} \frac{1}{K(K-1)} \sum_{k=1}^{K} \sum_{k' \neq k}^{K} \Delta(G(z_k, x_n; \theta), G(z_{k'}, x_n; \theta))$$ (5)

To summarise, we have an unbiased estimate of the DISCO Nets’ objective function of equation [4] defined as:

$$\hat{F}(\Delta, \theta) = \hat{\text{DIV}}_{\Delta}(P_T, P_G) - \gamma \hat{\text{DIV}}_{\gamma\Delta}(P_T, P_G)$$ (6)

where $y_n = G(z_k, x_n; \theta)$ is a candidate output sampled from DISCO Nets for $(x_n, z_k)$, and $\theta$ are the parameters of DISCO Nets. It is important to note that the second term of equation $[6]$ is summing over $k$ and $k' \neq k$ since $G(z_k, x_n; \theta)$ and $G(z_{k'}, x_n; \theta)$ must be two independent samples. The parameters $\theta$ are learned by Gradient Descent. Algorithm $[4]$ shows the training of DISCO Nets. In steps 4 and 5 of Algorithm $[4]$ we draw $K$ random noise vectors $(z_{n1}, ..., z_{nk})$ per input example $x_n$, and generate $K$ candidate outputs $G(z_{nk}, x_n; \theta)$ for this input. This allows us to compute an unbiased estimate of the gradient in step 7. For clarity, in the remainder of the paper we do not explicitly write the parameters $\theta$ and write $G(z_k, x_n)$. 


Generate \( K \) candidate outputs for updating parameters end

Sample minibatch of \( b \) training example pairs \( \{(x_1, y_1), \ldots, (x_b, y_b)\} \).

for \( n=1 \ldots b \) do
  Sample \( K \) random noise vectors \( (z_{n,1}, \ldots, z_{n,K}) \) for training example \( x_n \)
  Generate \( K \) candidate outputs \( G(z_{n,k}, x_n, \theta), k = 1 \ldots K \) for training example \( x_n \)
end

Update parameters \( \theta^t \leftarrow \theta^{t-1} \) by descending the gradient of equation \( (6) \): \( \nabla_{\theta} \hat{F}(\Delta, \theta) \).

Algorithm 1: DISCO Nets Training algorithm.

### Proper Scoring Rule.

Any non-negative symmetric loss function \( \Delta \) can be incorporated in the objective function of DISCO Nets. However, if the dissimilarity coefficient of equation \( (4) \) is a strictly proper scoring rule as defined in Gneiting and Raftery \( [5] \), it is ensured to be minimized only when \( P_G(y|x) \) is the true conditional distribution \( P_T(y|x) \). By theorem 5 in Gneiting and Raftery \( [5] \), this is the case, for example, if we take as loss function \( \Delta_\beta(y, y') = ||y - y'||_2^\beta = (\sum_{i=1}^n (y_i - y'_i)^2)^{\beta/2} \) with \( \beta \in [0, 2] \) excluding 0 and 2, and use \( \gamma = \frac{1}{2} \). In this setting, our training objective boils down to:

\[
\hat{F}(\Delta, \theta) = \frac{1}{N} \sum_{n=1}^N \left[ \frac{1}{K} \sum_k ||y_n - G(z_k, x_n)||_2^\beta - \frac{1}{2} \frac{1}{K(K-1)} \sum_k \sum_{k' \neq k} ||G(z_{k'}, x_n) - G(z_k, x_n)||_2^\beta \right]
\]

This specific case of our objective function is related to the Maximum Mean Discrepancy method (MMD) of Gretton et al. \( [7, 8] \). Indeed as shown in Proposition 3 of Schölkopf \( [23] \), the function \( k : \mathcal{Y} \times \mathcal{Y} \to \mathbb{R} \) defined as \( k(y, y') = ||y - y'||_2^\beta \) for \( 0 < \beta \leq 2 \) is a conditionally positive definite kernel, that is a specific type of positive definite kernel. It would be interesting to see if the properties of the MMD estimator extend to conditionally positive definite kernels and to our setting where the posterior distribution is conditioned on the input \( x \).

### 4 Experiments: Hand Pose Estimation

Given a depth image \( x \), which often contains occlusions and missing values, we wish to predict the hand pose \( y \). We use the NYU Hand Pose dataset of Tompson et al. \( [26] \) to estimate the efficiency of DISCO Nets for this task.

#### 4.1 Experimental Setup

**NYU Hand Pose Dataset.** The NYU Hand pose dataset of Tompson et al. \( [26] \) contains 8252 testing and 72,757 training frames of captured RGBD data with ground-truth hand pose information. The training set is composed of images of one person whereas the testing set gathers samples from two persons. For each frame, the RGBD data from 3 Kinects is provided: a frontal view and 2 side views. In our experiments we use only the depth data from the frontal view. While the ground truth contains \( J = 36 \) annotated joints, we follow the evaluation protocol of Oberweger et al. \( [16, 17] \) and use the same subset of \( J = 14 \) joints. We also perform the same data preprocessing as in Oberweger et al. \( [16, 17] \), and extract a fixed-size metric cube around the hand from the depth image. We resize the depth values within the cube to a \( 128 \times 128 \) patch and normalized them in \([−1, 1]\). Pixels deeper than the back of the cube and missing depth values are both set to a depth of 1.

**Methods.** We employ loss functions between two outputs of the form used in equation \( (7) \), that is, \( \Delta_{\text{training}} = \Delta_\beta(y, y') = ||y - y'||_2^\beta \). With this loss the dissimilarity coefficient is a strictly proper scoring rule when \( \gamma = \frac{1}{2} \). Our first goal is to assess the advantages of DISCO Nets with respect to non-probabilistic deep networks. To do so, we compare two models. One model, referred as DISCO\(_{\beta, \gamma} \), is a DISCO Nets probabilistic model, with \( \gamma \neq 0 \) in the dissimilarity coefficient of equation \( (6) \). The other model, referred as BASE\(_\beta \), is a non-probabilistic model, by taking \( \gamma = 0 \) in the objective function of equation \( (6) \) and no noise is concatenated. This corresponds to a classic deep network which for a given input \( x \) generates a single output \( y = G(x) \). Note that we write \( G(x) \) and not \( G(z, x) \) since no noise is concatenated.

**Evaluation Metrics.** We use the metrics of Table \( [3] \) These metrics include classic non-probabilistic metrics for hand pose estimation employed in Oberweger et al. \( [16, 17] \) and Taylor et al. \( [25] \), that
Table 2: Evaluation metrics. $N$ is the number of testing example pairs $(x_n, y_n)$ and $y_{\Delta_{\text{base}}.n}$ is the prediction, specific to the metric, for the $n^{th}$ input example $x_n$. $J$ is the number of joints of the hand.

| Shorthand & Definition | $\Delta_{\text{metrics}}$ | Formula |
|-----------------------|--------------------------|---------|
| ProbLoss: Probabilistic Loss of $K$ sampled poses. | $\Delta_{\text{ProbLoss}} = \sum_{j=1}^{J} ||y_{\Delta}^j||^2_{\text{joint}}$ | $\bar{F}(\Delta_{\text{ProbLoss}}, \theta)$ with $\gamma = 0.5$ |
| MeJEE (Mean Joint Euclidean Error) : per-joint Euclidean distance between the pointwise pose and the ground-truth, averaged by $J$ and $N$. | $\Delta_{\text{MeJEE}} = \frac{1}{J} \sum_{j=1}^{J} ||y_{\Delta}^j||^2_{\text{joint}}$ | $\frac{1}{N} \sum_{n=1}^{N} \frac{1}{J} \sum_{j=1}^{J} ||y_{\Delta}^j - y_{\Delta_{\text{base}}.n}||^2_{\text{joint}}$ |
| MaJEE (Max Joint Euclidean Error) per-joint maximal Euclidean distance between the pointwise pose and the ground-truth averaged by $N$. | $\Delta_{\text{MaJEE}} = \max_{j \in \{1, J\}} ||y_{\Delta}^j||^2_{\text{joint}}$ | $\frac{1}{N} \sum_{n=1}^{N} \max_{j \in \{1, J\}} ||y_{\Delta}^j - y_{\Delta_{\text{base}}.n}||^2_{\text{joint}}$ |
| FF($d$) (Fraction of Frames) : fraction of test examples that have all predicted joints of the pointwise pose below a given maximum Euclidean distance $d$ in mm from the ground-truth. | $\Delta_{\text{FF}} = 1 - \frac{1}{\sum_{n=1}^{N} \sum_{j \in \{1, J\}} ||y_{\Delta}^j - y_{\Delta_{\text{base}}.n}||^2}_{\text{joint}} \leq d$ | (note the minus sign since we want to maximise FF) |

are, the Mean Joint Euclidean Error (MeJEE), the Max Joint Euclidean Error (MaJEE) and the Fraction of Frames within distance (FF). Oberweger et al. [15, 17] present the metric FF as the most challenging metric for hand pose estimation. These metrics use losses based on the norm $||.||^2_{\text{joint}}$, that is the Euclidean distance between the prediction and the ground-truth for the joint $j$. They require a single pointwise prediction referred as $y_{\Delta}$, where $\Delta$ is specific to each metric. The pointwise $y_{\Delta}$ is chosen with the MEU method among the $K$ candidates. For the non-probabilistic model $\text{BASE}_{\beta,\gamma}$, only a single pointwise predicted output $y$ is available. Thus we artificially construct the $K$ candidates by adding some Gaussian random noise $\mathbf{F}(\mathbf{F}(\mathbf{x}))$ of mean 0 and diagonal covariance $\Sigma = \sigma I$. We use $\sigma \in \{1\text{mm}, 5\text{mm}, 10\text{mm}\}$ and refer to the model as $\text{BASE}_{\beta,\sigma}$. We added a probabilistic metric that we call ProbLoss, evaluated on $K$ candidate poses for a given depth image. As observed in Fukumizu et al. [3], kernel density estimation fails in this scenario due to the high dimensionality of $y$.

Architecture. The novelty of DISCO Nets resides in their objective function. They do not require the use of a specific network architecture. This allows us to design a simple network architecture inspired by Oberweger et al. [17]. The architecture is shown in Figure 2. The input depth image $x$ is fed to 2 convolutional layers, each having 8 filters, with kernels of size $5 \times 5$, with stride 1, followed by Rectified Linear Units (ReLU) and Max Pooling layers of kernel size $3 \times 3$. A third and last convolutional layer has 8 filters, with kernels of size $5 \times 5$, with stride 1, followed by a Rectified Linear Unit. The output of the convolution is concatenated to the random noise vector $z$ of size $d_z = 200$, drawn from a uniform distribution in $[-1, 1]$. The result of the concatenation is fed to 2 dense layers of output size 1024, with ReLUs, and a third dense layer that outputs the candidate pose $y \in \mathbb{R}^{3 \times J}$. For the non-probabilistic $\text{BASE}_{\beta,\sigma}$ model no noise is concatenated as only a pointwise estimate is produced.

Training. For DISCO$_{\beta,\gamma}$, we used $\gamma = 0.5$. We use $\beta = 1$ in the loss function $\Delta_{\text{training}}(y, y') = ||y - y'||^2_{\text{base}}$, that is the Euclidean distance. This is a relevant choice to hand pose estimation given the evaluation metrics employed. We use 10,000 examples from the 72,757 training frames to construct a validation dataset and train only on 62,757 examples. Back-propagation is used with Stochastic Gradient Descent with a batchsize of 256. The learning rate is fixed to $\lambda = 0.01$ and we use a momentum of $m = 0.9$ (see Polyak [18]). We also add L2-regularisation controlled by the parameter $C$. We use $C = [0.0001, 0.001, 0.01]$ which is a relevant range as the comparative model $\text{BASE}_{\beta}$ is best performing for $C = 0.001$. Note that DISCO Nets report consistent performances across the different values of $C$, contrarily to $\text{BASE}_{\beta}$. We use 3 different random seeds to initialize each model network parameters. We report the performance of each model with its best cross-validated seed and $C$. We train all models for 400 epochs as it results in a change of less than 3% in the value of the loss on the validation dataset for $\text{BASE}_{\beta}$. Each epoch takes $\sim 20$ seconds including monitoring of the loss on the validation set, that is more than 3000 training example frame per second on a single GPU NVIDIA-Titan X. We refer the reader to the appendix for details on the setting and the results. During training, $K = 2$ in Algorithm 1 and $K = 100$ for metrics evaluation.

---

2We also evaluate the non-probabilistic model $\text{BASE}_{\beta}$ using its pointwise prediction rather than the MEU method. Results are consistent and detailed in the appendix.
4.2 The Advantage of DISCO Nets over Non-Probabilistic Networks

Quantitative Evaluation. Table 3 reports performances on the test dataset, with parameters cross-validated using the validation set, of the two models we compare. We can see that our probabilistic model DISCO $\beta=1, \gamma=0.5$ outperforms the non-probabilistic models BASE $\beta=1, \sigma$ on all metrics. This confirms that by accurately modeling the uncertainty on the pose to output given the input depth image, DISCO Nets provide better prediction. Details in appendix show that the DISCO $\beta=1, \gamma=0.5$ is best-performing on all metrics for all values of $C$.

Qualitative Evaluation. In Figure 3, we show candidate poses generated by DISCO $\beta=1, \gamma=0.5$ for 3 testing examples. The top image shows the input depth image, and the bottom image shows the ground-truth pose (in grey) with 100 candidate outputs (superimposed in transparent red). The model predict the joint locations and we interpolate the joints with edges. If an edge is thinner and more opaque, it means the different predictions overlap and that the uncertainty on the location of the edge’s joints is low. We can see that DISCO $\beta=1, \gamma=0.5$ is able to capture relevant information on the structure of the hand.

(a) When there are no occlusions, DISCO Nets model low uncertainty on all joints. (b) When the hand is half-fisted, DISCO Nets model the uncertainty on the location of the fingertips. (c) Here the fingertips of all fingers but the forefinger are occluded and DISCO Nets model high uncertainty on them.

Figure 3: Visualisation of DISCO $\beta=1, \gamma=0.5$ predictions for 3 examples from the testing dataset. The top image shows the input depth image, and the bottom image shows the ground-truth pose in grey with 100 candidate outputs superimposed in transparent red. Best viewed in color.

4.3 Comparison with existing probabilistic models.

We consider the applications of the conditional Generative Adversarial Networks (cGAN) model from Mirza and Osindero [15] and to the best of our knowledge cGAN has not been applied to pose estimation. In order to compare cGAN to DISCO Nets on hand pose estimation, several issues must be overcome. First, we must design a network architecture for the Discriminator. This is a first disadvantage of cGAN compared to DISCO Nets which require no adversary. Second, as mentioned in Goodfellow et al. [6] and Radford et al. [20], GAN (and thus cGAN) require very careful design of the networks’ architecture and training procedure. In order to do a fair comparison, we followed the work in Mirza and Osindero [15] and practical advice for GAN presented in Larsen and Sønderby [12]. The Generator G has the same network architecture as DISCO $\beta=1, \gamma=0.5$. We add batch normalization to help the training. The Discriminator D is composed a convolutional part that also take a depth image $x$ as input. This part is the same as the convolutional part of G. The output of the convolutions is concatenated either the ground-truth pose for $x$ or the pose generated by G and fed to 2 dense layers with 200 hidden units. Dense layers apply dropout with a ratio of 0.5 and ReLUs. We use Maxout activation after the second dense layer. A third dense layer outputs a scalar value, on which we apply a sigmoid activation function. We try (i) cGAN, initialising all layers of D and G randomly, and (ii) cGAN$_{Init, fixed}$ initialising the convolutional layers of G and D.

The model ProbLoss (mm) MeJEE (mm) MaJEE (mm) FF (80mm)
BASE $\beta=1, \sigma=1$
103.8 ± 0.627 25.2 ± 0.152 52.7 ± 0.290 86.040
BASE $\beta=1, \sigma=5$
99.3 ± 0.620 25.5 ± 0.151 52.9 ± 0.289 85.773
BASE $\beta=1, \sigma=10$
96.3 ± 0.612 25.7 ± 0.149 53.2 ± 0.288 85.664
DISCO $\beta=1, \gamma=0.5$
83.8 ± 0.503 20.9 ± 0.124 45.1 ± 0.246 94.438

Table 3: Metrics values on the test set ± SEM. Best performances in bold.

The model ProbLoss (mm) MeJEE (mm) MaJEE (mm) FF (80mm)
cGAN 442.7 ± 0.513 109.8 ± 0.128 201.4 ± 0.320 0.000
cGAN$_{Init, fixed}$ 128.9 ± 0.480 31.8 ± 0.117 64.3 ± 0.230 78.454
DISCO $\beta=1, \gamma=0.5$
83.8 ± 0.503 20.9 ± 0.124 45.1 ± 0.246 94.438

Table 4: Metrics values on the test set ± SEM for cGAN and DISCO Nets. Best performances in bold.

4.3 Comparison with existing probabilistic models.

We consider the applications of the conditional Generative Adversarial Networks (cGAN) model from Mirza and Osindero [15] and to the best of our knowledge cGAN has not been applied to pose estimation. In order to compare cGAN to DISCO Nets on hand pose estimation, several issues must be overcome. First, we must design a network architecture for the Discriminator. This is a first disadvantage of cGAN compared to DISCO Nets which require no adversary. Second, as mentioned in Goodfellow et al. [6] and Radford et al. [20], GAN (and thus cGAN) require very careful design of the networks’ architecture and training procedure. In order to do a fair comparison, we followed the work in Mirza and Osindero [15] and practical advice for GAN presented in Larsen and Sønderby [12]. The Generator G has the same network architecture as DISCO $\beta=1, \gamma=0.5$. We add batch normalization to help the training. The Discriminator D is composed a convolutional part that also take a depth image $x$ as input. This part is the same as the convolutional part of G. The output of the convolutions is concatenated either the ground-truth pose for $x$ or the pose generated by G and fed to 2 dense layers with 200 hidden units. Dense layers apply dropout with a ratio of 0.5 and ReLUs. We use Maxout activation after the second dense layer. A third dense layer outputs a scalar value, on which we apply a sigmoid activation function. We try (i) cGAN, initialising all layers of D and G randomly, and (ii) cGAN$_{Init, fixed}$ initialising the convolutional layers of G and D.

The model ProbLoss (mm) MeJEE (mm) MaJEE (mm) FF (80mm)
cGAN 442.7 ± 0.513 109.8 ± 0.128 201.4 ± 0.320 0.000
cGAN$_{Init, fixed}$ 128.9 ± 0.480 31.8 ± 0.117 64.3 ± 0.230 78.454
DISCO $\beta=1, \gamma=0.5$
83.8 ± 0.503 20.9 ± 0.124 45.1 ± 0.246 94.438

Table 4: Metrics values on the test set ± SEM for cGAN and DISCO Nets. Best performances in bold.
with the trained best-performing DISCO\(_{\beta=1,\gamma=0.5}\) of Section 6.3 and keeping these layers fixed (hence training only the dense layers). That is, the convolutional parts of G and D are fixed feature extractors for the depth image. This is a setting similar to the one employed for tag-annotation of images in Mirza and Osindero [15]. We refer the reader to appendix for details. Table 4 shows that the cGAN model obtains relevant results only when the convolutional layers of G and D are initialised with our trained model and kept fixed, that is cGAN\(_{\text{init, fixed}}\). These results are still worse than DISCO Nets performances. Moreover, since cGAN model require the training of the additional Discriminator network, each epoch of training requires \(\sim 60\) seconds compared to \(\sim 20\) seconds for the DISCO Nets model. Finally, we showed that we can easily construct an unbiased estimate of our probabilistic objective function of equation (3). Thus we compute the probabilistic objective of DISCO Nets, that is the metric ProLoss. However it is not straightforward to estimate the objective function of cGAN, based on the Jensen-Shannon Divergence. As mentioned in Section 4.1 the output space size is too large to employ density estimation methods. While there may be a better architecture for cGAN, our experiments demonstrate the difficulty of training cGAN over DISCO Nets.

4.4 Reference state-of-the-art values.

We train the best-performing DISCO\(_{\beta=1,\gamma=0.5}\) of Section 6.3 on the entire dataset, and compare performances with state-of-the-art methods in Table 5 and Figure 4. These state-of-the-art methods are specifically designed for hand pose estimation. In Oberweger et al. [16] a constrained prior hand model, referred as NYU-Prior, is refined on each hand joint position to increase accuracy, referred as NYU-Prior-Refined. In Oberweger et al. [17], the input depth image is fed to a first network NYU-Init, that outputs a pose used to synthesize an image with a second network. The synthesized image is used with the input depth image to derive a pose update. We refer to the whole model as NYU-Feedback. On the contrary, DISCO Nets uses a single network whose architecture is similar to NYU-Prior (without constraining on a pose prior). By accurately modeling the distribution of the pose given the depth image, DISCO Nets are able to obtain comparable performances to NYU-Prior and NYU-Prior-Refined. Furthermore, without any extra effort, DISCO Nets could be embedded in the presented refinement and feedback methods, possibly boosting state-of-the-art performances.

Table 5: DISCO Nets compared to state-of-the-art performances ± SEM.

| Model               | MeJEE (mm) | MaJEE (mm) | FF (80mm) |
|---------------------|------------|------------|-----------|
| NYU-Prior           | 20.7±0.150 | 44.8±0.289 | 91.190    |
| NYU-Prior-Refined   | 19.7±0.157 | 44.7±0.327 | 88.148    |
| NYU-Init            | 27.4±0.152 | 55.4±0.265 | 86.537    |
| NYU-Feedback        | 16.0±0.096 | 36.1±0.208 | 97.334    |
| DISCO\(_{\beta=1,\gamma=0.5}\) | 20.7±0.121 | 45.1±0.246 | 93.250    |

Figure 4: Fractions of frames within distance \(d\) in mm (by 5 mm). Best viewed in color.

5 Discussion.

We presented DISCO Nets, a new family of probabilistic model based on deep networks. DISCO Nets employ a prediction and training procedure based on the minimisation of a dissimilarity coefficient. Theoretically, this ensures that DISCO Nets accurately capture uncertainty on the correct output to predict given an input. Experimental results on the task of hand pose estimation consistently support our theoretical hypothesis as DISCO Nets outperform non-probabilistic equivalent models, and existing probabilistic models. Furthermore, DISCO Nets can be tailored to the task to perform. This allows a possible user to train them to tackle different problems of interest. As their novelty resides mainly in their objective function, DISCO Nets do not require any specific architecture and can be easily applied to new problems.

We contemplate several directions for future work. First, we will apply DISCO Nets to other prediction problems where there is uncertainty on the output. Second, we would like to extend DISCO Nets to latent variables models, allowing us to apply DISCO Nets to diverse dataset where ground-truth annotations are missing or incomplete.

---

1 We also tried (iii) initialising the convolutional layers of G and D with the best DISCO\(_{\beta=1,\gamma=0.5}\) from Section 6.3, and do not keep them fixed, but training was always divergent.
References

[1] E.L. Denton, S. Chintala, A. Szlam, and R. Fergus. Deep generative image models using a Laplacian pyramid of adversarial networks. In NIPS, 2015.

[2] G. K. Dziugaite, D. M. Roy, and Z. Ghahramani. Training generative neural networks via maximum mean discrepancy optimization. In UAI, 2015.

[3] K. Fukumizu, L. Song, and A. Gretton. Kernel Bayes’ rule: Bayesian inference with positive definite kernels. JMLR, 2013.

[4] J. Gauthier. Conditional generative adversarial nets for convolutional face generation. Class Project for Stanford CS231N: Convolutional Neural Networks for Visual Recognition, 2014.

[5] T. Oonitani and A. E. Raftery. Strictly proper scoring rules, prediction, and estimation. Journal of the American Statistical Association, 2007.

[6] I. J. Goodfellow, J. Pouget-Abadie, M. Mirza, Bing Xu, D. Warde-Farley, S. Ozair, A. Courville, and Y. Bengio. Generative adversarial nets. In NIPS, 2014.

[7] A. Gretton, K. M. Borgwardt, M. J. Rasch, B. Scholkopf, and A. J. Smola. A kernel method for the two-sample problem. In NIPS, 2007.

[8] A. Gretton, K. M. Borgwardt, M. J. Rasch, B. Scholkopf, and A. J. Smola. A kernel two-sample test. In JMLR, 2012.

[9] D. P. Kingma and M. Welling. Auto-encoding variational Bayes. In ICLR, 2014.

[10] A. Radford, L. Metz, and S. Chintala. Unsupervised representation learning with deep convolutional generative adversarial networks. In ICLR, 2015.

[11] S. Lacoste-Julien, F. Huszar, and Z. Ghahramani. Approximate inference for the loss-calibrated Bayesian. In AISTATS, 2011.

[12] V. Premachandran, D. Tarlow, and D. Batra. Empirical minimum Bayes risk prediction: How to extract an extra few% performance from vision models with just three more parameters. In CVPR, 2014.

[13] A. Radford, L. Metz, and S. Chintala. Unsupervised representation learning with deep convolutional generative adversarial networks. In ICLR, 2015.

[14] C.R. Rao. Diversity and dissimilarity coefficients: A unified approach. Theoretical Population Biology, pages Vol. 21, No. 1, pp 24–43, 1982.

[15] S. Reed, Z. Akata, X. Yan, L. Logeswaran, H. Lee, and B. Schiele. Generative adversarial text to image synthesis. In ICML, 2016.

[16] B. Schölkopf. The kernel trick for distances. In NIPS, 2001.

[17] J. T. Springenberg. Unsupervised and semi-supervised learning with categorical generative adversarial networks. ICLR, 2016.

[18] J. Taylor, J. Shotton, T. Sharp, and A. Fitzgibbon. The vitruvian Manifold: Inferring dense correspondences for oneshot human pose estimation. In CVPR, 2012.

[19] J. Tompson, M. Stein, Y. Lecun, and K. Perlin. Real-time continuous pose recovery of human hands using convolutional networks. ACM Transactions on Graphics, 2014.

[20] X. Yan, J. Yang, K. Sohn, and H. Lee. Attribute2image: Conditional image generation from visual attributes. 2015. URL http://arxiv.org/abs/1512.00570
6 Appendix

In this supplementary material, we refer to equations and sections of the main paper, and to figures and tables of this supplementary material, unless otherwise stated.

6.1 Toy example experimental details.

In this section, we provide details on the toy example presented in Section 1. We used the following simple experimental setting. All covariances of the bidimensional distributions are diagonal, therefore all bidimensional Gaussian distributions are parametrised by 4 parameters \((\mu_1, \mu_2, \sigma_1, \sigma_2)\) where \(\mu, \sigma\) is a mean-variance pair on each dimension. We consider a data distribution that is a mixture of 2 bidimensional Gaussian distributions, referred as \(G_M G_M\). The first Gaussian of the mixture, \(G_1\), is parametrised by \((1, 1.5, 2, 0.8)\) and the second Gaussian \(G_2\) is parametrised by \((0, -0.5, 0.7, 0.6)\). The mixture weights are 0.7 and 0.3, such that \(G_M G_M = 0.7 \times G_1 + 0.3 \times G_2\). We consider two models to capture the true data distribution \(G_M G_M\). Each model is able to represent a bidimensional Gaussian distribution parametrised by \((\mu_1, \mu_2, \sigma_1, \sigma_2)\). The sets in which to search for the parameters are the same in both dimensions and both models. The set to search the means ranges from \(-3\) to \(3\) by \(1\), and the set to search the variances ranges \([0.1 \text{ to } 0.5]\). The training dataset is composed of \(N = 10000\) examples drawn randomly from \(G_M G_M\), denoted as \((x_1, \ldots, x_N)\). The testing dataset is composed of \(1000\) examples drawn randomly from \(G_M G_M\). During training, we draw \(K = 2\) samples from the model and estimate the probabilistic loss defined as:

\[
\frac{1}{N} \sum_{n=1}^{N} \left[ \frac{1}{K} \sum_{k} \Delta_M(x_n, G_M(z_k)) - \frac{1}{2K(K-1)} \sum_{k,k' \neq k} \Delta_M(G_M(z_k), G_M(z_{k'})) \right]
\]

where \(M\) indexes the model, \(\Delta_M\) is the model’s specific loss, and \(G_M(z_k)\) is the \(k^{th}\) sample drawn from \(M\) for the training data \(x_n\). During testing of a model, we draw \(K = 10\) samples from the model and choose a pointwise prediction with the MEU method. The MEU method employs the same loss as the evaluation loss.

6.2 Details on the MEU method

For an input \(x\), to choose a single prediction \(y\) among \(K\) candidate outputs sampled for \(x\), DISCO Nets use the principle of Maximum Expected Utility (MEU). The prediction \(y_{\Delta_k}\) maximises the expected utility, or rather minimises the expected task-specific loss \(\Delta_k\), estimated using the sampled candidates. Formally, the prediction is made as follows:

\[
y_{\Delta_k} = \arg\max_{k \in [1,K]} \text{EU}(y_k) = \arg\min_{k \in [1,K]} \sum_{k' = 1}^{K} \Delta_k(y_k, y_{k'})
\]

(9)

where \((y_1, \ldots, y_K)\) are the candidates output corresponding to the single input \(x\). For example, for a given input \(x\), we need to choose a pointwise output to evaluate the Mean Joint Euclidean Error (MeJEE). We sample \(K\) candidate outputs values for the input \(x\), and pick:

\[
y_{\text{MeJEE}} = \arg\min_{k \in [1,K]} \sum_{k' = 1}^{K} \Delta_k(y_k, y_{k'}) = \arg\min_{k \in [1,K]} \sum_{k' = 1}^{K} \frac{1}{J} \sum_{j = 1}^{J} ||y_{ij} - y_{k'j}||_2
\]

(10)

Then when we evaluate MeJEE, the loss encountered on the example \(x\) is \(\Delta_{\text{MeJEE}}(y_{\text{GT}}, y_{\Delta_{\text{MeJEE}}})\) where \(y_{\text{GT}}\) is the ground-truth output that corresponds to \(x\).

6.3 Experimental details

We provide in this section additional details on the Hand Pose experiment of Section 6.3. As in the main paper, we denote the training loss function \(\Delta_{\text{training}} = \Delta_{\beta}(y, y') = \|y - y'\|^2_2\).

Cross-validation procedure. We substract \(I = 10000\) examples from the 72757 training frames to construct a validation dataset. The validation examples are chosen at random and are the same for all experiments. Let us denote the examples pairs from the validation dataset as \(V = \{x_i, y_i, i = 1..I\}\), and \(y_{\Delta_{\text{training}}}\) is the prediction for the \(i^{th}\) example. During training, we monitor the value of the loss \(\Delta_{\text{training}}\) on the validation dataset. In details, this loss is:

\[
L_{\text{val}} = \frac{1}{I} \sum_{i=1}^{I} \|y_i - y_{\Delta_{\text{training}}(i)}\|^2_2
\]

(11)

In order to evaluate \(L_{\text{val}}\) for the model \(\text{BASE}_\beta\), we simply use for \(y_{\Delta_{\text{training}}}\) the pointwise prediction of \(\text{BASE}_\beta\). To monitor \(L_{\text{val}}\) for \(\text{DISCO}_\beta, \gamma\), we use the MEU method to pick \(y_{\Delta_{\text{training}}}\). In the MEU method, we draw \(K = 100\)
samples per validation example. In order to reduce variance, we draw the 100 random noise vectors per example, \( \{ z_{1,1}, ..., z_{1,K}, ..., z_{I,1}, ..., z_{I,K} \} \) once for all before starting the optimisation Algorithm 1. Indeed, contrarily to the random noise vector drawn to estimate the gradient of the training objective in step 4 of Algorithm 1 these noise vectors do not influence the training but are only used for monitoring purposes. They remain independent when the network’s parameters are optimised. Finally, we choose for each model the best value of \( C \) and the best seed by taking the setting that gives the lowest final value of \( L_{val} \).

**Training procedure** All network weights are initialised at random with a Gaussian distribution of mean 0 and standard deviation 0.01, all biases are initialised to 0. During training, the number of candidates outputs generated by the probabilistic models DISCO_{γ,β} is \( K = 2 \). This is sufficient to construct an unbiased estimate of the gradient in step 7 of Algorithm in the main paper. Note that in step 4 of Algorithm 1 we must draw new noise samples \( (z_{n,1}, ..., z_{n,K}) \) for each training example at each iteration. Indeed, let us consider the iteration \( t \) on a training example \( x_n \). The values of the noise sampled for \( x_n \) at iteration \( t - 1 \), \( (z_{t-1,n,1}, ..., z_{t-1,n,K}) \), were used in the estimation of the gradient, and thus influenced the update \( \theta_t \leftarrow \theta_{t-1} \). Thus, we need to draw new sample to ensure that the \( K \) candidate outputs for \( x_n, y_n = G(z_{n,k}, x_n; \theta_t), k = 1..K \), remain independent given \( x_n \) and \( \theta_t \). We train all models for 400 epochs as it results in a change of less than 3% in the value of \( L_{val} \) for BASE_β.

![Figure 5: \( L_{val} \) monitoring for the two models, for different values of \( C \), for seed 0.](image1)

![Figure 6: \( L_{val} \) monitoring for the two models, for different values of \( C \), for seed 1.](image2)

![Figure 7: \( L_{val} \) monitoring for the two models, for different values of \( C \), for seed 2.](image3)

**Detailed results** We list here all results of the experiment presented in Section for all values of \( C \), for the best seed for each \( C \) value. In the main paper we report performances using the MEU method for choosing a pointwise prediction \( y_\Delta \) for an input \( x \). However we can use 3 different methods that are:

- MEU method.
- MEAN method: pick for the non-probabilistic model BASE_{β,σ} its pointwise prediction. Therefore the value of the metrics MeJEE, MaJEE and FF are the same regardless of \( σ \). For DISCO_{γ,β}, pick the mean of the \( K \) candidates.
• RANDOM method: pick an output \( y \) at random among \( K \) candidates.

Detailed results show that the model DISCO\( _{\beta=1,\gamma=0.5} \) consistently outperforms BASE\( _{\beta=1,\sigma} \) for all values of \( C \) and all methods, and that results are consistent across the pointwise prediction method employed. Tables 6, 7, and 8 present the results when we use the MEU method to choose the pointwise estimate. Tables 9, 10, and 11 present the results when we use the MEAN method. Tables 12, 13, and 14 present the results when we use the RANDOM method.

**Table 6: Metrics values \( \pm \text{SEM} \) for \( C = 1e^{-2} \) using MEU.**

| Model     | ProbLoss | MeJEE (mm) | MrJEE (mm) | FF (80mm) |
|-----------|----------|------------|------------|-----------|
| BASE\( _{\beta=1,\sigma=1} \) | 210.1±0.793 | 51.9±0.202 | 92.5±0.325 | 38.451    |
| BASE\( _{\beta=1,\sigma=5} \) | 204.8±0.792 | 52.0±0.201 | 92.7±0.325 | 38.257    |
| BASE\( _{\beta=1,\sigma=10} \) | 201.1±0.791 | 52.2±0.201 | 92.8±0.324 | 37.215    |
| DISCO\( _{\beta=1,\gamma=0.5} \) | 87.8±0.506 | 23.5±0.129 | 50.9±0.264 | 88.900    |

**Table 7: Metrics values \( \pm \text{SEM} \) for \( C = 1e^{-3} \) using MEU.**

| Model     | ProbLoss | MeJEE (mm) | MrJEE (mm) | FF (80mm) |
|-----------|----------|------------|------------|-----------|
| BASE\( _{\beta=1,\sigma=1} \) | 100.8±0.586 | 24.5±0.142 | 51.0±0.271 | 88.742    |
| BASE\( _{\beta=1,\sigma=5} \) | 96.3±0.579 | 24.8±0.141 | 51.2±0.270 | 88.548    |
| BASE\( _{\beta=1,\sigma=10} \) | 93.3±0.571 | 25.1±0.140 | 51.5±0.268 | 88.488    |
| DISCO\( _{\beta=1,\gamma=0.5} \) | 80.4±0.490 | 20.6±0.123 | 44.5±0.245 | 94.292    |

**Table 8: Metrics values \( \pm \text{SEM} \) for \( C = 1e^{-4} \) using MEU.**

| Model     | ProbLoss | MeJEE (mm) | MrJEE (mm) | FF (80mm) |
|-----------|----------|------------|------------|-----------|
| BASE\( _{\beta=1,\sigma=1} \) | 103.8±0.627 | 25.2±0.152 | 52.7±0.290 | 86.040    |
| BASE\( _{\beta=1,\sigma=5} \) | 99.3±0.620 | 25.5±0.151 | 52.9±0.289 | 85.773    |
| BASE\( _{\beta=1,\sigma=10} \) | 96.3±0.612 | 25.7±0.149 | 53.2±0.288 | 85.664    |
| DISCO\( _{\beta=1,\gamma=0.5} \) | 83.8±0.503 | 20.9±0.124 | 45.1±0.246 | 94.438    |

**Table 9: Metrics values \( \pm \text{SEM} \) for \( C = 1e^{-2} \) using MEAN.**

| Model     | ProbLoss | MeJEE (mm) | MrJEE (mm) | FF (80mm) |
|-----------|----------|------------|------------|-----------|
| BASE\( _{\beta=1,\sigma=1} \) | 210.1±0.793 | 51.8±0.202 | 92.5±0.325 | 38.536    |
| BASE\( _{\beta=1,\sigma=5} \) | 204.8±0.792 | 51.8±0.202 | 92.5±0.325 | 38.536    |
| BASE\( _{\beta=1,\sigma=10} \) | 201.1±0.791 | 51.8±0.202 | 92.5±0.325 | 38.536    |
| DISCO\( _{\beta=1,\gamma=0.5} \) | 87.8±0.506 | 23.5±0.129 | 50.7±0.263 | 90.221    |

**Table 10: Metrics values \( \pm \text{SEM} \) for \( C = 1e^{-3} \) using MEAN.**

| Model     | ProbLoss | MeJEE (mm) | MrJEE (mm) | FF (80mm) |
|-----------|----------|------------|------------|-----------|
| BASE\( _{\beta=1,\sigma=1} \) | 100.8±0.586 | 24.4±0.142 | 50.9±0.271 | 88.657    |
| BASE\( _{\beta=1,\sigma=5} \) | 96.3±0.579 | 24.4±0.142 | 50.9±0.271 | 88.657    |
| BASE\( _{\beta=1,\sigma=10} \) | 93.3±0.571 | 24.4±0.142 | 50.9±0.271 | 88.657    |
| DISCO\( _{\beta=1,\gamma=0.5} \) | 80.4±0.490 | 20.6±0.123 | 44.3±0.244 | 94.535    |
As mentioned in Section 4.3, we tried 3 different training setting in order to apply cGAN to hand pose estimation.

C procedure presented in Section 6.3 of this supplementary to cross-validate L2-regularisation controlled by a parameter $\lambda$. We use the cross-validation method presented in Section 6.3 and keep these part fixed. That is, the convolutional parts of the Generator and the Discriminator are feature extractors that are not trained. This is a setting similar to the one employed for tag-annotation of images in Mirza and Osindero [15].

Table 11: Metrics values $\pm$ SEM for $C = 1e^{-4}$ using MEAN.

| Model        | ProbLoss | MeJEE (mm) | MaJEE (mm) | FF (80mm) |
|--------------|----------|------------|------------|-----------|
| BASE$\beta=1,\sigma=1$ | 103.8±0.627 | 25.1±0.152 | 52.7±0.290 | 85.991    |
| BASE$\beta=1,\sigma=5$ | 99.3±0.620 | 25.1±0.152 | 52.7±0.290 | 85.991    |
| BASE$\beta=1,\sigma=10$ | 96.3±0.612 | 25.1±0.152 | 52.7±0.290 | 85.991    |
| DISCO$\beta=1,\gamma=0.5$ | 83.8±0.503 | 20.9±0.124 | 45.0±0.246 | 94.619    |

Table 12: Metrics values $\pm$ SEM for $C = 1e^{-2}$ using RANDOM.

| Model        | ProbLoss | MeJEE (mm) | MaJEE (mm) | FF (80mm) |
|--------------|----------|------------|------------|-----------|
| BASE$\beta=1,\sigma=1$ | 210.1±0.793 | 51.9±0.202 | 92.5±0.325 | 38.500    |
| BASE$\beta=1,\sigma=5$ | 204.8±0.792 | 52.1±0.201 | 92.8±0.324 | 37.700    |
| BASE$\beta=1,\sigma=10$ | 201.1±0.791 | 52.4±0.201 | 93.1±0.325 | 37.082    |
| DISCO$\beta=1,\gamma=0.5$ | 87.8±0.506 | 24.1±0.130 | 52.1±0.266 | 88.572    |

Table 13: Metrics values $\pm$ SEM for $C = 1e^{-3}$ using RANDOM.

| Model        | ProbLoss | MeJEE (mm) | MaJEE (mm) | FF (80mm) |
|--------------|----------|------------|------------|-----------|
| BASE$\beta=1,\sigma=1$ | 100.8±0.586 | 24.6±0.142 | 51.0±0.271 | 88.694    |
| BASE$\beta=1,\sigma=5$ | 96.3±0.579 | 25.0±0.140 | 51.4±0.269 | 88.500    |
| BASE$\beta=1,\sigma=10$ | 93.3±0.571 | 25.5±0.138 | 51.8±0.267 | 88.585    |
| DISCO$\beta=1,\gamma=0.5$ | 80.4±0.490 | 20.9±0.123 | 45.0±0.246 | 94.062    |

Table 14: Metrics values $\pm$ SEM for $C = 1e^{-4}$ using RANDOM.

| Model        | ProbLoss | MeJEE (mm) | MaJEE (mm) | FF (80mm) |
|--------------|----------|------------|------------|-----------|
| BASE$\beta=1,\sigma=1$ | 103.8±0.627 | 25.3±0.151 | 52.8±0.290 | 86.028    |
| BASE$\beta=1,\sigma=5$ | 99.3±0.620 | 25.7±0.150 | 53.1±0.288 | 85.761    |
| BASE$\beta=1,\sigma=10$ | 96.3±0.612 | 26.1±0.148 | 53.5±0.286 | 85.822    |
| DISCO$\beta=1,\gamma=0.5$ | 83.8±0.503 | 21.0±0.124 | 45.3±0.246 | 94.510    |

Detailed comparison with cGAN. We aim at performing a fair comparison with the conditional Generative Adversarial Networks (cGAN) model presented in Mirza and Osindero [15]. However as mentioned in Section 4.3, we encountered several challenges. We present here additional details on the comparison with cGAN models.

As mentioned in Section 4.3, we tried 3 different training setting in order to apply cGAN to hand pose estimation. The setting cGAN initialises the convolutional layers of both the Discriminator and the Generator parameters. The setting cGAN$_{init}$ initialises the convolutional layers of both the Discriminator and the Generator with the trained convolutional layers of our best DISCO Nets from Section 6.3 without keeping it fixed. That is, we try to perform fine-tuning only. The setting cGAN$_{init, fixed}$ initialises the convolutional layers of both the Discriminator and the Generator with the trained convolutional layers of our best DISCO Nets from Section 6.3, and keep these part fixed. That is, the convolutional parts of the Generator and the Discriminator are feature extractors that are not trained. This is a setting similar to the one employed for tag-annotation of images in Mirza and Osindero [15].

Since cGAN model require the training of the additional Discriminator network, each epoch of training requires $\sim$ 60 seconds compared to $\sim$ 20 seconds for DISCO Nets. Therefore, we were only able to use one random seed for the initialisation of the network parameters. However, DISCO Nets present a consistent behavior regardless of the seed employed for initialisation. We could expect a similar behavior for the cGAN model. The experimental setting is similar to the one of Section 6.3. We use the exact same training and validation sets as in Section 6.3. Back-propagation was used with Stochastic Gradient Descent. The learning rate is fixed to $\lambda = 0.01$ and we use a momentum of $m = 0.9$. We use a batchsize of 256 samples. We also add L2-regularisation controlled by a parameter $C$. We use $C = [1e^{-4}, 1e^{-3}, 1e^{-2}]$. We use the cross-validation procedure presented in Section 6.3 of this supplementary to cross-validate $C$. We train all models for 400 epochs. For $C = 1e^{-4}$, we train for more epochs since 400 were not enough to have the final convergence of cGAN$_{init, fixed}$ (see Figure 80). However, this does not help the cGAN performances compared to the one.
(a) $L_{val}$ monitoring for cGan with $C = 1e^{-4}$ for 400 epochs.

(b) $L_{val}$ monitoring for cGan with $C = 1e^{-4}$. We trained for more than 1400 epochs as 400 epochs were not enough.

(c) $L_{val}$ monitoring for cGan with $C = 1e^{-3}$.

(d) $L_{val}$ monitoring for cGan with $C = 1e^{-2}$.

Figure 8: $L_{val}$ monitoring for cGan.

reported for 400 epochs, see Table 15. Figure 8 shows the training behavior of the different settings of cGAN. When the curve is missing, it means that the model has diverged after the few first iterations and thus we cannot show its behavior. This is always the case for cGANinit.

Table 15: Metrics values ± SEM for $C = 1e^{-4}$ for cGAN model.

| Model                     | ProbLoss (mm) | MeJEE (mm) | MaJEE (mm) | FF (80mm) |
|---------------------------|---------------|------------|------------|------------|
| cGANinit, fixed, 400 epochs of training | 128.9±0.480   | 31.8±0.117 | 64.3±0.230 | 78.454     |
| cGANinit, fixed, >1400 epochs of training | 134.5±0.591   | 31.8±0.146 | 64.0±0.253 | 74.261     |