UNIT GROUP OF $\mathbb{F}_p \mathbf{SL}(3,2), p \geq 11$

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Abstract. We provide the structure of the unit group of $\mathbb{F}_p \mathbf{SL}(3,2)$, where $p \geq 11$ is a prime and $\mathbf{SL}(3,2)$ denotes the $3 \times 3$ invertible matrices over $\mathbb{F}_2$.

1. Introduction

Let $q = p^k$ for some prime $p$ and $k \in \mathbb{N}$. Let $\mathbb{F}_q$ denote the finite field of cardinality $q$. For any group $G$, let $\mathbb{F}_q G$ denotes the group algebra of $G$ over $\mathbb{F}_q$. For basic notations and results on the subject of study, we refer the readers to the classic by Milies and Sehgal [MS1]. The group of units of $\mathbb{F}_q G$ has many applications. As an application of the unit groups of matrix rings, Hurley has proposed the constructions of convolutional codes (See [H1], [H2], [HH1], [HH2]). The structure of unit group can also be used to deal with some problems in combinatorial number theory as well (See [CGK]). This has encouraged a lot of researchers to find out the explicit structure of the group of units of $\mathbb{F}_q G$.

A substantial amount of work has been done to find the structure of the algebra $\mathbb{F}_q G$, and also of the group of units of these algebras. For example in [S], the author has described units of $\mathbb{F}_q G$, where $G$ is a $p$-group. In a recent paper [BLP] the authors have discussed the groups of units for the group algebras over abelian groups of order 17 to 20. However the complexity of the problem increases with increase in the size of the group and the number of conjugacy classes it has. For more, one can check [MSS2], [MA], [TG] et cetera.

Very little is known for $\mathbb{F}_q G$, when $G$ is a non-Abelian simple group. For the case $G = A_5$, this has been discussed in [MSS1]. The next group in the family of non-Abelian simple groups is the group $\mathbf{SL}(3,2)$. In Theorem 4.4 of this article we give a complete description of the unit group of $\mathbb{F}_q \mathbf{SL}(3,2)$ for $p \geq 11$.

Rest of the article is organized as follows. In section 2, we give the known results which we will be using in subsequent sections. In section 3 we discuss about some simple components of the Artin-Wedderburn decomposition of the group algebra. Next
in section 4, we deduce the main result. We discuss some observations and conclude the paper by mentioning some remarks, in section 5.

2. Preliminaries

First we fix some notations. We adopt already mentioned notations from section 1. For an extension field $E/F$, $\text{Gal}(E/F)$ denotes the Galois group of the extension. For $n \in \mathbb{N}$ the notation $M(n, R)$ denotes the full matrix ring of $n \times n$ matrices over $R$ where as $\text{GL}(n, R)$ will denote the set of all invertible matrices in $M(n, R)$. For a ring $R$, the set of units of $R$ will be denoted as $R^\times$. The center of a ring $R$ will be denoted as $Z(R)$. If $G$ is a group and $g \in G$, then $[g]$ will denote the conjugacy class of $g$ in $G$. For the group ring $F_q G$, the group of units will be denoted as $U(F_q G)$. For the notations on projective spaces, we follow [HI].

We say an element $g \in G$ is a $p'$-element if the order of $g$ is not divisible by $p$. Let $e$ be the exponent of the group $G$ and $\eta$ be a primitive $r$th root of unity, where $e = p^f r$ and $p \nmid r$. Let

$$I_{F_q} = \left\{ l \pmod{e} : \text{there exists } \sigma \in \text{Gal}(F_q(\eta)/F_q) \text{ satisfying } \sigma(\eta) = \eta^l \right\}.$$

**Definition 2.1.** For a $p'$-element $g \in G$, the cyclotomic $F_q$-class of $g$, denoted by $S_{F_q}(g)$ is defined as $\{ \gamma g^l : l \in I_{F_q} \}$ where $\gamma g^l \in F_q G$ is the sum of all conjugates of $g^l$ in $G$.

Then we have the following results which are crucial in determining the Artin-Wedderburn decomposition of $F_q G$.

**Lemma 2.2.** [F, Proposition 1.2] The number of simple components of $F_q G/J(F_q G)$ is equal to the number of cyclotomic $F_q$-classes in $G$.

**Lemma 2.3.** [F, Theorem 1.3] Let $n$ be the number of cyclotomic $F_q$-classes in $G$. If $L_1, L_2, \cdots, L_n$ are the simple components of $Z(F_q G/J(F_q G))$ and $S_1, S_2, \cdots, S_n$ are the cyclotomic $F_q$-classes of $G$, then with a suitable reordering of the indices,

$$|S_i| = [L_i : F_q].$$

**Lemma 2.4.** [MS2, Lemma 2.5] Let $K$ be a field of characteristic $p$ and let $A_1, A_2$ be two finite dimensional $K$-algebras. Assume $A_1$ to be semisimple. If $h : A_2 \to A_1$ is a surjective homomorphism of $K$-algebras, then there exists a semisimple $K$-algebra $l$ such that $A_2/J(A_2) = l \oplus A_1$.

We will be using various descriptions of $\text{SL}(3, 2)$ in the sequel, which are well known. From [CCNPW], it is known that

$$\text{SL}(3, 2) = \text{GL}(3, 2) \cong \text{PGL}(2, 7) \cong \text{PSL}(2, 7).$$
We have an embedding of $\text{SL}(3, 2)$ inside $S_8$ as follows:

$$\text{SL}(3, 2) \cong \langle (3, 7, 5)(4, 8, 6), (1, 2, 6)(3, 4, 8) \rangle.$$ 

This group has 7 conjugacy classes and using [GAP2021], we have the following table:

| Class | Representative | Order | No. of elements |
|-------|----------------|-------|-----------------|
| $C_1$ | $\alpha_1 = (1)$ | 1     | 1               |
| $C_2$ | $\alpha_2 = (1, 2)(3, 4)(5, 8)(6, 7)$ | 2     | 21              |
| $C_3$ | $\alpha_3 = (3, 5, 7)(4, 6, 8)$ | 3     | 56              |
| $C_4$ | $\alpha_4 = (1, 2, 3, 5)(4, 8, 7, 6)$ | 4     | 42              |
| $C_5$ | $\alpha_5 = (2, 3, 5, 4, 7, 8, 6)$ | 7     | 24              |
| $C_6$ | $\alpha_6 = (2, 4, 6, 5, 8, 3, 7)$ | 7     | 24              |

We note down the following relations

(2.1) $[\alpha_5] = [\alpha_5^2] = [\alpha_5^4].$

and

(2.2) $[\alpha_6] = [\alpha_6^3] = [\alpha_6^5] = [\alpha_6^6] = [\alpha_6].$

3. ON SOME SIMPLE COMPONENTS OF $\mathbb{F}_qG$

The next few lemmas are crucial for determining the different $n_i$’s occurring in the Artin-Wedderburn decomposition of $\mathbb{F}_q\text{SL}(3, 2)$.

**Lemma 3.1.** Let $G$ be a group of order $n$ and $\mathbb{F}$ be a field of characteristic $p > 0$. Let $G$ acts on a finite set $X = \{1, 2, \ldots, k\}$ doubly transitively. Set $G_i = \{g \in G : g \cdot i = i\}$ and $G_{i,j} = \{g \in G : g \cdot i = i, g \cdot j = j\}$. Then the $\mathbb{F}G$ module

$$W = \left\{ x \in \mathbb{F}^k : \sum_{i=1}^{k} x_i = 0, i \in X \right\}$$

is an irreducible $\mathbb{F}G$ module if $p \nmid k, p \nmid |G_{1,2}|$.

**Proof.** Let $U \subseteq W$ be a non-zero invariant space under the action of $G$. Since the action is doubly transitive, it is enough to show that we have $(1, -1, 0, \ldots, 0)^{(k-2)\text{times}} \in U$.

Let $x = (x_1, x_2, \ldots, x_n) \in U$ be nonzero. Then we can assume that $x_1 \neq 0$, since $G$ acts transitively on $X$. Considering the element $y = \sum_{g \in G_i} gx \in U$, we see that

$$y_1 = |G_1|x_1$$

$$y_2 = y_3 = \cdots = y_n$$

$$= |G_{1,2}| \sum_{i=2}^{n} x_i,$$
since $G$ permutes $X$. Note that $y_i \neq 0$ for all $1 \leq i \leq k$. Next taking a $g \in G$, which permutes $1, 2$ (this exists since the action is doubly transitive) we see that $(y_1 - y_2)(1, -1, 0, \ldots, 0) \in U$, which finishes the proof. \hfill \square

**Corollary 3.2.** The representation induced by the action of $\text{GL}(3, 2) = \text{PGL}(3, 2)$ on $\mathbb{P}^2(\mathbb{F}_2)$ has an irreducible degree 6 component over $\mathbb{F}_{p^k}$, for $p \geq 11$.

*Proof.* We know that the action of $\text{GL}(3, 2)$ on $\mathbb{P}^2(\mathbb{F}_2)$ is doubly transitive (see [HI, pp. 124]). Since $G_{1,2}$ is a subgroup of $\text{GL}(3, 2)$ and $p \nmid |G|$, the result follows from Lemma 3.1. \hfill \square

**Corollary 3.3.** The representation induced by the action of $\text{GL}(3, 2) \cong \text{PGL}(2, 7)$ on $\mathbb{P}^1(\mathbb{F}_7)$ has an irreducible degree 7 component over $\mathbb{F}_{p^k}$, for $p \geq 11$.

*Proof.* The action of the group $\text{PGL}(2, 7)$ on $\mathbb{P}^1(\mathbb{F}_7)$, is transitive, as well as doubly transitive (see [HI, pp. 157]). We see that $p \nmid |G_{1,2}|$, as $G_{1,2}$ is a subgroup of $\text{PGL}(3, 2)$ and $p \nmid 168$. \hfill \square

**Remark 3.4.** Note that in Corollaries 3.2 and 3.3 the prime $p$ can be chosen lesser than 11.

**Remark 3.5.** Using Lemma 3.1 it can be seen that the regular representation of the symmetric group $S_n$, decomposes into the trivial representation and an irreducible representation of degree $n - 1$ over the field $\mathbb{F}_{p^k}$, whenever $p > n$.

**Lemma 3.6.** Let $A_i$, $1 \leq i \leq n$ be a family of unital algebra with unit $1_i$ and $D_i$ be the set of representatives of simple $A_i$-modules. Then any simple $\bigoplus_{i=1}^{n} A_i$-module is of the form $\bigoplus_{i=1}^{n} M_i$, where not all $M_i$’s are zero and $M_i \in D_i$.

*Proof.* Since $1 = \bigoplus_{i=1}^{n} A_i = \sum_{i=1}^{n} 1_i A_i$ and hence for any $\bigoplus_{i=1}^{n} A_i$-module $M$, we have

$$M = M \cdot 1 = \bigoplus_{i=1}^{n} A_i \bigoplus_{i=1}^{n} A_i = \bigoplus_{i=1}^{n} M A_i.$$ \hfill \square

**Lemma 3.7.** [P, Example 3.3] For any division algebra (in particular field) $D$, the only simple $M(n, D)$ module is $D^n$ up to isomorphism.
Corollary 3.8. Let $G$ be a finite group, $k$ be a finite field of characteristic $p > 0$, $p \nmid |G|$. Then if there exists an irreducible representations of degree $n$ over $k$, then one of the component of $kG$ is of the form $M(n,k)$.

Proof. Since $p \nmid |G|$, by Maschke’s theorem $kG$ is semisimple. Hence by Artin–Wedderburn theorem we have that

$$kG = \bigoplus_{i=1}^{n} M(n_i, k_i),$$

where $k_i$’s are finite extensions of $k$ (hence a field). It follows from Lemma 3.6 and Lemma 3.7 that for some $i$, we have $n_i = n, k_i = k$. Hence the result follows. \qed

Corollary 3.9. Two of the components of the group algebra $\mathbb{F}_q\text{SL}(3,2)$ are $M(6,\mathbb{F}_q), M(7,\mathbb{F}_q)$.

Proof. This follows immediately from Corollaries 3.2, 3.3 and 3.8. \qed

4. Units in $\mathbb{F}_q\text{SL}(3,2)$

Proposition 4.1. Let $\mathbb{F}_q$ be a field of characteristic $p$ and $p \geq 11$ and $q = p^k$. Let $G$ be the group $\text{SL}(3,2)$. Then the Artin-Wedderburn decomposition of $\mathbb{F}_qG$ is one of the following:

$$\mathbb{F}_q \oplus \bigoplus_{i=1}^{5} M(n_i, \mathbb{F}_q),$$

$$\mathbb{F}_q \oplus \bigoplus_{i=1}^{3} M(n_i, \mathbb{F}_q) \oplus M(4, \mathbb{F}_q^2)$$

Proof. Since $p \nmid |G|$, by Maschke’s theorem we have $\mathbb{F}_qG$ is semisimple and hence $J(\mathbb{F}_qG)$ is zero. By its Wedderburn decomposition we have $\mathbb{F}_qG$ is isomorphic to $\bigoplus_{i=1}^{n} M(n_i, K_i)$, where $n_i > 0$ and $K_i$ is a finite extension of $\mathbb{F}_q$, for all $1 \leq i \leq n$.

Firstly from Lemma 2.4, we have

$$\mathbb{F}_qG \cong \mathbb{F}_q \bigoplus_{i=1}^{n-1} M(n_i, K_i),$$

taking $h$ to be the augmentation map. Now to compute these $n_i$’s and $K_i$’s we calculate the cyclotomic $\mathbb{F}_q$ classes of $G$. We do this in 6 cases, for $k = 6l + i$, $0 \leq i \leq 5$. Note that $p$ can have the following possibilities, being a prime

$$p \in \{\pm 1\} \mod 4,$$

$$p \in \{\pm 1\} \mod 3,$$

$$p \in \{\pm 1, \pm 2, \pm 3\} \mod 7.$$
(1) The case \((k = 6l)\): In this case \(p^k \equiv 1 \mod 7, p^k \equiv 1 \mod 4\) and \(p^k \equiv 1 \mod 3\), hence \(p^k \equiv 1 \mod 84\) (using Chinese Remainder theorem). Thus \(I_{F_q} = \{1\}\) and \(S_{F_q}(\gamma_g) = \{\gamma_g\}\) for all \(g \in G\). Thus by Lemma 2.2, Lemma 2.3 and Equation 4.1

\[
F_q G \cong F_q \oplus \bigoplus_{i=1}^5 M(n_i, F_q).
\]

When such a decomposition arises, we say that \((p, k)\) is of type 1.

(2) The case \((k = 6l + 1)\): In this case if \(p \equiv \pm 1 \mod 3\), \(p \equiv \pm 1 \mod 4\) and \(p \equiv 1, 2, -3 \mod 7, S_{F_q}(\gamma_g) = \{\gamma_g\}\) for all \(g \in G\), because we have

\[
[\alpha_2] = [\alpha_2^{-1}], [\alpha_3] = [\alpha_3^{-1}], [\alpha_4] = [\alpha_4^{-1}].
\]

Once again by Lemma 2.2 and Lemma 2.3 and Equation 4.1

\[
F_q G \cong F_q \oplus \bigoplus_{i=1}^5 M(n_i, F_q).
\]

i.e \((p, k)\) is of type 1. Now if \(p \equiv -1, -2, 3 \mod 7\), then we get that \(S_{F_q}(\gamma_g) = \{\gamma_g\}\) for \(g \in \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}\) and \(S_{F_q}(\gamma_g) = (\gamma_g, \gamma_g^{-1})\) when \(g \in \{\alpha_5, \alpha_6\}\) since \([\alpha_5] \neq [\alpha_5^{-1}]\). Hence in this case we have

\[
F_q G \cong F_q \oplus \bigoplus_{i=1}^3 M(n_i, F_q) \oplus M(n_4, F_{q^2}).
\]

When such a decomposition arises, we say that \((p, k)\) is of type 2.

It can be further shown using Equation 2.1 and Equation 2.2 that \((p, k)\) is either of type 1 or 2. The possibilities are listed in the table below.

| \(p \mod 7\) | \(k\) | Type of \((p, k)\) |
|----------------|------|------------------|
| \(\pm 1, \pm 2, \pm 3\) | \(6l\) | 1 |
| 1, 2, -3 | \(6l + 1\) | 1 |
| -1, -2, 3 | \(6l + 1\) | 2 |
| \(\pm 1, \pm 2, \pm 3\) | \(6l + 2\) | 1 |
| 1, 2, -3 | \(6l + 3\) | 1 |
| -1, -2, 3 | \(6l + 3\) | 2 |
| \(\pm 1, \pm 2, \pm 3\) | \(6l + 4\) | 1 |
| 1, 2, -3 | \(6l + 5\) | 1 |
| -1, -2, 3 | \(6l + 5\) | 2 |

\[\square\]

**Proposition 4.2.** We have \((n_1, n_2, n_3, n_4, n_5, n_6) = (1, 6, 7, 8, 3, 3)\) up to some permutation.
Proposition 4.3. Let $\mathbb{F}_q$ be a field of characteristic $p$ and $p \geq 11$ and $q = p^k$. Let $G$ be the group $\text{SL}(3, 2)$. Then the Wedderburn decomposition of $\mathbb{F}_q G$ is as follows:

$$
\mathbb{F}_q \oplus M(6, \mathbb{F}_q) \oplus M(7, \mathbb{F}_q) \oplus M(8, \mathbb{F}_q) \oplus M(3, \mathbb{F}_q)^2 \text{ if } (p, k) \text{ is of type 1}, \\
\mathbb{F}_q \oplus M(6, \mathbb{F}_q) \oplus M(7, \mathbb{F}_q) \oplus M(8, \mathbb{F}_q) \oplus M(3, \mathbb{F}_q^2) \text{ if } (p, k) \text{ is of type 2}.
$$

Proof. Follows immediately from Proposition 4.1 and Proposition 4.2.

Theorem 4.4. Let $\mathbb{F}_q$ be a field of characteristic $p$ and $p \geq 11$. Let $G$ be the group $\text{SL}(3, 2)$. Then the unit group $\mathcal{U}(\mathbb{F}_q G)$ is as listed in the following table:

| $p \mod 7$ | $k$ | $\mathcal{U}(\mathbb{F}_q \text{SL}(3, 2))$ |
|------------|-----|----------------------------------|
| $\pm 1, \pm 2, \pm 3$ | $6l$ | $\mathbb{F}_q^\times \oplus \text{GL}(6, \mathbb{F}_q) \oplus \text{GL}(7, \mathbb{F}_q) \oplus \text{GL}(8, \mathbb{F}_q) \oplus \text{GL}(3, \mathbb{F}_q)^2$ |
| $1, 2, 3$ | $6l + 1$ | $\mathbb{F}_q^\times \oplus \text{GL}(6, \mathbb{F}_q) \oplus \text{GL}(7, \mathbb{F}_q) \oplus \text{GL}(8, \mathbb{F}_q) \oplus \text{GL}(3, \mathbb{F}_q)^2$ |
| $-1, -2, 3$ | $6l + 1$ | $\mathbb{F}_q^\times \oplus \text{GL}(6, \mathbb{F}_q) \oplus \text{GL}(7, \mathbb{F}_q) \oplus \text{GL}(8, \mathbb{F}_q) \oplus \text{GL}(3, \mathbb{F}_q^2)$ |
| $\pm 1, \pm 2, \pm 3$ | $6l + 2$ | $\mathbb{F}_q^\times \oplus \text{GL}(6, \mathbb{F}_q) \oplus \text{GL}(7, \mathbb{F}_q) \oplus \text{GL}(8, \mathbb{F}_q) \oplus \text{GL}(3, \mathbb{F}_q^2)$ |
| $1, 2, 3$ | $6l + 3$ | $\mathbb{F}_q^\times \oplus \text{GL}(6, \mathbb{F}_q) \oplus \text{GL}(7, \mathbb{F}_q) \oplus \text{GL}(8, \mathbb{F}_q) \oplus \text{GL}(3, \mathbb{F}_q^2)$ |
| $-1, -2, 3$ | $6l + 3$ | $\mathbb{F}_q^\times \oplus \text{GL}(6, \mathbb{F}_q) \oplus \text{GL}(7, \mathbb{F}_q) \oplus \text{GL}(8, \mathbb{F}_q) \oplus \text{GL}(3, \mathbb{F}_q^2)$ |
| $\pm 1, \pm 2, \pm 3$ | $6l + 4$ | $\mathbb{F}_q^\times \oplus \text{GL}(6, \mathbb{F}_q) \oplus \text{GL}(7, \mathbb{F}_q) \oplus \text{GL}(8, \mathbb{F}_q) \oplus \text{GL}(3, \mathbb{F}_q^2)$ |
| $1, 2, 3$ | $6l + 5$ | $\mathbb{F}_q^\times \oplus \text{GL}(6, \mathbb{F}_q) \oplus \text{GL}(7, \mathbb{F}_q) \oplus \text{GL}(8, \mathbb{F}_q) \oplus \text{GL}(3, \mathbb{F}_q^2)$ |
| $-1, -2, 3$ | $6l + 5$ | $\mathbb{F}_q^\times \oplus \text{GL}(6, \mathbb{F}_q) \oplus \text{GL}(7, \mathbb{F}_q) \oplus \text{GL}(8, \mathbb{F}_q) \oplus \text{GL}(3, \mathbb{F}_q^2)$ |

Proof. This follows immediately from Proposition 4.3 and the fact that given two rings $R_1, R_2$, we have $(R_1 \times R_2)^\times = R_1^\times \times R_2^\times$.

Remark 4.5. Theorem 4.4 holds for $p = 5$ as well.

5. Concluding remarks

We have used some techniques of character theory to reduce the number of possibilities for $n_i$’s. The book [DL] deals a good portion of ordinary representation theory over finite field. From exercise at the end of §4, we have

Remark 5.1. Let $G$ be a finite group and $k$ is a field such that $\text{chark} \nmid |G|$. Assume $\{V_i : 1 \leq i \leq r\}$ to be full set of representatives of non-isomorphic irreducible $kG$-modules. Then $k$ is a splitting field of $G$ if and only if

$$|G| = \sum_{i=1}^{r} \dim_k(V_i)^2.$$
Using this we conclude that

**Remark 5.2.** For $G = \text{GL}(3, 2)$, the field $\mathbb{F}_q$, where $q = p^k$, where either $p = 5$ or $p \geq 11$ is a splitting field of $G$ if and only if $(p, k)$ is of type 1.

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