Fixed-Time Synergetic Approach for Biological Pest Control Based on Lotka-Volterra Model

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ABSTRACT Biological pest control has a strong advantage in its non-chemical effects on the environment. In this study, a fixed-time synergetic control scheme for the biological pest control problems represented by the \(n\)-dimensional Lotka-Volterra model was proposed. The proof of stability shows that the proposed controller can regulate the biological pest control systems with the characteristic of fixed-time convergence. The performance of the proposed scheme was demonstrated through simulation studies. The simulation results show that the pre-specified bound of the settling time can be satisfied regardless of the initial conditions, confirming a desired fixed-time convergence characteristic. Moreover, unlike the existing control policy based on the sliding mode control, the control inputs of the proposed policy are free from chattering.

INDEX TERMS Biological control, fixed-time stability, Lotka-Volterra model, nonlinear feedback control, pest regulation, synergetic control.

I. INTRODUCTION

The main concept of biological control is to use natural enemies to control population of plants and animals [1]–[4]. Employing natural enemies such as parasites, predators, and pathogens, helps reduce the damage caused by pests since the enemies can decrease the population of pests. A number of approaches for controlling pest populations are available for implementation, including conservation of local natural enemies, introduction of new natural enemies, and augmentations of natural enemies [1]–[4]. Previous works have shown that biological pest control is a challenging topic [1]–[7].

Researchers have investigated the stability and the control of the pest population dynamics using various mathematical representations [5]–[11]. The representation of the dynamics of prey-predator interaction is typically in the form of the Lotka Volterra model. In this model, pests and their natural enemies are considered as preys and predators, respectively [7], [11]–[13]. According to [15], dynamic optimization and feedback control allow the determining of biological control policies based on the mathematical models.

For the dynamic optimization, Goh [6] used the Lotka-Volterra model to represent the dynamic of the ecosystem containing one predator population and one pest population. He considered determining a control policy as a dynamic optimization problem and used dynamic programming to define the control policy by solving Hamilton Jacobi Bellman (HJB) equation. Molter and Rafikov [10] extended the use of the dynamic programming to obtain a control policy for an ecosystem with multiple species of predators and preys under multiple control inputs. This policy is applicable for biological pest control systems.

As for the feedback control, Meza et al. [9] proposed a nonlinear feedback control technique based on a control Lyapunov stability function to determine the control policy for a Lotka-Volterra model representing the one predator-one prey ecosystem. The control policy can stabilize the prey and predator populations at a coexistence equilibrium. Compared
with other techniques, including sliding mode, backstepping, and I&I control techniques, the technique proposed by Becerra et al. [9] is easier for implementation and requires only the information of the prey population. However, applying this technique to a multiple predator-multiple prey ecosystem is not straightforward since determining a control Lyapunov function is complicated. Rafikov et al. [7] presented a policy to control multiple species of pests using linear optimal feedback control for an ecosystem in the form of the Lotka-Volterra model. The model is an $n$-dimensional Lotka-Volterra system, including $p$ prey species and $p$ predator species. Based on the control policy by Rafikov et al., pest population densities converge to the desired levels. Additionally, the necessities to have multiple species of predators with corresponding control inputs become evident in controlling multiple species of pests. In [16], a controller based on tensor product model transformation (TPMT) was applied to set a control policy for a biological pest control system with two species of pests and two species of predators. The control policy was able to drive the pest population densities to the desired levels. The TPMT-based control policy was presented in a numerical form. Applying this technique for the fractional order biological pest control systems was also presented in [17]. The use of a single control input to control a single pest species for the biological pest control chain model involving $n$ populations was in the work by Puebla and colleagues [15], [18]. Puebla et al. [18] used super-twisting sliding mode control which reduced high frequency in the control input known as chattering phenomena. Furthermore, Puebla et al. [15] applied the model error compensate (MEC) control to formulate the control policy for the biological pest control chain model under uncertainties. As presented in [15], the feedback control allows a designer to formulate the pest control policy for an ecosystem under an inaccurate model caused by uncertainties of model parameters and disturbances efficiently. This is superior to the policy based on the dynamic optimization technique, which requires an accurate model. Thus, employing feedback control in this application is more suitable.

These previous works did not include the consideration of the convergence time of control ecosystems. As mentioned by Becerra et al. [19], convergence time plays an important role as an anticipated amount of time to achieve a certain objective of a control system. According to [20], in a finite-time stable system, the state variables of the system, where the initial condition of the state variables located in an open neighborhood, converge to the equilibrium at the origin in a finite time. The convergence time of the finite-time stable system is defined by settling time with its upper bound depending on the initial condition. Based on [21], applying a finite-time control provides the finite-time convergence property for a control system. In the finite-time control, a designer can specify the bound of settling time in advance. Then, controller parameters corresponding to the specified bound are selected. The finite time control is useful in formulating a pest control policy since the designer can set the control policy, which guarantees that the pest population densities converge to the desired levels within a desired time frame. This is achievable by defining the bound of settling time as the desired time frame. Even though Rafikov et al. [7] have presented the control policy for multiple pest species, the policy did not have the finite-time convergence property. Therefore, for higher versatility, it is beneficial to extend the concept of finite-time convergence to the biological pest control so that the policy becomes eligible for usage with multiple pest species using multiple control inputs. Apparently, difficulties in applying the nonlinear controller design for population control of an ecosystem represented by the Lotka-Volterra equations depend on control objectives and the appearance of the control inputs [7], [15], [18], [22]–[25]. Moreover, designing the controllers for the $n$-dimensional Lotka-Volterra system, which contains $p$ prey species and $p$ predator species with $p$ control inputs, is complex since it is a nonlinear multiple-input and multiple-output (MIMO) problem.

Therefore, the formulation of the control policy for multiple pest species requires a suitable nonlinear feedback control. The control policy needs to be applicable for high-order multiple-input and multiple-output systems. Additionally, it needs to be feasible to include the finite-time convergence property.

To achieve the aforementioned characteristics, the feedback control known as synergetic control [26]–[28], allows for the handling of nonlinear multiple-input and multiple-output problems in an efficient and effective way. This control method was introduced by Kolesnikov et al. [26]–[28]. As summarized by Kolesnikov et al. [29], Santi et al. [30], and Liu and Hsiao [31], the design procedure of the synergetic control consists of three following steps. The first step involves the selection of variables named macro variables, and the construction of a manifold according to the control objectives and control inputs considered as external controls. As presented by Veselov et al. [32], the error of a state variable and its desired reference signal is a feasible choice of the macro variables. The manifold construction is achieved by setting the selected macro variables equal to zero. In the second step, a designer defines a differential equation for constraining each macro variable known as a dynamic evolution. This dynamic constraint defines the convergence rate of each macro variable to reach its manifold. The last step is determining all control inputs to satisfy all the dynamic evolutions.

When the control inputs cannot directly affect the target state variables, the control law based on the synergetic control can be constructed from external and internal controls [28], [32]. The previous work by Veselov et al. [32] confirmed the effectiveness of the synergetic controller including both external controls and internal controls applying to a quadrotor which is a high-order multiple-input and multiple-output system.

Likewise, in Rafikov et al.’s ecosystem [7], the multiple control inputs directly affect the growth rates of the predator population densities, whereas the control objective is to
regulate the pest population densities. The internal controls refer to the required population densities of the predators used to control the pest population densities. The external control refers to each control input which affects each growth rate of the predator. The synergetic control method is applicable for various dynamical systems such as power systems [30], [33]–[37], mechanical systems [38]–[40], robotics [31], [32], and aircraft systems [41]. Boonyaprapasorn et al. [8] demonstrated feasibility in employing the synergetic control to formulate the biological pest control policy for the model which contains one predator species and one pest species as presented in [8]. However, they did not address the convergence time of the control ecosystem.

To fill in the aforementioned gap, the use of the synergetic control with finite-time convergence, as presented in [31] is beneficial. However, the bound of settling time of the control system depends on initial conditions. Thus, the specification of the bound of settling time may become inaccurate, if the information of the initial conditions is inaccurate or unavailable [42], [43]. Consequently, a designer may fail to pre-specify the accurate bound of settling time for the control system. The accurate tuning of the suitable parameters of the control policy may not be possible.

To address this, Polyakov [42] introduced the concept of fixed-time convergence to enhance the finite-time one. The fixed-time convergence property allows a designer to estimate or pre-specify the bound of settling time without the information of the initial conditions. According to [21] and [19], the fixed-time convergence requires a stronger condition of the bound of settling time since it requires the bound of settling time to be uniform globally. Different convergence times corresponding to each initial condition are not greater than this uniform bound. This property allows a designer to determine a control policy with the pre-specified bound of settling time regardless of the initial conditions. Nowadays, many researchers have become interested in the fixed-time control as seen in its application in various dynamical systems such as mechanical systems, [44], [45], multi-agent systems [46], [47], power systems [48]–[50], robotics [51], aircraft systems [52], and neural networks [43], [53]. It is feasible for the synergetic control to have the fixed-time convergence characteristic as presented by Wang et al. [54] and Al-Hussien et al. [55]. Wang et al. [54] applied the fixed-time synergetic control to the power system for controlling of chaotic oscillations which occur in the system. In [55], plasma glycemic level of the diabetic patient was controlled to the desired level by using the fixed-time synergetic controller.

According to [30]–[33], [40], and [41], the chattering-free characteristic can be acquired by using synergetic control. With this characteristic, the control policy of the biological pest control system can be implemented in practical situation since the control inputs do not oscillate with high frequency, thus, making synergetic control an advantage over the sliding mode control.

The main focus of this study is to formulate the control policy for multiple pest species with the pre-specified settling time using feedback control. The control policy is formulated based on the biological pest control system presented by the Lotka-Volterra model with n-total population under multiple control inputs. Thus, a control scheme obtained by the fixed-time synergetic control method for the biological pest control systems was proposed. To the best of the authors’ knowledge, applying the fixed-time synergetic control to this biological pest control problem has not been presented. The proposed control policy has superior characteristics as follows: (i) It is capable of controlling multiple pest species using multiple control inputs for the biological system represented by an n-dimensional Lotka-Volterra model. Except for the linear optimal control policy presented by Rafikov et al. [7], most control policies in analytical forms based on the feedback control in previous works such as [9], [15], and [18] can only be employed for ecosystems with a single pest species using single control input. (ii) It has the fixed-time convergence characteristic. Consequently, the time frame required to control the pest population densities to converge to the desired levels can be estimated or specified in advance based on the pre-specified bound of settling time regardless of the initial conditions. However, for the control policy presented by Rafikov et al. [7], the bound of settling time cannot be pre-specified. (iii) It is free from chattering, unlike the control policy based on the sliding mode control for the predator and prey system [9], [18].

The remaining of this paper is organized as follows. In Section II, some necessary preliminaries are provided. Then, the derivation of the proposed control scheme is presented in Section III. The simulation results of the control systems and discussion are provided in Section IV. Finally, concluding remarks are given in Section V.

II. PRELIMINARIES

A. THEOREM AND LEMMA

Theorem 1 ([43], Corollary 2): Consider dynamical systems represented by the differential equation

\[ \dot{x}(t) = f(x(t)), \quad x(0) = x_0, \]  

where \( x \in \mathbb{R}^n \) is the state vector and \( f : \mathbb{R}^n \to \mathbb{R}^n \) is a continuous vector function. Suppose that there exists a function \( V : \mathbb{R}^n \to \mathbb{R} \), such that \( V(x(t)) \) is regular, positive definite and radially unbound and it satisfies the following inequality

\[ V' \leq -aV + bV^\theta, \quad x(t) \in \mathbb{R}^n \backslash \{0\} \]  

for any solution \( x(t) \) of the system in (1) where \( a > 0, b > 0 \) and \( \delta > 1, 0 \leq \theta \leq 1 \). Then, the zero equilibrium of system in (1) is fixed-time stable with the bound of settling time \( T(x_0) \) given by

\[ T(x_0) \leq T_s = \frac{1}{b} \left( b \frac{1-\theta}{\theta} \right)^{1-\theta} \left( \frac{1}{1-\theta} + \frac{1}{\delta-1} \right). \]
Lemma 2 ([56], Lemma 2.3; [46], Lemma 3.4): If \( y_1, y_2, \ldots, y_n \) are positive real numbers and the exponent term, \( r \), is a positive real number, then the inequality in (4) is satisfied:

\[
\max(n^{r-1}, 1)(y_1^r + y_2^r + \ldots + y_n^r) \geq (y_1 + y_2 + \ldots + y_n)^r.
\]  

(4)

B. MODEL

The dynamic of a biological pest control system with total population \( n \) including the interaction between \( p \) species of pest population densities and \( p \) species of predator population densities can be represented by a Lotka-Volterra model [7], [10], [12], [13]:

\[
x'_q = x_q \left( r_q \sum_{j=1}^{p} a_{q,j} x_j - \sum_{j=p+1}^{n} a_{q,j} x_j \right),
\]

\[ q = 1, 2, \ldots, p, \]  

(5)

\[
x'_{p+k} = x_{p+k} \left( r_{p+k} \sum_{j=1}^{p} a_{p+k,j} x_j \right) + u_k,
\]

\[ k = 1, 2, \ldots, p, \]  

(6)

where \( x_1, x_2, \ldots, x_p \) represent the population densities of pest or prey species, and \( x_{p+1}, x_{p+2}, \ldots, x_n \) represent the population densities of predator species at time \( t \). For each \( i^{th} \) species, \( r_i \) denotes the rate of reproduction or mortality, whereas \( a_{i,j} \) represents one of rates associated with predation, competition, and conversion rates. The variables, \( u_1, u_2, \ldots, u_p \), denote the set of control inputs at time \( t \) affecting the growth rate of each predator. In accordance with [7], the variable \( n \) can be set as \( n = 2p \).

III. CONTROL POLICY DESIGN

A. OBJECTIVE

The objective of the pest control policy is to regulate the pest population densities to the desired levels based on economic criterion [7], [11]. The economic injury level is the minimum population density of pest that can cause economic damage. Crop producers set this level based on pest management cost, product market price, crop damage, and crop response [11], [57], [58].

The desired level \( x_{ir} \) for \( i = 1, 2, \ldots, p \) was defined to ensure that the population density of each pest always remains below the economic injury level after a given pre-specified bound of settling time \( T_x > 0 \) [7], [11]:

\[
\lim_{t \to T_x} x_i(t) = x_{ir},
\]

(7)

and

\[
x_i(t) = x_{ir}, \quad \forall t \geq T_x, \quad \text{for} \quad i = 1, 2, \ldots, p.
\]

(8)

B. PEST CONTROL POLICY

The procedure for formulating the pest control policy based on the synergetic controller design in [28], [29], [30]–[33] is presented as follows:

1) Select the macro variables in accordance with the control objectives and the control inputs. Define two sets of macro variables as follows. The first set of macro variables corresponding to the control objective is selected according to [28], [30], [32], and [33] as

\[
\psi_i = x_i - x_{ir}, \quad i = 1, 2, \ldots, p.
\]

(9)

Next, the second set of macro variables is chosen as

\[
\psi_{p+k} = x_{p+k} - \varphi_{p+k}, \quad k = 1, 2, \ldots, p,
\]

(10)

where \( \varphi_{p+k} \) is the required level of the corresponding predator population density to drive the pest population densities \( x_i \) to the corresponding desired level \( x_{ir} \). The term \( \varphi_{p+k} \), named by Kolesnikov [28], is referred to as internal controls in the synergetic control.

2) Define the dynamic evolutions constraining macro variables. The \( k^{th} \) dynamic evolution is

\[
\psi'_k = -\gamma_k \psi_k^g - \eta_k \psi_k^h,
\]

(11)

where \( g, h, l, \) and \( m \) are positive odd numbers, and \( \gamma_k \) and \( \eta_k \) are positive real numbers for \( i = 1, 2, \ldots, n \). The selected dynamic evolutions own the fixed-time convergence property [44], [46], [54], [55].

3) Utilize (5) to solve for the required internal control \( \varphi_{p+k} \) defined in (10) such that the macro variables in (9) satisfy the dynamic evolution in (11). Letting \( x_{p+k} = \varphi_{p+k} \) in (5) yields

\[
x'_q = x_q \left( r_q \sum_{j=1}^{p} a_{q,j} x_j - 2p \sum_{j=p+1}^{n} a_{q,j} \psi_j \right), \quad q = 1, 2, \ldots, p.
\]

(12)

Next, solve for \( \varphi_{p+k} \) to satisfy the first \( p \) dynamic evolutions as

\[
\psi'_q = -\gamma_q \psi_q^g - \eta_q \psi_q^h, \quad q = 1, 2, \ldots, p.
\]

(13)

Then, substituting \( \psi'_q = x'_q - x'_{qr} \) from (9) into (13) gives

\[
x'_q - x'_{qr} = -\gamma_q \psi_q^g - \eta_q \psi_q^h, \quad q = 1, 2, \ldots, p.
\]

(14)

Equation (14) can be determined by using (12) as (15):

\[
\sum_{j=p+1}^{n} a_{q,j} \psi_j = \sum_{k=1}^{p} a_{q,p+k} \varphi_{p+k}
\]

\[
= x_q^{-1} \left[ x_q \left( r_q \sum_{j=1}^{p} a_{q,j} x_j \right) - x'_{qr} + \gamma_q \psi_q^g + \eta_q \psi_q^h \right].
\]

(15)

Equation (15) can be expressed as

\[
A \varphi = A
\]

(16)
In order to ensure the fixed-time stability of the system in (5) and (6) over time, we modify the control inputs in (22) by adding $v_k$ for $k = 1, 2, \ldots, p$:

$$
u_k = \left[ -x_{p+k} \left( r_{p+k} - \sum_{j=1}^{p} a_{p+k,j} \bar{x}_j \right) + \psi'_{p+k} - \left( \gamma_{p+k} \psi_{p+k}^k + \eta_{p+k} \psi_{p+k}^l \right) \right] + v_k, \quad (23)$$

where

$$v_k = x_k \sum_{q=1}^{p} a_{k+p+q} \psi_q, \quad k = 1, 2, \ldots, p. \quad (24)$$

The following theorem gives the stability of the control system.

**Theorem 3**: The control inputs in (23) and (24) can fixed-time regulate the pest population densities of the biological pest control system in (5) and (6) with the uniform bound of settling time $T(\psi_0)$ as

$$T(\psi_0) \leq T_{\min} = \frac{1}{n_{\min}} \left( \frac{\eta_{\min}}{\bar{\eta}_{\min}} \right)^{\frac{1}{1-\theta} + \frac{1}{\delta - 1}}.$$  \quad (25)

where $\bar{\eta}_{\min} = (2^{(l+m)/2m})^{\min((2p)^{e^{l+1}/2m}-1, 1)}$, $\eta_{\min} = (2^{l/m} \eta_{\min})^{\max((2p)^{e^{l+1}/2m}-1, 1)}$, $\theta = (l + m)/(2m)$, and $\delta = (g + h)/(2h)$.

Proof of Theorem 3 is in Appendix.

**IV. SIMULATION EXAMPLE**

In order to present the capability and to evaluate the performance of the fixed-time synergetic control policy, we applied the proposed policy to the two-predator and two-prey and the one-predator and one-prey Lotka-Volterra models representing the biological control systems. These systems were presented by Rafikov et al. [7]. First, we evaluated the fixed-time convergence characteristic of the proposed control and showed the capability of controlling multiple pest species by applying the proposed control policy to the first model. Then, we compared the simulation results of the proposed control policy with those of the linear optimal control policy in [7] as shown in Section IV A. Second, for simplicity, we applied the proposed control policy to the second model to present the chatter-free characteristic of the proposed control policy and compared the simulation results with those of the sliding mode control policy in [61] and [62] as presented in Section IV B. All simulations were conducted by Simulink in MATLAB software.

**A. TWO-PREY AND TWO-PREDATOR BIOLOGICAL PEST CONTROL SYSTEM**

The control policy using control inputs in (23) and (24) was applied to the biological pest control problem presented in Rafikov et al. [7] and Molter and Rafikov [10]. In their model, the pests are two types of soybean caterpillars (Rachiplusia nu and Pseudoplusia includes) which are considered as the first
and the second species of the pests. According to [7] and [10], two candidate parasitoids are available as predators. Thus, we assumed that the candidate parasitoids of the model are the parasitoid A and the parasitoid B. Referring to [59], various dipterans and hymenopterans belonging to different families such as the Tachinidae family: Euphorocera sp., Lespezia sp., and Voria ruralis, Ichneumonidae family: Campopleis sonorensis, Casinaria plusiae, and Microbrachis binmaccuata, Braconidae family: Cotesia gregnensis and Meteorus sp., and Encyrtidae family: Copidosoma floridanus, can be selected as candidate parasitoids for both caterpillars of the model. From (5) and (6), the interaction between caterpillars and parasitoids is governed by

\[
x_1' = x_1 (r_1 - a_1,1 x_1 - a_1,2 x_2 - a_1,3 x_3 - a_1,4 x_4),
\]

\[
x_2' = x_2 (r_2 - a_2,1 x_1 - a_2,2 x_2 - a_2,3 x_3 - a_2,4 x_4),
\]

\[
x_3' = x_3 (-r_3 + a_3,1 x_1 + a_3,2 x_2) + u_1,
\]

\[
x_4' = x_4 (-r_4 + a_4,1 x_1 + a_4,2 x_2) + u_2.
\]

The state variable $x_1$ represents the population density of the first caterpillar ($Rachiplusia mo$); the state variable $x_2$ represents the population density of the second caterpillar ($Pseudoplusia includes$). The pest population densities are in the unit of (pests/m$^2$) at time $t$ (day). The state variable $x_3$ denotes the population densities of the first parasitoid; the state variable $x_4$ denotes the population densities of the second parasitoid. The parasitoid population densities are in the unit of (parasitoids/m$^2$) at time $t$ (day). The first control input $u_1$ affects the first parasitoid population density; the second control input $u_2$ affects the second parasitoid population density at time $t$ (day).

Using (23) and (24), we obtain the pest control policy according to biological control as

\[
u_1 = -x_3 (-r_3 + a_3,1 x_1 + a_3,2 x_2) + w_3' - w_3 + v_1,
\]

\[
u_2 = -x_4 (-r_4 + a_4,1 x_1 + a_4,2 x_2) + w_4' - w_4 + v_2,
\]

where

\[
w_3 = \gamma_3 \psi_3 + \eta_3 \psi_\frac{m}{l},
\]

\[
w_4 = \gamma_4 \psi_4 + \eta_4 \psi_\frac{m}{l},
\]

\[
\varphi_3' = \frac{a_3,2}{a_3,1} \xi_1' - \frac{a_1,4}{a_1,3 a_2,4 - a_1,4 a_2,3} \xi_2',
\]

\[
\varphi_4' = -\frac{a_4,3}{a_4,2} \xi_1' + \frac{a_2,1}{a_2,3 a_4,2 - a_2,4 a_3} \xi_2',
\]

\[
\xi_1' = \frac{1}{x_1^2} \left[ x_1 (x_1 (-a_1,1 x_1' - a_1,2 x_2') + x_1' (r_1 - a_1,1 x_1 - a_1,2 x_2) - x_1'' + w_1') - (x_1 (r_1 - a_1,1 x_1 - a_1,2 x_2) - x_1'' + w_1) x_1' \right],
\]

\[
\xi_2' = \frac{1}{x_2^2} \left[ x_2 (x_2 (-a_2,1 x_1' - a_2,2 x_2') + x_2' (r_1 - a_2,1 x_1 - a_2,2 x_2) - x_2'' + w_2') - (x_2 (r_2 - a_2,1 x_1 - a_2,2 x_2) - x_2'' + w_2) x_2' \right],
\]

The parameters of the model in (26)-(29) and their corresponding units presented in [7], [10], and [60] were used for simulation. Then, they are summarized as shown in Table 1.

| Parameter | Quantity | Unit |
|-----------|----------|------|
| $r_1$     | 0.17     | day$^{-1}$ |
| $r_2$     | 0.17     | day$^{-1}$ |
| $r_3$     | 0.17     | day$^{-1}$ |
| $r_4$     | 0.17     | day$^{-1}$ |
| $a_{1,1}$ | 0.00017  | m$^2$/pests-day |
| $a_{1,2}$ | 0.00017  | m$^2$/pests-day |
| $a_{1,3}$ | 0.00017  | m$^2$/parasitoids-day |
| $a_{1,4}$ | 0.00085  | m$^2$/parasitoids-day |
| $a_{2,1}$ | 0.000255 | m$^2$/pests-day |
| $a_{2,2}$ | 0.00017  | m$^2$/parasitoids-day |
| $a_{2,3}$ | 0.00017  | m$^2$/parasitoids-day |
| $a_{2,4}$ | 0.00085  | m$^2$/pests-day |
| $a_{3,1}$ | 0.00085  | m$^2$/pests-day |
| $a_{3,2}$ | 0.00085  | m$^2$/pests-day |
| $a_{3,3}$ | 0.00425  | m$^2$/pests-day |
| $a_{3,4}$ | 0.00425  | m$^2$/pests-day |

The parameters of the model in (26)-(29) and their corresponding units presented in [7], [10], and [60] were used for simulation. Then, they are summarized as shown in Table 1.

In accordance with [7] and [10], the desired levels of the pest population densities were set as $x_1r = 9$ (pests/m$^2$). These values correspond to the total population density of both caterpillars which is below the economic injury level of 20 (pests/m$^2$). Consequently, we obtained the corresponding values of $x_3 = 51.92$ (parasitoids/m$^2$) and $x_4r = 92.56$ (parasitoids/m$^2$) from (26) and (27) with $x_1' = x_4' = 0$ and $x_1r = x_4r = 9$ pests/m$^2$. We specified the bound of the settling time corresponding to (26)-(29) as $T_s = 50$ days.

The parameters of the model in (26)-(29) and their corresponding units presented in [7], [10], and [60] were used for simulation. Then, they are summarized as shown in Table 1.

### Table 1. System parameters of the simulation example.

The procedure in determining the parameters of $u_1(t)$ and $u_2(t)$ in (30) and (31) consists of three steps as follows. The first step is the selection of exponent terms in (32)-(33). According to [31] and [46], we selected odd numbers $g$ and $h$ satisfying $1 < g/h < 2$ as $g = 7$ and $h = 5$. Then, we chose odd numbers $l$ and $m$ satisfying $0 < l/m < 1$ as $l = 3$ and $m = 5$. The second step is determining $\tilde{\gamma}_{\text{min}}$ and $\tilde{n}_{\text{min}}$ for $T_s = 50$ days from (25). Referring to [46], $\tilde{\gamma}_{\text{min}}$ influences the convergence of $\psi_i$ for $i = 1, 2, 3, 4$ during $|\psi_i| > 1$, whereas $\tilde{n}_{\text{min}}$ has a higher effect on the convergence rate when $|\psi_i| < 1$. Thus, $\tilde{\gamma}_{\text{min}} = \tilde{\gamma}_{\text{min}} = 0.2$. The last step is determining the coefficients $\gamma_i$ and $\eta_i$ for $i = 1, 2, 3, 4$. The minimum values of $\gamma_i$ and $\eta_i$ could be determined from
For simplicity, the coefficients $\gamma_i$ and $\eta_i$ were defined as $\gamma_i = \gamma_{\text{min}} \text{day}^{-1}$ and $\eta_i = \eta_{\text{min}} \text{day}^{-1}$ for $i = 1, 2, 3, 4$.

In order to evaluate the performance corresponding to the fixed-time convergence characteristic of the proposed control policy, the simulation results of the proposed control policy were compared with those of the linear optimal control policy based on feedback control presented by Rafikov et al. [7].

According to [7], the input vector $u = [u_1, u_2]^T$ of the linear optimal control policy for the biological control system in (26)-(29) consists of two parts. The first part is the required control input vector $u_r = [u_{1r}, u_{2r}]^T$ corresponding to the equilibrium point. The second part is the control input vector $\bar{u} = [\bar{u}_1, \bar{u}_2]^T$ of the dynamic system of the error vector $y$ corresponding to the system in (26)-(29). Thus, the linear optimal control input vector in (26)-(29) can be expressed as

$$u = u_r + \bar{u}. \quad (42)$$

The control input vector $\bar{u}$ is synthesized to regulate the error which can be obtained as (43) [7]:

$$\bar{u} = -R^{-1}B_T Py, \quad (43)$$

where the dynamic error vector $y$ is denoted as $y = [y_1, y_2, y_3, y_4]^T = [x_1 - x_{1r}, x_2 - x_{2r}, x_3 - x_{3r}, x_4 - x_{4r}]^T$. The input matrix $B$ of the dynamic system of the error vector is defined as

$$B = \begin{bmatrix} 0_{4 \times 4} \\ I_{4 \times 4} \end{bmatrix}, \quad (44)$$

where $I$ is the identity matrix. The positive definite symmetric matrix $P$ is a solution of the Ricatti equation as

$$PA + A^TP - PBR^{-1}B_T P + Q = 0, \quad (45)$$

where $Q$ is a positive symmetric matrix, and $R$ is a positive definite. The system matrix $A$ is defined in [7].

Based on the parameters in Table 1 and the desired levels of the pest population densities, the components of control input vector $u_{1r}$ and $u_{2r}$ at the steady state could be calculated from the biological pest control system (26)-(29). For fair comparison, we selected the matrices $Q$ and $R$ so that the transient and steady state responses of the linear optimal control system had similar behaviors to those of the fixed-time synergetic control system. Thus, the matrices $Q$ and $R$ were selected as $Q = \text{diag}(Q_{ii})$ and $R = \text{diag}(R_{ii})$ where $Q_{ii} = 1$ and $R_{ii} = 1$ for $i = 1, \ldots, 4$.

For further details about the linear optimal control policy for the biological control system (26)-(29), readers can refer to Rafikov et al. [7].

In all simulations, Runge-Kutta method was used as numerical integration from $t = 0$ to $t = 250$ days with the time step of 0.005 day.

To show the superior performance of the fixed-time convergence of the proposed control policy, two different initial conditions, $x(0) = [15 \ 15 \ 55 \ 95]^T$ and $x(0) = [15 \ 15 \ 1 \ 1]^T$ were considered in simulation. Both conditions represent situations when the initial population densities of pests are higher than their desired levels. However, the former condition corresponds to the situation when the initial population density of each predator is greater than that of each pest, whereas the latter condition corresponds to the situation when the initial population density of each predator is less than that of each pest.

The simulation results of the fixed-time synergetic control and those of the linear optimal control policies are presented in the black solid lines and the red dash lines, respectively as shown in Fig. 1–Fig. 4.

For the first initial condition as shown in Fig. 1, the population densities of both caterpillars corresponding to the fixed-time synergetic control policy and those corresponding to the linear optimal control policy converged to their desired levels. However, the population densities of the parasitoids of the fixed-time synergetic control policy converged faster than those of the linear optimal control. The population densities of both parasitoids of the fixed-time synergetic control policy and those of the linear control policy increased to the maximum levels before they decreased and converged to the desired level. Fig. 2 shows that the first control input of the fixed-time synergetic control policy and that of the linear optimal control policy decreased from their corresponding initial values and converged. The second control input of the fixed-time synergetic control policy and that of the linear optimal control decreased from the initial values. Then, they increased and converged.

For the second initial condition as shown in Fig. 3, the population densities of the two caterpillars corresponding to the fixed-time synergetic and those corresponding to the linear optimal control policy increased to the maximum values at the beginning. Then, they decreased and converged to their desired levels. However, the maximum population densities of the linear optimal control were higher than those of the fixed-time synergetic control policy. Likewise, the population densities of the two parasitoids of both control policies increased to the maximum levels before they converged. However, both population densities of the parasitoids corresponding to the fixed-time synergetic control policy had the higher maximum values when compared with those corresponding to the linear optimal control policy. In Fig. 4, the first control inputs of the fixed-time synergetic control policy and that of the linear optimal control reduced from their corresponding values and converged. The second control inputs of the fixed-time synergetic control policy and that of the linear optimal control policy decreased to the minimum values before they increased and converged.

Based on the simulation results, the fixed-time synergetic control policy effectively manipulated the population densities of the caterpillars to the desired levels within the pre-specified bound of the settling time. Convergence times from the two cases of the initial conditions are different. Still, they are less than this pre-specified bound. Moreover, with the pre-specified bound of settling time regardless of the initial conditions, tuning of the control policy parameters is systematic. However, the uniform bound of the convergence times...
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FIGURE 1. Time responses of state variables representing population densities: *Rachiplusia nu* (*x_1*(t)), *Pseudoplusia inclina* (*x_2*(t)), parasitoid A (*x_3*(t)), and parasitoid B (*x_4*(t)) for the first initial condition, *x*(0) = [15 15 55 95]^T.

FIGURE 2. The control inputs *u_1*(t) and *u_2*(t) for the first initial condition, *x*(0) = [15 15 55 95]^T.

...of the linear optimal control system cannot be pre-specified since the control system only satisfies asymptotic stability.

The fixed-time synergetic control policy is superior to the linear optimal control policy in terms of the convergence characteristic. Consequently, the time frame required for driving the pest population densities to converge to the level below the economic injury levels can be approximately determined based on the pre-specified bound of settling time,
FIGURE 3. Time responses of state variables representing population densities: *Rachiplusia nu* ($x_1(t)$), *Pseudoplusia includes* ($x_2(t)$), parasitoid ($A x_3(t)$), and parasitoid *B* ($x_4(t)$) for the second initial condition, $x(0) = [15\ 15\ 1\ 1]^T$.

FIGURE 4. The control inputs $u_1(t)$ and $u_2(t)$ for the second initial condition, $x(0) = [15\ 15\ 1\ 1]^T$.

Even when the initial conditions are unknown or inaccurate, however, this beneficial characteristic is not achievable by using the linear optimal control policy. Thus, the implementation of the proposed biological pest control policy is promising in practical situations, especially when the initial conditions are inaccurate or unknown.
where synergetic control policy as control policies were compared. the system in (46) and (47). Then, the simulations of both policy and the sliding mode control policy were applied to characteristic of the proposed control policy, the proposed control was employed to synthesize the sliding mode control

According to [9], [61], and [62], backstepping sliding mode and one-prey Lotka-Volterra model as (46) and (47) [7]:

\[ x_1' = x_1(r_1 - a_{11}x_1) - a_{12}x_1x_2, \] (46)
\[ x_2' = x_2(-r_2 + a_{21}x_1) + u_1, \] (47)

where the state variables \( x_1 \) and \( x_2 \) are the population densities of the pest and the predator respectively. The control input is denoted by \( u_1 \).

In order to show the superior of the chattering-free characteristic of the proposed control policy, the proposed control policy and the sliding mode control policy were applied to the system in (46) and (47). Then, the simulations of both control policies were compared.

Based on (23) and (24), we can determine the fixed-time synergetic control policy as

\[ u_1 = -w_2 + \varphi_2' - x_2(-r_2 + a_{21}x_1) + a_{12}x_1 \psi_1, \] (48)

where

\[ \varphi_2 = (-a_{12}x_1)^{-1}(-w_1 - [x_1(r_1 - a_{11}x_1) - x_1']). \] (49)
\[ w_1 = \gamma_1 \psi_1^g + \eta_1 \psi_1^f, \] (50)

and

\[ w_2 = \gamma_2 \psi_2^g + \eta_2 \psi_2^f. \] (51)

According to [9], [61], and [62], backstepping sliding mode control was employed to synthesize the sliding mode control policy. The sliding mode control policy for the biological control system (54) and (55) can be obtained as (52):

\[
\begin{align*}
\dot{u}_{1,SMC} & = -\beta_1 s - \lambda_s \cdot \text{sign}(s) - x_2(-r_2 + a_{21}x_1) + a_{12}x_1z_1 \\
& + [(-a_{12}x_1)^{-1}[-\alpha_2z'_2 - x_1(-a_{11}x_1') - x_1'(r_1 - a_{11}x_1) + x_1'''] \\
& - [-\alpha_1z_1 - x_1(r - a_{11}x_1) + x_1'])(a_{12}x_1)^{-1}.
\end{align*}
\] (52)

where the error \( z_1 \) is denoted as \( z_1 = x_1 - x_1r \). The sliding surface \( s \) is defined as (53):

\[ s = x_2 - (-a_{12}x_1)^{-1}[-\alpha_2z_1 - x_1(r_1 - a_{11}x_1) + x_1']. \] (53)

The system parameters from Rafikov et al. [7], which are \( r_1 = 0.17, r_2 = 0.119, a_{11} = 0.0003825, a_{12} = 0.000935, \) and \( a_{21} = 0.000935, \) were used for simulation. The initial condition was selected as \( x_1(0) = 25 \) (pests/m²) and \( x_2(0) = 170 \) (parasitoids/m²). The control parameters of the fixed-time synergetic control were defined based on the pre-specified bound of the settling time of \( T_s = 50 \) days for \( g = 7, h = 5, l = 3, m = 5, \) and \( \bar{\eta} = 0.2. \) The desired value of the pest population density was selected as \( x_1r = 20 \) (pests/m²) [7], [18]. The parameters of the sliding mode control policy were chosen as \( \alpha_z = 0.15, \beta_z = 0.1 \) and \( \lambda_z = 10 \) so that the convergence occurred within 50 days.

Readers can find more details about backstepping sliding mode control for the biological control system in (46) and (47) from [9], [61], and [62].

Runge-Kutta method was used as numerical integration from \( t = 0 \) to \( t = 250 \) days with the time step of 0.005 day.

The time responses of the proposed control policy and those of the sliding mode control policy are presented in black solid lines and the red dash lines respectively as shown in Fig. 5. In Fig. 6, the control input of the proposed control policy is presented in the black solid line while that of the sliding mode control policy is presented in the red solid line.
The time responses of the pest population densities could be manipulated by both control policies to the desired level within the bound of the settling time of 50 days as shown in Fig. 5. The parasitoid population densities of both control policies converged to the corresponding steady state level as shown in Fig. 5. However, the control input of the sliding mode control policy had chattering phenomena, while the control input of the fixed-time synergetic control policy was free from chattering. The simulation results confirmed that using the fixed-time synergetic control policy for this application can solve the problem caused by the chattering phenomena in the sliding mode control policy.

The proposed control policy with the chattering-free characteristic is superior to the sliding mode control policy since it can be implemented without high frequency in the control input of the policy [30], [33], [41].

V. CONCLUSION

A control policy for the biological pest control systems represented by the $n$-dimensional Lotka-Volterra model was formulated by using the synergetic feedback controller design. In the controller design, two sets of macro variables corresponding to pest and predator population densities were defined. The dynamic evolutions with the fixed-time convergence property of macro variables were selected so that the population densities of pests converge to the desired levels before the pre-specified bound of settling time regardless of the initial conditions. The synthesis of the control policy is simple without applying any transformation to the system. The policy is also applicable for controlling multiple species of pests. The chattering-free characteristic can be achieved by using the proposed control policy. Simulation results confirmed that the proposed control policy provides better performances in terms of convergence and chattering-free characteristics.

APPENDIX

A. PROOF OF THEOREM 3

The Lyapunov function is selected as

$$V(\psi) = \frac{1}{2} \psi^T \psi,$$

where $\psi = [\psi_1 \psi_2 \ldots \psi_{2p}]^T$. From (54), $V'(\psi)$ can be obtained as

$$V'(\psi) = \psi^T \psi' = \sum_{i=1}^{2p} \psi_i \psi'_i = \sum_{q=1}^{p} \psi_q \psi'_q + \sum_{k=1}^{p} \psi_{p+k} \psi'_{p+k}.$$  

Equation (55) can be written in vector form as

$$V'(\psi) = [\psi_1 \psi_2 \ldots \psi_p] \begin{bmatrix} \psi'_1 \\ \psi'_2 \\ \vdots \\ \psi'_p \\ \psi'_{p+1} \\ \psi'_{p+2} \\ \vdots \\ \psi'_{2p} \end{bmatrix} + [\psi_{p+1} \psi_{p+2} \ldots \psi_{2p}] \begin{bmatrix} \psi'_{p+1} \\ \psi'_{p+2} \\ \vdots \\ \psi'_{2p} \end{bmatrix}.$$  

Substituting the macro variables in (9) and (10) into (56) gives

$$V'(\psi) = \begin{bmatrix} x'_1 - x'_{1r} \\ x'_2 - x'_{2r} \\ \vdots \\ x'_{p} - x'_{pr} \end{bmatrix} + \begin{bmatrix} \psi'_{p+1} \\ \psi'_{p+2} \\ \vdots \\ \psi'_{2p} \end{bmatrix}.$$  

FIGURE 6. The control inputs $u_1(t)$ for the initial condition, $x(0) = [25 \ 170]^T$. 

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From the dynamic models (5) and (6), \( V'(\psi) \) can be determined as

\[
V'(\psi) = \begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\vdots \\
\dot{x}_p
\end{bmatrix} = \begin{bmatrix}
x_1 \left( r_1 - \sum_{j=1}^{p} a_{1,j} x_j \right) - x'_1 \ 
\dot{x}_2 \\
\dot{x}_2 \\
\vdots \\
\dot{x}_p
\end{bmatrix}
\]

Substituting \( x_{p+k}(t) \) with \( \psi_{p+k}(t) + \psi_{p+k}(t) \) for \( k = 1, 2, \ldots, p \) into (58), we obtain

\[
V'(\psi) = \begin{bmatrix}
\psi_1 & \psi_2 & \cdots & \psi_p
\end{bmatrix}
\]

\[
\times \begin{bmatrix}
x_1 \left( r_1 - \sum_{j=1}^{p} a_{1,j} x_j \right) - x'_1 \\
\dot{x}_2 \\
\vdots \\
\dot{x}_p
\end{bmatrix}
\]

Rewriting (58) gives

\[
V'(\psi) = \begin{bmatrix}
\psi_1 & \psi_2 & \cdots & \psi_p
\end{bmatrix}
\]

\[
\times \begin{bmatrix}
x_1 \left( r_1 - \sum_{j=1}^{p} a_{1,j} x_j \right) - x'_1 \\
\dot{x}_2 \\
\vdots \\
\dot{x}_p
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
\psi_{p+1} & \psi_{p+2} & \cdots & \psi_{2p}
\end{bmatrix}
\]

\[
\times \begin{bmatrix}
\dot{x}_{p+1} \\
\dot{x}_{p+2} \\
\vdots \\
\dot{x}_{2p}
\end{bmatrix}
\]

\[
\sum_{k=1}^{p} a_{p+1,j} x_j
\]
We obtain \( \varphi = [\varphi_{p+1}, \varphi_{p+2}, \ldots, \varphi_p]^T \) from (16) as

\[
\begin{bmatrix}
\varphi_{p+1} \\
\varphi_{p+2} \\
\vdots \\
\varphi_p
\end{bmatrix} = \mathbf{A}^{-1}
\begin{bmatrix}
x_{p+1} \\
x_{p+2} \\
\vdots \\
x_p
\end{bmatrix},
\]

where \( \mathbf{A} = \begin{bmatrix}
a_{1,p+1} & a_{1,p+2} & \cdots & a_{1,2p} \\
a_{2,p+1} & a_{2,p+2} & \cdots & a_{2,2p} \\
\vdots & \vdots & \ddots & \vdots \\
a_{p,p+1} & a_{p,p+2} & \cdots & a_{p,2p}
\end{bmatrix} \). Substituting \( u_k(t) \) from (23) and \( \varphi_{p+k}(t) \) for \( k = 1, 2, \ldots, p \) from (61) into (60) gives

\[
V'(\psi) = \begin{bmatrix}
\psi_1 & \psi_2 & \ldots & \psi_p
\end{bmatrix}
\begin{bmatrix}
x_1 \left( r_1 - \sum_{j=1}^{p} a_{1,j}x_j \right) - x_1' \\
x_2 \left( r_2 - \sum_{j=1}^{p} a_{2,j}x_j \right) - x_2' \\
\vdots \\
x_p \left( r_p - \sum_{j=1}^{p} a_{p,j}x_j \right) - x_p'
\end{bmatrix}
\]

\[
- \begin{bmatrix}
\psi_1 & \psi_2 & \ldots & \psi_p
\end{bmatrix}
\begin{bmatrix}
\eta_1 \psi_1^{\frac{p}{2}} + \eta_1 \psi_1^{\frac{p}{2}} \\
\eta_2 \psi_2^{\frac{p}{2}} + \eta_2 \psi_2^{\frac{p}{2}} \\
\vdots \\
\eta_p \psi_p^{\frac{p}{2}} + \eta_p \psi_p^{\frac{p}{2}}
\end{bmatrix}
\times
\begin{bmatrix}
-a_{1,p+1}x_1 & -a_{1,p+2}x_1 & \cdots & -a_{1,2p}x_1 \\
-a_{2,p+1}x_2 & -a_{2,p+2}x_2 & \cdots & -a_{2,2p}x_2 \\
\vdots & \vdots & \ddots & \vdots \\
-a_{p,p+1}x_p & -a_{p,p+2}x_p & \cdots & -a_{p,2p}x_p
\end{bmatrix}
\begin{bmatrix}
\varphi_{p+1} \\
\varphi_{p+2} \\
\varphi_{p+3} \\
\vdots \\
\varphi_p
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\psi_1 & \psi_2 & \ldots & \psi_p
\end{bmatrix}
\begin{bmatrix}
x_1 \left( r_1 - \sum_{j=1}^{p} a_{1,j}x_j \right) - x_1' \\
x_2 \left( r_2 - \sum_{j=1}^{p} a_{2,j}x_j \right) - x_2' \\
\vdots \\
x_p \left( r_p - \sum_{j=1}^{p} a_{p,j}x_j \right) - x_p'
\end{bmatrix}
\]

\[
- \begin{bmatrix}
\psi_1 & \psi_2 & \ldots & \psi_p
\end{bmatrix}
\begin{bmatrix}
\eta_1 \psi_1^{\frac{p}{2}} + \eta_1 \psi_1^{\frac{p}{2}} \\
\eta_2 \psi_2^{\frac{p}{2}} + \eta_2 \psi_2^{\frac{p}{2}} \\
\vdots \\
\eta_p \psi_p^{\frac{p}{2}} + \eta_p \psi_p^{\frac{p}{2}}
\end{bmatrix}
\times
\begin{bmatrix}
-a_{1,p+1}x_1 & -a_{1,p+2}x_1 & \cdots & -a_{1,2p}x_1 \\
-a_{2,p+1}x_2 & -a_{2,p+2}x_2 & \cdots & -a_{2,2p}x_2 \\
\vdots & \vdots & \ddots & \vdots \\
-a_{p,p+1}x_p & -a_{p,p+2}x_p & \cdots & -a_{p,2p}x_p
\end{bmatrix}
\begin{bmatrix}
\varphi_{p+1} \\
\varphi_{p+2} \\
\varphi_{p+3} \\
\vdots \\
\varphi_p
\end{bmatrix}
\]

Defining \( w_i = \gamma_i \psi_i^{\frac{p}{2}} + \eta_i \psi_i^{\frac{p}{2}} \) for \( i = 1, 2, \ldots, n \), we can express (62) as

\[
V'(\psi) = \begin{bmatrix}
\psi_1 & \psi_2 & \ldots & \psi_p
\end{bmatrix}
\begin{bmatrix}
-w_1 \\
-w_2 \\
\vdots \\
-w_p
\end{bmatrix}
\]

\[
- \begin{bmatrix}
-a_{1,p+1}x_1 & -a_{1,p+2}x_1 & \cdots & -a_{1,2p}x_1 \\
-a_{2,p+1}x_2 & -a_{2,p+2}x_2 & \cdots & -a_{2,2p}x_2 \\
\vdots & \vdots & \ddots & \vdots \\
-a_{p,p+1}x_p & -a_{p,p+2}x_p & \cdots & -a_{p,2p}x_p
\end{bmatrix}
\begin{bmatrix}
\psi_{p+1} \\
\psi_{p+2} \\
\psi_{p+3} \\
\vdots \\
\psi_p
\end{bmatrix}
\]

Rewriting (63) gives

\[
V'(\psi) = \begin{bmatrix}
\psi_1 & \psi_2 & \ldots & \psi_p
\end{bmatrix}
\begin{bmatrix}
-w_1 \\
-w_2 \\
\vdots \\
-w_p
\end{bmatrix}
\]

\[
- \begin{bmatrix}
-a_{1,p+1}x_1 & -a_{1,p+2}x_1 & \cdots & -a_{1,2p}x_1 \\
-a_{2,p+1}x_2 & -a_{2,p+2}x_2 & \cdots & -a_{2,2p}x_2 \\
\vdots & \vdots & \ddots & \vdots \\
-a_{p,p+1}x_p & -a_{p,p+2}x_p & \cdots & -a_{p,2p}x_p
\end{bmatrix}
\begin{bmatrix}
\psi_{p+1} \\
\psi_{p+2} \\
\psi_{p+3} \\
\vdots \\
\psi_p
\end{bmatrix}
\]
Using (24), we determine
\[
\psi_1, \psi_2, \ldots, \psi_p \leq -\gamma \min_{i} \left( \psi_i^2 \right),
\]
where \( \rho_1 = \max((2p)^{\frac{g+h}{2}}, 1) \) and \( \rho_2 = \max((2p)^{\frac{i+m}{2}}, 1) \).

Using Lemma 2, we obtain
\[
V'(|\psi|) \leq -\gamma \min_{i} \frac{1}{\rho_1} \left( \sum_{i=1}^{2p} \left( \psi_i^2 \right) \right) - \eta \min_{i} \frac{1}{\rho_2} \left( \sum_{i=1}^{2p} \left( \psi_i^2 \right) \right) \leq -n \min_{i} \frac{1}{\rho_1} \left( \sum_{i=1}^{2p} \left( \psi_i^2 \right) \right) \leq -\gamma \min_{i} \left( \psi_i^2 \right),
\]
where \( \rho_1 = \max((2p)^{\frac{g+h}{2}}, 1) \) and \( \rho_2 = \max((2p)^{\frac{i+m}{2}}, 1) \).

Manipulating (66) gives
\[
V'(|\psi|) \leq -\gamma \min_{i} \left( \psi_i^2 \right).
\]

From Theorem 1, the control system is fixed-time stable and has the bound of settling time as
\[
T(\psi_0) \leq T_0 = \frac{1}{\gamma \min_{i} \left( \psi_i^2 \right)} \left( \frac{1}{1-\theta} + \frac{1}{\delta - 1} \right),
\]
where \( \theta = (l+m)/(2m) \) and \( \delta = (g+h)/(2h) \). This completes the proof. \( \square \)

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