Analysis of physical properties of granular materials on their numerical models (2D case)

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Abstract. The paper describes a method for determining the density and specific surface of the package of rock particles represented by discs with the size corresponding to the size of the rock fragments of real dump massifs (2D case). The packing method of the particles corresponds to the schemes of dumping used in open-pit mining - areal and peripheral ones. The influence of boundaries, in which the dump is located, on the physical characteristics of this massif - density and porosity - is revealed. It was revealed that the way of formation of the bulk of particles affects its density, especially at the boundaries of the massif. The massif of particles formed in accordance with the area technology of dumping has a denser composition and is less affected by the effect of its boundaries than the massif formed in accordance with the peripheral technology of dumping. The applied method of determining the physical characteristics of the bulk medium differs from the statistical methods (e.g., Monte Carlo) and is well-proven for static media, such as dumps and filtering massifs in open-pit mining.

1 Introduction

The physical properties of technogenic rock massifs are determined by one of the most essential factors - the geometrical structure of the pore space and the solid phase. The state of the system of solid particles is determined by its geometry: the size of the particles, their shape and way of packing. Moreover, the parameters of mutual arrangement of particles (intercenter distances, orientation, angles between segments connecting particle centers, coordination number, etc.) have a decisive influence on macroscopic characteristics of the medium (bulk density, specific surface, filtration characteristics, carrying capacity, etc.).

In this regard, there is a question of determining these parameters in order to quickly assess and manage the state of the technogenic rock massifs of overburden bedrock.

2 Methods

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Instrumental methods for studying the structural and physical-technical characteristics of technogenic rock massifs consisting of relatively large pieces of rock are in most cases inapplicable due to their high cost. In this regard, there is a need to turn to mathematical models of granular media [1-6, 11-20].

Using separately developed algorithms and programs, values of relative density, porosity, specific surface and coordination number of the mathematical model of the peripheral dump were calculated.

3 Results

In Fig. 1, which is a graphical representation of the structural model of a technogenic rock massif constructed using peripheral technology (a) and areal technology (b), we can see a non-uniform (a) and a uniform (b) distribution of particles of large, medium and small fractions in both the horizontal and vertical directions.

![Fig. 1. A non-uniform (a) and a uniform (b) structures of a peripheral and areal construction technology of rock dump respectively (2D case).](image)

In the general case, the relative density of granular media in the 2D case is the ratio of the total area of the circles $S_K$ to the total area covered by the circles and the space between them $S_0$:

$$\rho = \frac{S_K}{S_0}$$  \hspace{1cm} (1)

The porosity $n$ in this case is

$$n = 1 - \rho.$$ \hspace{1cm} (2)

The specific surface $A_s$ is the ratio of the sum of the lengths of the circles $\Sigma L_i$ to the area of the pore space not covered by the circles:

$$A_s = \frac{\Sigma L_i}{S_0 - S_K}$$ \hspace{1cm} (3)

The result of the simulation routine is a matrix of generalized particle coordinates, which is a numerical analogue of the stochastic structure of irregularly packed circles (with a fairly small, 1-2 maximum diameters, height of the examined layer), which forms ensembles with different local values of structural and physical-technical parameters.
Also, due to the segregation of the grain size distribution along the height of the dump, the structural parameters of the lower part of the dump massif will differ from the parameters of the middle and upper parts. Therefore, the methods of averaged description of structural parameters are inapplicable because of the high probability of discrepancy of local values of parameters within the massif to the average values.

The most convenient from this point of view is the geometrical approach, which has universality for calculation of the heterogeneous media properties. For the same reason, the probabilistic-geometric approach (Monte-Carlo method), based on the statement about equality of geometric and statistical probabilities for a large number of tests, is not reasonable to use for calculating heterogeneous media (in particular, fractioned by height dump massifs), because in this case the number of statistical tests increases sharply [7-10].

4 Results and Discussion

The geometric approach for determining the local values of relative density, specific surface and coordination number is implemented by the following algorithm.

The local value of relative density at some distance from the lower boundary (base) of the package can be determined as the ratio of the sum of circular layers and segments $\sum_{i=1}^{n} S_i$ (shaded areas, Fig. 2, a), formed as a result of intersection of n circles by a layer of thickness $h = z_1 - z_2$, to the total area of the layer $S_0$:

$$\rho = \frac{\sum_{i=1}^{n} S_i}{S_0},$$  \hspace{1cm} (4)

where $n$ is the number of particles crossed by the calculated layer.

The specific surface at some distance from the bottom of the package is defined as the ratio of the sums of the lengths of the arcs of circles $\sum_{i=1}^{n} l_{AB_i} + \sum_{i=1}^{n} l_{CD_i}$, caught in the layer of thickness $h = z_1 - z_2$, to the area of the space in the layer $h$ not covered by circles (Fig. 2, a):

$$A_S = \frac{\sum_{i=1}^{n} l_{AB_i} + \sum_{i=1}^{n} l_{CD_i}}{S_0 - \sum_{i=1}^{n} S_i},$$  \hspace{1cm} (5)

In the case of the location of the particle and the lines with apiculates $z_1$ and $z_2$ and radius $R = OA$, as shown in Fig. 2, b, the lengths of arcs $AB$ and $CD$ and area $ABCD$ are determined as follows. The central angle of the arc $AB$ is the difference of angles $\alpha_1$ and $\alpha_2$:

$$\alpha_1 = \arccos \left( \frac{z_0 - z_1}{R} \right); \quad \alpha_2 = \arccos \left( \frac{z_0 - z_2}{R} \right),$$  \hspace{1cm} (6)

Calculated (depending on the distance from the bottom of the dump model) values of density, porosity, specific surface and coordination number of the dump rock massif are shown in Fig. 3-5.
As can be seen from Fig. 3, the average values of relative density (taking into account the effect of the boundary effect) for the fractionated spoil increases from the lower part of the fractionated dump massif ($\rho = 0.82$) to the upper fine-grained part ($\rho = 0.865$).

Fig. 2. Accounting schemes for determining density, porosity and specific surface of the dump rock massifs.

Fig. 3. Distribution of values of relative density depending on the height: $a$ – peripheral and $b$ – areal construction technology of rock dump respectively.
Abnormal values at 1.5-2 m from the boundaries of the massif (at the base and the upper site) in the calculation of the average relative density were not taken into account, because these areas are subject to the influence of the boundary effect. For the dump with mixed structure (Fig. 1, b) there are some fluctuations of relative density values around the mean value of 0.85 caused by presence of big particles evenly distributed throughout the bulk. No decrease or increase in relative density with the height of the massif is observed, which indirectly confirms the uniformity of the structure of the dump dumped by the area method.

![Fig. 4. Distribution of values of relative porosity depending on the height: a – peripheral and b – areal construction technology of rock dump respectively.](image)

Since the value of porosity of the massif depends on the relative density, for the fractioned massif the porosity of the lower parts of the dump is higher than that of the upper parts (Fig. 4, a). For the dump with a homogenous structure, reduction or increase in porosity (as well as relative density) with the height of the dump is not observed (Fig. 4, b).

![Fig. 5. Distribution of values of specific surface $A_S$ of the dump massif depending on the height of the dump: a – peripheral and b – areal construction technology of rock dump respectively.](image)

Because of accumulation of particles of fine fractions in the upper third of the fractionated dump, the average value of specific surface of this part of the dump ($A_S = 90-100 \text{ m}^{-1}$) in 5-7 times exceeds corresponding value for the bottom parts of a dump ($A_S = 15-25 \text{ m}^{-1}$). Such sharp fluctuations are not observed for the dump with a homogenous structure, where average value of specific surface changes in the range of 50-100 $\text{m}^{-1}$ throughout the height of the dump massif.

### 5 Conclusions

Calculation methods of density, porosity, specific surface area can be used in calculations of heat transfer and mass transfer in various types of granular structures, as well as in management of these parameters of bulk rock masses formed during open-cast mining by selecting the optimal grainsize distribution or the technology of bulk material backfill.
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