Investigation of the transmission accuracy of ball screw considering errors and preloading level

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Abstract
Transmission accuracy is one of the most important parameters in ball screw mechanism (BSM); however, very few researches can be found on the transmission error modeling for BSM. Therefore, on the basis of the converting principle of the errors in the normal and axial direction proposed in this paper, this paper proposes a new model to predicate the transmission accuracy of BSM considering the manufacturing errors, installation errors, as well as the transmission error due to different loading status. After the error analysis and calibration of a transmission accuracy measuring system, the transmission accuracy measurement of a typical BSM under five different preloading levels is performed. The experimental results show that the difference compared with the analytical solution is 21.6% under no preload condition, and is less than 11% under preload condition, largely owing to the uneven distribution of clearance can increase the travel deviation. Further analysis shows that the eccentricity error is the dominant factor leading to the periodic fluctuation of the transmission error. More importantly, the travel deviation increases with increasing preload, which indicates the transmission accuracy of the BSM deteriorates when the load increases.

Keywords
Ball screw · Transmission accuracy · Error analysis · Contact deformation

Nomenclature

| Symbol | Description |
|--------|-------------|
| $F_a$  | The applied axial force on the ball screw |
| $Q_i$  | The contact force between the $i$th ball and the raceway |
| $F_{mi}$ | The load of the nut between the $i$th and the $(i+1)$th circle |
| $F_{si}$ | The load of the screw between the $i$th and the $(i+1)$th circle |
| $\delta_i$ | The normal deformation of the $i$th ball with screw and nut raceway corresponding to the $i$th ball |
| $\delta_{bi}$ | The variation of the ball center relative to the initial position |
| $\Delta L_{si}$ | The axial distance of screw between the adjacent balls |
| $\Delta L_{ni}$ | The axial distance of nut between the adjacent balls |
| $\Delta n_i$ | The deformation of the nut within the $i$th and $(i+1)$th circle along the axial direction |
| $\Delta s_i$ | The deformation of the screw within the $i$th and $(i+1)$th circle along the axial direction |
| $\delta_{si}$ | Axial displacement of the $i$th ball with screw |
| $\delta_{ni}$ | Axial displacement of the $i$th ball with nut |
| $\delta_{i,a}$ | The axial deformation of the $i$th ball with screw and nut raceway |
| $q$ | The gravity of the ball screw per unit length |
| $\Delta d_p$ | The travel error caused by the profile error |
| $\Delta d_l$ | The travel error caused by the lead error |
| $\Delta d_e$ | The travel error caused by the eccentricity error |
| $\Delta d_r$ | The travel error caused by the roundness error of raceway |
| $\delta_{sg}$ | The travel error caused by the support unit |
| $\delta_{in}$ | The travel error caused by the inclined installation error |
| $x_g$ | The deflection caused by the weight of ball screw |
| $I$ | The polar moment of inertia |
| $x_s$ | The travel caused by the screw support and weight |
| $x_h$ | The jacking height of the middle support unit |
| $E_n$ | Elastic modulus of the nut |
| $E_s$ | Elastic modulus of the screw |
| $A_n$ | The superficial area of the cross-section of the nut |
| $A_s$ | The superficial area of the cross-section of the screw |
| $M$ | Total ball number of ball screw |
| $z_b$ | The number of the balls in a circle |
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\( \varphi \) The helix angle of the ball screw

\( P_h \) Lead of ball screw

\( \theta \) The central angle corresponding to the point on the normal profile

\( \lambda \) The phase angle of the ball along the raceway

\( \lambda_\varphi \) The circumference angle of the screw around the axis

\( \theta_i \) The inclination angle of the sinusoidal curve caused by the eccentricity

\( \alpha_0 \) The initial contact angle of ball screw

\( \alpha_t \) The contact angle of \( i \)th ball

\( e \) The eccentricity of the arc on the normal section

\( e_{sc} \) The eccentricity of the screw

\( \theta_i \) The inclined angle of installation of screw

\( L_{ns} \) The center distance between the center of screw and nut

\( V_{n0} \) The projection distances of the initial center distance in \( n \) direction

\( V_{b0} \) The projection distances of the initial center distance in \( b \) direction

\( \delta_{ni} \) The displacement of screw center in \( n \) direction

\( \delta_{bi} \) The displacement of screw center in \( b \) direction

\( f_s \) Conformity of screw

\( f_n \) Conformity of nut

\( r_{Gs} \) The radius of the screw raceway

\( r_{Gn} \) The radius of the nut raceway

\( r_{LG} \) The radius of left arc of the raceway

\( r_{RG} \) The radius of right arc of the raceway

\( Y_s \) Intermediate variable of screw

\( Y_n \) Intermediate variable of nut

\( \rho_s \) Reciprocal of radius of curvature on the contact between ball and screw raceway

\( \rho_n \) Reciprocal of radius of curvature on the contact between ball and nut raceway

\( \tau_s \) Intermediate variable of screw

\( \tau_n \) Intermediate variable of nut

\( c_E \) Material constant

\( c_K \) Geometry factor

\( D_{pw} \) Pitch circle diameter of ball screw

\( r_m \) Half of the pitch diameter of the screw

\( D_b \) The diameter of the ball

\( r_b \) The radius of the ball

\( \Delta L_m \) The relative displacement under preload

\( L_{ns} \) The distance between the center of the nut and screw raceway

\( \delta_c \) The thermal deformation error

\( \delta_p \) The periodic error

\( \delta_t \) The accidental error

\( \theta_{AD} \) The angular displacement of the screw

\( E_{bs} \) The transmission error of the ball screw

\( E_s \) The transmission error of the screw

\( O_{ni} \) The initial center of the screw raceway corresponding to the \( i \)th ball

\( O_{ni} \) The initial center of the nut raceway corresponding to the \( i \)th ball

\( O_{bi} \) The initial ball center corresponding to the \( i \)th ball

\( O_{GR} \) The center of left arc

\( O_{ni} \) The center after deformation corresponding to the \( i \)th ball

\( O_{ni} \) The center of the screw raceway after deformation corresponding to the \( i \)th ball

\( e_1 \) The straightness error of the mobile platform

\( e_2 \) The abbe error during measurement

\( e_3 \) The laser measuring system error

\( e_4 \) The circular grating error

\( V_{ni} \) Radial distance of raceway centers after deformation

\( V_{bi} \) Axial distance of raceway centers after deformation

\( V_{n0} \) Initial radial distance of raceway centers

\( V_{b0} \) Initial axial distance of raceway centers

\( \epsilon_p \) The travel deviation

\( T \) The temperature compensation of the travel

1 Introduction

Ball screws, which are widely used in CNC machine tools, have a great performance on dynamic stability and reliability [1–3]. For a ball screw used in high-precision machining, transmission error is an important index of acceptance conditions to evaluate the transmission accuracy of the ball screw, which generates the velocity and position fluctuation of the worktable and affects its dynamic characteristics [4, 5]. Therefore, it is of significance to modeling the transmission accuracy of ball screw accurately, which can provide a theoretical basis for error forecast and compensation of feed system [6].

In the study of transmission accuracy for the functional elements of machine tools, existing researches mainly involve linear guide, spindle, and roller screw. Hu et al. provided a new method based on the parallel mechanism to study the motion errors of the linear guideway, and developed the relationship between the straightness error and the motion error [7]. Choi et al. proposed a modified volumetric error model that includes spindle error motions and geometric errors to predict the positioning errors at a given axis position [8]. Ma et al. studied the transmission accuracy of a planetary roller screw mechanism under different working conditions, considering the elastic deformations, manufacturing errors, and assembly errors [9]. Fu et al. proposed a kinematic model of the planetary roller screw, which accounted for the run-out errors of the screw, roller, nut, ring gear, and carrier, and the position errors of the nut.
and the pinhole in the carrier [10]. Zhang et al. developed a model of load distribution over threads of planetary roller screw according to the relationships of deformation compatibility and force equilibrium, and an improved approach is proposed to reach uniform load distribution and high transmission accuracy [11].

For the dynamic modeling of the ball screw drives system, the transmission accuracy of ball screw is usually considered as a constant under each specific accuracy grade, which is feedback to the CNC system for the position feedback and accuracy compensation in the while travel. While the transmission at each position of ball screw is different, which results in the fluctuation of stiffness and friction during the operation [12–16]. Therefore, the transmission analysis of ball screw before assembly is essential, which contributes to improving the positioning accuracy for the feed system. For the study of transmission accuracy of ball screw, Zhao et al. studied the position precision including travel variation and deviation by considering the load distribution of ball screw under axial and radial loads conditions, based on the measurement of initial precision without applied load [17]. Zhang et al. developed a predicted model by the linear regression considering positioning error and thermal error, which can compensate for the positioning error during the temperature rising of the ball screw [18]. Li et al. presented a feedforward compensation method of dynamic mechanical tracking errors to improve the tracking accuracy of ball screw drives [19]. Kamalzadeh et al. presented a method for mitigating the detrimental effect of elongation and compression of the ball screw to improve the translational accuracy of ball screw drives when only rotational feedback is available [20]. Since different load corresponds different contact status between ball and raceway, for the analysis of elastic deformation of ball screw under the axial load, a lot of research has been carried out on the load distribution of BSM under the effect of force, torque, and geometric errors [21–24]. However, the manufacturing errors and installation errors of BSM are not mentioned in the above references, and few scholars have studied the effect of preloading level on transmission accuracy.

Therefore, this paper provides a detailed analysis to estimate the transmission accuracy of ball screw caused by errors and preloading level. In “Theoretical analysis,” the transmission error caused by the manufacturing error, installation error and contact deformation is discussed, and the transmission accuracy models of the screw shaft and ball screw are analyzed. In “Experimental verification,” a measuring system for transmission accuracy of ball screw is used to verify the proposed model, and the error of the system and ball screw are analyzed and calibrated. In “Results and discussion,” an error simulation is conducted, and the experimental results are compared with the theoretical values. Finally, “Conclusions” is the conclusion of the paper.

## 2 Theoretical analysis

The transmission error of the ball screw refers to the difference between the theoretical position and the actual position of the nut during the screw rotation and nut axial movement [25]. Similarly, the transmission error of the screw shaft can be measured with the position difference of the measuring ball contacting with the raceway. Combined with the industrial application of ball screw, the transmission error may originate from machining and installation [26], and the preload can change the contact state, which affect the actual position of the nut. All factors affecting the transmission error can be analyzed after being converted to the axial direction.

### 2.1 Analysis of error source

#### 2.1.1 Manufacturing errors

**Profile error** Considering the relative length of the nut, the manufacturing error can be ignored compared with the screw shaft. During the machining of screw shaft, due to the grinding wheel with errors, and the relative motion of the grinding wheel and ball screw, the raceway error, contact angle, lead error may be affected [27]. As presented in Fig. 1, a gothic arc profile and a standard circle are in the $X_2O_2Z_2$ plane, where $O_2$ is at the ball center; $r_b$ is the ball radius; $O_{GL}$ and $O_{GR}$ are the centers of left and right arc respectively; $e$ is the eccentricity; $\alpha$ is the contact angle; $\theta$ is the central angle corresponding to the point on the profile; $r_G$ is the arc

![Fig. 1 Contact between ball and raceway on the normal section](image_url)
radius of the raceway. The contact angle can be obtained from the following:

\[ e = (r_G - r_b) \sin \alpha \tag{1} \]

The coordinate system in normal section is established according to the ball center, and the left arc of the raceway can be expressed by the function:

\[ A_{RC\bot}(\theta) = \begin{cases} (r_{LG} - r_b) \cos \alpha_0 - r_{LG} \cos \theta \\ 0 \\ e - r_{LG} \sin \theta \end{cases} \tag{2} \]

Accordingly, the points on the right arc of the raceway can be obtained. As presented in Fig. 2, a global coordinate system \( XYZ \), with its origin fixed on the screw axis, is established such that its \( Z \)-axis is along the screw axis, and the \( XOY \) plane is vertical to the screw axis. A local coordinate system \( X_2Y_2Z_2 \), with its origin on the ball center trace, is established such that its \( Y_2 \)-axis is tangent to the helical line of the ball center trace, and its \( X_2 \)-axis is along the line from screw axis to ball center. The raceway error in normal section is mainly in two cases. The raceway contact angle is constant, while the raceway arc radius has machining error \( \Delta r \), as shown in the arc \( \odot \); the raceway contact angle and raceway arc radius have errors at the same time, as shown in the arc \( \ominus \); the raceway error in axial section is the change of the helical line, as shown in the arc \( \odot \). For the error arc \( \odot \), the expression of left arc is changed, namely, the \( X \)-coordinate is \((r_{LG} + \Delta r - r_b) \cos \alpha_0 - (r_{LG} + \Delta r) \cos \theta\), and the \( Z \)-coordinate is \( e - (r_{LG} + \Delta r) \sin \theta \). For the error arc \( \ominus \), the expression of left arc is changed, namely, the \( X \)-coordinate is \((r_{LG} + \Delta r - r_b) \cos \alpha_1 - (r_{LG} + \Delta r) \cos \theta\), and the \( Z \)-coordinate is \((e + \Delta e) - (r_{LG} + \Delta r) \sin \theta \). For the error arc \( \ominus \), the expression of left arc is maintained, but the position of the origin \( O_2 \) is changed along \( Z \)-axis, which directly influences the travel error.

And the ball center can be obtained by the tangency relation between the ball and two arcs, which can be expressed as follows:

\[
\begin{align*}
(x_{Lc} - x_b)^2 + (y_{Lc} - y_b)^2 &= (r_{LG} - r_b)^2 \\
(x_{Rc} - x_b)^2 + (y_{Rc} - y_b)^2 &= (r_{RG} - r_b)^2 \\
\end{align*} \tag{3}
\]

where \( (x_{Lc}, y_{Lc}) \) and \( (x_{Rc}, y_{Rc}) \) are the center of the left and right arc of the raceway, respectively; \( r_{LG} \) and \( r_{RG} \) are the radii of the left and right arc respectively; \( (x_b, y_b) \) is the coordinate of the ball center. By combining the tangent condition, the variation \( \delta_{Ob} \) of the ball center \( O_2 \) relative to the initial position in the \( Z_2 \)-direction can be obtained. The travel error caused by the profile error can be calculated as follows:

\[ \Delta d_p = \frac{\delta_{Ob}}{\cos \varphi} \tag{4} \]
**Lead error** The lead error reflects the difference between the actual position and the theoretical position of ball center when the ball rolling through each raceway along the axial single generatrix, which has an impact on the load distribution and transmission accuracy of the ball screw. By measuring the raceway along a single generatrix with 300 mm or the whole effective travel, the accumulative error $\Delta P_L$ in the axial direction can be obtained, and the travel error caused by the lead error can be calculated as follows:

$$\Delta d_l = \Delta P_L / L$$

(5)

**Eccentricity error of screw shaft** During the machining of ball screw, the misalignment between the central line of the tip hole and the axis of the spiral raceways is inevitable. The error can cause the raceway runout during the screw rotation, which has a periodic impact on the transmission error. For the raceway at different positions, the phase angle $\lambda$ (rad) is introduced to represent the location of the ball center. The ball center trajectory in coordinate system $XYZ$ which follows the helical line is given by the function:

$$
\begin{align*}
X &= r_m \cdot \cos \lambda \\
Y &= r_m \cdot \sin \lambda \\
Z &= \frac{\lambda}{2\pi} P_h
\end{align*}
$$

(6)

where $r_m$ is half of the theoretical pitch diameter of the screw; $P_h$ is the lead. Since the projection of the helical line on the axial section is a sinusoidal curve, the projected sinusoidal curve will move along the axis as the screw rotates around the axis. Due to the axis eccentricity, the nut will move slightly forward or backward from the original position, that is, causing an error in the axial direction. As presented in Fig. 3, the error can be obtained by the trigonometric function.

$$\Delta d_e' = e_{sc} \sin \lambda_s \tan \theta_l$$

(7)

where $e_{sc}$ is the eccentricity of the screw, $\lambda_s$ is the circumference angle of the screw around the axis, $\theta_l$ is the inclination angle of the sinusoidal curve caused by the eccentricity. Considering the angle $\theta_l$ which changes with Z-coordinate is hard to determine, to accurately analyze the travel deviation, the relationship of transmission error is expressed by the curve difference. This paper takes the measuring rod connecting with the nut or ball as the measuring reference. The circular motion of the measuring reference around the axis can be regarded as an additional eccentricity is added to the $X$-coordinate. The travel affected by the eccentricity is expressed as follows:

$$x_e = r_m \cdot \sin(z \cdot 2\pi / P_h) + e_{sc} \cdot \sin \lambda_s$$

(8)

When $x_e$ is equal to 0, the corresponding Z-coordinate represents the current travel, and the difference with the nominal lead can be considered as the travel error caused by the eccentricity error.

$$\Delta d_e = z|_{x_e=0} - P_h$$

(9)

**Roundness error of raceway** Since the error of the machine tool spindle on the error sensitive direction, or the unreasonable support during processing, the helical line of the screw has a roundness error. Assuming the axial view is the ellipse shape, the radius of the helical line can be expressed as follows.

$$\rho_r = \frac{r_m(r_m + e_r)}{\sqrt{r_m^2 \cos^2 \lambda_s + (r_m + e_r)^2 \sin^2 \lambda_s}}$$

(10)

Combining with the influence of eccentricity, the travel and travel error caused by the roundness error of raceway are as follows:

$$x_r = \rho_r \cdot \sin(z \cdot 2\pi / P_h) + e_{sc} \cdot \sin \lambda_s$$

(11)

$$\Delta d_r = z|_{x_r=0} - P_h$$

(12)

**2.1.2 Installation errors**

**Error caused by the support unit** For the ball screw with a large aspect ratio, support is necessary to reduce the bending deformation. Considering the fluctuation in the radial direction, the bending trend can be regarded as a quasi-sinusoidal curve. As shown in Fig. 4, when the middle support unit is
higher than the allowable value, the measured screw will move upward as a whole. Since the raceway is right-handed, the measuring reference will move downward obliquely, which will cause the opposite trend of the travel error for the forward and reverse measurement.

Due to the influence of gravity, the deflection of ball screw is as follows:

\[
x_g = \frac{1}{EI} \left( \frac{q}{24} z^4 - \frac{q l^2}{12} z^2 + \frac{q l^3}{24} z \right)
\]

where \( q \) is the gravity of the ball screw per unit length; \( E \) is the elastic modulus; \( I \) is the polar moment of inertia. Then, the travel caused by the screw support and weight is as follows:

\[
x_s = r_m \cdot \sin(z \cdot \frac{2\pi}{P_h}) + (x_g - x_b) \cdot \sin(z \cdot \frac{2\pi}{2L})
\]

where \( x_b \) is the jacking height of the middle support unit. The travel error caused by the support unit is as follows:

\[
\delta_{is} = z_{is=0} - P_h
\]

**Inclined installation error** Since the ball screw is installed by the center hole, it will be inclined if the height of both apexes cannot be guaranteed. As shown in Fig. 5, to tilt coordinate system \( XOZ \) to coordinate system \( X'0'Z' \), the error coordinate system can be obtained by translation and rotation sequentially.

\[
\begin{bmatrix}
   Z' \\
   X'
\end{bmatrix} = \begin{bmatrix}
   \cos \theta_{xy} - \sin \theta_{xy} \\
   \sin \theta_{xy} \cos \theta_{xy}
\end{bmatrix} \begin{bmatrix}
   Z - Z_o \\
   X
\end{bmatrix} + \begin{bmatrix}
   Z_o \\
   0
\end{bmatrix}
\]

\[
\theta_{xy} = \arctan(\frac{e_{sc} \cdot \sin \lambda}{L})
\]

By changing \( Z_o \), the intersection coordinate between the curve and \( Z \)-axis is obtained. The difference between the travel and the lead at the corresponding time is the variation of the travel error.

\[
\delta_{in,y} = z_{is=0} - P_h
\]

Similarly, there is an inclined installation error in \( YOZ \), the coordinate transformation relationship is as follows:

\[
\begin{bmatrix}
   y' \\
   z'
\end{bmatrix} = \begin{bmatrix}
   \cos \theta_{ix} - \sin \theta_{ix} \\
   \sin \theta_{ix} \cos \theta_{ix}
\end{bmatrix} \begin{bmatrix}
   Y \\
   Z - Z_o
\end{bmatrix} + \begin{bmatrix}
   0 \\
   Z_o
\end{bmatrix}
\]

\[
\theta_{ix} = \arctan(\frac{\delta_i}{L})
\]

where \( \delta_i \) is the distance between the apex of headstock and tailstock. The difference between the travel and the lead at the corresponding time is the variation of the travel error. Considering the headstock and tailstock are installed positioning by the flat-\( V \) rail, the installation accuracy in the \( Y \)-direction can be guaranteed and the inclined installation error in \( YOZ \) can be ignored.

![Fig. 4 Diagram of bending error caused by the support unit](image)

![Fig. 5 Diagram of error caused by inclined installation](image)
2.1.3 Error caused by contact deformation

When the screw shaft is matched with the nut, the ball and raceway will be deformed under the axial force, which is different from merely taking the screw shaft into account. For a single-nut ball screw, assuming that the ball size is not large enough to produce preload, the load and deformation are analyzed as follows. As shown in Fig. 6, both the screw shaft and the nut are in compression condition, which corresponding to the C–C loading condition in BS ISO 3408-4.

Under the static loading state, the total contact load of balls can be expressed as follows:

\[ F_a = \sum_{i=1}^{M} Q_i \sin \alpha_i \cos \varphi \]  \hspace{1cm} (21)

where \( F_a \) is the applied axial force; \( Q_i \) is the contact load of the \( i_{th} \) ball; \( M \) is the total number of balls; \( \alpha_i \) is the contact angle between \( i_{th} \) ball and raceway; \( \varphi \) is the helical angle of ball screw.

According to Mei’s model, the nut and screw are not regarded as rigid bodies, and the deformations of balls under different positions are various due to the plastic deformation [22]. To analyze the contact load between the ball and raceway, the screw and nut can be divided into several parts, that is, each ball and its contact region with the nut and screw are regarded as a force unit. The length of the screw and nut between two adjacent balls is as follows:

\[ \Delta L_{si} = \frac{P_h}{z_b} \]  \hspace{1cm} (22)

where \( \Delta L_{si} \) and \( \Delta L_{ni} \) are the unit length of screw and nut; \( P_h \) is the lead of ball screw; \( z_b \) is the effective bearing number of balls in one circle.

As shown in Fig. 7, the force acting on the screw or nut between adjacent balls is unequal.

\[ F_{si} = \begin{cases} F_a - \sum_{j=1}^{i} Q_j \sin \alpha_j \cos \varphi, & i = 1, 2, 3, ..., M-1 \\ 0, & i = M \end{cases} \]  \hspace{1cm} (23)

\[ F_{ni} = \begin{cases} F_a - \sum_{j=i+1}^{M} Q_j \sin \alpha_j \cos \varphi, & i = 1, 2, 3, ..., M-1 \\ F_a, & i = M \end{cases} \]  \hspace{1cm} (24)

where \( F_{si} \) and \( F_{ni} \) are the axial force submitted to the \( i_{th} \) unit, respectively. According to the deformation coordination relation, the following correlation can be obtained.

\[ \Delta n_i - \Delta n_j = (\delta_{s,i+1} + \delta_{n,i+1}) - (\delta_{s,i} + \delta_{n,i}) \]  \hspace{1cm} (25)

where \( \Delta n_i \) and \( \Delta n_j \) are the axial deformations for the unit length of screw and nut; \( \delta_{s,i} \) and \( \delta_{n,i} \) are the axial projection of the contact deformation between the \( i_{th} \) ball and the screw, nut.

\[ \Delta n_i = \frac{\Delta L_{si}}{E_s A_s} \cdot F_{ni} \]  \hspace{1cm} (26)

\[ \Delta s_i = \frac{\Delta L_{ni}}{E_s A_s} \cdot F_{si} \]  \hspace{1cm} (27)
where $E_s$ and $E_n$ are the elastic moduli of the screw and nut; $A_s$ and $A_n$ are the cross-section area of the screw and nut.

For the contact between the ball and raceway, the deformation relation is constructed in Fig. 8. After the axial loading, the center of ball $O_{bl}$, raceway center of screw $O_{nl}$, and raceway center of nut $O_{nl'}$ move a distance to $O_{bl'}',O_{nl'}',O_{nl''}$, respectively. And the initial contact angle $\alpha^0$ will become $\alpha_r$

$$\alpha_r = \sin^{-1} \left( \frac{V_{bi}}{\sqrt{V_{ni}^2 + V_{bi}^2}} \right)$$  \hspace{1cm} (28)

where $V_{ni}$ and $V_{bi}$ are the projection distances of the center distance in $n$-direction and $b$-direction, respectively.

$$V_{ni} = V_{mi} + \delta_{ni} = L_{ni} \cos \alpha^0 + \left[ \delta_{ni,i} + (i - 1)\Delta L \sin \theta' \right] \cos \psi_i \hspace{1cm} (29)$$

$$V_{bi} = V_{bi0} + \delta_{bi} = L_{ni} \sin \alpha^0 + \delta_{bi} \hspace{1cm} (30)$$

$$L_{ns} = r_{Gi} + r_{Gn} - 2R_b$$ \hspace{1cm} (31)

where $V_{mi}$ and $V_{bi0}$ are the projection distances of the initial center distance in $n$-direction and $b$-direction, respectively; $\delta_{ni}$ and $\delta_{bi}$ are the displacements of screw center in $n$-direction and $b$-direction; $L_{ni}$ is the center distance between the center of screw and nut; $r_{Gi}$ and $r_{Gn}$ are the radii of screw and nut. According to Hertz’s contact and deformation theory, the relationship between axial and normal deformation is obtained.

$$\delta_{i,a} = \delta_{i,n}$$ \hspace{1cm} (32)

$$\delta_{i,a} = \frac{\delta_i}{\sin \alpha_i \cdot \cos \varphi} \hspace{1cm} (33)$$

$$\delta_i = c_K \cdot c_s^2 \cdot \hat{Q}_i$$ \hspace{1cm} (34)

$$c_K = Y_s \cdot \left( \sum \rho_s \right) \hat{Q} + Y_n \cdot \left( \sum \rho_n \right) \hat{Q}$$ \hspace{1cm} (35)

$$Y_s = 1.282 \left[ -0.154 \sin \tau_s^{1/4} + 1.348 \sin \tau_s^{1/2} - 0.194 \sin \tau_s \right]$$ \hspace{1cm} (36)

$$Y_n = 1.282 \left[ -0.154 \sin \tau_n^{1/4} + 1.348 \sin \tau_n^{1/2} - 0.194 \sin \tau_n \right]$$ \hspace{1cm} (37)

where $\delta_{i,a}$ is the sum of axial deformation of the contact point between screw and nut; $\delta_i$ is the sum of normal deformation of the contact point between screw and nut; $Y_s$ and $Y_n$ are the auxiliary values of the screw and nut for the first and second type elliptic integrals in Hertzian contact theory; $c_{s,E}$ is the material constant; $\rho_s$ and $\rho_n$ are the reciprocal of the curvature radius at the contact point with screw and nut respectively.

$$\sum \rho_s = \frac{4}{D_b} - \frac{1}{f_s D_b} + \frac{2 \cdot \cos \alpha_i}{D_{pw} - D_b \cos \alpha_i}$$ \hspace{1cm} (38)

$$\sum \rho_n = \frac{4}{D_b} - \frac{1}{f_n D_b} - \frac{2 \cdot \cos \alpha_i}{D_{pw} + D_b \cos \alpha_i}$$ \hspace{1cm} (39)

$$\cos \tau_s = \left| \frac{-\left( f_s \cdot D_b \right)^{-1} - 2 \cos \alpha_i \cdot (D_{pw} - D_b \cdot \cos \alpha_i)^{-1}}{\sum \rho_s} \right|$$ \hspace{1cm} (40)

$$\cos \tau_n = \left| \frac{-\left( f_n \cdot D_b \right)^{-1} + 2 \cos \alpha_i \cdot (D_{pw} + D_b \cdot \cos \alpha_i)^{-1}}{\sum \rho_n} \right|$$ \hspace{1cm} (41)

where $D_{pw}$ is the ball pitch circle diameter; $f_s$ and $f_n$ are the conformities of screw and nut. The contact load and deformation corresponding to each ball can be solved by iterations using the Newton–Raphson method, which is shown as follows:

$$x^{(t+1)} = x^{(t)} - J(x^{(t)})^{-1} \cdot f(x^{(t)})$$ \hspace{1cm} (42)

When the screw and nut are stressed at the end, to analyze the displacement of the nut relative to the shaft end, the deformation of the ball, the raceway and the deformation of the non-contact area of the screw and nut need to be considered. While for the double-nut ball screw under no axial load, the axial deformation of the ball in the nuts on both sides presents the situation of that in the single nut, and the relative displacement of the nut can be regarded
as that of the single-nut under axial load. The relative displacement under preload can be expressed as follows:

$$\Delta L_m = \delta_{M,m} + \sum_{i=1}^{M} (\Delta n_i + \Delta s_i)$$  \hspace{1cm} (43)

### 2.2 Modeling of transmission accuracy

The transmission error of the ball screw is affected by many factors, the most important of which is temperature. The thermal deformation error is approximately linear, which can be expressed as follows:

$$\delta_r(t) = a + bt$$  \hspace{1cm} (44)

Periodic error is a combination of multiple harmonic errors, which is mainly composed of the runout of the machine tool spindle, the master screw error of the feed ball screw, raceway error, eccentricity, shaft bending, etc. The periodic error can be expressed as the superposition of several sinusoidal functions:

$$\delta_p(t) = \sum_{i=0}^{N} A_i \sin(iwt + \theta_i)$$  \hspace{1cm} (45)

Accidental error is caused by workpiece surface defects and interference of external vibration to the grinding wheel, which can be expressed as follows:

$$\delta_r \sim N(0, \sigma^2)$$  \hspace{1cm} (46)

#### 2.2.1 Transmission error of screw shaft

Assuming that the manufacturing errors and installation errors of the screw are independent of each other, the transmission error of the screw can be expressed as the algebraic sum of each error in the axial direction:

$$E_s = \Delta d_p + \Delta d_i + \Delta d_r + \Delta s_g + \delta_{in,y}$$  \hspace{1cm} (47)

As shown in Fig. 9, considering the compensation for the thermal elongation, there is an irregular fluctuation trend for the actual travel error curve caused by various errors. The representative travel curve is calculated by linear regression, and the average travel deviation $e_s$ is calculated by the difference between the ordinate of representative travel and reference travel within a specific measuring travel.

$$\begin{align*}
e_s &= a + br \\
a &= \frac{\sum r_i^2 \sum e_i - \sum r_i \sum r_i e_i}{\sum r_i^2 - \sum r_i \sum r_i} \\
b &= \frac{\sum r_i e_i - \sum r_i \sum e_i}{\sum r_i^2 - \sum r_i \sum r_i}
\end{align*}$$  \hspace{1cm} (48)

#### 2.2.2 Transmission error of ball screw

When considering the transmission error with the nut, deformations between the ball, the screw, and the nut raceway needs to be taken into account. Combined with the screw shaft error, the nut needs to overcome the influence of the manufacturing errors, installation errors, and contact deformation during the axial movement. Therefore, the transmission error of the ball screw is as follows:

$$E_{bs} = \Delta d_p + \Delta d_i + \Delta d_r + \Delta s_g + \delta_{in,y} + \Delta L_m$$  \hspace{1cm} (50)

### 3 Experimental verification

#### 3.1 Transmission accuracy measurement

As shown in Fig. 10, to measure the transmission accuracy of ball screw, the rotation of the screw or the movement of the nut drives the worktable to move horizontally. The rotary position measurement is registered from the rotary encoder, denoted with $\theta_{AB}$[rad]. The rotary displacement has been converted to correspond to the equivalent worktable displacement, by using the relation $\theta_{AB} \cdot P_h/2\pi$. And the horizontal displacement of the worktable is obtained from the laser measuring system, shown with $z$[mm]. During
constant speed movement, the difference between the theoretical displacement and actual displacement of the worktable at each time instant is continuously recorded as the transmission error.

For the actual curve of transmission error shown in Fig. 9, the abscissa $Z$ is the measured horizontal distance from the length measuring system, and the ordinate $Y$ is the difference between the actual travel and theoretical travel.

\[
\begin{align*}
Z &= z \\
Y &= z - \frac{\theta_{AD} P_h}{2\pi}
\end{align*}
\]

(51)

According to the measuring principle, a transmission accuracy measuring system for the ball screw is constructed, as shown in Fig. 11. The measuring system is composed of headstock, tailstock, support unit, mobile platform, laser length measuring system, circular grating, industrial computer, etc. The laser length measuring system HP5528A (Hewlett-Paekard) adopts the principle of dual-frequency laser interferometry, which is composed of He–Ne laser source, spectrometer, reflector, and compensation unit. The ball screw is installed by the headstock and tailstock, and the apex spring is used in the tailstock to eliminate the effect on the thermal elongation of the ball screw. A chuck is used in the headstock rotating device to connect one end of the ball screw. During the measurement, the ball screw is driven by a servo motor inside the headstock, and its angular displacement is measured by a circular grating connecting with the rotating device.

When measuring the transmission accuracy of the ball screw, the measuring reference is rigidly connected with the nut flange, and the rotation of the screw drives the measuring
reference connected with the worktable to move along the axial direction. When measuring the transmission accuracy of the screw shaft, the same size ball on the measuring reference meshes with the raceway. The worktable moves with the rotation of the screw, and its horizontal displacement is measured by a laser measuring system. The angular and horizontal displacements are collected by an industrial control computer at the same time. The travel deviation of the ball screw can be obtained by software processing. For the ball screw with a large aspect ratio, a fixed support unit is used at the middle position when measuring the transmission error of the screw shaft. When measuring the transmission error of the ball screw, a movable support unit is placed under the nut to reduce the deflection caused by the weight.

For the gasket preloading ball screw R50-10, its structural parameters are shown in Table 1. The transmission error of the screw shaft and the ball screw are measured. The preload between two nuts can be changed by replacing the gaskets with incremental thickness, and the preload is calculated according to the friction torque of the ball screw. Combined with the proposed model, the predicted value of transmission error can be calculated. The difference between the experimental result and the theoretical value is compared to verify the validity of the calculated model.

### 3.2 Error analysis of measuring system

Considering the performance of the measuring system is significantly influenced by the accuracy of the component assembly, systematic errors are analyzed and calibrated. Systematic errors mainly come from static errors and motion errors, which are related to the accuracy of the sensors and their installation mechanism.

#### 3.2.1 Straightness error of the mobile platform

The measuring module is carried by the air-floating platform. This means that its vertical straightness will make the measuring reference move slightly along the raceway of the ball screw, resulting in the axial error and affecting the data accuracy. Since the measuring reference in the horizontal direction is compressed by the spring, the horizontal straightness of the platform will not affect the accuracy. Therefore, the air-floating platform is calibrated by Renishaw laser interferometer system XL-80, and the whole measurement curve is analyzed. Considering the lead of the measured ball screw is 10 mm, the mobile platform error is \( e_1 = 0.73 \mu m \).

#### 3.2.2 Abbe error

The length measuring laser should be parallel with the platform mobile direction during measurement, which enables the displacement record accurately. By calibrating the angle error of the laser and the axis of the ball screw, the displacement measured results can be compensated. The installation deflection angle can be ensured within 0.1° after calibration. Considering the measuring distance is within 1000 mm, and combining the relationship between real and measured displacement, the abbe error is \( e_2 = 0.015 \mu m \).

#### 3.2.3 Laser measuring system error

According to the index of the laser length measuring system, the error in the effective length is 0.7 \( \mu m \)/m. Considering the lead of the measured ball screw is 10 mm, the maximum measurement error in the axial direction is \( e_3 = 0.007 \mu m \).

#### 3.2.4 Circular grating error

According to the index of Heidenhain circular grating, the measurement accuracy is 5” for circumferential displacement. Considering the lead of the measured ball screw is 10 mm, the maximum measurement error in the axial direction is \( e_4 = 0.039 \mu m \).

Assuming that the errors are independent of each other, the accuracy of the measurement system can be calculated as \( \sqrt{e_1^2 + e_2^2 + e_3^2 + e_4^2} = 0.73 \mu m \).

Considering the accuracy grade of the ball screw is P2 and the measured travel is 1000 mm, the system measuring error 0.73 \( \mu m \) can meet the tolerance of measurement requirements [25, 27].

### 3.3 Error analysis of ball screw

#### 3.3.1 Measurement of the raceway profile of screw shaft

For the measured ball screw R50-10, the manufacturing errors and installation errors can be obtained only by measurement. As shown in Fig. 12, the axial profile of the screw was measured using a Hommel-Etamic T8000 roughness profilometer, at a speed of 12 mm min”1. The probe was set
to move the travel of 300 mm and the whole travel along the screw generatrix, and the lead error under a specific travel is analyzed. Furthermore, the normal profile of raceways in the middle and ends were measured at five different positions with the distance between two adjacent positions set as 200 mm. To reflect the roundness error of the raceways, the pitch diameter was measured by the NNY hand-held measuring instrument at the same measured raceway, and each raceway was measured at three positions with the angle between two positions set as 120°.

According to the measurement and analysis, the cumulative lead error is 0.004 mm, and the maximum roundness error of raceways is 0.003 mm. Maximum raceway asymmetry of arc and contact angle are 0.003 mm and 0.49° respectively, within the acceptable range.

### 3.3.2 Eccentricity error and installation error of screw shaft

As shown in Fig. 13, the eccentricity error and installation error can be measured on the workbench. The screw shaft was installed on the headstock and tailstock through the center hole. The dial gauge can move along the axial direction by fixing with the mobile platform.

During the measurement, the screw shaft was driven to rotate at the speed of 10r/min. Meanwhile, the dial gauge was set to contact with the precision surface of the screw shaft to record the runout. In the process of screw shaft rotation, half of the indication difference of the maximum and the minimum of the dial indicator was taken as the runout error, and the mean value of three times was taken as the eccentric error. The height indication of the apex for the headstock and tailstock was recorded respectively, and the difference was taken as the tilt installation error of the ball screw. The height difference of the middle support and two ends are regarded as the support error. The measuring results are listed in Table 2.

### 4 Results and discussion

#### 4.1 Simulation of transmission accuracy

Based on the above analysis, the simulation is conducted by substituting the measured errors into Eq. (48), and the results calculated by the program are shown in Fig. 14.

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**Table 2** Measuring results of eccentricity error and installation error (mm)

| Measuring indexes                      | 1     | 2     | 3     | Average |
|----------------------------------------|-------|-------|-------|---------|
| Eccentric error of screw               | 0.011 | 0.010 | 0.010 | 0.010   |
| Inclined error of ball screw           | 0.011 | 0.012 | 0.011 | 0.011   |
| Support error of ball screw            | 0.016 | 0.013 | 0.015 | 0.015   |
The theoretical transmission error curve within 10 cycles is given, which shows a descending periodic fluctuation of transmission error. The maximum amplitude of the curve is about 1.3 \( \mu \text{m} \), and the frequency is \( \frac{2\pi}{T} \), where \( T \) is the time consumption for one revolution of screw. The descending trend of the curve is mainly caused by the negative lead error. The periodic fluctuation of the sine-like curve is mainly caused by the eccentric error, which can produce the end jitter, and the transmission error caused by other errors affects the fluctuation based on the periodic variation.

### 4.2 Experimental results

In the experiment, considering the influence of the environment on the laser measuring system and the ball screw, all the related experiments were carried out in the constant temperature chamber with \( 20 \pm 0.5 \, ^\circ\text{C} \) and \( 50 \pm 10\% \) humidity. Considering the temperature is the main factor affecting the travel deviation of the ball screw, it is necessary to keep the real-time temperature of the ball screw at about \( 20 \, ^\circ\text{C} \). When the temperature of the ball screw is lower than the required value, it should be heated by a period of high-speed running. When the temperature of the ball screw is higher than the required value, it should be cooled by a fan.

The transmission error measurements of the ball screw under different preload levels were carried out at the speed of 20r/min, in which the preload is adjusted by inserting different gaskets \[28\]. After the measuring system running smoothly, the forward transmission error of the ball screw within 1000 mm was measured three times. After analyzing the average, the measuring curves under no preload are shown in Fig. 15, and the measuring curves under increased preload level (2 kN, 4 kN, 6 kN, 8 kN) are shown in Fig. 16. To compare the curve under each preload level more clearly, the first 10 cycles of the curve are shown as an enlarged picture inserted in Figs. 15 and 16. The curve of the first 10 cycles shows a regular periodicity, which is in accordance with the rotational cycle. Different from the theoretical curve in Fig. 14, the measuring curves present the characteristics of sharp peaks and burrs, which may be caused by the superposition of several eccentricity errors and the external vibration on the grinding wheel. Besides, the fluctuation of the transmission accuracy in Figs. 15 and 16 is mainly due to the uneven errors at different positions.

By the method of linear regression for the transmission error described in Fig. 9, the travel deviation of the ball screw is \( -2.68 \, \mu\text{m} \) under no preload. With the increase of preload level, the travel deviations are \( -7.61 \, \mu\text{m} \) at 2 kN, \( -9.92 \, \mu\text{m} \) at 4 kN, \( -12.7 \, \mu\text{m} \) at 6 kN, and \( -14.4 \, \mu\text{m} \) at 8 kN, which mainly due to the effect of the increasing contact deformation on the operation of ball and nut. When there is no preload, the balls in the nut stays in a relaxed balanced position, and the transmission error mainly reflects the manufacturing errors and the installation errors. Under the preload, the distance between the left and right nuts increases, and the balls in the left nut and right nut are appressed to the left and right sides of the gothic arc raceway during the movement. With the increase of the preload, the uneven load distribution of each circle of balls caused by the errors is aggravated. The effect of the increased contact deformation is superimposed on the effect caused by the manufacturing errors and installation errors. The comparison between the experimental results and the theoretical values is shown in Table 3. In absolute terms, the increased trend of the theoretical results is consistent with the experimental results, which shows the travel deviation increases with the increase of the preload, and the predicted values are smaller than the experimental results. The largest difference between the theoretical results and the experimental results is 21.6\% under no preload condition, which is probably because the
Fig. 15 Transmission accuracy of ball screw with no preload

Fig. 16 Transmission accuracy of ball screw with different preload
uneven distribution of clearance between ball and screw, nut can increase the travel deviation. The other predicted differences are less than 11%, which shows the accuracy of the proposed model.

5 Conclusions

In this paper, the transmission accuracy model of ball screw under no preload and different preload conditions considering the manufacturing errors, installation errors and contact deformation are proposed based on the error analysis, transformation, and measurement. All errors are analyzed and converted to the axial direction to characterize the transmission error of the BSM. The proposed model is experimentally validated under different preload conditions based on a measuring system. The main conclusions can be drawn as follows:

1. The error analysis shows the eccentric error is the dominant parameter resulting in the periodic fluctuation of transmission error, and the fluctuation period of the curve is in accordance with the screw rotation.
2. The experimental results indicate the fluctuation period of the transmission error is equal to the rotation cycle of the screw, and the travel deviation is related to the preload, that is, travel deviation increases with increasing the preload.
3. Under different preload conditions, the difference between the experimental results and the predicted values is less than 11%, which indicates the model well forecasts the transmission error. The largest difference between the theoretical results and the experimental results is 21.6% under no preload condition, which is probably because the uneven distribution of clearance between ball and screw, nut can increase the travel deviation.
4. The study can provide a reliable theoretical basis for further analysis concerning the effect of transmission error on the performance of CNC machine tools.

Table 3 Comparison of the experimental results with the theoretical results

| Preload level(N) | Travel deviation (μm) | Difference (%) |
|-----------------|-----------------------|----------------|
|                 | Experimental         | Predicted      |               |
| 0               | −2.68                | −2.1           | 21.6          |
| 2000            | −7.61                | −6.7           | 10.8          |
| 4000            | −9.92                | −9.4           | 5.2           |
| 6000            | −12.7                | −11.8          | 7.1           |
| 8000            | −14.4                | −13.8          | 4.2           |

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Data availability Data transparency.

Code availability Custom code.

Declarations

Conflict of interest The authors declare no competing interests.

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