On location of piezoelectric element in a smart-structure: numerical investigation and experiment

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Abstract. In this paper, based on some example problems it was demonstrated that in examining the possibilities of smart structure applications, the matter of considerable researchers' concern is the problem of location of piezoelectric elements in the structure to allow effective realization of its smart functions in the framework of the specified strategy of structure control and target purposes (vibration damping, defectoscopy, etc.) The numerical and experimental investigations have shown that for structures with the elements made of piezoelectric materials, it is more convenient to use as a parameter, specifying the best location of the piezoelectric element for damping the vibrations at the prescribed frequency, the coefficient of electromechanical coupling, which is evaluated by the values of eigenfrequencies of the structure in the short-circuit and open-circuit regimes. The values of eigenfrequencies of vibrations are evaluated by solving the problem of natural vibrations of electromechanical systems by the finite element method using the applied ANSYS package. The investigation were conducted for a thin-walled aluminum shell in the form of half-cylinder.

1. Introduction
In designing of smart structures, which involve elements made of piezoelectric materials, one of the most important question is how the piezoelectric elements can be located in the structure to ensure the most effective realization of the function of the smart structure in compliance with the target goal (control of dynamic behavior, monitoring of the structure state, collection and accumulation of energy, etc.).

In one of the first works devoted to determining the location of a piezoelectric element, which would allow the maximum damping of the oscillations at a given frequency [1], it was shown that the zones associated with the highest values of average strains are the most appropriate for this purpose. In [2] the main currently available approaches to optimization of piezoelectric element arrangement are systematized and analyzed, allowing using it most effectively to achieve the goal. It should be emphasized that for the most part, the works cited in the review paper [2] investigate smart systems that realize the strategy of active control, in which case the requirements on the operating efficiency of piezoelectric elements are very high. It should be noted that improper location of piezoelements in a structure can upset the regime of its steady operation and eventually leads to its failure. Although at present, there are hundreds of works devoted to this problem [2], the question of the optimal
piezoelement arrangement in structures is of immediate interest due to an abundance of possible application fields and purposes they are intended to serve.

At the same time, for systems designed to fulfill the strategy of passive control, the problem of determining the optimal location of piezoelectric elements in the system in most cases is reduced to determining the place where the structure deformation leads to the maximum value of the electromechanical coupling coefficient [3-5], which is defined by the eigenfrequencies of the structure under short-circuit and open-circuit regimes in the form proposed in [6]. Despite the fact that different authors apart from the above coefficient offer a number of other parameters for passive systems (deformation energy [3, 7], the level of the electric charge [3], the level of the electric potential [7], etc.), priority is given to the coefficient of electromechanical coupling [3-5, 6].

In this paper, we determine an optimal location of the piezoelectric element in the structure, which provides damping of its vibrations at a certain frequency, based on the coefficient of electromechanical coupling. The eigenfrequencies of structure vibrations are evaluated by solving the problem of natural vibrations of electroelastic bodies by the finite element method using the applied ANSYS package (license agreement ANSYS Academic Research Mechanical and CFD № 1064623). The optimal arrangement of the piezoelement in the structure, determined numerically, is confirmed experimentally.

2. The technique for determining the optimal location of piezoelectric element

The object of investigation is the structure with a piezoelectric element, connected to the external electric circuit. The task is to select a location for the piezoelectric element such, that provides the most intensive damping of structure vibrations at a certain frequency.

As it follows from the analysis of the literature, the deformation of the piezoelectric element in the structure specified by the vibration mode of the system at the prescribed frequencies depends on its location, which directly affects its possibilities for transforming the mechanical energy to the electrical one. Recent works [7, 8] showed that, as a characteristic parameter for determining the optimal location of the piezoelectric element in the structure, it is best to use a relative quantity of the electric potential generated on the ungrounded surface of the piezoelectric element under deformation. However, further investigations have shown that the selection of the best location for the piezoelectric element for spatial structures cannot always rely on the relative values of the electric potential, which have been obtained by solving the natural vibration problem. This is because of the fact that the relative value has meaning only within the confines of one particular computation performed for estimation of the operating efficiency of the piezoelectric element at different vibration modes. A comparison between the values of the electric potential obtained by solving the natural vibration problem for different locations, dimensions, physico-mechanical characteristics of the element, etc, is not always correct.

Therefore, to estimate the efficiency of operation of the piezoelectric element placed on the main structure, in [6] it has been suggested that the generalized coefficient of the electromechanical coupling $K$, determining the amount of the mechanical vibration energy transformed into the electric energy, can be used as a criterion of the optimal element location:

$$K = \sqrt{\frac{\omega_{osc}^2 - \omega_{sc}^2}{\omega_{osc}^2}}$$

where $\omega_{osc}$, $\omega_{sc}$ are the eigenfrequencies of the structure with piezoelectric elements in the open circuit regime and short-circuit regimes. The open-circuit regimes are realized in the case when one of the electrode-covered surfaces of the piezoelectric element is grounded (its potential is equal to zero), and the other surface is at no load. In the short-circuit regime, zero potentials are set at both surfaces.

In works [7-8], the correlation between the value of the electric potential and electromechanical coupling coefficient is demonstrated by finding the best variant of piezoelement arrangement allowing the most effective damping of several vibration modes. In this case, from the viewpoint of energy transformation, the coefficient $K$ has proved to be the most informative. With all this in mind, it has
been suggested that the evaluation of the optimal location of the piezoelectric element be made based on the maximum value of the electromechanical coupling coefficient $K$.

3. The obtained results

To demonstrate the applicability of the proposed condition, we conducted a series of numerical and experimental investigations with the aim of finding a piezoelement location such that could provide its most efficient work (i.e. provide maximum value of the received signal) when connected to an external electric circuit to provide damping of structure vibrations at a certain frequency. The investigations were made for a thin-walled shell in the form of a half cylinder (fig. 1). The generatrices of the shell were clamped along their entire length with the aid of flange plates.

![Experimental scheme (a). Computational scheme for modeling (b).](image)

**Figure 1.** Experimental scheme (a). Computational scheme for modeling (b).

The shell with geometrical dimensions: radius $R=0.044$ m, length $L=0.17$ m, thickness $h=15 \cdot 10^{-5}$ m (fig. 1) was made of 3104 aluminum. In computations, it was assumed that the material of the shell is elastic and isotropic and has the following physicomechanical characteristics: $E=6.5 \cdot 10^{10}$ Pa, $\nu=0.33$, $\rho=2800$ kg/m$^3$. For experiments we used piezoelectric elements of rectangular shape, which were made of LZT-19 (Lead zirconate titanate) piezoceramics and had the following standard dimensions: $70 \times 15 \times 0.72$ mm, $50 \times 20 \times 0.36$ mm. The upper and lower surfaces of piezoelectric elements were covered with electrodes. The polarization axis coincided with the direction of the normal to the electrode-covered surface.

In computation, the material of piezoelectric elements was assumed to be elastic. The physicomechanical characteristics claimed by the manufacturer, have the following values: elastic modulus $C_{11}=C_{22}=13.9 \cdot 10^{10}$ Pa, $C_{12}=7.78 \cdot 10^{10}$ Pa, $C_{13}=C_{23}=7.43 \cdot 10^{10}$ Pa, $C_{33}=11.5 \cdot 10^{10}$ Pa, $C_{44}=3.06 \cdot 10^{10}$ Pa, $C_{55}=C_{66}=2.56 \cdot 10^{10}$ Pa, piezoelectric coefficients $\beta_{31}=\beta_{32}=-5.2$ C/m$^2$, $\beta_{33}=15.1$ C/m$^2$, $\beta_{32}=\beta_{31}=12.7$ C/m$^2$, dielectric coefficients $e_{11}=e_{22}=6.45 \cdot 10^{-9}$ F/m, $e_{33}=5.62 \cdot 10^{-9}$ F/m, density $\rho=7700$ kg/m$^3$, the vacuum permittivity $\varepsilon_0=8.85 \cdot 10^{-12}$ F/m.

3.1. Evaluation of eigenfrequencies of the shell

At the first stage we performed simulation to evaluate eigenfrequencies of the structure vibrations in the absence of the piezoelectric element. The oscillations were excited by an impact applied to the
shell and were fixed by a laser vibrometer PDV-100 in a single-measurement regime during 6 seconds at the point \((z = L/2, \varphi = 15^\circ)\) (fig.1,a).

Table 1 presents the values of eigenfrequencies determined numerically by solving the natural vibration problem and experimentally in the case of impact-excited vibrations.

| Eigenfrequencies, Hz | Numerical computation | Experiment | Difference, % |
|---------------------|-----------------------|------------|--------------|
| 77                  | 79                    | 2.44       |
| 172                 | 191                   | 11.18      |
| 173                 | -                     | -          |
| 288                 | -                     | -          |
| 320                 | 305                   | 4.65       |
| 492                 | 458                   | 7.52       |

The results obtained numerically and experimentally for a shell without a piezoelectric element, demonstrate that the computational model describes quite adequately the actual experiment. It should be noted that a particular type of external action can not excite all vibration modes, in particular, in the example considered, the 4th mode of vibration is not fixed. In the spectrum of eigenfrequencies determined numerically there are three pairs of frequencies, the values of which are very close but not multiple (the vibration modes at these frequencies are different). In the realized experiment, these frequencies failed to separate from each other. Therefore, in the comparison of numerical and experimental results a pair of close frequencies aligned to a single frequency determined experimentally (for example, for frequencies 172.31 Hz and 173.55 Hz – 191.56 Hz, and for frequencies 492.90 Hz and 495.21 Hz – 457.96 Hz).

3.2. Determination of placement of the actuator

To ensure stability and repeatability of the experiment the excitation of vibrations is executed by a piezoelectric element, acting as an actuator, the possibility of which is demonstrated in [9]. As an actuator, we used a rectangular 30x15x0.72 mm piezoelectric element.

Figures 2. Patterns of distribution of the coefficients \(K\) in relation to the location of the actuator for the vibration modes corresponding to the frequencies of 172 Hz and 493 Hz.
Based on the solution of the natural vibration problem were determined eigenfrequencies of the structure in the open- and short-circuit regimes, the coefficients of electromechanical coupling $K$ and to obtain the distribution of its values depending on the location of the actuator for the first six eigenfrequencies.

As an example, fig.2 shows the pattern of distribution of the coefficient $K$ for vibration modes corresponding to the frequencies of 172 Hz and 493 Hz in relation to the location of the actuator. For illustration purpose, the cylindrical surface of the shell is unfolded in the plane; the angular coordinate $\varphi$, is plotted along the x-axis and the longitudinal coordinate $z$ is plotted along the y-axis. In the distribution pattern, each point corresponds to the location of the center of actuator mass.

Numerical simulation was used to select the location of the actuator, which provides the excitation of the largest number of vibration modes in the given frequency range. Based on the analysis of the obtained $K$ distribution, it was decided to place the coordinates of the center of the actuator mass are at points of 15°, 27 mm.

A series of experiments conducted by determining the frequency spectrum when the oscillations are excited by an actuator located in accordance with the results of a numerical study. A signal was set to actuator from the frequency generator through the E-413.D2 voltage amplifier for piezoelectric elements. The vibrations were registered by a laser vibrometer PDV-100.

Table 2. Eigenfrequencies of shells fitted with an actuator

| № of frequency | Frequency, Hz | Numerical computation | Experiment | Difference, % |
|----------------|--------------|-----------------------|------------|---------------|
| 1              | 83           | 80                    |            | 4.13          |
| 2              | 171          | 155                   |            | 9.49          |
| 3              | 183          | 178                   |            | 3.14          |
| 4              | 288          | 312                   |            | 8.23          |
| 5              | 310          | 338                   |            | 8.78          |
| 6              | 469          | 459                   |            | 2.14          |

Table 3. Resonance frequencies and vibration amplitudes registered in the experiment for a shell with the actuator

| № | Frequency (Hz) | Vibration velocity ($\mu$m/s) | Displacement (nm) | Electromechanical coupling coefficient $K*1000$ (calculated) |
|---|----------------|-----------------------------|------------------|----------------------------------------------------------|
| 1 | 80             | 15                          | 30               | 0.32                                                     |
| 2 | 155            | 43                          | 14               | 2.02                                                     |
| 3 | 178            | 654                         | 600              | 1.53                                                     |
| 4 | 312            | 1500                        | 550              | 3.48                                                     |
| 5 | 338            | 45                          | 17               | 1.41                                                     |
| 6 | 459            | 1400                        | 590              | 5.46                                                     |
Table 2 gives the values of eigenfrequencies of the shell with an actuator, which were calculated in the framework of the natural vibration problem in the short-circuit regime and determined experimentally in the case of vibration excitation by the piezoelectric element.

From the results presented in Table 2 it follows that connection of the piezoelectric element to the shell causes a frequency shift in the original spectrum. It is experimentally possible to obtain all frequencies in the considered range. The maximum difference between the frequency values obtained numerically and experimentally is not more than 11%.

Table 3 displays the experimentally obtained velocity of vibrations and displacements at corresponding resonances. Also Table 3 shows the calculated values of the electromechanical coupling coefficient $K$ for frequencies obtained numerically, with the chosen position of the piezoelectric element-actuator. In this case, the largest value of $K$ takes on the 6th frequency (in the experiment 459 Hz).

3.3. Determination of the placement of the damping piezoelectric element

In addition to being used as an actuator, the piezoelectric element can be used for passive damping of oscillations when to its electroded surfaces attached the external electrical circuit consisting of resistance, inductance and capacitance. In this embodiment, in order to provide the maximum damping at the corresponding frequency, it is necessary to find the location of the piezoelectric element, at which the maximum value of the electromechanical coupling coefficient $K$.

As an example, let us consider the problem of choosing the location of the piezoelectric element, which provides the maximum damping of the 6th frequency.

![Figure 3. Patterns of distribution of the electromechanical coupling coefficient for the 6th frequency, depending on the location of the damping piezoelectric element.](image)

To implement passive damping using an external electrical circuit, a rectangular piezoceramic element measuring 30x10x0.36 mm was chosen.

Figure 3 shows the results of numerical calculations of the electromechanical coupling coefficient $K$ for the sixth frequency for various arrangements of the damping piezoelectric element when the actuator corresponds to the results given in Table 3. The cut rectangular region in Fig. 3 corresponds to the position occupied by the actuator.

On the basis of an analysis of the distribution patterns of the coefficient $K$, a damping piezoelectric element was placed for experimental studies at a point with coordinates $(\phi = 52^\circ, z = 27 \, \text{mm})$, where $K$ takes on the largest value.
Figure 4. Frequency response of the displacement velocity (W), taken by a laser vibrometer at the center of the shell (from peak to peak).

Table 4. The amplitudes of the electrical voltage on the second piezoelectric element and the coefficient of electromechanical coupling at different frequencies.

| Frequency (Hz) | Amplitudes of voltage (V) | Electromechanical coupling coefficient $K*1000$ (calculated) |
|----------------|---------------------------|------------------------------------------------------------|
| 80             | 0.3                       | 5.01                                                       |
| 178            | 0.4                       | 8.90                                                       |
| 320            | 0.7                       | 10.84                                                      |
| 352            | 0.7                       | 6.64                                                       |
| 498            | 1.2                       | 13.30                                                      |

It should be noted that changing the spectrum of natural frequencies as a result of adding the second element does not change the qualitative conclusions obtained on the basis of the results presented in Table 3. This is confirmed by the frequency response of the displacement velocity (W) shown in Fig. 4, taken by a laser vibrometer, where the maximum values of W are attained at the sixth frequency.

Table 4 shows the experimental values of the electric potential at the second piezoelectric element and the values of the electromechanical coupling coefficient, obtained as a result of numerical calculations.

These results demonstrate the qualitative agreement of numerical and experimental conclusions about the optimum location of the piezoelectric element for damping a given (sixth) eigenfrequency.

Conclusions

The experimental and numerical investigations performed for smart-structures based on the application of piezoelectric elements have shown that it is essential to conduct a study on the
arrangement of piezoelectric elements, providing optimal damping of vibrations, energy harvesting, etc.

The numerical and experimental confirmed, that as a parameter, determining the best location of the piezoelectric element for damping the structure vibrations at the prescribed frequency, can be used the coefficient of electromechanical coupling, which is evaluated by the values of eigenfrequencies of the structure in the short-circuit and open-circuit regimes.

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