Research on Model and Related Parameters of Supercritical CO$_2$ Injection into Depleted Reservoir

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Abstract. On the basis of the research on CO$_2$ geological storage and enhanced oil recovery (EOR) technology, a random porosity distribution model which conforms to logarithmic normal distribution was adopted in order to describe the heterogeneous characteristics of pore structure. On this basis, the two-phase flow model of CO$_2$-formation water was established to describe the displacement process. Through the simulation of CO$_2$ injection into depleted reservoir, it confirmed that injection point pressure was associated with the temperature and depth of the formation except heterogeneity. But the saturation distribution was greatly influenced by formation heterogeneity and depth. Thus, the space utilization of the injection layer reduced gradually with the depth increasing. The related research is important for CO$_2$ storage, migration and evolution in depleted reservoir.

1. Introduction

With the rapid development of modern industrial economy, the increasing oil consumption results in many oil blocks losing economic exploitation value and becoming depleted reservoirs[1]. Meanwhile, as the main component which discharged from the burning of fossil fuels, carbon dioxide (CO$_2$) is considered to be one of the greenhouse gases that may contribute most to global warming on the earth[2-4]. Disposal of CO$_2$ from stationary sources into subsurface structures has been suggested as a possible means for reducing CO$_2$ emissions into the atmosphere. The geological storage has been studied as one of the effective ways of CO$_2$ emission reduction[5-6]. As one of the ideal storage place, depleted reservoir is selected for carbon dioxide sequestration, and its oil recovery also can be improved by CO$_2$-flooding during CO$_2$ injection. However, the oilfield has injection wells, production wells and equipments, which can greatly reduce the carbon dioxide capture and recycling costs in the early stage of storage. Depleted reservoir has a natural closed space and conditions, carbon dioxide in depleted reservoir can be mainly stored by dissolving into the residual oil and formation water sealed in the formation. In addition, carbon dioxide can react with the high valence cations of the formation water and sealed. Depleted reservoir which have lost its economic exploitation value, is mostly in a state of high water cut. Porous media of these reservoirs is mainly composed of formation water (or injected water) with highly scattered residual oil distributing[7]. Considering the oil-water contact, two-phase flow of CO$_2$ and water is studied firstly during CO$_2$ being injected into the depleted reservoir. The pore structure of formation changed a lot due to the long-time water flooding, physical and chemical reaction among CO$_2$ and water and rocks[8]. Thus, the original permeability and porosity of the reservoir are not accurate any more. Given the characterization of porosity can be characterized by average value, the random simulation method is used to describe the heterogeneous characteristics of porosity. A random porosity distribution model which conforms to logarithmic normal distribution is adopted in order to describe the heterogeneous characteristics of pore structure which had been changed in the
injection layer. On this basis, two-phase flow model of CO₂-formation water is established to describe the displacement process. Considering the operation parameters influence on the characteristics of CO₂ injecting into depleted reservoirs, the effects were analyzed through the numerical simulation. The impacts of porosity heterogeneity, injection depth and injection pressure were mainly discussed in this paper, in order to provide a theoretical basis for CO₂ sequestration and migration in the depleted reservoir. Meanwhile, the results can be referenced for the further studying on WAG.

1.1. Porosity Distribution Model
The logarithmic normal distribution function is used to describe the average porosity of the injection layer. As the porosity distribution model is built, heterogeneous pore structure characteristics of CO₂ injection layer can be studied.

Logarithmic normal distribution of probability density function can be expressed as follows:

\[ f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left[-\frac{\left(\ln x - \theta\right)^2}{2\sigma^2}\right] \]  

Among them, the parameter \( \theta \) and \( \sigma \) are the mean and standard deviation of the variables logarithm respectively. Its mathematical expectation is the average porosity of injection layer which can be expressed as: \( \phi = e^{\theta+\sigma^2/2} \). Its Variance is the variance of the porosity which can be expressed as: 

\[ \varepsilon = (e^{\sigma^2} - 1) e^{2\theta+\sigma^2} \]

The relationship between them can be expressed as:

\[ \theta = \ln \left( \frac{\phi^2}{\sqrt{\varepsilon + \phi^2}} \right) \]  

\[ \sigma = \sqrt{\ln \left( \frac{\varepsilon + \phi^2}{\phi^2} \right)} \]  

Rock porosity is 0.1~0.4 in commonly, considering the different reservoir conditions of Depleted reservoirs which has been developed by water flooding for long time, 0.25 is selected as average porosity and 0.01 as its variance in this study. Thus, the parameter \( \theta \) and \( \sigma \) can be calculated by formula (2) and (3): \( \theta = -1.4605, \sigma = 0.3853 \).

![Figure 1. Porosity distribution.](image)

The porosity distribution model can be solved through MATLAB7.0 as follows:

1. Function meshgrid() is used for meshing the regional of calculation model, to generate the compute nodes;
2. In the region of two-dimensional, the function lognrnd () is used to generate the porosity array which satisfy \( \theta \) and \( \sigma \) in the grid nodes and interpolation method is used on the outside space. Calculating the two-dimensional area so as to realize the porosity distribution of random logarithmic normal distribution. As shown in Figure 1. And the pore variance can be 0 if Pore formation is homogeneous.

2. Mathematical Model and Computation Conditions
The critical pressure of CO\(_2\) is 7.38 MPa, and the critical temperature is 31.04°C. Under the supercritical state, CO\(_2\) will become a supercritical fluid, which has as big as liquid density, but at the same time is as high as the gas dispersion coefficient and low viscosity.[9]

2.1. Research Area
The model of two phase flow in porous media was employed to develop a simulation code of CO\(_2\) injection. The two phases are assumed to be CO\(_2\) fluid as non-wetting phase and groundwater as wetting phase. The CO\(_2\) density was calculated by the Peng-Robinson equation, and CO\(_2\) viscosity was calculated by the Jossi-Stiel-Thodos equation. The formulation of the numerical model and theoretic explanation is as follows.
Established mathematical model based on the following assumptions: (1) CO\(_2\) is in the supercritical state.(2) Ignore the solubility of supercritical CO\(_2\) in formation water;(3) Ignore the capillary force in reservoir pore;(4) The geothermal gradient is only considered in supercritical CO\(_2\) density and the influence of viscosity;(5) Fluid and rocks are incompressible, the fluid flow conform to darcy law.

2.2. The Establishment of the Two-Phase Fluid Continuity Equation and the Equation of Motion
The mass conservation equation is as follows:

\[
\frac{\partial (\phi S_w \rho_w)}{\partial t} + \nabla \cdot (\rho_w v_w) = q_w \tag{4}
\]

\[
\frac{\partial (\phi S_n \rho_n)}{\partial t} + \nabla \cdot (\rho_n v_n) = q_n \tag{5}
\]

where the subscript w for the water phase and n for the CO\(_2\) phase, \(\phi\) is the porosity, \(S\) the saturation, \(q\) the density, \(t\) time and \(v\) the Darcy flux velocity.

Darcy’s law extended to two phase flow can be written for each phase as

\[
v_w = -\frac{K_{rw} \rho_w}{\mu_w} (\nabla p_w - \rho_w g) \tag{6}
\]

\[
v_n = -\frac{K_{rn} \rho_n}{\mu_n} (\nabla p_n - \rho_n g) \tag{7}
\]

where \(K\) is the absolute permeability, \(k_{rw}, k_{rn}\) the relative permeability, \(\mu_w, \mu_n\) the viscosity, \(p_w, p_n\) the pressure and \(g\) the acceleration due to gravity.

2.3. Equation of State for Two Phase
The multiphase system in porous media is characterized by parameters such as the capillary pressure, relative permeability and saturation relationship. The pressure difference across the interface between two immiscible fluids was neglected under the assumption that the injection pressure would be much higher than the capillary pressure (\(p_w = p_n\)), so the capillary pressure \(p_c = 0\).

The two phase saturation is satisfied as follows:

\[S_w + S_n = 1 \tag{8}\]

According to the van Genuchten model, the relationships between the relative permeabilities and the effective saturation are given by

\[k_{rw} = S_e^\alpha \left[1 - \left(1 - S_e^{1/\psi}\right)^\psi\right] \tag{9}\]

\[k_{rn} = (1 - S_e)^\gamma \left[1 - S_e^{1/\psi}\right]^\gamma \tag{10}\]

where \(S_e\) is the effective saturation, which can be described as follows:

\[S_e = \frac{S_w - S_{sw}}{1 - S_{sw} - S_{sw}} \tag{11}\]

Where \(S_w\) and \(S_{sw}\) are the water and CO\(_2\) residual saturation respectively, \(\alpha, \psi\) and \(\gamma\) are parameters that are determined by the shape of the pores. According to the literature’ typical values for the parameters \(\alpha, \psi\) and \(\gamma\) are \(1/2, 1/3\) and \(0.77\), respectively.
In order to describe the changes of density and viscosity of supercritical \( \text{CO}_2 \) caused by the changes of temperature and pressure in the injection layer, the Peng-Robinson equation is used to calculate the density of supercritical \( \text{CO}_2 \), and the Jossi-Stiel-Thodos equation is used to calculate the viscosity of supercritical \( \text{CO}_2 \) in this paper.

2.4. Boundary Conditions
For the study area and the establishment of model equations, the initial conditions and boundary conditions are as follows:

**Initial conditions:** when \( t=0 \), \( p=f(z) \), \( S_w=0.9 \), \( S_n=0.1 \)

The injection point is located in the centre of the calculation region, and \( \text{CO}_2 \) saturation is 1 in the injection point while simulating the \( \text{CO}_2 \) injection. In the calculation region, the initial saturation of \( \text{CO}_2 \) is 0.1, except for the injection point.

**Boundary conditions:** In the calculation, the pressure and initial saturation are given by the pressure gradient at the boundary, which is the first kind of boundary condition:

\[
\begin{align*}
p &= f(z), \quad x=\pm 500; \quad z=\pm 500 \\
S_w &= 0.9, \quad S_n = 0.1, \quad x=\pm 500; \quad z=\pm 500 \\
q &= q_0, \quad x=0; \quad z=0
\end{align*}
\]

2.5. Solving Model
The commercial simulation software of COMSOL Multiphysics is used in order to solve the mathematical model. Coupled with the Peng-Robinson equation, the Jossi-Stiel-Thodos viscosity equation and Porosity model conforms to logarithmic normal distribution. Table 1 is the part of the parameter values used in the model[5].

| attribute parameter                      | parameter values |
|------------------------------------------|------------------|
| Calculation area                         | 1000m×1000m      |
| Permeability K/10³μm²                    | 100              |
| Average porosity of injection layer \( \phi \) | 0.2              |
| Formation water density \( \rho_w/\text{kg}·\text{m}^3 \) | 1.0×10³         |
| Formation water viscosity \( \mu_w/\text{mPa}·\text{s} \) | 0.283           |
| \( \text{CO}_2 \) viscosity \( \mu_n/\text{mPa}·\text{s} \) | by Jossi-Stiel-Thodos |
| \( \text{CO}_2 \) density \( \rho_n/\text{kg}·\text{m}^3 \) | by Peng-Robinson |
| \( \text{CO}_2 \) residual saturation \( S_{nr} \) | 0.05             |
| Residual water saturation \( S_{wr} \)  | 0.05             |
| Pressure gradient /MPa·km⁻¹              | 10.5             |
| Geothermal gradient /K·km⁻¹               | 45               |
| Surface temperature /K                   | 293              |
| Injection rate \( q_0/\text{kg}·\text{s}^{-1} \) | 5                |
Through the simulation, the change of injection point pressure at different temperatures was obtained (Figure 2). At first, the results of this model are compared with the results of K Sasaki et al. In the calculation process, the initial conditions and relevant parameters used in this paper are most of the same as that used by Sasaki K(Table 1)[5], and the difference is that the viscosity of supercritical carbon dioxide, which is used by Sasaki K et al is a fixed value of 0.216×10^{-1} mPa·s, and in this paper, the Jossi-Stiel-Thodos equation is used to characterize the viscosity which adapted to the formation temperature and pressure, the calculated viscosity range is 0.206×10^{-1} mPa·s to 0.394×10^{-1} mPa·s.

As Figure 2 shows that, at the same temperature, the viscosity values obtained by different viscosity models have obvious effects on the injection pressure of carbon dioxide. When the injection temperature is 343K (70℃), the injection point pressure variation curve which corresponding to the viscosity value calculated by the equation of Jossi-Stiel-Thodos is lower than the injection pressure which corresponding to the fixed viscosity, and when the injection temperature is 383k (90℃), the result is just the opposite. Therefore, in the formation of large changes in temperature and pressure, in order to more accurately characterize the influence of the actual viscosity of carbon dioxide on injection point pressure, using viscosity equation which is adapted to local temperature and pressure formation to characterize is more in line with the actual situation of formation.

Under a certain temperature, the pressure of the injection point will increase gradually and tends to be stable over time. The higher the reservoir temperature, the greater the pressure of the injection point, which is due to the density of carbon dioxide decreases with the increase of temperature. The higher the temperature is, the longer the injection pressure is stable.

Figure 2. Comparison of pressure at the injection point of two different viscosity models.

Figure 3. Changes of injection point pressure within 24 hours.

Figure 4. Changes of injection point pressure in 10 days.

Figure 5. The edge distribution of CO₂ in homogeneous strata of different depth.
In previous studies, scholars have mostly assumed that the formation is homogeneous strata, and not made a note for whether the result can accurately express the change of the actual formation pressure. To this end, the injection point pressure of homogeneous and heterogeneous strata in different depth are simulated and compared in the study(Figure 3 and Figure 4). From the figure we can see that the heterogeneity of the formation has little effect on the pressure of the injection point. In both shallow and deep strata, the injection pressure is related to the formation depth, which is not related to the heterogeneity of the formation. The deeper the formation is, the greater the pressure of the injection point is, and the pressure increases rapidly with time and tends to stabilize to a relatively stable value. Due to the influence of geothermal gradient and pressure gradient, the flow characteristics of supercritical CO$_2$ in different depth strata will also change. Figure 5 is the saturation distribution characteristics of CO$_2$ in the injected layer after injecting 10 days into homogeneous strata of 1000m and 2500m depth with the injection rate of 5kg/s respectively. The closed line in the figure is the edge of CO$_2$ which is sealed in the strata. As can be seen, under the same conditions of injection rate and time, with the increase of the depth, the outer edge of CO$_2$ has increased. Showing that the space utilization of the injection layer reduced gradually with the depth increasing. Under the same volume, the amount of CO$_2$ stored in the deep reservoir is smaller than that in the shallow reservoir, but the injection pressure is larger than that of the latter. Considering the long-term storage of sealing, the deep reservoir still has a certain practical application value and advantage. At the same time, because of the density difference between formation water and supercritical CO$_2$, the influence of the buoyancy effect makes the CO$_2$ distribution shift to the surface direction (Figure 5). This phenomenon also exists in the homogeneous formation, but saturation distribution of CO$_2$ are greatly influenced by the heterogeneity of porosity. In the larger porosity region, the flow ability is stronger, and then produce irregular distribution characteristics, the simulation results will be further discussed in the later study.

3. Conclusions

(1)The CO$_2$ injection point pressure is little affected by the heterogeneity of porosity, and the injection point pressure is mainly related to the depth of the injection layer, the average porosity, the injection rate and formation temperature.

(2)The viscosity of CO$_2$ has obvious influence on the pressure of the injection point. Therefore, it is more consistent with the actual situation of the formation to be characterized by the viscosity equation adapted to the local temperature and pressure.

(3)Due to the existence of the density difference between the supercritical CO$_2$ and the formation water, the buoyancy effect makes the CO$_2$ to the surface of the earth's surface, which is great significance to grasp the migration and evolution of CO$_2$ after storage.

(4)At the same injection condition, affected by the geothermal gradient and pressure gradient, temperature and pressure in the deep strata are larger, which results in CO$_2$ viscosity decreases, and flow ability enhanced. In the deep injection layer, the space utilization rate of CO$_2$ is relatively low, and the injection point pressure greatly. But from the point of view of the effect of long-term storage, deep formation has certain advantages.

4. References

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