Defining of nonlocal damping model parameters based on composite beam dynamic behaviour numerical simulation results

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Abstract. The nonlocal model of internal damping is considered in this paper using the Euler-Bernoulli beam with fixed ends as an example. Equation of beam motion considering nonlocal damping is solved by Galerkin method to develop the model. The required number of eigenmodes is obtained for the beam under an instantly applied distributed load. The influence of nonlocal damping model parameters variation on the beam vibration process simulation results is shown. The beam is considered under a periodic deterministic distributed load. Defining of nonlocal damping model parameters is carried out using results of numerical three-dimensional finite element simulation. The parameters are defined for the glass fibre reinforced plastic beam with the fixed ends under the instantly applied distributed load. The parameters obtained for two different kernel functions and corresponding standard errors are compared. An opportunity of one-dimensional beam model use for the design of composite elements is shown and justified.

1. Introduction

Various composite materials find appropriate place in the contemporary construction industry. High strength under the different loads, low weight, high fire resistance could be named among the main advantages of such materials. For successful design of composite elements it is necessary to take into account that composite materials have inhomogeneous structure: physical properties of their matrix and reinforcement are significantly different. Generally, to provide required precision of calculations the composite beam elements behaviour is simulated using detailed 3D finite element modelling. This approach allows assigning material different material properties in different directions depending on reinforcement orientation.

However, in some cases one-dimensional beam models can be more convenient than detailed 3D. For example on the initial design stage when multiple variants should be compared and the best variant should be chosen. To use one-dimensional model of composite beam and not to sacrifice the calculation precision special hypotheses that approximate the orthotropic material properties are needed. The nonlocal hypothesis can be used to approximate the damping properties of composite material.

Nonlocal damping model is based on principals of nonlocal mechanics. [1] Damping in the certain point of structure with longitudinal coordinate x1 assumed to be dependent not only on local value of strain rate at this point, but also on the values of strain rates at the neighboring points. The bigger is the distance between the two points the lower influence one of them has on the other [3].

In this paper the nonlocal damping problem is solved on the example of glass fibre reinforced plastic beam with the fixed ends. The method of nonlocal model parameters defining is also shown on the beam of the same type.
2. Materials and methods

2.1. Nonlocal damping model for the beam with fixed ends

The Kelvin-Voigt material model is commonly used in engineering practice to describe the damping process:

\[ \sigma = E \varepsilon + \gamma E \dot{\varepsilon}, \] (1)

where \( \sigma, \varepsilon \) – normal stress and axial strain, \( \dot{\varepsilon} \) – rate of strain change, \( E \) – Young modulus, \( \gamma \) – damping ratio.

Dot indicates the time derivative.

Consider nonlocal damping equation (1) transforms to [3]:

\[ \sigma(x,t) = E \left[ \varepsilon(x,t) + \gamma \int_{0}^{\infty} C_{\gamma}(|x-\theta|) \dot{\varepsilon}(|\theta|,t) d\theta \right]. \] (2)

here \( C_{\gamma}(|x-\theta|) \) – the kernel function, which characterize the nonlocal damping. The \( C_{\gamma}(|x-\theta|) \) function responds to normalization requirement, which is:

\[ \int_{-\infty}^{\infty} C_{\gamma}(|x-\theta|) d\theta = 1. \] (3)

The kernel function can have rectangle shape, triangle shape, exponential shape or error function shape. [4,8] These shapes are just mathematical approximation and none of them is correlated with the internal structure of the material. Therefore none of them has an advantage before another. In this paper two types of kernel function are applied and compared:

a) Exponential kernel function:

\[ C_{\gamma}(|x-\theta|) = \frac{\mu}{2} e^{-\mu|x-\theta|}. \] (4)

b) Error kernel function:

\[ C_{\gamma}(|x-\theta|) = \frac{\mu}{\sqrt{2\pi}} e^{-\frac{(x-\theta)^2}{2}}. \] (5)

Here \( \mu \) is the parameter that characterizes the influence distance in the nonlocal damping model \( x \) and \( \theta \) are the longitudinal coordinate.

The equilibrium equation for an elementary part of the beam is:

\[ \frac{\partial^2 M(x,t)}{\partial x^2} = m \frac{\partial^2 w(x,t)}{\partial t^2} - q(x,t), \] (6)

where \( w(x,t) \) – beam deflection, \( m \)– distributed mass, \( q(x,t) \) – distributed load.

Considering that plane sections remain plain the bending moment expression is:

\[ M(x,t) = -EI \left[ \frac{\partial^2 w(x,t)}{\partial x^2} + \gamma \int_{0}^{\infty} C_{\gamma}(|x-\theta|) \frac{\partial^3 w(\theta,t)}{\partial \theta^2 \partial t} d\theta \right]. \] (7)

Here \( EI \) is beam bending stiffness.

Substitute the second derivative of the moment expression to the left part of the equation (5), and obtain the expression regarding the deflection function \( w(\theta,t) \):

\[ \frac{\partial^2 w(x,t)}{\partial t^2} + \frac{EI}{m} \left[ \frac{\partial^4 w(x,t)}{\partial x^4} + \gamma \frac{\partial^2 w(x,t)}{\partial x^2} \right] \frac{\partial^3 w(\theta,t)}{\partial \theta^2 \partial t} d\theta = \frac{q(x,t)}{m}. \] (8)

Solution of this equation should satisfy the boundary conditions for \( x=0 \) and \( x=l \).

Function \( w(x,t) \) is presented as normal modes of beam vibrations:
\[ w(x,t) = \sum_{i=1}^{n} f_i(t)V_i(x). \]  

Here \( f_i(t) \) – generalized displacements, \( V_i(x) \) – coordinate functions.

For the beam with fixed ends the coordinate functions are [5]:
\[ V_i(x) = (shk_l - \sin k_l)(ch k_i x - \cos k_i x) - (ch k_l - \cos k_l)(shk_l x - \sin k_l x). \]  

Here \( k_i \) is calculated as the \( i \)-root of the characteristic equation \( ch(kl) \cos(kl) = 1 \).

To obtain the generalized displacements \( f_i(t) \) the Galerkin method is used. The resulting system of the integral-differential equations with exponential function as kernel function is:
\[
\ddot{f}_j(\tau) + \left( k_j^{-1} \right)^4 f_j(\tau) + \frac{2}{a_j k_j^{-4}} \sum_{i=1}^{m} \int_{0}^{1} V_i^{-1}(z) \int_{0}^{1} e^{-\mu(x-z)} V_i^{-1}(y) dy dz \dot{f}_i(\tau) \\
= \frac{L^4}{a_j k_j^{4}} \int_{0}^{1} q(y,\tau) dy V_i(\tau),
\]  

where \( k_j^{4} = m\omega_j^{2}/EI \), \( \omega_j \) – natural frequency, \( a_j = \int_{0}^{l} V_j^{2}(x) dx \), \( z \) and \( y \) – dimensionless coordinates - \( z = x/l \), \( y = \theta/l \), \( \tau \) – dimensionless time \( \tau = \omega t \), \( \omega_j = E\lambda_j^{4}/m, 2 \varepsilon = \gamma\omega_j, \omega_j \) – minimum natural frequency.

The \( f_i(\tau) \) values are obtained by solving the equation system by numerical Runge-Kutta method of 4th order [6] that applied in MATLAB, and beam deflections - using expression (10).

3. Results

3.1. Influence of nonlocal damping consideration on the computer modelling results

First of all it is necessary to determine the number of vibration modes that need to be considered to obtain results of required precision [9]. For this purpose the beam with the fixed ends is considered under the instantly applied distributed load \( q = 1000\varepsilon \). The beam is 10 m long and has bending stiffness \( EI = 9.82 \cdot 10^7 t \cdot m^2 \). The deflection value is picked up in the middle point of the beam at the moment when the vibrations are totally damped and the deflection value is equal to static one.

As it is shown in table 1, when 5 and 7 eigenmodes are hold the results are equal up to the third sign.

Table 1. Deflection in the middle point of the beam corresponding to the number of eigenmodes.

| Number of Eigen modes | Deflection, m |
|-----------------------|--------------|
| 1                     | -0.0268      |
| 3                     | -0.0264      |
| 5                     | -0.0265      |
| 7                     | -0.0265      |

In case of periodic or stochastic load the number of eigenmodes should be taken according to the load frequency.

To figure out how variation of the influence distance \( \mu \), which is the main parameter of nonlocal model, affects the results of the beam vibration process computer modelling, the beam is loaded with the periodic distributed load:
\[ q = A\sin(\omega t), \]  

where \( A \) – load amplitude, \( \omega \) – load frequency, \( t \) – time.
The influence distance $\mu$ shows the level of nonlocal properties in material. The lower is $\mu$ the higher is the level. High $\mu$ values make the model close to local (figure 1).

![Figure 1. Exponential kernel function for different values of $\mu$.](image)

The vibration process of the beam under the periodic load (12) and $\mu = 0.2 \, l/m$ is shown on figure 2.

![Figure 2. Time history of deflection in the middle of the beam for $\mu=0.2 \, l/m$.](image)

The deflection amplitudes for different $\mu$ values are shown in table 2.

| Influence distance $\mu$, l/m | Deflection amplitude, m |
|-----------------------------|-------------------------|
| 0.2                         | 0.0350                  |
| 0.5                         | 0.0304                  |
| 1                           | 0.0237                  |

These amplitudes decrease as $\mu$ gets bigger, thus model with higher nonlocal properties gives the bigger deflection amplitude.

3.2. Defining of nonlocal model properties
The influence distance for the material has to be determined to make nonlocal model applicable for practical needs. For this purpose experimental or numerical simulation data can be used.

Consider GFRP beam with the fixed ends made of orthotropic thermoset vinyl ester class 1 FRP under instantly applied distributed load.

The characteristics of the material obtained experimentally in [7,2,10] are presented in table 3.

These characteristics were assigned to a 3D beam created in Abaqus SIMULIA. The beam is 6 m long and has 0.2x0.3m rectangular cross-section. The time history of vibrations under the instantly
applied load was obtained for this beam and imported to MATLAB. The difference between Abaqus results obtained with 3D orthotropic material model and the results obtained with one-dimensional beam where local Kelvin-Voigt model is used to describe damping is shown on figure 3. To make the damping model close to Kelvin-Voigt μ was assumed equal to 100.

Table 3. Characteristics of thermoset vinyl ester class 1 FRP

| Characteristics          | Value     |
|-------------------------|-----------|
| Young modulus (longwise)| 17.2 GPA  |
| Young modulus (crosswise)| 12.2 GPA  |
| Poisson ration (longwise)| 0.32     |
| Poisson ration (crosswise)| 0.15   |
| Density                 | 1.9 Mg/m3 |
| Damping ratio           | 0.042     |

Figure 3. Deflection of 6m long beam (0.2x0.3m) using local damping model in comparison to numerical simulation data.

Standard error for deflection can be calculated as follows:

\[
\sigma = \sqrt{\frac{\sum_{i=1}^{N} (y_i - f_i(\mu))^2}{N}}.
\]

Here  \(y_i\) is 3D orthotropic data value at a time step  \(i\),  \(f_i(\mu)\) – 1D model data value at a time step  \(i\),  \(N\) – number of time steps.

For Kelvin-Voight model the standard error is 16.7% of the deflection value at time step of 25 seconds.

For nonlocal-model the special script was created in MATLAB that allows defining \(\mu\) by the method of least squares. The kernel function was assumed exponential. In this case the results look as in figure 4.
Figure 4. Deflection of 6m long beam (0.2x0.3m) using nonlocal damping model in comparison to numerical simulation data.

For nonlocal model with parameters defined by method of least squares the standard error is 3.1% of deflection at time step of 25 seconds. 

The influence distance is a material property. Thus, the nonlocal model with defined \( \mu \) should work for any beam made of the same material. Two more beams with different geometry were modelled in 3D orthotropic and 1D nonlocal formulation to compare their vibration processes. It was assumed that the material is the same, so the influence distance does not change. The influence distance and standard errors for all three beams are presented in table 4, along with the influence distance and standard errors obtained using error kernel function.

Table 4. Comparison of 3D orthotropic and 1D nonlocal models results

| Kernel function          | Defined influence distance \( \mu \), 1/m | Beam length and X-section | Standard error, % |
|--------------------------|------------------------------------------|---------------------------|-------------------|
| Exponential kernel function | 1.07                                     | 6 m, 0.2x0.3              | 3.1               |
|                          |                                          | 7 m, 0.2x0.3              | 3.2               |
|                          |                                          | 7 m, 0.3x0.4              | 4                 |
|                          |                                          | 6 m, 0.2x0.3              | 3.1               |
| Error kernel function    | 0.96                                     | 7 m, 0.2x0.3              | 3.2               |
|                          |                                          | 7 m, 0.3x0.4              | 4                 |

The standard error stays less than 5% for beams with changed geometry, which means that nonlocal damping model can be used to simulate the beam vibration process with one-dimensional beam model. There is almost no difference between the results obtained by exponential and error kernel function. Both of them are just mathematical approximation and none of them is better than the other. To decrease the error it is necessary to carry out kernel function that will be based on the internal structure of composite material.

4. Discussion

The review of different studies of composite elements design has shown that their dynamic behaviour is often simulated using detailed 3D finite element modelling which requires significant amount of time and efforts. To apply one-dimensional models for the composite beam design it is possible to use the nonlocal damping model to describe the internal friction process in composite material. It is
possible to change the characteristics of the beam vibrations flexibly by varying the parameters of nonlocal damping model.

The nonlocal damping model with properties which are defined using the method of least squares properly fits the numerical simulation data. The relative error says less then 4%.

Comparison of two different kernel functions – exponential and error function – has shown almost no difference between results precision. Working out the kernel function based on the internal structure of material will be an object of further research.

Conclusion
The nonlocal damping model that can be used to simulate the composite beams dynamic behaviour is presented in this paper. The number of eigenmodes needed to obtain required calculation precision is determined and the influence of the nonlocal damping parameters on the beam vibrations is analysed.

It is shown in the paper that increasing the level of nonlocal properties in material (decreasing the influence distance $\mu$) leads to increasing of deflection amplitude.

The nonlocal damping model presented in this paper is more flexible in comparison to traditional local models. This allows modelling dynamic behaviour of beams that made of anisotropic and orthotropic materials in one-dimensional statement. This might be rewarding on the initial design stages when it is required to check and reject a significant amount of structure variants.

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