One-loop corrections to a scalar field during inflation

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Abstract. The leading quantum correction to the power spectrum of a gravitationally-coupled light scalar field is calculated, assuming that it is generated during a phase of single-field, slow-roll inflation.

Keywords: Inflation, Cosmological perturbation theory, Physics of the early universe, Quantum field theory in curved spacetime.
1. Introduction

Over the last several decades, our theories of the early universe have been promoted from an area of speculation to a field of intense scientific study. The most important developments in our knowledge concern the nature of the primordial curvature perturbation $\zeta$, which is widely believed to have seeded temperature variations in the cosmic microwave background (CMB). It is now understood that $\zeta$ must have had a spectrum which was close to scale invariance on the scales probed by the CMB [2, 3, 4].

Many theories have been proposed to explain how a set of primordial perturbations with an almost scale-invariant spectrum could have been generated in the early universe. The most widely-studied candidate is the suggestion that an era of inflation may have taken place at high energy [5, 6, 7, 8, 9, 10, 11], where “inflation” is defined to be any epoch in which the scale factor $a$ undergoes acceleration, $\ddot{a} > 0$. Under these conditions, local regions of the universe are exponentially driven to spatial flatness, homogeneity and isotropy [12], and each light bosonic field acquires a spectrum of perturbations generated by amplification of quantum-mechanical vacuum fluctuations [13, 14, 15, 16]. This spectrum is close to scale-invariance when the universe inflates at a rate $\dot{a}/a$ which is almost constant. The curvature perturbation observed in the CMB is taken to be a model-dependent mix of these fluctuations, giving rise to anisotropies in the temperature of the microwave sky which are compatible with observation. Therefore, inflation apparently provides a natural framework in which one can simultaneously understand both the large-scale regularity of the universe and its small-scale irregularity. For this reason, among others, it has become the dominant paradigm in which to model the very early universe.

Inflation is not a single model, but rather a whole collection of scenarios which fit into the above framework. The only necessary ingredients are: (i) a specification of the field content, which allows a division into ‘light’ and ‘heavy’ fields; (ii) a background evolution $a(t)$ which gives rise to $\ddot{a} > 0$ with the Hubble parameter $H \equiv \dot{a}/a$ slowly varying; and (iii) a rule for generating $\zeta$ from the light bosonic fields.

This prescription is extremely general and implies that very many models (perhaps with wildly different and mutually incompatible microphysics) may simultaneously be compatible with the observational data, since they may make equivalent predictions for the spectrum of $\zeta$. In view of this redundancy, we must expect that it will be difficult to learn about the microscopic physics which was operative during the very early universe. In particular, it will almost certainly be insufficient simply to study the spectrum of $\zeta$. In order to distinguish between wildly different models of the early universe it is necessary to find another source of data.

† There are two primordial perturbations commonly encountered in the literature. The first of these is the comoving curvature perturbation, written $R$, which is proportional to the laplacian of the Ricci curvature of comoving spatial slices. On the other hand, the uniform density curvature perturbation $\zeta$ is proportional to the laplacian of the Ricci curvature on spatial slices of uniform density. On superhorizon scales, $R$ and $\zeta$ are equivalent up to a convention for signs [1].
Fortunately, any detailed model of the inflationary era does not merely predict the spectrum of $\zeta$; it also implies a subtle but calculable network of correlations between the higher-order moments. These moments collectively measure the so-called non-gaussianity of $\zeta$ and arise from self-interactions among the quanta of the $\zeta$-field in the early universe. Such self-interactions are mandatory in any realistic theory of inflationary physics: since gravity is non-linear and couples universally to matter, there will always be interactions mediated by gravitational effects, quite apart from whatever interactions are explicitly postulated among the matter fields.

Non-gaussianities from self-interactions of $\zeta$ quanta have been investigated very extensively over the last few years, with the hope that observations of such effects may be able to discriminate between different models for physics in the early universe. However, much more is possible. Self-interactions do not only imply non-gaussian statistics in the three- and higher $n$-point correlation functions: they also imply quantum corrections to all correlation functions, and in particular the power spectrum or two-point function. It is possible that such corrections may be large in their own right, demanding that they be taken into account in accurate analyses of the observational data, as recently suggested by Sloth [49, 50]. However, regardless of their exact magnitude (provided they are detectable), by searching for signatures of such quantum corrections in the power spectrum and correlating the results with predictions for non-gaussian statistics in the higher $n$-point functions we obtain a more sensitive test of physics during inflation. This gives rise to the hope that it will eventually be possible to place restrictions on the effective field theory which was operative during inflation, although at present that hope is still somewhat distant.

A second powerful motivation for studying loop corrections is a simple point of principle. The tree-level formula for the spectrum of $\zeta$ is widely used to make predictions for the amplitude and scale-dependence of fluctuations generated in a very large class of early universe scenarios. Before deciding what degree of credence we should attach to any of these predictions, it is necessary to thoroughly investigate whether the tree-level amplitude is a genuine approximation to the full quantum result.

In this paper, the prospects for detecting quantum corrections to the power spectrum are assessed in the simplest model of inflationary physics, that of inflation with a single scalar $\phi$ and arbitrary potential $V(\phi)$. In view of the importance of accurate comparison with the precision measurements which are becoming available, this issue has already attracted considerable attention. Early work by Mukhanov, Abramo & Brandenberger [51, 52] and Abramo & Woodard [53] demonstrated that significant

‡ In a complete embedding of inflation within a self-consistent model of particle physics, the inflaton will most likely carry quantum numbers associated with some gauge group, as for example in the MSSM-based proposal discussed in Refs. [17, 18, 19]. The possible couplings between Standard Model matter and the inflaton are then restricted by gauge-invariance which can have important consequences, as described in the above references. However, in the present paper such couplings are ignored and the inflaton is treated as a gauge singlet with all significant loop corrections coming from the coupling to gravity.
effects were possible (see also Unruh [54]). Later estimates of loop effects were made in a large number of models [55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75], even at two-loop order [76]. Recently, Sloth [49, 50] determined the full fourth-order action for Einstein gravity coupled to a scalar field and used this to estimate the one-loop correction to the power spectrum of scalar field fluctuations. Although one would naively expect the loop correction to be suppressed by a factor of \((H/M_p)^2 \sim 10^{-10}\), where \(H\) is the Hubble parameter during inflation, this yielded a significant cumulative effect (as large as 70% in some models) which could affect the precision determination of cosmological parameters from CMB experiments. The unexpectedly large size of the loop correction in this estimate is due to an amplification by \(N\), the total number of e-folds of inflation which occur. Since \(N\) can be very large in models where inflation begins at around the Planck scale, it may dramatically modify the predictions of naïve dimensional analysis.

This paper attempts to readdress these issues using a formalism similar to that applied by Sloth [49, 50]. However, in complete contrast to all previous analyses, the estimate is divided into two parts, the first of which is the central subject of the present paper. First, the one-loop correction to the power spectrum of field fluctuations is computed soon after horizon exit, using the slow-roll approximation to control the calculation. This correction is not observable by itself; it must be combined with other correlators of the fields in a second step to yield the one-loop correction to the power spectrum of the observable perturbation \(\zeta\) long after horizon exit. The correct combination can be computed using the \(\delta N\) formula [77, 78, 22, 79, 26]. This two-step process has several advantages. We shall see that the loop correction is generally afflicted by divergences at late times and on large scales. The \(\delta N\) formalism naturally resums these late-time divergences into time evolution, which allows the slow-roll approximation to be kept under control. On the other hand, the divergences on large scales can be controlled by performing the calculation within a finite box. In analogy with the late-time divergences, it has recently been shown by Byrnes et al. [48] that these divergences can be resummed into spatial variation on large scales.

The present paper is concerned with the mostly technical issue of computing the loop correction for the power spectrum of the field fluctuations. This calculation involves the application of standard methods from quantum field theory, adapted to the case of an expanding spacetime. On the other hand, the assembly of field correlators into \(\zeta\) correlators is an essentially classical calculation using the \(\delta N\) formula. For clarity, this calculation will be presented separately elsewhere [80].

In §2 the background evolution and perturbation theory of the single scalar field are briefly described. The perturbations are characterized (as in more complex cases) by cubic and higher self-interactions which involve the time derivative of the perturbation, a fact which has important consequences for the calculation of quantum corrections. These corrections are introduced in §3. In §3.1 a path-integral expression for a general one-loop, single-vertex correction to the power spectrum is given in the Schwinger formalism, and in §3.2 the question of deriving a correct path integral expression for theories with
derivative interactions is considered. In such cases the correct path-integral formula is well-known to contain a ghost field, whose quanta do not appear in physical states but circulate in the loops which give rise to quantum corrections. The Feynman rules for this theory are written down in §3.3. In §4 the assembled formalism is used to compute the leading radiative correction to the one-point function of the field. This is of interest in its own right, but also provides a simple setting in which some subtle features of the calculational machinery can be resolved. The one-loop correction to the two-point function is computed in §5, and a brief discussion is given in §6.

§2 is introductory and merely serves to fix notation. The reader who is mostly interested in the computation of the two-point function \[\langle \delta \phi(k_1) \delta \phi(k_2) \rangle\] may wish to dispense with §§3.1–3.2 and §4. These sections are largely dominated by the question of setting up a correct formalism in which the one-loop correction may be computed.

Units are chosen throughout such that \(\hbar = c = M_P = 1\), where \(M_P^{-2} = 8\pi G\) is the reduced Planck mass. The metric convention is \((- , +, +, +)\), and the unperturbed background is written in cosmic time \(t\) as

\[
ds^2 = -dt^2 + a^2(t) \, dx^2.\tag{1}\]

It is frequently more convenient to employ a conformally rescaled time variable, defined by \(\eta = \int_0^\infty dt'/a(t')\). Indices labelling spacetime coordinates are chosen from the beginning of the Latin alphabet \((a, b, \ldots)\); indices labelling purely spatial coordinates are chosen from the middle of the alphabet \((i, j, \ldots)\). The different species of light bosonic fields are labelled with Greek indices \((\alpha, \beta, \ldots)\).

2. Inflation from a single scalar field

2.1. The background evolution

The simplest realistic microphysical model capable of supporting an inflationary epoch consists of Einstein gravity coupled to a single scalar field \(\phi\) with potential \(V(\phi)\), which can be taken to be arbitrary except that it must support inflation for some values of \(\phi\). The field \(\phi\) is known as the inflaton. The combined action for this system is

\[
S = -\frac{1}{2} \int d^4x \, \sqrt{-g} \left\{ R - \nabla^a \phi \nabla_a \phi - 2V(\phi) \right\}, \tag{2}
\]

where \(g \equiv \det g_{ab}\) is the metric determinant, \(d^4x \equiv dt \, d^3x\) is the product of the spacetime coordinate differentials, and \(R\) is the spacetime Ricci curvature. The background field \(\phi\) is taken to be spatially homogeneous and the background metric is parametrized by the scale factor \(a\), given in (1). The evolution of \(a\) is determined by the Friedmann equation

\[
3H^2 = \frac{1}{2} \dot{\phi}^2 + V(\phi), \tag{3}
\]

where \(H \equiv \dot{a}/a\) is the Hubble parameter, and \(\phi\) obeys the homogeneous Klein–Gordon equation

\[
\ddot{\phi} + 3H \dot{\phi} + V'(\phi) = 0, \tag{4}
\]
where a prime \( ' \) denotes a derivative with respect to \( \phi \). The condition that inflation occurs is \( \ddot{a} > 0 \). This can be rewritten as \( aH^2(1 - \epsilon) > 0 \), where the parameter \( \epsilon \) is defined by

\[
\epsilon \equiv -\frac{\dot{H}}{H^2}.
\]

Using Eqs. (3)–(4) one can show that an equivalent definition is \( \epsilon \equiv \dot{\phi}^2/2H^2 \). Thus \( \epsilon \) is manifestly positive. In terms of \( \epsilon \), inflation occurs whenever \( \epsilon < 1 \).

When \( \epsilon \) obeys the stronger condition \( \epsilon \ll 1 \), the rate of change of \( \phi \) is negligible in comparison with the expansion rate \( H \). In this case one says that the field is in the slow-roll régime. Although slow-roll is not mandatory for inflation to occur, the near scale-invariance of the power spectrum imprinted on the scales which are observed in microwave background experiments suggests that slow-roll was approximately satisfied if the CMB perturbation has an inflationary origin. When \( \epsilon \ll 1 \) applies, one can develop a useful perturbation expansion in \( \epsilon \), known as the slow-roll approximation. In this paper, we compute all effects to leading order in \( \epsilon \).

2.2. Scalar perturbations

Now consider the possibility of small spatially-dependent perturbations in the inflaton, \( \phi = \phi_0 + \delta\phi(t, x) \), where \( \phi_0 \) is the homogeneous background evolution and \( \delta\phi \) obeys the smallness condition \( |\delta\phi| \ll |\phi_0| \). Since \( \phi \) dominates the energy density of the universe by assumption, any perturbation in \( \phi \) will lead to a perturbation in the metric. These perturbations can be parametrized by a scalar \( N \) (the lapse), a spatial vector \( N^i \) (the shift), and a spatial metric \( h_{ij} \),

\[
d s^2 = -N(t, x)^2 \, dt^2 + h_{ij}(t, x) \left\{ dx^i + N^i(t, x) \, dt \right\} \left\{ dx^j + N^j(t, x) \, dt \right\}.
\]

Because of general coordinate invariance, not all choices of \( \{N, N^i, h_{ij}\} \) lead to different configurations of the gravitational field. This redundancy is removed by fixing a gauge, which requires a choice of slicing into spatial hypersurfaces accompanied by a prescription for threading these spatial slices together. We will choose to work in the spatially flat gauge, where \( h_{ij} \) is given by its background value \( h_{ij} = a^2(t)\delta_{ij} \). Having done so, the metric functions \( N \) and \( N^i \) are completely determined in terms of \( \delta\phi \) by the constraint part of the Einstein equations.

These constraint equations can be obtained by inserting the metric (6) in the Einstein–scalar action (2). One obtains

\[
S = -\frac{1}{2} \int dt \, d^3 x \, \sqrt{h} \left\{ N(\nabla^i \delta\phi \nabla_i \phi + 2V) - \frac{1}{N}(E^{ij} E_{ij} - E^2 + \pi^2) \right\},
\]

where \( E_{ij} = \frac{1}{2} \dot{h}_{ij} - \nabla_i N_j \nabla_j \) is the “gravitational momentum” associated with \( h_{ij} \), \( \nabla_i \) is the spatial covariant derivative compatible with \( h_{ij} \), and \( \pi = \dot{\phi} - N^j \nabla_j \phi \) is the field momentum. The equations of motion for the lapse and shift follow by varying \( S \) with

\[\dagger\] We are adopting a useful convention used throughout this paper in which repeated spatial indices in complementary raised and lowered positions are contracted with the spatial metric \( h_{ij} \), whereas a pair
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respect to \( N \) and \( N^i \) respectively, and do not involve time derivatives. Therefore they are not evolution equations but constraints and can be solved algebraically: \( N \) and \( N^i \) are not propagating fields. Once \( N \) and \( N^i \) are known they may be substituted in (7) to obtain a reduced action which depends only on \( \delta \phi \).

The \( N \) constraint is
\[
\nabla^i \phi \nabla_i \phi + 2V + \frac{1}{N^2} (E^{ij} E_{ij} - E^2 + \pi^2) = 0
\]
and the \( N^i \) constraint is
\[
\nabla_i \left\{ \frac{1}{N} (E^j_i - E \delta^j_i) \right\} = \frac{\pi}{N} \nabla_j \phi.
\]
One solves Eqs. (8)–(9) order by order in \( \delta \phi \). We write
\[
N = 1 + \sum_{m=1}^{\infty} \alpha_m, \quad \text{and} \quad N^i = \nabla_i \left( \sum_{m=1}^{\infty} \vartheta_m \right) + \sum_{m=1}^{\infty} \beta_{mi}
\]
where \( \alpha_m, \vartheta_m \) and \( \beta_{mi} \) are all \( m \)th order in \( \delta \phi \) and the \( \beta_{mi} \) are chosen to be divergenceless, so that \( \nabla^i \beta_{mi} \) for all \( m \). The expressions necessary to compute \( S \) to third order in \( \delta \phi \) were given by Maldacena in the comoving slicing \[21\] and rewritten in the flat slicing for multiple fields in Ref. \[24\]. The expressions necessary to compute \( S \) to fourth order were obtained in the flat slicing by Sloth \[49, 50\] in an approximation where all vector modes were absent, and given in complete generality in Ref. \[81\].

We work to leading order in the slow-roll approximation. At first order in \( \delta \phi \) the leading terms are \( o(\epsilon^{1/2}) \),
\[
\alpha_1 = \frac{1}{2H} \phi \delta \phi, \quad \partial^2 \vartheta_1 = -\frac{a^2}{2H} \dot{\phi} \delta \phi \quad \text{and} \quad \beta_{ii} = 0.
\]
At second order in \( \delta \phi \) the leading terms are \( o(\epsilon^0) \)
\[
\alpha_2 = \frac{1}{2H} \partial^2 \Sigma, \quad \frac{4H}{a^2} \partial^2 \vartheta_2 = -\frac{1}{a^2} \partial_r \delta \phi \partial_r \delta \phi - \delta \dot{\phi} \delta \dot{\phi} - 12H^2 \alpha_2,
\]
\[
\frac{1}{2a^2} \partial^4 \beta_{2i} = \delta^r_s (\partial_r \Sigma_{rs} - \partial_s (\Sigma_{rs})),
\]
where bracketed indices (\( \cdots \)) are symmetrized with total weight unity and \( \Sigma_{rs} \) is defined by
\[
\Sigma_{rs} \equiv \partial_r \delta \dot{\phi} \partial_s \delta \phi + \delta \dot{\phi} \partial_r \partial_s \delta \phi,
\]
with \( \Sigma = \text{tr } \Sigma_{rs} \) its trace in the Euclidean metric. Eqs. (11)–(13) can be inserted into the action, Eq. (7), after which one obtains an expansion of \( S \) in powers of \( \delta \phi \). The first non-trivial term is quadratic. At \( o(\epsilon^0) \) it is equal to
\[
S_2 = \frac{1}{2} \int dt \, d^3x \, a^3 \left\{ \delta \phi^2 - \frac{1}{a^2} (\partial \delta \phi)^2 \right\}
\]
of repeated indices which both appear in the lowered position are contracted with the Euclidean metric \( \delta_{ij} \). Thus, \( a^i b_i = \sum_{i,j} h^{ij} a_i b_j \), whereas \( a_i b_i = \sum_i a_i b_i \). Spacetime indices obey the usual Einstein convention, and always appear in complementary raised and lowered pairs which are contracted with the spacetime metric \( g_{ab} \).
There is a cubic interaction whose leading term enters at $o(\epsilon^{1/2})$ [21], which can be written

$$S_3 = \int dt \, d^3 x \, a^3 \frac{\dot{\phi}}{4H} \left\{ 2\delta \dot{\phi} \partial_t \partial^{-2} \delta \dot{\phi} \partial_j \partial \delta \phi - \delta \phi \left[ \frac{3}{2} \left( \partial \delta \phi \right)^2 + \frac{1}{a^2} \left( \partial \delta \phi \right)^2 \right] \right\}. \tag{16}$$

The quartic term has leading terms of order $o(\epsilon)$ [81]. These terms correspond to

$$S_4 = \int dt \, d^3 x \left\{ -\frac{1}{4a} \beta_2 \partial^2 \beta_j \partial_j - a^3 \frac{3}{4H} \partial^{-2} \sum \delta \phi^2 + \frac{1}{a^2} \left( \partial \delta \phi \right)^2 \right\} - \frac{3}{4} a^3 \left( \partial^{-2} \Sigma \right)^2 - a \delta \phi \beta_2 \partial_j \partial \delta \phi \right\}. \tag{17}$$

The free field action $S_2$ and the interactions $\{S_3, S_4\}$ as written here are all accompanied by terms of higher-order in slow-roll parameters, which we neglect. One must be careful to ensure that this approximation is accurate, and we will return to this at various points in the analysis (see also Ref. [80]).

2.3. Expectation values

A class of especially important observables in this theory are expectation values of products of $n$ factors of the perturbation $\delta \phi$, taken at a common time $t$, but at distinct spatial coordinates $\{x_1, \ldots, x_n\}$. However, it is often more convenient to work with momentum space expectation values which are obtained by taking Fourier transforms with respect to the $x_i$, giving $k$-space correlators which are functions of $\{k_1, \ldots, k_n\}$.

At tree level the one-point expectation value vanishes, $\langle \delta \phi(k) \rangle = 0$, since $\delta \phi$ is by definition a perturbation in the comoving region under consideration. However, the gravitational background is time dependent since the scale factor $a(t)$ varies with $t$, and therefore the vacuum state of the theory is changing continuously. This effect leads to the gravitational production of inflaton particles [82]. Therefore we must expect a non-zero one-point function to be generated radiatively, reflecting the emergence of $\phi$ quanta from the vacuum. This issue is discussed in more detail in §4 below.

The two-point expectation value defines the power spectrum $P(k)$,

$$\langle \delta \phi(k_1) \delta \phi(k_2) \rangle_\ast = (2\pi)^3 \delta(k_1 + k_2) P_\ast(k_1). \tag{18}$$

The subscript ‘$\ast$’ denotes evaluation at the time when the $k$-mode under consideration left the horizon. At tree-level, $P_\ast(k) = H_*^2/2k^3$. It is often useful to work instead with the so-called dimensionless power spectrum, which is related to $P(k)$ by the rule $P(k) = k^3 P(k)/2\pi^2$.

The three-point expectation value defines the bispectrum, $B(k_1, k_2, k_3)$, and the four-point expectation value defines the trispectrum, $T(k_1, k_2, k_3, k_4)$,

$$\langle \delta \phi(k_1) \delta \phi(k_2) \delta \phi(k_3) \rangle = (2\pi)^3 \delta(\sum_i k_i) B(k_1, k_2, k_3), \tag{19}$$

and

$$\langle \delta \phi(k_1) \delta \phi(k_2) \delta \phi(k_3) \delta \phi(k_4) \rangle = (2\pi)^3 \delta(\sum_i k_i) T(k_1, k_2, k_3, k_4). \tag{20}$$

Typically, $B$ and $T$ at tree-level are proportional to $P^2$ and $P^3$ respectively, multiplied by a momentum-dependent form-factor [21, 22, 23, 25, 83, 84, 24, 81, 85].
3. Quantum corrections

3.1. Loop corrections from Schwinger integrals

The appropriate formalism for computing expectation values in a quantum field theory was outlined by Schwinger [86]. Consider the vacuum expectation value of any observable $O$, observed at some time $t_*$, taken in some theory with light scalar fields $\{\phi^\alpha\}$. By inserting a complete set of states at any time $t_*$, taken in the correct vacuum, this expectation value can be written

$$\langle \Omega | O | \Omega \rangle_* = \int [d\varphi^\alpha] \langle \Omega | \phi^\alpha_2 = \varphi^\alpha \rangle \langle \phi^\alpha_2 = \varphi^\alpha | O | \Omega \rangle_*,$$  \hspace{1cm} (21)

where $|\Omega\rangle$ is the vacuum state at $t \rightarrow -\infty$, the subscript ‘*’ indicates that the fields in the expectation value are evaluated at $t_*$, and $\phi^\alpha_2$ denotes $\phi$ evaluated at $t_*$. Each factor in the product of transition amplitudes on the right-hand side of (21) can be expressed using the conventional Feynman path integral formula [87, 88, 89],

$$\langle \phi^\alpha = \varphi | O | \Omega \rangle = \int [d\varphi^\alpha] \exp \{ iS[\varphi] \} O \exp iS,$$

where $S$ is the action functional and the integral is taken over all field configurations which begin in the state $|\Omega\rangle$ and end in the state $|\phi^\alpha = \varphi\rangle$. These boundary conditions on $\phi$ are schematically denoted by the limits $\Omega$ and $\varphi$ attached to $[d\phi]$. 

The interacting vacuum In order to evaluate such integrals by the usual Feynman diagram expansion it is necessary to remove these boundary conditions, so that we integrate over all $\phi$ unrestrictedly. We follow the analysis of Weinberg [68]. To remove the restriction that the field must begin in the vacuum state one can integrate over all $\phi$ obeying an arbitrary boundary condition at $t \rightarrow -\infty$, after multiplying the integrand by the vacuum wavefunctional, $\Psi[\psi] = \langle \phi(t \rightarrow -\infty) = \psi|\Omega\rangle$. This has the desired effect of restricting the integral to field configurations which begin in the correct vacuum. The exact expression for $\Psi$ depends on what we assume about $|\Omega\rangle$, but because the theory is supposed to be free as $t \rightarrow -\infty$ it must be a gaussian in the fields [87, 68]. Therefore we assume

$$\Psi[\psi] \propto \prod_\alpha \exp \left\{ -\frac{1}{2} \int \frac{d^3q}{(2\pi)^3} \frac{d^3r}{(2\pi)^3} \delta(q + r) \Omega_\alpha(q) \psi^\alpha(q) \psi^\alpha(r) \right\} = \prod_\alpha \exp \left\{ -\frac{1}{2} \langle \psi^\alpha, \Omega_\alpha \psi^\alpha \rangle \right\}, \hspace{1cm} (22)$$

for some set of weight functionals $\{\Omega_\alpha(q)\}$, where $\langle \psi, \Omega \psi \rangle$ is a convenient abbreviation for the integral. The expectation value (21) can therefore be written [68]

$$\langle \Omega | O | \Omega \rangle \propto \left( \prod_\alpha \int [d\varphi^\alpha] \right) \left( \prod_\beta \int [d\varphi^\beta] \right) \exp \{ iS[\varphi] \} \prod_\beta \exp \left\{ -\frac{1}{2} \langle \psi^\beta, \Omega_\beta \psi^\beta \rangle \right\} \hspace{1cm} \uparrow$$

$$= \left( \prod_\gamma \int [d\varphi^\gamma_+] \right) O \exp \{ iS[\varphi_+] \} \prod_\gamma \exp \left\{ -\frac{1}{2} \langle \psi^\gamma_+, \Omega_\gamma \psi^\gamma_+ \rangle \right\} \hspace{1cm} (23)$$

where ‘$\uparrow$’ denotes Hermitian conjugation, and the fields in the two path integrals have been differentiated by the addition of subscripts ‘+’ and ‘−’. In addition, $S[\varphi_+]$ and
$S[\phi_-]$ represent the action evaluated on the + and − fields, respectively. The overall constant of proportionality is irrelevant. Since (23) requires an integral over final field configurations, it is possible to drop the restriction on the fields $\phi_\pm$ at $t_5$, provided we guarantee that the + and − fields for each species share a common value at this time. This can be accommodated by inserting a δ-function into the integrand which constrains the fields to agree [68]

$$\prod_\alpha \delta\{\phi^\alpha_+ (t_3) - \phi^\alpha_- (t_3)\} \propto \lim_{\varepsilon \to 0} \exp \left\{ -\frac{1}{\varepsilon} \sum_\alpha \left[ \phi^\alpha_+ (t_3) - \phi^\alpha_- (t_3) \right]^2 \right\},$$

where $\varepsilon$ is positive. Since the action is real by assumption, the only effect of the Hermitian conjugation in (23) is to flip the sign of the $iS$ term. Suppose that the action corresponds to a free field, so that it can be written $S = (2\pi)^{-3} \int d^3k_1 d^3k_2 dt_1 dt_2 \phi(t_1, k_1) \Delta \phi(t_2, k_2)/2$ for some differential kernel $\Delta$. Eq. (23) can then be written as an unrestricted path integral over the fields $\{\phi_+, \phi_-\}$ of the form

$$\left( \prod_\alpha \int [d\phi^\alpha_+ d\phi^\alpha_-] \right) \exp \left\{ \frac{i}{2} \int \frac{d^3k_1 d^3k_2}{(2\pi)^3} dt_1 dt_2 \sum_\alpha \left[ \phi^\alpha_+ (t_1, k_1) \right]^T K_{12}^{\alpha} \left[ \phi^\alpha_+ (t_2, k_2) \right] \right\},$$

where we have assumed that there are no linear couplings among the various species, $T$ denotes a transpose, and $K_{12}^{\alpha}$ is the $(2 \times 2)$ kernel

$$K_{12}^{\alpha} \equiv \delta(k_1 + k_2) \begin{pmatrix} \Delta_\alpha + \frac{2i}{\varepsilon} \delta_{12} \delta_{23}^i + i\delta_{1\infty} \delta_{2\infty} \Omega_\alpha & -\frac{2i}{\varepsilon} \delta_{12} \delta_{23}^i - \delta_{1\infty} \delta_{2\infty} \Omega_\alpha \\ -\frac{2i}{\varepsilon} \delta_{12} \delta_{23}^i - \delta_{1\infty} \delta_{2\infty} \Omega_\alpha & \Delta_\alpha + \frac{2i}{\varepsilon} \delta_{12} \delta_{23}^i + i\delta_{1\infty} \delta_{2\infty} \Omega_\alpha \end{pmatrix}.$$ (26)

In Eq. (26), $\delta_{ij}$ is the δ-function $\delta(t_j - t_2)$ and $\delta_{ij\infty}$ is the δ-function $\delta(t_j + \infty)$. We will also occasionally use the notation $\delta_{ij} \equiv \delta(t_i - t_j)$. The field propagator matrix for any particular species, consisting of propagators $\{G_{++}, G_{+-}, G_{-+}, G_{--}\}$ which connect the + and − fields, is found by inverting the quadratic term given in (25),

$$\int dt_2 d^3k_2 K_{12}^{\alpha} \begin{pmatrix} G_{++} & G_{+-} \\ G_{-+} & G_{--} \end{pmatrix}_{23} = i(2\pi)^3 \delta_{13} \delta(k_1 - k_3) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$ (27)

The subscript ‘23’ indicates that $G_{++}$ is a function of times and momenta in the form $G_{++}(t_2, k_2; t_3, k_3)$, etc., and similarly for the other $G$.

Eq. (27) splits into coupled equations for $G_{++}, G_{+-}, G_{-+}$ and $G_{--}$. It will shortly become apparent that the doublets $(G_{++}, G_{+-})$ and $(G_{-+}, G_{--})$ are to be regarded as forming complex conjugate pairs, so half of these equations are related to the other half by complex conjugation. In the application of interest, $\Delta$ is given to leading order in the slow-roll approximation by the laplacian of exact de Sitter space,

$$\Delta_{12} = \frac{\partial}{\partial t_1} \frac{\partial}{\partial t_2} \left\{ a^3 \delta_{12} \right\} + (k_1 \cdot k_2) a \delta_{12}.$$ (28)

Write $G_{++}^{12} = (2\pi)^3 \delta(k_1 + k_2) \tilde{G}_{++}^{12}$. With this choice, the $\tilde{G}_{++}$ equation reads

$$\frac{\partial^2}{\partial t_1^2} \tilde{G}_{++}^{12} + 3H(t_1) \frac{\partial}{\partial t_1} \tilde{G}_{++}^{12} + \frac{k_1^2}{a(t_1)^2} \tilde{G}_{++}^{12} - \frac{i}{a^3} \delta_{1\infty} \Omega(k_1) \tilde{G}_{++}^{12} = -\frac{i}{a^3} \delta_{12}.$$ (29)
\( \tilde{G}_{++} \) obeys the homogeneous version of (29), whereas \( \tilde{G}_{--} \) obeys the complex conjugate of (29) and \( \tilde{G}_{+-} \) its homogeneous complex conjugate. In addition, Eq. (27) gives \( \tilde{G}_{+-} \) and \( \tilde{G}_-- \) the boundary conditions

\[
\delta_{12} \tilde{G}^{22}_{++} = \delta_{12} \tilde{G}^{22}_{--} \quad \text{and} \quad \delta_{12} \tilde{G}^{22}_{+-} = \delta_{12} \tilde{G}^{22}_{--}. \tag{30}
\]

Consider any solution, say \( \tilde{G} \), to the homogeneous version of (29). Any such solution is a function of the single variable \( t \), which after changing to conformal time \( \eta \) can be written in the form \( \tilde{G}(\eta) \equiv \zeta_k(\eta)/a(\eta) \) for some function \( \zeta(\eta) \) to be determined. [The dependence of the mixed propagators on a second time argument, \( t_2 \), enters only through the boundary conditions \( (30) \).] The mode function \( \zeta_k \) must obey

\[
\zeta''_k + \left\{ k^2(1 - 2i\nu) - (aH)^2(2 - \epsilon) \right\} \zeta_k = 0 \tag{31}
\]

where \( k \) is the common magnitude of \( k_1 \) and \( k_2 \), a prime \( ' \) denotes a derivative with respect to \( \eta \), and \( \nu \) satisfies

\[
\nu \equiv \delta_{k\infty} \frac{\Omega(k)}{2ak^2} > 0. \tag{32}
\]

Eq. (31) is equivalent to the condition \( \zeta''_k + \{ k^2 - (aH)^2(2 - \epsilon) \} \zeta_k = 0 \) almost everywhere, together with the boundary condition \( \zeta_k/a^2 \to 0 \) as \( \eta \to -\infty \). Heuristically, this boundary condition can be accommodated most naturally by redefining the range of \( \eta \) to include some evolution in imaginary time, \( \eta \to \eta(1 + i\nu) \). Although strictly speaking this contour is singular, owing to the presence of the \( \delta \)-function, it can be approached as a limit of regular contours. It will be seen below that when integrations over \( \eta \) are required the integrands in question are holomorphic, at least at tree level. Therefore, any one of these regular contours suffices for calculation and we may as well take \( \eta \to \eta(1 + i\nu) \) for fixed \( \nu \). This prescription was used in Refs. [21, 24, 81] to compute tree-level correlation functions of the \( \{ \delta \phi^n \} \) in the interacting vacuum.

One can now construct an explicit solution for \( G_{++} \), which satisfies (in conformal time with arguments \( \eta_1 \) and \( \eta_2 \))

\[
G^\text{12}_{++}(k_1, k_2) = (2\pi)^3 \delta(k_1 + k_2) \times \begin{cases} \xi_k(\eta_1, \eta_2) & \text{if } \eta_1 < \eta_2 \\ \xi^*_k(\eta_1, \eta_2) & \text{if } \eta_2 < \eta_1 \end{cases} \tag{33}
\]

where \( * \) denotes complex conjugation, \( k \) is the common magnitude of \( k_1 \) and \( k_2 \), and \( \xi_k(\eta_1, \eta_2) \) is defined by

\[
\xi_k(\eta_1, \eta_2) \equiv \frac{iW^{-1}(\zeta^*_k, \zeta_k)}{a(\eta_1)a(\eta_2)} \zeta_k(\eta_1)\zeta^*_k(\eta_2), \tag{34}
\]

in which \( W(f, g) \) is the Wronskian \( W(f, g) \equiv fg' - gf' \). Note that \( iW^{-1}(\zeta^*_k, \zeta_k) \) is real and time-independent, in virtue of Abel’s identity, but may depend on \( k \). The propagator \( G_{--} \) is obtained by complex conjugation of Eq. (33); the mixed propagator \( G_{+-} \) is obtained from a homogeneous equation and therefore is smooth at \( \eta_1 = \eta_2 \). The boundary condition (30) implies that it must satisfy

\[
G^\text{12}_{+-}(k_1, k_2) = (2\pi)^3 \delta(k_1 + k_2)\zeta_k(\eta_1, \eta_2) \tag{35}
\]
and $G_{-+}$ is given by its complex conjugate. [Note that there is no ambiguity in deciding which propagator should be assigned to a mixed pair $\langle \delta \phi_+ \delta \phi_- \rangle$, because the mode $\zeta$ is always assigned to the argument of the $+$ field, and $\zeta^*$ is always assigned to the argument of the $-$ field.]

The above analysis was carried out for a single field, but where more than one species of light field is present similar results apply, with a mode function $\zeta_k^\alpha$ for each species which obeys a vacuum boundary condition of the form $(\zeta_k^\alpha/a^2) \to 0$ in the far past. The propagators which connect two fields of different species which do not couple linearly, then it follows that $\zeta_k^\alpha = \zeta_k^\beta \zeta_k^\gamma$. If the fields do not couple linearly, then it follows that $\zeta_k^{\alpha\beta} = \delta^{\alpha\beta} \zeta_k^\alpha$.

One-vertex, one-loop amplitudes In the remainder of this paper, we shall be concerned with computing expectation values in which a set of $n$ external fields $\{\phi(k_n)\}$, observed at some time $\eta_*$, and carrying momenta $\{k_n\}$, are paired with a single $(n + 2)$-valent internal vertex with coupling constant $g$. Applying Schwinger’s formula shows that the term in such an expectation value of leading order in $g$ is given by

$$i(2\pi)^3 \int dq_1 \cdots dq_n dq_{n+1} dq_{n+2} \delta(\sum q_i) \int_{-\infty}^{\eta_*} d\eta \ gM,$$

where $M$ is defined by

$$M \equiv \left\langle \phi^\alpha_+(k_1) \cdots \phi^\beta_+(k_n) \phi^\gamma_+(q_1) \cdots \phi^\delta_+(q_n) \phi^\sigma_+(q_{n+1}) \phi^\sigma_+(q_{n+2}) \right\rangle$$

$$- \left\langle \phi^\alpha_+(k_1) \cdots \phi^\beta_+(k_n) \phi^\gamma_-(q_1) \cdots \phi^\delta_-(q_n) \phi^\sigma_-(q_{n+1}) \phi^\sigma_-(q_{n+2}) \right\rangle.$$  \hspace{1cm} (37)

Greek indices label the species of fields, which are here allowed to run over field derivatives as well as the fields themselves; the issue of obtaining a correct path integral for theories with derivative interactions causes no difficulties for the purposes of Eqs. (36)–(37), but will be taken up again in more detail in the next section. Any amplitude of the type given in (36)–(37) is automatically of one-loop order, because the two field operators left over after all $n$ external fields have been paired with $n$ of the vertex fields must contract amongst themselves, leaving a single unconstrained integral over momentum.

The time integral in (36) has been carried to some arbitrary late time $\eta_\delta$ which satisfies $\eta_\delta > \eta_*$. Using Eqs. (33)–(35) together with their complex conjugates in Eq. (36), it follows that the expectation value can be written

$$i(2\pi)^3 \delta(k_1 + \cdots + k_n) \int \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{\eta_*} d\eta \ \xi_1^{\alpha\gamma}(\eta, \eta_*) \cdots \xi_{k_n}^{\beta\delta}(\eta, \eta_*) \xi_\eta^{\rho\sigma}(\eta, \eta)$$

$$- i(2\pi)^3 \delta(k_1 + \cdots + k_n) \int \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{\eta_*} d\eta \ \xi_1^{\alpha\gamma^*}(\eta, \eta_*) \cdots \xi_{k_n}^{\beta\delta^*}(\eta, \eta_*) \xi_\eta^{\rho\sigma^*}(\eta, \eta)$$

$$+ \text{permutations}, \hspace{1cm} (38)$$

in which the second term is the complex conjugate of the first, and all permutations likewise assemble into complex conjugate pairs. Observe that the internal term $\xi_\eta(\eta, \eta)$
in the first line comes from pairing two $+$ fields, whereas in the second line it comes from pairing two $-$ fields. Eq. (34) shows that for the $\delta\phi$ propagator, $\xi_q(\eta, \eta)$ is real, so that $\xi_q(\eta, \eta)$ and $\xi^*_q(\eta, \eta)$ are in fact equal.

The part of the integration over times between $\eta_\ast$ and $\eta_\sharp$ has cancelled out, since in this region Eq. (35) is given by the same expression as Eq. (33), whereas in the region $\eta < \eta_\ast$ it is given by its complex conjugate. Note that for interactions which contain more than a single vertex and a single loop the process of deriving expressions such as (38) using the path integral technology described above becomes increasingly cumbersome. For such interactions, some form of the diagrammatic operator formalism recently elaborated by Musso [90] is likely to prove superior (see also Ref. [91]).

3.2. Theories with derivative interactions

An important feature of the interactions (16) and (17) is that they include time derivatives of the perturbation, $\delta\dot{\phi}$ [51]. This means that the lagrangian can not be written in the canonical form $L(\delta\phi, \delta\dot{\phi}) = \frac{1}{2}\delta\dot{\phi}\Delta\delta\phi + V(\delta\phi)$ (where the operator $\Delta$ is field independent), in which there is only a quadratic dependence on $\delta\dot{\phi}$. As a result the textbook construction of the path integral formula based on $L$ does not work.

In the standard construction one identifies a momentum, $\pi$, canonically conjugate to $\delta\phi$ and writes the lagrangian as a Legendre transformation of the hamiltonian function $H$,

$$L(\delta\phi, \delta\dot{\phi}) = \pi\delta\dot{\phi} - H(\delta\phi, \pi).$$

(39)

In the quantum theory $\delta\phi$ and $\pi$ cannot be specified simultaneously. Since it is $H$ that generates time evolution, when one constructs the path integral one naturally arrives at a functional integration that involves independent integrals over $\delta\phi$ and $\pi$. If $L$ depends at most quadratically on $\delta\dot{\phi}$ then $H$ depends at most quadratically on $\pi$ and the momentum integral can be performed immediately. This has the effect of setting the value of $\pi$ equal to the one stipulated by Hamilton’s equations and results in the standard lagrangian path integral formula [87]. However, when $H$ has a more complicated dependence on $\pi$ the momentum integral must be treated more carefully.

The properties of lagrangians with derivative interactions have been studied extensively in the context of the non-linear $\sigma$-model. (See, eg., Coleman [92]; a path integral treatment is given in Ref. [87], whereas the canonical approach was followed in Ref. [93].) After inspection of Eqs. (16)–(17) it is clear that no term contains as many as four time derivatives, although there are terms containing one, two or three. Let us parametrize a general action for a field $\theta$ with arbitrary interactions containing as many as three time derivatives in form

$$S = (2\pi)^3 \int d\eta \left( \frac{1}{2}\gamma_{a\beta}\theta^a\theta^\beta - \frac{1}{2}\tilde{\gamma}_{a\beta}\partial\theta^a\partial\theta^\beta - V(\theta) + \lambda_\alpha\dot{\theta}^\alpha + \frac{1}{3}\omega_{a\beta\gamma}\dot{\theta}^a\dot{\theta}^\beta\dot{\theta}^\gamma \right).$$

(40)

In order to keep this and subsequent expressions manageable, Eq. (40) has been written in an abbreviated “de Witt” notation, where contraction over indices implies not only a summation over species, but also an integration over momentum variables with measure
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\[ d^3k/(2\pi)^3. \] With these conventions the object \( \delta_{\alpha\beta} = \delta(k_\alpha + k_\beta) \) is a “pseudo-metric” on \( k \)-space which is numerically identical to its index-raised counterpart, \( \delta^{\alpha\beta} \). Note that we have taken any interactions involving exactly two factors of \( \dot{\theta} \) to be included with the kinetic term in \( \gamma_{\alpha\beta} \). Moreover, without loss of generality \( \gamma_{\alpha\beta} \) and \( \omega_{\alpha\beta\gamma} \) can be supposed to be symmetric under exchange of their indices. We assume that \( \gamma_{\alpha\beta} \) is invertible, with inverse \( \gamma^{\alpha\beta} \).

The momentum conjugate to \( \dot{\theta}^\alpha \) is \( \pi_\alpha \),

\[ \pi_\alpha \equiv \frac{\delta S}{\delta \dot{\theta}^\alpha} = (2\pi)^3 \left( \gamma_{\alpha\beta} + \lambda_\alpha + \omega_{\alpha\beta\gamma} \dot{\theta}^\beta \dot{\theta}^\gamma \right) \]  

(41)

where we have used the assumed symmetry under index exchange to simplify this expression. In order to apply this formalism to the cubic and quartic interactions \( (16) \) and \( (17) \) it is only necessary to compute to \( O(\theta^4) \), where we formally assume that \( \pi \sim O(\theta) \) in order of magnitude. Since there are no three-derivative interactions in \( S_3 \), this implies that \( \omega \sim O(\theta) \) and it will be sufficient to work to leading order in \( \omega \). To this order, the hamiltonian can be written

\[ H = \frac{1}{2} \left( \frac{1}{(2\pi)^3} \gamma^{\alpha\beta} \pi_\alpha \pi_\beta + \frac{1}{2} (2\pi)^3 \delta_{\alpha\beta} \dot{\theta}^\alpha \dot{\theta}^\beta + \frac{1}{2} (2\pi)^3 \gamma_{\alpha\beta} \lambda_\alpha \lambda_\beta + (2\pi)^3 V \right) - \frac{1}{3} \left( \frac{1}{(2\pi)^6} \omega_{\alpha\beta\gamma} \gamma^{\alpha\rho} \gamma^{\beta\sigma} \gamma^{\gamma\tau} \pi_\rho \pi_\sigma \pi_\tau - \gamma^{\alpha\beta} \pi_\alpha \pi_\beta \right). \]  

(42)

This hamiltonian can be used to construct a path integral for \( \theta \), giving

\[ \int [d\theta^\alpha d\pi_\beta] \exp \left\{ i \int d\eta \left( \pi_\alpha \dot{\theta}^\alpha - H \right) \right\}. \]  

(43)

The fields \( \dot{\theta}^\alpha \) and \( \pi_\beta \) are now variables of integration, and therefore independent, so we are free to redefine the momentum field by a shift,

\[ \pi_\alpha \mapsto (2\pi)^3 (\pi_\alpha + \chi_\alpha) \]  

(44)

with \( \chi_\alpha \) chosen to eliminate the term in Eq. \( (43) \) which is linear in \( \pi_\alpha \),

\[ \chi_\alpha \equiv \gamma_{\alpha\beta} \dot{\theta}^\beta + \lambda_\alpha + \omega_{\alpha\beta\gamma} \dot{\theta}^\beta \dot{\theta}^\gamma. \]  

(45)

This shift leaves the path integral measure \( [d\pi_\beta] \) invariant. Having done so, one may rearrange terms to find a simplified path integral expression

\[ \int [d\theta^\alpha d\pi_\beta] \exp \left\{ i(S_\theta + S_{gh}) \right\}, \]  

(46)

where \( S_\theta \) is the original \( \theta \) action \( (40) \) with all derivative interactions in their original form, and \( S_{gh} \) is an effective action for the “ghost” field \( \pi \),

\[ S_{gh} = (2\pi)^3 \int \left( -\frac{1}{2} \gamma^{\alpha\beta} \pi_\alpha \pi_\beta + \omega_{\alpha\beta\gamma} \gamma^{\beta\sigma} \gamma^{\gamma\tau} \dot{\theta}^\alpha \pi_\sigma \pi_\tau + \frac{1}{3} \omega_{\alpha\beta\gamma} \gamma^{\alpha\rho} \gamma^{\beta\sigma} \gamma^{\gamma\tau} \pi_\rho \pi_\sigma \pi_\tau \right). \]  

(47)

The quanta associated with \( \pi \) do not appear in physical states, although they couple to \( \theta \) and so affect its expectation values when loop corrections are taken into account.

† Although it is tempting to regard \( \delta_{\alpha\beta} \) as an object for raising and lowering indices, it is not a true metric because with our conventions, the object obtained by mixing indices, \( \delta_{\alpha\beta} \delta_{\beta\gamma} \), is not the identity operator \( (2\pi)^3 \delta(k_\alpha - k_\gamma) \), although it is proportional to it.
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This explains why it has been permissible to ignore such ghosts in previous tree-level calculations; the $\pi$ integral makes no contribution to tree-level expectation values. Note that unlike the more familiar Fadeev–Popov ghost, the $\pi$ field is a spacetime scalar and not a spin-1/2 fermion.

Eq. (47) is not yet in a form suitable for perturbative calculations. In particular the ghost kinetic term involves the inverse $\gamma^{\alpha\beta}$. This will be a complicated object even for relatively simple choices of $\gamma_{\alpha\beta}$, but it is pointless to compute beyond $O(\theta^4)$ since the action to which we wish to apply this formalism [namely Eqs. (15)–(17)] was truncated at this level. For a canonically normalised scalar field in an almost-de Sitter spacetime $\gamma_{\alpha\beta}$ can be written

$$\gamma_{\alpha\beta} = a^2 \delta(k_\alpha + k_\beta) + 2\Gamma_1(k_\alpha, k_\beta) + 2\Gamma_2(k_\alpha, k_\beta) + \cdots$$  \hspace{1cm} (48)

where the $\Gamma_m$ are taken to be $o(\theta^m)$, the factors of two have been inserted for future convenience, and ‘$\cdots$’ denotes higher order terms which have been omitted.

The identity operator with our conventions is

$$\delta \gamma_{\alpha\beta} \equiv (2\pi)^{3/2} \delta(k_\alpha - k_\beta).$$  \hspace{1cm} (49)

The o($\theta$) equation implies that $\psi_1$ satisfies

$$\psi_1(k_\alpha, k_\beta) \equiv -\frac{2}{a^4} \Gamma_1(-k_\alpha, -k_\beta),$$  \hspace{1cm} (51)

whereas the o($\theta^2$) equation implies that $\psi_2$ satisfies

$$\psi_2(k_\alpha, k_\beta) \equiv -\frac{2}{a^4} \Gamma_2(-k_\alpha, -k_\beta) - \frac{4}{a^4} \int d^3 q \Gamma_1(-k_\alpha, q) \Gamma_1(-q, -k_\beta).$$  \hspace{1cm} (52)

The ghost action can therefore be written

$$S_{gh} = (2\pi)^9 \int d\eta \left\{ -\frac{1}{2a^2} \hat{\delta}^{\alpha\beta} \pi_\alpha \pi_\beta - \psi_1^{\alpha\beta} \pi_\alpha \pi_\beta - \psi_2^{\alpha\beta} \pi_\alpha \pi_\beta + \frac{(2\pi)^6}{a^4} \omega_{\alpha\beta\gamma} \hat{\delta}^{\beta\sigma} \hat{\delta}^{\gamma\tau} \hat{\omega}^{\alpha\sigma\tau} \pi_\sigma \pi_\tau \right.$$

$$+ \frac{(2\pi)^{12}}{3a^6} \omega_{\alpha\beta\gamma} \hat{\delta}^{\alpha\rho} \hat{\delta}^{\beta\sigma} \hat{\delta}^{\gamma\tau} \pi_\rho \pi_\sigma \pi_\tau \left\}$$  \hspace{1cm} (53)

The first term is $o(\theta^2)$ and can be taken as the free-field part of the ghost action, whereas the remainder is $O(\theta^3)$ and can be taken as the interaction term. In this form, the ghost action is suitable for perturbative evaluation.
3.3. Feynman rules for the interacting scalar/ghost theory

We are now in a position to write down the Feynman rules for the \( \delta \phi \) theory, including the effect of the ghost field. In this section we will not be obliged to carry out any of the complicated manipulations which characterized §3.2 and so we will revert to a notation in which momentum labels and integrals are written explicitly.

The propagators for the pure \( \delta \phi \) theory were written down in §3.1. The free part of the ghost action can be inverted immediately to find the ghost propagator. For the + fields this gives

\[
\langle \pi_+(\eta_1, k_1) \pi_+(\eta_2, k_2) \rangle = -\frac{i}{(2\pi)^3} a(\eta_1) a(\eta_2) \delta(\eta_1 - \eta_2) \delta(k_1 + k_2),
\]

from which the \(--\) propagator can be obtained by complex conjugation. Eq. (54) is the propagator for a so-called static ultra-local field [94, 88]. Its \( k \) and \( \eta \) dependence is constrained by the appearance of \( \delta \)-functions, so the ghost does not propagate: its purpose is to provide corrections to the vertices of the theory which account for the presence of coincident time derivatives there. At one-loop order, we do not require the mixed propagator; the ghost field only appears in loops, but at one-loop the only mixed contractions involve pairings of external fields with internal fields and these cannot occur for the ghost.

In the one-loop, single-field vertex formula, Eq. (58), the ghost propagator always appears in the role of the propagator evaluated at equal arguments, associated with the internal momentum \( q \). Note that the special simplification, which occurred for the \( \delta \phi \) propagator, where \( \xi_q(\eta, \eta) \) and \( \xi_q^*(\eta, \eta) \) were equal, does not apply for the ghost field.

It remains to identify the interaction terms \( V \), \( \lambda \), \( \Gamma_1 \), \( \Gamma_2 \) and \( \omega \). Reading these off from Eqs. (16)–(17) we obtain

\[
V = a^2 \frac{\phi}{4H} \int \frac{d^3 q_1 d^3 q_2 d^3 q_3}{(2\pi)^9} \delta \left( \sum_{i=1}^3 q_i \right) \left\{ \prod_{j=1}^3 \delta(\phi(q_j)) \right\} (q_2 \cdot q_3) \quad (55)
\]

\[
\lambda(q_1) = \frac{a}{4H} \int \frac{d^3 q_2 d^3 q_3 d^3 q_4}{(2\pi)^9} \delta \left( \sum_{i=1}^4 q_i \right) \left\{ \prod_{j=2}^4 \delta(\phi(q_j)) \right\} (q_2 \cdot q_3) \frac{\sigma(q_1, q_4)}{q_{14}^2}, \quad (56)
\]

\[
\Gamma_1(q_1, q_2) = -a^2 \frac{\phi}{4H} \int \frac{d^3 q_3}{(2\pi)^3} \delta \left( \sum_{i=1}^3 q_i \right) \delta(\phi(q_3)) \left( 1 + 2 \frac{\sigma(q_2, q_3)}{q_1^2} \right) \quad (57)
\]

\[
\Gamma_2(q_1, q_2) = -a^2 \int \frac{d^3 q_3 d^3 q_4}{(2\pi)^9} \delta \left( \sum_{i=1}^4 q_i \right) \left\{ \prod_{j=3}^4 \delta(\phi(q_j)) \right\} \frac{z(q_1, q_3) \cdot z(q_2, q_4)}{q_{13}^2 q_{24}^2} + \frac{3 \sigma(q_1, q_3) \sigma(q_2, q_4)}{4 q_{13}^2 q_{24}^2} - 2 \frac{q_1 \cdot z(q_2, q_3)}{q_{23}^2} \quad (58)
\]

\[
\omega(q_1, q_2, q_3) = -\frac{a}{4H} \int \frac{d^3 q_4}{(2\pi)^3} \delta \left( \sum_{i=1}^4 q_i \right) \delta(\phi(q_4)) \frac{\sigma(q_1, q_4)}{q_{14}^2}, \quad (59)
\]
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where \( q_{ij} = q_i + q_j \). The functions \( \sigma \) and \( z \) are defined by

\[
\sigma(a, b) \equiv a \cdot b + b^2
\]

and

\[
z(a, b) \equiv \sigma(a, b)a - \sigma(b, a)b.
\]

These are the momentum-space counterparts of Eqs. (13)–(14). Note that as written, Eqs. (57)–(58) for \( \Gamma \) and Eq. (59) for \( \omega \) are not symmetric under exchange of their arguments. For \( \Gamma \) this is immaterial, because (40) and (53) show that it always appears in a symmetric contraction. On the other hand, \( \omega \) does appear once in an asymmetric contraction, namely \( \omega_{\alpha\beta\gamma} \delta^\alpha_\dot{\theta} \delta^\beta_\dot{\varphi} \delta^\gamma_\dot{\pi} \pi_\delta \pi_\tau \). To avoid an unnecessarily tripling of the length of (59) we leave it in asymmetric form, carrying out an explicit symmetrization when computing amplitudes involving the asymmetric vertex.

Diagrammatic representation

Eqs. (55)–(59) lead to a rather complicated diagrammatic formalism in which the vertices produce a number of related terms, depending on the number of derivatives which apply to the lines entering the vertex. In order to keep track of these related contributions it is useful to introduce a refinement of the Feynman rules in which the lines of scalar propagators to which derivatives are applied are decorated with a dot.

For the pure \( \delta \phi \) vertices, the resulting diagrams are depicted in Fig. 1. For the mixed \( \delta \phi/\text{ghost} \) vertices, the resulting diagrams are shown in Fig. 2.

4. The one-point function

In §2.3 we observed that at tree-level the one-point function of \( \delta \phi \) is zero, \( \langle \delta \phi(k) \rangle = 0 \). This is not merely a question of convention; if the one-point function was not zero then so-called ‘tadpole’ diagrams such as Fig. 3 would mean that \( \delta \phi \) quanta would emerge from the vacuum. Conservation of momentum forces such particles to appear in the zero-momentum mode, and the accumulation (or “condensation”) of such particles causes the zero-momentum classical background field to change. Such an instability implies that any perturbation theory based on the original unstable vacuum state could
Figure 2. Scalar/ghost vertices. Solid lines represent the scalar field $\delta\phi$, whereas dashed lines represent the ghost. A dot on a scalar line entering a vertex shows that a time derivative is applied to the field at the point of interaction. Time derivatives are never applied to ghost fields. In terms of Eq. (53) the diagrams correspond to the vertices produced by (a) the $\psi\pi^{2}$ interaction; (b) the $\psi_2\pi^{2}$ interaction; (c) the $\omega\theta\pi\pi$ interaction; and (d) the $\omega\pi^{3}$ interaction.

Figure 3. Instability of the vacuum due to condensation. $\delta\phi$ particles emerge from the vacuum (represented by the hatched condensate) in a zero momentum state. The accumulation of such particles changes the homogeneous classical field configuration associated with the vacuum.

not give meaningful answers. This problem can be avoided by ensuring that the vacuum which we take as the basis of our perturbation theory is stable, at least at tree-level.

In an inflationary universe, the emergence of $\delta\phi$ quanta from the vacuum is exploited to produce small density fluctuations on superhorizon scales. Therefore we may expect to encounter some symptoms of vacuum instability when quantum corrections are taken into account. These symptoms manifest themselves as a radiatively generated one-point function,

$$\langle \delta\phi(k) \rangle = (2\pi)^3 \delta(k) O,$$

where $O \neq 0$ is a dimensionless quantity. Although it is not the observable in which we are principally interested, the present section is devoted to a calculation of $O$. This is important for two reasons. The first is that it provides a consistency check on $\delta N$ calculations [24, 81, 26, 37, 85, 47, 48] which typically assume $O = 0$, even beyond tree level. The second is that it allows us to develop some aspects of the calculational formalism in a simpler setting than the computation of the two-point function.

4.1. Ghost diagrams

Consider the one-point function associated with some wavenumber $k$. We aim to compute this at the time $\eta_*$ when $k$ crosses the horizon, which is roughly defined by the condition $-k\eta_* = 1$. Eventually conservation of momentum will force us to set $k = 0$, but in order to regularize the calculation we compute for finite $k$ and then study the
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limit \( k \to 0 \).

We deal first with the diagrams which contain a ghost loop. There is only one such diagram, which arises from the \( \psi_1 \pi^2 \) coupling,

\[
\langle \delta \phi(k) \rangle_* \subseteq \langle \delta \phi \rangle.
\]

This diagram makes a contribution to \( \langle \delta \phi(k) \rangle_* \) equal to

\[
\langle \delta \phi(k) \rangle_* \supseteq -\left(2\pi\right)^3 \delta(k) \int_{-\infty}^{\eta_*} d\eta \int \frac{d^3 q}{(2\pi)^3} \frac{H_* H}{2k^3} \delta(0)(1 - ik\eta \epsilon^{ik\eta}) \frac{\dot{\phi}}{4H} \left(1 + 2\frac{\sigma(-q, k)}{q^2}\right)
\]

+ complex conjugate,

where the symbol ‘\( \supseteq \)’ indicates that \( \langle \delta \phi(k) \rangle \) contains the indicated contribution (among others), and in deference to the vacuum prescription outlined in \( \text{§}3.1 \) we should deform the contour of the \( \eta \) integral to include some evolution in imaginary time for large \( |\eta| \). In this region the exponential factor is strongly decaying (cf. the discussion in \( \text{§}3.1 \)), so there is very little contribution to the integral from very early times; if \( \eta_* \) is not too late, the integral receives its dominant contributions from times around horizon crossing, where \( \eta \sim -1/k \). We may therefore approximate the slowly varying factors \( H_* H \) and \( \dot{\phi}/H \) by their values at the time of horizon crossing, which are equal to \( H_*^2 \) and \( \dot{\phi}_*/H_* \) respectively. In this simple example the \( \eta \) integral and the integral over the internal momentum \( q \) factorize, leaving a final result

\[
\langle \delta \phi(k) \rangle_* \supseteq -\left(2\pi\right)^3 \delta(k) P_*(k) \frac{\dot{\phi}_*}{4H_*} \int_{-\infty}^{\eta_*} d\eta \delta(0)(1 - ik\eta \epsilon^{ik\eta}) \int \frac{d^3 q}{(2\pi)^3} e^{i k \eta \Sigma}, +\text{complex conjugate}
\]

where \( P_*(k) \) is the tree-level power spectrum evaluated at \( \eta_* \). The object \( \delta(0) \) is the \( \eta \) delta-function evaluated at zero argument, and is badly divergent. In the present case, however, this is not material. The \( \eta \) integral can be rotated to imaginary time, leaving a result which is purely imaginary. Hence, although divergent, this diagram makes no contribution to the one-point function.

4.2. Pure \( \delta \phi \) diagrams

Now consider the pure \( \delta \phi \) diagrams.

The vacuum prescription and renormalization In order to assess the contribution that such diagrams make to \( O \), it is convenient to adopt the approximation made above that early times make almost no contribution to the \( \eta \) integral, so that slowly varying quantities such as \( H \) and \( \dot{\phi}/H \) can be evaluated at \( \eta_* \). A generic pure-\( \delta \phi \) contribution to \( O \) will then take the form

\[
O \supseteq i P_*(k) \frac{\dot{\phi}_*}{4H_*} \int_{-\infty}^{\eta_*} d\eta \int \frac{d^3 q}{(2\pi)^3} e^{ik\eta \Sigma} +\text{complex conjugate}
\]

(\ref{eq:Oapprox})
where $\Sigma$ is a $\mathbf{k}$- and $\mathbf{q}$-dependent quantity which is to be calculated. In evaluating $\Sigma$ we will encounter instances where a $\delta \phi$ propagator begins and ends at the same vertex, giving it coincident time arguments. We will choose to set such a propagator equal to

$$
\langle \delta \phi(\mathbf{q}_1, \eta) \delta \phi(\mathbf{q}_2, \eta) \rangle = (2\pi)^3 \delta(\mathbf{q}_1 + \mathbf{q}_2) \frac{H^2}{2q^3} (1 + q^2 \eta^2),
$$

where the exponential factors $e^{i\eta \mathbf{q}} e^{-i\eta \mathbf{q}}$ have cancelled among themselves, and $q$ is the common magnitude of $\mathbf{q}_1$ and $\mathbf{q}_2$.

The discussion of vacuum boundary conditions in §3.1 emphasized that the fields entering the Schwinger path integral must be chosen to begin in the appropriate interacting vacuum, and that this could be achieved heuristically by deforming the contour of integration to include some evolution in imaginary time. In view of this, one may question whether (67) is the correct choice, or whether it should be modified to read

$$
\langle \delta \phi(\mathbf{q}_1, \eta) \delta \phi(\mathbf{q}_2, \eta) \rangle = (2\pi)^3 \delta(\mathbf{q}_1 + \mathbf{q}_2) \frac{H^2}{2q^3} |1 - i\mathbf{q} \eta|^2 e^{-2q \Im(\eta)}, \quad \text{where} \ \Im(\eta) > 0.
$$

This would apparently have the very desirable effect of decoupling our prediction for $\langle \delta \phi(\mathbf{k}) \rangle$ from the deep ultra-violet regime where $q \to \infty$; in this limit, Eq. (67) leads to divergent integrals, whereas the modified propagator (68) is strongly decaying due to the exponential factor. Therefore one might have some reservations that the choice of (67) would yield the possibility of unphysical divergences, arising from an incorrect treatment of the vacuum. On the other hand, Eq. (68) has the undesirable feature that it leads to a non-holomorphic integrand. This means that it would be necessary to rescind the possibility of contour rotation in evaluating the $\eta$ integral. A loop amplitude computed using (68) would therefore depend sensitively upon the entirely arbitrary value we assign to $\Im(\eta)$. Such a sensitive dependence would have catastrophic consequences for sensibly interpreting the loop amplitude.

This situation can be understood as follows. It was explained in §3.1 that the trick of contour rotation can only be expected to reliably account for the vacuum boundary conditions when the integrand is holomorphic. This works successfully at tree-level, where all integrands are automatically holomorphic, and contain exponential factors of the form $e^{i\mathbf{k} \eta}$ for fixed external momenta $\mathbf{k}$. Since there are typically no poles in $\eta$, it is a simple consequence of Cauchy’s integral formula that all contours of integration beginning at $|\eta| = \infty$ and ending at $\eta = \eta_*$ are equivalent, provided that $\Im(\eta) > 0$ (strictly) over the whole contour. Hence, at tree-level, as argued in §3.1 enforcing the vacuum prescription by contour rotation gives completely unambiguous results which are independent of the deep ultra-violet region, owing to exponential decay of the $e^{i\mathbf{k} \eta}$ factors.

At one-loop level the situation is different. Together with exponentials of fixed momenta of the form $e^{i\mathbf{k} \eta}$, one now encounters two potential sources of non-holomorphicity. Eq. (68) shows that exponential factors of the form $e^{-2q \Im(\eta)}$ may

\[\dagger\] It makes no difference if we allow $\eta$ to develop a small imaginary component in the prefactor $|1 - i\mathbf{q} \eta|^2$, since the exponential term is so strongly decaying for large $|\eta|$.
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be present, together with the absolute-value $|1 - iq\eta|^2$. Integrands containing such factors are not holomorphic. Therefore, the trick of accounting for vacuum boundary conditions by contour rotation becomes invalid. It is this fact which accounts for the sensitive dependence of the resulting loop amplitudes on the arbitrary value of $\text{Im}(\eta)$. One should therefore adopt the holomorphic expression Eq. (67) for the propagator, and adopt some other method of regularizing the $q$ integral. The divergence associated with the limit $q \to \infty$ can then be subtracted by conventional methods of renormalization, after which the contour-rotation prescription gives a finite, contour-independent result which correctly incorporates the vacuum boundary conditions.

This leaves open the question of how the $q$-integral should be regulated. Since the Einstein action is supposed to be an effective theory of gravity for energies less than the Planck scale $M_P \approx 10^{18}$ GeV, and inflation is usually supposed to occur at energies at least a few orders of magnitude less than $M_P$, one might imagine applying a cutoff on the loop momenta of order the Planck scale. However this in itself is ambiguous since the Planck scale, unlike the speed of light, is not a Lorentz invariant and varies between locally inertial frames. In particular, the comoving Planck scale at a given instant $\eta$ is given by $a(\eta)M_P$. Therefore a momentum cutoff of this form entangles the $\eta$ and $q$ integrations, and for this reason it seems preferable to use a method of regularization, such as dimensional regularization, which does not depend on the explicit use of a cutoff.

Unfortunately, the integrals which we will encounter, especially in the computation of the two-point function (to be considered in §5 below) do not lend themselves to evaluation by dimensional regularization (see, for example, the example calculation given in Ref. [68]). In the present paper we will compute expectation values using a fixed momentum cutoff both in the infra-red and ultra-violet. However, in the case of the one-point function where dimensional regularization can be usefully applied, it may be checked that when the ultra-violet region has been discarded the two methods yield comparable predictions for the leading infra-red divergences (up to numerical factors and irrelevant logarithms of the IR cutoff).

This is sufficient for the purposes of the present paper, since it is the behaviour in the infra-red rather than the ultra-violet which is of principal interest in a cosmological context. Divergences in the ultra-violet come from the behaviour of the fields at high energies and small scales. Such small scale modes exist far inside the horizon, where the equivalence principle suggests that flat spacetime quantum field theory is expected to be a good approximation. The subtraction of these modes has recently been considered by Finelli et al., who argue that no special treatment is required for the power spectrum [95] (see also Ref. [96]). On the other hand, the infra-red behaviour comes from low energies and large scales, where the field modes are well outside the horizon. On such scales, flat spacetime field theory is a very poor approximation and we are obliged to take account of the gravitational background.

This does not preclude the appearance of new ultra-violet divergences in our expectation values. Indeed, many of the integrals we shall encounter do contain ultra-violet divergences, of which the ultra-violet divergent quantity $\delta(0)$ which appears in
Eq. (65) is an example. Such divergences do not interfere with our ability to perform accelerator or laboratory particle physics experiments on earth, which are characterized by time- and length-scales that are small compared to the expansion time-scale and horizon length-scale of the universe. On such small scales the ultra-violet divergences we shall encounter (none of which are present in the $\delta\phi$ theory in Minkowski space) are presumably subdominant with respect to divergences from the pure matter theory and therefore do not interfere with our ability to perform terrestrial experiments, or with the success of the principle of equivalence.

In what follows we shall generally assume that the ultra-violet divergences can be correctly subtracted, and focus our attention on the infra-red behaviour.

**Zero-derivative interactions** We now return to the $\delta\phi$ diagrams. It is simplest to classify these diagrams according to the number of derivatives applied to propagators entering the vertex.

There is a zero-derivative interaction from the vertex in Fig. 1(a),

$$\langle \delta\phi(k) \rangle \supset \circ. \quad \text{(69)}$$

This term makes a contribution to $\Sigma$ which equals

$$\Sigma \supset -\left( \frac{1}{2q\eta^2} - \frac{k \cdot q}{q^2\eta^2} \right) (1 - ik\eta)(1 + q^2\eta^2). \quad \text{(70)}$$

The term involving $k \cdot q$ is not rotationally invariant and disappears in the integral over $q$. Let us introduce an ultra-violet cutoff $k\Lambda$, where $\Lambda$ is a dimensionless number, and a comparable infra-red cutoff $km$ for some dimensionless quantity $m$. Evaluating the $q$ and $\eta$ integrals as described above gives

$$O_\ast \supset P_\ast \frac{\dot{\phi}_\ast}{4H_\ast} \left( -\frac{k^3}{4\pi^2} \left[ \frac{1}{2} \Lambda^4 - \frac{1}{2} m^4 + \Lambda^2 - m^2 \right] \right). \quad \text{(71)}$$

**Two derivatives, both derivatives on internal leg** The next class of diagrams contain two derivative operators, and divide naturally into two sorts: those where the derivatives are applied to both ends of the internal loop, and those where one derivative is applied to the loop but the other is applied to the external leg.

The first sort give rise to diagrams of the form

$$\langle \delta\phi(k) \rangle \supset \bigcirc \bigcirc; \quad \text{(72)}$$

such diagrams contribute an amount to $\Sigma$ corresponding to

$$\Sigma \supset -\frac{q}{2} (1 - ik\eta) \left( 1 + 2 \frac{\sigma(q, -k)}{q^2} \right). \quad \text{(73)}$$

Evaluating the integrals by the method described above, one arrives at

$$O_\ast \supset P_\ast(k) \frac{\dot{\phi}_\ast}{4H_\ast} \left( -\frac{k^3}{4\pi^2} \left[ \frac{1}{2} \Lambda^4 - \frac{1}{2} m^4 + 2\Lambda^2 - 2m^2 \right] \right). \quad \text{(74)}$$
Two derivatives, single derivative on internal leg

The final class of diagrams contain a single derivative on the internal line, and apply the remaining derivative to the external leg. These diagrams are of the form

\[ \langle \delta \phi(k) \rangle_s \supseteq \quad \bigcirc \]

and contribute to \( \Sigma \) according to the rule

\[ \Sigma \supseteq -\frac{k^2}{2q}(1 - i k \eta) \left( 1 + 2 \frac{\sigma(-k, q)}{q^2} \right) \]

This class of diagrams makes a contribution to \( O \), which equals

\[ O_s \supseteq P_s(k) \frac{\dot{\phi}_s}{4 H_s} \left( -\frac{k^3}{4 \pi^2} \left[ \frac{4}{3} \Lambda^3 - \frac{4}{3} m^3 + 2 \Lambda^2 - 2 m^2 \right] \right) \]

4.3. Infra-red behaviour

We now collect terms in Eqs. (71), (74) and (77). The ultra-violet divergences will be cancelled by renormalization, but without a definite prescription for the necessary counterterms it is not possible to assign a value to the finite remainder. Therefore, the residue of the UV divergences will be manifest in a renormalization-scheme dependent number \( \alpha \). Accordingly, the one-point amplitude behaves in the infra-red like

\[ \langle \delta \phi(k) \rangle_{s,m} \sim (2\pi)^3 \delta(k) \frac{P_s}{8 \rho_s} \left( \alpha + \frac{3}{2} m^4 + \frac{4}{3} m^3 + 5 m^2 \right) \]

As discussed above, we have computed \( O \) as if any value of \( k \) were allowed, whereas in reality momentum conservation restricts us to the zero-momentum case \( k = 0 \). This leads to Eq. (78) for vanishingly small \( m \), with the horizon-crossing time \( \eta_s \) in the infinitely far past. In practice, inflation does not last for an indefinite number of e-folds and the region of the universe described by the inflationary patch will not be unboundedly large. One should identify the \( k = 0 \) mode with the spatial average of \( \delta \phi \) within this patch. If it is non-zero, this spatial average can be absorbed into a redefinition of the background field \( \phi(t) \) by enforcing the renormalization condition \( \langle \delta \phi(k) \rangle = 0 \), as discussed (for example) in Refs. [65, 66, 49, 50]. It follows that when \( \delta \phi \) is defined in this way one may take \( O = 0 \), as usually assumed.

5. The two-point function

We now turn to the central purpose of this paper, the computation of the leading loop correction to the two-point function \( \langle \delta \phi(k_1) \delta \phi(k_2) \rangle_s \).

It is first necessary to decide which classes of diagrams are to be included in the computation. In general, the one-loop correction to the two-point function of the \( \delta \phi \) will be given by a sum of diagrams, the leading terms of which are of the form

loop correction \( \supseteq \quad \bigcirc \quad + \quad \bigcirc \quad + \cdots. \)
Eqs. (16)–(17) show that the leading contribution from the first diagram is $O(\epsilon^0)$ in slow-roll, whereas the leading contribution from the second diagram is $O(\epsilon)$. Therefore, provided $\epsilon \ll 1$ and there are no large logarithms which can compensate for small slow-roll parameters, the expectation value will be dominated by the lowest-order part of the first diagram. This is opposite to the case considered by Sloth [49, 50], where a large logarithm was used to compensate for the smallness of $\epsilon$. In this régime the first diagram will be dominated by its subleading slow-roll part, and for consistency one should also take into account sub-leading slow-roll terms from the second diagram, and possibly from other sources. In the present paper, we wish to use the slow-roll approximation to simplify the calculation and therefore we will retain only the contribution from the leading part of the first diagram. The question of when this is a good approximation, together with a more general analysis of any possible large logarithms, will be postponed to another publication [80].

5.1. Ghost diagrams

The relevant vertices here come from the $\psi_2\pi\pi$ and $\omega\delta\phi\pi\pi$ terms in the ghost action. There is no contribution from the $\omega\pi\pi\pi$ interaction because this involves three ghost fields, which must appear in loops, and at one-loop order there is always one ghost field which is left unpaired. Therefore this term can be disregarded, although it would play a role in a two- or higher-loop calculation. To determine the $\psi_2$ contribution explicitly, consider Eq. (52) which gives $\psi_2$ in terms of the known functions $\Gamma_1$ and $\Gamma_2$. We are computing to leading order in slow-roll, so the term involving $\Gamma_2^2$ can be discarded, because Eq. (57) shows that it is proportional to the slow-roll parameter $\epsilon \sim \dot{\phi}^2/H^2$, whereas the leading terms in the fourth-order interaction are $o(\epsilon^0)$.

The relevant ghost diagrams are

$$\langle \delta\phi(k_1)\delta\phi(k_2) \rangle = \underbrace{\hat{\Gamma}^+} + \underbrace{\hat{\Gamma}^-}.$$ (79)

Both these diagrams are purely imaginary and cancel between the $++$ and $--$ propagators in exactly the same manner described in §4 for the computation of the one-point function.

The ghost diagrams have therefore entirely cancelled out in both the one- and two-point functions. This leads to expressions which agree with those reported in Refs. [49, 50]. However, one should not immediately conclude that the ghost diagrams always sum to zero. Although this issue deserves more detailed attention, at two-loop order and above one can presumably expect the factors of $i$ to combine to give non-vanishing contributions. This will apparently occur whenever there are an even number of ghost propagators in the diagram.

5.2. Pure $\delta\phi$ diagrams

As in the one-point calculation, it is convenient to classify the pure $\delta\phi$ diagrams according to the number of derivatives they contain.
Single derivative  There are no zero-derivative interactions, because the gravitational interactions responsible for generating the vertices in Fig. I make no contribution to the potential at \( o(\delta \phi^4) \), and the cubic contribution which is generated would make a contribution to the loop correction which is subleading in slow-roll.

The first non-trivial term contains a single derivative, which can be applied to an internal or external line,

\[
\langle \delta \phi(k_1) \delta \phi(k_2) \rangle = \sum \quad \sum .
\]

As in the case of the one-point function, it is useful to parametrize the contribution each diagram makes to \( \langle \delta \phi(k_1) \delta \phi(k_2) \rangle \) in terms of a function \( \Pi \), which is defined by

\[
\langle \delta \phi(k_1) \delta \phi(k_2) \rangle = i(2\pi)^3 P_s(k)^2 \int d\eta \int \frac{q^3}{(2\pi)^3} e^{2i k\eta} \Pi + \text{complex conjugate},
\]

where \( P_s(k)^2 \) is the square of the tree-level power spectrum, and the quantity \( \Pi \) (to be calculated in this section) depends on the external momenta \{\( k_1, k_2 \)\} and the loop momentum \( q \).

The class of diagrams where the derivative is applied to the external leg makes a contribution to \( \Pi \) which corresponds to

\[
\sum \quad \sum : \quad \Pi \supset (1 - i k\eta)(1 + q^2 \eta^2) \left( \frac{k^2}{4q^2} (q \cdot k_2) \frac{\sigma(-k_1, q)}{|k_1 + q|^2} + \frac{k^2}{8q} \frac{\sigma(-k_1, k_2)}{k_{12}^2} \right) + [k_1 \leftrightarrow k_2],
\]

where \([k_1 \leftrightarrow k_2]\) denotes the same term with \( k_1 \) and \( k_2 \) interchanged. The ratio \( \sigma(-k_1, k_2)/k_{12}^2 \) is obviously singular when the momentum conservation condition \( k_1 + k_2 = 0 \) is enforced and must be treated carefully to avoid an unwanted divergence.

Consider the non-singular quotient \( \sigma(a, b)/|a + b|^2 \) where \( a \) and \( b \) approach \( k \) and \( -k \) respectively,

\[
\lim_{\epsilon, \delta \to 0} \frac{\sigma(k + \delta, -k + \epsilon)}{|\epsilon + \delta|^2} = \lim_{\epsilon, \delta \to 0} \frac{k \cdot (\delta + \epsilon) + \epsilon^2 + \delta \cdot \epsilon}{\delta^2 + \epsilon^2 + 2\delta \cdot \epsilon}
\]

This is not symmetric between \( \delta \) and \( \epsilon \), because \( \sigma \) is not a symmetric function of its arguments; as a result, the limits do not commute. Moreover, as \( \delta \) and \( \epsilon \) approach zero the numerator of (83) vanishes linearly, as fast as \( O(\epsilon, \delta) \), whereas the denominator is vanishing quadratically, like \( O(\epsilon^2, \delta^2) \). Therefore (82) is naïvely divergent. In fact, the value of (83) depends on what is assumed about \( k \cdot \delta \) and \( k \cdot \epsilon \); if we demand that the limit is approached along a sequence of vectors of magnitude \( k = |k| \) then it follows that \( |k + \delta| = | -k + \epsilon| = k \) and therefore

\[
k \cdot \epsilon = \frac{\epsilon^2}{2} \quad \text{and} \quad k \cdot \delta = -\frac{\delta^2}{2}.
\]

With this choice, Eq. (83) evaluates to 1/2 and the limits become commuting. This prescription was used implicitly in §4.2 of Ref. [81], but there does not seem to be any compelling reason to demand that the limit is approached along such a specific
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sequence of vectors. Fortunately a catastrophic divergence is averted, since Eq. (82) requires symmetrization over the exchange $k_1 \leftrightarrow k_2$. The problematic term $k \cdot (\delta + \epsilon)$ is antisymmetric under this exchange and cancels out of the expectation value (82), leaving a finite limit. The result of this procedure gives the same answer as if we had adopted Eq. (84), which can be regarded as an a posteriori justification for the analysis presented in Ref. 81.

After performing the symmetrization over $k_1$ and $k_2$ and integrating over $q$ and $\eta$, this class of diagrams make a contribution to the two-point function of the form

$$\langle \delta \phi(k_1) \delta \phi(k_2) \rangle \cong (2\pi)^3 \delta(k_1 + k_2) \frac{P_s(k)^2}{\pi^2} \left( -\frac{3}{16} k^3 \ln k - \frac{1}{120} k^3 + \cdots \right)$$

where ‘…’ denotes ultra-violet divergent terms which have been omitted, together with terms which vanish in the limit $m \to 0$.

Now consider the diagrams in which the derivative is applied to the internal line. Such diagrams contribute to $\Pi$ according to

$$\Pi \cong \langle 1 - i k \eta \rangle^2 (1 - i q \eta) \left( \frac{k_1 \cdot q \sigma(-q, -k_2)}{4q |k_2 + q|^2} + \frac{k^2 \sigma(q, -q)}{8q |q - q|^2} \right) + [k_1 \leftrightarrow k_2],$$

This class of diagrams contains a similar ill-defined ratio, $\sigma(q, -q)/|q - q|$. Consider Eq. (83) again, with $k$ replaced by $q$. Although there is no longer any injunction to symmetrize over $q \equiv -q$, the non-rotationally-invariant part $q \cdot (\delta + \epsilon)$ will vanish underneath the integral and does not give rise to any divergence. In order to assign a definite value to the remaining limit, we must assume something about $\delta$ and $\epsilon$. Since $q$ is merely a variable of integration and can be freely replaced by $-q$, we assume that $\sigma(q, -q)$ is to be regularized by taking its symmetric part. With this prescription, the ratio $\sigma(q, -q)/|q - q|$ evaluates to $1/2$.

Symmetrizing over $k_1$ and $k_2$ and omitting ultra-violet divergent terms together with any terms which vanish in the limit $m \to 0$, we find

$$\langle \delta \phi(k_1) \delta \phi(k_2) \rangle \cong (2\pi)^3 \delta(k_1 + k_2) \frac{P_s(k)^2}{\pi^2} \left( \frac{1}{8} k^3 \ln k - \frac{3}{20} k^3 + \cdots \right)$$

Two derivatives We have now exhausted all diagrams with only a single derivative. The next set of diagrams all involve two derivatives and break naturally into three sets: the first class includes all diagrams with the derivatives applied to both external legs of the two-point function; the second set includes all diagrams where one derivative is applied to an external leg while the other applies to the internal propagator; and the third set includes all diagrams with both derivatives applied to the internal propagator:

$$\langle \delta \phi(k_1) \delta \phi(k_2) \rangle \cong \langle \phi \rangle + \langle \phi \rangle + \langle \phi \rangle$$

(88)
Consider first the set of diagrams with both derivatives on the external legs. We obtain
\[
\begin{align*}
\langle \rightarrow \rangle & : \quad \Pi \supseteq -\frac{k^4}{2q^2} (1 + q^2 \eta^2)Q,
\end{align*}
\] where \( Q \) is the quantity
\[
Q = \frac{z(-k_1, q) \cdot z(-k_2, q)}{|q - k_1|^2 |q + k_2|^2} + \frac{3 \sigma(-k_1, q) \sigma(-k_1, -q)}{4 |q - k_1|^2 |q + k_2|^2} + 2 \frac{q \cdot z(-k_2, q)}{|q - k_2|^2} + [k_1 \equiv k_2].
\] (90)

Unlike the previous examples, none of the ratios which appear in \( Q \) are ill-defined. However, this result can still be significantly simplified using the symmetry properties of \( z \) and \( \sigma \). In particular, we observe that \( \sigma \) is a quadratic form, and therefore
\[
\sigma(-a, -b) = \sigma(a, b) \quad \text{and} \quad z(-a, -b) = z(b, a).
\] (91)

These identities can be used together with the obvious antisymmetry of \( z \) \( i.e. \), \( z(a, b) = -z(b, a) \). After performing the symmetrization over \( k_1 \) and \( k_2 \), \( Q \) can be reduced to the simpler form
\[
Q = -2 \frac{z(q, k)^2}{|q + k|^6} + \frac{3 \sigma(q, q)^2}{2 |q + k|^4} + 4 \frac{q \cdot z(q, k)}{|q + k|^4}.
\] (92)

In this expression, \( k \) can be taken to be either \( k_1 \) or \( k_2 \); after integration, the result depends only on the magnitude \( k \) and not its orientation, and we obtain
\[
\langle \delta \phi(k_1) \delta \phi(k_2) \rangle \supseteq (2\pi)^3 \delta(k_1 + k_2) \frac{P_s(k)^2}{\pi^2} \left( \frac{15}{16} k^3 \ln k + k^3 + \cdots \right).
\] (93)

The set of diagrams with one derivative on an external leg and one derivative on the internal propagator are the most complicated. To evaluate them, we write
\[
\begin{align*}
\langle \rightarrow \rangle & : \quad \Pi \supseteq -\frac{k^2}{2q^2} (1 - ik \eta)(1 - iq \eta) R,
\end{align*}
\] where \( R \) can be expressed as
\[
R \equiv 4 \frac{z(q, k)^2}{|q + k|^6} + \frac{3 \sigma(q, k) \sigma(q, q)}{|q + k|^4} + 3 \frac{3 |q + k|^4}{4 |q + k|^4} + 4 \frac{k \cdot z(q, k)}{|q + k|^4}.
\] (95)

in which we have used the symmetry properties of \( z \) and \( \sigma \), and the same convention that \( k \) may be chosen as either \( k_1 \) or \( k_2 \) applies. In arriving at this expression for \( Q \), we have discarded a number of contributions of the form \( X \cdot \{z(-k, k) + z(k, -k)\} \) for some vector \( X \), which may itself require regularization. However, no matter how we choose to regularize the bracket \( \{\cdot\} \), the antisymmetry of \( z \) guarantees that it sums to zero, and therefore that such contributions cancel out of the observable expectation value.

After integration, one obtains
\[
\langle \delta \phi(k_1) \delta \phi(k_2) \rangle \supseteq (2\pi)^3 \delta(k_1 + k_2) \frac{P_s(k)^2}{\pi^2} \left( 3k^3 \ln k - \frac{5}{2} k^3 + \cdots \right).
\] (96)

The final class of diagrams of this type involve both derivatives applied to the internal propagator,
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\[ \Pi \supseteq -\frac{q}{2} (1 - ik\eta)^2 \left( -2 \frac{z(q, k)^2}{|q + k|^2} + \frac{3 \sigma(q, k)^2}{2 |q + k|^6} + 4 \frac{k \cdot z(q, k)}{|q + k|^4} \right), \]

(97)

which does not require regularization. After integration, one obtains

\[ \langle \delta \phi(k_1) \delta \phi(k_2) \rangle_* \supseteq (2\pi)^3 \delta(k_1 + k_2) \frac{P_*(k)^2}{\pi^2} \left( -\frac{15}{16} k^3 \ln k + \frac{1}{4} k^3 + \cdots \right). \]

(98)

Three derivatives

The only remaining class of diagrams are those containing three derivatives at the vertex. These diagrams break into two groups: those in which one end of the internal propagator is free of a derivative, and those in which an external leg is free of a derivative:

\[ \langle \delta \phi(k_1) \delta \phi(k_2) \rangle_* \supseteq \quad + \quad \]

(99)

Both types give rise to comparatively simple expressions. For the first we obtain

\[ \Pi \supseteq \frac{k^4}{8q} \eta^2 (1 - iq\eta) \left( 4 \frac{\sigma(k, q)}{|q + k|^2} + 1 \right); \]

after integration this class of diagrams give contributions totalling

\[ \langle \delta \phi(k_1) \delta \phi(k_2) \rangle_* \supseteq (2\pi)^3 \delta(k_1 + k_2) \frac{P_*(k)^2}{\pi^2} \left( -\frac{1}{24} k^3 \ln k + \frac{1}{18} k^3 + \cdots \right). \]

(101)

On the other hand, for the second type of diagram we obtain

\[ \Pi \supseteq \frac{k^2 q}{8} \eta^2 (1 - ik\eta) \left( 4 \frac{\sigma(q, k)}{|q + k|^2} + 1 \right). \]

(102)

After integration, we find

\[ \langle \delta \phi(k_1) \delta \phi(k_2) \rangle_* \supseteq (2\pi)^3 \delta(k_1 + k_2) \frac{P_*(k)^2}{\pi^2} \left( \frac{1}{48} k^3 \ln k - \frac{1}{90} k^3 + \cdots \right). \]

(103)

5.3. Infra-red behaviour

Having obtained the relevant contributions to the two-point function, given by Eqs. (85), (87), (93), (96), (98), (101) and (103), we may collect these quantities to obtain an estimate of the total loop correction. It can be written

\[ \langle \delta \phi(k_1) \delta \phi(k_2) \rangle_* \sim (2\pi)^3 \delta(k_1 + k_2) P_*(k) P_* \left( \frac{35}{6} \ln k - \frac{491}{180} + \hat{\beta} \right) \]

\[ \simeq (2\pi)^3 \delta(k_1 + k_2) P_*(k) P_* \left( 5.83 \ln k + \beta \right), \]

(104)

where \( \hat{\beta} \) is an unknown renormalization-scheme dependent quantity left over from cancellation of the ultra-violet divergences. In the last equality this has been combined with the constant term to give an overall unknown constant \( \beta \). The coefficient of the logarithm, however, is scheme-independent \([68, 87]\).
6. Discussion

In this paper, I have computed estimates for the infra-red behaviour of the leading radiative corrections to the one- and two-point expectation values of the inflaton field perturbation during a phase of single-field, slow-roll inflation. After suitable ultra-violet renormalization, the loop correction to the one-point function was found to be given on large scales by an unknown renormalization-scheme dependent quantity $\alpha$, whereas the loop correction to the two point function gave a correction to the power spectrum of the form

$$P_{\text{loop}} = P_\ast \left\{ 1 + P_\ast \left( \frac{35}{6} \ln k + \beta \right) \right\}.$$  \hspace{1cm} (105)

where $\beta$ is a similar scheme-dependent constant. The presence of such unknown constants is not really a limitation, because on large scales one would expect that these are negligible in comparison with the $\ln k$ term.

Although the amplitude of the $\delta \phi$ power spectrum itself is not observable, the amplitude of $\zeta$ is accurately known to be of order $10^{-10}$. At tree-level, the two are related via the approximate relation $P_\zeta \sim P_\ast / \epsilon$, where $\epsilon$ is the slow-roll parameter introduced in Eq. \hspace{1cm} (5). Since $\epsilon$ is expected to be of order $10^{-2}$ or less, we can conservatively suppose that $P_\ast \lesssim 10^{-10}$. The loop correction given by Eq. \hspace{1cm} (105) is therefore extremely small provided that $k$ is not too large in comparison with the infra-red cutoff scale.

This does not allow us to conclude that loop corrections are too small to be observable in the CMB, because it is the loop corrections in $\zeta$ rather than the $\delta \phi$ themselves which are accessible to experiment. Therefore, the prediction \hspace{1cm} (105) needs to be translated into a prediction for $P_{\text{loop}}^\zeta$ before a final determination concerning the magnitude of loop corrections can be made. This calculation will be presented elsewhere \hspace{1cm} \cite{80}. However, it is already clear from Eq. \hspace{1cm} (105) that quantum effects do not greatly disturb the magnitude of the fluctuations imprinted in $\delta \phi$ as successive $k$-modes pass outside the horizon. It is only the accumulation of fluctuations on superhorizon scales, where the fields are in an effectively classical régime, which can give rise to a large loop correction.

Eq. \hspace{1cm} (105) is entirely consistent with previous estimates which have been made in the literature. In particular, Weinberg has estimated a correction to $P_\ast$ from matter loops in a multi-field theory \hspace{1cm} \cite{68} which has the same functional form as \hspace{1cm} (105). Sloth \hspace{1cm} \cite{49, 50} has given a similar estimate, based on the same action given in Eqs. \hspace{1cm} (25)–\hspace{1cm} (17), but evaluated several tens of e-folds after horizon crossing when large infra-red divergences can compensate for a suppression in powers of slow-roll parameters; in this limit a different set of terms extracted from Eq. \hspace{1cm} (17) dominate the loop correction. This loop correction is proportional to $\langle \delta \phi^2 \rangle \sim P_\ast \ln(k)$ for a flat spectrum, which reproduces the logarithmic $k$-dependence described by \hspace{1cm} (105).
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References

[1] D. Wands, K. A. Malik, D. H. Lyth, and A. R. Liddle, A new approach to the evolution of cosmological perturbations on large scales, Phys. Rev. D62 (2000) 043527, astro-ph/0003278.
[2] WMAP Collaboration, D. N. Spergel et al., Wilkinson Microwave Anisotropy probe (WMAP) three year results: Implications for cosmology, astro-ph/0603449.
[3] J. Martin and C. Ringeval, Inflation after WMAP3: Confronting the slow-roll and exact power spectra to CMB data, JCAP 0608 (2006) 009, astro-ph/0605367.
[4] W. H. Kinney, E. W. Kolb, A. Melchiorri, and A. Riotto, Inflation model constraints from the Wilkinson Microwave Anisotropy Probe three-year data, Phys. Rev. D74 (2006) 023502, astro-ph/0605338.
[5] A. A. Starobinsky, A new type of isotropic cosmological models without singularity, Phys. Lett. B91 (1980) 99–102.
[6] K. Sato, First order phase transition of a vacuum and expansion of the universe, Mon. Not. Roy. Astron. Soc. 195 (1981) 467–479.
[7] A. H. Guth, The inflationary universe: A possible solution to the horizon and flatness problems, Phys. Rev. D23 (1981) 347–356.
[8] S. W. Hawking and I. G. Moss, Supercooled phase transitions in the very early universe, Phys. Lett. B110 (1982) 35.
[9] A. Albrecht and P. J. Steinhardt, Cosmology for grand unified theories with radiatively induced symmetry breaking, Phys. Rev. Lett. 48 (1982) 1220–1223.
[10] A. D. Linde, A new inflationary universe scenario: A possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems, Phys. Lett. B108 (1982) 389–393.
[11] A. D. Linde, Chaotic inflation, Phys. Lett. B129 (1983) 177–181.
[12] R. W. Wald, Asymptotic behavior of homogeneous cosmological models in the presence of a positive cosmological constant, Phys. Rev. D28 (1983) 2118–2120.
[13] J. M. Bardeen, P. J. Steinhardt, and M. S. Turner, Spontaneous creation of almost scale-free density perturbations in an inflationary universe, Phys. Rev. D28 (1983) 679.
[14] A. H. Guth and S. Y. Pi, Fluctuations in the new inflationary universe, Phys. Rev. Lett. 49 (1982) 1110–1113.
[15] S. W. Hawking, The development of irregularities in a single bubble inflationary universe, Phys. Lett. B115 (1982) 295.
[16] S. W. Hawking and I. G. Moss, Fluctuations in the inflationary universe, Nucl. Phys. B224 (1983) 180.
[17] R. Allahverdi, A. Jokinen, and A. Mazumdar, Sub-eV hubble scale inflation within gauge mediated supersymmetry breaking, hep-ph/0610243
[18] R. Allahverdi, A. Kusenko, and A. Mazumdar, A-term inflation and the smallness of the neutrino masses, JCAP 0707 (2007) 018, hep-ph/0608138.
One-loop corrections to a scalar field during inflation

[19] R. Allahverdi, K. Enqvist, J. Garcia-Bellido, and A. Mazumdar, *Gauge invariant MSSM inflaton*, Phys. Rev. Lett. 97 (2006) 191304, hep-ph/0605035.

[20] N. Bartolo, E. Komatsu, S. Matarrese, and A. Riotto, *Non-gaussianity from inflation: Theory and observations*, Phys. Rept. 402 (2004) 103–266, astro-ph/0406398.

[21] J. M. Maldacena, *Non-gaussian features of primordial fluctuations in single field inflationary models*, JHEP 05 (2003) 013, astro-ph/0210603.

[22] G. I. Rigopoulos and E. P. S. Shellard, *Non-linear inflationary perturbations*, JCAP 0510 (2005) 006, astro-ph/0405188.

[23] G. I. Rigopoulos, E. P. S. Shellard, and B. W. van Tent, *A simple route to non-gaussianity in inflation*, Phys. Rev. D72 (2005) 083507, astro-ph/0410486.

[24] D. Seery and J. E. Lidsey, *Primordial non-gaussianities from multiple-field inflation*, JCAP 0509 (2005) 011, astro-ph/0506056.

[25] G. I. Rigopoulos, E. P. S. Shellard, and B. W. van Tent, *Non-linear perturbations in multiple-field inflation*, Phys. Rev. D73 (2005) 083521, astro-ph/0504508.

[26] D. H. Lyth and Y. Rodríguez, *The inflationary prediction for primordial non-gaussianity*, Phys. Rev. Lett. 95 (2005) 121302, astro-ph/0504045.

[27] D. H. Lyth and I. Zaballa, *A bound concerning primordial non-gaussianity*, JCAP 0510 (2005) 005, astro-ph/0507608.

[28] I. Zaballa, Y. Rodríguez, and D. H. Lyth, *Higher order contributions to the primordial non-gaussianity*, JCAP 0606 (2006) 013, astro-ph/0603534.

[29] T. Battefeld and R. Easther, *Non-gaussianities in multi-field inflation*, JCAP 0703 (2007) 020, hep-th/0610296.

[30] S. A. Kim and A. R. Liddle, *Nflation: non-gaussianity in the horizon-crossing approximation*, Phys. Rev. D74 (2006) 063522, astro-ph/0608186.

[31] L. Alabidi, *Non-gaussianity for a two component hybrid model of inflation*, JCAP 0610 (2006) 015, astro-ph/0604611.

[32] S. Gupta, A. Berera, A. F. Heavens, and S. Matarrese, *Non-gaussian signatures in the cosmic background radiation from warm inflation*, Phys. Rev. D66 (2002) 043510, astro-ph/0205150.

[33] S. Gupta, *Dynamics and non-gaussianity in the weak-dissipative warm inflation scenario*, Phys. Rev. D73 (2006) 083514, astro-ph/0509676.

[34] D. H. Lyth, C. Ungarelli, and D. Wands, *The primordial density perturbation in the curvaton scenario*, Phys. Rev. D67 (2003) 023503, astro-ph/0208055.

[35] L. Boubekeur and D. H. Lyth, *Detecting a small perturbation through its non-gaussianity*, Phys. Rev. D73 (2006) 021301, astro-ph/0504046.

[36] D. H. Lyth and Y. Rodríguez, *Non-gaussianity from the second-order cosmological perturbation*, Phys. Rev. D71 (2005) 123508, astro-ph/0502578.

[37] D. H. Lyth, *Non-gaussianity and cosmic uncertainty in curvaton-type models*, JCAP 0606 (2006) 015, astro-ph/0602285.

[38] K. A. Malik and D. H. Lyth, *A numerical study of non-gaussianity in the curvaton scenario*, JCAP 0609 (2006) 008, astro-ph/0604387.

[39] K. Enqvist and S. Nurmi, *Non-gaussianity in curvaton models with nearly quadratic potential*, JCAP 0510 (2005) 013, astro-ph/0508573.

[40] K. Enqvist, A. Jokinen, A. Mazumdar, T. Multamaki, and A. Vaihkonen, *Non-gaussianity from preheating*, Phys. Rev. Lett. 94 (2005) 161301, astro-ph/0411394.

[41] Q.-G. Huang and K. Ke, *Non-gaussianity in KKLMMT model*, Phys. Lett. B633 (2006) 447–452, hep-th/0504137.
One-loop corrections to a scalar field during inflation

[44] N. Barnaby and J. M. Cline, Nongaussian and nonscale-invariant perturbations from tachyonic preheating in hybrid inflation, Phys. Rev. D73 (2006) 106012, astro-ph/0601481.

[45] N. Barnaby and J. M. Cline, Large nongaussianity from nonlocal inflation, arXiv:0704.3426 [hep-th].

[46] J. Valiviita, M. Sasaki, and D. Wands, Non-gaussianity and constraints for the variance of perturbations in the curvaton model, astro-ph/0610001.

[47] C. T. Byrnes, M. Sasaki, and D. Wands, Diagrammatic approach to non-gaussianity from inflation, arXiv:0705.4096 [hep-th].

[48] C. T. Byrnes, K. Koyama, M. Sasaki, and D. Wands, Non-gaussianity and constraints for the variance of perturbations in the curvaton model, astro-ph/0610001.

[49] M. S. Sloth, On the one loop corrections to inflation and the CMB anisotropies, Nucl. Phys. B748 (2006) 149–169, astro-ph/0604488.

[50] M. S. Sloth, On the one loop corrections to inflation. II: The consistency relation, Nucl. Phys. B775 (2007) 78–94, hep-th/0612138.

[51] V. F. Mukhanov, L. R. W. Abramo, and R. H. Brandenberger, On the back reaction problem for gravitational perturbations, Phys. Rev. Lett. 78 (1997) 1624–1627, gr-qc/9609026.

[52] L. R. W. Abramo, R. H. Brandenberger, and V. F. Mukhanov, The energy-momentum tensor for cosmological perturbations, Phys. Rev. D56 (1997) 3248–3257, gr-qc/9704037.

[53] L. R. W. Abramo and R. P. Woodard, One loop back reaction on chaotic inflation, Phys. Rev. D60 (1999) 044010, astro-ph/9811430.

[54] W. Unruh, Cosmological long wavelength perturbations, astro-ph/9802323.

[55] T. Prokopec, O. Tornkvist, and R. P. Woodard, One loop vacuum polarization in a locally de Sitter background, Ann. Phys. 303 (2003) 251–274, gr-qc/0205130.

[56] R. H. Brandenberger, Back reaction of cosmological perturbations and the cosmological constant problem, hep-th/0210165.

[57] V. K. Onemli and R. P. Woodard, Super-acceleration from massless, minimally coupled $\phi^4$, Class. Quant. Grav. 19 (2002) 4607, gr-qc/0204065.

[58] T. Prokopec and R. P. Woodard, Vacuum polarization and photon mass in inflation, Am. J. Phys. 72 (2004) 60–72, astro-ph/0303358.

[59] G. Geshnizjani and R. Brandenberger, Back reaction of perturbations in two scalar field inflationary models, JCAP 0504 (2005) 006, hep-th/0310265.

[60] R. Brandenberger and A. Mazumdar, Dynamical relaxation of the cosmological constant and matter creation in the universe, JCAP 0408 (2004) 015, hep-th/0402205.

[61] R. H. Brandenberger and J. Martin, Back-reaction and the trans-planckian problem of inflation revisited, Phys. Rev. D71 (2005) 023504, hep-th/0410223.

[62] R. H. Brandenberger and C. S. Lam, Back-reaction of cosmological perturbations in the infinite wavelength approximation, hep-th/0407048.

[63] V. K. Onemli and R. P. Woodard, Quantum effects can render $w < -1$ on cosmological scales, Phys. Rev. D70 (2004) 107301, gr-qc/0406098.

[64] T. Brunier, V. K. Onemli, and R. P. Woodard, Two loop scalar self-mass during inflation, Class. Quant. Grav. 22 (2005) 59–84, gr-qc/0408080.

[65] D. Boyanovsky, H. J. de Vega, and N. G. Sanchez, Quantum corrections to slow roll inflation and new scaling of superhorizon fluctuations, Nucl. Phys. B747 (2006) 25–54, astro-ph/0503669.

[66] D. Boyanovsky, H. J. de Vega, and N. G. Sanchez, Quantum corrections to the inflaton potential and the power spectra from superhorizon modes and trace anomalies, Phys. Rev. D72 (2005) 103006, astro-ph/0507596.

[67] E. O. Kahya and R. P. Woodard, Charged scalar self-mass during inflation, Phys. Rev. D72 (2005) 104001, gr-qc/0508015.

[68] S. Weinberg, Quantum contributions to cosmological correlations, Phys. Rev. D72 (2005) 043514, hep-th/0506238.

[69] P. Martineau and R. H. Brandenberger, The effects of gravitational back-reaction on cosmological
One-loop corrections to a scalar field during inflation

70. K. Chaicherdsakul, Quantum cosmological correlations in an inflating universe: Can fermion and gauge fields loops give a scale free spectrum?, Phys. Rev. D75 (2007) 063522, astro-ph/0611352.

71. B. Losic and W. G. Unruh, Long-wavelength metric backreactions in slow-roll inflation, Phys. Rev. D72 (2005) 123510, gr-qc/0510078.

72. E. O. Kahya and V. K. Onemli, Quantum stability of a \( w < -1 \) phase of cosmic acceleration, gr-qc/0612026.

73. A. Bilandžić and T. Prokopec, Quantum radiative corrections to slow-roll inflation, astro-ph/0505236.

74. E. O. Kahya and R. P. Woodard, One loop corrected mode functions for SQED during inflation, Phys. Rev. D74 (2006) 084012, astro-ph/0608049.

75. S. Kim, Quantum fluctuations in the inflationary universe, Phys. Rev. D75 (2007) 063522, hep-th/0611352.

76. B. Losic and W. G. Unruh, Long-wavelength metric backreactions in slow-roll inflation, Phys. Rev. D72 (2005) 123510, gr-qc/0510078.

77. E. O. Kahya and V. K. Onemli, Quantum stability of a \( w < -1 \) phase of cosmic acceleration, gr-qc/0612026.

78. E. O. Kahya and R. P. Woodard, One loop corrected mode functions for SQED during inflation, Phys. Rev. D74 (2006) 084012, gr-qc/0608049.

79. S. P. Kim, Quantum fluctuations in the inflationary universe, astro-ph/0701399.

80. T. Prokopec, N. C. Tsamis, and R. P. Woodard, Two loop scalar bilinears for inflationary SQED, Class. Quant. Grav. 24 (2007) 201–230, gr-qc/0607094.

81. A. A. Starobinsky, Multicomponent de Sitter (inflationary) stages and the generation of perturbations, JETP Lett. 42 (1985) 152–155.

82. M. Sasaki and E. D. Stewart, A general analytic formula for the spectral index of the density perturbations produced during inflation, Prog. Theor. Phys. 95 (1996) 71–78, astro-ph/9507001.

83. G. I. Rigopoulos, E. P. S. Shellard, and B. J. W. van Tent, Large non-gaussianity in multiple-field inflation, Phys. Rev. D73 (2006) 083522, astro-ph/0506704.

84. G. I. Rigopoulos, E. P. S. Shellard, and B. J. W. van Tent, Quantitative bispectra from multifield inflation, astro-ph/0511041.

85. D. Seery and J. E. Lidsey, Non-gaussianity from the inflationary trispectrum, JCAP 0701 (2007) 008, astro-ph/0611034.

86. J. Schwinger, Brownian motion of a quantum oscillator, J. Math. Phys. 2 (1961) 407–432.
the spectrum of curvature perturbations during inflation. \[\text{arXiv:0707.1416 [hep-th]}\]

[96] A. Ashoorioon and R. B. Mann, \textit{On the tensor/scalar ratio in inflation with UV cut off}, \textit{Nucl. Phys. B716} (2005) 261–279, \[\text{gr-qc/0411056}\]