Finite sensor selection algorithm in distributed MIMO radar for joint target tracking and detection

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Abstract: Due to the requirement of anti-interception and the limitation of processing capability of the fusion center, the subarray selection is very important for the distributed multiple-input multiple-output (MIMO) radar system, especially in the hostile environment. In such conditions, an efficient subarray selection strategy is proposed for MIMO radar performing tasks of target tracking and detection. The goal of the proposed strategy is to minimize the worst-case predicted posterior Cramer-Rao lower bound (PCRLB) while maximizing the detection probability for a certain region. It is shown that the subarray selection problem is NP-hard, and a modified particle swarm optimization (MPSO) algorithm is developed as the solution strategy. A large number of simulations verify that the MPSO can provide close performance to the exhaustive search (ES) algorithm. Furthermore, the MPSO has the advantages of simpler structure and lower computational complexity than the multi-start local search algorithm.

Keywords: distributed multiple-input multiple-output (MIMO) radar, subarray selection, target tracking, target detection, particle swarm optimization (PSO).

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1. Introduction

1.1 Background and motivation

Enjoying the superiorities of anti-stealth and parameter identification, the multiple-input multiple-output (MIMO) radar with distributed antennas has drawn much attention in recent years [1,2]. For brevity, we call the MIMO radar with distributed antennas as the MIMO radar in the rest of the paper, unless doing so causes confusion.

Theoretically speaking, for the parameter estimation and target detection, more antennas or higher power will gain better performance [3]. However, the increased antennas will increase the computational cost of the fusion center. In addition, especially in the hostile environment, it is necessary to reduce the transmit power to meet the requirement of low possibility of interception [4]. Therefore, how to allocate the finite resource to achieve better performance is a crucial issue in the MIMO radar system [5–17].

Generally, the resource allocation in the MIMO radar is divided into two types: the transmit parameter selection and the system structure configuration. The first item contains the transmit power, bandwidth, etc. The latter one is associated with the selection of subarrays and their placement. For example, Godrich et al. formed two power optimization models for the target localization [5]. One is to minimize the power subject to the given localization accuracy, and the other is to minimize the Cramer-Rao lower bound (CRLB) under the constraint of the predetermined power budget. Reference [9] regarded the sensor location as the variable and put forward a subset selection algorithm for the single target localization. In [17], the joint sensor selection and power allocation problem was studied, and the multipath effect was integrated in the target tracking.

Though the above research seems fruitful to the MIMO radar resource allocation, they all pay attention to the implementation of parameter estimation. However, particularly in the military application, diversiform tasks are required to be executed by a MIMO radar, where the multitarget tracking and detection for a certain region are two typical ones. Furthermore, Radmard et al. demonstrated that the antenna placement has a great impact on the detection performance of the MIMO radar [18]. Therefore, it is of interest to study the optimal antenna allocation under a new background, i.e., the joint of multi-target tracking and detection. While an antenna placement method was proposed in [18] for the MIMO radar detection, two points should be emphasized. (i) The antenna placement is not
practical in applications, because the MIMO radar antennas are often stationary, or their locations cannot be dynamically changed. As such, the sensor selection model is more appropriate. (ii) The proposed exhaustive search (ES) method in [18] possesses a large computational burden, and it is only suitable for small-scale problems.

1.2 Methodology

The cognition technique [19–21] integrates the perception with the decision to improve the system performance. Since the posterior CRLB (PCRLB) is predictive, some researches utilize the predicted PCRLB as the criterion, so that the resource allocation can be adaptively tuned [14–17].

On the other hand, the sensors election issue is known to be NP-hard [22]. Though the ES [23] is feasible, its computational complexity will increase exponentially with the number of active sensors. References [9,16] put forward the multi-start local search (MSLS) as an alternative, but the performance is unstable due to its shortsighted essence. The particle swarm optimization (PSO) algorithm [24] is a nature-inspired optimization method. Owing to the advantages of easy implementation and efficiency, it is widely adopted in many optimization problems. Furthermore, many methods have been put forward aiming at its global optimization ability enhancement [25–27], and their performance is demonstrated through enormous simulations.

1.3 Main contributions

The main contributions of this paper are listed as follows:

(i) A finite sensor selection strategy for the joint of multi-target tracking and detection in the MIMO radar is proposed. By using the predicted PCRLB in the worst case as the tracking part of the objective function, the target information at current time is used to approximate the PCRLB at the next time. Moreover, the detection probability for a certain region is used to characterize the detection performance and is combined with the PCRLB to form the optimization criterion. By obtaining the probabilistic understanding of the environment, the subarray selection can be adaptively adjusted, rendering a closed loop feedback system.

(ii) A modified PSO (MPSO) algorithm is put forward to solve the subarray selection optimization model. As mentioned before, the subarray selection problem is known to be NP-hard. Solutions by the ES or MSLS method [9,16] are either computational demanding or only local-optimal in performance [14]. Therefore, a PSO variant is developed for the problem solving. The attributes of ergodicity and randomness of chaotic sequences are used to optimize the initialized solutions. The hierarchy penalty functions are designed to handle different degrees of constraint violation and to facilitate the exploitation precision. Additionally, the crossover operation and mutation operation are introduced to enhance the PSO global exploration ability. Thereby, both the exploration capability and the efficiency are improved.

(iii) A large scale of simulations demonstrate that the proposed PSO algorithm carries with a simpler structure and offers comparable performance to the ES algorithm. Furthermore, the proposed sensor selection strategy is very effective in gaining the overall performance improvement for the MIMO radar.

The remainder of this paper is organized as follows. Section 2 depicts the signal model. Section 3 focuses on the establishment of the target tracking problem. Section 4 structures the Neyman-Pearson detector for the MIMO radar. The optimization model and a modified PSO are formulated in Section 5. Section 6 presents the simulations and analysis. Section 7 concludes the paper.

2. Signal model

A MIMO radar with $M$ transmitting arrays and $N$ receiving arrays is chosen for analysis. The locations of the $m$th transmitting array and the $n$th receiving array are $(x_{tm}, y_{tm})$ and $(x_{rn}, y_{rn})$, respectively, for $m \in \{1, \ldots, M\}$, and $n \in \{1, \ldots, N\}$. Q targets move in the radar surveillance region. At the $k$th sampling interval, the state of the $q$th target is $[x_q^k, y_q^k, \dot{x}_q^k, \dot{y}_q^k]^T$. The transmit waveform [5] is normalized as

$$\int_{-\infty}^{+\infty} |\tilde{s}_m(t)|^2 dt = 1$$

and orthogonal to each other

$$\int_{-\infty}^{+\infty} \tilde{s}_m(t) \tilde{s}_{m'}(t - \tau) dt = \begin{cases} 0, & m \neq m' \\ 1, & m = m' \end{cases}$$

where $(\cdot)^*$ is the conjugate operator. Additionally, all transmit signals are narrowband with individual equivalent bandwidth [5]

$$\beta_m = \int |f|^2 |S_m(f)|^2 df$$

and individual equivalent time duration

$$T_m = \int |t|^2 |\tilde{s}_m(t)|^2 dt.$$
Therefore, the baseband signal reflected by the qth target via the \((m,n)\)th path [12] is
\[ \tilde{r}_n^q(t) = \sum_{m=1}^{M} \tilde{h}_{m,n,k}^q \tilde{v}_m(t - \tau_{m,n,k}^q)e^{-j2\pi v_f^q t} + \tilde{n}_{m,n}(t) \]  
(5)
where \( h_{m,n,k}^q \) is the path loss effect; \( \tilde{h}_{m,n}^q \) is the complex amplitude of the target and \( \tilde{h}_{m,n}^q \sim N(0, \sigma_h^2) \); \( \tau_{m,n,k}^q \) is the time-delay, and \( \nu_{m,n,k}^q \) is the Doppler frequency; \( \tilde{n}_{m,n}(t) \sim N(0, \sigma_n^2) \) is the complex and Gaussian noise.

\( h_{m,n,k}^q \) [14,18] satisfies
\[ h_{m,n,k}^q \propto \frac{\sqrt{P_t}}{R_{m,k}^q R_{n,k}^q} \]  
(6)
where \( R_{m,k}^q \) is the distance from transmitting array \( m \) to target \( q \), \( R_{n,k}^q \) is the distance from target \( q \) to receiving array \( n \), and \( P_t \) is the transmit power. Here, we assume that the transmit powers of each array are equivalent. The relationship between \( R_{m,k}^q, R_{n,k}^q \) and \( \tau_{m,n,k}^q \) is
\[ \begin{cases} \tau_{m,n,k}^q = \frac{R_{m,k}^q + R_{n,k}^q}{c} \\ R_{m,k}^q = \sqrt{(x_k^m - x_{tm})^2 + (y_k^m - y_{tm})^2} \\ R_{n,k}^q = \sqrt{(x_k^n - x_{rn})^2 + (y_k^n - y_{rn})^2} \end{cases} \]  
(7)
where \( c \) is the speed of light. The Doppler frequency is calculated as
\[ \nu_{m,n,k}^q = -\frac{2\pi}{\lambda} [\tilde{\phi}_m^q \cos(\phi_{m,k}^q + \phi_{n,k}^q) + \tilde{\phi}_m^q \sin(\phi_{m,k}^q + \phi_{n,k}^q)] \]  
(8)
where \( \phi_{m,k}^q \) and \( \phi_{n,k}^q \) denote the target bearing angle at transmitter \( m \) with respect to \( x \) axis and at receiver \( n \) with respect to \( x \) axis, respectively.

\[ \begin{cases} \phi_{m,k}^q = \arctan \left[ \frac{y_k^q - y_{tm}}{x_k^m - x_{tm}} \right] \\ \phi_{n,k}^q = \arctan \left[ \frac{y_k^n - y_{rn}}{x_k^n - x_{rn}} \right] \end{cases} \]  
(9)

### 3. Target tracking model

#### 3.1 Target dynamics

The target motion is assumed to comply with the constant velocity (CV) model [16].
\[ x_k^q = F x_k^{q-1} + w_k^q \]  
(10)
where \( x_k^q = [x_k^q, y_k^q, \dot{x}_k^q, \dot{y}_k^q]^T \) is the state of the qth target. \( F \) and \( w_k^q \) are the state transition matrix and the process noise, respectively. \( w_k^q \sim N(0,Q) \). Specific expressions are
\[ F = \begin{bmatrix} 1 & T_s & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]  
(11)
\[ Q = \kappa_f \begin{bmatrix} T_3^3 & T_2^2 \\ T_3^2 & T_s \end{bmatrix} \]  
(12)
with the noise density \( \kappa_f \) and sampling interval \( T_s \), where \( \otimes \) denotes the Kronecker product.

#### 3.2 Measurement model

By signal processing techniques, the time-delay and Doppler frequency can be extracted from the receive signals. Thus, the measurement model is
\[ z_k^q = h(x_k^q) + v_k^q \]  
(13)
where \( h(\cdot) \) denotes the nonlinear transformation with \( h(x_k^q) = [\tau_{m,n,k}^q, v_{m,n,k}^q]^T \). Here, we adopt the square-root cubature filter (SCKF) [28,29] to obtain an accurate estimate. \( v_k^q \sim N(0, R_k^q) \), and \( R_k^q \) is the CRLB of joint location and velocity, which will be shown in the next subsection.

#### 3.3 CRLB derivation

The CRLB provides a lower bound for any unbiased estimator [13]. For the target measurement model depicted in Subsection 3.2, we have (14) on the condition of assuming \( f(y_k^q) \) is the unbiased estimate for \( x_k^q \):
\[ E_{x_k^q,y_k^q} \{ [f(y_k^q) - x_k^q] [f(y_k^q) - x_k^q]^T \} \geq J^{-1}(x_k^q) \]  
(14)
where \( E \) denotes the expectation operation to the target state \( x_k^q \) and measurement \( y_k^q \). \( J(x_k^q) \) is the Bayesian Fisher information matrix (BIM) about the target state, and it can be calculated by
\[ J(x_k^q) = -E_{x_k^q,y_k^q} [\nabla_x \nabla_y \ln p(y_k^q, x_k^q)] \]  
(15)
where the notation \( \nabla_x \nabla_y = \nabla_x \nabla_y^T \) is the second order partial derivative vector. The joint probability density function (PDF) in (15) can be factorized as
\[ p(y_k^q, x_k^q) = p(y_k^q | x_k^q) p(x_k^q) \]  
(16)
where \( p(y_k^q, x_k^q) \) is the joint PDF of \( (y_k^q, x_k^q) \), \( p(x_k^q) \) is the PDF of the target state \( x_k^q \), and \( p(y_k^q | x_k^q) \) is the joint conditional PDF. By blocking BIM, Tichavsky et al. [13] proposed an elegant recursive method for the discrete-time nonlinear filtering, which avoids the manipulation on the large matrix:
\[ J(x_k^q) = J_l(x_k^q) + J_D(x_k^q) \]  
(17)
where $J_I(x_k^q)$ and $J_D(x_k^q)$ are the Fisher information matrices of the prior information and the data, respectively:

\[
\begin{align*}
J_I(x_k^q) &= -\mathbb{E}_{x_k^q}[\nabla_{x_k^q}^T \ln p(x_k^q)] = \\
&= Q + F J^{-1}(x_{k-1}^q) F^T]^{-1} \tag{18}
\end{align*}
\]

\[
\begin{align*}
J_D(x_k^q) &= -\mathbb{E}_{x_k^q,y_k^q}[\nabla_{x_k^q}^T \ln p(y_k^q|x_k^q)] \\
&= E_{x_k^q} \{E_{y_k^q|x_k^q}[-\nabla_{x_k^q}^T \ln p(y_k^q|x_k^q)] = \\
&= -\frac{x_k^q \sin^2 \phi_{m,k}^q + \frac{1}{2} \dot{y}_k^q \sin 2 \phi_{m,k}^q + \frac{1}{2} \dot{y}_k^q \sin 2 \phi_{n,k}^q}{R_{m,k}^q} \\
&+ \frac{\dot{y}_k^q \cos^2 \phi_{m,k}^q + \frac{1}{2} \dot{x}_k^q \sin 2 \phi_{m,k}^q + \frac{1}{2} \dot{x}_k^q \sin 2 \phi_{n,k}^q}{R_{n,k}^q} \\
&- \frac{\cos \phi_{m,k}^q \sin \phi_{n,k}^q}{R_{m,k}^q} - \frac{\cos \phi_{n,k}^q \sin \phi_{m,k}^q}{R_{n,k}^q} \\
&= \left[ \begin{array}{c}
\frac{\dot{x}_k^q \sin^2 \phi_{m,k}^q + \frac{1}{2} \dot{y}_k^q \sin 2 \phi_{m,k}^q + \frac{1}{2} \dot{y}_k^q \sin 2 \phi_{n,k}^q}{R_{m,k}^q} \\
\frac{\dot{y}_k^q \cos^2 \phi_{m,k}^q + \frac{1}{2} \dot{x}_k^q \sin 2 \phi_{m,k}^q + \frac{1}{2} \dot{x}_k^q \sin 2 \phi_{n,k}^q}{R_{n,k}^q} \\
- \frac{\cos \phi_{m,k}^q \sin \phi_{n,k}^q}{R_{m,k}^q} - \frac{\cos \phi_{n,k}^q \sin \phi_{m,k}^q}{R_{n,k}^q}
\end{array} \right]. \tag{20}
\end{align*}
\]

As to $J_D(x_k^q)$, we have the following equation when substituting (16) into (18):

\[
\begin{align*}
J_D(x_k^q) &= E_{x_k^q,y_k^q}[\nabla_{x_k^q}^T \ln p(y_k^q|x_k^q)] = \\
&= E_{x_k^q} \{E_{y_k^q|x_k^q}[-\nabla_{x_k^q}^T \ln p(y_k^q|x_k^q)] = \\
&= \frac{8\pi^2}{\sigma_n^2} \begin{bmatrix}
\Omega_{\text{diag}}(\beta_m^2 |\bar{a}_{m,n}|^2 h_{m,k}^q)^2 & \mathbf{0} \\
\mathbf{0} & \Omega_{\text{diag}}(|\bar{a}_{m,n}|^2 h_{m,k}^q)^2
\end{bmatrix} \tag{23}
\end{align*}
\]

where $\Omega_{\text{diag}}(\cdot)$ is the $MN \times MN$ diagonal matrix. Finally, we have the BIM expression:

\[
\begin{align*}
J(x_k^q) &= [Q + F J^{-1}(x_{k-1}^q) F^T]^{-1} + \\
&= E_{x_k^q}[(H_k^q)^T(R_{k}^{-1})^{-1} H_k^q]. \tag{24}
\end{align*}
\]

To achieve the PCRLB, we must implement large numbers of Monte Carlo trials [25,26]. That will consume too much time. To satisfy the real-time demand, we use $\hat{H}_k^q$ and $(R_k^{-1})^{-1}$ to approximate $H_k^q$ and $(R_k^{-1})^{-1}$, respectively, where $\hat{H}_k^q$ and $(R_k^{-1})^{-1}$ are the Jacobian matrix and measurement covariance matrix evaluated around $x_{k|k-1}$. $x_{k|k-1}$ denotes the predicted state at the $(k-1)\text{th}$ interval. Therefore, (24) is rewritten as

\[
\begin{align*}
J(x_k^q) &= [Q + F J^{-1}(x_{k-1}^q) F^T]^{-1} + (\hat{H}_k^q)^T(R_k^{-1})^{-1} \hat{H}_k^q. \tag{25}
\end{align*}
\]

4. Neyman-Pearson detector

The detection performance can be characterized by the probability of detection under the constraint of the predefined false alarm ratio, which is also called as the Neyman-Pearson detector. Therefore, we establish the Neyman-Pearson model in this section, which provides a basis for the optimal subarrays selection.

In the hypothesis of target existence $H_1$ or not $H_0$, the detection model is given [18] by

\[
y_{m,n} = \begin{cases}
\tilde{r}_m(t) \bar{s}_n(t) dt = h_{m,n} \bar{a}_{m,n} x_{m,n} + \bar{n}_{m,n}, & H_1 \\
\bar{n}_{m,n}, & H_0
\end{cases} \tag{26}
\]

where $\bar{a}_{m,n} = a_R + j a_I$, which is composed of the real part $a_R$ and the imaginary part $a_I$; $\bar{n}_{m,n}$ is the complex and additive white noise.

The observation and state from all paths are all aggregated into

\[
y = [y_{1,1}, y_{2,1}, \ldots, y_{M,1}, y_{1,2}, y_{2,2}, \ldots, y_{M,2}, \ldots, y_{1,N}, y_{2,N}, \ldots, y_{M,N}]^T, \tag{27}
\]

\[
x = [x_{1,1}, x_{2,1}, \ldots, x_{M,1}, x_{1,2}, x_{2,2}, \ldots, x_{M,2}, \ldots, x_{1,N}, x_{2,N}, \ldots, x_{M,N}]^T. \tag{28}
\]
Therefore, the likelihood ratio in the Neyman-Pearson result. Moreover, the joint PDF of
\[ p(y|H_1) = \left\{ \begin{array}{ll}
\int p_1(y|H_1, a_R, a_1)p(a_R, a_1)da_Rda_1 =
\end{array} \right. \]
\[ k_0 \exp \left\{ -\sum_{m=1}^{M} \sum_{n=1}^{N} (y_{m,n} - h_{m,n}\tilde{a}_{m,n}x_{m,n})^2 \right\}p(a_R, a_1)da_Rda_1 =
\]
\[ k_0 \exp \left( -\frac{y^H y}{\sigma_n^2} \right) \exp \left( \frac{\sum_{m=1}^{M} \sum_{n=1}^{N} |x_{m,n}y_{m,n}|^2}{k_1} \right) \]
where \( \alpha = [\tilde{a}_{1,1}, \tilde{a}_{2,1}, \ldots, \tilde{a}_{M,1}, \tilde{a}_{1,2}, \tilde{a}_{2,2}, \ldots, \tilde{a}_{M,2}, \ldots, \tilde{a}_{1,N}, \tilde{a}_{2,N}, \ldots, \tilde{a}_{M,N}]^T = \alpha_n + j\alpha_1, \) and \( \alpha \sim N(0, \sigma_n^2 I_{MN}). \) In (29), we have the following expression in the integration operation:
\[ p(y_{m,n}|H_0) = k_0 \exp \left( -\frac{y_{m,n}^*y_{m,n}}{\sigma_n^2} \right) \exp \left( \frac{\sum_{m=1}^{M} \sum_{n=1}^{N} |x_{m,n}y_{m,n}|^2}{k_1} \right) \]
where \( k_0 \) and \( k_1 \) are constants that do not influence the result. Moreover, the joint PDF of \( H_0 \) hypothesis is
\[ p(y|H_0) = k_0 \exp \left( -\frac{y^H y}{\sigma_n^2} \right). \]
Therefore, the likelihood ratio in the Neyman-Pearson detector is given by
\[ \ln L(y) = \eta_0 + \sum_{m=1}^{M} \sum_{n=1}^{N} |x_{m,n}y_{m,n}|^2. \]
After removing \( \eta_0, \) the sufficient statistic is
\[ T_1(y) = \sum_{m=1}^{M} \sum_{n=1}^{N} |x_{m,n}y_{m,n}|^2. \]
In the \( H_0 \) and \( H_1 \) hypothesis, the variances of \( x_{m,n}y_{m,n} \) are \( \sigma^2_n \) and \( h_0^2\sigma^2_n + \sigma^2_n, \) respectively, where \( h_0 \) is the mean path loss via multiple paths.
\[ h_0^2 = \frac{1}{MN} \sum_{m=1}^{M} h_{m,n}^2 \]
Therefore, the distribution of \( T_1(y) \) is
\[ \begin{cases}
H_0 : & \frac{T_1(y)}{\sigma_n^2} \sim \chi^2_{2NM} \\
H_1 : & \frac{T_1(y)}{h_0^2\sigma_n^2 + \sigma^2_n} \sim \chi^2_{2NM}
\end{cases} \]
The false alarm probability is expressed as
\[ P_{fa} = P(T_1(y) > \eta|H_0). \]
According to (36), the detection threshold \( \eta \) is calculated by
\[ \eta = \frac{\sigma^2_n}{2} F^{-1}_{\chi^2_{2NM}}(1 - P_{fa}) \]
where \( F^{-1}_{\chi^2_{2NM}} \) is the inverse function of \( F_{\chi^2_{2NM}} \), and \( F_{\chi^2_{2NM}} \) is the function which follows \( \chi^2 \) distribution with the freedom of \( 2NM. \) Finally, we have the expression of the probability of detection:
\[ P_d = P(T_1(y) > \eta|H_1) = \frac{1}{F_{\chi^2_{2NM}} \left( \frac{\sigma^2_n}{h_0^2\sigma_n^2 + \sigma^2_n} F^{-1}_{\chi^2_{2NM}}(1 - P_{fa}) \right)}. \]

5. Sensor selection strategy

5.1 Optimization criterion
To describe the subarray selection problem, two vectors, which denote whether the subarray is selected or not, are firstly introduced.
\[ \begin{align*}
t_s &= [t_{s1}, t_{s2}, \ldots, t_{sM}] \\
r_s &= [r_{s1}, r_{s2}, \ldots, r_{sN}]
\end{align*} \]
where \( t_{sm} \in \{0, 1\} \) and \( r_{sn} \in \{0, 1\}. \)

For the target tracking problem, the PCRLB is usually chosen as the criterion [14 – 16], because it not only provides a tight performance bound for the state estimation, but also is predictive. Therefore, the deduced PCRLB is exploited as the tracking part of the objective function:
\[ J(x^q_k, t_s, r_s) = J_1(x^q_k) + J_D(x^q_k, t_s, r_s) \]
\[ J_D(x^q_k, t_s, r_s) = \sum_{m=1}^{M} \sum_{n=1}^{N} t_{sm}r_{sn}(H^q_{m,n,k})^T J_D(\theta^q_{m,n,k})H^q_{m,n,k}. \]
In the multi-target tracking case, the limited resource should be prior allocated to the target in the most urgent demand. Therefore, we utilize the PCRLB in the worst case to form the tracking part of the objective function. Additionally, as shown in (38), the detection probability can also be improved by suitable antenna selections. Therefore, the objective function is formed as
\[ F(t_s, r_s) = \max_q \left\{ \sqrt{\text{tr}(J^{-1}(x^q_k, t_s, r_s))}, \frac{1}{P_d(t_s, r_s)} \right\}. \]
Equation (42) shows that the subarray selection problem for joint tracking and detection is a bi-objective optimization one. There are a set of solutions located at the Pareto front. However, the enumeration of all feasible solutions is out of this paper’s scope. Therefore, we transform it into a single objective function through the weighted form

\[
F(t_s, r_s) = \left\{ \max_q \sqrt{\text{tr}(J^{-1}(x^q_k, t_s, r_s))} + \frac{\lambda_1}{P_d(t_s, r_s)} \right\}
\]

(43)

where \(\lambda_1 \in [0, 1]\), which balances the proportion between the tracking and detection performance, meanwhile, unifies their meanings [14].

5.2 Problem formulation

In the implementation of the sensor selection, first, the total number of active subarrays is limited, to meet the requirement of anti-interception:

\[
\sum_{m=1}^{M} t_{sm} + \sum_{n=1}^{N} r_{sn} = K.
\]

(44)

At each time, there needs to be at least one active transmitter and one active receiver:

\[
\begin{cases}
\sum_{m=1}^{M} t_{sm} \geq 1 \\
\sum_{n=1}^{N} r_{sn} \geq 1
\end{cases}
\]

(45)

Additionally, the detector performance is characterized by the predefined false alarm probability:

\[P_{fa} = P_{fa0}.\]

(46)

As such, the optimization model is established as

\[
\begin{align*}
\min & \quad F(t_s, r_s) \\
\text{s.t.} & \quad \sum_{m=1}^{M} t_{sm} + \sum_{n=1}^{N} r_{sn} = K \\
& \quad \sum_{m=1}^{M} t_{sm} \geq 1, \sum_{n=1}^{N} r_{sn} \geq 1 \\
& \quad t_{sm} \in \{0, 1\}, \quad r_{sn} \in \{0, 1\} \\
& \quad P_{fa} = P_{fa0}.
\end{align*}
\]

(47)

5.3 MPSO algorithm

On account of the binary constraint, the optimization model described in (47) is NP-hard [9]. Though the best performance can be provided via exhaustive examinations of all possibilities of \(t_s\) and \(r_s\), such a search method requires an exponential complexity for computation. In order to solve this problem efficiently, an MPSO algorithm is proposed, which carries with the following salient characteristics. First, the properties of ergodicity and randomness in chaotic sequences are exploited to improve the quality of initialized particles. Second, different penalty functions are designed in the fitness calculation to facilitate the algorithm convergence. Last but not least, the crossover operator and mutation operator are introduced to drive the algorithm to jump out from local optima. Thereby, the MPSO will gain better exploration ability and exploitation ability. The framework of MPSO is presented as follows.

Pseudo code of the MPSO

1: Parameter initialization: swarm cardinality \(N_{\text{pop}}\), maximum iteration number for chaotic sequences \(t_{\text{max}}\), inertia weight \(w\), maximum iteration number for swarm \(t_{\text{max}}\), preset number \(t_{\text{max}}\), crossover probability \(P_c\) and mutation probability \(P_m\)
2: Initialize the particles with chaotic sequences by (50) within the problem range
3: for \(i = 1: N_{\text{pop}}\) do
4: Round operation to \(x_i\)
5: end
6: Calculate the fitness value by (51), assign \(x_i(t)\) to be \(p_{\text{best}}(t)\) and find out \(g_{\text{best}}(t)\) among the swarm
7: \(t = 1\)
8: while \(t < t_{\text{max}}\) do
9: for \(i = 1: N_{\text{pop}}\) do
10: if \(i_{\text{num}} < i_{\text{num}}\) then
11: Update the velocity and position of the particle by (48) and (49) respectively
12: Round operation to \(x_i(t)\)
13: else
14: \(i_{\text{num}} = 0\)
15: Update the velocity and position of the particle by (48), (49), and the crossover operation and mutation operation
16: Round operation to \(x_i(t)\)
17: end if
18: Evaluate fitness value for new particle \(x_i(t)\)
19: if \(x_i(t)\) is better than \(p_{\text{best}}(t)\) then
20: \(i_{\text{num}} = 0\)
21: Set \(x_i(t)\) to be \(p_{\text{best}}(t)\)
22: else
23: \(i_{\text{num}} = i_{\text{num}} + 1\)
24: end if
25: if \(x_i(t)\) is better than \(g_{\text{best}}(t)\) then
26: Set \(x_i(t)\) to be \(g_{\text{best}}(t)\)
In many swarm search algorithms, a traditional approach for the constraint handling is to use the penalty function, by which the constrained optimization problem is transformed into unconstrained ones. However, many PSOs fail to consider different degrees of constraint violation. They regard the individuals that do not satisfy the constraints as equivalent, and endow them the unified penalty function. Such a uniform treating easily misses the search for the boundary space [35]. Thus, we devise the hierarchy penalty function as

$$ f_{pen}(\bar{x}_i(t)) = \begin{cases} 
10^2, & \left\{ K - K' \leq \sum_{m=1}^{M} t_{sm} + \sum_{n=1}^{N} r_{sn} \leq K + K' \right\} \cap \left\{ \sum_{m=1}^{M} t_{sm} \geq 1 \right\} \cap \left\{ \sum_{n=1}^{N} r_{sn} \geq 1 \right\} \\
10^3, & \left\{ K - K' \leq \sum_{m=1}^{M} t_{sm} + \sum_{n=1}^{N} r_{sn} \leq K + K' \right\} \cap \left\{ \sum_{m=1}^{M} t_{sm} \geq 1 \right\} \cap \left\{ \sum_{n=1}^{N} r_{sn} = 0 \right\} \\
10^3, & \left\{ K - K' \leq \sum_{m=1}^{M} t_{sm} + \sum_{n=1}^{N} r_{sn} \leq K + K' \right\} \cap \left\{ \sum_{m=1}^{M} t_{sm} = 0 \right\} \cap \left\{ \sum_{n=1}^{N} r_{sn} \geq 1 \right\} \\
10^8, & \left\{ \sum_{m=1}^{M} t_{sm} + \sum_{n=1}^{N} r_{sn} = 0 \right\} \cup \left\{ \sum_{m=1}^{M} t_{sm} + \sum_{n=1}^{N} r_{sn} = 1 \right\} \\
10^4, & \text{otherwise}
\end{cases} $$

where $K'$ is a preset scalar. The aim of the penalty function is to abandon bad solutions in order to select feasible ones. Since different severities of the penalties are applied, the infeasible solutions that approach to the feasible solution space will gain a higher probability to be reserved and transformed into feasible ones by the crossover and mutation operation (they will be shown in the next subsection).

It not only widens the exploration space, but also facilitates the exploration progress.

### 5.3.4 Genetic algorithm introduction

In order to break such stagnation of PSO, the crossover operation and the mutation operation in the genetic algorithm are introduced. The selection, crossover and muta-
tion operations are three basic components in the genetic algorithm. We often call the selected individuals as parents, the offspring as children, and the candidate sub-solutions lied on the individual as genes. The crossover and the mutation operations will diversify the swarm, and drive the algorithm to search unknown areas.

The crossover operation procedure is described as follows: first, $|N_{\text{pop}} \times P_r|$ ([$\cdot$] is the round operator) particles are selected as parents; then, the one-point crossover operation is applied to two different particles to exchange the genes located behind a specific point; after that, two children are available.

Similar initial procedure can be applied to the mutation operation, and just replace $P_r$ by $P_m$. However, the mutation operation is based on one particle, and each individual is updated by

$$\bar{x}_i(t) = \bar{x}_i(t) \odot (1 + \text{rand})$$  \hspace{1cm} (52)

where rand is a pseudorandom scalar which complies with the standard normal distribution.

By the crossover and the mutation operations, the MPSO is impelled to get rid of local optima. Thus, the algorithm will gain better solutions.

5.4 Computational complexity

In this subsection, we evaluate the computational complexity of the MPSO. The MPSO is one of the swarm search based algorithms, and the computational complexity is mostly affected by two parameters: the cardinality $N_{\text{pop}}$ and the maximum iteration $t_{\text{max}}$. For a specific run, it requires $O(N_{\text{pop}} t_{\text{max}})$ iterations to achieve a given threshold. In contrast, the ES algorithm requires an exponential complexity of $O(2^{M+N})$ [9]. The MSLS algorithm [9] carries with the lower computational burden, which is $O(KMN(M + N))$. Thus, we have the complexity comparison, as shown in Table 1. It is noticeable that in the MPSO, $N_{\text{pop}}$ and $t_{\text{max}}$ can be tuned to adapt to different size problems, bringing about more flexibility for the problem solving.

| Algorithm | ES | MSLS | MPSO |
|-----------|----|------|------|
| Computational complexity | $O(2^{M+N})$ | $O(KMN(M + N))$ | $O(N_{\text{pop}} t_{\text{max}})$ |

6. Simulations and analysis

6.1 Performance evaluation criterion

We adopt the ES method [22] as the evaluation benchmark, and assume that the selection result by this method is $(t_{s,\text{opt}}, r_{s,\text{opt}})$, and the corresponding objective function is $F(t_{s,\text{opt}}, r_{s,\text{opt}})$. The MSLS algorithm [9] and the standard PSO algorithm [30] are also presented for comparison. The normalized error in the ith simulation is defined [9] as

$$\varepsilon_i = \frac{F_i(t_{s,\text{oth}}, r_{s,\text{oth}}) - F_i(t_{s,\text{opt}}, r_{s,\text{opt}})}{F_i(t_{s,\text{opt}}, r_{s,\text{opt}})}$$  \hspace{1cm} (53)

where $F_i(t_{s,\text{oth}}, r_{s,\text{oth}})$ is the objective function that the three other algorithms have achieved. The mean error is

$$\varepsilon_{\text{ave}} = \frac{1}{N_{\text{sim}}} \sum_{i=1}^{N_{\text{sim}}} \varepsilon_i$$  \hspace{1cm} (54)

where $N_{\text{sim}}$ is the number of simulations. Since the sub-array selection problem is a minimization one, the lower value means the better performance.

6.2 Parameter designation

Consider a MIMO radar with 10 transmitters and 10 receivers, which is responsible for the target tracking and the detection for a specific region. Two targets with CV models move in the surveillance region, whose initial states are $x_0^1 = [-3 000, 3 000, 200\sqrt{1/2}, -200\sqrt{1/2}]^T$ and $x_0^2 = [-4 000, -2 000, 200/\sqrt{5}, 200/\sqrt{5}]^T$, respectively. The geometric relationship between the MIMO radar and two targets is shown in Fig. 1. The detection center is located at $[8 000, 9 000]^T$. In the MIMO radar, $P_t = 1$ kW, $\lambda = 0.3$ m, $\beta = 1$ MHz, $T_m = 10^{-6}$ s, $T_s = 4$ s, and $P_{\text{rad}} = 10^{-7}$. For simplicity, we assume that the target radar cross section (RCS) to all subarrays is constant and equal to 1. The 16 frame observation data are used in each simulation. As to the MPSO, $N_{\text{pop}} = 50$, $w = 0.8$, $t_{\text{max}} = 100$, $q_{\text{max}} = 3 000$, $i_{\text{max}} = 10$, $P_c = 0.6$, and $P_m = 0.3$.

![Fig. 1 Geometric relationship between MIMO radar and targets](image_url)
6.3 Results and analysis

6.3.1 Subarray selection result

At first, we assume that the number of subarrays to be selected is $K = 8$. Fig. 2 – Fig. 5 show the optimal antenna selection results by the ES algorithm, MPSO, MSLS and PSO in once simulation, respectively.

Obviously, the result by the MPSO is very close to the optimal antenna selection, except for the 5th frame. The performance of the MSLS is intermediate. By contrast, the result by the PSO is far from satisfactory. The active subarrays are mismatched with the ES algorithm in many frames.
The PCRLB and the probability of detection achieved by the four algorithms are plotted in Fig. 6(a) and Fig. 6(b), respectively. Moreover, the normalized errors are shown in Fig. 6(c). It can be seen that in the 5th frame, the error achieved by the MPSO is very little. That conforms to the result shown in Fig. 3. As to the MSLS, there are three-frame errors, but all of them are below 5%. However, the normalized errors achieved by the PSO are much higher than the other two algorithms, indicating its worst performance.

To eliminate the influence of randomness, we show the results averaged over 500 simulations. Table 2 shows the mean normalized error by all the algorithms. It can be seen that the mean error achieved by the proposed MPSO is well controlled below 2%. The result by the MSLS is a little higher, but is below 5%. However, the PSO provides the worst performance, and the normalized error ranges from 3.74% to 14.58%. Meanwhile, the highest error occurs in the terminal phase of the PSO.

That is because in this phase, the PSO easily suffers from the deteriorated population diversity. By contrast, the MPSO behaves the best with persistent performance. That is because many optimization methods are adopted, such as chaos initialization, hierarchy penalty function, crossover operation and mutation operation.

To intuitively show the advantages of the proposed MPSO, the convergence graphs of MPSO and PSO are presented in Fig. 7. Meanwhile, we also supplement the MPSO without chaotic sequences to demonstrate the effectiveness of chaos initialization.

Fig. 7(a) to Fig. 7(c) compare the convergence graph of the three algorithms in the 5th, the 10th and the 15th frame, respectively. The subfigures show the details. It is evident that the proposed MPSO with chaotic sequences owns the merits of higher-quality initialized solutions, quicker con-
vergence and global exploration capability. Though the MPSO without chaotic sequences has worse initialized solutions, it can explore near-global optima by the crossover operation and mutation operation. By contrast, the PSO displays a torpid behavior and is easily trapped into local optima.

6.3.3 Computational complexity comparison

Table 3 gives the comparison of iterations with different $K$ values. Evidently, the MPSO and the PSO carry with the lowest computational complexity. It should be noted that when $K \leq 6$ and $K \geq 14$, the MPSO only needs $N_{\text{pop}} = 50$ and $t_{\text{max}} = 50$ to achieve the near-optimal solution. However, the performance of the PSO cannot be guaranteed, just as shown in Table 2. The computational complexity of ES is quite large. For example, it requires 125 880 iterations to solve the eight antennas selection problem with ten transmitters and ten receivers. Therefore, such an algorithm is impracticable. Though the MSLS offers solutions with lower computational complexity, the result (shown in Table 2) is only comparable with the MPSO, sometimes even worse. As such, the proposed MPSO possesses the merits of a simpler structure and better performance. It is noticeable that the chaotic sequences can be generated offline, since the initial particles are all generated randomly. Therefore, the time consumption of chaotic sequences is not taken into account in this paper.

| $K$ | 4   | 6   | 8   | 10  | 12  | 14  | 16  |
|-----|-----|-----|-----|-----|-----|-----|-----|
| ES  | 425 | 384 | 340 | 125 | 780 | 184 | 754 |
| MSLS| 500 | 660 | 930 | 116 | 000 | 135 | 500 |
| MPSO| 250 | 250 | 500 | 350 | 000 | 750 | 000 |
| PSO | 250 | 250 | 500 | 500 | 500 | 250 | 250 |

6.3.4 Result from random subarray location

The result above is based on the circular subarray locations. To demonstrate the robustness of the proposed MPSO, we randomly generate the equal-size subarrays (ten transmitters and ten receivers) in a 20 km x 20 km region, and repeat the simulations 500 times. After testing 50 kinds of subarray locations, we obtain the mean normalized errors of each algorithm, shown in Table 4.

It can be seen that the performance gap between the MPSO and the MSLS is very close, and their normalized errors are well controlled below 3%. By contrast, the PSO algorithm cannot provide high-quality solutions in consistent manners, and its normalized error is also the highest.

| Frame | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 11    | 12    | 13    | 14    | 15    | 16    |%
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| MPSO  | 1.34  | 0.81  | 0.32  | 0.44  | 1.12  | 0.34  | 1.81  | 1.24  | 0.31  | 0.96  | 1.18  | 0.31  | 0.97  | 0.43  | 0.65  |
| MSLS  | 0.94  | 0.96  | 0.48  | 0.55  | 0.91  | 0.88  | 2.08  | 1.19  | 1.44  | 0.88  | 1.43  | 0.56  | 0.86  | 0.84  | 1.88  |
| PSO   | 4.52  | 10.23 | 8.64  | 7.46  | 9.27  | 7.35  | 8.31  | 18.68 | 10.44 | 12.63 | 7.82  | 5.45  | 8.36  | 12.48 | 19.38 | 25.46 |
7. Conclusions

The effective utilization of subarray resources in the MIMO radar is very important in military applications. For the scenario of joint target tracking and detection, an efficient subarray selection strategy is put forward. The optimization model is formed as minimizing the predicted PCRLB in the worst case and maximizing the probability of detection subject to the predetermined subarray size. Since such a selection problem is NP-hard, an MPSO algorithm is proposed for the solution exploration. Numerical simulations demonstrate that the MPSO can provide close-to-optimal subarray selection results. Meanwhile, compared with the MSLS algorithm, it possesses merits of simpler structure and lower computational complexity.

Potential future work includes detecting targets in unknown locations and solving this problem via other metaheuristic algorithms with better performance.

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