Based on ADRC Strategy for Tracking Trajectory of Undamped Plants

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Abstract. This paper mainly actualizes the precise trajectory tracking for the direct-drive servo system with undamped plant. Using the nominal model of the undamped object in this paper is to realize the precise control and obtain the better tracking effects by adopting the ADRC strategy which is introduced to avoid the overshoot causing by design of typical PID control law and improve the robustness of servo system. Simultaneously, it can also achieve the fast tracking and obtain the better convergence effects. Extensive simulations indicate that it practically validates the superiority of adopting the ADRC strategy to control these typical undamped plant. Both average placement tracking error and velocity tracking error for the sinusoidal waves and multi-step signal are converging to the value fluctuating around zero or gradually approaching to zero. This all-around control strategy to achieve the precise and fast trajectory tracing for the direct-drive servo system with undamped object is effective and feasible.

1. Introduction
Some servo system like direct-drive servo system has applied widely in all industrial field, especially in military-industrial weapons because of the necessary for manufacturing precision instruments and equipment or using the servo actuators in robots which can offer the high performance and precision[1-3]. MLPM (Magnetically Levitated Planar Motor) in advanced precise instruments [4-6], they can produce little motion damping when the mover is moving in any direction, and there is a vacuum between mover and stator or the air gaps [7]. Conventional PID control strategy can’t satisfy necessary of the high performance when researchers want to achieve the more precise trajectory tracking such as MLPM or naval gun control precisely which always require nanoscale tracking accuracy to defend its security[8, 9]. Professor Han [9] proposed the advanced strategy, namely ADRC (Active Disturbance Rejection Controller), has widely applied into some high precise field so as to avoid some typical engineering matter including overshoot or bad robustness causing of the typical control technology and the purpose they were taking is to integrate the new strategy and combine the advantages of typical PID control to realize the better controlling effects and higher performance [8, 9, 10], etc.

In this paper, the trajectory tracking for undamped plant is researched. In Section 2, the nominal model of the undamped plant is given and its several features are analyzed, the ADRC strategy is denoted by equations to realize the trajectory tracking for direct-drive servosystem with undamped plant. In Section 3, both the sinusoidal wave and the multi-step signal trajectory tracking are arranged for servo
system in order to verify the effectiveness of proposed control strategy. Section 4 concludes the full paper.

2. Model of Undamped Plant and Control Structure Design

2.1 Determine the model of undamped controlled Servo system under investigation, its mechanical parameters as the control object, is an automatic control system. For the sake of that, the object in this paper is described as:

\[
J \frac{d^2 q(t)}{dt^2} = \tau_n
\]

where \(J\) is the whole rotary inertia of motor, \(q\) represents the angular position, \(\tau_n\) denotes the controlled electromagnetic torque. By the Laplace transformation, the equation (1) is written as

\[
G(s) = \frac{Q(s)}{\tau_n(s)} = 1/(J \cdot s^2)
\]

On the basis of aforesaid analyses, some objects with the similar characteristic as above like levitated degrees of freedom (DOF) in MLPM, has been also studied. Owing to the gravitational effect in this direction, so the levitated force/torque \(z\) is showing a different features from other 5-DOF. The dynamical model in this direction after gravity compensation is denoted as:

\[
m \frac{d^2 z(t)}{dt^2} + Kz(t) = K_0 f_i(t)
\]

where \(m\) is weight of the mover in MLPM, both \(K\) and \(K_0\) are the relative factor dependent upon the levitated balance point \(z_0\) (\(z_0 = 1.0mm\)). The form of the Laplace transformation [10] in equation (2) is deduced as follow.

\[
G(s) = \frac{Z(s)}{F_i(s)} = \frac{K_0}{ms^2 + K}
\]

This levitated DOF is just a nominal model for MLPM. In light of this way, the model of direct-drive servo system without damping factor \(\zeta\) as shown above is simplified so as to verify the validity of this strategy and obtain the better performance.

2.2 Active Disturbance Rejection Controller Strategy

The basic principles of ADRC is consisted by NTD (Nonlinear Trace Derivator) which is a transient process to avoid the overshoot and improve the robustness of control system, NFC (Nonlinear Feedback Combiner) and ESO (Extended States Observer) as shown in figure 1. NFC can calculate the initial control law as denoted in equation (4) which can combine states \(z_3\) of ESO to obtain the control law namely \(u_t\). The discrete form of NTD is represented in equation (5).

\[
u_0 = K_p \cdot fal(\epsilon_1, a_1, \rho) + K_d \cdot fal(\epsilon_2, a_1, \rho)
\]

\[
\begin{align*}
\epsilon_1 &= v_i - z_i \\
\epsilon_2 &= v_2 - z_2
\end{align*}
\]

\[
\begin{align*}
\rho, a_2 &> 1 \\
\mu, \lambda &> 1 \\
r_1(k + 1) &= r_1(k) + h(r_1(k)) \\
r_2(k + 1) &= r_2(k) + h \cdot fst(r_1(k) - r(k), r_2(k), \mu, \lambda)
\end{align*}
\]

where \(fst(\cdot)\) is the steepest control synthesis function, \(\mu\) is the velocity factor, \(\lambda\) is the filtering factor acting on the noise with a certain frequency. \(\epsilon_i (i = 1, 2)\) is error between \(v_i (i = 1, 2)\) and \(z_j (j = 1, 2)\). \(a_i (i = 1, 2)\) is the exponential power parameter, \(\rho (\rho = 0.001)\) is the nonlinear coefficient. And the
working mechanism of ESO is indicated by equation (6) as follow:

\[
\begin{align*}
\varepsilon &= z_1 - y \\
z_1 &= z_1 + h \ast (z_2 - \beta_1 \cdot \varepsilon) \\
z_2 &= z_2 + h \ast (z_3 - \beta_2 \cdot \text{fal}(\varepsilon, \alpha, \delta) + b \cdot ut) \\
z_3 &= z_3 + h \ast (\beta_2 \cdot \text{fal}(\varepsilon, \alpha, \delta))
\end{align*}
\]

where \(z_i (i = 1, 2, 3)\) is the states of ESO which estimates the position, velocity and uncertain part of plant, namely \(z_i (i = 1, 2, 3)\) approaching to \(x_i (i = 1, 2)\) , \(z_3\) approaching to \(f(x_1, x_2)\) which is the uncertain part given as in equation (7). \(\text{fal}()\) is the saturation function to avoid the high frequency oscillation. \(\delta\) denotes the linear section of the interval length. \(\varepsilon\) is the error. The ADRC strategy for trajectory tracking of direct-drive servo system with undamped plant can be configured as in figure 1.

\[
\begin{align*}
\text{Input Signal} & \rightarrow \text{Transient Profile Generator} \\
& \rightarrow \text{Nonlinear Feedback Combiner} \\
& \rightarrow \text{Control Law} \\
& \rightarrow \text{Plant} \\
& \rightarrow \text{Velocity} \\
& \rightarrow \text{Placement y(k)}
\end{align*}
\]

\(f_l(a) = \begin{cases} g & |\varepsilon| \geq \delta \\ \varepsilon & \varepsilon \leq \delta \end{cases} \)

\(f(a)\) is the saturation function to avoid the high frequency oscillation. \(\delta\) denotes the linear section of the interval length. \(\varepsilon\) is the error. The ADRC strategy for trajectory tracking of direct-drive servo system with undamped plant can be configured as in figure 1.

3. Validation of Trajectory Tracking

As shown in equation (3), the state-space description of two order system is denoted as shown in equation (7). When \(z_0\) is setting 1.0mm, then \(K = 39.5\) and \(K_0 = 0.7425\) by calculating, respectively.

\[
\begin{align*}
\dot{x}_1(t) &= x_2(t) \\
x_2(t) &= f(x_1, x_2) + \frac{K}{m} f_x, \quad f(x_1, x_2) = -\frac{K}{m} x_1 \\
y &= x_1(t)
\end{align*}
\]

These parameters as in equation (7) has been paraphrased. Besides, \(x_1(t)\) is the placement of the mover in MLPM and its speed of the mover is represented by \(x_2(t)\).

3.1 Trajectory tracking of sinusoidal waves

Both sinusoidal waves’ angle frequency and its amplitude are set as 0.2 \(\pi\) rad/s and 1mm, respectively. Figure 2 shows the control strategy applied in this servo system works well. The effects of trajectory tracking is up to the expectation. And the tracking error of both is lower and approaches to 0.1mm and 0.1mm/s from figure 2 (c) and (d), respectively. Figure 3 (a) shows the tracking effects of both practical estimated position and real placement by ESO method. Figure 3 (b) shows the velocity estimation by state \(z_2\) with the output of real speed. Figure 3 (c) shows the estimation for unknown part and external disturbance of controlled object.
Figure 2. Tracking the sine wave by ADRC strategy: (a) shows ideal position, transient process $v_1$ and real position output; (b) shows the trajectory tracking effect of ideal speed and real output speed; (c) shows the position tracking error; (d) is the speed tracking error.

Figure 3. Tracking the sine wave by ESO in ADRC strategy: (a) is the states $z_1$ and real position output; (b) is the states $z_2$ and difference of position tracking; (c) the states $z_3$ for estimating the unknown part by ESO.

On the basis of aforesaid shown in figure 2, the actual trajectory is gradually approaching to the ideal given signal. In the figure 2 (d), the local subplot shows that there is a light shake in the initial time owing to the big initial error or some unknown reasons. It can be observed from figure 3, these states of ESO have a better performance of trajectory tracking though there is a little flaws in figure 3 (b) which estimates the velocity state variable approaching the real speed with the increase of the time.

3.2 Trajectory tracking of multi-step waves
Multi-step waves’ frequency is set as $0.2 \pi \text{ rad/s}$ which is shown as in figure 4 (a). Figure 4 (a) also shows the transient process for tracking the multi-step trajectory. On the basis of aforesaid analysis in section two, the TD should be designed in ADRC strategy to realize the transient process for multi-step signal which is of catastrophe characteristic. Trajectory tracking error is shown as in figure 4 (c), and the speed tracking and its error is shown in figure 4 (b) and (d) which has some picture-in-picture (PiP) so as to indicate the details that is necessary to show.
Figure 4. Tracking the multi-step wave by ADRC strategy: (a) shows ideal position, transient process $v_1$ and real position output; (b) shows the trajectory tracking effect of ideal speed and real output speed; (c) shows the position tracking error; (d) is the speed tracking error.

Figure 5. Tracking the multi-step wave by ESO in ADRC strategy: (a) is the states $z_1$ and real position output; (b) is the states $z_2$ and difference of position tracking; (c) Estimating uncertain part by ESO.

According to the figure 5, these states of ESO is tracked well by the output of servo system. Figure 5 (c) indicates the uncertain part and external disturbance is estimated by the third state $z_3$ in ESO and its precision is considerably effective in trajectory tracking. Compared these two typical trajectory tracking of sinusoidal waves and multi-step waves, the smooth sine wave has better tracking performance which is faster than multi-step waves. Nevertheless, multi-step signal has abrupt features that will cause vibration in the rising point and its response as shown in figure 5 (a) which marked by blue line.

4. Conclusion
This paper mainly verifies the performance of the trajectory tracking for these direct-drive servo system using the active disturbance rejection control strategy by sine wave and multi-step wave, respectively. The nominal model exampled by the levitated freedom degree of MLPM and its several variables are given. Although conventional PID control strategy adopts the weight sum consisted by coefficient and error to design the control law, the overshoot and several ripples in initial stage or in the process can’t be guaranteed and there is no way to satisfy the precise instrument control. Taking these reality into comprehensive consideration, the advanced ADRC strategy is adopted to control the servo system and these controlling parameters is adjusted to realize the better tracking effects. Simulation results indicate that both the placement and speed tracking error are gradually converging to the value fluctuating around zero or approaching to zero except the abrupt influence by multi-step signal. In next research, the work...
platform will be setup and the ADRC control strategy will be continually verified by many experiments.

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