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Covid-19 PPE distribution planning with demand priorities and supply uncertainties

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A B S T R A C T

The recent Covid-19 outbreak put healthcare resources under enormous pressure. Governments and healthcare authorities faced major challenges in securing and delivering critical supplies such as personal protective equipment (PPE) and test kits. As timely distribution of critical supplies exceeded government resources, certain sectors, negatively impacted by the pandemic, offered their storage and distribution capabilities; both helping with the crisis and creating economic revenue. We investigate the problem of optimally leveraging the capacity and efficiency of underutilized distribution networks to enhance the capability of government supply networks to meet healthcare needs for critical supplies. We model the problem as a dynamic distribution planning problem that decides on the re-purposing of storage facilities, the allocation of demand, and the timely distribution of limited PPE supplies to different jurisdictions. From a resource provider's perspective, the goal is to maximize demand fulfillment based on priorities set out by the government, as well as maximize economic value to participating networks. As uncertainty is a prevalent feature of the problem, we adopt a robust framework due to the lack of historical data on such supply uncertainties. We provide a mixed integer programming formulation for the adversarial problem and present a cutting plane algorithm to solve the robust model efficiently under both polyhedral and ellipsoidal uncertainty sets. We build a case study for the province of Ontario, Canada, and run extensive analysis of the service and economic value trade-off, and the effects of modeling demand priorities and supply uncertainties.

1. Introduction

During the Covid-19 pandemic, healthcare authorities faced major challenges in meeting demand for critical supplies such as personal protective equipment (PPE) and test kits. As inventories were depleted and demand rose, it was essential to make optimal use of these scarce and critical resources. While some distribution channels were overwhelmed with increased demand from online deliveries, other sectors, with established infrastructure for storage and distribution, were negatively impacted by the pandemic and were operational at reduced capacity. With ample capacity at hand, service providers were ready to put their resources at the disposal of the government to help fight the pandemic, and generate revenue for themselves.

The current work is motivated by an industrial collaboration aiming at leveraging spare capacity and efficiency of the automotive aftermarket distribution network to help deliver PPEs. Motivated by the fact that the automotive aftermarket industry was operating at about 50% of its pre-Covid levels, our partners asked for help to leverage such spare capacity in the distribution of PPEs. With quick and far-reaching distribution channels and idle capacity due to the pandemic, the automotive aftermarket industry has resources that the government could immediately deploy. The research will address the situation where logistics providers offer to share capacity with the government in return for income. On the one hand, this leverages the capacity and efficiency of such distribution networks to optimally enhance the capability of existing government supply networks to meet health care needs for critical supplies. On the other hand, the income generated helps alleviate an aggravated economic situation due to the pandemic.

The government is expected to welcome such settings as it achieves the dual goal of meeting urgent supply chain needs and of providing economic income for service providers, negatively impacted by the pandemic. As authorities are faced with the immediate challenge of meeting demand for essential supplies, solving this immediate problem and helping economic sectors survive the economic impact of the pandemic is ideal. For instance, the Government of Canada has recently signed an agreement with Amazon to make use of its distribution network for delivery of PPEs (Public Services and Procurement Canada, 2020). In U.S., FedEx partnered with the federal government to...
expedite the transport of PPEs and other critical medical supplies (Action News, 2020).

In classical supply chain design, the focus is typically on matching demand with supply and on optimizing the resources required. At times of crisis, however, resources are limited and time is pressing, which necessitates optimal and wise use of existing resources. The Covid-19 pandemic, despite its challenges, provides an opportunity to pool resources from different sectors to respond to an unfortunate situation. Many industries and services are ready to put their spare capacity at the disposal of authorities to help fight the pandemic.

From a practical perspective, the ultimate goal is to match supply, demand and capacity to meet critical demand needs for PPEs, test kits, and essential supplies. Supply chain and logistics network design were historically driven by many objectives depending on the circumstances: cost efficiency with make-to-stock strategies dominated the early work. Responsiveness with make-to-order and assemble-to-order strategies have helped respond to changing market conditions in sectors such as the computer and IT markets. Resiliency in the form of creating redundancy to mitigate the risk of disruptions has been studied under resilient or humanitarian supply chains. Environmental impact and carbon policies have been investigated under green and sustainable supply chains.

The current research focuses on a different set of objectives motivated by and tailored to the needs arising from the pandemic. The first is maximizing demand fulfillment under limited supply while accounting for priorities dictated by the severity for the pandemic at certain regions, such as hospitals, long term homes, and front-line workers. The second is the re-purposing of logistics capabilities from various sectors to enable immediate deployment of resources and to create income for sectors, greatly in need of a revenue boost. All this while taking into account uncertainty in supply, one of the major planning obstacles faced by governments in the early months of the pandemic.

The Contributions of the current work are three-fold.

- First we, present a distribution planning model where service providers put spare capacity, created due to an unfortunate situation, at the disposal of authorities to help fight the cause and to generate income. By doing so, the government can leverage spare capacity in existing supply chain, logistics and distribution networks that is created due to a pandemic to help fight it. This has the dual benefit of using resources and creating economic benefit to help alleviate the cause that led to the situation. It may include some objectives related to cost efficiency or responsiveness, but the bigger objective is to mitigate the immediate and future risk of an aggravated situation, in case of Covid-19, the propagation of the disease because of shortages in critical supplies.

- Second, we provide a dynamic distribution planning model to maximize profits for capacity providers and timely demand fulfillment health authorities. In addition to the multiple objectives, the model also incorporates demand priorities which may be set exogenously by the planner in function of pandemic characteristics or the demand source itself.

- Third, we address what is probably the most challenging hurdle faced by governments, namely significant uncertainties in supply because of the supply chain interruptions, lockdowns and closures of production facilities among others. To account for supply uncertainty and due to the lack of historical data, we propose a robust framework where supply is not exactly known but rather lies within a polyhedral uncertainty set. The latter restricts percentage deviation from nominal values using a set of budgeted uncertainty constraints. We model the budgeted uncertainty such that the level of conservatism set by the decision maker lies between 0 and 1 allowing for interpretable results. The resulting formulation, however, leads to a nonlinear adversarial problem and an exact robust counterpart does not exist. We present a fast cutting plane algorithm that is able to solve the real-life instances exactly within 40 s, on average.

- Fourth, we build a case study based on the Canadian province of Ontario, using real Covid-19 demand data. An extensive scenario analysis is carried out to derive managerial insights and a Python-based graphical user interface is developed to visualize results including network layout, open store locations, and service levels in various regions.

The paper is organized as follows. In Section 2, we review the literature on logistics and distribution planning during a pandemic. In Section 3, we present the dynamic distribution planning model with multiple objectives and demand priorities. In Section 4, we incorporate supply uncertainty into the model using a robust framework and develop a cutting plane algorithm the resulting robust optimization model. In Section 5, we build a realistic case study, perform numerical testing, and derive managerial insights. In Section 6, a graphical user interface is developed to visualize the solutions. Finally, in Section 7, we conclude and discuss future research directions.

2. Literature review

During a pandemic, the demand for PPE and other critical medical resources depends on disease progression over time. As a result, a periodic allocation and distribution decision model is often considered. A vast body of literature uses compartmental models to forecast disease progression in order to make optimal resource allocation decisions to minimize total infections. In this stream, a multi-period vaccine allocation and distribution problem for smallpox is studied by Ren et al. (2013) with the objective of minimizing deaths, which is modeled using a Susceptible–Exposed–Infected–Recovered (SEIR) compartmental model. A similar modeling framework is followed by Long et al. (2018) who consider optimizing location and timing of Ebola treatment units across different geographical regions to minimize the total number of infected cases. He and Liu (2015) use a modified SEIR model to forecast demand for medical relief in different geographical regions over time. The forecasts are then used within a dynamic logistics model to decide on optimal distribution of multiple relief resources from emergency reserve centers to epidemic areas. The objective is to minimize psychological fragility of affected people which is a function of priority score associated with each relief resource type. Rachaniotis et al. (2012) model the problem of assigning a mobile medical team to different regions as a deterministic resource scheduling problem with the objective of minimizing the number of infections over a planning period. The problem is later extended to include multiple resources by Rachaniotis et al. (2017). Yin et al. (2021) study a ventilator allocation problem to minimize the expected number of infected and deceased Covid-19 cases. They present a multi-stage stochastic compartmental disease model that accounts for uncertainty in testing among asymptomatic individuals and short-term migration between regions.

The previously mentioned papers focus on reducing health risks without accounting for economic benefits and follow a push-based approach to allocate and distribute resources. Our modeling approach is rather pull-based where allocation and distribution decisions are based on demand requests originating from affected regions. In addition, we account for health benefits through a weighted service level objective that attempts to allocate resources based on priority. A similar pull-based approach is proposed by Ekici et al. (2014) who model the food distribution problem during an influenza pandemic as an agent-based continuous-time stochastic disease spread model, which estimates the spread of infection over space and time and decides on facility location and resource allocation during the planning horizon. Meal demand is first estimated and then used as an input to the model. Similarly, Blackmon et al. (2021) develop a decision-support tool to implement the “Food Box Program” launched by the United States Department of Agriculture in which food items are directly purchased from farmers and delivered to people in need, in response to food insecurity caused by Covid-19 pandemic.
On the logistics planning side, Liu and Zhang (2016) develop a dynamic logistics model, with decisions made at three levels: plants, distribution centers, and hospitals, to optimize the allocation of resources under capacity constraints at hospitals and plants, while Liu and Zhao (2012) propose a heuristic approach for a dynamic logistics planning model for emergency response. The presented mathematical models minimize distribution costs under the assumption that the demand is fully met. However, during a pandemic, there is often an imbalance between supply and demand. Disruptions decrease supply while demand rises as the pandemic progresses. Hence, it may not be possible to meet the required demand. Instead, resources should be allocated on a priority basis. Our modeling approach takes this into account by setting demand priorities and maximizing a priority-weighted service level metric. Dasaklis et al. (2017), however, model the demand for vaccines as a function of disease progression using a Susceptible–Infected–Recovered (SIR) disease propagation model. They consider a lot-sizing problem for vaccine distribution under the objective of minimizing unmet demand while accounting for various capacity constraints such as storage, loading, and unloading capacities. Eisenhandler and Tzar (2019) study a humanitarian pickup and distribution problem where food has to be collected from suppliers to be delivered to food relief agencies in order to serve individuals in need. The modeling requires simultaneous vehicle routing and resource allocation decisions. The authors consider two objectives: maximizing total food distributed and balancing equity in distribution. Ivanov (2021) investigates the after-shock effects of the Covid-19 pandemic and uses discrete event simulation to identify supply chain disruption risks and to devise optimal post-pandemic recovery strategies. It is found that gradual ramping up of capacity is effective in responding to demand peaks. In a similar work, Ivanov (2020) proposes a new viable supply chain model focused on supply chain flexibility and its capability to react to positive and negative changes in the post-pandemic era.

During the pandemic and as resources were limited, new research problems surfaced. For example, as the US Federal Aviation Authority has granted approval to use of unmanned aerial vehicles to deliver PPEs and other medical items during Covid-19 pandemic in North Carolina, Gellich et al. (2021) present an optimization model to schedule drone trips to deliver PPEs, medicines, testing kits, and vaccines from urban facilities to rural areas. Similarity, and in order to meet varying Covid-19 testing requirements, Liu et al. (2021) propose a two-phase optimization model to optimize testing facility locations and distribution of test kits, and present a dynamic allocation policy that allows capacity to be adjusted dynamically as demand changes. Another example is related to increased demand and readmission of canceled surgeries caused by Covid-19 pandemic. Naderi et al. (2021) study the operating room planning and scheduling problem at Toronto General Hospital under this context and use mixed-integer and constrained programming models to optimally allocate Intensive Care Unit beds for scheduled surgeries and Covid-19 patients.

Maximizing health benefits and ensuring allocation equity are the main objectives from an ethical perspective. Achieving these objectives, however, depends on profit-driven distributors and manufacturers who must be incentivized to ensure timely and adequate delivery of critical resources. In this regard, Chick et al. (2008) study supply chain coordination issues for influenza vaccination where manufacturers attempt to maximize their profits while the government tries to balance cost and potential health benefits associated with vaccinations. The authors pose the problem as a game theoretic model to design a cost-sharing contract to minimize costs for both parties. Dai et al. (2016) study a game-theoretic contracting problem between retailers and manufacturers for vaccine distribution. Nagurney et al. (2021) propose a Nash Equilibrium framework to model competition for PPE supplies. In this work, we maximize profits for distributors alongside health benefits captured by a weighted service level metric. Although, not explicit in the objective function, the model implicitly maximizes on-time deliveries by incentivizing distributors. We model unit revenue earned by the distributor as a function of delivery time relative to delivery due date. The amount per unit paid by the government decreases in a step-wise fashion once the due date has passed, thus encouraging distributors to make fast deliveries to maximize their profits.

A key difference between our proposed modeling approach and the literature surveyed so far, is the incorporation of uncertainty and robustness. The Covid-19 pandemic disrupted supply chains across multiple sectors, especially the PPE supply chain (Queiroz et al., 2020). In normal situations, procurement decisions for PPEs are driven by cost. Such a cost-effective supply chain, however, fails to respond to sudden increase in demand, mainly due to disruptions in supply (Feinmann, 2020). This, along with measures such as border closure, lock down, restricted international trade, and labor shortages introduced major uncertainty in the PPE supply chain (Golan et al., 2020). Uncertainty could be handled via stochastic optimization (e.g. Bhuiyan et al., 2020; Gholami-Zanjani et al., 2021) or robust optimization (e.g. Rahmani, 2019; Ramezanian and Behboodi, 2017; Wang and Chen, 2020). In this work, we follow a robust optimization approach. We do not assume any distribution of parameters and, instead, seek a solution under worst-case scenario within an uncertainty set of possible scenarios. The literature on robust logistics planning is vast. Related to the pandemic, Jabbarzadeh et al. (2016) present a hybrid robust-stochastic optimization approach to design a supply chain that is resilient against facility disruptions. The modeling approach adds uncertainty to facility capacity which then effects the supply. During a pandemic, however, there is no disruption in facility capacity but rather in the overall supply chain which creates a lot of uncertainty in terms of quantity and timing of shipments. Zokaee et al. (2016) consider a robust three-level relief network model with uncertainty in demand, supply, and costs. The uncertainty is incorporated using a budgeted uncertainty set. The modeling only deals with strategic decisions and operational distribution planning is not considered. From a modeling perspective, our work closely relates to Mirzapour Al-E-Hashem et al. (2011) propose a multi-objective production planning problem with uncertainty and under two objectives, minimizing cost and the maximum shortage. To solve the resulting multi-objective formulation, the authors use a weighted-metric method similar to the one used in this paper where the model is solved with each objective separately and then a single objective is formulated that minimizes the weighted sum of deviations from optimal solutions for both objectives. de Mattos et al. (2019) propose a hierarchical robust optimization approach for demand and supply uncertainty in a distribution planning problem for insecticide-treated bed nets for malaria prevention. The uncertainty set presented restricts budgeted uncertainty constraints by a user-defined global robustness level. In addition to the global robustness level, we define the budgeted uncertainty constraint over the sum of the right-hand side of constraints as opposed to row-wise uncertainty. This results in a non-convex “max–max” adversarial problem for which an exact robust counterpart does not exist. We devise a cutting plane algorithm that iteratively adds constraints to the master problem based on worst-case scenarios generated from solving the adversarial problem. The proposed solution approach is fast and converges to an optimal solution in few iterations.

3. Distribution planning of personal protective equipment

The problem under study may be defined as follows. During the pandemic and due to lockdowns, a number of sectors with established distribution networks, were negatively impacted by the pandemic. These providers, aware of the challenges faced by the government and driven by the desire to help and to generate income, propose to put their distribution networks at the disposal of the government. The government on the other hand sees the opportunity in using resources they could immediately deploy and in helping certain sectors of the economy. Specifically, the government requires delivery capabilities to all regions as well as storage facilities at certain regions. Service
providers will essentially rent out local storage, re-purposed for storage of PPEs at a cost and their delivery capabilities to regions they serve. The government remunerates service providers for delivery of demand. The government ultimate objective is to fulfill as much as possible of the demand for PPEs. But with limited supply, only a fraction is possible. Hence priorities are given to highly affected demand sources, for example hospitals, hot spots, or outbreaks within a region.

To model the PPE distribution planning problem, let node 0 be the supply point and indices \( i \in I, j \in J, k \in K, m \in M, p \in P \) and \( t \in T \) to denote distribution centers, local stores, demand sources, types of demand, PPE types, time periods. The type of demand \( m \) for PPE \( p \) at demand source \( k \) reflects its priority and how critical it is to fulfill. In other words, the same source may have multiple demand requests with distinct priorities. For instance, a hospital may have demand requests originating from four different groups such as patients, physicians, nurses, and non-medical staff. PPEs are supplied from supply point 0, to distribution center \( i \in I \) at unit delivery cost \( c_{i}^{0} \) \( \forall i \). The total stock to be delivered by the supplier for PPE \( p \in P \) in period \( t \in T \) is \( A_{p} \). PPEs are delivered to local store \( j \in J \) with unit delivery cost of \( c_{j}^{i} \). PPEs may be stocked at local stores through proper certification by the government at cost \( L \). Each local store has an existing limited storage capacity \( C_{j} \) and daily delivery capacity \( D_{j} \). Delivery to demand source \( k \in K \) is made through local stores in \( J \). At each demand source \( k \), demand \( d_{k}^{m} \) for PPE \( p \in P \) in period \( s \in T \) originates from demand source type \( m \in M \) with unit delivery cost from local store \( j \) to be \( c_{j}^{m} \). In addition to delivery costs, we define \( r_{s}^{m} \) to be the unit revenue earned by the distributor for fulfilling the demand for PPE \( p \) at demand source \( k \) for type \( m \) in period \( s \). The unit revenue is a function of delivery time relative to delivery due date and decreases as the due date is exceeded which pushes timely deliveries to maximize their profits. The overall distribution network is illustrated in Fig. 1 and the modeling parameters and decision variables are summarized in Table 1.

![Fig. 1. The PPE distribution planning network.](image)

Table 1
Summary of modeling variables.

| Parameters | \( c_{i}^{0} \) | Unit delivery cost from government to distribution center \( i \) for PPE \( p \) |
|---|---|---|
| \( c_{j}^{i} \) | Unit delivery cost from distribution center \( i \) to local store \( j \) for PPE \( p \) |
| \( r_{s}^{m} \) | Revenue earned per unit delivery of PPE \( p \) to demand source type \( m \) for demand period \( s \) in period \( t \) |
| \( L \) | Local store certification cost |
| \( A_{p} \) | Stock available to government for PPE \( p \) in time period \( t \) |
| \( C_{j} \) | Inventory holding capacity at local store \( j \) |
| \( d_{k}^{m} \) | Demand for PPE \( p \) in period \( i \in T \) originating from demand source \( k \) and of type \( m \) |
| \( l_{0} \) | Delivery (or lead) time from government to distribution center \( i \) |
| \( l_{j} \) | Delivery (or lead) time from distribution center \( i \) to local store \( j \) |
| \( l_{k} \) | Delivery (or lead) time from local store \( j \) to demand source \( k \) |

| Decision variables | \( x_{ip}^{m} \) | \# units of PPE \( p \) delivered from node 0 to distribution center \( i \) during time period \( t \) |
|---|---|---|
| \( y_{ij}^{k} \) | \# units of PPE \( p \) delivered from distribution center \( i \) to local store \( j \) during time period \( t \) |
| \( z_{jkm}^{m} \) | \# units of PPE \( p \) delivered from local store \( j \) to demand source \( k \) of type \( m \) during time period \( t \) |
| \( a_{j} \) | Equals 1 if local store \( j \) is used |
| \( P_{i}^{m} \) | Stock level for PPE \( p \) in period \( t \) at distribution center \( i \) |
| \( P_{j}^{m} \) | Stock level for PPE \( p \) in period \( t \) at local store \( j \) |

Mathematically, the two objectives are defined as

\[ z_{c} = \sum_{j,k,m,s,t} r_{s}^{m} x_{ij}^{k} - \sum_{j,k,m} \beta_{j}^{m} d_{k}^{m} - \sum_{i,p} \epsilon_{i}^{m} g_{0i}^{m} \]
respectively. In other words, \( z \) yields pareto optimal solutions for 0 < \( w \) < 1. The advantage of the objective in [MO] is that it standardizes weights between 0 and 1, allowing an easier interpretation of the two objectives.

4. Robust PPE distribution planning

In the first few months of the pandemic, governments faced major shortages of PPE supply due to closures of production facilities, the interruption of the global supply chain, and a sudden and immense increase in demand (Tumilty, 2021). They rushed to put in measures like increasing local production by re-purposing manufacturing capabilities and putting in place additional distribution channels (Cochrane and Harris, 2020). Yet, all these measures did not effectively deal with the shortage and uncertainty in PPE supply. Supply did not arrive on time and in the expected quantities and decision makers had to make allocation and distribution decisions taking into account these uncertainties. The lack of historical data on such supply uncertainties due to the little knowledge of the pandemic and its progression makes robust optimization the best approach. We extend model [MO] to incorporate uncertainty in supply. For some PPE \( p \), government expects to receive a shipment \( A_p = \sum_{b \in T} \pi_{pb} \), where \( \pi_{pb} \) is the quantity expected to be received in period \( t \), see Fig. 2. Due to uncertainty, we may not receive the exact requested quantity. Let \( \delta_p \) be the actual number of units received in period \( t \) for PPE \( p \). The uncertainty is captured by \( \delta = [\delta_p] \) which is not exactly known but rather lies within an uncertainty set \( U \). As such, constraint (4) is changed to

\[
\sum_{i \in T} \delta_i \leq \delta_p \quad \forall \ p \in P, b \in T, \delta \in U.
\]  

We are interested in a solution that is feasible for all possible realizations of \( \delta \) in \( U \), i.e.,

\[
\max_{b \in U} \left( \max_{p \in P, b \in T} \left\{ \sum_{i \in T} x_{i0} - \delta_p \right\} \right) \leq 0,
\]
where the inner problem maximizes the infeasibility of original constraints (4). We define the uncertainty set as

\[
\delta' = \left\{ \delta : \sum_{p \in P} \frac{1}{L_p} \left| \frac{\delta_p}{\bar{\delta}_p} - 1 \right| \leq \beta_p \prod_{p \in P} \mu_p \right\},
\]

(13)

The first set of inequalities ensures that the total units received over the planning period is \(\bar{A}_p\) for each PPE \(p \in P\). We assume that the total ordered quantity \(A_p\) will be received during the planning period, but the government faces uncertainty in terms of when (during the planning period) and how much of the ordered quantity is expected to be received on a given day. If an uncertainty set is defined using the first set of constraints only, the worst-case scenario will always be that the government receives shipments on the last day of the planning period (i.e., supply \(\delta_p\) on day \(T\)).

**Fig. 2.** An illustration of expected supply over the planning horizon.

The resulting formulation is quadratic due to the bilinear term \(\delta_p \mu_p\) which may be linearized using McCormick relaxation (McCormick, 1976) by introducing variable \(\delta_{\mu}\) and the following set of constraints

\[
\begin{align*}
\Delta_{p,t} & \geq 0, & & \Delta_{p,t} \geq \delta_{\mu} + M_p \mu_p - M_{-\mu} - \delta_{\mu} - M_{-\mu} \mu_p, & & \Delta_{p,t} \leq \delta_{\mu}, & & \Delta_{p,t} \leq M_{-\mu} \mu_p, \\
\end{align*}
\]

(14)

The resulting program is linear due to the bilinear term \(\delta_{p,t} \mu_{p,t}\) which may be linearized using auxiliary variable \(\delta_{\mu,t}\). The adversary problem (14) is then formulated as a mixed-integer program

\[
\text{[AP]: max } \sum_{p \in P} \sum_{t \in T} x_{p,t}^0 \mu_p - \sum_{p \in P} \sum_{t \in T} \Delta_{p,t} \]

s.t.

\[
\begin{align*}
\Delta_{p,t} & \geq \delta_{\mu} + M_{\mu} \mu_p - M_{-\mu} - \delta_{\mu} - M_{-\mu} \mu_p, & & \forall p \in P, t \in T, \\
\Delta_{p,t} & \leq \delta_{\mu}, & & \forall p \in P, t \in T, \\
\Delta_{p,t} & \leq M_{\mu} \mu_p, & & \forall p \in P, t \in T, \\
\sum_{p \in P} \mu_p & = 1, \\
\Delta_{p,t} & \leq \bar{\delta}_p, & & \forall p \in P, \\
v_{p,t} & \leq \delta_{\mu} - \bar{\delta}_p, & & \forall p \in P, t \in T, \\
v_{p,t} & \geq \bar{\delta}_p - \delta_{\mu}, & & \forall p \in P, t \in T, \\
v_{p,t} & \leq 0, & & \forall p \in P, t \in T, \\
\sum_{t \in T} v_{p,t} & \leq \bar{\delta}_p, & & \forall p \in P, \\
\Delta_{p,t}, v_{p,t} & \geq 0, & & \forall p \in P, t \in T.
\end{align*}
\]

(16)

4.2. A cutting plane solution approach

The proposed uncertainty set leads to an adversarial problem with binary constraints. As such, dual formulation of model [AP] does not exist and an exact robust counterpart cannot be derived. One possible solution could be to relax the binary requirement and replace the left-hand side of constraint (12) by the relaxed dual formulation. This results in a conservative approximate robust counterpart. However, numerical experiments show that the resulting solutions are overly conservative. We therefore use an exact cutting plane algorithm to solve the robust problem where the deterministic model [MO] is the master problem and the adversarial problem is the subproblem. The algorithm dynamically adds cuts to the master problem until an optimal solution

\[
\begin{align*}
\text{[AP]: max } \sum_{p \in P} \sum_{t \in T} x_{p,t}^0 \mu_p - \sum_{p \in P} \sum_{t \in T} \Delta_{p,t} \\
\end{align*}
\]

s.t.

\[
\begin{align*}
\Delta_{p,t} & \geq \delta_{\mu} + M_{\mu} \mu_p - M_{-\mu} - \delta_{\mu} - M_{-\mu} \mu_p, & & \forall p \in P, t \in T, \\
\Delta_{p,t} & \leq \delta_{\mu}, & & \forall p \in P, t \in T, \\
\Delta_{p,t} & \leq M_{\mu} \mu_p, & & \forall p \in P, t \in T, \\
\sum_{p \in P} \mu_p & = 1, \\
\Delta_{p,t} & \leq \bar{\delta}_p, & & \forall p \in P, \\
v_{p,t} & \leq \delta_{\mu} - \bar{\delta}_p, & & \forall p \in P, t \in T, \\
v_{p,t} & \geq \bar{\delta}_p - \delta_{\mu}, & & \forall p \in P, t \in T, \\
v_{p,t} & \leq 0, & & \forall p \in P, t \in T, \\
\sum_{t \in T} v_{p,t} & \leq \bar{\delta}_p, & & \forall p \in P, \\
\Delta_{p,t}, v_{p,t} & \geq 0, & & \forall p \in P, t \in T.
\end{align*}
\]

(16)

where constraints (17)–(19) are McCormick relaxation constraints while constraints (22) and (23) defines the absolute term in the second and third inequalities of the uncertainty set denoted by constraints (24) and (25) in the adversarial problem. Under ellipsoidal uncertainty, constraint (25) is replaced by \(\sum_{t \in T} v_{p,t} \leq \bar{\delta}_p \bar{\delta}_p\).
to the robust problem is found. Let $x^k = [x^k_0]$ be the solution obtained from model [MO] at $k$th iteration. Given $x^k$, model [AP] is solved to determine $\delta^k$ that maximizes the infeasibility of the current solution $x^k$. Let $Z_{AP}$ be the optimal objective function value of model [AP]. When $Z_{AP} > 0$, there exists a solution $\delta \in U$ such that at least one of the constraints (11) is violated and the following cuts are added to model [MO]

$$\sum_{i \in I} x^k_i \leq \delta^k_p \quad \forall p \in P, t \in T \quad (27)$$

This procedure continues until $Z_{AP} = 0$, the current solution $x^k$ is feasible to all supply scenarios in uncertainty set $U$ and is therefore optimal to the robust problem. The overall iterative procedure is summarized in Algorithm 1.

5. Numerical testing

In this section, we report on numerical testing on a case study with the purpose of analyzing the trade-offs between profits and demand fulfillment goals, the impact of incorporating demand priorities and supply uncertainties, and the effects of modeling parameters. The case study is based on real PPE demand requests in the province of Ontario. The case study data is summarized in Section 5.1 and the base case scenario uses the profit maximization objective with no uncertainty in PPE supply. The analysis of base case scenario is given in Section 5.2 where we carry out extensive sensitivity analyses over model parameters including minimum service level requirement $a$, cost of opening a local store, $L$, and unit delivery cost. In Section 5.3, the bi-objective function that maximizes both profit and priority-weighted service level is used to study the trade-off between the two objectives under different priority scores. Finally, the effect of supply uncertainty on network design and the resulting distribution planning is examined in Section 5.4. The optimization models and cutting plane algorithm are coded in C++ Visual Studio 2012 and all instances are solved using CPLEX version 12.6.1 on Windows 10 (64 bit) with Intel core i5-4790 3.60 GHz processors and 8 GB of random-access memory (RAM). Computational performance of the deterministic and robust problems is discussed in Section 5.5.

5.1. The case of Ontario

To determine demand for PPE we used the COVID PPE HELP tool at https://www.covidppehelp.ca/. The tool serves as a marketplace where those in need of PPE list the requested quantity, location and due date. A snapshot of the available data is given in Table 2.

Fig. 3 shows customer requests for 4 PPE types over a four-month time period, where gowns and facemasks are the top two most requested PPEs. We consider customer requests originating within Ontario during May 2020. Fig. 4 plots total daily demand for PPEs originating in Ontario in the May 2020. We use facemasks, the PPE with highest demand, for the purpose of testing where the unit is boxes of 200 facemasks each. This results in a total of 71 demand requests for facemasks from 67 distinct demand sources with total demand equaling 50,069 as shown in Fig. 5. Out of the 67 demand sources, we randomly select 9 locations as potential local stores.

To generate supply schedules, we randomly select 7 days on which supply is received and we randomly generate supply $A_\mu$ from a uniform

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**Table 2** Covid PPE Help data.

| Category     | PPE                  | Quantity | Required date | Expiration date | Location     | Notes                        | Posted On | Customer Contact   |
|--------------|----------------------|----------|---------------|-----------------|--------------|-----------------------------|-----------|-------------------|
| Facemask     | Adult disposable     | 200      | 5/4/2020      | 5/1/2021        | Edmonton     | 5/1/2020                    | InfHome Inc |
| Facemask     | Adult disposable     | 100      | 7/6/2020      | 9/19/2020       | Red Deer     | Day care — $100             | 5/20/2020 | Wonderflow school house |
| Facemask     | Adult disposable     | 150      | 5/15/2020     | 7/1/2020        | Calgary      | Can pay up to 90 cents/mask | 4/30/2020 | Radiant physiotherapy |
| Hand sanitizer | Liquid => 1 L   | 5        | 5/10/2020     | 6/1/2020        | Calgary      | Reasonable                  | 4/30/2020 | Radiant physiotherapy |
| Booties      | Knee high           | 1        | 7/24/2021     | 7/25/2021       |              |                             | 4/27/2020 | NewCo              |

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**Algorithm 1** Pseudocode for the cutting plane algorithm

Require: $\beta_\mu, \tau_\mu$ ▷ User sets level of conservatism

Initialization
1: $Z^* \leftarrow -\infty$ ▷ adversarial problem objective value
2: Solve deterministic model [MO] to obtain $x^*$

Main Loop
3: \( \text{while } Z^* \neq 0 \text{ do} \)
4: Solve the adversarial problem [AP] given $x$ ▷ obtain solution $\delta^k$ and $Z_{AP}$
5: $Z^* \leftarrow Z_{AP}$
6: \( \text{if } Z^* > 0 \text{ then} \)
7: Add cuts (27) to model [MO]
8: Solve model [MO] to obtain $x^*$
9: \( \text{end if} \)
10: \( \text{end while} \)
Table 3
Summary of data used.

| Physical network characteristics | 
|---------------------------------|
| $|I||$ = 1 | #, Distribution Center |
| $|J||$ = 9 | #, Local stores |
| $|K||$ = 67 | #, Demand sources |
| $|P||$ = 1 | #, PPEs (facemask) |
| $|M||$ = 1 | #, Customer types |
| $|T||$ = 31 | #, Time periods |

| Cost parameters | 
|-----------------|
| $c^0_p(i) = \text{dist}(0,i) \times c$ | Unit delivery cost to distribution center $i \in I$ |
| $c^1_p(i) = \text{dist}(i,j) \times c$ | Unit delivery cost from distribution center $i \in I$ to local store $j \in J$ |
| $c^2_p(j) = \text{dist}(j,k) \times c$ | Unit delivery cost from local store $j \in J$ to demand source $k \in K$ |
| $\tau = 0.01$ | Distance to unit delivery cost conversion constant |
| $r^0_p = 0.01$ | Unit profit margin, a piece-wise function (see Fig. 6) |
| $L = 2000$ | Fixed cost for using a local store |

| Other parameters | 
|-----------------|
| $\alpha = 0.0$ | Minimum service level requirement |
| $C_j = 2000$ | Daily storage capacity at local stores |
| $D_j = 200$ | Daily delivery capacity at local stores |
| $\sum_{p,t} A_{pt} = 54,456$ | Total supplies in $T$ |
| $\sum_{k,m,p,s} d_{pskm} = 50,069$ | Total demand in $T$ |

Fig. 5. Daily demand and Supply for facemasks during May, 2020.

Fig. 6. Profit margins per unit $r^0_p(t-s)^+$ as a function of due date and delivery date.

distribution $\text{unif}(6000,10000)$ resulting in a total of 54,456 units as shown in the time series plot colored blue in Fig. 5.

Unit delivery cost between two locations $i$ and $j$ is calculated as $\text{dist}(i,j) \times \tau$ where $\text{dist}(i,j)$ is the distance between the locations and $\tau$ is a distance-to-cost conversion parameter. The non-trivial part is to estimate $r^0_p(t-s)^+$. For hypothetical testing, we assume that the distributor is paid $2.0$ for each unit delivery made within the due date, i.e., $r^0_m(t-s)^+ = 2.0$ when $t \leq s$. Fig. 6 plots $r^0_p(t-s)^+$ used for computational testing as a function of $(t-s)^+$ where $(t-s)^+ = 0$ if $t \leq s$.

In addition, the fixed cost associated with using a local store is assumed to be $L = $2000 which includes certification cost as well as any other costs incurred for repurposing the store to carry PPEs. We assume each local store has a storage capacity of 2000 units with delivery capacity of 200 units per day. A summary of the data used is given in Table 3.

5.2. Effects of modeling parameters

We analyze the deterministic setting with the objective of maximizing profit and carry out extensive sensitivity analysis over modeling parameters including minimum service level requirement $\alpha$, facility (or certification) costs, distance-to-cost conversion constant $\tau$, and daily delivery capacity, $D_j$. Our goal is to see how modeling parameters effect optimal network configuration, overall profits, percentage of demand sources served, demand fulfilled, and delivery dates. Table 4 reports the revenues, delivery and facility costs, demand fulfilled, open stores, average deviation from delivery due date, and number of demand sources served under three service level requirements. At 0% service level, the distributor is able to earn a profit of $31,770 by serving 56% of the total demand using 5 local stores. In total, 34 out of 67 demand sources are served. The weighted average deviation from delivery due date is −2.93 days, i.e., on average demand is fulfilled 2.93 days prior to the due date. However, as the service level requirement increases, we observe that profit decreases due to increasing the number of open local stores and serving non-profitable demand sources.
Next, we vary $\alpha$ between 0 and 1 with increments of 0.1. As service level requirement $\alpha$ increases, network profit decreases due to increasing number of open facilities resulting in higher fixed costs as shown in Fig. 7. With increasing $\alpha$, more local stores have to be used for PPE distribution to increase overall delivery capacity. Although a higher service level requirement increases the percentage of demand fulfilled, it also results in deliveries being made closer to the due date.

When varying facility cost $L$ between $0$ and $10,000$ and as $L$ increases, network profits decrease and fewer number of stores are certified and used for PPE distribution as illustrated in Fig. 8. As a result, the percentage of demand sources served and demand fulfilled is reduced due to the limited delivery capacity of the network.

The effect of varying store daily delivery capacity $D_j \in \{100, 200, 400, 800, 1200, 2400, 3600\}$ is illustrated in Fig. 9. As $D_j$ increases, network profits are increasing at a decreasing rate. When a single local store has higher delivery capacity, a smaller number of local stores is required to achieve the minimum service level requirement. Note that the percentage of demand sources served and demand fulfilled also increase with increasing delivery capacity. This is due to the fact that a smaller number of local stores results in reduction of network fixed costs so it becomes profitable to serve demand sources with lower profit margins.

Fig. 10 illustrates the effect of the distance-to-cost conversion constant $\tau$ in $[0.01, 0.05, 0.1]$ on optimal network configuration. As $\tau$ increases, unit delivery costs are increased and the number of profitable demand source decreases. As such, fewer number of demand sources
are served which results in reduced overall daily delivery capacity required and therefore a smaller number of stores are used. We also carried out sensitivity analysis over the storage capacity by varying $C_j$ between 0 and 6000. It turns out that storage capacity does not affect the optimal network configuration due to the possibility of direct deliveries from the distribution center to customers as well as just-in-time delivery where products are shipped to customers via local stores as soon as they are received.

### 5.3. Profit and service level trade-offs

We use the same parameter settings as in Table 3 except that initial inventory levels at distribution centers are set to 2500 units. In addition, we assign a demand source type with priority score $P_m$ to each requested demand as shown in Table 5. For the base case scenario, we set the same priority score to all demand types. To obtain $z^*_c$, we use profit maximization as the objective to solve model (1). Similarly, service level maximization is used as objective in model (1) to obtain $z^*_s$. Once $z^*_c$ and $z^*_s$ are obtained, we solve model (1).

We vary weight $w$ between 0 and 1 with an increment of 0.1 resulting in a total of 11 instances. Fig. 11 plots the effect of $w$ on network profits, service level, delivery date and open stores. The trade-off curve between profits and service level show that optimizing both objectives leads to more balanced solutions compared to optimizing either one and ignoring the other. Consider the case when profits are maximized where $z^*_c = $33,613 and service level $\alpha = 67\%$. Compromising on profits by $3.0\%$ to $32,605$ can improve service level by $6.8\%$ from $67\%$ to $72\%$. Similarly, a maximum service level of $99\%$ could be achieved with profits $z^*_c = $4484. Compromising on service level by $4\%$ from $99\%$ to $95\%$ increases profits by $142\%$ to $10,857$. The decision maker should consider a balance between the two conflicting objectives.

We consider two levels of supply: $75\%$ and $100\%$. At $100\%$, we have sufficient supplies to achieve a service level of $99\%$ while at $75\%$, the maximum service level that can be achieved is $82\%$. Fig. 12 illustrates the effect of supply on profit and service level trade-offs. Under limited supply, a maximum of $29\%$ increase in service level can achieved by compromising over $44\%$ of profits. However, when supply is sufficient, $32\%$ gain in service level results in profit loss of $90\%$.

Recall that priority score is an exogenous parameter set by the government to reflect the relative importance of satisfying a demand request. For example the priority score can be set based on how critical the demand is or based on the situation of the pandemic as captured by its reproduction number or the number of cases in a specific region. The service level objective function is a weighted function of percentage demand fulfilled and priority score and it is expected that varying priority scores should change the distribution plan. In order to explore how the demand is fulfilled for varying priority scores, we consider three types of priority scores: (1) same \{(1,1,1,1,1)\}; where each demand

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### Table 4

Summary results for profit maximizing setting.

| Performance measures | Service level requirement | $\alpha = 0\%$ | $\alpha = 75\%$ | $\alpha = 90\%$ |
|-----------------------|---------------------------|----------------|----------------|----------------|
| Value %               | Value %                   | Value %        | Value %        | Value %        |
| Revenue               | 55,912 176%               | 70,897 223%    | 81,870 258%    |                |
| Delivery costs        | 14,142 45%                | 31,346 99%     | 54,843 173%    |                |
| Facility costs        | 10,000 31%                | 14,000 44%     | 18,000 57%     |                |
| Profits               | 31,770 100%               | 25,551 100%    | 9,027 100%     |                |
| Demand fulfilled      | 28,306 57%                | 37,552 75%     | 45,062 90%     |                |
| Open local stores     | 5 56%                     | 7 78%          | 9 100%         |                |
| Avg. delivery date ($t-s$) | $-2.93$            | $-2.13$        | $-1.11$        |                |
| Demand sources served | 34 51%                    | 50 75%         | 58 87%         |                |

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### Table 5

Customer types and respective priorities for base case scenario.

| Type, $m$ | Demand | Percent | Priority score, $P_m$ |
|-----------|--------|---------|-----------------------|
| 1         | 3742   | 7%      | 1                     |
| 2         | 7540   | 15%     | 1                     |
| 3         | 7845   | 16%     | 1                     |
| 4         | 12,345 | 25%     | 1                     |
| 5         | 18,597 | 37%     | 1                     |

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Fig. 11. Effect of weight $w$ on profits, service level, and open facilities.

Fig. 12. Effect of supply on profit-service level trade off.
type is equally important, (2) increasing \{1,3,8,21,55\}; where nodes with lower demand are given a smaller priority score, and (3) decreasing \{55,21,8,3,1\}; where nodes with lower demand are given higher priority score. Fig. 13(a) illustrates the effect of priority score on service level achieved by each demand type when weight \(w = 0.5\), i.e., equal weight is given to service level and profit maximization objective. As expected, going from decreasing to increasing priority scores, there is a shift in demand fulfilled from lower indexed demand types to higher indexed ones. Demand type 1 achieves highest service level under decreasing priority while demand type 5 has highest percentage of demand fulfilled in increasing. The effect of priority scores is higher when there is limited supply. For instance, when supply level is 50\% and \(w = 0.5\), the service level for demand type 1 is 100\% under decreasing priority and 3\% under increasing priority. On the other hand, when supply level is 100\%, the change in service level is from 100\% to 20\%. When the objective is solely to maximize service level, i.e., \(w = 1.0\), priority scores do not have a noticeable impact on service levels for different demand types under 100\% supply as shown in Fig. 13(b). While we consider three simple priority score patterns, the analysis shows that our modeling is effective in capturing the impact of priority scores.

Decision makers can use the modeling to answer such questions as: is available distribution capacity sufficient to deal with changes in demand priorities due to an outbreak in specific region? How much should the government be willing to pay for additional distribution capacity in order to respond to potential high priority demand occurring later in the horizon? Similarly for other parameters, the significance of the analysis is to show that the modeling is comprehensive and does take into account several logistics features like capacities, locations, revenues, costs, and delivery dates. The modeling maybe used to identify under-served regions where the government should focus its efforts to secure additional distribution channels. It may also be used by the distributors to identify areas where increasing storage and delivery capacity maybe economically viable and at the same time contributes towards achieving government goals.

5.4. Effects of supply uncertainty

In this section, we assess the impact of incorporating supply uncertainty in PPE allocation and distribution planning. We solve model \([MO]\) under different values of weight \(w\) to obtain the profit-service level trade off curve at varying levels of uncertainty defined by parameters \(\beta_p\) and \(\theta_p\) in \( \mathcal{U} \). Recall that \(\beta_p\) restricts the percentage deviation from the nominal daily supply \(a_{pt}\) while \(\theta_p\) denotes the total percentage deviation. Parameter \(\beta_p\) is set at 6 levels between 0.1 and 1.0, and \(\theta_p\) takes five distinct values from 0.0 to 0.20. Fig. 14 plots the effect of uncertainty parameters on profits and service level for each objective. Under profit maximization, the profit and service level decrease with increasing uncertainty. For instance, the optimal profit equals $35,073 with weighted service level of 69\% when supply is assumed known. These numbers decrease to $2766 and 8.5\% for \(\theta_p = 0.05\) and \(\beta_p = 1.0\). This phenomenon is not observed under service level maximization as shown in Fig. 14(b). For \(\theta_p = 0.05\), increasing \(\beta_p\) from 0.2 to 1.0 increases profits from $13,504 to $16,444 at the expense of 2.1\% decrease in service level from 99.7\% to 97.6\%.

We also analyze the effect of uncertainty on profit-service level trade-off curve as illustrated in Fig. 15 where increased uncertainty not only shifts the trade-off curve downward but it also limits the feasible region. For instance, when \(\theta_p = 0.05\) and \(\beta_p = 0.1\), the distributor may achieve service levels of up to 100\% as shown in Fig. 15(a). However, when \(\theta_p = 0.15\) and \(\beta_p = 1.0\), the distributor is unable to achieve service levels of up to 100\% as shown in Fig. 15(b). These numbers decrease to $2766 and 8.5\% for \(\theta_p = 0.05\) and \(\beta_p = 1.0\).
level greater than 31%. Our analysis show that supply uncertainty not
only negatively impacts service level but it also leads to decreased
profits for distributors.

5.5. Computational efficiency

In this section, we evaluate the computational efficiency of the
proposed model [MO] and the cutting plane algorithm. We generate
a total of 27 instances by varying supply, priority scores, and weight
$w$. Supply is set at three levels at \{50\%, 75\%, 100\%\} of the base-case
supply. Priority score types decreasing, increasing, and same are used.
Weight $w$ is varied between 0 and 1 at increments of 0.2 resulting in
6 distinct values. The results are summarized in Table 6 where column
"Obj.Value" refers to the optimal objective function values of [MO].
The results show that the proposed model is computationally efficient
and off-shelf solvers are able to handle real-life instances. Cplex is able
to solve all instances within 1400 s.

When examining the effect of supply level and priority scores on
CPU time, the results show that the problem is easy to solve when
supply is either too low or too high. For supply level 75\%, the average
CPU time is 238.8 s which decreases to 18.0 s and 45.8 s for supply
levels 50\% and 100\%, respectively. Similar trends are seen for priority
scores.

The deterministic model [MO] requires solving a total of three
optimization problems while solving for its robust version using the
cutting plane algorithm would require solving a total of $(3 + 1) \times$
the number of iterations. Therefore, for the cutting plane algorithm we use
the objective of maximizing the weighted sum of profits and service
levels where seven distinct weight values are used. For each weight, we
solve the model for different uncertainty sets $\mathcal{U}$ by varying parameters
$\beta_p$ and $\theta_p$ at 6 and 5 distinct levels, respectively. This results in a total
of 210 instances. Results are summarized in Table 7 where column "\#.
iterations" denotes the total number of iterations between the master
problem and adversarial problem. Columns "AP" and "MP" refers to
the total time spent in solving the adversarial problem and model MO,
respectively. The results show that the proposed solution approach is
very fast and all instances are solved to optimality within 40 s. On
average, the algorithm makes 5 iterations with total time to solve
the adversarial problem is 0.09 s. Most of the computational effort is put
in solving the model [MO] with CPU time of 24 s, on average. We also
observe that CPU time is sensitive to uncertainty set parameters $\beta_p$
and $\theta_p$. This is because increasing the values for these parameters increases
the uncertainty set size which results in increased number of iterations.
Table 6
Effect of supply level & priority score on model \([\text{MO}]\) computational efficiency.

| Weight | Supply level 50% | Supply level 75% | Supply level 100% |
|--------|------------------|------------------|------------------|
| \(w\)  | Obj. Value | CPU time (s) | Gap (%) | Obj. Value | CPU time (s) | Gap (%) | Obj. Value | CPU time (s) | Gap (%) |
| 0      | 0.00% | 17.33 | 0.00% | 17.19 | 0.00% | 15.72 | 0.00% | 17.52 | 0.00% |
| 0.2    | 8.09% | 18.80 | 7.66% | 1344.03 | 7.14% | 193.90 | 7.16% | 16.62 | 0.00% |
| 0.4    | 9.16% | 18.39 | 7.88% | 17.77 | 7.16% | 16.62 | 7.16% | 16.62 | 0.00% |
| 0.6    | 8.41% | 18.09 | 6.82% | 17.83 | 5.90% | 16.30 | 5.90% | 16.30 | 0.00% |
| 0.8    | 5.81% | 17.56 | 5.11% | 17.92 | 4.08% | 16.42 | 4.08% | 16.42 | 0.00% |
| 1      | 0.00% | 17.56 | 0.00% | 18.14 | 0.00% | 16.19 | 0.00% | 16.19 | 0.00% |
| Average | 5.24% | 18.02 | 4.58% | 238.81 | 4.05% | 45.86 | 4.05% | 45.86 | 0.00% |

(a) Supply level effect for priority score type “Decreasing”

| Weight | Decreasing priority | Same priority | Increasing priority |
|--------|---------------------|--------------|---------------------|
| \(w\)  | Obj. Value | CPU time (s) | Gap (%) | Obj. Value | CPU time (s) | Gap (%) | Obj. Value | CPU time (s) | Gap (%) |
| 0      | 0.00% | 17.19 | 0.00% | 17.44 | 0.00% | 17.37 | 0.00% | 17.37 | 0.00% |
| 0.2    | 7.66% | 1344.03 | 7.75% | 274.03 | 4.37% | 17.94 | 4.37% | 17.94 | 0.00% |
| 0.4    | 7.88% | 17.77 | 10.76% | 17.95 | 4.64% | 17.59 | 4.64% | 17.59 | 0.00% |
| 0.6    | 6.82% | 17.83 | 7.77% | 17.98 | 4.45% | 17.92 | 4.45% | 17.92 | 0.00% |
| 0.8    | 5.11% | 17.92 | 4.44% | 17.86 | 3.61% | 18.12 | 3.61% | 18.12 | 0.00% |
| 1      | 0.00% | 18.14 | 0.00% | 102.26 | 0.00% | 18.11 | 0.00% | 18.11 | 0.00% |
| Average | 4.58% | 238.81 | 5.12% | 74.59 | 2.85% | 17.84 | 2.85% | 17.84 | 0.00% |

(b) Effect of priority score Under Supply level of 75%

Table 7
Computational performance under polyhedral uncertainty set.

| \(\sigma_p\) | \(\beta_p\) | \(\#\) | CPU time (in s) | Optimality gap (%) |
|-------------|-------------|-------|-----------------|-------------------|
| Iterations  | AP | MP | Total |
| 0.1 | 1 | 0.00 | 3.04 | 3.04 | 0.00 |
| 0.2 | 1 | 0.00 | 3.02 | 3.02 | 0.00 |
| 0.4 | 1 | 0.00 | 2.96 | 2.96 | 0.00 |
| 0.6 | 1 | 0.00 | 2.95 | 2.95 | 0.00 |
| 0.8 | 1 | 0.00 | 2.96 | 2.96 | 0.00 |
| 1 | 1 | 0.00 | 2.94 | 2.95 | 0.00 |

| 0.05 | 0.1 | 5 | 0.10 | 28.51 | 28.61 | 0.00 |
| 0.2 | 5 | 0.12 | 28.81 | 28.93 | 0.00 |
| 0.4 | 6 | 0.21 | 34.03 | 34.24 | 0.00 |
| 0.6 | 6 | 0.28 | 33.76 | 34.04 | 0.00 |
| 0.8 | 6 | 0.27 | 33.36 | 33.63 | 0.00 |
| 1 | 6 | 0.26 | 33.70 | 33.96 | 0.00 |

| 0.1 | 0.1 | 5 | 0.10 | 28.51 | 28.61 | 0.00 |
| 0.2 | 5 | 0.07 | 27.60 | 27.67 | 0.00 |
| 0.4 | 7 | 0.15 | 39.71 | 39.86 | 0.00 |
| 0.6 | 8 | 0.14 | 37.47 | 37.60 | 0.00 |
| 0.8 | 8 | 0.16 | 38.08 | 38.24 | 0.00 |
| 1 | 10 | 0.24 | 47.46 | 47.70 | 0.00 |

| 0.15 | 0.1 | 5 | 0.06 | 23.04 | 23.10 | 0.00 |
| 0.2 | 5 | 0.07 | 24.50 | 24.57 | 0.00 |
| 0.4 | 7 | 0.13 | 34.66 | 34.79 | 0.00 |
| 0.6 | 8 | 0.12 | 37.77 | 37.90 | 0.00 |
| 0.8 | 8 | 0.17 | 40.23 | 40.39 | 0.00 |
| 1 | 9 | 0.15 | 43.50 | 43.65 | 0.00 |

| 0.2 | 0.1 | 5 | 0.07 | 24.58 | 24.65 | 0.00 |
| 0.2 | 5 | 0.07 | 27.88 | 27.95 | 0.00 |
| 0.4 | 7 | 0.11 | 40.70 | 40.81 | 0.00 |
| 0.6 | 8 | 0.12 | 38.54 | 38.67 | 0.00 |
| 0.8 | 8 | 0.14 | 37.83 | 37.98 | 0.00 |
| 1 | 8 | 0.11 | 38.09 | 38.20 | 0.00 |
| Average | 5.53 | 0.12 | 28.08 | 28.20 | 0.00 |

Table 8
Computational performance under ellipsoidal uncertainty set.

To evaluate the effectiveness of the proposed cutting-plane algorithm for different types of uncertainty sets, we carried out computational experiments with an ellipsoidal uncertainty set where \(\sigma_p\) is varied between 0 and 0.2 at increments of 0.05. The results, summarized in Table 8, confirm the effectiveness of the proposed solution approach for both polyhedral and ellipsoidal uncertainty sets. Compared to polyhedral uncertainty, the average CPU time for ellipsoidal uncertainty has increased from 24.14 s to 28.20 s, which is due to the increased number of iterations from 5.14 to 5.53.

5.6. Managerial insights

The proposed framework is comprehensive in that it accounts for logistics resources such as capacities, deliveries, locations, revenues, costs and delivery due dates. It is also operational as it equips decision...
makers with a tool to allocate critical resources dynamically in face of future uncertainties. It can help answer such questions as: is available distribution capacity sufficient to deal with changes in demand priorities due to an outbreak in a specific region? How much should the government be willing to pay for additional distribution capacity in order to respond to critical needs? How should supply be allocated over the planning horizon to respond to potential future demand spikes? Along the same lines, the framework can be used to identify underserved regions where the government should focus its efforts to secure additional distribution channels. It may also be used by the distributors to identify areas where increasing storage and delivery capacity is economically viable and at the same time contributes towards achieving government goals. Numerical testing reveals that higher service levels require more facilities for PPE distribution and higher daily delivery capacity. Therefore, the government should provide incentives for distributors based on minimum service level requirements. In addition, the distributor’s profit and customer service levels are greatly affected by the uncertainty in supply, which calls for decisions to be robust, in particular delivery schedules and network designs that are robust against uncertainties and or disruptions in PPE supply.

6. A graphical user interface

For better display and interpretation of results by regulating authorities, we develop a Python-based graphical user interface to display the flow of PPEs, the local stores used, and the demand fulfilled. The results are visualized on a map using the “Folium” package, routes are displayed on maps using the “ors” (OpenRouteServices) package and “OpenRouteServices API”, and the graphical user interface is developed using the “PyQt5” package. Three main visualizations are provided. The first is an overview and provides a summary of demand sources along with distribution centers and potential local store locations. An illustration is provided in Fig. 16. Demand sources are represented by green circles and the size indicates the total PPE demand during the planning period. The maps are interactive, allowing the user to click on nodes for detailed information.

The second visualization is the distribution network and displays the demand allocation at different service level requirements $\alpha$. It allows the assessment of the effect of the service level on the logistics plan, which includes identifying the stores that are open and the percentage of demand that is satisfied under varying service requirements. It also illustrates the percentage demand fulfilled at any given time and the distribution plan for a particular day. An illustration is provided in Fig. 17.

The third visualization is the distribution routes and displays the daily delivery routes to demand sources. A work order based on delivery schedule can also be generated. An illustration is provided in Fig. 18.

7. Conclusions

We considered the problem of leveraging spare capacity in existing supply chain, logistics and distribution networks that is created due to a pandemic, to help authorities with the optimal storage and distribution of PPE supplies to demand sources. The goal was to create revenue for sectors negatively impacted by the pandemic, and to respond effectively to the rising demand. We provided a logistics network planning model that incorporates that dual objective and accounts for uncertainty through a robust framework where supply lies within a polyhedral uncertainty set. We proposed an efficient cutting plane algorithm to solve the resulting nonlinear adversarial problem, capable of solving real-life instances within an average of 40 s. To test the approach, we built a case study based on the Canadian province of Ontario, using real Covid-19 demand data. We derived managerial insights and provided a Python-based graphical user interface to visualize the results for potential use and interpretation by healthcare decision makers. We assessed the effect of key parameters such as service level, fixed and variable costs, and storage and delivery costs on the logistics network configuration, profits, demand fulfillment, number of local stores
utilized, and the average delivery date. We also analyzed the effect of supply and priority scores on service levels for different demand types. Our results indicate that priority scores play a key role in the PPE allocation. Under supply shortages, the priority scores should be carefully set to favor deliveries to the neediest, in order to help curb the spread of the virus. For future research, one possible extension is to incorporate PPE demand as a function of the number of Covid-19 cases which is then integrated within the optimization model. Another interesting future research direction is to include the minimization of infection risk in the objective, which may require explicit modeling of infection risk as a function of PPE demand satisfied.

Fig. 17. Distribution network for day 15 at 0% and 75% service level.
Fig. 18. Distribution network for day 15 at 0% service level.

**CRediT authorship contribution statement**

**Gohram Baloch:** Conceptualization, Methodology, Writing – review & editing, Coding and implementation. **Fatma Gzara:** Conceptualization, Methodology, Writing – review & editing, Supervision, Work initiation. **Samir Elhedhli:** Conceptualization, Methodology, Writing – review & editing, Supervision, Work initiation.

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