Prediction of subsidence deformation of subway project by improved markov chain and optimized wavelet neural network model

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Abstract. In view of the fact that the factors affecting the subsidence deformation of subway engineering are complex and the subsidence monitoring data are random, this paper constructs an optimized wavelet neural network model with markov chain improvement to improve the accuracy of the predicted data. Firstly, the wavelet neural network is optimized by adding momentum and adjusting learning rate adaptively. Finally, based on markov chain theory, the residual between the predicted value and the actual monitoring value of the optimized wavelet neural network is improved. Taking the settlement data of a settlement monitoring point of xi'an metro line 14 as the research object, the prediction accuracy of the model was compared and analyzed by C++ program. The results of the program show that the additional momentum and the adaptive learning rate are effective for the optimization of the wavelet neural network model, and the prediction accuracy of the optimized wavelet neural network model improved by the markov chain is higher than that of the single model, which is suitable for the prediction of the subway engineering settlement.

1. Introduction
With the rapid development of urbanization, metro has rapidly become the first choice for large and medium-sized cities to solve traffic problems at home and abroad due to its advantages of large volume, punctuality and environmental protection. During the construction of the subway, the excavation of the subway tunnel destroys the original geological stress balance, so the subway tunnel structure will be deformed \(^{[1]}\). In order to avoid the influence of deformation on the safety of subway construction and operation, it is necessary to carry out settlement monitoring on subway engineering, and establish a high-precision settlement prediction model based on the measured settlement data, so as to provide accurate data support for the prediction and prediction of subway settlement trend. Due to the complicated settlement mechanism of subway engineering, the nonlinear relationship between the influencing factors and the settlement amount is difficult to construct the influencing factor-settlement amount model. The wavelet neural network model with generalization ability is used in this paper. Wavelet neural network model to solve in the process of actual application shows that the slow convergence speed, easy to fall into local minimum and other shortcomings, in this paper, the additional momentum method and the adaptive adjustment vector method are introduced to optimize the wavelet neural network model to form the optimal wavelet neural network. According to the characteristics of the randomness of the subway settlement, the Markov theory is taken as the research object to study the random dynamic change process. The transfer probability matrix can reflect the random fluctuation of the data sequence, so it can be used to study the subway settlement trend \(^{[2]}\).
Based on the theory of optimized wavelet neural network and markov theory, this paper establishes a combined prediction model of markov chain modified optimized wavelet neural network for subsidence prediction, and tests the validity of the model and analyzes the accuracy of the prediction results by using the measured settlement data of a subsidence monitoring point of Xi’an metro line 14.

2. Optimize the wavelet neural network model

2.1. Topology of wavelet neural network

Wavelet Neural Network (WNN) is the product of combining Wavelet analysis theory and Neural Network algorithm, and belongs to feedforward Neural Network. From the structural form, the wavelet neural network can be divided into two types: loose (auxiliary) and compact (embedded): (1) Loose. The excellent denoising function of wavelet analysis is used to preprocess the data and then the processed data is used as the input data of the neural network model. (2) Compact. On the basis of the neural network topology, the excitation function of the hidden layer is replaced by the wavelet basis function with good time-frequency localization property, and the weight from the corresponding input layer to the hidden layer and the threshold value of the hidden layer are respectively replaced by the scaling factor and translation factor of the wavelet basis function. In this paper, a 3-layer compact wavelet neural network model is adopted to realize the approximation of arbitrary nonlinear functions, which is composed of the input layer, the single hidden layer and the output layer. The excitation function of the hidden layer selects the Morlet wavelet basis function.

The expression of Morlet wavelet basis function is:

$$h(x) = \cos(rx)e^{-\frac{(x)^2}{2}}$$  \hspace{1cm} (1)

In the formula, the parameter r is generally 5 or 1.75. In this paper, the value r=1.75.

![Figure 1 Topology of wavelet neural network](image)

2.2. Optimize the wavelet neural network

In order to improve the performance of the wavelet neural network, improve the prediction accuracy of the model and accelerate the convergence speed of the algorithm, the improved method in BP neural network, the additional momentum method and the adaptive learning rate method are introduced. \(^3\) The operation steps of optimizing the wavelet neural network are as follows:

(1) Build the model structure. From the perspective of practical problems, the number of nodes in the output layer is determined to be 1. For the reality that it is difficult to determine the number of nodes in the input layer m and the number of nodes in the hidden layer n of the wavelet neural network, this paper adopts C++ programming, sets the precision of $10^{-4}$ and iterates through the loop to get the optimal number of nodes in the input layer and hidden layer. Determine the number of nodes m, n and N in the input layer, hidden layer and output layer of the network topology.

(2) Initialization. Set parameters of wavelet neural network \(\{\omega_{jk}, \omega_{ij}, b_j, a_j, \eta, \alpha\}\) by random assignment method to determine the initial value.

(3) Data preprocessing. In order to eliminate the influence of singular values in the data on the prediction results, all data are normalized to the interval of [0,1], and the expression is:
Type: $x_i$ represents the original observation data; $x_{\text{min}}$ and $x_{\text{max}}$ respectively correspond to the minimum and maximum values in the original observation data. $X_{\text{normalization}}$ represents normalized data.

(4) Forward propagation process. The learning and training pairs are provided to the network. After the training sample data are transferred to the wavelet neural network, they are processed by the middle layer and output from the network output layer, and the output value is inversely normalized. Calculate the global error $E$. If the number of iterations exceeds the maximum number of iterations, but $E > \varepsilon$, then the number of hidden layer nodes is corrected, and $n=n+1$ is set to re-execute the forward propagation process. If $E < \varepsilon$, reserves parameter and exit the training mode, and in turn into the forecast link, given the sample of noise is bigger, avoid weight correction when shock, choose batch learning method [4], namely to head to add tail method [5].

35 period before data can be divided into 25 groups, each group of 12 period data, the first 11 period as the network input samples, after phase 1 as the output. The inverse normalization formula is:

$$X_{\text{normalization}} = \frac{x_i - x_{\text{min}}}{x_{\text{max}} - x_{\text{min}}}$$  \hspace{1cm} (2)

The expression of output layer $y_i$ is:

$$y_i = \sum_{j=1}^{n} \omega_{ij} h \left( \frac{\sum_{k=1}^{m} \omega_{jk} x_k - b_j}{a_j} \right)$$  \hspace{1cm} (4)

Make $X=\sum_{k=1}^{m} \omega_{jk} x_k , A=\frac{x-b_j}{a_j} , B= h(A)'$. Then the above equation can be simplified as:

$$y_i = \sum_{j=1}^{n} \omega_{ij} h(A)$$  \hspace{1cm} (5)

The expression of the total error is:

$$E = \frac{1}{p} \sum_{p=1}^{P} (y_i^p - y_i)^2$$  \hspace{1cm} (6)

Type: P as the training sample, $\bar{y}_i$ expectations, $y_i$ output value for the network.

(5) Error back propagation process. If $E > \varepsilon$ and number of iterations is than the largest number of iterations, is the parameter set $R\{\omega_{jk}, \omega_{ij}, b_j, a_j, \eta\}$ adjustment and correction, the gradient correction method [6]. Then go back to step 4.

Formula for parameter set modification:

$$\omega_{ij}(t+1) = \omega_{ij}(t) - \eta \frac{\partial E}{\partial \omega_{ij}(t)} + \alpha \Delta \omega_{ij}(t)$$  \hspace{1cm} (7)

$$\omega_{ij}(t+1) = \omega_{ij}(t) - \eta \frac{\partial E}{\partial \omega_{ij}(t)} + \alpha \Delta \omega_{ij}(t)$$  \hspace{1cm} (8)

$$\omega_{jk}(t+1) = \omega_{jk}(t) - \eta \frac{\partial E}{\partial \omega_{jk}(t)} + \alpha \Delta \omega_{jk}(t)$$  \hspace{1cm} (9)

$$b_j(t+1) = b_j(t) - \eta \frac{\partial E}{\partial b_j(t)} + \alpha \Delta b_j(t)$$  \hspace{1cm} (10)

$$a_j(t+1) = a_j(t) - \eta \frac{\partial E}{\partial a_j(t)} + \alpha \Delta a_j(t)$$  \hspace{1cm} (11)

$$\eta(t+1) = \begin{cases} 1.05\eta(t), E(t+1) < E(t) \\ 0.70\eta(t), E(t+1) \geq 1.04E(t) \\ \eta(t), \text{else} \end{cases}$$  \hspace{1cm} (12)

$$\alpha = \begin{cases} 0, E(t+1) > 1.04E(t) \\ 0.95, E(t+1) < E(t) \\ \alpha, \text{else} \end{cases}$$  \hspace{1cm} (13)

$$\frac{\partial E}{\partial \omega_{ij}(t)} = - \sum_{p=1}^{P} (\bar{y}_i^p - y_i^p) * h(A)$$  \hspace{1cm} (14)

$$\frac{\partial E}{\partial \omega_{jk}(t)} = - \sum_{p=1}^{P} (\bar{y}_i^p - y_i^p) * \omega_{ij} * B * x_k / a_j$$  \hspace{1cm} (15)

$$\frac{\partial E}{\partial b_j(t)} = \sum_{p=1}^{P} (\bar{y}_i^p - y_i^p) * \omega_{ij} * B / a_j$$  \hspace{1cm} (16)
\[ \frac{\partial E}{\partial a_j(t)} = \sum_{p=1}^{P} (y^P_1 - \bar{y}^P_1) * \omega_{ij} * B * A/a_j \]  

(17)

3. Overview of markov chain

Markov chain is a discrete stochastic process with markov properties. Markov prediction method is a method that applies the theory and method of markov chain in probability theory to study the change of random events and to analyze and predict the trend of future changes \[ ^7 \]. In the prediction of settlement deformation trend, the random process is the series of measured settlement amount. According to the prediction of the probability of the last period and the one-step transition probability matrix, the state and predicted value interval of the settlement amount of the current period can be calculated, and the predicted value of the current period can be obtained by taking the average value of the predicted value interval. The most important steps of using markov chain optimization are state partition and the calculation of transition probability matrix.

3.1. State classification

Calculate the relative error according to the measured and predicted values, and the formula is:

\[ \xi(t) = \frac{X(t) - \bar{X}(t)}{X(t)} * 100\% \quad (t = 1,2, \ldots n) \]  

(18)

Type of X(t) is the observed value, \( \bar{X}(t) \) is a wavelet neural network model prediction, n is number of observations.

According to the relative error, the golden section method \[ ^8 \] identify three state interval: \([0, \Psi_1(t)], [\Psi_1(t), \Psi^{-1}_1(t)], [\Psi^{-1}_1(t), 1] \), the \( \Psi \) value 0.618, \( \bar{X}(t) \) is a normalized after the average relative error.

After the above three state intervals are inversely normalized, the state interval set of relative errors can be obtained.

\[ E = (E_1, E_2, E_3) = ([\alpha \ b], [\beta \ c], [\gamma \ d]) \]  

(19)

3.2. Construction of transition probability matrix

In the actual prediction, the one-step transition probability matrix \[ ^9 \] is generally considered, and its construction method is as follows:

\[ P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix} \]  

(20)

In the above formula, \( P_{ij} = \frac{m_{ij}}{M_i} \) where \( M_i \) is the number of times the state \( E_i \) appears in the process, and \( m_{ij} \) is the number of times the state \( E_i \) moves to \( E_j \) after 1 step. The one-step transition matrix is the product of the initial state matrix and \( P \). The initial state \((0,0,0)\) means that we are currently in the second state. In that state, the value is 1. It's basically the first row of the matrix. The state corresponding to the maximum probability is the state corresponding to the predicted value.

4. Application analysis of engineering examples

Xi 'an subway line 14 runs from east to west, and the first phase of the project is heshao village to north passenger station. It is planned to be completed and open to traffic in June, 2021. In order to ensure the safety of the foundation pit and surrounding pipelines during the excavation of each subway station, observation points for the foundation pit settlement are specially set up. In this paper, DBC17-1, the pit settlement observation point of xuefu station, which is located in the northeast of the pit, has been monitored for multiple periods and is still being monitored. The accumulated settlement amount of 50 periods at this point is selected for analysis. The first 35 periods are taken as sample data, and the last 15 periods are taken as test and comparison data. The graph of DBC17-1 observation point is shown in figure 2.
The predicted values and errors of the wavelet neural network and the optimized wavelet neural network are shown in table 1. It can be seen that the prediction accuracy of the optimized wavelet neural network model is improved to some extent.

![Figure 2 DBC17-1 cumulative settlement line diagram](image)

Table 1 Prediction values and relative errors of wavelet neural network model and optimized wavelet neural network model

| Period | Measured value /mm | Wavelet neural network predicted value /mm | Relative error / (%) | Optimized wavelet neural network predicted value /mm | Relative error / (%) |
|--------|--------------------|------------------------------------------|---------------------|--------------------------------|---------------------|
| 36     | 1.08               | 1.1238                                   | -4.0556             | 1.1118                          | -2.9444             |
| 37     | 0.82               | 0.8309                                   | -1.3293             | 0.8115                          | 1.0366              |
| 38     | 0.50               | 0.5136                                   | -2.7200             | 0.4936                          | 1.2800              |
| 39     | 0.58               | 0.5705                                   | 1.6379              | 0.5805                          | -0.0862             |
| 40     | 0.45               | 0.4739                                   | -5.3111             | 0.4639                          | -3.0889             |
| 41     | 0.78               | 0.7535                                   | 3.3974              | 0.7735                          | 0.8333              |
| 42     | 1.21               | 1.1702                                   | 3.2893              | 1.2002                          | 0.8099              |
| 43     | 0.81               | 0.8186                                   | -1.0617             | 0.8186                          | -1.0617             |
| 44     | 0.69               | 0.7085                                   | -2.6812             | 0.6885                          | 0.2174              |
| 45     | 1.14               | 1.1443                                   | -0.3772             | 1.1443                          | -0.3772             |
| 46     | 1.07               | 1.0908                                   | -1.9439             | 1.0908                          | -1.9439             |
| 47     | 1.25               | 1.2161                                   | 2.7120              | 1.2161                          | 2.7120              |
| 48     | 0.96               | 1.002                                    | -4.3750             | 0.942                           | 1.8750              |
| 49     | 1.40               | 1.3383                                   | 4.4071              | 1.4183                          | -1.3071             |
| 50     | 1.49               | 1.4017                                   | 5.9262              | 1.4917                          | -0.1141             |

On the basis of the optimized wavelet neural network model, the first 10 phases (36-45) of the predicted value of the optimized wavelet neural network model were taken as the markov optimization samples. The mean error is -0.3382% and 0.4598 after normalization. The golden section method calculated the three state intervals of the 10-phase relative errors as [0.0.2842], [0.2842, 0.7440], [0.7440, 1].
[0.7440, 1.]. After inversely normalizing the three state intervals, the final state interval of relative error is obtained: E1=[-3.0889, -1.8473], E2=[-1.8473, 0.1616], E3=[0.1616, 1.2800]. The state distribution of the 10 relative errors is shown in table 2.

### Table 2 States of each relative error

| Monitoring periods | In state |
|--------------------|----------|
| 36                 | 1        |
| 37                 | 3        |
| 38                 | 3        |
| 39                 | 2        |
| 40                 | 1        |
| 41                 | 3        |
| 42                 | 3        |
| 43                 | 2        |
| 44                 | 3        |
| 45                 | 2        |

The fitting value of the relative error in phase 45 is located in the state E2 then its state vector 

\[ P_0 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \]

so the state vector of the fitting value of the relative error in phase 46 is:

\[ P_1 = P_0 P = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1/3 & 0 \\ 1/3 & 0 & 2/3 \\ 0 & 2/3 & 2/5 \end{bmatrix} \]

So that the relative error in phase 46 is in the state interval of E3, and the predicted value in phase 46 is calculated:

\[ \hat{y}(t) = \frac{1}{2} \left( \frac{x(t)}{1-\frac{1}{100}} + \frac{x(t)}{1-\frac{1}{100}} \right) = 1.0769 \]

Using the principle of rolling prediction method [10], the prediction value of the 35th phase is removed, the prediction value of the 36-46 phase is used, and the prediction value of the 47th phase is calculated according to the calculation method of the prediction value of the 46th phase. By analogy, the predicted values of the 48th, 49th and 50th phases were calculated. The results are shown in table 3.

In order to further evaluate and compare the prediction results of different prediction models, MAPE, MAE and MSE were used as the evaluation criteria of the model in this paper. The results are shown in table 4.

Table 3 Predicted value and relative error of markov chain improvement

| Predict periods | Measured value /mm | Wavelet neural network predicted value /mm | Relative error / (%) | Optimized wavelet neural network predicted value /mm | Relative error / (%) |
|-----------------|-------------------|------------------------------------------|---------------------|------------------------------------------|---------------------|
| 46              | 1.07              | 1.0908                                   | -1.9439             | 1.0769                                   | -0.6449%            |
| 47              | 1.25              | 1.2161                                   | 2.7120              | 1.2478                                   | 0.1760%             |
| 48              | 0.96              | 0.942                                    | 1.8750              | 0.9506                                   | 0.9792%             |
| 49              | 1.40              | 1.4183                                   | -1.3071             | 1.3985                                   | 0.1071%             |
| 50              | 1.49              | 1.4917                                   | -0.1141             | 1.4905                                   | -0.0336%            |
Table 4 Comparison of prediction accuracy of three models

| Model type                           | MAPE   | MAE   | MSE   |
|--------------------------------------|--------|-------|-------|
| Wavelet neural network model         | 0.2049 | 0.5814| 0.3852|
| Optimization of wavelet neural network model | 0.1124 | 0.5169| 0.3318|
| Markov chain improvement model       | 0.0956 | 0.1787| 0.2563|

5. Conclusion
In this paper, based on the basic principle of wavelet neural network, the additional momentum method and adaptive vector adjustment method is introduced into the gradient descent method, the wavelet neural network learning algorithm was improved, and markov chain to optimize wavelet neural network model of the residual value was improved. The improved wavelet neural network settlement prediction model of Markov chain is established, and the subway settlement is predicted. Through the comparison of calculation accuracy evaluation indexes, the improved wavelet neural network model of Markov chain has higher prediction accuracy, and the subway settlement has a good prediction effect with certain reference value.

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