Supplement of

STEMMUS-UEB v1.0.0: Integrated Modelling of Snowpack and Soil Mass and Energy Transfer with Three Levels of Soil Physical Process Complexities
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1. Overview of Coupled Soil-Snow Modelling Framework: STEMMUS-UEB

STEMMUS-UEB simulates water and energy fluxes between the land surface and the atmosphere accounting for the water and energy exchange across various interfaces, i.e., root-soil, soil-atmosphere, vegetation-atmosphere, soil-snow, snow-atmosphere. The model is specialized in solving the vadose zone physical process by interpreting it with multi-level complexity. It describes the vadose zone processes including soil water, vapor, dry air, and energy transfer, root water uptake, and freeze-thaw (STEMMUS-FT component). Moreover, snowpack processes, snow accumulation, melting, ablation, are implemented via the UEB module. Multiple processes are interactively represented in the model, reproducing the underlying physics of the soil-snow-atmosphere system. The interactive dynamics of water and energy across different interfaces are numerically solved by STEMMUS-UEB with the local meteorological forcing, boundary conditions, and soil/snow/vegetation properties. The operational time scale is flexible from minutes to daily, and further long term simulations. Currently, local scale simulation is resolved while it has the potential to conduct large scale simulations taking advantage of the remote sensing and reanalysis data. The conceptual coupling soil-snow-atmosphere framework is illustrated in Figure 1.1. An outline of the simulated physical processes and model structure is presented in Figure 1.2. The general development and application of soil and snowpack submodules are briefly introduced in Section 1.1 and 1.2.

Figure 1.1. The conceptual figure of coupled soil-snow-atmosphere modelling framework. The UEB module is adapted from Tarboton and Luce (1996). $\Delta T$, $\Delta h$, $\Delta P_a$ are the vertical gradient of soil temperature, matric potential, and air pressure, respectively.
1.1 Soil module

The detailed physically based two-phase flow soil model (Simultaneous Transfer of Energy, Momentum and Mass in Unsaturated Soil, STEMMUS) was first developed to investigate the underlying physics of soil water, vapor, and dry air transfer mechanisms and their interaction with the atmosphere (Zeng et al., 2011b, a; Zeng and Su, 2013). It is realized by simultaneously solving the balance equations of soil mass, energy, and dry air in a fully coupled way. The mediation effect of vegetation on such interaction was latterly incorporated via the root water uptake sub-module (Yu et al., 2016) and furthermore by coupling with the detailed soil and vegetation biogeochemical processes (Wang et al., 2020; Yu et al., 2020a). Implementing the freeze-thaw process (hereafter STEMMUS-FT, for applications in cold regions), it facilitates our understanding of the hydrothermal dynamics of respective components in frozen soil medium (i.e., soil liquid water, water vapor, dry air, and ice) (Yu et al., 2018; Yu et al., 2020b; see Section 2).

1.2 Snowpack module

The Utah energy balance (UEB) snowpack model (Tarboton and Luce, 1996) is a single-layer physically-based snow accumulation and melt model. The snowpack is characterized as the conservation of mass and energy using two primary state variables, snow water equivalent $W_{SWE}$ and the internal energy $U$ (see Section 3). Snowpack temperature is expressed diagnostically as the function of $W_{SWE}$ and $U$, together with the states of snowpack (i.e., solid, solid and liquid mixture, and liquid). Given the insulation effect of the snowpack, snow surface temperature differs from the snowpack bulk temperature, which is mathematically considered.
using the equilibrium method (i.e., balances energy fluxes at the snow surface). The age of the snow surface, as the auxiliary state variable, is utilized to calculate snow albedo (see Section 3.2.4). The melt outflow is calculated using Darcy’s law with the liquid fraction as inputs.

UEB is recognized as one simple yet physically-based snowmelt model, which can capture the first order snow process (e.g., diurnal variation of meltwater outflow rate, snow accumulation, and ablation, see a general overview of UEB model development and applications in Table S3). It requires little effort in parameter calibration and can be easily transportable and applicable to various locations (e.g., Gardiner et al., 1998; Schulz and de Jong, 2004; Watson et al., 2006; Sultana et al., 2014; Pimentel et al., 2015; Gichamo and Tarboton, 2019) especially for data scarce regions as for example Tibetan Plateau.

1.3 Structures

In the following sections, STEMMUS-FT module, including its governing equations, constitutive equations, underlying physics, and the difference among three level of model complexities, is first introduced in Section 2. The description of snowmelt module UEB is followed by in Section 3. Section 4 presents the coupling procedure of STEMMUS-UEB model and its structure, subroutines and input data. The following Section 5 shows the model capability in understanding the water and heat transfer mechanisms in frozen soils.

2 STEMMUS-FT model

The STEMMUS (Simultaneous Transfer of Energy, Momentum and Mass in Unsaturated Soil), detailed in (Zeng et al., 2011b, a; Zeng and Su, 2013), taking into account the soil Freeze-Thaw process (STEMMUS-FT, Yu et al., 2018) was developed. Three levels of complexity of mass and heat transfer physics are made available in the current STEMMUS-FT modelling framework (Yu et al., 2020b). First, the 1-D Richards equation and heat conduction were deployed in STEMMUS-FT to describe the isothermal water flow and heat flow (termed BCD). In the BCD model, the interaction of soil water and heat transfer is only implicitly via the parameterization of heat capacity, thermal conductivity and the water phase change effect. For the advanced coupled water and heat transfer (ACD model), the water flow is affected by soil temperature regimes. The movement of water vapor, as the linkage between soil water and heat flow, is explicitly characterized. STEMMUS-FT further enables the simulation of temporal dynamics of three water phases (liquid, vapor and ice), together with the soil dry air component (termed ACD-Air model).

In the following sections, we first present the governing equations, underlying physics, and constitutive equations of liquid water flow, vapor flow, air flow, and heat flow for the complete STEMMUS-FT (ACD-Air) model in Section 2.1 and 2.2. The description of BCD, ACD model and the different physics among three levels of model complexities are given in Section 2.3.
2.1 Governing Equations

2.1.1 Soil water transfer

\[
\frac{\partial}{\partial t}(\rho_L \theta_L + \rho_v \theta_v + \rho_i \theta_i) = -\frac{\partial}{\partial z}(q_{Lh} + q_{LT} + q_{La} + q_{Vh} + q_{VT} + q_{Va}) - S
\]

\[
= \rho_L \frac{\partial}{\partial z} \left[ K \left( \frac{\partial h}{\partial z} + 1 \right) + D_{TD} \frac{\partial T}{\partial z} + \frac{K}{\gamma_w} \frac{\partial P_g}{\partial z} \right] + \frac{\partial}{\partial z} \left[ D_{Vh} \frac{\partial h}{\partial z} + D_{VT} \frac{\partial T}{\partial z} + D_{Va} \frac{\partial P_g}{\partial z} \right] - S
\]

where \( \rho_L, \rho_v \) and \( \rho_i \) (kg m\(^{-3}\)) are the density of liquid water, water vapor and ice, respectively; \( \theta_L, \theta_v \) and \( \theta_i \) (m\(^3\) m\(^{-3}\)) are the volumetric water content (liquid, vapor and ice, respectively); \( z \) (m) is the vertical space coordinate (positive upwards); \( S \) (s\(^{-1}\)) is the sink term for the root water extraction. \( K \) (m s\(^{-1}\)) is hydraulic conductivity; \( h \) (m) is the pressure head; \( T \) (°C) is the soil temperature; and \( P_g \) (Pa) is the mixed pore-air pressure. \( \gamma_W \) (kg m\(^{-2}\) s\(^{-3}\)) is the specific weight of water. \( D_{TD} \) (kg m\(^{-1}\) s\(^{-1}\) °C\(^{-1}\)) is the thermal vapor diffusion coefficient. \( D_L \) (kg m\(^{-2}\) s\(^{-1}\)) is the isothermal vapor conductivity; and \( D_{VT} \) (kg m\(^{-1}\) s\(^{-1}\) °C\(^{-1}\)) is the thermal vapor diffusion coefficient. \( D_{la} \) is the advective vapor transfer coefficient (Zeng et al., 2011b, a). \( q_{Lh}, q_{LT}, \) and \( q_{La}, \) (kg m\(^{-2}\) s\(^{-1}\)) are the liquid water fluxes driven by the gradient of matric potential \( \frac{\partial h}{\partial z} \), temperature \( \frac{\partial T}{\partial z} \), and air pressure \( \frac{\partial P_g}{\partial z} \), respectively. \( q_{Vh}, q_{VT}, \) and \( q_{Va} \) (kg m\(^{-2}\) s\(^{-1}\)) are the water vapor fluxes driven by the gradient of matric potential \( \frac{\partial h}{\partial z} \), temperature \( \frac{\partial T}{\partial z} \), and air pressure \( \frac{\partial P_g}{\partial z} \), respectively.

2.1.2 Dry air transfer

\[
\frac{\partial}{\partial t} \left[ \varepsilon \rho_{da}(S_a + H_c S_i) \right] = \frac{\partial}{\partial z} \left[ D_a \frac{\partial \rho_{da}}{\partial z} + \frac{S_a K_g}{\mu_a} \frac{\partial P_g}{\partial z} - H_c \rho_{da} \frac{q_L}{\rho_L} + \theta_a D_{Vg} \frac{\partial \rho_{da}}{\partial z} \right]
\]

where \( \varepsilon \) is the porosity; \( \rho_{da} \) (kg m\(^{-3}\)) is the density of dry air; \( S_a (=1-S_i) \) is the degree of air saturation in the soil; \( S_i (=\theta_i/\varepsilon) \) is the degree of saturation in the soil; \( H_c \) is Henry’s constant; \( D_a \) (m\(^2\) s\(^{-1}\)) is the molecular diffusivity of water vapor in soil; \( K_g \) (m\(^2\)) is the intrinsic air permeability; \( \mu_a \) (kg m\(^{-2}\) s\(^{-1}\)) is the air viscosity; \( q_L \) (kg m\(^{-2}\) s\(^{-1}\)) is the liquid water flux; \( \theta_a (=\theta_L) \) is the volumetric fraction of dry air in the soil; and \( D_{Vg} \) (m\(^2\) s\(^{-1}\)) is the gas phase longitudinal dispersion coefficient (Zeng et al., 2011a, b).

2.1.3 Energy transfer

\[
\frac{\partial}{\partial t} \left[ \rho_s \theta_s C_s + \rho_L \theta_L C_L + \rho_v \theta_v C_v + \rho_{da} \theta_a C_a + \rho_i \theta_i C_i \right] (T - T_r) + \rho_v \theta_v L_0 - \rho_i \theta_i L_f = \frac{\partial}{\partial z} \left( \lambda_{eff} \frac{\partial T}{\partial z} \right) - \frac{\partial}{\partial z} [q_L C_L(T - T_r) + q_v (L_0 + C_v(T - T_r)) + q_{da} C_a(T - T_r)] - C_L S(T - T_r)
\]

where \( C_s, C_L, C_v, C_a \) and \( C_i \) (J kg\(^{-1}\) °C\(^{-1}\)) are the specific heat capacities of solids, liquid, water vapor, dry air and ice, respectively; \( \rho_s \) (kg m\(^{-3}\)) is the density of solids; \( \theta_i \) is the volumetric fraction of solids in the soil; \( T_r \) (°C) is the reference temperature; \( L_0 \) (J kg\(^{-1}\)) is the latent heat of vaporization of water at temperature \( T_r \); \( L_f \) (J kg\(^{-1}\)) is the latent heat of fusion; \( W \) (J kg\(^{-1}\)) is the differential heat of wetting (the amount of heat released when a small amount of free water is added to the soil matrix); and \( \lambda_{eff} \) (W m\(^{-1}\) °C\(^{-1}\)) is the effective thermal
conductivity of the soil; \( q_L, q_V \), and \( q_a \) (kg m\(^{-2}\) s\(^{-1}\)) are the liquid, vapor water flux and dry air flux.

### 2.1.4 Underlying physics and calculation procedure

#### 1) Underlying physics of STEMMUS-FT

When soil water starts freezing, soil liquid water, ice, vapor, and gas coexist in soil pores. A new thermodynamic equilibrium system will be reached and can be described by the Clausius Clapeyron equation (Fig. 2.1). In combination with soil freezing characteristic curve (SFCC), the storage variation of soil water can be partitioned into the variation of liquid water content \( \theta_L \) and ice content \( \theta_i \), and then vapor content \( \theta_V \).

![Figure 2.1. The underlying physics and calculation procedure of STEMMUS-FT expressed within one time step. \( n \) is the time at the beginning of the time step, \( n+1 \) is the time at the end. The variables with the superscript \((n+1)/2\) are the intermediate values.](image)

With regard to a unit volume of soil, the change of water mass storage with time can be attributed to the change of liquid/vapor fluxes and the root water uptake \( S \) (Eq. 2.1). The fluxes, in the right hand side of Eq. 2.1, can be generalized as the sum of liquid and vapor fluxes. The liquid water transfer is expressed by a general form of Darcy’s flow \( (-\rho_L K \frac{\partial h}{\partial z} + \rho_L g \frac{\partial \theta_L}{\partial z}) \). According to Kay and Groenevelt (1974), the other source of liquid flow is induced by the effect of the heat of wetting on the pressure field \( (-\rho_L D_T \frac{\partial P}{\partial z}) \).

The vapor flow is assumed to be induced in three ways: i) the diffusive transfer (Fick’s law), driven by a
vapor pressure gradient \((-D_v \frac{\partial p_v}{\partial z})\). ii) the dispersive transfer due to the longitudinal dispersivity (Fick’s law, 
\(-\theta_L D_v g \frac{\partial p_v}{\partial z}\)). iii) the advective transfer, as part of the bulk flow of air \((\rho_v \frac{q_a}{\rho_{da}})\). As the vapor density is a function of temperature \(T\) and matric potential \(h\) (Kelvin’s law, Eq. 2.18), the diffusive and dispersive vapor flux can be further partitioned into isothermal vapor flux, driven by the matric potential gradient \((D_{v h} \frac{\partial h}{\partial z})\), and the thermal vapor flux, driven by the temperature gradient \((D_{vT} \frac{\partial T}{\partial z})\). The advective vapor flux, driven by the air pressure gradient, can be expressed as \((D_{v a} \frac{\partial p_a}{\partial z})\) in Equation 2.1.

Dry air transfer in soil includes four components (Eq. 2.2): 1) the diffusive flux (Fick’s law) \(D_e \frac{\partial \rho_{da}}{\partial z}\), driven by dry air density gradient; 2) the advective flux (Darcy’s law, \(\rho_{da} \frac{s_a k_a \rho p_a}{\mu_a} \frac{\partial p_a}{\partial z}\), driven by the air pressure gradient; 3) the dispersive flux (Fick’s law, \((\theta_a D_{v g}) \frac{\partial \rho_{da}}{\partial z}\)); and 4) the advective flux due to the dissolved air (Henry’s law, \(H_c \frac{p_a}{\rho_L}\)). According to Dalton’s law of partial pressure, the mix soil air pressure \(P_a\) is the sum of the dry air pressure and water vapor pressure. Considering dry air as an ideal gas, the dry air density \(\rho_{da}\), can be expressed as the function of air pressure \(P_a\), water vapor density \(\rho_V\), thus the function of three state variables \((h, T, P_a)\) (see Eqs. 2.20 &2.21).

Heat transfer in soils includes conduction and convection. The conductive heat transfer contains contributions from liquid, solid, gas and ice \((\lambda_{eff} \frac{\partial T}{\partial z})\). The convective heat is transferred by liquid flux \(-C_L q_L(T - T_r)\), vapor flux \(-[L_0 q_V + C_V q_V(T - T_r)]\) and air flow \(q_a C_a(T - T_r)\). The heat storage in soil, the left hand side of Equation 2.3, includes the bulk volumetric heat content \((\rho_i \theta_i C_i + \rho_L \theta_L C_L + \rho_v \theta_v C_V + \rho_i \theta_i C_i)(T - T_r)\), the latent heat of vaporization \((\rho_v \theta_v L_0)\), the latent heat of freezing/thawing \((-\rho_i \theta_i L_f)\) and a source term associated with the exothermic process of wetting of a porous medium (integral heat of wetting) \((-\rho_i W \frac{\partial \theta_i}{\partial t})\).

2) Calculation procedure of STEMMUS-FT

The mutual dependence of soil temperature and water content makes frozen soils a complicated thermodynamic equilibrium system. The freezing effect explicitly considered in STEMMUS-FT includes three parts: i) the blocking effect on conductivities (see Eq. 2.11); ii) thermal effect on soil thermal capacity/conductivity (see Section 2.2.8); iii) the release/absorption of latent heat flux during water phase change. The calculation procedure of STEMMUS-FT can be summarized as Fig. 2.1.

Step 1. Partition of the soil mass storage

Firstly, applying the Clausius Clapeyron equation, soil temperature \(T\) at time step \(n\) was utilized to achieve the initial soil freezing water potential. Given the pre-freezing water matric potential \(h\) and liquid water matric potential \(h_L\), the SFCC and SWRC are applied to obtain pre-freezing water content \(\theta\) and liquid water content \(\theta_L\). Then the soil ice content \(\theta_i\) can be derived via total water conservation equation \(\theta\) and liquid water content \(\theta_L\). The volumetric fraction of soil vapor \(\theta_v\) in soil pores is the difference of soil porosity and the total water content.
Step 2. Solving the mass balance equation

Taking the soil mass storage variables and matric potentials as inputs, we can solve the mass balance equation successfully. Then a new matric potential can be achieved. Applying Darcy’s law with consideration of the blocking effect of soil ice on the hydraulic conductivity, we can get liquid water flux $q_L$. The liquid water matric potential can be updated by applying Clausius Clapeyron equation. Applying the Kelvin’s law (Eq. 2.18), we can update the vapor density $\rho_v$ at the end of time step. Then the dispersive and diffusive vapor flux are possible to be calculated according to Fick’s law. Another component of vapor flux is considered as part of the bulk flow of air, which is driven by the air pressure according to Darcy’s law.

Step 3. Solving the dry air balance equation

When considering soil dry air as an independent component in soil pores, the dry air balance equation is utilized, whose solution provides the new air pressure $P_g^{n+1}$. Applying Dalton’s law, air pressure can be partitioned into vapor pressure and dry air pressure. Given the updated vapor density, the dry air density can be expressed as the function of air pressure, and vapor density (Eqs. 2.20 & 2.21). Applying Fick’s law, we can calculate the diffusive and dispersive components of dry air flux. Applying Darcy’s law, the advective flux is derived from the air pressure. To maintain the mechanical and chemical equilibrium, a certain amount of air will dissolve into liquid, such effect is described by Henry’s law. Finally, we can achieve the dry air flux $q_a$ by the sum of the aforementioned effects.

Step 4. Solving the energy balance equation

Given the inputs, updated values of liquid water flux $q_{L}^{n+1}$, water vapor flux $q_{V}^{n+1}$, soil liquid water content $\theta_L^{n+1/2}$, vapor content $\theta_V^{n+1/2}$, ice content $\theta_i^{n+1/2}$, and dry air flux $q_a^{n+1}$, we can update the thermal parameters, calculate the latent heat of water phase change, then solve the energy balance equation. A successful estimate of soil temperature will be obtained, which can be used as inputs for the next time step.

2.2 Constitutive Equations

2.2.1 Unfrozen water content

As the fixed freezing point methods is not physically realistic, the freezing point depression theory was employed in deriving the soil freezing characteristic curve (SFCC) for estimating the unfrozen water content (Koopmans and Miller, 1966; Dall’Amico, 2010). In combination with Clapeyron equation and two soil water retention curve models, two different kinds of SFCC are given below.

Clapeyron + Van Genuchten (Van Genuchten, 1980)

$$\theta_{tot}(h) = \begin{cases} \theta_r + \frac{\theta_s - \theta_r}{1 + |\alpha h|^{n}} m, & h < 0 \\ \theta_s, & h \geq 0 \end{cases} \tag{2.4}$$

where $\alpha$ is related to the inverse air-entry pressure. $\theta_{tot}$, $\theta_s$, and $\theta_r$ are the total water content, saturated water content and the residual water content, respectively; $h$ (m) is the pre-freezing soil water potential; $m$ is the empirical parameter. The parameter $m$ is a measure of the pore-size distribution and can be expressed...
as \( m = 1-1/n \), which in turn can be determined by fitting van Genuchten’s analytical model (Van Genuchten, 1980).

The unfrozen water content was estimated by employing soil freezing characteristic curve (SFCC) (Dall’Amico, 2010)

\[
\theta_L(h, T) = \theta_r + \frac{\theta_s - \theta_r}{[1 + |a(h + h_{Fz})|]^m},
\]

(2.5)

where \( \theta_L \) is the liquid water content, \( L_f \) (J kg\(^{-1}\)) is the latent heat of fusion, \( g \) (m s\(^{-2}\)) is the gravity acceleration, \( T_0 \) (273.15 \(^{\circ}\)C) is the absolute temperature. \( h \) (m) is the pre-freezing pressure and \( \alpha, n, \) and \( m \) are the van Genuchten fitting parameters. \( h_{Fz} \) (m) is the soil freezing potential.

\[
h_{Fz} = \frac{L_f}{g T_0} (T - T_0) H(T - T_{CRIT}),
\]

(2.6)

where \( T \) (\(^{\circ}\)C) is the soil temperature. \( H \) is the Heaviside function, whose value is zero for negative argument and one for positive argument. \( T_{CRIT} \) (\(^{\circ}\)C) is the soil freezing temperature.

\[
T_{CRIT} = T_0 + \frac{\alpha HT_0}{L_f},
\]

(2.7)

**Clapeyron + Clapp and Hornberger** (Clapp and Hornberger, 1978)

\[
\theta_L(h, T) = \theta_s \left( \frac{L_f}{g \psi_s} \right)^{1/b},
\]

(2.8)

where \( \psi_s \) (m) is the air-entry pore water potential, \( b \) is the empirical Clapp and Hornberger parameter.

### 2.2.2 Hydraulic conductivity

According to the pore-size distribution model (Mualem, 1976), the unsaturated hydraulic conductivity using Clapp and Hornberger, van Genuchten method can be expressed as,

\[
K_{Lh} = K_s \left( \frac{\theta / \theta_s}{\theta / \theta_s} \right)^{3+2/\beta},
\]

(2.9)

\[
K_{Lh} = K_s S_e^l \left[ 1 - \left( 1 - S_e^{1/m} \right)^m \right]^2,
\]

(2.10a)

\[
S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r},
\]

(2.10b)

\[
m = 1 - 1/n,
\]

(2.10c)

where \( K_{Lh} \) and \( K_s \) (m s\(^{-1}\)) are the hydraulic conductivity and saturated hydraulic conductivity. \( \beta = 1/b \) is the empirical Clapp and Hornberger parameter. \( S_e \) is the effective saturation. \( l, n, \) and \( m \) are the van Genuchten fitting parameters.

The block effect of the ice presence in soil pores on the hydraulic conductivity is generally characterized by a correction coefficient, which is a function of ice content (Taylor and Luthin, 1978; Hansson et al., 2004),

\[
K_{FLh} = 10^{-EQ} K_{Lh},
\]

(2.11a)

\[
Q = (\rho_i \theta_i / \rho_s \theta_s),
\]

(2.11b)

where \( K_{FLh} \) (m s\(^{-1}\)) is the hydraulic conductivity in frozen soils, \( K_{Lh} \) (m s\(^{-1}\)) is the hydraulic conductivity in unfrozen soils at the same negative pressure or liquid moisture content, \( Q \) is the mass ratio of ice to total water, and \( E \) is the empirical constant that accounts for the reduction in permeability due to the formation of ice (Hansson et al., 2004).
2.2.3 Temperature dependence of matric potential and hydraulic conductivity

Soil matric potential and hydraulic conductivity are dependent on soil temperature in STEMMUS (Zeng and Su, 2013), which is related to soil water surface tension and viscous flow effects. The temperature dependence of matric potential can be expressed as

\[ h_{\text{Cor,T}} = h e^{-C_\psi(T-T_r)} \]  

(2.12)

where, \( h_{\text{Cor,T}} \) is the soil matric potential considering temperature effect; \( C_\psi \) is the temperature coefficient, assumed to be constant as 0.0068 °C⁻¹ (Milly, 1982); \( T_r \) is the reference temperature (20 °C).

Hydraulic conductivity, taken into account the temperature effect, can be written as

\[ K(\theta, T) = K_s K_r(\theta) K_T(T) \]  

(2.13)

where \( K_r(\theta) \) is the relative hydraulic conductivity, \( K_T(T) \) is the temperature coefficient of hydraulic conductivity, expressed as

\[ K_T(T) = \frac{\mu_w(T_r)}{\mu_w(T)} \]  

(2.14)

where \( \mu_w \) is the viscosity of water. The dynamic viscosity of water can be written as

\[ \mu_w(T) = \mu_{w0} \exp \left[ \frac{\mu_1}{R(T + 133.3)} \right] \]  

(2.15)

where \( \mu_{w0} \) is the water viscosity at reference temperature, \( \mu_1 = 4.7428 \text{ (kJ mol}^{-1}) \), \( R = 8.314472 \text{ (J mol}^{-1} \text{ °C}^{-1}) \), \( T \) is temperature in °C.

2.2.4 Gas conductivity

According to Darcy’s law, the gas conductivity can be expressed as

\[ K_g = K_{rg}(S_a) K_s \mu_w \]  

(2.16)

where \( \mu_g \) is gas viscosity, and the air viscosity; \( K_{rg} \) is the relative gas conductivity, which is a function of effective gas saturation and is defined by Van Genuchten-Mualem model,

\[ K_{rg} = (1 - S_a^{0.5})[1 - (1 - S_a^{1/2})^m]^{1/2} \]  

(2.17)

2.2.5 Gas phase density

The gas in the soil pores includes water vapor and dry air. The water vapor density, according to Kelvin’s law, is expressed as (Philip and Vries, 1957)

\[ \rho_V = \rho_{V0} H_r, \quad H_r = \exp \left( \frac{g h}{R_V T} \right) \]  

(2.18)

where \( \rho_{V0} \) is the density of saturated water vapor; \( H_r \) is the relative humidity; \( R_V (461.5 \text{ J kg}^{-1} \text{ K}^{-1}) \) is the specific gas constant for vapor; \( g \) is the gravitation acceleration; \( T \) is temperature.
The gradient of the water vapor density with respect to \( z \) can be expressed as
\[
\frac{\partial \rho_V}{\partial z} = \rho_{sv} \frac{\partial H_t}{\partial h} \bigg|_h + \rho_{sv} \frac{\partial H_t}{\partial \theta} \bigg|_T + H_r \frac{\partial \rho_V}{\partial T} \bigg|_{\theta}.
\] (2.19)

Assuming that the pore-air and pore-vapor could be considered as ideal gas, then soil dry air and vapor density can be given as
\[
\rho_{da} = \frac{P_m}{R_{da}} \rho_s, \quad \rho_V = \frac{P_m}{R_{V}} \rho_s.
\] (2.20)

where \( R_{da} (287.1 \text{ J kg}^{-1} \text{ K}^{-1}) \) is the specific gas constant for dry air; \( P_m \) and \( P_{v} \) (Pa) are the dry air pressure and vapor pressure. Following Dalton’s law of partial pressure, the mixed soil air pressure is the sum of the dry air pressure and the vapor pressure, i.e. \( P_g = P_{da} + P_{v} \). Thus, combining with Eq. 2.20, the soil dry air density can be derived as
\[
\rho_{da} = \frac{P_g}{R_{da}} - \frac{\rho_{v} R_{v}}{R_{da}},
\] (2.21)

The derivation of dry air density with respect to time and space are
\[
\frac{\partial \rho_{da}}{\partial t} = X_{aa} \frac{\partial \rho_d}{\partial t} + X_{at} \frac{\partial \theta}{\partial t} + X_{ah} \frac{\partial h}{\partial t},
\] (2.22)
\[
\frac{\partial \rho_{da}}{\partial z} = X_{aa} \frac{\partial \rho_d}{\partial z} + X_{at} \frac{\partial \theta}{\partial z} + X_{ah} \frac{\partial h}{\partial z},
\] (2.23)

where
\[
X_{aa} = \frac{1}{R_{da}},
\] (2.24)
\[
X_{at} = \left[ \frac{P_m}{R_{da}} \left( H_r \frac{\partial \rho_{v}}{\partial T} + \rho_{sv} \frac{\partial H_t}{\partial T} \right) \right],
\] (2.25)
\[
X_{ah} = - \frac{\partial \rho_{v}}{\partial h}.
\] (2.26)

### 2.2.6 Vapor diffusivity

The isothermal vapor diffusivity is followed the simple theory and expressed as
\[
D_{V,iso} = D_v \frac{\partial \rho_{v}}{\partial h} = D_{atm} \nu T \theta_a \frac{\partial \rho_{v}}{\partial h},
\] (2.27)

where \( \nu \) is set to 1, \( \tau = \theta_a^{2/3} \), and \( D_{atm} = 0.229(1 + \frac{T}{273})^{1.75} \) (m2 s-1).

The thermal vapor diffusivity is given by considering the enhancement factor as
\[
D_{V,Noniso} = D_v \frac{\partial \rho_{v}}{\partial \theta} = D_{atm} \eta \frac{\partial \rho_{v}}{\partial \theta},
\] (2.28)

where \( \eta \) is the thermal enhancement factor.

### 2.2.7 Gas dispersivity

According to Bear, the gas phase longitudinal dispersivity \( D_g \) is expressed as
\[
D_g = \alpha_{L, \text{gas}}, \quad i = \text{gas or liquid},
\] (2.29)

where \( q_i \) is the pore fluid flux in phase \( i \), and \( \alpha_{L,i} \) is the longitudinal dispersivity in phase \( i \), which can be related to the soil saturation as
\[
\alpha_{L,i} = \alpha_{L,\text{Sat}} \left[ 13.6 - 16 \times \frac{q_i}{\epsilon} + 3.4 \times \left( \frac{q_i}{\epsilon} \right)^5 \right].
\] (2.30)
Following Grifoll’s work, the saturation dispersivity can be set to 0.078 m in case of lacking dispersivity values.

2.2.8 Thermal properties

1) Heat capacity

The volumetric heat capacity is the average of the soil component capacity weighted by its fraction.

\[ C = \sum_{j=1}^{6} C_j \theta_j \]  

(2.31)

Where \( C_j \) and \( \theta_j \) are the volumetric heat capacity and volumetric fraction of the \( j \)th soil constituent (J cm\(^{-3} \) °C\(^{-1} \)). The components are (1) water, (2) air, (3) quartz particles, (4) other minerals, (5) organic matter, and (6) ice (see Table 2.1).

2) Thermal Conductivity

The method used to calculate the frozen soil heat conductivity can be divided into three categories: i) empirical method (e.g., Campbell method as used in Hansson et al., 2004), ii) Johansen method (Johansen, 1975), and iii) de Vries method (de Vries, 1963). Due to the necessity in the calibration of parameters, the empirical Campbell method is not easy to adapt and rarely employed in LSMs and thus not discussed in the current context. The other variations of Johansen method and de Vries method, in which the parameters are based on soil texture information, i.e., Farouki method (Farouki, 1981) and the simplified de Vries method (Tian et al., 2016), were further incorporated into STEMMUS-FT.

**Johansen method** (Johansen, 1975)

The soil thermal conductivity is the weighted function of soil dry and saturated thermal conductivity,

\[ \lambda_{\text{eff}} = K_e (\lambda_{\text{sat}} - \lambda_{\text{dry}}) + \lambda_{\text{dry}}, \]  

(2.32)

where the \( \lambda_{\text{sat}} \) (W m\(^{-1} \) °C\(^{-1} \)) is saturated thermal conductivity, \( \lambda_{\text{dry}} \) (W m\(^{-1} \) °C\(^{-1} \)) is the dry thermal conductivity, \( K_e \) is the Kersten number, which can be expressed as

\[ K_e = \begin{cases} \log (\theta/\theta_s) + 1.0, & \theta/\theta_s > 0.05 \\ 0.7 \log (\theta/\theta_s) + 1.0, & \theta/\theta_s > 0.1 \\ \theta/\theta_s, & \text{frozen soil} \end{cases} \]  

(2.33)

The saturated thermal conductivity \( \lambda_{\text{sat}} \) is the weighted value of its components (soil particles \( \lambda_{\text{soil}} \) and water \( \lambda_w \)),

\[ \lambda_{\text{sat}} = \lambda_{\text{soil}}^{1-\theta_s} \lambda_w^{\theta_s}, \]  

(2.34)

where the solid soil thermal conductivity \( \lambda_{\text{soil}} \) can be described as

\[ \lambda_{\text{soil}} = \lambda_{\text{qtz}}^{1-\theta_{\text{qtz}}} \lambda_o^{\theta_{\text{qtz}}}, \]  

(2.35)

where the \( \lambda_{\text{qtz}} \) and \( \lambda_o \) (W m\(^{-1} \) °C\(^{-1} \)) are the thermal conductivity of the quartz and other soil particles, \( \theta_{\text{qtz}} \) is the volumetric quartz fraction.

The dry soil thermal conductivity is a function of dry soil density \( \rho_d \),
\[ \lambda_{dry} = \frac{0.135 \rho_d + 64.7}{2700 - 0.947 \rho_d}, \]  
\[ \rho_d = (1 - \theta_s) \cdot 2700. \]  
\[ (2.36) \]

**Farouki method** (Farouki, 1981)

Similar to Johansen method, the weighted method between the saturated and dry thermal conductivities is utilized by Farouki method to estimate soil thermal conductivity. The difference between Farouki method and Johansen method is to express the dry thermal conductivity and solid soil thermal conductivity as the function of soil texture. Equation can be replaced with,

\[ \lambda_{soil} = 8.80(\%sand) + 2.92(\%clay) \]

where \%sand, \%clay are the volumetric fraction of sand and clay.

\[ (2.38) \]

**de Vries method** (de Vries, 1963)

\[ \lambda_{eff} = \left( \sum_{j=1}^{6} k_j \theta_j \lambda_j \right) \left( \sum_{j=1}^{6} k_j \theta_j \right)^{-1}, \]

where \( k_j \) is the weighting factor for each components; \( \theta_j \) the volumetric fraction of the \( j \)-th constituent; \( \lambda_j \) (W m\(^{-1}\) °C\(^{-1}\)) the thermal conductivity of the \( j \)-th constituent. The six components are: 1. water, 2. air, 3. quartz particles, 4. clay minerals, and 5. organic matter, 6, ice. (see Table 2.1).

\[ (2.39) \]

\[ k_j = \left( \frac{2}{3} \left[ 1 + \left( \frac{\lambda_j}{\lambda_1} - 1 \right) g_j \right] \right)^{-1} + \frac{1}{3} \left[ 1 + \left( \frac{\lambda_j}{\lambda_1} - 1 \right) (1 - 2 g_j) \right]^{-1}, \]

and \( g_j \) is the shape factor of the \( j \)-th constituent (see Table 2.1), of which the shape factor of the air \( g_2 \) can be determined as follows,

\[ g_2 = \left\{ \begin{array}{ll}
0.013 + \left( \frac{0.022}{\theta_{witting}} + \frac{0.299}{\theta_s} \right) \theta_L, & \theta_L < \theta_{witting} \\
0.035 + \frac{0.299}{\theta_s} \theta_L, & \theta_L \geq \theta_{witting}
\end{array} \right. \]

\[ (2.40) \]

**Table 2.1 Properties of Soil Constituents** (de Vries, 1963)

| Substance          | \( j \) | \( \lambda_j \) (mcal cm\(^{-1}\) s\(^{-1}\) °C\(^{-1}\)) | \( C_j \) (mcal cm\(^{-1}\) s\(^{-1}\) °C\(^{-1}\)) | \( \rho_j \) (g cm\(^{-3}\)) | \( g_j \) |
|--------------------|--------|---------------------------------|---------------------------------|-----------------|------|
| Water              | 1      | 1.37                            | 1                               | 1               | ...  |
| Air                | 2      | 0.06                            | 0.0003                          | 0.00125         | ...  |
| Quartz             | 3      | 21                              | 0.48                            | 2.66            | 0.125|
| Clay minerals      | 4      | 7                               | 0.48                            | 2.65            | 0.125|
| Organic matter     | 5      | 0.6                             | 0.6                             | 1.3             | 0.5  |
| Ice                | 6      | 5.2                             | 0.45                            | 0.92            | 0.125|

**Simplified de Vries model** (Tian et al., 2016)

Tian et al. (2016) proposed the simplified de Vries method as an alternative method of traditional de Vries method. In this method, the thermal conductivity of soil particles component can be directly estimated based on the relative contribution of measured soil constitutes.

\[ \lambda_{eff} = \frac{\theta_{w} \lambda_w + k_1 \theta_1 \lambda_1 + k_2 \theta_a \lambda_a + k_{min} \theta_{min} \lambda_{min}}{\theta_w + k_1 \theta_1 + k_2 \theta_a + k_{min} \theta_{min}}, \]

\[ (2.42) \]
where $k_{\text{min}}$, can be derived by Eq. 2.40, is the weighting factor of soil minerals, $\theta_{\text{min}}$ is the volumetric fraction of soil minerals, $\lambda_{\text{min}}$ (W m$^{-1}$ °C$^{-1}$) is the thermal conductivity of soil minerals, can be expressed as the weighted value of its components,

$$\lambda_{\text{min}} = f_{\text{sand}}\lambda_{\text{sand}} + f_{\text{silt}}\lambda_{\text{silt}} + f_{\text{clay}}\lambda_{\text{clay}}, \quad (2.43)$$

where $f_{\text{sand}}$, $f_{\text{silt}}$, and $f_{\text{clay}}$ are the volumetric fraction of soil sand, silt and clay, respectively. The shape factor of soil minerals is determined as the volumetrically weighted arithmetic mean of the constituent shape factors,

$$g_{\text{min}} = g_{a,\text{sand}}f_{\text{sand}} + g_{a,\text{silt}}f_{\text{silt}} + g_{a,\text{clay}}f_{\text{clay}}, \quad (2.44)$$

where $g_{a,\text{sand}}$, $g_{a,\text{silt}}$, $g_{a,\text{clay}}$ are the shape factors of soil sand, silt and clay, their values are 0.182, 0.0534 and 0.00775, respectively (Tarnawski and Wagner, 1992; Tarnawski and Wagner, 1993; Tian et al., 2016).

3) **Differential Heat of Wetting**

The differential heat of wetting, $W$ is the amount of heat released when a small amount of free water is added to the soil matrix and expressed by Edlefsen and Anderson (1943) as

$$W = -\rho L_c (\psi - T_T) = -0.01 g (h + T a h) = -0.01 g h (1 + T a), \quad (2.45)$$

where Prunty (2002) expressed the differential heat of wetting as

$$W = -0.293 h, \quad (2.46)$$

4) **Transport coefficient for adsorbed liquid flow**

The transport coefficient for adsorbed liquid flow due to temperature gradient is expressed as Groenevelt and Kay (1974)

$$D_{Ta} = \frac{H_w e}{b n_T} (1.5548 \times 10^{-15}), \quad (2.47)$$

where $H_w$ is the integral heat of wetting (J m$^{-2}$); $b = 4 \times 10^{-8}$ (m); $T$ is temperature in °C.

### 2.2.9 Calculation of surface evapotranspiration

The one step calculation of actual soil evaporation ($E_s$) and potential transpiration ($T_p$) is achieved by incorporating canopy minimum surface resistance and actual soil resistance into the Penman-Monteith model (i.e., the ET$_{dir}$ method in Yu et al., 2016). LAI is implicitly used to partition available radiation energy into the radiation reaching the canopy and soil surface.

$$T_p = \frac{\Delta (R_{n}^{c} - G) + \rho_a c_p \frac{(e_s - e_a)}{r_a^c}}{\lambda (\Delta + \gamma (1 + \frac{r_{c,\text{min}}}{r_a^c}))}, \quad (2.48)$$

$$E_s = \frac{\Delta (R_{n}^{s} - G) + \rho_a c_p \frac{(e_s - e_a)}{r_a^s}}{\lambda (\Delta + \gamma (1 + \frac{r_{c,\text{min}}}{r_a^s}))}, \quad (2.49)$$

where $R_n^c$ and $R_n^s$ (MJ m$^{-2}$ day$^{-1}$) are the net radiation at the canopy surface and soil surface, respectively; $\rho_a$ (kg m$^{-3}$) is the air density; $c_p$ (J kg$^{-1}$ K$^{-1}$) is the specific heat capacity of air; $r_a^c$ and $r_a^s$ (s m$^{-1}$) are the aerodynamic resistance for canopy surface and soil surface, respectively; $r_{c,\text{min}}$ (s m$^{-1}$) is the minimum canopy
surface resistance; and \( r_s \) (s m\(^{-1}\)) is the soil surface resistance.

The net radiation reaching the soil surface can be calculated using the Beer’s law:

\[
R_n^s = R_n \exp(-\tau LAI)
\]  

(2.50)

And the net radiation intercepted by the canopy surface is the residual part of total net radiation:

\[
R_n^c = R_n (1 - \exp(-\tau LAI))
\]  

(2.51)

The minimum canopy surface resistance \( r_{c,min} \) is given by:

\[
r_{c,min} = \frac{r_{l,min}}{LAI_{eff}}
\]  

(2.52)

320 where \( r_{l,min} \) is the minimum leaf stomatal resistance; \( LAI_{eff} \) is the effective leaf area index, which considers that generally the upper and sunlit leaves in the canopy actively contribute to the heat and vapor transfer.

The soil surface resistance can be estimated following van de Griend and Owe (1994),

\[
r_s = \begin{cases} 
  r_{sl} & \theta_1 > \theta_{min}, h_1 > -100000 \text{ cm} \\
  r_{sl} e^{a(\theta_{min} - \theta_1)} & \theta_1 \leq \theta_{min}, h_1 > -100000 \text{ cm} \\
  \infty & h_1 \leq -100000 \text{ cm}
\end{cases}
\]  

(2.53)

where \( r_{sl} \) (10 s m\(^{-1}\)) is the resistance to molecular diffusion of the water surface; \( a \) (0.3565) is the fitted parameter; \( \theta_1 \) is the topsoil water content; \( \theta_{min} \) is the minimum water content above which soil is able to deliver vapor at a potential rate.

The root water uptake term described by Feddes et al. (1978) is:

\[
S(h) = \alpha(h)S_p
\]  

(2.54)

where \( \alpha(h) \) (dimensionless) is the reduction coefficient related to soil water potential \( h \); and \( S_p \) (s\(^{-1}\)) is the potential water uptake rate.

\[
S_p = b(z)T_p
\]  

(2.55)

where \( T_p \) is the potential transpiration in Eq. 2.48. \( b(z) \) is the normalized water uptake distribution, which describes the vertical variation of the potential extraction term, \( S_p \), over the root zone. Here the asymptotic function was used to characterize the root distribution as described in (Gale and Grigal, 1987; Jackson et al., 1996; Yang et al., 2009; Zheng et al., 2015).

### 2.3 STEMMUS-FT model framework with three levels of complexity

On the basis of STEMMUS modelling framework, the increasing complexity of vadose zone physics in frozen soils was implemented as three alternative models (Table 2.2). Firstly, STEMMUS enabled isothermal water and heat transfer physics (Eqs. 2.56 & 2.57). The 1-D Richards equation is utilized to solve the isothermal water transport in variably saturated soils. The heat conservation equation took into account the freezing/thawing process and the latent heat due to water phase change. The effect of soil ice on soil hydraulic and thermal properties was considered. It is termed the basic coupled water and heat transfer model (BCD).

Secondly, the fully coupled water and heat physics, i.e., water vapor flow and thermal effect on water flow, was explicitly considered in STEMMUS, termed the advanced coupled model (ACD). For the ACD physics, the extended version of Richards equation (Richards, 1931) with modifications made by Milly (1982) was
used as the water conservation equation (Eq. 2.58). Water flow can be expressed as liquid and vapor fluxes driven by both temperature gradients and matric potential gradients. The heat transport in frozen soils mainly includes: heat conduction (CHF, $\lambda_{eff} \frac{\partial T}{\partial z}$), convective heat transferred by liquid flux (HFL, $-C_L q_L(T - T_r)$), vapor flux (HFV, $-C_V q_V(T - T_r)$), the latent heat of vaporization (LHF, $-q_V L_0$), the latent heat of freezing/thawing ($-\rho_i \theta_i L_f$) and a source term associated with the exothermic process of wetting of a porous medium (integral heat of wetting) ($-\rho_L W \frac{\partial \theta_l}{\partial t}$). It can be expressed as Eq. 2.59 (De Vries, 1958; Hansson et al., 2004).

Lastly, STEMMUS expressed the freezing soil porous medium as the mutually dependent system of liquid water, water vapor, ice water, dry air and soil grains, in which other than air flow all other components kept the same as in ACD (termed ACD-Air model) (Eqs. 2.60, 2.61, & 2.62, Zeng et al., 2011b, a; Zeng and Su, 2013). The effects of air flow on soil water and heat transfer can be two-fold. Firstly, the air flow-induced water and vapor fluxes ($q_L$, $q_V$) and its corresponding convective heat flow ($HFa$, $-q_L C_L (\frac{\partial q_l}{\partial z})$) were considered. Secondly, the presence of air flow alters the vapor transfer processes, thus can considerably affects the water and heat transfer in an indirect manner.

STEMMUS utilized the adaptive time-step strategy, with maximum time steps ranging from 1s to 1800s (e.g., with 1800s as the time step under stable conditions). The maximum desirable change of soil moisture and soil temperature within one time step was set as 0.02 cm$^3$ cm$^{-3}$ and 2 °C, respectively, to prevent too large change in state variables that may cause numerical instabilities. If the changes between two adjacent soil moisture/temperature states are less than the maximum desirable change, STEMMUS continues without changing the length of current time step (e.g., 1800s). Otherwise, STEMMUS will adjust the time step with a deduction factor, which is proportional to the difference between the too large changes and desirable allowed maximum changes of state variables. Within one single time step, the Picard iteration was used to solve the numerical problem, and the numerical convergence criteria is set as 0.001 for both soil matric potential (in cm) and soil temperature (in °C).

**Table 2.2. Governing equations for different complexity of water and heat coupling physics (See Section 4.4 for notations)**

| Models | Governing equations (water, heat and air) | Number |
|--------|------------------------------------------|--------|
| BCD    | $\frac{\partial \theta}{\partial t} = -\frac{\partial q_l}{\partial z} - S = \rho_L \frac{\partial}{\partial z}[K \left(\frac{\partial \psi}{\partial z} + 1\right)] - S$ | (2.56) |
|        | $\frac{C_{soil}}{H_C} \frac{\partial T}{\partial t} - \rho_L \frac{\partial \theta_l}{\partial t} = \frac{\partial}{\partial z} \left(\lambda_{eff} \frac{\partial T}{\partial z}\right)$ | (2.57) |
| ACD    | $\frac{\partial}{\partial t} (\rho_1 \theta_L + \rho_V \theta_V + \rho_i \theta_i) = -\frac{\partial}{\partial z} (q_L + q_V) - S$ |        |
|        | $= -\frac{\partial}{\partial z} (q_L h + q_L L + q_V h + q_V L) - S$ |        |
|        | $= \rho_L \frac{\partial}{\partial z} [K_{lh} \left(\frac{\partial \psi}{\partial z} + 1\right) + K_L \frac{\partial T}{\partial z}] + \frac{\partial}{\partial z} \left[D_{vh} \frac{\partial \psi}{\partial z} + D_{VT} \frac{\partial T}{\partial z}\right] - S$ | (2.58) |
\[
\frac{\partial}{\partial t} \left[ \rho_a \theta_a C_\rho + \rho_l \theta_l C_L + \rho_v \theta_v C_v + \rho_i \theta_i C_i \right] (T - T_r) + \rho_v \theta_v L_a - \rho_i \theta_i L_f - \rho_L W \frac{\partial q_L}{\partial t} \right] \\
= \frac{\partial}{\partial z} \left[ \frac{\partial}{\partial z} \left( \lambda_{eff} \frac{\partial T}{\partial z} \right) \right] - \frac{\partial}{\partial z} \left[ q_L L_a + q_v L_v(T - T_r) \right] \cdot \frac{\partial}{\partial z} \left[ q_L C_L(T - T_r) - C_L S(T - T_r) \right] \\
= \rho_L \left[ \frac{\partial}{\partial z} \left( \lambda_{eff} \frac{\partial T}{\partial z} \right) \right] + \frac{\partial}{\partial z} \left[ q_L L_a + q_v L_v(T - T_r) \right] \cdot \frac{\partial}{\partial z} \left[ q_L C_L(T - T_r) - C_L S(T - T_r) \right] \\
= \rho_L \left[ \frac{\partial}{\partial z} \left( \lambda_{eff} \frac{\partial T}{\partial z} \right) \right] - \frac{\partial}{\partial z} \left[ q_L C_L(T - T_r) - C_L S(T - T_r) \right] \\
= \frac{\partial}{\partial z} \left[ \epsilon \rho_d \left( S_a + H_c S_L \right) \right] = \frac{\partial}{\partial z} \left[ D_e \frac{\partial \rho_d}{\partial z} + \rho_d S_a K_a \frac{\partial T}{\partial z} - H_c \rho_d q_L C_v \frac{\partial \rho_d}{\partial z} + \left( \theta_d D_f \right) \frac{\partial \rho_d}{\partial z} \right] \\
\text{(2.62)}
\]

The main difference of underlying soil physical processes considered by three level of model complexity is summarized in Table 2.3. For the BCD model, soil water and heat transfer is independent during the unfrozen period, the coupling between water and heat transfer only can be induced by the freezing/thawing process. Such coupling is mainly: i) the ice effect (thermal effect) on soil hydraulic properties; ii) latent heat flux due to phase change. For the ACD model, it enables not only frozen soil physics but also additional processes and most importantly the vapor flow transfer, which links the soil water and heat flow to implement the tight coupling of water and heat effects. In addition to the ice blocking effect as presented in BCD, the thermal effect on water flow is also expressed with the temperature dependence of hydraulic conductivity and matric potential (Section 2.2.3). Furthermore, not only the latent heat due to phase change, but also the convective heat due to liquid/vapor flow can be simulated. For the ACD-Air model, the dry air is considered as an independent component of soil pores and interactively coupled with soil water and heat transfer. The airflow induced convective heat is calculated. Although it contributes little to the total heat budgets while indeed can affect the relative contribution of other heat flux components (see Section 5.2).

| Model complexity | Soil Physical Processes | Frozen period | Model Components |
|------------------|-------------------------|---------------|-----------------|
| BCD              | Independent water and heat transfer | FT induced water and heat transfer coupling; Ice effect on soil properties; Latent heat due to phase change; | Eqs. 2.56 & 2.57 |
| ACD              | Tightly coupled water and heat transfer | Convective heat due to liquid/vapor flow. | Eqs. 2.58 & 2.59 |

Table 2.3. The underlying soil physical processes considered by STEMMUS-FT with various model complexities
ACD-Air  Tightly coupled water, dry air, and heat transfer  
Tightly coupled water, dry air, and heat transfer;  
Ice effect on soil properties;  
Latent heat due to phase change;  
Convective heat due to liquid/vapor/air flow.  

Note:  
Independent water and heat transfer: Soil water and heat transfer process is independent.  
FT induced water and heat transfer coupling: Soil water and heat transfer process is coupled only during the freezing/thawing (FT) period. Soil water flow is affected by temperature only through the presence of soil ice content (the impedance effect).  
Tightly coupled water and heat transfer: Soil water and heat transfer process is tightly coupled; vapor flow, which links the soil water and heat flow, is taken into account; thermal effect on water flow is considered (the hydraulic conductivity and matrix potential is dependent on soil temperature; when soil freezes, the hydraulic conductivity is reduced by the presence of soil ice, which is temperature dependent); the convective/advective heat due to liquid/vapor flow can be calculated.  
Tightly coupled water, dry air, and heat transfer: On the basis of “Tightly coupled water and heat transfer”, the soil dry air transfer is taken into account and simultaneously simulated with water and heat transfer; the convective/advective heat due to liquid/vapor/air flow can be calculated.  
Ice effect on soil properties: the explicit simulation of ice content and its effect on the hydraulic/thermal properties.

3 UEB snowmelt module

The Utah energy balance (UEB) snowmelt model is a physically-based snow accumulation and melt model (Fig. 3.1). The snowpack is characterized mainly using two primary state variables, snow water equivalent $W$ and the internal energy $U$. The snow age is considered as the ancillary state variable. The conservation of mass and energy, forms the basis of UEB (Tarboton and Luce, 1996), is presented in Section 3.1. The relevant constitutive equations are given in Section 3.2.

![Figure 3.1. The schematic of (a) energy flux involved in snowmelt and snowpack ablation (b) related variables in UEB model. Adapted from Tarboton and Luce (1996).](image)

3.1 Governing Equations

3.1.1 Mass balance equation

The increase or decrease of snow water equivalence with time equals the difference of income and outgoing water flux:

$$\frac{dW_{SWE}}{dt} = P_r + P_s - M_r - E$$  \hspace{1cm} (3.1)

where $W_{SWE}$ (m) is the snow water equivalent; $P_r$ (m s$^{-1}$) is the rainfall rate; $P_s$ (m s$^{-1}$) is the snowfall rate; $M_r$
(m s\(^{-1}\)) is the meltwater outflow from the snowpack; and \(E\) (m s\(^{-1}\)) is the sublimation from the snowpack.

### 3.1.2 Energy balance equation

The energy balance of snowpack can be expressed as:

\[
\frac{dU}{dt} = Q_{sm} + Q_{lt} + Q_p + Q_{\theta} - Q_{le} + Q_h + Q_e - Q_m
\]  

(3.2)

where \(Q_{sm}\) (W m\(^{-2}\)) is the net shortwave radiation; \(Q_{lt}\) (W m\(^{-2}\)) is the incoming longwave radiation; \(Q_p\) (W m\(^{-2}\)) is the advected heat from precipitation; \(Q_{\theta}\) (W m\(^{-2}\)) is the ground heat flux; \(Q_{le}\) (W m\(^{-2}\)) is the outgoing longwave radiation; \(Q_h\) (W m\(^{-2}\)) is the sensible heat flux; \(Q_e\) (W m\(^{-2}\)) is the latent heat flux due to sublimation/condensation; and \(Q_m\) (W m\(^{-2}\)) is the advected heat removed by meltwater.

Equations (3.1) and (3.2) form a coupled set of first order, nonlinear ordinary differential equations. Euler predictor-corrector approach was employed in UEB model to solve the initial value problems of these equations (Tarboton and Luce, 1996).

### 3.2 Constitutive Equations

#### 3.2.1 Mass balance

The observed precipitation rate \(P\), can be partitioned into rain \(P_r\), and snow \(P_s\), (both in terms of water equivalence depth) based on air temperature \(T_a\)

\[
P_r = P\quad T_a \geq T_r
\]

\[
P_r = P(T_a - T_b)/(T_r - T_b)\quad T_b < T_a < T_r
\]

\[
P_r = 0\quad T_a < T_b
\]

\[
P_s = F(P - P_r)
\]

(3.3)

where \(T_r\) is a threshold air temperature above which all precipitation is rain and \(T_b\) is a threshold air temperature below which all precipitation is snow. \(F\) is employed to account for the wind redistribution effect on the accumulation of snow.

The amount of water sublimate from the snowpack is

\[
E = \rho_a(q_s - q_a)K_e
\]

(3.5)

where \(\rho_a\) is air density, \(q_s\) is the surface specific humidity, \(q_a\) is the air humidity. \(K_e\) is turbulent transfer conductance for latent heat.

The meltwater outflow from the snowpack can be expressed as

\[
M_r = K_{sat}S^{s-3}
\]

(3.6)
where $K_{sat}$ is the snow saturated hydraulic conductivity and $S^*$ is the relative saturation in excess of water retained by capillary forces. $S^*$ is given by:

$$S^* = \frac{\text{liquid water volume - capillary retention}}{\text{pore volume - capillary retention}}$$  \hspace{1cm} (3.7)

### 3.2.2 Energy balance

The net shortwave radiation is calculated from incident shortwave radiation $Q_{si}$ and albedo $\alpha$, which is a function of snow age and solar illumination angle.

$$Q_{sn} = (1 - \alpha)Q_{si}$$  \hspace{1cm} (3.8)

The Stefan–Boltzmann equation is used to estimate the incoming longwave radiation $Q_{le}$ and outgoing longwave radiation $Q_{li}$ based on air temperature $T_a$ and snow surface temperature $T_{SS}$, respectively.

$$Q_{le} = \varepsilon_s \sigma T_{SS}^4$$  \hspace{1cm} (3.9)

$$Q_{li} = \varepsilon_a \sigma T_a^4$$  \hspace{1cm} (3.10)

where $\varepsilon_s$ is emissivity of snow, $\sigma$ is the Stefan Boltzmann constant, $\varepsilon_a$ is the air emissivity, which is based on air vapor pressure, air temperature and cloud cover.

The latent heat flux, $Q_e$ and sensible heat flux, $Q_h$ are modeled using bulk aerodynamic formulae:

$$Q_h = \rho_a c_p(T_a - T_{SS})K_h$$  \hspace{1cm} (3.11)

$$Q_e = \rho_a h_v(q_s - q_a)K_e = K_e \frac{0.622 h_v}{R_d T_a} (\varepsilon_a - \varepsilon_s(T_{SS}))$$  \hspace{1cm} (3.12)

$K_h$ and $K_e$ are turbulent transfer conductance for sensible and latent heat respectively. Under neutral atmospheric conditions $K_h$ and $K_e$ can be given by

$$K_e = K_h = \frac{k_v^2 u}{[\ln (z_m/z_0)]^2}$$  \hspace{1cm} (3.13)

where $z_m$ is the measurement height for wind speed, air temperature, and humidity, $u$ is the wind speed, $k_v$ is von Kármán’s constant (0.4), and $z_0$ is the aerodynamic roughness.

The heat advected with the snow melt outflow, relative to the solid reference state is:

$$Q_{m} = \rho_w h_f M_f$$  \hspace{1cm} (3.14)

The advected heat $Q_p$ is the energy required to convert precipitation to the reference state ($0 \, ^\circ C$ ice phase). The temperature of rain and snow is taken as the greater and lesser of the air temperature and freezing point. With different temperature inherent to snow and rain, this amount of energy can be described as
\[
Q_p = \rho_w C_s P_s \cdot \min(T_a, 0) + P_r \left[\rho_w h_f + \rho_w C_w \cdot \max(T_a, 0)\right]
\] (3.15)

3.2.3 Snow temperatures

1) Snowpack temperature, \(T_{SN}\)

Snowpack temperature \(T_{SN}\), a quantity important for energy fluxes into the snow, is determined diagnostically from the state variables energy content \(U\), and water equivalence \(W_{SWE}\), as follows, recalling that energy content \(U\) is defined relative to 0°C ice phase.

\[
T_{SN} = \frac{U}{\rho_w W_{SWE} C_i + \rho_g D_e C_g}, \quad U < 0, \quad \text{all solid phase}
\] (3.16)

\[
T_{SN} = 0, \quad 0 < U < \rho_w W_{SWE} h_f, \quad \text{solid and liquid mixture}
\] (3.17)

\[
T_{SN} = \frac{U - \rho_w W_{SWE} h_f}{\rho_w W_{SWE} + \rho_g D_e C_g}, \quad U > \rho_w W_{SWE} h_f, \quad \text{all liquid phase}
\] (3.18)

where \(\rho_w W_{SWE} C_i\) is the heat capacity of the snow (kJ °C\(^{-1}\) m\(^{-2}\)), \(\rho_w\) is the density of water (1000 kg m\(^{-3}\)) and \(C_i\) is the specific heat of ice (2.09 kJ kg\(^{-1}\) °C\(^{-1}\)). \(\rho_g D_e C_g\) is the heat capacity of the soil layer (kJ °C\(^{-1}\) m\(^{-2}\)), \(\rho_g\) is the soil density and \(C_g\) the specific heat of soil. \(D_e\) is the depth of soil that interacts thermally with the snowpack. These together determine snowpack temperature \(T_{SN}\) when energy content \(U < 0\).

Otherwise, \(\rho_w W_{SWE} h_f\) is the heat required to melt all the snow water equivalence at 0 °C (kJ m\(^{-2}\)), \(h_f\) is the heat of fusion (333.5 kJ kg\(^{-1}\)) and \(U\) in relation to this determines the solid-liquid phase mixtures. The liquid fraction \(L_f = U / (\rho_w W_{SWE} h_f)\) quantifies the mass fraction of total snowpack (liquid and ice) that is liquid.

Although in Equation (3.17) \(W_{SWE}\) is always 0 as a completely liquid snowpack cannot exist, we present this equation for completeness to keep track of energy content during periods of intermittent snow cover. \(\rho_w W_{SWE} C_w\) is the heat capacity of liquid water, \(C_w\) is the specific heat of water (4.18 kJ kg\(^{-1}\) °C\(^{-1}\)), is included for numerical consistency during time steps when the snowpack completely melts.

2) Snow Surface Temperature, \(T_{SS}\)

Snow surface temperature \(T_{SS}\) is in general different from snowpack temperature \(T_{SN}\) due to the snow insulation effect. We take into account such temperature difference using an equilibrium approach that balances energy fluxes at the snow surface. Heat conduction into the snow is calculated using the temperature gradient and thermal diffusivity of snow, approximated by:

\[
Q_{SN} = \frac{\kappa \rho_s C_s (T_{SS} - T_{SN})}{Z_e} = K_{SN} \rho_s C_s (T_{SS} - T_{SN})
\] (3.19)

where \(\kappa\) is snow thermal diffusivity (m\(^2\) hr\(^{-1}\)) and \(Z_e\) (m) an effective depth over which this thermal gradient acts. \(K_{SN} (\kappa/Z_e)\) is termed snow surface conductance, analogous to the heat and vapor conductance. Here \(K_{SN}\) is used as a tuning parameter, with this calculation used to define a reasonable range. Then assuming equilibrium at the surface, the surface energy balance gives:
\[ Q_{SN} = Q_{sm} + Q_{li} + Q_{h}(T_{SS}) + Q_{e}(T_{SS}) + Q_{p} - Q_{le}(T_{SS}) \]  

(3.20)

where the dependence of \( Q_h, Q_e, \) and \( Q_{le} \) on \( T_{SS} \) is through equations (3.11), (3.12) and (3.9) respectively.

Analogous to the derivation of the Penman equation for evaporation the functions of \( T_{SS} \) in this energy balance equation are linearized about a reference temperature \( T^* \), and the equation is solved for \( T_{SS} \):.

\[ \frac{\partial S_{SS}}{\partial t} = \frac{Q_{sn} + Q_{li} + Q_{p} + K_{T_a} \rho_a C_p - 0.622 K_h \rho_a (e_s(T^* - e_a - T^* \Delta))}{K_{SN} \rho_s C_s + K \rho_a C_p + 0.622 K_h \rho_a \frac{4 \epsilon_s \sigma T^*}{P_a} + 4 \epsilon_s \sigma T^* - 3 \} \]

(3.21)

where \( \Delta = \frac{d e_s}{d T} \) and all temperatures are absolute in (K). This equation is used in an iterative procedure with an initial estimate \( T^* = T_a \), in each iteration replacing \( T^* \) by the latest \( T_{SS} \). The procedure converges to a final \( T_{SS} \) which, if less than freezing, is used to calculate surface energy fluxes. If the final \( T_{SS} \) is greater than freezing it means that the energy input to the snow surface cannot be balanced by thermal conduction into the snow. Surface melt will occur and the infiltration of meltwater will account for the energy difference and \( T_{SS} \) is then set to 0°C.

### 3.2.4 Albedo calculation

**1) Ground albedo**

Instead of the constant bare soil albedo in the original UEB model, the bare soil albedo is expressed as a decreasing linear function of soil moisture in STEMMUS-UEB.

\[ \alpha_{g,v} = \alpha_{sat} + \min \{ \alpha_{sat}, \max \{ (0.11 \cdot 0.4 \theta), 0 \} \} \]

(3.22)

\[ \alpha_{g,ir} = 2 \alpha_{g,v} \]

(3.23)

where \( \alpha_{g,v} \) and \( \alpha_{g,ir} \) are the bare soil/ground albedo for the visible and infrared band, respectively. \( \alpha_{sat} \) is the saturated soil albedo, depending on local soil color. \( \theta \) is the surface volumetric soil moisture.

**2) Vegetation albedo**

The calculation of vegetation albedo is developed to capture the essential features of a two-stream approximation model using asymptotic equation. It approaches the underlying surface albedo \( \alpha_{g,\lambda} \) or the thick canopy albedo \( \alpha_{c,\lambda} \) when the \( L_{SAI} \) is close to zero or infinity.

\[ \alpha_{Veg,b,\lambda} = \alpha_{c,\lambda} \left[ 1 - \exp\left(-\frac{\omega_c \beta L_{SAI}}{\mu \alpha_{c,\lambda}}\right) \right] + \alpha_{g,\lambda} \exp\left[-\left(1 + \frac{0.5}{\mu}\right) L_{SAI}\right] \]

(3.24)

\[ \alpha_{Veg,d,\lambda} = \alpha_{c,\lambda} \left[ 1 - \exp\left(-\frac{2 \omega_c \beta L_{SAI}}{\alpha_{c,\lambda}}\right) \right] + \alpha_{g,\lambda} \exp\left[-2 L_{SAI}\right] \]

(3.25)

where subscripts \( Veg,b,d,c,g \) and \( \lambda \) represent vegetation, direct beam, diffuse radiation, thick canopy,
ground, and spectrum bands of either visible or infrared bands. $\mu$ is the cosine of solar zenith angle; $\omega_\lambda$ is the single scattering albedo, 0.15 for visible and 0.85 for infrared band, respectively; $\beta$ is assigned as 0.5; $L_{SAI}$ is the sum of leaf area index LAI and stem area index SAI; $\alpha_{c, \lambda}$ is the thick canopy albedo dependent on vegetation types.

The bulk snow-free surface albedo, averaged between bare ground albedo and vegetation albedo, then is written as:

$$\alpha_{\eta, \lambda} = \alpha_{veg, \lambda} f_{veg} + \alpha_{g, \lambda} (1 - f_{veg})$$

(3.26)

where $\alpha_{\eta, \lambda}$ is the averaged bulk snow-free surface albedo; $f_{veg}$ is the fraction of vegetation cover.

3) Snow albedo

According to Dickinson et al. (1993), snow albedo can be expressed as a function of snow surface age and solar illumination angle. The snow surface age, which is dependent on snow surface temperature and snowfall, is updated with each time step in UEB. Visible and near infrared bands are separately treated when calculating reflectance, which are further averaged as the albedo with modifications of illumination angle and snow age.

The reflectance in the visible and near infrared bands can be written as:

$$\alpha_{vd} = (1 - C_v S_{age}) \alpha_{vo}$$

(3.27)

$$\alpha_{ird} = (1 - C_{ir} S_{age}) \alpha_{iro}$$

(3.28)

where $\alpha_{vd}$ and $\alpha_{ird}$ represent diffuse reflectance in the visible and near infrared bands, respectively. $C_v (= 0.2)$ and $C_{ir} (=0.5)$ are parameters that quantify the sensitivity of the visible and infrared band albedo to snow surface aging (grain size growth), $\alpha_{vo} (=0.85)$ and $\alpha_{iro} (=0.65)$ are fresh snow reflectance in visible and infrared bands, respectively. $S_{age}$ is a function to account for aging of the snow surface, and is given by:

$$S_{age} = \frac{\tau}{1 + \tau}$$

(3.29)

where $\tau$ is the non-dimensional snow surface age that is incremented at each time step by the quantity designed to emulate the effect of the growth of surface grain sizes.

$$\Delta \tau = \frac{r_1 + r_2 + r_3}{r_o} \Delta t$$

(3.30)

where $\Delta t$ is the time step in seconds with $r_o = 10^6s$. $r_1$ is the parameter to represent the effect of grain growth due to vapor diffusion, and is dependent on snow surface temperature:

$$r_1 = \exp [5000 \left( \frac{1}{273.16} - \frac{1}{T_s} \right)]$$

(3.31)

$r_2$ describes the additional effect near and at the freezing point due to melt and refreeze:

$$r_2 = \min (r_1^{10}, 1)$$

(3.32)
The reflectance of radiation with illumination angle (measured relative to the surface normal) is computed as:

\[
\alpha_v = \alpha_{vd} + 0.4 f(\varphi)(1 - \alpha_{vd})
\]

(3.33)

\[
\alpha_{ir} = \alpha_{ird} + 0.4 f(\varphi)(1 - \alpha_{ird})
\]

(3.34)

where \( f(\varphi) = \begin{cases} 
\frac{1}{b} \left[ \frac{\frac{b+1}{1+2b \cos(\varphi)}}{\cos(\varphi)} - 1 \right], & \text{for } \cos(\varphi) < 0.5 \\
0, & \text{otherwise}
\end{cases} \)

When the snowpack is shallow (depth \( z < h = 0.01 \) m), the albedo is calculated by interpolating between the snow albedo and bare ground albedo with the exponential term approximating the exponential extinction of radiation penetration of snow.

\[
A_{v/ir} = r \alpha_{p,v/ir} + (1 - r) \alpha_{v/ir}
\]

(3.35)

where \( r = \left(1 - \frac{z}{h}\right) e^{-z/h} \).

4 STEMMUS-UEB: Coupling structure, Subroutines and Input Data

4.1 Coupling procedure

The coupled process between the snowpack model (UEB) and the soil water model (STEMMUS-FT) is illustrated in Figure 4.1. The one-way sequential coupling is employed to couple the soil model with the current snowpack model. The role of the snowpack is explicitly considered by altering the water and heat flow of the underlying soil. The snowpack model takes the atmospheric forcing as the input (precipitation, air temperature, wind speed and direction, relative humidity, shortwave and longwave radiation) and solves the snowpack energy and mass balance (Eqs. 3.1 & 3.2, Subroutines: ALBEDO, PARTSNOW, PREDICORR), provides the melt water flux and heat flux as the surface boundary conditions for the soil model STEMMUS-FT (Subroutines: h_sub and Enrgy_sub for ACD models; Diff_Moisture_Heat for BCD model). STEMMUS-FT then solves the energy and mass balance equations of soil layers in one timestep. To highlight the effect of snowpack on the soil water and vapor transfer process, we constrained the soil surface energy boundary as the Dirichlet type condition (take the specific soil temperature as the surface boundary condition). Surface soil temperature was derived from the soil profile measurements and not permitted to be higher than zero when there is snowpack. To ensure the numerical convergence, the adapted timestep strategy was used. The half-hourly meteorological forcing measurements were linearly interpolated to the running timesteps (Subroutine Forcing_PARM). The precipitation rate (validated at 3-hour time intervals) was regarded uniformly within the 3-hour duration (see Table S1 for detail). The general
description of the subroutines in STEMMUS-UEB, including the main functions, input/output, and its connection with other subroutines, was presented in Table 4.1 & 4.2 (linked with Table S1 and S2 for the description of model input parameters and outputs for this study).

**Figure 4.1.** The overview of the coupled STEMMUS-FT and UEB model framework and model structure. SFCC is soil freezing characteristic curve; \( \theta_L \) and \( \theta_i \) are soil liquid water and ice content; \( K_{th} \) is soil hydraulic conductivity; \( \lambda_{ef} \) is thermal conductivity. \( \psi, T, P_g \) are the state variables for soil module STEMMUS-FT (matric potential, temperature, and air pressure, respectively). U, SWE, and \( \tau \) are the state variables for snow module UEB (snow energy content, snow water equivalent, and snow age, respectively). UEB, Utah Energy Balance module. Precip, Ta, HRa, Rn, and u are the meteorological inputs (precipitation, air temperature, relative humidity, radiation and wind speed). Model subroutines are in red fonts.

### 4.2 Subroutines and Inputs/Outputs

Table 4.1 and Table 4.2 summarize the main functions, input/output, and code inter-connections of the primary subroutines and secondary subroutines, which presents the complete Input-Primary Subroutine-Secondary Subroutine-Output loop of STEMMUS-UEB modelling framework.

STEMMUS-UEB model subroutines can be generally divided into four groups as identified by different calling sequential orders or roles/functions in the main program: Initialization Group, Parameterization Group, Processing Group, and Post-process Group. Note that some subroutines can be categorized into more than one group, we made the classification based on the functions of the subroutine here. For example, subroutine SOIL2 is called by subroutine StartInit, which belongs to the Initialization Group. Nevertheless,
according to the function of SOIL2, it falls into the Parameterization Group. We then label SOIL2 as Parameterization Group.

Table 4.1. Primary subroutines in STEMMUS-UEB

| Model Subroutines | Main functions | Main inputs | Main outputs | Subroutine-Connections | Remarks |
|-------------------|----------------|-------------|--------------|------------------------|---------|
| **Soil module**   |                |             |              |                        |         |
| Air_sub           | Solves soil dry air balance equation | Water vapor density, diffusivity, dispersion coefficient; dry air density, gas conductivity, flux; liquid water flux; top and bottom boundary conditions | Soil air pressure profile | CondV_DVg, CondL_h, Cond_k_g, Density_V, h_sub -->; --> Enrgy_sub, | Processing Group |
| CondL_h           | Calculates soil hydraulic conductivity | Soil hydraulic parameters; soil matric potential; soil temperature | Soil hydraulic conductivity; soil water content | StartInit -->; --> h_sub, Air_sub; Enrgy_sub, | Parameterization Group |
| CondT_coeff       | Calculates soil thermal capacity and conductivity | Thermal properties of soil constituents; soil texture; soil water content; volumetric fraction of dry air; dry air density; vapor density | Soil thermal capacity and conductivity | StartInit, CondL_h, Cond_k_g, Density_V, EfeCapCond -->; --> Enrgy_sub, | Parameterization Group |
| CondV_DVg         | Calculates flux of dry air and vapor dispersity | Gas conductivity, dry air pressure, volumetric fraction of dry air; saturated soil water content | Dry air flux and vapor dispersion coefficient | StartInit, CondL_h, Cond_k_g -->; --> h_sub; Air_sub; Enrgy_sub, | Parameterization Group |
| CondL_Tdisp       | Calculates transport coefficient for adsorbed liquid flow | Soil porosity, soil water content, temperature, matric potential, volumetric fraction of dry air | Transport coefficient for adsorbed liquid flow and the heat of wetting | StartInit, CondL_h, Cond_k_g -->; --> h_sub; Enrgy_sub, | Parameterization Group |
| Cond_k_g          | Calculates gas conductivity | Soil porosity, saturated hydraulic conductivity, volumetric fraction of dry air | Gas conductivity | StartInit, CondL_h -->; --> CondV_DVg, | Parameterization Group |
| Density_DA        | Calculates dry air density | Calculates vapor density and its derivative with respect to temperature and matric potential | Density of dry air | StartInit, CondL_h, Density_V -->; --> Cond_k_g, Enrgy_sub, | Parameterization Group |
| Density_V         | | Soil temperature, matric potential | Vapor density and its derivative with respect to temperature and matric potential | CondL_h -->; --> Density_DA, CondT_coeff, h_sub, Air_sub, Enrgy_sub, | Parameterization Group |
| EfeCapCond        | Calculates soil thermal capacity and conductivity | Thermal properties of soil constituents; soil texture; soil water content; volumetric fraction of dry air; dry air density; vapor density | Soil heat capacity; thermal conductivity | StartInit, CondL_h, Density_V, Density_DA -->; --> CondT_coeff, | Parameterization Group |
| Enrgy_sub         | Solves soil energy balance equation | Soil thermal properties, soil hydraulic conductivity, soil matric potential, soil water content, soil temperature, soil dry air pressure, density of dry air, heat of wetting, vapor density, liquid water flux, vapor flux, dry air flux, meteorological forcing, top and bottom boundary conditions | Soil temperature profile, liquid water flux, vapor flux, and dry air flux, surface and bottom energy fluxes | Air_sub, h_sub, CondL_h, CondV_DVg, CondL_Tdisp, CondT_coeff, Density_D, Density_DA, PREDICORR -->; | Processing Group |
| **Forcing_PA**    | Disaggregates the meteorological forcing into the required time steps | Observed meteorological forcing at hourly/daily time scale | Meteorological forcings at model required time scale | StartInit -->; --> h_sub, Enrgy_sub, | Initialization Group |

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| Subroutine | Task | Inputs | Outputs | Processing Group |
|------------|------|--------|---------|------------------|
| h_sub      | Solves soil water balance equation | Soil temperature, soil water content, matric potential, soil hydraulic conductivity, heat of wetting, soil dry air pressure, vapor density, diffusivity, dispersivity, volumetric fraction of vapor, meteorological forcing, top and bottom boundary conditions | Soil matric potential profile and top and bottom water evaporation | Processing Group |
| StartInit  | Initializes model setup | Soil texture, thermal properties of soil constituents, initial soil water content and temperature, top and bottom boundary condition settings | - | Initialization Group |
| Diff_Moisture_Heat | Solves soil water and energy balance equations independently | Soil thermal properties, soil hydraulic conductivity, soil matric potential, soil water content, soil temperature, meteorological forcing, top and bottom boundary conditions | Soil water content and temperature profile, liquid water flux, surface and bottom water and energy fluxes | Processing Group |

**Snowpack module**

| Subroutine | Task | Inputs | Outputs | Processing Group |
|------------|------|--------|---------|------------------|
| agesn      | Calculates snow age | Snow surface temperature, snowfall, Fresh snow reflectance at visible and near infrared bands, snow age, bare ground albedo, albedo extinction parameter, snow water equivalent | Updated snow age | Parameterization Group |
| ALBEDO     | Calculates snow albedo | Precipitation, air temperature, temperature thresholds for rainfall/snowfall | Snow albedo | Parameterization Group |
| PARTSNO W  | Partitions precipitation into rainfall and snowfall | Air temperature, snow albedo, wind speed, relative humidity, rainfall/snowfall, shortwave/longwave radiation, site parameters | Rainfall, snowfall | Parameterization Group |
| PREDICOR R | Solves the snow mass and energy balance equations and updates state variables SWE and U | Snow energy content, water equivalent, snow albedo, snow surface temperature, meltwater outflow rate, snow sublimation, snowfall/rainfall | Snow energy content, water equivalent, snow albedo, snow surface temperature, meltwater outflow rate, snow sublimation, snowfall/rainfall | Processing Group |

Note:

--> means the relevant subroutines which are incoming to the current one,
---> means the relevant subroutines for which the current subroutine is output to;
agesn$^2$ and ALBEDO$^2$, means the use of subroutines agesn and ALBEDO after solving the snowpack energy and mass conservation equations, to update the snow age and albedo.
### Table 4.2. Secondary subroutines in STEMMUS-UEB

| Model Subroutines | Main functions | Main inputs | Main outputs | Subroutine-Connections | Remarks |
|-------------------|----------------|-------------|--------------|------------------------|---------|
| **Soil module**   |                |             |              |                        |         |
| **Constants**     | Set the constants | Water vapor density, diffusivity, dispersion coefficient; dry air density, gas conductivity, flux; liquid water flux; top and bottom boundary conditions | Soil air pressure profile | Initializing the following subroutines | Initialization Group |
| **Dtrmn_Z**       | User define the vertical discretization Δz | Soil column depth, layer number | Thickness of each soil layer | - | - |
| **SOIL2**         | Calculate soil moisture $θ_L$ | Soil hydraulic parameters; soil matric potential; soil temperature | Soil hydraulic conductivity; soil water content | Initializing Group | Parameterization Group |
| **Latent**        | Calculate the latent heat $L$ | Soil temperature | Latent heat | --> Constants | - |
| **Evap_Cal**      | Calculate albedo, evaporation, and root water uptake | Soil moisture, temperature, meteorological forcing, time | Soil evaporation, resistance, albedo, root water uptake, transpiration | --> h_BC | Parameterization Group |
| **SOIL1**         | Update the wetting history | Soil moisture at previous and current time step, indicator of the wetting/drying status | Updated indicator of the wetting/drying status | --> MainLoop | Processing Group |
| **hPARAM**        | Calculate the matrices coefficient for liquid equation | Soil temperature, soil water content, matric potential, soil hydraulic conductivity, vapor density, diffusivity, dispersivity, volumetric fraction of vapor | Matrices coefficient for liquid equation | hPARAM --->; --> h_EQ, h_sub, | Processing Group |
| **h_MAT**         | Assemble the global coefficient matrices of the Galerkin expressions for liquid equation | Matrices coefficient for liquid equation | Global coefficient matrices for liquid equation | h_MAT --->; --> h_EQ, h_sub, | Processing Group |
| **h_EQ**          | Perform the finite difference of the time derivatives in the matrix equation | Global coefficient matrices for liquid equation | Updated right-hand side values | h_MAT --->; --> h_EQ, h_sub, | Processing Group |
| **h_BC**          | Set the boundary condition for solving liquid equation | Soil temperature, soil water content, matric potential, soil hydraulic conductivity, meteorological forcing, top and bottom boundary conditions | Global coefficient matrices at boundary nodes | h_EQ, h_BC --->; --> h_Bdry_Flux, h_sub, | Processing Group |
| **h_Solve**       | Solve the matrix equation for liquid conservation | Global coefficient matrices of all nodes | Updated soil matric potential profile | h_EQ, h_BC --->; --> h_Bdry_Flux, h_sub, | Processing Group |
| **h_Bdry_Flux**   | Calculate liquid flux of the boundary node | Updated soil matric potential profile | Top and bottom water fluxes | h_EQ, h_BC --->; --> h_sub, | Processing Group |
| **AirPARAM**      | Calculate the matrices coefficient for dry air equation | Dry air pressure, density, gas conductivity, flux; water vapor density, diffusivity, dispersion coefficient; soil matric potential, water content, temperature, conductivity | Matrices coefficient for dry air equation | Matrices coefficient for dry air equation | Processing Group |
| **Air_MAT**       | Assemble the global coefficient matrices of the Galerkin expressions for dry air equation | Matrices coefficient for dry air equation | Global coefficient matrices for dry air equation | AirPARAM --->; --> Air_EQ, Air_sub, | Processing Group |
| Module         | Description                                                                                                         | Details                                                                                      | Group            |
|---------------|---------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------|------------------|
| Air_EQ        | Perform the finite difference of the time derivatives in the matrix equation for dry air                           | Global coefficient matrices for dry air equation                                             | Processing Group |
| Air_BC        | Set the boundary condition for solving dry air equation                                                              | Top and bottom boundary conditions                                                          | Processing Group |
| Air_Solve     | Solve the matrix equation for dry air conservation                                                                  | Global coefficient matrices at boundary nodes                                              | Processing Group |
| EnergyPAR     | Calculate the matrices coefficient for energy equation                                                               | Soil temperature, soil water content, matric potential, soil hydraulic conductivity, vapor density, diffusivity, dispersivity, volumetric fraction of vapor, soil thermal properties, soil dry air pressure, conductivity, air flux | Processing Group |
| Energy_MAT    | Assemble the global coefficient matrices of the Galerkin expressions for energy equation                            | Matrices coefficient for energy equation                                                    | Processing Group |
| Energy_EQ     | Perform the finite difference of the time derivatives in the matrix equation for energy                            | Global coefficient matrices for energy equation                                             | Processing Group |
| Energy_BC     | Set the boundary condition for solving energy equation                                                               | Top and bottom boundary conditions                                                          | Processing Group |
| Energy_Solve  | Solve the matrix equation for energy conservation                                                                   | Global coefficient matrices at all nodes                                                    | Processing Group |
| Energy_Bndry_Flux | Calculate energy flux of the boundary node                                                                           | Soil temperature profile                                                                  | Processing Group |
| TimestepCHK   | Assessing the change in boundary conditions after one time step                                                     | Surface and bottom energy fluxes                                                            | Post-processing Group |
| CnvrngCHK     | Check the convergence                                                                                               | Surface boundary conditions, time step, indicators of the boundary condition change          | Post-processing Group |
| PlotResults   | Plot the results                                                                                                     | Soil state variables, convergence criteria, time step, indicators of the boundary condition change | Post-processing Group |
| Snowpack      | Calculate the atmospheric transmissivity                                                                            | Date, Campbell coefficient                                                                 | Parameterization Group |
| atf           | Date, Campbell coefficient                                                                                           | Atmospheric transmissivity                                                                  | Parameterization Group |
| Cosen         | Calculate the hourly radiation index                                                                                  | Date, slope, latitude                                                                       | Parameterization Group |
| hyri          | Date, slope, latitude                                                                                                | Hourly radiation index                                                                      | Parameterization Group |
| FMELT         | Calculate the melt rate and outflow                                                                                    | Energy content, snow water equivalent, snow saturated hydraulic conductivity, precipitation | Parameterization Group |
| JULIAN        | Convert the real date to julian date                                                                                   | Date (mm, dd)                                                                              | Parameterization Group |
### 4.3 Setup and Running the model

The current STEMMUS-UEB is tested with MATLAB 2019b. STEMMUS-UEB is executed in MATLAB by simply running `MainLoop.m` after you finish all the model setup and give the input data to STEMMUS-UEB. Several steps are necessary to build up the model setup.

1. Setting the temporal information and model domain;
2. Setting soil properties and snow properties;
3. Setting the initialization condition for soil and snow submodules, respectively;
4. Inputting the meteorological forcing information;
5. Setting the surface/bottom conditions.

Then you are ready to run STEMMUS-UEB by running `MainLoop.m`.

---

| Subroutine | Description |
|------------|-------------|
| PREHELP | Correct energy and mass fluxes when numerical overshoots |
| QFM | Calculate snow mass and energy fluxes |
| qlf | Compute the incoming longwave radiation |
| QPF | Calculate the heat advected to the snowpack |
| RKINST | Compute no neutral turbulent transfer coefficient |
| SRFTMP | Compute snow surface temperature |
| surfbe | Solve the surface energy balance for surface temperature |
| SVP | Calculate the vapor pressure over water or ice |
| SVPI | Calculate the vapor pressure over ice |
| SVPW | Calculate the vapor pressure over water |
| TAVG | Calculate the average temperature of snow and the interacting soil layer |
| TURBFLUX | Calculate the turbulent heat fluxes |
| UPDATETIME | Update time for each time step |

Note:  
---> means the relevant subroutines which are incoming to the current one, --> means the relevant subroutines for which the current subroutine is output to.
4.4 List of model variables

Table 4.3 summarizes the main model parameters/variables and divides them into input and output parameters/variables. Some of the value for the input parameters are also listed.

### Table 4.3. The descriptions of the main model input/output variables

| Symbol | Parameter | Unit | Value |
|--------|-----------|------|-------|
| **Main inputs** | | | |
| $a$ | Fitted parameter for soil surface resistance | - | 0.3565 |
| $b(z)$ | Normalized water uptake distribution | m$^{-1}$ | |
| $C_a$ | Specific heat capacity of dry air | J kg$^{-1}$ °C$^{-1}$ | 1.005 |
| $C_{app}$ | Apparent heat capacity | J kg$^{-1}$ °C$^{-1}$ | |
| $C_i$ | Specific heat capacity of ice | J kg$^{-1}$ °C$^{-1}$ | 2.0455 |
| $C_L$ | Specific heat capacity of liquid | J kg$^{-1}$ °C$^{-1}$ | 4.186 |
| $C_s$ | Specific heat capacity of soil solids | J kg$^{-1}$ °C$^{-1}$ | |
| $C_{soil}$ | Heat capacity of the bulk soil | J kg$^{-1}$ °C$^{-1}$ | |
| $C_V$ | Specific heat capacity of water vapor | J kg$^{-1}$ °C$^{-1}$ | 1.87 |
| $c_p$ | Specific heat capacity of air | J kg$^{-1}$ K$^{-1}$ | |
| $D_e$ | Molecular diffusivity of water vapor in soil | m$^2$ s$^{-1}$ | |
| $D_{TD}$ | Transport coefficient for adsorbed liquid flow due to temperature gradient | kg m$^{-1}$ s$^{-1}$ °C$^{-1}$ | |
| $D_{va}$ | Adveive vapor transfer coefficient | s | |
| $D_{vg}$ | Gas phase longitudinal dispersion coefficient | m$^2$ s$^{-1}$ | |
| $D_{vh}$ | Isothermal vapor conductivity | kg m$^2$ s$^{-1}$ | |
| $D_{VT}$ | Thermal vapor diffusion coefficient | kg m$^{-1}$ s$^{-1}$ °C$^{-1}$ | |
| $H_e$ | Henry’s constant | - | 0.02 |
| $K$ | Hydraulic conductivity | m s$^{-1}$ | |
| $K_g$ | Intrinsic air permeability | m$^2$ | |
| $K_{lh}$ | Isothermal hydraulic conductivities | m s$^{-1}$ | |
| $K_{LT}$ | Thermal hydraulic conductivities | m$^2$ s$^{-1}$ °C$^{-1}$ | |
| $K_s$ | Soil saturated hydraulic conductivity | m s$^{-1}$ | |
| $L_0$ | Latent heat of vaporization of water at the reference temperature | J kg$^{-1}$ | |
| $LAI_{eff}$ | Effective leaf area index | - | |
| $L_f$ | Latent heat of fusion | J kg$^{-1}$ | 3.34E+05 |
| $n$ | Van Genuchten fitting parameters | - | |
| $r_a^c$ | Aerodynamic resistance for canopy surface | s m$^{-1}$ | |
| $r_a^s$ | Aerodynamic resistance for bare soil | s m$^{-1}$ | |
| $r_{c_{min}}$ | Minimum canopy surface resistance | s m$^{-1}$ | |
| $r_{l_{min}}$ | Minimum leaf stomatal resistance | s m$^{-1}$ | |
| $r_s$ | Soil surface resistance | s m$^{-1}$ | |
| $r_{st}$ | Resistance to molecular diffusion of the water surface | s m$^{-1}$ | 10 |
| $R_n$ | Net radiation | MJ m$^2$ day$^{-1}$ | |
| $R_n^c$ | Net radiation at the canopy surface | MJ m$^2$ day$^{-1}$ | |
| $R_n^s$ | Net radiation at the soil surface | MJ m$^2$ day$^{-1}$ | |
| Symbol | Description                                             | Unit       |
|--------|---------------------------------------------------------|------------|
| $S_a$  | Degree of saturation of the soil air                    | -          |
| $S_L$  | Degree of water saturation in the soil                  | -          |
| $S_p$  | Potential water uptake rate                             | s$^{-1}$   |
| $t$    | Time                                                   | s          |
| $T_p$  | Potential transpiration                                 | m s$^{-1}$ |
| $T_r$  | Arbitrary reference temperature                         | °C 20      |
| $W$    | Differential heat of wetting                             | J kg$^{-1}$|
| $z$    | Vertical space coordinate (positive upwards)            | m          |
| $\alpha$ | Air entry value of soil                                | m$^{-1}$   |
| $\alpha(b)$ | Reduction coefficient related to soil water potential | -         |
| $\varepsilon$ | Porosity                                                    | -         |
| $\lambda_{eff}$ | Effective thermal conductivity of the soil             | W m$^{-1}$°C$^{-1}$ |
| $\theta_i$ | Volumetric fraction of solids in the soil               | m$^3$ m$^{-3}$ |
| $\theta_{sat}$ | Saturated soil water content                           | m$^3$ m$^{-3}$ |
| $\theta_r$ | Residual soil water content                             | m$^3$ m$^{-3}$ |
| $\theta_t$ | Topsoil water content                                  | m$^3$ m$^{-3}$ |
| $\theta_{min}$ | Minimum water content above which soil is able to deliver vapor at a potential rate | m$^3$ m$^{-3}$ |
| $\rho_o$ | Air density                                             | kg m$^{-3}$ |
| $\rho_{da}$ | Density of dry air                                      | kg m$^{-3}$ |
| $\rho_i$ | Density of ice                                          | kg m$^{-3}$ 920 |
| $\rho_L$ | Density of soil liquid water                            | kg m$^{-3}$ 1000 |
| $\rho_s$ | Density of solids                                       | kg m$^{-3}$ |
| $\rho_v$ | Density of water vapor                                  | kg m$^{-3}$ |
| $\gamma_W$ | Specific weight of water                               | kg m$^2$ s$^{-2}$ |
| $\mu_a$ | Air viscosity                                           | kg m$^2$ s$^{-1}$ |

**Snow model component (UEB)**

| Symbol | Description                                             | Unit       |
|--------|---------------------------------------------------------|------------|
| $T_r$  | Air temperature above which precipitation is all rain   | °C 3.5     |
| $T_{sn}$ | Air temperature below which precipitation is all snow | °C 0       |
| $\varepsilon_{sn}$ | Emissivity of snow                                      | 0.99        |
| $C_g$  | Ground heat capacity                                    | J kg$^{-1}$°C$^{-1}$ 2.09 |
| $z_o$  | Snow surface aerodynamic roughness                       | m 0.001    |
| $L_c$  | Liquid holding capacity of snow                         | - 0.05     |
| $K_{sn}$ | Snow saturated hydraulic conductivity                    | m h$^{-1}$ 160 |
| $\alpha_{tn}$ | Visual new snow albedo                                 | - 0.95     |
| $\alpha_{tn}$ | Near-infrared new snow albedo                           | - 0.65     |
| $\alpha_{bg}$ | Bare ground albedo                                     | - Eqs. 3.22-3.26 |
| $D_c$  | Thermally active depth of soil                          | m 0.4      |
| $\lambda_{sn}$ | Snow surface thermal conductivity                       | m h$^{-1}$ 0.02 |
| $\rho_s$ | Snow density                                            | kg m$^{-3}$ 450 |
| $\rho_{ed}$ | Albedo extinction depth                                 | m 0.0001   |
| $F_c$  | Forest cover fraction                                   | - 0        |
| $D_f$  | Drift factor                                            | - 1        |
| $\rho_s$ | Soil density                                            | kg m$^{-3}$ 1700 |
| Main outputs                                                                 |
|----------------------------------------------------------------------------|
| Soil model component ( STEMMUS-FT )                                        |
| $\psi$  | Soil water potential            | m                                      |
| $P_s$   | Mixed pore-air pressure         | Pa                                     |
| $T$     | Soil temperature                | °C                                     |
| $\theta$ | Volumetric water content        | m$^3$ m$^{-3}$                          |
| $\theta_i$ | Soil ice volumetric water content | m$^3$ m$^{-3}$                         |
| $\theta_L$ | Soil liquid volumetric water content | m$^3$ m$^{-3}$                       |
| $\theta_V$ | Soil vapor volumetric water content | m$^3$ m$^{-3}$                      |
| $\theta_a$ | Volumetric fraction of dry air in the soil | m$^3$ m$^{-3}$                     |
| $q$     | Water flux                      | kg m$^2$ s$^{-1}$                      |
| $q_a$   | Dry air flux                    | kg m$^2$ s$^{-1}$                      |
| $q_L$   | Soil liquid water fluxes (positive upwards) | kg m$^2$ s$^{-1}$                  |
| $q_{La}$ | Liquid water flux driven by the gradient of air pressure | kg m$^2$ s$^{-1}$              |
| $q_{Lh}$ | Liquid water flux driven by the gradient of matric potential | kg m$^2$ s$^{-1}$           |
| $q_{LT}$ | Liquid water flux driven by the gradient of temperature | kg m$^2$ s$^{-1}$            |
| $q_V$   | Soil water vapor fluxes (positive upwards) | kg m$^2$ s$^{-1}$                 |
| $q_{Va}$ | Water vapor flux driven by the gradient of air pressure | kg m$^2$ s$^{-1}$           |
| $q_{Vh}$ | Water vapor flux driven by the gradient of matric potential | kg m$^2$ s$^{-1}$            |
| $q_{VT}$ | Water vapor flux driven by the gradient of temperature | kg m$^2$ s$^{-1}$            |
| $S$     | Sink term for transpiration     | s$^{-1}$                               |
| $S_h$   | Latent heat flux density        | W m$^{-3}$                             |

| Snow model component (UEB)                                                 |
|----------------------------------------------------------------------------|
| $P_r$   | Precipitation in the form of rain | m s$^{-1}$                             |
| $P_s$   | Precipitation in the form of snow | m s$^{-1}$                             |
| $\text{SWE}$ | Snow water equivalent          | m                                      |
| $Q_h$   | Surface Sensible Heat Flux      | W m$^{-2}$                             |
| $Q_e$   | Surface Latent Heat Flux        | W m$^{-2}$                             |
| $E$     | Surface Sublimation             | m s$^{-1}$                             |
| $T_{SS}$ | Snow Surface Temperature        | °C                                     |
| $U$     | Energy Content                  |                                        |
| $M_r$   | Melt outflow rate               | m s$^{-1}$                             |
| $A_{alb}$ | Surface Albedo                 | -                                      |
| $Q_m$   | Heat advected by melt outflow   | W m$^{-2}$                             |
| $Q_{sh}$ | Net shortwave radiation        | W m$^{-2}$                             |
| $Q_{li}$ | Net longwave radiation         | W m$^{-2}$                             |
| $\tau$  | No-dimensional snow age        | -                                      |
5 Additional results: Understanding the water/heat transfer mechanisms in frozen soil

This section presents the example modelling results, illustrating the model capability in terms of detailed interpretation of water/heat transfer mechanisms. The analysis of water fluxes is shown in Section 5.1 (see Yu et al., 2018 for detail). Section 5.2 conducted the heat budget analysis (see Yu et al., 2020b for detail).

5.1 Water flux analysis

Figure 5.1. Observed latent heat flux and simulated (a) latent heat flux and (b) surface soil (0.1cm) thermal and isothermal liquid water and vapor fluxes \( (LE, q_{VT}, q_{Vh}, q_{LT}, q_{Lh}) \) (c) surface soil (0.1cm) advective liquid water and vapor fluxes \( (q_{La}, q_{Va}) \) of a typical five-day freezing period (from 8th to 12th Days after Dec. 1, 2015). LE is the latent heat flux, \( q_{VT}, q_{Vh} \) are the water vapor fluxes driven by temperature and matric potential gradients, \( q_{LT}, q_{Lh} \) are the liquid water fluxes driven by temperature and matric potential gradients, \( q_{La}, q_{Va} \) are the liquid and vapor water fluxes driven by air pressure gradients. Positive/negative values indicate upward/downward fluxes.
Figure 5.2. Simulated vertical profiles of the thermal and isothermal liquid water and vapor fluxes, soil ice content at 1200 and 0000 h of a typical freezing period during 11th and 12th Days after Dec. 1. 2015. Positive/negative values indicate upward/downward fluxes. Solid lines and dot lines represent for the fluxes and soil moisture, temperature and ice content profile on the 11th and 12th Days after Dec. 1. 2015, respectively.
Figure 5.3. Simulated vertical profiles of the air pressure induced liquid water and vapor fluxes, soil air pressure gradient, soil ice content, liquid water content and soil temperature at 1200 and 0000 h of a typical freezing period during 11<sup>th</sup> and 12<sup>th</sup> Days after Dec. 1. 2015. Positive/negative values indicate upward/downward fluxes. Solid lines and dot lines represent for the fluxes and soil moisture, temperature and ice content profile on the 11<sup>th</sup> and 12<sup>th</sup> Days after Dec. 1. 2015, respectively.
Figure 5.4. Spatial and temporal variations of (a) temperature gradient, (b) matric potential gradient and (c) air pressure gradient at surface soil layers (top 2cm, upper figure) and deeper soil layers (2-30cm, bottom figure), respectively, of a typical freezing period during 8th and 12th Days after Dec. 1, 2015.
Figure 5.5. The spatial and temporal distributions of (a, and b) thermal liquid water, and vapor fluxes, (c, and d) isothermal liquid water, and vapor fluxes, (e, and f) advective liquid water, and vapor fluxes, at surface soil layers (top 2cm, upper figure) and deeper soil layers (2-30cm, bottom figure), respectively, of a typical freezing period during 8th and 12th Days after Dec. 1, 2015. Note that the unit for the fluxes is g cm$^{-2}$ s$^{-1}$. 
Figure 5.6. Same as Figure 8 but for a typical five-day thawing period (from 87th to 91th Days after Dec. 1. 2015).
Figure 5.7. Simulated vertical profiles of the thermal and isothermal liquid water and vapor fluxes, soil ice content at 1200 and 0000 h of a typical freezing period during 90th and 91th Days after Dec. 1, 2015. Positive/negative values indicate upward/downward fluxes. Solid lines and dot lines represent for the fluxes and soil moisture, temperature and ice content profile on the 90th and 91th Days after Dec. 1, 2015, respectively.
5.2 Heat budget analysis

Figure 5.8. Time series of model simulated heat budget components at the soil depth of 5cm using (a &d) Basic Coupled Model (BCM), (b &e) Advanced Coupled Model (ACM), and (c &f) Advanced Coupled Model with Air flow (ACM-AIR) simulations during the typical 6-day freezing (left column) and freezing-thawing transition (right column) periods. HC, change rate of heat content, CHF, conductive heat flux divergence, HFL, convective heat flux divergence due to liquid water flow, HFV, convective heat flux divergence due to air flow, LHF, latent heat flux divergence. Note that for graphical purposes, HFL, HFV, HFa, and LHF were enhanced by a factor of 10 during the freezing period.
Figure 5.9. The spatial and temporal distributions of model estimated soil latent heat flux density using (a &d) Advanced Coupled Model (ACM), (b &e) Advanced Coupled Model with Air flow (ACM-AIR) and (c &f) the difference between ACM and ACM-AIR simulations ($S_h^{ACM-AIR} - S_h^{ACM}$) during the typical 6-day freezing and freezing-thawing transition periods. The left and right column are for the freezing and freezing-thawing transition period, respectively. Note that figures for the Basic Coupled Model (BCM) are absent as it can not simulate the subsurface soil latent heat flux density.
CLAPP, R. B., and HORNBERGER, G. M.: Empirical equations for some soil hydraulic properties, Water Resour Res, 14, 601-604, https://doi.org/10.1029/WR014i004p00601, 1978.

CLARK, M. P., NIJSSEN, B., LUNDQUIST, J. D., KAVETSKI, D., RUPP, D. E., WOODS, R. A., FREER, J. E., GUTTMANN, E. D., WOOD, A. W., BREKKE, L. D., ARNOLD, J. R., GOCHIS, D. J., and RASMUSSEN, R. M.: A unified approach for process-based hydrologic modeling: I. Modeling concept, Water Resour Res, 51, 2498-2514, https://doi.org/10.1002/2015wr017198, 2015.

DALL'AMICO, M.: Coupled water and heat transfer in permafrost modeling, University of Trento, 2010.

DE VRIES, D. A.: Simultaneous transfer of heat and moisture in porous media, Eos, Transactions American Geophysical Union, 39, 909-916, https://doi.org/10.1029/TR039i005p00909, 1958.

de Vries, D. A.: Thermal properties of soils, Physics of Plant Environment, edited by: van Wijk, W. R., North-Holland Publishing Company, Amsterdam, 210-235 pp., 1963.

DICKINSON, R. E., HENDERSON-SELLERS, A., and KENNEDY, P. J.: Biosphere-atmosphere Transfer Scheme (BATS) Version 1c as Coupled to the NCAR Community Climate Model (No. NCAR/TN-387-STR), University Corporation for Atmospheric Research, 1993.

EDLEFSEN, N., and ANDERSON, A.: Thermodynamics of soil moisture, Hilgardia, 15, 31-298, 1943.

FAROUKI, O. T.: The thermal properties of soils in cold regions, Cold Regions Sci Tech, 5, 67-75, https://doi.org/10.1016/0165-232X(81)90041-0, 1981.

FEDDES, R. A., KOWALIK, P. J., and ZARADNY, H.: Simulation of field water use and crop yield, Centre for Agricultural Publishing and Documentation, Wageningen, the Netherlands, 189 pp., 1978.

GALLE, M. R., and GRIGAL, D. F.: Vertical root distributions of northern tree species in relation to successional status, Canadian Journal of Forest Research, 17, 829-834, https://doi.org/10.1139/x87-131, 1987.

GARDNER, M. J., ELLIS-EVANS, J. C., ANDERSON, M. G., and TRANTER, M.: Snowmelt modelling on Signy Island, South Orkney Islands, Ann Glaciol, 26, 161-166, https://doi.org/10.3189/1998aog26-1-161-166, 1998.

GICHAMO, T. Z., and TARBOTON, D. G.: Ensemble Streamflow Forecasting Using an Energy Balance Snowmelt Model Coupled to a Distributed Hydrologic Model with Assimilation of Snow and Streamflow Observations, Water Resour Res, 55, 10813-10838, https://doi.org/10.1029/2019wr025472, 2019.

HANSSON, K., SIMÅNEK, J., MIZOGUCHI, M., LUNDIN, L. C., and VAN GENUCHTEN, M. T.: Water flow and heat transport in frozen soil: Numerical solution and freeze-thaw applications, Vadose Zone J, 3, 693-704, 2004.

JACKSON, R. B., CANADELL, J., EHLERINGER, J. R., MOONEY, H. A., SALA, O. E., and SCHULZE, E. D.: A Global Analysis of Root Distributions for Terrestrial Biomes, Oecologia, 108, 389-411, 1996.

JOHANSEN, O.: Thermal conductivity of soils, PhD, University of Trondheim, 236 pp., 1975.

KAY, B. D., and GROENEVELT, P. H.: On the Interaction of Water and Heat Transport in Frozen and Unfrozen Soils: I. Basic Theory; The Vapor Phase, Soil Sci Soc Am J, 38, 395-400, https://doi.org/10.2136/sssaj1974.03615995003800030011x, 1974.

KOOPMANS, R. W. R., and MILLER, R. D.: Soil Freezing and Soil Water Characteristic Curves, Soil Sci Soc Am J, 30, 680-685, https://doi.org/10.2136/sssaj1966.03615995003000060011x, 1966.

MILLY, P. C. D.: Moisture and heat transport in hysteretic, inhomogeneous porous media: A matric head-based formulation and a numerical model, Water Resour Res, 18, 489-498, https://doi.org/10.1029/WR018i003p00489, 1982.

MUALEM, Y.: New model for predicting the hydraulic conductivity of unsaturated porous media, Water Resour Res, 12, 513-522, 1976.

PHILIP, J. R., and VRIES, D. A. D.: Moisture movement in porous materials under temperature gradients, Eos, Transactions American Geophysical Union, 38, 222-232, https://doi.org/10.1029/TR038i002p00222, 1957.

PIMENTEL, R., HERRERO, J., ZENG, Y., SU, Z., and POLO, M. J.: Study of Snow Dynamics at Subgrid Scale in Semiarid Environments Combining Terrestrial Photography and Data Assimilation Techniques, J Hydrometeorol, 16, 563-578, https://doi.org/10.1175/jhm-d-14-0046.1, 2015.

PRUNTY, L.: Soil water heat of transport, Journal of Hydrologic Engineering, 7, 435-440, 2002.

RICHARDS, L. A.: Capillary Conduction of Liquids Through Porous Mediums, Physics, 1, 318, 1931.

SCHULZ, O., and DE JONG, C.: Snowmelt and sublimation: field experiments and modelling in the High Atlas Mountains of Morocco, Hydrol Earth Syst Sci, 8, 1076-1089, https://doi.org/10.5194/hess-8-1076-2004, 2004.
Sultana, R., Hsu, K. L., Li, J., and Sorooshian, S.: Evaluating the Utah Energy Balance (UEB) snow model in the Noah land-surface model, Hydrol Earth Syst Sci, 18, 3553-3570, https://doi.org/10.5194/hess-18-3553-2014, 2014.

Tarboton, D. G., and Luce, C. H.: Utah Energy Balance Snow Accumulation and Melt Model (UEB), Computer model technical description and users guide, Utah Water Research Laboratory and USDA Forest Service Intermountain Research Station, 1996.

Tarnawski, V. R., and Wagner, B.: A new computerized approach to estimating the thermal properties of unfrozen soils, Can Geotech J, 29, 714-720, https://doi.org/10.1139/t92-079, 1992.

Tarnawski, V. R., and Wagner, B.: Modeling the thermal conductivity of frozen soils, Cold Regions Sci Tech, 22, 19-31, https://doi.org/10.1016/0165-232X(93)90043-8, 1993.

Taylor, G. S., and Luthin, J. N.: A model for coupled heat and moisture transfer during soil freezing, Can Geotech J, 15, 548-555, https://doi.org/10.1139/t78-058, 1978.

Tian, Z., Lu, Y., Horton, R., and Ren, T.: A simplified de Vries-based model to estimate thermal conductivity of unfrozen and frozen soil, Eur J Soil Sci, 67, 564-572, https://doi.org/10.1111/ejss.12366, 2016.

Van Genuchten, M. T.: A closed-form equation for predicting the hydraulic conductivity of unsaturated soils, Soil Sci Soc Am J, 44, 892-898, https://doi.org/10.2136/SSSAJ1980.03615995004400050002X, 1980.

Vandegriend, A. A., and Owe, M.: BARE SOIL SURFACE-RESISTANCE TO EVAPORATION BY VAPOR DIFFUSION UNDER SEMIARID CONDITIONS, Water Resour Res, 30, 181-188, https://doi.org/10.1029/93wr02747, 1994.

Wang, Y., Zeng , Y., Yu, L., Yang, P., Van de Tol, C., Cai, H., and Su, Z.: Integrated Modeling of Photosynthesis and Transfer of Energy, Mass and Momentum in the Soil-Plant-Atmosphere Continuum System, Geosci Model Dev Discuss, 2020, 1-37, https://doi.org/10.5194/gmd-2020-85, 2020.

Watson, F. G. R., Newman, W. B., Coughlan, J. C., and Garrott, R. A.: Testing a distributed snowpack simulation model against spatial observations, J Hydrol, 328, 453-466, https://doi.org/10.1016/j.jhydrol.2005.12.012, 2006.

Yang, K., Chen, Y. Y., and Qin, J.: Some practical notes on the land surface modeling in the Tibetan Plateau, Hydrol Earth Syst Sci, 13, 687-701, https://doi.org/10.5194/hess-13-687-2009, 2009.

Yu, L., Zeng, Y., Su, Z., Cai, H., and Zheng, Z.: The effect of different evapotranspiration methods on portraying soil water dynamics and ET partitioning in a semi-arid environment in Northwest China, Hydrol Earth Syst Sci, 20, 975-990, https://doi.org/10.5194/hess-20-975-2016, 2016.

Yu, L., Zeng, Y., Yang, P., Van de Tol, C., Cai, H., and Su, Z.: Understanding the mass, momentum, and energy transfer in the frozen soil with three levels of model complexities, Hydrol Earth Syst Sci, 24, 4813-4830, https://doi.org/10.5194/hess-24-4813-2020, 2020.

Zheng, D., Van der Velde, R., Su, Z., Wen, J., and Wang, X.: Under-canopy turbulence and root water uptake of a Tibetan meadow ecosystem modeled by Noah-MP, Water Resour Res, 51, 5735-5755, https://doi.org/10.1002/2015wr017115, 2015.
## Supplement

### Table S1. The description of measurements and its temporal resolution deployed as inputs/outputs of the model

| Model/Measurements | Time Period | Time Interval | Notes |
|--------------------|-------------|---------------|-------|
| **Meteorological Inputs** | | | |
| Precipitation | 2015/12/1 - 2016/3/15 | 3 hourly | From weather station, about 12 km away from the study site. In order to meet the input requirement for the adaptive time step simulation, the precipitation was evenly distributed within the three hours. |
| Air Temperature | 2015/12/1 - 2016/3/15 | 30 min | From the in situ meteorological station. The time disaggregated values, to meet the requirement for the adaptive time step simulation (1 s - 30 mins), were obtained by the linear interpolation between the half-hour measurements. |
| Air Relative Humidity | 2015/12/1 - 2016/3/15 | 30 min | From the in situ meteorological station. The time disaggregated values, to meet the requirement for the adaptive time step simulation (1 s - 30 mins), were obtained by the linear interpolation between the half-hour measurements. |
| Wind Speed | 2015/12/1 - 2016/3/15 | 30 min | From the in situ meteorological station. The time disaggregated values, to meet the requirement for the adaptive time step simulation (1 s - 30 mins), were obtained by the linear interpolation between the half-hour measurements. |
| Air pressure | 2015/12/1 - 2016/3/15 | 30 min | From the in situ meteorological station. The time disaggregated values, to meet the requirement for the adaptive time step simulation (1 s - 30 mins), were obtained by the linear interpolation between the half-hour measurements. |
| Four component downwelling and upwelling solar and thermal radiation | 2015/12/1 - 2016/3/15 | 30 min | From the in situ meteorological station. The time disaggregated values, to meet the requirement for the adaptive time step simulation (1 s - 30 mins), were obtained by the linear interpolation between the half-hour measurements. |
| **Model** | | | For all simulations, the adaptive time step was deployed. |
| STEMMUS/UEB | 2015/12/1 - 2016/3/15 | From 1 s to 30 mins | |
| **Outputs** | | | |
| Soil Moisture | 2015/12/1 - 2016/3/15 | 15 min | From the in situ 5TM ECH2O sensors, installed at 5 cm, 10 cm, 20 cm, 40 cm and 80 cm. |
| Soil Temperature | 2015/12/1 - 2016/3/15 | 15 min | From the in situ 5TM ECH2O sensors, installed at 5 cm, 10 cm, 20 cm, 40 cm and 80 cm. |
| Albedo | 2015/12/1 - 2016/3/15 | 30 min | The albedo was derived as the ration of half-hourly upwelling shortwave radiation to downwelling shortwave radiation measurements. The data during the nighttime was filtered out. |
| Latent heat flux | 2015/12/1 - 2016/3/15 | 30 min | From the installed Eddy Covariance (EC150) system |
| Parameter                                | Unit       | Value                      | Remarks                                                                 |
|------------------------------------------|------------|----------------------------|--------------------------------------------------------------------------|
| Soil Clay content                        | %          | 9.00 @ 0-10 cm; 10.12 @ 10-40 cm; 5.59 @ 40-160 cm; 44.13 @ 0-10 cm; | Soil texture, site-specific (can be obtained from the in-situ measurements, global soil texture maps) |
| Soil sand content                        | %          | 44.13 @ 0-10 cm; 44.27 @ 10-40 cm; 65.55 @ 40-160 cm; 1.45 @ 0-10 cm; |                                                                   |
| Soil saturated conductivity $K_s$         | $10^{-6}$ m s$^{-1}$ | 0.94 @ 10-40 cm; 0.68 @ 40-160 cm; | Soil hydraulic parameters, site-specific (can be obtained from in-situ/laboratory measurements, or derived from soil texture information) |
| Soil saturated volumetric content $θ_s$  | m$^3$ m$^{-3}$ | 0.5                        |                                                                   |
| Soil residual water content $θ_r$        | m$^3$ m$^{-3}$ | 0.035                      |                                                                   |
| Air entry value                          | m$^3$      | 0.041                      |                                                                   |
| VG fitting parameter $n$                 | -          | 1.332                      |                                                                   |
| Specific heat of water                   | KJ Kg$^{-1}$ K$^{-1}$ | 4.18                      |                                                                   |
| Specific heat of ice                     | KJ Kg$^{-1}$ K$^{-1}$ | 2.09                      |                                                                   |
| Specific heat of air                     | KJ Kg$^{-1}$ K$^{-1}$ | 1.005                     | Thermal properties of soil constituents, Constant                   |
| Water heat conductivity                  | W m$^{-1}$ K$^{-1}$ | 0.6                       |                                                                   |
| Ice heat conductivity                    | W m$^{-1}$ K$^{-1}$ | 2.2                       |                                                                   |
| Air heat conductivity                    | W m$^{-1}$ K$^{-1}$ | 0.026                     |                                                                   |
| Temperature threshold for rainfall       | °C         | 3.5                        | Partition precipitation, can be adjusted                             |
| Temperature threshold for snowfall       | °C         | 0                          | For the calculation of meltwater outflow, default value              |
| Snow density                             | Kg/m$^3$   | 450                        | Snow energy balance components, default value                         |
| Snow emissivity                          | -          | 0.99                       |                                                                   |
| Reflectance for new snow at visual bands | -          | 0.95                       | For the calculation of snow albedo, calibrated locally               |
| Reflectance for new snow at near-infrared bands | -          | 0.65                       |                                                                   |
| Snow surface roughness                   | m          | 0.001                      | For the calculation of energy balance components, calibrated locally |
| Snow saturated hydraulic conductivity    | m h$^{-1}$ | 160                        | For the calculation of the meltwater outflow, calibrated             |
| Snow surface thermal conductance         | m h$^{-1}$ | 0.02                       | For the calculation of snow energy balance components, default value |
| Thermally active depth of soil           | m          | 0.4                        | For the calculation of snow energy balance components, default value |


### Table S3. A general overview of Utah energy balance (UEB) snowmelt model related researches from the perspective of model development and applications

| Study | Research aim, modelling/application perspective | Method/Data used | Study region | Model capability/utilities/focus/highlights |
|-------|-----------------------------------------------|------------------|--------------|--------------------------------------------|
| **UEB model development/extension** | | | | |
| Tarboton et al. (1995); Tarboton and Luce (1996) | Developing a distributed snowmelt model UEB | Meteorological inputs: air temperature, wind speed, humidity, precipitation and total incoming solar and longwave radiation; site information | Central Sierra Snow Laboratory, California, USA; Reynolds Creek Experimental Watershed, Boise Idaho, USA; and the Utah State University drainage and evapotranspiration research farm, Logan, Utah, USA | Snow surface temperature, bulk temperature, snow water equivalent, melt outflow; snow sublimation/ablation, |
| Hellstrom (2000) | Developing the forest cover algorithms in UEB and test its performance for coniferous and deciduous forest | Meteorological inputs; canopy architecture measurements: vegetation area index (VAI), sky view factor (SVF), forest canopy closure (FC); site information | Northern Michigan, USA | Canopy processes including attenuation of solar radiation and wind speed, the mixed sky and canopy components of longwave irradiance, and precipitation interception by canopy elements; more realistic atmospheric stability algorithm, |
| Mahat and Tarboton (2012) | Better estimating the radiation energy within and beneath the forest canopy in UEB | Meteorological inputs, vegetation properties, site information | Rocky Mountains in Utah, USA | Two stream radiation transfer model that explicitly accounts for canopy scattering, absorption and reflection, |
| Mahat and Tarboton (2014) | Representing the canopy snow interception, unloading and melt in UEB | Meteorological inputs, vegetation properties, site information | Rocky Mountains in Utah, USA | New UEB model algorithms that represent the processes of canopy snow interception, sublimation, mass unloading and melt, Modified force-restore approach; adjust effective conductivity considering the presence of ground near to a shallow snow surface; representing the penetration of the refreezing front following melt, |
| You et al. (2014) | Improve snow surface temperature modelling | Meteorological inputs; site information | Central Sierra Snow Laboratory, CA, Utah State University experimental farm, USA, and subnivean snow laboratory at Niwot Ridge, USA | Hydrological model with topographical effect, surface water and streamflow, |
| Sen Gupta et al. (2015) | Developing a modelling framework facilitating the integration of UEB, hydrologic model BASINS, and GeoSFM | Gridded meteorological forcing, DEM, vegetation variables, land cover, glacier outlines and albedo, hydrological data | Langtang Khola watershed (Himalaya), Nepal | UEB snowmelt model with assimilation of SWE using ensemble Kalman filter, Sacramento Soil Moisture Accounting (SAC-SMA), rutpix7 stream routing model with assimilation of streamflow observation using particle filter, Two parallel versions of UEB model, one using the Message Passing Interface (MPI) and the other using NVIDIA’s CUDA code on Graphics Processing Unit (GPU), |
| Gichamo and Tarboton (2019) | Coupling UEB to hydrologic model SAC-SMA together with assimilation of snow and streamflow observations | Gridded meteorological forcing, vegetation properties, watershed domain variables (e.g., slope, aspect), hydrological data, and SWE & discharge data for assimilation | Green River watershed, Salt Lake City, USA | |
| Gichamo and Tarboton (2020) | Developing UEB parallel for the simulation of snow process using parallel computing | Gridded meteorological forcing, vegetation properties, watershed domain variables (e.g., slope, aspect), in NetCDF format | Logan River watershed, Utah, USA | First application of UEB in Antarctic, |
| **UEB model applications** | | | | |
| Gardiner et al., (1998) | Testing UEB in terms of SWE | Meteorological inputs, site information | Paternoster Valley, Signy Island, South Orkney Islands, Antarctic | Snowmelt and sublimation/ablation, |
| Schulz and de Jong (2004) | Testing UEB in terms of snowmelt and sublimation | Meteorological variables, site information | High Atlas Mountains of Morocco, Morocco | UEB considering glacier ice melt over clean and debris-covered tongues, Geospatial Stream Flow Model (GeoSFM), BASINS model, streamflow, |
| Brown et al. (2014) | Estimating the contribution of glacier and snowmelt to stream flow using integrated modelling system (UEB, GeoSFM, BASINS) | Downscaled NASA satellite based and earth system data products, in-situ hydrologic data | Langtang Khola watershed (Himalaya), Nepal | Snow surface temperature, snowmelt event, SWE, |
| Sultana et al. (2014) | Resolve the underestimation of SWE | Meteorological forcing from NLDAS-2, site information | NRCS SNOTEL stations, California, USA; T.W. Daniel | |

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| Study                          | Methodology                                                                 | Site Information                                                                                                                                                                                                 |
|-------------------------------|-----------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Pimentel et al. (2015)         | Improving snow cover simulation over mountainous regions with highly irregular distribution | High-frequency images were combined with UEB model to reproduce snow evolution at cell scale (30 m × 30 m) by means of the assimilation of the snow cover fraction observation dataset obtained from terrestrial photography. Sierra Nevada, southern Spain |
| Raleigh et al. (2015)          | Diagnosing the sensitivity/impact of forcing error characteristics on snow simulations | Site information, meteorological forcing with various error characteristics. Innovait Creek site in Alaska, USA; the maritime Col de Porte site in the Rhône-Alpes of France, France; the intermountain Reynolds Mountain East sheltered site in the Owyhee Range in Idaho, USA; the continental Swamp Angel Study Plot site in the San Juan Mountains of Colorado, USA |
| Watson et al. (2006)           | Testing distributed UEB                                                    | Daily precipitation and temperature data, and 28.5-m maps of mean annual precipitation, terrain, vegetation, and geothermal heat flux. Meteorological data and remotely sensed data from Landsat ETM+, IRS P-6 LISS-III and MODIS 8-day snow cover data product. SNOTEL sites, USA |
| Khanduri and Thakur (2020)     | Testing UEB in terms of snowmelt runoff                                   | Meteorological data and remotely sensed data from Landsat ETM+, IRS P-6 LISS-III and MODIS 8-day snow cover data product. Spatial downscaling of the China meteorological forcing dataset (CMFD) coupled with other parameters, the model simulates the total surface water balance using surface water input from snowmelt, glacial melt and rainfall. Himachal Pradesh state, India |
| Liu et al. (2020)              | Testing UEB in terms of glacier- and snowmelt-driven streamflow           | Spatial SWE, requires improvements of snow interception, and snowpack thermal dynamics for tested regions. Middle Tianshan Mountains, China |

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Figure S1. Observed latent heat flux and simulated (a, c & e) latent heat flux and (b, d & f) surface soil liquid water content \( \theta_L \) with/without snow module of a typical five-day freezing period (from 10\textsuperscript{th} to 15\textsuperscript{th} Day after Dec. 1, 2015) with precipitation. LE is the latent heat flux.
Figure S2. Observed latent heat flux and simulated (a, c & e) latent heat flux and (b, d & f) surface soil liquid water content $\theta_L$ with/without snow module of a typical five-day thawing period (from 100th to 105th Day after Dec. 1. 2015) with precipitation. LE is the latent heat flux.
Reference

Brown, M. E., Racoviteanu, A. E., Tarboton, D. G., Gupta, A. S., Nigro, J., Policelli, F., Habib, S., Tokay, M., Shrestha, M. S., Bajracharya, S., Hummel, P., Gray, M., Duda, P., Zaitchik, B., Mahat, V., Artan, G., and Tokar, S.: An integrated modeling system for estimating glacier and snow melt driven streamflow from remote sensing and earth system data products in the Himalayas, J Hydrol, 519, 1859-1869, https://doi.org/10.1016/j.jhydrol.2014.09.050, 2014.

Gardiner, M. J., Ellis-Evans, J. C., Anderson, M. G., and Tranter, M.: Snowmelt modelling on Signy Island, South Orkney Islands, Ann Glaciol, 26, 161-166, https://doi.org/10.3189/1998aog26-1-161-166, 1998.

Gichamo, T. Z., and Tarboton, D. G.: Ensemble Streamflow Forecasting Using an Energy Balance Snowmelt Model Coupled to a Distributed Hydrologic Model with Assimilation of Snow and Streamflow Observations, Water Resour Res, 55, 10813-10838, https://doi.org/10.1029/2019WR025472, 2019.

Gichamo, T. Z., and Tarboton, D. G.: UEB parallel: Distributed snow accumulation and melt modeling using parallel computing, Environ Model Software, 125, https://doi.org/10.1016/j.envsoft.2019.104614, 2020.

Hellstrom, R. A.: Forest cover algorithms for estimating meteorological forcing in a numerical snow model, Hydrol Processes, 14, 3239-3256, https://doi.org/10.1002/1099-1085(20001230)14:18<3239::aid-hyp201>3.0.co;2-o, 2000.

Liu, Y., Xu, J. H., Lu, X. Y., and Nie, L.: Assessment of glacier- and snowmelt-driven streamflow in the arid middle Tianshan Mountains of China, Hydrol Processes, 34, 2750-2762, https://doi.org/10.1002/hyp.13760, 2020.

Mahat, V., and Tarboton, D. G.: Canopy radiation transmission for an energy balance snowmelt model, Water Resour Res, 48, https://doi.org/10.1002/2011WR010438, 2012.

Mahat, V., and Tarboton, D. G.: Representation of canopy snow interception, unloading and melt in a parsimonious snowmelt model, Hydrol Processes, 28, 6320-6336, https://doi.org/10.1002/hyp.10116, 2014.

Pimentel, R., Herrero, J., Zeng, Y., Su, Z., and Polo, M. J.: Study of Snow Dynamics at Subgrid Scale in Semi-arid Environments Combining Terrestrial Photography and Data Assimilation Techniques, J Hydrometeorol, 16, 563-578, https://doi.org/10.1175/jhm-d-14-0046.1, 2015.

Raleigh, M. S., Lundquist, J. D., and Clark, M. P.: Exploring the impact of forcing error characteristics on physically based snow simulations within a global sensitivity analysis framework, Hydrol Earth Syst Sci, 19, 3153-3179, https://doi.org/10.5194/hess-19-3153-2015, 2015.

Schulz, O., and de Jong, C.: Snowmelt and sublimation: field experiments and modelling in the High Atlas Mountains of Morocco, Hydrol Earth Syst Sci, 8, 1076-1089, https://doi.org/10.5194/hess-8-1076-2004, 2004.

Sen Gupta, A., Tarboton, D. G., Hummel, P., Brown, M. E., and Habib, S.: Integration of an energy balance snowmelt model into an open source modeling framework, Environ Model Software, 68, 205-218, https://doi.org/10.1016/j.envsoft.2015.02.017, 2015.

Sultana, R., Hsu, K. L., Li, J., and Sorooshian, S.: Evaluating the Utah Energy Balance (UEB) snow model in the Noah land-surface model, Hydrol Earth Syst Sci, 18, 3553-3570, https://doi.org/10.5194/hess-18-3553-2014, 2014.

Tarboton, D. G., Chowdhury, T. G., and Jackson, T. H.: A spatially distributed energy balance snowmelt model, Biogeochemistry of seasonally snow-covered catchments Proc symposium, Boulder, 1995, 228, 141-155, 1995.

Tarboton, D. G., and Luce, C. H.: Utah Energy Balance Snow Accumulation and Melt Model (UEB), Computer model technical description and users guide, Utah Water Research Laboratory and USDA Forest Service Intermountain Research Station, 1996.

Watson, F. G. R., Newman, W. B., Coughlan, J. C., and Garrott, R. A.: Testing a distributed snowpack simulation model against spatial observations, J Hydrol, 328, 453-466, https://doi.org/10.1016/j.jhydrol.2005.12.012, 2006.

You, J., Tarboton, D. G., and Luce, C. H.: Modeling the snow surface temperature with a one-layer energy balance snowmelt model, Hydrol Earth Syst Sci, 18, 5061-5076, https://doi.org/10.5194/hess-18-5061-2014, 2014.