Power system reserve scheduling with wind farm integration considering robust security constraints

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Abstract. The fluctuation of real-time wind power output and the capacity limits of transmission network have been identified as bottlenecks to integrate wind power in power system operation. This paper proposes a novel model of robust linear optimization for the reserve scheduling considering the secondary frequency power network security constraints. In this model, the linearization decision rule is introduced to describe the allocation of the secondary frequency control reserves; the mean value and unsymmetrical interval bound instead of the probability distribution are used to describe the wind power uncertainty. The stochastic robust linear optimization model is converted into a deterministic linear programming, thus achieving the tractability. Case studies are carried out to verify the effectiveness of the proposed method.

1. Introduction
By the end of 2013, the installed wind capacity in China has been the largest among all the countries in the world, and wind power has become the third-largest electrical power source in China [1]. The fluctuation and intermittency of real-time wind power output affect the quality of power supply seriously [2]. To ensure the security of power system operation, reserves provided by the thermal generators need to be scheduled in advance. As the penetration of wind power increases, more reserves are needed, and it is more difficult to ensure the economic and stable operation of power systems [3]. Therefore, it is of importance to develop reliable and economic reserve scheduling strategies.

The economic dispatch and reserve scheduling with wind power integration have been well studied in a lot of Literature\textsuperscript{s} [4-6]. The scheduling methods can be mainly divided into two categories: the method of deterministic reserve margins and stochastic programming. However, the first method is economically conservative in many cases [7, 8], and the second one relies on the accurate probability distribution of the uncertainty, which is hard to obtain in practical situations [9].

To handle the wind power uncertainty, another method called robust linear optimization (RLO) has attracted considerable attention in recent years, which can exhibit an adjustable conservativeness and do not require a given probability distribution for analysis in advance [10, 11]. To our best knowledge, Kang’s robust optimization (KRO) in [12] has already been used for power system decision-making considering wind power integration. For example, in both [10] and [11], the mean value and unsymmetrical interval bound instead of probability distribution are used to describe the wind power.
uncertainty, and the original stochastic linear optimization problem is converted into a robust counterpart problem which is still linear.

In this paper, a novel security constrained reserve scheduling model considering the capacity limits of transmission network and the KRO method are introduced to obtain the minimum reserve consumption and appropriate allocation of the reserve capacity among units to meet the integration requirement of all the wind power. Taking account of the network topology, this model can also be used to analyze the influence of active power imbalance and transmission capability limits in power system with multiple wind farms integrated. To avoid the use of accurate probability distribution, this paper proposes an uncertain set, defined by the linear functional inequalities according to the mean value and the interval bound of wind power output, which can reflect the wind power uncertainty. KRO method and linearization decision rule are introduced to convert the stochastic programming model into deterministic linear formulation. Case study results show the effectiveness of the proposed approach.

2. Mathematical formulation

2.1. Reserves representation

The problem formulation of this section is based on the following assumptions: 1) perfect load forecasts can be obtained; 2) wind generators are connected to the transmission network from a single bus; 3) the unit commitment problem has been solved in advance. The first and second assumptions are included to simplify the analysis of the problem. The last assumption can be removed by referring to the recent work about a unit commitment problem in [8].

With the above assumptions, reserves are allocated to balance generation-load mismatches in real-time operation, which occurs mainly due to the inaccurate forecasts of wind power output. Such mismatches induce frequency deviations further activates the primary and secondary frequency controllers - automatic generation control (AGC). In this paper, based on the assumption that the primary frequency control functionality is able to compensate any fast time-scale power deviation, the output of the steady-state behavior of AGC mainly refers to the secondary frequency control reserves, which needs to be assigned to each participating adjustable conventional generator at a certain percentage \( d_i, i=1, 2, \ldots, n \) [13]. The vector contains all the reserve percentages, \( d = [d_1, \ldots, d_n]^T \), is defined as the distribution vector.

![Figure 1. Schematic diagram illustrating the AGC functionality required for the security-constrained reserve scheduling model.](image-url)
2.2. Reserve scheduling model
The existing setup of the AGC loop is shown in Figure 1, which shows that the secondary frequency reserve is allocated to the AGC generators according to the distribution vector. Typically, the reserve scheduling does not take the network constraints into account and the allocation does not differ between up-spinning and down-spinning reserves [13]. To deal with the above shortcomings, the unsymmetrical bound and the mean value are introduced to denote the wind power fluctuation and the network security constraints are taken into account to optimize the distribution vector $d$.

The objective of the security constrained reserve scheduling aims to minimize the cost of generation and procurement of reserve capacities. In this type of problems, the production cost is usually in terms of a piecewise linear functions or a quadratic form. To simplify the approach, a linear cost for the production and reserves is used as follows:

$$\min \mathbf{C}^T \mathbf{P}_G + \mathbf{C}_{up}^T \mathbf{R}_{up} + \mathbf{C}_{down}^T \mathbf{R}_{down}$$

where $\mathbf{C}_G^T, \mathbf{C}_{up}^T, \mathbf{C}_{down}^T \in \mathbb{R}^{N_t}$ are the cost coefficient vectors of conventional generation, up spinning reserve and down spinning reserve, respectively. $\mathbf{P}_G$ denotes the generation dispatch; $\mathbf{R}_{up}, \mathbf{R}_{down} \in \mathbb{R}^{N_t}$ are the up-spinning and down-spinning reserves to be determined, respectively; $N_t$ is the number of thermal generators.

The security constraints are as follows.
1) Nodal power balance constraints:

$$-\mathbf{B}\mathbf{\theta} + \mathbf{g}_s + (\mathbf{g}_f + \Delta \mathbf{g}_f) + \mathbf{p}_w = \mathbf{d}$$

Equation (2) is based on a standard dc power flow approximation [13], where the matrix $\mathbf{B}$ denotes the nodal admittance matrix of transmission network; $\mathbf{\theta}$ represents the phase angle vector; $\mathbf{g}_s$ and $\mathbf{g}_f$ correspond to the scheduled output vectors of non-adjustable units and adjustable units, respectively; and $\mathbf{g}_f + \mathbf{g}_s = \mathbf{P}_G$; $\Delta \mathbf{g}_f$ denotes the power compensation corresponding to the real-time mismatch; $\mathbf{p}_w$ and $\mathbf{d}$ denote the random variable of the wind power output and the forecast value of the load.

2) Bounds of wind power fluctuation:

$$\mathbf{p}_w^l \leq \mathbf{p}_w \leq \mathbf{p}_w^u$$

where $\mathbf{p}_w^l, \mathbf{p}_w^u \in \mathbb{R}^n$ denotes the lower and upper bound of the wind power vector, respectively.

Considering the time scale of reserve scheduling and the practical engineering requirement, the random variable $\mathbf{p}_w$ can be described as a combination of the mean value and bounds:

$$\mathbf{p}_w = \mathbf{\mu}_w + \Delta \mathbf{p}_w$$

where $\mathbf{\mu}_w$ denotes the mean value of the wind power random variable, which can be replaced by the wind power forecast value; $\Delta \mathbf{p}_w$ denotes the deviation of the real-time wind power from the mean value, $-\omega^u \Delta \mathbf{p}_w \leq \Delta \mathbf{p}_w \leq \omega^f$, with

$$\begin{cases}
\omega^f = \mathbf{p}_w^u - \mathbf{\mu}_w \\
\omega^b = \mathbf{\mu}_w - \mathbf{p}_w^l
\end{cases}$$

where $\omega^f$ and $\omega^b$ denote the maximum value of upward and downward deviation of the real-time wind power.

3) Generation limits:

$$\mathbf{g}_f^{min} \leq \Delta \mathbf{g}_f + \mathbf{g}_f \leq \mathbf{g}_f^{max}$$

$$\mathbf{g}_s^{min} \leq \Delta \mathbf{g}_s + \mathbf{g}_s \leq \mathbf{g}_s^{max}$$

where the vectors $\mathbf{g}_f^{min}$ and $\mathbf{g}_f^{max}$ denote the lower generation limits of the non-adjustable units and adjustable units; the vectors $\mathbf{g}_s^{min}$ and $\mathbf{g}_s^{max}$ denote the upper generation limits of the non-adjustable units and adjustable units.
4) Reserve allocation constraints:

\[ \Delta g_f = d(I^T \Delta p_w) \]  \hspace{1cm} (8)
\[ -R_{down} \leq \Delta g_f \leq R_{up} \]  \hspace{1cm} (9)
\[ R_{down}, R_{up} \leq \eta(g_f^{max} - g_f^{min}) \]  \hspace{1cm} (10)
\[ I^T d = 1 \]  \hspace{1cm} (11)

where \( I^T \) denotes a vector that each element is one; the element of \( d \) is zero if corresponds to a non-adjustable unit, is a decision variable if corresponds to an adjustable unit; The linearization decision rule [12] is introduced to demonstrate the distribution for the fluctuation of wind power and then to determine the reserve capacity in the second Constraint (9); the third Constraint (10) indicates that the reserve capacity should be within the ramp rate limits where \( \eta \) corresponds to the maximum ability of the power correction; the last Constraint (11) ensures that the power mismatch can be fully compensated.

5) Branch power flow constraints:

\[ F = A_L \theta \]  \hspace{1cm} (12)
\[ -f_{max} \leq F \leq f_{max} \]  \hspace{1cm} (13)

where \( F \) is the branch power flow vector; \( A_L \) denotes the imaginary part of the admittance of network branches; \( f_{max} \) denotes the capacity limits vector. The random variables of wind power are included in Constraints (2), so the branch power flow is also a random variable. Constraints in (12) and (13) guarantee that the standard transmission capacity constraints will be satisfied in all cases.

Following these formulations, the reserve scheduling considering secondary frequency control, as an additional AGC functionality, is introduced [13]. The operating controller of the power system may monitor the deviation of the wind power from its forecast and use the distribution vector \( d \) respecting Constraints (8-11), as a lookup table, to allocate the proper reserves required, as is computed in the optimization problem (1) - (13) (see Figure 1).

The above model (1) - (13) is a stochastic linear programming model. The difficulty of solving the problem arises from the presence of uncertain variables \( \Delta p_w \) and the equation Constraint (2).

3. Main idea of KRO

For a linear programming problem:

\[
\min \begin{cases} cx \\ s.t. \ A x \leq b \\ l \leq x \leq u \end{cases}
\]  \hspace{1cm} (14)

Where \( x \in \mathbb{R}^n \) is the decision vector; \( u, l \in \mathbb{R}^n \) denote the upper and lower bound of decision vector; \( c \in \mathbb{R}^n \) is the coefficient vector; \( b \in \mathbb{R}^m \) is a consistent matrix, and the coefficient matrix \( A \in \mathbb{R}^{m \times n} \) has full uncertain elements \( a_{ij} \in [a_{ij}^l, a_{ij}^u] \), \( i = 1, \ldots, m; j = 1, \ldots, n \); the mean value of \( a_{ij} \) is denoted by \( \bar{a}_{ij} \); moreover, the random variables in two different inequality constraints are assumed to be independent of each other. Let \( t_{ij}^b = \bar{a}_{ij} - a_{ij}^l \), \( t_{ij}^u = a_{ij}^u - \bar{a}_{ij} \); \( J_i \) denotes the set of the random parameters in row \( i \) of matrix \( A \), and \( |J_i| \) represents the number of elements in set \( J_i \).

To handle the uncertainty, a robustness parameter \( \Gamma_i \) (\( \Gamma_i \leq |J_i| \)) is introduced and an uncertainty set can be defined accordingly based on the RLO theory [9]:

\[
\mathcal{R}_i (\Gamma_i) = \left\{ p_k \in \left[ \bar{a}_{ik} - \beta_k t_{ik}^b, \bar{a}_{ik} + \beta_k t_{ik}^u \right] : 0 \leq \beta_k \leq \Gamma_i, \sum_{k \in J_i} \beta_k \leq \Gamma_i \right\}
\]  \hspace{1cm} (15)
where \( a_i \) denotes the random parameters vector in row \( i \) of matrix \( A \); the value of variable \( \beta_k \) depends on robustness index. Here \( \Gamma_i \) serves as adjusting the conservative level of the uncertainty set \( \mathcal{R}(\Gamma_i) \), i.e., the larger the value of \( \Gamma_i \), the more conservative will \( \mathcal{R}(\Gamma_i) \) be.

Therefore, the robust counterpart of the linear programming problem (14) can be written as:

\[
\begin{align*}
\min \quad & c^T x \\
\text{s.t.} \quad & \sum_{j=1}^n a_{ij} x_j + \Gamma_i z_i + \sum_{i \in \mathcal{E}} p_{ik} \leq b_i, i = 1, \ldots, m \\
& z_i + p_{ik} \geq \max \left( t_{ik}^+ x_i - t_{ik}^0 x_i \right), i = 1, \ldots, m, \forall k \in J_i \\
& z_i \geq 0, p_{ik} \geq 0, i = 1, \ldots, m, \forall k \in J_i \\
& I \leq x \leq u
\end{align*}
\]

(16)

where \( z_i \) and \( p_{ik} \), \( i = 1, \ldots, m \) are auxiliary variables with no actual physical meaning [11]. The robust counterpart is a determined linear programming, which can be solved by ordinary linear programming solution. Changing the robustness index \( \Gamma_i \) in a certain range, we can coordinate the conservative level of the solution and the optimal value of the objective function.

The linear programming (16) has the same optimal solution with the uncertain linear programming (14), which is proved in [12].

4. Case studies

The modified Garver’s 6-bus system [8] is used to verify the effectiveness of the proposed method. As shown in Figure 1 in Reference [8], four wind farms are connected to the power grid form different buses. Parameters of the load (L1-L5), generators, wind farms, branches and the cost coefficients are shown in Tables 1-5 of Reference [14], respectively. All of the thermal units are engaged in the reserve scheduling; \( n_{ij} \) denotes the number of transmission lines between node \( i \) and \( j \); \( x_{ij} \) denotes the imaginary part of admittance of each branch; \( f_{ij} \) corresponds to the active power limit of one line between node \( i \) and \( j \).

1) The effect of robust budget

As can be seen in Table 1, the total cost of the dispatch scheduling, the total up-spinning reserves and the total down-spinning reserves all decrease as the robustness index \( \Gamma \) decreases. The reason for the results in Table 1 is that the increasing of \( \Gamma \) corresponds to the level of wind power fluctuation, which means a larger power mismatch caused by uncertain wind power is considered in the scheduling. Figures 2-3 describe the reserve curves of every participating unit in the simulation system. G2 provides the largest reserve capacity for uncertain wind power in the 6-bus system, since the two largest wind farms are located nearest and changing the mean point of the other units will cost more than the reserves cost of G2.

**Table 1.** Total cost and reserve requirement with \( \eta = 0.17 \).

| \( \Gamma \) | Total cost/$ | \( R_{up}/MW \) | \( R_{down}/MW \) |
|---|---|---|---|
| 4 | 22897 | 55 | 95 |
| 3.5 | 22827 | 50 | 90 |
| 3 | 22758 | 45 | 85 |
| 2.5 | 22683 | 40 | 75 |

**Figure 2.** Down-spinning reserves provide by each unit ( \( \eta = 0.17 \)).
Figure 3. Up-spinning reserves provide by each unit ($\eta = 0.17$).

(2) The effect of ramping rate $\eta$

Figures 4-5 and Table 3 of Reference [14] show the results with an adjusted parameter $\eta$, which increases to 0.25 from 0.17 in the above table and figures. It can be seen from the comparison between the results of $\eta = 0.17$ and $\eta = 0.25$ that a bigger value of $\eta$, which corresponds to a faster ramp rate applied by the participating units, will reduce the total cost in Table 3 of Reference [14], and increase the reserves provided by unit 2 in Figures 4 and 5. The reason of the above results is that a faster ramp rate ensures the power mismatch caused by wind farms. This could be accommodated more by the nearest units, which is a cheaper approach in this simulation system.

Figure 4. Down-spinning reserves provide by each unit ($\eta = 0.25$).

Figure 5. Up-spinning reserves provide by each unit ($\eta = 0.25$).

Table 2. Total cost and reserve requirement with $\eta = 0.25$.

| $\gamma$ | Total cost/$ | R_{up}$/MW | R_{down}$/MW |
|----------|--------------|-------------|--------------|
| 4        | 22897        | 55          | 95           |
| 3,5      | 22827        | 50          | 90           |
| 3        | 22758        | 45          | 85           |
| 2.5      | 22683        | 40          | 75           |

In addition, the total up and down reserves are equal in Tables 1-2. The reason for this result is that the total reserves are determined by the worst case fluctuation caused by the wind power. As can be seen in the simulation system, the total down reserve requirement at the worst case is 95MW when the robustness index is 4 - the number of the wind generators, as is obvious from Table 3 of Reference [14], which presents the probable maximum deviation between the real-time wind power output and
the forecast output – mean value. In other words, the total reserve is determined by the parameter $\Gamma$, but the distribution vector can reflect the influence on reserve scheduling in other aspects, such as the network topology and energy price.

5. Conclusions
A novel robust linear optimization method is proposed in this paper as an effective quantitative analysis tool for reserves scheduling associated with security constraint and the secondary frequency reserves allocation. The stochastic wind power output is described as a robust uncertain set with the mean value and unsymmetrical bound, so that the robust counterpart by the KRO method remains a linear form.

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