Research on factors influencing the positioning accuracy of four-quadrant detector

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Abstract. In order to analyze the positioning accuracy of the four-quadrant detector, this paper deeply studies the positioning principle of the four-quadrant detector, establishes the mathematical model of azimuth standard deviation based on Gaussian facula, and deeply analyzes the influence of each factor on the positioning accuracy from the three influencing factors of facula position, facula radius and signal-to-noise ratio. The results show that the positioning accuracy can be effectively improved by improving the signal-to-noise ratio, using smaller light spot and making the light spot as close to the center of the detector as possible.

1. Introduction

Compared with CCD and PSD, the four-quadrant detector has the advantages of high sensitivity, fast response speed and simple hardware circuit. Therefore, it is widely used in laser guidance [1], atomic force microscope [2], laser tracking [3], space laser communication [4], sub-nano measurement [5] and many other fields.

However, the positioning accuracy of the four-quadrant detector is affected by many factors. According to the basic principles of the positioning system, it can be divided into four categories: the characteristics of the light spot, the limitation of the four-quadrant detector, noise and environmental interference. In recent years, a large number of scholars have studied the factors affecting the four-quadrant detector. Literature [6] compared several laser spot center positioning algorithms and concluded that the spot based on Gaussian distribution has higher sensitivity. Literature [7] studied the influence of the non-uniformity of the four-quadrant detector on the positioning accuracy, and proposed a soft correction algorithm, which effectively improved the detection accuracy. Literature [8] analyzed the influence trend of detector noise and amplifier noise on the positioning accuracy of four-quadrant detectors through experiments, but did not establish an effective mathematical model for systematic numerical analysis of their influence. In reference to the problem of reduced position detection accuracy caused by low signal-to-noise ratio, Literature [9] proposed a cyclic autocorrelation processing method, which effectively improved the system's signal-to-noise ratio and positioning accuracy. Literature [10] proposed a method based on Kalman filter to improve the spot center positioning accuracy under the condition of low signal-to-noise ratio detection, which greatly reduces the positioning error.

In this paper, based on the spot model of Gaussian energy distribution, the principle of spot center positioning of the four-quadrant detector is deeply studied. The expression of the positioning algorithm when the spot energy obeys the Gaussian distribution is derived, and the position detection sensitivity under different energy distributions is compared. Aiming at the influence of noise on the
positioning accuracy, from the analysis of the mathematical standard deviation model, the mathematical model between the positioning accuracy of the four-quadrant detector and the spot radius, centroid coordinates and signal-to-noise ratio ratio is derived. The established model is used to simulate and analyze the main factors affecting the positioning accuracy of the four-quadrant detector. The simulation results show that the mathematical model can be a good measure of the detection accuracy of the center position of the light spot, which has guiding significance for engineering applications.

2. Four-quadrant detector positioning principle

2.1. Four-quadrant detector basic principle

In the laser positioning system, the target echo signal is received by the optical system to form a spot on the four-quadrant detector. Due to the photovoltaic effect, the four quadrants will generate photocurrents proportional to the optical power received by each quadrant. According to the relationship of the four currents, the position of the center of mass of the spot can be judged. When the photocurrents generated by the four quadrants are equal, the center of the spot Located at the origin, and when the four currents are not equal in size, the spot is off-center, as shown in Figure 1.

![Figure 1. Schematic diagram of the working principle of the four-quadrant detector.](image)

The commonly used algorithms to solve the spot center position are normalized sum difference algorithm, diagonal algorithm, logarithmic algorithm and ΔΣ algorithms, and each algorithm has its own scope of application. In this paper, the normalized sum difference algorithm is selected for analysis. The specific expression is as follows:

\[
\begin{align*}
    d_x &= \frac{(I_A + I_B) - (I_C + I_D)}{I_A + I_B + I_C + I_D} \\
    d_y &= \frac{(I_A + I_B) - (I_C + I_D)}{I_A + I_B + I_C + I_D}
\end{align*}
\]  

(1)

where, \(d_x\) and \(d_y\) is the relative position of the spot centroid in the x direction and y direction. \(I_A, I_B, I_C, I_D\) is the current corresponding to each quadrant.

2.2. Location algorithm

The spot center positioning accuracy is related to the energy distribution of the spot. Different spot energy distributions correspond to different positioning expressions. Commonly used spot energy distribution models include uniform distribution and Gaussian distribution. Figure 2 shows the relationship between the actual position and the normalized position for different energy distributions. It can be seen from Figure 2 that the slope of the curve corresponding to the Gaussian spot at the...
origin is greater than the slope of the uniform spot, which shows that the Gaussian spot is relative to the uniform spot has higher sensitivity. However, it can also be seen from Figure 2 that the detection range of a Gaussian spot is smaller than that of a uniform spot. In actual engineering, the laser beam is received and imaged by the optical system. The energy of the spot on the photosensitive surface of the four-quadrant detector is usually Gaussian.

![Figure 2. Comparison of different energy distributions.](image)

The energy expression of the Gaussian distribution spot is:

\[
h(x, y) = \frac{2h_0}{\pi \omega^2} \exp \left\{ -\frac{1}{2} \left( \frac{(x-x_0)^2 + (y-y_0)^2}{\omega^2} \right) \right\}
\]  

where, \( h_0 \) is the total energy of light spot, \( x_0, y_0 \) is the center position of light spot, \( \omega \) is the Gaussian spot radius. When calculating the current of each quadrant, considering that \( \omega \) is far less than the radius of the photosensitive surface of the four-quadrant detector, the upper and lower limits of the integral can be taken as infinite, and the following expressions are given:

\[
I_A + I_B = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x, y) \, dx \, dy = \int_{-\infty}^{\infty} h(x, y) \, dy \int_{-\infty}^{\infty} h(x, y) \, dx
\]  

\[
I_B + I_C = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x, y) \, dx \, dy = \int_{-\infty}^{\infty} h(x, y) \, dy \int_{-\infty}^{\infty} h(x, y) \, dx
\]

Substituting Formulas (3), (4), (5) into Formula (1), we can get:

\[
d_x = \text{erf} \left( \frac{\sqrt{2}x_0}{\omega} \right)
\]

where, \( \text{erf}(x) \) is the error function. From the symmetry of the four-quadrant detector, the centroid coordinates in the \( y \) direction can be obtained:

\[
d_y = \text{erf} \left( \frac{\sqrt{2}y_0}{\omega} \right)
\]

The actual spot centroid position is the inverse function solved above, and the specific expression is as follows:

\[
\begin{align*}
x_0 &= \frac{\omega}{\sqrt{2}} \text{erf}^{-1}(d_x) \\
y_0 &= \frac{\omega}{\sqrt{2}} \text{erf}^{-1}(d_y)
\end{align*}
\]
2.3. Angle measurement principle
As shown in Figure 3, the laser echo signal forms a spot on the four-quadrant detector after passing through the lens. The spot center is \((x_0, y_0)\), and \(L\) is the distance from the lens to the detector in the figure, \(\theta\) is the azimuth angle of the target relative to the detector.

\[
\theta = \arctan \frac{\sqrt{x_0^2 + y_0^2}}{L} \approx \frac{\sqrt{x_0^2 + y_0^2}}{L}
\]  

Figure 3. Azimuth measurement geometry.

Then the azimuth angle of the target can be obtained according to the light path diagram as follows:

\[
\theta = \arctan \frac{\sqrt{x_0^2 + y_0^2}}{L} \approx \frac{\sqrt{x_0^2 + y_0^2}}{L}
\]  

3. Mathematical model of standard deviation of target bearing detection

3.1. Model derivation
Formula (1) is a solution without considering noise, but in practical engineering, the accuracy of spot location is affected by noise. Then Formula (1) should be rewritten as follows:

\[
d_s = \frac{(I_{A} + I_{B} + I_{C} + I_{D}) - (I_{B} + I_{A} + I_{C} + I_{D})}{I_{A} + I_{B} + I_{C} + I_{D}} = \frac{I_{A} + I_{B} + I_{C} + I_{D} + I_{A} + I_{B} + I_{C} + I_{D}}{I_{A} + I_{B} + I_{C} + I_{D}}
\]  

where, \(I_{A}\) is the noise current of each quadrant. Define \(I_1 = I_{A} + I_{D}\) as the sum of the currents in the right quadrant, \(I_{a1} = I_{A} + I_{D}\) as sum of noise current in the right quadrant, \(I_2 = I_{B} + I_{C}\) as the sum of the currents the left quadrant, \(I_{a2} = I_{B} + I_{C}\) as the sum of noise current in the left quadrant. Then Formula (10) can be rewritten as:

\[
d_s = \frac{(I_1 + I_{a1}) - (I_2 + I_{a2})}{I_1 + I_{a1} + I_2 + I_{a2}} = \frac{I_1 - I_2 + I_{a1} - I_{a2}}{I_1 + I_2 + I_{a1} + I_{a2}} \frac{1}{I_1 + I_2}
\]  

The second term in Formula (11) can be expanded by Taylor, then the above formula can be simplified to:

\[
d_s \approx \frac{I_1 - I_2 + 2I_{a1}I_{a2} - I_1I_2}{I_1 + I_2} \frac{1}{(I_1 + I_2)^2}
\]  

In Formula (12), the first term can be regarded as the solution value of the light spot, and the second term can be regarded as the influence of noise on the solution value. It can also be seen from
this formula that when noise exists, the solution value of the spot is not a fixed value. Define the random error of the solution value as $d_{sn}$, it can be expressed as:

$$d_{sn} = 2 \frac{I_{n1}I_2 - I_1I_{n2}}{(I_1 + I_2)^2}$$

(13)

Assuming that the noise of each quadrant is independent of each other, according to the theory of variance composition, the variance of the solution value can be expressed as:

$$\text{var}(d_{sn}) \approx \frac{4I_2^2}{(I_1 + I_2)^4} \text{var}(I_1) + \frac{4I_1^2}{(I_1 + I_2)^4} \text{var}(I_2)$$

(14)

where, $\text{var}(I_1)$ and $\text{var}(I_2)$ is the variance of $I_1$ and $I_2$ respectively. Ignore the non-uniformity of the four-quadrant detector itself, assuming the same noise level in the four quadrants, then there is $\text{var}(I_1) = \text{var}(I_2) = i_n^2$, $i_n^2$ is the current noise variance of each quadrant. The Formula (14) can be written as:

$$\text{var}(d_{sn}) \approx 8 \frac{I_1^2 + I_2^2}{(I_1 + I_2)^4} i_n^2 = \frac{K}{\text{SNR}}$$

(15)

In Formula (15), SNR is the total signal-to-noise ratio, $K = 1 + d_{sn}^2$. It can be seen from Formula (6) that there is a non-linear relationship between the solution value and the actual value, and the standard deviation of the center position of the spot cannot be directly calculated by Formula (6) and Formula (15). Therefore, using the central approximation method to perform Taylor expansion of Formula (6) and make a first-order approximation, then we can get $x_n = (\sqrt{\pi} \omega / 2\sqrt{2})d_{sn}$. Combining Formula (15) and Formula (6), the variance at the center position $x_0$ of the spot can be obtained as:

$$\text{var}(x_0) = \left(\frac{\sqrt{\pi} \omega}{2\sqrt{2}}\right)^2 \frac{1}{\text{SNR}} \left(1 + \text{erf}^2\left(\frac{\sqrt{2}x_0}{\omega}\right)\right)$$

(16)

In the same way, the variance of the spot center position at $y$ can be obtained. According to the error synthesis theory, the variance of the azimuth angle can be expressed as:

$$\text{var}(\theta) = \left(\frac{\partial \theta}{\partial x_0}\right)^2 \text{var}(x_0) + \left(\frac{\partial \theta}{\partial y_0}\right)^2 \text{var}(y_0)$$

(17)

Then the standard deviation of the azimuth angle can be expressed as:

$$\text{std}(\theta) = \sqrt{\text{var}(\theta)} = \frac{\sqrt{\pi} \omega}{2\sqrt{2}\lambda \text{SNR}} \left[ x_0^2 \text{erf}^2\left(\frac{\sqrt{2}x_0}{\omega}\right) + y_0^2 \text{erf}^2\left(\frac{\sqrt{2}y_0}{\omega}\right) \right]^{1/2}$$

(18)

3.2. Model simulation and analysis

The standard deviation model of the azimuth angle is derived in the previous article. From the formula of the model, the standard deviation of the azimuth angle is related to the spot radius, the spot centroid position, the signal-to-noise ratio and the distance from the lens to the detector. In engineering, the distance from the lens to the detector is often fixed, so this article only conducts simulation analysis for the other three influencing factors.

3.2.1. Standard deviation of azimuth angle at different spot positions. Fix the signal-to-noise ratio and spot radius, and simulate the influence of spot centroid position on positioning accuracy. Assuming that the spot radius is 1.5mm, the signal-to-noise ratio is 60dB, and the distance between the lens and
the detector is 20mm. Figure 4 shows the standard deviation curve of the azimuth angle detection in the x direction [-1mm~1mm] and y direction [-1mm~1mm]. It can be seen from Figure 4 that the more the spot center shifts from the center, the greater the azimuth error.

3.2.2. Standard deviation of azimuth angle at different spot radius. The SNR is defined as 60dB. Assuming that the spot center is located at (0.1 mm, 0.1 mm), the azimuth standard deviation curve of spot radius from 1.5 mm to 2.0 mm is simulated. It can be seen from Figure 5 that the larger the spot radius is, the larger the azimuth standard deviation is, and the lower the positioning accuracy is. In practical engineering, the appropriate spot radius should be selected according to the engineering needs.

3.2.3. Standard deviation of azimuth angle at different SNR. Assuming that the spot centroid is located at (0.1 mm, 0.1 mm) and the spot radius is 1.5 mm, the signal-to-noise ratio is increased from 10 dB to 60 dB, and the standard deviation curve of light azimuth angle is obtained. As can be seen from Figure 6, the higher the SNR is, the smaller the azimuth standard deviation is, and the higher the positioning accuracy is.
4. Conclusions
In this paper, the positioning accuracy of four quadrant detector is deeply studied. Aiming at the influence of noise on positioning accuracy, the azimuth standard deviation model is established, and the three main factors affecting the standard deviation are simulated and analyzed. The results show that the main factors affecting the positioning accuracy of the four-quadrant detector are: spot centroid position, spot radius and signal-to-noise ratio. In practical engineering, the positioning accuracy can be improved by increasing the signal-to-noise ratio, reducing the spot radius and making the spot close to the center of the detector.

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