ABSTRACT: We show that charged Eguchi-Hanson instantons provide a concrete and calculable new source of intrinsic Peccei-Quinn symmetry breaking by quantum gravity. The size of this breaking is shown to depend sensitively on the short-distance details of a given theory, but is generically suppressed by fermion zero modes. Demanding that these gravitational effects not affect the axion solution to the strong CP problem, we find that at least two sets of quarks with differing Peccei-Quinn charges are required. In addition, these effects obviate the cosmological axion domain wall problem but leave unchanged problems associated with coherent axion oscillations.

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The invisible axion\textsuperscript{1} was introduced more than a decade ago as a phenomenologically viable solution to the weak ‘strong-CP’ problem. The associated Peccei-Quinn (PQ) symmetry\textsuperscript{2} is an exact symmetry to all orders in perturbation theory, but is broken intrinsically by nonperturbative QCD effects. This breaking induces a mass of order \( m_a \approx m_\pi f_\pi / f_a \) for the invisible axion. At temperatures larger than \( \Lambda_{\text{QCD}} \), it is known that the axion mass diminishes rapidly\textsuperscript{3}.

It has always been assumed that the low energy physics of the axion was dictated by the chiral symmetries of the light quarks. Thus what was happening at high energies (other than the spontaneous breaking of the PQ symmetry) appears to be essentially irrelevant in determining low-energy axion dynamics. This assumption, however, rests heavily on having the colored matter content be as in the minimal standard model, e.g. having QCD be asymptotically free. In this case, only instantons of size \( \sim \Lambda_{\text{QCD}}^{-1} \) contribute to the axion potential. It has been pointed out\textsuperscript{4,5,6} that if extra colored particles are present in the theory above the electroweak scale, the contribution of small QCD instantons to axion physics cannot be neglected. Indeed in some situations, it is, in fact, the dominant contribution to the axion potential. This example clearly shows that short-distance physics can indeed have a significant effect on axion physics. We are then led to contemplate if and how non-QCD physics, e.g. semiclassical gravity near the Planck scale, can alter the axion physics in a similar way.

Since gravitational interactions are CP conserving, perturbative quantum gravity should leave the PQ symmetry and axion physics intact. On the other hand, the CP symmetry could conceivably be affected by nonperturbative quantum gravity effects. Indeed, this possibility was recently pointed out in Ref.\textsuperscript{7}, where the effects of higher dimension operators which break the PQ symmetry (possibly induced by the exchange of virtual black holes or by wormhole physics\textsuperscript{8}) were investigated. However, in Ref.\textsuperscript{7}, only plausibility arguments for the existence of PQ symmetry breaking operators, based on the classical black hole
no-hair theorem and wormhole physics were given. There, it was impossible to estimate the size of PQ symmetry breaking operators reliably. Thus, the issue whether and how the PQ symmetry and axion physics are modified by non-QCD short-distance physics remains unsettled.

In this Letter, we attempt to provide a more concrete analysis of the question of whether Planck-scale physics will disturb the PQ mechanism and low energy axion physics. We do this by making use of certain well-known and well-studied self-dual gravitational instantons\(^9\). We assume they saturate the Euclidean path integral, and hence should be included in the partition function describing low energy physics. While there may be many other potential quantum gravity effects that might give rise to PQ breaking effects, the approximation we use has the benefit of being both well-motivated and above all, controllably calculable.

The relevant interactions to our discussion is described by the action:

\[
S_0 = S_{\text{grav}} + S_{\text{gauge}} + S_{\text{fermion}} + S_{\theta}
\]

\[
S_{\text{grav}} = \int d^4x \sqrt{g} \left[ -\frac{M_{\text{pl}}^2}{16\pi^2} R + \oint dn[K - K_0] \right]
\]

\[
S_{\text{gauge}} = \int d^4x \sqrt{g} \left[ \frac{1}{4e^2}(F_{\mu\nu})^2 + \frac{1}{2g^2}\text{Tr}(G_{\mu\nu})^2 \right]
\]

\[
S_{\text{ferm}} = \int d^4x \sqrt{g} \left[ \sum_{a=1}^{N_f} \bar{\psi}_a \gamma^\mu e^\mu_a (\nabla_\mu + Q^a_{\text{em}} A_\mu + B_\mu(R^a)) \psi_a \right]
\]

\[
S_{\theta} = \int d^4x \sqrt{g} \left[ \frac{i\theta_{\text{QCD}}}{16\pi^2} \text{Tr}(G_{\mu\nu} \tilde{G}_{\mu\nu}) + \frac{i\theta_{\text{em}}}{32\pi^2} F_{\mu\nu} \tilde{F}_{\mu\nu} + \frac{i\theta_{\text{grav}}}{384\pi^2} (\tilde{R} \tilde{R}) \right].
\]

Here, \(A_\mu\) and \(B_\mu\) comprise the electromagnetic and gluon gauge fields, \(K\) the trace of the extrinsic curvature, \(e^\mu_a\) the vierbein, and \(\nabla_\mu = \partial_\mu + \omega_\mu\) the covariant derivative for gravity. This action supports the existence of not only the usual QCD-Einstein instanton but also a self-dual configuration in both the gravitational curvature \(R_{\mu\nu} \equiv R_{\mu\alpha\nu\beta} \Sigma^{\alpha\beta}\) and the electromagnetic field strength (or that of any other abelian (sub)group in the theory): \(R_{\mu\nu} = \pm \tilde{R}_{\mu\nu}\) and \(F_{\mu\nu} = \pm \tilde{F}_{\mu\nu}\).\(^9\,10\) The simplest such configuration was known as the
Eguchi-Hanson (EH) instanton. However, as we will see later, an instanton with nonzero $F_{\mu\nu}$ turns out to be the most relevant to our subsequent discussion of axion physics. We will call these configurations Abelian Eguchi-Hanson (AEH) instantons. The explicit solution with core size $\rho$ (where $x^2 \equiv x^\mu x^\mu \geq \rho^2$) is:

$$
\begin{align*}
g^{\mu\nu} &= \delta^{\mu\nu} - \frac{\rho^4}{x^4} \frac{x^\mu x^\nu}{x^2} + \frac{\rho^4}{x^4 - \rho^4} \frac{\tilde{x}^{\mu} \tilde{x}^{\nu}}{x^2}, \\
A_\mu &= \frac{2P\rho^2 \tilde{x}_\mu}{x^4}.
\end{align*}
$$

(2)

The geometry of the manifold described by the above metric is such that $\tilde{x}^\mu \equiv (y, -x, t, -z)$ is equivalent to $x^\mu$ due to the $\mathbb{RP}^3$ global topology of the instanton. The instanton $U(1)$ charges $P$ are to be chosen compatible with the existence of spin structures.

The action of the AEH instanton is nonzero and given by

$$
S_{AEH} = \frac{1}{4e^2} \int d^4x \sqrt{-g} F^2_{\mu\nu}
$$

$$
= \frac{4\pi^2 P^2}{e^2} = \frac{\pi P^2}{\alpha_e(\rho)}.
$$

(3)

Note that the action is independent of Newton’s constant, and rather similar to that of the Yang-Mills instanton except for a factor of two. In fact, the analogy with Yang-Mills instanton goes much deeper as we will see. Consider a Dirac fermion $\psi$ in Eq.(1) of electric charge $Q$ and color representation $R$. The axial current $J_5^{\mu} \equiv \bar{\psi} \gamma_5 \gamma^\mu \psi$ is anomalous:

$$
\nabla^{\mu} J_5^\mu = \frac{2N_c Q^2_{em}}{16\pi^2} F_{\mu\nu} \tilde{F}_{\mu\nu} + \frac{2C_2(R)}{8\pi^2} \text{Tr} G_{\mu\nu} \tilde{G}_{\mu\nu} - \frac{2N_c}{384\pi^2} \text{Tr} R_{\mu\nu} \tilde{R}_{\mu\nu}.
$$

(4)

This implies a chiral charge asymmetry in the background of an AEH-instanton 11:

$$
\Delta Q_5(AEH) = \int [F \tilde{F} - \text{Tr} \tilde{R} R] = \begin{cases} 
2P^2Q^2_{em}, & \text{if } P \text{ is an integer} \\
2(P^2Q^2_{em} - \frac{1}{4}) & \text{if } P \text{ is not an integer,}
\end{cases}
$$

(5)

i.e. there is no axial charge asymmetry contribution from the purely gravitational sector, but only from the $U_{em}(1)$ field. The $-\frac{1}{4}$ contribution to the axial charge asymmetry in case $P$ is not an integer is due to different boundary conditions fermions have to satisfy across the antipodal points of the identified spacetime described by the EH instanton.
Since the minimal electric charge of the standard model fermions is $-1/3$, the instanton charge should be restricted to $P \in 3\mathbb{Z}$.

We now analyze the effects of AEH instantons on PQ symmetry breaking when an invisible axion is introduced. For definiteness, we consider an ‘extended’ invisible axion model, with isosinglet heavy quarks transforming under the representation $R$ of color and carrying nonzero PQ charges. The size of the QCD induced axion potential is estimated to be:

$$V_{QCD} \approx 2K_{QCD} \cos[N_{QCD} a + \theta_{QCD}]$$

where

$$K_{QCD} = \left( \frac{1}{m_u} + \frac{1}{m_d} + \frac{1}{m_s} \right)^{-1} \cdot \langle q\bar{q} \rangle.$$  

The integer $N_{QCD}$ counts the QCD vacuum multiplicity and is given by

$$N_{QCD} = \frac{2\pi}{T_{QCD}} (2\text{Tr}[Q_{pq} T_a^2(R)])$$

where $T_{QCD}$ is the periodicity of the QCD theta parameter $^{12,13}$, $Q_{pq}$ denotes the PQ charge operator, $T_a$ is a QCD color generator, and the trace taken over all fermions.

AEH instantons of arbitrary neck size $\rho$ are all equally good semiclassical solutions. Therefore, in the semiclassical approximation to the path integral, we need to sum over all instanton sizes $\rho \geq M_{pl}^{-1}$. Although a precise form of the measure $d\mu[\rho]$ is not known, let us assume $d\mu[\rho] = d\ln \rho$ as in Yang-Mills case.

As is evident from Eq.(5), the $P = 0$ EH instanton does not give rise to an intrinsic breaking of PQ symmetry, neither does a nontrivial axion potential. The leading PQ breaking effect comes from the $P = 3$ AEH instanton. It produces $18Q_{em}^2$ chiral zero modes for each fermion, and gives rise to an induced local operator of the form

$$O_{AEH}[\rho] = \frac{1}{\rho^4} \text{Det} \left[ \left( E_R E_L \right)^9 \prod_{\text{color}} (U_R U_L)^4 (\bar{D}_R D_L)^4 \right]^{N_g} \prod_{\text{heavy}} \left( \bar{Q}_R Q_L \right)^9 Q_{em}^2$$

$$\cdot \exp \left[ -\frac{9\pi}{\alpha_e(\rho)} + iN_{AEH} a + i\theta_{em} \right].$$

Here, $N_g$ is the number of generations, while $N_{AEH} \equiv 9 N_c \sum_{\text{quark}} Q_{pq} Q_{em}^2 + 9 \sum_{\text{lepton}} Q_{pq} Q_{em}^2$. $Det$ denotes the totally antisymmetrized, color, isospin and hypercharge singlet products.
of quark and charged lepton zero modes. The AEH instanton induced operator is strikingly similar to that induced by QCD instantons.

In order to calculate the axion potential at low-energies, we tie up the fermion zero modes via Yukawa interactions with the Higgs fields. Thus, \( \bar{\psi}_L \psi_R \to \lambda_i H_{1,2} \Phi \) in which \( \lambda, H_{1,2} \) and \( \Phi \) denote specific Yukawa coupling constants, and the weak isodoublet and isosinglet Higgs fields, respectively. From each Higgs loop integral in a size \( \rho \) instanton background and the fact that \( \langle \Phi \rangle \approx f_a \), we get factors of \( \lambda / 2\pi \) and \( 2\pi / \rho f_a \) for \( \rho \leq f_a \) or \( \rho f_a / 2\pi \) for \( \rho \geq f_a \) respectively. A more precise estimate of these factors and loop integrals can be done following Dine and Seiberg\(^5\) and Flynn and Randall\(^6\) (The possibility of increasing the axion mass using small QCD instantons was originally suggested by Holdom and Peskin\(^4\)). Tying up the fermion zero modes through Yukawa couplings, we find a new contribution to the axion potential due to \( P = 3 \) AEH instantons:

\[
V_{AEH}[a] \approx 2K_{AEH} \cos(N_{AEH} \frac{a}{f_a} + \theta_{em}),
\]

\[
K_{AEH} = \left( \frac{9\pi}{\alpha_e(\mu)} \right)^2 \frac{7}{2} e^{-9\pi/\alpha_e(\mu)} \left\{ \int_{f_a^{-1}}^{f_m^{-1}} \frac{d\rho}{\rho^2} \left( \mu \rho \right)^{b_0} \prod \left[ \lambda \frac{f_a \rho}{4\pi^2} \right] 
\right. 
+ \left. \int_{f_a^{-1}}^{M_{pl}^{-1}} \frac{d\rho}{\rho^2} \left( \mu \rho \right)^{b_0} \prod \left[ \frac{\lambda}{f_a \rho} \right]. \right\}
\]

Here, \( b_0 \) denotes the first coefficient of the \( \alpha_e \) beta function, \( \mu \) the renormalization scale (the inclusion of which renders the result renormalization group invariant). The largest contribution to the instanton size integrations come from \( \rho \approx f_a^{-1} \) for both terms in \( K_{AEH} \). The new AEH instanton contribution is thus very sensitive to the matter content of the theory near and above the PQ scale!

For illustration, let us take a model in which four color triplet heavy quarks with electric charge \( Q_{em} = 2/3 \) and mass \( 10^{10} \) GeV are introduced. Taking \( \mu \approx f_a \), Eq.(9) gives

\[
\frac{K_{AEH}}{K_{QCD}} \approx 2 \times 10^{-10} \left[ \frac{e^{-9\pi/\alpha_e(f_a)}}{e^{-90\pi}} \right] \left[ \frac{m_Q}{10^{10} \text{GeV}} \right]^9 \frac{Q_{em}^2}{10^{12} \text{GeV}^2} \left[ \frac{f_a}{10^{10} \text{GeV}} \right]^2.
\]

We have normalized the product of the light quark and lepton masses to the standard model values with \( m_t \sim 150 \) GeV, and \( \alpha_e(f_a) \) to 1/10 (as occurs in many supersymmetric
models containing a number of heavy charged scalar fields). With these assumptions, we find that the new small instanton induced potential is about one billionth of the QCD instanton induced potential! Our estimate is quite conservative, and varying the gauge and Yukawa coupling constants certainly can and will make the size of this contribution vary over a wide range. As we will see later, the estimate in Eq.(10) with $\alpha_e$ and $m_t$ as above is roughly the upper bound to any small instanton contributions to the axion potential based on current experimental bounds on the neutron electric dipole moment (NEDM).

We first discuss how the invisible axion couples to hadrons and photons at low energy. Below the QCD chiral symmetry breaking scale, the low-energy dynamics of the Goldstone bosons, the invisible axion and the photon is described by an effective Lagrangian. Let us realize the $SU_L(3) \times SU_R(3) \times U_A(1)$ chiral symmetry nonlinearly using $\Sigma \equiv \exp\left(\frac{2i}{f_\pi}(\Pi + \frac{\pi^0}{\sqrt{6}}I)\right) \in U(3)$ and let $M = \text{diag}(m_u, m_d, m_s)$ represent the current quark mass matrix. Then, assuming $f_\pi = f_{\eta'}$ for simplicity, the low-energy chiral Lagrangian reads:

$$L_{\text{chiral}} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a + \frac{1}{4} f_\pi^2 \text{Tr}(\partial \Sigma^\dagger \partial \Sigma) + \frac{1}{2} (\partial a)^2 + \cdots + <q\bar{q}> \text{Tr}(M \Sigma^\dagger + h.c.)$$

$$+ \frac{\text{Tr}[\text{Iln}(\Sigma e^{ia/f_\pi + i\theta_{\text{QCD}}})]}{32\pi^2} G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a$$

$$+ \frac{\text{Tr}[Q^2 \text{ln}(\Sigma e^{ia/f_\pi + i\theta_{\text{em}}})]}{32\pi^2} F_{\mu\nu} \tilde{F}_{\mu\nu}. \quad (11)$$

where the ellipses denote higher derivative terms in $\Sigma$ and $a$, $Q_{\text{em}} \equiv \text{diag.}(2/3, -1/3, -1/3)$ is the electric charge matrix and the traces are taken over flavor space. The last two terms come from the Wess-Zumino term which arises after integrating out the quarks coupled to gluons and photon. It ensures the correct anomalous $U_A(1)$ Ward identity Eq.(4) involving both $U_{\text{em}}(1)$ and QCD color anomalies. For simplicity, we have ignored additional axion-photon interactions, e.g. those coming from nonzero current quark masses.

Integrating out the QCD and AEH instanton fluctuations (in the dilute gas approxi-
ation), the last two terms in Eq.(11) give:

\[ L_{\text{inst}} \approx K_{\text{QCD}} \exp(\text{Tr}(\ln \Sigma) + iN_{\text{QCD}} \frac{a}{f_a} + i\theta_{\text{QCD}}) \]

\[ + K_{\text{AEH}} \exp(\text{Tr}(9Q^2_{\text{em}} \ln \Sigma) + iN_{\text{AEH}} \frac{a}{f_a} + i\theta_{\text{em}}) \]

\[ + h.c. \] (12)

The QCD and AEH instanton amplitudes \( K_{\text{QCD}} \) and \( K_{\text{AEH}} \) are given as in Eqs.(6) and (9), and the PQ charge weights are denoted by \( N_{\text{QCD}} \) and \( N_{\text{AEH}} \) respectively.

Having two independent sources for the axion potential, one from small AEH instantons and the other from large QCD instantons, the axion field may not settle to \(-\theta_{\text{QCD}}\) at the hadronic scale. From Eqs.(6) and (12), and ignoring a small mixing (\( \sim O(f_\pi / f_a) \)) between the \( \eta' \) and the axion, we find the new axion minima at:

\[ \bar{\theta}_{\text{QCD}} \equiv N_{\text{QCD}} \frac{<a>}{f_a} + \theta_{\text{QCD}} \]

\[ = -N_{\text{AEH}} \cdot \frac{N_{\text{AEH}}}{N_{\text{QCD}}} \cdot \frac{K_{\text{AEH}}}{K_{\text{QCD}}} \cdot (\frac{\theta_{\text{QCD}}}{N_{\text{QCD}}} - \frac{\theta_{\text{em}}}{N_{\text{AEH}}}). \] (13)

Typically, we expect \( \Delta \theta \equiv \theta_{\text{QCD}} - \theta_{\text{em}} \approx O(\pi) \) and \( N_{\text{QCD}} \approx N_{\text{AEH}} \). Then, \( \bar{\theta}_{\text{QCD}} \approx K_{\text{AEH}}/K_{\text{QCD}} \cdot \Delta \theta \). Thus, demanding that \( \bar{\theta}_{\text{QCD}} \) be less than \( 10^{-9} \), as required by NEDM measurements, yields \( K_{\text{AEH}}/K_{\text{QCD}} \lesssim 10^{-10} \! \!. \) This bound is indeed met by Eq.(10) as long as not too many charged and colored PQ charge carrying fermions are introduced at short distances. The invisible axion still can solve the ‘strong-CP’ problem.

The physical axion mass is obtained from Dashen’s theorem:

\[ m_a^2 f_a^2 \approx [Q_a, [Q_a, L_{\text{inst}}]] \] (14)

in which \( Q_a \equiv \int d^3 \vec{x} J^0_a \) is the physical axion charge operator. Let us see how the AEH instanton induced potential modifies the invisible axion and other Goldstone boson mass spectrum. These modifications are obtained most easily by diagonalizing the total Goldstone boson mass matrix from the second line of Eq.(11) and QCD and AEH instanton induced potentials in Eq.(12). From this we find the physical axion mass:

\[ m_a^2 f_a^2 \approx K_{\text{QCD}} + K_{\text{AEH}} + O(\frac{K_{\text{AEH}}^2}{K_{\text{QCD}}}) \] (15)
In addition, we find that both the $\eta$ and the $\eta'$ receive mass corrections due to small AEH instantons proportional to $K_{\text{AEH}}/f_{\pi}^2$.

If the gravitational effects discussed above saturate the bounds from NEDM measurements, they can also solve the cosmological domain wall problem of axion theories.\(^{12}\) Recall that domain walls arise due to the fact that a $Z_{N_{\text{QCD}}}$ subgroup of the $U(1)_{pq}$ symmetry may be anomaly free with respect to QCD, but maybe broken spontaneously by both the Higgs vacuum expectation values and the quark condensate. We now repeat the same calculation for the gravity induced potential. Using the fact that all fermions couple equally to gravity, we can easily see that the subgroup of $U(1)_{pq}$ free from gravitational anomalies is $Z_{N_{\text{grav}}}$ where $N_{\text{grav}}$ is defined by: $N_{\text{grav}} \equiv \text{Tr}[Q_{pq}]$. Furthermore, $Z_{N_{\text{em}}}$, where $N_{\text{em}} \equiv 9\text{Tr}[Q_{pq}Q_{\text{em}}^2]$ is the subgroup of $U(1)_{pq}$ which is anomaly-free with respect to the $U_{\text{em}}(1)$. The full axion potential takes the form:

\[
V(a) = 2K_{\text{QCD}} \cos(N_{\text{QCD}} \frac{a}{f_a} + \bar{\theta}_{\text{QCD}}) \\
+ 2K_{\text{grav}} \cos(N_{\text{grav}} \frac{a}{f_a} + \bar{\theta}_{\text{grav}}) \\
+ 2K_{\text{em}} \cos(N_{\text{em}} \frac{a}{f_a} + \bar{\theta}_{\text{em}}).
\] (16)

As we argued earlier, for the matter content of the standard model, the purely gravitational contribution can be neglected compared to the other two.

It is clear that if $N_{\text{QCD}}$ does not divide $N_{\text{em}}$, then the $N_{\text{QCD}}$ vacua of pure QCD which were originally degenerate will have their degeneracy split by an amount $\sim 2K_{\text{em}}$. For $\frac{K_{\text{em}}}{K_{\text{QCD}}} \sim 10^{-10}$, this splitting is enough to bias the $N_{\text{QCD}}$ multiple vacua so that the lowest energy vacuum percolates. Hence, new but tiny nonperturbative effects due to small AEH instantons can solve the cosmological domain wall problem.

Our analysis up to this point was made under the assumptions that the matter content is that of a certain extended class of the standard model and that the AEH instantons represent the characteristic size of nonperturbative semiclassical gravity effects. It should
always be kept in mind that these assumptions may not hold. In such cases, the axion solution to ‘strong CP’ problem may be endangered by short-distance physics. If this occurs, the axion solution can still be saved by decoupling the fermion PQ current from the gravity and $U(1)$ gauge field, but not from QCD. This implies

$$\text{Tr} [Q_{pq} T_d^2] \neq 0,$$

$$\text{Tr} [Q_{pq} Q_{em}] = 0,$$

$$\text{Tr} [Q_{pq}] = 0. \quad (17)$$

These require, in general, that the quarks must be at least in two different color and $U_{em}(1)$ representations. We note that a similar observation was made by Georgi and Wise\textsuperscript{14} in the context of grand unified theory axion models. To be free of cosmological domain wall problem, $N_{QCD} = 1$, or inflation is also required.

Since size of the gravity-induced axion potential is rather small, there is virtually no change in the cosmological coherent axion oscillation and associated axion energy density problem. Since the new contribution to the axion potential by small instantons is most pronounced near $f_a$, finite-temperature effects are insignificant at the time of potential coherent oscillations. Even if the new induced axion potential were sizeable, e.g. $\theta_{QCD} \approx \theta_{AEH}$ and $K_{AEH} \gg K_{QCD}$, one would still have to overcome finite-temperature thermal damping. The damping rate is larger than that of the universe expansion as long as $T \geq 10^4$ GeV.\textsuperscript{15} Due to this overdamping, coherent axion oscillations will not start any sooner than around the electroweak scale or $K_{AEH}/f_a^2$, whichever is larger. Since the generated coherent axion energy is sufficiently redshifted away by now, the PQ scale $f_a$ is still bounded above by $\approx 10^{12}$ GeV.

Finally, a natural place for AEH instantons to appear is in four-dimensional compactified superstring theories.\textsuperscript{16} However, there are a few minor differences in that case. First, there exists a nontrivial dilaton field which grows stronger near the core of the instanton. Thus, the above instanton sum should be modified correspondingly. The final form is a
nontrivial action for the zero mode of the dilaton field. Also, it is necessary to keep higher
derivative terms in calculating the instanton action. Since the sign in front of curvature
squared term is negative, the resulting instanton action may actually be much smaller than
that in the pure Einstein theory discussed in the present paper. These two aspects add
together in a way to increase the total axion potential size. This will be discussed in detail
in a separate paper\textsuperscript{16}.

To conclude, we have given a concrete and calculable realization of the possibility
that short distance effects such as semiclassical gravity might disturb the PQ mechanism.
Indeed, the very same argument goes through for any global symmetries spontaneously
broken at a relatively low energy scale. It is clear that axion physics does generically
depend sensitively on the details of physics at short distance scales. While we have looked
at effects of short-distance physics such as gravity on the axion, these effects would manifest
themselves equally strongly on low-energy pseudo-Goldstone bosons in QCD: for example,
the mass of the $\eta$ and the $\eta'$ were seen to depend sensitively on short-distance physics, and
hence may serve as a potential probe of QCD at short-distances.

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