Poster Abstract: StocHy - automated verification and synthesis of stochastic processes

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ABSTRACT

Stochastic hybrid systems (shs) are a rich mathematical modelling framework capable of describing complex systems, where uncertainty and hybrid (that is, both continuous and discrete) components are relevant. We introduce a new software tool - StocHy - aimed at simplifying both the modelling of shs and their analysis. StocHy can (i) perform verification tasks, e.g., compute the probability of staying within a certain region of the state space from a given set of initial conditions; (ii) automatically synthesise strategies maximising this probability, and (iii) simulate the system evolution over time. We highlight the performance of StocHy, via a set of experiments that are run on a standard laptop, with an Intel Core i7-8550U CPU at 1.80GHz × 8 and with 8 GB of RAM. StocHy is available at gitlab.com/natchi92/StocHy.

CCS CONCEPTS

• Mathematics of computing → Markov processes.

KEYWORDS

formal methods, verification, synthesis, stochastic hybrid systems

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1 IMPLEMENTATION

StocHy is implemented in C++ and employs manipulations based on vector calculus, the symbolic construction of probabilistic kernels, and multi-threading. Shs are described by parsing well-known and used state-space models from which StocHy generates a standard shs model automatically and formats it to be analysed. StocHy is modular, and has separate simulation, verification and synthesis engines, which are implemented as independent libraries.

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2 DISCRETE-TIME STOCHASTIC HYBRID SYSTEMS

A shs [1] is a discrete-time model defined as the tuple

\[ H = (Q, n, U, T_x, T_q), \]

where

- \( Q = \{q_1, q_2, \ldots, q_m\}, m \in \mathbb{N}, \) represents a finite set of modes (locations);
- \( n \in \mathbb{N} \) is the dimension of the continuous space \( \mathbb{R}^n \) of each mode; the hybrid state space is then \( \mathcal{D} = \bigcup_{q \in Q} \{q\} \times \mathbb{R}^n; \)
- \( U \) is a continuous set of actions, e.g., \( \mathbb{R}^2; \)
- \( T_q : \mathcal{D} \times U \to [0, 1] \) is a discrete stochastic kernel on \( Q \) given \( \mathcal{D} \) and \( U, \) which assigns to each state \( s = (q,x) \in \mathcal{D} \) and action \( u \in U, \) a probability distribution over \( Q \);
- \( T_x : \mathcal{B}(\mathbb{R}^n) \times \mathcal{D} \times U \to [0, 1] \) is a Borel-measurable stochastic kernel on \( \mathbb{R}^n \) given \( \mathcal{D} \) and \( U, \) which assigns to each state \( s \in \mathcal{D} \) and action \( u \in U, \) a probability measure on the Borel space \( (\mathbb{R}^n, \mathcal{B}(\mathbb{R}^n)) \).

In this model the discrete component takes values in a finite set \( Q \) of modes (a.k.a., locations), each endowed with a continuous domain (the Euclidean space \( \mathbb{R}^n \)). The semantics of transitions at any point over a discrete time domain, are as follows: given a point \( \xi \in \mathcal{D}, \) the discrete state is chosen from \( T_q, \) and depending on the selected mode \( q \in Q \) the continuous state is updated according to the probabilistic law \( T_x \). Non-determinism in the form of actions can affect both discrete and continuous transitions.

3 FORMAL VERIFICATION

StocHy performs formal verification of shs via either of two abstraction techniques: (i) for discrete-time, continuous-space models with additive disturbances, and possibly with multiple discrete modes, we employ formal abstractions as general Markov chains (MC) or Markov decision processes (MDP); StocHy improves techniques the state-of-the-art Faust2 tool [4] by simplifying the input model description and by reducing the computational time needed to generate the abstractions; and (ii) for models with a finite number of actions, we employ interval Markov decision processes (IMDP) and the model checking framework in [3]; StocHy incorporates a novel abstraction algorithm allowing for efficient computation [2].

3.1 Comparison of verification methods

We consider a simple shs, consisting of one discrete mode \( Q = \{q_0\} \) with two continuous variables \( x \in \mathbb{R}^2 \) which evolve according to

\[ T_x = \mathcal{N}(\cdot; A_q x, G_q). \] (2)

Here, \( \mathcal{N}(\cdot; \eta, \Sigma) \) denotes a Gaussian density function with mean \( \eta \) and covariance matrix \( \Sigma. A_q = \begin{bmatrix} 0.945 & 0 \\ 0 & 0.945 \end{bmatrix} \) and \( G_q = \begin{bmatrix} 1.782 & 0 \\ 0 & 0.511 \end{bmatrix}. \)

We are interested in computing the probability of remaining within
We consider a stochastic process with two modes $X = \{q_0, q_1\}$ and with two continuous variables $x \in \mathbb{R}^2$ evolve using (2), with,

$$A_{q_0} = \begin{bmatrix} 0.43 & 0.52 \\ 0.65 & 0.12 \end{bmatrix}, \quad G_{q_0} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \quad A_{q_1} = \begin{bmatrix} 0.65 & 0.12 \\ 0.52 & 0.45 \end{bmatrix}, \quad G_{q_1} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}.$$

For this example, the actions are associated to a deterministic selection of locations. The continuous variables $x$ are bounded within the domain shown Fig. 2a. We would like to synthesise a trajectory such that given any initial condition we avoid the "purple" region until we reach the "green" region. This requirement can be expressed as an LTL formula $\phi_2 := (\neg \text{green}) U \text{ purple}$ where $U$ is the "until" operator and the atomic propositions (purple, green) denote regions within the set $X = [-1.5, 1.5]^2$. We synthesise a strategy, $\pi^*$, using IMPD algorithm within StocHy. This generates an abstraction with a total of 2410 states, a maximum probability of 1, a maximum abstraction error of 0.21 and takes 1639.3 [s]. The lower probabilities of satisfying $\phi_2$ for each mode are shown in Fig. 2b and Fig. 2c. Fig. 2a shows the simulation of a trajectory under $\pi^*$ with a starting point of $(-0.5, -1)$ in $q_0$.

5. SIMULATION

StocHy allows simulation of complex stochastic processes by means of Monte Carlo techniques; StocHy automatically generates statistics from the simulations in the form of histograms, visualising the evolution of both the continuous random variables and the discrete modes. We consider a $\text{shs}$ consisting of $Q = \{q_0, q_1, q_2, q_3\}$ with the continuous dynamics are characterised using the continuous kernel $T_x = N(\cdot; A_q x + B_q u + x \sum_{i=1}^n N_q(i) u_i + F_q, G_q)$, where $A_q, B_q, G_q$ are appropriately sized matrices, $N_q(i)$ represents the bilinear influence of the $i$-th input component $u_i$. The actual values of the matrices $A_q, B_q, G_q, N_q$ are provided within the tool distribution. We depict the $\text{shs}$ for the discrete modes and the input control signal $u$ within Fig. 3a. We simulate the evolution of this dynamical model over a fixed time horizon $K = 32$ steps, with an initial $x_1 \sim N(450, 25)$ and $x_2 \sim N(17, 25)$. The generated histograms depicting the range of values the continuous variables can be in during each time step and the associated count are shown in Fig. 3c for $x_1$ and Fig. 3d for $x_2$; and a histogram showing the likelihood of being in a discrete mode within each time step in Fig. 3e. The total time taken to generate the simulations is 48.6 [s].

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