Research Article

Motion Control of a 4WS4WD Path-Following Vehicle: Dynamics-Based Steering and Driving Models

Zhonghua Zhang,1 Caijin Yang,1 Weihua Zhang,1 Yanhai Xu,2 Yiqiang Peng,2 and Maoru Chi1

1State Key Laboratory of Traction Power, Southwest Jiaotong University, Chengdu 610031, China
2School of Transportation and Automotive Engineering, Xihua University, Chengdu 610039, China

Correspondence should be addressed to Caijin Yang; ycj78_2012@163.com

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This paper deals with a four-wheel-steering four-wheel-driving (4WS4WD) vehicle under the path-following control. Focuses are placed on the motion control of the vehicle, and the drive forces and steering angles for achieving accurate path-following by the vehicle are determined. In this research, a nonlinear vehicle model of three degrees of freedom (DOFs) is used. The vehicle path-following dynamics are modeled using the classical mass-damper-spring vibration theory, which is described by three ordinary differential equations of second order with lateral, heading and velocity deviations, and control parameters. Combined with the vehicle path-following dynamic model, the nonlinear vehicle dynamic model is decoupled in generalized coordinate space. The required drive forces and steering angles for the vehicle path-following controllers are thus calculated and control models are obtained. Theoretical analysis for steering and driving control models is also carried out. It discloses that control models can maintain good performance against uncertainties. The vehicle path-following control is exhibited by dynamic simulation in CarSim with consideration of a complex vehicle model and a variable-curvature planned path. Numerical results obtained are analyzed and show control models have capable of dealing with a complex path-following problem. This paper provides a new insight into understanding path-following control of a 4WS4WD vehicle at the generalized vibration level.

1. Introduction

The demand for good acceleration and maneuvering performances of a road vehicle is ever increasing in the modern automotive industry. To meet this demand, novel vehicles such as four-wheel-steering (4WS) vehicles [1–4] and four-wheel-driving (4WD) vehicles [5–9] have thus been developed. Compared with conventional road vehicles, these vehicles are often equipped with multiple steering and/or driving devices so that they have the advantages of smaller turning radius, higher maneuverability, and better traction. During the past several years, the 4WS vehicles and 4WD vehicles have been received extensive researches and rich achievements are obtained. Li [1] proposed a new LVP control strategy with robustness for 4WS vehicles under velocity-varying motion. Chen [2] investigated nonlinear input-output decoupling control for 4WS vehicles with an observer. Marlene Kreutz [3] presented two design strategies for an active rear-wheel steering control system to improve the maneuverability and stability of four-wheel steering vehicles. In Ref. [5], Chen considered modeling errors and complex driving scenarios, and further studied the path-following control of autonomous four-wheel-independent-drive electric vehicles by means of second-order sliding mode and nonlinear disturbance observer techniques. Zhang [7] discussed the actuator fault detector design problem for an electric ground vehicle equipped with an active 4WS system. Nguyen [8] aimed at the shared lateral control between the human driver and lane-keeping assist system and developed a driver-automation cooperative control approach for intelligent vehicles that may be equipped with multiwheel steering and/or driving systems.
In Ref. [9], this researcher also investigated the path-following control problem of autonomous intelligent vehicles and proposed a fuzzy static output feedback control method.

In recent years, another promising vehicle has been received considerable attention from researchers and engineers, namely, the four-wheel-steering four-wheel-driving (4WS4WD) vehicles [10, 11] with excellent maneuverability and strong traction. The 4WS4WD vehicle is an over-actuated system where each of the four wheels can independently steer and drive. It can, therefore, combine with the benefits of 4WS vehicles and 4WD vehicles and have superiority over conventional road vehicles, 4WS vehicles, and 4WD vehicles in engineering applications. In particular, a 4WS4WD vehicle under the path-following control can be used to perform various tasks or missions. For example, a path-following vehicle is able to be an autonomous mobile carrier for heavy goods and human transportation. It can even carry out scientific research activities in outer space due to excellent performances and complex space conditions. Hence, it is of great significance for studying the path-following 4WS4WD vehicle, both theoretically and practically.

This paper deals with the path-following control of the 4WD4WS vehicle. This topic has been studied by some researchers and engineers. Representative studies could be found in the literature [12–23]. Majura F. [12] investigated the path-following control of ground robotic vehicles with four independently steered and driven wheels. Closed-loop kinematic constraints of the vehicle were established using the path geometry and the vehicle speed only. With established constraints, steering angles and speeds of individual wheels of the vehicle were determined, and then, a path-tracking controller was developed. Elie Maalouf [13] designed a fuzzy logic path-following controller for a wheeled-mobile robot. This controller is highly robust and flexible. Moreover, it can control the robot at a higher level and automatically follow a sequence of discrete waypoints. No interpolation of the waypoints was needed to generate a continuous reference trajectory in controller design. Martin Udengaard [14] presented a kinematic analysis and control method for an omnidirectional mobile robot, whose average isotropy was analyzed as a function of wheel module geometry on both flat and rough terrain. A simple kinematic controller with the effects of terrain unevenness was presented, and the performance of the presented algorithm was studied by simulation in Ref. [14]. Reza Oftadeh [15] provided a motion control law that can make the base follow a given smooth path and heading profile. This law was successfully used to solve the problem of motion control for a mobile robot with four individual steers and drive wheels.

Farbod Fahimi [16] proposed a 3DOFs dynamic model-based controller for four-wheel-steer and all-wheel-drive vehicles with consideration of considered parameter uncertainty. The controller proposed was capable of regulating longitudinal, lateral, and yaw motions of the vehicle simultaneously. Simulation results showed that the controller was robust. By integrating sliding mode control and particle swarm optimization, Dai [17] presented a novel control method for the path following a 4WS4WD vehicle. This control method had the ability to resist nonlinear, highly coupled and overactuated characteristics of the 4WS4WD vehicle. The robustness of this method was demonstrated by simulations. Liang [18] proposed a comprehensive control method for the 4WS4WD vehicle, which integrated active steering and direct yaw moment control systems. The proposed integrated control method can effectively improve the lateral dynamics performance of the vehicle at high speeds as compared to previous methods. Peng [19] presented an approach of wheel slip constraint to control an autonomous 4WS4WD vehicle. An integral compensation with a low-and-high gain technique was exploited to simultaneously eliminate the steady-state error of the path tracking and enhance the utilization of the constrained wheel slip. Simulation results showed that the proposed scheme was effective. Ramprasad Potluri [20] studied the path-tracking control of an autonomous 4WS4WD electric vehicle using its natural feedback loops. A trajectory tracking control scheme is developed for a four-wheel-independent steering and four-wheel-independent driving mobile robot by Yang [21], where both nonlinear kinematic control and dynamic sliding-mode control are designed. Lee [22] investigated the path-tracking problem for 4WS4WD electric vehicles with input constraints, actuator faults, and external resistance. A hybrid fault-tolerant control approach was proposed, which combines the linear-quadratic control method and the control Lyapunov function technique. This method can not only maintain the vehicle’s tracking performance in spite of faults, input constraints, and external resistance but also reduce the cost of the fault-tolerant process. Li [23] investigated the path-tracking control problem of 4WS4WD road vehicles. An adaptive and fault-tolerant tracking control scheme was proposed in [23], which had capable of compensating vehicle uncertain dynamics/disturbances and actuation failures.

Though great efforts were made and some achievements were obtained in previous studies [12–23], the path-following control of the 4WS4WD vehicle may be worthy of further studying because of its complexity and unsolved problems. As well known, the 4WS4WD vehicle is an over-actuated system with six or eight control inputs. To maintain the good handling performance of the vehicle, all controls need to be accurate and cooperative at the high level. As for the path-following controllers [12–23] as concerned, some [12–15] are designed based on kinematic vehicle models while the others [16–23] are developed in vehicle dynamics. In general, kinematics-based control models [12–15] are simple and easy to be implemented. However, they are usually unable to afford dynamic disturbances and valid at low vehicle velocities. By contrast, dynamics-based control models [16–23] are often complex and may cause expensive computations despite that they are more accurate.

Motivated by mentioned-above problems, this paper aims at developing a dynamics-based method for fast determining steering angles and driving forces for achieving accurate path-following by a 4WS4WD vehicle. In this research, a 3DOFs nonlinear coupled dynamic model is used to describe the vehicle motions. The path-following problem of the vehicle is characterized using lateral, heading, and
velocity deviations. The mass-damper-spring model is adopted to form self-adaptive and self-stable zero-convergences of lateral, heading, and velocity deviations. The vehicle path-following dynamics are then modeled by three second-order ordinary differential equations of lateral, heading, and velocity deviations. Combined with the vehicle path-following dynamics model, the nonlinear motion model of the vehicle is decoupled by common approximation techniques. Linear equations with steering angles and driving forces of vehicle wheels are obtained and control input variables are thus determined. Compared with previous methods, the present method can maintain the main advantages of kinematics- and dynamics-based path-following control models. It is fast and robust with the nonlinear effects of vehicle dynamics. Original contributions of this paper are as follows: (1) the path-following dynamics of the vehicle are ascribed to the classical mass-damper-spring vibration problem. In this way, coupled nonlinear motion equations of the vehicle are successfully decoupled in generalized coordinate space. (2) A fast and robust method for determining all control input variables of a path-following 4WS4WD vehicle is proposed. The proposed method is analytical with nonlinear effects of vehicle dynamics and can be implemented in controllers without expensive computations. (3) Steering and driving control models of the vehicle obtained are examined and validated by means of dynamic simulation in CarSim with a complex vehicle model and planned path.

The rest part of this paper is organized as follows: after a brief description of a 4WS4WD vehicle system considered in Section 2, a 3DOFs nonlinear motion model of the vehicle is presented, and the vehicle path-following dynamics are modeled subsequently. The path-following problem is addressed using three uncoupled mass-damper-spring systems of single DOF with lateral, heading, and velocity deviations of the vehicle. In Section 3, a decoupling method is presented to determine steering angles and driving forces for the path-following vehicle from the nonlinear motion model of the vehicle. Important theoretical analysis for control models is carried out in this section. In Section 4, control models are examined and validated by using dynamic simulation in CarSim with consideration of a variable-curvature planned path and a complex vehicle model, instead of 3DOFs simple vehicle model used for the controller design. Dynamic results are obtained and analyzed. Conclusions are given finally.

2. Dynamic Modeling

A 4WS4WD vehicle system considered is shown in Figure 1. It is mainly composed of a vehicle body, four in-wheel motors, and two steering systems. Each motor can be driven independently. Each steering system consists of a steering servo motor and a mechanical device, which connects the wheels on the left and right sides of the vehicle. In this section, the modeling of the considered vehicle is presented first and the path-following dynamics of the vehicle are described next. Relevant details are provided below.

2.1. Vehicle Motion Model. For the sake of studying the motion control of the vehicle, the vehicle is routinely treated as a rigid body in a plane motion. Hence, the vehicle system can be modeled using three degrees of freedom represented by the coordinates $X_C, Y_C$ and $\theta$. Note that the coordinates $X_C, Y_C$ and $\theta$ denote the X and Y components of displacement of the vehicle mass center and the yaw angle of the vehicle, respectively, which are defined in a global coordinate system $OXY$. Using Newton’s second law of motion, motion equations of the vehicle can be established in terms of the mentioned-above coordinates. They are as follows:

The motion equation of the vehicle in the horizontal direction can be expressed as

$$m\ddot{X}_C = F_x(\delta_i, F_y, F_f, F_w, \dot{F}_f, \dot{\theta}),$$

where $m$ is the mass of the vehicle and $F_x$ is the resultant of all forces exerted on the vehicle along with the horizontal direction. In classical vehicle dynamics, the force $F_x$ associates with the steering angle $\delta_i$, the lateral force $F_y$, and the longitudinal force $F_f$ of the $i$th wheel, in which a subscript $i = fl, fr, rl, and rr$, respectively, denotes the left and right wheels at front and rear axles of the vehicle, as well as the aerodynamic resistance $F_w$ and the rolling resistance $F_r$ of the vehicle. Note that these forces are formulated in the following text. The force $F_x$ can be thus calculated as

$$F_x = f_x \cos \theta - f_y \sin \theta,$$

where the forces $f_x$ and $f_y$ are, respectively, defined as

$$f_x = F_l^f \cos \delta_j - F_r^s \sin \delta_j - F_y^l \cos \delta_r + F_y^r \sin \delta_r - F_w - F_f,$$

$$f_y = F_l^f \sin \delta_j + F_r^s \cos \delta_j + F_y^l \sin \delta_r + F_y^r \cos \delta_r,$$

in which there are [17]

$$\delta_j = \delta_{jl} - \delta_{jr}, \delta_r = \delta_{rl} = \delta_{rr},$$

and

$$F_l^f = F_{l_{fl}}^f + F_{l_{fr}}^f,$$

$$F_r^s = F_{r_{rl}}^s + F_{r_{rr}}^s,$$

$$F_y^l = F_{y_{fl}}^l + F_{y_{fr}}^l,$$

$$F_y^r = F_{y_{rl}}^r + F_{y_{rr}}^r.$$

Similarly, the motion equation of the vehicle in the vertical direction can be expressed as

$$m\ddot{Y}_C = F_y(\delta_i, F_y, F_f, F_w, \dot{F}_f, \dot{\theta}),$$

where $F_y$ is the resultant of all forces exerted on the vehicle along with the vertical direction and can be defined as

$$F_y = f_x \sin \theta + f_y \cos \theta.$$

Moreover, the equation of the yaw motion of the vehicle can be expressed as

$$\ddot{\theta} = M_\theta(\delta_i, F_y, F_f, F_w, \dot{F}_f, \dot{\theta}),$$
where \( J \) is the inertia moment of the yaw motion of the vehicle and the resultant moment \( M_z \) is defined as
\[
M_z = (F_f \sin \delta + F_r \cos \delta)b + (F_r \sin \delta + F_f \cos \delta)a,
\]
in which \( a \) and \( b \) denote half of the front and rear track widths, respectively.

For a general purpose, equations (1), (7), and (9) are rewritten in the matrix form as
\[
M \ddot{X} = \mathbf{F},
\]
where the coordinate vector \( \mathbf{X} = [X_C, Y_C, \theta]^T \), in which a superscript “T” denotes the transpose of a vector or matrix, and the mass matrix \( M \) and the force vector \( \mathbf{F} \) are, respectively, expressed as
\[
M = \begin{bmatrix}
m & 0 & 0 \\
0 & m & 0 \\
0 & 0 & J
\end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix}
f_x \cos \theta - f_y \sin \theta \\
f_x \sin \theta + f_y \cos \theta \\
M \zeta
\end{bmatrix}.
\]

It is remarkably noted that the vehicle model in equation (11) is described in the global coordinate system \( OXY \). It can be identical with the same one used in Refs. [16, 24] by means of a coordinate transformation.

### 2.1.1. Tire Model
The dynamic performance of a road vehicle largely depends on the characteristics of the tire. Various tire models are developed for the vehicle dynamics analysis and available in the literature, e.g., simple linear model [16–18, 24, 25] and complex magic formula [26, 27]. A classical linear tire model [16, 17] is adopted here for controller design routinely. In Refs. [18, 24], the lateral force \( F_y \) of the tire is calculated as
\[
F_y = C_{a} \alpha,
\]
where \( C_{a} \) is the cornering stiffness of the tire and \( \alpha \) is the slip angle of the tire. According to classical tire dynamics, the slip angle \( \alpha \) is defined as
\[
\alpha = \zeta - \delta,
\]
where \( \delta \) is the tire steering angle and \( \zeta \) is the tire velocity angle. The tire velocity angle \( \zeta \) is expressed as
\[
\zeta = \tan^{-1}\left(\frac{v_y}{v_x}\right),
\]
where \( v_x \) and \( v_y \) are two components of the velocity of the wheel center defined in the vehicle-body coordinate system \( OXY \).

#### 2.1.2. Resistances to the Vehicle Motion
The aerodynamic and rolling resistances are included in the vehicle model. In general, the aerodynamic resistance \( F_w \) depends on the air density \( \rho \), the drag coefficient \( C_d \), the frontal cross-sectional area \( A \) of the vehicle, and the longitudinal velocity \( V_d \) of the vehicle. In Refs. [1, 16, 24, 28], the aerodynamic resistance \( F_w \) is calculated as
\[
F_w = C_d \rho V_d^2.
\]
Rolling resistance mainly results from tire deformation. The tire rolling resistance \( F_f \) can be simply calculated as
\[
F_f = C_f mg,
\]
where \( C_f \) is the rolling drag coefficient and \( g \) is the gravitational acceleration constant.

### 2.2. Vehicle Path-Following Dynamics
Sometimes, the vehicle is expected to follow a planned path. But, it inevitably deviates from the planned path due to some reasons, e.g., road disturbances. In this section, the vehicle path-following problem is addressed based on vehicle deviation dynamics together with the classical mass-damper-spring model. Important details are presented below.

#### 2.2.1. Vehicle Path-Following Deviations
The path-following deviations of the vehicle in the current state from its
desired state can be measured using the lateral deviation $\varepsilon_d$, the heading deviation $\varepsilon_\theta$, and the velocity deviation $\varepsilon_v$, as illustrated in Figure 2. The lateral deviation $\varepsilon_d$ is defined as a minimal distance from a control point in the vehicle, e.g., the mass center $C$, to the planned path. The heading deviation $\varepsilon_\theta$ actually represents the angle between the vehicle centerline and the road tangent line. The velocity deviation $\varepsilon_v$ is a difference between the instantaneous and planned velocities of the vehicle in the vehicle longitudinal direction.

2.2.2. Vehicle Deviation Dynamics. There is no doubt that the vehicle is in a state of three zero-deviations defined above when it accurately follows the planned path as expected. In that sense, the path-following of the vehicle is such a process that three-vehicle deviations converge zeros. Zero-convergence dynamics of lateral, heading, and velocity deviations of the vehicle are characterized in the following text.

Take zero-convergence dynamics of the vehicle lateral deviation as an example for illustrative purposes. Considering the mass-damper-spring model in classical vibration theory, the zero-convergence of the vehicle lateral deviation $\varepsilon_d$ can be governed by

$$\dot{\varepsilon}_d + c_d \ddot{\varepsilon}_d + k_d \varepsilon_d = 0,$$  
(19)

where constants $c_d$ and $k_d$ are control parameters introduced. It is obvious that nonzero lateral deviation $\varepsilon_d$ in the system of equation (19) can quickly converge to nearly zero in classical vibration theory. In a similar way, zero-convergence dynamics for the vehicle heading and velocity deviations are addressed as

$$\dot{\varepsilon}_\theta + c_\theta \ddot{\varepsilon}_\theta + k_\theta \varepsilon_\theta = 0,$$  
(20)

$$\dot{\varepsilon}_v + c_v \ddot{\varepsilon}_v = 0,$$  
(21)

where constants $c_\theta$, $k_\theta$, and $c_v$ are control parameters.

Next, equations (19)–(21) are rewritten in terms of vehicle coordinates. Considering a smooth planned path, $\varphi(X, Y) = 0$, where the curve $\varphi$ is assumed to be continuous and differentiable to at least second order and knowing the vehicle state at any time, the deviations $\varepsilon_d$, $\varepsilon_\theta$, and $\varepsilon_v$ can be specified as follows. Let $P$ be the projection point of the mass center $C$ onto the curve $\varphi$. Without loss of generality, the position $(X_P$ and $Y_P)$ of point $P$ can be expressed in an implicit form as

$$\begin{cases} 
X_P = \mathcal{T}(X_C, Y_C), \\
Y_P = \mathcal{G}(X_C, Y_C), 
\end{cases}$$  
(23)

where the functions $\mathcal{T}$ and $\mathcal{G}$ are relation to the curve $\varphi$ and the mass center $C (X_C$ and $Y_C)$. In terms of the differential geometry theory, the normal vector $\mathbf{n}$ and the tangential vector $\mathbf{t}$ of the curve at projection point $P (X_P, Y_P)$ are, respectively, expressed as

$$\mathbf{n} = [\varphi_X, \varphi_Y]^T,$$  
(24)

$$\mathbf{t} = [-\varphi_Y, \varphi_X]^T,$$  
(25)

where $\varphi_X$ and $\varphi_Y$ denote partial derivatives of the equation of the curve $\varphi$ with respect to coordinates $X$ and $Y$, respectively. The normalization of equation (25) leads to

$$\mathbf{n}_r = \frac{\mathbf{t}}{\|\mathbf{t}\|} = \frac{1}{\sqrt{\varphi_X^2 + \varphi_Y^2}} [-\varphi_Y, \varphi_X]^T,$$  
(26)

where $\mathbf{n}_r$ is an unit vector. Let $\mathbf{h}$ be a unit vector that indicates the longitudinal direction of the vehicle. Considering the definition of the vehicle yaw angle $\theta$, the unit vector $\mathbf{h}$ is then given as

$$\mathbf{h} = [\cos \theta, \sin \theta]^T.$$  
(27)

Let $V_d$ and $V_t$ be the instantaneous and planned velocities of the vehicle in the longitudinal direction, respectively. The longitudinal velocity $V_d$ can be expressed as

$$V_d = X_C \cos \theta + Y_C \sin \theta.$$  
(28)

Thus, the lateral deviation, the heading deviation, and the velocity deviation are, respectively, given by

$$\varepsilon_d = \sqrt{(X_C - X_P)^2 + (Y_C - Y_P)^2} = f_d(X_C, Y_C, \theta),$$  
(29)

$$\varepsilon_\theta = \cos^{-1}([\mathbf{h}^T \mathbf{n}_r]) = f_\theta(X_C, Y_C, \theta),$$  
(30)

$$\varepsilon_v = V_d - V_t = \dot{X}_C \cos \theta + \dot{Y}_C \sin \theta - V_t = f_v(\dot{X}_C, \dot{Y}_C, \theta, t).$$  
(31)

Substituting equations (29)–(31) into equations (19)–(21) and rearranging them in terms of vehicle coordinates $X_C$, $Y_C$, and $\theta$, the vehicle path-following dynamics can be described as

$$\overline{M} \ddot{\mathbf{X}} = \mathbf{F},$$  
(32)

where the matrix $\overline{M}$ is defined as

$$\overline{M} = \begin{bmatrix} 
\frac{\partial f_d}{\partial X_C} & \frac{\partial f_d}{\partial Y_C} & 0 \\
\frac{\partial f_\theta}{\partial X_C} & \frac{\partial f_\theta}{\partial Y_C} & \frac{\partial f_v}{\partial \theta} \\
\frac{\partial f_v}{\partial X_C} & \frac{\partial f_v}{\partial Y_C} & \frac{\partial f_v}{\partial \theta} 
\end{bmatrix},$$  
(33)

and the force vector $\mathbf{F}$ is defined as

$$\mathbf{F} = - \begin{bmatrix} 
\frac{\partial f_v}{\partial \theta} \dot{\theta} + c_v \varepsilon_v + V_t \\
X^T \mathbf{H}_d \dot{X} + c_d \varepsilon_d + k_d \varepsilon_d \\
X^T \mathbf{H}_\theta \dot{\mathbf{X}} + c_\theta \varepsilon_\theta + k_\theta \varepsilon_\theta 
\end{bmatrix},$$  
(34)

in which $\mathbf{H}_d$ and $\mathbf{H}_\theta$, respectively, have forms of
3. Steering and Driving Control Models

The current vehicle system steers by two servo motors and drives by four in-wheel motors. There are, therefore, six control inputs and redundant controls exist. For the sake of simplicity, it is assumed that longitudinal forces of tires on the front and rear vehicle axes have a proportional relationship of the constant ratio $k_{fr}$. The following equation is then obtained as

$$F_l^f = k_{fr}F_l^r.$$  \hspace{1cm} (36)

Thus, four control inputs are required in the vehicle motion control. They are steering angles $\delta_f$ and $\delta_r$ and longitudinal tire forces $F_l^f$ and $F_l^r$. A decoupling method is presented below to determine these control variables from the nonlinear vehicle model in equation (11).

$$H_d = \begin{bmatrix}
\frac{\partial^3 f_d}{\partial X_C^2} & \frac{\partial^2 f_d}{\partial Y_C \partial X_C} & \frac{\partial^2 f_d}{\partial \theta \partial X_C} & \frac{\partial^2 f_d}{\partial \theta^2} \\
\frac{\partial^2 f_d}{\partial X_C \partial Y_C} & \frac{\partial^2 f_d}{\partial Y_C^2} & \frac{\partial^2 f_d}{\partial \theta \partial Y_C} & \frac{\partial^2 f_d}{\partial \theta^2} \\
\frac{\partial^2 f_d}{\partial X_C \partial \theta} & \frac{\partial^2 f_d}{\partial Y_C \partial \theta} & \frac{\partial^2 f_d}{\partial \theta^2} & \frac{\partial^2 f_d}{\partial \theta^2}
\end{bmatrix}, \quad H_\theta = \begin{bmatrix}
\frac{\partial^3 \theta}{\partial X_C^2} & \frac{\partial^2 \theta}{\partial Y_C \partial X_C} & \frac{\partial^2 \theta}{\partial \theta \partial X_C} & \frac{\partial^2 \theta}{\partial \theta^2} \\
\frac{\partial^2 \theta}{\partial X_C \partial Y_C} & \frac{\partial^2 \theta}{\partial Y_C^2} & \frac{\partial^2 \theta}{\partial \theta \partial Y_C} & \frac{\partial^2 \theta}{\partial \theta^2} \\
\frac{\partial^2 \theta}{\partial X_C \partial \theta} & \frac{\partial^2 \theta}{\partial Y_C \partial \theta} & \frac{\partial^2 \theta}{\partial \theta^2} & \frac{\partial^2 \theta}{\partial \theta^2}
\end{bmatrix}. \hspace{1cm} (35)

3.1. Dynamic Decoupling and Determination of Control Inputs. Multiplying equation (12) by an inverse matrix $M^{-1}$ leads to

$$\dot{X} = M^{-1}F.$$ \hspace{1cm} (37)

Substituting equation (37) into equation (32) yields

$$\dot{M}M^{-1}F = \bar{F}. \hspace{1cm} (38)$$

Equation (32) is useful in the sense that it provides dynamic constraints on control input variables. The combination of equation (36) with equation (38) leads to

$$Ax = b. \hspace{1cm} (39)$$

where $x = [F_l^f, \delta_f, F_l^r, \delta_r]^T$ and a four by four matrix $A$ is expressed as

\begin{table}
\begin{tabular}{|c|c|}
\hline
Notation & Description \\
\hline
$\varepsilon_d$ & Lateral deviation \\
$\varepsilon_\phi$ & Heading deviation \\
$\varepsilon_v$ & Velocity deviation \\
\hline
\end{tabular}
\end{table}
as determined by the sensor measurement. For a small sample the vehicle dynamics in equation (11) are nonlinearly dynamics and vehicle path-following dynamics. Clearly, mean

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where $\Delta \mathbf{M}$ and $\Delta \mathbf{F}$ are dynamic terms caused by approximations taken, and the perturbation term $\Delta \mathbf{F}$ is defined as

$$\Delta \mathbf{F} = \mathbf{M}^{-1} \Delta \mathbf{F} + \Delta \mathbf{M} \left( \mathbf{M}^{-1} \mathbf{F} + \mathbf{M}^{-1} \Delta \mathbf{F} \right) - \Delta \mathbf{F}.$$  

(47)

By contrast with equations (19)–(21), the separation of equation (46) leads to

$$\epsilon_d + c_d \epsilon_d + k_d \epsilon_d + \Delta f_d = 0,$$  

(48)

$$\epsilon_\theta + c_\theta \epsilon_\theta + k_\theta \epsilon_\theta + \Delta f_\theta = 0,$$  

(49)

$$\epsilon_v + c_v \epsilon_v + \Delta f_v = 0,$$  

(50)

where $\Delta f_d$, $\Delta f_\theta$, and $\Delta f_v$ are nonlinear functions of $\epsilon_d$, $\epsilon_\theta$, and $\epsilon_v$, as well as parameters and variables mentioned above. In the classical vibration theory, lateral, heading, and velocity deviations govern by equations. (48)–(50) may converge to nonzero constant values $\bar{\epsilon}_d$, $\bar{\epsilon}_\theta$, and $\bar{\epsilon}_v$ at the steady state. The values $\bar{\epsilon}_d$, $\bar{\epsilon}_\theta$, and $\bar{\epsilon}_v$ can be calculated as

$$k_d \bar{\epsilon}_d + \Delta f_d = 0,$$  

(51)

$$k_\theta \bar{\epsilon}_\theta + \Delta f_\theta = 0,$$  

(52)

$$c_v \bar{\epsilon}_v + \Delta f_v = 0.$$  

(53)

Based on the above analysis, it can be concluded that current control models have capable of achieving the path-following motion of the vehicle against approximations taken. However, the approximations taken have an active influence on the vehicle path-following accuracy.

Equations (48)–(53) are reconsidered and further analyzed. The analysis may discover that the path-following accuracy of the vehicle can be improved by means of adjusting control parameters. For illustration purpose, the vehicle is assumed to be in a stable state that it laterally deviates from a planned path. A simple and feasible method for improving the path-following accuracy of the vehicle is presented here according to equation (51). Without loss of generality, the term $\Delta f_d$ in equation (51) can be written as

$$\Delta f_d = \Delta k_d \epsilon_d - O_d,$$  

(54)

where $\Delta k_d$ can be a constant and the truncation error $O_d$ is close to zero. Substituting equation (54) into equation (48) yields

$$\epsilon_d + c_d \epsilon_d + (k_d + \Delta k_d) \epsilon_d = O_d \approx 0.$$  

(55)

It is thus concluded from equation (55) that the path-following accuracy of the vehicle can be improved by proper control parameter $k_d + \Delta k_d$ to some extent. By considering equations (52) and (53), similar conclusions are obtained. Importantly note that dynamic term $\Delta k_d \epsilon_d$ is caused by many reasons, e.g., unknown road conditions, inaccuracy of vehicle and tire models used, and even some uncertainties. In that sense, current control models are robust and effective against uncertainties.

4. Model Validation and Dynamic Results

Steering and driving control models in equation (43) are examined by dynamic simulation in this section. The vehicle system shown in Figure 1 is modeled in CarSim using simulation parameters listed in Table 1. A variable-curvature path is considered and illustrated in Figure 3. As shown in Figure 3, the planned path is composed of line-straight segments and circular segments.

In simulation, control parameters are taken as: $c_d = 15.0$, $k_d = 5.0$, $c_\theta = 15.0$, $k_\theta = 5.0$, and $c_v = 1.5$. The vehicle is initially at rest and deviates from the path with a lateral distance of 0.5 m. After starting, the vehicle experiences four distinct stages. In the first stage, the longitudinal velocity of the vehicle increases from zero to 5.0 m/s with a constant acceleration of 1.0 m/s$^2$. Then, the longitudinal velocity of the vehicle maintains constantly in the second stage. After simulation time $t = 30$ s, the longitudinal velocity of the vehicle decreases up to 3.0 m/s with the acceleration of $-1.0$ m/s$^2$ in the third stage. In the last stage, the vehicle runs along the path at another constant velocity of 3.0 m/s. Under the above conditions, the path-following dynamics of the vehicle are simulated. Dynamic results are obtained and presented below.

Figure 4 shows time-domain variations of lateral, heading, and velocity deviations of the vehicle under the path-following control. Figure 5 shows comparisons of the trajectory of the vehicle controlled with the planned path. It can be seen in Figures 4 and 5 that control models take effect at the higher level. The vehicle controlled quickly enters into the planned path and then better runs along the path against with discontinuous curvature variations of the path. The maximum lateral deviation is about 0.077 m, the maximum heading deviation is about 13 degrees, and the maximum velocity deviation is about 1.0 km/h. As observed in Figure 5, the vehicle path-following deviations may fluctuate slightly as the vehicle enters into or leaves off the straight-curved/curved-straight sections of the path. This phenomenon is caused by discontinuous variations of the path curvature at these regions. Control models are free from these fluctuations and continuously effective.

Figure 6 shows time-domain variations of steering angles of wheels and longitudinal forces of tires of the vehicle controlled. From the curves plotted in Figure 6(a), one can see that the vehicle initially turns in such a way that all wheels steer towards the same direction. Thus, the initial lateral deviation of the vehicle can be quickly diminished. By contrast, the rear wheels steer the opposite direction of the front wheels in the case that the vehicle stably runs along the circular sections of the path. There are two distinct steering modes. The phenomena show that control models are able to autonomously determine steering modes to fit into various circumstances. It just demonstrates the excellent maneuvering performance of the 4WS4WD vehicle. Moreover, it is interesting to note that steering angles of the front and rear wheels reversely vary with a decrease of the curvature.
Table 1: Vehicle parameters used in simulation.

| Notation                              | Value  | Unit      |
|---------------------------------------|--------|-----------|
| Vehicle mass $m$                      | 1060   | Kg        |
| Inertia moment $J$                    | 1523   | kg·m²     |
| Cornering stiffness of front tire $C_f \alpha$ | $-61060$ | N/rad    |
| Cornering stiffness of rear tire $C_r \alpha$ | $-97290$ | N/rad    |
| Half of front track width $a$         | 0.79   | M         |
| Half of rear track width $b$          | 0.84   | M         |
| The distance from mass center of vehicle to front axles $L_f$ | 1.539 | M |
| The distance from mass center of vehicle to rear axles $L_r$ | 1.539 | M |
| Air density $\rho$                   | 1.2258 | N/s²/m⁴   |
| Air drag coefficient $C_d$            | 0.3    | —         |
| Frontal cross-sectional area $A$      | 2.5    | m²        |
| Tire rolling resistance $C_f$         | 0.01   | —         |
| Gravitational constant $g$            | 9.8    | m/s²      |
| Friction coefficient $\mu$           | 0.8    | —         |

Figure 3: Schematic of the planned path.

Figure 4: Time histories of path-following deviations of the vehicle under control. (a) Lateral deviation (m). (b) Heading deviation (deg). (c) Velocity deviation (km/h).
radius of the path as the vehicle is stable in circular sections of the path. It can be understood in this sense that larger lateral forces of the vehicle are required as it runs along the circular path with a smaller radius at the same velocity, corresponding to larger steering angles of wheels. Figure 6(b) shows several jumps in the curves of longitudinal forces of tires. As observed in Figure 6(b), these jumps mainly arise in the locations where the
vehicle is near straight-curved/curved-straight sections of the path. Thus, it can be included that these jumps are caused by discontinuous curvature variations of the path in the above locations. Figure 7 shows time-domain responses of longitudinal and lateral velocities and yaw rate of the vehicle controlled. As clearly shown in Figure 7(a), the vehicle controlled runs along the path with the desired velocities as expected.

Steering and driving control models are further examined by considering variations of vehicle mass with an increment of 65 kg. Figure 8 shows the change of maximum lateral deviation of the vehicle with vehicle mass under different velocities. It can be seen in Figure 8 that there is a trend towards an increased maximum lateral deviation of one vehicle with the larger longitudinal velocity of the vehicle. Moreover, the maximum lateral deviation may become larger with an increase of vehicle mass in the case of a certain longitudinal velocity of the vehicle. Similar phenomena are found in the curves plotted in Figure 9. Figure 9 shows the variations of root mean square of lateral deviation of the vehicle against vehicle mass under different vehicle velocities. From Figures 8 and 9, one can conclude that the control models of this paper are robust, to some extent [30].

5. Conclusions

The path-following motion control of a 4WS4WD vehicle is studied. A fast and robust method is developed to determine control input variables for achieving an accurate path-following of the vehicle. Steering and driving control models are validated both theoretically and numerically. Important results obtained are as follows:

(1) Motion controls of the path-following vehicle can be regarded as such a process that three deviations
converge to zeros. The vehicle path-following dynamics can be modeled using the classical mass-damper-spring vibration theory and characterized by three second-order ordinary differential equations of parameters \( k_d, c_d, k_v, c_v \), and \( k_r \).

(2) Control models of this paper have good quality. Theoretical analysis shows that control models are effective against uncertainties. Numerical studies show that control models still take effect and afford to a complex path-following problem in Sect. 4.

Data Availability
The figure and table of simulation results used to support the findings of this study are included within the article.

Conflicts of Interest
The authors declare that they have no conflicts of interest.

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