Reply to the Comment on "Theorem on the proportionality of inertial and gravitational masses in classical mechanics"

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In a preceding Comment, the author declares that we claim that the ratio of inertial mass to gravitational mass can be derived ex nihilo and that our paper was published by mistake. In this "Reply" we dispute the point of view of the author.

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In the preceding Comment [1], B. Jancovici tries to argue why in his opinion our proof makes no sense: "Reading the paper, one realizes that it contains irrelevant calculations... aiming to prove that is a constant for one given body: how could the ratio of two constants be something else?" The point is that after reading our paper, anyone can realize that we do not use a postulate of classical mechanics that mass of a body is a constant. In other words, we prove that ratio of inertial mass to gravitational mass is a constant without postulating that masses and are absolute constants. To be more specific, let us compare postulates used by a generally accepted classical mechanics (GACM) with the postulates used in our proof.

Postulates of GACM:

(a) any body with non-zero inertial mass possesses also non-zero gravitational mass;
(b) (the Equivalence principle) the inertial mass of the body is proportional to the gravitational mass of the same body and a constant of the proportionality is the same for all bodies;
(c) masses of bodies (inertial and gravitational) are absolute constants (invariant);
(d) masses obey the principle of additivity.

Note: In GACM the claim that

\[ \text{from } (a_0) + (b) + (c) \quad (a) \]

is not quite obvious from a theoretical point of view: for example, let us consider two concrete bodies with the different masses and . Their constant masses , , , obviously, must obey the relations
\[ m_{i1} = m_{g1} = 1 \quad \text{and} \quad m_{i2} = m_{g2} = 2 \]  

(2)

but, generally speaking, it is not obvious that

\[ 1 = 2 \]  

(3)

So in GACM one considers Eq. (3) as an experimental fact. However, after one applies our arguments (see Eqs. (15)-(21) in [7]) one can prove the validity of the claim (a) (or Eq. (3)).

An important remark about the postulate (b): in his famous book [2] E. Mach convincingly shows (analyzing Newton’s well-known experiment with the ‘revolving pail’’) that all experimentally verifiable equations of Newtonian classical mechanics do not change if one supposes that inertial mass of a body is not an absolute constant (invariant) and, generally speaking, it can depend on the location of a body in space. So many scientists (see, e.g., [3-5]), following Mach’s ideas, expect that the principle of the proportionality of \( m_i \) and \( m_g \) may not be valid for classical mechanics. That is why when constructing general relativity, Einstein started with the Mach principle, but had to reject it thereafter (e.g., see [6]) because of its disagreement (as Einstein believed) with the Equivalence principle. And that is why a proof of the postulate (a) also in the framework of Mach’s ideas is of great interest.

Our postulates:

(a) any body with non-zero inertial mass possesses also non-zero gravitational mass;

(b) the masses of bodies (inertial and gravitational) do not depend explicitly on time but they can depend on their location in space;

(c) both inertial and gravitational masses obey the principle of additivity.

It is obvious that our postulates are weaker than the former ones. One can see that we do not conserve the point (a) as a postulate, but what’s more, our postulate (b) sufficiently differs from the postulate (b). The validity of the point (a) from the postulates (a); (b); (c) also (compare with Eq. (1)) is not obvious, because \( m_i \) and \( m_g \) may depend on location in different ways. This is so, because \( m_i \) and \( m_g \) have a different origin in classical mechanics.

In our paper we proved that from our postulates (a); (b) and (c) one infers the claim (a). Actually, we proved that if according to the Mach principle, the inertial mass
of a body can change from point to point in space, then the gravitational mass of the same body must also change by the same law, i.e. \(m_i\) and \(m_g\) are linear dependent (proportional) one-to-one functions. In other words, we show that even in the framework of the Mach principle the proportionality of inertial and gravitational masses must take place.

In the last paragraph of the Comment, the author advances his most serious critical remark. However, at this point he misses an implicit but very important factor:

Indeed, after one applies our arguments (see [7], Eqs. (6)-(11)), and provided that one neglects electromagnetic radiation, we can formally obtain an expression

\[m_i = \text{const} \cdot q_m;\]  

(4)

(where \(q_m\) is a charge of the given body \(m\)) for a concrete body with a given inertial mass and a given velocity. But we cannot apply the subsequent arguments (see [7], Eqs. (15)-(21)) in order to prove that all particles have the same charge-to-mass ratio. The point is that in our speculations we implicitly use the postulate \(a_0\), namely, "any body with non-zero inertial mass possesses also non-zero gravitational mass". In other words, after Eq. (11) (in [7]) we can claim that for bodies having the same mass, their \(m_i\) satisfy Eq.(11) from [7]. But we cannot claim this in the discussion concerning "inertial mass-charges" relation. In the latter case one should postulate that any body with non-zero inertial mass possesses also non-zero charge. It is obvious that such a "postulate" does not follow from the experiment.

As a finishing remark, let us note that reading the Abstract of the "Comment", a reader might come to the conclusion that our paper was published by mistake, after being rejected by the Editorial Board. In fact, the paper was published with minor omissions following a positive report by an anonymous referee selected by the Board. However, we recognize that the critical remarks contained in the "Comment" are partly justified: we should have explicitly defined our postulates.

[1] Jancovici B 1998 "Comment on ..." Eur. J. Phys.
[2] Mach E 1960 "The Science of Mechanics. A Critical and Historical Account of Its Development" (Open Court Publishing Co., La Salle)

[3] Sachs M 1976 "On the Logical Status of Equivalence Principles in General Relativity Theory" Brit. J. Phil. Sci. Vol 27 225

[4] Graneau P 1990 "The Riddle of Inertia" Electronics W orld and W ireless W orld Vol 96 60

[5] Assis A 1989 "On Mach’s Principle" Found. Phys. Lett. Vol 2 301

[6] Einstein A 1949 Autobiographical notes (Illinois).

[7] Chubykalo A and Vlaev S 1998 Eur. J. Phys. Vol 19 1