Di-neutron elastic transfer in the 
$^4$He($^6$He,$^6$He)$^4$He reaction

Dao T. Khoa$^{a,*}$ and W. von Oertzen$^{b,1}$

$^a$Institute for Nuclear Science and Technique, VAEC, P.O. Box 5T-160,
Nghia Do, Hanoi, Vietnam.

$^b$Hahn-Meitner-Institut GmbH, Glienicker Str. 100, D-14109 Berlin, Germany.

Abstract

Elastic $^6$He+$^4$He data measured at $E_{c.m.} = 11.6$, 15.9, and 60.3 MeV have been analyzed within the coupled reaction channels (CRC) formalism, with the elastic-scattering and two-neutron ($2n$) transfer amplitudes coherently included. Contributions from the direct (one-step) and sequential (two-step) $2n$-transfers were treated explicitly based on a realistic assumption for the $2n$-transfer form factor. The oscillatory pattern observed in $^4$He($^6$He,$^6$He)$^4$He angular distribution at low energies was found to be due to an interference between the elastic scattering and $2n$-transfer amplitudes. Our CRC analysis shows consistently that the direct $2n$-transfer strongly dominates over the sequential transfer and thus confirms the dominance of $2n-^4$He configuration over the $n-^5$He one in the $^6$He wave function. This result suggests a strong clusterization of the two valence neutrons and allows, therefore, a reliable estimate for the di-neutron spectroscopic amplitude.

Key words: NUCLEAR REACTIONS, $^4$He($^6$He,$^6$He)$^4$He, $E_{c.m.} = 11.6$, 15.9, and 60.3 MeV, microscopic CRC analysis, direct and sequential di-neutron transfer, deduced di-neutron spectroscopic amplitude.

With $^6$He beams becoming available with high intensity and good resolution, this Borromean nucleus is now one of the most studied unstable nuclei. It is well established that $^6$He consists of an inert $^4$He core and two valence
neutrons, with the two-neutron (2n) separation energy of 0.975 MeV [1]. The Borromean binding mechanism implies that the two valence neutrons cannot bind to the \(^{4}\)He core separately but only as a pair and a strong di-neutron correlation is necessary for the formation of the 2n-halo [2,3]. However, a direct experimental confirmation about the neutron-neutron correlation in this nucleus is not a simple task. For example, from the measurement of the total reaction (or interaction) cross section (see Ref. [4] and references therein) one can only deduce the long tail of the \(^{6}\)He ground-state (g.s.) density which is due to the low binding energy. Elastic and inelastic scattering of \(^{6}\)He beams on proton target [5,6] is more informative for testing the halo tail, especially when measured with high precision over a large angular range [7], but also cannot provide any information on the di-neutron correlation in \(^{6}\)He.

In difference to the above experiments, neutron transfer reaction induced by \(^{6}\)He beams can provide us with valuable information about the two valence neutrons in \(^{6}\)He [3,8]. In the same way as a single-nucleon transfer reaction delivers reliable estimate for the spectroscopic factor of a single-nucleon configuration, it is expected that the 2n-transfer reaction induced by \(^{6}\)He beam will also provide the spectroscopic information about 2n wave function. For this purpose, elastic \(^{4}\)He(\(^{6}\)He,\(^{6}\)He)\(^{4}\)He reaction has been measured at \(E_{c.m.} = 60.3\) MeV by Ter-Akopian et al. [9] and the data show a rise of the elastic cross section at backward angles which is likely due to the elastic 2n-transfer process [10]. An analysis of these data in the distorted wave Born approximation (DWBA) by Oganessian et al. [3] has shown that from the two configurations (“di-neutron” and “cigar”) predicted for \(^{6}\)He [1] the di-neutron configuration is dominant. It should be noted, however, that the conclusion made in Ref. [3] is meaningful only if the direct one-step transfer dominates over the sequential transfer. The experience with the 2n-transfer reactions measured with heavy ions is such that the direct and sequential transfer amplitudes are of similar magnitude, especially at low energies (see, in particular, Sect. 16.6.5 of Ref. [11]). Therefore, the contribution of sequential two-step transfer (\(^{6}\)He,\(^{5}\)He;\(^{5}\)He,\(^{4}\)He) to the total (\(^{6}\)He,\(^{4}\)He) transfer amplitude should be carefully investigated before making conclusion about the di-neutron configuration in \(^{6}\)He. In addition, the 2n-transfer might also include an indirect route [12,13] through the \(2^+\) excitation of \(^{6}\)He.

Elastic \(^{4}\)He(\(^{6}\)He,\(^{6}\)He)\(^{4}\)He reaction has been measured recently at lower energies of \(E_{c.m.} = 11.6\) and 15.9 MeV by Raabe et al. [14,15]. The most accurate are the data at \(E_{c.m.} = 11.6\) MeV which were measured in two separate experiments using the static \(^{4}\)He gas [14] and \(^{4}\)He-implanted Al foil [15] as targets. Elastic 2n-transfer contribution in the \(^{4}\)He(\(^{6}\)He,\(^{6}\)He)\(^{4}\)He reaction at \(E_{c.m.} = 11.6\) MeV was estimated [15] within DWBA, assuming the direct transfer of a structureless 2n-cluster in the \(S\)-state. It was also shown in Ref. [15] that these data are well reproduced in a simple optical model (OM) calculation using the double-folding potential obtained by Baye et al. [16] added by
a parity-dependent term to simulate elastic 2n-transfer, in the same way as discussed in Ref. [10]. Although these DWBA and OM results deliver some estimate of the 2n-transfer strength in the $^4$He($^6$He,$^6$He)$^4$He reaction, they can be of little help in extracting finer details about the $^6$He structure. The situation becomes further confused by a coupled-discretized-continuum channels (CDCC) analysis of the $^4$He($^6$He,$^6$He)$^4$He data at $E_{\text{c.m.}} = 11.6, 15.9, \text{and } 60.3 \text{ MeV}$ by Rusek and Kemper [17], where the rise of elastic $^6$He+$^4$He scattering cross section at backward angles is reproduced by using a weakly absorptive optical potential (OP) and taking into account only the coupling between the elastic scattering and breakup channels.

The purpose of our investigation is, therefore, to include a realistic reaction mechanism into the analysis of the $^4$He($^6$He,$^6$He)$^4$He reaction and to estimate as accurate as possible the 2n-transfer contribution. We analyze the elastic $^6$He+$^4$He data at $E_{\text{c.m.}} = 11.6, 15.9, \text{and } 60.3 \text{ MeV}$ by Rusek and Kemper [17], where the rise of elastic $^6$He+$^4$He scattering cross section at backward angles is reproduced by using a weakly absorptive optical potential (OP) and taking into account only the coupling between the elastic scattering and breakup channels.

The basic ingredients of the present CRC calculation are the (diagonal) OP and (off-diagonal) nonlocal coupling potentials due to the one- and two-neutron transfers. Dropping the indices of angular momenta for simplicity, the coupling potential of the direct (one-step) 2n-transfer between $^6$He and $^4$He can be written as

$$< \Psi_{^6\text{He}}(r'_{\alpha_2}, r'_1, r'_2) \Psi_{^4\text{He}}(r'_{\alpha_1}) | V_{\text{direct}} | \Psi_{^6\text{He}}(r_{\alpha_1}, r_1, r_2) \Psi_{^4\text{He}}(r_{\alpha_2}) >, \quad (1)$$

where $r_{\alpha_1,2}$ and $r'_{\alpha_1,2}$ are the centers-of-mass coordinates of the two $^4$He cores before and after collision, respectively, and similarly for the coordinates $r_{1,2}$ of the two valence neutrons in $^6$He. The transfer interaction is further determined, in the post form [11], as

$$V_{\text{direct}} = V_{1-\alpha} + V_{2-\alpha} + (U_{4\text{He}-^4\text{He}} - U_{6\text{He}-^4\text{He}}), \quad (2)$$

where $V_{1,2-\alpha}$ are the potentials binding each of the two valence neutrons to the $^4$He core in $^6$He, $U_{4\text{He}-^4\text{He}}$ and $U_{6\text{He}-^4\text{He}}$ are the core-core and $^6$He+$^4$He optical potentials at the considered energy. For the sequential (two-step) transfer $^6$He+$^4$He$\rightarrow^5$He+$^5$He$\rightarrow^4$He+$^6$He, one needs to determine the (post-form) transfer interactions

$$V_{\text{seq}}^{(1)} = V_{n-^5\text{He}} + (U_{5\text{He}-^4\text{He}} - U_{5\text{He}-^5\text{He}})$$

(3)
\[ V_{\text{seq}}^{(2)} = V_{n-\alpha} + (U_{5\text{He}-4\text{He}} - U_{6\text{He}-4\text{He}}) \]

for the first- and second-step transfers, respectively. We treat the \(^4\text{He}\) core in our CRC calculation as a structureless particle with spin \(J_{\text{4He}} = 0^+\), and both the direct and sequential \(2n\)-transfer amplitudes are properly \textit{symmetrized} with respect to the \(^4\text{He}\) exchange. In such an approximation, the transfer operators (2), (3) and (4) do not act on the internal coordinates of \(^4\text{He}\), and the coupling matrix element (1) turns out to be directly proportional to the overlap \(<\Psi_{4\text{He}}|\Psi_{6\text{He}}>\) and vertex \(<\Psi_{4\text{He}}|V + V_{2\alpha}|\Psi_{6\text{He}}>\) for the direct transfer. Similarly, the coupling terms for the sequential transfer are determined by the overlaps \(<\Psi_{4\text{He}}|\Psi_{5\text{He}}>\) and \(<\Psi_{5\text{He}}|\Psi_{6\text{He}}>,\) and vertices \(<\Psi_{4\text{He}}|V_{n-5}\Psi_{5\text{He}}>,\) \(<\Psi_{4\text{He}}|V_{n-5}\Psi_{6\text{He}}>,\)

We discuss now our choice of the wave functions \(|\Psi_{5,6\text{He}}>\). Since g.s. of \(^5\text{He}\) is a \(p^3_2\) resonance of 0.6 MeV width and unbound by 0.89 MeV, we have adopted the same \textit{quasi-bound} approximation as that used in Ref. [13] for the \(p^3_2\) valence neutron in \(^5\text{He}\). This is a reasonable approximation which produces a fast decaying tail of this \(l = 1\) state. The standard \textit{core + valence neutron} option of the code \textsc{Fresco} [18] was used for the coupling potentials of the sequential transfer, where the \(p^3_2\) valence neutron in \(^5\text{He}\) is bound by the potential \(V_{n-\alpha}\) consisting of a central Woods-Saxon (WS) potential with \(r_0 = 1.35\) fm, \(a = 0.65\) fm and a spin-orbit term of the Thomas form [11]. The WS depth was fixed to reproduce the quasi-binding energy of 0.01 MeV and strength of the spin-orbit potential was taken to be 17 MeV. Note that such a quasi-bound approximation is used in the construction of one-neutron transfer form factor only, and the CRC calculation includes the correct \textit{(negative)} \(Q\)-value for the sequential transfer channel.

The same WS geometry, as that of \(V_{n-\alpha}\) potential, was used for \(V_{n-5}\text{He}\) binding potential to generate \(p^3_2\) wave function for each of the two valence neutrons in \(^6\text{He}\), but with the WS depth adjusted to reproduce the experimental one-neutron separation energy of 1.86 MeV. Although the Borromean binding mechanism does not necessarily leads to the single-neutron wave function with the asymptotic defined by the one-neutron separation energy of \(^6\text{He}\), this approximation has been proven to be reasonable in the CRC analysis of proton-induced reactions on \(^6\text{He}\) [8] or in the calculation of \(^6\text{He}\) g.s. density [19]. Then, \(|\Psi_{6\text{He}}>\) is modelled by a \textit{“core + 2n”} bound-state wave function

\[
|\Psi_{6\text{He}}> = |\Psi_{4\text{He}} \otimes (p^3_2)^2> = \sum_N |N L(nl J); 0^+>,
\]

where \(J\) is the internal angular momentum of the \(2n\)-cluster and \(L\) is its orbital angular momentum with respect to the \(^4\text{He}\) core. The relative motion of the two \(p^3_2\) neutrons coupled to \(J = 0\) and 1 in the cluster frame is described.
by $|nlJ>$ [see Eq. (3.23) in Ref. [18] for the explicit expression of (5)]. As a
result, we have taken into account all configurations with $l = L = 0$ and 1,
where $l$ is the relative orbital angular momentum between the two neutrons.
These configurations were shown to give the most important contributions to
the $^6\text{He}$ binding energy [20]. Thus, the g.s. wave function (5) consists only of
two parts: $S$-wave (with $J = L = 0$) and $P$-wave (with $J = L = 1$). Since
(5) is not a solution of a structure model, we need to assign the amplitudes of
the $S$- and $P$-waves as accurate as possible for the CRC analysis. Given the
results of the microscopic structure study of $^6\text{He}$ [1,2] using the hyperspherical
basis which give $P_p \approx 10 - 14\%$, as well as those of the three-cluster model calculation [16] which give $P_p \approx 17\%$, we have included
explicitly into the CRC calculation such $S$- and $P$-wave amplitudes that give
$P_s = 85\%$ and $P_p = 15\%$. Note that $P$-wave does not contribute to the
"di-neutron" configuration because of the centrifugal barrier and corresponds
more likely to the "cigar" configuration [21]. A probe of the $S$- and $P$-wave
contributions to the 2$n$-transfer cross section is, therefore, necessary before a
conclusion about the di-neutron and cigar configurations is made.

We discuss now our choice of He-He potentials used in the coupling terms (2),
(3), and (4) for 2$n$-transfer. In general, realistic complex OP's for $^6\text{He}+^4\text{He},$
$^5\text{He}+^5\text{He},$ $^5\text{He}+^4\text{He},$ and $^4\text{He}+^4\text{He}$ systems should be used in the CRC calcu-
lation. Since the double-folding model (DFM) [22] for the real part of nucleus-
nucleus OP has been proven to be quite accurate at low and medium energies,
we use the latest version of DFM [23,24] to calculate the real OP ($V_{\text{Fold}}$) based
on the CDM3Y6 density dependent interaction [25] and realistic choice of
the g.s. densities for $^4,^5,^6\text{He}$. Such a folding approach was used recently by
Avrigeanu et al. [26] to successfully predict real OP’s for $^6\text{He}+^4\text{He}$ and $^6\text{He}+p$
systems. The imaginary OP is due to the coupling of the elastic channel to
all nonelastic channels, and to calculate it microscopically will be a task far
beyond the scope of DFM [22], especially for a weakly bound projectile like
$^6\text{He}$. Therefore, the standard WS shape is used for the imaginary OP and the
total OP is determined as

$$U(R) = N_V V_{\text{Fold}}(R) + V_C(R) + iW(R),$$

where

$$W(R) = -W \left[1 + \exp \left(\frac{R - R_W}{a_W}\right)\right]^{-1}.$$  

Here $V_C$ is the Coulomb potential between a point charge and a uniform charge
distribution of radius $R_C = 1.25(A_1^{1/3} + A_2^{1/3})$ fm, $R_W = r_W(A_1^{1/3} + A_2^{1/3})$.
Parameters of the WS imaginary OP (Table 1) and renormalization factor
of the energy-dependent real folded potential ($N_V = 1.15$) were chosen to
reproduce the data points at the most forward angles, where elastic scattering
dominate.
Table 1
WS parameters of $^6\text{He}+^4\text{He}$ imaginary OP (7) used in the CRC analysis of $^4\text{He}(^6\text{He},^6\text{He})^4\text{He}$ reaction with or without the coupling to the $2^+$ excitation of $^6\text{He}$. $\sigma_R$ and $\sigma_{2^+}$ are the total reaction cross section and integrated $2^+$ inelastic cross section, $SA_{g.s.}$ and $SA_{2^+}$ are the di-neutron spectroscopic amplitudes of $^6\text{He}_{g.s.}$ and $^6\text{He}_{2^+}$, respectively.

| $E_{c.m.}$ | $J_V$ | Coupling | $W$ | $r_W$ | $a_W$ | $\sigma_R$ | $\sigma_{2^+}$ | $SA_{g.s.}$ | $SA_{2^+}$ |
|---------|-----|---------|-----|-----|-----|-------|-------|------|-------|
| MeV     | MeV fm$^3$ | g.s. ↔ $2^+$ | MeV | fm | fm | mb | mb | |
| 11.6    | 443 | No      | 29.0 | 1.30 | 0.25 | 859  | 0    | 1.15 | 0      |
| 11.6    | 443 | Yes     | 23.0 | 1.30 | 0.28 | 801  | 53.8 | 0.95 | 1.3    |
| 15.9    | 436 | No      | 29.0 | 1.15 | 0.20 | 756  | 0    | 1.15 | 0      |
| 60.3    | 376 | No      | 9.00 | 1.25 | 0.75 | 829  | 0    | 1.00 | 0      |
| 60.3    | 376 | Yes     | 7.50 | 1.25 | 0.75 | 797  | 36.1 | 0.85 | 1.3    |

$^4\text{He}$ density was taken in the Gaussian form adopted in Ref. [22], which has been proven by a folding analysis of $\alpha$-nucleus elastic scattering [24] as the most realistic. The independent-particle model (IPM) [22,27] (which generates each single-nucleon orbital using an appropriate WS potential added by a spin-orbit term of the Thomas form) was used to calculate g.s. densities of $^5\text{He}, ^6\text{He}$. Since $^6\text{He}$ can be produced by picking up a proton from $^7\text{Li}$ [3], we have used the same $s_{1/2}$ binding potential as that used for $^7\text{Li}$ by Satchler [22] ($r_0 = 1.25$ fm, $a = 0.65$ fm for $s_{1/2}$ neutrons and protons which are bound by $S_n = 25$ MeV and $S_p = 23$ MeV, respectively), but with the recoil effect [27] properly taken into account. For the valence $p_{3/2}^2$ neutrons in $^5\text{He}, ^6\text{He}$, the WS parameters are the same as those used above for the $p_{3/2}^2$ component in the “core + valence neutron” wave function of $^5\text{He}, ^6\text{He}$. This choice of the $^6\text{He}$ g.s. density was made in a recent study [19] of the interaction cross section induced by $^6\text{He}$ beams at high energies. In addition to the IPM density, we have also used the $^6\text{He}$ density calculated in a realistic three-body model [28] and both densities give actually the same CRC results.

To complete the CRC input, one needs to give explicitly the spectroscopic amplitudes (SA) of one- and two-neutron configurations that enter the overlaps $<\Psi_{4\text{He}}|\Psi_{5\text{He}}>$, $<\Psi_{3\text{He}}|\Psi_{6\text{He}}>$, $<\Psi_{4\text{He}}|\Psi_{6\text{He}}>$, and the corresponding transfer vertices. Without the coupling to the $2^+$ excitation of $^6\text{He}$, the most sensitive to the $^4\text{He}(^6\text{He},^6\text{He})^4\text{He}$ data is the di-neutron spectroscopic amplitude of $^6\text{He}$ in the ground state ($SA_{g.s.}$) and it has been adjusted in each case to reproduce the large-angle data points. Other SA values do not affect the calculation strongly and were kept unchanged as taken from Ref. [13].

Results of our CRC calculations, which take into account two-way coupling between all the considered channels of the $^4\text{He}(^6\text{He},^6\text{He})^4\text{He}$ reaction at $E_{c.m.} = 11.6$ and 15.9 MeV, are plotted in Fig. 1. They show that the interference
between the elastic scattering and $2n$-transfer amplitudes gives rise to the observed oscillations of the cross section. One can also see that the direct one-step transfer is dominant and contribution from the sequential two-step transfer can be neglected at these low energies. The deepest minimum of the cross section at $\Theta_{c.m.} \approx 100^\circ$ (best seen in the data at 11.6 MeV) is due mainly to the direct transfer. The data at 15.9 MeV are of much poorer quality, but they have a minimum at about the same angle, which is reproduced in our CRC calculation by the same interference mechanism. In difference from our results, the CDCC calculation of elastic $^6$He+$^4$He scattering [17] (which reproduces
Fig. 2. The same as Fig. 1 for $E_{\text{c.m.}} = 60.3$ MeV, but with the di-neutron spectroscopic amplitude $SA_{g.s.} = 1.0$, in comparison with the data obtained by Ter-Akopian et al. [9].

The rise of cross section at backward angles by taking into account only the coupling between the elastic scattering and $^6$He breakup channels) does not describe consistently the observed oscillation pattern (see Fig. 4 in Ref. [17]). Moreover, adding the transfer amplitude to the elastic scattering was reported to deteriorate the CDCC description of the data. We conclude, therefore, that the $2n$-transfer is the main physics process responsible for the rise and broad oscillations of the $^4$He($^6$He,$^6$He)$^4$He cross section at backward angles.

The $^4$He($^6$He,$^6$He)$^4$He data at $E_{\text{c.m.}} = 60.3$ MeV are compared with the CRC results in Fig. 2. In difference from the data at lower energies, these data consist of two parts: the data points at forward angles which are purely elastic scattering events and those at backward angles which are due entirely to the $2n$-transfer process. The OP parameters at $E_{\text{c.m.}} = 60.3$ MeV were chosen, therefore, to reproduce data points at forward angles only. Our scenario for the di-neutron transfer becomes more convincing after the backward part of the data at 60.3 MeV is well reproduced by the coupling potential (1) obtained with the same $\Psi_{^6\text{He}}$ structure as that used at lower energies. Although the contribution from the sequential transfer becomes more sizable at 60.3 MeV,
the direct transfer remains dominant and, given a rather high experimental uncertainty, one can still neglect the sequential transfer and assume a direct transfer mechanism at this energy. Different choices of the WS imaginary OP might give different cross-section shapes, but the magnitude of the sequential transfer is always negligible compared to that of the direct transfer. This effect is common for three considered energies and is likely due to the unbound nature of $^5\text{He}+^5\text{He}$ system, since a negative $Q$-value for breaking the neutron pair in $^6\text{He}$ makes the two-step transfer less probable [29]. In a semiclassical consideration [11], a negative $Q$-value might also narrow the “window” open for the multi-step transfer.

As already mentioned above, the $P$-wave part of the $^6\text{He}$ wave function (5) belongs to the “cigar”-type configuration [21], and it is necessary to check relative contributions by the $S$- and $P$-wave components of (5) to the $2n$-transfer before concluding about the “di-neutron” configuration. We have, therefore, made further CRC analysis by omitting either the $S$- or $P$-wave part of (5) in the calculation of the coupling potential (1). Such a decomposition of the direct $2n$-transfer into the contributions by the $S$- and $P$-waves is shown in Fig. 3. One can see that the $P$-wave contribution is indeed negligible if it contributes up to 15% to the norm of $\Psi_{^6\text{He}}$.

Given the dominance of the direct $2n$-transfer, coupling potential (1) needs to be evaluated as accurately as possible using realistic $V_{n-\alpha}$ and $^6\text{He}+^4\text{He}$ OP’s. We tried first to fit the forward part of the $^4\text{He}(^6\text{He},^6\text{He})^4\text{He}$ data at 60.3 MeV within the OM, using the real folded potential $V_{\text{Fold}}$ and WS imaginary potential. The OM results shown in Fig. 2 were obtained with $V_{\text{Fold}}$ renormalized by a factor $N_V \approx 1.15$ and a WS imaginary potential close to that found in Ref. [13]. Such a $N_V$ factor agrees reasonably well with the folding analysis of $\alpha$-nucleus elastic scattering [24], where a factor of $N_V \approx 1.1$ was needed for light targets. We decided, therefore, to use the same $N_V$ factor at lower energies and adjust the WS parameters of the imaginary OP to fit the $^4\text{He}(^6\text{He},^6\text{He})^4\text{He}$ data over the whole angular range by the CRC results. The volume integral $J_V$ of the renormalized folded potential turned out to be closer to the global systematics established for $^4\text{He}+^4\text{He}$ [30] than to that for light heavy-ion systems [31]. For consistency, we have also used $N_V = 1.15$ for the folded $^4\text{He}+^4\text{He}$ potentials. Using a OP systematics from Ref. [30], we found that the contribution from $^4\text{He}+^4\text{He}$ imaginary OP’s is small and, therefore, was not included in the final CRC calculation. Concerning the core-core potential, we have used at the two low energies also a deep $^4\text{He}+^4\text{He}$ potential [32] (representing microscopic results of the resonating group method) and a shallow potential parametrized by Ali and Bodmer [33]. These two potentials were known to give equally good description of $^4\text{He}+^4\text{He}$ phase shifts at low energies, and the CRC results given by them are almost indistinguishable from those given by the folded $^4\text{He}+^4\text{He}$ potential. The stability of our CRC results with respect to the choice of different core-core potentials as well as the
Fig. 3. Results of CRC calculations, with the direct (one-step) 2n-transfer amplitude added coherently to that of the elastic scattering, in comparison with the $^4\text{He}(^6\text{He},^6\text{He})^4\text{He}$ data measured at $E_{\text{c.m.}} = 11.6$ and 60.3 MeV by Raabe et al. [14,15] and Ter-Akopian et al. [9], respectively. Contributions by the $S$-wave ($L = J = 0$) and $P$-wave ($L = J = 1$) parts of $^6\text{He}$ wave function (5) to the direct transfer are shown explicitly.

dominance of the direct 2n-transfer show consistently the “core + 2n-cluster” structure of $^6\text{He}$.

With a dominance of the direct 2n-transfer, especially at low energies, a reliable estimate for the spectroscopic amplitude $S_{A_{\text{g.s.}}}$ of the 2n-configuration in $^6\text{He}_{\text{g.s.}}$ can be made. In a simple shell model limit, with the “core + 2n-
Fig. 4. Results of CRC calculations, with the direct (one-step) $2n$-transfer amplitude added coherently to that of the elastic scattering, in comparison with the $^4\text{He}(^6\text{He},^6\text{He})^4\text{He}$ data measured at $E_{\text{c.m.}} = 11.6$ and 60.3 MeV by Raabe et al. [14,15] and Ter-Akopian et al. [9], respectively. Different di-neutron spectroscopic amplitudes $SA_{\text{g.s.}}$ were used in the calculation of the coupling potential (1).

Cluster wave function (5) normalized to unity, $SA_{\text{g.s.}}$ is expected to be unity if the transfer process exhausts all the available $2n$-strength in the total wave function. However, $SA_{\text{g.s.}}$ was shown by Timofeyuk [21] to be increased by a factor of $(25/16)^{1/2} = 1.25$ if one takes into account explicitly the center-of-mass motion as well as the antisymmetrization between all individual nucleons (including those in $^4\text{He}$ core). In the present work, we have treated $SA_{\text{g.s.}}$ as
a parameter to be found from the best fit to the transfer data by the CRC results. From the results obtained for the low energies of 11.6 and 15.9 MeV we can deduce the best fit value $S_{A_{g.s.}} \approx 1.15 \pm 0.15$, while the CRC fit to the data at 60.3 MeV gives $S_{A_{g.s.}} \approx 1.00 \pm 0.15$ (see Table 1 and Fig. 4).

Fig. 5. Results of CRC calculations, with the $2n$-transfer amplitude added coherently to that of the elastic scattering, in comparison with the $^4$He($^6$He,$^6$He)$^4$He data measured at $E_{c.m.} = 11.6$ and $60.3$ MeV by Raabe et al. [14,15] and Ter-Akopian et al. [9], respectively. Contributions by the direct and indirect (via $2^+$ excitation of $^6$He) $2n$-transfers are shown explicitly (see text for more details).

As noted above, the indirect $2n$-transfer route via the $2^+$ excitation of $^6$He needs also to be investigated before making conclusion about the di-neutron
configuration in $^6\text{He}_{\text{g.s.}}$. The coupling between the $^6\text{He}_{\text{g.s.}}+^4\text{He}$ and $^6\text{He}^*+^4\text{He}$ partitions was investigated earlier in the CDCC analysis of elastic $^6\text{He}+^4\text{He}$ scattering [17] and CRC study of the $^4\text{He}(^6\text{He},^6\text{He})^4\text{He}$ reaction at 60.3 MeV [13]. While suggesting different scenarios for the $^4\text{He}(^6\text{He},^6\text{He})^4\text{He}$ reaction, these works do show that the (g.s. $\rightarrow 2^+$) coupling is strong and affects significantly the calculated elastic cross section. Actually, the second-order coupling effect in this case is two-fold and due to

i) inelastic scattering $^6\text{He}_{\text{g.s.}} \rightarrow ^6\text{He}^* \rightarrow ^6\text{He}_{\text{g.s.}}$,

ii) indirect 2n-transfers like $^6\text{He}_{\text{g.s.}} \rightarrow ^6\text{He}^* \rightarrow ^4\text{He}$ or $^4\text{He} \rightarrow ^6\text{He}^* \rightarrow ^6\text{He}_{\text{g.s.}}$.

As a result, the change in the $^4\text{He}(^6\text{He},^6\text{He})^4\text{He}$ cross section is a mixed effect of the changes caused by both the couplings (i) and (ii), and one needs to separate the coupling (i) from the total coupling in order to probe the strength of the indirect 2n-transfer. Since such an analysis has not been done so far, we present here our first attempt to address this interesting problem using a reasonable choice of the inelastic scattering form factor (FF). We have first added the inelastic scattering $^4\text{He}(^6\text{He},^6\text{He}^*)^4\text{He}$ channel and refitted parameters of the WS imaginary OP (Table 1) to obtained about the same elastic scattering cross section at forward angles with the CRC calculation including the elastic, inelastic $2^+$ scattering and direct 2n-transfer only. The DFM [23] was used to calculate the real inelastic scattering FF, which has been proven to be more accurate than the standard collective-model FF [23,34]. The proton and neutron parts of the ($^6\text{He}_{\text{g.s.}} \rightarrow ^6\text{He}^*_{2^+}$) transition density used in the DFM calculation was given by the Bohr-Mottelson prescription [35] of deforming proton and neutron parts of the $^6\text{He}$ g.s. density with the deformation lengths $\delta_{2^+}^{(p)}$ and $\delta_{2^+}^{(n)}$, respectively. While the proton deformation length can be fixed [23] at $\delta_{2^+}^{(p)} \approx 1.75$ fm by the empirical transition rate of $B(E2) \approx 3.2$ e²fm⁴ [36], $\delta_{2^+}^{(n)}$ is not necessarily to be the same. For example, $\delta_{2^+}^{(n)}$ was found to be more than double $\delta_{2^+}^{(p)}$ for the lowest $2^+$ excitation in $^{20}\text{O}$ [37]. To get a realistic estimate of $\delta_{2^+}^{(n)}$ using the prescription suggested in Ref. [37], we have performed a coupled channel analysis of the inelastic scattering $p(^6\text{He},^6\text{He}^*)p$ data at 41 MeV/nucleon [6]. We found that $\delta_{2^+}^{(n)} \approx 2.55$ fm which is about 50% stronger than $\delta_{2^+}^{(p)}$ and, thus, indicates a strong contribution by the valence neutrons to the $2^+$ excitation of $^6\text{He}$. The imaginary part of the inelastic FF was given by deforming the WS imaginary OP with an isoscalar deformation length $\delta_{2^+}^{(0)} \approx 2.35$ fm, which was directly obtained [37] from the above values of $\delta_{2^+}^{(p)}$ and $\delta_{2^+}^{(n)}$. In this way, we have gained control of some important parameters for our CRC analysis which, otherwise, can be obtained only when the (purely) inelastic $^6\text{He}+^4\text{He}$ scattering data are available.

After the coupling (i) is properly taken into account, we could estimate the coupling strength (ii) by adding the indirect 2n-transfer channel to the CRC
calculation using the new imaginary OP. The same structureless 2n-cluster in a (quasi-bound) \( N = 0, \ L = 2 \) state as that used in Ref. [13] has been assumed for \(^6\text{He}^*_{2+}\). The only remaining parameter is the di-neutron spectroscopic amplitude \( SA_{2+} \) of \(^6\text{He}^*_{2+}\), which determines the strength of the indirect 2n-transfer. Since the largest-angles data points at 60.3 MeV were shown in Ref. [13] to be strongly sensitive to the indirect 2n-transfer and could not be described by the CRC calculation including the coupling (i) only, we have adjusted \( SA_{2+} \) to reproduce these data points when both the couplings (i) and (ii) were included. As a result, a perfect agreement with the large-angle data at 60.3 MeV was obtained with \( SA_{2+} \approx 1.3 \) (see Fig. 5). With the direct and indirect transfer routes now included in equal footing, \( SA_{g.s.} \) has also been readjusted for an optimal agreement of the final CRC results with the data and it turned out to be slightly reduced (\( SA_{g.s.} \approx 0.95 \) and 0.85 at 11.6 and 60.3 MeV, respectively). To test the sign effect, we have performed the CRC analysis using two sets of the inelastic FF’s which were given by the positive and negative deformation lengths. We found that the \(^4\text{He}(^6\text{He},^6\text{He})^4\text{He} \) data at 60.3 MeV is much better reproduced with the negative deformation (\( \delta_{2+}^{(p)} = -1.75, \delta_{2+}^{(n)} = -2.55 \) and \( \delta_{2+}^{(0)} = -2.35 \) fm). Such a preference of the negative deformation is in an agreement with the earlier CRC analysis [13] of the same data, but using a simpler ansatz for the inelastic FF. This sign effect is especially important when the interference between the direct and indirect transfer amplitudes is not negligible [38]. In fact, this interference was shown here to be vital for the full agreement of the CRC results with the data at 60.3 MeV. While the sequential \((^6\text{He},^6\text{He};^6\text{He},^4\text{He}) \) transfer is negligible, one can see from the relative contributions by the direct and indirect transfers shown in Fig. 5 that the indirect \((^6\text{He},^6\text{He}^{*}_{2+};^6\text{He}^{*}_{2+},^4\text{He}) \) transfer via the \( 2^+ \) excitation of \(^6\text{He} \) is not negligible. It should be noted that the strength of the indirect transfer (\( SA_{2+} \approx 1.3 \)) has been fixed by the CRC fit to the last 3 data points at 60.3 MeV. A future measurement of the inelastic \(^4\text{He}(^6\text{He},^6\text{He}^{*}_{2+})^4\text{He} \) reaction at about the same energies would be very valuable for a more accurate estimate of \( SA_{2+} \) from the direct \(^6\text{He}^{*}_{2+} \leftrightarrow ^4\text{He} \) transfer process. The most realistic values of the di-neutron spectroscopic amplitudes should be deduced from a CRC analysis including both the direct and indirect 2n-transfers if the indirect transfer is significant. From the present CRC results we can suggest \( SA_{g.s.} \approx 0.9 \pm 0.1 \) and \( SA_{2+} \approx 1.3 \pm 0.1 \). In this case, instead of the structure model for \(^6\text{He} \) used by Oganessian et al. [3], one should use a more consistent structure model for both \(^6\text{He}_{g.s.} \) and \(^6\text{He}^{*}_{2+} \) in the CRC analysis of the \(^4\text{He}(^6\text{He},^6\text{He})^4\text{He} \) reaction for a direct probe of the di-neutron and cigar configurations in the \(^6\text{He} \) wave function.

In conclusion, a microscopic CRC analysis of the \(^4\text{He}(^6\text{He},^6\text{He})^4\text{He} \) reaction at \( E_{c.m.} = 11.6, 15.9, \) and 60.3 MeV which includes coherently the pure elastic scattering, direct (one-step) and sequential (two-step) 2n-transfer processes, has been performed for the first time using semi-microscopic optical potentials for the involved He-He systems and reasonable choices of the “core + valence
neutrons” wave functions of $^{5,6}\text{He}$. Our analysis showed consistently that the $2n$-transfer is the main physics process responsible for the rise of the elastic $^{6}\text{He}+^{4}\text{He}$ cross section at large angles. The direct $2n$-transfer was found to be dominating over the sequential transfer and due mainly to the contribution from the $S$-wave component of the $^{6}\text{He}$ wave function. We found further some indication that the indirect $2n$-transfer via the $2^+$ excitation of $^{6}\text{He}$ is significant, especially at $E_{c.m.} = 60.3$ MeV, and the most reliable estimate for the di-neutron spectroscopic amplitudes $S_{g.s.}$ and $S_{2^+}$ can be made only if the interference between the direct and indirect $2n$-transfer amplitudes are taken into account properly. The dominance of the (direct + indirect) $2n$-cluster transfer shows a strong $2n$-correlation at the nuclear surface and, thus, confirms the dominance of the $2n-^{4}\text{He}$ configuration over the $n-^{5}\text{He}$ one in the $^{6}\text{He}$ wave function.

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References

[1] M.V. Zhukov, B.V. Danilin, D.V. Fedorov, J.M. Bang, I.J. Thompson, and J.S. Vaagen, Phys. Rep. 151 (1993) 299.

[2] E. Nielsen, D.V. Fedorov, A.S. Jensen, and E. Garrido, Phys. Rep. 347 (2001) 373.

[3] Yu.Ts. Oganessian, V.I. Zagrebaev, and J.S. Vaagen, Phys. Rev. Lett. 82 (1999) 4996; Phys. Rev. C 60 (1999) 044605.

[4] A. Ozawa, T. Suzuki, and I. Tanihata, Nucl. Phys. A 693 (2001) 32.

[5] V. Lapoux et al., Phys. Lett. B 517 (2001) 18.

[6] A. Lagoyannis et al., Phys. Lett. B 518 (2001) 27; S.V. Stepansov et al., Phys. Lett. B 542 (2002) 35.

[7] P. Egelhof, Nucl. Phys. A 722 (2003) 254c.

[8] N.K. Timofeyuk and I.J. Thompson, Phys. Rev. C 61 (2000) 044608.

[9] G.M. Ter-Akopian et al., Phys. Lett. B 426 (1998) 251.

[10] W. von Oertzen and H.G. Bohlen, Phys. Rep. 19C (1975) 1.

[11] G.R. Satchler, Direct Nuclear Reactions, Clarendon Press, Oxford, 1983.

[12] W. von Oertzen and A. Vitturi, Rep. Prog. Phys. 64 (2001) 1191.
[13] I.V. Krouglov, M. Avrigeanu, and W. von Oertzen, Eur. Phys. J. A 12 (2001) 399.

[14] R. Raabe et al., Phys. Lett. B 458 (1999) 1.

[15] R. Raabe et al., Phys. Rev. C 67 (2003) 044602.

[16] D. Baye, L. Desorgher, D. Guillain, and D. Herschkowitz, Phys. Rev. C 54 (1996) 2563.

[17] K. Rusek and K.W. Kemper, Phys. Rev. C 61 (2000) 034608.

[18] I.J. Thompson, Comp. Phys. Rep. 7 (1988) 167.

[19] Dao T. Khoa, H.S. Than, T.H. Nam, M. Grasso, and N. Van Giai, Phys. Rev. C 69 (2004) 044605.

[20] S. Funada, H. Kameyama, and Y. Sakuragi, Nucl. Phys. A 575 (1994) 93.

[21] N.K. Timofeyuk, Phys. Rev. C 63 (2001) 054609.

[22] G.R. Satchler and W.G. Love, Phys. Rep. 55 (1979) 183.

[23] Dao T. Khoa and G.R. Satchler, Nucl. Phys. A 668 (2000) 3.

[24] Dao T. Khoa, Phys. Rev. C 63 (2001) 034007.

[25] Dao T. Khoa, G.R. Satchler, and W. von Oertzen, Phys. Rev. C 56 (1997) 954.

[26] M. Avrigeanu, G.S. Anagnostatos, A.N. Antonov, and J. Giapitzakis, Phys. Rev. C 62 (2000) 017001.

[27] G.R. Satchler, Nucl. Phys. A 329 (1979) 233.

[28] J.S. Al-Khalili, J.A. Tostevin, and I.J. Thompson, Phys. Rev. C 54 (1996) 1843.

[29] W. von Oertzen et al., Phys. Lett. 151 B (1985) 195; H. Lenske, H. Wolter, and H.G. Bohlen, Phys. Rev. Lett. 62 (1989) 1457.

[30] K.A.G. Rao, A. Nadasen, D. Sisan, W. Yuhasz, D. Mercer, Sam M. Austin, P.G. Roos, and R.E. Warner, Phys. Rev. C 62 (2000) 014607.

[31] M.E. Brandan and G.R. Satchler, Phys. Rep. 285 (1997) 143.

[32] B. Buck, H. Friedrich, and C. Wheatley, Nucl. Phys. A 275 (1977) 246.

[33] S. Ali and A.R. Bodmer, Nucl. Phys. 66 (1966) 99.

[34] J.R. Beene, D.J. Horen, and G.R. Satchler, Nucl. Phys. A 596 (1996) 137.

[35] A. Bohr and B.R. Mottelson, Nuclear Structure (Benjamin, New York, 1975), Vol.2.

[36] T. Aumann et al., Phys. Rev. C 59 (1999) 1252.

[37] Dao T. Khoa, Phys. Rev. C 68 (2003) 011601(R).

[38] R.J. Ascuitto and E. Seglie, in Treatise on Heavy Ion Science, ed. by D.A. Bromley, Vol. 1 (Plenum Press, N.Y., 1984) p. 463.