Hadronic uncertainties in the $B \to K^*\mu^+\mu^-$ decay

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Motivated by the persisting “anomaly” in the measurement of $P_5'$, we review hadronic uncertainties entering the angular observables of the decay $\bar{B} \to \bar{K}^*\mu^+\mu^-$. We argue that hadronic uncertainties could account for the present measurements. We discuss how to extract information on the non-factorizable hadronic contribution from experimental data exploiting its $q^2$ dependence and propose a parametrization optimized for this purpose. While no clear conclusion can be drawn with present experimental uncertainties, we show that future measurements should be able to pin down many hadronic parameters that we define in our parametrization.

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1. Introduction

Anomalies in $B$ physics are attracting a lot of attention as of late. Deviations from the Standard Model (SM) predictions exceeding $3\sigma$ persist in several observables, including exciting hints of lepton flavour universality (LFU) violation [1–4]. Furthermore, the emerging pattern may have a simple explanation in terms of new physics (NP) contributions to the Wilson coefficients of one/few operator(s), which are obtained for instance in models with $Z'$ or leptoquarks, for a recent review see the contributions of F. Feruglio and A. Greljo, or e.g. ref. [5].

In these proceedings, we focus on the prediction of the observable $P_5'$ obtained from the angular analysis of the decay $B \rightarrow K^* \mu^+ \mu^-$ in the low $q^2$ region, arguing that non-factorizable hadronic contributions coming from four-quark operators are not fully under control and could be responsible for the observed deviation from the SM. Elaborating on our previous results [6, 7], we point out that the $q^2$ dependence can be used to extract from data interesting information on the hadronic contribution, even though it cannot be fully disentangled from the NP contribution in the absence of an adequate theoretical control. We then propose a variant of our previous parametrization of the hadronic contribution optimized for extracting information from data and show that, while no sound result can be obtained with present experimental uncertainties, yet future measurements should be able to pin down most hadronic parameters.

These proceedings are organized as follows. In section 2 we set up the scene for the calculation of $P_5'$. In section 3 we focus on the non-factorizable hadronic contributions and present our new parametrization. The extraction of the hadronic parameters using present and (expected) future data are collected in section 4, while conclusions are drawn in section 5.

2. Calculating $P_5'$

The angular analysis of the decay $B \rightarrow K^* \mu^+ \mu^-$ allows to extract the coefficients $I_i$ of the fully differential decay rate

$$\frac{d^3\Gamma}{dq^2 d\cos \theta_L d\cos \theta_K d\phi} = \frac{9}{32\pi} \left( I_1 \cos^2 \theta_K + I_2 \cos^2 \theta_K + (I_3 \sin^2 \theta_K) \cos 2\theta_L 
+ I_4 \sin^2 \theta_K \cos 2\phi + I_5 \sin 2\theta_K \sin 2\theta_L \cos \phi + I_6 \sin 2\theta_K \sin \theta_L \cos \phi 
+ (I_7 \sin^2 \theta_K + I_8 \cos^2 \theta_K) \cos \theta_L + I_9 \sin 2\theta_K \sin \theta_L \sin \phi 
+ I_{10} \sin 2\theta_K \sin 2\theta_L \sin \phi + I_{11} \sin^2 \theta_K \sin^2 \theta_L \sin 2\phi \right). \tag{2.1}$$

These coefficients are conveniently recast in terms of the optimized variables $P_i$ [8–10] written in terms of the CP-averaged angular coefficients $\Sigma_i = (I_i + \tilde{I}_i)/2$. In particular, one defines

$$P_5' = \frac{\Sigma_5}{2\sqrt{\Sigma_2\Sigma_2}}. \tag{2.2}$$

In the helicity basis [11] adopted in these proceedings, the angular coefficients can be computed in terms of seven helicity amplitudes [7, 12]:

$$H_V^\lambda = i\frac{4G_F m_B}{\sqrt{2}} \frac{e^2}{16\pi^2} \lambda_i \left( c_{\tilde{V}_L,\lambda} + \frac{m_B^2}{m_B^2} C_{\tilde{V}_L,\lambda} - 16\pi^2 h_\lambda \right),$$

$$H_A^\lambda = i\frac{4G_F m_B}{\sqrt{2}} \frac{e^2}{16\pi^2} \lambda_{i10} \tilde{V}_L, \quad H_P = -i\frac{4G_F m_B}{\sqrt{2}} \frac{e^2}{16\pi^2} \lambda_i \frac{2m_B}{q^2} C_{10} \left( \tilde{S}_L - \frac{m_i}{m_B^2} \tilde{S}_R \right), \tag{2.3}$$
with \( \lambda = 0, \pm \). The CKM factor \( \lambda_t = V_{ts} V_{tb}^{*} \) are Wilson coefficients of the \( \Delta B = 1 \) effective weak Hamiltonian, \( \tilde{V}_{L\lambda}, \tilde{T}_{L\lambda}, \tilde{S}_{L}, \tilde{S}_{R} \) are form factors entering the factorized part of the amplitudes (as defined in Appendix A of ref. [7]), while \( h_{\lambda} \) are the genuine non-factorizable hadronic contributions.

The detailed discussion of hadronic uncertainties related to form factors is beyond the scope of these proceedings, but we briefly comment on the current status. Although only a light-cone sum rules (LCSR) calculation of the form factors is available in the large recoil (low \( q^2 \)) region [13], it matches reasonably well the extrapolation of lattice QCD calculations at low recoil [14]. Moreover, LCSR results for the form factors are provided together with the full correlation matrix to allow taking into account correlations induced by the heavy quark symmetry. Thus the uncertainty of 10–15% attached to the form factors looks credible and moreover it is further reduced in the optimized observables, making this contribution to the theoretical uncertainty quite smaller than the present experimental error.

In the next section we focus on the main topic of these proceedings, namely the non-factorizable contributions \( h_{\lambda} \).

3. Non-factorizable hadronic contributions

The non-factorizable hadronic contribution

\[
h_{\lambda}(q^2) = \frac{\varepsilon_{\mu}^\lambda(\lambda)}{m_B^2} \int d^4xe^{i\mu x} \langle \bar{K}^* | T \{ j_{\mu}^{em}(x) \mathcal{H}^{\text{had}}_{\text{eff}}(0) \} | B \rangle
\]

(3.1)
is generated by the insertion in the matrix element of the four-quark operators present in the \( \Delta B = 1 \) effective weak Hamiltonian, denoted here as \( \mathcal{H}^{\text{had}}_{\text{eff}} \), together with an electromagnetic quark current. Details on the definition of the effective Hamiltonian can be found for instance in ref. [7]. The largest contribution is given by the current-current operators

\[
Q^1_c = (\bar{s}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma_\mu T^a b_L), \quad Q^2_c = (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma_\mu b_L),
\]

(3.2)

with the two charm quark fields closed in a loop. These contributions are notoriously troublesome to estimate, as they can produce on-shell intermediate hadronic states which give rise to strong phases, non-local amplitudes, etc. Many years ago, similar charm-loop contributions [15,16] stimulated an intense debate about the validity of factorization in the infinite mass limit for heavy-to-light non-leptonic \( B \) decays [17–20]. For \( B \rightarrow V\ell\ell \), factorization of the amplitudes in the infinite mass limit has been proven in ref. [21] at low \( q^2 \). Yet the issue of computing the non-factorizable contribution in eq. (3.1), albeit power suppressed, remains open.

The only estimate of \( h_{\lambda} \) presently available can be found in ref. [22]. Using LCSR, the authors of ref. [22] were able to compute \( h_{\lambda}(q^2) \) for \( q^2 \ll 4m_c^2 \), where the single soft gluon approximation used in the calculation is applicable. These results were then extended to all \( q^2 \) with a dispersion relation using a spectral function including the \( J/\psi \) and \( \psi' \) resonances plus an additional pole modeling the contribution from higher resonances and continuum. This combination of methods and approximations testifies the complexity of the calculation which is reflected in the large uncertainty quoted by the authors, albeit intrinsic limitations of the adopted methods (e.g. lack of strong

\footnote{In the following we use the notation of ref. [12].}
phases) and model dependence can hardly be quantified. The correction to $P_{3}^{c}$ induced by the result of ref. [22] is not large, rather flat in $q^{2}$, and goes in the direction of increasing the anomaly.

Recently, attempts at confirming or improving the results of ref. [22] have appeared in the literature [23, 24]. The empirical model of ref. [23] assumes that $h_{\lambda}$ can be obtained as a sum of relativistic Breit-Wigner functions and uses resonance data to fix the parameters, although some of them, notably strong phases, cannot be fixed with present data. The result is remarkably in agreement with ref. [22] for vanishing strong phases, but quite different for other choices, showing the importance of controlling strong phases. A more theoretical approach was followed in ref. [24]: the authors studied the analytic properties of $h_{\lambda}$, isolated the resonance poles and proposed a $z$ expansion for the remainder function, mapping the contribution of the cut at the boundary of the region of convergence. The coefficients of the expansion are fixed using both resonance data and LCSR results at negative $q^{2}$ provided by the authors of ref. [22]. Results compare well with ref. [22], but the coefficients obtained at different orders show a poor convergence of the series.

Given what is at stake, we consider the present theoretical knowledge of $h_{\lambda}$ not fully satisfactory. LCSR estimates could be in the right ballpark, but the unsatisfying control over strong phases, the theoretical problems pointed out in ref. [25], and the fact that the $P_{3}$ anomaly lies in the $q^{2}$ region approaching the $J/\psi$ resonance call for extra care. We therefore decided to expand $h_{\lambda}$ in powers of $q^{2}$ in the region $q^{2} \in [0,8]$ GeV$^{2}$ and use the $B \to K^{*} \mu^{+}\mu^{-}$ and $B \to K^{*} \gamma$ data to fix the coefficients, considering two cases: a “standard” scenario, denoted as PMD (phenomenological model driven), where the results of ref. [22] are used to constrain the coefficients of the expansion in the whole low $q^{2}$ region, and a “conservative” scenario, denoted as PDD (phenomenological data driven), where only the actual LCSR results computed at $q^{2} = 0$ and 1 GeV$^{2}$ in ref. [22] are used to constrain the absolute values of the $h_{\lambda}$, while their phases and $q^{2}$ dependence are inferred from the experimental data. In the second scenario, the constraining power of $B \to K^{*} \mu^{+}\mu^{-}$ on NP is lost, as some coefficients of the $h_{\lambda}$ expansion are indistinguishable from NP contributions. However, one can still determine most coefficients of $h_{\lambda}$ and look for “unexpected” hadronic contributions, to either invalidate or gain more confidence in the available estimates.

To this end, in these proceedings we propose a variation of the simple Taylor expansion of $h_{\lambda}$ we used in previous publications [6, 7] that reads $^{2}$

\begin{align*}
h_{-}(q^{2}) &= -\frac{m_{b}}{8\pi^{2}m_{b}} \tilde{T}_{L-}(q^{2})h_{-}^{(0)} - \frac{1}{16\pi^{2}m_{b}^{2}} \tilde{V}_{L-}(q^{2})h_{-}^{(1)}q^{2} + h_{-}^{(2)}q^{4} + O(q^{6}) \\
&= -\frac{1}{16\pi^{2}} \left[ 2m_{b} \tilde{T}_{L-}(0)h_{-}^{(0)} + \left( \frac{1}{m_{b}^{2}} \tilde{V}_{L-}(0)h_{-}^{(1)} + \frac{2m_{b}}{d} d\tilde{T}_{L-}(0)h_{-}^{(0)} \right) q^{2} \\
&\quad \quad + \left( \frac{m_{b}}{m_{b}} \frac{d^{2}\tilde{T}_{L-}(0)h_{-}^{(0)}}{d^{2}q^{2}} + \frac{1}{m_{b}^{2}} \frac{d\tilde{V}_{L-}(0)h_{-}^{(1)}}{d^{2}q^{2}} - 16\pi^{2}h_{-}^{(2)} \right) q^{4} \right] + O(q^{6}), \\
h_{+}(q^{2}) &= h_{+}^{(0)} - \frac{m_{b}}{8\pi^{2}m_{b}} \tilde{T}_{L+}(q^{2})h_{+}^{(0)} + \left( h_{+}^{(1)} - \frac{\tilde{V}_{L+}(q^{2})}{16\pi^{2}m_{b}^{2}} h_{+}^{(1)} \right) q^{2} + h_{+}^{(2)}q^{4} + O(q^{6}) \\
&= -\frac{1}{16\pi^{2}} \left[ 2m_{b} \tilde{T}_{L+}(0)h_{+}^{(0)} - 16\pi^{2}h_{+}^{(0)} + \left( \frac{1}{m_{b}^{2}} \tilde{V}_{L+}(0)h_{+}^{(1)} + \frac{2m_{b}}{d} d\tilde{T}_{L-}(0)h_{+}^{(0)} \right) q^{2} \right] + O(q^{6}).
\end{align*}

$^{2}$The two definitions of each $h_{\lambda}$ are equivalent up to higher order terms in the $q^{2}$ expansion. Notice the different $q^{2}$ behaviour of $h_{0}$ [26].
The equations above clearly show that the impact of our approach on the NP interpretation of the experimental data. We discuss the perspective of this analysis with improved data and comment on the impact of our approach on the NP interpretation of the B anomalies.

4. Present fit and extrapolations

We present results from a global analysis of the B anomalies, along the lines of the one pre-
Before concluding, we comment on the effect of the non-factorizable hadronic contributions (denoted as $\Delta C_{\gamma}$ in the plot), as a consequence of imposing the constraint from the theoretical estimate of ref. [22] at $q^2 = 0$. There is however an interesting correlation between $|h_0^{(1)}| = |\Delta C_{\gamma}|$ and $|h_2^{(2)}|$: the present anomaly can be reproduced either with a constant shift of the Wilson coefficient $C_9^{\text{eff}}$ (due to hadronic contribution or NP, no way to disentangle them) or with a $q^4$ term in $h_\lambda$. If $|\Delta C_{\gamma}|$ is small, $|h_2^{(2)}|$ is found to be different from zero at more than $2\sigma$, in agreement with the finding of ref. [7].

We then repeated the analysis by reducing the experimental error by a factor of six, with the central values given by the global mode of the SM fit. The errors obtained in this simple way are in the ballpark of what is expected from the future LHCb upgrade. From the right panel of fig. 2, it can be seen that many coefficients $|h_{i}^{(1)}|$ can be extracted from data once the experimental error is reduced. In particular, the correlation between $|\Delta C_{\gamma}|$ and $|h_{2}^{(2)}|$ is much reduced, allowing to distinguish a constant shift of the Wilson coefficient from a rise of $h_\lambda$ for $q^2 = 6$–8 GeV$^2$. A more detailed analysis, including a discussion of the phases, will be presented in a forthcoming publication.

Before concluding, we comment on the effect of the non-factorizable hadronic contributions on the NP interpretation of the $B$ anomalies. In fig. 3, we plot the NP coefficients

\begin{equation}
C_{9,\pm}^{\text{NP}} = \frac{1}{2} \left( C_{9,\mu}^{\text{NP}} \pm C_{9,e}^{\text{NP}} \right), \quad C_{10,\pm}^{\text{NP}} = \frac{1}{2} \left( C_{10,\mu}^{\text{NP}} \pm C_{10,e}^{\text{NP}} \right),
\end{equation}

as fitted in the $\text{PMD}$ and $\text{PDD}$ cases. It is shown that the LFU-conserving coefficients $C_{9,\pm}^{\text{NP}}$ and $C_{10,\pm}^{\text{NP}}$ are affected by the different treatment of the charm-loop contribution, while the LFU-violating coefficients $C_{9,\pm}^{\text{NP}}$ and $C_{10,\pm}^{\text{NP}}$ are not, as expected. For $C_{9,\pm}^{\text{NP}}$, both $\text{PMD}$ and $\text{PDD}$ cases

\begin{table}[h]
\begin{tabular}{|c|c|c|c|}
\hline
$f_{K^*}^{\parallel}$ [MeV] & $f_{K^*\perp}$ [MeV] & $a_1(K^*)_{\perp,\parallel}$ & $a_2(K^*)_{\perp,\parallel}$ \\
\hline
204 ± 7 & 159 ± 6 & 0.04 ± 0.03 & 0.11 ± 0.09 \\
\hline
\end{tabular}
\caption{Inputs of our global analysis which has been updated with respect to ref. [6].}
\end{table}
provide a good fit (the IC value of PMD is smaller, reflecting the more economical description of
the anomalies in terms of NP contributions only), but the evidence for a deviation of $C_{9+}^{NP}$ from zero
is much less significant in the PDD case, as the measurement of $P_3'$ is accommodated by hadronic
contributions. As for the explanation of the $B$ anomalies in terms of $C_{10,1}$, the effect of the charm-
loop contribution is more striking: this scenario produces a bad fit in the PMD case, as $C_{10}$ alone
cannot account for the $P_3'$ anomaly, but is perfectly viable in the PDD case, where NP is not needed
to reproduce $P_3'$ (see IC’s in fig. 3), as pointed out in ref. [6].

5. Conclusions

We have reviewed hadronic uncertainties entering the angular observables of the decay
$B \to K^* \mu^+ \mu^-$, arguing that the non-factorizable hadronic contribution could account for the present
measurements. We have proposed a new parametrization of this contribution optimized to fit the
new parameters from data, exploiting the $q^2$ dependence of the correction. While a fit to present
data produces no clear determination of many of these parameters, we have shown how future mea-
urements could be able to pin down many of them, improving our knowledge of the theoretically
challenging charm-loop contribution. Finally, we have emphasized once more that the NP inter-
pretation of the $B$ anomalies is affected by hadronic uncertainties, showing how an explanation in
terms of $C_{10}$ becomes viable if the charm-loop contribution is treated as we have suggested.

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Figure 2: Determination of the coefficients $|\beta_3^{(i)}|$ from present (left panel) and future (right panel) data. See
text for details.
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Figure 3: Upper row: global fit for $C_{9,\pm}^{NP}$ in the PMD (left) and PDD (right) case. Lower row: global fit for $C_{10,\pm}^{NP}$ in the PMD (left) and PDD (right) case.

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