THE 2PPI EXPANSION: DYNAMICAL MASS GENERATION AND VACUUM ENERGY

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We discuss the 2PPI expansion, a summation of the bubble graphs up to all orders, by means of the 2D Gross-Neveu toy model, whose exact mass gap and vacuum energy are known. Then we use the expansion to give analytical evidence that a dimension two gluon condensate exists for pure Yang-Mills in the Landau gauge. This \( \langle A^a_\mu A^a_\mu \rangle \) condensate consequently gives rise to a dynamical gluon mass.

1. Introduction

Lately, there was growing evidence for the existence of a condensate of mass dimension two in Yang-Mills (YM) theories in the Landau gauge. An obvious candidate for such a condensate is \( \langle A^a_\mu A^a_\mu \rangle \). The phenomenological background of this type of condensate can be found in 1,2,3. Also lattice simulations indicated a non-zero condensate \( \langle A^a_\mu A^a_\mu \rangle \). See 5 for an overview of recent results.

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Thinking of simpler models like massless $\lambda\phi^4$ or Gross-Neveu and the role played by their quartic interaction in the formation of a (local) composite (in particular, containing two fields) condensate and the consequent dynamical mass generation for the originally massless fields, it is clear the possibility exists that the quartic gluon interaction gives rise to a two field composite operator condensate in YM (QCD) and mass generation for the gluons too.

In section 2, we give a short review of the $2PPI$ expansion by means of the $2D$ Gross-Neveu model. In section 3, we present our results for $\langle A^2 \rangle$ for $SU(N)$ Yang-Mills theory in the Landau gauge.

2. The Gross-Neveu model

The $U(N)$ invariant Gross-Neveu Lagrangian in $2D$ Euclidean space time reads

$$\mathcal{L} = \overline{\psi} \gamma^5 \psi - \frac{1}{2} g^2 (\overline{\psi} \psi)^2.$$ (1)

This model possesses a discrete chiral symmetry $\psi \rightarrow \gamma_5 \psi$, imposing $\langle \overline{\psi} \psi \rangle = 0$ perturbatively. We focus on the topology of vacuum diagrams. We can divide them into 2 disjoint classes: those diagrams falling apart in 2 separate pieces when 2 lines meeting at the same point are cut. We call those 2-point-particle-reducible or $2PPR$. The second diagram of Figure 1 is a $2PPR$ diagram. The other type is the complement of the $2PPR$ class, we call these diagrams 2-point-particle-irreducible ($2PPI$) diagrams. The first and third diagram of Figure 1 are $2PPI$ ones. We could now remove all $2PPR$ bubbles from the diagrammatic sum building up the vacuum energy by summing them in an effective mass $m$. Defining $\Delta = \langle \overline{\psi} \psi \rangle$, it can be shown that

$$m = -g^2 \left(1 - \frac{1}{2N} \right) \Delta.$$ (2)

Then the $2PPI$ vacuum energy $E_{2PPI}$ is given by the sum of all $2PPI$ vacuum diagrams, now with a mass $m$ running in the loops. It is important to notice that $E_{2PPI}$ is not the vacuum energy due to a double counting ambiguity, already visible in the second diagram of Figure 1: each diagram can be seen as an insertion on the other one. This can be solved by considering $\frac{dE}{dg^2}$ instead of $E$. The $g^2$ derivative can hit $2PPR$ or $2PPI$ vertex. Hence
Figure 1. The vacuum energy as a sum of $2PPI$ and $2PPR$ diagrams.

$$E = \begin{array}{c}
\begin{array}{c}
\text{2PPI} \\
\text{2PPR}
\end{array}
\end{array} + \begin{array}{c}
\begin{array}{c}
\text{2PPI} \\
\text{2PPR}
\end{array}
\end{array} + \cdots$$

Figure 2. The $2PPI$ vacuum energy $E_{2PPI}$.

$$-\frac{1}{2} \left(1 - \frac{1}{2N}\right) \Delta^2$$

$$\frac{\partial E_{2PPI}}{\partial g^2}$$

This can be integrated using the ansatz

$$E = E_{2PPI} + cg^2\Delta^2.$$  (4)

It remains to determine the unknown constant $c$. Diagrammatically, it is easy to show that one has the following gap equation.

$$\frac{\partial E_{2PPI}}{\partial m} = \Delta.$$  (5)

Combination of the above formulae finally gives

$$E = E_{2PPI} + \frac{1}{2}g^2 \left(1 - \frac{1}{2N}\right) \Delta^2.$$  (6)
An important point is the renormalizability of the 2\text{PPI} expansion. Two possible problems could be mass renormalization and vacuum energy renormalization, since originally there was no external mass scale present. The proof is quite technical, but all formulae remain correct and are finite when the conventional counterterms of the Gross-Neveu model are included. Essentially, the proof is based on coupling constant renormalization and the separation of 2\text{PPI} and 2\text{PPR} contributions.

It can be shown that

$$\frac{\partial E_{\text{2PPI}}}{\partial \Delta} = m \Leftrightarrow \frac{\partial E}{\partial m} = 0.$$  \hfill (7)

However, this does not mean that $E(\Delta)$ has the meaning of an effective potential, since $E(\Delta)$ is meaningless if the gap equation (7) is not fulfilled.

In Table 1, we list the numerical deviations in terms of percentage between our optimized 2-loop results\textsuperscript{7} for the mass gap $M$ and the square of minus the vacuum energy $\sqrt{-E}$ and the exact known values. We conclude that the 2\text{PPI} results are in relative good agreement with the exact values and converge to the exact $N \to \infty$ limit.

| $N$ | deviation $M$ (%) | deviation $\sqrt{-E}$ (%) |
|-----|-----------------|-------------------|
| 2   | ?               | ?                 |
| 3   | -4.5            | 47.7              |
| 4   | -6.5            | 27.9              |
| 5   | -6.1            | 19.9              |
| 10  | -3.5            | 8.4               |
| $\infty$ | 0         | 0                |

3. \textit{SU}(N) Yang-Mills theory in the Landau gauge

Next, we consider the Euclidean Yang-Mills action in the Landau gauge where $A^a_\mu$ denotes the gauge field. Repeating the analysis of section 2 leads
to

\[ \Delta = \langle A^\mu_\mu A^\mu_\mu \rangle, \]

\[ m^2 = g^2 \frac{N}{N^2 - 1} \frac{d - 1}{d - \Delta}, \]

\[ E = E_{2PPI} - g^2 \frac{N}{4} \frac{d - 1}{d} \Delta^2, \]

\[ \frac{\partial E_{2PPI}}{\partial m^2} = \frac{\Delta}{2} \Leftrightarrow \frac{\partial E}{\partial m^2} = 0. \]

After some manipulation, the 2-loop results became

\[ \frac{g^2 N}{16\pi^2} \approx 0.131 \quad \sqrt{\Delta} \approx 536 \text{MeV} \quad E \approx -0.002 \text{GeV}^4. \]

We notice that the relevant expansion parameter, \( \frac{g^2 N}{16\pi^2} \), is relatively small. As such, our results should be qualitatively trustworthy.

As a comparison, the value found by Boucaud et al from lattice simulations and an OPE treatment was \( \langle A^2 \rangle_{\mu=10 \text{GeV}} \approx 1.64 \text{GeV} \). To find a value for the gluon mass \( m_g \) itself (as the pole of the gluon propagator) within the 2PPI framework, the diagrams relevant for mass renormalization of \( m \) should be calculated.

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