Relation between Loading Rate and Fracture Velocity on Limestone

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Abstract: The relation between loading rate and fracture velocity is the key to determining the fracture toughness of rock mass under dynamic loading. While designing an optimal blast design for any limestone mines, understanding the relationship between blast detonation pressure and rock fragmentation can increase the energy utilisation in any limestone mine blast. The detonation pressure is directly related to dynamic loading rate and fracture velocity is directly related to stress wave propagation speed during blasting. This paper discusses the relationship between dynamic loading rate and fracture velocity for limestone samples. It was observed that crack propagation velocity increases with fracture toughness of rock samples. It may be concluded that as the dynamic loading increases, the fracture velocity increases.

Key words: Fracture velocity, split Hopkinson pressure bar (SHPB), loading rate.

1. Introduction

In a limestone mine dynamic load to rock mass is applied by detonating explosives, which could be site mixed slurry (SMS), site mixed emulsion (SME), ammonium nitrate fuel oil (ANFO), heavy ANFO (HANFO) or non-permitted large diameter explosives. Once the explosives detonate in blast hole, stress wave is released and fresh fracture in rock is generated when explosive impedance exceeds rock impedance. In another words, the equation of state (EOS) of rock must match with EOS of rock for the fracture to propagate. Eleven (11) rock samples were selected to investigate the impact of dynamic load on rock specimen. The selected rock sample blocks are labelled as SCW1, SCW2, SCW3, SCW4, SCW5, SCW6, SCW7, SCW8, SCW9, SCW12 and SCW13. In detail, SCW1 is dolomitic limestone; SCW3 is pegmatite; SCW8 is siliceous limestone; and the rests are limestone. Split Hopkinson pressure bar system (SHPB) was used to provide dynamic loads.

2. Materials and Methods

The setup and principle of SHPB system are explained in this section along with basic formula for the data analysis. Since force balance on both sides of the specimen is necessary for a valid SHPB test, pulse shaping technique was applied for each test [1]. Such technique is also discussed in this section.

2.1 Experimental Apparatus

SHPB is adopted to test the dynamic properties of limestone rock samples, and its setup is shown in Fig. 1. SHPB system consists three parts: striker bar, incident bar and transmitted bar. These three bars are made of steel and have the same diameter of 25.4 mm but different length as shown in Fig. 1. Three bars are carefully aligned to ensure the accuracy of measurement, and specimen was sandwiched between the incident bar and transmitted bar.

In the test, the striker bar is launched by the compressive gas in the gas gun, which impacts the incident bar in high speed to create a dynamic load and subsequently generates an incident wave, propagated along the incident bar. When the stress wave reaches the end of incident bar and contacts the specimen, part
of wave would reflect back called reflected wave, and part of wave transmits through the specimen called transmitted wave [3]. Two strain gauges SG1 and SG2 were tightly pasted on the incident bar and transmitted bar to record the elastic deformations of the two bars due to the stress wave. The strain signals from SG1 and SG2 are recorded in the oscilloscope. Theoretically, sum of the incident wave and reflected wave should equal to the transmitted wave; therefore, the force acted in the two ends of the specimen should be identical in the test. Such force equilibrium state is called force balance which is baseline for a valid SHPB test.

2.1 Dynamic Mode-I Fracture Toughness Test

Notched semi-circular bend (NSCB) specimen is used for dynamic mode-I fracture test. The geometry of such specimen is shown in Fig. 2. To prepare such specimens, 40-mm-diameter core samples were cut to 18 mm thick disk. The cylindrical surface was ensured to be free from any marks and any irregularities. The thickness of the specimen should not exceed 0.025 mm, and end faces shall be flat to 0.25 mm. Subsequently, the disk was cut and split along the diameter into two semi-circular samples. A notch was subsequently machined to the semi-circular sample, located on the centre of the original disk and perpendicular to the diametrical cut. The photograph of prepared specimens is shown in Fig. 2.

Figs. 3 and 4 show the dynamic mode-I fracture toughness of each rock specimen responding to different loading rates. Dynamic mode-I toughness is in unit of MPa·m$^{1/2}$, while loading rate is in MPa·m$^{(1/2)}$/s. It is evident from Figs. 3 and 4 that, when the dynamic load is low (on the left), the specimen splits equally into two semi-circular disks.

Anderson [2] developed an empirical relationship between fracture velocity and fracture propagation toughness as shown below.

$$K_f^c = \frac{K_{I0}}{1 - \left(\frac{\varepsilon_f}{V_t}\right)^m}$$  \hspace{1cm} (1)

$K_f^c$ is the dynamic fracture toughness at a specific crack propagation velocity $V_c$ in a dynamic test, $m$ is a constant, $V_t$ is the limiting fracture velocity, and $K_{I0}$ is the stress intensity factor at zero fracture velocity, or arrest fracture toughness. The other constants $m$, $V_t$, and $K_{I0}$ are estimated from above equation [8].

On the other hand, fracture toughness has a linear relationship with tensile strength as expressed below [5, 7].

$$K_i = \eta \sigma_t$$ \hspace{1cm} (2)

where $\eta$ is a constant, $K_i$ is the static fracture toughness and $\sigma_t$ is the static tensile strength. On differentiation on both sides the above equation is converted to following equation [4].

$$K_i = \eta \dot{\sigma}_t$$ \hspace{1cm} (3)

where, $\dot{K}_i$ and $\dot{\sigma}_t$ are the loading rate.

Moreover, according to the test data, there is a linear relationship between $K_f^c$ and the loading rate $\dot{K}_i$ as shown below:

$$K_f^c = a + b \dot{K}_i$$ \hspace{1cm} (4)

On combining all equations, the following equation is obtained [3].

$$\frac{K_{I0}}{1 - \left(\frac{\varepsilon_f}{V_t}\right)^m} = a + b \frac{\dot{\sigma}_t}{\sigma_t} K_i$$ \hspace{1cm} (5)

where $\eta = \sigma_t/K_i$ and parameter $a$ and $b$ are the constants obtained by fitting data into equations [6].

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Fig. 1  Schematics of a split Hopkinson pressure bar (SHPB) system.
Fig. 2  Prepared NSCB specimens.

Fig. 3  Example of one of the dynamic mode-I fracture toughness test results.
3. Results and Discussion

The relationship between loading rate and fracture velocity under various loading rate for all rock samples is shown in Tables 1-11.

Table 1 Relationship between loading rate and fracture velocity for sample KCW-1.

| $\sigma_t$ (GPa/s) | $V_c$ (m/s) |
|-------------------|-------------|
| 100               | 18          |
| 200               | 325         |
| 300               | 502         |
| 400               | 611         |
| 500               | 685         |

Table 2 Relationship between loading rate and fracture velocity for sample KCW-2.

| $\sigma_t$ (GPa/s) | $V_c$ (m/s) |
|-------------------|-------------|
| 100               | 89          |
| 200               | 249         |
| 300               | 360         |
| 400               | 437         |
| 500               | 492         |

Table 3 Relationship between loading rate and fracture velocity for sample KCW-3.

| $\sigma_t$ (GPa/s) | $V_c$ (m/s) |
|-------------------|-------------|
| 100               | 57          |
| 200               | 262         |
Table 3 to be continued

| Load Rate (GPa/s) | Fracture Velocity (m/s) |
|------------------|-------------------------|
| 100              | 208                     |
| 200              | 355                     |
| 300              | 430                     |
| 400              | 475                     |
| 500              | 505                     |

Table 4  Relationship between loading rate and fracture velocity for sample KCW-4.

| Loading Rate (GPa/s) | Fracture Velocity (m/s) |
|----------------------|--------------------------|
| 100                  | 208                      |
| 200                  | 355                      |
| 300                  | 430                      |
| 400                  | 475                      |
| 500                  | 505                      |

Table 5  Relationship between loading rate and fracture velocity for sample KCW-5.

| Loading Rate (GPa/s) | Fracture Velocity (m/s) |
|----------------------|--------------------------|
| 100                  | 242                      |
| 200                  | 346                      |
| 300                  | 412                      |
| 400                  | 456                      |
| 500                  | 488                      |

Table 6  Relationship between loading rate and fracture velocity for sample KCW-6.

| Loading Rate (GPa/s) | Fracture Velocity (m/s) |
|----------------------|--------------------------|
| 100                  | 293                      |
| 200                  | 454                      |
| 300                  | 556                      |
| 400                  | 624                      |
| 500                  | 674                      |

Table 7  Relationship between loading rate and fracture velocity for sample KCW-7.

| Loading Rate (GPa/s) | Fracture Velocity (m/s) |
|----------------------|--------------------------|
| 100                  | 260                      |
| 200                  | 396                      |
| 300                  | 481                      |
| 400                  | 538                      |
| 500                  | 579                      |

Table 8  Relationship between loading rate and fracture velocity for sample KCW-8.

| Loading Rate (GPa/s) | Fracture Velocity (m/s) |
|----------------------|--------------------------|
| 100                  | 110                      |
| 200                  | 227                      |
| 300                  | 314                      |
| 400                  | 380                      |
| 500                  | 430                      |
Table 9  Relationship between loading rate and fracture velocity for sample KCW-9.

| KCW9 | \( \sigma_c \): loading rate (GPa/s) | \( V_c \): fracture velocity (m/s) |
|------|---------------------------------|---------------------------------|
|      | 100                             | 364                             |
|      | 200                             | 547                             |
|      | 300                             | 627                             |
|      | 400                             | 671                             |
|      | 500                             | 699                             |

Table 10  Relationship between loading rate and fracture velocity for sample KCW-12.

| KCW12 | \( \sigma_c \): loading rate (GPa/s) | \( V_c \): fracture velocity (m/s) |
|-------|---------------------------------|---------------------------------|
|       | 100                             | 146                             |
|       | 200                             | 225                             |
|       | 300                             | 297                             |
|       | 400                             | 360                             |
|       | 500                             | 414                             |

Table 11  Relationship between loading rate and fracture velocity for sample KCW-13.

| KCW13 | \( \sigma_c \): loading rate (GPa/s) | \( V_c \): fracture velocity (m/s) |
|-------|---------------------------------|---------------------------------|
|       | 100                             | 97                              |
|       | 200                             | 236                             |
|       | 300                             | 370                             |
|       | 400                             | 486                             |
|       | 500                             | 584                             |

Table 12  Values of constants obtained during dynamic testing.

|       | \( K_{1d} \) (MPa\cdot m^{1/2}) | \( V_i \) (m/s) | \( m \) | \( a \) (MPa\cdot m^{1/2}) | \( b \) (s) | \( K_f \) (MPa\cdot m^{1/2}) | \( \sigma_1 \) (MPa) |
|-------|-------------------------------|----------------|------|-----------------------------|-----|-----------------------------|-----------------|
| KCW1  | 3.56                          | 1,061          | 0.84 | 1.71                        | 1.08E-04 | 3.51                        | 19.31           |
| KCW2  | 1.98                          | 820            | 0.48 | 1.50                        | 7.28E-05 | 1.50                        | 7.17            |
| KCW3  | 2.01                          | 811            | 0.72 | 1.10                        | 1.87E-04 | 1.11                        | 16.59           |
| KCW4  | 1.32                          | 652            | 0.52 | 1.03                        | 2.75E-04 | 1.12                        | 16.06           |
| KCW5  | 1.26                          | 682            | 0.54 | 1.77                        | 4.82E-05 | 1.77                        | 7.26            |
| KCW6  | 1.69                          | 968            | 0.49 | 2.17                        | 1.25E-04 | 2.17                        | 16.48           |
| KCW7  | 1.89                          | 826            | 0.68 | 2.14                        | 1.03E-04 | 2.14                        | 16.51           |
| KCW8  | 1.99                          | 796            | 0.62 | 1.95                        | 7.11E-05 | 1.95                        | 16.02           |
| KCW9  | 1.75                          | 825            | 0.61 | 0.44                        | 5.31E-04 | 1.01                        | 15.55           |
| KCW12 | 1.02                          | 1,052          | 0.17 | 2.73                        | 3.21E-05 | 2.73                        | 10.36           |
| KCW13 | 1.81                          | 1,500          | 0.23 | 2.52                        | 8.27E-05 | 2.52                        | 15.42           |

4. Conclusions

The relation between the dynamic loading rate and the fracture velocity can help in achieving optimal blast design for improved rock fragmentation. It may be also be concluded that knowledge of fracture velocity, fracture toughness, stress intensity factor can be a harbinger in explosives selection so that maximum energy utilization for fragmenting rock uniformly can be achieved. The study shows that with increase in dynamic load, the crack propagation velocity increases along with dynamic fracture toughness. It may also be observed that fracture velocity increases with dynamic load suggesting that a very high dynamic load will result in optimal rock fragmentation.
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