Methods of system composition and decomposition as a tool for efficient processing of large amounts of experimental data

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Abstract: The industrial age led to the emergence of geographically distributed industrial complexes operating in conditions of limited resources and internal competition. Accordingly, the problem of optimizing the regional distribution of industrial objects arose, which was fully solved only with the development of computer technology, which made it possible to quickly and efficiently process large amounts of experimental data. Mathematical approach to solving the economic problems helped to solve some difficult problems. To solve these problems, in the second half of the last century, a whole set of mathematical methods was developed, which made it possible to finally turn Economics into an exact science. When solving large-scale national economic problems, a systematic approach and mathematical methods such as stochastic programming, discrete programming, convex programming, nonlinear and linear programming were used. The decomposition method is one of the most popular ways to solve tasks and goals of any type, which are based on their detailed analysis and division of the process into several stages. This method is most often used in such fields as Economics, mathematics, etc. There is such a method as the approximation-combinatorial method of decomposition and composition of systems. The considered approximation method offers a modification of the listed methods, which can lead to a decrease in their “sensitivity” to changes in the problem conditions, which certainly makes it possible to expand the class of problems that can be solved using the listed methods.

1. Introduction

It is known that many actual problems of making optimal and optimal-compromise decisions in various spheres of scientific and practical human activity can be formulated in terms of mathematical models. The use of various methods allowed expanding the range of tasks that were set for the planned economy. Now this number includes the task of long-term planning and territorial planning, the task of developing targeted integrated scientific and technical tasks, social and economic programs, as well as the task of developing regional programs and programs for the development of territorial and industrial complexes.

The impetus for the emergence of these tasks was the beginning in the post-war period of effective development of new, previously unused, territories in Siberia and the far East of the country, which required increased efficiency in the use of material and technical costs and human resources. The planned economy of the Soviet Union was a guarantee of organizational solutions to these large-scale tasks.

It was at this time that the concept of a regional national economic complex appeared, which in turn consists of separate enterprises and territorial-industrial complexes connected by a common transport infrastructure and located on the same territory. Moreover, the regional national economic complex, in turn, was an integral part of the unified national economic complex of the country. A peculiar hierarchical structure has emerged, which can be called fractal with some stretch: a separate enterprise, a territorial national economic complex, a regional national economic complex, and a single national economic complex of the country as a whole.
An interesting mathematical problem required an adequate solution. Meanwhile, it is known that the computational efficiency of information processing algorithms is largely determined by the volume of initial statistical data and decreases as it increases. At the time, to get out of the situation, several General and quite effective approaches to solving applied problems on discrete structures based on combinatorial elements were proposed, such as the method of sequential analysis with subsequent screening of options, proposed by V. S. Mikhalevich and N. Z. Shor; V. A. sequential schemes. Emelichev; local algorithms Y. I. Zhuravlev; approximation-combinatorial algorithms.

We should not forget that one of the reasons that prompted researchers to develop effective approximation methods in the last quarter of the last century was the limited capabilities and resources of electronic computers. In the conditions of limited memory space and relatively low speed of operations compared to modern capabilities, researchers were forced to develop original solutions that allowed to obtain models that describe real systems with a high degree of accuracy and allow to optimize the latter. Many of these solutions are still successfully used today. Among them is an approximation-combinatorial method for solving the optimization problem.

2. The essence of the method of composition and decomposition
The method of decomposition and composition of systems was proposed by V. R. Khachaturov in 1974. This method solves the problem of finding the extremum of a functional that is defined on subsets of a finite set of elements.

The decomposition method is one of the most popular ways to solve tasks and goals of any type, which are based on their detailed analysis and division of the process into several stages. This method is most often used in such fields as Economics, mathematics, etc. There is such a method as the Approximation-combinatorial method of decomposition and composition of systems.

To solve problems using the approximation-combinatorial approach of decomposition and composition of systems, as a rule, the main methodological approaches listed below are used. Let’s look at them on the example of designing complex systems.

1. Decomposition and composition problems. The problem is that it is often impossible to solve the problem in General because the problem has a large dimension, as well as because it has specific conditions and a number of restrictions. If we consider a design problem, then the original problem is divided into a group of problems, each of which is dedicated to the design of one of the many existing technological systems (the decomposition of the design problem occurs). In accordance with the method under consideration, a set of projects that are "close" to the optimal functional value is determined.

2. Approximating problems. There may be a situation when the problem of designing each of the considered set of technological systems turns out to be unsolvable. If we are considering a construction project, the reason for the unsolvability can be concluded both in the complex form of cost functions of construction of objects and communications leading to them, and in the presence of specific restrictions.

3. Multi-Extreme tasks. For discrete optimization problems (the problem of placing enterprises, the problem of determining the structure of networks), multi-extremality is characteristic.

4. To get a final decision on the project, it is usually necessary to solve the problems of dynamic and simulation design.

In conclusion of this section, we cannot fail to mention one limitation of the approximation-combinatorial method of decomposition and composition of systems. The fact is that it only considers systems whose elements are described by a single set of possibly heterogeneous characteristics. Thus, heterogeneous and heterogeneous systems were excluded from consideration. However, this does not detract from the advantages of the method, which has proven its effectiveness over the past forty years.

3. Justification of the approximation-combinatorial method of decomposition and composition of systems
Consider below the mathematical formulation of the optimization problem for a system consisting of a set of homogeneous elements. In this case, uniformity implies that each of the set of elements is described by the same set of variables. However, there is no condition for all variables in the set to have the same dimension.

As an object of consideration, we will take some system D. It is necessary to solve the problem of defining such a multidimensional element \( X^* \in D \), which would be in some (not necessarily formalized) sense the best.
Let's write this property $X^\circ$:

$$E(X^\circ) = \frac{\text{extr}}{\text{extr} E(X)},$$

where $\text{extr} E(X)$ is the operator rule the comparison of the various elements $X \in D$, with which we can determine the best element $X^\circ \in D$. In other words, we need to establish that $X^\circ$ is the best element precisely in the sense of the operator $E(X)$.

If $\text{extr} = \min$ (or $\max$), and $E(X)$ is some formalized evaluation rule, then we get a mathematical programming problem. In order to facilitate the search for the element $X^\circ \in D$ when solving specific problems, it is necessary to make some assumptions about the properties of the operator $E(X)$, which facilitate the search for the desired element. As a rule, these assumptions are based on the specifics of the problem under consideration.

We will say that system D:

- investigated, if found $X^\circ$;
- partially studied if a set is found $D_0$ such that $X^\circ \in D_0 \subset D$, $D_0 \neq D$;
- has not been studied if $D_0 = D$.

In order to study the system D and, an approximation-combinatorial method of decomposition and composition of systems was proposed.

We will briefly describe some of its stages. Let's start with decomposition.

Let's introduce $n$ systems into consideration $D_j(\omega_j)$, where $j \in J = \{1,2,...,n\}$, $\omega_j \subset I$, $\omega_j \neq \emptyset$, $D_j(\omega_j) \subset A(\omega_j) = \Pi A_j$, $i \in \omega_j$.

The elements of the systems $D_j(\omega_j)$, respectively, are $X_j = X_j(\omega_j)$, where $X_j(\omega_j) = (x_{1j}, ..., x_{|\omega_j|})$.

For each subset $\omega_j$, $k = 1,2,...,|\omega_j|$, $\omega_j = \{i_1 < \cdots < i_{|\omega_j|}\}$.

For each system $D_j(\omega_j)$, we set a formalized rule for selecting the best element $X_j^\circ(\omega_j) \in D_j(\omega_j)$, i.e. setting the optimization criterion $E_j(X_j(\omega_j))$.

The value of one of the indicators $E_j(X_j(\omega_j))$ can be taken as such a criterion $x_{ik(j)}$, where $i_{k(j)} \in \omega_j$.

Let's put $\text{extr} = \min$ for certainty. Then

$$E_j(X_j^\circ(\omega_j)) = \min_{X_j(\omega_j) \in D_j(\omega_j)} E_j(X_j(\omega_j)).$$

If we solve $n$ optimization problems, we define $X_j^\circ(\omega_j)$ and the corresponding values $E_j(X_j^\circ(\omega_j))$. We can then use the obtained values to define some $\tilde{X}(I) = \tilde{X} \in D$, which we take as an approximate solution of the problem.

At the same time, the methods for obtaining $\tilde{X}$ can be very diverse, depending on the specifics of the task and the properties of auxiliary systems $D_j(\omega_j)$.

Let's move on to the composition, which will consist of defining $D_0 \subset D$ and analyzing it to determine $X^\circ$.

For each $j \in J$ we set the values $R_j \geq 0$ and define the sets

$$\Omega_j(R_j, \omega_j) = \{X_j(\omega_j) \in D_j(\omega_j) \mid E_j(X_j(\omega_j)) \leq E_j\{X_j^\circ(\omega_j)\} + R_j\}.$$  

The set $\Omega_j(R_j, \omega_j)$ is called the set of optimal solutions and all solutions that are close to the optimal one, that is, differ from the optimal functional by no more than the value $R_j \geq 0$. In other words, the set $\Omega_j(R_j, \omega_j)$ is called the domain of $R_j$-stability of the optimal value of the functional we introduced earlier $E_j(X_j(\omega_j))$.

For each subset $\nu \subset J$ we match a certain set $q(\nu) \subset D$ according to the rule:

$$q(\nu) = \{X \in D \mid X \in \Omega_j, \forall j \in \nu \cup X \in \Omega_j, \forall j \in J \setminus \nu\}.$$  

For the sake of certainty let's put $q(\emptyset) = \{X \in D \mid X \in \Omega_j, \forall j = 1,2,...,n\}$.

As a result, the original set $D$ is divided into $2^n$ disjoint subsets $q(\nu)$.

These sets $q(\nu)$ are called combinatorial elements. If $X \in q(\nu)$, we will say that X has the property v.

Denote by V the set of all subsets of the set J, including the empty set. In this case, each element $v \in V$ corresponds to only one composite element, and the total number of composite elements is equal to $|V| = 2^n$.  

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From the considered construction \( q(v) \)

a) \( q(v_1) \cap q(v_2) = \emptyset \), if \( v_1 \neq v_2 \).

b) \( D = \bigcup_{v \in V} q(v) \), since an empty subset is also an element \( V \).

We conclude that \( X^0 \) belongs to one and only one composite element \( q(v) \).

If the property is known in advance \( v \), which it possesses \( X^0 \), then \( X^0 \in q(v) \) and \( D_0 = q(v) \).

If the exact property \( X^0 \) is not known in advance, but a subset of the properties \( \delta_\emptyset \subset V \), is known, one of which is sure to satisfy \( X^0 \), then

\[
X^0 \in \bigcup_{v \in \delta_0} q(v) \cap D_0 = \bigcup_{v \in \delta_0} q(v).
\]

Accordingly subsets can be constructed for all \( \delta \subset V \)

\[
Q(\delta) = \bigcup_{v \in \delta} q(v)
\]

note that \( Q(V) = D \) in accordance with the previously considered property b).

The set \( Q(\delta) \) is called a composite subsystem \( Q(\delta) \), or a composition composed of the set \( \delta \) of composite elements \( q(v), v \in \delta \).

Put \( Q(\emptyset) = \phi \) and call \( Q(\emptyset) \) an empty composition.

The task of the composition operation is to determine, based on the expected properties of \( X \), the set \( \delta_\emptyset \) and the corresponding composition subsystem \( Q(\delta_\emptyset) \), which contains \( D_0 \). Consequently, \( X^0 \).

Let us consider in detail one method of composition, namely, we will assume that the desired element \( X^0 \) must have the property \( \delta = f \) (\( \delta = v = f \)). In this case, the composition \( Q(J) \) will match its composition element \( q(J) \):

\[
Q(J) = q(J) = \bigcup_{j \in J} q_j
\]

Such a composition contains all elements of \( X \in D \), for which all their projections on the sets \( A(\omega_j) \) (\( j \in J \)) are in the corresponding regions of \( R_j \) stability \( \Omega_j(R_j, \omega_j) \), if \( X \in Q(J) \), then

\[
E_j \left( X^0(\omega_j) \right) \leq E_j \left( X(\omega_j) \right) \leq E_j \left( X^0(\omega_j) \right) + R_j
\]

and if \( X \not\in Q(J) \), then

\[
E_j \left( X(\omega_j) \right) > E_j \left( X^0(\omega_j) \right) + R_j
\]

It turns out that \( Q(J) \) consists of elements \( X \in D \), for each of which the value of the criterion \( E_j (X) \) for all \( j \in J \) differs from \( \omega_j \) by no more than a predetermined value \( R_j \geq 0 \). The composition problem is solved.

4. Examples of using the approximation-combinatorial method of decomposition and composition of systems

In our country at the end of the last century, in the conditions of the planned Soviet economy and the possibility of centralized regulation of the territorial distribution of industrial facilities, an approach to optimizing the hierarchical territorial industrial complex using mathematical modeling methods was developed.

The development of territorial models of industrial systems that could vary in scale – from a complex of several similar industrial facilities to the entire planned national industrial structure (national economic complex of the country) – posed a number of problems and prompted researchers to search for solutions to these problems. Widespread models and methods, such as the balance method and the method of linear programming, could no longer effectively solve the problems faced by the planned economy.

The difficulty of solving a new type of problem arose from the complex, territorial and intersectoral nature of regional problems. Thus, the unified territorial planning program should have taken into account and linked the interests and activities of several branches of the national economy at once. In these large-scale projects, issues related to the optimal development of infrastructure, transport system, rational use of material, natural and labor resources, and environmental protection were to be solved in a complex. The methods mentioned above (linear programming method, balance method) could no longer effectively take into account all the factors that characterize the task of comprehensive development of territories. The problem was complicated by the fact that these factors were nonlinear and discrete in nature, and the amount of data that needed to be processed significantly exceeded the capabilities of existing electronic computers.

To effectively solve complex multi-criteria problems in the presence of a large array of source data and limited computing power for their processing, a number of original approximation solutions were developed that provide the necessary accuracy of territorial planning.
These solutions allowed to solve a complex public problems such as the problem of optimal placement of enterprises, the agglomeration effect into account when placing the territorial-production complex optimization tasks of placement companies, taking into account existing between them of communication, recording several different steps in the problems of optimal allocation, the optimal structure of the transport network and optimizing existing network of transport and communications.

As already mentioned, the approximation-combinatorial method of decomposition and composition of systems was developed to solve a wide range of economic problems, such as finding the optimal structure of the regional economy. The article considers in detail the application of the approximation-combinatorial method for solving a variety of national economic problems, such as the problem of optimal placement of enterprises and territorial production complexes, the problem of building an optimal network structure and the problem of optimizing the existing network structure.

For example, when designing the development of oil fields, this method allows you to solve several problems sequentially.

We can distinguish several characteristic features of the problem of constructing an optimal network that do not depend on the type of the considered technological system – tree, a fractal type network, which connects multiple sources of raw materials collection system, the rising cost of a private link network in the case of increasing flow of raw materials in this connection, the problem of accounting for the already existing parts of your network when choosing the optimal network configuration plan.

We can confidently say that in the case of multidimensional data, approximate and heuristic methods allowed us to solve problems that were not possible for exact methods. Especially since it was possible to do this in conditions of limited machine resources (speed, memory).

However, the approximation-combination method of decomposition and composition of systems finds its application in areas far from Economics.

An essential parameter for the characterization of multilayer nanostructures is the quality of the interface, which strongly depends on the dynamics and mechanisms of growth of structures. To determine the statistical characteristics of inhomogeneities of the internal interface, the analysis of the angular spectrum of the intensity of reflected x-ray radiation is used.

Mathematical models and methods that allow us to determine the amount of interlayer roughness from the angular spectrum of the intensity of reflected x-ray radiation were considered.

To solve the inverse problem, an approximation-combinatorial method was proposed, as a result of which the problem of estimating the roughness value was successfully solved. This suggests a possible expansion of the range of problems that can be solved using the approximation-combination method of decomposition and composition of systems.

5. Conclusion

The development of applied mathematics, the widespread use of mathematical methods in Economics, combined with the capabilities of modern computer technology, laid the Foundation for the successful solution of a wide class of problems of optimization of the national economy. The peculiarity of these problems is the presence of a large data array that needs to be subjected to mathematical processing. This task is often difficult to solve at the current stage of computer technology development. It was even more difficult to solve it forty or fifty years ago in the conditions of limited resources and speed of electronic computers.

It was at this time that many brilliant approximation methods were developed that allowed us to successfully and efficiently solve optimization problems. One of these methods is the approximation-combinational method of decomposition and composition of systems, developed by V. R. Khachaturov to solve problems of optimization of regional economy.

Meanwhile, the potential of the approximation-combination method of decomposition and composition of systems is far from exhausted due to the fact that it has recently been used in a number of physical problems for determining the parameters of nanostructures using optical methods.

The decomposition method can also be used in various areas of everyday life, business, and science. A simpler way to find solutions to the most complex problems has not yet been found. The main thing is to fully master the tricks of dividing a large task or problem into small ones that are easier to solve, and then add the resulting results into a single whole.
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