Magnetic Moments of 70-plet Baryons

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Abstract

Magnetic moments of the positive parity 70-plet baryons are estimated within the non relativistic quark model and QCD sum rules method. It is found that the magnetic moments of the 70-plet baryons can be expressed in terms of the \( D \) and \( F \) couplings and exhibit unitary symmetry. The QCD sum rules for the magnetic moments of the 70-plet octet baryons are formulated. Comparison of magnetic moments of 56-plet and 70-plet baryons predicted from QCD sum rules are presented.
INTRODUCTION

Study of the electromagnetic properties of hadrons represents very important source of information about their internal structure and can provide valuable insight in understanding the mechanism of strong interactions at low energies, i.e. about nonperturbative aspects of QCD. Particular interest deserves magnetic moments of baryons as a subject of permanent study due to growing experimental information [1].

Magnetic moments of the positive parity octet and decuplet baryons are studied in framework of different approaches, such as nonrelativistic quark model (NRQM) [1], static quark model [2], QCD string approach [3], chiral perturbation theory [4], skyrme model [5], traditional QCD Sum rules [6], light-cone version of QCD sum rules [7, 8], lattice QCD [9].

Although the magnetic moments of the positive baryons are widely studied experimentally and theoretically, however about their negative parity counterparts there very limited information. Magnetic moments of these states can be extracted through the bremsstrahlung processes in photo and electro production reactions.

There exist comparatively few theoretical works on the magnetic moments of the negative parity baryons in framework of LCSR [10–12], effective Hamiltonian approach in QCD [13], constituent quark model [14], chiral quark model [15], unitarized chiral perturbation theory [16], chiral constituent quark model [17] and lattice QCD [18].

The positive parity octet and decuplet baryons usually enter the 56-plet representation of the SU(6), which is the group-theoretical basis of the NRQM. In the unitary SU(3) model coupling constants of photons (and vector mesons) with baryons are expressed in terms of the F and D constants. In chiral model [15] mixing of 56- and 70-plets are studied for $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons. Instead negative parity baryons are put usually in 70-plet.

The basic element of QCD sum rules calculation is the interpolating current of the corresponding hadron which can be reduced to 56-plet wave functions in nonrelativistic limit. Here we note that in studying the properties of the $\frac{1}{2}^-$ baryons the interpolating current for the positive parity baryons have been used although it has no usual NRQM limit for the 70-plet baryons. Despite this difficulty we shall try to write QCD sum rules in such a way as to respect the NRQM limit.
F AND D COUPLINGS IN QUARK-DIQUARK MODEL

Although the NRQM results on magnetic moments can not be expressed in terms of F and D couplings, in [19] it was shown that it can be achieved in framework of diquark-quark model. The characteristic property of this model is that the photon or vector boson field interact with diquark and single quark in a different way.

Let us discuss the magnetic moments of 56-plet baryons in NRQM in framework of diquark-quark model. As an example let consider proton from 56-plet. Its wave function can be written as

\[ \sqrt{18} |p, s = 1/2 > = |2u_1 u_2 - u_1 d_2 - d_1 u_2 + 2u_1 d_2 u_1 - u_2 u_1 d_1 - u_2 d_1 u_1 > \]

where subindices 1 and 2 means quark spin up and spin down respectively. The wave functions of other members of octet baryons can be obtained from proton wave function by appropriate replacements of quark fields.

The results for the magnetic moments of the 56-plet baryons in terms of F and D couplings can be achieved by using their wave functions and introducing following four different matrix elements:

\[ < q_1 q_2, q_1 q_2' | \hat{w}_q | q_1 q_1, q_1 q_1' > = w^q_{11} \]
\[ < q_1 q_2, q_1 q_2' | \hat{w}_q | q_1 q_2, q_1 q_1 > = w^q_{12} \]
\[ < q_1 q_2, q_1 q_2' | \hat{w}_q | q_1 q_2, q_1 q_2 > = v^q_{11} \]
\[ < q_1 q_2, q_1 q_2' | \hat{w}_q' | q_1 q_2, q_1 q_2 > = v^q_{12} \]

Using these matrix elements with \( e_q \hat{w}_q \sigma^q_z \) as the modified magnetic moment operator the magnetic moments of \( \Sigma^0 \) and \( \Lambda \) baryons are obtained [19]

\[ \mu(\Sigma^0) = (e_u + e_d) \frac{2}{3} w_{11} + \frac{1}{3} e_s (2v_{11} - v_{12}) \]
\[ = (e_u + e_d) F + e_s (F - D) \]
\[ \mu(\Lambda) = (e_u + e_d) (F - \frac{2}{3} D) + e_s (F + \frac{1}{3} D) \]

where \( \frac{2}{3} w_{11} = F, w_{12} = D \) and \( (2v_{11} - v_{12}) = 3(F - D) \) and also \( v_{11} = D, v_{12} = 3F - D \). These results show that the non relativistic quark-diquark model reveals the unitary symmetry pattern and the magnetic moments of baryons can be formulated in terms of the \( F \) and \( D \) couplings [19], [20].
If we put $e_u = 2/3$ and $e_d = e_s = -1/3$ from Eq. (3), one can easily obtain well known $SU(3)$ relations between magnetic moments of $(1/2)^+$ baryons [21]:

\[\begin{align*}
\mu(p) &= \mu(\Sigma^+) = F + \frac{1}{3}D \\
\mu(\Sigma^-) &= \mu(\Xi^-) = -F + \frac{1}{3}D \\
\mu(n) &= \mu(\Xi^0) = -\frac{2}{3}D \\
\mu(\Lambda) &= -\frac{1}{3}D.
\end{align*}\] (4)

The NRQM results can be found if we take $F = \frac{2}{3}D$, $D = 1$ and replace the electric charge of quarks by their magnetic moments, i.e. $e_q \rightarrow \mu_q$.

Now let us analyze magnetic moment of baryons entering to the 70-plet representation $SU(6)$ in framework of NRQM. The 70-plet in NRQM has following decomposition $70 = (8, 2) + (10, 2) + (8, 4) + (1, 2)$. The wave function of 70-plet within the NRQM obtained in numerous works (see [17] and references therein). Following to the [17], the wave function of $N^{*+}$ state in 70-plet with positive parity can be written as;

\[\sqrt{18}|N^{*+}> = 2u_1u_1d_2 - u_1d_1u_2 - d_1u_1u_2 + 2d_1u_2u_1 - u_1u_2d_1 - u_1d_2u_1 \\
+ 2u_2d_1u_1 - u_2u_1d_1 - d_2u_1u_1 > \] (5)

Using this wave function, within the quark-diquark model for the magnetic moment of $N^{*+}$ we get

\[\mu_{N^{*+}} = e_u \frac{2}{3}w_{12} + e_d \frac{1}{3}(2v_{12} - v_{11})\] (6)

Using definitions $w_{11}, w_{12}, v_{11}, v_{12}$ (see redefinitions after Eq.(3)) we have for magnetic moment of $N^{*+}$

\[\mu_{N^{*+}} = e_u F + e_d (2F - D) \] (7)

Performing similar analysis for the magnetic moments of $\Sigma^{0*}$ and $\Lambda^*$ baryons we obtain;

\[\begin{align*}
\mu_{\Sigma^{0*}} &= \frac{1}{2}(e_u + e_d)F + e_s (2F - D) \\
\mu_{\Lambda^{0*}} &= (e_u + e_d) \frac{1}{6}(9F - 4D) + e_s \frac{1}{3}D
\end{align*}\] (8)
Transition moment $\mu_{\Sigma^0,\Lambda^0}$ is obtained immediately from group-theoretical relation [19, 20]

$$< \Sigma_{d\leftrightarrow s}|O|\Sigma_{d\leftrightarrow s} > - < \Sigma_{u\leftrightarrow s}|O|\Sigma_{u\leftrightarrow s} >= \sqrt{3} < \Sigma^0|O|\Lambda >$$ (9)

where $O$ is appropriate operator and reads

$$\mu_{\Sigma^0,\Lambda^0} = -\frac{3}{2}(e_u - e_d)(F - \frac{2}{3}D).$$ (10)

and in the NRQM yields zero.

Magnetic moments of other members of octet baryons can be found with the help of appropriate replacements of quark fields. In the limit $F = \frac{2}{3}D$, $D = 1$ and replace $e_q \rightarrow \mu_q$ we get the NRQM results for the magnetic moments of 70-plet baryons. When in Eqs. (7),(8) take the relevant quark charges we get unitary symmetry results for the octet baryons in the 70-plet, similar to the relations for the 56-plet baryons see Eq.(4):

$$\mu(N^+) = \mu(\Sigma^+) = \frac{1}{3}D$$
$$\mu(N^0) = \mu(\Xi^0) = F - \frac{2}{3}D$$
$$\mu(\Sigma^-) = \mu(\Xi^-) = -F + \frac{1}{3}D$$
$$\mu(\Sigma^0) = -\mu(\Lambda^+) = -\frac{1}{2}(F - \frac{2}{3}D)$$
$$\mu(\Sigma^0\Lambda^0) = -\frac{3}{2}(F - \frac{2}{3}D)$$ (11)

Finally, magnetic moments of 70-plet octet baryons can be obtained from results of 56-plet respectively with the help of simple replacements by comparing Eqs. (3),(7) and (8). For example, the magnetic moment of $\Sigma^0$ in 70-plet can be achieved from 56-plet one by replacing coefficients of $F$ and $D$ terms:

for $\Sigma^0$ baryons;

$F : (e_u + e_d) \rightarrow \frac{1}{2}(e_u + e_d + 2e_s)$

$(F - D) : e_s \rightarrow e_s$ (12)

These relations constitute one of the main results of the present work.
These results for the magnetic moments of octet baryons can also be obtained from the QCD sum rules method which exhibits unitary symmetry pattern in a sense that they can be represented in terms of only two independent F and D type functions. Note that the conclusion is true not only for photon but for any vector field too (see for example [22]). The key object of the QCD sum rules method is the interpolating currents. In the non relativistic limit the interpolating current of baryons can be reduced to their wave functions [23].

The magnetic moments of the octet baryons in framework of traditional and light cone versions of QCD sum rules are calculated in various works (see for example [6, 8] and references therein.)

The QCD sum rules method is based upon the correlation function

\[ \Pi = i \int d^4xe^{ipx} < 0 | T \{ \eta_B(x)\bar{\eta}_B(0) \} | 0 > \gamma \]

where \( T \) is the time ordering operator, \( \gamma \) means external electromagnetic field and \( \eta_B \) is the interpolating current carrying the same quantum numbers as the corresponding baryon \( B \).

As an example we present the interpolating current for \( \Sigma^0 \) baryon

\[ \eta_{\Sigma^0} = \sqrt{\frac{1}{2}} e^{abc} \{(u^a T C s^b)\gamma_5 d^c - (s^a T C d^b)\gamma_5 u^c + \beta(u^a T C s^b)\gamma_5 d^c - \beta(s^a T C d^b)u^c \} \] (13)

where \( a, b, c \) are color indices, \( C \) is the charge conjugation operator and \( \beta \) is the arbitrary parameter ( \( \beta = -1 \) corresponds to the so called Ioffe current).

In order to construct QCD sum rules for magnetic moments of the octet baryons the correlation function is calculated in terms of hadrons and quark-gluon degrees of freedom. By matching these two representations the QCD sum rules for the octet baryons magnetic moments are obtained [7]. The magnetic moments of octet baryons in general form can be written as

\[ \lambda_B^2 \mu_B e^{-m_B^2/M^2} = e_u \Pi_1^B(u, d, s, M^2) + e_d \Pi_1^B(d, u, s, M^2) + e_s \Pi_2(u, d, s, M^2) \] (14)

where \( \lambda_B \) and \( \mu_B \) are the residue and magnetic moment of corresponding baryon, respectively and \( \Pi_i \) is the invariant function in the coefficient of \( \gamma_5 \) Lorentz structure and their expressions can be found in [6, 8]. Using the \( SU(2) \) symmetry Eq. (14) can be written as

\[ \lambda_B^2 \mu_B e^{-m_B^2/M^2} = (e_u + e_d) \Pi_1^B(u, d, s, M^2) + e_s \Pi_2^2(u, d, s, M^2) \] (15)
Using the relation obtained in [24] which connected $\Lambda$ and $\Sigma^0$ interpolating current one can easily find the expression for the magnetic moment $\Lambda$-hyperon:

$$3\mu(\Lambda) = 2\mu(\Sigma^0(d \leftrightarrow s)) + 2\mu(\Sigma^0(u \leftrightarrow s)) - \mu(\Sigma^0)$$  \hspace{1cm} (16)

We can now predict the magnetic moments of the octet in 70-plet with positive parity if we assume that the transformations obtained in NRQM (see Eqs.(3),(8) and (12)) hold in the case of QCD sum rules, i.e., at the level of correlation functions $\Pi$’s.

In this case even when the explicit expressions for interpolating currents of octet baryons belonging to the 70-plet representation are not known one can predict the magnetic moments of these baryons.

Using Eqs. (8),(12) as well as Eqs (15) and (16) for the magnetic moments of $\Sigma$ and $\Lambda$ baryons in 70-plet we get following sum rules

$$\lambda_{\Sigma^*}^2\mu_{\Sigma^*}^2 e^{-m_{\Sigma^*}^2/M^2} = \frac{1}{2}[e_u + e_d + 2e_s]\Pi_1^B + e_s\Pi_2^B$$

$$\lambda_{\Lambda^*}^2\mu_{\Lambda^*}^2 e^{-m_{\Lambda^*}^2/M^2} = \frac{3}{2}(e_u + e_d)\Pi_1^B + [2(e_u + e_d) - e_s]\Pi_2^B$$  \hspace{1cm} (17)

Comparing these equations with the sum rules for the $\Sigma$ and $\Lambda$ baryons from 56-plet we see that they have changed drastically.

In order to get the idea about the magnitude of the magnetic moments of 70-plet baryons, in the obtained sum rules we put their experimental mass and for residues we put their values for 56-plet baryons. Under this assumption, the magnetic moments of the positive parity 70-plet baryons are given in Table 1 (the 2nd column). For completeness we also put magnetic moments of 56-plet baryons in this table (the 1st column).

**CONCLUSION**

It is shown that octet baryons in the 70-plet can be analyzed in the way similar to those of 56-plet. In particular magnetic moments are written in terms of the $D$ and $F$ quantities characteristic for octet coupling. Moreover the main formulas for the magnetic moments are written in such a way as to obtain the NRQM results as well as unitary symmetry ones. Borel QCD sum rules are constructed for the magnetic moments of the 70-plet octet. Comparison of magnetic moments for $1/2^+$ 56-plet and 70-plet baryons from QCD sum rules are presented.
| $\mu(B^*)$ | QCD SR $\frac{1}{2}^+$ [8] | This Work ($\frac{1}{2}^+,$70-plet ) |
|----------|-----------------|-----------------|
| $\mu(N^{*+})$ | 2.72 | 0.83 |
| $\mu(N^{*0})$ | -1.65 | 0.22 |
| $\mu(\Sigma^{*+})$ | 2.52 | 0.70 |
| $\mu(\Sigma^{-})$ | -1.13 | -1.13 |
| $\mu(\Sigma^{0})$ | 0.70 | 0.11 |
| $\mu(\Lambda)$ | -0.50 | -0.11 |
| $\mu(\Xi^{0})$ | -0.89 | 0.69 |
| $\mu(\Xi^{-})$ | -1.18 | -1.18 |

Table I: Magnetic moments of 70-plet and 56-plet baryons.
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