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Particle with spin 2 and anomalous magnetic moment in external electromagnetic and gravitational fields

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Tensor 50-component form of the first order relativistic wave equation for a particle with spin 2 and anomalous magnetic moment is extended to the case of an arbitrary curved space – time geometry. An additional parameter considered in the presence of only electromagnetic field as related to anomalous magnetic moment, turns to determine additional interaction terms with external geometrical background through Ricci $R_{kl}$ and Riemann $R^{klmn}$ tensors.

Theory of massive and massless fields of spin 2, starting from Pauli and Fierz investigations [1, 2], always were attracted attention: de Broglie [3], Pauli [4], Gel’fand–Yaglom [5], Fradkin [6], Fedorove et al. [7, 8, 9, 11, 12], Fainberg [13], Regge [14], Buchdah [15, 16], Velo – Zwanziger [17, 18], Cox [22], Barut [23], Loide [27], Vasiliev [29], Buchbinder et al. [30, 31, 32, 33, 34, 35]. Most of investigations were performed within second order wave equations approach. However, it is known that many of problems arising for fields of higher spin may be avoided if one starts with the theory of first order equations. One of the earliest considerations of the theory for spin 2 particle was given by Fedorov [7]. It turned out that such a description requires 30 component wave function. Afterwards Fedorov proposed else one description for spin 2 particle, 50-component model, [10, 21]. The primary question is about relation between two models. Bogush and Kisel [28] demonstrated (within spinor formalism) that 50-component model describes a spin 2 particle with additional electromagnetic characteristics, anomalous magnetic moment

$^1$ More detailed analysis of this spinor description was given in [24, 25, 25, 26, 28]. In [34, 35], that theory was transformed to more a simple tensor technique.

In the present paper we consider a 50-component model for a massive spin 2 particle in presence of external electromagnetic and gravitational fields. The primary question is about additional intrinsic structure of the particle manifests itself in any curved space-time background.

We start with tensor equations given in [34, 35] for flat Minkowski space-time, and extend them by changing ordinary derivative into covariant ones ($\partial_b \rightarrow \nabla_\beta$), so we arrive at

$$2 \lambda_1 D^a \Psi^{(1)}_a + 2 \lambda_2 D^a \Psi^{(2)}_a + iM \Psi = 0 ,$$

(1a)

$^1$Here one may see analogy with the known Petras [36, 37, 38, 40] theory for a spin 1/2 particle, or Shamali – Capri [13, 44, 45, 16, 47, 48] theory for a particle with spin 1, when increase in the number of field variables permits us to introduce an additional parameter for a particle, anomalous magnetic moment.
\[
\begin{align*}
\lambda_3 D_a \Psi + 2 \lambda_4 D^b \Psi_{(ba)} + iM \Psi_a^{(1)} &= 0, \\
\lambda_5 D_a \Psi + 2 \lambda_6 D^b \Psi_{(ba)} + iM \Psi_a^{(2)} &= 0, \\
\frac{\lambda_7}{2} \left( D_a \Psi_b^{(1)} + D_b \Psi_a^{(1)} - \frac{1}{2} g_{ab} D^c \Psi_c^{(1)} \right) \\
+ \frac{\lambda_8}{2} \left( D_a \Psi_b^{(2)} + D_b \Psi_a^{(2)} - \frac{1}{2} g_{ab} D^c \Psi_c^{(2)} \right) &+ 2 \lambda_9 D^c \Psi_{(abc)} - 2 \lambda_{10} \left( D^c \Psi_{a[bc]} + D^c \Psi_{b[ac]} \right) + iM \Psi_{(ab)} = 0, \\
\frac{\lambda_{11}}{2} \left( D_c \Psi_{(ab)} - D_b \Psi_{(ac)} - \frac{1}{3} g_{ca} D^m \Psi_{(mb)} + \frac{1}{3} g_{ba} D^m \Psi_{(mc)} \right) + iM \Psi_{a[bc]} = 0, \\
\frac{\lambda_{12}}{3} \left( D_a \Psi_{(bc)} + D_b \Psi_{(ca)} + D_c \Psi_{(ab)} \\
- \frac{1}{3} g_{ac} D^m \Psi_{(mb)} - \frac{1}{3} g_{cb} D^m \Psi_{(ma)} - \frac{1}{3} g_{ba} D^m \Psi_{(mc)} \right) + iM \Psi_{(abc)} = 0.
\end{align*}
\]

Here \( D_a = \nabla_a + ieA_a \), where \( \nabla_a \) is a covariant derivative, \( A_a \) stands for electromagnetic potential; \( \lambda_1, \ldots, \lambda_{12} \) are 12 numerical constants obeying additional restrictions:

\[
2 \lambda_{10} \lambda_{11} - \frac{2}{3} \lambda_9 \lambda_{12} = 1, \quad \lambda_4 \lambda_7 + \lambda_6 \lambda_8 + \frac{8}{9} \lambda_9 \lambda_{12} = \frac{1}{3}, \\
\lambda_1 \lambda_3 + \lambda_2 \lambda_5 = \frac{1}{4}, \quad (\lambda_1 \lambda_4 + \lambda_2 \lambda_6) (\lambda_3 \lambda_7 + \lambda_5 \lambda_8) = -\frac{1}{12}.
\]

In 50-component model for a spin 2 particle, we employ one scalar, two vectors, and three tensors:

\[
\Psi, \quad \Psi_a^{(1)}, \quad \Psi_a^{(2)}, \quad \Psi_{(ab)}, \quad \Psi_{a[bc]}, \quad \Psi_{(abc)}.
\]

Recall that in 30-component model there involved scalar, vector, and two tensors:

\[
\Phi, \quad \Phi_a, \quad \Phi_{(ab)}, \quad \Phi_{[ab]c} ;
\]

with 30 independent variables

\[
\Phi(x) \implies 1, \quad \Phi_a \implies 4, \\
\Phi_{(ab)} \implies (10 - 1) = 9, \quad \Phi_{[ab]c} \implies 6 \times 4 - 4 - 4 = 16;
\]

and equations (compare with (1a)–(1f))

\[
D^a \Phi_a - M \Phi = 0, \quad \frac{1}{2} D_a \Phi - \frac{1}{3} D^b \Phi_{(ab)} - M \Phi_a = 0.
\]

\[\text{Footnote: Below it will be clear that only one parameter has physical sense, referring to anomalous magnetic moment, all other can be eliminated from the model.}\]
\[ D_a \Phi_b + D_b \Phi_a - \frac{1}{2} g_{ab} D^k \Phi_k + \frac{1}{2} (D^k \Phi_{[ka]} b + D^k \Phi_{[kb]} a - \frac{1}{2} g_{ab} D^k \Phi_{[kn]}) - M \Phi_{(ab)} = 0 , \]

\[ D_a \Phi_{(bc)} - D_b \Phi_{(ac)} + \frac{1}{3} (g_{bc} D^k \Phi_{(ak)} - g_{ac} D^k \Phi_{(bk)}) - M \Phi_{[ab]c} = 0 . \]  

(3b)

Below we will show that excluding from the 50-component models superfluous variables (formally they consist a 4-vector and 3-rank tensor) and introducing new field variables, one can get a 30-component model modified by presence additional interaction terms with electromagnetic and gravitational fields.

To this end, first instead of \( \Psi_a^{(1)} \), \( \Psi_a^{(2)} \) let us introduce new variables

\[
\begin{vmatrix}
B_a \\
C_a
\end{vmatrix} = \begin{vmatrix}
\lambda_1 & \lambda_2 \\
\lambda_7 & \lambda_8
\end{vmatrix}
\begin{vmatrix}
\Psi_a^{(1)} \\
\Psi_a^{(2)}
\end{vmatrix},
\begin{vmatrix}
\Psi_a^{(1)} \\
\Psi_a^{(2)}
\end{vmatrix} = \begin{vmatrix}
\frac{1}{\lambda_1 \lambda_8 - \lambda_2 \lambda_7} & \lambda_8 - \lambda_2 \\
-\lambda_7 & \lambda_1
\end{vmatrix}
\begin{vmatrix}
B_a \\
C_a
\end{vmatrix}.
\]

The system (1) can be presented as follows

\[ 2 D^a B_a + i m \Psi = 0 , \]

\[ -\frac{1}{4} D_a \Psi + 2 (\lambda_1 \lambda_4 + \lambda_2 \lambda_6) D^b \Psi_{(ba)} + i M B_a = 0 , \]

\[ (\lambda_7 \lambda_3 + \lambda_8 \lambda_5) D_a \Psi + 2 (\lambda_7 \lambda_4 + \lambda_8 \lambda_6) D^b \Psi_{(ba)} + i M C_a = 0 , \]

\[ \frac{1}{2} \left( D_a C_b + D_b C_a - \frac{1}{2} g_{ab} D^c C_c \right) + \]

\[ + 2 \lambda_9 D^c \Psi_{(abc)} - 2 \lambda_{10} \left( D^c \Psi_{a[bc]} + D^c \Psi_{b[ac]} \right) + i M \Psi_{(ab)} = 0 , \]

\[ \frac{\lambda_{11}}{2} \left( D_c \Psi_{(ab)} - D_b \Psi_{(ac)} - \frac{1}{3} g_{ca} D^m \Psi_{(mb)} + \frac{1}{3} g_{ba} D^m \Psi_{(mc)} \right) + i M \Psi_{a[bc]} = 0 , \]

\[ \frac{\lambda_{12}}{3} \left[ D_a \Psi_{(bc)} + D_b \Psi_{(ca)} + D_c \Psi_{(ab)} \right] - \frac{1}{3} g_{ac} D^m \Psi_{(mb)} - \frac{1}{3} g_{ab} D^m \Psi_{(ma)} - \frac{1}{3} g_{ba} D^m \Psi_{(mc)} \right] + i M \Psi_{(abc)} = 0 . \]

(4f)

Multiplying eq. (4c) by \( \lambda_1 \lambda_4 + \lambda_2 \lambda_6 \) and taking into account (2a), we get

\[ -\frac{1}{12} D_a \Psi + 2 (\lambda_1 \lambda_4 + \lambda_2 \lambda_6) (\lambda_7 \lambda_4 + \lambda_8 \lambda_6) D^b \Psi_{(ba)} + i M (\lambda_1 \lambda_4 + \lambda_2 \lambda_6) C_a = 0 . \]

Substituting expression for \( D_a \Psi \) from (4b), we arrive at

\[ -\frac{2}{3} (\lambda_1 \lambda_4 + \lambda_2 \lambda_6) D^b \Psi_{(ba)} - \frac{i M}{3} B_a \]
+2(\lambda_1 \lambda_4 + \lambda_2 \lambda_6)(\lambda_7 \lambda_4 + \lambda_8 \lambda_6) D^b \Psi_{(ba)} + iM (\lambda_1 \lambda_4 + \lambda_2 \lambda_6) C_a = 0 ,

from whence it follows

\[ C_a = \frac{1}{3(\lambda_1 \lambda_4 + \lambda_2 \lambda_6)} B_a - \frac{2}{iM} \left[ (\lambda_7 \lambda_4 + \lambda_8 \lambda_6) - \frac{1}{3} \right] D^a \Psi_{(na)} . \quad \text{(4c') \hfill (4c') \hfill (4c') \hfill (4c')} \]

This identity permits to exclude a superfluous vector \( C_a \). In particular, then eq. (4d) gives

\[ \frac{1}{6(\lambda_1 \lambda_4 + \lambda_2 \lambda_6)} (D_a B_b + D_b B_a - \frac{1}{2} g_{ab} D^c B_c) \]

\[ - \frac{1}{iM} \left[ (\lambda_7 \lambda_4 + \lambda_8 \lambda_6) - \frac{1}{3} \right] \left( D_a D^a \Psi_{(ab)} + D_b D^a \Psi_{(na)} - \frac{1}{2} g_{ab} D^c D^a \Psi_{(nc)} \right) \]

\[ + 2\lambda_9 D^c \Psi_{(abc)} - 2\lambda_{10} (D^c \Psi_{a[bc]} + D^c \Psi_{b[ac]}) + iM \Psi_{(ab)} = 0 . \quad \text{(4d')} \]

Therefore, instead of eqs. (4) we can use an equivalent one

\[ 2 D^a B_a + iM \Psi = 0 , \quad \text{(4a)} \]

\[ - \frac{1}{4} D_a \Psi + 2(\lambda_1 \lambda_4 + \lambda_2 \lambda_6) D^b \Psi_{(ba)} + iM B_a = 0 , \quad \text{(4b)} \]

\[ C_a = \frac{1}{3(\lambda_1 \lambda_4 + \lambda_2 \lambda_6)} B_a - \frac{2}{iM} \left[ (\lambda_7 \lambda_4 + \lambda_8 \lambda_6) - \frac{1}{3} \right] D^a \Psi_{(na)} , \quad \text{(4c')} \]

\[ \frac{1}{6(\lambda_1 \lambda_4 + \lambda_2 \lambda_6)} (D_a B_b + D_b B_a - \frac{1}{2} g_{ab} D^c B_c) \]

\[ - \frac{1}{iM} \left[ (\lambda_7 \lambda_4 + \lambda_8 \lambda_6) - \frac{1}{3} \right] \left( D_a D^a \Psi_{(mb)} + D_b D^a \Psi_{(na)} - \frac{1}{2} g_{ab} D^c D^a \Psi_{(nc)} \right) \]

\[ + 2\lambda_9 D^c \Psi_{(abc)} - 2\lambda_{10} (D^c \Psi_{a[bc]} + D^c \Psi_{b[ac]}) + iM \Psi_{(ab)} = 0 , \quad \text{(4d')} \]

\[ \frac{\lambda_{11}}{2} (D_c \Psi_{(ab)} - D_b \Psi_{(ac)} - \frac{1}{3} g_{ca} D^m \Psi_{(mb)} + \frac{1}{3} g_{ba} D^m \Psi_{(mc)}) + iM \Psi_{a[bc]} = 0 , \quad \text{(4e)} \]

\[ \frac{\lambda_{12}}{3} \left[ D_a \Psi_{(bc)} + D_b \Psi_{(ca)} + D_c \Psi_{(ab)} - \frac{1}{3} g_{ac} D^m \Psi_{(mb)} - \frac{1}{3} g_{cb} D^m \Psi_{(ma)} - \frac{1}{3} g_{ba} D^m \Psi_{(mc)} \right] + iM \Psi_{(abc)} = 0 . \quad \text{(4f)} \]

With the help of (4e) and (4f), let us express tensors \( \Psi_{a[bc]} \) and \( \Psi_{(abc)} \) through the 2-rank tensor:

\[ \Psi_{a[bc]} = \frac{i\lambda_{11}}{2M} \left( D_c \Psi_{(ab)} - D_b \Psi_{(ac)} - \frac{1}{3} g_{ca} D^m \Psi_{(mb)} + \frac{1}{3} g_{ba} D^m \Psi_{(mc)} \right) , \quad \text{(5a)} \]
\[ \Psi_{(abc)} = \frac{i \lambda_{12}}{3M} \left( D_a \Psi_{(bc)} + D_b \Psi_{(ca)} + D_c \Psi_{(ab)} \right) - \frac{1}{3} g_{ac} D^m \Psi_{(mb)} - \frac{1}{3} g_{cb} D^m \Psi_{(ma)} - \frac{1}{3} g_{ba} D^m \Psi_{(mc)} \right). \]  

(5b)

Substitution it into eq. (4d'), we get

\[ \frac{1}{6(\lambda_1 \lambda_4 + \lambda_2 \lambda_6)} (D_a B_b + D_b B_a - \frac{1}{2} g_{ab} D^c B_c) \]
\[ + \frac{i}{M} \left[ (\lambda_7 \lambda_4 + \lambda_8 \lambda_6) - \frac{1}{3} \right] \left( D_a D^c \Psi_{(cb)} + D_b D^c \Psi_{(ca)} - \frac{1}{2} g_{ab} D^c D^m \Psi_{(mc)} \right) \]
\[ + i \frac{2 \lambda_9 \lambda_{12}}{3M} D^c (D_a \Psi_{(bc)} + D_b \Psi_{(ca)} + D_c \Psi_{(ab)} \right) \]
\[ - \frac{1}{3} g_{ac} D^m \Psi_{(mb)} - \frac{1}{3} g_{cb} D^m \Psi_{(ma)} - \frac{1}{3} g_{ba} D^m \Psi_{(mc)} \]
\[ - i \frac{\lambda_{10} \lambda_{11}}{M} \left[ D^c \left( D_c \Psi_{(ab)} - D_a \Psi_{(ab)} - \frac{1}{3} g_{ac} D^m \Psi_{(mb)} + \frac{1}{3} g_{ba} D^m \Psi_{(mc)} \right) \right] + iM \Psi_{(ab)} = 0. \]

Now, allowing for (see (2a))

\[ \lambda_{10} \lambda_{11} = \frac{1}{2} + \frac{1}{3} \lambda_9 \lambda_{12}, \quad \lambda_4 \lambda_7 + \lambda_6 \lambda_8 - \frac{1}{3} = - \frac{8}{9} \lambda_9 \lambda_{12}. \]

and using the notation \( \lambda_9 \lambda_{12} = \mu \), we obtain

\[ \frac{M}{6i(\lambda_1 \lambda_4 + \lambda_2 \lambda_6)} \left( D_a B_b + D_b B_a - \frac{1}{2} g_{ab} D^c B_c \right) \]
\[ - \frac{8}{9} D_a D^c \Psi_{(cb)} - \frac{8}{9} D_b D^c \Psi_{(ca)} + \frac{4}{9} g_{ab} D^c D^m \Psi_{(mc)} \]
\[ + \frac{2}{3} D^c D_a \Psi_{(bc)} + \frac{2}{3} D^c D_b \Psi_{(ca)} + \frac{2}{3} D^c D_c \Psi_{(ab)} - \frac{2}{9} g_{ac} D^c D^m \Psi_{(mb)} \]
\[ - \frac{2}{9} g_{cb} D^c D^m \Psi_{(ma)} - \frac{2}{9} g_{ba} D^c D^m \Psi_{(mc)} \]
\[ - \frac{1}{2} D^c D_c \Psi_{(ab)} + \frac{1}{2} D^c D_b \Psi_{(bc)} + \frac{1}{6} g_{ca} D^c D^m \Psi_{(mb)} - \frac{1}{6} g_{ba} D^c D^m \Psi_{(mc)} \]
\[ - \frac{1}{2} D^c D_c \Psi_{(ab)} + \frac{1}{2} D^c D_a \Psi_{(bc)} + \frac{1}{6} g_{cb} D^c D^m \Psi_{(ma)} - \frac{1}{6} g_{ab} D^c D^m \Psi_{(mc)} \]
\[ - \frac{1}{3} D^c D_c \Psi_{(ab)} + \frac{1}{3} D^c D_a \Psi_{(bc)} + \frac{1}{9} g_{ca} D^c D^m \Psi_{(mb)} - \frac{1}{9} g_{ba} D^c D^m \Psi_{(mc)} \]
\[ - \frac{1}{3} D^c D_c \Psi_{(ab)} + \frac{1}{3} D^c D_a \Psi_{(bc)} + \frac{1}{9} g_{cb} D^c D^m \Psi_{(ma)} - \frac{1}{9} g_{ab} D^c D^m \Psi_{(mc)} \]
\[ + M^2 \Psi_{(ab)} = 0. \]  

(4d'')
From whence, after simple manipulations, we arrive at (commutator will be noted as \([\ , \ ]_-\))

\[
\frac{1}{6i (\lambda_1 \lambda_4 + \lambda_2 \lambda_6)} \left( D_a B_b + D_b B_a - \frac{1}{2} g_{ab} D^c B_c \right)
\]

\[
- \frac{1}{M} \left[ D^c D_c \Psi_{(ba)} - \frac{1}{2} \left( D^c D_b \Psi_{(ac)} + D^c D_a \Psi_{(bc)} \right) \right]
\]

\[
+ \frac{1}{3} g_{ab} D^n D^m \Psi_{(nm)} - \frac{1}{6} \left( D_a D^m \Psi_{(mb)} + D_b D^m \Psi_{(ma)} \right) \right]
\]

\[
+ \frac{\mu}{M} \left( [D^c, D_a] \Psi_{(bc)} + [D^c, D_b] \Psi_{(ac)} \right) + M \Psi_{(ab)} = 0 .
\]

(4d'')

Now, let us introduce a new variable (constant \(\gamma\) will be specified below)

\[
\Phi_{[bc]a} = - \frac{1}{M} \frac{\gamma}{2} \left( D_c \Psi_{(ab)} - D_b \Psi_{(ac)} + \frac{1}{3} g_{ab} D^m \Psi_{(mc)} - \frac{1}{3} g_{ac} D^m \Psi_{(mb)} \right),
\]

then we derive an identity

\[
\frac{1}{\gamma} \left( D^c \Phi_{[bc]a} + D^c \Phi_{[ac]b} \right) = - \frac{1}{M}
\]

\[
\times \left[ \frac{1}{2} \left( D^c D_c \Psi_{(ab)} - D^c D_b \Psi_{(ac)} + \frac{g_{ab}}{3} D^c D^m \Psi_{(mc)} - \frac{g_{ac}}{3} D^c D^m \Psi_{(mb)} \right) \right]
\]

\[
+ \frac{1}{3} \left( D^c D_c \Psi_{(ba)} - D^c D_a \Psi_{(bc)} + \frac{g_{ba}}{3} D^c D^m \Psi_{(mc)} - \frac{g_{bc}}{3} D^c D^m \Psi_{(ma)} \right) \right]
\]

\[
= - \frac{1}{M} \left( D^c D_c \Psi_{(ab)} - \frac{1}{2} D^c D_b \Psi_{(ac)} - \frac{1}{3} D^c D_a \Psi_{(bc)} + \frac{g_{ab}}{3} D^c D^m \Psi_{(mc)} \right)
\]

\[
- \frac{g_{ac}}{6} D^c D^m \Psi_{(mb)} - \frac{g_{bc}}{6} D^c D^m \Psi_{(ma)} \right)
\]

which coincides with the expression in rackets in (4d''). Therefore, eq. (4d'') may be presented as (let it be \(\gamma = \sqrt{2}\))

\[
\frac{1}{6i (\lambda_1 \lambda_4 + \lambda_2 \lambda_6)} \left( D_a B_b + D_b B_a - \frac{1}{2} g_{ab} D^c B_c \right) + \frac{1}{\sqrt{2}} \left( D^c \Phi_{[bc]a} + D^c \Phi_{[ac]b} \right)
\]

\[
+ \frac{\mu}{M} \left( [D^c, D_a] \Psi_{(bc)} + [D^c, D_b] \Psi_{(ac)} \right) + M \Psi_{(ab)} = 0 .
\]

(4d'''')

In the following, it will be convenient to use two variables

\[
\Phi = - \frac{1}{4 \sqrt{3} (\lambda_1 \lambda_4 + \lambda_2 \lambda_6)} \Psi, \quad \Phi_a = \frac{i}{\sqrt{6} (\lambda_1 \lambda_4 + \lambda_2 \lambda_6)} B_a .
\]

Thus, from 50-component system, we have arrives at a modified 30-component model

\[
\frac{1}{\sqrt{2}} D^a \Phi_a + M \Phi = 0 ,
\]
\[ \frac{1}{\sqrt{2}} D_a \Phi + \sqrt{\frac{2}{3}} D^b \Psi_{(ba)} + M \Phi_a = 0 , \]
\[ - \frac{1}{\sqrt{6}} ( D_a \Phi_b + D_b \Phi_a - \frac{1}{2} g_{ab} D^c \Phi_c ) + \frac{1}{\sqrt{2}} ( D^c \Phi_{[bc]} + D^c \Phi_{[ac]b} ) \]
\[ + \frac{\mu}{M} ( [D^c, D_a]-\Psi_{(bc)} + [D^c, D_b]-\Psi_{(ac)} ) + M \Psi_{(ab)} = 0 , \]
\[ \frac{1}{\sqrt{2}} ( D_c \Psi_{(ab)} - D_b \Psi_{(ac)} + \frac{1}{3} g_{ab} D^m \Psi_{(mc)} - \frac{1}{3} g_{ac} D^m \Psi_{(lb)} ) + M \Phi_a[b]c = 0 . \]

By the simple linear transformations
\[ \Phi = -\tilde{\Phi} , \quad \Psi_a = \sqrt{2} \tilde{\Phi}_a , \quad \Phi_{(ab)} = \frac{1}{\sqrt{3}} \tilde{\Phi}_{(ab)} , \quad \Phi_{[bc]a} = \frac{1}{\sqrt{6}} \tilde{\Phi}_{[bc]a} \quad (5) \]
it becomes simpler
\[ \begin{aligned} & D^a \tilde{\Phi}_a - M \Phi = 0 , \\
& - \frac{1}{2} D_a \tilde{\Phi} - \frac{1}{3} D^b \Psi_{(ba)} - M \tilde{\Phi}_a = 0 , \\
& ( D_a \tilde{\Phi}_b + D_b \tilde{\Phi}_a - \frac{1}{2} g_{ab} D^c \tilde{\Phi}_c ) + \frac{1}{2} ( D^c \tilde{\Phi}_{[ca]b} + D^c \tilde{\Phi}_{[eb]a} ) \\
& - \frac{\mu}{M} ( [D^c, D_a]-\tilde{\Phi}_{(bc)} + [D^c, D_b]-\tilde{\Phi}_{(ac)} ) - M \tilde{\Phi}_{(ab)} = 0 , \\
& D_c \tilde{\Phi}_{(ba)} - D_b \tilde{\Phi}_{(ca)} + \frac{1}{3} g_{ba} D^m \tilde{\Phi}_{(mc)} - \frac{1}{3} g_{ca} D^m \tilde{\Phi}_{(mb)} - M \tilde{\Phi}_{[cb]a} = 0 . \end{aligned} \quad (7) \]

If \( \mu = 0 \), we obtain a 30-component theory (the sign of \( \sim \) is taken away):
\[ \begin{aligned} & D^a \Phi_a - M \Phi = 0 , \\
& - \frac{1}{2} D_a \Phi - \frac{1}{3} D^b \Phi_{(ab)} - M \Phi_a = 0 , \\
& D_a \Phi_b + D_b \Phi_a - \frac{1}{2} g_{ab} D^c \Phi_c + \frac{1}{2} ( D^c \Phi_{[ca]b} + D^c \Phi_{[eb]a} ) - M \Phi_{(ab)} = 0 , \\
& D_c \Phi_{(ba)} - D_b \Phi_{(ca)} + \frac{1}{3} g_{ba} D^m \Phi_{(mc)} - \frac{1}{3} g_{ca} D^m \Phi_{(mb)} - M \Phi_{[cb]a} = 0 . \end{aligned} \quad (8) \]

Comparing it with (3), we note differences in 3-d equation. However it is easily demonstrated their equivalence. Indeed, from 4-th equation we derive
\[ \begin{aligned} & g^{ac} \Phi_{(bc)} - g^{ac} \Phi_{(ac)} + \frac{1}{3} ( g^{ac} g_{bc} D^k \Phi_{(ak)} - g^{ac} g_{ac} D^k \Phi_{(bk)} ) - M g^{ac} \Phi_{[ab]c} = 0 . \end{aligned} \]

Because \( g^{ac} \Phi_{(ac)} = 0 , g^{ac} g_{ac} = 4 \), then
\[ \Phi^{\Phi}_{[ab]} = 0 , \quad (9) \]
which means that the term $\frac{1}{2}g_{ab}D^c\Phi^c_{[cn]}$ in 3-d equation in (3) vanishes identically. Thus, systems (3) and (8) are equivalent.

Let us find an explicit form for (see 4-th eq. in (7))

$$\mu M^{-1} \left( [D^c, D_a] - \Phi_{(bc)} + [D^c, D_b] - \Phi_{(ac)} \right).$$

(10)

It suffices to consider the first term

$$[D^c, D_a] - \Phi_{(bc)} = [\nabla_c + ieA_c, \nabla_a + ieA_a] - \Phi^c_b$$

$$= (\nabla_c \nabla_a - \nabla_a \nabla_c) \Phi^c_b + ieF_{ca} \Phi^c_b$$

with the help of known rules

$$(\nabla_c \nabla_a - \nabla_a \nabla_c) A_{bk} = -A_{nk} R^n_{b ca} - A_{bn} R^n_{k ca}.$$ from whence it follows

$$(\nabla_c \nabla_a - \nabla_a \nabla_c) A^c_b = -A^c_n R^n_{b ca} - A_{bn} R^{nc}_{ca}.$$ Further, allowing for symmetry of curvature tensor we find

$$(\nabla^c \nabla_a - \nabla_a \nabla^c) A_{bc} = R_{ca bn} A^{nc} + A^c_n R_{na},$$

we derive

$$(\nabla^c \nabla_a - \nabla_a \nabla^c) \Phi_{bc} = R_{ca bn} \Phi^c_n + R_{ac} \Phi^c_b.$$ (12)

Therefore,

$$[D^c, D_a] - \Phi_{(bc)} = ieF_{ca} \Phi^c_b + R_{ca bn} \Phi^c_n + R_{ac} \Phi^c_b,$$

(13) and additional interaction term is specified by

$$\frac{\mu}{M} \left( [D^c, D_a] - \Phi_{(bc)} + [D^c, D_b] - \Phi_{(ac)} \right)$$

$$= \frac{\mu}{M} ie (\Phi^c_a F_{cb} + \Phi^c_b F_{ca}) + \frac{\mu}{M} (R_{ca bn} \Phi^c_n + R_{cb an} \Phi^c_n)$$

$$+ \frac{\mu}{M} (R_{ac} \Phi^c_b + R_{bc} \Phi^c_a).$$ (14)

Relation (14) means that the parameter $\mu$, initially interpreted as defining anomalous magnetic moment, also determines additional interaction with geometrical background, through Ricci $R_{kl}$ and Riemann curvature tensor $R_{klmn}$.

It should be noted that in the case of spin 1/2 particle with anomalous magnetic moment [36, 37, 38, 39, 40] there arises additional interaction through Ricci scalar [41, 42]; in the case of spin 1 particle [43, 44, 45, 46, 47, 48] there arises an additional interaction through Ricci tensor [49]. In other words, sensitivity of the anomalous magnetic moment to the space-time geometry substantially depends on spin of the particle.

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References

[1] W. Pauli. Über relativistische Feldgleichungen von Teilchen mit beliebigem Spin im elektronmagnetischen Feld. Helv. Phys. Acta. 1939. Bd. 12. S. 297–300.

[2] M. Fierz. On relativistic wave equations for particles of arbitrary spin in an electromagnetic field. Proc. Roy. Soc. London. A. 1939. Vol. 173. P. 211–232.

[3] L. De Broglie. Sur l’interprétation de certaines équations dans la théorie des particules de spin 2. C. R. Acad. Sci. Paris. 1941. Vol. 212. P. 657–659.

[4] W. Pauli. Relativistic field theories of elementary particles Rev. Mod. Phys. 1941. Vol. 13. P. 203–232.

[5] I.M. Gel’fand, A.M. Yaglom General relativistic-invariant equations and infinite-dimensional representations of the Lorentz group. JETF. 1948. Vol. 18. P. 703–733.

[6] E.E. Fradkin. To the theory of particle of high spin LETP. 1950. Vol. 20, no 1. P. 27–38.

[7] F.I. Fedorov. To the theory of a particle with spin 2. Uchenye zapiski BGU. fiz.-mat. 1951. no 12. P. 156–173.

[8] B.V. Krylov, F.I. Fedorov. First order equations for a graviton. DAN BSSR. 1967. Vol. 11, no. 8. P. 681–684.

[9] A.A. Bogush, B.V. Krylov, F.I. Fedorov. On matrices for equations for particles with spin 2. Vesti AN BSSR, fiz.-mat. 1968. no 1. P. 74–81.

[10] F.I. Fedorov. First order equations for gravitational field. DAN SSSR. 1968. Vol. 179, no. 4. P. 802–805.

[11] B.V. Krylov. On systems of first order equations for a graviton Vest AN BSSR. fiz.-mat. 1972. no. 6. P. 82–89.

[12] V.V. Kisel. On relativistic wave equations for massive particle with spin 2. Vesti AN BSSR. fiz.-mat. 1986. no. 5. P. 94–99.

[13] V.Ya. Fainberg. To the theory of interaction of particle with higher spins with electromagnetic and meson fields. Trudy FIAN SSSR. 1955. Vol. 6. P. 269–332.

[14] T. Regge. On properties of the particle with spin 2. Nuovo Cimento. 1957. Vol. 5, no. 2. P. 325–326.

[15] H.A. Buchdahl. On the compatibility of relativistic wave equations for particles of higher spin in the presence of a gravitational field. Nuovo Cim. 1958. Vol. 10. P. 96–103.

[16] H.A. Buchdahl. On the compatibility of relativistic wave equations in Riemann spaces. Nuovo Cim. 1962. Vol. 25. P. 486–496.

[17] G. Velo, D. Zwanziger. Noncausality and other defects of interaction Lagrangians for particles with spin one and higher. Phys. Rev. 1969. Vol. 188, no. 5. P. 2218–2222.
[18] G. Velo. Anomalous behavior of a massive spin two charged particle in an external electromagnetic field. Nucl. Phys. B. 1972. Vol. 43. P. 389–401.

[19] C.R. Hagen. Minimal electromagnetic coupling of spin-two fields. Phys. Rev. D. 1972. Vol. 6, no. 4. P. 984–987.

[20] C.R. Hagen. Scale and conformal transformations in Galilean-covariant field theory. Phys. Rev. D. 1972. Vol. 5, no. 2. P. 377–388.

[21] F.I. Fedorov, A.A. Kirilov. First order equations for gravitational field in vacuum. Acta Physica Polonica. B. 1976. Vol. 7. no. 3. P. 161 – 167.

[22] W. Cox. First-order formulation of massive spin-2 field theories. J. Phys. A. 1982. Vol 15. P. 253–268.

[23] A.O. Barut, B.W. Xu. On conformally covariant spin-2 and spin-3/2 equations J. Phys. A. 1982. Vol. 15, no. 4. P. 207–210.

[24] A.A. Bogush, V.V. Kisel. Description of a free particle by different wave equations. DAN SSSR. 1984. Vol. 28, no. 8. P. 702–705.

[25] A.A. Bogush, V.V. Kisel, F.I. Fedorov. On interpretation of additional components of the wave functions in electromagnetic interaction. DAN SSSR. 1984. Vol. 277, no 2. P. 343–346.

[26] V.V. Kisel. Relativistic wave equations with extended sets of representations. Dissertation. Institute of Physics, Minsk, 1984.

[27] R.K. Loide. On conformally covariant spin-3/2 and spin-2 equations. J. Phys. A. 1986. Vol. 19, no 5. P. 827–829.

[28] A.A. Bogush, V.V. Kisel On description of anomalous magnetic moment of a massive particle with spin 2 in the theory of relativistic wave equations. Izvestia Vuzov. SSSR. Fizika. 1988. Vol. 31, no. 3. P. 11 – 16.

[29] M.A. Vasiliev. More on equations of motion for interacting massless fields of all spins in (3+1)-dimensions. Phys. Lett. B. 1992. Vol. 285. P. 225–234.

[30] I.L. Buchbinder, V.A. Krykhtin, V.D. Pershin. On consistent equations for massive spine-2 field coupled to gravity in string theory. Phys. Lett. B. 1999. Vol. 466. P. 216–226.

[31] I.L. Buchbinder et al. Equations of motion for massive spin 2 field coupled to gravity Nucl. Phys. B. 2000. Vol. 584. P. 615–640.

[32] A.A. Bogush, V.V. Kisel, N.G. Tokarevskaya, V.M. Red’kov. On equation for a particle with spin 2 in external electromagnetic and gravitational fields. Proceedings of National Academy of Sciences of Belarus. ser. phys.-mat. 2003. no. 1. P. 62–67.

[33] V.M. Red’kov, N.G. Tokarevskaya, V.V. Kisel. Graviton in a curved space-time background and gauge symmetry Nonlinear phenomena in complex systems. 2003. Vol. 6, no. 3. P. 772–778.
[34] V.V. Kisel, V.M. Red’kov. System of tensor equations for a particle with spin 2 and description of an anomalous magnetic moment. I. Vesti BDPU. named after M. Tank. ser. 3. 2010. no. 1(63). P. 3–6.

[35] V.V. Kisel, V.M. Red’kov. System of tensor equations for a particle with spin 2 and description of an anomalous magnetic moment. I. Vesti BDPU. named after M. Tank. ser. 3. 2010. no. 2(64). P. 8–10.

[36] M. Petras. A contribution of the theory of the Pauli-Fierz’s equations a particle with spin 3/2 / Czech. J. Phys. 1955. Vol. 5, no. 2. P. 169–170.

[37] M. Petras. A note to Bhabha’s equation for a particle with maximum spin 3/2. Czech. J. Phys. 1955. Vol. 5, no. 3. P. 418–419.

[38] J. Formanek. On the Ulehla – Petras wave equation. Czech. J. Phys. B. 1955. Vol. 25, no. 8. P. 545–553.

[39] A.A. Bogush, V.V. Kisel. Equations with extended representations of the Lorentz group and interaction of the pauli type. Proceedings of National Academy of Sciences of Belarus. ser. phys.-mat. 1979. no. 3. P. 61–65.

[40] A.A. Bogush, V.V. Kisel. Equations for a spin 1/2 particle with anomalous magnetic moment. Izvestia Vuzov. SSSR. Fizika. 1984. no 1. P. 23–27.

[41] A.A. Bogush, V.V. Kisel, N.G. Tokarevskaya, V.M. Red’kov. Petras theory for a particle with spin 1/2 in a curved space-time. Proceedings of National Academy of Sciences of Belarus. ser. phys.-mat. 2002. no. 1. P. 63–68.

[42] V.V. Kisel, N.G. Tokarevskaya, V.M. Red’kov. Petras theory for a particle with spin 1/2 in a curved space-time. Minsk, 2002. 25 pages. Preprint no 737.

[43] A.Z. Capri. Non uniqueness of the spin 1/2 equation. Phys. Rev. 1969. Vol. 178, no. 5. P. 1811–1815.

[44] A.Z. Capri. First-order wave equations for half-odd-integral spin. Phys. Rev. 1969. Vol. 178, no. 5. P. 2427–2433.

[45] A. Shamaly. First-order wave equations for integral spin. Nuovo Cim. B. 1971. Vol. 2, no. 2. P. 235–253.

[46] A.Z. Capri. Electromagnetic properties of a new spin-1/2 field. Progr. Theor. Phys. 1972. Vol. 48, no. 4. P. 1364–1374.

[47] A. Shamaly, A.Z. Capri. Unified theories for massive spin 1 fields. Can. J. Phys. 1973. Vol. 51, no 14. P. 1467–1470.

[48] C.R. Hagen. Consistency of the anomalous-magnetic-moment coupling of a vector-meson field. Phys. Rev. D. 1974. Vol. 9, no. 2. P. 498–499.

[49] V.V. Kisel, V.M. Red’kov. Shamali–Kapri equation and additional interaction of a vector particle with gravitational field. Kovarian methods in theoretical physics. Physics of elementary particles and relativity theory. Institute of Physics, National Academy of Sciences of Belarus. Minsk, 2001. – P. 107–112.