Long-term investigations on reduction of constraint forces caused by imposed deformation by creep

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Abstract
External constraint is an action that arises when a structure cannot deform freely. This deformation may be caused by imposed deformation. These deformations lead to support reactions and internal forces only in statically indeterminate systems and are proportional to the absolute values of the structural stiffness. The structural stiffness is significantly reduced by creep of concrete. The classical models for calculating the creep impact on constraint forces assume linear-elastic or linear-viscoelastic behavior. The influence of the reinforcement on the creep deformation is neglected as well as the effect of cracking. For this reason, experiments were carried out to obtain results on the creep-induced degradation of constrained forces caused by imposed deformations. Long-term experiments were performed on a two-span beam on which the constraint force was applied by lifting the middle support. Dimensions of the test bodies were l/w/h = 7.3 m/0.4 m/0.2 m. Measurements were carried out on two test bodies, one conventional reinforced test body and one post-tensioned test body. Thus, the stiffness of a cracked and non-cracked test body was given. As a result, the influence of the reinforcement as well as effect of crack formation can be taken into account. The experiment lasted over a period of 500 days. The aim is to compare the results of the theoretical models taking into account different stiffness and different creep behavior with the measured test results.

Keywords
constraint forces, creep, effective stiffness, experiments, long-term test, restrained forces, stiffness, post-tensioning

1 | INTRODUCTION

The exact prediction of constraint forces is almost impossible due to multiple reasons. In particular because of the link with the stiffness which strongly depends on cracking and plasticization of individual areas in reinforced concrete members. Creep of the concrete also...
significantly reduces the stiffness. By coupling the absolute value of the constraint forces with the stiffness, time-dependent influences can be expected. The classical models for calculating the creep effects on constraint forces according to Trost assume linear-elastic or linear-viscoelastic behavior of an aging material. The influence of the reinforcement is neglected, as well as the effect of crack formation.

Nevertheless, assumptions are valid in good approximation for post-tensioned concrete structures that generally show few cracks under service conditions. The time-dependent behavior has significant effects on constraint forces. Due to the proportionality of the constraint forces to the stiffness, long-lasting constraint is reduced by creep to a considerable extent. The assumption of homogeneously elastic behavior does not apply to reinforced concrete structures.

2 | TIME-DEPENDENT DEVELOPMENT OF CONSTRAINT FORCES IN CASE OF SUDDEN SUPPORT DISPLACEMENT

Due to the proportionality of the constrained forces and the stiffness, long-lasting constraint is reduced by creep to a considerable extent. As creep ability of concrete is reduced with increasing loading age, suddenly occurring constraint is reduced to a greater extent compared with gradually developing constraint. Sudden or rapidly occurring constraint is caused, for example, by foundation settlement in non-cohesive soils, while gradually increasing constraint is caused by shrinkage or settlement in cohesive soils. The reduction of rapidly occurring constraint stresses due to creep is shown on a two-span beam stressed by imposed deformation, see Figure 1. The determination of statically indeterminate quantities is performed with the aid of the flexibility method (force method). The compatibility condition for calculating the statically indeterminate \( X_1 \) immediately after the foundation settlement at time \( t_0 \) is \( \delta_{10} = \Delta s^4 \):

\[
\delta_{10}(t_0) = X_1(t_0) \cdot \delta_{11}(t_0)
\]

With constant force \( X_1 \) the deformation would increase to \( \Delta s(1 + \varphi) \). Since this is not possible, the fictitious deformation increase \( \Delta s \cdot \varphi \) must be compensated by \( \Delta X_1 \). The complete compatibility condition, see Figure 1, is therefore:

\[
\delta_{10}(t_0) = X_1(t_0) \cdot \delta_{11}(t_0) \cdot (1 + \varphi) + \Delta X_1(t) \cdot \delta_{11}(t_0) \cdot (1 + \rho \varphi)
\]

\( X_1 \) corresponds to the force with which deformation \( \Delta s \) can be imposed on the beam without central support. With constant force \( X_1 \) the deformation would increase to \( \Delta s(1 + \varphi) \). Since this is not possible, the fictitious deformation increase \( \Delta s \cdot \varphi \) must be compensated by \( \Delta X_1 \). The complete compatibility condition, see Figure 1, is therefore:

\[
\delta_{10}(t_0) = X_1(t_0) \cdot \delta_{11}(t_0) \cdot (1 + \varphi) + \Delta X_1(t) \cdot \delta_{11}(t_0) \cdot (1 + \rho \varphi)
\]
The variation of the statically indeterminate value results to:

$$\Delta X_1(t) = \frac{\delta_{10}(t_0) - X_1(t_0)}{\delta_{11}(t_0)} \left(1 + \rho \varphi \right) = -X_1(t_0) \frac{\varphi}{1 + \rho \varphi}$$  \hspace{1cm} (4)

The time-dependent reaction force due to support lowering is:

$$X_1(t) = X_1(t_0) + \Delta X_1(t)$$  \hspace{1cm} (5)

$$= X_1(t_0) \cdot \left(1 - \frac{\varphi}{1 + \delta \varphi} \right)$$  \hspace{1cm} (6)

$$= X_1(t_0) \cdot (1 - \psi)$$  \hspace{1cm} (7)

Equation 7 also applies directly to the moment due to settlement:

$$M_{\Delta s}(t) = M_{\Delta s}(t_0) \cdot (1 - \psi)$$  \hspace{1cm} (8)

where $\psi$ is the relaxation value that describes the creep-induced stress reduction at constant strain.

3 | INFLUENCE OF CREEP ON THE STIFFNESS

Creep of concrete leads to a considerable increase in bending deformation. Tests showed that crack formation has a considerable influence on the time-dependent deformation. In non-cracked cross sections, both the compression zone and the tensile zone creep under long-term loads, whereas in cracked cross sections, essentially only the compression zone shows creep deformation, but the steel elongation remains virtually unchanged.$^4$ The decrease in stiffness due to crack formation considerably reduces the constraint forces. The ratio of the effective stiffnesses of the times $t_0$ and $t$ is essential for the reduction or rearrangement of the constraint forces. The actual value of the constraint and its time-dependent development can be determined, for example, by a nonlinear calculation. In Zilch and Fritsche$^7$ a method for estimating the rearrangement of internal forces aimed at changing the structural system of precast elements was presented, which can also be used for approximate calculation of the constraint reduction. The crack formation and the obstruction of shrinkage deformations by the reinforcement can be taken into account with the aid of the effective creep ratio $\varphi_{\text{eff}}$ in the explained Equation (6) according to Trost.$^5$

The deflection increases by creep to $(1 + \varphi)$. The following therefore applies to the bending stiffness:

$$E_{II}(t) = \frac{EI(t_0)}{1 + \varphi} = E_{\text{c,eff}} I$$  \hspace{1cm} (9)

Conversely, the effective creep index of the unreinforced, uncracked cross section according to Equation (9) naturally applies to:

$$\varphi_{\text{eff}} = \frac{E_{II}(t_0)}{E_{II}(t)} - 1$$  \hspace{1cm} (10)

A distinction must be made between the uncracked cross sections ($E_{II}$) and the mean behavior of cracked cross sections ($E_{III}$):

$$\varphi_{\text{eff}}^I = \frac{E_{II}(t_0)}{E_{II}(t)} - 1 \cong \varphi$$  \hspace{1cm} (11)

$$\varphi_{\text{eff}}^{II} = \frac{E_{III}(t_0)}{E_{III}(t)} - 1 \ll \varphi$$  \hspace{1cm} (12)

4 | EFFECTS OF TIME-DEPENDENT BEHAVIOR

Under normal conditions of use, concrete already has a pronounced time-dependent material behavior.$^8,9$ As a result, the strains occurring over time can be many times greater than the elastic strains.$^{10,11}$ Normally, these properties are subdivided as follows:

- Creep
- Relaxation
- Shrinkage

The first two properties generally describe the viscous material behavior of concrete. Creep and relaxation problems can often be treated together in practice. Shrinkage, on the other hand, must be considered separately, which also makes calculations easier.$^{12,13}$

4.1 | Creep ratio

Concrete already has a pronounced time-dependent material behavior under normal conditions of use. As a result, the strains occurring over time can be many times greater than the elastic strains. In the range of linear
creep, the creep strain is calculated using the creep index \( \varphi(t,t_0) \) as a multiple of the elastic strain \( \varepsilon_{cl,28} \):

\[
\varepsilon_{cl}(t,t_0) = \varepsilon_{cl,28} \cdot \varphi(t,t_0)
\]

For calculation of the creep index \( \varphi(t,t_0) \) two basic parameters are required. First, the basic creep index \( \varphi_0 \), which consists of the components of drying and basic creep and the concrete age at the start of loading. The second component \( \beta_c(t,t_0) \) describes the time variable of the function.

\[
\varphi(t,t_0) = \frac{\varphi_0}{C_1} \beta_c(t,t_0)
\]

### 4.2 | Effective Young’s modulus

\( E_{c,eff} \) describes the effective modulus of elasticity, which facilitates the calculation of creep deformations. The relaxation caused by the constraint can be considered in the formula with the relaxation coefficient \( \rho(t,t_0) \) as follows:

\[
E_{c,eff}(t) = \frac{E_c}{1 + \rho(t,t_0) \cdot \varphi(t,t_0)}
\]

The relaxation coefficient \( \rho(t,t_0) \) is set to 0.8. More information about the relaxation coefficient can be found in References [16,17].

### 4.3 | Time-dependent second moment of area

The calculation of the time-dependent second moment of area is carried out with \( \alpha_{s,eff}(t) \), taking into account the effective Young’s modulus:

\[
\alpha_{s,eff}(t) = \frac{E_s}{E_{c,eff}(t)}
\]

\[
I_{II}(t) = \frac{b^* x^3}{3} + \alpha_{s,eff}(t) \left[ A_{s1}^* (d-x)^2 + A_{s2}^* (x-d_s)^2 \right]
\]

### 4.4 | Time-dependent mean flexural stiffness

The effective, time-dependent mean flexural stiffness \( E_{m}^{II}(t) \) is obtained using the distribution coefficient:

\[
E_{m}^{II}(t) = \frac{1}{\zeta_{II}(t) + (1-\zeta)_{II}(t)}
\]

where \( \zeta \) is a distribution coefficient that is described in OENORM EN/B 1992.

### 5 | EFFECTIVE CREEP RATIO

Cracked structures show significantly reduced creep deformations; the reduction of constrained internal forces is thus considerably smaller. In the following, the relationships are derived separately for the uncracked and cracked cross sections. The ratio of the flexural stiffness and an approximate formula will be used to calculate the effective creep ratio \( \varphi_{eff} \).

#### 5.1 | Creep in an uncracked cross section

The second moment of area of an unreinforced cross section \( I^I \) is not affected by creep. The following relationship applies:

\[
I^I(t_0) = I^I(t) = I^I
\]

#### 5.1.1 | Ratio of flexural stiffness (PT1)

To determine the effective creep ratio, the flexural stiffness of the uncracked cross section (post-tensioned beam PT) at the start of stressing \( t_0 \) and at the time \( t \) are compared. Via the effective modulus of elasticity \( E_{c,eff} \) according to Equation (15) results:

\[
\varphi_{eff}^{I}(t,t_0) = \frac{E_c \cdot I^I(t_0)}{E_{c,eff}(t) \cdot I^I(t)} - 1
\]

#### 5.1.2 | Approximate formula (PT2)

For a rough estimation of the creep behavior, an approximate formula according to Zilch and Fritsche can be used. The approximation formulas (PT2 and RC2), which neglect the degree of reinforcement, were derived from the results of numerical analyses taking into account crack formation and the development of internal forces over time. The following relationship is given:
\[ \frac{E_{II}(t)}{E_{II}(t_0)} \approx \frac{1}{1 + \phi} \]  
(21)

(21) inserted in (20) results in:

\[ \phi_{eff}^{II}(t, t_0) = \frac{1 + \phi(t, t_0)}{1} - 1 = \phi(t, t_0) \]  
(22)

The approximate formula for the uncracked cross section does not provide any change to the creep ratio according to the Equation (14).

5.2 | Creep in a cracked cross section

In cracked cross sections, only the compression zone creeps. Since the distribution of the strains over the cross section must remain plain, the zero strain line moves closer to the reinforcement. Due to the increased compression zone, the concrete stress drops, while the steel stress increases slightly to compensate for the slightly reduced lever arm for reasons of equilibrium. The effects of creep in cracked cross sections can be approximated by \( E_{c,eff} \). Stresses and cross section values can still be determined with \( \alpha_{s,eff} = E_s/E_{c,eff} \). The flexural stiffness of the cracked cross section (pure state II) decreases significantly less by creep compared to state I.

5.2.1 | Ratio of flexural stiffness (RC1)

The second moment of area for the reinforced cross section (RC) \( I_{m}^{II} \) is determined according to Equation (17). In addition, the mean flexural stiffness \( E_{m}^{II} \), see Equation (18), must also be taken into account. For the Young’s modulus of elasticity \( E \) the tangent modulus \( E_c \) is used at \( t_0 \) and the effective modulus of elasticity at \( t \). This results in the following equation for \( \phi_{eff}^{II} \):

\[ \phi_{eff}^{II}(t, t_0) = \frac{E_c \cdot I_{m}^{II}(t_0)}{E_{c,eff}(t) \cdot I_{m}^{II}(t)} - 1 \]  
(23)

5.2.2 | Approximate formula (RC2)

For the cracked cross section, there is also an approximate formula that can be used to calculate the effective creep ratio.\(^7\) The reinforcement in the cross section is neglected. The following relationship is given:

\[ \frac{E_{II}(t)}{E_{II}(t_0)} \approx \frac{2 + \phi}{2 \cdot (1 + \phi)} \]  
(24)

(24) inserted in (23) results in:

\[ \phi_{eff}^{II}(t, t_0) = \frac{2 \cdot (1 + \phi)}{2 + \phi} - 1 = \frac{\phi(t, t_0) \cdot (2 + \phi)}{2 + \phi(t, t_0)} \]  
(25)

6 | EXPERIMENTS

6.1 | Production of test bodies

Two test bodies with the dimensions \( l/w/h = 7.3 \text{ m}/0.4 \text{ m}/0.2 \text{ m} \) were produced. The longitudinal reinforcement of both test specimens consisted of \( 4\Omega 14 \) reinforcing steel BSt 550B which were arranged on the top and bottom of the cross section, see Figure 2. For one test body, a non-bonded monosstrand tendon (\( A_p = 1.5 \text{ cm}^2 \)) was placed centrically. After removing the effects of wedge slip and friction, the remaining post-tensioning force was \( F_p = 200 \text{ kN} \).

6.2 | Material properties

The concrete strength was \( f_{cm,cyl,28d} = 43 \text{ MPa} \). The concrete mix is given in Table 1. The development of the modulus of elasticity over time is shown in Table 2.

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**Figure 2** Cross sections and reinforcement arrangement
6.3 | Test procedure

The age of the concrete was $t_0 = 50$ days at the beginning of the test. The reinforced test body was preloaded with two forces of $F = 45$ kN at a distance of 1.4 m from the middle support to create cracks. The crack pattern can be seen in Figure 3. The maximum load achieved in an ultimate limit test with the same configuration was $F = 105$ kN.$^{21}$ Thus, the load level at preloading was $F/F_{\text{max}} = 0.42$. The crack pattern was thus generated from a stress at service load level.

The long-term test consisted of a two-span beam in which the middle support was lifted for the imposed deformation, see Figure 4. The superelevation and the resulting constraint forces determined according to linear elasticity theory and analyzed experimentally are shown in Table 3. The constraint moment for the uncracked test body, determined according to linear elasticity theory, agrees well with the test results. For the cracked test body, the ratio of the constraint moment calculated according to linear elasticity theory and experiment is 3.7, which is due to the reduction in stiffness caused by crack formation.

6.4 | Measuring methods

Load cells were installed at the supports to document the support reactions and their development. In addition, strain measurements were carried out at the middle bearing and at a distance of 2 m from the middle bearing. Potentiometric displacement transducers with a measuring length of $l_0 = 260$ mm for the tension side and strain gauges with a measuring grid length of 50 mm for the compression side were used. To determine compensation values for the effects of temperature, shrinkage, strain gauges were also applied to an unloaded specimen. After installation of the measuring equipment, the test bodies were covered with insulating boards to keep temperature changes small, see Figure 5. The temperature differences were $\pm 1.5^\circ\text{C}$ over the test period and the humidity was 40% on average with fluctuations of $\pm 10\%$.

7 | TEST RESULTS

7.1 | Comparison of the test series

Figure 6 compares the measured results of the constraint moment reduction for the reinforced and the post-tensioned test specimen. It can be seen that the reduction of constraint moment is higher in the post-tensioned test body than in the reinforced one. This result agrees well with the theory. By subtracting the two test series, it can be determined that

| TABLE 1  | Concrete mix
| --- | --- |
| Aggregate | 
| 0/4 | 1,230 kg (60%) |
| 4/8 | 206 kg (10%) |
| 8/16 | 617 kg (30%) |
| Cement | CEM II/A-M(S-L) | 260 kg |
| Effective water content | W/B = 0.68 | 177 kg |
| Superplasticizer | dynamiQ flow L-08 | 1.56 kg |

| TABLE 2  | Young’s modulus development
| --- | --- |
| Time (d) | Young’s modulus (MPa) |
| 28 | 34,200 |
| 107 | 36,400 |
| 500 | 39,600 |

FIGURE 3 Crack pattern—reinforced test body

FIGURE 4 Structural system
7.2 Comparison of the test results with the calculation models

The measured results of the reduction of constraint moment are compared to the presented theoretical models using the effective creep ratios, see Section 5. The time-dependent moment at the support (hogging) due to imposed deformation is determined analytically with the (effective) creep ratios, see Table 4.

To interpret the accuracy of the calculation model for the constraint moment reduction, the value $\mu_{CT,i}(t)$ is introduced, where the calculated constraint moment is compared to the constraint moment from the test. This value is calculated as follows:

$$\mu_{CT,i}(t) = \frac{\Delta M_{CT,calc}(t)}{\Delta M_{CT,test}(t)}$$

7.2.1 Post-tensioned test body (PT)

Figure 7 shows the results of the calculation models for the post-tensioned test specimen. It can be seen that the calculation of the constraint moment reduction with the creep ratio $\varphi$ overestimates the experimental result by $\mu_{CT,\varphi,PT1}(t = 500 \text{ days}) = 140\%$. A better result is obtained by a calculation with the effective creep ratio $\varphi_{eff}^1$ taking into account the time-dependent bending stiffness with $\mu_{CT,PT1}(t = 500 \text{ days}) = 116\%$. Since the second moment of area $I$ of an unreinforced cross section is not affected by creep, the effective creep ratio depends only on the time-dependent material properties of the concrete. These are the creep ratio and the modulus of elasticity, where deviations can be seen.

7.2.2 Reinforced test body (RC)

For the reinforced test body the analysis with the creep ratio $\varphi$ overestimates the experimental result by $\mu_{CT,\varphi,RC1}(t = 500 \text{ days}) = 165\%$, see Figure 8. The calculation with the effective creep ratio $\varphi_{eff}^2$ underestimates the degradation ($\mu_{CT,RC1}(t = 500 \text{ days}) = 42\%$ and $\mu_{CT,RC2}(t = 500 \text{ days}) = 77\%$) whereas the model RC2 (approximate model) comes closest to the test results. The parameters for the effective creep ratio are the second moment of area and the material properties, where deviations can be seen. According to Berger et al., it was experimentally found that the stiffness at low load levels is underestimated by analytical calculations. Furthermore, it has already been determined for the post-tensioned test body, see Section 7.2.1, that the theoretical material properties do not correspond exactly to the experimental values. These two points can be used as arguments for the deviation of the theoretical models from the experimental results.

7.2.3 Comparison

Table 5 provides a summary of the constraint moment reduction and shows the deviation of the calculation models from the experiments.
In addition to the analysis of the constraint moment development, which was carried out by means of measurements of the support reactions by load cells, an attempt was made to obtain information on the constraint moment development by means of strain measurements. Since these were large-scale experiments, this was a challenging task, because the following factors had an influence on the strain development:

**TABLE 4**  Used creep ratios for the calculation models

| Creep ratio | Post-tensioned test body (PT) | Reinforced test body (RC) |
|-------------|-----------------------------|---------------------------|
| Creep ratio | \( \varphi(t, t_0) \)       | \( \varphi(t, t_0) \)    |
| Effective creep ratio | Ratio of flexural stiffness | \( \varphi_{\text{eff}}^{\text{PT1}} \) | \( \varphi_{\text{eff}}^{\text{RC1}} \) |
| Approximate formula | \( \varphi_{\text{eff}}^{\text{PT2}} \) | \( \varphi_{\text{eff}}^{\text{RC2}} \) |

**FIGURE 6**  Comparison of the constraint moment development of the test series

**FIGURE 7**  Constraint moment development (PT): experimental—analytical

**8 | STRAIN MEASUREMENTS**

In addition to the analysis of the constraint moment development, which was carried out by means of measurements of the support reactions by load cells, an
Temperature
Shrinkage
Creep (dead load, post-tensioning, constraint moment reduction)

Due to the compensation measurements, the influences of temperature, shrinkage, and creep (post-tensioning) could be determined and by subtraction only the strain caused by creep due to deadweight and constraint moment reduction could be obtained.

The strain resulting from temperature and shrinkage is shown in Figure 9. The measured temperature and shrinkage strain are in the range of the elongation strain measurements resulting from dead weight and constraint moment reduction. Since there is no obstruction of deformation in the horizontal direction, no constraint forces from temperature

\[ \varphi(\tau, t_0) \]

\[ \varphi_{eff}(RC1) \]

\[ \varphi_{eff}(RC2) \]

\[ \mu_{CT,i}(t) \]

\[ \text{TABLE 5} \]

Comparison of the accuracy of the calculation models with the experiments—\( \mu_{CT,i}(t) \)

|                         | Post-tensioned test body (PT) % | Reinforced test body (RC) % |
|-------------------------|-------------------------------|-----------------------------|
| Experiment              | 100                           | 100                         |
| Creep ratio             | 140                           | 165                         |
| Effective creep ratio   | Ratio of flexural stiffness (1) 116 | 42                          |
|                         | Approximate formula (2)       | 140                         |
|                         |                               | 77                          |

\[ \text{FIGURE 8} \]
Constraint moment development (RC): experimental—analytical

\[ \text{FIGURE 9} \]
Strain development on the compression and tension side at the location of the middle support

- Temperature
- Shrinkage
- Creep (dead load, post-tensioning, constraint moment reduction)
or shrinkage are to be expected. Due to the thermal housing, one-sided heating of the body is prevented. For this reason, no curvature of the test specimen can occur due to one-sided temperature stress. Shrinkage strain is also very small, as the loading started after 50 days.

Figure 9 shows the strain on the compression and tension side of the cross section at the location of the central support for the post-tensioned (PT) and reinforced (RC) test specimen. It can be seen that on the tension side there is a significant increase of the elongation over time. For the reinforced test body, the increase is greater than for the post-tensioned. The development of the compressions is significantly smaller. These experimental results contradict the current model. For an uncracked cross section, the elongation and compression strain should be approximately equal. For a cracked cross section, creep should occur predominantly in the compression zone and only slightly in the tension zone. Basically, it can be said that the results of the strain measurements should be treated with caution and that they do not provide essential information for the reduction of constraint moments.

9 | SUMMARY

Experimental investigations on the reduction of constraint forces caused by support displacement by creep were carried out and the results were compared with theoretical calculation models. In the analytical calculation models, not only the creep ratio but also the effective creep ratios were considered. These are dependent on the cross sectional and time-dependent material characteristics. The stiffness of the uncracked and cracked cross section was used as a parameter for cross section characteristics. This means that one test body was post-tensioned and the other made of reinforced concrete. The dimensions of the test bodies were l/w/h = 7.3 m/0.4 m/0.2 m. The static system was a two-span beam and the constraint force was generated by lifting the middle bearing. The experiments lasted over a period of 500 days. The analysis of the results has shown that the post-tensioned test body has a greater creep ability than the reinforced concrete test body. This was shown by the greater constraint moment reduction. When comparing the experimental results with the calculation models, it was found that the analytical determination of the time-dependent constraint moment development using the creep ratio $\phi(t_0)$ greatly overestimates the constraint force reduction. Better agreement was achieved in calculations using the effective creep ratio $\phi^{\text{eff}}(t_0)$. Deviations were nevertheless present, whereby the reduction is overestimated by up to 16% or underestimated by up to 60% in the exact calculation model. Parameters that lead to deviations between the test and the calculation model are the bending stiffness and the time-dependent material behavior.

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NOTATION

- $X_1$: statically indeterminate value
- $\Delta X_1$: the time-dependent change of $X_1$ caused by creep
- $\delta_{10}$: deformation value due to load
- $\delta_{11}$: deformation value due to the unit stress state
- $\Delta S$: differential settlement
- $t_0$: time of loading
- $t$: time being considered
- $\varphi$: creep coefficient
- $\rho$: relaxation coefficient
- $\psi$: relaxation value that describes the creep induced stress drop at constant strain
- $\varphi^{\text{eff}}$: effective creep coefficient
- $\varphi^{\text{0 eff}}$: effective creep coefficient for the uncracked cross section
- $\varphi^{\text{1 eff}}$: effective creep coefficient for the cracked cross section
- $M_{\Delta s}$: moment due to differential settlement
- $\varepsilon_{c,28}$: short term elastic strain in the concrete
- $\varepsilon_{cc}$: creep strain
- $\beta(t_0)$: function of the time course
- $E$: modulus of elasticity
- $l$: second moment of area of concrete section
- $E_c$: modulus of elasticity of concrete
- $E_{c,\text{eff}}$: effective modulus of elasticity of concrete
- $E_s$: modulus of elasticity of reinforcing steel
- $E_{\text{II}}$: bending stiffness of the uncracked cross section
- $E_{\text{II}}$: bending stiffness of the cracked cross section with mean behavior
- $\alpha_{s,\text{eff}}$: effective ratio of the moduli of elasticity of reinforcing steel and concrete
- $d$: effective depth of a cross section
- $b$: overall width of a cross section
- $x$: neutral axis depth
- $l$: length/span
- $w$: width of the cross section
- $h$: height
- $A_{\text{SL}}$: reinforcement in the bending tensile zone
- $A_{\text{S2}}$: reinforcement in the compression zone
- $\zeta$: distribution coefficient
- $f_{\text{cm},cyl,28d}$: mean value of concrete cylinder compressive strength at 28 days
- $F$: action
- RC: reinforced concrete
- PT: post-tensioned concrete
**DATA AVAILABILITY STATEMENT**
Data available on request from the authors

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