Reply to Comment on “Null weak values and the past of a quantum particle” by D. Sokolovski

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Abstract

We discuss the preceding Comment and conclude that the arguments given there against the relevance of null weak values as representing the absence of a system property are not compelling. We give an example in which the transition matrix elements that make the projector weak values vanish are the same ones that suppress detector clicks in strong measurements. Whether weak values are taken to account for the past of a quantum system or not depend on general interpretational commitments of the quantum formalism itself rather than on peculiarities of the weak measurements framework.
I. INTRODUCTION

The meaning of weak values has been debated since their inception [1]. Although the weak measurements framework is entirely derived from standard quantum mechanical quantities, the axiomatics of quantum theory do not prescribe any meaning to the weak values. The very basic question is whether weak values can be taken as a generalized form of eigenvalues, and hence refer to values taken by properties of the system on par with the values obtained from projective measurements. A recent focal point of this debate has been the implications when assessing the past evolution of a quantum particle from weak values. Vaidman noted [2] that by inserting a small Mach-Zehnder interferometer (MZI) along the arm of a larger MZI, the particle’s presence as inferred from the weak values was detected inside the nested MZI, but not along the entrance or exit arms, where the relevant weak values vanished. Discontinuous trajectories were also noted in a 3 paths interferometer [3], though seen to depend on the observable chosen to weakly probe the particle. The paradoxes appearing when employing the ”weak trace criterion” – Vaidman’s suggestion of inferring [2] the presence of a quantum particle from the traces left on weakly coupled meters – led several authors (see Refs [12-23] of [4] and [6–8] for more recent work) to entirely question the weak values approach.

Against this backdrop we proposed in our paper [4], henceforth P, an analysis of null weak values. We argued that it is consistent to maintain that vanishing weak values are indicative of the absence of the corresponding system property in a pre and post-selected system provided (i) the property is understood as relevant to a transition to the postselected state and (ii) the quantum system is thought of an extended undulatory entity, not as a point-like particle. A null weak value of an observable $A$ thus indicates the local value taken by the property represented by $A$ as the system evolves and is finally detected in the postselected state.

In the preceding Comment [5], Sokolovski asserts that even understood in the liberal sense advocated in P, weak values do not represent physical properties: they are related to probability amplitudes that are just computational tools. In the Comment (henceforth C), Sokolovski further attempts to explain how computational quantities can nevertheless be determined experimentally from the response of a weak pointer – this was indeed one of the main arguments we had given in P in favor of the physical relevance of vanishing weak values.
In the present Reply, we first examine the novel arguments put forward in the Comment (Sec. II). We then give an example comparing vanishing transition amplitudes as measured in a weak measurement scheme and as inferred from projective measurements (Sec. III). On the basis of this example, we reexamine in Sec. IV the position outlined in C, and compare it to the weak trace criterion as well as to our interpretation put forward in P. Our conclusions given in P remain unchanged.

II. WEAK VALUES AND TRANSITION AMPLITUDES: PHYSICAL PROPERTY OR COMPUTATIONAL TOOL?

In P (Sec. IV.C.2) we had criticized Sokolovski’s position [9] according to which an experimentally measurable quantity is no more than a computational tool, on the ground that asserting that an experimentally measurable quantity was a computational tool was a peculiar position. The main merit of the Comment, in our view, is to clarify this point.

In C, it is reaffirmed that only strong projective measurements lead to real paths and that transition amplitudes concern virtual paths and hence as such they belong to the realm of computational quantities, indicating how certain terms add up or cancel. Although Sokolovski recognizes that the achievement of the weak measurement framework is to have discovered a scheme measuring the response of a weakly perturbed system, weak values are no more, in his view, than a consequence of perturbation theory. The conclusion is that if no probabilities are produced, attributing any reality to the transition amplitudes will lead to unwanted and unnecessary paradoxes.

We agree on one point: attributing the property of a weak value to a property of a localized particle will indeed lead to paradoxes. The particle aspect is intimately linked to projective measurements that produce probabilities. What we proposed in P is that the transition amplitudes could be regarded as local properties of a pre-post selected system understood as a sort of undulatory entity extended all over space. We did not assert that this was the only consistent approach to account for these experimentally measurable transition amplitudes, and we had further specified that the choice of the approach depended on whether one endorsed the assumption linking properties of a quantum system to projective measurements.

From this point of view, we disagree with the statement made in C asserting that the
approach relying on strong measurements would be the only consistent way to understand vanishing weak values. We note in particular that Sokolovski has not shown in C that our proposal was inconsistent. We now turn to a simple example aimed at emphasizing the link between vanishing transition amplitudes in strong and weak measurements.

III. TRANSITION AMPLITUDES WITH STRONG MEASUREMENTS: AN EXAMPLE

Let us go back to the 3 path interferometer presented in Fig. 1 of P (identical to Fig. 1c of C). The weak values along the path are given by

\[ t = t_1 : \Pi^w_E = 1 \quad \Pi^w_F = -1 \quad (1) \]
\[ t = t_2 : \Pi^w_D = 1 \quad \Pi^w_O = 0 \quad (2) \]
\[ t = t_3 : \Pi^w_{E'} = 1 \quad \Pi^w_{F'} = -1 \quad (3) \]
\[ t = t_4 : \Pi^w_O' = 0 \quad (4) \]

(see Sec. III.C of P, and [3] for computational details). The transition amplitudes at \( O \) and \( O' \) vanish, \( \langle \chi_f(t_2) | \Pi_O' | \psi(t_2) \rangle = \langle \chi_f(t_4) | \Pi_O' | \psi(t_4) \rangle = 0 \), explaining why the weakly coupled pointers at \( O \) and \( O' \) do not pick up a shift.

Let us now replace the weakly coupled pointers by pointers having a strong coupling. The initial state of the pointer located at \( X \), \( |\psi_X(0)\rangle \), shifts to \( |\psi_X(s)\rangle \) after the interaction, the strong interaction imposing \( \langle \phi_X(s) | \phi_X(0) \rangle = 0 \). As in P, \( |\psi\rangle \) and \( |\Psi\rangle \) designate the system and total state vector. Let us start by placing a single strong pointer at any of the positions \( X = E, F, D \ldots \) shown in Fig. 1. The initial state

\[ |\Psi(t_i)\rangle = |\psi(t_i)\rangle |\phi_X(0)\rangle \quad (5) \]

evolves right after the interaction time \( t_X \) [corresponding to the relevant interaction time as per Eqs. (1)-(4)] to

\[ |\Psi(t)\rangle = |\psi_X(t)\rangle |\phi_X(s)\rangle + |\psi_X(t)\rangle |\phi_X(0)\rangle \quad (6) \]

where we use the notation \( |\psi(t)\rangle \equiv \sum_X |\psi_X(t)\rangle \) as a shorthand equivalent to Eq. (15) of P (with \( |\psi_X(t)\rangle = d_X(\alpha) |\alpha_X = X\rangle |\xi_X(t)\rangle \)). \( |\psi_X(t)\rangle \) is the fraction of the state that evolves without passing through \( X \) and thus does not interact with the pointer. Eq. (6) is rewritten
FIG. 1: The 3 path interferometer for spin-1 particles discussed in P is shown here with strongly coupled pointers placed at D and O. The pointer at D always detects the particle, while the pointer at O remains in its ready state. Strong measurements coincide with the weak values because the transition amplitudes accounting for the strongly coupled pointers are the same ones that enter the definition of the weak values.

introducing the projectors $\Pi_X$, $\bar{\Pi}_X$ as

$$|\Psi(t)\rangle = \Pi_X |\psi(t)\rangle |\varphi_X(s)\rangle + \Pi_{\bar{X}} |\psi(t)\rangle |\varphi_X(0)\rangle$$

(7)

The system then evolves unitarily ($U$ is the evolution operator) until $t = t_f$ at which point the system is measured and found, say, in state $|\chi_f\rangle$. With $Q_X$ labeling the position of the pointer, we have at $t = t_f$

$$\langle Q_X | \langle \chi_f | \Psi(t_f) \rangle = \langle \chi_f | U(t_f,t_X) \Pi_X |\psi(t_X)\rangle |\varphi_X(Q_X,s)\rangle + \langle \chi_f | U(t_f,t_X) \Pi_{\bar{X}} |\psi(t_X)\rangle |\varphi_X(Q_X,0)\rangle.$$  

(8)

We see that if a single strongly coupled pointer is inserted at $X$, it will undergo a shift and detect the particle with certainty provided the transition amplitude $\langle \chi_f | U(t_f,t_X) \Pi_X |\psi(t_f)\rangle$ vanishes (assuming $\langle \chi_f | U(t_f,t_X) \Pi_X |\psi(t_X)\rangle \neq 0$). The transition amplitudes appearing in Eq. (8) are the same that appear in the expression of the weak values, Eqs. (1)-(4). Hence a null weak value also implies that a single strongly coupled pointer will never detect the particle: the property ascription (that is, giving a value to the observable that is being measured) can be straightforwardly extended from the strong to the weak coupling case.

As an example, assume we place strong pointers at D and O (see Fig. 1). Then the initial state

$$|\Psi(t_i)\rangle = |\psi(t_i)\rangle |\varphi_D(0)\rangle |\varphi_O(0)\rangle$$

(9)

evolves right after the interaction time $t_2$ to

$$|\Psi(t)\rangle = |\psi_D(t)\rangle |\varphi_D(s)\rangle |\varphi_O(0)\rangle + |\psi_O(t)\rangle |\varphi_D(0)\rangle |\varphi_O(s)\rangle,$$  

(10)
that can be rewritten as

$$|\Psi(t)\rangle = \Pi_D |\psi(t)\rangle |\varphi_D(s)\rangle |\varphi_O(0)\rangle + \Pi_O |\psi(t)\rangle |\varphi_D(0)\rangle |\varphi_O(s)\rangle.$$  

(11)

The postselected state $|\chi_f\rangle$ is chosen so that $\langle\chi_f(t_f)|U(t_f,t_4)|\psi_{O'}(t_4)\rangle = 0$ also implying

$$\langle\chi_f(t_f)|U(t_f,t_2)\Pi_O |\psi(t_2)\rangle = 0.$$  

(12)

When $|\Psi(t)\rangle$ evolves up to $t_f$ and the system is measured and found in $|\chi_f\rangle$, the final state becomes

$$\langle Q_D, Q_O | \langle \chi_f | \Psi(t_f) \rangle = \langle \chi_f | U(t_f,t_2)\Pi_D |\psi(t_2)\rangle \varphi_D(Q_D, s)\varphi_O(Q_O, 0)$$

$$+ \langle \chi_f | U(t_f,t_2)\Pi_O |\psi(t_2)\rangle \varphi_D(Q_D, 0)\varphi_O(Q_O, s).$$  

(13)

(14)

Since the transition amplitude $\langle \chi_f | U(t_f,t_2)\Pi_O |\psi(t_f)\rangle$ vanishes, the path followed by the system is revealed unambiguously by the strongly coupled pointers. The pointer at $D$ has shifted, while the pointer at $O$ has remained in the initial state. The system, when found in state $|\chi_f\rangle$, has always gone through $D$ and will never be found at $O$. Note that an additional pointer with a dynamical variable $Q_{O'}$ can be added at $O'$ without changing the narrative: since the transition amplitude $\langle \chi_f | U(t_f,t_2)\Pi_O' |\psi(t_f)\rangle$ vanishes the extra term appearing in $\langle Q_D, Q_O, Q_{O'} | \langle \chi_f | \Psi(t_f) \rangle$ relative to Eq. (14) vanishes and only the single term $\varphi_D(Q_D, s)\varphi_O(Q_O, 0)\varphi_O(Q_{O'}, 0)$ remains. We conclude that due to the fact that the transition amplitudes are the same in the strong coupling case and in the weak coupling case examined in P, although the physics is different, extending the property ascription from the strong to the weak coupling case is straightforward.

Let us now place strongly coupled pointers at $D$, $O$, $E'$ and $F'$ (see Fig. 2). Shortly after $t = t_2$ the quantum state is again given by Eq. (10), factored by the ready states $|\varphi_{E'}(0)\rangle |\varphi_{F'}(0)\rangle$. However after $t_3$ we have

$$|\Psi(t)\rangle = |\psi_D(t)\rangle |\varphi_D(s)\rangle |\varphi_O(0)\rangle |\varphi_{E'}(0)\rangle |\varphi_{F'}(0)\rangle$$

$$+ \langle |\psi_{E'}(t)\rangle |\varphi_{E'}(s)\rangle |\varphi_{F'}(0)\rangle + |\psi_{F'}(t)\rangle |\varphi_{E'}(0)\rangle |\varphi_{F'}(s)\rangle) |\varphi_D(0)\rangle |\varphi_O(s)\rangle.$$  

(15)

After postselection and measurement of the 4 pointers’ positions, we see that either only $D$ clicks, or alternatively $O$ and $E'$, or $O$ and $F'$ click. The transition amplitude (12) that ensured the $O$ pointer did not move does not appear anymore. Indeed, the loss of coherence due to the pointers at $E'$ and $F'$ being in different states now correlate a particle
FIG. 2: Same interferometer displayed in Fig. 1 but shown here with strongly coupled pointers placed at $D$, $O$, $E'$, and $F'$. In the case pictured in the Figure, only the pointers at $O$, and $E'$ click (see the main text for the other possibilities). The strong measurements at $E'$ and $F'$ disturb the quantum state and the transition amplitudes are now different than in the weak case; in particular the vanishing transition amplitude yielding $\Pi^w_O = 0$ does not appear when strong measurements disturb the quantum state.

detected either at $E'$ or at $F'$ with a particle detected at $O$. We therefore conclude that that a negative strong measurement coincides with a vanishing weak value (here, $\Pi^w_O = 0$) if the quantum state is not disturbed by the strong measurements. Otherwise the transition amplitude [here, Eq. (12)] that accounts for vanishing weak values or the lack of clicks in strong measurements that do not disturb the overall quantum state does not even appear.

IV. DISCUSSION AND CONCLUSION

The example given above indicates that strong and weak measurements can be set on the same footing regarding the absence of a property provided the strong measurements do not disturb the quantum state. The transition amplitudes that account for the strong pointers motion are the same quantities that enter the definition of the weak values $\Pi^- \Pi^+$. For example in Eq. (14), the $O$ pointer does not detect the particle because $\langle \chi_f(t_f)|U(t_f,t_2)\Pi_O|\psi(t_2)\rangle = 0$, and from there follows the conclusion that the particle is only detected along the upper path of Fig. 1 (through $D$). For the same reason the weak value $\Pi^w_O$ vanishes and given the same configuration (one weakly coupled pointer at $D$, another at $O$), reaching the same conclusion as in the strong measurement case doesn’t seem to be a problem (although strictly speaking no real pathways are generated). Setting additional weakly coupled pointers along the lower path $E/F \rightarrow O \rightarrow E'/F' \rightarrow O'$ does not change the result or the meaning of $\Pi^w_O$ but the system spatial presence, as captured by the weakly coupled pointers at $E,F,E'F'$, appears as discontinuous. Understood in this
manner, extending the concept of property ascription from the outcomes of strong measurements to weak values is consistent, provided one keeps in mind that real pathways endowed with a given probability are not created, and therefore we are not dealing with properties of a point-like particle. This caveat suffices to avoid dealing with paradoxes.

Extending property ascription based on weak values and transition amplitudes disagrees with Sokolovski’s position interpreting a vanishing transition amplitude as a simple numerical cancellation without any deeper meaning (Secs. V and VIII of C). Sokolovski further explains that the weakly coupled pointers motion should be understood as the pointers’ reaction to a perturbation, but this does not entail that the pointer reflects the value of the property possessed by the system. As we have discussed in P, keeping the standard quantum mechanical assumption requiring strong measurements and real pathways in order to ascribe properties to a quantum system is consistent. But then, by this assumption, the possibility to ascribe a value to a system observable as the system evolves from a preselected to a postselected state is discarded. We do not see any compelling argument to discard this possibility. Depending on how the quantum formalism is interpreted (in particular the interplay between the state vector, the measurement problem and wave-particle duality) one may feel more comfortable with the standard approach to quantum properties. On the other hand the fact that a weakly coupled pointer reacts to the interaction with the system by picking up a universal quantity related to the transition matrix element to the final system state, rather than by some arbitrary perturbation depending on the specifics of the pointer, paves the way for a consistent extension of quantum properties to pre-post selected systems.

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