Parton Energy Loss with Detailed Balance

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Stimulated gluon emission and thermal absorption in addition to induced radiation are considered for an energetic parton propagating inside a quark-gluon plasma. In the presence of thermal gluons, stimulated emission reduces while absorption increases the parton’s energy. The net effect is a reduction of the parton energy loss. Though decreasing asymptotically as $T/E$ with the parton energy, the relative reduction is found to be important for intermediate energies. The modified energy dependence of the energy loss will affect the shape of suppression of moderately high $p_T$ hadrons due to jet quenching in high-energy heavy-ion collisions.

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Introduction. — Gluon radiation induced by multiple scattering for an energetic parton propagating in a dense medium leads to medium induced parton energy loss or jet quenching. Such a phenomenon, if occurring in high-energy heavy-ion collisions, will suppress large transverse jet quenching. Such a phenomenon, if occurring in high-medium leads to medium induced parton energy loss or studies of high proportional to the gluon density [2], the experimental same energy [1]. Since medium-induced energy loss is reduction decreases with the parton energy loss induced by multiple scattering. The relative mal absorption results in a net reduction of parton en-
tial cancellation between stimulated emission and ther-
$\omega_{\text{c}} \sim T/E$}

Final-state absorption. — Jet production in a hard scattering is always accompanied by final-state radiation. In an axial gauge and in the leading-log approximation, the radiation amplitude off a quark can be factorized from the hard scattering and has the form,

$$R^{(0)} = 2\bar{q}g T c \frac{k_\perp \cdot \epsilon}{k_\perp} ,$$

in the limit of $z = |\omega|/E \to 0$, where $k = (\omega, \mathbf{k})$ is the four-momentum of the radiated gluon with polarization $\epsilon = [0, (\mathbf{k}_\perp \cdot \epsilon_\perp)/k^+, \epsilon_\perp]$, $T_c$ the color matrix and $\alpha_s = g^2/4\pi$ strong coupling constant. We assume that the hot medium is in thermal equilibrium shortly after the production of the hard parton. Therefore, taking into account of both stimulated emission and thermal absorption in a thermal medium with finite temperature $T$, one has the probability of gluon radiation with energy $\omega$,

$$\frac{dP^{(0)}}{d\omega} = \frac{\alpha_s C_F}{2\pi} \int \frac{dz}{z} \int \frac{dk_\perp^2}{k_\perp^2} \left[ N_g(zE) \delta(\omega + zE) \right] P(\frac{\omega}{E}),$$

where, $N_g(|\mathbf{k}|) = 1/\exp(|\mathbf{k}|/T) - 1$ is the thermal gluon distribution and $C_F$ is the Casimir of the quark jet in the fundamental representation. We have also included the splitting function $P_{gg}(z) \equiv P(z)/z = [1 + (1 - z)^2]/z$ for $q \to gg$. The first term is from thermal absorption and the second term from gluon emission with the Bose-Einstein enhancement factor. For $E \gg T$, one can neglect the quantum statistical effect for the leading parton. Note that the vacuum part has a logarithmic infrared divergency while the finite-temperature part has a linear divergency, since $N_g(|\mathbf{k}|) \sim T/|\mathbf{k}|$ as $|\mathbf{k}| \to 0$. These infrared divergences will be canceled by the virtual corrections which also contain a zero-temperature and a finite-temperature part \cite{5}. In addition, the virtual corrections are also essential to ensure unitarity and momen-
tum conservation in the QCD evolution of the fragmentation functions. However, they do not contribute to the effective parton energy loss. The remaining collinear divergency in the above spectrum can be absorbed into a renormalized fragmentation function that follows the QCD evolution equations.

To define the effective parton energy loss, we only consider gluon radiation outside a cone with $|k_\perp| > \mu$. Here, $\mu$ should be a nonperturbative scale for parton fragmentation in the vacuum. In a hot QCD medium, it can be replaced by the gluon’s Debye screening mass. Assuming the scale of the hard scattering as $Q^2 = 4E^2$, one has the kinematic limits of the gluon’s transverse momentum,

$$\mu^2 \leq k_{\perp \max}^2 \leq 4|\omega|(E - \omega). \quad (3)$$

Subtracting the gluon radiation spectrum in the vacuum, one then obtains the energy loss due to final-state absorption and stimulated emission,

$$\Delta E_{\text{abs}}^{(0)} = -\int d\omega \omega \left( \frac{dP^{(0)}}{d\omega} - \frac{dP^{(0)}}{d\omega} \bigg|_{T=0} \right) = \frac{\alpha_s C_F}{2\pi} E \int d\omega \int \frac{d^2k_\perp}{k_\perp^2} \left[ P(-z)N_g(zE) - P(z)N_g(zE)\bar{\theta}(1-z) \right]. \quad (4)$$

Even though the stimulated emission cancels part of the contribution from absorption without rescattering is still dominated by the final-state thermal absorption, resulting in a net energy gain. For asymptotically large parton energy, $E \gg T$, one can complete the above integration approximately and have,

$$\Delta E_{\text{abs}}^{(0)} \approx \frac{\pi \alpha_s C_F}{3} \frac{T^2}{E^2} \left[ \ln \frac{4ET}{\mu^2} + 2 - \gamma_E + \frac{6\zeta'(2)}{\pi^2} \right], \quad (5)$$

where, $\gamma_E \approx 0.5772$ and $\zeta'(2) \approx -0.9376$. Terms that are proportional to $\exp(-E/T)$ or beyond the order of $(T/E)^2$ are neglected. The quadratic temperature dependence of the leading contribution is a direct consequence of the partial cancellation between stimulated emission and thermal absorption, each having a leading contribution linear in $T$.

Rescattering-induced absorption. — During the propagation of the hard parton after its production, it will suffer multiple scattering with the medium. The multiple scattering in turn can also induce gluon radiation which has been the focus of recent theoretical studies of radiative energy loss. Here we will investigate the stimulated emission and thermal absorption associated with multiple scattering in a hot QCD medium.

Assuming a hard parton produced at $y_0 = (y_0, y_{0\perp})$ inside the medium with $y_0$ being the longitudinal coordinate, we model the interaction between the jet and target partons by a static color-screened Yukawa potential as in Gyulassy-Wang (GW),

$$V_n = 2\pi \delta(q^0) v(q_n)e^{-\frac{\gamma_\perp}{y_0}T_{a_n}(j)T_{a_n}(n)}, \quad (6)$$

$$v(q_n) = \frac{4\pi \alpha_s}{q_n^2 + \mu^2}. \quad (7)$$

Here $q_n$ is the momentum transfer from a target parton $n$ at $y_n = (y_n, y_{\perp n})$, $T_{a_n}(j)$ and $T_{a_n}(n)$ are the color matrices for the jet and target parton.

We will also follow the framework of opacity expansion developed by Gyulassy, Lévai and Vitev (GLV) and Wiedemann. However, we will only consider contributions to the first order in the opacity expansion. It was shown by GLV that the higher order corrections contribute little to the radiative energy loss. The opacity is defined by the mean number of collisions in the medium, $\bar{n} \equiv L/\lambda = N\sigma_{el}/A_{\perp}$. Here $N$, $L$ and $A_{\perp}$ are the number, thickness and transverse area of the targets, and $\lambda$ is the average mean-free-path for the jet.

The radiation amplitude associated with a single rescattering is

$$R^{(S)} = 2ig \left[ HT_aT_e + B_1 e^{i\omega y_{10}}[T_e, T_a] + C_1 e^{i(\omega_{10}-\omega_1) y_{10}}[T_e, T_a] \right] \cdot \epsilon_\perp, \quad (8)$$

where, $y_{10} = y_1 - y_0$.

$$\omega_0 = \frac{k_\perp^2}{2\omega}, \quad \omega_1 = \frac{(k_\perp - q_\perp)^2}{2\omega}, \quad H = \frac{k_\perp}{k_\perp^2}, \quad C_1 = \frac{k_\perp - q_\perp}{(k_\perp - q_\perp)^2}, \quad B_1 = H - C_1. \quad (9)$$

The first, third and second terms correspond to the hard final-state radiation, radiation induced by the rescattering and the interference, respectively.

To the first order in opacity, one should also include the interference between the processes of double and no rescattering. Assuming no color correlation between different targets, the double rescattering corresponds to the “contact limit” of double Born scattering with the same target. The radiation amplitude can be found as

$$R^{(D)} = 2igT_e e^{i\omega_{10} y_{10}} \left( -\frac{C_F + C_A}{2} \frac{H}{2} e^{i\omega_1 y_{10}} + \frac{C_A}{2} B_1 + \frac{C_A}{2} C_1 e^{-i\omega_1 y_{10}} \right) \cdot \epsilon_\perp. \quad (11)$$

Multiplying with the radiation amplitude with no rescattering $R^{(0)}$, the first term in $R^{(D)}$ cancels exactly the hard radiation contribution in the single scattering amplitude $R^{(S)}$.

Similarly to the case of final-state absorption, one can also include stimulated emission and thermal absorption when calculating the radiation probability at the first order in opacity,
where \( C_2 \) and \( C_A \) are Casimirs of the target parton in the fundamental and adjoint representations in \( d_R \) and \( d_A \) dimension, respectively. The factor \( 1 - \exp(\omega t) \) reflects the destructive interference arising from the non-Abelian LPM effect [10]. Averaging over the longitudinal target profile is defined as \( \langle \cdots \rangle = \int dp y(p) \cdots \). The target distribution is assumed to be an exponential form \( \rho(y) = 2 \exp(-2y/L)/L \).

Again, one should in principle include contributions from virtual corrections [11] which will cancel the infrared divergences in the real emission (absorption) processes. However, they do not contribute to the effective parton energy loss and can be neglected here. Similarly to the final-state absorption, the contribution from thermal absorption associated with rescattering is larger than that of stimulated emission, resulting in a net energy gain. However, the zero-temperature contribution corresponds to the radiation induced by rescattering which will lead to an effective energy loss by the leading parton. We denote this part as \( \Delta E^{(1)}_{rad} \), which should be the same as obtained by previous studies [8]. The remainder or temperature-dependent part of energy loss induced by rescattering at the first order in opacity is then defined as

\[
\Delta E^{(1)}_{abs} = - \int d\omega \omega \left( \frac{dP^{(1)}}{d\omega} - \frac{dP^{(1)}}{d\omega} \big|_{T=0} \right)
\]

which mainly comes from thermal absorption with partial cancellation by stimulated emission in the medium. According to Eq. (13),

\[
\Delta E^{(1)}_{rad} = - \frac{\alpha_s C_F}{\pi} \frac{L}{\lambda_g} E \int dz \int \frac{dk^2}{k^2} \int d^2 q_\perp |\vec{v}(q_\perp)|^2 \frac{k_\perp \cdot q_\perp}{(k_\perp - q_\perp)^2} P(z) \langle Re(1 - e^{i\omega_1 y_{10}}) \rangle \theta(1 - z),
\]

\[
\Delta E^{(1)}_{abs} = - \frac{\alpha_s C_F}{\pi} \frac{L}{\lambda_g} E \int dz \int \frac{dk^2}{k^2} \int d^2 q_\perp |\vec{v}(q_\perp)|^2 \frac{k_\perp \cdot q_\perp}{(k_\perp - q_\perp)^2} N_g(z_E) \left[ P(-z) \langle Re(1 - e^{i\omega_1 y_{10}}) \rangle - P(z) \langle Re(1 - e^{i\omega_1 y_{10}}) \rangle \theta(1 - z) \right],
\]

where \( \lambda_g = C_F \lambda / C_A \) is the mean-free-path of the gluon.

In the limit \( q_{\perp max} \to \infty \), the angular integral can be carried out by partial integration [8]. These contributions to the energy loss become

\[
\Delta E^{(1)}_{rad} \approx - \frac{\alpha_s C_F}{\pi} \frac{L}{\lambda_g} E \int dz P(z) h(\gamma) \theta(1 - z),
\]

\[
\Delta E^{(1)}_{abs} \approx - \frac{\alpha_s C_F}{\pi} \frac{L}{\lambda_g} E \int dz N_g(z_E) h(\gamma) \left[ P(-z) - P(z) \theta(1 - z) \right],
\]

where, \( \gamma = \mu^2 L/(4zE) \) and

\[
h(\gamma) = \begin{cases} \frac{2\pi}{\sqrt{4\gamma^2 - 1}} \arcsin(2\gamma), & \gamma < 1/2 \\ \frac{2\pi}{\sqrt{4\gamma^2 - 1}} \ln[2\gamma + \sqrt{4\gamma^2 - 1}], & \gamma > 1/2 \end{cases}
\]

One can approximate \( h(\gamma) \) with \( \pi \gamma + (11/4 - 2\pi)^2 \gamma^2 + (5/2) \gamma^3 \) for \( \gamma < 1/2 \) and \( \ln(4\gamma) + 0.1/\gamma + 0.028/\gamma^2 \) for \( \gamma > 1/2 \). In the limit of \( EL \gg 1 \) and \( E \gg \mu \), one can then get the approximate asymptotic behavior of the energy loss.

\[
\frac{\Delta E^{(1)}_{rad}}{E} \approx - \frac{\alpha_s C_F \mu^2 L^2}{4\lambda_g E} \left[ \ln \frac{2E}{\mu^2 L} - 0.048 \right],
\]

\[
\frac{\Delta E^{(1)}_{abs}}{E} \approx \frac{\pi \alpha_s C_F}{3} \frac{LT^2}{\lambda_g E^2} \left[ \ln \frac{\mu^2 L}{T} - 1 + \gamma E - \frac{6\zeta(2)}{\pi^2} \right].
\]

Our analytic approximation of the GLV zero-temperature result [7] also agrees with the improved limit by Zakharov [12]. However, our result is accurate through the order of \( 1/E \). In Eq. (22), we have assumed \( \mu^2 L/T \gg 1 \) and kept only the first two leading terms. In this limit, the average formation time for stimulated emission or thermal absorption is much smaller than the total propagation length. Therefore, the energy gain, \( \Delta E^{(1)}_{abs} \), by thermal
absorption (with partial cancellation by the stimulated emission) is linear in \( L \), as compared to the quadratic dependence in the zero-temperature case. However, the logarithmic dependence on \( \mu^2 L/T \) as compared to the factor \( \log(4 E T/\mu^2) \) in Eq. (3) for no rescattering is still a consequence of the LPM interference in medium. A quadratic \( L \)-dependence of \( \Delta E_{\text{abs}}(1) \) will arise when \( \mu^2 L/T \ll 1 \).

**Numerical results.** — To study the significance of the thermal absorption relative to the induced radiation, we evaluate Eqs. (4), (14) and (15) numerically. We assume the Debye screening mass to be \( \mu^2 = 4\pi\alpha_s T^2 \) from the perturbative QCD at finite temperature [13]. The mean-free-path for a gluon \( \lambda_g \) in the GW model is \([3]\),

\[
\lambda_g^{-1} = \langle \sigma_{gg} \rho_g \rangle + \langle \sigma_{gq} \rho_g \rangle \approx \frac{2\pi\alpha_s^2}{\mu^2} 9 \times 7\zeta(3) \frac{T^3}{\pi^2},
\]

where \( \zeta(3) \approx 1.202 \). With fixed values of \( L/\lambda_g \) and \( \alpha_s \), \( \Delta E/\mu \) should be a function of \( E/\mu \) only.

![Graph showing the ratio of effective parton energy loss with and without stimulated emission and thermal absorption as functions of \( E/\mu \) for \( L/\lambda_g = 3.5 \) and \( \alpha_s = 0.3 \). The thermal absorption reduces the effective parton energy loss by about 30-10\% for intermediate values of parton energy. This will increase the energy dependence of the effective parton energy loss in the intermediate energy region. However, for partons with very high energy the effect of the gluon absorption is small and can be neglected. Shown in the inserted box are the energy gain via gluon absorption with \( \Delta E_{\text{abs}}(1) \) and without \( \Delta E_{\text{abs}}(0) \) rescattering. The result is an energy gain via absorption which reduces the effective parton energy loss. Such a reduction is found to be important for intermediate parton energy but can be neglected at very high energies. For large distance \( \mu^2 L/T \gg 1 \), we find that the energy gain due to induced absorption with rescattering is linear in \( L \), modulo a logarithmic dependence. As in the case of QCD evolution of the fragmentation functions in the vacuum, one should resum higher order contributions of gluon absorption and stimulated emission. Such a study in QED [14] has found the resummation to be important in the final radiation spectrum. We expect the same in the QCD case with and without multiple rescattering. This might further reduce the effective parton energy loss in a hot QCD medium.

**Conclusions.** — In summary, we have considered the effect of stimulated emission and thermal absorption in the calculation of the effective energy loss for an energetic parton propagating in a quark-gluon plasma. Even with partial cancellation by stimulated emission, the net

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[1] X.-N. Wang and M. Gyulassy, Phys. Rev. Lett. 68, 1480 (1992).
[2] M. Gyulassy and X.-N. Wang, Nucl. Phys. B 420, 583 (1994); X.-N. Wang, M. Gyulassy and M. Plumer, Phys. Rev. D 51, 3436 (1995).
[3] A. Drees, [nucl-ex/0105019]. Proceedings of Quark Matter 2001, Stony Brook, New York, 15-20 Jan 2001.
[4] X.-N. Wang, [nucl-th/0105053]; P. Levai, et al., [nucl-th/0104035]. Proceedings of Quark Matter 2001, Stony Brook, New York, 15-20 Jan 2001.
[5] R. Baier, Y. L. Dokshitzer, S. Peigne and D. Schiff, Phys. Lett. B 345, 277 (1995); R. Baier, Y. L. Dokshitzer, A. H. Mueller, S. Peigne and D. Schiff, Nucl. Phys. B 484, 265 (1997).
[6] B. G. Zakharov, JETP Lett. 63, 952 (1996); JETP Lett. 65, 615 (1997).
[7] M. Gyulassy, P. Levai and I. Vitev, Phys. Rev. Lett. 85, 5535 (2000); Nucl. Phys. B 594, 371 (2001).
[8] X. Guo and X.-N. Wang, Phys. Rev. Lett. 85, 3591 (2000); [hep-ph/0102238]. Nucl. Phys. A, in print.
[9] U. A. Wiedemann, Nucl. Phys. B 588, 303 (2000); [hep-ph/0008241].
[10] L. D. Landau and I. Pomeranchuk, Dokl. Akad. Nauk Ser. Fiz. 92 (1953) 735; A. B. Migdal, Phys. Rev. 103, 1811 (1956).
[11] E. Wang and X.-N. Wang, to be published.
[12] B. G. Zakharov, JETP Lett. 73, 49 (2001).
[13] E. Braaten and R. D. Pisarski, Nucl. Phys. B 337, 569 (1990).
[14] H. A. Weldon, Phys. Rev. D 49, 1579 (1994).