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Do We Really Need Inertia?

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Abstract—The emphasis on inertia for system stability has been a long-held tradition in conventional grids. The fast and flexible controllability of inverters opens up new possibilities. This letter investigates inertia-free power systems driven by inverter-based resources. We illustrate that by replacing inertia with fast primary control, the first-swing stability region is greatly extended compared to the classic equal area criterion. The new criterion is fully decentralised and independent of network topology, and therefore is readily scalable to complex systems. This enables simplified system operation and new business models of stability services from distributed resources. The findings of the letter are rigorously proved and verified by simulation on the IEEE 68-bus system.

Index Terms—Power system stability, first swing, inertia, fast primary control, inverter based resources

I. INTRODUCTION

Rotating inertia has been the cornerstone of power system stability ever since the early days of ac electricity more than a hundred years’ ago. The reasons behind this are twofold: (i) conventional generators have large intrinsic inertias due to the mechanical property of turbines; (ii) the speed governors for turbines respond rather slowly and need inertias to serve as short-term energy buffers. When power systems are moving towards inverter-based renewable resources, none of the above two conditions holds. First, inverters do not have intrinsic inertias that are directly coupled to the grid. The energy storage components in the renewable resources (e.g. wind turbines and batteries) are interfaced to the grid indirectly so their frequency regulation services can be shaped in a more flexible way, not necessarily limited to emulated inertias. Second, inverters respond much faster than speed governors so it is no longer necessary to use inertias as energy buffers.

Despite the change of circumstances, the emphasis on inertia has been carried over to inverter-based power systems, rather as a tradition than as a necessity of technology. The requirement of virtual or synthetic inertia has been suggested in grid codes, and the technologies to provide the virtual inertia have been heavily studied in literature [1]. This letter takes an opposite perspective by investigating the possibility of an inertia-free power system. We illustrate that it is not only feasible but also beneficial to run an inverter-based power system with very low inertias or no inertias at all.

The reduction of inertia can be compensated by fast primary control as the alternative way of frequency regulation and power balancing. The replacement of inertia by fast primary control extends the first-swing stability region beyond the equal area criterion (EAC) to the boundary of unstable equilibrium point (UEP). This is because the primary control is fast enough, inertia is no longer needed. The new criterion is fully decentralised and independent upon the topology of the network. This relieves the obligation of a central operator to oversee the transient stability across the whole system and enables a new paradigm where simple stability services can be contracted and deployed to distributed resources.

The claims of the letter are presented with rigorous mathematical proof as well as systemic simulation verification. Our results agree with the current practice in industry. For example, the system operator in UK has launched a new suite of faster-acting frequency response services, including Dynamic Regulation, Dynamic Moderation, and Dynamic Containment, all of which can be categorised as fast primary responses with different droop coefficients at different frequency ranges [2]. Thus our work provides a theoretical justification for the ongoing exploration of the best form of frequency regulation of inverter-based renewable power systems.

II. INERTIA-FREE SYSTEM

There are two complementary mechanisms to regulate system frequency and maintain power balancing. The first is rotating inertia which responds to the derivative of frequency, and the second is primary control (or droop control) which responds to the deviation (error) of frequency. If primary control is fast enough, inertia is no longer needed. The first-swing stability of a low-inertia system is significantly improved compared to a high-inertia system. As illustrated in Fig. 1, if inertia is sufficiently small and primary control is sufficiently fast, the transient stability region can be pushed beyond the equal area criterion (EAC) to the boundary of unstable equilibrium point (UEP). This is because the primary control has an equivalent damping effect to dissipate the fault-induced kinetic energy almost instantaneously. An inertia-free system can be implemented via grid-forming (GFM) inverters by tuning down the virtual inertia, or by increasing the bandwidth of the droop control [1]. Thus inverters can be endowed with superior rather than inferior stability performances to synchronous generators, by exploiting their controllability.

The intuitive reasoning above can be formalised and generalised to multi-inverter systems by the theorem below.

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Theorem 1. For a power system with GFM inverters and inductive transmission lines, the region of attraction of the SEP extends to the UEP if the equivalent inertia of all inverters is sufficiently small.

Proof. The state equations for the power system is

\[ J_m \dot{\omega}_m = -D_m \omega_m + P^* - \sum_n K_{mn} \sin(\theta_m - \theta_n) \]

\[ \dot{\theta}_m = \omega_m \]

(1)

where \( \theta \) is angle, \( \omega \) is frequency deviation (with respect to the nominal frequency), \( J \) is inertia, \( D \) is the droop coefficient of the primary control, \( P^* \) is the reference power, \( K \) is the synchronisation power coefficient determined by the network topology, and the subscripts \( m, n \) denote the index of inverters.

The system of (1) can be divided into two coupled sub-systems, one with states \( \omega_m \), the other with states \( \theta_m \). The sub-system has a time constant of \( \tau_m = J_m/D_m \). With \( J_m \) sufficiently small, \( \tau_m \) is also small indicating that \( \omega_m \) responds sufficiently fast to the variation of \( \theta_m \). This results in a two-timescale system with fast-scale \( \omega_m \) and slow-scale \( \theta_m \). According to the singular perturbation theory, the fast-scale sub-system can be treated as instantaneous in the slow-scale sub-system [3]. To this end, we let \( J_m \to 0 \).

\[ D_m \omega_m = P^* - \sum_n K_{mn} \sin(\theta_m - \theta_n) \]

(2)

from which follows the reduced system of \( \theta_m \)

\[ \dot{\theta}_m = D_m^{-1} \left( P^* - \sum_n K_{mn} \sin(\theta_m - \theta_n) \right) \]

(3)

Define the Lyapunov function \( H \) for the reduced system as

\[ H = \sum_m \frac{1}{2} D_m (\theta^* - \theta_m)^2 \]

(4)

where \( \theta^* \) is the SEP of \( \theta \). The time-derivative of the Lyapunov function is

\[ \dot{H} = \sum_m (\theta_m - \theta^*) (P^* - \sum_n K_{mn} \sin(\theta_m - \theta_n)) \]

\[ = \sum_m (\theta_m - \theta^*) \sum_n K_{mn} (\sin \delta^*_{mn} - \sin \delta_{mn}) \]

\[ = \sum_{m\leq n} K_{mn} (\delta_{mn} - \delta^*_{mn}) (\sin \delta^*_{mn} - \sin \delta_{mn}) \]

(5)

where \( \delta_{mn} = \theta_m - \theta_n, \delta^*_{mn} = \theta^* - \theta_n, \) and we make use of the symmetry \( K_{mn} = K_{nm} \). It is clear that

\[ \dot{H} < 0, \forall \delta_{mn} \in (-\pi - \delta^*_{mn}, \pi - \delta^*_{mn}) \backslash \{\delta^*_{mn}\} \]

(6)

Remark 1. The inertia is sufficiently small if the \( \omega_m \) and \( \theta_m \) sub-system is separated in timescale, that is, \( \tau_m \ll \lambda^{-1} \) where \( \lambda \) is the Lyapunov characteristic exponent of the reduced system (3) around the transient trajectory (\( \lambda \) is an indicator of timescale for non-linear systems) [4]. The time constant \( \tau_m \) can be decreased by either decreasing \( J_m \) or increasing \( D_m \).

Remark 2. Theorem 1 has important implications in practice. The SEP \( \delta^*_{mn} \) is bounded within 90° due to the property of static power flow, so \( (-\pi - \delta^*_{mn}, \pi - \delta^*_{mn}) \ni (-\pi/2, \pi/2) \), which means that the transient angle difference can go beyond 90° without losing stability. Moreover, the stability region applies on each pair of angles disregarding the topology of the network. Thus an inertia-free power system has a significantly extended stability region and a decentralised stability criterion. This may mitigate the stability barrier of power systems and greatly simplify system operation. It also introduces a pathway towards new business models of contracting and deploying stability services to distributed resources.

III. Simulation Results

The IEEE 68-bus NETS-NYPS power system in Fig. 2 is tested here. All synchronous generators are replaced by GFM inverters which are controlled as virtual synchronous generators with configurable virtual inertias and droop coefficients. Matlab Simscape PowerSystem is used for electromagnetic transient (EMT) simulation of the test system. The parameters and simulation models are available online [5]. Faults are applied near bus 2 and bus 60 to trigger local and inter-area angle swing, and the EMT simulation results are displayed in Fig. 3 and Fig. 4 respectively. Excessive fault-clearing times (10 cycles for local mode and 0.5s for inter-area mode) are used to test the stability limit.

All inverters are configured to have a uniform time constant \( \tau \). Three cases (a)-(c) are tested in Fig. 3 and Fig. 4 with different combinations of inertia \( J \) and primary coefficient \( D \). The case (a) with high inertia and low droop coefficient (\( \tau = 79.6s \)) experiences loss of synchronisation for GFM2 near the fault in the local mode (Fig. 3), or grid-splitting along the failed interconnector in the inter-area mode (Fig. 4). In contrast, the system remains stable subject to the same faults if inertias are reduced and droop coefficients are increased, as illustrated in cases (b)-(c). The angles are pulled back from beyond 90° and re-synchronised after faults for both the local and inter-area swing modes. This verifies the superior performance of angle stability for the inertia-free power system. The stability is consistent for local and inter-area modes and thus scalable to complex networks.

A side effect is observed for case (b) (almost zero inertia, \( \tau = 1.33ms \)), which is the fundamental cycle ripple in the frequency during the fault transients. This ripple is induced by the transient dc currents on transmission lines after faults [6], which maps to power ripples under ac voltages. This power ripple is filtered by inertia but feeds through to frequency when inertia is low. We can mitigate the fundamental cycle frequency ripple by slightly increasing the inertia, as is illustrated in case (c). In this case, the time constant is set to \( \tau = 13.3ms \) which is still sufficiently small compared to the timescale of angle swing (on the scale of a second seen from Fig. 3 and Fig. 4), but is large enough to filter the fundamental cycle ripple. The inertia of this scale can be provided by the dc-link capacitors of inverters with no extra energy storage component needed, and thereby is called quasi-inertia-free.

IV. Conclusions

An inertia-free power system driven by inverter-based resources is both feasible and beneficial. Replacing inertia by fast primary control as the major mechanism of frequency regulation extends the first-swing stability region and decentralises the stability criterion. This may potentially lay the basis for a new paradigm of system operation and a new business model of stability services from distributed resources.
Fig. 2. Layout of the IEEE 68-bus NETS-NYPS power system. All synchronous generators are replaced by GFM inverters with configurable inertias and droop coefficients.

Fig. 3. Local mode transient responses subject to a short-circuit fault near bus 2. (a) High inertia and low droop coefficient, $\tau = 79.6s$. (b) Inertia-free, $\tau = 1.33\text{ms}$. (c) Quasi inertia-free, $\tau = 13.3\text{ms}$.

Fig. 4. Inter-area mode transient responses subject to a short-circuit fault near bus 60. (a) High inertia and low droop coefficient, $\tau = 79.6s$. (b) Inertia-free, $\tau = 1.33\text{ms}$. (c) Quasi inertia-free, $\tau = 13.3\text{ms}$.

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