NUMERICAL MODELING OF FLOW NATURAL CONVECTION IN A THERMOSIPHON: THERMAL PANACHE.

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Abstract

This present work follows a series of experimental studies on thermosiphon flows. In order to numerically highlight these experimental studies, we realized a geometric model from a vertical cylinder open at both ends, including a heated heat source with imposed heat flow. This source is placed at the entrance of the cylinder. The vertical walls of the cylinder are maintained adiabatic. The confinement of the fluid inside the cylinder causes a suction of the fresh air from below and thus transfers a thermal power absorbed by the system. There is a thermosiphon flow around the thermal plume. The profiles of the dynamic and thermal fields have shown that the structure of a plume generated by a heated source is closely influenced, mainly by the properties of the flow around this source. During the vertical evolution of the thermal plume, we have identified three different zones: An area, used to supply the plume with fresh air, is distinguished by temperature and velocity profiles, a zone of development of the flow of the buoyancy effect characterized by temperature profiles having a maximum on the plume axis and an area where the profiles flatten out, thus reflecting the establishment of the flow. The results showed an improvement in the energy absorbed by the fluid and an increase in the volume flow rate of the flow inside the cylinder characterized by the recirculation of the fluid.

Introduction:

The flows of thermosiphon types generated by a heat source can simulate several cases in the industrial field. This is the case, for example, of hot air emissions from industrial chimneys, fires in high-rise buildings, stairwells or elevators, cooling of photovoltaic panels or printed circuits (Yahya and Mbow, 2015). The mastery and the understanding of the phenomena related to these flows, contributes to the development of the techniques of prevention and security to the fire and to the atmospheric nuisances (Mahmoud, 2006). In such configurations, the thermosiphon effect appears predominant; the flow locally has properties close to those of a forced convection flow (Ben 1995). This phenomenon has been the subject of several studies throughout the world. Exploring the structure of the flow inside a heated cylinder, (Dring and Gebhart, 1966; Yahya and Mbow, 2015) have shown that at the entrance of the system, the vertical velocity is important in the central part, and that at As the height increases, the...
speed becomes more and more intense close to the wall and lower and lower in the central part of the cylinder. In addition, other works such as those of (Dalbert.et al., 1981; Ben 1995) have highlighted the importance of the volume flow of air sucked by the thermosiphon. These authors have subsequently proposed empirical laws connecting the flow rate of the circulating fluid in the cylinder and Rayleigh number for different flow regimes. The change in the boundary conditions of the source of the plume due to the effect of the thermosiphon could therefore be at the origin of a possible modification of the flow structure. (Agator and Doan,1982; Agator 1983; Brahimi and Doan l 986; Brahimi,1987; Brahimi et al.,1989; Dehmani et al.,l 996; Brahimi et al.,1998; Agator et al.1998) have simulated turbulent flows in the laboratory, by studying the structure of a plume generated by a hot spring whose fresh air supply is carried laterally and evolving in a free and unlimited environment. The results obtained in this case made it possible to distinguish, during the vertical evolution of the plume, two different zones: the first which characterizes the development of turbulence and the second which is the seat of a fully established turbulent flow. Several other numerical and experimental studies have been carried out separately for free and unlimited thermal plume flows (Agator and Doan,1983; Guillou,1984), or thermosiphon flows (Kettleborough,1972; Wirtz and Stutzman, 1982; Ramanathan and Kumar,1991). Our study is inspired by the experimental study of (Mahmoud, et al.,2006) which experimentally studied the effects of thermosiphon on the dispersion of pollutants. In this work, we studied the flow of fluid inside an open cylinder at both ends and subjected to an imposed heat flow heating plate. The cylinder is of height H, factor A = 0.5, and the radius of the source is R * = 0.125. Our calculations were made from a Fortran code developed for this purpose. Originpro.8 software was then used for the exploitation of the numerical results. The objective of this study is to evaluate the various flow control parameters in the vertical cylinder open to both for industrial applications. The results obtained are overall consistent and revealing.

**Method and materials:-**

**Theoretical model:-**

To model the transfer mechanisms involved in the thermosiphon, we considered the cylinder shown schematically in Figure 1 below. Our goal is to predict what happens in the cylinder when setting boundary conditions at the inlet and outlet of the cylinder and the conditions on the vertical walls.

![Figure 1: Geometric configuration of the problem](image)

**Simplifying hypotheses:-**

In order to facilitate equation and numerical processing, we have formulated the following simplifying assumptions: Fluid is an incompressible Newtonian fluid. The generated flow is laminar; the flow satisfies the Boussinesq approximation, this approximation assumes that the thermo-physical quantities of the fluid are constant with the exception of the density in the term of natural convection generating force where the density is given by:
\( \rho = \rho_L \left[ 1 - \beta_r (T - T_c) \right] \)  

Where:  
\( \beta_r = -\frac{1}{\rho_L} \left( \frac{\partial \rho}{\partial T} \right)_c \), the coefficient of thermal expansion of the fluid, the temperature, magnitude value at the reference state. The general fluid conservation equations are written:

\[
\frac{\partial \rho}{\partial t} + \rho \nabla \cdot \mathbf{V} = 0
\]

\[
\frac{\partial}{\partial t} (\rho \mathbf{V}) + \rho \nabla \mathbf{V} = \rho g - \nabla P + \mu \nabla^2 \mathbf{V}
\]

\[
dT = -\frac{k}{\rho C_p} \nabla^2 T = 0
\]

The evolution of the fields (speed \( \mathbf{V} \), temperature \( T \)) is constrained by the laws of conservation (of the quantity of movement, the mass, the energy). For an inert incompressible fluid, these equations are obtained by using the dimensionless variables defined as follows:

\[
r' = \frac{r}{H}, \quad z' = \frac{z}{H}, \quad t' = \frac{V r}{R}, \quad \theta = \frac{T - T_0}{\Delta T_r}, \quad u' = \frac{U}{V_r}, \quad v' = \frac{V}{V_r}
\]

The conservation equations of mass, momentum, concentration and adimensional energy in bidimensional cylindrical coordinate are then written:

\[
\frac{1}{r} \frac{\partial (rU)}{\partial r} + \frac{\partial V}{\partial z} = 0
\]

\[
\frac{\partial U}{\partial t} + \left( U \frac{\partial U}{\partial r} + V \frac{\partial U}{\partial z} \right) = -\frac{\partial P}{\partial r} + \frac{1}{\text{Re}} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial U}{\partial r} \right) + \frac{\partial^2 U}{\partial z^2} \right]
\]

\[
\frac{\partial V}{\partial t} + \left( U \frac{\partial V}{\partial r} + V \frac{\partial V}{\partial z} \right) = -\frac{\partial P}{\partial z} + \frac{1}{\text{Re}} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial V}{\partial r} \right) + \frac{\partial^2 V}{\partial z^2} \right] - \frac{\text{Gra}_r}{\text{Re}^2} \frac{\partial \theta}{\partial z}
\]

\[
\frac{\partial \theta}{\partial t} + \left( U \frac{\partial \theta}{\partial r} + V \frac{\partial \theta}{\partial z} \right) = \frac{1}{\text{Pe}_r} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \theta}{\partial r} \right) + \frac{\partial^2 \theta}{\partial z^2} \right]
\]

The vorticity-current function formulation offers the advantage of eliminating the pressure and automatically checking the continuity equation. A current function is introduced by asking:

\[
U = -\frac{1}{r} \frac{\partial \psi}{\partial z}; \quad V = \frac{1}{r} \frac{\partial \psi}{\partial r}; \quad \omega = \frac{\partial U}{\partial z} - \frac{\partial V}{\partial r}
\]

\[
\frac{1}{r} \frac{\partial^2 \psi}{\partial z^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial \psi}{\partial r} \right) = -\omega
\]
\[
\frac{\partial \omega}{\partial t} - \frac{\partial \omega}{\partial r} \left( \frac{1}{r} \frac{\partial \psi}{\partial z} \right) + \frac{\partial \omega}{\partial z} \left( \frac{1}{r} \frac{\partial \psi}{\partial r} \right) = \frac{1}{\text{Re}} \left[ \frac{\partial}{\partial r} \left( r \frac{\partial \omega}{\partial r} \right) + \frac{\partial^2 \omega}{\partial z^2} - \frac{\omega}{r^2} \right] + \frac{\text{Grat}}{\text{Re}^2} \frac{\partial \theta}{\partial r}
\]

(12)

\[
\frac{\partial \theta}{\partial t} - \frac{1}{r} \frac{\partial \psi}{\partial z} \frac{\partial \theta}{\partial r} + \frac{1}{r} \frac{\partial \psi}{\partial r} \frac{\partial \theta}{\partial z} = \frac{1}{\text{pe}_i} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \theta}{\partial r} \right) + \frac{\partial^2 \theta}{\partial z^2} \right]
\]

(13)

Conditions to the limits:-

- Initial conditions
  \[ t=0, \quad \theta(r, z, 0) = 0, \quad \psi(r, z, 0) = 0 \]
  \[ V(r, 0, t) \]
  \[ \frac{\partial \theta}{\partial z} = -1 \]

- Condition at the entrance
  \[ \theta(r, 0, t) = 0 \]

- Condition on the walls of the pipe
  \[ \frac{\partial \theta}{\partial z} = 0 \]

- Thermal conditions
  \[ s_i \psi e_z \leq 0, \quad \theta = 0 \]

- Conditions on speed, current function and vorticity
  \[ s_i \psi e_z \geq 0, \quad \frac{\partial U}{\partial z} = \frac{\partial V}{\partial z} = \frac{\partial \theta}{\partial z} \]

- Hydrodynamic conditions
  \[ \frac{\partial U}{\partial r} = \frac{\partial V}{\partial r} = \frac{\partial \psi}{\partial r} = \frac{\partial \omega}{\partial r} = 0 \]

(14)

(15)

(16)

(17)

(18)

(19)

Results and discussions:-

The results that we present during this work were made with air as fluid passing through the cylinder. These results were obtained thanks to a calculation code developed from Fortran calculation software. The Grashof number is taken at \( \text{Grat} = 105 \), the shape ratios and the radius of the hot source are respectively \( A = 0.5 \) and \( R^* = 0.125 \). The physical properties of the fluid were taken at the inlet temperature of the (ambient) flow.

Variable Z thermal and dynamic fields:-

Figure 1. (a) presents the radial distribution of the adimensional temperature of the flow at different levels \( Z \). Indeed, in the zone near the hot source (\( Z \leq 0.2475 \)), the profile has a maximum located on the axis of the plume. The appearance of this maximum is due to a strong development of the plume on the area of the hot spring. At intermediate levels (\( 0.2475 < \ Z < 0.9975 \)), this figure shows temperature profiles that are decreasing in relation to the vertical. This is explained by a decrease in the thermal effect. In the last zone (\( Z = 0.9975 \)), the temperature profiles lower, become self-similar and almost identical to show thus the total establishment of the flow. The radial evolution of the adimensional vertical velocity of the flow at different levels \( Z \) is given in Fig. 1 (b). As previously shown, these profiles present three different aspects of the overall flow structure. Indeed, for the first levels close to the hot source (\( Z < 0.2475 \)) the velocity profiles evolve very little, widen with a maximum located largely at the center of the
cylinder at \((r_e = 0.125)\). This structure is due to the fact that the fluid layers arriving from outside, at room temperature and more dense, are located above the warmer and lighter layers, adjacent to the hot surface of the source. This is a potentially unstable stratification, which gives rise to a convective motion. This maximum results from the interaction between the plume and the thermosiphon surrounding it. At the intermediate levels, \((0.2475 < Z < 0.9975)\), this figure shows the appearance of a new flow structure where the velocity profiles have a maximum away from the plume axis and located in the vicinity, the radius of the source \((r_e = 0.125)\). In the upper part of the cylinder \((0.2475 < Z \leq 0.9975)\), the temperature gradients become smaller and smaller, slowing down the flow rate until the flow is fully established.

Figure 1:- Evolution of temperature and adimensional radial velocity

**Thermal and dynamic fields with variable \(t\):-**

Figure 2 shows the isovalues of the fields of temperature and velocity. Overall we find the same structure found in the first case, despite the strong penetration of air at room temperature from below. We show the existence of three different zones. Figure (c) shows zone 1 at \(t = 0.0005\), characterized by the formation of an insuperable envelope with a strong thermal plume. Figure 2 (d) shows area 2 at \(t = 0.2085\); the fluid heats up rapidly and reaches relatively high temperatures, with high thermal gradients resulting in a sudden transformation of the fresh fluid into a plume. Figure 2 (e) corresponds to zone 3 at \(t = 0.325\). An attenuation of the lateral expansion of the plume in the space between the source and the vertical walls of the channel, the circulation of the fresh fluid dominates the flow and the temperature becomes uniform.

Figure 2_c:- Zone 1 at \(t = 0.00005\)
Figure 2_d: Zone 2 at t = 0.2085

Figure 2_e: Zone 3 at t = 0.325

Figure 2: Thermal and dynamic field with variable t indicating the different zones

Symbols

| Symbol          | Description                                      |
|-----------------|--------------------------------------------------|
| $C_p$           | Constant pressure thermal capacity               |
| $k$             | Thermal conductivity ($\text{W.m}^{-1}.\text{K}^{-1}$) |
| $H$             | Dimensionless length                             |
| $(r, z)$        | Dimensionless coordinates                        |
| $t$             | Time ($\text{s}$)                                |
| $V_r$           | Velocity of entry                               |
| $g$             | Gravity intensity, $\text{m.s}^{-2}$             |
| $\beta_T$       | Coefficient of thermal expansion, $\text{K}^{-1}$ |
| $\nabla$         | Operator Nabla                                   |
| $P_{e_T}$       | Number of thermal Peclet                         |
| $\rho_f$        | Reference density of fluid, $\text{kg.m}^{-3}$   |
| $Re$            | Reynolds number                                  |
| $\psi$          | Stream function                                  |
| $\omega$        | Vorticity                                        |
| $T$             | Temperature ($\text{K}$)                        |

Conclusion:
In this article, we studied the flow of fluid inside an open cylinder at both ends and subjected to an imposed heat flow heating plate. The cylinder is of height $H$, factor $A = 0.5$, and the radius of the source is $R^* = 0.125$. For calculation purposes, we used the Fortran code programmed for this purpose. Once the results were obtained, the OriginPro.8 software was then used for their graphical operations. The objective of this study is to evaluate the natural convection flows in the vertical cylinder open at both ends from different control parameters for industrial applications. The results obtained are overall consistent and revealing. The radial distribution of the temperature allowed identifying: a zone with strong development of the plume; an area marked by a decrease of the thermal...
effect and a zone marked by the total establishment of the flow. The radial evolution of vertical velocity has highlighted three different aspects of the global flow structure: a potentially unstable stratification giving rise to a convective motion; the appearance of a new flow structure where the velocity profiles have a maximum and slowing of the flow velocity until the total establishment of the flow. The isovales of the fields of temperature and velocity have shown the existence of three different zones: an area characterized by the formation of an impassable envelope with a strong thermal plume; a zone where the fluid heats up rapidly and reaches relatively high temperatures, strong thermal gradients translating a sudden transformation of the fresh fluid into a plume and an area marked by an attenuation of the lateral expansion of the plume in the space between the source and the vertical walls of the channel, the flow of fresh fluid dominates the flow and the temperature becomes uniform. Compared to the experimental study of MAO. Mahmoud et al we managed numerically to obtain similar results. The similarities of these results result in an evolution of the flow in three different zones: a first zone of instability serving to feed the plume, an intermediate zone of development of the flow and a last zone of establishment of the 'flow. However, there are differences in numerical values such as, Grashof number, shape ratio and source radius, compared to experimental values of AOM Mohamoud which used as experimental value \( Gr = 1.56 \times 10^7, A = 0, 15, R^* = 0.46. \) Although this work promises opportunities for the industry, it should also be noted that the model used in this study is two-dimensional for laminar flow. As the natural flows are mostly turbulent, it would be interesting to confirm these results using a three-dimensional numerical model, in the context of turbulence that can understand the physical phenomenon related to the flow.

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