Challenges to the field of nuclear clustering

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Abstract. This contribution reviews some of the challenges that nuclear cluster science has taken on focussing on those surrounding nuclear molecules for example in $^{24}$Mg. A view is taken that the most exciting challenges link to the structure of light nuclei, where they may be used to inform ab inito theories which attempt to utilise realistic nuclear forces. In some cases the limit of these theories is the nucleus $^{12}$C. The experimental progress in understanding this crucial nucleus is examined.

1. Introduction

The subject of clustering has, for many years, sat at the edge of nuclear physics – a curiosity rather than the main stream. There are many reasons for this, but perhaps the most obvious is that the subject has often promised significant discoveries or new ways of understanding nuclear phenomena and subsequently not delivered.

Between the 1960s and 1990s a colossal effort was devoted to the study of molecular structures populated in resonant scattering [1]. These measurements populated the intermediate short-lived systems at excitation energies typically in excess of 20 MeV, far from the ground-states, where the number degrees of freedom are high and the correspondingly complexity is large. Given this, it is surprising that simple-modes are resistant to damping into the reservoir of the compound system. The best known of these are the $^{12}$C+$^{12}$C resonances where close to the barrier 100 keV wide structures indicate the formation of an intermediate $^{24}$Mg complex which survives for a period which significantly longer than the collision time. But as indicated in Fig. 1, such resonances are not only limited to this single system but are widespread. Such a dramatic phenomenon has attracted many, just as the Sirens in Greek mythology, but a definitive understanding remains to be grasped. In some ways the field has struggled to extract itself from this grasp. Some aspects are clear; the resonances are only observed when the underlying absorption is low (the number of open reaction channels is limited [2]), the energy-spin systematics follow a rotational behavior (but with a number of resonances of the same spin), often the resonances are associated with very large partial widths for the entrance channel [1]. This has been interpreted as the formation of a cluster-cluster intermediate complex associated with shape-isomeric minima in the potential energy of the composite system [3]. The resonances are then rotational states formed in the
second minimum with the fragmentation arising from the coupling with states of the same
spin in the first. The demonstration that such a picture is accurate lies at the limits of
experimental possibilities – for example the measurement of the electromagnetic transition
rates, $B(E2)$, between the rotational states. The one valiant attempt to detect the gamma-rays
from transitions between the states produced, realistically, only produced an upper limit [4].
An alternative approach to characterize the states is through their radiative decay to the
ground-state; so-called radiative capture. This is a technique pioneered by Nathan and
Sandorfi using NaI detectors [5], but has more recently been pursued using high resolution Ge
detectors (and reported in the present conference [6]). But again an unambiguous
understanding has not emerged. Complexity still dominates.

In recent times, there has been a refocusing of the efforts of the cluster community on the
outstanding questions in lighter systems. Here as indicated in Fig. 2, the clustering degree of
freedom moves closer to the ground state and the competing degrees of freedom diminish.
Indeed here it has been possible to observe the electromagnetic decay of the $4^+$ to $2^+$ state in
the nucleus $^8\text{Be}$ – confirming its deformation, consistent with a $2\alpha$ cluster structure [7].
Similarly, the quality of information available for the states in $^{20}\text{Ne}$ point very strongly to its
$^{16}\text{O}+\alpha$ structure [8]. The attraction of the study of light nuclear systems is that they are also
the dominion of the ab initio type calculation. Such models utilize realistic nucleon-nucleon
forces in a bid to replicate nuclear properties. These include the Greens Function Model Carlo
(GFMC) [9], No Core Shell Model (NCSM) [10] and Effective Field Theories (EFT) [11]. In
the context of clustering the images of the density of the $^8\text{Be}$ nucleus calculated in the
framework of the GFMC approach are particularly stimulating [9].

Structural changes in nuclei are often driven by the valence particles, interactions between
these particles and the core then can induce collective changes in shape or deformation. In
light systems the valence nucleons dominate and correlations are extremely important.
Understanding the correlations, both spatial and momentum, that drive the formation of
various nuclear structures is strongly entwined with understanding the nature of the strong
interaction at play within the nucleus. It is here that perhaps the understanding of nuclear correlations or clusters has its greatest impact, as understanding the nature of the strong interaction in nuclei is currently one of the most important frontiers for nuclear science.

It has not gone unnoticed (and also reported in the present conference [12]), that one of the other great conquests for nuclear science, the determination of the abundance of the elements, is strongly linked to the nature of threshold states. This is depicted in both the Ikeda and modified-Ikeda diagrams, which indicate that clusterisation occurs close to the threshold for the formation of a nucleus from its cluster constituents. Correspondingly, structural clusterisation is intimately entwined with the formation of the elements and even life itself.

Remarkably, these two aspects, nucleosynthesis and the nature of the strong interaction, intersect in $^{12}$C and the Hoyle-state. Within the field, understanding the nature of this state is the foremost challenge and is perhaps the reason why so many contributions to the conference focused on this element. The structure of this state has variously been described in terms of a Bose condensate, a cluster state and an alpha-particle gas. It is not clear yet which picture is appropriate and even if the various descriptions are non-degenerate. Similarly, the spectroscopy of $^{12}$C above the alpha-decay threshold remains to be determined definitively.

The challenges in determining the structure of light nuclei both from a theoretical and experimental perspective require a new approach; one which is coherent rather than disjointed and has a genuine strategy in terms of its approach.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{threshold_diagrams.png}
\caption{Threshold diagrams – revealing potential excitation energies at which clustering should appear. \textit{Left} The Ikeda diagram [13] which reveals the cluster thresholds for alpha-particle like nuclei. \textit{Right} Modified Ikeda diagram [14] for systems in which covalent neutrons may be exchanged between the cluster cores.}
\end{figure}
For the remainder of this contribution, I will focus on the nature of $^{12}\text{C}$ as an illustration of the advances that have been made in this single nucleus and the questions that remain.

2. Structure of $^{12}\text{C}$

The $^{12}\text{C}$ nucleus has, potentially, a symmetry which is related to a three-alpha structure. The ground-state has an oblate character which may be understood in terms of the arrangement of the three alpha-particles in a plane. The collective excitations of this structure would result in the population of $0^+, 2^+$ and $4^+$ ...... states associated with rotations around an axis which lies within the plane of alpha-particles and $3^+, 4^+, 5^+, ....$ corresponding to a rotation perpendicular to the plane, corresponding to $K=0$ and $K=3$ bands, respectively. Within the ground-state the alpha-particle structure is expected to be strongly suppressed, though the symmetry remains. This suppression would coincide with the fact that the 3alpha decay threshold lies some 7 MeV higher in energy and hence the cluster structure should not be strongly developed. Indeed, quasi free proton scattering $^{12}\text{C}(p,p\alpha)$, reported in the present conference [15], indicates that the alpha-particle spectroscopic factor is close to that which would be predicted by the shell model. The $3^-$ state on the other hand lies at 9.6 MeV which is above the alpha-decay threshold and hence could in principle display a more developed 3alpha cluster structure. An analysis of the width of the $3^-$ resonance in the high resolution inelastic scattering data shown in Fig. 3, would indicate a width of 40(2) keV. This would, for a channel radius of $1.35(8^{1/3}+4^{1/3})$ fm, suggest a width which is 25% of the Wigner limit.

The complexity of the Greens Function Monte Carlo (GFMC) approach prohibits its extension currently to nuclei with $A>12$, and $^{12}\text{C}$ is currently the focus of the calculations with some preliminary results for the 7.65 MeV, $0^+$, Hoyle-state just being reported. The Hoyle-state was proposed by Sir Fred Hoyle some 50 years ago [16]. This state is the gateway through which carbon is synthesized via what is known as the triple-alpha process. Without this state there would be no organic life. Calculations of the properties of the Hoyle-state using the GFMC approach are still on-going. However, calculations using the

![Figure 3](image-url)

**Figure 3.** Left-hand-side. $^{12}\text{C}$ excitation energy spectrum showing the broad $2^+$ state required to reproduce the shape of the experimental spectrum, from Ref. 8. Right-hand-side. Possible arrangements of the alpha-clusters in either a triangle (top) or chain (bottom). The experimental data indicate that the triangular structure is preferred.

Antisymmetrized Molecular Dynamics (AMD) [17] and its cousin the FMD approach
(Fermionic Nuclear Dynamics) [18] reveal a highly clustered state. Its’ very dilute nature, together with the 3alpha cluster structure has been interpreted in terms of an alpha-condensate by Schuck and co-workers [19], this would be a new bosonic state of nuclear matter. Very recent calculations using the framework of chiral effective field theory have been able to closely reproduce the energy of the Hoyle-state [11]. Although they offer little detailed structural information, they do suggest a larger charge radius than the ground-state, as observed experimentally. As with the $^8$Be ground-state, the width of the 7.65 MeV state indicates a significant cluster structure and, in agreement with theoretical predictions, a large charge radius (see Ref. [17]) indicating a volume which is 3-4 times that of the $^{12}$C ground state.

Revealing the collective excitations of such a state could have significant implications for understanding its structure. In the alpha-condensate model this would correspond to a $2^+$ state which is associated with the conversion of one of the bosons from s-wave to d-wave. In a more traditional cluster model it would correspond to a collective rotation and the moment of inertia would then reveal the details of the cluster arrangement. The closest $2^+$ state which has been tabulated with these characteristics is at 11.16 MeV [20]. This state was observed in the $^{11}$B($^3$He,d)$^{12}$C reaction, but has not been observed in measurements subsequently.

![Figure 4. $^{12}$C excitation energy spectrum from the $^{11}$B($^3$He,d) reaction. The measurement clearly demonstrates the absence of the 11.16 MeV state in $^{12}$C [9].](image)

A recent re-measurement of this reaction using the K600 spectrometer at iThemba demonstrates that the earlier observation of a state at 11.16 MeV was almost certainly an experimental artifact and no such state exists [21]. The existence of the $2^+$ excitation is then an open question.

Recent studies of the $^{12}$C($\alpha,\alpha'$) [22] and $^{12}$C($p,p'$) [23,24] (see Fig. 3) reactions indicate the presence of a $2^+$ state close to 9.6-9.7 MeV with a width of 0.5 to 1 MeV. The state is only weakly populated in these reactions, presumably due to its underlying cluster structure, and is broad. Consequently, its distinction from other broad-states and dominant collective excitations (e.g. the 9.6 MeV, 3$^-$) makes its unambiguous identification challenging. Further evidence for such an excitation comes from measurements of the $^{12}$C($\gamma,3\alpha$) reaction performed at the HIGS facility, TUNL [25]. Here a measurable cross section for this process was observed in the region of 9-10 MeV which cannot be attributed to known states in this region. Furthermore, the angular distributions of the alpha-particles are consistent with an $L=2$
pattern, indicating a dominant $2^+$ component. The alpha-particle inelastic scattering measurements also included a multipole decomposition analysis of the angular distributions [22] the amplitudes of the $L=2$ component is shown in Fig. 5. The variation of this amplitude with energy matches well that deduced from an R-matrix type analysis of the component found in the excitation energy spectra [26].

Based on a rather simple description of this state in terms of three alpha-particles with radii given by the experimental charge radius, it is possible to use the 2 MeV separation between the Hoyle-state and the proposed $2^+$ excitation to draw some conclusions as to the arrangements of the clusters (see Fig. 3). This would indicate that rather than a linear arrangement of the three clusters that a more appropriate description would be a loose arrangement of the alpha-particles in something approaching a triangular.

A natural extension of such a conclusion is that there should also be a collective $4^+$ state. Using the simple $j(j + 1)$ scaling, a $4^+$ excitation close to $E_x(^{12}\text{C}) = 14 \text{ MeV}$ would be expected. We have performed recent measurements of the two reactions $^9\text{Be}(\alpha,3\alpha)n$ and $^{12}\text{C}(\alpha,3\alpha)^4\text{He}$ [27]. In these measurements three alpha-particles were detected in an array of four silicon strip detectors (shown in Fig 6). The analysis required that two of the three alpha-particles came from the decay of the ground-state of $^8\text{Be}$. For the decay of $^{12}\text{C}$ to $^8\text{Be}+\alpha$ this ensures that the decay process can proceed through only natural parity states (i.e. $0^+, 1^-, 2^+$ ....). This restricts the complexity of the excitation energy spectrum. The measurements for both the $^9\text{Be}$ and $^{12}\text{C}$ targets reveal the known $3^-, 1^-$ and $4^+$ states at $9.64, 10.84 \text{ and } 14.08 \text{ MeV}$, respectively. However, there is an additional component to the spectrum close to 13.3 MeV with a width estimated to be 1.7 MeV (Fig. 6). It is believed that this is not a contaminant and is observed with similar properties in all spectra. Angular correlation measurements made using the $^{12}\text{C}$ target are not definitive, but indicate a $4^+$ assignment.

Figure 5. The $2^+$ strength function extracted from $^{12}\text{C}(\alpha,\alpha')$ inelastic scattering measurements. The various curves correspond to R-matrix line shapes and show that there is a consistent agreement between the $2^+$ line shape found in the excitation energy spectra and extracted from the $L$-dependence of the angular distributions. Ref. [26]
Such measurements are challenging and it is difficult to be absolutely certain that the feature does not correspond to an experimental artifact. Nevertheless, the appearance of a $4^+$ state close to the energy predicted based on the extension of the Hoyle-state and proposed $2^+$ excitation. It is clear that further measurements are required to confirm, or otherwise, this new state in $^{12}$C. Moreover, analysis of beta-decay measurements populating alpha-decay states in $^{12}$C indicate that there may be broad $2^+$ strength in $^{12}$C but at somewhat higher energies than suggested in the proton and alpha inelastic scattering measurements [20]. This contradiction between the two types of measurements remains to be reconciled and hence a firm conclusion regarding the excitation structure of $^{12}$C above the alpha-decay threshold remains to be established.

3. Summary

The characterization of clustering is extremely challenging, but there is a broad spectrum of evidence for this behavior. Providing a detailed characterization of the cluster states has been made for a number of systems. However, even for nuclei as simple as $^{12}$C there remain unanswered questions. We have shown here that we are moving towards a more detailed understanding, but have not finally arrived. This will require precise experimental measurements coupled with state-of-the-art theory.

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