DYNAMICS OF THE
INTERMEDIATE-AGE ELLIPTICAL LMC CLUSTER NGC 1978

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ftp 130.113.0.111 Name: anonymous Password: ¡username¿ ftp ¿ cd pub ¿ ftp ¿ get fig1978.ps ¿ ftp ¿ quit
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ABSTRACT

BV CCD images of the elliptical LMC cluster NGC 1978 out to a projected radius \( R \sim 100 \) pc were obtained using the 1.0m telescope at Las Campanas. In addition, radial velocities with a precision of 1.5 km s\(^{-1}\) were measured for 35 member giants using the echelle spectrographs and 2D-Frutti detectors on the Las Campanas 2.5m and the Cerro Tololo 4.0m telescopes.

After star-subtraction and median-filtering the ellipticity of the surface brightness distribution was determined to be \( \epsilon = 0.30 \pm 0.03 \) and the major axis position angle to be \( \text{PA} = 152 \pm 7^\circ \). The stellar radial velocities indicate that NGC 1978 has a systemic velocity of \( \mathbf{v} = 293.3 \pm 1.0 \) km s\(^{-1}\). NGC 1978 appears to be several times older than its central relaxation time but considerably younger than its half-mass relaxation time.

Single and multi-mass anisotropic King-Michie models and single-mass rotating and non-rotating oblate spheroid models were fitted to both the surface luminosity profiles and the radial velocity data. The total cluster luminosity is \( L_B = 3.1 - 3.7 \pm 0.2 \times 10^5 \) L\(_\odot\) and \( L_V = 3.0 - 3.5 \pm 0.2 \times 10^5 \) L\(_V\odot\) (for an assumed LMC distance of 50 kpc), where the range results from different model extrapolations of the brightness profile. The multi-mass models, while very effective at constraining the central mass-to-light ratios (hereafter, M/L’s) at about \( (M/L)_0 = 0.13 \pm 0.06 \) M\(_\odot\)/L\(_\odot\), yielded global M/L’s which ranged over a factor of 5; \( M/L = 0.3 - 1.5 \) M\(_\odot\)/L\(_\odot\) for a sample of mass function slopes. The best agreement between population and dynamical M/L’s is seen for the cases \( x = 0.0 \) for the B band \([(M/L)_0 = 0.14 \pm 0.06 \) M\(_\odot\)/L\(_B\odot\) and M/L = 0.35 \pm 0.15 M\(_\odot\)/L\(_B\odot\)] and \( x = 0.5 \) for the V \([(M/L)_0 = 0.13 \pm 0.06 \) M\(_\odot\)/L\(_V\odot\) and M/L = 0.40 \pm 0.15 M\(_\odot\)/L\(_V\odot\)]. The single-mass models tended to give better agreement with the luminosity profiles but produced M/L’s (i.e. \( M/L = 0.20 \pm 0.08 \) M\(_\odot\)/L\(_V\odot\)) that were difficult to reconcile with simple population studies without invoking a rather high low-mass cut-off (i.e. 0.8 M\(_\odot\)).

We found no significant differences between the M/L’s derived with oblate spheroid models and those derived with spherical models. While the non-rotating (anisotropic)
models were in better agreement with the kinematic data, it was impossible to completely rule out the rotating models. As well, there is no morphological evidence for a merger.

1. INTRODUCTION

Globular clusters present a unique opportunity to study the internal dynamics of resolved stellar systems in which the two-body relaxation timescales are similar to the current ages. One can kinematically examine clusters to dynamically determine masses and mass-to-light ratios (hereafter, M/L’s) in order to constrain the initial mass function (IMF). Attempts can also be made to determine the internal cluster dynamics at various stages in their evolution to try to form a coherent picture of formation, energy equipartition, and mass segregation. At later evolutionary stages, gravothermal catastrophe and the resulting core collapse can be studied.

In the Milky Way, these studies can be broken down into two subsets: 1) those which utilize measurements of central radial velocity dispersions from integrated spectra (c.f., Illingworth 1976), and 2) radial velocity measurements of individual member stars (c.f., Gunn and Griffin 1979, Meylan and Mayor 1986, Lupton et al. 1987, Pryor et al. 1989 and 1991 and others).

The Large Magellanic Cloud (LMC) clusters occupy a much wider range in parameter space (i.e. age, metallicity, morphology, etc., see Olszewski et al. 1991) than their Milky Way counterparts and, hence, provide a more complete cluster sample. To date there have been several dynamical studies of LMC clusters including integrated spectra for several old clusters (Elson and Freeman 1985; Dubath et al. 1990; and Mateo et al. 1991) and individual stellar velocity measurements of mostly young clusters (Lupton et al. 1989, Mateo et al. 1991, Fischer et al. 1992).

An interesting aspect of the cluster age distribution is that it does not appear to be a continuum. There are approximately eight old (i.e. \( > 10^{10} \) yrs) clusters and then a large number of clusters younger than \( 3 \times 10^9 \) yrs, possibly indicating two major epochs of star formation interrupted by a more quiescent period (Olszewski et al. 1991). Another
interesting feature of some LMC clusters is the existence of projected ellipticities as large as \( \epsilon \approx 0.3 \) (Geisler and Hodge 1980) - a feature not restricted to the young clusters. For this reason, we have embarked upon a project to dynamically study a sample of LMC clusters covering a range of ages and ellipticities.

NGC 1978 is an intermediate age cluster \((\tau = 2 \times 10^9\) years, Olszewski 1984 and Mould and Da Costa 1988) which also happens to be among the most highly elliptical clusters known (\( \epsilon = 0.3 \), Geisler and Hodge 1980). Three explanations for this ellipticity immediately suggest themselves: rotation, an anisotropic velocity dispersion tensor, or a recent cluster-cluster merger. In this paper we will investigate the relative likelihoods of these three scenarios and attempt to constrain the dynamics of this largely unrelaxed object (except, perhaps, in the innermost regions).

A discussion of the the surface photometry data and reductions will be presented in §2, with the modeling results appearing in §3. §4 contains a description of the spectroscopic observations and reductions and in §5 we calculate evolutionary timescales. §6 details the M/L determinations and constraints on the internal dynamics, and in §7 we compare our findings with two previous studies.

2. SURFACE PHOTOMETRY

2.1 Observations and Reductions

In order to derive a surface brightness profile for NGC 1978, BV CCD frames were obtained on the Las Campanas Observatory (LCO) 1.0 m telescope on 1991 January 22. The TEK2 1024\(^2\) chip was used (readout noise = 6 e\(^-\), gain = 2 e\(^-\)/ADU, and angular scale = 0.61" pix\(^-1\)). The integration times were 200s and 100s for the B and V frames, respectively.

There are two complications to measuring the surface brightness profile of NGC 1978. First, the cluster is in an area densely populated with LMC field stars, and second, elliptical apertures are necessary.

To maximize the surface brightness intensity range, it is necessary to get an accurate
estimate of the field contribution. The apparent BV color-magnitude diagram (Fig. 1) illustrates the richness of the NGC 1978 field. One can see a clearly delineated red giant branch consisting of both cluster and LMC field stars. However, there are also a similar number of blue, main sequence LMC field stars. The presence of these bright stars complicates the background determination and therefore it is advantageous to remove them. This was accomplished using the profile-fitting photometry package DAOPHOT (Stetson 1987). All stars bluer than B-V = 0.48 and all stars brighter than V = 15.1 were removed (above and to the left of the solid line in Fig. 1). We found that after star-subtraction the mean background brightness was only reduced by about 10-15% but that its uncertainty decreased by more than a factor of ten.

Because NGC 1978 is one of the most elliptical clusters known, it requires the use of elliptical annuli for surface photometry which in turn requires knowledge of the mean cluster ellipticity and major axis position angle. The many resolved stars greatly complicate attempts to derive ellipticities, tending to skew them towards high values and yielding rapidly varying position angles as a function of radius. Therefore, we first subtracted out the resolved (both background and cluster) stars using DAOPHOT and then median filtered the star-subtracted image using a 9″ radius circular filter. The result was a relatively smooth light distribution with the contribution from the bright giants largely eliminated. It was then possible to use the ELLIPSE task in the IRAF STSDAS package, which uses the technique of Jedrzejewski (1987), to fit elliptical contours to a smooth light distribution. We were only able to employ this technique reliably in the range 2.5 ≤ R (pc) ≤ 25 (Throughout this paper, we assume a distance to NGC 1978 of 50 kpc and hence an angular scale of 4.12″ pc⁻¹) due to crowding at smaller radii and a lack of light at larger radii.

1 IRAF is distributed by the National Optical Astronomy Observatories, which is operated by the Association of Universities for Research in Astronomy, Inc., under contract to the National Science Foundation.
Table 1 lists the ellipse parameters as a function of major axis radius. Column 1 is the radius of the projected major axis in pc, while columns 2 and 4 are the projected ellipticities and columns 3 and 5 are the major axis position angles (PA) for the B and V frames, respectively. The tabulated uncertainties represent the formal fitting errors. Fig. 2 shows isophotal contour diagrams for the star-subtracted median-filtered BV frames. Superimposed on the contours are the best-fit ellipses. The ellipses appear to provide very reasonable models for the isophotes in the range shown. There is no evidence for subclustering, indicating that NGC 1978 is unlikely to be a recent merger of two clusters. Finally, 14 stars from the Guide Star Catalog (Lasker et al. 1990, Russell et al. 1990, and Jenkner et al. 1990) that appeared on the V frame were used to accurately align the frames to the J2000.0 equinox. The rms residuals for the positional fit were less than 0.8 arcsec. The cluster center is located at $\alpha(2000) = 5:28:44.8$ and $\delta(2000) = -66:14:09.9$.

For both frames, a mean representative ellipticity and position angle were determined and these are shown at the bottom of Table 1. These values agree well with those obtained photographically by Geisler and Hodge (1980) ($\epsilon = 0.30 \pm 0.06$ and $\text{PA} = 159^\circ \pm 7.0^\circ$) and were used to construct the elliptical annuli for the flux measurements. Surface photometry was performed in a manner similar to Djorgovski (1988). The frames were broken up into a series of concentric elliptical annuli centered on the cluster. The annuli were further divided into eight azimuthal sectors. The average pixel brightness was determined for each sector in a given annulus and the median of the eight separate measurements was taken as the representative brightness at the area-weighted average radius of the annulus (i.e., the mean radius of all the pixels within the annulus which is approximately equal to the geometric mean). The standard error of the median of the eight sectors is equal to the standard error of the mean multiplied by $\sqrt{\pi/2}$ and this was adopted as the photometry uncertainty in each annulus. Comparisons between photometry measured with elliptical and circular apertures revealed that, while the circular apertures did not introduce significant systematic errors, they tended to have substantially higher sector-to-sector variations.
which resulted in greater scatter in the surface photometry.

A background level (a combination of sky light and Galactic foreground and remaining LMC field stars) was estimated from regions at large projected distances from the cluster. We found that the surface brightness profiles tended to level out beyond 65 pc for the B frame and about 55 pc for the V (both of these distances are along the major axis). By “levelling out” we don’t necessarily mean that the cluster light does not extend beyond this point but simply that fluctuations in the background dominate to such an extent that it is no longer possible to observe the profile declining in intensity. Therefore, it was this region that was used for the background determinations.

The cluster was reobserved on 1991 Dec 7 in order to calibrate the photometry. Fourteen BV observations of 8 E-region standards (Menzies et al. 1989) were observed on the same night covering an airmass range of 1.1 - 2.0 and a color range of 0.1 mag ≤ B-V ≤ 1.6 mag. The rms of the adopted solution was less than 0.02 mag and the zero point had a similar accuracy. E_{B-V} = 0.06 mag, consistent with Olszewski (1984), was adopted. The background-subtracted surface photometry data is presented in Table 2 [assuming M_V⊙ = 4.83 and (B-V)_⊙ = 0.65, Mihalas and Binney 1981, p. 60]. Columns 1 and 4 are the projected area-weighted radii, columns 2 and 5 are the projected major axis radii and columns 3 and 6 are the B and V surface profiles, respectively. The background values for the B and V frames were, respectively, 134.3 ± 0.4 L_B⊙ pc^{-2} and 148.1 ± 0.7 L_V⊙ pc^{-2}. Fig. 3 is a plot of the B and V surface brightness profiles. Also shown is a typical stellar profile which has a FWHM approximately a factor of ten smaller than the core radius (see §3); hence seeing will have a negligible effect on measurements of this quantity (Mihalas and Binney 1981, p. 315). We looked for a radial B-V color gradient in the cluster but concluded that no significant effect is present within our measurement uncertainties.

3. KING-MICHIE MODELS FOR THE SURFACE PHOTOMETRY

Despite the fact that the highly elliptical shape of NGC 1978 is a clear indication that its dynamics cannot be adequately described within the standard framework of the
King-Michie (KM) formulation (King 1966 and Michie 1963), we fitted multi-component KM models to the photometry data. The reasons for this are: 1) These models provide us with classification parameters adhering to a widely used scheme and thus enable potentially fruitful comparisons to other clusters. 2) The fit provides for a reasonably accurate means of determining the total cluster light and 3) the fit, in conjunction with the radial velocity data (§4), gives a first approximation to the cluster mass. Alternate models will be discussed in §6.2.

For the KM models, the stellar mass spectrum is sub-divided into mass classes; nine were used for NGC 1978 (see Table 3). Each mass class has an energy and angular momentum per unit mass (\(E\) and \(J\), respectively) distribution function given by

\[
f_i(E = -0.5v_s^2 + W, J) \propto e^{-[J/(2v_s r_a)]^2}[e^{A_i E} - 1],
\]

where \(v_s\) is the scale velocity, \(W\) is the reduced gravitational potential, and \(r_a\) is the anisotropy radius beyond which stellar orbits become increasingly radial. The \(A_i\) are constants discussed below. The details are thoroughly described in Gunn and Griffin (1979), but briefly, the shape of the density distribution for the KM models, \(\rho(r)\), is determined by solving Poisson’s equation and is dependent on three parameters: the reduced central potential \(W_0\), the anisotropy radius \(r_a\), and the slope of the mass function, \(x\), given by

\[
\phi(m) = m^{-(x+1)} \, dm \quad m \geq 0.3M_\odot,
\]

\[
\phi(m) = m \, dm \quad m < 0.3M_\odot.
\]

Scaling is applied in both the radial (\(r_s\)) and luminosity dimensions to yield the best fit to the surface photometry data. \(A_i\) is forced to be proportional to the mean mass of stars in the \(i^{th}\) mass class, which approximates equipartition of energy in the cluster center.

The reason that choosing the \(A_i\) in the above manner does not yield true equipartition of energy is that the lower mass stars are more affected by the energy cut-off implicit in the model than are the higher mass stars (Pryor et al. 1986). It turns out that the deviations
in equipartition seen in the KM models are qualitatively similar to what is calculated in theoretical multi-mass evolutionary models. That is, they both exhibit a tendency for the high-mass stars to have higher kinetic energies. For example, in the $x = 0.0$ isotropic KM model described below, the lowest mass class has roughly one-third the kinetic energy of the highest mass class at the cluster center. In Fokker-Plank models, the cluster relaxes and equipartition begins to occur. Specifically, the kinetic energy of the higher mass stars decreases while that of the lower mass stars remains fairly constant. This results in mass segregation, with the high mass stars migrating to smaller radii and hence an effective decoupling of the different mass classes. With the greatly lowered rate of interactions, it becomes difficult for energy exchange between different mass stars to occur and core equipartition cannot be fully achieved (Inagaki and Saslaw 1985, see also Spitzer 1969 for an analytical treatment of a two-mass system).

In order to apply the multi-mass KM models to the NGC 1978 data we adopted stellar BV mass-luminosity relationships which were a combination of Bergbusch & VandenBerg (1992) in the range $0.15 \leq m (M\odot) \leq 0.70$ and Bertelli et al. (1990) in the range $0.70 \leq m (M\odot) \leq 1.65$. We assumed an age of $2 \times 10^9$ years and a metallicity of $z = 0.004$ (Mould and Da Costa 1988, and Olszewski 1984). The models of Bertelli et al. incorporate convective overshooting and mass-loss, both associated with high mass stars, while the VandenBerg models incorporate neither of these and should be valid in the low mass regime. The treatment of Pryor et al. 1986 was adopted for remnants: stars with initial masses of $1.65 - 4.0 \ M\odot$ and $4.0 - 8.0 \ M\odot$ become white dwarfs with masses of $0.7 \ M\odot$ and $1.2 \ M\odot$, respectively. These objects are added to the corresponding mass bins. More massive stars, which have presumably evolved into neutron stars, are assumed to be ejected from the cluster in agreement with the typically large velocities observed for these objects in the field (Gunn and Griffin 1979). Clearly, this assumption is not strictly correct as pulsars have been detected in several galactic globular clusters.

Tables 4 and 5 (B and V, respectively) contain the fitted KM parameters for models
with parameters ranging from isotropic orbits to \( r_a = 5 \, r_s \) and \( 0.0 \leq x \leq 2.0 \). Column 1 is the anisotropy radius, column 2 is the mass function slope, column 3 is the reduced central potential, column 4 is the scale radius, column 5 is the ratio of the tidal radius to the scale radius and columns 6 and 7 are the reduced chi-squared for the fit (B has 18 and V has 17 degrees of freedom) and the probability of exceeding this value, respectively. These probabilities are based on 1000 Monte Carlo simulations for each parameter set, each using a surface profile generated from the best fit model with errors drawn from the uncertainties shown in Table 2. Hence, the probabilities are somewhat dependent on the accuracy of the photometry uncertainties and for this reason it is safer to view them in the relative sense. All the KM parameters are based on using surface brightness profiles expressed as a function of area-weighted radius (from columns 1 and 4 of Table 2). The remaining columns of the two tables will be discussed in §6.1.

Tables 6 and 7 contain the derived model parameters: columns 1 and 2 specify the anisotropy radius and mass function slope, while columns 3 and 4 contain the central luminosity density and total cluster luminosity, respectively. Columns 7 and 8 are the central and global population M/L’s given by

\[
\frac{M}{L_V} = \frac{\int_{m_l}^{m_u} m \phi(m) dm}{\int_{m_l}^{m_u} l(m) \phi(m) dm},
\]

(4)

where \( l(m) \) is the luminosity of a star of mass \( m \) given by a theoretical mass-luminosity relationship for main-sequence and evolved stars and \( m_l \) and \( m_u \) are the lower and upper mass cut-offs, respectively. The remaining columns will be discussed in §6.1.

We have plotted two of the KM models in Fig. 3; isotropic single-mass models and isotropic \( x = 1.0 \) models. The difference in the quality of the fits results from the tendency of the multi-mass models to have shallower density fall-offs which is not seen in the data.

The results of the KM surface brightness modeling can be summarized as follows: 1) The parameters derived from the two surface profiles are, in general, consistent. The B band profile, despite having smaller tabulated uncertainties, appears to yield substantially lower \( \chi^2_V \). Clearly, the B profile is much less sensitive to local luminosity fluctuations caused
by cluster giants. 2) The single-mass models give the best quality fits to the data for all values of $r_a$ with the best agreement observed for the isotropic model. 3) For the multi-mass models there is a trend toward poorer quality fits with steeper mass functions but the surface brightness distributions are not sufficiently well-determined to confidently choose a specific model. 4) The surface photometry is inadequate to unambiguously discriminate between models with differing degrees of anisotropy. 5) For the multi-mass models the global population M/L’s are not well constrained, varying by a factor of five. However, the central population M/L’s occupy a much narrower range, only varying between 0.08 - 0.14 $M_\odot/L_{B\odot}$ and 0.10 - 0.17 $M_\odot/L_{V\odot}$. Because the integrated cluster luminosity is overwhelmingly attributable to the giants, the total number of giant stars is fairly insensitive to the slope of the mass function (i.e. one needs the same number of giants to produce the cluster luminosity). As a result of the equipartition of energy, the low mass stars tend to be located at large radii and, due to their relatively low luminosities, have little effect on the measured surface brightness profile; the total cluster luminosity varies by only 10% for the various models. Hence the composition (and therefore the M/L) of the cluster core is largely independent of the mass function slope. As we shall see when we discuss the KM kinematic modeling in §6.1 the dynamical M/L’s exhibit similar behavior.

4. RADIAL VELOCITIES

Spectra of 36 red giants were obtained during 1991 January 18-20 using the 4m at Cerro Tololo Inter-American Observatory (CTIO) and during January 30 -February 1 and February 14-20 at the 2.5m at LCO. Echelle spectrographs with 2D-Frutti detectors were employed at both telescopes.

The observation and reduction procedures for a previous run at LCO have been discussed extensively in Welch et al. 1991 and remain largely unchanged for this data. The CTIO data were obtained and reduced in a similar manner. Unfortunately, due to technical problems involving the dither on the CTIO 2D-Frutti, only half the available observing time was productive. Furthermore, the spectral resolution was about 50% lower than the
LCO spectrograph, resulting in velocities with uncertainties about 50% larger. The observing procedure consisted of exposures with integration times of 500 - 1500s and Th-Ar arcs approximately every 45 minutes. A representative LCO spectrum is shown in Fig. 2 of Côté et al. (1991). The reduction utilizes the IRAF ECHELLE and RV packages (Tody 1986) to obtain both velocities and velocity uncertainties. The velocity zero-point is tied to the IAU velocity standard 33 Sex as described in Fischer et al. 1992 and is believed to be accurate to better than 1 km s$^{-1}$.

The radial velocity data are presented in Table 8. Column 1 contains the stellar identifications, column 2 indicates the observatory, column 3 has the projected radius, column 4 the equinox J2000.0 position angle, column 5 contains the radial velocities and column 6 contains the mean velocity for stars with repeated measurements. Column 7 is the Heliocentric Julian Date – 2448000 for the velocity measurements. Columns 8 and 9 are V and B–V for the stars. The velocity uncertainties returned by the RV package seem to agree fairly well with the observed scatter in stars with repeated measurements. Ten stars have been measured at least twice (a total of 24 spectra), yielding $\chi^2 = 13.2$ for 14 degrees of freedom. Closer examination of Table 8 reveals that Star 13 has two measurements which are significantly discrepant. Further, the higher precision velocity is more than 5 km s$^{-1}$ larger than any other cluster star. The radial velocity implies that this star is in the LMC and, therefore, definitely a giant. The large velocity change over about 24 hours argues against the star being a binary and further observations are required to determine its true nature. Alternatively, it might simply be a case of measurement error. Regardless, we choose not to include it in the following analysis. Removing Star 13 reduces $\chi^2$ to 6.5 for 13 degrees of freedom. This rather low value of $\chi^2$ is a strong indication that we have not underestimated the velocity uncertainties and that there are no significant zero-point differences between spectra taken on different nights or on different telescopes. Fig. 4a is a finder chart for stars 1 through 36, while Fig 4b shows the positions of the stars relative to the cluster center along with a line indicating the photometric minor axis.
Fig. 5 shows mean radial velocity vs. projected radius (upper panel) and versus position angle (lower panel). The solid line is the mean velocity ( §6), \( \bar{v} = 293.3 \pm 1.0 \text{ km s}^{-1} \), which is consistent with the two lower precision velocities obtained by Olzsewski et al. (1991) of 292.0 km s\(^{-1}\), indicating no serious zero-point problems. There are no obvious trends present in the data such as one might expect. Typically, cluster velocity data will exhibit a decreasing velocity dispersion with increasing projected radius and, if rotation is present and the cluster inclination favorable, a sinusoidal functional dependence on position angle (Fischer et al. 1992). The reason we do not observe either of these phenomena may simply be due to the sparseness of the data and the large values of the velocity uncertainties relative to both the velocity dispersion and the rotation amplitude.

5. EVOLUTIONARY TIMESCALES

Implicit in the adoption of “mass segregation” models is the assumption that there has been sufficient time for energy exchange between different mass classes to occur. Two important relaxation timescales based on energy exchange through distant two-body encounters are the central relaxation time

\[
t_{r0} = (1.55 \times 10^{7} \text{yr}) \left( \frac{r_s}{\text{pc}} \right)^2 \left( \frac{v_s}{\text{km s}^{-1}} \right) \left( \frac{M_{\odot}}{\langle m \rangle} \right) \left[ \log(0.5M/\langle m \rangle) \right]^{-1} = 2.9 - 7.9 \times 10^{8} \text{yr},
\]

(Lightman and Shapiro 1978) and the half mass relaxation time

\[
t_{rh} = (8.92 \times 10^{8} \text{yr}) \left( \frac{M}{10^{6}M_{\odot}} \right)^{1/2} \left( \frac{r_h}{\text{pc}} \right)^{3/2} \left( \frac{M_{\odot}}{\langle m \rangle} \right) \left[ \log(0.4M/\langle m \rangle) \right]^{-1} = 6.0 - 16.0 \times 10^{9} \text{yr},
\]

(Spitzer and Hart 1971), where the numerical values are for mass functions slopes ranging from 0.0 \( \leq x \leq 2.0 \). As was mentioned earlier, the best age estimate for NGC 1978 is \( 2 \times 10^{9} \) yrs which is several times greater than the central relaxation time, but significantly younger than the half-mass relaxation time. We conclude, therefore, that there are significant portions of the cluster which have not had sufficient time to relax and it is important to keep this in mind when interpreting the results of the KM model analysis.
6. MASS DETERMINATIONS

6.1 King-Michie Models

The mass of a multi-mass KM model is given by

\[ M = \frac{9 v_s^2}{4\pi G} \int \frac{\rho}{\rho_0} r^2 dr \]  

(Illingworth (1976), where \( r_s \) is given in Tables 4 and 5, and \( v_s \) is the scale velocity. The run of \( \sigma_{r,i}^2(r) \) and \( \sigma_{t,i}^2(r) \) are determined from

\[ \sigma_{(r,t),i}^2(r) = \frac{\int_{\sigma_i \leq W(r)} f_i(\sigma_i, W) \sigma_i^2 \sigma_i}{\int_{\sigma_i \leq W(r)} f_i(\sigma_i, W) d^3 \sigma_i}, \]  

where \( W \) is the reduced potential (\( W = 0 \) at the tidal radius), \( \sigma_k = \sigma_i \cos \theta \) or \( \sigma_i \sin \theta \) for \( \sigma_{r,i} \) or \( \sigma_{t,i} \), respectively, and the \( i \) subscript refers to the \( i^{th} \) mass class. Comparisons were made between the observed velocities and scaled model velocity dispersions projected along the line of sight,

\[ \sigma_{p,i}^2(R) = \frac{2}{\mu_i(R)} \int_R^\infty \rho_i(r) \frac{[(r^2 - R^2)\sigma_{r,i}^2(r) + R^2 \sigma_{t,i}^2(r)]}{r(r^2 - R^2)^{1/2}} dr, \]  

(Binney and Tremaine 1987, p. 208), where \( \mu_i \) is the surface density of the \( i^{th} \) mass class. The optimal scaling was derived using the maximum likelihood technique outlined in Gunn and Griffin (1979). Simply put, the probability density function for \( v_{x,k} \), an observed stellar velocity, is a Gaussian with standard deviation equal to the model dispersion added in quadrature to the velocity uncertainty:

\[ P_k \sim \frac{1}{\sqrt{\sigma_{err,k}^2 + v_{s}^2 \sigma_{p,k,i}^2}} e^{-\left(v_{x,k} - \mathbf{\nabla}\right)^2/2(v_{s}^2 \sigma_{p,k,i}^2 + \sigma_{err,k}^2)}. \]  

This function is minimized with respect to \( v_s \) and \( \mathbf{\nabla} \) resulting in two equations which can be solved simultaneously for the most probable values of the two parameters.

The values of \( v_s \) thus obtained are displayed in column 8 of Tables 4 and 5. The corresponding dynamical masses and M/L’s are in columns 9 and 10, respectively of Tables
Monte-Carlo orbit simulations were used to determine the uncertainties in the fitted and derived parameters and to search for possible systematic effects. We started with the known projected radii \( R_k \) of the program stars. The true radii are in the range \( R_k \leq r \leq r_t \). If \( x \) is the displacement from the mean cluster position along the line-of-sight such that \( r = \sqrt{R_k^2 + x^2} \), then the probability that a star is at \( x \) is \( p(x) \), where

\[
p(x) \sim \rho_i(\sqrt{R_k^2 + x^2}).
\]  

Three-dimensional positions, along with corresponding model-dependent radial and tangential velocities were drawn at random from their respective probability distributions. The velocity component along the line-of-sight was then determined, and an error term, drawn from a Gaussian distribution with standard deviation equal to the velocity uncertainty, as tabulated in Table 8, was added. This process was repeated, producing 10000 sets of data each with a given mass, \( r_a \), \( x \) and the same projected positions and velocity measurement errors as the original data set. Finally the maximum likelihood technique was applied to each of the artificial data sets and the results compared to the input values for the models. From this we noticed that the maximum likelihood method resulted in scale velocities that were biased systematically too low by approximately 4\% (the values for \( v_s \) in Tables 4 and 5 have already been corrected for this effect).

A goodness-of-fit statistic

\[
\zeta^2 = \sum \frac{(v_{x,k} - \overline{\nabla})^2}{(v_s^2 \sigma_{p,k} + v_{err,k}^2)}
\]  

was generated for each model and is shown in column 9 of Tables 4 and 5 (33 degrees of freedom). The distribution of this statistic can be extracted from the Monte Carlo simulations and column 10 shows the probability of exceeding the observed \( \zeta^2 \) assuming that the cluster velocities are specified by the model parameters indicated and have the uncertainties tabulated in Table 8. The greater this probability the higher the likelihood that the cluster velocities are drawn from the specified distribution.
The results of the KM kinematic modeling can be summarized as follows: 1) The results for the two different bandpasses are consistent. 2) As can be seen from the $P(>\zeta^2)$, the radial velocity data are too sparse to provide a means of discriminating between the different sets of dynamical parameters with any confidence. However, the multi-mass models are in marginally better agreement with the kinematic data than the single-mass models. 3) A similar trend is seen in the dynamical M/L’s as was seen for the population M/L’s: the global M/L is very poorly determined while the central M/L is much more tightly constrained. As $x$ is increased, the number of low-mass stars at large radii is increased. The resultant change in the gravitational potential at points where we have measured stellar velocities is minor, and therefore there is very little change in $v_s$. As was mentioned in §3, there is also very little change in the central luminosity density and, therefore, a relatively model-independent central M/L. 4) The best agreement between population and dynamical M/L’s is seen for the cases $x = 0.0$ for the B band $[(M/L)_0 = 0.14 \pm 0.06 \, M_\odot/L_{B\odot}$ and $M/L = 0.34 \pm 0.15 \, M_\odot/L_{B\odot}]$ and $x = 0.5$ for the V $[(M/L)_0 = 0.13 \pm 0.06 \, M_\odot/L_{V\odot}$ and $M/L = 0.40 \pm 0.15 \, M_\odot/L_{V\odot}]$, independent of $r_a$. However, there is agreement at fairly high confidence levels for all values of $x$. 5) Because the velocity uncertainties are about 70% of the derived velocity dispersion, the cluster mass estimates are somewhat dependent upon the accuracy of these uncertainties. As discussed earlier, the $\chi^2$ for the 10 stars with multiple measurements came out lower than average indicating that the uncertainties are probably not underestimated. If, however, they are overestimated then one would get an underestimate for both the velocity dispersion and the mass. The worst possible (not to mention unreasonable) case is that the uncertainties are actually zero. In this case, we get a slightly less than 30% increase in the velocity dispersion and about a 60% increase in the cluster mass and M/L.

6.2 Oblate Spheroids

In order to determine the cause of flattening we have decided to model NGC 1978 as both a rotating and non-rotating (i.e. anisotropically supported) oblate spheroid. Because
of the sparseness of the kinematic data, the goal will be to construct models with *extreme* sets of parameters and see which provide the greatest consistency with the data. This will also yield a mass range for the cluster.

For an axisymmetric system, the relevant Jeans’ equations (velocity moments of the collisionless Boltzmann equation) in cylindrical coordinates are:

\[
\frac{\partial (\rho \sigma_R^2)}{\partial R} + \frac{\partial (\rho \sigma_{Rz})}{\partial z} + \rho \left( \frac{\sigma_R^2 - \sigma_\phi^2 - v_\phi^2}{R} \right) + \rho \frac{\partial \Phi}{\partial R} = 0, \tag{13}
\]

and

\[
\frac{\partial (\rho \sigma_{Rz})}{\partial R} + \frac{\partial (\rho \sigma_z^2)}{\partial z} + \frac{\rho \sigma_{Rz}}{R} + \rho \frac{\partial \Phi}{\partial z} = 0, \tag{14}
\]

where \((R, \phi, z)\) are the cylindrical coordinate axes, \((\sigma_R, \sigma_\phi, \sigma_z)\) are the corresponding velocity dispersions, and \(v_\phi\) is the rotation velocity. \(\Phi\) is the gravitational potential.

Both the rotating and non-rotating models which were used have velocity ellipsoids aligned with the cylindrical coordinate axes (i.e. \(\sigma_{Rz} = 0\)) and, as well, both have

\[
\sigma_\phi = \frac{\sigma_R}{\sqrt{1 + (R/R_a)^2}}, \tag{15}
\]

where \(R_a\) is varied from 5 pc to \(\infty\). The rotating models also have the condition

\[
\sigma_R = \sigma_z, \tag{16}
\]

implying

\[
v_\phi^2 = R \frac{\partial \Phi}{\partial R} + \frac{R}{\rho} \frac{\partial}{\partial R} \int_z^\infty \rho \frac{\partial \Phi}{\partial z} dz + \frac{1}{\rho} \left[ 1 - \frac{1}{\sqrt{1 + (R/R_a)^2}} \right] \int_z^\infty \rho \frac{\partial \Phi}{\partial z} dz. \tag{17}
\]

The models are constructed by assuming that the mass distribution is equivalent to the deprojected light distribution (constant M/L). Equations 14 and 17 can then be solved directly to obtain \(\sigma_z\), and \(v_\phi\). Once \(v_\phi\) is known it can be substituted into equation 13 which, in turn, can be solved for \(\sigma_R\) and \(\sigma_\phi\). This is outlined in Binney and Tremaine (1987) for the \(\sigma_R = \sigma_\phi = \sigma_z\) case.
Fig. 6 shows isovelocity maps for the models with $R_a = \infty$. The top panel is $\sigma_R$ for the rotating model and the middle panel is the corresponding $v_\phi$. The bottom panel is $\sigma_R$ for the non-rotating model. Unfortunately, it is impossible to determine the inclination of NGC 1978 so we assume that it is $90^\circ$. We believe this to be reasonable as it is already among the most elliptical clusters known and, of course, if it is inclined then it is intrinsically even more elliptical.

Once the models have been generated, it is simply a matter of scaling them using a similar maximum likelihood method to that employed for the KM models. Table 9 shows the results of this exercise; column 1 is $R_a$, while columns 2, 3, 4 and 5 are, respectively, the total mass, the goodness-of-fit parameter $\zeta^2$, the probability of exceeding it (as described in §6.1) and the cluster M/L$_V$ for the non-rotating case. Similarly, columns 6, 7, 8, and 9 represent the rotating case.

There are four points worth noting: 1) The distribution of $\zeta^2$ is much broader for the rotating models. Fig. 7 shows histograms of $\zeta^2$ based on sets of 10000 Monte Carlo simulations for two $R_a = \infty$ models. The solid line is the non-rotating model while the dashed line is the rotating model. It is because of this broader distribution that we cannot completely reject the rotating models. 2) The rotating models produce higher masses than the non-rotating models, opposite to what is seen in NGC 1866 (Fischer et al. 1992), a cluster which has rotation detected at the 97% confidence level. In fact, higher masses should result from the application of a rotating model to a non-rotating system since the removal of a non-existent rotation field causes an increase in the apparent velocity dispersion. 3) The oblate spheroid masses are consistent with the spherical single-mass KM model masses derived in §6.1. Even with $\epsilon \approx 0.3$ a spherical approximation is excellent. 4) The non-rotating (anisotropic velocity dispersion) models appear to represent a better fit to the data, although it is impossible to exclude the rotating models, especially those with higher values of $R_a$. This trend to higher P($> \zeta^2$) with increasing $R_a$ is seen in the non-rotating case as well. The precision and size of our radial velocity data set is
insufficient to strongly constrain the flattening mechanism.

There is very little evidence to support the hypothesis of a recent cluster-cluster merger. The cluster light distribution is very smooth, the isophotes agree well with ellipses with no evidence for subclustering, and there is no sign of tidal interaction. Furthermore, except in very restricted cases, a recent merger would tend to give the cluster a net rotation for which there is no strong indication.

Because these are single mass models (i.e. the mass scales as the luminosity) they do not explicitly yield information about the mass function. However, it is possible to test which mass function is consistent with the dynamical M/L\_V. We maintain the same assumptions about the form of the mass function (i.e. a power-law with a flattening at 0.3 M\_☉), the mass-luminosity relationship and the mass of the stellar remnants that were used for the KM modeling.

The population M/L\_V is given by equation 4. We conclude that it is very difficult to reconcile the low M/L\_V’s with the adopted form of the mass function without invoking a high low-mass cut-off. With the KM low-mass cut-off (0.15 M\_☉), the lowest population M/L\_V that can be achieved is M/L\_V = 0.40 M\_☉/L\_V\_☉ at x ≈ 0.2. It is necessary to raise the low-mass cut-off to 0.8 M\_☉ (with x = 1.6) in order to get a population M/L\_V of about 0.2 M\_☉/L\_V\_☉. This seems unreasonably high for a low-mass cut-off and leads us to believe that either our adopted mass function is an oversimplification or the assumption that mass follows light is unreasonable. Certainly, we see from the multi-mass KM models that it is possible to get excellent agreement between population and dynamical M/L\_V’s supporting the latter supposition.

7. COMPARISON WITH PREVIOUS RESULTS

There have been two previous kinematic studies of NGC 1978, and we consider these in turn. Meylan et al. (1991) find a central velocity dispersion of 5.8 ± 1.2 km s\(^{-1}\) based upon an integrated spectrum of the cluster center (they use a region that does not overlap with our radial velocity data set). Seitzer (1991) finds a dispersion of 3.7 ± 0.8 km s\(^{-1}\).
based upon 8 stellar velocities within two core radii of the cluster center. Seitzer further claims that the central velocity dispersion is a model-dependent 10 - 40% higher than this number. Our central velocity dispersion is also model-dependent and is about \( \sigma_0 = 2.2 \pm 0.5 \text{ km s}^{-1} \).

Our result disagrees with Meylan et al. at the 2.8\( \sigma \) level and, even if we assume that our radial velocity uncertainties are zero, we get an upper limit of \( \sigma_0 = 2.8 \pm 0.6 \text{ km s}^{-1} \), still more than 2.2\( \sigma \) lower. Perhaps their result is affected by a large rotation velocity at the cluster center, but it is not possible to rule out a central mass density cusp. If so, the surface photometry does not reveal an accompanying luminosity density cusp. The disagreement with Seitzer is at approximately the 2\( \sigma \) level and it is not possible to definitively resolve this discrepancy without knowledge of which stars he measured.

We conclude by stating that when measuring a velocity dispersion the sources of error one encounters, such as binaries, field stars, slit errors, zero-point drift, etc. will tend to bias the result too high. One possible source of error than can cause the opposite effect occurs when two (or more) stars fall in the slit. This will tend to give the mean velocity for the two objects, which will, on average, be lower than the residual velocities of the individual stars. We feel that this has not been a problem with our sample of giants which are relatively isolated and significantly brighter than the underlying cluster light.

8. CONCLUSIONS

In this paper we have examined the internal dynamics of the elliptical LMC cluster NGC 1978 using BV CCD images and echelle spectra of 35 giants. Projected radii for the giants range from \( 1.4 \leq R \text{ (pc)} \leq 20.0 \) and the mean estimated stellar velocity uncertainty is \( \sigma_{\text{err}} \approx 1.6 \text{ km s}^{-1} \). The mean cluster velocity is \( \overline{v} = 293.3 \pm 1.0 \text{ km s}^{-1} \).

1) BV luminosity profiles were constructed out to projected radii of \( R > 100 \text{ pc} \). Despite the large ellipticity, single and multi-mass King-Michie models with \( 5.0 \leq r_a \) \( (r_s) \leq \infty \) and \( 0.0 \leq x \leq 2.0 \) were applied to the data. The single-mass models provided better agreement with the surface photometry which is perhaps not surpris-
ing since NGC 1978 is considerably younger than its half-mass two-body relaxation time. Among multi-mass models, there is (slightly) better agreement seen for the models with shallow mass functions. The total cluster luminosity is model dependent; $L_B = 3.1 - 3.7 \pm 0.2 \times 10^5 L_{B\odot}$ and $L_V = 3.0 - 3.5 \pm 0.2 \times 10^5 L_{V\odot}$.

2) The single mass KM models yielded $M/L = 0.20 \pm 0.08 M_\odot/L_\odot$. For the multi-mass KM models, we found that while the central $M/L$’s were relatively tightly constrained to be around $(M/L)_0 = 0.13 \pm 0.06 M_\odot/L_\odot$ the global $M/L$’s ranged over more than a factor of five (i.e., $M/L = 0.3 - 1.5 M_\odot/L_\odot$). The best agreement between the population and dynamical $M/L$’s is seen for the cases $x = 0.0$ for the $B$ band $[(M/L_B)_0 = 0.14\pm 0.06 M_\odot/L_{B\odot}$ and $M/L_B = 0.34 \pm 0.15 M_\odot/L_{B\odot}]$ and $x = 0.5$ for the $V$ $[(M/L)_0 = 0.13 \pm 0.06 M_\odot/L_{V\odot}$ and $M/L = 0.40 \pm 0.15 M_\odot/L_{V\odot}]$, independent of $r_a$. The kinematic data were too sparse to place strong constraints on dynamical parameters such as the anisotropy radius or the mass function.

3) Non-rotating single-mass oblate spheroid models produced $M/L$’s consistent with the single-mass KM models while the rotating models had marginally higher $M/L$’s. We found that the non-rotating model was in better agreement with the kinematic data but that it was impossible to completely rule out the rotating models. As well, there is very little morphological evidence for a merger; the light distribution is quite smooth, the isophotes are very elliptical (i.e. no subclustering) and there is no sign of tidal interaction.

4) In order to get consistency between the single-mass dynamical $M/L$ and a simple power-law mass function requires an unusually high low-mass cut-off. A more probable solution invokes the multi-mass models or perhaps a more complex form for the mass function.

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FIGURE CAPTIONS

Fig. 1– Apparent BV color-magnitude diagram for NGC 1978. Stars above and to the left of the solid line were subtracted prior to surface brightness measurements.

Fig. 2– Plot of elliptical contours fit to star-subtracted and median-filtered BV CCD images. The solid lines are the isophotes and the dashed lines are the best fit ellipses. The elliptical parameters are tabulated in Table 1.

Fig. 3– CCD BV luminosity profiles for NGC 1978. The solid lines are the single-mass isotropic KM models, while the short-dashed lines are the multi-mass \( x = 1.0 \) isotropic models. The long-dashed lines are typical stellar profiles (FWHM approximately one tenth the cluster core radius).

Fig. 4– A finder chart for stars with measured radial velocities. North is at the top, and east is to the left.

Fig. 4b – Positions with respect to the cluster center for stars with measured radial velocities. The straight line is the photometric minor axis.

Fig. 5– Plot of observed radial velocity versus position angle (lower panel) and projected radius (upper panel). The solid lines indicate the cluster mean velocity.

Fig. 6– Isovelocity maps for oblate spheroid models. The top panel is \( \sigma_R \) for the rotating model and the middle panel is the corresponding \( v_\phi \). The bottom panel is \( \sigma_R \) for the non-rotating model.

Fig. 7– Histograms of \( \zeta^2 \) based on sets of 10000 Monte Carlo simulations for two \( R_a = \infty \) oblate spheroid models. The solid line is the non-rotating model while the dashed line is the rotating model.
Table 1
Ellipticity and Position Angle

| a (pc) | $\epsilon$  | PA ($^\circ$) | $\epsilon$  | PA ($^\circ$) |
|-------|-------------|---------------|-------------|---------------|
| 2.9   | 0.33 ± 0.01 | 164.0 ± 2.0   | 0.29 ± 0.01 | 161.0 ± 2.0   |
| 5.2   | 0.33 ± 0.02 | 147.0 ± 2.0   | 0.28 ± 0.02 | 158.0 ± 3.0   |
| 7.6   | 0.25 ± 0.03 | 151.0 ± 4.0   | 0.20 ± 0.03 | 154.0 ± 4.0   |
| 10.1  | 0.28 ± 0.02 | 148.0 ± 2.0   | 0.25 ± 0.02 | 148.0 ± 2.0   |
| 13.4  | 0.31 ± 0.04 | 146.0 ± 4.0   | 0.30 ± 0.02 | 148.0 ± 3.0   |
| 17.8  | 0.31 ± 0.04 | 146.0 ± 4.0   | 0.33 ± 0.03 | 148.0 ± 3.0   |
| 21.6  | 0.34 ± 0.03 | 147.0 ± 3.0   | 0.34 ± 0.03 | 147.0 ± 3.0   |
| Mean  | 0.32 ± 0.02 | 151.0 ± 7.0   | 0.29 ± 0.03 | 153.0 ± 6.0   |
Table 2
Surface Photometry

\[
\begin{array}{cccc|cccc}
R & a & L_B & R & a & L_V \\
(pc) & (pc) & (L_B \odot \text{ pc}^{-2}) & (pc) & (pc) & (L_V \odot \text{ pc}^{-2}) \\
\hline
0.4 & 0.5 & 1886.0 \pm 140.0 & 0.4 & 0.5 & 2406.0 \pm 305.0 \\
0.6 & 0.7 & 1668.0 \pm 137.0 & 0.6 & 0.7 & 2142.0 \pm 199.0 \\
0.8 & 1.0 & 1697.0 \pm 92.0 & 0.8 & 0.9 & 1813.0 \pm 141.0 \\
1.0 & 1.2 & 1599.0 \pm 88.0 & 1.0 & 1.2 & 1678.0 \pm 88.0 \\
1.3 & 1.5 & 1479.0 \pm 107.0 & 1.3 & 1.4 & 1469.0 \pm 189.0 \\
1.6 & 1.9 & 1429.0 \pm 121.0 & 1.6 & 1.8 & 1674.0 \pm 146.0 \\
2.0 & 2.4 & 1478.0 \pm 72.0 & 2.0 & 2.3 & 1806.0 \pm 170.0 \\
2.6 & 3.0 & 1195.0 \pm 58.0 & 2.5 & 2.9 & 1266.0 \pm 77.0 \\
3.2 & 3.8 & 1018.0 \pm 38.0 & 3.2 & 3.6 & 1049.0 \pm 38.0 \\
4.1 & 4.8 & 829.0 \pm 47.0 & 4.0 & 4.6 & 910.0 \pm 73.0 \\
5.1 & 6.0 & 687.0 \pm 24.0 & 5.0 & 5.8 & 755.0 \pm 41.0 \\
6.5 & 7.6 & 496.0 \pm 22.0 & 6.3 & 7.3 & 507.0 \pm 30.0 \\
8.2 & 9.5 & 320.0 \pm 11.0 & 7.9 & 9.1 & 324.0 \pm 27.0 \\
10.3 & 12.0 & 201.0 \pm 8.2 & 10.0 & 11.5 & 213.0 \pm 15.0 \\
12.9 & 15.1 & 134.0 \pm 9.6 & 12.6 & 14.5 & 136.0 \pm 12.0 \\
16.3 & 19.0 & 73.0 \pm 2.5 & 15.8 & 18.2 & 78.0 \pm 7.4 \\
20.5 & 24.0 & 35.0 \pm 5.4 & 20.0 & 23.0 & 38.0 \pm 7.7 \\
25.8 & 30.2 & 15.0 \pm 3.1 & 25.1 & 28.9 & 15.0 \pm 3.7 \\
32.4 & 38.0 & 8.6 \pm 3.7 & 31.6 & 36.4 & 7.2 \pm 3.3 \\
40.9 & 47.8 & 3.5 \pm 2.3 & 39.8 & 45.8 & 3.6 \pm 1.6 \\
51.4 & 60.2 & 0.2 \pm 1.9 & & & \\
\end{array}
\]
| Bin | $m_{min}$ (M$_\odot$) | $m_{max}$ (M$_\odot$) |
|-----|----------------------|----------------------|
| 1   | 0.16                 | 0.30                 |
| 2   | 0.30                 | 0.45                 |
| 3   | 0.45                 | 0.60                 |
| 4   | 0.60                 | 0.75                 |
| 5   | 0.75                 | 0.90                 |
| 6   | 0.90                 | 1.05                 |
| 7   | 1.05                 | 1.20                 |
| 8   | 1.20                 | 1.43                 |
| 9   | 1.43                 | 1.65                 |
Table 4
King-Michie - B Band Fitted Parameters

| \( r_a \) (\( r_s \)) | Photometry | Velocities |
|-----------------|------------|------------|
|                 | \( W_0 \)  | \( r_s \) (pc) | \( c \)     | \( \chi^2_{\nu} \) (\( \nu = 18 \)) | \( P(> \chi^2_{\nu}) \) | \( v_s \) (km s\(^{-1}\)) | \( \zeta^2 \) | \( P(> \zeta^2) \) |
| ISO 0.0         | 5.9 ± 0.2  | 4.6 ± 0.2 | 17.0 ± 2.0 | 0.75 | 0.74 | 2.33 ± 0.5 | 33.79 | 0.71 |
| ISO 0.0         | 6.9 ± 0.4  | 5.6 ± 0.2 | 21.0 ± 4.0 | 1.21 | 0.20 | 2.31 ± 0.5 | 33.00 | 0.84 |
| ISO 0.5         | 7.5 ± 0.4  | 5.7 ± 0.2 | 24.0 ± 4.0 | 1.33 | 0.13 | 2.28 ± 0.5 | 32.89 | 0.85 |
| ISO 0.5         | 7.6 ± 0.4  | 5.7 ± 0.2 | 29.0 ± 8.0 | 1.36 | 0.12 | 2.28 ± 0.5 | 32.96 | 0.85 |
| ISO 0.5         | 7.8 ± 0.1  | 5.7 ± 0.1 | 90.0 ± 30.0 | 1.34 | 0.16 | 2.30 ± 0.5 | 33.15 | 0.82 |
| ISO 1.0         | 9.0 ± 0.4  | 5.6 ± 0.2 | 33.0 ± 6.0 | 1.47 | 0.09 | 2.22 ± 0.5 | 32.77 | 0.87 |
| ISO 1.0         | 9.3 ± 0.2  | 5.4 ± 0.1 | 61.0 ± 13.0 | 1.39 | 0.17 | 2.22 ± 0.5 | 32.85 | 0.86 |
| ISO 1.5         | 11.0 ± 0.2 | 5.5 ± 0.1 | 51.0 ± 4.0 | 1.51 | 0.11 | 2.18 ± 0.5 | 32.71 | 0.87 |
| ISO 1.5         | 10.6 ± 0.2 | 5.6 ± 0.1 | 93.0 ± 26.0 | 1.30 | 0.20 | 2.19 ± 0.5 | 32.81 | 0.86 |
| ISO 1.5         | 9.1 ± 0.1  | 6.3 ± 0.1 | 101.0 ± 30.0 | 1.94 | 0.01 | 2.31 ± 0.5 | 33.01 | 0.84 |
| ISO 2.0         | 13.4 ± 0.2 | 5.4 ± 0.1 | 72.0 ± 5.0 | 1.40 | 0.21 | 2.16 ± 0.5 | 32.70 | 0.87 |
| ISO 2.0         | 11.8 ± 0.3 | 6.0 ± 0.1 | 105.0 ± 36.0 | 1.54 | 0.10 | 2.23 ± 0.5 | 32.78 | 0.86 |
| ISO 2.0         | 9.9 ± 0.1  | 6.8 ± 0.1 | 102.0 ± 15.0 | 2.97 | 0.00 | 2.35 ± 0.5 | 32.95 | 0.84 |
| $r_a$ (rs) | x | $W_0$ | $r_s$ (pc) | c | $\chi^2_{\nu}$ (ν = 17) | $P(>\chi^2_{\nu})$ | $v_s$ (km s$^{-1}$) | $\zeta^2$ | $P(>\zeta^2)$ |
|-----------|---|--------|------------|---|----------------|-------------------|------------------|--------|-------------|
| ISO       |   | 6.0 ± 0.2 | 4.3 ± 0.2 | 18.0 ± 2.0 | 1.27 | 0.18 | 2.35 ± 0.5 | 33.84 | 0.70 |
| 10        |   | 5.9 ± 0.2 | 4.4 ± 0.2 | 19.0 ± 3.0 | 1.29 | 0.17 | 2.35 ± 0.5 | 33.91 | 0.69 |
| 5         |   | 5.7 ± 0.2 | 4.6 ± 0.2 | 21.0 ± 6.0 | 1.37 | 0.12 | 2.40 ± 0.5 | 34.10 | 0.65 |
| ISO 0.0   |   | 6.7 ± 0.4 | 5.5 ± 0.2 | 20.0 ± 4.0 | 1.64 | 0.04 | 2.35 ± 0.5 | 33.05 | 0.83 |
| 10 0.0    |   | 6.7 ± 0.5 | 5.5 ± 0.2 | 22.0 ± 7.0 | 1.67 | 0.03 | 2.35 ± 0.5 | 33.12 | 0.82 |
| 5 0.0     |   | 7.3 ± 0.4 | 5.4 ± 0.2 | 65.0 ± 30.0 | 1.74 | 0.03 | 2.35 ± 0.5 | 33.26 | 0.80 |
| ISO 0.5   |   | 7.3 ± 0.5 | 5.5 ± 0.2 | 22.0 ± 5.0 | 1.71 | 0.03 | 2.30 ± 0.5 | 32.93 | 0.85 |
| 10 0.5    |   | 7.2 ± 0.6 | 5.6 ± 0.2 | 24.0 ± 8.0 | 1.73 | 0.03 | 2.30 ± 0.5 | 33.01 | 0.84 |
| 5 0.5     |   | 7.7 ± 0.3 | 5.5 ± 0.2 | 71.3 ± 40.0 | 1.72 | 0.03 | 2.30 ± 0.5 | 33.19 | 0.81 |
| ISO 1.0   |   | 8.2 ± 0.6 | 5.7 ± 0.3 | 25.0 ± 6.0 | 1.82 | 0.02 | 2.30 ± 0.5 | 32.81 | 0.86 |
| 10 1.0    |   | 9.1 ± 0.6 | 5.3 ± 0.3 | 50.0 ± 24.0 | 1.79 | 0.02 | 2.25 ± 0.5 | 32.87 | 0.86 |
| 5 1.0     |   | 8.4 ± 0.2 | 5.7 ± 0.2 | 94.5 ± 35.0 | 1.74 | 0.03 | 2.30 ± 0.5 | 33.10 | 0.83 |
| ISO 1.5   |   | 10.5 ± 0.4 | 5.5 ± 0.2 | 43.0 ± 6.0 | 1.87 | 0.02 | 2.20 ± 0.5 | 32.72 | 0.87 |
| 10 1.5    |   | 10.4 ± 0.4 | 5.4 ± 0.2 | 73.0 ± 35.0 | 1.73 | 0.03 | 2.20 ± 0.5 | 32.82 | 0.84 |
| 5 1.5     |   | 9.1 ± 0.1 | 6.0 ± 0.2 | 89.3 ± 35.0 | 1.93 | 0.02 | 2.30 ± 0.5 | 33.03 | 0.83 |
| ISO 2.0   |   | 12.9 ± 0.3 | 5.3 ± 0.2 | 64.0 ± 8.0 | 1.80 | 0.02 | 2.20 ± 0.5 | 32.70 | 0.87 |
| 10 2.0    |   | 11.7 ± 0.2 | 5.8 ± 0.2 | 93.0 ± 25.0 | 1.78 | 0.02 | 2.25 ± 0.5 | 32.79 | 0.86 |
| 5 2.0     |   | 9.9 ± 0.1 | 6.5 ± 0.2 | 99.0 ± 17.0 | 2.33 | 0.00 | 2.35 ± 0.5 | 32.98 | 0.84 |
| $r_a$ ($r_s$) | $x$ | $L_{B_0}$ ($L_{B_0}$ pc$^{-3}$) | $L_B$ ($10^5 L_{B_0}$) | $\rho_0$ (M$_\odot$ pc$^{-3}$) | $M$ ($10^3 M_\odot$) | Population ($M/L_B$) | Dynamical ($M/L_B$) |
|-------------|-----|-------------------------------|----------------|------------------|----------------|-------------------|-------------------|
| ISO 200.0 ± 10.0 | 0.10 | 3.10 ± 0.08 | 52.0 ± 20.0 | 0.70 ± 0.30 | | 0.21 ± 0.08 | 0.21 ± 0.08 |
| 10 197.0 ± 10.0 | 0.10 | 3.14 ± 0.08 | 52.0 ± 20.0 | 0.70 ± 0.30 | | 0.20 ± 0.08 | 0.20 ± 0.08 |
| 5 192.0 ± 10.0 | 0.11 | 3.21 ± 0.11 | 50.0 ± 20.0 | 0.70 ± 0.30 | | 0.20 ± 0.08 | 0.20 ± 0.08 |
| ISO 0.0 182.0 ± 8.0 | 0.12 | 3.30 ± 0.12 | 28.0 ± 10.0 | 1.20 ± 0.45 | | 0.15 ± 0.06 | 0.34 ± 0.15 |
| 10 0.0 178.0 ± 8.0 | 0.13 | 3.32 ± 0.13 | 28.0 ± 10.0 | 1.20 ± 0.45 | | 0.15 ± 0.06 | 0.34 ± 0.15 |
| 5 0.0 174.0 ± 11.0 | 0.07 | 3.64 ± 0.07 | 28.0 ± 10.0 | 1.45 ± 0.50 | | 0.13 ± 0.06 | 0.38 ± 0.15 |
| ISO 0.5 178.0 ± 8.0 | 0.14 | 3.33 ± 0.13 | 27.0 ± 11.0 | 1.40 ± 0.50 | | 0.12 ± 0.06 | 0.40 ± 0.15 |
| 10 0.5 177.0 ± 8.0 | 0.14 | 3.37 ± 0.14 | 27.0 ± 11.0 | 1.45 ± 0.50 | | 0.12 ± 0.06 | 0.40 ± 0.15 |
| 5 0.5 171.0 ± 11.0 | 0.07 | 3.58 ± 0.07 | 27.0 ± 11.0 | 1.75 ± 0.50 | | 0.11 ± 0.06 | 0.44 ± 0.20 |
| ISO 1.0 178.0 ± 8.0 | 0.14 | 3.50 ± 0.14 | 26.0 ± 10.0 | 2.10 ± 0.85 | | 0.10 ± 0.06 | 0.55 ± 0.25 |
| 10 1.0 183.0 ± 7.0 | 0.09 | 3.61 ± 0.09 | 28.0 ± 11.0 | 2.35 ± 0.95 | | 0.10 ± 0.06 | 0.61 ± 0.25 |
| 5 1.0 162.0 ± 11.0 | 0.08 | 3.48 ± 0.08 | 25.0 ± 10.0 | 2.10 ± 0.85 | | 0.10 ± 0.06 | 0.57 ± 0.25 |
| ISO 1.5 183.0 ± 7.0 | 0.09 | 3.64 ± 0.09 | 26.0 ± 10.0 | 3.55 ± 1.40 | | 0.09 ± 0.06 | 0.91 ± 0.40 |
| 10 1.5 173.0 ± 10.0 | 0.07 | 3.60 ± 0.07 | 26.0 ± 10.0 | 3.55 ± 1.40 | | 0.09 ± 0.06 | 0.91 ± 0.40 |
| 5 1.5 150.0 ± 10.0 | 0.06 | 3.33 ± 0.06 | 22.0 ± 9.0 | 2.80 ± 1.10 | | 0.10 ± 0.06 | 0.78 ± 0.30 |
| ISO 2.0 183.0 ± 7.0 | 0.08 | 3.70 ± 0.08 | 27.0 ± 11.0 | 6.15 ± 2.45 | | 0.09 ± 0.06 | 1.54 ± 0.65 |
| 10 2.0 157.0 ± 10.0 | 0.10 | 3.40 ± 0.10 | 23.0 ± 9.0 | 4.95 ± 1.95 | | 0.10 ± 0.06 | 1.35 ± 0.55 |
| 5 2.0 139.0 ± 8.0 | 0.06 | 3.17 ± 0.06 | 20.0 ± 8.0 | 3.70 ± 1.45 | | 0.14 ± 0.05 | 1.07 ± 0.45 |
### Table 7
King-Michie - V Band Derived Parameters

| r_a (r_s) | x | L_V0 (L_V⊙ pc⁻³) | L_V (10⁵L_V⊙) | ρ₀ (M⊙ pc⁻³) | M (10²M⊙) | Population (M/L_V)₀ | (M/L_V)₀ | Population (M/L_V) | (M/L_V) | Dynamical (M/L_V)₀ | (M/L_V) |
|-----------|---|-------------------|--------------|----------|----------|---------------------|----------|---------------------|----------|--------------------|----------|
| ISO       | 222.0 ± 14.0 | 3.07 ± 0.09       | 48.0 ± 20.0  | 0.65 ± 0.25 | 0.20 ± 0.08 | 0.20 ± 0.08         |
| 10        | 222.0 ± 14.0 | 3.08 ± 0.09       | 48.0 ± 20.0  | 0.65 ± 0.25 | 0.20 ± 0.08 | 0.20 ± 0.08         |
| 5         | 213.0 ± 14.0 | 3.10 ± 0.14       | 46.0 ± 20.0  | 0.65 ± 0.25 | 0.20 ± 0.08 | 0.20 ± 0.08         |
| ISO       | 200.0 ± 11.0 | 3.13 ± 0.14       | 30.0 ± 12.0  | 1.10 ± 0.45 | 0.18 ± 0.08 | 0.12 ± 0.06         |
| 10        | 198.0 ± 11.0 | 3.14 ± 0.14       | 30.0 ± 12.0  | 1.10 ± 0.45 | 0.18 ± 0.08 | 0.12 ± 0.06         |
| 5         | 202.0 ± 12.0 | 3.41 ± 0.14       | 30.0 ± 12.0  | 1.30 ± 0.50 | 0.16 ± 0.08 | 0.12 ± 0.06         |
| ISO       | 198.0 ± 11.0 | 3.16 ± 0.14       | 30.0 ± 12.0  | 1.30 ± 0.50 | 0.15 ± 0.06 | 0.13 ± 0.06         |
| 10        | 195.0 ± 11.0 | 3.16 ± 0.16       | 30.0 ± 12.0  | 1.30 ± 0.50 | 0.14 ± 0.06 | 0.13 ± 0.06         |
| 5         | 197.0 ± 13.0 | 3.38 ± 0.12       | 30.0 ± 12.0  | 1.60 ± 0.50 | 0.13 ± 0.06 | 0.13 ± 0.06         |
| ISO       | 192.0 ± 11.0 | 3.19 ± 0.14       | 26.0 ± 10.0  | 1.70 ± 0.70 | 0.13 ± 0.05 | 0.12 ± 0.05         |
| 10        | 203.0 ± 11.0 | 3.39 ± 0.17       | 30.0 ± 12.0  | 2.15 ± 0.85 | 0.12 ± 0.06 | 0.13 ± 0.06         |
| 5         | 183.0 ± 14.0 | 3.34 ± 0.11       | 27.0 ± 11.0  | 2.05 ± 0.85 | 0.12 ± 0.06 | 0.13 ± 0.06         |
| ISO       | 198.0 ± 10.0 | 3.37 ± 0.14       | 27.0 ± 11.0  | 3.05 ± 1.20 | 0.11 ± 0.07 | 0.12 ± 0.05         |
| 10        | 198.0 ± 11.0 | 3.39 ± 0.14       | 27.0 ± 11.0  | 3.20 ± 1.25 | 0.11 ± 0.07 | 0.12 ± 0.05         |
| 5         | 170.0 ± 13.0 | 3.22 ± 0.09       | 25.0 ± 10.0  | 2.70 ± 1.10 | 0.12 ± 0.06 | 0.13 ± 0.06         |
| ISO       | 202.0 ± 9.0  | 3.48 ± 0.14       | 28.0 ± 11.0  | 5.40 ± 2.15 | 0.11 ± 1.19 | 0.12 ± 0.05         |
| 10        | 181.0 ± 11.0 | 3.29 ± 0.10       | 25.0 ± 10.0  | 4.55 ± 1.80 | 0.12 ± 1.20 | 0.12 ± 0.05         |
| 5         | 160.0 ± 10.0 | 3.08 ± 0.09       | 22.0 ± 9.0   | 3.50 ± 1.40 | 0.14 ± 1.22 | 0.12 ± 0.05         |
Table 8  
Radial Velocities  

| ID | Telescope | $R$ (pc) | $\Theta$ (°) | $v_r$ (km s$^{-1}$) | $\langle v_r \rangle$ (km s$^{-1}$) | HJD $(-2448000)$ | V (mag) | B-V (mag) |
|----|-----------|---------|-------------|-------------------|-------------------|-----------------|---------|-----------|
| 1  | CTIO      | 1.4     | 346.7       | 295.0 ± 2.1       | 294.9 ± 1.8       | 276.7601        | 16.88   | 0.82      |
|    | CTIO      |         |             |                   |                   | 275.7578        |         |           |
| 2  | CTIO      | 1.5     | 273.3       | 296.4 ± 2.1       | 294.8 ± 1.0       | 276.7251        | 15.94   | 1.36      |
|    | LCO       |         |             |                   |                   | 287.5633        |         |           |
|    | CTIO      |         |             | 294.6 ± 1.2       |                   |                 |         |           |
| 3  | CTIO      | 2.2     | 119.1       | 292.4 ± 2.0       | 292.8 ± 1.5       | 277.5507        | 15.89   | 1.38      |
|    | CTIO      |         |             |                   |                   | 275.7289        |         |           |
|    | CTIO      |         |             | 293.3 ± 2.2       |                   |                 |         |           |
| 4  | LCO       | 2.2     | 316.6       | 294.8 ± 1.5       | 294.2 ± 1.2       | 288.5598        | 16.31   | 1.26      |
|    | CTIO      |         |             |                   |                   | 276.7102        |         |           |
|    | CTIO      |         |             | 293.0 ± 2.5       |                   |                 |         |           |
|    | CTIO      |         |             | 293.5 ± 2.9       |                   |                 |         |           |
| 5  | CTIO      | 2.4     | 353.7       | 291.0 ± 2.9       | 292.8 ± 1.6       | 275.7691        | 16.09   | 1.01      |
|    | CTIO      |         |             |                   |                   | 276.7798        |         |           |
|    | CTIO      |         |             | 293.6 ± 1.9       |                   |                 |         |           |
| 6  | LCO       | 2.5     | 89.0        | 291.4 ± 1.3       | 291.4 ± 1.3       | 287.6484        | 16.35   | 1.08      |
| 7  | CTIO      | 3.0     | 139.4       | 291.4 ± 1.7       | 291.2 ± 1.4       | 277.5654        | 16.48   | 1.44      |
|    | CTIO      |         |             |                   |                   | 275.8273        |         |           |
|    | CTIO      |         |             | 290.8 ± 2.4       |                   |                 |         |           |
| 8  | CTIO      | 3.4     | 44.4        | 293.4 ± 3.3       | 296.3 ± 2.2       | 275.7979        | 16.92   | 1.03      |
|    | CTIO      |         |             |                   |                   | 277.7268        |         |           |
|    | CTIO      |         |             | 298.6 ± 2.9       |                   |                 |         |           |
| 9  | CTIO      | 3.5     | 284.0       | 291.4 ± 2.2       | 291.4 ± 2.2       | 277.6689        | 16.35   | 1.54      |
| 10 | CTIO      | 3.6     | 358.1       | 293.6 ± 1.9       | 293.6 ± 1.9       | 277.7086        | 16.57   | 1.01      |
| 11 | LCO       | 3.9     | 327.2       | 296.7 ± 1.0       | 296.7 ± 1.0       | 287.5977        | 17.09   | 1.39      |
| 12 | LCO       | 4.0     | 4.2         | 292.9 ± 1.4       | 292.9 ± 1.4       | 287.6338        | 16.57   | 1.01      |
| 13 | CTIO      | 4.1     | 36.7        | 304.3 ± 2.6       | Not Used          | 276.7952        | 16.11   | 1.52      |
|    | CTIO      |         |             |                   |                   | 275.7854        |         |           |
|    | CTIO      |         |             | 292.7 ± 3.6       |                   |                 |         |           |
| 14 | CTIO      | 4.3     | 16.1        | 290.7 ± 2.1       | 290.7 ± 2.1       | 277.6874        | 16.59   | 1.46      |
| 15 | LCO       | 4.3     | 234.7       | 295.2 ± 1.5       | 295.2 ± 1.5       | 287.5769        | 16.75   | 1.25      |
Table 8 (cont.)
Radial Velocities

| ID | Telescope | $R$ (pc) | $\Theta$ (°) | $v_r$ (km s$^{-1}$) | $\langle v_r \rangle$ (km s$^{-1}$) | HJD (−2448000) | V (mag) | B-V (mag) |
|----|-----------|---------|-------------|----------------|----------------|---------------|--------|----------|
| 16 | CTIO      | 4.3     | 340.1       | 296.2 ± 2.8    | 291.9 ± 0.9    | 275.7405      | 16.02  | 1.38     |
|    | LCO       |         |             | 291.3 ± 1.1    |               | 302.5609      |        |          |
| 17 | LCO       | 4.6     | 120.1       | 295.5 ± 2.1    | 295.5 ± 2.1    | 289.6231      | 16.98  | 1.10     |
| 18 | CTIO      | 5.0     | 308.3       | 294.2 ± 3.4    | 294.2 ± 3.4    | 277.6038      | 16.70  | 1.42     |
| 19 | LCO       | 5.0     | 104.0       | 292.6 ± 1.5    | 292.6 ± 1.5    | 289.6057      | 16.90  | 1.04     |
| 20 | LCO       | 5.2     | 172.1       | 292.8 ± 0.9    | 292.8 ± 0.9    | 289.5814      | 16.96  | 1.27     |
| 21 | CTIO      | 5.3     | 346.8       | 297.5 ± 2.6    | 297.5 ± 2.6    | 277.7824      | 16.76  | 1.25     |
| 22 | CTIO      | 5.4     | 163.2       | 294.4 ± 1.8    | 294.4 ± 1.8    | 277.8052      | 16.70  | 1.34     |
| 23 | LCO       | 5.6     | 21.7        | 291.8 ± 1.6    | 291.8 ± 1.6    | 287.6130      | 16.64  | 1.31     |
| 24 | CTIO      | 5.6     | 70.3        | 291.8 ± 1.7    | 291.8 ± 1.7    | 277.5862      | 16.31  | 1.11     |
| 25 | LCO       | 8.1     | 67.5        | 291.1 ± 1.6    | 291.1 ± 1.6    | 288.6356      | 16.87  | 1.51     |
| 26 | CTIO      | 8.4     | 325.4       | 291.3 ± 2.9    | 291.3 ± 2.9    | 277.6291      | 17.02  | 1.52     |
| 27 | LCO       | 8.9     | 142.4       | 290.6 ± 0.9    | 290.6 ± 0.9    | 288.6536      | 16.35  | 1.60     |
| 28 | LCO       | 9.1     | 2.8         | 290.1 ± 1.3    | 290.1 ± 1.3    | 288.5932      | 16.91  | 1.41     |
| 29 | LCO       | 9.5     | 40.8        | 296.6 ± 1.3    | 296.6 ± 1.3    | 288.6113      | 16.90  | 1.64     |
| 30 | LCO       | 11.1    | 11.3        | 297.3 ± 0.9    | 297.4 ± 0.7    | 288.5716      | 15.79  | 1.89     |
|    | LCO       |         |             | 297.6 ± 1.0    |               | 287.6762      |        |          |
|    | CTIO      |         |             | 296.7 ± 3.2    |               | 275.8437      |        |          |
| 31 | LCO       | 12.1    | 133.7       | 295.3 ± 1.0    | 295.3 ± 1.0    | 289.5633      | 16.56  | 1.77     |
| 32 | LCO       | 12.5    | 86.6        | 295.7 ± 1.2    | 295.7 ± 1.2    | 289.6661      | 16.82  | 1.56     |
| 33 | LCO       | 12.7    | 285.1       | 291.9 ± 1.4    | 291.9 ± 1.4    | 289.6897      | 16.68  | 1.74     |
| 34 | LCO       | 12.7    | 116.2       | 290.3 ± 1.3    | 290.3 ± 1.3    | 289.6481      | 16.89  | 1.22     |
| 35 | LCO       | 14.0    | 325.4       | 287.5 ± 1.9    | 287.5 ± 1.9    | 289.7078      | 16.84  | 2.10     |
| 36 | LCO       | 19.9    | 155.8       | 294.0 ± 1.4    | 294.0 ± 1.4    | 302.6129      | 16.66  | 1.80     |
Table 9
Oblate Spheroid Models

| $R_a$ (pc) | $M_\infty$ ($10^4 M_\odot$) | $\zeta^2$ | $P(>\zeta^2)$ | $M/L_V$ ($M_\odot/L_\odot$) | $M_\infty$ ($10^4 M_\odot$) | $\zeta^2$ | $P(>\zeta^2)$ | $M/L_V$ ($M_\odot/L_\odot$) |
|-----------|-----------------|----------|-------------|-----------------|-----------------|----------|-------------|-----------------|
| $\infty$  | 6.5 ± 2.5       | 33.75    | 0.72        | 0.20 ± 0.08     | 7.6 ± 2.5       | 39.14    | 0.23        | 0.23 ± 0.09     |
| 100       | 6.5 ± 2.5       | 33.76    | 0.71        | 0.20 ± 0.08     | 7.6 ± 2.5       | 39.28    | 0.23        | 0.23 ± 0.09     |
| 50        | 6.5 ± 2.5       | 33.82    | 0.71        | 0.20 ± 0.08     | 7.8 ± 2.5       | 39.69    | 0.22        | 0.24 ± 0.09     |
| 25        | 6.5 ± 2.5       | 34.07    | 0.66        | 0.20 ± 0.08     | 7.9 ± 3.0       | 41.14    | 0.18        | 0.24 ± 0.09     |
| 10        | 6.6 ± 2.5       | 35.41    | 0.44        | 0.20 ± 0.08     | 9.0 ± 3.0       | 48.04    | 0.09        | 0.27 ± 0.10     |
| 5         | 7.1 ± 2.5       | 36.98    | 0.30        | 0.22 ± 0.09     | 9.8 ± 3.5       | 59.18    | 0.03        | 0.29 ± 0.10     |
