A Modified Quantum Renormalization Group

for
xxz Spin Chain

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Abstract

A simple modification of the standard Renormalization Group (RG) technique for the study of quantum spin systems is introduced. Our method which takes into account the effect of boundary conditions by employing the concept of superblock, may be regarded as a simple way for obtaining first estimates of many properties of spin systems. By applying this method to the xxz spin $\frac{1}{2}$ Heisenberg chain, we obtain the ground state energy with much higher accuracy than the standard RG. We have also obtained the staggered magnetization and the z-component of spin-spin correlation function which confirms the absence of long-range order in the massless region of the 1D xxz model.

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1 Introduction

Soon after the introduction of real space Renormalization Group (RG) by Wilson and it’s application to the Kondo problem\(^1\), it was found that this type of RG does not always produce accurate numerical results, as compared with other powerful techniques. One of the weak performances of the RG method was found in Lee’s work on 2D Anderson localization\(^2\). Using numerical RG method he concluded that there is a critical amplitude for the random potential which induces localization. However, not long after Lee’s work, Lee and Fisher\(^2\), using a different approach, found that the 2D model is logarithmically localized even for arbitrary small randomness. This result is now generally accepted. Such unsatisfactory behavior of the RG method compared with the other methods such as Quantum Monte Carlo, was one of the reasons that the RG method for quantum lattice problems remained undeveloped during the 80’s.

Let us first describe the standard RG method\(^3\) (see ref.4 for a modern treatment and references therein). First, one divides the whole lattice into small blocks (fig.1) and obtains the lowest states (\(|i\rangle\)) of each isolated block for a particular Boundary Condition (BC). The effect of inter-block interactions is taken into account by constructing an effective Hamiltonian \(H^{\text{eff}}\) which now acts on a smaller Hilbert space \(H^{\text{eff}}\) embedded in the original one. In this new Hilbert space, each of the former blocks is treated as a single site.

The technical way of implementing this idea is to construct an embedding operator

\[
T : \mathcal{H}^{\text{eff}} \longrightarrow \mathcal{H}
\]

and a truncation operator

\[
T^\dagger : \mathcal{H} \longrightarrow \mathcal{H}^{\text{eff}}
\]

and demand the commutativity of the following diagram\(^4\):

\[
\begin{array}{c}
\mathcal{H}^{\text{eff}} \\
\mathcal{H}^{\text{eff}}
\end{array} \xrightarrow{T} \begin{array}{c}
\mathcal{H} \\
\mathcal{H}
\end{array}
\]

\[
\begin{array}{c}
\mathcal{H}^{\text{eff}} \\
\mathcal{H}^{\text{eff}}
\end{array} \xrightarrow{T} \begin{array}{c}
\mathcal{H} \\
\mathcal{H}
\end{array}
\]
i.e. : $TH^{eff} = HT$. From the last relation one obtains the effective Hamiltonian as:

$$H^{eff} = T^\dagger HT \quad (1)$$

Note that the operators $T$ and $T^\dagger$ satisfy the relation $T^\dagger T = 1_{H^{eff}}$ but $TT^\dagger \neq 1_H$. More precisely one divides $H$ as $H = H^B + H^{BB}$ (fig.1) where $H^B = \sum_I h_I^B$ is the sum of block Hamiltonians and $H^{BB} = \sum_{I,J} h_{I,J}^{BB}$ is the sum of inter-block Hamiltonians and constructs $T = \prod_I T_I$ where $T_I$ is constructed as follows:

$$T_I = \sum_{i=1}^m |i\rangle \langle i| \quad (2)$$

where $m$ is the number of low energy states kept. Note that each $T_I$ acts trivially on all the other blocks $J \neq I$ consequently $[T_I, T_J] = 0$. Repeating the RG steps one hopes to restrict himself to spaces of lower and lower energy and finally arrives at the ground state energy. In the general case this process induces a renormalization of coupling constants:

$$H^{eff} \equiv H(k'_1, k'_2, ... ) = T^\dagger H(k_1, k_2, ...)T \quad (3)$$

where $K = (k_1, k_2, ...)$ is the coupling constant space.

In principle the form the Hamiltonian changes in a such RG program rendering analytical treatment, rather difficult if not possible. In the standard RG treatment of some models including $xxz$ Heisenberg chain, it is possible to choose the embedding operator so that no new terms appear in the renormalized Hamiltonian$^3$. In such cases, repeating the RG steps, leads to the following general form:

$$T^\dagger H(K)T^n = \sum_{p=0}^{n-1} \frac{N}{l^{p+1}} e_{\alpha}^B(K^{(p)}) + H_{\alpha}(K^{(n)}) \quad (4)$$

here $H_{\alpha}$ is the Hamiltonian of the $N$-sites lattice, $l$ is the number of sites in each block, $n$ is the number of RG steps performed, $e_{\alpha}^B(K^{(p)})$ is the ground state energy of each block after the $p$-th step and under RG : $K^{(n)} \rightarrow K^{(n+1)}$, then the ground state of the whole lattice is then given by:

$$E_{\alpha} = \sum_{p=0}^{\infty} \frac{N}{l^{p+1}} e_{\alpha}^B(K^{(p)}) \quad (5)$$

However the main difficulty of the method is that by fixing a particular BC on an isolated block one may loose a number of states which contribute to the ground state of the whole
lattice due to the interaction of the block with its surrounding. Stated in another way, the
ground state wave function of the whole lattice for a particular BC is not a simple juxtapo-
sition of the ground state wave functions of the blocks. This point is clearly highlighted in
1D Tight Binding model in which the standard RG fails for some type of BCs. It must be
noticed that this difficulty is not removed by increasing the size of blocks.

In the Density Matrix Renormalization Group one embeds each block into a larger(super)
block and considers the block as a quantum system in interaction with a reservoir(the rest
of the superblock). The block can then be described by a reduced density matrix, whose
eigenkets with the highest eigenvalues are used to construct the embedding operator(T).
Provided that one keeps a large number of states in each step of truncation(i.e. m = 16, 24, 36
and 44 in four different runs), this method leads to very accurate results for energies and
correlation functions of the isotropic Heisenberg chain.

However keeping a large number of states renders the problem of identification of coupling
constants of the effective Hamiltonian very difficult, which means that one should make a
compromise in obtaining accurate results for energies and correlation functions on the one
hand and coupling constant trasformations on the other, the later objective is of course
important if one is interested in studying the critical properties of these systems. In fact while
the DMRG method is quite appropriate for obtaining the former properties, the standard
RG appears to give better results for the later properties. One should adapt one or another
of these RG schemes to obtain reasonable results in both directions. In ref.[9] a strategy has
been suggested for the adaptation of DMRG so that one can also derive the transformation
of coupling constants and the critical properties of the Heisenberg chain.

The present paper has a different starting point in that it adapts the standard RG and
incorporates in it the interaction of the block with its environment, to do the same general
task. The method presented in this paper and in ref.[9] are to be regarded as complementary.
Depending on the problem and the desired accuracy either of them can be used. The
simplicity of our method is such that one may hope to apply it to two dimensional systems.
Although it does not yield as much accuracy as in DMRG, is much simpler practically, so that
for obtaining first estimates of many properties of spin or fermion systems one may try this
For the \textit{xxz} chain, with blocks of 3 sites in the infinite chain and with retaining only \( m = 2 \) states in each block, we have obtained the ground state energy with an accuracy much better than the standard RG method. The results are compared in table-1 and table-2. We have also obtained the staggered magnetization and spin-spin correlation functions in the massless region of the \textit{xxz} model. These results verify the absence of long-range order in this region which is in agreement with the known results.

2 A modified RG for the \( s = \frac{1}{2} \) \textit{xxz} chain

The system under consideration is the anti-ferromagnetic \( s = \frac{1}{2} \) \textit{xxz} chain with the following Hamiltonian:

\[
H_N(J, \Delta) = \left( \frac{J}{4} \right) \sum_{i=1}^{N-1} (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \Delta(\sigma_i^z \sigma_{i+1}^z))
\]  

(6)

where \( J \) is positive and \( \Delta \) is the anisotropy parameter which is limited by \( 0 \leq \Delta \leq 1 \) in the massless region.

Fig.2 shows the decomposition of the chain into isolated blocks and super blocks with an odd number of sites per each block. Thus the block Hamiltonian \( h^B \) and the superblock Hamiltonian \( h^{SB} \) are

\[
h^B = \left( \frac{J}{4} \right)(\sigma_2^x \sigma_3^x + \sigma_3^x \sigma_4^x + \sigma_2^y \sigma_3^y + \sigma_3^y \sigma_4^y + \Delta(\sigma_2^z \sigma_3^z + \sigma_3^z \sigma_4^z))
\]  

(7)

\[
h^{SB} = \left( \frac{J}{4} \right)(\sigma_1^x \sigma_2^x + \sigma_1^y \sigma_2^y + \Delta \sigma_1^z \sigma_2^z) + h^B + \left( \frac{J}{4} \right)(\sigma_4^x \sigma_5^x + \sigma_4^y \sigma_5^y + \Delta \sigma_4^z \sigma_5^z)
\]  

(8)

The basic steps of our method are follows:

\textbf{step-1}) First one diagonalizes the superblock Hamiltonian \( (h^{SB}) \). Although \( h^{SB} \) does not have su(2) symmetry, it has a number of symmetries which helps in it’s diagonalization. Defining the z-component of total spin \( S^z = \frac{1}{2}(\sigma_1^z + \sigma_2^z + \sigma_3^z + \sigma_4^z + \sigma_5^z) \), the flip operator \( \sigma = \sigma_1^x \sigma_2^x \sigma_3^x \sigma_4^x \sigma_5^x \), and the parity operator (\( \Pi \)), \( \Pi|\alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5\rangle = |\alpha_5 \alpha_4 \alpha_3 \alpha_2 \alpha_1\rangle \). It is easy to check that, \( H \) commutes with \( \Pi, \sigma \) and \( S^z \). Furthermore \([\Pi, S^z] = 0, [\Pi, \sigma] = 0\), which means that the energy eigenstates of \( h^{SB} \) can be chosen to have definite parity and the z-component of spin. The ground state is doubly degenerate, one with the z-component of the spin equal
to $+\frac{1}{2}$ and the other equal to $-\frac{1}{2}$.

\[ h^{SB}|\pm\frac{1}{2}\rangle_{SB} = e_{o}^{SB}(J,\Delta)|\pm\frac{1}{2}\rangle_{SB} \]  

where $|\pm\frac{1}{2}\rangle_{SB}$ are the doubly degenerate ground state of $h^{SB}$ and $e_{o}^{SB}$ is their corresponding energy. We note that the states of lowest energy are even parity states with their z-component of spin equal to $\pm\frac{1}{2}$. Thus the generic form of $|\pm\frac{1}{2}\rangle_{SB}$ is as follows.

\[ |\pm\frac{1}{2}\rangle_{SB} = a_{1}|+++\rangle + a_{2}|++-\rangle + a_{3}|+-+\rangle + a_{4}|--+\rangle + a_{5}|-+-\rangle + a_{6}|--+\rangle \]  

The state $|\mp\frac{1}{2}\rangle_{SB}$ is obtained from $|\pm\frac{1}{2}\rangle_{SB}$ by flipping all the spins. All the coefficients $a_{i}$; $i=1,\ldots,6$ can be expressed as functions of $J$ and $\Delta$ although we do not need to explicitly write them at this step, and $e_{o}^{SB}$, itself can be calculated numerically to any desired accuracy.

**step-2) **Knowing that the ground state of the block is a 2-dimensional subspace with $s_{z} = \pm\frac{1}{2}$, we then project the $|\pm\frac{1}{2}\rangle_{SB}$ states onto the $s_{z} = \pm\frac{1}{2}$ subspaces of the block Hilbert space. It is this step which effectively smoothes out the sharp effects of the boundary conditions, by immersing the block into a superblock (which to some extent simulates the effect of the rest of the lattice), the projected state does not restrict to any particular boundary conditions of an isolated block (open, periodic, ...). Moreover, in this way the form of the Hamiltonian does not change, as we will see later. What we do is that, after projection onto the block Hilbert space we implement the projection operator $P^{+}$ which is shown below, to reach the $s_{z} = +\frac{1}{2}$ subspaces of the block Hilbert space.

\[ P^{+} = |++\rangle\langle++| + |+-\rangle\langle+-| + |-+\rangle\langle-+| + |--\rangle\langle--| \]  

Then the resulting normalized states are

\[ |\pm\frac{1}{2}\rangle_{B} \equiv P^{\pm}|\pm\frac{1}{2}\rangle_{SB} = (1/A)((a_{1} + a_{4})|++\rangle + 2a_{3}|+-\rangle + (a_{1} + a_{4})|--\rangle) \]

\[ |\mp\frac{1}{2}\rangle_{B} \equiv P^{-}|-\frac{1}{2}\rangle_{SB} = (1/A)((a_{1} + a_{4})|--\rangle + 2a_{3}|+-\rangle + (a_{1} + a_{4})|++\rangle) \]  

where $A$ is a normalization factor. Straightforward calculations give the following results

\[ \alpha \equiv a_{1} + a_{4} = \frac{2\epsilon^{2} + \epsilon \Delta \Delta - 2}{2(\epsilon^{2} + \epsilon \Delta \Delta - 1)} + \frac{\epsilon}{2} \]

\[ \beta \equiv 2a_{3} = \frac{(\epsilon - \Delta)(2\epsilon^{2} + \epsilon \Delta - 2)}{\epsilon(\epsilon^{2} - \Delta^{2} - 1)} \]  

(13)
where

\[ \varepsilon = (2/J)e^S_B. \] (14)

Numerical calculation gives the following result for \( e^S_B \) \((0 \leq \Delta \leq 1)\),

\[ e^S_B(J, \Delta) = J(a + b\Delta + c\Delta^2 + d\Delta^3 + e\Delta^4 + f\Delta^5 + g\Delta^6 + h\Delta^7 + k\Delta^8 + l\Delta^9) \] (15)

with

\[ a = -1.36623 \quad b = -0.63707 \quad c = 2.92264 \quad d = -22.07630 \quad e = 84.52129 \]
\[ f = -187.92379 \quad g = 251.64014 \quad h = -200.06805 \quad k = 86.98911 \quad l = -15.92981. \]

**step-3)** The embedding operator is now constructed in the following form

\[ T_I = (|+\rangle_B^\dagger \langle +|_{B} + |-\rangle_B^\dagger \langle -|_{B}) \] (16)

where \(|+\rangle\) and \(|-\rangle\) are the renamed base kets of the effective Hamiltonian Hilbert space of each block.

Having the form of the embedding operator, one can calculate the projection of any operator onto the effective Hilbert space, as in eq.(1) i.e.

\[ \sigma_i^{\mu,eff} = T_I^\dagger \sigma_i^\mu T = (\frac{2\alpha\beta}{\Lambda_T})\sigma_i'^\mu \quad i = 1, 3; \mu = x, y \]
\[ \sigma_i^{z,eff} = T_I^\dagger \sigma_i^z T = (\frac{\beta^2}{\Lambda_T})\sigma_i'^z \quad i = 1, 3 \] (17)

where \(\sigma_i'\) acts as pauli matrices in the new Hilbert space. Fig.1 shows that the interaction between blocks are in the following form

\[ h_{BB}^{I,J} = (J/4)(\sigma_{3I}^x \sigma_{1J}^x + \sigma_{3I}^y \sigma_{1J}^y + \Delta \sigma_{3I}^z \sigma_{1J}^z). \] (18)

Then the effective Hamiltonian between the new sites are

\[ h_{I,J}^{eff} = T_J^\dagger T_I^\dagger h_{BB}^{I,J} T_I T_J \] (19)

where I and J are two neighbouring blocks while \( T_I \) and \( T_J \) are their corresponding embedding operators. Doing so we arrive at:

\[ h_{I,J}^{eff} = (J'/4)(\sigma_{I}^{x'} \sigma_{J}^{x'} + \sigma_{I}^{y'} \sigma_{J}^{y'} + \Delta' \sigma_{I}^{z'} \sigma_{J}^{z'}). \] (20)
where
\[
J' = \left(\frac{2\alpha\beta}{A^2}\right)^2 J
\]
\[
\Delta' = \left(\frac{\beta}{2\alpha}\right)^2 \Delta
\] (21)

Eq.(21) describes the RG flow of $J$ and $\Delta$. The RG flow goes to the origin of $J - \Delta$ plane which verifies that we are in the massless region.

**step-4)** The crudest estimate for the ground state energy is just the sum of ground state energies of all the blocks or superblocks. Thus
\[
\left(\frac{E}{N}\right)^{(0)} = \frac{1}{5} e_{0}^{SB}(J, \Delta)
\] (22)

or
\[
\left(\frac{E}{N}\right)^{(0)} = \frac{1}{3} e_{0}^{B}(J, \Delta)
\] (23)

where $(\frac{E}{N})^{(0)}$ is the ground state energy of the whole system in the zeroth order approximation. Eq.(23) in fact defines $e_{0}^{B}$ as :
\[
e_{0}^{B} = \frac{3}{5} e_{0}^{SB}
\] (24)

However this estimate clearly neglects the contribution to the energy from interaction between blocks. To take this missing contribution into account, we add to $(\frac{E}{N})^{(0)}$, the ground state energy of the new lattice, and obtain :
\[
\left(\frac{E}{N}\right)^{(1)} = \frac{1}{3} e_{0}^{B}(J, \Delta) + \frac{1}{9} e_{0}^{B}(J^{(1)}, \Delta^{(1)})
\] (25)

iterating this procedure we finally obtain
\[
\frac{E}{N} = \sum_{n=0}^{\infty} \frac{1}{3^{n+1}} e_{0}^{B}(J^{(n)}, \Delta^{(n)})
\] (26)

or
\[
\frac{E}{N} = \sum_{n=0}^{\infty} \frac{1}{3^{n+1}} \left(\frac{3}{5} e_{0}^{SB}(J^{(n)}, \Delta^{(n)})\right)
\] (27)

where $J^{(0)} = J$, $\Delta^{(0)} = \Delta$ and $J^{(n)}$, $\Delta^{(n)}$ are the renormalized coupling constants after n-steps of RG. In fact the above argument describes the physical interpretation of the formal limiting process defined in ref.4, in which the ground state energy is obtained as :
\[
E = \lim_{n \to \infty} T^{1n} HT^{n}
\]
However in standard RG the terms \( e_0^B(J, \Delta) \) is the result of diagonalization of \( h^B \), i.e: \( T^\dagger h^B T = e_0^B \), while in our method \( e_0^B \) is just another expression for \( \frac{1}{2} e_0^{SB} \) where \( e_0^{SB} \) is the eigenvalue of \( h^{SB} \), \( l' \) and \( l \) are the sizes of the superblocks and blocks respectively. The results for the ground state energy per site, obtained by this modified RG method are collected in table-1, for a series of anisotropy parameter \( 0 \leq \Delta \leq 1 \). The results are compared both with the exact results (obtained by the Bethe ansatz for xxx\(^{11}\) and xxz\(^{12}\) spin \( \frac{1}{2} \) chains) and with those obtained from the standard RG\(^{13}\). It is clearly seen that this type of modified RG method yields much better results than the standard RG method.

A remark is in order here, concerning the non-variational nature of our results, which is caused by our crude estimate for the ground state energy of each block in terms of that the superblock \( (e_0^B = \frac{3}{5} e_0^{SB}) \). However one should note that this behaviour is not unusual in RG treatment of lattice systems. For example in standard RG at second order perturbation of free-fermion model\(^3\) and isotropic Heisenberg model\(^{14}\), one obtains non-variational results.

We have also obtained the ground state energy in the \( \Delta = 0 \) (xx model) case for other \( \frac{n_B}{n_{SB}} \) ratios, namely for the ratios \( \frac{5}{7}, \frac{7}{9}, \frac{9}{11} \) and \( \frac{11}{13} \), the results are collected in table-2. It is interesting to note that our results in these cases are above the exact values of ground state energy, which indicates that the non-variational result in the \( \frac{3}{5} \) case is due to the very small block and superblock sizes. Therefore we expect that for \( n_B \) and \( n_{SB} \) not very small, our method gives indeed variational results. Our data in table-2 show much accuracy compared with the standard RG results for any value of block sizes. These results show that the computed ground state energy indeed converges to the exact result faster than the standard RG method by increasing the \( \frac{n_B}{n_{SB}} \) ratios.

### 3 Correlation Functions

In this section we want to gain more insight on the effective Hamiltonian constructed by this modified RG. The xxz Hamiltonian in the massless region \( (0 \leq \Delta \leq 1) \) has a non-degenerate ground state with a zero staggered magnetization in the thermodynamic limit \( N \rightarrow \infty \).
staggered magnetization is defined as

\[ m_{st} = \langle 0 | \frac{1}{2N} \sum_{i=1}^{N} (-1)^i \sigma_i^z | 0 \rangle \]  (28)

In the RG formalism the ground state \( |0\rangle \) is replaced by \( T|0'\rangle \), where \( |0'\rangle \) is the ground state of the effective Hamiltonian Hilbert space. Here we consider a chain of length \( 3^p \) and let \( p \) go to infinity. Then the staggered magnetization can be written in the following form

\[ m_{st} = \frac{1}{2N} \sum_{i=1}^{N} (-1)^i \langle 0'| T^i \sigma_i^z T | 0' \rangle \]  (29)

By dividing the chain into 3-sites blocks and consider \( T \) as the product of the embedding operator of each block, we can use eq (17) and write it again as

\[ T^i \sigma_i^z T = \gamma_i \sigma_i^z \]

where

\[ \gamma_1 = \gamma_3 = (\frac{\beta^2}{A^2}) ; \quad \gamma_2 = (\frac{2\alpha^2 - \beta^2}{A^2}) \].  (30)

Here we can continue this relation and replace \( |0'\rangle = T|0''\rangle \), and so on. Then the staggered magnetization will be

\[ m_{st} = \frac{1}{2N} \sum_{i=1}^{N} (-1)^i \langle 0| \gamma_i^p \sigma_i^z | 0 \rangle \]  (31)

which gives \( m_{st} = 0 \), when \( p \) goes to infinity, since \( (0 \leq \gamma_i < 1) \) for \( (0 \leq \Delta \leq 1) \).

Another quantity which verifies the absence of long-range order in the massless region of the \textbf{xxz} model is the z-component of spin-spin correlation functions. It is defined as

\[ g(r) = \langle 0 | (-1)^r \sigma_i^z \sigma_{i+r}^z | 0 \rangle \]  (32)

for the anti-ferromagnetic ordering. In the classical Neel order, \( g(r) \) is a constant for any distance \( r \), but quantum fluctuations can destroy Neel ordering. Despite the 2D Heisenberg model in the 1D \textbf{xxz} model quantum fluctuations destroy the long-range order.

We show that the constructed effective Hamiltonian will produce the above results. Again we assume a \( 3^p \) sites chain and let \( p \rightarrow \infty \). In this procedure \( s_i^z \) is considered at the center of chain and the other blocks are expanded in both sides of it symmetrically. To calculate \( g(r) \), the ground state \( |0\rangle \) is replaced by \( T|0'\rangle \) and eq.(30) is used for the effective operator
in the effective Hilbert space. The obtained $g(r)$ is plotted in fig.3a for $xx$ model ($\Delta = 0$) and compared with the exact results$^{15}$. We have also plotted $g(r)$ for $\Delta = 0.5$ and $\Delta = 1$ in fig.3b. All of these graphs show good agreement with the exact results$^{15}$. The good agreement of the correlation functions verify that the constructed effective Hamiltonian is a good approximate one for the low energy spectrum of the $xxz$ model.

4 Conclusions

We have shown how by a simple modification of the standard RG method one can take care of the effect of boundary conditions in treating quantum lattice systems. We have applied this method to the spin $\frac{1}{2}$ $xxz$ model in the massless region. We have shown our method give more accurate results than the standard RG for different value of block sizes. Our data of the ground state energy converge to the exact one faster than the standard RG results by increasing the block sizes. We have also calculated the staggered magnetization and the $z$-component of spin-spin correlation function in the massless region. These data confirm that we have obtained a good approximate Hamiltonian for the low energy spectrum which leads to no long-range order where, $0 \leq \Delta \leq 1$. These results are in good agreement with the exact results$^{11,12,15}$. Our method although not as exact as the more sophisticated DMRG method has the merit that it can be implemented very easily on personal computers. One may hope that for a first estimate of many properties of these systems, our method or it's improvements (or adaptation for other lattice systems) may give acceptable results. This method can also be applied to other spin systems, to fermion models$^{10}$, and more interestingly to 2-dimensional models (work on 2D anti-ferromagnetic Heisenberg model is in progress).

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Tables

Table-1. Ground state energy per site for standard RG, exact results and modified RG for $xxz$ spin-$\frac{1}{2}$ chain.

| $\Delta$ | $\langle \frac{E}{JN} \rangle_{standardRG}$ | $\langle \frac{E}{JN} \rangle_{exact}$ | $\langle \frac{E}{JN} \rangle_{modifiedRG}$ |
|----------|------------------------------------------|--------------------------------------|------------------------------------------|
| 0        | -0.28284295                              | -0.31830988                          | -0.32529130                              |
| 0.1      | -0.29208390                              | -0.32869220                          | -0.33583683                              |
| 0.2      | -0.30163311                              | -0.33956430                          | -0.34603469                              |
| 0.3      | -0.31150403                              | -0.35091277                          | -0.35700253                              |
| 0.4      | -0.32171198                              | -0.36272723                          | -0.36813752                              |
| 0.5      | -0.33227452                              | -0.37500000                          | -0.37947617                              |
| 0.6      | -0.34321210                              | -0.38772591                          | -0.39120956                              |
| 0.7      | -0.35454900                              | -0.40090224                          | -0.40325429                              |
| 0.8      | -0.36631503                              | -0.41452883                          | -0.41546686                              |
| 0.9      | -0.37854863                              | -0.42860840                          | -0.42793815                              |
| 1        | -0.39130452                              | -0.44314718                          | -0.44076268                              |

Table-2. Ground state energy per site of standard RG, exact results and modified RG for $\Delta = 0$ at different value of block sizes.

| Exact result $-0.3183\left(\frac{-1}{\pi}\right)$ | Standard RG | Modified RG | Type of SB and B |
|-----------------------------------------------|-------------|-------------|------------------|
| $n_B = 3, -0.2828$                           | $-0.3253$   | $n_{SB} = 5, n_B = 3$ |
| $n_B = 5, -0.2927$                           | $-0.3113$   | $n_{SB} = 7, n_B = 5$ |
| $n_B = 7, -0.2983$                           | $-0.3096$   | $n_{SB} = 9, n_B = 7$ |
| $n_B = 9, -0.3019$                           | $-0.3098$   | $n_{SB} = 11, n_B = 9$ |
| $n_B = 11, -0.3044$                          | $-0.3103$   | $n_{SB} = 13, n_B = 11$ |
Figure Captions:

Fig.1) Decomposition of the lattice to isolated blocks and considering neglected bonds as the effective interactions in the new Hilbert space.

Fig.2) Block and Super Block for the lattice chain.

Fig.3a) $z$-component of spin-spin correlation function $g(r) = \langle (-1)^r s_i^zs_{i+r}^z \rangle$ versus $r$ for $xx$ model ($\Delta = 0$), modified RG and exact results.

Fig.3b) $z$-component of spin-spin correlation function $g(r) = \langle (-1)^r s_i^zs_{i+r}^z \rangle$ versus $r$ for anisotropy parameter ($\Delta$) equal to 0.5 and 1, obtained by modified RG.
Figure 1

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