Top quark associated production of the neutral top-pion at high energy $e^+e^-$ colliders

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In the context of topcolor-assisted technicolor (TC2) models, we calculate the associated production of the neutral top-pion $\pi_0^t$ with a pair of top quarks via the process $e^+e^- \rightarrow t\bar{t}\pi_0^t$. We find that the production cross section is larger than that of the process $e^+e^- \rightarrow t\bar{t}H$ both in the standard model (SM) and in the minimal supersymmetric SM. With reasonable values of the parameters in TC2 models, the cross section can reach 20 fb. The neutral top-pion $\pi_0^t$ may be directly observed via this process.

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The mechanism of electroweak symmetry breaking (EWSB) remains the most prominent mystery in elementary particle physics despite the success of the standard model (SM) tested by the high energy experimental data. The present and next generation of colliders will help explaining the nature of the EWSB and the origin of fermion masses. The LHC is expected to directly probe possible new physics (NP) beyond the SM up to a scale of a few TeV, while the high energy linear $e^+e^-$ collider (LC) is required to complement the probe of the new particles with detailed measurements. Further more, some kinds of NP predict the existence of new particles that would be manifested as rather spectacular resonance in the LC experiments, if the achievable centre-of-mass energy is sufficient. Even their masses exceed the centre-of-mass energy, it also retains an indirect sensitivity through a precision study of the virtual corrections to electroweak observable. A LC represents an ideal laboratory for studying this kind of NP [1].

Until a Higgs boson with large coupling to gauge boson pair is discovered, the possibility of strong EWSB must be entertained [2]. The most commonly studied class of theories is technicolor (TC) [3], which dynamical break the electroweak symmetry. Although TC models have many theoretical problems as well as conflicts with data and broad classes of these models have been ruled out, there are still viable models worthy of investigation in light of the capabilities of the current generation of experiments.

It is widely believed that the top quark will be a sensitive probe into physics beyond the SM. The properties of the top quark could reveal information about flavor physics, EWSB, as well as NP [4]. Given the large value of the top quark mass and the sizable splitting between the masses of the top and bottom quarks, it is natural to wonder whether $m_t$ has a different origin from the masses of the other quarks and leptons. There may be a common origin for EWSB and top quark mass generation. Much theoretical work has been carried out in connection to the top quark and EWSB. The TC2 models [5], the top see-saw models [6], and the flavor universal TC2 models [7] are three of such examples. Such type of models generally predict a number of scalars with large Yukawa coupling to the third generation. For example, TC2 models predict the existence of the top-pions ($\pi^\pm_t, \pi^0_t$). These new particles are most directly related to the EWSB. Thus, studying the possible signatures of these new particles at present and future high energy colliders would provide crucial information for the EWSB and can be used to test TC2 theory.

The production of a Higgs boson predicted by the SM or the minimal supersymmetric SM (MSSM) in association with a pair of top-antitop quarks is of extreme interest for two reasons [8]. First, the $t\bar{t}H$ production mode can be important for discover of a Higgs boson around 120–130GeV at the LHC, and even at the Run II of the Tevatron with high enough luminosity [9]. Although it has a small event rate($1 \sim 5 fb$) for a SM-like Higgs boson, and even lower for an MSSM Higgs boson, the signature is quite spectacular. Second, this production mode offers a direct handle on the Yukawa coupling of the top quark, supposedly the most relevant one to understand the nature of fermion masses. Both the LHC and LC can use the second feature to try a precision measurement of the $t\bar{t}H$ coupling.

The signals of the $t\bar{t}H$ production mode has been studied quite extensively in the LC [10], Tevatron and LHC [11] in the context of the SM and MSSM. Recently, Ref [11] has investigated the associated production of a neutral scalar predicted by TC2 models with a pair of top quarks at hadron colliders. They find that the neutral scalar may be observed.
at the LHC via the process $p\bar{p} \rightarrow t\bar{t}\phi$. In this paper, we will calculate the production cross section of the process $e^+e^- \rightarrow t\bar{t}\pi^0_t$ and discuss the possibility of detecting $\pi^0_t$ via this process in the future LC experiments. We find the production cross section is larger than that of the process $e^+e^- \rightarrow t\bar{t}H$. In most of the parameter space of the TC2 models, the production cross section is larger than $5fb$ which may be detected in the future LC experiments.

TC2 models generate the large top quark mass through the formation of a dynamical $t\bar{t}$ condensation and provide possible dynamical mechanism for breaking electroweak symmetry. In TC2 models, the EWSB is driven mainly by TC interaction, ETC interactions give contribution to all ordinary quark and lepton masses including a very small portion of the top quark mass, namely $m_t' = \epsilon m_t$ with $\epsilon \ll 1$. The topcolor interactions also make small contributions to the EWSB and give rise to the main part of the top quark mass $m_t - m_t' = (1 - \epsilon)m_t$ similar to the constituent masses of the light quark in QCD. This means that the associated top-pions $\pi^0_t$, $\pi_t^\pm$ are not the longitudinal bosons $W$ and $Z$, but separately, physically observable objects.

To maintain electroweak symmetry between top and bottom quarks and get not generate $m_b \approx m_t$, the topcolor gauge group is usually take to be a strongly coupled $SU(3) \otimes U(1)$. At the $\Lambda \sim 1TeV$, the dynamics of a general TC2 model involves the following structure [7,12]:

$$SU(3)_1 \otimes SU(3)_2 \otimes U(1)\gamma_1 \otimes U(1)\gamma_2 \otimes SU(2)_L \rightarrow SU(3)_{QCD} \otimes U(1)_{EM},$$

where $SU(3)_1 \otimes U(1)\gamma_1 (SU(3)_2 \otimes U(1)\gamma_2)$ generally preferentially to the third (first and second) generations. The $U(1)\gamma_2$ are just strongly rescaled versions of electroweak $U(1)\gamma$. The above breaking scenario gives rise to the topcolor gauge bosons including the color-octet coloron $B^A_\mu$ and color-singlet extra $U(1)$ gauge bosons $Z'$. The relevant couplings of the new gauge boson $Z'$ to ordinary fermions can be written as:

$$\frac{1}{2}g_1[\frac{1}{3}\cot \theta' Z'_\mu(\bar{e}_L\gamma^\mu t_L + 2\bar{t}_R\gamma^\mu t_R) + \tan \theta' Z'_\mu(\bar{e}_L\gamma^\mu e_L + 2\bar{e}_R\gamma^\mu e_R)],$$

where $g_1$ is the $U(1)\gamma_1$ coupling constant at the scale $\Lambda_{TC}$ and $\theta'$ is the mixing angle with $\tan \theta' = g_1/(2\sqrt{\pi k_1})$. To obtain proper vacuum tilting (the topcolor interactions only condense the top quark but not the bottom quark), the coupling constant $k_1$ should satisfy certain constraint, i.e. $k_1 \leq 1$ [7,12]. In the following estimation, we will take $k_1 = 1$. The choice $k_1 = 1$ corresponds to $\tan^2 \theta' = 0.01$ [12].

For TC2 models, the underlying interactions, topcolor interactions, are non-universal and therefore do not posses a GIM mechanism. The non-universal gauge interactions can result in the flavor changing (FC) coupling vertices when one writes the interactions in the quark mass eigen-basis. Thus, the top-pions have large Yukawa couplings to the third family quarks and can induce the new FC scalar couplings. For the neutral top-pion $\pi^0_t$, the relevant coupling can be written as [3,13]:

$$\frac{m_t}{\sqrt{2}F_t} \sqrt{\nu_w^2 - F_t^2} [K_{UR}^{tt} K_{UL}^{tt} f_{LR} t_R \pi^0_t + K_{UR}^{te} K_{UL}^{te} f_{CR} c_R \pi^0_t + h.c.],$$

where $F_t$ is the top-pion decay constant and $\nu_w = v/\sqrt{2} = 174GeV$. It has been shown that the values of these coupling parameters can be taken as:
the topcolor gauge boson $Z$

From above discussion, we can see that the $t\bar{t}\pi^0_t$ production channels can be represent by the processes:

$$e^+e^- \rightarrow \gamma^*, Z^*, Z'^* \rightarrow t\bar{t}\pi^0_t$$

Using Eq.(2)-(4) and other relevant Feynman rules, we can give the invariant scattering amplitude of the process $e^+e^- \rightarrow t\bar{t}\pi^0_t$.

$$M = M_\gamma + M_Z + M_{Z'},$$

with

$$M_\gamma = \bar{v}(p_-)(-i)e\gamma_\mu u(p_+)\frac{-ig^{\mu\nu}}{s}\times \frac{i}{\not{p}_t + \not{p}_e - m_t} \bar{u}(p_t)i\beta\gamma_5 i\epsilon_{\nu}^\gamma v(p_t),$$

$$M_Z = \bar{v}(p_-)i\gamma_\mu(v_e + a_\pi\gamma_5)u(p_+)\frac{-ig^{\mu\nu}}{s- m_Z^2 + i m_Z \Gamma_Z}\times \frac{i}{\not{p}_t + \not{p}_e - m_t} \bar{u}(p_t)i\beta\gamma_5 i\epsilon_{\nu}^\gamma v(p_t),$$

$$M_{Z'} = \bar{v}(p_-)i\gamma_\mu(v_e' + a_{\pi}'\gamma_5)u(p_+)\frac{-ig^{\mu\nu}}{s- m_{Z'}^2 + i m_{Z'} \Gamma_{Z'}}\times \frac{i}{\not{p}_t + \not{p}_e - m_t} \bar{u}(p_t)i\beta\gamma_5 i\epsilon_{\nu}^\gamma v(p_t),$$

where

$$\beta = \frac{m_t}{\sqrt{2} F_t} \frac{\sqrt{v^2_w - F_t^2}}{v_w} K_{UR}^{tt} K_{UL}^{tt*}, \quad v_e = \frac{e}{4 \sin \theta \cos \theta} (4 \sin^2 \theta - 1),$$

$$a_\pi = \frac{e}{4 \sin \theta \cos \theta}, \quad v_t = \frac{e}{4 \sin \theta \cos \theta} (1 - \frac{8}{3} \sin^2 \theta),$$

$$a_{\pi}' = -\frac{1}{4} \sqrt{4\pi k_1 \tan^2 \theta'}, \quad v_e' = \frac{3}{4} \sqrt{4\pi k_1 \tan^2 \theta'}, \quad v_e' = \frac{5}{12} \sqrt{4\pi k_1} \tan^2 \theta',$$

$$a_{\pi}' = -\frac{1}{4} \sqrt{4\pi k_1 \tan^2 \theta'}, \quad v_e' = \frac{5}{12} \sqrt{4\pi k_1} \tan^2 \theta', \quad a_{\pi}' = -\frac{1}{4} \sqrt{4\pi k_1}.$$

Where $\sqrt{s}$ is the centre-of-mass energy of the LC experiments and $M_{Z'}$ is the mass of the topcolor gauge boson $Z'$. The decay width $\Gamma_{Z'}$ is dominated by $t\bar{t}$, $bb$ and we have [13]:

$$\Gamma_{Z'} \approx g_1^2 \cot^2 \theta' \frac{M_{Z'}}{12 \pi} = \frac{1}{3} M_{Z'},$$

where corresponds to $\tan^2 \theta' = 0.01$.

To obtain numerical results we take the SM parameters as $\sin^2 \theta = 0.2315$, $\alpha = \frac{1}{128.8}$, $m_Z = 91.2 GeV$, $\Gamma_Z = 2.495 GeV$ and $m_t = 175 GeV$ [15]. For TC2 models, the parameters
$m_{\pi_t}$, $\epsilon$ and $M_{Z'}$ are the free parameters. To see how those parameters affect our numerical results, we take $200 GeV \leq m_{\pi_t} \leq 450 GeV$, $0.01 \leq \epsilon \leq 0.1$, and $2 TeV \leq M_{Z'} \leq 6 TeV$.

The $t\bar{t}\pi_t^0$ production cross section $\sigma$ is plotted in Fig.1 as a function of $m_{\pi_t}$ for the centre-of-mass energy $\sqrt{s} = 1.5 TeV$, $M_{Z'} = 2.5 TeV$ and three values of the parameter $\epsilon$. From Fig.1 we can see that $\sigma$ is not sensitive to the value of parameter $\epsilon$ and decreases with $m_{\pi_t}$ increasing. Thus, in the following numerical estimation, we will take $\epsilon = 0.05$. For $\epsilon = 0.05$, $\sigma$ varies between $10.7 fb$ and $48 fb$ for the neutral top-pion mass $m_{\pi_t}$ in the range of $200 - 400 GeV$.

To see the effect of the extra $U(1)_Y$ gauge bosons $Z'$ mass $M_{Z'}$ on the production cross section $\sigma$, we plot the $\sigma$ as function of $M_{Z'}$ for $\sqrt{s} = 1.5 TeV$ in Fig.2, in which the full, broken and dotted-dashed curves stand for $m_{\pi_t} = 250 GeV$, $350 GeV$, and $450 GeV$, respectively. One can see from Fig.2 that $\sigma$ decreases with $m_{\pi_t}$ increasing. In most of the parameter space, the production cross section $\sigma$ is larger than $5.8 fb$.

Recently, Ref.[16] has shown that $B\bar{B}$ mixing provides strong lower bounds on the masses of $B^0_\mu$ and $Z'$ bosons, i.e. there must be larger than $4 TeV$. In Fig.3 we take $M_{Z'} = 5 TeV$ and plot $\sigma$ as a function of $\sqrt{s}$ for three values of the top-pion mass. From Fig.3 we can see that, when $\sqrt{s}$ is larger than $2 TeV$, the cross section $\sigma$ is not sensitive to the value of the centre-of-mass energy $\sqrt{s}$. For $\sqrt{s} = 2 TeV$ and $\epsilon = 0.05$, $\sigma$ increase from $6.8 fb$ to $21.8 fb$ as $m_{\pi_t}$ decreasing from $450 GeV$ to $250 GeV$.

The process $e^+e^- \rightarrow t\bar{t}H$ has been extensively studied and calculated both in the SM and in the MSSM at $O(\alpha_s)$ [14]. The cross section turns out to be highly sensitive to the top Yukawa coupling $g_{t\bar{t}H}$ over most of the parameter space. Although the cross section is very smaller than $2 fb$ for $m_{H} = 120 - 130 GeV$, the signature for $t\bar{t}H$ production mode is spectacular. It has been shown that the top Yukawa coupling $g_{t\bar{t}H}$ can be measured with a precision of the order of $7 - 15\%$ at $\sqrt{s} = 1 TeV$ for $M_{H} = 120 - 130 GeV$, assuming a b-tagging efficiency between 0.6 and 1 [17] and with a precision of $5.5\%$ at $\sqrt{s} = 800 GeV$, when optimal b-tagging efficiency is assumed [15]. The cross section of the process $e^+e^- \rightarrow t\bar{t}\pi_t^0$ is larger than $5 fb$, even for $M_{Z'} = 5 TeV$ and $\epsilon = 0.01$, which is significantly larger than that of the process $e^+e^- \rightarrow t\bar{t}H$. Thus, the top Yukawa coupling $g_{t\bar{t}\pi_t^0}$ can be precisely measured in the future LC experiments. The signatures for $t\bar{t}\pi_t^0$ production mode may be detected.

Since the negative top-pion corrections to the $Z \rightarrow b\bar{b}$ branching $R_b$ become smaller when the top-pion is heavier, the LEP/SLD data of $R_b$ give lower bound on the top-pion mass [19]. Ref.[20] has shown that the top-pion mass is allowed to be in the range of a few hundred GeV depending on the values of the parameters in the TC2 models. So we have take the mass of the top-pion to vary in the range of $200 GeV - 450 GeV$. For $200 GeV \leq m_{\pi_t} \leq 350 GeV$, the dominate decay channel is $\pi_t^0 \rightarrow t\bar{c}$. If we take $K_{UR}^{tc} = 0.05$, $m_{\pi_t} = 300 GeV$, we have that the branching ratio $B_r(\pi_t^0 \rightarrow t\bar{c})$ is approximately equal to $60\%$ and $B_r(\pi_t^0 \rightarrow gg)$ is only equal to $16\%$ [24]. This is different from that of Ref.[11], which has assumed that the top-pion can not induce the FC scalar coupling. Thus, the produce mode $t\bar{t}\pi_t^0$ should be detected via the dominate decay channel $\pi_t^0 \rightarrow t\bar{c}$, which has been extensively studied in Ref.[13]. For $m_{\pi_t} > 350 GeV$, the branching ratio of the decay channel $\pi_t^0 \rightarrow t\bar{t}$ is close to $100\%$, all other decay channels may be ignored. In this case, the associated production of the $\pi_t^0$ with a pair of top quarks induces an four top quark final state, which may be experimentally observable [22].

From triviality, we see that the SM can only be an effective theory valid below some
high energy scale $\Lambda$, the strong EWSB theories might be needed \[23\]. The strong top dynamics models, such as TC2 models, are the modern dynamical models of EWSB. Such type of models generally predict a number of scalars with large top Yukawa couplings. Direct observation of these new particles via their large top Yukawa coupling would be confirmation that the EWSB sector realized in nature is not the SM or part of the MSSM. In this paper, we investigate the top quark associated production with the neutral top-pion $\pi_t^0$ and calculated the cross section of the $t\bar{t}\pi_t^0$ production mode via the process $e^+e^- \rightarrow t\bar{t}\pi_t^0$ in the context of TC2 models. Our results show that the cross section is significantly larger than that of the process $e^+e^- \rightarrow t\bar{t}H$ and is larger $5\, fb$ in most of the parameter space. With reasonable values of the parameters in TC2 models, the cross section can reach $20\, fb$. Thus, the neutral top-pion $\pi_t^0$ may be direct observed via the process $e^+e^- \rightarrow t\bar{t}\pi_t^0$. 
Figure captions

**Fig.1:** The cross section $\sigma$ of the process $e^+ e^- \rightarrow t\bar{t}\pi_t$ as a function of the mass $m_{\pi_t}$ for $\sqrt{s} = 1.5 TeV$, $M_{Z'} = 2.5 TeV$ and $\epsilon = 0.03$ (full curve), 0.05 (broken curve), and 0.08 (dotted-dashed curve).

**Fig.2:** The cross section $\sigma$ versus the mass of $M_{Z'}$ for $\epsilon = 0.05$, $\sqrt{s} = 1.5 TeV$ and $m_{\pi_t} = 250GeV$ (full curve), $m_{\pi_t} = 350GeV$ (broken curve) and $m_{\pi_t} = 450GeV$ (dotted-dashed curve).

**Fig.3:** The cross section $\sigma$ versus the center-of mass $\sqrt{s}$ for $\epsilon = 0.05$, $M_{Z'} = 5 TeV$ and $m_{\pi_t} = 250GeV$ (full curve), $m_{\pi_t} = 350GeV$ (broken curve) and $m_{\pi_t} = 450GeV$ (dotted-dashed curve).
[1] M. Battaglia et al., hep-ph/0101114; D. Dominici, hep-ph/0110084; M. Battaglia et al., hep-ph/0112270.

[2] T. L. Barklow, hep-ph/0112286.

[3] For recent reviews, see K. Lane, "Technicolor 2000", hep-ph/0007304; C. T. Hill and E. H. Simmons, "Top physics", hep-ph/0011244.

[4] M. Beneke, et al. and A. Ahmsdov, et al., "Top quark physics", hep-ph/0003033; E. H. Simmons, "Top physics", hep-ph/0011244.

[5] C. T. Hill, Phys. Lett. B345, 483(1995); K. Lane and E. Eichten, Phys. Lett. B352, 383(1995); K. Lane, Phys. Lett. B433, 96(1998); G. Cvetic, Rev. Mod. Phys. 71, 513(1999).

[6] B. A. Dobrescu and C. T. Hill, Phys. Rev. Lett. B81, 263(1998); R. S. Chivukula, B. A. Dobrescu and C. T. Hill, Phys. Rev. D59, 075003(1999).

[7] M. B. Popovic, and E. H. Simmons, Phys. Rev. D58, 095007(1998).

[8] S. Dawson and L. Reina, Int. J. Mod. Phys. A16SIA, 375(2001).

[9] ATLAS Collaboration, CERN/LHCC/99-15, ATLAS TDR 15(1999); E. Richter-Was, M. Sapiniski, Acta Phys. Pol. B30, 1001(1999); J. Goldstein, et al., Phys. Rev. Lett. 86, 1694(2001); L. Reina and S. Dawson, Phys. Rev. Lett. 87 201804(2001); W. Beenakker et al., Phys. Rev. Lett. 87 201805(2001).

[10] S. Dawson and L. Reina, Phys. Rev. D59, 054012; Phys. Rev. D60, 015003(1999); S. Dittmaier, et al., Phys. Lett. B441, 383(1998); Phys. Lett. B 478, 247(2000).

[11] A. K. Leibovich and D. Rainwater, Phys. Rev. D65, 055012(2002).

[12] G. Buchalla, G. Burdman, C. T. Hill, D. Kominis, Phys. Rev. D53, 5185(1996).

[13] H. -J. He and C. -P Yuan, Phys. Rev. Lett. 83, 28(1999); G. Burdman, Phys. Rev. Lett. 83, 2888(1999).

[14] C. T. Hill, S. T. Parke, Phys. Rev. D49, 4454(1994).

[15] Particle Data Group, D. Green et al., Eur. Phys. J. C115, 1(2000).

[16] G. Burdman, K. Lane and T. Rador, Phys. Lett. B514, 41(2001).

[17] H. Baer, S. Dawson, L. Reina, Phys. Rev. D61, 013002(2000).

[18] A. Juste and G. Merino, hep-ph/9910301; F. Maltoni, D. Rainwater, S. Willenbrok, hep-ph/0202203.

[19] G. Burdman and D. Komninos, Phys. Lett. B403, 101(1997); W. Loinaz, T. Takuchi, Phys. Rev. D60, 015005(1999).

[20] Chongxing Yue, et al., Phys. Rev. D62, 055005(2000).

[21] Chongxing Yue, et al., Phys. Rev. D63, 115002(2001).

[22] M. Spira and J. D. Wells, Nucl.Phys. B523, 3(1998).

[23] E. H. Simmons, hep-ph/0101916; R. S. Chivukula and C. Holbling, hep-ph/0110214.
Fig. 1

\[ M_{\pi^+} \text{ (GeV)} \]

\[ \sigma ( fb ) \]

- \( \varepsilon = 0.03 \)
- \( \varepsilon = 0.08 \)
- \( \varepsilon = 0.08 \)
Fig. 2

$\sqrt{s} = 1.5 \text{ TeV}$

- $m_{\pi t} = 250$
- $m_{\pi t} = 350$
- $m_{\pi t} = 450$

$\sigma$ (fb)

$M_{Z'}$ (TeV)
Fig. 3