Constraints on mirror fermion mixing angles from anomalous magnetic moment data

F. Csikor and Z. Fodor
Institute for Theoretical Physics, Eötvös University
Budapest, Hungary

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Abstract

The new contributions to the electron (muon) anomalous magnetic moment arising in mirror fermion theories have been calculated. Imposing the experimental constraint lowers the current upper bound on the ordinary - mirror lepton mixing angles by a factor of 50 making predictions for mirror lepton production at HERA undetectably small. A way out is to allow for different mixing angles of the L and R field components. Choosing very small right mixing angles compatibility with the anomalous magnetic moment measurement may be easily maintained, while choosing left mixing angles close to the upper limits yields still reasonable HERA cross-sections.
1. Mirror fermions are hypothetical particles (for a review see [1]) which should have opposite chirality properties as compared to the known ordinary, light fermions. Thus right handed mirror fermions form weak SU(2) doublets, while the left handed fields are singlets. Since mirror fermions have not been observed directly one has to search for their effects indirectly through the modification of ordinary fermion-vector boson couplings. Such an investigation has been performed in [2], [3] where the hermitian mirror fermion model of [4] has been explicitly considered. A recent update of essentially the same analysis has been published in [5]. In both analyses the authors allow for different mixing angles of the L and R components of ordinary and mirror fermion fields. The result of [5] is that the upper bounds on the sine squares of leptonic mixing angles are around 0.10 - 0.15, while the somewhat higher bounds for quarks are not less than 0.02. Since [5] did not equate the L and R mixing angles the bounds valid in the hermitian mirror fermion model may be even slightly higher.

Another source of information on the mixing angles comes from higher order electroweak corrections. A particularly sensitive observable is the anomalous magnetic moment of the electron (muon). This quantity has already been studied from the point of view of mirror fermion mixing angles [6]. However, the range of allowed values of mirror fermion masses has changed dramatically, therefore a reanalysis is timely. The new bounds on the masses come from LEP experiments, restricting the number of light neutrinos to three and imposing a lower bound on the mirror masses of about half the Z boson mass. If mixing angles are not unexpectedly small even masses up to the Z mass are excluded. Therefore we shall consider the case of heavy mirror fermions with masses of the order of 100GeV. In order not to get into trouble with one loop electroweak corrections [7] we shall also assume almost degenerate doublets. In this case LEP experiments exclude more than three heavy mirror fermion generations at the 90% C.L. [8]. The tree unitarity constraints on the mirror masses are rather low, of the order of a few hundred GeV (for more details see [9]). A new feature of our analysis is that we have found a relation between the masses and mixing angles, which has not been considered in [6]. Therefore we get a completely new situation and restrictive bounds on the mixing angles, which seriously modify mirror lepton production expectations [10, 11] at future ep colliders like HERA.

2. The mirror fermion model we consider has the standard model gauge
group and symmetry breaking, the only modification being that mirror partners of all ordinary standard model fermions are introduced. Ordinary and mirror fermions may mix. In principle the mixing angles of the L and R fields may be different, equating them we obtain the hermitian mirror fermion model of [4]. The leptonic weak interaction part of the model is written down in detail in [6], the notations of which will be used in the following. Some consequences of such a model have been worked out (especially for electron proton colliders) in [9].

The calculation of the contribution arising from mirror fermions to the anomalous magnetic moment of the electron (muon) is straightforward. The graphs to be taken into account are shown in Fig.1. Wherever a vector boson exchange occurs a Goldstone boson should be also exchanged, when the calculation is performed in a renormalizable gauge. Our result reads:

\[ \Delta \mu_e = \frac{G_F m_e \mu_B}{32 \pi^2 \sqrt{2}} \times \]

\[ \left( M_Z h_1 \left( \frac{M_E}{M_Z} \right) \sin^2(2\Theta) - 2 M_W h_2 \left( \frac{m_N}{M_W} \right) \sin(2\Theta) \cdot \sin(2\Phi) \right) + \Delta \mu_e^{\text{Higgs}}, \]

(1)

where

\[ h_1(x) = - \frac{6 x^3}{(x^2 - 1)^3} \cdot \ln x^2 + \frac{x^5 + x^3 + 4x}{(x^2 - 1)^2} \]

\[ h_2(x) = \frac{6 x^5}{(x^2 - 1)^3} \cdot \ln x^2 + \frac{x^5 - 11x^3 + 4x}{(x^2 - 1)^2} \]

and

\[ \Delta \mu_e^{\text{Higgs}} = \frac{G_F m_e \mu_B}{64 \pi^2 \sqrt{2}} \cdot M_H h_3 \left( \frac{m_E}{M_H} \right) \sin^4(4\Theta), \]

(2)

\[ h_3(x) = \left( - \frac{1}{(x^2 - 1)} \cdot \ln x^2 + \frac{3 - x^2}{2} \right) \cdot \frac{x^3}{(1 - x^2)^2}. \]

\(\Theta\) and \(\Phi\) are the electron - mirror electron, neutrino - mirror neutrino mixing angles, respectively, \(\mu_B\) the Bohr magneton. \(m_e, m_E, m_N\) denote the
masses of the electron, mirror electron and mirror neutrino. Other notation is standard.

Note the small change as compared to the formula of $[3]$. In particular the relative sign of the first two terms has been changed and a misprint in the function $h_2$ has been corrected. The above formula applies to mirror fermions. However, in a general theory the formula still applies. The mixing angle factors correspond to $v^2 - a^2$ of the neutral (charged) current coupling vertex of fermion-exotic fermion-vector boson. Therefore a modification of the formula is straightforward.

As well known the standard model prediction for the anomalous magnetic moment of the electron and muon is in good agreement with experiment. Thus the new contribution should be small, essentially of the order of the experimental error. The experimental error on electron (muon) anomalous magnetic moment is $10^{-11} \mu_B$ $(8.1 \cdot 10^{-9} \frac{e\hbar}{2m_e})$ $[11]$. With the usual values of the mixing angles (i.e. $\sin^2 \Theta$, $\sin^2 \Phi \approx 0.02$) one gets much larger values than the experimental error. The Higgs contribution is small so it can be safely neglected. The only way to achieve a reasonable value is to allow for the cancellation of the two terms containing $h_1$ and $h_2$. That would lead to a relation between the two mixing angles. On the other hand, a detailed investigation of the fermion mass generation via the usual electroweak symmetry breaking shows that the mixing angles can not be chosen arbitrarily.

We get the relation

$$-m_\nu \sin \Phi \cos \Phi + m_N \cos \Phi \sin \Phi = -m_e \sin \Theta \cos \Theta + m_E \cos \Theta \sin \Theta,$$

(3)

which makes the necessary cancellation of the two terms possible only if the $N, E$ mass splitting is unreasonably large. (E.g. for exact cancellation $m_N/m_E \approx 13$.) Fixing the value of $\Delta \mu_e$ to $10^{-11} \mu_B$, and using Eqs. (1,3) one gets the curves of Fig.2, specifying the mixing angles and masses which yield acceptably small contribution to the anomalous magnetic moment. (The mass of the mirror electron neutrino is determined by Eq.(3).) We see that the mixing angles have to be very small, with the square of the order of 0.0004. This corresponds to $m_E \approx m_N$ as required by consistency with the LEP results $[7, 8]$. (The muon anomalous magnetic moment error constrains the mixing angles of the second generation lepton mixing angles in a similar way, with approximately the same upper limit.) This is a factor 1/50 lower
that the values assumed in [9]. Since the HERA production cross-sections are proportional to the mixing angle squares, the predictions of [9] decrease roughly by a factor of 1/50, making mirror lepton productions at HERA assuming the usual $100pb^{-1}$ yearly integrated luminosity essentially hopeless.

Let us now consider the case of a mixing scheme with different L and R mixing angles. In this case the following replacements should be made in Eq.(1).

$$\sin(2\Theta) \sin(2\Phi) \to 4 \cos \Theta_L \cos \Phi_R \sin \Phi_L$$

$$\sin^2(2\Theta) \to \sin(2\Theta_R) \sin(2\Theta_L).$$

From the structure of the symmetry breaking mechanism one gets the relation:

$$-m_{\nu} \sin \Phi_R \cos \Phi_L + m_N \cos \Phi_R \sin \Phi_L = -m_e \sin \Theta_R \cos \Theta_L + m_E \cos \Theta_R \sin \Theta_L$$

(5)

Neglecting $m_{\nu}$ and $m_e$ one gets a relation sensitive only to the L mixing angles. Neglecting the Higgs contribution and inserting into Eq.(1) we get:

$$\Delta \mu_e = \frac{G_F m_e \mu_B}{32 \pi^2 \sqrt{2}} \times$$

$$\sin(2\Theta_R) \sin(2\Theta_L) \left( M_Z h_1 \left( \frac{m_E}{M_Z} \right) - 2 M_W h_2 \left( \frac{m_N}{M_W} \right) \cdot \frac{m_E}{m_N} \right).$$

(6)

This shows that either the R or the L angle has to be small in order to suppress the contribution. Generalizing the formulae of [9] we may write in the small mixing angle approximation the HERA mirror lepton production cross-sections:

$$\frac{d\sigma_{e^-u \to N_e d}}{dQ^2} \simeq \frac{g^4}{64\pi(M_W^2 + Q^2)^2} \times$$

$$\left[ \Phi_L^2 \left( 1 - \frac{M_W^2}{xs} \right) + \Theta_R^2(1 - y) \left( 1 - y - \frac{M_W^2}{xs} \right) \right]$$

(7)
\[
\frac{d\sigma}{dQ^2} \simeq \frac{(g^2 + g'2)^2}{1024\pi (M_Z^2 + Q^2)^2}.
\]

\[
\begin{align*}
&\Theta_L^2 \left( (1 - 4 \cdot \frac{M^2}{x_s}) + (4Q_A^2 \sin^2 \Theta_W)^2 \frac{1 - y}{(1 - y - \frac{M^2}{x_s})} \right) + \\
&\Theta_R^2 \left( (1 - 4 \cdot \frac{M^2}{x_s}) + (4Q_A^2 \sin^2 \Theta_W)^2 \frac{1 - y}{(1 - y - \frac{M^2}{x_s})} \right)
\end{align*}
\]

(8)

We see\[^1\] that choosing \(\Theta_R\) to be small both the mirror neutrino and mirror electron cross-sections have an unconstrained L mixing angle factor. Assuming the upper limits we get the numbers of Table 1, which are only about a factor 2 lower than the predictions presented in [3].

3. Let us now discuss how Eqs. (3,5) follow from the electroweak symmetry breaking. It is sufficient to consider the first family ordinary and mirror leptons. Assuming a standard scalar sector the most general \(SU(2) \otimes U(1)\) Yukawa couplings and mass mixing terms of the Lagrangian may be written as follows. Introduce the notation

\[
\psi_L^o = \left( \begin{array}{c} \nu_L^o \\ e_L^o \end{array} \right), \quad \chi_R^o = \left( \begin{array}{c} N_R^o \\ E_R^o \end{array} \right),
\]

\[
e_R^o, \nu_R^o, N^o_L, E^o_L,
\]

\[
\phi = \left( \begin{array}{c} \phi^+ \\ \phi^- \end{array} \right), \quad \tilde{\phi} = \left( \begin{array}{c} \phi^o \\ -\phi^o \end{array} \right).
\]

(9)

Then

\[
\mathcal{L}_{Yukawa} = g_1 \bar{\psi}_L^o \tilde{\phi} e_R^o - g_1 \bar{\psi}_L^o \phi \nu_R^o + g_2 \bar{\chi}_R^{\sigma} \tilde{\phi} E_R^o - g_2 \bar{\chi}_R^{\sigma} \phi N_L^o
\]

\[
+ g_3 \bar{\chi}_R^{\sigma} \bar{\psi}_L^o + g_4 \bar{\nu}_R^o E_L^o + g_5 \bar{\nu}_R^o N_L^o + h.c.
\]

(10)

Besides the Yukawa-couplings we have as well included the \(SU(2) \otimes U(1)\) invariant contact terms responsible for the ordinary - mirror fermion mixing. Inserting the vacuum expectation value of the scalar field, we get as usual the mass matrices:

\[
\begin{pmatrix}
eg_R^o \ E_R^o \\
\end{pmatrix}
\begin{pmatrix}
g_1 \sqrt{2} & g_4 \\
g_3 & -g_2 \sqrt{2} \\
\end{pmatrix}
\begin{pmatrix}
e_L^o \\
E_L^o \\
\end{pmatrix} + h.c.
\]

\[^1\]This possibility has been pointed out to us by I. Montvay.
Assuming CP conservation \( g_4 = g_4^* \), \( g_5 = g_5^* \) and \( g_3 = g_3^* \) we can diagonalize with real unitary matrices \((i=L,R)\)

\[
\begin{pmatrix}
\nu_R^o & N_R^o
\end{pmatrix}
\begin{pmatrix}
g_1 v / \sqrt{2} & g_5 \\
g_3 & -g_2 v / \sqrt{2}
\end{pmatrix}
\begin{pmatrix}
\nu_L^o \\
N_L^o
\end{pmatrix}
\]

+ h.c. \quad (11)

The relations

\[
g_3 = -m_e \sin \Theta_R \cos \Theta_L + m_E \sin \Theta_L \cos \Theta_R
\]

\[
g_3 = -m_\nu \sin \Phi_R \cos \Phi_L + m_N \sin \Phi_L \cos \Phi_R,
\]

arise from the electron and neutrino mass matrices, respectively. Equating the two expressions of \( g_3 \), we get Eq.(5), mentioned before. The origin of the relation is that the \( g_3 \bar{\chi}_R \psi_L^o + h.c. \) term makes a connection between the parameters of the electron and neutrino mass matrices.

Excluding \( \nu_R^o \) from the initial fields we get instead of Eq. (13)

\[
g_3 = m_N \sin \Phi_L,
\]

which coincides with Eq. (13) provided \( m_\nu = 0, \Phi_R = 0 \) are choosen. Since \( m_E, m_N \gg m_e, m_\nu \) and the mixing angles are small, this possibility does not much differ from the previous one. It is clear that similar relations do arise for all particle pairs, whose L components form a doublet and have mirror partners.

4. We have studied the constraints on mirror fermion mixing angles arising from a comparison with anomalous magnetic moment data. We have found a new relation Eq.(3) (or Eq.(5)) connecting mirror mixing angles and fermion masses. Taking this relation into account makes reconciliation of anomalous magnetic moment data with not too small mixing angles difficult in the case when left and right mixing angles are equal. An appealing possibility leading to reasonably large mirror lepton production cross-sections in electron proton collisions is to assume a very small right electron mixing
angle, with left mixing angles close to the experimental limits of $2, 3$.

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Table 1. Total production cross-section $\sigma$ of the mirror electron-neutrino and mirror electron in electron-proton collisions at $\sqrt{s}=314\text{GeV}$ as a function of the mirror lepton mass. Left mixing angle squares are assumed to be 0.02, while the right mixing angles are zero. The cross-sections are in $10^{-2}\text{pb}$, masses are in GeV.

| M (GeV) | $\sigma(ep \to NX)$ | $\sigma(ep \to EX)$ |
|--------|---------------------|---------------------|
| 100    | 19.5                | 4.7                 |
| 120    | 11.8                | 2.7                 |
| 140    | 6.8                 | 1.5                 |
| 160    | 3.5                 | 0.7                 |
| 180    | 1.7                 | 0.3                 |
| 200    | 0.7                 | 0.13                |
Figure Captions

Fig. 1. Diagrams for the calculation of the mirror fermion contribution to the magnetic moment of the electron.

Fig. 2. Curves showing the possible values of the mixing angles for different mirror electron masses. The value of the mirror fermion contribution to the anomalous magnetic moment of the electron is set equal to the experimental error.