Thermophoresis phenomenon in radiative flow about vertical movement of a rotating disk in porous region

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Abstract
This study examines the time-dependent, three-dimensional, laminar, incompressible, and inclined hydromagnetic flow of viscous fluid above a rotating disk. The disk is saturated in a porous space and moves vertically along axial direction. Heat transfer process is analyzed by the thermal radiation and convective boundary condition. Thermophoresis process is illustrated in the flow stream by mass equation. The similarity functions are exerted to transform the system of partial differential equations into a one independent variable expressions. The finite differences scheme is applied to the resultant dimensionalized system and then solved in iterative manner using Successive over Relaxation (SOR) method. The physical quantities are illustrated graphically in terms of velocity, thermal and concentration fields. Two-dimensional contour phenomenon and three-dimensional flow visualizations for some parameters are also sketched. The skin-friction co-efficient, frictional torque, Nusselt number and Sherwood number against selected parameters are discussed in tabular format.

Keywords
Viscous fluid, permeable disk, porous medium, thermal radiation, thermophoresis, numerical solutions

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Introduction
Flow phenomenon of viscous fluid across disk systems is a classical topic to study due to its practical/technological applications in flight mechanics, aircrafts engines, turbo machinery, spin coating, multi-pore distributors, rotor-stator systems to stabilize the effectiveness of turbine disks, medical equipment, oceanography, and microclimate systems. The rotating flows are relevant to chemically vapor deposition reactor used for the sedimentation of optical thin films in the electrochemical engineering. Turkyilmazoglu¹ obtained the analytic expressions for the compressible viscous fluid flow across a rotating porous disk and discussed the flow thermal characteristics. Asghar et al.² executed the flow and thermal features of viscous fluid through a rotating disk which stretches in radial direction with power-law velocity. Turkyilmazoglu³ investigated the axisymmetric activated liquid flow about a rotating sphere in the occurrence of latitudinal stretching. Hafeez et al.⁴ explored the flow phenomenon of Oldroyd-B fluid

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subjected to disk rotation by adopting BVP Midrich software. Doh et al.\textsuperscript{5} investigated the silver nanoparticle influence in chemically reactive fluid flow produced by the disk rotation. They described that the flow velocity is enhanced by the disk thickness. Rauf et al.\textsuperscript{6} addressed the supremacy of variable properties in compressible flow of viscous fluid across rotating disk. Shehzad et al.\textsuperscript{7} elaborated the impact of Hall current in compressible fluid flow about rotating disk. The normalized form of flow model is tackled iteratively through SOR numerical procedure. Acharya et al.\textsuperscript{8} reported the features of hybrid nanoparticles and Hall current in viscous fluid about rotating disk. Devi and Mabood\textsuperscript{9} discussed the entropy anamorization and activation energy in Marangoni Maxwell nanofluid about a rotating disk. Abbas et al.\textsuperscript{10} examined the flow of elastico-viscous nanofluid comprising of motile microorganisms through a rotating disk. Naz et al.\textsuperscript{11} analyzed the flow transportation for Eyring-Powell nanofluid across a rotating disk. A decreased in thermal field is observed due to increase in bioconvective Rayleigh number. Thermophoresis and Coriolis forces influence on peristaltic movement of viscous nanofluid through a rotating frame was studied by Mabood et al.\textsuperscript{12} The exploration of physical phenomenon of fluid movements about disk movements specifically across vertically upward/downward movement of rotating disk has been of considerable interest now adays due to its significant industrial applications. Turkyilmazoglu\textsuperscript{13} studied the fluid flow development and resultant thermal phenomenon caused due to rotating disk describing upward and downward motion vertically. Khan et al.\textsuperscript{14} examined the non-linear thermal radiation analysis in Maxwell fluid about vertical upward/downward movement of rotating disk by bvp-4c technique. Manjunatha et al.\textsuperscript{15} computed the numerical solution for flow model of magnetized nanofluid about a vertically moving spinning disk through RKF-45 technique. Kumar et al.\textsuperscript{16} presented the thermal analysis for Casson hybrid nanofluid flow by a vertically upward/downward movement of rotating disk. Kumar and Sharma\textsuperscript{17} analyzed the Soret and Dufour’s effects in flow of hybrid nanofluid over a vertically moving disk.

Convective thermal transport in fluids saturated by the porous space plays major role in petroleum engineering, geophysics, geothermal energy, hydrology, heat pipes, grain storage and coal combustors. Combustion efficiency is also based on porous medium. Combustion is implemented in surface burner and advanced boiler. Furthermore, uniform heat flux can also be acquired by implementing porous media through combustion systems and is useful in various industrial applications like paper drying processes. The analysis of fluid phenomenon across porous medium deals with the differential equation, which is based upon Darcy’s law.\textsuperscript{18} Krishna and Chamkha\textsuperscript{19} discussed the magnetic field implementation in elastico-viscous fluid between rotating rigid plates through porous space. Abbas et al.\textsuperscript{20} considered the magnetic field impact in time-dependent oscillatory flow about rotating disk, which experience porous medium supremacy. Mabood and Das\textsuperscript{21} analyzed the thermal melting features in hydromagnetic Casson fluid via linear horizontal surface through porous medium. They recognized that permeability and magnetic parameters are turned down the velocity field. Liu et al.\textsuperscript{22} distinguished the convective-diffusion in magneto-hydrodynamic fluid flow about unsteady stretched surface occupies heterogeneous porous space. MHD flow phenomenon with nanofluid about rotating disk was presented by Reddy et al.\textsuperscript{23} The fluid model experienced the effect of porous medium and solved in numerical way by finite element method.

Thermal radiation has important features in technological processes at large temperature scales and hence the effects of thermal radiation cannot be eliminated in heat transfer processes. Recent advancements of thermal radiation in the missiles combustion chambers, power plants, hypersonic flights, solar technologies, space vehicles and satellites have focused the realization of the scientists that energy can be transferred through electromagnetic waves. Reddy and Ferdows\textsuperscript{24} discussed the behavior of thermal radiation and species transfer in dusty micropolar fluid through paraboloid revolution. Heat transfer rate is found as declining function of radiation parameter by RK integration method. Khan et al.\textsuperscript{25} analyzed the impact of radiation in chemically reactive flow of Maxwell nanofluid across vertically moving disk. The influence of thermal radiation in three-dimensional revolving flow of Maxwell fluid about exponential surface was prescribed by Rashid et al.\textsuperscript{26} using Homotopy procedure. Mishra and Kumar\textsuperscript{27} considered the hydro-thermal nanofluid over permeable surface and scrutinized the thermal radiation impacts in flow behavior. Mahanthes et al.\textsuperscript{28} addressed the numerical analysis of non-linear convective Oldroyd-B fluid flow past a bi-directional stretching surface. Zhu et al.\textsuperscript{29} studied the radiative flow of nanofluid with temperature jump and velocity slip boundary conditions using HAM.

The thermophoretic depositions of the particles is a major process in aerosol and electronic industries. The particles depositions can be influenced by various include turbulence, inertia effect, surface, geometry, thermophoretic and electrophoretic. The particles depositions by the thermophoretic factor are accounted in this investigation. Thermophoretic is a force, which is accounted by the small aerosol particles through temperature gradient and velocity gained by the particle is familiar as “thermophoretic velocity.” The process of thermophoresis in Newtonian fluid flow about speedily rotating disk was analyzed by Rahman and
Postelnicu. The results indicated that increased thermophoretic coefficient enhances the thermophoretic velocity, which also intensifies significantly by increase in relative temperature difference parameter. Alam et al. investigated the particle’s thermophoretic deposition in flow phenomenon of viscous fluid across rotating disk. They elaborated that the axial thermophoretic velocity is of decreasing nature against increased Lewis number. Khan and Khan discussed the thermophoretic effects in Burgers fluid flow about bidirectional surface. Concentration field was observed to be declining function of thermophoretic parameter. Doh and Muthamiliselvan examined the flow features of micropolar fluid through rotating disk under the impact of thermophoresis. Ho et al. discussed the combined influence of thermophoresis-electrophoresis on aerosol particle depositions rates across a wavy medium. The vertical speed of the disk is a declining function of thermophoretic parameter. Shehzad et al. investigated the particle’s thermophoretic coefficient enhances the thermophoretic velocity, which also intensifies significantly by increase in temperature gradient. Shehzad et al. examined the incompressible flow of Maxwell fluid about spinning disk under the impacts of Cattaneo-Christov expressions and thermophoresis phenomenon. Irfan and Farooq analyzed the thermophoretic hydromagnetic flow with variable liquid features by adopting the exponential surface.

In view of the above literature, the thermophoresis phenomenon in electrically conducting thermal flow stream of viscous fluid about a revolving and orthogonally moving permeable disk is not investigated yet. Thermophoresis influence finds key applications in the process of microscale thermophoresis that is useful in the characterization phenomenon of drug discovery screening process in pharmaceutical industry and biomolecular interactions. The radiative flow has numerous features in manufacturing processes like energy devices, hybrid engines, fuel cells, power systems and biomedicine procedures. Furthermore, the convective thermal transport involves in various engineering processes such as nuclear plants and thermal energy storage. The flow model in normalized form is solved through SOR method. Graphical and tabular descriptions are presented to elaborate the numerical results for various physical parameters.

**Problem formulation**

A vertically moving permeable rotating disk is considered. The fluid movement is generated by the upward/downward movement of the disk. The disk also revolves with angular speed $\Omega(t)$ and saturates in a porous medium. The vertical speed of the disk is $\omega = \dot{a}(t)$.

Initially, at $t = 0$ the disk is located at the position $a(0) = h$ and after some time the disk attains its vertical position at $z = a(t)$. Magnetic field with strength $\beta_0^2$ is implemented with acute angle $\gamma$ along r-direction. The magnetic field acts in transverse direction, that is, perpendicular to the revolving disk with $\gamma = 90^\circ$. The induced magnetic field is negligible comparison to the imposed field. The magnetic Reynolds number is considered to be small. It is also supposed that there is no implication of polarization voltage such that the electric field is zero. The permeable surface is heated owing to the contact with hot fluid having temperature $T_f$ and temperature at free-stream is $T_\infty$. The uniform concentration at disk surface is $C_w$, whereas the fluid ambient concentration is $C_w$. The impressions of thermophoresis and thermal radiation are encountered in the flow stream through mass and energy equations. The mass flow is substantially slow so that the main flow and thermal stream are not affected by the thermophysical process. For the flow model, the accurate choice for the co-ordinate system is cylindrical co-ordinate system $(r, \theta, z)$ with velocity components $(u, v, w)$. The physical flow phenomenon is sketched through Figure 1.

The flow constitutive equations are follows as:

\[ \frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} + \frac{u}{r} = 0, \] (1)

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{v^2}{r} + \frac{\partial w}{\partial r} = v \left( \frac{\partial^2 u}{\partial z^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2} - \frac{u}{r^2} \right) + \frac{P}{\rho} \frac{\partial \rho}{\partial r} - \frac{\mu \phi_0}{\rho k_0} u - \frac{\beta_0^2 g}{\rho} \sin^2 \gamma u, \] (2)
\[ \frac{\partial \nu}{\partial t} + \frac{w \nu}{r} + \frac{\nu w}{r} + u \frac{\partial \nu}{\partial r} = \nu \left( \frac{\partial^2 \nu}{\partial z^2} + 1 \frac{\partial \nu}{\partial r} - \frac{\nu}{r} + \frac{\partial^2 \nu}{\partial r^2} \right) \]
\[ - \frac{\mu}{\rho} \frac{\nu}{\partial z} + \beta_\nu \nu \sin^2 \gamma \nu, \] (3)
\[ \frac{\partial \nu}{\partial t} + \frac{w \partial \nu}{\partial z} + \frac{\nu \partial \nu}{\partial r} = \nu \left( \frac{\partial^2 \nu}{\partial z^2} + 1 \frac{\partial \nu}{\partial r} + \frac{\partial^2 \nu}{\partial r^2} \right) \]
\[ - \frac{1}{\rho} \frac{\partial \rho}{\partial z} - \frac{\mu}{\rho} \frac{\nu}{\partial z} w, \] (4)
\[ \frac{\partial T}{\partial t} + \frac{u \partial T}{\partial r} + \frac{w \partial T}{\partial z} = \frac{k}{\rho c_p} \left( \frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) \]
\[ - \frac{1}{\rho c_p} \frac{\partial}{\partial z} (\varphi_r), \] (5)
\[ \frac{\partial C}{\partial t} + \frac{u \partial C}{\partial r} + \frac{w \partial C}{\partial z} = D \left( \frac{\partial^2 C}{\partial z^2} + \frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} \right) \]
\[ - \frac{1}{\partial z} \left( C \varphi_r \right) \] (6)

in which \( p, \rho, \varphi_0, \mu, k_0, \nu, k, q_r, c_p, C, D, T, U_T, \sigma_e, \) and \( W_T \) specifies the pressure, fluid density, porosity of permeable space, fluid viscosity, porous medium permeability, kinematic viscosity, thermal conductivity, radiative heat flux, specific heat, concentration, mass diffusivity, temperature, electrical conductivity, and thermophoretic velocities.

The radiative heat flux is calculated as:
\[ q_r = - \left( \frac{4 \sigma^*}{3 k^*} \right) \frac{\partial}{\partial z} (T^4). \] (7)

Where \( \sigma^* \) and \( k^* \) represent the Stefan-Boltzmann factor and mean absorption co-efficient. The temperature differences are considered small enough such that \( T^4 \) appear as a linear function. Taylor’s theorem is used for expansion of \( T^4 \) at \( T_\infty \) and omitting higher order terms, we get
\[ T^4 \approx T_\infty^4 + 4 T_\infty^4 (T - T_\infty) = 4 T_\infty^4 - 3 T_\infty^4. \] (8)

The thermophoretic velocities are defined by:
\[ U_T = - \frac{\nu k_1}{T} \left( \frac{\partial T}{\partial r} \right) \text{ and } W_T = - \frac{\nu k_1}{T} \left( \frac{\partial T}{\partial z} \right), \] (9)
in which \( k_1 \) and \( \nu k_1 \) determines the thermophoretic co-efficient and thermophoretic diffusivity. The interval of \( k_1 \) is 0.2–1.2.

The suitable boundary conditions under flow assumptions are expressible as:
\[ u = 0, \quad v = r \Omega(t), \quad w = \beta_2 \hat{u}(t), \quad -k \left( \frac{\partial^2}{\partial x^2} \right) = h_1(T_f - T), \]
\[ C = C_w = 0 \text{ (clean and completely absorbing surface),} \]
\[ p = \text{ const.} = 0 \text{ at } z = a(t), \]
\[ u \rightarrow 0, \quad v \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \text{ as } z \rightarrow \infty, \] (10)

where \( h_1 \) and \( \beta_2 \) are the heat transfer co-efficient and the wall permeability strength. Furthermore \( \beta_2 > 1, \beta_2 < 1 \) and \( \beta_2 = 1 \) determine the wall injection, wall suction and impermeable surface.

The below similarity functions are considered:
\[ u = \frac{\nu}{\sigma^*} F(\eta), \quad v = \frac{\nu}{\sigma^*} G(\eta), \quad w = \frac{\nu}{\sigma^*} P(\eta), \]
\[ T = T_\infty + (T_f - T_\infty) \theta(\eta), \quad C = C_\infty \varphi(\eta), \quad \eta = \frac{r}{\sigma^*} - 1. \] (11)

Switching equation (11) into the equations (1)–(6) and making use of equations (8) and (9), we have the following system:
\[ 2F + H' = 0, \]
\[ F'' - HF' - F^2 + G^2 + S \left( \frac{\eta + 1}{2} F' + F \right) \]
\[ - P_1F - M^2 \sin^2 \gamma F = 0, \]
\[ G'' - HG' - 2FG + S \left( \frac{\eta + 1}{2} G' + G \right) \]
\[ - P_1G - M^2 \sin^2 \gamma G = 0, \]
\[ H'' - HH' + S \left( \frac{\eta + 1}{2} H' + H \right) - P_1H = P', \]
\[ \left( \frac{4}{3} R_d + 1 \right) \theta'' + PrS \left( \frac{\eta + 1}{2} \right) \theta' - Pr \theta H' = 0, \] (12)
\[ \phi'' - \left( k_1 \text{Sc} N_t (1 - N_t \theta)^{-1} \theta' + \text{Sc} H - \text{Sc} S \left( \frac{\eta + 1}{2} \right) \right) \phi' \]
\[ - k_1 \text{Sc} N_t (1 - N_t \theta)^{-1} \theta'' + N_t (1 - N_t \theta)^{-2} \theta^2 \phi = 0, \] (13)

The boundary conditions (10) in view of (11) extract the following form:
\[ F(0) = 0, \quad G(0) = \Gamma_0, \quad H(0) = \beta_1 \frac{\eta}{2}, \quad \theta'(0) \]
\[ = - \beta_1 (1 - \theta(0)), \quad \phi(0) = 0, \quad P(0) = 0, \]
\[ F(\infty) \rightarrow 0, \quad G(\infty) \rightarrow 0, \quad \theta(\infty) \rightarrow 0, \quad \phi(\infty) \rightarrow 1. \] (14)

Where \( S = \frac{2 \nu}{\sigma^*}, \quad M = \sqrt{\frac{\beta_\nu \sigma^* \rho (\gamma)}{\rho w}}, \quad P_1 = \frac{\sigma^* \rho (\gamma)}{\rho w}, \quad \text{Pr} = \frac{\nu \sigma^* \rho (\gamma)}{\rho w}, \quad \text{Sc} = \frac{v}{\nu}, \quad \beta_1 = \frac{k_1 v}{R_d}, \quad R_d = \frac{4 \pi^2 T_\infty}{k_0 k_1}, \quad \Gamma_0 = \frac{\Omega \sigma^*}{v}, \quad \text{and } N_t = \frac{T_\infty - T_f}{T_\infty}, \)
determine the contracting/expanding parameter,
magnetic parameter, porosity parameter, Prandtl number, Schmidt number, heat transfer Biot number, radiation parameter, rotation parameter and relative temperature difference parameter, which have negative values against heated surface and positive value for cooled surface.

After integrating equation (15), the pressure distribution can be evaluated as:

\[
P(\eta) = \frac{H'}{2} + S \left( \frac{\eta + 1}{2} \right) H + \frac{S}{2} \int_0^\eta H(\eta) \, d\eta - P_i \eta H(\eta) \, d\eta + p_0.
\]

(19)

Here \(p_0\) stands for the unknown constant pressure.

The most notable quantities are \(G'(0), F'(0), - (1 + \frac{1}{4} \delta d) \theta'(0)\) and \(-\phi'(0)\), which are related with the torque represented for disk rotation, the local skin-friction calculated for the drag, the Nusselt number to quantify heat transfer and the Sherwood number to compute mass transfer.\(^{13,14}\)

**Numerical solution**

The essence of finite difference procedure is very close to numerical methods which are used to solve the differential equations. The derivatives are replaced in the differential equation through differential quantities in order to approximate the differential operator. The domain is split up in time and space. The solution approximations are calculated at time points or space. To solve the differential system (12)–(18), we adopted the following finite difference schemes:\(^{6,7}\):

\[
\frac{\partial F}{\partial \eta} \approx \frac{F_{i+1, j} - F_{i-1, j}}{2 \delta \eta}, \quad \frac{\partial G}{\partial \eta} \approx \frac{G_{i+1, j} - G_{i-1, j}}{2 \delta \eta},
\]

\[
\frac{\partial H}{\partial \eta} \approx \frac{H_{i+1, j} - H_{i-1, j}}{2 \delta \eta},
\]

\[
\frac{\partial \theta}{\partial \eta} \approx \frac{\theta_{i+1, j} - \theta_{i-1, j}}{2 \delta \eta}, \quad \frac{\partial \phi}{\partial \eta} \approx \frac{\phi_{i+1, j} - \phi_{i-1, j}}{2 \delta \eta}.
\]

(20)

\[
\frac{\partial^2 F}{\partial \eta^2} \approx \frac{F_{i+1, j} - 2 F_{i, j} - F_{i-1, j}}{\delta \eta^2},
\]

\[
\frac{\partial^2 G}{\partial \eta^2} \approx \frac{G_{i+1, j} - 2 G_{i, j} - G_{i-1, j}}{\delta \eta^2},
\]

\[
\frac{\partial^2 \theta}{\partial \eta^2} \approx \frac{\theta_{i+1, j} - 2 \theta_{i, j} - \theta_{i-1, j}}{\delta \eta^2},
\]

\[
\frac{\partial^2 \phi}{\partial \eta^2} \approx \frac{\phi_{i+1, j} - 2 \phi_{i, j} - \phi_{i-1, j}}{\delta \eta^2}.
\]

(21)

Here \((i, j)\) is examined along spatial and time level accordingly.

The semi-infinite domain \([0, \infty]\) is truncated into finite one \([0, n]\). The last point \(n\) of the domain is revealed at infinity for the boundary conditions. The spatial variable \(\eta\) is discretized uniformly through \(r + 1\) points and having step size \(\delta \eta = \frac{\delta}{r}\). After that the SOR method is implemented to the obtained system for the faster convergence. The SOR method is an advanced form of the Gauss-Seidel iterative method where a weighted average parameter or relaxation parameter \(\omega\) is used to accelerate the rate of convergence of the solution. The SOR method is more efficient and memory storage efficient. The range of the relaxation parameter \(1 < \omega < 2\) gives the accelerated convergence of the SOR iterative method. In our numerical computations, we fixed \(\omega = 1.95\). The value of relaxation parameter \(\omega \geq 2\) is not acceptable as it leads to the divergence of the current method. The convergence criterion of a spectral radius is expressed by:\(^{41}\):

\[
\rho(G_\omega) = \frac{1}{4} (\omega p + \sqrt{(\omega p)^2 - 4(\omega - 1)^2}) \quad \text{for } 0 < \omega \leq \omega_{opt}
\]

\[
\rho(G_\omega) = \frac{\omega - 1}{1 + \sqrt{1 - (\rho(G_{Jac}))^2}} \quad \text{for } \omega_{opt} < \omega < 2.
\]

(22)

Where the optimal relaxation parameter is determined as:

\[
\omega_{opt} = 1 + \left( \frac{\rho(G_{Jac})}{1 + \sqrt{1 - (\rho(G_{Jac}))^2}} \right)^2.
\]

(23)

Here \(\rho(G_{Jac})\) determines spectral radius of Jacobi method.

**Results and discussion**

Numerical study is analyzed for the viscous fluid across orthogonally moving permeable rotating disk. The flow model incorporates the impacts of inclined magnetic field, porous medium and thermophoresis. A constant injection/suction is utilized at the surface of the permeable disk. The governing system is normalized through similarity functions. The normalized system consists upon the physical quantities \(S, P_1, M, Pr, k_1, Sc, Nt, \Gamma_0, \beta_2\) and \(\beta_1\), which are discussed for the suitable choice of values\(^{42}\) against the radial velocity \(F(\eta)\), axial velocity \(H(\eta)\), tangential velocity \(G(\eta)\), temperature field \(\theta(\eta)\), and concentration field \(\phi(\eta)\) by using SOR method. To simply the problem, the numerical analysis is performed for the wall injection \((\beta_2 > 1)\) and the disk expansion \((S > 0)\).
Figures 2 to 4 examine the behavior of contracting/expanding parameter on flow velocities $F(\eta)$, $G(\eta)$, and $H(\eta)$ respectively. Figure 2 explains that initially the velocity profiles are enhanced and then a decreasing phenomenon in radial velocity is occurred. This generates a blowing-like influence for the upward wall movement and a suction-like impact for the downward act of the spinning disk. Figure 3 demonstrated the effect of $S$ on $G(\eta)$. The velocity $G(\eta)$ is accountable for the influence of centrifugal forces. It is seen that the wall upward motion augments the rotational influence of the liquid particles while the disk movement in downward direction declines the fluid rotational velocity. The azimuthal velocity curves against $S$ are portrayed through Figure 4. The velocity field $H(\eta)$ is enlarging function of the expanding/expanding parameter. For the asymptotic behavior of the flow field, we fix $\eta = 7$

in Figures 4 to 7. Figure 5 describes the effect of porosity parameter on $F(\eta)$. The increasing behavior of porosity parameter resulted into reduction of the velocity curves. An enhancement in the porosity parameter generates the resistive force between the fluid layers due to which retardation in the flow movement is observed. As a result, a reduction in velocity field along radial direction is observed. Furthermore, the first case $P_1 = 0$ characterizes the non-porous medium. The characterization of the magnetic parameter on $F(\eta)$ is reported through Figure 6. The velocity profiles delineating reducing trend against the magnifying magnetic parameter values. Lorentz force is generated in the flow field by enhancement in magnetic parameter due to which friction force increases among the fluid layers. Therefore, profiles are exhibiting reducing nature (see Figure 6).
Figure 7 elaborates the influence of rotation parameter on velocity field $G(\eta)$. The profiles are enhanced against larger values of the rotation parameter. The torsional velocity is enhanced due to the increase in rotational parameter. An increase in the torsional velocity is then responsible in the enhancement of the momentum transportation process along the tangential direction. Therefore, an uplift in the curves $G(\eta)$ is observed against enlarged rotation parameter. In Figure 7, the upper boundary is adjusted by adopting $\eta = 6$. Figures 8 to 10 are elaborated to discuss the consequences of wall strength permeability parameter for the impermeable surface, wall suction and wall injection on $F(\eta)$, $G(\eta)$, and $H(\eta)$. The velocity profiles are lower down due to the suction ($\beta_2 < 1$) at moving surface while the velocity curves are increases against the wall injection ($\beta_2 > 1$) case (see Figures 8–10). Furthermore, the flow phenomenon for the wall suction/injection is similar to the disk contraction/expansion effect onto the flow field.

The phenomenon of Prandtl number on temperature field is prescribed through Figure 11. Larger Prandtl number contributes to the reduction of thermal filed along with allied boundary layer thickness. As Prandtl number is inversely proportional to the thermal diffusivity. For larger Pr values, the thermal diffusivity is decreased and hence a reducing trend of the thermal curves is noticed in Figure 11. Radiation parameter has opposite nature on temperature field in comparison to the effect of Pr on $\theta(\eta)$. Due to increase in radiation parameter, more heat is added in the thermal flow field as a result, the thermal curves are enlarged by strengthen the radiation parameter as elucidated in Figure 12. Figure 13 demonstrates the thermal
phenomenon against heat transfer Biot number. Physically, the Biot number has directly relationship with the energy transmission coefficient which enlarges with the increase in $b_1$. Such modification in the heat transfer coefficient results into the enhancement of fluid temperature. Therefore, a boost in temperature curves are examined against increased Biot number.

Schmidt number effect is elucidated on concentration field by Figure 14. The concentration profiles exhibit increasing trend due to the enlargement in Schmidt number. The boundary conditions on to concentration field are also satisfied by the profiles $\phi(\eta)$. Furthermore, for the asymptotic behavior of the concentration field, we fixed $\eta = 12$. The relative temperature difference parameter effect in terms of concentration field is presented in Figure 15.

Concentration profiles are close to each other however a decreasing nature in concentration curves is observed against increased $Nt$ values. The demonstration of thermophoretic co-efficient on concentration filed is elaborated via Figure 16. Thermophoretic co-efficient is responsible in the reducing phenomenon of concentration profiles. Moreover, the concentration field is not significantly influenced by the thermophoretic co-efficient.

The contour graphs of the stream functions $F$, $G$, $H$, $\theta$ and $\phi$ under the variation of physical parameters have been presented in Figures 17–21 respectively. The contours graphs are coded with two basic colors such as blue and yellow. The blue color is indicating the minimum value in the graph while the yellow color is representing the maximum value in the graph.

![Figure 10. Effect of $b_2$ on $H(\eta)$.](image)

![Figure 11. Effect of $Pr$ on $\theta(\eta)$.](image)

![Figure 12. Effect of $Rd$ on $\theta(\eta)$.](image)

![Figure 13. Effect of $b_1$ on $\theta(\eta)$.](image)
The intermediate values between minimum and maximum values are demonstrating with the gradient of two colors as mentioned above.

Figure 17 represents the contour graph of the stream function \( F \) with the variation in the values of the parameter \( S \). We note that the maximum values of the contour’s curves are found within the range 0.5 and 1 of the variable \( \eta \). There is a significant increase in the values of the contour curves along the incremental values of the parameter \( S \). The effect of contracting/expanding parameter on the stream function \( G \) is illustrated in Figure 18. The values of the contour curves decrease with the rising values in the variable \( \eta \) while the increasing values in \( S \) has insignificant effect on the contour’s curves. For the case of the stream function \( H \), the values in the contours curves increases with the increasing values of contracting/expanding parameter as shown in Figure 19. The growing values of \( \beta_1 \) has the increasing effect on the contour curves of temperature \( \theta \) (see Figure 20). The contours graph of the stream function \( \varphi \) is demonstrated in Figure 21. We observed that the contour curves have increasing values with the increment in the values of \( \eta \) while the parameter \( Nt \) has insignificant effects on the contour curves of \( \varphi \).

The three-dimensional visualization of the stream functions \( F, G, H, \theta \) and \( \varphi \) against some selected physical quantities is demonstrated in Figures 22 to 26. Figure 22 is showing the influence of the parameter \( S \) on the three-dimensional graph of the stream function \( F \). Along the \( \eta \), the stream function \( F \) has a rising trend from 0 to about 1 and then it gets retardation after that. The rising values of the contracting/expanding
parameter increases the amplitude of the graph of the stream function $F$. The three-dimensional graph of $G$ is illustrated in Figure 23, which has maximum value at $\eta = 0$ and then it starts decreasing with the increasing values of $\eta$. However, the contracting/expanding parameter has negligible influence on the graph of the stream function $G$. The similar behavior of the three-dimensional graph of the stream function $H$ has been noted in Figure 24. Figure 25 is depicting the three-dimensional graph of the temperature $u$ under the influence of permeability strength parameter. This graph has also maximum values at $\eta = 0$ and has retardation trend after that value of $\eta$. The three-dimensional graph of $\phi$ is displayed in Figure 26 which has minimum value at $\eta = 0$ and has significant increasing trend about $\eta = 6$. However, the parameter $Nt$ has insignificant effects on the graph of $\phi$.

Table 1 revealed the effects of magnetic and porosity parameters on the local skin-friction and frictional torque. A decreasing trend in local skin-friction is observed against the magnetic parameter porosity parameters whereas magnitude of the frictional torque is enhanced at the disk surface by such parameters $(M P_1)$. Table 2 determines the heat and mass transfer rates at the surface of moving disk against relative temperature difference parameter and thermophoretic coefficient. A modification in mass transfer rate is experienced for increased $Nt$ and $k_1$ values while both the parameters have reverse phenomenon on heat transfer rate. Table 3 is presented for the validation of our
numerical scheme with the literature work in limiting scenario for local skin friction against expanding/contracting parameter. Excellent agreement in the numerical results is observed.

Conclusions

An incompressible, laminar, and electrically conducting flow of viscous fluid is examined about vertical upward/downward moving disk. The disk also rotates along tangential direction and occupied space in a porous medium. The expression of thermal radiation is utilized in the energy equation. The convective condition of thermal transport is implemented through thermal boundary condition at the disk surface. The thermophoresis effects are visualized through mass equation. The mathematical model is treated numerical by applying first finite differences schemes and then iteratively solved through SOR method. Following are the main findings:
1. A decreasing trend in radial velocity curves is obtained against increased values of the porosity parameter.

2. The radial velocity profiles against contracting/expanding parameter act in similar way as the effects of wall blowing and suction at the rotating disk surface. The profiles along radial directions are first increased and then a reducing trend is noticed in such profiles.

3. The tangential velocity curves preserve increasing nature against the rotation parameter.

4. The temperature profiles with related thermal boundary layer thickness become larger due to the enhancement in the thermal Biot number.

5. The increased values of the temperature difference parameter \( N_t \) is responsible in the reduction of concentration field.

6. Local skin-friction is reduced, and frictional torque is enhanced by the modified values of the porosity parameter \( P_1 \).

7. The thermophoretic co-efficient is accountable in the increasing phenomenon of mass transfer rate.

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**References**

1. Turkyilmazoglu M. Purely analytic solutions of the compressible boundary layer flow due to a porous rotating disk with heat transfer. *Phys Fluids* 2009; 21: 106104.

2. Asghar S, Jalil M, Hussan M, et al. Lie group analysis of flow and heat transfer over a stretching rotating disk. *Int J Heat Mass Transf* 2014; 69: 140–146.

3. Turkyilmazoglu M. Latitudinally deforming rotating sphere. *Appl Math Model* 2019; 71: 1–11.

4. Hafeez A, Khan M and Ahmed J. Flow of Oldroyd-B fluid over a rotating disk with Cattaneo–Christov theory for heat and mass fluxes. *Comput Methods Programs Biomed* 2020; 191: 105374.

5. Doh DH, Muthtamilselvan M, Swathene B, et al. Homogeneous and heterogeneous reactions in a nanofluid flow due to a rotating disk of variable thickness using HAM. *Math Comput Simul* 2020; 168: 90–110.

6. Rauf A, Abbas Z, Shehzad SA, et al. Characterization of temperature-dependent fluid properties in compressible viscous fluid flow induced by oscillation of disk. *Chaos Solitons Fractals* 2020; 132: 109573.

7. Shehzad SA, Abbas Z, Rauf A, et al. Effectiveness of Hall current and thermophysical properties in compressible flow of viscous fluid thorough spinning oscillatory disk. *Int Commun Heat Mass Transf* 2020; 116: 104678.

8. Acharya N, Bag R and Kundu PK. Influence of Hall current on radiative nanofluid flow over a spinning disk: a hybrid approach. *Physica E Low Dimens Syst Nanosyst* 2019; 111: 103–112.

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**Table 1. Shear stresses against \( M \) on \( P_1 \).**

| \( M \) | \( P_1 \) | \( F'(0) \) | \( -G'(0) \) |
|---|---|---|---|
| 0.0 | 0.3 | 0.409757 | 0.738237 |
| 0.2 | 0.3 | 0.405143 | 0.747431 |
| 0.3 | 0.3 | 0.399499 | 0.758901 |
| 0.4 | 0.3 | 0.391824 | 0.774914 |
| 0.5 | 0.3 | 0.382337 | 0.795412 |
| 0.5 | 0.0 | 0.513246 | 0.565534 |
| 0.5 | 0.2 | 0.453992 | 0.657447 |
| 0.5 | 0.4 | 0.404003 | 0.749727 |
| 0.5 | 0.7 | 0.344991 | 0.884880 |
| 0.5 | 1.0 | 0.301354 | 1.012536 |

**Table 2. Heat and mass transfer rates against \( N_t \) and \( k_1 \).**

| \( N_t \) | \( k_1 \) | \( -(1 + \frac{4}{3} R_d) \theta'(0) \) | \( -\phi'(0) \) |
|---|---|---|---|
| 1.2 | 0.5 | 0.232273 | 0.661869 |
| 1.5 | 0.5 | 0.116316 | 0.675037 |
| 1.8 | 0.5 | 0.087102 | 0.676890 |
| 2.2 | 0.5 | 0.070972 | 0.686089 |
| 2.5 | 0.5 | 0.064520 | 0.702147 |
| 1.5 | 0.2 | 0.011061 | 0.663348 |
| 1.5 | 0.5 | 0.027652 | 0.665566 |
| 1.5 | 0.8 | 0.044242 | 0.667782 |
| 1.5 | 1.0 | 0.055303 | 0.669259 |
| 1.5 | 1.2 | 0.066364 | 0.670734 |

**Table 3. Comparison of the local skin friction \( F'(0) \) against expanding/contracting parameter.**

| \( S \) | \( F'(0) \) Turkyilmazoglu13 | \( F'(0) \) [Present] |
|---|---|---|
| 0.1 | 0.00087459 | 0.00087460 |
| 1   | 0.06317625 | 0.06317626 |
| 5   | 0.74000467 | 0.74000469 |
| 10  | 1.34016519 | 1.34016520 |
9. Devi SSU and Mabood F. Entropy anatomization on Marangoni Maxwell fluid over a rotating disk with non-linear radiative flux and Arrhenius activation energy. *Int Commun Heat Mass Transf* 2020; 118: 104857.

10. Abbasi A, Mabood F, Farooq W, et al. Bioconvective flow of viscoelastic nanofluid over a convective rotating stretching disk. *Int Commun Heat Mass Transf* 2020; 119: 104921.

11. Naz R, Mabood F, Sohail M, et al. Thermal and species transportation of Eyring–Powell material over a rotating disk with swimming microorganisms: applications to metallurgy. *J Mater Res Technol* 2020; 9: 5577–5590.

12. Mabood F, Abbasi A, Farooq W, et al. Thermophoresis effect on peristaltic flow of viscous nanofluid in rotating frame. *J Therm Anal Calorim* 2021; 143: 2621–2635.

13. Turkyilmazoglu M. Fluid flow and heat transfer over a rotating and vertically moving disk. *Phys Fluids* 2018; 30: 063605.

14. Khan M, Ahmed J and Ali W. Thermal analysis for radiative flow of magnetized Maxwell fluid over a vertically moving rotating disk. *J Therm Anal Calorim* 2021; 143: 4081–4094.

15. Manjunatha PT, Punith Gowda RJ, Naveen Kumar R, et al. Numerical simulation of carbon nanotubes nanofluid flow over vertically moving disk with rotation. *Partial Differ Equ Appl Math* 2021; 4: 100124.

16. Kumar RN, Gowda RJP, Gireesha BJ, et al. Non-Newtonian hybrid nanofluid flow over vertically upward/downward moving rotating disk in a Darcy–Forchheimer porous medium. *Eur Phys J Spec Top* 2021; 230: 1227–1237.

17. Kumar S and Sharma K. Entropy optimized heat transfer of hybrid nanofluid over vertical moving rotating disk with partial slip. *Chin J Phys* 2022; 77: 861–873.

18. Darcy HRPG. *Les Fontaines Publiques de la volle de Dijon.* Paris: Vector Dalmont, 1856.

19. Krishna MV and Chamkha AJ. Hall and ion slip effects on MHD rotating flow of elasto-viscous fluid through porous medium. *Int Commun Heat Mass Transf* 2020; 113: 104494.

20. Abbas Z, Rauf A and Shehzad SA. Aspects of heterogeneous and homogenous reactions on hydromagnetic oscillatory rotating flow in porous medium. *J Porous Media* 2020; 23: 837–850.

21. Mabood F and Das K. Outlining the impact of melting on MHD Casson fluid flow past a stretching sheet in a porous medium with radiation. *Heliyon* 2019; 5: e01216.

22. Liu C, Zheng L, Pan M, et al. Effects of fractional mass transfer and chemical reaction on MHD flow in a heterogeneous porous medium. *Comput Math Appl* 2019; 78: 2618–2631.

23. Reddy PS, Sreedevi P and Chamkha AJ. MHD boundary layer flow, heat and mass transfer analysis over a rotating disk through porous medium saturated by Cu-water and Ag-water nanofluid with chemical reaction. *Powder Technol* 2017; 307: 46–55.

24. Reddy MG and Ferdows M. Species and thermal radiation on micropolar hydromagnetic dusty fluid flow across a paraboloid revolution. *J Therm Anal Calorim* 2021; 143: 3699–3717.

25. Khan M, Ahmed J, Ali W, et al. Chemically reactive swirling flow of viscoelastic nanofluid due to rotating disk with thermal radiations. *Appl Nanosci* 2020; 10: 5219–5232.

26. Rashid S, Khan MI, Hayat T, et al. Darcy–Forchheimer flow of Maxwell fluid with activation energy and thermal radiation over an exponential surface. *Appl Nanosci* 2020; 10: 2965–2975.

27. Mishra A and Kumar M. Thermal performance of MHD nanofluid flow over a stretching sheet due to viscous dissipation, Joule heating and thermal radiation. *Int J Appl Comput Math* 2020; 6: 1–17.

28. Mahanthesh B, Gireesha BJ, Shehzad SA, et al. Non-linear three-dimensional stretched flow of an Oldroyd-B fluid with convective condition, thermal radiation, and mixed convection. *Appl Math Mech* 2017; 38: 969–980.

29. Zhu J, Zheng L, Zheng L, et al. Second-order slip MHD flow and heat transfer of nanofluids with thermal radiation and chemical reaction. *Appl Math Mech* 2015; 36: 1131–1146.

30. Rahman MM and Postelnicu A. Effects of thermophoresis on the forced convective laminar flow of a viscous incompressible fluid over a rotating disk. *Mech Res Commun* 2010; 37: 598–603.

31. Alam MS, Chapal Hossain SM and Rahman MM. Transient thermophoretic particle deposition on forced convective heat and mass transfer flow due to a rotating disk. * Ain Shams Eng J* 2016; 7: 441–452.

32. Khan WA and Khan M. Impact of thermophoresis particle deposition on three-dimensional radiative flow of Burgers fluid. *Results Phys* 2016; 6: 829–836.

33. Doh DH and Mutthamisvela M. Thermophoretic particle deposition on magnetohydrodynamic flow of micropolar fluid due to a rotating disk. *Int J Mech Sci* 2017; 130: 350–359.

34. Ho PY, Chen CK and Huang KH. Combined effects of thermophoresis and electrophoresis on particle deposition in mixed convection flow onto a vertical wavy plate. *Int Commun Heat Mass Transf* 2019; 101: 116–121.

35. Lu H, Zhang LZ, Lu L, et al. Numerical investigation on monodispersed particle deposition in turbulent duct flow with thermophoresis. *Energy Proc* 2019; 158: 5711–5716.

36. Shehzad SA, Mabood F, Rauf A, et al. Forced convective Maxwell fluid flow through rotating disk under the thermophoretic particles motion. *Int Commun Heat Mass Transf* 2020; 116: 104693.

37. Irfan M and Farooq MA. Thermophoretic MHD free stream flow with variable internal heat generation/absorption and variable liquid characteristics in a permeable medium over a radiative exponentially stretching sheet. *J Mater Res Technol* 2020; 9: 4855–4866.

38. Ashraf M and Wehgal AR. MHD flow and heat transfer of micropolar fluid between two porous disks. *Appl Math Mech* 2012; 33: 51–64.
39. Doh DH, Cho GR, Ramya E, et al. Cattaneo-Christov heat flux model for inclined MHD micropolar fluid flow past a non-linearly stretchable rotating disk. Case Stud Therm Eng 2019; 14: 100496.

40. Saif RS, Muhammad T and Sadia H. Significance of inclined magnetic field in Darcy–Forchheimer flow with variable porosity and thermal conductivity. Phys A Stat Mech Appl 2020; 551: 124067.

41. Hachbusch W. Iterative solution of large sparse systems of equations. Cham: Springer International Publishing, 2016.

42. Turkyilmazoglu M. Determination of the correct range of physical parameters in the approximate analytical solutions of nonlinear equations using the Adomian decomposition method. Mediterr J Math 2016; 13: 4019–4037.

Appendix

Notation

\( (u, v, w) \) velocity components
\( z = a(t) \) vertical position \([m]\)
\( \sigma = \dot{a}(t) \) vertical speed \([m/s]\)
\( T_f \) hot fluid temperature
\( C_w \) wall concentration
\( \rho \) density \([kg/m^3]\)
\( \phi_0 \) porosity of permeable space
\( k_0 \) porous medium permeability
\( \sigma_e \) electrical conductivity
\( k \) thermal conductivity \([W/mK]\)
\( c_p \) specific heat \([J/KgK]\)
\( T \) temperature \([K]\)
\( U_T \) thermophoretic velocity \([m/s]\)
\( k_1 \) thermophoretic co-efficient
\( h_1 \) heat transfer co-efficient
\( S = \frac{2n_i}{\theta_0} \) contracting/expanding parameter
\( P_i = \frac{k_1}{h_1} \) porosity parameter
\( Sc = \frac{r}{h_1} \) Schmidt number
\( Rd = \frac{4\sigma^2 T_f^2}{k_2} \) radiation parameter
\( Nt = \frac{T_f - T_i}{T_i} \) temperature difference parameter
\( G'(0) \) torque
\( \delta \eta \) spatial and time level
\( \sigma \) step size
\( G(A) \) spectral radius of Jacobi method
\( \theta(A) \) tangential velocity
\( \Omega(t) \) temperature field
\( \beta_2 \) cylindrical co-ordinate system
\( r, \theta, z \) angular speed \([m/s]\)
\( \beta_0 \) magnetic field strength
\( T \) free stream temperature
\( C_w \) ambient fluid concentration
\( \rho \) pressure
\( \mu \) fluid viscosity \([kg/ms]\)
\( \nu \) kinematic viscosity \([m^2/s]\)
\( \gamma \) angle
\( \phi_r \) radiative heat flux
\( C \) concentration
\( D \) mass diffusivity \([m^2/s]\)
\( W_T \) thermophoretic velocity \([m/s]\)
\( k \) thermpophoretic diffusivity
\( \beta_2 \) wall permeability strength
\( M = \sqrt{\frac{Rd}{\rho_0 a^2(t)}} \) magnetic parameter
\( Pr = \frac{\mu c_p}{k} \) Prandtl number
\( \beta_1 = \frac{\eta u}{\nu} \) heat transfer Biot number
\( \Gamma_0 = \frac{k_1}{\nu} \) rotation parameter
\( P_o \) constant pressure
\( F(0) \) local skin-friction
\( \phi(0) \) Sherwood number
\( \eta \) spatial variable
\( \phi(\eta) \) relaxation parameter
\( \omega \) axial velocity

Prandtl number
heat transfer Biot number
rotation parameter
constant pressure
local skin-friction
Sherwood number
spatial variable
relaxation parameter
axial velocity
concentration field