Theoretical Overview of CP Violation in \( B \)-Meson Decay

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**Abstract**

After a brief look at CP violation in kaon decays, a short overview of CP violation in the \( B \)-meson system and of strategies to determine the angles of the unitarity triangles of the CKM matrix is given. Both general aspects and some recent developments are discussed, including \( B_\pm \rightarrow D_\pm D \) and \( B \rightarrow \pi K \) decays, as well as the \( B_d \rightarrow \pi^+\pi^- \), \( B_s \rightarrow K^+K^- \) system.

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Theoretical Overview of CP Violation in $B$-Meson Decay

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After a brief look at CP violation in kaon decays, a short overview of CP violation in the $B$-meson system and of strategies to determine the angles of the unitarity triangles of the CKM matrix is given. Both general aspects and some recent developments are discussed, including $B^\pm \to D^\pm D$ and $B \to \pi K$ decays, as well as the $B_d \to \pi^+\pi^-$, $B_s \to K^+K^-$ system.

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1. Introduction

The non-conservation of the CP symmetry, which was discovered in 1964 in neutral kaon decays [1], is one of the central aspects of modern particle physics, and is still one of the least well experimentally constrained phenomena. In particular the $B$-meson system provides a very fertile testing ground for the Standard-Model (SM) description of CP violation. This feature is also reflected in the tremendous effort put in the experimental programmes to explore $B$ physics. The BaBar and BELLE detectors are already taking data, HERA-B has seen its first events, and CLEO-III, CDF-II and D0-II will follow in the near future. Although the physics potential of these experiments is very exciting, it may well be that the “definite” answer in the search for new physics will be left for second-generation $B$-physics experiments at hadron machines, such as LHCb or BTeV [2].

Within the framework of the SM, CP violation is closely related to the Cabibbo–Kobayashi–Maskawa (CKM) matrix, connecting the electroweak eigenstates of the down, strange and bottom quarks with their mass eigenstates. As far as CP violation is concerned, the central feature is that – in addition to three generalized Cabibbo-type angles – also a complex phase is needed in the three-generation case to parametrize the CKM matrix. This complex phase is the origin of CP violation within the SM. Concerning tests

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of the CKM picture of CP violation, the central targets are the unitarity triangles of the CKM matrix. The unitarity of the CKM matrix leads to a set of 12 equations, consisting of 6 normalization and 6 orthogonality relations. The latter can be represented as 6 triangles in the complex plane, all having the same area. However, in only two of them, all three sides are of comparable magnitude $O(\lambda^3)$, while in the remaining ones, one side is suppressed relative to the others by $O(\lambda^2)$ or $O(\lambda^4)$, where $\lambda \equiv |V_{us}| = 0.22$ denotes the Wolfenstein parameter [3]. The two non-squashed triangles agree at leading order in the Wolfenstein expansion ($O(\lambda^3)$), so that we actually have to deal with a single triangle at this order, which is usually referred to as “the” unitarity triangle of the CKM matrix. However, in the era of second-generation experiments, starting around 2005, we will have to take into account the next-to-leading order terms of the Wolfenstein expansion, and will have to distinguish between the unitarity triangles shown in Fig. 1.

2. A brief look at the $K$-meson system

Although the discovery of CP violation goes back to 1964 [1], so far this phenomenon has been observed only within the neutral $K$-meson system, where it is described by two complex quantities, called $\varepsilon$ and $\varepsilon'$, which are defined by the following ratios of decay amplitudes:

$$
\frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = \varepsilon + \varepsilon', \quad \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} = \varepsilon - 2\varepsilon'.
$$

While $\varepsilon = (2.280 \pm 0.013) \times 10^{-3}$ parametrizes “indirect” CP violation, originating from the fact that the mass eigenstates of the neutral kaon system are not CP eigenstates, the quantity $\text{Re}(\varepsilon'/\varepsilon)$ measures “direct” CP violation in $K \to \pi\pi$ transitions. The CP-violating observable $\varepsilon$ plays an important role to constrain the unitarity triangle [5, 6] and implies
in particular a positive value of the Wolfenstein parameter $\eta$. In 1999, new measurements of $\text{Re}(\varepsilon'/\varepsilon)$ have demonstrated that this observable is non-zero, thereby excluding “superweak” models of CP violation [7]:

$$\text{Re}(\varepsilon'/\varepsilon) = \begin{cases} (28 \pm 4.1) \times 10^{-4} & \text{(KTeV Collaboration [8])}, \\ (14 \pm 4.3) \times 10^{-4} & \text{(NA48 Collaboration [9])}. \end{cases}$$

Unfortunately, the calculations of $\text{Re}(\varepsilon'/\varepsilon)$ are very involved and suffer at present from large hadronic uncertainties [10]. Consequently, this observable does not allow a powerful test of the CP-violating sector of the SM, unless the hadronic matrix elements of the relevant operators can be brought under better control.

In order to test the SM description of CP violation, the rare decays $K_L \rightarrow \pi^0 \nu\bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu\bar{\nu}$ are more promising and may allow a determination of $\sin(2\beta)$ with respectable accuracy [11]. Yet it is clear that the kaon system by itself cannot provide the whole picture of CP violation, and therefore it is essential to study CP violation outside this system. In this respect, $B$-meson decays appear to be most promising.

3. The central target: the $B$-meson system

In order to determine the angles of the unitarity triangles shown in Fig. 1, and to test the SM description of CP violation, the major role is played by non-leptonic $B$ decays, which can be divided into three decay classes: decays receiving both “tree” and “penguin” contributions, pure “tree” decays, and pure “penguin” decays. There are two types of penguin topologies: gluonic (QCD) and electroweak (EW) penguins. Because of the large top-quark mass, also EW penguins play an important role in several non-leptonic $B$-decay processes [12].

3.1. CP violation in neutral $B$-meson decays

A particularly simple and interesting situation arises if we restrict ourselves to decays of neutral $B_q$-mesons ($q \in \{d,s\}$) into CP self-conjugate final states $|f\rangle$, satisfying the relation $(\mathcal{CP})|f\rangle = \pm |f\rangle$. In this case, the corresponding time-dependent CP asymmetry can be expressed as

$$a_{\text{CP}}(t) \equiv \frac{\Gamma(B_q^0(t) \rightarrow f) - \Gamma(B_q^0(t) \rightarrow f)}{\Gamma(B_q^0(t) \rightarrow f) + \Gamma(B_q^0(t) \rightarrow f)} = 2 e^{-\Gamma_q t} \left[ A_{\text{CP}}^{\text{dir}}(B_q \rightarrow f) \cos(\Delta M_q t) + A_{\text{CP}}^{\text{mix}}(B_q \rightarrow f) \sin(\Delta M_q t) \right] \left( e^{-t \Gamma_0^{(q)} t} + e^{-t \Gamma_0^{(\bar{q})} t} + A\Delta\Gamma(B_q \rightarrow f) \left( e^{-t \Gamma_0^{(q)} t} - e^{-t \Gamma_0^{(\bar{q})} t} \right) \right),$$

(3)
where $\Delta M_q \equiv M_{H}^{(q)} - M_{L}^{(q)}$ is the mass difference between the $B_q$ mass eigenstates, and the $\Gamma_{H,L}^{(q)}$ denote their decay widths, with $\Gamma_q \equiv (\Gamma_{H}^{(q)} + \Gamma_{L}^{(q)})/2$. In Eq. (3), we have separated the “direct” from the “mixing-induced” CP-violating contributions, which are described by

$$A_{\text{dir}}^{\text{CP}}(B_q \to f) \equiv \frac{1 - |\xi_f^{(q)}|^2}{1 + |\xi_f^{(q)}|^2} \quad \text{and} \quad A_{\text{mis}}^{\text{CP}}(B_q \to f) \equiv \frac{2 \text{Im} \xi_f^{(q)}}{1 + |\xi_f^{(q)}|^2},$$

respectively. Whereas the width difference $\Delta \Gamma_q \equiv \Gamma_{H}^{(q)} - \Gamma_{L}^{(q)}$ is negligibly small in the $B_d$ system, it may be sizeable in the $B_s$ system (for a recent calculation, see [13]), thereby providing the observable

$$A_{\Delta \Gamma}(B_q \to f) \equiv \frac{2 \text{Re} \xi_f^{(q)}}{1 + |\xi_f^{(q)}|^2}.$$

Essentially all the information needed to evaluate the CP asymmetry (3) is included in the following quantity [12]:

$$\xi_f^{(q)} = \mp e^{-i\phi_q} \frac{A(B_q^0 \to f)}{A(B_q^0 \to f)} = \mp e^{-i\phi_q} \frac{\sum_{j=u,c} V_{jr}^* V_{jb} \mathcal{M}_{jr}^r}{\sum_{j=u,c} V_{jr}^* V_{jb} \mathcal{M}_{jr}^r},$$

where the $\mathcal{M}_{jr}^r$ denote hadronic matrix elements of certain four-quark operators, the label $r \in \{d, s\}$ distinguishes between $b \to d$ and $b \to s$ transitions, and

$$\phi_q = \begin{cases} +2\beta & (q = d) \\ -2\delta \gamma & (q = s) \end{cases}$$

is related to the weak $B_q^0 \to \bar{B}_q^0$ mixing phase. In general, the quantity $\xi_f^{(q)}$ suffers from hadronic uncertainties, which are due to the hadronic matrix elements $\mathcal{M}_{jr}^r$. However, if the decay $B_q \to f$ is dominated by a single CKM amplitude, the corresponding matrix elements cancel, and the convention-independent observable $\xi_f^{(q)}$ takes the simple form

$$\xi_f^{(q)} = \mp \exp \left[ -i \left( \phi_q - \phi^{(f)}_D \right) \right],$$

where $\phi_D^{(f)}$ is a weak decay phase, which is given by

$$\phi_D^{(f)} = \begin{cases} -2\gamma & \text{for dominant } b \to \pi d \pi \text{ CKM amplitudes,} \\ 0 & \text{for dominant } \bar{b} \to \bar{c} \pi \tau \text{ CKM amplitudes.} \end{cases}$$
3.1.1. The “gold-plated” mode $B_d \to J/\psi K_S$

The most important application of the simple formalism discussed above is the decay $B_d \to J/\psi K_S$, which is dominated by the $\bar{b} \to \bar{c}c\bar{s}$ CKM amplitude (for a detailed discussion, see [12]), implying

$$A_{\text{CP}}^{\text{mix}}(B_d \to J/\psi K_S) = + \sin[-(2\beta - 0)].$$

(10)

Another non-trivial prediction of the SM is vanishingly small direct CP violation. Since (8) applies with excellent accuracy to $B_d \to J/\psi K_S$ – the point is that penguins enter essentially with the same weak phase as the leading tree contribution – it is referred to as the “gold-plated” mode to determine the CKM angle $\beta$ [14]. First attempts to measure $\sin(2\beta)$ through the CP asymmetry (10) were already performed [15]:

$$\sin(2\beta) = \begin{cases} 
3.2^{+1.8}_{-2.0} \pm 0.5 & \text{(OPAL Collaboration)} \\
0.79^{+0.41}_{-0.44} & \text{(CDF Collaboration)} \\
0.93^{+0.36}_{-0.88-0.24} & \text{(ALEPH Collaboration)}
\end{cases}$$

(11)

Although the experimental uncertainties are still very large, it is interesting to note that these results favour the SM expectation of a positive value of $\sin(2\beta)$ [6]. In the $B$-factory era, an experimental uncertainty of $\Delta \sin(2\beta)|_{\text{exp}} = 0.05$ seems to be achievable, whereas second-generation experiments of the LHC era aim at $\Delta \sin(2\beta)|_{\text{exp}} = \mathcal{O}(0.005)$ [2]. This tremendous experimental accuracy raises the question of hadronic uncertainties due to penguin contributions. An interesting channel in this context is $B_s \to J/\psi K_S$, allowing us to control the – presumably very small – penguin effects in the determination of $\phi_d = 2\beta$ from $B_d \to J/\psi K_S$, and to extract the CKM angle $\gamma$ [16].

3.1.2. The decay $B_d \to \pi^+\pi^-$

If this mode would not receive penguin contributions, its mixing-induced CP asymmetry would allow a measurement of $\sin(2\alpha)$:

$$A_{\text{CP}}^{\text{mix}}(B_d \to \pi^+\pi^-) = - \sin[-(2\beta + 2\gamma)] = - \sin(2\alpha).$$

(12)

However, this relation is strongly affected by penguin effects, which were analysed by many authors [17]. There are various methods to control the corresponding hadronic uncertainties. Unfortunately, these strategies are usually rather challenging from an experimental point of view.

The best known approach was proposed by Gronau and London [18]. It makes use of an SU(2) isospin relation between the $B^+ \to \pi^+\pi^0$, $B^0_d \to \pi^+\pi^-$ and $B^0_d \to \pi^+\pi^-$ decay amplitudes, as well as their CP conjugates,
which can be represented as two triangles in the complex plane. Unfortunately, the Gronau–London approach suffers from a serious experimental problem, since the measurement of $\text{BR}(B_d \to \pi^0\pi^0)$ is very difficult.

Alternative methods to control the penguin uncertainties in the extraction of $\alpha$ from $B_d \to \pi^+\pi^-$ are very desirable. An important one for the $e^+e^-$ $B$-factories is provided by $B \to \rho \pi$ modes [19]. Here the isospin triangle relations are replaced by pentagonal relations, and the corresponding approach is rather complicated. As we will see in more detail below, an interesting strategy for hadron machines to employ the CP-violating observables of $B_d \to \pi^+\pi^-$ is offered by $B_s \to K^+K^-$, allowing a simultaneous determination of $\beta$ and $\gamma$ without any penguin uncertainties [20].

### 3.1.3. The $B_s$-meson system

Since the $e^+e^-$ $B$-factories operating at $\Upsilon(4S)$ are not in a position to explore the $B_s$ system, it is of particular interest for hadron machines. There are important differences to the $B_d$ system: the $B_s^0$–$\bar{B}_s^0$ mixing phase $\phi_s = -2\lambda^2\eta = \mathcal{O}(0.03)$ is negligibly small in the SM, and a large mixing parameter $x_s \equiv \Delta M_s/\Gamma_s = \mathcal{O}(20)$ is expected. Moreover, the expected sizeable width difference $\Delta \Gamma_s$ provides interesting strategies to extract CKM phases from “untagged” $B_s$ data samples, where the rapid oscillating $\Delta M_s t$ terms cancel [21]. Among the $B_s$ benchmark modes are $B_s \to D^{\pm}K^\mp$, allowing a theoretically clean determination of the CKM phase $\gamma - 2\delta\gamma$ [22], and $B_s \to J/\psi \phi$. This decay offers interesting strategies to extract $\Delta M_s$, $\Delta \Gamma_s$ and $\phi_s$ from the angular distribution of the $J/\psi[\to l^+l^-] \phi[\to K^+K^-]$ decay products [23]. Since $B_s \to J/\psi \phi$ modes exhibit, moreover, very small CP-violating effects in the SM, they represent an interesting probe for new-physics contributions to $B_s^0$–$\bar{B}_s^0$ mixing [24, 25].

### 3.2. CP violation in charged $B$-meson decays

Since there are no mixing effects present in charged $B$-meson decays, non-vanishing CP asymmetries $A_{CP}$ would give us unambiguous evidence for “direct” CP violation in the $B$ system. Such CP asymmetries arise from the interference between decay amplitudes with both different CP-violating weak and different CP-conserving strong phases. In the SM, the weak phases are related to the phases of the CKM matrix, whereas the strong phases are induced by FSI processes. In general, the strong phases introduce severe theoretical uncertainties into the calculation of $A_{CP}$, thereby destroying the clean relation to the CP-violating weak phases.

An important tool to overcome these problems is provided by amplitude relations between certain non-leptonic $B$ decays. The prototype of this approach, which is due to Gronau and Wyler [26], uses $B^{\pm} \to K^{\pm}D$ decays.
If the $D$-meson is observed as a CP eigenstate, amplitude triangles can be constructed, allowing a theoretically clean determination of $\gamma$. Unfortunately, these triangles turned out to be highly stretched, and are – from an experimental point of view – not very useful to determine $\gamma$. Further difficulties were pointed out in [27]. As an alternative, the decays $B_d \to K^{*0}D$ were proposed [28] because the triangles are more equilateral. But all sides are small because of various suppression mechanisms. In another paper, the triangle approach to extract $\gamma$ [26] was also extended to the $B_c$ system [29]. At first sight, here everything is completely analogous to $B_u \to K^{\pm}D$. However, there is an important difference [30]: in the $B_c \to D_s^{\pm}D$ system, the amplitude with the rather small CKM matrix element $V_{ub}$ is not colour suppressed, while the larger element $V_{cb}$ comes with a colour-suppression factor. Therefore, the two amplitudes are similar in size! In contrast to this favourable situation, in the $B_u \to K^{\pm}D$ system, the matrix element $V_{ub}$ comes with the colour suppression factor, resulting in a very stretched triangle, while in the decays $B_d \to K^{*0}D$, all amplitudes are colour suppressed. Decays of the type $B_c^{\pm} \to D_s^{\pm}D$ – the $U$-spin counterparts of $B_c^{\pm} \to D_s^{\pm}D$ – can be added to the analysis, as well as channels, where the $D_s^{\pm}$- and $D^{\pm}$-mesons are replaced by higher resonances. At the LHC, one expects about $10^{10}$ untriggered $B_c$s per year of running. Provided there are no serious experimental problems, the $B_c^{\pm} \to D^{(s)}_sD$ approach should be very interesting for the corresponding $B$-physics programme.

3.3. Probing $\gamma$ with $B \to \pi K$ decays

In order to obtain direct information on $\gamma$, $B \to \pi K$ decays are very promising [31], and have received a lot of attention during the recent years [32]. Because of the small ratio $|V_{us}V_{ub}^*/(V_{ts}V_{tb}^*)| \approx 0.02$, these modes are dominated by penguin topologies and are hence very sensitive to new-physics contributions [33]. Interestingly, already CP-averaged $B \to \pi K$ branching ratios may imply highly non-trivial constraints on $\gamma$ [34]. So far, the studies of these bounds have focussed on the following two systems: $B_d \to \pi^\pm K^\pm$, $B^\pm \to \pi^\pm K$ [34], and $B^\pm \to \pi^0 K^\pm$, $B^\pm \to \pi^\pm K$ [35]. Recently, it was pointed out that also the neutral decays $B_d \to \pi^\mp K^\pm$ and $B_d \to \pi^0 K$ may be very interesting in this respect [36, 37].

The $B \to \pi K$ strategies to probe $\gamma$ make use of flavour-symmetry arguments ($SU(2)$ or $SU(3)$), and rely, in addition, on dynamical assumptions, concerning mainly the smallness of certain rescattering processes, such as $B^+ \to \{\pi^0 K^+\} \to \pi^+ K^0$. The theoretical understanding of such FSI processes is poor at present [38]. However, there are important experimental indicators for possible large rescattering effects, e.g. $B^+ \to K^+K^0\pi^0$ or $B_d \to K^+K^-\pi^0$, and methods to include them in the strategies to probe $\gamma$. 
In order to constrain \( \gamma \) through \( B \to \pi K \) decays, the key quantities are ratios \( R_{(c,n)} \) of CP-averaged branching ratios, which can be constructed for the “mixed”, charged and neutral \( B \to \pi K \) systems listed above. Employing the theoretical ingredients sketched in the previous paragraph, we obtain

\[
R_{(c,n)} = R_{(c,n)}(\gamma, q_{(c,n)}, r_{(c,n)}, \delta_{(c,n)}),
\]

where \( q_{(c,n)} \) denotes the ratio of EW penguins to trees, \( r_{(c,n)} \) is the ratio of trees to QCD penguins, and \( \delta_{(c,n)} \) is the CP-conserving strong phase between tree and QCD penguin amplitudes. Whereas \( q_{(c,n)} \) can be fixed through theory, and \( r_{(c,n)} \) with the help of additional experimental information, e.g. on \( \text{BR}(B^{\pm} \to \pi^{\mp}\pi^{0}) \), \( \delta_{(c,n)} \) suffers from large hadronic uncertainties and is essentially unknown. However, we can get rid of \( \delta_{(c,n)} \) by keeping it as a “free” variable, yielding minimal and maximal values for \( R_{(c,n)} \):

\[
R_{(c,n)}^{\text{ext}} \bigg|_{\delta_{(c,n)}} = \text{function}(\gamma, q_{(c,n)}, r_{(c,n)}).
\]

Keeping in addition \( r_{(c,n)} \) as a free variable, we obtain another – less restrictive – minimal value for \( R_{(c,n)} \):

\[
R_{(c,n)}^{\text{min}} \bigg|_{r_{(c,n)}, \delta_{(c,n)}} = \kappa(\gamma, q_{(c,n)}) \sin^{2} \gamma.
\]

Since values of \( \gamma \) corresponding to \( R_{(c,n)}^{\text{exp}} < R_{(c,n)}^{\text{min}} \) or \( R_{(c,n)}^{\text{exp}} > R_{(c,n)}^{\text{max}} \) imply an allowed range for \( \gamma \). Although it is too early to draw definite conclusions, it is interesting to note that the most recent CLEO results on \( R_{(c,n)} \) are in favour of strong constraints on \( \gamma \), where the second quadrant, i.e. \( \gamma \geq 90^\circ \), is preferred. Such a situation would be in conflict with the standard analysis of the unitarity triangle, yielding \( 38^\circ \leq \gamma \leq 81^\circ \) [6].

The observables \( R_{(c,n)} \) imply also constraints on \( \delta_{(c,n)} \), where the present CLEO data are in favour of \( \cos \delta_{c} > 0 \) and \( \cos \delta_{n} < 0 \), which would be in conflict with the theoretical expectation of equal signs for \( \cos \delta_{c} \) and \( \cos \delta_{n} \) [37]. If future data should confirm this “puzzle”, it may be an indication for new-physics contributions to the EW penguin sector, or a manifestation of large non-factorizable \( SU(3) \)-breaking effects. In order to distinguish between these possibilities, detailed studies of the various patterns of new-physics effects in all \( B \to \pi K \) decays are essential, as well as critical analyses of possible sources for \( SU(3) \) breaking. As soon as CP asymmetries \( A_{CP}^{(c,n)} \) in \( B_{d} \to \pi^{+}K^{\mp} \) or \( B^{\pm} \to \pi^{0}K^{\pm} \) are observed, we may go beyond the bounds and may determine \( \gamma \) and \( \delta_{(c,n)} \). The physics potential of \( B \to \pi K \) decays is very interesting and plays a central role for the \( B \)-factories.
3.4. Extracting $\beta$ and $\gamma$ from $B_d \rightarrow \pi^+\pi^-$ and $B_s \rightarrow K^+K^-$

There are interesting strategies to extract CKM phases with the help of $U$-spin-related $B$ decays, where all down and strange quarks are interchanged with each other [39]. A particularly interesting one is provided by the decays $B_d \rightarrow \pi^+\pi^-$ and $B_s \rightarrow K^+K^-$, allowing a simultaneous determination of $\beta$ and $\gamma$ [20]. This new strategy is not affected by any penguin topologies – it rather makes use of them – and does not rely on certain “plausible” dynamical or model-dependent assumptions. Moreover, FSI effects, which led to considerable attention in the context of the determination of $\gamma$ from $B \rightarrow \pi K$ decays, as we have noted in Subsection 3.3, do not lead to any problems. The theoretical accuracy is only limited by $U$-spin-breaking effects, which vanish in the factorization approximation in the present case. This strategy is ideally suited for LHCb ($\Delta\gamma = \mathcal{O}(1^\circ)$) [2], and is also very promising for CDF-II [40]. Conceptually similar approaches are provided by $B_{s(d)} \rightarrow J/\psi K_S$ or $B_{d(s)} \rightarrow D_{s(d)}^+ D_{s(d)}^-$ decays [16].

4. Conclusions and outlook

The phenomenology of non-leptonic $B$ decays is very rich and provides a fertile testing ground for the SM description of CP violation. As a by-product, interesting insights into hadronic physics can be obtained. There is no doubt that an exciting future – the $B$-physics era of particle physics – is ahead of us. Hopefully, it will shed light on the physics beyond the SM.

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