Cosmology in a locally scale invariant gravity

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ABSTRACT
A ‘bouncing’ cosmological model is proposed in the context of a Weyl-invariant scalar-tensor (WIST) theory of gravity. In addition to being Weyl-invariant the theory is U(1)-symmetric and has a conserved global charge. The entire cosmic background evolution is accounted for by a complex scalar field that evolves in the static ‘comoving’ frame. Its (dimensional) modulus $\chi$ regulates the dynamics of masses and the apparent space expansion. Cosmological redshift is essentially due to the cosmic evolution of the Rydberg constant in the comoving frame. The temporal evolution of $\chi$ is analogous to that of a point particle in the presence of a central potential $V(\chi)$. The scalar field sources the spacetime curvature; as such it can account for the (cosmological) Dark Sector. An interplay between the energy density of radiation and that of the kinetic energy associated with the phase $\alpha$ of the scalar field (which are of opposite signs) results in a classical non-singular stable and nearly-symmetric bouncing dynamics deep in the radiation-dominated era. This encompasses the observed redshifting era which preceded by a ‘bounce’ that follows a blushing era. The model is essentially free of the horizon or flatness problems. Big Bang nucleosynthesis sets a lower 1-10 MeV bound on the typical energy scale at the ‘bounce’.

Key words: cosmology: theory, dark energy

1 INTRODUCTION

General relativity (GR), the backbone of the standard cosmological model, has successfully passed numerous tests within our solar system. However, it is not comparably successful on larger, galactic and supergalactic scales, unless cold dark matter (CDM) and dark energy (DE) are introduced into the cosmic energy budget. The latter are foreign to the standard model (SM) of particle physics and only appear in our cosmological model as nearly perfect fluids with characteristic equations of state (EOS).

In addition, this classical field theory of gravitation is genuinely plagued by singularities. A few singularity theorems imply that curvature is inevitably singular unless certain plausible ‘energy conditions’ are violated. For the latter to take place some form of exotic matter is needed. Curvature or energy density singularities are encountered either at the centers of black holes or at the Big Bang, even in the presence of a very early inflationary phase in the latter case Borde et al. (2003). It is widely hoped that a would-be quantum theory of gravity will ameliorate these unwelcome singularities, thereby constituting a major thrust behind the quest for a quantum theory of gravity.

The largest physical scales, over which the short-range nuclear interactions and the highly screened electromagnetic interaction are irrelevant, are an ideal testbed for alternative theories of gravitation, and indeed a few major persistent anomalies of $\Lambda$CDM may possibly indicate that GR requires modifications on cosmological scales.

The GR-based $\Lambda$CDM, with an early inflationary phase and a dominant dark sector [the latter contains CDM and DE components that determine the background evolution, large scale structure (LSS) formation history, and gravitational potential wells on galactic and supergalactic scales], has clearly proved to be a very successful paradigm that provides a compelling interpretation of essentially all currently available cosmic microwave background (CMB) and LSS observational data, as well as of light element abundances based on Big Bang nucleosynthesis (BBN). It is remarkable that $\Lambda$CDM provides a very good fit to this extensive observational data, that sample a wide range of phenomena over a vast dynamical range, using less than a dozen free parameters.

However, $\Lambda$CDM also lacks in a few ways. A major drawback is that the essence of DE and CDM remains elusive. Another problem is that, the currently leading inflationary scenario, ‘eternal inflation’, seems to lack predictive power as it is most naturally realized in the multiverse. In addition, conceptually, the existence of the Big Bang, which essentially signals the breakdown of the underlying theory of gravitation, is also a major problem.

Moreover, a few mild to strong inconsistencies between various datasets comparison to $\Lambda$CDM have been found, e.g. Addison et al. (2016). These include the existence of a persistent relative deficit in power of density perturbations on superhorizon scales, e.g. Hinshaw et al. (1996), Copi et al.
(2009), Schwarz et al. (2016), an anomalously large weak lensing of the CMB anisotropy by the intervening large scale structure between the present and the last scattering surface [Planck Collaboration et al. (2020)], a statistically significant ‘Hubble tension’ between local and high-z inferences of the local expansion rate, e.g. Freedman (2017), Riess et al. (2018), Riess et al. (2019), Wong et al. (2020), and others.

The main objective of the present work is to demonstrate the viability of an alternative, classical, non-singular ‘bouncing’ cosmological model within a physical framework based on a globally U(1)-symmetric and locally Weyl-invariant scalar-tensor (WIST) theory of gravity, while the SM of particle physics is left unchanged. Unlike in quantum gravity-inspired bouncing models classical bounces may take place at typical energies that are well within the range of well-established physics.

The terminology that will be used in this work is somewhat different from the one used in the standard cosmological model. In the latter, the observed redshift on cosmological scales is synonymous to space expansion, and in bouncing cosmological models it is actually meant that space contraction itself momentarily halts at the ‘bounce’ followed by space expansion. Since in the proposed model space is static and cosmic evolution is regulated by the temporal evolution of mass we will employ the more appropriate notions of ‘turning point’, ‘blueshifting’- and ‘redshifting-phase’ instead of the commonly used parlance of ‘bounce’, ‘contraction’ and ‘expansion’, respectively.

Throughout, we adopt a mostly-positive signature for the spacetime metric (−1, 1, 1, 1). Our units convention is ℏ = c = 1. We outline the theoretical approach adopted in the present work in section 2, and the cosmological model is presented in section 3. The main results are summarized in section 4. Stability analysis at and around the ‘bounce’ is carried out in Appendix A.

2 THEORETICAL FRAMEWORK

The gravitational sector of the fundamental interactions is described by the following WIST action which is in addition to the Poisson equation in the weak field limit within, e.g. the GR-based ΛCDM model where gravitation is not exclusively determined by $L_{_{SM}}$, the matter lagrangian density of the SM of particle physics; in order for ΛCDM to provide a reasonably good fit to observational data $L_{_{SM}}$ is amended by CDM and DE which together comprise up to $\sim 95\%$ of the cosmic energy budget at present. However, a notable conceptual difference is that in the framework described by Eqs. (1)-(3) a single scalar field, $\chi$, is responsible to the evolution of the Planck mass, active gravitational masses, and potentially also to the existence of what is normally interpreted as CDM and DE on cosmological scales. In contrast, in ΛCDM the Planck mass is fixed and the evolution is determined by space expansion, which in turn is driven by the energy budget of which CDM and DE are key building blocks. The former is believed to be in the form of some exotic beyond-the-SM particles, and the latter is believed to be the manifestation of some slow-rolling quintessence field.

The field $\chi$ is the analog of $a(t)$, in ΛCDM, but in the latter case the dynamics represents space expansion and energy conservation of the individual species. The latter are described by effective perfect fluids. In other words, the total energy density, $\rho$, in the standard ΛCDM model is $\rho = \sum_i \rho_i$ with $\rho_i \propto a^{-3(1+w_i)}$ being the energy density of the $i$'th species which is characterized by an EOS, $w_i$. It is assumed that they do not interact, i.e. energy-momentum is separately conserved for the individual species, and consequently each species evolves independently of the others. In contrast, in the WIST-based model, the energy-momentum is not the source of curvature, yet we obtain analogous evolution to that of ΛCDM simply because it is assumed that the potential, $V$, is an analytic polynomial in $\chi$, which is arguably one of the simplest possible shapes that a potential could possibly have.

Since all quantities that would be normally referred to as ‘active gravitational masses’, as well as the ‘Planck mass’, are now governed by a single scalar field $\chi$, the dimensionless ratios of active gravitational masses of any two bodies, as well as ratios of active masses to the Planck mass, are fixed to their standard values.

If the scalar field is fixed $\chi = \sqrt{\frac{4}{3\pi G}}$ and $V(\chi)$ is renamed $L_{_{M}}$ then Eq. (1) reduces to the Einstein-Hilbert (EH) action. The specific constant $G$ that appears in the EH action guarantees that the resulting gravitational field equations reduce to the Poisson equation in the weak field limit within, e.g. the solar system, where the ‘universality’ of $G$ has been reasonably established. In addition, even if we favor the idea that DM exists in the form of some exotic, beyond-the-SM particles, we still lack Cavendish-like experimental evidence that...
they ‘couple’ gravitationally via the same ‘Universal’ strength $G$ either to each other or to baryons.

General properties of the theory described by Eqs. (1)-(3) are discussed in Shimon (2021b). For example, the tensor

$$S_{\mu\nu} \equiv -\frac{2}{3} \frac{\delta(V)}{\delta g_{\mu\nu}},$$

satisfies

$$S_{\mu\nu} = \phi_{\mu} \frac{\partial V}{\partial \phi} + \phi_{\nu} \frac{\partial V}{\partial \phi^*},$$

(4)

i.e. in general it is not conserved. This is expected since the Planck mass varies in space and time. In addition, the requirement that $I_S$ described by Eq. (3) is WI implies that

$$\phi \frac{\partial V}{\partial \phi} + \phi^* \frac{\partial V}{\partial \phi^*} = S,$$

(5)

where $S \equiv S_{\mu\nu}^\mu$, and assuming that only variations of $g_{\mu\nu}$ and the scalar field are considered. In analogy with the energy-momentum tensor $T_{\mu\nu} = -\rho \cdot \text{diag}(1,-w,-w,-w)$ of a perfect fluid characterized by an EOS $w$, that is employed in the standard cosmological model, we have $S_{\mu\nu}^\mu = -V \cdot \text{diag}(1,-w,-w,-w)$. The perfect fluid assumption in cosmology is the embodiment of the idea that there is no energy and momentum flow in an isotropic and homogeneous Universe. The microphysical meaning of $w \equiv P/\rho$, where $P$ is the pressure, is irrelevant in $\Lambda$CDM. In practice, the EOS $w$ is simply a parameter that determines the evolution rate of the energy density, and so is the case in the model that is proposed here.

Since $S_{00}^0 = -V$ it immediately follows from Eq. (5) that

$$V \propto \chi^{1-3w},$$

(6)

in the case of an effective single fluid with an EOS $w$, Shimon (2021b). Consequently, the potential is quartic in the case $w = -1$, is independent of $\chi$, i.e. of active masses, in the case $w = 1/3$, and is linear in masses in case of non-relativistic (NR) matter, i.e. the case $w = 0$. Although there is no fundamental principle that requires $V$ to be an analytic function of the field $\chi$ we do impose this restriction. In particular, we assume that $V$ is an analytic polynomial. This by itself corresponds to a constraint on the EOS, $w \leq 1/3$, in case that $V$ was a monomial. As is generically the case, there is no prescription for choosing the form of the potential. The latter is designed, subject to certain symmetry requirements, to recover (along with the kinetic terms) the required dynamics of the fields, as determined by experiments/observations. Therefore, and following the foregoing discussion, the ‘coefficients’ $V_i(\psi)$ in the potential

$$V(\phi; \{\psi\}) = \sum_{i=0}^{m_{\alpha,\lambda}} V_i(\{\psi\}) \chi^i,$$

(7)

are determined on cosmological scales by the observed cosmic evolution. The effective EOS associated with the $i$th contribution is

$$w_i \equiv \frac{1 - i}{3}.$$

(8)

Consequently, the only allowed terms are characterized by parameters $w_i$ which are integer multiples of $1/3$ subject to the constraint $w_i \leq 1/3$ for all $i$. $V$ is a polynomial in $\chi$ that contains only non-negative powers of $\chi$ and there is no ‘anisotropy problem’. The latter is a well-known problem Misner (1969) that is generic to bouncing models Levy (2017). In practice, the constraint the $w_i \leq 1/3$ is implicitly assumed to hold in the standard $\Lambda$CDM model as well. The ‘mixmaster’ model of Misner (1969) typically arises from allowing for different evolutions along three principal axes. This would require, in particular, that these three functional degrees of freedom are manifested in the matter lagrangian density. In the limit of small anisotropic evolution, this is represented by an effective stiff matter contribution to the energy budget that dominates the cosmic energy budget prior to radiation. This dynamics does not have an analog in the present model because the potential $V$ is ‘protected’ against this ‘shearing’ dynamics by the assumed $U(1)$ symmetry.

In the same fashion that effective energy density and pressure of the various species in $\Lambda$CDM depend on a single $a(t)$ function, commensurate with isotropic expansion, so does $V$ in Eq. (7) depends on $\chi$, and not on $\chi \cos \alpha$ and $\chi \sin \alpha$. In other words, the underlying $U(1)$ symmetry of Eq. (7) reflects isotropic expansion in the $\Lambda$CDM model.

3 COSMOLOGICAL MODEL

The modulus of the scalar field, $\chi$, delineates essentially the same dynamics that the scale factor $a(\eta)$ does in $\Lambda$CDM (insofar $\alpha$ is dynamically insignificant). However, unlike $a(\eta)$ which is part of the Friedmann-Robertson-Walker (FRW) metric, $\chi(\eta)$ is a scalar field living on a static spacetime. If the time variation of $\alpha$ is sufficiently slow, the entire observable cosmic evolution, from BBN (taking place at typical $\sim 1$ MeV energies) onward, is essentially indistinguishable from that of $\Lambda$CDM, thereby retaining its merits. However, the very early Universe can be much different, e.g. there is no initial singularity in the proposed model and possibly also no primordial phase transitions that in $\Lambda$CDM are expected to have taken place at energy scales of $O(100)$ MeV and $O(100)$ GeV, and possibly also at the GUT scale.

3.1 Evolution of the cosmological background

The equivalence between the FRW action and Eq. (1) in the non-vacuum homogeneous and isotropic case, in the case of a real $\phi$ field, has been discussed in, e.g. Shimon (2021b). As mentioned above, according to the alternative cosmological model proposed here, invoking a complex scalar field evades the initial singularity problem thereby resolving in effect the cosmological horizon Rindler (1956), Misner (1969), and flatness Dicke (1969) problems. In the following we derive the background evolution equations.

Before focusing on the case of Minkowski spacetime background we consider a more general spacetime which is described by the static metric $g_{\mu\nu} = \text{diag}(-1, 1, r^2, r^2 \sin^2 \theta)$. The latter is the FRW metric in the ‘comoving frame’ where the time coordinate is conformal time $d\eta = dt/a$, and $K$ is the spatial curvature parameter. This spacetime trivially satisfies the Cosmological Principle. The Ricci curvature scalar associated with this metric is \(R = 6K\). Considering $\phi = \chi e^{a\alpha}$ and assuming $\xi = 1/6$, Eq. (1) then reads

$$I = \int \left[K \chi^2 + \chi'^2 + \chi^2 a'^2 + V(\chi)\right] \sqrt{-g} d^4x.$$  

(9)
Variation with respect to \( \alpha \) and \( \chi \), respectively, results in
\[
\alpha' = \frac{c_\alpha}{\chi^2},
\]
\[
2\chi'' = 2K\chi + 2\alpha'^2\chi + \frac{dV}{d\chi},
\]  
(10)
where \( c_\alpha \) is an arbitrary integration constant. Employing Eq. (10) in (11), multiplying by \( \chi' \), and integrating we obtain the analog of the Friedmann equation
\[
Q^2 + \alpha'^2 + K = \frac{V}{\chi^2},
\]  
(12)
where an integration constant has been absorbed in \( V \). This contribution to \( V \) corresponds to an EOS \( w \equiv P/\rho = \frac{1}{3} \) as is evident from Eqs. (6) and (8).

Here, \( P \) and \( \rho \) are the pressure and energy density, respectively, and \( Q \equiv \frac{\chi}{\alpha} \) the analog of the conformal Hubble function, \( \mathcal{H} \equiv \frac{2}{\chi} \), has been defined. Making the replacement \( \chi \rightarrow \alpha \) and substituting \( \alpha = 0 \) in Eqs. (9) and (12) we recover the EH action and standard Friedmann equation, respectively. Here, instead of space expansion, the entire cosmic evolution is due to the time-dependence of \( \chi \), i.e. of active gravitational masses and the Planck mass, or equivalently due to the evolution of inertial masses, i.e. of the Rydberg ‘constant’. The evolution of the latter, on a static background, is then due to the monotonic growth of inertial masses in the redshifting era. Upon combining Eq. (12) and its derivative with Eq. (6) we obtain
\[
2Q' + Q^2 + K = -\frac{3wV}{\chi^2} + 3\alpha'^2.
\]  
(13)

Back to the ‘Friedmann equations’, Eqs. (12) and (13). These are augmented with \( \alpha\chi^2 \) terms that effectively behave as a negative energy with a ‘stiff’ EOS \( (\omega_s = 1) \), with an effective contribution \(-c_\alpha\chi^2\) to the effective potential. Assuming this contribution is negligible at present, as well as at any observationally accessible cosmological era, then at sufficiently small \( \chi \) this term competes with the radiation term, \( V_0 \propto \chi^4 \), and at the ‘turning point’ \( \chi_t = c_\alpha/\sqrt{3} \) the rate \( Q \) momentarily vanishes and transition from \( Q < 0 \) to \( Q > 0 \) ensues.

Light element abundances set a limit on the redshift at the turning point, \( z_t > 10^9 \). Since \( c_\alpha/(\chi^2V_0) \propto (1 + z)^2 \) it then follows that if \( z_t > 10^9 \) then already by recombination \( c_\alpha^2/\chi^2 \) dropped at best to a trillionth of \( V_0 \) and therefore has virtually no impact on the standard \( \Lambda \)CDM description of structure formation history.

Other bouncing models with similar behavior near the bounce, that contain a negative energy effective stiff matter component (that are sourced by other mechanisms), have been considered in Barcelo and Visser (2000) and Barrow et al. (2004). We emphasize that in the present model the energy condition is not violated, i.e. \( V - \chi \) the analog of \( \mathcal{L}_M \) – still includes only positive contributions, and it is only the contribution of the \( U(1) \)-symmetric kinetic term that makes an effective negative contribution, \(-c_\alpha\chi^{-2}\), to the source term in Eq. (12). This case is somewhat similar to the effective energy density associated with spatial curvature in the standard GR-based FRW spacetime, \( \rho_K \equiv -K/H_0^2 \). In case of a closed Universe, \( K > 0 \), the effective \( \rho_K \) is negative, and so a hypothetical radiation-only closed Universe (a legitimate solution of the Friedmann equation) will consequently undergo a bounce incurring no violation of the energy condition.

Here, the negative kinetic energy associated with \( \alpha \) is not counted as a contribution to the source, in the same fashion that \( Q^2 \), the kinetic energy associated with \( \chi \) (which in the standard Friedmann equation is represented by \( a \)) is not.

Accounting for the vacuum-like, NR, radiation and effectively stiff energy densities, Eqs. (10) and (12) combine to
\[
Q^2 = V_4\chi^2 + V_1/\chi + V_0/\chi^2 - c_\alpha^2/\chi^2.
\]  
(14)

Two interesting limits of this equation will suffice for our purposes. The first is obtained by neglecting the vacuum-like and NR terms near the turning point. In this case, the Friedmann-like equation, Eq. (14), integrates to
\[
\chi^2 = V_0\eta^2 + c_\alpha^2/V_0,
\]  
(15)
where \( \chi \) attains its minimum at \( \eta = 0 \). It could be readily integrated to yield the analog of the cosmic time around the turning point, \( t \propto \int \chi(\eta)d\eta \), and is easily verified to be non-singular as well. In other words, the effective time coordinates of both massless and massive particles can be extended through the turning point, i.e. spacetime is geodesically complete as is generically the case in non-singular bouncing cosmological models, Ijjas and Steinhardt (2018). In the absence of turning point (e.g. in case that \( \alpha \) is a fixed constant) the scalar field would have scaled \( \chi \propto \eta \) as does the scale factor \( a(\eta) \) in the radiation-dominated (RD) era. The scalar field then vanishes at \( \eta = 0 \). Whereas this does not represent a curvature singularity, it is a topological singularity and it is not entirely clear that the theory can be extended past beyond \( \eta = 0 \) as this would (at least formally) imply negative (active gravitational) masses, i.e. negative modulus of the scalar field, \( \chi < 0 \).

In the other extreme – sufficiently far from the turning point, where the background dynamics is dominated by the quartic potential – Eq. (12) integrates to \( \chi = \left(\sqrt{\chi(\eta_c \mp \eta)}\right)^{-1} \). The integration constant \( \eta_c > 0 \) represents the start and end of the (conformal) time coordinate in the proposed nearly symmetric model, i.e. \( \eta \in (-\eta_c, \eta_c) \). Again, since \( \eta \) is bounded from below, this time due to the presence of the vacuum-like energy density component, then observed radial distances are bounded at \( \eta = -\eta_c \) where the scalar field diverges and the model breaks down \( (\eta = \mp \eta_c \) correspond to \( t = \pm \infty \), i.e. past and future infinity). Specifically, since in the case \( K = 0 \) incoming null geodesics satisfy \( r(\eta) = \eta_0 - \eta \) and since \( \eta > -\eta_c \) then the maximal observable distance at the present horizon time is \( r_{\text{max}} = \eta_0 + \eta_c \). The latter can be much larger than the Hubble scale if \( \eta_c \gg \eta_0 \), thereby potentially addressing the horizon problem. In this regime \( Q = \frac{\chi}{\alpha} \) is constant and the scalar field is \( \chi \propto \exp(Qt) \). Recall that \( Q \) flips sign at the turning point and so \( \chi \) diverges at \( t \rightarrow \pm \infty \) exactly as does \( a(t) \) in the standard FRW space-time in the DE-dominated phase. Although we focus here on DE-dominated asymptotics it is clear that a similar breakdown is expected in any model which is asymptotically dominated by \( w < -1/3 \) as Eq. (10) integrates in this case to \( \chi = c(\eta_0 \mp \eta)^{\frac{1}{1-w}} \) (in the cases \( \eta < 0 \) and \( \eta > 0 \) respectively) where \( c \) and \( \eta_c \) are positive integration constants. Note that this is also the solution of the (equivalent) Friedmann equation, the scale factor \( a(\eta) \), and this and scalar field singularity is present in \( \Lambda \)CDM as well (as expected; only the small \( \chi \) dynamics is affected by introducing the phase \( \alpha \)). However, in the latter it is only a future singularity (where \( \chi \) diverges), whereas in the proposed
model it is also a past singularity taking place during the blueshifting epoch, thereby rendering the causally-connected Universe finite, although possibly much larger than naively thought based on the singular ΛCDM model.

Regardless of whether the ‘flatness problem’ is a genuine fine-tuning problem of the standard cosmological model or not, e.g. Helbig (2012), Carroll (2014), Helbig (2020), we lay out the problem as it is normally presented in the literature followed by its resolution by the symmetric ‘bouncing’ scenario in general, and the proposed model in particular. The ‘flatness problem’ is often claimed to arise in the hot Big Bang model due to the monotonic expansion of space and the consequent faster dilution of the energy density of matter (either relativistic or NR) compared to the effective energy density dilution associated with curvature. It is thus hard to envisage how could space be nearly flat [as is indeed inferred from observations, e.g. Planck Collaboration et al. (2020)] if not for an enormous fine-tuning taking place at the very early Universe, or alternatively for an early violent inflationary era. In the proposed scenario the matter content of the Universe has always been the same, and in particular the present (η0) ratio of matter- to curvature-energy densities has been exactly the same at time tη0. However, in the blueshifting era matter domination over curvature is actually an attractor point of the Big Bang model in the absence of inflation), i.e. at \(\chi \propto \frac{\rho}{\Omega} \approx \frac{\rho}{\rho_0} \ll 1\), implies in particular that the scalar and metric field transform as \(\chi \rightarrow \chi/\Omega \approx \chi(1+\delta_\chi)\) and \(g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}\), respectively. Consequently, the new, ‘shifted’, perturbation variables, e.g. \(\varphi \equiv \varphi + \delta_\chi, \psi \equiv \psi + \delta_\chi, \rho_{\text{m}} \equiv \rho_{\text{m}} + 4\delta_\chi\), etc., obey the same perturbation equations that are satisfied by the old perturbation quantities. The fact that the structure of the perturbation equations is unchanged under Weyl transformations is crucial in the context of stability near the bounce because perturbations of the scalar field in bouncing scenarios based on scalar-tensor theories of gravity are a potential cause for instability near the bounce, e.g. Gratton et al. (2004).

By virtue of the U(1) symmetry we have two new perturbation variables \(\delta_\chi\) and \(\delta_\rho\) in addition to the standard perturbation variables \(\varphi, \psi, \rho_{\text{m}}\) and \(\psi\). However, in the long wavelength limit, and ignoring coupling with other perturbation variables, Eq. (10) \(\alpha' = c_\alpha / \chi^2\) implies that \(\delta_\rho / \rho_{\text{m}} \rightarrow -2\delta_\chi\), and consequently the only new independent perturbation variable is, e.g. \(\delta_\rho\) in this limit. Possible implications of redefining the gravitational potential to CDM on galactic and cosmological scales in this framework are discussed in Shimon (2021a) and Shimon (2022a). Here we point out that the same rational can be applied on cosmological scales to the point that particle CDM may not be required at the perturbation level for explaining the clustering and growth of structure because \(\delta_\chi\) is a priori an arbitrary function of space and time. Specifically, the spectrum of metric perturbations at recombination, with an rms level of \(O(10^{-5})\), may simply reflect a similar perturbation in \(G\), gauged by \(\delta_\chi\) rather than the existence of particulate CDM. And indeed, \(G\) has never been directly measured on cosmological scales, surely not at the \(O(10^{-5})\) precision, and the fact that CDM seems at all to be required on these scales heavily relies on the premise that \(G\) is a Universal constant. All this implies that on the largest scales, and ignoring coupling between the various perturbation variables at the leading perturbation order both \(\varphi\) and \(\rho_{\text{m}}\) are perturbed at the \(O(10^{-5})\) level at the turning point and consequently the perturbations are stable. A more detailed stability analysis is carried out in Appendix A. It has been argued that, quite generally, modified theories of gravity that include no CDM component ought to be naturally contrived, exhibiting strong features in the transfer function Pardo and Spergel (2020). In arriving at this conclusion it has been assumed that the energy-momentum is conserved and that acceleration is caused by baryon-matter-only overdensities. However, none of these assumptions apply to the framework proposed in the present work.

In any case, we emphasize that the model proposed here by no means relies on the premise that CDM phenomena are merely a manifestation of variations of \(\chi\). CDM can be equally well taken to be particulate, exactly as in ΛCDM [Eq. (12) is agnostic to the microphysics of CDM insofar it is char-
acterized by $w = 0$. Rather, it is only mentioned in passing that at both the background and perturbation levels CDM phenomena can be accounted for by $\varphi$ and its perturbations.

In $\Lambda$CDM the linear gravitational potential has two modes in the matter-dominated (MD) era, $\varphi = \text{constant}$ and $\varphi \propto \eta^{-5}$. The latter dies off very quickly in an expanding Universe and is therefore conventionally ignored in practice. However, in a contracting Universe $\varphi \propto \eta^{-5}$ is the fastest growing mode, and if we assume that primordial scalar perturbations are set sometime during the contracting MD phase or even at $-\eta_k$, deep in the DE era of the contracting phase, they grow by a factor $(1 + z_{eq})^{3/2}$ during the MD era (and essentially freeze during the DE and RD pre-bounce epochs). Assuming the model is approximately symmetric around the bounce and employing the observationally favored value for the redshift at radiation-matter equality, $z_{eq} \approx 3400$, we obtain that $\varphi$ amplifies at most by a factor billion over the time period spanned between $-\eta_0$ and $-\eta_{eq}$. Consequently, scalar perturbations which started anywhere in the range $\varphi \in (10^{-14} - 10^{-5})$ (depending on their formation epoch) would end up at the observed $\varphi = O(10^{-5})$ level by the time of recombination, $\eta_{re}$.

4 SUMMARY

While $\Lambda$CDM has clearly been very successful in phenomenologically interpreting a wide range of observations, it still lacks a microphysical explanation of several key components, primarily the nature of CDM and DE. It also suffers from a few outstanding conceptual problems such as the initial singularity, in addition to a few ‘coincidence’ or ‘naturality’ problems.

Direct spectral information on the CMB is unavailable (due to opacity) in the pre-recombination era ($z \gtrsim 1100$). From the observed cosmic abundance of light elements, BBN at redshifts $O(10^3)$ could be indirectly probed. Earlier on, at $z = O(10^5)$ and $z = O(10^3)$ [energy scales of $O(200)$ MeV and $O(100)$ GeV, respectively], the quantum chromodynamics (QCD) and electroweak phase transitions had presumably took place, although their (indeed weak) signatures in, e.g. the CMB, have not been found.

In addition, inflation, a cornerstone of the standard cosmological model, is clearly beyond the realm of well-established physics; its ultimate detection via the B-mode polarization that it imprints on the CMB could be achieved only if inflation took place at energy scales ~trillion times larger than currently achievable in colliders. Moreover, theoretical expectations for the amplitude of this B-mode as a function of the energy scale of inflation rely on the assumption that gravitation is genuinely quantized. The latter assumption lacks empirical basis at present. By itself, inflation is plagued by the the $\eta$-problem, trans-Planckian problem, and the ‘measure problem’ in the multiverse, essentially a lack of predictive power.

Ideally, an alternative cosmological model that agrees well with $\Lambda$CDM at BBN energies and lower, i.e. $z < 10^{10}$, while still addressing the classical problems of the hot Big Bang model that inflation was originally designed to undertake, as well as avoiding the initial curvature singularity, and all this in the $\lesssim 10PeV$ range of energies, will be an appealing alternative that relies on experimentally established physics.

One conclusion of the present work is that this could be in principle achieved, at least in part, with a (relatively late classical) non-singular ‘bounce’ that also removes the technically and conceptually undesirable initial singularity problem of GR-based cosmological models. In order to achieve such a bounce within GR, or a conformally-related theory, certain ‘energy conditions’ have to be effectively violated. One specific realization of this program has been the focus of the present work.

Symmetries play a key role in our current understanding of the inner workings of the fundamental interactions. For example, the SM of particle physics is based on a local $U(1) \times SU(2) \times SU(3)$ gauge group with quantized gauge fields. In addition, our favorite theory of gravitation, GR, is diffeomorphism-invariant. In this work we entertained the possibility that in addition to diffeomorphism-invariance, GR is endowed with Weyl invariance, i.e. local scale invariance, as well as an internal $U(1)$ symmetry with a global charge. The SM of particle physics is assumed as is, with an explicitly broken Weyl symmetry. Only the salient merits of the cosmological model based on this alternative theory of gravitation have been discussed in the present work.

In the proposed model spacetime is described by the FRW metric in comoving frame, which in the absence of spatial curvature reduces to the Minkowski spacetime. Here, the role of the scale factor in $\Lambda$CDM as the regulator of cosmic history is played by the modulus, $\chi$, of a complex scalar field $\phi$ that lives on a static background. The phase, $\alpha$, plays a crucial role near the turning point and is largely irrelevant elsewhere. There is no analog to $\alpha$ in $\Lambda$CDM. Here, $\chi$, which regulates the evolution of (dynamical) active gravitational masses starts infinitely large, first monotonically decreases until it ‘bounces’, then grows again without bound. Put alternatively, the Planck length starts out infinitely small, increases until it peaks at the turning point, then decreases again. Described in terms of these length ‘units’ the Universe is said to undergo a ‘contraction’ phase of evolution, followed by a ‘bounce’ and ‘expansion’. Since spacetime is described by the FRW metric in the comoving frame, i.e. the time coordinate is conformal, then inertial masses evolve exactly as do active gravitational masses and the Planck mass. Cosmological redshift is then a manifestation of evolving Rydberg ‘constant’ rather than space expansion.

The alternative cosmological scenario explored in this work starts with a deflationary evolution which culminates in a turning point when the (absolute value of the negative) energy density associated with the effective ‘stiff matter’ (provided by the kinetic term of $\alpha$) momentarily equals that of radiation. In the vacuum-like-dominated epoch the energy density of the Universe is dominated by a $\propto \chi^4$ term in the potential which is genuinely classical with no quantum fluctuations. Therefore, DE according to the present scenario is not zero-point energy but rather a manifestation of the self-coupling of the scalar field, i.e. a term in the potential of the form $V_4\chi^4$ with $V_4$ being a dimensionless parameter. This DE contribution is characterized by a non-dynamical EOS with no recourse to, e.g. a new quintessence, field; here, the same scalar field accounts for both ‘G’ and DE, and possibly also CDM.

One of the most notable achievements of inflation was the realization that a slightly red-tilted primordial spectrum of gaussian perturbations can be generated by quantum fluctua-
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In a vacuum-like expanding Universe. It should be noted that a decade before the inflationary scenario has been proposed it has been realized by Harrison and Zeldovich, as well as by others, that the primordial spectrum of perturbations should be nearly flat for the rms perturbations on the wide dynamical range of cosmological scales not to have infrared or ultraviolet divergences that mark the breakdown of linear perturbation theory. In any case, no mechanism has been proposed in the present work for the generation of primordial density perturbations. Whatever this mechanism turns out to be, it does not involve quantum fluctuations of the metric field, unlike in inflation. Again, it is assumed here that gravitation is a genuine classical interaction.

In addition, the ‘anisotropy problem’ that in general plagues bouncing scenarios does not exist in our construction. As discussed above, Weyl symmetry and the consequent absence of any dimensional parameter in the action, in addition to the postulated global U(1) symmetry, protects the model from running into a chaotic, anisotropy-dominated, evolution phase, unless we are willing to consider non-canonical terms in the potential, $V$, of the form, e.g. $V_{\text{an}} \propto (\psi \bar{\psi})^2\chi^{-2}$ and $C_{\alpha\beta}C_{\gamma\delta}C_{\rho\sigma}^\alpha\chi^{-2}$, where $\psi$ is e.g., a Dirac field, and $C_{\rho\sigma}^\alpha$ is the Weyl tensor.

Conformal time is both past- and future-bounded in this scenario, i.e. $\eta \in (-\eta_c, \eta_c)$. In principle, any ‘horizon problem’ could be avoided if $\eta_c \gg \eta$. Specifically in this scenario, cosmic history starts with very large (and in principle infinite) particle masses and therefore the causal horizon is much larger than would be naively expected from monotonically growing masses (that corresponds to the redshifting era), i.e. essentially $\eta_0 \ll \eta_c \gg \eta_0$. Likewise, the ‘flatness problem’ afflicting the hot Big Bang scenario stems from the slower decay of the energy density associated with curvature as compared to that of matter in a monotonically expanding Universe. In bouncing scenarios the situation is reversed before the turning point; starting at infinitely large $\chi$ (masses) one typically expects to find that the energy density in the forms of NR matter and radiation largely exceeds that of curvature at any finite $\chi$ value in the blueshifting era. Since this adiabatic model is nearly symmetric in $\chi$ around the turning point (barring possible entropy processing effects at around the RD era, the efficacy of which depends on the redshift at which such a bounce takes place), one generally expects the Universe to look spatially flat at any finite $\chi$ after the would-be singularity (actually a non-singular ‘bounce’).

From this perspective flatness is an attractor-, rather than an unstable-point that requires fine-tuning of the initial conditions.

The model we considered is falsifiable in several ways: First, $w_{\text{DE}} = -1$ due to Weyl invariance and any observationally inferred $w_{\text{DE}} \neq -1$ would either rule out this particular model or alternatively either imply soft breakdown of WI at very early and late times or the existence of a non-canonical DE term in the potential of the form, e.g. $V_{\text{DE}} \propto (\psi \bar{\psi})^{-\varepsilon/3}$ (where $\varepsilon$ is some dimensionless parameter), that involves non-integer, and possibly irrational, powers of the fields. Second, within the framework adopted here, if a scale-invariant B-mode polarization is ultimately measured, it would provide a compelling evidence that gravity is quantized, in contradiction to the assumption made here that gravity is a genuinely classical interaction which implies that its perturbations are not subject to the Bunch-Davies vacuum condition. Consequently, unlike the inflationary-induced B-mode polarization of $\Lambda$CDM, it does not follow from any fundamental principle that B-mode polarization have to be characterized by a flat spectrum. Third, signals from primordial phase transitions as well as leptogenesis or baryogenesis that ought to be imprinted in the CMB anisotropy and polarization (perhaps too weak to be detected) may not have taken place at all in the proposed model, depending on the typical temperature at the turning point.

We believe that, in addition to addressing the cosmological horizon and flatness problems, the framework proposed here provides important insight on the nature of DE, and initial singularity and stability near the turning point. Even so, the work presented here is by no means exhaustive, and indeed a few of its basic aspects will be further elucidated in future works.

### APPENDIX A: STABILITY ANALYSIS

As discussed in section 3.2 the initial scalar metric perturbations deep in the pre-bounce era should be at the $O(10^{-14}) - O(10^{-5})$ level (depending on their formation time in the blueshifting phase) so that by the time of recombination they have grown to $O(10^{-5})$. In this Appendix we analyze the dynamics of scalar perturbations at and near $\eta = 0$ and show that all linear perturbation variables smoothly transform through this point.

Near this point the energy budget is dominated by radiation and the kinetic term associated with $\alpha$. The Friedmann-like equation, Eq. (12), then integrates to

$$\chi^2 = A\eta^2 + B,$$

where according to Eq. (15) $A \equiv V_0$ and $B \equiv c_s^2/V_0$. The analog of conformal Hubble function in this case becomes $Q = A\eta/(A\eta^2 + B)$. Linear perturbation equations [e.g., Hwang and Noh (2001)] result in coupled equations for $\delta\alpha$ and $\varphi$

$$\delta\alpha'' + \frac{2A\eta}{A\eta^2 + B} \delta\alpha' + k^2\delta\alpha = -\frac{4\sqrt{AB}}{A\eta^2 + B} \varphi',$$

$$\varphi'' + \frac{4A\eta}{A\eta^2 + B} \varphi' + \frac{k^2\varphi}{3} = \frac{2\sqrt{AB}}{A\eta^2 + B} \delta\alpha',$$

where it is understood that fractional perturbations of the modulus scalar field, $\delta\alpha$, are absorbed in the gravitational potential as described in section 3.2. Eq. (18) is a generalized Bardeen equation that is obtained from combination of the Arnowitt-Deser-Misner (ADM) energy constraint and the Raychaudhuri equation. In the long wavelength limit this system of equations is satisfied by

$$\varphi = c_1 + \frac{c_2\eta}{(B + A\eta^2)^2} + \frac{c_3(A\eta^2 - B)}{(B + A\eta^2)^2},$$

where $c_1$, $c_2$ and $c_3$ are three integration constants. It is clear from Eqs. (19) that both $\varphi$ and $\delta\alpha'/\alpha'$ are well-behaved at $\eta = 0$ due to the non-vanishing $B$, i.e. the non-vanishing of $\alpha'$, and perturbation theory does not break down there.

To solve for the short-wavelength limit we differentiate Eq.
(17) with respect to η and then substitute for $\delta \alpha'$ from Eq. (18). The resulting fourth-order equation for $\varphi$ is

$$
\left( \frac{Ax^2}{k^2} + B \right) \varphi + \frac{18A}{k^2} x \varphi_x + \left( \frac{60A}{k^2} + \frac{4A}{k^4} x^2 + 4B \right) \varphi_{xx} + \frac{30A}{k^2} x \varphi_{xxx} + 3 \left( B + \frac{Ax^2}{k^2} \right) \varphi_{xxxx} = 0, \tag{20}
$$

where e.g., $\varphi_x \equiv \frac{\partial \varphi}{\partial x}$, in the large-$k$ limit, which has no closed-form solution. Here $x \equiv k \eta$. In the limit $x \ll 1$, i.e. $\eta \to 0$, the potential $\varphi$ decouples from both $A$ and $B$, and the equation considerably simplifies to

$$
3\varphi_{xxxx} + 4\varphi_{xx} + \varphi = 0, \tag{21}
$$

with the general solution

$$
\varphi = c_1 \cos x + c_2 \sin x + c_3 \cos \left( x/\sqrt{3} \right) + c_4 \sin \left( x/\sqrt{3} \right), \tag{22}
$$

illustrating the fact that perturbations are manifestly stable on small scales as well.

Slightly off $\eta = 0$, $A \eta^2 \gtrsim B$ perturbations smoothly approach their short wavelength limit standard behavior in the RD era. In the latter, within $\Lambda$CDM, $\varphi = \frac{1}{3} \left[ c_1 j_1(x) + c_2 y_1(x) \right]$ where $j_1$ and $y_1$ are the spherical Bessel and the modified spherical Bessel functions, respectively, of the first kind. In $\Lambda$CDM, $c_2 = 0$ since $y_1(0)$ diverges, and the phase of the oscillating potential is $0$, i.e. $\varphi \propto \sin x$. This needs not be the case in the proposed model as is evident from this discussion insofar $B \neq 0$. The implications for CMB observables of not neglecting the ‘diverging’ mode have been discussed recently by Kodwani et al. (2019).

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