Research Article

Modelling and Analysis of Propagation Behavior of Computer Viruses with Nonlinear Countermeasure Probability and Infected Removable Storage Media

Xulong Zhang and Yong Li

School of Computer and Network Engineering, Shanxi Datong University, Datong 037009, China

Correspondence should be addressed to Xulong Zhang; zxl-095@163.com

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1. Introduction

The continual emergence of computer viruses, especially with the growing popularity of the Internet, has brought great troubles and threats to our daily work and life (e.g., [1]). Besides, removable storage media, such as compact disk, removable hard disk, USB flash disk, flash memory card, and so on, which are often used in our daily work provide another spreading route for computer viruses except the Internet. Antivirus software, patches, and firewall are the main technical means of defending against computer viruses, which can weed out all viruses they can recognize that stay in individual electronic devices such as personal computer (PC) and removable storage media. Unfortunately, these techniques seem powerless to the outbreak of a new virus. In order to effectively contain virus spread, one needs to understand the propagation laws of computer viruses, which may provide a theoretical basis for decision making, as well as to use technical measures.

A multitude of propagation models of computer viruses have been presented since 1991, specifically, SIS (susceptible-infected-susceptible) models (e.g., [2, 3]), SIRS (susceptible-infected-recovered-susceptible) models (e.g., [4, 5]), SLBS (susceptible-latent-breaking-susceptible) model (e.g., [6]), SICS (susceptible-infected-countermeasured-susceptible) models (e.g., [7–9]), and SDIRS (susceptible-delitescent-infected-recovered-susceptible) model (e.g., [10]).

In the field of computer viruses, countermeasures such as warnings, firewall, and software patches can provide a practical approach to avoid virus infection problems. In 2004, Chen and Carley [11] addressed the countermeasure competing (CMC) strategy, which shows that the CMC strategy is more available compared to previous strategies. Inspired by this work and in order to macroscopically describe the mixed transmission of computer viruses and countermeasures, Zhu et al. [7] presented the first compartment model in this aspect, named as the SICS model, and its global dynamics was fully examined. Afterwards, Yang and Yang [8] extended this model by incorporating the effect of infected external computers (i.e., computers outside the Internet) and removable storage media. However, these two models ignore two important facts. On the one hand, they overlook the fact that the linear infection probability is fit well for the real-world situations only when the
countermeasured (or immune) nodes are few. On the other hand, they neglect the fact that countermeasures may be disseminated through networks at different rates, which has been mentioned in Reference [11]. Thus, the assumption of linear countermeasure probability is unreasonable.

To remedy these defects and considering the influences of general countermeasure and infected removable storage media on viral diffusion, this paper studies a new propagation model incorporating generic countermeasure probability and infected removable storage media. The main result, the global stability of the unique (viral) equilibrium, is proved, which is also examined by some numerical experiments. Furthermore, the numerical experiments of different countermeasure probabilities are conducted.

The paper is organized as follows. The model formulation is made in Section 2. Section 3 determines the (viral) equilibrium and investigates its global stability. Numerical experiments are presented in Section 4. This work is summarized in Section 5.

### 2. Model Description

In this paper, a computer is called external or internal computer determined by whether it is disconnected from or connected to the Internet. All internal computers may have three states: susceptible, infected, and immune (with countermeasures). For brevity, let $S(t)$, $I(t)$, and $C(t)$ ($S$, $I$, and $C$, for short) denote the average numbers of susceptible, infected, and countermeasured computers at time $t$, respectively. Their entering rates are $\mu_1 > 0$, $\mu_2 > 0$, and $\mu_3 > 0$, respectively. Besides, the following basic assumptions of the model are made.

1. Each internal computer leaves the Internet with probability $\delta > 0$.

2. Each susceptible internal computer becomes infected by connecting with infected internal computer (or infected removable storage media) with probability $\beta_1 > 0$ (or $\beta_2 > 0$).

3. Each infected or susceptible internal computer gains the latest countermeasure with probability $\gamma_1(C(t))$ at time $t$, where $\gamma_1$ is twice continuously differentiable, $\gamma_1' > 0$, $\gamma_1'' < 0$, and $\gamma_1(0) = 0$. The concavity assumption seizes well the saturability of the countermeasure probability.

4. By reinstalling the operating system, each countermeasured (or infected) internal computer becomes susceptible with probability $\alpha > 0$ (or $\gamma_2 > 0$).

Now, one can derive the mathematical representation of the model as follows (also see Figure 1):

\[
\begin{align*}
\frac{dS}{dt} &= \mu_1 - \beta_1 IS - \beta_2 S - \gamma_2 I + \alpha C - \delta S, \\
\frac{dI}{dt} &= \mu_2 + \beta_1 IS + \beta_2 S - \gamma_2 I - \gamma_2 I - \delta I, \\
\frac{dC}{dt} &= \mu_3 + \gamma_1(C)I - \alpha C - \delta C,
\end{align*}
\]

with initial condition $(S(0), I(0), C(0)) \in \mathbb{R}^3$.

### 3. Theoretical Analysis

Let $N = S + I + C$, and $\mu = \mu_1 + \mu_2 + \mu_3$. Adding up the three equations of system (1), it is easy to get that $\lim_{t \to \infty} N = (\mu/\delta)$. It follows by the asymptotically autonomous system theory [12] that system (1) is equivalent to the following reduced limiting system:

\[
\begin{align*}
\frac{dI}{dt} &= \mu_2 + \frac{\beta_2 \mu}{\delta} + \left(\frac{\beta_1 \mu}{\delta} - \beta_2 - \gamma_2 - \delta\right)I - \beta_2 C - \beta_1 I^2 - \beta_1 IC - \gamma_1(C)I, \\
\frac{dC}{dt} &= \mu_3 + \gamma_1(C)(\frac{\mu}{\delta} - C) - (\alpha + \delta)C,
\end{align*}
\]

with initial condition $(I(0), C(0)) \in \Omega$, where

\[
\Omega = \{(I, C) \in \mathbb{R}^2_+: I + C \leq \frac{\mu}{\delta}\},
\]

and $\Omega$ is positively invariant for system (2).

In the following sections, we just need to investigate the dynamical behavior of system (2).

#### 3.1. Equilibrium

**Theorem 1.** There exists a unique (viral) equilibrium $E^* = (I^*, C^*)$ for system (2), where $E^*$ is the single positive solution to the following system:

\[
\begin{align*}
\mu_2 + \frac{\beta_2 \mu}{\delta} + \left(\frac{\beta_1 \mu}{\delta} - \beta_2 - \gamma_2 - \delta\right)x - \beta_2 y - \beta_1 x^2 - \beta_1 xy - \gamma_1(y)x &= 0, \\
\mu_3 + \gamma_1(y)(\frac{\mu}{\delta} - y) - (\alpha + \delta)y &= 0,
\end{align*}
\]


with the initial condition \((x(0), y(0)) \in \Omega\).

**Proof.** Let us suppose that \(E^* = (I^*, C^*)\) is an equilibrium of system (2). Clearly, \(E^*\) satisfies system (4). Thus, it suffices to prove that system (4) has a unique positive solution.

Firstly, let us prove that the second equation of system (4) has a unique positive root. Let

\[
g(x) = \mu_2 + \beta_2 \left( \frac{\mu}{\delta} - C^* \right) + \left( \frac{\beta_1}{\delta} \right) - \beta_2 - \gamma_2 - \frac{\alpha C}{\delta} - \frac{\beta_1}{\delta} C^* - \gamma_1 (C^*) x - \beta_1 x^2.
\]

As \(g(0) = \mu_2 + \beta_2 ((\mu/\delta) - C^*) > 0\) and \(g((\mu/\delta) - C^*) = -\mu_1 - \alpha C^* - \gamma_2 ((\mu/\delta) - C^*) < 0\), \(g\) does have a (positive) zero located in \((0, (\mu/\delta) - C^*)\). Besides, note that

\[
g'(x) = -\beta_1 \frac{\mu}{\delta} - C^* - (\beta_2 + \gamma_2 + \beta_1 (C^*)) < 0,
\]

\[
g''(x) = -2\beta_1 < 0.
\]

We shall also proceed by distinguishing two possibilities depending upon whether \(g'(0)\) is positive or negative.

**Case 1:** \(g'(0) > 0\). Let

\[
\gamma = \max \left\{ y \in \left[0, \frac{\mu}{\delta}\right] : g'(y) > 0 \right\}.
\]

Thus, \(g\) is strictly increasing in \([0, \gamma]\) and strictly decreasing in \([\gamma, (\mu/\delta)]\), which implies that \(g\) has a unique zero in \([\gamma, (\mu/\delta)]\).

**Case 2:** \(g'(0) \leq 0\). So, \(g\) is decreasing and has a unique zero.

It is easily obtained from the above discussions that \(g\) does have a single zero. Then, \(y = C^*\). Besides, \(f'(C^*) < 0\).

Next, let us prove that the first equation of system (4) has a single positive root. Let

\[
\delta = \max \left\{ x \in \left[0, \frac{\mu}{\delta} - C^*\right] : g'(x) > 0 \right\}.
\]

Then, \(g\) is strictly increasing in \([0, \delta]\) and decreasing in \([\delta, (\mu/\delta) - C^*]\), meaning that \(g\) has a single zero in \([\delta, (\mu/\delta) - C^*]\).

**Case 2:** \(g'(0) \leq 0\). Hence, \(g\) is decreasing and has a unique zero. Then, \(g\) always has a single zero \(x = I^*\).

Thus, the claimed result follows. \(\square\)

### 3.2. Local Stability

**Theorem 2.** \(E^*\) is locally asymptotically stable.
Proof. The corresponding Jacobian matrix of system (2) at $E^*$ is as follows:

$$\begin{pmatrix}
\frac{\beta_1\mu}{\delta} - \beta_2 - \gamma_2 - \delta - \beta_1C^* - 2\beta_1I^* - \gamma_1(C^*) & 0 \\
0 & -\beta_2 - \beta_1I^* - \gamma_1(C^*) \mu (C^*) - \gamma_1(C^*) - (\alpha + \delta)
\end{pmatrix}$$

and its two eigenvalues are

$$\lambda_1 = \frac{\beta_1\mu}{\delta} - \beta_2 - \gamma_2 - \delta - \beta_1C^* - 2\beta_1I^* - \gamma_1(C^*) = g'(I^*) < 0,$$

$$\lambda_2 = 1 (C^*) \mu (C^*) - \gamma_1(C^*) - (\alpha + \delta) = f'(C^*) < 0.$$

Thus, the claimed result follows from the Lyapunov stability theorem [13]. □

3.3. Global Stability

**Lemma 1.** System (2) has no periodic orbit.

**Proof.** Let

$$h_1(I, C) = \mu + \beta_2\mu \delta + \left( \frac{\beta_1\mu}{\delta} - \beta_2 - \gamma_2 - \delta \right) I - \beta_2 C$$

$$- \beta_1 I^2 - \beta_1 IC - \gamma_1(C) I,$$

$$h_2(I, C) = \mu_2 + \gamma_1(C) \mu (C) - (\alpha + \delta) C,$$

$$D(I, C) = \frac{1}{IC}$$

It can be obtained in the interior of $\Omega$ that

$$\frac{\partial (Dh_1)}{\partial I} + \frac{\partial (Dh_2)}{\partial C} = \frac{\beta_1}{C} \gamma_1'(C) + \frac{\mu_2}{I^2} - \frac{\mu_2}{IC^2} - \frac{\beta_2}{IC} \left( \frac{\mu}{\delta} - C \right)$$

$$+ \frac{\mu_2}{\delta IC^2} \left( \gamma_1(C) C - \gamma_1(C) \right).$$

Let

$$k(x) = \gamma_1'(x)x - \gamma_1(x).$$

As $k(0) = 0$ and $k'(x) = \gamma_1'(x) x < 0$ for all $x > 0, k(C) < 0.$ Thus, we have $(\partial (Dh_1)/\partial I) + (\partial (Dh_2)/\partial C) < 0.$

Hence, in the interior of $\Omega,$ system (2) has no periodic orbit according to the Bendixson–Dulac criterion [13].

On the boundary of $\Omega$, let $(\bar{I}, \bar{C})$ denote an arbitrary point. Thus, three possibilities may occur.

Case 1: $0 < \bar{C} < (\mu/\delta),$ $\bar{I} = 0.$ Then, $(\partial I/\partial r)|_{(\bar{I}, \bar{C})} = \mu_2 + \beta_2 ((\mu/\delta) - \bar{C}) > 0.$

Case 2: $0 < \bar{I} < (\mu/\delta),$ $\bar{C} = 0.$ Then, $(\partial C/\partial r)|_{(\bar{I}, \bar{C})} = \mu_1 > 0.$

Case 3: $\bar{I} + \bar{C} = (\mu/\delta),$ $\bar{C} \neq 0, \bar{I} \neq 0.$ Hence,

$$(\partial (I + C)/\partial r)|_{(\bar{I}, \bar{C})} = -\mu_1 - \gamma_2 \bar{I} - a\bar{C} < 0.$$

Thus, system (2) has no periodic orbit across the arbitrary point $(\bar{I}, \bar{C}).$ The proof is completed.

In what follows, the main result of this paper will be given as follows. □

**Theorem 3.** $E^*$ is globally asymptotically stable.

**Proof.** Based on Theorem 1, Lemma 1, and Theorem 2, the claimed result follows from the generalized Poincaré–Bendixson theorem [13]. □

4. Numerical Experiments

To illustrate the main result of this paper and the impacts of different countermeasure probabilities on viral spread, some numerical experiments are presented in this section.

**Example 1.** Consider system (1) with $\mu_1 = 0.55,$ $\mu_2 = 0.25,$

$$\mu_3 = 0.2, \alpha = 0.02, \beta_1 = 0.05, \beta_2 = 0.03, \gamma_1 = 0.02, \delta = 0.1,$$ and $\gamma_1(C) = 0.05C(1 + C).$ The initial condition is $S(0), I(0), C(0) = (3, 1, 5).$ In Figure 2, a comparison between the new proposed SICs model and the original SICS model is shown, from which it can be seen that the new proposed model is more reasonable in predicting virus prevalence because computer viruses would not go extinct (i.e., $I \geq 1$), which demonstrates that the linear countermeasure probability overestimates the suppression of countermeasures on virus diffusion when compared to the nonlinear one.

**Example 2.** Consider system (1) with $\mu_1 = 55,$ $\mu_2 = 38,$

$$\mu_3 = 7, \alpha = 0.01, \beta_1 = 0.52, \beta_2 = 0.015, \gamma_2 = 0.01, \delta = 0.02,$$ and $\gamma_1(C) = 0.006C^{0.15}.$ Six different initial conditions are listed below.

(1) $(S(0), I(0), C(0)) = (325, 25, 10).$

(2) $(S(0), I(0), C(0)) = (925, 125, 90).$

(3) $(S(0), I(0), C(0)) = (1525, 225, 170).$

(4) $(S(0), I(0), C(0)) = (2125, 325, 250).$
Figure 3 shows six orbits of system (1) with different initial conditions for a common system. It can be seen from this figure that no matter where the initial state starts, computer viruses would always exist and tend to a steady state, which coincides with the main result. This also reveals that the global stability is independent of the initial state.

**Example 3.** Consider system (1) with the common initial condition \((S(0), I(0), C(0)) = (1050, 450, 105)\), and six sets of parameters are given in Table 1. Six orbits of system (1) with different system parameters for a common initial condition are shown in Figure 4, from which it can be seen that computer viruses would remain present and tend to a steady state, which accords with the main result. Additionally, this figure reveals that even starting from the same initial state the system would approach to different states for different parameters, which is distinct from the phenomenon in Example 2.

**Example 4.** Consider system (1) with \(\mu_1 = 5.5, \mu_2 = 3.8, \mu_3 = 0.7, \alpha = 0.01, \beta_1 = 0.52, \beta_2 = 0.015, \gamma_1 = 0.01,\) and \(\delta = 0.02.\) The initial condition is \((S(0), I(0), C(0)) = (325, 25, 10)\). Figure 5 shows the influences of the varied countermeasure probabilities on the number of infected computers, where \(\gamma_1(C) = 0.08C^{0.2}, \gamma_2(C) = (0.08C/(1+0.416C)),\) and \(\gamma_3(C) = (0.08C/(1+C)).\) This figure also demonstrates that the nonlinear countermeasure probabilities which are continuously differentiable up to the second order may have many forms and pose different impacts on viral spread.

5. **Summary and Outlook**

A new SICS model has been proposed and analyzed in this paper. The global stability of the unique (viral) equilibrium has been proved and illustrated completely. Besides, a comparison between the new proposed model and the original SICS model has been shown, and the effects of varied countermeasure probabilities have also been revealed. The numerical experiments demonstrate that the nonlinear countermeasure probability is more reasonable than the linear one.

Additionally, the follow-up work arrangement is as follows. Firstly, time delays (e.g., [14, 15]), pulses (e.g., [16]), random fluctuations (e.g., [17, 18]), and optimal control strategies (e.g., [19]) can be considered in the new model. Secondly, the new model may be extended on wireless sensor networks (e.g., [20–22]). With the popularity of social networks, individuals’ participation has a particularly important effect on information diffusion including propagation of computer viruses. For example, Alduaij et al. [23] developed an influence propagation model for clique-based community detection in social networks. Li et al. [24] proposed a metric to measure the community-diversified influence in social networks. Therefore, the new model may also be extended in social networks. Finally, the new

\[(5) (S(0), I(0), C(0)) = (2725, 425, 330).\]
\[(6) (S(0), I(0), C(0)) = (3325, 525, 410).\]
The proposed model can be formulated for cloud computing security (e.g., [25]).

**Data Availability**

The data included in this study are available from the corresponding author upon request.

**Conflicts of Interest**

The authors declare that there are no conflicts of interest with regard to the publication of this paper.

**Authors’ Contributions**

Xulong Zhang conceived and designed the study and reviewed and edited the manuscript. Yong Li performed the numerical experiments. All authors read and approved the manuscript.

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