Hall Coefficient of a Dilute 2D Electron System in Parallel Magnetic Field.

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Measurements in magnetic fields applied at a small angle with respect to the 2D plane of the electrons of a low-density silicon MOSFET indicate that the Hall coefficient is independent of parallel field from $H = 0$ to $H > H_{sat}$, the field above which the longitudinal resistance saturates and the electrons have reached full spin-polarization. This implies that the mobilities of the spin-up and spin-down electrons remain comparable at all magnetic fields, and suggests there is strong mixing of spin-up and spin-down electron states.

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Dilute, strongly interacting two-dimensional systems of electrons and holes have drawn intensive recent attention due to their anomalous behavior as a function of temperature and magnetic field\[1\]: the resistance exhibits metallic temperature-dependence above a critical density, $n_c$, raising the possibility of an unexpected metallic phase in two dimensions\[2\]. An additional intriguing characteristic of these systems is their enormous response to magnetic fields applied in the plane of the electrons\[3,4\] or holes\[5,6\]: the resistivity increases up to several orders of magnitude (depending on density, temperature, and the mobility of the sample), and saturates to a new value above a density-dependent characteristic magnetic field $H_{sat}$. Recent experiments\[7,8\] have shown that the field $H_{sat}$ corresponds to the onset of full spin polarization of the 2D electron system. With increasing parallel magnetic field the system thus evolves from zero net spin polarization, with equal numbers of spin-up and spin-down electrons, to a completely spin-polarized system. Based on straightforward arguments, one expects different screening\[9\] and different field-dependent mobilities for the spin-up and spin-down electrons. The purpose of the Hall effect measurements reported in this paper was to find evidence for these two distinct sets of carriers. Our results indicate that the Hall coefficient does not, in fact, vary with a parallel magnetic field ranging from 0 to well above $H_{sat}$. Within a simple single-particle interpretation\[10\] this indicates that, despite very different parameters such as Fermi velocities, the transport mobilities of the spin-up and spin-down electrons are comparable over the entire range of magnetic fields. The fact that the Hall coefficient is independent of the degree of polarization of the electrons implies that frequent spin-flip scattering events cause substantial mixing of the spin-up and spin-down electron bands, highlighting the importance of electron-electron interactions in these dilute 2D electron systems.

FIG. 1. (A): Longitudinal resistivity versus in-plane magnetic field of a silicon MOSFET of electron density $n_s = 1.44 \times 10^{11} \text{ cm}^{-2} \approx 1.7n_c$, where $n_c$ is the critical density for the metal-insulator transition in zero field. The temperature is 0.25 K. (B) The Hall resistance plotted as a function of the normal component of the magnetic field, $H_{\perp} = H \sin 2.4^\circ$. (C) The Hall coefficient, $R_H = R_{xy}/H_{\perp}$, as a function of parallel magnetic field.
Measurements were performed on three silicon MOSFETs of comparable mobilities $\mu \approx 20,000 \text{ V/(cm}^2\text{s})$ at $T = 4.2 \text{ K}$. Contact resistances were minimized by using a split-gate geometry which allows a higher electron density in the vicinity of the contacts than in the 2D system under investigation. Data were taken with standard four-probe techniques using AC phase-sensitive detection to eliminate thermoelectric as well as other parasitic rectified signals. Small unavoidable misalignment of the transverse Hall potential contacts introduces an admixture of the longitudinal resistance, which is known to be large and strongly field-dependent in these dilute 2D systems. The longitudinal component was eliminated by averaging the voltages measured for two opposite directions of the magnetic field. Data were taken in the linear regime with AC currents typically below 5 nA, at frequency 3 Hz.

The sample was mounted on a rotating platform at the end of a low temperature probe in a 4 He Oxford Heliox system and measurements were obtained for temperatures between 0.235 K and 1.66 K in magnetic fields $H$ up to 12 T. The angle $\phi$ between the magnetic field $H$ and the 2D plane was determined by measuring the Hall resistance; $\phi$ was chosen to be very small in order to limit the size of the perpendicular field component to avoid quantum oscillations of the longitudinal resistance and Hall signal.

For a silicon MOSFET with density $n_s = 1.44 \times 10^{11} \text{ cm}^{-2}$, Fig. 1 (A) shows the longitudinal resistivity $\rho_{xx}$ at temperature $T = 0.25 \text{ K}$ as a function of magnetic field applied at an angle $\phi = 2.4^\circ$; here $n_s \approx 1.7 n_c$, where $n_c$ is the critical density for the zero-field metal-insulator transition. For an angle of 2.4$^\circ$, the in-plane component of the magnetic field differs from the total field by only 0.1%; $H_{\perp} = H \cos \phi = 0.999H$, while the perpendicular magnetic field remains small: $H_{\parallel} = H \sin \phi = 0.0419H$. When the in-plane component has reached 12 T the perpendicular component is only 0.5 T; no Shubnikov-de Haas oscillations are apparent in either component of the resistivity, $\rho_{xx}$ or $R_{xy}$. In agreement with earlier measurements for densities well above $n_c$, Fig. 1 (A) shows that the resistance increases by a factor of approximately four and saturates to a constant value above a magnetic field $H_{\text{sat}} \approx 6 \text{ T}$. Earlier measurements have shown that the resistance varies weakly with temperature for this electron density in a large magnetic field $H = 10.8 \text{ T} > H_{\text{sat}}$, so that the sample has not entered the insulating phase in the highest field used in our experiments [11]. For the same sample under the same conditions, Fig. 1 (B) shows that the transverse Hall resistance is a clean linear function of perpendicular magnetic field up to $H_{\perp} = 0.5 \text{ T}$, for in-plane magnetic field increasing to about 12 T. Fig. 1 (C) shows the Hall coefficient, $R_H = R_{xy}/H_{\perp}$, normalized to its value in zero field plotted as a function of in-plane magnetic field. The Hall coefficient is constant to within 1–3%, the experimental error of our measurements, while the parallel component of the magnetic field changes from 0 to well above $H_{\text{sat}}$. The Hall coefficient is thus independent of the degree of polarization of the 2D electron system.

Temperature dependent screening of charged impurities [12,13] has been proposed as a possible explanation for the unusual metallic temperature dependence of the resistivity of dilute 2D systems. Dolgopolov and Gold [9] have recently argued that the screening properties of the electrons also depend strongly on in-plane magnetic field, causing the resistance to increase with field and to saturate when the electrons reach full spin polarization at $H > H_{\text{sat}}$, as observed experimentally [14,15]. We note that the effect of screening on the behavior of the Hall coefficient has not been considered. In what follows, we first discuss the dependence of the longitudinal resistance on magnetic field due to field-induced changes in screening and compare it with the measured magnetoresistance. We then extend these concepts to consider the effect of field-dependent screening on the Hall coefficient.

In the Born approximation, the probability of electron scattering depends directly on the dielectric function $\epsilon$. Due to the sharp edge of the electron distribution at $E = E_F$, the dielectric function $\epsilon(q)$ has a singularity at $q = 2K_F$ in the limit $T = 0$. Increasing the temperature causes smearing of the Fermi distribution, giving rise to a temperature-dependent resistance [12]. The effect of parallel field on electron screening was recently considered by Dolgopolov and Gold [13]. A strong in-plane magnetic field decreases the energy of electrons with spins aligned along the magnetic field (spin-up electrons) and increases the energy of electrons with opposite spin (spin-down electrons). The radius of the Fermi circle of spin-up (spin-down) electrons, $K_F^\uparrow$ ($K_F^\downarrow$), increases (decreases) with field. The ability of the electron sea to screen the Fourier components of the external potential $V(q > 2K_F^\parallel)$ with wavevectors $q > 2K_F^\parallel$ is reduced because the spin-down particles with wavevector $K < K_F^\parallel$ cannot interact with the potential $V(q)$ due to momentum conservation: $|\vec{K}_{\text{fin}} - \vec{K}_{\text{f}}| \leq 2K_F^\perp < q$. In other words, the short wavelength components of the external potential $V(q > 2K_F^\parallel)$ cannot be screened effectively by electrons with long de-Broglie wavelengths. The magnetic field thus reduces the screening of the short-wavelength components of the external potential by the spin-down electrons, causing an increase of scattering of the spin-up electrons. We thus expect the spin-up and spin-down electrons to have different mobilities in a magnetic field.

Results of calculations [16] of the longitudinal resistivity as a function of in-plane magnetic field, performed in the clean limit ($kT \gg \hbar/2\tau$), are compared with experiment in Fig. 2. Although the shapes of the curves are different, due perhaps to approximations made in the calculations, there is reasonable agreement between theory and experiment in this range of electron densities.
We point out, however, that agreement between theory and experiment breaks down at lower densities near the metal-insulator transition, where the magnetoresistance is substantially larger (up to several orders of magnitude instead of a factor 4).

We now extend these concepts to consider the effect of screening on the Hall resistance. As discussed above, the spin-up and spin-down electrons are expected to have different field-dependent mobilities in the presence of a parallel magnetic field. If two distinct types of carriers with different, field-dependent mobilities contribute to the conductivity, the Hall coefficient $R_H$ is no longer expected to be constant as a function of magnetic field $H$. For a two component ($↑, ↓$) Fermi system the Hall coefficient is given by

$$R_H = \frac{(\sigma_{↑μ↑} + \sigma_{↓μ↓})}{(\sigma_{↑} + \sigma_{↓})^2},$$

where $\sigma_{↑↓}$ and $μ_{↑↓}$ are the conductivities and mobilities, respectively, of the spin-up and spin-down electrons.

Dolgopolov and Gold [9] calculated the average scattering probabilities, $1/τ^{↑↓}$, corresponding to spin-up and spin-down electrons in a parallel magnetic field. The longitudinal conductivity is the sum of contributions of spin-up and spin-down bands: $\sigma = \sigma^{↑} + \sigma^{↓} = n_s^{↑}μ_1^{↑} + n_s^{↓}μ_1^{↓}$, where $n_s^{↑↓} = n_s/(1 \pm \xi)$ is the density of the spin-up (spin-down) electrons and $ξ = H/H_{sat},(H < H_{sat})$ is the degree of spin polarization of the 2D system. Using their expression for the conductivity $σ(H)$ [10] to calculate the mobilities $μ^{↑↓}(H)$, the Hall coefficient $R_H$ was obtained as a function of parallel magnetic field $H$.

The calculated and measured Hall coefficients are shown as a function of parallel magnetic field in Fig. 2. The experimental curve represents data obtained for three different densities: 1.91, 2.47, and $2.75 \times 10^{11}$ cm$^{-2}$. A single-particle model which considers contributions from two bands of electrons that have different mobilities due to field-dependent screening predicts a Hall coefficient which varies substantially with magnetic field. In contrast, the measured Hall coefficient is constant to within the $1 - 3\%$ over the entire range of magnetic field up to well above $H_{sat}$. This implies that the spin-up and spin-down electrons have mobilities that remain comparable at all fields, including very high magnetic field where the wave vectors of the spin-up and spin-down electrons $K^{↑↓}_F$ differ considerably. Yet, we know that the average mobility of the electrons at $H = 0$ depends strongly on the electron density and, therefore, on the value of $K_F$, especially near the transition.

One way to resolve this paradox is to consider the possibility that there are strong interactions between spin-up and spin-down electrons. Frequent spin-flip scattering forces electrons to spend half the time in a spin-up state and half the time in a spin-down state. Because of conservation of total momentum of the electron system, the strong scattering between spin-up and spin-down bands is not the same as the scattering responsible for the measured transport of the dilute 2D system. The transport scattering prob-

![FIG. 2. (A) The longitudinal resistance normalized to its zero-field value as a function of normalized in-plane magnetic field, $H/H_{sat}$. The theoretical curve denotes the Dolgopolov-Gold theory for field-dependent screening of spin-up and spin-down carriers. The three experimental curves are for densities 1.91, 2.47, and $2.75 \times 10^{11}$ cm$^{-2}$; the saturation field $H_{sat}$ is chosen to match the theoretical results of Dolgopolov and Gold. (B) The Hall coefficient normalized to its zero-field value as a function of normalized in-plane magnetic field, $H/H_{sat}$. The experimental curve is compared with the Hall coefficient expected from field-dependent screening (see text).]
ability is nevertheless strongly affected by the spin-flip electron-electron scattering, which results in electrons in either spin state having the same average transport scattering probability. The strong mixing of spin-up and spin-down electron states may provide an explanation for our observation that the Hall coefficient is independent of in-plane magnetic field. We note that we recently suggested that interactions between spin-up and spin-down electrons are also responsible for the unusual phase relation between the first and the second harmonics of the Shubnikov-de Haas oscillations in magnetic fields below saturation, $H < H_{sat}$.

To summarize, measurements at low temperature of the longitudinal resistance and Hall coefficient as a function of in-plane magnetic field of silicon MOSFETs with high electron densities $n_s \geq 1.7 \times n_c$ (where $n_c$ is the critical density for the zero-field metal-insulator transition) are compared with expectations based on field-induced changes in the screening of spin-up and spin-down electrons. Fair agreement between theory and experiment is found for the longitudinal resistance at high densities well above $n_c$. However, the Hall coefficient obtained by considering a two-band model of spin-up and spin-down electrons with different, field-dependent mobilities predicts a sizable dependence on in-plane magnetic field which is in clear contradiction with experiment. We suggest that the Hall coefficient $R_H$ is independent of parallel magnetic field as a result of substantial mixing of spin-up and spin-down states due to strong electron-electron interactions in these dilute two-dimensional electron systems.

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