Plasmon assisted transport through disordered array of quantum wires

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Abstract. – Phononless plasmon assisted transport through a long disordered array of finite length quantum wires is investigated analytically. Generically strong electron plasmon interaction in quantum wires results in a qualitative change of the temperature dependence of thermally activated resistance in comparison to phonon assisted transport. At high temperatures, the thermally activated resistance is determined by the Luttinger liquid interaction parameter of the wires.

Hopping transport in a quasi-one-dimensional system formed by a parallel arrangement of conducting wires is of much relevance to a number of experimental setups, including quantum wire arrays in heterojunctions [1], carbon nanotube films [2], atomic wires on silicon surface [3], and stripe phases [4]. At finite length of constituent wires, such systems represent particular examples of granular arrays, where a one-dimensional wire plays the role of a grain. Considered as a granular array, the array of parallel quantum wires is rather peculiar because of the very long charge relaxation time in a one-dimensional wire. Due to this peculiarity, the theoretical description of thermally activated transport in arrays of long quantum wires requires taking into account the charge dynamics and treatment of the interactions beyond the capacitive model adopted in recent theoretical investigations of transport through disordered granular arrays [5].

In this letter we show that charge-density fluctuations (plasmons) in the array can act as the agent promoting thermally activated transport, thus providing the possibility for phononless inelastic transport. As the result of generically strong plasmon-electron coupling in a quantum wire, the features of plasmon and phonon assisted transport are qualitatively different. We provide a qualitative explanation of plasmon assisted transport, identify the transport regimes, where the features of plasmon and phonon assisted transport are either similar or substantially different, and derive analytic expressions for the temperature dependence of the thermally activated resistance for a special model of a strongly correlated disordered array of quantum wires.

The model we formulate below is special, because it combines two seemingly incompatible features: i) it is strongly disordered for single electron transport; ii) it is much weaker disordered for propagation of plasmons.
Consider a one-dimensional array of parallel identical quantum wires of length $L$ and diameter $a$ placed regularly with the interwire distance $d$, $L \gg d \gg a$ (see Fig. 1). We investigate transport in the direction perpendicular to the wires. The spectrum of low-energy plasmons in a single isolated wire is equidistant with energies $E_{i,n} = \frac{\pi u_0}{L} n$, where $L$ is the length of the wire, and $v_0$ is the plasmon velocity along the wire $i$. For identical wires at regular positions, the intra- and interwire interactions between the charge density fluctuations do not change along the array. Then each localized plasmon level broadens into a plasmon band with truly continuous spectrum, quite analogously to the formation of electronic bands in the tight binding model. The role of the hopping in the tight-binding model for plasmons is played by the matrix element of the charge-density interactions between the neighbor wires. The formation of plasmon bands is reflected by the dependence of the plasmon velocity along each wire on the wave vector $p$ along the array (that is, perpendicular to the wires) $v_0 \rightarrow u(p)$. The particular form of plasmon dispersion depends on details of the interwire interactions, yet the function $u(p)$ should be periodic with a period of one Brillouin zone. That is why we choose the specific form

$$u_p = v_0 + v_1 \cos(\pi p), \quad -1 < p \leq 1.$$  

(1)

The chosen form of dispersion can be considered as the first two terms of the Fourier expansion of some general dispersion law. The plasmon energy within a band centered around the level $n$ is given by $\epsilon_n(p) = \frac{\pi u_0}{L} n$.

Furthermore, if the potential energy barriers between the wires are high, the electronic bands for the motion along the array (in $Y$-direction in Fig. 1) are very narrow with the width $t \approx \frac{h^2 d^3}{2 m_e (a+1)} e^{-\lambda d}$, where $\lambda$ describes the decay of the single particle wave function inside the barrier, $m_e$ is the effective mass of an electron. Now let us introduce the disorder as a random height of the energy barriers between the neighbor wires. Such a disorder induces fluctuations of the decay parameter $\lambda$ thus rendering $t$ random. As the result, the single particle wave functions become localized. At $\langle \delta \lambda^2 \rangle \approx \frac{\pi^2}{8 \pi^2} e^{-2\lambda d}$, the localization length of the single particle wave function is of order $d$, that is, an electron is localized within a single wire.

The resistance of a long disordered one-dimensional array is determined by so-called breaks, the junctions between two neighboring wires with exponentially high resistance [7]. Let us denote the energy cost to transfer an electron over the break as $E_a$. To facilitate the transport over the break, the energy $E_a$ should be borrowed by absorption of a bosonic excitation. Now we show that due to the transfer of energy between charge density excitations in different wires, a bosonic bath with continuous energy spectrum can be created at the break. Then any energy deficit $E_a$ can be matched by absorption of a plasmonic excitation, which constitutes the microscopic mechanism of plasmon assisted transport [8]. For two isolated wires forming the break, the matching condition $E_{i,n} = E_a$ cannot be satisfied for arbitrary $E_a$ because of discreteness of $E_{i,n}$. However, due to the charge-density interwire interaction $H_{int} = \sum_i \sum_{n,n'} V_{i,i+1}^{nn'} \hat{p}_i(n) \hat{p}_{i+1}(n')$, where $n,n'$ mark the plasmon modes, the energy can be transferred between excitations localized in different wires. Treating the interwire interaction perturbatively, we deduce, that the transfer of plasmon energy from the nearest neighbor of the wire $i$ leads to splitting of the plasmon level $E_{i,n}$ into $M$ sublevels with energies $\epsilon'_{i,n} = E_{i,n} - \frac{|V_{i,i+1}^{nn'}|^2}{E_{i,i+1} - E_{i,n}}$, where $l = 1, M$ runs over the plasmon levels of the wire $i+1$, and $M$ denotes the number of levels that have a nonvanishing interaction strength with the level $(i,n)$. Each split energy sublevel corresponds to a complex plasmon excitation, which is a superposition of plasmons localized in the two wires. Note that all the split sublevels are situated in the energy window $|V|^2 \delta_1^2$ around $E_{i,n}$, where $V$ is a typical value of the interwire interaction, and $\delta_1$ is the plasmon interlevel distance in a single wire, $|V| \ll \delta_1$. Now we take

$$\delta_1 \approx \frac{\pi}{v_0} \delta_{\lambda \lambda} \approx \frac{h^2 d^3}{2 m_e (a+1)} e^{-\lambda d} \approx \frac{h^2 d^3}{2 m_e (a+1)} e^{-\lambda d} \frac{\pi}{v_0} \frac{\lambda^2}{\delta_{\lambda \lambda}} \approx \frac{h^2 d^3}{2 m_e (a+1)} e^{-\lambda d} \frac{\pi}{v_0} \frac{\lambda^2}{\delta_{\lambda \lambda}}$$

This relation is a typical value of the interwire interaction.
into account that each energy level of the wire $i+1$ is in turn split due to the interaction with the next-nearest wire $i+2$. Repeating the previous calculation with $E_{i+1,l}$ substituted by the energy of a split level $\varepsilon_{i+1,l}$, we conclude that the energy interval $|V|^2/\delta_1$ around $E_{i,n}$ is now filled with $M^2$ levels. By recursion we obtain that the transfer of energy from $n$ neighbors of the break results in $M^n$ plasmon sublevels within the energy interval $|V|^2/\delta_1$ around $E_{i,n}$. Therefore, due to the transfer of energy from plasmons localized in distant wires to the break, a bath of bosonic excitations at the break is formed that allows to match the energy $E_a$ with the higher accuracy the more localized plasmons are involved in the creation of a bosonic excitation.

In the disordered array the number of wires that can supply energy to the break is limited by the plasmon localization length that can be evaluated as $\xi_p = u_g \tau_p$, where $\tau_p$ denotes the plasmon mean free time and $u_g$ is the plasmon group velocity. The mean free time $\tau_p$ is related to the fluctuations of the matrix element of the interwire charge density interactions 

$$\langle (\delta V_{i,i+1})^2 \rangle = \frac{\nu_0}{(2\pi)^2 \omega_p \nu_p d^4}$$

where $\nu_p = \frac{L}{\pi \nu_{1,1}}$ is the plasmon density of states in the middle of the band $n$, and $\omega$ is the plasmon frequency. Furthermore, the random height of interwire tunneling barriers affects the interwire charge density interactions $V_{i,i+1} \sim \epsilon_a^2 |\psi_i|^2 |\psi_{i+1}|^2 / d$ through the randomness of the wave functions $\psi_i$. Another source of randomness in the model is a random charging energy due to the random environment (local concentration of charged impurities) around each wire. Random energy barriers are directly felt only by the exponentially decaying tail of the single particle wave function within the barrier, whereas random fluctuations of the charge density interactions are mostly determined by the part of the single particle wave function within the barrier. That is the reason why the disorder that leads to the localization of single particle wave functions is felt only weakly by the charge density fluctuations. Using the normalization condition $|\psi_i|^2 (a + \int_0^\infty e^{-2\lambda x^2} dx) = 1$ one can relate the fluctuations of the decay factor $\lambda_i$ with the fluctuations of $|\psi_i|^2$ that in turn determines the fluctuations of the charge density interactions and the plasmon mean free time. One obtains

$$\tau_p = \frac{d(a\lambda + 1)^2 \nu_0^2}{16 \pi \omega_p \epsilon_a^2 (a\lambda)^2 \lambda_x^2}.$$  

In the regime, when electrons are localized on the length $d$, we obtain

$$l_p / d = \frac{\gamma_0 (a\lambda + 1)^4 \epsilon_a^2 \lambda_x d}{(2\omega \lambda_x d)^2}.$$  

For $\lambda_x \gg \log \left( \frac{a\lambda_x d}{\nu_0 (a\lambda + 1)^2} \right)$, the plasmon localization length greatly exceeds the single particle localization length. If the length of array is less than $l_p$, the plasmons propagate freely along the array, whereas the electrons are strongly localized. In that regime, the dispersion (1) can be used to describe the propagation of plasmons.

Treating the interwire charge density interaction perturbatively, we can write the transition rate with the absorption of a plasmon similarly to a transition with the absorption of a phonon using the Fermi golden rule [9]

$$\gamma \propto \int dp \sum_{n,m,k=0}^{\infty} \left| V_n(p) N_B(\varepsilon_n(p)) f (-E_m) \right| \left[ 1 - f (E_a + E_k) \right] \delta (\varepsilon_n(p) - E_a - E_m - E_k).$$

Here $V_n(p)$ is the strength of interwire charge density interaction for the plasmon mode $n$, $N_B(\varepsilon_n(p))$ is the occupation number of the plasmon mode, $f(E_m)$ denotes the Fermi distribution and describes the occupation of the $m$-th single-particle energy level in the wire, $E_m = \frac{\pi m^2}{2L}$. For narrow plasmon energy bands, the perturbative approach suggests that if the energy $E_a$ lies in the gap between the plasmon bands the hopping over the break is blocked. This suggestion turns out to be wrong because of a conceptual difference between the plasmon and phonon transport mechanisms. Whereas the phonons represent a bath of bosonic excitations that is independent of electrons, the plasmons are “made” of electrons themselves. Consequently, while the electron-phonon interaction can generally be treated perturbatively,
the perturbative treatment of plasmons is possible only under special conditions. The applicability of the perturbative treatment of plasmons is determined by the relation of two time scales: the characteristic time of plasmon dynamics \( t_p \) and the characteristic time of a single electronic hop \( t_h \). If the Coulomb interaction in a grain is well-screened or the plasmons are strongly localized, then \( t_p \) is the characteristic relaxation time of a plasmon excitation within a single grain. For \( t_p \ll t_h \) the plasmons can be neglected in transport. The description of interactions thus reduces to the capacitive model [5]. For the delocalized undamped plasmons, the time \( t_p \) is associated with the formation of an extended in space plasmonic excitation. In that case, \( t_p \ll t_h \) correspond to the regime of a strongly nonlinear coupling between plasmons and electrons, and the perturbative treatment of plasmons is incorrect. Plasmons in one-dimensional wires represent a profound example for that regime. For the model considered in this paper \( t_p \sim L/v_1 \), and at the break the condition \( t_p \ll t_h \) is always satisfied. As we show below, due to the strong electron-plasmon coupling, the nonlinear effects lead to the creation of plasmon complexes with energies covering the whole spectrum continuously, even though the plasmon bands initially are very narrow. This in turn leads to plasmon assisted transport with a temperature dependence that is qualitatively different from the case of phonon assisted transport. In the regime \( t_p \gg t_h \), the effective interaction time is limited by \( t_h \). Then the plasmon dynamics is essentially independent of the electron dynamics and, in the case of a continuous plasmon spectrum, the plasmon assisted transport is quite analogous to the phonon assisted one.

A unique feature of the chosen model is the applicability of the bosonized description that allows exact treatment of interactions and hence nonperturbative treatment of plasmons. Precisely, the plasmon dynamics is described by the action

\[
S = \int_{-1}^{1} dp \int_0^\beta d\tau \int_{-L/2}^{L/2} \int_{-\frac{K_p}{2}}^{\frac{K_p}{2}} \frac{d\chi}{2K_p} \left\{ \frac{1}{u_p} |\partial_x \Theta_p|^2 + u_p |\partial_x \Theta_p|^2 \right\},
\]

representing a finite size generalization of the sliding Luttinger liquid model [10]. The relation of the plasmon velocity \( u_p \) and the Luttinger liquid constant \( K_p \) with inter- and intrawire interactions has been calculated in [10]. A fermion annihilation operator in the wire \( n \), \( \Psi_n(x) \), is represented as

\[
\hat{\Psi}_n(x) \sim \hat{F}^R_n \exp \left[ -i \int_{-1}^{1} dp \phi^R_p(x) e^{-ipm} \right],
\]

where \( \chi = R, L \) denotes the chirality, \( \phi^R_p(x) \) is a chiral bosonic field, and \( \hat{F}^R_n \) is a Klein factor. The chiral field \( \phi^R_p(x) \) is in turn expressed through the field \( \Theta_p(x) \) and its dual \( \Phi_p(x) \),

\[
\phi^R_p(x) = (\Theta_p(x) \pm \Phi_p(x)) \sqrt{\pi}.
\]

The resistance of the array is calculated along the lines of Ref. [7]. Let us parameterize the tunneling matrix element between two wires in the form \( t_{i,i+1} = \exp(-|y_{i,i+1}|/d) \). The parameter \( y \) can be associated with an effective distance between the two wires. This effective distance is random, its distribution follows from the distribution of the heights of potential barriers. Since a break, being a junction with exponentially large resistance, is not shorted by other resistances connected in parallel, we can write the resistance of a break in the form \( R_1 = R_0 \exp[2|y_{i,i+1}|/d + f(E_a, T)] \). Here \( E_a \) denotes an addition energy to transfer an electron over the break. We remind that the disorder enters the model only as a random distribution of addition energies \( E_a \) and effective distances \( y_{i,i+1} \). The function \( f(E_a, T) \) accounts for the effect of thermally activated plasmons in the resistance of the break. According to Ref. [7], the probability density \( p(u) \) for the resistance \( R/R_0 = e^u \) is proportional to \( e^{-uA} \), where \( A \) is the area in the \((y, E_a)\) phase space that results in the resistance \( e^u \), and \( g \) is the linear density
of localized single particle states. The resistance is calculated as $R = R_0 l_y \int_0^\infty du \, e^{u - g A(u)}$, where $l_y$ is the length of the array. Therefore, in order to calculate the resistance of the array in the localized regime, we have to obtain an expression for the resistance of the break $R_1$. Since the break is not shorted by other resistances, we conclude $R_1 = 1/\sigma_1$, where $\sigma_1$ is the conductance of a break. Assume that the break is formed by a junction between the wires with numbers 0 and 1. We take the position of a pinhole connecting the two wires as $x$.

In the linear response approximation the current through the break $I_1$ is determined by the correlation function $[11]$

$$X(\tau) = |t_{01}|^2 \langle T_\tau \left( \Psi_0(x, \tau) \Psi_1^\dagger(x, \tau) \Psi_1(x, 0) \Psi_0^\dagger(x, 0) \right) \rangle$$

(5)

that characterizes the probability of a single hop over the break, $X_+ (\tau) = X (\tau > 0)$ and $X_- (\tau) = X (\tau < 0)$. Here $t_{01}$ is the tunneling matrix element. In the bosonized representation $[4]$, the correlation function $X(\tau)$ factorizes in the correlator of Klein factors and the correlator of bosonic exponents that we denote as $X_\delta (\tau)$. The time dependence of the Klein factors $F_n (\tau)$ is given by the ground state energy of the wire $n$ that includes the capacitive interaction between the wires. The Klein factors are proportional to $e^{-E_n \tau}$.

Furthermore, calculating the correlators of bosonic exponents, we cast the expression for the current into the form $I(eV) \propto \int_{-\infty}^\infty d\omega' J_0(eV + \omega') Z (-\omega')$. Here $J_0 (\omega) \equiv \delta(\omega - E_n) - \delta(\omega + E_n)$ defines the zero-temperature current, whereas the influence of thermally activated plasmons is contained in the factor $Z(\omega)$ that is given by the Fourier transform of

$$Z(t) = \exp \left[ - \left( \kappa_p \sum_{\sigma = \pm 1} \sum_{m = 1}^\infty \ln \left( 1 - e^{-\frac{\pi u}{L} \left( m^{\beta + i\sigma} \right) + \frac{n \pi}{L} \right) \right) \right]$$

(6)

where $\kappa_p = K_p + 1/K_p$, and the average over the plasmon wave vector $p$ is defined by $\langle \cdot \rangle_p \equiv \int_0^1 (1 - 2 \cos(\pi p) + \cos(2\pi p)) dp$. Further we assume the coupling constant to be $p$-independent, $\kappa_p = \kappa$ and use the simplified dispersion law $[1]$. Note, that in approximate evaluations it is much more important to keep the $p$-dependence of the velocity $u_p$ that reflects the formation of plasmon bands than the $p$-dependence of the coupling constant $\kappa_p$. To the lowest order in $p$-dependent terms, the latter just leads to the averaging of the single Luttinger liquid result over the coupling constant. The basic average to be used in subsequent calculations reads

$$\langle e^{-b u_p} \rangle_p = e^{-b v_0} \left[ I_0 (b v_1) + \left( 1 - \frac{1}{b v_1} \right) I_1 (b v_1) \right],$$

(7)

where $b = (m \beta \pm i t) \pi / L$, and $I_\nu (z)$ denotes the Bessel function of complex argument. For large times, $t$, (7) gives asymptotically

$$\langle e^{-\frac{\pi u}{\sqrt{2} |v_1|} (m \beta \pm i t)} \rangle_p \approx \frac{2L}{\pi^2 |v_1| (m \beta \pm i t)} e^{-\frac{\pi}{2} w (m \beta \pm i t)},$$

(8)

where $w = v_0 - v_1$. Despite being obtained for $v_1 < v_0$, $[6]$ is essentially nonperturbative in $v_1$. The relevant values of the transport time are restricted by the hopping time $\tau_h$. For $v_1 |m \beta \pm i n \tau_h| < 1$ the correlations giving rise to plasmon bands do not develop, and the short time expansion of (7) has to be used instead of (8). The latter is equivalent to the perturbative treatment of plasmons, leading to a result similar to the phonon mechanism of hopping.

At low temperatures, $T \ll \frac{e}{\pi v_1}$, the major contribution to $Z(t)$ in (6) is given by the term with $m = 1$. Leaving only that term, expanding the logarithms, and substituting (8) in (6)
we obtain
\[
Z(\omega) \approx \sum_{n,l=0}^{\infty} \frac{2\kappa^{1+n}}{n!l!} \left( \frac{2L}{\pi^2|v_1|} \right)^{\frac{1}{2}} e^{-\beta(\frac{\pi\omega}{\kappa} (n+l)+|\omega-\epsilon_{ln}|)} \left( |\omega-\epsilon_{ln}| \right)^{\frac{1}{2} - 1} \frac{\sin \left( \frac{\omega}{2} \right) \Gamma \left( 1 - \frac{1}{2} \right)}{(2\beta)^{\frac{1}{2} - 1}},
\]
where \(\epsilon_{ln}(\omega) = \frac{\pi\omega}{\kappa} (l-n)\), and \(\nu = l, n\) for \(\omega-\epsilon_{ln} \geq 0\) respectively. Each term in (9) describes a thermal excitation of a multiparticle plasmon complex with a continuous density of states. The plasmon complexes described mathematically by (9) form the bath of bosonic excitations that facilitate transport over the break. Higher values of \(m\) in (6) would correspond to excitations involving progressively more plasmon modes. The leading terms in (9) at low temperatures are given by \(l, n = 0, 1\). Those terms result in the leading low-temperature contribution to \(\sigma_1\) in the form
\[
\sigma_1 \approx \frac{e^2}{h} \kappa^2 L T \left( \frac{1}{\beta E_a - \frac{3}{2}} + 2|\beta E_a|^{-\frac{3}{2}} \right) e^{-\beta(E_a + \frac{\pi\omega}{\kappa})}.
\]
Further calculations closely follow Ref. [7] leading to the temperature dependence of the resistance
\[
R \approx \frac{h}{e^2 \pi^2 |v_1| l_y T^2} \exp \left[ \left( \frac{1}{2g_d T} \right)^{\frac{2\pi d}{LT}} \right].
\]
Since in the low-temperature regime the temperature broadening of discrete plasmon levels in a single wire is less than the interlevel separation, the finiteness of the wires essentially determines the physics of transport. In particular, the LL interaction parameter \(\kappa\) does not enter the temperature dependence of the resistance, except as a general prefactor in (11). For comparison, in the case of phonon-assisted transport in the same model, as well as by a perturbative treatment of the plasmon-electron interactions the preexponential factor in (11) goes like \(T^{1/2}\). The result (11) also differs from the thermally activated resistance of a single disordered wire obtained recently in Ref. [6].

At high temperatures, when \(T > \frac{\pi\omega}{\kappa} \), the temperature broadening exceeds the interlevel separation in a single wire. Then the plasmons in the array behave similarly to plasmons in an array of infinite length wires. The conductance of a single junction depends on the addition energy not exponentially but as a power law \(\sigma_1 \sim E_a \cdot \max (T, E_a)^{\kappa - 3}\). Therefore, the resistance of the array is not determined by the resistance of a single break, it is rather given by the average over the resistances of all junctions. The fact that the leading contribution to the conductance at high temperatures is given by the result for infinitely long wires implies that the coherence of the single particle motion is broken already by a single hop between the two neighbor wires. The latter justifies the averaging over all junctions in calculation of the resistance. The expression for the resistance can be written as a Drude formula with the interaction and temperature dependent mean free time \(\tau_f\). The expression for the mean free time can be organized as an expansion in powers of the small parameter \(e^{-\frac{\pi\omega}{\kappa}}\). At a typical charging energy \(E_a > T\), the leading term in the expression for the mean free time is temperature independent, \(\tau_f \propto 1/(E_a^{2-\kappa})\). For comparison, phonon assisted transport in that temperature regime still has a thermally activated character with the preexponential factor \(T^{-1/2}\) in (11). At temperatures even larger than the typical charging energy, \(E_a < T\), \(\tau_f\) exhibits the power-low temperature dependence typical for transport across a sliding Luttinger liquid, \(\tau_f \propto T^{3-\kappa}/(E_a)\) [10]. Therefore, at high temperatures, the mean free time is determined by the Luttinger liquid interaction parameter \(\kappa\). The calculation of the phonon assisted resistance in that regime using the Fermi golden rule leads to the Drude formula with the logarithmic temperature dependence of the mean free time \(\tau_f \propto \ln T\).
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Fig. 1 – Geometry of the model. The arrow shows the direction of the current.

In conclusion, we demonstrated the possibility of plasmon assisted inelastic transport in the particular case of a disordered granular array, an array of parallel quantum wires. Due to large charge relaxation time in a wire, the plasmon-electron interaction has to be treated nonperturbatively. In the result, thermally activated resistance has qualitatively different temperature dependence for plasmon assisted transport as compared to phonon assisted transport. Despite the specific quasi one-dimensional geometry of the grains in this model, the described phenomenon is believed to be of general importance for granular arrays with delocalized or weakly localized plasmons.

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