Vibration effect on the Soret-induced convection of ternary mixture in a rectangular cavity heated from below

T P Lyubimova¹² and N A Zubova¹
¹ Institute of Continuous Media Mechanics UB RAS, 1, Acad. Korolev St., Perm, 614013, Russia
² Perm State National Research University, 15, Bukirev St., Perm, 614990, Russia

E-mail: lubimova@psu.ru

Abstract. This paper presents the results of numerical simulation of the Soret-induced convection of ternary mixture in the rectangular cavity elongated in horizontal direction in gravity field. The cavity has rigid impermeable boundaries. It is heated from the bellow and undergoes translational linearly polarized vibrations of finite amplitude and frequency in the horizontal direction. The problem is solved by finite difference method in the framework of full unsteady non-linear approach. The procedure of diagonalization of the molecular diffusion coefficient matrix is applied, allowing to eliminate cross-diffusion components in the equations and to reduce the number of the governing parameters. The calculations are performed for model ternary mixture with positive separation ratios of the components. The data on the vibration effect on temporal evolution of instantaneous and average fields and integral characteristics of the flow and heat and mass transfer at different levels of gravity are obtained.

1. Introduction

The Soret effect can make a significant effect on the convection in mixtures caused by the density inhomogeneity in the gravity field. The thermodiffusion properties of a mixture consisting of $n$ components are described by the vector of $n-1$ the elements called the separation ratios vector:

$$\psi = -\beta_\tau^{-1} B D^{-1} D_\tau,$$

where $B$ is the diagonal matrix of the coefficients of the concentration dependence on the density, $\beta_\tau$ is the thermal expansion coefficient, $D$ is the matrix of the molecular diffusion coefficients, $D_\tau$ is the vector of thermal diffusion coefficients. When subjected to the temperature gradient, the molecules of the components with negative separation ratio move to more heated domain of the cavity and the molecules of the components with a positive separation ratio move to the cold domain.

Linear stability analysis carried out for convection of ternary mixtures with fixed positive separation ratio of one component in horizontal layer and square cavity in static gravity field has shown the existence of monotonic instability at the positive separation ratio of other component and the oscillatory instability at the negative separation ratio in the case of heating from the bellow. The monotonic instability occurs at the negative separation ratio of the other component in the case of heating from above [1, 2].

In the presence of vibrations, convection may exist even in zero gravity conditions. In gravity field vibrational and gravitational mechanisms of convection interact, intensifying or reducing each other.
For example, the longitudinal high-frequency vibrations make destabilizing effect on the horizontal layer of binary mixtures with positive separation ratios and stabilizing effect on binary mixtures with negative separation ratios [3]. Vibrations applied in the direction perpendicular to the layer play stabilizing role independently of the separation ratio sign [4]. In square cavity at side heating in conditions of low gravity the residual flow can be damped by applying an appropriate vibration intensity in the appropriate direction [5]. Direct numerical simulation of convection in the water-isopropanol mixtures in cubic cavity subjected to the gravity and vibrations perpendicular to the temperature gradient is carried out in [6].

Linear stability analysis [7] for horizontal layer of ternary liquid mixtures with the negative separation ratios of the first component has shown that longitudinal low-amplitude high-frequency vibrations reduce the area of the convectionless state stability in the case of heating from below and to extend stability area in the case of heating from above. Numerical simulations [8] have demonstrated that for the ternary mixture in the rectangular cavities in zero gravity conditions at the increasing the vibration intensity there is a transformation of the average flow structure associated with the instability of quasi-equilibrium state in the central part of the cavity.

The present paper presents the results of the direct numerical simulation of the Soret-induced convection of ternary mixture in the rectangular cavity subjected to the heating from below, horizontal vibrations and gravity.

2. Formulation of the problem

Let us consider Soret-induced convection of ternary mixture in the horizontally elongated rectangular cavity with the rigid impermeable boundaries. The cavity performs the longitudinal harmonic vibration with amplitude \( a \) and angular frequency \( \omega \). The vertical boundaryless of the cavities are adiabatic and on the horizontal boundaries the constant different temperatures corresponding to the heating from below are maintained.

Unsteady non-linear equations of the Soret-induced convection of ternary in the Boussinesq approximation in the reference frame of oscillating cavity are [9]:

\[
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \nabla \mathbf{u} = -\nabla p + \nabla^2 \mathbf{u} + \frac{\text{Ra}}{\text{Pr}} (T + \mathbf{I} \cdot \mathbf{C}) \mathbf{\hat{y}} + \frac{\text{Ra}}{\text{Pr}} \cos(\Omega t) (T + \mathbf{I} \cdot \mathbf{C}) \mathbf{\hat{n}} ,
\]

\[
\frac{\partial \mathbf{C}}{\partial t} + \mathbf{u} \nabla \mathbf{C} = \frac{1}{\text{SC}} \left( \nabla^2 \mathbf{C} - \mathbf{\psi} \nabla^2 T \right) ,
\]

\[
\frac{\partial T}{\partial t} + \mathbf{u} \nabla T = \frac{1}{\text{Pr}} \nabla^2 T ,
\]

\[
\nabla \cdot \mathbf{u} = 0 .
\]

Here \( \mathbf{u} \) is the velocity vector, \( p \) is the pressure, \( \mathbf{C} = (C_1, \ldots, C_n) \) is the transpose vector of concentrations, \( \mathbf{I} = (1, \ldots, 1) \) is the unit vector, \( \mathbf{\hat{y}} \) is the unit vertical vector and \( \mathbf{\hat{n}} \) is the unit vector in the direction of vibrations.

The equations are written in the dimensionless form using the following quantities as the scales: the height of the cavity \( H \) for the length, \( \nu/H \) for the velocity, \( H^2/\nu \) for the time, \( \rho_0 \nu/H \) for the pressure, temperature difference between the lower and upper boundaries \( \Delta T \) for the temperature, and \( \beta \Delta T \mathbf{B}^{-1} \) for the vector of concentrations. Here \( \nu \) is the viscosity of the mixture and \( \rho_0 \) is the initial mixture density. Equations (2)-(5) contain the following dimensionless parameters: \( \text{Pr} = \nu/\chi \) is the Prandtl number (\( \chi \) is the thermal diffusivity), \( \mathbf{\psi} \) is the separation ratios vector, \( \text{SC} = \nu^2 \mathbf{B}^{-1} \) is the matrix of Schmidt numbers, \( \text{Ra} = g \beta \Delta T H^3/(\nu \chi) \) is the Rayleigh number, \( \Omega = \omega H^2/\nu \) is the dimensionless vibration frequency, \( \text{Ra}_v = a \omega^2 \beta \Delta T H^3/(\nu \chi) \) is the vibrational analogue of the
Rayleigh number (equals to the usual Rayleigh number multiplied by the ratio of vibrational acceleration to gravity acceleration and related to the vibrational Rayleigh number used in the average approach as \( Ra_v = \Omega \left( 2 \Pr Ra_w \right)^{1/2} \)).

The boundary conditions are:

\[
x = 0, L: \bar{u} = 0, \frac{\partial T}{\partial x} = \frac{\partial C}{\partial x} = 0, \tag{6}
\]

\[
y = 0, H: \bar{u} = 0, T = \pm 1/2, \frac{\partial}{\partial y} \left( C - \varphi T \right) = 0. \tag{7}
\]

3. Numerical method
The problem (2)-(7) was rewritten in terms of stream function and vorticity and solved numerically by finite difference method. The spatial derivatives were approximated by central differences. Unsteady equations were solved using explicit finite difference scheme with a constant time step \( h^2/8 \), where \( h \) is the constant spatial step (the grid \( 200 \times 40 \) was used in main calculations). The Poisson equation for the stream function was solved by successive over-relaxation method. The vorticity values at the boundaries of the cavity were calculated using the Thom formula [10].

For ternary mixtures the matrix of molecular diffusion coefficients \( D \) consists of four elements two nondiagonal elements of which are responsible for the cross-diffusion effect. This leads to the appearance of additional terms in the equation and complicates the calculations. The diagonalization procedure of matrix \( D \) allows to eliminate the cross diffusion terms in the equations and to decrease the number of governing parameters, in this case the equations structure does not change [11]. Such transformation applied to the problem (2)-(7) in dimensionless form can be written as:

\[
C = BV (BQ)^{-1} \tilde{C} , \quad \varphi = BV (BQ)^{-1} \tilde{\varphi},
\]

where \( V \) is the matrix whose columns are the eigenvectors \( v_j = (v_{i1}, \ldots, v_{in})^T \) of the matrix \( D \), \( Q = \text{diag} \{ q_1, \ldots, q_n \} \), \( q_i = \beta_i \sum_{j=1}^{n-1} \beta_j v_{ij} \). After the transformation for dimensionless problem the matrix of the Schmidt numbers becomes diagonal.

The calculations were carried out for rectangular cavities with aspect ratio 5:1. The initial conditions corresponded to the uniform vertical gradients of temperature and solutes concentrations:

\[
T = 1/2 - y \quad \text{and} \quad C = \varphi \left( 1/2 - y \right).
\]

The calculations were carried out for ternary mixtures with the following dimensionless parameters: the Prandtl number \( \Pr = 10 \), the Schmidt numbers \( Sc_1 = 100 \) and \( Sc_2 = 1000 \), separation ratios \( \varphi_1 = 0.3 \) and \( \varphi_2 = 0.1 \). The chosen parameters are typical for the liquid mixtures. The dimensionless frequency of vibrations was fixed \( \Omega = 445 \). For the cavity of 1 cm height filled by ternary liquid mixture with the viscosity \( \nu \times 10^{-6} \text{ m}^2/\text{s} \) this value corresponds to the dimensional vibration frequency approximately equal to 1 Hz. The Rayleigh number was varied in the range \( Ra = 10^2 - 10^4 \) and the vibrational Rayleigh number in the range \( Ra_v = 10^7 - 2 \times 10^8 \).

4. Results

4.1. Soret-induced convection in gravity field in the absence of vibrations
As known [12], in the case of single-component fluid in the horizontal layer heated from below with rigid perfectly conductive boundaries the crisis of conductive state occurs at the Rayleigh number.
equal to $Ra \approx 1.7 \cdot 10^3$, the wave number of the most dangerous perturbations is approximately equal to 3.1. The calculations carried out in the present work for ternary mixture with the parameters specified above, in section 3, in the absence of vibrations, have shown that in this case the onset of convection takes place at $Ra \approx 2.5 \cdot 10^3$ and the spatial period of the emerging convective structures is $\lambda \approx 2.5$ (the corresponding wave number $k \approx 2.5$), (see fig. 1a, $Ra \approx 3 \cdot 10^3$).

![Stream function](image1)

![Stream function](image2)

![Stream function](image3)

![Stream function](image4)

**Figure 1.** Streamlines of flow and solute concentration fields in the absence of vibration ($Ra_v = 0$): (a) $Ra = 3 \cdot 10^2$, (b) $Ra = 1 \cdot 10^3$, (c) $Ra = 2 \cdot 10^3$, (d) $Ra = 5 \cdot 10^3$.

With the increase of the Rayleigh number the spatial period of the convective structures decreases (see Fig. 1b, for $Ra = 1 \cdot 10^3$, where $\lambda \approx 1.7$ ($k \approx 3.8$)). When the Rayleigh number reaches the value $Ra \approx 1.7 \cdot 10^3$, the thermogravitational mechanism of instability takes the floor and the spatial period of the convective structures increases to $\lambda \approx 2.0$ ($k \approx 3.1$) (Fig. 1b), which is accompanied by the sharp increase in the intensity of the flow and heat and mass transfer and the change of the inclination of the curve $Nu/L(Ra)$ (Fig. 2). With further increase of the Rayleigh number the spatial period again decreases (Fig. 1d).

![Figure 2](image5)

**Figure 2.** Dimensionless heat flux per unit length versus Rayleigh number in the absence of vibrations ($Ra_v = 0$).

### 4.2. Vibrational convection of the mixture in the absence of gravity

Figure 3 shows the structure of the average flow and solute concentration fields under vibrations in the absence of gravity. The average values were obtained by averaging the instantaneous values over the vibration period. For small values of the vibrational Rayleigh number (Fig. 3a, $Ra_v = 1 \cdot 10^3$) the
average flow in the form of the four vortexes located in pairs near the sidewalls of the cavity is generated and in the central part of the cavity the flow is nearly absent, the flow intensity is rather weak and the deformation of the solute concentration fields by the convective flow is not visible. When the vibrational Rayleigh number approaches $5 \cdot 10^4$, the qualitative transformation of the average flow structure occurs due to the crisis of the quasi-equilibrium state in the central part of the cavity. The new regime corresponds to the multi-vortex steady average flow (Fig. 3b, $Ra_v = 5 \cdot 10^4$), the spatial period of the vibrational convective structures is approximately equal to $2.0$ ($k \approx 3.1$). This flow transformation is accompanied by a sharp increase in the intensity of the heat transfer (Fig. 4).

![Streamlines of average flow and solute concentration fields in zero gravity conditions](image1)

**Figure 3.** Streamlines of average flow and solute concentration fields in zero gravity conditions ($Ra = 0$): (a) – $Ra_v = 1 \cdot 10^3$, (b) – $Ra_v = 5 \cdot 10^4$, (c) – $Ra_v = 1 \cdot 10^5$.

![Dimensionless heat flux per unit length versus vibrational Rayleigh number in zero gravity conditions](image2)

**Figure 4.** Dimensionless heat flux per unit length versus vibrational Rayleigh number in zero gravity conditions ($Ra = 0$).

4.3. The effect of vibration on Soret-induced convection in gravity field

Figures 5, 6 present the average fields of stream function and solute concentrations for steady convective regimes in the presence of gravity and vibrations. As one can see, in the range of vibrational Rayleigh numbers $3 \cdot 10^5 - 1 \cdot 10^6$ the spatial period of steady average flow is constant and equal to approximately $2.5$ ($k \approx 2.5$) (fig. 5a-c, fig. 6a-c). Further growth of the vibration intensity leads to decrease of the spatial period (fig. 5d, fig. 6d).
Figure 5. Streamlines of average flow and solute concentration fields at $Ra = 3\times 10^3$: (a) $Ra_v = 1\times 10^3$, (b) $Ra_v = 5\times 10^3$, (c) $Ra_v = 10^5$, (d) $Ra_v = 2\times 10^5$.

Figure 6. Streamlines of average flow and solute concentration fields at $Ra = 5\times 10^3$: (a) $Ra_v = 1\times 10^3$, (b) $Ra_v = 5\times 10^3$, (c) $Ra_v = 10^5$, (d) $Ra_v = 2\times 10^5$.

Figure 7. Dimensionless heat flux per unit length versus Rayleigh number at different vibrational Rayleigh numbers.

The dimensionless heat flux per unit length through the lower boundary of cavity monotonously increases with the increase of vibration intensity (fig. 7). At low vibration intensity (fig. 7, curve 1), as in absence of vibrations (see section 4.1), at $Ra = Ra^* \approx 1\times 10^7$ the sharp increase of the flow intensity and heat and mass transfer related to the activation of thermogravitational mechanism of instability is observed; it is accompanied by the sharp change of inclination of the curve $Nu/L(Ra)$. In this case the
destabilizing effect of vibrations leads to the fact that this change occurs at lower Rayleigh number than in the absence of vibrations (at \( \text{Ra}_v = 10^3 \) the critical Rayleigh number for the onset of thermogravitational convection in horizontal layer of single-component fluid is approximately 907). The increase of \( \text{Ra}_v \) leads to the further decrease \( \text{Ra}^* \) (Fig. 7, curve 2). Further growth of the vibration intensity leads to a smoothing of curve \( \text{Nu}/L(\text{Ra}) \) (the change of the curve \( \text{Nu}/\text{Ra} \) inclination associated with the activation of thermogravitational mechanism becomes less pronounced (fig. 7, curves 3, 4)).

5. Conclusions
The horizontal vibration effect on the Soret-induced convection of ternary mixture with positive separation ratios of both solutes in horizontally elongated rectangular cavity with rigid boundaries heated from below is studied numerically. It is found that in the absence of vibrations in the cavity the multi-vortexes flow with a spatial period decreasing with increasing gravity (Rayleigh number) occurs in a threshold manner. When the Rayleigh number reaches the value which corresponds to the onset of thermogravitational convection, the flow transformation occurs which is accompanied by the sharp increase in intensity of flow and change the inclination of the Nusselt number dependence on the Rayleigh number curve. Vibrations make destabilizing effect, as a result, the Rayleigh number, where the sharp increase in the intensity of the convective flow of ternary mixture is observed, decreases.

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