Random transition between two temperatures profiles in magnetized plasma

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Abstract. A set of Langevin equations are used to describe the transport of the guiding centers of charged particles perpendicularly to the main magnetic field and the stochastic transition between the two temperature states is described by a randomly interrupted noise. Using this model, the variation of the running diffusion coefficients in the radial direction is analyzed for two cases of temperature profiles with applications to fusion plasma.

1. Introduction
The Langevin equations was used before to study the temporal variation of the diffusion coefficients in one temperature plasma and to analyze the regime of diffusion of particles in fluctuating magnetic field, see e.g. [1]. Here we aim to study the spatial variation in perpendicular direction of the equilibrium magnetic field in random transition between two temperature state of the plasma using a randomly interrupted noise [2]. We give here two examples when the electron temperature presents transition between two temperature values: in pre-ELM-phases, see [3], and in pellet fuelling of Ohmic and Lower Hybrid - driven discharges in Tore Supra, see [4]. We can find many examples of random transitions between two given states both in hot plasmas and in cold plasmas where this model can be used.

2. Characteristics of the model
The characteristics of the model, sumarized here, are given in detail in [5].

The transport of the guiding centers of charged particles is described by using the following Langevin equations:

\[
\begin{align*}
\frac{dx}{dt} (t) &= \beta b [x, z] v_z (t) \\
\frac{dz}{dt} (t) &= v_z (t) \\
\frac{dv_z}{dt} (t) &= -\nu z v_z (t) + \alpha z (t)
\end{align*}
\]

(1)

(2)

(3)
The stochastic process of jumps between two given temperatures at random times is describing by using a randomly interrupted noise [2]

\[ \alpha_z(t) = \frac{1}{b-a} [b - \eta(t)] \alpha_{za}(t) + \frac{1}{a-b} [a - \eta(t)] \alpha_{zb}(t) \tag{4} \]

where \(a\) and \(b\) refer to the temperature \(T_a\) (low) and \(T_b\) (high), respectively. With the stationary solution for the probability transitions we obtain

\[ \langle \eta(t) \rangle = 0; \quad \langle \eta(t) \eta(u) \rangle = a^2 \left( \frac{1}{\lambda \tau_0} - 1 \right) \exp \left[ -\frac{|t-u|}{\tau_0} \right] \tag{5} \]

The components \(\alpha_{za}(t)\) and \(\alpha_{zb}(t)\) verify the properties, e.g. [1], \(\langle \alpha_{zi}(t) \rangle = 0, \langle \eta(t) \alpha_{zi}(t) \rangle = 0\) and

\[ \langle \alpha_{zi}(t) \alpha_{zj}(u) \rangle = 2 \delta_{ij} v_{Ti}^2 \nu_i \delta(t-u) \tag{6} \]

where \(i, j = a, b\) and \(v_{Ti}\), \(\nu_i\) are respective the thermic velocity and collision frequency at temperature \(T_i\). Then \(\alpha_z(t)\) is such that \(\langle \alpha_z(t) \rangle = 0\) and

\[ \langle \alpha_z(t) \alpha_z(u) \rangle = 2 \left\{ (1 - \lambda \tau_0) \nu_{za} v_{Ti}^2 + \lambda \tau_0 \nu_{zb} v_{Tb}^2 \right\} \delta(t-u) \tag{7} \]

where \(\lambda \tau_0\) is the weighting factor of collisions of kind \(b\) and \(1 - \lambda \tau_0\) for the collisions of kind \(a\).

2.1. Model for the magnetic field

The magnetic field is stochastic

\[ \vec{B} = B_0 \left[ \vec{e}_z + \beta b_x(z) \vec{e}_x + \beta b_y(z) \vec{e}_y \right] \tag{8} \]

Stochastic process for the components \(b_x, b_y\) is described by the equation of second order correlations:

\[ \left\langle \hat{b}_m(k) \hat{b}_n(k') \right\rangle = \vec{B}_\parallel(k) \delta(k + k') \delta_{mn} \tag{9} \]

where the spectral density of the magnetic field fluctuations is

\[ \vec{B}_\parallel(k) = \lambda_\parallel (2\pi)^{-1/2} \exp \left( -\frac{1}{2} \lambda_\parallel^2 k^2 \right) \tag{10} \]

2.2. Second order correlation of parallel velocity

With \(f = T_b/T_a\) and \(f > 1\) it result that \(v_{Ta}^2 = f v_{Tb}^2\). The relation between collision frequencies in the hot plasma limit read as, e.g. [6], \(\nu_{zb} = f^{-3/2} \nu_{za}\) in the limit approximation \(n_a = n_b\). The stationary correlation (in the limit \(t \rightarrow \infty\)) is

\[ \langle v_z(t) v_z(t + \tau) \rangle = \frac{1}{2} v_{Ta}^2 \exp(-\nu_{za} |\tau|) C(\tau; \lambda \tau_0, f) \tag{11} \]

where

\[ C(\tau; \lambda \tau_0, f) = \frac{1 - \lambda \tau_0 \left( 1 - f^{-1/2} \right)}{1 - \lambda \tau_0 \left( 1 - f^{-3/2} \right)} \exp \left[ \lambda \tau_0 \left( 1 - f^{-3/2} \right) \nu_{za} |\tau| \right] \tag{12} \]
2.3. Perpendicular transport

The perpendicular transport is given by the mean square displacement,

\[ \langle (\delta x_m(\tau))^2 \rangle = \int_0^\tau d\tau_1 \int_0^\tau d\tau_2 \langle b_m[z(\tau_1)] v_z(\tau_1) b_m[z(\tau_2)] v_z(\tau_2) \rangle \]  

(13)

We assume that the magnetic field fluctuations and the velocity fluctuations are statistically independent: \( \langle b_m(z) v_z(t) \rangle_b v_z = 0 \). The parallel collisional diffusion coefficient \( \chi_\parallel \) and the anomalous perpendicular running diffusion coefficient \( D_\infty(t) \) in our case read as, see [5],

\[ \chi_\parallel = \frac{\nu_T}{2v_\parallel} \frac{1 - \lambda_\parallel (1 - f^{-1/2})}{[1 - \lambda_\parallel (1 - f^{-3/2})]^2}, \quad D_\infty(t) = \chi_\parallel \beta^2 \frac{\phi(\nu_\parallel t)}{[1 + 2\theta_\parallel \mu(\nu_\parallel t)]^2} \]  

(14)

where \( \mu(\theta) = \theta - \phi(\theta), \phi(\theta) = 1 - \exp(-|\theta|), \theta = \nu_\parallel \tau \).

3. Discussions and conclusions

We apply the model in the following two cases:

Case 1: We consider the electron temperature profiles (see figure 1):

\[
T_a = T_0 \exp[-3 \cdot \tanh[7(x-0.8)]] \\
T_b = T_0 \exp[-3 \cdot \tanh[7.8(x-0.82)]]
\]  

(15) (16)

Figure 1. Temperature profiles: \( T_a \) given in (15) - (dashed line) and \( T_b \) given in (16) - (full line).

The difference between the two temperatures has an amplitude of about 1.5 keV, - see figure 2- corresponding to the experimental observations, e.g. [3].

Case 2: We consider the following temperature profiles (see figure 3):

\[
\frac{T_a}{T_0} = \exp[-3 \cdot \tanh[3(r/a - 0.84)]] \\
\frac{T_b}{T_0} = \exp[-3 \cdot \tanh[3.5(r/a - 0.8)]] \\
+ 1.5 \exp[-40 \cdot (r/a - 0.8)^2] + 0.5 \exp[-480 \cdot (r/a - 0.9)^2]
\]  

(17) (18)

with \( T_b - T_a \) plotted in figure 4. We will draw the graph for the dimensionless running diffusion coefficient \( D(\theta) = \left( \chi_{\parallel,a} \beta^2 \right)^{-1} D_\infty(\theta / \chi_{\parallel,a}) \) for \( \chi_{\parallel,a} = 0.01 \) which corresponds to the strong collisionality regime.

In the case 1, the running diffusion coefficient (see figure 5) is maximum for about \( r \approx 0.8a \) corresponding to the maximum for \( T_b - T_a \) at \( r \approx 0.7a \). In the case 2, the running diffusion...
Figure 3. Temperature profiles: $T_a$ given in (17) - (dashed line) and $T_b$ given in (18) - (full line).

Figure 4. Profile of $T_b - T_a$ in the case 2.

Figure 5. Dimensionless running diffusion coefficient $D$ for $\chi_{||,a} = 0.01$ with $\lambda \tau_0 = 0.2$ (dashed line) and $\lambda \tau_0 = 0.8$ (full line).

Figure 6. The dimensionless running diffusion coefficient $D$ for $\chi_{||,a} = 0.01$ (corresponding to strong collisionality) is plotted for $\lambda \tau_0 = 0.8$.

coefficient (see figure 6) has one maximum value for about $r \approx 0.9a$ and $T_b - T_a$ has two maximum values, for $r \approx 0.3a$ and for $r \approx 0.85a$.

From here we conclude that the running diffusion coefficient is less sensible to the difference between the two temperatures in the central region than the difference between temperatures in the plasma edge region.

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