The gravitational-wave event GW170817 and the near-simultaneous corresponding gamma-ray burst (GRB 170817 A) falsify modified gravity theories in which the gravitational geometry differs non-conformally from physical geometry. Thus, the observations of this event definitively rule out theories, such as TeVeS, a suggested relativistic extension of Milgrom’s modified Newtonian dynamics (MOND), that predict a significantly different Shapiro delay for electromagnetic and gravitational radiation. While not falsifying MOND per se, GW170817 severely constrains relativistic extensions of MOND to theories that do not rely on additional matter-coupling fields but rather upon modified field equations for one universal gravitational and physical metric. Here I mention a simple preferred-frame theory as an example.
Modified Newtonian dynamics (MOND) is an alternative to dark matter proposed by Milgrom 35 years ago [1]. The basic idea is that the modification of dynamics or gravity occurs below a critical acceleration, $a_0 \approx cH_0/6$, that appears to have a cosmological significance. In its original manifestation, MOND was essentially an algorithm permitting the calculation of the effective gravitational acceleration in an astronomical object from the observed distribution of baryonic matter. And it works on the scale of galaxies as judged by the successful prediction of the observed form and amplitude of galaxy rotation curves with, in a number of cases, no free parameters, apart from the new universal constant $a_0$ [2]. The fact that such an algorithm exists is challenging for non-interacting dark matter that does not naturally permit such a parameter-free prediction. However, the absence of a credible relativistic extension of MOND has frustrated considerations of cosmology and cosmogony; MOND in original form is clearly incomplete.

The first step toward a more fundamental theory was made by Bekenstein and Milgrom [3] who wrote down a non-relativistic Lagrangian from which a modified Poisson equation may be derived:

\[ \nabla \cdot \left[ \mu \left( \frac{\left| \nabla \Phi \right|}{a_0} \right) \nabla \Phi \right] = 4\pi G\rho. \tag{1} \]

Here $\mu$ is an unspecified function that must have the asymptotic limits $\mu(x) = 1$ where $x >> 1$ (the Newtonian limit) and $\mu(x) = x$ where $x << 1$ (the modified gravity limit).

A relativistic extension as a modified scalar-tensor theory is immediately suggested. To general relativity one adds a scalar field with a non-standard Lagrangian; i.e.,

\[ L = \frac{\sqrt{-g}}{16\pi} \left[ R - a_0^2 F(\xi) \right], \tag{2} \]

where $F$ is a function of $\xi$, the standard scalar field invariant in terms of $a_0$, i.e. $\xi = \Phi_0 \Phi^\alpha/\sqrt{a_0^2}$. In the modified Poisson equation $\mu = dF/d\xi$ and can be chosen such that the theory becomes a weakly coupled Brans-Dicke theory where $\nabla \Phi >> a_0$; i.e., $\mu \to \omega >> 10000$ with $\omega$ as the Brans-Dicke parameter. In the opposite limit, the force about a point mass $M$ becomes $\nabla \Phi = \sqrt{GMa_0/r}$ exceeding the Newton-Einstein force, $\nabla \phi$, at accelerations below $a_0$.

Consistency with the Equivalence Principle in its weak form (the universality of free fall) is typically guaranteed in scalar-tensor modifications by assuming that matter couples to the scalar field jointly with the gravitational or Einstein metric ($g_{\mu\nu}$) via a conformal factor to form a new metric – a “physical metric” $g_{\mu\nu} = f^2(\Phi)g_{\mu\nu}$. Particles follow geodesics of the physical metric, and as usual the paths of photons and other relativistic particles trace null geodesics. But it is trivial to see that null geodesics of the two conformally related metrics coincide. In other words, photons
are not affected by the scalar field, and this has an immediate observational consequence if an anomalous force is to replace dark matter. Photons, in contrast to the slowly moving constituents of an astronomical object, do not respond to the putative dark matter but only to the baryonic content of the object. Then in gravitational lensing of a distant source by an intervening massive structure (a cluster of galaxies, for example), the effective lensing mass should be significantly smaller than the traditional Newtonian dynamical mass – in stark contradiction to the observations.

The lensing contradiction led Bekenstein to consider a more general relation between physical and gravitational geometry: the so-called disformal transformation. Basically, a conformal transformation takes the geometry described a metric and stretches it in a manner that is isotropic but space-time dependent. A disformal transformation, on the other hand, picks a preferred direction for additional stretching. Because one wishes space to be isotropic, that direction is generally taken to be the time coordinate in some preferred frame – which is to say, at a fundamental level the theory violates Lorentz invariance of gravitational phenomena (not particle dynamics).

The mechanism for choosing this special direction is taken to be a normalized vector field, $A^\mu$, that is dynamical but points in the positive time direction of the preferred frame. This is the basis of Bekenstein’s Tensor-Vector-Scalar theory or TeVeS where the relation between the physical and gravitational metrics is given by

$$\tilde{g}_{\mu \nu} = \exp(-2\Phi) g_{\mu \nu} - 2 \sinh(2\Phi) A_{\mu} A_{\nu}. \tag{3}$$

For such a transformation null geodesics of the physical metric no longer coincide with null geodesics of the gravitational metric; the photons respond to the scalar field (or dark matter) and the relationship between the deflection of photons due to the total weak-field force is identical to that provided by general relativity with dark matter.

But while photons track null geodesics of the physical metric (and therefore “feel” the dark matter), gravitational waves follow null geodesics of the Einstein metric and are not affected by the scalar field or putative dark matter. Given a source of the gravitational radiation within a galaxy, such as an inspiraling and coalescing binary neutron star, the potential well from which the gravitational waves emerge is that due only to the baryonic content of the galaxy and thus significantly more shallow than that experienced by the emerging photons that see the baryonic galaxy plus scalar field (“dark halo”). Thus the Shapiro delay for photons from any such event is typically greater than that of the gravitational radiation by several hundred days.

As we all know such a burst of gravitational radiation, GW170817, has been detected by
LIGO/Virgo; the frequency and observed flux is consistent with a coalescing pair of binary neutron stars at a distance of about 40 Mpc. A corresponding gamma-ray burst was detected by the Fermi satellite in the direction of an early-type galaxy also at a distance of 40 Mpc; the gamma-ray burst followed the gravitational-radiation event by less than 2 seconds – inconsistent (by a factor of at least $10^7$) with that expected if the gravitational and electromagnetic radiation track null geodesics of two distinct disformally related metrics. The implication is that $\tilde{g}_{\mu\nu} \equiv g_{\mu\nu}$. Gravitational geometry corresponds to the physical geometry of space-time to high precision (as Einstein assumed). So while the observation does not falsify MOND as a non-relativistic theory, it seriously constrains relativistic extensions of MOND. The theory cannot rely upon adding an additional field that couples to matter disformally with the Einstein metric, as TeVeS, but rather upon a modification of the field equation itself and hence of the Einstein metric.

There are several existent theories that might fit this bill [8, 9], but here I focus on one in particular because of its simplicity: that is, a modification connected to a preferred timelike foliation of space-time provided by a dynamical scalar field, the “Khronon”, that does not directly couple to matter [10]. The unit normal to this foliation defines a vector field that may contribute various terms to the field Lagrangian – terms that are quadratic in first derivatives of the vector (an Einstein-Aether theory [11, 12]). But the unique aspect here is that only one of these – the term becoming the three-dimensional gravitational acceleration in the preferred frame – is included and assumed to vanish at high gravitational accelerations ($\gg a_0$) thus restoring general relativity in this limit [13, 14].

Following Blanchet and Marsat, the field Lagrangian of the theory is given by

$$L = \frac{\sqrt{-g}}{16\pi} [R - a_0^2 F(\chi)] \quad (4)$$

where matter couples directly to the gravitational metric as in general relativity. This appears to be identical to the Lagrangian in the relativistic version of the original Bekenstein-Milgrom theory (eq. 2), but in fact $\chi$ is formed by the ordinary three-dimensional acceleration in the preferred frame, i.e., $\chi = \phi^i \phi^j a_0^2$ where $\phi$ is the first Newtonian potential ($g_{00} = -1 - 2\phi$) in the weak-field expansion; it is not an additional scalar field but part of the gravitational metric. Therefore, the theory modifies the Einstein equation and the gravitational metric.

Taking the weak field to first order one finds that the two Newtonian potentials, $\phi$ and $\psi$ ($g_{ij} = \delta_{ij}(1 - 2\psi)$), are the same. This provides gravitational lensing equal to that of general relativity; there is no need to construct a physical metric disformally related to the gravitational metric. We also find the Bekenstein-Milgrom modified Poisson equation (eq. 1), but now with
\[ \mu = 1 + a_0^2 \frac{dF}{d\chi}. \] The form of this function can be chosen to achieve MOND phenomenology where \( \nabla \phi < a_0 \) but general relativity at high accelerations (including a cosmological constant on the natural order of \( a_0^{-2} \)).

This may or may not be the correct theory (several issues are swept under the carpet in the brief description given here), but it does suggest a qualitative scenario. Observationally there is clearly a preferred frame – the frame at rest with respect to the cosmic microwave background radiation. We do not detect any dynamical effects of this preferred frame locally because at the high accelerations prevailing in the Solar System, the world becomes Lorentz-invariant to high precision and described by general relativity. In the outer parts of galaxies, the transition between local general relativity and preferred-frame cosmology, the phenomenology of MOND appears – a phenomenology commonly ascribed to dark matter.

**ACKNOWLEDGMENTS**

I have had many discussions on this subject with Jacob Bekenstein over three decades and benefited greatly from his deep and intuitive understanding of relativity. I also gratefully acknowledge Phillip Helbig for a critical reading of this manuscript.

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