Numerical solution of direct and inverse problems of heat transfer in oil reservoirs exploiting with horizontal well

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Abstract. In this paper, a mathematical model is constructed and its grid analog for modeling the thermohydrodynamic processes occurring in the oil reservoir and horizontal wellbore after its launch. On the basis of the proposed model and the theory of ill-posed problems, a numerical method is proposed for solving the inverse coefficient problem for determining the filtration properties of an oil reservoir.

1. Introduction

In the last decade, in connection with the creation of sensitive downhole measuring equipment, new technologies for the field experiment have been emerged as well as methods for interpreting its results. It was shown in [1, 2] that the use of temperature measurements under certain conditions makes it possible to determine the filtration parameters of the formation. The paper [1] considers the problem of determining the filtration properties of a layered formation opened by a vertical well from the measurements of bottomhole pressures and temperature in each interlayer using multi-sensor technology. In describing the process of heat and mass transfer in the "formation - well" system it is assumed that the process of pressure distribution in the wellbore is quasi-stationary, the filtration in the formation is nonstationary, single-phase and non-isothermal. A method for determining the filtration properties of a formation opened by a horizontal well is proposed in [2], according to data on the distribution of temperature and pressure along the wellbore under steady-state conditions. In this paper, we propose a numerical method for solving the inverse coefficient problem of estimating the filtration properties of an inhomogeneous oil reservoir from the results of thermohydrodynamic studies of horizontal wells. As initial information, the data on temperature and pressure changes obtained simultaneously by several deep instruments installed at different sections of the horizontal part of the wellbore are used. Using the proposed method, the heterogeneity of the formation along the horizontal part of the wellbore is evaluated. Interpretation of the curves of temperature and pressure changes recorded by deep instruments in the horizontal well No. 18326 of the Romashkinskoye deposit is carried out.

2. Direct Problem Statement

Measurements of temperature and pressure in the wellbore of a horizontal well by deep equipment make it possible to obtain a fairly complete picture of the thermohydrodynamic processes occurring in the formation and the wellbore. The temperature in the wellbore is sensitive to changes in the filtration
properties of the formation, especially in the bottomhole zone [3]. The change in temperature in the wellbore of a horizontal well is an integral indicator of heat and mass transfer processes occurring both in the well itself and in the formation. Therefore, the use of the results of thermohydrodynamic studies makes it possible to evaluate not only the thermophysical parameters of the formation, but also the filtration parameters.

When constructing a mathematical model, it is assumed that the wellbore of a horizontal well is parallel to the roof and the base of the formation, the inflow of fluid to the well at launch is radial, fluid flow in the wellbore is one-dimensional. Under these assumptions, from the laws of conservation of mass, momentum, and energy, it follows:

\[
\frac{\partial v}{\partial x} = -\frac{2w}{r_c}, \quad w = -\frac{k(r)}{\mu} \left. \frac{\partial p_2}{\partial r} \right|_{x=r_c}, \quad 0 < x \leq L,
\]

\[
\frac{\partial p_1}{\partial x} = \rho \frac{\partial (v^2)}{\partial x} + \frac{\psi}{4r_c} \rho |v|, \quad 0 < x \leq L,
\]

\[
\frac{\partial T_1}{\partial t} + v \left( \frac{\partial T_1}{\partial x} + \frac{\partial p_1}{\partial r} \right) = \frac{2(\alpha_m-wC_p)}{\rho C_p r_c} \left( T_2 \big|_{x=r_c} - T_1 \right), \quad 0 < x \leq L, \quad 0 < t \leq t_{exp},
\]

\[
\beta^* \frac{\partial p_2}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{k(r)}{\mu} \frac{\partial p_2}{\partial r} \right), \quad 0 \leq x \leq L, \quad r_c < r < R_k, \quad 0 < t \leq t_{exp},
\]

\[
C_p r \frac{\partial T_2}{\partial t} = \rho C_p \frac{k(r)}{\mu} \frac{\partial p_2}{\partial r} \left( \frac{\partial T_1}{\partial x} + \frac{\partial p_2}{\partial r} \right), \quad 0 \leq x \leq L, \quad r_c \leq r < R_k, \quad 0 < t \leq t_{exp},
\]

with initial

\[
p_2(x,r,0) = p_0(x,r), \quad T_2(x,r,0) = T_0(x,r), \quad 0 \leq x \leq L,
\]

and boundary conditions

\[
\int_S \frac{k}{\mu} \frac{\partial p_2}{\partial r} dS = q, \quad 0 < t \leq t_{exp},
\]

\[
p(x,R_k,t) = p_k, \quad T_2(x,R_k,t) = T_k.
\]

Here \( k(r) = \begin{cases} k, & r > r_{si}, \quad i=1,N, \\ k_{si}, & r \leq r_{si}, \end{cases} \)

\( p_i = p_i(x), \quad T_i = T_i(x,t) \) – pressure and temperature in the wellbore, \( p = p(x,r,t), \quad T_2 = T_2(x,r,t) \) – pressure and temperature in the formation, \( p_k \) – reservoir pressure, \( T_k \) – reservoir temperature, \( q \) – horizontal well flow rate, \( S \) – horizontal wellbore surface, \( k \) – reservoir permeability, \( k_{si} \) – permeability and bottomhole zone radius in the vicinity of the gauge location, \( r_c \) – the borehole radius, \( R_k \) – the reservoir radius, \( \beta^* \) – the elastic capacity of the formation, \( v(x) \) – the fluid velocity in the wellbore of the horizontal well, \( \rho \) – the fluid density, \( \varepsilon \) – the Joule-Thomson coefficient, \( \psi \) – the hydraulic factor, \( \alpha_m \) – the coefficient of heat transfer of the horizontal wellbore, \( C_p \) – the specific heat of the fluid, \( w \) – the filtration rate, \( L \) – the length of the horizontal wellbore, and \( t_{exp} \) – the operating time of the well.

The derivation of the equation of conservation of momentum with the assumptions made is given in the monograph [4]. Equations (4) - (5) describe non-isothermal fluid filtration in the vicinity of a
horizontal well. The method for solving the boundary value problem (1) - (8) is based on the conjugation of the external (layer) and internal (wellbore) tasks. The system (1) - (8) is solved numerically by the method of finite differences. The filtration area is covered by a non-uniform grid, which thickens to the well. The construction of such a grid is carried out by means of coordinate transformation $\xi = \ln r$ [5]. The resulting nonlinear system of difference equations is solved iteratively.

2.1. Analysis of the solution of the direct problem
A model oil reservoir, opened by a horizontal well, is considered. A horizontal well is put into operation with a constant selection of fluid from the formation.

In each zone of formation homogeneity there is a downhole gauge (Figure 1). Two variants with the following parameters are considered:

1. $r_{s1} < r_{s2}$, $k_1 = 0.01 \mu m^2$, $k_2 = 0.05 \mu m^2$, $k = 0.1 \mu m^2$, $r_{s1} = 0.5 m$, $r_{s2} = 1 m$.

2. $r_{s1} > r_{s2}$, $k_1 = 0.01 \mu m^2$, $k_2 = 0.05 \mu m^2$, $k = 0.1 \mu m^2$, $r_{s1} = 1 m$, $r_{s2} = 0.5 m$.

![Figure 1. Arrangement of downhole gauges in a non-uniform reservoir](image)

Fluid comes to the wellbore of a horizontal well from the zones of uniformity of the oil reservoir with different temperatures due to the Joule-Thomson effect (Figure 1). The temperature in the wellbore of a horizontal well changes due to the calorimetric effect.

The results of numerical calculations have shown that the temperature values recorded by downhole gauges in the wellbore of a horizontal well vary with time unequally. In Figure 2 the results of calculating the temperature variation for variants 1, 2 at time = 100 hours are shown.

![Figure 2. Temperature distribution along the wellbore: dashed red – Var. 1, solid green – Var. 2.](image)

![Figure 3. Distribution of inflow in the wellbore: red circle – Var. 1, green square – Var. 2.](image)
The intensity of fluid inflow to the horizontal wellbore depending on the values of the radius of bottomhole zones and permeabilities is shown in Figure 3.

As the calculation results show, the permeability and size of the homogeneity zones have a dominant influence on the temperature distribution, the flow velocity in the horizontal wellbore and the intensity of the fluid inflow to the horizontal wellbore.

3. Statement and solution of the inverse coefficient problem

Data on changes in pressure and temperature during the start-up period at different sections of the horizontal wellbore are used as initial information for solving the inverse problem. The location of the deep measuring equipment, as well as the radii of the bottomhole zones in each zone of homogeneity, are determined on the basis of geophysical investigations of the well. The technology of conducting thermohydrodynamic studies of a horizontal well with the help of several autonomous downhole gauges is described in [6,7].

Let us assume that in the locations of downhole gauges in the wellbore of a horizontal well with coordinates $x_i$, $i=1, N$, a change in pressure and temperature is registered:

$$T_{i,t}(t) = T_i(t) = \phi_i(t), \quad p_{i,t}(t) = p_i(t) = \zeta_i(t), \quad i=1, N, \quad 0 < t \leq t_{exp}.$$  (9)

The inverse coefficient problem is formulated as follows: determine the coefficient of permeability $k = k(x,r)$, when the thermohydrodynamic processes in the oil reservoir and the wellbore of a horizontal well are described by equations (1) - (8). As the initial information, the values of pressure and temperature measured by downhole gauges are used.

An estimate of the permeability coefficient is sought in the class of piecewise constant functions, $k(x,r) = k_n$, $(x,r) \in V_n$, $\bigcup_{n=1}^{N} V_n = V$, $n=1, N$, where $V_n$ - homogeneity regions (Figure 1).

The numerical solution of the inverse coefficient problem (1) - (9) is sought from minimizing the quadratic deviation between the observed and calculated values:

$$F(\alpha) = \sum_{i=1}^{N} \left[ T_{i,t}(t) - \phi_i(t) \right]^2 dt + \varepsilon^2 \sum_{i=1}^{N} \left[ p_{i,t}(t) - \zeta_i(t) \right]^2 dt,$$  (10)

where $\phi_i(t)$ and $\zeta_i(t)$ are the observed pressure and temperature at time $t$, $p_{i,t}(t)$, $T_{i,t}(t)$ - the calculated pressure and temperature obtained from the numerical solution of equations (1) - (8), $\alpha = (k_1, k_2, \ldots, k_N)$ is the vector of unknown permeabilities, $0 < m_0 \leq k_n \leq M_n$.

The computational algorithm for minimizing (10) is based on the Levenberg-Marquardt method.

4. Well test analysis

Investigation of horizontal well No. 18326 of the Romashkinskoye deposit of the Republic of Tatarstan is provided. Thermohydrodynamic studies on multi-sensor technology were carried out in the well. In Figure 4 has shown the location of the downhole gauges. Upon completion of the underground repair, the well was started up with a flow rate of 7.8 m³/day. Geophysical studies have shown that near the location of the gauge No. 1721 the wellbore passes through a low permeable inclusion, and in the region of the gauge No. 1726, through an inclusion poorly saturated with oil. The results of the studies carried out by “Permneftegeofizika” company showed that in the zones of location of these gauges a low inflow to the horizontal wellbore is observed [8]. The proposed computational algorithm is used to interpret the pressure and temperature variation curves taken with the downhole gauges Nos. 1879, 1721, 1726 and 1885. For this purpose, the grid model of the reservoir is divided into five zones of homogeneity, gauges Nos. 1879, 1721, 1726 and 1885 are located in four of them, and the fifth zone is remote (Figure 4).
Figure 4. Scheme of the horizontal well No 18326 trajectory and the location of the downhole gauges.

Figure 5. HW No. 18326. Temperature distribution along the wellbore

Figure 6. HW No 18326. Distribution of fluid flow along the wellbore

Distributions of temperature and of fluid flow along the wellbore are shown in Figures 5 and 6. As it is seen from the table, the zones, where gauges Nos. 1721, 1726 are located, have low permeability and, as a consequence, the inflow to the horizontal wellbore is the smallest on these sections (Figure 6).

**Table.** Estimates of the filtration parameters.

| Gauge area | gauge No.1879 | gauge No.1721 | gauge No.1726 | gauge No.1885 |
|------------|--------------|---------------|---------------|---------------|
| $k/\mu$ ($\mu m^2/mPa\cdot s$) | $7.12 \times 10^{-4}$ | $4.00 \times 10^{-3}$ | $3.85 \times 10^{-4}$ | $6.81 \times 10^{-4}$ |
| $r_{si}$ (m) | 1 | 0.3 | 2 | 1 |

In Figures 7 and 8, the calculated and observed curves for the temperature changes according to the gauges No. 1879, 1885 are given. The values of the radii of the near-wellbore zones in the vicinity of the gauges were chosen on the basis of geophysical studies. Estimates of the conductivity values in these zones are given in the table. The conductivity of the remote zone is $1.25 \times 10^{-2} \mu m^2 / mPa\cdot s$. 
The analysis of the obtained results shows that the proposed method makes it possible to evaluate the filtration parameters of a zone-inhomogeneous reservoir from the data on changes in temperature and pressure in the wellbore of a horizontal well, and it agrees well with the results of geophysical studies.

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