

Research Article

Due-Window Assignment and Resource Allocation Scheduling with Truncated Learning Effect and Position-Dependent Weights

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This paper studies single-machine due-window assignment scheduling problems with truncated learning effect and resource allocation simultaneously. Linear and convex resource allocation functions under common due-window (CONW) assignment are considered. The goal is to find the optimal due-window starting (finishing) time, resource allocations and job sequence that minimize a weighted sum function of earliness and tardiness, due window starting time, due window size, and total resource consumption cost, where the weight is position-dependent weight. Optimality properties and polynomial time algorithms are proposed to solve these problems.

1. Introduction

Scheduling models and problems with learning effects (see Biskup [1]; Lu et al. [2]; Azzouz et al. [3]; Wang et al. [4]) and/or resource allocations (see Shabtay and Steiner [5]; Yang et al. [6]) have become popular topics for scheduling researchers in recent years. Scheduling with learning effects and resource allocations simultaneously was introduced by Wang et al. [7], who focused on single-machine scheduling problems. Lu et al. [8] studied single-machine due-date assignment scheduling with learning effects and resource allocations. They proved that several problems can be solved in polynomial time. Wang and Wang [9] and Li et al. [10] considered common and slack due-window assignment problems with learning effects and resource allocations. Wang and Wang [11] considered single-machine scheduling problems with learning effects and convex resource allocation function. For the scheduling criterion (the total resource compression criterion) minimization subject to the constraint that the total resource compression criterion (the scheduling criterion) is less than or equal to a fixed constant, they proved that the problems can be solved in polynomial time. Wang et al. [12] and Liu and Jiang [13] considered due-date assignment scheduling with job-dependent learning effects and resource allocation. Liu and Jiang [14] considered flow shop due-date assignment scheduling with resource allocation and learning effect. Shi and Wang [15] considered flow shop due-window assignment scheduling with resource allocation and learning effect.

In recent years, many researchers focused on the study of scheduling with due-window, where a time interval is assumed, such that jobs completed within this interval are not penalized (Janiak et al. [16] and Wang et al. [17]). Wang et al. [18] considered the single-machine due-window scheduling problems with position-dependent weights. For the weighted sum of earliness and tardiness, due window starting time, and due window size, where the weight only dependent on its position in a sequence (i.e., a position-dependent weight), they proved that the problems can be solved in polynomial time. In this study, we continue the work of Wang et al. [18], i.e., we consider the due-window assignment scheduling problems with learning effect and resource allocation in the single-machine environment. The goal is to find the optimal due-window starting (finishing) time, resource allocations, and job sequence such that a sum of scheduling cost (including weighted sum function of
earliness and tardiness, due window starting time, due window size, where the weight is position-dependent weight) and total resource consumption cost is minimized.

The contributions of this paper are given as follows. (1) The structural properties of single-machine scheduling problems are derived. (2) For the linear resource allocation, we proved that the sum of scheduling cost and total resource consumption cost can be solved in polynomial time. For the convex resource allocation, three versions of scheduling cost and total resource consumption cost can be solved in polynomial time respectively. (3) It is further extended the model to the case with slack due-window (SLKW) assignment model.

The rest of the article is organized as follows: In Section 2, we introduce the problem. In Sections 3 and 4, we provide some properties to optimally solve these problems under linear and convex resource allocation. In Section 5, we conclude the paper.

2. Problem Formulation

We study a scheduling problem consisting of a set of $n$ independent jobs $N = \{J_1, J_2, \ldots, J_n\}$ that need to be processed on a single machine. For the linear resource allocation, the actual processing time of job $J_j$ is

$$P^A_j = \bar{p}_j \max\{r^{\alpha_i}, \delta\} - \beta_j u_j,$$

where $\bar{p}_j$ is the basic processing time of job $J_j$ (i.e., the processing time without any resource allocation and truncated learning rate (Mosheiov and Sidney [19]) of job $J_j$, $0 < \delta < 1$ is a truncation parameter (Wang et al. [20]), $\beta_j$ is the compression rate of job $J_j$, and $u_j$ is the amount of resource allocated to job $J_j$ and satisfies $0 \leq u_j \leq \bar{u}_j \leq (\bar{p}_j \max\{r^{\alpha_i}, \delta\})/\beta_j$.

For the convex resource allocation, the actual processing time of job $J_j$ is

$$P^C_j = \left(\frac{\bar{p}_j \max\{r^{\alpha_i}, \delta\}}{u_j}\right)^\eta,$$

where $\eta > 0$ is a constant, i.e., $P^C_j$ is a convex decreasing function of resource $u_j$.

Let $[d_1, d_2]$ be the common due-window for all jobs, where $d_1 \geq 0$ is $d_2$, $d_1 \leq d_2$. Both $d_2$ and $d_2$ are decision variables in this paper. The goal of this paper is to find jointly the optimal due-window location, the optimal resource allocation and sequence $\pi$ such that the following objective function is minimized:

$$Z(d_1, d_2, \pi) = \sum_{j=1}^{n} w_j L_{[j]} + w_0 \sum_{j=1}^{n} \alpha_j U_{[j]} + \sum_{j=1}^{n} v_{[j]} U_{[j]}.$$  

(3)

where $[j]$ denotes the job scheduled in $j$th position, $w_j (j = 0, 1, 2, \ldots, n, n + 1)$ denotes a position-dependent weight, $L_{[j]}$ is the earliness-tardiness of job $J_j$, and

$$L_{[j]} = \begin{cases} 0, & \text{for } d_1 \leq C_{[j]} \leq d_2, \\ \delta_j, & \text{for } d_1 \leq C_{[j]} \leq d_2, \\ \delta_j, & \text{for } C_{[j]} > d_2, \\ \delta_j, & \text{for } C_{[j]} < d_1, \end{cases}$$  

(4)

where $C_{[j]}$ is the completion time of job $J_j$, $j = 1, 2, \ldots, n$. Using the three-field classification, the problem can be denoted as $1|\text{CONW}, P^A_j|w_j L_{[j]} + w_0 d_1 + w_0 d_2 + \sum_{j=1}^{n} v_{[j]} U_{[j]}$, where $P^A_j \in \{\tilde{p}_j \max\{r^{\alpha_i}, \delta\} - \beta_j u_j, (\tilde{p}_j \max\{r^{\alpha_i}, \delta\})/\beta_j\}$ (Graham et al. [21]), where CONW denotes the common due-window assignment. Wang et al. [18] considered single-machine scheduling problems with CONW and slack due-window (SLKW) assignments problems $1|\text{CONW}|\sum_{j=1}^{n} w_j L_{[j]} + w_0 d_1 + w_0 d_2 + \sum_{j=1}^{n} v_{[j]} U_{[j]}$ for the SLKW model, $[d'_{j}, d'_j]$ is the due-window of job $J_j$ such that $d'_{j} \leq d'_j$, where $d'_{j} = d^A_{j} + q', d''_{j} = a^A_{j} + q''$, $j = 1, 2, \ldots, n$ limit, $q'$ and $q''$ are decision variables and $D = q'' - q'$. Wang et al. [18] proved that these both problems can be solved in $O(n \log n)$ time, respectively.

3. Linear Resource Allocation

Lemma 1 [Wang et al. [18]]. For any given sequence $\pi$, there exists an optimal sequence in which $d_{k} = C_{[k]}$ for some $k$ and $d_{k} = C_{[j]}$ for some $l, l \geq k$, where $\sum_{i=0}^{l-1} w_i \leq w_{n+1} \leq \sum_{i=0}^{l} w_i$, and $\sum_{j=1}^{n} w_j \leq w_{n+1} \leq \sum_{j=1}^{n} w_j$.

Lemma 2. The objective function of the problem $1|\text{CONW}, P^A_j|w_j L_{[j]} + w_0 d_1 + w_0 d_2 + \sum_{j=1}^{n} v_{[j]} U_{[j]}$ can be written as

$$\sum_{j=1}^{n} w_j L_{[j]} + w_0 d_1 + w_0 d_2 + \sum_{j=1}^{n} v_{[j]} U_{[j]} = \sum_{j=1}^{n} \xi_j P^A_{[j]} + \sum_{j=1}^{n} v_{[j]} U_{[j]},$$  

(5)

where

$$\xi_j = \begin{cases} \sum_{h=0}^{j-1} w_h, & \text{for } j = 1, 2, \ldots, k; w_{n+1}, & \text{for } j = k + 1, k + 2, \ldots, l; \sum_{h=0}^{j-1} w_h, & \text{for } j = l + 1, l + 2, \ldots, n. \end{cases}$$  

(6)
Proof. From Lemma 1, we have

\[
Z(d_1, d_2, \pi) = u_0 C_{[k]} + \sum_{j=1}^{k-1} w_j (C_{[k]} - C_{[j]}) + \sum_{j=k+1}^{n} w_j (C_{[j]} - C_{[i]}) + u_{n+1} (C_{[i]} - C_{[k]}) + \sum_{j=1}^{n} v_{j} u_{j}^{*}
\]

\[
= u_0 \sum_{j=1}^{k} p_{[j]}^{A} + \sum_{j=1}^{k-1} w_j \left( \sum_{h=j+1}^{k} p_{[h]}^{A} \right) + \sum_{j=k+1}^{n} w_j \left( \sum_{h=j+1}^{n} p_{[h]}^{A} \right) + u_{n+1} \left( \sum_{h=n+1}^{n} p_{[h]}^{A} \right) + \sum_{j=1}^{n} v_{j} u_{j}^{*}
\]

\[
= \sum_{j=1}^{n} \xi_{j} p_{[j]}^{A} + \sum_{j=1}^{n} v_{j} u_{j}^{*}
\]

where \( \xi_{j} \) (\( j = 1, 2, \ldots, n \)) are given by (6).

From Lemma 2, we have

\[
Z(d_1, d_2, \pi) = \sum_{j=1}^{n} w_j L_{[j]} + w_0 d_1 + w_{n+1} D + \sum_{j=1}^{n} v_{j} u_{j}^{*}
\]

\[
= \sum_{j=1}^{n} \xi_{j} \left( \bar{p}_{[j]} \max\{r^{u_1}, \delta\} - \beta_{[j]} u_{j}^{*} \right) + \sum_{j=1}^{n} v_{j} u_{j}^{*}
\]

\[
= \sum_{j=1}^{n} \xi_{j} \bar{p}_{[j]} \max\{r^{u_1}, \delta\} + \sum_{j=1}^{n} (v_{j} - \xi_{j} \beta_{[j]}) u_{j}^{*}.
\]

(8)

From (8), for a given sequence, the optimal resource allocation \( u_{j}^{*} \) with \( v_{j} - \xi_{j} \beta_{[j]} < 0 \) should be \( \pi_{[j]} \); otherwise, \( u_{j}^{*} = 0 \), i.e., the optimal resource allocation of job \( J_{[j]} \) is

\[
u_{j}^{*} = \begin{cases} 0, & \text{if } v_{j} - \xi_{j} \beta_{[j]} \geq 0, \\ \pi_{[j]}, & \text{if } v_{j} - \xi_{j} \beta_{[j]} < 0. \end{cases}
\]

(9)

For a given sequence, from (8), we can obtain the optimal resource allocation. In order to determine the optimal

\[
\Psi_{j} = \begin{cases} \xi_{j} \bar{p}_{[j]} \max\{r^{u_1}, \delta\}, & \text{if } v_{j} - \xi_{j} \beta_{[j]} \geq 0, j, r = 1, 2, \ldots, n, \\ \xi_{j} \bar{p}_{[j]} \max\{r^{u_1}, \delta\} + (v_{j} - \xi_{j} \beta_{[j]}) \pi_{[j]}, & \text{if } v_{j} - \xi_{j} \beta_{[j]} < 0, j, r = 1, 2, \ldots, n.
\end{cases}
\]

(14)

And \( \xi_{r} \) (\( r = 1, 2, \ldots, n \)) are given by (6).

Based on the above analysis, the problem 1|CONW, \( P_{j}^{A} = \bar{p}_{[j]} \max\{r^{u_1}, \delta\} - \beta_{[j]} u_{j} \sum_{j=1}^{n} w_{j} L_{[j]} + w_0 d_1 + w_{n+1} D + \sum_{j=1}^{n} v_{j} u_{j}^{*} \) can be optimally solved by the following algorithm.

Algorithm 1

Step 1. Calculate the indices \( k \) and \( l \) according to Lemma 1.

Step 2. Calculate the values \( \Psi_{j} \) by using (14).

Step 3. Solve the assignment problem (10)–(13) to determine the optimal job sequence.

Step 4. Calculate the optimal resource allocation by (7).

Step 5. Calculate \( d_1 = C_{[k]}, d_2 = C_{[l]} \).

Theorem 1. The problem 1|CONW, \( P_{j}^{A} = \bar{p}_{[j]} \max\{r^{u_1}, \delta\} - \beta_{[j]} u_{j} \sum_{j=1}^{n} w_{j} L_{[j]} + w_0 d_1 + w_{n+1} D + \sum_{j=1}^{n} v_{j} u_{j}^{*} \) can be solved by Algorithm 1 in \( O(n^3) \) time.

Proof. The correctness of Algorithm 1 follows from the above analysis. The time complexity of Step 1 is \( O(n) \) time, Step 2 is \( O(n^2) \) time, Step 3 is \( O(n^3) \) time, Step 4 is \( O(n) \), and Step 5 is \( O(n) \) time. Thus, the overall computational complexity of Algorithm 1 is \( O(n^3) \).
In order to illustrate Algorithm 1 for \(1|\text{CONW}, P_j^A = \bar{p}_j \max\{r^{p_j}, \delta\} - \beta_j u_j \sum_{j=1}^n w_jL_{ij} + w_jd_i + w_{n+1}D + \sum_{j=1}^n v_{ij} u_{ij}\), we present the following example.

**Example 1.** Data: \(n = 7, \delta = 0.6, w_9 = 19, w_1 = 20, w_2 = 12, w_3 = 7, w_4 = 14, w_5 = 24, w_6 = 22, w_7 = 15, w_8 = 22\), and the other corresponding parameters shown in Table 1.

Solution:

Step 1. According to Lemma 1, \(k = 1, l = 6\).

Step 2. From (5), \(x_1 = 19, x_2 = x_3 = x_4 = x_5 = 22, x_6 = 22\), and the values \(\Psi_j\) are given in Table 2.

Step 3. Stemming from the assignment problem (8)–(11), the optimal job sequence is \(\pi = (J_4, J_7, J_2, J_6, J_1, J_5, J_3)\).

Step 4. From (7), the optimal resource allocation is

\[
\begin{align*}
\bar{u}_1 &= 1, u_2 = 5, u_3 = 3, u_4 = 4, u_5 = 5, u_6 = 2, u_7 = 0.
\end{align*}
\]

Step 5. Calculate \(d_1 = C_{[1]} = C_4 = 7, d_2 = C_{[6]} = C_5 = 22.67617\), and \(\sum_{j=1}^n w_jL_{ij} + w_jd_i + w_{n+1}D + \sum_{j=1}^n w_jL_{ij} = 886.8757\).

4. Convex Resource Allocation

4.1. Problem 1|CONW, \(P_j^A = (\bar{p}_j \max\{r^{p_j}, \delta\}/u_j)| \sum_{j=1}^n w_jL_{ij} + w_jd_i + w_{n+1}D + \sum_{j=1}^n v_{ij} u_{ij}\). From Lemma 2 and

\[
P_j = (\bar{p}_j \max\{r^{p_j}, \delta\}/u_j)\eta,
\]

we have

\[
\sum_{j=1}^n w_jL_{ij} + w_jd_i + w_{n+1}D + \sum_{j=1}^n v_{ij} u_{ij} = \sum_{j=1}^n \xi_j \left( \frac{\bar{p}_j \max\{r^{p_j}, \delta\}}{u_j} \right) \eta + \sum_{j=1}^n v_{ij} u_{ij},
\]

where \(\xi_j (j = 1, 2, \ldots, n)\) are given by (6).

By taking the first derivative of the objective given by (15) with respect to \(u_{ij}\), equating it to zero and solving it for \(j_{ij}\), we have (16).

**Theorem 2.** Algorithm 2 solves the problem \(1|\text{CONW}, P_j^A = (\bar{p}_j \max\{r^{p_j}, \delta\}/u_j)| \sum_{j=1}^n w_jL_{ij} + w_jd_i + w_{n+1}D + \sum_{j=1}^n v_{ij} u_{ij}\) in \(O(n^3)\) time.

In order to illustrate Algorithm 2 for \(1|\text{CONW}, P_j^A = (\bar{p}_j \max\{r^{p_j}, \delta\}/u_j)| \sum_{j=1}^n w_jL_{ij} + w_jd_i + w_{n+1}D + \sum_{j=1}^n v_{ij} u_{ij}\), we present the following example.

**Example 2.** Consider \(n = 7, \delta = 0.6, \eta = 2, w_9 = 9, w_1 = 8, w_2 = 12, w_3 = 7, w_4 = 14, w_5 = 24, w_6 = 5, w_7 = 15, w_8 = 22\), and the other corresponding parameters shown in Table 3.

Solution:

Step 1. According to Lemma 1, \(k = 2, l = 5\).

Step 2. From (5), \(x_1 = 9, x_2 = 17, x_3 = x_4 = x_5 = 22\), \(\xi_6 = 20, \xi_7 = 15\), and the values \(\Psi_j\) are given in Table 4.

Step 3. Stemming from the assignment problem (8)–(11), the optimal job sequence is \(\pi = (J_2, J_1, J_3, J_7, J_6, J_5, J_4)\).

Step 4. From (14), the optimal resource allocation is

\[
\bar{u}_1 = 10.91533, u_1 = 10.71178, \bar{u}_3 = 11.53352, u_7 = 5.841858, u_6 = 9.501238, u_5 = 9.251873, u_4 = 5.013141.
\]
Table 2: Values $\Psi_{jr}$ for Example 1.

| $jr$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|------|---|---|---|---|---|---|---|
| $\psi_{jr}$ | 262.0000 | 200.3414 | 151.0178 | 119.7068 | **98.60000** | 98.60000 | 72.00000 |
| 2 | 194.0000 | 160.6827 | **131.3178** | 112.1496 | 98.16679 | 87.28498 | 66.85148 |
| 3 | 361.0000 | 332.5343 | 290.8883 | 264.5431 | 250.80000 | 250.80000 | **171.0000** |
| 4 | **142.0000** | 127.9972 | 110.1639 | 98.56349 | 90.12287 | 83.56748 | 56.21822 |
| 5 | 202.0000 | 162.1420 | 127.1396 | 104.6077 | 88.33854 | **75.77993** | 54.00000 |
| 6 | 212.0000 | 169.9128 | 137.1665 | **116.2334** | 101.1959 | 95.20000 | 84.00000 |
| 7 | 171.0000 | **151.9446** | 133.9793 | 122.4548 | 114.1549 | 108.8000 | 81.00000 |

Bold numbers are the optimal solution.

Table 3: Data for Example 2.

| $J_j$ | $J_1$ | $J_2$ | $J_3$ | $J_4$ | $J_5$ | $J_6$ | $J_7$ |
|-------|-------|-------|-------|-------|-------|-------|-------|
| $\bar{p}_j$ | 13 | 17 | 12 | 10 | 18 | 16 | 9 |
| $\alpha_j$ | -0.32 | -0.24 | -0.33 | -0.25 | -0.28 | -0.3 | -0.29 |
| $v_j$ | 3 | 4 | 2 | 9 | 6 | 5 | 8 |

Table 4: Values $\Psi_{jr}$ for Example 2.

| $jr$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|------|---|---|---|---|---|---|---|
| $\psi_{jr}$ | 45.20903 | **48.20302** | 48.17622 | 45.30844 | 43.32309 | 41.96834 | 38.13077 |
| 2 | 65.49197 | 72.45895 | 74.00176 | 70.67272 | 68.19401 | 64.16226 | 56.87507 |
| 3 | 32.70817 | 34.71350 | **34.60055** | 32.47854 | 31.34372 | 30.36358 | 27.58714 |
| 4 | 78.94848 | 86.94426 | 88.55797 | 84.49099 | 81.32839 | 76.42716 | **67.67740** |
| 5 | 89.15204 | 96.82953 | 97.82774 | 92.71289 | 88.93040 | **83.26686** | 75.19374 |
| 6 | 72.98642 | 78.54253 | 78.92439 | 74.51135 | 71.25929 | 67.75459 | 61.55913 |
| 7 | 68.03574 | 73.55408 | 74.11175 | 70.10229 | 67.14231 | 63.15878 | 57.38356 |

Bold numbers are the optimal solution.

4.2. Problem 1|CONW, $P^\lambda=\left(\bar{p}_j, J_j\right)$ are given by (6). $\sum_{j=1}^n v_j u_j(j) \leq U[\sum_{j=1}^n w_j L_j(j)] + w_0 d_1 + w_{v+1} D$ where $U$ is a limitation on the total resource consumption cost. Obviously, in an optimal solution for the problem $\mathcal{P}_A = \left(\bar{p}_j, \max\{r^\alpha, \delta\}/u_j\right)$, $\sum_{j=1}^n v_j u_j(j) \leq U[\sum_{j=1}^n w_j L_j(j)] + w_0 d_1 + w_{v+1} D$ the constraint will be satisfied as $\sum_{j=1}^n v_j u_j(j) = U$.

**Lemma 4.** For a given sequence, the optimal resource allocation of the problem $\mathcal{P}_A = \left(\bar{p}_j, \max\{r^\alpha, \delta\}/u_j\right)$, $\sum_{j=1}^n v_j u_j(j) \leq U[\sum_{j=1}^n w_j L_j(j)] + w_0 d_1 + w_{v+1} D$, $j = 1, 2, \ldots, n$, $\xi_j$ are given by (6).

**Proof.** For a given sequence $\pi = \langle J_1, J_2, \ldots, J_n \rangle$, the Lagrange function is

$$L(u, \lambda) = \sum_{j=1}^n \xi_j p^A_j + \lambda\left(\sum_{j=1}^n v_j u_j(j) - U\right) + \delta \left(\sum_{j=1}^n v_j u_j(j) - U\right),$$

where $\lambda$ is the Lagrangian multiplier. Deriving (20) with respect to $u_j(j)$ and $\lambda$, we have

$$\frac{\partial L(u, \lambda)}{\partial u_j} = \lambda v_j - \eta \delta j \times \left(\frac{\bar{p}_j \max\{r^\alpha, \delta\}}{u_j} \right) = 0. \quad (21)$$

It follows that

**(22)**
Theorem 3. Problem 1 |CONW, \( P^A = ((\tilde{p}_j, \max\{r^a, \delta\})/u_j))^n \), \( \sum_{j=1}^n v_j u_{[j]} \leq U \), \( \sum_{j=1}^n w_j L_{[j]} + w_0 d_1 + w_{m+1} D \) can be solved in \( O(n^2) \) time.

Similarly to Section 4.1, we have the following. \( \square \)

\[
\sum_{j=1}^n \xi_j P^A_{[j]} = U^{-n} \left( \sum_{j=1}^n (v_j \tilde{p}_j \max\{f^{a,\delta}, \delta\})^{n/(\eta+1)} \right)^{\eta+1}.
\]  

(24)

Similarly to Section 4.2, we have.

Lemma 5. For a given sequence, the optimal resource allocation of the problem 1 |CONW, \( P^A = ((\tilde{p}_j, \max\{r^a, \delta\})/u_j))^n \), \( \sum_{j=1}^n w_j L_{[j]} + w_0 d_1 + w_{m+1} D \leq V \), \( \sum_{j=1}^n v_j u_{[j]} \) is

\[
u_* = V^{-1/\eta}(\xi_j)^{(1/\eta + 1)}(v_j)^{-(1/\eta + 1)}(\tilde{p}_j \max\{f^{a,\delta}, \delta\})^{n/(\eta+1)} \left( \sum_{j=1}^n (\xi_j)^{(1/\eta + 1)}(\tilde{p}_j v_j \max\{f^{a,\delta}, \delta\})^{n/(\eta+1)} \right)^{(1/\eta)}.
\]  

(25)

5. Conclusions

This paper considered the single-machine due-window assignment scheduling problems with learning effect and resource allocation. For the linear and convex resource allocation, we showed that some different models are polynomially solvable, respectively. Future research may focus on the flow shop scheduling problems with learning effect and resource allocation or study the Pareto-optimal solutions with respect to the criterion \( \sum_{j=1}^n w_j L_{[j]} + w_0 d_1 + w_{m+1} D \) and the resource compression cost \( \sum_{j=1}^n v_j u_{[j]} \).

Data Availability

No data were used in the study.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

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