DENSITY DISTRIBUTION OF PHOTOSPHERIC VERTICAL ELECTRIC CURRENTS IN FLARE-ACTIVE REGIONS OF THE SUN

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Electric currents flow in active regions of the sun. Information on the distribution of the currents is important for understanding energy release processes on the sun’s surface and in overlying layers. This is an analysis of the probability density function (PDF) of the absolute value of the density of photospheric vertical electric currents $|j_z|$ in 48 active regions from 2010 through 2015 at times before and after flares. $|j_z|$ is calculated by applying a differential form of the magnetic field circulation theorem (Ampere’s law) to photospheric vector magnetograms from the Helioseismic and Magnetic Imager (HMI) on board the Solar Dynamics Observatory (SDO). It is shown that for the active regions studied here PDF $|j_z|$ can be calculated in a first approximation by a model consisting of a folded normal distribution at low values ($|j_z| \leq 9 \cdot 10^3$ statampere/cm²) and a falling power law function at higher values. A least squares method yields the model parameters for all regions, histograms of their distributions are plotted, and the mathematical expectations and mean square deviations are calculated. No systematic changes in the model parameters over the time of a flare were observed. Neither an explicit relation of the parameters to the class of a flare, nor to the Hale magnetic class was found in terms of the approach used for the limited sample of flares and active regions examined here. Arguments are presented in favor of the proposition that a folded normal distribution at low values represents noise in the data, while a power-law “tail” may reflect the nature of the processes that generate the currents in active regions of the sun.

Keywords: active regions of the sun: electric currents

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Original article submitted September 6, 2019; accepted for publication June 24, 2020. Translated from Astrofizika, Vol. 63, No. 3, pp. 463-477 (August 2020)
1. Introduction

Magnetic fields determine the processes of solar activity, heating the corona, and acceleration of the solar wind. At present, magnetic fields are routinely measured at the level of the photosphere. It is clear that active regions are permeated by fields concentrated in magnetic flux tubes [1,2]. Based on Ampere’s law (the theorem of circulation of the magnetic field) it has been found that electric currents can flow in these tubes [3,4]. Since vector magnetograms are usually accessible only for a single narrow layer, most of the information is about the vertical component \( j_z \) of the electric current in the photosphere. It should be noted that attempts have also been made to evaluate the horizontal component of the electric currents [3,5-7].

Studies of the electric currents in active regions are important for a number of reasons [3,4,8,9]. First, the free magnetic energy released in such phenomena of solar activity as coronal jets, flares, and coronal mass ejections (CME) is associated with electric currents. The dissipation of electric currents, both longitudinal to the magnetic field and in the form of current layers, leads to the transformation of free magnetic energy into kinetic energy of plasma and of populations of accelerated particles, the energy of electromagnetic radiation over a wide spectral range, and into the energy of waves. Second, Joule dissipation of currents can contribute to the thermal balance in different layers of the solar atmosphere. Third, the existence of currents can influence the way Alfvén waves propagate and are dissipated in active regions and this may be important for heating the corona and acceleration of the solar wind.

On the whole, it has been established that there is a relationship between \( j_z \) and the flare productivity of active regions [3,5,10-12]. Detailed studies are needed for further clarification of how \( j_z \) is specifically related to flares. The traditional approach to studying electric currents in active regions involves constructing charts of the density \( j_z \) in the photosphere based on vector magnetograms and analyzing the relationship of the spatial structure of \( j_z \) with sources of electromagnetic radiation for solar activity processes. A number of studies have been made of the relationship of the radiation sources (microwave, H\( \alpha \), ultraviolet, x-ray) of flares with photospheric \( j_z \) [5, 13-22]. It has been found that a large percent (>70%) of the flare centers observed in H\( \alpha \) coincide with maxima of \( j_z \) [5,13]. The sources of hard x-rays of flares directly associated with sites of the injection of accelerated electrons into dense layers of the solar atmosphere, however, have a tendency to appear at the edges of regions of strong \( j_z \) and avoid their local maxima [17,22]. In addition, no quantitative coupling between the flux of hard x-rays and \( j_z \) under the sources has been observed [22]. Thus, since the relationship between the photospheric \( j_z \) and flare sources of hard x-rays is still not fully understood, it makes sense to continue studying and try to find additional behavior, drawing on the techniques of statistical analysis.

Despite a fairly large number of studies of \( j_z \), we know of no work in which the probability density function (PDF) of the density \( j_z \) on the photosphere has been studied systematically along with the relationship of its characteristics to energy release processes in active regions. This has been done, for example, for the density of electric currents in the corona based on modeling and extrapolation of the magnetic field from the photosphere for single active regions [23,24]. It was found that the PDF of the electric current density in the corona can be represented as a power-law function or a double power-law function (with a discontinuity). It should be noted, however, that extrapolations of magnetic fields are not unique. They depend on the method of extrapolation and the quality of
the boundary data that are used. [23] also shows an example of a PDF(\(|j_z|\)) for a single flare-active region (AR12158; SOL2014-09-10) and the visual difference from the PDF for the coronal currents is shown. There a quantitative analysis of PDF(\(|j_z|\)) was not made and its shape was not studied.

In this paper the form of PDF \(\langle |j_z|\rangle\) for a number of flare-active regions is studied in a first approximation. We regard it as an interesting and natural step to check whether there are systematic differences between PDF \(\langle |j_z|\rangle\) before and after a flare and whether there is a correlation between the parameters of PDF \(\langle |j_z|\rangle\) and the x-ray class of a flare, as well as with the Hale magnetic class of an active region according to the Mount Wilson classification [25].

2. Data and methods

We note first of all that the idea of this paper arose during a statistical study of the relationship of flare x-ray sources and photospheric vertical electric fields \(j_z\) [22], when it was necessary to calculate PDF\((j_z)\) to estimate the error in determining \(j_z\) using data from the Helioseismic and Magnetic Imager (HMI) [26] on board the Solar Dynamics Observatory (SDO). This is the reason for the choice of active regions for analysis. 48 active regions were chosen over the time period from May 2010 through October 2017 in which flares of different x-ray classes occurred near the center of the solar disk (helioprojection coordinates -60° < \((x, y)\) < +60°, i.e., the heliolongitude and heliolatitude of the flares are within ±40°) and for which it was possible to determine the coordinates of the hard x-ray sources at energies of 50-100 keV using data from the Reuven Ramaty High-Energy Solar Spectroscopic Imager (RHESSI) spacecraft [27]. The time interval examined here was determined by simultaneous observations of the sun by SDO and RHESSI. Information on the active regions being studied and the flares is given in [22].

In this paper we use prepared vector photospheric magnetograms from the HMI/SDO, that are freely accessible at (http://jsoc.stanford.edu/) in the form of output from the Spaceweather HMI Active Region Patch (SHARP) [28,29]. Standard data files of the form “hmi.sharp_cea_720s.fits” with a time step of 12 min were used. A special algorithm was used to isolate a limited segment (patch) corresponding to the active region and its surroundings for each time interval out of the entire field of view of the HMI. Each patch was assigned its own number HARPNUM. In these files the magnetic field vector \(B = (B_r, B_\theta, B_\phi)\) in a spherical coordinate system was projected onto a cylindrical Lambert grid \((x = \varphi, y = (180^o/\pi)\sin\theta)\) with the same cell area of \(1.33 \times 10^4\) km² on the photosphere (Lambert cylindrical equal-area projection) [30,31].

The \(180^o\)-uncertainty in the transverse (to line of sight) magnetic field component, \(B_\perp\), is eliminated in these data. The program package “WCS” in the SolarSoftWare (SSW) was used to transform the CEA coordinates into a Stonyhurst heliographic system and then, for convenience in the calculations, into a spherical coordinate system with its origin at the sun’s center.

The density of the photospheric vertical electric currents was calculated numerically in the spherical coordinate system using Ampere’s law in differential form:
\[ j_z = j_r = \frac{c}{4\pi\mu} (\nabla \times \mathbf{B})_r = \frac{c}{4\pi R_s \sin\theta} \left( \frac{\Delta B_\theta}{\Delta \phi} \sin\theta + B_\phi \cos\theta - \frac{\Delta B_\theta}{\Delta \phi} \right), \]  

(1)

where \( c \) is the speed of light in vacuum, \( R_s \) is the sun's radius at the level of the photosphere, and the magnetic permeability coefficient \( \mu = 1 \). For each active region a chart of \( j_z \) was constructed for two times: immediately before the flare onset in soft x-ray emission and after the end of the impulsive phase of the flare, when the hard x-ray emission (> 25 keV) falls to the pre-flare level. This makes it possible to study possible variations in the PDF of \( j_z \) over the time of the flare, avoiding variations in \( j_z \) which may arise owing to perturbations of the photosphere by beams of accelerated particles, hydrodynamic flows, shock waves, and fluxes of electromagnetic radiation during the impulsive phase of the flare [32].

Based on the data sets obtained here, histograms of \( j_z \) with a bin width of 2.5 x 10^3 statampere/cm^2 (1 statampere/cm^2 = 1/3 x 10^{-5} A/m^2) were constructed. This fixed bin width was chosen, first, to maintain a unified approach in analyzing all the events and, second, so there would be more than 15 bins for each event, but here the number of bins with low values of \( j_z \) would not be very large and the number of empty bins with high values of \( j_z \) would be low. Empty bins in which zero values of \( j_z \) fell were excluded. Then, from the resulting values of the centers of the bins (the x axis) the absolute values are taken and the natural logarithm of the absolute values of the bins and the number of data points in each bin is taken. For two data vectors, the values of the positive and negative \( j_z \), a Kolmogorov-Smirnov test is run, the results of which do not conflict with the zero hypothesis that

Fig. 1. Examples of the distributions of \(|j_z|\) on a log-log scale for two active regions: NOAA 12172 before and after the flare SOL2014-09-24T17:45 (left) and NOAA 11263 before and after the flare SOL2011-08-03T04:29 (right). The distributions of \(|j_z|\) before and after a flare are indicated by circles and crosses, respectively. The model approximations before and after the flare by a least squares method are indicated by the grey and black continuous curves, respectively. The black and grey curves are close to one another and almost overlap; thus, it is hard to distinguish them. The vertical dashed lines indicate the positions of the transition points between the folded normal distribution and the power-law function.
these two vectors belong to a single distribution at a significance level of 1%. This indicates that we can deal with the absolute value $|j_z|$, rather than treat the positive and negative values separately. This approach increases the number of points in the histogram by a factor of two, which is important for adequate fitting of the region of high values of $|j_z|$. Ultimately, the histograms contain from 30 to 40 nonzero bins for each event, which is considerably higher than the number of free parameters of the functions used for fitting (see below). The histograms were normalized to the maximum before fitting. These histograms can be regarded as an approximation to the PDF of the absolute value of the density of the photospheric vertical electric currents $|j_z|$. We note that to obtain an approximation of the PDF it is necessary to normalize the set of $|j_z|$ that is used to the overall number of points. However, we decided to normalize to the maximum so that the values of the histograms ranged from 0 to 1. This had no effect on the results of this work.

Figure 1 shows some examples of these histograms (of the distributions) of $|j_z|$ on a log-log scale for two active regions at times before and after a flare. The histograms are similar for all the other events. For low values of $|j_z|$ the distribution has a Gaussian “bell” shape, and for higher values, an inclined decaying “tail.” Given the shape of the distribution of $|j_z|$, we set up an approximation with a model consisting of a folded normal distribution for low values of $|j_z|$ and a falling power-law function for higher values. The data separate into two sets: in the first $n > 5$ points (bins), as $x (|j_z|)$ increases and in the second, the next $(N-n) > 5$, where $N$ is the total number of points. Here the two sets of data have a single common point which we refer to below as the transition point (tp). The first set of data is approximated (on a log-log scale) by a folded normal distribution for the modulus of the random variable [33,34] and the second, by the power-law function

$$f(x) = \begin{cases} 
0, & x < 0 \\
\frac{a e^{-|x|/c} + a e^{-|x+b|/c}}{dx}, & 0 \leq x \leq x_p \\
x \geq x_p.
\end{cases}$$

The transition point $x_p$ is chosen by minimizing the absolute value of the linear deviation of the model from the data, $|y_{\text{data}} - y_{\text{model}}|$.

All the approximations were made using the MATLAB function “nlinfit”, employing a Levenberg-Marquardt algorithm to solve the least squares problem. For each case the quality of the approximation was determined with the aid of a corrected determination coefficient:

$$R^2_{\text{adj}} = 1 - \frac{SS_{\text{res}}/(n-k)}{SS_{\text{tot}}/(n-1)},$$

where $SS_{\text{res}}$ is the sum of the squares of the regression residuals, $SS_{\text{tot}}$ is the overall sum of the squares, $n$ is the number of observations, and $k$ is the number of model parameters. When $R^2_{\text{adj}}$ is closer to 1, the model is closer to the data.
After the model parameters were determined for all the active regions examined here, we tested their correlations with the x-ray class of the flares (according to data from the Geostationary Operational Environmental Satellite, GOES spacecraft), before the start of which the distributions of $|j_z|$ were counted, as well as the Hale magnetic class of the parent active regions (Mount Wilson classification). This auxiliary information was taken from the site https://solarmonitor.org/.

Fig. 2. Histograms of the model parameters obtained for 96 distributions of $|j_z|$ (48 each before and after a flare): a) mathematical expectation of the folded normal distribution, b) standard deviation of the folded normal distribution, c) transition point between the folded normal distribution and the power-law function, d) absolute value of the power-law exponent. The results of an approximation of the histograms by Gaussian functions are shown by thick black curves. The mathematical expectation $\mu$ and standard deviation $\sigma$ of the Gaussian function are given in the upper right hand corner of the corresponding panels.
3. Results

A visual analysis showed that the model (2) provides an adequate approximation of the structure of the distribution of $|j_z|$ for all 48 of the regions examined here. Of the 96 distributions of $|j_z|$ (for 48 before and after a flare), only 34 (35%) turned up with $R_{adj}^2 < 0.95$. Figure 1 shows some typical examples of using the model for two events: 1) SOL2014-09-24T17:45 in the active region NOAA 12172 and 2) SOL2011-08-03T04:29 in the active region NOAA 11263. For the active region NOAA 12172 two SHARP vector magnetograms were used for the times 17:36 UT before and 18:00 UT after the flare; for NOAA 11263, the two times were 04:24 and 04:36 UT. It may be noted that for NOAA 12172 (Fig. 1, left frame) there is a deviation of some of the data points from the model in the region of high $\ln|j_z| > 10$. This may be related both to the low statistics of the data points in the “tail” of

![Graphs showing model parameters](image)

**Transition point before [statamperes/cm²]**

**Power-law exponent before**

**Math. expectation after [statamperes/cm²]**

**Standard deviation after**

**Math. expectation before [statamperes/cm²]**

**Standard deviation before**

**Transition point after [statamperes/cm²]**

**Power-law exponent after**

Fig. 3. Model parameters for the distributions of $|j_z|$ of the 48 active regions examined here before (x axis) and after (y axis) the flare: a) mathematical expectation of the folded normal distribution, b) standard deviation of the folded normal distribution, c) transition point between the folded normal and the power-law distributions, and d) the absolute value of the power-law exponent. The errors in the determination (except a) of the parameters are indicated by thin horizontal and vertical segments. The dashed line is the function $y = x$. The correlation coefficients $cc$ are indicated in the lower right corner of all the figures.
the distribution and to real deviations from the power-law for high currents. A study of this question goes beyond the scope of this paper.

Figure 2 shows histograms of the distributions of the major parameters of the model: (a) the mathematical expectation of the Gaussian distribution (numerically equal to the parameter \( b \) from Eq. (2)), (b) the mean square (standard) deviation of the Gaussian distribution (numerically equal to \( c/\sqrt{2} \) from Eq. (2)), (c) the transition point (\( x_{tp} \) from Eq. (2)), and (d) the absolute value of the power law exponent (\(|p|\) from Eq. (2)). The resulting distributions are approximated by Gaussians with the mathematical expectation \( \mu \) and mean square deviation \( \sigma \) as parameters shown in the labels of the corresponding graphs (Fig. 2): (a) the mathematical expectation of the Gaussian distribution 19 ± 4 statamperes/cm², (b) the mean square deviation of the Gaussian distribution \( (2.9 \pm 0.3) \times 10^3 \) statamperes/cm², (c) the transition point to the power-law function \( (9.1 \pm 2.7) \times 10^3 \) statamperes/cm², and (d) the absolute value of the power law exponent 3.72 ± 0.78, respectively. We have also calculated \( \mu \) and \( \sigma \) for these model parameters directly without the Gaussian approximation: (a) 18 ± 6 statamperes/cm², (b) \( (2.9 \pm 0.3) \times 10^3 \) statamperes/cm², (c) \( (8.9 \pm 1.9) \times 10^3 \) statamperes/cm², and (d) 3.89 ± 0.96. The values of the model parameters \( \mu \) and \( \sigma \) calculated with the Gaussian approximation are close to the average and mean square deviations calculated directly without approximation.

For all 48 of the active regions examined here we have compared the above model parameters before and after the flares (Fig. 3) and calculated the Pearson linear correlation coefficients for: (a) the mathematical expectation of the Gaussian 0.43 [0.17, 0.64]; (b) the mean square deviation of the Gaussian distribution 0.83 [0.71, 0.90]; (c) the transition point 0.80 [0.67, 0.88]; and, (d) the modulus of the power law exponent 0.92 [0.86, 0.95]. The 95% confidence intervals for the estimates of the correlation coefficients of the parameters are given in square brackets.

![Graph](image.png)

**Fig. 4.** The absolute value of the power-law model exponent used to approximate the distribution of \(|j_z|\) in the active regions before a flare as functions of the base-ten logarithm of the peak x-ray flux of the sun in the 1-8 Å channel of GOES during a flare, i.e., the class of the flare with a negative sign (left), and of the Hale magnetic class of the active regions (right). The errors in the parameter determinations are indicated by thin vertical segments.
The strongest correlation is for the model power law function. No systematic variation in the model parameters is observed during the time of the flare for the set of active regions examined here.

We also checked whether there is an explicit relationship between the parameters of the models and the x-ray class of the flares, as well as the Hale magnetic class of the active regions. To do this the dependences of the x-ray classes of the flares and the Hale classes of the regions on the model parameters were plotted. As an example, Fig. 4 shows plots of the absolute value of the power-law exponent for the model as a function of the flare class (left) and Hale class (right). A visual analysis of the plots revealed no explicit relationship. It may be noted that a large part (29 or 60%) of these active regions had $\gamma$ magnetic class. As it is known that $\gamma$-regions have a tendency to produce large flares, including powerful ones [35], this seems entirely natural, since the selected flares were fairly powerful and accompanied by hard x-ray emission $>$50 keV.

4. Discussion

Based on SHARP_CEA vector magnetograms obtained from observational data from HMI/SDO for a sample of 48 active regions in which flares of different x-ray classes took place (see [22]), we have constructed the distributions of the absolute magnitude of the density of photospheric vertical electric currents $PDF(j_z)$. Beginning with a visual analysis of the form of $PDF(j_z)$, an analytic model has been chosen and a least squares approximation taken. In a first approximation, the shape of $PDF(j_z)$ can be approximated by a folded normal distribution at low values of $j_z$ and a decaying power-law function at higher values. The transition point between the two functions has an average value of $310^{-\tilde{t}p zj}_{stat} \text{statamperes/cm}^2$. We assume that the distribution at low values is determined by the noise in the vector magnetograms that have been used, while the power-law “tail” can be close to the real distribution of $j_z$ and may be related to the physics of the magnetic fields and electric currents in the solar active regions.

To justify the assumption of an instrument (noise) origin of the distribution at low values the distribution of $j_z$ for the entire region determined in SHARP is compared with the distribution calculated only for the outer part of this region. In the outer parts there is no significant magnetic field ($\leq 50$ G), hence $|j_z|$, and they may represent a region of the quiescent sun, where the readouts from HMI/SDO are, at least partially, noise. This was done by examining a plane with a width of 50 pixels along the perimeter of the region. Examples of the distributions of $|j_z|$ for the two regions NOAA 12172 and 11263 are shown in Fig. 5. The distribution of $|j_z|$ for a background region can be represented by a folded normal distribution and has no explicit “tail”, while for an entire active region the distribution has the form of a folded normal distribution at low values and a distinct power-law “tail” at higher values. Here the folded normal distribution for the background region is close to the folded normal distribution for the entire region.

For additional confirmation that the folded normal distribution represents the distribution of the noise in $|j_z|$,
we estimate the error of the transverse to the line of sight magnetic field component $\sigma(B_\perp)$ from the obtained mean square deviation of the Gaussian distribution of $|j_z|$, $\sigma(|j_z|) = (2.9 \pm 0.3) \times 10^3$ statamperes/cm$^2$: $\sigma(B_\perp) = |4\pi \Delta l \sigma(|j_z|)|/[c \sqrt{2}] = 31 \pm 3$ G, where $\Delta l = 3.6 \times 10^7$ cm is the linear size of an HMI/SDO pixel on the photosphere and $c$ is the speed of light in vacuum. The resulting value of $\sigma(B_\perp)$ lies between the threshold values of 20 G (up to 2014) and 50 G (afterward) determined from the transverse component of the magnetic field on the HMI/SDO vector magnetograms (the parameter DOFFSET [29]). This is an important argument in favor of the assumption made here.

One argument that the power-law “tail” in the distribution of $|j_z|$ is not noise may be the fact that the transition point from the Gaussian to the power-law distribution is observed at values ($|<j_z>|_p = (8.9 \pm 1.9) \times 10^3$ statamperes/cm$^2$) close to three times the standard deviation of the gaussian $3\sigma(|j_z|) = (8.7 \pm 0.9) \times 10^3$ statamperes/cm$^2$. This indicates that when studying the vertical currents in the photosphere using HMI/SDO data it is necessary to use the “three sigmas” rule and consider only values exceeding $3\sigma(|j_z|)$, while lower values of $|j_z|$ must be treated with extreme caution.

The presence of a power-law “tail” in the distributions of $|j_z|$ for the active regions of the sun is an interesting fact. This may indicate a specific turbulent character of the processes for formation of the electric currents. Essentially, this is unsurprising, since it is known that the distributions of various characteristics of the photospheric magnetic field have a power-law form, in particular the magnetic flux [36], the derivative of which is $j_z$. The power

![Fig. 5. Distributions of $|j_z|$ on a log-log scale in two active regions NOAA 12172 before the flare SOL2014-09-24T17:45 (left) and NOAA 11263 before the flare SOL2011-08-03T04:29 (right). The circles denote SHARP data obtained for the entire region and the crosses, for the background segment of the quiescent sun. The approximation for the background segment by a folded normal distribution is indicated by a black curve and for the whole region, by a grey curve representing the combined folded normal distribution below the transition point (denoted by the vertical dashed line) and a power-law function above the transition point.](image_url)
spectrum of the magnetic field also has a power-law form [37,38]. A power-law dependence is also inherent in the spatial characteristics of the current helicity of active regions [39]. Power-law distributions are typical for fractal-cluster systems, which may include active regions of the sun (e.g., see Refs. [40] and [41]). We found that for the sample of 48 active regions examined here, the modulus of the power-law exponent in the distribution of $|j_z|$ has a value of $3.89\pm0.96$. The question of which specific processes are responsible for this value requires further study.

We conclude by noting that, in terms of the approach used here, no explicit correlation was found between the parameters of this model of PDF $|j_z|$ for the limited sample of active regions examined here and the x-ray class of the flares that took place in them. This can be interpreted in terms of the fact that the model parameters are determined by the distribution of $|j_z|$ for an entire active region with scales of several hundred angular seconds, while the flare is a local process that usually occupies a small part of the parent active region (a few angular seconds or tens of angular seconds). In the future it will be interesting to study the statistics of the coupling between flare characteristics and the parameters of the local distributions of $|j_z|$ in flare regions, in particular, those adjacent to flare ribbons in the neighborhood of photospheric magnetic polarity inversion lines. In addition, flares usually occur in the vicinity of extrema in $j_z$ [5,13,14], which values contribute to the very “tail” of the distribution of $|j_z|$ where deviations from a power-law dependence may be observed (see Fig. 1). These deviations may be related both to fluctuations owing to the low statistics of the points with extremal values and to the physics of the formation of extremal currents in active regions. A simple approximation of the distribution of $|j_z|$ by a power-law function may be insufficient for clarifying the variations in the distribution during the time of flares. This question requires further study. Finally, the absence of an explicit coupling between the parameters of the distribution of $|j_z|$ and the magnetic class of active regions may be explained by the insufficient variety of the sample being studied here (more than half of the regions being studied have class $\beta\gamma\delta$) or even the extremely descriptive (qualitative) nature of the Hale classification of active regions, without detracting from its advantages.

We thank the team of the Helioseismic and Magnetic Imager experiment on board the Solar Dynamics Observatory spacecraft for the vector photospheric magnetograms of the SHARP series, which are freely available on the Internet. We also thank anonymous referees for a number of useful comments which improved the quality of this article. The initial phase of this study was supported by the President’s International Fellowship Initiative (Grant No. 2018VMB0007) of the Chinese Academy of Sciences. The main part of this work (data processing and analysis, interpretation of results, preparation of the article) was supported by a grant from the Russian Science Foundation (project No. 17-72-20134).

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