Morphological Detector
for Multilevel Signals in \( \epsilon \) - Noise

Sandy Stepanov, Anastasios Venetsanopoulos

Ryerson University
Toronto, Canada

Email: sandy@ee.ryerson.ca

Abstract

The novel approach was developed for multilevel signal detection in channels with impulsive non-Gaussian noise. This approach consists of using morphological nonlinear image filtration principles for two-dimensional signals. It is a new method of signal demodulation, using three-dimensional image processing algorithms. Successful results of this morphologic detector encourage more investigation towards using image processing theory and algorithms for two-dimensional signal processing. As can be seen in the example in section IV, this new approach of reusing well-developed and extensively developing image processing has significantly improved performance.

I. INTRODUCTION

The widely used Gaussian approach for noise pdf relies on assuming that by using limiters the impulsive noise can be transformed to approximately a Gaussian one. This approach works for the two-level case, but for multilevel signals this approach should be reviewed. Consequently, it is not surprising that literature about the comparison between actually achieved and expected communication system performance is very rare, since the expected difference has a dramatic drop in performance. Reference [1] highlights a remarkable analytical example of performance deterioration due to impulsive non-Gaussianity. Known practical ways to achieve the impulse noise immunity are: empirical blanking or cutting noise realization by limiters, using nonparametric statistics theory, and detectors based on Robust Statistic Theory. The suggested approach belongs to the nonparametric statistical theory branch, and specifically to nonlinear filtering. The essential
methodological principle is to use three-dimensional image processing for signals conventionally described by a two-dimensional model. It is advantageous to use a large bank of image processing algorithms in the decoder design. We show here that a tremendous improvement of performance can be achieved even if we are attempting a proof of principle for the first time. Further investigations in this direction are expected to bring more performance gain by a variety of image processing approaches. Our intention of opening a new way for demodulation design is successful and promising as it can be seen from achieved morphological detector performance. The realization complexity is moderate, since all morphological filter calculations are accomplished for logical operations for numbers 1 and 0. Proof of principle is done by using a two-level signal, when it is assumed that the limiter around these signal levels can not be used, since other signal levels can also be expected. Such simplicity is convenient for detecting observing and analyzing process features, when general game rules for conventional multilevel signal demodulation are adhered to: the signal and noise should pass through a receiver filter; after a nonliner signal is detected by a conventional matches filter detector. But the signal can be at a wide range (much more than an unnoisy signal range).

Morphological filtering (MoF) is used as specific signal transformation to cut impulsive realizations from the signal. As can be seen from the performance curves below, as the impulsivity of noise increases the MoF signal BER approaches the ideal one, when an optimal MAP detector is placed at receiver input and the full statistical signal description is known. The result is: significant a high level of efficiency is achieved, despite unknown noise parameters. In other words, an efficient invariant to noise impalsivity detection is attained. It is remarkable that the performance margin is minor for only Gaussian noise. If necessary, the switcher between impulsive and nonimpulsive channel environments can be used to eliminate this shortage for practical use. Last but not least, the gain from morphological detector usage is increased significantly when the error correction coding is used, since coding BER is dramatically expanded even with minor BER improvement at the detection level. What is more, the gain from shifting from one area where the code is inefficient to an area where the code is efficient can not be expressed by the BER improvement measure, since the improvement is qualitative rather than quantitative. In particular this improvement is achieved by using the suggested signal processing.
II. System Model

The mixed Gaussian noise ($\epsilon$-noise) model is a simple approach [2], [3] to the Middleton general Gaussian noise description [4]. It is shown analytically [5] that this noise model is a realistic choice. Analytically, $\epsilon$-noise pdf for noise realization $\xi$, at receiver antenna input is described by

$$\rho(\xi) = (1 - \epsilon)\varphi(\xi, 0, \sigma_1) + \epsilon\varphi(\xi, 0, \sigma_2),$$  \(1\)

where

- $\rho(\xi)$ is the pdf of the stochastic process $\xi$;
- $\varphi(\xi, 0, \sigma_1)$ is the gaussian pdf of the background noise stochastic process with mean 0 and std $\sigma_1$;
- $\epsilon$ is the probability of the impulsive noise; relatively, (1-$\epsilon$) is the probability of background noise only;
- $\varphi(\xi, 0, \sigma_2)$ is the gaussian pdf of the stochastic process of the sum of background and impulse noise with mean 0 and std $\sigma_2 \gg \sigma_1$.

Physically, this model has a clear interpretation: with probability (1-$\epsilon$) the background noise is generated by a noise source; with probability $\epsilon$, impulse noise and background noise are generated. Both pdfs are Gaussian; therefore, the overall pdf for the sum of background and impulse noise is Gaussian, too. In the case of impulse noise generation by military jamming [6], the description is similar. The linear filtration in receiver input is a realistic scenario, so detector input noise is the convolution of noise with impulse response $h(t)$ of the receiver filter Matlab was used for $h(t)$ calculation, so $\sum_i h_i = 1$. The effective noise rectification by a limiter can be used for only two signal levels, whereas modern communication systems use multilevel signals. WiFi and WiMax standards are examples of such systems. It is natural to look for algorithms with limiter abilities applicable to multilevel signals, too. Where whitening relates to the whitening operation of color noise, the term ”gaussining” applies to the approach taken in this paper. The filtering process makes the $\epsilon$-noise vary gradually according to $h(t)$ length. This effect means that now we have ”mountains” and ”valleys” with durations approximating $h(t)$ length. For example, we used $h(t)$ with a number of significant samples, amounting to nearly 10 percent of the number of symbol samples (see Fig.1). In this case, we used 70 samples.

The critical issue of the model is to find the potential bound. This bound should be used
for algorithm effectiveness estimation and for trade-off design in practical use, when a designer optimizes the algorithm by balancing effectiveness of mutual criteria against cost. The first straight-forward decision is to use Maximum a Posteriori (MAP) detector for pdf (1) when noise is not distorted by a filter. This results in a nonfeasible detector at antenna input. When the parameters $\epsilon, \sigma_1, \sigma_2$ are known, the detector determines that the transmitted symbol is more probable if

$$\prod_i \rho(\xi_i) > \prod_i \rho(\xi_i + e_i)$$

(2)

where $e_i$ is the difference between sample number $i$ of the transmitted symbol and its alternative.

The BER curve for this detector seems to be good enough to be potentially achieved. The parameter $\epsilon$ is difficult to estimate, so the next candidate for potential reference is the detector, which somehow knows whether the background noise or the sum of background noise and impulsive noise is at an input of the antenna. The decision is made in favor of the transmitted symbol if

$$\prod_i (k_i(1 - \epsilon)\varphi(\xi, 0, \sigma_1) + (1 - k_i)\epsilon\varphi(\xi, 0, \sigma_2)) >$$

$$\prod_i (k_i(1 - \epsilon)\varphi(\xi + e_i, 0, \sigma_1) + (1 - k_i)\epsilon\varphi(\xi + e_i, 0, \sigma_2))$$

(3)
where \( k_i \) is the indicator, whether the noise is only background noise or the sum of background noise and impulsive noise. Last but not least, the model is designed for proof of the principle case, so it should be as general as possible. For this reason, a rectangular symbol shape was chosen. The test case investigates model behavior for two signal levels. Here, limiting or blanking by using information about possible signal levels is impossible, since it is assumed that other signal levels exist.

### III. Morphological Detector

As mentioned above, the signal after the input filter looks like a mountain landscape image, so it is logical to use image processing methods. Moreover, there is impulse noise; therefore, image processing methods for nonGaussian noise can be a good choice. Analysis of leads to the conclusion that MoF can be used for signal smoothing, in order to eliminate distractive impulse noise influence. The first view decision is to use a conventional MoF consisting of a sequence of opening and closing operations

\[
\tilde{S} = (S \circ SE) \bullet SE,
\]

where

- \( S \) is signal image;
- \( SE \) is structure element, in the current example a line with length 15 is used for \( SE \).

However, as shown below, the double use of MoF is required when the second use is for the symmetric version of the signal image. We need two interfaces: one to convert the data signal \( s \) to image \( S \) and the second to convert filtered \( S \) to a signal, in order to make a decision. The first interface starts from

\[
S(i, j) = 1, i = 1, 2, ...N \text{ and } j = 1, 2, ...M,
\]

where \( N \) is the number of signal discretization levels and \( M \) is the number of samples for symbol. In our example case \( N = 300 \) and \( M = 70 \). It is important to stress that in OFDM systems, values of \( M \) tend to be large. Therefore, there is no requirement for dramatic hardware changes to implement MoF in OFDM systems, which use one-carrier signal, too. The next step is

\[
S[i, j = 1 : (S_{int}(i) + V/2)] = 0, \quad i = 1, 2, ...M,
\]
where \( V = N/2 \), \( S_{\text{int}} = \text{round}(s \cdot K) \), i.e. \( S_{\text{int}} \) is the quantized version of the signal, using scale factor \( K \).

The second interface calculates the filtered signal

\[
s_1(i) = \left[ \sum_j \tilde{S}(i, j) \right] - V. \tag{6}
\]

One of the main problems is the asymmetry of MoF. We overcome this by calculating the symmetrical image \( S_s \) as described below, and applying MoF once again to this new image converting the result to signal \( s_2 \). Mathematically, the symmetrical image \( S_s \) is calculated, according to following two steps. The first step is

\[
S_s(i, j) = 1, i = 1, 2, \ldots N \text{ and } j = 1, 2, \ldots M.
\]

The second step is

\[
S_s[i, j = 1 : (-S_{\text{int}}(i) + V/2)] = 0, i = 1, 2, \ldots M. \tag{7}
\]

Then, MoF is applied:

\[
\tilde{S}_s = (S_s \circ SE) \bullet SE. \tag{8}
\]

The transformation to a signal is carried out with analogy to (6) using as answer the value \( s_2 \). The final operation is to find the average signal

\[
s_r = (s_1 + s_2)/2 \tag{9}
\]

and to find which level 1 or -1 is more probable, by using as answer the value

\[
R = \text{sign} \sum_i s_r(i). \tag{10}
\]

IV. SIMULATION RESULTS

General understanding of MoF detection can be obtained from looking at Fig. 2, showing the detecting process. From Fig. 2, it can be seen how strong noise pulses in the signal can be cut by the MoF. Each of these pulses can shift the decision to the wrong polarity, but all of them are easily removed by MoF. In spite of the minor negative pulse which is not aligned, the decision that the overall signal is negative is right. The additional resource - to make algorithm
The detecting process illustration for right decision that level "1*K" was transmitted ($K = 10$), solid line corresponds to MoF applied to signal image, dashed line corresponds to MoF applied to symmetrical signal image, the dashed line with dots corresponds to average decision.

enable side pulses to be cut - can be simply applied to the waveform in Fig. 2. For example the signal can be synthetically prolonged to the left and right, but this is beyond the scope of this paper, since we only show the proof of principle for MoF detection. From a theoretical point of view, it is important to investigate the tendency of BER change variations in (a) impulse noise probability; (b) Gaussian noise level; (c) impulse noise level. The relations between noise and signal levels may be conveniently investigated when BER curves are depicted for noise std values, rather than for the relation between powers of noise and signal. The reason for this is that it is important for the morphological filter to be able to cut the strong noise pulses. Therefore, the BER curves are represented by BER dependency on the std of the sum of the impulse noise and background noise for a fixed background noise level in contrast to the more conventional method of BER performance representation. On the one hand, the impulse noise level must be large enough to be able to cause errors. Therefore, it is much greater than the worst background noise level from Fig. 3, where only Gaussian noise is applied. On the other hand, the impulse noise level significantly but not greatly exceeds 256 QAM signal maximum levels. The analysis of Fig. 3 shows modest a margin for the MoF detector for only Gaussian noise conditions. For nonGaussian noise as the support point, the set $\epsilon = 0.01$ and $\sigma_1 = 2$ is chosen and then: (a) impulse noise probability was reduced to $\epsilon = 0.001$; (b) the background noise level was reduced to $\sigma_1 = 1$. The initial impulse noise set was represented by a typical value of impulse
noise probability, when the background noise role is to provide reasonable BER for practical use (see Fig. 4). The gain from using MoF is obvious. The next point is to check how noise impulsiveness, increased by reduction of epsilon, influences performance. It is possible to see from Fig. 5, that MoF gives further BER closure to optimal detectors and its behavior is the same as the optimal one, when $\epsilon$ is reduced by a factor of 10. The BER for MoF is limited by MoF performance in only Gaussian noise. In Fig. 5 MoF BER is approximately $10^{-4}$ and in Fig. 3 there is the same BER for the same $\sigma_1 = 2$. The behavior of MoF is similar to that of an optimal detector. Optimal detection performance at Fig. 5 is defined by BER in only Gaussian noise for
the same $\sigma_1 = 2$. Both optimal detectors have the same performance, which is defined only by background noise. This shows that an efficient detector can not be given parameter $\epsilon$, but only should recognize whether the noise generated results from background noise or impulse noise. This once again shows that empirical limiting has potentially near optimal detector abilities, since the use of the limiter enables the detector to make a decision only by analyzing low level background noise. One important conclusion follows from this observation: influence of impulse noise can be potentially eliminated, so further improvement of MoF is possible and will bring significant gain. Further noise impulsivity increased by background noise influences reduction, leading to further detection efficiency of MoF, as can be seen in Fig. 6. As a result, all mistakes

Fig. 6. Performance for $\epsilon$ noise for $\epsilon = 0.01$ and $\sigma_1 = 1$. 

March 4, 2015 DRAFT
are caused by impulse noise, and MoF is efficient particularly for this kind of noise. The optimal
detector performance can not be received by simulation, since background noise influences is
negligible for the condition of Fig. 6. The demonstration of quantitative gain is very impressive.
Where as MoF is not given any information about noise pdf parameters when optimal detectors
are given all the information. One more impressive note is that MoF accomplishes only binary
numbers operations in comparison with the very complicated calculation of optimal detectors.

The simulation was carried out until at least 100 mistake symbols occurred to ensure reliable
results.

V. CONCLUSION

A new nonparametric detection approach for using image signal processing for multilevel
signal which bears digital data is suggested. The high potential ability of this approach was
shown using a detector based on a Morphological filter. This successful proof of the principle
encourages more investigation of image processing usage in the area of conventional detection.

REFERENCES

[1] Kim Yoo-Shin, “Performance of high level QAM in the presence of impulsive noise and co-channel interference in multipath
fading environment,” IEEE Trans. Broadcasting, vol. 36, pp.170-174, June 1990.
[2] B. Aazhang and V. Poor, "Performance of DS/SSMA Communications in Impulsive Channels-Part I: Linear Correlation
Receivers," IEEE Trans. Commun., vol. 35, pp. 1179-1181, Nov. 1987.
[3] S. A. Kassam, Signal Detection in non-Gaussian noise. New York: Springer Verlog, 1988.
[4] D. Middleton, "Non-Gaussian Noise Models in Signal Processing for Telecommunications: New Methods and Results for
Class A and Class B Noise Models," IEEE Trains. Inform. Theory, vol. 45, pp. 1129 - 1149, May 1999.
[5] J. H. Miller and J.B. Thomas, “The detection of signals in impulse noise modelled as mixture process,” IEEE Trans.
Commun., vol. 24, pp. 559-562, 1976.
[6] J. K. Holmes, Spread Spectrum Systems for GNSS and Wireless Communications. Artech House Publishers, 2007.
[7] I. Pitas and A. N. Venetsanopoulos, Nonlinear Filters in Image Processing: Principles and Applications. Boston: Kluwer
Academic Publishers, 1990.