Pre-service teachers’ challenges in presenting mathematical problems

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Abstract. The purpose of this study was to analyzed how pre-service teachers prepare and assigned tasks or assignments in teaching practice situations. This study was also intended to discuss about kind of tasks or assignments they gave to students. Participants of this study were 15 selected pre-service mathematics teachers from mathematics education department who took part on microteaching class as part of teaching preparation program. Based on data obtained, it was occasionally found that there were hidden errors on questions or tasks assigned by pre-service teachers which might lead their students not to be able to reach a logical or correct answer. Although some answers might seem to be true, they were illogical or unfavourable. It is strongly recommended that pre-service teachers be more careful when posing mathematical problems so that students do not misunderstand the problems or the concepts, since both teachers and students were sometimes unaware of errors in problems being worked on.

1. Introduction
Mathematicians as well as mathematics educators have turned their attention to teacher’s knowledge. In the last decade, researchers in mathematics education have begun to specify and assess teachers’ knowledge in new ways [1]. At the same time, there is a general expectation that teachers have enough mathematical knowledge to teach. In mathematics teaching, knowledge for teaching is necessary because it will direct a teacher in deciding what to do in class, including considering examples to be given, the form of presentation to be used. This knowledge will also help teachers determine what relation or connection will be used during the learning process, including how to respond to students’ thinking. Knowledge for teaching will also affect teachers’ preparation in organizing class.

For mathematics teachers, teacher development is an extensive and thriving research area around the world. It faces multidimensional challenges as regards both pre-service and in-service teachers’ programs [2]. Focus on pre-service mathematics teachers, professional development can be supported by all parties, and university mathematics education play an important role when teaching their students as prospective mathematics teachers. One aspect that can be developed during their teaching practice is their skill of posing mathematical problems.

2. Mathematical Problems
Problems abound in mathematics education at all level [3]. There are many kinds of mathematical problems described in the literature. Many researchers also classified problems in different ways. Souviney in [4] classified as routine story or word problems and non-routine problems. If word or
story problems can be solved by applying previously learned concepts and skills, non-routine problems, on the other hands, cannot be solved by only selecting and applying one operation as world problems. Solving non-routine problems requires flexible thinking.

Another way to classify mathematical word problems is to group them into standards and non-standard problematic (parallel) problems. Standard word problems are those that can be solved by applying the most obvious arithmetic operations using given numbers. Non-standard word problems can be solved by using arithmetical operations, but it requires the problem solvers to use their real-life knowledge into account in problems’ context. For non-standard word problems, appropriate mathematical model are less obvious and mathematical modeling assumption is problematic [4].

Students can only be asked to solve mathematical problems if the problems to be solved exist. Even though almost all mathematics textbooks for any level provide mathematical problems, it does not mean that it should depend on textbooks based problems. One aspect of problem solving that school mathematics teachers need to engage in is the art of problem posing [5]. Teachers are also suggested to think of self-generated mathematics problems. If students need to practice how to solve mathematics problems, teachers need to provide good mathematical problems. It leads to the fact that school mathematics teachers need to develop their problems posing skill.

3. Challenge in Posing Mathematical Problems

It has been quite difficult to literarily find the exact definition of problem posing. Some sources describe posing problems in indirect ways. Problem posing is recognized as an important component of mathematics teaching and learning [6]. Similar to this, problem posing has been identified by some distinguish leaders in mathematics and mathematics education as an important aspect of mathematical education [7]. Stoyanora and Ellerton [8] described problem posing as a process of creating meaningful mathematics problem from interpretation established throughout concrete cases and based on mathematical experiences.

Problem posing referred to both generator of new problems and the re-formulating of given problems. Thus, posing can occur before, during and after of the solution of the problems. Problem posing has long been viewed as a characteristic of creativity or exceptional talent. For this reason, Gonzales in [9] noted that teachers have an important role in the implementation of problem posing in the curriculum. Since problem posing has been embedded in the assessment of creativity or mathematical talent, it is reasonable to assume that there are some links between posing and creativity [7]. Crespo in [10] identified three different approaches that pre-service teachers used in posing problems to pupils: i) making problems easier to solve, ii) posing similar problem, and iii) posing problems blindly.

If we referred to Crespo, problems which are easy to solve, could be kind of forms where hints were provided to lead someone to as solution strategy or answers. Familiar problems could be kind of problems as quick translation story problems or computational exercise, and blindly problems refer to problems without solving beforehand or deeply understanding mathematics.

Problem solving can also be considered as necessary part for pre-service teachers because teaching entails posing good question would impact students’ development of mathematical understanding. It means that one excellent way for pre-service teachers to practice posing good questions is to apply problem posing in their teaching practice. Many researchers in mathematics education have also been highlighted on relationship between pre-service teachers and their classroom practices. However, for pre-service teachers who have been prepared to be mathematics teachers, practicing to pose mathematical problem could be kind of challenge of learning to teach mathematics. Pre-service teachers may learn this essential practice when they took part on practice teaching during teaching preparation program. Pre-service mathematics teachers can get many benefits when generating problems. First, since problem posing can be viewed as a window into mathematical understanding, pre-service teachers will be more motivated to master the concept better before posing problems in their class. Second, in posing mathematical problems, pre-service teachers are required to be able to
create problem related to the facts or conditions that exist around. Third, problem posing might also help pre-service teacher in reducing their dependency on their textbooks.

It has been asserted in several studies that pre-service teachers have some issues related to problem posing activities [4]. In order that pre-service teacher will gain the knowledge and required confident for incorporating problem posing activities in their next classes, they have to experience it first [9]. However, the challenges faced by pre-service teachers in posing mathematical have not been widely discussed. Considering that posing mathematical problems are not always easy, it is important to find out the challenges pre-service mathematics teachers face in posing mathematical problems during teacher education as a means of educating them. Within this context, the study about how pre-service mathematics teachers prepared to give mathematical problems during their teaching practices, are getting important. In this sense, the answers of the following questions were searched.

1. What was the main sources of the mathematical problems the pre-service teachers provide in the classroom?
2. What approaches pre-service mathematical teachers used to pose mathematical problems
3. What types of tasks of posing problems the pre-service teachers during their teaching practice?
4. What are difficulties and challenges facing the pre-service students in posing mathematical problems?

4. Methods
This was a descriptive research to identify how pre-service teacher posed what challenges they faced in posing mathematical problem. Participants of the study were 15 pre-service mathematics teachers enrolled in mathematics teaching practice or micro-teaching class at University of Bung Hatta during February – June 2017 term. All participants had already completed coursework in mathematics teaching method course the earlier semester. Each of 15 pre-service mathematics teachers had a chance to practice twice. For each class, one mathematical problem provided by teachers was selected for analysis. The total 30 of mathematical problems were picked up. Data of the study were collected during mathematics teaching practice course. The main sources of the data for the study were: (i) the collected mathematical problems the pre-service teachers posed during their practice; (ii) the observation sheets described how pre-service mathematics teachers posed mathematical problem during their practice teaching; (iii) discussion’s note written by mathematics educator as course’s supervisor.

5. Results and Discussion
By asking those pre-service teachers about the sources from which of the selected mathematical problem provided during the class, their response can be classified as follow:

| Sources of mathematical problems | Frequency | Percentage |
|---------------------------------|-----------|------------|
| Original from textbooks         | 3         | 10         |
| Slightly revised from textbooks | 7         | 23.3       |
| Self-generated                  | 20        | 66.67      |

Based on the information provided in Table 1, it was found out that the sources pre-service teachers referred when posing problems could be derived from two main sources, course textbook and self-generated. However, part of the problems was taken from textbooks have been slightly revised to be adjust into appropriate situation in the class.

In order to be more focused on the posing problems, the only self-generated problems to be analyzed in more detail. The approach pre-service teacher used in the total of 20 posed mathematical problems can be described as follows
Table 2. The approaches pre-service teachers used in posing mathematical problems.

| Approach used               | Frequency | Percentage |
|----------------------------|-----------|------------|
| Posing problems easy to solve | 9         | 45         |
| Posing familiar problems   | 6         | 30         |
| Posing problems blindly     | 5         | 25         |

From Table 2 above, majority of pre-service teachers tend to use easy problem. They also preferred to provide simple algorithm so it will be easy to check students’ answers. Based on our discussion with pre-service teachers as research participants, it was known that even though some of them were eager to pose familiar problems, but in majority, pre-service teachers tended to pose easier problem for two reasons: i) lack confidents to pose more complicated mathematical problems, and ii) to avoid their own mistake in front of class.

By categorizing as standard and non-standard problems, distribution of mathematical problems which were posed by pre-service teachers can be described in Table 3 as follow:

Table 3. Types of tasks (mathematical problems) given

| Types of Problems/Tasks  | Frequency | Percentage |
|--------------------------|-----------|------------|
| Standard Problems        | 15        | 75         |
| Non-standard Problems    | 5         | 25         |

Based on data provided in Table 3, if we referred to the standard/non-standard problems classified by Kılıç discussed earlier, it was figured out that majority of posed problems were categorized as standard, since those problems could be solved by applying simple arithmetic operations. It also showed that most mathematical problems post by pre-service mathematics teachers reflected that mainly, lower mathematical thinking level needed to solve those problems. These standard problems could be solved by applying the most obvious arithmetical operations using the given numbers.

The language (text) pre-service teacher used in posing problems was analyzed regarding to level of easiness to understand by reading the problems. The result was described in the following table.

Table 4. The language levels of mathematical problems provided by pre-service teachers.

| Language Level | Frequency | Percentage |
|----------------|-----------|------------|
| Not clear      | 4         | 30         |
| Relatively clear | 9       | 45         |
| Clear          | 7         | 35         |

Table 4 told us that most pre-service teachers were able to pose problems which were relatively understandable. However, some posed problems were incomplete and need to be more explained since they do not specifically indicate what was being asked. The example of the case was noted in the following example of posed problem.

Solve the following problem: \( \sin 2x = \cos x, \; 0 \leq x \leq 360 \)

There was no specific clue was given by pre-service teacher regarding to problem \( \sin 2x = \cos x \) above, so students had no idea how to solve problem since there was no specific question to declare.
Based on the observation and note discussion, pre-service teachers faced a plenty of challenge in posing mathematical problems. There were mainly four challenges they faced were related to various aspects of which can be described in the following table:

| Challenges                          | Number of pre-service teachers face it | Percentage |
|-------------------------------------|----------------------------------------|------------|
| Lack of experience                  | 11                                     | 73         |
| Lack of mastering content knowledge | 10                                     | 67         |
| Eager to find problem issues        | 15                                     | 100        |
| Time allocation                     | 7                                      | 46         |

The Table 5 above describes the fact that majority of students did not face single challenge in posing problems. Data also showed us that all pre-service teachers in this study admitted that finding related issues is part of their challenges; they even argued that the process on how to find related issue was the biggest challenge. They also admitted that other challenges took into account to pose problems. 11 out of 15 pre-service teachers thought of their lack of experiences and lack of mastering content knowledge also influenced their ability in posing mathematical problems.

One unconsidered factor that came up was that time allocation factor. This factor also arose due to the time limitation of teaching practice provided to them. Almost half of them pointed out that their short time in front of class to teach could also be limiting them to be creative in posing problems. This situation should be overcome by having enough preparation before they come to teach.

By looking at more deeply on posed mathematical problems, there were some interesting facts that need to be discussed since there were various errors in the problems.

One interesting thing that appeared what was called a ‘hidden error’, it referred to the existence of the posed problem that did not look like a mistake, so all the works on automatically according to the formula that might apply. However, it was not likely to happen. That kind of posed mathematical problems had the unaware error. When problem was seen there was no problem at a glance, in fact, there was a hidden error in posed problem. The examples of posed problem given by one pre-service teacher were restated as follows:

_Determine the circumference of the symmetric trapezoid in the next figure_

![Trapezoid](image)  

_Figure 1. Trapezoid posed by pre-service teacher to find its circumference._

Figure 1 describes the posing problem which was, at first glance, seemed obvious. The language was clear and understandable as students would easily understood what they were supposed to do. The problem also looked pretty good because it required students to think how to use Pythagorean’s
Theorem to determine the length of the unknown side before adding all the side to get its circumference. However, if the problem was examined more carefully, it would be recognized that according to Pythagorean’s theorem, the trapezoid could in fact not exist since the length of the opposite side was 15 instead of 13. When the particular pre-service teacher who posed that problem was asked whether he was careless about the side which length was 15, he said that it did not affect anything, because circumference was just the sum of all sides of trapezoid that were already known. It also showed us that pre-service teachers who have inadequate meaningful mathematical content knowledge often posed problems with contextual error. The measurement had also no specific dimension or unit, which was not suggested in general mathematics, because circumference was supposed to have its unit.

Another thing that came up from the data was the presence of posed mathematical problems that impressed less rational/realistic. This was seen from the following problem:

*If one paving block has its thickness of 8 cm, what is the height of 55 paving blocks which are arranged vertically?*

At a glance, there was nothing wrong with the problem above. It was just kind of simple problem with simple calculation. The language was also clear that students were asked to do direct multiplication. But if viewed further, this problem seemed less realistic, since it will be difficult to arranged 55 paving blocks vertically in the form of single line

In posing mathematical problems, pre-service teachers sometimes also less concerned with things in real world that seems trivial. For example, the following bathtub problem

*A bath in a shape of cube with the outside wall has its length of 50 cm. If only half-filled, how much volume of water in the tub?*

There was one issue to be addressed in the problem above. One might think that this problem seemed easy to solve directly by calculating the volume of the tub and do some other simple calculations. However, it would seem less logical if the bathtub did not have a thickness or just as thick as a piece of paper. This data also showed that pre-service teachers were sometimes less concerned with the essence of real world problems.

Regarding to posing world problems, it was also found that pre-service teachers were less confident in posing world problems. Majority of posed problems posed by pre-service teachers were not world problems, even if the subject being though was geometry, pre-service teachers were still eager to find issues in real world problems. This because one factor or more combination of challenges the faced we discussed earlier, including lack of experience and/or did not master subject content knowledge.

Some errors in mathematical problems above related to the challenges the pre-service teachers faced in posing problems. Since problem posing could be used as a task to assess students’ mathematical thinking, then this approach would be a good alternative for pre-service teachers to realize their mathematical knowledge and learning from their experiences to resolve and reduces their errors in posing next mathematical problems.

6. Conclusion
This study investigated pre-service mathematics teachers’ challenge and how they pose mathematical problems. Based on the data and discussion, some important aspect could be picked up. Many pre-service teachers felt lack of experiences, and lack of realizing their lack of mastering content knowledge which had influences to their posing problems’ ability. Some of them experienced difficulties in finding related issues so most of them tended to pose non-real world problems. This was also because of what many teachers and textbook perceived to be real life mathematics were not always the same as what people experienced in their daily life. Their focused on how to pose problems were still constrained by what type of questions should be given regarding to the subject, as
the result, they were not aware of the errors appeared in their posed problems. Pre-service teachers still needed to some guidance from their mathematics educators on how to pose mathematical problems that they would use as task or assignment in their class practices.

The main goals of teachers’ preparation and professional development program are in helping pre-service teachers master mathematics content they teach, understand how their students learn, and know how they facilitate their students to learn. For these reasons, pre-service teachers’ ability in posing problems need to be the main concern. This because in posing mathematical problems, pre-service teachers’ venture may have multiple approaches and solutions can also be adjusted to real world situations. The pre-service teachers may pose open-ended tasks which need some explorations by themselves before giving it to their students.

Since it is impossible for someone to teach what he/she does not know, it reminds us that the only way to achieve better mathematics education is to improve mathematics teachers’ capabilities. Professional development of pre-service mathematics teachers becomes necessary in helping them be better prospective mathematics teachers.

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