Asymptotic model of size-exclusion grouting

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Abstract. In the construction of foundations of buildings and structures on fragile ground, various technologies of soil grouting are used. When pouring fine-grained concrete into the porous soil, the concrete grains are filtered in the pores of the soil. The filtration process depends on the ratio of the pore sizes of the soil and the solid particles of the injected concrete mortar.

Injection of a carrier fluid with small solid particles in a porous medium forms a dynamic concentrations front of suspended and retained particles, separating the suspended particles and the hollow part of the porous frame. The purpose of the study is to construct and calculate an asymptotic model near the concentrations front for the filtration of monodisperse suspension in a porous medium with size-exclusion mechanism of particles retention.

The classical mathematical model for one-dimensional filtration of suspensions and colloids in a porous medium is based on the geometric ratio of the particles and pores sizes: the particles freely pass through the large pores and get stuck in the pore throats with sizes smaller than the particles diameter. The model is determined by a system of two quasilinear first-order partial differential equations with the gap between boundary and initial conditions. To construct an asymptotic expansion in the vicinity of the concentrations front, a special small parameter is used that specifies the distance to the front. This parameter provides direct determination of the asymptotic terms from the recurrent system of ordinary linear differential equations of the first order.

Near the concentrations front of the suspended and retained particles, a nonlinear high-order asymptotics is constructed for the filtration problem of solid particles transported by a carrier fluid in a porous medium. The obtained solution is zero before the front and nonzero behind the front. Approximation of the asymptotic expansion is carried out. It is shown that, for a linear blocking filtration coefficient, the asymptotics coincides with the exact solution.

The asymptotic model of deep bed filtration makes it possible to obtain exact formulas for the high-order asymptotic expansion near the dynamic concentrations front of suspended and retained particles. The asymptotics improves the ability to fine-tune the filtration model depending on the properties of the porous soil and the grout.

1. Introduction

Strengthening the foundation is an important stage in the construction of structures on loose ground. Injection of grout slurry into soil is one of modern technologies of soil grouting [1, 2]. When pouring fine-grained concrete into the porous soil, the concrete grains are filtered in the pores of the soil. After solidification the soil becomes solid and waterproof. The filtration process depends on the ratio of the pore sizes of the soil and the solid particles of the injected concrete mortar. Theoretical calculation of the filtration model allows choosing the best parameters of the grout.
In this paper, a mathematical model of particle transport in a porous medium is considered. It is based on the mechanical-geometric interaction of particles with a porous medium called size-exclusion [3, 4]. Other types of solid particles interaction with the porous medium, including diffusion, the influence of the fluid viscosity and electric forces are neglected. A monodisperse suspension - a fluid with identical suspended solids flows through pores of various sizes. It is assumed that the capture of particles by pores is carried out geometrically: if a fluid flow brings a particle to a small pore, it gets stuck in its throat. If the size of a pore is greater than the particle size, then the particle moves freely through the pore. The retained particles cannot be knocked out of the pores by a fluid flow or other particles, remain in the pores forever. The particles trapped in the pores form a deposit – a grout. One particle clogs only one small pore and vice versa, one pore can hold only one particle.

The mathematical model of the filtration process determines the concentrations of suspended and retained particles as functions of time and coordinate. The suspended particles concentration at the porous medium outlet is of particular interest, since its value can be compared with the experimental data [5].

For some filtration models, exact solutions are obtained [6, 7]. In the absence of exact solutions, the asymptotics of the problem can be constructed [8–10].

Assume that the suspension is injected into an empty porous medium. The mobile boundary of the two phases – the concentrations front of the suspended and retained particles moves from the inlet to the outlet of the porous medium. At the boundary, the suspended particles concentration has a gap. The high-order asymptotics of a solution behind the concentrations front is constructed in the article.

Section 2 presents a one-dimensional mathematical model of the particle transport in a porous medium. The asymptotic model in the vicinity of the concentrations front is constructed in Section 3. Section 4 is devoted to the comparison of the asymptotics with the exact solution for linear and quadratic filtration coefficients. Discussion and Conclusion in sections 5 and 6 finalize the article.

2. Mathematical model

The standard mathematical model of filtration in a homogeneous porous medium consists of two first-order partial differential equations: the mass transfer equation and the kinetic equation of deposit growth. These equations form a quasilinear hyperbolic system with unknown concentrations of suspended \( C(x,t) \) and retained particles \( S(x,t) \)

\[
\frac{\partial C}{\partial t} + \frac{\partial C}{\partial x} + \frac{\partial S}{\partial t} = 0; \quad (1)
\]

\[
\frac{\partial S}{\partial t} = \Lambda(S)C. \quad (2)
\]

Here \( \Lambda(S) \) is the filtration coefficient.

The initial and boundary conditions

\[
C|_{t=0} = p, \quad p = \text{const}; \quad (3)
\]

\[
C|_{x=0} = 0; \quad S|_{x=0} = 0 \quad (4)
\]

provide the uniqueness of the solution.

The problem (1)–(4) is considered in the domain

\[
\Omega = \{(x,t) : 0 < x < 1, t > 0\}.
\]

A rigorous derivation of the equations (1), (2) on the basis of microstochastic diffusion equations with subsequent macro-averaging is given in [11].
Note the characteristic features of the system (1)-(4). The boundary conditions (3) and (4) do not match at the origin, hence the solution \( C(x,t) \) has a discontinuity on the concentrations front \( t = x \).

Concentrations of suspended and retained particles have the form

\[
C(x,t) = \begin{cases} 
0, & t < x; \\
> 0, & t > x;
\end{cases}
\quad S(x,t) = \begin{cases} 
0, & t < x; \\
> 0, & t > x.
\end{cases}
\]

(5)

For \( \Lambda(S) = \lambda = \text{const} \) the equation (2) is linear and the system has an exact solution

\[
C(x,t) = \begin{cases} 
0, & t < x; \\
e^{-\lambda t}, & t > x;
\end{cases}
\quad S(x,t) = \begin{cases} 
0, & t < x; \\
p\lambda e^{-\lambda (t-x)}, & t > x.
\end{cases}
\]

(6)

Consider the physical meaning of the particles filtration in a porous medium. In each cross section of the porous medium, the concentration of free small pores decreases with the deposit growth. The retained particles concentration increases with time and reaches its maximum value after a long period of filtration, when all small pores are clogged by deposit particles. If the concentration of small pores decreases, the growth of the deposit slows down, and the suspended particles concentration differs little from the initial concentration at the inlet \( x = 0 \). For \( t \to \infty \) the solution of the system (1)-(5) satisfies the relations

\[
\lim_{t \to \infty} C(x,t) = 1; \quad \lim_{t \to \infty} S(x,t) = S_{\text{max}}; \quad 0 < x < 1,
\]

(7)

where \( S_{\text{max}} = \text{const} \) is the concentration of the maximum limit deposit.

The solution (6) does not satisfy the limiting relations (7). Such a solution is suitable only in a small vicinity of the characteristic \( t = x \), that is, near the concentrations front of the moving suspended particles. It is shown in Sec. 4 that the solution (6) is a zero approximation of the solution of the nonlinear problem in the vicinity of concentrations front.

Thus, the nonlinearity of problem (1)-(4) is essential and is related to the physical meaning of filtration. For the linear filtration coefficient \( \Lambda(S) = \lambda(S_{\text{max}} - S) \) the system (1)-(4) has an exact solution [12]

\[
C_{\text{lin}}(x,t) = \begin{cases} 
0, & t < x; \\
\frac{pe^{\lambda(p-1)x}}{e^{\lambda p t} + e^{\lambda S_{\text{max}} t} - 1}, & t > x;
\end{cases}
\quad S_{\text{lin}}(x,t) = \begin{cases} 
0, & t < x; \\
\frac{S_{\text{max}} (e^{\lambda(p-1)x} - 1)}{e^{\lambda p t} + e^{\lambda S_{\text{max}} t} - 1}, & t > x.
\end{cases}
\]

(8)

It is obvious that the solution (8) satisfies the limiting relations (7).

For an arbitrary smooth function \( \Lambda(S) \), an asymptotic solution is obtained below.

3. Asymptotic model
The asymptotic solution of the problem (1)-(4) is constructed in the domain

\[
\Omega_S = \{0 < x < 1, t > x\},
\]

where the solution (5) is continuous.

The line \( t = 0 \) on which the boundary conditions (4) are imposed, is located outside the domain \( \Omega_S \). To construct a solution in the domain, it is necessary to determine a new condition on the boundary of \( \Omega_S \). A new boundary condition is set on concentrations front \( t = x \)

\[
S(x,t) \bigg|_{t=x} = 0.
\]

(9)

The line \( t = x \) is a characteristic of equation (1). The problem (1)-(3), (9) is called the Goursat problem. The solutions of the systems (1)-(4) and (1)-(3), (9) coincide in the domain \( \Omega_S \) [13].


The first four terms of the asymptotic expansion of the problem (1)–(3), (9) are obtained below. Assume that the function $\Lambda(S)$ can be expanded in powers of $S$

$$\Lambda(S) = \lambda_0 + \lambda_1 S + \lambda_2 S^2 + \lambda_3 S^3 + \ldots, \quad \lambda_0 \neq 0. \quad (10)$$

In the vicinity of the concentrations front, the solution of problem (1)–(3), (9) is obtained in the form of a series in powers of the small parameter $(t-x)$ with coefficients depending only on the variable $x$

$$C(x,t) = c_0(x) + (t-x)c_1(x) + (t-x)^2c_2(x) + (t-x)^3c_3(x) + \ldots; \quad (11)$$

$$S(x,t) = (t-x)v_0(x) + (t-x)^2v_1(x) + (t-x)^3v_2(x) + (t-x)^4v_3(x) + \ldots. \quad (12)$$

Substitution of (11), (12) into equations (1) and (2) and equation of the expressions for the same powers of $(t-x)$ gives the sequence of equations

$$(t-x)^0: \quad c_0' + v_0 = 0; \quad (13)$$

$$(t-x)^1: \quad c_1' + 2v_1 = 0; \quad (14)$$

$$(t-x)^2: \quad c_2' + 3v_2 = 0; \quad (15)$$

$$(t-x)^3: \quad c_3' + 4v_3 = 0. \quad (16)$$

Similarly, substitution of the expansions into equation (2) yields

$$(t-x)^0: \quad v_0 = \lambda_0 c_0; \quad (17)$$

$$(t-x)^1: \quad 2v_1 = \lambda_1 c_1 + \lambda_1 v_0 c_0; \quad (18)$$

$$(t-x)^2: \quad 3v_2 = \lambda_2 c_2 + \lambda_2 (v_0 c_1 + v_1 c_0) + \lambda_2 v_0^2 c_0; \quad (19)$$

$$(t-x)^3: \quad 4v_3 = \lambda_3 c_3 + \lambda_3 (v_0 c_2 + v_1 c_1 + v_2 c_0) + \lambda_3 (v_0^3 c_1 + 2v_0 v_1 c_0) + \lambda_3 v_0^3 c_0. \quad (20)$$

The boundary conditions for the unknown functions $c_i(x)$ are determined from condition (3)

$$c_i\big|_{x=0} = p; \quad (21)$$

$$c_i\big|_{x=0} = 0, \quad i = 1, 2, 3. \quad (22)$$

The solution of the recurrent system (13)–(22) is obtained successively. Substitution of the solution into the expansions (11), (12) gives the asymptotics of the problem (1)–(3), (9)

$$C_{\text{asym}}(x,t) = p e^{-\lambda_0 x} + p^2 \lambda_1 (e^{2\lambda_1 x} - e^{-\lambda_1 x}) (t-x)$$

$$+ p^3 \left( \lambda_1^2 + \frac{1}{2} \lambda_2 \lambda_0 e^{-3\lambda_0 x} - \frac{3}{2} \lambda_1^2 e^{-2\lambda_1 x} \right) (t-x)^2$$

$$+ p^4 \left( \lambda_1^3 + \frac{4}{3} \lambda_2 \lambda_1 \lambda_0 + \frac{1}{3} \lambda_2 \lambda_0^2 e^{-3\lambda_0 x} \right) (t-x)^3 \ldots; \quad (23)$$
\begin{equation}
S^{\text{asymp}}(x,t) = p\lambda_0 e^{-\lambda_0 t} (t-x) + \lambda_1 \lambda_0 p^2 (e^{-2\lambda_0 t} - \frac{1}{2} e^{-\lambda_0 t})(t-x)^2
\end{equation}

\begin{equation}
+ p^3 \left( \lambda_1^2 \lambda_0^2 + \frac{1}{2} \lambda_2 \lambda_0^2 \right) e^{-3\lambda_0 t} - \lambda_1^2 \lambda_0 e^{-2\lambda_0 t} + \frac{1}{6} (\lambda_1^2 \lambda_0 - \lambda_2 \lambda_0^2) e^{-\lambda_0 t} \right) (t-x)^3
\end{equation}

\begin{equation}
+ p^3 \left( \lambda_1^2 \lambda_0 + \frac{4}{3} \lambda_2 \lambda_0^2 + \frac{1}{3} \lambda_3 \lambda_0^3 \right) e^{-4\lambda_0 t} - \frac{3}{2} \lambda_1^2 \lambda_0 + \frac{3}{4} \lambda_2^2 \lambda_0^2 e^{-3\lambda_0 t}
\end{equation}

\begin{equation}
+ \left( \frac{7}{12} \lambda_1^3 \lambda_0 - \frac{2}{3} \lambda_3 \lambda_0^2 \right) e^{-2\lambda_0 t} - \left( \frac{1}{24} \lambda_1^3 \lambda_0 - \frac{1}{4} \lambda_2 \lambda_0^2 \lambda_3 + \frac{1}{12} \lambda_3 \lambda_0^3 \right) e^{-\lambda_0 t} \right) (t-x)^4 + \ldots.
\end{equation}

\section{Results of numerical asymptotic modelling}

Asymptotic expansions of different orders are compared with the exact solution for the linear and quadratic filtration coefficients and constant suspended particles concentration \( p = 1 \) at the porous medium inlet. In the linear case the asymptotic formulas (23), (24) take the form

\begin{equation}
C^{\text{asymp}}(x,t) = e^{-\lambda_0 t} \left( e^{\lambda_0 t} - e^{2\lambda_0 t} \right) (t-x) + \frac{1}{2} \left( e^{\lambda_0 t} - 3 e^{2\lambda_0 t} + 2 e^{3\lambda_0 t} \right) (t-x)^2
\end{equation}

\begin{equation}
+ \frac{1}{6} \left( e^{\lambda_0 t} - 7 e^{2\lambda_0 t} + 12 e^{3\lambda_0 t} - 6 e^{4\lambda_0 t} \right) (t-x)^3 + \frac{1}{24} \left( e^{\lambda_0 t} - 15 e^{2\lambda_0 t} + 50 e^{3\lambda_0 t} - 60 e^{4\lambda_0 t} + 24 e^{5\lambda_0 t} \right) (t-x)^4;
\end{equation}

\begin{equation}
S^{\text{asymp}}(x,t) = e^{-\lambda_0 t} (t-x) + \frac{1}{2} \left( e^{\lambda_0 t} - 2 e^{2\lambda_0 t} \right) (t-x)^2
\end{equation}

\begin{equation}
+ \frac{1}{6} \left( e^{\lambda_0 t} - 6 e^{2\lambda_0 t} + 6 e^{3\lambda_0 t} \right) (t-x)^3 + \frac{1}{24} \left( e^{\lambda_0 t} - 14 e^{2\lambda_0 t} + 36 e^{3\lambda_0 t} - 24 e^{4\lambda_0 t} \right) (t-x)^4.
\end{equation}

In Fig. 1–4 graphs of the exact solution (8) and asymptotics of different orders for the linear filtration coefficient \( \Lambda_1(S) = 1-S \) are presented. Fig. 1, 2 show the time dependence of the suspended and retained particles concentrations at the porous medium outlet \( x = 1 \). Fig. 3, 4 show the dependence of the solution on the coordinate \( x \) for a fixed time \( t = 2 \).

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{figure1.png}
\caption{\( C_{\text{lin}}(x,t) \big|_{t=1} \).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{figure2.png}
\caption{\( S_{\text{lin}}(x,t) \big|_{t=1} \).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{figure3.png}
\caption{\( C_{\text{lin}}(x,t) \big|_{t=2} \).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{figure4.png}
\caption{\( S_{\text{lin}}(x,t) \big|_{t=2} \).}
\end{figure}

Exact solution for the quadratic filtration coefficient \( \Lambda_2(S) = 1-S^2 \) in the domain \( \Omega_x \) is given by the formulas [12]

\begin{equation}
C^{\text{quad}}(x,t) = \frac{e^{-\lambda_0 t} \left(1 + e^{2(t-x)}\right)}{\sqrt{4 e^{-2(t-x)} + e^{-2x} \left(1 - e^{-2(t-x)}\right)^2}}; \quad S^{\text{quad}}(x,t) = \frac{e^{-\lambda_0 t} \left(1 - e^{2(t-x)}\right)}{\sqrt{4 e^{-2(t-x)} + e^{-2x} \left(1 - e^{-2(t-x)}\right)^2}}.
\end{equation}

The asymptotic expansions of the solution (27) are obtained with an accuracy \( O(t-x)^6 \)

\begin{equation}
C^{\text{asymp}}_{\text{quad}}(x,t) = e^{-\lambda_0 t} + \frac{1}{2} \left( e^{\lambda_0 t} - e^{-\lambda_0 t} \right) (t-x)^2 + \frac{1}{24} \left( e^{\lambda_0 t} - 10 e^{-\lambda_0 t} + 9 e^{-3\lambda_0 t} \right) (t-x)^4;
\end{equation}

\begin{equation}
S^{\text{asymp}}_{\text{quad}}(x,t) = e^{-\lambda_0 t} \left(1 + e^{2(t-x)}\right).
\end{equation}
\[ S_{\text{quad}}^{\text{asy}}(x,t) = e^{-x}(t-x) + \frac{1}{6}(e^{-x} - 3e^{-3x})(t-x)^3 + \frac{1}{120}(e^{-x} - 30e^{-3x} + 45e^{-5x})(t-x)^5. \] (29)

Fig. 5–8 show graphs of the exact solution (27) and asymptotics (28), (29) of different orders for the quadratic filtration coefficient \( \Lambda_2(S) = 1 - S^2 \).

Calculations prove that with an increase in the expansion order, the asymptotics approaches an exact solution. For a fixed \( x = 1 \), the asymptotics with respect to time is applicable for \( t \leq 3 \), and for a fixed \( t = 2 \), the asymptotics with respect to coordinate – for \( x \geq 0.6 \). Graphs of the solutions in Fig. 1–8 satisfy the limit conditions (7).

5. **Discussion**

The phenomenological large-scale model of size-exclusion deep bed filtration can be obtained by averaging the stochastic one-dimensional population balance model with pore size distribution and slow fines migration [14]. Therefore, any solution of the large-scale system allows for downscaling, including explicit formula for timely evolution of pore size distribution during the injection.

The concentrations front is a shock front for suspended particles concentration and a weak discontinuity for retained particles concentration. The constant speed of the concentrations front is equal to the carrier fluid velocity.

At the first stage after the breakthrough moment, the linear approximation gives a good result. As the time increases, the filtration process becomes nonlinear and higher order approximations have to be used.

The model of the filtration problem assumes that the porosity and permeability of the porous medium are constant with long-term deep bed filtration. However, the accumulation of deposit in the small pores changes the structure of the porous medium. More complex filtration models suggest that the porosity and permeability depend on the retained particles concentration [15, 16]. In this case the mass transfer equation is quasilinear. Formulas for the asymptotic solution become cumbersome, and only the first order approximation can be used [17]. To obtain a global solution numerical methods are used [18–20].

The size-exclusion model of deep bed filtration with discontinuous solutions on the concentrations front is one of the approximations of a real filtration process in a porous medium. Assumption for the diffusion of particles leads to a mathematical problem with a continuous solution that changes rapidly near the front. The discontinuous solution is a good approximation of the physical process outside a small vicinity of the concentrations front.

6. **Conclusions**

The mathematical model of filtration determines the transport and deposition of solid suspended particles in a porous medium. The geometric mechanism of particle capture, called size-exclusion, determines the process of clogging the pores with sizes smaller than the particles diameter.
The asymptotics of the filtration problem of a suspension in a porous medium is constructed behind the concentrations front. Before the front, the solution is zero, behind the front the solution is positive. On the front, the solution has a gap.

The high-order asymptotics well approximates the exact solution of the problem not only in a narrow strip near the concentrations front, but also in the entire domain $\Omega_S$ behind the front. Asymptotic methods can be effectively used for solving filtration problems that do not have an exact analytic solution.

The asymptotics of the filtration problem determines the dependence of the solution on the controlled parameters of the suspension in an explicit form. Variation of parameters allows the construction engineers to select the right way to grout the soil and optimally choose the composition of concrete mortar [2].

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