On Newton’s third law and its symmetry-breaking effects

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Abstract
The law of action–reaction, considered by Ernst Mach as the cornerstone of physics, is thoroughly used to derive the conservation laws of linear and angular momentum. However, the conflict between momentum conservation law and Newton’s third law, on experimental and theoretical grounds, calls for more attention. We give a background survey of several questions raised by the action–reaction law and, in particular, the role of the physical vacuum is shown to provide an appropriate framework for clarifying the occurrence of possible violations of the action–reaction law. Then, in the framework of statistical mechanics, using a maximizing entropy procedure, we obtain an expression for the general linear momentum of a body particle. The new approach presented here shows that Newton’s third law is not verified in systems out of equilibrium due to an additional entropic gradient term present in the particle’s momentum.

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(Some figures in this article are in colour only in the electronic version.)

1. Introduction

The law of action–reaction, or Newton’s third law (Hawking 2002), is thoroughly used to derive the conservation laws of linear and angular momentum. Ernst Mach considered the third law as ‘his most important achievement with respect to the principles’ (Mach 1960, Jammer 1999)). However, the reasoning used primarily by Newton applies to point particles without structure and is not concerned with the motion of material bodies composed of a large number of particles, in or out of thermal equilibrium.

Ernst Mach sustained that the concept of mass and Newton’s third law were redundant; that in fact it should be enough to define operationally the mass of a given body as the unit of mass to be sure that ‘If two masses 1 and 2 act on each other, our very definition of mass asserts that they impart to each other contrary accelerations, which are to each other respectively as 2:1’ (Mach 1960). Yet philosophy has delivered us extraordinary new insights into a basic understanding of the underlying physics of force. For example, Ravaisson (1999) in the 19th century sustained that within the realm of the inorganic world action equals reaction; they are the same act perceived by two different viewpoints. But in the organic world, whenever more complex systems are at work, ‘Ce n’est pas assez d’un moyen terme indifférent comme le centre des forces opposées du levier; de plus en plus, il faut un centre qui, par sa propre vertu, mesure et dispense la force’1. So, there is in Nature the need for an ‘agent’ that controls and delivers the action from one body to another and this is, as we will see, the role of the physical vacuum or just barely the environment of a body.

We can find in Cornille (1999) a review of the applications of the action–reaction law in several branches of physics. In addition, Cornille (2003) introduced the concepts of spontaneous force (obeying Newton’s third law) and stimulated force (which violates it), clarifying the nature of spontaneous emission with respect to electron accelerators and lasers.

1 ‘It is not enough an indifferent middle agent, like the center of opposed forces acting on the lever; it is necessary an agent that, by its own virtues, measure and control the force’ (translated by the author).
In this paper, we review major aspects of the action–reaction law in the frame of classical mechanics and electrodynamics, as described by the skew rank 2 field tensor $F_{ij} = \partial_i A_j - \partial_j A_i$, which is not affected by the gauge transformation $A_i \mapsto A_i + \text{ie} \partial_i \Lambda$, invariant under the symmetry $U(1)$ group (group of all rotations about a given axis) with Abelian commutation relations (extension to non-abelian SU(2) group (Edmonds 1978, Khvorostenko 1992, Barrett and Terence 1995)) and higher-symmetry forms (Baum et al 1995) may lead to symmetry breaking and the existence of longitudinal electric fields, and these topics are outside the scope of this paper). Also, we intend to show that, in general, for any system out of equilibrium with velocity-dependent entropy terms, Newton’s third law is violated. The need for re-examination of these problems is pressing, since long-term exploitation of the cosmos faces serious problems due to outdated spacecraft technologies mankind possess. And this principle is fundamental and instrumental in understanding physics.

Section 2 offers methodological notes related to the action–reaction law, as it appears in mechanics and electrodynamics. Section 3 discusses the possible role of physical vacuum as a third agent which might explain action–reaction law violations. Sections 4 and 5 discuss the intrinsic violation of Newton’s third law for systems out-of-equilibrium. Section 6 presents the conclusions that follow logically from the previous discussion.

2. Background survey

The usual derivation of the laws governing the linear and angular momenta presented in textbooks is as follows. The equation of motion for the $i$th particle is given by

$$F_i + \sum_{j \neq i} F_{ij} = \frac{dp_i}{dt}, \quad (1)$$

which is Newton’s second law, and where $F_i$ denotes the external force acting on the $i$th particle (due to an external source), $F_{ij}$ represents the internal force exerted on particle $i$ by particle $j$ and $p_i = m_i v_i$. For a single particle, if the internal force $\mathbf{F}$ derives from a potential function $U(r, t)$, then the equation of motion is written as

$$m \frac{d\mathbf{v}}{dt} = -\nabla U. \quad (2)$$

Multiplying by the velocity $\mathbf{v}$, we have

$$m \frac{d\mathbf{v}}{dt} \cdot \mathbf{v} = -\nabla U \cdot \mathbf{v}. \quad (3)$$

From equation (1) we may conclude that, if we assume the validity of the action–reaction law, equation (3) can be written in the form of the law of conservation of energy:

$$\frac{d}{dt} \left( \frac{1}{2} m \mathbf{v}^2 + U \right) = 0. \quad (4)$$

Thus, we can infer that the validity of the law of conservation of energy depends on two assumptions: (i) the internal force is conservative, $\mathbf{F} = -\nabla U$; (ii) the action–reaction law is observed, e.g. for two particles, $F_{12} = -F_{21}$. We can talk of mutual interaction only when Newton’s third law is verified. Along the same line of thought, we define a closed system as one that does obey Newton’s third law; an open system is one that is acted on by external force(s) that by definition does not obey Newton’s third law. When external forces are zero, we say that the system is closed or isolated. These statements will be instrumental in clarifying different situations (see also Cornille 1999).

In the case of central forces, the relation $F_{ij} = -F_{ji}$ is indeed verified, in fact a manifestation of Newton’s third law. Summing up all the particles belonging to the system, we have from equation (1)

$$\sum_i F_i = \sum_i \frac{dp_i}{dt}. \quad (5)$$

Podolsky (1966) called our attention to the discrepancies obtained when directly using Newton’s second law, or by using instead the invariance of the Lagrangian under rotations. In the case of non-central forces, like a system subject to a potential function of the form $V = r^{-1} \cos \vartheta$, we might expect a deviation from Newton’s third law. Indeed, angle-dependent potentials and long-range (van der Waals) forces describe rigorously the physical properties of molecular gases. One can but wonder from which mechanism comes the unbalance of forces.

We might expect that thermodynamics and statistical mechanics both provide a more complete description of macroscopic matter. The internal energy and, in particular, the average total energy of a system $E = \sum_i U_i$, which includes summing up all the particles constituting the system and all storage modes, play a fundamental role together with the entropy of the system. Interesting enough, a microscopic model of friction showed that the irreversible entropy production is drawn from the increase of Shannon information (Diósi 2002).

This question is related to the fundamental one, still not answered by physicists and biophysicists: how can chaos in various natural systems spontaneously transform to order? The observation of various physical and biological systems shows that a feedback is induced according to: \textquoteleft‘The medium controls the object—the object shapes the medium’ (Ivanitskiǐ et al 1991). At the microscopic level, a large class of systems generating directed motion through the interaction of a moving object with an inhomogeneous substrate periodically structured has been studied (Popov 2002). This is the ratchet-and-pawl principle.

The apparent violation of Newton’s third law at the microscopic scale is well known; it occurs, e.g., when two equal-charged bodies having equal velocities in magnitude and opposing directions cross each other. The Lorentz force acting on both electric charges does not cancel each other since the magnetic forces do not actuate along a common line (see also Onoochin’s paradox (McDonald 2006)). The paradox is solved introducing the electromagnetic momentum $[\mathbf{E} \times \mathbf{H}] / c^2$ (values in SI units will be used throughout the text) (Keller 1942).

In the domain of astrophysics the same problem appears again. For instance, based on unexplained astrophysical
observations, such as the high rotation of matter around the center of the Galaxy, a modification of Newton’s equations of dynamics was proposed (Milgrom 1983), while more recently a new effect was reported, about the possibility of a violation of Newton’s second law with bodies experimenting spontaneous acceleration (Ignatiev 2007). In the frame of statistical mechanics, studying the effective forces exerted between two fixed big colloidal particles immersed in a bath of small particles, it has been shown that the nonequilibrium force field is nonconservative and violates the action–reaction law (Dzubiella et al. 2003).

An ongoing debate on the validity of the electrodynamic force law is still raging (Wesley 1996), with experimental evidence that the Biot–Savart law does not obey the action–reaction law (see, e.g., Gerjuoy 1949, Graneau 1982, Graneau and Graneau 2001 and references therein). The essence of the problem stands on two different laws that exist in magnetostatics, giving the force between two infinitely thin line-current elements ds1 and ds2 through which pass currents i1 and i2. Ampère’s law states that this force is given by

$$d^2 F_{2,A} = -\frac{\mu_0 i_1 i_2}{4\pi} \frac{\mathbf{r}_{12}}{r_{12}^3} \left[2(ds_1 \times ds_2) - \frac{3}{r_{12}^2} (ds_1 \cdot \mathbf{r}_{12}) (ds_2 \cdot \mathbf{r}_{12}) \right].$$

(6)

This means that the force between two current elements depends not only on their distance, as in the inverse square law, but also on their angular position (in particular, implicating the existence of a longitudinal force, experimentally confirmed by Saumont (1968) and Graneau (1987) and discussed by Costa de Beauregard (1993) and Martins and Pinheiro (2009)). The other force, generally considered, is given by the Biot–Savart law, also known as Grassmann’s equation in its integral form:

$$d^2 F_{2,BS} = -\frac{\mu_0 i_1 i_2}{4\pi} \frac{1}{r_{12}^3} [(ds_2 \times (ds_1) \times \mathbf{r}_{12})].$$

(7)

Here, r12 is the position vector of element 2 relative to 1. While Ampère’s law obeys Newton’s third law, the Biot–Savart law does not obey it (e.g. Christrodoulides 1988, Graneau 1994, Guala-Valverde and Achilles 2008a,b). The theory developed by Lorentz was criticized by Poincaré (1900), because it sacrificed the action–reaction law.

The problem of linear momentum of a stationary system of charges and currents is far from being resolved too. Costa de Beauregard (1967) pointed out a violation of the action–reaction law in the interaction between a current loop I flowing on the boundary of area A with moment M = IA and an electric charge, concluding that when the moment of the loop changes in the presence of an electric field, a force must act on the current loop, given by F = [E × δR]/c². Shockley and James (1967b) have attributed F to a change in the ‘hidden momentum’ Gh = [E × δR]/c², carried within the current loop by the steady state power flow, necessary to balance divergence of the Poynting’s vector. The total momentum is p = Gf + Gb, where Gb = m(rCM) is the body momentum associated with the center of mass m (Haus and Penfield 1968, Shockley 1968). In particular, it was shown (Shockley 1968) that the ‘hidden linear momentum’ has as the quantum mechanical analogue the term α · E, where α are Dirac matrices appearing in the Hamiltonian form Hψ = iℏ∂ψ/∂t, where H = −icℏα · ∇ is the Hamiltonian operator (e.g. Sakurai 1987). Although certainly an important issue, the concept of ‘hidden momentum’ needs further clarification (Boyer 2005).

Calkin (1971) has shown that the net linear momentum for any closed stationary system of charges and currents is zero, and it can be written as

$$\mathbf{p} = \int d^3 r \left(\frac{\hat{u}}{c^2} \right) = M \mathbf{r}_{CM},$$

(8)

where u is the energy density, M is the total mass M = \int d^3 r (u/c²) and rCM is the radius vector of the center of mass. He has shown, however, that the linear mechanical momentum PME in a static electromagnetic field is nonzero and is given by

$$\mathbf{p}_{ME} = -\int d^3 r \rho A^T.$$

(9)

Here, A^T denotes the transverse vector potential given by A^T = (µ0/4π) \int d^3 r J^T/r. Equation (9) shows that ρA is a measure of momentum per unit volume.

Similar conclusions were obtained by Aharonov et al. (1988) showing, in particular, that the neutron’s electric dipole moment in an external static electric field \mathbf{E}_0 experiences a force given by ma = −(v · ∇)(v × \mathbf{E}_0). The experimental verification of the Aharonov–Cashner effect would confirm total momentum conservation when interactions of magnets and electric charges occur (Goldhaber 1989).

Breitenberger (1968) thoroughly discusses this question, showing the delicate intricacies behind the subject, pointing out the conservation of canonical momentum and the ‘extremely small’ effect of magnetic interactions, making an analysis based on Darwin’s Lagrangian, derived in 1920 (Darwin 1920). Boyer (2006), applying Darwin’s Lagrangian to the system of a point charge and a magnet, has shown that the center-of-energy has uniform motion. Darwin’s Lagrangian is correct to the order 1/c² (remaining Lorentz-invariant) and the procedure to obtain it eliminates the radiation modes and, thus, describes the interaction of charged particles in the frame of an action-at-a-distance electrodynamics. However, it can lead to unphysical solutions (Bessonov 1999).

Hnizdo (1992) has shown that at nonrelativistic velocities, Newton’s third law is verified in the interactions between current-carrying bodies and charged particles because the electromagnetic field momentum is equal and opposite to the hidden momenta, held by the current-carrying bodies; the mechanical momentum of the entire closed system is conserved. Hnizdo has also shown, however, that the field angular momentum in a system is not compensated for by hidden momentum, and thus the mechanical angular momentum is not conserved alone, but has to be summed with the field angular momentum, in order to become a conserved quantity.

In fact, the ‘magnetic current force’, produced by magnetic charges that ‘flow’ when magnetism changes, given by \mathbf{f}_m = ε0 \mathbf{E} × (\mathbf{B} − μ_0 \mathbf{H}) (Shockley and James 1967a), is the ‘Abraham term’, appearing in the Abraham density force \mathbf{f}_A
which differs from the Minkowsky density force $f_M$ through the equation

$$f_M = \frac{\partial}{\partial t}[g^M - g^A].$$  \hspace{1cm} (10)

Here, $g^M = [D \times B]$ is the Minkowsky momentum density of the field and $g^A = [E \times H]/c^2$ is the Abraham momentum density.

3. Interaction with the vacuum

Although Newton’s third law of motion apparently does not comply for some situations, action and reaction are likely to occur by pairs and a kind of accounting balance such as $F = - F'$ holds.

According to Maxwell’s theorem, the resultant of $K$ forces applied to bodies situated within a closed surface $S$ is given by the integral over the surface $S$ of the Maxwell stress tensor:

$$\int_S T(n) dS = \int_V f dv = K. \hspace{1cm} (11)$$

Here, $f$ is the ponderomotive force density and $dv$ is the volume element. The tensor $T(n)$ under the integral in the left-hand side (lhs) of the equation is the tension force acting on a surface element $dS$, with a normal $n$ directed toward the exterior, and it is assumed that the integration is done over a constant volume. In Cartesian coordinates, each component of $T(n)$ is defined by

$$T_x(n) = t_{xx} \cos(n, x) + t_{xy} \cos(n, y) + t_{xz} \cos(n, z). \hspace{1cm} (12)$$

with similar expressions for $T_y$ and $T_z$. The four-dimensional (4D) electromagnetic momentum–energy tensor (in flat spacetime) of rank 2 (with respect to the 3D rotations) is a generalization of the 3D (Maxwell’s) stress tensor $\sigma_{\alpha\beta}$ (in cgs-Gaussian units):

$$\sigma_{\alpha\beta} = \frac{1}{4\pi} \left[ E_\alpha E_\beta + B_\alpha B_\beta - \frac{\delta_{\alpha\beta}}{2} (E^2 + B^2) \right]. \hspace{1cm} (13)$$

The indices $\alpha$ and $\beta$ refer to the coordinates $x$, $y$, and $z$, and $\delta_{\alpha\beta}$ is the Kronecker delta. Since Maxwell, stress has become one of the field properties, in addition to energy, power and momentum, consistent with experimental observations and widely used in numerical field solutions. Usually fields and matter interact, and the stress–energy tensor must be a summation of their respective contributions, $T = T_{\text{matter}} + T_{\text{fields}}$. For convenience, we may here recall that for a viscous fluid, the stress–energy tensor is given by Landau and Lifchitz (1987):

$$T_{ij}^{\text{fluid}} = p \delta_{ij} + \rho \nu_{ij} v_j - \eta \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial v_l}{\partial x_l} \right) \delta_{ij} \frac{\partial v_i}{\partial x_j} \hspace{1cm} (14)$$

Here, $\eta$ and $\zeta$ are the viscous coefficients. For an isotropic body, the stress tensor $\sigma_{ij}^{\text{is}}$ is given by Landau and Lifchitz (2007):

$$\sigma_{\alpha\beta}^{\text{is}} = K u_{\gamma\gamma} \delta_{\alpha\beta} + 2\mu (u_{\alpha\beta} - \frac{1}{3} \delta_{\alpha\beta} u) \hspace{1cm} (15)$$

where $u_{\gamma\gamma}$ is the deformation tensor; $K$ and $\mu$ are, respectively, the moduli of compression and rigidity.

If electric charges are inside a conducting body in vacuum, in the presence of electric $E$ and magnetic $H$ fields, then equation (11) must be modified to the form

$$\int_S T(n) dS - K = \int_V \frac{1}{4\pi c} \left( \frac{\partial[E \times H]}{\partial t} \right) dv. \hspace{1cm} (16)$$

In the right-hand side (rhs) of the above equation, there now appears the temporal derivative of $G = \int g \omega d\Omega$, the electromagnetic momentum of the field in the entire volume contained by the surface $S$ (with $g$ denoting its momentum density). The integrals have to be done over a sufficiently large volume $V(r, t)$ bounded by a closed surface $S(r, t)$ containing all particles and fields.

In the case where the surface $S$ is filled with a homogeneous medium without true electric charges, Abraham proposed to write the following equation:

$$\int_S T(n) dS = \int_V \frac{\partial}{\partial t} \left( \frac{\varepsilon \mu}{4\pi c} [E \times H] \right) dv, \hspace{1cm} (17)$$

with $\varepsilon$ and $\mu$ being the dielectric constant of the medium and its magnetic permeability, and assuming constant volume of integration.

As remarked by Selak et al (1989) and Cornille (2003), if the volume of integration is not constant, equation (16) should be written in the form

$$K + \int_V \frac{1}{4\pi c} \left( [E \times H] \right) dv = \int_{S(t)} \mathbf{T}_{\text{eff}}(n) dS, \hspace{1cm} (18)$$

where the effective stress–energy tensor is given by

$$\mathbf{T}_{\text{eff}} = T - c P, \hspace{1cm} (19)$$

Here, $P_r = \frac{1}{2\pi c} [(E \times B) + PE]$, and $P$ denotes the polarization vector (see Cornille 2003). This transformation is necessary because it is not permissible to substitute a convective time derivative for a Eulerian time derivative when we have a nonconstant and finite volume of integration. The wrong assessment of this problem may lead to contradictions when, e.g., a moving vacuum–plasma boundary is modeled (Bellan 1986). This problem was discussed in Pinheiro (2007), where it has been shown that with the convective derivative, Lorentz’s equation is just an outcome of Maxwell’s equations, and not a necessary condition to complete the system of fundamental equations of the electromagnetic field.

Equation (17) can be written in the form of a general conservation law:

$$\frac{\partial \sigma_{\alpha\beta}}{\partial x^\beta} - \frac{\partial g_{\alpha}}{\partial t} = f_\alpha, \hspace{1cm} (20)$$

where $\alpha, \beta = 1, 2, 3$, $\sigma_{\alpha\beta}$ is the stress tensor, $g_{\alpha}$ is the momentum density of the field and $f_\alpha$ is the total force density. After some algebra, this equation can take the final form (e.g. Ginzburg and Ugarov 1976):

$$\frac{\partial \sigma_{\alpha\beta}}{\partial x^\beta} = f_\alpha + \frac{1}{4\pi c} \frac{\partial}{\partial t} [D \times B]_\alpha + f_{m,\alpha}. \hspace{1cm} (21)$$

Here, $f_m^\alpha$ is the total force acting in the medium (see Ginzburg and Ugarov 1976), $f_\alpha = \rho_e E + \varepsilon [j \times B]$ is the Lorentz force density with $\rho_e$ denoting the charge density and $j$ the current.
density. The second term in the rhs of the above equation could be called the vacuum-interactance term (Cleveland 1996)—in fact, it is the Minkowski term. According to an interpretation of Einstein and Laub (1908), when integrating the above equation over all space, the derivative over the stress tensor gives a null integral, and the Lorentz forces summed over all the Universe must be balanced by the quantity \( \int S dV \) in order to verify Newton’s third law (Cornille 2003). It is important to remark that the field momentum \( \{D \times B\} \) is equivalent to \( \rho \dot{A} \), the first term is related to the stress–tensor representation, while the second one is related to the ‘fluid-flow’ representation (Carpenter 1989). Hence, the last remark drives us to the Machian view of the origin of mass, which had fascinated Einstein to such a degree that he sought to build his general theory of relativity on that ground. Einstein gave the first published reference to Mach’s principle in Einstein (1912): ‘… the entire inertia of a point mass is the effect of the presence of all other masses, deriving from a kind of interaction with the latter’. In this sense, Mach’s principle (supported by Einstein during the early years of his work on general relativity, but not in his later period) seeks to restore the action–reaction law in the entire Universe. Of course, field, matter and physical vacuum together form a closed system and it is usual to catch the momentum conservation law in the general geometric form (Thirring 1927, Lee 1981, Landau and Lifchitz 1987) (see figure 1):

\[
\frac{\partial}{\partial t} \left( T^{\text{Field}}_{\alpha\beta} + T^{\text{Matter}}_{\alpha\beta} + T^{\text{Vacuum}}_{\alpha\beta} \right) = 0.
\]  

(21)

Table 1 shows the different expressions for the energy–momentum tensors of Minkowsky, \( T^{\text{M}}_{\alpha\beta} \) and Abraham, \( T^{\text{A}}_{\alpha\beta} \).

The general relation between Minkowski and Abraham momentum, free of any particular assumption, holding particularly for a moving medium, is given by

\[
P^M = P^A + \int \mathbf{f}^A \, dt \, dv.
\]  

(22)

For clarity, we shall distinguish the following different parts of a system: (i) the body carrying currents and the currents themselves (i.e. the structure, denoted here by \( \mathcal{R} \)), (ii) fields and (iii) the physical vacuum (or the medium).

Table 1. Expressions for the energy–momentum tensors of Minkowsky \( T^{\text{M}}_{\alpha\beta} \) and Abraham \( T^{\text{A}}_{\alpha\beta} \), using \( i, k = 1, 2, 3, 4; \alpha, \beta = 1, 2, 3; x_1 = x, x_2 = y, x_3 = z, x_4 = ct \). The Poynting’s vector is \( S = \mathbf{E} \times \mathbf{H} \) and the energy for a system at rest is \( W = \frac{1}{2} \left( \varepsilon \mathbf{E}^2 + \mu \mathbf{H}^2 \right) \).

| Minkowsky | Abraham |
|-----------|---------|
| \( T^{\text{M}}_{\alpha\beta} = \left( \sigma_{\alpha\beta} - icg^{\alpha\beta} \right) \) | \( T^{\text{A}}_{\alpha\beta} = \left( \sigma_{\alpha\beta} - icg^{\alpha\beta} \right) \) |
| \( g^M = \frac{\varepsilon}{c} (\mathbf{E} \times \mathbf{H}) \) | \( g^A = \frac{1}{c} (\mathbf{E} \times \mathbf{H}) \)

Figure 1. Conservation law for the closed system: matter + field + physical vacuum.

Here, \( \mathbf{f}^A \) denotes the Abraham’s force density Abraham (1910a, 1910b, Pfeifer et al 2007):

\[
f^A = \frac{\varepsilon_r \mu_r - 1}{4\pi c} \frac{\partial [\mathbf{E} \times \mathbf{H}]}{\partial t}.
\]  

(23)

This is in agreement with experimental data (Jones and Richard 1954) and was proposed by others (Gordon 1973, Tangherlini 1975). As this force acts over the medium, it is expected that nonlinearities appear related to the behavior of the dielectric to different applied frequencies, temperature, pressure, and large amplitudes of the electric field, when a pure dielectric response of the material is no longer proportional to the electric field (see, e.g., Böttger (2005) on this topic).

As is well known, Maxwell’s classical theory introduces the idea of a real vacuum medium. After being considered useless by Einstein in his special theory of relativity, the ‘ether’ (actually replaced by the term vacuum or physical vacuum) was rehabilitated by Einstein (1920). In fact, the general theory of relativity describes space as possessing physical properties by means of ten functions \( g_{\mu\nu} \) (see also Ginzburg and Frolov 2002). According to Einstein:

The ‘ether’ of general relativity is a medium that by itself is devoid of all mechanical and kinematic properties but at the same time determines mechanical (and electromagnetic) processes.

Dirac felt the need to introduce the idea of ‘ether’ in quantum mechanics (Dirac 1951). In fact, according to quantum field theory, the particles can condense in vacuum giving rise to space–time dependent macroscopic objects, for example, of ferromagnetic type. Besides, stochastic electrodynamics has shown that the vacuum contains measurable energy, called zero-point energy (ZPE), described as a turbulent sea of randomly fluctuating electromagnetic fields. Quite interestingly, it was recently shown that the interaction of atoms with the zero-point field (ZPF) guarantees the stability of matter and, in particular, the energy radiated by an accelerated electron in circular motion is balanced by the energy absorbed from the ZPF (Kozłowski and...
Marcia-Kozlowska 2002). An attempt to replace a field by a finite number of degrees of freedom was made by Pearle (1971). In this theory, a set of \( N \) particles are supposed to not interact directly with each other, but to interact directly with a number of dynamical variables (called the ‘medium’) carrying the ‘information’ from one particle to another.

Graham and Lahoz have made three important experiments (Graham and Lahoz 1979, 1980, Walter and Lahoz 1975). While the first experiment provided an experimental observation of the Abraham force in a dielectric, the second one has provided evidence of a reaction force which appears in magnetite. The third one gave the first evidence of free electromagnetic angular momentum created by quasistatic and independent electromagnetic fields \( E \) and \( B \) in physical vacuum\(^2\). Whereas the referred paper by Lahoz and co-workers provided experimental evidence for the Abraham force at low-frequency fields, it remains to gather evidence of its validity at a higher-frequency domain, although some methods have recently been outlined (Antoci and Miéhichi 1998).

In view of the above, we will write the ponderomotive force density acting on the composite body of arbitrarily large mass (formed by the current configuration and its supporting structure) in the form (here in SI units)

\[
\rho \frac{\partial \mathbf{V}}{\partial t} = \nabla \cdot \mathbf{T} = \frac{\partial}{\partial t} (\epsilon_0 \mu_0 [(\mathbf{E} \times \mathbf{H})]).
\] (26)

Here, \( \mathbf{T} \) is a dyadic representation of the electromagnetic (stress) force per unit area acting on the surface \( S \); \( -T_{ij} \) is the momentum in the \( i \) direction crossing a surface oriented in the \( j \) direction, per unit area, per unit time. Equation (26) and equation (21) both assume that the energy and momentum density are continuously distributed over the region of space occupied by fields. This gives rise to difficulties with the problem of absorption of light, in particular, when localized discrete particles are considered. For this reason, the above-described continuity equations must be written in integral form. Accordingly, integrating equation (26) over the entire volume of the structure and fields gives

\[
\frac{d\mathbf{P}_{\text{mec}}}{dt} = \int_{V(t)} \mathbf{T} \cdot d\mathbf{S} - \frac{d}{dt} \oint_{V(t)} (\epsilon_0 \mu_0 [(\mathbf{E} \times \mathbf{H})]) dV.
\] (27)

The last integral represents the momenta stored in the electromagnetic field. The surface integral tends towards zero when the radius \( R \) tends to infinity, but when the near-field is taken into account, this may not be true, as they decrease as \( R^{-2} \) (see, e.g., Obara and Baba (2001) for an analytical example), the integral tending to a finite value (Cornille (2003)) since the surface elements \( dS = R^2 \partial\Omega \) increase as \( R^2 \). Hence, the surface integral is not necessarily null, as stated in several textbooks (Cohen-Tannoudji et al 1987, Ginzburg 1989, Landau and Lifchitz 1970), but it is correctly assessed in others (Plonsey and Collin 1961, Becker 1964) (see also Cornille (2003) and references therein). The stress–energy tensor constitutes a powerful tool when studying problems such as levitation (Brant 1989), or the action of the radiation pressure exerted by light on cells, particles and atoms (Ashkin et al 1986), manipulating the concentrated electromagnetic energy in sub-wavelength regions near tips, objects or surfaces.

### 3.1. Examples

#### 3.1.1. Force exerted on an interface between two different media.

The force exerted on an interface between two different media can be obtained by integrating the stress tensor over a cylindrical surface with its base parallel to the interface and subsequently the height of the cylinder tending to zero. This force is given by

\[
f_i = \iiint_S \left[ \epsilon_2 E_{2i} E_{2j} - \epsilon_1 E_{1i} E_{1j} \right. \\
- \frac{1}{2} \delta_{ij} \left( E_2^2 \left( \epsilon_2 - \frac{d\epsilon}{d\eta} \right) \gamma_E^2 \left( \epsilon - \frac{d\epsilon}{d\eta} \right) \right)
\] (28)

where \( \epsilon, \mu, \eta \) are, respectively, the permittivity, permeability and mass density of the medium. When considering non-uniform periodic fields of the form \( \mathbf{E}(r, t) = \mathbf{E}_0(r) e^{i\omega t} \) (most experiments are conducted at optical frequencies), and using the identity \( \Re \langle \mathbf{A} | \mathbf{B} \rangle = 1/2 \Im \langle \mathbf{A} | \mathbf{B}^* \rangle \), with \( \Re \) denoting the real part, equation (28) may be written in the form

\[
\mathcal{T}_i = \frac{1}{2} \Re \iint \left[ \epsilon_2 E_i E_j^* - \epsilon_1 E_{1i} E_{1j} \right. \\
- \frac{1}{2} \delta_{ij} \left( \epsilon_2 |E_2|^2 - \epsilon_1 |E_1|^2 \right) \left]
\] (29)

where \( \mathcal{T}_i \) denotes the time average as given by \( \mathcal{T} = \lim_{T \rightarrow \infty} \int_{-T}^{T} \mathcal{T}(f) df \). Its application to the problem of an oscillating charge \( q = q_0 e^{i\omega t} \) facing a semi-infinite dielectric gives the following average force transmitted by the fields across the dielectric interface (Giner et al 1995, Chaumet et al 2009):

\[
\mathcal{T} = \frac{q_0^2}{32\pi \varepsilon_0 \varepsilon d} \Re \left( \frac{\epsilon - \varepsilon_0}{\epsilon + \varepsilon_0} \right),
\] (30)

where \( d \) is the distance between the oscillating charge and its image.

The role of the stress–energy tensor is made comprehensible considering that the \( E \) and \( B \) near-fields both take place on the physical space and, when a charge is accelerated, there occurs a bending of the lines of force, which subsequently becomes an independent physical entity, detached from the electric charge but not accelerated with the charge (Soker and Harpaz 2004, Martins and Pinheiro 2008). The effect of the self-field on an extended charged particle was shown to contribute to inertia (Martins and Pinheiro 2008).

Hence, the composite body is acted on by the Minkowski force in such a way that

\[
M \mathbf{V} = \mathbf{P}^M - \mathbf{P}^A.
\] (31)

The Minkowski momentum is transferred only to the field in the structure and not to the structure and field in the medium (Skobel’tsyn 1974, Ginzburg and Ugarov 1976, Graham and Lahoz 1980). In summary, to move a spacecraft forward,
the spacecraft must push ‘something’ backwards; and this ‘something’ might be the physical vacuum. This effect was shown to be made feasible, Abraham’s force representing the reaction of the physical vacuum fluctuations to the motion of dielectric fluids in crossed electric and magnetic fluids imparting velocities of the order of 50 nm s$^{-1}$ (Feigel 2004), although this result was contested by van Tiggelen and Rikken (2004). However, the resulting tiny forces produced by the electromagnetic field momentum (or the associated Poynting’s vector) make it difficult to experimentally measure Abraham’s force and weaken the possibility of application of it in field propulsion concepts.

3.1.2. The Graham and Lahoz experiment. Another cornerstone of electrodynamics is the equation of conservation of angular momentum (e.g. Chow 2006):

$$\frac{d\mathbf{L}_m}{dt} = -\frac{d}{dt} \int_{V(t)} \frac{1}{c^2} [\mathbf{r} \times \mathbf{S}] d\Omega - \oint_{\partial S(t)} \left( \mathbf{r} \times \frac{\partial \mathbf{E}}{\partial t} \right) \cdot dS,$$ (32)

where we assumed that the shape of $S(t)$ depends on time. Here, $\mathbf{L}_m$ is the angular momentum of the charges (matter), $[\mathbf{r} \times \mathbf{S}]/c^2$ is the field angular momentum density, and the last term on the rhs is the angular momentum flux of the field with density (tensor) $[\mathbf{r} \times \frac{\partial \mathbf{E}}{\partial t}]$. The component $\beta$ of the surface integral can also be represented in the form $\oint_{\delta S} \mathbf{r} \times \frac{\partial \mathbf{E}}{\partial t} \cdot dS$, with $\delta S$ denoting the totally antisymmetric Levi–Civita symbol (normalized by $\epsilon_{123} = 1$) and $n_\zeta$ is the $\zeta$ component of the unit vector outward normal to the 2D surface $S$. It is worth noting that this is a governing equation similar to equation (20). The so-called Feynman’s paradox (Feynman 1964) has been experimentally reproduced by Graham and Lahoz (1980). In their experiment the torque on a cylindrical capacitor apparently gave evidence of a reaction acting on physical (empty) space. We may note that when the integral on the stress–energy tensor is naturally occurs a violation of the action–reaction law. This result in propulsion (see Obara and Baba (2011)) for a concrete analytical example). Also, propulsion based on Maxwell’s stress tensor has been proposed by Slepian (1949) and Corum et al (1999).

4. Deducing the linear momentum of a body on the basis of statistical physics

When two bodies of matter collide, the repulsive force exerted on them is equal whenever no dissipative process is at play. When a ball rebounds on the floor it has the same total mechanical energy before and after the collision, except for a loss term that is due to the fact that the bodies have internal structure. At a microscopical level, bodies are aggregates of molecules. When the body collides, molecules gain an internal (random) kinetic energy. Macroscopically this generates heat and therefore raises the system entropy. In global terms, some fraction of heat does not return to the particle’s collection constituting the ball and the entropy of the Universe ultimately increases.

Let us consider an isolated material body composed of a large number of macroscopic particles (let us say $N$) possessing an internal structure with a great number of degrees of freedom (to validate the entropy concept) with momentum $\mathbf{p}_i$, energy $E_i$ and with intrinsic angular momentum $\mathbf{J}_i$, all constituted by classical charged particles with charge $q_i$ and inertial mass $m_i$. Using the procedure outlined in Pinheiro (2002, 2004) we can show that the entropy gradient in momentum space is given by

$$\mathbf{p}_i = m_i \mathbf{v}_i + q_i \mathbf{A} + m_i (\omega \times \mathbf{r}_i) - m_i T_i \frac{\partial \mathbf{S}}{\partial \mathbf{p}_i}. \quad (33)$$

It was assumed that all particles have the same drift velocity and they turn all at the same angular velocity $\omega$. The center of mass of the body moves with the same macroscopic velocity and the body turns at the same angular velocity (Landau and Lifschitz 1987). The last term of equation (33) represents the gradient of the entropy in a nonequilibrium situation and $\mathbf{S}$ is the transformed function defined by

$$\mathbf{S} = \sum_{i=1}^{N} \left( S_i \left[ E_i - \frac{p_i^2}{2m_i} - \frac{J_i^2}{2I_i} - q_i V_i + q_i (A_i \cdot v_i) \right] + (a \cdot \mathbf{p}_i) + b \cdot [(\mathbf{r}_i \times \mathbf{p}_i) + J_i] \right), \quad (34)$$

where $a$ and $b$ are the Lagrange multipliers.

Whenever the system is in thermodynamic equilibrium the canonical momentum is obtained for each composing particle:

$$\mathbf{p}_i = p_{i\|} + m_i (\omega \times \mathbf{r}_i) + q_i \mathbf{A}_i. \quad (35)$$

Otherwise, when the system is subjected to forced constraints in such a way that entropic gradients in momentum space do exist, a new expression for the particle momentum must be taken into account, i.e. equation (33).

Summing up over all the constituent particles of a given thermodynamical system pertaining to the same aggregate (e.g. a body or Brownian particle), we obtain

$$\mathbf{P} = \mathbf{M} \mathbf{v}_c + \sum_i m_i (\omega \times \mathbf{r}_i) + Q \mathbf{A} - \sum_i m_i T_i \frac{\partial \mathbf{S}}{\partial \mathbf{p}_i}. \quad (36)$$

To simplify, we can assume that all particles inside the system share the same random kinetic energy, $T_i = \zeta$;

$$\mathbf{P} = \mathbf{M} \mathbf{v}_c + \sum_i m_i (\omega \times \mathbf{r}_i) + Q \mathbf{A} - \zeta \sum_i \frac{\partial \mathbf{S}_{mc}}{\partial \mathbf{r}_i}. \quad (37)$$
where we denote by $\overline{S}_{ne}$ the entropy when the system is in a state out of equilibrium. The first term on the rhs is the bodily momentum associated with the motion of the center of mass $M$; the second term represents the rotational momentum; the third is the momentum of the joint electromagnetic field of the moving charges (Scanio 1980, Fowles 1980); finally, the last term is a new momentum term, physically understood as a kind of 'entropic momentum' since it is ultimately associated with the information exchanged with the medium from the physical system viewpoint (e.g., momentum that is eventually radiated by the charged particle). Lorentz's equations do not change when time is reversed, but when retarded potentials are applied they time the delay of electromagnetic signals on different parts of the system do not allow perfect compensation of internal forces, introducing irreversibility into the system (Ritz 1908). This is always true whenever there is time-dependent electric or/and magnetic fields (Jefimenko 2000). Cornish (1986) obtained a solution of the equation of motion of a simple dumbbell system held at fixed distance and has shown that the effect of radiation reaction on an accelerating system induces a self-accelerated transverse motion. Obara and Baba (2004) have discussed the electromagnetic propulsion mechanism obtained from an electric dipole system, showing that the propulsion effect results from the delay action of the static and inductive near-field created by one electric dipole on the other. These are examples of irreversible (out of equilibrium) phenomena that do not comply with the action–reaction law.

4.1. Example

4.1.1. Missing symmetry. At this stage, we can argue that the momentum is always a conserved quantity provided that we add the appropriate term, in order that Newton’s third law can be verified. This apparent ‘missing symmetry’ might result because matter alone does not form a closed system, and we need to include the physical vacuum in order to restore lost symmetry. So, when we have two systems 1 and 2 interacting via some kind of force field $F$, the reaction from the vacuum must be included as a sort of bookkeeping device:

$$F_{12}^{\text{matter}} = -F_{21}^{\text{matter}} + F_{\text{vacuum}}.$$  \hspace{1cm} (38)

We may assume the existence of a physical vacuum probably well described by a spin-0 field $\phi(x)$ whose vacuum expectation value is not zero:

$$\text{vacuum} \sim \phi(x),$$  \hspace{1cm} (39)

and at its lowest-energy state to have zero 4-momentum, $k_\mu = 0$ (e.g. Lee 1981).

This new state out of equilibrium can be constrained by applying an external force to the system (e.g. set all of the system into rotation about its central axis at the same angular velocity $\omega$).

It was shown that the entropy must increase with a small displacement from a previous referred state (Lavenda 1974, Landau and Lifchitz 1987). Considering that the entropy is proportional to the logarithm of the statistical weight $\Omega \propto \exp(\overline{S}/k_B)$ and considering that $\overline{S} = \overline{S}_0 + \overline{S}_{ne}$, we can expect an increase of the nonequilibrium entropy $\overline{S}_{ne}$ with a small increase of the $i$th particle’s velocity $v_i = r_i$, since with an increase of the particle’s speed (although in random motion) the entropy must increase altogether. Therefore, we must always have

$$T \frac{\partial \overline{S}_{ne}}{\partial r_i} \geq 0, \quad \forall i = 1, \ldots, N.$$  \hspace{1cm} (40)

In the conditions of mechanical equilibrium the equality must hold; otherwise condition (40) can be considered a universal criterion of evolution. Considering that the entropy is an invariant (Rengui 1996) there is no extra similar term when the momentum is transferred to another inertial frame of reference.

In this regard, there is an important theorem derived by Baierlein (1968) showing that the Gibbs entropy for a system of free particles with kinetic energy $K$, density $\rho$ and absolute temperature $T$, $S(K, \rho, T)$, is greater than the entropy associated with the same system subject to arbitrary velocity-independent interactions $V$, $S(K + V, \rho, T)$, such that $S(K + V, \rho, T) \leq S(K, \rho, T)$.

At the electromagnetic level, Maxwell conceived a dynamical model of a vacuum with hidden matter in motion. As is well known, Einstein’s theory of relativity eradicated the notion of ‘ether’ but later revived interest in it in order to give some physical meaning to $g_{ij}$. Minkowski obtained, as a mathematical consequence of Maxwell’s mechanical medium, that Lorentz’s force should be exactly balanced by the divergence of Maxwell’s tensor in vacuum $T_{\text{vac}}$ minus the rate of change of Poynting’s vector:

$$\rho E + \mu_0 [J \times H] = \nabla \times T_{\text{vac}} = \frac{\partial}{\partial t} \epsilon_0 \mu_0 [E \times H].$$  \hspace{1cm} (41)

Einstein and Laub (1908) have remarked that when equation (12) is integrated all over the entire Universe the term $\nabla \cdot T_{\text{vac}}$ must vanish, which means that the sum of all Lorentz forces in the Universe must be equal to the quantity $\int_{\text{vac}} \epsilon_0 \mu_0 \partial/\partial t [E \times H] \text{d}v$ in order to comply with Newton’s third law (see Graham and Laoh 1980). But this long-range force depends on the constant of gravitation $G$. Einstein accepted Faraday’s viewpoint on the reality of fields, and this gravitational field according to him would propagate all over the entire space without loss, locally obeying the action-reaction law. But nothing can reassure us that the propagating wave through the vacuum will be lost at infinite distances (Brillouin 1970). Poincaré (2003b) also argues about the possible dissipation of the action on matter due to the absorption of the propagating wave in the context of Lorentz’s theory.

Newton’s laws are valid, generally, for large scales. When the scale tends to the mesoscopic level or even smaller scales, all three Newton’s laws will become invalid. Newton’s third law is acceptable in most observable scales, but when the scale tends to the microscopic realm or extremely large scale, difficulties with Newtonian mechanics will arise (Vujčić and He 2004). In particular, according to Vujčić and He (2004), the third law becomes invalid for electron interaction (e.g. Onočin’s paradox). To better handle with a possible fractal nature of spacetime, El-Naschie’s $E$-infinity theory (El Naschie 2007) regards discontinuities of space and time in a transfinite way, through the introduction of a Cantorian spacetime.
By Noether’s theorem, energy conservation is related to translational invariance in time \((t \rightarrow t + a)\) and momentum conservation is related to translational invariance in space \((r_1 \rightarrow r_1 + b_1)\). This important theorem thus implies that the law of conservation of momentum (not equivalent to the action-equals-reaction principle) is always valid, while the law of action and reaction does not always hold, as shown in the previous examples. Some kind of relationship must therefore exist between entropy and Newton’s third law, since it was through the first and second laws of thermodynamics combined that our main result was obtained. This idea was verified recently through a standard Smoluchowski’s approach, and on the Brownian dynamic computer simulation of two fixed big colloidal particles in a bath of small Brownian particles, drifting with uniform velocity along a given direction. It was shown that, in striking contrast to the equilibrium case, the nonequilibrium effective force violates Newton’s third law, implying the presence of nonconservative forces with a strong anisotropy (Dzubiella et al 2003), in concordance with our equation (38).

5. Is the action-equals-reaction in an out-of-equilibrium thermodynamical system verified?

The maximizing entropy procedure proposed in Pinheiro (2002, 2004) suggests the following ‘gedankenexperiment’, which bears some resemblance to Leo Szilard’s thermodynamical engine, made up of a one-molecule fluid (e.g. Leff and Rex 1990), although we are not concerned here with negentropy issues.

5.1. Example

5.1.1. Self-accelerated engine. Let us consider a physical system consisting of a spherical body built of \(N\) number of particles closed in a box, moving along one direction (see figure 2). The left side is at temperature \(T_2\), the right side is at temperature \(T_3\), while all the particles inside the body itself are at temperature \(T_1\) (and in equilibrium with their photonic environment). Furthermore, let us assume that both surfaces and the body particle are all thermal reservoirs, and hence their respective temperatures do not change. Let us suppose that the onset of nonequilibrium dynamics can be forced by some means in the previously described device. When the particle collides with the left surface, its momentum varies according to

\[
\delta p_i = -mv_i'' + mv_i' + (T_3 - T_1)\partial_iS. \tag{42}
\]

Here, \(\partial_iS\) denotes the (nonequilibrium) entropy gradient in velocity space. After the collision, the particle goes back to hit the right-side surface at temperature \(T_3\). The momentum variation after the second collision is given by

\[
\delta p_i = mv_i' - mv_i'' + (T_2 - T_1)\partial_iS. \tag{43}
\]

We assume that the body attains thermal equilibrium with the environment (which must remain at constant temperature \(T_1\)) fast enough before the next hit against the wall of the thermal reservoir. The total balance after a back and forth complete cycle is given by

\[
\delta p_i = -\delta p_i - \partial_i(S(T_2 + T_3 - 2T_1)) = -\delta p_i - \Delta\xi \nabla_v S. \tag{44}
\]

To make it more clear, we might write equation (44) under the form

\[
\delta p_i = -\delta p_i - \delta p_i^o, \tag{45}
\]

where we denote by \(\delta p_i^o \equiv \Delta\xi \nabla_v S\) the change in momentum by the physical vacuum (or, more appropriately, we should call ‘inertial space’). Therefore, it is clear from the above analysis that in systems out of equilibrium Newton’s third law is not verified, but the conservation of canonical momentum is well verified, however, as it must be according to Noether’s theorem. Otherwise, when the temperatures are equal to all the thermal bath in contact, such as \(T_1 = T_2 = T_3\), Newton’s third law is satisfied:

\[
\delta p_i = -\delta p_i. \tag{46}
\]

In the frame of the nonlinear dynamics and statistical approach, Denisov has shown (Denisov 2002) that a rigid shell and a nucleus with internal dynamic asymmetry can perform self-unidirectional propulsion. Also, it seems now certain that depletion forces exerted between two big colloidal particles in a bath of small particles exhibit nonconservative strongly anisotropic forces that violate the action-to-reaction law (Dzubiella et al 2003) (see also Wang et al 2002). In addition, internal Casimir’s forces exerted between a circle and a plate in the nonequilibrium situation violate Newton’s law (Buenzli and Soto 2008).

5.1.2. Stimulated emission versus Newton’s third law. Considering the radiation as a reservoir, Einstein (1917) introduced master equations seeking to describe the effect of absorption, stimulated emission and spontaneous emission processes between two levels of an atom immersed in the black-body radiation field. These equations read

\[
\begin{align*}
\frac{dN_b}{dt} &= -A_{ba}N_b + u(\omega)(B_{ba}N_a - B_{ba}N_b), \\
\frac{dN_a}{dt} &= A_{ba}N_a + u(\omega)(B_{ba}N_b - B_{ba}N_a). 
\end{align*} \tag{47}
\]

Here, \(N_a\) and \(N_b\) are the numbers of atoms in states \(a\) and \(b\) (with \(E_a > E_b\)); \(A_{ba}\) is the spontaneous emission rate from \(b \rightarrow a\); \(B_{ab}\) is the absorption rate from \(a \rightarrow b\);
... is the stimulated (or induced) emission rate from $b \rightarrow a$; $u(\omega)$ is the energy density of the radiation field at the frequency $\omega = (E_b - E_d)/\hbar$. The first part of the Einstein paper (Einstein 1917) deals with energy transformations and the $A$ and $B$ rates of absorption and emission for processes in an atom or molecule in equilibrium with the radiation in a cavity. Incidentally, in this paper, for the probability that an atom decays spontaneously from state $b \rightarrow a$, Einstein takes $dW = \lambda_{ba} \, dt$, and he quotes radioactive $\gamma$ decay and Hertzian oscillators as physical analogues. In the second part of his work, he addresses the momentum conservation in the radiation process concluding that in the spontaneous emission process (ausstrahlung) the atom should recoil with magnitude $\hbar v/c$ in a direction ‘... determined only by “chance”’ (Greenberger et al 2007) (Einstein introduced, in this way, an element of chance in quantum mechanics). Spontaneous emission may be understood as the result of action of the particle as a whole, an immanent cause, occurring even if the system is closed (notwithstanding the possible role of the ZPF (Milonni 1994)). In contrast, stimulated emission (einsteinstrahlung) occurs when the atom is an open system, interacting with the medium (Cornille 2003), the initial and final states in the transition are defined by an external variable (i.e. the incident electric field), and the distinction between closed systems and open systems explains to a certain extent the existence of two types of radiation. Stimulated emission can be re-enforced (by means of ‘optical pumping’) by making the input wave with intensity $I$, traverse an inverted medium ($N_2 > N_1$), so that the radiation decays (or amplifies) according to $I_s(z) = I_s(0) e^{-\alpha z}$, with $\alpha = (N_1 - N_2) h \cdot g(\nu)/8\pi n^2 \gamma_{\text{spont,}}$, with $\lambda$ being the wavelength of the radiation, $\gamma_{\text{spont}}$ the spontaneous lifetime for $2 \rightarrow 1$ transitions, $n$ is the medium index of refraction and $g(\nu)$ the lineshape function. Stimulated emission does not conserve energy, since atoms are open systems in a radiant medium. When the atom is submitted to a beam of plane waves propagating within the divergence angle of the beam, the momentum of the photon can be changed by stimulated absorption by the atom of a photon from one plane wave and subsequent stimulated emission into another plane wave; although the two photons involved in these two processes have the same energy, they differ in their propagation direction, resulting in a gradient force that can pull the atom into or out of the laser beam; there is violation of the action–reaction force. This effect is used in optical tweezers.

6. Conclusion

The purpose of this study is to examine how the action–reaction law is presented in the literature, particularly concerning mechanics, electrodynamics and statistical mechanics, and to offer a methodological approach in order to clarify the fundamental aspects of the problem, in particular suggesting that a third system must be included in the analysis of forces, which we call here, for the sake of conciseness, the physical vacuum. Furthermore, our procedure leads to a generalization of the general linear canonical momentum of a body particle in the framework of statistical mechanics. Theoretical arguments and numerical computations suggest that Newton’s third law is not verified in out-of-equilibrium systems, due to an additional term, an entropic gradient term, which must be in the particle’s canonical momentum. Although Noether’s theorem guarantees the conservation of canonical momentum, the action-equals-reaction principle can be restored in nonequilibrium conditions only if a new force term, representing the action of the medium on the particles, is taken into account.

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