Dark energy from a geometrical gauge scalar-tensor theory of gravity

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Abstract

In this paper we obtain some cosmological solutions that describe the present period of accelerating expansion of the Universe in the framework of a geometrical gauge scalar-tensor theory of gravity. The background geometry in the model is the Weyl integrable and we found a class of power law solutions for the Weyl scalar field when an invariant metric is employed in a power law expanding universe. We obtain a deceleration and an equation of state parameters (EoS) in agreement with PLANCK 2018 observational data for some specific parameters of the model. The deceleration parameter tends asymptotically to $-1/2$ and the phantom divide line can be crossed by the EoS parameter in this model.

1. Introduction

Since 1998 it was discovered that in the present epoch the Universe is expanding in an accelerated manner [1–3]. The origin of such acceleration has become one of the greatest challenges of modern cosmology. A positive cosmological constant has been one of the first options that appeared to explain such acceleration at least in general relativity [4]. However, a cosmological constant can be viewed as the existence of a vacuum energy, called dark energy, responsible for the acceleration [5]. When this vacuum energy is described by the quantum field theory then appears the well-known cosmological constant problem [6, 7]. Despite this problem the concordance model $\Lambda CDM$ has been considered the simplest and robust model of dark energy. However, the robustness of the model may be altered by the 5-$\sigma$ Hubble tension problem. An intense research to investigate this issue is being doing, including the proposal of an early dark energy to alleviate the tension [8–11].

With the intention to avoid the cosmological constant problem many proposals to explain the cosmic acceleration have appeared. Dynamical cosmological constant [12–15] and different versions of quintessence models [16–18] can be found within these proposals. An interesting feature of the class of quintessence models is that dark energy is driven by a scalar field similar to what happens in the early inflationary epoch of the Universe. Some other models of late time cosmology employing scalar fields (canonical/phantom) in the context of scalar-tensor theories of gravity have been proposed in [19, 20]. Interactions between the dark energy and dark matter sectors are other interesting proposals we can find in the literature [21, 22]. However one important question around this is about the origin of the scalar field. Inspired in this issue some cosmological models in which the scalar field employed is part of the affine structure of the geometry have recently appeared. A particular class of these models is based on the so called geometrical scalar-tensor theories of gravity.

Geometrical scalar-tensor theories of gravity have been first proposed by C. Romero and collaborators [23–26]. They started considering a typical scalar-tensor theory of gravity and instead of imposing the background geometry they adopted the Palatini’s variational principle to determine such geometry. The result is that the natural background geometry of this class of theories is determined by the Weyl-integrable compatibility condition. An extension of this idea was introduced by J E Madriz-Aguilar and collaborators by considering that the action of the theory must have the same symmetry group than the background geometry. To achieve it they
introduce a gauge covariant derivative and construct a new invariant action. This theory is also known as geometrical gauge scalar-tensor theory of gravity [13, 17]. Different topics have studied in the light of these frameworks, for example interacting quintessence scenarios [17], scalar fluctuations of the metric during inflation [18], generation of the seeds of cosmic magnetic fields during inflation [25], Higgs inflation frameworks [18, 26], (1 + 2)-dimensional gravity [27] and viscous cosmology [28], among others.

The idea of considering scalar fields geometrical in origin have also been employed in other models. For example in some approaches in which the Palatini’s principle is employed [29, 30], and in the so named conformal equivalence principle [31]. Geometrical scalar fields have been even used in some theories of gravity with extra dimensions. Induced matter theory and relativistic quantum approaches in non-Riemannian geometries are good examples [32, 33]. The interest in non-Riemannian geometries and in particular in the Weyl class of geometries has recently increased due to their proposal to solve, for example, the frame equivalence issue. In traditional approaches of scalar-tensor theories the Einstein frame is obtained by implementing a conformal transformation of the metric. However, this operation results in chancing the background geometry of the theory generating that observers in different frames (Jordan and Einstein) are not geodesic due to the appearance of a fifth force in one of the frames. This fact may lead to different physical properties for a same phenomena in each frame. An example of this situation in the context of some dark energy models can be seen in [34]. What makes attractive the geometrical gauge scalar-tensor theory is that it does not suffers of this problem. A disformal generalization of a Weyl structure is studied in [35]. Moreover, in this geometrical approach a Higgs inflationary period can be modeled without the unitarity problem [18, 26]. Thus results of interest to find out if a late time cosmological scenario can be described in this theory. Some different cosmological models using scalar fields in Weyl geometry can be also in [36].

In this paper we obtain cosmological solutions in the framework of a geometrical gauge scalar-tensor theory of gravity capable to describe the present period of accelerating expansion in the Universe. The paper is organized as follows. Section 1 is left for the introduction. In section 2 we derive the field equations of the geometrical gauge scalar-tensor theory of gravity. In section 3 we reduce the field equations to the case of cosmological scales. Section 4 is devoted to obtain solutions of the cosmological field equations that describe the present period of accelerating expansion. Finally in section 5 we give some final comments as conclusions.

2. The field equations

Let us start writing the general action of a scalar-tensor theory of gravity in the form [17]

\[ S = \int d^4x \sqrt{-h} \left[ e^{-\phi} \left( \frac{R}{16\pi G} + \frac{1}{2} \omega(\varphi) h^{\alpha\beta} \partial_{\alpha} \varphi \partial_{\beta} \varphi - V(\varphi) \right) \right], \]  

with \( \omega(\varphi) \) being a well behaved function of the scalar field \( \varphi \), \( V(\varphi) \) is the scalar potential, \( h_{\alpha\beta} \) is the metric tensor, \( h = \text{det} \left( h_{\alpha\beta} \right) \) and \( R(\Gamma) \) is the Ricci scalar of curvature that we consider depending on the affine connection. Palatini variational principle leads to the compatibility condition

\[ \nabla_{\mu} h_{\alpha\beta} = \varphi_{,\alpha} h_{\beta\mu}, \]  

which corresponds to a Weyl-Integrable background geometry and \( \nabla_{\mu} \) is denoting the covariant derivative with respect the Weyl connection coordinate components

\[ (w)\Gamma_{\mu\nu}^{\alpha} = (R)\Gamma_{\mu\nu}^{\alpha} - \frac{1}{2} (\varphi_{,\nu} K_{\mu}^{\alpha} + \varphi_{,\mu} K_{\nu}^{\alpha} - \varphi_{,\mu} h^{\alpha\sigma} h_{\mu\nu}), \]  

where \((R)\Gamma_{\mu\nu}^{\alpha}\) is the Levi-Civita connection. The expression (2) is invariant under the transformations

\[ h_{\alpha\beta} = e^f h_{\alpha\beta}, \] \[ \varphi = \varphi + f, \]  

with \( f(x^\alpha) \) is a well-behaved function of the space-time coordinates. Notice that under (4) and (5) applied simultaneously the Ricci tensor remains invariant

\[ R_{\mu\nu}(h_{\alpha\beta}, \varphi) = R_{\mu\nu}(h_{\alpha\beta}, \varphi), \]  

whereas the scalar curvature changes according to the rule

\[ \hat{R}(h, \varphi) = e^{-f} R(h, \varphi). \]
In order to study some cosmological implications of the setting both

As we have mentioned the background geometry of these equations is of the Weyl-integrable type and in this manner the action

where we have introduced the gauge Weylian covariant derivative

\( \varphi_{\mu\nu} = \nabla_\mu \varphi + \gamma B_\mu \varphi, \)

with \( \gamma \) being an imaginary constant, \( B_\mu \) is a gauge field defined in every point of the space-time manifold and

\( H_{\alpha\beta} = \partial_\alpha (\partial_\beta \varphi) - \partial_\beta (\partial_\alpha \varphi) \)

is a strength field. The action (8) is invariant only when the next transformations are valid

\( \varphi B_\mu = \varphi B_\mu - \gamma \gamma B_\mu \)

\( \nabla (\varphi) = V (\varphi - f) = V (\varphi), \)

\( \omega (\varphi) = \omega (\varphi - f) = \omega (\varphi). \)

The expression (10) corresponds to the transformation of the elements of the algebra of the group \( U(1) \). Thus, we can associate the field \( W_\mu = \varphi B_\mu \) with an electromagnetic potential defined on the Weyl-integrable background geometry. In this manner the action (8) is invariant under the set of transformations (4), (5) and

\( W_\mu = W_\mu - \gamma \gamma B_\mu. \)

The field equations derived from the action (8) read

\[ e^{-\varphi} (w)_G_{\alpha\beta} + 2e^{-\varphi} (\nabla_\alpha \nabla_\beta \varphi - 2 \nabla_\alpha \nabla_\beta \varphi + 2h_{\alpha\beta} \nabla_\nu \varphi \nabla^\nu \varphi - h_{\alpha\beta} \omega (\varphi) = 8\pi Ge^{-\varphi} \left[ \omega (\varphi) \varphi_{\alpha\beta} \right] \]

\[ - \frac{1}{2} h_{\alpha\beta} (\omega \varphi) \nabla^\nu \varphi \nabla_\nu \varphi + 2 V (\varphi) e^{-\varphi} \right) \]

\[ + 8\pi G \left( h_{\alpha\beta} h_{\mu\nu} - \frac{1}{4} h_{\alpha\beta} H_{\mu\nu} H_{\alpha\beta} \right). \]

\( (\omega^\prime (\varphi) - \omega (\varphi)) \nabla^\alpha \varphi \nabla_\alpha \varphi + \varphi (\varphi) \nabla^\alpha \varphi A_\alpha + \gamma \varphi \omega (\varphi) D_\alpha A^\alpha - \frac{1}{16\pi G} \)

\[ + \frac{1}{2} (\omega (\varphi) - \omega^\prime (\varphi)) \varphi^\alpha \nabla_\alpha + e^{-\varphi} (V^\prime (\varphi) - 2 V (\varphi)) = 0, \]

\( D_\alpha H^{\alpha\beta} = \gamma e^{-\varphi} \omega (\varphi) \varphi_{\beta}, \)

where \( (w)_G_{\alpha\beta} \) is the Weylian Einstein tensor, \( \omega^\prime R \) is the Weylian scalar curvature, \( \nabla_\alpha \) is denoting the Weylian covariant derivative, \( D_\alpha \) stands for the Riemannian covariant derivative, \( \omega \square \) is the Weylian D'Alambertian operator, \( \square \) denotes the Riemannian D'Alambertian operator and the prime is denoting derivative with respect \( \varphi \). Taking the trace of (14), the equation (15) written in terms of the field \( W_\mu \)

\[ \left( \omega^\prime (\varphi) - \omega (\varphi) - \frac{5}{8\pi G} \right) \nabla_\alpha \varphi \nabla^\alpha \varphi + \left( \omega (\varphi) - \frac{3}{8\pi G} \right) \varphi \]

\[ + \varphi \omega (\varphi) W_\alpha + \gamma \omega (\varphi) (D_\alpha W_\alpha - \nabla_\alpha (\ln \varphi) W_\alpha) - \left( \omega (\varphi) - \frac{1}{2} \omega^\prime (\varphi) \right) \varphi_{\alpha} \]

\[ + e^{-\varphi} V^\prime (\varphi) = 0. \]

As we have mentioned the background geometry of these equations is of the Weyl-integrable type and in this setting both \( \varphi \) and \( W_\mu \) can be considered geometrical fields that form part of the affine structure of the space-time. In the next section we shall obtain cosmological applications of the system (14) (16), in particular we will focus on those that describe the current acceleration in the expansion of the Universe.

### 3. The cosmological field equations

In order to study some cosmological implications of the field equations (14)–(16), we consider that the Weyl scalar field can be expressed by the sum of two contributions, one on cosmological scales that respects the cosmological principle and another one valid on smaller scales, thus we assume the separation formula

\[ \varphi (t, x^i) = \phi (t) + \zeta (t, x^i), \quad |\zeta| < |\phi|, \]

where \( \phi (t) \) is the cosmological part of the field and \( \zeta (t, x^i) \) accounts for the contribution of the Weyl field on non-cosmological scales. The gauge field \( W_\mu \) is a vector field, thus in order to maintain valid the cosmological principle we will consider that its contribution is only on non-cosmological scales. Hence, it follows from (14)–(16) and (18) that the field equations valid on cosmological scales acquire the form

\[ \omega_{\alpha\beta} - 4 \nabla_\alpha \phi \nabla_\beta \phi - h_{\alpha\beta} \nabla_\rho \phi \nabla^\rho \phi + 2 \nabla_\alpha \nabla_\beta \phi - 2 h_{\alpha\beta} \omega (\phi) \nabla_\alpha \phi \nabla^\beta \phi \]

\[ - \frac{1}{2} h_{\alpha\beta} \omega (\phi) \nabla^\sigma \phi \nabla_\sigma \phi - 2 e^{-\phi} V (\phi)), \]

\[ \omega_{\alpha\beta} - 4 \nabla_\alpha \phi \nabla_\beta \phi - h_{\alpha\beta} \nabla_\rho \phi \nabla^\rho \phi + 2 \nabla_\alpha \nabla_\beta \phi - 2 h_{\alpha\beta} \omega (\phi) \nabla_\alpha \phi \nabla^\beta \phi \]

\[ - \frac{1}{2} h_{\alpha\beta} \omega (\phi) \nabla^\sigma \phi \nabla_\sigma \phi - 2 e^{-\phi} V (\phi)), \]
Now, in order to introduce matter sources in our cosmological model it is important to remember that in this geometrical formalism gravity is described at the same time by the metric $h_{\alpha\beta}$ and the scalar field $\varphi$ \cite{17}. Thus, it follows that under the consideration that gravity couples to matter then both fields do. This fact is expressed in the matter action

$$S_m = \int \sqrt{-h} \ e^{-2\varphi} L_m(\varphi, h_{\alpha\beta}, \chi^{(w)} \nabla \chi),$$

(21)

where $\chi$ denotes a matter field and $L_m$ is a matter Lagrangian constructed in the same way that it is done in special relativity. In this manner the energy momentum tensor $T_{\mu\nu}(\varphi, h_{\alpha\beta}, \chi^{(w)} \nabla \chi)$ is determined by means of the formula

$$\delta \int d^4x \sqrt{-h} \ e^{-2\varphi} L_m(\varphi, h_{\alpha\beta}, \chi^{(w)} \nabla \chi) = \int d^4x \sqrt{-h} \ e^{-2\varphi} T_{\mu\nu}(\varphi, h_{\alpha\beta}, \chi^{(w)} \nabla \chi) \delta(e^{\varphi}h^{\mu\nu}),$$

(22)

where $\delta$ is denoting variation with respect to $h_{\mu\nu}$ and $\varphi$.

On the other hand, it is not difficult to see that due to the Weyl transformations the differential line element defined with the metric $h_{\alpha\beta}$ is not an invariant. Thus, we introduce the invariant metric $g_{\alpha\beta} = e^{-\varphi}h_{\alpha\beta}$ which on cosmological scales reduces to $g_{\mu\nu} = e^{-\varphi}h_{\mu\nu}$. Hence, in terms of the invariant metric and in presence of matter sources the field equations (19) and (20) read

$$G_{\alpha\beta} = 4(D_{\alpha} \phi D_{\beta} \phi - g_{\alpha\beta} D_{\alpha} \phi D_{\beta} \phi) + 2D_{\alpha} \phi \phi - 2g_{\alpha\beta} \Box \phi = 8\pi GT_{\alpha\beta}^{(m)} + 8\pi GT_{\alpha\beta}^{(\varphi)},$$

(23)

$$\omega(\phi) \Box \phi + \frac{1}{2} \omega'(\phi) e^{\omega(\phi)} D_{\alpha} \phi D_{\beta} \phi + \frac{dV(\phi)}{d\phi} = 0,$$

(24)

where the energy-momentum tensor for matter sources is denoted by $T_{\alpha\beta}^{(m)}$ and

$$T_{\alpha\beta}^{(m)} = \omega(\phi) D_{\alpha} \phi D_{\beta} \phi - \frac{1}{2} g_{\alpha\beta} \omega(\phi) D_{\alpha} \phi D_{\beta} \phi - 2e^{-\phi} V(\phi),$$

(25)

stands for the energy-momentum tensor associated with the scalar field $\phi$. Now, we consider the Friedmann-Lemaître-Robertson-Walker (FLRW) line element for a spatially-flat universe as

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$$

(26)

being $a(t)$ the cosmological scale factor and $t$ the cosmic time. As usual we assume that the matter content of the Universe can be described by an energy-momentum tensor corresponding to a perfect fluid

$$T^{(m)}_{\mu\nu} = \text{diag}(\rho_m, -\rho_m, -\rho_m, -\rho_m)$$

(27)

where $\rho_m = \rho_m + \rho_r$ and $\rho_r = p_r$ stand for the total energy density and the total pressure of the cosmic fluid, respectively. The energy density of matter and radiation are denoted respectively by $\rho_m$ and $\rho_r$ whereas $p_r$ accounts for the radiation pressure. Thus for a comoving class of observers $U^\mu = \delta^\mu_0$ related to the metric background (26) the equations (23) yield

$$3H^2 = 8\pi G(\rho_m + \rho_r) + 8\pi G \left( \rho_\phi + \frac{6H \dot{\phi}}{8\pi G} \right),$$

(28)

$$2\frac{\ddot{a}}{a} + H^2 = -8\pi G p_r - 8\pi G \left( \rho_\phi - \frac{2\ddot{\phi}}{8\pi G} + 4H \dot{\phi}^2 - 4\phi^2 \right),$$

(29)

where $H = \dot{a}/a$ is the Hubble parameter, the dot denotes derivative with respect the cosmic time $t$ and

$$\rho_\phi = \frac{1}{2} \omega(\phi) \dot{\phi}^2 + V(\phi),$$

(30)

$$\rho_r = \frac{1}{2} \omega(\phi) \dot{\phi}^2 - V(\phi),$$

(31)

are the energy density and pressure associated to the scalar field $\phi$. Similarly, (24) now reads

$$\omega(\phi) (\ddot{\phi} + 3H \dot{\phi}) + \frac{1}{2} \omega'(\phi) \dot{\phi}^2 + \frac{dV(\phi)}{d\phi} = 0.$$  

(32)

It follows from (28) and (29) that the dark energy sector is governed by the $\rho_m$, $p_r$ and extra terms. Thus we can introduce the quantities

$$\rho_{de} = \rho_\phi + \frac{6H \dot{\phi}}{8\pi G}$$

(33)
\[
P_{de} = P_\phi - \frac{2\ddot{\phi} + 4H\dot{\phi} - 4\dot{\phi}^2}{8\pi G},
\]

describing the energy density and pressure for the dark energy sector in the model.

4. Cosmological solutions exhibiting accelerated expansion

Now, let us to obtain the conditions for accelerated expansion solutions of the model. Thus, it follows from (28) and (29) the acceleration equation

\[
\frac{\ddot{a}}{a} = -4\pi G \left[ p_G + \frac{1}{3}(\rho_m + \rho_r) + p_\phi + \frac{1}{3}\rho_\phi - \frac{2\ddot{\phi} - 4H\dot{\phi} + 4\dot{\phi}^2}{8\pi G} \right].
\]

It can be easily seen from (35) that to achieve \( \ddot{a} > 0 \) solutions, must be valid the condition

\[
\frac{2}{3}\Omega_r + \frac{1}{3}\Omega_m + \left(\frac{1}{3}\Omega_{de}\right) < 0,
\]

where \( \Omega_r, \Omega_m \) and \( \Omega_{de} \) are the radiation, matter and dark energy density parameters, respectively. The equation of state parameter for the dark energy sector is given by

\[
\omega_{de} = \frac{8\pi G \left(\frac{1}{2} \omega(\phi) \dot{\phi}^2 - V(\phi)\right) - 2\ddot{\phi} - 4H\dot{\phi} + 4\dot{\phi}^2}{8\pi G \left(\frac{1}{2} \omega(\phi) \dot{\phi}^2 + V(\phi)\right) + 6H\dot{\phi}}.
\]

Given that \( 2/3\Omega_r + 1/3\Omega_m > 0 \), the inequality (36) is satisfied when

\[
\frac{1}{3}\Omega_{de} < 0,
\]

The condition (38) can be put in the form

\[
\frac{8\pi G \left(\frac{1}{2} \omega(\phi) \dot{\phi}^2 - V(\phi)\right) - 2\ddot{\phi} - 4H\dot{\phi} + 4\dot{\phi}^2}{8\pi G \left(\frac{1}{2} \omega(\phi) \dot{\phi}^2 + V(\phi)\right) + 6H\dot{\phi}} < -\frac{1}{3}.
\]

On the other hand, by means of the auxiliary field

\[
\Phi = \int \sqrt{\omega(\phi)} \, d\phi,
\]

the equation (32) becomes

\[
\ddot{\Phi} + 3H\dot{\Phi} + \frac{dU(\Phi)}{d\Phi} = 0,
\]

where \( U(\Phi) = V(\Phi) \) is the potential in terms of the new field \( \Phi \). Now, usually a potential is chosen to obtain the scale factor from the field equations. However, in this paper instead of choosing the potential we consider a power law for the scale factor and thus the potential will be determined by (42). It is important to note that other expansion scenarios can be also included in our formalism, the election of a power law expansion is just to illustrate how the model works. With this in mind the Hubble parameter and the scale factor read respectively as

\[
H(t) = \frac{p}{t}, \quad a(t) = a_0 \left(\frac{t_0}{t}\right)^p,
\]

being \( p > 1 \) a parameter characterizing the expansion, \( t_0 \) is the present time and \( a_0 = a(t_0) \). In view of (43) and as it is usually done in scalar-tensor theories we consider a power-law form for the scalar field \( \Phi \) as follows

\[
\Phi(t) = \Phi_0 \left(\frac{t_0}{t}\right)^n,
\]

where \( \Phi_0 = \Phi(t_0) \) and \( n \) is an integer parameter. For the choices made to satisfy the equation (42), inserting (43) and (44) in (42) we obtain the potential

\[
U(\Phi) = U_0 \left(\frac{\Phi}{\Phi_0}\right)^{\frac{2(1+n)}{p}} ,
\]

being \( U_0 = U(\Phi_0) \). Now, the simplest case we can study is when \( \omega(\phi) = \omega_0 = \text{const} \). In this particular example the expressions (41) and (45) lead to the potential
\[ V(\phi) = U_0 \left( \frac{\phi}{\phi_0} \right)^{2(1+n-m)}. \] (46)

With the help of (28), (29), (43), (44) and (45) the deceleration parameter results
\[ q = -1 - \frac{\Omega_m + \frac{4}{3} \Omega_r + \left(1 + \frac{1}{2\Omega_m} \right) \frac{8\pi G}{3H^2_0} n^2 \phi_0^{2n} t^{-(2(1+n))} - \frac{2n(n + n + 1)}{3H^2_0} \phi_0^{2} t^{-(2(1+n))} \left(1 + \frac{8\pi G}{3H_0^2 t^4} \right)}, \] (47)
which tends asymptotically to \(-\frac{1}{2}\). Evaluating in the present time \(t_0\) the expression (47) leads to
\[ q_0 = -1 - \frac{\Omega_{m0} + \frac{4}{3} \Omega_{r0} + \left(1 + \frac{1}{2\Omega_{m0}} \right) \frac{8\pi G}{3H_{00}^2} n^2 \phi_0^{2n} t_0^{-(2(1+n))} - \frac{2n(n + n + 1)}{3H_{00}^2} \phi_0^{2} t_0^{-(2(1+n))} \left(1 + \frac{8\pi G}{3H_0^2 t_0^4} \right)}, \] (48)
where \(\Omega_{m0}\) and \(\Omega_{r0}\) are the present density parameters for matter and radiation respectively.

Now, considering a slow roll of the field \(\phi\), the condition (40) reduces to
\[ -\frac{8\pi G}{3} V(\phi) \sim 2H\dot{\phi} + 4\dot{\phi}^2 < 0. \] (49)

In terms of the time, the expression (49) reads
\[-\left(\frac{8\pi G}{3} U_0 t_0^2 - 4n^2\phi_0^2\right) t_0^{2n} t^{-(2(1+n))} - \left(2n + (n + n + 1)\right) \phi_0^{2n} t^{-(2(1+n))} < 0. \] (50)
For \(\phi_0 > 0\) it can be reduced to the system
\[ \frac{8\pi G}{3} U_0 t_0^2 - 4n^2\phi_0^2 > 0, \] (51)
\[ n(n + 1) - 2p > 0. \] (52)

Thus, it follows from (51) that
\[ U_0 > \frac{3n^2\phi_0^2}{2\pi G\phi_0^2} \] (53)
and (52) has for solutions \(n < -2\) and \(n > 1\) for \(p > 1\), the latest corresponding to an accelerating expansion. The present value for the equation of state parameter for dark energy \(\omega_{de}\) according to (37), (43), (44), (45) and (46) reads
\[ \omega_{de} = -\frac{8\pi GU_0 + n(n + 1)(H_0 t_0)^{-2}H_0^2 \phi_0 - 4(H_0 t_0)^{-4}H_0^2 \phi_0 - 4n^2(H_0 t_0)^{-2}H_0^2 \phi_0^2}{8\pi G U_0 - 4(H_0 t_0)^{-2}H_0^2 \phi_0}. \] (54)

The equation (54) can be written in the form
\[ 4n^2(H_0 t_0)^{-2}H_0^2 \phi_0^2 + (H_0 t_0)^{-2}H_0^2 \left(4(1 + \omega_{de}) - n(n + 1)\right) \phi_0 - 8\pi G(1 + \omega_{de}) = 0. \] (55)

Now, according to observational data Planck + SNe + BAO: \(\omega_{de} = -0.95 \pm 0.080\) and from TT, TE, EE + lowE + lensing: \(\Omega_{m0} = 0.3111 \pm 0.0056, \Omega_{r0} = 0.6889 \pm 0.0056, \Omega_{c0} = 2.47 \times 10^{-3} h^{-2}, H_0 = 67.4 \pm 0.5 \frac{km}{s} Mpc^{-1} \) [37]. Thus from (55) and employing \(H_0 t_0 \approx \frac{2}{3} \Omega_{m0} = 0.2046, 0.0036\) and taking the intermediate values for the observational parameters we arrive to the equation
\[ \phi_0 = \frac{2.016 \times 10^{-9}}{n^2} (-1.066 \times 10^7 + 6.2011 \times 10^7 n^2 + 6.2011 \times 10^7 n + \sqrt{\Delta_1(n)\phi_0}), \] (56)
where
\[ \Delta_1(n) = 1.1376 \times 10^{14} + 2.5225 \times 10^{15} n^2 - 1.3228 \times 10^{15} n + 3.8453 \times 10^{15} n^4 + 7.6908 \times 10^{15} n^5 \]
\[ + 6.12719 \times 10^{11} n^2 U_0. \] (57)

On the other hand, it follows from (33) that
\[ \Omega_{de} = \frac{(8\pi G)^2}{3H^2_0} U_0 - \frac{32\pi G}{3H_0^2 t_0^4} \phi_0. \] (58)
Solving (58) for $U_0$ we obtain

$$U_0 = \frac{3H_0^2}{(8\pi G)^2} \left( \Omega_{d_0} + \frac{32\pi G}{3(H_0t_0)} \phi_0 \right). \quad (59)$$

Inserting the observational data the equation (59) yields

$$U_0 = 14.8633 + 3535.7293 \phi_0. \quad (60)$$

With the help of (57) and (60) the value of $\phi_0$ in order to achieve the observational value for $\omega_{d_0}$ is

$$\phi_0 = \frac{1}{n^2} (-0.01709 + 0.12501n^2 + 0.12251n + 6.4512 \times 10^{-3} \Delta_2(n)), \quad (61)$$

where

$$\Delta_2(n) = 7.0305 \times 10^{24} + 2.7366 \times 36n^2 - 1.02746 \times 10^{16}n + 3.75517 \times 10^{16}n^4 + 7.51054 \times 10^{36}n^3. \quad (62)$$

Finally, it follows from (48) that the $\omega_0$ parameter of the model is given by

$$\omega_0 = -\frac{3(H_0t_0)^2}{8\pi G} \frac{1}{n^2\phi_0^2} \left[ 2(1 + q_0) \left( \frac{\Omega_m}{3(H_0t_0)^2} \frac{\Omega_{d_0}}{3(H_0t_0)^2} + \frac{8\pi G}{9H_0^2} U_0 \right) - \left( \frac{\Omega_m}{3} + \frac{4}{3} \Omega_r \right) \right]$$

$$= -\frac{4n^2\phi_0^2}{3(H_0t_0)^2} \left( n(n + 1) + H_0t_0 \frac{2\phi_0}{3(H_0t_0)^2} \right). \quad (63)$$

Taking into account that according to Planck 2018 observational data $q_0 = -0.5581^{+0.0273}_{-0.0267}$ the expression (63) yields

$$\omega_0 = 1 \times 10^5[-5.016 \times 10^{14}n^2 + 2.193 \times 10^{14}n^3 + 1.46 \times 10^{14}n - 1.5832 \times 10^{9}n \Delta_3(n) - 1.7669 \times 10^{13} + 1.6685 \times 10^{9} \Delta_3(n) + 3.4194 \times 10^{14}n^4 + 4.4156 \times 10^{14}n \sqrt{\Delta_3(n)} + 7.7438 \times 10^{9}n^2 + 7.7438 \times 10^{9}n$$

$$- 1.0586 \times 10^9 + 1 \times 10^3 \sqrt{\Delta_3(n)}]^{-2}, \quad (64)$$

Figure 1. This plot shows the EoS parameter $\omega_{d_0}$ versus $n$ and the scalar field $\phi_0$. It can be seen that $\omega_{d_0}$ crosses the boundary $\omega_{d_0} = -1$ for certain values of $n$ and $\phi_0$. The former is given in Planckian masses $M_p$. 

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where

$$\Delta_3(n) = 5.9965 \times 10^9 n^4 + 1.1993 \times 10^{10} n^3 + 4.37 \times 10^9 n^2 - 1.6407 \times 10^9 n + 1.1226 \times 10^8. \quad (65)$$

For example, when $n=3$, we obtain $\phi_0 = 0.3295 \, M_p$ and $\omega_0 = 0.19925$. With these values our model is compatible with observational data. Moreover, by using (54) and (59) the figure [1] shows values for the dark energy EOS parameter: $\omega_{DE} < -1$. In particular for $n = 3$ and $\phi_0 = 0.001979 \, M_p$, we obtain $\omega_{DE} = -1.03$, and for $\phi_0 = 0.002 \, M_p$, it gives $\omega_{DE} = -1$. Thus the EoS parameter $\omega_{DE}$ can cross the boundary $\omega_{DE} = -1$ in the present model. The figure 2 shows the variation of $\omega_{DE}$ versus $\phi_0$ for the particular case $n = 3$.

5. Conclusion

In this paper, in the framework of a recently introduced geometrical scalar-tensor theory of gravity, we propose a cosmological model that exhibits solutions describing the present period of accelerating expansion of the Universe. In traditional scalar-tensor theories of gravity the background geometry is assumed apriori to be Riemannian. This fact leads to the physical equivalence frame problem (the Jordan and Einstein frames) [31]. In the geometrical gauge scalar-tensor theory the equivalence frame problem is avoided as a consequence of determining the background geometry of the theory by imposing the Palatini’s variational principle, instead of regarding the background geometry as Riemannian, apriori. Thus for a traditional scalar-tensor theory of gravity the background geometry resulting is of the Weyl-Integrable type, whose symmetry group is the Weyl group of transformations. The problem is that the original scalar-tensor action is not an invariant under the new geometrical symmetry group. Hence, to solve the problem it is proposed a new scalar-tensor action that generates a new kind of scalar-tensor theory called geometrical gauge scalar-tensor theory of gravity.

Working with an invariant metric under the Weyl group, we have derived a cosmological model for the present epoch of accelerating expansion in which the dark energy sector is described by the Weyl scalar field, which is geometrical in origin. Notice that only for the class of observers that use the invariant metric the Weyl scalar field becomes physical, because for these observers the background geometry can be seen as Riemannian effective.

Figure 2. This is a plot of $\omega_{DE}$ versus $\phi_0$ depicted for $n = 3$. The phantom line $\omega_{DE} = -1$ is crossed for $\phi_0 = 0.002 \, M_p$. 

![Figure 2](image-url)
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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files). See reference [35] and [36].

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References

[1] Riess A G et al 1998 Astron. J 116 1009
[2] Perlmutter S et al 1998 Nature 391 51
[3] Spergel D N et al 2007 Astrophys. J. Supp. 170 377
[4] Betoule M et al (SDSS collaboration) 2014 Astron. Astrophys. 568 A22
[5] Frieman J, Turner M and Huterer D 2008 Ann. Rev. Astron. Astrophys. 46 385–432
[6] Martin J 2012 Comptes Randus Physique 1356–665
[7] Percival J S 2022 The cosmological constant problem and running vacuum in the expanding universe arXiv:2203.13757 [gr-qc]
[8] Escudero H G, Kuo J-L, Keeley R E and Abazajian K N 2022 Early or phantom dark energy, self-interacting, extra, or massive neutrinos, primordial magnetic fields, or a curved universe: an exploration of possible solutions to the H0 and s8 problems Phys. Rev. D 106 103517
[9] Dainotti M G, Simone B D, Schiavone T, Montani G, Rinaldi E, Lambiase G, Bogdan M and Ugale S 2022 On the evolution of the hubble constant with the SNe ia pantheon sample and baryon acoustic oscillations: a feasibility study for GRB-cosmology in 2030 Galaxies 10 24
[10] Gómez-Valent A, Zheng Z, Amendola L, Wetterich C and Pettorino V 2022 Coupled and uncoupled early dark energy, massive neutrinos, and the cosmological tensions Phys. Rev. D 106 103522
[11] Brissenden L, Dimopoulos K and Sánchez-López S Non-oscillating early dark energy and quintessence from α -attractors arXiv:2301.03572v2 [astro-ph]
[12] Madriz Aguilar J E, Bellini M and Cruz M A S 2008 Grav. Cosmol. 14 286–91
[13] Madriz-Aguilar J E, Zamarripa J, Peraza A and Licea J A 2017 Phys. Dark. Univ. 18 11–6
[14] Alcaniz J S 2006 Braz. J. Phys. 36 1109
[15] Madriz-Aguilar J E, Zamarripa J, Montes M, Licea J A, de Loza C and Peraza A 2022 The European Phys. J. Plus 137 135
[16] de Putter R and Linder E V 2007 Astropart. Phys. 28 263–72
[17] Madriz-Aguilar J E and Montes M 2018 Phys. Dark Univ. 21 47–54
[18] Madriz Aguilar J E, Bernal A, Montes M, Zamarripa J and Aceves E 2022 Phys. Dark. Univ. 35 100988
[19] Elizalde E, Nojiri S and Odintsov S D 2004 Phys. Rev. D 70 043539
[20] Elizalde E, Nojiri S, Odintsov S D, Saez-Gomez D and Faraòini V 2008 Phys. Rev. D 77 106005
[21] Copeland E J, Sáez-Gómez D and Sahu A 2007 Int. J. Mod. Phys. D 15 1753–936
[22] Oks E 2021 New Astron. Rev. 93 101632
[23] Almeida T S, Pucheu M L, Romero C and Formiga J B 2014 From Brans-Dicke gravity to a geometrical scalar-tensor theory Phys Rev D 89 064047
[24] Pucheu M L, Alves Junior F A P, Barreto A B and Romero C 2016 Cosmological models in Weyl geometrical scalar-tensor theory Phys. Rev. D 94 064010
[25] Montes M, José Edgar Madriz Aguilar and Granados V 2019 Can. J. Phys. 97 517–23
[26] Madriz-Aguilar J E, Zamarripa J, Montes M and Romero C 2020 Phys. Dark. Univ. 28 100480
[27] Madriz Aguilar J E, Romero C, Fonseca Neto J B, Almeida T S and Formiga J B 2015 Class. Quant. Grav. 32 215003
[28] Madriz Aguilar J E, Ocanzana A G, Montes M and Zamarripa J 2020 Phys. Dark. Univ. 30 100706
[29] Liu Y 2021 Eur. Phys. J. Plus 136 901
[30] Rosa J L and Lemos J P S 2021 Phys. Rev. D 104 124076
[31] Quiros I, García-Salcedo R, Madriz-Aguilar J E and Matos T 2013 Gen. Relat. Grav. 45 489–518
[32] Bellini M, Madriz-Aguilar J E, Montes M and Sánchez P A 2019 Phys. Dark. Univ. 25 100309
[33] Ríos L S and Bellini M 2015 Phys. Lett. B 741 565–71
[34] Nozari K and Sadatian S D 2009 Mod. Phys. Lett. A 24 3143–55
[35] Delhom A, Lobo P P, Olmo G and Romero C 2019 Eur. Phys. J. C 79 878
[36] Scholz E and Weyl A 2022 A Weyl geometrical scalar field approach to dark sector arXiv:2202.13467 [gr-qc]
[37] Aghanim N et al (Planck Collaboration) 2018 arXiv:1807.06209
[38] Particle Data Group 2018 Phys. Rev. D 98 030001