Analysis of rainfall patterns in Kalimantan using fast fourier transform (FFT) and empirical orthogonal function (EOF)

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Abstract. Rainfall patterns in Kalimantan are generally divided into 2 types, namely monsoonal and equatorial. The pattern can be determined by analyzing the 6-month frequency of rainfall signal. This analysis has been carried out on general data in Indonesia, but no one has yet examined it in detail in Kalimantan. Therefore, this study will analyze the 6-month frequency signal and rainfall patterns spatially and temporally in Kalimantan using TRMM 3B42RT as the main data. The Fast Fourier Transform (FFT) method is applied to analyze the 6-month frequency of rainfall signal, while the Empirical Orthogonal Function (EOF) method is applied to reduce data and obtain the main pattern of rainfall in Kalimantan. The results of FFT analysis in 15 cities of Kalimantan show that the rainfall pattern in Samarinda, Sendawar, Tarakan, Tanjungselor, Malinau, Pangkalanbun, Pontianak, Ketapang, and Sintang are an equatorial type, while a monsoonal type appear in Balikpapan, Palangkaraya, Purukcahu, Banjarmasin, Kotabaru and Barabai. Moreover, based on the results of FFT and EOF analysis, most areas in West, East and North Kalimantan have an equatorial rainfall pattern. Meanwhile, most areas in Central and South Kalimantan have a monsoonal rainfall pattern.

1. Introduction
Climate is a weather characteristic of a place or area, but not the average of weather. The weather characteristics of a place are determined based on the criteria of frequency or probability value of one or more defined climate elements, such as rain, temperature, wind, temperature and rain, temperature and wind, rain and evaporation, and others [1]. Rain is a form of precipitation, which is the process of water deposition from the atmosphere to the earth's surface in liquid (rain) and solid (snow) form [2]. Rainfall is an atmospheric parameter that is difficult to predict because it has very high spatial and temporal diversity [3]. Rain intensity is influenced by many factors such as geographic location, topography, wind direction and others. In addition, Indonesia's geographic location causes rainfall in Indonesia to be influenced by global climate phenomena such as the Asia-Australia monsoon system, the El-Nino Southern Oscillation (ENSO), the Indian Ocean Dipole (IOD), the Walker circulation, the Hadley Circulation and local circulation [4,5].

Rainfall variability over some provinces in Indonesia is more dominantly controlled by ENSO than the Indian Dipole Mode (IOD), that mostly drive the seasonal and inter-annual variability of rainfall and extreme events in Indonesia [6]. The Asia-Australia monsoon interaction causes most of the seasonal rainfall in Indonesia to be clearly categorized, that are the rainy season in December-January-February (DJF) and the dry season in June-July-August (JJA). However, the rainfall variation in Indonesia is not equal for every place. In some areas which are close to the equator, such as Sumatra, Kalimantan and...
Sulawesi, the component of the 6-month rain cycle is quite strong which is characterized by the presence of two maxima values around the equinox period, that are in March-April and October-November [7].

There are 2 patterns of rainfall in Kalimantan, namely monsoonal and equatorial as can be seen in Figure 1 [8]. Figure 1 explains that A is an area with a monsoonal pattern, B is an area with an equatorial pattern, and C is an area with a local pattern. On the one hand, the equatorial pattern has a monthly average rain with 2 maximum peaks (double maxima) around March and November, so it has a 6-month rain cycle or semi-annual oscillation (SAO) and the peak of rainfall usually occurs when the sun is above the Intertropical Convergence Zone (ITCZ). On the other hand, the monsoonal type has a monthly average rain pattern with one peak of maximum rain, namely in January or December, so that it has a 12-month rain cycle or annual oscillation (AO). In this type, the peak of rainfall usually occurs when the dominant west monsoon system crosses the area [2,9,10].

One method that can be used to examine the effect of the 6-month rain cycle is the fast Fourier transform (FFT) method. This method will convert the data signal from time domain to frequency domain [3,10]. This method has been applied to analyze the effect of rainfall signals in Indonesia [3]. However, this study only examined the effect of the 6-month rainfall cycle in general in Indonesia. Additionally, another method that can be used to analyze spatial and temporal patterns of climate data is the empirical orthogonal function (EOF) method [3,11,12]. This method has been applied to analyze data on forest fires in Indonesia [11]. This method reduces large data, so that there are several dominant modes that can represent variances of the data.

Based on these studies, this paper will examine the effect of 6-month and 12-month rainfall signal frequencies to determine rainfall patterns and analyze the dominant patterns of rainfall data spatially and temporally in Kalimantan. This paper is expected to answer the public’s need for rainfall patterns to estimate the intensity of rainfall in Kalimantan.

2. Data and methods
2.1. Fourier Transform
Fourier transform is used to convert time domain signals into frequency domain signals. The Fourier transform is defined using the following equation.

\[ F(v) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi vx} dx \]  \hspace{1cm} (1)

\[ F(x) = \int_{-\infty}^{\infty} f(v) e^{-j2\pi vx} dv \]  \hspace{1cm} (2)

where \( f(x) \) is a function in the time domain, \( f(v) \) is a function in the frequency domain, and \( e^{-j2\pi vx} \) is a kernel function. The functions \( f \) and \( F \) often are referred to as a Fourier integral pair or Fourier
The multiplication matrix \( U \Sigma \) is the principal component matrix. Meanwhile, the matrix \( V \) is an EOF matrix or vector component. The principal component value is given by

\[
\mathbf{z}_i = \mathbf{u}_i \sigma_i
\]

where \( i = 1,2,3,4,5,\ldots,r \) and the variance of \( i \) principal component is given by

\[
\mathbf{\mu}_i = \frac{\sigma_i^2}{\sum_{i=1}^{r} \sigma_i^2}
\]
where $\sigma_i : i = 1, 2, ..., r$ is the singular values of matrix $A$. Practically, the first EOF mode explains that EOF 1 has the largest variance. Meanwhile, EOF 2 is a linear combination of all variances and is orthogonal to EOF 1 mode and has the second largest variance. Moreover, the $k$-th EOF mode has a maximum variance to the $k$-th and does not correlate with the previous EOF mode. EOF analysis is used to find the $m \times n$ component score matrix of $m$ observation objects and $n$ times $[17, 18]$.

2.4. Types and sources of data
The precipitation data used is TRMM 3B43RT (Tropical Rainfall Measuring Mission 3B42 Real Time) which is measured every three hours per day. TRMM data has $0.25^\circ \times 0.25^\circ$ spatial resolution and was downloaded from 2001-2019. Data is downloaded from the NASA website in the Network Common Data Form (NetCDF) format in the Indonesian astronomic location. The variable taken from the TRMM 3B42RT data is precipitation. The total data size after download is 2.65 GB.

2.5. Research step
There are three steps in this research, which are the preparation and data extraction step, the FFT analysis step and the empirical orthogonal function analysis step.

2.5.1. Preparation and data extraction. The purpose of this step is to prepare the raw data that has been downloaded into data that is ready to be analyzed. The TRMM 3B42RT data is a high-accuracy rainfall estimation data in the form of a grid that has been downloaded in the Indonesian region. The process of data extraction includes reading data from the network common data format (netCDF) into a rainfall data matrix, cutting the data area from the Indonesian region to the Kalimantan region, converting data from 3-hours data into monthly data format, and compiling a 3-dimensional data matrix for fast Fourier transform and 2-dimensional data matrix for empirical orthogonal function analysis. In the process, some empty data were found in Kalimantan region, so that them filled using the singular value decomposition approach.

2.5.2. Fast Fourier Transform analysis. The second stage is to analyze the 3-dimensional rainfall data that has been extracted in the previous stage using the fast Fourier transform method. This stage is divided into 2 parts, which are FFT analysis for rainfall data in 15 cities spread across Kalimantan and FFT analysis for rainfall data in Kalimantan in general. The result of this step is the effect of the signal frequency of 6 monthly rainfall in Kalimantan in a power spectral density (PSD) plot.

2.5.3. Empirical Orthogonal Function analysis. The last step is to analyze the 2-dimensional rainfall data using the empirical orthogonal function method. This method will reduce the large rainfall matrix data into several modes that can represent variances of the rainfall data matrix. The result of this step is a spatial and temporal plot of the first 3 EOF modes that have the largest singular values.

3. Results and discussions
3.1. Data extraction
TRMM 3B42RT is a grid global rainfall data that has been downloaded for Indonesian region. Data extraction is started by determining the Kalimantan region domain to be analyzed, which is $7^\circ 2'N - 4^\circ 10'S$ latitude and $108^\circ 50'E - 118^\circ 59'E$ longitude. In the TRMM netCDF file, there are several variables involving the intensity of rainfall (precipitation), the location of latitude (nlat) and the location of longitude (nlon). Furthermore, the data was extracted according to the predetermined Kalimantan domain by adjusting the lowest and highest value of nlat and nlon variable. After extracting data, some empty data with Nan (not a number) entries were found, which is them filled using the singular value decomposition approach. After that, the 3-hour resolution data is converted to monthly resolution data. Finally, data is stored in 3-dimensional data, where the data has a dimension of $nlon \times nlat \times time$, which is $50 \times 45 \times 228$ and 2-dimensional data by combining the dimensions of nlon and nlat into one dimension so the data has a dimension of space $\times$ time size, which is $2250 \times 228$. 
3.2. Fast Fourier Transform analysis

Fast Fourier Transform (FFT) is used to convert signals from the time domain into the frequency domain. FFT is used to determine the effect of the signal frequency for 6-month and 12-month/annual rainfall. The effect of the signal frequency for 6-month and 12-month/annual rainfall was analyzed using 4 parameters, which are Param1 (power spectral density value at the 6-month period), Param2 (power spectral density value at the 12-month period), Param3 (the ratio between the power spectral density value at 6-month period against the overall signal), and Param4 (the ratio between the power spectral density value at 6-month period against power spectral density value at 12-month period) [3]. Param3 and Param4 are needed to normalize the 6-month signal against the overall signal and the 12-month signal.

3.2.1. FFT analysis in 15 cities in Kalimantan.

The first part of the FFT analysis step is to apply FFT analysis for rainfall data in 15 cities spread across Kalimantan. In this process, several sample cities consist of 3 cities for 5 provinces in Kalimantan, which are the cities of Samarinda, Balikpapan, and Sendawar for East Kalimantan, the cities of Tarakan, Tanjungselor, and Malinau for North Kalimantan, the cities of Palangkaraya, Purukcahu, and Pangkalanbun for Central Kalimantan, the cities of Banjarmasin, Kotabaru, and Barabai for South Kalimantan, and the cities of Pontianak, Ketapang, and Sintang for West Kalimantan.

The rainfall data for each city is transformed using the FFT algorithm. The results of the transformation are visualized in a periodogram where the x-axis represents the period (months/cycle) and the y-axis represents the power spectral density (mm²/month) of the rainfall data at period x. The results of the FFT analysis over 15 cities in Kalimantan can be seen in Figure 2.

![Figure 2. Power spectral density for rainfall data over 15 cities in Kalimantan from 2001-2019](image)

Based on Figure 2, most cities in West, East and North Kalimantan have a semi-annual oscillation (SAO) or equatorial rainfall pattern which is indicated by the existence of 2 peaks of PSD values at 6 and 12 months/cycle periods. Otherwise, most cities in Central and South Kalimantan have an annual oscillation (SAO) or monsoonal rainfall pattern which is indicated by the low PSD value at 6 months/cycle period.
months/cycle period [9,10]. To see the effect of the PSD value at 6-month to 12-month period, Figure 3 shows the Param4 value for each city.

![Figure 3](image)

**Figure 3.** The ratio between the PSD value at 6-month to the PSD value at 12-month period (Param4) over several cities in Kalimantan.

Based on Figure 3, the Param4 value in Balikpapan, Palangkaraya, Purukcahu, Banjarmasin, Kotabaru, and Barabai is low, so it can be concluded that these cities have a monsoonal rainfall pattern, which means that there is only one rain cycle in a year. Otherwise, the Param4 value in Samarinda, Sendawar, Tarakan, Tanjungselsor, Malinau, Pontianak, Ketapang, Pangkalanbun, dan Sintang is high which is over 0.5 especially the cities in West and North Kalimantan which are Tarakan, Tanjungselsor, Pontianak, and Ketapang have scores above 1, so it can be concluded that these cities have an equatorial rainfall pattern, which means that there is two rain cycles in a year.

### 3.2.2 FFT analysis in Kalimantan generally.

The second part of the FFT analysis is to apply FFT analysis for rainfall data in Kalimantan. In this part, the FFT algorithm is applied to all spatial points of the rainfall data matrix in Kalimantan and obtained a periodogram as in the previous part but now all grid points have a periodogram. Therefore, FFT analysis at all data points in Kalimantan will be very difficult and complicated if the results are analyzed one by one. Consequently, the values of the 4 parameters described previously, which are Param1, Param2, Param3, and Param4 are shown in Figure 4.

![Figure 4](image)

**Figure 4.** Colormap for the result of FFT analysis in Kalimantan (a) Param1 (the PSD values at 6-month period), (b) Param2 (the PSD values at 12-month period), (c) Param3 (the ratio between Param1 against the overall signal), and (d) Param4 (the ratio between Param1 against Param2).
Param2, Param3, Param4 are computed and arranged in a matrix for each grid points. After that, the results of each parameter matrix are visualized on a colormap as can be seen in Figure 4. Figure 4(a) and 4(b) show the power spectral density values of the rainfall data in Kalimantan at 6-month/cycle and 12-month/cycle period, respectively. Based on Figure 4(a), the PSD value of 6-month/cycle period is high in most areas of West, East and North Kalimantan and a small area of Central and South Kalimantan. Otherwise, Figure 4(b) highlight that the areas of Central and South Kalimantan have high PSD values in the 12-month/cycle period. Moreover, Figure 4(c) and 4(d) illustrate the ratio between the PSD value at 6-month period against the overall signal and the ratio between the PSD value at 6-month against the PSD value at 12-month period, respectively. Based on Figure 4(c) and 4(d), the power frequency of the 6-month rainfall signal is very affecting in most areas of West, East and North Kalimantan as well as small areas of Central and South Kalimantan. For more details, Figure 5 shows the ratio between the PSD value at 6-month/cycle against the PSD value at 12-month/cycle period (Param4) which has been filtered for values above 0.5. The value of 0.5 is determined based on several experiments on the boundary value between the monsoonal and equatorial patterns, in order to obtain a rainfall pattern map that is suitable for references. Thus, the pattern map in Figure 5 corresponds to the Indonesian rainfall pattern in Figure 1.

![Figure 5. Colormap for the ratio between the PSD value at 6-month/cycle against the PSD value at 12-month/cycle period which has been filtered for values above 0.5](image)

Based on Figure 5, the colored areas have a high effect of the 6-month frequency of rainfall signal, which is the values is more than 0.5, so that these areas have a semi-annual oscillation (SAO) or equatorial rainfall pattern which are in most areas of West, East and North Kalimantan and a small part of Central and South Kalimantan. Otherwise, the white area shows that the effect of the 6-month frequency of rainfall signal is not significant so that these areas have an annual oscillation (AO) or monsoonal rainfall pattern. This result is slightly different from Figure 1 because Figure 5 shows that several areas in Central Kalimantan have a high effect of the 6-month frequency of rainfall signal, so that these areas have an equatorial rainfall pattern.

3.3. Empirical Orthogonal Function analysis
The empirical orthogonal function (EOF) method based on singular value decomposition (SVD) is used to find dominant patterns in a large data matrix with a resolution of space and time. The used data matrix in this step is the 2-dimensional data matrix generated in the data extraction step. This step uses only rainfall matrix data in the mainland area of Kalimantan. Therefore, data was deleted for the ocean area by zeroing the rainfall data in the sea around Kalimantan. After that, the SVD method was applied to obtain some dominant EOF modes. This process generates 228 EOF modes from the rainfall data matrix with singular values and explain cumulative variance as shown in Figure 6. The higher variance value of EOF modes will result a higher contribution of the components to the observation.
Figure 6. (a) Singular value $\sigma_k$ and (b) cumulative variance explain in the first $k$ modes.

In this study, the rainfall matrix data in Kalimantan were analyzed based on the three largest singular mode values resulted from data reduction. Furthermore, Figure 7 shows the spatial and temporal patterns of the 3 largest singular values of the EOF modes which show the highest contribution to the variance of rainfall patterns in Kalimantan. The spatial pattern is the result of visualization of the principal component scores of each EOF mode, while the temporal plot is the result of the FFT of the principal component [3].

Figure 7. The (a) first (b) second and (c) third EOF mode spatial plot and the (d) first (e) second and (f) third EOF mode temporal plot for rainfall data in Kalimantan from 2001-2019.

Figure 7(a) and 7(d) show the spatial and temporal patterns of the first EOF mode. The variance explain value of the first EOF mode against the first 6 principal component scores is 64.44%. The first mode can explain the rainfall patterns in most of West Kalimantan and parts of Central, East and North Kalimantan. These areas have an equatorial or a 6-month rainfall pattern, which is shown by the high PSD value during the 6-month/cycle period in the first temporal mode plot.
Figure 7(b) and 7(e) show the spatial and temporal patterns of the second EOF mode. The variance explain value of the second EOF mode against the first 6 principal component scores is 11.72%. The second mode can explain the rainfall pattern in North Kalimantan. This area has an equatorial or a 6-month rainfall pattern, which is shown by the high PSD value during the 6-month/cycle period in the second temporal mode plot.

Figure 7(c) and 7(f) show the spatial and temporal patterns of the third EOF mode. The variance explain value of the third EOF mode against the first 6 principal component scores is 7.9%. The third EOF mode can explain the rainfall patterns in most of South and Central Kalimantan which are not explained in the previous two modes. These areas have a monsoonal or an annual rainfall pattern, which is shown by the low PSD value during the 6-month/cycle period in the third temporal mode plot.

4. Conclusions

Based on the results of the FFT analysis, Kalimantan has 2 types of rainfall patterns, which are monsoonal and equatorial. The power of the 6-month frequency of rainfall signal has no significant effect on the overall signal in the monsoonal type area. Otherwise in the equatorial type area, the power of the 6-month frequency value is high, so that the 6-month frequency has significant effect on the overall signal. The results of FFT analysis in 15 cities around Kalimantan show that the rainfall pattern in Samarinda, Sendawar, Tarakan, Tanjungselor, Pontianak, Ketapang, Malinau, Pangkalanbun, and Sintang are an equatorial type, while the rainfall pattern in Balikpapan, Kotabaru, Palangkaraya, Barabai, Purukcahu, and Banjarmasin are a monsoonal type. Moreover, the FFT results on all Kalimantan rainfall data show that most areas in West, East and North Kalimantan have an equatorial rainfall pattern. Meanwhile, most areas in Central and South Kalimantan have a monsoonal rainfall pattern.

The results of the FFT analysis were supported by the result of the empirical orthogonal function (EOF) analysis based on singular value decomposition (SVD). Based on the first 3 EOF modes, the first and second modes explain the equatorial rainfall patterns in West, East, and North Kalimantan, and some areas in Central Kalimantan, while the third mode explains the monsoonal rainfall patterns in Central and South Kalimantan and some areas in East Kalimantan.

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