Multivariate capability indices in inventory control

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Abstract. Inventory control in the industry has an important role in supply chain management to produce products according to management standards or quality standards based on consumer demand. In the apparel industry, inventory control is adjusted to the type of apparel product according to consumer demand. The research aims are to derive a multivariate inventory control model with multivariate capability analysis based on several types of apparel products as quality characteristics. The method in the research uses the $T^2$ Hotelling multivariate control chart and the multivariate capability indices for analysis apparel inventory control by management specification standards. We derived the $T^2$ Hotelling multivariate control chart and the multivariate capability indices using data with the assumption that multivariate data are controlled and has multivariate normal distribution Variable research uses quality characteristics that are grouped into two types of apparel products. This model is useful as a multivariate inventory control model that can manage the amount of production and supply of products based on probabilistic consumer demand.

1. Introduction

In the current industrial era, market changes in the apparel industry are developing very fast which is marked by very dynamic and complex change [1]. Industry players must always be ready to face market changes every day [2]. Industrial or company management actions must be able to anticipate to face these changes [3, 4]. The actions taken include predicting and preparing the business for every day that is experiencing change [5, 6]. Dynamic changes in the apparel industry must be followed by management who can anticipate business processes that require various stages, and each stage is accompanied by an element of uncertainty caused by uncertain market demand [7].

At present every apparel industry prepares some product types according to what consumers want. The industry also makes several products based on the type of apparel. This will have an impact on apparel product inventory models [2]. Each type of apparel product needs different from one another. The number of types of apparel products is very dependent on consumer demand. With the existence of different inventories according to the type of product, a control model is needed for each inventory.

The model of multivariate inventory control is part of supply chain management which has an important role to control the company's inventory by the types of products in all supply chains [1]. For the apparel industry to be successful in a very tight competitive business environment, the apparel industry must be able to carry out supply chain management to be able to continuously improve supply chain performance [7]. Many companies are faced with a tight, competitive environment and the emergence of uncertainty as a result of information technology innovation and changing customer needs [2]. In this case, inventory control can be used to measure consistent and sustainable performance in each of the links in the supply chain which makes a key role in the company's success in achieving
business objectives with certain profitability [1]. Supply chain performance also needs to be evaluated by market changes.

Multivariate inventory control (MIC) is part of the multivariate statistical control or multivariate process control [8]. The purpose of MIC in the apparel industry is to keep the number of supplies in the industry for each type of apparel by management standards related to the efficiency of resource management, especially inventory resources. Besides that, it is also to maintain the amount of inventory by the amount needed by consumers.

One important method in MIC is a multivariate control chart, which is a control chart that uses several variables in the control process [8]. The control chart for the MIC serves to control the number of products needed using quality characteristics according to the type of apparel product [9, 10]. The results of MIC control are used to state that the inventory process is in a stable or unstable condition. The process is called stable, if the observation points in the control model are at the control limit, namely the upper and lower control limits. In the case of MIC, many types of products are used in inventory as quality characteristics, where the variables are correlated.

In the production process often uses several correlated quality characteristics, so that in the process control system is carried out simultaneously based on some quality characteristics used simultaneously in multivariate control charts or the capability model [7]. Multivariate control charts can use the $T^2$ Hotelling chart that is useful for detecting a shift in process averages. The research aims to design the MIC control model and an analysis of multivariate capability in the apparel industry based on two types of apparel products so that it can regulate the amount of production and supply of products for each type of apparel based on probabilistic market or consumer demand.

2. Research methods
The research is an applied multivariate statistical research on the MIC model, using consumer demand as probabilistic quality characteristics. Research material uses problems in case of studies for the apparel industry, and use observation data on the apparel industry for 61 days. The research method uses statistical and probability approaches to design MIC models with two types of apparel products as quality characteristics that are mutually correlated.

Two methods are used in MIC, first is the MIC chart with the $T^2$ Hotelling chart to state that the observation data is in controlled condition or not for the production process by involving two types of apparel products together. The second, the multivariate capability indices for inventory capability analysis grouped by two types of apparel together according to management standards. The multivariate control model is part of the supply chain innovation that can make inventory planning according to needs or demands of consumers that are used as the basis for regulating the amount of production and supply of products based on consumer demand based on several types of apparel.

Research material includes hypothesis testing of observational data to prove data observation is meeting assumptions for correlated and multivariate normal distribution. Testing hypothesis for multivariate normal distribution using the q-q plot method based on the Mahalanobis distance at each development point with average as the midpoint. Testing hypothesis for multivariate normal distribution also using the Kolmogorov-Smirnov method. Inspection of observer data to meet the assumption that multivariate data is controlled using $T^2$ Hotelling chart. Data processing techniques using SPSS and Minitab, as well as other software needed for computing and data analysis.

3. Multiple demands in inventory control
The inventory control model in this research uses the consumer demand variable as a multivariate variable quality characteristic for analysis by involving more than one variable and each correlating with each other. The data structure multiple customer control is given in Table 1.
Table 1. Data structure multiple customer control

| Observation | Quality characteristics |
|-------------|-------------------------|
|             | X₁                      |
| 1           | x₁₁, x₁₂, ..., x₁j, ..., x₁q₁ |
| 2           | x₁₂, x₁₂, ..., x₁j, ..., x₁q₂ |
| 3           | x₁₃, x₁₃, ..., x₁j, ..., x₁q₃ |
| ...         | ...                     |
| n           | xᵣᵢₙ, xᵣᵢₙ, ..., xᵣᵢ, ..., xᵣᵢₚ |

Mean  \( \bar{X} = [\bar{X}_1, \bar{X}_2, ..., \bar{X}_q] \)

Variance  \( S^2 = \)  

where  \( x_{ij} \) is the observation of quality characteristic denoting the  \( j \)th variable and the  \( i \)th observation.

The quality characteristic in Table 1 can be represented by 
\[
X = [X_1, X_2, ..., X_q]
\]

The  \( j \)th sample means and variances are calculated from each observation:

\[
\bar{X}_j = \frac{1}{n} \sum_{i=1}^{n} x_{ij}
\]

\[
S_{jj} = S^2_i = \frac{1}{n-1} \sum_{i=1}^{n} (x_{ij} - \bar{X}_j)^2
\]

Sample covariance for  \( l \)th and  \( k \)th quality characteristics are calculated by:

\[
S_{lk} = \frac{1}{n-1} \sum_{i=1}^{n} (x_{li} - \bar{X}_l)(x_{ki} - \bar{X}_k); \ l = 1, 2, ..., q; k = 1, 2, ..., q
\]

Then set off the quality characteristic mean is represented by 
\[
\bar{X} = [\bar{X}_1, \bar{X}_2, ..., \bar{X}_q]
\]

and sample covariance is represented by
\[
S = \begin{bmatrix}
S_{11} & S_{12} & \cdots & S_{1q} \\
S_{21} & S_{22} & \cdots & S_{2q} \\
\vdots & \vdots & \ddots & \vdots \\
S_{q1} & S_{q2} & \cdots & S_{qq}
\end{bmatrix}
\]

Furthermore, the population mean and covariance of the quality characteristic mean is represented by:

\[
\mu = [\mu_1, \mu_2, ..., \mu_q]
\]

\[
\Sigma = \begin{bmatrix}
\sigma_{11} & \sigma_{12} & \cdots & \sigma_{1q} \\
\sigma_{21} & \sigma_{22} & \cdots & \sigma_{2q} \\
\vdots & \vdots & \ddots & \vdots \\
\sigma_{q1} & \sigma_{q2} & \cdots & \sigma_{qq}
\end{bmatrix}
\]

Mean vector \( \mu \) in (6) and covariance matrix \( \Sigma \) in (7) can be computed by sample mean vector \( \bar{X} \) as formula (4) and sample covariance matrix \( S \) as formula (5).
The normal distribution is one of the continuous distributions that are important in controlling quality statistical control. In univariate statistical quality control, the normal distribution is used with the normal probability function expressed by:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma^2} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, -\infty < x < \infty$$

with mean $\mu$ and variance $\sigma^2$. Whereas the probability function for a multivariate normal distribution is expressed by:

$$f(x) = \frac{1}{(2\pi)^{p/2}|\Sigma|^{1/2}} e^{-\frac{1}{2}(x-\mu)'\Sigma^{-1}(x-\mu)}, -\infty < x_j < \infty, j = 1,2,..., p$$

Testing the multivariate normal distribution from sample data or observations using a Q-Q plot based on quadratic distance and the Kolmogorov-Smirnov test [11].

4. Multivariate control charts

MIC is used as a method for coordinating and overseeing the supply, storage, distribution, and recording of products from different types of products (Figure 1). MIC in supply chain management aims to regulate the production and supply of products based on the determination of safe stock and product stock renewal by market needs or consumer demand [12]. Data on consumer demand are the basis for the material in inventory control. The MIC model is expected to generate maximum profits with minimal inventory investment without reducing the level of customer satisfaction or the level of filling demand or orders.

Figure 1 shows that the data from consumer demand or retail for several types of apparel products are the basis for making estimates of the products that must be produced. Therefore it is necessary to determine the multivariate inventory allocation by the estimates of each type of product as a basis for preparing production. Data of demand is packaged in the request data within the waiting time. In this case, the multiple demands as a quality characteristic is a random variable and the waiting time is considered constant.

![Multivariate inventory control in the supply chain](image)

**Figure 1.** Multivariate inventory control in the supply chain

Multivariate control charts are one of the main techniques in the statistical quality control process that is used to reduce variations in the process with quality characteristics variables being examined more than one variable or commonly called multivariate [13, 14]. In general, multivariate control charts are divided into two, namely multivariate attribute control charts for qualitative data and multivariate variable control charts for quantitative (measurement) data. The most common and familiar procedure in multivariate statistical quality control processes is to use the $T^2$ Hotelling control chart to see the average vector of the process.

Multivariate control charts for observational data with some quality characteristics can use the $T^2$ Hotelling control chart. The formula $T^2$ Hotelling for $ith$ observation is expressed by the formula:
\[ T_i^2 = (X_{ji} - \bar{X}_j)^T S^{-1} (X_{ji} - \bar{X}_j) \]  

(8)

Hotelling control chart has control limit as upper control limit (UCL) and lower control limit (LCL) which can be expressed by the following formula

\[ UCL = \frac{p(n-1)}{n-p} F_{a,p,n-p} ; \quad LCL = 0 \]

where \( F_{a,p,n-p} \) is F-distributed with \( p \) and \( (n - p) \) degrees of freedom. An observation of multivariate quality characteristics said to be controlled if the value of \( T_i^2 < UCL \).

5. Multivariate capability indices

Multivariate capability indices can be used if using data with the assumption that multivariate observation data shows that it is controlled and has multivariate normal distribution [15, 16]. The multivariate capability indices is an index that shows the value of the ratio between the spread (variability) of permitted product specifications and the actual process deployment that involves more than one variable [17]. The multivariate capability index value is used to measure process capability in cases with many variables. In the multivariate case, the specification limits and the process of dissemination are more difficult to visualize.

Let \( X = (X_1, X_2, \ldots, X_q) \) is a q-dimensional quality characteristic with a mean vector \( \mu \) and a variance-covariance matrix \( \Sigma \). Let \( S \) is the sample variance-covariance matrix. For analysis of MIC, each quality characteristic of univariate \( X_j \) has a given specification interval \((\text{LSL}_j, \text{USL}_j)\), \( j = 1, 2, \ldots, q \). The tolerance area for \( X_j \) is the hyperrectangular area [17]. The multivariate capability index is defined by the formula

\[ MC_p = \frac{\text{Vol. (modified tolerance region)}}{\text{Vol. (modified process region)}} = \frac{\text{Vol. (modified tolerance region)}}{\text{Vol.}(X - \mu)^T \Sigma^{-1} (X - \mu) \leq k(q)} \]

(9)

where \( k(q) \) is \((1-\alpha)100\% \) of \( \chi^2 \) distribution with \( q \) degrees of freedom, \( |\Sigma| \) is the determinant of covariance matrix \( \Sigma \) and \( \Gamma(\cdot) \) is the gamma function. The multivariate capability index can be defined based on a vector of target values \( T \):

\[ MC_{pm} = \frac{MC_p}{D} \]

where \( D = [1 + (\mu - T)^T \Sigma^{-1} (\mu - T)]^{1/2} \)

The volume of modified tolerance region is formed by upper and lower limits of specification, relative to the characteristics of quality:

\[ \prod_{j=1}^{p}(\text{LSL}_j - LLS_j) \]

(10)

In general case, volume (modified tolerance region) with q quality characteristic can be expressed by volume:

\[ \text{Vol.}(\text{modified tolerance region}) = \frac{2 \prod_{j=1}^{p} L_j \, q^{q/2}}{q \, \Gamma(q/2)} \]

(11)

where \( L_j \) is the lengths of semi-axes [18].

Based on the assumption that the q-dimensional quality characteristic has a multivariate normal distribution, then MCI () can be stated by the formula:

\[ MC_p = \frac{\text{vol. (modified tolerance region)}}{(\pi q/2)^{q/2} \, |\Sigma|^{1/2} \, \Gamma(q/2)} \]

\[ \left[ \frac{q}{2} + 1 \right]^{-q/2} \]

The estimation of \( MC_p \) and \( MC_{pm} \) can be estimated by an estimator from sampling as follow
\[
\hat{MC}_p = \frac{\text{vol(modified tolerance region)}}{(\pi \bar{X}^{2n-1})^{1/2}|S|^{1/2}[\Gamma(q/2+1)]^{-1}}
\]  
(12)

\[
\hat{MC}_{pm} = \frac{MC_p}{\hat{D}}
\]  
(13)

where \(S\) is the sample covariance matrix and \(|S|\) is the determinant of \(S\), and

\[
\hat{D} = \left[1 + \left(\frac{n}{n-1}\right)(\bar{X} - T)'S^{-1}(\bar{X} - T)\right]^{1/2}
\]  
(14)

If the value of \(\hat{MC}_p\) or \(\hat{MC}_{pm}\) > 1, then the process has smaller variation than the specification limits, so it can be said the process has gone well. Conversely, if the index value of \(\hat{MC}_p < 1\) indicates the process variation is greater than the company's management specification limits. This means that the process produces many production processes that are not by specifications.

6. Case study
In the research, a case study for analysis of apparel product inventory control is based on probabilistic consumer demand data. Case studies use two quality characteristics in the form of apparel product types, namely types I and II. The data used in this case study uses customer demand data per day for 61 days. Sales management determined that the specification limits of products sold every day for the type of apparel I and II are respectively are [425, 175] and [350, 850]. The specification of the target for two quality characteristics is \(T = [300, 600]\).

Based on sample data, two quality characteristics correlate with correlation coefficients is 0.472 with \(p\)-value = 0.00. The sample mean vector and sample covariance matrix for two quality characteristics based on data observation for 61 days are given in Table 2. We derived distribution \(F_{0.05,2,61-2} = 3.15\). From (9), we derived :

\[
UCL = \frac{2(61-1)}{61-2}F_{0.05,2,61-2} = 6.407
\]

Furthermore, based on data observation and (8), a multivariate control chart for two quality characteristics can be expressed in Figure 2. Figure 2 shows that all points on the \(T^2\) Hotelling control chart are below the UCL value and above the LCL. Therefore, it can be concluded sample data in stable condition.

| Type of product | n  | Min | Max | Mean | Std. Dev |
|-----------------|----|-----|-----|------|----------|
| Type 1          | 61 | 150 | 415 | 272  | 55       |
| Type 2          | 61 | 345 | 698 | 532  | 76       |

![Figure 2. \(T^2\) Hotelling control chart](image-url)
The sample mean vector (4) and the sample covariance matrix (5) of multivariate customer demand is

\[ \bar{X}^T = \begin{bmatrix} 272 & 532 \end{bmatrix} \quad \text{and} \quad S = \begin{bmatrix} 2988 & 1971 \\ 1971 & 5840 \end{bmatrix}. \]

Furthermore, we derived the inverse of sample covariance \( S \) and determinant \( |S| \) as follows:

\[ S^{-1} = \begin{bmatrix} 0.000431 & 0.000145 \\ 0.000145 & 0.000220 \end{bmatrix} \quad \text{and} \quad |S| = 13567585.5. \]

Based on formula (10), (11) and (12), for \( q = 2, \alpha = 0.05, \chi^2_{2,0.95} = 5.99, \quad \Gamma(2) = 1 \), the estimation of \( MC_p \) can be computed as follows:

\[
\hat{MC}_p = \frac{\text{vol}(\text{modified tolerance region})}{\left( \pi \chi^2_{q,0.95} \right)^{q/2} |S|^{q/2} \Gamma(q/2+1)}
\]

\[
= \frac{\pi \times \left[ \frac{125-175}{2} \times \frac{850-350}{2} \right]}{\pi \chi^2_{2,0.95} |S|^{1/2} \Gamma(2)^{-1}} = 1.416
\]

And from (14), we have

\[ \hat{D} = \left[ 1 + \left( \frac{61}{1-\alpha} \right) (\bar{X} - T)' S^{-1} (\bar{X} - T) \right]^{1/2} = 1.99 \]

And from (13), we have

\[ \hat{MC}_{pm} = \frac{MC_p}{\hat{D}} = 1.289 \]

The computational results for the estimation of multivariate capability index \( \hat{MC}_p = 1.289 \). It can be concluded that the multivariate capability index for the production process of apparel products using inventory control is greater than 1 at the 95% level. Therefore, it can give a decision that apparel products produced are close to the target by management standards.

7. Conclusions

The model of multivariate inventory control is designed as part of supply chain management which functions to regulate the number of production and supply of products consisting of several types of clothing products together based on probabilistic consumer demand. This control model uses multiple customer demands as multiple quality characters that are controlled together. By assuming that multiple quality characteristics have a multivariate normal distribution, multivariate inventory control chart can be used using T Hotelling to detect whether multivariate quality characteristics are controlled or not. If multivariate quality characteristics are controlled, then multivariate capability indices can be determined using multivariate statistical control theory. Multivariate models of inventory control are part of supply chain management has an important role to control the resources of companies in all of the chain. With the functioning of the inventory control system, the apparel industry can carry out supply chain management to continuously improve company performance. With this inventory control model, it can be used as a consistent and sustainable performance measurement tool.

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