Suction effect on an unsteady Casson hybrid nanofluid film past a stretching sheet with heat transfer analysis

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Abstract. This paper studies the heat transfer in the blood fluid-based copper and alumina nanoparticles over an unsteady permeable stretching sheet. The model is governed by the governing equations consist of a series of ODEs that are reduced from PDEs by implementing the similarity transformations subjected to mixed boundary conditions. The results of the transformed equations has been obtained by using the Keller-box method in MATLAB software. This paper focuses on the characteristics of thin-film flow and heat transfer through the governing parameters; unsteadiness parameter, nanoparticles volume fraction, Casson parameter, and intensity of suction. From this study, it is observed that the behavior of both fields for nanofluid is lower than hybrid nanofluid under the suction effect. It is noticed that enhance the physical parameters increase the velocity field of the fluid. Further, increase the physical parameter also deteriorate the temperature field except for nanoparticles volume fraction.

Keywords: Thin-film; Nanoparticles; Stretching Sheet; Keller-box Method.

1. Introduction

The development of technologies nowadays attracted researchers to explore and analyze the issues on heat transfer. The passage of particles from different temperatures is regarded as the transfer of heat. As far as the thermodynamic system is concerned, heat transfer is the flow of heat across the device boundary due to the difference in temperature between the system and the environment. The heat transfer in the system can also be performed at different points inside the system because of temperature differences. Numerous researchers studied and explored the issues of heat transfer in theoretical and numerical works included the following literature: Anitha et. al. [1], Bai et. al. [2], Hashimoto, Kurazono and Yamauchi [3], Hussein et. al. [4], Izadi et. al. [5] and Waini, Ishak and Pop [6].

The presence of nanoparticles in the fluid additionally influenced the heat transfer of fluid. The mixes of nanoparticles known as hybrid nanoparticles will be improved the heat transfer contrasted with regular fluids [7]. Hashimoto et al. [4] and Ali [8] discussed the presence of hybrid nanofluid that would affect the heat transfer. The perfect combination of various nanoparticle properties has demonstrated an outstanding heat transfer coefficient enhancement with a low-pressure decrease limit. The host fluid with the hybrid nanoparticles studied by Yazid et al [9] increased the heat flux of the fluid. The numerical studied of natural or free convection heat embedded with hybrid nanoparticles in a nanofluid also has been explored by Bai [5]. Besides that, the increase of the material industrial productions attracted the researcher to explore and study the similar phenomena in the non-Newtonian fluids flow since the fluids
are widely used in that industry. Firstly, the numerical behavior of Casson fluid due to a stretching sheet has been done by Abas et al. [10]. The empirical and theoretical analysis of the Casson fluid with the inclusion of nanoparticles used in the industry has been cross-examined by several scientists, including [11-14]. Rawi et al. [12] claimed that the enhancement of the temperature profile in Casson fluid is due to an increase in the volume of composite in the fluid. Furthermore, second-grade fluid type, also known as viscoelastic fluid, is contained in silicone fluids where these fluids show both viscous and elastic properties. Goyal and Bhargava [15] studied the numerical method which is the FEM to solve the viscoelastic solution of flow.

Recently, coating technologies such as wire and fiber coatings are one of the thin-film stretch layer used in industrial applications. The best nature in the process’s cycle is controlled by changing the amount of heat in a certain fluid when a sheet is stretched [16]. From the literature review, Carragher and Crane [17], Crane [18] and Sakiadis [19] imposed the theoretical technique to solve the issues of heat transfer in the same geometry. Motivated by the published paper [17-19], the first problem of thin-film flow has been explored by Wang [20]. This author applied the HAM to solve the governing equations analytically. Andersson et. al [21] improved the work of [20] and introduced the dimensionless for temperature.

For the first time, the power-law fluid in a thin film along the stretching sheet was examined in 1996 by Andersson et al. [21]. The heat transfer analysis of Carreau fluid in the same geometry was observed by Tsehla et. al [22], Nagatharan et. al. [23] and Khan et. al. [24]. The MHD and thermal radiation effects on thin film in Maxwell nanofluid were observed by Fareesha et. al. [25] and Hameed et. al. [26] joined the MHD and electric field effects on the same fluid. Ali et al [27] investigated the numerical method, fourth-order Runge-Kutta (RK4) of MHD Casson fluid in a liquid film. Rehman et. al. [16] explored the both analytical (HAM) method and numerical method (bvp4c) on the Casson fluid together with slip and suction/injection effects in uniform film thickness over a stretching sheet. Both methods show a good agreement through the calculation of the total average square residual error with the help of MATHEMATICA. The combinations of several effects on MHD Casson film fluid flow have been cross-examined by Vijaya et. al. [28]. Next, the slip parameter on Casson fluid flow affected the film thickness as claimed by Mahmoud and Megahed [29]. Ray et. al [30] added the nanoparticles to the magneto-bioconvection in a similar type of fluid flow. Azis and Afify [31] investigated the MHD Casson liquid film with the variable of thermal conductivity and thermal radiation. The comparisons of the numerical results with the analytical result from a published paper show an excellent agreement. Megahed [32] investigated the variable of heat flux and viscous dissipation on the fluid flow.

Considering the above broad examinations, it is apparent that the blend of mixture nanoparticles and suction impact in a uniform thin film thickness past an unsteady stretching sheet has not been considered in the writing. This is the fundamental focal point of the current work. The PDEs form of governing equations will be transformed into the simple set of ODEs by employing the similarity transformations from the published paper and then deal with the Keller-box method.

2. Method and materials
This study considered an unsteady 2-D incompressible hybrid nanofluid of Casson flow. The film has a uniform thickness \( h(t) \) as displays in Figure 1 [20]. The horizontal plane is placed at \( x=0 \) axis from the narrow slit. \( y=\) axis is perpendicular to \( x=\) axis. The plane is stretching with velocity \( U_w(x,t)=bx/(1-\alpha t) \). The constants appearing in the stretched rate are \( b>0 \) and \( \alpha>0 \) where \( b \) is an initial stretching rate. The fluid is being sucked, \( V_w>0 \) and injected, \( V_w<0 \) at the boundary layer with velocity \( V_w=(V_w)_0/(1-\alpha t)^{1/2} \) where \((V_w)_0\) is an initial concentration of the reactant. The slit temperature, \( T_0 \) and reference temperature, \( T_{ref} \) at \( y=0 \) can be defined as
\[
T_w(x,t)=T_0+T_{ref}\left[\frac{bx^2}{2\nu}\right](1-\alpha t)^{1.5}.
\]
Figure 1: Physical diagram

The rheological logical equation of state for an isotropic and incompressible flow of a Casson fluid can be defined as [31].

$$
\tau_{ij} = \begin{cases}
2 \left( \mu_B + \frac{P_y}{\sqrt{2\pi}} \right) e_{ij} & \tau > \tau_c \\
2 \left( \mu_B + \frac{P_y}{\sqrt{3\pi}} \right) e_{ij} & \tau < \tau_c
\end{cases}
$$

Where $\mu_B$ is the plastic dynamic viscosity on the Non-newtonian, $P_y$ is the yield stress of fluid, $\tau$ is the product of the component of deformation rate with itself, namely, $\tau = e_{(i,j)}$, $e_{ij}$ is the component of the deformation rate and $\tau_c$ is the critical values of $\tau$ based on the non-Newtonian model. The governing equations can be written as [33]

$$
\begin{align*}
u_x + u_y &= 0 \quad (1) \\
\rho_{\text{hvf}} \left( u_t + uu_x + vu_y \right) &= \mu_{\text{hvf}} \left[ 1 + \frac{1}{\beta} \right] u_{xy} \quad (2) \\
\left( \rho c_p \right)_{\text{hvf}} \left( T_t + uu_T + vT_v \right) &= k_{\text{hvf}} T_{xy} \quad (3)
\end{align*}
$$

Where velocity segments along $x$ and $y$ direction is $(u,v)$. $k_{\text{hvf}}$ represents the conduction by the heat of nanodispersion, the density for the hybrid nanofluids is $\rho_{\text{hvf}}$, $(c_p)_{\text{hvf}}$ indicates the energy capacity of nanodispersion and $\mu_{\text{hvf}}$ demonstrates an effective dynamic viscosity and $\beta$ is Casson fluid parameter. This is subjected to

$$
\begin{align*}
y &= 0; u = U, v = V, T = T_s \quad (4) \\
y &= h; u_y = 0, v = \frac{dh}{dt} \quad (5)
\end{align*}
$$

where $U$ and $T_s$ denote the velocity and temperature. Introducing the following transformations vector $f(\eta)$ and $\theta(\eta)$[20].
\[
\psi = \left[ vb (1-\alpha t)^{1} \right]^{1/2} x \delta f (\eta)
\]
\[
T = T_0 - T_{ref} \left(\frac{bx^2}{2v}\right)(1-\alpha t)^{3/2} \theta(\eta)
\]
\[
\eta = \left(\frac{b}{v}\right)^{1/2} (1-\alpha t)^{1/2} \delta^{-1} y
\]

(6)

Where \( \eta \) is the similarity variable, \( \delta \) is an undefined constant which denotes the film’s dimensionless thickness and stream function, \( \psi(x,y,t) \). The first Equation (6) automatically satisfies the mass conservation equation (1). At the free surface, \( \eta = 1 \) and from the last equation (6), when \( y = h(t) \) we have:

\[
\delta = \left(\frac{b}{v}\right)^{1/2} (1-\alpha t)^{1/2} h(t)
\]

(7)

Which gives

\[
\frac{dh}{dt} = -\frac{1}{2} \alpha \delta \left(\frac{v}{b}\right)^{1/2} (1-\alpha t)^{1/2}
\]

(8)

The equations (1), (2), and (3) with the constraints (4) and (5) can be written in the new variables \( f(\eta), \theta(\eta) \) and \( \eta \) as a series of ODEs as follows:

\[
\left(\frac{A_1}{A_2}\right) \left(1+\frac{1}{\beta}\right) (f''(\eta)) + \gamma \left[ f(\eta) f''(\eta) - (f'(\eta))^2 - S \left[ f'(\eta) + \frac{1}{2} \eta f''(\eta) \right] \right] = 0
\]

(9)

\[
\left(\frac{A_1}{A_2}\right) \theta''(\eta) + Pr \gamma \left[ -S \left(\frac{3}{2} \theta(\eta) + \frac{1}{2} \eta \theta'(\eta) \right) - 2 (f'(\eta) \theta(\eta)) + (f(\eta) \theta'(\eta)) \right] = 0
\]

(10)

\[
f'(0) = 1, f(0) = w, f(1) = 0.5S, f''(1) = 0
\]

(11)

\[
\theta'(1) = 0, \theta(0) = 1
\]

(12)

Where \( A(i=1,2,3,4) \) is the hybrid nanofluid constants, \( \gamma = \delta^2 \) is thin film thickness, unsteadiness of the dimensionless measure is \( S = \frac{\alpha}{b} \), Prandtl number, \( Pr = \left(\frac{\mu c_p}{k_f}\right)_{bf} \) and suction/injection parameter is

\[
w = -\frac{(V_{w0})}{\beta \sqrt{\nu b}}.
\]

The hybrid nanofluid constants are demonstrated in [7] and the characteristics of the hybrid nanofluids, copper and alumina as well as blood as base fluids are described in [34] and [7].

3. Numerical procedure

The boundary value problem in the form of ODEs, Eqs (9) and (10) with related boundary conditions, Eqs (11) and (12) are solved with the help of the numerical method via the Keller-box method in MATLAB software. The book of Cebeci and Bradshaw [35] explained the details of this method. The Keller box method that implements the finite difference method (FDM) is extremely powerful to get the
estimated results to an arrangement of a nonlinear differential system like other FDM. Among various other mathematical strategies, the FDM is more versatile for the clarification that underlying approximations control the convergence rate. The four fundamental advances are included to get the mathematical arrangements through the Keller-box technique and are:

a) Reduce the governing equations (Eqs 9 and 10) into a first-order equations’ system;
b) The first-order system is written as a finite difference method by applying central differences;
c) Linearize the subsequent equations by using the Newton technique and utilized on the coefficient of a matrix of the finite difference equations;
d) The obtained linear system is understood utilizing the block elimination method by employing a block-tridiagonal factorization scheme.

The solutions can gain when imposing an excellent preliminary guess with a uniform step size $\Delta h = 0.001$. Then, the Prandtl number for blood, $Pr = 30$ [36] has been fixed throughout the computational process with convergence criterion $10^{-5}$ is used to obtain the solutions.

4. Results and discussion

The behavior of the hybrid Casson nanofluid flow and heat transfer in a thin-film is obtained by separating into two cases; nanofluid of Cu/blood and hybrid nanofluid of Cu-Al$_2$O$_3$/blood through a porous medium when the sheet is stretching. The numerical solutions are obtained for various physical parameters values to explain in-depth the pattern of the physical problems. Validation of the numerical code is observed by comparing the numerical results of the present study with Wang [20], which solved the problem using HAM. Table 1 provides a comparison of the numerical findings with previous publications [20] in the absence of nanoparticles volume fraction $\phi = \phi_2 = 0$, Casson parameter $\beta \to \infty$, suction parameter $w = 0$, and unsteadiness parameter $S = 0.8$ and $S = 1.2$ for different values of Prandtl number. A good agreement was found in a comparison with them despite different methods.

| $Pr$ | $S = 0.8$ | $\theta(\eta)$ | $S = 1.2$ | $\theta(\eta)$ |
|------|-----------|-----------------|-----------|-----------------|
| 0.01 | Wang [20] | Present study   | Wang [20] | Present study   |
| 0.1  | -0.090474 | -0.090478       | -0.037734 | -0.037734       |
| 1    | -3.595970 | -3.595574       | -1.999590 | -1.999429       |

Figure 2 until Figure 5 exhibit the velocity profile on the investigated parameters listed as unsteadiness parameter of $S$, nanoparticles volume fraction parameter of $\phi_2$, Casson parameter of $\beta$ and suction parameter of $w$. Figure 2 unveils the variations of the $S$ towards $f'(\eta)$. The velocity of fluid for Cu/blood is higher than Cu-Al$_2$O$_3$/blood with increasing the values $S$ from 1.0 to 2.5. It causes a decrement of the thickness momentum boundary layer of the fluid flow in a thin film for nanofluid and hybrid nanofluid. The diminishing of fluid speed because of the transverse force makes a drag force when the presence of unsteadiness alongside the stretching sheet. The influence of $\phi_2$ against velocity profile is demonstrated in Figure 3. The increase of the copper concentration ($\phi_2 = 0.01$ to $\phi_2 = 0.04$) leads to a decline in the velocity profile. However, the velocity profile with the presence of hybrid nanofluid is more advanced than regular nanofluid. The momentum boundary layer is more approached to $\eta = 0$. Furthermore, the momentum boundary layer for nanofluid is thinner compared to
hybrid nanofluid when the fluid behaves like Newtonian as increases $\beta$ as noticed in Figure 4. Usually, the raising of $\beta$ would boost the volume of plastic dynamic viscosity which made the resistance in the fluid motion. Therefore, the lessening of the velocity profile due to the slow movement of the fluid. Figure 5 and indicates the same result as in Figure 4 for various $w$. The intensity of suction increases as increases the value of $w$ at the surface of a permeable stretching sheet. Enhancement of the $w$ was slow down the molecules in the fluid regime and improve the slow movement of the fluid on the stretching sheet. Therefore, it reduces the momentum boundary layer.

Next, Figure 6 to Figure 9 express the variation of the governing parameters with respect to $\theta(\eta)$ when the sheet is stretched. The Figs illustrate that the temperature profile for hybrid nanofluid is higher when contrasted with regular nanofluid. The temperature profile in Figure 6 declines with decreases in the value of the $S$ of the fluid and it produces a thinner thermal boundary layer. The effect of $\phi_2$ towards temperature profile is viewed in Figure 7. Physically, the nanoparticles scatter energy as warmth. In this way, more energy will be utilized when the expansion of nanoparticles in the fluid and simultaneously causes an increment in temperature and thickness of the thermal boundary layer. The more noteworthy the estimation of a non-Newtonian fluid which is the Casson fluid parameter, $\beta$ the higher the temperature and the thicker the thermal boundary layer as shown in Figure 8. Figure 9 demonstrates the opposite pattern for temperature profile as increasing the intensity of suction, $w$. The upsurge in the suction attracts the amount of fluids particles into the wall then reduces the thermal boundary layer of the fluid in the thin film.

![Figure 2. Variation of $S$ against $f'(\eta)$](image)

![Figure 3. Variation of $\phi_2$ against $f'(\eta)$](image)

![Figure 4. Variation of $\beta$ against $f'(\eta)$](image)

![Figure 5. Variation of $w$ against $f'(\eta)$](image)
Figure 6. Variation of $S$ against $\theta(\eta)$

Figure 7. Variation of $\phi_2$ against $\theta(\eta)$

Figure 8. Variation of $\beta$ against $\theta(\eta)$

Figure 9. Variation of $w$ against $\theta(\eta)$

5. Conclusion
The problem of Casson hybrid nanofluid in a thin film over an unsteady stretching sheet has been investigated numerically by using a collocation technique. The model is used together with the suction effect and hybrid nanoparticles. The velocity profile diminishes as raises the investigated parameter. Besides, it found that the increment of the unsteadiness parameter and intensity of suction leads to a decline in the temperature field. The opposite behavior is displayed for nanoparticle volume fraction and Casson parameter.

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