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CCII based fractional filters of different orders

Ahmed Soltan a, Ahmed G. Radwan b, *, Ahmed M. Soliman c

a Electronics and Communications Engineering Department, Faculty of Engineering, Fayoum University, Egypt
b Engineering Mathematics Department, Faculty of Engineering, Cairo University, Egypt
c Electronics and Communications Engineering Department, Faculty of Engineering, Cairo University, Egypt

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ABSTRACT

This paper aims to generalize the design of continuous-time filters to the fractional domain with different orders and validates the theoretical results with two different CCII based filters. In particular, the proposed study introduces the generalized formulas for the previous fractional-order analysis of equal orders. The fractional-order filters enhance the design flexibility and prove that the integer-order performance is a very narrow subset from the fractional-order behavior due to the extra degrees of freedom. The general fundamentals of these filters are presented by calculating the maximum and minimum frequencies, the half power frequency and the right phase frequency which are considered a critical issue for the filter design. Different numerical solutions for the generalized fractional order low pass filters with two different fractional order elements are introduced and verified by the circuit simulations of two fractional-order filters: Kerwin–Huelsman–Newcomb (KHN) and Tow-Tomas CCII-based filters, showing great matching.

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Introduction

Generally, the classical linear circuit theory is based on integer order differential equations which reflect the behavior of the three well-known elements: the resistor, the capacitor and the inductor. However, these integer order elements are a very narrow subset of the real arbitrary-orders. During the last decade, a dramatic shift has taken place and many scientific researchers have been concerned with fractional order calculus [1–16]. Consequently, modeling real-world phenomena using fractional order calculus has received increasing attention. Many applications based on fractional-order systems have been recently discussed such as in the fields of bioengineering [12–14], chaotic systems [14], agriculture [15], electromagnetics, Smith-chart [14] and control theorems [16]. In addition, many fundamentals in the conventional circuit theories and stability techniques have been generalized into the fractional-order domain [2–4]. Moreover, fractional order electrical circuits such as filters [5–9] and oscillators [10–12] gained a large part of the attention of researchers.

The Caputo definition [1] of the fractional derivative of order \( \alpha \) can be written as:

\[
\mathcal{D}_t^\alpha f(t) := \begin{cases} 
\frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^\alpha} \, d\tau & n-1 < \alpha < n, \\
\frac{d^n}{dt^n} f(t) & \alpha = n.
\end{cases}
\]

where \( a \) is the initial time and \( t \) is the required time of calculation (independent time-variable). This definition is considered
the generalization of the conventional integer-order definition due to the extra degree of freedom \( x \). Applying the Laplace transform to (1) assuming zero initial conditions yields:

\[
L\{e^{\alpha f(t)}\} = s^\alpha F(s)
\]

(2)

In the analog domain such an operation is called the fracture device [17–19]. The expression for the impedance function of the fracture device is given by:

\[
Z(s) = k_o s^x = k_o (j\omega)^x
\]

(3)

where \( k_o \) is a constant and \( x \) is the fractional order. Then, the magnitude and phase of \( Z \) becomes

\[
|Z| = k_o s^x \quad \angle Z = \frac{\pi}{2}
\]

(4)

From (3), it can be observed that for \( x = 1 \), \( Z \) is an inductor; for \( x = 0 \), \( Z \) is a resistor; and for \( x = -1 \), \( Z \) is a capacitor. The phase angle of the impedance of the inductor, the resistor, and the capacitor is \( \pi/2 \), 0 and \(-\pi/2\), respectively. So a passive circuit element that gives a constant phase angle with frequency can be called a Fractional-Order Element (FOE) which is the generalized element of the already existing electrical circuit elements [17–19]. Four decades ago, some researchers investigated realizing a fractional-order capacitor. A finite element approximation of the special case \( Z = 1/(C\sqrt{s}) \) was reported by Saito and Sugii [19]. This finite element approximation is based on the possibility of emulating a fractional-order capacitor via semi-infinite RC trees [18]. The technique was later developed by some authors [17,18] to include any order. Finite element approximations offer a valuable tool by which the effect of a fracture device can be simulated using a standard circuit simulator, or studied experimentally. However, they do not offer a simple practical two-terminal device. Therefore, many practical realizations have been made for the fracture element by using the frequency dependent dielectric properties of some materials like \( \text{LiN}_2\text{H}_4\text{SO}_4 \) or using chemical reaction probe [20].

Moreover, analog circuit design using the current mode approach has recently gained considerable attention. This stems from its inherent advantages of wide bandwidth, high slew rate, low power consumption and simple circuitry [21–25]. So, as filters represent one of the most important and popular analog blocks [26–33], this work presents here a fractional order filter based on the CCII [21,22]. Due to the importance of filters, some of the previous work aimed to generalize the filter design to the fractional domain [5–9]. The main advantage in the fractional order filter is that the analog designer can obtain the exact requirements of the filter. Besides that, the design degree of freedom is also increased where the frequencies of interest become dependent not only on the circuit components but also on the fractional-order parameters [2–11]. So, this work aims to generalize the procedure described by Radwan et al. [6] using two fractional elements but of different orders (\( \alpha, \beta \)).

Now it is important to mention that, there are some critical frequency points which are necessary to improve the filter design. This paper will study the general formulas of the three frequencies which are: the maximum or minimum frequency (\( \omega_m \)), the half power frequency (\( \omega_h \)), and the right phase frequency (\( \omega_{rp} \)) [5,6]. This paper seeks to generalize the design of classical second-order filters to the fractional-order domain. In addition, two fractional elements of different non-integer order \( \alpha \) and \( \beta \) are used which are considered the generalized case of equal orders [5–7] and also of the integer-order case.

This paper is organized as follows: the next section demonstrates the proposed design procedure for the fractional order filters for independent values of \( \alpha \) and \( \beta \). Then the numerical analysis and ADS simulation results of the KHN and Tow Tomas fractional-order filters are presented including a subsection on the fractional order frequency scaling. Finally, the main contribution points will be summarized in the conclusion.

**Proposed design procedure**

To simplify the discussion, the design equations for the low pass filter response of two different fractional elements of orders \( \alpha \) and \( \beta \) are demonstrated where any other response can be obtained using frequency transformation techniques. Consequently, the proposed design procedures can be used to obtain any filter response. The fractional order low pass filter transfer function will be given by:

\[
T(s) = \frac{d}{s^{\alpha + \beta} + a s^\alpha + c} = \frac{d}{D(s, \alpha, \beta)}
\]

(5)

where \( a, c, d \) are constants and \( \alpha, \beta \) are the fractional-orders. Therefore the frequency response of the characteristic equation of (5) and its magnitude can be obtained (after neglecting the transient response) by substituting \( s = j\omega \) as follows:

\[
|D(j\omega, \alpha, \beta)| = \omega^{2(\alpha+\beta)} + \alpha^{2} \omega^{2\alpha} + 2 \alpha c \omega^{2\alpha+\beta} \cos(0.5\beta\pi)
\]

(6a)

\[
+ 2 \omega c \cos(0.5\alpha\pi) + 2 \omega c \alpha \cos(0.5(\alpha + \beta)\pi) + c^2
\]

(6b)

As a special case when \( \alpha = \beta \) then (6b) will return back to the special case presented in [6]. Note that, the previous Eqs. (6a) and (6b) are valid only if the poles of this system lie in the left-half plane which requires studying the stability condition which represents one of the most important parameters of the filter design [4]. Therefore the following subsections will study the general formulas of the three critical frequencies of interest mentioned before \( \{ \omega_m, \omega_h, \text{ and } \omega_{rp} \} \).

**The maximum and minimum frequencies (\( \omega_m \))**

The maximum and minimum frequency points determine the attenuation in the pass-band of the filter response. From (6b), the maximum and minimum frequencies can be obtained by solving the following non-linear equation:

\[
\omega_m^{2(\alpha+\beta)} + \frac{2\alpha + \beta}{\alpha + \beta} \omega_m^{2\alpha} \cos\left(\frac{\beta\pi}{2}\right) + c \omega_m^{2\alpha} \cos\left(\frac{(\alpha + \beta)\pi}{2}\right)
\]

\[
+ \alpha c \omega_m^{2\alpha+\beta} + \frac{\alpha \omega_m^{2\alpha+\beta} \cos(0.5\beta\pi)}{\alpha + \beta} = 0
\]

(7)

In this case, the design flexibility is increased because of the new variables \( \alpha \) and \( \beta \). According to (7), if the parameters \( a, c \), \( \alpha \) and \( \beta \) are given, then the value of the maximum and minimum frequency \( \omega_m \) can be calculated as illustrated in Fig. 1. As seen in Fig. 1a, the filter will have two values of \( \omega_m \) with
the change in the value of $a$ for $b > 1$. But on the other hand, the solution also does not produce any valid values for $\omega_m$ under all conditions. Fig. 1b proves that $\omega_m$ takes more than one value at certain ranges of $b$, and these ranges depend also on the value of $x$. It is apparent that for $x = 1.2$ and $b > 1.7$ in (7), this produces three values for $\omega_m$, which means that there are ripples in the pass-band. Also from Fig. 1c the value of $\omega_m$ takes sometimes more than one solution with the change in the values of $c$ at fixed values of $x$ and $b$ and the same happens with the change in $a$ too. This indicates that the design flexibility is increased where the design equation becomes dependent on the parameters $a$ and $b$ besides the original parameters $a$ and $c$. Moreover, Fig. 1d displays the number of values available for $\omega_m$ for all the combinations of $x$ and $a$. It is interesting to mention here that, Fig. 1d confirms the previous discussion of (7) as it shows that the filter can have one, two and three values or no valid values at all for $\omega_m$. Finally a summary of the numerical analysis introduced in Fig. 1 is presented in Table 1 where the important points and regions of operation for the cases of interest are displayed. From the previous discussion, there are three regions as follows: region1 where $\omega_m$ is undefined, region2 where $\omega_m$ is defined and has a value but the filter response is unstable, and region3 where $\omega_m$ is defined and the filter is stable. It’s clear that the value of the design parameters must be chosen carefully to make the filter work in region3.

The half-power frequency

The half power frequency represents the most important frequency point as most of the filter designers depend on it to design the filter bandwidth. So, the next step is to calculate the half power frequency $\omega_h$. By using the definition of $\omega_h$ introduced in [6,7] then $\omega_h$ will be given by solving the following non-linear equation:

$$\omega_h^2 + a^2\omega_h^2 + 2a\omega_h^2\cos\frac{\beta\pi}{2} + 2ac\omega_h^2\cos\frac{\beta\pi}{2} + 2c\omega_h^2\cos\frac{(x + \beta)\pi}{2} - c^2 = 0 \quad (8)$$

Consequently, the half power frequency depends on the value of $a$, $c$, $b$, and $x$, which adds an extra degree of design freedom. The analysis of (8) is shown in Fig. 2 under different conditions. It is clear from Fig. 2 that Eq. (8) always results in a value for $\omega_h$ under any condition. But some of these values lie in the unstable region of the filter as will be shown later. It is clear that the filter response suffers from a strong change in the pass band of the filter under certain conditions, because $\omega_h$ takes more than one value at these conditions as shown in Fig. 2a and b for $b > 1$. Also, the effect of $a$ on the calculation of $\omega_h$ is illustrated in Fig. 2c for the case $(x, \beta, a, c) = (0.6, 1.7, 6, 8)$ which gives three values of $\omega_h$ and
reflects that the filter suffers from very strong ripples. This happens because the filter poles lie near the unstable region [4]. Table 2 shows a summary for the critical regions when \(c = 8\) for different cases of \(a\) and \(\alpha\). In addition to the values presented here, the value of \(\omega_h\) can also be calculated for any given combination of the parameters by using (8). Another common practice is to design the filter at \(\omega_h = 1\) rad/s then frequency scaling is done to fulfill the required filter response. So, (8) will be rewritten at unity half power frequency as follows:

\[
a^2 + 2(\cos(0.5\beta\pi) + c \cos(0.5\pi))a + 2c \cos(0.5(\beta + \alpha)\pi) - c^2 + 1 = 0
\]  

(9)

It is clear that (9) is very critical in the filter design where the frequency scaling can be used to meet the required filter specifications. The numerical analysis of (9) at different conditions is shown in the 3D plot of Fig. 2d where the rate of change in the value of \(c\) is large with respect to \(\alpha\). But on the other hand, the change of \(\alpha\) due to \(\beta\) is small. It is clear from

Table 1 Critical points for the cases of interest.

| \(a\)       | \(\alpha\) | \(\beta\) | No. of roots                  |
|-------------|------------|-----------|-------------------------------|
| 0.01–3.9    | 8          | 1         | One root in the stable region |
| \(a > 3.9\) | 8          | 1         | \(\omega_h\) Is undefined     |
| 0.01–5.5    | 8          | 1.2       | One root but in the unstable region |
| 5.5 \(\leq a \leq 18.6\) | 8          | 1.2       | Three roots in the stable region |
| \(a > 18.6\) | 8          | 1.2       | One root in the stable region  |
| Whole tested range | 8          | 1.2       | One root in the stable region  |
| \(a < 8.4\) | 8          | 0.6       | Two roots in the stable region |
| \(a \geq 8.4\) | 8          | 0.6       | \(\omega_h\) Is undefined     |
| 6           | \(c < 18.1\) | 1         | One root in the stable region  |
| 6           | 18.1 \(\leq c \leq 80\) | 1         | \(\omega_h\) Is undefined     |
| 6           | Whole tested range | 1.2       | One root in the stable region  |
| 6           | 9.1 \(> c\) | 1.2       | One root in the stable region  |
| 6           | 9.1 \(\leq c \leq 9.5\) | 1.2       | One root in the stable region  |
| 6           | \(c > 9.5\) | 1.2       | One root in the stable region  |
| 6           | 4.9 \(> c\) | 0.6       | \(\omega_h\) Is undefined     |
| 6           | 4.9 \(\leq c \leq 80\) | 0.6       | Two roots in the stable region |

Fig. 2 (a) Change in \(\omega_h\) with respect to \(\alpha\) at different values of \(\beta\) at \(c = 8\) and \(a = 6\), (b) change in \(\omega_h\) with respect to \(\beta\) at different values of \(\alpha\) at \(c = 8\) and \(a = 6\), (c) change in \(\omega_h\) with respect to \(\alpha\) at different values of \(\alpha\) and \(\beta\) at \(c = 8\), and (d) change of \(c\) with respect to \(\alpha\) and \(\beta\) at \(a = 6\) at \(\omega_h = 1\).
Table 2 Summary of the critical regions for the cases of interest for \( c = 8 \).

| \( a \) | \( \alpha \) | \( \beta \) | No. of roots |
|-------|-------|-------|--------------|
| 0–6.6 | 0.5  | 1.7   | Three roots (damping exist) in the stable region |
| 6     | 0.67–2| 1.7   | One root in the stable region |
| 6     | 0.4   | 0.01–1.39 | One root in the stable region |
| 6     | 0.4   | 1.4–2 | Three roots (damping exist) in the stable region |
| 0.01–6.9 | 0.8 | 1.6   | One root but in the stable region |
| 7–9   | 0.8   | 1.6   | Three roots (damping exist) in the stable region |
| \( a > 9 \) | 0.8 | 1.6   | One root in the stable region |

However, (11) can be used only for the traditional filters or filters with fractional order elements of the same orders. On the other hand, for the fractional order filter with two different orders, scaling can be made for the transfer function parameters \((a, c)\), and then the component values can be calculated. So, in case of a fractional order filter with two fractional elements of different orders, the frequency scaling can be made as follows:

\[
\alpha_{new} = \alpha^{a/b}, \quad c_{new} = c_{old}^{a/b}, \quad d_{new} = d_{old}^{a/b}
\]  

Circuit simulation

We investigate here some of the famous RC second-order filters assuming their two normal capacitors to be replaced by two fractional capacitors of different orders \( x \) and \( \beta \). Then as a popular block in the analog design KHN and Tow-Tomas filters [28–35] are used here to illustrate the proposed design procedure. The finite element approximating circuit given by Sugi et al. [17] is used to simulate a capacitor of fractional order. The equivalent circuit of the fractional element is presented in the elliptical Fig. 4a. The values of the resistances and capacitors of the equivalent circuit composed of fractional order capacitor with \( Y_F = s^x C_F \) are given by the following relations [17]:

\[
R_{fi} = \left[ \frac{Y_F(x) \sin(\pi x) \sigma^n A(\ln \sigma)}{\pi} \right]^{-1}
\]  

(13a)

\[
C_{fi} = \frac{Y_F(x) \sin(\pi x) \sigma^{n-1} A(\ln \sigma)}{\pi}
\]  

(13b)

where \( Y_F \) is the admittance value, \( \sigma \) is the relaxation rate corresponding to the pole of the \( n \)th branch and \( A(\ln \sigma) = \ln \sigma_{n+1} - \ln \sigma_{n} \) is the pole interval taken in a logarithmic scale.

CCII based KHN filter

This section will focus on the simulation of the CCII based KHN filter [28–30] where its transfer function is similar to (5). The KHN biquad modified to include two fractional capacitors is shown in Fig. 4a. It simultaneously provides low-pass, band-pass, and high-pass behavior, although here we are interested only in the low-pass response. The transfer function of the KHN filter will be given by:

\[
\frac{V_{LP}^{\pm}}{V_{in}} = \frac{R_1}{s^{a+b} + \frac{R_2}{C_2 R_1} + \frac{R_3}{C_3 R_1 R_2 C_2} + \frac{R_4}{C_4 R_1 R_3 R_2 C_2}}
\]  

It is apparent that the transfer function of the KHN filter is similar to the transfer function in (5). Hence
For \( R_5 = R_6 \) and \( R_3 = R_4 \) the values of \( a \) and \( c \) will be reduced to \( a = 1/(2R_1C_1) \) and \( c = 1/(R_1R_2C_1C_2) \). So, the proposed design procedure can be applied here to design the KHN filter when \( \omega_h = 1000 \text{ rad/s} \). Then (9) can be used to calculate the value of \( c \) for a given value of \( a \) at the required cut-off frequency. And by substituting in (15), the required circuit ele-
ments will be determined and used in the simulations of the circuit shown in Fig. 4a for different values of $\alpha$ and $\beta$. The simulation results of the CCII based KHN filter is presented in Fig. 4b for different values of $\alpha$ and $\beta$.

**CCII based Tow-Tomas filter**

The history of the Tow-Thomas (TT) second order filter using operational amplifiers (op-amps) has been reviewed previously [31,32]. Passive and active compensation methods to improve the circuit performance for high $Q$ designs were also reviewed. It is well known that the classical TT circuit using op-amps has frequency limitations due to the finite gain-bandwidth of the op-amps. So, to improve these limitations of the op-amp based TT, many realizations for the TT filter were proposed based on the current mode building blocks [28]. One of the most important realizations of the TT filter using the CCII is presented by Soliman [28]. This circuit has the advantage of high input impedance and all resistors and fractional elements are grounded. Then, the TT is modified to include two fractional capacitors as shown in Fig. 4c. The fractional order filter transfer function is given by:

$$\frac{V_{LP}}{V_{in}} = \frac{1}{s^{\alpha} + \frac{1}{C_1 R_1} s + \frac{1}{C_1 C_2 R_2 R_3}}$$

(16)

By comparing the transfer function of (16) with that of (5), then

$$a = \frac{1}{C_1 R_1}, \quad c = \frac{1}{C_1 C_2 R_2 R_3}$$

(17)

Then by using (9) to calculate the value of the parameters $a$ and $c$ at certain fractional order, it will be easy to calculate the circuit values from (17). After that, a frequency scaling is made to fulfill the required frequency response. The circuit simulation of the fractional order filter for the cases $(\alpha, \beta) = (1.7, 1.2), (1.2, 0.4)$ and $(0.8, 0.7)$ is shown in Fig. 4d. It is worthy to note here that, the simulations are made using the parameters presented in Table 4 but after frequency scaling is made.

**Table 3** Summary of the simulation parameters.

| $(\alpha, \beta)$ | $a$ | Poles                  |
|------------------|-----|-----------------------|
| (0.8, 0.7)       | 6.1775 | $-6.768 \pm 2.889i$   |
| (0.6, 1.7)       | 5.769  | $-0.195 \pm 3.509i$   |
| (1.2, 0.4)       | 10.535 | $-0.6394 \pm 0.4204i$ |
| (1.7, 1.2)       | 18.4286 | $-1.342 \pm 5.279i$   |
|                  |       | $-0.4368 \pm 0.244i$  |

**Fig. 4** (a) CCII based KHN filter using two elements of different orders, (b) simulation results for the KHN filter at different values of $\alpha$ and $\beta$ for $\omega_0 = 1 kHz$, (c) CCII based TT filter using two fractional elements of different orders, and (d) circuit simulation at different values for $\alpha$ and $\beta$. 
By comparing the simulation results of Fig. 4b and d and the numerical analysis of Fig 3, a great similarity will appear. For example, the magnitude response in the case of \((\alpha, \beta) = (0.8, 0.7)\) has the lowest roll off rate as expected in Fig. 3 where this case has the lowest filter order in the cases of interest. Also, the fractional order filter with the orders \((\alpha, \beta) = (1.2, 0.4)\) has a small peaking in the simulation as indicated in the numerical analysis presented in Fig. 3. In addition, the cut off frequency for this simulation result is 160 Hz which is the scaled version from the numerical result. Therefore, the circuit simulation results of the proposed design procedure produce very close results to their theoretical study.

Conclusions

In this work, we have generalized classical continuous time filter networks to the fractional-order domain by using two fractional elements of different orders. We have driven the expressions for the pole frequencies, the right-phase frequencies, and the half-power frequencies. It is clear that more flexibility in shaping the filter response can be obtained via a fractional-order filter. Besides that fractional order circuit simulations of the KHN and Tow-Thomas filters design examples are introduced to validate the theoretical study.

Conflict of interest

The authors have declared no conflict of interest.

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