Continuum EoS for QCD with \( N_f = 2+1 \) flavors

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28.11.2012 | Stefan Krieg
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• Conclusions
Motivation: status 2012
Simulation: ensembles/statistics

# trajectories

$10^5$ $10^4$ $10^3$ $10^2$

$32^4$ $32^3 \times 64$ $40^3 \times 64$ $48^4$ $48^3 \times 64$ $64^4$

$a [fm]$ 0.16 0.14 0.12 0.1 0.08 0.06

# configurations

$10^6$ $10^4$ $10^3$ $10^2$

$N_t = 6$ $N_t = 8$ $N_t = 10$ $N_t = 12$ $N_t = 16$

$T [MeV]$ 120 140 160 180 200 220 240
Simulation: scale setting

2-stout scale function

- scale from \( w_0 \) along the LCP
- scale from \( w_0 \)-based step scaling
  - fit to the \( w_0 \) scale
  - fit to the \( f_K \) scale

\( w_0 \): JHEP 1209 010

**Aim:** estimate error of scale setting procedure

- original LCP
- step-scaling procedure like in 2010, \( w_0 \)-based observable
- deviations on the 2% level
- Included in analysis
Simulation: T/2 subtraction

• Reaching large temperatures requires small lattice spacings
• Algorithmically T=0 runs have difficulties to reach a<0.05 fm (frozen topology, diverging autocorrelation times).
• Solution: T/2 subtraction:

\[ I_{sub}(T) = ( I(T) - I(T/2) )_{\beta(a_0)} + ( I(T/2) - I('T=0') )_{\beta(2a_0)} \]

• Requires new simulations, however these are still in the high-temperature phase (\(N_t=8 \rightarrow N_t=16, \ldots\))
Systematics

- Finite volume effects
  - Studied explicitly in 2010 (see also Lattice 2011):
    - no effects (larger than statistical errors) seen.
  - This study includes larger volumes
    → Finite volume effects will be negligible compared to other systematic uncertainties
- Scale setting and lattice spacing artifacts
  - We vary the range of lattice spacings in our fits:
    - $N_t=6$ is included or left out
  - We use different scale settings
  - We include $O(a^4)$ in our fit procedure
Simulation: systematics, histogram method

- vacuum fits
  - 7 different fit models (incl. direct subtr. w. interp.)
- continuum extrapolation
  - Vary node points (8 different sets)
  - Include or leave out leave $N_t=6$
  - With or without improvement factors
  - We use two different scale settings ($f_k$ vs. $w_0$)
  - Fit includes $a^2$ or $a^2$ and $a^4$ terms

→ This results in $7 \times 8 \times 2 \times 2 \times 2 \times 2 = 896$ different fits
- Weighting: we consider AICc, Q, or unweighted histograms
Results: trace anomaly
Results: trace anomaly @ 215 MeV
Results: pressure

\[
p/T^4
\]

- **lattice continuum limit**
- **HRG**
- **HTL NNLO** [1103.2528]
- **SB**
- **215 MeV**

- **T [MeV]**

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Lattice 2013 – Stefan Krieg
Results: pressure @ 215 MeV

![Graph showing results at 215 MeV pressure]

- $N_t$, $dlog(Z)/dlog(m_{\text{phys}})$
- $T=214$ MeV
- $N_t=12$, $N_t=16$
- Light quark
- Strange quark
Results: pressure @ 215 MeV

\[
p/T^4 \text{ at } T=215 \text{ MeV}
\]

- tree-level corection
- no corection

\[
1/N_t^2
\]

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Results: entropy et al.
Results: trace anomaly

\[(\varepsilon - 3p)/T^4\]

2stout continuum limit
WB 2010
HRG
215 MeV

HISQ action [1211.1678]

- \(N_t = 6\)
- \(N_t = 8\)
- \(N_t = 10\)
- \(N_t = 12\)

\(T [\text{MeV}]\)
Conclusions

- We have performed a continuum extrapolation of the EoS for $N_f=2+1$ QCD
- We carefully studied and included systematic uncertainties
- Within our error the discrepancy to the hotQCD/HISQ results remains
- Final conclusion requires continuum extrapolation of HISQ data combined with a study of systematic uncertainties.
- In any case above $T\approx 300$ MeV charm effects become important.
Thank You for Your attention!
Simulation: ensembles/statistics

- Vacuum (T=0) runs:
  - Renormalization (& $w_0$ scale setting):
    - Volumes: $32^4$, $48^4$, $64^4$
    - #traj.: $O(10^4)$ for $32^4$
      $O(10^5)$ for $48^4$
      $O(10^3)$ for $64^4$
  - Scale setting ($f_k$):
    - Volumes: $32^3 \times 64$, $40^3 \times 64$, $48^3 \times 64$
    - #traj.: $O(10^4)$ for $32^3 \times 64$,
      $O(10^3)$ for $40^3 \times 64$ and $48^3 \times 64$
Simulation: ensembles/statistics

• Used available ensembles (see e.g. 1305.5161)
• Added ensembles:
  • $32^3 \times 6$, $32^3 \times 8$, with $13-50 \times 10^3$ trajectories
• Use sufficiently large volumes only
  • $L>2$ fm for all $T$
  • $L>5.3$ (12); 4.2 (10); 5.2 (8) @ 150 MeV
    • $48^3 \times 8$
    • $64^3 \times 10$ @ $O(10^4)$ trajectories each
    • $64^3 \times 12$
• Additional $48^3 \times 16$, $64^3 \times 20$, $64^3 \times 24$ ensembles
  (T/2 subtraction, 215 MeV point)

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Results: entropy @ 215 MeV

![Graph showing entropy at 215 MeV]