Bounding $b \to s\mu^+\mu^-$ tensor operators from $B \to K^*(X_s)\gamma$

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Tensor operators are often invoked as specific new physics operators beyond the standard model in an effort to explain anomalies in rare B-decays and CP asymmetries. Specifically, $b \to s\mu^+\mu^-$ tensor operators are invoked in the study of semi-leptonic decays, both inclusive and exclusive. In this note we use the data on $b \to s$ radiative decay modes and CP asymmetries to tightly constrain the tensor operators. It is found that constraints thus obtained are tighter than those from semi-leptonic modes. We also comment on $b \to s\bar{s}s$ tensor operators that help in explaining the $B \to \phi K^*$ polarization puzzle, and $b \to s\tau^+\tau^-$ operators with tensor structure.

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Absence of flavour changing neutral currents at the tree level within the standard model (SM) makes them very sensitive to quantum corrections due to heavy particles in the loops. Rare decays, both inclusive and exclusive, of the type $b \to s\gamma$, $b \to sg$ and $b \to s\ell^+\ell^-$ are among the most promising channels in our search for possible new physics (NP) beyond SM. The study of CP violation, and its origin, has been one of the main aims of the B-factories. Thanks to excellent experimental precision reached at the B-factories, and also at CLEO and TeVatron, we now have accurate measurements of branching ratios and CP asymmetries for many rare decay processes. LHCb has to be added to this list and has already begun to yield competitive results even with small amount of data collected till now (see for example [1], [2], [3]). The situation is expected to improve drastically over the next few years [4]. It is also not improbable that the first glimpse of NP or the absence of it at the electroweak scale will be from rare B-decays rather than the direct searches at LHC.

Till date SM has turned out to be consistent with almost all the available experimental data, though there are some anomalies or unexplained features that seem to call for physics beyond SM (see for example [5]). Experimental observations and measurements have established the dominance of Cabibbo-Kobayashi-Maskawa (CKM) phase as the prominent source of CP violation as far as the low energy sector is concerned. Any attempt to infer hints of new physics (NP) need to ensure that we have quantitatively exhausted all the possibilities within SM, including sub-leading effects and any other neglected contributions based on some assumptions. Semi-leptonic and radiative decays of B-mesons offer a unique opportunity to explore the possibility of NP, including new sources of CP violation beyond the CKM phase. The inclusive decays, radiative and semileptonic, are theoretically more under control while the exclusive decays are relatively easier experimentally. At present, the inclusive rate $BR(B \to X_s\gamma)$ and the exclusive rate $BR(B \to K^{(*)}\ell^+\ell^-)$ and associated lepton forward-backward asymmetry provide the most stringent constraints on any new physics model, even in a quite model independent manner. Latest experimental results indicate good agreement with SM expectations for these modes but also leave a bit of a room for NP.

When going beyond SM, new and heavier particles (more massive than the electroweak scale) can bring in totally new contributions not present in SM. An example could be left-right symmetric models which naturally lead to operators in the low energy theory that have different chiral structure than SM (see for example [6]). Other popular examples include supersymmetric theories which not only have different chiral structures for the operators but also bring along completely new contributions not present in SM. An example could be left-right symmetric models which naturally lead to operators in the low energy theory that have different chiral structure than SM (see for example [6]). The aim of the current and future experiments is to accurately measure all possible observables and thereby end up tightly constraining the possible structures/operators, and if possible revealing the specific type of NP present.

The effective Hamiltonian responsible for the semi-leptonic and radiative $b \to s$ transitions within SM is given by

$$\mathcal{H}_{eff}^{SM} = \frac{-4G_F}{\sqrt{2}} V_{tb}V_{ts} \left[ \sum_{i=1}^{10} C_i Q_i + \left\{ C_{7\gamma} Q_{7\gamma} + C_{8\gamma} Q_{8\gamma} + C_{9\gamma} Q_{9\gamma} + C_{10\gamma} Q_{10\gamma} + C_S Q_S + C_P Q_P + \sum_X C_X^\prime Q_X^\prime \right\} \right] + H.C$$

where $C_i$'s are the relevant Wilson coefficients while $Q_i$'s are four fermion operators. Here, $Q_{1,2}$ are the current-current operators, while $Q_{3-6}$ and $Q_{7-10}$ are the QCD penguin and electroweak (EW) penguin operators. Operators $Q_5$, $Q_6$, $Q_7$ and $Q_8$ have $(V - A) \otimes (V + A)$ structure while all others have $(V - A) \otimes (V - A)$ structure. $Q_{7\gamma}$ and $Q_{8\gamma}$ are the electromagnetic and chromo-magnetic dipole operators while $Q_{9\gamma}$ and $Q_{10\gamma}$ are the vector and axial-vector semi-leptonic operators. $Q_S$ and $Q_P$ are the scalar and pseudoscalar semi-leptonic operators. The primed operators can be obtained from the unprimed ones by making the replacement $L \leftrightarrow R$. The terms proportional to $V_{us}^* V_{us}$ are usually neglected due

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to the smallness of $|V_{ub}|$. For what concerns us here, the explicit form of some of the operators is:

\[ Q_1 = (\bar{c}L_\beta\gamma^\mu b_L)(\bar{s}L_\alpha\gamma^\mu c_L_\beta), \]
\[ Q_2 = (\bar{c}L_\alpha\gamma^\mu b_L)(\bar{s}L_\beta\gamma^\mu c_L_\beta), \]
\[ Q_7 = \frac{g_s}{16\pi^2} m_b (\bar{s}_\alpha\sigma^{\mu\nu} Rb_\alpha) F_{\mu\nu}, \]
\[ Q_{8g} = \frac{g_s}{16\pi^2} m_b (\bar{s}_\alpha\sigma^{\mu\nu} R^\alpha_\beta b_\beta) G^4_{\mu\nu}, \]
\[ Q_{9V} = \frac{e^2}{16\pi^2} m_b (\bar{s}_\alpha\gamma^\mu Lb_\beta)(\bar{c}_\mu\ell), \]
\[ Q_{10A} = \frac{e^2}{16\pi^2} m_b (\bar{s}_\alpha\gamma^\mu Lb_\beta)(\bar{c}_\mu\gamma_5\ell), \]
\[ Q_S = \frac{e^2}{16\pi^2} m_b (\bar{s}_\alpha Rb_\beta)(\bar{\ell}), \]
\[ Q_P = \frac{e^2}{16\pi^2} m_b (\bar{s}_\alpha Rb_\beta)(\bar{c}_5\gamma_5\ell) \]  

The SM Wilson coefficients at scale $\mu = m_b$ (approximately) read:

\[ C_1 \sim -0.3, \quad C_2 \sim 1.14, \quad C_3 \sim 6 \sim O(10^{-2}), \]
\[ C_{7,8} \sim O(10^{-4}), \quad C_9 \sim -1.28\alpha, \quad C_{10} \sim 0.33\alpha \]
\[ C_{7,8} \sim -0.31, \quad C_{8g} \sim -0.15, \quad C_{9V} \sim 4.2, \quad C_{10A} \sim -4.1 \]

SM Wilson coefficients are real save for the small imaginary contributions due to $V_{ub}$, which we neglect here. Due to the extreme smallness of $C_{S, P} \sim m_t m_b/m^2_W$ within SM, the corresponding operators are usually neglected. The primed Wilson coefficients typically read $C'_{X} \sim (m_s/m_b)C_X$, implying that they are expected to be suppressed and hence neglected. Specifically, within SM

\[ C'_{7,8} \sim -0.006, \quad C'_{8g} \sim -0.003, \quad C'_{9V} = 0 = C'_{10A} \]

The scalar and pseudo-scalar operators can have enhanced coefficients in many extensions of NP beyond SM, and can be cleanly probed in modes like $B_s \rightarrow \mu^+\mu^-$. In any extension of SM, either the Wilson coefficients of the operators already present get new non-negligible contributions or there are new operators induced in the low energy theory or both. In many analyses of semi-leptonic decays, tensor operators are considered $^{10, 11}$. Tensor operators are also invoked to explain the polarization puzzle in $B \rightarrow \phi K^*$ since they have the capability to significantly enhance the transverse polarization fraction and hence explain the experimental data $^{12}$. We consider the following tensor operators with the scale of new physics denoted by $\Lambda$

\[ Q'_{LL} = (\bar{s}_\alpha\sigma^{\mu\nu} Lb_\alpha)(\bar{f}\sigma_{\mu\nu} Lf), \]
\[ Q'_{LL} = (\bar{s}_\alpha\sigma^{\mu\nu} Lb_\alpha)(\bar{f}\sigma_{\mu\nu} Rf), \]
\[ Q'_{LL} = (\bar{s}_\alpha\sigma^{\mu\nu} Rb_\alpha)(\bar{f}\sigma_{\mu\nu} Lf), \]
\[ Q'_{LL} = (\bar{s}_\alpha\sigma^{\mu\nu} Rb_\alpha)(\bar{f}\sigma_{\mu\nu} Rf) \]  

such that the additional terms in the effective Hamiltonian read

\[ H_{eff}^{NP} = -\frac{1}{\Lambda^2} \sum_{AB} C_{AB}^T Q_{AB}^T \]
\[ = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts} \sum_{AB} C_{AB}^T Q_{AB}^T \]  

In the above equations, $f$ refers to any light charged fermion. We shall be specifically interested in $f = \mu$. The Wilson coefficients are in general complex quantities but to simplify the analysis we assume them to be real here. This then broadly refers to Minimal Flavour Violation scenario where it is assumed that CKM is the only source of CP violation (see for example $^{12}$). This should only be taken as a simplifying assumption when trying to make model independent statements. Else, we should write all possible relevant operators and allow for complex coefficients. Then a detailed fit to the data would yield the best fit values for the coefficients. This however requires a very large data set and a complicated analysis. In the meantime, one could just focus on a smaller sub-set of operators and try to constrain them in a somewhat independent fashion. This is the typical approach that is followed generally and we also adhere to that for the present study. Given the new tensor operators, the next task would be to study their effect on various processes. The obvious ones $f = \mu$ are the semileptonic channels like $B \rightarrow K^{(*)}(X_s)\mu^+\mu^-$. By comparing the theoretical branching ratios and other observables like forward-backward, CP and any other possible asymmetries with the experimentally available data, constraints on the coefficients of the new operators are obtained. For example, the rate of the inclusive semileptonic process $B \rightarrow X_s\mu^+\mu^-$ leads to a relation $|C_T|^2 + 4|C_{TE}|^2 < 1$ $^{11}$, where $C_T$ and $C_{TE}$ are the coefficients of the following two operators that are usually considered in the literature:

\[ O_T = (\bar{s}_\mu a_{\mu\nu\lambda}(\bar{\mu}\sigma_{\mu\lambda}\mu) \]
\[ O_{TE} = i(\bar{s}_\mu a_{\mu\nu\lambda}(\bar{\mu}\sigma_{\mu\lambda}\mu) \]

It is clear that the above two operators in Eq.5 when added in suitable combinations are equivalent to the four tensor operators listed in Eq.3. There is however one small but potentially crucial difference. The Wilson coefficients of operators generated via suitable linear combinations of the operators $O_T$ and $O_{TE}$ are not all different, and therefore much more tightly constrained, although there is a priori no reason for some of the coefficients to be equal.

We now study the effect of the tensor operators on $b \rightarrow s\gamma$ processes. It is clear that due to the dipole structure of the operators involved, these operators will directly contribute to $b \rightarrow s\gamma$. Fig. 1 shows the Feynman diagrams for operator insertions leading to new contributions to the process. For the present case, we study the effects when $f = \mu$. Thus only the left diagram
contributes (other possible diagrams where the photon is attached to the external quark lines are not shown). To evaluate the effect of all the four tensor operators listed in Eq. (3), we consider the following general structure

\[ Q^T = (\bar{\sigma}_a \sigma^{\mu \nu} \frac{1}{2} (1 - a \gamma^5) 2 b_3) (f \sigma^{\mu \nu} \frac{1}{2} (1 - a' \gamma^5) f) \]  
with \( a, a' = \pm 1 \).

FIG. 1: Feynman diagrams (drawn using the package JaxoDraw [14]) for generating \( b \to s \gamma \) via the operator insertions (crossed circles denote the operator insertions). The right hand diagram gives the second insertion possible when the light fermion \( f \) is the strange quark.

On evaluating the diagram with the operator in Eq. (6) one finds that the chirality factors involved yield a non-zero contribution only if \( a = a' \). This simply implies that only \( Q^T_{LL} \) and \( Q^T_{RR} \) contribute and can thus be bounded. The other two operators are totally unconstrained from the present analysis. This is precisely the potentially important difference between the basis considered in Eq. (3) and the one usually employed in the study of semileptonic decays, i.e., the one in Eq. (4). On evaluating the loop diagram, one finds for both \( a = a' = +1 \) and \( a = a' = -1 \), the same factor

\[ F_{\text{Loop}} = 16 Q_f \frac{m_f}{m_b} \ln \left( \frac{m_f^2}{\mu_R^2} \right) \]

where \( Q_f, m_f \) and \( \mu_R \) are the light fermion charge, mass and renormalization scale respectively. We set \( \mu_R = m_b = 4.8 \text{ GeV} \). The extra explicit factor of \( m_b \) in the above expression has been introduced for convenience such that the new contribution finally takes the familiar form of \( Q^T_{77} \). Denoting by \( \Delta C^T_{77} \) and \( \Delta C^T_{77}' \) the contributions to respective Wilson coefficients due to the new physics effects, one has

\[ \Delta C^T_{77} = F_{\text{Loop}} C^T_{RR(LL)} \]  

For the case of muon, \( F^\mu_{\text{Loop}} \sim 2.5 \). For the case of strange quark, the \( F^\mu_{\text{Loop}} \sim 0.8 \). However, for the tau lepton or charm quark, \( F^\tau_{\text{Loop}} \sim \mathcal{O}(10) \). For the present analysis, we set \( F^\mu_{\text{Loop}} = 2 \). It may be worthwhile to mention that the Wilson coefficients for the tensor operators at scales \( m_b \) and \( m_W \) are related by \[ C^T_{AB}(m_b) = \left( \frac{\alpha_s(m_W)}{\alpha_s(m_b)} \right)^{1/3} C^T_{AB}(m_W) \sim \mathcal{O}(1) \]

where \( \beta_s = 11 - 2N_f^{\text{active}}/3 \). This then implies that as a first approximation, the changes due to running may be neglected. One therefore has the following at the scale \( m_b \):

\[ C_{77} \to C_{77} + \Delta C_{77}; \quad C_{77}' \to C_{77}' + \Delta C_{77}' \]  

We note in passing that if the light fermion \( f \) is a quark, then the same set of operators will also contribute to the chromomagnetic dipole operators, i.e., to \( \Delta C_{77}' \). The answers can be easily read off after making appropriate changes in \( \Delta C_{77}' \). At the scale \( m_b \), the Wilson coefficients mix and read [16]

\[ C_{77}(m_b) \sim -0.31 + 0.67 C_{77}'(m_W) + 0.09 C_{77}(m_W) \]

\[ C_{7}(m_b) \sim -0.15 + 0.70 C_{77}'(m_W) \]  

It is the extra contribution which is labeled \( \Delta C_{77,s} \) and when translating into constraints on the coefficients of specific tensor operators, these relations could be inverted and the constraints read off.

The branching ratio and time dependent CP asymmetry have been measured for \( B \to K^* \gamma \). Also available are the very precise measurement of the inclusive branching fraction \( BR(B \to X_s \gamma) \). The direct CP asymmetry in the inclusive radiative mode can also be considered. The experimental situation is summarised in Table 1.

We consider the exclusive mode first. The decay rate (or the branching fraction) reads [17]

\[ BR(B \to K^* \gamma) = (V_{tb}V_{ts})^2 G^2_F m_t^2 m_B^2 \left( 1 - \frac{m_{K^*}^2}{m_B^2} \right)^3 \times |T_1(0)|^2 (|C_{77}|^2 + |C_{77}'|^2) \]  

where \( T_1 \) is the form factor at \( q^2 = 0 \), while the time dependent CP asymmetry reads [18]

\[ S_{K^* \gamma} \sim \frac{2}{|C_{77}|^2 + |C_{77}'|^2} Im(e^{-i\phi_4} C_{77} C_{77}') \]  

In the above equation, \( \phi_4 \) describes the mixing in the \( B_d \) sector, i.e., \( \sin(\phi_4) = S_{\psi K_S} \), time dependent mixing induced CP asymmetry. We employ the experimentally measured value \( S_{\psi K_S} = 0.67 \pm 0.02 \) [3] in the numerical analysis. We consider 1\sigma and 2\sigma ranges for the branching ratio and mixing induced CP asymmetry respectively. The reason for choosing 2\sigma range for \( S_{K^* \gamma} \) is to include possible error due to \( S_{\psi K_S} \), and we then employ the central value in the analysis. Fig. 2 shows the constraints in \( \Delta C_{77} \times \Delta C_{77}' \) plane. The constraints on \( C^T_{LL,RR} \) are readily obtained from Eq. (7). From Fig. 2 it is clear that

| Observable                  | HFAG average [5] |
|-----------------------------|------------------|
| \( BR(B \to K^* \gamma) \)  | \((42.7 \pm 1.8) \times 10^{-6}\) |
| \( BR(B \to X_s \gamma) \) | \((3.55 \pm 0.26) \times 10^{-4}\) |
| \( A_{CP}(b \to s \gamma) \) | \((-0.012 \pm 0.028)\%\) |
| \( S_{K^* \gamma} \)        | \(-0.16 \pm 0.22\) |
demanding that $BR(B \to K^*\gamma)$ and $S_{K^*\gamma}$ are within
the experimental ranges yields very tight constraints on
the Wilson coefficients, more stringent than the maxi-
mally allowed ones from the semi-leptonic processes. As
an example and a check, we looked at the representative
values of $C_T$ and $C_{TE}$ employed in [11] and check whether
they yield consistent values for both $BR(B \to K^*\gamma)$ and $S_{K^*\gamma}$. We find that for most of the representative
pairs $(C_T, C_{TE})$ either or both the observables fail to fall
within the experimentally allowed ranges. At this point
it is rather important to clearly mention that the values
employed in [11] are the ones that give maximal deviation
from SM expectations for the observables studied. How-
ever, smaller values are also consistent with their anal-
ysis [11]. In no way this invalidates the analysis in [11]
but the main point of this exercise was to illustrate that
once the tensor operators are appropriately contracted in
order to obtain additional contributions to $C_{\gamma}$ and $C'_{\gamma}$,
only a restricted region of parameter space survives. This
therefore shows the power and importance of combining
the constraints from a direct analysis like semi-leptonic
modes and indirect ones like radiative modes.

Such operators are invoked in order to explain the po-
larization puzzle in $B \to \phi K^*$ modes. Authors of [12]
study a host of observables available in $B \to \phi K^*$ modes
and obtain the following best fit values: $C_{LL}^T(f = s) \sim
2 \times 10^{-4}e^{i\delta_{LL}}e^{i\delta_{RR}}$ with $\phi_{LL} = -0.12$,
$\delta_{LL} = 1.15$ and $C_{RR}^T(f = s) \sim 1.7 \times 10^{-4}e^{i\delta_{RR}}e^{i\delta_{RR}}$ with $\phi_{RR} =
0.14, \delta_{RR} = 2.36$ where $\phi$'s and $\delta$'s are the weak
and strong phases expressed in radians. We use these and
find that they yield consistent values for both $BR(B \to K^*\gamma)$
and $S_{K^*\gamma}$.

We have explicitly checked that the inclusive $b \to s\gamma$
rate also yields similar constraints. Another powerful
observable is the direct CP asymmetry in $B \to X_s\gamma$.
Following [10], for a cut on photon energy, $E_\gamma > (1
- \delta_\gamma)E_{\gamma}^{\max}$ with $\delta_\gamma = 0.3$

$$A_{CP}(b \to s\gamma) \sim \frac{0.01}{(|C_{\gamma}|^2 + |C'_{\gamma}|^2)}[1.17Im(C_{2}\gamma_{\gamma})
-9.51Im(C_{s}\gamma_{\gamma} + C_{s}\gamma_{\gamma}^*)
+0.12Im(C_{2}\gamma_{s\gamma}) - 9.40Im(\epsilon_2 C_2
\times (C_{\gamma}^* - 0.0138C_{s\gamma} + C_{\gamma}^*-0.0138C_{s\gamma}^*))]
$$

where $\epsilon_2 = V_{ub}^*V_{us}/(V_{ub}V_{us})$.

We have checked that our results are consistent with
$A_{CP}(b \to s\gamma)$, and so are the values for Wilson coeffi-
cients obtained by [12]. It is interesting to notice that
seemingly small Wilson coefficients as in [12] which as
expected would be consistent with the constraints from
radiative modes still are able to explain the polarization
puzzle in $B \to \phi K^*$. Very recently, a similar situation
has arisen in $B_s \to K^{*0}\bar{K}^{*0}$ where again the longitudinal
fraction is found to be much lower than the expectations
[12]. This is a $b \to s\bar{d}d$ penguin dominated mode and is
expected to be an important channel in search of new
physics. It would be interesting to see if similar tensor
operators can explain the polarization puzzle and remain
consistent with the constraints from radiative modes. We
leave this for a separate study. It is also noteworthy that
to explain the polarization puzzle other type of opera-
tors are also considered, eg right handed currents [20],
(pseudo-)scalar operators [21]. We would like to emphasis
that any operator of the form $(s\Gamma)(\bar{q}\Gamma')q$, where
$\Gamma, \Gamma'$ are Dirac structures would in principle generate
new contributions to dipole operators and one should ex-
plitly check whether they pass the simple tests discussed
above.

In this note we have studied the impact of tensor op-
terators corresponding to physics beyond SM, that are
invoked in the study of semi-leptonic decays $b \to s\ell^+\ell^-$
and $b \to s\bar{s}s$ to explain polarization puzzle in $B \to \phi K^*$
modes, on radiative modes $b \to s\gamma$ and CP asymmetries.
We have shown that two of the tensor operators with chiral-
ity $LL$ and $RR$ can be stringentlly constrained. The
other two operators with chiral structure $LR$ and $RL$ do
not contribute to the radiative mode and therefore are
left unconstrained from the present analysis. We have
also eluded to a potential difference between the case
when all the Wilson coefficients for these operators are taken as free parameters and the case when due to specific choice of the operators some of the coefficients are equal to each other. This according to us may be over restrictive. We have found that the tensor operators end up generating new contributions to dipole operators with both the chiralities. The Wilson coefficients have been assumed to be real but extension to complex coefficients is straightforward. It is known that complex coefficients yield a far more richer phenomenology (see for example \[22\] for effects of complex coefficients on \(B \to K^{*}(\ell^{+}\ell^{-})\) and in fact presence of new phases may fake the effects naively expected due to the presence of new class of operators. Also, if the operator under consideration has the light fermion as a quark, then a similar contribution is expected due to the presence of new phases within very narrow bands. Similar remarks would apply to any other operator (any Dirac and chiral structure) with the light fermion being a quark and in many cases it may be plausible that some of the operators get tightly constrained or almost (practically) ruled out.

[1] R. Aaij et al. [LHCb Collaboration], arXiv:1112.1600 [Unknown].
[2] R. Aaij et al. [LHCb Collaboration], arXiv:1111.4183 [Unknown].
[3] P. Koppenburg, arXiv:1111.6829 [hep-ex].
[4] B. Meadows et al., arXiv:1109.5028 [hep-ex]; B. Adeva et al. [The LHCb Collaboration], arXiv:0912.4179 [hep-ex].
[5] D. Asner et al. [Heavy Flavor Averaging Group], arXiv:1010.1589 [hep-ex] (online update at http://www.slac.stanford.edu/xorg/hfag).
[6] G. Beall, M. Bander and A. Soni, Phys. Rev. Lett. 48, 848 (1982); P. Langacker and S. Uma Sankar, Phys. Rev. D 40, 1569 (1989); P. L. Cho and M. Misiak, Phys. Rev. D 49, 5894 (1994) arXiv:hep-ph/9310332.
[7] S. Bertolini, F. Borzumati, A. Masiero and G. Ridolfi, Nucl. Phys. B 353, 591 (1991); R. Barbieri and G. F. Giudice, Phys. Lett. B 309, 86 (1993) arXiv:hep-ph/9303270; V. D. Barger, M. S. Berger, P. Ohmann and R. J. N. Phillips, Phys. Rev. D 51, 2438 (1995) arXiv:hep-ph/9407273; P. L. Cho, M. Misiak and D. Wyler, Phys. Rev. D 54, 3329 (1996) arXiv:hep-ph/9601360; A. J. Buras, P. Gambino, M. Gorbahn, S. Jager and L. Silvestrini, Nucl. Phys. B 592, 55 (2001) arXiv:hep-ph/0007313.
[8] A. J. Buras, arXiv:hep-ph/9806471; C. Bobeth, M. Misiak and J. Urban, Nucl. Phys. B 574, 291 (2000) arXiv:hep-ph/9901220.
[9] S. R. Choudhury and N. Gaur, Phys. Lett. B 451, 86 (1999) arXiv:hep-ph/9810307; C. Hamzaoui, M. Pospelov and M. Toharia, Phys. Rev. D 59, 095005 (1999) arXiv:hep-ph/9807350; K. S. Babu and C. E. Kolda, Phys. Rev. Lett. 84, 228 (2000) arXiv:hep-ph/9909476; C. Bobeth, T. Ewerth, F. Kurger and J. Urban, Phys. Rev. D 64, 074014 (2001) arXiv:hep-ph/0104284.
[10] S. Fukae, C. S. Kim, T. Morozumi and T. Yoshikawa, Phys. Rev. D 59, 074013 (1999) arXiv:hep-ph/9807254; A. K. Alok, A. Dighe, D. Ghosh, H. London, J. Matias, M. Nagashima and A. Szymkowiak, JHEP 1002, 053 (2010) arXiv:0912.1382 [hep-ph]; A. K. Alok, A. Dighe and S. Uma Sankar, Phys. Rev. D 78, 114025 (2008) arXiv:0810.3779 [hep-ph]; T. M. Aliev, C. S. Kim and Y. G. Kim, Phys. Rev. D 62, 014026 (2000) arXiv:hep-ph/9910501.
[11] A. K. Alok, A. Datta, A. Dighe, M. Duraisamy, D. Ghosh, D. London and S. U. Sankar, JHEP 1111, 121 (2011) arXiv:1008.2397 [hep-ph].
[12] P. K. Das and K. C. Yang, Phys. Rev. D 71, 094002 (2005) arXiv:hep-ph/0412313.
[13] A. J. Buras, Acta Phys. Polon. B 34, 5615 (2003) arXiv:hep-ph/0310208.
[14] D. Binosi and L. Theussl, Comput. Phys. Commun. 161, 76 (2004) arXiv:hep-ph/0309015; D. Binosi, J. Collins, C. Kaufhold and L. Theussl, Comput. Phys. Commun. 180, 1709 (2009) arXiv:0811.4113 [hep-ph].
[15] C. Bobeth and U. Haisch, arXiv:1109.1826 [hep-ph].
[16] A. L. Kagan and M. Neubert, Phys. Rev. D 58, 094012 (1998) arXiv:hep-ph/9803368; K. Kiers, A. Soni and G. H. Wu, Phys. Rev. D 62, 116004 (2000) arXiv:hep-ph/0006280.
[17] A. Ali and A. Y. Parkhomenko, Eur. Phys. J. C 23, 89 (2002) arXiv:hep-ph/0105302; S. W. Bosch and G. Buchalla, Nucl. Phys. B 621, 459 (2002) arXiv:hep-ph/0106081; P. Ball, G. W. Jones and R. Zwicky, Phys. Rev. D 75, 054004 (2007) arXiv:hep-ph/0612081.
[18] D. Atwood, M. Gronau and A. Soni, Phys. Rev. Lett. 79, 185 (1997) arXiv:hep-ph/9704272; P. Ball and R. Zwicky, Phys. Lett. B 642, 478 (2006) arXiv:hep-ph/0606937.
[19] A. Dighe, private communication.
[20] A. L. Kagan, hep-ph/0407076.
[21] C.-H. Chen and C.-Q. Geng, Phys. Rev. D 71, 115004 (2005) hep-ph/0504145; H. Hatanaka and K.-C. Yang, Phys. Rev. D 77, 053015 (2008) arXiv:0711.3086 [hep-ph].
[22] A. Hovhannisyan, W. -S. Hou and N. Mahajan, Phys. Rev. D 77, 014016 (2008) hep-ph/0701046.
[23] A. L. Kagan and J. Rathmann, hep-ph/0701300; A. Kagan, In *Santa Barbara 1997, Heavy flavor physics* 215.
