Pancharatnam Phase and Photon Polarization Optics

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Abstract

Parallel transport of a vector around a closed curve on the surface of a sphere leads to a direction holonomy which can be related with a geometric phase that is equal to the solid angle subtended by the closed curve. Since Pancharatnam phase is half of the solid angle subtended by the polarization cycle on the Poincare sphere, quantum parallel transport law takes recourse o spin-half wave function to obtain this result. A critique is offered on this factor of half anomaly in the geometric phase, and a natural resolution using Riemann sphere polarization representation is suggested. It is argued that spin angular momentum of photon is fundamental in polarization optics, and new insights are gained based on the hypothesis that two helicity states correspond to two distinct species of photon. This approach leads to the concept of a physical Poincare sphere: nonlinearity and jumps in the Pancharatnam phase find a simple physical explanation while novel features pertaining to the discrete and pulsating sphere are predicted. Paired photon spin zero structure of unpolarized light is also discussed. An outline of possible experimental tests is presented.

PACS numbers: 42.50.Dv, 03.65.CA, 042.50.Dv, 03.65.CA, 03.65.Vf
I. INTRODUCTION

Recent advances in the quantum information science have in a subtle way led to a paradigm shift on the quantum versus classical debate and controversies; now typical counter-intuitive aspects of quantum theory are viewed as resources for novel applications. It is recognized that single photon states, quantum vacuum and entangled photon pairs constitute one of the most important resources in this endeavor, however polarization quantum optics still depends on the classical notions developed prior to the emergence of the electromagnetic theory of light. One can marvel on the ingenuity of Stokes [1] that the vector nature of light could be operationally defined solely based on the intensity (a scalar quantity) measurements. In fact, even today introducing polarization states of light without any recourse to the electromagnetic fields gives useful insights [2]. Stokes vectors have found vast applications in the light scattering experiments, and suitable density matrix representation of Stokes four-vector is used to analyze polarization-sensitive cross sections in particle physics [3]. In quantum optics the Stokes parameters are defined as the expectation or mean values of the Stokes operators constructed from the canonical field operators [4]. Note that polarization is simply an index attached to the field operators corresponding to a quantized simple harmonic oscillator. In the basis of circular polarization, the spin angular momentum operator for a plane wave is diagonal in the number states, and the difference between the number of right circularly polarized (RCP) and left circularly polarized (LCP) photons determines the spin angular momentum, see Ch. 10 of [4]. Though mechanical interpretation of classical electromagnetic fields relates polarization with angular momentum, and Poynting’s suggestion [5] found unequivocal support in the Beth experiment [6], there exist delicate questions related to gauge invariance and manifest Lorentz covariance. The problem of time-like and longitudinal field excitation is circumvented in quantum optics working in the radiation gauge. However, the separation of angular momentum of radiation into orbital and spin parts is not unambiguous. In most cases plane wave approximation and polarization index associated with the field operators imply that it is only the spin of quantized field that is of significance. In recent past studies on optical vortices and singular laser light beams have brought into focus the significance of the orbital angular momentum of light, see the review [7] and references cited therein.

A comprehensive critique on the concept of photon [8] concludes that in spite of the
enormous advances in quantum optics the physical reality of photon remains undecidable. Should we go back to the basics and look afresh on the physical interpretation of the Maxwell field equation? In plausible arguments are put forward to suggest that the Maxwell action represents the rotational dynamics of photon fluid. I believe that spin angular momentum and polarization hold the secrets of photon structure, and going beyond the co-existence of quantum and classical descriptions of light there is a need to develop a unitary picture for optical phenomena. Recently there has been a renewed intense activity to develop insightful approaches to the polarization of light see and the cited literature. Karassiov incorporates SU(2) polarization symmetry at the quantization level. Lehner et al. investigate unpolarized light employing rotational and retardation invariance, and offer a critique to the new class of unpolarized light discussed in. Luis in a series of papers has developed a formalism based on the probability distribution on the Poincare sphere to characterize the degree of polarization of light.

The aim of the present paper is to revisit Pancharatnam phase which is an ‘all polarization effect’, and gain new insights into the nature of light. It is well known that Pancharatnam’s work was rediscovered after the Berry phase, and since then quantum as well as classical explanations to the geometric phases in optics have been discussed in the literature. Experimentally numerous studies have confirmed the existence of this effect in polarization optics, therefore we proceed in the other way and ask: what could be learnt on the properties of light from Pancharatnam phase? The main results of our study are as follows. First it is pointed out in the next section that there is a discrepancy of a factor of half in the phase obtained in modern approaches, and invoking spin-half for two-level polarization system is an artifact. In Sec. III a satisfactory formal resolution is established based on the Riemann sphere representation of polarization. Postulating that LCP and RCP photons are distinct species, spinor wave function and geometrical mapping of polarization states are analyzed to seek a probability distribution of the number of photons corresponding to the states on the Poincare sphere in Sect. IV. Note that elementary particle physics oriented photon and anti-photon idea was earlier discussed by Good, and in the multivector language advocated by Hestenes plane electromagnetic wave solutions having positive (negative) frequency correspond to RCP (LCP) light. In contrast to them, in our extended space-time model of photon, the internal structure of photon determines its spin angular momentum and intrinsic frequency (energy). The significance of the results obtained
in the context of nonlinearity and singularity of Pancharatnam phase \[19\], unpolarized light and spin angular momentum transfer as a physical mechanism for geometric phase \[20\] is discussed in Sect. V.

II. POINCARE SPHERE AND PANCHARATNAM PHASE

A. Pancharatnam’s original approach

Pancharatnam’s motivation in \[13\] is to understand the physics of crystal optics: interference of polarized light beams, geometrical approach to the polarization phenomenon, and spherical trigonometry are the principal ingredients in his approach. It is remarkable that the operational definition based on the measurement of intensity of light after its passage through the polarizer and analyzer settings pioneered in 19\textsuperscript{th} century is followed by Pancharatnam. The following proposition is proved by him: the interference between mutually coherent light beams of intensities \(I_1\) and \(I_2\) in the polarization states \(P_1\) and \(P_2\) respectively is given by the expression (that defines the phase difference \(\delta\))

\[
I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos P_1P_2\cos \delta
\]

Here polarization states are described on the Poincare sphere, and \(P_1, P_2\) is the angular separation between the points \(P_1\) and \(P_2\) on the surface of the sphere. Next, the most important geometrical result is obtained: a phase of pure geometric origin, \(\Gamma\) depends on the solid angle \(\Omega\) subtended by the triangle \(P_1P_2P\) on the Poincare sphere and is given by

\[
\Gamma = \pm \frac{1}{2}\Omega(P_1P_2P)
\]

In the modern derivations the parallel transport of a vector on sphere naturally leads to the value of \(\Gamma\) equal to \(\Omega\), and it is this anomaly of a factor of half that we wish to elaborate since it is not sufficiently recognized in the literature. For this purpose it is important to realize how a vectorial property of polarization is mapped on to the surface of a sphere defined in terms of the scalar quantities: the intensity \(I\), and the Stokes parameters (\(M, C, S\)). Adopting the four-vector notation \((I, M, C, S)\) can be written as \(S_\mu\), \(\mu = 0, 1, 2, 3\). For a perfectly polarized light

\[
S_\mu S^\mu = 0 \quad \text{or} \quad I^2 = M^2 + C^2 + S^2
\]
Eq. (3) defines the Poincare sphere, and the cartesian coordinates of a point on the sphere are given by the Stokes vector $\mathbf{S}$ (or $S_i, i = 1,2,3$). To make transparent the relationship with polarization, consider a general polarization state that is described by the orientation of, say major axis of the ellipse, and the ratio of semi-minor axis to semi-major axis ($b/a$). Let the propagation direction of light be fixed, and it is assumed along z-axis, and the orientation is specified by the angle $\lambda$ made by the major axis with the x-axis. Define the ellipticity by an angle $\xi$ such that $\tan \xi = b/a$. The angles $\lambda$ and $\xi$ called azimuth and ellipticity uniquely represent the polarization state; here $0 \leq \lambda \leq \pi$ and $-\pi/4 \leq \xi \leq \pi/4$.

On the surface of the unit sphere shown in Fig.1, standard geometric definitions are introduced: by virtue of Eq.(3) only two coordinates ($\theta, \phi$) of spherical polar coordinate system are sufficient to specify any point P on the sphere. Polar angle $\theta$ is the great circle arc length ZP, and the azimuthal angle $\phi$ is the spherical angle XZP. The diametrically opposite points Z and Z’ represent the poles of the great circle termed the equator. The great circle arc ZP and the spherical angle AZP define the latitude [(π/2)-ZP] and longitude respectively. The meridian ZPBZ’ intersects the equator at B: all points on meridian have the same longitude. The points on the small circle KPL with poles (Z, Z’) have the same latitude, and it is called the parallel of latitude.
On the Poincare sphere, latitude and longitude have the values $2\xi$ and $2\lambda$ of the polarization ellipse shown as the inset in Fig.1. Poles $Z(Z')$ represent RCP (LCP) light, and points on the equator represent linearly polarized light. Elliptical polarization corresponds to rest of the points on the surface, and orthogonal states lie on the antipodal points on the sphere. Now it is not necessary to go into the details of the derivation of expression (2) for $\Gamma$ which can be found in [13], the crucial point is that a polarization rotation in real space corresponds to a rotation twice of that value on the Poincare sphere and Pancharatnam’s approach takes into account this naturally while considering decomposition of a coherent polarized light beam into two parts having arbitrary polarizations. This results into a geometric phase acquired by a light beam traversing a cycle along the geodesic path on the Poincare sphere equal to half of the solid angle subtended by the closed cycle.

B. Discrepancy of a factor of half

In a lucid paper [21] Berry brought out the significance of Pancharatnam phase in the light of quantum mechanical adiabatic phase discovered by him in 1984 [14] establishing a close relation of light polarization description on the Poincare sphere with the spinor evolution for the Hamiltonian

$$H(\mathbf{r}) = \mathbf{r} \cdot \sigma$$  \hspace{1cm} (4)

Here $\mathbf{r}$ is a unit vector parametrized by polar angles $(\theta, \phi)$ and $\sigma$ is the Pauli spin matrix. In the concluding section of [21] spin-half representation for getting $\Gamma$ for photon which has spin one is sought to be justified with the argument that photons have only two states of helicity. On the other hand, the phase acquired on the closed path on the surface of the sphere equal to the solid angle subtended by the path can be identified with the spin redirection phase on the sphere of directions of the propagation vector, $\mathbf{k}$. The question arises: could SU(2) symmetry associated with spinor be used for spin one photon? In [22] we made a brief comment on this question and rather naively suggested that since phase corresponds to electric field vector one has to take the square root of phase factor obtained on the sphere. In the next section an attempt is made to give a sound basis for this suggestion. But first we elaborate the problem.

It is known that non-Euclidean geometry of the sphere results into the change of direction of a vector under parallel transport: it can be proved calculating the Christoffel symbol
and using geodesic equation that a vector parallel transported around a curve gets rotated through an angle equal to the solid angle subtended by the area enclosed by the curve. In another description, one can use the Fermi-Walker parallel transport for curved geometry \[23\] and obtain the same result for the sphere. In the context of geometric phase in optics, Chiao and Wu \[24\] proposed k space as the parameter space and predicted rotation of polarization for a light beam propagating through a helically wound optical fiber, and immediately this was experimentally demonstrated by Tomita and Chiao \[25\]. In the light of earlier anticipations due to Rytov and Vladimirskii, we term this spin redirection phase as RVCW phase \[26\]. Haldane explained this phase in terms of the geometry of fiber relating it with the work of Ross \[27\]. However, Segert \[23\] and others have explained the RVCW phase purely in geometric terms i.e. tangent bundles and parallel transport laws \[28\]. In \[29\] Berry discusses parallel transport of a vector on the surface of a sphere, and to translate the direction holonomy to phase holonomy he defines a complex unit vector and chooses local basis vectors along the parallel of latitude and meridian of longitude. He obtains the expected result, namely the phase holonomy is equal to the solid angle subtended by the closed curve on the sphere. Obviously the RVCW phase finds natural geometric explanation in all these modern approaches \[21, 23, 28, 29\].

To derive Pancharatnam phase Berry replaces the complex unit vector by a quantum state, and assuming the spinor quantum state for photon arrives at a factor of half. Jordan \[30\] similarly states that, ‘since the spin eigenvalue is 1/2, the phase difference is Ω/2’. Bhandari \[31\] asks the question: Is there any paradox in a spin 1 particle like a photon behaving as a spin 1/2 particle? He answers, ‘there is in fact none’. The reasoning behind this assertion is two-fold: spin-half representation holds for any two-level system, and Jones calculus has been effectively in use for light waves in polarization optics. Note that complex representation of the components of electric field vector in Jones calculus does not in any way imply its interpretation as a wavefunction of photon, and as shown by Jiao et al \[32\] without using any quantum rendition Pancharatnam phase can be derived using Jones calculus. At a basic level, there is an intricate problem in defining a wavefunction for photon \[8\]. As for the two-level system, spinor form is merely an analogy; there is nothing quantum mechanical in it; the fact partly admitted by Bhandari \[31\] when he says that quantum statistics is different for spin-half particle. The inevitable conclusion is that phase holonomy on the sphere (in k space) unambiguously corresponds to the RVCW phase, and Pancharatnam
phase derived using spinor wavefunction on the Poincare sphere is an artifact.

III. RIEMANN SPHERE

Penrose [15] has remarked that the fundamental role of Riemann sphere is not well recognized for any two level quantum system. In the case of photon spin the description is abstract, but it is worth considering to understand Pancharatnam phase. Let us note some mathematical properties of the sphere [33] which have importance for the present discussion. The sphere is defined on the real space i.e. field of real numbers $\mathbb{R}^3$ and it is orientable. Orientability could be proved considering the parallel transport of a frame where frame is a set of linearly independent vectors $(u,v)$. The atlas defined on the sphere assigns different set of coordinates on the two hemispheres. Physically for the Poincare sphere, $\mathbb{R}^3$ is described by $(M, C, S)$, and orientability is responsible for the sign of the Pancharatnam phase in Eq. (2). There cannot exist a continuous, regular vector field on the sphere. In a simple illustration this is reflected in the stereographic projection where infinity is mapped on to the pole of the sphere. The sphere is not a group manifold, and the group manifold of SU(2) is 3-sphere defined on $\mathbb{R}^4$. Further the group of rotations SO(3) has a relationship with SU(2): the mapping $\text{SU}(2) \rightarrow \text{SO}(3)$ is a two-to-one homomorphism. Aitchison [34] gives a nice discussion on the monopole problem and absence of singularity-free vector potential in terms of the property of the sphere that continuous regular vector field cannot be defined on it. He also shows that the Berry phase-monopole relationship can be made more clear using the fact that 3-sphere is a group manifold of SU(2). Since SU(2) is double-covering of the rotation group in $\mathbb{R}^3$ i.e. SO(3), the spin-half particle can be described by SU(2); Berry phase for the neutron using spinor wavefunction on the sphere would be unambiguous.

We propose that parallel transport of a vector on the Riemann sphere representation of light polarization leads to the Pancharatnam phase. Note that this phase can be obtained from the geometry of the Poincare sphere (without making use of spinor wavefunction and quantum parallel transport); see [32] besides Pancharatnam’s derivation [13]. In [33] it is proved that the complex sphere is homeomorphic to the tangent bundle of the (real) sphere: could one relate Riemann sphere with the Poincare sphere in a similar fashion? We leave aside this question here, and proceed to describe Riemann sphere construction following [15].
Let us consider two complex quantities $z$ and $w$, and define their ratio

$$q = \frac{z}{w} \quad (5)$$

In the Argand plane, a complex number is geometrically represented as a point with real and imaginary parts as coordinates along the two orthogonal axes (R,I). Let us construct a unit sphere whose centre is taken to be the origin of the Argand plane lying horizontally as shown in Fig.2. The points $(1, 0), (0, i), (-1, 0)$ and $(0, -i)$ lie on the equator of the sphere. Assuming that one of the poles represents infinity, the projections from this point to the Argand plane map all of the numbers $q$ (including the one where $w = 0$) to the surface of the sphere uniquely; i.e. the stereographic projection.

To represent polarized light on the Riemann sphere, let us write an arbitrary polarized state as a linear combination of RCP and LCP light

$$P = RCP + qLCP \quad (6)$$

Assuming that the poles represent RCP and LCP light, the complex number $q$ on the Riemann sphere represents an arbitrary polarization state specified by its square root

$$p = \sqrt{q} \quad (7)$$
As shown in the inset of Fig. 2 direction of propagation of light is vertically upwards, and the polarization ellipse is obtained as follows. The plane normal to the line joining the centre to a point on the sphere intersects the sphere in a circle. Projection of this circle on to the horizontal plane gives the ellipse which represents the elliptically polarized state at that point.

It is now straightforward to obtain Pancharatnam phase: square root of the parallel transport phase holonomy on the Riemann sphere immediately gives the correct value given by the Eq.(2). To end this section we quote an interesting observation made by Penrose in a footnote (p.272) \[15\]: “The square root has to do with the fact that the photon is a massless particle of spin one, i.e. twice the fundamental unit $\hbar/2$. For a graviton – the yet undetected massless quantum of gravity – the spin would be two, i.e. four times the fundamental unit, and we should need to take the fourth root of $q$ in the above description”.

IV. PHOTON SPIN AND LIGHT POLARIZATION

The term ‘photon spin’ is freely used by Penrose, however in the light of conceptual problems and elusive physical realization of photon \[2, 4, 8\] we have used light polarization in the preceding section to describe the Riemann sphere. It is known that to picture linearly polarized photon – a single photon, one has to rely on the counter-intuitiveness of quantum mechanics. Feynman in his characteristic style discussed this issue \[35\] and quantum probability amplitude comes in handy for this purpose. That it does not make sense to imagine a fraction of one photon means interpreting the thought experiment of the passage of a single photon through polarizer/analyzer differently, e.g. he says, ‘Quantum mechanics tells us it is all there 3/4 of the time’. We argue in this section that spin angular momentum of $\hbar$ per photon (demonstrated in numerous experiments) in the light beams ought to be accorded a fundamental role in the physical model of a single photon. Before that we draw attention to the geometric phase unwittingly found by Feynman. In the discussion on what he calls a ‘curious point’, an interesting result is obtained: RCP (LCP) photon though remains RCP(LCP) photon if viewed from any arbitrary rotated frame, the photon acquires a phase factor that distinguishes the frames. Note that this observation predates Byrne’s analysis \[36\] that we highlighted in \[20\].

We adopt a heuristic approach based on the hypothesis: spin of photon is a characteristic
property of its internal structure, and spin+ \(\hbar\) (or RCP) photon is a distinct object than
spin-\(\hbar\) (or LCP) photon. As a consequence there does not exist a linearly or elliptically po-
larized photon, and RCP photon cannot change to LCP photon in a passive optical process.
This hypothesis does not depend on any specific model of the internal structure of photon
and therefore inconclusive state of the photon model discussed in [9] is unimportant. In the
present paper the polarization optics is discussed based on the ensemble of the constituent
quantum objects: RCP and LCP photons. Intensity of light and polarization state descrip-
tion are the main ingredients for this polarization optics. Note that we are not using second
quantized formulation of optics here. For simplicity we consider light propagation along a
fixed direction, say z-axis. Total average number of photons \(N\) determines the light intensity,
\(I\). A reasonable assumption is that \(I\) is proportional to \(N\). Let \(N_r\) and \(N_l\) be the number of
RCP and LCP photons, then the problem is to characterize the state of light. In a stream of
photons possessing on the average equal number of RCP and LCP photons, the net spin of
the light beam is zero. However, the difference in the distribution of photons will give rise to
the different states of what we call linearly polarized light in this case. The intensity being
a scalar is a constant quantity for a fixed \(N\) therefore to account for this difference 'phase'
seems a natural choice. To understand this, let us go back to the elementary description of
a simple harmonic oscillator. To describe its state, say displacement, one can just record
its instantaneous position at different points of time or alternatively describe the motion in
terms of a constant amplitude and a phase variable. Intensity is analogous to the constant
amplitude of the oscillator, and one needs another variable, i.e. phase to describe the photon
stream. Thus probability distribution of \(N_r\) and \(N_l\),and phase would represent the light in
an arbitrary polarized state.

Based on our hypothesis we reanalyze the geometrical description of polarization, i.e. the
Poincare sphere. The RCP has \(N = N_r\), and the LCP light \(N = N_l\), therefore the poles have
fundamental significance representing these states. We restrict the domain of the surface of
the sphere in two hemispherical regions \(S_+(0 \leq \theta < \pi/2+\epsilon)\) and \(S_-(\pi/2-\epsilon < \theta \leq \pi)\), and the
overlap equatorial region has non-trivial topology. To characterize probability distribution
we reinterpret the spinor functions

\[
\psi_+ = \begin{bmatrix}
\cos \theta/2 & e^{i\phi/2} \\
\sin \theta/2 & e^{-i\phi/2}
\end{bmatrix}
\]

(8)

\[
\psi_- = \begin{bmatrix}
\sin \theta/2 & e^{i\phi/2} \\
-\cos \theta/2 & e^{-i\phi/2}
\end{bmatrix}
\]

(9)

Multiplying both sides of Eqs. (8) and (9) by \(\sqrt{N}\) we have

\[
P_+ = \sqrt{N}\psi_+ = \sqrt{N_r}e^{i\phi/2} + \sqrt{N_l}e^{-i\phi/2}
\]

(10)

\[
P_- = \sqrt{N}\psi_- = \sqrt{N_r}e^{i\phi/2} - \sqrt{N_l}e^{-i\phi/2}
\]

(11)

where on \(S_+\) and \(S_-\) respectively \(N_r\) and \(N_l\) are given by

\[
\sqrt{N_r} = \sqrt{N}\cos \theta/2, \quad \sqrt{N_l} = \sqrt{N}\sin \theta/2
\]

(12)

\[
\sqrt{N_r} = \sqrt{N}\sin \theta/2, \quad \sqrt{N_l} = \sqrt{N}\cos \theta/2
\]

(13)

Conventionally diametrically opposite points on the Poincare sphere represent orthogonal (elliptical) polarization states; in the new interpretation these correspond to the light consisting of the mean number of RCP (LCP) photons interchanged with LCP(RCP) photons. The equator has a special significance since near the transition overlap region the difference between the numbers \(N_r\) and \(N_l\) tends to 1, and the cross-over implies a jump in handedness. For, what one usually refers to as classical light beams, besides the averaged field quantities and corresponding Stokes parameters, one can go a step further and introduce second order correlations of the fields. The fluctuations in fields (and intensities), both spatial and temporal, are inherent in the light phenomena; field correlations at space-time points in the coherence theory give information regarding such fluctuations. The 2x2 coherence matrix defined for vector waves embodies the polarization property of light in this second order coherence theory [4]. In quantum optics, quantum field operators have to be used, however one may simplify the discussion using photon number fluctuations. Evidently such effects would be more pronounced near the equatorial region of the Poincare sphere. In the next section some of the physical consequences of the present approach are discussed.
V. NEW INSIGHTS AND PROPOSED TESTS

The approach proposed in the preceding section is based on the quantum nature of light whose constituents are two species of photon distinguished by their uniquely defined spin (or handed-ness). Note that we are not using the philosophy of quantum mechanics (a la single particle interpretation of the wavefunction), and do not adopt second quantized field theory. The photon polarization optics (PPO), being developed, is inspired by the geometrical considerations and the extended space-time model of photon. Known polarization properties of light are incorporated by construction in the PPO, however new insights are also gained that are presented in the following:

(i) **Unpolarized light**: The definition of unpolarized light is not simple in spite of the fact that natural light is unpolarized. The present work suggests a definition: uniformly weighted paired states \( (P_+, P_-) \) spanning the whole surface of the Poincare sphere characterize the unpolarized light. Recall that in any paired state \( (P_+, P_-) \) the number of RCP (LCP) photons in \( P_+ \) is equal to the number of LCP (RCP) photons in \( P_- \). The pairing gives rise to net spin zero, and the resulting distribution of paired photons represents unpolarized light. In \cite{9} it was argued that a random ensemble of quantum entangled photons could be viewed as a new state of light. Disregarding quantum mechanical interpretation, the unpolarized light defined here essentially corresponds to this ‘new state of light’ – let us call it paired photons unpolarized light (PUL).

Luis defines unpolarized light \cite{12} in terms of the Q function having the value

\[
Q(\theta, \phi) = \frac{1}{4\pi} \tag{14}
\]

This corresponds to a uniform distribution for fields. In his work SU(2) coherent states are used. Formally our definition also entails \( 4\pi \), i.e. the solid angle subtended by the whole surface of the Poincare sphere, however the physical ideas are different. Lehner et al \cite{11} define two types of unpolarized light. Type II polarized light is defined by a distribution function of fields that is rotationally invariant and symmetric with respect to RCP→LCP transformation. Type I unpolarized light besides these requirements possesses phase retardation invariance. The PUL defined here satisfies all the conditions necessary for type I unpolarized light, however, PUL has a polarization structure of paired photons. Karassiov \cite{10} speculates on the hidden polarization structure of unpolarized light. Though relating
PUL with the formal P-scalar light proposed by him [10] is not clear, we disagree with Lehner et al that polarization structure of unpolarized light is paradoxical. Interestingly Simmons and Guttman [2], (p. 91), also find an unusual representation for completely unpolarized light. Using the property that diametrically opposite points represent orthogonal polarization states the Stokes four-vector for unpolarized light can be rewritten as

\[
\begin{bmatrix}
I \\
0 \\
0 \\
0
\end{bmatrix}
= \begin{bmatrix}
I/2 \\
M \\
C \\
S
\end{bmatrix}
+ \begin{bmatrix}
I/2 \\
-M \\
-C \\
-S
\end{bmatrix}
\] (15)

Thus unpolarized light is an incoherent superposition of any two polarized states.

The preceding discussion should not be construed to imply that photon pairing is a basic requirement for unpolarized light. If we relax the plane wave approximation and a fixed direction of propagation then randomly oriented spinning photons in the beam such that the net average spin is zero, would also represent unpolarized light. This kind of unpolarized light cannot be said to have a polarization structure. How to distinguish the two kinds of unpolarized light? It can be easily guessed that the PUL has the structure of the correlated quantum entangled pairs, therefore the difference between the polarization correlation measurements should throw light on these states. Mandel and Wolf [4], p.649, point out that each one of the photon pair in quantum entangled state is unpolarized as defined by the coherence matrix. Thus it would seem that these questions deserve deeper analysis, and further study.

(ii) The nature of Pancharatnam phase: The Poincare sphere used in the work of Pancharatnam retains its mathematical idealization for continuous field variables representing light. The quantum nature embodied in the PPO, on the other hand, gives rise to the idea of a physical Poincare sphere in which two poles representing RCP (LCP) light are indispensable, while the equator relates the topological property of the sphere. Pancharatnam phase for a polarization cycle on either hemi-sphere is geometrical, while near the equator due to the constraint that fractional photon is unphysical, one would expect peculiar behaviour. Schmitzer et al [19] found such an ‘exotic’ nonlinearity in the Pancharatnam phase. The cross-over from the excess of \(N_r\) to the excess of \(N_l\) (or \(N_l\) to \(N_r\)) in the polarization path reveals in a topological effect of phase jumps. Bhandari’s observations [19] are proposed to
be due to this effect. Thus nonlinearity and singularity in Pancharatnam phase could be given a physical origin in PPO.

The physical mechanism responsible for the geometric phases (including the RVCW phase) was suggested to be the transfer of angular momentum in [20]. We have elaborated this suggestion recently [26] and proposed experiments to test these ideas. In the context of Pancharatnam phase, the question that we ask is the following. Let us consider a beam of light that has specified average values of $N_r$ and $N_l$; after the passage through optical elements that change its polarization let it return to the state with the same $N_r$ and $N_l$. How to distinguish the two states? Of course, Pancharatnam phase represents a criterion to distinguish the final state from the initial state. In [20, 26] the notion of the uniform level of angular momentum has been discussed, and it is argued that this level is changed that manifests as a geometric phase. A quantitative estimate is lacking, however in the light of PPO a plausible connection with the rearrangement of the distribution of photons could be made. Though average $N_r$ and $N_l$ are same in the initial and final states, the local distribution and correlations of photons are changed in the rearranged final state of the light. Therefore, we expect that physical Poincare sphere has a granular structure (or discretized pairs on the surface) and fluctuating radius. To probe the spin transfer mechanism and the hidden structure of the sphere, a great deal of experimental ingenuity would be required. It is suggested that second-order correlations, and higher-order interference effects for low intensity light beams undergoing polarization cycles could probe these features.

An intriguing observation is that of frequency shift as an evolving Pancharatnam phase [37]: a rotating half-wave plate seems to induce a frequency shift. Breteanaker and Le Floch [38] argued that the experimental observation could be explained in terms of the energy conservation. They assume that the light beam consists of $N\sigma^+$ (RCP) and $N\sigma^-$ (LCP) photons, and calculate the energy transfer via rotating half-wave plate. Angular momentum exchange (torque imparted to the wave plate by the incident beam) leads to the energy transfer, and energy conservation law gives the frequency shift. It is known that for a monochromatic radiation no passive electronic/optical element can change its frequency, therefore the frequency shift by a rotating wave plate is intriguing. An alternative explanation consistent with the hypothesis that internal frequency of RCP and LCP photons cannot be changed unless these are exchanged by photons with different frequency, would be to interpret the observed intensity oscillation in terms of the time-dependent photon
numbers \( N_r \) and \( N_l \) in Eqs. (10) and (11). The question whether the time-varying phase change is to be interpreted as frequency shift or oscillations in the number of photons cannot be decided solely on the intensity measurements. On the physical Poincare sphere the time-dependence of \( N_r \) and \( N_l \) should manifest as a pulsating sphere, therefore to observe this feature measurements on all the Stokes parameters (M, C, S) would be required. Another possibility is to seek a detection scheme for the output beam intensity that distinguishes frequency effects and the effect of photon numbers.

VI. DISCUSSION AND CONCLUSIONS

In this paper we have proposed an approach to understand the polarization property of light based on a physical model of photon, and yet did not consider single photon effects. The reason is two-fold: first physical model of photon is at present tentative, and secondly the approach outlined here is in a developing state. There has been a great deal of understanding on the classical light, and quantum optics for large number of photons. For a single photon there are many conceptual problems \[8\], and the role of quantum vacuum is very significant. Localized photon and the spin angular momentum in more physical terms (i.e. the structure of photon) than a polarization index have not found a satisfactory solution in quantized field theory. The vacuum field becomes crucial in quantum optics even in the case of a beam splitter, see Sec. 10.9.5 in \[4\]. The significance of Pancharatnam phase to gain insights has been stressed here. In view of the recent speculations on what has come to be known as `quantum-vacuum geometric phase’, see \[39\]; it would seem that in the context of our hypothesis the very idea of Pancharatnam phase for a single photon would involve quantum vacuum. Shen \[39\] discusses RVCW phase for RCP and LCP photons, and argues that the phase at quantum-vacuum level could be envisaged. Our hypothesis, on the other hand, implies that a single photon can be represented only at one of the poles of the physical Poincare sphere, and any notion of a polarization cycle for a single photon depends on the existence of a hidden source of photons which could be identified with aether or quantum vacuum. Obviously instead of special optical media to realize quantum-vacuum geometric phase \[39\], here it is suggested that single photon polarization cycle, if observed carefully, would reveal the quantum-vacuum effect for Pancharatnam phase and similar to the Casimir force, quantum vacuum torque would come into the play.
In conclusion, we have elaborated a conceptual problem in the derivation of Pancharatnam phase using quantum parallel transport that depends on the spin-half treatment for photon having spin one, and suggested that the square root of phase factor obtained on the Riemann sphere gives correct result. A heuristic approach for polarized light is outlined introducing the hypothesis that RCP and LCP photons are distinct species, and spin angular momentum originates due to the structure of photon. The notion of physical Poincare sphere, new insights on the unpolarized light, and the implications of these ideas on the Pancharatnam phase are discussed. Qualitative arguments are also presented to test some of the ideas experimentally. The role of spin angular momentum to address fundamental questions seems inevitable, both for photon, and electron discussed elsewhere [18, 40].

VII. ACKNOWLEDGEMENTS

The library facility of the Banaras Hindu University, Varanasi is acknowledged.

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