A new view on the superposition of quantum states and the wave-particle duality of particles

Yong-Jun Qiao and Guo-Feng Zhang

(Dated: July 22, 2021)

We construct a coupled quantum vortex superposition state (CVSS), since in actual physical systems, linear Schrödinger equations will not be available because of a nonlinear effect. By studying the dynamic evolution of CVSS both analytically and numerically, we show that the superposition of vortex states is not only a mathematical algebraic sum, but also corresponds to a physical process of formation. Moreover, a new method to generate quantum vortex lattice in CVSS research is given. By comparing with the density profiles and phase distributions of quantum vortex state, we have a new understanding of vortex state \(e^{\pm i\theta} \rightarrow (e^{\pm i\theta})^2\), \(L_z = \pm \hbar = (\pm h)_1 + (\pm h)_2 + \cdots (\pm h)_t\), which means that there is spatial degeneracy of angular momentum of a particle. According to this idea, a free particle can be understood as the center of mass of a ring-shaped matter in space. Thus, we revisit the double-slit interference experiment and give a new interpretation.

The collapse caused by measuring the quantum superposition state of micro particles, the non-locality caused by quantum entanglement and the wave-particle duality reflected by the double-slit interference of particles are three major problems in the world of quantum mechanics. Schrödinger gave a general equation for describing micro particles, and thus leads to the emergence of the quantum state superposition hypothesis in 1926 [1]. Furthermore, the application of entanglement theory is already brilliant before its principles are fully understood [2] [3]. And in order to clarify the principle of the double-slit interference experiment of particles, Bohm proposed the guided wave theory [3], but its fatal flaw is that the guided wave speed is greater than the speed of light.

Strictly speaking, the linear quantum superposition state only satisfies the linear Schrödinger equation, but the generation mechanism and stability of the vortex-antivortex superposition state are studied in the Bose-Einstein condensate (BEC) with non-zero scattering between particles which cause a nonlinear effect [10] [12], and experimentally there have been successful cases by way of Josephson coupling [13] [17]. So we suggest a coupled vortex superposition state (CVSS) which contains a hidden variable \(\kappa\) (Josephson coupling coefficient). In the study of its dynamic evolution, we find that the collapse caused by quantum measurement is no longer so dramatic, and the non-local entangled system is localized into a vortex space. Furthermore, the double-slit experiment of particles shows that the particle source can emit particles with a intrinsic angular momentum (IAM) of \(\frac{h}{2}\cos 2\kappa t\) or \(-\frac{h}{2}\cos 2\kappa t\), which determining whether the interference fringes appear or not on screen.

In this letter, we investigate the dynamic evolution of CVSS both analytically and numerically. It is shown that the superposition of vortex states is not only a mathematical algebraic sum, but also corresponds to a physical process of formation. Meanwhile, we discover a new method of generating quantum vortex lattice in BEC, and get a new understanding of the quantum vortex state, namely \(e^{\pm i\theta} \rightarrow (e^{\pm i\theta})^2\) and angular momentum \(L_z = \pm \hbar = (\pm h)_1 + (\pm h)_2 + \cdots (\pm h)_t\). That is to say, the vortex angular momentum of a particle has degenerate nature of spatial position. According to this idea, we find that the particles in entangled state in CVSS have an IAM of \(\frac{h}{2}\cos 2\kappa t\) or \(-\frac{h}{2}\cos 2\kappa t\), so there is a hidden variable \(\kappa\) in the entangled state. In addition, we also show that a free particle have an IAM of \(\frac{h}{2}\) or \(-\frac{h}{2}\), and it can be understood as the center of mass of a ring-shaped matter in space, we design a scheme and give the numerical simulation of the double-slit interference of particles as a solid proof.

**Coupled vortex superposition state**—We suggest there is a linear vortex superposition state (LVSS) \(\psi = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2)\) to particles, which satisfy the linear Schrödinger equation. However, the preparation of quantum superposition state by Josephson coupling is a common method for researchers, so we make a new analysis of dynamics evolution of LVSS as shown in Fig. 1. Here vortices are \(\psi_{1,j}(r, \theta, t) = f_{j}(r)e^{i\theta_j}\) with \(f_j(r) = A_j e^{-r^2/2\sigma_j^2}(r/\sigma_j)^{|l_j|}\), and \(A_j^2 = \frac{1}{\pi\sigma_j^2l_j}\) (\(j = 1, 2\)) being a normalization constant as \(\iint |\psi|^2 r dr d\theta = 1\), \(l_j\) is the angular momentum quantum number of a vortex, \(\sigma_j\) and \(\mu_j\) are the size of the particles distribution scale in space and the chemical potential of particles, respectively. Now we assume \(\psi_1\) and \(\psi_2\) satisfy Eq. (1) with \(\iint |c_{j,i}(t)\psi_1 + c_{j,i,2}(t)\psi_2|^2 r dr d\theta = 1\), \(\iint |\psi_1|^2 r dr d\theta = 1\) and \(\psi_r\) is the actual superposition state name the coupled vortex superposition state (CVSS).
Here $\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} (r \frac{\partial}{\partial r}) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}$ is the 2D operator in a $x-y$ plane that we have adopted plane-polar coordinates $(r, \theta)$, and $V(r) = V_0 r^2$ is a trapping potential. Here $g_1$, $g_2$, $g_{12}$ are non-linear constants which characterize the scattering interaction between particles. Note that $\psi_1$ and $\psi_2$ are two states of particles which are in one-component particle system, rather than describing a two-component particle system. Through careful and detailed analytical derivation in supplemental material, we can get

$$\psi_1 = \frac{\sqrt{2}}{2} e^{-i\chi} (\psi_{1\uparrow} \cos \kappa t + i \psi_{1\downarrow} \sin \kappa t),$$

$$\psi_2 = \frac{\sqrt{2}}{2} e^{-i\chi} (i \psi_{1\uparrow} \sin \kappa t + \psi_{1\downarrow} \cos \kappa t),$$

and

$$\psi_r = \frac{\sqrt{2}}{2} e^{-i\chi} (\psi_{1\uparrow} + \psi_{1\downarrow}) e^{i\sigma t}.$$  

The specific expression of $\chi$ is in the supplemental material. The initial wave functions are $\psi_1(0) = \frac{\sqrt{2}}{2} \psi_{1\uparrow}(0)$, $\psi_2(0) = \frac{\sqrt{2}}{2} \psi_{1\downarrow}(0)$ and $\psi_r(0) = \frac{\sqrt{2}}{2} [\psi_{1\uparrow}(0) + \psi_{1\downarrow}(0)]$ of particles with CVSS in Eqs. (2) and (3). This indicates $\psi_1$ and $\psi_2$ are coupled to form $\psi_1$ and $\psi_2$ linearly. It is shown that the linear superposition between quantum states is not only a mathematical operation, but also corresponds to a real physical process. The difference between $\psi$ and $\psi_r$ is the latter shows a formation process of the superposition state, and LVSS is the initial state of CVSS. A measurement of particles breaks the coupling between quantum states, resulting in $\kappa \to 0$, then $\psi_1 \to \frac{\sqrt{2}}{2} e^{-i\chi} \psi_{1\uparrow}$, $\psi_2 \to \frac{\sqrt{2}}{2} e^{-i\chi} \psi_{1\downarrow}$ and CVSS decays to LVSS. This is what the Copenhagen School says that a measurement will causes collapse of a superposition state. This idea is also true for CVSS with different proportional coefficients of $\psi_1$ and $\psi_2$ which is described in supplemental material. Note that the real motion of particles should be described by $\psi_1$ or $\psi_2$. Moreover, numerical solutions have been taken with $\frac{\sqrt{2}}{2} \psi_{1\uparrow}$ and $\frac{\sqrt{2}}{2} \psi_{1\downarrow}$ as initial wave functions in Eq. (1) and shown in Fig. S1 in supplemental material.

Degeneracy of angular momentum in vortex—
We study the dynamics evolution of a particle system with CVSS with $l_1 = l_2 = 0$ numerically and give the diagrams of $|\psi_r|^2$ as shown in Fig. 2. We can see that there are $l$ quantum vortices in $|\psi_r|^2$ profiles, but such quantum vortex lattice were always generated in rotating BECs [18–26], this means that we find a new method for preparing quantum vortex lattice of BEC. In order to understand the physical mechanism of the quantum vortex lattice produced by this way, we investigate the dynamics evolution of a particle system with a quantum vortex state $\psi_1$ numerically which satisfies $i \frac{\partial \psi_1}{\partial t} = [-\nabla^2 + V(r)]\psi_1$, and we have given the numerical solution diagrams of $|\psi_1|^2$ and $\arg(\psi_1)$ in Fig. 2.

![Figure 2](image-url)

|\psi_1|^2 profiles exhibit a single vortex which has been realized experimentally [27], and phase $\arg(\psi_1)$ distributions appear $l$ vortices, which prompts us to establish a new idea, that is to do the transformation $e^{\pm i\theta} \to (e^{\pm i\theta})^l$, $e^{\pm i\theta}$ characterizes the real space perimeter of $\psi_{\pm l}$ state could accommodate an integer number $l$ of de Broglie wavelengths of a particle. However, $(e^{\pm i\theta})^l$ is re-
fer to the real space perimeter of $\psi_{\ell l}$ state could accommodate an integer number $l$ of particles which moving in a circular motion, or a particle has $l$ degeneracy spatial positions, and $l = 0$ can be regarded as a vacuum state. $e^{\pm i\theta} \rightarrow (e^{\pm i\theta})^l$ also cause $\pm \hbar \rightarrow \Sigma_l^1 (\pm \hbar)$, the significance of this operation is that we regard $\pm \hbar$ as an angular momentum level, the particles at this level have angular momentum $\pm \hbar$ and $l$ degree degenerate spatial positions, we call this is degeneracy of angular momentum in vortex (DAMV). A particle with $\psi_1$ state makes a circular motion in real space with angular momentum $\hbar$ which named intrinsic angular momentum (IAM), and by $l$ degenerate spatial positions on this circle for the particle. So $|\psi_l|^2$ profiles display a single vortex and phase arg($\psi_l$) distributions shown $l$ vortices. IAM is relative to the spatial center of the particle system with CVSS, so it has the characteristic of “spin without rateate” relative to a particle itself. We think that the continuous movement of the mass element in the BEC is the core to produce a quantum vortex lattice. When the mass element has an angular momentum $\hbar$ relative to the center of the condensate a small vortex is formed, so the number of vortex core in the vortex lattice is $l$.

Now we study the actual movement of particles with CVSS, according to the average momentum formula of a particle with $\psi_1$ or $\psi_2$ state in Eq. (2), we can get

$$L_{1z} = \frac{1}{2} [l_1|c_{1,1,l}(t)|^2 + l_2|c_{1,2,l}(t)|^2] \hbar$$
$$= l_1 \hbar - (l_1 - l_2) \frac{\hbar}{2} \sin^2 \kappa t,$$
$$L_{2z} = \frac{1}{2} [l_1|c_{2,1,l}(t)|^2 + l_2|c_{2,2,l}(t)|^2] \hbar$$
$$= l_1 \hbar - (l_1 - l_2) \frac{\hbar}{2} \cos^2 \kappa t. \quad (4)$$

1) As $l_1 = l_2 = 0$, then $L_{1z} = l (\frac{k}{2} \cos^2 \kappa t) = \Sigma_l^1 (\frac{k}{2} \cos^2 \kappa t)$. $L_{2z} = l (\frac{\kappa}{2} \sin^2 \kappa t) = \Sigma_l^1 (\frac{\kappa}{2} \sin^2 \kappa t)$. This shows that the particles which in the quantum vortex lattice have an IAM of $\frac{\kappa}{2} \cos^2 \kappa t$ or $\frac{\kappa}{2} \sin^2 \kappa t$. Now we consider the effect of the interaction which is described by $\kappa \rightarrow 0$ between any one finite ($\sigma \neq \infty$), closed particle system and the vacuum ($l = 0$, $\sigma = \infty$). According to Eq. (S7) (See the supplemental material for specific derivation) we can obtain

$$\psi_\tau = e^{-i\frac{k}{2}(\alpha+\beta\sigma-\mu)t} \psi_{\pm l}. \quad (5)$$

We can see from Eq. (5) that due to the coupling between the system and the vacuum, there is an additional phase in its wave function, and this phase indicates that the system has a vortex phase space [28], here $\alpha = \frac{l_1+l_2}{\sigma}$. 2) If $l_1 = -l_2 = l$, then $L_{1z} = l (\frac{\kappa}{2} \cos 2\kappa t) = \Sigma_l^1 (\frac{\kappa}{2} \cos 2\kappa t)$. $L_{2z} = l (-\frac{k}{2} \cos 2\kappa t) = \Sigma_l^1 (-\frac{\kappa}{2} \cos 2\kappa t)$. This shows that the particle with CVSS have an IAM of $\frac{\kappa}{2} \cos 2\kappa t$ or $-\frac{\kappa}{2} \cos 2\kappa t$, this just illustrates the characteristics of arg($\psi_1$) and arg($\psi_2$) distributions as shown in Fig. S1 in supplemental material, they also show that there are coupled particle pairs with IAM of $\frac{\kappa}{2} \cos 2\kappa t$ and $-\frac{\kappa}{2} \cos 2\kappa t$ in multi-particle system with CVSS. The formation and existence of these pairs are not limited by the spatial location and the influence of scattering interaction between particles. According to Eq. (1) and its initial conditions, we know that this is a method for preparing entangled state [29]. This means that we find a hidden variable $\kappa$ in the entanglement of particles, which corresponds to a linear interaction. In addition, due to the distance between entangled particles is not infinite, we can consider this entangled system to be a finite and closed system, so that the non-local entangled system is localized into a finite vortex space. See the supplemental material for detail.

**FIG. 3.** Numerical simulation of a free particle with $l_1 = l_2 = 13$, $r_0 = r_0(2, 0.2)$, $\sigma_1 = \sigma_2 = 0.5$, $\kappa = 15\pi$, for different $\varphi = 0.3, -3$, there are the same diagrams. The yellow circle in $|\psi_1|^2$ profile is regard the particle density distribution in space as a whole, the yellow arrow points to the particle’s spatial position, and the blue arrows show the possible direction of the movement of the particle.

**Free particle and double-slit interference experiment**—We now consider another special case, that is, $l_1 = l_2 = \pm l$, then $L_{1z} = L_{2z} = \Sigma_l^1 (\pm \hbar/2)$. This situation reflects the continuous movement which is described by $\psi_1 = \psi_2 = \frac{\sqrt{2}}{2} e^{i\pi t} e^{-i(\sigma-\mu)l} e^{i\kappa t} \psi_{\mp l}$ of a particle on the circumference of a eigen vortex $\psi_{\pm l}$, $\varphi$ denotes the circular frequency of the particle. In fact, here we close the external potential well and take the scattering interaction between particles to be zero, this means that our research object is a free particle. In addition, from the perspective of classical mechanics, a particle moving in a circular motion must be subjected to a centripetal force. We suggest that the provider is the continuous and symmetrical nature of space for micro particle, according to this idea, it can be known that the IAM of a particle has nothing to do with the direction of measurement, and its root cause is that the measurement surface has symmetric property about the direction of motion for a particle. Note that the IAM value of a free particle is determined and will not change due to measurement. And the IAM of $\hbar/2$ or $-\hbar/2$ of a particle is independent on the moving direction that described by $\varphi$ and even whether or not moving in space. In order to express this idea intuitively, we have designed a scheme and given relevant numerical solution diagrams as shown in Fig. 3. The specific analysis diagram Fig. S3 and numerical solution scheme are in the supplemental material. Furthermore, considering...
profile as a whole, the free particle motion can be understood as the motion of the mass center of a ring-shaped matter, and its wave function is a vortex function as shown in Fig. S3(b). This may be the reason for the A-B effect \[30\] in the double-slit interference experiments of particles.

Our analysis of free particles is very novel, so it is necessary to verify its correctness. The best way is to carry out double-slit interference (DSI) experiment of “free particles”, various versions of which-way experiment show that as long as the path information of particles is obtained, the interference fringes on screen will disappear \[31\–35\]. Now we give a new explanation about this phenomenon. We think the distribution pattern of particles on the screen is dependent on the nature of the particle source which is in front of screen, but the slits are the paths that the particles reach the screen. We consider the case of \(l_1 = -l_2 = l\) in CVSS and extend \(\sigma_1 = \sigma_2 = \sigma\), \(\sigma\) denotes the size of the spatial scale that particles can be emitted after they are emitted from the particle source. We think that the particles emitted from the particle source have an IAM of \(\frac{\hbar}{2} \cos 2\kappa t = -\frac{\hbar}{2} \cos 2\kappa L\), so the distribution of particles in space meet the distribution rules similar to \(|\psi_1|^2 \oplus |\psi_2|^2\) and \(|\psi_1|^2\) profiles as shown in Fig. S1 in supplemental material. When particles pass through the left or right slit to the screen, the distribution pattern is interference fringes. Here \(|\psi_1|^2 \oplus |\psi_2|^2\) characterizes entangled two-particles interference but not two particle interference \[35\], and \(|\psi_1|^2\) is one-particle interference. The meaning of taking the straight sum is that the two particles in the entangled state have their own spaces. When the particles are measured, it results in \(\kappa \rightarrow 0\) and \(\psi_1 \rightarrow \frac{\sqrt{2}}{2} e^{-i\kappa} \psi_{l_1}, \psi_2 \rightarrow \frac{\sqrt{2}}{2} e^{-i\kappa} \psi_{l_2}\), which in turn produce new particle sources \(\psi_{l_1}\) and \(\psi_{l_2}\). Combined with the analysis of free particles, the particles emitted from this particle source have an IAM of \(\frac{\hbar}{2}\) or \(-\frac{\hbar}{2}\), and the distribution of particles in space is similar to the pattern which marked with yellow arrows as shown in Fig. 2. Therefore, quantum measurement is performed on the particles before or after the double slit that named QMBDSI and QMADSI respectively, the interference fringes on the screen will disappear. We find the dynamic law of “free particles” which is different from the guided wave theory \[37\–40\]. We have given numerical simulation diagrams of one-particle and entangled two-particles dynamic evolution in the DSI, QMBDSI and QMADSI as shown in Fig. 4. The specific analysis diagram Fig. S4 and scheme of numerical simulation is described in detail in the supplemental material.

\[|\psi_1^{l_1}|^2\] and \(|\psi_2^{l_2}|^2\) profiles at moment \(t = 10, 40\) indicate that the particles pass through the left or right slit individually. Comparing the profiles of \(|\psi_1^{l_1} + \psi_2^{l_2}|^2\) and \(|\psi_1^{l_1} \oplus \psi_2^{l_2}|^2\) in the DSI at \(t = 15, 35, 40\), we know that the line width of the interference fringe of entangled two-particles is half of that of one-particle, this rule has been confirmed in the photon interference experiment \[41\]. Both \(|\psi_1^{l_1} + \psi_2^{l_2}|^2\) and \(|\psi_1^{l_1} \oplus \psi_2^{l_2}|^2\) profiles in the QMBDSI exhibit a normal distribution, while \(|\psi_1^{l_1} \oplus \psi_2^{l_2}|^2\) profiles in the QMADSI show a distribution pattern which corresponding to double slits. The numerical simulation diagrams of particle interference in DSI, QMBDSI, and QMADSI are consistent with our analysis of the double-slit interference experiment. This means that the IAM of particles determine whether interference fringes appear on the screen. In addition, when we consider a free particle as the mass center of a ring-shaped matter, the double-slit interference becomes the interference of two eigen vortices. The double vortex interference shows the same law as the double-slit interference, and points out the existence of single-slit diffraction of particles. The specific implementation scheme and numerical simulation diagrams are in the supplemental material.

\[\frac{|\psi_1^{l_1} + \psi_2^{l_2}|^2}{|\psi_1^{l_1} \oplus \psi_2^{l_2}|^2}\] respectively with \(r_1 = r_1(0, 3), r_2 = r_2(0, 9), l_1 = -l_2 = 13, \sigma = 2, \kappa_0 = 50\pi, \kappa = 50\pi\) for DSI and QMBDSI, and \(\kappa = 0.05\pi\) for QMADSI. \(\frac{|\psi_{l_1}^{R_1} + \psi_{l_2}^{R_2}|^2}{|\psi_{l_1}^{R_1} \oplus \psi_{l_2}^{R_2}|^2}\) profiles which in the dashed box is not allowed physically. (Due to \(\kappa \rightarrow 0\), the particles can only be with \(\psi_{l_1}^{R_1}\) or \(\psi_{l_2}^{R_2}\) state.) although the interference fringe distribution can be obtained in math.

**Conclusion**—This letter have studied the analytical solution of CVSS in detail, and we find that there is a formation process for superposition state, not just the algebraic sum of states. The essence of superposition state collapse is the coupling coefficient \(\kappa \rightarrow 0\) in CVSS caused by quantum measurement. Secondly, we propose DAMV to clarify the fundamental principle of the appearance of vortex lattice in BEC, and localized the non-local quantum entanglement system into a vortex space. In addi-
tion, we suggest that a free particle have a vortex wave function, which is verified by the numerical simulation of a double-slit and double vortex interference experiment of particles.

This work was supported by NSFC under grants Nos.12074027.

[1] E. Schrödinger, Phys. Rev. 28, 1049 (1926).
[2] R. Horodecki, P. Horodecki, M. Horodecki, K. Horodecki, Rev. Mod. Phys. 81, 865 (2009).
[3] X. L. Wang, Y. H. Luo, H. L. Huang, M. C. Chen, Z. E. Su, C. Liu, C. Chen, W. Li, Y. Q. Fang, X. Jiang, J. Zhang, L. Li, N. L. Liu, C. Y. Lu, and J. W. Pan, Phys. Rev. Lett. 120, 260502 (2018).
[4] D. Bouwmeester, J. W. Pan, K. Mattle, M. Eibl, H. Weinfurter and A. Zeilinger, Nature 390, 575 (1997).
[5] A. S. Rab, E. Polino, Z. X. Man, N. B. An, Y. J. Xia, N. Spagnolo, R. L. Franco and F. Sciarrino, Nature 8, 915 (2017).
[6] J. C. Lee, K. K. Park, T. M. Zhao and Y. H. Kim, Phys. Rev. Lett. 117, 250501 (2016).
[7] J. Tang, Y. Ming, Z. X. Chen, W. Hu, F. Xu and Y. Q. Lu, Phys. Rev. A 94, 012313 (2016).
[8] A. Einstein, B. Podolsky, N. Rosen, Phys. Rev. 47, 777 (1935).
[9] D. Bohm, Phys. Rev. 85, 166 (1952).
[10] M. Liu, L. H. Wen, H. W. Xiong and M. S. Zhan, Phys. Rev. A 73, 063620 (2006).
[11] K. T. Kapale and J. P. Dowling, Phys. Rev. Lett. 95, 173601 (2005).
[12] L. H. Wen, Y. J. Qiao, Y. Xu and L. Mao, Phys. Rev. A 87, 033604 (2013).
[13] K. C. Wright, L. S. Leslie, and N. P. Bigelow, Phys. Rev. A 77, 041601(R) (2008).
[14] M. F. Andersen, C. Ryu, P. Clad´e, V. Natarajan, A. Vaziri, K. Helmerson, and W. D. Phillips, Phys. Rev. Lett. 97, 170406 (2006).
[15] K. C. Wright, L. S. Leslie, A. Hansen, and N. P. Bigelow, Phys. Rev. Lett. 102, 030405 (2009).
[16] M. Scherer, B. Lücke, G. Gebreyesus, O. Topic, F. Deuretzbacher, W. Ertmer, L. Santos, J. J. Arlt, and C. Klempt, Phys. Rev. Lett. 105, 135302 (2010).
[17] S. Thanvanthri, K. T. Kapale and J. P. Dowling, Phys. Rev. A 77, 053825 (2008).
[18] M. Tsubota, K. Kasamatsu, M. Ueda, Phys. Rev. A 65, 023603 (2002).
[19] A. Rakonjac, A. L. Marchant, T. P. Billam, J. L. Helm, M. M. H. Yu, S. A. Gardiner, and S. L. Cornish, Phys. Rev. A 93, 013607 (2016).
[20] X. F. Zhang, L. Wen, C. Q. Dai, R. F. Dong, H. F. Jiang, H. Chang and S. G. Zhang, Sci. Rep. 6, 19380, (2016).
[21] D. Haag, D. Dast, H. Cartarius and G. Wunner, Phys. Rev. A 97, 033607 (2018).
[22] K. Kasamatsu and K. Sakashita, Phys. Rev. A 97, 053622 (2018).
[23] J. J. Jin, W. Han and S. Y. Zhang, Phys. Rev. A 98, 063607 (2018).
[24] S. K. Adhikari, Phys.: Condens. Matter. 31, 275401 (2019).
[25] M. N. Tengstrand, P. Sturmer, E. O. Karabulut and S. M. Reimann, Phys. Rev. Lett. 123, 160405 (2019).
[26] S. B. Prasad, T. Bland, B. C. Mulkerin, N. G. Parker and A. M. Martin, Phys. Rev. A 100, 023625 (2019).
[27] M. R. Matthews, B. P. Anderson, P. C. Haijuan, D. S. Hall, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. 83, 2498 (1999).
[28] B. Simon, Phys. Rev. Lett. 51, 2167 (1983).
[29] S. Choi and N. P. Bigelow, Phys. Rev. A 72, 033612 (2005).
[30] Y. Aharonov, D Bohm, Phys. Rev. 115, 485, (1959).
[31] Y. H. Kim, R. Yu, S. P. Kulik, Y. Shih and M. O. Scully, Phys. Rev. Lett. 84, 1 (2000).
[32] X. S. Ma, J. Koller, and A. Zeilinger, Rev. Mod. Phys. 88, 015005 (2016).
[33] Y. Xiao, H. M. Wiseman, J. S Xu, Y. Kedem, C. F. Li, and G. C. Guo, Sci. Adv. 5, eaav9547 (2019).
[34] J. Q. Quach, Phys. Rev. A 95, 042129 (2017).
[35] E. Bagan, J. A. Bergou and M. Hillery, Phys. Rev. A 102, 022224 (2020).
[36] T. B. Pittman, D. V. Strekalov, A. Migdall, M. H. Rubin, A. V. Sergienko and Y. H. Shih, Phys. Rev. Lett. 77, 1917 (1996).
[37] D. Mahler, L. Rozema and K. Fisher, L. Vermeyden, K. J. Resch, H. M. Wiseman and A. Steinberg, Sci. Adv. 2, e1501466 (2016).
[38] S. Kocsis, B. Braverman, S. Ravets, M. J. Stevens, R. P. Mirin, L. K. Shalm and A. M. Steinberg, Science 332, 1170 (2011).
[39] K. Mathew and M. V. John, Foundations of Physics 47, 873 (2017).
[40] Y. Xiao, Y. Kedem, J. S. Xu, C. F. Li and G. C. Guo, Optics Express 25, 14463 (2017).
[41] K Edamatsu, R Shimizu, T Itoh, Phys. Rev. Lett. 89, 213601 (2002).