\(^{12}\text{C}+^{12}\text{C}\) and \(^{16}\text{O}+^{16}\text{O}\) fusion reactions at low energies

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Abstract.

The \(^{12}\text{C}+^{12}\text{C}\) and \(^{16}\text{O}+^{16}\text{O}\) fusion reactions are investigated in a folding model. We use a multichannel approach involving the \(^{12}\text{C}(0^+, 2^+, 0^+\!_{2^+}, 3^-)\) states. The \(^{12}\text{C}\) densities (including transition densities) are taken from the RGM calculation of Kamimura. For the nucleon-nucleon interaction, we use the DDM3Y density-dependent interaction. Owing to the explicit presence of inelastic channels, the imaginary part of the optical potential only contains a short-range fusion contribution. The \(S\)-factor is then virtually insensitive to the precise value, and the model is free of any fitting parameter. From the coupled-channel system, we determine the elastic and fusion cross sections simultaneously. For the \(^{16}\text{O}+^{16}\text{O}\) fusion cross section, we show preliminary results obtained in a single-channel approximation, where the \(^{16}\text{O}\) densities is obtained from an \(\alpha+^{12}\text{C}\) cluster calculation.

1. Introduction

The \(^{12}\text{C}+^{12}\text{C}\) and \(^{16}\text{O}+^{16}\text{O}\) fusion reactions play an important role in stellar nucleosynthesis [1] and in the evolution of massive stars [2]. The extrapolation of the \(S\)-factor at low energies is made difficult by the presence of resonances whose interpretation is not clear (see a recent review in [2]). Most fusion calculations to date are performed in a single-channel model, i.e. involving the ground-state only, while the absorption is simulated by a phenomenological imaginary potential [4]. In light systems, however, it is known that inelastic channels may be important and require to be explicitly included in the calculation. In Ref. [3], the authors suggest that mutual excitations play an important role even at low energies, where excited channels are closed. At first sight, this effect may seem surprising since only a single channel is open. It is explained by distortion effects in the wave functions: the cross section is mostly sensitive to the inner part of the wave functions, where closed channels may have a significant amplitude.

The aim of our calculation is to investigate the \(^{12}\text{C}+^{12}\text{C}\) and \(^{16}\text{O}+^{16}\text{O}\) fusion in a folding method [5]. We include the \(^{12}\text{C}(0^+_1, 2^+, 0^+_{2^+}, 3^-)\) and \(^{16}\text{O}(0^+_1, 0^+_{2^+}, 3^-, 2^+, 1^-)\) states and also the corresponding mutual excitations for both fusion reactions. A folding method is performed using the density-dependent M3Y (DDM3Y) potential [7] as nucleon-nucleon (NN) interaction. The \(^{12}\text{C}\) densities are taken from the RGM values of Kamimura [6], while \(^{16}\text{O}\) densities are obtained from an \(\alpha+^{12}\text{C}\) cluster calculation. An important aspect is that our calculation is free of parameter, except for a weak dependence on the absorption potential. It provides a simultaneous description of elastic scattering and of fusion.
2. Theoretical framework

In a coupled-channel formalism, the potentials are defined as

\[ V_{a_1a_2\alpha'\alpha_2}(r) = \int \int dr_1 dr_2 v_{NN}(r - r_1 + r_2) \rho_1^{\alpha_1\alpha_1'}(r_1) \rho_2^{\alpha_2\alpha_2'}(r_2), \]  

where \( v_{NN}(r) \) represents the nucleon-nucleon nuclear interaction, \( r = (r, \Omega_r) \) is the relative coordinate, \( \rho_k^{\alpha\alpha'}(r_k) \) are the nuclear densities, and labels \( \alpha_k \) refer to different states. The same formalism is applied to the Coulomb interaction. In the present work, we include \( ^{12}\text{C}(0^+, 2^+, 0^+_1, 3^-) \) and \( ^{16}\text{O}(0^+_1, 0^+_2, 3^-, 2^+, 1^-) \) states, which means that ten \( ^{12}\text{C}+^{12}\text{C} \) and fifteen \( ^{16}\text{O}+^{16}\text{O} \) channels are introduced in the coupled-channel systems. In practice, the evaluation of the double integral (1) is performed by using Fourier transforms for the nuclear as well as for the Coulomb interactions [5].

As usual, the densities \( \rho_k^{\alpha\alpha'}(r) \) are expanded in multipoles [6], and the radial wave functions \( g_{cc}^{J\pi}(r) \) are obtained from the coupled-channel system

\[ -\frac{\hbar^2}{2\mu} \left[ \frac{d^2}{dr^2} - \frac{\ell(\ell + 1)}{r^2} \right] g_{cc}^{J\pi}(r) + \sum_{c'} V_{cc'}^{J\pi}(r) g_{cc'}^{J\pi}(r) = (E - E_1^c - E_2^c) g_{cc}^{J\pi}(r), \]  

where \( \mu \) is the reduced mass of the system, and \( E_1^c \) are the \( ^{12}\text{C} \) or \( ^{16}\text{O} \) energies. In this equation, label \( c \) stands for \( c = (\alpha_1, \alpha_2, I, \ell) \), where \( I \) is the channel spin and \( \ell \) the relative orbital momentum.

In this approach, the absorption to other channels is simulated by adding an imaginary component to the potentials as (see Ref. [12] for details)

\[ V_{cc'}^{J\pi}(r) \rightarrow V_{cc'}^{J\pi}(r) + iW_{cc'}^{J\pi}(r). \]  

In the present calculation, the coupled-channel system (2) explicitly includes inelastic channels in a wide energy range. To define the fusion component of the potential, a short-range absorption potential [11] is included as

\[ W_{cc'}(r) = -\frac{W_0}{1 + \exp((r - R_0)/a)} \delta_{cc'}. \]  

The range \( R_0 \) is chosen smaller than the barrier radius, and this potential acts at short distances only. In this multichannel calculation, the values were taken as \( W_0 = 10 \text{ MeV} \), \( R_0 = 3 \text{ fm} \), and \( a = 0.1 \text{ fm} \), and the same conditions were employed to investigate the elastic-scattering and fusion processes. The calculations are stable within 1–2% when these parameters are modified. An important consequence is that the model is free of parameters, and that all cross sections are obtained without any adjustment.

The coupled-channel system (2) is solved with the R-matrix method [9]. In the internal region \( (r \leq a) \), the radial functions \( g_{cc'}^{J\pi}(r) \) are expanded over a Lagrange basis [10]. In the external region, they are given by linear combinations of Coulomb functions. The matching provides the collision matrix \( U^{J\pi} \). The elastic cross sections are computed from the collision matrices by using standard formulas [4].

3. Results and Discussions

The present folding model was first applied to \( ^{12}\text{C}+^{12}\text{C} \) elastic scattering at energies around the Coulomb barrier, where experimental data are available [14]. The idea was to assess the accuracy of the model on elastic scattering, and then apply it to the calculation of fusion cross sections. The procedure was started from a single-channel approximation, and we progressively
Figure 1. (A) Ratios of the elastic and Rutherford cross sections around the Coulomb barrier and (B) modified $S$ factor for increasing numbers of $^{12}\text{C}+^{12}\text{C}$ inelastic channels, respectively. Experimental data are taken from Refs. [14, 16].

included additional channels. The comparison between theory and experiment is presented in Fig.1(A).

When the energy increases, and in particular at $E = 10$ MeV, inelastic channels significantly improve the theoretical cross section. At 6 MeV, the physics of the problem is essentially determined by the Coulomb interaction, and the role of the inelastic channels is hardly visible. The most sensitive angular range is beyond $\theta = 70^\circ$, where the single-channel approximation provides a poor fit of the data. Including the $2^+$ state improves the overall agreement, but adding further the $0^+_2$ Hoyle state provides an excellent agreement with the data. Note that good fits can be obtained even in the single-channel approximation [15], but after fitting the imaginary potential to optimize the agreement with experiment.

For the $^{12}\text{C}+^{12}\text{C}$ reaction, the fusion cross section is traditionally converted in a modified $S$ factor as $\tilde{S}(E) = \sigma_F(E)E \exp(2\pi\eta + 0.46E)$, where $\eta$ is the Sommerfeld parameter and $\sigma_F(E)$ is the fusion cross section [13].

The $^{12}\text{C}+^{12}\text{C}$ modified $S$ factor is displayed in Fig.1(B), where the experimental data have been corrected as suggested by Aguilera et al. [17]. Above the Coulomb barrier ($\approx 6.5$ MeV) the data are well reproduced by the calculation, and the role of inelastic channels is minor. When the energy decreases, the sensitivity with respect to the number of excited channels is more and more important, in agreement with Ref.[3]. At $E = 1$ MeV, the multichannel calculation provides an enhancement of about a factor of three, in comparison with the single-channel approach.

Preliminary results for the $^{16}\text{O}+^{16}\text{O}$ system are shown in Fig.2. We limit the calculations to a single-channel approach, and investigate the sensitivity of the cross sections to a renormalization factor $N_r$. From Fig. 2, we conclude that the elastic cross sections are weakly sensitive to this factor. However the fusion cross section below 10 MeV is underestimated with $N_r = 1$. Using a factor $N_r = 1.3$ provides a reasonable agreement with the data. Multichannel calculations are in progress.

4. Conclusion
In the present work, the $^{12}\text{C}+^{12}\text{C}$ fusion process was investigated in a multichannel model. The coupling potentials are generated from $^{12}\text{C}$ densities obtained in a microscopic cluster model.
The calculation does not contain any fitting parameter, and provides simultaneously the fusion and elastic cross sections. Around the Coulomb barrier the elastic data are well reproduced by the model provided that all inelastic channels, and in particular those involving the $0^+_2$ state, are included. The results confirm the conclusion of Ref.[3]. For the $^{16}$O+$^{16}$O system, we have shown preliminary results, obtained with a single-channel approximation. Multichannel calculations are in progress.

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