Can the Kappa-distributed Electron Energies Account for the Intensity Ratios of O II Lines in Photoionized Gaseous Nebulae?

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Abstract

A vexing puzzle in the study of planetary nebulae and H II regions is that the plasma diagnostic results based on collisionally excited lines systematically differ from those based on recombination lines. A fairly speculative interpretation is the presence of nonthermal electrons with the so-called \( \kappa \) energy distributions, yet there is little observational evidence to verify or disprove this hypothesis. In this paper, we examine the influence of \( \kappa \)-distributed electrons on the emissivities of O II recombination lines using an approximate method, where the rate coefficients for a \( \kappa \) distribution are computed by summing Maxwellian–Boltzmann rate coefficients with appropriate weights. The results show that if invoking \( \kappa \)-distributed electrons, the temperatures derived from the \([\text{O III}] \lambda 4959 + \lambda 5007)/\lambda 4363\) ratios could coincide with those estimated from the O II \( \lambda 4649/\lambda 4089\) ratios. However, the estimated temperatures and \( \kappa \) values are not in agreement with those obtained through comparing the \([\text{O III}] \lambda 4959 + \lambda 5007)/\lambda 4363\) ratios and the hydrogen recombination spectra, suggesting that the electron energy is unlikely to follow the \( \kappa \)-distributions over a global scale of the nebular regions. Nevertheless, based on this observation alone, we cannot definitely rule out the presence of \( \kappa \)-distributed electrons in some microstructures within nebulae.

Unified Astronomy Thesaurus concepts: H II regions (694); Planetary nebulae (1249); Atomic spectroscopy (2099); Plasma astrophysics (1261)

1. Introduction

Plasma diagnostics using emission lines are of fundamental importance to understand the physical conditions and chemical compositions of photoionized gaseous nebulae such as planetary nebulae (PNe) and H II regions. A major problem in nebular physics, sometimes called the temperature and abundance discrepancy problem, is that the electron temperatures and elemental abundances derived from collisionally excited lines (CELs) are significantly different from those derived from recombination spectra (see, e.g., Peimbert 1967; Liu et al. 2000). These discrepancies have motivated studies of new physical scenarios. For instance, Nemer et al. (2019) recently suggested that a process called Rydberg Enhanced Recombination, which was never considered before, may contribute to the C II and O II recombination lines (RLs) in low-temperature photoionized plasma. Among many proposals for solving the temperature and abundance discrepancy problem, a rather disputable one is the presence of nonthermal electrons (Nicholls et al. 2012, 2013; Dopita et al. 2013). In this scenario, the energy of free electrons is assumed to follow a \( \kappa \) function that has a superthermal tail in the otherwise Maxwell–Boltzmann (MB) energy distribution, while the usually assumed MB distribution is a special case of the \( \kappa \) distribution in the limit of an infinite \( \kappa \) index. The intensities of CELs will be increased by the superthermal tail, leading to inappropriate results of the MB-based diagnostics.

The \( \kappa \) distribution function has been commonly used to fit the electron energy distribution in space plasma populated in the planetary magnetospheres, heliospheres, solar wind, and solar corona (e.g., Vasyliunas 1968; Feldman et al. 1975; Seely et al. 1987; Nicolaou et al. 2018). It was found that the \( \kappa \) distribution can be theoretically deduced from the statistical mechanics of out-of-equilibrium system that are applicable for systems subject to long-range interactions (Livadiotis & McComas 2009). The origin of the \( \kappa \) distribution has been a hot topic for debate in the solar physics community (Testa et al. 2014; Dudík et al. 2015; Nicolaou & Livadiotis 2019).

However, the presence of \( \kappa \)-distributed electrons is not supported by the classical theory of photoionized gaseous nebulae. Ferland et al. (2016) showed that the nonthermal electrons in nebulae will be quickly relaxed to the MB energy distribution before they are able to influence the excitation of CELs. The theoretical calculations of Draine & Kreisch (2018) suggested that the fraction of nonthermal electrons in PNe and H II regions is too small to affect the diagnostic results. These authors claimed that the temperature discrepancy problem must be caused by the spatial variations of electron temperatures. However, it should be noted that no theoretical model can produce temperature variations to the level required to explain the observed temperature discrepancy (Śtasińska 2017). From the theoretical perspective, it is fair to say that the issue of the temperature variation hypothesis is not less pronounced than that of the \( \kappa \)-distribution hypothesis.

Seldom efforts have been made to observationally determine the electron energy distribution in photoionized gaseous nebulae. C II dielectronic RLs and H I continuum emission spectra have been used to trace the electron energy distributions in PNe (Storey & Sochi 2013, 2014; Zhang et al. 2014), but the reported results are inconclusive or even conflicting due to rather large uncertainties. Under the assumption of \( \kappa \) distributions, Zhang et al. (2016) estimated the \( \kappa \) values for a sample of PNe and H II regions by comparing the \([\text{O III}] \lambda 4959 + \lambda 5007)/\lambda 4363\) ratios and the H I Balmer jump. There is no obvious correlation between the resultant \( \kappa \) values and various physical properties of the nebulae, providing no support for the hypothesis that \( \kappa \)-distributions can be pumped by known physical mechanisms.
Certainly, more observational evidences are needed to confirm or refute the $\kappa$-distribution hypothesis. If the $\kappa$-distributed electrons are present throughout the nebula, a consistent $\kappa$ value would be derived by comparing the intensity ratios of different diagnostic lines. Recently improved calculations of atomic data have confirmed that the MB temperatures derived from the O II RLs are remarkably lower than those derived from the [O III] CELs (Storey et al. 2017). The purpose of the present paper is to investigate whether the O II/[O III] temperature discrepancy conforms with the $\kappa$-distribution hypothesis.

Such an investigation usually requires the energy-tabulated rate coefficients, which are hardly ever available in the literature. Except for Storey & Sochi (2015a, 2015b), which presented the collision strengths for [O III] CELs and the recombination coefficients for hydrogen under the $\kappa$-distributions, prior research mostly reported MB-integrated rate coefficients. A similar situation has arisen in the study of solar plasma. In order to investigate the atomic processes important for solar physics, Hahn & Savin (2015) developed an easy method to approximately obtain the rate coefficients of $\kappa$-distribution plasma by summing up the MB rate coefficients with appropriate weights. In this work we apply this method to investigate the O II RLs in nebulae with $\kappa$-distributed electrons.

This paper is structured as follows: Section 2 describes the methods used to obtain the recombination coefficients of O II RLs for $\kappa$ distributions. Section 3 presents an uncertainty analysis and the $\kappa$ values derived from the observed O II RLs. In Section 4 we compare the results with those derived from H I Balmer jump and discuss the implications. A summary is given in Section 5.

2. Methodology

The calculations of recombination rate coefficients are based on the energy-dependent recombination cross sections integrating over the electron energy distribution function $f$. The isotropic $\kappa$ distribution has the form

$$f_\kappa(E, T_U, \kappa) = \frac{2}{\sqrt{\pi}} \frac{\Gamma(\kappa + 1)\sqrt{E}}{(\kappa - 1.5)^2\Gamma(\kappa - 0.5)} \times \left(\frac{1}{k_bT_U}\right)^\frac{2}{\kappa} \left[1 + \frac{E}{(\kappa - 1.5)k_bT_U}\right]^{\kappa - 1},$$

(1)

where $E$ is the electron energy, $\Gamma$ is the Gamma function, and $k_b$ is the Boltzmann constant. The low-energy part of this function can be described by an MB function with a temperature of $T$, and the high-energy part resembles a power law. The unitless $\kappa$ index characterizes the deviation from the MB distribution with smaller value corresponding to a larger deviation, and is always larger than 1.5. In the limit of $\kappa \rightarrow \infty$, Equation (1) reduces to the MB function. The temperature $T_U$ characterizes the mean kinetic energy, and has the expression $T_U = \kappa T/(\kappa - 1.5)$. Given the fact that $T_U$ is larger than $T$, MB-based plasma diagnostics could result in higher CEL temperatures than RL temperatures as the high- and low-energy parts of $f$ more effectively contribute to the excitation of CELs and RLs, respectively.

The recombination rate coefficients take the form

$$\alpha(T) = \int \sigma(E)f(E, T) \frac{2E}{m_\mu} dE,$$

(2)

where $m_\mu$ is the reduced mass, and $\sigma(E)$ is the cross section. Overwhelming published atomic data are for the plasma with a MB electron energy distribution $f_{\text{MB}}(E, T)$. In principle, the $\kappa$-distribution rate coefficients, $\alpha_\kappa(T_U, \kappa)$, can be accurately obtained by substituting $f_\kappa(E, T_U, \kappa)$ into Equation (2). However, the tabulated cross sections are commonly unavailable. An approximate method was presented by Hahn & Savin (2015) to derive the $\kappa$-distribution rate coefficients from published MB rate coefficients $\alpha_{\text{MB}}(T)$. They found that $f_\kappa(E, T_U, \kappa)$ can be decomposed to a series of MB distribution functions, i.e.,

$$f_\kappa(E, T_U, \kappa) = \sum_j c_j(\kappa)f_{\text{MB}}(E, T_j),$$

(3)

with $T_j = a_j(\kappa)T_U$, where the fitting parameters $a_j(\kappa)$ and $c_j(\kappa)$ are independent of $T_U$ and satisfy $\sum_j c_j(\kappa) = 1$. The uncertainties associated with this method increase with increasing $E/T_U$ and decreasing $\kappa$ index. However, as shown in Figure 1, even for the electron energy distribution with relatively low $\kappa$ indexes, the results are accurate within 3%. From Equations (2) and (3), we have

$$\alpha_\kappa(T_U, \kappa) = \sum_j c_j(\kappa)\alpha_{\text{MB}}(T_j),$$

(4)

namely, the $\kappa$-dependent rate coefficients can be decomposed into several weighted MB rate coefficients.

The purpose of Hahn & Savin (2015) is to investigate the collision processes in solar plasmas, such as the solar corona, which are generated by collisional ionization. Dissimilarly, PNes and H II regions are the photoionized gas characterized by much lower temperature and density, where radiative decay predominates over collisional decay for hydrogen. Nevertheless, the decomposition approach does not depend on the specific physical conditions and atomic processes, and applies to the calculation of any atomic data that can be expressed by Formula (2) (e.g., recombination coefficients and collision strengths). In low-density nebulae, because the effect of collisional transitions is likely negligible, captures and
downward-radiative transitions are the only processes to produce RLs, and thus the effective recombination coefficient to the i-th level is a linear weighted sum of the recombination coefficients to the levels of \( \geq i \). As a result, the approximate method can be employed to compute the effective recombination coefficients (and thus the emissivities) of RLs from the plasma with \( \kappa \)-distributions. In this work we focus on the commonly detected H I and O II RLs in photoionized gaseous nebulae.

3. Results

3.1. Viability of the Decomposition Approach in Determining the Emission Coefficients of H I RLs

Taking the \( a_f(\kappa) \) and \( c_f(\kappa) \) parameters provided in Hahn & Savin (2015), we computed the \( \kappa \)-dependent emission coefficients \( \epsilon(N_e, T_U, \kappa) \) for the hydrogen Balmer and Paschen lines using the decomposition approach by replacing \( \alpha_e(T_U, \kappa) \) and \( \alpha_{MB}(T_j) \) in Equation (4) with \( \epsilon(N_e, T_U, \kappa) \) and \( \epsilon(N_e, T_U, \kappa, \kappa) \) (Storey & Sochi 2015b).

Storey & Sochi (2015b) reported \( \epsilon(N_e, T_U, \kappa) \) of H I lines obtained from first-principle calculations, allowing us to validate the results of this approximate approach. When the \( \kappa \) index is sufficiently large, \( \epsilon(N_e, T_U, \kappa) \) is practically the same with the corresponding MB value. Thus, in the calculations with Equation (4) we have used \( \epsilon(N_e, T_U, 10^6) \) given by Storey & Sochi (2015b) as a substitute of the MB emission coefficients. In summing the weighted MB emission coefficients, we have ignored the terms of \( T_j > 10^4 \) K in that the recombination is essentially insignificant at such high temperatures.

The uncertainties of \( \epsilon(N_e, T_U, \kappa) \) can be estimated through comparing our results with those given by Storey & Sochi (2015b), as illustrated in Figures 2 and 3. An inspection of Figure 2 shows that the errors increase with increasing electron density, suggesting that the impact of collisions is becoming increasingly important and cannot be ignored in very dense conditions. Nevertheless, for the typical physical conditions of PNe and H II regions \( (N_e < 10^4 \text{ cm}^{-3} \text{ and } T_U < 2 \times 10^4 \text{ K}) \), the errors are up to 5%. Figure 3 presents the errors of \( \epsilon(N_e, T_U, \kappa) \) for a few Balmer and Paschen lines. The errors tend to decrease for the lines with higher upper levels, and dramatically increase with decreasing \( \kappa \) values. At the range of \( \kappa > 5 \), the errors are typically less than 3%. For extreme \( \kappa \)-distribution \( (\kappa < 3) \), it could be up to 10%. Figure 3 also plots the errors for the H I 3–2 line at various temperatures and densities, from which we can clearly see that even under extreme conditions the results are relatively reliable at the range of \( \kappa > 3 \). Consequently, the uncertainty analysis based on H I RLs strongly suggests that in typical nebular conditions, the decomposition approach can provide a close approximation to investigate the \( \kappa \)-dependent emissivities of RLs. But one should take caution when applying this method to the plasma with extremely low \( \kappa \) index.

3.2. The Kappa Index Derived from the O II RL Ratios

In order to investigate the influence of \( \kappa \)-distributed electrons on the temperature diagnostics of RLs, we computed the \( \kappa \)-dependent emissivities of O II RLs using the decomposition approach. In the calculations, we have set a constant density of \( 10^3 \text{ cm}^{-3} \) and taken the MB recombination coefficients of O II lines in case B recently reported by Storey et al. (2017). The O II \( \lambda 4649/\lambda 4089 \) intensity ratio has been commonly used to determine the electron temperatures of PNe (Wesson et al. 2003; Fang & Liu 2013; McNabb et al. 2016). As stated by Storey et al. (2017), the O II \( \lambda 4649/\lambda 4089 \) lines originate from high-\( J \) levels, and thus both strongly depend on the population of the \( ^3P_2 \) level of O\( ^{3+} \). As a result, their intensity ratio is rather insensitive to density, and can serve as a good temperature indicator. In Table 1, we present the theoretical O II \( \lambda 4649/\lambda 4089 \) intensity ratio for different \( \kappa \) and \( T_U \) values, which can be readily used to determine the physical conditions of \( \kappa \) distribution plasma. As illustrated in Figure 4, the O II \( \lambda 4649/\lambda 4089 \) ratio increases with increasing \( \kappa \) and \( T_U \). It is apparent that this ratio is more sensitive to \( \kappa \) in high-temperature and low-\( \kappa \) ranges, and is virtually indistinguishable between MB and \( \kappa \) distributions in the range of \( \kappa > 20 \). Therefore, when we use the O II \( \lambda 4649/\lambda 4089 \) ratio to determine the \( \kappa \) index, the uncertainties dramatically increase with increasing \( \kappa \).

In a homogeneous nebula, the [O III] CELs at 4959, 5007, and 4363 Å arise from the same region with the O II RLs. Storey & Sochi (2015a) reported the effective collision strengths for excitation and de-excitation of [O III] lines with \( \kappa \)-distributed electron energies, which can be used to derive the theoretical \( [\text{O III}] (\lambda 4959 + 5007)/4363 \) intensity ratio as functions of \( \kappa \) and \( T_U \). Then \( \kappa \) and \( T_U \) can be simultaneously
index and the ADF. Despite this, the PNe with extremely large ADF, such as Hf 2-2 (Liu et al. 2006), exhibit relatively lower $\kappa$ values.

4. Discussion

In the present study, we investigated the effect of $\kappa$-distributed elections on the emissivities of H I and O II RLs as well as on the plasma diagnostics based on RLs by using a decomposition approach. In typical nebular conditions, the emissivities of H I RLs obtained by this approximate method are in excellent agreement with the ab initio calculations, demonstrating the viability of this method. We found that the presence of $\kappa$-distributed elections can reduce the O II $\lambda 4649/\lambda 4089$ intensity ratio, and hence lead to significantly underestimated electron temperatures for MB distributions. McNabb et al. (2016) used the O II $\lambda 4649/\lambda 4089$ ratio to determine the electron temperatures of a PN sample, which are about 3000 K on average, while the electron temperatures derived from the [O III] CELs are typically about 10,000 K. Our results indicate that the [O III] and O II temperatures can reach a common value through tuning the $\kappa$ index. The $\kappa$-distribution hypothesis, therefore, seems to provide a plausible explanation for the CEL/RL temperature discrepancy problem. As shown in Figure 6, the relationship between the $\kappa$ index and the [O III]/O II MB-temperature difference $\delta T$ can be approximated with an empirical function

$$\kappa = 1.5 \exp \left( \frac{3.25}{0.40} \delta T \right),$$

where $\delta T$ equals to $T_{MB}([\text{O III}]) - T_{MB}(\text{O II})$ in unit of $10^3$ K. This formula can be used to estimate the $\kappa$ values from the previously reported O II temperatures.

However, contrary to the expectation of the $\kappa$-distribution hypothesis, the $\kappa$ values derived from the [O III] CELs and O II RLs are not consistent with those derived by Zhang et al. (2016) from the [O III] CELs and H I Balmer jump. The latter showed that the average $\kappa$ values of PNe are about 30, much larger than the current results. The $\kappa$ and $T_{UL}$ values derived by us and those by Zhang et al. (2016) are compared in Figures 7 and 8, respectively. If the superthermal electrons are homogeneously distributed over the whole nebula, a consistent $\kappa$ value should be derived from different diagnostics. However, from Figures 7 and 8, we can see that $\kappa$ and $T_{UL}$ derived from the O II ratio are systematically smaller than those derived from the H I Balmer jump. This is a strong evidence against the presence of globally distributed superthermal electrons, and thus is compatible with current theoretical propositions that no known physical mechanism can pump a $\kappa$ electron energy

| $\kappa$ | 2 | 3 | 5 | 7 | 10 | 15 | 20 | 50 | MB |
|----------|---|---|---|---|----|----|----|----|----|
| $T_U$ (K) | 1000 | 2.0601 | 2.1106 | 2.1566 | 2.1764 | 2.1912 | 2.2023 | 2.2079 | 2.2175 | 2.2241 |
|         | 2000 | 2.1221 | 2.2469 | 2.3378 | 2.3730 | 2.3986 | 2.4188 | 2.4289 | 2.4461 | 2.4571 |
|         | 5000 | 2.3172 | 2.5696 | 2.7400 | 2.8037 | 2.8508 | 2.8848 | 2.9041 | 2.9350 | 2.9556 |
|         | 7500 | 2.4586 | 2.7843 | 3.0003 | 3.0804 | 3.1401 | 3.1830 | 3.2067 | 3.2457 | 3.2719 |
|         | 10000 | 2.5794 | 2.9697 | 3.2288 | 3.3231 | 3.3936 | 3.4467 | 3.4730 | 3.5191 | 3.5499 |
|         | 12500 | 2.6891 | 3.1371 | 3.4404 | 3.5481 | 3.6293 | 3.6931 | 3.7220 | 3.7744 | 3.8089 |
|         | 15000 | 2.7969 | 3.2982 | 3.6500 | 3.7715 | 3.8647 | 3.9394 | 3.9725 | 4.0306 | 4.0682 |

Table 1

The Computed O II $\lambda 4649/\lambda 4089$ Intensity Ratios

Figure 4. Theoretical O II $\lambda 4649/\lambda 4089$ intensity ratio as a function of $T_U$ for various $\kappa$ indexes. $N_e$ is assumed to be $10^3$ cm$^{-3}$.
distribution over the whole photoionized gaseous nebula (Ferland et al. 2016; Draine & Kreisch 2018).

A hypothesis that cannot be ruled out is that $\kappa$-distributed electrons perhaps exist only at a small spatial scale. Zhang et al. (2016) obtained a similar fitting function for $\kappa$ versus $T_{\text{MB}}$([O III])—$T_{\text{MB}}$(H I). Comparing their formula with Equation (5), we can conclude that $T_{\text{MB}}$(O II) more sensitively depends on $\kappa$ than does $T_{\text{MB}}$(H I). It follows that if there was $\kappa$-distribution plasma embedded within the “normal” nebula, the O II $\lambda 4649/\lambda 4089$ ratio would be affected significantly more than the H I Balmer jump. This provides a possible explanation for the smaller $\kappa$ values obtained by us. If this is the case, the $\kappa$ indices listed in Table 2 are only upper limits and, in principle, a comparison of two pairs of O II RLs can impose a more stringent constraint on the $\kappa$-index of the nonthermal component. To develop a better understanding of nebular physical conditions, we need to construct models comprising adjustable parameters representing the properties of the nonthermal component (temperature, density, $\kappa$-index, filling factor, etc.) to match spectroscopic observations in particular of RLs. For this purpose, the computations of $\kappa$-based atomic data for more RLs from O$^+$ and other ions are required, which are beyond the scope of this paper.

| Object | $T_{\text{MB}}$(K) | References |
|--------|-----------------|------------|
| Cn 2-1 | 5000-10000      | (Liu et al. 2018; Wesson et al. 2015; Zhang et al. 2016) |
| H1-73  | 5000-10000      | (Bohigas et al. 2015; McNabb et al. 2016; Madonna et al. 2017; García-Rojas et al. 2018) |
| NGC 5307 | 10000-200000 | (Zhang et al. 2016; Wesson et al. 2015; Zhang et al. 2016) |

Note. References. (1) Liu et al. (2001), (2) Ruiz et al. (2003), (3) Tsamis et al. (2003), (4) Wesson et al. (2005), (5) Zhang et al. (2005), (6) Wang & Liu (2007), (7) Fang & Liu (2011), (8) Bohigas et al. (2015), (9) McNabb et al. (2016), (10) Madonna et al. (2017), (11) García-Rojas et al. (2018).
It is well established that the $\kappa$-distribution plasma in the solar system is mostly confined in local regions. If such plasma could be preserved until the PN phase or the same pumping mechanism works in photoionized gaseous nebulae, the plasma diagnostics of PNe would be greatly influenced. Although this is admittedly a highly speculative conjecture, further theoretical studies are desirable.

5. Conclusion

In this work we computed the $\kappa$-dependent emissivities of RLs using a decomposition method, aiming to find observational evidence supporting or rejecting the postulated presence of $\kappa$-distributed electrons in PNe and H II regions. The validity of this approximate method was verified through an analysis of H I RLs, which suggests that reliable results can be obtained in the range of $\kappa > 3$. We showed that the O II $\lambda 4649/\lambda 4089$ intensity ratio has a moderate dependence on the $\kappa$ index in the range of $\kappa < 20$, and the introduction of $\kappa$ electron energy distribution allows us to reconcile the incongruent MB temperatures derived from [O III] CELs and O II RLs. However, the $\kappa$ values estimated from the [O III]/O II temperature discrepancies are generally lower than those obtained from the [O III]/H I ones, disfavoring the global presence of $\kappa$-distributed electrons over the whole nebulae. Therefore, we can conclude that if the $\kappa$-distribution hypothesis holds, the superthermal electrons must be distributed within small-scale regions. Further investigation of $\kappa$-dependent emissivities of other RLs such as He I is strongly recommended.
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References

Bobigas, J., Escalante, V., Rodríguez, M., & Dufour, R. J. 2015, MNRAS, 447, 817
Dopita, M. A., Sutherland, R. S., Nicholls, D. C., Kewley, L. J., & Vogt, F. 2013, ApJS, 208, 10
Draine, B. T., & Kreisch, C. D. 2018, ApJ, 862, 30
Dudík, J., Mackovjak, Š., Dzifčáková, E., et al. 2015, ApJ, 807, 123
Fang, X., & Liu, X.-W. 2011, MNRAS, 415, 181
Fang, X., & Liu, X. W. 2013, MNRAS, 429, 2791
Feldman, W. C., Asbridge, J. R., Bame, S. J., Montgomery, M. D., & Gary, S. 1975, JGR, 80, 4181
Ferland, G. J., Henney, W. J., O’Dell, C. R., & Peimbert, M. 2016, RMxAA, 52, 261
García-Rojas, J., Delgado-Inglada, G., García-Hernández, D. A., et al. 2018, MNRAS, 473, 4476
Hahn, M., & Savin, D. W. 2015, ApJ, 809, 178
Liu, X. W., Barlow, M. J., Zhang, Y., Bastin, R. J., & Storey, P. J. 2006, MNRAS, 368, 1959
Liu, X.-W., Luo, S.-G., adn Barlow, M. J., Danziger, I. J., & Storey, P. J. 2001, MNRAS, 327, 141
Liu, X.-W., Storey, P. J., Barlow, M. J., et al. 2000, MNRAS, 312, 585
Livadiotis, G., & McComas, D. J. 2009, JGR, 114, A11105
Madonna, S., García-Rojas, J., Sterling, N. C., et al. 2017, MNRAS, 471, 1341
McNabb, I. A., Fang, X., & Liu, X.-W. 2016, MNRAS, 461, 2818
Nemer, A., Sterling, N. C., Raymond, J., et al. 2019, ApJ, 887, L9
Nicholls, D. C., Dopita, M. A., & Sutherland, R. S. 2012, ApJ, 752, 148
Nicholls, D. C., Dopita, M. A., Sutherland, R. S., Kewley, L., & Palay, E. 2013, ApJS, 207, 21
Nicolaou, G., & Livadiotis, G. 2019, ApJ, 884, 52
Nicolaou, G., Livadiotis, G., Owen, C. J., Verscharen, D., & Wicks, R. T. 2018, ApJ, 864, 3
Peimbert, A., & Peimbert, M. 2013, ApJ, 775, 89
Peimbert, M. 1967, ApJ, 150, 825
Ruiz, M. T., Peimbert, A., Peimbert, M., & Esteban, C. 2003, ApJ, 595, 247
Seely, J. F., Feldman, U., & Doschek, G. A. 1987, ApJ, 319, 541
Stasinska, G. 2017, CaPh, 95, 821
Storey, P. J., & Sochi, T. 2013, MNRAS, 430, 599
Storey, P. J., & Sochi, T. 2014, MNRAS, 440, 2581
Storey, P. J., & Sochi, T. 2015a, MNRAS, 449, 2974
Storey, P. J., & Sochi, T. 2015b, MNRAS, 446, 1864
Storey, P. J., Sochi, T., & Bastin, R. 2017, MNRAS, 470, 379
Testa, P., de Pontieu, B., Allred, J., et al. 2014, Sci, 346, 315
Tsamis, Y. G., Barlow, M. J., Liu, X.-W., Danziger, I. J., & Storey, P. J. 2003, MNRAS, 345, 186
Vasyliunas, V. M. 1968, JGR, 73, 2839
Wang, W., & Liu, X.-W. 2007, MNRAS, 381, 669
Wesson, R., Liu, X.-W., & Barlow, M. J. 2003, MNRAS, 340, 253
Wesson, R., Liu, X.-W., & Barlow, M. J. 2005, MNRAS, 362, 424
Zhang, Y., Liu, X.-W., Luo, S. G., Péquignot, D., & Barlow, M. J. 2005, A&A, 442, 249
Zhang, Y., Liu, X.-W., & Zhang, B. 2014, ApJ, 780, 93
Zhang, Y., Zhang, B., & Liu, X.-W. 2016, ApJ, 817, 68