Analysis of the abilities of algebraic laminar-turbulent transition models

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Abstract. Due to their simplicity and numerical stability, algebraic transition models seem to be a good alternative to differential transition models. The paper considers two promising algebraic transition models $k-\varepsilon$ KD and SA BC and compares their results with the results of the most popular differential model - SST $\gamma-\text{Re}_\theta$. Three test cases are considered: flat plate boundary layer, NLF(1)-0414F airfoil and two NACA-0012 airfoils in tandem. Results show that although algebraic models are simpler and easier to use, their accuracy compared to the differential model leaves much to be desired.

1. Introduction

Despite most practical flows having Reynolds numbers large enough for turbulence to arise, laminar zones are quite common. In these cases, the accuracy of the description of laminar-turbulent transition is very important to acquire precise magnitudes of quantities such as lift and drag coefficients.

Differential transition models contain additional equations for transitional parameters along with “standard” turbulent variables ($k$, $\varepsilon$, $\omega$, $n_i$). In particular, the SST $\gamma-\text{Re}_\theta$ [1] model, which is the most popular and precise in its class at the moment (see for example [2]), along with basic SST equations for $k$ and $\omega$ contains two additional differential equations for intermittency $\gamma$ and transition onset Reynolds number $\text{Re}_\theta$. The model is very flexible which allows it to somewhat accurately predict location and the type of transition for different types of transitional flows. However, despite its good sides, the model requires large amounts of computational power and time to achieve a converged solution, making it hard to use in engineering computations.

Algebraic transition models don’t rely on differential equations for transitional characteristics, but use algebraic relations. They have been gaining popularity because they are much simpler to implement and to work with, while using less computational resources. However, due to the algebraic models’ novelty there is little data to assert their quality and universality. Two particular models, SA BC [3] and $k-\omega$ KD [4] look promising based on their results presented in the corresponding papers. Both models use intermittency as the main transitional characteristic.

The goal of this paper is to establish whether models [3] and [4] are capable of showing acceptable results on different transitional flows. The flows under consideration are the T3 flat plate boundary layer cases [5], the flow around the NLF(1)-0414F airfoil, representing the “laminar drag bucket effect” [6] and the...
flow around NACA-0012 airfoils in tandem [7]. Computational results are compared with experimental data and the results of the SST $\gamma$-$Re_\theta$ model.

2. Model formulations and numerical aspects

2.1. The SA BC model [3]

The SA BC model consists of the equation for intermittency $\gamma_{bc}$ which is used as a multiplier in the SA model [8] without the $f_{12}$ function:

$$\gamma_{bc} = 1.0 - \exp\left(-\sqrt{\text{Term}_1} - \sqrt{\text{Term}_2}\right);$$

where

$$\text{Term}_1 = \max\left(\text{Re}_\theta - \text{Re}_{\theta c}, 0\right); \quad \text{Term}_2 = \max\left(\nu_{bc} - \chi_2, 0\right);$$

are the triggering functions. Term$_1$ triggers the onset of transition when $\text{Re}_\theta > \text{Re}_{\theta c}$ and the Term$_2$ function triggers the growth of intermittency inside the boundary layer. The auxiliary relations are

$$\text{Re}_\theta = \frac{\text{Re}_c}{2.193}; \quad \text{Re}_c = \frac{\rho d^2_w}{\mu}; \quad \text{Re}_{\theta c} = 803.73\left(Tu_v + 0.6067\right)^{-1.027}; \quad \nu_{bc} = \frac{V_s}{U_d}.$$}

The empirical constants defined in the paper [3] are $\chi_1 = 0.002$, $\chi_2 = 5.0$. However, initial testing shows that the model does not predict transition with this set of constants and it is most likely a misprint. The constants were adjusted ( $\chi_1 = 1.000$, $\chi_2 = 0.0005$ ) and this set of constants ensures good agreement with the data from the paper (based on the T3A case) and were used for all computations.

2.2. The k-$\omega$ KD model [4]

In this model, intermittency is used in the production term in “k” equation of the 2008 k-$\omega$ Wilcox model [9] which takes the form

$$P = \gamma P_k + (1 - \gamma) P_{sep};$$

The model uses the following equation for the intermittency

$$\gamma = \min\left(\max\left(\frac{\sqrt{k} \nu}{A \nu} - 1.0, 0.0\right), 1.0\right);$$

where

$$P_{sep} = C_{sep} F_{sep} \nu S^2;$$

$$F_{sep} = \min\left(\max\left(\frac{R_v}{2.2A_v} - 1.0, 0.0\right), 1.0\right); \quad R_v = \frac{y^2 S}{\nu};$$
Eddy viscosity is defined as a sum of “small-scale” and “large-scale” parts

\[ \nu_T = \nu_s + \nu_i; \]

where

\[ \nu_i = k_i / \tilde{\omega}; \quad \tilde{\omega} = \max \left[ \omega, \frac{C_{lim}S}{a_s} \right]; \quad \nu = \max \left[ \omega, \frac{C_{lim}S}{a_i} \right]; \quad k_i = k - k_i; \quad R_{\nu} = \frac{y^2S}{\nu}. \]

The model constants are as follows:

\[ A_v = 12.0, \quad C_s = 21.0, \quad C_A = 1.0, \quad C_\psi = 10.0, \quad C_w = 10.0, \]
\[ C_{sep} = 2.0, \quad A_v = 550.0, \quad a_i = 0.45, \quad a_s = 0.3, \quad C_{lim} = 7.0 / 8.0 \]

2.3. Computational aspects

Incompressible numerical computations were carried out in an academic code NTS (Numerical Turbulence Simulation) using RANS equations. For all transport equations, including the equations for turbulent variables, the second order discretization scheme for the convective terms was selected. For each test case all simulations were started from the same uniform initial field and the simulations were performed until the normalized maximum residuals of all the equations were below the value of $10^{-5}$. All solutions are grid independent.

It is also important to point out that algebraic transition models are less sensitive to the parameters of the solver and often need significantly less iterations to converge than the SST $\gamma$-Re, which requires lower Courant numbers and/or relaxation parameters.

3. Considered tasks and computational results

3.1. Flat plate transitional boundary layer

Experiments with transitional boundary layers at different turbulent intensities and pressure gradients (T3 series) were conducted in Rolls Royce [5]. Three cases with no pressure gradient from these series were considered in the present work, namely T3A- with 0.87% freestream intensity, T3A with 3.3% and T3B with 6.2% with corresponding Reynolds numbers of $1.0 \cdot 10^5$, $3.0 \cdot 10^5$ and $3.3 \cdot 10^5$. Even at the lowest intensity bypass transition takes place.

The computational domain contains a freeflow region upstream of the plate’s leading edge to minimize the pressure gradient effect. The boundary conditions are as follows: uniform velocity profile was set at the inlet boundary, constant pressure at outlet and upper boundaries, no-slip wall at the plate. Turbulence characteristics at the start of the plate were chosen to coincide with experimental values. A computational grid of $305 \times 146$ cells was used.
Experimental and computational skin friction coefficients distribution for all cases and models are presented on figure 1. One can see that there is a significant difference in the transition location between models and it is most pronounced for the lowest turbulence intensity.

![Figure 1](image1)

**Figure 1.** Comparison of experimental skin friction coefficient with results of computations of three considered cases: T3A- (a), T3A (b), T3B (c)

The best agreement with experimental data is obtained using SST $\gamma$-$Re_0$ model, which is not surprising since it was calibrated using T3 series experiments. The model also captures the magnitudes of skin friction in the fully turbulent region better. Results of algebraic models are in less consistency with experimental data. In T3A- case they predict transition too early and in other cases, while able to predict the position of transition, they fail to predict the specifics. The $k$-$\omega$ KD model predicts a characteristic peak in skin friction right after the transition which is not observable in the experiment data, the SA BC model predicts too low values of skin friction at the transition location and downstream from it.

### 3.2. Flow around NLF(1)-0414F airfoil

This flow was studied experimentally in [6]. The airfoil was set up in a rectangular wind tunnel with the height equal to 2.5 chord length. The Reynolds numbers considered are $3.0 \times 10^6$, $6.0 \times 10^6$ and $1.0 \times 10^7$. The characteristic of this flow is the “laminar drag bucket phenomena”, which a peculiarity in the $C_L$-$C_D$ graph (see fig. 2, circles). At low angles of attack a large portion of the airfoil surface can maintain laminar flow regime resulting in low drag coefficients.

![Figure 2](image2)

**Figure 2.** The $C_L$-$C_D$ dependencies for the NLF(1)-0414F airfoil for three considered Reynolds numbers: $3E6$ (a), $6E6$ (b), $1E7$ (c)

As figure 2 shows, at the lowest Reynolds number, $3.0 \times 10^6$ SST $\gamma$-$Re_0$ and SA BC models are able to predict low drag observed in the experiment while the $k$-$\omega$ KD model, while still being closer to experimental values than fully turbulent SST solution, fails to predict the laminar drag bucket effect. SST $\gamma$-$Re_0$ and SA BC show too low drag though and the “bucket” predicted by them covers a wider range of lift coefficient values than in the experiment and doesn’t have a local maximum around $C_L = 0.4$. At $Re = 6.0 \times 10^6$ only the SST $\gamma$-$Re_0$ model shows good agreement with the experiment results while algebraic
models predict almost fully turbulent solutions. At the highest Reynolds number none of the models predict the laminar drag bucket effect, but the differential model results are still slightly closer to experimental.

3.3. Flow around NACA-0012 tandem

The CFD setup of the flow around NACA-0012 airfoils in a tandem corresponds to the experiment of Lee and Kang [7]. Two airfoils are placed horizontally one after another in a wind tunnel. The distance between the airfoils and Reynolds numbers were different (see table 1). In the experiment strips of sandpaper were pasted on both sides of the upstream airfoil between $X/C=0.58$ and 0.65 to generate a turbulent wake flow. In order to achieve the same effect in the simulations (Figure 2) $\gamma$ was equated to 1.0 for $X/C = 0.58 \div 1$ for all transition models. More detailed description of the computational setup can be found in [10].

Table 1. Flow configurations

| Case    | Distance between airfoils, G | Reynolds Number ($\times 10^5$) |
|---------|------------------------------|---------------------------------|
| Case-1  | 1.00                         | 6.00                            |
| Case-2  | 1.00                         | 4.00                            |
| Case-3  | 1.00                         | 2.00                            |
| Case-4  | 0.50                         | 2.00                            |
| Case-5  | 0.25                         | 2.00                            |

Figure 3. Distribution of skin friction coefficient on the downstream airfoil predicted by different transition models
The comparison of computational and experimental distributions of skin friction coefficient over the downstream airfoil is shown in Figure 3. At higher Reynolds numbers and distances between the airfoils (Cases 1 and 2) SST $\gamma$-$Re_\theta$ and $k$-$\omega$ KD models tend to transition more rapidly ($k$-$\omega$ KD showing fast, bypass-type transition) while SA BC models shows fully turbulent behavior. In cases 3-5 $k$-$\omega$ KD and SA BC show more slow, natural-type transition which is too close to the leading edge and the skin friction coefficient is too low while the position of the transition predicted by SST $\gamma$-$Re_\theta$ is close to experimental and skin friction values being slightly higher.

4. Conclusion
Two algebraic transition models were tested on three transitional flows against the differential SST $\gamma$-$Re_\theta$ model and experimental results. Both models demonstrate much better convergence than the differential one, however the quality of their results is considerably worse. In particular, in most cases algebraic models predict transition to turbulence too early. In conclusion, while algebraic transition models remain a promising alternative to differential ones, at the present time their low accuracy precludes their use in engineering applications.

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