INTRODUCTION

Oil and gas are extremely significant energy, which have heavy demand in the whole world. Efficient drilling operation can make huge contribution to exploitation and development of oil and gas. Clay shale is a common geological formation, which is often encountered in drilling. Clay shale formation has high risk of wellbore collapsing, impairing drilling...
efficiency and causing huge economic losses. More importantly, clay shale formation is located above oil and gas reservoir. Severe wellbore collapse in clay shale can block the approach of reaching oil and gas reservoir. Major reason of causing wellbore collapse is shale hydration. It is well known that when shale interacts with drilling fluid, hydration damage occurs, influencing shale mechanical property and causing instability. Numerous researches have studied this mechanical influence from hydration damage. These researches found out that rock constitutive relation is variable in hydration evolutionary process. Correspondingly, damage models of constitutive relation have been established. In particular, due to this constitutive relation change, strength and deformation parameters will be affected, incurring rock instability. Meanwhile, hydration is able to cause damage on rock structure, which offers easier approach for fluid invasion, aggravating hydration damage.

In order to evaluate hydration in drilling operation, extensive researches have been conducted to establish evaluation method for hydration damage. Most of these researches are based on damage mechanics theory. According to the ratio of damage volume to original volume, hydration damage index can be established. Especially, CT scanning and nuclear magnetic resonance (NMR) have been used to acquire rock microstructure in hydration. Based on these images from CT and NMR, fracture initiation and propagation can be detected in hydration. This gradually increasing void space in rock can be used to evaluate hydration damage. Considering hydration damage on rock mechanical property, hydration damage has been discussed on the basis of elastic modulus, strength, and strain. Furthermore, hydration capacity of clay shale has been assessed from its mineral composition, physicochemical properties, and hydration stress. According to above statements, there are already lots of researches about hydration damage, and different kinds of evaluation methods have been established. However, shale hydration is a tricky problem and wellbore collapse still frequently happens. The limitation of current hydration evaluation is the lack of practicability in oilfield. Like hydration damage index made by CT technology or NMR, their evaluations for hydration damage are mainly conducted in laboratory test. It is very difficult to directly put these indexes into drilling operation. Thus, in this paper, we proposed a new hydration damage index using clay shale acoustic property, which can be directly obtained from acoustic logging. In fact, acoustic property is a vital information in petroleum engineering. By using acoustic test, Tutuncu and Mese applied acoustic velocity to evaluate shale structure anisotropy. Besides, based on wave theory of continuous media, Chen et al. and Xu et al. established numerical model to analyze the influence of rock structure (porosity, fracture and bedding plane) on acoustic velocity. Based on the interaction between water and rock, Al-Bazali et al. and Zhu et al. conducted research on acoustic velocity and mechanical property with variable water content. Yan et al. and Li et al. combined acoustic test and uniaxial test to acquire acoustic velocity in the loading, which can be used to reflect stress damage evolution in uniaxial test. In a word, acoustic test has been widely used in researches about rock structure characteristics, rock strength, rock damage, etc. Whereas, when it comes to acoustic test, these researches are all limited to acoustic velocity. Acoustic data contain abundant information, like acoustic attenuation, amplitude, and frequency. Acoustic velocity is not able to fully represent acoustic information. Thus, in this paper, acoustic frequency spectrum is added into hydration index. Besides, based on this new index, hydrated constitutive model has been built to analyze mechanical effect of hydration damage. This study can offer a new method for hydration evaluation. Outcomes in this paper can be beneficial for drilling in clay shale formation.

2 | SAMPLE AND EXPERIMENTAL METHOD

2.1 | Clay shale sample

In this study, mineral composition of clay sample has been tested using XRD, as shown in Tables 1 and 2. Clay shale is composed of quartz, feldspar (orthoclase and plagioclase), carbonates (calcite and dolomite), clay (smectite, illite, illite/smectite layer, kaolinite, and chlorite), and siderite. It is important to note that clay is dominated in mineral composition. Water-sensitive minerals, such as smectite, illite/smectite layer, and illite, are abundant in clay shale. Meanwhile, to further know its water sensitivity, cation exchange capacity (CEC) has been measured by methylene blue test, as shown

| TABLE 1 | Results of shale mineral composition |
|---------|-------------------------------------|
| No      | Quartz/% | Orthoclase/% | Plagioclase/% | Calcite/% | Dolomite/% | Siderite/% | Clay/% |
| 1       | 15.1     | 5.3         | 2.4          | 8.3       | 3.2        | 0.5        | 65.2   |
| 2       | 20.3     | 4.2         | 2.3          | 10.2      | 4.1        | 0.3        | 58.6   |
| 3       | 18.6     | 3.9         | 3.2          | 9.2       | 2.6        | 0.3        | 62.2   |
| 4       | 21.3     | 4.3         | 4.2          | 11        | 3.6        | 0.4        | 55.2   |
| 5       | 19.2     | 3.6         | 3.8          | 9.6       | 3.5        | 0.2        | 60.1   |
in Figure 1. CEC value is associated with water adsorption capacity and surface hydration, meaning high CEC represents stronger hydration expansion capacity. For clay shale in this work, average CEC is 119.4 mmol/kg, which is high value according to rock CEC distribution. Because of high content of water-sensitive mineral and high CEC value, clay shale has strong hydration ability, impairing wellbore stability in drilling. Hence, investigation of hydration is vital for drilling in clay shale formation.

### 2.2 Uniaxial compressive test

In order to discuss rock constitutive relation, stress-strain curve needs to be obtained from uniaxial compressive test. In this paper, equipment of uniaxial test is RTR-1000 compressive apparatus and its schematic is shown in Figure 2. This uniaxial compressive apparatus is mainly composed of loading column, pressure frame, pressure pump, pressure controller, foundation, upper and lower block, axial strain gauge, compressive test software, and computer. For this equipment, it can have 1000KN loading capacity with 0.5 ~2.5 MPa/s loading rate. In uniaxial test, core sample has 25 mm diameter and 50 mm height. In this loading process, axial stress is increased until sample has failure. Strain gauge and loading system will transfer data to computer, drawing stress-strain curve. Except for stress-strain curve, this test can give mechanical parameters, such as compressive strength, elastic modulus, and strain, which are important parameters for analyzing mechanical influence of hydration damage.

### 2.3 Acoustic test method

In acoustic test, all core samples are standard cylinder with 25 mm diameter and 50 mm height. The acoustic test is conducted in room condition, that is, room temperature (25°C) and without confining pressure. Before conducting acoustic test, to reduce difference among rock samples, all core samples were selected based on similar physical properties, such as density and porosity. Subsequently, according to ultrasonic pulse penetration method, acoustic equipment has been used to obtain acoustic wave. The acoustic equipment consists of signal source, ultrasonic emission probe and receiving probe, oscilloscope, and computer, as shown in Figure 3. By using the adhesive, core sample is closely placed between emission and receiving probe. From signal source, P-wave with 50KHz is created and passed to core sample by using ultrasonic emission

### Table 2

Results of shale clay composition

| No | Illite/% | Smectite/% | I/S | Kaolinite/% | Chlorite/% |
|----|---------|-----------|-----|-------------|------------|
| 1  | 20.6    | 42.2      | 13.4| 14.6        | 9.2        |
| 2  | 18.6    | 40.1      | 12.2| 18.2        | 10.9       |
| 3  | 19.3    | 45.2      | 11.2| 16.6        | 7.7        |
| 4  | 19.8    | 38.9      | 16.6| 15.6        | 9.1        |
| 5  | 21.3    | 39.5      | 10.2| 17.6        | 11.4       |
At another side of core sample, ultrasonic receiving probe obtains acoustic data after going through core sample and transfers acoustic information to oscilloscope and computer. Consequently, acoustic wave is captured on the oscilloscope and data of acoustic wave are saved in computer. For these acoustic data, sampling interval is 0.1 us.

Based on the acoustic test, acoustic data in time domain can be acquired, as shown in Figure 4. Data in time domain cannot fully reflect acoustic frequency spectrum characteristics. Thus, Fourier transform has to be conducted to process these data. In geotechnical engineering area, lots of scholars have applied it to process acoustic data when acoustic technology is used into evaluation of rock integrity. Based on the acoustic signal from above equipment, we apply Fourier transform to acquire acoustic frequency spectrum. The equation of Fourier transform can be written as:

\[
x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)e^{i\omega t} d\omega \\
X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-i\omega t} dt
\]

where \(x(t)\) is acoustic signal at time domain; \(X(\omega)\) is acoustic signal at frequency domain; \(\omega\) is frequency, KHz; and \(t\) is time, ms.

\(X(\omega)\) is plural form, written as:

\[
X(\omega) = R(\omega) - iS(\omega) = A(\omega)e^{-i\phi(\omega)}
\]

where \(R(\omega)\) and \(S(\omega)\) are real part and imaginary part in Fourier transform. \(\phi(\omega)\) is phase spectrum. \(A(\omega)\) is Fourier amplitude, which can be expressed as:
For acoustic signal data, Fourier series of the periodic signal \( x(t) \) can be written as:

\[
x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n}{T} t + b_n \sin \frac{n}{T} t \right)
\]

where \( n \) is number of signal data. \( a_0 \), \( a_n \), and \( b_n \) are Fourier coefficients, expressed as:

\[
a_0 = \frac{2}{T} \int_{0}^{T} x(t) dt
\]

\[
a_n = \frac{2}{T} \int_{0}^{T} x(t) \cos \frac{n}{T} t dt \quad n = 1,2,3 \ldots
\]

\[
b_n = \frac{2}{T} \int_{0}^{T} x(t) \sin \frac{n}{T} t dt
\]

where \( T \) is period of signal data.

Combining Equations 1, 4, and 5, connection between acoustic time domain and frequency domain can be established on the basis of Fourier transform. Consequently, acoustic frequency spectrum characteristics can be analyzed, as shown in Figure 5. There are two significant parameters in acoustic frequency spectrum, which are maximum acoustic amplitude and dominant frequency. Maximum acoustic amplitude and dominant frequency can be used as indicator of rock structure since these parameters are related to acoustic energy attenuation when acoustic wave has propagation inside rock. For shale sample without hydration damage, maximum acoustic amplitude and dominant frequency are measured, shown in Table 3. Maximum acoustic amplitude is 347.1-389.5 mV, and average value is 362.4 mV. Dominant frequency is 8.9-9.7 KHz with 9.3 KHz average value.

\[
A(\omega) = \sqrt{R(\omega)^2 + S(\omega)^2}
\]

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\]

\[
b_n = \frac{2}{T} \int_{0}^{T} x(t) \sin \frac{n}{T} t dt
\]
Figure 6 gives acoustic frequency spectrum of shale sample with different soaking time. Based on this acoustic frequency spectrum, acoustic maximum amplitude and dominant frequency in hydration process are obtained, as shown in Figure 8. With increasing soaking time, acoustic maximum amplitude and dominant frequency both show decline. This decreasing phenomenon can be explained by increasing void space caused by hydration damage. When sonic pulse goes through rock, due to fracture initiation and propagation, acoustic wave has increment of reflection, refraction, and scattering, thus lengthening propagation path of acoustic wave in rock. Consequently, in this longer path, acoustic energy attenuation becomes larger, showing relatively small amplitude. Furthermore, according to acoustic propagation mechanism, these fracture plane can be regarded as typical filters, which tend to absorb high frequency part in acoustic wave, eventually increasing the proportion of low frequency part. Thus, dominant frequency has decreasing tendency.

Meanwhile, it can be found that the variation of acoustic frequency spectrum is consistent with rock structure change in Figure 6. In initial stage (0 hours–12 hours), it is not easy to detect the change of acoustic frequency spectrum. Similarly, at late stage (after 72 hours), the acoustic frequency spectrum has no obvious change since fracture initiation and propagation almost stop. In contrast, at the middle stage with clear fracture initiation and propagation (12 hours–72 hours), decline tendency of amplitude and dominant frequency are clearly noticed, indicating strong hydration effect occurs at this period.

3.2 Establishment of hydration damage index

According to above analysis, hydration damage can be detected in acoustic frequency spectrum. Due to the decreasing trend of maximum amplitude and dominant frequency, triangle area ($S$) can be formed in acoustic frequency spectrum, as shown in Figure 9. Obviously, with stronger hydration damage, two lines of this triangle (decline of maximum amplitude and dominant frequency) become large, increasing the area of this triangle. Furthermore, we consider two limiting conditions. Firstly, before hydration damage occurs, no modification happens in acoustic frequency spectrum and this triangle area can be regarded as zero ($S = 0$). Secondly, we assume that, with continuous decline, in complete hydration damage condition, maximum amplitude and dominant frequency both become zero, forming this largest triangle ($S_m$) in acoustic frequency spectrum. The variation of triangle area ($S$) is related to hydration damage degree. Therefore, we apply triangle area to evaluate this hydration damage and built a new hydration damage index, shown as:

$$D = \frac{S}{S_m} = \frac{(M_a - M_c) \cdot (F_a - F_b)}{M_a \cdot F_a}$$

($M_a$, $F_a$: maximum amplitude and dominant frequency before hydration damage; $M_c$, $F_c$: maximum amplitude and dominant frequency after hydration damage; $M_a$, $F_a$: maximum amplitude and dominant frequency at hydration damage occurrence.)
where $D$ is hydration damage index. $S$ is triangle area made by maximum amplitude and dominant frequency. $S_{m}$ is triangle area in complete hydration damage condition. $M_a$ and $M_c$ are maximum amplitude of shale before and after soaking, mV. $F_a$ and $F_b$ are dominant frequency of shale before and after soaking, KHz.

Based on this hydration damage index and experimental results, hydration evolutionary process can be obtained, as shown in Figure 10. Each point in Figure 10 is made from average experimental data of three shale samples. Based on these data, equation of fitting curve can be established, as shown in Equation 7. This tendency is typical $S$ curve with three different stages, which are slow increase stage (0 hour–24 hours), rapid increase stage (24 hours–72 hours), and stable stage (after 72 hours). Fitting curves are consistent with experimental data, proving its capability of expressing hydration damage.

$$D = \frac{0.359}{1+55.08 \cdot e^{-0.0815 \cdot t}} - 0.0054$$

(Figure 7) Acoustic frequency spectrum with different soaking time

$D$ is hydration damage index. $S$ is triangle area made by maximum amplitude and dominant frequency. $S_{m}$ is triangle area in complete hydration damage condition. $M_a$ and $M_c$ are maximum amplitude of shale before and after soaking, mV. $F_a$ and $F_b$ are dominant frequency of shale before and after soaking, KHz.

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methods have been established, as shown in Table 4. To obtain these existing indexes in Table 4, elastic modulus, density, and compressive strength of clay shale sample with varying soaking time have been measured. Based on these parameters, correlations between these existing indexes and index in this paper are acquired, as shown in Figure 11. Results demonstrate hydration damage index of this paper has good correlation with $D_e$, $D_d$, and $D_\sigma$. Large declines of compressive strength, elastic modulus, and density represent strong hydration damage, which are fit for current researches’ conclusions, proving the practicability of this new index.

### 3.3 Hydration damage index in variable conditions

In above sections, tests of hydration damage are performed under room condition, which is different from drilling situation in subsurface. For instance, in deep formation, interaction between fluid and shale occurs in high temperature. Also, due to difference between mud pressure in fluid and pore pressure in shale, drilling pressure difference is formed.
at the wall of borehole when shale contacting with fluid. Furthermore, since the hydration is typical chemical reaction, fluid composition is directly associated with hydration damage. Thus, in this section, hydration damage in subsurface condition has been discussed from three aspects: (a) high temperature; (b) drilling pressure difference, and (c) drilling fluid composition.

To simulate hydration damage in subsurface condition, based on geological and drilling data, temperature and drilling pressure difference are mainly located at 90°C–120°C and 4.5–7.5 MPa, respectively. Besides, since there are numerous ingredients for drilling fluid, it is impossible to discuss all these ingredients. In this part, we only use brine solution (5% KCl + 5%NaCl and 10% KCl + 10% NaCl) to simulate drilling fluid with different inhibition. The whole experimental process is illustrated: sample is firstly put into container with different types of fluid. Subsequently, by injecting nitrogen gas into container, pressure difference between fluid and sample has been created, as shown in Figure 12. After that, container is sealed and put into oven to create high temperature.

Figure 13 demonstrates hydration evolutionary process in different conditions. Each point is average value of three shale samples. High temperature and pressure difference stimulate hydration reaction, meaning hydration index is increased and its increasing rate becomes large. This is because high temperature is able to intensify physicochemical reaction between drilling fluid and shale. Besides, drilling pressure difference is the force of pushing fluid into shale, leading to stronger hydration damage. In addition, due to brine ion, sample shows relatively small damage index. This can be explained by the inhibition of brine solution. Based on absorption and ion exchange effect, brine ion can combine with clay to change its

| Equations | Nomenclature | References |
|----------|--------------|-----------|
| $D_e = 1 - \frac{E_e}{E}$ | $E_e, E$ are elastic modulus in hydration and intact condition respectively, GPa. | Cao et al.33 |
| $D_d = 1 - \frac{Den}{Den}$ | $Den, Den$ are density in hydration and intact condition respectively, g/cm$^3$. | Wang and Qu.34 |
| $D_\sigma = 1 - \frac{\sigma_{ch}}{\sigma_c}$ | $\sigma_{ch}, \sigma_c$ are uniaxial compressive strength in hydration and intact condition respectively, MPa. | Gui et al.35 |

**FIGURE 11** Correlation between $D$ and existing indexes
hydration activity, thus inhibiting hydration. To sum up, hydration damage is associated with external conditions and its evaluation should include these influence factors.

4 | HYDRATED CONSTITUTIVE MODEL OF CLAY SHALE

The main reason why hydration attracts attention in drilling operation is its influence on rock mechanical property. It is well known that rock mechanical property is a vital factor of wellbore stability and drilling engineering design. When it comes to rock mechanical property, constitute relation is the foundation, which will be affected by hydration damage. Therefore, in this section, considering this interaction between rock and fluid, hydrated constitute relation has been investigated.

4.1 | Hydrated constitutive model

According to the strain equivalence assumption made by Lemaitre, stress on damage material is equal to effective stress on intact material. Therefore, constitutive relation under damage is derived, shown as:

\[ \sigma = \sigma^* (1 - D_m) = E(1-D_m)\varepsilon \quad (8) \]

where \( \sigma \) is stress on damage material, MPa; \( \sigma^* \) is effective stress on intact material, MPa; \( \varepsilon \) is strain, %; and \( E \) is elastic modulus of intact sample, MPa. \( D_m \) is total damage index.

For conventional constitutive model, it is acquired on the basis of compressive test (triaxial test or uniaxial test). The damage is merely from stress (axial stress or confining stress). Whereas, in this paper, since sample firstly goes through soaking, total damage index \( D_m \) involves stress damage and hydration damage, illustrated as:

\[ D_m = D_s + D \quad (9) \]

where \( D_s \) is stress damage index.

The hydration damage index can be acquired from above section using acoustic spectrum. For stress damage, it is regarded as macro-embodiment of initiation, propagation, and aggregation of the microstress damage on microelement. Thus, we have established a stress damage index using statistical damage method under the assumption that microunit strength of rock is randomly distributed, which can apply to Weibull distribution.

Since strength of the microunit accords with the Weibull distribution, its function of probability density is written as:

\[ \phi(\varepsilon) = \frac{m}{n} \left( \frac{\varepsilon}{\theta} \right)^{m-1} e^{-\left(\frac{\varepsilon}{\theta}\right)^n} \quad (10) \]

where \( \phi(\varepsilon) \) is stress probability of microelement at \( \varepsilon \); \( m \) and \( n \) are parameters of Weibull distribution.

When stress has been loaded on rock, strain of microelement becomes \( \varepsilon \) and some microelements enter into failure. With gradual accumulation of microelement failure, macro-damage has been formed, expressed as:

\[ D_s = \int_0^\varepsilon \phi(\varepsilon) d\varepsilon = 1 - e^{-\left(\frac{\varepsilon}{\theta}\right)^n} \quad (11) \]

Ignoring anisotropy of confining pressure (\( \sigma_2 = \sigma_3 \) and \( \varepsilon_2 = \varepsilon_3 \)), based on linear elastic theory and Hooke law, stress and strain in the loading can be illustrated as:

\[ \left\{ \begin{array}{l} \varepsilon_1 = \frac{\sigma_1 - 2\mu\sigma_3}{E} \\
\varepsilon_3 = \frac{\sigma_3 - \mu(\sigma_1 + \sigma_3)}{E} \end{array} \right. \quad (12) \]

where \( \varepsilon_1 \) and \( \varepsilon_3 \) are strain along axial and radial direction, %, \( \sigma_1 \) and \( \sigma_3 \) are axial stress and confining stress under stress damage, MPa. \( \mu \) is Poisson’s ratio.

Merely considering stress damage \( D_s \) and combining Equations 8, 11, and 12, constitutive relation can be acquired, shown as:
In above equations, rock elastic parameters ($E$ and $\mu$) can be directly obtained by compressive test. For Weibull parameters ($m$ and $n$), their values are determined by characteristics of peak point at stress-strain curve. The geometric feature of peak point gives following stress-strain relation:

$$
\begin{align*}
\sigma_1 &= E\varepsilon_f \exp \left[ -\left( \frac{\varepsilon}{\sigma_f} \right)^n \right] + 2\mu\sigma_3 \\
\sigma_3 &= \frac{E\varepsilon_f \exp \left[ -\left( \frac{\varepsilon}{\sigma_f} \right)^n \right]}{1-\mu} + \frac{\mu\sigma_1}{1-\mu}
\end{align*}
$$

(13)

Additionally, the differential equation of Equation 13 is written as:

$$
\begin{align*}
d\sigma_1 &= A_1 d\varepsilon_f + A_2 d\varepsilon + A_3 d\mu + A_4 d\mu + 2\mu d\sigma_3 \\
d\sigma_3 &= B_1 d\varepsilon_f + B_2 d\varepsilon + B_3 d\mu + B_4 d\mu + \frac{\mu}{1-\mu} d\sigma_1
\end{align*}
$$

(16)

where $A_i$ and $B_i (i = 1, 2, 3, 4)$ are shown as:

$$
\begin{align*}
A_1 &= \frac{\partial \sigma_1}{\partial \varepsilon_f}, & A_2 &= \frac{\partial \sigma_1}{\partial \varepsilon}, & A_3 &= \frac{\partial \sigma_1}{\partial \mu}, & A_4 &= \frac{\partial \sigma_1}{\partial \mu} \\
B_1 &= \frac{\partial \sigma_3}{\partial \varepsilon_f}, & B_2 &= \frac{\partial \sigma_3}{\partial \varepsilon}, & B_3 &= \frac{\partial \sigma_3}{\partial \mu}, & B_4 &= \frac{\partial \sigma_3}{\partial \mu}
\end{align*}
$$

(17)

According to Hooke law and Von Mises criterion, strain of microelement in Equation 13 can be expressed as:

$$
\begin{align*}
\varepsilon &= \varepsilon_f - \frac{(1-2\mu)\sigma_3}{E} \\
\varepsilon &= \frac{\varepsilon_f}{E} + \frac{(1-2\mu)\sigma_3}{\mu E}
\end{align*}
$$

(18)

Based on Equation 18, after conducting the differential, expression of strain can be written as:
Once \( m \) and \( n \) are confirmed, stress damage index can be calculated, shown as:

\[
D_i = 1 - \exp \left\{ \frac{\varepsilon_i - (1 - 2\mu)\sigma_3}{E_i} \cdot m \left( \frac{\sigma_j - 2\mu\varepsilon_3}{E_j} \right) \ln \left( \frac{\sigma_j - 2\mu\varepsilon_3}{E_j} \right) \right\} \tag{26}
\]

Note that this paper only considers uniaxial stress condition (\( \sigma_3 = 0 \)). Thus, by substituting Equation 26 into Equation 9, total damage index and hydrated constitutive model can be written as:

\[
\begin{align*}
D_m &= 1 - \exp \left[ \left( \frac{\varepsilon_i}{E_i} \right)^m \ln \left( \frac{\sigma_j}{E_j} \right) \right] + D \\
\sigma &= E\varepsilon \left[ \exp \left( \frac{\varepsilon_i}{E_i} \right)^m \ln \left( \frac{\sigma_j}{E_j} \right) - D \right] \tag{27}
\end{align*}
\]

### 4.2 Verification of hydrated constitutive model

In order to verify this new hydrated constitutive model, stress curves from model have been calculated. Basic mechanical parameters of model are given by uniaxial test for intact sample, as shown in Table 5. It is shown that for intact clay shale sample, its uniaxial strength, elastic modulus, and strain at peak point are 116.5 MPa, 54.2 GPa, and 2.27%, respectively. Comparing to existing researches on clay shale,\(^{42}\) sample in this paper shows relatively high uniaxial strength. There are numerous factors of causing rock strength distinction. The high uniaxial strength is mainly because sample is obtained from outcrop, which avoids damage from coring in drilling operation.

Stress curves of constitutive model and experimental test are depicted in Figure 14. It is shown hydrated constitutive model is consistent with experimental curves. Only in postfailure stage, since we do not consider residual strength in wellbore stability analysis, stress-strain curves have some discrepancies. Furthermore, in order to quantitatively verify this model, mechanical parameters (elastic modulus, peak strain, peak stress) in hydrated constitutive model and experimental test have been illustrated, as shown in Figure 15. By comparison, mechanical parameters of model have small deviation from experimental data. Average deviations of elastic modulus, peak strain, and peak stress are 4.7%, 4.1%, and 1.4%, showing good accuracy of this hydrated constitutive model.

### 4.3 Mechanical property with hydration damage

After verifying this hydrated constitutive model, in this section, we apply this model to depict stress-strain curves with variable hydration damage, as shown in Figure 16-18. It can
be seen that with increasing hydration time, peak stress, peak strain, and elastic modulus all show decline. These declines represent smaller ability of bearing pressure and lower resistance to deformation. All these phenomena show the increment of rock mechanical instability. Meanwhile, since hydration damage is stronger in high temperature, large drilling pressure difference, and low concentration of brine, large declines exist in these conditions.

Based on stress-strain curves in Figures 16-18 and uniaxial test for samples in variable soaking conditions (Figure 13), mechanical parameters in varying degree of hydration damage have been illustrated, as shown in Figure 19. For x-axis in Figure 19, variable hydration damage index is caused by different soaking conditions, such as time, temperature, pressure difference, and brine concentration. According to data from model and experimental test, in stronger hydration damage, higher decline of mechanical parameters exists. Maximum hydration damage index is approximately 0.65; correspondingly, decline of peak stress, peak strain, and elastic modulus is 83 MPa, 0.18%, and 35.3GPa, respectively. More importantly, decreasing tendency of model is in good agreement with experimental data. Thus, we can conclude
FIGURE 16 Stress curves with different temperatures based on hydrated constitutive model

(A) Room temperature  
(B) 90°C  
(C) 120°C

FIGURE 17 Stress curves with different pressure difference based on hydrated constitutive model

(A) 0 MPa  
(B) 4.5 MPa  
(C) 7.5 MPa

FIGURE 18 Stress curves with different fluid based on hydrated constitutive model

(A) Water  
(B) 5%Kcl+5%NaCl  
(C) 10%Kcl+10%NaCl
FIGURE 19 Mechanical properties in hydration damage

(A) Peak stress

(B) Peak strain

(C) Elastic modulus
that no matter what hydration condition or hydration damage sample has, mechanical parameters under hydration damage can be precisely predicted by using this model.

5 | DISCUSSION

5.1 | Hydration mechanism of clay shale

Clay shale hydration is basically relied on its mineral composition. Since high content of clay mineral exists, making shale has strong water absorption and high hydration stress. In a word, this mineral composition makes shale has water sensitivity. When clay shale contacts with fluid, hydration damage exhibits typical S curve, having clear three stages, that is, slow increase stage, rapid increase stage, and stable stage. This increasing trend is consistent with crack propagation. For instance, damage degree increases fast at middle stage, which is also the rapid development of fracture induced by hydration. This hydration damage index in the subsurface condition has obvious discrepancy with index under room condition. High temperature can boost the chemical-physical reaction between shale and water, highly elevating hydration damage. Positive pressure difference is able to push water into shale, which enlarges contacting area between shale and water. Thus, for hydration evaluation, precise results will not be acquired unless subsurface condition is fully considered.

Besides, no matter what condition rock is in, influence of hydration on rock structure exists. Fracture initiation and propagation create convenient approach for water invasion. Once this water invades into shale, reaction between fluid and water-sensitive mineral (smectite, illite/smectite layer and illite) will happen inside rock, aggravating hydration damage and decreasing rock mechanical stability. Therefore, for drilling fluid applied into clay shale formation, hydrate inhibition and blocking are both necessary requirements.

5.2 | Hydration damage index in wellbore stability evaluation

The rock structure will have tremendous change in hydration evolutionary process, affecting acoustic wave propagation in rock. As a result, acoustic property will have corresponding change, which can be used to evaluate hydration damage. The establishment of this index has been clearly illustrated in above sections. Based on that, further application has been discussed in this section.

The illustration of application has been given in Figure 20. Firstly, hydration damage evolutionary process \( D(t) \) can be established using the method in this paper. Secondly, since logging is conducted after drilling, its data can be treated as data at \( t = t_d \) (\( t_d \) is the time of ending of drilling) with variable depth \( H \). By inserting acoustic logging into hydration damage evolutionary process, acoustic property in original formation \( (t = 0) \) and hydrated condition \( (0 < t \leq t_d) \) can be computed. Correspondingly, rock structural and mechanical property in original and hydrated condition can be calculated, respectively. These two properties are necessary parameters in wellbore stability. Mechanical property reflects rock strength...
and the resistance of deformation, which are directly associated with rock stability. Rock structure is related to rock failure type, affecting rock strength and wellbore collapsing shape. With change of rock structure under hydration, the interaction between rock and drilling fluid is dynamically modified, affecting capillary force, ion exchange, osmosis pressure difference, pore pressure transport, and so on. Rock property of original formation is vital in wellbore stability. These parameters of original formation are key factors of determining mud pressure in the beginning of drilling. If initial mud pressure is wrongly chosen, shape of borehole will be destroyed, forming irregular shape. In that case, controlling wellbore stability becomes more tricky, especially in formation with high risk of wellbore collapsing Furthermore, according to rock property in hydrated condition, drilling engineering parameters (mainly mud pressure) can be adjusted with time so that wellbore stability can be maintained in the whole drilling process.

For the application of hydration damage index in this paper, there are still some limitations. Firstly, we only consider time, temperature, drilling pressure difference, and drilling fluid composition in the evaluation of hydration. There are still many other influence factors, such as in situ stress, rock anisotropy, and structure characteristics. Hydration evaluation can be further improved by adding these factors into hydration damage index. Meanwhile, in drilling operation, numerous physical-chemical reactions will happen when drilling fluid contacts with clay shale, such as inhibition of brine ion, ion exchange, electrostatic interaction, and semipermeable membrane effect. More works have to be done to establish good wellbore stability technology by including all these influences factors.

6 | CONCLUSION

1. A new hydration damage index has been built based on shale acoustic frequency spectrum. In hydration evolutionary process, rock structure will be damaged, having fracture initiation and propagation, influencing acoustic wave propagation, and changing shale acoustic frequency spectrum. Based on that, acoustic test has been conducted for shale sample with variable soaking time. By using hydration damage index and experimental results, hydration evolutionary process can be obtained. This hydration process has three different stages, which are slow increase stage (0 hour–12 hours), rapid increase stage (12 hours–72 hours), and stable stage (after 72 hours). Meanwhile, hydration damage index is different with varying conditions. High temperature and pressure difference accelerate the hydration process. In contrast, hydration damage is decreased in fluid with high brine concentration. Even though hydration damage degree is changed with conditions, evolutionary tendency remains typical S curve with time.

2. In combination with new hydration damage index and constitutive relation, hydrated constitutive model can be obtained. This hydrated constitutive model clearly depicts stress-strain relation in hydration evolutionary process. Stress-strain curves from uniaxial compressive test show good consistency with curves of hydrated constitutive model, proving this model can precisely simulate constitutive relation. By using this model, mechanical parameters under hydration damage can be acquired. With increasing hydration time, peak stress (uniaxial compressive strength), peak strain, and elastic modulus all show decline, representing increment of wellbore instability. In particular, these declines are stronger in high temperature, large drilling pressure difference, and low concentration of brine ion. Meanwhile, no matter what hydration condition shale has, mechanical parameters can be accurately calculated by using this hydrated constitutive model.

3. Acoustic property is a significant and practical parameter. Rock structure will have tremendous change in hydration evolutionary process, affecting acoustic wave propagation in rock. The corresponding change of acoustic property can be used to evaluate hydration damage. In particular, with increment of void space inside rock, acoustic frequency spectrum characteristics will have obvious modification because of the increment of reflection, refraction, and scattering. This relation between acoustic property and shale hydration offers theoretical guidance for hydration evaluation on the basis of acoustic logging and eliminates hydration interference on acoustic logging. Consequently, based on acoustic data with and without hydration, parameters of rock structure and mechanical properties in original and hydrated formation can be acquired, respectively, which are meaningful for evaluating wellbore stability in the whole drilling process.

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