Heat transfer on the outer surface of vertical longitudinally finned tubes

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Abstract. This paper analyses the heat transfer process on the outer surface of a vertical tube with eight longitudinal aluminium fins situated central-symmetrically along its perimeter. The goal of this experimental study is to determine the local heat transfer coefficients on the outer surface of vertical tubes with longitudinal fins during airflow under natural and forced convection conditions. The analysis concerned the distribution of heat transfer coefficients on the perimeter and along the pipe. It looked at various ranges of air velocity \( w = 0–2.3 \text{ m/s} \) and a variety of air refrigerant temperatures \( \Delta T = 24–40 \text{ K} \). Measurements taken for non-zero air flow velocity indicated the need to take into account the effect of the temperature difference between the heat transfer coefficients obtained for conditions of forced convection in the analysed range of flow rates. In addition, the study involved the determination of temperature distribution on the ribs, and their efficiency was verified numerically by CFD modelling of the heat exchange process for the ribs. Numerical calculations confirmed the correctness of the method for determining the efficiency of the ribs and of the method of measurement.

1. Introduction

Heat exchanger design is determined by the type of heat transfer (natural or forced convection) and the intervening media. For instance, heat pump evaporators may be designed without a fan which would force the flow of external air [1]. Such evaporators are designed as freely arranged systems of longitudinally finned vertical tubes. The operating conditions of the exchangers vary and may correspond to natural, forced or mixed convection conditions, the latter being the most frequent. The applicability of equations which may be used to calculate the air-side heat transfer coefficients depends on flow type and geometry. For instance, the equations used for horizontal lamellated or transversely finned tubes include those proposed by: Schmidt [2], Norris-Spoford [3], Stasiulevicius [4] and Briggs-Young [5]. Papers [6] and [7] present the results of measurements of local and mean heat transfer coefficients during transverse airflow around horizontal membrane tubes or tubes with two ribs (fins) under forced convection conditions. The authors of papers [8] and [9] looked at the heat transfer processes during forced transverse flow of exhaust gas around a horizontal bank of tubes with three fins.

The numerical analysis of natural convective heat transfer processes on the surface of longitudinal fins is often performed for flow inside longitudinally-finned tubes [10,11].

Experimental determination of mean or local values of heat transfer coefficients, especially for finned surfaces, is a complex measurement problem. This paper presents the results of experimental studies of local values of heat transfer coefficients obtained using the authors’ own measurement method.
method, which may be used for any finned surfaces with fins with a prismatic cross-section. The measured local heat transfer coefficients were verified using independent measurements of averaged heat transfer coefficients for the analyzed heat transfer processes.

2. Experimental studies

The study looks at the heat transfer process on the outer surface of a vertical bimetallic tube with eight aluminum longitudinal fins arranged centrally symmetrically along the circumference of the tube with an inner copper tube core (Figure 1). Tubes of this type are used to build heat pump evaporators without fans. As the evaporators operate under natural external conditions, the heat transfer conditions on the outer surface of the tubes may correspond to both natural and forced convection with low flow velocities and longitudinal and transverse flow around the tubes.

![Figure 1. Distribution of temperature sensors on the surface of the tube.](image1.png)

![Figure 2. Experimental setup: 1-radial fan, 2-flow stream equalizer, 3- Pitot tube, 4-studied tubes, 5-condensing unit, 6-electric heaters.](image2.png)

The experimental setup consisted of a system of two vertical finned tubes with a height of 2m, whose axes were located at a distance of $S_0=0.2306$ m from each other. The experiments were carried out in a horizontal channel with a 2x0.464 m rectangular flow cross-section and a length of $L_p=2.5$ m (Figure 2). Airflow in the channel was forced using a radial fan. The heat exchanger tubes were fed with ice water at a temperature of 7°C or with refrigerants (R407C and R507) with evaporation temperatures of: $-25 \leq T_R \leq -5$ (°C). Calibrated NiCr-NiAl thermocouples were placed on the outer surface of one of the tubes to enable the measurement of the temperature of the outer tube surface and of the fin tips in the upper and lower part of the exchanger (Figure 1). The temperature was measured for both forced and natural convection conditions. The heat flux transferred from the air to the heat transfer surface was calculated using the heat balance equation for the cooling medium (water or refrigerant) [12].

The experimentally identified distribution of temperature values on the surface of the fin was used to determine the local values of the air-side heat transfer coefficients. Figure 3 shows sample measured temperature distributions on the fin surface.
2.1. Description of the method used to determine the local values of heat transfer coefficients

The following solved energy equation for a straight prismatic fin was used to determine the local values of air-side heat transfer coefficients:

\[
\Delta T(x) = T(x) - T_a = F_1 \cdot e^{m \cdot x} + F_2 \cdot e^{-m \cdot x}
\]  

(1)

Constants \( F_1 \) and \( F_2 \) in equation (1) were determined for boundary conditions corresponding to the measurement conditions:

\[
\Delta T(x = 0) = T_w - T_a
\]  

(2)

\[
\Delta T(x = h) = T_f - T_a
\]  

(3)

Equations (1...3) made it possible to determine the constants of integration \( F_1 \) and \( F_2 \):

\[
F_1 = \frac{(T_a - T_w) \cdot e^{-m \cdot h} + (T_f - T_a)}{2 \cdot \sinh(m \cdot h)}
\]  

(4)

\[
F_2 = -\frac{(T_a - T_w) \cdot e^{m \cdot h} + (T_f - T_a)}{2 \cdot \sinh(m \cdot h)}
\]  

(5)

The local value of the heat transfer coefficient for air was determined on the basis of the condition of equality of the sum of the heat fluxes absorbed by the fin \( \dot{Q}_{\text{fin}} \) and the non-finned part of the outer surface of the tube \( \dot{Q}_{\text{o}} \), and the heat flux transferred to the cooling medium. The above assumption written down for the tube and referred to a single fin is as follows:

\[
\dot{Q}_{\text{fin}} / n_{\text{fin}} = \dot{Q}_{\text{fin}} + \dot{Q}_{\text{o}}
\]  

(6)

Where the heat fluxes \( \dot{Q}_{\text{fin}} \) and \( \dot{Q}_{\text{o}} \) were calculated from the equations:
\[
\dot{Q}_{\text{fin}} = -\lambda \cdot A_{\text{fin1}} \cdot \frac{dT}{dx}_{x=0} = -\lambda \cdot A_{\text{fin1}} \cdot m \cdot (F_1 - F_2)
\]

(7)

\[
\dot{Q}_o = RCJ \cdot \alpha_{\text{loc}} \cdot A_{\text{a1}} \cdot (T_w - T_a)
\]

(8)

Figure 4. Distribution of local values of air-side heat transfer coefficients for natural convection: a) for \(\Delta T = T_a - T_w = 28K\), b) for \(\Delta T = T_a - T_w = 35K\).

Figure 5. Distribution of local values of air-side heat transfer coefficients for forced convection: a) \(w_a=2.3 \text{ (m/s)}\) and \(\Delta T=22 \text{ K}\), b) \(w_a=2.3 \text{ (m/s)}\) and \(\Delta T=31 \text{ K}\).

The sought value \(\alpha_{\text{loc}}\) is determined by solving, for each measurement point, the system of equations (1...8), complemented with equations for the calculation of the parameter \(m\) and coefficient \(RCJ\) [13].

Figures 4a and 4b present examples of the determined local values of the heat transfer coefficient of a natural convection heat exchanger. Figures 5a and 5b present the determined local values of the heat transfer coefficients for a forced convection heat exchanger. Figure 6 presents a verification of the averaged local values of the heat transfer coefficients, with the mean values of these coefficients being determined on the basis of another independent measurement method, presented in [12]. The high conformity of the results obtained using the two different independent measurement methods suggests the accuracy of the measurement method adopted to determine the local heat transfer coefficients. Figure 6 indicates that in the investigated range of airflow velocities, the values of forced convective
heat transfer coefficients are determined both by air velocity and by the difference in temperature between the air and the heat exchange surface. In this paper, the accuracy of the assumptions made and the measurement method adopted were also verified using CFD modelling of the heat transfer process between the finned surface and the air.

\[ \alpha_{a} = f(T_{a} - T_{wo}) \]

**Figure 6.** Equation \( \alpha_{a} = f(T_{a} - T_{wo}) \) for natural and forced convection and two measurement methods: \( \alpha_{loc} \) - averaged local values, \( \bar{\alpha} \) - measured mean values.

### 2.2. Numerical modelling of the air-fin heat transfer processes

The heat transfer process between the finned tube and external air was modelled using Fluent software. The shape of the fin and the transverse direction of airflow (for forced convection) required the modelling to be carried out in a 3D space. Air was modelled as a perfect gas and the dynamic viscosity coefficient was determined on the basis of Sutherland’s equation, determined by two coefficients. The equations of motion and energy complemented by a k-epsilon turbulence model (realizable) were solved for natural and forced convection processes with the right boundary conditions being assumed.

Natural convective airflow around a finned tube was modelled as a cylindrical volume filled with a finned tube and air. Due to the large number of elements of the grid, the model was divided into four sections. During the modelling of particular sections, velocity, pressure and temperature profiles were read for air in the bottom part of the fin. They were subsequently entered as the boundary conditions for the next section of the fin.

The assumed boundary conditions are therefore as follows (Figure 7a, 7b):

- constant temperature on the outer tube wall: \( T(A_{1})=T_{w} \),
- constant air temperature at a large distance from the fin: \( T(A_{2})=T_{a} \),
- changes in air pressure above the fin and in the bottom part of the tube: \( p(A_{3})=p(A_{5})=0 \),
- symmetry on the lateral surfaces of the tube cross-section and the air: \( A_{4}=\text{symmetry} \),
- condition of velocity change at the inlet: \( w(A_{6})=w(A_{5}) \), (\( w(A_{3})=0 \)).

| Table 1. Sample data adopted for the modelling of the temperature distribution on the fin. |
|---------------------------------|----------|----------|----------|----------|
| Ambient temperature \( T_{a} \) (K) | 293.00 | 296.00 | 294.00 | 292.00 |
| Temperature of the inner fin surface \( T_{w} \) (K) | 245.83 | 259.31 | 266.45 | 269.3 |
| Temperature difference \( \Delta T = T_{a} - T_{w} \) (K) | 47.17 | 36.69 | 28.55 | 22.7 |
The temperature distribution along the fin (the temperature difference between the base and the tip of the fin) was analyzed and fin efficiency was calculated. Sample calculation results were shown for the assumed values of air temperature and the temperature of the inner tube wall listed in Table 1.

Distribution of temperatures were presented in Figure 7a; 7b. Results of calculations for natural convection conditions were presented in Figure 8a; 8b.

![Figure 7](image1.png)

**Figure 7.** Boundary conditions and temperature distribution on the surface of the fin: a) initial part; b) other parts of finned tube.

![Figure 8](image2.png)

**Figure 8.** Results of CFD calculations: a) temperature distribution along the height of the fin for \( \Delta T=47.17 \) K (top of the fin, \( \varphi=\pi/8 \)); b) temperature distribution along the height of the fin for \( \Delta T=36.69 \) K (top of the fin, \( \varphi=\pi/8 \)).

For forced convection, the studies were performed with transverse airflow and air velocity of \( w_a=2.3 \) m/s. Numerical modelling in this case was performed for the entire cross-section of the finned surface. The width of the cuboid assigned to the conventional air layer corresponded to the spacing between the investigated tubes. In this case, the following boundary conditions were assumed (Figure 9a, table. 1):

- constant temperature of the inner tube surface: \( T(A_1)=T_w \),
- constant air temperature at the inlet: \( T(A_2)=T_a \),
- condition of constant pressure at the air outlet \( p(A_3)=0 \),
- condition of velocity at the inlet \( w(A_2)=2.3 \) m/s,
- condition of symmetry for the surface \( A_4 \),
Sample distribution of temperatures and results of calculations for forced convection conditions were presented in Figures 9b and 9c.

**Figure 9.** CFD calculations, forced convection $w_p=2.3$ m/s: a) boundary conditions and temperature distribution for a finned tube; b) comparison of the measured and calculated temperature distribution for $\Delta T=23K$; c) temperature distribution for a single fin for $\Delta T=23 K$ ($\phi=3\pi/8$).

Results of measurements and simulation calculations made it possible to compare the mean values of fin efficiency calculated using equations (9) and (10).

\[
\eta_f = \frac{\tanh(mh)}{mh} \quad (9)
\]

\[
\eta_f = \frac{T_{\text{fin}} - T_a}{T_w - T_a} \quad (10)
\]

The temperature distributions obtained from CFD modelling and calculated analytically on the basis of measurements from equations (1,4,5) were used to determine fin efficiency using definition (10) and then compared with the values calculated using equation (9) (Figure 10).
3. Summary
The paper presents an analysis of temperature distribution on the outer surface of longitudinally finned tubes. The obtained results of the experimental studies were compared with the temperature distribution results obtained by numerical modelling of the heat exchange process using the finite volume method (Fluent). The discrepancies between the obtained values were small. Regardless of whether natural or forced convection was considered, the relative difference between the measured and calculated temperatures along the height of the fin was smaller than 2%. The temperature distribution was used to compare fin efficiency results, calculated using three different methods. The obtained results were also similar and the relative differences in the measured and calculated fin efficiency values did not exceed 4%. The consistency of the results of numerical analyses and experimental studies confirms the correctness of the model and its potential usefulness e.g. in the process of fin geometry optimization. Moreover, the paper provides the obtained local values of heat transfer coefficients for natural and forced convection processes for various flow rate and temperature ranges. The average values measured with this method were within the range of 4-7 W/(m²K) and 14-20 W/(m²K), accordingly, for natural and forced convection. The measurement results indicate that in the analyzed airflow velocity range, heat transfer coefficients are determined by both natural and forced convection processes.

Appendix

\[ A_{st} = \left(\frac{\pi d^2}{4} / n_{fin} \cdot s\right) L, \]  
\[ d \quad \text{diameter, (m)} \]
\[ \dot{G} \quad \text{refrigerant mass flux density, (kgm}^{-2}s^{-1}) \]
\[ h \quad \text{equivalent height of a straight fin with the surface area of the actual fin, } h = A_{fin} \left(\frac{n_{fin} \cdot L}{A_{fin}}\right), \]  
\[ L \quad \text{tube length, (m)} \]
\[ L_m \quad \text{measurement channel length, (m)} \]
\[ m \quad \text{parameter in equations (1) and (4) and (5)} \]
\[ n \quad \text{number of fins, (-)} \]
\[ RCJ \quad \text{degree of process openness [13], (-)} \]
\[ \dot{Q} \quad \text{heat flux, (W)} \]
\[ s \quad \text{fin thickness, (m)} \]
\[ S_d \quad \text{tube spacing, (m)} \]
\[ T \quad \text{temperature, (°C)} \]
$T_{\text{wall}}$ temperature of the outer tube wall, temperature at the fin base where the “i” sensor is installed

$w$ velocity, (ms$^{-1}$)

$V$ volumetric flow rate, (m$^3$s$^{-1}$)

**Symbols**

$\Delta$ increment,

$\alpha$ heat transfer coefficient, (Wm$^{-2}$K$^{-1}$)

$\varepsilon$ fin efficiency, (-)

$\phi$ measurement channel diameter, (m)

$\lambda$ thermal conduction coefficient, (Wm$^{-1}$K$^{-1}$)

$\varphi$ angle, (rad)

**Subscripts**

$a$ air

$f$ fin tip

$fin$ fin

$in$, $out$ inlet, outlet

$loc$ local value

$R$ refrigerant (cooling agent or water)

$w$ wall

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