Partially Massless Spin-2 Fields in String Generated Models

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Abstract

In cosmological backgrounds, there can be 'partially massless' higher spin fields which have fewer degrees of freedom than their massive partners. The equations for the partially massless spin-2 fields are usually taken to be the linearized Einstein equations augmented with a 'tuned' Pauli-Fierz mass. Here, we add more powers of curvatures and show that for the string-generated Einstein-Gauss-Bonnet model, partially massless spin-2 fields have real mass in AdS, in contrast to the Einstein level result. We discuss the implication of this for the AdS/CFT applications and briefly study the $C^4$-corrected $AdS_5 \times S^5$ solution in type IIB SUGRA.

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In Anti-de-Sitter and de-Sitter spacetimes, massive higher spin fields \((s \geq 2)\) exhibit a novel phenomenon: partial masslessness (or the appearance of extra gauge invariances), if the masses of the fields are ‘tuned’ to the background cosmological constant \([1, 2, 3]\). A massless spin-2 field (which we shall exclusively employ with here) in a flat \(D\)-dimensional background has \(D(D-3)/2\) degrees of freedom (DOF) and a massive one has \((D+1)(D-2)/2\) DOF. On the other hand, in a constant curvature background there is a third possibility, for which the spin-2 field has one fewer than the generic massive case: Hence the name partially massless. A second derivative, scalar gauge invariance appears at the partially massless point. For these fields, there is a crucial difference between dS and the AdS spaces: The tuned \(m^2\) is positive for the former and negative for the latter.

In the literature, the original analysis involves the equations of the linearized gravity augmented with the linear Pauli-Fierz mass term. In this note, we shall slightly generalize by considering also (the linearizations of) higher curvature terms that are generated in certain string models and study their effects on the partially massless fields.

The motivation for this work comes from the possible applications of partially massless spin-2 fields to AdS/CFT duality \([4, 5, 6]\). As is well-known, bulk fields in AdS correspond to operators in the boundary conformal field theory. According to the AdS/CFT dictionary, the masses of the bulk fields determine the conformal dimensions of the boundary fields. In particular, it was shown by Dolan \textit{et. al.} \([7]\), that partially massless spin-2 fields \((h_{\mu\nu})\) in the bulk of AdS space correspond, on the boundary, to fields \((L_{ij})\), which obey a specific conformally-invariant differential equation. If the boundary of the AdS is flat, the differential operator reduces to partial conservation equation of a tensor field (operator) : \(\partial_i \partial_j L^{ij} = 0\). As demonstrated in \([7]\), due to \(m^2\) being negative for partially massless fields in AdS, the first descendant of the corresponding partially conserved operator has a negative norm, rendering the boundary theory unphysical. Therefore, at the linear level there seems to be no room for the partially massless fields in AdS. For dS, on the other
hand, we do not have a proper dS/CFT correspondence to make use of these fields. What happens beyond the (already studied) quadratic Einstein level, in AdS, is our main aim in this work.

Here we suggest that, in certain cases (though, not in the most famous $AdS_5 \times S^5$ example), stringy higher curvature corrections flip the sign of $m^2$ in AdS. In particular, we will show that if the equations of motion for the massive spin-2 field in an AdS background are taken to be the linear version of the Einstein plus Gauss-Bonnet theory, the partially massless fields have positive mass.

We also study the (Weyl)$^4$ corrected spin-2 action in the maximally supersymmetric $AdS_5 \times S^5$ case, and show that partially massless fields are ruled out here, supporting the conclusion of [7]. [Strictly speaking, by imposing the Pauli-Fierz term, which does not arise in string theory, we are changing the background geometry.]

We first review the main ingredients of the partially massless spin-2 fields in a cosmological background, and then consider the spin-2 field equations derived from higher curvature theories. Finally, we study the $AdS_5 \times S^5$ background as a solution to Type IIB String/SUGRA equations with $R^4$ quantum corrections.

Our conventions are: signature $(-, +, +, \ldots +)$, $[\nabla_\mu, \nabla_\nu] V_\lambda = R_{\mu\nu\lambda} V_\sigma$, $R_{\mu\nu} \equiv R_{\mu\lambda\nu}^\lambda$.

We derive the equations of motions of a spin 2-field by linearizing the gravity part of the low energy string-generated gravity equations around the relevant background. Let us start with the lowest order Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 0,$$

linearized around $\bar{g}_{\mu\nu}$, for which the Ricci tensor is $\bar{R}_{\mu\nu} = \frac{2}{D-2} \Lambda \bar{g}_{\mu\nu}$. We obtain the following covariantly conserved equation for the massless spin-2 field $h_{\mu\nu} \equiv g_{\mu\nu} - \bar{g}_{\mu\nu}$

$$\mathcal{G}^L_{\mu\nu} \equiv R^L_{\mu\nu} - \frac{1}{2} \bar{g}_{\mu\nu} R^L - \frac{2}{D-2} \Lambda h_{\mu\nu} = 0,$$
where \( R^L = (g^\mu\nu R_{\mu\nu})_L \) and the linear part of the Ricci tensor is defined as

\[
R^L_{\mu\nu} \equiv R_{\mu\nu} - \bar{R}_{\mu\nu} = \frac{1}{2}(-\Box h_{\mu\nu} - \nabla_\mu \nabla_\nu h + \nabla^\sigma \nabla_\nu h_{\sigma\mu} + \nabla^\sigma \nabla_\mu h_{\sigma\nu}).
\] (3)

All derivatives are with respect to the background metric, which also raises and lowers the indices. Here: \( h = \bar{g}^{\mu\nu} h_{\mu\nu} \) and \( \Box = \bar{g}^{\mu\nu} \nabla_\mu \nabla_\nu \). To get a massive spin-2 field, we add the usual ghost-free Pauli-Fierz mass, with the correct relative sign:

\[
\mathcal{G}^L_{\mu\nu} + \frac{1}{2} m^2 (h_{\mu\nu} - \bar{g}_{\mu\nu} h) = 0
\] (4)

For \( m^2 \neq 0 \), taking the double divergence and the trace of this equation, one obtains, respectively:

\[
\nabla_\mu \nabla_\nu h_{\mu\nu} - \Box h = 0,
\]

(5)

\[
\left[ \Lambda - \frac{(D-1)}{2}m^2 \right] h = 0.
\]

(6)

Unlike the generic \((m^2, \Lambda)\) theory, \( h_{\mu\nu} \) does not have to be traceless \((i.e. h \neq 0)\) at the partially massless point for which the mass is tuned as

\[
m^2 = \frac{2}{D-1} \Lambda.
\]

(7)

One condition on the field drops out, yielding instead the following (higher derivative) scalar gauge invariance

\[
\delta h_{\mu\nu} = \nabla_\mu \nabla_\nu \xi + \frac{2}{(D-1)(D-2)} \bar{g}_{\mu\nu} \Lambda \xi
\]

(8)

Needless to say (7) assigns a negative \( m^2 \) for the partially massless fields in AdS. This by itself does not pose a threat since AdS allows negative \( m^2 \) for various fields, including massive spin-2 field as long as the corresponding Brietenlohner-Freeden [8] bounds are satisfied.

In [7], whose notations and results we follow, partially massless spin-2 field was employed in the context of AdS/CFT. The authors showed that such a field in the bulk of
AdS$_D$ corresponds to a symmetric tensor $L_{ij}$ on the boundary, which satisfies the following conformally invariant differential equation

$$\left[\nabla_i \nabla_j - \frac{1}{D-3} \tilde{R}_{ij}\right] L^{ij} = 0,$$  

(9)

where $\tilde{g}_{ij}$ is the conformal metric on the boundary. This differential equation was studied before by Eastwood and collaborators [9]. As in the simplest examples of AdS/CFT, let us assume that the boundary is conformally flat, then (9) reduces to the more transparent partial conservation law

$$\partial_i \partial_j L^{ij} = 0.$$  

(10)

Suppose now $|L^{ij}|$ is a symmetric primary state on which the $D-1$ dimensional conformal group acts. Let $P_i$ be the momenta; as a result of (10), we require $P_i P_j |L^{ij}|$ to be a null state. This then implies the existence of a negative norm state: $||P_i L^{ij}|^2 < 0$. In the language of the bulk fields, this negative norm state corresponds to $m^2$ being negative for the partially massless AdS fields (7).

At the linear level, the above result signals an impasse for the partially massless fields in the context of AdS/CFT. But here, I will argue that higher curvature corrections can drastically change the linear level results in certain theories. In fact, though rare, it is not unheard-of that higher derivative terms alter the sign of the linear level theory. To warm up, let us recall the non-trivial example of the Topologically Massive Gravity (TMG) in 2+1 dimensions [13]. This model is constructed by adding a third derivative Cotton tensor (or Chern-Simons term) to the Einstein one $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \frac{1}{4!} C_{\mu\nu} = 0$. Unlike pure 3D Einstein gravity, TMG is a dynamical theory: Linearized equations show that there is a massive scalar degree of freedom with mass $|M|$. For this scalar mode to be non-ghost with positive Hamiltonian, the sign of the Einstein action has to be flipped from its usual one once the higher derivative topological term is introduced [13]. Even though TMG is
quite distinct from the theories we are about to consider, it nevertheless provides us with a concrete example of how higher derivative terms can change linear level signs.

Let us now turn to Einstein-Gauss-Bonnet model [10], which is expected to arise in some string theories [or string theory compactifications].

$$I = \int d^D x \sqrt{-g} \left\{ \frac{R}{\kappa} + \gamma ( R_{\mu\nu\rho\sigma}^2 - 4 R_{\mu\nu}^2 + R^2 ) \right\}.$$  (11)

String theory dictates the sign of $\gamma$ to be positive. Even without an explicit cosmological constant, AdS [not dS!] is a solution [11] to the equations of motion derived from (11)

$$0 = \frac{1}{\kappa} ( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R ) + 2\gamma \left\{ R R_{\mu\nu} - 2 R_{\mu\sigma\rho\nu} R^{\sigma\rho} + R_{\mu\sigma\rho\tau} R_{\nu}^{\sigma\rho\tau} - 2 R_{\mu\sigma} R_{\nu}^{\sigma} - \frac{1}{4} g_{\mu\nu} ( R_{\gamma\lambda\rho\sigma}^2 - 4 R_{\sigma\rho}^2 + R^2 ) \right\}.$$  (12)

The cosmological constant of the AdS space that solves the equations is determined by $(\gamma, \kappa)$

$$\Lambda = - \frac{2(D - 1)(D - 2)}{\kappa \gamma (D - 3)(D - 4)}.$$  (13)

Linearizing (12) about AdS, one finds [12] how the presence of $\gamma$ modifies the massive spin-2 equation,

$$G^L_{\mu\nu} \left[ \frac{1}{\kappa} + \frac{4 \Lambda \gamma (D - 4)(D - 3)}{(D - 2)(D - 1)} \right] + \frac{1}{2 \kappa} m^2 ( h_{\mu\nu} - \bar{g}_{\mu\nu} h ) = 0.$$  (14)

subject to the condition (13). Thus one actually obtains the sign-flipped version of (4)

$$G^L_{\mu\nu} - \frac{1}{2} m^2 ( h_{\mu\nu} - \bar{g}_{\mu\nu} h ) = 0.$$  (15)

The crucial and non-trivial point here is that both (4) and (14) come from proper, non-ghost Lagrangians, despite ‘contradicting’ each other. It is, by now, clear that the partially massless states in AdS will be non-tachyonic for Einstein-Gauss-Bonnet theory. Namely for (15): $m^2 = -2\Lambda/(D - 1) > 0$. It also follows that if this model describes spin-2 fields in...
the bulk of $AdS_D$, for $D \geq 5$, partially massless fields correspond to conformally invariant boundary operators with positive descendants. Thus adding the Gauss-Bonnet term would circumvent the objection of [7] (at least beyond four dimensions).

On the other hand, if we include an explicit cosmological constant ($\Lambda_0$, whose sign is arbitrary), the above analysis branches into several directions. For given $(\Lambda_0, \kappa, \gamma)$ there are two different AdS spacetimes which solve the equations. [See the related work [14]]. The cosmological constant of these spaces are

$$\Lambda_\pm = \frac{-1 \pm \sqrt{1 + 8\kappa\tilde{\gamma}\Lambda_0}}{4\kappa\tilde{\gamma}}$$

(16)

where $\tilde{\gamma} = \gamma(D-4)(D-3)/[(D-2)(D-1)]$ and $8\kappa\Lambda_0\tilde{\gamma} \geq -1$. The equation (15) becomes

$$G^L_{\mu\nu} \pm \frac{1}{2} \frac{m^2}{\sqrt{1 + 8\kappa\Lambda_0\tilde{\gamma}}} (h_{\mu\nu} - \bar{g}_{\mu\nu} h) = 0$$

(17)

and the partially massless fields have the mass

$$m^2 = \pm \frac{2\Lambda_\pm}{D-1} \sqrt{1 + 8\kappa\Lambda_0\tilde{\gamma}}$$

(18)

One, now, has various choices, depending on the sign of $\Lambda_0$ and on the choice of $\Lambda_+$ or $\Lambda_-$. For $\Lambda_0 < 0$, we have $\Lambda_\pm < 0$, but only $\Lambda_-$ gives real mass to partially massless fields.

Having pointed out that certain higher curvature spin-2 theories have non-tachyonic masses for the partially massless fields, can we now make use of this result in AdS/CFT? Unfortunately, I do not know any examples of latter where there are Gauss-Bonnet corrections to bulk gravity. One would expect that less supersymmetric forms of AdS/CFT duality might allow Gauss-Bonnet terms [See [15] and the references therein.] But, for now, let us briefly study the well-understood maximally supersymmetric case. In low energy Type IIB SUGRA the next to leading order corrections (in the $\alpha'$ expansion) around the $AdS_5 \times S^5$ vacuum are of the $C^4$ type (here $C^4$ schematically represents the two inequivalent contractions of four Weyl tensors $C_{\mu\nu\alpha\beta}$ [16]. Switching off the dilaton and all the fields but spin-2, the action reads

$$I = \frac{1}{\alpha''} \int d^{10} \sqrt{-g} \left( R + k\alpha'^2 f_4(\rho, \bar{\rho}) C^4 \right)$$

(19)
where \( \rho \) is the usual complex coupling and \( f_4 \) is given by an Eisenstein series, whose details will not be relevant to us, but can be found in [16]. \( k \) is a constant. \( AdS_5 \times S^5 \) is a background solution. (Due to the flux from the self-dual five form \( F_5 \), a cosmological constant is generated.) \( AdS_5 \times S^5 \), with the same radius on \( AdS_5 \) and \( S^5 \) is conformally flat. This vacuum is expected to be a solution to full string theory: Thus the appearance of \( C^4 \), which vanishes for the conformally flat geometries, is no surprise. To see if the higher order terms can affect the partially massless fields, one can linearize the equations derived from (19) around \( AdS_5 \times S^5 \) and add a Pauli-Fierz mass term. But it is immediately clear that there will be no contributions from the \( C^4 \) terms since, at \( O(h^2) \) level one has \( \overline{C}^2 \delta(\sqrt{-g}C^2) \), which vanishes for all conformally flat spaces. Thus \( C^4 \) corrections cannot change the structure of the partially massless fields. With negative \( m^2 \), since the latter predict the existence of negative norm states in the boundary \( \mathcal{N} = 4 \) \( SU(N) \) Yang-Mills theory, they ought to be ruled out in the bulk. It is important to note that if the corrections were of the generic \( R^n \)-type, where \( R \) represents scalar, Ricci, or Riemann curvatures, the equations for the spin-2 field and its partially massless point would have changed drastically. On the other hand \( C^3 \), and \( C^2 \) corrections do not change the picture nor do arbitrary power of \( C \). Up to \( O(h^2) \), \( C^3 \) terms vanish just like the \( C^4 \) ones for AdS. Even though \( C^2 \) does not vanish at this order, it does not contribute to the trace of the equations, and thus does not affect the partially massless point.

To summarize, we have shown that, unlike the linear spin-2 theory, partially massless fields have positive \( m^2 \) in AdS, if Gauss-Bonnet corrections are added for \( D \geq 5 \). In the context of AdS/CFT, these partially massless fields in the bulk correspond to operators with positive norm descendants in the boundary.

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