Analytical solitons for the space-time conformable differential equations using two efficient techniques

Ahmad Neirameh\(^ 1 \, ^\#\) and Foroud Parvaneh\(^ 2 \)

\#Correspondence: a.neirameh@gonbad.ac.ir

\(^ 1 \)Department of Mathematics, Faculty of Sciences, Gonbad Kavous University, Gonbad, Iran

Full list of author information is available at the end of the article

Abstract

Exact solutions to nonlinear differential equations play an undeniable role in various branches of science. These solutions are often used as reliable tools in describing the various quantitative and qualitative features of nonlinear phenomena observed in many fields of mathematical physics and nonlinear sciences. In this paper, the generalized exponential rational function method and the extended sinh-Gordon equation expansion method are applied to obtain approximate analytical solutions to the space-time conformable coupled Cahn–Allen equation, the space-time conformable coupled Burgers equation, and the space-time conformable Fokas equation. Novel approximate exact solutions are obtained. The conformable derivative is considered to obtain the approximate analytical solutions under constraint conditions. Numerical simulations obtained by the proposed methods indicate that the approaches are very effective. Both techniques employed in this paper have the potential to be used in solving other models in mathematics and physics.

Keywords: Generalized exponential rational function method; Extended sinh-Gordon equation expansion method; Conformable derivative; Fokas equation; Burgers equation; Cahn–Allen equation

1 Introduction

Despite the recent extensive advances in the theory of differential equations, it can generally be said that it is still a complex task to determine an analytical solution for many ordinary and partial differential equations [1–9]. One of the events that led to the introduction of a wide range of new methods was the emergence and use of computers. So today it is almost impossible to use most of the existing techniques in solving differential equations, numerically or analytically, without the use of suitable computer software [10–19].

In recent years, the search for accurate solutions to differential equations has become a popular research topic. The natural result of this volume of attention has been the provision of efficient and powerful techniques. For example, the auxiliary equation method [20], the simplest equation method [21], the Hirota bilinear method [22], the homotopy analy-
sis method [23], the Jacobi elliptic method [24, 25], the complex transform [26], the bilinear form approach [27], the $G'/G$ expansion method [28], the $\exp(-\phi)$-expansion method [29], the generalized logistic equation method [30], the modified Kudryashov method, the extended tanh-coth method, the modified simple equation method and soliton ansatz method [31], the Hirota bilinear method [32], the modified form of an auxiliary equation approach [33]. Some more examples of differential equations and their applications can be followed in [34–52].

Khalil in [53] proposed an interesting definition of a derivative, namely the conformable derivative that generalizes the classical concept of derivative. This definition is well-behaved and obeys the Leibniz rule and the chain rule. Nonlinear conformable differential and integral equations have been the focus of many studies due to their applications in various applications in physics, biology, engineering, signal processing, control theory, finance, etc. [54–58]. More precisely, the extended Zakharov–Kuznetsov equation with conformable derivative using the generalized exponential rational function method was solved in [59]. In [60], a generalized type of conformable local fractal derivative (GCFD) was employed to investigate some nonlinear evolution equations. They also set up a general technique to find exact solutions for their under studied PDEs. In [61] the first integral method was employed to construct the solutions to the conformable Burgers equation, modified Burgers equation, and Burgers–Korteweg–de Vries equation. In [62], several wave solutions for Burgers’ type equations in the sense of conformable fractional derivative have been obtained via the residual power series method. Moreover, in [63] the auxiliary equation method has been employed to solve $(2 + 1)$-dimensional time-fractional Zoomeron equation and the time-fractional third order modified KdV equation. Abundant solitary wave solutions to an extended nonlinear Schrödinger’s equation with conformable derivative using an efficient integration method called the generalized exponential rational function method have been reported in [64]. Very recently, the conformable derivative and adequate fractional complex transform have been implemented to discuss the conformable higher-dimensional Ito equation [65].

In this paper, we apply both the generalized exponential rational function method and the extended sinh-Gordon equation expansion method for solving space-time conformable partial differential equations. Approximate analytical solutions for the coupled Cahn–Allen equation, coupled Burgers equation, and Fokas equation are obtained. Several exact solutions for them are successfully established. The solutions obtained by the methods indicate that they are easy to implement and effective. This article has been arranged as follows. In Sect. 2, we propose some mathematical definitions and prerequisites required later in the article. The section also illustrates general principles of the conformable derivative along with basic steps of techniques. In Sect. 3, three equations including the space-time conformable coupled Cahn–Allen equation, the space-time coupled Burgers equation, and the space-time conformable Fokas equation are examined, and the exact solution for them is determined using two techniques. This section also contains several numerical simulations of acquired solutions. Finally, the article ends with some conclusions.

2 Preliminaries and definitions
In this section, we review some of the necessary prerequisites that will be employed in the article.
2.1 The conformable derivative

Khalil proposed an interesting definition of derivative called conformable derivative [53]. This derivative can be considered to be a natural extension of the classical derivative. Furthermore, the conformable derivative satisfies all the properties of the standard calculus, for instance, the chain rule.

Definition 1 Let \( f : [0, \infty) \to \mathbb{R} \), the conformable derivative of a function \( f(t) \) of order \( \alpha \) is defined as [53]

\[
D_\alpha^\alpha f(t) = \lim_{\epsilon \to 0} \frac{f(t + \epsilon t^{1-\alpha}) - f(t)}{\epsilon}, \quad \alpha \in (0,1], \quad t > 0. \tag{1}
\]

It should be noted that taking \( \alpha = 1 \) in this derivative yields the standard definition for derivative. Therefore, this method can be considered a natural generalization for the conventional derivative.

This new definition satisfies the following properties. Let \( \alpha \in (0,1] \), \( f \) and \( g \) be \( \alpha \)-differentiable at a point \( t \), then

- \( D_\alpha^\alpha (af(t) + bg(t)) = aD_\alpha^\alpha (f(t)) + bD_\alpha^\alpha (g(t)) \) for \( a, b \in \mathbb{R} \).
- \( D_\alpha^\alpha (t^\mu) = \mu t^{\mu-\alpha} \) for \( \mu \in \mathbb{R} \).
- \( D_\alpha^\alpha (fg) = f(t)D_\alpha^\alpha (g(t)) + g(t)D_\alpha^\alpha (f(t)) \).
- \( D_\alpha^\alpha \left( \frac{f(t)}{g(t)} \right) = \frac{g(t)f'(t) - f(t)g'(t)}{g(t)^2} \).
- If \( f(t) \) is a differentiable function (in standard sense), then we obtain \( D_\alpha^\alpha (f(t)) = t^{1-\alpha} \frac{df(t)}{dt} \) holds.

As stated in [64], many of the existing definitions for derivative do not meet some of these mentioned properties. Enjoying these features is one of the valuable and distinctive points for the conformable derivative.

2.2 The generalized exponential rational function method

In 2018, an integration method called the generalized exponential rational function method (GERFM) was introduced by Ghanbari et al. to solve the resonance nonlinear Schrödinger equation [66]. Following their work, the technique has been used successfully many times to handle other partial equations [67–82]. In this part, we outline the main steps of GERFM as follows.

1. Let us take the following problem with the conformable derivative:

\[
\mathcal{L}(\psi, D_\alpha^\alpha \psi, D_\alpha^\alpha \phi, D_\alpha^\alpha x \psi, \ldots) = 0. \tag{2}
\]

2. Using the transformations \( \psi = \psi(\xi) \) and \( \xi = \sigma \frac{t^{1/\alpha}}{t^{1/\alpha}} - \xi \frac{t^{1/\alpha}}{t^{1/\alpha}} \), we reduce the nonlinear partial differential equation to the following ordinary differential equation:

\[
\mathcal{L}(\psi, \psi', \psi'', \ldots) = 0, \tag{3}
\]

where the values of \( \sigma \) and \( l \) will be found later.

3. Now, consider that Eq. (3) has the solution of the form

\[
\psi(\xi) = A_0 + \sum_{k=1}^{M} A_k \Psi(\xi)^k + \sum_{k=1}^{M} B_k \Psi(\xi)^{-k}, \tag{4}
\]
where

\[ \Psi(\xi) = \frac{p_1 e^{p_1 \xi} + p_2 e^{p_2 \xi}}{p_3 e^{p_3 \xi} + p_4 e^{p_4 \xi}}. \]  

(5)

The values of constants \( p_i, q_i \) (1 ≤ \( i \) ≤ 4), \( A_0, A_k \), and \( B_k \) (1 ≤ \( k \) ≤ \( M \)) are determined in such a way that solution (4) always persuades Eq. (3). By considering the homogenous balance principle, the value of \( M \) is determined.

4. Putting Eq. (4) into Eq. (3) and collecting all terms, the left-hand side of Eq. (3) gives us an algebraic equation \( P(Z_1, Z_2, Z_3, Z_4) = 0 \) in terms of \( Z_i = e^{q_i \xi} \) for \( i = 1, \ldots, 4 \).

By considering the homogenous balance principle, the value of \( M \) is determined.

5. By solving the above system of equations using any symbolic computation software, the values of \( p_i, q_i \) (1 ≤ \( i \) ≤ 4), \( A_0, A_k \), and \( B_k \) (1 ≤ \( k \) ≤ \( M \)) are determined, replacing these values in Eq. (4), we obtain the solutions of Eq. (2).

2.3 The extended sinh-Gordon equation expansion method

The extended sinh-Gordon equation expansion method (EShGEEM) is a robust method that may easily derive dark, bright, combined dark-bright, singular, combined singular soliton, and other trigonometric function solutions to nonlinear PDEs of an integer or noninteger order [83]. This technique has had many successful applications in solving various problems. For example, the authors of [84] used EShGEEM to study the conformable version of Biswas–Milovic equation with the Kerr law and parabolic law nonlinearity. Another application of EShGEEM can be found in [85], where they considered a nonlinear partial differential equation describing the wave propagation in nonlinear low-pass electrical transmission lines.

Following the works of [84, 85], we outline the main steps of EShGEEM as follows.

1. Let us take the following problem with the conformable derivative:

\[ L(\psi, D_x^\alpha \psi, D_t^\alpha \phi, D_x^{2\alpha} \psi, \ldots) = 0. \]  

(6)

Using the transformations \( \Psi = \Psi(\xi) \) and \( \xi = \sigma \tau^{\alpha} - l \tau^{\alpha/2} \), it is possible to reduce the NPDE to the following ordinary differential equation:

\[ L(\Psi, \Psi', \Psi'' \ldots) = 0, \]  

(7)

where the values of \( \sigma \) and \( l \) will be found later, and the prime notation means the derivative of \( \Psi \) with respect to \( \xi \).

2. Consider Eq. (7) has the solution of the form

\[ \Psi(\theta) = A_0 + \sum_{j=1}^{M} \cosh^{j-1}(\theta)[B_j \sinh(\theta) + A_j \cosh(\theta)], \]  

(8)

where \( A_0, A_j, B_j \) (\( j = 1, 2, \ldots, M \)) are constants to be determined later and \( \theta \) is a function of \( \xi \) that satisfies the following ordinary differential equation:

\[ \theta' = \sinh(\theta). \]  

(9)
By considering the homogenous balance principle in (7), the value of $M$ can be determined.

Equation (9) possesses the following solutions:

$$\sinh(\theta) = \pm \text{csch}(\xi), \quad \text{or} \quad \sinh(\theta) = \pm i \text{sech}(\xi)$$

(10)

and

$$\cosh(\theta) = - \coth(\xi), \quad \text{or} \quad \cosh(\theta) = - \tanh(\xi),$$

(11)

where $i = \sqrt{-1}$.

3. Substituting Eq. (8) along with Eqs. (10) and (11) into Eq. (7) and collecting all terms, we obtain a polynomial in terms of $\theta^l \sinh(\theta) \cosh(\theta)$ for $l = 0, 1, i, j = 0, 1, 2, \ldots$. Setting each coefficient of such a polynomial equal to zero, a system of nonlinear equations in terms of $\sigma, l, A_0, A_j, B_j (1 \leq k \leq M)$ is generated.

4. Solving the above algebraic equations using any symbolic computation software, the values of $\sigma, l$ and $A_0, A_j, B_j (1 \leq k \leq M)$ are determined.

5. Based on Eqs. (10) and (11), one can obtain the soliton solutions of Eq. (6) as follows:

$$\Psi(\xi) = A_0 + \sum_{j=1}^{M} (-\tanh(\xi))^{l-1} \left[ \pm iB_j \text{sech}(\xi) - A_j \tanh(\xi) \right],$$

(12)

$$\Psi(\xi) = A_0 + \sum_{j=1}^{M} (-\coth(\xi))^{l-1} \left[ \pm B_j \text{csch}(\xi) - A_j \coth(\xi) \right].$$

(13)

3 Applications of techniques and the main results

In this section, to illustrate the applicability of the generalized exponential rational function method and the extended sinh-Gordon equation expansion method to solve nonlinear conformable partial differential equations, three examples are considered.

3.1 The space-time conformable coupled Cahn–Allen equation

Consider the space-time conformable Cahn–Allen equation [86]

$$D_\alpha u - u_{xx} + u^3 - u = 0.$$ (14)

Using the transformation

$$u(x, t) = U(\xi), \quad \xi = c \left( x - \frac{vt}{\Gamma(\alpha)} \right),$$

(15)

where $c$ and $v$ are two nonzero constants.

Utilizing the wave transformation (15) converts Eq. (14) into the following NODE:

$$-cvU' - \xi^2 U'' - U + U^3 = 0.$$ (16)

Using the balance principle on the terms $U^3$ and $U''$ in Eq. (16), we have $M + 2 = 3M$, so $M = 1$. 
Application of GERFM for (14)

Using Eq. (5) together with $M = 1$, we have

$$U(\xi) = A_0 + A_1 \Psi(\xi) + \frac{B_1}{\Psi(\xi)},$$  \hspace{1cm} (17)

Proceeding as outlined in the second section, we acquire the following sets of solutions to Eq. (14).

Set 1: One obtains $r = [-1, 0, 1, 1]$ and $s = [1, 0, 1, 0]$, so Eq. (5) turns into

$$\Psi(\xi) = -\frac{1}{1 + e^\xi}. \hspace{1cm} (18)$$

Case 1: We obtain

$$c = \frac{\sqrt{2}}{2}, \quad v = -\frac{3\sqrt{2}}{2}, \quad A_0 = 0, \quad A_1 = -1, \quad B_1 = 0.$$

Putting values in Eqs. (17) and (18) yields the following solution:

$$U(\xi) = \frac{1}{1 + e^\xi}.$$  \hspace{1cm} (19)

Consequently, we get the solution of Eq. (14) as

$$u_1(x, t) = \frac{1}{1 + e^{\frac{\sqrt{2}}{2}(\sqrt{2} + \frac{3\sqrt{2}}{2}t)}},$$

Figure 1 depicts the dynamic behavior of solution $u_1(x, t)$ presented in (19).

Case 2: We obtain

$$c = \frac{\sqrt{2}}{2}, \quad v = -\frac{3\sqrt{2}}{2}, \quad A_0 = 1, \quad A_1 = 1, \quad B_1 = 0.$$

Putting values in Eqs. (17) and (18) yields the following solution:

$$U(\xi) = \frac{e^\xi}{1 + e^\xi}.$$
Consequently, we get the solution of Eq. (14) as
\[
 u_2(x, t) = \frac{e^{\sqrt{2}2(x + \frac{3\sqrt{2}}{2\Gamma(\alpha)})}}{1 + e^{\sqrt{2}2(x + \frac{3\sqrt{2}}{2\Gamma(\alpha)})}}. \tag{20}
\]
Figure 2 depicts the dynamic behavior of solution \( u_2(x, t) \) presented in (19).

Set 2: One obtains \( r = [-3, -1, 1, 1] \) and \( s = [1, -1, -1, 1] \), so Eq. (5) turns into
\[
 \Psi(\xi) = -2 \cosh(\xi) - \sinh(\xi) \cosh(\xi). \tag{21}
\]
We obtain
\[
 c = \frac{\sqrt{2}}{4}, \quad \nu = -\frac{3\sqrt{2}}{2}, \quad A_0 = -\frac{3}{2}, \quad A_1 = 0, \quad B_1 = -\frac{3}{2}.
\]
Putting values in Eqs. (17) and (21) yields the following solution:
\[
 U(\xi) = \frac{-3 \cosh(\xi) - 3 \sinh(\xi)}{4 \cosh(\xi) + 2 \sinh(\xi)}.
\]
Consequently, we get the solution of Eq. (14) as
\[
 u_3(x, t) = \frac{3 \cosh(\sqrt{2}2(x + \frac{3\sqrt{2}}{2\Gamma(\alpha)}) ) + 3 \sinh(\sqrt{2}2(x + \frac{3\sqrt{2}}{2\Gamma(\alpha)}) )}{4 \cosh(\sqrt{2}2(x + \frac{3\sqrt{2}}{2\Gamma(\alpha)}) ) + 2 \sinh(\sqrt{2}2(x + \frac{3\sqrt{2}}{2\Gamma(\alpha)}) )}. \tag{22}
\]
Figure 3 depicts the dynamic behavior of solution \( u_3(x, t) \) presented in (22).

Set 3: One obtains \( r = [1 - i, 1 + i, 1, 1] \) and \( s = [-i, -i, i, i] \), so Eq. (5) turns into
\[
 \Psi(\xi) = -\sin(\xi) + \cos(\xi) \cos(\xi). \tag{23}
\]
We obtain
\[
 c = \frac{\sqrt{2}}{4}, \quad \nu = \frac{3\sqrt{2}}{2}, \quad A_0 = -\frac{1}{2} - \frac{i}{2}, \quad A_1 = \frac{i}{2}, \quad B_1 = 0.
\]
Putting values in Eqs. (17) and (23) yields the following solution:

$$U(\xi) = \frac{-\cosh(\xi) + \sinh(\xi)}{2 \cosh(\xi)}.$$  

Consequently, we get the solution of Eq. (14) as

$$u_4(x, t) = -\frac{\cosh\left(\sqrt{2} x - \frac{3\sqrt{2} \alpha}{2\Gamma_1(\alpha)} t\right) - \sinh\left(\sqrt{2} x - \frac{3\sqrt{2} \alpha}{2\Gamma_1(\alpha)} t\right)}{2 \cosh\left(\sqrt{2} (x - \frac{3\sqrt{2} \alpha}{2\Gamma_1(\alpha)} t)\right)}. \quad (24)$$

Set 4: One obtains \( r = [1, 1, 1, 1] \) and \( s = [1, -1, 1, -1] \), so Eq. (5) turns to

$$\Psi(\xi) = -\frac{\cosh(\xi)}{\sinh(\xi)}. \quad (25)$$

We obtain

$$c = \frac{\sqrt{2}}{8}, \quad v = \frac{3\sqrt{2}}{2}, \quad A_0 = -\frac{1}{2}, \quad A_1 = -\frac{1}{4}, \quad B_1 = -\frac{1}{4}.$$  

Putting values in Eqs. (17) and (25) yields the following solution:

$$U(\xi) = \frac{(\coth(\xi) - 1)^2}{4\coth(\xi)}.$$  

Consequently, we get the solution of Eq. (14) as

$$u_5(x, t) = \frac{(\coth\left(\sqrt{2} x - \frac{3\sqrt{2} \alpha}{2\Gamma_1(\alpha)} t\right) - 1)^2}{4\coth\left(\sqrt{2} (x - \frac{3\sqrt{2} \alpha}{2\Gamma_1(\alpha)} t)\right)}. \quad (26)$$

Figure 4 depicts the dynamic behavior of solution \( u_5(x, t) \) presented in (26).

Set 5: One obtains \( r = [3, 2, 1, 1] \) and \( s = [1, 0, 1, 0] \), so Eq. (5) turns into

$$\Psi(\xi) = \frac{3e^\xi + 2}{e^\xi + 1}. \quad (27)$$
We obtain
\[ c = \frac{\sqrt{2}}{2}, \quad \nu = -3\sqrt{2}, \quad A_0 = -3, \quad A_1 = 0, \quad B_1 = 6. \]

Putting values in Eqs. (17) and (27) yields the following solution:
\[ U(\xi) = -\frac{3e^\xi}{3e^\xi + 2}. \]

Consequently, we get the solution of Eq. (14) as
\[ u_6(x, t) = -\frac{3e^{\sqrt{2}(x + \frac{3\sqrt{2}}{2\alpha}t)}}{3e^{\sqrt{2}(x + \frac{3\sqrt{2}}{2\alpha}t)} + 2}. \] (28)

**Application of EShGEEM for (14)**

According to what was discussed above, we obtain \( M = 1 \). Taking \( M = 1 \) into account in Eqs. (8), (12), and (13), we respectively obtain
\[ U(\theta) = A_0 + B_1 \sinh(\theta) + A_1 \cosh(\theta) \] (29)

and
\begin{align*}
U_1(\xi) &= A_0 \pm iB_1 \text{sech}(\xi) - A_1 \tanh(\xi), \\
U_2(\xi) &= A_0 \pm B_1 \text{csch}(\xi) - A_1 \coth(\xi).
\end{align*} (30)

Inserting Eq. (29) into Eq. (16) gives a polynomial in powers of hyperbolic functions. Summing each coefficient of the hyperbolic functions of the same power and equating each summation to zero, we get a group of over-determined nonlinear algebraic equations. For each set, if we substitute the values of the parameters into any of Eqs. (30), the solutions to Eq. (14) are constructed as follows.

**Set 1:**
\[ c = 1/4\sqrt{2}, \quad \nu = -3/2\sqrt{2}, \quad A_0 = -1/2, \quad A_1 = 1/2, \quad B_1 = 0. \]
Using these values, the following solution for (16) is obtained:

\[ U_1(\xi) = \frac{\cosh(\xi) + \sinh(\xi)}{2 \cosh(\xi)}, \]
\[ U_2(\xi) = \frac{-\cosh(\xi) + \sinh(\xi)}{2 \sinh(\xi)}. \]  

(31)

Consequently, we get the solution of Eq. (14) as

\[ u_7(x,t) = \cosh\left(\sqrt{2}x/\Gamma(\alpha)/\Gamma(1/\alpha) + 3t/\alpha\right) - \frac{\sinh\left(\sqrt{2}x/\Gamma(\alpha)/\Gamma(1/\alpha) + 3t/\alpha\right)}{2 \cosh\left(\sqrt{2}x/\Gamma(\alpha)/\Gamma(1/\alpha) + 3t/\alpha\right)}, \]
\[ u_8(x,t) = \cosh\left(\sqrt{2}x/\Gamma(\alpha)/\Gamma(1/\alpha) + 3t/\alpha\right) - \frac{\sinh\left(\sqrt{2}x/\Gamma(\alpha)/\Gamma(1/\alpha) + 3t/\alpha\right)}{2 \sinh\left(\sqrt{2}x/\Gamma(\alpha)/\Gamma(1/\alpha) + 3t/\alpha\right)}. \]  

(32)

Set 2:

\[ c = 1/4\sqrt{2}, \quad \nu = 3/2\sqrt{2}, \quad A_0 = 1/2, \quad A_1 = 1/2, \quad B_1 = 0. \]

Using these values, the following solution for (16) is obtained:

\[ U_1(\xi) = \frac{\cosh(\xi) + \sinh(\xi)}{2 \cosh(\xi)}, \]
\[ U_2(\xi) = \frac{-\cosh(\xi) + \sinh(\xi)}{2 \sinh(\xi)}. \]  

(33)

Consequently, we get the solution of Eq. (14) as

\[ u_9(x,t) = \cosh\left(\sqrt{2}x/\Gamma(\alpha)/\Gamma(1/\alpha) + 3t/\alpha\right) - \frac{\sinh\left(\sqrt{2}x/\Gamma(\alpha)/\Gamma(1/\alpha) + 3t/\alpha\right)}{2 \cosh\left(\sqrt{2}x/\Gamma(\alpha)/\Gamma(1/\alpha) + 3t/\alpha\right)}, \]
\[ u_{10}(x,t) = -\cosh\left(\sqrt{2}x/\Gamma(\alpha)/\Gamma(1/\alpha) + 3t/\alpha\right) - \frac{\sinh\left(\sqrt{2}x/\Gamma(\alpha)/\Gamma(1/\alpha) + 3t/\alpha\right)}{2 \sinh\left(\sqrt{2}x/\Gamma(\alpha)/\Gamma(1/\alpha) + 3t/\alpha\right)}. \]  

(34)

Figure 5 depicts the dynamic behavior of solution \( u_{10}(x,t) \) presented in (34).

Set 3:

\[ c = 1/2\sqrt{2}, \quad \nu = 3/2\sqrt{2}, \quad A_0 = 1/2, \quad A_1 = 1/2, \quad B_1 = 1/2. \]

Using these values, the following solution for (16) is obtained:

\[ U_1(\xi) = \frac{i + \cosh(\xi) - \sinh(\xi)}{2 \cosh(\xi)}, \]
\[ U_2(\xi) = \frac{-\cosh(\xi) + \sinh(\xi) + 1}{2 \sinh(\xi)}. \]  

(35)
Consequently, we get the solution of Eq. (14) as

\[
\begin{align*}
\dot{u}_{11}(x, t) &= i + \cosh\left(\frac{\sqrt{2}x \Gamma(\alpha)-3 \mu}{2 \Gamma(\alpha)}\right) - \sinh\left(\frac{\sqrt{2}x \Gamma(\alpha)-3 \mu}{2 \Gamma(\alpha)}\right), \\
\dot{u}_{12}(x, t) &= -\cosh\left(\frac{\sqrt{2}x \Gamma(\alpha)-3 \mu}{2 \Gamma(\alpha)}\right) + \sinh\left(\frac{\sqrt{2}x \Gamma(\alpha)-3 \mu}{2 \Gamma(\alpha)}\right) + 1.
\end{align*}
\]

(36)

Figure 6 depicts the dynamic behavior of solution \( u_5(x, t) \) presented in (36).

3.2 The space-time coupled Burgers equation

Consider the space-time conformable coupled Burgers equations [87]

\[
\begin{align*}
D_t^\alpha u - D_x^{2\alpha} u + 2u D_x^\alpha u + pD_x^\alpha (uv) &= 0, \\
D_t^\alpha v - D_x^{2\alpha} v + 2v D_x^\alpha v + qD_x^\alpha (uv) &= 0.
\end{align*}
\]

(37)
Using the transformation

\[ u(x, t) = U(\xi), \quad v(x, t) = V(\xi), \quad \xi = \frac{x^\alpha}{\Gamma(\alpha)} + \frac{ct^\alpha}{\Gamma(\alpha)}, \tag{38} \]

where \( c \) is a nonzero constant.

Utilizing the wave transformation (38) converts Eq. (37) into the following NODE:

\[
\begin{align*}
cl' - Ul'' + 2Ul' + p(UV)' &= 0, \\
cV' - V'' + 2VV' + q(UV)' &= 0.
\end{align*}
\tag{39}
\]

Using the balance principle on the terms \( Ul' \) and \( Ul'' \) in Eq. (39), we have \( M + 2 = M + M + 1 \), so \( M = 1 \).

**Application of GERFM for (37)**

Using Eq. (5) together with \( M = 1 \), we have

\[
\begin{align*}
U(\xi) &= A_0 + A_1 \Psi(\xi) + \frac{B_1}{\Psi'(\xi)}, \\
V(\xi) &= A'_0 + A'_1 \Psi(\xi) + \frac{B'_1}{\Psi'(\xi)}. \tag{40}
\end{align*}
\]

Proceeding as outlined in the second section, we acquire the following sets of solutions to Eq. (37).

**Set 1**: One obtains \( r = [1, 1, -1, 1] \) and \( s = [1, -1, 1, -1] \), so Eq. (5) turns into

\[
\Psi(\xi) = -\frac{\cosh(\xi)}{\sinh(\xi)}. \tag{41}
\]

We obtain

\[
\begin{align*}
c &= -\frac{2A_0(pq - 1)}{p - 1}, \\
A_0 &= A_0, \quad A_1 = \frac{p - 1}{pq - 1}, \quad B_1 = B_1, \\
A'_0 &= \frac{A_0(q - 1)}{p - 1}, \quad A'_1 = \frac{q - 1}{pq - 1}, \quad B'_1 = B'_1.
\end{align*}
\]

Putting values in Eqs. (40) and (41) yields the following solution:

\[
\begin{align*}
U(\xi) &= \frac{pqA_0 - \coth(\xi)p + \coth(\xi) - A_0}{pq - 1}, \\
V(\xi) &= \frac{pqA_0 - \coth(\xi)q + \coth(\xi) - A_0}{pq - 1}. \tag{42}
\end{align*}
\]

Consequently, we get the solution of Eq. (37) as

\[
\begin{align*}
u_1(x, t) &= \frac{pqA_0 - \coth(\xi)p + \coth(\xi) - A_0}{pq - 1}, \\
v_1(x, t) &= \frac{pqA_0 - \coth(\xi)q + \coth(\xi) - A_0}{pq - 1}. \tag{43}
\end{align*}
\]
Figure 7 depicts the dynamic behavior of solution $u_1(x, t)$, $v_1(x, t)$ presented in (43).

Set 2: One obtains $r = [-2 - i, -2 + i, 1, 1]$ and $s = [i, -i, i, -i]$, so Eq. (5) turns into

$$
\Psi(\xi) = -2 \cos(\xi) + \sin(\xi) \cos(\xi).
$$

(44)

We obtain

$$
c = \frac{-2pA_0q + 4p + 2A_0 - 4}{p - 1}, \quad A_0 = A_0, \quad A_1 = \frac{p - 1}{pq - 1}, \quad B_1 = B_1,
$$

$$
A'_0 = \frac{A_0(q - 1)}{p - 1}, \quad A'_1 = \frac{q - 1}{pq - 1}, \quad B'_1 = B'_1.
$$

Putting values in Eqs. (40) and (44) yields the following solution:

$$
U(\xi) = \frac{(pqA_0 - 2p - A_0 + 2) \cos(\xi) + \sin(\xi)(p - 1)}{(pq - 1)\cos(\xi)},
$$

$$
V(\xi) = \frac{(pqA_0 - 2q - A_0 + 2) \cos(\xi) + \sin(\xi)(q - 1)}{(pq - 1)\cos(\xi)}.
$$

(45)

Consequently, we get the solution of Eq. (37) as

$$
u_2(x, t) = \frac{(pqA_0 - 2q - A_0 + 2) \cos(\xi) + \sin(\xi)(q - 1)}{(pq - 1)\cos(\xi)}.
$$

(46)

Figure 8 depicts the dynamic behavior of solution $u_2(x, t)$, $v_2(x, t)$ presented in (46).

Set 3: One obtains $r = [1, 0, 1, 1]$ and $s = [1, 0, 1, 0]$, so Eq. (5) turns into

$$
\Psi(\xi) = \frac{e^\xi}{1 + e^\xi}.
$$

(47)
We obtain

\[ c = \frac{-2pA_0q + q + 2A_0' - 1}{q - 1}, \quad A_0 = \frac{(p - 1)A_0'}{q - 1}, \quad A_1 = \frac{-p + 1}{pq - 1}, \quad B_1 = B_1, \]

\[ A_0' = A_0', \quad A_1' = \frac{-q + 1}{pq - 1}, \quad B_1' = B_1'. \]

Putting values in Eqs. (40) and (47) yields the following solution:

\[ U(\xi) = \frac{(p - 1)((pqA_0' - q - A_0' + 1)e^\xi + A_0'(pq - 1))}{(q - 1)(pq - 1)(1 + e^\xi)}, \]

\[ V(\xi) = \frac{(q - 1)((pqA_0' - p - A_0' + 1)e^\xi + A_0'(pq - 1))}{(p - 1)(pq - 1)(1 + e^\xi)}. \]

Consequently, we get the solution of Eq. (37) as

\[ u_3(x,t) = \frac{(p - 1)((pqA_0' - q - A_0' + 1)e^\xi + A_0'(pq - 1))}{(q - 1)(pq - 1)(1 + e^\xi)}, \]

\[ v_3(x,t) = \frac{(q - 1)((pqA_0' - p - A_0' + 1)e^\xi + A_0'(pq - 1))}{(p - 1)(pq - 1)(1 + e^\xi)}. \]

Figure 9 depicts the dynamic behavior of solution \(u_3(x,t), v_3(x,t)\) presented in (49).

**Application of EShGEEM for (37)**

The initial assumption of the solution structure of (39) is taken to be:

\[ U(\theta) = A_0 + B_1 \sinh(\theta) + A_1 \cosh(\theta), \]

\[ V(\theta) = A_0' + B_1' \sinh(\theta) + A_1' \cosh(\theta), \]

\[ U_1(\xi) = A_0 \pm iB_1 \operatorname{sech}(\xi) - A_1 \tanh(\xi), \]

\[ V_1(\xi) = A_0' \pm iB_1' \operatorname{sech}(\xi) - A_1' \tanh(\xi), \]
and

\[ U_2(\xi) = A_0 \pm B_1 \text{csch}(\xi) - A_1 \text{coth}(\xi), \]
\[ V_2(\xi) = A'_0 \pm B'_1 \text{csch}(\xi) - A'_1 \text{coth}(\xi). \]

Applying the extended EShGEEM with the help of Eqs. (50)–(52), the following new exact soliton solutions of the space-time conformable coupled Burgers equations (37) are obtained.

Set 1:

\[ c = -\frac{2A_0(pq - 1)}{p - 1}, \quad A_0 = A_0, \quad A_1 = \frac{p - 1}{2pq - 2}, \quad B_1 = \frac{p - 1}{2pq - 2}, \]
\[ A'_0 = \frac{A_0(q - 1)}{p - 1}, \quad A'_1 = \frac{q - 1}{2pq - 2}, \quad B'_1 = \frac{q - 1}{2pq - 2}. \]

Using these values, the following solution for (16) is obtained:

\[ U_1(\xi) = \frac{(2pqA_0 - 2A_0) \cosh(\xi) + (i - \sinh(\xi))(p - 1)}{2(pq - 1) \cosh(\xi)}, \]
\[ V_1(\xi) = \frac{(q - 1)(2pqA_0 - 2A_0) \cosh(\xi) + (i - \sinh(\xi))(p - 1)}{2(pq - 1)(p - 1) \cosh(\xi)}, \]

and

\[ U_2(\xi) = \frac{(2pqA_0 - 2A_0) \sinh(\xi) - (\cosh(\xi) - 1)(p - 1)}{2(pq - 1) \sinh(\xi)}, \]
\[ V_2(\xi) = \frac{(q - 1)(A_0(pq - 1) \sinh(\xi) - 1/2(\cosh(\xi) - 1)(p - 1))}{(pq - 1)(p - 1) \sinh(\xi)}. \]

Consequently, we get the solution of Eq. (14) as

\[ u_4(x, t) = \frac{(2pqA_0 - 2A_0) \cosh(\xi) + (i - \sinh(\xi))(p - 1)}{2(pq - 1) \cosh(\xi)}, \]
\[ v_4(x, t) = \frac{(q - 1)(2pqA_0 - 2A_0) \cosh(\xi) + (i - \sinh(\xi))(p - 1))}{2(pq - 1)(p - 1) \cosh(\xi)}. \]
and
\[
\begin{align*}
    u_5(x,t) &= \frac{(2pqA_0 - 2A_0) \sinh(\xi) - (\cosh(\xi) - 1)(p - 1)}{2(pq - 1) \sinh(\xi)}, \\
    v_5(x,t) &= \frac{(q - 1)(A_0(pq - 1) \sinh(\xi) - 1/2(\cosh(\xi) - 1)(p - 1))}{(pq - 1)(p - 1) \sinh(\xi)}.
\end{align*}
\]

where \( \xi = \frac{1}{\Gamma(\alpha)} (x^\alpha + \frac{2A_0(pq - 1)}{p-1} t^\alpha) \).

Figure 10 depicts the dynamic behavior of solution \( u_5(x,t), v_5(x,t) \) presented in (56).

Set 2:
\[
\begin{align*}
    c &= \frac{-2A_0(pq - 1)}{p-1}, & A_0 &= A_0, & A_1 &= \frac{p-1}{pq-1}, & B_1 &= 0, \\
    A_0' &= \frac{A_0(q - 1)}{p-1}, & A_1' &= \frac{q - 1}{pq - 1}, & B_1' &= 0.
\end{align*}
\]

Using these values, the following solution for (16) is obtained:
\[
\begin{align*}
    U_1(\xi) &= \frac{A_0(pq - 1) \cosh(\xi) - (p - 1) \sinh(\xi)}{(pq - 1) \cosh(\xi)}, \\
    V_1(\xi) &= \frac{(A_0(pq - 1) \cosh(\xi) - (p - 1) \sinh(\xi))(q - 1)}{(pq - 1)(p - 1) \cosh(\xi)},
\end{align*}
\]

and
\[
\begin{align*}
    U_2(\xi) &= \frac{A_0(pq - 1) \sinh(\xi) - \cosh(\xi)(p - 1)}{(pq - 1) \sinh(\xi)}, \\
    V_2(\xi) &= \frac{(q - 1)(A_0(pq - 1) \sinh(\xi) - \cosh(\xi)(p - 1))}{(pq - 1)(p - 1) \sinh(\xi)}.
\end{align*}
\]

Consequently, we get the solution of Eq. (14) as
\[
\begin{align*}
    u_6(x,t) &= \frac{A_0(pq - 1) \cosh(\xi) - (p - 1) \sinh(\xi)}{(pq - 1) \cosh(\xi)}, \\
    v_6(x,t) &= \frac{(A_0(pq - 1) \cosh(\xi) - (p - 1) \sinh(\xi))(q - 1)}{(pq - 1)(p - 1) \cosh(\xi)}.
\end{align*}
\]
and

\[
\begin{align*}
  u_7(x, t) &= \frac{A_0(pq - 1) \sinh(\xi) - \cosh(\xi)(p - 1)}{(pq - 1) \sinh(\xi)}, \\
  v_7(x, t) &= \frac{(q - 1)(A_0(pq - 1) \sinh(\xi) - \cosh(\xi)(p - 1))}{(pq - 1)(p - 1) \sinh(\xi)},
\end{align*}
\]

where \( \xi = \frac{1}{\Gamma_{(\alpha)}}(x^\alpha + \frac{2A_0(pq - 1)}{p - 1} t^\alpha). \)

Set 3:

\[
\begin{align*}
  c &= -\frac{2A_0(pq - 1)}{p - 1}, \quad A_0 = A_0, \quad A_1 = \frac{p - 1}{2pq - 2}, \quad B_1 = \frac{-p + 1}{2pq - 2}, \\
  A'_0 &= \frac{A_0(q - 1)}{p - 1}, \quad A'_1 = \frac{q - 1}{2pq - 2}, \quad B'_1 = \frac{-q + 1}{2pq - 2}.
\end{align*}
\]

Using these values, the following solution for (16) is obtained:

\[
\begin{align*}
  U_1(\xi) &= \frac{(2pqA_0 - 2A_0) \cosh(\xi) - (i + \sinh(\xi))(p - 1)}{2(pq - 1) \cosh(\xi)}, \\
  V_1(\xi) &= \frac{(q - 1)((-2pqA_0 + 2A_0) \cosh(\xi) + (i + \sinh(\xi))(p - 1))}{2(pq - 1)(p - 1) \cosh(\xi)},
\end{align*}
\]

and

\[
\begin{align*}
  U_2(\xi) &= \frac{(2pqA_0 - 2A_0) \sinh(\xi) - (\cosh(\xi) + 1)(p - 1)}{2(pq - 1) \sinh(\xi)}, \\
  V_2(\xi) &= \frac{(A_0(pq - 1) \sinh(\xi) - 1/2(\cosh(\xi) + 1)(p - 1)(q - 1))}{(pq - 1)(p - 1) \sinh(\xi)}.
\end{align*}
\]

Consequently, we get the solution of Eq. (14) as

\[
\begin{align*}
  u_8(x, t) &= \frac{(2pqA_0 - 2A_0) \cosh(\xi) - (i + \sinh(\xi))(p - 1)}{2(pq - 1) \cosh(\xi)}, \\
  v_8(x, t) &= \frac{(q - 1)((-2pqA_0 + 2A_0) \cosh(\xi) + (i + \sinh(\xi))(p - 1))}{2(pq - 1)(p - 1) \cosh(\xi)},
\end{align*}
\]

and

\[
\begin{align*}
  u_9(x, t) &= \frac{(2pqA_0 - 2A_0) \sinh(\xi) - (\cosh(\xi) + 1)(p - 1)}{2(pq - 1) \sinh(\xi)}, \\
  v_9(x, t) &= \frac{(A_0(pq - 1) \sinh(\xi) - 1/2(\cosh(\xi) + 1)(p - 1)(q - 1))}{(pq - 1)(p - 1) \sinh(\xi)},
\end{align*}
\]

where \( \xi = \frac{1}{\Gamma_{(\alpha)}}(x^\alpha + \frac{2A_0(pq - 1)}{p - 1} t^\alpha). \)

Figure 11 depicts the dynamic behavior of solution \( u_9(x, t), v_9(x, t) \) presented in (64).
3.3 The space-time conformable Fokas equation

Consider the space-time conformable Fokas equation \[ 88 \]

\[
\begin{align*}
4 \frac{\partial^{2\alpha} u}{\partial t^{\alpha} \partial x_1^2} &- \frac{\partial^{4\alpha} u}{\partial x_1^{2\alpha} \partial x_2^{2\alpha}} + 12 \frac{\partial^{2\alpha} u}{\partial x_1^{\alpha} \partial x_2^{2\alpha}} + 12\frac{\partial^{2\alpha} u}{\partial x_1^{2\alpha} \partial x_2^{2\alpha}} + 6 \frac{\partial^{2\alpha} u}{\partial y_1^{\alpha} \partial y_2^{\alpha}} = 0, \\
0 < \alpha &\leq 1. 
\end{align*}
\]  

(65)

Let us introduce the wave transformation as

\[
u(x_1,x_2,y_1,y_2,t) = U(\xi), \quad \xi = \frac{ct^\alpha}{\Gamma(\alpha)} + \frac{k_1 x_1^{\alpha}}{\Gamma(\alpha)} + \frac{k_2 x_2^{\alpha}}{\Gamma(\alpha)} + \frac{l_1 y_1^{\alpha}}{\Gamma(\alpha)} + \frac{l_2 y_2^{\alpha}}{\Gamma(\alpha)},
\]

(66)

where \( c, k_1, k_2, l_1, l_2 \) are nonzero constants.

Utilizing Eq. (66) converts Eq. (65) into the following NODE:

\[
4ck_1U'' - k_1^2 k_2U'''' + k_2^2 k_1U'''' + 12k_1 k_2 (U')^2 + 12k_1 k_2 \ddot{U} U'' - 6l_1 l_2 \ddot{U}'' = 0.
\]

(67)

If we apply the balance principle on the terms \( \ddot{U}U'' \) and \( U'''' \) in Eq. (67), we have \( 2M = M + 2 \), so \( M = 2 \).

Application of GERFM for (65)

Using Eq. (5) together with \( M = 2 \), we have

\[
U(\xi) = A_0 + A_1 \Psi(\xi) + A_2 \Psi^2(\xi) + \frac{B_1}{\Psi(\xi)} + \frac{B_2}{\Psi^2(\xi)}.
\]

(68)

Proceeding as outlined in the second section, we acquire the following sets of solutions to Eq. (65).

Set 1: One obtains \( r = [-1, 3, 1, -1] \) and \( s = [1, -1, 1, -1] \), so Eq. (5) turns into

\[
\Psi(\xi) = \frac{\cosh(\xi) - 2 \sinh(\xi)}{\sinh(\xi)}.
\]

(69)
Case 1: We obtain

\[
c = \frac{20k_1^3k_2 - 20k_1k_2^3 - 6k_1k_2A_0 + 3l_1l_2}{2k_1},
\]
\[
A_0 = A_0, \quad A_1 = 4k_1^2 - 4k_2^2, \quad A_2 = k_1^2 - k_2^2, \quad B_1 = 0, \quad B_2 = 0,
\]
\[
k_1 = k_1, \quad k_2 = k_2, \quad l_1 = l_1, \quad l_2 = l_2.
\]

Putting values in Eqs. (68) and (69) yields the following solution:

\[
U(\xi) = \frac{(-3k_1^2 + 3k_2^2 + A_0) \cosh^2(\xi) + 4k_1^2 - 4k_2^2 - A_0}{\sinh^2(\xi)}.
\]

Consequently, we get the solution of Eq. (65) as

\[
u_1(x_1, x_2, y_1, y_2, t) = \frac{(-3k_1^2 + 3k_2^2 + A_0) \cosh^2(\xi) + 4k_1^2 - 4k_2^2 - A_0}{\sinh^2(\xi)}, \tag{70}
\]

where

\[
\xi = \frac{x_1^\alpha}{\Gamma(\alpha)} + \frac{k_1 x_2^\alpha}{\Gamma(\alpha)} + \frac{k_2 x_2^\alpha}{\Gamma(\alpha)} + \frac{l_1 y_1^\alpha}{\Gamma(\alpha)} + \frac{l_2 y_2^\alpha}{\Gamma(\alpha)}.
\]

Case 2: We obtain

\[
c = -\frac{3}{\sqrt{9k_2^2 + B_2}} \left( k_2 \left( A_0 - \frac{10B_2}{27} \right) \sqrt{9k_2^2 + B_2 - 3/2l_1l_2} \right),
\]
\[
A_0 = A_0, \quad A_1 = 0, \quad A_2 = 0, \quad B_1 = 4/3B_2, \quad B_2 = B_2,
\]
\[
k_1 = 1/3 \sqrt{9k_2^2 + B_2}, \quad k_2 = k_2, \quad l_1 = l_1, \quad l_2 = l_2.
\]

Putting values in Eqs. (68) and (69) yields the following solution:

\[
U(\xi) = \frac{(27A_0 - 9B_2) \cosh^4(\xi) + (-72A_0 + 29B_2) \cosh^2(\xi) + 4B_2 \sinh(\xi) \cosh(\xi) + 4B_2 \sinh(\xi) \cosh(\xi) + 48A_0 - 20B_2}{3(3 \cosh^2(\xi) - 4)^2}.
\]

Consequently, we get the solution of Eq. (65) as

\[
u_2(x_1, x_2, y_1, y_2, t) = \frac{(27A_0 - 9B_2) \cosh^4(\xi) + (-72A_0 + 29B_2) \cosh^2(\xi) + 4B_2 \sinh(\xi) \cosh(\xi) + 4B_2 \sinh(\xi) \cosh(\xi) + 48A_0 - 20B_2}{3(3 \cosh^2(\xi) - 4)^2}.
\]

\[
(71)
\]

Set 2: One obtains \( r = [1, 1, 1, -1] \) and \( s = [1, -1, 1, -1] \), so Eq. (5) turns into

\[
\Psi(\xi) = \frac{\cosh(\xi)}{\sinh(\xi)}. \tag{72}
\]

Case 1: We obtain

\[
c = -\frac{4k_1^3k_2 + 4k_1k_2^3 - 6k_1k_2A_0 + 3l_1l_2}{2k_1},
\]
Putting values in Eqs. (68) and (69) yields the following solution:

\[ U(\xi) = \frac{(k_1^2 - k_2^2 + A_0) \cosh^2(\xi) - k_1^2 + k_2^2}{\cosh^2(\xi)}. \]

Consequently, we get the solution of Eq. (65) as

\[ u_3(x_1, x_2, y_1, y_2, t) = \frac{(k_1^2 - k_2^2 + A_0) \cosh^2(\xi) - k_1^2 + k_2^2}{\cosh^2(\xi)}. \] (73)

**Case 2:** We obtain

\[ c = -\frac{4k_1^3 k_2 + 4k_1 k_2^3 - 6k_1 k_2 A_0 + 3l_1 l_2}{2k_1}, \]

\[ A_0 = A_0, \quad A_1 = 0, \quad A_2 = k_1^2 - k_2^2, \quad B_1 = 0, \quad B_2 = k_1^2 - k_2^2, \]

\[ k_1 = k_1, \quad k_2 = k_2, \quad l_1 = l_1, \quad l_2 = l_2. \]

Putting values in Eqs. (68) and (69) yields the following solution:

\[ U(\xi) = \frac{(2k_1^2 - 2k_2^2 + A_0) \cosh^4(\xi) + (-2k_1^2 + 2k_2^2 - A_0) \cosh^2(\xi) + k_1^2 - k_2^2}{\sinh^2(\xi) \cosh^2(\xi)}. \]

Consequently, we get the solution of Eq. (65) as

\[ u_4(x_1, x_2, y_1, y_2, t) = \frac{(2k_1^2 - 2k_2^2 + A_0) \cosh^4(\xi) + (-2k_1^2 + 2k_2^2 - A_0) \cosh^2(\xi) + k_1^2 - k_2^2}{\sinh^2(\xi) \cosh^2(\xi)}. \] (74)

**Set 2:** One obtains \( r = [1, 1, 1, -1] \) and \( s = [1, -1, 1, -1] \), so Eq. (5) turns into

\[ \Psi(\xi) = \frac{\cosh(\xi)}{\sinh(\xi)}. \] (75)

**Case 1:** We obtain

\[ c = -\frac{4k_1^3 k_2 + 4k_1 k_2^3 - 6k_1 k_2 A_0 + 3l_1 l_2}{2k_1}, \]

\[ A_0 = A_0, \quad A_1 = 0, \quad A_2 = 0, \quad B_1 = 0, \quad B_2 = k_1^2 - k_2^2, \]

\[ k_1 = k_1, \quad k_2 = k_2, \quad l_1 = l_1, \quad l_2 = l_2. \]

Putting values in Eqs. (68) and (69) yields the following solution:

\[ U(\xi) = \frac{(k_1^2 - k_2^2 + A_0) \cosh^2(\xi) - k_1^2 + k_2^2}{\cosh^4(\xi)}. \]
Consequently, we get the solution of Eq. (65) as
\[ u_5(x_1, x_2, y_1, y_2, t) = \frac{(k_1^2 - k_2^2 + A_0) \cosh^2(\xi) - k_1^2 + k_2^2}{1 + \cosh^2(\xi)}. \] (76)

Set 3: One obtains \( r = [-1, 0, 1, 1] \) and \( s = [0, 0, 1, 1] \), so Eq. (5) turns into
\[ \Psi(\xi) = \frac{1}{1 + e^\xi}. \] (77)

We obtain
\[ c = \frac{k_1^3 k_2 - k_1 k_2^3 - 12 k_1 k_2 A_0 + 6l_1 l_2}{4 k_1}, \]
\[ A_0 = A_0, \quad A_1 = k_1^2 - k_2^2, \quad A_2 = k_1^2 - k_2^2, \quad B_1 = 0, \quad B_2 = 0, \]
\[ k_1 = k_1, \quad k_2 = k_2, \quad l_1 = l_1, \quad l_2 = l_2. \]

Putting values in Eqs. (68) and (77) yields the following solution:
\[ U(\xi) = \frac{e^{2\xi} A_0 + (-k_1^2 + k_2^2 + 2A_0)e^\xi + A_0}{(1 + e^\xi)^2}. \]

Consequently, we get the solution of Eq. (65) as
\[ u_6(x_1, x_2, y_1, y_2, t) = \frac{e^{2\xi} A_0 + (-k_1^2 + k_2^2 + 2A_0)e^\xi + A_0}{(1 + e^\xi)^2}. \] (78)

Set 4: One obtains \( r = [-3, -2, 1, 1] \) and \( s = [0, 1, 0, 1] \), so Eq. (5) turns into
\[ \Psi(\xi) = \frac{-3 - 2e^\xi}{1 + e^\xi}. \] (79)

We obtain
\[ c = \frac{1}{\sqrt{36k_2^2 + B_2}} \left( k_2 \left( A_0 - \frac{73B_2}{432} \right) \sqrt{36k_2^2 + B_2 - 3l_1 l_2} \right), \]
\[ A_0 = A_0, \quad A_1 = 0, \quad A_2 = 0, \quad B_1 = 5/6B_2, \quad B_2 = B_2, \]
\[ k_1 = 1/6 \sqrt{36k_2^2 + B_2}, \quad k_2 = k_2, \quad l_1 = l_1, \quad l_2 = l_2. \]

Putting values in Eqs. (68) and (79) yields the following solution:
\[ U(\xi) = \frac{(24A_0 - 4B_2)e^{2\xi} + (72A_0 - 13B_2)e^\xi + 54A_0 - 9B_2}{6(3 + 2e^\xi)^2}. \]

Consequently, we get the solution of Eq. (65) as
\[ u_7(x_1, x_2, y_1, y_2, t) = \frac{(24A_0 - 4B_2)e^{2\xi} + (72A_0 - 13B_2)e^\xi + 54A_0 - 9B_2}{6(3 + 2e^\xi)^2}. \] (80)
Set 5: One obtains \( r = [-2 - i, 2 - i, 1, -1] \) and \( s = [-i, i, -i, i] \), so Eq. (5) turns into

\[
\Psi(\xi) = \frac{\cos(\xi) + 2 \sin(\xi)}{\sin(\xi)}.
\]  

(81)

We obtain

\[
c = \frac{28k_1^3k_2 - 28k_1k_2^3 - 6k_1k_2A_0 + 3l_1l_2}{2k_1},
\]

\[
A_0 = A_0, \quad A_1 = -4k_1^2 + 4k_2^2, \quad A_2 = k_1^2 - k_2^2, \quad B_1 = 0, \quad B_2 = 0,
\]

\[
k_1 = k_1, \quad k_2 = k_2, \quad l_1 = l_1, \quad l_2 = l_2.
\]

Putting values in Eqs. (68) and (81) yields the following solution:

\[
U(\xi) = \frac{(5k_1^2 - 5k_2^2 - A_0) \cos^2(\xi) - 4k_1^2 + 4k_2^2 + A_0}{\sin^2(\xi)}.
\]  

Consequently, we get the solution of Eq. (65) as

\[
u_9(x_1, x_2, y_1, y_2, t) = \frac{2 \sin(\xi)(-2k_1^2 + 2k_2^2 + A_0) \cos(\xi) + A_0}{2 \cos(\xi) \sin(\xi) + 1}.
\]  

(82)

Set 6: One obtains \( r = [1 - i, -1 - i, 1, -1] \) and \( s = [-i, i, -i, i] \), so Eq. (5) turns into

\[
\Psi(\xi) = \frac{\cos(\xi) + \sin(\xi)}{\sin(\xi)}.
\]  

(83)

We obtain

\[
c = \frac{10k_1^3k_2 - 10k_1k_2^3 - 6k_1k_2A_0 + 3l_1l_2}{2k_1},
\]

\[
A_0 = A_0, \quad A_1 = 0, \quad A_2 = 0, \quad B_1 = -4k_1^2 + 4k_2^2, \quad B_2 = 4k_1^2 - 4k_2^2,
\]

\[
k_1 = k_1, \quad k_2 = k_2, \quad l_1 = l_1, \quad l_2 = l_2.
\]

Putting values in Eqs. (68) and (83) yields the following solution:

\[
U(\xi) = \frac{2 \sin(\xi)(-2k_1^2 + 2k_2^2 + A_0) \cos(\xi) + A_0}{2 \cos(\xi) \sin(\xi) + 1}.
\]  

Consequently, we get the solution of Eq. (65) as

\[
u_9(x_1, x_2, y_1, y_2, t) = \frac{2 \sin(\xi)(-2k_1^2 + 2k_2^2 + A_0) \cos(\xi) + A_0}{2 \cos(\xi) \sin(\xi) + 1}.
\]  

(84)

Set 7: One obtains \( r = [2, 0, 1, -1] \) and \( s = [1, 0, 1, -1] \), so Eq. (5) turns into

\[
\Psi(\xi) = \frac{\cosh(\xi) + \sinh(\xi)}{\sinh(\xi)}.
\]  

(85)
We obtain
\[
c = \frac{2k_1^3k_2 - 2k_1k_2^3 - 6k_1k_2A_0 + 3l_1l_2}{2k_1},
\]
\[
A_0 = A_0, \quad A_1 = -2k_1^2 + 2k_2^2, \quad A_2 = k_1^2 - k_2^2, \quad B_1 = 0, \quad B_2 = 0,
\]
\[
k_1 = k_1, \quad k_2 = k_2, \quad l_1 = l_1, \quad l_2 = l_2.
\]

Putting values in Eqs. (68) and (85) yields the following solution:
\[
U(\xi) = \frac{\cosh^2(\xi)A_0 + k_1^2 - k_2^2 - A_0}{\sinh^2(\xi)}.
\]

Consequently, we get the solution of Eq. (65) as
\[
u_{10}(x_1, x_2, y_1, y_2, t) = \frac{\cosh^2(\xi)A_0 + k_1^2 - k_2^2 - A_0}{\sinh^2(\xi)}. \tag{86}
\]

Set 8: One obtains \(r = [i, -i, 1, 1]\) and \(s = [i, -i, i, -i]\), so Eq. (5) turns into
\[
\Psi(\xi) = -\frac{\sin(\xi)}{\cos(\xi)}. \tag{87}
\]

We obtain
\[
c = \frac{4k_1^3k_2 - 4k_1k_2^3 - 6k_1k_2A_0 + 3l_1l_2}{2k_1},
\]
\[
A_0 = A_0, \quad A_1 = 0, \quad A_2 = 0, \quad B_1 = 0, \quad B_2 = k_1^2 - k_2^2,
\]
\[
k_1 = k_1, \quad k_2 = k_2, \quad l_1 = l_1, \quad l_2 = l_2.
\]

Putting values in Eqs. (68) and (87) yields the following solution:
\[
U(\xi) = \frac{(k_1^2 - k_2^2 - A_0)\cos^2(\xi) + A_0}{\sin^2(\xi)}.
\]

Consequently, we get the solution of Eq. (65) as
\[
u_{11}(x_1, x_2, y_1, y_2, t) = \frac{(k_1^2 - k_2^2 - A_0)\cos^2(\xi) + A_0}{\sin^2(\xi)}. \tag{88}
\]

**Application of EShGEEM for (65)**

Firstly, we assume that the solution of Eq. (67) takes the following form:
\[
U(\theta) = A_0 + B_1 \sinh(\xi) + A_1 \cosh(\xi) + \cosh(\xi)(B_2 \sinh(\xi) + A_2 \cosh(\xi)). \tag{89}
\]

and
\[
U_1(\xi) = A_0 + iB_1 \text{sech}(\xi) - A_1 \tanh(\xi) - \tanh(\xi)(iB_2 \text{sech}(\xi) - A_2 \tanh(\xi)),
\]
\[
U_2(\xi) = A_0 + B_1 \text{csch}(\xi) - A_1 \coth(\xi) - \coth(\xi)(B_2 \text{csch}(\xi) - A_2 \coth(\xi)). \tag{90}
\]
Now, the extended EShGEEM with the help of Eqs. (89)–(90) can introduce the following new exact soliton solutions of the space-time conformable Fokas equation given by (65).

Set 1:

\[
c = c, \quad A_0 = \frac{-4k_1^3k_2 - 2ck_1 + 3l_1l_2}{6k_1k_2}, \quad A_1 = 0, \quad A_2 = k_1^2,
\]

\[B_1 = B_2 = 0, \quad k_1 = k_1, \quad k_2 = k_2, \quad l_1 = l_1, \quad l_2 = l_2.
\]

Using these values, the following solution for (67) is obtained:

\[
U_1(\xi) = \frac{(2k_1^3k_2 - 2ck_1 + 3l_1l_2)(\cosh(\xi))^2 - 6k_1^3k_2}{6k_1k_2(\cosh(\xi))^2},
\]

\[
U_2(\xi) = \frac{(2k_1^3k_2 - 2ck_1 + 3l_1l_2)(\cosh(\xi))^2 + 4k_1^3k_2 + 2ck_1 - 3l_1l_2}{6k_1k_2(\sinh(\xi))^2}.
\]

Consequently, we get the solution of Eq. (65) as

\[
u_{12}(x_1,x_2,y_1,y_2,t) = \frac{(2k_1^3k_2 - 2ck_1 + 3l_1l_2)(\cosh(\xi))^2 - 6k_1^3k_2}{6k_1k_2(\cosh(\xi))^2},
\]

\[
u_{13}(x_1,x_2,y_1,y_2,t) = \frac{(2k_1^3k_2 - 2ck_1 + 3l_1l_2)(\cosh(\xi))^2 + 4k_1^3k_2 + 2ck_1 - 3l_1l_2}{6k_1k_2(\sinh(\xi))^2},
\]

where

\[
\xi = \frac{ct^\alpha}{\Gamma(\alpha)} + \frac{k_1x_1^\alpha}{\Gamma(\alpha)} + \frac{k_2x_2^\alpha}{\Gamma(\alpha)} + \frac{l_1y_1^\alpha}{\Gamma(\alpha)} + \frac{l_2y_2^\alpha}{\Gamma(\alpha)}.
\]

Set 2:

\[
c = c, \quad A_0 = \frac{-5k_1^3k_2 - 4ck_1 + 6l_1l_2}{12k_1k_2}, \quad A_1 = 0, \quad A_2 = B_2 = k_1^2/2, \quad B_1 = 0,
\]

\[k_1 = k_1, \quad k_2 = k_2, \quad l_1 = l_1, \quad l_2 = l_2.
\]

Using these values, the following solution for (67) is obtained:

\[
U_1(\xi) = \frac{(k_1^3k_2 - 4ck_1 + 6l_1l_2)(\cosh(\xi))^2 - 6(i \sinh(\xi) + 1)k_2k_1^3}{12k_1k_2(\cosh(\xi))^2},
\]

\[
U_2(\xi) = \frac{(k_1^3k_2 - 4ck_1 + 6l_1l_2)\cosh(\xi) - 512k_1^3k_2 - 4ck_1 + 6l_1l_2}{k_1k_2(\cosh(\xi) + 1)}.
\]

Consequently, we get the solution of Eq. (65) as

\[
u_{12}(x_1,x_2,y_1,y_2,t) = \frac{(k_1^3k_2 - 4ck_1 + 6l_1l_2)(\cosh(\xi))^2 - 6(i \sinh(\xi) + 1)k_2k_1^3}{12k_1k_2(\cosh(\xi))^2},
\]

\[
u_{13}(x_1,x_2,y_1,y_2,t) = \frac{(k_1^3k_2 - 4ck_1 + 6l_1l_2)\cosh(\xi) - 512k_1^3k_2 - 4ck_1 + 6l_1l_2}{k_1k_2(\cosh(\xi) + 1)}.
\]
where

\[ \xi = \frac{ct^\alpha}{\Gamma(\alpha)} + \frac{k_1 x_1^\alpha}{\Gamma(\alpha)} + \frac{k_2 x_2^\alpha}{\Gamma(\alpha)} + \frac{l_1 y_1^\alpha}{\Gamma(\alpha)} + \frac{l_2 y_2^\alpha}{\Gamma(\alpha)}. \]

Set 3:

- \( c = c_1 \)
- \( A_0 = -5k_1^3k_2 - 4ck_1 + 6l_1l_2 \)
- \( A_1 = 0 \)
- \( A_2 = -B_2 = k_1^2/2 \)
- \( B_1 = 0 \)
- \( k_1 = k_1 \)
- \( k_2 = k_2 \)
- \( l_1 = l_1 \)
- \( l_2 = l_2 \)

Using these values, the following solution for (67) is obtained:

\[ U_1(\xi) = \left( \frac{k_1^3k_2 - 4ck_1 + 6l_1l_2}{12k_1k_2} \right)(\cosh(\xi))^2 \]
\[ + 6\left( i\sinh(\xi) - 1 \right) \frac{k_2k_3}{12k_1k_2}(\cosh(\xi))^2, \]

\[ U_2(\xi) = \left( \frac{k_1^3k_2 - 4ck_1 + 6l_1l_2}{12k_1k_2} \right) \cosh(\xi) + 5\frac{k_3k_2}{12k_1k_2}(\cosh(\xi) - 1), \]

Consequently, we get the solution of Eq. (65) as

\[ u_{14}(x_1,x_2,y_1,y_2,t) = \left( \frac{k_1^3k_2 - 4ck_1 + 6l_1l_2}{12k_1k_2} \right)(\cosh(\xi))^2 \]
\[ + 6\left( i\sinh(\xi) - 1 \right) \frac{k_2k_3}{12k_1k_2}(\cosh(\xi))^2, \]

\[ u_{15}(x_1,x_2,y_1,y_2,t) = \left( \frac{k_1^3k_2 - 4ck_1 + 6l_1l_2}{12k_1k_2} \right) \cosh(\xi) + 5\frac{k_3k_2}{12k_1k_2}(\cosh(\xi) - 1), \]

where

\[ \xi = \frac{ct^\alpha}{\Gamma(\alpha)} + \frac{k_1 x_1^\alpha}{\Gamma(\alpha)} + \frac{k_2 x_2^\alpha}{\Gamma(\alpha)} + \frac{l_1 y_1^\alpha}{\Gamma(\alpha)} + \frac{l_2 y_2^\alpha}{\Gamma(\alpha)}. \]

The correctness of all the solutions obtained in the paper has been examined by placing them directly in the main equation, and it has been found that they satisfy the main equation.

**4 Conclusion**

Pursuing new concepts in mathematics provides a promising framework for describing many complex phenomena and structures in the real world. Many of these structures cannot be described by the existing classical definitions. This is an incentive for researchers to explore new definitions in differential calculus. In this paper, based on the generalized exponential rational function method and the extended sinh-Gordon equation expansion method, we have obtained several new exact solutions of the space-time conformable coupled Cahn–Allen equation, coupled Burgers equation, and Fokas equation. Both schemes are easy to implement in computer programs and take small memory. On the other hand, they require less computational cost compared to other techniques. Numerical results clearly indicate the reliability and efficiency of the proposed method. To the best of our knowledge, the solutions obtained for these nonlinear equations considering the GERFM and EShGEEM are new and have not been reported in the literature. It is important to note that a wide range of solutions, such as exponential, triangular, dark, and light solitons, periodic solutes, for the equations considered in this paper are determined by two methods
that have not been previously explored in previous references. Since the techniques are direct, powerful, and efficient, they can be efficiently used to find the exact solutions of different nonlinear differential equations in several branches of nonlinear sciences.

Acknowledgements
The authors gratefully thank the referee for the constructive comments and recommendations which definitely helped to improve the readability and quality of the paper.

Funding
This research work is not supported by any funding agencies.

Availability of data and materials
Not applicable.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally and significantly in writing this paper. All authors have read and approved the final paper.

Author details
1Department of Mathematics, Faculty of Sciences, Gonbad Kavous University, Gonbad, Iran. 2Department of Mathematics, Kermanshah Branch, Islamic Azad University, Kermanshah, Iran.

Publisher's Note
Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Received: 11 March 2021 Accepted: 26 May 2021 Published online: 06 June 2021

References
1. Nabti, A., Ghanbari, B.: Global stability analysis of a fractional SVEIR epidemic model. Math. Methods Appl. Sci., 1–21 (2021). https://doi.org/10.1002/mma.7285
2. Wang, W.B., Lou, G.W., Shen, X.M., Song, J.Q.: Exact solutions of various physical features for the fifth order potential Bogoyavlenski–Schiff equation. Results Phys. 18, 103243 (2020)
3. Ghanbari, B., Kumar, S.: A study on fractional predator–prey–pathogen model with Mittag-Leffler kernel-based operators. Numer. Methods Partial Differ. Equ. (2021). https://doi.org/10.1002/num.22689
4. Gao, W., Baskonus, H.M., Shi, L.: New investigation of bats–hosts–reservoir–people coronavirus model and application to 2019-nCoV system. Adv. Differ. Equ. 2020, 391 (2020)
5. Ghanbari, B., Djilali, S.: Mathematical and numerical analysis of a three-species predator–prey model with herd behavior and time fractional-order derivative. Math. Methods Appl. Sci. 43(4), 1736–1752 (2020)
6. Alharbi, A., Almatrafi, M.B.: Numerical investigation of the dispersive long wave equation using an adaptive moving mesh method and its stability. Results Phys. 16, 102670 (2020)
7. Ghanbari, B.: On the modeling of the interaction between tumor growth and the immune system using some new fractional and fractional-fractal operators. Adv. Differ. Equ. 2020, 585 (2020)
8. Gao, W., Veeresha, P., Prakash, D.G., Baskonus, H.M.: New numerical simulation for fractional Benney–Lin equation arising in falling film problems using two novel techniques. Numer. Methods Partial Differ. Equ. 37(1), 210–243 (2021)
9. Ghanbari, B., Atangana, A.: Some new edge detecting techniques based on fractional derivatives with non-local and non-singular kernels. Adv. Differ. Equ. 2020, 435 (2020)
10. McCue, S.W., El-Hachem, M., Simpson, M.J.: Exact sharp-fronted travelling wave solutions of the Fisher-KPP equation. Appl. Math. Lett. 114, 106918 (2021)
11. Djilali, S., Ghanbari, B.: The influence of an infectious disease on a prey–predator model equipped with a fractional-order derivative. Adv. Differ. Equ. 2021, 20 (2021)
12. Gao, W., Veeresha, P., Baskonus, H.M., Prakash, D.G., Kumar, P.: A new study of unreported cases of 2019-nCOV epidemic outbreaks. Chaos Solitons Fractals 138, 109929 (2020)
13. Ghanbari, B.: A fractional system of delay differential equation with nonsingular kernels in modeling hand–foot–mouth disease. Adv. Differ. Equ. 2020, 536 (2020)
14. Ghanbari, B.: On approximate solutions for a fractional prey–predator model involving the Atangana–Belieou derivative. Adv. Differ. Equ. 2020, 679 (2020)
15. Ertaç, Y.S., Kumar, P.: Solution of a COVID-19 model via new generalized Caputo-type fractional derivatives. Chaos Solitons Fractals 139, 110280 (2020)
16. Herron, I., McCalla, C., Mickens, R.: Traveling wave solutions of Burgers equation with time delay. Appl. Math. Lett. 107, 106946 (2020)
17. Munusamy, K., Ravichandran, C., Nisar, K.S., Ghanbari, B.: Existence of solutions for some functional integro-differential equations with nonlocal conditions. Math. Methods Appl. Sci. 43(17), 10319–10331 (2020)
18. Goyal, M., Baskonus, H.M., Prakash, A.: Regarding new positive, bounded and convergent numerical solution of nonlinear time fractional HIV/AIDS transmission model. Chaos Solitons Fractals 139, 110096 (2020)
19. Kudryashov, N.A.: Traveling wave solutions of the generalized Gerdjikov–Ivanov equation. Optik 219, 165193 (2020)
20. Akbulut, A., Kaplan, M.: Auxiliary equation method for time-fractional differential equations with conformable derivative. Comput. Math. Appl. 75(3), 876–882 (2018)
21. Chen, C., Jiang, Y.L.: Simplest equation method for some time-fractional partial differential equations with conformable derivative. Comput. Math. Appl. 75(8), 2978–2988 (2018)
22. Ghanbari, B., Kuo, C.K.: A variety of solitary wave solutions to the (2 + 1)-dimensional bidirectional SK and variable-coefficient SK equations. Results Phys. 18, 103266 (2020)
23. Kurt, A., Tasbozan, O., Cenesiz, Y.: Homotopy analysis method for conformable Burgers–Korteweg-de Vries equation. Bull. Math. Sci. Appl. 17, 17–23 (2016)
24. Ghanbari, B., Kumar, S., Nivas, M., Baleanu, D.: The Lie symmetry analysis and exact Jacobi elliptic solutions for the Kawahara–Kdv type equations. Results Phys. 23, 104006 (2021)
25. Ghanbari, B., Baleanu, D.: A novel technique to construct exact solutions for nonlinear partial differential equations. Eur. Phys. J. Plus 134(10), 506 (2019)
26. Cenesiz, Y., Kurt, A.: New fractional complex transform for conformable fractional partial differential equations. J. Appl. Math. Stat. Inform. 12(2), 41–47 (2016)
27. Cao, Y., Tian, H., Ghanbari, B.: On constructing of multiple rogue wave solutions to the (3 + 1)-dimensional Korteweg–de Vries Benjamin–Bona–Mahony equation. Phys. Sci. 96(3), 035226 (2021)
28. Kurt, A., Tasbozan, O., Baleanu, D.: New solutions for conformable fractional Nizhnik–Novikov–Veselov system via G/G expansion method and homotopy analysis method. Opt. Quantum Electron. 49(10), 1–23 (2017)
29. Khater, M.M., Ghanbari, B.: On the solitary wave solutions and physical characterization of gas diffusion in a homogeneous medium via some efficient techniques. Eur. Phys. J. Plus 136(4), 447 (2021)
30. Pinar, Z., Rezazadeh, H., Eslami, M.: Generalized logistic equation method for Kerr law and dual power law Schrödinger equations. Opt. Quantum Electron. 52, 504 (2020)
31. Savas, T., Geng, B., Rezazadeh, H., Bekir, A., Duka, S.Y.: Exact optical solitons to the perturbed nonlinear Schrödinger equation with dual-power law of nonlinearity. Opt. Quantum Electron. 52, 318 (2020)
32. Liu, J.G., Eslami, M., Rezazadeh, H., Mirzaadeh, M.: The dynamical behavior of mixed type lump solutions on the (3 + 1)-dimensional generalized Kadomtsev–Petviashvili–Boussinesq equation. Int. J. Nonlinear Sci. Numer. Simul. 21(7–8), 661–665 (2020)
33. Rezazadeh, H., Korkmaz, A., Eslami, M., Mirhosseini-Alizamini, S.M.: A large family of optical solutions to Kundu–Eckhaus model by a new auxiliary equation method. Opt. Quantum Electron. 51, 81 (2019)
34. Ghanbari, B.: A new model for investigating the transmission of infectious diseases in a prey–predator system using a nonsingular fractional derivative. Math. Methods Appl. Sci., 1–20 (2021). https://doi.org/10.1002/mma.7412
35. Ghanbari, B.: Chaotic behaviors of the prevalence of an infectious disease in a prey and predator system using fractional derivatives. Math. Methods Appl. Sci., 1–16 (2021). https://doi.org/10.1002/mma.7386
36. Rahman, G., Nisar, K.S., Ghanbari, B., Abdeljawad, T.: On generalized fractional integral inequalities for the monotone weighted Chebyshev functions. Adv. Differ. Equ. 2020, 368 (2020)
37. Ghanbari, B.: On forecasting the spread of the COVID-19 in Iran: the second wave. Chaos Solitons Fractals 140, 110176 (2020)
38. Ghanbari, B., Gómez-Aguilar, J.F.: Two efficient numerical schemes for simulating dynamical systems and capturing chaotic behaviors with Mittag-Leffler memory. Eng. Comput., 1–29 (2020). https://doi.org/10.1007/s10366-020-01170-0
39. Ghanbari, B., Djalil, S.: Mathematical analysis of a fractional-order predator–prey model with prey social behavior and the fractional predator population. Chaos Solitons Fractals 138, 109960 (2020)
40. Djalil, S., Ghanbari, B.: Coronavirus pandemic: a predictive analysis of the peak outbreak epidemic in South Africa, Turkey, and Brazil. Chaos Solitons Fractals 138, 109971 (2020)
41. Ghanbari, B., Günerhan, H., Srivastava, H.M.: An application of the Atangana–Baleanu fractional derivative in mathematical biology: a three-species predator–prey model. Chaos Solitons Fractals 138, 109910 (2020)
42. Ghanbari, B., Cattani, C.: On fractional predator and prey models with mutualistic predation including non-local and non-singular kernels. Chaos Solitons Fractals 136, 109823 (2020)
43. Ghanbari, B., Atangana, A.: A new application of fractional Atangana–Baleanu derivatives: designing ABC-fractional masks in image processing. Phys. A, Stat. Mech. Appl. 542, 123516 (2020)
44. Allahviranloo, T., Ghanbari, B.: On the fuzzy fractional differential equation with interval Atangana–Baleanu fractional derivative approach. Chaos Solitons Fractals 130, 109397 (2020)
45. Ghanbari, B., Gomez-Aguilar, J.F.: Analysis of two avian influenza epidemic models involving fractal-fractional derivatives with power and Mittag-Leffler memories. Chaos Solitons Fractals 127, 312–317 (2019)
46. Djalil, S., Ghanbari, B.: Dynamical behavior of two predators–one prey model with generalized functional response and time-fractional derivative. Adv. Differ. Equ. 2021, 235 (2021)
47. Polyaniy, A.D., Sorokin, V.G.: A method for constructing exact solutions of nonlinear delay PDEs. J. Math. Anal. Appl. 494(2), 124619 (2021)
48. Veeresha, P., Prakasha, D.G., Kumar, D., Baleanu, D., Singh, J.: An efficient computational technique for fractional model of generalized Hirota–Satsuma-coupled Korteweg–de Vries and coupled modified Korteweg–de Vries equations. J. Comput. Nonlinear Dyn. 15, 071003 (2020)
49. Singh, J., Kumar, D., Purohit, S.D., Mishra, A.M., Bohra, M.: An efficient numerical approach for fractional multi-dimensional diffusion equations with exponential memory. Numer. Methods Partial Differ. Equ. 37(2), 1631–1651 (2021)
50. Veeresha, P., Prakasha, D.G., Singh, J., Kumar, D., Baleanu, D.: Fractional Klein–Gordon–Schrödinger equations with Mittag-Leffler memory. Chin. J. Phys. 68, 65–78 (2020)
51. Ghanbari, B., Kumar, D., Singh, J.: Exact solutions of local fractional longitudinal wave equation in a magnetoeleeto-electro-elastic circular rod in fractal media. Indian J. Phys. (2021). https://doi.org/10.1007/s12648-021-02043-y
52. Khalil, R., Al Horani, M., Yousef, A., Sababheh, M.: A new definition of fractional derivative. J. Comput. Appl. Math. 264, 65–70 (2014)
53. Kaplan, M., Bekir, A., Ozar, M.N.: A simple technique for constructing exact solutions to nonlinear differential equations with conformable fractional derivative. Opt. Quantum Electron. 49, 266 (2017)
55. Abdeljawad, T.: On conformable fractional calculus. J. Comput. Appl. Math. 279, 57–66 (2015)
56. Zhou, H.W., Yang, S., Zhang, S.Q.: Conformable derivative approach to anomalous diffusion. Phys. A, Stat. Mech. Appl. 491, 1001–1013 (2018)
57. Cenesiz, Y., Kurt, A., Nane, E.: Stochastic solutions of conformable fractional Cauchy problems. Stat. Probab. Lett. 124, 126–131 (2017)
58. Vasilan, H.C.: New analytic solutions of the conformable space-time fractional Kawahara equation. Optik, Int. J. Light Electron Opt. 140, 123–126 (2017)
59. Ghanbari, B., Osman, M.S., Baleanu, D.: Generalized exponential rational function method for extended Zakharov–Kuznetsov equation with conformable derivative. Mod. Phys. Lett. A 34(20), 1950155 (2019)
60. Hider, A.A., Soliman, A.H.: Exact solutions of space-time local fractal nonlinear evolution equations: a generalized conformable derivative approach. Results Phys. 17, 103135 (2020)
61. Cenesiz, Y., Baleanu, D., Kurt, A., Tsobozan, O.: New exact solutions of Burgers-type equations with conformable derivative. Waves Random Complex Media 27, 103–116 (2017)
62. Senol, M., Tsobozan, O., Kurt, A.: Numerical solutions of fractal Burgers-type equations with conformable derivative. Chin. J. Phys. 58, 75–84 (2019)
63. Akbulut, A., Kaplan, M.: Auxiliary equation method for time-fractional differential equations with conformable derivative. Comput. Math. Appl. 75(3), 876–882 (2018)
64. Ghanbari, B., Nisar, K.S., Aldhaifallah, M.: Abundant solitary wave solutions to an extended nonlinear Schrödinger’s equation with conformable derivative using an efficient integration method. Adv. Differ. Equ. 2020, 328 (2020)
65. Az-Zo’bi, E.A., Alzoubi, W.A., Akinyemi, L., Şenol, M., Masaedeh, B.S.: A variety of wave amplitudes for the resonance nonlinear Schrödinger equation. Eur. Phys. J. Plus 133, 142 (2018)
66. Osman, M.S., Ghanbari, B.: New optical solitary wave solutions of Fokas–Lenells equation in presence of perturbation terms by a novel approach. Optik 175, 328–333 (2018)
67. Ghanbari, B., Kuo, C.K.: Abundant wave solutions to two novel KP-like equations using an effective integration method. Phys. Scr. 96(4), 045203 (2021)
68. Ghanbari, B., Rada, L., Inc, M.: Solitary wave solutions to the Tzitzeica type equations obtained by a new efficient approach. J. Appl. Anal. Comput. 9(2), 568–589 (2019)
69. Ghanbari, B., Baleanu, D.: New optical solutions of the fractional Gerdjikov–Ivanov equation with conformable derivative. Front. Phys. 8, 167 (2020)
70. Ghanbari, B., Baleanu, D., Al Qurashi, M.: New exact solutions of the generalized Benjamin–Bona–Mahony equation. Symmetry 11(1), 20 (2019)
71. Ghanbari, B.: Abundant soliton solutions for the Hirota–Maccari equation via the generalized exponential rational function method. Mod. Phys. Lett. A 33(9), 1950106 (2019)
72. Ghanbari, B.: On novel nondifferentiable exact solutions to local fractional Gardner’s equation using an effective technique. Math. Methods Appl. Sci. 44(6), 4673–4685 (2021)
73. Ismael, H.F., Bulut, H., Baskonus, H.M.: W-shaped surfaces to the nematic liquid crystals with three nonlinearity laws. Soft Comput. 25, 4513–4524 (2021)
74. Ghanbari, B., Yusuf, A., Inc, M., Baleanu, D.: The new exact solitary wave solutions and stability analysis for the (2 + 1)-dimensional Zakharov–Kuznetsov equation. Adv. Differ. Equ. 2019, 49 (2019)
75. Srivastava, H.M., Gunerhan, H., Ghanbari, B.: Exact traveling wave solutions for resonance nonlinear Schrodinger equation with intermodal dispersions and the Kerr law nonlinearity. Math. Methods Appl. Sci. 42(18), 7210–7221 (2019)
76. Ghanbari, B., Gunerhan, H., Ilhan, O.A., Baskonus, H.M.: Some new families of exact solutions to a new extension of nonlinear Schrodinger equation. Phys. Scr. 95(7), 075208 (2020)
77. Kumar, D., Joardar, A.K., Hoque, A., Paul, G.C.: Investigation of dynamics of nematicons in liquid crystals by extended sinh-Gordon equation expansion method. Opt. Quantum Electron. 51(7), 212 (2019)
78. Foroutan, M., Kumar, D., Manafian, J., Hoque, A.: New explicit soliton and other solutions for the conformable fractional Biswas–Milovic equation with Kerr and parabolic nonlinearity through an integration scheme. Optik 170, 170–192 (2018)
79. Kumar, D., Seadawy, A.R., Haque, M.R.: Multiple soliton solutions of the nonlinear partial differential equations describing the wave propagation in nonlinear low-pass electrical transmission lines. Chaos Solitons Fractals 115, 62–76 (2018)
80. Tascan, F., Bekir, A.: Travelling wave solutions of the Cahn–Allen equation by using first integral method. Appl. Math. Comput. 207(1), 279–282 (2009)
81. Aksoy, E., Kaplan, M., Bekir, A.: Exponential rational function method for space-time fractional differential equations. Waves Random Complex Media 26(2), 142–151 (2016)
82. Zhang, S., Zhang, H.Q.: Fractional sub-equation method and its applications to nonlinear fractional PDEs. Phys. Lett. A 375(7), 1069–1073 (2011)