Extended Inflation from Strings

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Abstract

We study the possibility of extended inflation in the effective theory of gravity from strings compactified to four dimensions and find that it strongly depends on the mechanism of supersymmetry breaking. We consider a general class of string–inspired models which are good candidates for successful extended inflation. In particular, the $\omega$–problem of ordinary extended inflation is automatically solved by the production of only very small bubbles until the end of inflation. We find that the inflaton field could belong either to the untwisted or to the twisted massless sectors of the string spectrum, depending on the supersymmetry breaking superpotential.

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It is nowadays commonly believed by most cosmologists that the inflationary paradigm may solve most of the problems of the standard cosmological model. However, it is not yet clear how the inflationary scenario can be successfully implemented. In fact, the first proposed inflationary model (known as 'old' inflation) \[1\], based on a first order phase transition, could not provide a satisfactory explanation of how to get out from the inflationary phase without disturbing the good properties of the standard cosmological model \[1, 2\]. The first models proposed to solve this 'graceful exit' problem, known as 'new' inflation \[3\], with a second order phase transition, were plagued with severe fine–tunings. Soon after, a different solution without phase transition, known as 'chaotic' inflation \[4\], was proposed. (Chaotic inflation has been recently shown to considerably soften the fine–tuning problems of new inflation \[5\].)

Recently, La and Steinhardt \[6\] proposed an inflationary model, known as 'extended' inflation, based again on a first order phase transition, where the graceful exit problem was solved by using a Jordan–Brans–Dicke \[7\] theory of gravity with a scalar field, instead of General Relativity \[8\]. It was soon realized that the anisotropy at decoupling produced by large bubbles \[9\] made extended inflation incompatible with the post–Newtonian bounds \[10\] of General Relativity. This disease could be cured either by using a more general scalar–tensor theory of gravity \[11\] or by means of a cosmological constant with a runaway dependence on the scalar field \[12\].

Most particle physicists believe that the theory of gravity at low energies (General Relativity, scalar–tensor theories or whatever) is an effective approximation of some fundamental theory of quantum gravity at scales beyond the Planck mass (\(M_P\)). The only reliable candidates for such a fundamental theory are superstrings \[13\]. They are known to describe gravity as a low energy effective theory. It is therefore of interest to know whether or not strings could lead to any kind of cosmological inflation in the low energy effective theory.

The effective theory of superstrings exhibits three properties that makes it a good candidate for some kind of extended inflation. First of all, the scalar fields of the gravitational multiplet (the dilaton and the moduli) are always coupled to the curvature scalar of the four–dimensional metric, in the same way as the scalar field in scalar–tensor theories of gravity. Second, the dilaton and moduli are also coupled to the matter sector giving rise to a scalar field dependent potential. Finally, the existence of flat directions in the potential
follows from very general principles \[13\]. In the presence of supersymmetry breaking terms and a positive vacuum energy, flat directions become runaway fields, and so are good candidates for solving the anisotropy/post–Newtonian bounds conundrum of extended inflation.

In a previous paper \[14\] we studied the conformal properties and cosmological solutions in the low energy effective theory of gravity from closed strings compactified to four dimensions, for different supersymmetric and non–supersymmetric string scenarios, during the radiation and matter dominated eras. In this paper we study the possibility of extended inflation in string scenarios with spontaneously broken supersymmetry. This problem has been recently studied under some assumptions (in particular, constant values for the moduli) with negative results \[15\]. However, we will show that the possibility of extended inflation strongly relies on the mechanism of supersymmetry breaking and find the conditions under which it could happen. We will argue that a general necessary condition is the existence of a positive ‘metastable’ minimum with some runaway direction along the scalar fields. This runaway direction should become flat at the true minimum in order to solve the cosmological constant problem. In this case, the same symmetry principle (if any) that could help solving the cosmological constant problem, could also help extended inflation. A non–constant value of the moduli along the runaway direction will help overcoming the problems found in Ref. \[15\].

At string tree level, and keeping only linear terms in the string tension $\alpha'$, the effective Lagrangian for the dilaton ($\phi$), the modulus ($\sigma$) $^\dagger$ and the matter fields ($C_n$), can be written in the Einstein frame as $^\ddagger$ $^{16, 17}$

$$L_{\text{eff}} = \sqrt{-\tilde{g}} \left[ \bar{R} - \frac{1}{2} (\partial_\mu \phi)^2 - 6 (\partial_\mu \sigma)^2 - \sum_{n=1}^{3} \frac{\alpha_n}{2^n} e^{-n(\sigma + \frac{1}{2} \phi)} | D_\mu C_n |^2 - V(\phi, \sigma, C_n, C_n^*) \right]$$

(1)

where $C_1$ are untwisted matter fields, $C_2$ twisted moduli (blowing up modes) and $C_3$ twisted matter fields, and $\alpha_n$ are some positive constants ($\alpha_1 = 3$, $\alpha_2 = \alpha_3 = 1$).

For the purpose of this paper, in order to establish the necessary condi-

\[1\] We take the diagonal direction in the space of moduli fields.

\[2\] We will work hereafter, unless explicit mention, in units in which $M_P \equiv 1$.  

2
tions for extended inflation, it will be enough to expand the potential \( V \) in (1) as
\[
V(\phi, \sigma, C_n, C_n^*) = V_o(\phi, \sigma) + \sum_{n=1}^{3} V_n(\phi, \sigma) | C_n |^2 + ...
\] (2)

In fact, the Lagrangian (1) and the potential (2) can be put in the standard supergravity form \([18]\) by means of the Kähler potential \([16, 17]\)
\[
K = -\ln(S + S^*) - 3\ln(T + T^*) + \sum_{n=1}^{3} \alpha_n (T + T^*)^{-n} | C_n |^2 + ...
\] (3)

and the superpotential
\[
W(S, T, C_n) = W_o(S, T) + \sum_{n=1}^{3} W_n(S, T) C_n^3 + ...
\] (4)

where
\[
\begin{align*}
ReS &= e^{3\sigma - \frac{1}{2} \phi} \\
ReT &= e^{\sigma + \frac{1}{2} \phi}
\end{align*}
\] (5)

It is important to stress that a superpotential \( W_o \) different from zero, necessary for supersymmetry breaking, and a non–constant superpotential \( W_n \) could be generated by string non–perturbative effects. Also note that we are consistently studying the Lagrangian along the (strong CP–conserving) real directions \( (ImS = ImT = 0) \).

The properties of the potential (4), and in particular its ability to produce extended inflation, will depend in general on the form of the superpotential (1). We will first give some (by no means sufficient) conditions on the potential (2), and their implications on the superpotential \( W_o \), in order to produce extended inflation:

a) We assume that the potential \( V_o \) has a minimum along some field direction, e.g.
\[
\sigma = -b \phi + d
\] (6)

3Notice that a minimum along a different direction would just amount to a field redefinition and so the general results of this paper should remain valid.
(with $b$ and $d$ some real parameters) and a runaway direction along the orthogonal field. This condition can be fulfilled depending on the functional form of the superpotential. For instance, if $W_o = W_o(X)$ with $X = S^\alpha T^{3\beta}$ ($\alpha$ and $\beta$ real), then

$$b = \frac{3\beta - \alpha}{6(\alpha + \beta)}$$

and the potential $V$ takes the form

$$V_o = \frac{1}{16} e^{-6\sigma - \phi} v_o(\sigma + b\phi)$$

$$V_n = \frac{1}{16 e^{2\phi}} e^{-(6+n)\sigma - (1-\frac{b}{2})\phi} v_n(\sigma + b\phi)$$

where

$$v_o(\sigma + b\phi) = f_o^2 + 3f_o^2 + 3f_o^2$$

$$v_n(\sigma + b\phi) = f_o^2 + (3-n)f_o^2 - 2f_o^2$$

with

$$f_\lambda(\sigma + b\phi) \equiv W_o - 2\lambda X \frac{\partial W_o}{\partial X}.$$  

The minimization of $v_o$ in (8) should provide the condition (6).

Notice that condition (6) is not essential for extended inflation. It is just a simplifying assumption where one direction in the $(\sigma, \phi)$ configuration space is fixed to its vacuum expectation value and so the remaining theory of gravity has only one scalar field. However, more general situations suitable for extended inflation are thinkable. For instance, the case where both $\sigma$ and $\phi$ are runaway directions (no field is fixed to its vacuum expectation value) can be easily realized in many models. In particular, in the simple

\[4\] Otherwise $\phi$ and $\sigma$ would be fixed to their vacuum expectation values and no extended inflation could be present. Since we are concerned in this paper with extended inflation from strings, we will not consider the latter case. On the other hand, the possibility of new inflation from strings was studied some years ago and shown to require a huge amount of fine-tuning \[19\]. Although these negative results are not general enough to exclude other kinds of inflation based on General Relativity (e.g. chaotic) which could arise from string theories, they make us search for inflation in more general theories of gravity.

\[5\] The case $\alpha = 0$, $\beta = 1/3$, giving $b = 1/2$, has recently been considered \[20\] and shown to be consistent with target space modular invariance. However, we will consider a more general case since non-perturbative effects could break modular invariance \[21\].
case where $W_o = \text{constant}$. (A constant superpotential can be triggered by the vacuum expectation value of some field.) In this case, $v_o = |W_o|^2$ and $v_n = (2 - n) |W_o|^2$.

b) There should be a positive cosmological constant at the minimum, i.e.

$$v_o(d) > 0 .$$

(11)

In particular, this implies that supersymmetry is broken at the minimum in such a way that

$$f^2_\alpha(d) + 3 f^2_\beta(d) > 3 f^2_\alpha(d) .$$

(12)

In the case $W_o = \text{constant}$, condition (11) is automatically satisfied.

c) The last condition is that the minimum is required to be stable along the inflaton field direction $C_n$, i.e.

$$v_n(d) > 0$$

(13)

or

$$f^2_\alpha(d) + (3 - n) f^2_\beta(d) > 2 f^2_\alpha(d)$$

(14)

where $n$ is the sector to which the inflaton belongs. In this way, the inflaton potential can trigger a first order phase transition from the false vacuum at $C_n = 0$ to the true physical vacuum at $C_n \neq 0$, which we assume to correspond to a zero cosmological constant.

In the simple case of $W_o = \text{constant}$, condition (13) is always satisfied for $n = 1$ (untwisted matter sector) but never satisfied for $n = 3$ (twisted matter sector). For $n = 2$ (blowing–up modes) $v_2 \equiv 0$ and so the stability along the inflaton direction $C_2$ would rely upon higher order derivatives of the potential and therefore upon the superpotential $W_2$.

In what follows we will assume that conditions (9), (11) and (13) hold and therefore will write the Lagrangian (1) for $\phi$ and the inflaton field $C_n$ as

$$\mathcal{L}_{\text{eff}} = \sqrt{-\tilde{g}} \left[ \tilde{R} - (6b^2 + \frac{1}{2}) (\partial_\mu \phi)^2 - e^{-n(\frac{2}{3} - b)\phi} (\partial_\mu C_n)^2 - e^{-(1-6b)\phi} \rho_0 + ... \right]$$

(15)

\text{Of course this would impose extra conditions on the total superpotential $W$, which we will not study here.}
where \( \rho_o \) is a constant energy density, we have used Eq.\((6)\) and absorbed all constant coefficients in the definition of \( C_n \). Notice that the energy density \( \rho(\phi) \) in \((13)\) is proportional to \( m_{3/2}^2 \), the scale of supersymmetry breaking (the gravitino mass), as expected,

\[
m_{3/2}^2 \propto e^{-(1-6\beta)\phi} |W_o|^2 .
\] (16)

The mass of the observable fields at the true vacuum depends on the global structure of the potential \( V \) in \((1)\), which is very poorly known in most cases. In fact, it depends on the total structure of the Kähler potential \((3)\) and the superpotential \((4)\), which could in turn depend on non–perturbative effects at high energy scales (string effects) and/or at low energy scales (QCD condensates, ...). We will assume for the masses a simple dependence

\[
m^2 \sim e^{-a\phi} m_o^2
\] (17)

where \( m_o \) is a constant mass and \( a \) is a real coefficient parametrizing our ignorance on the details of supersymmetry breaking in string theory and the low energy non–perturbative effects. The case of constant masses \( (a=0) \), considered in the analysis of Ref.\([15]\), is particularly interesting and will be commented later on.

Under a conformal redefinition \[22, 23, 13, 14\] of the metric

\[
\tilde{g}_{\mu\nu} = e^{c\phi} g_{\mu\nu}
\] (18)

\[
\tilde{R} = e^{-c\phi} \left[ R - c(D-1)D^2\phi - \frac{1}{4}c^2(D-1)(D-2)g^{\mu\nu}\partial_{\mu}\phi\partial_{\nu}\phi \right]
\] (19)

the masses transform as

\[
m^2 = e^{c\phi} \tilde{m}^2 
\] (20)

It is therefore convenient to make the conformal redefinition of \( g_{\mu\nu} \) \[13\] with parameter \( c = a \) such that the mass of the observable fields, see Eqs.\((17)\) \[23\], become \( \phi \)–independent \[14\]. Then \((13)\) can be written as \[13\]

\[
\mathcal{L} = \sqrt{-\tilde{g}} \left[ \Phi R - \frac{\omega}{\Phi} (\partial_{\mu}\Phi)^2 - \frac{1}{2} \Phi^{1-\beta'} (\partial_{\mu}C_n)^2 - \Phi^{2(1-\beta)} \rho_o \right]
\] (21)

\[7\]Recall that under a conformal redefinition of the Robertson–Walker metric, the scale factor and the time variable transform as \( \tilde{a}(\tilde{t}) = \Phi(t)^{1/2}a(t) \) and \( d\tilde{t} = \Phi(t)^{1/2}dt \) respectively.
where
\[ \Phi = e^{a\phi} \]  
(22)
and the parameters \( \omega, \beta \) and \( \beta' \) in (21) are defined as functions of \( a \) and \( b \) as
\[ 2\omega + 3 = \frac{1 + 12b^2}{a^2} \]  
(23)
\[ \beta = \frac{1 - 6b}{2a} \]  
(24)
\[ \beta' = n \left( \frac{1 - 2b}{2a} \right) \]  
(25)
Written in terms of a Robertson–Walker metric, \( \Phi(t) \) is a dimensionless scalar related to the variation of the Plank mass
\[ \Phi(t) = \frac{M^2_P(t)}{M^2_P} \]  
(26)
where \( M^2_P \) stands for \( M^2_P(t_o) \equiv 1/G_N \) (\( t_o \) is the present age of the universe), and we assume \( \Phi(t_e) \approx 1 \) at the end of inflation. We can also define the scales \( M \) and \( m_P \) through
\[ \rho(0) = \Phi(0)^{2(1-\beta)} \rho_o \equiv M^4 \]  
(27)
\[ \Phi(0) = \frac{m^2_P}{M^2_P} \]  
(28)
The equations of motion then read
\[ \left( \frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} = \frac{\rho_o}{6} \Phi^{1-2\beta} + \frac{\omega}{6} \left( \frac{\dot{\Phi}}{\Phi} \right)^2 - \frac{\dot{a}}{a} \frac{\dot{\Phi}}{\Phi} \]  
(29)
\[ \ddot{\Phi} + 3\dot{\Phi} = \frac{2\beta}{2\omega + 3} \rho_o \Phi^{2(1-\beta)} \]
with solutions for \( k = 0 \)
\[ a(t) = a(0)(1 + Bt)^p, \quad p = \frac{2\omega + 3 - 2\beta}{2\beta(2\beta - 1)} \]  
(30)
\[ \Phi(t) = \Phi(0)(1 + Bt)^q, \quad q = \frac{2}{2\beta - 1} \]
where
\[ B^2 = \frac{2\beta^2(2\beta - 1)^2}{(2\omega + 3)(6\omega + 9 - 4\beta)} \rho \Phi(0)^{1-2\beta} = \frac{2\beta^2(2\beta - 1)^2}{(2\omega + 3)(6\omega + 9 - 4\beta)} \left( \frac{M}{M_P} \right)^4 \left( \frac{m_P}{m_M} \right)^2. \]

We now raise the question of the sufficient conditions for extended inflation and whether or not a 'graceful exit' can be achieved.

First of all, we require that quantum gravity effects be negligible. In other words, that the kinetic energy due to de Sitter fluctuations (maximal at beginning of inflation [24, 6]) be less than \( \rho_0 \), see Eqs.(27–29), i.e.
\[ H^4(0) \approx M^8 \left( \frac{M_P}{m_P} \right)^4 < \rho(0). \]
This gives the constraint
\[ m_P > M. \]

We are assuming that the universe at \( T_c \) goes through a first order phase transition in which the high-temperature phase remains metastable down to \( T = 0 \) [2], where bubble nucleation is dominated by quantum mechanical tunneling [23]. Bubbles are assumed to be formed with zero radius at \( t_B \) and then expand at the speed of light. A bubble radius at a later time \( t > t_B \) is given by
\[ r(t, t_B) = \int_{t_B}^{t} \frac{dt'}{a(t')} . \]

We now define the asymptotic radius of a bubble nucleated at \( t \) as
\[ r_{as}(t) = \int_{t}^{\infty} \frac{dt'}{a(t')} = \frac{p}{p - 1} \frac{1}{a(t)H(t)} , \]
where \( H(t) \) is the Hubble expansion factor
\[ H(t) = pB \left( \frac{\Phi(0)}{\Phi(t)} \right)^{\beta - 1/2} = pB \left( \frac{a(0)}{a(t)} \right)^{1/p}. \]

The end of inflation is determined by the competition between the bubble nucleation rate and the cosmic expansion rate. The dimensionless parameter which controls the percolation of the phase transition can be computed as
\[ \epsilon(t) = \int_{t_B}^{t} dt' \lambda(t')a^3(t') \frac{4\pi}{3} r^3(t, t') \simeq \frac{\lambda(t)}{H^4(t)} \quad (p \gg 1) \]
where $\lambda(t)$ is the bubble nucleation rate per unit volume. In our model, $\lambda(t)$ is time dependent since the energy density which drives inflation is itself time dependent through $\Phi(t)$, see Eq.(21). Holman et al. [26] compute this dependence to be

$$\lambda(t) = \lambda_o \Phi(t)^{2(1-\beta')} e^{-B_o \left[ \Phi(t)^{2(\beta-\beta')} - 1 \right]}$$

(37)

where $\lambda_o = A e^{-B_o}$. $B_o$ is the Euclidean bounce action [25, 2], which depends on the inflaton potential and can acquire very big values $O(10^2)$, while the prefactor $A$ is of order one and has dimensions of $T^4_c$, where $T_c \sim M$ is the mass scale of the phase transition.

The epsilon parameter can then be written as

$$\epsilon(t) = \epsilon_o \Phi(0)^{2(1-2\beta)} \Phi(t)^{2(2\beta-\beta')} e^{-B_o \left[ \Phi(t)^{2(\beta-\beta')} - 1 \right]}$$

(38)

$$= \epsilon(t_e) \Phi(t)^{2(2\beta-\beta')} e^{-B_o \left[ \Phi(t)^{2(\beta-\beta')} - 1 \right]}$$

where $\epsilon_o \equiv \frac{\lambda_o}{H^4(0)}$ is the usual parameter considered in the literature.

A measure of the progress of the transition is the fraction of space which remains in the high temperature phase (‘false vacuum’), $p(t) = e^{-\epsilon(t)}$. We need a very small epsilon parameter at the beginning of inflation which increases very quickly to above a critical value, thus allowing for percolation of the low temperature phase (‘true vacuum’). Therefore we require

$$\epsilon(t_e) = \epsilon_o \Phi(0)^{2(1-2\beta)} > \epsilon_{cr}$$

(39)

where $1.1 \times 10^{-6} < \epsilon_{cr} < \frac{3}{4\pi}$ was computed in Ref.[2] for solving the ’graceful exit’ problem. Thus

$$\epsilon_o \geq \left( \frac{M}{M_P} \right)^{4(2\beta-1)} \epsilon_{cr}$$

(40)

which gives ample room for very small values of $\epsilon_o$, provided that $2\beta > 1$ (which is anyhow necessary for an increasing $\Phi(t)$). We must be sure,
however, that $\epsilon(t)$ is increasing, that is
\[
\frac{\dot{\epsilon}(t)}{\epsilon(t)} = 2(\beta' - \beta) \frac{\dot{\Phi}(t)}{\Phi(t)} \left[ B_0 \Phi(t)^{-2(\beta' - \beta)} - \frac{\beta' - 2\beta}{\beta' - \beta} \right] > 0
\] (41)
which is satisfied for
\[
\beta' > \beta
\] (42)
and
\[
B_0 \Phi(t)^{-2(\beta' - \beta)} > \frac{\beta' - 2\beta}{\beta' - \beta}.
\] (43)
This condition is very easily satisfied as we will see.

We are now ready to analyze our string model for inflation, see Eq.(21). The peculiarity of this model is the fact that $\omega$, $\beta$ and $\beta'$ are not independent but determined by the string effective action, see Eqs.(23–25). This dependence corresponds to the conformal redefinition of the metric tensor for which observable matter have constant masses, as discussed above. We will now impose further constraints on our model.

A necessary condition for inflation is that $\ddot{a} > 0$, or $p > 1$, which becomes
\[
0 < b < \frac{1}{2}.
\] (44)
We must also impose that $\Phi(t)$ increases, which gives the condition
\[
a < 1 - 6b.
\] (45)
The condition that $\epsilon(t)$ increases, see Eqs.(12, 13), then becomes
\[
a \geq 0
\] (46)
\[
B_0 > 1
\] (47)
which are both sufficient conditions for all values of $n$, see Eq.(25).

Assuming $N$ orders of magnitude increase in the scale factor,
\[
10^N = \frac{a(t_e)}{a(0)} = \left( \frac{\Phi(t_e)}{\Phi(0)} \right)^{p/q} < \left( \frac{M_p}{M} \right)^{p/q}
\] (48)
imposes the constraint
\[ a < \left( \frac{1 + 12b^2}{1 - 6b} \right) \frac{\log \left( \frac{M_T}{M} \right)}{N + \log \left( \frac{M_T}{M} \right)} \].

(49)

Furthermore, in order to solve the horizon problem we need sufficient inflation such that \[ d_{H_o} < d_{H(0)} \frac{a_o}{a(0)} \]. However, since \( H(t) \sim t^{-1} \sim T^2 \) and \( aT \sim \text{constant} \) during the post–inflationary period, and assuming 'good reheating' for recovering all the latent energy density of the phase transition \( (T_e \equiv T(t_e) \sim T_c \sim M) \), we obtain the condition
\[ N > p \frac{\rho}{p - 1} \log \left( \frac{M}{T_o} \right) \].

(50)

Therefore, the required number of orders of magnitude of inflation depends crucially on the energy scale of the phase transition \( M \).

Inflation must occur after the production of monopoles or any other topological defects whose densities might affect cosmology. For the same reason, the universe must also reheat before baryogenesis. These conditions place the constraint \( 10^2 \text{ GeV} < M < 10^{18} \text{ GeV} \).

However, solving the horizon and monopole problems is not enough. We must be sure that the phase transition ends and that all the bubbles percolate without disturbing too much the isotropy and homogeneity of the cosmic background radiation. Therefore, we expect that the volume fraction contained in bubbles with radius greater than a given comoving radius \( r = r(t_e, t) \) at the end of inflation be less than \( 10^{-n} \) at a temperature \( T \):
\[ \mathcal{V}(r, t_e) = 1 - p(t) \simeq \epsilon(t) = \epsilon(t_e) \left( \frac{T}{M} \right)^{\delta} e^{-B_o \left[ (\Phi')^{-1} \right]} < 10^{-n} \]

(51)

where we have used
\[ \Phi(t)^{2(2\beta' - \beta')} = \left( \frac{r_o}{r} \right)^{\delta} \simeq \left( \frac{T}{M} \right)^{\delta} \]

(52)

where \( r_o \equiv r_{as}(t_e) \) is the asymptotic radius of a bubble nucleated at the end of inflation, \( \delta \equiv \frac{8\beta(2\beta - \beta')}{2\omega + 3 - 4\beta^2} \) and \( \delta' \equiv \frac{8\beta(\beta' - \beta)}{2\omega + 3 - 4\beta^2} > 0 \). In particular,

\(^9\text{We use here the notation } H_o = H(t_o), a_o = a(t_o) \text{ and } T_o = T(t_o).\)
for the cosmic background radiation, we require that \( n \simeq 5 \) at \( T \simeq 1 \text{ eV} \) in (51). This condition is trivially satisfied thanks to the exponential, using \( \frac{M}{T} > 10^{11} \) and condition (47). In this way, the extended inflation problem of anisotropy at decoupling produced by large bubbles is successfully solved in this kind of models \(^4\).

We still have to be sure of reestablishing a common Robertson–Walker frame in all the bubble clusters that will coalesce to form our universe. There must be some way to remember the original (pre–bubble–nucleation) coordinates; such a record can be found in the evolution of \( a(t) \) or \( \Phi(t) \). Since constant \( H(t) \) corresponds in General Relativity to a de Sitter universe with no distinguished frame, we must require sufficient variations of this quantity, \( e.g. \) \( m \) orders of magnitude in \( H(t) \) \(^3,\, 12\). In particular, we expect that homogeneity and isotropy must hold by the time of nucleosynthesis \( (T_{\text{NS}} \simeq 1 \text{ MeV}, \, m \simeq 1) \), thus

\[
\frac{H(t)}{H(t_e)} = \left( \frac{r + r_o}{r_o} \right)^{\frac{1}{p-1}} \simeq \left( \frac{M}{T} \right)^{\frac{1}{p-1}} > 10^m
\]

corresponding to

\[
p < 1 + \log \left( \frac{M}{T_{NS}} \right) \equiv p_o
\]

which is an explicit bound on the power of the scale factor and gives an extra condition on our parameters

\[
a < \frac{p_o}{p_o - 1} (1 - 6b) - \frac{1}{p_o - 1} \left( \frac{1 + 12b^2}{1 - 6b} \right).
\]

Furthermore, quantum fluctuations of the scalar field \( \Phi \) create a spectrum of adiabatic fluctuations, which can be computed for power–law solutions in the Einstein frame \(^27\)

\[
\frac{\delta \tilde{\rho}}{\tilde{\rho}} \simeq \frac{\tilde{H}^2}{\pi \tilde{\phi}} \simeq \frac{\tilde{p}^{3/2}}{\pi} \cdot \frac{1}{\tilde{t}}
\]

and must be bounded in the conformal frame \( \rho = \Phi^2 \tilde{\rho} \) to be compatible with the observed density perturbations \(^28\)

\[
\frac{\delta \rho}{\rho} \simeq \left( \frac{M}{M_P} \right)^2 \tilde{p} \cdot \frac{k}{\pi} < 10^{-4} \quad (\tilde{p} \gg 1)
\]

\(^10\)In fact, this solution was proposed on general grounds in Ref.\(^{15}\).
where $k$ is the dimensionless physical scale of reentering perturbations and $\tilde{p}$ is the power of the scale factor in the Einstein frame \( \tilde{a}(\tilde{t}) \sim \tilde{t}^{\tilde{p}} \), \( \tilde{p} = \frac{2\omega + 3}{4\beta^2} \).

This imposes a very mild constraint on $M$

\[
M < \left( \frac{1 - 6b}{\sqrt{1 + 12b^2}} \right) 10^{-2}M_P < 10^{-2}M_P .
\]

Finally, the most stringent bounds will come from the post–Newtonian experiments of time delay [28, 10] and the nucleosynthesis bound [29] on the $\omega$ parameter

\[
2\omega + 3 > 500 \quad (2\sigma)
\]

which gives a very strong constraint on our parameters

\[
a < \sqrt{\frac{1 + 12b^2}{500}} .
\]

It is interesting to notice that the anisotropy of the cosmic background radiation, which was the main problem for extended inflation, does not impose any significant bound on our model, see Eq.(51). The most stringent bound comes from the post–Newtonian experiments and nucleosynthesis bound, see Eq.(60), which constrain the parameter $a$. On the other hand, the strongest constraint on $b$ comes from the isotropy and homogeneity at nucleosynthesis, see Eqs.(54, 55).

Most of the previous bounds depend on the energy scale $M$ of the phase transition. We have studied those bounds for two typical values of $M$.

For a phase transition driven by phenomenological supersymmetry breaking \( (m_{3/2} \simeq 1 \text{ TeV}) \) we have $M = (m_{3/2}M_P)^{1/2} \simeq 10^{11}$ GeV, while for the usual grand unified theory we take $M = 10^{16}$ GeV. The inflationary scenario is characterized by two parameters, the power $p$ of the scale factor and the number $N$ of orders of magnitude increase during inflation. Both parameters depend on the energy scale of the phase transition, see Eqs.(54, 54). For $M = 10^{11}$ GeV we have $p < 16$ and $N > 26$, while for $M = 10^{16}$ GeV, $p < 21$ and $N > 31$ to solve the horizon problem without disturbing the isotropy and homogeneity at nucleosynthesis. The actual value of $N$ depends on the parameters of the theory. Using the bounds (54), (58) and (60) we obtain $N > 45$, which widely solves the flatness problem.
In Fig.1 we show the region in parameter space \((a, b)\) allowed by all the inflationary conditions, for \(M = 10^{11} \text{ GeV}\) and \(M = 10^{16} \text{ GeV}\) (dashed and dotted curves respectively). The allowed region is the one below the curves. The condition associated with neglecting the quantum gravity effects \([32, 49]\) strongly depends on the energy scale of the phase transition, as expected, and as we can see from the lines labelled QG. Other conditions depend slightly on \(M\), like those associated with reestablishing the isotropic and homogeneous Robertson–Walker frame \([53, 55]\), and labelled RW in Fig.1. Finally, there are those conditions which do not depend at all on the energy scale of the phase transition, like the post–Newtonian bounds \([59, 60]\) and the condition \([57]\) that \(\Phi\) increases from \(m_P\) to \(M_P\), labelled pN–NS and \(\Phi\) respectively. However, as we can see from Fig.1, the final allowed region in parameter space does not depend much on the scale \(M\) since it is bounded by the post–Newtonian and nucleosynthesis bound and the isotropy and homogeneity condition at nucleosynthesis.

As we can see from Fig.1, the case of constant observable masses \((a = 0)\) is consistent with all inflationary and post–Newtonian bounds. This can be easily obtained by taking the limit \(a \to 0\) in our explicit solution \((30)\), which corresponds to

\[
\begin{align*}
a(t) &\sim t^{\frac{1+12b^2}{2}+\frac{14}{1-6b}} \\
\Phi(t) &\sim 1
\end{align*}
\]

On the other hand the direction \(b = 0\), see Eq.\((3)\), is incompatible with the necessary condition for inflation, Eq.\((44)\), and corresponds to the case of constant moduli. We thus agree with the negative results found in Ref.\([15]\).

In conclusion, we have studied in this paper the general conditions under which the effective theory of gravity from strings compactified to four dimensions could lead to extended inflation. We have found that a necessary condition is the existence of runaway directions in the space of fields coupled to the curvature scalar (dilaton and moduli fields). However, the existence of runaway directions is a usual feature of the effective theory of strings (through classical invariance arguments \([13]\)). It is satisfied for many supersymmetry breaking potentials. In the simplest case of supersymmetry breaking, a constant \(W_o\) superpotential in \((4)\), all moduli and the dilaton are runaway fields with a positive potential \((\sim |W_o|^2)\) and extended inflation may follow. However, to simplify the study of the equations of motion, we have assumed just one runaway direction and parametrized it by a real
parameter $b$. This is just a simplifying hypothesis since extended inflation might occur under much more general circumstances.

A second necessary condition for extended inflation is the existence of a metastable minimum along some (matter) inflaton field. This condition is necessary to enforce a first order phase transition. It also depends on the particular structure of the supersymmetry breaking superpotential, but this condition (see Eq. (13)) is easily satisfied in many models. For instance, in the simple case $W_\phi = \text{constant}$, it holds when the inflaton belongs to the untwisted sector ($n = 1$), and does not hold if it belongs to the twisted sector ($n = 3$). The case of the inflaton as a blowing–up mode ($n = 2$) would require the precise knowledge of the total superpotential.

Third, we assume a simple behaviour of the mass of observable fields on the runaway direction, and parametrize this behaviour with a real parameter $a$. (The case of constant masses corresponds to $a = 0$.) We make a conformal redefinition of the metric in order to go into the ‘physical’ frame, where the masses of the observable fields are constant. Of course, if the functional dependence of masses were more complicated, we would have needed a more general conformal transformation and the theory would look different, in particular it would have a non–constant $\omega$ parameter. However, we should stress here that a conformal redefinition is not essential since Physics cannot depend on it. In other words, we could redefine the physical scale factor by taking its ratio with respect to the Compton wavelength [30] which is then manifestly independent of the conformal transformation [15, 14].

Finally, we have imposed all the conditions for successful extended inflation on the solution of our model and found an allowed region in parameter space $(a, b)$. Our results are summarized in Fig.1. The direction $b = 0$ (the region of constant moduli) is excluded from the allowed region, while $a = 0$ (the case of constant masses) is inside the permitted region and therefore consistent with all experimental bounds. Notice that our model successfully solves the $\omega$–problem of extended inflation (namely, that the condition of isotropy of the cosmic background radiation at decoupling is in conflict with the post–Newtonian bounds on $\omega$) by producing very small bubbles until the end of inflation when the epsilon parameter increases exponentially up to the critical value.
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Figure Captions

Fig.1 Plot of the region in parameter space \((a, b)\) allowed by the inflationary conditions in the text. The solid lines represent those bounds which do not depend on the scale \(M\) of the phase transition. The dashed curve correspond to the bounds associated with the scale \(M = 10^{11}\) GeV and the dotted curve to those related to \(M = 10^{16}\) GeV. The allowed region is the one below the curves. The border \(b = 0\) is excluded from it. We have labelled the curves as follows: QG corresponds to the condition associated to neglecting quantum gravity effects, \(\Phi\) corresponds to the condition for an increasing scalar field, RW corresponds to the bound on isotropy and homogeneity at the time of nucleosynthesis and pN–NS corresponds to the bounds from post–Newtonian experiments and the nucleosynthesis bound on \(\omega\).