Atom-Based RF Electric Field Measurements:  
An Initial Investigation of the Measurement Uncertainties

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Abstract: We discuss a new method for the measurement of electric (E) fields that will lead to a self-calibrating, direct SI-traceable E-field probe. The technique is based on radio frequency E-field interactions with alkali atoms placed in glass cells. After we present the concept of this approach and present some experimental data to show its validity, we give a discussion of the different types of uncertainties that are associated with this new approach. We discuss how the uncertainties of this approach compare to commonly used E-field measurement techniques.

I. Introduction

A stated goal of international metrology organizations to make all measurements quantum traceable is to the SI. To date, methods to make electric (E) field measurements require a complex indirect traceability paths. E-field measurements based on the atom allow a simple direct traceability path. Measurement standards based on atoms have been used for a number of years for a wide array of measurements; most notable are time, frequency, and length. Here we discuss how the atom can be used for E-field measurements.

In recent work, we have demonstrated a fundamentally new approach for E-field measurements [1]-[6]. The new approach is based on the interaction of RF E-fields with Rydberg atoms, where alkali atoms are excited optically to Rydberg states and the applied RF E-field alters the resonant state of the atoms. The new approach discussed here allows for a direct SI-traceable measurement of an E-field. For this technique, the Rydberg atoms are placed in a glass vapor cell. This vapor cell acts like an RF-to-optical transducer, converting an RF E-field to an optical frequency response. The approach utilizes the concept of electromagnetically induced transparency (EIT), where the RF transition in the four-level atomic system causes a splitting of the transmission spectrum for a probe laser. This splitting is easily measured and is directly proportional to the applied RF E-field amplitude (through Planck’s constant and the dipole moment of the atom). Therefore, by measuring this splitting we get a direct measurement of the RF E-field strength.

Unlike the Rydberg atom approach discussed here, previous work has been done utilizing ground-state atomic transitions in alkali atoms for power standards and magnetic field measurements [7-11]. The significant dipole response of Rydberg atoms over the GHz regime indicates that the EIT technique can be used for traceable measurements over a large frequency band, roughly 1 GHz to 500 GHz [1]. The new approach has several benefits over existing techniques, including, 1) a direct SI units linked E-field measurement, 2) a self-calibrating measurement due to atomic resonances, 3) large bandwidth, 4) a technique that is independent of current approaches, 5) a very small spatial resolution (optical fiber and chip-scale), and 6) a technique with vastly improved sensitivity and dynamic range over current E-field methods.

For the new technique to be viable, it is important to understand its measurement uncertainties. In this paper, we discuss how the uncertainties can be grouped into two different categories: (a) quantum based uncertainties (i.e., parameters and issues related to the atomic physics aspect of the technique) and (b) RF based uncertainties (i.e., parameters and issues related to the RF aspect of the technique). We discuss the dominant contributions for these two types of uncertainties. The uncertainties of the new measurement technique can be smaller than existing measurement approaches, as such, enabling the benefits discussed above.
II. The EIT Technique

The basic concept uses vapor of alkali atoms (placed in a glass cell, referred to as a “vapor” cell) as the active medium of the radio frequency (RF) E-field probe. The EIT technique involves using a laser (referred to as the “probe” laser) to probe the atoms placed in the glass cell, see Fig. 1. A detailed explanation both from an atomic physics viewpoint and from an effective media viewpoint is given in [1]. In effect, the RF E-fields alter the susceptibility of the atomic vapor seen by the “probe” laser as it propagates through the vapor cell. By detecting the probe light propagating through the cell, the RF E-field strength can be determined.

![Fig. 1: Illustration of a vapor cell setup for measuring EIT, with counter-propagating probe and coupling beams.](image)

When the frequency of the light matches the ground state atomic resonance (“probe” laser), the atoms scatter light from the incident beam and reduce the transmitted light intensity. If a second strong (“coupling”) light field is applied that matches a second atomic resonance, a destructive quantum interference of these excitation pathways occurs. As such, a transparency window is opened for the probe laser light: probe light transmission is increased. This is the phenomenon known as EIT [12-14]. If one then applies an RF E-field which couples to an additional atomic state, a constructive interference in the probe light absorption occurs. This has the effect of splitting the EIT resonance in two, see Fig. 2. In this figure, \( \Delta p \) is the detuning of the probe laser from the ground state transition of the atom (where \( \Delta p = \omega_p - \omega_0 \); \( \omega_0 \) is the on-resonance angular frequency of the ground state transition and \( \omega_p \) is the angular frequency of the probe laser).

The separation between the two peaks (\( \Delta f \)) in the probe laser spectrum is measured in Hertz, and is related to the amplitude of the applied RF E-field by [1,2]:

\[
|E| = \frac{2\pi \hbar}{\phi} \frac{\lambda_p}{\lambda_c} \Delta f ,
\]

where \( \hbar \) is Planck’s constant, \( \phi \) is the atomic dipole moment of the RF atomic transition; \( \lambda_p \) and \( \lambda_c \) are the wavelengths the probe and coupling lasers, respectively. This expression assumes the probe and coupling lasers are counter propagating in the cell [1].

We consider a measurement based on (1) a direct SI traceable measurement in that it is related to Planck’s constant. To estimate \( |E|, \Delta f \) is obtained from a measurement, Planck’s constant is known, and \( \phi \) is calculated using first principles (see [1] for discussion on determining \( \phi \)).

A large range of atomic transitions can be selected, allowing measurements of RF E-fields over a wide selection of frequencies. In essence, the atoms act as highly-tunable, resonant RF detectors. We used this fact to show how the technique can perform broadband measurements of RF E-fields, ranging from 1 GHz to 500 GHz [1]. The technique has also been used for sub-wavelength imaging [3, 6], for millimeter wave and sub-terahertz wave traceable calibrations (not previously possible) [4], and for full-vector field measurements [5].

![Fig. 2: Illustration of the EIT signal (i.e., probe laser transmission through the cell) as a function of probe laser detuning \( \Delta p \).](image)
A representative experimental setup is shown in Fig. 3 and Fig. 4 shows a set of experimental data for the measured field strength $|E|$. In this experiment three different horn antennas were used depending on the RF E-field measured. These results were obtained for a vapor cell filled with cesium ($^{133}$Cs) atoms. The probe laser is an 852 nm laser scanned over the ground state transition of $^{133}$Cs and the coupling laser is approximately 510 nm (the wavelength of this laser determines which RF frequency is measured, see [1] for details). This figure illustrates that the experimental data follow a linear trend as predicted from theory.

Fig. 5 shows a comparison of the atom-based field measurement to results obtained from far-field calculations and to full wave numerical simulations. These results were obtained for a vapor cell filled with rubidium-85 ($^{85}$Rb) atoms. The probe laser is a 780 nm laser scanned over the ground state transition of $^{85}$Rb and the coupling laser is around 480 nm (the wavelength of this laser determines which RF frequency is measured, see [1] for details). The far-field calculations were obtained by taking into account the distance from the antenna to the cell, the gain of the antenna, and cable losses. For the higher frequency comparisons, the numerical simulations were obtained from a commercially available finite-element code. We see very good agreement between the far-field calculated, the numerical simulations, and the atom-based E-field measurements. The results in Figs. 4 and 5 illustrate the validity of the atom-based measurement approach.

Fig. 3: Experimental setup for an E-field measurement using EIT.

Fig. 4: Experimental data for a 12-mm $^{133}$Cs filled vapor cell as a function of the square-root of power at the RF signal generator.

Fig. 5: Experimental data for a $^{85}$Rb filled vapor cell as a function of the square-root of power at the RF signal generator. These data compare the measured $|E|$ to values obtained from both far-field calculations and from numerical simulations.
III. Uncertainties

For this type of measurement technique to be widely accepted, an understanding of the uncertainties in this type of measurement is needed. The uncertainties can be grouped into two different categories: (a) quantum based uncertainties, and (b) RF based uncertainties. Ref. [2] discussed various quantum based uncertainties. The major contribution to the quantum based uncertainties is the determination of the atomic dipole moment. The atomic dipole moment is obtained from two numerical calculations, see [1] for details. The calculation requires one to first numerically solve Schrödinger’s equation for the atomic wavefunctions, and then a numerical evaluation of the radial overlap integrals involving the wavefunctions for a set of atomic states. For a given atomic transition, these numerical simulations require one to use the quantum defects for $^{133}$Cs or Rb. Fig. 6 shows the results of such a calculation for a particular RF atomic transition for both $^{85}$Rb and $^{133}$Cs atoms ($\varphi \propto Q$, where $Q$ is a normalized parameter, see [1] for details). Using the best available quantum defects to perform a numerical calculation of $\varphi$ it is believed that $\varphi$ can be determined to less than 0.1 %. Direct measurements of dipole moments confirming the accuracy and uncertainty of the calculation method are desirable. Individuals are currently looking at methods to obtain more accurate values for $\varphi$. With that said, we believe our worst case quantum-based uncertainty is on the order of 0.1 %.

The largest source of measurement uncertainty is RF based, mainly due to the fact that we are using a glass (or dielectric) cell to hold the atoms. As a result, the RF field interacts with the cell itself. The dominant interaction occurs because when an RF wave is incident onto a hollow glass cell, standing waves can develop on the inside of the cell due to the internal reflection from the glass walls of the cell. The distribution of the E-fields inside the cell will vary depending on the frequency of the RF E-field and on the size of the cell. This may result in the field inside the cell being different than the incident field (the desired measured quantity). We first demonstrated this phenomenon in [3] where we showed that by scanning two lasers along the cross-section of a cell, the RF E-field distribution in the cell can be imaged and mapped. Fig. 7 shows the field distribution inside the cell for two different RF frequencies (17 GHz and 104 GHz). The data were obtained for a $^{85}$Rb filled cylinder cell of diameter 25 cm and length 75 cm. The results in this figure show that the field varies inside the cell by as much as 20 %. This variability of the fields inside the cell results in a 20 % uncertainty in the estimated E-field strength.

This perturbation can be reduced by making the cell as small as possible. To illustrate this we performed measurements on a 12 mm $^{133}$Cs filled vapor cell. In these measurements, the cell was placed on a micrometer and scanned across two counter-propagating lasers (850 nm and 510 nm). This allows us to image the fields inside the cell for various RF sources. The magnitude of the RF field strength inside the cell is shown in Fig 8. As expected, the large variability in the fields inside the cell is reduced for relatively small cell sizes, i.e., as long as the cell size is much less than the wavelength. In these measurements we see the spatial variability of the electric-field stabilize when the cell size is below $\lambda/4$ (where $\lambda$ is the RF wavelength).
The field variability inside the cell can be approximated with a simple one-dimensional 5-layer material model: the region inside the cell, two glass walls and the surrounding free-space. For a given incident field, the field variability between the two glass walls can be determined with the 5-layer model. The field variability between two glass walls obtained from this model is shown in Fig. 9 for different $D/\lambda$ (where $D$ is the cell dimension, 8 mm in this example). The results in this figure and those in Fig. 8 illustrate that once $D/\lambda<0.05$ the variability inside the cell is less than 2.5%. This corresponds to less than 0.2 dB uncertainties in estimating (and/or measuring) the magnitude of the E-field.

Anechoic chambers or transverse electromagnetic (TEM) cells are used to generate a known field in order to calibrate commonly used E-field probes, which leads to a chicken-or-egg dilemma. That is, to calibrate a probe, we need a known field. But, to know the field, we need a calibrated probe. Due to the uncertainties in this approach, the “known” field is typically known to within 5% (or 0.5 dB). This results in an uncertainty in a field measurement of 0.5 dB. Thus, from the estimated uncertainties in the previous paragraph, the atom-based technique can be controlled and reduced to be less than the current E-field measurement uncertainties.

Although not discussed in detail here, the inhomogeneity in the field that the probe laser see as it propagates across the cell can also affect the uncertainties of the atom approach. Information on the inhomogeneous field distributions inside the cells can be obtained directly from the line shapes, multiplicity, and signal strengths of the atomic spectra [15]. The uncertainties due to field inhomogeneities in the cell can therefore in principle also be accounted for and will be investigated later.

IV. Summary

We have discussed a fundamentally new approach for measuring E-fields. This approach is based on alkali atoms placed in glass vapor cells. We have demonstrated good agreement between the atom-based approach and values obtained from both far-field calculations and numerical simulations. We have done an initial investigation on the types of uncertainties related to this approach. We have concluded that the probe based on atomic vapor cells can be designed to obtain uncertainties less than the uncertainties in commonly used probe technology. While more work is needed to fully understand all the uncertainties related to this type of measurement, we can argue that this atom-based field probe is a viable approach for self-calibrating, SI traceable E-field measurements.
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