Data Association for a Hybrid Metric Map Representation

Shugen Ma, Senior Member, IEEE, Shuai Guo, Minghui Wang and Bin Li

Abstract—This paper presents an approach to solve the data association problem for a hybrid metric map representation. The hybrid metric map representation uses Voronoi diagram to partition the global map space into a series of local subregions, and then a local dense map is built in each subregion. Finally the global feature map and the local maps make up of the hybrid metric map, which can represent all the observed environment. In the proposed map representation, there exists an important property that global feature map and local maps have clear one-to-one correspondence. Benefited from this property, an identifying rule of the data association based on compatibility testing is proposed. The identifying rule can efficiently reject the wrong data association hypothesis in the application of dense environment. Two experiments validated the efficiency of data association approach and also demonstrated the feasibility of the hybrid metric map presentation.

I. INTRODUCTION

SLAM solves the problem that a robot simultaneously constructs a map of its nearby environment and localizes itself in this map, while it explores an unknown environment. In 1987 Smith, Self and Cheeseman [1] firstly explained the need for simultaneous localization and mapping (SLAM) and presented a solution based on extended Kalman filter (EKF). Since then, SLAM has obtained wide interest and a number of achievements have been published [2]–[6]. Currently, dense map representations have been adopted widely in the field of SLAM, and they can be classified into two categories: unitary metric map representations and hybrid map representations. The former includes grid map [6], sampled environment map (SEM) adopted in SLAMSEM [7] and BS-SLAM [8]. Grid map can describe the complex environment, but it lacks an explicit meaning of the environment. In SLASEM and BS-SLAM the sensor’s observation and features stored in map have not a clear one-to-one correspondence. Compared with the unitary metric map representation, Hybrid map representations extract obvious environment features and store the rest information of environment in local maps. This map structure is compact and efficient. Blanco thus proposed a hybrid metric-topological map based SLAM algorithm [9]. The map consists of nodes and local maps. Slightly differently, DenseSLAM [10] adopted a hybrid metric map.

The map consists of a global metric feature map and metric local maps. Our interest in hybrid map representations is the aspect of data association. In the hybrid metric-topological map, nodes are not real environment’s contour. They are not specific geometry elements, and thus cannot be used to solve the data association problem without help of local maps. The problem of the DenseSLAM is that the multi-to-multi correspondence between features and local maps prevents any benefit brought to data association from local maps. In our previous works [11] [12] we proposed a hybrid metric map representation and introduced the preliminary implementation. In this paper we propose an identifying rule benefited from the structure of the proposed hybrid metric map. The proposed identifying rule can efficiently increase the accuracy and robustness of data association.

Moreover, the data association (DA) is an important part of SLAM algorithms. Most DA algorithms are based only on the Mahalanobis distance. The classic DA algorithms include NN (Nearest Neighbor) algorithm [13] and JCBB (Joint Compatibility and Branch and Bound) algorithm [14]. NN computes the Mahalanobis distance between one feature stored in feature map and one current observed feature (individual compatibility). The experiment results in section V show that NN is not adequate to solve the DA problem. JCBB computes the Mahalanobis distance between a group features stored in the feature map and a group current observed features (joint compatibility). As an advantage of JCBB, it can reject the ambiguous DA hypothesis in the application of cluster environments. However, JCBB algorithm does not guarantee to solve our DA problem if two features or only one feature have been observed at many time instants. It has been proven that our proposed map representation could represent all the observed environment. An important property of this representation is a clear one-to-one corresponding relationship between global feature map and local maps. Local maps contain rich information of environment. Furthermore each local map is an invariant of its corresponding feature. Benefited from this property, an identifying rule based on compatibility testing is proposed to increas DA's accuracy.

II. A HYBRID METRIC MAP

In the proposed hybrid metric map representation [11], the global map $M^f$ consists of “corner” features ($f_i \in 1,2, \cdots$), which contain both position and orientation information $f_i = [x_i, y_i, \theta_i]^T$. By operating Voronoi diagram on the feature map (only the position is used), the map space ($\mathbb{R}^2$ for 2D case) is partitioned into a series of subregions $VD(M^f) \equiv \text{Voronoi}(M^f)$ (1)

This work is supported by the National Natural Science Foundation of China (60905058).

Shugen Ma is with Department of Robotics, Ritsumeikan University, 1-1-1 Noji-Higashi, Kusatsu-Shi, Shiga 525-8577, Japan and he is also with the State Key Laboratory of Robotics, Shenyang Institute of Automation, Chinese Academy of Sciences, Shenyang, 110016, China. {shugen@se.ritsumei.ac.jp

Shuai Guo, Minghui Wang and Bin Li are with the State Key Laboratory of Robotics, Shenyang Institute of Automation, Chinese Academy of Sciences, Shenyang, 110016, China. {guoshuai, mhwang, libin}@siae.cn. Shuai Guo is also with the Graduate School of the Chinese Academy of Sciences, Beijing 100039, China.
Each subregion contains only one feature. We define
\[
VP_i \triangleq gVoronoi(M^f, f_i)
\]
(2)
It denotes the subregion that includes feature \(f_i\). The property of Voronoi diagram is
\[
\|q, f_i\| \leq \|q, f_j\|, \quad j \neq i \quad \forall q \in VP_i
\]
where \(\|a, b\|\) denotes the Euclidean distance between points \(a\) and \(b\).

Voronoi diagram is a fundamental technique in SLAM area. In [15], [16] the generalized Voronoi graph (GVG) was built as a topological map of the environment. GVG contains edges and nodes. They together indicate the accessibility, connectivity and departability of the environment. In our work, the standard Voronoi diagram is adopted to partition the global map into subregions. Voronoi is not a map but is just a tool to partition global map into local maps.

Voronoi diagram allocs a subregion \(VP_i\) to each feature \(f_i\). On each feature a local coordinate system \(L_i\) is fixed. A local map \(m_i\) is built in each \(L_i\) and it describes the environment in \(VP_i\). Finally the global feature maps and the set of local maps make up of the hybrid metric map \(M = \{M^f, M^l\} = \{\{f_i, m_i\}|i = 1, \cdots, n\}\). Figure 1 is a schematic drawing of the hybrid metric map representation. The feature map consists of corners \(M^f = \{C1, C2, \cdots, C6\}\), which are used to partition the map space into six subregions by the Voronoi diagram (green lines). The local map \(m_1\) (blue dashed lines) is represented in coordinate system \(L_1\) (the red arrows) and it describes the detailed environment in \(VP_1\). The other five coordinate systems and local maps are built in the same way.

The hybrid metric map representation needs that “corner” features indeed exist in the environment. Except this requirement there is no limitation to the environment. So the local maps can describe complexity environment. This ability is verified by the experiment in section V. The clear one-to-one correspondence exists between features and local maps. This brings benefits to data association process. Compared with other hybrid map representations, this is an advantage of this hybrid metric map representation.

III. SLAM USING THE HYBRID METRIC MAP

First, the implementation of the SLAM algorithm using the hybrid metric map is introduced in this section. Then, a theorem proves that the local map is intact and the set of local maps can describe all the observed environment in feature-rich environment.

A. Preprocessing of Raw Sensor Data

The preprocessing of raw sensor data includes feature extraction, data segmentation and coordinate transformation of raw sensor data.

1) Feature Extraction: In this work the feature is a corner, which is defined as a point of intersection of two adjacent lines. The iterative end point fit (IEPF) algorithm [17] is used to extract lines from raw sensor data \(Z^0\). If both of the two adjacent lines are long enough, a corner \(g_j\) (unknown data association results) is extracted. Its direction parallels with the angular bisector of the two adjacent lines.

\[
S^g = \{g_j|j = 1 : k\} \leftarrow^\text{feature extraction} (Z^0)
\]
(4)

2) Data Segmentation: Once features \(S^g = \{g_j|j = 1 : k\}\) have been extracted, Voronoi partitions the observation plane into subregions. The sensor data in each subregion is segmented from \(Z^0\)

\[
z^g = \{d|d \in Z^0 \& d \in gVoronoi(S^g, g_j)\}
\]
(5)
where \(d\) is a 2D point, \(d = [x, y]^T\).

3) Coordinate Transformation: Each time a feature \(g_j\) is extracted, a local coordinate system \(L_j\) is inherently built. \(L_j\) locates on \(g_j\) and its X axis parallels with the direction of \(g_j\).

Currently \(z^g\) is represented in sensor’s coordinate system. Its equivalent representation in \(L_j\) is \(^1\)

\[
z^g_l = g_j \oplus z^g_j
\]
(6)
\(^1\ominus\): compound operator, \(\oplus\): compound operator. See reference [1]
After the above preprocessing of raw sensor data, feature \( g_j \) and sensor data \( z_i^{g_j} \) constitute a one-to-one correspondence. Figure 2 shows the above processes.

B. Procedures of the SLAM Algorithm

The SLAM algorithm includes the process of global map’s building and process of local maps’ building. The global feature map is built through standard EKF-SLAM. Local maps building is the important part of this algorithm.

1) Global Feature Map’s Building: In EKF-SLAM, the robot’s pose \( X_r = [x_r, y_r, \theta_r]^T \) and the feature map \( M^f = \{ f_i | i = 1, \ldots, n \} \) make up of the state of EKF

\[
X = [X_r^T, f_1^T, f_2^T, \cdots, f_n^T]^T
\]

The covariance matrix is

\[
P = \begin{bmatrix}
    p_{r_r} & p_{r_{f_1}} & p_{r_{f_2}} & \cdots & p_{r_{f_n}} \\
p_{f_1r} & p_{f_1f_1} & p_{f_1f_2} & \cdots & p_{f_1f_n} \\
p_{f_2r} & p_{f_2f_1} & p_{f_2f_2} & \cdots & p_{f_2f_n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
p_{f_nr} & p_{f_nf_1} & p_{f_nf_2} & \cdots & p_{f_nf_n}
\end{bmatrix}
\]

EKF estimates the state by two step: observation update and motion predict. They update the state \( X \) and its covariance matrix \( P \), while considering a frame of sensor data and robot’s one-step-movement respectively.

The update of global feature map is relatively independent from local maps. However, the feature’s calibration discussed in section IV shows that it is interactive with the update of local maps.

2) Local Dense Map’s Building: Local maps’ generation and their update are simultaneous with features’ generation and their update. The process is described as follows.

a) Generating of a new local map

If \( g_j \) has not been represented in feature map \( M^f \), a new feature \( f_i \) is added into \( M^f \). At the same time, a new local map \( m_i \) is generated:

\[
\begin{cases}
    z_i^{g_j} \mapsto m_i \\
    L_j \mapsto L_i \\
    z_i^s, Z_i = \Theta g_j + Z^0 \\
    z_i^s, X_i = \Theta g_j
\end{cases}
\]

where \( z_i^s \) saves the current sensor data \( Z^0 \) and robot’s relative pose in coordinate system \( L_i \). It is not a part of the hybrid metric map, but it will be used in the data association process.

b) Update of a local map

If \( g_j \) is an environment corner \( f_i \) represented already in \( M^f \), the feature map \( M^f \) is updated through the observation update step in EKF. At the same time, the local map \( m_i \) is updated through fusing the current sensor data \( z_i^{g_j} \) into \( m_i \),

\[
m_i = f(m_i, z_i^{g_j})
\]

Adding \( z_i^{g_j} \) directly to \( m_i \) is the most simple implementation of function \( f \). However, the error occurred in the feature extraction will make the final \( m_i \) be inconsistency with the environment. This problem will be discussed in the data association step.

C. Proof of the Hybrid Metric Map

In the procedures above, because Voronoi operates only on the current observed features \( S^g = \{ g_j | j = 1, 2, \cdots, k \} \) rather than on the whole global feature map \( M^f = \{ f_i | i = 1, 2, \cdots, n \} \), the final hybrid metric map is not exactly the same as the one described in section II. However, with a reasonable precondition, the theorem presented below ensures that the final hybrid metric can represent all the observed environment in feature-rich environment.

Theorem. Local map \( m_i \) can describe all the observed environment in \( gVoronoi(M^f, f_i) \).

Proof. At a time instant \( f_i \) is observed, feature map \( M^f \) will be updated through the observation-update step of EKF-SLAM and all the current observed features \( S^g = \{ g_j | j = 1, 2, \cdots, k \} \) will be represented in \( M^f \). This process means that after feature map’s update \( S^g \subseteq M^f \).

\[
S^g \text{ is re-formulared as } S^g = \{ f_i, f_k | \delta \in [1, 2, \cdots, n] \}. m_i
\]

is updated by fusing the segmented data \( z_i^{f_k} \) into local map \( m_i \). \( z_i^{f_k} \) contains all the sensor data \( d \) that has the following property:

\[
||d - f_i|| \leq ||d - f_k||, \delta \in [1, 2, \cdots, n] \quad \delta \neq i
\]

A local map \( m_i \) is assumed. It represents the observed environment that is limited in \( gVoronoi(M^f, f_i) \) exactly. \( \forall d \in m_i \), it has:

\[
||d, f_i|| \leq ||d, f_j||, j = 1, 2, \cdots, n \quad j \neq i
\]

Because \( S \subseteq M^f \), it is deduced that the sensor data \( d \) satisfying equation (12) must satisfy equation (11). This conclusion means that \( z_i^{f_k} \) contains at least the observation of the environment limited in \( gVoronoi(M^f, f_i) \).

Local map \( m_i \) has fused all the \( z_i^{f_k} \)’s which are produced when feature \( f_i \) is observed. If all the environment contour in \( gVoronoi(M^f, f_i) \) and feature \( f_i \) can be observed at the same time, at least one time. \( m_i \) will represent the observed environment in \( gVoronoi(M^f, f_i) \) completely. \( \square \)

The precondition in the proof is not satisfied in some environments, however, there are no sensor data been deserted in the generating and update of local maps. This character consequentially makes the hybrid metric map capable of representing all the observed environments.

IV. DATA ASSOCIATION

Equation (13) shows the structure of the hybrid metric map. The map representation keeps clear one-to-one correspondence between features and local maps. In the hybrid metric map, \( m_i \) represents the nearby environment of \( f_i \). \( m_i \) will not change its position and shape, since it is represented in coordinate system \( L_i \). Similarly, \( z_i^{g_j} \) is represented in \( L_i \) and it represents \( g_j \)’s nearby environment. So the one-to-one corresponding relationship between features and local maps (between \( m_i \) and \( f_i \) between \( g_j \) and \( z_i^{g_j} \)) straightforwardly brings benefits to data association.

\[
M = \left\{ \begin{array}{ll}
M^f = \{ f_1, f_2, f_3, \cdots, f_n \} \\
M^g = \{ m_1, m_2, m_3, \cdots, m_n \}
\end{array} \right\}
\]
Assuming that there are no errors in features’ extraction process, \( m_i \) and \( z_i^{(j)} \) are compatible in some sense if \( f_i \) and \( y_j \) represent one of the same environment feature.

Benefited from the one-to-one corresponding relationship between features and local maps, we present an identifying rule of data association based on compatibility testing.

### A. Feature Calibration

For each potential DA hypothesis \( [f_i, g_j] \), the error occurred in \( g_j \)'s extraction process is calibrated by iterative closest point (ICP) algorithm [18].

\[
\delta = \arg \min_{\delta^*} \sum_q ||d^q(m_i) - (R(\delta^*) \ast d^q(z_i^{(j)\*}) + t(\delta^*))||^2 
\]

where \( \{d^q(m_i), d^q(z_i^{(j)\*})\} \) is a pair of closest points in ICP algorithm, \( R(\delta^*) \) and \( t(\delta^*) \) are the equivalent rotation matrix and translation vector of \( \delta \).

![Fig. 3. The principle of feature calibration. After feature calibration, the error of feature extraction decreases greatly. The calibration procedure brings benefits to the compatibility testing because it prevents the right DA hypothesis from obtaining a big \( R_u \) and \( \alpha_u \). It also increases the accuracy of both feature map and local maps’ building.](image)

Figure 3 illustrates the principle of the calibration process. Compared with \( m_i \), \( z_i^{(j)}.Z \) can be regarded as the initial map of \( m_i \) and it does not contain the error accumulated from the fusion step of equation (10). So \( z_i^{(j)}.Z \) is chosen as the reference of calibration.

\[
\delta = \arg \min_{\delta^*} \sum_q ||d^q(z_i^{(j)}.Z) - (R(\delta^*) \ast d^q(z_i^{(j)\*}) + t(\delta^*))||^2 
\]

If \( [f_i, g_j] \) is finally determined as a right DA hypothesis, \( g_j \) and \( z_i^{(j)} \) are calibrated as well.

\[
\begin{align*}
    g_j &= g_j \oplus (\oplus \delta) \\
    z_i^{(j)} &= \delta \oplus z_i^{(j)}
\end{align*}
\]

In the observation update of EKF-SLAM, \( g_j \) is the observation of feature \( f_i \). This calibration increases the accuracy of features’ observation and furthermore increases the accuracy of feature map \( M^f \). Similarly the calibrated \( z_i^{(j)} \) increases the accuracy of local maps’ fusion step in equation (10).

### B. Compatibility Testing

Feature calibration decreases the error of feature extraction greatly. After aligning \( L_i \) with \( L_i, m_i \) and \( z_i^{(j)} \) should overlap together if \( [f_i, g_j] \) is a right DA hypothesis. Figure 4 is an example of wrong DA hypothesis. \( z_i^{(j)} \) is observed from current robot’s pose \( \oplus g_j \), \( z_i^{(j)} \) and \( \oplus g_j \) together determines a part of free area (white color) of the environment (grey color for unknown area). In the testing process of DA hypothesis \( [f_i, g_j] \), a part of local map \( m_i \) appears in the free area. This part of \( m_i \) should shadow the visual line of sensor and prevent the sensor from exploring the free area at a father distance. Areas A and B in figure 4 are two examples of this incompatible phenomenon. Area C is also a sample of this incompatible phenomenon. However, area C cannot be recognized from the knowledge of \( z_i^{(j)} \) and \( m_i \), because this part of \( m_i \) does not appear in the free area discussed above. The final solution is to test the compatibility between \( \oplus g_j \oplus Z^0 \) and \( z_i^{(j)}.X_i, z_i^{(j)}.Z_i \). As shown in figure 4, in the area C a part of \( \oplus g_j \oplus Z^0 \) appears in the free area determined by \( z_i^{(j)}.X_i \) and \( z_i^{(j)}.Z_i \).

Two indexes are adopted to represent the incompatibility. They are the shadowed area \( R_u \)

\[
\begin{align*}
    R_{u1} &= g(\oplus g_j \oplus Z^0, z_i^{(j)}.Z_i) \\
    R_{u2} &= g(z_i^{(j)}.X_i, z_i^{(j)}.Z_i, \oplus g_j \oplus Z^0) \\
    R_u &= R_{u1} + R_{u2}
\end{align*}
\]

and shadowed angle \( \alpha_u \):

\[
\begin{align*}
    \alpha_{u1} &= h(\oplus g_j, \oplus g_j \oplus Z^0, z_i^{(j)}.Z_i) \\
    \alpha_{u2} &= h(z_i^{(j)}.X_i, z_i^{(j)}.Z_i, \oplus g_j \oplus Z^0) \\
    \alpha_u &= \alpha_{u1} + \alpha_{u2}
\end{align*}
\]

where \( \alpha_{u1} \) and \( \alpha_{u2} \) are calculated by treating currently observation \( Z^0 \) and saved history observation \( z_i^{(j)}.Z_i \) as exploring free area of the environment respectively.

The function \( g \) and \( h \) are implemented by splitting the shadowed area as sum of small sectors. The angle of each sector equals the resolution of laser range finder. The long radius is the distance between the sensor and boundary of free area, and the short radius is the distance between the sensor and shadowing obstacle. The sum of areas and angles
of these small sectors are $R_u$ and $\alpha_u$.

\[
\begin{aligned}
R_{u1} &= \sum_{\Omega} 0.5 \ast (r_1^2 - r_2^2) \ast \pi/512 \\
\alpha_{u1} &= \sum_{\Omega} \pi/512
\end{aligned}
\]  \hspace{1cm} (19)

where $\Omega$ is the area existing the incompatibility, $r_1$ is the observed distance directly from $Z_1^0$, $r_2$ is predicted from $z_i^s \cdot Z_1$ and $g_j$. Similarly $R_{u2}$ and $\alpha_{u2}$ are calculated.

If $f_i$ and $g_j$ describe one of the same environment corner, $R_u$ and $\alpha_u$ need to stay in a low level. When $R_u$ and $\alpha_u$ exceed a threshold, $[f_i, g_j]$ is supposed wrong.

C. DA Approach

The proposed DA approach is a union of relaxation of NN and the identifying rule based on the compatibility testing.

1) Relaxation of NN: For each observed $g_j$, if the mahalanobis distance between $f_i$ and $g_j$ is smaller than a threshold and smaller than the distance between $g_j$ and any other features stored in $M^f$, the standard NN algorithm chooses $[f_i, g_j]$ as a right DA hypothesis,

\[
[f_i, g_j] = \{f_i | d^m(f_i, g_j) < d_{\text{threshold}}, \ \& \ d^m(f_i, g_j) > d^m(f_i, g_j), \forall g_j \neq i \}
\]  \hspace{1cm} (20)

where $d^m(f_i, g_j)$ represents the mahalanobis distance between $f_i$ and $g_j$. To overcome the limitation of the NN algorithm, the strict conditions of NN are relaxed. A group of potential DA hypothesis is produced,

\[
H = \{f_i, g_j \} = \{f_i | d^m(f_i, g_j) < d_{\text{threshold}} \}
\]  \hspace{1cm} (21)

2) Feature Calibration for Each Potential DA Hypothesis: For each $[f_i, g_j] \in H$, $g_j$ and $z^g_i$ are calibrated through the process in section IV-A. The calibration brings benefits to the compatibility testing, because it will prevent the correct DA hypothesis from obtaining a big $R_u$ and $\alpha_u$. Currently the calibrated $g_j$ and $z^g_i$ are regarded as a group of intermediate temporary data.

3) Identifying Rule based on Compatibility Testing: After calibration, each $[f_i, g_j] \in H$ is checked by the identifying rule based on compatibility testing in section IV-B. The hypothesis with the smallest $R_u$ and $\alpha_u$ is supposed as the final DA hypothesis. At the same time, the correspondingly calibrated $g_j$ and $z^g_i$ are ascertained to increase the accuracies of both of feature map $M^f$ and local map $m_i$’s building.

The identifying rule based on compatibility testing utilizes the particular property of the one-to-one corresponding relationship between features and local maps.

V. EXPERIMENTS

To validate the proposed data association approach, two experiments have been carried out.

A. Experiment I

This experiment setup was described in detail in [7]. The built hybrid metric map is shown in figure 5. All the local maps represented in global coordinate system have described all the observed dense environment successfully. The grey lines represent the boundaries of $VD(M^f)$.

The proposed DA approach has been compared with the standard NN algorithm. For each observed $g_j$, NN searches the nearest feature $f_i$ in $M^f$. Figure 6a is an example of DA hypothesis produced by NN algorithm. The mahalanobis distance of this DA hypothesis is 3.595. In the proposed DA approach, we have relaxed the NN algorithm by searching a group of potential DA hypothesis, and then chosen the correct hypothesis by using the identifying rule based on compatibility testing. Another possible DA hypothesis is plotted in figure 6b. It has a larger mahalanobis distance (3.7096), but the smaller shadowed area and angle ($R_u = 0.0069452, \alpha_u = 0.073631$ for figure 6b and $R_u = 0.59353, \alpha_u = 1.1536$ for figure 6a) show that NN has made a mistake and the correct DA hypothesis is the one shown in figure 6b.

B. Experiment II

Figure 7 is the schematic diagram of the environment. A shape-shifting robot, AMOEBA-I [19], equipped with URG-04LX, started from position A, then explored the environment clockwise along the annular path shown by the red curve, and finally returned to position A.
As shown in figure 8, we know that the proposed DA approach has successfully made AMOEBA-I return its start position. The feature map is updated greatly. Global positions of local maps have been updated equally. The experiment of loop closure shows that the DA approach proposed for the hybrid metric map is efficient.

Based on the settlement of DA problem, the experiment shows the advantage of the proposed hybrid metric map.

VI. CONCLUSIONS

We have proposed a data association approach for a hybrid metric map representation. In this hybrid metric map representation, there is an important one-to-one corresponding relationship between features and local maps. Each local map is represented in a local coordinate system, and it is inherently generated from the corresponding feature, so the local map represents its corresponding feature’s nearby environment. Utilizing this property we have proposed an identifying rule based on compatibility testing. The principle of compatibility testing is independent from principle of Mahalanobis distance, and the final data association approach is a combination of the relaxation of NN algorithm and the compatibility testing.

Two experiments have validated the efficiency of the proposed DA approach. The feasibility and advantage of the proposed hybrid metric map representation has been also validated in the experiments.

REFERENCES

[1] R. Smith, M. Self, and P. Cheeseman, “A stochastic map for uncertain spatial relationships,” in Proceedings of the 4th international symposium on Robotics Research, Santa Cruz, CA, United states, 1987, pp. 467–474.
[2] H. Durrant-Whyte and T. Bailey, “Simultaneous localization and mapping: Part I,” IEEE Robotics and Automation Magazine, vol. 13, no. 2, pp. 99 – 108, 2006.
[3] T. Bailey and H. Durrant-Whyte, “Simultaneous localization and mapping (SLAM): Part II,” IEEE Robotics & Automation Magazine, vol. 13, no. 3, pp. 108–117, 2006.
[4] S. Thrun, Y. Liu, D. Koller, A. Y. Ng, Z. Ghahramani, and H. Durrant-Whyte, “Simultaneous localization and mapping with sparse extended information filters,” International Journal of Robotics Research, vol. 23, no. 7-8, pp. 693 – 716, 2004.
[5] M. Montemerlo, S. Thrun, D. Koller, and B. Wegbreit, “FastSLAM: A factored solution to the simultaneous localization and mapping problem,” in Proceedings of the National Conference on Artificial Intelligence, Edmonton, Aka, Canada, 2002, pp. 593 – 598.
[6] A. I. Elnazar and R. Parr, “Op-slam 2.0.,” in Proceedings of IEEE International Conference on Robotics and Automation, 2004, no. 2, New Orleans, LA, United States, 2004, pp. 1314 – 1320.
[7] R. Sun, S. Ma, B. Li, M. Wang, and Y. Wang, “A Simultaneous Localization and Mapping Algorithm in Complex Environments: SLASEM,” Advanced Robotics, vol. 25, no. 7, pp. 941–962, 2011.
[8] L. Pedraza, D. Rodriguez-Losada, F. Matia, G. Dissanayake, and J. Miró, “Extending the limits of feature-based slam with b-splines,” IEEE Transactions on Robotics, vol. 37, no. 9, pp. 1095–1104, 2011.
[9] J. Blanco, J. González, and J. Fernández-Madrigal, “Subjective local maps for hybrid metric-topological slam,” Robotics and Autonomous Systems, vol. 57, no. 1, pp. 64–74, 2009.
[10] J. Nieto, J. Guivant, and E. Nebot, “Denseslam: Simultaneous localization and dense mapping,” International Journal of Robotics Research, vol. 25, no. 8, pp. 711–744, 2006.
[11] S. Guo, S. Ma, B. Li, R. Sun, and Y. Wang, “Vorslam: A new solution to simultaneous localization and mapping,” in IEEE International Conference on Information and Automation, 2010, pp. 1896–1901.
[12] S. Guo, S. Ma, B. Li, M. Wang, and Y. Wang, “Simultaneous localization and mapping through a voronoi-diagram-based map representation,” Acta Automation Sinica, vol. 37, no. 9, pp. 1095–1104, 2011.
[13] T. Bailey, “Mobile robot localisation and mapping in extensive outdoor environments,” 2002.
[14] J. Neira and J. Tardós, “Data association in stochastic mapping using the joint compatibility test,” IEEE Transactions on robotics and automation, vol. 17, no. 6, pp. 890–897, 2001.
[15] H. Choset and K. Nagatani, “Topological simultaneous localization and mapping (slam): toward exact localization without explicit localization,” IEEE Transactions on Robotics and Automation, vol. 17, no. 2, pp. 125–137, 2001.
[16] N. Doh, W. Chung, S. Lee, S. Oh, and B. You, “A robust general voronoi graph based slam for a hyper symmetric environment,” in International Conference on Intelligent Robots and Systems (IROS), 2003, vol. 1, 2003, pp. 218–223.
[17] G. A. Borges and M.-J. Aldon, “Line extraction in 2d range images for mobile robotics,” Journal of Intelligent and Robotic Systems: Theory and Applications, vol. 40, no. 3, pp. 267 – 297, 2004.
[18] J. Minguéz, L. Montesano, and F. Lamiraux, “Metric-based iterative closest point scan matching for sensor displacement estimation,” IEEE Transactions on Robotics, vol. 22, no. 5, pp. 1047–1054, 2006.
[19] B. Li, S. Ma, J. Liu, M. Wang, T. Liu, and Y. Wang, “Amoeba-i: a shape-shifting modular robot for urban search and rescue,” Advanced Robotics, vol. 23, no. 9, pp. 1057–1083, 2009.