Unconventional Superconductivity in Semiconductor Artificial Graphene

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Unconventional superconductivity featuring large pairing energies has attracted immense interest, yet tractable microscopic theories have proven elusive. A major breakthrough has been the advent of twisted bilayer graphene (TBG), which serves as a simple model system to 'look under the hood' of unconventional superconductivity. We propose a new model, within current experimental reach, to investigate the microscopics of strong-binding superconductivity. Our proposed device is semiconductor artificial graphene (AG), a two dimensional electron gas overlaid with a periodic potential (superlattice). We demonstrate a new mechanism for superconductivity that originates solely from the repulsive Coulomb interaction. The superlattice promotes certain interactions, which are antiscreened, cause attractive p-wave pairing and – in contrast to graphene – can be strongly enhanced through device engineering. The strength of the pairing energy is similar to TBG, and we find within the accuracy of our calculations $T_c$ up to 20 K for InAs heterostructures.

I. INTRODUCTION

Two dimensional semiconductor systems have provided striking manifestations of both the quantum behavior of single electrons and a variety of paradigmatic interacting states of matter [1]. Over several decades, experimental technology has advanced to allow remarkable control and tunability over these systems, and access to a variety of fundamental physical effects. Despite these advances, no supercon ducting state of any kind has been demonstrated to exist in a system consisting purely of semiconductor materials.

Designer superlattices such as artificial graphene – a semiconductor heterostructure patterned with a periodic triangular potential – seek to combine the novel physics of materials like graphene with the high degree of control in semiconductor devices [2–9]. As in conventional graphene, the periodic potential in AG gives rise to a pair of hand crossings near which the single-electron dynamics may be described by a $2 + 1$ dimensional Dirac fermionic theory with emergent relativistic invariance. Motivated by substantial recent improvements in the quality of these superlattices [5, 6], we propose that such a system is capable of supporting unconventional superconductivity at relatively high temperatures, and across a range of experimentally achievable parameters.

Our mechanism relies on the interplay between the superlattice potential and the emergent Dirac physics. We identify certain interactions, mediated by charge density oscillations between the two sublattices, which in combination with the Berry phase associated with the Dirac cones can mediate superconducting chiral $p$-wave pairing. Unlike in conventional graphene, by engineering the AG superlattice spacing, charge density and potential strength, these interactions become significant, and drive the system into a superconducting phase.

Since the single-particle physics in semiconductor heterostructures can be engineered with great control, the known structure of the wavefunctions provides a reliable starting point for a perturbative examination of interaction effects. We predict critical temperatures $T_c \approx 20$ K in InAs quantum wells with lattice spacing $L = 10 \text{ nm}$, and $T_c \approx 10 \text{ K}$ for GaAs. Superconductivity occurs within a broad range of densities and for a broad range of potential strengths; the model is therefore not a result of fine tuning. The ratio of the critical temperature to the electron density suggests that the electron pairing strength ranks among cuprates, iron pnictides, and TBG [10] – offering a new class of systems to study the elusive physics of strongly bound Cooper pairs.

II. THEORETICAL MODEL

The single-particle physics of AG is described by a two dimensional electron gas (2DEG) in the presence of an electrostatic superlattice potential. There are several existing approaches to implementing the superlattice, including patterning the upper layer of a semiconductor heterostructure [5, 6] or depositing a metallic top-gate using standard lithographic techniques [7]. Accounting for the Coulomb interaction, we may model the system by the second quantised Hamiltonian

$$H = \int \Psi^\dagger(r) \left[ \frac{\mathbf{p}^2}{2m^*} + \mathbf{U}(r) \right] \Psi(r) d^2r + \frac{e^2}{2\varepsilon_r} \int \frac{\Psi^\dagger(r')\Psi^\dagger(r)\Psi(r)\Psi(r')}{|r - r'|} d^2r d^2r'$$

where $m^*$ and $\varepsilon_r$ are the effective mass and dielectric constant of the semiconductor, $r = (x, y)$ is the in-plane coordinate vector, $p$ is the magnitude of the in–plane
electron momentum, \( \Psi(\mathbf{r}) \) is the spin-\( \frac{1}{2} \) electron operator and \( U(\mathbf{r}) \) is a superlattice potential with triangular symmetry and lattice spacing \( L \). We assume the system is distance from gates by \( D \gg k_F^{-1} \). The form of the potential is subject to the particular design of the system, however these details are not conceptually important to our mechanism and we may treat a generic system via a simplified model involving three reciprocal lattice vectors \( \mathbf{G}_1 = \frac{2\pi}{a}(3, \sqrt{3}) \), \( \mathbf{G}_2 = \frac{2\pi}{a}(0, 2\sqrt{3}) \), \( \mathbf{G}_3 = \mathbf{G}_1 - \mathbf{G}_2 \), and parametrised by an effective energy constant \( W \),

\[
U(\mathbf{r}) = 2W \sum_i \cos(\mathbf{G}_i \cdot \mathbf{r}) \tag{2}
\]

As in graphene, the band structure features two band crossings at the valley momenta \( \mathbf{K} = \frac{4\pi}{3L}(0, 1) \) and \( \mathbf{K}' = -\mathbf{K} \), near which the single–electron dynamics is described by the Dirac Hamiltonian

\[
\mathcal{H}_0 = \sum_k \psi_k^\dagger \left( \begin{array}{c} v_F^2 k \cdot \sigma \end{array} \right) \psi_k \tag{3}
\]

where \( v \) is the effective velocity, \( \psi_k \) is an 8-component spinor possessing spin, valley (\( \tau \)) and an additional pseudospin (\( \sigma \)) degrees of freedom. The eigenstates of the pseudospin operator \( \sigma^z \) correspond to electronic states with charge density residing primarily on either the A or B sublattices of the honeycomb structure surrounding the antidot sites.

The wavefunctions of (1) allow a direct computation of the matrix elements of the unscreened Coulomb interaction, in the basis of valley and pseudospin. The interactions near the Dirac points are

\[
\mathcal{H}_{\text{int}} = \frac{1}{2} \sum_{k, p, q} \frac{2\pi e^2}{\varepsilon r \cdot q} \left( \psi_k^\dagger \psi_{k+q} \psi_p^\dagger \psi_{p-q} \right)
+ V_{\mu\nu\rho\lambda} \left( \psi_k^\dagger \psi_{k+q} \sigma^\mu \sigma^\nu \sigma^\rho \psi_k \right) \left( \psi_p^\dagger \psi_{p-q} \sigma^\tau \sigma^\lambda \psi_p \right) \tag{4}
\]

where the pseudospin and valley indices run over \( \mu, \nu, \rho, \lambda \in \{0, x, y, z\} \) (with \( \sigma^0 \) and \( \tau^0 \) denoting the identity operator in pseudospin and valley space).

The interaction (4) contains both the long range \( 1/q \) repulsion as well an additional pseudospin and valley dependent repulsion \( V_{\mu\nu\rho\lambda} \propto 1/K \). We consider the situation where the chemical potential is tuned close to the Dirac points \( k_F \ll K \), so the single–electron dynamics is well described by the Dirac theory. The pseudospin-dependent interactions at the Fermi surface are therefore suppressed compared to \( 1/q \) by a factor \( k_F/K \ll 1 \).

However, when the size of the Wannier orbitals induced by the superlattice becomes small in comparison to the lattice spacing, the interactions \( V_{0000} \) and \( V_{zzzz} \) become strongly enhanced while the remaining short range interactions become negligibly small, as illustrated in the Supplemental Material. For the potential (2), this occurs when the ratio \( W/E_K > 1 \) where \( E_K = K^2/2m^* \), which results in the electron wavefunctions becoming highly localised at the minima of the potential (Fig. 1). This situation is impossible in graphene, since the lattice spacing \( L = 2.54 \text{ Å} \) is comparable to the atomic radius, and the effect of pseudospin dependent interactions is insignificant. Interaction effects in AG can therefore differ dramatically to graphene.
To accurately analyse many-body effects, it is necessary to account for screening in the Random Phase Approximation (RPA). The RPA can be justified here by the fact that the Dirac cones have fourfold \((N = 4)\) spin and valley degeneracy; corrections to the RPA are relatively suppressed by the factor \(1/N\). The intravalley scattering vertex for electron pairs with zero total momentum and energy above the Dirac cones is

\[
\Gamma_{\tau\sigma}(p,k) = \frac{V_{00;00} + \frac{2\pi e^2}{\epsilon |q|} \left(1 + e^{-i\theta}\right)^2}{1 - \Pi^{00}(\omega,q)} \left(1 + \frac{2}{1 - \Pi^{zz}(\omega,q)V_{zzzz} \left(1 - e^{-i\theta}\right)^2}ight)
\]

where \(q = k - p, \omega = \nu(|k| - |p|)\), \(\theta\) is the scattering angle, \(\Pi^{00}\) and \(\Pi^{zz}\) are the intravalley polarisation operators (given in the Supplemental Material). The pseudospin independent interactions undergo conventional Thomas–Fermi screening, with \(\Pi^{00} < 0\) near the Fermi level. Significantly, \(\Pi^{zz}\) has opposite sign, leading to antiscreening of the pseudospin dependent interaction \(V_{zzzz}\). Physically, \(\Pi^{zz}\) measures the response of the system to a local perturbation of \((\sigma^z,\rho)\), the relative electron density on sublattices \(A\) and \(B\). The perturbation of \((\sigma^z)\) induces a ferromagnetic pseudospin polarisation of the nearby environment, which enhances the response and hence corresponds to antiscreening. Formally, the origin of this effect is that the operator \(\sigma^z\) introduced by the interactions \((4)\) anticommutes with the original single–particle Hamiltonian \((3)\), causing \(\Pi^{zz} > 0\).

There are two important pieces of physics in the expression \((5)\): (i) The second term in \((5)\) has a negative \(\ell = 1\) partial wave amplitude due to the negative prefactor of \(\epsilon \theta^0\), which results in an effective attractive interaction in the \(\ell = 1\) scattering channel. Projecting the electron operators to the upper band of the Dirac dispersion, there is a relative phase winding of the electronic densities on the two sublattices, associated with the Berry phase surrounding the Dirac point, so that upon projection \((\rho_A + \rho_B)^2 \rightarrow \frac{1}{4}(1 + e^{-i\theta})^2 \rho_A^2\), where \(\rho_+\) is the density in the upper band. Hence, while the interactions are proportional to \((\rho_A + \rho_B)^2\), which is always positive, the phase winding of the Dirac wavefunction causes the interaction to separate into negative and positive pieces with different angular dependence. The effect is analogous to how in conventional metals, the antisymmetry of the spatial wavefunction leads to the attractive exchange interaction in the triplet channel responsible for ferromagnetism. (ii) The susceptibility \(\Pi^{00} < 0\) leads to a weakening of the repulsive \(V_{00}\) interaction, while \(\Pi^{zz} > 0\) leads to an enhancement of the attractive interaction. These two physical features constitute the mechanism for unconventional superconductivity in AG.

We emphasise that the combination of Berry phase and antiscreening is a new mechanism, distinct from other proposals for superconductivity due to repulsive interactions, which may require flat bands, nesting, or the Kohn–Luttinger mechanism which relies on singularities in the interaction due to backscattering \([11]\). Our mechanism also applies at weak coupling, and the ‘pairing glue’ is the fluctuations of the pseudospin density \((\psi^\dagger \sigma^z \psi)\).

In Figure 2a we plot the \(\ell = 1\) partial wave amplitude \(\Gamma^{\ell=1}(p,k_F)\). The low–frequency attractive part of the intravalley interaction is due to the antiscreened \(V_{zzzz}\) term, while at high–frequencies, both the screened and antiscreened interactions are repulsive. The same step–like frequency dependence appears in phonon–mediated BCS superconductors, in which the interaction consists of a screened repulsive part (the so–called Anderson–Morel pseudopotential) and an antiscreened attractive part beneath the Debye frequency due to phonons \([12]\).

We therefore solve the gap equations using standard methods from BCS theory.

### III. SUPERCONDUCTING GAP AND CRITICAL TEMPERATURE

As shown in Figure 2a, the (dimensionless) intravalley scattering amplitude can be well approximated by a simplified form consisting of step functions in \(k, p\) with three positive parameters \((g_{1,2,3})\),

\[
\nu_0 \Gamma^{\ell=1}(p,k) = \begin{cases} 
2g_2(\Omega - p) - g_3(\Omega - p), & k < \Omega \\
2g_2(\Omega - p) - g_3(p - \Omega), & k > \Omega
\end{cases}
\]

where \(\nu_0\) is the single–particle density of states at the Fermi energy \(E_F\). An attractive interaction between electrons within the same valley in the \(\ell = 1\) channel implies a spin triplet \(p_x + i\tau p_y\) superconducting gap,

\[
\Delta_{s,x';\tau}\tau'(k) = \sum_p \Gamma^{\ell=1}(p,k) \langle \psi_{s',\tau'} \psi_{s,\tau} \rangle 
\]

\[
= \Delta^{\ell=1}(k) k^{-1}(k_x + i\tau k_y)_{\tau'}(id \cdot \sigma^S S^S)_{s,s'}
\]

where \(d\) is a real three–dimensional vector associated with the spin triplet ordering, \(\sigma^S\) are the Pauli matrices acting on spin, and \(\Delta^{\ell=1}\) satisfies the BCS gap equation,

\[
\Delta^{\ell=1}(k) = \sqrt{\frac{\Delta^{\ell=1}(p,k)^2}{2T} - \frac{d^2 p}{(2\pi)^2}}
\]
FIG. 2. (a) The ℓ = 1 partial wave component of the Cooper channel scattering amplitude \( \Gamma^{ℓ=1}(p, k_F) \) for parameters corresponding to a 10 nm InAs quantum well with \( W/E_K = 7.3 \) and \( k_F/K = 0.25 \). The solid line indicates the full momentum dependence in Eq. (5), while the blue dashed line is the step function approximation Eq. (6). (b) Heat plot of critical temperature as a function of Fermi momentum \( k_F/K \) and potential strength \( W/E_K \). The lower (upper) dashed line marks \( \Delta/E_F = 0.1 \) (0.2), while the unshaded region \( \Delta/E_F > 0.2 \) is strongly coupled and contains competing instabilities. (c) Critical temperature as a function of \( W \) for InAs and GaAs (inset) quantum wells at doping \( k_F/K = 0.25 \). The red shaded region indicates that the ratio of gap to Fermi energy \( 0.1 < \Delta/E_F < 0.2 \). (d) Experimental data for \( T_c \) versus carrier density \( n \) of various known superconductors; data adapted from [10]. The dashed lines mark \( T_c/T_F = \{10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}\} \). Enlarged square labels mark the predicted \( T_c/n \) for InAs and GaAs.

the solution of which is presented in the Supplemental Material.

In Figure 2b, we show \( T_c \) as a function of electron density and potential strength \( W/E_K \) where \( E_K = K^2/2m^* \) using parameters for InAs, \( \varepsilon_r = 14.6 \) and \( m^* = 0.0229 \) [13], and setting \( L = 10 \) nm. For any doping within the range shown, there is a corresponding range of potential strengths for which superconductivity exists. This is an important result, demonstrating that superconductivity is a not a result of fine tuning.

In the region \( \Delta/E_F < 0.1 \) beneath the dashed lines, our calculations are completely reliable, as determined by the coupling constants \( g_i \ll 1 \). The dashed lines mark the region \( 0.1 < \Delta/E_F < 0.2 \), at which point the effective coupling constants approach order unity. In this region, it is reasonable to expect that superconductivity will persist and that \( T_c \) will continue to increase, but our theory loses reliability. Above the dashed line \( \Delta/E_F > 0.2 \), the effective coupling constants become \( g_i \gg 1 \). In this region, Stoner–type instabilities including ferromagnetism and density wave ordering become important. Despite the lack of theoretical predictions for this region of the phase diagram, we expect the system to exhibit interesting physics to be explored experimentally.

Importantly, the Stoner–type instabilities do not appear at weak coupling because the Fermi surface does not exhibit the nesting phenomenon, which is required for these instabilities to compete with superconductivity [11].

In Figure 2c, we fix the doping ratio at \( k_F/K = 0.25 \), and plot \( T_c \) against potential strength \( W \) for both InAs, and GaAs (inset) based AG. Taking \( L = 10 \) nm for InAs we find \( T_c \approx 20 \) K. For GaAs, with \( \varepsilon_r = 12.4 \) and \( m^* = 0.067 \), for \( L = 10 \) nm we find \( T_c \approx 10 \) K. The shaded regions on the two curves indicate entry into the strong coupling regime, again determined by \( 0.1 < \Delta/E_F < 0.2 \). We note that \( T_c \) for GaAs is lower than in InAs, due the larger effective mass and hence lower \( E_K \).

As demonstrated for InAs and GaAs, the critical temperatures possible in AG are remarkably large. To further gain an understanding of the electron–electron pairing...
strength in these systems versus other well known unconventional/high temperature superconductors, in Figure 2d we plot the critical temperature $T_c$ against the carrier density, $n$. The ratio $T_c/n$ indicates the strength of the pairing [10]; from this criterion we predict that semiconductor AG hosts superconducting pairing stronger than many previously known unconventional/high temperature superconductors.

We note that the superconducting portion of the phase diagram requires a large ratio $W/E_K \gtrsim 6$ (2.4) in InAs (GaAs), which implies $W \gtrsim 2$ eV (0.25 eV). The large potential strength is a result of the simplified model (2) with only a single energy parameter $W$, chosen for a simple conceptual illustration of the theory. In a situation where the superlattice is engineered to effectively increase the antidot size relative to the lattice spacing, additional cosine terms must be added in (2) which preserve the lattice symmetry, and similar values of $V_{zzzz}$ may be obtained with a significantly shallower potential variation, as we discuss in the Supplemental Material.

IV. DISCUSSION

The parameters we used stay within the limits of perturbation theory $q_t \ll 1$, and at the same time the RPA is controlled by the small parameter $1/N$. Unlike phenomenological models of existing high temperature superconductors, we have a microscopic theory of pairing that is controlled within perturbation theory. Our model also exhibits superconductivity within an experimentally achievable parameter range.

In order to understand the experimental feasibility of our proposal, it is crucial to determine whether our mechanism survives the disorder expected to be present in a nanofabricated device: impurity scattering, as well as superlattice disorder, i.e. shape, size and position variations of the antidots. It has been shown that antidot size variation is the dominant long–wavelength disorder, and generates variations in the Fermi energy across the sample [14]. Even though the gap function (7) is time reversal symmetric, pairing does not occur between time–reversed states and Anderson’s theorem [15, 16] does not apply, so we expect that $T_c$ is reduced by regions of the sample deviating from optimal doping [17]. It has been suggested that $T_c$ can be enhanced by disorder under some conditions [18, 19], and the ability to artificially tune the extent of disorder in AG allows for an experimental probe of its precise effect. Weak disorder may in fact stabilise superconductivity by pinning vortices which would otherwise create dissipation [20]. We finally note that it is possible for disorder to promote another superconducting state over $p_z + i\tau p_y$ intravalley pairing – Anderson’s theorem would apply to s-wave intervalley pairing, for which we found lower $T_c$ in the absence of disorder.

Promisingly, very recent experimental work has reported the creation of low disorder AG, as indicated by clear signatures of the Dirac dispersion [5, 6]. These low disorder realisations of AG possess superlattice spacings $L \sim 70$ nm; the main experimental challenge of our proposal is achieving smaller values of $L \sim 10$ nm while maintaining low superlattice disorder. While we have plotted results for this ambitious scenario, superconductivity can still be realised with larger lattice spacings: for GaAs and InAs superlattices with $L = 50$ nm our theory can predict up to $T_c \approx 1$ K for $k_F/K = 0.25$.

While the superconducting phase discussed is among several explored previously in honeycomb lattices [21–25], our analysis demonstrates that a pairing interaction mediated by antiscreened pseudospin fluctuations ($\sigma_z$) exists generically as a feature of interacting Dirac systems, and is provided concretely by the sublattice degree of freedom in honeycomb lattices. This mechanism might be applicable to other artificial lattices [26–29] or possibly a twisted bilayer system [30], which would have the advantage of much lower superlattice disorder. We stress, however, that our theory does not require tuning to the ‘magic twist angle’; our mechanism can exist in the weak coupling regime, as in phonon–mediated superconductivity, and does not require flat bands.

The tunability of AG presents the opportunity to test many simple variations of this theory, in much the same way that cold atomic gases have allowed experimentalists a platform to implement a large host of toy models. Alternative lattice geometries can be imposed on the 2DEG, and future studies may also wish to investigate the role of higher bands beyond the first two Dirac points. In particular, it has been shown that AG can be used to engineer topological flat bands with Chern number $C = 3$, and may host exotic correlated phases such as fractional Chern insulators [9].

An alternative avenue to chiral superconductivity in AG is to increase the density to a van Hove singularity, causing $d$-wave superconductivity alongside competing magnetic order, a scenario first proposed in the context of graphene [31–33]. A crucial distinction in our mechanism is its validity over a large range of densities, and the absence of nesting and competing instabilities in the weak coupling regime. The ability to tune AG to the strong coupling regime, outside the parameter range for which we are confident superconductivity dominates, allows access to a significantly richer phase diagram in which density wave and magnetic order compete with chiral superconductivity – an interesting scenario reminiscent of TBG and cuprate superconductors [10]. Experiments will be necessary to understand this section of the phase diagram.

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Supplemental Material

DERIVATION OF THE LOW-ENERGY INTERACTING HAMILTONIAN

The generic single-particle Hamiltonian describing a superlattice with triangular symmetry is given by

\[
H = \frac{p^2 + \gamma^2}{2m} + W_0(z) + \sum_{n \neq 0} W_n(z) \cos (G_n \cdot r)
\]  

(S1)

where \( r = (x, y) \) are the in-plane coordinates, \( z \) is the out-of-plane coordinate and \( n \) indexes the reciprocal lattice vectors \( G_n \). At the \( K, K' \) points, the space of single-particle wavefunctions \( |\Psi_{k,\sigma,\tau} \rangle = e^{ik \cdot r} |\sigma, \tau \rangle \) is spanned by the basis of pseudospin eigenstates \( |\sigma, \tau \rangle \) (with \( \tau = +1, -1 \) corresponding to states near the \( K \) and \( K' \) points respectively) whose coordinate representation has the structure

\[
\langle r, z | \sigma, \tau \rangle = \varphi(z) \sum_n C_{\sigma,n} e^{i(G_n - \tau K) \cdot r}
\]  

(S2)

where \( K = \frac{4\pi}{3} (0, 1) \) is the valley momentum.

The many-body Hamiltonian describing physics near the \( K, K' \) points may be expressed in terms of the fermionic creation operators \( \psi_{k,\sigma,\tau,s}^\dagger \) with quasimomentum \( k \) relative to the valley momentum, pseudospin \( \sigma \) and spin \( s \) near the \( K \) or \( K' \) point, in the form

\[
H = \sum_k \psi_{k,\sigma,\tau,s}^\dagger (v \tau k \cdot \sigma) \sigma \psi_{k,\sigma',\tau',s'} + \frac{1}{2} \sum_{k,p,q} U_{\sigma_1,\sigma_2,\sigma_3,\sigma_4}(q) \psi_{k+q,\sigma_3,\tau_3,s}^\dagger \psi_{k,\sigma_1,\tau_1,s} \psi_{p-q,\sigma_4,\tau_4,s'}^\dagger \psi_{p,\sigma_2,\tau_2,s'}
\]  

(S3)

where

\[
U_{\sigma_1,\sigma_2,\sigma_3,\sigma_4}(q) = \sum C_{\sigma_1,n}^* C_{\sigma_1,n} C_{\sigma_2,m}^* C_{\sigma_2,m} V (q + (\tau_3 - \tau_1)K_1) + G_{n'} - G_n
\]

\[
V(q) = \frac{e^2}{\epsilon_r} \int \left[ \frac{1}{\sqrt{|r|^2 + (z - z')^2}} - \frac{1}{\sqrt{|r|^2 + (z + z')^2 + 2D^2}} \right] e^{i q \cdot r} \varphi(z)^2 \varphi(z')^2 dzd'dr
\]  

(S4)

where the second term in the brackets is the contribution from an image charge, resulting from a metallic gate at distance \( D \) from the system. The sum in the first line is taken over values of \( n, n', m, m' \) satisfying

\[
G_n + G_m - G_{n'} - G_{m'} + (\tau_1 + \tau_2 - \tau_3 - \tau_4)K_1 = 0
\]  

(S5)

and therefore vanishes unless \( \tau_1 + \tau_2 = \tau_3 + \tau_4 \).

The interaction matrix elements depend on the vertical profile of the charge density, which is sensitive to the harmonics of the electrostatic potential \( W_n(z) \). For simplicity, however, we will consider the limit of a narrow well, so the transverse wavefunctions \( \varphi(z) \) are highly localized near \( z = 0 \) and the matrix elements may be replaced by

\[
V(q) = \frac{e^2}{\epsilon_r} \int \left[ \frac{1}{|r|} - \frac{1}{\sqrt{|r|^2 + 4D^2}} \right] e^{i q \cdot r} d^2r = \frac{2\pi e^2}{\epsilon_r |q|} \frac{2}{1 - e^{-2|q|D}}.
\]  

(S6)

We may neglect the term \( \propto e^{-2|q|D} \) if the gate is far from the system, \( D > 1/k_F \).

In the main text we consider a model in which the only harmonics of the superlattice potential \( W_n(z) \) involve reciprocal lattice vectors \( |G_n| = \sqrt{3} K \) connecting points within the first Brillouin zone. We assume the potential is vertically uniform, so \( W_n(z) = 2W \) is constant.

The matrix elements of the interaction vanish unless either \( \tau_1 = \tau_3, \tau_2 = \tau_4 \) or \( \tau_1 = \tau_4 = -\tau_2 = -\tau_3 \), and in these cases the interactions are of the form

\[
U_{\sigma_1,\tau_1,\sigma_{1'},\tau_{1'},\sigma_3,\tau_3}(q) = U_{00}(q) \delta_{\sigma_1,\sigma_{1'}} \delta_{\sigma_2,\sigma_4} + V_{ij}(q)(\sigma_i)(\sigma_{1'i})\sigma_2\sigma_4 + \tau_{1'i} V_{zz}(q)(\sigma_z)(\sigma_1\sigma_{1'}(\sigma_z)\sigma_2\sigma_4)
\]

\[
U_{\sigma_1,\tau_2,\sigma_{-3},\tau_{-3},\sigma_4,\tau_4}(q) = V_{00}'(q) \delta_{\sigma_1,\sigma_{1'}} \delta_{\sigma_2,\sigma_4} + V_{xx}(q)(\sigma_x)(\sigma_{1'i})\sigma_2\sigma_4 + V_{yy}(q)(\sigma_y)(\sigma_{1'i})\sigma_2\sigma_4
\]  

(S7)
In our analysis of the superconducting state, only the matrix elements corresponding to $\tau_1 = \tau_2 = \tau_3 = \tau_4$ appear, since we consider the case of Cooper pairs formed from electrons within the same valley. We take the leading order in the ratio $q \ll K$, which allows us to write

$$V_{00}(q) = \frac{2\pi e^2}{\epsilon_r} \left( \frac{1}{q} + \frac{v_{00}}{K} \right)$$
$$V_{zz}(q) = v_{zz} \frac{2\pi e^2}{\epsilon_r K}$$
$$V_{ij}(q) = v_{xx} \frac{2\pi e^2}{\epsilon_r K} \delta_{ij}$$

where $i, j$ run over $x$ and $y$, and $v_{00}, v_{zz}, v_{xx}$ are purely functions of the parameter $2m^*W/K^2$. These functions are plotted in Fig. S1. In the regimes of interest, $2m^*W/K^2 > 2$, we have $v_{ii} \ll v_{zz}, v_{00}$ and can be neglected.

While for the results in the main text we have considered the simplified potential (2), in the experimental situation terms of the form $\cos(\mathbf{G} \cdot \mathbf{r})$ will appear where $\mathbf{G}$ is any reciprocal lattice vector, and terms in which $|\mathbf{G}| > \sqrt{3}K$ may be comparable to the terms previously considered. It is therefore useful to consider the effects of possible higher harmonics on the strengths of the interactions. Experimentally, these additional harmonics become more important when the ratio of the antidot size to the unit cell is increased, and therefore the electron wavefunctions are localised more effectively. In Figure S2, results are presented for $v_{zz}$ for a model superlattice potential with an additional set of harmonics

$$U'(r) = W \sum_{|\mathbf{G}|=\sqrt{3}K} \cos(\mathbf{G} \cdot \mathbf{r}) - W' \sum_{|\mathbf{G}'|=3K} \cos(\mathbf{G}' \cdot \mathbf{r})$$

The figures show the dimensionless interaction $v_{zz}$ as a function of $W/E_K$ for fixed ratios $W'/W = 0, 0.5, 1$. We see that the additional harmonics greatly reduce the value of $W$ required to achieve the superconducting state. Note that the additional harmonics $\propto W'$ do not affect the total variation of the potential.
FIG. S2. The interaction constant $v_{zz}$ defined in (S8), as a function of the dimensionless parameter $W/E_K = 2m^*W/K^2$, for various values of the second harmonic potential (S9), $W'/W = 0, 0.5, 1$.

SCREENED INTERACTIONS

The RPA equations for the screened intravalley interactions are

$$\tilde{V}_{\mu\nu}(q, \omega) = V_{\mu\nu}(q) + V_{\mu\alpha}(q, \omega)\Pi_{\alpha\beta}(q, \omega)\tilde{V}_{\beta\nu}(q, \omega)$$  \hspace{1cm} (S10)

where $\Pi_{\alpha\beta}(q, \omega)$ is the intravalley polarisation operator given by

$$\Pi_{\alpha\beta}(q, \omega) = -i \int \text{Tr}(J_{\alpha}G(k + q, E + \omega)J_{\beta}G(k, E)) \frac{dE d^2k}{(2\pi)^2}$$  \hspace{1cm} (S11)

and $\alpha, \beta = 0, x, y, z$ such that $J_{\alpha} \in \{\sigma_0 \otimes \tau_0, \sigma_x \otimes \tau_0, \sigma_y \otimes \tau_0, \sigma_z \otimes \tau_z\}$. From previous results, we may set $V_{\mu\nu}(q) \rightarrow 0$ unless $\mu = \nu = 0, z$ which gives

$$\tilde{V}_{00}(q, \omega) = \frac{V_{00}(q)}{1 - \Pi_{00}(q, \omega)V_{00}(q)}, \quad \tilde{V}_{zz}(q, \omega) = \frac{V_{zz}}{1 - \Pi_{zz}(q, \omega)V_{zz}}$$  \hspace{1cm} (S12)

Evaluation of the polarisation operators $\Pi_{\alpha\beta}$ is straightforward, and we merely present the results:

$$\Pi_{00}(q, \omega) = -\frac{q^2}{4\sqrt{v^2q^2 - \omega^2}} - \frac{2k_F}{\pi v} \frac{1}{\pi \sqrt{v^2q^2 - \omega^2}} \sum \left\{ p_i \sqrt{\frac{q^2 - p_i^2}{4} + \frac{q^2}{4} \sin^{-1} \frac{2p_i}{q}} \right\}$$

$$\Pi_{zz}(q, \omega) = \frac{\sqrt{v^2q^2 - \omega^2}}{2v^2} + \frac{2k_F}{\pi v} - \sqrt{\frac{v^2q^2 - \omega^2}{2\pi v^2}} \sum \sin^{-1} \frac{2p_i}{q} \right\}$$  \hspace{1cm} (S13)

where $p_i = \{\min(k_F + \frac{\omega}{2v}, \frac{q}{2}), \min(k_F - \frac{\omega}{2v}, \frac{q}{2})\}$. Calculation of the polarisation operator for intervalley scattering is similarly straightforward, but our calculations found the intravalley scattering to be the dominant effect, so we omit these results.

ZERO TEMPERATURE GAP

The gap equation is solved by adopting a simplified form of the interaction (6), where the constants $g_1, g_2, g_3$ may be taken to be averages of the scattering amplitude (5) over the ranges $0 < p < \Omega$ and $\Omega < p < K$. The solution to the gap equation is then of the form

$$\Delta(k) = \Delta_1(\Omega - k) + \Delta_2(\Omega - k)$$  \hspace{1cm} (S14)
where the effective coupling is

\[ g_T = \frac{g_1}{2\nu_0} \int_0^\Omega \frac{d^2 p}{\sqrt{\epsilon_p^2 + \Delta_1^2}} - \frac{g_2}{2\nu_0} \int_0^\Lambda \frac{d^2 p}{\epsilon_p}, \]

\[ \Delta_2 = -\frac{g_2g_1}{2\nu_0} \int_0^\Omega \frac{d^2 p}{\sqrt{\epsilon_p^2 + \Delta_1^2}} - \frac{g_3g_2}{2\nu_0} \int_0^\Lambda \frac{d^2 p}{\epsilon_p}, \]

(S15)

where at large \( p \), we make the replacement \( \sqrt{\epsilon_p^2 + \Delta_2^2} \to \epsilon_p \), and the single–particle density of states is \( \nu_0 = k_F/(2\pi\nu) \).

The second line gives

\[ \Delta_2 = \left(1 + \frac{g_3}{2\nu_0} \int_0^\Lambda \frac{d^2 p}{\epsilon_p}\right)^{-1} \left(-\frac{g_2g_1}{2\nu_0} \int_0^\Omega \frac{d^2 p}{\sqrt{\epsilon_p^2 + \Delta_1^2}}\right), \]

(S16)

from which it follows

\[ \Delta_1 = -\frac{g^*\Delta_1}{2\nu_0} \int_0^\Omega \frac{d^2 p}{\sqrt{\epsilon_p^2 + \Delta_1^2}} \]

where the effective coupling \( g^* \) is

\[ g^* = g_1 - \frac{\frac{g_2^2}{2\nu_0} \int_\Omega^\Lambda \frac{d^2 p}{\epsilon_p}}{1 + \frac{g_1}{2\nu_0} \int_\Omega^\Lambda \frac{d^2 p}{\epsilon_p}} = \frac{g_1 + \frac{g_2g_1}{2\nu_0} \int_\Omega^\Lambda \frac{d^2 p}{\epsilon_p}}{1 + \frac{g_1}{2\nu_0} \int_\Omega^\Lambda \frac{d^2 p}{\epsilon_p}}. \]

(S17)

In the weak-coupling limit we may set \( g^* \to g_1 \). The magnitude of the coupling \( g_1 \) (and hence \( g^* \)) may be varied arbitrarily between the weak and strong coupling regimes by tuning of the density within a narrow range. We may therefore nominally choose \( g^* = g_1 = \nu_0 \Gamma(k_F,k_F) \) since any corrections to this value can be simply compensated by small changes in the density.

Thus the gap equation to leading order in the coupling depends only on \( g_1 \),

\[ 1 = -\frac{g_1}{2\nu_0} \int_0^\Omega \frac{1}{\sqrt{\epsilon_p^2 + \Delta_1^2}} \frac{p dp}{2\pi} \]

(S18)

Defining \( \Delta_1 = \pi k_F x, \Omega = k_F \kappa_0 \) and changing variables \( p - k_F = k_F \kappa \), we may express the implicit formula for the gap \( x = \Delta/E_F \) in the form

\[ 1 = -\frac{g_1k_F}{4\pi\nu\nu_0} \int_{-1}^{\kappa_0-1} \frac{\kappa + 1}{\sqrt{\kappa^2 + x^2}} d\kappa = -\frac{g_1}{2} \left( \sinh^{-1} \frac{\kappa_0 - 1}{x} + \sinh^{-1} \frac{1}{x} + \sqrt{(\kappa_0 - 1)^2 + x^2} - \sqrt{1 + x^2} \right) \]

Solution of this implicit equation yields the zero temperature gap.

**CRITICAL TEMPERATURE**

In order to obtain the critical temperature, we take the limit \( \Delta \to 0 \) in Eq. (8) which gives

\[ \Delta(k) = -\frac{1}{2\nu_0} \int_0^\kappa \frac{\Gamma_{\kappa=1}(p,k)\Delta(p)}{\epsilon_p} \tanh \frac{\epsilon_p}{2T} \frac{p dp}{2\pi} \]

(S19)

Similarly to the \( T = 0 \) gap solution, we obtain coupled equations for \( \Delta_1, \Delta_2 \). Using the same manipulations we find that to leading order we may keep only the first term in the equation for \( \Delta_1 \). Writing \( T = E_F t \) and \( p - k_F = k_F \kappa \),

\[ 1 = -\frac{g_1}{2} \int_{-1}^{\kappa_0-1} \frac{\tanh \frac{\kappa}{k_F}}{\kappa} (\kappa + 1) d\kappa \]

(S20)

The critical temperature \( T_c \) is plotted in Figure 2b. We note that the ratio remains \( T_c/\Delta \approx 1.8 \) for all \( 0 < \Delta < 0.2E_F \).
SYMMETRY ANALYSIS OF THE SUPERCONDUCTING GAP

The structure of the superconducting order parameter can be obtained by a minimising the free energy obtained from mean field theory. The generic mean field Hamiltonian to account for all pairing possibilities is

$$\mathcal{H}_{MF} = \sum_{k,s,\sigma} \varepsilon_k c_{ks\sigma}^\dagger c_{ks\sigma} + \sum_{k,s,\sigma,s',\tau} \sum_{\mu} \tilde{c}_{kss's'\sigma\tau}^\dagger (\Delta_k i \sigma_y \tau_x)_{ss's'\sigma\tau} \tilde{c}_{kss's'\sigma\tau} + \text{h.c.}$$  \hspace{1cm} (S21)

where $\sigma, \tau$ account for physical spin and valley. We write the gap function as

$$\Delta_k = \sum_{\alpha,\beta=\pm} d_{\alpha\beta}^\mu \sigma^\mu (k_x + i \alpha k_y) (\tau_x + i \beta \tau_y) = \sum_{\alpha,\beta=\pm} \sum_{\mu} d_{\alpha\beta}^\mu \eta_{\alpha\beta}^\mu \varepsilon_{\alpha\beta}$$  \hspace{1cm} (S22)

The basis functions $\tilde{k}_x \pm i \tilde{k}_y$ and $\tau_x \pm i \tau_y$ account for $p$-wave pairing within the two valleys; the analysis can be easily generalised to the case of point group rather than full rotational symmetry – for more detail see [S1]. Intravalley pairing is accounted for by the presence of $\tau_x$ in the meanfield Hamiltonian, since the basis $(\tau_x \pm i \tau_y)_{\tau x}$ is diagonal in valley space. Finally, the vector $\mu$ components of $d_{\alpha\beta}^\mu$ account for the triplet spin structure in the usual way,

$$d_x = \frac{1}{2} (|\uparrow\uparrow⟩ - |\downarrow\downarrow⟩) \quad d_y = \frac{1}{2i} (|\uparrow\downarrow⟩ + |\downarrow\uparrow⟩) \quad d_z = -\frac{1}{2} (|\uparrow\downarrow⟩ + |\downarrow\uparrow⟩).$$  \hspace{1cm} (S23)

We collect all degrees of freedom into a triplet of $2 \times 2$ matrices

$$\hat{d} = (\hat{d}_x^\dagger, \hat{d}_y^\dagger, \hat{d}_z^\dagger); \quad \hat{d}^\mu = \begin{pmatrix} d_{++}^\mu & d_{+-}^\mu \\ d_{-+}^\mu & d_{--}^\mu \end{pmatrix}. \hspace{1cm} (S24)$$

The corresponding symmetry operations then take the following representation

$$SU(2) : \quad d_{\alpha \beta} \mapsto R d_{\alpha \beta}$$  \hspace{1cm} (S25)

$$U(1)_x : \quad \hat{d} \mapsto \hat{d} e^{i \phi x}$$  \hspace{1cm} (S26)

$$U(1) : \quad \tilde{\hat{d}} \mapsto \tilde{\hat{d}} e^{i \phi x}$$  \hspace{1cm} (S27)

$$\Theta : \quad \hat{d} \mapsto \sigma_x (\hat{d}^\dagger)^* \sigma_x. \hspace{1cm} (S28)$$

The action at mean field level is given by

$$\mathcal{S} = \frac{1}{2} \sum_{k} \begin{pmatrix} \varepsilon_{k} c_{k-}\dagger c_{k-} \\ \varepsilon_{k} \end{pmatrix}^T \begin{pmatrix} \omega - \varepsilon_{k} & -\Delta_k i \sigma_y \tau_x \\ i \sigma_y \tau_x \Delta_k & \omega + \varepsilon_{k} \end{pmatrix} \begin{pmatrix} \varepsilon_{k} c_{k-}\dagger c_{k-} \\ \varepsilon_{k} \end{pmatrix}. \hspace{1cm} (S29)$$

Integrating out the electronic degrees of freedom gives the free energy in terms of the order parameter $d_{\alpha \beta}^\mu$ [S2, S3],

$$\mathcal{F} = \mathcal{F}^{(2)} + \mathcal{F}^{(4)}$$  \hspace{1cm} (S30)

$$\mathcal{F}^{(2)} = -a_1 \left( d_{++}^\dagger d_{++} + d_{--}^\dagger d_{--} \right) + a_2 \left( d_{+-}^\dagger d_{+-} + d_{-+}^\dagger d_{-+} \right)$$  \hspace{1cm} (S31)

$$\mathcal{F}^{(4)} = \sum_{\beta} \left\{ \frac{1}{2} b \left( \sum_{\alpha} d_{\alpha \beta}^\dagger d_{\alpha \beta} \right)^2 + 2 b |d_{++}^\dagger d_{--}^\dagger|^2 - 2 b |d_{++}^\dagger d_{--}^\dagger|^2 - \frac{1}{2} b \sum_{\alpha} |d_{\alpha \beta}^\dagger d_{\alpha \beta}|^2 \right\}$$  \hspace{1cm} (S32)

$$= \sum_{\beta} \left\{ \frac{1}{2} b \left( d_{++}^\dagger d_{++} \right)^2 + \frac{1}{2} b \left( d_{--}^\dagger d_{--} \right)^2 + 2 b \left( d_{++}^\dagger d_{--} \right) \left( d_{--}^\dagger d_{--} \right) \right\}$$

$$+ 2 b |d_{++}^\dagger d_{--}^\dagger|^2 + \frac{1}{2} b \sum_{\alpha} |d_{\alpha \beta}^\dagger d_{\alpha \beta}|^2 \right\}$$

where $a_1, a_2 > 0$. Importantly, the coefficients of the quartic terms are all known and are simple ratios of each other, and $b$ is positive,

$$b \propto \sum_{\omega, k} \left[ \frac{k^4}{(\omega^2 - \varepsilon_{k}^2)^2} \right] > 0. \hspace{1cm} (S33)$$
The lowest energy superconducting state (S37) is a time reversal invariant combination of opposite chirality states at each Dirac point. The electrons undergo \( p_x + ip_y \) pairing in one valley, and \( p_x - ip_y \) pairing in the other.

The off-diagonal terms in \( d_{\alpha\beta} \) with \( \alpha = -\beta \) are energetically costly and can be set to zero. Since \( \alpha = \beta \), this implies that condensation within each valley involves a single chirality. The cross products make non-unitary states energetically unfavourable. We find that the free energy for \( d_{++} \) and \( d_{--} \) decouple. Minimizing (S30), we find that

\[
\begin{align*}
    d_{++} &= R d_0 (1, 0, 0), & d_{+-} &= 0, & d_0^2 &= \frac{a_1}{b} \\
    d_{-+} &= 0, & d_{--} &= R d_0 (1, 0, 0), & d_0^2 &= \frac{a_1}{b}
\end{align*}
\]  

where \( R \) is an arbitrary \( SO(3) \) rotation matrix. Setting \( d_{++} = d_{--} \equiv d/2 \), the gap function then takes the form

\[
\Delta_k i\sigma_y \tau_x = \sum_{\alpha\pm} \sum_{\mu} d_{\alpha\beta \mu \nu} \sigma_i \sigma_y (k_x + i\alpha k_y) (\tau_0 + \alpha \tau_z)
\]

\[
= (d \cdot \sigma_i \sigma_y) (k_x \tau_0 + i k_y \tau_z)
\]

This is a unitary spin triplet state, with chiral \( p_x + ip_y \) pairing in one valley and \( p_x - ip_y \) pairing in the other – analogous to two (opposite chirality) copies of the A–phase of \(^3\)He [S4]. This state preserves time reversal symmetry, but spontaneously breaks \( SU(2) \) spin rotation symmetry down to \( U(1) \) rotations about the vector \( d \) (which is arbitrary).

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