Stability analysis of carbon emission reduction based on evolutionary game model

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Abstract. Based on the limited rationality of game participants, this paper discusses the root of complexity in cooperative competition game, and then discusses the evolutionary model of cooperative competition game by using evolutionary game theory. This paper analyzes the emission reduction behavior in carbon emissions, and establishes a corresponding evolutionary game model. Based on the assumption of incomplete information and the idea of "replication dynamics" in the process of biological evolution, the asymptotical stability analysis of asymmetric evolutionary game equilibrium is carried out. The results show that the carbon reduction strategy depends on the comparison the numerical value of two key factors.

1. Introduction
Carbon trading is an effective mechanism to reduce greenhouse gas emissions through government regulation and market trading. The government allocates carbon emission rights to various enterprises and allows enterprises to trade carbon emission rights. The initial carbon emission quota obtained by enterprises is usually less than the actual carbon emissions needed. Enterprises that achieve carbon emission reduction can gain benefits by selling excess carbon emission rights, and other enterprises can also avoid sanctions for excess emissions in this way. Therefore, the enterprises will combine carbon emission reduction with their business objectives, and fully consider the carbon emission restrictions when making decisions.

In recent years, many scholars have studied the carbon trading mechanism by means of game theory [1-3]. But they all assume that enterprises and governments are completely rational and can fully grasp all the information in the game process, so as to achieve Nash equilibrium. This is inconsistent with the actual situation. Due to the complexity of real environments, it is difficult for the players to make the optimal decision in a game, but only to achieve limited rationality. Evolutionary game theory provides a good analysis method for the study of the trading mechanism of limited rational enterprises participating in carbon trading. It holds that individuals in reality gradually achieve dynamic equilibrium through imitation, learning, mutation and other processes rather than achieving Nash equilibrium after a single game. At present, the research on carbon emission trading based on evolutionary game mainly focuses on the game analysis between government and enterprises [4-7], while the game analysis between enterprises is less.
Based on the existing research, this paper takes the limited rational enterprises participating in the carbon trading as the research object, uses the evolutionary game method to the enterprises' emission reduction behavior, and provides suggestions for the governments according to the results.

2. Model Establishment

Consider that there are many large groups of enterprises participating in carbon trading market, which are mainly divided into two types: enterprise A and enterprise B. Each enterprise in the group is independent of each other, and each time enterprise A and enterprise B are randomly matched. All enterprises are limited rationality and constantly adjust their strategies to maximize their own interests through learning and imitation. According to the rules of carbon trading mechanism, the enterprise's emission reduction strategy space can be expressed as (direct purchase, self-emission reduction), abbreviated as (S, R). Suppose that the profits of enterprises before they did not participate in carbon trading are \( \pi_1 \) and \( \pi_2 \), \( \pi_1 > 0, \pi_2 > 0 \). The following conditions can be met.

(1) If both enterprise A and B adopt the direct purchase strategy, their revenues are \( \pi_1 = p(E_1 - Q_1) \) and \( \pi_2 = p(E_2 - Q_2) \) respectively. Where \( p (p > 0) \) is the trading price of carbon emission rights, which is determined by the market; \( E_1 \) and \( E_2 \) are the carbon emissions of the two companies; \( Q_1 \) and \( Q_2 \) are the initial carbon emission quotas of the two, \( E_1 > Q_1 > 0, E_2 > Q_2 > 0 \).

(2) If enterprise A adopts self-emission reduction strategy and enterprise B adopts direct purchase strategy, the revenues of enterprise A and B are \( \pi_1 = -c_1e_1 + p(e_1 + Q_1 - E_1) + \Delta \pi_1 \) and \( \pi_2 = -p(E_2 - Q_2) \). Where \( e_1 \) is the carbon emission reduction amount of enterprise A; \( c_1 \) is the unit carbon emission reduction cost, \( e_1 + Q_1 > E_1 \); \( \Delta \pi_1 \) is additional income, since only enterprise A adopts self-emission reduction strategy in the market.

(3) If enterprise A adopts direct purchase strategy and enterprise B adopts self-emission reduction strategy, the revenues of enterprise A and B are \( \pi_1 = -p(E_1 - Q_1) \) and \( \pi_2 = -c_2e_2 + p(e_2 + Q_2 - E_2) \) +\( \Delta \pi_2 \).

(4) If both enterprise A and B adopt the self-emission reduction strategy, their revenues are \( \pi_1 = -c_1e_1 + p(e_1 + Q_1 - E_1) \) and \( \pi_2 = -c_2e_2 + p(e_2 + Q_2 - E_2) \).

3. Evolutionary game analysis of the model

3.1. Equilibrium point of evolution process

Suppose that the proportions of enterprises adopting direct purchase strategy and self-emission reduction strategy in enterprise A group are \( x \) and \( 1 - x \), while that of enterprise B group are \( y \) and \( 1 - y \) respectively. \( x \in [0,1], y \in [0,1] \).

According to evolutionary game theory, simultaneous the replication dynamic equations of enterprise A and B. A two-dimensional dynamic system (1) is obtained as follow.

\[
\begin{align*}
\frac{dx}{dt} &= x(1-x)[(c_1 - p)e_1 - \Delta \pi_1]y \\
\frac{dy}{dt} &= y(1-y)[(c_2 - p)e_2 - \Delta \pi_2]x
\end{align*}
\]

From \( \frac{dx}{dt} = 0 \) and \( \frac{dy}{dt} = 0 \), the following propositions can be obtained.
Proposition 1: The equilibrium points of system (1) are (0,0), (0,1), (1,0), (1,1). When \(0 < c_1 < p + \Delta \pi_1 / e_1\) and \(0 < c_2 < p + \Delta \pi_2 / e_2\), \((x^*, y^*)\) is also the equilibrium point.

3.2. Stability analysis of equilibrium point

The equilibrium points obtained by replication dynamic equations are not necessarily the evolutionary stability strategy (ESS) of system. The stability of the evolution equilibrium point can be obtained from the local stability analysis of the Jacobian matrix of the system. If the Jacobian determinant \(\det J\) of the equilibrium point is positive and the Jacobian trace \(\text{tr} J\) is negative, then the equilibrium point is ESS. Jacobian matrix of System (1) is as follow.

\[
J = \begin{bmatrix}
(1-2x)[(c_1 - p)e_1 - \Delta \pi_1 g y]
-\Delta \pi_2 g(1-y)
-\Delta \pi_1 g(1-x)
(1-2y)[(c_2 - p)e_2 - \Delta \pi_2 g x]
\end{bmatrix}
\]

PROPOSITION 2: (1) If \(0 < c_1\) and \(c_2 < p\), the ESS of system (1) is \((R, R)\).
(2) If \(0 < c_1 < p < c_2 < p + \Delta \pi_2 / e_2\), the ESS of system (1) is \((R, S)\).
(3) If \(0 < c_1 < p < c_2 < p + \Delta \pi_1 / e_1\), the ESS of system (1) is \((S, R)\).
(4) If \(0 < c_1 < p + \Delta \pi_1 / e_1\) and \(0 < c_2 < p + \Delta \pi_2 / e_2\), the ESS of system (1) is \((S, R)\) or \((R, S)\).
(5) If \(c_1 > p + \Delta \pi_1 / e_1\) and \(c_2 > p + \Delta \pi_2 / e_2\), the ESS of system (1) is \((S, S)\).

Proof: According to the above judgment methods, the evolution stability analysis of 5 situations is shown in Table 2. The result is the same as Proposition.

### Table 1. Local stability of equilibrium point in 5 situations

| Equilibrium points | (0,0) | (0,1) | (1,0) | (1,1) | (x*,y*) |
|--------------------|-------|-------|-------|-------|---------|
| Situation (1) | \(\det J\) | + | - | - | + | non-equilibrium point |
| | \(\text{tr} J\) | - | uncertain | uncertain | + | unstable point |
| | result | ESS saddle point | saddle point | ESS saddle point | |
| Situation (2) | \(\det J\) | uncertain | - | + | + | unstable point |
| | \(\text{tr} J\) | - | uncertain | saddle point | ESS | unstable point |
| | result | saddle point | ESS saddle point | ESS | |
| Situation (3) | \(\det J\) | uncertain | uncertain | - | + | unstable point |
| | \(\text{tr} J\) | - | uncertain | saddle point | ESS | unstable point |
| | result | saddle point | ESS saddle point | ESS | |
| Situation (4) | \(\det J\) | - | + | + | + | non-equilibrium point |
| | \(\text{tr} J\) | + | uncertain | saddle point | ESS | unstable point |
| | result | unstable point | ESS saddle point | ESS | |
| Situation (5) | \(\det J\) | + | - | - | + | |
| | \(\text{tr} J\) | + | uncertain | saddle point | ESS | |
3.3. Impact of parameter changes on emission reduction strategy

According to the above results, in situation(4), self-emission reduction costs of both enterprise A and B are greater than the revenues of carbon emission right, but less than the sum of the additional income and the income from selling carbon emission right, the ESS of system is $(R,S)$ or $(S,R)$. In this situation, the probability of ESS being $(R,S)$ can be expressed as follow.

$$S_i = \frac{1}{2} \left[ \frac{(c_2 - p)e_2 + \Delta \pi_1 - (c_1 - p)e_1}{\Delta \pi_1} \right]$$  \hspace{1cm} (3)

From equation (3), there are 7 factors affecting $S_i$. So the following propositions can be obtained.

Proposition 3: When the unit carbon emission reduction cost of enterprise A is smaller, and that of enterprise B is greater, enterprise A is more inclined to adopt self-emission reduction strategy while enterprise B is more inclined to adopt direct purchase strategy.

Proof: The partial derivatives of $S_i$ with respect to $c_1$ and $c_2$ can be obtained as follows.

$$\frac{\partial S_i}{\partial c_1} = -\frac{e_1}{2g \Delta \pi_1} < 0, \quad \frac{\partial S_i}{\partial c_2} = -\frac{e_1}{2g \Delta \pi_1} > 0 \hspace{1cm} (4)$$

Therefore, $S_i$ is a monotonically decreasing function of $c_1$ and an increasing function of $c_2$. As the unit carbon emission reduction cost of enterprise A decreases and that of enterprise B increases, the probability of ESS being $(R,S)$ is greater.

Proposition 4: When the carbon emission reduction amount of enterprise A is smaller, and that of enterprise B is greater, enterprise A is more inclined to adopt self-emission reduction strategy while enterprise B is more inclined to adopt direct purchase strategy.

Proof: The partial derivatives of $S_i$ with respect to $e_1$ and $e_2$ can be obtained as follows.

$$\frac{\partial S_i}{\partial e_1} = -\frac{(c_1 - p)}{2g \Delta \pi_1} < 0, \quad \frac{\partial S_i}{\partial e_2} = -\frac{(c_1 - p)}{2g \Delta \pi_2} > 0 \hspace{1cm} (5)$$

Therefore, $S_i$ is a monotonically decreasing function of $e_1$ and an increasing function of $e_2$. As the carbon emission reduction amount of enterprise A decreases and that of enterprise B increases, the probability of ESS being $(R,S)$ is greater.

Proposition 5: When the additional income of enterprise A is smaller, and that of enterprise B is greater, enterprise A is more inclined to adopt self-emission reduction strategy while enterprise B is more inclined to adopt direct purchase strategy.

Proof: The partial derivatives of $S_i$ with respect to $\Delta \pi_1$ and $\Delta \pi_2$ can be obtained as follows.

$$\frac{\partial S_i}{\partial \Delta \pi_1} = \frac{(c_1 - p)e_1}{2g \Delta \pi_1^2} > 0, \quad \frac{\partial S_i}{\partial \Delta \pi_2} = \frac{(c_1 - p)e_2}{2g \Delta \pi_2^2} < 0 \hspace{1cm} (6)$$

Therefore, $S_i$ is a monotonically increasing function of $\Delta \pi_1$ and an decreasing function of $\Delta \pi_2$. As the additional income of enterprise A increases and that of enterprise B decreases, the probability of ESS being $(R,S)$ is greater.

Proposition 6: When the trading price of carbon emission rights is smaller, enterprises with greater additional income are more inclined to adopt self-emission strategy. Enterprises with less additional income are more likely to adopt direct purchase strategy.

Proof: The partial derivatives of $S_i$ with respect to $\Delta \pi_1$ and $\Delta \pi_2$ can be obtained as follows.

$$\frac{\partial S_i}{\partial p} = \frac{1}{2} \left( \frac{e_1 - e_2}{\Delta \pi_1} \right) \hspace{1cm} (7)$$
If \( \frac{e_2}{\Delta \pi_2} > \frac{e_1}{\Delta \pi_1} \), then \( \frac{\partial S_i}{\partial p} > 0 \), \( S_i \) is a monotonically increasing function of \( p \). It shows that if the additional income of enterprise A is less than that of enterprise B, then as the trading price of carbon emission rights decreases, the probability of ESS being \((S, R)\) is greater. Vice versa. Therefore, as the trading price of carbon emission rights decreases, the enterprises with greater additional income are more inclined to adopt self-emission strategy and those with less additional income are more inclined to adopt direct purchase strategy.

Based on the above proposition, it can be seen that in the process of carbon trading, enterprises' self-emission reduction behavior is affected by the unit carbon emission reduction cost, carbon emission reduction amount, additional income and the trading price of carbon emission rights.

4. Conclusion

The results show that the enterprise's emission reduction strategy depends on the comparison between the unit emission reduction cost and the trading price, and it is related to the additional income obtained when only one enterprise adopts self-emission reduction strategy. If the unit emission reduction cost is less than the trading price, the enterprises will reduce their own emission actively. Otherwise, most of the enterprises will choose to purchase the carbon emission rights. If the additional income is greater than the self-emission reduction cost, the enterprise has a tendency to reduce the carbon emission by itself. When the trading price of carbon emission right is fixed, as the unit carbon emission reduction costs of the two companies gradually increase, there will be an evolutionary stable equilibrium in which (self-emission reduction, self-emission reduction), (self-emission reduction, direct purchase), (direct purchase, self-emission reduction), (direct purchase, direct purchase) and other strategies coexist.

Based on the above results, governments can use subsidy policy to encourage the adoption of self-emission reduction strategy. For enterprises that choose to reduce emissions by themselves, the costs of self-emission reduction can be lowered by several measures such as tax exemption, bonus and other subsidies. This will enable companies to invest more in low-carbon technology and improve energy efficiency, and thus realize the carbon emission reduction targets.

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