Inclusive Two–Photon Reactions at TRISTAN

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Abstract

After briefly reviewing past accomplishments of TRISTAN experiments in the field of inclusive two–photon reactions, I discuss open problems in the Monte Carlo simulation of such reactions. The main emphasis is on multiple scattering, i.e. events where at least two pairs of partons scatter within the same $\gamma\gamma$ collision to form at least four (mini)jets. The cross section for such events might just be observable at TRISTAN. While theoretical arguments for the existence of such events are strong, they have not yet been directly observed experimentally, thereby potentially opening a new opportunity for TRISTAN experiments.

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1. Introduction

In this talk I will attempt to cover some topics relevant for the understanding of inclusive (mini)jet production in quasi–real two–photon production at TRISTAN and elsewhere. In my opinion the study of such reactions is interesting for at least two reasons. First, it increases our knowledge of the perturbative structure of the photon, described by the parton densities inside the photon [1]. This is necessary to improve our ability to predict (background) cross sections at higher energy ep [2] and e+e− [3] colliders. An accurate determination of these parton densities should also allow to test predictions [1, 4] for these densities based on certain dynamical assumptions.

This latter point is connected to the second main motivation for studying two–photon reactions: They are an excellent testing ground for our understanding of those aspects of semi–hard and non–perturbative QCD that are relevant for collider phenomenology [5]. On the one hand γγ collisions “ought” to be more easily treatable than fully hadronic collisions, since intuitively a photon should be a simpler object than a proton. On the other hand, the current description [3] of non–diffractive γγ reactions in terms of direct, single resolved and double resolved contributions, or the even more complicated classification scheme of ref. [5], makes the complete understanding of γγ reactions appear considerably more challenging than that of pp or pp scattering.

In recent years TRISTAN experiments have contributed greatly to our understanding of inclusive two–photon reactions. In particular, in 1991 the AMY collaboration for the first time established [6] the existence of resolved photon contributions. This was also the first experiment that succeeded in describing their data with a QCD–based Monte Carlo program; previous codes had not included hard resolved photon contributions, and had consequently not been able to reproduce PEP and PETRA data. A little later, the TOPAZ collaboration presented [8] a first measurement of cross sections for the production of fully reconstructed jets, using a cone algorithm. This is a great improvement over the previously used definition of jets as “thrust hemispheres”, which obscured the relation between partons and jets. At the same time, and approximately simultaneous with HERA experiments [9, 2], TOPAZ directly observed the spectator or remnant jets characteristic for resolved photon processes. Most recently, TOPAZ has begun to use [10] this ability to tag remnant jets to disentangle direct and resolved photon contributions to inclusive charm and K⁰ production. Finally, it should be mentioned that the LEP experiments ALEPH [11] and DELPHI [12] have published first results on inclusive no–tag two–photon reactions, and VENUS is also entering the fray [13].

On the theoretical side, progress has been made in the calculation of next–to–leading order (NLO) corrections. Full NLO calculations for single–jet inclusive cross sections (for massless partons) [14], and for inclusive charm production [14], are available. Improved estimates for the photon flux factors relevant for resolved photon contributions have been presented in [14]. Finally, the JETSET/PYTHIA program package has been extended to include all classes of inclusive γγ reactions [5].

In spite of this progress, open problems remain. The for practical purposes most urgent problem is probably the lack of a reliable “standard” MC code, which is necessary to link parton–level calculations [5, 14] to measured quantities. At present all four experimental groups active in this field use their own MC generators. One measure of the differences between these codes is the value of the cut–off parameter $p_{T,\text{min}}$ (the minimal allowed partonic
transverse momentum in “hard” resolved photon collisions) determined from their respective data, even when assuming the same parton densities in the photon: While AMY and TOPAZ now both quote [8] values around 2.0 GeV for the DG parametrization [17], DELPHI finds [12] a value of about 1.45 GeV, while ALEPH gives [11] a value as large as 2.5 GeV. These groups all use different trigger criteria, and also use different methods to determine the optimal value of \( p_{T,\text{min}} \). Nevertheless this quite substantial discrepancy indicates that (some) current generators are not yet complete.

In fact, to the best of my knowledge none of the generators used by experiments to date includes initial state radiation (ISR). The implementation if ISR in standard PYTHIA apparently leads to very poor agreement with the data [11]. I have argued elsewhere [18] that ISR in resolved photon interactions should be cut off sooner than in \( pp \) scattering, at least whenever the “pointlike” component of the photon structure functions is involved. I believe the program developed in ref.[5] allows to do so at least as an option. I would like to stress here that some amount of ISR has to exist; switching it off completely is certainly an over–simplification. One possibility to investigate this experimentally is to study the opening angle in the transverse plane \( \Delta \phi \) between the two jets in two–jet events. On the parton level (and in the absence of multiple interactions, to be discussed below), \( \Delta \phi = 180^\circ \) unless final and initial state radiation are included. The more \( \Delta \phi \) deviates from 180\(^\circ\), the more ISR is present.\footnote{Note that the integrated double resolved contribution to the jet cross section for \( p_T > p_{T,\text{min}} \) scales approximately like \( p_T^{3.5} \); increasing \( p_{T,\text{min}} \) from 1.45 to 2.5 GeV therefore decreases this contribution by a factor of approximately 6.5!}

Another potentially important ingredient of a successful MC code is the treatment of multiple partonic scatterings in the same \( \gamma\gamma \) scattering; see Fig. 1. For example, PYTHIA makes use of multiple scattering to reproduce features of the “underlying event” at \( pp \) colliders [19]. This effect is included in the treatment of ref.[3], but has otherwise not been discussed in any detail in the context of two–photon reactions. I therefore decided to use this Contribution to provide a first quantitative estimate of multiple scattering event rates in \( \gamma\gamma \) collisions. To that end, I briefly describe in Sec. 2 the relation between multiple scattering and calculations of the total \( \gamma\gamma \) cross section at high energies in the minijet picture. Sec. 3 contains estimates for multiple scattering event rates at TRISTAN, and Sec. 4 is devoted to a brief summary and conclusions.

2. Multiple scattering and \( \sigma_{\text{tot}}(\gamma\gamma \to \text{hadrons}) \)

In leading order QCD, the inclusive cross section for the production of (at least) one jet pair with \( p_T \geq p_{T,\text{min}} \) in \( \gamma\gamma \) collisions is given by the well–known expression

\[
\sigma_{\gamma\gamma}^{\text{jet}} = \sum_{i,j,k,l} \int_{x_{\text{min}}}^{1} dx_1 \int_{x_{\text{min}}}^{1} dx_2 f_i|_{\gamma}(x_1)f_j|_{\gamma}(x_2) \int_{p_{T,\text{min}}}^{\sqrt{s}/2} \frac{d\hat{s}}{dp_T} \frac{d\hat{\sigma}_{ij\rightarrow kl}(\hat{s})}{dp_T} dp_T, \tag{1}
\]

with \( x_{\text{min}} = 4p_{T,\text{min}}^2/s \), \( s \) being the squared \( \gamma\gamma \) centre–of–mass energy, and \( \hat{s} = x_1x_2s \). \( f_i|_{\gamma} \) is the density of parton \( i \) in the photon, and the \( \hat{\sigma}_{ij\rightarrow kl}(\hat{s}) \) are the hard QCD 2 \( \rightarrow \) 2 scattering cross

\footnote{Final state radiation can also give \( \Delta \phi < 180^\circ \). However, I see no reason why standard prescriptions for FSR should fail for resolved photon events; FSR can therefore be subtracted, or included, using standard MC codes.
sections [21]. Eq. (1) is straightforward to evaluate numerically for given parton distribution functions \( f_i \mid \gamma \) and given \( p_{T,\text{min}} \). For example, using the DG parametrization [17] one finds approximately [21]

\[
\sigma_{\gamma\gamma}^{\text{jet}}(\text{DG}) \simeq 270 \text{ nb} \left( \frac{\sqrt{s}}{50 \text{ GeV}} \right)^{1.4} \left( \frac{1.6 \text{ GeV}}{p_{T,\text{min}}} \right)^{3.6},
\]

where I have used \( Q^2 = \hat{s}/4 \) as scale in \( \alpha_S \) and \( f_i \mid \gamma \).

Notice that the cross section (2) increases like a power of the cms energy \( \sqrt{s} \). At sufficiently high energy it will therefore exceed the usual VDM estimate [22] for the total cross section for \( \gamma\gamma \rightarrow \text{hadrons} \),

\[
\sigma_{\gamma\gamma}^{\text{VDM}} \simeq 250 \text{ nb} + \frac{300 \text{ nb GeV}}{\sqrt{s}}.
\]

One possibility is, of course, that the total hadronic cross section at high energies is indeed much larger than the VDM estimate (3). However, given the rather modest increase of the total \( \gamma p \) cross section as measured at HERA [23], which can be described quite well [24] by the “universal” asymptotic \( s^{0.08} \) behaviour also found in total \( pp \) and \( pp \) cross sections, a rapid increase of \( \sigma_{\text{tot}}(\gamma\gamma \rightarrow \text{hadrons}) \) now seems implausible.

It is important to notice here that eqs. (1,2) refer to an inclusive cross section, which by definition contains a factor of the average jet pair multiplicity:

\[
\sigma_{\gamma\gamma}^{\text{jet}} = \langle n_{\text{jet\ pair}} \rangle \cdot \sigma_{\gamma\gamma \rightarrow \text{hadrons}}.
\]

This means that whenever eq. (2) exceeds the total hadronic cross section, there must be on average more than one jet pair per \( \gamma\gamma \) collision. Since we are still working in leading order in QCD, the only possibility to produce additional jet pairs is to have several parton–parton scatterings in the same \( \gamma\gamma \) scattering.\(^*\) Hence there is very strong evidence from perturbative QCD that multiple partonic interactions within one \( \gamma\gamma \) event must occur at high energies.\(^\dagger\)

The simplest quantitative estimates of multiple interaction rates are based on the eikonal formalism [25, 19]. One writes the total interaction cross section as

\[
\sigma_{\text{tot}}(\gamma\gamma \rightarrow \text{hadrons}) = P_{\text{had}}^2 \int d^2b \left[ 1 - e^{-\chi(s)A(b)/P_{\text{had}}^2} \right].
\]

Here \( \vec{b} \) is the (two–dimensional) impact parameter, \( A(b) \) describes the distribution of scatter centers (i.e., partons) in the transverse plane, and the dynamical information about the individual scattering processes is contained in the eikonal \( \chi(s) \). The quantity \( P_{\text{had}} \), first introduced in ref. [27], describes the probability for a photon to go into a hadronic state, and is thus of order \( \alpha_{\text{em}} \).

\(^*\)Higher order QCD corrections will not change this conclusion, since here the cross section for the production of many jets is suppressed by powers of \( \alpha_S \); this suppression is not compensated by any enhancement factors, except in the relatively rare cases where \( \hat{s} \gg 4p_T^2 \). In contrast, the comparison of eqs. (2) and (4) shows that \( \langle n_{\text{jet\ pair}} \rangle \) must grow like some power of the cms energy, if \( \sigma_{\text{tot}} \) grows only slowly with energy.

\(^\dagger\)The only way around this conclusion is to make \( p_{T,\text{min}} \) grow with \( \sqrt{s} \). However, intuitively \( p_{T,\text{min}} \) should describe a cut–off due to confinement effects. It is difficult to understand why this should depend on the energy.
It is customary to split the eikonal into a soft (nonperturbative) part $\chi_{\gamma\gamma}^{\text{soft}}$ and the perturbative contribution, which is nothing but the minijet contribution $\chi_{\gamma\gamma}^{\text{jet}}$:

$$\chi(s) = \chi_{\gamma\gamma}^{\text{soft}}(s) + \chi_{\gamma\gamma}^{\text{jet}}(s).$$  \hfill (6)

The physical meaning of the ansatz $\chi_{\gamma\gamma}^{\text{jet}}$ then becomes more transparent when it is re-written as

$$\sigma_{\text{tot}}(\gamma\gamma \rightarrow \text{hadrons}) = P_{\text{had}}^2 \int d^2b \ e^{-\sigma_{\gamma\gamma}^{\text{jet}}(s)A(b)/P_{\text{had}}^2} \left[ e^{\sigma_{\gamma\gamma}^{\text{jet}}(s)A(b)/P_{\text{had}}^2} - e^{-\chi_{\gamma\gamma}^{\text{soft}}(s)A(b)/P_{\text{had}}^2} \right] \hfill (7)$$

Notice that each term in the sum, when multiplied with the exponential in front of the square brackets, is equivalent to the Poisson probability to have $n$ independent hard scatters at impact parameter $b$, the average number of scatters being $\sigma_{\gamma\gamma}^{\text{jet}}(s)A(b)/P_{\text{had}}^2$. Note that $\sigma_{\gamma\gamma}^{\text{jet}}$ is of order $\alpha_{\text{em}}^2$, since the parton densities $f_{i/q}$ in eq.(1) are of order $\alpha_{\text{em}}$. The presence of the factor $P_{\text{had}}^2$ in the denominator then ensures that the probability for additional hard scatters is not suppressed by additional powers of $\alpha_{\text{em}}$; this is reasonable, since the transition of the incident photons into hadronic states only has to occur once, independent of the number of hard scatters. Furthermore, the fact that we obtain a Poisson distribution for the number of scatters at fixed impact parameter means that we have assumed that these reactions occur independently of each other; we will see later that this assumption might be questionable in case of $\gamma\gamma$ collisions.

The fact that the perturbative contribution to the eikonal $\chi$ of eq.(1) should indeed be the jet cross section (1) can be seen by computing the inclusive cross section for the production of (at least) $k$ jet pairs from eq.(7). To that end, one simply has to include all terms with $n \geq k$ in the sum, and multiply them with the combinatorics factor $\binom{n}{k}$ to pick $k$ jet pairs out of a total of $n$ pairs:

$$\sigma(\geq k \text{ jet pairs}) = P_{\text{had}}^2 \int d^2b \ e^{-\sigma_{\gamma\gamma}^{\text{jet}}(s)A(b)/P_{\text{had}}^2} \sum_{n=k}^{\infty} \frac{1}{n!} \binom{n}{k} \left( \frac{\sigma_{\gamma\gamma}^{\text{jet}}(s)A(b)}{P_{\text{had}}^2} \right)^n$$

$$= P_{\text{had}}^2 \frac{1}{k!} \int d^2b \ e^{-\sigma_{\gamma\gamma}^{\text{jet}}(s)A(b)/P_{\text{had}}^2} \sum_{n=k}^{\infty} \frac{1}{(n-k)!} \left( \frac{\sigma_{\gamma\gamma}^{\text{jet}}(s)A(b)}{P_{\text{had}}^2} \right)^{n-k+k}$$

$$= P_{\text{had}}^2 \frac{1}{k!} \int d^2b \left( \frac{\sigma_{\gamma\gamma}^{\text{jet}}(s)A(b)}{P_{\text{had}}^2} \right)^k. \hfill (8)$$

This just gives $\sigma_{\gamma\gamma}^{\text{jet}}$ for $k=1$, since $\int d^2A(b) = 1$ by definition. Of more interest for us is the cross section for having at least a second independent partonic collision within the same $\gamma\gamma$ event, which is given by eq.(3) with $k=2$:

$$\sigma(\geq 2 \text{ jet pairs}) = \frac{1}{2} \left[ \sigma_{\gamma\gamma}^{\text{jet}}(s) \right]^2 \int d^2b \frac{A(b)^2}{P_{\text{had}}^2}$$

$$= \frac{\sigma_{\gamma\gamma}^{\text{jet}}(s)^2}{\sigma_0}, \hfill (9)$$
where I have introduced

$$\sigma_0 = \frac{2P^2_{\text{had}}}{\int d^2 b A(b)^2}. \quad (10)$$

Obviously the quantity $\sigma_0$ will play a crucial role in estimates of multiple scattering event rates. Intuitively it is something like the probability $P^2_{\text{had}}$ for the two incident photons to go into hadronic systems, multiplied with the geometrical cross section of these systems. Clearly the same quantities that determine the numerical value of $\sigma_0$, i.e. $P_{\text{had}}$ and $A(b)$, also enter the prediction (3) for the total hadronic cross section at high energies. In order to make this connection more quantitative let us consider the simple Gaussian ansatz

$$A(b) = \frac{1}{\pi b_0^2} e^{-b^2/b_0^2} \quad (11)$$

for the transverse distribution of partons in the photon. It is then quite easy to see that all physical quantities will only depend on the product $P_{\text{had}} \cdot b_0$. In particular, this ansatz gives

$$\sigma_0 = 4\pi b_0^2 P^2_{\text{had}}. \quad (12)$$

We can therefore somewhat arbitrarily fix $P_{\text{had}} = 1/200$ and explore the model dependence by varying $b_0$, which characterizes the transverse size of the hadronic system.

Fig. 2 shows predictions for the energy dependence of the total $\gamma\gamma$ cross section as computed from eq.(3) using the DG parametrization [17] with $p_{T,\text{min}} = 1.6$ GeV and three different values of $b_0$. For this calculation I have assumed

$$\chi_{\gamma\gamma}(s) = \chi_0 + \frac{\chi_1}{\sqrt{s}}, \quad (13)$$

as indicated by the VDM prediction (3); numerically, $\chi_0 = 0.375$ (0.5, 1.0) $\mu$b and $\chi_1 = 1.1$ (2.5, 12.5) $\mu$b-GeV for $b_0 = 2.3$ (1.9, 1.5) GeV$^{-1}$. The largest value of $b_0$ shown, 2.3 GeV$^{-1} = 0.46$ fm, would lead to a substantial increase of the total $\gamma\gamma$ cross section already at $\sqrt{s} = 100$ GeV, while the smallest choice, $b_0 = 1.5$ GeV$^{-1} = 0.3$ fm, leads to a very slow rise of this cross section, in rough agreement with the universal $s^{0.08}$ behaviour postulated by Donnachie and Landshoff [24]. It is crucial to keep in mind that smaller values of $b_0$ give smaller total cross sections (3) at high energies, but also smaller values for $\sigma_0$ (12) and hence larger rates for multiple scattering events (3); the three values of $b_0$ shown in Fig. 2 correspond to $\sigma_0 = 645, 440, \text{and} 275$ nb, respectively. This inverse relation between the total cross section and the rate of multiple scattering events is a direct consequence of eq.(4), i.e. it is independent of the specific ansatz for $A(b)$, or even of the eikonal ansatz (3) for $\sigma_{\text{tot}}(\gamma\gamma \rightarrow \text{hadrons})$. I repeat, the smaller the total cross section at high energies, the larger the rate for multiple scattering events at a given energy.

### 3. Multiple Scattering Rates at TRISTAN

In order to compute rates at $e^+e^-$ colliders for events containing multiple partonic scatters within one $\gamma\gamma$ collision, one has to convolute the $\gamma\gamma$ cross section with photon flux factors
in the standard way. In particular, the cross section \( \sigma_{e^+e^-}(\geq 2 \text{ jet pairs}) \) for the production of at least two jet pairs becomes:

\[
\sigma_{e^+e^-}(\geq 2 \text{ jet pairs}) = \frac{1}{\sigma_0} \sum_{\text{partons}} \int_{x_1,\min}^{1} dx_1 f_{\gamma\gamma}(z_1) \int_{x_2,\min}^{1} dx_2 f_{\gamma\gamma}(z_2) \int_{x_{1,\min}}^{1} dx_1 f_{\gamma}(x_1)
\]

\[
\cdot \int_{x_{2,\min}}^{1} dx_2 f_{\gamma}(x_2) \int_{p_{T,\min}}^{\sqrt{s}/2} dp_T \frac{d\hat{\sigma}_{ij\to kl}(s)}{dp_T}
\]

\[
\cdot \int_{x_{1,\min}}^{1-x_1} dx_1' f_{\gamma}(x_1') \int_{x_{2,\min}}^{1-x_2} dx_2' f_{\gamma}(x_2') \int_{p_{T,\min}}^{\sqrt{s}/2} dp_T \frac{d\hat{\sigma}_{ij\to kl}(s)}{dp_T}. \tag{14}
\]

Here, \( z_{1,\min} = 4p_T^2/s, \) \( z_{2,\min} = z_{1,\min}/z_1, \) \( x_{1,\min} = z_{2,\min}/z_2, \) \( x_{2,\min} = z_{1,\min}/x_1, \) \( x_{1,\min}' = x_{1,\min}/x_1', \) \( \hat{s} = z_1 z_2 x_1 x_2 s \) and \( \hat{s}' = z_1 z_2 x_1' x_2' s. \) Notice that even for given \( z_1 \) and \( z_2 \) the expression (14) does not factorize into two independent cross sections, as was indicated in eq. (8). The reason is that I modified the upper boundaries of the integrations over \( x_1' \) and \( x_2' \) in order to enforce energy–momentum conservation, i.e. \( x_1 + x_1' \leq 1 \) and \( x_2 + x_2' \leq 1. \) A slightly different method to do this has been used in ref. [19]. However, this requirement is much more important for \( \gamma\gamma \) collisions than for \( p\bar{p} \) collisions, since the quark densities inside the photon are much harder than those inside the proton, i.e. they remain sizable for \( x \) quite close to 1. I will come back to this point later.

In principle the sum in eq. (14) contains 64 independent terms (combinations of parton densities and hard subprocess cross sections \( \hat{\sigma} \)). The calculation can be greatly simplified by using the observation of ref. [27] that the sums over parton species can be treated approximately by introducing the effective parton density

\[
f_{\text{eff}}(x) = \frac{9}{4} f_{G}(x) + 2 \sum_i f_{q_i}(x), \tag{15}
\]

where the factor of two takes care of anti–quarks. In the same approximation all hard scattering cross sections are replaced by the cross section for the elastic scattering of two different quarks, \( \hat{\sigma}(qq' \to qq') \). The sum in eq. (14) then collapses to a single term. I checked that this approximation reproduces the “exact” (leading order) prediction for the single jet pair inclusive cross section, eq. (1), to better than 10\%, which is considerably smaller than the overall theoretical uncertainty of this estimate of multiple scattering rates.

Eq. (14) describes the production of (at least) four jets. However, not all of them have to fall within the angular region covered by a given detector, i.e. have rapidity \( |y| \leq y_{\text{cut}}. \) At the parton level, the jet rapidities are given by the usual relations

\[
y_{1,2} = \log \left[ \frac{x_1 z_1}{x_T} \left( 1 \pm \sqrt{1 - \frac{x_T^2}{z_1 z_2 x_1 x_2}} \right) \right], \tag{16}
\]

where \( x_T = 2p_T/\sqrt{s}; \) the rapidities \( y'_{1,2} \) of the second pair of jets are given by eq. (14) with \( (x_1, x_2, x_T) \to (x_1', x_2', x_T'). \) For a given \( y_{\text{cut}} \) one can then compute five independent cross sections from eq. (14), depending on the number of jets that satisfy \( |y_i| \leq y_{\text{cut}}, \) which I denote by \( \sigma_{n4} (1 \leq n \leq 4); \) moreover, for \( \sigma_{24} \) I distinguish contributions where both detected jets come from the same partonic scattering (\( \sigma_{24a} \)) from those where both jet pairs contribute
one jet each ($\sigma_{22}$). Notice that these $\sigma_{n4}$ are (approximately) exclusive cross sections, i.e. $\sigma_{14}$ is the cross section for having exactly one jet with $|y| \leq y_{\text{cut}}$, and so on.

Fig. 3 shows the dependence of these five cross sections on $p_{T,\text{min}}$, where I have taken $\sqrt{s} = 58$ GeV, $y_{\text{cut}} = 1$, and $\sigma_0 = 300$ nb (corresponding to $b_0 = 1.6$ GeV$^{-1}$; see Fig. 2). I have included the anti-tagging condition $\theta_e \leq 5^\circ$ for the outgoing electron and positron when computing the photon flux functions $f_{\gamma|e^\pm}$, and have estimated the suppression due to the virtuality of the photon as described in ref. [1]. For comparison this figure also shows single pair inclusive cross sections, split into contributions where exactly one ($\sigma_{12}$) or both ($\sigma_{22}$) jets pass the rapidity cut. These contributions, represented by the dotted curves, are nothing but the standard (LO) predictions [3] for double resolved jet production at TRISTAN. If single resolved and direct contributions were added, the dotted curves would have to be pushed up by a factor of 3 to 5. However, recent results on spectator jet tagging [10] indicate that this might not be necessary.

Evidently rates for events with multiple hard scatterings are not very large at TRISTAN, not even for $p_{T,\text{min}} = 1.6$ GeV, the smallest value shown. Moreover, a fraction of these events, given by $\sigma_{14}$ and $\sigma_{24a}$, have the same partonic final state within the given rapidity range as the “standard” contributions $\sigma_{12}$ and $\sigma_{22}$, respectively. Indeed, those parts of $\sigma_{14}$ and $\sigma_{24a}$ where the detected jet(s) come(s) from the “unprimed” (first) partonic collision in eq. (14) are already included in $\sigma_{12}$ and $\sigma_{22}$. Recall that these are inclusive cross sections as far as additional partonic scatterings are concerned; e.g., $\sigma_{22}$ is the cross section for having both jets produced in the first partonic scattering inside the acceptance region, independent of whether or not additional hard scatters occur in the same event.

Assuming that nothing at all is known about particle flows in the region $|y| > y_{\text{cut}}$ (other than perhaps the existence of spectator jets), information about multiple scattering events must therefore come from the contributions described by $\sigma_{24b}$, $\sigma_{34}$ and $\sigma_{44}$. Clearly the most distinctive signature would be the detection of all four jets. Except for (small) corrections due to initial and final state radiation, one expects these four jets to occur in two back-to-back pairs with equal and opposite transverse momentum, i.e. $\vec{p}_T(j1) \simeq -\vec{p}_T(j2)$ and $\vec{p}_T(j3) \simeq -\vec{p}_T(j4)$. Moreover, and in sharp contrast to multi-jet final states produced by higher order QCD processes from a single parton pair, the angular distribution in the transverse plane between these two jet pairs should be flat, i.e. the pairs should be uncorrelated. Unfortunately Fig. 3 shows that, given TRISTAN’s integrated luminosity of a few hundred pb$^{-1}$, each group will at best find a handful of such events with $y_{\text{cut}} = 1$, unless $\sigma_0$ is substantially smaller than 300 nb; notice that no allowance for finite jet reconstruction efficiencies has yet been made.

These cross sections are only approximately exclusive since in the derivation of eq. (3), which led to eq. (14), all terms with $n \geq 2$ were included. In other words, eq. (14) includes contributions with three, four, . . . , independent scatters, some of which might produce additional jets in the acceptance region. However, we will see below that, at least at TRISTAN energies, the cross section for the simultaneous production of at least two independent jet pairs at any rapidity is almost certainly substantially smaller than that for the production of a single jet pair. This indicates that for two-photon cms energies of relevance for TRISTAN, the sum in eq. (4) is still dominated by its first term, so that the rate for producing at least two jet pairs is very close to that for producing exactly two jet pairs.

It should be clear that multiple interactions can only occur in double resolved $\gamma\gamma$ reactions, since a pointlike (direct) photon is “used up” after a single interaction.

Similarly, $\sigma_{34}$ is included partly in $\sigma_{12}$ and partly in $\sigma_{22}$, and $\sigma_{44}$ is included entirely in $\sigma_{22}$.
On the other hand, the rate of three–jet events that are due to multiple interactions, described by \( \sigma_{34} \), might well be detectable. Up to ISR and FSR effects, these events should have the configuration \( p_T(j1) \simeq -p_T(j2) \), with a flat \( \Delta \phi \) distribution of the third jet with respect to the other two. Clearly a full MC analysis will be necessary to decide whether these properties make such events sufficiently distinguishable from "ordinary" QCD 3–jet events. A good understanding of initial and final state radiation will certainly be crucial for this study.

Finally, the cross sections for events with at least two independent partonic scatters clearly drop much faster with increasing \( p_{T,\text{min}} \) than the single jet pair inclusive cross sections do. This is not surprising. Eq.(14) shows that additional hard scatters mean additional factors of \( d\hat{\sigma}/dp_T \) (which decreases rapidly with increasing \( p_T \)) and additional factors of \( f_{\text{eff}}(x_i) \) (which decrease with increasing \( x_i \); recall that the lower bound on the \( x_i \) scales like \( p_{T,\text{min}}^2 \)). Clearly finding any evidence for multiple scattering at TRISTAN will be hopeless if one requires \( p_{T,\text{min}} \geq 2.5 \text{ GeV} \) or so for all detected jets. This indicates that most present analyses [8], which ignored multiple scattering and focussed on relatively large \( p_T \), will remain unaffected. It also means that such events with comparatively high \( p_T \) can be used to study initial and final state radiation without having to worry about multiple interaction effects, as described in Sec. 1. Once ISR and FSR are understood, one can go back to smaller \( p_T \) and look for the contribution \( \sigma_{34} \) as described above. One might then even be able to find evidence for the contribution \( \sigma_{234} \), where one has two jets which are usually neither back–to–back nor have equal \( |p_T| \); these events should show up in the tails of the \( \Delta \phi \) distribution discussed in Sec. 1.

Fig. 4 shows the dependence of the various cross sections on the rapidity cut, for \( p_{T,\text{min}} = 1.6 \text{ GeV} \). In the limit \( y_{\text{cut}} \to \infty \) only \( \sigma_{22} \) and \( \sigma_{44} \) remain finite since all produced jets are now detected; the fact that \( \sigma_{22} > 5\sigma_{44} \) even in this limit, and for the small value of \( p_{T,\text{min}} \) chosen, once again indicates that multiple scattering events are indeed quite rare at TRISTAN. However, if the rapidity coverage could be extended to 1.5 or even 2, the chances of detecting four–jet events due to independent partonic scatters would improve dramatically compared to the case with \( y_{\text{cut}} = 1 \) shown in Fig. 3. Of course, it is no longer possible to modify TRISTAN detectors; however, it might be possible to reconstruct jets using calorimetric information only. Although this will presumably come at the cost of substantially increased errors on the \( p_T \) of jets outside the core region of the detector, it could still greatly facilitate the study of multiple interactions.

It might be appropriate to briefly discuss some of the uncertainties of my estimates of multiple interaction rates here. As emphasized in Sec. 2, the overall rate scales like \( 1/\sigma_0 \), which in turn largely determines the high–energy behaviour of \( \sigma_{\text{tot}}(\gamma\gamma \to \text{hadrons}) \). The value \( \sigma_0 = 300 \text{ nb} \) used for my numerical estimates corresponds to \( b_0 = 1.6 \text{ GeV}^{-1} \), and thus to a rather modest growth of the total cross section, as shown in Fig. 2. Of course, the multiple interaction rate also depends on the same quantities that determine the usual (leading order) jet cross sections, i.e. the parton distribution functions, the value of the QCD scale parameter \( \Lambda \), and the momentum scale to be used in \( \alpha_S \) and in the parton densities; in fact, eq.(14) shows that the cross section for multiple interactions depends twice as strongly on these parameters as ordinary (inclusive) jet cross sections do.

Finally, there is the question of how reliable the eikonal ansatz [8] is, on which my estimates are based. As already mentioned earlier, this ansatz only holds if multiple interactions
occur independently of each other. This might be a good approximation for $p\bar{p}$ collisions, where such an ansatz has been tested most extensively [19], since here one deals with real hadrons that do certainly contain several partons, even though the exact relation between partons and constituent quarks might be quite complicated. In contrast, in the final analysis the entire parton content of the photon can be traced back to the $\gamma q\bar{q}$ vertex. One would therefore expect dynamical correlations to exist between different partons “in” the photon. These will probably be stronger at large Bjorken–$x$, since partons at large $x$ cannot be far removed from the primary vertex, i.e. not many parton splittings can have occurred, starting from the original $\gamma q\bar{q}$ vertex, to produce a parton at large $x$. One might therefore expect the assumption of independent scatters to work better if one sticks to relatively soft partons. Unfortunately, requiring $x_1 + x'_1 \leq 0.5$ and $x_2 + x'_2 \leq 0.5$ in eq.(14) reduces the total multiple interaction rate by more than a factor of 4, making them very difficult to detect experimentally.\footnote{For $y_{\text{cut}} = 1$, this restriction of the Bjorken–$x$ variables only reduces $\sigma_{44}$ by about a factor of two, since events with at least one large Bjorken–$x$ in the initial state tend to be more strongly boosted, i.e. to produce jets at large rapidities. I should also mention that in ref.[3] the eikonal ansatz is used only for the “hadronic” component of the photon structure function. There the contribution from the “pointlike” or “anomalous” component of the $f_{1\gamma}$ is regularized by increasing the cutoff $p_T,\text{min}$ linearly with the cms energy. I find this treatment not very satisfying. For one thing, the sharp separation in “hadronic” and “pointlike” pieces is clearly an over–simplification. Also, as mentioned earlier, I find it hard to understand why $p_T,\text{min}$ should depend on the beam energy. Nevertheless the generator of ref.[3] is the only existing two–photon MC code that treats multiple interactions at all.}

As mentioned earlier, the ansatz [5] (with $P_{\text{had}} \equiv 1$) has been used to describe $p\bar{p}$ collisions. One can certainly reproduce the observed behaviour of the total cross section in this way [28], but other successful descriptions exist as well [24]. In ref.[19] independent multiple interactions were used to describe details of the event structure at $p\bar{p}$ colliders, some of which are difficult to understand otherwise. One example is the so–called pedestal effect, i.e. the observation that events with a very hard interaction, say a pair of jets with $x_T > 0.1$, have a higher multiplicity and transverse energy flow even away from these jets than minimum bias events do. This is expected in the eikonal picture, since such events are more likely to be very central, i.e. to have impact parameter $b$ close to zero, which enhances the chance for additional partonic interactions. Furthermore, we heard at this meeting [2] that HERA data also have some features that might be explainable in terms of multiple interactions. Nevertheless no direct evidence for the existence of events with multiple partonic interactions has yet been found. In particular, no clear signal for the production of two independent jet pairs (equivalent to the contribution $\sigma_{44}$ discussed here) has yet been observed anywhere. Therefore questions about the reliability of the eikonal ansatz remain even in case of the proton.

Why might two–photon experiments succeed where $p\bar{p}$ experiments at much higher energy failed? There are two reasons to be optimistic, which are actually related to each other. In spite of the hadronic nature of the photon, which gives rise to spectator jets in resolved photon processes, $\gamma\gamma$ collisions are considerably “cleaner” than $p\bar{p}$ collisions. This leads to the possibility, demonstrated during this workshop [24], to reconstruct jets with very low transverse momentum, below 2 GeV; this is inconceivable at hadron colliders [31], and is probably impossible even at HERA [31]. Secondly, $\gamma\gamma$ collisions are considerably more “jetty” than $p\bar{p}$ events, that is the fraction of events with identifiable jets is larger. This can partly

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be explained by the hardness of the quark densities inside the photon, which allows to funnel a large fraction of the photon’s energy into a single parton, thereby allowing jet production (from resolved photons!) at relatively small two–photon cms energy $W_{\gamma\gamma}$. Moreover, the soft (non–jet) cross section seems to be anomalously small for photons, or large for protons; that is, the ratio of total $p\bar{p}$ to $\gamma\gamma$ cross sections exceeds the naive expectation of $1/\alpha_{em}^2$ by almost an order of magnitude. In other words, effectively the normalization $1/\sigma_0$, and hence the rate for multiple interactions, is about an order of magnitude smaller for $p\bar{p}$ than for $\gamma\gamma$ collisions.

However, experiments looking for multiple interactions in two–photon events at $e^+e^-$ colliders do face one obstacle not encountered at hadron colliders: The presence of $e^+e^-$ annihilation events makes it necessary to impose upper limits on the energy and/or invariant mass of the observed hadronic system. Recall that the cross section for multiple interactions grows very quickly with $W_{\gamma\gamma}$; see eqs.(9) and (2) in Sec. 2. These cuts might therefore reduce the signal significantly. This is demonstrated in Fig. 5, which shows the normalized distributions in $W_{\gamma\gamma}$ and in the summed energy of all jets, for four–jet events. Here I have chosen $y_{cut} = 2$, so that the sum over energies should approximate the total energy deposition from the high–$p_T$ partons in the calorimeter of a typical TRISTAN detector. Notice that some parts of the spectator jets are usually also detected, which shifts this distribution to the right. On the other hand, some parts of the spectator jets will almost always be lost in the beam pipes, so that the measured $W_{\gamma\gamma}$ distribution will be somewhat softer than the one shown in Fig. 5. Nevertheless it should be clear from this plot that the typical selection cuts, $W_{vis} \leq 15$ (20) GeV for TOPAZ (AMY), will reduce the signal for multiple interactions significantly. It might therefore be worthwhile to try and relax this cut, or to replace it, e.g. by a cut on the transverse energy in the event, which will be quite small for all $\gamma\gamma$ events, including those with multiple interactions.

4. Summary and Conclusions

TRISTAN experiments have contributed greatly to our understanding of multi–hadron production in $\gamma\gamma$ collisions, and thus of the hadronic structure of the photon. Recent theoretical progress, especially concerning NLO QCD calculations, should allow to fully exploit these measurements. The major stumbling block at present seems to be the lack of a reliable event generator that allows to estimate effects due to parton showering and fragmentation. Among other things, this is crucial for comparing results from different experiments, which have different trigger criteria, acceptance cuts etc. As mentioned in Sec. 1, one weakness of existing generators is the (lack of) treatment of initial state radiation; another, related, effect that is specific for resolved photon interactions is the relatively large intrinsic $k_T$ of partons in the photon $[32]$.

The main focus of this contribution was on events with multiple partonic interactions in the same $\gamma\gamma$ collision, since they might offer another great opportunity for TRISTAN experiments. Indeed, in my view the proof of the existence of such events would be even more important than the proof $[7]$ of the existence of resolved photon interactions. As I argued in Sec. 2, multiple interactions are expected to occur in all hadronic collisions ($p\bar{p}$, $\gamma p$ and $\gamma\gamma$) at sufficiently high energies, yet they have never been observed directly.
The rate for such events is intimately linked to the behaviour of total hadronic cross sections at high energies, a subject of much debate (without definite conclusion!) for about thirty years. The numerical estimates of Sec. 3 indicate that detection of such events at TRISTAN will not be easy, but might be possible. Not only the “gold plated” four–jet events, but also certain classes of three– and even two–jet events might be utilized for demonstrating the existence of multiple interactions. I argued that it should be advantageous, and might be necessary, to extend previous analyses of two–photon reactions by trying to reconstruct jets using calorimetric information only (in order to increase the angular coverage, which is crucial for four–jet events), and/or by relaxing the upper limits on $E_{\text{vis}}$ and $W_{\text{vis}}$ in the definition of the two–photon event sample.

As discussed in Sec. 3, the numerical estimates presented here are quite uncertain. In my opinion this should serve as additional stimulus for experimenters to try and find such events. After all, a large theoretical uncertainty means that little is known, so all information is helpful, including negative one. In particular, a meaningful lower bound on the quantity $\sigma_0$ might indicate that the total $\gamma \gamma$ cross section grows faster at high energies than presently anticipated, which can have ramifications for the planning of future experiments. Positive evidence for multiple interactions might indicate that previous estimates [3] of the transverse energy flow in typical $\gamma \gamma$ events at high energies have to be revised upwards. To my mind much more important than these rather mundane considerations of backgrounds at future colliders is the exciting possibility to learn something about an aspect of strong interactions about which little is known to date. I eagerly look forward to the first experimental study of multiple partonic interactions in two–photon collisions, at TRISTAN or elsewhere.

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Figure Captions

Fig.1 A Feynman diagram for a $\gamma\gamma$ collision with two separate parton–parton collisions. All combinations of partonic reactions ($qq$, $qG$ and $GG$) contribute to the total rate for events with multiple interactions.

Fig.2 The dependence of the total hadronic two–photon cross section on the $\gamma\gamma$ cms energy $W_{\gamma\gamma}$, as calculated in the simple eikonal model of Sec. 2, for three different values of $b_0$.

Fig.3 The dependence of the multiple interaction cross section on the transverse momentum cut–off $p_{T,\min}$, as computed from eq.(14). The total cross section for the production of at least four jets is split into five different contributions, depending on the number of jets with $|y| \leq y_{\text{cut}} = 1$, as described in the text. The double resolved contribution to the single jet pair inclusive cross sections are shown for comparison by the dotted curves. All cross sections have been computed using an anti–tag cut $\theta_e \leq 5^\circ$ for the outgoing electron and positron.

Fig.4 The dependence of jet cross sections on the acceptance region $y_{\text{cut}}$. The notation is as in Fig. 3.

Fig.5 The normalized distribution in the sum over high–$p_T$ jet energies (dashed) and in the $\gamma\gamma$ cms energy $W_{\gamma\gamma}$ (solid) of multiple interaction events with four jets with rapidity $|y| \leq y_{\text{cut}} = 2$. This value of $y_{\text{cut}}$ has been chosen to approximate the angular coverage of the calorimeter of a typical TRISTAN detector. Notice that more than 50% of the events have $W_{\gamma\gamma} \geq \sqrt{s}/2 = 29$ GeV.