Proximity-Aware Calculation of Cable Series Impedance for Systems of Solid and Hollow Conductors

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Abstract—Wide-band cable models for the prediction of electromagnetic transients in power systems require the accurate calculation of the cable series impedance as a function of frequency. A surface current approach was recently proposed for systems of round solid conductors, with inclusion of skin and proximity effects. In this paper we extend the approach to include tubular conductors, allowing to model realistic cables with tubular sheaths, armors and pipes. We also include the effect of a lossy ground. A noteworthy feature of the proposed technique is the accurate prediction of proximity effects, which can be of major importance in three-phase, pipe type, and closely-packed single-core cables. The new approach is highly efficient compared to finite elements. In the case of a cross-bonded cable system featuring three phase conductors and three screens, the proposed technique computes the required 120 frequency samples in only six seconds of CPU time.

Index Terms—Electromagnetic transients, broadband cable modeling, series impedance, skin effect, proximity effect.

I. INTRODUCTION

INSULATED cables are increasingly being used in all areas of modern high-voltage power systems. As the presence of cables has a strong impact on the transient behavior of a given power system, accurate cable models should be used when analysing the system performance during transient events following circuit breaker operations, fault situations and lightning discharges. Such analyses are typically performed using suitable Electro-Magnetic Transient Programs (EMTP) [1], [2]. As the transients may span a very broad frequency range, from a few Hz up to the MHz range, wide-band models should be used in the modeling of all relevant system components, including cables.

The input parameters for all broadband cable models [3], [4], [5] are the per-unit-length matrices of series impedance and shunt admittance [6]. The calculation of the series impedance is difficult, due to frequency-dependent phenomena in conductors and earth such as skin and proximity effects.

In existing EMTP tools, the series impedance is obtained with analytic formulas [7], [8], which however assume a circularly-symmetric current distribution on the conductors. This assumption becomes inaccurate in configurations combining non-coaxial arrangements and small lateral distances, like three-phase cables, pipe-type cables, and closely packed single-core cables. Here, proximity effects leads to a non-circular current distribution on conductors which is not accounted for. It has been demonstrated in numerous works [9], [10] that proximity effect can significantly affect transient voltages and should therefore be taken into account.

In a recent work [11], the authors introduced a new method for calculating the series impedance of systems of round solid conductors which takes into account both skin and proximity effects. The new method, called MoM-SO, relies upon a Surface Operator (SO) [12] and the Method of Moments (MoM) [13]. This surface-based approach requires only the discretization of the surface of the conductors, in contrast with volume-based approaches that mesh the entire cable cross-section, such as finite elements [14], [15], [16] and conductor partitioning [17], [18], [19], [20], [21].

The surface-based approach has been proposed in the literature for cables with conductors of rectangular [12], triangular [22], and solid round shape [11]. Recently, an extension of the approach to hollow conductors was presented without proofs in [23]. In this paper, we present the complete derivation of the surface admittance operator for hollow conductors, and we also include the effect of ground by an approximate formulation [10] where the proximity correction is added to a conventional solution which considers only skin effect. Hollow round conductors are useful in modeling realistic cable systems with tubular sheaths, armors, and pipes. The extended MoM-SO method is validated against a finite element (FEM) computation. Finally, we demonstrate the complete procedure with the modeling and simulation of a cross-bonded cable system involving three closely-packed single-core cables.

II. PROBLEM STATEMENT

We consider a cable made by $P$ round conductors oriented along the $z$-axis and surrounded by a lossless medium of permittivity $\varepsilon_o$ and permeability $\mu_o$. The cross section of each conductor can be either solid, as in the left panel of Fig. 1, or hollow, as in the left panel of Fig. 2. We denote the outer radius of the $p$-th conductor with $a_p$. If the conductor is hollow, we denote its inner radius with $\bar{a}_p$. The conductors have conductivity $\sigma$, permittivity $\varepsilon$, and permeability $\mu$. 

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From the geometry of the cable, we aim to compute the p.u.l.\(^1\) resistance \(\mathbf{R}(\omega)\) and inductance \(\mathbf{L}(\omega)\) matrices which appear in the Telegrapher’s equation [6]

\[
\frac{\partial \mathbf{V}}{\partial z} = -[\mathbf{R}(\omega) + j\omega \mathbf{L}(\omega)] \mathbf{I},
\]

where we collect the potential \(V_p\) and the current \(I_p\) of each conductor in the column vectors \(\mathbf{V} = [V_1 \ldots V_p]^T\), and \(\mathbf{I} = [I_1 \ldots I_p]^T\), respectively.

### III. Surface Admittance Formulation

We calculate the p.u.l. resistance and inductance of the cable with the surface method introduced in [12], where a surface admittance operator is used to replace all conductors with an equivalent current on their boundary. We first present the surface admittance representation focusing on a single solid or hollow conductor. Then, in Sec. III-C, the representation is extended to all conductors in the cable, and used in Sec. IV to compute the cable parameters.

#### A. Surface Admittance Operator for a Solid Conductor

We let conductor \(p\) be solid with the cross section shown in the left panel of Fig. 1. Using cylindrical coordinates, we trace the boundary \(c_p\) of the conductor with the position vector

\[
\mathbf{r}_p(\theta) = (x_p + a_p \cos \theta) \hat{x} + (y_p + a_p \sin \theta) \hat{y},
\]

where \((x_p, y_p)\) is the position of the conductor’s center and \(\hat{x}, \hat{y}\) denote the unit vectors along the \(x-\) and \(y-\)axis, respectively. The longitudinal component of the electric field on the conductor boundary is denoted with \(E_z^p(\theta)\), and is expanded in truncated Fourier series

\[
E_z^p(\theta) = \sum_{n=-N_p}^{N_p} E_n^p e^{jn\theta},
\]

where \(N_p\) controls the number of harmonics taken into account. The case \(N_p = 0\) corresponds to assuming a circularly-symmetric current distribution in the conductor. This assumption, made by analytic formulas used in existing EMTP tools, is accurate only for well-separated conductors. When spacing is comparable to the conductors size, proximity effects become significant and lead to a non-uniform field distribution that calls for \(N_p > 0\). Numerical tests [11] show that a \(N_p\) of 3 or 4 is sufficient in most cases. The choice of the parameter \(N_p\) can be performed automatically [24], and will not be discussed here.

Given the field (3) on the boundary, the electric field inside the conductor \(E_z^p(\rho, \theta)\) can be found by solving the Helmholtz equation [25] and reads

\[
E_z^p(\rho, \theta) = \sum_{n=-N_p}^{N_p} \frac{E_n^p}{J_{|n|}(k a_p)} J_{|n|}(k \rho e^{jn\theta}),
\]

where \(k = \sqrt{\omega \mu (\varepsilon - j\sigma)}\) is the wavenumber inside the conductor, \(\rho \in [0, a_p]\) is the radial coordinate, and \(J_{|n|}(\cdot)\) is the Bessel function of the first kind of order \(|n|\) [26].

We next use the equivalence theorem [27], [12] to replace the conductor with the surrounding medium. On its boundary \(c_p\), we introduce an equivalent surface current density \(J_s^p(\theta)\) in order to keep the electromagnetic field outside the conductor unchanged. This transformation is illustrated in Fig. 1. The value of the equivalent current \(J_s^p(\theta)\) is given by the equivalence theorem [27], [12] as

\[
J_s^p(\theta) = H_t^p(a_p^-, \theta) - \bar{H}_t^p(a_p^-, \theta),
\]

where \(H_t^p(a_p^-, \theta)\) and \(\bar{H}_t^p(a_p^-, \theta)\) denote the magnetic field tangential to the conductor boundary respectively before and after the application of the equivalence theorem. Both fields are evaluated just inside the conductor boundary (i.e. for \(\rho = a_p^+\)).

The magnetic fields in (5) can be related to the longitudinal electric field as [12]

\[
H_t^p(a_p^-, \theta) = \frac{1}{j\omega \mu} \frac{\partial E_z^p}{\partial \rho} \Big|_{\rho=a_p^-},
\]

\[
\bar{H}_t^p(a_p^-, \theta) = \frac{1}{j\omega \mu_0} \frac{\partial \bar{E}_z^p}{\partial \rho} \Big|_{\rho=a_p^-},
\]

where \(E_z^p(\rho, \theta)\) is the electric field inside \(c_p\) after application of the equivalence theorem. This field can be written as

\[
E_z^p(\rho, \theta) = \sum_{n=-N_p}^{N_p} \frac{E_n^p}{J_{|n|}(k a_p)} J_{|n|}(k \rho e^{jn\theta}),
\]

which is (4) where \(k\) has been replaced with the wavenumber of the surrounding medium \(k_o = \omega \sqrt{\mu_o \varepsilon_o}\). By substituting (6) and (7) into (5), we obtain

\[
J_s^p(\theta) = \frac{1}{j \omega} \left[ \frac{1}{\mu} \frac{\partial E_z^p}{\partial \rho} - \frac{1}{\mu_o} \frac{\partial \bar{E}_z^p}{\partial \rho} \right] \Big|_{\rho=a_p^-}.
\]

Equation (9) defines a surface admittance operator that relates the equivalent current density to the electric field on the conductor boundary. If we adopt the equivalent current density \(J_s^p(\theta)\) a truncated Fourier expansion analogous to (3)

\[
J_s^p(\theta) = \frac{1}{2\pi a_p} \sum_{n=-N_p}^{N_p} J_n^p e^{jn\theta},
\]
we can conveniently express the surface admittance operator (9) in terms of the Fourier coefficients \( J_n^{(p)} \) and \( E_n^{(p)} \) as [12]

\[
J_n^{(p)} = Y_n^{(p)} E_n^{(p)},
\]

where

\[
Y_n^{(p)} = \frac{2\pi}{j\omega} \left[ \frac{k_\alpha a_p J_n'(k_\alpha a_p)}{\mu_\alpha J_n(k_\alpha a_p)} - \frac{k_\beta a_p J_n'(k_\beta a_p)}{\mu_\beta J_n(k_\beta a_p)} \right],
\]

and where \( J_n'(\cdot) \) is the derivative of \( J_n(\cdot) \). Before exploiting (11) for the computation of the series impedance, we extend the surface operator to hollow conductors.

**B. Surface Admittance Operator for a Hollow Conductor**

We now consider a hollow conductor with the cross section depicted in the left panel of Fig. 2. In addition to the outer boundary \( c_p \), we now have an inner boundary, denoted by \( \tilde{c}_p \). The outer boundary is traced by the position vector (2), while the inner boundary is traced by

\[
\tilde{r}_p(\theta) = (x_p + \tilde{a}_p \cos \theta) \hat{x} + (y_p + \tilde{a}_p \sin \theta) \hat{y}.
\]

The electric field on the inner and outer boundaries are denoted with \( \tilde{E}_n^{(p)}(\theta) \) and \( E_n^{(p)}(\theta) \), respectively, and are approximated by Fourier series as

\[
\tilde{E}_n^{(p)}(\theta) = \sum_{n=-N_p}^{N_p} \tilde{E}_n^{(p)} e^{jn\theta},
\]

\[
E_n^{(p)}(\theta) = \sum_{n=-N_p}^{N_p} E_n^{(p)} e^{jn\theta}.
\]

Given the boundary conditions (14) and (15), the electric field \( E_n^{(p)}(\rho, \theta) \) inside the conductor can be found by solving the Helmholtz equation in a hollow region [28], and reads

\[
E_n^{(p)}(\rho, \theta) = \sum_{n=-N_p}^{N_p} \left( C_n(k) \mathcal{H}_n(\rho) + D_n(k) \mathcal{K}_n(\rho) \right) e^{jn\theta},
\]

where the constants \( C_n(k) \) and \( D_n(k) \) are found from the boundary conditions. Imposing (14) and (15) on the two boundaries, we obtain

\[
C_n(k) = \frac{E_n^{(p)} \mathcal{K}_n(\rho_0)}{m_n(k \rho_0, k \rho)}, \quad D_n(k) = \frac{E_n^{(p)} \mathcal{N}_n(\rho_0)}{m_n(k \rho_0, k \rho)},
\]

where \( m_n(\alpha, \beta) = \mathcal{H}_n(\alpha) \mathcal{K}_n(\beta) - \mathcal{K}_n(\alpha) \mathcal{H}_n(\beta) \). In these formulas, \( \mathcal{H}_n(\cdot) \) and \( \mathcal{K}_n(\cdot) \) denote the Hankel functions of order \( |n| \) of, respectively, first and second kind [26].

In analogy with the solid conductor case, we replace the current densities on the outer boundary with \( \tilde{J}_n^{(p)}(\theta) \), and on the inner boundary with \( J_n^{(p)}(\theta) \). Both current densities are approximated by truncated Fourier series

\[
J_n^{(p)}(\theta) = \frac{1}{2\pi a_p} \sum_{n=-N_p}^{N_p} J_n^{(p)} e^{jn\theta}, \quad \tilde{J}_n^{(p)}(\theta) = \frac{1}{2\pi a_p} \sum_{n=-N_p}^{N_p} \tilde{J}_n^{(p)} e^{jn\theta}.
\]

Using the equivalence theorem [27], we size the equivalent currents in order to preserve the original electric field both inside the cavity (\( \rho < \tilde{a}_p \)) and beyond the outer boundary (\( \rho > a_p \)). With a derivation analogous to the one presented in Sec. III-A for a solid conductor, we obtain the following expression for the equivalent currents

\[
J_n^{(p)}(\theta) = \frac{1}{j\omega} \left[ \frac{1}{\mu} \frac{\partial \tilde{E}_n^{(p)}}{\partial \rho} - \frac{1}{\mu_0} \frac{\partial \tilde{E}_n^{(p)}}{\partial \rho} \right]_{\rho=\tilde{a}_p},
\]

\[
\tilde{J}_n^{(p)}(\theta) = \frac{1}{j\omega} \left[ \frac{1}{\mu_0} \frac{\partial \tilde{E}_n^{(p)}}{\partial \rho} - \frac{1}{\mu} \frac{\partial \tilde{E}_n^{(p)}}{\partial \rho} \right]_{\rho=a_p},
\]

where \( \tilde{E}_n^{(p)}(\rho, \theta) \) is the electric field inside the conductor after the application of the equivalence theorem. Its value is given by (16) with \( k \) replaced by the wavenumber of the surrounding medium \( k_0 \). Equations (21) and (22) define the surface admittance operator for a hollow conductor. This result is a generalization of the operator given in [12] for solid conductors.

We now rewrite (21) and (22) in terms of the Fourier coefficients \( J_n^{(p)} \), \( \tilde{J}_n^{(p)} \), \( E_n^{(p)} \) and \( \tilde{E}_n^{(p)} \). By substituting (16) into (21) and (22), we obtain

\[
\begin{bmatrix} \tilde{J}_n^{(p)} \\ J_n^{(p)} \end{bmatrix} = \begin{bmatrix} Y_n^{(p)} & \tilde{E}_n^{(p)} \\ E_n^{(p)} & \tilde{E}_n^{(p)} \end{bmatrix},
\]

where

\[
Y_n^{(p)} = \begin{bmatrix} Y_{11,n} & Y_{12,n} \\ Y_{21,n} & Y_{22,n} \end{bmatrix}
\]

is a 2×2 matrix which generalizes (12) to the hollow conductor.
case. The matrix entries are given by

\[
Y_{11,n} = \frac{2\pi}{j\omega} \left[ \chi_n(k_0 a_p, k_0 a_p) - \frac{\chi_n(k_0 a_p, k_0 a_p)}{m_n(k_0 a_p, k_0 a_p) \mu} - \frac{\chi_n(k_0 a_p, k_0 a_p)}{m_n(k_0 a_p, k_0 a_p) \mu_o} \right],
\]

\[
Y_{12,n} = \frac{2\pi}{j\omega} \left[ \chi_n(k_0 a_p, k_0 a_p) - \frac{\chi_n(k_0 a_p, k_0 a_p)}{m_n(k_0 a_p, k_0 a_p) \mu} - \frac{\chi_n(k_0 a_p, k_0 a_p)}{m_n(k_0 a_p, k_0 a_p) \mu_o} \right],
\]

\[
Y_{21,n} = \frac{2\pi}{j\omega} \left[ \chi_n(k_0 a_p, k_0 a_p) - \frac{\chi_n(k_0 a_p, k_0 a_p)}{m_n(k_0 a_p, k_0 a_p) \mu} - \frac{\chi_n(k_0 a_p, k_0 a_p)}{m_n(k_0 a_p, k_0 a_p) \mu_o} \right],
\]

\[
Y_{22,n} = \frac{2\pi}{j\omega} \left[ \chi_n(k_0 a_p, k_0 a_p) - \frac{\chi_n(k_0 a_p, k_0 a_p)}{m_n(k_0 a_p, k_0 a_p) \mu} - \frac{\chi_n(k_0 a_p, k_0 a_p)}{m_n(k_0 a_p, k_0 a_p) \mu_o} \right],
\]

with \(\chi_n(\alpha, \beta) = \beta [\mathcal{H}^i_{11}(\beta) K^c_{11}(\alpha) - \mathcal{H}^c_{11}(\alpha) K^c_{11}(\beta)]\), where \(\mathcal{H}^i_{11}(\cdot)\) and \(K^c_{11}(\cdot)\) are the derivatives of the Hankel functions \(\mathcal{H}_{11}(\cdot)\) and \(K_{11}(\cdot)\), respectively.

C. Surface Admittance Operator for Multiple Conductors

We now apply the surface admittance operator to all conductors in the cable, introducing equivalent currents on their boundaries. In order to simplify the notation for upcoming formulas, we gather all field Fourier coefficients related to conductor \(p\) in a vector \(E^{(p)}\). If conductor \(p\) is solid, we let

\[
E^{(p)} = \begin{bmatrix} E_{p}^{(p)} & \ldots & E_{0}^{(p)} & \ldots & E_{N_p}^{(p)} \end{bmatrix}. 
\]

If conductor \(p\) is hollow, we set

\[
E^{(p)} = \begin{bmatrix} \tilde{E}_{p}^{(p)} & \ldots & \tilde{E}_{N_p}^{(p)} & \ldots & \tilde{E}_{N_p}^{(p)} \end{bmatrix}. 
\]

Furthermore, all electric field coefficients are collected in the global vector of unknowns

\[
E = \begin{bmatrix} E^{(1)} & E^{(2)} & \ldots & E^{(p)} & \ldots & E^{(P)} \end{bmatrix}^T.
\]

In a similar manner, we cast all current coefficients in the vector \(J\). The current coefficients in \(J\) are related to the electric field coefficients (27) by the surface admittance operators (11) and (23). All these relations can be summarized in matrix form as

\[
J = Y_s E,
\]

where the block diagonal matrix \(Y_s\) can be interpreted as the surface admittance operator of the whole system of conductors.

IV. IMPEDANCE COMPUTATION

A. Electric Field Integral Equation

The surface admittance operator describes the field-current relation imposed by the conductors. The effect of the surrounding medium is instead modeled with the electric field integral equation [29], [12]

\[
E_z(r) = -j\omega A_z(r) - \frac{\partial V}{\partial z},
\]

where \(V\) is the scalar potential and \(A_z(r)\) is the \(z\)-component of the vector potential. We can write the vector potential \(A_z(r)\) as

\[
A_z(r) = \sum_{q=1}^{P} A_q(r),
\]

where \(A_q(r)\) is the contribution of the current that replaced conductor \(q\). If conductor \(q\) is solid we have

\[
A_q(r) = -\mu_o \int_0^{2\pi} J_q^{(q)}(\theta') G(r, r_q(\theta')) a_q d\theta',
\]

while if conductor \(q\) is hollow we have

\[
A_q(r) = -\mu_o \int_0^{2\pi} \tilde{J}_q^{(q)}(\theta') G(r, \tilde{r}_q(\theta')) \tilde{a}_q d\theta',
\]

since we have to superimpose the effect of the equivalent current on both the inner and outer contours. In (31) and (32), \(G(r, r')\) is the Green’s function of a two-dimensional infinite homogeneous space [27]

\[
G(r, r') = \frac{1}{2\pi} \ln |r - r'|.
\]

When \(r\) belongs to the outer boundary of conductor \(p\), we can rewrite (29) as

\[
E_z^{(p)}(\theta) = -j\omega A_z(r_p(\theta)) + \sum_{q=1}^{P} \left[ R_{pq}(\omega) + j\omega L_{pq}(\omega) \right] I_q,
\]

where \(E_z(r)\) has been replaced by its Fourier expansion (3), and the term \(\frac{\partial V}{\partial z}\) has been written through (1). The symbols \(R_{pq}(\omega)\) and \(L_{pq}(\omega)\) represent the \((p, q)\) entry of the matrices \(R(\omega)\) and \(L(\omega)\), respectively. Equation (34) is written for all conductors, both solid and hollow. In addition, if \(p\) is a hollow conductor, we also evaluate (29) on the inner boundary \(\tilde{r}_p\), obtaining

\[
\tilde{E}_z^{(p)}(\theta) = -j\omega A_z(\tilde{r}_p(\theta)) + \sum_{q=1}^{P} \left[ R_{pq}(\omega) + j\omega L_{pq}(\omega) \right] \tilde{I}_q.
\]

The integral equations (34) and (35) can be solved numerically with the method of moments [13], using the Fourier expansions (14), (15), (19) and (20) for the unknown fields and currents. This process was presented in [11] and is here omitted due to the limited space. It finally leads to

\[
E = j\omega \mu_o G J + U [R(\omega) + j\omega L(\omega)] U^T J,
\]

which is an algebraic approximation of (34) and (35). This system of equations relates the Fourier coefficients \(E\) and \(J\) of the unknowns, and combined with (28) will lead to the series impedance. In (36), the matrix \(G\) is the discretization of the Green’s function (33). The entries of \(G\) are given by a double integral involving the Green’s function (33). This integral can be solved analytically with the approach we proposed in [11]. Analytic integration significantly reduces the CPU time needed to set up the matrix \(G\), which is dense, and makes the proposed algorithm very efficient. Indeed, all coefficient matrices in (36) can be computed analytically. Finally, the matrix \(U\) in (36) relates the conductor currents \(I\) and the equivalent current coefficients \(J\) through

\[
I = U^T J.
\]

The matrix \(U\) has \(P\) columns. In the \(p\)-th column, we have a “1” in the row corresponding to the position of \(J_0^{(p)}\) in \(J\). If conductor \(p\) is hollow, there is a “1” also in the row corresponding to \(\tilde{J}_0^{(p)}\). All other entries of \(U\) are zeros.
B. Computation of the p.u.l. Impedance

By combining (36) with (28) we finally obtain, with a few algebraic manipulations [11], the p.u.l. resistance and inductance

\[ R(\omega) = \Re \left\{ \left[ U^T (1 - j\omega\mu_0 Y_s G)^{-1} Y_s U \right]^{-1} \right\}, \]
\[ L(\omega) = \omega^{-1} \Im \left\{ \left[ U^T (1 - j\omega\mu_0 Y_s G)^{-1} Y_s U \right]^{-1} \right\}. \]

(V) GROUND RETURN

We now show how we include the effect of lossy ground in the proposed technique. We decompose the series impedance \( Z \) of a buried cable as

\[ Z = (Z_c + Z_g) + \Delta Z_{\text{prox}}, \]

where \( Z_c \) and \( Z_g \) denote, respectively, the contributions of the cables and of the ground evaluated neglecting proximity effects, which are instead represented by the term \( \Delta Z_{\text{prox}} \).

Conventional EMTP tools compute the series impedance of cables using analytical formulae which account for skin effect in both conductors and earth, but ignore any proximity effect [7]. Therefore, they only return the first two terms of (39).

On the other hand, the proposed method estimates \( \Delta Z_{\text{prox}} \) very accurately, but does not incorporate the effect of the ground return \( (Z_g) \) since it has been developed assuming a lossless medium around the conductors. In what follows, we show an easy approach [10] which permits to properly include ground return in the proposed technique.

We capitalize on the fact that with MoM-SO one can easily exclude proximity effects by setting \( N_p = 0 \) for all conductors. If we calculate the impedance matrix twice, with \( N_p > 0 \) and with \( N_p = 0 \), we can estimate the contribution of proximity as

\[ \Delta Z_{\text{prox}} = Z_{\text{MoM-SO}}(N_p > 0) - Z_{\text{MoM-SO}}(N_p = 0). \]

Next, we calculate \( Z = Z_c + Z_g \) using a conventional approach (ex: Cable Constants [7], [8] or analytic formulas [30]) and add \( \Delta Z_{\text{prox}} \) to the result according to (39). The proposed approach assumes that conductors and ground are separated by an infinitesimally-thin insulation layer. Since the thickness of the insulation layer in a real power cable is much smaller than skin depth in ground, it can be safely neglected in the computation of the cable impedance.

This simplified approach is valid as long as the penetration depth in ground is much larger than the distance between the conductors. To see this, consider the correction term \( \Delta Z_{\text{prox}} \) in (40). This correction is independent of the chosen return path (reference conductor) provided that the same return is used in the two calculations \( (N_p > 0) \) and \( (N_p = 0) \), and that the return path is far away from the conductors. This implies that one would get the same result if one had chosen the return path to be that of the classical ground return formula for \( Z_g \) in (39).

VI. NUMERICAL RESULTS

A. Validation against Finite Elements

We first validate the proposed MoM-SO approach against FEM computation [15] for a system of three uniformly-spaced coaxial shells surrounded by lossless medium. The center-to-center distance between the shells is 45 mm. Shells have a diameter of 40 mm and thickness of 4 mm. The conductivity of each shell is 58·10^6 S/m. We calculate the positive-sequence resistance and inductance using MoM-SO with orders \( N_p = 0 \) (no proximity effects) and \( N_p = 4 \) (with proximity effects). Result, illustrated in Fig. 3, demonstrate an excellent agreement between MoM-SO and FEM. By comparing the two curves obtained with MoM-SO, one can appreciate the influence of proximity effects on the parameters of this cable, which becomes significant at medium/high frequency. Neglecting proximity leads to an overestimation of the series inductance, and of an underestimation of losses.

Timing results, presented in Table I, show that the proposed method is 34 times faster than FEM. This remarkable speed up arises from two differences between MoM-SO and FEM:

- finite element methods have to mesh the entire cross-section of the cable, instead of the sole surface which is sufficient for MoM-SO. This difference is particularly significant at high frequency, where the small skin depth imposes a very fine mesh in FEM;
- MoM-SO uses very few unknowns per conductor. For example, when \( N_p = 4 \), the field/current Fourier series have only 9 coefficients for solid conductors, and 18 for hollow conductors.
TABLE I

| Computation of G | Proposed (MoM-SO) | FEM |
|-----------------|-------------------|-----|
| Np = 0          | 0.011 s           | 0.254 s |
| Np = 4          | 0.025 s           | 0.240 s |

All computations were performed on a system with a 2.5 GHz CPU and 16 GB of memory.

*Positive sequence only. Mesh size: 41,562 triangles.

B. Validation of Ground Return

We demonstrate the adequacy of the approach proposed in Sec. V for the inclusion of ground return by a direct comparison against a FEM computation [15]. We consider two close conductors that are buried in an infinite earth with \( \sigma_o = 0.1 \) S/m. The radius of each conductor is \( a = 25 \) mm, while separation is \( D = 70 \) mm. The conductivity of each conductor is \( \sigma = 58 \cdot 10^5 \) S/m. We wish to calculate the impedance matrix \( \mathbf{Z} \) at 10 kHz. The impedance by the classical approach [30] is obtained as

\[
\mathbf{Z}_{\text{analytic}} = \begin{bmatrix}
Z_1 & Z_{g,s} & Z_{g,m} \\
Z_{g,m} & Z_1 & Z_{g,m}
\end{bmatrix},  \quad (41)
\]

where

\[
Z_1 = \frac{m}{2\pi\sigma} \mathcal{I}_0(ma), \quad (42)
\]

\[
Z_{g,s} = \frac{m_o}{2\pi\sigma_o} \mathcal{L}_0(m_oa), \quad (43)
\]

\[
Z_{g,m} = \frac{m_o}{2\pi\sigma_o} \left( \mathcal{L}_1(m_oa) \right)^2, \quad (44)
\]

for \( m = \sqrt{j\omega\mu\sigma} \), \( m_o = \sqrt{j\omega\mu_o\sigma_o} \), and where \( \mathcal{I}_n(\cdot) \) and \( \mathcal{L}_n(\cdot) \) are the modified Bessel functions of first and second kind [26] of order \( n \).

When calculating the correction (40) using MoM-SO, we use as return a tubular conductor of 10-m radius and 1-mm wall thickness with \( \sigma = 58 \cdot 10^5 \) S/m.

The result is validated against a FEM computation [15]. Since the penetration depth in earth is \( \delta = 15.9 \) m at the given frequency and soil resistivity, it is sufficient to use a boundary of radius \( 3\delta = 48 \) m.

Table II shows the impedance values calculated in the various steps, presented in the form of common mode and per-phase loop impedances. It is observed that the simplified approach agrees with the FEM result with an error smaller than 0.1% for both the real and imaginary part of (41).

The per-phase loop-mode inductance and resistance are plotted over frequency in Fig. 4. The results validate the proposed approach for ground return inclusion, since the obtained results match closely those computed with FEM. A similar agreement was obtained for the common mode inductance and resistance.

C. Transient Overvoltages in a Crossbonded Cable System

1) Cable Data: We consider the modeling of three single core cables buried in a homogeneous soil as shown in Fig. 5. The cables are touching, leading to a significant proximity effect for waves that propagate external to the sheaths. The cable geometry and material properties are listed in Table III.

We compute the series impedance matrix \( \mathbf{Z} \) in two alternative ways: using Wedepohl’s analytical approach which considers skin effects and ground return [8], and using MoM-SO. With MoM-SO, we use as reference conductor a tubular conductor of 10-m radius, 1-mm thickness and conductivity equal to that of the core conductor. The shunt admittance \( \mathbf{Y} \) is established by standard analytical formulas [8]. In both cases, we evaluate the impedance at 120 logarithmically spaced points distributed from 1 Hz to 1 MHz.
TABLE III
SINGLE CORE CABLES OF SEC. VI-C: GEOMETRICAL AND MATERIAL PARAMETERS.

| Core       | Outer diameter = 39 mm, \(\rho = 3.365 \times 10^{-8} \Omega \cdot m\) |
|------------|---------------------------------------------------------------------|
| Insulation | \(t = 18.25 \text{ mm}, \varepsilon_r = 2.85\)                      |
| Sheath     | \(t = 0.22 \text{ mm}, \rho = 1.718 \times 10^{-8} \Omega \cdot m\) |
| Jacket     | \(t = 4.53 \text{ mm}, \varepsilon_r = 2.51\)                      |

TABLE IV
TIMING RESULTS FOR THE EXAMPLE OF SEC. VI-C.

| Computation of \(G\) | MoM-SO: \(N_p = 0\) | MoM-SO: \(N_p = 4\) |
|-----------------------|---------------------|---------------------|
| Impedance computation (per frequency sample) | 0.019 s | 0.068 s |
|                       | 0.030 s             | 0.047 s             |

2) Timing Results: Timing results are presented in Table IV for the two runs of MoM-SO necessary to evaluate 40 and consequently (39). In total, for computing the cable impedance at 120 frequency points, the proposed approach takes less than 10 s. This result confirms the efficiency of MoM-SO, that can provide, in a few seconds, cable parameters with the accuracy of a FEM simulation.

3) Modal Analysis: Figure 6 compares the modal velocities of propagation obtained when \(Z\) is computed with and without the inclusion of proximity effects. Analytical approach considers skin effect but neglects proximity effect. To capture the proximity effect we use MoM-SO with order \(N_p = 4\). By combining the MoM-SO results with Wedepohl’s analytical formulas [8], we get by (39) an impedance matrix which accounts for skin, proximity and earth return effects. Clearly, proximity effect increases the propagation speed of the intersheath waves.

4) Modeling for Transient Calculations: We use the computed series impedance to perform a transient simulation. For this purpose, we calculate the parameters of the Universal Line Model [3], [31] using the series impedance \(Z\) and shunt admittance \(Y\) found in the previous section. The model is formulated in terms of the phase-domain characteristic admittance \(Y_c\) and propagation matrix \(H\). We used 12 poles for the fitting of \(Y_c\) and 14 poles for fitting each of the four modal delay groups of \(H\).

5) Transient Overvoltages: We wish to simulate transient overvoltages within a major section of a cross-bonded cable system. The obtained cable model was exported to the PSCAD simulation tool [2] and utilized in a transient simulation with crossbondings and terminal conditions as shown in Fig. 7. The simulation was done with a unit step voltage excitation. Figs. 8 show the simulation results for the sheath voltage at nodes #1 in Fig. 7. The result is shown when \(Z\) has been obtained with the analytical approach [8], which neglects proximity effects, and with the proposed approach. Clearly, the proximity effect has a very strong impact on the voltage waveforms.

VII. Discussion
In Section VI, we demonstrated the extended MoM-SO method for the modeling of a typical cable system that includes three solid phase conductors and three tubular screens, as well as earth return. In order to compute the 120 samples required for this system, the proposed approach took about 10 s. Although this is slower than standard analytical approaches, it is in our opinion fast enough to be effectively used in EMTP-type tools, in particular when proximity effects are suspected to be of concern.

We have also applied the new MoM-SO method for the modeling of pipe-type cables with similar results [23]. Again, proximity effect was found to have a strong influence on the computed series impedance, which was accurately captured by the proposed approach with an acceptable CPU time.

VIII. Conclusion
The MoM-SO approach is an efficient method to compute the series impedance of power cables including skin and proximity effects. In this paper, we extended the methodology to hollow round conductors, useful to efficiently represent...
proximity effects in cables with closely-space conductors. The obtained results validate the

to account. Compared to other proximity-aware

techniques, such as finite elements, the proposed method is

much faster, thanks to a surface-based formulation. Finally,

the method has been used to predict a transient overvoltage

in a cross-bonded cable system. The obtained results validate

the technique and remark on the importance of accounting for

proximity effects in cables with closely-space conductors.

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