A Quark Transport Theory to describe Nucleon–Nucleon Collisions *†

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Abstract

On the basis of the Friedberg–Lee model we formulate a semi-classical transport theory to describe the phase-space evolution of nucleon–nucleon collisions on the quark level. The time evolution is given by a Vlasov–equation for the quark phase-space distribution and a Klein–Gordon equation for the mean-field describing the nucleon as a soliton bag. The Vlasov equation is solved numerically using an extended testparticle method. We test the confinement mechanism and mean-field effects in $1 + 1$ dimensional simulations.
1 Introduction

In this paper we extend the established transport theories, which describe dynamics of heavy–ion–collisions on the basis of nucleons ([1] – [7]), to a theory whose basic ingredients are quarks which are the relevant degrees of freedom at bombarding energies from some GeV up to some TeV.

The fundamental theory for the interaction of quarks is QCD. There are derivations of transport equations for quarks and gluons based on QCD in the literature [8], [9], [10]. The problem is the complexity of the resulting equations which makes them rather tedious for practical applications. All other approaches start with some approximations to QCD. For bombarding energies larger than $\approx 100\text{GeV}$ QCD can be treated perturbatively. This is the motivation for many parton cascade models (Fritjof [11], Venus [12], RQMD [13], HIJING [14], parton cascade [15]).

We are interested in the low energy regime of a few GeV where the QCD coupling constant is too large to treat QCD perturbatively. The non–perturbative QCD effects can be modeled by a so–called mean–field, which provides the confinement and governs the dynamics of the quarks.

For the description of properties of the nucleon we have many well-established static quark models at our disposal. As a starting point for a dynamical theory we need a model, which generates dynamically a surface and is able to simulate absolute confinement. The last point is essential because there will be excitations of quarks in a collision which could lead to deconfinement if the confining potential is finite. The simplest model which fulfills our requirements is the well–known Friedberg–Lee–soliton model with a field–dependent coupling constant (for a review see [16]). Starting from this model we will derive transport equations for quarks moving in a mean–field. In this paper we study the resulting model in 1+1 dimensions to check the numerical methods and the behavior of the model in nucleon–nucleon collisions.

Zhang and Wilets [17] have derived transport equations based on the Nambu–Jona–Lasinio model in order to estimate chiral symmetry effects in heavy–ion collisions; this is conceptionally close to our work. However, besides the absence of confinement in their model, these authors do not actually perform dynamical simulations.

This paper is organized as follows: In section 2 we review the static Friedberg-Lee model and present results in one spatial dimension. In section
we derive equations of motion for the phase–space evolution of the quarks in this model. Section 4 discusses the extension of the usual testparticle ansatz to include particle and antiparticle degrees of freedom. The initialization of a stationary nucleon in the semiclassical approximation is discussed in section 5. First simulations of nucleon–nucleon collisions in 1+1 dimensions are presented in section 7. Finally, we summarize and conclude in section 8.

2 The Friedberg–Lee Model

The Friedberg–Lee soliton model in its basic version was formulated in 1977 and 1978 by Friedberg and Lee [18],[19]. The quark–quark interaction in this model is mediated by a selfinteracting scalar $\sigma$ field. The scalar field is interpreted as a summation of all nonperturbative gluonic interactions between the quarks. Because of its color neutral nature, the $\sigma$–field can only model many–gluon–exchange. Therefore, the model has been extended to include absolute color confinement by introducing a color–dielectric function [27]. In this paper we use the following simplified version of this model [28]

$$L = i\bar{\Psi} \left( \gamma_\mu \partial^\mu - m_0 \right) \Psi - \bar{\Psi} g_{eff}(\sigma) \Psi + \frac{1}{2} (\partial_\mu \sigma)^2 - U(\sigma),$$

which describes quarks with a rest mass $m_0$ coupled to a scalar field $\sigma$. This coupling together with the mass term leads to an explicit breaking of chiral symmetry. For the effective coupling to the scalar field we use the form given by Fai et al. [28] (a similar coupling is discussed in [29])

$$g_{eff}(\sigma) = g_0 \sigma_{vac} \left[ 1 - \frac{1}{\kappa(\sigma)} \right]$$

and

$$\kappa(\sigma) = 1 + \theta(x)x^n[nx - (n + 1)]$$

with $x = \frac{\sigma}{\sigma_{vac}}$ and $n = 3$.

It is shown in [28] that this form of the effective coupling gives a good approximation to the effects of the gluon field in the chirally invariant color dielectric extension of the soliton–bag model. It guarantees absolute confinement, because the quarks acquire an infinite effective mass $m_0 + g_{eff}(\sigma)$.
in the vacuum. The $\theta$– function in (3) guarantees that the effective quark mass is larger than the quark rest mass and always positive ($g > 0$), which is essential for our semiclassical treatment of this model. Therefore we start with the Lagrangian (1) to formulate a transport theory.

For the scalar field $\sigma$, a nonlinear self-interaction $U(\sigma)$ is assumed, which is necessary to allow for solitonic solutions of the field. This nonlinear potential is parametrized as

$$ U(\sigma) = \frac{a}{2!} \sigma^2 + \frac{b}{3!} \sigma^3 + \frac{c}{4!} \sigma^4 + B. \tag{5} $$

The free parameters of the model $g_0, a, b, c, B$ can be adjusted to reproduce the basic properties of the nucleon, namely mass, RMS radius, magnetic moment and the ratio $g_V/g_A$.

Solutions of the Friedberg–Lee model have been extensively studied during the last decade [16], [20] – [27].

In this paper we restrict our studies to $1 + 1$ dimensions. In order to ascertain that the essential properties of the model are still present in $1 + 1$ dimensions we have compared the solutions of the Friedberg–Lee model in 1 spatial dimension with the 3-dimensional results from the literature. It has turned out, that the results are essentially identical. In order to show this, we present a 1-dimensional result for the model (1) with the effective coupling given by (2). In one space–dimension the ansatz for the spinless quark spinors is

$$ \Psi = \begin{pmatrix} u \\ iv \end{pmatrix}, \tag{6} $$

which gives the following equations of motion in the usual mean–field approximation

$$ \frac{du}{dx} = -(\varepsilon + g\sigma + m_0) v, \tag{7} $$

$$ \frac{dv}{dx} = (\varepsilon - g\sigma - m_0) u, \tag{8} $$

$$ \frac{d^2}{dx^2} \sigma - \frac{dU(\sigma)}{d\sigma} = N \frac{dg(\sigma)}{d\sigma} (u^2 - v^2) \tag{9} $$
with the occupation number \( N = 3 \) for the valence orbital. A typical solution for the parameters \( a = 222.91, b = -5347.05, c = 38610, g_0 = 1 \) with a total energy of \( 931 MeV \) and an RMS radius of \( 0.56 fm \) is shown in figs. 1 and 2.

3 Derivation of the Transport Equation

The aim of this section is to derive equations of motion which describe the phase–space evolution of a moving nucleon. We therefore follow the spirit of the well known RBUU model [1, 6], which has been applied very successfully to heavy ion collisions [6]. The following steps are similar to the derivation in [30, 6]. Starting with a static model for the nucleon, the Friedberg–Lee model, we derive an equation for the time evolution of the Wigner–function, which is the quantum mechanical analog of the classical phase–space density. The Wigner–function is defined as

\[
W^{r_i, r_j}(x, p) = \frac{1}{(2\pi)^4} \int d^4R e^{-ip\cdot R} \overline{\Psi}^{r_i}(x + \frac{R}{2}) \otimes \Psi^{r_j}(x - \frac{R}{2}),
\]

(10)

where the indices \( r_i, r_j \) label internal degrees of freedom, such as color, flavor, etc; in the following they are suppressed. Densities and particle number can be easily calculated by integrating over the Wigner–function:

\[
\rho(x) = \int d^4p \ tr(\gamma_0 W(x, p))
\]

(11)

\[
\rho(p) = \int d^4x \ tr(\gamma_0 W(x, p))
\]

(12)

\[
N = \int d^4x \int d^4p \ tr(\gamma_0 W(x, p))
\]

(13)

In general, expectation–values of one–particle operators \( \hat{O} \) are given by

\[
\langle \hat{O}\rangle = \int d^4x \int d^4p \ tr(\hat{O}W(x, p))
\]

(14)

The equation of motion for the Wigner–function can be derived by calculating

\[
[\gamma_\mu (\partial_\mu - 2ip^\mu)] W(x, p) = \frac{2}{(2\pi)^4} \int d^4R e^{-ip\cdot R} \overline{\Psi}(x_1) \otimes \gamma_\mu \partial_{x_2} \Psi(x_2)
\]

(15)

where

\[
x_1 = x + \frac{R}{2}, \quad x_2 = x - \frac{R}{2}.
\]

(16)
On the r.h.s. of equation (15) one can use the Dirac–equation of the Friedberg–Lee–model to replace \( \gamma_\nu \partial_\nu x^2 \Psi(x^2) \) which gives after some algebraic transformations:

\[
\left[ \gamma_\mu \left( \hbar \partial_\mu - 2ip^\mu \right) \right] W(x, p) = -2i e^{\frac{i}{\hbar} \partial_\mu \partial_x^\mu} \left[ m_0 + g\sigma(x) \right] W(x, p) .
\]

The derivative \( \partial_\mu p \) acts on the Wigner–function and \( \partial_x^\mu \) acts on the field \( \sigma(x) \). The factors \( \hbar \) are inserted explicitly. Up to now, equation (17) together with the equation for the mean field

\[
\partial_\mu \partial^\mu \sigma - \frac{dU(\sigma)}{d\sigma} = \frac{dg(\sigma)}{d\sigma} \int d^4p \, tr W(x, p)
\]

is equivalent to the equations of motion for the Friedberg–Lee model in the mean–field approximation. In the semiclassical approximation, one expands the Wigner–function and the exponential function in equation (17) in orders of \( \hbar \):

\[
W = W_0 + i\hbar W_1 + \cdots
\]

\[
e^{\frac{i}{\hbar} \partial_\mu \partial_x^\mu} = 1 + \frac{i}{2} \hbar \partial_\mu \partial_x^\mu + \cdots
\]

This expansion is expected to converge the better the less the fields vary over the Compton wavelength of the quarks. In lowest order \( \hbar \) this leads to the equation

\[
(\gamma_\mu p^\mu - m^*) W_0(x, p) = 0 ,
\]

with \( m^* = m_0 + g_{eff}(\sigma(x)) \).

This is the well known mass–shell constraint. In first order \( \hbar \) one has:

\[
\left[ \gamma_\mu \partial_\mu + \partial_\mu \partial_x^\mu m^*(x) \right] W_0(x, p) = 2 \left[ \gamma_\mu p^\mu - m^* \right] W_1(x, p) .
\]

We now take the trace on both sides of this equation. If one requires baryon current conservation in lowest order in \( \hbar \), \( W_1 \) has to fulfill a constraint so that the trace of the r.h.s. of equation (21) vanishes. We are then left with an equation of motion for \( W_0 \), the well known Vlasov–equation.

All equations are so far derived for 3 + 1 dimensions. From now on we study the model in 1 + 1 dimensions in order to test its main features and the
numerical methods for its solution. The generalization to $3 + 1$ dimensions will be discussed in a future work. In $1 + 1$ dimensions we introduce Dirac-Matrices $\Gamma_\mu$, which have to fulfill the usual anti-commutation relations:

$$\{ \Gamma_\mu, \Gamma_\nu \} = g_{\mu\nu} \quad \mu, \nu \in \{0, 1\}$$

with

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$  

(22)

(23)

One possible representation for the $\Gamma$–matrices is

$$\Gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \Gamma_1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad \quad \quad (24)$$

One also defines the product

$$\Gamma_2 = \Gamma_0 \Gamma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

(25)

which is analogous to $\gamma_5$ in $3+1$ dimensions.

The relations

$$\{ \Gamma_2, \Gamma_\mu \} = 0 \quad , \quad \Gamma_0^2 = I \quad , \quad \Gamma_1^2 = -I \quad , \quad [I, \text{and} \Gamma_\mu] = 0,$$

(26)

hold for these matrices, where $I$ is the $2 \times 2$ unit matrix.

In the 1–dimensional notation the Vlasov–equation can be written in the following form:

$$tr \left( [\Gamma_0 \partial_t + \Gamma_1 \partial_{x^1} + I(\partial_t m^* \partial E - \partial_{x^1} m^* \partial p_1)] W_0(x,p) \right) = 0,$$

(27)

with $x = (t, x^1)$ and $p = (E, p^1)$.

This equation incorporates energy– and particle–number conservation; it approximates the quantum–mechanical solution only if the fields vary slowly, a condition which we will discuss later.

In the following we deal with $W_0$ only and suppress the subscript. In view of the mass–shell constraint eq. (20), one takes the following ansatz for the Wigner matrix:

$$W(x,p) = (\Gamma_\mu p^\mu + m^*) f(x,p),$$

(28)
where \( f(x, p) \) is a scalar function of \( x = (t, x^1) \) and \( p = (E, p^1) \). This is equivalent to the usual spinor decomposition neglecting the pseudoscalar part. Insertion of this ansatz in equation (27) finally gives the Vlasov–equation:

\[
[p_\mu \partial_\mu + m^* \partial_x \partial_\mu \partial_\mu] f(x, p) = 0. \tag{29}
\]

The mass–shell constraint

\[
(\Gamma_\mu p^\mu - m^*) W(x, p) = (\Gamma_\mu p^\mu - m^*) (\Gamma_\mu p^\mu + m^*) f(x, p) = (p^2 - m^{*2}) f(x, p) = 0 \tag{30}
\]

will be fulfilled by a using a Dirac \( \delta \)–function \( \delta \left(p^2 - m^{*2}\right) \) in \( f(x, p) \). For the numerical treatment of this equation we use the so called testparticle ansatz for the scalar phase–space distribution function \( f(x, p) \) [31]:

\[
f(x, p) = \delta \left(p^2 - m^{*2}\right) \theta(E) \sum_n \delta(x - x_n(t)) \delta(p^1 - p^1_n(t)). \tag{31}
\]

The \( \theta \)–function expresses the restriction to positive energy states. This approximation will be discussed in detail in section 4. Inserting this ansatz into the Vlasov–equation (29) shows that the so called testparticles have to move like classical particles according to Hamilton equations of motion:

\[
\dot{x}_n = \frac{p^1_n}{E_n} \tag{32}
\]

\[
\dot{p}^1_n = -\frac{m^*_n \partial_x m^*_n}{E_n}. \tag{33}
\]

with

\[
E_n = \sqrt{p^1_n^2 + m^{*2}(x_n)} \tag{34}
\]

4 Extended testparticle ansatz

The usual testparticle ansatz [31] which is restricted to positive energy solutions implies:

\[
\int dE \ tr W(x, p) = W_{\text{scalar}}(x, p^1) = \frac{m^*}{E(x, p^1)} f(x, p^1) \tag{35}
\]

\[
\int dE \ tr \left(\Gamma_0 W(x, p)\right) = W_{\text{baryon}}(x, p^1) = f(x, p^1). \tag{36}
\]
Therefore, at an initial time instant a given baryon–density can be reproduced by using eq. (36). The scalar density is then given by eq. (35). In nuclear transport theories the rest mass of the testparticles is of the order of 1GeV. In this case this ansatz works quite well. On the quark level the rest mass of the quarks is of the order of $5 - 10\text{MeV}$, which is reflected in the fact that the lower component of the quantum mechanical wave functions is nearly as large as the upper component (see fig. 3). A projection on negative energy states is not negligible. It is therefore necessary to extend the testparticle ansatz in order to fit scalar and baryon density simultaneously. The importance of the scalar density is obviously clear because it determines the dynamics of the system through its coupling to the $\sigma$–field, whereas the baryon density determines the charge radius of the nucleon.

Our suggested extension of (31) is given by

$$f(x, p) = f^{\text{particle}}_{\text{pos}}(x, p)\theta(E) + \left(1 - f^{\text{hole}}_{\text{neg}}(x, p)\right)\theta(-E), \quad (37)$$

where $f^{\text{particle}}_{\text{pos}}$ represents the occupation of positive energy particle states and $(1 - f^{\text{hole}}_{\text{neg}})$ the occupation of negative energy hole states relative to the filled Dirac–sea. With this ansatz the Wigner–function is written as

$$W(x, p) = \left(\Gamma \mu p^{\mu} + m^*\right) \frac{2\pi \delta(E - \omega)}{2|E|} f^{\text{particle}}_{\text{pos}}(t, x, p) +$$

$$+ \left(\Gamma \mu p^{\mu} + m^*\right) \theta(E) \frac{2\pi \delta(E + \omega)}{2|E|} \left(1 - f^{\text{hole}}_{\text{neg}}(t, x, p)\right)\theta(-E), \quad (38)$$

with $\omega = \sqrt{p^2 + m'^2}$, and $x$ and $p$ denote the second component of the 2 vectors $x^\mu, p^\mu$. The energy dependence of the on-shell distribution functions $f(t, x, p)$ is implicit.

Performing the energy integration we end up with:

$$\int \frac{dE}{2\pi} W(t, x, E, p) = \frac{\Gamma_0\omega - \Gamma_1p + m^*}{2\omega} f^{\text{particle}}_{\text{pos}}(t, x, p) +$$

$$+ \frac{-\Gamma_0\omega - \Gamma_1p + m^*}{2\omega} \left(1 - f^{\text{hole}}_{\text{neg}}(t, x, p)\right), \quad (38)$$
where $\omega$ is the on–shell energy. Defining
\[ f_{\text{pos}}(t, x, p) = f(t, x, p) \]
\[ (1 - f_{\text{hole}}(t, x, p)) = \bar{f}(t, x, -p) \]
this can be rewritten as
\[ \int \frac{dE}{2\pi} W(t, x, p, E) = \frac{\Gamma_0 \omega - \Gamma_1 p + m^*}{2\omega} f(t, x, p) - \frac{\Gamma_0 \omega + \Gamma_1 p - m^*}{2\omega} \bar{f}(t, x, -p). \]

The meaning of $f$ and $\bar{f}$ becomes more clear if one calculates the scalar and the baryon density:
\[ \rho_s(x) = \int dE dp \text{tr} W = \int dp \frac{M}{E} (f + \bar{f}) \]
\[ \rho_B(x) = \int dE dp \text{tr} (\Gamma_0 W) = \int dp (f - \bar{f}). \]

While $f$ and $\bar{f}$ add up in calculating the scalar density, their difference yields the baryon density. In view of this $f$ can be interpreted as particle distribution function whereas $\bar{f}$ is the anti particle distribution function.

For a given Wigner–function $W$ $f$ and $\bar{f}$ will then be constructed according to
\[ f(x, p) = \frac{1}{2} \left( \frac{\omega}{m^*} W_{\text{scalar}} + W_{\text{baryon}} \right) \]
\[ \bar{f}(x, p) = \frac{1}{2} \left( \frac{\omega}{m^*} W_{\text{scalar}} - W_{\text{baryon}} \right) \]
with
\[ W_{\text{scalar}} = \int dE \text{tr} W(t, x, E, p) \quad , \quad W_{\text{baryon}} = \int dE \text{tr} (\Gamma_0 W(t, x, E, p)) \]
and $\omega = \sqrt{p^2 + m^*}$. 

For the numerical realization we initialize $N_1$ testparticles according to the distribution $f(x, p)$ and $N_2$ testparticles according to $\bar{f}(x, p)$, where
\[ N_1 = \frac{I_1}{I_1 + I_2} \quad , \quad N_2 = \frac{I_2}{I_1 + I_2} \]
with \( I_1 = \int dx dp \ f(x,p) \)

and \( I_2 = \int dx dp \ \bar{f}(x,p). \) (46)

All testparticles follow the Hamiltonian equations of motion (32), (33). This can be easily understood, because in a scalar field particles and antiparticles feel the same forces. The baryon density and the scalar density are given by

\[
\rho_B(x) = \frac{1}{N} \sum_{n=1}^{N} \delta(x-x_n) b_n \\
\rho_s(x) = \frac{1}{N} \sum_{n=1}^{N} \frac{m_n}{E_n} \delta(x-x_n),
\]

(47) \ (48)

where \( N = N_1 + N_2, \ \tilde{N} = N_1 - N_2, \)

\[ b_n = \begin{cases} 
1 & \text{if testparticle from } f \\
-1 & \text{if testparticle from } \bar{f}.
\end{cases} \]

On a spatial grid with spacing \( \Delta x \) the \( \delta \)-functions are evaluated as

\[
\delta(x-x_n) = \begin{cases} 
1/\Delta x & \text{if } x_n \in [x - \frac{\Delta x}{2}, x + \frac{\Delta x}{2}] \\
0 & \text{otherwise}.
\end{cases}
\]

(49)

With this extended testparticle ansatz, it is possible to reproduce a given baryon density and a scalar density independent of each other. In the semiclassical approximation the phase–space evolution of the Friedberg–Lee model is determined by the equations of motion for the testparticles (32) and (33) and by the mean–field equation by

\[
\partial_\mu \partial^\mu \sigma - \frac{dU(\sigma)}{d\sigma} = \frac{dg(\sigma)}{d\sigma} \rho_s(x).
\]

(50)

5 \ A semiclassical nucleon

We first tried to initialize testparticles as described in the last section to reproduce the baryon and scalar phase–space distributions calculated from the quantum mechanical solution (fig. 2). Performing the time evolution of the testparticles and the \( \sigma \)-field, it turned out that the nucleon initialized in
this way is not a stable solution of the semiclassical equation of motion \([27]\). This failure indicates that the conditions of weakly varying fields necessary for the derivation of the transport equation are actually not met. Similar difficulties show up when we calculate the semiclassical energy–distribution from the quantummechanical solution. The latter, is of course, a \(\delta\)–function whereas the former turns out to be a broad distribution.

We therefore drop now the connection between the semiclassical and the quantummechanical solution and from now on work consistently in the transporttheoretical framework. In doing so we take the potential function \(U(\sigma)\) as an effective classical potential that incorporates all higher order corrections to the quantummechanical one and determine it by adjusting the properties of the semiclassical solution to empirical properties.

First, we look for solutions of the stationary Vlasov–equation. A stationary solution can be characterized by

\[
\partial_t f(x, p) = 0 \quad \text{and} \quad \partial_t m^* = \partial_t g_{eff}(\sigma(t)) = 0 .
\]

(51)

The Vlasov–equation reduces to

\[
\frac{E}{m^* \partial_x m^*} \partial_x f(x, p) = \frac{E}{p} \partial_p f(x, p) ,
\]

(52)

where \(E = \sqrt{p^2 + m^*}^2\). Every function \(f(E)\) fulfills this equation. This means that every distribution function which does not explicitly depend on position and momentum, but only on the combined quantity energy is a stationary solution of the Vlasov--equation. For the initialization of a stable nucleon we have to find a distribution which reproduces the properties of the nucleon.

We start with a functional ansatz:

\[
f_{total}(x, p) = f(E) + \tilde{f}(E)
\]

(53)

where

\[
f(E) = f_0 e^{-kE^2} \quad \text{and} \quad \tilde{f}(E) = \tilde{f}_0 e^{-\tilde{k}E^2}
\]

(54)

are two Gaussians in energy with four free parameters; the amplitudes and the widths. The soliton solution is then calculated by starting with a Wood-Saxon shape for the \(\sigma\)–field. With the Gaussian ansatz we can also calculate
energy and density contributions:

\[ \rho_s = \int dp \frac{M}{E} \left( f_0 e^{-kE^2} + \bar{f}_0 e^{-\bar{k}E^2} \right), \]
\[ \rho_B = \int dp \left( f_0 e^{-kE^2} - \bar{f}_0 e^{-\bar{k}E^2} \right), \]
\[ E_{\text{quark}} = \int dp \int dx E \left( f_0 e^{-kE^2} + \bar{f}_0 e^{-\bar{k}E^2} \right), \]
\[ E_\sigma = \int dx \left( U(\sigma) + \frac{1}{2} \left( \frac{d\sigma}{dr} \right)^2 \right), \]
\[ E_{\text{total}} = 3E_{\text{quark}} + E_\sigma, \] (55)

with \( E = \sqrt{p^2 + m^*^2} \).

The \( \sigma \)-field is then calculated selfconsistently with the scalar density. The parameters are varied to give reasonable values for the energy and RMS-radius of the nucleon. In principle also the parameters for the potential could be varied to find the optimal parameter set for this 1+1 dimensional case, but this was not done here.

In fig. 3 we show a typical solution. Baryon density and the \( \sigma \)-field are similar to the quantum mechanical solution. The baryon density is volume centered. The shape of the scalar density is quite different from that of the baryon density and surface–peaked, reminiscent of the SLAC bag solutions to the Friedberg–Lee soliton bag model [18]. The surface–peaking is here a clear consequence of the strong increase of the coupling constant \( g \) towards the surface. The maxima at the surface are stabilizing, because the testparticle mass increases at the surface which results in a deceleration, meaning that the testparticles stay longer in the surface region. Fig. 4 and 5 show the stability of the semiclassical nucleon in the time evolution. The stability is almost perfect.

6 The boost

After the discussion of a stable initialization for a nucleon we now describe the Lorentz–boost of a given testparticle distribution, necessary for the preparation of the initial state of a collision. The Lorentz–boost has to
give the correct transformation properties for the baryon density, the scalar density, the $\sigma$-field and the Lorentz scalar distribution functions $f$ and $\bar{f}$.

We initialize the field and distribution functions at a time $t' = 0$ in the moving frame. The transformation is therefore given by:

$$
t' = 0, \quad x' = \frac{x}{\gamma} \quad (56)
$$

$$
p' = \gamma(p - \beta E), \quad E' = \gamma(E - \beta p) \quad (57)
$$

To calculate the $\sigma$-field and the distribution functions $f$ and $\bar{f}$ at the time $t' = 0$, we need to know the configuration in the rest frame for different times $t$. In this frame, however, the nucleon is stationary, i.e. $f(t) = f(0)$ and $\sigma(t) = \sigma(0)$ for all times $t$. Therefore, we can set $t = 0$.

The assumption that the distribution function is a scalar implies that

$$f'(x', p', t' = 0) = f(x, p, t = 0) = f(\gamma x', \gamma(p' + \beta E'), t = 0) \quad (58)$$

Inserting the testparticle ansatz:

$$f(x, p, 0) = \sum_n \delta(x - x_n(0))\delta(p - p_n(0)) \quad (59)$$

we get

$$f'(x', p', 0) = \sum_n \delta(\gamma x' - x_n(0)) \delta(\gamma(p' + \beta E') - p_n(0)) \quad (60)$$

We express this sum using the coordinates in the moving system:

$$f'(x', p', 0) = \sum_n (1 - \beta v_n) \delta(x' - x_n'(0)) \delta(p' - p_n'(0)) \quad (61)$$

with $x_n' = \frac{x_n}{\gamma}$ and $p_n' = \gamma(p_n - \beta E_n) \quad (62)$

In addition to the usual transformation of the testparticle coordinates and momenta often used in the literature we obtain a factor $(1 - \beta v_n)$ in the testparticle sum which results from the transformation of the arguments in the $\delta$-functions.

As a check for the expression for the boost (61), we calculate the transformed densities:

$$\rho_B'(x') = \int f'(x', p')dp' = \sum_n \delta(x - x_n)\gamma(1 - \beta v_n) = \gamma(\rho_B(x) - \beta j_1(x))$$

$$\rho_s'(x') = \int \frac{m^*}{E'} f'(x', p')dp' = \sum_n \delta(x - x_n)\frac{m^*}{E_n} = \rho_s(x) \quad (63)$$
Here, \( j_1 \) denotes the baryon current, which vanishes for a stable nucleon at rest. The densities have the correct transformation properties. The antiparticle distribution \( \bar{f}'(x', p') \) will be correspondingly transformed.

If one inserts the testparticle sum (62) into the Vlasov–equation, the resulting testparticle equations contain terms generated by the momentum derivatives acting on the factor \( (1 - \beta v_n) \). To avoid this change, we use a new testparticle distribution with coordinates \( \hat{x}, \hat{p} \), which have to fulfill:

\[
\sum_n \delta(x' - \hat{x}_n')\delta(p' - \hat{p}_n') = \sum_n (1 - \beta v_n)\delta(x' - x_n')\delta(p' - p_n') .
\]  

This can be guaranteed, if the new testparticles located at \( \hat{x}, \hat{p} \) are distributed according to the modified distribution function in the rest frame

\[
f(\hat{x}, \hat{p}) = (1 - \beta v) f(x, p) .
\]  

Finally, a moving nucleon has to be initialized in the following way: The testparticle coordinates are distributed according to (65) and the transformation to the moving coordinates is given by (62). The \( \sigma \)-field remains unchanged with the transformed \( x \) coordinates.

\section*{7 Collisions}

To simulate collisions of two nucleons, we initialize semiclassical nucleons as described in section 5 and transform them to the center of mass system by applying the boost of section 6. For each of the nucleons the center of the boosted phase–space distribution in momentum is shifted to \( p_c' = \gamma \beta m_0 \); due to the small quark rest mass, it stays close to zero momentum. This means that the boosted phase–space distributions always overlap in momentum space around \( p \approx 0 \). The boosted phase–space distribution is symmetric in \( x \) direction. In momentum direction one half of the distribution is contracted and the other is stretched, because of the factor \( (1 - \beta v_n) \) in (64). This leads to a very asymmetric shape in momentum space.

All simulations which are shown in this paper are calculated using the testparticle method. The testparticle propagation and the mean field time evolution are calculated using predictor–corrector techniques. As a check we also integrated the Vlasov equation numerically on a phase–space grid using an alternating direction implicit procedure (ADIP) which leads to the
same results. The direct numerical integration can be performed only in the 1+1 dimensional case for reasons of computing time. For the extension to 3 spatial dimensions the testparticle method is clearly superior for practical applications. All calculations are done in the c. m. system. In the simulations presented here we use 20000 testparticles for each nucleon. This relatively large number in the 1–dimensional case is necessary to obtain good statistics for the calculation of the scalar density. Due to the increasing coupling–constant at the surface of the soliton, the $\sigma$–field is very sensitive to small changes of the density in this region which requires good spatial resolution that cannot be obtained by the rather coarse–grained smearing methods.

First, we have tested the stability of one moving nucleon. It is perfectly stable for times ($\approx 20 \text{fm/c}$) larger than the typical collision times ($\approx 1 \text{fm/c}$). The time evolution of the quark distribution and the sigma field is shown in fig. 4 and fig. 5, respectively.

We have calculated collisions for bombarding energies of $1−25 \text{GeV}$ in the laboratory frame. In the simulations we initialize two nucleons which are well separated in space; the propagation of a few $\text{fm}$ until the collision takes place ensures that we have stable incoming nucleons. For a boost velocity of $\beta > 0.9$ the collision time is of the order of $1 \text{fm/c}$. To be sure to observe stable outgoing nucleons, we continue the calculation for $5–10 \text{fm/c}$ after the collision.

For very low bombarding energies ($T_{lab} \approx 1 \text{GeV}$) we observe a fusion of the two incoming bags. This is due to the slow motion of the nucleons which leads to a long overlap time. The testparticles are decelerated and loose kinetic energy which results in a gain of potential mean–field energy. The result is one extended bag with constant density inside. Due to the mixing of testparticles this bag does not break up into separate bags. The fusioned bag reaches a maximal extension and shrinks again converting potential energy back to kinetic testparticle energy. This behaviour is shown in fig. 8. The oscillation is undamped because there is no mechanism for energy loss in the model. We note that Schuh et. al. [32] estimate the potential for nucleon–nucleon scattering in the Friedberg–Lee soliton bag model and also end up with an attractive potential which may be responsible for the fusion that we observe for slowly moving bags. However, the surface energy in our 1+1 dimensional case is different from the real 3+1 dimensional world so that it is not sure that the fusion effect will survive in 3 spatial dimensions.

In collisions with bombarding energies $E_{lab}$ from $5–25 \text{GeV}$ the nucleons
are transparent on the mean–field level. The phase–space evolution of the baryon distribution \((f - \bar{f})\) is shown in fig. 6 for a bombarding energy \(E_{lab} = 15 GeV\). Since the scalar field acts in the same way on testparticles and antitestparticles, the distribution functions \(f\) and \(\bar{f}\) show the same behaviour.

In fig. 6 we display the corresponding \(\sigma\–field and the scalar source density. In the moment the two nucleons collide, the scalar density pushes the \(\sigma\–field barrier down. The \(\sigma\–field overshoots to negative values (timestep \(t = 1.1 fm/c\), but due to the \(\theta\–function in the effective quark–\(\sigma\) coupling this does not lead to negative effective masses. The field decouples from the density in the region where it is negative and oscillates up again (timestep \(t = 1.2 fm/c\)). In the following time–step (\(t = 1.3 fm/c\)) the \(\sigma\–field comes actually very close to the vacuum value. Here the coupling to the quarks then becomes large as shown by the large values of the scalar density. The quarks thus become very massive and stay confined. Remembering the physical motivation for the ingredients of our model we may say that at this point the explicit gluon–exchange forces take over from the color background field. The correlation then swings back to the background field thus reestablishing the soliton. The velocity of the nucleons is too large to allow for any significant one–body dissipation during the short overlap time. The result is a dip in the density in the moment the nucleons have passed through each other (timestep \(t = 1.6 fm/c\)). The \(\sigma\–field then comes close to the vacuum value and the two solitons separate (timestep \(t = 1.9 fm/c\)). If we continue the simulation, the small deformations, which can be seen in phase–space at \(t = 1.9 fm/c\), vanish and we end up with two stable nucleons. The time evolution of the energy is shown in fig. 8. There is no significant exchange of energy from the testparticles to the mean–field. This is most probably due to the restricted geometry in the present version of our model. A realistic surface in a 3 + 1 dimensional calculation would clearly lead to a larger coupling between mean–field modes and the quark motion.

We conclude from these simulations that the model is able to describe moving quark systems and absolute confinement in a collision of nucleons. Our model is in this sense a basis for extensions in the direction of direct quark–quark collisions, which we will put on top of the stable colliding nucleons we describe so far.
8 Summary and Conclusion

In this paper we have formulated a transport theory on the quark level which includes nonperturbative aspects of QCD. This is done by including a soliton mean-field which governs the dynamics. As a starting point we chose a phenomenological quark model for the nucleon, namely the Friedberg–Lee model. We have derived equations of motion for the time evolution of the Wigner–function which leads in the semiclassical expansion to the well known Vlasov–equation combined with a mass–shell constraint. For the Wigner–function we have used a testparticle ansatz which results in classical equations of motion for the testparticles.

Considering different extensions to the original Friedberg–Lee model it has turned out that the formulation with an effective quark–σ coupling, which comprises non–color–singlet many–gluon–exchange effects, can be used to model absolute confinement on the testparticle level. Inside the dynamically generated bag the quarks are nearly massless (‘asymptotic freedom’).

The usual testparticle ansatz was extended including also negative energy states which was necessary because of the small rest mass of the quarks. With the extended ansatz it is possible to describe scalar and baryon density independently of each other.

For the initialization of a stable nucleon we have used a consistent solution of the stationary Vlasov–equation. It was shown that any distribution function which depends only on energy and not explicitly on position and momentum is a solution of the stationary Vlasov–equation. After the discussion of the semiclassical nucleon we have described how to boost the initialized nucleon to give boosted distribution functions with the correct Lorentz transformation properties.

Having all these ingredients, we have performed some first collisions in 1+1 dimensions in order to test the numerical methods and the behavior of the model. It has turned out that on the pure mean field level the nucleons are totally transparent for high bombarding energies. The confinement is realized for each nucleon separately. For very low bombarding energies (≤ 1GeV in the lab. frame) we observe a fusion of the bags with a following oscillation of the six quark bag. This may be an artifact of the 1–dimensional treatment.

At the moment we are extending our model to 3+1 dimensions. This is essential to allow for non–central collisions and sideways flow. The next step is then to include a direct quark–quark collision term and cross sections.
for particle production.
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Figure 2: Upper component $u$ (solid) and lower component $v$ (dashed) of the quark spinor for the 1–dimensional solution of the Friedberg–Lee model with effective coupling

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