$a \rightarrow e^+e^-$ decay in a model with induced coupling to leptons

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Abstract

The process of electron-positron pair production by axion propagating in an external magnetic field is investigated in a model with only induced axion coupling to leptons. Contributions of the lowest and higher Landau levels are analyzed. The results we have obtained demonstrate a strong catalyzing influence of the field.
The pseudo-Goldstone boson associated with Peccei-Quinn symmetry $U_{PQ}(1)$ [1], the axion [2], is of interest not only in theoretical aspects of elementary particle physics, but in some astrophysical and cosmological applications as well [3, 4, 5]. It is also known that astrophysical and cosmological considerations have been very effective in obtaining restrictions on the axion mass [1] 

$$10^{-5} \text{eV} \leq m_a \leq 10^{-2} \text{eV}. \quad (1)$$

In some astrophysical considerations it is important to take into account both dense star matter and intensive electromagnetic fields. Of particular interest become investigations in strong external magnetic fields when the field strength can be of order or even more the critical value $B > B_e$, $B_e = m_e^2/e \simeq 0.44 \cdot 10^{14} \text{G}$. A possible existence of astrophysical objects with such extremely strong magnetic fields as $B \sim 10^{15} \div 10^{17} \text{G}$ was pointed out, for example, in [8, 9]. On the other hand, in such strong magnetic fields otherwise negligible processes are not only opened kinematically but become substantial ones as well (for example, the photon splitting into electron-positron pair $\gamma \rightarrow e^+e^-$ [10], Cherenkov process $\nu \rightarrow \nu\gamma$ [11, 12]).

In recent paper [13] we have studied the field-induced axion decay $a \rightarrow e^+e^-$ in DFSZ model [14], where axions couple to leptons at tree level. It was shown that the axion lifetime could be reduced to $10^{5}$ s in case of the decaying axion energy $\sim 1 \text{MeV}$ and the magnetic field strength $\sim 10^{15}$ G. In this paper we investigate the field-induced axion decay into electron-positron pair via a photon intermediate state $a \rightarrow \gamma \rightarrow e^+e^-$ in KSVZ model [15] in which axions have not direct coupling to leptons. The reason for which this channel is opened in the magnetic field is that $e^+e^-$-pair can have in the field both time-like and space-like total momentum as it occurs in photon splitting $\gamma \rightarrow e^+e^-$ [10]. A diagram describing $a \rightarrow \gamma \rightarrow e^+e^-$ is shown in Fig. 1 where double lines imply the influence of the magnetic field in the electron wave functions. The Lagrangian describing the effective axion-photon coupling is:

$$\mathcal{L}_{a\gamma} = g_{a\gamma} \partial_\mu A_\nu \tilde{F}_{\nu\mu} a, \quad (2)$$

where $g_{a\gamma}$ is a constant with the dimension $(\text{energy})^{-1}$; $A_\mu$ is the 4-potential of the quantized electromagnetic field, $\tilde{F}$ is the dual external field tensor. In the second order of the perturbation theory a matrix element can be presented in the form:

$$S = ie g_{a\gamma} \int d^4x d^4y \left( \tilde{F}_{\alpha\mu}^{\text{ext}} \partial_\mu a(y) \right) \left( \bar{\psi}(x) \gamma_\nu \psi(x) \right) G_{\alpha\nu}(x-y)$$

$$= -\frac{e g_{a\gamma}}{\sqrt{2E_a V}} \int d^4x \left( \bar{\psi}(x) \hat{h} \psi(x) \right) e^{-iqx}, \quad (3)$$

$$h_\alpha = (q \tilde{F}G(q))_\alpha = q_\mu \hat{F}_{\mu\nu}G(q)_{\nu\alpha},$$

where $e > 0$ is the elementary charge; $\psi(x)$ is the known solution of the Dirac equation in a magnetic field; $q_\alpha$, $E_a$ are the 4-momentum and the energy of the decaying axion, respectively. The expression for the photon propagator $G_{\alpha\beta}$ can be presented in a diagonal form [16]:

\footnote{However in paper [1] a possibility to solve the CP problem of QCD with a heavy axion $M_a \leq 1 \text{ TeV}$ is considered.}
\[ G_{\alpha\beta} = \sum_{\lambda=1}^{3} \frac{b^{(\lambda)}_{\alpha} b^{(\lambda)}_{\beta}}{(b^{(\lambda)})^2} \frac{1}{q^2 - \omega^{(\lambda)}}, \quad b^{(\lambda)}_{\alpha} b^{(\lambda)}_{\alpha} = \delta_{\lambda\lambda'} \left( b^{(\lambda')}_{\alpha} \right)^2 \] (4)

in a basis:

\[ b^{(1)}_{\alpha} = (qF)_{\alpha}, \]
\[ b^{(2)}_{\alpha} = (q\tilde{F})_{\alpha}, \]
\[ b^{(3)}_{\alpha} = q^2(qFF)_{\alpha} - q_{\alpha} \cdot (qFFq), \]
\[ b^{(4)}_{\alpha} = q_{\alpha}. \] (5)

We note that the basis vectors \( b^{(\lambda)}_{\alpha} \) are the eigenvectors of the photon polarization tensor with the eigenvalues \( \omega^{(\lambda)} \). As it is seen from (3) only the basis vector \( b^{(2)}_{\alpha} = (q\tilde{F})_{\alpha} \) gives a contribution to the decay, as \( (F\tilde{F})_{\alpha\beta} = 0 \).

One can obtain the decay probability carrying out a non-trivial integration over the phase space of the electron-positron pair taking the specific kinematics of charged particles in the magnetic field into account. However the probability of \( a \rightarrow e^+ e^- \) decay can be obtained from the imaginary part of \( a \rightarrow \gamma \rightarrow a \) transition amplitude via the virtual photon:

\[ E_a W_{a \rightarrow e^+ e^-} = \text{Im} M_{a \rightarrow a} = -g_{a\gamma}^2 (q\tilde{F} (\text{Im} G) \tilde{F} q). \] (6)

The result of our calculations is:

\[ W = -\frac{g_{a\gamma}^2}{4\pi \alpha E_a} \cdot \frac{e^2(q\tilde{F} \tilde{F} q) \cdot \text{Im} \omega^{(2)}}{\left( m_a^2 - \text{Re} \omega^{(2)} \right)^2 + \left( \text{Im} \omega^{(2)} \right)^2}, \] (7)

where \( \omega^{(2)} \) has a form of a double integral:

\[ \omega^{(2)} = -\frac{e^2}{4\pi^2} \int_0^1 du \int_0^\infty dt \left\{ \frac{\beta t}{\sin \beta t} \left[ \frac{q_{\parallel}^2}{2} \cos \beta t (1 - u^2) \right. \right. \]
\[ - \frac{q_{\perp}^2}{2} \left( \cos \beta ut - u \sin \beta ut \cos \beta t \right) \sin \beta t \right]\left. \right\} e^{-i\Phi} - \frac{q_{\parallel}^2}{2} (1 - u^2) e^{-i\Phi_0} \right\}, \] (8)

\[ \Phi = t \left( m_e^2 - q_{\parallel}^2 - \frac{u^2}{4} \right) + \frac{q_{\perp}^2}{2\beta} \frac{\cos \beta ut - \cos \beta t}{\sin \beta t}, \]
\[ \Phi_0 = \Phi(B = 0) = t \left( m_e^2 - \frac{q_{\parallel}^2}{4} (1 - u^2) \right), \]
\[ q_{\parallel}^2 = \frac{(q\tilde{F} \tilde{F} q)}{B^2}, \quad q_{\perp}^2 = \frac{(qFFq)}{B^2}, \]
\[ \beta = eB = \sqrt{e^2(FF)/2}. \]
The expression (7) is significantly simplified in two limiting cases which are of interest in some astrophysical applications:

1. The field invariant $|e^2(\mathbf{F})|^{1/2}$ is relatively small parameter ($E_a^2 \gg eB$), so electron and positron are born in states corresponding to the highest Landau levels;

2. In the strong field limit $eB \gg E_a^2$ the field invariant $|e^2(\mathbf{F})|^{1/2}$ appears to be the largest physical parameter, so the electron and the positron are born only on the lowest Landau level.

In the case $E_a^2 \gg eB$ the eigenvalue $\xi^{(2)}$ may be described by the expression in the crossed field limit ($\mathbf{E} \perp \mathbf{B}, E = B$) [17]:

$$\xi^{(2)} \simeq \frac{9 \cdot 3^{1/6} \Gamma^4(\frac{2}{3})}{14\pi^2} \alpha (1 - i\sqrt{3}) (e^2 qFFq)^{1/3}.$$ (9)

Using (9) we obtain the following expression for the decay probability:

$$W \simeq \frac{7 \cdot 3^{1/3} \pi}{8 \cdot 9 \cdot \Gamma^4(\frac{2}{3})} \frac{g_{a\gamma}^2}{\alpha^2 E_a} (e^2 qFFq)^{2/3}$$

$$= 2 \cdot 46 \cdot 10^3 g_{a\gamma}^2 (eB)^{4/3} E_a^{1/3} \sin^{4/3} \theta$$

(θ is the angle between the vectors of the magnetic field strength $\mathbf{B}$ and the momentum of the axion $\mathbf{q}$) and the axion lifetime:

$$\tau_{KSVZ} \simeq 1,16 \left( \frac{10^{-10}}{g_{ae} GeV} \right)^2 \left( \frac{E_a}{10 \text{ MeV}} \right)^{-1/3} \left( \frac{10^{15} G}{B \sin \theta} \right)^{4/3} s.$$ (11)

For comparison we present here the lifetime for DFSZ axions [13] (Fig. 2) in analogous case of the decaying axion energy $\sim 10 \text{ MeV}$ in the field of strength $\sim 10^{15} \text{ G}$:

$$\tau_{DFSZ} \simeq 1,03 \cdot 10^6 \left( \frac{10^{-13}}{g_{ae}} \right)^2 \left( \frac{E_a}{10 \text{ MeV}} \right)^{1/3} \left( \frac{10^{15} G}{B \sin \theta} \right)^{2/3} s,$$ (12)

where $g_{ae}$ is a dimensionless Yukawa coupling constant.

In another limiting case, $eB \gg E_a^2$, when the field strength $B$ appears to be the largest physical parameter, the eigenvalue $\xi^{(2)}$ has a form:

$$\xi^{(2)} \simeq \frac{2\alpha}{\pi} eB \left( 1 - i \frac{2\pi m_e^2}{E_a^2 \sin^2 \theta} \right).$$ (13)
With $\tilde{\alpha}^{(2)}$ the decay probability can be significantly simplified:

$$W \simeq \frac{\pi g_{a\gamma}^2}{4\alpha^2} \frac{eBm_a^2}{E_a},$$

and the axion lifetime is reduced to seconds as in previous case:

$$\tau^{KSVZ} \simeq 0.29 \left( \frac{10^{-10}}{g_{a\gamma} \text{GeV}} \right)^2 \left( \frac{E_a}{1 \text{MeV}} \right) \left( \frac{10^{16} \text{G}}{B} \right) \text{s.}$$

Let us also give here the results in DFSZ model \cite{13} for the same axion and field parameters:

$$\tau^{DFSZ} \simeq 1.4 \cdot 10^4 \left( \frac{10^{-13}}{g_{ae}} \right)^2 \left( \frac{E_a}{1 \text{MeV}} \right) \left( \frac{10^{16} \text{G}}{B} \right) \text{s.}$$

The expressions for the axion lifetime both for KSVZ hadronic axions Eqs. (11), (15) and DFSZ axions Eqs. (12), (16) demonstrate the strong catalyzing influence of the external field on $a \rightarrow e^+ e^-$ decay forbidden in vacuum by the momentum conservation.

Notice that the hadronic axions lifetime in the external field is reduced to seconds for the appropriate axion and field parameters (see Eqs. (11) and (15)) while the relativistic axion lifetime in vacuum \cite{11} is gigantic one:

$$\tau^{(0)}(a \rightarrow 2\gamma) \sim 10^{44} \left( \frac{10^{-10}}{g_{a\gamma} \text{GeV}} \right)^2 \left( \frac{10^{-3} \text{eV}}{m_a} \right)^4 \left( \frac{E_a}{10 \text{MeV}} \right) \text{s.}$$

The results we have presented here are of interest in such astrophysical objects where from both components of the active medium, a magnetic field and plasma, the magnetic component dominates. Such conditions can be realized, for example, in a supernova explosion or in a coalescence of neutron stars when a region of order of hundred kilometers outside the neutrinosphere with strong magnetic fields $\sim 10^{14} \div 10^{16} \text{G}$ \cite{9} and rather rarified plasma can exist.

The work was supported by Grant INTAS 96-0659.

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Figure 1

Figure 2