The Existence of Entire Positive Solutions to Monge-Ampère Type Equations

Shuangshuang Bai

Department of Mathematics and Physics, North China Electric Power University, Beijing, 102206, PR China.

Author’s contribution

The calculation, proof and writing of this article are completed by the author independently.

Article Information

DOI: 10.9734/ARJOM/2021/v17i530303

Editor(s):

(1) Dr. Nikolaos D. Bagis, Aristotle University of Thessaloniki, Greece.

Reviewers:

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Complete Peer review History: [http://www.sdiarticle4.com/review-history/72033](http://www.sdiarticle4.com/review-history/72033)

Received: 02 June 2021
Accepted: 06 August 2021
Published: 12 August 2021

Abstract

Aims/ Objectives: In this paper, we study the Monge-Ampère type equation

\[ \det D^2 u + \alpha \Delta u = p(|x|)f(u)(x \in \mathbb{R}^n). \]

In the previous articles, the equation with Monge-Ampère operator or Laplace operator has been studied extensively. However, the research about the combination of two kinds of operator is scarce. We would like to do some research on this topic. We obtain a sufficient condition of the existence of entire positive solutions for the equation.

Study Design: Study on the existence of solutions.

Place and Duration of Study: Department of Mathematics and Physics, North China Electric Power University, September 2019 to present.

Methodology: We prove the existence of the solution by constructing Euler’s break line, combining the idea of transformation and the method of mathematical analysis.

Results: We obtain a sufficient condition of the existence of entire positive solutions for the equation.

*Corresponding author: E-mail: baishuang105@163.com;
Conclusion: We prove the existence of entire positive solutions to Monge-Ampère type equation \( \det D^2u + \alpha \Delta u = p(|x|)f(u)(x \in \mathbb{R}^n) \) and obtain the sufficient condition for the existence of solutions.

Keywords: Monge-Ampère type equation; entire positive solutions; sufficient condition; existence.

2010 Mathematics Subject Classification: 53C25; 83C05; 57N16.

1 Introduction

In this paper, we consider a class of Monge-Ampère type equation

\[
\det D^2u + \alpha \Delta u = p(|x|)f(u)(x \in \mathbb{R}^n),
\]

where \( \alpha > 0 \), \( \det D^2u \) is the standard Monge-Ampère operator and \( \Delta u \) is the Laplace operator.

Monge-Ampère equation, as a kind of second order fully nonlinear partial differential equation, is widely used in many fields. Up to now, standard Monge-Ampère equations have attracted many interests and many results have been obtained, see [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11] and the references therein. Recently, some Monge-Ampère type equations also have been studied by researchers. In [12], Dai and Li studied the Monge-Ampère type equation

\[
\det^{\frac{1}{n}}(D^2u + \alpha I) = f(u) \text{ in } \mathbb{R}^n
\]

and proved that there exists a subsolution \( u \) if and only if

\[
\int_0^\infty (\int_0^\tau f^n(t)dt)^{-\frac{2}{n+2}}d\tau = \infty
\]

holds.

For equation

\[
\Delta u = f(u) \text{ in } \mathbb{R}^n,
\]

when \( f(u) = u^p(p > 1) \), (1.4) has no positive solution (see Keller [13], Osserman [14] and Brezis [15]). For general \( f(u) \), Osserman [14] showed that (1.4) has a solution if and only if

\[
\int_0^\infty (\int_0^\tau f^n(t)dt)^{-\frac{2}{n+2}}d\tau = \infty
\]

holds. It is well known that (1.5) is called Keller-Osserman condition.

The combination of Laplace operator and Monge-Ampère operator has not been considered. Based on the previous results, we will consider the problem in this paper.

We suppose that \( f, p \) satisfy

\[\text{(s1)} \; f : (0, \infty) \to (0, \infty) \text{ is continuous and nondecreasing}; \]
\[\text{(s2)} \; p : (0, \infty) \to (0, \infty) \text{ is continuous and nondecreasing},\]

We define a function

\[
\theta(s) = s^n + s, \; s \geq 0.
\]
It is obviously that \( \theta \) is strictly increasing for \( s \geq 0 \) and \( \theta(\infty) := \lim_{s \to \infty} \theta(s) = \infty \). Let \( \Theta \) be the inverse function of \( \theta \). We have \( \Theta(\infty) := \lim_{s \to \infty} \Theta(s) = \infty \) and for \( s > 0 \),

\[
\Theta'(s) = \frac{1}{n(\Theta(s))^{n-1} + 1} > 0, \quad \Theta''(s) = -\frac{n(n-1)(\Theta(s))^{n-2}}{(n(\Theta(s))^{n-1} + 1)^3} < 0. \tag{1.7}
\]

The main result of this paper is the following theorem.

**Theorem 1.1.** Suppose (s1)-(s2) hold. If

\[
\int_{-\infty}^{\infty} \left( \int_{0}^{r} f(t) dt \right)^{-\frac{1}{n-1}} dr = \infty
\]

holds, then there exists an entire positive solution \( u \) of (1.1).

## 2 Preliminary Lemmas

In this section, we give some lemmas for the radial functions before proving the main results.

Let \( r = |x| = \sqrt{x_1^2 + \cdots + x_n^2} \) and let \( B_R := \{ x \in \mathbb{R}^n : |x| < R \} \) for \( R \in (0, \infty) \).

**Lemma 2.1.** Assume \( \varphi \in C^2([0, R]) \) with \( \varphi'(0) = 0 \). Then for \( u(x) = \varphi(r) \), we have \( u(x) \in C^2(B_R) \), and the eigenvalues of \( D^2u \) are

\[
\lambda(D^2u) = \begin{cases} 
(\varphi''(r), \frac{\varphi'(r)}{r}, \ldots, \frac{\varphi'(r)}{r}), & r \in (0, R), \\
(\varphi''(0), \varphi'(0), \ldots, \varphi'(0)), & r = 0,
\end{cases}
\]

and so

\[
det D^2u + \alpha \Delta u = \begin{cases} 
\varphi''(r) \left( \frac{\varphi'(r)}{r} \right)^{n-1} + \alpha \left( \frac{\varphi'(r)}{r} \right)^{n-1}, & r \in (0, R), \\
(\varphi''(0))^n + n \alpha \varphi'^2(0), & r = 0.
\end{cases}
\]

The proof of Lemma 2.1 is similar to the proof of Lemma 2.1 in [11], which is the case of \( k = 1 \) and \( k = n \), so we omit the proof here.

By Lemma 2.1, we can conclude that \( u(x) = \varphi(r) \) is a \( C^2 \) radial solution of (1.1) if and only if \( \varphi(r) \) satisfies

\[
\varphi''(r) \left( \frac{\varphi'(r)}{r} \right)^{n-1} + \alpha \left( \frac{\varphi'(r)}{r} \right)^{n-1} = p(r) f(\varphi(r)). \tag{2.1}
\]

**Lemma 2.2.** Suppose (s1)-(s2) hold. Let \( \varphi(r) \in C^2([0, R]) \) satisfy (2.1) with \( \varphi'(0) = 0 \). Then \( \psi'(r) > 0 \) and \( \varphi''(r) > 0 \).

**Proof.** From (2.1), we have

\[
(\varphi'(r))^{n-1} \varphi''(r) + \alpha (r^{n-1} \varphi'(r))^{n-1} = p(r) f(\varphi(r)), \quad r > 0.
\]

It follows that

\[
(\varphi'(r))^n + \alpha r^{n-1} \varphi'(r) = n \int_{0}^{r} s^{n-1} p(s) f(\varphi(s)) ds, \quad r > 0. \tag{2.2}
\]

Define

\[
\omega(r) = \frac{\varphi'(r)}{\alpha r^\alpha}, \quad \alpha_n = (\alpha n)^{\frac{1}{n-1}},
\]
then we can write (2.2) in the form
\[(\omega(r))^{n} + \omega(r) = n\alpha_{n}^{-n}r^{-n}\int_{0}^{s} s^{n-1}p(s)f(\varphi(s))ds, \ r > 0. \tag{2.3}\]

From the definition of \(\Theta\), we see that (2.3) is equivalent to
\[\omega(r) = \Theta(n\alpha_{n}^{-n}r^{-n}\int_{0}^{s} s^{n-1}p(s)f(\varphi(s))ds), \ r \geq 0. \tag{2.4}\]

It follows that
\[
\varphi'(r) = \alpha_{n}r\Theta(n\alpha_{n}^{-n}r^{-n}\int_{0}^{s} s^{n-1}p(s)f(\varphi(s))ds), \ r \geq 0. \tag{2.5}\]

For convenience, let
\[M(r) = n\alpha_{n}^{-n}r^{-n}\int_{0}^{r} s^{n-1}p(s)f(\varphi(s))ds. \]

Obviously, combining with properties of \(\Theta\), we can easily get that
\[\varphi'(r) > 0, \ r > 0. \]

From (2.5), we have
\[
\varphi''(r) = \alpha_{n}\Theta(M(r)) + n\alpha_{n}^{1-n}r\varphi'(M(r)) \left[p(r)f(\varphi(r))r^{-1} - nr^{-n-1}\int_{0}^{r} s^{n-1}p(s)f(\varphi(s))ds\right]
\geq \alpha_{n}\Theta(M(r)) + n\alpha_{n}^{1-n}\varphi'(M(r)) \left[p(r)f(\varphi(r)) - nr^{-n}p(r)f(\varphi(r))\frac{r^{n}}{n}\right]
\geq \alpha_{n}\Theta(M(r)) > 0. \tag{2.6}\]

Lemma 2.3. Suppose (s1)-(s2) hold. Assume that \(\varphi(r) \in C[0, R] \cap C^{1}(0, R)\) satisfies (2.5). Then \(\varphi(r) \in C^{2}[0, R]\) and satisfies (2.1) with \(\varphi'(0) = 0\).

Proof. It is clear that
\[
\lim_{r \to 0} M(r) = \lim_{r \to 0} n\alpha_{n}^{-n}r^{-n}\int_{0}^{r} s^{n-1}p(s)f(\varphi(s))ds = \alpha_{n}^{-n}p(0)f(\varphi(0)).
\]

So, we obtain
\[
\lim_{r \to 0} \varphi'(r) = \lim_{r \to 0} \alpha_{n}r\Theta(M(r)) = \lim_{r \to 0} \alpha_{n}r\Theta(n\alpha_{n}^{-n}r^{-n}\int_{0}^{r} s^{n-1}p(s)f(\varphi(s))ds)
= \lim_{r \to 0} \alpha_{n}r\Theta(\alpha_{n}^{-n}p(0)f(\varphi(0)))
= 0 = \varphi'(0).
\]

\[
\varphi''(0) = \lim_{r \to 0} \frac{\varphi'(r) - \varphi'(0)}{r - 0} = \lim_{r \to 0} \frac{\alpha_{n}r\Theta(M(r))}{r} = \lim_{r \to 0} \frac{\alpha_{n}r\Theta(n\alpha_{n}^{-n}r^{-n}\int_{0}^{r} s^{n-1}p(s)f(\varphi(s))ds)}{r} = \alpha_{n}\Theta(\alpha_{n}^{-n}p(0)f(\varphi(0))). \tag{2.8}\]
It is clear that $\varphi(r) \in C^2(0, R)$. For $r \in (0, R)$, by (2.6) and (2.8) we have
\[
\lim_{r \to 0} \varphi''(r) = \lim_{r \to 0} \left\{ \alpha_n r \Theta(M(r)) + n \alpha_n^{1-n} r \Theta'(M(r)) \left[ p(r) f(\varphi(r)) r^{1-n} - r^{n-1} \int_0^r s^{n-1} p(s) f(\varphi(s)) ds \right] \right\}
\]
\[
= \varphi''(0) + n \alpha_n^{1-n} \Theta'(a_n^{1-n} p(0) f(\varphi(0))) p(0) f(\varphi(0)) - \lim_{r \to 0} n \alpha_n^{1-n} \Theta'(M(r)) \frac{1}{r^n} \int_0^r s^{n-1} p(s) f(\varphi(s)) ds
\]
\[
= \varphi''(0) + n \alpha_n^{1-n} \Theta'(a_n^{1-n} p(0) f(\varphi(0))) p(0) f(\varphi(0)) - n \alpha_n^{1-n} \Theta'(a_n^{1-n} p(0) f(\varphi(0))) p(0) f(\varphi(0))
\]
\[
= \varphi''(0).
\]
Therefore, $\varphi(r) \in C^2(0, R)$. Also, by (2.5) and (2.6), we know that $\varphi$ satisfies (2.1).

From (2.5) and Lemma 2.3, we know that $\varphi(r) \in C^2(0, R)$ and satisfies (2.1) with $\varphi'(0) = 0$ if and only if $\varphi(r) \in C^2(0, R) \cap C^3(0, R)$ and $\varphi$ satisfies (2.5).

**Lemma 2.4.** Suppose (s1)-(s2) hold. For any positive constant $a$, there exists a positive number $R$ such that (2.5) with the initial value
\[
\varphi(0) = a, \quad \varphi'(0) = 0
\]
has a solution $\varphi \in C^2(0, R) \cap C^3(0, R)$.

**Proof.** As the proof of Lemma 2.3 in [11], we define a function
\[
F(r, \varphi) = \alpha_n r \Theta(a_n^{1-n} r^{-n} \int_0^r s^{n-1} p(s) f(\varphi(s)) ds),
\]
on
\[
R = [0, l] \times \{ \varphi \in C^2[0, l] : a - h < \varphi < a + h \}, \quad (2.9)
\]
where $l$ and $h$ are positive constants small enough. Then (2.5) can be rewritten as
\[
\varphi'(r) = F(r, \varphi).
\]
By Lemma 2.2, we know $F > 0$ for $r > 0$.

Define an Euler’s break line on $[0, l]$ as
\[
\psi(0) = a,
\]
\[
\psi(r) = \psi(r_{i-1}) + F(r_{i-1}, \psi(r_{i-1}))(r - r_{i-1}), \quad r_{i-1} < r \leq r_i,
\]
where $0 = r_0 < r_1 < ... < r_m = l$. Then $\psi \in C^2[0, l]$. We claim that $(r, \psi) \in R$.

Indeed, for any $(r, \psi) \in R$, we have
\[
F(r, \varphi(r)) \leq \alpha_n r \Theta(a_n^{1-n} p(l) f(a + h)). \quad (2.10)
\]
It implies
\[
N := \max_{R} F(r, \varphi) \leq \alpha_n r \Theta(a_n^{1-n} p(l) f(a + h)).
\]
Then for the break line $\psi$, we have
\[
a - h \leq \psi(r) \leq a + \frac{\alpha_n}{2} r \Theta(a_n^{1-n} p(l) f(a + h)), \quad r \in [0, l].
\]
Thus if $h$ is fixed, we can choose $l$ sufficiently small such that
\[
a - h \leq \psi(r) \leq a + h.
\]
Next, we prove that Euler’s break line \( \psi \) is an \( \varepsilon \)-approximation solution of (2.5). For this, we only need to prove that for any \( \varepsilon > 0 \), we can choose points \( \{r_i\}_{i=1,\ldots,m} \) such that the break line satisfies

\[
\left| \frac{d\psi(r)}{dr} - F(r, \psi(r)) \right| < \varepsilon, \quad r \in [0, l].
\] (2.11)

In fact, from (2.7) we can easily get that

\[
\lim_{r \to \varphi} F(r, \varphi) = 0
\]

holds for any \( \varphi \in C^2[0, l], \ a - h \leq \varphi \leq a + h \). So for any \( \varepsilon > 0 \), there exists \( \bar{r} \in (0, l) \) such that for \( 0 \leq r < \bar{r} \), we have

\[
F(r, \psi) < \frac{\varepsilon}{2}.
\]

Then

\[
\left| \frac{d\psi(r)}{dr} - F(r, \psi(r)) \right| = \left| F(r_{i-1}, \psi(r_{i-1})) - F(r, \psi(r)) \right| < \varepsilon.
\]

For \( \bar{r} \leq r \leq l \), let

\[
|r' - r''| < \delta(\varepsilon) < \frac{\varepsilon}{\alpha_n \Theta(\alpha_n^{-\eta} p(l) f(a + h))}
\]
for \( r', r'' \in [\bar{r}, l] \).

Suppose \( r_1 = \bar{r} \) and

\[
\max_{2 \leq i \leq m} |r_{i-1} - r_i| < \min\{\bar{r}, \delta(\varepsilon)\},
\]

combining with \( r_{i-1} < r < r_i \) and \( \Theta' > 0 \), we have

\[
\left| \frac{d\psi(r)}{dr} - F(r, \psi(r)) \right| = \left| F(r_{i-1}, \psi(r_{i-1})) - F(r, \psi(r)) \right|
\]
\[
= \left| \alpha_n r_{i-1} \Theta(\alpha_n^{-\eta} r_{i-1}^{-\eta} \int_0^{r_{i-1}} s^{n-1} p(s) f(\psi(s))ds) - \alpha_n r \Theta(\alpha_n^{-\eta} r^{-\eta} \int_0^r s^{n-1} p(s) f(\psi(s))ds) \right|
\]
\[
\leq \left| \alpha_n r_{i-1} \Theta(\alpha_n^{-\eta} r_{i-1}^{-\eta} \int_0^{r_{i-1}} s^{n-1} p(s) f(\psi(s))ds) - \alpha_n r \Theta(\alpha_n^{-\eta} r^{-\eta} \int_0^r s^{n-1} p(s) f(\psi(s))ds) \right|
\]
\[
= \alpha_n \Theta(\alpha_n^{-\eta} r^{-\eta} \int_0^r s^{n-1} p(s) f(\psi(s))ds) \left| r - r_{i-1} \right|
\]
\[
< \alpha_n \Theta(\alpha_n^{-\eta} p(l) f(a + h)) \cdot \frac{\varepsilon}{\alpha_n \Theta(\alpha_n^{-\eta} p(l) f(a + h))} = \varepsilon.
\]

Thus, Euler’s break line \( \psi \) is an \( \varepsilon \)-approximation solution of (2.5).

The next step is to find a solution of Cauchy problem (2.5) by the Euler break line we defined. Assume \( \{\varepsilon_j\}_{j=1}^\infty \) is a positive constant sequence converging to 0. For \( \varepsilon_j \), there is an \( \varepsilon_j \)-approximation solution \( \psi_j \) on \([0, l]\), defined as above. It is easy to know that

\[
|\psi_j(r') - \psi_j(r'')| \leq M' |r' - r''|
\]
where \( r', r'' \in [0, l] \). That is to say, \( \psi_j \) is equicontinuous and uniformly bounded. Therefore by Ascoli-Arzelà Lemma, we can find a uniformly convergent subsequence, still denoted as \( \{\psi_j\} \), without loss of generality.

Assume \( \lim_{j \to \infty} \psi_j = \varphi\). Then \( \varphi(0) = a \), and \( \varphi'(0) = 0 \).

Since \( \psi_j \) is an \( \varepsilon_j \)-approximation solution, we have

\[
\frac{d\psi_j(r)}{dr} = F(r, \psi_j(r)) + \Delta_j(r),
\] (2.12)
where $|\Delta_j(r)| < \varepsilon_j$, for $r \in [0, l]$. Integrating (2.12) from 0 to $r(\leq l)$, we have

$$
\psi_j(r) = a + \int_0^r F(s, \psi_j(s))ds + \int_0^r \Delta_j(s)ds.
$$

Let $j \to \infty$,

$$
\varphi(r) = a + \lim_{j \to \infty} \left( \int_0^r F(s, \psi_j(s))ds + \int_0^r \Delta_j(s)ds \right)
= a + \int_0^r F(s, \varphi(s))ds.
$$

(2.13)

Since $\psi_j$ is continuous, we know that $\varphi$ is continuous. By (2.11), $\varphi$ is continuously differentiable. Differentiating (2.11), we can see that $\varphi$ satisfies equation (2.5) in $[0, l]$.

In fact, a local solution also exists for any real number $a$ if we do not consider only the positive ones. Once $a$ is positive, it is easy to know the solution $\varphi$ is positive, too.

### 3 Proof of the Main Results

In this section, we prove the main results.

**Lemma 3.1.** Suppose (s1)-(s2) hold, then there exists a solution $u$ of (1.1) for all $x \in \mathbb{R}^n$ if and only if (2.5) has a solution $\varphi \in C^2[0, \infty)$ satisfying $\varphi'(0) = 0$ and $\varphi(0) = a$ for any constant $a$.

**Proof.** Sufficiency. If (2.5) has a solution $\varphi$, then obviously $u(x) = \varphi(r) = \varphi(|x|)$ is the desired solution of (1.1).

Necessity. Conversely, assume that there exists no such function $\varphi(r)$ existing globally. By Lemma 2.4 and Lemma 2.3, (2.5) possesses a $C^2$ solution $\varphi$ on some interval with $\varphi(0) = a$, $\varphi'(0) = 0$. Then there is a maximal interval $[0, R]$ in which the solution exists. By Lemma 2.2, we know that $\varphi'(r) > 0$ for $r \in (0, R)$, then $\varphi(r) \to \infty$ as $r \to R$. So it is a contraction, Lemma 3.2 is proved.

**Lemma 3.2.** Suppose (s1)-(s2) hold. If (1.3) holds, then (2.5) has a solution $\varphi \in C^2[0, \infty)$ satisfying $\varphi'(0) = 0$.

**Proof.** Suppose there exists no such solution of (2.5). Just as the proof of Lemma 2.3, there is a $C^2$ solution $\varphi(r)$ of (2.5) with $\varphi'(0) = 0$ and $\varphi(0) = 0$ on the maximal interval $[0, R]$ and $\varphi(r) \to \infty$ as $r \to R$. So $\varphi$ satisfies

$$
\varphi''(r)(\varphi'(r))^{n-1} + \alpha(r^{n-1}\varphi'(r))' = r^{n-1}p(r)f(\varphi(r)).
$$

Since $\varphi'(r) > 0$ and $\varphi''(r) > 0$, we have

$$
\varphi''(r)(\varphi'(r))^{n-1} + \alpha(r^{n-1}\varphi'(r))' \geq \varphi''(r)(\varphi'(r))^{n-1}.
$$

Therefore,

$$
\varphi''(r)(\varphi'(r))^{n-1} \leq r^{n-1}p(r)f(\varphi(r)).
$$

Then,

$$
\varphi''(r)(\varphi'(r))^n \leq r^{n-1}p(r)f(\varphi(r))\varphi'(r).
$$

For $0 < r < R$, we know

$$
\varphi''(r)(\varphi'(r))^n \leq R^{n-1}p(R)f(\varphi(r))\varphi'(r).
$$
Intergrating from 0 to \( r \), we have

\[
(\varphi'(r))^{n+1} \leq (n + 1)R^{n-1} p(R) \int_0^r f(\varphi(s)) \varphi'(s) ds
\]

\[
= (n + 1)R^{n-1} p(R) \int_{\varphi(0)}^{\varphi(r)} f(t) dt.
\]

Thus,

\[
\varphi'(r) \leq ((n + 1)p(R))^{\frac{1}{n+1}} R^{\frac{n-1}{n+1}} \left( \int_{\varphi(0)}^{\varphi(r)} f(t) dt \right)^{\frac{1}{n+1}}.
\]

As a result,

\[
\left( \int_{\varphi(0)}^{\varphi(r)} f(t) dt \right)^{-\frac{n+1}{n+4}} d\varphi \leq ((n + 1)p(R))^{\frac{1}{n+1}} R^{\frac{n-1}{n+1}} dr.
\]

Noticing that \( \varphi(0) = 0 \) and \( \varphi(R) = \infty \) and integratng on \( r \) from 0 to \( R \), we have

\[
\int_0^\infty \left( \int_0^r f(t) dt \right)^{-\frac{n+1}{n+4}} dr \leq ((n + 1)p(R))^{\frac{1}{n+1}} R^{\frac{2n}{n+1}} < \infty,
\]

So

\[
\int_0^\infty \left( \int_0^r f(t) dt \right)^{-\frac{n+1}{n+4}} d\tau < \infty,
\]

which contradicts with (1.3).

**Proof of Theorem 1.1.** By Lemma 3.1 and Lemma 3.2, Theorem 1.1 is proved.

### 4 Conclusion

Suppose (s1)-(s2) hold. If

\[
\int_0^\infty \left( \int_0^r f(t) dt \right)^{-\frac{n+1}{n+4}} d\tau = \infty
\]

holds, then there exists an entire positive solution \( u \) of (1.1).

### Acknowledgement

This paper can be completed, thanks to my teacher’s help in research methods, modification and so on. At the same time, I would like to thank the good learning conditions provided by North China Electric Power University.

### Competing Interests

Author has declared that no competing interests exist.

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