The Possibility of the Kelvin–Helmholtz Instability during Sedimentation of Dust Grains in the Protoplanetary Disk

Yukihiro HASEGAWA and Toru TSURIBE
Department of Earth and Space Science, Graduate School of Science, Osaka University, Toyonaka, Osaka 560-0043
hasegawa@vega.ess.sci.osaka-u.ac.jp, tsuribe@vega.ess.sci.osaka-u.ac.jp

(Received 2012 September 18; accepted 2012 December 11)

Abstract

We reexamine the possibility of shear-driven turbulence during the sedimentation of dust grains in the protoplanetary disk. Shear-driven turbulence is expected to occur before the onset of a gravitational instability for the MMSN model. However, according to previous studies that did not account of the growth of dust grains, with a larger abundance of dust grains, the gravitational instability is indicated to occur before shear-driven turbulence. In this paper, the case including dust growth is considered, and it is found that a Kelvin–Helmholtz instability tends to occur before the gravitational instability, even in the case with a large abundance of dust grains. This is different from previous results without the dust growth.

Key words: instabilities — methods: numerical — planetary systems: protoplanetary disk — solar system: formation

1. Introduction

For the formation of planetesimals, the growth and motion of dust grains in the protoplanetary disks are essential. In a typical scenario, dust grains settle toward the midplane, and a thin dust layer is supposed to form (Nakagawa et al. 1981, 1986). A gravitational instability (GI) of a disk (Goldreich & Ward 1973; Sekiya 1983) is expected to occur when the density in the disk exceeds a critical density, that is given by

\[
\rho_c = \frac{0.606 M_\odot}{r^3},
\]

where \( r \) is the heliocentric distance (Sekiya 1983). In order to resolve the problem of radial drift of meter-sized dust (Adachi et al. 1976), GI in the thin dust layer is essential. However, as dust grains settle toward the midplane, the vertical dust density gradient increases. In such a case, the vertical shear of the rotational velocity in the dust layer becomes strong. This strong shear is expected to induce a Kelvin–Helmholtz instability (KHI) (Chandrasekhar 1961). As a result, KHI is expected to induce shear-driven turbulence. It is considered that turbulence due to KHI prevents dust grains from settling further toward the midplane (Sekiya 1998; Sekiya & Ishitsu 2001; Michikoshi & Inutsuka 2006). Sekiya (1998) showed that GI does not occur in the MMSN (minimum mass solar nebula) model (Hayashi 1981). On the other hand, Sekiya (1998) and Sekiya and Ishitsu (2001) showed that turbulence induced by KHI becomes weaker if the dust abundance in the protoplanetary disk is much larger than that in the MMSN model. If the turbulence is weak, GI will occur, and planetesimals may form. To discuss the possibility of shear-driven turbulence, the Richardson number is known as an indicator of KHI (Chandrasekhar 1961). If the Richardson number is smaller than a critical value, KHI is expected to occur. Chandrasekhar (1961) showed that the critical value is 0.25, but Gómez and Ostriker (2005) as well as Johansen et al. (2006) showed that inclusion of the Coriolis force yields much higher critical Richardson numbers. Other than KHI, the dynamics of the midplane has been shown to be dominated by the streaming instability (Youdin & Goodman 2005; Johansen & Youdin 2007).

Investigations of the turbulence are essential for understanding the processes of the planetesimal formation, and there were many previous studies (Weidenschilling 1980; Cuzzi et al. 1993; Dobrovolskis 1999; Johansen & Youdin 2007; Bai & Stone 2010). These previous studies confirm the result, that shows that GI is expected to occur before KHI if the dust abundance is large. However, the above-mentioned previous studies did not take into account dust growth. Not only gas turbulence, but also dust growth are essential to understand planetesimal formation. Dust grains grow due to dust–dust collisions while they settle toward the midplane (Nakagawa et al. 1981, 1986). Nakagawa, Sekiya, and Hayashi (1986) showed that the law of gas drag force on dust grains is changed during the process of sedimentation of dust grains because of dust growth. They also show that the change in the law of gas drag influences the sedimentation of dust grains. Dust growth is strongly related to gas drag. However, it is difficult to simulate both dust growth and gas turbulence numerically because of computer performance.

In the present work, we numerically calculated the sedimentation and growth of dust grains. The possibility of KHI is discussed based on the distribution of the dust density that is consistent with their sedimentation in the disk. In section 2, the models of gas and dust grains of the protoplanetary disk described in this paper are summarized. In section 3, we show numerical results for sedimentation of dust grains without growth. In section 4, results are shown for the case with dust growth. In section 5, we discuss the effects neglected in this paper. In section 6, we summarize our results.
2. Models of the Protoplanetary Disk

2.1. Disk Model

For simplicity, we restrict ourselves to the case with \( r = 1 \) AU. A protoplanetary disk is composed of gas and dust. In this paper, the gas surface density, \( \Sigma_g \), and the dust surface density, \( \Sigma_d \), are assumed to be

\[
\Sigma_g = 1.7 \times 10^3 f_g \left( \frac{r}{1 [\text{AU}]} \right)^{-\frac{1}{2}} \text{ [g cm}^{-2}] ,
\]

\[
\Sigma_d = 7.1 f_d \xi_{\text{ice}} \left( \frac{r}{1 [\text{AU}]} \right)^{-\frac{1}{2}} \text{ [g cm}^{-2}] ,
\]

where \( f_g \) and \( f_d \), and \( \xi_{\text{ice}} \) are parameters of abundance for gas, dust, and condensed water ice, respectively (Hayashi 1981). At \( r = 1 \) AU, \( \xi_{\text{ice}} = 1 \). The case with \( f_g = 1 \) and \( f_d = 1 \) corresponds to the MMSN model. We consider cases with \( f_g = 1 \) and various \( f_d \). In the MMSN model, temperature \( T \) is given by

\[
T = 280 \left( \frac{r}{1 [\text{AU}]} \right)^{-\frac{1}{2}} \text{ [K]}.
\]

2.2. Gas Properties

We assume that the gas component is in hydrostatic equilibrium in the vertical direction without self-gravity. In this case, the gas density is given by

\[
\rho_g(z) = \frac{\Sigma_g}{\sqrt{\pi} H_g} \exp \left[ -\left( \frac{z}{H_g} \right)^2 \right] ,
\]

where \( z \) is the height from the midplane of the disk and \( H_g \) is the scale height of the disk, given by

\[
H_g = \frac{\sqrt{2} \Sigma_g}{\Omega_K} = 4.7 \times 10^{-2} \left( \frac{r}{1 [\text{AU}]} \right)^{\frac{1}{2}} \text{ [AU]} .
\]

The symbol \( \Omega_K \) is the Keplerian angular velocity. The symbol \( c_s \) is the sound velocity,

\[
c_s = \sqrt{\frac{k_B T}{m_\mu}} = 0.99 \left( \frac{r}{1 [\text{AU}]} \right)^{-\frac{1}{2}} \text{ [km s}^{-1}] ,
\]

where \( k_B \) is the Boltzmann constant and \( m_\mu(=3.9 \times 10^{-24} \text{ g}) \) is the mass of gas molecules (the mean molecular weight is 2.34). The mean free path of gas molecules, \( l_g \), is given by \( l_g = 1.44 \text{ cm at } r = 1 \text{ AU} \) (Nakagawa et al. 1986). From equation (6), the protoplanetary disk is geometrically thin.

2.3. Dust Properties

In this paper, dust grains are assumed to be compact and have a spherical shape with radius \( s \). In the case with \( s \leq 3 l_g/2 = 2.2 \text{ cm} \), the gas drag force is given by Epstein’s law (Epstein 1924). In this case, at \( r = 1 \) AU and \( z = 0 \), the stopping time, \( t_{\text{stop}} \), is given by

\[
t_{\text{stop}} = \frac{\rho_d}{\rho_g(z)} \frac{s}{c_s} = 1.5 \times 10^{-3} \left( \frac{\rho_g}{3 \text{[g cm}^{-3}] \right)} \left( \frac{s}{2.2 \text{[cm]}} \right) \text{[yr]} ,
\]

where \( \rho_d \) is the internal density of the dust grains. In the case with \( s \geq 3 l_g/2 \), gas drag is given by Stokes’ law, and the stopping time is given by

\[
t_{\text{stop}} = \frac{2 \rho_d}{3 \rho_g(z)} \frac{s^2}{l_g c_s} = 1.5 \times 10^{-3} \left( \frac{\rho_g}{3 \text{[g cm}^{-3}] \right)} \left( \frac{s}{2.2 \text{[cm]}} \right) ^2 \text{[yr]} ,
\]

at \( r = 1 \) AU and \( z = 0 \). We adopt the criterion \( s \geq 3 l_g/2 \) for the validity of the Epstein regime, while there were also previous studies that used \( s \leq 9 l_g/4 \) (e.g., Youdin & Shu 2002).

In the present work, we assumed that the radial motion of dust grains could be neglected (Nakagawa et al. 1981). The equation of motion of a dust grain in the vertical direction is given by

\[
\frac{dv_z}{dt} = - \frac{v_z}{t_{\text{stop}}} - \Omega_K^2 z ,
\]

where \( t \) is the time and \( v_z \) is the vertical velocity of the dust grain. We assume that gas is not affected by dust grains, and that gas density is always given by equation (5). We approximate the vertical velocity of the dust grains by the terminal velocity (Nakagawa et al. 1981). By setting \( dv_z/dt = 0 \) in equation (10), we obtain a terminal velocity of a dust grain as

\[
v_z(z) = - t_{\text{stop}} \Omega_K^2 z .
\]

2.4. The Mixture of Gas and Dust Grains

We use a single-fluid approximation for the azimuthal motion. The rotational velocity of a mixed fluid of gas and dust is given as

\[
v_\phi = \left[ 1 - \frac{\rho_g(z)}{\rho_g(z) + \rho_d(z)} \eta \right] v_K .
\]

In equation (12), \( \rho_d \) is the dust density, \( v_K \) is the circular Keplerian velocity, and

\[
\eta = - \frac{1}{4} \frac{H_g^2}{r^2} \frac{\partial \ln P}{\partial \ln r} ,
\]

where \( P = c_s^2 \rho_g \) is the gas pressure. Using equations (2), (5), (6), and (7), we have

\[
\eta = \frac{13}{16} \left( \frac{H_g}{r} \right)^2 = 1.8 \times 10^{-3} \left( \frac{r}{1 [\text{AU}]} \right)^{\frac{1}{2}} \ll 1 .
\]

2.5. Richardson Number

To discuss the possibility of shear-driven turbulence, we calculate the Richardson number, which is an indicator of KHI (Chandrasekhar 1961). The Richardson number is given by

\[
J = - \frac{g_z}{\rho_g(z) + \rho_d(z)} \left[ \frac{\partial \rho_g(z)}{\partial z} + \frac{\partial \rho_d(z)}{\partial z} \right] \left( \frac{\partial v_g}{\partial z} \right) ^{-2} ,
\]

where \( g_z \) is the gravitational acceleration, given by \( g_z = \Omega_K^2 z \) (Sekiya & Ishitsu 2001). Equation (15) is based on an assumption that the disk is composed of an incompressible one-component fluid, for simplicity. We use equation (15) as
an indicator of KHI for simplicity. If \( J \) is smaller than the critical value, \( J_c \), KHI is expected to be induced in the protoplanetary disk, and it is expected that the laminar flow of the mixed fluid in the disk becomes turbulent. Chandrasekhar (1961) showed that \( J_c = 0.25 \), but Gómez and Ostriker (2005) as well as Johansen et al. (2006) showed that including of the Coriolis force yields much higher critical Richardson numbers. In the present work, we used the value \( J_c = 0.25 \), but in a subsequent section we discuss the case with \( J_c = 0.8 \) (Johansen et al. 2006).

According to equations (12) and (15), \( J \) depends on the distribution of the dust density. We calculated the Richardson number for the dust density given by numerical calculations at each time.

### 2.6. Growth and Sedimentation of Dust Grains

The growth and sedimentation of dust grains are described by

\[
\frac{\partial}{\partial t}n(m,z) + \frac{\partial}{\partial z}[n(m,z)v_z(m,z)] = 0
\]

which is the continuity equation for dust grains treated as fluid, where the mean sedimentation velocity of dust fluid at \( z \) is given by

\[
v_z(z) = \frac{\int_0^\infty v_z(m,z)mn(m,z)dm}{\int_0^\infty mn(m,z)dm}.
\]

### 3. Sedimentation of Dust Grains without Growth

In this section, in order to clarify the effect of dust growth, we first consider the case at \( r = 1 \text{ AU} \) without growth.

#### 3.1. The Initial Condition

In this paper, dust density \( \rho_d(z) \) is assumed to be symmetric to the midplane, so we examine only the region of \( z \geq 0 \). As an initial condition, the initial density of dust is assumed to be

\[
\rho_d(z) = \frac{\Sigma_d}{\sqrt{\pi}H_d}\exp\left[-\left(\frac{z}{H_d}\right)^2\right],
\]

where \( H_d \) is the time-dependent scale height of the dust density profile with \( H_d = H_g \) at \( t = 0 \).

We assume the same initial functional form for \( \rho_g(z) \) and \( \rho_d(z) \), and we neglect the dependence of \( v_g \) on \( z \). Note that \( J = \infty \) at \( t = 0 \) from equation (15), i.e., initial state is stable against KHI. This is because the rotational velocity of the mixed fluid of gas and dust grains \( v_g \) at \( t = 0 \) is independent of \( z \) [cf. equation (12)], i.e., \( \frac{\partial v_g}{\partial z} = 0 \) in equation (15).

#### 3.2. Sedimentation of Dust Grains with Single Size

First, we consider the sedimentation of dust grains with a single size. We assume that all dust grains are small enough, and that the drag force is given by Epstein’s law. The characteristic time scale for sedimentation of dust grains is given by

\[
t_{\text{sed}} \equiv \frac{z}{|v_z(z)|} = \frac{1}{t_{\text{stop}}\Omega_K^2}.
\]

From equations (8) and (25), it is shown that the characteristic time scale of sedimentation is much larger than the Keplerian period, \( t_{\text{sed}}\Omega_K^2 \gg 1 \), in the case when the stopping time is much smaller than the Keplerian period, \( t_{\text{stop}}\Omega_K^2 \ll 1 \). In such a case with a single size without dust growth, the profile of the dust density evolves in a self-similar manner (Garaud & Lin 2004). Here, during the sedimentation of dust, the distribution of the gas density assumed to be constant remains as equation (5). When the time evolution of the dust density proceeds in a self-similar manner, the dust density profile in \( t > 0 \) is also given by equation (24) with a temporally decreasing \( H_d(t) \) with \( 0 < H_d(t) \leq H_g \). Using equations (5), (12), and (24), we have

\[
\frac{\partial v_g}{\partial z} = 2\eta_z v_K \left( \frac{1}{H_g^2} - \frac{1}{H_d^2} \right) \frac{\rho_g(z)\rho_d(z)}{[\rho_g(z) + \rho_d(z)]^2}.
\]

Substituting (14) and (26) into (15), an analytical formula for Richardson number can be derived.

Figure 1 shows results of the distribution of the dust density and the Richardson number when \( \rho_d(0) \approx 16\rho_d(0) \) for the case with \( f_d = 1 \), where \( H_d \) is 0.0062 \( H_g \). We define the initial value of the dust density at the midplane
as $\rho_d(0) = \Sigma_d/(\sqrt{\pi} H_g)$. In figure 1, it can be seen that dust density at the midplane is much smaller than the critical density, $\rho_c = 6.3 \times 10^4 \rho_d(0)$, and that the Richardson number is smaller than $J_c = 0.25$ around the midplane ($z/H_g \lesssim 0.0075$). Thus, KHI is expected before GI in this case with $f_0 = f_d = 1$ at $r = 1$ AU.

Assuming an equilibrium condition for KHI, previous studies showed that GI tends to occur if the dust abundance in the protoplanetary disk is much larger than that according to the MMSN model (Sekiya 1998; Sekiya & Ishitsu 2001). Now, using a non-equilibrium time-dependent density profile during sedimentation, we consider the possibility for GI in the case with a large $f_d$. We seek the condition of the dust abundance, $f_d$, by which GI occurs before the onset of KHI. In figure 1a, it can be seen that the distribution of dust density has a maximum value at the midplane. When $f_d$ is larger than 1, the dotted line in figure 1a moves to left, because the abscissa is inversely proportional to $\rho_d(0) \propto f_d$. The dust density at the midplane is the first to reach the critical density for GI, because the time development of the dust density proceeds in a self-similar manner. A characteristic $H_g$ when dust density at the midplane attains $\rho_c$ can be obtained from equation (24). We define the characteristic $H_g$ as $H_c$. With substituting 0, $H_d$ and $\rho_c$ for $z$, $H_g$ and $\rho_d(z)$ in equation (24), respectively, $H_c$ is given by

$$H_c = \frac{\Sigma_d}{\sqrt{\pi} \rho_c} = 1.6 \times 10^{-5} f_d H_g \propto f_d. \quad (27)$$

From figure 1b, it is found that the distribution of the Richardson number has a local minimum value at $z/H_g = 3.5 \times 10^{-3}$. The height where the distribution of the Richardson number takes the local minimum value is defined as $z_c$. At $z = z_c$, from $\partial J/\partial z|_{z=z_c} = 0$, we find

$$\rho_c(z_c) = 2 \rho_d(z_c), \quad (28)$$

by assuming $H_d/H_g \ll 1$. From equation (28), $z_c$ is given as

$$z_c = \left[ \ln \left( \frac{2 \Sigma_d}{\Sigma_g H_d} \right) \right]^{1/2} H_g. \quad (29)$$

The Richardson number at $z = z_c$ is given by

$$J(z = z_c) = \frac{27}{8} \left( \frac{H_d}{\eta r} \right)^2 \quad (30)$$

by assuming $H_d/H_g \ll 1$. From equations (27), (28), and (30), the condition of the dust abundance, $f_d$, which is necessary for GI to occur before KHI is derived as

$$f_d = \left[ \frac{8 \pi}{27} J(z = z_c) \right]^{1/3} \eta r \rho_c \left( \frac{\Sigma_d}{f_d} \right)^{-1} \geq 6.6 \times 10^2, \quad (31)$$

at $r = 1$ AU and $f_0 = 1$. Note that $\Sigma_d \propto f_d$.

In figure 2, results for the case with $f_d = 6.6 \times 10^2$ are shown. In figure 2, it can be seen that the dust density at the midplane indeed attains the critical density, $\rho_c$, and that Richardson number remains marginally larger than the critical value, $J_c$. Thus, in the case when dust grains have a single size and don’t grow at $r = 1$ AU with $f_g = 1$, GI is expected to occur before KHI only if the dust surface density is much larger than the gas surface density.

This tendency is similar to the result of Sekiya (1998), but the value of $f_d$ given in this paper ($f_d = 6.6 \times 10^2$) is larger than the value given in Sekiya (1998) ($f_d = 16.8$). The value of $f_d$ in Sekiya (1998) corresponds to the condition that the protoplanetary disk is in a quasi-equilibrium state for KHI. On the other hand, the value given in this paper corresponds to the condition that GI occurs before KHI during sedimentation, and that is more stringent. Therefore, the value of $f_d$ used in this paper is larger than the value in Sekiya (1998).

### 3.3. Effect of Size Distribution

Next, we consider the sedimentation of dust grains with the initial size distribution, but without growth. The result given by equation (31) is independent of the dust radius, as long as the stopping time is given by Epstein’s law. We now consider the effect of the initial size distribution of the dust grains. The characteristic time scale for the sedimentation of the dust grains in equation (25) depends on size of the dust grains. Thus, even for the same time, the scale height of the dust density profile...
for different sizes of dust grains is different. Therefore, it is expected that the total dust density profile will change from the initial Gaussian profile in the case when dust grains have a size distribution. We consider \(N_d\) kinds of dust grains with different radius. For simplicity, we consider the case when \(s\) is given by integer multiples of the minimum radius of dust grains, \(s_0\), and when the maximum value, \(s_{\text{max}}\), is given by \(N_d s_0\).

In order to derive the scale height, \(H_{d,s}\), of the dust density profile for dust grains with radius \(s\), we assume that \(\rho_d(z)\) is uniform for simplicity. The stopping time of dust grains, \(t_{\text{stop}} = (\rho_s/\rho_d)(s/c_s)\), is proportional to \(s\), and is constant with \(z\). The vertical velocity of dust grains, \(v_z(z) \propto t_{\text{stop}},\) is proportional to \(s\) and to \(z\). From the time evolution of \(z(t)\) with different radius of dust grains with \(z = H_g\) at \(t = 0\), the formula for \(H_{d,s}\) can be derived as

\[
H_{d,s} = \left(\frac{H_{d,s_0}}{H_g}\right)^{s/s_0}.
\]  

(32)

In equation (32), it is seen that \(H_{d,s}\) is determined only by \(H_{d,s_0}\) and \(s/s_0\), instead of \(s\), because the radii of all dust grains are normalized by the smallest dust grains.

The distribution of the total dust density, \(\rho_d(z)\), is given as

\[
\rho_d(z) = \sum_{s/s_0=1}^{N_d} \rho_d(s,z).
\]  

(33)

where \(\rho_d(s,z)\) is the density of dust grains with radius \(s\) at \(z\). For simplicity, initial size distribution is assumed to be a power law of dust radius,

\[
n_d(s,z) = n_d(s_0,z) \left(\frac{s}{s_0}\right)^p,
\]  

(34)

where \(n_d(s,z)\) is the initial condition of the number density for radius \(s\) at \(z\) with a power index \(p\). We assume \(p = -3\) for simplicity. In the case with \(p = -3\), \(s^3 n_d(s,z)\) is equal to \(s_0 s^3 n_d(s_0,z)\) from equation (34). Then, initial condition of \(\rho_d(s,z)\) is given by

\[
\rho_d(s,z) = \frac{4}{3} \pi \rho_s s^3 n_d(s,z) = \frac{4}{3} \pi \rho_s s_0^3 n_d(s_0,z) = \rho_d(s_0,z).
\]  

(35)

In the case with \(p = -3\), equation (35) shows that the initial density of dust with different radius is the same. In this case, \(\rho_d(s,z)\) is given by

\[
\rho_d(s,z) = \frac{1}{N_d} \rho_d(z) = \frac{1}{N_d} \frac{\sum n_d}{\sqrt{\pi H_{d,s}}} \exp \left[ -\left( \frac{z}{H_{d,s}} \right)^2 \right].
\]  

(36)

In the case without dust growth, \(\rho_d(s,z)\) with different \(s\) evolves independently, with different \(H_{d,s}\). From equations (32), (33), and (36), we can derive analytical solutions of \(\rho_d(z)\). We assume \(N_d = 1000\).

Figure 3 shows the dust density and the Richardson number for the case with \(f_d = 50\). In figure 3, it can be seen that the dust density at the midplane attains the critical density, \(\rho_c\), and that the Richardson number remains larger than the critical value. This demonstrates that a dust fraction 50-times larger than that of the MMSN model induces GI before KHI in the case without dust growth. Note that this dust fraction is about 10-times smaller than that in figure 2. We checked the dependence on \(N_d\), and found that this is the case with for \(N_d \gtrsim 10\). Thus, it can be suggested that KHI tends to be inhibited before GI if dust grains have a size distribution even with the same \(f_d\). The reason is that the vertical gradient of the dust density becomes smaller as a result of the continuous size distribution.

Figure 4 shows dust density in the midplane at the onset of KHI for both cases without and with a size distribution. The symbol \(\rho_{d,KH}\) is the dust density in the midplane at the onset of KHI. It is seen that dust density in the midplane at the onset of KHI increases with increasing the dust abundance in both cases with and without any size distribution. Especially, it is seen that the condition \(\rho_{d,KH} = \rho_c\) is attained by a smaller dust abundance, \(f_d\), in the case with a size distribution of \(f_d > 50\), rather than without a size distribution \(f_d \gtrsim 700\).

The above results are based on \(J_c = 0.25\) (Chandrasekhar 1961). However, Johansen et al. (2006) showed that \(J_c = 0.8\). We also investigated the case with \(J_c = 0.8\). In the case without a size distribution, the required abundance of dust for GI is about \(1.2 \times 10^3\). On the other hand, in the case with a size distribution, the required abundance is smaller than 100. This abundance is much smaller than that for the case without a size distribution; this result is qualitatively the same as that for the
Fig. 3. Dust density and the Richardson number at \( r = 1 \) AU for the case when dust grains have a size distribution with \( f_d = 50 \). The lines, abscissas, and ordinates show the same as ones in figure 1.

Fig. 4. Dust density in the midplane and in \( r = 1 \) AU at the onset of KHI for the case without a size distribution of the dust grains (triangles), and for the case with a size distribution of dust grains (circles). The abscissa shows dust abundance, \( f_d \), and the ordinate shows the dust density, \( \rho_d/\rho_c \). The approximated curves are also drawn (dotted lines).

case with \( J_c = 0.25 \). Thus, below we concentrate on the case with \( J_c = 0.25 \).

In the case without dust growth, the results are summarized as follows:

1. In the case when the abundance of dust grains is given as MMSN model, KHI is expected to occur when the dust density at the midplane is still much smaller than the critical density for GI.
2. GI tends to occur if the abundance of dust grains is larger.
3. If dust grains have an initial size distribution, the required abundance of dust for GI has the possibility to be smaller than that in the case without a size distribution.

4. The Case with Dust Growth

4.1. Initial Condition and Numerical Method

We consider the sedimentation and growth of dust grains at \( r = 1 \) AU in the case with \( f_g = 1 \). We solved equation (16) numerically. We assumed that the internal density of dust grains, \( \rho_s \), is 3.0 g cm\(^{-3}\). As an initial condition, we assumed that all dust grains have the initial size \( s_0 = 1.0 \times 10^{-4} \) cm and the initial density given by equation (24).

In numerical calculations, the TVD scheme (Roe 1986) was applied to calculate the sedimentation of dust grains; the method of WS89 (Wetherill & Stewart 1989) was applied for dust growth. Mass coordinates, \( m_i \), were logarithmically divided into 400 mass bins. Our numerical method was tested using an analytical solution (Trubnikov 1971). Figure 5 shows the results with our numerical method for the growth of dust grains. In figure 5, it can be seen that our numerical method had satisfactory accuracy to calculate the growth of dust grains. The \( z \) coordinates \( z_j \)'s were logarithmically divided into 106 spaced grids. The thickness of the nearest grid to the midplane was \( 1.4 \times 10^{-8} \) H\(_g\). Using this spatial resolution, the scale height at GI was sufficiently resolved.

4.2. Possibilities of KHI in the Early Stage

Figure 6 shows the distribution of the dust density and the Richardson number at \( t = 25 \) yr. In figure 6, although the distribution of the dust density changes little from the initial state in this short period, it can be seen that the Richardson number becomes small enough for small \( z \), especially at \( z/H_g \approx 10^{-4} \ll 1 \). This rapid decline of \( J \) did not appear in the case without the growth of dust grains, described in section 3. We suppose that the growth of dust grains is the origin of this rapid decline in the Richardson number at \( z/H_g \ll 1 \) and at \( t/t_{\text{sed}} \ll 1 \).

Equations (12) and (15) show that the Richardson number is a function of the gradient of the dust density, \( \partial \rho_d(z)/\partial z \), so we now consider \( \partial \rho_d(z)/\partial z \). First, we look at the case without the growth of dust grains in order to analyze the effect of dust growth. We consider the case when all dust grains have a single
size as an initial condition, because we assume that all dust grains have an initial radius \( s_0 \) as an initial condition for calculating in the case with dust growth. In the case without any growth and size distribution of dust grains, the typical radius of the dust grains, \( \bar{s}(z) \), which is defined as

\[
\bar{s}(z) = \frac{\int_0^\infty s mn(m,z) dm}{\int_0^\infty mn(m,z) dm},
\]

is equal to \( s_0 \), and is independent of \( z \). Garaud and Lin (2004) showed that the time dependence of the dust density proceeds in a self-similar manner in the case without the growth of dust grains. In this case, the gradient of the dust density is given by

\[
\frac{\partial \rho_d(z)}{\partial z} = -\frac{2\Sigma_d z}{\sqrt{\pi} H_d^3} \propto z \to 0 \quad (z \to 0).
\]

In order to compare with the case including the growth of dust grains, we consider the sedimentation velocity for the gravity center of the dust system at \( z, \bar{v}_s(z) \), which is derived from equations (11), (23), and (37). At \( z/H_g \ll 1 \) and at \( t/t_{sed} \ll 1 \), \( \bar{v}_s(z) \) is given by

\[
\bar{v}_s(z) = \frac{\rho_g}{\rho_g(z)} \frac{\bar{s}(z)}{c_s} \Omega K^2 s.
\]

In the case without the growth of dust grains, \( \bar{v}_s(z) \propto z \) at \( z/H_g \ll 1 \), because \( \bar{s}(z) = s_0 \).

Second, we consider the case with the growth of dust grains. In this case, the typical radius of dust grains, \( \bar{s}(z) \), is a linear function of \( z \) at \( z/H_g \ll 1 \) and at \( t/t_{sed} \ll 1 \), owing to collisions due to sedimentation (see the Appendix for the reason). From equation (39), \( \bar{v}_s(z) \) has a second-order term of \( z \) because the typical radius of dust grains is a linear function of \( z \). By comparing the functional form of \( \bar{v}_s(z) \) with that in the case without dust growth, at \( z/H_g \ll 1 \) and at \( t/t_{sed} \ll 1 \), it is suggested that the gradient of the dust density is given by

\[
\frac{\partial \rho_d(z)}{\partial z} = \delta_1 z + \delta_2 \to \delta_2 \quad (z \to 0),
\]

where \( \delta_1(>0) \) and \( \delta_2(>0) \) are appropriate values.

Figure 7 shows the distribution of the gradient of the dust density at \( z/H_g \ll 1 \) and at \( t = 25 \text{ yr} \) for cases with and without dust growth. From figure 7, it is confirmed that the distribution of the gradient of the dust density can be approximated by equation (40) with \( \delta_2 > 0 \) in the case with dust growth. From figure 7, we can approximate \( \partial \rho_d(z)/\partial z \propto [-2.0(z/H_g) + 1.0 \times 10^{-5}][\rho_{d0}(0)/H_g] \). This indicates that the distribution of the dust density has a local maximum value at \( z/H_g \sim 5 \times 10^{-6} \) in the case with the growth of dust grains. This can be confirmed by noting that figure 8 that shows the distribution of the dust density in \( z/H_g \ll 1 \) and at \( t = 25 \text{ yr} \) in the case with dust growth. By approximating \( \rho_d(z) \) as \( \rho_{d0}(0) \) and \( \rho_{d0}(z) \) as \( \rho_{d0}(0) \), the Richardson number for \( z/H_g \ll 1 \) is given by

\[
J = \left( \frac{16}{13} \right)^2 \left( \frac{r}{H_g} \right)^2 \left( \frac{\Sigma_g}{\Sigma_d} \right)^{-2} \left( \frac{\Sigma_g}{\Sigma_d} + 1 \right)^3 \times \frac{z}{H_g} \left[ \frac{2 \Sigma_g}{\Sigma_d} \frac{\partial \rho_d(z)}{\partial z} \frac{H_g}{\rho_{d0}(0)} \right]^{-2} \times \left[ \frac{2 \Sigma_g}{\Sigma_d} \frac{\partial \rho_d(z)}{\partial z} \frac{H_g}{\rho_{d0}(0)} \right]^{-2}.
\]

Figure 9 shows the distribution of the Richardson number at \( z/H_g \ll 1 \) and at \( t = 25 \text{ yr} \) in the case with the growth of
Fig. 7. Distribution of the gradient of dust density in $r = 1$ AU and $z/H_g \ll 1$ at $t = 25$ yr. The abscissa shows the gradient of the dust density $\partial \rho_d(z)/\partial z$ in units of $\rho_d(0)/H_g$. The ordinate shows $z$ coordinate in units of $H_g$. Circles show the case with dust growth, and triangles show that without dust growth.

Fig. 8. Distribution of dust density in $r = 1$ AU and $z/H_g \ll 1$ at $t = 25$ yr in the case with the growth of dust grains. The line, abscissa, and ordinate show the same situation as in figure 1a.

dust grains. In figure 9, it can be seen that the numerical solutions of the Richardson number are sufficiently close to the approximate equation (41). Figure 9 shows that the Richardson number drawn by the solid line is smaller than the critical value, $J_c$, around the midplane at $t = 25$ yr.

However, it is doubtful whether KHI occurs. Figure 8 shows that the distribution of the dust density has a local maximum value at $z \neq 0$. Under the local maximum point, the Rayleigh–Taylor instability (RTI) is suspected to occur because the distribution of the gradient of dust density becomes positive (Chandrasekhar 1961). If RTI occurs, the distribution of the dust density in this region is expected to be adjusted to be constant by RTI in the region $z \lesssim z_{RTI}$. Assuming this RTI, we modify the distribution of dust density as figure 10b with mass conservation. Note that this treatment for RTI is crude, and that a more accurate treatment should be addressed in future. Hereafter, the same modification is always applied for the density near the midplane.

Figure 11 shows the modified dust density profile at the onset of KHI. Figure 11 corresponds to figure 4 with dust growth. Since it is difficult to simulate the case with large $f_d$ numerically because of the computer performance, results only for the case with $f_d = 1$ are plotted. In figure 11, it can be seen that dust density at KHI, $\rho_{d,KHI}$, increases with increasing dust abundance for the case with $f_d < 2$. On the other hand, in the case with $f_d > 2$, the dust density at KHI decreases with increasing dust abundance. This tendency is qualitatively different from figure 4. In figure 11, our results show that dust density in the midplane at the onset of KHI for the case with $f_d = 4$ is about the same as that for the case with $f_d = 1$. As the physical origin of the decline of $\rho_{d,KHI}$ for $f_d > 2$, we consider the difference in property of gas drag. After dust grains grow,
the law of gas drag changes from Epstein’s law to Stokes’ law. Figure 12 shows the mass function at the onset of KHI at the maximum density for the case with \( f_\text{d} = 1 \) and \( f_\text{d} = 4 \), respectively. For the case with \( f_\text{d} = 1 \), the mass function has a peak at \( m/m_0 \sim 10^{12} \), and the typical size of dust grains is 0.9 cm. On the other hand, for the case with \( f_\text{d} = 4 \), the mass function has a peak at \( m/m_0 \sim 10^{14} \), and the typical radius of dust grains is 4 cm. In the case with \( s \geq 3/\rho_c/2 = 2.2 \) cm, the stopping time is given by Stokes’ law. By a comparison of these results, it is suggested that the decrease of the dust density at KHI for \( f_\text{d} > 2 \) in figure 11 originates from a change in the law of gas drag due to dust growth.

To confirm this possibility, for reference, we recalculated the evolution using Epstein’s law for all sizes. The solids line in figure 13 shows the dust density at the onset of KHI for this case. It can be clearly seen that the dust density at the onset of KHI increases with increasing \( f_\text{d} \). By comparing the two lines in figure 13, it is clear that the physical origin for the decline of \( \rho_{d, \text{KHI}} \) for \( f_\text{d} > 2 \) is the change of the gas drag from Epstein’s law to Stokes’ law. Therefore, it is significant for us to take into account the dust size dependence of the stopping time as well as dust growth when we investigate the shear-driven turbulence in the protoplanetary disk.

5. Discussion

5.1. Dependence on the Heliocentric Distance

We now estimate the dependence of the required abundance of dust for GI on the heliocentric distance, \( r \). First, we consider the case without dust growth. For simplicity, we consider the case when all dust grains have a single size. The scale height of the dust density profile at the onset of KHI can be obtained from equation (30), and the definition of the scale height is given by

\[
H_{\text{KHI}} = \left( \frac{8 J_c}{27} \right)^{1/2} \eta r = 1.0 \times 10^{-2} H_g \left( \frac{r}{1 \text{ [AU]}} \right)^{1/2}.
\]  

Equation (43) shows that \( H_{\text{KHI}} \) is independent of \( f_\text{d} \). The dependence of \( H_c \) [defined in equation (27)] on \( r \) is given by

\[
H_c = 1.6 \times 10^{-5} f_\text{d} \xi_{\text{ice}} H_g (r/1 \text{ [AU]})^{1/4},
\]

and we have

\[
\frac{H_{\text{KHI}}}{H_c} = 6.6 \times 10^2 \left( \frac{f_\text{d}}{T} \right)^{-1} \left( \frac{\xi_{\text{ice}}}{T} \right)^{-1}.
\]  

Equation (44) shows that the required abundance of dust for GI is smaller outside the snow line than inside. Next, we consider the case with dust growth. At the onset of KHI, most of the dust grains with the typical size calculated in subsection 4.3 [defined in equation (37)] have settled near the midplane. Thus, the typical size of dust at the onset of KHI can be obtained as the size of dust grains that have settled from high altitudes to the midplane. We have

\[
dm = 4\pi s^2 \rho_d ds \sim -p_a \rho_c (2s^3) \rho_d(z) dz.
\]

Assuming that \( s = s_0 \) at \( z = +\infty \) and that \( s = s_1 \gg s_0 \) at \( z = 0 \), we derive \( s_1 \) as
and the path of gas molecules scales as disk, even for a large given by Epstein’s law in the outer region of a protoplanetary disk is more suitable for GI. They are based on the discussion in Takeuchi et al. (2012), which shows that the inner part of the protoplanetary disk might be more suitable for GI than the inner region. Epstein regime. Therefore, the outer region of the protoplanetary disk is more suitable for GI. They are based on a different condition from ours. They used the scale height of the dust layer, which is determined by the balance between sedimentation and the diffusion of dust grains, while we used the scale height of the dust density profile at the onset of KHI.

5.2. The Linear Analysis of the Shear Instability

In this paper, we used the Richardson number as an indicator of KHI in order to discuss the possibility of shear-driven turbulence. However, although KHI occurs, there is a possibility that some dust grains continue to settle toward the midplane if the shear-induced turbulence is weak. An analysis with only the Richardson number is insufficient in order to understand the effect of the shear-induced instability in the protoplanetary disk. Sekiya and Ishitsu (2001) and Michikoshi and Inutsuka (2006) calculated the growth rate of the shear-induced instability by solving the linear perturbation equations. In these studies, the growth rate of the shear-induced instability depends on the assumed distribution of the dust density, which is not consistent with the formation process of the dust layer. Thus, it would be better to calculate the growth rate of KHI as well as RTI with the consistent density distribution that is given in this paper. This will be addressed in a forthcoming paper.

5.3. Possibilities of the Streaming Instability and the Fractal Growth of Dust

Youdin and Goodman (2005) and Johansen and Youdin (2007) showed that the dynamics in the midplane is dominated by the streaming instability. Bai and Stone (2010) showed that dust grains with $\tau_s = \Omega_{\text{KHI,stop}} > 0.01$ trigger a streaming instability before KHI. In our model, at $r = 1$ AU, $\tau_s > 0.01$ corresponds to $s > 2$ cm. In our calculations, for the case with dust growth and $f_d > 2$, the typical size of dust grains at the maximum density is larger than 2 cm. Therefore, in the dust layer governed by Stokes’ law, the streaming instability would occur before KHI and GI.

In this paper, we assumed compact and spherical dust grains. However, since dust grains grow due to dust–dust collisions, large dust grains are aggregates of small dust grains. Both labo-
or thermal Brownian motion. For the dust–dust collisions, it is assumed that dust grains collide and coalesce by sedimentation for the azimuthal motion. For the dust–dust collisions, it is assumed that dust grains collide and coalesce by sedimentation or thermal Brownian motion.

Our study shows the following results: (1) Shear-driven turbulence is expected to occur before the onset of gravitational instability in MMSN model at $r = 1$ AU. (2) In the case without dust growth, at the onset of KHI, the dust density in the midplane is large for a larger dust abundance. This tendency holds for the same initial size distribution of dust grains. (3) In the case with dust growth, the dust density at the onset of KHI decreases for increasing dust abundance for $f_d > 2$. This result is qualitatively different from that in the case without dust growth. The reason is that gas drag changes from Epstein’s law to Stokes’ law for larger dust grains that grow up in advance. Thus, we stress that, for studying of shear-driven turbulence, the change of the law of gas drag from Epstein’s law to Stokes’ law as well as dust growth is required to be taken into account.

For the formation of planetesimals, it has been suggested that in order to occur GI before an inward drift of dust, dust grains have to settle toward the midplane, and it is suggested to be possible with large dust abundance. However, in this paper, it is suggested that shear-driven turbulence is certain to occur even when the dust abundance in the protoplanetary disk is larger than that in the MMSN model. In future work, we should develop a more realistic model by calculating the effect of KHI and RTI directly.

We thank Fumio Takahara for fruitful discussions and continuous encouragement. We also acknowledge discussions with I. Sugawara during the early stage of this work.

**Appendix. The Typical Radius of Dust Grains in the Case with Dust Growth**

We explain the reason that the typical radius of dust grains $\langle r \rangle$ is a linear function of $z$ at $z/H_g \ll 1$ and at $t/t_{\text{sed}} \ll 1$ owing to collisions due to sedimentation in the case with the growth of dust grains. We now estimate the mean collision time by the same method as that used in Nakagawa et al. (1981). The mean collision time is given by

$$t_{\text{coll}} = \frac{1}{n_d \sigma \Delta v}, \quad (A1)$$

where $n_d$ is the number density of dust grains, $\sigma$ is the collisional cross section and $\Delta v$ is the relative velocity of the dust–dust collision. We assume that the radii of dust grains are given by the typical radius, and that masses of dust grains are given by the typical mass of dust grains. The typical mass is given by

$$m(z) = \frac{4}{3} \pi \rho_d \langle s(z) \rangle^3, \quad (A2)$$

and we regard that $\sigma = \pi \langle s \rangle^2$. We treat $n_d$ as $\rho_d / m$ or $(\Sigma_d / \Sigma_g)[\rho_d / m]$ at $z/H_g \ll 1$ and at $t/t_{\text{sed}} \ll 1$. For the collision due to sedimentation, we simply put $|s - s'| = \langle s \rangle$ and $\Delta v = \Delta \langle v \rangle$. The mean collision time for sedimentation is defined as $t_{\text{coll,s}}$. At $z/H_g \ll 1$ and at $t/t_{\text{sed}} \ll 1$, $t_{\text{coll,s}}$ is obtained by

$$t_{\text{coll,s}} = \frac{2 \sqrt{2} \Sigma_d}{3 \Sigma_g} \frac{1}{H_g} \left( \frac{z}{H_g} \right)^{-1} = \frac{6}{5} \left( \frac{z}{H_g} \right)^{-1} \text{[yr]}. \quad (A3)$$

In equation (A3), it can be seen that $t_{\text{coll,s}}$ is independent of $\langle s \rangle$, and that $t_{\text{coll,s}} \propto z^{-1}$ at $z/H_g \ll 1$ and at $t/t_{\text{sed}} \ll 1$. For collisions due to the thermal motion, we simply put $m = m' = m$ and $\Delta v = \Delta v_T$. The mean collision time for the thermal motion, $t_{\text{coll,B}}$, is obtained by

$$t_{\text{coll,B}} = \frac{4}{3} \sqrt{\frac{2 \pi H_g}{3 \Sigma_d} \rho_d \langle s \rangle^2} \left( \frac{\langle s \rangle}{s_0} \right)^{3/2} \exp \left( \left( \frac{z}{H_g} \right)^2 \right) \text{[yr]}, \quad (A4)$$

For collisions due to the thermal motion, it is found that $t_{\text{coll,B}} > t_{\text{coll,s}}$, and that collisions due to sedimentation are dominant. The growing speed of dust grains in sedimentation is expected to be proportional to $z$, because $t_{\text{coll,s}} \propto z^{-1}$. Thus, it is supposed that the typical mass of dust grains at $z$, $m(z)$, is given by $m(z) = [a_1(z/H_g) + a_2] \langle m_0 \rangle$, where $a_1$ and $a_2$ are appropriate values. Then, the typical radius of dust grains at $z$ is given by $\langle r \rangle = [a_1(z/H_g) + a_2] \langle s \rangle / s_0$. If $a_1 z / a_2 H_g \ll 1$, it is supposed that $\langle s \rangle$ is approximated by $[a_2(z/H_g) + a_4 s_0]$, with a Taylor expansion. Symbols $a_3$ and $a_4$ are appropriate values.

Figure 14 shows the distribution of the typical mass and radius of dust grains at $z/H_g \ll 1$ and at $t = 25$ yr. From figure 14, it is confirmed that $\langle s \rangle = [a_3(z/H_g) + a_4 s_0]$, with a Taylor expansion. Symbols $a_3$ and $a_4$ are appropriate values.

The above discussions assume that collisions due to sedimentation become more dominant than those due to the thermal motion immediately. However, it is not confirmed that collisions due to sedimentation are dominant to the growth of dust grains before $t = 25$ yr. We should now confirm that this assumption is appropriate for the case that we investigated.
Fig. 14. (a) Distribution of the typical mass of dust grains in $r = 1$ AU at $t = 25$ yr. The abscissa, where $m_{\text{typical}}$ means $\tilde{m}$, shows the typical mass $\tilde{m}$ in units of $m_0$. The ordinate shows $z$ coordinate in units of $H$. (b) Distribution of the typical radius of dust grains at $t = 25$ yr. The abscissa, where $s_{\text{typical}}$ means $\tilde{s}$, shows the typical radius, $\tilde{s}$, in units of $s_0$. The ordinate shows $z$ coordinate in units of $H$. 

At $z/H \ll 1$ and $t \approx 0$, it is expected that collisions due to the thermal motion is dominant, and that $t_{\text{coll}, \text{H}}$ is independent of $z$. In this case, it is supposed that the typical radius of dust grains is independent of $z$, i.e., $a_3$ in the formula for $\tilde{s}(z)$ is temporally constant. However, in a certain time, it is expected that the dominant effect in the growth of dust grains changes from collisions due to the thermal motion to those due to sedimentation because of dust growth. Therefore, the time for this change can be determined by investigating the time development of $a_3$ for $\tilde{s}(z)$.

Figure 15 shows the time development of $a_3$. From figure 15, $a_3$ is approximated by $a_3 = 6.3 \times 10^{-5}(t/1 \text{ yr})^2 + 7.0 \times 10^{-3}(t/1 \text{ yr}) - 3.0 \times 10^{-2}$. This shows that $a_3 > 0$ at $t \gtrsim 4$ yr, so it is considered that collisions due to sedimentation dominate in the growth of dust grains at $t \gtrsim 4$ yr. Therefore, we show that the assumption that collisions due to sedimentation become more dominant to the growth of dust grains than those due to the thermal motion immediately is proper for the case that we consider.

Fig. 15. The time development of $a_3$ (circles) in $r = 1$ AU and $z/H \ll 1$ at $t/t_{\text{sed}} \ll 1$. The abscissa shows time and the ordinate shows $a_3$ that is derived from fitting $\tilde{s}(z)$ into $[a_3(z/H) + a_4]s_0$. The approximated curve is also drawn (dotted line).

References

Adachi, I., Hayashi, C., & Nakazawa, K. 1976, Prog. Theor. Phys., 56, 1756
Bai, X.-N., & Stone, J. M. 2010, ApJ, 722, 1437
Chandrasekhar, S. 1961, Hydrodynamic and Hydromagnetic Stability (Oxford: Clarendon Press), 491
Cuzzi, J. N., Dobrovolskis, A. R., & Champney, J. M. 1993, Icarus, 106, 102
Dobrovolskis, A. R., Dacles-Mariani, J. S., & Cuzzi, J. N. 1999, J. Geophys. Res., 104, 30805
Epstein, P. S. 1924, Phys. Rev., 23, 710
Garaud, P., & Lin, D. N. C. 2004, ApJ, 608, 1050
Goldreich, P., & Ward, W. R. 1973, ApJ, 183, 1051
Gómez, G. C., & Ostriker, E. C. 2005, ApJ, 630, 1093
Hayashi, C. 1981, Prog. Theor. Phys. Suppl., 70, 35
Johansen, A., Henning, T., & Klahr, H. 2006, ApJ, 643, 1219
Johansen, A., & Youdin, A. 2007, ApJ, 662, 627
Kempf, S., Pfalzner, S., & Henning, T. K. 1999, Icarus, 141, 388
Michikoshi, S., & Inutsuka, S. 2006, ApJ, 641, 1131
Nakagawa, Y., Nakazawa, K., & Hayashi, C. 1981, Icarus, 45, 517
Nakagawa, Y., Sekiya, M., & Hayashi, C. 1986, Icarus, 67, 375
Okuzumi, S. 2009, ApJ, 698, 1122
Okuzumi, S., Tanaka, H., & Sakagami, M. 2009, ApJ, 707, 1247
Roe, P. L. 1986, Ann. Rev. Fluid Mech., 18, 337
Sekiya, M. 1983, Prog. Theor. Phys., 69, 1116
Sekiya, M. 1998, Icarus, 133, 298
Sekiya, M., & Ishitsu, N. 2001, Earth, Planets, Space, 53, 761
Takeuchi, T., Muto, T., Okuzumi, S., Ishitsu, N., & Ida, S. 2012, ApJ, 744, 101
Trubitsyn, B. A. 1971, Soviet Phys. Dokl., 16, 124
Weidenschilling, S. J. 1980, Icarus, 44, 172
Wetherill, G. W., & Stewart, G. R. 1989, Icarus, 77, 330
Wurm, G., & Blum, J. 1998, Icarus, 132, 125
Youdin, A. N., & Goodman, J. 2005, ApJ, 620, 459
Youdin, A. N., & Shu, F. H. 2002, ApJ, 580, 494
Zsom, A., Ormel, C. W., Görtler, C., Blum, J., & Dullemond, C. P. 2010, A&A, 513, A57