Quantum-discord-induced superradiance and subradiance in a system of two separated atoms

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We investigate collective radiant properties of two separated atoms in X-type quantum states. We show that quantum correlations measured by quantum discord (QD) can induce and enhance superradiance and subradiance in the two-atom system even though in the absence of interatomic quantum entanglement. We also explore quantum statistical properties of photons in the superradiance and subradiance by addressing the second-order correlation function. In particular, when the initial state of the two separated atoms is the Werner state with non-zero QD, we find that radiation photons in the superradiant region exhibit the nonclassical sub-Poissonian statistics and the degree of the sub-Poissonian statistics increases with increasing of the QD amount, while radiation photons in the subradiant region have either the sub-Poissonian or super-Poissonian statistics depending on the amount of QD and the directional angle. In the subradiant regime we predict the QD-induced photon statistics transition from the super-Poissonian to sub-Poissonian statistics. These results shed a new light on applications of QD as a quantum resource.

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I. INTRODUCTION

Quantum discord (QD) [1, 2] is a measure of nonclassical correlations which include not only quantum correlations with entanglement but also entanglement-free quantum correlations that may occur in separable states. QD captures the nonclassical correlations, more general than entanglement, that can exist between parts of a quantum system even if the corresponding quantum entanglement does vanish. QD is considered to be a more general resource of quantum advantage than quantum entanglement in quantum information processing [3–11]. QD has been investigated in a wider context including entanglement-free quantum computation [3], quantum communication [10, 13–16], and quantum phase transitions in many-body physics [17–22]. In this paper, we add superradiance and subradiance to the list of QD applications. We show that QD can induce superradiance and subradiance in a system of two separated atoms.

Superradiance, first introduced by Dicke [25], is a coherent spontaneous emission generated by the cooperative effect of many atoms, molecules, or nuclei. At this point, all the atoms simultaneously interact with a common radiation field and exhibit a collective behavior due to the coherent superposition. Ordinary spontaneous emission is usually incoherent spontaneous radiation, because the phases of radiations of different atoms have no any correlation with each other. As a consequence, the emitted light pulse is a simple algebraic addition of the independent radiation intensity, and its shape decays exponentially. The shape of superradiant pulse, however, does not take on exponential decay and has a peak after a certain delay. Note that superradiant emission in certain directions is accompanied by subradiant emission in other different directions [25, 28]. In the original Dicke model, the superradiance was shown to be a consequence of the macroscopic dipole moment produced by the collective coherent behavior of numerous atoms in a local system. It has been shown that the larger the macroscopic dipole moment is, the more visible the superradiance phenomenon will be. Since the superradiance was observed firstly by Skribanowitz et al [29], it has attracted considerable interest and been extensively investigated [30–42], due to its wide applications such as generating X-ray lasers with higher power. Much of the aforementioned literatures were focused on the ensembles of atoms with inherent complexities associated with ensembles, where the interatomic distance was considered to be shorter than the emission wavelength. Recently, Wiegner, Zanthier, and Agarwal [43] investigated the superradiance phenomenon of an N-atom system prepared in the canonical W state [44]. It was shown that superradiance occurs even if the interatomic distance is larger than the wavelength of the emitted photons, due to the existence of quantum entanglement among these atoms. It should be pointed out that such a system has no dipole moment due to the fact that the distance between any two atoms is larger than the emission wavelength and the dipole-dipole interactions are negligible.

In this paper, we study the radiative characteristics of emission for a two-atom system with quantum correlations characterized by the QD. We show that two atoms in a nonzero-QD state can create superradiance and subradiance even without interatomic entanglement, that is, entanglement-free QD can induce superradiance and subradiance. The rest of this paper is organized as follows. In Sec. I we investigate the radiative characteristics of the emission from the two two-level atoms initially in a X-type state, and demonstrate the existence of the QD-induced super- and sub-radiance phenomena. In Sec. III we analyze quantum statistical properties of emitted photons and indicate that photons in super- and sub-radiance may display different statistical statistics. In particular, it is found that the QD effect can induce nonclassical photon statistics and the...
transition from the super-Poissonian to sub-Poissonian statistics. Finally, this work is concluded in Sec. [V]
FIG. 2: (Color online) Plot of the radiation intensity \( I \) with respect to the QD amount \( D \) and the direction angle parameter \( \sin \beta \), where \( kl = \pi \).

Obviously, the concurrence is zero for \( 0 \leq c \leq 1/3 \). This indicates that the Werner state in the parameter regime of \( 0 \leq c \leq 1/3 \) is a separable quantum state without entanglement. However, from Eq. (9) we can find that the QD of the Werner state in the parameter regime of \( 0 \leq c \leq 1/3 \) is nonzero. Hence, the Werner state with \( 0 \leq c \leq 1/3 \) is such a quantum state with quantum correlation but without quantum entanglement. In particular, at the entanglement-transition point of \( c = 1/3 \), we have the QD with \( D = D_c \approx 0.126 \). The numerical analysis indicates that both the QD amount \( D \) and the concurrence \( C \) increase with the increasing of the state parameter \( c \) in the range \( c > 1/3 \) \([47, 48]\).

For the Werner state in Eq. (7), the radiation intensity given by Eq. (5) becomes

\[
I = 1 - c \cos (kl \sin \beta),
\]

from which we can see that the radiation intensity depends on not only the interatomic distance \( l \) and the detection angle \( \beta \) but also the state parameter \( c \) for a given wave vector \( k \). We have plotted the radiation intensity with respect to the QD and the direction angle parameter \( \sin \beta \) in Fig. 2 which indicates the existence of the QD-induced superradiance and subradiance in different angle regimes. From Fig. 2 we can see that the nearby region along the \( \pm x \)-axis direction with \( |\sin \beta| < 0.5 \) is the subradiance region with \( I < 1 \) while the nearby region along the \( \pm z \)-axis direction with \( |\sin \beta| > 0.5 \) is the superradiance region with \( I > 1 \).

In order to clearly see the QD influence on the radiation intensity, in Fig. 3 we have plotted the radiation intensity with respect to the QD for the directional parameter \( \sin \beta \) taking 0 and 1, respectively. From Fig. 3 we can clearly see the QD-induced superradiant and subradiant phenomena. The superradiance appears in the branch of \( \sin \beta = 1 \) while the subradiance occurs in the branch of \( \sin \beta = 0 \). The superradiance (subradiance) is enhanced with the increase of the QD of the Werner state since the radiation intensity \( I \) monotonically increases (decreases) with increasing of the QD value \( D \). This shows that the QD can enhance the superradiance and subradiance.

A notable consequence could be obtained from Fig. 3 that quantum correlation is at least a necessary condition for generating superradiant and subradiant emission for the two-separated-atom system under our consideration. Note that when \( 0 < c \leq 1/3 \), the Werner state has no entanglement while its quantum discord is larger than zero \((0 < D < D_c)\) \([47, 48]\). This implies that entanglement is not a necessary condition for yielding superradiance and subradiance, in contrast to the case where the atoms are initially in a pure entangled state, supporting the necessity of entanglement \([43]\) to produce superradiance and subradiance. Therefore, we can conclude that quantum correlations without entanglement, i.e., the entanglement-free QD, can induce and enhance the superradiance and subradiance of two separated atoms.

III. QUANTUM STATISTICAL PROPERTIES OF RADIATED PHOTONS

This section aims at revealing the quantum statistical behavior of radiated photons by analyzing the second-order correlation function of the radiation intensity. We only pay attention to the photon statistics in superradiance and subradiance. It will be shown that quantum statistical properties of photons in the superradiance are different from those in the subradiance. More interestingly, we find that the QD can induce nonclassical photon statistics \([51]\) and the statistical property could jump from the classical super-Poissonian to the nonclassical sub-Poissonian in some angle regimes.

The second-order correlation function of the radiation in-
In order to observe how the quantum correlations affect quantum statistical properties of the radiation photons in the superradiant and subradiant regimes, we perform numerical simulations for the influence of the QD of the atomic initial state on the second-order correlation function of the radiation intensity. In Fig. 4, we have plotted the second-order correlation function with respect to the amount of QD $D$ and the directional angle parameter $\sin \beta$. Combining Fig. 4 with Fig. 2, we can see that photons in the superradiant regime of $|\sin \beta| > 0.5$ exhibit the sub-Poissonian statistics with $g^{(2)}(0) < 1$. In the subradiant regime of $|\sin \beta| < 0.5$, the quantum statistical properties of the radiated photons become more complicated as shown below. The second-order correlation function may have an inflexion and the photon statistics may change sharply with the variation of the QD amount. These effects lead to an interesting phenomenon that QD can induce the photon statistics transition from the classical super-Poissonian to the sub-Poissonian statistics. Such a phenomenon can be observed more clearly in Fig. 5 which gives the second-order correlation function as a function of the QD amount $D$ when $\sin \beta = 0.2$. It can be seen that the transition point happens at the position of $D = D_t = 0.87$. Photon statistics in the range $0 < D < D_t$ is the super-Poissonian while photon statistics in the range $D_t < D < 1$ is the sub-Poissonian. At the transition point $D_t$, photon statistics is the Poissonian. Therefore, we can conclude that quantum statistical behaviors of radiation photons can be manipulated by changing the interatomic quantum correlations for some fixed direction angles, and that the QD can induce the photon statistics transition from classical super-Poissonian to the sub-Poissonian statistics.

### IV. CONCLUDING REMARKS

In conclusion, we have studied collective radiant properties of two separated atoms in X-type quantum states. It has been shown that QD can induce and enhance superradiance and subradiance in the two-atom system. More interestingly, we have found that quantum correlations without any entanglement can induce and enhance superradiance and subradiance. This sheds a new light on applications of QD as a quantum resource with quantum advantages. Furthermore, we have investigated quantum statistical properties of emitted photons in both regions of superradiance and subradiance by addressing the second-order correlation function. When the initial state of the two separated atoms is the Werner state with non-zero QD, it has been found that radiation photons in the superr-
diant region exhibit the nonclassical sub-Poissonian statistics and the degree of the sub-Poissonian statistics increases with increasing of the QD amount. However, radiation photons in the subradiant region have different statistical characteristics, they are either the sub-Poissonian or the super-Poissonian statistics, depending on the amount of QD and the directional angle. A particularly interesting finding is that in the subradiance region the QD can induce photon statistics transition from the super-Poissonian to sub-Poissonian statistics.

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