About the quark currents in leptonic decays of 
pseudoscalar mesons

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Abstract

With usage of obvious mechanisms of quark currents the amplitudes of 
leptonic decays of pseudoscalar mesons are investigated. The estimation for 
a constant of leptonic decay of the $D^+$ meson is obtained, $f_{D^+} \simeq 0.23$ GeV.

In the paper [1] by the calculation of form factors in $K^{\mu 3}$ decay in spectator 
nearing the research of obvious mechanisms of quark currents was conducted. Thus 
the output for frameworks of the conventional phenomenological description of weak 
decays of hadrons was carried out. In the present paper are investigated quark 
currents in leptonic decays of pseudoscalar mesons.

Let us consider $\pi^+ \rightarrow \mu^+ \nu_\mu$ decay (Fig. 1). The amplitude of the process has a 
kind:

$$M = \frac{G_F}{\sqrt{2}} V_{ud}^* \bar{u}_\mu \gamma^\alpha (1 - \gamma_5) v_\mu < 0 \mid J_\alpha \mid \pi^+ >,$$  

(1)

where the phenomenological matrix element of a pseudovector current can be submitted as

$$< 0 \mid J_\alpha \mid \pi^+ > = \varphi_\pi f_\pi p_\alpha.$$  

(2)

Here $f_\pi$ is a constant of a charged pion, $f_\pi = 0.131 GeV$ [2], $\varphi_\pi$ is a wave function 
of a pion, and $p_\alpha$ is its four-momentum.

As well as in paper [1] we record a pseudovector current as a current of constituent 
quarks

$$< 0 \mid J_\alpha \mid \pi^+ > = \bar{v}_d \gamma_\alpha \gamma_5 u_u F_\pi,$$  

(3)

here $\bar{v}_d, u_u$ are four-spinors of constituent quarks, $F_\pi$ is parameter of an annihilation 
in vacuum, having dimension of mass. $F_\pi$ corresponds to an annihilation of a
virtual pair of $u\bar{u}$. And the degree of this process (departure from a mass surface) corresponds to energy of a pair lepton-antilepton or mass of a pion.

In the rest frame of a pion from (3) by simple calculation is received

$$< 0 \ | \ J_\alpha \ | \pi^+ > = F_\pi 2m_u,$$

where $m_u$ is mass of a constituent $u(d)$ quark, as well as in the paper [1] and also in the papers [3, 4]. From expressions (2) and (4) is received

$$f_\pi m_\pi = F_\pi 2m_u,$$

(5)

and $f_\pi = 0.131 \ GeV$ [2].

Similarly for $K^+ \to \mu^+ \nu_\mu$ decay we have

$$f_K m_K = F_K 2\sqrt{m_u m_s},$$

(6)

and $f_K = 0.160 \ GeV$ [2].

And also for $D_s^+ \to \mu^+ \nu_\mu$ decay is

$$f_{D_s^+} m_{D_s^+} = F_{D_s^+} 2\sqrt{m_c m_s},$$

(7)

and $f_{D_s^+} \simeq 0.288 \ GeV$ [2].

Using (5, 6, 7) and also masses of constituent quarks

$$m_u = 0.305 \ GeV,$$

$$m_s = 0.487 \ GeV,$$

$$m_c = 1.400 \ GeV$$

is received

$$F_{\pi^+} = \frac{f_\pi m_\pi}{2m_u} = 0.030 \ GeV,$$

$$F_{K^+} = \frac{f_K m_K}{2\sqrt{m_u m_s}} = 0.103 \ GeV,$$

$$F_{D_s^+} = \frac{f_{D_s^+} m_{D_s^+}}{2\sqrt{m_c m_s}} = 0.343 \ GeV.$$

(8)

We are interested by a prediction for a constant of $D^+ \to \mu^+ \nu$ decay. In paper [2] for parameter $f_{D^+}$ the high bound is given only

$$f_{D^+} < 310 \ MeV.$$  

(9)

We have suspected that parameter $F_{M^+}$ depends only on mass of a meson. We search for this relation as

$$F_{M^+} = aM^2 + bM + c.$$  

(10)

With usage of (8) we discover

$$F_{M^+} = -0.0238M^2 + 0.2213M - 0.0005 \ (GeV).$$  

(11)
The behavior of parameter $F_{M^+}$ depending on mass is submitted in (Fig. 2).

Let us remark that a matrix element in (3) is half outside of a mass surface. And is approximated as on mass surface. The effect of departure from a mass surface is actuated in parameter of an annihilation $F_{M^+}$. The difference of these effects for pairs $u\bar{u}$ and $s\bar{s}$ is supposed miner.

From the formula (11) is received

$$F_{D^+} = 0.33 \text{ GeV}. \quad (12)$$

Then from expression

$$F_{D^+}m_{G^+} = F_{D^+}2\sqrt{m_cm_u} \quad (13)$$

we have

$$f_{D^+} = 0.23 \text{ GeV}, \quad (14)$$

that does not contradict (9).

The obtaining of the new data of constants of decay of pseudoscalar mesons will help to explain a phisical essens of parameter of an annihilation $F_{M^+}$. The introducing of this parameter encourages division of series stages weak and strong interactions in decaies of pseudoscalar mesons and refinement of mechanism of these decaies.
References

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Figure 1:
Figure 2:
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Fig. 1. The diagram of $\pi^+ \rightarrow \mu^+ \nu_\mu$ decay on a quark level.

Fig. 2. Relation of parameter of an annihilation $F_{M^+}$ to mass of a pseudoscalar meson.