S=I=0 PION PAIRS
IN THE $A(\pi, 2\pi)X$ REACTION

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Abstract

Recent experimental results show a large enhancement of the $2\pi$ emission on the scalar isoscalar channel, which had been predicted by some theoretical estimates. We present here a detailed calculation of the $A(\pi, 2\pi)X$ process incorporating a microscopic model for the elementary reaction in vacuum, pion-pion final state interaction in the nuclear medium, and other nuclear medium effects.

1 Introduction

The large, $A$ dependent enhancement of the cross section at low invariant masses of the dipion system found in Refs. [1, 2, 3, 4], could be a signal of very strong nuclear medium effects on correlated pion pairs in the $I = J = 0$ ("$\sigma$") channel. This strength accumulation had been predicted in Ref. [5], and similar results were obtained by Hatsuda et al. [6], where the enhancement at low invariant masses of the spectral function in the $\sigma$ channel appears as a consequence of the partial restoration of chiral symmetry in the nuclear medium.

In the last few years, several non perturbative models have been developed, which describe very successfully the $\pi\pi$ interaction in vacuum [7, 8]. When nuclear medium effects were included, some accumulation of strength was found close to the two pion threshold [9, 10, 11], which could be consistent with the experimental results. However, a full calculation of the $A(\pi, 2\pi)X$ process was needed in order to take into account all other nuclear effects and the detailed structure of the scattering amplitude.

A first attempt was presented in ref. [12]. In that work, a very simple model for the elementary $\pi N \rightarrow \pi\pi N$ amplitude was used, and the most important medium effects were included. The results showed a clear peak on the $M_{\pi\pi}$ distribution slightly above threshold, in good agreement with experimental
data for medium nuclei. However, the agreement was not as satisfactory for deuterium, where the $M_{\pi\pi}$ distribution was overestimated at threshold.

In this paper we will present a new study of the reaction, including a more realistic $\pi N \rightarrow \pi\pi N$ amplitude, which reproduces very well the cross section on hydrogen and deuterium, and we shall also consider some nuclear effects omitted previously, like the reduction of the incoming pion flux due to absorption and quasielastic scattering, which modifies drastically the effective density at which the reaction occurs.

2 $\pi\pi$ scattering in the scalar isoscalar channel

The $\pi\pi$ scattering amplitude is obtained solving the Bethe-Salpeter (BS) equation

$$T = V + V\mathcal{G}T.$$  

(1)

Fully detailed formulas and many technicalities can be found in Refs. [13, 11]. The $|\pi\pi, I = 0 >$ and $|K\bar{K}, I = 0 >$ states are included in the coupled channels calculation. The potential $V$ is obtained from the lowest order chiral lagrangians and $\mathcal{G}$ is the two meson propagator. A cutoff of 1 GeV is used to regularize the momentum integral appearing in the calculation of $\mathcal{G}$. The method guarantees both unitarity and consistency with chiral perturbation theory at low energies. This theoretical $\pi\pi$ scattering amplitude agrees well with experimental phase shifts and inelasticities from threshold up to energies around 1.2 GeV, and therefore provides a good starting point for our analysis.

To account for nuclear effects, the BS equation is modified by the substitution of vacuum meson propagators by the medium ones. Namely,

$$\tilde{\mathcal{G}} = i \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m^2 - \Pi(k)} \frac{1}{(P - k)^2 - m^2 - \Pi(P - k)},$$  

(2)

where $\Pi(k)$ is the meson selfenergy in nuclear matter, which accounts for the particle-hole and $\Delta$-hole excitations. The resulting $\pi\pi$ scattering amplitude shows a strong dependence on the baryon density, as can be appreciated in Fig. 1. Whereas at high energies (around 600 MeV) the imaginary part of $T_{\pi\pi}$ is reduced, there is a large enhancement around 300 MeV, where the CHAOS data show a well marked peak. Similar results have been found using quite different approaches. See for instance Ref. [10] and Fig. 3 of Ref. [14].

Finally, let us mention that other isospin channels either have a very small contribution to the $A(\pi, \pi\pi)X$ reaction at low energies, ($I = 1$), or show a weak interaction between the pions ($I = 2$), thus, for them, we will not consider the interaction between the pions.
Fig. 1. $\text{Im} \ T_{\pi\pi}$ in the $S=I=0$ scalar channel as a function of the invariant mass of the pion pair and the nuclear density. Labels correspond to the nucleons Fermi momentum.

3 Elementary $\pi N \rightarrow \pi\pi N$ reaction

In order to be able to compare detailed effects on the differential cross section a high quality description of the elementary cross section is clearly required. Fortunately, such models are readily available in the literature [15, 16, 17, 18, 19]. The model used in this paper follows closely that of ref. [15], although with some improvements to accommodate it to new resonance data in the PDG book [20], and to properly include the final state $\pi\pi$ interaction in the scalar isoscalar channel. The Lagrangians and coupling constants used can be found in the appendix of ref. [13].

Although the model is quite complex, and includes many mechanisms, it has no free parameters. Some coupling constants, related to the Roper resonance, have an uncertainty band associated to the uncertainties quoted in the PDG book. In those cases we have always taken the central values.

The results agree well with the experimental data for total and differential cross sections for all isospin channels [13], including of course the two-pions invariant mass distributions measured by CHAOS.
4 $\pi A \rightarrow \pi\pi X$ reaction

Many different nuclear effects modify the pion production cross sections. First, the initial and final pions undergo a strong distortion. This is implemented in the calculation following the methods of refs. [21, 22]. The incoming pion flux is reduced by absorption and quasielastic scattering. Both are very large because we are close to the $\Delta$ resonance peak. The pions scattered quasielastically are simply removed because they lose energy, and thus the probability to participate in a pion production process is drastically reduced. Distortion is less important for the final pions because of their lower energy. Only absorption has been considered for them. Second, the incoming pion collides with a nucleon which is moving in a Fermi sea, and the emitted nucleon is Pauli blocked, therefore only momenta above certain value are allowed to contribute, and this is implemented by means of a local density approximation. Third, the intermediate resonances ($\Delta$'s, $N^*$'s) also see their properties modified by the medium. Also, new reaction mechanisms like meson exchange currents, could play some role, although it has been shown in Ref. [3] that the reaction is essentially quasifree, and these possible mechanisms are not included in our calculation. Finally, the pion-pion final state interaction in the nuclear medium is considered. We select the part of the amplitude in which the two final pions are in the scalar isoscalar channel, and then we modify this part by incorporating the nuclear medium $\pi\pi$ interaction [13].

5 Results and discussion

As shown in Fig. 2, we find that our model describes fairly well the $\pi^+ \rightarrow \pi^+\pi^+$ reaction both in deuterium and in heavier nuclei. Details and comments on normalization can be found in ref. [13]. This gives us much confidence on our treatment of the nuclear medium effects. Note that they are the same for this and the $\pi^+\pi^-$ channel except for the the two-pions final state interaction, which is pure isospin 2 in $\pi^+\pi^+$, and mostly isospin 0 in the $\pi^+\pi^-$ case. However, although the $\pi^- \rightarrow \pi^+\pi^-$ reaction is well reproduced in deuterium, the model fails for this channel in heavier nuclei (see Fig. 3). Furthermore, we find the effect of in-medium final state interaction of the pions to be rather small. Similar results are obtained for heavier nuclei.

The main reason for the small enhancement found is the very small effective density (see Fig. 4) at which the pion production process occurs. As explained before, the initial pion has a large probability of being absorbed or quasielastically scattered. As a consequence, the flux reaching the center of the nucleus is small and the reaction occurs mainly at the surface. An esti-
mation of the average density gives $\rho_{av} = \rho_0/4$, considerably lower than those used in Ref. [12]. Imposing a high fixed average density we get a much larger although yet insufficient enhancement, and that at the price of destroying the nice agreement found for the $\pi^+\pi^+$ case, due to the too large Fermi motion of the nucleons. Better agreement with the nuclear data can be reached by selecting a simplified version of the model used for the elementary $\pi N \rightarrow \pi\pi N$ reaction, that overestimates the low mass region for the deuteron case.

There are several possibilities which could explain the large discrepancy between data and our model. One could be a much stronger pion-pion interaction in the medium in the scalar isoscalar channels, and some recent works point into that direction[23]. On the other hand, more trivial effects could be playing an important role. Probably, apart from concentrating on the peak appearing in medium and heavy nuclei, one should look more carefully to the very low values at low invariant masses of the cross section in deuteron and

Fig. 2. Two pion invariant mass distributions in the $\pi^+ + Ca \rightarrow \pi^+\pi^+X$ (upper box), and $\pi^+ + ^2H \rightarrow \pi^+\pi^+X$ (lower box) reactions. Experimental points are from ref. [2].
Fig. 3. Two pion invariant mass distributions in the $\pi^- + Ca \rightarrow \pi^+\pi^-X$ (upper box), and $\pi^- + ^2H \rightarrow \pi^+\pi^-X$ (lower box) reactions. Solid line, full calculation; dashed line, no medium effects in the FSI of the two pions. Experimental points are from ref. [2].

hydrogen. According to our model, this is due to destructive interference between large pieces of the amplitude. If some of these pieces are substantially modified in nuclei, the interference could disappear, and the spectral function would have some additional strength close to threshold.

Some of these questions could be answered soon. There are new experimental data being analyzed, which have measured a wider phase space than CHAOS, and also other CHAOS measurements studying the energy dependence (meaning the incoming pion energy) of the peak. In our model, this is important because the interference effects are smaller at lower energies.

Finally, we think that lepton induced reactions, which are free from the initial state interaction and would allow the pion production to happen at higher densities could be a better probe. In particular, $(\gamma, \pi\pi)$ is currently under theoretical investigation.
Fig. 4. Solid line: Profile function showing the probability of a pion production event as a function of the radius for Calcium (arbitrary units). Dashed line: nuclear density.

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