Controlling Physical Properties on Interfaces Using Parametrised Level Set Methods and Extended Finite Element Method

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The stress field information states a main point of interest, when regarding structural optimisation of bimaterial structures. The introduction of stress criteria along the volume is crucial in the development of new designs. In the case of bimaterials, the stress field along the interface deserves additional attention e.g. to prevent delamination. Tracing the interface through shape optimisation in CAD-based methods is rather expensive due to the high cost in remeshing techniques. Therefore, level set methods with fixed background meshes are used as in [1].

In this work an approach for controlling strains and stresses on the interface in a framework of shape optimisation is introduced. The geometry description is managed by parametrised level set functions and a sub-meshing technique is coupled with the extended finite element method. The parametrisation with superellipses allows to reduce the number of design variables to a minimum of six variables per introduced ellipse while holding up a sufficient precision in the geometry description (see [2]). Moreover, it simplifies the shape derivatives as it provides an implicit description for the moving interfaces. The sub-meshing technique makes it possible to keep existing strategies from homogeneous structures and to transform them on a discontinuous material using enriched shape functions provided by the standard extended finite element method (see [3]). Shape sensitivities are evaluated on the sub-elements and extrapolated to the interface introducing pseudo nodes. The sensitivity information of the stress field along these pseudo nodes can be used in the framework of stress minimisation as well as for a side condition in a volume minimisation setup.

1 Introduction

The development of new materials in the field of engineering requires the knowledge of stress states within the structure. The evaluation and not least the control of those is a main goal in the process of finding new designs. Considering multimaterial structures, the discontinuities between different domains deserve special investigation because in many cases these interfaces are the weak point in the design and are at high risk of delamination or plastic deformations. This work introduces a method for controlling stresses and strains on multimaterial and material-void interfaces within a framework of structural optimisation.

2 A Modified XFEM Approach

The extended finite element method (XFEM) can represent unsteady field quantities in the domain without fitting the discretisation mesh to these discontinuities. This is realised by an enhancement which fits the desired behaviour, i.e. unsteadiness in the quantity itself or its first derivative. For structural mechanics, the enriched displacements can be approximated within the element as

\[ u^h(x) = \sum_{i \in I} N_i(x) u_i + \sum_{j=1}^{N} \sum_{i \in I_j} N_i^*(x) \psi_j(x) a_j^i, \]  

(1)

with the standard shape functions \( N_i \) and an enriched shape function \( M_i^j = N_i \psi_j \) which is build up from the level set function. In this work, the level set function is a superellipse. That provides six geometric parameters which will be the design variables later on. The modified approach used in this work divides the element \( e \) (later on referred to as master element) into sub elements \( ej \). Within these, standard techniques from the finite element method can be used. The displacements of the emerging sub nodes \( u_{ej} \) are extrapolated to the master nodes \( u_e = [u_e, a_e] \) by the extended shape functions from Eq.1. The stiffness matrix can then be derived from the sum over all sub elements \( ne,j \) as

\[ k_e = \frac{\partial \Pi^j}{\partial u_e, \partial u_e} = \sum_{ne,j} \frac{\partial \Pi^j_{ne,j}}{\partial u_e, \partial u_e} = \sum_{ne,j} \frac{\partial u_{ej}}{\partial u_e} \frac{\partial \Pi^j_{ne,j}}{\partial u_{ej}} \frac{\partial u_{ej}}{\partial u_e} = \sum_{ne,j} T^\top k_{ej} T. \]  

(2)

We can identify the middle term as a standard FEM stiffness matrix and define the outer terms as a mapping function derived from Eq. 1. The sub nodal displacement is interpolated within the element \( u_{ej} = u^h(x_{ej}) \). The stress field along the interface can then be computed from the gauss point stresses of sub elements lying at the interface.

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3 Structural Optimisation

Two concepts of stress control within the framework of structural optimisation are proposed here. One will use it as a constraint while minimizing a certain objective function, e.g. the volume. The other one will set the stress along the interface as the objective function while holding up to a certain constraint. The latter one requires a scalar measurement of the interface stresses which is on a first attempt chosen as the p-norm in this work. The two formulations for the optimisation problem are stated below.

\[
\begin{align*}
\min_{s \in \mathcal{D}} & \quad V(s), \\
\text{s.t.} \quad & \sigma(s) \leq \sigma_{\text{max}}
\end{align*}
\]

(3)

\[
\begin{align*}
\min_{s \in \mathcal{D}} & \quad J(s), \quad \text{with } J(s) = \left[ \int_{\partial \Omega} \sigma(s)^p \, dx(s) \right]^{1/p}, \\
\text{s.t.} \quad & V_{\text{incl}}(s) \leq V_{\text{max}}
\end{align*}
\]

(4)

The design variables \( s \) are stated by the parameters of the level set function \( \phi(s) \). For both optimisation techniques the sensitivity of the stresses with respect to the design is required. The variation of the stresses reads

\[
\delta_s \sigma_{ip} = \delta u_{ej} \frac{\partial \sigma_{ip}}{\partial u} \frac{du}{ds} + \delta x_{ej} \frac{\partial \sigma_{ip}}{\partial s} = A_u TS + A_x W = \begin{bmatrix} A_u^T & A_x^T \end{bmatrix} \begin{bmatrix} T \\ S \\ W \end{bmatrix}.
\]

(5)

The variation of stresses w.r.t. displacement and coordinates is known from [4]. The partial derivatives of sub nodal displacements can be rewritten as the transformation matrix \( T \). The velocity field of sub nodal coordinates is defined as the matrix \( W \) and the total derivative of field quantities w.r.t. design is known as the sensitivity matrix \( S \). It is noted that the variation of stresses can be strictly split up into sub element terms on the left-hand side and mapping terms on the right-hand side. This makes it possible to utilise existing techniques for non-cut elements in the XFEM framework, which is the main benefit of this modified technique.

4 Example

In this example a plate with a soft inclusion is loaded under uniaxial tension. The aim is to minimise the 2-norm of the Von Mises stresses along the interface, while holding the inclusion volume at least at its initial value, see Eq.4.

![Fig. 1: Objective and Constraint](image1)

![Fig. 2: Initial Von Mises Stress](image2)

![Fig. 3: Final Von Mises Stress](image3)

The stress along the interface converges to a minimum while the constraint is satisfied, see Fig. 1. The final stress around the interface is much lower than the initial.

References

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