Indirect Constraints on the Triple Gauge Boson Couplings from

\[ Z \rightarrow b\bar{b} \] Partial Width: An Update.

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(March 27, 2022)

Abstract

We update the indirect bounds on anomalous triple gauge couplings coming from the non–universal one–loop contributions to the \( Z \rightarrow b\bar{b} \) width. These bounds, which are independent of the Higgs boson mass, are in agreement with the standard model predictions for the gauge boson self-couplings since the present value of \( R_b \) agrees fairly well with the theoretical estimates. Moreover, these indirect constraints on \( \Delta g_1^Z \) and \( g_5^Z \) are more stringent than the present direct bounds on these quantities, while the indirect limit on \( \lambda_Z \) is weaker than the available experimental data.
I. INTRODUCTION

The predictions of the standard model of electroweak interactions (SM) agree extremely well with the available experimental data [1]. The LEP I and SLC collaborations studied in great detail the couplings of the $Z$ to fermions, validating the $SU(2)_L \times U(1)_Y$ invariant interactions between fermions and gauge bosons at the level of 1%. At LEP II and at Tevatron we are starting to probe the triple couplings among the weak gauge bosons with a precision of the order of 10% [2]. This is an important test of the SM since these couplings are completely determined by the non-abelian gauge symmetry of the model.

The $Z \to b\bar{b}$ width receives non-universal one-loop corrections due to the presence of heavy particles running in the loop [3], and it is an important source of information on new physics beyond the standard model. After the $R_b$ crisis has been solved, it is important to revisit all the phenomenological analyses that are strongly based on this quantity. In particular, the precise measurement of the $Z \to b\bar{b}$ width is able to constrain possible deviations of the triple gauge–boson couplings with an accuracy that, in some cases, are even better than the direct measurements of these interactions. In this letter, we make an update on our previous analysis taking into account recent experimental data on the electroweak parameters. We present our results in terms of effective Lagrangians for the anomalous gauge interactions both for linear as well as non-linear realization of the $SU(2)_L \times U(1)_Y$ symmetry [4,5].

The non-universal contributions to the $Zb\bar{b}$ couplings have been parametrized, in a model independent way, in terms of the parameter $\epsilon_b$ [6], defined as

$$\epsilon_b \equiv \frac{g_A^b}{g_A^\ell} - 1 ,$$

(1)

where $g_A^b \ (g_A^\ell)$ is the axial coupling of the $Z$ to $b\bar{b} \ (\ell\bar{\ell})$ pairs. An important feature of this parameter is that its SM value is basically independent of the Higgs boson mass. Therefore, the bounds withdrew from it have less uncertainties. The anomalous contribution to the $Zf\bar{f}$ vertex can be written in terms of the form factor $F(m_j)$ as,
\[ \Gamma_{\text{anom}}^\mu(Z \bar{f} f) = i \frac{e}{4s_W c_W} \sum_i V_{if} V_{ij}^\dagger F(m_j) \gamma^\mu (1 - \gamma^5), \]  

where \( V_{if} \) is the Cabibbo–Kobayashi–Maskawa mixing matrix in the case of quarks and \( V_{if} = \delta_{if} \) for leptons. This amplitude is the same for all external fermions but the \( b \) quark when we neglect the mixings \( V_{tb(s)} \) and all the internal fermions masses but \( m_{\text{top}} \). Therefore, \( \epsilon_b \) takes the form

\[ \epsilon_b = \Delta F \equiv F(m_{\text{top}}) - F(0). \]

The contribution of anomalous \( W^+ W^- Z \) couplings to the \( Z \to b \bar{b} \) partial width was evaluated previously in Refs. [7,8]. In this process, the non–universal effect are enhanced, as expected, by powers of the top quark mass due to the virtual top quark running in the loop vertices corrections [3].

**II. EFFECTIVE LAGRANGIANS**

The usual Lorentz invariant and CP conserving parametrization of the \( W^+ W^- V \) vertex, with \( V = \gamma \) or \( Z \), is given by the effective Lagrangian [3],

\[ \mathcal{L}_{\text{eff}}^{\text{WWV}} = -ig_{\text{WWV}} \left[ g_1 V(W_{\mu \nu}^+ W_{-\mu}^- - W_{\mu \nu}^- W_{-\mu}^+) V_\nu \right. 
\[ \left. + \kappa V_{W^+ W^-} V_{\mu \nu}^+ W_{-\nu}^+ W_{-\mu}^+ + \frac{\lambda V}{M_W^2} W_{\mu}^+ W_{\nu}^- W_{\rho}^- V_{\mu \rho}^+ - ig_5 \epsilon^{\mu \rho \sigma} (W_{\mu}^+ \partial_\rho W_{\sigma}^- - W_{\nu}^- \partial_\rho W_{\mu}^+) V_\sigma \right] \]  

where \( V_{\mu \nu} = \partial_\mu V_\nu - \partial_\nu V_\mu \), \( g_{\text{WW}} = e \), and \( g_{\text{WWZ}} = e c_W / s_W \), with \( s_W(c_W) = \sin(\cos) \theta_W \). The first three terms in Eq. (3) are C and P invariant while the last one violates both C and P.

Since the standard model is consistent with the available experimental data, it is natural to parametrize the anomalous triple gauge boson couplings in terms of an effective Lagrangian which exhibits the \( SU(2)_L \times U(1)_Y \) gauge invariance. The particular way this symmetry is realized depends on the particle content at low energies. If a light Higgs boson is present, the symmetry can be realized linearly [4,10], and the leading effects of new interactions are described by eleven dimension–6 operators \( \mathcal{O}_i \),
\[ L_{\text{linear}}^{\text{eff}} = \sum_i \frac{f_i}{\Lambda^2} \mathcal{O}_i , \]  

(4)

at energies below the new physics scale \( \Lambda \). Three of these operators \( [4] \), namely,

\[ \mathcal{O}_B = (D_\mu \Phi) ^\dagger \bar{B}^{\mu\nu} (D_\nu \Phi) , \]

\[ \mathcal{O}_W = (D_\mu \Phi) ^\dagger \bar{W}^{\mu\nu} (D_\nu \Phi) , \]

\[ \mathcal{O}_{WW} = \text{Tr} \left[ \hat{W}_{\mu\rho} \hat{W}^{\nu\rho} \hat{W}^{\mu}_\rho \right] , \]

(5)

modify the triple gauge boson couplings without affecting the gauge boson two–point functions at tree level (“blind” operators). In our notation, \( \hat{B}_{\mu\nu} = i (g'/2) B_{\mu\nu} \) and \( \hat{W}_{\mu\nu} = i (g/2) \sigma^a W^a_{\mu\nu} \) with \( B_{\mu\nu} \) and \( W^a_{\mu\nu} \) being the \( U(1)_Y \) and \( SU(2)_L \) full field strengths and \( \sigma^a \) representing the Pauli matrices. In this framework, it is expected that \( g_5^Z \) should be suppressed since it is related to a dimension 8 operator \( [7] \).

The anomalous couplings of the parametrization (3) are related to the coefficients of the linearly realized effective Lagrangian by

\[ \Delta g_1^Z = f_W \frac{m_Z^2}{2\Lambda^2} , \]

(6)

\[ \Delta \kappa_Z = [f_W - s_W^2 (f_B + f_W)] \frac{m_Z^2}{2\Lambda^2} , \]

(7)

\[ \lambda_Z = f_{WWW} \frac{3m_W^2 g^2}{2\Lambda^2} , \]

(8)

where \( \Delta \kappa_V \equiv \kappa_V - 1, \Delta g_1^Z \equiv g_1^Z - 1, \) and \( \lambda_V \) are all zero in the SM at tree level. It is interesting to notice that these effective operators lead to the following relation between the coefficients of Lagrangian (3):

\[ \Delta \kappa_\gamma = \frac{s_W^2}{s_W^2} \left( \Delta g_1^Z - \Delta \kappa_Z \right) , \]

(9)

\[ \lambda_\gamma = \lambda_Z . \]

(10)

In the scenario where the \( SU(2)_L \times U(1)_Y \) gauge symmetry is non–linearly realized \([3]\), a chiral Lagrangian can be constructed from the dimensionless unitary matrix \( U \) that belongs to the \( (2, 2) \) representation of the group \( SU(2)_L \times SU(2)_C \),
\[ \mathcal{L}_{\text{non-lin.}} = \sum_i \alpha_i \mathcal{O}_i . \quad (11) \]

The “blind” directions that appear in the lowest order of the chiral expansion are described by the Lagrangians \[5\],

\[ \mathcal{O}_2 = \frac{ig'}{2} B^{\mu\nu} \text{Tr} \left( T \left( (D_\mu U)^\dagger, (D_\nu U)^\dagger \right) \right) , \]

\[ \mathcal{O}_3 = \frac{ig}{2} \text{Tr} \left( W^{a\mu\nu} \sigma^a T \left( (D_\mu U)^\dagger, (D_\nu U)^\dagger \right) \right) , \]

\[ \mathcal{O}_9 = \frac{ig}{4} \text{Tr} \left( TW^{a\mu\nu} \sigma^a \right) \text{Tr} \left( T \left( (D_\mu U)^\dagger, (D_\nu U)^\dagger \right) \right) , \]

\[ \mathcal{O}_{11} = \frac{g}{2} \epsilon^{\mu\nu\lambda\rho} \text{Tr} \left( T(D_\mu U)^\dagger \right) \text{Tr} \left( (D_\nu U)^\dagger W^a_{\lambda\rho} \sigma^a \right) . \]

where the custodial symmetry breaking operator \( T \equiv U_3 \) and the covariant derivative of \( U \) is defined as \( D_\mu U \equiv \partial_\mu U + ig/2 \sigma^a W^a_\mu U - ig'/2 U_3 B_\mu \).

The contribution of the above chiral operators can be expressed in terms of the standard parametrization as \[5\]

\[ \Delta g_1^Z = \alpha_3 \frac{g^2}{c_W^2} , \quad (13) \]

\[ \Delta \kappa_Z = \left[ c_W^2 (\alpha_3 + \alpha_9) - s_W^2 \alpha_2 \right] \frac{g^2}{c_W^2} , \quad (14) \]

\[ g_5^Z = \alpha_1 \frac{g^2}{c_W^2} . \quad (15) \]

III. RESULTS AND CONCLUSIONS

The contribution of the anomalous couplings \[3\] to \( \epsilon_b \) can be written as

\[ \epsilon_b^{\text{ano}} \equiv \epsilon_b - \epsilon_b^{\text{SM}} = \Delta \kappa_Z \Delta F_{\kappa_Z} + \Delta g_1^Z \Delta F_{g_1^Z} + \lambda_Z \Delta F_{\lambda_Z} + g_5^Z \Delta F_{g_5^Z} \quad (16) \]

where the form factors \( \Delta F_{\kappa_Z}, \Delta F_{g_1^Z}, \Delta F_{\lambda_Z}, \) and \( \Delta F_{\lambda_Z} \) were presented elsewhere \[3\].

In order to obtain our numerical results we used the most recent data for the electroweak parameters and masses \[4\]: \( \alpha(M_Z) = 1/128.896, s_W^2 = 0.2321, M_Z = 91.1867 \text{ GeV}, M_W = \)
80.37 GeV, and \( m_{\text{top}} = 173.8 \) GeV. Substituting these parameters into the expressions for the form factor \[7\], we obtain

\[
e_{b}^{\text{ano}} = \Delta \kappa_{Z} \left[ -3.1 \times 10^{-3} \log \left( \frac{\Lambda^{2}}{M_{W}^{2}} \right) \right] + \Delta g_{1}^{Z} \left[ -1.4 \times 10^{-2} \log \left( \frac{\Lambda^{2}}{M_{W}^{2}} \right) \right] + \lambda_{Z} \left( -2.6 \times 10^{-3} \right) + g_{5}^{Z} \left( -8.3 \times 10^{-3} \right) .
\]

(17)

The form factors \( \Delta F_{\lambda_{Z}} \) and \( \Delta F_{g_{5}^{Z}} \) are independent of the cutoff \( \Lambda \), while the form factors \( \Delta F_{\kappa_{Z}} \) and \( \Delta F_{g_{1}^{Z}} \) are ultra-violet divergent which indicate a logarithmic dependence in \( \Lambda \). Since the \( \log \Lambda \) terms are dominant in these form factors, we dropped the constant term from their expressions.

The SM prediction for \( \epsilon_{b} \) is practically independent of the Higgs boson mass, and for \( m_{\text{top}} = 173.8 \) GeV its value is \( \epsilon_{b}^{\text{SM}} = -6.51 \times 10^{-3} \) \[11\]. On the other hand, a global fit to the available data leads to \( \epsilon_{b}^{\text{exp}} = (-3.9 \pm 2.1) \times 10^{-3} \) \[11\]. The constraints on the couplings of the effective Lagrangian \[3\] can be easily obtained using the SM and experimental values of \( \epsilon_{b} \) and expression \( (17) \). We present in Table \[4\] our 1-\( \sigma \) limits on \( \Delta g_{1}^{Z} \), \( \Delta \kappa_{Z} \), \( \lambda_{Z} \), and \( g_{5}^{Z} \), assuming that only one coupling at a time is allowed to deviate from zero and taking \( \Lambda = 1 \) TeV.

At this point it is interesting to compare our indirect bounds with the present direct limits on the anomalous triple gauge boson couplings. Taking into account both LEP and DØ data, the allowed range of the parameters \( \Delta \kappa_{\gamma} \), \( \Delta g_{1}^{Z} \), and \( \lambda_{\gamma} \) are \[1\]

\[
\Delta \kappa_{\gamma} = 0.13 \pm 0.14 , \\
\Delta g_{1}^{Z} = 0.00 \pm 0.08 , \\
\lambda_{\gamma} = -0.03 \pm 0.07 .
\]

Therefore, our indirect bounds on \( \Delta g_{1}^{Z} \) is a factor of 4 more stringent than the present direct limit. Moreover, the above experimental results assumed the \( SU(2) \) invariant relations \( (9) \) and \( (10) \). Using this hypothesis, our indirect constraint on \( \lambda_{\gamma} (= \lambda_{Z}) \) turns out to be a factor of 15 looser than the available direct bound.
The relations (6)–(8) and (13)–(15) allow us to derive bounds on the “blind” operators (5) and (12). We also present our constraints on these couplings in Table I, where we assumed $\Lambda = 1$ TeV and that only one coupling is non-vanishing at a time. For the sake of comparison we show here the combined LEP limits on some of these operators

$$\frac{M_W^2}{2\Lambda^2} f_W = -0.05 \pm 0.06 ,$$

$$\frac{M_W^2}{2\Lambda^2} f_B = -0.04^{+0.33}_{-0.24} ,$$

$$\frac{3M_W^2 g^2}{2\Lambda^2} f_{WWW} = -0.09^{+0.13}_{-0.12} .$$

As we can see, the indirect limits on $f_W$ and $f_B$ are of the same order of the experimental ones while the direct bound on $f_{WWW}$ is much better. It is interesting to notice that the indirect limits of operators, which lead to divergent one-loop contributions to the vertex and consequently are enhanced by factors $\log(\Lambda/M_W)$, are the only ones competitive with the present experimental results.

It is also possible to constrain the triple gauge boson couplings via the analysis of rare $B$ and $K$ decays [12,13]. Recently, Burdman has obtained the $1-$σ limits $|\Delta g^Z_1| < 0.10$ and $|\Delta \kappa_\gamma| < 0.20$, for a new physics scale $\Lambda = 2$ TeV. In order to compare our results with this work we derive the $1-$σ bounds for $\Lambda = 2$ TeV:

$$-0.051 < \Delta g^Z_1 < -0.0055 ,$$

$$-0.24 < \Delta \kappa_\gamma < -0.026 ,$$

when just one anomalous coupling is non-vanishing in each analyses. Taking into account the relation given in Eq. (8), we can also derive an indirect constraint on $\Delta \kappa_\gamma$ when just $\Delta g^Z_1$ is different from zero, i.e. $-0.17 < \Delta \kappa_\gamma < 0.018$. This shows that the limits obtained from the analysis of the data on $Z \to b\bar{b}$ is in some cases, more than one order of magnitude better than the one coming from the $B$ and $K$ decays.
ACKNOWLEDGMENTS

M.C. G-G is very grateful to the Instituto de Física Teórica da Universidade Estadual Paulista for their kind hospitality. This work was supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP), and by Programa de Apoio a Núcleos de Excelência (PRONEX).
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TABLE I. One-σ allowed regions of the anomalous triple gauge–boson couplings in different parametrizations, assuming Λ = 1 TeV.

|                  | Eq. (3)         | Eq. (5)         | Eq. (12)        |
|------------------|-----------------|-----------------|-----------------|
| Δg^Z_i           | −0.036 ± 0.029  | 0.56 ± 0.45     | α_2 1.3 ± 1.1   |
| Δκ_Z             | −0.17 ± 0.13    | −0.024 ± 0.019  | α_3 −0.057 ± 0.046 |
| λ_{γ,Z}          | −1.0 ± 0.81     | −1.0 ± 0.81     | α_9 −0.40 ± 0.32 |
| g^Z_S            | −0.31 ± 0.23    |                 | α_{11} −0.57 ± 0.46 |