A GLOBAL DESCRIPTOR OF SPATIAL PATTERN INTERACTION IN THE GALAXY DISTRIBUTION

MARTIN KERSCHER,1 MARÍA JESÚS PONS-BORDERÍA,2 JENS SCHMALZING,1,3 ROBERTO TRASARTI-BATTISTONI,1,4 THOMAS BUCHERT,1 VICENT J. MARTÍNEZ,5 AND RICCARDO VALDARNINI6

Received 1997 December 5; accepted 1998 October 15

ABSTRACT

We present the function $J$ as a morphological descriptor for point patterns formed by the distribution of galaxies in the universe. This function was recently introduced in the field of spatial statistics, and is based on the nearest-neighbor distribution and the void probability function. The $J$ descriptor allows us to distinguish clustered (i.e., correlated) from regular (i.e., anticorrelated) point distributions. We outline the theoretical foundations of the method, perform tests with a Matern cluster process as an idealized model of galaxy clustering, and apply the descriptor to galaxies and loose groups in the Perseus-Pisces Survey. A comparison with mock samples extracted from a mixed dark matter simulation shows that the $J$ descriptor can profitably be used to constrain (or in this case reject) viable models of cosmic structure formation.

Subject headings: galaxies: clusters: general — large-scale structure of universe — methods: statistical

1. INTRODUCTION

Three-dimensional patterns formed by the spatial distribution of galaxies in the universe have already been described and quantified by various methods: correlation functions, counts-in-cells (Peebles 1993), the void probability function (White 1979), the genus (Melott 1990), the multifractal spectrum (Martínez et al. 1990), skewness and kurtosis (Gaztañaga & Frieman 1994), and Minkowski functionals (Mecke, Buchert, & Wagner 1994; Schmalzing & Buchert 1997). Some of these descriptors are complementary, and suggest a physical interpretation of cosmic patterns by emphasizing different spatial features of the galaxy distribution.

The treatment of the galaxy distribution as a realization of a spatial point process promises useful insights through the application of methods from the field of spatial statistics. The forthcoming three-dimensional galaxy catalogs, offering more than half a million redshifts, further motivate the development of new statistical techniques.

In this article we want to reinforce a morphological measure for the study of the distribution of galaxies, the $J(r)$ function, which has recently been introduced into the field of spatial statistics by van Lieshout & Baddeley (1996), and which is related to the nearest neighbor distribution $G(r)$ and the spherical contact distribution $F(r)$. Indeed, the $J(r)$ function is equal to the first conditional correlation function (Strattonovich 1963; White 1979), and was used by Sharp (1981) to test a hierarchical Ansatz for $n$-point correlation functions. We will focus on different features of the $J(r)$ function, showing its discriminative power as a measure of the strength of clustering.

Our article is organized as follows. In § 2 we present the distribution functions $F$ and $G$ and show how the $J$ function is constructed. A Matern cluster process is considered as a simple example of a clustering point distribution. In § 3 we study the clustering properties of a galaxy sample and of galaxies in groups extracted from the Perseus-Pisces redshift survey (PPS). We compare the observed galaxy distribution with mock samples extracted from a mixed dark matter (MDM) simulation in § 4. We summarize and conclude in § 5.

2. THE $J$ FUNCTION

In the theory of spatial point processes, the distribution of a point’s distance to its nearest neighbor is a common tool for analyzing point patterns (Stoyan, Kendall, & Mecke 1995). We consider the redshift space coordinates $\{x_i\}_{i=1}^N \in \mathbb{R}^3$ of $N$ galaxies inside a region $D \subseteq \mathbb{R}^3$ as a realization of the point process describing the spatial distribution of galaxies in the universe.

The nearest neighbor distribution $G(r)$ is the distribution function of the distance $r$ of a point of the process to the nearest other point of the process. Similarly, the spherical contact distribution $F(r)$ is the distribution function of the distance $r$ of an arbitrary point in $\mathbb{R}^3$ to the nearest point of the process. $F(r)$ is equal to the volume fraction occupied by the set of all points in $D$ that are closer than $r$ to a point of the process. Hence, $F(r)$ coincides with the volume density of the first Minkowski functional (Mecke et al. 1994; Kerscher et al. 1997), and is related to the void probability function $P_\theta(r)$ via $F(r) = 1 - P_\theta(r)$.

For a homogeneous Poisson process, we have

$$F_\theta(r) = 1 - \exp \left(-\frac{4\pi}{3} r^3 n \right) = G_\theta(r),$$

(1)

where $n$ is the number density.

Boundary-corrected estimators for both the nearest neighbor distribution and the spherical contact distribution used in our studies are provided by minus (reduced sample)
tractable behavior of want to test it on a model with nontrivial yet analytically sphere of radius allowed. of such a process. Note that overlapping clusters are pretered as a relic of the uniform distribution of the cluster centers. In three dimensions, estimators (Stoyan et al. 1995; see also detailed in Kerscher et al. 1998).

In a recent paper, van Lieshout & Baddeley (1996) have suggested using the quotient

\[ J(r) = \frac{1 - G(r)}{1 - F(r)} \]  

(2)
to characterize a point process; in that way, the surroundings of a point belonging to the process and the neighborhood of a random point are compared. They consider several point process models and provide limits and exact results on \( J(r) \) (see also § 2.1).

If the process under consideration is clustered, an arbitrary point will usually lie farther away from a point of the process than in the case of a Poisson process. Hence, clustering is indicated by \( F(r) < F_{\rho}(r) \). Consistently, \( G(r) > G_{\rho}(r) \), since clustered points tend to lie closer to their nearest neighbors than randomly distributed points. So, for a clustered point distribution, \( J(r) < 1 \).

In the case of anticorrelated, “regular” structures, the situation is the opposite: on average, a point of a regular process is farther away from the nearest other point of the process, so \( G(r) < G_{\rho}(r) \), and a random point is closer to a point of the process, resulting in \( F(r) > F_{\rho}(r) \). Therefore, regular structures are indicated by \( J(r) > 1 \).

For a homogeneous Poisson process, we obtain \( J_{\rho}(r) = 1 \), separating regular from clustering structures.

### 2.1. The Matérn Cluster Process

Before attempting to apply \( J(r) \) to galaxy samples, we want to test it on a model with nontrivial yet analytically tractable behavior of \( J(r) \).

In order to describe the clustering of galaxies, Neyman & Scott (1958) suggested a class of point processes that was subsequently named after them. We concentrate on a subclass called Matérn cluster processes. These are constructed by first uniformly distributing \( M \) cluster centers. Around each cluster center, which is itself not included in the final point distribution, \( m \) galaxies are placed randomly within a sphere of radius \( R \), where \( m \) is a Poisson-distributed random variable with mean \( \mu \). In Figure 1 we show a sketch of such a process. Note that overlapping clusters are allowed.

For a Matérn cluster process, van Lieshout & Baddeley (1996) proved that \( J(r) \) is monotonically decreasing from 1 at \( r = 0 \) and attains a constant value for \( r > 2R \), where \( R \) is the radius of a cluster. This constant value can be interpreted as a relic of the uniform distribution of the cluster centers. In three dimensions,

\[ J_{\text{M}}(r) = \begin{cases} \frac{1}{\text{Vol} (B_r)} \int_{B_r} e^{-\mu V(x, r, R)} d^3x & \text{for } 0 \leq r \leq 2R, \\ \exp(-\mu) & \text{for } r > 2R, \end{cases} \]

(3)

where

\[ V(x, r, R) = \frac{\text{Vol} (B_m(x) \cap B_r)}{\text{Vol} (B_r)} \]

(4)
denotes the ratio of the volume of the intersection of two balls to the volume of a single ball. Here \( B_m(x) \) is a ball of radius \( r \) centered at the point \( x \), while \( B_r \) is a ball of radius \( R \) centered at the origin. This quantity can be calculated from basic geometric considerations, in both two (Stoyan & Stoyan 1994) and three dimensions, where the result is

\[ V(x, r, R) = \begin{cases} c_3 x^3 + c_1 x + c_0 + c_{-1} x^{-1} & \text{for } 0 \leq r < R \text{ and } R - r < x < R, \\ c_3 x^3 & \text{for } 0 \leq r < 2R \text{ and } r - x < r, \\ c_3 x^3 + c_1 x + c_0 + c_{-1} x^{-1} & \text{for } 0 \leq x \leq R, \\ 1 & \text{for } R \leq r \leq 2R \text{ and } 0 \leq x \leq r - R. \end{cases} \]

(5)

where \( x = |x| \) and

\[ c_3 = \frac{1}{16 R^3}, \quad c_1 = -\frac{3}{8} \left( \frac{r^2}{R^3} + \frac{1}{R} \right), \quad c_0 = \frac{1}{2} \left( \frac{r^3}{R^3} + 1 \right), \quad c_{-1} = \frac{3}{16} \left( \frac{2r^3}{R} - \frac{r^4}{R^3} - R \right). \]

(6)

In Figure 2, we show \( J_{\text{M}}(r) \) for \( R = 1.5 \ h^{-1} \text{ Mpc} \) and several values of \( \mu \); this represents typical situations of galaxy clustering. Obviously, \( J(r) \) discriminates between the varying richness classes of the Matérn cluster processes.

---

7 Throughout this article, distances are given in \( h^{-1} \text{ Mpc} \), where \( h \) denotes the value of the Hubble parameter measured in units of 100 km s\(^{-1}\) Mpc\(^{-1}\).
3. GALAXY SAMPLES

In this section, we want to go one step farther by applying $J(r)$ to catalogs of galaxies and groups of galaxies, and comparing them with a Matérn cluster process.

3.1. Description of the PPS Galaxy and Group Samples

The PPS database was compiled in the last decade (Giovanelli & Haynes 1991; Wegner, Hanyes, & Giovanelli 1993). The full redshift survey is magnitude limited down to a Zwicky magnitude of $m_Z \leq 15.7$ (Zwicky et al. 1961–1968), and is at least 95% complete to $m_Z \leq 15.5$ (see Fig. 1 of Iovino et al. 1993). We extract a volume-limited subsample with $M_Z \leq -19$ and radius $79$ $h^{-1}$ Mpc, confined to $-1^{50} \leq z \leq 2^{00}$ and $0^\circ \leq \delta \leq 40^\circ$, i.e., a solid angle of 0.76 sr. Redshifts are corrected for the motion of the Sun relative to the rest frame of the cosmic microwave background (CMB) as in Peebles (1993), and we also correct Zwicky magnitudes for interstellar extinction as in Burstein & Heiles (1978). The final volume-limited sample PPS79 contains 817 galaxies.

To find groups, we use the redshift-space friends-of-friends algorithm of Huchra & Geller (1982), suitably adapted to our case. This is a truncated percolation algorithm with two independent linking parameters, $D$ and $V$. Briefly, two galaxies are “friends” if their transverse and radial separations, $r_{ij}^T$ and $r_{ij}^R$, satisfy $r_{ij}^T \leq D$ and $r_{ij}^R \leq V/H_0$, respectively. Friendship is transitive, and a set of three or more friends is called a loose group of galaxies.

Usually, loose groups are identified in magnitude-limited samples. Here we consider only volume-limited samples. Values of $D = 0.52$ $h^{-1}$ Mpc and $V = 600$ km s$^{-1}$ give very good agreement of global properties (e.g., the total fraction of galaxies in groups, the ratio of groups to galaxies, or the median velocity dispersion) between our volume-limited samples. Here we consider only volume-limited samples. With $J(r)$ we measure the strength of clustering, which is emphasized when we consider galaxies in groups only and less pronounced when we look at the whole sample with field galaxies included. Similarly, the value of $J(r)$ for the subsampled PPS79 is higher than $J(r)$ for the whole PPS79, because random subsampling (thinning) tends to increase $J(r)$ toward the Poisson value.

3.2. $J(r)$ for the Galaxy Samples

We calculated $J(r)$ for all galaxies from the PPS79 sample; the results are shown in Figure 3. With $J(r)$ lying outside the area occupied by realizations of a Poisson process, one can clearly see that galaxies are strongly clustered—not a particularly surprising result. In § 4.3, somewhat more interesting comparisons with galaxy mock samples extracted from $N$-body simulations are performed.

Figure 4 displays the results for grouped galaxies. Since each group contains at least three members, the nearest neighbor of a grouped galaxy is certainly found within the largest link length used in the friends-of-friends procedure. Hence, we observe $G(r) = 1$ and subsequently $J(r) = 0$ for $r > 5.6$ $h^{-1}$ Mpc in the grouped sample. $J(r)$ is in general not invariant under changes in the number density (van Lieshout & Baddeley 1996). To compare the $J(r)$ for grouped galaxies with the $J(r)$ for all galaxies, we subsample the denser PPS79. $J(r)$ is calculated from 50 subsamples of 230 galaxies randomly selected from the whole PPS79 sample. With $J(r)$ we measure the strength of clustering, which is emphasized when we consider galaxies in groups only and less pronounced when we look at the whole sample with field galaxies included. Similarly, the value of $J(r)$ for the subsampled PPS79 is higher than $J(r)$ for the whole PPS79, because random subsampling (thinning) tends to increase $J(r)$ toward the Poisson value.
The centers of loose groups show a strong correlation themselves (Trasarti-Battistoni, Invernizzi, & Bonometto 1997); therefore, a Matérn cluster process can only serve as a rough approximation to the true distribution of galaxies in groups. Despite this, a Matérn cluster process with $\mu = 5$ galaxies per group (cluster) and a group radius of $R = 1.5 h^{-1}$ Mpc shows a $J(r)$ comparable to the $J(r)$ obtained from the galaxies in groups, where in the mean 4.8 galaxies reside in a group (see Fig. 2). We see a low, almost constant value of $J(r)$ for $r > 2.5 h^{-1}$ Mpc. This suggests that we are indeed looking at highly clustered galaxies, with small contamination by “field” galaxies.

The $J_{\text{M}}(r)$ of a Matérn cluster process becomes constant for radii twice as large as the cluster radius. Already, van Lieshout & Baddeley (1996) have expressed their hope to be able to deduce a cluster scale $R$ in a point distribution from $J(2R) \approx \text{const}$. However, this must be taken with extreme caution. As can be seen from Figure 2, we may be fooled to a factor of 3 by fluctuations in the estimated $J(r)$. The uncertainty becomes even worse when we consider certain Cox processes, in which $J(r)$ decreases strictly monotonically toward a constant value (van Lieshout & Baddeley 1996); in principle, no scale can be deduced from the comparison with the oversimplified Matérn cluster process. We must either restrict ourselves to qualitative statements, or come up with more refined and realistic models.

4. COMPARISON WITH $N$-BODY SIMULATIONS

The preceding section showed that the qualitative features of the galaxy distribution are well described by the $J$ function. In this section we demonstrate that the $J$ function is also suitable for a quantitative comparison, and allows us to constrain cosmological models.

4.1. $N$-Body Simulations and Mock Catalogs

We extract 64 mock PPS catalogs from a cosmological $N$-body simulation of a MDM model.

We consider a MDM model with one species of massive neutrinos, a dimensionless Hubble parameter $h = 0.5$, and density parameters $\Omega_b = 0.8$ and $\Omega_m = 0.2$ for cold and hot dark matter, respectively. The analytical expression for the MDM power spectra, $P(k)$, was taken from Ma (1996). The initial $P(k)$ was normalized to the COBE 4 yr data (Bunn & White 1997), giving a corresponding value of $\sigma_8 = 0.82$ for the rms mass fluctuation in an $8 h^{-1}$ Mpc sphere.

The simulation was run from an initial expansion factor $a_i = 1$ down to $a_f = 4.5$ using a P$^4$M code with 100$^3$ particles of mass $1.49 \times 10^{13} M_\odot$, on a cubic grid of 256$^3$ cells, with a force-softening radius of 0.32 $h^{-1}$ Mpc, in a box with side length $300 h^{-1}$ Mpc. The integration was performed in commoving coordinates using $a(t)$ as time variable for a total of 225 steps.

We identify “galaxies” in our simulation with a method similar to the one discussed by Little & Weinberg (1994). First, we associate with each particle a number $n_i$ of galaxy-scale peaks calculated from the initial density contrast field, $\delta(x)$. In the peak-background split approximation (Bardeen et al. 1986; White et al. 1987; Park 1991), $n_i$ is the number of galaxy peaks with height $\delta_i(x) \geq \nu_h \sigma_8$, where $\delta_i(x)$ denotes the field smoothed with a Gaussian kernel of width $R_* = 0.55 h^{-1}$ Mpc, and $\sigma_8^2$ gives the smoothed field’s variance. The field is subject to the constraint that it take the value $\nu_h \sigma_8$ when smoothed on a scale of $R_g > R_*$ (see Park 1991 for more details). Choosing $\nu_h = 0.05$, at $a = 4.5$ the particle two-point correlation function, weighted according to $n_i$, matches in slope and amplitude the galaxy two-point correlation function. For the adopted parameters, the total number of peaks in the box is $\sum n_i \approx 690,000$.

We then select the $i$th particle as a galaxy if $An_i > p$, where $p \in (0, 1)$ is a uniformly distributed random variable and $A$ is a constant of proportionality. The latter is set by the requirement that the mean number density of galaxies in the box matches the mean density of $M = -19 + 5 \log(h)$ galaxies expected from the Schechter luminosity function with the $\alpha = -1.15, M_s = -19.3 + 5 \log(h)$, and $\phi_\ast = 0.02 h^3$ Mpc$^{-3}$ appropriate for the PPS (Trasarti-Battistoni 1998; Marzke, Huchra, & Geller 1994). This Monte Carlo procedure makes the implicit assumption that the higher the peak, the more luminous the associated galaxy.

The mock PPS catalogs were built as follows. The simulation cube was divided into 64 subcubes of side length $75 h^{-1}$ Mpc. Within each subcube, we fitted a PPS-like wedge of radius $79 h^{-1}$ Mpc. Redshift-space coordinates $x, \delta$, and $cz$ were assigned to all the “galaxies” of the subcubes. Finally, we kept only the “galaxies” within the redshift-space boundaries of the mock PPS catalogs.

Although we are looking at a large volume, with a depth of 79 $h^{-1}$ Mpc and a solid angle of 0.76 sr, we observe large fluctuations of 25% in the number of points per mock sample (Fig. 5). This is consistent with the large-scale fluctuations of the clustering properties of IRAS galaxies, as found by Kerscher et al. (1998) out to scales of 200 $h^{-1}$ Mpc, and expresses cosmic variance in agreement with expected sample-to-sample variations (Buchert & Martinez 1993). As we will see, this slightly complicates the analysis.

4.2. $J(r)$ for the Mock Samples

First we investigate the mock samples selected in redshift space. If we use all the 64 mock samples, our results are dominated by the fluctuations between samples with different number densities (see Fig. 6). Therefore, we restrict ourselves to mock samples with approximately the same

![Fig. 5.—Histogram displaying the number of points per mock sample; solid lines give the values in redshift space, dashed lines corresponds to the selection in real space.](image-url)
number of points as in the observed galaxy sample: \( N_{\text{gal}} - \Delta \leq N_{\text{mock}} \leq N_{\text{gal}} + \Delta \), with \( N_{\text{gal}} = 817 \). For \( \Delta = 30 \) only six samples are eligible, while for \( \Delta = 100 \) we already have 17 mock samples to analyze. The mean value of \( J(r) \) hardly changes between samples with different \( \Delta \). Obviously, samples with low densities tend to be centered on voids, and high-density samples typically include large, Coma-like clusters. Therefore, large fluctuations in the number density lead to large fluctuations in the clustering properties measured by \( J(r) \), but cancel in the mean. These fluctuations decrease for smaller \( \Delta \) (see Fig. 6; this was confirmed by inspecting samples with \( \Delta = 50 \) and \( \Delta = 200 \)). In order to look at structures comparable to the PPS sample, we consider mock samples with a similar number density as in the observed galaxy sample, and do not subsample the mock samples with high number densities.

In Figure 7 the results of the mock samples in real and redshift space are compared. The mock samples selected in redshift space show a weaker clustering than mock samples selected in real space on small scales out to at least \( 2h^{-1}\text{Mpc} \), as can be deduced from the higher \( J(r) \). This can be traced back to redshift-space distortions. The peculiar motions act by erasing small-scale clustering; therefore, the \( J \)-value of redshift-space samples is larger (less clustering) than that of real-space samples. This effect changes at a given distance (\( 2h^{-1}\text{Mpc} \)). The same effect was found by Martínez et al. (1993) in volume-limited subsamples extracted from CfA I by means of the two-point correlation function.

4.3. Comparison of the PPS Galaxies with the Mock Samples

In Figure 8, the results of the mock samples in redshift space are compared with the results of the observed galaxy distribution in the PPS. The mock samples show insufficient clustering on small scales out to at least \( 3h^{-1}\text{Mpc} \), as can be deduced from the higher \( J(r) \). This is probably the result of the high velocity dispersion in MDM models (Jing et al. 1994). In real space, which is not directly comparable to the PPS data, the mock samples reproduce the clustering on small scales out to \( 1h^{-1}\text{Mpc} \), but again do not show enough clustering, even though they become marginally consistent with the observed galaxy distribution on larger scales. We are forced to conclude that this MDM simulation is unable to reproduce the observed strong clustering of galaxies on small scales. Of course, this result depends on our method of galaxy identification. A different biasing prescription might change this.

On large scales a definitive answer is not possible, since for \( r \) larger than \( 6h^{-1}\text{Mpc} \) an estimation of \( J(r) \) becomes
unreliable; the empirical $G(r)$ and $F(r)$ approach unity, and the quotient $J(r)$ is ill defined.

5. CONCLUSION AND OUTLOOK

We have highlighted promising properties of the global morphological descriptor $J(r)$. It connects the distribution functions $F(r)$ and $G(r)$, and hence incorporates all orders of correlation functions. $J(r)$ measures the strength of clustering in a point process and distinguishes between correlated and anticorrelated patterns. The example of a Matérn cluster process illustrates that $J(r)$ sensitively depends on the richness of the clusters or groups.

Since $J(r)$ is built from cumulated distribution functions, we do not encounter spurious results caused by binning. This becomes particularly important on small scales.

The application of the $J$-function to galaxies in a volume-limited sample and to a sample of galaxies in loose groups clearly showed the stronger clustering of galaxies in groups. In a comparison with a Matérn cluster, we found that internal properties, such as the richness of loose groups, are satisfactorily modeled. However, for the large-scale distribution of galaxies, the Matérn cluster process is clearly an oversimplification.

We used the $J$-function for a comparison of the observed galaxy distribution with galaxy mock samples. Although the mock samples extracted from a MDM simulation cover a large volume, we detected large fluctuations, of the order of 25%, in the number of points per sample. On small scales, out to $1 \, h^{-1} \text{Mpc}$, the clustering in real space is as strong as in the observed galaxy distribution, but the comparable redshift-space mock samples show clustering that is too weak. On larger scales, from 2 to $6 \, h^{-1} \text{Mpc}$, both real- and redshift-space mock samples show clustering that is too weak. Hence, this MDM simulation is not able to reproduce the observed strong clustering of the galaxies on small scales.

The function $J(r)$ has proved capable of achieving discriminative power comparable to the Minkowski functionals (Kerscher et al. 1998), and is most suitable for addressing the question of regularity on large scales, as demonstrated in an analysis of the distribution of superclusters (Kerscher 1998). In this article we have shown that the $J(r)$ function is a useful tool for quantifying the clustering of galaxies on small scales and is capable of constraining cosmological models of structure formation.

It is a pleasure to thank Adrian Baddeley, Bryan Scott, Claus Beisbart, and Herbert Wagner for useful discussions and comments. We thank Simon D. M. White for pointing out the relation to the conditional correlation function. This work was partially supported by the EC network of the program Human Capital and Mobility, No. CHRX-CT93-0129, the Acción Integrada Hispano-Alemana HA-188A (MEC), the Sonderforschungsbereich SFB 375 für Astroteilchenphysik der Deutschen Forschungsgemeinschaft, and the Spanish DGES (project PB96-0797).

REFERENCES

Bardeen, J. M., Bond, J. R., Kaiser, N., & Szalay, A. S. 1986, ApJ, 304, 15
Buchert, T., & Martinez, V. J. 1993, ApJ, 411, 485
Bunn, E. F., & White, M. 1997, ApJ, 480, 6
Burstein, D., & Heiles, C. 1978, ApJ, 225, 40
Gaztañaga, E., & Frieman, J. A. 1994, ApJ, 437, L13
Giovanelli, R., & Haynes, M. P. 1991, ARA&A, 29, 499
Huchra, J., & Geller, M. 1982, ApJ, 257, 423
Iovino, A., Giovanelli, R., Haynes, M. P., Chincarini, G., & Guzzo, L. 1993, MNRAS, 265, 21
Jing, Y., Mo, H., Börner, G., & Fang, L. Z. 1994, A&A, 284, 703
Kerscher, M. 1998, A&A, 336, 29
Kerscher, M., Schmalzing, J., Buchert, T., & Wagner, H. 1998, A&A, 333, 1
Kerscher, M., Schmalzing, J., Retzlaff, J., Borgani, S., Buchert, T., Gottlöber, S., Müller, V., Plionis, M., & Wagner, H. 1997, MNRAS, 284, 73
Little, B., & Weinberg, D. 1994, MNRAS, 267, 605
Ma, C.-P. 1996, ApJ, 471, 13
Martínez, V. J., Jones, B. J. T., Dominguez-Tenreiro, R., & van de Weygaert, R. 1990, ApJ, 357, 50
Martínez, V. J., Portilla, M., Jones, B. J. T., & Paredes, S. 1993, A&A, 280, 5
Marzke, R., Huchra, J., & Geller, M. 1994, ApJ, 428, 43
Mecke, K. R., Buchert, T., & Wagner, H. 1994, A&A, 288, 697
Melott, A. L. 1990, Phys. Rep., 193, 1
Neyman, J., & Scott, E. L. 1958, J. R. Stat. Soc., 20, 1
Park, C. 1991, MNRAS, 251, 167
Peebles, P. J. E. 1993, Principles of Physical Cosmology (Princeton: Princeton Univ. Press)
Schmalzing, J., & Buchert, T. 1997, ApJ, 482, L1
Sharp, N. 1981, MNRAS, 195, 857
Stoyan, D., Kendall, W. S., & Mecke, J. 1995, Stochastic Geometry and Its Applications (2d ed; Chichester: Wiley)
Stoyan, D., & Stoyan, H. 1994, Fractals, Random Shapes and Point Fields (Chichester: Wiley)
Strattonovich, R. L. 1963, Topics in the Theory of Random Noise, Vol. 1 (New York: Gordon & Breach)
Trasarti-Battistoni, R. 1998, A&AS, 130, 341
Trasarti-Battistoni, R., Invernizzi, G., & Bonometti, S. 1997, ApJ, 475, 1
van Lieshout, M. N. M., & Baddeley, A. J. 1996, Stat. Neerlandica, 50, 344
Wegner, G., Haynes, M. P., & Giovanelli, R. 1993, AJ, 105, 1251
White, S. D. M. 1979, MNRAS, 186, 145
White, S. D. M., Frenk, C. S., Davis, M., & Efstathiou, G. 1987, ApJ, 313, 505
Zwicky, F., Herzog, E., Wild, P., Karpowicz, M., & Kowal, C. 1961–1968, Catalog of Galaxies and Clusters of Galaxies, Vol. 1–6 (Pasadena: California Inst. Technology)