CHIRAL SYMMETRY AND $U_A(1)$ ANOMALY
IN AN EFFECTIVE THEORY OF QCD

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ABSTRACT
We show on the basis of an effective theory of QCD that a wide variety of observables in the hadron world is governed by the chiral symmetry together with an interplay between the axial anomaly and the explicit symmetry breaking due to the current quark mass. We also discuss the nature of the chiral transition at finite temperature and related dynamical phenomena using the effective Lagrangian. Some phenomenological implications of the small vector coupling ("vector limit") at high temperatures are suggested.

1. Introduction

The main topic of this conference is the confinement of colored quarks and gluons. However, one may notice that even apart from the confinement, the basic properties of the hadron world include (i) chiral symmetry and its dynamical and (small) explicit breaking, (ii) $U_A(1)$ anomaly, (iii) approximate $SU_f(3)$ symmetry, (iv) the OZI rule and its violation and so on. Some of these are interrelated. The purpose of the present talk is not to pin down the basic mechanism for the confinement, (i) and (ii) and so on as other talks here do, but to show that an effective theory which embodies (i) and (ii) but not the confinement can well describe a wide variety of phenomena and observables in the hadron world, and then apply the effective theory to $T \neq 0$ and/or $\rho \neq 0$ systems. Thereby we shall elucidate the governing roles of (i) and (ii) in the hadron world and get insight into the nature of chiral transition at $T \neq 0$ and/or $\rho \neq 0$. We shall also give heuristic discussions about phenomenological implications of the small coupling in the vector channel at high temperatures.

2. The Model Lagrangian

Our model Lagrangian is the generalized Nambu-Jona-Lasinio (NJL) model with the anomaly term:

$$ \mathcal{L} = \bar{q}i \gamma \cdot \partial q + \sum_{a=0}^{8} \frac{g_s}{2} \left[ (\bar{q} \lambda_a q)^2 + (\bar{q}i \gamma_5 \lambda_a q)^2 \right] - \bar{q} m_q + g_\rho \text{det} \bar{q} (1 - \gamma_5) q + h.c., $$

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*A good reference for this report may be found in the review article.
\[ \equiv \mathcal{L}_0 + \mathcal{L}_{SB} + \mathcal{L}_S + \mathcal{L}_D, \]  

where the quark field \( q_i \) has three colors (\( N_c = 3 \)) and three flavors (\( N_f = 3 \)), \( \lambda^a (a=0 \sim 8) \) are the Gell-Mann matrices with \( \lambda_0 = \sqrt{\frac{2}{3}} \mathbf{1} \). Here \( \mathcal{L}_0 + \mathcal{L}_S \equiv \mathcal{L}_{NJL} \) is the \( U(3) \) generalization of the NJL model and has manifest flavor-\( U_L(3) \otimes U_R(3) \) invariance. \( \mathcal{L}_{SB} \) is the explicit \( SU_V(3) \) breaking part with \( m_i (i=u, d, s) \) being the current quark mass. Finally, \( \mathcal{L}_D \) in Eq.(1) denotes the term which has \( SU_L(3) \otimes SU_R(3) \) invariance but breaks the \( U_A(1) \) symmetry; this term is a reflection of the axial anomaly in QCD. While \( \mathcal{L}_S \) does not cause the flavor mixing, the anomaly term does; with the dynamical breaking of chiral symmetry, it induces effective 4-fermion vertices such as \( \langle \bar{u}d \rangle \langle \bar{u}u \rangle \langle \bar{s}s \rangle \) and \( -\langle \bar{d}d \rangle (\bar{u}i\gamma_5 u)(\bar{s}i\gamma_5 s) \), where the former (latter) gives rise to a flavor mixing in the scalar (pseudo-scalar) channels. The fact that \( \mathcal{L}_D \) represents the \( U_A(1) \) anomaly can be seen in the anomalous divergence of the flavor singlet axial current \( J_5^\mu = \bar{u}\gamma^\mu \gamma_5 u + \bar{d}\gamma^\mu \gamma_5 d + \bar{s}\gamma^\mu \gamma_5 s \) as

\[ \partial_\mu J_5^\mu = -4N_fg_D \text{Im}(\text{det}\Phi) + 2i\bar{q}m\gamma_5 q, \]

which is to be compared with the usual anomaly equation written in terms of the topological charge density of the gluon field, \( \partial_\mu J_5^\mu = 2N_f {g^2 \over 32\pi} F_\mu^a \tilde{F}_\mu^a + 2i\bar{q}m\gamma_5 q. \) Thus one may say that the determinantal 6-fermion operator \( -g_D {g^2 \over 32\pi} F_\mu^a \tilde{F}_\mu^a \) in the quark sector.

Table 1. Comparison of the theoretical estimates and the experimental/empirical values of the basic physical quantities. * indicates the quantity used as input.

| Theory | Empirical values |
|--------|-----------------|
| \( M_u (M_s) \) | 335 (527) | 336 (540) MeV |
| \( \langle \bar{u}u \rangle^{NP} \) | \(-245^3\) | \(-225 \pm 25)^3 \text{MeV}^3\) |
| \( \langle \bar{s}s \rangle^{NP} \langle \bar{u}u \rangle^{NP} \) | 0.78 | 0.8 \pm 0.1 |
| \( m_\pi (m_K) \) | 138* (496*) | 138 (496) MeV |
| \( m_\eta (m_{\eta'}) \) | 487 (958*) | 549 (958) MeV |
| \( m_\sigma (m_{\sigma'}) \) | 668 (1348) | \( \sim 700 (\sim 1400) \) MeV |
| \( \Gamma_{\sigma \to 2\pi} \) | \( \sim 900 \) | \( \sim \text{Re} m_\sigma \) |
| \( f_\pi (f_K) \) | 93.0* (97.7) | 93 (113) MeV |
| \( f_\eta (f_{\eta'}) \) | 94.3 (90.8) | 93\pm9 (83\pm7) MeV |
| \( \theta_\eta (\varphi_\sigma) \) | \(-21^\circ (\sim -6.8^\circ)\) | \( \sim -20^\circ (-) \) |
| \( G_{\pi\eta} (G_{K\eta}) \) | 3.5 (3.6) | \( \sim 3.5 (-) \) |
| \( G_{\pi N} (G_{\sigma N}) \) | 12.7 (7 - 10) | 13.4 (\sim 10.0) |
| \( \Sigma_{\pi N} \) | 49 \pm 7 | 45 \pm 10 MeV |

In Table 1, we summarize basic physical quantities calculated in this model in the mean-field plus ring approximation\[10\], the corresponding empirical values are also quoted for comparison. The numerical values are obtained with the following...
parameter set; $\Lambda = 631.4\text{MeV}, \quad g_s \Lambda^2 = 3.67, \quad g_D \Lambda^5 = -9.29, \quad m_s = 135.7\text{MeV}$, where we have used a three-momentum cutoff scheme. Here we remark that the mixing angle $\theta_\eta$ of the $\eta$ and $\eta'$ mesons is in a nice agreement with the experimental value; the mixing angle is a good measure of the strength of the axial anomaly. We also notice that the $\pi$-N sigma term $\Sigma_{\pi N}$ agrees with the “empirical” value. To calculate the baryon quantities such as $\Sigma_{\pi N}$, we have employed the chiral quark model where the NJL model Lagrangian is adopted as a chiral Lagrangian. The constituent quark mass is identified as that generated by the dynamical chiral symmetry breaking. Then a successful constituent quark model (actually, Isgur-Karl’s) was employed. Comments on other quantities listed in the table are given in [1]. Table 1 tells us that the $SU(3)$ NJL model reproduces the fundamental physical quantities in the accuracy of $O(10\%-15\%)$.

3. Static Properties at Finite $T$ and $\rho$

So much for the vacuum and hadron properties at zero temperature. Now let us go to finite temperature case. Lattice simulations show that the order and even the existence of the phase transition(s) are largely dependent on the number of the flavors especially when the physical current quark masses are used: For $m_u \sim m_d \sim 10\text{MeV} \ll 100\text{MeV} \approx m_s$, the phase transition may be weak 1st order or 2nd order or not exist.

The gross feature of the $T$ dependence and the striking difference between the condensates of u (d) quark and the s quark can be well described by the NJL model. It is noteworthy that at high temperatures, the flavor $SU(3)_f$-symmetry gets worse badly, which may reflect in the baryon and the vector meson spectra, because they are well described by the constituent quark models.

Some people may be interested in the nature of the chiral transition in the chiral limit in this model. The numerical calculation shows that the phase transition is of 2nd order, although the thermodynamical potential is asymmetric with respect to the zero condensate due to the cubic term coming from the determinantal term.

How about the phase transition at finite density? A numerical calculation shows that at low temperatures lower than about 50 MeV, the phase transition is of strong first order in the density direction. Note that our model Lagrangian has no vector term like $g_v(q\gamma_\mu q)^2$. As a matter of fact, the strength and even the existence of the 1st order transition are strongly dependent on the strength of the vector coupling $g_v$; the vector term prevents a high-density state.
4. Dynamic Properties

4.1. hadrons at $T < T_c$

One can calculate meson properties at finite temperature by computing the response functions. In our model\[1\], the $\sigma$ meson mass decreases as $T$ increases till $T_c$, the critical temperature\[b\]. $m_\pi$ is found to be constant as long as $T < T_c$. The lattice simulations on the meson masses\[12\] show that both $m_\pi$ and $m_\sigma$ show similar behaviors with those given in the NJL model. One should note however that the masses in the lattice simulation are so called the screening mass $m_{sc}(T)$, the exponent describing the mesonic correlations in the spatial direction; namely for the hadronic operator $O_H$

$$
\int dxdy\tau \langle \langle O_H(\vec{x}, \tau)O_H(\vec{0}, 0) \rangle \rangle \rightarrow \exp[-m_{sc}(T) | z |] \quad (| z | \rightarrow \infty). \quad (3)
$$

A detailed calculation has shown that the two kind masses have a similar temperature dependence both in the Wigner and the Nambu-Goldstone phases even off the chiral limit\[13\]; see also Appendix of\[8\]. This implies that the lattice results on the screening masses may suggest that the dynamical masses also behave like those given in the NJL model.

4.2. Hadronic excitations at $T > T_c$

It is remarkable that there seem exist colorless hadronic excitations even in the high-$T$ phase\[14\] contrary to the naive picture of it.\[c\]

Hatsuda and the present author noted that there should exist precursory soft modes in the high temperature phase prior to the phase transition if the chiral transition is of second order or weak first order: The soft modes are actually fluctuations of the order parameter of the phase transition, $\langle \langle (\bar{q}q)^2 \rangle \rangle$ and hence $\langle \langle (\bar{q}\gamma_5 q)^2 \rangle \rangle$ due to the chiral symmetry. They demonstrated these using an effective theory of QCD\[14\].

The subsequent measurements of the screening masses on the lattice QCD supported the existence of such hadronic modes in the high-$T$ phase\[15\]: Vector-mesonic and baryonic modes were obtained as well as the scalar and the pseudo-scalar mesons. The following comments are in order here: (i) The screening masses of the pion and the sigma meson are both well below $2\pi T$, which indicates that the interactions between $q$-$\bar{q}$ in the pseudo-scalar and the scalar channels are still rather strong even in

\[b\] $T_c$ may be defined as the temperature at which $m_\pi$ starts to go high or the temperature where the quark condensate takes a half of its zero-temperature value.

\[c\] As for the gluon sector, it was shown that non-perturbative effects seem significant in the low energy regime at temperatures $T_c < T < (2 \sim 3)T_c$.\[14\]
the high-$T$ phase, as suggested in\textsuperscript{3}. (ii) The screening masses of the vector modes coincide with $2\pi T$ within the error bars soon after the chiral restoration, which may simply indicate that the interactions between $q$-$\bar{q}$’s in this channel are absent or greatly suppressed\textsuperscript{3}. (iii) The nucleons exist as a parity doublet\textsuperscript{18}, which had been considered to be unrealistic because such a model admitting a parity-doubled nucleons leads to the \textit{vanishing} $\pi$-$N$ coupling in the NG phase\textsuperscript{3}.

We have seen that there is a reason d’etre of the pionic and sigma-meson excitations near the transition point even in the high-$T$ phase; they are the fluctuations of the order parameter of the chiral restoration.

Then how hadronic excitations in the vector channel? The approach of the hidden local symmetry\textsuperscript{20} seems to claim the existence of the vector mesons are intimately related with the chiral symmetry and its spontaneous breaking. The point (ii) above suggests that there exist no strong correlations in the vector channel in the high-$T$ phase. The present author showed\textsuperscript{21} that the great suppression seen in the screening mass is consistent with the $T$-dependence of the quark-number susceptibility $\chi_q(T)$ obtained by the lattice simulations\textsuperscript{22}: As $T$ is raised $\chi_q$ increases very rapidly around the critical point of the chiral transition. We shall give more discussions on the susceptibility.

4.3. Quark-number Susceptibility

The quark-number susceptibility $\chi_q$ is the measure of the response of the quark number density to infinitesimal changes in the chemical potentials $\mu_i (i = u, d)$\textsuperscript{2};

$$
\chi_q(T, \mu) = \left[ \frac{\partial}{\partial \mu_u} + \frac{\partial}{\partial \mu_d} \right] (\rho_u + \rho_d),
$$

$$
= \beta \int d\mathbf{x} \langle \bar{q}(0, \mathbf{x}) \gamma_0 q(0, \mathbf{x}) \bar{q}(0, \mathbf{0}) \gamma_0 q(0, \mathbf{0}) \rangle,
$$

where $\mu = (\mu_u, \mu_d)$, $\hat{N}_q$ is the quark-number operator, $\rho_i$ the i-th quark-number density, $V$ the volume of the system and $\beta = 1/T$. In the following, we treat the case where $\mu_u = \mu_d \equiv \mu_q$.

It is important that the number susceptibility at finite density is directly related to the (iso-thermal) compressibility $\kappa_T$ as $\kappa_T = \chi_q/\rho^2$. Therefore if $\chi_q$ of a system is large, the system is easy to compress, which may be a reflection of a weak (if exists) repulsion between the constituents of the system.

In the simple free-quark gas model, one sees that $\chi_q^{(0)}(T)$ increases as the constituent quark mass $M(T)$ decreases owing to the partial chiral restoration and reaches $N_f T^2$ at $M(T) = 0$. The enhancement is, however, found to be merely about 1.6 with $M(T)$ as described in the NJL model\textsuperscript{8}. Thus there must be an additional mechanism to increase $\chi_q$ to realize the anomalous (relative) enhancement obtained in the lattice simulations.

We note here that $\chi_q$ is the density-density correlation which is nothing but the 0-0 component of the vector-vector correlations or fluctuations. It means that the
quark-number susceptibility is intimately related with the properties of the vector mesons and the fluctuations in the vector channel.

Using the NJL model with the vector coupling $g_V$, one can show that at $\mu_q = 0$

$$\chi_q = \frac{\chi_q^{(0)}(T)}{1 + 2g_V \chi_q^{(0)}(T)}$$  \hspace{1cm} (5)$$

where $\chi_q^{(0)}(T)$ is the susceptibility for the free-quark gas.

In general, $\chi_q$ is suppressed with the vector coupling because the denominator is larger than unity for positive $g_V$. One would find it reasonable for a system at finite $\mu_q$ because it implies that the system has a small compressibility; recall that $\chi_q$ is proportional to $\kappa_T$. One thus sees the anomalous enhancement of $\chi_q$ as given by the lattice simulations can be accounted for by a possible change (decrease) of the vector coupling, i.e., the vanishing or abrupt decrease after the chiral transition.$^d$

When $\mu_q \neq 0$ there arises a coupling between $\chi_q$ and the scalar-density susceptibility $\chi_s$ owing to the non-vanishing “vector-scalar susceptibility” $\chi_{VS}$, which are defined by

$$\chi_s = -\frac{d\langle\bar{q}q\rangle}{dm} = \beta \int d\mathbf{x} \langle\langle\bar{q}(0,\mathbf{x})q(0,\mathbf{x})\bar{q}(0,\mathbf{0})q(0,\mathbf{0})\rangle\rangle, \hspace{1cm} (6)$$

$$\chi_{VS} = \frac{\partial\langle\bar{q}q\rangle}{\partial\mu_q} = \beta \int d\mathbf{x} \langle\langle\bar{q}(0,\mathbf{x})\gamma_0 q(0,\mathbf{x})\bar{q}(0,\mathbf{0})q(0,\mathbf{0})\rangle\rangle. \hspace{1cm} (7)$$

Thus when $\mu_q \neq 0$, the fluctuation of the order parameter reflects in $\chi_q$. This should give an enormous effect on $\chi_q$ when the chiral transition is of 2nd order or weak first order where the fluctuation of the order parameter becomes huge near the critical point. It would be intriguing to see this in lattice calculations.

Phenomenologically, the rise of $\chi_q$ means that the system is easy to compress as noted before, which implies that there develops a large density fluctuation near and above critical temperature, especially in a finite-density system.$^e$ The large density fluctuation may be reflected in the distribution of, say, pions produced in the relativistic heavy ion collisions, and may become a seed for the generation of heavy elements in the early universe.

5. Summary and future problems

We have seen that (to a surprise) the NJL model with the determinantal interaction well describes the low-energy hadron world. This may imply that the low-energy $^d$ Recently, Brown and Rho$^{23}$ have indicated that the chiral restoration may imply the decrease of the vector coupling, thereby realize the “vector limit” of Georgi$^{24}$ on the basis of the work by Harada and Yamawaki$^{25}$ on the renormalization of the Lagrangian of the hidden local symmetry.

$^e$When $\mu_q \neq 0$, it can be shown that $\chi_q \sim 1/g_v$ hence blows up at $T$ near $T_c$. interactions.
behavior of the hadron world is mainly determined by the chiral symmetry, and the
axial anomaly and the explicit breaking due to the current quark masses give some
variations to it. We have also seen that the model is useful to explore the finite
temperature and/or density systems.

As future problems, one should, of course, incorporate the effect of the confinement
explicitly together with the chiral symmetry, and see how and why the effect of
the confinement did not affect so much the hadron phenomenology at low energy
once the color-singlet states are projected out as in the NJL model. The monopole
condensation is enthusiastically advocated in this conference as the mechanism of the
color confinement. The chiral symmetry breaking is probably related with the other
topological object, instanton. One may be interested in how the monopole instanton
configuration is affected with the presence of the dynamical quarks, especially light
quarks: The proposed picture of the confinement based on the monopole condensation
seems to be extracted when quarks are heavy.

In describing baryons, we have employed a chiral quark model where the NJL
model is used as a chiral Lagrangian together with the perturbative gluon exchange. It
would be interesting to apply the renormalization group to it and see if the Lagrangian
becomes the perturbative QCD with the NJL coupling being vanishingly small at
high-energy scales.

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