Simulation of wave refraction in the western waters of Aceh, Indonesia

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Abstract. A two-dimensional numerical model which is the solution of a shallow water equation without advection is used for long wave refraction simulations in the western waters of Aceh. The model is discretized with the resolutions of Δx = Δy = 925 meters and Δt = 10 seconds. The bathymetry data used is obtained from SRTM30. As an external force, we use amplitude of 0.3 m and a period of 1 hour. The numerical simulation results are discussed every quarter of the period (T/4, T/2, 3T/4, and T). The comparison between the two scenarios is that the gradient of the sea level with the period of one hour is greater than that of two hours, especially at T/2 and T. For the model with one hour period, when T/2 and T, the currents and gradients converge to Sikandang village in the west coast. While for the model with two hour period, when T / 2 and T currents and gradient of sea level experience a little convergence. According to the theory, the cape and bay form a convergent and divergent zone, respectively. This study confirms this theory.

1. Introduction

Wave refraction is a physical process that is important for determining wave motion around the beach. The phenomenon of refraction or bending of the wave direction occurs when the wave enters shallow water. Refraction waves occur at the depths of transitional water and shallow water because wave celerity decreases with a decrease in water depth. As a result, part of the wave crest from shallow water propagates forward at a slower speed than the part in deeper water. The result is a bend from the peak of the wave so that the wave peaks approach the orientation that corresponds to the depth contour. Wave orthogonals, to remain normal to the wave crest, will also curve so that the orthogonals are aligned in deep water. This event can take place converging or diverging when refraction occurs. The orthogonal convergence or divergence of this wave will cause an increase or decrease in local wave energy and result in wave height. The wave celerity relative to the seafloor will also depend on the possibility of the presence of ocean currents. If the train of waves crosses an area where there is a horizontal gradient in terms of current velocity, then wave refraction occurs [1].

The energy and direction of the waves that approach the coast will affect longshore drift. At constant energy, the maximum capacity occurs when a wave forms a 45° angle with the coastline. An increase or decrease in angle values results in a decrease in longshore flow capacity. Its close relationship occurs
between the net longshore drift and coastline orientation, and this is related to the main direction of incident waves [2].

Thus, the wave can form a convergence zone around the headland [3]. The western waters of Aceh include Nagan Raya Regency and Southwest Aceh, directly adjacent to the Indian Ocean. Waves in these waters are affected by the Indian Ocean and Monsoon throughout the year [4, 5]. Wave conditions play a major role in coastal dynamics such as sediment transport, shoreline changes [6, 7, 8], waste pollution [9] and fishing activities [5, 10].

Wave refraction is quite fluctuating because the disturbances around the coast have different periods. The purpose of this study is to study a long period wave refraction in the western waters of Aceh. Long period wave categories are waves with periods longer than 30 seconds. These waves can be formed by wind, seiches, storm surges, earthquakes, and tsunamis. Different from the tidal wave, the long period has a period that is not fixed and is always changing. This research is quite interesting because it can be a basic knowledge in understanding the possibility of extreme waves around the coast of Aceh and for the process of transporting coastal sediments.

2. Materials and Methods
2.1 The motion equations
This study uses a 2D linear hydrodynamics model [11]. Seawater dynamics are derived from the Navier-Stokes equation with the finite-difference method:

\[ u_{j,k}^{n+1} = u_{j,k}^{n} - \Delta t g (\eta_{j,k+1}^{n} - \eta_{j,k}^{n})/\Delta x \]  \hspace{1cm} (1)

\[ v_{j,k}^{n+1} = v_{j,k}^{n} - \Delta t g (\eta_{j,k+1}^{n} - \eta_{j,k}^{n})/\Delta y \]  \hspace{1cm} (2)

\[ \eta_{j,k}^{n} = \eta_{j,k}^{n} - \Delta t \{(u_{j,k}^{n+1}h_e - u_{j,k-1}^{n+1}h_w)/\Delta x - (v_{j,k}^{n+1}h_n = v_{j,k-1}^{n+1}h_n)/\Delta y \} \]  \hspace{1cm} (3)

Equations (1) and (2) are used to find the velocities of current u and v in cartesian coordinates, where \( u(j,k) \) and \( v(j,k) \) are the velocities of the west-east and north-south currents, \( n \) is the number of iterations, \( \Delta t \) is the time step, \( \eta(j,k) \) is sea level elevation, \( \Delta x \) and \( \Delta y \) are interval in the x and y direction, respectively, and \( h \) is the depth of sea water. Equation (3) is used to find sea level elevation from the results of current velocities (1) and (2). For stability and smoothness numerical simulation results, the Shapiro filter equation (4) is used. This equation serves to minimize extreme sea level gradients:

\[ \eta_{j,k}^{n+1} = (1 - \epsilon)\eta_{j,k}^{n} + 0.25\epsilon (\eta_{j,k-1}^{n} + \eta_{j,k+1}^{n} + \eta_{j-1,k}^{n} + \eta_{j+1,k}^{n}) \]  \hspace{1cm} (4)

2.2 Material and Methods
SRTM30 data has been successfully applied to several shallow water model simulations. For example, in the Malacca Strait [12]; Gulf of Thailand [13] and the northern of Aceh waters [14]. In this study SRTM30 (https://dds.cr.usgs.gov/srtm/version2_1/SRTM30/) was used as bathymetry input. This data is filtered and discretized according to the domain model.

The model domain covers the western Aceh waters [96. 20° – 97.00°E dan 3.5° – 4.1°N]. The domain is discretized as \( \Delta x = \Delta y = 30 \) seconds or 925 meters so that it has a high resolution. The time interval is determined by the criterion of Courant-Friedichi-Lewy (CFL), namely \( \Delta t = 10 \) seconds.

In this numerical simulation, the external force of sinusoidal waves with an amplitude of 0.3 meters and a 1 hour period is applied to the open plane. Furthermore, numerical results in the form of sea level and current are sampled every \( 1/4 \) wave period.
Figure 1 shows the bathymetry of the western waters of Aceh. Depth along the coastline ranges from 0 to 50 meters. Deeper depths are in the southwest part (until 300 m depth).

3. Results and Discussion

Numerical simulation results with an amplitude of 0.3 meters and a 1 hour period are shown in figures 2 until 5. When the wave at the highest point (at \( t = T/4 \)) (figure 2) the swash is seen on the west and south, while the backwash is seen in the southeast part. Swash is the current that moves towards the beach while the backwash is the current that moves away from the beach.

The dominant swash in the west cause convergence around Tanjung or Sikandang waters which is marked by a high sea level gradient and the sea level is quite high, reaching 2 meters. The process of increasing sea level due to changes in depth is indicated by shoaling. Shoaling usually occurs along with wave refraction. Refraction of waves at \( t = T/4 \) shows the current tends to be perpendicular to the coast in the west and the parallel currents on the inside of the bay.
Figure 2. The vector indicates the direction of the current and the contour line indicates the value of the sea level (meters) at the highest point at $t = T/4$.

Figure 3. Same as figure 2, but for the middle point (at $t = T/2$).
Figure 4. Same as figure 2, but for the lowest point (at $t = 3T/4$).

Figure 5. Same as figure 2, but for the middle point (at $t = T$).

At time $t = T/2$, as shown in Figure 3, the backwash is seen along the coastline. The current is directed to the west and south of the coast perpendicularly. But at the same time, there is a slight flow from the western boundary which is directed towards the coast so that it forms a convergent zone near the western boundary plane. While shoaling is still visible in Sikandang waters. Sea level height in Sikandang waters reaches about 1.5 meters.

The wave at time $t = 3T/4$, see Figure 4, the backwash, in general, is visible along the coastline and can reach up to the western and southern boundaries. These currents are generally directed perpendicular to the coast towards the west or south. Different from the situation at $t = T/4$ and $t = T/2$, at $t = 3T/4$ sea levels in Sikandang waters is much lower than those of the surrounding waters, which is around -1.5 meters. It means the sea level 1.5 meters below the mean sea level.
When the wave at $t = T$, see Figure 5, the backwash is still visible along the coastline but there are currents from the western and southern boundary fields directed towards the beach. So that it produces clear convergent lines on the western and southern part. This convergent line resembles the coastline contours of the western waters of Aceh.

The results obtained as shown in Figures 2 until 5 show that wave refraction dynamics are influenced by the coastline and wave period.

4. Conclusions
Two-dimensional numerical simulations with linear shallow water equations have been applied to the waters of western Aceh. Numerical simulations show that the depth contours and wave periods affect the direction of the current and sea level. Likewise, the convergent and shoaling zones around the coast are also influenced by the depths and wave periods.

In general, wave refraction formed in the western waters of Aceh produces parallel longshore currents along the headland and change into perpendicular coastal currents when entering the bay. This process is able to form abrasion on cape and sedimentation in the bay.

Convergent zones around the cape have considerable potential energy. This condition is relatively in accordance with the dynamics of the waters on the headland of India [3].

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