BLACK HOLE INFORMATION *

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Abstract

Hawking’s 1974 calculation of thermal emission from a classical black hole led to his 1976 proposal that information may be lost from our universe as a pure quantum state collapses gravitationally into a black hole, which then evaporates completely into a mixed state of thermal radiation. Another possibility is that the information is not lost, but is stored in a remnant of the evaporating black hole. A third idea is that the information comes out in nonthermal correlations within the Hawking radiation, which would be expected to occur at too slow a rate, or be too spread out, to be revealed by any nonperturbative calculation.

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1. Hawking’s Proposed Loss of Information

Hawking’s 1974 calculation [1, 2] of the emission from a stationary classical black hole was soon shown to give uncorrelated thermal emission in each mode [3-5]. If a semiclassical approximation were used so that the black hole shrinks in a quasistationary way during the evaporation, one would still expect nearly thermal emission with high entropy, though even this calculation (without quantizing the geometry) has not been done precisely in four dimensions. As a result of these calculations and expectations, Hawking argued [3] that the semiclassical approximation should be good until the black hole shrunk near the Planck mass, and then there would not be enough energy left for the information that collapsed into the black hole to come back out. Thus a pure quantum state that underwent gravitational collapse into a black hole that subsequently evaporated away would end up as a mixed quantum state of Hawking radiation. Hawking proposed that the process would be described by a superscattering operator $\mathcal{S}$ that would take initial density matrices into final density matrices in a nonunitary way, generically increasing the fine-grained entropy $S \equiv -Tr(\rho \ln \rho)$:

$$\rho_{ab}^{\text{final}} = \mathcal{S}_{ab} \rho_{cd}^{\text{initial}},$$

with a sum over the repeated indices $c$ and $d$, where $\mathcal{S}_{ab}$ would not have the usual unitary form $S_{ac} S_{bd}$ in terms of a unitary $S$ matrix.

This process would correspond to the loss of information in the sense that from an initial pure state, one could not predict any single final pure state with certainty. A pure state may be represented by a normalized vector $|\psi\rangle$ in the Hilbert space of the quantum system (here, the universe, or at least our connected component of it). A pure state can also be represented by a statistical state or density matrix (in general, a positive-semidefinite Hermitian unit-trace operator or matrix acting on the vectors in the Hilbert space) which when pure has the form $\rho = |\psi\rangle\langle\psi|$. Hence in this pure case $\rho$ is a rank-one projection operator with $\rho^2 = \rho$ and therefore with $Tr\rho^2 = 1$ and entropy $S = 0$ as well as the general normalization (unit-trace) requirement $Tr\rho = 1$. A pure state may be contrasted with a mixed state, which cannot be represented by a single vector in the Hilbert space, but which can be represented by a statistical state or density matrix $\rho = \sum_n p_n |\psi_n\rangle\langle\psi_n| \neq \rho^2$ with more than one nonzero eigenvalue $p_n$, each of which can be interpreted to be the probability of measuring the mixed state to be in one of the corresponding orthonormal pure states $|\psi_n\rangle$. (It can be misleading, e.g., in the EPR ‘paradox,’ to say that a system in a mixed state is actually in one of its component pure states $|\psi_n\rangle$ with probability $p_n$, because the system might instead actually be correlated with another system in
a composite system, and the composite system as a whole could even be in a pure state.) By the positive-semidefinite and unit-trace properties of general density matrices, the $p_n$’s are nonnegative and add up to unity, and one can readily see that a mixed state has $Tr\rho^2 < 1$ and $S \equiv -Tr(\rho \ln \rho) > 0$.

If one makes a complete measurement of a system, by which I mean the measurement of a nondegenerate observable (represented by some Hermitian operator with totally nondegenerate eigenvalues), a pure state is the only kind that can give a definite result with certainty (unit probability). A pure state, and only a pure state, has the property that one can predict with certainty the result of some complete measurement of the system. For one indeed to be able to predict with certainty, one needs the measured observable to have one of its eigenvectors proportional to the Hilbert space vector $|\psi\rangle$ representing the pure state, so of course not every observable will give a uniquely predictable result. (This is a manifestation of the quantum uncertainty that applies even to pure states.) However, for any pure state, there do exist nondegenerate observables (in the sense of Hermitian operators but not, in general, in the sense of what is experimentally and practically possible) which would give definite results.

For example, suppose one had access to a sufficiently large ensemble of identical systems in the same (initially unknown) pure state and could in principle measure enough observables. The first observable one randomly chose to measure would generally not give definite results (i.e., one would get different individual results when one measured different members of the ensemble with it). Nevertheless, one could eventually find some observable which would always give the same definite result when applied to members of the ensemble. (This would not be a unique observable, since only its eigenvector proportional to the pure-state vector would be uniquely determined, up to a complex multiplicative constant. Furthermore, this member of the preferred class of observables would only be determined to some finite accuracy if only a finite sequence of measurements were made to find a finitely good empirical approximation to one of these preferred observables.)

It is in this sense that one can say that complete information (the maximum allowed by quantum mechanics) exists for a system in a pure state.

On the other hand, for a mixed or impure state there is no nondegenerate observable that would have a unique value with unit probability. In other words, one cannot predict with certainty the result of any complete measurement of a system in a mixed state. In this sense one says that mixed states have less than maximal information. Of course, in another sense one could say that the actual density matrix gives all of the information possible about the state of a system (at least if one considers the
system by itself, ignoring any quantum correlations it may have with other systems). In this latter sense, if one knows the actual density matrix (not just an estimate based on additional uncertainty about the actual statistical state of the system), one has complete information about the system in whatever statistical state it is actually in. However, it is in the former sense, of how much information is possible about the system (which depends on the actual statistical state rather than on how well that is known), that one says that the evolution of a pure state to a mixed state by Hawking’s proposed Eq. 1 for black hole formation and evaporation would be a loss of information. A measure of the possible amount of information in a system with Hilbert-space dimension \( m \) and statistical state \( \rho \) is

\[
I = S_{\text{max}} - S = \ln m + Tr(\rho \ln \rho).
\]

(2)

This loss of information does not necessarily mean that a knowledge of the final mixed state would be insufficient to reconstruct the initial state. (The final mixed state could in principle be learned to arbitrarily high statistical accuracy from the results of repeated measurements of a sufficiently large set of observables if one had a sufficiently large ensemble of systems with that same mixed state.) To illustrate this claim and the points made above, consider a spin-1/2 system, so that the indices \( a, b, c, d \) range from 1 to 2. The general statistical state of the system can be written as

\[
\rho = \rho_{11} |\uparrow\rangle\langle\uparrow| + \rho_{12} |\uparrow\rangle\langle\downarrow| + \rho_{21} |\downarrow\rangle\langle\uparrow| + \rho_{22} |\downarrow\rangle\langle\downarrow|,
\]

(3)

where \( \rho_{11} \) and \( \rho_{22} \) must be nonnegative real numbers that sum to unity and \( \rho_{12} \) and \( \rho_{21} \) must be complex conjugates with \( \rho_{12}\rho_{21} \leq \rho_{11}\rho_{22} \) in order that \( \rho \) be a positive-semidefinite Hermitian unit-trace operator. This statistical state can be characterized by the polarization vector \( P \) with Cartesian components \( (\rho_{12} + \rho_{21}, i\rho_{12} - i\rho_{21}, \rho_{11} - \rho_{22}) \) and gives a probability \((1 + P)/2\), where \( P \) is the magnitude of \( P \), of finding the spin in the direction of \( P \).

Now an example of a superscattering matrix for this system is

\[
\delta_{ab} = \lambda \delta_a \delta_b + \frac{1}{2}(1 - \lambda) \delta_{ab} \delta_{cd},
\]

(4)

which simply multiplies the polarization vector by the real number \( \lambda \), where \( 0 \leq \lambda \leq 1 \). If \( \lambda = 1 \), this gives the trivial unitary transformation which takes a pure state (one with \( \rho_{12}\rho_{21} = \rho_{11}\rho_{22} \) or \( P = 1 \), which has the spin pointing in the direction of the unit vector \( P \) with unit probability) to the same pure state. However, if \( \lambda < 1 \), \( \$ \) takes a pure state to a mixed state. But unless \( \lambda = 0 \), a determination of the final state (e.g., by a successive determination of the expectation values of the three
Cartesian components of the spin for a large ensemble of systems with this identical final state) would readily give the initial state, simply by dividing the polarization vector by $\lambda$. (Of course, I am always assuming that the law of evolution, in this case parametrized by $\lambda$, has already been determined, as it could have been by measuring the results of the evolution of an ensemble of systems in different initial states.) Only in the special case $\lambda = 0$ does the final statistical state not uniquely determine the initial statistical state.

Although the simple case just given is quite special, the property appears to be generic, that a superscattering matrix is invertible within the restricted set of density matrices comprising its range, if the dimensions of both the initial and final Hilbert spaces are the same finite integer $m$. That is, for the set of $(m^2 - 1)^2$ real parameters defining the generic superscattering matrix, all but a set of measure zero, given by one or more hypersurfaces of codimension one, or dimension $m^2(m^2 - 2)$, in the $(m^2 - 1)^2$-dimensional space of all the parameters, gives an invertible $. Of course, if $\$ increases the entropy $S$ and decreases $Tr\rho^2$, there are hypothetical positive-semidefinite final density matrices (e.g., pure states) that have no pre-images by $\$ in the space of positive-semidefinite initial density matrices. (The inverse of $\$ would map them to matrices with one or more negative eigenvalues.) However, the generic $\$ would have an inverse within the smaller set of final density matrices given by the range of $\$ acting on the set of general positive-semidefinite initial density matrices of the same finite dimension. On the other hand, if the dimension of the final Hilbert space were smaller than that of the initial one (a rather violent violation of CPT), no superscattering matrix could be invertible.

Thus one might say that a nonunitary superscattering operator does not generally lead to an absolute loss of information about the initial state, assuming the Hilbert spaces stay the same finite dimensions. However, it does lead to a degradation of partial information in the sense that the empirical accuracy of the measured state is reduced in extrapolating back (e.g., in dividing the final approximately-determined polarization vector by $\lambda$). Nevertheless, I shall continue to follow the usual convention of defining a loss of information as an increase in the fine-grained entropy $S \equiv -Tr(\rho \ln \rho)$ of the complete system.

This loss of information proposed by Hawking would be a new feature of quantum gravity, not seen in other quantum field theories in a fixed globally hyperbolic spacetime. Gibbons speculated that it might be related to the indefiniteness of the Einstein action of general relativity for positive-definite metrics (Riemannian or ‘Euclidean’ as opposed to pseudo-Riemannian or Lorentzian).
2. Initial Objections and Alternatives to Hawking’s Proposal

To the best of my knowledge, the first objection to Hawking’s proposal of a loss of information was made by the referee \[7\], who apparently forced Hawking to change the title from “Breakdown of Physics . . .” to “Breakdown of Predictability . . . .” However, I am not aware of whatever detailed objections he may have given. So far as I know now, the first published objection was given by Zel’dovich \[8\], who asked whether Hawking’s “very radical” conclusion is “connected with the fact that he considers a macroscopic black hole? . . . Could not this new and greater indeterminacy arise as a result of this macroscopic and semiclassical treatment of the situation? . . . Must one not treat the emission of a black hole at the quantum level? Can one not, and should one not formulate the theory with black holes in such a way that additional indeterminacy and incoherence do not arise?”

Unaware of Zel’dovich’s objection, I independently objected to Hawking’s proposed nonunitary evolution Eq. (1) on the grounds that it is not CPT invariant, since it takes pure states to mixed states, but it does not give the CPT-reversed process of mixed states going to pure states \[9\]. After making that new observation, I raised the same objection that Zel’dovich had \[8\], that Hawking’s proposal was based on the semiclassical approximation (SCA). I explicitly showed how the SCA would be expected to break down in a noticeable way from fluctuations of the black hole momentum, long before the black hole shrunk to the Planck size, if momentum is conserved in detail and not just on average. Although this particular fluctuation effect by itself would not help restore any of the information Hawking believed would be lost, it did at least illustrate the uncertainty of relying on the SCA for the question of whether the results of black hole evaporation are predictable in the sense that a pure quantum state would be. Therefore, I listed a number of open possibilities that occurred to me at that time, which it may be useful for me to rewrite and comment on as I see them today:

(A) Evolution by an S matrix. This would say that in a quantum theory of everything, including the gravity of the black hole, a pure state of the complete system would always go to a pure state, and no information would be lost. At the time, I wrote that “in the absence of further information, it would seem most productive to pursue the most conservative possibility (A).” Today I might be somewhat inclined to delete the word “most,” but I have not yet seen any strong evidence that (A) does not remain an open possibility, for reasons I shall partially discuss below. In some sense it would be the simplest possibility, though I must admit I still have little idea how it might be actually realized and yet be consistent with what we think we know
about gravity. More recent arguments for this viewpoint include [10-49], as will be discussed below.

(B) Evolution by a CPT-noninvariant superscattering matrix. This was Hawking’s proposal [3]. It seemed undesirable to me to believe in a violation of CPT invariance, despite the previous arguments of Penrose [50], but Hawking, though not one to believe Penrose’s arguments on this, concurred [51] with Wald [52-54] that it would be enough to have CPT in the weak form of CPT-invariant transition probabilities

\[ p(c \rightarrow a) \equiv S_{aa}^{cc} = p(\Theta a \rightarrow \Theta c) \tag{5} \]

between an initial pure state \( c \) and a final pure state \( a \) (no sum on these repeated indices), by using Eq. 1 as an intermediate tool but not interpreting the final density matrix given there as literally the actual final state of the system. I have found this hard to swallow in my naïvely realist view of density matrices as being the more basic objects, and of probabilities as being derived from them, rather than the other way around. However, Hawking’s argument [55] that one should interpret Eq. 1 as merely an intermediate tool for calculating conditional probabilities (given a measurement of a particular initial pure state, what is the conditional probability of measuring a particular final pure state?) now makes more sense to me [56]. Then the asymmetry may indeed be more in the conditional nature of the probability than in any time asymmetry (e.g., CPT noninvariance).

(C) Evolution backward in time by a CPT-noninvariant superscattering matrix. I threw this in as a counterpoint to (B) and wrote, “Then the past would be predictable from the future, but the future would not be predictable from the past. This possibility would suit historians better than physicists.” I suppose that if the superscattering matrix were merely used to calculate conditional probabilities, and if (B) were applicable when the condition were to the past of the result whose conditional probability were to be calculated, then (C) would apply whenever the result were to the past of the condition. This indeed appears to be roughly the case when historians ask, “What is the probability that \( x \) happened in the past, given our present records?”

(D) Evolution by a superscattering matrix which is not of the form Hawking proposed, i.e., not

\[ S_{ab}^{cd} = S_{aa}^{cc} \bar{S}_{dd}^{bb}, \tag{6} \]

where \( A \) denotes an orthonormal basis of states (to be summed over) of the “Hilbert space of all possible data on the hidden surface” which is to surround “either singularities (as in the Schwarzschild solution) or ‘wormholes’ leading to other space-time...
regions about which the observer has no knowledge (as in the Reissner-Nordström or other solutions) [4]. $S_{cA}^c$ would be an $S$-matrix from a state $c$ on an initial surface (before a black hole formed) to a state $aA$ with $a$ on a final surface (after the black hole evaporated) and $A$ on the hidden surface. In my Letter I explicitly assumed an $S$ with this form in (B), and also in (C), except with “initial” and “final” reversed in (C). I didn’t have any motivation for considering other forms of a possible super-scattering matrix, which would be of no help in avoiding violating $CPT$ invariance in my strong sense. However, it now appears to be necessary if the initial and final Hilbert spaces have the same finite dimension (see below).

(E) Evolution of density matrices deterministically but nonlinearly. I don’t see any clear motivation for this, but why not leave it on the table as an open possibility? Nonlinear generalizations of the quantum mechanical evolution of pure states have been considered [57-60], but I am not aware of much discussion of nonlinear evolution of density matrices that do not keep pure states pure [61]. The apparent linearity of quantum mechanics seems to me to be the main reason why we do not notice any influence from other Everett worlds, which surely must exist unless quantum mechanics is modified in a very particular nonlinear way (e.g., by the collapse of the wavefunction somehow very precisely into just the quasiclassical components we observe). I would think that any proposed nonlinearities of quantum mechanics would be very strongly limited by our nonobservance of such other-worldly effects.

(F) Evolution in which black holes or naked singularities form but do not disappear. These would now be called remnants or various other terms [62-65], but this term is also taken to mean other massive objects that I did not think to consider [66], and also objects that can decay unitarily after a very long time [67, 68], which I would have counted as an intermediate state in possibility (A) but did not consider explicitly. As far as the absolutely stable remnants go, I uncritically accepted Hawking’s argument that “Because black holes can form when there was no black hole present beforehand, $CPT$ implies that they must also be able to evaporate completely; they cannot stabilize at the Planck mass, as has been suggested by some observers” [3, 69]. However, Horowitz [70] led me to realize that the only requirement from $CPT$ is that a $CPT$-reversed remnant should be able to combine with $CPT$-reversed Hawking radiation to form a large $CPT$-reversed black hole (i.e., a white hole) which can convert into the $CPT$ reverse of whatever collapsed to form the original black hole; if there is no $CPT$-reversed Hawking radiation impinging on the $CPT$-reversed remnant, it can be absolutely stable and yet be consistent with $CPT$ invariance.

It would seem that there could be several possibilities for a set of absolutely
stable (when isolated) remnants resulting from black hole evaporation that would be consistent with $CPT$: (1) The set could be empty. Then stable remnants would not exist. (2) The set could be nonempty and include the $CPT$ reverse of every element of the set (or at least a quantum superposition thereof). Then each remnant could in principle be made to go away by combining it with the $CPT$ reverse of the Hawking radiation that accompanies the formation of the $CPT$-reversed remnant. (3) The set of remnants could be nonempty but distinct from the $CPT$-reversed set of anti-remnants. (For example, they could be cornucopia geometries [62-65] with internal regions that are expanding toward internal future null or timelike infinities, whereas anti-remnants would have internal regions contracting from internal past null or timelike infinities.) Then remnants could never be destroyed (as anti-remnants could by the $CPT$ reverse of the process of remnant formation), although presumably they could merge or be swallowed by black holes (which could later become remnants again).

In this latter possibility, it is presumably true [69, 71] that a fixed finite energy in a box would never evolve into an absolutely $CPT$-invariant thermal state, since there would eventually tend to be more remnants than anti-remnants. However, contrary to [69, 71], I see nothing violating $CPT$-invariant evolution in this scenario. Furthermore, if from the outside remnants don’t look much different from anti-remnants (as would be the case for cornucopia that tend toward static configurations as seen from the outside), the state of the box at late times could appear very nearly $CPT$ invariant, particularly since one can readily estimate that it would be exponentially rare to have more than one remnant in the box (assuming they can merge or fall into a black hole).

Possibility (3) might be subdivided into case (a) in which information could never be retrieved from inside the remnants, case (b) in which some, but not all, of the information could in principle be retrieved, and case (c) in which all of the information could in principle be retrieved. In case (a) all of the remnants presumably would be distinct from anti-remnants, in (b) some, but not all, of the remnants would be, and in (c) apparently there would be only a single remnant state that would be distinct from its $CPT$-reversed anti-remnant. Cornucopia with internal future null or timelike infinities, where information can go and never be retrieved, would presumably fall into case (a), unless some of the internal information does not go to the future null or timelike infinity, in which case they might fall into case (b).

(G) Evolution in which the disappearance of black holes results in mixed states that are unpredictable. At the time, I did not have any proposed model for this,
but later I realized \cite{72, 73} it could occur for a $CPT$-invariant model in which our universe is an open system, and information can both leave and enter. An analogue would be a room with a window: from the density matrix of the inside of the room alone at one time, one cannot know what light might come in from the outside, and hence one cannot predict even the density matrix inside the room at a later time. Unlike the case of deterministic evolution of the density matrix by a generic superscattering matrix, in the case of an open system one generally cannot extrapolate backward from the later density matrix to a unique earlier one, so information would be truly lost in an even more fundamental way.

(H) Replacement of density matrices by something more fundamental. I had no proposals for this, but in the Letter I did say, “In view of the historical developments in the concept of nature, one might say that the most radical, (H), is the most realistic.” The interpretation of taking the superscattering matrix as merely being a tool to calculate conditional probabilities would in some sense require this, so perhaps Hawking’s proposal of information loss would fit better here than under (B). It is now also extremely interesting to see a whole new formal approach being developed, the decoherent histories reformulation of quantum mechanics, particularly the generalized quantum mechanics without states \cite{74-78}. If it is indeed the correct approach, it should have something to tell us about black holes and information.

In the debate I thus initiated with Hawking, informal discussions showed me that relativists tended to side with Hawking’s viewpoint, often arguing that the event horizon should be an absolute barrier to the recovery of information, whereas particle physicists were much more sympathetic to the possibility of a unitary $S$-matrix. For example, Witten \cite{79} dismissed the idea of the horizon as a barrier, since the uncertainty principle applied to gravity would prevent its localization. However, in the early years there were very few papers about the subject.

One interesting earlier paper that was never even published was Dyson’s suggestion that if information were lost from our universe, it might simply go into what we would now call a baby universe rather than being destroyed at a singularity \cite{80}. (Around the same time, Zel’dovich \cite{81} suggested that baryons could leave our universe and go into a closed space by the Hawking process, but he did not discuss information there, and, as noted above, he later argued \cite{8} against Hawking’s proposed loss of information.) I have already cited Wald’s papers \cite{52-54} that argued that $CPT$ should indeed be broken in a strong sense and pointed out how it could be preserved in a weak sense by a superscattering operator.

A conference abstract of mine \cite{10} noted that another problem with the semiclassical approximation is that if one feeds a black hole for a sufficiently long time
at the Hawking emission rate, the SCA gives an arbitrarily large number of internal configurations for a black hole of a given size, not just the exponential of the entropy Hawking had derived for it \[2, 82\]. Furthermore, it seemed to me that even without CPT invariance in the strong sense, it would be most natural to assume that the Hilbert spaces on the initial and final surfaces would have the same dimension, and then (at least if those dimensions were finite) the postulated existence of the 3-index $S$ matrix of Eq. 6 would imply that the hidden hypersurface could only have a trivial Hilbert space (one unique state $A$). Then the sum over $A$ in Eq. 6 would collapse to a single term, giving a unitary superscattering matrix.

The lack of CPT invariance in the strong sense for a superscattering operator and the problem with the dimensions of the Hilbert spaces motivated me to consider the example of possibility (G), that our universe is an open system, with not one but two hidden Hilbert spaces, one from which states can come (say baby universes in the past), and one to which states can go (say baby universes in the future) \[72, 73\]. One could postulate that there is an $S$ matrix, from the product Hilbert space of our past universe and the past baby universes, to the product Hilbert space of our future universe and the future baby universes. If the initial density matrix on the past product Hilbert space were a tensor product of a fixed density matrix for the past baby universes and an arbitrary density matrix for our past universe (so the baby universes started uncorrelated with our universe), then the final density matrix of our universe would indeed be given by a superscattering matrix (depending on the initial baby universe density matrix, but that is assumed fixed) acting on the initial density matrix of our universe, which would be possibility (B). This would be like the case of a room with a pure state outside, say the completely dark vacuum state, or perhaps a fixed thermal state which is completely uncorrelated with what is in the room. In this case, from knowing the initial state of what is inside, one can give the density matrix of what will bounce back from or enter the window. At the other extreme, if the final density matrix on the future product Hilbert space were a tensor product with a fixed density matrix for the future baby universes, then there would be evolution backward in time by a reversed superscattering matrix, possibility (C). But in general, if the baby universes are correlated with our universe in both the past and the future, or if their statistical state is unknown, one would not have a superscattering matrix at all, possibility (G).

This possibility in some sense sounds the most likely, especially if quantum gravity can allow connections to baby universes that can branch off or join on. However, it raises some questions that are not yet very clear. For example, if the dimension of the Hilbert space of our universe stays the same from past to future, then the two
hidden Hilbert spaces should also have the same dimension in order that there be an $S$ matrix between the two product Hilbert spaces (at least the argument would be valid if all these dimensions were finite). That would mean that there would be in principle as many ways for information to enter our universe as to leave it. And yet the semiclassical approximation seems to show many ways for old information to leave our universe (e.g., by going down a black hole), but the only place it seems to allow for new information to enter is at a possible naked singularity at the end of the black hole evaporation, where the semiclassical approximation breaks down. One might even expect quantum gravity to heal the naked singularity so that no new information enters the universe from it, a possibility I termed Quantum Cosmic Censorship [9]. In other words, the semiclassical approximation suggests that the dimension of the future hidden Hilbert space is large, but that the dimension of the past hidden Hilbert space is small or perhaps even zero. If the two are actually equal, which suggestion is correct? Taking the large dimension supports the view that pure states go to mixed states, but taking the small dimension suggests that little or no information is lost, and that pure states may stay pure. This is one of the strongest reasons for me to think that possibility (A), unitary evolution, is at least reasonably likely, and to resist what often seem to me to be premature reasons to dismiss it.

On the other hand, it could turn out that even if the dimensions of the two hidden Hilbert spaces are identical and nontrivial, some principle influencing the states on those two spaces might make it so that in actuality more information leaves our universe than enters it. This is apparently happening in my office room now at night as I type this, for outside it is dark, and little information in the visual band of photon modes is coming in, whereas there is much more information going out from the light inside. From the inside, I can more easily predict the light I now see (reflected) in the window, whereas in the daytime, I cannot predict the light entering from the clouds outside I see floating past. So in this language the question would be, why do past baby universes seem to be dark?

Perhaps the answer is that something like the Hartle-Hawking no-boundary proposal [83-86], the Vilenkin tunneling proposal [87-94], or the Linde inflationary proposal [95-98] makes the state of small past baby universes simple, just as the state of our past universe seems to have been simple when it was small. Now our universe has grown to be large and complicated, and so if it connects to the Hilbert space of small baby universes in initially simple states, information would naturally tend to go from our universe into the baby universes rather than the other way around.

It would be interesting to try to formulate the problem of black hole information
in terms of the no-boundary proposal, at least if one could avoid being stymied with all of the problems of the path integral for gravity. Assuming that the path integrals are over some contours of complex geometries, one would have to reformulate the concepts of “initial” and “final,” “past” and “future,” etc. so that they are not in terms of the classical Lorentzian concept of time. And then there is the question of whether it is better to treat the possible “loss” of information as a process to be seen in a decohering set of histories [99-103, 74-78, 104], or as something to be found in the records existing in a single “marvelous moment” [105-122]. An interesting recent paper by Smolin [123] takes a quantum-cosmological approach to the problem and conjectures that if quantum effects do not eliminate singularities, “loss of information is a likely result because the physical operator algebra that corresponds to measurements made at late times must be incomplete.”

3. Further Arguments for Four-Dimensional Black Holes

After these rather few responses to Hawking’s original proposal for information loss, some new interest was generated by a new paper [55] in which Hawking attempted to put the idea in an axiomatic framework and apply it to processes involving tiny virtual black holes. He proposed a set of axioms for scattering in quantum gravity with asymptotically flat boundary conditions. These included most of the usual axioms but omitted the axiom of asymptotic completeness, so that a pure initial state would not give a unique pure final state. Alvarez-Gaumé and Gomez [12] gave a rigorous derivation of CPT from Hawking’s axioms and pointed out some difficulties with Hawking’s idea of asymptotic incompleteness. Gross [13] showed that nontrivial topologies in the path integral for quantum gravity need not give asymptotic incompleteness and a loss of information, though Hawking [124] argued that more complicated examples than the ones Gross considered would. Ellis, Hagelin, Nanopoulos, and Srednicki [14] noted that with a superscattering matrix rather than an $S$ matrix, symmetries no longer imply the usual conservation laws, so that the latter would need to added independently. Banks, Peskin, and Susskind [15] showed that making the superscattering operator act locally would lead to a violation of energy-momentum conservation. Hawking’s response [124] was that it should not be made into a local operator. More recent studies [29, 63] have reaffirmed and intensified various aspects of this problem but do not seem to have conclusively shown that no formulation with loss of information could be consistent with energy-momentum conservation.

In addition to the proposals for loss of information, and the ensuing counterargu-
ments, a program on the other side, pursuing possibility (A) above, was launched by ’t Hooft [16, 21-23, 25-28, 31]. He made a bold analogy between string worldsheets and event horizons and attempted to work out the principles for an $S$-matrix for black hole processes. This has provided some innovative ideas of how information might be preserved, but it is unfortunately probably too optimistic to expect this program to be brought to completion in the foreseeable future, because of the severe difficulties of quantum gravity.

Another development that appeared to tip the balance somewhat toward possibility (A) is the work on quantum wormholes and baby universes [125-127, 11, 128-138, 112, 139-146]. This primarily addressed the question of the cosmological constant, but it also addressed the question of whether information can get lost into wormholes. The somewhat surprising answer was that although there may be different superselection sectors of the theory (each with different low-energy effective coupling constants that would have to be determined experimentally rather than theoretically from some fundamental ‘theory of everything’), in each sector one would get an $S$ matrix with no loss of information (though Strominger has recently argued [147] that in each sector one would get an $S$ matrix). Few of the experts claimed that black hole formation and evaporation could be described by wormholes, but Hawking did [133], which seemed to undermine his proposal for loss of information. He did try to argue that the uncertainty of the coupling constants represented the loss of information [138, 145]. However, it would really be different from his previous proposal, in that once one did enough repeated black hole collapse and evaporation experiments to measure the relevant effective coupling constants (assuming there were only a finite number that had any significance for the process at hand), one could predict final pure states for any subsequent experiments. An interesting question would be the number of relevant effective coupling constants for a certain process, and only infinity would correspond to the continual uncertainty of Hawking’s original proposal.

To illustrate this difference, consider a simplified process in which there is a finite box containing either a “red” or a “green” particle of the same mass, each of which would be absolutely stable in the absence of gravity. With gravity, suppose each could collapse to form a black hole which could then emit either kind of particle. (If you do not believe one particle could do this, replace it by two particles or whatever you think the minimum number is.) For simplicity, assume that the black hole can emit its energy in no other forms. In Hawking’s original proposal, an initial pure state of red, say, would become mixed and would tend toward the mixture of 50% probability for red (but never lower) and 50% for green (but never higher).
However, in the wormhole analysis, there would presumably be an effective coupling constant for the transition rate from red to green and vice versa, leading to a coherent oscillation between red and green in each superselection sector. \textit{A priori}, one might not know this rate and therefore not be able to predict better than a density matrix for the final state. However, after sufficient measurements of the rate to find out which superselection sector one is in, one could predict thereafter the oscillation rate. In particular, one could predict times at which the originally red particle has a probability of unity to be green (to the accuracy of the measured coupling constant), a situation that would never occur for Hawking’s superscattering matrix.

Here I have been assuming that there is only one relevant coupling constant for the transition rate. Of course, there might be more, say depending on the state of the particle in the box. If there were were an infinite number of relevant coupling constants, and if they coupled to states outside the box (as they presumably would have to if there are only a finite number of relevant possible states within the box), then one might never be able to predict a probability greater than 50\% for a green particle. This might happen, for example, if the coupling were sufficiently strong to the records of the previous measurements so that they become anti-self-fullfilling prophecies: the results of repeated experiments would be identical only if there were no records to mess up the subsequent experiments, but if no records were kept, then no one could predict the repetition.

At one time I thought Hawking’s suggestion of describing black hole formation and evaporation by a wormhole process was reasonable, and then it seemed that there would be no loss of information once enough measurements were made (with various speculations of how many might be needed) \cite{148}. However, it now seems to me that the standard wormhole calculus is probably not applicable \cite{149}, since it involves integrating over length scales up to the wormhole length in order to get an effective theory at larger scales. With black hole formation and evaporation that would take a timescale of order $M^3$ in Planck units (assuming no long-lived remnant at the end, which would only intensify the problem), it would seem that one would need to integrate over length scales up to about $M^3$, and the effective theory would apply only on length scales larger than that. But then the effective theory could not describe what collapsed to form the hole (in a much shorter time) or the individual quanta being emitted (with wavelength of order $M$).

Bekenstein has recently conjectured \cite{32} that since Hawking radiation is not precisely blackbody but rather is greybody (because of the partially reflecting curvature and angular momentum barriers around a black hole), this nonthermal aspect could code the information in the black hole. However, it does not seem that this effect,
or another similar nonthermal effect (such as stimulated emission when incident radiation is present [36]), could by itself ever lead to a pure final state [150], since each of them occurs already in the semiclassical or even classical approximation. Indeed, the generalized second law for quasistationary black holes [152-155, 1, 2, 156-184] implies that under these approximations the entropy of the radiation would always be at least as large as one quarter the area lost by the black hole. The information actually in the radiation simply means that it is not completely random. For example, it might be such that from the complete final state, even if it is an impure density matrix, one could in principle deduce the complete initial (possibly pure) state, as was discussed above. However, this does not exclude the possibility that the information in the sense of Eq. (2) above might decrease.

Another attempt to avoid a loss of information in black holes is to postulate that black holes never really form. For example, Frolov and Vilkovisky [185, 186] and Hájíček [17, 20] conjectured that gravitational collapse might lead to no singularities or event horizons (only apparent horizons), and so no true black holes. Nevertheless, there would be a very large time delay before ingoing null rays become outgoing null rays, and there would be Hawking radiation, so the quantum-corrected system would appear much like a true semiclassical black hole, thus fulfilling the correspondence principle. Unfortunately, our understanding of quantum gravity is too meagre at present to confirm or refute this conjecture, at least in four dimensions.

An even more direct way to try to eliminate black holes is to assume a different classical theory of gravity. For example, Moffat [187, 188] has postulated that if the correct theory of gravity were NGT rather than GRT, the NGT charge could prevent black holes from forming. But even if NGT were a consistent theory of gravity [189-193], it would allow black holes to be formed from pure radiation without NGT charge, and so it would not really succeed in circumventing the problem. It would probably be very difficult for any simple consistent classical theory of gravity (which agrees with Newtonian gravity and with special relativity in the appropriate limits) to avoid producing black holes in all circumstances.

Other arguments have been given [194-197] that black holes have their area \( A \) quantized in units of \( CL_{Pl}^2 \), where \( L_{Pl} \) is the Planck length (set equal to unity), and \( C \) is a numerical constant of order unity (e.g., \( 8\pi \), [194], \( 4\ln 2 \), [192, 196], or \( 16/(3\pi) \) [197]). This quantization of the black hole would seem to make most sense if there were unitary evolution, with the black hole as part of the intermediate quantum state, and with no loss of information. However, these proposals appear to imply [193] that an uncharged black hole with zero angular momentum and mass \( M \gg 1 \) (which has an area \( A = 16\pi M^2 \), at least classically) would have the
nearest different energy level differing by an energy very nearly $C/(32\pi M)$. Then the black hole could absorb or emit single quanta of radiation with energies only integer multiples of this (i.e., wavelengths equal to $64\pi^2 M/C$, which is of the order of the Schwarzschild radius $2M$ of the hole, divided by an integer, for radiation quanta of zero rest mass). This line spectrum \[195\] would be significantly different from the expected semiclassical limit in which one gets quantum field theory in the classical spacetime of the black hole, which would allow absorption or emission of a continua of frequencies or wavelengths (e.g., wavelengths longer than $64\pi^2 M/C$).

A loophole in this argument is the possibility that the classical relation between area and mass is invalid, so that an area quantization does not imply the naively corresponding mass quantization. But in any case, if the black hole mass is quantized, I would expect the levels generally to be discrete and be separated, on average, by the inverse of the level density (e.g., by roughly $e^{-A/4}$, or $e^{-4\pi M^2}$ for uncharged black holes with zero angular momentum), rather than being clumped in highly degenerate levels that are much further separated (e.g., by $C/(32\pi M)$). (Actually, I would expect the levels to be discrete only in the case that one put the black hole in a finite box and considered the energy levels of the entire system in the box, e.g., the black hole plus the surrounding radiation. For a black hole in infinite space, the instability to evaporation would smear out the levels by amounts much greater than their separations if the separations were indeed of order $e^{-A/4}$.) If my expectation were true, it would seem quite possible to get, in the semiclassical limit, ordinary quantum field theory in the curved spacetime of a classical black hole, with no noticeable line spectra or departures from the expected ordinary thermal spectra.

There has also been a long sequence of analyses of black hole evaporation in string theory, and related processes, by Ellis, Mavromatos, and Nanopoulos [198-210]. The latest conclusion seems to be [210] that there is an $\$ matrix with loss of information. However, I am not competent to judge this work, and there doesn’t seem to be much other comment on it in the literature that I am aware of.

4. Two-Dimensional Black Hole Models

After nearly sixteen years in which only a few papers per year were written directly about Hawking’s proposal, the subject experienced a strong upsurge of interest as a result of a paper by Callen, Giddings, Harvey, and Strominger [211]. This paper replaced the presently intractable problem of four-dimensional gravity with a two-dimensional model theory, motivated by extreme dilatonic black holes [212, 213] and by string black holes [214-218]. This two-dimensional model is much simpler
and yet seems to capture many of the aspects of four-dimensional gravitational collapse and evaporation. One might object that this toy model might miss the heart of the problem in four dimensions, but since we do not yet have any consistent understandable theory of quantum gravity in four dimensions, any attempt to understand the problem must make some simplification or truncation of the unknown complete theory, and so one might as well start first with the simplest such model that apparently has enough of the realistic features. Although the two-dimensional model does not really have independent gravitational degrees of freedom, it does have black holes and matter degrees of freedom (the dilaton and minimally coupled scalars), which can give Hawking radiation and lead to at least some measure of black hole evaporation when the back reaction of this radiation is included.

This simplified model can be solved exactly classically. However, its extension to a consistent quantum theory appears to have an infinite number of arbitrary parameters [219-236, 30, 237-247, 46]. In some cases the quantum theory may be be solved exactly, and in a wider class of cases the semiclassical equations are analytically soluble, but no completely satisfactory quantum model has been yet found. Much of the work has concentrated on solving the semiclassical equations of the original CGHS [211] model. At first it was thought [211] that there would be nonsingular evolution with an S matrix, but then it was shown [62, 220, 248-254] that the semiclassical equations generally lead to singularities. It is not yet known what this model, or the various modifications of it that have been proposed, would really give in a full quantum analysis concerning information loss. However, this model has inspired much new thought about the subject, some of which I shall now summarize.

One approach, which is somewhat motivated by and patterned after ’t Hooft’s program [16, 21-23, 25-28, 31] in four dimensions, is to look for a unitary S-matrix for two-dimensional black hole formation and evaporation [30, 34, 37, 45, 46]. This can apparently be done, for example [34, 37], by imposing reflecting boundary conditions at a critical value of the dilaton field, though there are some subtleties (e.g., when this critical value occurs on a spacelike line), and of course there is the question of whether these boundary conditions are realistic. However, it is interesting that this work seems to give an explicit example of a process that looks very much like black hole formation, followed by nearly thermal Hawking radiation, and yet is described by an S-matrix with no loss of information, possibility (A), that I have continually argued has remained a viable alternative. Unfortunately, it appears that the full S-matrix of [34, 37] may not be unitary, and the part that is may merely represent the part in which no black hole forms [255].
A second series of papers patterned after ’t Hooft’s program is [38, 47, 49], which proposes that there is a “stretched horizon,” a membrane just outside the global event horizon which appears to be physically real to an outside observer. From the outside viewpoint, “the stretched horizon is a boundary surface equipped with microphysical degrees of freedom that appear in the quantum Hamiltonian used to describe the observable world.” As a quantum object, the stretched horizon may be able to store and return all of the information of the gravitational collapse to the outside.

It is not clear in this approach how the information gets from the infalling fields to the stretched horizon. It is not proposed that an infalling observer would feel his or her information getting bleached, so that it would all immediately go onto the stretched horizon as he or she crosses it. Instead, the claim is that “there is complementarity between observations made by infalling observers who cross the event horizon and those made by distant observers.” Preskill’s view of this [250, 253] is that any apparent contradiction which arises when one tries to combine the viewpoints of infalling and outside observers will actually involve assumptions about physics at energies above the Planck scale [19]. Nevertheless, it would be useful to have a mechanism for seeing how in detail the information can get onto the stretched horizon.

Another approach, which lends some support for the opposite conclusion, is the study of scattering by extremal black holes [257, 211, 258, 62, 259-261, 63, 64, 262, 263, 65]. It does seem from these analyses that if one had a theory with absolutely stable large extremal black holes (e.g., electrically charged holes in a theory with no charged particles, such as Einstein-Maxwell theory with only neutral other fields, or magnetically charged holes in a theory with no magnetic monopoles), then there may well be an infinite number of arbitrarily low-energy perturbation states deep down in the throat of these black holes. One could presumably lose an arbitrary amount of information into these states, though it would be a bit ambiguous whether one said this information were lost from our universe or persisting in the remnant.

To be relevant to the problem of black hole formation and evaporation, one would have to assume that these stable black holes are not merely eternal parts of a background spacetime but can be produced by some process. If the black holes are charged in a theory with no charged matter, one could not form them from the gravitational collapse of matter. One might instead imagine forming them by pair production. This raises the problem that if there are an infinitely large number of these remnant states below a fixed finite energy, then it would appear that they should be produced at an infinite rate [67] (e.g., in the Hawking radiation of a larger
black hole). This has appeared to be a very strong argument against an arbitrary amount of information from gravitational collapse going into a remnant of bounded mass (e.g., the Planck mass).

However, possible reasons have been given why the standard argument might be wrong and that an infinite degeneracy of remnants can be produced at a merely finite rate. The first reason given \[67\] was that the remnant form factors might vanish when the momentum transfer is timelike. This possibility, which I am not competent to judge, seems to have been generally ignored in the literature. Another reason given is that the remnants may have infinite internal volumes which can carry an arbitrarily large amount of information while being pair produced at a finite rate. \[63, 64, 202, 65\]. On the other hand, there are counterarguments \[203\] that unless there is what would be described from the effective remnant theory as a “strong coupling conspiracy,” there would still be an infinite production rate. The possibility of an arbitrary amount of information remaining in remnants seems to be more open than I would have thought it was two years ago, but it is yet by no means convincing to me.

Another remnant scenario is Giddings’ proposal \[66\] that the information from gravitational collapse ends up in a remnant whose mass and size depend on its information content. This would avoid the problem of infinite production, since there would only be a finite number of states up to any given mass (in a finite volume), and so any process with a finite amount of energy available could only access a finite number of states. However, even Giddings recognizes that these remnants “would seem to require new physics at weak curvatures and runs afoul of causality” \[263\].

Bekenstein \[183\] has raised a related objection to information-bearing remnants based on his conjectured limit \[158\] on the entropy \(S\) of a system of bounded linear size \(R\) and energy \(E\), \(S \leq 2\pi RE\). (This conjecture is supported by a number of examples in flat spacetime \[158, 162, 165, 264-266, 173\] if one chooses judicious definitions for \(S\), \(R\), and \(E\), and it has somewhat weaker support \[159, 175-177, 181\] for self-gravitating systems, but it by no means seems to be proved in general \[160, 267, 161, 166\].) If the thermodynamic entropy \(S\) of a remnant is indeed limited, but if the information capacity (the number of possible internal states) is not limited, one would have the same problem \[10\] discussed above for the semiclassical approximation applied to black holes of finite thermodynamic entropy that are fed radiation for a sufficiently long time at the Hawking emission rate. However, if those who believe an arbitrarily large amount of information can be put into a black hole in this way without an equal amount coming back out have an answer for this
mystery (why the thermodynamic entropy remains bounded while the information content of the hole grows indefinitely), then the same answer would presumably allow remnants to contain an arbitrarily large amount of information, even if their thermodynamic entropy is bounded, say by Bekenstein’s conjectured limit.

Even if large information-storing remnants can be shown to be possible and truly consistent in some model theory, I am sceptical that they would occur in a realistic theory of gravity. Extremal electrically charged black holes in the real universe are unstable to emitting charged particles, such as electrons and positrons, by essentially the Schwinger process \[268\] (assuming that such black holes are sufficiently large that a semiclassical analysis suffices for the emission rate) \[269-271\]. Extremal magnetically charged black holes would presumably be unstable by the analogous process \[272\] to emitting magnetic monopoles, which in typical grand unified theories exist with magnetic charge greater than mass and therefore could energetically be emitted by an extremal hole. Once the magnetically charged hole emits enough of them to get small enough, there is even a classical instability for it to convert into a monopole-like configuration with a small black hole at the center which then gets hotter and hotter as it emits \[273, 274\]. Only if no such magnetic monopoles existed would extremal magnetically charged black holes be absolutely stable. I suppose that it is conceivable that the true theory of the world would allow some sort of extremal black holes but no suitably charged matter that could be emitted from them. However, until there is more evidence for such hopes, it would seem more natural to investigate theories in which black holes can decay at least down to the Planck mass.

Since we do not know how to analyze what might happen then, one could postulate that the information remains in a Planck size remnant (if they somehow can avoid the infinite-production disaster), but we can as yet do no calculation supporting this hypothesis. In fairness, one could say this is no worse than the hypotheses that black holes evaporate away completely and that information either is or is not lost, neither of which can be presently supported by calculations in domains where they can be believed. Nevertheless, in the absence of a clear mechanism for keeping Planck mass remnants absolutely stable, it would seem more natural to suppose that they would simply decay away. After all, particles that are believed to be absolutely stable all have some symmetry principle preventing their decay (such as charge conservation for the stability of the lightest charged particles, electrons and positrons). Unless there is some unknown symmetry principle protecting a Planck mass remnant, it would seem highly surprising for it not to decay. Indeed, Zel’dovich \[81\] and Hawking \[55\] argued that remnants of primordial black holes would unac-
ceptably dominate the mass density of the universe. However, I am not aware that this argument was ever presented in the days before black hole evaporation was discovered, when it would have presumably have been an even more serious problem. Furthermore, inflation would most likely make the density of the remnants of primordial black holes quite acceptable even if they are massive. Still, if I had to bet, I would lay low odds on the possibility that the information contained in gravitational collapse goes into stable massive remnants.

5. Can the Information Come Out in the Hawking Radiation?

If one examines the alternative hypothesis, that all black holes eventually evaporate completely (assuming that the universe lasts long enough and that it is not a $k = 0$ universe in which black hole coalescence always continues to dominate, on average, over Hawking evaporation [275, 276]), then we still have the uncertainty of whether the information is lost or not. Furthermore, if the information is not lost, there is the question of whether it comes out throughout the Hawking emission or whether it comes out only after the black hole gets down to near the Planck mass. Hawking argued [5] that it would be impossible for the information to come out then, for there would not be enough energy, but that is only valid under what may have been the implicit assumption that the remaining energy comes out in a short time. Aharonov, Casher, and Nussinov [67] proposed that the remnant would decay in an exponentially long time, as Hawking also did later [55]. Carlitz and Willey [277], and later Preskill by a more general argument [68], showed that the actual lower bound for the lifetime of a Planck mass remnant, which contained all the information originally in a black hole of mass $M$, would be of order $M^4$ in Planck units, which is much longer than the period of order $M^3$ for the black hole to get down to near the Planck mass by Hawking radiation, but not exponentially longer.

This slow decay of a remnant which contained all of the original information seems to me no more reasonable than the absolute stability of the remnant. It is hard for me to see why a Planck mass object could last so long. I can see that it would be required if the remnant indeed had a huge amount of information which has to escape as the remnant decays, so what actually seems unlikely to me is the possibility that a small remnant can contain a huge amount of information that eventually comes back out. If it could rigorously be shown that the information in gravitational collapse does not almost entirely come out before the black hole shrinks to a small size, I would think it more reasonable to suppose that the remaining small object disappears in a short time, without the emission of the information, than for
the small object to spit out all the remaining information in the long time required.

Therefore, it seems most probable to me that the black hole decays away completely, and that either the information comes out slowly during the entire emission process, or else it does not come out at all. Various problems have been noted recently with the suggestion that the information comes out, some of which sound serious, so I shall now examine them. It turns out that none of them appear airtight to me.

Giddings [263] argues that this version of possibility (A) “would require that the Hawking radiation extracts all information from the ingoing matter, e.g. through scattering as they cross near the horizon. In particular, matter that crosses the horizon and falls towards \( r = 0 \) must therefore have essentially zero information content. For this reason this option seems...farfetched to many.” If there really is an absolute global event horizon, his argument seems valid, but I think it more likely that the quantum uncertainty applied to the causal structure of the spacetime makes it impossible to define exactly an absolute horizon. The information might be taken out of the matter near what is interpreted classically as \( r = 0 \) and yet not be, with 100% quantum probability, within any putative absolute horizon. Or, the Principle of Black Hole Complementarity [38, 47, 49] may somehow allow the information to fall towards \( r = 0 \) as seen by a freely infalling observer and yet appear to an outside observer to stay outside the horizon.

Harvey and Strominger [231] allow for the fact that a global event horizon may never form but say, “The big difference for black holes (as stressed in [278]) is that until the final Planckian stage of the evaporation they are surrounded by an apparent horizon which is very nearly null. The infalling particles therefore carry the information into a region causally shielded from that part of future null infinity which precedes (in retarded time) the final stage of evaporation. Thus the information cannot come back out without violating macroscopic causality until the black hole has evaporated down to the Planck size. It is conceivable that quantum coherence could be restored by radiation emitted in the final stage of evaporation which is governed by unknown laws of quantum gravity. However, since the total available energy is bounded and small (relative to the initial black hole mass), this is possible only is the radiation is emitted over an extremely long period [67].” Although this argument allows for back reaction so that the apparent horizon is not truly an event horizon, it still takes an essentially semiclassical view with a definite classical metric (whose form presumably depends on some quantum average of the Hawking emission). True quantum fluctuations in the geometry, and hence in the resulting causal structure, could invalidate the argument and allow information to come out.
long before the apparent black hole has shrunk to near the Planck size.

Preskill [68], building on discussions with Susskind and earlier work by Susskind and Thorlacius [249], gives a longer argument that I shall only briefly summarize here. He notes, “On the spactime of an evaporating black hole, it is possible to draw a single spacelike slice that crosses most of the outgoing Hawking radiation, and also crosses the collapsing body, well inside the (apparent) horizon.” If the Hawking radiation on the part of the slice outside the horizon is to be a pure state for each possible pure initial collapse state, the state of the body on the part of the slice inside the horizon must be a unique state (or else for a generic initial collapse state, one would get correlations between the radiation and the body, so neither would be in a pure state). (This argument, a more detailed version of what I had pointed out in [10], as discussed above, was apparently first given in this particular form by Susskind [279], is also discussed in further detail in [150, 38], and is related to the fact that “a single quantum cannot be cloned” [280].)

To get this unique state, Preskill continues [68], “a mysterious force must bleach” the information out of the body before it crosses the horizon, which “is hard to imagine any reasonable way to achieve.” Again, a conceivable way out is the quantum nature of the causal structure. The slice considered here must be nearly null (in the semiclassical picture) to cross most of the Hawking radiation, and it may be impossible in the quantum gravity picture to say that it is definitely spacelike. If it is not, one would not expect to have a tensor product structure of Hilbert spaces for the radiation outside and the body inside, on which the detailed argument depends. (A slightly different-appearing way out is the Principle of Black Hole Complementarity [38, 47, 49] discussed briefly above.)

Nevertheless, if the information almost all comes out before the apparent black hole gets down near the Planck mass (which I am arguing is still a viable possibility), then I would agree with Preskill’s final caveat about this possibility: “At the very least, the semiclassical picture of the causal structure must be highly misleading.”

Giddings and Nelson [281], and later Giddings alone [282], repeated some of the other arguments against the possibility that the information comes out gradually throughout the emission (“Objection: this would appear to imply that either all of the information has been extracted from the infalling matter by the time it crosses the horizon, or that information propagates acausally from behind the horizon to outside” [282]) that I have already countered here, but they also gave a more detailed version of one of Hawking’s original arguments [5]. Starting from a semiclassical calculation for two-dimensional black holes in which information apparently would be lost, they noted that this analysis “may, of course, be invalidated once higher-
order quantum corrections are taken into account. However, these corrections are expected to be unimportant until the weak-coupling approximation breaks down. This only happens in the final stages on the black-hole evaporation. The above arguments therefore strongly suggest that within the present model information does not escape until the black hole is very small. Making these rigorous will therefore rule out one suggested resolution of the black-hole information problem, namely, that the information escapes over the course of black-hole evaporation if the effects of the back reaction are included" [281]. Later Giddings toned this down to a more tentative claim, that “working order-by-order in $1/N$, it is probable that one can construct an argument...analogous to stating that the information doesn’t come out of four-dimensional black holes until they reach the Planck scale” [282].

However, it seems to me more likely that if the information does indeed come out gradually over the entire emission process, initially the rate of information outflow may be so low that it would not show up in order-by-order (perturbative) analysis. The two-dimensional moving mirror model [283] analyzed by Carlitz and Willey [284] and by Wilczek [35] show that it is theoretically possible to have the early Hawking radiation exactly thermal, in a maximally mixed state with no information, but then entirely correlated with the late radiation so that the total state is pure. (The Carlitz-Willey model had the late radiation coming out only after the black hole had shrunk to a small remnant, but one could have the late radiation be simply the second half of the Hawking radiation at the usual rate, so that the information almost entirely comes out in the correlations before the black hole gets extremely small.) An order-by-order analysis of the information in the early radiation would show no information getting out, but the conclusion that the information cannot get out until the black hole gets small would be invalidated by the nonanalytic change in the information rate at the beginning of the correlated late radiation.

One might object that the case of exactly thermal local radiation, with correlations only between the first half and the second half, is an extreme case that is not at all plausible. Therefore, I did a calculation [39] of the correlations one might expect in the most crude approximation that the state of the total radiation from the black hole (once it has completely evaporated) is a random pure state consistent with the macroscopic expectations (conservation of energy, momentum, and angular momentum, in modes that appear to come from where the black hole was, etc.). To calculate the information in an early part of the radiation, I assumed that this radiation and the black hole at that stage were subsystems making up a combined system in a pure state. (This is reminiscent of the entropy inside an
imaginary sphere for a quantum field in the vacuum state \([285, 286]\). I took the exponential of the coarse-grained black hole entropy \((s = A/4, \text{given by Hawking’s semiclassical calculation \([2, 82]\), using a lower-case } s \text{ to denote a coarse-grained entropy or logarithm of the dimension of the Hilbert space of states})\) as an estimate of the dimension of the Hilbert space of black hole states of the same macroscopic characteristics, and the exponential of the initial coarse-grained black hole entropy \((s_0 = A_0/4)\) as an estimate of the dimension of the total accessible Hilbert space of the combined system. Then I calculated the average fine-grained entropy of entanglement, \(S \equiv -Tr(\rho \ln \rho)\), of the radiation subsystem, averaged over all possible pure states of the combined system. The difference between this average entropy and the maximum entropy possible for this subsystem, as given by Eq. 1 above, represents the average information of the subsystem.

If the two subsystems have Hilbert space dimensions \(m\) and \(n\), with \(m \leq n\), then, based on previous work by \([287, 288]\), I found \([289]\) evidence for a conjecture that the average entropy of each subsystem is

\[
S_{m,n} = \sum_{k=n+1}^{mn} \frac{1}{k} - \frac{m-1}{2n}. \tag{7}
\]

(Note added after publication: This conjectured formula was shortly thereafter proved by Foong and Kanno \([290]\).) For \(1 \ll m \leq n\), I found (by reasoning independent of, but consistent with, the conjecture) that the average information in the smaller subsystem is

\[
I_{m,n} \simeq \frac{m}{2n}. \tag{8}
\]

This means that the smaller subsystem typically has very little information in it. That is, for a typical pure quantum state of a large system, the smaller subsystem is very nearly maximally mixed, showing little signs that the total system is pure.

Another way of summarizing this result is to say that if the two subsystems, with large Hilbert space dimensions \(m\) and \(n\), are broken up into tiny sub-subsystems, which typically would each be very nearly maximally mixed, there would be virtually no information is the sub-subsystems considered separately. For quantum information, the whole system contains more information than the sum of the information in the separate parts, and in this case almost all the information giving the precise pure state of the entire system, \(\ln m + \ln n\), is in the correlations of the sub-subsystems. The result above shows that for a typical pure state of the entire system, very little of the information, roughly \(m/(2n)\), is in the correlations within the smaller subsystem itself, roughly \(\ln n - \ln m + m/(2n)\) is in the correlations within the larger
subsystem itself, and the remaining amount of information, roughly $2 \ln m - m/n$, is in the correlations between the larger and smaller subsystems.

Suppose we apply this to the case of a black hole which has lost less than half its initial coarse-grained entropy $s_0 = A_0/4$ to radiation, so

$$m \simeq e^{(s_0 - s)} \leq n \simeq e^s.$$  (9)

Then, using the average above as a typical value, a typical value for the information in this early part of the radiation would be

$$I_{rad} \simeq \frac{1}{2} e^{-(2s - s_0)},$$  (10)

which is exponentially small. For example, if a Schwarzschild black hole decays primarily by emitting massless Hawking radiation so that its mass decreases at the rate

$$- \frac{dM}{dt} = \frac{\alpha}{M^2},$$  (11)

then the initial rate of information outflow from the hole (when $M = M_0$) is

$$\frac{dI_{rad}}{dt} \simeq 4\pi \alpha M^{-1} e^{-4\pi M^2} \simeq 4\pi \alpha x e^{-4\pi/x^2},$$  (12)

where $x = M_{Planck}/M$ is a natural expansion parameter of how good the semiclassical approximation is for large four-dimensional black holes.

For two-dimensional dilatonic black holes, when the quantum corrections are small so that the classical equations of [211] provide good approximations for the various thermodynamical quantities, the Hawking temperature is $T = \lambda/2\pi$ (independent of the mass $M$), so the semiclassical coarse-grained entropy is $s = 2\pi M/\lambda = 2\pi e^{-2\phi_H}$, where $\phi_H$ is the value of the dilaton at the horizon. With $N$ minimally coupled scalar fields, the quantum-corrected equations [211] are valid outside the horizon for $e^{-2\phi_H} \geq N/24$ or $s \geq \pi N/12$, so the minimum coarse-grained entropy of the black hole is bounded below by a constant times the number $N$ of scalar fields.

Therefore, the information in the radiation when its coarse-grained entropy is $s_{rad} < s$ is

$$I_{rad} = s_{rad} - S_{rad} \simeq \frac{1}{2} e^{s_{rad} - s} < e^{s_{rad} - \pi N/12} = e^{s_{rad} - 1/x},$$  (13)

where now the small parameter is $x = 12/(\pi N)$.

Thus we see that the initial rate of information outflow is not analytic in the small expansion parameter $x$ at $x = 0$, so one would never find it by the order-by-order (perturbative) analysis that Giddings and Nelson advocate. That is, even
if they succeed in their goal of proving that the information does not come out at
any finite order of the perturbation, it would not be a convincing argument that the
information is not actually coming out in a nonperturbative way, since that seems to
be the typical behavior for a random joint pure state of a black hole plus radiation.

Even if one looked at most of the radiation so that $I_{rad}$ is not exponentially
small in $1/x$, one would not see the information unless one made at least of order
$e^s$ measurements, i.e., more than of order $e^N$ measurements in the case of radiation
from a two-dimensional dilatonic black hole. The $1/N$ expansion may indeed still
be good until $s$ gets down of order $N$ \cite{291}, so that one can in principle calculate
each typical measurement to an accuracy that is an arbitrarily high power of $1/N$.
However, predicting the more than $e^N$ measurements necessary would presumably
be impossible to enough accuracy by this perturbative method. I.e., suppose that
the rms error of each measurement could be made smaller than any finite power of
$1/N$. But when one squares and sums the errors for more than $e^N$ measurements, one
does not have a result that can be controlled by making $N$ large in this perturbative
analysis. Therefore, this perturbative analysis simply could not say whether the
information is there or not.

Of course, it must be admitted that although there does not seem to be any way
at present to rule out the possibility that the information in gravitational collapse
comes out slowly during the entire period of radiation, no really plausible mecha-
nisms have been proposed for this either. The proposals of 't Hooft \cite{16, 21-23, 25-28, 31}, Miković \cite{30, 46}, the Verlindes \cite{34}, Schouten and the Verlindes \cite{37}, and Russo
\cite{45} are possible mechanisms, and that of \cite{38, 47, 49} is a framework without yet a
mechanism, but it is by no means certain that any of these would work in realistic
situations.

Since we do not yet have a definitive mechanism for getting the information out
from a black hole, permit me to give a few of my own speculations. One idea would
be that the information comes out in wormholes from the high curvature region (i.e.,
near $r = 0$) to just outside the (apparent) horizon. (Such a suggestion of wormholes
with one end inside a black hole has been made by Frolov and Novikov \cite{292}, though
they discuss the case of a small number of macroscopic wormholes, whereas I wish
to consider the possibility of a huge number of microscopic quantum wormholes.)
One must of course ask why the information-transmitting wormholes would tend to
have ends at those locations, since by the wormhole calculus, they are supposed to
have equal amplitudes of attaching on anywhere \cite{135-137, 112, 139, 141}. However,
the amplitudes can be affected by the \textit{conditions} at the locations, and one could
easily imagine that the high curvature could increase the amplitude for wormhole
ends to be there. At the other putative ends, near the horizon, there do not appear to be any special local conditions to increase the amplitude for having wormhole ends there. However, one can have pairs of particles coming out of wormhole ends there without violating the conservation of energy, since one particle can escape to infinity with positive energy, while another carries a corresponding amount of negative energy down the hole. That is, in the path integral the integration over time that leads to the conservation of energy cancels the effects of all wormhole ends that emit but do not absorb particles far outside any horizon (or ergosphere) but leaves the effect of such wormhole ends inside and at the surface of the black hole. Wormhole ends entirely inside the apparent black hole are ineffective in bringing information to the outside, so only wormhole ends near the horizon are unabated by energy conservation and can bring information out from deep down inside a black hole.

The pairs of particles coming out of a putative wormhole end at the black hole horizon must be different from the pairs produced by the gravitational field that lead to the ordinary Hawking radiation, since in the latter, each particle is precisely correlated with its corresponding antiparticle, so that the positive-energy particles going out actually decrease the information in the outside. The particles escaping to infinity from a wormhole end at the horizon would have to be more correlated with other particles going to infinity than with their negative-energy partners that go down the hole in order to increase the information outside. This correlation with other particles going to infinity would need to be brought about by the antiparticles of these other particles going to the other end of the wormhole (the end in the high curvature region).

Without knowing how to do any relevant calculations, its sounds rather unlikely for everything to work out for wormholes to be able to bring out all the information that goes down an apparent black hole, but maybe in some sense any remaining information just persists in the high curvature region until it has been brought back out to the outside by wormholes. A somewhat bigger question I have with this is that if the topological changes of wormholes are allowed, why should the wormholes bring all the information back out to our universe, rather than dumping some of it in another universe (e.g., a baby universe)? Considerations such as these do lead me to feel that a loss of information from our universe may be about as plausible as its preservation, but it seems that both are open possibilities.

For example, instead of wormholes bringing the information out from inside black holes, it might be done by tiny threads or tubes or energy conduits which are narrow regions where the metric and causal structure are much different from that of the
surrounding spacetime, but which do not change the topology. If there were some principle preventing amplitudes for different topologies, such as wormholes, then perhaps information could be prevented from escaping to other universes. However, if amplitudes were allowed for these tubes, through which the future timelike direction might be from the high curvature region near \( r = 0 \) to just outside what would be the horizon in the absence of these tubes, then perhaps, just perhaps, all of the information would always be brought back into outgoing radiation as the apparent black hole evaporates away.

6. Conclusions

At an Open Discussion on June 24, 1993, at the Conference on Quantum Aspects of Black Holes held at the Institute for Theoretical Physics of the University of California at Santa Barbara, Joe Polchinski asked the conference participants to express their opinions about what happens to the information that falls into a black hole. 25 voted for option a), “It’s lost.” 39 voted for b), “It comes out with the Hawking radiation.” 7 voted for c), “It remains (accessible) in a black hole remnant.” Finally, 6 voted for d), “Something else.”

Of course, this voting for specific options represents a renormalization of the varying opinions held within each individual, such as John Preskill, who had just previously concluded his talk \[61\] by saying that in his heart he believed in an S-matrix, whereas in his mind he believed in information loss. To illustrate my own varying opinions, permit me to repeat the subjective likelihoods I listed at the same conference for the various possibilities \[44\]:

\[
\begin{align*}
\text{a) It's lost.} & \quad 25 \% \\
\text{b) Hawking radiation} & \quad 39 \% \\
\text{c) Remains accessible} & \quad 7 \% \\
\text{d) Something else} & \quad 6 \%
\end{align*}
\]
### Possibilities for Information

| A. It’s lost | $30\%$ |
|--------------|---------|
| 1. Evolution forward by $\%$ | $9\%$ |
| 2. Evolution backward by $\%$ | $1\%$ |
| 3. Nonlinear evolution of $\rho$ | $1\%$ |
| 4. Unpredictable evolution (open system) | $14\%$ |
| 5. Inaccessible in a remnant | $5\%$ |

| B. It comes out with the Hawking radiation | $35\%$ |
|-------------------------------------------|---------|
| 1. Unitary $S$ matrix | $33\%$ |
| 2. Nonunitary $S$ matrix | $2\%$ |

| C. It remains accessible in a remnant | $5\%$ |
|-------------------------------------|---------|
| 1. Long-lived decaying remnant | $2\%$ |
| 2. Stable remnant that can be destroyed | $2\%$ |
| 3. Stable remnant that can’t be destroyed | $1\%$ |

| D. Something else | $30\%$ |
|------------------|---------|
| 1. Marvelous moment QM | $5\%$ |
| 2. Gell-Mann–Hartle QM | $5\%$ |
| 3. Other generalized QM | $10\%$ |
| 4. Overthrow of QM | $5\%$ |
| 5. Other | $5\%$ |

Even in my own mind, these subjective estimates of the likelihoods are uncertain by factors of order two (though the sum is not). I should also clarify that the likelihoods under “Something else” were not my estimates of the likelihoods that these possibilities are correct, but that they are correct and give results that cannot be listed under the previous classifications. Actually, after a bit more reflection, I am now inclined to give these possibilities higher likelihoods, so perhaps I should have voted for d) rather than b).

In conclusion, most of the original possibilities still seem to be open, with various ones of these amplified or expanded into new possibilities, such as the various types of remnants that have now been suggested. The introduction of interesting two-dimensional models to this problem has opened up a wide range of new attacks, but it appears to me that no approach we now know will solve the problem in the foreseeable future. Here is certainly a worthwhile challenge for the twenty-first century.
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