Analysis of Flexural Fatigue Strength of Self Compacting Fibre Reinforced Concrete Beams

G Murali*, J P Arul Sudar Celestina, N Subhashini, M Vigneshwari
School of Civil Engineering, SASTRA University, Thanjavur-613 401. India

*Email: murali_220984@yahoo.co.in

Abstract. This study presents the extensive statistical investigation of variations in flexural fatigue life of self-compacting Fibrous Concrete (FC) beams. For this purpose, the experimental data of earlier researchers were examined by two parameter Weibull distribution. Two methods namely Graphical and moment were used to analyse the variations in experimental data and the results have been presented in the form of probability of survival. The Weibull parameters values obtained from graphical and method of moments are precise. At 0.7 stress level, the fatigue life shows 59861 cycles for a reliability of 90%.

Keywords: Weibull parameters, Fibre, Concrete, fatigue, beams.

1. Introduction
According to ACI [1]self-compacting concrete is an innovative technique and is characterised by its ability to consolidate under its own weight without any mechanical consolidation [2]. Addition of fibres in self-compacting concrete (SCC) improves its mechanical properties when compared to conventional concrete [2-6]. The imminent benefits of self-compacting fibrous concrete (SCFC) makes it suitable for wide range of application, especially, in road pavements, off shore platforms under deep sea to produce oil, and bridges where the flexural fatigue loading is predominant [7]. The phenomenon of progressive permanent internal structural change is known as fatigue failure and is caused by repetitive stresses[8-9]. Numerous studies have been carried out in the past few decades to evaluate the fatigue behaviour of concrete and its failure probability. The fatigue life of concrete depends on various factors viz., the typical stress level [10], stress ratio [11], loading frequency [12] and presence of fibre reinforcement [13].

Substantial researches has been accomplished to investigate the flexural fatigue life of SCFC. However, the understanding on the flexural fatigue life in terms of probability of survival, is still lacking although it is important in design of concrete structures and therefore, one is encouraged to work in this direction. Thus, the present investigation deals with the analysis of flexural fatigue life data of an earlier researcher [7] using Weibull distribution and assessment of probability distributions of SFRC.

2. Weibull distribution
The concrete is not homogeneous material, although the level of stress is the same in flexural fatigue test, the experimental data are very scattered. Hence, it is essential to adopt a suitable method to analyse the testing data. Weibull, in 1939, pioneered a probability density function to analyse the
fatigue test data [14-15]. The further study exhibited that the Weibull distribution has convincing backgrounds and may be sufficient to describe the behaviour of concrete under fatigue loading [16-17]. Therefore, in this paper, the Weibull distribution was selected to examine the fatigue performance of SFRC is shown in Table 1.

| S. No | Stress level ‘S’ | 0.9  | 0.85 | 0.8  | 0.75 | 0.7  |
|-------|-----------------|------|------|------|------|------|
| 1     | 1114            | 5685 | 17414| 96251| 282014|
| 2     | 1240            | 6106 | 19650| 119562| 380132|
| 3     | 1327            | 6832 | 23237| 131198| 472329|
| 4     | 1522            | 7229 | 24363| 150947| 518928|
| 5     | 1617            | 7987 | 31927| 178217| 635492|
| 6     | 1726            | 8921 | 34256| 191326| 724120|
| 7     | 1810            | 9542 | 37236| 211319| 812596|
| 8     | 1918            | 10146| 42035| 235282| 893415|
| 9     | 2103            | 13240| 51089| 346603| 1022520|
| 10    | 2685            | 14538| 63549| 394352| 1698642|
| 11    | 2948            | 15831| 72632| 468581| 1882857|

| Mean (N) | 1114 | 5685 | 17414 | 96251 | 282014 |
| Standard Deviation (σ) | 577  | 3473 | 18043 | 121727 | 517441 |

The cumulative distribution function can be expressed as follows [7,13,18].

\[ L_N(n) = e^{\exp \left[ \frac{n}{u} \right]^\alpha} \]  \hspace{1cm} (1)

where ‘n’ represents the fatigue life data N; ‘\( \alpha \)’ is shape parameter at different stress level; ‘\( u \)’ is fatigue life at particular stress level. The fatigue failure life \( N_R \) is based on probability of survival (R) can be written as.

\[ N_R = u(-\ln(R))^\frac{1}{\alpha} \]  \hspace{1cm} (2)

2.1 Graphical method

The natural logarithm transformation for both sides of equation (1) gives

\[ \ln \left[ \ln \left( \frac{1}{L_N} \right) \right] = \alpha \ln n - \alpha \ln u \]  \hspace{1cm} (3)

The equation (3) can be written as follows

\[ Y = \alpha X - \beta \]  \hspace{1cm} (4)

Where, \( Y = \ln[\ln(1/L_N)] \), \( X = \alpha \ln(n) \), and \( \beta = \alpha \ln u \).

Initially, the flexural fatigue failure data should be placed in ascending order and the empirical survivorship function \( L_N \) of each fatigue failure data is obtained from the following expression as [19-20]:

\[ L_N(n) = 1 - \frac{i}{k+i} \]  \hspace{1cm} (5)

where ‘i’ is the failure order number; and ‘k’ denotes the number of fatigue data.

\[ u = \exp \left( \frac{\text{slope}}{\text{intercept}} \right) \]  \hspace{1cm} (6)

2.2 Method of Moments (MOM)

A prominent technique called MOM is commonly used in the engineering research area for assessing Weibull parameters. It depends on the numerical iteration of average fatigue life data and its standard deviation. The ‘\( \alpha \)’ and ‘\( u \)’ can be determined as follows [21]:
The gamma function is defined by
\[ \Gamma(x) = \int_0^\infty t^{x-1} \exp(-t) \, dt \] (9)

### 3. Results and discussions

Using the graphical method, the Weibull parameter value can be assessed from the \( \ln[\ln(1/LN(n))] \) vs \( \ln(n) \) graph in Fig. 1. The slope of line gives the values of ‘\( \alpha \)’ and ‘\( u \)’ can be calculated from equation (6). It can be seen from Figure 1 that the coefficient of determination (\( R^2 \)) for all stress levels are hitting the value greater than 0.90, indicating the linear relationship of \( \ln[\ln(1/LN(n))] \) and \( \ln(N) \), thus proving that the mixes follow the two parameter Weibull distribution. In the case of method of moments, the parameter values attained are \( \alpha = 3.48 \) and \( u = 2022.34 \) at a stress level \( S = 0.9 \). The Weibull parameters values obtained at other stress levels i.e., \( S = 0.8 \) to \( S = 0.7 \) are given in Table 2.

![Figure 1. Graphical analysis of fatigue life data of SCFRC with 0.5% steel fibres.](image)

| Stress level | Parameters | GM | MOM | Average |
|--------------|------------|----|-----|---------|
| S-0.9        | \( \alpha \) | 3.23| 3.48| 3.35    |
|              | \( u \)    | 2034.55| 2022.34| 2028.44 |
| S-0.85       | \( \alpha \) | 2.80| 3.03| 2.91    |
|              | \( u \)    | 10897.01| 10793.18| 10845.09|
| S-0.80       | \( \alpha \) | 2.11| 2.23| 2.17    |
|              | \( u \)    | 43522.91| 42845.94| 43184.42|
| S-0.75       | \( \alpha \) | 1.93| 1.98| 1.95    |
|              | \( u \)    | 263710.78| 258824.05| 261267.41|
| S-0.7        | \( \alpha \) | 1.69| 1.70| 1.69    |
|              | \( u \)    | 973401.07| 949741.76| 961571.41|

The shape and scale parameter values estimated using the graphical method and method of moments are reported in Table 2. It can be noticed from the Table 2, that the average ‘\( \alpha \)’ value for the fatigue life data of SCFC decreases corresponding to the decrease in stress level, thereby indicating lesser deviations at higher stress levels[19]. This lesser deviation in the fatigue life distribution of SCFC might be due to the elimination of conventional vibration techniques that possess the influence
of inherent deficiencies and material defects due to bleeding or segregation. Thus, the SCFC is highly homogenous and predominantly increased fatigue life. Also, the orientation of steel fibres along the direction of concrete flow, enhances the flexural fatigue properties of SCFC significantly when compared to SCC, as reported by few researchers [22-23].

By using Weibull parameters, \( \alpha \) and \( \beta \), the fatigue life of SCFC at different reliability levels (i.e. \( R = 0.99, 0.9 \) to \( 0.1 \) and \( 0.01 \)) corresponding to a stress level \( S \), can be determined using equation (2) and it is shown in Table 3. The fatigue life of SCFC at 0.7 stress level, shows that the cycles of loading decreases to 59861 for 90% reliability (\( R = 0.9 \)). Similarly, for each specific stress level, the fatigue life decreases with increasing reliability level.

| Table 3. Flexural fatigue life data (N) in form of reliability. |
|-------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( R \)          | S-0.9           | S-0.85          | S-0.80          | S-0.75          | S-0.70          |
| 0.01             | 2785            | 17158           | 91623           | 616332          | 2616448         |
| 0.1              | 1393            | 8579            | 45811           | 308166          | 1308224         |
| 0.2              | 973             | 5996            | 32021           | 215399          | 914409          |
| 0.3              | 728             | 4486            | 23954           | 161133          | 684043          |
| 0.4              | 554             | 3414            | 18230           | 122632          | 520595          |
| 0.5              | 419             | 2582            | 13791           | 92767           | 393815          |
| 0.6              | 309             | 1903            | 10163           | 68366           | 290228          |
| 0.7              | 216             | 1329            | 7096            | 47736           | 202646          |
| 0.8              | 135             | 831             | 4440            | 29864           | 126780          |
| 0.9              | 64              | 393             | 2096            | 14101           | 59861           |
| 0.99             | 6               | 37              | 200             | 1345            | 5710            |

4. Conclusions

Based on the statistical analysis the following conclusions can be drawn,
- Using the two parameter Weibull distribution, the fatigue life of SCFC at a particular stress level can be modelled precisely with the correlation coefficient value higher than 0.9.
- The shape parameter of SFRC decreases with decrease in stress level (S-0.9 to S-0.7), thereby denoting lesser deviation in the fatigue life distribution of SCFC at higher stress levels.
- The Weibull parameter values obtained from graphical and method of moments yields almost similar results.
- Fatigue life distribution for SFRC in forms of probability of survival have been obtained by using Weibull distribution function. The fatigue life at 0.7 stress level (considering 59861 cycles of loading) has a reliability of 90%. The survival probability (%) of fatigue life can be found easily corresponding to any stress level. Safe design life is considered much important for brittle structured composites (Concrete). These methods of statistical analysis can be considered as reliable with higher safety limits in identification of the fatigue life under any stress level.

Acknowledgements

The authors would like to gratefully acknowledge the support of SASTRA University.

References

[1] ACI Committee 237 2007 Self-consolidating concrete. ACI Emerging. Technol. Series 1–30. 237R-07.
[2] Yehia S, DoubaA, AbdullahiO and Farrag S2016 Mechanical and durability evaluation of fiber-reinforced self-compacting concrete. Constr. Build. Mater. 121 120-33.
[3] Cunha V M C F, Barros J A O and Sena-Cruz J M 2010 Pullout behaviour of steel fibers in self-compacting concrete. J. Mater. Civil Eng.-ASCE 22 1-9.

[4] Khaliq W and Kodur V.K,R 2011 Effect of high temperature on tensile strength of different types of high-strength concrete. ACI Mater. J. 108(4) 394-402.

[5] Akcay B and Tasdemir M A 2012 Mechanical behaviour and fibre dispersion of hybrid steel fibre reinforced self-compacting concrete. Constr. Build. Mater. 28(1) 287–93.

[6] Sideres K K and Manita P 2013 Residual mechanical characteristics and spalling resistance of fiber reinforced self-compacting concretes exposed to elevated temperatures. Constr. Build. Mater. 41(4) 296–302.

[7] Goel S, Singh S P and Singh P 2012 Residual mechanical characteristics and spalling resistance of fiber reinforced self-compacting concretes exposed to elevated temperatures. Eng. Struct. 40 131–40.

[8] ACI 215R-74 1997 Considerations for design of concrete structures subjected to fatigue loading, ACI Committee Report.

[9] Zandi Y and Akpinar M V 2012 Evaluation of internal resistance in asphalt concretes. Int. J. Concr. Struct. Mater. 6(4) 247-50.

[10] Lee M K and Barr B I G 2004 An overview of the fatigue behavior of plain and fibre reinforced concrete. Cem. Concr. Compos. 26(4) 299–305.

[11] Tepfers R and Kutti T 1979 Fatigue strength of plain, ordinary, and lightweight concrete. ACI J. Proceeding 76(5) 635–52.

[12] Saucedo L, Yu R C, Medeiros A, Zhang X and Ruiz G 2013A probabilistic fatigue model based on the initial distribution to consider frequency effect in plain and fiber reinforced concrete. Int. J. Fatig. 48 308–18.

[13] Mohammadi Y and Kaushik S K 2005 Flexural fatigue-life distributions of plain and fibrous concrete at various stress levels. J. Mater. Civ. Eng. 17(6) 650–8.

[14] Kaitai F and Jianlun X 1987 Statistical distribution. Beijing: Science Press.

[15] Gao Zhentong 1986 Fatigue application statistics. Beijing: National Defence Industry Press.

[16] Oh B H 1986 Fatigue analysis of plain concrete in flexure. J. Struct. Eng. 112(2).

[17] Oh B H 1991 Fatigue-life distributions of concrete for various stress levels. ACI Mater. J.

[18] Gumble E J 1963 Parameters in distribution of fatigue life. J. Eng. Mech. ASCE 45–63.

[19] Ganesan N, Bharati Raj J and Shashikala A P 2013 Flexural fatigue behavior of self-compacting rubberized concrete. Constr. Build. Mater. 44 7–14.

[20] Arora S and Singh S P 2016 Analysis of flexural fatigue failure of concrete made with 100% Coarse Recycled Concrete Aggregates. Constr. Build. Mater. 102 782–91.

[21] Kumar K S P and Gaddada S 2015Statistical scrutiny of Weibull parameters for wind energy potential appraisal in the area of northern Ethiopia. Renewables 2(14) 1-15.

[22] Grunewald S and Walraven J C 2001 Parameter-study on the influence of steel fibers and coarse aggregate content on the fresh properties of self-compacting concrete. Cem. Concr. Res. 31(12) 1793–8.

[23] Goel S, Singh S P and Singh P 2012 Fatigue analysis of plain and fiber reinforced self-consolidating concrete. ACI Mater. J. 109(5) 573–82.