Application of pseudo-spectral full waveform inversion in underground pipeline detection

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Abstract. An accurate description of urban underground pipelines can effectively avoid blind construction and damage to underground pipelines during urban planning, design, construction, and management. Full waveform inversion could reconstruct the underground velocity structures with high resolution and has been widely used in urban engineering geophysical prospecting. The pseudo-spectral method combines the efficiency and accuracy of seismic forward modeling. Using the pseudo-spectral method for full waveform inversion can effectively improve this method's detection accuracy for urban pipelines. In this paper, we test the pseudo-spectral full waveform inversion method in underground pipeline detection. The synthetic test shows that compared with conventional full wave inversion, the pseudo-spectral method is easier to converge and has higher accuracy. The method has great potential to be used to detect the urban underground pipelines.

1. Introduction
Accurate imaging of urban underground pipelines can avoid damaging underground pipelines and even causing disasters during related pipeline planning, design, construction, and management. The ground penetrating radar imaging method can perform high-precision imaging of other underground pipelines. However, in a complex urban environment, its imaging ability is poor for deeper pipeline targets. The seismic exploration method is a high-resolution exploration method that has deep exploration depth and high structure resolution. Full waveform inversion (FWI) is a relatively new method for reconstructing the underground velocity field using seismic data sets. FWI has been widely used in urban engineering geophysical prospecting [1-3]. It could give an image with a resolution below 0.5 meters at a depth of 10 meters to help identify underground pipelines. FWI is closely related to its forward modeling method. A suitable forward modeling method can significantly improve the inversion’s efficiency and accuracy for specific inversion problems. Conventional FWI usually uses the finite difference algorithm to perform forward modeling, which is fast and straightforward. But the dispersion effect is the apparent disadvantage of this method. Refining the grid can solve the frequency dispersion problem, but it will increase calculation and reduce efficiency [4]. Compared with the finite difference method, the pseudo-spectral method can use a larger grid spacing to obtain the same modeling accuracy [5]. This paper tests the pseudo-spectral FWI method using the velocity model generated from the urban underground pipe examples. The results show that compared with conventional FWI, the pseudo-spectral FWI has better converge speed and higher accuracy than the urban underground pipeline imaging.
2. Theory

2.1 Pseudo-spectral method

The pseudo-spectral method’s basic principle is to transform the data from the spatial domain to the frequency wavenumber domain through a fast Fourier transform. Perform Fourier derivation of the data in the frequency wavenumber domain, and then transform it back into the space domain through inverse Fourier transform for calculation. Compared with the finite difference method, the pseudo-spectral method’s main advantage is that it uses Fourier transform to find the space reciprocal and use a large grid spacing to obtain the same simulation accuracy. For the 2D acoustic wave equation:

\[
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{V^2} \frac{\partial^2 u}{\partial t^2}
\]  

(1)

We could get the numerical simulation flow of the pseudo-spectral method in Figure 1.

![Figure 1](image)

**Figure 1.** The working flow of the pseudo-spectral modeling method.

2.2 FWI method

FWI aims to find a suitable physical model to make the synthetic modeling data and field data achieve the best match. It is widely used in petroleum exploration and engineering exploration in recent years. It was introduced in the 1980s [6-7] and developed by many researchers [8-14]. The FWI working flow used in this paper is shown in Figure 2.
3. Synthetic data sets
To test the method, we generate two underground pipelines model based on a real field case from shanghai. Each model contains 4 different types pipelines including concrete pipeline (with velocity 4000 m/s), PVC pipeline (1500 m/s), iron pipeline (5200m/s) and copper pipeline (3500m/s). The background velocity of both models is 1000m/s. In model 1, the pipeline materials from top to bottom are concrete, PVC, iron, and copper. In model 2, the pipeline materials from left to right are concrete, PVC, iron, and copper. The thickness of the pipelines is 50 cm. The model size is 2002*886, and the grid size is 0.01m. Figure 3 shows the two pipelines, velocity models. The source wavelet we use here is an 800Hz ricker wavelet. This source frequency is relatively high for normal seismic exploration. However, we could get a broadband seismic data set by using portable high-frequency vibrators and MEMS receivers. The modeling parameters are shown in Table 1. We generate one data set for each model. The shot spacing is 2m, and 10 shots were generated in total. The receiver spacing is 0.1m, and 20 receivers were recorded in total.

![Figure 3. The true pipeline velocity for model 1 and model 2.](image)

| Table 1: The modeling parameters |
|----------------------------------|
| dx | dz | nx | nz | Source pick frequency |
| 0.01m | 0.01m | 2002 | 886 | 800Hz |

4. Inversion results
We applied both conventional FWI and pseudo-spectral FWI on the synthetic data sets of model 1 and model 2. The starting model is the background velocity model with 1000m/s. 220 iterations were done for model 1 and 140 iterations for model 2 for both conventional and pseudo-spectral FWI. In figure 4, we could find the values of the objective function for each iteration. The pseudo-spectral FWI has a better coverage rate and less misfit function values. Figure 5 shows the inversion results for model 1. The left side of figure 5 is conventional FWI results, and the right side is pseudo-spectral FWI results. From the results, we could find that after 60 iterations, the pseudo-spectral FWI result shows a clear pipeline structure. The conventional FWI fails to give correct structures of some pipeline. After 140 iterations, both methods provide almost the correct pipeline’s structures. The pseudo-spectral FWI gives better velocity match with the true model compared with the conventional FWI. Figure 6 shows
the pseudo-spectral (right) and conventional (left) FWI results for model 2. After 60 iterations, both methods show a good structure match with the true pipeline model. The conventional FWI still has some artifacts in the lower parts of the model, and these artifacts did not disappear as the number of iterations increased. The pseudo-spectral FWI gives a more precise image compared with conventional FWI. The same as model 1, both methods fail to provide correct velocity values. But the pseudo-spectral FWI results show a better velocity match with correct velocity.

**Figure 4.** The values of the objective function of each inversion iterations for model 1(left) and model 2 (right).

**Figure 5.** The conventional FWI results (left) compared with pseudo-spectral FWI results (right) with different iterations for model 1.
5. Conclusions

In this paper, the underground pipeline model’s forward modeling experiment shows that the pseudo-spectrum method has higher simulation accuracy than the traditional finite difference method under the same grid conditions. When simulating the underground pipeline model with a relatively small size, the pseudo-spectrum method is a more suitable forward modeling method. The accuracy of full waveform inversion depends on the forward modeling. The full waveform inversion experiment of the underground pipeline model shows that the waveform inversion based on the pseudo-spectral method can reconstruct a more precise and stable underground velocity structure. The pseudo-spectral method has great potential to be used to detect urban underground pipelines.

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