Research on Inrush Current Mechanisms of Single Phase Transformers

Jianqin Feng¹, Pei Zhang¹* and Zhifei Chen²

¹School of Electrical and Information Engineering, Zhengzhou University of Light Industry, Dongfeng Rd 5, Zhengzhou, China
²School of Management and Engineering, Zhengzhou University, Science Avenue 100, Zhengzhou, China

Email: zhangpei_zzuli@163.com, 1161679542@qq.com

Abstract. Based on a transformer equivalent circuit which is nonlinear but approximately piecewise linear, a set of differential equations are given to constrain the change of flux in the transient process after a no-load transformer is switched on. By solving the differential equations, a group of mathematical expressions are obtained to depict the change law of flux during the transient process. Thereafter, the mathematical expressions are changed into an alternative form of iterative formulae describing change of flux at different time periods, for digital simulation of the inrush current. According to the simulation result, the change curves of flux and current and their components are given, and their typical local change characteristics and corresponding reasons are analyzed. Finally, the reason resulting in decaying of inrush current, the change law of decaying rate of inrush current, and the factors affecting decaying rate of inrush current are respectively analyzed by means of mathematical derivation. Some of the conclusions of the existing researches are confirmed in theory, some incorrect or inaccurate viewpoints are corrected and some new conclusions are obtained in this paper.

1. Introduction

A large excitation current can be generated when the transformer is empty load, and the numerical value can be 4 ~ 8 times that of the rated current. Excitation inrush current leads to transformer winding deformation, the damage of transformer insulation, the lower the quality of power supply, and cause of transformer differential protection misoperation [1][2] [3], thus widely attention for a long time. At present, the research on transformer field flow is mainly focused on the identification method of excitation current flow, which produces a series of research results [4] [5] [6] [7] [8]. However, due to the non-linearity, randomness, confusion and diversity of the excitation current, the existing solutions are not perfect. Therefore, it is necessary to thoroughly study the mechanism of excitation flow and fully grasp its change law, so as to facilitate the correct recognition of excitation current flow.

At present, the research on the mechanism of excitation current flow is relatively small, and it is not thorough enough, either qualitative or oversimplified. Literature analyzed the excitation flow of nondestructive transformers without taking into account the attenuation of flow. Literature only
qualitatively analyses the effect of transformer winding resistance and system resistance on the attenuation of excitation current, but lacks strict theoretical derivation. Obtained by mathematical deduction [9][10] and literature to the same conclusion, analysis on the excitation inrush current decay is still qualitative, and in this paper, through calculating the nonlinear inductance of transformer excitation loop inductance average linearization method is too simplified, not suitable for descriptions of excitation inrush current attenuation law. In addition, the literature [11][12] indicates that the transformer winding resistance and system resistance result in the decrease of the core deflator, which is the reason for the attenuation of excitation current, and the calculation method of the flow attenuation is given. This paper argues that bias only within the duration of the inrush current by the index attenuation law, its attenuation speed depends on the excitation circuit time constant, bias during flow pulsation for basically remain unchanged, as a result the attenuation of bias flow duration is the only reason why excitation inrush current decay to zero, these views worthy of further discussion.

In this paper, starting from the nonlinear equivalent circuit of single-phase transformer excitation circuit, through the study of the piecewise linearization of transformer magnetizing curve, through solving the differential equation, the mathematical description about the change rule of excitation inrush current is obtained, and further analyzes the change mechanism of excitation inrush current, the different conclusions.

2. Mathematical Model
The equivalent circuit of single-phase transformer airload is shown in Figure 1.

![Figure 1. Equivalent circuit for single phase transformers when switching on without load.](image)

In Figure 1, u as the power supply voltage, i as the excitation current, R1σ for transformer winding resistance, L1σ as the leakage inductance of the transformer primary winding, the Rm to reflect transformer core eddy current loss and magnetic hysteresis loss equivalent resistance, L for excitation inductance of the transformer.

Since the eddy current loss and hysteresis loss are very small, the resistance Rm is very large, so it is ignored. Excitation inductance is non-linear inductance. In order to simplify the problem, the magnetization curve is divided into piecewise linearization, as shown in Figure 2.

![Figure 2. Approximate magnetization curve for transformers.](image)

Figure 2, the Lm for transformer core, the excitation inductance is not satisfied when Lμ is iron core saturation excitation inductance, φ as the core of main magnetic flux, Φsat for saturation magnetic flux, Isat for corresponding saturation current saturation magnetic flux. The nonlinear excitation inductance can be approximated using equation (1).
\[ L = \begin{cases} L_m, & |\phi| \leq \Phi_{sat} \\ L_\mu, & |\phi| \geq \Phi_{sat} \end{cases} \]  

(1)

As can be seen from Figure 2, the relationship between magnetic flux and current is

\[ i = \begin{cases} \phi / L_m, & \phi \leq \Phi_{sat} \\ (\phi - \Phi_{sat}) / L_\mu + I_{sat}, & \phi \geq \Phi_{sat} \\ (\phi + \Phi_{sat}) / L_\mu + I_{sat}, & \phi \leq -\Phi_{sat} \end{cases} \]  

(2)

Given that \( I_{sat} \) is small, it is ignored. Make

\[ \Phi_s = \begin{cases} \Phi_{sat}, & \phi \geq \Phi_{sat} \\ -\Phi_{sat}, & \phi < -\Phi_{sat} \end{cases} \]  

(3)

There are

\[ i = \begin{cases} \phi / L_m, & |\phi| \leq \Phi_{sat} \\ (\phi - \Phi_s) / L_\mu, & |\phi| > \Phi_{sat} \end{cases} \]  

(4)

Set the transformer at the time of \( t=0 \), the initial phase Angle of the power supply voltage is \( \alpha \), and the power supply voltage \( u=U_m \sin(\omega t + \alpha) \), and the parameters of each parameter are indicated by the scalar value, and the voltage equation of the excitation circuit is

\[ \frac{d\phi}{dt} + L_\sigma \frac{di}{dt} + R_{\sigma} i = U_m \sin(\omega t + \alpha) \]  

(5)

In equation (4) and equation (5), the magnetic flux equation in the transient process of the transformer no-load is obtained, as shown in equation (6).

\[ \begin{cases} (1 + \frac{L_\sigma}{L_m}) \frac{d\phi}{dt} + \frac{R_{\sigma}}{L_m} \phi = U_m \sin(\omega t + \alpha), |\phi| \leq \Phi_{sat} \\ (1 + \frac{L_\sigma}{L_\mu}) \frac{d\phi}{dt} + \frac{R_{\sigma}}{L_\mu} \phi = U_m \sin(\omega t + \alpha) + \frac{R_{\sigma}}{L_\mu} \Phi_s, |\phi| \geq \Phi_{sat} \end{cases} \]  

(6)

3. The Change Law of Excitation Current Flow

3.1. Mathematical Description

The differential equation (6) can be used to describe the mathematical expression of the change of ferromagnetic flux. See type (7).
\[
\phi = \begin{cases} 
U_m \cos(\omega t + \alpha) - \\
& \left( R_{\sigma \sigma} + \frac{\omega L_{m}}{L_{m}} \right) + \Phi_0 e^{R_{\sigma \sigma} \left( t_m + \frac{1}{T_{m}} \right)} + \Phi_1 e^{R_{\sigma \sigma} \left( t_m + \frac{1}{T_{m}} \right)} 
\end{cases}
\]

Make

\[
\xi_m = \sqrt{\left( \frac{R_{\sigma \sigma}}{\omega L_{m}} \right)^2 + \left( 1 + \frac{L_{m}}{L_{m}} \right)^2}
\]

\[
\xi_{\mu} = \sqrt{\left( \frac{R_{\sigma \sigma}}{\omega L_{\mu}} \right)^2 + \left( 1 + \frac{L_{n}}{L_{n}} \right)^2}
\]

\[
\Phi_m = U_m / \omega \xi_m
\]

\[
\Phi_{\mu \mu} = U_{\mu} / \omega \xi_{\mu}
\]

\[
\beta_m = \arctan \left( \frac{\omega (L_{m} + L_{\sigma \sigma})}{R_{\sigma \sigma}} \right)
\]

\[
\beta_{\mu} = \arctan \left( \frac{\omega (L_{\mu} + L_{\sigma \sigma})}{R_{\sigma \sigma}} \right)
\]

\[
\tau_m = \left( L_{m} + \frac{1}{T_{m}} \right) / R_{\sigma \sigma}
\]

\[
\tau_{\mu} = \left( L_{\mu} + \frac{1}{T_{\mu}} \right) / R_{\sigma \sigma}
\]

The equation (7) can be expressed as
\[
\Phi = \begin{cases} 
\Phi_m \sin(\omega t + \alpha - \beta_m) + \Phi_p e^{-t/\tau_m}, & |\phi| \leq \Phi_{sat} \\
\Phi_{m\mu} \sin(\omega t + \alpha - \beta_m) + \phi_s + \Phi_p e^{-t/\tau_m}, & |\phi| > \Phi_{sat} 
\end{cases}
\]  
(16)

Considering the \(\omega L \gg R_1 \sigma\), \(L_m \gg L_1 \sigma\), \(L_m + L_1 \sigma \gg R_1 \sigma\), so \(\xi_m \approx 1\), \(\beta_m \approx 0.5\pi\), \(\tau_m\) can be thought of infinity, \(e^{-t/\tau_m} \approx 1\), (16) can be simplified to

\[
\phi = \begin{cases} 
\Phi_m \sin(\omega t + \alpha - \pi/2) + \Phi_0, & |\phi| \leq \Phi_{sat} \\
\Phi_{m\mu} \sin(\omega t + \alpha - \beta_m) + \phi_s + \Phi_0 e^{-t/\tau_m}, & |\phi| > \Phi_{sat} 
\end{cases}
\]  
(17)

Make

\[
\phi_p = \begin{cases} 
\Phi_m \sin(\omega t + \alpha - \pi/2), & |\phi| \leq \Phi_{sat} \\
\Phi_{m\mu} \sin(\omega t + \alpha - \beta_m), & |\phi| > \Phi_{sat} 
\end{cases}
\]  
(18)

\[
\phi_{np} = \begin{cases} 
\Phi_0, & |\phi| \leq \Phi_{sat} \\
\phi_s + \Phi_0 e^{-t/\tau_m}, & |\phi| > \Phi_{sat} 
\end{cases}
\]  
(19)

The transformer main flux can be expressed as

\[
\phi = \phi_p + \phi_{np}
\]  
(20)

According to equation (20), ferroflux is composed of two parts, \(\phi_p\) and \(\phi_{np}\), during the transient process after the empty load of the transformer.

\(\phi_p\) is the sinusoidal quantity of the frequency with the power supply voltage, which is generated by the power supply excitation, which is called the mandatory flux component, and its variation rule is shown in equation (18). \(\phi_{np}\) is the free flux component, which makes the transformer magnetic waveform biased to the time axis side, so it is also called partial magnetic field, and its variation rule is shown in equation (19). It can be seen from equation (18) that the forced magnetic flux component changes according to two different sinusoidal rules during the duration of flow and the duration of flow. During the flow pulsation amplitude and phase Angle of \(\phi_m\) and \(\alpha-\beta_m\) respectively to the duration respectively \(\Phi_m\mu\) and \(\alpha-\beta_m\). It can be seen from equation (19) that the free flux component is not periodic component, and the variation law of the duration of flow and flow duration is different. During the flow pulsation, free flux component is a dc component, its value is \(\Phi_0\). In flow duration, free flux component includes a dc component and an exponentially decaying aperiodic component law, the value of the dc component for \(\Phi_s\), aperiodic component of the initial value of \(\Phi_1\), decay time constant for \(\tau_m\). \(\Phi_0\) and \(\Phi_1\) according to enter the flow pulsation periods and \(\phi\) into the initial flow duration time of the initial conditions determine, represented as a unity

\[
\phi(t-0) = \phi(t+0)
\]  
(21)

The change law of excitation flow is obtained by equation (2) and equation (17)
\[
\begin{aligned}
    i &= \begin{cases} 
        \frac{\Phi_m}{L_m} \sin(\omega t + \alpha - \pi/2) + \frac{\Phi_0}{L_m}, & |\phi| \leq \Phi_{sat} \\
        \frac{\Phi_{mu}}{L_{\mu}} \sin(\omega t + \alpha - \beta_{\mu}) + \frac{\Phi_{p0} e^{-t/\tau_{\mu}}}{L_{\mu}}, & |\phi| > \Phi_{sat}
    \end{cases} 
\end{aligned}
\]  

(22)

Because \(L_m\) is large, the excitation current \(i \approx \infty\) during the flow. Make

\[
    I_{mu} = \frac{\Phi_{mu}}{L_{\mu}} 
\]

(23)

\[
    I_{np0} = \frac{\Phi_{p0}}{L_{\mu}} 
\]

(24)

The equation (22) can be simplified

\[
    i = \begin{cases} 
        0, & |\phi| \leq \Phi_{sat} \\
        I_{mu} \sin(\omega t + \alpha - \beta_{\mu}) + I_{np0} e^{-t/\tau_{\mu}}, & |\phi| > \Phi_{sat}
    \end{cases} 
\]

(25)

Make

\[
    i_p = \begin{cases} 
        0, & |\phi| \leq \Phi_{sat} \\
        I_{mu} \sin(\omega t + \alpha - \beta_{\mu}), & |\phi| > \Phi_{sat}
    \end{cases} 
\]

(26)

\[
    i_{np} = \begin{cases} 
        0, & |\phi| \leq \Phi_{sat} \\
        I_{np0} e^{-t/\tau_{\mu}}, & |\phi| > \Phi_{sat}
    \end{cases} 
\]

(27)

There are

\[
i = i_p + i_{np}
\]

(28)

According to the equation (28), the excitation current flow is composed of the force component \(i_p\) and the free component \(i_{np}\). The force current component corresponds to the forced magnetic flux component, and its variation rule is shown in equation (26). The free current component corresponds to the free flux component, and its variation rule is shown in equation (27). During the flow interval, the force component, the free component and the flow of the excitation current are 0. In the duration of flow, the force component is changed by sine, the amplitude is \(I_{mu}\) and the phase Angle is \(\alpha - \beta_{\mu}\). The free component is the aperiodic component of the exponential decay, whose initial value is \(I_{np0}\) and the time constant is \(\tau_{\mu}\). The full current of the current is superimposed by the two. The initial value of the aperiodic component \(I_{np0}\) can be determined by equation (24) or by equation (29).

\[
i(t-0) = i(t+0)
\]

(29)

3.2. Change Curve

Sets the remanence of transformer core before switching to \(\Phi_r\), by type (21), the closing moment core flux is still \(\Phi_r\), namely \(\varphi = \Phi_r\). Because \(\Phi_r < \Phi_{sat}\), \(|\varphi| < \Phi_{sat}\), so the transformer no-load closing the first
core for a period of time will not be saturated, the transformer in phase flow pulsation, the core flux according to the type (17) of the first law of function change. As the change of magnetic flux, when |\(\phi|\leq\Phi_{sat}\), transformer iron core saturation, the transformer into the flow duration, the core flux according to the type (17) of the second kind of function change. When magnetic flux changes to |\(\phi|\leq\Phi_{sat}\), transformer core out of saturation state, and enter the flow pulsation period, until once again into the flow duration. Since then, the flow interval and the duration of continuous flow are repeated alternately, and the main magnetic flux of iron core alternates with two different functions.

Alternating moment for \(t_j(j=0,1,\ldots,m)\), \(t_0\) for transformer no-load closing moments when the first phase flow pulsation of the initial moment, \(t_1\) as the flow for the first time duration of the initial point, \(t_2\) for the second phase flow pulsation of the initial time, \(t_3\) for a second flow duration of initial point, and the \(t_m\) for the last time the end of the flow duration time is end of the entire excitation inrush current, the \(t_2k\) acuities were \(t_2k\leq t < t_2k+1\) is the flow pulsation stage, \(t_2k+1\leq t < t_2(k+1)\) as the continuous phase flow. Here, the indices are going to be \(k=0, 1, \ldots, m/2-1\).

According to different time periods of transformer flux variation, a set of iterative formula can be obtained, as shown in formula (30) ~ (40).

\[
\phi(t) = \phi_p(t) + \phi_{np}(t)
\]

\[
\phi_p(t) = \begin{cases} 
\Phi_m \sin(\omega t + \alpha - \pi/2), & 0 \leq t < t_{2k+1} \\
\Phi_{mu} \sin(\omega t + \alpha - \beta_\mu), & t_{2k+1} \leq t < t_{2(k+1)} \\
\Phi_0(t_{2k}), & t_{2k} \leq t < t_{2k+1} 
\end{cases}
\]

\[
\phi_{np}(t) = \begin{cases} 
\Phi_m \cos \alpha + \Phi_r, & k = 0 \\
\Phi_{mu} \sin(\omega t + \alpha - \beta_\mu) - \\
\Phi_m \sin(\omega t + \alpha - \pi/2) + \\
\Phi'_s + \Phi_i(t_{2k}) e^{-\omega(t-t_{2k})/\tau'}, & k = 1, 2, L, m/2-1
\end{cases}
\]

\[
\Phi_0(t_{2k}) = \begin{cases} 
\Phi_m \cos \alpha + \Phi_r, & k = 0 \\
\Phi_{mu} \sin(\omega t + \alpha - \beta_\mu) - \\
\Phi_m \sin(\omega t + \alpha - \pi/2) + \\
\Phi_i(t_{2k+1}) e^{-\omega(t-t_{2k+1})/\tau'}, & k = 1, 2, L, m/2-1
\end{cases}
\]

\[
\Phi_i(t_{2k+1}) = \Phi_m \sin(\omega t - \pi/2) - \\
\Phi_{mu} \sin(\omega t + \alpha - \beta_\mu) + \\
\Phi_0(t_{2k}) - \Phi'_s, & k = 0, 1, L, m/2-1
\]
\[
\begin{align*}
t_0 &= 0, \\
t_{2k+1} &= \left[ \arcsin(\Phi_i^0 - \Phi_i(t_{2k})) / \Phi_m - \alpha + \pi / 2 \right] / \omega \\
k &= 0, 1, L , m / 2 - 1 \\
\Phi_{m\mu} \sin(\omega t_{2k} + \alpha - \beta_\mu) + \Phi_i(t_{2k+1}) e^{-(t_{2k+1} - t_{2k})/r_t} &= 0 \\
k &= 1, 2, L , m / 2
\end{align*}
\]

(35)

\[
i(t) = i_p(t) + i_{np}(t)
\]

(36)

\[
i_p(t) = \begin{cases} 
0, & t_{2k} \leq t < t_{2k+1} \\
I_{m\mu} \sin(\omega t + \alpha - \beta_\mu), & t_{2k+1} \leq t < t_{2(k+1)} 
\end{cases}
\]

(37)

\[
i_{np}(t) = \begin{cases} 
0, & t_{2k} \leq t < t_{2k+1} \\
I_{np0}(t_{2k+1}) e^{-(t-t_{2k+1})/r_t}, & t_{2k+1} \leq t < t_{2(k+1)} 
\end{cases}
\]

(38)

\[
I_{np0}(t_{2k+1}) = \Phi_i(t_{2k+1}) / L_p
\]

(39)

\[
I_{np0}(t_{2k+1}) \text{ can also be determined by formula (40).}
\]

(40)

According to the type (30) ~ (40) flow flux forced component can be obtained, free weight, total
flux and excitation inrush current force components, freedom, all current numerical solution, and then
draw the transformer magnetic flux and the change of excitation inrush current curve. The change
curve of magnetic flux and current is shown in FIGURE 3.
Can be seen from Figure 3, mandatory stable flux component changes, positive amplitude for $\Phi_{m\mu}$, slightly less than the opposite amplitude $\Phi_m$, its the width of the upper and lower envelope for $\Phi_m+\Phi_{m\mu}$. The bias magnetic field shows the volatility and attenuation. The fluctuation range of the bias is smaller and smaller, until it goes to zero, then it will become bigger and bigger and eventually stabilize. Although the bias is in the fluctuation, the decrease and the increase alternate, but from the envelope, the trend of continuous attenuation, and gradually to zero. The whole flux envelope also shows a tendency of attenuation, and the width of the upper and upper envelope gradually becomes larger, until finally reaching stability. Envelope of the initial width is greater than $2\Phi_{m\mu}$ less than $\Phi_m+\Phi_{m\mu}$ stable when the width of the $2\Phi_m$.

Can be seen from Figure 3, mandatory current component varies at different stages of change rule: in supersaturated phase, partly to timeline side, positive amplitude remains unchanged, peak reverse gradually decay to zero; In the undersaturated phase, it is completely biased to the time axis, and the amplitude is always stable. The free current component is positive in the supersaturation phase and gradually attenuates to zero. In the unsaturated phase, the value is negative and gradually increases, and eventually stabilizes. The full current is completely partial to the time axis in the whole current flow, and the peak shows the continuous attenuation trend and gradually goes to zero.

Points out, by the way, transformer no-load closing moments of the initial bias for $\phi_{np}(0)=\Phi_m\cos\alpha+\Phi_r$, are sufficient and necessary conditions of excitation inrush current

$$|\phi_{np}(0)| > \Phi_{sat} - \Phi_m$$

(41)

The excitation current is generated only when the initial flux satisfies formula (41). If $|\phi_{np}(0)|<\Phi_{sat}$, excitation inrush current process includes undersaturated phase only, the greater the absolute value of $\phi_{np}(0)$, the longer the duration of undersaturated stage. If $|\phi_{np}(0)|>\Phi_{sat}$, excitation inrush current process will include supersaturated phases and undersaturated stage, the greater the absolute value of $\phi_{np}(0)$, the longer the duration of supersaturated phases, then undersaturated phase duration is certain. Because of the initial bias by residual magnetism and no-load switching-in moment power transformer phase Angle to determine, there is a great deal of randomness, so that the
The generation of excitation inrush current, size, and duration have great randomness.

The typical changes of magnetic flux and current and its components in the excitation current flow are shown in Figure 4.

![Graph showing changes in flux and current components](image_url)

**Figure 4.** Local change characteristics of flux and current and their components.

4. **Attenuation of Excitation Current**

Because the bias magnetism determines the generation, size and attenuation of the excitation flow, the attenuation of the excitation current can be studied by the attenuation of bias magnetic.

4.1. **The Reason for the Bias Attenuation**

The change of magnetic bias can be divided into three situations in each workweek. In the duration of flow, the bias is changed by exponential rule. During the alternation of the current and the duration of flow, the bias changes. It can be seen that the factors that lead to partial magnetic attenuation are not external to two: the exponential rule change of the flow duration and the variation of the time bias of the current period.

1. the influence of the change of flow in the current period

By type (32), the flow duration, bias by the constant term and index of two parts, the change of the bias depends on the index of the quantity, the first cycle flow continued during the period bias of the
power frequency variation is

$$\Delta \phi_{np1,l} = -\Phi_{l(t_{z1}, l)}[1 - e^{-(t_{z2} - t_{z1})/\tau_z}]$$  \hspace{1cm} (42)$$

Considering formula (23), formula (39) and formula (40), there are

$$\Delta \phi_{np1,l} = \Phi_{\mu, \omega} \sin(\omega t_{2l-1} + \alpha - \beta \mu)[1 - e^{-(t_{z2} - t_{z1})/\tau_z}]$$  \hspace{1cm} (43)$$

Make \(0_j=\omega t_j+\alpha,(j=0,1,\cdots,m)\). There are

$$\Delta \phi_{np1,l} = \Phi_{\mu, \omega} \sin(\theta_{2l-1} - \beta_j)[1 - e^{-(\theta_j - \theta_{2l-1})/\tan \beta \mu}]$$  \hspace{1cm} (44)$$

In supersaturated phase, \(\sin(\theta_{2l-1} - \beta \mu)<0\), then \(\Delta \phi_{np1,l}<0\), so the bias. In undersaturated phase, \(\sin(\theta_{2l-1} - \beta \mu)>0\), then \(\Delta \phi_{np1,l}>0\), so the bias. Thus, the change of the bias in the supersaturated phase flow duration will cause the attenuation of bias, but instead cause bias in undersaturated phase increases, it is clearly not lead to the root of the excitation inrush current decay to zero. The reason for this phenomenon is that during the supersaturated phase, the free current component is consistent with the partial magnetic direction, so the attenuation of the free current component causes the attenuation of the bias magnetic field. In the unsaturated phase, the free current component is the opposite of the partial magnetic field, and the attenuation of the free current component leads to the increase of the bias magnetic field.

It can be seen from equation (19) that when time goes to infinity, the exponential term tends to zero and the partial magnetic flux tends to saturate the flux. Therefore, if we ignore the change of the bias magnetic variation and only consider the variation of the continuous period of flow, the trend of bias magnetic variation is shown in Figure 5.

![Figure 5. The change trend of bias flux if only considering the influence of exponential term.](image)

Figure 5 shows that the bias in only considering flow duration changes, if the excitation inrush current process from the beginning of the supersaturated phases, the bias in the supersaturated phase continuous attenuation, at the end of the supersaturated phases, bias attenuation to the saturated magnetic flux, after constant; If starting from the undersaturated phase, the bias increases gradually until the saturation flux is reached and then remains constant. Thus it can be seen that the variation of the continuous phase bias is inclined to make the bias towards saturated magnetic flux instead of approaching zero.

(2) the influence of partial magnetic jump on the bias attenuation

The forced magnetic flux component changes continuously in the continuous period and the duration of the flow duration respectively. During the alternating current period and the duration of current flow, the forced magnetic flux component will change from one sinusoid to another sine curve, and then the magnetic flux component will be mutated. Since the total flux cannot be mutated, the bias will be changed accordingly, and its mutation is the same as that of the forced flux component, in the opposite direction.
It is not considered that the change of the magnetic flux component and the partial magnetic variation are shown in FIGURE 6.

![Diagram showing magnetic flux components](image)

**Figure 6.** Mutations of forced flux and bias flux.

It can be seen from FIGURE 6 that the magnetic bias occurs twice in each phase of the cycle, and the total number of mutations in the l frequency cycle is

\[
\Delta \beta = \beta_m - \beta_\mu
\]

(45)

It can be seen from FIGURE 6 that the waveform of the forced magnetic flux component of the duration of flow is symmetric about \( \theta = \pi - \Delta \beta \), so it is

\[
\theta_{2l} = 2(\pi - \Delta \beta) - \theta_{2l-1}
\]

(46)

The equation (42) and (43) can be obtained

\[
\Delta \phi_{nc2,l} = -2\Phi_m \sin \Delta \beta \sin(\theta_{2l-1} + \Delta \beta)
\]

(47)

Because \( \Delta \beta \) usually is very small, \( \theta_{2l-1} \approx \Delta \beta \), \( \theta_{2l-1} + \Delta \beta \approx \theta_{2l-1} \), there are

\[
\Delta \phi_{nc2,l} = -2\Phi_m \sin \Delta \beta \sin(\theta_{2l-1})
\]

(48)

By type (48), when \( \beta_\mu = \beta_m, \Delta \beta = 0 \), \( \Delta \phi_{nc2,l} = 0 \), a bias the total variation in the cycle is zero, bias is not decay; When \( \beta_\mu < \beta_m, \Delta \beta > 0 \), \( \Delta \phi_{nc2,l} < 0 \), the change of the bias amount is negative, partial magnetic attenuation will happen.

Thus draw the conclusion: \( \Delta \beta > 0 \), bias amount of mutations in each cycle power frequency always negative, bias will be diminishing, until the end of the excitation inrush current.

It is shown in FIGURE 7 that the variation trend of bias magnetic variation is only considered when the deflection of the magnetic field is ignored.
(3) influence of transformer winding parameters

If you ignore the transformer primary winding resistance and leakage reactance, namely the $R_{1}\sigma=0$, $L_{1}\sigma=0$, $\beta_{\mu}=\beta_{m}=\pi/2$, $\Delta\beta=0$, $\tau_{m}=\infty$, $\tan\beta_{\mu}=0$, $\Phi_{m}\mu=\Phi_{m}$, by type (44) with type (45) $\Delta\phi_{np1.l}=0$, $\Delta\phi_{np2.l}=0$, so the bias will not decay. At this time, the bias of the duration of the current flow remains the same, and due to the same force flux waveform in the continuous period and the duration of flow, the deflection of the magnetic flux will not occur at the time of alternating current. Therefore, in the whole flow process, the partial magnetic flux is constant, and the whole magnetic flux is changed by the sinusoidal law with the bias axis, corresponding to the so-called non-destructive transformer.

If only to ignore the resistance of the transformer winding, namely the $R_{1}\sigma=0$, $\beta_{\mu}=\beta_{m}=\pi/2$, $\Delta\beta=0$, $\tau_{m}=\infty$, $\tan\beta_{\mu}=\infty$, $\Phi_{m}\mu<\Phi_{m}$, then there is still $\Delta\phi_{np1.l}=0$, $\Delta\phi_{np2.l}=0$, bias is still will not decay. In this case, although the flow pulsation periods and flow flux component of the same phase duration of mandatory, but the amplitudes are not equal, when forced to magnetic flux component from a sine curve changes will still occur on a sine curve to another jump, is just a period of two jump are equal in size, in the opposite direction, the total amount of mutations still is zero. Accordingly, the bias magnetic field will fluctuate, but not attenuate, and its change curve is shown in Figure 8.

![Figure 7](image_url)  
**Figure 7.** The change trend of bias flux if only considering its mutations.

![Figure 8](image_url)  
**Figure 8.** Leakage reactance’s influence on bias flux.

In the case of $R_{1}\sigma\neq0$, $\beta_{\mu}<\beta_{m}$, $\Delta\beta>0$, then $\Delta\phi_{np1.l}\neq0$, $\Delta\phi_{np2.l}<0$ At this time, bias in flow duration according to the index law of change, change in on both sides of the boundary flow duration time, and under the joint action of both, gradually decay to zero, corresponds to the actual situation, the curve is shown in Figure 3.

To sum up, due to the existence of the transformer winding resistance (including resistance) system, the forced component in flow pulsation periods and the phase Angle difference of flow duration $\Delta\beta$ is not zero, the bias in the flow duration on both sides of the border moment of jump in transformer bias continues to decline, leading to continuous attenuation of excitation inrush current.

4.2. The Decay Rate of Bias

As mentioned above, the variation within a cycle consists of two parts: the variation of the duration of flow and the total number of mutations in the two boundary moments of flow duration.

The attenuation rate of partial magnetic field is denoted by the attenuation of the partial magnetic field in each period, and the attenuation rate of the $l$ cycle is due to the exponential term
\[ v_{np1,l} = \left( \frac{\Phi_m}{\xi_\mu} \right) \cos \theta_{2l-1} \left[ 1 - e^{-\left(\theta_{2l-1} - \theta_{2l-1} \right)/\tan \Delta \beta} \right] \]  

(49)

The attenuation velocity caused by a bias magnetic mutation is

\[ v_{np2,l} = 2\Phi_m \sin \Delta \beta \sin \theta_{2l-1} \]

(50)

The total attenuation rate is zero

\[ v_{np,l} = v_{np1,l} + v_{np2,l} \]

(51)

By type (49), The velocity component is related to vnp1,l, \( \theta_{2l-1} \), \( \Delta \beta \) and \( \xi_\mu \). \( \theta_{2l-1} \) depends on the stage during which the current cycle is in the excitation flow, in the over-saturation stage, \( 0 < \theta_{2l-1} < \pi/2 \), \( \cos \theta_{2l-1} > 0 \) and the velocity component vnp1,l. At the beginning of the flow, \( \theta_{2l-1} \) is the smallest, and the \( \cos \theta_{2l-1} \) is the largest, so the velocity component vnp1,l is also the largest. As \( \theta_{2l-1} \) increases, the cosine \( \cos \theta_{2l-1} \) decreases and the velocity component vnp1,l becomes smaller and smaller. At the critical saturation stage, because\( \theta_{2l-1} = \pi/2 \), \( \cos \theta_{2l-1} = 0 \), the velocity component vnp1,l=0. After entering the undersaturated phase, \( \pi/2 < \theta_{2l-1} < \pi \), \( \cos \theta_{2l-1} < 0 \), so the speed component vnp1,l<0. As \( \theta_{2l-1} \) goes from \( \pi \) over \( \pi/2 \), cosine theta of \( \cos \theta_{2l-1} \) goes from 0 to -1, so the absolute value of the velocity component vnp1,l. L is getting bigger and bigger. \( 2(\pi - \theta_{2l-1}) \) represents the width of the flow duration, its maximum value at the beginning of flow, reduce gradually, after therefore \( 2(\pi - \theta_{2l-1}) \) make the influence of velocity component vnp1. At the beginning of the freshness of the absolute value of l is the largest, with the passage of time gradually reduced after, more and more small. Parameter \( \Delta \beta \) will affect the size of the velocity components vnp1,l the size of the time constant of the \( c \tan \Delta \beta \), The smaller \( \Delta \beta \), the greater the time constant of \( c \tan \Delta \beta \), velocity component vnp1,l. The greater the absolute value of l. The influence of parameter \( \xi_\mu \) on vnp1,l is more intuitive. The larger the \( \xi_\mu \), the smaller the absolute value of vnp1,l. \( \theta_{2l-1} \), \( \Delta \beta \) and \( \xi_\mu \) of vnp1,l. The influence is shown in Figure 9.

![Figure 9](image_url)

Figure 9. Influence of factor \( \theta_{2l-1} \) and \( \Delta \beta \) and \( \xi_\mu \) on velocity component vnp1,l.

Figure 9 shows that the parameter is deduced \( \xi_\mu \) of velocity component vnp1,l the influence of the relative to the \( \Delta \beta \) effect is very small, can be ignored. \( \Delta \beta \) is vnp1,l mainly.

By type (50), the velocity component vnp2,l and \( \Delta \beta \) and \( \theta_{2l-1} \). In the whole field of excitation.
flow, 0<θ2l−1<π, sinθ2l−1>0, so the velocity component \( v_{np2.l} \) is always positive. In the supersaturated phase, \( θ2l−1 \) from small to large tends to \( \pi/2 \), \( \sin θ2l−1 \) monotonically increasing, gradually tending to 1, so the velocity component \( v_{np2.l} \) is getting bigger and bigger. At the critical saturation stage, because \( θ2l−1=\pi/2 \), \( \sin θ2l−1=1 \), the velocity component \( v_{np2.l} \) reaches its maximum value. In the undersaturated phase, \( θ2l−1 \) tends to \( \pi \) over \( \pi/2 \), and the \( \sin θ2l−1 \) decreases monotonically, and the 1 tends to 0, so the velocity component \( v_{np2.l} \) is getting smaller and smaller. Since \( Δβ \) is very small, usually between 0 and \( \pi/2 \), \( \sin Δβ \) monotone increasing, so the \( Δβ \), the greater the velocity component \( v_{np2.l} \). \( θ2l−1 \) and \( Δβ \) to \( v_{np2.l} \). The influence is shown in Figure 10.

![Figure 10](image1.png)

**Figure 10.** Influence of factor \( θ2l−1 \) and \( Δβ \) on velocity component \( v_{np2.l} \).

Figure 10 shows that \( Δβ \), the greater the velocity component \( v_{np2.l} \). The faster the bias attenuation, the shorter the duration of excitation flow.

According to the velocity component \( v_{np1.l} \) and \( v_{np2.l} \), the total velocity \( v_{np.l} \) of partial magnetic decrement is obtained by equation (51). The relationship between the three is shown in Figure 11.

![Figure 11](image2.png)

**Figure 11.** Comparison between velocity component \( v_{np1.l} \) and \( v_{np2.l} \) and velocity \( v_{np.l} \).

As you can see from Figure 11, except that the value of the velocity component \( v_{np1.l} \) is roughly equivalent to \( v_{np2.l} \) at the beginning of the flow, In the flow process, it is shown that \( v_{np1.l} \) is less than \( v_{np2.l} \). It can be seen that \( v_{np2.l} \) is the main factor determining the speed of partial magnetic decay.

\( Δβ \) as mentioned above, the parameters of \( v_{np1.l} \) and \( v_{np2.l} \) is larger, the influence of inevitable on total speed \( v_{np.l} \) has very big effect. The influence of \( ξμ \) on the secondary factor \( v_{np1.l} \) without affecting the main factor \( v_{np2.l} \), and the effect on \( v_{np1.l} \) is very small, so the effect on the total speed \( v_{np.l} \) will be smaller and can be ignored. \( Δβ \) to \( v_{np.l} \). The influence is shown in Figure 12.

![Figure 12](image3.png)

**Figure 12.** Influence of factor \( Δβ \) on velocity \( v_{np.l} \).
It can be seen from FIGURE 12 that the velocity of partial magnetic decrement is the largest at the beginning of the current flow, and then the monotonousness decreases, and the velocity of the oversaturation phase decreases rapidly and the speed of the unsaturated phase decreases slowly. $\Delta \beta$ have bigger influence on the attenuation of bias, $\Delta \beta$, the greater the attenuation of bias the greater speed, and vice versa.

Need to point out is, (49) type (50) is under the assumption that the forced flow duration waveform of magnetic component symmetrical about $\omega t=\pi-\Delta \beta$, under the condition of considering the influence of the bias flow duration attenuation, the condition is not established, so the need for them to make correction. The influence of bias magnetic attenuation on magnetic flux waveform is shown in Figure 13.

As can be seen from Figure 13 in supersaturated phase, caused by exponential term attenuation will lower bias flux waveforms, causing $\theta_{2l}$ reach, in undersaturated phase, caused by exponential term bias increase will raise the flux waveforms, causing $\theta_{2l}$.

Sets the offset of $\theta_{2l}$ to $\Delta \theta$, and moved forward positive, ward is negative, there are

$$\theta_{2l} = 2\pi - 2\Delta \beta - \theta_{2l-1} - \Delta \theta$$  \hspace{1cm} (52)

According to the derivation, the velocity component $v_{np 1.1}$ still can be approximated by formula (49), and $v_{np 2.1}$ can be determined by equation (53).

$$v_{np 2.1} = 2\Phi_m [\sin(\Delta \beta + \Delta \theta / 2) - \frac{\sin(\Delta \theta / 2)}{\frac{\pi}{2}}] \sin \theta_{2l-1}$$  \hspace{1cm} (53)

In fact, due to the transformer saturation excitation circuit time constant is still large, in a flow of power frequency cycle duration bias caused by item index variation is very small, $\Delta \theta \approx 0$, ignore mandatory flux waveforms of asymmetric flow duration is acceptable. Ignore $\Delta \theta$ bias attenuation speed before and after the contrast as shown in Figure 14.

Figure 13. Influence of exponential term of bias flux on transformer flux waveform.

As can be seen from Figure 13 in supersaturated phase, caused by exponential term attenuation will lower bias flux waveforms, causing $\theta_{2l}$ reach, in undersaturated phase, caused by exponential term bias increase will raise the flux waveforms, causing $\theta_{2l}$.

Sets the offset of $\theta_{2l}$ to $\Delta \theta$, and moved forward positive, ward is negative, there are

$$\theta_{2l} = 2\pi - 2\Delta \beta - \theta_{2l-1} - \Delta \theta$$  \hspace{1cm} (52)

According to the derivation, the velocity component $v_{np 1.1}$ still can be approximated by formula (49), and $v_{np 2.1}$ can be determined by equation (53).

$$v_{np 2.1} = 2\Phi_m [\sin(\Delta \beta + \Delta \theta / 2) - \frac{\sin(\Delta \theta / 2)}{\frac{\pi}{2}}] \sin \theta_{2l-1}$$  \hspace{1cm} (53)

In fact, due to the transformer saturation excitation circuit time constant is still large, in a flow of power frequency cycle duration bias caused by item index variation is very small, $\Delta \theta \approx 0$, ignore mandatory flux waveforms of asymmetric flow duration is acceptable. Ignore $\Delta \theta$ bias attenuation speed before and after the contrast as shown in Figure 14.

Figure 14. The difference of decaying rate of bias flux between considering factor $\Delta \theta$ and ignoring it.

Figure 14, $v'_{np 1.1}$ to ignore the bias before $\Delta \theta$ attenuation speed, by type (49), (51) and (53) to determine; $v_{np 1.1}$ for ignoring $\Delta \theta$ after partial magnetic attenuation speed, by type (49), (50), and (51).
As can be seen from FIGURE 14, the difference between the two values is not significant, and the trend of change is also roughly the same. Therefore, it is not possible to ignore the asymmetry of the magnetic waveform in the duration of the current flow and not affect the correctness of the foregoing conclusion.

5. Conclusion
Based on the content of this paper, the following conclusions can be drawn:

(1) the sufficient and necessary condition for generating excitation current flow is the difference between the absolute value of initial partial magnetic flux and the amplitude of the forced flux component.

(2) the trend of partial magnetism is changed by the exponential rule in the duration of the current flow. The undersaturated stage increases exponentially.

(3) there is a tendency towards saturation flux in the continuous flow of the bias magnetic flux; During the flow interval, the bias is basically the same. Mutations occur at alternating periods of continuous flow and continuous flow.

(4) because of the existence of transformer winding resistance makes mandatory flux flow component of the flow pulsation period and duration of phase is not zero, cause a bias in the flow duration on both sides of the border moment of jump, lead to transformer bias continues to decline, which cause the continuous attenuation of excitation inrush current, and eventually lead to the end of the excitation inrush current.

(5) the change of partial magnetic flux in the duration of flow will only affect the attenuation velocity of the bias magnetic field, which will not cause the continuous attenuation of the bias magnetic, so it is not the decisive factor that causes the excitation current to end.

6. Acknowledgements
The work here is supported by the National Natural Science Foundation of China(51507157), and the work here is supported by the National Natural Science Foundation of China(51607158). Supported by the Graduate’s Scientific Research Foundation of Zhengzhou University of Light Industry.

7. References
[1] Yuhuan Liu, Yuping Lu and Yubo Yuan 2007 A Novel Scheme Based on Flux Restraint Theory Used in Distinguishing Inrush Currents for UHV Transformers J. Proceedings of the CSEE. 34 52-58
[2] Xuesong Zhang and Benteng He 2005 A New Method To Identify Inrush Current By Phasor Comparison J. Proceedings of the CSEE. 19 43-47
[3] Hongliang Zong, Huafeng Jin, Zhengfei Zhu and Shaojun Zhang 2001 Transformer Inrush Detected By The Variation of Magnetizing Impedance J. Proceedings of the CSEE. 7 91-94
[4] Xue Wang 2012 Synthetic transformer inrush identification based on characteristic space J. Electric Power Automation Equipment. 11 83-86
[5] Jiangping Tan 2008 Identification of transformer magnetization inrush current based on magnetic flux track J. Electric Power Automation Equipment. 12 77-80
[6] Zhengqing Han and Shuping Liu 2007 Magnetizing inrush identification based on differential current characteristics J. Electric Power Automation Equipment. 9 51-54
[7] Zhijuan Lu, Wanshun Liu, Shiwu Xiao and Tao Zheng 2006 A Novel Scheme to Fast Discriminate Inrush Current from Internal Fault of Power Transformer J. Proceedings of the CSEE. 2 47-51
[8] Haifeng Li, Gang Wang, Xiaohua Li and Shaopeng Hu 2005 Distinguish Between Inrush and Internal Fault of Transformer Based on Adaptive Wavelet neural network J. Proceedings of the CSEE. 7 144-150
[9] Yubo Yuan, Dejia Li and Yuping Lu 2005 Physical Mechanism of Sympathetic Inrush of Transformer and Its Influence on Differential Protection. *J. Automation of Electric Power Systems.* **6** 9-14

[10] Hongchun Shu, Xun He and Lixin Li 2006 Research on sympathetic inrush in operating transformer *J. Electric Power Automation Equipment.* **10** 7-12

[11] Jiansong Zhang, Benteng He and Xuesong Zhang 2005 Approach to Calculate Attenuated Magnetizing Inrush Current in Transformer *J. Automation of Electric Power Systems.* **12** 57-60

[12] Xuesong Zhang, Benteng He and Jiansong Zhang 2005 Principle and Influencing Factors of the Transformer SymPathetic Inrush *J. Automation of Electric Power Systems.* **6** 15-19