Anomalous suppression of the shot noise in a nanoelectromechanical system

Federica Haupt\textsuperscript{1}, Fabio Cavaliere\textsuperscript{1}, Rosario Fazio\textsuperscript{2,3}, and Maura Sassetti\textsuperscript{1}

\textsuperscript{1} Dipartimento di Fisica, Universit\`a di Genova and LAMIA-INFM-CNR, Via Dodecaneso 33, 16146 Genova, Italy
\textsuperscript{2} International School for Advanced Studies (SISSA), Via Beirut 2-4, 34014 Trieste, Italy
\textsuperscript{3} NEST-INFM-CNR and Scuola Normale Superiore, Piazza dei Cavalieri 7, 56126 Pisa, Italy

(Dated: July 3, 2006)

In this paper we report a relaxation–induced suppression of the noise for a single level quantum dot coupled to an oscillator with incoherent dynamics in the sequential tunneling regime. It is shown that relaxation induces qualitative changes in the transport properties of the dot, depending on the strength of the electron-phonon coupling and on the applied voltage. In particular, critical thresholds in voltage and relaxation are found such that a suppression below 1/2 of the Fano factor is possible. Additionally, the current is either enhanced or suppressed by increasing relaxation, depending on bias being greater or smaller than the above threshold. These results exist for any strength of the electron–phonon coupling and are confirmed by a four states toy model.

PACS numbers: 73.50.Td,73.23.-b,85.85.+j

I. INTRODUCTION

In the last years, nanoelectromechanical systems (NEMS) have been a hot research topic both from the theoretical and the experimental point of view.\textsuperscript{1,2} Combining electronic and mechanical degrees of freedom, NEMS have potentially important applications as fast and ultra-sensitive detectors\textsuperscript{3,4,5,6} as well as being interesting dynamical systems in their own right. In these devices, current can be used both to create and to detect vibrational excitations. Clear evidence of phonon excitations induced by single electron tunneling has been reported in a number of different systems, including semiconducting phonon cavities,\textsuperscript{7} molecules\textsuperscript{8} and suspended carbon nanotubes.\textsuperscript{9,10} At finite bias electrons tend to drive phonons out of equilibrium; signatures of non equilibrium phonon distribution were observed in a suspended carbon nanotube.\textsuperscript{10,11}

On the theoretical side, NEMS are often described with simple phenomenological models involving a single electron device coupled to an harmonic oscillator.\textsuperscript{12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28} Even within these simple models, many peculiar features such as negative differential conductance,\textsuperscript{12,13,14} shuttling instability,\textsuperscript{15,16} and strong mechanical feedback\textsuperscript{23,24} have been predicted in the case of an underdamped oscillator. It is then a physically relevant question whether the vibrational energy is reduced by relaxation processes induced by coupling to an external environment or rather because tunneling itself.\textsuperscript{18,19,20} Up to now, theoretical works have focused mostly on the case of negligible relaxation, taking the opposite case of strong relaxation as a reference term. Significant differences between these two cases have been found both for weak and strong electron–phonon (e–ph) coupling\textsuperscript{20,21}.

Many recent theoretical works have focused on the study of current noise on NEMS.\textsuperscript{19,20,21,22,23,24,25} In particular, the Fano factor \( F \), which is the ratio between the zero frequency component of the noise and the average current, has proven to be very sensitive to the e–ph interaction and to the details of the phonon distribution.\textsuperscript{20,21} A giant enhancement of Fano factor \(( F \sim 10^2 \sim 10^3 \)) has been predicted for strong coupling and negligible phonon relaxation.\textsuperscript{21} In the opposite limit of strong relaxation, i.e. when the phonons are thermally distributed, a NEMS behaves essentially as a single electron transistor (SET).\textsuperscript{17} Shot noise in SET has been extensively studied\textsuperscript{30,31,32} and \( F \) was always found to be larger or equal to 1/2. However, Fano factors below this limit in the single electron tunneling regime have been predicted in more complicated systems. For instance, coupling to internal degrees of freedom\textsuperscript{33} can induce \( F \) slightly below 1/2 \(( F \sim 0.45 \)) . A strong suppression of the noise has been predicted for the quantum shuttle.\textsuperscript{25,26} In this case, very low values of the Fano factors stem from an highly ordered charge transfer mechanism given by strong correlations between charge and mechanical motion.

In this work we discuss how intermediate phonon relaxation influence the transport properties of a SET coupled to a mechanical oscillator. We focus on the sequential tunneling regime and we adopt a rate equation to describe the dynamics of the system. This approach is justified when the characteristic frequency of the oscillator is much bigger than the tunneling rate\textsuperscript{14,13,20,27} which is the typical experimental situation. We find that finite relaxation rate affects the dynamics in a highly non trivial way. Both current and noise can be either enhanced or suppressed by relaxation, depending on the e–ph coupling and on the considered voltage range. In
particular, for voltages higher than a certain critical value, the Fano factor can be even suppressed below 1/2. This suppression is observed in a completely incoherent regime as a consequence of the interplay between vibration assisted tunneling and direct relaxation of different vibrational states.

The paper is organized as follows. The model Hamiltonian is defined in Sec. II while in Sec. III we introduce the rate equation and the formal expressions for the current and the noise. In Sec. IV numerical results for the current and Fano factor are presented: in particular, the suppression of the Fano factor is discussed in detail for a wide range of parameter. Finally, analytic expressions for the current and the Fano factor are derived within a toy model employing few phononic states.

II. MODEL

In several experimental realizations, either using lithographically defined quantum dots molecules or nanotubes electron transport is dominated by single electron tunneling. In this regime, the system is essentially a SET coupled to a harmonic oscillator. Describing the SET as a single electronic level, the Hamiltonian of the system is

\[ H_s = H_n + H_b + H_{n,b} \]

where\( \hbar = 1 \)

\[ H_n = \varepsilon n, \]

\[ H_b = \omega_0 (b^\dagger b + 1/2), \]

\[ H_{n,b} = \lambda \omega_0 (b^\dagger + b) n. \]

The operator \( n = d^\dagger d \) represents the occupation number of the single level, whose energy \( \varepsilon = \varepsilon(V_g) \) can be tuned with the aid of an external gate voltage \( V_g \). Vibrational excitations are created by \( b^\dagger \) and their ground state is defined as the zero–phonon state when \( n = 0 \). The frequency of the oscillator \( \omega_0 \) can range from the hundreds of MHz of a nanometrical cantilever to a dozen of THz in the case of molecular devices or suspended nanotubes. The dimensionless parameter \( \lambda \) in the coupling term \( H_{n,b} \) represents the strength of the e–ph interaction. For example, \( \lambda \sim 1 \) was reported for the C_{60} devices while values of \( \lambda \) between 0.4 and 3 have been found in different C_{140} samples.

The SET is coupled to external leads by a tunneling Hamiltonian

\[ H_t = \sum_{k,\alpha=1,2} t_{\alpha}(c_{k,\alpha}^\dagger d + d^\dagger c_{k,\alpha}), \]

where the operators \( c_{k,\alpha}^\dagger \) create electrons with momentum \( k \) in lead \( \alpha = 1, 2 \). The leads are described as non interacting Fermi liquids with

\[ H_{leads} = \sum_{k,\alpha=1,2} \varepsilon_{k,\alpha} c_{k,\alpha}^\dagger c_{k,\alpha} \]

and their chemical potential can be shifted by a bias voltage \( V \). For simplicity, in the following we will assume symmetric voltage drops and symmetric barriers \( t_1 = t_2 = t_0 \).

Finally, the oscillator is coupled a dissipative environment that we describe as a set of harmonic oscillators

\[ H_{env} = \sum_{j} \omega_j (a_j^\dagger a_j + 1/2), \]

\[ H_{b,env} = \sum_{j} \omega_j (a_j^\dagger + a_j)(b^\dagger + b). \]

Here \( a_j^\dagger \) are the creation operators of the bosonic bath modes. The environmental coupling is usefully characterized by its spectral function

\[ \mathcal{J}(\omega) = 2\pi \sum_{j} \omega_j^2 \chi_j^2 \delta(\omega - \omega_j). \]

III. RATE EQUATION

The eigenstates of \( H_s \) can be written as \( |n, l\rangle \), where \( n \) denotes the occupation of the single level and \( l \) the phonon number. The coupling to the leads and to the environment induces an energy broadening of these eigenstates. This broadening is the smallest energy scale of the problem, a perturbative treatment for \( H_t \) and \( H_{b,env} \) is appropriate and a master equation for the reduced density matrix of the system can be derived in the sequential tunneling regime. At lowest order, the reduced density matrix is diagonal in \( n \) but may still be off–diagonal in \( l \) because of the e–ph coupling \( H_{n,b} \).

In the following we will consider the case where \( \omega_0 \) is much larger than the bare tunneling rate \( \Gamma^{(0)} = 2\pi \nu F^2 \) (with \( \nu \) the density of states of the leads). In this “diabatic” regime, which is the typical experimental situation, the elements of the density matrix that are non–diagonal in phonon number become negligible. Then, the master equation reduces to a rate equation for the occupation probabilities \( P_{nl} \) of the state \( |n, l\rangle \)

\[ \frac{d}{dt} P_{nl} = \sum_{n' \neq n, l'} \left[ P_{n'l'} \Gamma_{n'\rightarrow l'n} - P_{nl'} \Gamma_{l'\rightarrow l'n'} \right] + \sum_{l'} \left[ P_{n'l} \Gamma_{l'\rightarrow l} - P_{nl} \Gamma_{l'\rightarrow l} \right]. \]

The coefficients \( \Gamma_{n'\rightarrow l'n} \) represent the tunneling rates through the \( \alpha \)-th barrier while \( \Gamma_{l'\rightarrow l} \) are the relaxation rates.

In order to evaluate such rates, it is convenient to eliminate the coupling term \( H_{n,b} \) from \( H_s \) by means of a canonical transformation. Due to the coupling term \( H_{b,env} \), this transformation must include both the operators of the oscillator and those of the environment

\[ \tilde{O} = e^{A_n} O e^{-A_n}, \quad A = \kappa(b^\dagger - b) - 2\kappa \sum_j \chi_j (a_j^\dagger - a_j), \]
where  
\[ \kappa = \frac{\lambda}{1 - 4 \sum_j \chi_j^2 \omega_j / \omega_0}. \]  
(10)

The total Hamiltonian is transformed into  
\[ \tilde{H} = \tilde{H}_n + H_b + H_{\text{leads}} + H_{\text{env}} + \tilde{H}_t + H_{b,\text{env}} \]
where
\[ \tilde{H}_t = \sum_{k,\alpha=1,2} t_0 (e_{k,\alpha}^\dagger e^{-\lambda d} + d^\dagger e^\lambda c_{k,\alpha}). \]  
(11)
and \( \tilde{H}_n = \varepsilon n \) with \( \varepsilon = \varepsilon - \lambda \kappa \omega_0 \). As the energy of the SET is renormalized by a factor proportional to \( \lambda^2 \), this represents the relevant parameter for the e–ph interaction.

The transition rates can now be easily calculated using Fermi golden rule, giving rise to tunneling rates proportional to \( t_0^2 \), and relaxation rates, which depend on \( \chi_j^2 \).

The relaxation rates represent transitions between vibrational excitations without change of the electronic state (\( \beta^{-1} = k_B T \))

\[ \Gamma_{\sigma l}^{\alpha l} = e^{\beta \omega_0} \Gamma_{\sigma l}^{\alpha l} e^{-\beta \omega_0} \]  
(12)
where \( \Gamma_{\sigma l}^{\alpha l} \) is the spectral density of the phonon bath Eq. 5, evaluated at the frequency of the oscillator. Treating \( H_{b,\text{env}} \) at second order allows only transitions between neighboring states (i.e. \( |l' - l| = 1 \)). Transitions with \( |l' - l| \geq 1 \) can be included considering different relaxation mechanisms.22

The charge transfer rates are induced by \( \tilde{H}_t \). Assuming the electrons in the leads are at equilibrium with their chemical potential, one obtains the following expressions

\[ \Gamma_{\sigma l}^{0 l} = \Gamma_{\sigma l}^{0,l'} f_{\sigma}(\omega_0(l' - l)), \]
\[ \Gamma_{\sigma l}^{1 l} = \Gamma_{\sigma l}^{0,l'} [1 - f_{\sigma}(\omega_0(l' - l'))], \]
where \( f_{\sigma}(x) \equiv f(x + \varepsilon - \delta \mu_\sigma) \), \( f(x) \) is the Fermi function and \( \delta \mu_\sigma = (-1)^{\alpha+1} eV/2 \) is the shift of the chemical potential of the leads induced by the bias voltage. The coefficients \( X_{ll'} \) are given by

\[ X_{ll'} = \frac{|\langle n, l | e^{-\lambda(l'-l)} | n, l' \rangle|^2}{e^{-\lambda^2 l'^2 - l^2 - (l'-l) \sum_j \langle L_{\sigma l'}^j \rangle} (\lambda^2)^2}, \]

(15)

where \( l_c = \min\{l, l'\}, l_s = \max\{l, l'\} \) and \( L_{\sigma l'}^j(x) \) is a generalized Laguerre polynomial. These terms are called Franck–Condon factors and are well known from molecular spectroscopy. The effect of the e–ph interaction on transport is two–fold: on one hand it suppresses the effective tunneling rate (because of the factor \( e^{-\lambda^2} \)), on the other it induces a non–trivial dependence on the phononic indices \( l, l' \). Up to moderate e–ph coupling (\( \lambda^2 \leq 1 \)), transitions which conserve or change slightly \( l \) have the largest amplitude and those between states with low vibrational number are dominant. Vice versa, the latter are exponentially suppressed for \( \lambda^2 \gg 1 \), while transitions which change \( l \) considerably become favored.

Within the rate equation approach, the current and noise can be evaluated by means of standard techniques.23,24 It is convenient to adopt a matrix formalism and write the rate equation as

\[ |\dot{P}\rangle = \mathcal{M} |P\rangle, \]

(16)
where the vector \( |P\rangle \equiv \{P_{nl}\} \) represents the time dependent occupation probability distribution. Calling \( |P^{(st)}\rangle \) the stationary solution of Eq. (10), the steady current through the \( \alpha \)-th barrier is

\[ \langle I_\alpha \rangle = e \langle 1 | |\mathcal{I}| | P^{(st)} \rangle, \quad \alpha = 1, 2 \]

(17)
where \( \langle 1 \rangle \equiv (1, 1, \ldots, 1) \), \( e \) is the the charge of the electron and \( \mathcal{I} \) is a matrix representing all the possible transitions through the considered barrier \( \mathcal{I}_{\alpha l,v'} \equiv (-1)^{\alpha+1} (n - n') \mathcal{I}_{l,v'}^{n,n'} \). Following Ref. 31, the zero frequency component of the noise is given by

\[ S_{\alpha\beta}(0) = -2e^2 \langle 1 | (\delta \mathcal{I}_{\alpha} \mathcal{M}^{-1} \delta \mathcal{I}_{\beta} + \delta \mathcal{I}_{\beta} \mathcal{M}^{-1} \delta \mathcal{I}_{\alpha}) | P^{(st)} \rangle + 2e^2 \delta_{\alpha\beta} \langle 1 | |\mathcal{I}| | P^{(st)} \rangle, \]

(18)
where \( \delta \mathcal{I}_{\alpha} = \mathcal{I}_\alpha - \langle \mathcal{I}_\alpha \rangle/e \). Because of charge conservation, the steady current and the zero frequency current correlators are independent of barrier index: \( \langle I_\alpha \rangle = I \), \( S_{\alpha\beta}(0) = S \). In the following we will mostly refer to the Fano factor \( F = S/2eI \).

IV. RESULTS

A. Full solution of the rate equation

The dynamics of the system is characterized by two competing time scales: the average time spent by an electron in the dot \( \tau_\text{el} \) and the phonon relaxation time \( \tau_\text{ph} \). If \( \tau_\text{el} \gg \tau_\text{ph} \), the vibrational excitations tend to relax between each tunneling event to the thermal Bose distribution \( P_l^{(eq)} = e^{-\beta \omega_0} (1 - e^{-\beta \omega_0}) \). In this limit, charge and vibrational degrees of freedom decouple \( P_{nl} = P_n P_l^{(eq)} \) and the dynamics of the system reduces to an effective two–state sequential tunneling process.22 The analytic expressions for current and noise are well known22 and, for \( k_B T \ll eV \), are respectively given by

\[ f^{(eq)} = e \frac{\tilde{\Gamma}_1 \tilde{\Gamma}_2}{\tilde{\Gamma}_1 + \tilde{\Gamma}_2}, \quad F^{(eq)} = \frac{\tilde{\Gamma}_1^2 + \tilde{\Gamma}_2^2}{(\tilde{\Gamma}_1 + \tilde{\Gamma}_2)^2}. \]

(19)
Here \( \tilde{\Gamma}_1 = \Gamma^{(0)} \sum_j a_j f_j(\omega_0) \) and \( \tilde{\Gamma}_2 = \Gamma^{(0)} \sum_j a_j [1 - f_j(-\omega_0)] \) are the renormalized rates for tunneling in and out of the dot and \( a_j \) are Poissonian weight factors.25 \( a_l = \theta(l) e^{-\lambda^2} \lambda^{2l}/l! \). In this case the smallest possible value of the Fano factor is \( F^{(eq)} = 1/2 \).
Vice versa, if $\tau_{el} \ll \tau_{ph}$ the tunneling electrons drive the vibrations out of equilibrium and peculiar features such as negative differential conductance (NDC)\textsuperscript{12,14,27} and super-Poissonian shot-noise\textsuperscript{21} have been predicted.

In our model, a rough estimate of $\tau_{el}^{-1}$ is given by $\tau_{el}^{-1} = \Gamma^{(0)} e^{-\lambda^2}$, i.e. by the effective transparency of the barrier set by the e-ph coupling, while $\tau_{ph}^{-1}$ is determined by environment spectral density $\tau_{ph}^{-1} = \mathcal{J}(\omega_0)$. It is useful to define a dimensionless parameter for the relaxation strength

$$w = \mathcal{J}(\omega_0)/\Gamma^{(0)}.$$\hspace{1cm} (20)

In terms of $w$, the condition for equilibrated phonons $\tau_{el} \gg \tau_{ph}$ reads $w \gg \exp(-\lambda^2)$. It is then evident that the e-ph coupling defines a characteristic scale for relaxation: the stronger is the coupling, the more sensitive is the system to phonon relaxation.

This is reflected by the stationary phonon distribution $P^{(st)}_l = P^{(st)}_l + P^{(st)}_l$. For increasing relaxation strength $w$, $P^{(st)}_l$ tends monotonically to $P^{(eq)}_l$ but with $\lambda^2$-depending speed (see Fig.\textsuperscript{1}). For strong e-ph coupling, the phonon distribution is narrow already in the non-relaxed case $w = 0$ and it reaches equilibrium for values of $w$ which are sensibly smaller than for weak $\lambda^2$.

Since $P^{(st)}_l$ converges monotonically to $P^{(eq)}_l$ for growing $w$, one expects most of the features of the non-equilibrated case to be washed out by increasing relaxation. This is particularly evident in the case of the giant Fano factor observed at low voltages for strong interaction ($\lambda^2 \gg 1$), which is strongly suppressed even by weak relaxation (see Fig.\textsuperscript{2}). This behavior can be easily understood observing that $F \gg 1$ depends dramatically on the non-equilibrium distribution of the vibrational excitations induced by tunneling\textsuperscript{21,28}. In fact, for large $\lambda^2$ transitions between low lying phonon states are exponentially suppressed (see Eq. (19)). Therefore, the main contribution to the current comes from high excited vibrational states (states with large $l$) but at low voltages the occupation probability of those states is strongly suppressed\textsuperscript{14,20}. These conditions leads to avalanches of tunneling processes which, in turn, are responsible for the huge values of $F$\textsuperscript{21,28}. Direct phonon relaxation inhibits this mechanism reducing even further the occupation of states with large $l$ and, consequently, both the current and the Fano factor are strongly suppressed. For very strong relaxation ($w \rightarrow \infty$), $F \rightarrow 1/2$ as one would expect for equilibrated phonon on resonance ($\bar{\varepsilon} = 0$).

![FIG. 1: Stationary phonon probability distribution $P^{(st)}_l$ for different values of $w$. Upper panels: $\lambda^2 = 0.4$, lower: $\lambda^2 = 7$. The rightmost panels ($w = \infty$) represent the thermal Bose distribution $P^{(eq)}_l = e^{-\beta \omega_0}(1 - e^{-\beta \omega_0})$. Other parameters: $V = 3 \omega_0$, $\bar{\varepsilon} = 0$ and $k_B T = 0.02 \omega_0$.](image1)

![FIG. 2: Fano factor as a function of voltage for $\lambda^2 = 16$ and for different values of the relaxation strength $w$: red $w = 0$, green $w = 0.1$, blue $w = 1$, magenta $w = 10$. Dashed line, $F = 1/2$. Other parameters: $\bar{\varepsilon} = 0$ and $k_B T = 0.02 \omega_0$.](image2)

Similarly, relaxation has a destructive effect on NDC (not shown) as this is also a consequence of the peculiarity of the nonequilibrium phonon distribution induced by tunneling itself\textsuperscript{14,27}.

One could be tempted to conclude that considering explicitly the effects of relaxation simply results in an “interpolating” behavior between the opposite limits of no relaxation and thermally distributed phonons. However, we find that finite relaxation rate can induce unexpected features.

Let’s first consider the case of moderate coupling $\lambda^2 = 3$. In Fig.\textsuperscript{3} we plot the Fano factor as a function of voltage for different values of $w$ and for $\bar{\varepsilon} = 0$. It appears that $F$ has a non systematic dependence on $w$: it can be either enhanced or suppressed by relaxation depending on the considered voltage range. For $eV < 6 \omega_0$, it is always $F \geq 1/2$. In particular, for $eV < 2 \omega_0$ it is $F = 1/2$ as the tunneling electrons cannot excite vibrations and the system behaves as an ordinary single level. More interestingly, for $eV > 6 \omega_0$, relaxation can suppress $F$ even below $1/2$.

It is worthwhile to stress that we are dealing with a single electronic level in the sequential tunneling regime,
coupled to a harmonic oscillator with a completely incoherent dynamics. Therefore, this unexpected suppression of the Fano below 1/2 can only be ascribed to the interplay between vibration–assisted tunneling and direct relaxation of the phononic excitations. Indeed, relaxation induces a tendency to ordered transfer of electrons thought the SET via emission–absorption of phonons.

From a numerical analysis, it emerges that this peculiar behavior can be found for any value of the e–ph coupling. In particular, we observed that it exists a voltage threshold set by the e–ph interaction

$$eV_{t}(\lambda) = 2\omega_{0} \text{int}[\lambda^2]$$  \hspace{1cm} (21)$$

such that, for $V > V_{t}(\lambda)$, relaxation larger than a certain threshold value $w_{t}(V, \lambda)$ suppresses the Fano factor below 1/2. This is shown in Fig. 4 which represents a grayscale plot of the Fano factor in the $(V, w)$-plane, for different values of $\lambda^2$. The white contour line corresponds to $F = 1/2$ and separates two different regions in the $(V, w)$-plane: the one to the right of the contour, where $F < 1/2$ and the other one where $F > 1/2$. In other words, the white line denotes $w_{t}$ as a function of $V$ at given $\lambda^2$. The threshold voltage $V_{t}(\lambda)$ corresponds to the position of the vertical asymptote of $w_{t}(V, \lambda)$. For $\lambda^2 < 2$ the critical voltage coincides with the onset of vibration assisted tunneling $eV_{t}(\lambda) = 2\omega_{0}$; vice versa for strong e–ph coupling ($\lambda^2 \gg 1$), $V_{t}(\lambda)$ becomes very large and this is why $F$ is always higher than 1/2 in Fig. 4.

The minimal value assumed by the Fano factor $F_{\text{min}}$ depends itself on the e–ph coupling (see Fig. 5). For weak coupling, $F_{\text{min}}$ differs only slightly from 1/2. For stronger coupling ($\lambda^2 > 1$) it decreases logarithmically and it only reaches the value $F_{\text{min}} \sim 0.4$ for considerably strong interactions. Note that each point in Fig. 5 corresponds to different values of voltage and relaxation strength, as the position of $F_{\text{min}}$ in the $(V, w)$-plane depends on $\lambda^2$. The inset shows the voltage $V_{\text{min}}$ where the minimum is found.

Finally, let’s observe that for $\lambda^2 > 2$, the threshold voltage $V_{t}(\lambda)$ corresponds to the onset of the transition $l : 0 \rightarrow \text{int}[\lambda^2]$. In this case $V_{t}(\lambda)$ is a characteristic voltage also for the current which can be either suppressed or enhanced by relaxation depending on $V$ being smaller or larger than $V_{t}(\lambda)$ (see Fig. 6). Relaxation contributes to populate the low lying phonon states and then, at low voltages, it inhibits the current as the transitions between those states have exponentially suppressed rates. However, for $V > V_{t}(\lambda)$ the transition $l : 0 \rightarrow \text{int}[\lambda^2]$ is allowed and, as it correspond the greatest Franck–Condon factor, it gives a substantial contribution to

FIG. 3: Fano factor as a function of voltage at $\lambda^2 = 3$ and for different values of the relaxation strength $w$: red $w = 0$, green $w = 5$, blue $w = 12$, magenta $w = 100$. Dashed line, $F = 1/2$. Other parameters: $\bar{\varepsilon} = 0$ and $k_{B}T = 0.02\omega_{0}$.

FIG. 4: Density plot of the Fano factor as a function of bias $V$ and relaxation $w$ for different values of $\lambda$. In all the panels: dark gray $F < 1/2$, medium gray $F = 1/2$ (indicated by the arrow in the color map) and light gray $F > 1/2$. The white line, corresponding to $F = 1/2$, represents $w_{t}(V)$. The black line in the 4th panel delimits the region where noise is superpoissonian, $F > 1$. Other parameters: $\bar{\varepsilon} = 0$ and $k_{B}T = 0.02\omega_{0}$.

FIG. 5: Main panel: $F_{\text{min}}$ as a function of $\lambda^2$. Each point corresponds to different values of $w$ and $V$. Inset: voltage $V_{\text{min}}$ where the minimum is found as a function of $\lambda^2$.\[\text{\hspace{1cm}}\]
the current. In this case relaxation has the opposite effect and it sustains the current “feeding” the population of the vibrational ground state. For \( V \sim V_I(\lambda) \), these two mechanisms coexist and, consequently, the current depends only weakly on relaxation (see inset in Fig. 6).

This observation fits nicely what is reported in literature. In fact, for \( \lambda^2 < 2 \) the critical voltage is smaller than the energy required to have phonon-assisted tunneling and then the current is enhanced by phonon relaxation at any voltage, consistently to what was observed in Ref. 19. Vice versa for very strong e–ph coupling the enhancement of the current due to relaxation can be hardly seen as \( V_I(\lambda) \) shifts to very large voltages.

\[ 2w_t(\lambda) = \Delta - 2X_{01} + \sqrt{\Delta^2 + 4X_{01}^2}, \quad (24) \]

such that for \( w > w_t \) the Fano factor is smaller than 1/2. For stronger e–ph coupling \( \lambda^2 > 2 \) (\( \Delta < 0 \)), it is always \( F > 1/2 \). This confirms the numerical estimate \( eV_I(\lambda) = 2\omega_0 \) as the threshold voltage for any \( \lambda^2 < 2 \).

Despite the coarseness of the model, Eq. (24) accords qualitatively with the exact numerical solution for \( eV < 4\omega_0 \) (see Fig. 7). The agreement is reasonably good even in the case of weak e–ph coupling, where the phonon distribution is mostly broadened and one expects the four state approximation to be more inaccurate. A better agreement can be obtained considering a six state model with \( n = 0, 1 \) and \( l = 0, 1, 2 \) but, in this case, the analytic solutions become quite cumbersome and we don’t report them here for simplicity. The agreement of the four states model with numerical result suggests that \( F < 1/2 \) rather depends on the interplay between relaxation and vibration–assisted tunneling, than on the possibility to access an high number of vibrational states.

\[ I = e\Gamma(0) \left[ \frac{X_{00}}{2} + \theta(eV-2\omega_0) \frac{X_{01}(w+2X_{01} - \Delta)}{2(w+2X_{01})} \right], \quad (22) \]

and

\[ F = \frac{1}{2} \frac{\theta(eV-2\omega_0)X_{01}\Delta[w^2 + w(2X_{01} - \Delta) - X_{01}\Delta]}{(w+2X_{01})^2 K}, \quad (23) \]

where \( \Delta = X_{00} - X_{11} \) and \( K = [w(X_{01} + X_{11} + \Delta) + X_{01}(2X_{01} + 2X_{11} + \Delta)] \).

From Eq. (24) it is easy to show that the current is an increasing function of \( w \) only for \( \Delta > 0 \) (that is, for \( \lambda^2 < 2 \), see Eq. 15). Vice versa, for \( \Delta < 0 \) (\( \lambda^2 > 2 \)) the current decreases for increasing relaxation, in agreement with what was previously observed. Moreover Eq. (24) tells that \( \Delta > 0 \) (\( \lambda^2 < 2 \)) is the necessary condition to have \( F < 1/2 \) in the region \( 2\omega_0 < eV < 4\omega_0 \). In fact only in this case, it exist a threshold value for relaxation.
non–monotonous behavior of the Fano factor, which can be suppressed even below $1/2$. This relaxation–induced tendency to order of the electronic transfer through the dot is unexpected, since we are dealing with an oscillator with incoherent dynamics coupled to a SET in the sequential tunneling regime. The onset of this oscillator with incoherent dynamics coupled to a SET the dot is unexpected, since we are dealing with an tendency to order of the electronic transfer through $V_0$ below $1$. Sapmaz, P. Jarillo-Herrero, Ya. M. Blanter, C. Dekker, B. J. LeRoy, S. G. Lemay, J. Kong, and C. Dekker, Nature 304, 74 (2004).

E. M. Weig, R. H. Blick, T. Brandes, J. Kirschbaum, W. Wegscheider, M. Bichler, and J. P. Kotthaus, Phys. Rev. Lett. 92, 046804 (2004).

H. Park, J. Park, A. K. Lim, E. H. Anderson, A. P. Alivisatos, and P. L. McEuen, Nature (London) 407, 57 (2000).

A.N. Pasupathy, J. Park, C. Chang, A. V. Soldatov, S. Lebedkin, R. C. Bialczak, J. E. Grose, L. A. K. Donev, J. P. Sethna, D. C. Ralph, and P. L. McEuen, Nano Lett. 5, 203 (2005).

B. J. LeRoy, S. G. Lemay, J. Kong, and C. Dekker, Nature 432, 371 (2004).

S. Sapmaz, P. Jarillo-Herrero, Ya. M. Blanter, C. Dekker, and H. S. J. van der Zant, Phys. Rev. Lett. 96, 026801 (2006).

D. Boese and H. Shoeller, Europhys. Lett. 54, 668 (2001).

K.D. McCarthy, N. Prokof’ev, and M. T. Tuominen, Phys. Rev. B 67, 245415 (2003).

K. C. Nowack, and M. R. Wegewijs. cond-mat/0506552.

L. Y. Gorelik, A. Isacsson, M. V. Voitova, B. Kasemo, R. I. Shekhter, and M. Jonson, Phys. Rev. Lett. 80, 4526 (1998).

T. Novotny, A. Donarini, and A.-P. Jauho, Phys. Rev. Lett. 90, 256801 (2003).

S. Braig, and K. Flensberg, Phys. Rev. B 68, 205324 (2003).

J. Koch, M. Semmelhack, F. von Oppen, and A. Nitzan, Phys. Rev. B 73, 155306 (2006).

A. D. Armour, Phys. Rev. B 70, 165315 (2004).

A. Mitra, I. Aleiner, and A.J. Millis, Phys. Rev. B 69, 245302 (2004).

J. Koch, and F. von Oppen, Phys. Rev. Lett 94, 206804 (2005).

N. M. Chchelkatschew, W. Belzig, and C. Bruder, Phys. Rev B 70, 193305 (2004).

Ya. M. Blanter, O. Usmani, and Yu. V. Nazarov, Phys. Rev. Lett. 93, 136802 (2004).

O. Usmani, Ya. M. Blanter, and Yu. V. Nazarov, cond-mat/0603017.

T. Novotny, A. Donarini, C. Flindt, and A.-P. Jauho, Phys. Rev. Lett. 92, 248302 (2004).

F. Pistolesi, Phys. Rev. B 69, 245409 (2004).

A. Zazunov, D. Feinberg, and T. Martin, Phys. Rev. B 73, 115405 (2006).

J. Koch, M. E. Raikh, and F. von Oppen, Phys. Rev. Lett. 95, 056801 (2005).

G.-L. Ingold, and Yu. V. Nazarov, in Single Charge Tunneling, edited by H. Grabert and M. Devoret (Plenum, New York, 1992).

Ya. M. Blanter, and M. Büttiker, Phys. Rep. 336, 1 (2000).

A. N. Korotkov, Phys. Rev. B 49, 10381 (1994).

E. V. Sukhorukov, G. Burkard, and D. Loss, Phys. Rev. B 63, 125315 (2001).

J. C. Egues, S. Hershfield, and J. W. Wilkins, Phys. Rev. B 49, 13517 (1994).

M. L. Roukes, Technical Digest of the 2000 Solid State Sensor and Actuator Workshop (Transducer Research Foundation, Cleveland, 2000).

A. O. Caldeira, and A. J. Leggett, Ann. Phys. 149, 374 (1983).

see e.g., H.-P. Breuer, and F. Petruccione, Theory of open quantum systems (Oxford, New York,2002).

G. Herzberg, Molecular Spectra and Molecular Structure: I. Spectra of Diatomic Molecules, 2nd ed. (Van Nostrand Reinhold, New York, 1950).

The general expression for the weight factors is: $\alpha_l = \exp[-\lambda^2 \coth(\beta \omega_0/2)] \exp[i(\beta \omega_0/2) I_l (\lambda^2 / \sinh(\beta \omega_0/2))$, where $I_l$ is the modified Bessel function of first kind.

Financial support by the EU via Contract No. MCRN-CT2003-504574 and by the Italian MIUR via PRIN05 is gratefully acknowledged.