CP–odd Higgs boson production in association with Neutral gauge boson in High-Energy $e^+e^-$ Collisions

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Abstract

We study the associated production of a CP–odd Higgs boson $A^0$ with a neutral gauge boson (Z or photon) in high–energy $e^+e^-$ collisions at the one–loop level in the framework of Two Higgs Doublet Models (THDM). We find that in the small $\tan\beta$ regime the top quark loop contribution is enhanced leading to significant cross–sections (about a few fb), while in the large $\tan\beta$ regime the cross–section does not attain observable rates.
1. Introduction

The discovery of a Higgs boson \([1]\) is one of the major goals of the present searches in particle physics. The Higgs boson of the Standard Model (SM) is until now undiscovered but direct or indirect results give stringent lower bounds on its mass. Moreover, the problematic scalar sector of the SM can be enlarged and some simple extensions of the SM with several Higgs bosons have been intensively studied for many years. The simplest extension is the Two Higgs Doublet Model (THDM) \([2]\), the two most commonly studied versions being classified as type I and II. They differ in how the Higgs bosons are coupled to the fermions, although both versions possess identical particle spectra after electroweak symmetry breaking \([3]\). From the 8 degrees of freedom initially present in the 2 Higgs doublets, 3 correspond to masses of the longitudinal gauge bosons, leaving 5 degrees of freedom which should be manifested as 5 physical Higgs particles \(2 \text{charged Higgs } H^\pm, 2 \text{CP–even } H^0, h^0 \text{and one CP–odd } A^0\). Model type II is the structure found in the Minimal Supersymmetric Standard Model (MSSM). Until now no Higgs boson has been discovered, and from the null searches one can derive direct and indirect bounds on their masses. The latest such limits are \(M_{H^\pm} > 69 \text{ GeV} \) \([4]\) and \(M_{h^0} + M_{A^0} > 90 \text{ GeV} \) \([5]\). We note that in the general THDM which will be considered in this paper the existence of a very light \(h^0\) or \(A^0\) is not excluded by the current searches \([6]\) and refs therein).

Higgs boson production in association with gauge bosons has been extensively studied in the literature:

- Higgsstrahlung \((e^+e^- \rightarrow Z\Phi)\) both for the SM Higgs boson \([7]\) or for Higgs bosons originating from extended Higgs sectors \([8]\) (THDM and MSSM).
- Associated production with a photon \((e^+e^- \rightarrow \gamma H^0/h^0/A^0)\) \([9, 10]\)
- Associated production of charged Higgs bosons with a W gauge boson both at \(e^+e^-\) colliders \([11, 12]\), and at Hadron colliders \([13]\).

CP–odd Higgs bosons can be produced at \(e^+e^-\) colliders \([14]\) via \(e^+e^- \rightarrow h^0 A^0\) and \(e^+e^- \rightarrow b\bar{b}A^0, t\bar{t}A^0\) \([15, 13]\). Pair production \(e^+e^- \rightarrow A^0A^0\) is small \([17]\), as is the analogous process at \(\gamma\gamma\) colliders \([18]\). In this paper we consider the rare decay \(e^+e^- \rightarrow \gamma^*, Z^* \rightarrow ZA^0\) in the context of the THDM. Such a process is forbidden at tree–level and proceeds via loops, with the rate at hadron colliders evaluated in Ref. \([19]\). Calculations of the loop mediated decay \(A^0 \rightarrow ZZ\) have appeared in Refs. \([20, 21]\), but not for the full process \(e^+e^- \rightarrow \gamma^*, Z^* \rightarrow ZA^0\). Although not expected to give a rate comparable to the mechanisms above, if observable this decay would be a test of the THDM at the one–loop level, since this mechanism would be sensitive to new physics entering in the loop. In addition it is important to have an accurate prediction for \(e^+e^- \rightarrow ZA^0\) in order to distinguish between possible signals from a CP–violating and CP–conserving THDM. In a CP–violating THDM tree–level mixing is allowed between the pure CP–even states and pure CP–odd state, giving rise to 3 neutral Higgs scalars \((h_i)\) with no definite CP quantum numbers. Thus all three neutral scalars may be produced in the Higgsstrahlung process \(e^+e^- \rightarrow Z^* \rightarrow Zh_i\), via a tree–level vertex \(ZZh_i\). Therefore it is important to know the possible magnitude of \(e^+e^- \rightarrow ZA^0\) in the CP–conserving THDM in order to be sure if a (say) detected boson in the Higgsstrahlung channel with a low rate could be

\[1\] There are still some regions which allow \(50 \text{ GeV} \leq M_{h^0} + M_{A^0} \leq 90 \text{ GeV}\).
consistent with $A^0$, or is actually evidence for a $h_i$ from a CP-violating THDM with a small component of CP-even scalar field. Such scalar–pseudoscalar mixing may also arise in the MSSM, being generated radiatively if one allows CP-violating phases in the model \cite{22}. The paper is organized as follows. In section 2, we review all the CP–odd Higgs boson interactions (with gauge bosons and fermions), section 3 contains the notation, conventions and one–loop calculations while in section 4 we present our numerical results. Finally section 5 contains our conclusions.

2. Notation, relevant couplings and One–Loop structure

2.1 Notation and One–Loop structure

We will use the following notation and conventions. The momenta of the incoming electron and positron, outgoing gauge boson $V$ and outgoing CP–odd Higgs boson $A^0$ are denoted by $p_{e^-}, p_{e^+}, p_V$ and $p_{A^0}$, respectively. Neglecting the electron mass $m_e$, the momenta in the center of mass of the $e^+e^-$ system are given by:

$$p_{e^-,e^+} = \frac{\sqrt{s}}{2} (1, 0, 0, \pm 1)$$

$$p_{V,A^0} = \frac{\sqrt{s}}{2} (1 \pm \frac{m_V^2 - M_A^2}{s}, \pm \kappa \sin \theta, 0, \pm \kappa \cos \theta),$$

where $\sqrt{s}$ denotes the center of mass energy, $\theta$ the scattering angle between $e^+$ and $A^0$ and

$$\kappa^2 = \frac{(s - (M_A + m_V)^2)(s - (M_A - m_V)^2)}{s^2};$$

$m_V$ and $M_A$ are the masses of the neutral gauge boson and of the CP–odd Higgs boson.

At the one–loop order (Fig.1.a–d), the differential cross–section reads:

$$\frac{d\sigma}{d\Omega} (e^+e^- \rightarrow VA^0) = \frac{\kappa}{256\pi^2s} \sum_{P_{ol}} |M_1|^2.$$  \hspace{1cm} (2.1)

The one–loop amplitude $M_1$ can be written as a sum of vertex and box contributions as follows:

$$M_1 = M_V^\gamma + M_V^Z + M_B^h + M_B^H$$

where $M_V^\gamma$ ($M_V^Z$) denotes the vertex correction s–channel photon exchange (s–channel Z exchange), depicted in Fig.1.a + Fig.1.b, and $M_B^{h,H}$ denote the box contribution with $h^0$.
and $H^0$ exchange (Fig.1.c). All these contributions fully project onto six invariants as follows:

$$ \mathcal{M}^1 = \sum_{i=1}^{6} \mathcal{M}_i \mathcal{A}_i $$

where the invariants $\mathcal{A}_i$ are given by:

$$ \begin{align*}
\mathcal{A}_1 &= \bar{v}(p_+^-) \ell(p^-) \frac{1 + \gamma_5}{2} u(p^-) \\
\mathcal{A}_2 &= \bar{v}(p_+^-) \ell(p^-) \frac{1 - \gamma_5}{2} u(p^-) \\
\mathcal{A}_3 &= \bar{v}(p_+^-) \not{p} \not{v} \frac{1 + \gamma_5}{2} u(p^-) (p_e - \epsilon(p_v)) \\
\mathcal{A}_4 &= \bar{v}(p_+^-) \not{p} \not{v} \frac{1 - \gamma_5}{2} u(p^-) (p_e - \epsilon(p_v)) \\
\mathcal{A}_5 &= \bar{v}(p_+^-) \not{p} \not{v} \frac{1 + \gamma_5}{2} u(p^-) (p_e + \epsilon(p_v)) \\
\mathcal{A}_6 &= \bar{v}(p_+^-) \not{p} \not{v} \frac{1 - \gamma_5}{2} u(p^-) (p_e + \epsilon(p_v))
\end{align*} \tag{2.2} $$

with $\epsilon$ the polarization of the vector boson $V$. Summing over the $V = \gamma, Z$ gauge boson polarizations, the squared amplitude may be written as:

**Case V=Z**

$$ \sum_{Z Pol} |\mathcal{M}^1|^2 = 2s(|\mathcal{M}_1|^2 + |\mathcal{M}_2|^2) - \frac{(m_Z^2 t - M_1^2)}{4m_Z^2} \{4|\mathcal{M}_1|^2 + 4|\mathcal{M}_2|^2 \\
+ 4(m_Z^2 - t) Re[\mathcal{M}_1\mathcal{M}_3^* + \mathcal{M}_2\mathcal{M}_1^*] + (m_Z^2 - t)^2 |\mathcal{M}_3|^2 + |\mathcal{M}_4|^2 \\
+ 4(m_Z^2 - u) Re[\mathcal{M}_1\mathcal{M}_5^* + \mathcal{M}_2\mathcal{M}_6^*] + (m_Z^2 - u)^2 |\mathcal{M}_5|^2 + |\mathcal{M}_6|^2 \\
- 2(m_A^2 m_Z^2 + m_Z^2 s - tu) Re[\mathcal{M}_3\mathcal{M}_5^* + \mathcal{M}_4\mathcal{M}_6^*] \} \tag{2.3} $$

**Case V=γ**

$$ \sum_{\gamma Pol} |\mathcal{M}^1|^2 = 2s(|\mathcal{M}_1|^2 + |\mathcal{M}_2|^2) + st(Re[\mathcal{M}_1^*\mathcal{M}_3 + \mathcal{M}_2^*\mathcal{M}_4]) + \\
su(Re[\mathcal{M}_1^*\mathcal{M}_5 + \mathcal{M}_2^*\mathcal{M}_6]) - stu(Re[\mathcal{M}_3^*\mathcal{M}_5 + \mathcal{M}_4^*\mathcal{M}_6]) \tag{2.4} $$

**2.2 Relevant couplings for our study**

**CP–odd Higgs boson interaction with fermions**

In the two Higgs doublet extensions of the Standard Model, there are different ways of coupling the Higgs fields to matter. The most popular are labelled as model type I and model type II. In the former, the quarks and leptons couple only to the second Higgs doublet $\Phi_2$, while in the latter, in order to avoid the problem of Flavor Changing Neutral Current (FCNC) [23], one assumes that $\Phi_1$ couples only to down quarks (and charged leptons) and $\Phi_2$ couples only to up quarks (and neutral leptons). The type II model is the pattern found in the MSSM.

In general, the CP–odd Higgs interaction with fermions is given by:

$$ A^0 u\bar{u} = Y_{u\bar{u}} \gamma_5, \quad A^0 d\bar{d} = Y_{d\bar{d}} \gamma_5 \tag{2.5} $$
Here \( u(d) \) may refer to any generation of up quark (down quark or charged leptons) and the \( Y \) couplings are defined as follows:

\[
Y_{uu} = -\frac{gm_u}{2M_W \tan \beta}, \quad Y_{dd} = \frac{gm_d}{2M_W \tan \beta} \quad \text{Model I}
\]

\[
Y_{uu} = -\frac{gm_u}{2M_W \tan \beta}, \quad Y_{dd} = -\frac{gm_d \tan \beta}{2M_W} \quad \text{Model II} \quad (2.6)
\]

It is worth noting the models I and II are not very different for the top–bottom loop corrections at low \( \tan \beta \) because the term \( m_t/ \tan \beta \) will dominate and it is common to both types. In the case of large \( \tan \beta \) the effects of down quarks and tau–lepton are enhanced (suppressed) in Model type II (Model type I).

### 2.3 Higgs boson interaction with gauge bosons

In a general two Higgs doublet model, the interactions between Higgs bosons and gauge bosons are completely dictated by the local gauge invariances and so they are model independent regardless of whether the model is supersymmetric or not. These interactions originate from the square of the covariant derivative in the Higgs lagrangian which is given by:

\[
\sum_i (D_\mu \Phi_i)^+ (D_\mu \Phi_i) = \sum_i [(\partial_\mu + ig\vec{T}_a W_\mu^a + ig' \frac{Y_{\Phi_i}}{2} B_\mu) \Phi_i]^+ (\partial_\mu + ig\vec{T}_a W_\mu^a + ig' \frac{Y_{\Phi_i}}{2} B_\mu) \Phi_i (2.7)
\]

where: \( \vec{T}_a \) is the isospin operator, \( Y_{\Phi_i} \) the hypercharge of the Higgs fields, \( W_\mu^a \) the \( SU(2)_L \) gauge fields, \( B_\mu \) the \( U(1)_Y \) gauge field, and \( g \) and \( g' \) are the associated coupling constants.

After expanding eq. (2.7) and deriving all the interaction vertices, one finds, in accordance with the conservation of electromagnetic current, that the \( \gamma-A^0-Z \) vertex vanishes at tree–level while the vertex \( Z-Z-A^0 \) vanishes by virtue of CP–invariance [24, 25].

Therefore, at tree–level, the contributions to \( e^+e^- \rightarrow ZA^0 \) and/or \( e^+e^- \rightarrow \gamma A^0 \) only come from the \( t \)–channel via electron exchange and from the following \( s \)–channels: \( e^+e^- \rightarrow h^* \rightarrow ZA^0 \) (\( e^+e^- \rightarrow H^* \rightarrow ZA^0 \)) where \( H^0 \) and \( h^0 \) are the two CP–even neutral Higgs fields. All these contributions are proportional to the electron mass \( m_e \) which we will neglect in our work. As a major consequence, in this limit, the studied process will be only due to loop effects.

From eq.(2.7) one can deduce the following vertices which are needed for our study:

\[
Z_\mu Z_\nu h^0 = g \frac{m_Z}{c_W} \sin (\beta - \alpha) g_{\mu\nu}, \quad Z_\mu Z_\nu H^0 = g \frac{m_Z}{c_W} \cos (\beta - \alpha) g_{\mu\nu}
\]

\[
Z_\mu A^0 h^0 = g \frac{\cos (\beta - \alpha)}{2c_W} (p_h - p_A)_\mu, \quad Z_\mu A^0 H^0 = -g \frac{\sin (\beta - \alpha)}{2c_W} (p_H - p_A)_\mu \quad (2.8)
\]

where \( c_W \) and \( s_W \) are the cosine and sine of the Weinberg angle \( \theta_W \). We need also:

\[
Z_\mu q \bar{q} = \gamma_\mu \left(g_q^L \frac{1 - \gamma_5}{2} + g_q^R \frac{1 + \gamma_5}{2}\right), \quad Z_\mu \epsilon \bar{e} = \gamma_\mu (g_V - g_A \gamma_5)
\]

\[
\gamma_\mu q \bar{q} = \gamma_\mu (-e_q), \quad \gamma_\mu \epsilon \bar{e} = \gamma_\mu (-1) \quad (2.9)
\]
where \( q \) may refer to \( u \) or \( d \), and

\[
\begin{align*}
g_V &= \frac{(1 - 4s_W^2)/(4s_Wc_W)}{g_A} = 1/(4s_Wc_W), \\
g_U^L &= \frac{(1 - 2s_W^2e_u)}{4s_Wc_W}, \quad g_d^L = \frac{(1 + 2s_W^2e_d)}{4s_Wc_W}, \quad g_{u,d}^R = \frac{2s_W^2e_{u,d}}{4s_Wc_W}
\end{align*}
\]

### 2.4 CP–Violating scenario

In a THDM, CP–violation (explicit or spontaneous) is possible in the Higgs sector if one allows the discrete symmetry to be broken softly by a term of dimension 2 (\( \mu_1 \Phi_1 \Phi_2 + h.c. \)). In such a case mixing is permitted between the pure CP–even states \( h^0, H^0 \) and the pure CP–odd state \( A^0 \), leading to three neutral Higgs mass eigenstates which cannot be assigned a definite CP quantum number. In the notation of Ref. \[20\] these are referred to as \( h_1, h_2, h_3 \), whose couplings obey various sum rules \[20, 27\] which in the CP–conserving case reduce to the familiar sum rules of the THDM/MSSM e.g. one has (where the couplings are normalized to SM Higgs boson strength)

\[
C_i^2 + C_j^2 + C_{ij}^2 = 1; \quad C_1^2 + C_2^2 + C_3^2 = 1 \tag{2.10}
\]

Here \( i, j \) run from 1 to 3, \( C_i \) is the coupling \( ZZh_i \), and \( C_{ij} \) is the coupling \( Zh_i h_j \). Ref. \[20\] explained how CP–violation may be probed in the Higgs sector by measuring non–zero values for each of \( Zh_1h_2, Zh_2h_3 \) and \( Zh_3h_1 \), or for each of \( Zh_1h_j, ZZh_iZ \) \( Z \) \( h_j \) (where \( i \neq j \)). A third way would be a positive signal in all three Higgsstrahlung channels \( e^+e^- \to h_1Z, h_2Z, h_3Z \), and it is possible that the Higgs masses are arranged such that the latter production mechanisms would be all open kinematically (i.e. \( \sqrt{s} > m_Z + m_{h_i} \)) while pair production of Higgs bosons are not. Therefore the CP–violating THDM may provide measurable signals in the \( e^+e^- \to h_iZ \) channel for all three neutral Higgs bosons. Hence in order to distinguish between a CP–violating THDM and a CP–conserving THDM in the above channels it is important to know the attainable value of \( e^+e^- \to ZA^0 \) in the CP–conserving case.

In Refs. \[28, 29\], using effective lagrangians, it was shown that a CP–odd Higgs scalar with effective point like couplings \( ZZ A^0 \) and \( Z \gamma A^0 \) would give rise to angular distributions in the channel \( e^+e^- \to \gamma^*, Z^* \to ZA^0 \) which would differ significantly from those in \( e^+e^- \to Z^* \to Zh(H) \). Our analysis essentially determines the possible magnitude of these effective couplings \( ZZ A^0 \) and \( Z \gamma A^0 \) in the context of the THDM. In the notation of \[29\] this would correspond to evaluating the arbitrary couplings \( \tilde{b}_y, \tilde{b}_Z \). Given the expected smallness of the rate for \( e^+e^- \to A^0Z \), any angular distribution analysis for this channel would most likely be hampered by small statistics. If in the CP–violating THDM all three \( ZZh_i \) couplings have a reasonable strength (i.e. significant pseudoscalar–scalar mixing), a detectable signal would be possible which could not be mimicked by the CP–conserving THDM. In addition there would be sufficient events to show that the angular distribution proceeded via the CP–even scalar component. The problematic case of interest to us is when a \( h_i \) is dominantly pseudoscalar and thus has a smaller (but still detectable) rate in the \( e^+e^- \to h_iZ \) channel. In this case the signal from \( e^+e^- \to A^0Z \) is background to any possible interpretation of a CP–violating signal in this channel. It is also this scenario when angular distributions would be affected by low statistics.


3. One-Loop Corrections

We have evaluated the pure radiative effects of the rare decay $e^+e^- \rightarrow ZA^0$ ($e^+e^- \rightarrow \gamma A^0$) at the one-loop level in the ‘t Hooft - Feynman gauge. The sum of all the one-loop effects are ultra–violet (UV) convergent but since some Feynman diagrams are UV divergent we will use the dimensional regularization scheme \[30\] to deal with them.

The typical Feynman diagrams for the virtual corrections of order $\alpha^2$ are drawn in figure 1. In the THDM, the $\gamma – Z – A^0$ and the $Z – Z – A^0$ vertices receive corrections only from fermion exchanges (Fig.1-a-b). There are also box diagrams and their crossed analogies (fig.1-c). Note that there is no mixing coming from the $Z – A^0$ ($Z – G^0$) $s$-channel self–energy because the vertices $\gamma – A^0 – A^0$ ($\gamma – G^0 – A^0$) and $Z – A^0 – A^0$ ($Z – G^0 – A^0$) vanish, while the mixing $Z – A^0$ has to be considered in the $t$–channel (fig.1.d). Owing to Lorentz invariance the $Z – A^0$ self energy is proportional to $p^\mu_A (p^++p^+-p^\nu)^\mu$; then, since the vector boson $V=\gamma$ or $Z$ is on–shell, the $t$–channel amplitude will be proportional to $m_e$ and consequently vanishes. Note also that the tadpole diagrams have vanishing contributions.

In the on–shell scheme defined in \[31\], it is found that there is no counter–term for the $Z–Z–A^0$, $Z–\gamma–A^0$ and $\gamma–\gamma–A^0$ vertices. Consequently the one–loop vertices $Z–Z–A^0$, $Z–\gamma–A^0$ and $\gamma–\gamma–A^0$ have to be separately UV finite and this feature will provide us with a good check of our calculations.

All the Feynman diagrams are generated and computed using FeynArts \[33\] and FeynCalc \[34\] packages supplied with some Mathematica ”know-how”. We also use the fortran FF–package \[35\] in the numerical analysis.

**Vertex correction: Fig.1.a + Fig.1.b:** $e^+e^- \rightarrow A^0 Z$

In terms of the $B_0$, $C_0$ and $D_i$ Passarino–Veltman functions, $M_i^V$ and $M_i^Z$ are given by\[36\]:

$$M_i^V = 2N_C\epsilon_q m_q Y_{qq} \frac{\alpha^2}{s} \{2(A_3 – A_4 + A_5 + A_6) + (t – u)(A_1 – A_2)\}(g_q^L + g_q^R)$$

$$C_0(M_A^2, m_Z^2, s, m_q^2, m_{\bar{q}}^2)$$

(4.1)

$$M_i^Z = \frac{-2N_C\epsilon^2_m Y_{qq}}{(s – m_Z^2)k^2} \{(g_A – g_V)(2A_3 – 2A_5 + (t – u)A_1)$$

$$+ (g_A + g_V)(2A_4 – 2A_6 + A_2(t – u))(g_q^L – g_q^R)^2[2M_A^2 B_0(M_A^2, m_q^2, m_{\bar{q}}^2)$$

$$- (M_A^2 + m_Z^2 – s)B_0(m_Z^2, m_q^2, m_{\bar{q}}^2) – (M_A^2 – m_Z^2 + s)B_0(s, m_q^2, m_{\bar{q}}^2)]$$

$$+ [M_A^2 m_Z^2 – m_A^4 + M_A^2 s + 2m_Z^2 s – s^2](g_q^L + g_q^R)^2$$

$$+ 2[-M_A^4 + M_A^3 m_Z^2 + M_A^2 s g_q^L g_q^R]C_0(M_A^2, m_Z^2, s, m_q^2, m_{\bar{q}}^2)$$

(4.2)

Here $\epsilon_q$ and $m_q$ denote the charge and the mass of the specific quark; $g_V$, $g_A$, $g_q^L$ and $g_q^R$ are the couplings defined before.

As defined in the appendix A, the $B_0$ functions are UV divergent whereas the $C_0$ are UV convergent. Consequently it is easy to check that the one–loop vertices defined below are UV convergent, as they should be.

\[2\]Note that the one–loop vertex $A^0 \rightarrow ZZ$ has been evaluated in \[30\].
In a similar way we found:

\[ \mathcal{M}^g_V = -4N_C e_q^2 m_q Y_{qq} \frac{\alpha^2}{s} \left\{ 2(A_3 - A_4 - A_5 + A_6) + (t - u)(A_1 - A_2) \right\} \]

\[ C_0(M_A^2, 0, s, m_q^2, m_q^2, m_q^2) \]  

(4.3)

\[ \mathcal{M}^Z_V = -2N_C e_q \frac{\alpha^2 m_q Y_{qq}}{s - m_Z^2} \left\{ (g_A + g_V)(2A_4 - 2A_6 + (t - u)A_2) + (g_A - g_V)(2A_3 - 2A_5 + (t - u)A_1) \right\} (g_q^L + g_q^R)C_0(M_A^2, 0, s, m_q^2, m_q^2, m_q^2) \]  

(4.4)

**Box corrections: Fig.1.c:**

Box corrections are only present for $e^+ e^- \to A^0 Z$, because the photon does not couple to neutral particles.

\[ \mathcal{M}^b_B = \frac{\alpha^2 \sin 2(\beta - \alpha) m_W}{4 s_W^2 c_W^3} \left\{ 4A_3(g_A - g_V)^2 D_1 + 4A_4(g_A + g_V)^2 D_1 - A_1(g_A - g_V)^2 \right\} \]

\[ C_0(m_e^2, m_e^2, s, m_Z^2, m_e^2, m_Z^2) + (m_h^2 - t)D_0 + (m_Z^2 - t)D_1 + (t - M_A^2)D_3 \]  

\[ + (t - M_A^2)D_3 \]  

(4.5)

All the $D_i (i = 0, 1, 2, 3)$ have the same arguments: $(m_e^2, m_e^2, m_A^2, s, t, s, m_h^2, m_Z^2, m_e^2, m_Z^2)$. The amplitude of a crossed box can be obtained from the direct one by changing: $t \to u$, $A_3 \to A_5$, $A_4 \to A_6$ and a global sign. Moreover $\mathcal{M}^H_B$ can be obtained from $\mathcal{M}^b_B$ just by changing $M_h$ in $M_H$ and a global sign.
4. Numerical results and discussion

In this section we focus on the numerical analysis. We take the following experimental input for the physical parameters \([36]\):

- the fine structure constant: \(\alpha = \frac{e^2}{4\pi} = 1/137.03598\).
- the gauge boson masses: \(m_Z = 91.187\, \text{GeV}\) and \(m_W = 80.41\, \text{GeV}\). In the on–shell scheme we consider, \(\sin^2 \theta_W\) is given by \(\sin^2 \theta_W \equiv 1 - \frac{m_W^2}{m_Z^2}\), which is not modified by loop corrections.
- the top–bottom quark masses are taken to be: \(m_t = 175\, \text{GeV}\) and \(m_b = 4.5\, \text{GeV}\). The tau lepton mass is taken to be \(m_\tau = 1.77\, \text{GeV}\).

As pointed out in the introduction, in the general THDM the CP–odd Higgs mass is not constrained by experiment, and only the sum \(M_h + M_A\) is constrained to be heavier than 90 GeV. Therefore in our analysis we are free to consider a very light CP–odd Higgs mass. Taking into account perturbative requirements on the Yukawa couplings \(\tan \beta\) is constrained to be in the range: \(0.1 \leq \tan \beta \leq 80\).

Let us first discuss briefly the THDM radiative contributions to the CP–odd \(A^0\) associated production with a photon which has been considered before in \([10]\). We stress here that we are in perfect agreement with \([10]\) both on analytical result and numerical result. We would like to highlight in Fig.2.a the enhancement of the cross–section for small \(\tan \beta\). One can reach a cross–section of about 0.8 fb for \(\tan \beta = 0.3\) (8 fb for \(\tan \beta = 0.1\)).

Fig.2.b shows the total cross–section against \(\tan \beta\) for CP–odd \(A^0\) associated production with a Z gauge boson. One can see also the enhancement of the cross–section in the small \(\tan \beta\) regime. This enhancement for small \(\tan \beta\) is common to both Model I and Model II. The most important contribution in the fermionic case comes from top quark loops. In the large \(\tan \beta\) regime in Model II the corrections start to be sensitive to the bottom, light quarks and tau lepton loops. As mentioned in section 3, in the case of associated CP–odd Higgs production with a Z gauge boson, there are extra box contributions to the cross–section. In all Figures 2a, 2b, 3a and 3b we have chosen \(M_h = 95\, \text{GeV}\), \(M_H = 180\, \text{GeV}\) and \(\tan \alpha = +2.1\). It is found that box contributions to the cross section are rather small. For large \(\tan \beta\) the fermionic contributions also become small and of the same order of magnitude as the box contributions.
In Fig.3.a we show the dependence of the cross-section on the CP-odd Higgs mass for $\sqrt{s} = 500$ GeV in the case where $\tan \beta = 0.2, 0.5$ and 1.6. One can see that the cross-section is maximized for both low $\tan \beta$ and small $M_A$ mass. The kink in the figure corresponds to $M_A > 2m_t$, i.e. when the decay $A^0 \rightarrow t\bar{t}$ opens. At $\sqrt{s} = 500$ GeV the cross-section for the SM Higgs boson in the Higgsstrahlung channel falls from 50 fb to 20 fb as the Higgs mass increases from 200 GeV to 300 GeV. Fig 3a shows that for $\tan \beta = 0.2$ one finds $\sigma(e^+e^- \rightarrow AZ)$ falling from 0.2 fb to 0.1 fb in the same mass region. If one wished to interpret a third (by definition the smallest) Higgsstrahlung signal as evidence for a CP-violating THDM, a cross-section comfortably in excess of 1 fb would be required. We recall from eq. 2.10 that $\sum_{i=1}^{3} C_i^2 = 1$ (normalized to SM strength), and so $\sigma(e^+e^- \rightarrow AZ)$ constitutes a non-negligible background.

In Fig.3.b we show the total cross-section as a function of the center of mass energy $\sqrt{s}$ for $M_A = 50, 150, 270, 400$ GeV and for $\tan \beta = 0.5$. One can find a total cross-section of about 0.04 fb only for the CP-odd Higgs mass in the intermediate range ($M_A=50–150$ GeV) and for $\sqrt{s} \approx 500$ GeV. At high center of mass energy the rate of production is rather small.

5. Summary

To conclude, we have computed the associated production of a CP-odd Higgs boson with a gauge boson $V = Z, \gamma$ in high energy $e^+e^-$ collisions in the framework of the general two Higgs doublet model. The calculation is performed within the dimensional regularisation scheme.

We have shown that in the small $\tan \beta$ regime the top quark loop contribution is enhanced leading to significant cross-sections (about a few fb), while in the large $\tan \beta$ regime the cross-section does not attain observable rates.

The smallness of the cross-section means that there is no chance of seeing such associated production at LEP–II, while such production could be relevant for a Next Linear Collider machine and would require a very high luminosity option.

Acknowledgment: A. Akeroyd was supported by the Japan Society for Promotion of Science (JSPS). A. Arhrib acknowledges the Abdus Salam International Centre for Theoretical Physics for the kind hospitality during his visit where part of this work has been done.

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Appendix A: Passarino–Veltman Functions

Let us briefly recall the definitions of scalar and tensor integrals we use. The inverses of the propagators are denoted by
\[ d_0 = q^2 - m_0^2, \quad d_i = (q + p_i)^2 - m_i^2 \]
where the \( p_i \) are the momenta of the external particles.

**Two point function:**
\[
B_0(p_1^2, m_0^2, m_1^2) = \frac{(2\pi\mu)^{(4-D)}}{i\pi^2} \int \frac{1}{d_0 d_1}
\]
\( \mu \) is an arbitrary renormalization scale and \( D \) is the space–time dimension.

**Three point function:**
\[
C_0(p_{12}^2, p_0^2, m_0^2, m_1^2, m_2^2) = \frac{(2\pi\mu)^{(4-D)}}{i\pi^2} \int \frac{1}{d_0 d_1 d_2}
\]
where \( p_{ij}^2 = (p_i - p_j)^2 \).

**Four point functions:**
\[
D_{0\mu}(p_1^2, p_{12}^2, p_2^2, p_{23}^2, p_3^2, p_{13}^2, m_0^2, m_1^2, m_2^2, m_3^2) = \frac{(2\pi\mu)^{(4-D)}}{i\pi^2} \int \frac{1}{d_0 d_1 d_2 d_3}
\]  \( A.1 \)

Using Lorentz invariance, one can write the vectorial function \( D_\mu \) in terms of new scalar functions \( D_i \) as follows:
\[
D_\mu = p_{1\mu}D_1 + p_{2\mu}D_2 + p_{3\mu}D_3
\]  \( A.2 \)

Moreover one can always get all the ”new” functions \( D_i \) in terms of the fundamental ones, \( A_0, B_0, C_0 \) and \( D_0 \). The analytical expressions of all the scalar functions can be found in Ref. 37.
Figure Captions

Fig. 1 One–Loop diagram contributions to \( e^+ e^- \to A^0 V \) \((V = \gamma \text{ or } Z)\). Fig.1.a + Fig.1.b are fermionic vertex corrections. Fig.1.c are box diagrams and Fig.1.d is the t–channel self energy.

Fig. 2 Fig.2.a: Total cross–section for \( e^+ e^- \to A^0 \gamma \) (vertex only) as a function of \( \tan \beta \) for \( \sqrt{s} = 500 \text{ GeV} \) and several values of \( M_A \).

Fig.2.b: Total cross–section for \( e^+ e^- \to A^0 Z \) (vertex + boxes) as a function of \( \tan \beta \) for \( \sqrt{s} = 500 \text{ GeV} \) and several values of \( M_A \).

Fig. 3 Fig.3.a: Total cross–section for \( e^+ e^- \to A^0 Z \) (vertex + boxes) as a function of \( M_A \) for \( \sqrt{s} = 500 \text{ GeV} \) and several values of \( \tan \beta \).

Fig.3.b: Total cross–section for \( e^+ e^- \to A^0 Z \) (vertex + boxes) as a function of \( \sqrt{s} \) for \( \tan \beta = 0.5 \) and for several values of \( M_A \).
$e^+e^- \rightarrow A^0 \gamma$

$\sqrt{s} = 500$ GeV Model-II

$\sigma(fb)$ vs. $\tan \beta$

$e^+e^- \rightarrow A^0 Z$

$\sqrt{s} = 500$ GeV Model-II

$\sigma(fb)$ vs. $\tan \beta$
Fig. 3.a

\[ \sigma(f b) \]
\[ M_A \text{ (GeV)} \]
\[ e^+e^- \rightarrow A^0Z \]
\[ \sqrt{s} = 500 \text{ GeV} \]

\[ \tan \beta = 0.2 \]
\[ \tan \beta = 0.5 \]
\[ \tan \beta = 1.6 \]

Fig. 3.b

\[ \sigma(f b) \]
\[ \sqrt{s} \text{ (GeV)} \]
\[ e^+e^- \rightarrow A^0Z \]
\[ M_A = 50 \text{ GeV} \]
\[ M_A = 150 \text{ GeV} \]
\[ M_A = 270 \text{ GeV} \]
\[ M_A = 400 \text{ GeV} \]
\[ \tan \beta = 0.5 \]

Figure 3