Non-locality and destructive interference of matter waves

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Abstract. Quantum mechanics with massive particles becomes an important tool for fundamental research and applied science since many previously named “Gedanken” experiments become feasible. Neutrons are massive particles which couple to gravitational, nuclear and electro-magnetic interactions and they are sensitive to topological effects as well. Therefore they are proper tools for testing quantum mechanics where several previously named “hidden” parameters become measurable. Widely separated coherent beams can be produced by means of perfect crystal interferometers and they can be influenced individually. Spinor symmetry, spin superposition and quantum beat effect experiments have been performed and topological phases have been observed. Recent experiments related to the decoherence problem have shown that interference effects can be revived even when the overall interference pattern seems to be incoherent. All retrieval processes involve inherently unavoidable losses which stem partly from the theory itself and partly from an imperfect environment. Related post-selection experiments shed a new light on questions of quantum non-locality and support the request for more complete quantum measurements in the future. A more rational explanation of non-locality effects may be obtained when the plane wave components outside the wave packets are included in the discussion. This can also help to discuss entanglement and contextuality effects in a new light. In all quantum experiments more information can be extracted by more complete quantum experiments which will be important in the future to get a better understanding of quantum physics. An example may be the consideration of the Compton frequency and of proper time effects of matter waves.

1. Introduction
Neutrons exhibit wave and particle features and are a proper tool for testing quantum phenomena with massive particles (e.g. Rauch and Werner [1]). With perfect crystal interferometers widely separated coherent beams can be produced and influenced by nuclear, electroweak and gravitational interactions. Highly efficient polarizers, spin rotators and detectors are available. Most experiments use thermal neutrons with velocities in the order of 2000 m/s and wavelengths in the order of about 2 Å. The spin state of neutrons can be described by typical two-level systems where transitions can be induced by electromagnetic interaction. The two coherent beam paths through the interferometer can also be seen as a two-level system and related entanglements can be achieved and used for novel quantum measurements. High degree of coherence, expressed by the visibility of the interference pattern (up to 95%) and high order interferences (up to 200) can be observed. The coherence features are usually described in terms of the well-known Glauber [2] formalism. The coherence function is defined as the auto-correlation function of the...
wave function, which reads for a stationary situation as

\[ \Gamma^{(1)}(\vec{\Delta}) = \left< \psi^*(\vec{r})\psi(\vec{r} + \vec{\Delta}) \right> = \int g(\vec{k}) e^{i(\vec{k}\cdot\vec{\Delta})} d^3k, \]  

(1)

where \( \vec{\Delta} \) denotes the spatial shift of the two wave fields and \( g(\vec{k}) \) the momentum distribution in the related direction. The characteristic widths of this function define the coherence lengths \( \Delta_i \) which are related to the widths \( \delta k_i \) of the related momentum distribution as: \( \Delta_i \delta k_i \leq 1; i = x, y, z \). From measurements of the momentum distributions and the loss of contrast at high order interference, which is described by \(|\Gamma^{(1)}(\vec{\Delta})|\), one obtains coherence lengths in the order of about 200-500 Å, which represents also the size of the related wave packets (Rauch et al. [3]).

Pre- and post-selection experiments indicate that coherence exists even when the coherence pattern is smeared out due to inhomogeneities along the beam paths or statistical motions of the interferometer crystal during the time-of-flight the neutron spends within the interferometer. Since the beam cross section is usually much larger than the size of the wave packets one has to average Eq. (1) classically over the beam cross section which then also includes variations of the parameters of the apparatus. The complementarity and duality feature of quantum physics is an essential part for its understanding. In this context the Englert-Greenberger-Yasin relation can be considered as a further “two level” system where pure particle features exist at the north and south poles of the Bloch sphere and pure wave properties along the equator (Greenberger and Yasin [4], Englert [5]). This relation combines path predictabilities \( P \) with wave properties \( V \), which equals the visibility of the interference pattern

\[ P^2 + V^2 \geq 1. \]  

(2)

This provides the basis for further entanglement experiments with other two level systems.

2. Momentum post-selection

In the course of several neutron interferometer experiments it has been established that smoothed out interference properties at high interference order can be restored even behind the interferometer when a proper spectral filtering is applied. This is shown schematically in Fig. 1 (Rauch [6]). The interference pattern reads as:

\[ I_0(\chi) \propto 1 + |\Gamma(\chi)| \cos \chi \]  

(3)

where \( \chi = K \cdot \Delta = nk\Delta = -Nb_c\lambda D_{eff} \) is the phase shift given by the particle density \( N \) and the coherent scattering length \( b_c \) of the phase shifter, the wavelength of the neutrons \( \lambda \) and the effective path length within the phase shifter \( D_{eff} \). For Gaussian wave packets the coherence function \(|\Gamma(\chi)|\) has also a Gaussian shape which reduces the interference fringes at high order. The related momentum distribution becomes modulated as

\[ I_0(\chi) \propto \exp \left[ \frac{-(k - k_0)^2}{2\delta k^2} \right] \left\{ 1 + \cos \left( \frac{\chi k_0}{k} \right) \right\}. \]  

(4)

Related measurements have been done by Jacobson et al. [7], who showed that interference fringes can be restored when a proper momentum filter is applied. Fig. 2 shows more recent measurements where the intensity wiggles have been observed for various phase shifts (Baron and Rauch [8]). In this case the imperfectness of the neutron beam (wavelength spread) causes the contrast reduction at high order.

The increasing modulation at high order is visible. This indicates a forceless beam modulation due to interference. The corresponding spatial distribution can be calculated for Gaussian beams.
as well and show that at high order (phase shifts larger than the coherence length) a double humped intensity distribution arises which can be written as (δxδ̂k = 1/2; Fig. 1)

\[ I_0(x) = |\psi(x) + \psi(x + \Delta)|^2 \propto e^{-x^2/2\delta x^2} + e^{-(x+\Delta)^2/2\delta x^2} + 2e^{-x^2/4\delta x^2}e^{-(\Delta^2/4\delta x^2)} \cos \chi \]  

This means that the neutron occupies at the same time two separate regions of space, i.e. so-called Schrödinger cat-like states. Fig. 3 shows such a double humped distribution measured with a double loop interferometer (Baron and Rauch [8]). \( \Delta_1 \) produces the Schrödinger cat-like state within the first loop and \( \Delta_2 \) scans the spatial distribution within the second loop.

As a conclusion of this section we can say that a loss of visibility of the interference pattern at high order does not mean a loss of coherence but a shift of coherence features from ordinary space into momentum space. By a proper post-selection procedure the interference pattern can be retrieved. In all cases a complete retrieval is impossible since unavoidable losses occur at any interaction the system experiences (Rauch [9]).

3. Visibility loss due to noisy interaction

Magnetic fields of strength \( B \) and length \( L \) produce a phase shift of half of the Larmor rotation angle \( \alpha \)

\[ \chi = \frac{\alpha}{2} = \frac{\mu BL}{\hbar v} \]  

where \( \mu \) is the magnetic moment of the neutrons and \( v \) their velocity. Within a quasi-static approximation which may be valid when the time-of-flight through the field \( \tau_{\text{of}} = L/v \leq 1/\nu_{\text{field}} \) is shorter than the typical time change of the field the loss of contrast for a Gaussian fluctuating field (\( \Delta B \)) can be written as

\[ I_0 \propto 1 + e^{-\left(\frac{\mu L}{\hbar v}\right)(\Delta B)^2/2} \cos \chi. \]  

Related measurements with unpolarized neutrons have shown the surprising effect that the interference pattern can be restored completely when the same noise field is applied to both beams at the same distance from the splitting (Baron and Rauch [8]; Fig. 4). When the
Figure 2. Experimental arrangement (top) with a third crystal as a momentum (wavelength) filter and typical momentum scans at different interference order (Baron and Rauch [8]).

Figure 3. Double loop interferometer (left) and measured spatial distribution for different phase shifts in the first loop (Baron and Rauch [8]).
Figure 4. Interference pattern with (full lines) and without magnetic noise field (dashed lines) with frequencies between 0 and 20kHz and a mean amplitude of 9 G (Baron and Rauch [8]). The noise field has been applied to the beams separately (top and middle) and to both beams synchronously (bottom).

relative position of the coils is varied, i.e. a time delay $\Delta t = \Delta x/v$ is introduced, the autocorrelation function of the noise signal can be measured.

At high order when the contrast disappears due to the wavelength dispersion a smearing out effect on the modulated momentum distribution occurs, as shown in Fig. 5. In this case the dephasing factor of Eq. (7) adds to Eq. (4).

Within the experimental errors the loss of contrast is independent of the spatial separation of the Schrödinger cat-like states. The reversibility features when the same noisy field is applied to both beams indicate that we deal with a dephasing rather than a decoherence effect (Stern et al. [10]). The question of how “real” decoherence (irreversibility) can be achieved remains open. As long as the same disturbance is applied to both beams the coherence features
Figure 5. Reduction of the momentum beam modulation when noisy fields are applied, shown on an absolute scale (a) and in a background-corrected way (b). The general loss of contrast as a function of the field strength is shown below (Sulyok et al. [11])

are preserved and that holds even when absorption processes occur with the same probabilities in both beams (Summhammer et al. [12]).

4. Inelasticity of noisy fields
In time-dependent fields like those discussed in section 3 photon exchanges between the field and the neutron occur. In the case of a Rabi resonance flipper a one photon exchange occurs and the neutron polarization changes from spin-up to spin-down and vice versa (Badurek et al. [13]). For non-resonance magnetic fields multi photon exchange has been observed (Summhammer et al. [14]). Here we want to see whether these inelastic processes can yield decoherence.

One assumes oscillating fields within a region $0 < x < L$ which is described as

$$\vec{B}(\vec{r}, t) = [B_0 + B(t) \cdot (\Theta(x) - \Theta(x - L))] \vec{z}$$

$$B(t) = \sum_{i=1}^{N} B_i \cos(\omega_i t + \varphi_i).$$

Taking into account that the kinetic energy of the neutrons ($\sim 20$ meV) is much higher than the maximal potential barrier ($\mu B_i \sim 0.5$ neV) one obtains after some analytical efforts the wave field behind the field region (Sulyok et al. [15])

$$\Psi_{III}(x, t) = \sum_{\vec{n}} J_{R_1}(\beta_1) \cdots J_{R_N}(\beta_N) e^{-\vec{n}\cdot \vec{x}} e^{-i k_n x} e^{-i \omega n t}$$
with
\[ \omega_{n_i} = \omega_0 + \pi \omega \]
\[ \vec{k}_{n_i}^2 = k_0^2 - \frac{2m}{\hbar^2} \mu B_0 + \frac{2m}{\hbar} \pi \omega \]
\[ \eta_i = \varphi_i + \frac{\omega_i T + \pi}{2} \]
\[ \beta_i = 2\alpha_i \sin \frac{\omega_i T}{2} \]
\[ \alpha_i = \frac{\mu B_i}{\hbar \omega_i} \]
\[ T = \frac{L}{v_0} \]
\[ \vec{n} = (n_1, \ldots, n_N), \quad \vec{\varphi} = (\varphi_1, \ldots, \varphi_N), \quad \vec{\omega} = (\omega_1, \ldots, \omega_N), \quad \vec{\eta} = (\eta_1, \ldots, \eta_N), \]

and where \( J_{n_i}(\beta_i) \) denote the Bessel functions of order \( n_i \), which determine the transition amplitudes. From that one gets the interference pattern as:

\[ I_0(x, t) = \frac{1}{2} |\Psi_I(x, t) + e^{i\chi} \Psi_{III}(x, t)|^2 = 1 + \text{Re} \left\{ e^{i\chi} \sum_{\vec{n}} J_{n_1} \cdots J_{n_N} e^{i\vec{n}(\xi + \vec{\omega} t)} \right\} \]  (11)

with
\[ \xi_i = \eta_i - \frac{\omega_i x}{v_0}. \]

If the fundamental frequency of all frequencies is \( \omega_f \), the interference pattern can be expressed in a Fourier series

\[ I_0(x, t) = \sum_{m=-\infty}^{\infty} c_m(x) e^{im\omega_f t} \]  (12)

where a comparison with Eq. (9) gives for unpolarized neutrons where only even \( m \)-terms remain:

\[ c_m = \delta_{m0} + \sum_{\vec{n}; \vec{n} \omega = m \omega_f} J_{n_1}(\beta_1) \cdots J_{n_N}(\beta_N) e^{i\vec{n} \cdot \vec{\xi}} \cos \chi \]  (13)

which shows that the Fourier coefficient belonging to the frequency \( m \omega_f \) contains the same product of Bessel functions as the transition amplitudes for an energy exchange \( m \hbar \omega_f \). The arguments of the Bessel functions also contain a \( \sin(\omega t/2) \)-term defining a “resonance”-condition. If the time-of-flight \( T = L/v \) through the field region fulfills \( \omega_i T = 2\pi \cdot L \) \((L = 1, 2, 3, \ldots)\) no resulting energy exchange occurs.

Related experiments have been done with a time resolved analysis of the interference pattern, where the periodicity of the applied magnetic field \( (\omega_f = 2\pi f_f; f_f = 1 \text{ kHz}) \) also manifests itself in the interference pattern (Sulyok et al. [15]). The related energy transfers lie in the order of 1 peV and energy gain and energy loss processes are equal, which is caused by the symmetric sinusoidal fields. The calculated results show good agreement with the measured values (Fig. 6). One notices that absorption and emission probabilities are equal which results from the symmetric interaction potential (Eq. (8)).

This procedure has been extended experimentally up to a 10 mode field and further by numerical methods up to a 1000 mode field. If random phases are assumed, and Gaussian variations of the amplitude and different frequency spectra, one simulates a noisy field. Fig. 7 shows the results for a rather low frequency and a higher frequency spectrum.

As a conclusion of this section it should be mentioned that a loss of visibility in an interference experiment does a priori not indicate a loss of coherence, but indicates that decoherence is rather difficult to achieve and probably as difficult as to preserve coherence. More and more complete quantum measurements may roll back decoherence to more sophisticated investigations, but some decoherence will stay as a residual of all interactions the system experienced, due to unavoidable losses (Rauch [9]).
Figure 6. Example of a time-resolved interference pattern for a two mode field $f_1 = 3$kHz and $f_2 = 3$kHz and $B_1 = 30$ G (left) and extracted multi-photon transition probabilities (right) (Sulyok et al. [15]).

Figure 7. Calculated loss of contrast for a low frequency and a high frequency field as a function of the mean field amplitude (Sulyok et al. [15]).

5. Discussion
Neutron interference experiments have shown that washed out interference pattern can be restored up to a high degree when proper post-selection procedures are applied. One may conclude that the particle-wave duality relation Eq. (2) has to be modified accordingly. Even in the case of a multi-mode noise field with multi-photon exchange the contrast can be retrieved when the same noise field is applied to the other beam or a time-resolved measurement is done. Whenever the reason for dephasing is known retrieval up to a certain degree is possible, but unavoidable quantum losses of coherence remain from any interaction the quantum system experienced (Rauch [9]). More complete measurements may show how much more information is inherent in any experiment and usually a small part is extracted. It also means that a complete isolation from the environment is impossible and a residual entanglement between the quantum object and the environment remains causing causal connections between the micro- and macro world (Zeh [16], Zurek [17]). This is not in contradiction with the idea that irreversibility is a fundamental property of nature and reversibility is only an approximation, a conclusion stated by several authors (e.g. Prigogine [18], Haag [19], Blanchard and Jadczyk [20]). At the same time it is closely connected to the quantum measurement problem, because a complete retrieval becomes impossible since in any interaction small loss mechanisms are unavoidable and
they are caused by boundary and initial conditions (Giulini et al. [21], Namiki et al. [22]). The appearance of entropy associated with decoherencing effects reflects the presence of an arrow in time in quantum theory, i.e. a fundamental irreversibility in the formalism of the theory itself (Rauch [9]). This irreversibility comes into play only through initial and boundary conditions in our universe.

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