Elliptic flow of the proton in $^{197}$Au + $^{197}$Au collisions reaction at intermediate energy by using a quantum molecular dynamics model

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Abstract. The elliptic flow ($v_2$) of the proton in $^{197}$Au + $^{197}$Au collisions reaction at intermediate energy by using a quantum molecular dynamics model (QMD) is investigated. We analyse the elliptic flow of the proton in $^{197}$Au + $^{197}$Au collisions reaction at incident energy 0.09, 0.12, 0.15 and 0.25 A GeV with the impact parameter from 0.25 to 0.45 (0.25 < $b_0$ < 0.45) by including a nuclear equation of state (EoS). The theoretical calculated results are compared with the FOPI data. We observe that the EoS effects obviously to the elliptic flow as a function of the centrality transverse component $t$ of the 4-velocity ($u_0$). After taking into account a soft nuclear equation of state (Soft EoS), a theoretical is in a good agreement with the experimental data this indicates that the elliptic flow as a function of the $u_0$ is one of sensitive probes to extract information on the EoS properties at high density.

1. Introduction

Recently, a nuclear matter under exclusive conditions has been investigated in the major research on topics nuclear and high energy physics [1-5]. The relativistic heavy-ion collision at high energy of 1-2 A GeV provides a unique opportunity to study the nuclear equation of state (EoS) at high energy and high temperature [6-7]. Although, the result of relativistic heavy-ion collisions is not only interest in itself, but also useful in understanding other filed in physics, such as the properties of the core of compact stars, the evolution of the early universe and the formation of element in stellar nucleosynthesis. The relativistic Boltzmann-equation calculated and compared with elliptic flow data.

According to the previous research [8], the elliptic flow excitation as a function of mid-central collision of $^{197}$Au + $^{197}$Au at 2, 4, 6 and 8 A GeV has been studied. The result demonstrated that the excitation function was changed from negative to positive elliptic flow with beam energy at 4 A GeV. The comparisons recommended that the evolution from a stiff EoS ($K \approx 380$ MeV) at low beam energies ($\leq 2$ A GeV) to a softer EoS ($K \approx 210$ MeV) at the higher beam energy (4
\[ \leq E_{\text{beam}} \leq 11 \text{ A GeV} \] Thus, a softening of the EoS could result from a number of effects and the possible onset of a nuclear phase change. Another research on the elliptic flow of fragment for different reacting systems at incident energies between 50 and 100 MeV/nucleon using the isospin-dependent quantum molecular dynamics (IQMD) model was investigated. The results showed that the elliptic flow displayed a transition from positive (in-plane) to negative (out-of-plane) value in the mid-rapidity region at incident energy. (known as transition energy). This transition energy was found to depend on the model ingredients and the size of the fragments and composite mass of the reacting system as well as on the impact parameter of the reaction. A reasonable agreement was observed for the excitation function of elliptic flow between the data and theoretical calculations. The transition energy was found to exhibit a power law mass dependence [9]. From the above research, the elliptic flow has been generally recognized and observed in the nuclear equation of state at high densities and high temperature.

In this work we presented the elliptic flow of protons in \(^{197}\text{Au} + ^{197}\text{Au}\) collisions as a function the centrality transverse component \(t\) of the 4-velocity at 0.09, 0.12, 0.15 and 0.25 \(\text{A GeV}\) and the impact parameter from 0.25 to 0.45 (0.25 \(< b_0 < 0.45\)) by using a quantum molecular dynamics model, which the quantum molecular dynamics model has been a frequently used to simulate the collision of heavy ion collision at intermediate energies and then the theoretical calculation result was compared with the FOPI experimental[10].

2. Theories

2.1. Quantum Molecular Dynamics (QMD) Model

The quantum molecular dynamics [11] in which each nucleon is represented by a coherent state of the form

\[ \psi(r, p, t) = \frac{1}{(2\pi L)^{3/2}} \exp \left\{ -\frac{(r - r_0)^2}{4L} \right\} \exp \{i p \cdot (r - r_0)\}, \]  

where \(r_0\) is the center of a Gaussian wave pocket and \(L = 1.08 \text{ fm}^2\) is the width of the wave pocket. Consequently, the density of the system with \(N\) nucleons in coordinate space is given as follows:

\[ \rho(r, t) = \sum_{i=1}^{N} \frac{1}{(2\pi L)^{3/2}} \exp \left\{ -\frac{(r - r_{i0})^2}{2L} \right\}. \]  

The time evolution of the \(N\)-body distribution is determined by the motion of the centroid of Gaussian \(\{r_{i0}, p_{i0}\}\), which is propagated by the Poisson brackets,

\[ \dot{r}_{i0} = \{p_{i0}, H\}, \]  

\[ \dot{p}_{i0} = \{r_{i0}, H\}, \]  

with \(H\) is the nuclear Hamiltonian

\[ H = \sum_{i=1}^{N} \sqrt{p_{i0}^2 + m_i^2} + \sum_{i<j}^{N} \left( U_{ij}^{\text{Str}} + U_{ij}^{\text{Coul}} \right). \]  

Here \(U_{ij}^{\text{Str}}\) is a nuclear mean field. \(U_{ij}^{\text{Coul}}\) is the Coulomb interaction.

The strength of the nuclear compression is quoted normally in terms of the incompressibility by value constant \(K\) (compressibility), which the compressibility defined as [12]:
\[ K = 9\rho^2 \frac{\partial^2}{\partial \rho^2} \left( \frac{E}{A} \right). \]  \hspace{1cm} (6)

For the explanation of the energy per nucleon \( \frac{E}{A} \) as a function of density, usually Skyrme parameterizations \( (U) \) are used in equation (7). Two different equations of state are commonly used: A hard equation of state (Hard EoS) with a compressibility of \( K = 380 \text{ MeV} \) and a soft equation of state (Soft EoS) with a compressibility of \( K = 200 \text{ MeV} \) [11]. The Skyrme parameterizations \( (U) \) are followed by

\[ U = \alpha \left( \frac{\rho}{\rho_0} \right) + \beta \left( \frac{\rho}{\rho_0} \right)^\gamma, \]  \hspace{1cm} (7)

where \( \rho \) is the nuclear density which is measured in units of the saturation density \( (\rho_0 \approx 0.16 \text{ fm}^{-3}) \). The relation of the \( \alpha, \beta \) and \( \gamma \) is shown in table 1, which the differential value of \( K \) constant is plotted in figure 1.

**Table 1.** Parameters in the equations (6) and (7) for the soft and hard nuclear equation of state (EoS) [11].

| \( K \) (MeV) | \( \alpha \) | \( \beta \) | \( \gamma \) | EoS   |
|--------------|-------------|-------------|-------------|-------|
| 200          | -356       | 303         | 7/6         | Soft  |
| 380          | -124       | 70.5        | 2           | Hard  |

**Figure 1.** The density dependence of the energy per particle in nuclear matter at temperature \( T=0 \) is displayed for the four different sets of parameters show in table 1 [11].
2.2. Elliptic flow

The phenomenon of collective flow could be quantitatively described in terms of anisotropies of the azimuthal emission pattern, expressed by a Fourier series \[13\]

\[
\frac{dN}{d\phi} (\phi) \propto 1 + 2v_1 \cos (\phi) + 2v_2 \cos (2\phi),
\]

when \(v_1\) and \(v_2\) are the Fourier coefficient, \[9\]

\[
-v_2 = -\left\langle \left( \frac{p_x}{p_t} \right)^2 - \left( \frac{p_y}{p_t} \right)^2 \right\rangle = -\langle \cos (2\phi) \rangle,
\]

whereas \(\phi\) is the azimuthal angle of the outgoing particle with respect to the reaction plane. \(p_t = \sqrt{p_x^2 + p_y^2}\) is called the transverse momentum. The second order the Fourier coefficient \[14\], \(v_2\) describes the elliptic flow as shown in figure 2 below:

**Figure 2.** Isotropic elliptic as the elliptic flow: if \(v_2\) is large, the circle will move on the Y axis.

2.3. Centrality transverse component \(t\) of the 4-velocity

The magnitude of the velocity of the reference frame on the transverse coordinates is described by

\[
u_{t0} = \frac{u_t}{u_p},
\]

where \(u_t\) is the velocity of reference frame on the transverse coordinates, described to

\[
u_t = \beta_t \gamma_t,
\]

\[
\beta_t = \frac{p_t}{E},
\]
\[ \gamma_t = \frac{1}{\sqrt{1 - \beta_t^2}}, \]  

(13)

and \( u_t \) can be proved by \( \gamma_t \) and \( \beta_t \) which \( \beta_t \) is the velocity of the transverse plane and where \( u_t \) is the velocity of reference frame on the reaction coordinates, related to

\[ u_t = \beta_p \gamma_p, \]  

(14)

\[ \beta_t = \frac{p_z}{E}, \]  

(15)

\[ \gamma_p = \frac{1}{\sqrt{1 - \beta_p^2}}, \]  

(16)

and \( u_p \) can be verified by \( \gamma_p \) and \( \beta_p \) which \( \beta_p \) is the velocity of the reaction plane. Both of equation have \( \gamma \), that is the Lorenz factor, which helps to connect two experiments frame of the particle physics system together [15].

3. Results and Discussion

![Figure 3.](image)

Figure 3. (a), (b), (c) and (d) show the elliptic flow of the proton \((v_2)\) as a function of the centrality transverse component \( t \) of the 4-velocity at the incident energy 0.09, 0.12, 0.15 and 0.25 A GeV for the impact parameter from 0.25 to 0.45 by using the nuclear equation of state (EoS; Soft and Hard). The theoretical results calculation is compared with the FOPI data [10].
Figures 3 (a), (b), (c) and (d) display the elliptic flow of the proton ($v_2$) as a function of the centrality transverse component $t$ of the 4-velocity in heavy ion collisions of $^{197}$Au + $^{197}$Au at incident energy 0.09, 0.12, 0.15 and 0.25 A GeV for the impact parameter from 0.25 to 0.45 with the nuclear equation of state (EoS; soft and hard). It is perceived from this figure that the centrality transverse component $t$ of the 4-velocity increase higher than 1.6, which the fluctuations of the theoretical calculation are also increasing. The problem is explained by a momentum dependence of the nucleon mean field that is too hard at large momenta, which pushes the high-momentum of the protons too soon from the central of the collision before the elliptic flow is measured [16,17]. Conversely, the momentum dependence of the nucleon mean field of suitable protons lead to a good fit to the experimental data. The result from the figures show that the elliptic flow as a function of the centrality transverse component $t$ of the 4-velocity is based on the soft nuclear equation of state; it is in a good agreement with the FOPI experimental and has smallest RMSE (table 2). This indicates that the result calculated by using the soft EoS is the best one for describing the experimental data. In the previous research which the quantum molecular dynamic is used and predicted in the other work that can found in the reference [15,18] and our results similar to the above-mentioned work, represent that the QMD model can use with other experiments (beam energy less than 3 A GeV).

Table 2. The root mean square errors [19] (RMSE) for calculated result of the elliptic flow of proton as a function the centrality transverse component $t$ of the 4-velocity in $^{197}$Au + $^{197}$Au collision at incident energy 0.09, 0.12, 0.15 and 0.25 A GeV for the impact parameter from 0.25 to 0.45.

| Energies (A GeV) | Soft EoS | Hard EoS |
|-----------------|----------|----------|
| 0.09            | 0.0380   | 0.0401   |
| 0.12            | 0.0246   | 0.0897   |
| 0.15            | 0.0401   | 0.1418   |
| 0.15            | 0.0747   | 0.1203   |

4. Conclusion
We use the quantum molecular dynamic model to simulate the proton production, to analyze the elliptic flow of proton in $^{197}$Au + $^{197}$Au as a function of the centrality transverse component $t$ of the 4-velocity at the incident energy 0.09, 0.12, 0.15 and 0.25 A GeV respectively, the impact parameter from 0.25 to 0.45 ($0.25 < b_0 < 0.45$) and to compare calculated results with the FOPI data. We observe that the elliptic flow as a function of the centrality transverse component $t$ of the 4-velocity by including the soft nuclear equation of state ($K$ compression is 200 MeV) is in a good agreement with experimental data. This means that the elliptic flow of the proton is one of sensitive probe to extract information on the nuclear equation of state properties at high density.

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