Magnetic anisotropy of BaCu$_2$Si$_2$O$_7$: theory and antiferromagnetic resonance

R. Hayn,$^1$ V.A. Pashchenko,$^2$ A. Stepanov,$^1$ T. Masuda,$^3$ and K. Uchinokura$^3$

$^1$Laboratoire Matériaux et Microélectronique de Provence associé au CNRS, Faculté de Saint-Jérôme, 13397 Marseille Cedex 20, France
$^2$ Institute for Low Temperature Physics and Engineering, National Academy of Sciences of Ukraine, 310164 Kharkov, Ukraine
$^3$Department of Advanced Materials Science, The University of Tokyo, 6th Engineering Bldg., 7-3-1 Hongo, Bunkyo-ku, Tokyo, 113-8656, Japan

(Dated: November 7, 2018)

Antiferromagnetic resonance (AFMR) of BaCu$_2$Si$_2$O$_7$ and a microscopic theory for the magnetic anisotropy of spin 1/2 chain compounds with folded CuO$_3$ geometry being in good agreement with the available data are presented. The AFMR studies at 4.2 K show the existence of two gaps (40 and 76 GHz) at zero magnetic field and of two spin re-orientation transitions for $H \parallel c$. The microscopic origin of the two gaps is shown to be Hund’s rule coupling which leads to a “residual anisotropy” beyond the compensation of the Dzyaloshinskii-Moriya term by the symmetric anisotropy which would be valid without Hund’s coupling.

There exist several cuprate compounds with CuO$_3$ corner-sharing chains that are known as nearly ideal model compounds for one-dimensional spin 1/2 systems. Those with folded CuO$_3$ zigzag chains, like BaCu$_2$Ge$_2$O$_7$ (Néel temperature of $T_N = 8.8$ K) [1] or BaCu$_2$Si$_2$O$_7$ ($T_N = 9.2$ K) [2,3,4,5] open the fascinating possibility to study noncollinear magnetism or the influence of a Dzyaloshinskii-Moriya (DM) term in a quantum spin chain. Especially BaCu$_2$Si$_2$O$_7$ attracted much interest recently in connection with the finding of two consecutive spin re-orientation transitions for a magnetic field applied along the easy c-axis, which were discussed controversially, however [2,3]. Furthermore, one [2] or two gaps [3] in the spin-wave spectrum measured by neutron scattering were reported. And also the microscopic origin of the magnetic interaction energies was not clarified up to now. The present work addresses all these points by a combined experimental and theoretical study, using antiferromagnetic resonance (AFMR) measurements and perturbation theory with respect to spin-orbit (SO) coupling in a Cu-O-Cu cluster with folded geometry.

Our experimental study shows clearly the existence of two different gaps at zero magnetic field in BaCu$_2$Si$_2$O$_7$ and a magnetic structure evolution with increasing field $H \parallel c$ with two re-orientation transitions corresponding to a spin rotation towards the middle $b$-axis followed by a rotation towards the hard $a$-axis which confirms the neutron scattering structure of Ref. [3].

Theoretically we show that the deviation from a straight bonds in a Cu-O-Cu cluster leads to a DM interaction and a spin canting of neighboring spins at $T < T_N$. The canting angle is rather large and the DM interaction alone would lead to much larger gaps in spin wave spectrum and AFMR than actually observed. It is the symmetric anisotropy part which compensates the DM one, as proposed in Refs. [4,5] and applied here to the given situation. A clear experimental confirmation of this effect was found in Ba$_2$CuGe$_2$O$_7$ [5]. We argue that Hund’s coupling leads to a “residual anisotropy”, beyond this compensation, which agrees well with the experimental data and explains the observed easy axis behavior and the two gaps mentioned above. Naturally, our theory is also valid for BaCu$_2$Ge$_2$O$_7$ which differs from BaCu$_2$Si$_2$O$_7$ by the interchain couplings. The more simple AFM structure of the Ge-compound allows for a weak ferromagnetic moment which gives direct information on the spin canting angle [5].

The crystal structure of BaCu$_2$Si$_2$O$_7$ and BaCu$_2$Ge$_2$O$_7$ belonging to the orthorhombic space group $Pnma$ is made of almost isolated CuO$_3$ corner-sharing chains running along the c-axis [6]. The Cu-O-Cu bond angle is found to be $124^\circ$ in the case of BaCu$_2$Si$_2$O$_7$ which is smaller than that of BaCu$_2$Ge$_2$O$_7$ ($135^\circ$). For

![FIG. 1: The frequency vs field diagram ($H \parallel c$) of AFMR modes in BaCu$_2$Si$_2$O$_7$ at $T = 4.2$ K. Dashed lines: AFMR modes of a biaxial antiferromagnet without the DM interaction in a collinear (phase 1) and a canted (phase 2) phase according to Ref. [5].](http://example.com/fig1.png)
both compounds a broad maximum was observed in the temperature dependence of the magnetic susceptibility indicating 1D magnetic behavior and allowing to extract the interchain exchange constants \( J = 47 \text{ meV} \) \((J = 24 \text{ meV})\) for the Ge (Si) compound. The single crystals of BaCu$_2$Si$_2$O$_7$ used in an AFMR experiment were grown by a floating-zone method. Since the \(a\)- and \(c\)-axis lengths are almost the same, particular attention was paid to the crystal orientation. The AFMR study of BaCu$_2$Si$_2$O$_7$ was done using a simple millimeter-range video spectrometer \([11]\). The microwave sources were either Gunn’s diodes or back-ward wave tubes. The field dependence of AFMR modes in BaCu$_2$Si$_2$O$_7$ at \( T = 4.2 \text{ K} \) is shown in Fig. 1. Two gaps are clearly seen at \( H = 0 \), \( \nu_{LF} = 40 \text{ GHz} \) (low-frequency mode) and \( \nu_{HF} = 76 \text{ GHz} \) (high-frequency mode). The gaps values are about 0.21% smaller as compared to the earlier reported values 0.21 meV and 0.36 meV measured at lower temperature \( T = 2 \text{ K} \) \([3]\). The observed frequency softening of the low-frequency mode at \( H_{\alpha} \) and \( H_{\beta} \) confirms the existence, for this particular magnetic field orientation, of two consecutive spin re-orientation transitions. As it was first proposed in Ref. \([10]\) this behavior is characteristic of an AFM with the DM interaction in a magnetic field applied along the easy axis and the DM vector orientation along the middle axis (\(c\-) and \(b\)-axis respectively in the case of BaCu$_2$Si$_2$O$_7$; see Fig. 4 of Ref. \([10]\) for comparison). Furthermore, our observation of quadratic field dependence of the high-frequency AFMR mode for \( H \parallel a \) (not presented in Fig. 1) allows the conclusion that the \(a\)-axis is the hard axis of the magnetic anisotropy tensor of BaCu$_2$Si$_2$O$_7$ \([12]\).

To understand the microscopic origin of the observed behavior in Fig. 1 and also other experimental facts already established for the two compounds in question we investigate the special Cu-O-Cu bond sketched in Fig. 2 with an arbitrary bond angle \( \pi - \phi \). The isotropic exchange, the DM term and the symmetric anisotropy are calculated by perturbation theory with respect to SO coupling and kinetic energy. The Hamiltonian contains all the 3\(d\) orbitals at the 2 Cu-sites (denoted by \(m\) or \(m'\) = \(\{0(d_{x^2-y^2}), z(d_{xy}), x(d_{xz}), y(d_{xz}), 1(d_{3z^2-r^2})\}\), where the usual notation is given in parenthesis) and the three \(2p\) orbitals \((n \text{ or } n' = \{p_x, p_y, p_z\})\) at the oxygen site in between. The local, unperturbed part contains the Coulomb terms

\[
\hat{H}_0 = E_p n_p^p + \frac{U_p}{2} \left( n_p^p n_p^p - n_p^p \right) + \sum_{\alpha m} E_m n^d_{\alpha m} + \frac{U_d}{2} \sum_{\alpha} \left( n^d_{\alpha m} n^d_{\alpha m} - n^d_{\alpha m} \right),
\]

with \(n^d_{\alpha m} = \sum n^d_{\alpha m} = \sum_{m m'} d^\dagger_{\alpha m} d_{\alpha m'} d_{\alpha m} d_{\alpha m'}\) and \(n^p = \sum_{\sigma} n^p_{\alpha m} = \sum_{m m'} d^\dagger_{\alpha m} d_{\alpha m'}\) where \(\alpha = A\) or \(B\) (sort of Cu), and \(\sigma\) are spin indices. The charge transfer energy \(\Delta_p = E_p - E_0\) has been set to 4 eV \([13]\) and \(E_m - E_0 = \epsilon_d = 2\) eV shall be the same for all \(m = \{x, y, z, 1\}\) \([13]\). We use in the following the standard parameters of cuprates, i.e. \(U_p = 4 \text{ eV}\) and \(U_d = 10 \text{ eV}\). All the remaining terms of the Hamiltonian are treated as a perturbation. That concerns especially the kinetic energy

\[
\hat{H}_t = \sum_{\alpha m \sigma} t_{\alpha m \sigma} (d^\dagger_{\alpha m \sigma} p_{\sigma \alpha} + h.c.) ,
\]

and the spin-orbit interaction

\[
\hat{H}_{SO} = \frac{\lambda}{2} \sum_{\alpha m \sigma \sigma'} \sum_{m' m'' \sigma' \sigma''} \epsilon_{\alpha m \sigma}^d \epsilon_{\alpha m' \sigma'}^d \epsilon_{\alpha m'' \sigma''}^d \epsilon_{\alpha m'' \sigma''}^d ,
\]

(\(\lambda = 0.1 \text{ eV}\)) with the only nonzero matrix elements \(L^2_{m m'} = 2i\), \(L^2_{m m'} = -i\), and \(L^2_{m m'} = -i (L^2_{m m'} = (L^2_{m m'})^*\), where \(b\) is a Cartesian coordinate and \(\sigma\) the corresponding Pauli spin matrix. The Hund’s exchange interaction is given by

\[
\hat{H}_H = -J_H d \sum_{\sigma \sigma'} \sum_{m \not= m'} d^\dagger_{\sigma m} d_{\sigma' m} d^\dagger_{\sigma' m'} d_{\sigma m'},
\]

\[
-J_{HF} \sum_{\sigma \sigma'} \sum_{n \not= n'} \epsilon_{n m}^p \epsilon_{n m'}^p \epsilon_{n m}^p \epsilon_{n m'}^p.
\]

In a first step, the intermediate oxygen orbitals are excluded and an effective spin Hamiltonian

\[
\hat{H}^{ex} = \frac{1}{2} \sum \sum_{\{m\} \sigma \sigma'} \epsilon_{m m'}^d \epsilon_{m m'}^d \epsilon_{m m'}^d \epsilon_{m m'}^d \epsilon_{m m'}^d \epsilon_{m m'}^d
\]

\[
\cdot J (m_{1 A} m_{2 B} m_{3 A} m_{4 B})
\]

is derived. The necessary exchange terms are calculated in 4th order perturbation theory with respect to the kinetic energy \(\hat{H}_t\). The magnetic anisotropy terms arise due to the SO interaction. Up to second order, the result can be written as

\[
\hat{H}_{DM} + \hat{H}_A = \hat{D} \left( \hat{S}_A \times \hat{S}_B \right) + \hat{S}_A \hat{S}_B \hat{S}_A \hat{S}_B ,
\]

with \(\Omega_{\alpha \beta} = \Gamma_{\alpha \beta}^{\alpha \beta} - \delta_{\alpha \beta} \sum \Gamma_{\alpha \beta}^{\alpha \beta}\). The first order term corresponds to a DM interaction:

\[
\hat{D} = \sum \sum \sum \alpha \beta \gamma \delta (J(m_{1 A} m_{2 B} m_{3 A} m_{4 B}) - J(m_{1 A} m_{2 B} m_{3 A} m_{4 B}))
\]

\[
\cdot \frac{1}{E_m - E_0} .
\]
and the second order describes a symmetric spin anisotropy

\[ \Gamma_{ab} = \left( \frac{\lambda}{2} \right)^2 \sum_{mm'} \left\{ \frac{L_{0m}^a L_{0m'}^b j(m_A, 0_B; m'_A, 0_B)}{(E_m - E_0)(E_{m'} - E_0)} + \frac{L_{0m}^a L_{0m'}^b j(m_A, m'_B; 0_A, 0_B)}{(E_m - E_0)(E_{m'} - E_0)} + A \leftrightarrow B \right\} . \]

There we neglected those terms in second order \( \sim \lambda^2 \) which lead only to a correction of the isotropic exchange interaction. To apply the general theory to the given situation one has to specify the transfer terms that couple \(|A, 0\rangle\) and \(|B, 0\rangle\) due to the different perturbations. First, we neglect Hund’s exchange. The oxygen \( p_x, p_y \) and \( p_z \) orbitals are defined in the local coordinate system of CuA (\( x'y'z' \) in difference to the \( x'y'z' \) system at CuB) and the relevant transfer terms between CuA and the intermediate oxygen are \( t_{A,0,px} = t_{pd} \) and \( t_{A,y,p_z} = t_{pd}/\sqrt{3} \), where we used the ratio between \( \sigma \) and \( \pi \) transfer to be \( 1 : \sqrt{3} \) which is valid with good approximation. To obtain the transfer integrals of oxygen with the CuB orbitals we have to rotate the \(|p'_x\rangle\) and \(|p'_y\rangle\) orbitals to \(|p_x\rangle\) and \(|p_y\rangle\). Then we get the transfer terms

\[
\begin{align*}
    t_{B,0,px} &= \cos \phi t_{pd}, \\
    t_{B,0,pz} &= -\sin \phi t_{pd}, \\
    t_{B,y,pz} &= \sin \phi \frac{t_{pd}}{\sqrt{3}}, \\
    t_{B,y,px} &= \cos \phi \frac{t_{pd}}{\sqrt{3}}.
\end{align*}
\]

The next step consists in calculating the exchange integrals in \( \tilde{H}^{ex} \). Let us start with the isotropic exchange \( J = J(0_A, 0_B; 0_A, 0_B) \). We use perturbation theory with respect to \( \tilde{H}_t \) in 4th order. There are two possible intermediate states, the doubly occupied oxygen state \(|p_x\rangle\) with an energy \( 2\Delta_p + U_p \) and the doubly occupied copper state with the energy \( U_d \). Collecting all the possible paths we get

\[ J = 4G\tilde{b}^2 \quad \text{with} \quad G = \frac{1}{\Delta_p^2} \left( \frac{1}{U_d} + \frac{2}{2\Delta_p + U_p} \right) \]

and \( \tilde{b} = t_{pd}^2 \cos \phi \). With the standard value for cuprates \( t_{pd} = 1.3 \text{eV} \) we obtain for BaCu2Ge2O7 with \( \phi = 45^\circ \) the estimate \( J \approx 95 \text{meV} \) being roughly twice times larger than the experimental value. The reason is most probably the insufficiency of 4th order perturbation theory which is also known for the standard CuO2 plane. For the Si-compound \( \phi = 56^\circ \) we estimate 60 meV, whereas the experimental value is 24 meV. Now, the DM vector \( \tilde{B} \) shall be calculated. The two ground states \(|A, 0\rangle\) and \(|B, 0\rangle\) are coupled by SO and \( \tilde{H}_t \) only via the \(|B, y\rangle\) or the \(|A, y\rangle\) orbitals. Therefore, only \( D^y \) is different from zero and the corresponding exchange paths (in 4th order) are shown in Fig. 3 with

\[ J(y_A, 0_B; 0_A, 0_B) = 4G \left( \frac{t_{pd}^2 \sin \phi}{\sqrt{3}} \right)^2 . \]

Summing both contributions we get

\[ \Gamma^{yy} = 4Ge^2 . \]

Now, we see that we can combine the isotropic exchange \( \tilde{B} \), the DM term \( \tilde{S}_A \tilde{S}_B \) and the symmetric anisotropy \( \tilde{B} \)

\[ 4G \left( (\tilde{b}^2 - e^2)\tilde{S}_A \tilde{S}_B + 2bc(\tilde{S}_A \times \tilde{S}_B)_y + 2c^2 S^y_A S^y_B \right) = \tilde{J} \tilde{S}_A \tilde{S}_B \]

to an isotropic exchange of canted spins \( \tilde{S}_{A/B} \) with the canting angle \( \Theta \). In the zigzag chain one

FIG. 3: Exchange paths for the DM term, contributions to \( J(y_A, 0_B; 0_A, 0_B) \) (a) and \( J(0_A, y_B; 0_A, 0_B) \) (b). The initial state is indicated by full arrows, the final state by dashed arrows.

being equal to \(-J(y_A, 0_B; 0_A, 0_B)\). We find

\[ D^y = 8G\tilde{b}c \quad \text{where} \quad c = \frac{\lambda}{\varepsilon_d} \left( \frac{t_{pd}}{\sqrt{3}} \right) \tan \phi \]

results in 0.058 for BaCu2Ge2O7 whereas 0.033 was estimated on the basis of the magnetization data.

The contribution in second order of the SO coupling corresponds to a symmetric anisotropy term. The two relevant nonzero exchange terms are \( J(y_A, 0_B; y_A, 0_B) = -J(y_A, y_B; 0_A, 0_B) \). The corresponding exchange paths are very similar to those shown in Fig. 3. So, the first term is characterized by an exchange between \(|A, y\rangle\) and \(|B, 0\rangle\) via the \(|p_x\rangle\) orbital. The second, mixed exchange goes from \(|A, y\rangle\) via \(|p_z\rangle\) to \(|B, 0\rangle\) and from \(|B, y\rangle\) via \(|p_x\rangle\) to \(|A, 0\rangle\). The result can be given as

\[ J(y_A, y_B; 0_A, 0_B) = 4G \left( \frac{t_{pd}^2 \sin \phi}{\sqrt{3}} \right)^2 . \]
can subsequently rotate all the spins. As a consequence, we would obtain a Hamiltonian which is rotationally invariant in spin space and no crystallographic direction would be preferred. That is exactly the compensation as proposed in Refs. [3, 4]. Now, we are going to show that Hund’s exchange interaction (4) leads to a “residual anisotropy”. These symmetric anisotropy terms exist already for $\phi = 0$. The anisotropy terms (14) and (15) can be eliminated by the rotation (13) and only the additional contributions due to (9) are considered further on. Starting with Hund’s exchange at oxygen $J_{HH}$, new exchange integrals become possible in 4th order of $\hat{H}_t$ and first order of $\hat{H}_H$. For example, the exchange paths for $J^p(\varepsilon_A, 0; \varepsilon_A, 0) \beta$ are realized due to the transfer term $t_{Az, p} (= t_{Ay, p} \beta)$. It turns out that it is identical to $J^p(\varepsilon_A, 0; \varepsilon_A, 0) \beta = J^{(p)}$:

$$J^{(p)} = -\frac{8 |\Gamma_{pd}|^2 \cos^2 \phi}{\Delta \tau^2} \cdot \frac{J_{t}}{(2 \Delta \tau + U_{px})^2}.$$  

(16)

We have no contribution via $|A, x \rangle$ or $|B, x \rangle$, and therefore, $\Gamma_{xx} = 0$. Next, consider Hund’s exchange at copper $J_{Hd}$. Then the transfer terms $t_{Bz, p} = t_{B1, p} = \cos \phi t_{pd} / \sqrt{3}$ become important. One obtains

$$J^{(d)} = \frac{4 |\Gamma_{pd}|^2 \cos^2 \phi}{9 \Delta \tau^2} \cdot \frac{J_{t}}{U_{d}}.$$  

(17)

Both terms (10, 17) together result in the “residual anisotropy” contribution

$$\Gamma_{yy} = \frac{\Gamma_{zz}}{4} = \frac{\lambda^2}{\varepsilon_d^2} \left( J^{(d)} + J^{(p)} \right)$$  

(18)

of one plaquette. To connect the cluster of Fig. 2 with the real crystallographic structure (see Fig. 1 in [2]), one has to rotate the coordinate axes $\hat{e}_z = (z_A, z_B, z_C) = (0.84, 0.38, 0.40)$ and $\hat{e}_y = (y_A, -y_B, -y_C) = (0.48, -0.85, -0.21)$ (and also $\hat{e}_x$) to the crystallographic ones. We obtain the diagonal elements

$$\Gamma_{vv} = \frac{\Gamma_{yy}}{2} = \frac{\lambda^2}{\varepsilon_d^2} \left( 4 z_A^2 + y_A^2 \right)$$  

(19)

with $\nu = \{a, b, c\}$ and only one non-zero off-diagonal element $\Gamma_{ab}$. With the characteristic parameters $J_{t} = 0.4$ eV and $J_{Hd} = 1$ eV and setting $\Gamma_{cc}$ to zero, we get $\Gamma_{aa} = -2 \mu$eV and $\Gamma_{bb} = -0.5 \mu$eV. These numbers are extremely small, but we have checked that they are still larger than the classical dipole-dipole interaction. For an antiferromagnetic arrangements of spins along the chain, the preferred spin direction is the $c$-direction which is indeed the case for BaCu$_2$Ge$_2$O$_7$ [4] and BaCu$_2$Si$_2$O$_7$ [3]. From the theoretical analysis follows that we can expect for BaCu$_2$Si$_2$O$_7$ an easy $c$-axis, a middle $b$-axis and a hard $a$-axis. The diagonal anisotropy energies (13) are directly related to the two gaps seen in Fig. 1 (and also in neutron scattering [3]) according to the standard formulas $\nu_{LF} = \sqrt{2\lambda H_{A1}H_E}$ and $\nu_{HF} = \sqrt{2\lambda H_{A2}H_E}$ with $H_E = zJS$ ($S$ - spin, $z$ - number of nearest neighbors) and $H_{A1} = 2z|\Gamma_{pd}|(S, H_{A2} = 2z|\Gamma_{pd}|(S$. This yields 0.44 meV and 0.22 meV in reasonable agreement with the experimental values 0.36 meV and 0.21 meV.

In conclusion, our theory is applicable to folded CuO$_3$ chains and gives the isotropic exchange, the DM term and the spin canting angle, the compensating symmetric exchange and the “residual anisotropy” due to Hund’s coupling in good accord with the available experimental data of BaCu$_2$Ge$_2$O$_7$ and BaCu$_2$Si$_2$O$_7$ especially with the two gaps measured by AFMR for the latter compound. Possible differences between the two compounds might be caused by different interchain couplings which were not included in the present study.

We thank J.-L. Richard for discussions and NATO (PST.CLG.976416) and DFG (UKR 113/49/0-1) for support.

[1] I. Tsukada, J. Takeya, T. Masuda, and K. Uchinokura, Phys. Rev. B 62, R6061 (2000).
[2] I. Tsukada, Y. Sasago, K. Uchinokura, A. Zheludev, S. Maslov, G. Shirane, K. Kakurai, and E. Ressouche, Phys. Rev. B 60, 6601 (1999).
[3] M. Kenzelmann, A. Zheludev, S. Raymond, E. Ressouche, T. Masuda, P. Böni, K. Kakurai, I. Tsukada, K. Uchinokura, and R. Coldea, Phys. Rev. B 64, 054422 (2001).
[4] I. Tsukada, J. Takeya, T. Masuda, and K. Uchinokura, Phys. Rev. Lett. 87, 127203 (2001).
[5] A. Zheludev, E. Ressouche, I. Tsukada, T. Masuda, and K. Uchinokura, Phys. Rev. B 65, 174416 (2002).
[6] T.A. Kaplan, Z. Phys. B 49, 313 (1983).
[7] L. Shekhtman, O. Entin-Wohlman, and A. Aharony, Phys. Rev. Lett. 69, 836 (1992).
[8] A. Zheludev, S. Maslov, I. Tsukada, I. Zaliznyak, L.P. Regnault, T. Masuda, K. Uchinokura, R. Erwin, and G. Shirane, Phys. Rev. Lett. 81, 5410 (1998).
[9] J.A.S. Oliviera, Ph.D. thesis, Ruprecht-Karls-Universität, Heidelberg, 1993.
[10] A.I. Zvyagin, M.I. Kobets, V.N. Kivoruchko, A.A. Stepanov, and D.A. Yablonskii, Sov. Phys. JETP 62, 1328 (1986).
[11] T. Nagamiya, K. Yoshida, R. Kubo, Adv. in Phys 4, 1 (1955).
[12] A more detailed discussion of the AFMR in BaCu$_2$Si$_2$O$_7$ will be published elsewhere.
[13] M.S. Hybertsen, E.B. Stechel, M. Schlüter, and D.R. Jennison, Phys. Rev. B 41, 11068 (1990).
[14] We take a value for $\xi d$ that is known for the related cuprate Ba$_2$Cu$_2$O$_2$Cl$_2$: V. Yushankhai, M. Wolf, K.-H. Müller, R. Hayn, and H. Rosner, Phys. Rev. B 62, 14229 (2000).
[15] A. Aharony, O. Entin-Wohlman, and A.B. Harris in Dynamical Properties of Unconventional Magnetic Systems edited by A.T. Skjeltorp and D. Sherrington, Kluwer Academic (1998).
[16] By inspection of the equations of motion it is easily seen that the anisotropy in a direction influences the magnetic susceptibility in b-direction and vice versa.