CAN THE PIONEER ANOMALY BE INDUCED BY
VELOCITY-DEPENDENT FORCES? TESTS IN THE OUTER
REGIONS OF SOLAR SYSTEM WITH PLANETARY DYNAMICS

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In this paper we analyze the impact on the orbital motions of the outer planets of the solar system from Jupiter to Pluto of some velocity-dependent forces recently proposed to phenomenologically explain the Pioneer anomaly, and compare their predictions (secular variations of the longitude of perihelion \( \omega \) or of the semimajor axis \( a \) and the eccentricity \( e \)) with the latest observational determinations by E.V. Pitjeva with the EPM2006 ephemerides. It turns out that while the predicted centennial shifts of \( a \) are so huge that they would have been easily detected for all planets with the exception of Neptune, the predicted anomalous precessions of \( \omega \) are too small, with the exception of Jupiter, so that they are still compatible with the estimated corrections to the standard Newton-Einstein perihelion precessions. As a consequence, we incline to discard those extra-forces predicting secular variations of \( a \) and \( e \), also for some other reasons, and to give a chance, at least observationally, to those models predicting still undetectable perihelion precessions. Of course, adequate theoretical foundations for them should be found.

Keywords: Experimental tests of gravitational theories; Modified theories of gravity; Celestial mechanics

1. Introduction
The Pioneer anomaly (PA) consists of an unmodelled almost constant and uniform acceleration approximately directed towards the Sun of magnitude

\[ A_{\text{Pio}} = (8.74 \pm 1.33) \times 10^{-10} \text{ m s}^{-2} \]  

detected in the radiometric data from the Pioneer 10 (launched in March 1972) and Pioneer 11 (launched in April 1973) spacecraft after they passed the \( \approx 20 \) AU threshold moving with speed \( v_{\text{Pio}} \approx 1.2 \times 10^4 \text{ m s}^{-1} \) along roughly antiparallel escape hyperbolic paths undertaken after their previous encounters with Jupiter (\( \approx 5 \) AU) and Saturn (\( \approx 10 \) AU), respectively. Concerning the possibility that PA started to manifest itself at shorter heliocentric distances, efforts to retrieve and analyze early data from Pioneer 10/11 are currently being performed.
The Pioneer spacecraft were particularly well suited for radioscience celestial mechanics experiments because they were spin-stabilized\(^a\); in practice, they could be regarded as gyroscopes so that only a few orientation maneuvers, easily modeled, were needed every year to keep the antenna pointed towards the Earth. On the contrary, 3-axis stabilized spacecraft like Voyager 1/2 undergo continuous, semi-autonomous, small gas jet thrusts to maintain the antenna facing the Earth; as a consequence, their navigation is not as precise as that of the Pioneer 10/11.

The attempts performed so far to explain PA in terms of known effects of gravitational\(^3\) and/or non-gravitational\(^10\,11\) origin were found to be not satisfactory\(^12\,13\), so that a vast number of exotic explanations based on modified models of gravity were proposed (see, e.g., Ref. 3\(^14\,15\,16\) and references therein). If PA is due to some modifications of the known laws of gravity, this must be due to a radial extra-force affecting the orbits of the planets as well, especially those moving in the region in which PA manifested itself in its presently known form. The impact of a Pioneer-like additional acceleration on the motion of major and minor bodies in the outer regions of the solar system was recently studied by numerous authors with different approaches\(^17\,18\,19\,20\,21\,22\): it turned out that a constant and uniform extra-acceleration with the magnitude of Eq. (1) would produce huge secular effects which are neatly absent in the planetary data.

It was recently suggested\(^23\) that, from a purely phenomenological point of view, test bodies moving in the (outer) solar system could experience velocity-dependent extra-accelerations of the form

\[
A_v = -|v_r| \left( \frac{A_{Pio}}{v_{Pio}} \right), \quad A_v = -v_r \left( \frac{A_{Pio}}{v_{Pio}} \right),
\]

(2)

and

\[
A_{v^2} = -v_r^2 \left( \frac{A_{Pio}}{v_{Pio}^2} \right), \quad A_{v^2} = -|v_r| v_r \left( \frac{A_{Pio}}{v_{Pio}^2} \right),
\]

(3)

where \(v_r\) is the radial component of the test particle’s velocity \(v\); Eq. (2) and Eq. (3) would reduce to Eq. (1) for the Pioneer 10/11 spacecraft whose velocities can be assumed entirely radial in the outer regions of the solar system in which PA was detected. Standish in Ref. 22 put on the test such a hypothesis by fitting huge planetary data sets with the dynamical force models of the latest Jet Propulsion Laboratory (JPL) DE ephemerides modified ad hoc according to Eq. (2) and Eq. (3) and examining the results in terms, e.g., of the reliability of the estimated parameters. His conclusion was that the existence of extra-accelerations like those of Eq. (2) and Eq. (3) at heliocentric distances \(\gtrsim 20\) AU cannot be ruled out by the present-day available data of the outer planets because Eq. (2) and, especially, Eq. (3) would induce orbital effects on them too small to be detected. Their existence in the inner regions of solar system is, instead, ruled out.

\(^a\)This was due to the fact that Pioneer 10/11 were equipped with Radioisotope Thermoelectric Generators (RTG) placed at the end of long booms to be away from the spacecraft and thereby avoid any radiation damage.
In this paper we will follow a different approach by using the EPM2006 ephemerides produced by E.V. Pitjeva\textsuperscript{24} at the Institute of Applied Astronomy (IAA) of the Russian Academy of Sciences (RAS). First, we will analytically work out the secular effects of small perturbing accelerations like those of Eq. (2) and Eq. (3) on the Keplerian orbital elements of a planet in order to gain as clear as possible insights about the modifications which the orbits would undergo if Eq. (2) and Eq. (3) were real; should some implausible physical feature turn out, it would be more difficult to trust such proposed anomalous forces. Then, we will compare some of such predictions with the latest observational determinations for the outer planets estimated by Pitjeva with the EPM2006 ephemerides in a purely phenomenological way as corrections to the known effects due to usual Newton-Einstein laws, without modelling any additional force. In Table 1 we quote some quantities we will use. They are the outcome of a global fit of more than 400,000 data points (1913-2006) performed by Pitjeva\textsuperscript{25,24} with the EPM2006 ephemerides; about 230 parameters were estimated. It must be noted that the uncertainties $\delta\Delta \dot{\varpi}$ in the estimated corrections to the perihelion precessions are the formal ones re-scaled by a factor 10 in order to obtain realistic evaluations for them.

\begin{table}
\begin{tabular}{lcccc}
\hline
Planet & $\delta a$ (m) & $\Delta \dot{\varpi}$ (arcsec cy$^{-1}$) & $\delta\Delta \dot{\varpi}$ (arcsec cy$^{-1}$) \\
\hline
Jupiter & 615 & 0.0062 & 0.036 \\
Saturn & 4,256 & -0.92 & 2.9 \\
Uranus & 40,294 & 0.57 & 13.0 \\
Neptune & 463,307 & N.A. & N.A. \\
Pluto & 3,412,734 & N.A. & N.A. \\
\hline
\end{tabular}
\end{table}

2. The orbital effects of velocity-dependent perturbing forces yielding Pioneer-type accelerations

2.1. Forces linear in velocity

According to the classification of Ref.\textsuperscript{22}, the first two kinds of extra-forces are linear in $v_r$ being

\begin{align}
A^{(2)}_v &= -|v_r| \mathcal{K}, \\
A^{(3)}_v &= -v_r \mathcal{K},
\end{align}
with
\[ K = \frac{A_{\text{Pio}}}{v_{\text{Pio}}} = 7.3 \times 10^{-14} \text{ rad s}^{-1} = 47.4 \text{ arcsec cy}^{-1}. \] (6)

The radial acceleration of Eq. (4) is constantly inward, i.e. directed towards the Sun, while the one of Eq. (5) is directed towards the Sun when \( v_r > 0 \), i.e. when the planets gets farther from the Sun, while is directed away the Sun when \( v_r < 0 \), i.e. when the planet gets closer to the Sun. Indeed, for an unperturbed Keplerian ellipse\(^{26}\)
\[ v_r = \frac{nae \sin f}{\sqrt{1 - e^2}} = \frac{nae \sin E}{1 - e \cos E}, \] (7)
where \( a \) is the semimajor axis, \( e \) is the eccentricity, \( n = \sqrt{GM/a^3} \) is the mean motion, \( f \) is the true anomaly counted anticlockwise from the perihelion, and \( E \) is the eccentric anomaly. \( v_r > 0 \) for \( 0 < f < \pi \), i.e., from the perihelion to the aphelion, and \( v_r < 0 \) for \( \pi < f < 2\pi \), i.e. from the aphelion back to the perihelion.

In view of the smallness of Eq. (4) and Eq. (5) for the planets of the solar system, we will treat them perturbatively. Indeed, the radial velocities for the outer planets amount to \( 10^{-1} - 10^{-3} \text{ m s}^{-1} \) only, so that \( A_v \approx 10^{-11} \text{ m s}^{-2} \), while the Newtonian attraction of the Sun is for them of the order of \( 10^{-4} - 10^{-6} \text{ m s}^{-2} \).

Let us work out the secular precession of the longitude of perihelion \( \dot{\omega} \). The Gauss equation for its variation due to an entirely radial perturbing acceleration \( A_r \) is\(^{26}\)
\[ \frac{d\omega}{dt} = -\frac{\sqrt{1 - e^2}}{nae} A_r \cos f. \] (8)
By inserting Eq. (4) into Eq. (8), evaluating the r.h.s over the unperturbed Keplerian ellipse characterized by
\[ r = a(1 - e \cos E), \]
\[ dt = \left( \frac{1 - e \cos E}{n} \right) dE, \]
\[ \cos f = \frac{\cos E - e}{1 - e \cos E}, \] (9)
\[ \sin f = \frac{\sqrt{1 - e^2} \sin E}{1 - e \cos E}, \]
and averaging over one orbital period one gets
\[ \langle \dot{\omega} \rangle = -\frac{K \sqrt{1 - e^2}}{\pi} \left[ 2e - (1 - e^2) \ln \left( \frac{1 + e}{1 - e} \right) \right] < 0. \] (10)
We have used the fact that
\[ |v_r| = v_r, \ 0 < f < \pi; \ |v_r| = -v_r, \ \pi < f < 2\pi. \] (11)
Instead, Eq. (5) yields no perihelion precession. Indeed,
\[
\langle \dot{\varpi} \rangle = -K \sqrt{1 - e^2} \int_0^{2\pi} \frac{(\cos E - e) \sin E}{1 - e \cos E} dE = 0.
\] (12)

The Gauss equation for the variation of the semimajor axis due to a radial perturbing acceleration \([20]\)
\[
\frac{da}{dt} = \frac{2e}{n \sqrt{1 - e^2}} A_r \sin f.
\] (13)

By proceeding as before it turns out that Eq. (11) does not yield secular variations of \(a\); instead, Eq. (5) induces a secular decrease of \(a\) according to
\[
\langle \dot{a} \rangle = 2Ka \left( \frac{1}{\sqrt{1 - e^2}} - 1 \right).
\] (14)

Note that for circular orbits, i.e. \(v_r = 0\), \(\langle \dot{a} \rangle = 0\). Since
\[
\frac{de}{dt} = \left( \frac{1 - e^2}{2ae} \right) \frac{da}{dt}
\] (15)

when \(A = A_r\), also the eccentricity decreases:
\[
\langle \dot{e} \rangle = K(1 - e^2) \left( \frac{1}{\sqrt{1 - e^2}} - 1 \right).
\] (16)

As expected for a central force, the orbital angular momentum \(L = \sqrt{GMa(1 - e^2)}\) is conserved, on average: indeed, Eq. (14) and Eq. (16) yield
\[
\left\langle \frac{dL^2}{dt} \right\rangle = GM \left[ \langle \dot{a} \rangle (1 - e^2) - 2ae \langle \dot{e} \rangle \right] = 0.
\] (17)

Instead, the energy \(E = -GM/2a\) is not conserved; according to Eq. (14),
\[
\left\langle \dot{E} \right\rangle = \frac{GM}{2a^2} \langle \dot{a} \rangle = \frac{GMK}{a} \left( \frac{1}{\sqrt{1 - e^2}} - 1 \right) < 0.
\] (18)

Such a result is certainly suspect from a physical point of view.

In order to independently check the results obtained analytically we performed two numerical integrations of the equations of motion adding to the Newtonian monopole term the perturbing accelerations of Eq. (4) and Eq. (5): the qualitative features of the resulting motions are depicted in Figure 1 and Figure 2.

Let us now consider the problem of the existence of the accelerations of Eq. (4) and Eq. (5) from a phenomenological point of view according to the present-day planetary data available. In Table 2 we quote the predictions for the outer planets of the centennial shifts in \(m\) of the semimajor axis, according to Eq. (14), and of the secular perihelion precessions in arcsec \(cy^{-1}\), according to Eq. (10). Such predictions must be compared with the observationally determined parameters quoted in Table 1. Concerning the semimajor axis, the present-day accuracy in determining them would clearly allow to detect shifts as large as those of Table 2 for all planets from Jupiter to Pluto with the exception of Neptune, even by re-scaling the values of
Fig. 1. Numerically integrated trajectory for the radial acceleration of Eq. (4) proportional to $-|v_r|$. The longitude of perihelion $\varpi$ undergoes a retrograde precession while neither the semimajor axis $a$ nor the eccentricity $e$ experience secular variations.

Fig. 2. Numerically integrated trajectory for the radial acceleration of Eq. (5) proportional to $-v_r$. Both the semimajor axis $a$ and the eccentricity $e$ secularly decrease, while the longitude of perihelion $\varpi$ remains fixed.

Table 2. Second column: shift $\Delta a$, in m, of the semimajor axis of the outer planets over 1 cy according to Eq. (14) induced by the acceleration of Eq. (5) proportional to $-v_r$. Third column: secular precessions $\dot{\varpi}$ of the perihelia of the outer planets, in arcsec cy$^{-1}$, according to Eq. (10) due to the acceleration proportional to $-|v_r|$. Such values are to be compared with those in Table 1.

| Planet | $\Delta a$ (m) | $\dot{\varpi}$ (arcsec cy$^{-1}$) |
|--------|----------------|-------------------------------|
| Jupiter| -419,726        | -0.973                        |
| Saturn | -1,030,797      | -1.1                          |
| Uranus | -1,470,544      | -0.9                          |
| Neptune| -76,220         | -0.1                          |
| Pluto  | -88,154,057     | -4.9                          |

$\delta a$ of Table 1 by a factor 10 or more. The situation is less neat for the perihelion precessions. Indeed, it turns out that the present-day accuracy in determining them does not allow to rule out Eq. (10), with exception of Jupiter. Thus, we conclude that the acceleration of Eq. (5) proportional to $-v_r$ is to be considered ruled out.
by observations; it is true that, in principle, adjusting the ephemerides (without modifying their dynamical force models as done with EPM2006) may absorb the exotic signatures, but we do not believe that could occur because of the huge size of them. Instead, the effects induced by Eq. (4), proportional to $-|v_r|$, are still compatible with data. Of course, in drawing such conclusions we are tacitly assuming that the Pioneer-type anomalous accelerations of Eq. (4) and Eq. (5) exist since one century at least. Let us assume that they act since much longer time, say 500 Myr; Eq. (5) and Eq. (14) tell us that, in this case, 500 Myr ago the semimajor axes of the outer planets were equal to 19 AU (Jupiter), 44 AU (Saturn), 68 AU (Uranus), 32 AU (Neptune) and 2,983 AU (Pluto). We have used the simple formula

$$a_0 = a - \dot{a} \Delta t,$$

in which $a_0$ represents the semimajor axis in the past while $a$ denotes its current value. Concerning the eccentricities, they would have been larger than 1 according to Eq. (10) and $e_0 = e - \dot{e} \Delta t$. With regard to the future evolution of the orbits of the outer planets, the time required to circularize their orbits with respect to the present-day values of the eccentricities is of the order of $8 \times 10^5$ yr, provided that the Pioneer-type forces considered here will continuously act upon the planets for a so long time span. Of course, issues concerning a theoretical justification for Eq. (4) and Eq. (5) remain: suffices it to say that they are not, in general, Lorentz-invariant, as can be straightforwardly shown by using

$$r' = \Gamma (r - Vt) + (\Gamma - 1) \frac{(r \times V) \times V}{V^2}, \quad t' = \Gamma \left( t - \frac{r \cdot V}{c^2} \right),$$

$$v' = \frac{1}{1 - \frac{v}{c}} \left[ v - V + \left( \frac{\Gamma - 1}{\Gamma V^2} \right) (v \times V) \times V \right],$$

with $\Gamma = 1/\sqrt{1 - V^2/c^2}$. The conclusions by Standish\cite{22} are that Eq. (4) and Eq. (5) cannot exist for planets up to Jupiter and Saturn, while their existence at heliocentric distances $\gtrsim 20$ AU is virtually undetectable from the motion of Uranus, Neptune and Pluto.

2.2. Forces quadratic in velocity

The other two anomalous radial accelerations examined in Ref.\cite{22} quadratic in the radial velocity, are

$$A_{v^2}^{(4)} = -v_r^2 \mathcal{H},$$

$$A_{v^2}^{(5)} = |v_r| v_r \mathcal{H},$$

with

$$\mathcal{H} = \frac{A_{\text{Pio}}}{v_{\text{Pio}}^2} = 6.07 \times 10^{-18} \text{ m}^{-1}.$$
The acceleration of Eq. (22) is always directed towards the Sun, while the one of Eq. (23) is inward when the planet moves away from the Sun, while it is directed outwards when the planet approaches the Sun, as in the case of Eq. (5).

An acceleration like Eq. (22) was theoretically obtained by Jaekel and Reynaud in the framework of their linear [27,28] and non-linear [29] metric extensions of general relativity. Its orbital effects were worked out in Ref. 19: neither the semimajor axis nor the eccentricity undergo secular variations, while the longitude of perihelion precesses according to

$$\langle \dot{\varpi} \rangle = \frac{\mathcal{H}n a \sqrt{1 - e^2}}{e^2} \left( -2 + e^2 + 2 \sqrt{1 - e^2} \right) < 0.$$ (25)

In Figure 3 we show the results of the numerical integration of the equations of motion with Eq. (22) added to the Newtonian monopole: the results obtained analytically in Ref. 19 are confirmed.

In Ref. 19 it was shown that the inner planets’ perihelion precessions predicted by Eq. (26) are neatly ruled out by the corrections to the perihelion precessions estimated by Pitjeva [30] with the EPM2004 ephemerides. In Table 3 we quote the predictions for the outer planets; it turns out that they are compatible with the results of Table 1; apart from Jupiter. Note that it is true also by considering the formal uncertainties in the estimated corrections to the perihelion precessions, i.e. the values of $\delta \Delta \dot{\varpi}$ in Table 1 reduced by 10 times. Such a conclusion substantially agrees with that by Standish [22].

Eq. (23), contrary to Eq. (22), induces no secular perihelion precession and a secular variation of the semimajor axis and the eccentricity which decrease according to

$$\langle \dot{a} \rangle = \frac{4\mathcal{H}n a^2}{\pi} \left[ 2e + \ln \left( \frac{1 - e}{1 + e} \right) \right],$$ (26)
Table 3. Secular precessions $\dot{\omega}$ of the perihelia of the outer planets, in arcsec cy$^{-1}$, according to Eq. (25) due to the acceleration of Eq. (22) proportional to $-v_r^2$. Such values are to be compared with those in Table 1.

| Planet | $\dot{\omega}$ (arcsec cy$^{-1}$) |
|--------|----------------------------------|
| Jupiter | -0.030                           |
| Saturn  | -0.029                           |
| Uranus  | -0.015                           |
| Neptune | -0.0004                          |
| Pluto   | -0.308                           |

$$\langle \dot{e} \rangle = \frac{2\mathcal{H}a(1-e^2)}{\pi e} \left[ 2e + \ln \left( \frac{1-e}{1+e} \right) \right].$$

(27)

Note that $\langle \dot{a} \rangle = 0$ for circular orbits. Thus, also Eq. (23) does not conserve the total energy. Such analytical results are confirmed by a numerical integration of the equations of motion showed in Figure 4.

![Fig. 4. Numerically integrated trajectory for the radial acceleration of Eq. (23) proportional to $-|v_r|v_r$. Both the semimajor axis $a$ and the eccentricity $e$ secularly decrease, while the longitude of perihelion $\omega$ remains unchanged.](image)

In Table 4 we quote the predictions for the centennial semimajor shifts according to Eq. (26). A comparison with the Table 1 shows that the formal uncertainties $\delta a$ are always quite smaller than such anomalous shifts, apart from Neptune. However, it must taken into account that realistic errors may be up to one order of magnitude larger: if so, it would not be possible to rule out the results of Table 4, apart from Jupiter. In this case, our conclusions would agree with those by Standish. Of course, serious issues concerning theoretical justifications of Eq. (23) and the temporal extent of its existence remain open. Indeed, given the present-day values of the planetary semimajor axes and eccentricities and assuming that Eq. (23) existed unchanged in the deep past, about 100 Myr-1Gyr ago $e = 1$ for the planets from...
Table 4. Shifts of the semimajor axes $a$ of the outer planets, in m, according to the acceleration of Eq. (23) proportional to $-|v_r|v_r$. Such values are to be compared with those in Table 1.

| Planet | $\Delta a$ (m) |
|--------|----------------|
| Jupiter | -18,753 |
| Saturn  | -39,362 |
| Uranus  | -33,347 |
| Neptune | -251   |
| Pluto   | -7,283,499 |

Jupiter to Pluto. Since, instead, the semimajor axes would have remained almost unchanged, this means that the perihelion distances vanished.

3. Conclusions

An ingenious attempt recently proposed to explain the Pioneer anomaly as due to a modification of the usual Newton-Einstein laws of gravitation consists in postulating the existence of some velocity-dependent extra-forces linear or quadratic in the radial component $v_r$ of the velocity of a test body. We put on the test such empirical models in the outer regions of the solar system in which the Pioneer anomaly manifested itself in its presently known form with the latest observational determinations of the planetary motions obtained by E.V. Pitjeva with the EPM2006 ephemerides. It turns out that the models yielding anomalous perihelion precessions cannot yet be ruled out, at least phenomenologically, for heliocentric distances larger than 5 AU. On the contrary, the models predicting secular variations of the semimajor axis $a$ and the eccentricity $e$ are much more difficult to be trusted not only because they would violate the conservation of energy but also because the centennial shifts for $a$ predicted by them are so large that they should have been detected, given the present-day accuracy in determining such orbital element. However, it must be considered that sound theoretical justifications for such models must be given.

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