Modeling of the height-diameter relationship in eucalyptus in integrated crop-livestock systems

Abstract – The objective of this work was to compare the height-diameter relationship, described by nonlinear biological models, in *Eucalyptus benthamii* in monoculture forestry and in three different integrated crop-livestock systems (ICLS): crop-forestry, livestock-forestry, and crop-livestock-forestry. The trees were evaluated during seven years after planting. Five nonlinear biological models were fitted to evaluate the height-diameter relationship, and Gompertz’s model was selected to describe the data, although all models described satisfactorily the height-diameter relationship of the trees in the ICLS. The analysis of the data showed that there is no similarity between monoculture forestry and the ICLS as to the height-diameter relationship. In addition, the height-diameter relationship in *E. benthamii* changes between the different ICLS. Particularly, two systems with cattle provide the same values of maximum growth rate, asymptote, and inflection point of diameter at breast height. Furthermore, with the integration of cattle into the tree component, the produced trees show lower asymptotic heights, with larger diameters when the average tree heights of the ICLS are equal.

Index terms: *Eucalyptus benthamii*, agroforestry systems, agrossilvipastoral, biological models, Gompertz, nonlinear models.

Modelagem da relação altura-diâmetro de eucalipto em sistemas de integração lavoura-pecuária

Resumo – O objetivo deste trabalho foi comparar a relação altura-diâmetro, descrita por modelos biológicos não lineares, de *Eucalyptus benthamii* em monocultura florestal e em três diferentes sistemas de integração lavoura-pecuária (ILP): lavoura-floresta, pecuária-floresta e lavoura-pecuária-floresta. As árvores foram avaliadas durante sete anos após o plantio. Cinco modelos biológicos não lineares foram ajustados para avaliar a relação altura-diâmetro, e o modelo de Gompertz foi selecionado para descrever os dados, embora todos os modelos tenham descrito satisfatoriamente a relação altura-diâmetro das árvores nos ILP. A análise dos dados mostrou que não há nenhuma semelhança entre a monocultura florestal e os ILP quanto à relação altura-diâmetro. Além disso, a relação altura-diâmetro de *E. benthamii* muda entre os diferentes ILP. Particularmente, dois sistemas com bovinos fornecem os mesmos valores de taxa máxima de crescimento, assíntota e ponto de inflexão do diâmetro à altura do peito. Além disso, com a integração do gado no componente arbóreo, as árvores produzidas apresentam menores alturas assintóticas, com maiores diâmetros quando as alturas médias das árvores dos ILP são iguais.

Termos para indexação: *Eucalyptus benthamii*, sistemas agroflorestais, agrossilvipastoral, modelos biológicos, Gompertz, modelos não lineares.
Introduction

Integrated crop-livestock systems (ICLS) are production models that allow the integration of agricultural, livestock, and forestry components, being recognized as an excellent alternative for sustainable intensification (Sekaran et al., 2021). ICLS, based on a conservationist use of natural and energy resources and coupled with the diversification of farm animal/crop, are more environmentally sustainable and less dependent on inputs, besides ensuring economic stability over time due to the improvement of chemical, physical, and biological soil characteristics (Garrett et al., 2017).

The arboreal component promotes an increased carbon sequestration per arable area, an increased biodiversity, and a reduced heat stress for grazing animals, contributing to thermoregulation and body homeostasis (Aranha et al., 2019). The component can also mitigate climate change effects by offering protection against solar radiation in areas with reduced rainfall and cloudiness, decreasing net radiation and wind speed, and slowing the evapotranspiration of understory plants in silvopastoral systems (Bosi et al., 2020). However, trees, as the longest standing component of agroforestry systems, require planned implementation and constant monitoring to minimize negative interactions, while maximizing the positive ones with the environment and production systems (Porfirio-da-Silva, 2018). Moreover, estimating the yield of forest plantations allows improving forestry planning and management, contributing to a reduction in the yield gaps that occur in different production regions (Elli et al., 2019).

The diameter and height of trees are variables frequently used in forest inventories to estimate the wood volume produced. Although these measurements reduce costs and evaluation time in the field, they are subject to methodological errors due to uncalibrated equipment, difficulty in visualizing the tree canopy, and phenotypic variations conditioned by spacing, soil type, sociological position of the tree, among others (Téo & Silva, 2020).

The strong relationship between tree diameter and height, however, is a useful indicator for forest management and can be described by mathematical models that make it possible to estimate unmeasured heights in the forest when using representative sample data, given the difficulty of measuring an entire forest population (Nascimento et al., 2020).

Biological models specifically allow relating the behavior of variables based on biological assumptions (Pienaar & Turnbull, 1973), usually in a sigmoidal shape. These relationships allow establishing growth curves for trees of different ages, in the juvenile, adult, and senescence stages (Fekedulegn et al., 1999). Among the commonly used nonlinear biological models stand out the logistic model and the models of Gompertz, von Bertalanffy, Mitscherlich, and Chapman & Richards.

Despite not being widespread, the application of hypsometric models to estimate the volume of wood produced in integrated production systems has proved to be an interesting alternative when compared with calculations by volumetric relations. Cunha et al. (2022), analyzing nine hypsometric models to estimate tree height in an ICLS with Eucalyptus urophylla (Eucalyptus urophylla S.T.Blake x Eucalyptus grandis W.Hill ex Maiden), in the municipality of Ipameri, in the state of Goiás, Brazil, found that all tested models presented a coefficient of determination greater than 0.7 and that Prodan’s model resulted in the best values for the analyzed statistics, showing a $R^2$ of 0.89, a trend observed in most studies related to integrated systems.

In the literature, there are several studies on the application of height-diameter relationships to estimate the volume of trees in monoculture (Soares & Tomê, 2002; Leduc & Golz, 2009; Paulo et al., 2011; Paula et al., 2013; Retslaff et al., 2015; Téo et al., 2017; Téo & Silva, 2020; Cunha et al., 2022); however, works on trees in ICLS are scarce (Abrantes et al., 2019), despite being important for the expansion of knowledge of the assessed components particularly in a subtropical climate region. The hypothesis here is that the height-diameter relationship of trees in ICLS differs from that of those in monoculture forestry, as well as between those under the various types of ICLS.

The objective of this work was to compare the height-diameter relationship, described by nonlinear biological models, in Eucalyptus benthamii Maiden et Cambage in monoculture forestry and in three different ICLS: crop-forestry, livestock-forestry, and crop-livestock-forestry.

Materials and Methods

The experiment was carried out in “Área de Proteção Ambiental do Iraí”, the environmentally protected area of the Iraí River Basin, located in the municipality of
Pinhais, in the state of Paraná, Brazil (25°24'S, 49°07'E, at an altitude > 930 m above sea level). This experimental area is part of a long-term project that assesses ICLS and is described in Dominschek et al. (2018). More details about the climate, soil, and tree management there can be found in Kruchelski et al. (2021).

In the experiment, a land-use intensification gradient of agricultural systems under no-tillage was evaluated, being determined by the levels of temporal and spatial diversity of agricultural components, including monoculture crop, pasture, and forestry, as well as the integration of these land uses (integrated/mixed systems). Four treatments were studied: livestock-forestry, with *E. benthamii* integrated to winter and summer pastures of oat (*Avena strigosa* Schreb.) and 'Áries' Guinea grass [*Megathyrsus maximus* (Jacq.) B.K.Simon & S.W.L.Jacobs] with cattle grazing; crop-forestry, *E. benthamii* integrated with summer corn (*Zea mays* L.) crop and winter oat cover crop without grazing; crop-livestock-forestry, *E. benthamii* integrated with three years of cattle grazing (as in the livestock-forestry treatment) and one year of crop rotation (as in the crop-forestry treatment); and monoculture forestry, *E. benthamii* monoculture. Tree rotation times were kept constant for all treatments, and only tree spacing varied between the monoculture and the agroforestry systems. The experiment was carried out in a randomized complete block design, with the four treatments and three replicates, totaling 12 experimental units. The blocks were separated according to the variation of soil types and topography (Figure 1).

All forestry managements until 2019, i.e., 72 months after eucalyptus planting, are described in Kruchelski et al. (2021). During this period, two pruning and one thinning were performed. The continuity of this description, from 2019 until 2020 (84 months after planting), is presented here.

The second thinning took place 78 months after planting by removing all trees from alternate rows in the agroforestry systems, as well as some trees in the remaining rows until the final average spacing of 28.0x6.0 m was reached. During this thinning, in the monoculture forestry, trees were eliminated to obtain a similar percentage to that of the trees remaining in the agroforestry systems, while maintaining a well-distributed spacing between trees. A tree mortality of 15.9% was recorded during the first three years post-planting due to falls caused by wind, damage by animals of the area such as the European hare (*Lepus europaeus* Pallas, 1778) and sawing beetles of the Cerambycidae family, and other unidentified causes.

The trees were evaluated for: total height, using the ECII clinometer (Häglof – Sweden, Långsele, Sweden); and diameter at breast height (DBH), obtained by measuring tree circumference at 1.30 m above ground with a measuring tape. Data were collected through seven inventories using a minimum of 10% and a maximum of 20% total trees, from the first to the seventh year after planting, with a census on the fifth year.

To assess the height-diameter relationship, biological models (Tjørve & Tjørve, 2017) were fitted by the Levenberg-Marquardt method (Elzhov et al., 2016) for the entire database (general model) and each production system (specific model) using the R language (R Core Team, 2021). These models were chosen due to the opportune relationship between model parameters and forest attributes such as maximum height and height growth rate. From the literature (Fekedulegn et al., 1999; Hulshof et al., 2015; Abrantes et al., 2019), five nonlinear biological models were selected and tested for modelling of *E. benthamii* total height:

Logistic
\[
    h = \frac{\beta_0}{1 + e(\beta_1 - \beta_2 DBH)} + \varepsilon
\]

Gompertz
\[
    h = \beta_0 \cdot e^{(\beta_1 - \beta_2 DBH)} + \varepsilon
\]

von Bertalanffy
\[
    h = \beta_0 \cdot (1 - e^{(-\beta_1(\beta_2 DBH))}) + \varepsilon
\]

Mitscherlich
\[
    h = \beta_0 \cdot (1 - e^{(\beta_1 DBH)}) + \varepsilon
\]

Chapman & Richards
\[
    h = \beta_0 \cdot (1 - e^{(\beta_1 DBH)])^{\beta_2} + \varepsilon
\]

where \(h\) is total height (m), DBH is diameter at breast height (cm), \(\beta\) are the model parameters, and \(\varepsilon\) is the residual random error.

The selection of the best regression model was based on the following statistical criteria: Akaike’s information criterion (AIC); root mean square error (RMSE); coefficient of determination (\(R^2\)), calculated as the squared correlation between the observed and estimated height; and graphical analysis.

AIC (Akaike, 1974) was defined by:
\[
    AIC = 2k - 2\ln(L)
\]
where $\hat{L}$ is the maximum likelihood estimate and $k$ is the number of estimated model parameters.

Lower values for AIC indicate a better explanation of data variability by the model. In addition, the RMSE is a widely used criterion to assess the quality of the fit of dendrometric equations and can be used for both absolute dimensions (RMSE) and percentages (RMSE %) (Sanquetta et al., 2014), as follows:

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n} (h_i - \hat{h}_i)^2}{n}}$$

where $h$ and $\hat{h}$ are the observed and estimated heights, respectively (m); $n$ is the number of observations; and $\bar{h}$ is the average observed height (m).

As previously mentioned, $R^2$ was calculated as the square of the correlation between the observed and estimated height, as:

$$R^2 = \text{Cor}(h, \hat{h})^2$$

where

$$\text{RMSE\%} = \frac{\text{RMSE}}{\bar{h}} \times 100$$

Figure 1. Experimental area located in the environmentally protected area of the Iraí River Basin, in the municipality of Pinhais, in the state of Paraná, Brazil, highlighting the four evaluated treatments: monoculture forestry (F), crop-forestry (CF), livestock-forestry (LF), and crop-livestock-forestry (CLF). Source: Kruchelski et al. (2021).
where Cor is the correlation between variables, and \( h \) and \( \hat{h} \) are the observed and estimated heights (m).

The model selected as best performing was then subjected to a likelihood ratio test (Regazzi, 2003) to verify the equality of the parameters and the identity of the model fitted to each production system. The test consists of calculating the ratio between the maximum likelihood of the complete model (different parameters for each group tested) and the maximum likelihood of the reduced model (restriction of equality of one or more parameters between treatments). If all parameters are restricted, then the test is considered an identity test between the models of the evaluated treatments. Otherwise, the test assesses whether constrained parameters are equivalent.

For large samples, the test statistic follows a chi-square distribution – with \( v = k_\Omega - k_w \) degrees of freedom, where \( k_\Omega \) and \( k_w \) are the number of parameters of the complete and reduced model, respectively – and can be calculated as follows (Regazzi, 2003):

\[
\chi^2_{calc} = n \ln \left( \frac{SQR_\Omega}{SQR_w} \right)
\]

where SQR is the residual sum of squares, \( \Omega \) is the complete model, \( w \) is the reduced model (with parameter restriction), \( n \) is the number of observations, and \( \ln \) is the natural logarithm.

If the calculated value is greater than the tabulated value (\( \alpha=0.05 \)), then the null hypothesis is rejected and it is concluded that there is no equality between the restricted parameters.

**Results and Discussion**

The models were fitted for the entire data set (general models) (Table 1). Although all fitted models showed a similar performance, Gompertz’s model was selected due to its slightly better values – lower values of RMSE, RMSE%, and AIC, as well as higher values of \( R^2 \). Gompertz’s model fitted well to the data of the present study and has been widely preferred in the literature to describe animal and plant growth, among other biological variables, due to its interpretable parameters and efficiency both in describing the growth of most types of organisms and in comparing fitted parameter values (Tjørve & Tjørve, 2017).

After fitted for each system (Table 2 and Figure 2), Gompertz’s model showed the best performance for the monoculture forestry and crop-forestry treatments. Therefore, the fit of the model showed a different behavior for each production system, especially for

| Table 1. Estimated parameters and statistical adjustment of the nonlinear biological models fitted for all production systems – crop-forestry, livestock-forestry, crop-livestock-forestry, and monoculture forestry – of *Eucalyptus benthamii* during seven years after planting in subtropical Brazil. |
|-------------------------------|-----------------|-----------------|-------------|-------------|-------------|
| **Model**                     | **Estimated parameter** | **RMSE (m)** | **RMSE (%)** | **AIC**     | **R^2**     |
| Logistic                      | \( b_0 \)         | 22.5444        | 3.2058      | 19.4616     | 30,296.4    | 0.7938      |
|                               | \( b_1 \)         | 2.2311         |             |             |             |             |
|                               | \( b_2 \)         | 0.1906         |             |             |             |             |
| Gompertz                      | \( b_0 \)         | 23.9365        | 3.1936      | 19.3879     | 30,252      | 0.7953      |
|                               | \( b_1 \)         | 1.0674         |             |             |             |             |
|                               | \( b_2 \)         | 0.1202         |             |             |             |             |
| Bertalanffy                   | \( b_0 \)         | 24.8462        | 3.1973      | 19.4103     | 30,265.5    | 0.7948      |
|                               | \( b_1 \)         | 0.6749         |             |             |             |             |
|                               | \( b_2 \)         | 0.0969         |             |             |             |             |
| Chapman & Richards            | \( b_0 \)         | 26.0411        | 3.2127      | 19.5036     | 30,321.7    | 0.7932      |
|                               | \( b_1 \)         | 0.0753         |             |             |             |             |
|                               | \( b_2 \)         | 1.4293         |             |             |             |             |
| Mitscherlich                  | \( b_0 \)         | 30.68455       | 3.2419      | 19.6807     | 30,425.6    | 0.7896      |
|                               | \( b_1 \)         | 0.04371        |             |             |             |             |

\( \text{RMSE} \), root mean square error; AIC, Akaike’s information criterion; \( R^2 \), coefficient of determination; and \( b_i \), model parameters.
monoculture forestry, where trees grew taller and earlier than in the ICLS. The equality hypothesis of three parameters for all system combinations was rejected, indicating that each integrated production system requires its own equation to describe the height-diameter relationship.

Conversely, Abrantes et al. (2019) found that, among the eight nonlinear models tested, the one of Chapman & Richards showed the best fit for modeling of the height, diameter, and volume of a hybrid clone of *E. urophylla* x *E. grandis* planted in two silvopastoral systems at two spacing (14.0x2.0 and 22.0x2.0 m). When analyzing the trees individually, the authors did not observe any differences in the height-diameter relationship between the spacing, probably because it was the only difference between treatments; this was not the case in the present study, where trees with equal spacing were evaluated in systems with crop, livestock, and both, as well as with monoculture forestry. Paula et al. (2013) assessed the growth of a *Eucalyptus camaldulensis* Dehnh. clone by testing two nonlinear models in five spatial arrangements – 3.6x2.5 and 3.3x3.3 m for monoculture forestry and (2.0x2.0) + 10 m, (3.0x3.0) + 9.0 m, and 9.0x3.0 m for the silvopastoral systems – and concluded that the arrangements did not affect plant height growth until 50 months, differently from what was observed in the present study for older trees. However, the average diameter was affected by plant proximity in the planting line, being smaller in the (2.0×2.0) + 10 m and 3.6×2.5 m arrangements, and larger in the 9.0×3.0 m arrangement (with the largest area per tree). This result was similar to those obtained in the present work, as Gompertz’s model was also selected because it presented a better fit to describe total height and diameter.

According to the selected model, no identity was found between any production system (Table 3), with restriction of all three parameters. However, an equality of one or two parameters was observed between the crop-forestry, crop-livestock-forestry, and livestock-forestry systems.

The relationship between height and diameter is plastic, not fixed, and depends on climatic variables,

![Figure 2. Gompertz’s model-specific fitted curves to total data (All treatments) and to each production system – crop-forestry (CF), livestock-forestry (LF), crop-livestock-forestry (CLF), and monoculture forestry (F) – of *Eucalyptus benthamii* during seven years after planting in subtropical Brazil. DBH, diameter at breast height at 1.30 m above ground.](image)

Table 2. Estimation of the parameters of Gompertz’s model and statistical adjustment applied to the production systems – crop-forestry (CF), crop-livestock-forestry (CLF), livestock-forestry (LF), and monoculture forestry (F) – of *Eucalyptus benthamii* during seven years after planting in subtropical Brazil\(^{(1)}\).

| Production system | \(\beta_0\) | \(\beta_1\) | \(\beta_2\) | RMSE (m) | RMSE (%) | \(R^2\) |
|------------------|-------------|-------------|-------------|----------|----------|--------|
| F                | 30.2901     | 1.1167      | 0.1166      | 2.7581   | 15.2191  | 0.8764 |
| CF               | 28.0854     | 1.03756     | 0.08835     | 2.4257   | 13.7952  | 0.8779 |
| CLF              | 28.8115     | 0.93669     | 0.08104     | 2.0697   | 13.4793  | 0.9015 |
| LF               | 25.8602     | 0.97667     | 0.08426     | 1.9899   | 13.0980  | 0.9027 |

\(^{(1)}\)\(\beta\)_i, model parameters; RMSE, root mean square error; and \(R^2\), coefficient of determination.
management, phylogenetic history, and environmental limitations at a biogeographic scale (Hulshof et al., 2015). In the present study, the unequal forms of management influenced the differences found in the height-diameter relationship between treatments. According to the carried out analysis, monoculture forestry shares no similarity with the other production systems, with a greater asymptote (Figure 2 and Table 4) and trees that reach greater heights with a smaller DBH. This difference between the management of the monoculture and the agroforestry systems has already been observed in other studies (Oliveira et al., 2009; Paula et al., 2013; Ferreira et al., 2020). The obtained results, together with the highest maximum growth rate and lowest inflection point of DBH (Table 4), show the effect of competition due to the smaller spacing between trees, which causes a greater height growth at the expense of a smaller increment in diameter.

The relationships between the parameters of Gompertz’s model and dendrometric measurements are presented in Table 4. According to the height-diameter model used, parameter represents the asymptote (Tjørve & Tjørve, 2017) or, in this case, the maximum height that the trees can reach. The crop-forestry and the crop-livestock-forestry systems have an equivalence of this parameter and, therefore, trees under both would

### Table 3. Calculated values of the likelihood ratio test and parameter restriction of Gompertz’s model for the production systems – crop-forestry (CF), livestock-forestry (LF), crop-livestock-forestry (CLF), and monoculture forestry (F) – of *Eucalyptus benthamii* during seven years after planting in subtropical Brazil(1).

| System       | β₀, β₁, β₂ (H₀) | β₀, β₁ (H₁) | β₀, β₃ (H₂) | β₁ (H₃) | β₂ (H₄) | β₃ (H₅) | β₄ (H₆) |
|--------------|-----------------|--------------|--------------|----------|----------|----------|----------|
| All          | 3777.68         | 163.97       | 1485.57      | 103.55   | 57.67    | 38.00    | 88.95    |
| F, CF, CLF   | 2522.70         | 101.96       | 1035.18      | 84.64    | 26.29    | 30.18    | 69.69    |
| F, CF, LF    | 2974.82         | 113.25       | 1162.95      | 79.50    | 44.36    | 18.19    | 64.29    |
| F, CLF, LF   | 3453.18         | 162.50       | 1464.40      | 91.77    | 58.01    | 38.31    | 88.42    |
| CF, CLF, LF  | 428.53          | 41.72        | 174.68       | 11.44    | 14.20    | 10.89    | 3.68     |
| F, CF        | 1231.95         | 28.24        | 448.88       | 49.89    | 9.66     | 3.90     | 32.73    |
| F, CLF       | 2053.86         | 92.57        | 932.82       | 61.82    | 21.39    | 30.01    | 61.40    |
| F, LF        | 2525.76         | 107.54       | 1079.56      | 59.73    | 41.68    | 18.25    | 55.10    |
| CF, CLF      | 197.95          | 24.09        | 97.41        | 10.48    | 3.60     | 9.99     | 3.37     |
| CF, LF       | 380.99          | 29.91        | 146.19       | 3.95     | 13.18    | 3.64     | 1.14     |
| CLF, LF      | 34.45           | 3.44         | 5.17         | 2.42     | 2.56     | 2.38     | 0.85     |

(1) Marked values (gray cells) indicate non-rejection of the null hypothesis (α=0.05), configuring equivalence between the parameters for the tested production systems. βᵢ, model parameters; and Hᵢ, tested parameter restriction hypotheses.

### Table 4. Relationship between the parameters of Gompertz’s model and the forest attributes of the production systems – crop-forestry (CF), crop-livestock-forestry (CLF), livestock-forestry (LF), and monoculture forestry (F) – of *Eucalyptus benthamii* during seven years after planting in subtropical Brazil(1).

| Measure                  | Relationship (restriction) | System               |
|--------------------------|---------------------------|----------------------|
| Asymptote (m)            | β₀ (H₁)                   | F                    |
|                          |                           | CF                   |
|                          |                           | CLF                  |
|                          |                           | LF                   |
| Inflection point of DBH (cm) | β₁/β₂ (H₃)               | 9.5772C              |
|                          |                           | 11.7437A             |
|                          |                           | 11.5911AB            |
| Maximum growth (m cm⁻¹)  | β₀. (β₂/e) (H₄)           | 1.2993A              |
|                          |                           | 0.9128B              |
|                          |                           | 0.7993C              |
|                          |                           | 0.8016C              |

(1) Equal letters, in the same line, represent equality of measure, based on equality between the parameters of the maximum likelihood test. Hᵢ, tested parameter restriction hypotheses; and e = 2.7182818.
theoretically reach the same maximum average height. Similarly, the crop-livestock-forestry and livestock-forestry systems also presented a statistically equal inflection point of DBH and maximum growth rate. In addition to the asymptote, the crop-livestock-forestry and livestock-forestry systems share the same DBH (inflection DBH) for highest growth and the same rate of maximum growth in height (Figure 2 and Table 4). Although there is no model identity between these systems, both have equivalent model parameters, suggesting that the presence of livestock in each of them is preponderant for the development of the height-diameter relationship in the studied forest species.

The tree effects on the animal component in ICLS are well documented, including shade for animal welfare, windbreak effect, decrease in temperature, regulation of air humidity, and improvement of forage quality, which are all favorable changes for animal behavior, together with adequate tree arrangements to provide a greater live weight (Silva et al., 2008; Porfírio-da-Silva, 2018). The results of the present study are indicative that there is also an animal effect on the arboreal component, with trees showing a larger diameter when the average tree heights of the ICLS are equal, not only due to planting spacing, but also to cattle grazing. This finding may be related to the fact that, when seeking shade, cattle gather closer to the tree line, which leads to a greater deposition of manure and urine in the area and, consequently, to a better nutrient cycling (Carpinelli et al., 2021) and a greater nutritional intake than usually found in the livestock-forestry system with a subsequent improvement in soil fertility. However, further research is needed to determine: how animal presence influences the yield of trees with lower asymptotic heights and a larger DBH; and what level of interaction between animal and forestry components can benefit both of them. This research is important because it allows obtaining a greater profitability with increased wood and pasture yields (Paula et al., 2013), defining the best tree arrangements that benefit livestock, and taking advantage of the stimulus given by the cattle to favor the growth of trees with a larger diameter.

Conclusions

1. Although Gompertz’s model shows the best fit, all biological nonlinear models tested (logistic, Von Bertalanffy, Mitscherlich, and Chapman & Richards) satisfactorily describe the height-diameter relationship in *Eucalyptus benthamii* in integrated crop-livestock systems (ICLS).

2. Based on Gompertz’s model, which was selected to describe the height-diameter relationship, monoculture forestry shows no similarity with the tested ICLS.

3. The height-diameter relationship in *E. benthamii* changes in the different ICLS evaluated, particularly in the systems with livestock (livestock-forestry and crop-livestock-forestry), which generate the same values of maximum growth, asymptote, and inflection point of diameter at breast height, producing trees with lower asymptotic heights, as well as with larger diameters when the average tree heights of the ICLS are equal.

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