I. INTRODUCTION

Quantum chromodynamics (QCD) is nowadays widely accepted as the fundamental theory to describe hadrons and their interactions. Conventional hadrons are composed of either a valence quark $q$ and an antiquark $\bar{q}$ (mesons) or three valence quarks (baryons) on top of the sea of $q\bar{q}$ pairs and gluons. One of the long-standing challenges in hadron physics is to establish and classify genuine multiquark states other than conventional hadrons because multiquark states may contain more information about the low-energy QCD than that of conventional hadrons. In the past several years, a charged charmonium-like $Z_c^+$ family, $Z_c^+(4340)$, $Z_c^+(4050)$, $Z_c^+(4250)$, $Z_c^+(3890)$, $Z_c^+(3865)$, $Z_c^+(3900)$, $Z_c^+(4020)$, $Z_c^+(4025)$, $Z_c^+(4070)$, and $Z_c^+(4150)$, has been successively observed by experimental Collaborations [1–9].

A systematic study of charged states $[cu][\bar{c}\bar{d}]$ is employed to extract important information on future experimental search for the missing higher orbital excitations in the $Z_c^+$ family. This is the goal of the present work. In our approach, a phenomenological model, color flux-tube model with a multi-body confinement potential instead of a two-body one in traditional quark model, is employed to explore the properties of excited charged tetraquark states $c\bar{c}u\bar{d}$ systematically. The model has been successfully applied to the ground states of charged tetraquark states $[Qq][Q'q']$ ($Q, Q' = c, b$ and $q, q' = u, d, s$) in our previous work [13].

This work is organized as follows: the color flux-tube model and the model parameters are given in Sec. II. The numerical results and discussions of the charged tetraquark states are presented in Sec. III. A brief summary is given in the last section.

II. COLOR FLUX-TUBE MODEL AND PARAMETERS

The details of the color flux-tube model basing on traditional quark models and lattice QCD picture can be found in our previous work [13], the prominent characteristics of the model are just presented here. The model Hamiltonian for the state $[cu][\bar{c}\bar{d}]$ is given as follows,

$$H_i = \sum_{i=1}^{4} \left( m_i + \frac{D_i^2}{2m_i} \right) - T_c + \sum_{i<j}^{4} V_{ij} + V_{ij}^{C} + V_{ij}^{C,SL},$$

$$V_{ij} = V_{ij}^{B} + V_{ij}^{B,SL} + V_{ij}^{S} + V_{ij}^{S,SL} + V_{ij}^{G} + V_{ij}^{G,SL}. \quad (1)$$
$T_c$ is the center-of-mass kinetic energy of the state, $\mathbf{p}_i$ and $m_i$ are the momentum and mass of the $i$-th quark (antiquark), respectively. The codes of the quarks (antiquarks) $c$ and $u$ ($\bar{c}$ and $\bar{d}$) are assumed to be 1 and 2 (3 and 4), respectively, their positions are denoted as $\mathbf{r}_1$ and $\mathbf{r}_2$ ($\mathbf{r}_3$ and $\mathbf{r}_4$).

The quadratic confining potential, which is believed to be flavor independent, of the tetraquark state with a diquark-antidiquark structure has the following form,

$$V_C = K \left( \left( \mathbf{r}_1 - \mathbf{y}_{12} \right)^2 + \left( \mathbf{r}_2 - \mathbf{y}_{12} \right)^2 + \left( \mathbf{r}_3 - \mathbf{y}_{34} \right)^2 + \left( \mathbf{r}_4 - \mathbf{y}_{34} \right)^2 + \kappa_d (\mathbf{y}_{12} - \mathbf{y}_{34})^2 \right),$$ (2)

The positions $\mathbf{y}_{12}$ and $\mathbf{y}_{34}$ are the junctions of two Y-shaped color flux-tube structures. The parameter $K$ is the stiffness of a three-dimension flux-tube, $\kappa_d K$ is other compound color flux-tube stiffness. The relative stiffness parameter $\kappa_d$ of the compound flux-tube is [19]

$$\kappa_d = \frac{C_d}{C_3},$$ (3)

where $C_d$ is the eigenvalue of the Casimir operator associated with the $SU(3)$ color representation $d$ at either end of the color flux-tube, such as $C_3 = \frac{2}{3}$, $C_6 = \frac{3}{2}$, and $C_8 = 3$.

The minimum of the confinement potential $V_{C_{\text{min}}}^{ \prime}$ can be obtained by taking the variation of $V_C$ with respect to $\mathbf{y}_{12}$ and $\mathbf{y}_{34}$, and it can be expressed as

$$V_{C_{\text{min}}}^{ \prime} = K \left( \mathbf{R}_1^2 + \mathbf{R}_2^2 + \frac{\kappa_d}{1 + \kappa_d} \mathbf{R}_3^2 \right),$$ (4)

The canonical coordinates $\mathbf{R}_i$ have the following forms,

$$\mathbf{R}_1 = \frac{1}{\sqrt{2}} (\mathbf{r}_1 - \mathbf{r}_2), \quad \mathbf{R}_2 = \frac{1}{\sqrt{2}} (\mathbf{r}_3 - \mathbf{r}_4),$$

$$\mathbf{R}_3 = \frac{1}{\sqrt{4}} (\mathbf{r}_1 + \mathbf{r}_2 - \mathbf{r}_3 - \mathbf{r}_4),$$

$$\mathbf{R}_4 = \frac{1}{\sqrt{4}} (\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4).$$ (5)

The use of $V_{C_{\text{min}}}^{ \prime}$ can be understood here that the gluon field readjusts immediately to its minimal configuration. It is worth emphasizing that the confinement $V_{C_{\text{min}}}^{ \prime}$ is a multi-body interaction in a multiquark state rather than the sum of many pairwise confinement interactions,

$$V_C = \sum_{i<j} \lambda_i \cdot \lambda_j \mathbf{r}_{ij}^n,$$ (6)

in Isgur-Karl quark model and chiral quark model with $n = 1$ or 2.

The central parts of one-boson-exchange $V_{ij}^B$ and $\sigma$-meson exchange $V_{ij}^\sigma$ only occur between $u$ and $\bar{d}$, and that of one-gluon-exchange $V_{ij}^G$ is universal. $V_{ij}^B$, $V_{ij}^\sigma$ and $V_{ij}^G$ take their standard forms and are listed in the following,

$$V_{ij}^B = V_{ij}^1 \sum_{k=1}^3 \mathbf{F}^k_i \mathbf{F}^k_j + V_{ij}^2 \sum_{k=1}^7 \mathbf{F}_i^k \mathbf{F}_j^k + V_{ij}^3 \cos \theta_p - \sin \theta_p,$$ (7)

$$V_{ij}^\sigma = \frac{g^2_\sigma}{4\pi} \frac{m^3}{12 m_i m_j} \Lambda^2 \frac{\Lambda^3}{m^2} \mathbf{r}_{ij},$$ (8)

$$V_{ij}^G = \frac{\alpha_s}{4} \frac{\lambda_i^c \cdot \lambda_j^c}{r_{ij}} - \frac{2\pi \delta(r_{ij})}{3 m_i m_j},$$ (9)

$$V_{ij}^G = -\frac{g^2_\sigma}{4\pi} \frac{m^2}{\Lambda^2 - m^2} \left( 2 \chi(m_i r_{ij}) - \frac{\Lambda^4}{m^2} Y(\Lambda r_{ij}) \right).$$ (10)

Where $\chi$ stands for $\pi$, $K$ and $\eta$, $Y(x) = e^{-x}/x$. The symbols $\mathbf{F}$, $\lambda$ and $\sigma$ are the flavor $SU(3)$, color $SU(3)$ Gell-Mann and spin $SU(2)$ Pauli matrices, respectively. $\theta_p$ is the mixing angle between $\eta_1$ and $\eta_8$ to give the physical $\eta$ meson. $g^2_\sigma / 4\pi$ is the chiral coupling constant. $\alpha_s$ is the running strong coupling constant and takes the following form [20],

$$\alpha_s(\mu_{ij}) = \frac{\Lambda_0}{\ln ((\mu^2_{ij} + \Lambda^2_\sigma)/\Lambda^2_\sigma)},$$ (11)

where $\mu_{ij}$ is the reduced mass of two interacting particles $q_i$ (or $\bar{q}_i$) and $q_j$ (or $\bar{q}_j$). $\Lambda_0$, $\alpha_0$ and $\mu_0$ are model parameters. The function $\delta(r_{ij})$ in $V_{ij}^G$ should be regularized [22],

$$\delta(r_{ij}) = \frac{1}{4\pi r_{ij}} \frac{\rho_0(\mu_{ij})}{\rho_0(\mu_{ij})} e^{-r_{ij}/\rho_0(\mu_{ij})},$$ (12)

where $\rho_0(\mu_{ij}) = \rho_0/\mu_{ij}$, $\rho_0$ is a model parameter.

The diquark $[cu]$ and antidiquark $[cd]$ can be considered as compound objects $Q$ and $\bar{Q}$ with no internal orbital excitation, and the angular excitation $L$ are assumed to occur only between $Q$ and $\bar{Q}$ in the present work and the parity of the state $[cu][\bar{c}d]$ is therefore simply related to $L$ as $P = (-1)^L$. In this way, the state $[cu][\bar{c}d]$ has lower energy than that of the states with additional internal orbital excitation in $Q$ and $\bar{Q}$. In order to facilitate numerical calculations, the spin-orbit interactions are approximately assumed to just take place between compound objects $Q$ and $\bar{Q}$, which is consensus with the work [21]. The related interactions can be presented as follows

$$V_{12,34}^{G,LS} \approx \frac{\alpha_s}{4} \frac{\lambda_{12}^c \cdot \lambda_{34}^c}{8 M_{12} M_{34}} \frac{3}{X^3} L \cdot \mathbf{S},$$ (13)

$$V_{12,34}^{\sigma,LS} \approx -\frac{g^2_\sigma}{4\pi} \frac{m^3}{\Lambda^2 - m^2} \frac{2}{2 M_{12} M_{34}} \left( G(m_\sigma X) - \frac{\Lambda^4}{m^2} G(\Lambda_\sigma X) \right),$$ (14)

$$V_{12,34}^{C,LS} \approx \frac{K}{8 M_{12} M_{34}} \frac{\kappa_d}{1 + \kappa_d} L \cdot \mathbf{S}. \quad (15)$$
where \( M_{12} = M_{34} = m_c + m_{u,d} \), \( G(x) = Y(x)(\frac{1}{2} + \frac{1}{\sqrt{x}}) \), and \( S \) stands for the total spin angular momentum of the tetraquark state \([cu][\bar{c}\bar{d}]\).

The model parameters are determined as follows. The mass parameters \( m_{\sigma}, m_K \) and \( m_{\eta} \) in the interaction \( V_{ij}^\beta \) are taken their experimental values, namely, \( m_{\sigma} = 0.7 \text{ fm}^{-1} \), \( m_K = 2.51 \text{ fm}^{-1} \) and \( m_{\eta} = 2.77 \text{ fm}^{-1} \). The cutoff parameters take the values, \( \Lambda_\sigma = \Lambda_\sigma = 4.20 \text{ fm}^{-1} \) and \( \Lambda_\eta = \Lambda_K = 5.20 \text{ fm}^{-1} \), the mixing angle \( \theta_P = -15^\circ \) \([20]\).

The mass parameter \( m_\sigma \) in the interaction \( V_{ij}^\gamma \) is determined through the PCAC relation \( m_\sigma^2 \approx m_{\sigma,\eta}^2 + 4m_{u,d}^2 \) \([23]\), \( m_{u,d} = 280 \text{ MeV} \) and \( m_{\sigma} = 2.92 \text{ fm}^{-1} \). The chiral coupling constant \( g_{ch} \) is determined from the \( \pi N N \) coupling constant through

\[
\frac{g_{ch}^2}{4\pi} = \left( \frac{3}{5} \right)^2 \frac{g_{\pi NN}^2 m_{u,d}^2}{4\pi} = 0.43. \tag{16}
\]

The other adjustable parameters and their errors are determined by fitting the masses of the ground states of mesons using Minuit program, which are shown in Table I. The mass spectrum of the ground states of mesons, which is listed in Table II, can be obtained by solving the two-body Schrödinger equation

\[
(H_2 - E_2)\Phi_{IJ}^{\text{Meson}} = 0. \tag{17}
\]

The mass error of mesons \( \Delta E_2 \) introduced by the parameter uncertainty \( \Delta x_i \) can be calculated by the formula of error propagation,

\[
\Delta H_2 = \sum_{i=1}^{8} \frac{\partial H_2}{\partial x_i} \Delta x_i, \tag{18}
\]

\[
\Delta E_2 \approx \left( \frac{\Phi_{IJ}^{\text{Meson}} [H_2] \Phi_{IJ}^{\text{Meson}}}{\Delta H_2} \right) . \tag{19}
\]

where \( x_i \) and \( \Delta x_i \) represent the \( i \)-th adjustable parameter and its error, respectively, which are listed in Table I.

### III. NUMERICAL RESULTS AND DISCUSSIONS

Within the framework of the diquark-antidiquark configuration, the wave function of the state \([cu][\bar{c}\bar{d}]\) can be written as a sum of the following direct products of color \( \chi_c \), isospin \( \eta_i \), spin \( \eta_h \) and spatial \( \phi \) terms,

\[
\Phi_{IJ,M,LM,J}^{[cu][\bar{c}\bar{d}]} = \sum_{\alpha} \xi_\alpha \left[ [\phi_{G,M,a}^G(r)\chi_{s,a}^{[cu]}[\phi_{B,M,b}^B(R) \times \chi_{s,b}^{[\bar{c}\bar{d}]}]_{J_{ab}}\right]_{J_{LM}(X)} \right]_{IJ,M,J}^{[cu][\bar{c}\bar{d}]} \tag{20}
\]

In which \( \mathbf{r}, \mathbf{R} \) and \( \mathbf{X} \) are relative spatial coordinates,

\[
r = r_1 - r_2, \quad \mathbf{R} = r_3 - r_4
\]

\[
\mathbf{X} = \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2}{m_1 + m_2} - \frac{m_3 \mathbf{r}_3 + m_4 \mathbf{r}_4}{m_3 + m_4}. \tag{21}
\]

### TABLE I: Adjustable model parameters. (units: \( m_\pi, m_c, m_{\bar{c}}, \mu, \Lambda_0, \text{MeV}; K, \text{MeV-fm}^{-1}; r_0, \text{MeV-fm}; \alpha_0, \text{dimensionless})

| Parameters | \( x_i \) | \( \Delta x_i \) | | Parameters | \( x_i \) | \( \Delta x_i \) |
|------------|-------------|-----------------| | |-------------|-----------------|
| \( m_\pi \) | 511.78 | 0.228 | \( \mu_0 \) | 4.554 | 0.018 |
| \( m_c \) | 1601.7 | 0.441 | \( \Lambda_0 \) | 9.173 | 0.175 |
| \( m_{\bar{c}} \) | 4936.2 | 0.451 | \( \mu_0 \) | 0.0004 | 0.540 |
| \( K \) | 217.50 | 0.230 | \( r_0 \) | 35.06 | 0.156 |

### TABLE II: Ground state meson spectra, unit in MeV.

| States | \( E_2 \) | \( \Delta E_2 \) | PDG | States | \( E_2 \) | \( \Delta E_2 \) | PDG |
|--------|---------|-----------|-----|--------|---------|-----------|-----|
| \( \pi \) | 142 | 26 | 139 \( \eta \) | 2912 | 5 | 2980 |
| \( K \) | 492 | 20 | 496 \( J/\psi \) | 3102 | 4 | 3097 |
| \( \rho \) | 826 | 4 | 775 \( B^0 \) | 5259 | 5 | 5280 |
| \( \omega \) | 780 | 4 | 783 \( B^+ \) | 5301 | 4 | 5325 |
| \( K^+ \) | 974 | 4 | 892 \( B^0 \) | 5377 | 5 | 5366 |
| \( \phi \) | 1112 | 4 | 1020 \( B^+ \) | 5430 | 4 | 5416 |
| \( D^\pm \) | 1867 | 8 | 1880 \( B_c \) | 6261 | 7 | 6277 |
| \( D^* \) | 2002 | 4 | 2007 \( B_c^+ \) | 6357 | 4 | ... |
| \( D_s^* \) | 1972 | 9 | 1968 \( \eta \) | 9441 | 8 | 9391 |
| \( D_s \) | 2140 | 4 | 2112 \( \Upsilon(1S) \) | 9546 | 5 | 9460 |

The other details of the construction of the wave function can be found in our previous work \([17]\). Subsequently, the converged numerical results can be obtained by solving the four-body Schrödinger equation

\[
(H_4 - E_4)\Phi_{IJ,M1,MJ}^{[cu][\bar{c}\bar{d}]} = 0. \tag{22}
\]

with the Rayleigh-Ritz variational principle.

The energies \( E_4 \pm \Delta E_4 \) of the charged states \([cu][\bar{c}\bar{d}]\) with \( n^{2S+1}L_J \) and \( J^P \) under the assumptions of \( S = 0, \ldots, 2 \) and \( L = 0, \ldots, 3 \) are systematically calculated and presented in Table III. The mass error of the states \( \Delta E_4 \) can be calculated just as \( \Delta E_2 \), they are around several MeV except for that of the state \( 1^1S_0 \). The spin-orbit interactions are extremely weak, less than 2 MeV, therefore the energies of excited states with the same \( L \) and \( S \) but different \( J \) are almost degenerate, see the energies of the excited states with \( 1^5D_0, 1^5D_1 1^5D_2, 1^5D_3 \) and \( 1^3D_4 \) in Table III, which is consistent with the conclusion of the work \([24]\). Other spin-related interactions are stronger and bring about a larger energy difference than spin-orbital interactions, especially for the ground states with \( 1^3S_0, 1^3S_1 \) and \( 1^3S_2 \). The energy difference among excited states mainly comes from the kinetic energy and confinement potential, which are proportional to the relative orbital excitation \( L \). However, the relative kinetic energy between two clusters \([cu]\) and \([\bar{c}\bar{d}]\) is inversely proportional to \( (X^2) \) while confinement potential is proportional to \( (X^2) \) so that they compete each other to reach an optimum balance.

The rms \( \langle r^2 \rangle^{\frac{1}{2}}, \langle \mathbf{R}^2 \rangle^{\frac{1}{2}} \) and \( \langle (X^2) \rangle^{\frac{1}{2}} \) stand for the size of the diquark \([cu]\), the antidiquark \([\bar{c}\bar{d}]\) and the distance between the two clusters, respectively, which are also calculated and listed in Table III. One can find that the
diquark \([cu]\) and antidiquark \([\bar{c}\bar{d}]\) share the same size in every \(Z^+_c\) state. The sizes of the diquark \([cu]\) and antidiquark \([\bar{c}\bar{d}]\) are determined by the total spin \(S\), the relative orbital excitation \(L\) of the states has a minor effect on them. However, the sizes do not vary largely with the total spin \(S\), especially for higher orbital excited states. So the diquark \([cu]\) and antidiquark \([\bar{c}\bar{d}]\) are rather rigid against the rotation. For examples, the sizes of the two groups \(1^1S_0-1^3S_1-1^5S_2\) and \(1^1F_3-1^3F_3-1^5F_3\) changes gradually with the total spin \(S\), 0.85-0.90-1.03 fm and 0.96-0.99-1.02 fm, respectively. And the sizes of the two groups \(1^3S_0-1^1P_1-1^3D_2-1^5D_2\) and \(1^3S_1-1^1P_1-1^3D_1-1^5D_1\) vary slightly with relative orbital excitation \(L\), 0.85-0.94-0.95-0.96 fm and 0.90-0.96-0.98-0.99 fm, respectively. On the contrary, the distance between the diquark \([cu]\) and antidiquark \([\bar{c}\bar{d}]\) \((\langle X^2 \rangle^\frac{1}{2}\rangle\) changes remarkably with the relative orbital excitation \(L\) between the two clusters and is irrelevant to the total spin of the system, see the sizes of \(1^3S_1-1^3P_1-1^3D_1-1^3F_2\) and \(1^1S_0-1^3S_1-1^5S_2\) in Table III. The sizes of the diquark \([cu]\), antidiquark \([\bar{c}\bar{d}]\) and the distance between the two clusters are helpful to understand the changing tendency of energies of charged states \(Z^+_c\) with quantum numbers \(S\) and \(L\).

In order to make clear the spatial configuration of charged states \([cu][\bar{c}\bar{d}]\), the distances in four states between any two particles are given in Table IV. The ground state \((1^1S_0\text{ and }1^+\rangle\) of charged tetraquark \([cu][\bar{c}\bar{d}]\) possesses a three-dimensional spatial configuration due to the competition of the confinement and the kinetic energy of the systems \([17]\), which is similar to a rugby ball. The diquark \([cu]\) and antidiquark \([\bar{c}\bar{d}]\) in the ground state have a large overlap because of the small \(\langle X^2 \rangle^\frac{1}{2}\rangle\), so the picture of the diquark or antiquark is not extremely distinct. However, all distances except for the sizes of the diquark and antidiquark \((\langle r_{ij}^2 \rangle^\frac{1}{2}\rangle\) and \((\langle r_{ij}^2 \rangle^\frac{1}{2}\rangle\) evidently augment with the increasing of the orbital angular momentum \(L\) in the excited states, see Table IV, which means that the picture of the diquark or antiquark is more and more clear with the raising of the orbital angular momentum \(L\). The spatial configuration of the excited states is still similar to a rugby ball, the higher orbital angular momentum \(L\), the more prolate of the shape of the excited states. The multibody color flux-tube basing on lattice QCD picture, a collective degree of freedom, plays an important role in the formation of these charged tetraquark states, it should therefore be the dynamical mechanism of the tetraquark systems.

Next, let us turn to argue the properties of the charged states \(Z^+_c\) observed in experiments and their possible candidates in the color flux-tube model, which are presented in Table IV. It can be seen from the Table IV that the spin and parity of the \(Z^+_c\) (3900) are still unclear up to now. The \(Z^+_c\) (3900) may correspond to the same state as the \(Z^+_c\) (3885) with \(1^+\). The charged state \([cu][\bar{c}\bar{d}]\) with \(1^+\) and \(1^3S_1\) has a mass of \(3858 \pm 10\) MeV in the

### Table III: The energy \(E_4 + \Delta E_4\) and rms \(\langle r^2 \rangle^\frac{1}{2}\), \(\langle R^2 \rangle^\frac{1}{2}\) and \(\langle X^2 \rangle^\frac{1}{2}\) of charged tetraquark states \([cu][\bar{c}\bar{d}]\) with \(J^P\) and \(n^{2S+1}L_J\), unit of energy: MeV and unit of rms: fm.

| \(J^P\) |  \(n^{2S+1}L_J\) |  \(1^1S_0\) |  \(1^3P_0\) |  \(1^5D_0\) |  \(1^3S_1\) |  \(1^3P_1\) |  \(1^3P_1\) |  \(1^5P_1\) |  \(1^3D_1\) |  \(1^5D_1\) |
|---|---|---|---|---|---|---|---|---|---|---|
| \(0^+\) | \(1S_0\) | \(4387 \pm 7\) | \(4001 \pm 7\) | \(4096 \pm 8\) | \(4152 \pm 7\) | \(4212 \pm 8\) | \(4235 \pm 7\) | \(4273 \pm 7\) | \(4354 \pm 7\) | \(4387 \pm 7\) | \(4150 \pm 7\) |
| \(0^-\) | \(1^3P_0\) | \(1^1S_0\) | \(1^3P_0\) | \(1^5D_0\) | \(1^3S_1\) | \(1^3P_1\) | \(1^3P_1\) | \(1^5P_1\) | \(1^3D_1\) | \(1^5D_1\) |
| \(0^+\) | \(1^5D_0\) | \(1^3S_1\) | \(1^3P_1\) | \(1^3P_1\) | \(1^5P_1\) | \(1^3D_1\) | \(1^5D_1\) |
| \(1^+\) | \(1^3S_1\) | \(1^3P_1\) | \(1^3P_1\) | \(1^5P_1\) | \(1^3D_1\) | \(1^5D_1\) |
| \(1^-\) | \(1^3P_1\) | \(1^3P_1\) | \(1^5P_1\) | \(1^3D_1\) | \(1^5D_1\) |
| \(1^-\) | \(1^3P_1\) | \(1^3P_1\) | \(1^5P_1\) | \(1^3D_1\) | \(1^5D_1\) |
| \(1^+\) | \(1^3P_1\) | \(1^3P_1\) | \(1^5P_1\) | \(1^3D_1\) | \(1^5D_1\) |

### Table IV: The average distances \(\langle r_{ij}^2 \rangle^\frac{1}{2}\rangle\) of the states \([cu][\bar{c}\bar{d}]\) with \(1^1S_0, 1^1P_1, 1^3D_2, 1^1F_3\), \(r_{ij} = r_i - r_j\), unit in fm.

| \(n^{2S+1}L_J\) | \(\langle r_{ij}^2 \rangle^\frac{1}{2}\rangle\) | \(\langle r_{ij}^2 \rangle^\frac{1}{2}\rangle\) | \(\langle r_{ij}^2 \rangle^\frac{1}{2}\rangle\) | \(\langle r_{ij}^2 \rangle^\frac{1}{2}\rangle\) |
|---|---|---|---|---|
| \(1^1S_0\) | 0.85 | 0.85 | 0.85 | 0.85 |
| \(1^1P_1\) | 0.94 | 0.94 | 0.94 | 0.94 |
| \(1^3D_2\) | 0.95 | 0.95 | 0.95 | 0.95 |
| \(1^1F_3\) | 0.96 | 0.96 | 0.96 | 0.96 |
The pair $D$ and $P$ can be excluded that the main component of $\eta_c(4025)$ as a tetraquark state [5], $\eta_c(4050)$ [1] 4051 $+ 2^+$, $P_c(4475)$ favors the spin-parity $J^P = 1^+$ with $P_c(4475)$ as a tetraquark state $[cu][\bar{c}\bar{d}]$ with $1^+$ and $1^3S_1$. The charged tetraquark states $[cu][\bar{c}\bar{d}]$ are systematically investigated from the perspective of the color flux-tube model with a four-body confinement potential. The investigation demonstrates that the charged charmoniumlike states $Z_c^+(3900)$ or $Z_c^+(3885)$, $Z_c^+(3930)$, $Z_c^+(4020)$ or $Z_c^+(4025)$, $Z_c^+(4050)$, $Z_c^+(4250)$, and $Z_c^+(4200)$ can be uniformly identified as tetraquark states $[cu][\bar{c}\bar{d}]$ with the quantum numbers $1^3S_1$ and $1^+$, $2^3S_1$ and $1^+$, $1^1D_2$ and $2^+$, $1^3P_1$ and $1^-$, $1^3D_1$ and $1^-$, in the color flux-tube model. The model predictions would shed light on looking for possible charmoniumlike charged states in the future at the BESIII, LHCb and Belle-II. They favor three-dimensional spatial structures which is similar to a rugby ball, the higher orbital angular momentum $L$, the more prolate of the shape of the states. Those charged charmoniumlike states may be so-called “color confined, multiquark resonance”. However, the two heavier charged states $Z_c^+(4430)$ and $Z_c^+(4475)$ can not be described as tetraquark states $[cu][\bar{c}\bar{d}]$ in the color flux-tube model.

The multibody color flux-tube is a collective degree of heavy charged states, which is left for future. The alternative configuration for the two states may be meson-meson molecular states, which is suggested by several theoretical methods.\[12\]

From the above analysis and Table III, we can see that the most of the low energy theoretical states can be matched with the experimental ones. One of the exception is the state with $0^+$ and $1^1S_0$, which has a mass of $3780 \pm 10$ MeV. The experimental search of the $\eta_c$-like charged state will give a crucial test of the present approach. Our calculation also suggests that there are two negative parity states around 4100 MeV. More information on the the states around this energy is expected. The model assignments of the $Z_c^+$ states are completed just in term of the proximity to the experimental masses, the more stringent check of the assignment is to study the decay properties of the states. These charged states should eventually decay into several color singlet mesons due to their high energy. In the course of the decay, the color flux-tube structure should break down first which leads to the collapses of the three-dimension spatial configuration, and then through the recombination of the color flux tubes the particles of decay products formed. The decay widths of the charged states $[cu][\bar{c}\bar{d}]$ are determined by the transition probability of the breakdown and recombination of color flux tubes. The calculations are in progress. This decay mechanism is similar to compound nucleus decay and therefore should induce a resonance, which we called it as “color confined, multiquark resonance” state before 28.

### IV. SUMMARY

The charged tetraquark states $[cu][\bar{c}\bar{d}]$ are systematically investigated from the perspective of the color flux-tube model with a four-body confinement potential. The investigation demonstrates that the charged charmoniumlike states $Z_c^+(3900)$ or $Z_c^+(3885)$, $Z_c^+(3930)$, $Z_c^+(4020)$ or $Z_c^+(4025)$, $Z_c^+(4050)$, $Z_c^+(4250)$, and $Z_c^+(4200)$ can be uniformly identified as tetraquark states $[cu][\bar{c}\bar{d}]$ with the quantum numbers $1^3S_1$ and $1^+$, $2^3S_1$ and $1^+$, $1^1D_2$ and $2^+$, $1^3P_1$ and $1^-$, $1^3D_1$ and $1^-$, in the color flux-tube model. The model predictions would shed light on looking for possible charmoniumlike charged states in the future at the BESIII, LHCb and Belle-II. They favor three-dimensional spatial structures which is similar to a rugby ball, the higher orbital angular momentum $L$, the more prolate of the shape of the states. Those charged charmoniumlike states may be so-called “color confined, multiquark resonance”. However, the two heavier charged states $Z_c^+(4430)$ and $Z_c^+(4475)$ can not be described as tetraquark states $[cu][\bar{c}\bar{d}]$ in the color flux-tube model.

The multibody color flux-tube is a collective degree of...
freedom, which acts as a dynamical mechanism and plays an important role in the formation and decay of those compact states. Just as colorful organic world because of chemical bonds, multiquark hadron world should be various due to the diversity of color flux-tube structure. The well-defined the charged state $Z_c^+(3900)$ and dibaryon $d^*_s$ resonance have been opening the gate of abundant multiquark hadronic world.

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