Coupled vibration study of the blade of the flexible wind wheel with the low-speed shafting

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Abstract. Movement and deformation of flexible wind wheel has a profound effect on dynamics of the low-speed shafting in Megawatt wind turbine. The paper is based on the power production 1.2 MW wind turbine, vibration characteristics of elastic wind wheel with the low-speed shafting were studied. In order to obtain the finite element model, the author created a physical model of this coupled system and used the minimum energy principle to simplify the model. While its single blade simplified as cantilever. Using modal superposition method for solving the coupled system model. Structural mechanics equations were used to solve the simple blade finite element model. Analyzing the natural frequency of the coupled system and the stress diagram, the results indicate that in the coupling system, low frequency vibration occurs in the low-speed shaft bearing, while the high-frequency vibration happens on wind turbine blades. In the low-frequency vibration process, blades vibration and low-speed shaft vibration there is a strong correlation. Contrast inherent frequency of the wind wheel with natural frequency of a single blade, the results show that the frequency of the wind wheel slightly less than it in the single blade.

1. Introduction
Wind wheel which is the key components of the wind turbine is the important part of the energy conversion[1]. Today most of the wind turbine failure problems have occurred in the vulgar wind wheel or axle systems, which is led to severe vibration in wind turbine. And output power, stability, and power grid characteristics, and many other physical characteristics were affected. In order to solve the above problems, so it is important to study the dynamics characteristic of the wind wheel with coupling components.

However, the dynamics characteristic of the wind wheel with coupling components research is not perfect. Today scholars to study a single component in multimedia dynamics method, finite element and modal analysis methods. JongWon Lee[2] established a new type of flexible blades physical model of the wind turbine model that is based on thin plate theory. Yi.Guo[3] made a study of the dynamic characteristics of the low speed shaft that is supported bearing. In the time of the building coupling system, the dynamic characteristics of the gear meshing and the bearing gap variation have been taken into account the entire model. Li Deyuan[4] made a study of the frequency of vibration of the rotating wind wheel and the factors impact of wind wheel vibration. At the same time he also studied dynamic stiffening effect of rotating wind wheel in multi-body system dynamics theory.
In order to get accurately free vibration characteristics of the blade, as well as the relationship between vibration characteristics of the blade and low speed deformation. The author wants to make a study on the natural vibration characteristics of the wind wheel with low speed shaft coupling system. Using modal superposition method for solving the coupled system model, and analyzing the natural frequency of the coupled system and the stress diagram. Accurately reveals between vibration characteristics of the blade and low speed deformation relationships.

2. The blade dynamics modelling

2.1. The blade finite element model

The beam structure refers to the rod or beam, which have a great long-chord ratio. Since the blade has the characteristics of large aspect ratio, so we consider the blade as a cantilever. Along the spanwise direction the blades were discretized by using two-noded beam elements, and the node of each elements have four dofs. The blade is discretized into several beam elements as shown in Fig.1. The fig 2 shows forces diagram in each element.

\[
\begin{align*}
\{\delta\} &= [v_1, \alpha_1, e_1, \beta_1, v_j, \alpha_j, e_j, \beta_j]^T \\
\{F\} &= [F_{ix}, M_{ix}, F_{ix}, M_{ix}, F_{jx}, M_{jx}]^T
\end{align*}
\]

Where \(L\) and \(\rho\) are the length and density of the blade element. \(N\) is the Hermite element shape function for primary unit. \(I\) and \(A\) are the moment of inertia and cross-sectional area. \(M\) and \(K\) are the mass matrix and stiffness matrix due to the blade properties, which is computed by [6]:

\[
M = \int \rho N^T N d\nu
\]

\[
K = \int_{0}^{L} EI \left(\frac{d^2 N}{dx^2}\right)^2 \left(\frac{d^2 N}{dx^2}\right) dx
\]

The mass matrix can be obtained by the formula (3):

\[
M = \begin{bmatrix}
m_{11} & m_{12} \\
m_{21} & m_{22}
\end{bmatrix}
\]
the stiffness matrix can be obtained by the formula (4):

\[
K = \begin{bmatrix}
k_{11} & k_{12} \\
k_{21} & k_{22}
\end{bmatrix}
\]  

(6)

\[
k_{11} = \begin{bmatrix}
\frac{12EI_z}{l^3} & \frac{-6EI_z}{l^2} & 0 & 0 \\
\frac{-6EI_z}{l^2} & \frac{4EI_z}{l} & 0 & 0 \\
0 & 0 & \frac{12EI_z}{l^3} & \frac{-6EI_z}{l^2} \\
0 & 0 & \frac{-6EI_z}{l^2} & \frac{-4EI_z}{l}
\end{bmatrix}
\]

\[
k_{12} = \begin{bmatrix}
\frac{12EI_z}{l^3} & \frac{6EI_z}{l^2} & 0 & 0 \\
\frac{6EI_z}{l^2} & \frac{2EI_z}{l} & 0 & 0 \\
0 & 0 & \frac{12EI_z}{l^3} & \frac{6EI_z}{l^2} \\
0 & 0 & \frac{6EI_z}{l^2} & \frac{2EI_z}{l}
\end{bmatrix}
\]

\[
k_{21} = \begin{bmatrix}
\frac{12EI_z}{l^3} & \frac{6EI_z}{l^2} & 0 & 0 \\
\frac{6EI_z}{l^2} & \frac{2EI_z}{l} & 0 & 0 \\
0 & 0 & \frac{12EI_z}{l^3} & \frac{6EI_z}{l^2} \\
0 & 0 & \frac{6EI_z}{l^2} & \frac{2EI_z}{l}
\end{bmatrix}
\]

\[
k_{22} = \begin{bmatrix}
\frac{12EI_z}{l^3} & \frac{-6EI_z}{l^2} & 0 & 0 \\
\frac{-6EI_z}{l^2} & \frac{-4EI_z}{l} & 0 & 0 \\
0 & 0 & \frac{12EI_z}{l^3} & \frac{-6EI_z}{l^2} \\
0 & 0 & \frac{-6EI_z}{l^2} & \frac{-4EI_z}{l}
\end{bmatrix}
\]

2.2. The blade structural dynamic equation
After getting the overall stiffness and mass matrix, the blade vibration equation for changing the load can be obtained in instantaneous kinetic energy principle [7,8]:

\[
m\ddot{x}(t) + c\dot{x}(t) + kx(t) = p(t)
\]  

(7)

Where \( m \) and \( c \) are the mass matrix and damping matrix. \( k \) is the stiffness matrix. Where \( x(t) \) and \( p(t) \) is the distance of element nodes and changing load. When the changing load \( p(t) = 0 \), blades in the free vibration state, damping matrix is not consider. Eigenvalues and eigenvectors is computed by :

\[
(k - \omega^2 m)\phi = 0
\]  

(8)

Where \( \phi \) and \( \omega \) are the structure matrix and natural frequency.
3. The coupling system dynamics modelling

3.1. Finite element model of coupled systems

The figure 3 shows the physical model of the coupled system, gearbox and generator equivalent quality point on the spindle by the instantaneous kinetic energy principle. Figure 4 shows equivalent coupling system structure, which was modelled by using the finite element method. The blades using shell elements for finite element modelling, and low speed shaft finite element modelling can be defined by beam188 element Hub and bearing are defined by mass21 and combin214 finite elements. We consider that blades and hub are rigidly connected, bearing and low speed shaft are connected by spring-damper unit. At the bottom of the bearing applied to the fully constrained.

3.2. Modal superposition method

Using modal superposition method for solving the coupled system model. In the multi-body system model of the wind wheel and low-speed coupled, arbitrary point total displacement is computed by:

\[
q = \phi_1 Y_1 + \phi_2 Y_2 + \ldots + \phi_n Y_n = \sum_{i=1}^{n} \phi_i Y_i
\]

(9)

with matrix expressed as:

\[
q = \phi Y
\]

(10)

After substituting Eq.(10) into Eq.(11):

\[
[m] \{\ddot{q}(t)\} + [c] \{\dot{q}(t)\} + [k] \{q(t)\} = \{Q(t)\}
\]

(11)

Where \([m]\) and \([c]\) are the mass matrix and damping matrix. \([k]\) is the stiffness matrix. \(q(t)\) and \(Q(t)\) are the generalized displacement column vector and the generalized force column vector. Equation both sides multiply by the n-th of the formation of the transposed matrix \(\phi^T_n\):

\[
\phi^T_n m \phi Y(t) + \phi^T_n c \phi \dot{Y}(t) + \phi^T_n k \phi Y(t) = \phi^T_n Q(t)
\]

(12)

Obtained by the model orthogonal condition:

\[
\phi^T_i m \phi_j = 0, \quad \phi^T_i c \phi_j = 0, \quad \phi^T_i k \phi_j = 0 \quad (i \neq j)
\]

(13)

After substituting Eq.(13) into Eq.(14):

\[
m_{n_n} \ddot{Y}_n(t) + c_{n_n} \dot{Y}_n(t) + k_{n_n} Y_n(t) = Q_n(t)
\]

(14)
Where $m_n$ and $c_n$ are the n-th order modal mass and the n-th order modal damping. $k_n$ and $Q_n(t)$ are modal stiffness and modal load. This linear damping equation of motion is converted into uncoupled equations of motion of a single degree of freedom. Solving each equation of motion and getting the response of coupled model in each model.

4. Numerical example
Wind wheel and low-speed shafting of 1.2MW Wind Turbine in Gansu as the research object. The blade length is 29 meter. Fig6 and Fig7 shows blade stiffness and mass distribution. The other main parameters of the various components shown in Table1 [9,10].

| Table 1. The main components of the coupled system parameters |
|-------------------------------------------------------------|
|                | Inertia /($kg\cdot m^2$) | Elastic /($Gpa$) | Poisson's ratio | Mass /$kg$ |
| hub            | 688                       | ---             | ---             | 550        |
| shaft          | 386                       | 0.0205          | 0.3             | 7010       |
| Gearbox        | ---                       | ---             | ---             | 15000      |
| Bearing        | ---                       | 1.764           | ---             | ---        |
| Bearing        | ---                       | 2.8             | ---             | ---        |

Using the finite element method to solve a single blade, Table2 gives the natural frequency and deformation of a single blade.

| Table 2. The blade free vibration Frequency and Deformation. |
|-------------------------------------------------------------|
| Frequency(Hz) | Deformation |
|----------------|-------------|
| 1th frequency  | 0.9807      |
| 2th frequency  | 2.15        |
| 3th frequency  | 3.52        |

Figure 6. Blade section quality maps.  

Figure 7. Blade section stiffness maps.

Analysis showed that the low-level free vibration of the coupled system occur in low-speed shafting. When the wind wheel in the free vibration deformation, the low-speed shafting deformation is more obvious. When the coupled system in free vibration of the first frequency, support bearing a larger force and deformation of the more obvious. In free vibration of the second frequency, Support
bearing deformation is more obvious. Fig 8 shows that the relationship of the deformation of the wind wheel with the wind wheel speed curve.

For the study of wind wheel, the natural frequency of wind turbine and the deformation were obtained by the modal superposition method as shown in fig 9. The fig 9 shows coupling system deformation maps of frequency of the first order analysis showed that freedom vibration frequency of the wind wheel of the first order between 0.98 hz and 1.1 hz. The blade deformation mainly is waving deformation. Freedom vibration of wind wheel of the second order is swing vibration, and the third order is waving vibration. The wind turbine blade vibration frequency and the vibration frequency of a single comparative analysis, wind turbine blade each order vibration values slightly less the vibration frequency of a single blade.

From the wind wheel force analysis and installation features can obtained. The wind wheel vibration major is waving and swing. And from the theoretical analysis of vibration, wind wheel vibration energy is concentrated in low frequencies, so the wind wheel vibration mainly waving. Results of two analytical methods agree with each other, so it is result that the correctness of such research methods. When the wind wheel in the free vibration, the low speed shaft deformation is small, they have weak coupling.

![Figure 8. Coupled system modal maps.](image)

![Figure 9. Speed and natural frequency maps.](image)

5. Conclusion
This paper based of the power production 1.2 MW wind turbine and studied vibration characteristics of flexible wind wheel with the low-speed shafting, mass unit, spring elements, beam elements was combined to build coupling system finite model. Using modal superposition method for solving the coupled system model. The results indicate that in the coupling system. 1) That the low-level free vibration of the coupled system occur in low-speed shafting. When the wind wheel in the free vibration deformation, the low-speed shafting deformation is more obvious. 2) For the study of wind wheel, from the deformation maps of frequency of the first showed that the blade deformation mainly is waving, The wind turbine blade vibration frequency and the vibration frequency of a single comparative analysis, wind turbine blade each order vibration values slightly less the vibration frequency of a single blade. 3) Study of the blade, from the wind wheel force analysis and installation features can obtained. The wind wheel vibration major is waving and swing. And from the theoretical analysis of vibration, wind wheel vibration energy is concentrated in low frequencies, Results of two analytical methods agree with each other, so it is result that the correctness of such research methods.

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