Metal-Insulator Transition in a System of Superconducting Vortices Caused by a Metallic Gate.

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We address a recent experiment in which a strong decrease of the resistance of a superconducting film has been observed when a remote unbiased gate was placed above the film. Here we explain the experimental finding as a suppression of the vortex tunneling due to the Orthogonality Catastrophe of the electrons inside the gate. We interpret the change in the resistance of the film as a "metal-insulator" transition in the system of vortices induced by the gate.

The dissipationless flow of electrons, which is the hallmark of superconductivity, can be destroyed in the presence of a magnetic field by the motion of vortices [1]. The dissipation is caused by nonsuperconducting electrons located inside the vortex core. Paradoxically, the superconducting properties can be restored by increasing the amount of disorder because imperfections create a pinning potential $\mathcal{V}$ that opposes the motion of vortices. The vortices may still hop between the minima of the pinning potential by thermal activation $\mathcal{T}$, or at low temperatures by quantum tunneling $\mathcal{Q}_G$. Therefore, the observation of a strong decrease of the resistance when an unbiased metallic gate is placed above an amorphous superconducting film $\mathcal{G}$ is of great interest.

Here we show how the response of electrons inside the metallic gate to a change in the vortex positions can suppress the tunneling of the vortices, thereby reducing the resistance of the film. We attribute the change in the resistance to a magnetic coupling between the film and the gate. The magnetic flux of a vortex penetrating inside the gate scatters the electrons in a way similar to Aharonov-Bohm (A-B) scattering $\mathcal{A}$. The response of the electrons to a sudden change in the vortex positions leads to the Orthogonality Catastrophe (OC) that manifests itself in the vanishing overlap $\langle \Psi_f | \Psi_i \rangle$ of the two wave functions describing the macroscopic electron system before and after the change in the scattering potential $\mathcal{G}$. The tunneling vortices force the electrons inside the gate to adjust themselves to the change in the vortex positions. In response, the electrons oppose the tunneling and can even localize the vortices restoring the perfect superconducting properties of the film at low temperatures. We interpret the experiment $\mathcal{G}$ as a "metal-insulator" transition in a system of tunneling vortices induced by the gate.

Little is known about the quantum motion of the vortex density at low temperatures $\mathcal{R}$. Fortunately, for studying the role of the gate it is sufficient to assume that the change in the vortex positions is a rare tunneling event. This can be tunneling of a single vortex, a bundle of vortices, or topological defects such as dislocation pairs in the case of a vortex lattice. Phenomenologically, the change in the vortex positions can be described by the hopping Hamiltonian

$$H = \sum_i \varepsilon_i a_i^+ a_i + \sum_{\langle i,j \rangle} (\Gamma_{ij} a_i^+ a_j + h.c.). \quad (1)$$

According to the standard criterion of the metal-insulator transition [14] the ratio of the variation of the potential minima $\varepsilon = \langle \varepsilon_i \rangle$ to the typical value of the tunneling rates $\Gamma = \langle \Gamma_{ij} \rangle$ specifies whether vortices are itinerant or localized. The finite resistance at low temperature in the absence of the gate indicates that the vortices are mobile, i.e., the tunneling rate in the ungated film $\Gamma_{unG} > \varepsilon$.

The dissipationless nature of the superconducting film is revived when the tunneling rate is reduced by the gate to $\Gamma_G < \varepsilon$; see Fig. 1 which illustrates the two cases.

Following the experiment $\mathcal{G}$, we assume that the gate does not change the superconducting properties (such as $T_c$), but rather affects only the motion of vortices by renormalizing the tunneling rate:

$$\Gamma_G = \Gamma_{unG} \langle \Psi_f | \Psi_i \rangle. \quad (2)$$

The overlap $\langle \Psi_f | \Psi_i \rangle$ accounting for the response of the electron gas can be analyzed in terms of low energy electron-hole excitations "decorating" the vortex tunneling. One may consider the cloud of virtual excitations as part of the tunneling process that lasts long after the change in the vortex positions occurs; as a result of the

![FIG. 1: The "metal-insulator" transition. The vortex in the effective potential landscape is represented by a hopping "particle". (a) The tunneling rate in the absence of the gate $\Gamma_{unG}$ exceeds the energy difference between the potential minima $\varepsilon$; the vortices are in a "metallic" phase (b) The tunneling rate is reduced by the gate to $\Gamma_G < \varepsilon$, and the system of vortices becomes an "insulator"; the dissipationless property of the superconducting film is restored.](cond-mat.supr-con/0609527v1)
substantial increase of the overall time of motion along the tunneling trajectory, the tunneling rate is suppressed. Formally, the overlap factor in Eq. (2) can be presented in terms of the operators $\hat{S}_i$ and $\hat{S}_f$ describing the scattering of the electrons by the magnetic field of the tunneling vortices in their initial and final positions: [15]

$$|\langle \Psi_f | \Psi_i \rangle| = N^{-K}; \quad K = -\frac{1}{8\pi^2} \text{Tr} \left\{ \ln^2 (\hat{S}_f \hat{S}_i^{-1}) \right\}. \quad (3)$$

Here, $N$ is the number of electrons and hence the overlap factor vanishes unless there is a mechanism that limits the effectiveness of the $OC$. At finite temperatures, the large parameter $N$ should be substituted by another large parameter $(\max \{ T, \Gamma_G \} \tau_{\text{tun}})^{-1}$. Here $\tau_{\text{tun}}$ is the time of vortex tunneling in the absence of the gate; $\tau_{\text{tun}}^{-1}$ acts as the high-energy cutoff because only slow excitations that cannot follow adiabatically the tunneling particle reduce the overlap factor. On the other hand, the temperature enters as a low-energy cutoff because the excitations with energy smaller than $T$ being thermally activated do not contribute to the orthogonality of the initial and final states. In addition, electrons with energies smaller than $\Gamma_G$ cannot react to the tunneling events as they are too frequent for them. This is why the tunneling rate determines its own renormalization in a self-consistent way:

$$\Gamma_G(T) = \Gamma_{\text{un}G}(\max \{ T, \Gamma_G \} \tau_{\text{tun}})^K. \quad (4)$$

For $K < 1$, the tunneling rate remains finite at low temperatures $\Gamma_G(T \to 0) = \Gamma_{\text{un}G}(\Gamma_{\text{un}G} \tau_{\text{tun}})^K/(1-K)$, while for $K > 1$ the tunneling rate $\Gamma_G(T)$ goes to zero with the temperature as $T^K$.

The renormalization of $\Gamma_G$ induced by the gate can explain the localization of vortices (i.e. a transition from $\Gamma_{\text{un}G} > \varepsilon$ to $\Gamma_G < \varepsilon$) if the exponent $K$ is comparable to or larger than one. In the following part of this Letter we show that for a superconducting film magnetically coupled to a metallic gate (see Fig. 2) the exponent $K$ is

$$K = \zeta(\alpha \delta r)^2 \frac{(k_F^{\text{gate}})^2 d}{64 R}. \quad (5)$$

Here $\delta r$ is the typical distance that vortices have to tunnel, which is approximately the coherence length $\delta$, $\delta r \sim \xi$. The parameter $\alpha$ is the total flux measured in units $\Phi_0 = 2\pi \hbar c/e$ that moves a distance $\delta r$ as a result of a tunneling event. For a single vortex, $\alpha = 1/2$; the same holds for interstitials or vacancies. When a bundle (or a pair of dislocations) is tunneling, $\alpha$ should be multiplied by the number of vortices. The result is universal and valid as far as $\delta r \ll R$, where $R$ is the radius of the area inside the gate occupied by the magnetic field of a vortex, which in the present geometry is about the superconducting penetration depth, $R \approx \lambda$. The expression for $\lambda$ is determined by the vortex solution specific for thin-film superconductors, known as the Pearl vortex [13, 14]. Other factors determining $K$ are the gate thickness $d$, and the Fermi momentum of the electrons in the gate $k_F^{\text{gate}}$. The prefactor $\zeta$ is evaluated numerically as $\approx 0.4$.

Using the expressions for $\lambda$ and $\xi$ in disordered thin films the exponent can be rewritten as:

$$K \sim \zeta \alpha^2 \frac{c^2}{48\pi} \left( \frac{e^2}{c} \right)^2 \frac{v_F^{\text{sc}}}{c^2} \frac{d}{k_F^{\text{gate}}a}(k_F^{\text{gate}}a)(k_F^{\text{g} \text{ate}}c)^2. \quad (6)$$

The index $\text{sc}$ refers to the electrons in the superconducting film; $v_F^{\text{sc}}$ is their mean free path (in the normal state) and $v_F^{\text{c}}$ is the Fermi velocity; $a$ is the thickness of the film, and $\hbar = 1$. Interestingly, $T_c$ drops out from $K$ so that it depends only on the geometrical factors and the nonsuperconducting properties of electrons. We see that the value of the exponent $K$ is determined by a small factor $\sim 10^{-7}$ opposed by a product of a few large factors. The condition for the vortex localization can be easily fulfilled for a not too thin gate and not too disordered superconducting film.

Next, we briefly sketch the steps in the derivation of Eq. (5) starting from Eq. (3). Let us consider the $OC$ in response to a single vortex tunneling. The cylindrical symmetry of the vortex allows us to analyze the scattering of electrons using the basis of cylindrical waves, $|\ell, q, k_z\rangle$; here $\ell$ is the angular momentum along the $z$ axis, while $q$ and $k_z$ are the magnitudes of the in-plane and $z$ components of the momentum. In this basis the scattering operator is diagonal and can be described in terms of the phase shifts

$$\langle \ell, q, k_z | S | \ell', q', k_z' \rangle = e^{2i\delta \ell} e^{i\delta q} e^{i\delta k_z} e^{i\delta k_z'}. \quad (7)$$

Since the exponent in Eq. (6) contains a product of two scattering operators corresponding to the different vortex positions shifted by $\delta r$, we have to use the transformation matrix between the two bases of cylindrical
waves centered at these positions: 
\[ f(\ell, q, k_z|\ell'|q', k_z')_i = J_{|\ell - \ell'|}(q\delta r) J_{|\ell - \ell'|}(q\delta r) \delta_{\ell,q'} \delta_{k_z,k_z'} \], 
where \( J_\nu(z) \) is the Bessel function. The elements of the matrix \( S_i S_i^{-1} \) can be easily calculated as
\[ f(\ell) S_i S_i^{-1}|\ell'|_f = \sum_n e^{2i\delta_{\ell} - 2i\delta_{n+1}} J_n(q\delta r) J_{n-\ell}(q\delta r). \tag{8} \]

To proceed further, we need to find the phase shifts specific for the scattering by the vortex. An analogy to classical scattering, where the angular momentum is related to the impact parameter \( b = |\ell|/q \), helps elucidate the behavior of the phase shift as a function of \( \ell \). For \( b \gg R \), the scattering by the vortex is similar to the \( A-B \) scattering by a flux \( \alpha \Phi_0 \). In the \( A-B \) scattering \([10, 11]\) electrons acquire the phase \( \delta_{\ell}^{A-B} = \frac{\pi}{2}(|\ell| - |\ell - \alpha|) \).

The uniqueness of this scattering is in its infinite range: \( \delta_{\ell} \) does not vanish when \( |\ell| \to \infty \). For scattering by the vortex, the jump in the \( A-B \) phase shifts is smeared out, but the infinite range character of this scattering is preserved. Hence, \( \delta_{\ell} \) varies monotonically as a function of \( \ell \) between the two limits:
\[ \delta_{\ell} \xrightarrow{\ell \gg qR} \frac{\pi}{2} \text{sgn} \ \ell. \tag{9} \]

Naturally, for \( qR \gg 1 \) the phase shift depends on \( b \) and \( R \) only through the dimensionless combination \( b/R = \ell/qR \) such that \( \delta_{\ell} = \frac{\pi}{2} g(\ell/qR) \); see Fig. 4 for illustration.

\[ \text{FIG. 3: The phase shift for an electron scattered by the magnetic field of a superconducting vortex as a function of the angular momentum. The phase shift is calculated for } qR = 50, 100, \text{ and } 150. \text{ The three curves are rescaled to } qR = 100 \text{ to demonstrate the universality of the scattering. The insert shows the asymptotic behavior of the phase shift at large angular momenta, } \delta_{\ell} \to \pm \pi/4. \]

We now notice that the sum determining the elements of \( f(\ell) S_i S_i^{-1}|\ell'| \) is accumulated at \(-q\delta r \lesssim n \lesssim q\delta r\). This is because the Bessel Functions \( J_\nu(z) \) decay exponentially with their order when \( \nu > z \). Therefore, since for thin superconductors \( \delta r/R \sim \xi/\lambda \ll 1 \), the phase shifts difference in Eq. (8) can be approximated as:
\[ \delta_{\ell} - \delta_{\ell+1} \xrightarrow{n \to \infty} -n\delta_{\ell}; \quad \delta_{\ell} \approx \frac{\alpha\pi}{2qR} g(\frac{\ell}{qR}) \ll 1. \tag{10} \]

The final step of the calculation is to expand in \( \delta r/R \) the logarithm in Eq. (9), and take the trace over \( \ell \) and the momentum on the Fermi surface. The outcome of the calculation is given in Eq. (10). The gate thickness \( d \) appears here as a result of taking the trace. The specifics of the vortex solution enter only through \( q(x) \), with the integral yielding \( \xi \approx \int dx (dg/dx)^2 \approx 0.4 \).

The expression in Eq. (10) can be applied for any bundle with a total flux \( \alpha \Phi_0 \) that moves a distance \( \delta r \) as a result of a tunneling event. This is because the magnetic field of the tunneling vortices extends over a large distance, so that their exact configurations before and after the tunneling are not important. The only relevant quantity is the product \( \alpha\delta r \).

Finally, note that although we invoke the expansion in terms of \( \delta r/R \ll 1 \), we get \( K \propto (\delta r)^2/R \). This is a typical feature of the \( OC \) in the case of an extended scattering potential \([20]\) when a large number of harmonics is involved. This can be understood from the following arguments. It has been shown that the \( OC \) in the discussed problem is determined by \( \sum_{\ell} (\delta_{\ell} - \delta_{\ell+1})^2 \approx \sum_{\ell} (\delta'_{\ell})^2 \). Since the phase shifts approach asymptotically the limit \( \pm \pi/2 \), then the sum
\[ \sum_{\ell} (\delta_{\ell} - \delta_{\ell+1}) \approx \sum_{\ell} \delta'_{\ell} = \pi\alpha. \tag{11} \]

Therefore, the result obtained for the exponent \( K \) corresponds to the differences \( \delta_{\ell} - \delta_{\ell+1} \) that are distributed almost equally between \( L \sim qR \) channels:
\[ \sum_{\ell} (\delta_{\ell} - \delta_{\ell+1})^2 \sim L \left( \frac{\pi\alpha}{L} \right)^2 \sim \frac{(\pi\alpha)^2}{qR}. \tag{12} \]

The above arguments helps to understand the effect of disorder in the gate on the \( OC \). One may expect that the randomization of the phase shifts due to the disorder can only increase the value of the exponent \( K \). In the general case, \( \ell \) should be substituted by an index \( i \) of the scattering channel (i.e., the index of the states diagonalizing the scattering matrix). The scattering by impurities leads to the randomization of the phase differences, while the asymptotic limits of the phase shifts remain the same, \( \pm \pi/2 \). Therefore, the value of the exponent \( K \), which is determined by the squares of the phase differences, should increase in the presence of disorder. [Under the condition \([11]\), the obtained exponent \( K \propto (\delta r)^2/R \) is close to the minimum possible value which is at equal phase differences.] Our conclusion that disorder increases the effect of the gate in suppressing the tunneling rate \( \Gamma_C \) is in accordance with the existing theoretical results.
about the enhancement of the \( \alpha \)-exponent by not too strong disorder \([21, 22]\).

The idea to use a double layer system to study dynamics of vortices is well known \([2, 24, 25]\). The peculiarity of the discussed experiment \([4]\) is that the resistance of a superconducting film has been measured at various magnetic fields both with and without the gate. In the absence of the gate, the resistance initially decreases with lowering the temperature but eventually saturates at finite values, strongly supporting the possibility of vortex tunneling. On the other hand, when the film is gated the resistance drops with no indication of saturation. The two behaviors begin to deviate at the same temperatures at which the saturation of the resistance in the ungated film occurs. In other words, the gate affects the vortex motion in the tunneling regime only. This supports our claim that the gate reduces the tunneling rate of the vortices. (The gate is not effective in slowing down a continuous flow of vortices as the electrons can follow the vortices adiabatically.) We interpret the marked difference in the low temperature behavior of the film with and without the gate as a "metal-insulator" transition in a system of tunneling vortices induced by the gate.

In addition to the magnetic coupling between the film and the gate, one may consider a capacitive coupling between them. In the case of the Josephson junction arrays (or granular superconductors) the capacitive coupling reduces the fluctuations of the phase of the superconducting order parameter \([26, 27]\). As a result, the system may undergo a transition from an insulating to a superconducting state. However, for a homogenous film with a relatively small resistance \( \sim 1.5 \, k\Omega/\square \) used in Ref. \([6]\) the phase fluctuations are not so effective \([27]\). This is confirmed by the observed insensitivity of the critical magnetic field \( H_c \) to the presence of the gate. Furthermore, in homogeneous superconductors the motion of vortices is not accompanied by the redistribution of the charge density. Therefore, there are good reasons to ignore the capacitive coupling between the film and the gate.

The Eddy currents (Foucault currents) generated inside the gate by the emf as a result of the magnetic coupling can also contribute to the suppression of \( \Gamma_G \). We find that under the conditions of the experiment \([6]\) the \( \alpha \) is dominant. Experimentally, one can identify the main mechanism of the gate response to the vortex tunneling by changing the conductivity of the gate.

In conclusion, vortices are in the core of any physical picture describing the quantum phase transitions in superconducting films. The gated system discussed here can be used as an effective tool for investigating the microscopics of the vortex motion at low temperatures. It provides a unique opportunity to study the vortex tunneling in thin superconducting films by such simple means as varying the characteristics of the gate, in particular the gate thickness.

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