Power spectrum of large-scale magnetic fields from Gravitoelectromagnetic inflation with a decaying cosmological parameter

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Introducing a variable cosmological parameter $\Lambda(t)$ in a geometrical manner from a 5D Riemann-flat metric, we investigate the origin and evolution of primordial magnetic fields in the early universe, when the expansion is governed by a cosmological parameter $\Lambda(t)$ that decreases with time. Using the gravitoelectromagnetic inflationary formalism, but without the Feynman gauge, we obtain the power of spectrums for large-scale magnetic fields and the inflaton field fluctuations during inflation. A very important fact is that our formalism is naturally non-conformally invariant.

Keywords: extra dimensions, variable cosmological parameter, inflationary cosmology, large-scale magnetic fields

I. INTRODUCTION

The origin of the primordial magnetic fields has been subject of a great amount of research[1]. The existence, strength and structure of these fields in the intergalactic plane, within the Local Supercluster, has been scrutinized recently[2]. Many spiral galaxies are endowed with coherent magnetic fields of $\mu G$ (micro Gauss) strength[3, 4, 5, 6, 7, 8], having approximately the same energy density as the cosmic microwave background radiation (CMBR). In particular, the field strength of our galaxy is $B \approx 3 \times 10^{-6} G$, similar to that detected in high redshift galaxies[9] and damped Lyman alpha clouds[10]. Limits imposed by the high isotropy of CMB photons, obtained from the COBE data[11] restrict the present day strength of magnetic fields on cosmological scales to $10^{-9} G$. It is very mysterious that magnetic fields in clusters of galaxies [i.e., on scales $\sim$ Mpc], to be coherent. The origin of these magnetic fields is not well understood yet. The seeds of these fields could be in the early inflationary expansion of the universe, when these fields were originated. Therefore, the study of its origin and evolution in this epoch should be very important to make predictions in cosmology[12]. During inflation the extension of the causally connected regions grows as the scale factor and hence faster than in the decelerated phase. This solves the horizon problem. Furthermore, during inflation the contribution of the spatial curvature becomes very small. The way inflation solves the curvature problem is by producing a very tiny spatial curvature at the onset of the radiation epoch taking place right after inflation. The spatial curvature can well grow during the decelerated phase of expansion but it will be always subleading provided inflation lasted for sufficiently long time. It is natural to look for the possibility of generating such a large-scale magnetic field during inflation. However, the FRW universe is conformal flat and the Maxwell theory is conformal invariant. Therefore, the conformal invariance must be broken to generate non-trivial magnetic fields. Various conformal symmetry breaking mechanisms have been proposed so far[13].

Gravitoelectromagnetic inflation was developed very recently with the aim to describe, in an unified manner, the inflaton, gravitatory and electromagnetic fields during inflation[14, 15]. In this formalism all the 4D sources have a geometrical origin. This formalism can explain the origin of seed magnetic fields on cosmological scales observed today. Gravitoelectromagnetic inflation was constructed from a 5D vacuum state on a $R^{A}_{\ BCD} = 0$ globally flat metric. As in all Space Time Matter (STM) models[16], the 4D sources are geometrically induced when we take a foliation on the fifth coordinate which is spacelike and noncompact. However, in the previous works was used the Feynman gauge in order to simplify the structure of the field equations.

In this letter we shall extend this formalism without using the Feynman gauge. As we shall see, the field equations become coupled, which has interesting physical consequences. We shall study the origin and evolution of the seed magnetic fields in a $\Lambda(t)$ (with $\Lambda < 0$) dominated early universe, from a 5D vacuum state. Finally, we shall try to explain why (since have been observed) the large-scale magnetic fields are coherent.

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II. VECTOR FIELDS IN 5D VACUUM

We begin considering a 5D manifold \( \mathcal{M} \) described by a symmetric metric \( g_{AB} = g_{BA} \). This manifold \( \mathcal{M} \) is mapped by coordinates \( \{ x^A \} \).

\[
dS^2 = g_{AB} dx^A dx^B, \tag{1}\]

which, we shall consider as Riemann-flat \( R_{BCD}^A = 0 \). To introduce the fields we can define an action in \( \mathcal{M} \).

\[
S = \int d^5x \sqrt{-g} \left[ (5) \frac{R}{16\pi G} + \mathcal{L}_{(A_B, \nabla_B A_C)} \right], \tag{2}\]

where \( A_B = (A_\mu, \varphi) \) and \( A^B = (A^\mu, -\varphi) \) are respectively the covariant and contravariant 5-vector potentials, \((5) R \) is the 5D scalar curvature and \( \nabla_B \) denotes de covariant derivative. We shall consider these fields as minimally coupled to gravity. In this space the fields will be free of potential energy, so the lagrangian density is of the form

\[
\mathcal{L} = -\frac{1}{4} Q_{BC} Q^{BC} = -\frac{1}{4} F_{BC} F^{BC} - \frac{5\gamma^2}{4} (\nabla D A^B)^2, \tag{3}\]

with \( Q_{BC} \equiv F_{BC} + \gamma g_{BC}(\nabla D A^D)^2 \). Notice the importance of the last term in \( Q_{BC} \), which has been included to add the symmetries of gravity through \( g_{AB} \). This term is the responsible for the rupture of conformal invariance in the Lagrangian. The Faraday tensor is antisymmetric \( F_{BC} = \nabla_B A_C - \nabla_C A_B \), and the last term in \( (3) \) is a 5D “gauge-fixing” term. The Lagrange equations are then

\[
\nabla^F \nabla_F A^B - R^B_{\phantom{B}F, A} F^F - \alpha \nabla^B \nabla_F A^F = 0, \tag{4}\]

where \( \alpha = 1 - \frac{2}{\sqrt{\gamma}}. \) We shall consider the following non commutative algebra for \( A^C \) and \( \bar{\Pi}^B = \frac{\partial \mathcal{L}}{\partial (\nabla A^B)} = F^B_{\phantom{B}0} - g^B_0 \nabla_C A^C \)

\[
\begin{align*}
[A^C(t, \vec{r}, \psi), \bar{\Pi}^B(t, \vec{r}', \psi)] &= i g^{CB} g^{tt} \frac{(5)}{(5)g} \delta^{(3)}(\vec{r} - \vec{r}'), \\
[A^C(t, \vec{r}, \psi), A^B(t, \vec{r}', \psi)] &= [\bar{\Pi}^C(t, \vec{r}, \psi), \bar{\Pi}^B(t, \vec{r}', \psi)] = 0.
\end{align*} \tag{5}\]

Here, \( \bar{\Pi}^t = -g^{tt} (\nabla C A^C) \) and \( \frac{(5) g_{0B}}{(5)g} \) is the inverse of the normalized volume of the manifold \( (1) \).

In this work we shall choose the generalized Lorentz gauge, so that the field equations in \( (4) \) will become

\[
\begin{align*}
\nabla_F A^F &= 0, \tag{7} \\
\nabla^F \nabla_F A^B - R^B_{\phantom{B}F, A} F^F &= 0. \tag{8}
\end{align*}
\]

The equation \((7)\) describes the conservation of the vectorial field \( A[A^F(x^B)] \) on the 5D Riemann-flat metric we shall consider in this work. The equation \((8)\), describes the motion of the components \( A^F \) on a metric with Ricci-tensor components: \( R_{SL} \). In our case will be null on the 5D flat metric we shall use.

The observers are constrained to see only a hypersurface of the manifold. We can define an hypersurface \( \Sigma_f \) using a function \( f = f(x^A) \) and taking the constrain equation \( f(x^A) = cte \). The hypersurface is then mapped by coordinates \( \{ y^\nu \} \) with \( \nu = 0, 1, 2, 3 \) and has an induced metric \( g_{\nu\nu} \). The direction normal to \( \Sigma_f \) is \( n_A \) defined by \( \nabla_A f \).

The 5D quantities in \( \mathcal{M} \) have their counterparts in the 4D hypersurface \( \Sigma_F \). Physical quantities in the brane are defined by identifying the parallel parts to the hypersurface and the normal ones. On the other hand, geometrical quantities in the brane are constructed from the induced metric \( g_{\nu\nu} \).

\[\text{1} \) In our conventions capital Latin indices run from 0 to 4, greek indices run from 0 to 3 and latin indices run from 1 to 3.}
A. The 5D Riemann-flat metric with decaying parameter

In particular, in this letter we are interested to deal with the following Riemann-flat metric \([17]\)

\[
dS^2 = \psi^2 \frac{\Lambda(t)}{3} dt^2 - \psi^2 c^2 \int_0^t d\tau \sqrt{\Lambda(\tau)/3} d\nu^2 - d\psi^2,
\]

where \(d\nu^2 = dx^i \delta_{ij} dx^j\) is the euclidean line element in cartesian coordinates and \(\psi\) is the space-like extra dimension. Adopting natural units \((\hbar = c = 1)\) the cosmological parameter \(\Lambda(t)\) (with \(\Lambda < 0\)), has units of \((\text{length})^{-2}\). The metric \((9)\) is very interesting to study the evolution of the gravito-emagnetic (vectorial) field, because is Riemann-flat, but has some connections \(\Gamma^C_{DE} \neq 0\). This fact is very important when we consider the covariant derivative of \(A^F\).

The equations of motion for the components of the vectorial field \(A\) are

\[
\frac{\partial^2 A_4}{\partial t^2} + \left[ 3 \sqrt{\frac{\Lambda}{3}} - \frac{\Lambda}{2\psi^2} \right] \frac{\partial A_4}{\partial t} - \frac{\Lambda}{3} e^{-2f} \sqrt{\Lambda}\Lambda dt \nabla^2 A_4 - \frac{\Lambda}{3} \left[ \psi^2 \frac{\partial^2 A_4}{\partial \psi^2} + 6\psi \frac{\partial A_4}{\partial \psi} + 4A_4 \right] = 0,
\]

where \(\partial^2 A_0/\partial t^2\) is do not depends on the other fields. Furthermore, if we consider the particular gauge \(A_0(\psi, \nu, \psi) = 0\), we obtain the following relevant equations:

\[
\frac{\partial^2 A_4}{\partial t^2} + \left[ 3 \sqrt{\frac{\Lambda}{3}} - \frac{\Lambda}{2\psi^2} \right] \frac{\partial A_4}{\partial t} - \frac{\Lambda}{3} e^{-2f} \sqrt{\Lambda}\Lambda dt \nabla^2 A_4 - \frac{\Lambda}{3} \left[ \psi^2 \frac{\partial^2 A_4}{\partial \psi^2} + 6\psi \frac{\partial A_4}{\partial \psi} + 4A_4 \right] = 0,
\]

These are our equations of motion on the metric \((9)\), once we consider the generalized Lorentz gauge \(\nabla_F A^F = 0\), with \(A^0 = 0\).

III. EFFECTIVE 4D DYNAMICS WITH A STATIC FOLIATION \(\psi = \psi_0\)

In order to study the effective evolution of the 4D universe, we shall consider the metric \((9)\) on the hypersurface \(\psi = \psi_0\). From the point of view of a relativistic observer, we are saying that the pentavelocity \(U^4 = \frac{d\psi}{d\tau} \equiv U^\psi = 0\), such that \(g_{CD} U^C U^D = 1\). Furthermore, using the changes of variables \(\psi_0 = \frac{\Lambda}{\Lambda_0}\), we obtain the following effective 4D metric

\[
dS^2|_{eff} = \frac{\Lambda(t)}{\Lambda_0} dt^2 - \frac{3}{\Lambda_0} e^{-\int_0^t \sqrt{\Lambda(\tau)/3} d\tau} d\nu^2,
\]
where $\Lambda_0 = \Lambda(t = t_0)$ is some constant of $\Lambda$ at the initial time $t = t_0$ (which can be the Planckian time). If now we require that the components $A_i$ describe photons on the metric (13), we obtain $3\Box \Lambda^{ij} - R^i_j A_j = 0$, where $\Box$ denotes the D’Alambertian operator on the metric (10). The resulting equations on the 4D hypersurface, are

\[
\begin{align*}
\frac{\partial^2 A_i}{\partial t^2} + \left[3\sqrt{\frac{\Lambda}{3}} - \frac{\dot{\Lambda}}{2a}\right] \frac{\partial A_i}{\partial t} - \frac{\Lambda}{3} e^{-2f \sqrt{\frac{\Lambda}{3}} t} \nabla^2 A_i - \frac{\Lambda}{3} \left[\dot{\psi}^2 \frac{\partial^2 A_i}{\partial \psi^2} + 6\dot{\psi} \frac{\partial A_i}{\partial \psi} + 4A_i\right] \right|_{\psi = \psi_0} = 0, \\
\frac{\partial^2 A_i}{\partial t^2} + \left[3\sqrt{\frac{\Lambda}{3}} - \frac{\dot{\Lambda}}{2a}\right] \frac{\partial A_i}{\partial t} - \frac{\Lambda}{3} e^{-2f \sqrt{\frac{\Lambda}{3}} t} \nabla^2 A_i - R^i_j A_j = 0, \\
\end{align*}
\]

where $R^i_j = 3/\psi_0^2$ and (19) play the role of ligadure equations such that their solutions must be well defined $\forall \psi$. Notice that this equations are arbitrary and were introduced to obtain an equation of motion for photons in (13).

The components $A^\mu|_{\psi = \psi_0}$ on the 4D hypersurface can be written as

\[
\begin{align*}
\varphi(t, \vec{r}, \psi_0) = \frac{1}{(2\pi)^{3/2}} \int d^3k \left[ \xi_k(t, \psi_0) e^{i\vec{k}.\vec{r}} + \xi^*_k(t, \psi_0) e^{-i\vec{k}.\vec{r}} \right], \\
A_\mu(t, \vec{r}, \psi_0) = \frac{1}{(2\pi)^{3/2}} \int d^3k \sum_{\sigma = 1, 2} \left[ \epsilon_{\mu}(\vec{k}, \sigma) \xi_k^{(\sigma)}(t, \psi_0) e^{i\vec{k}.\vec{r}} + \epsilon^*_{\mu}(\vec{k}, \sigma) \left( \xi^*_k(t, \psi_0) \right)^* e^{-i\vec{k}.\vec{r}} \right],
\end{align*}
\]

where the two “physical” polarization 4-vectors satisfy $\epsilon^{(\sigma)} \epsilon^{(\sigma')} = \delta^{\sigma \sigma'}$. Furthermore, we shall define the modes of the fields as

\[
\begin{align*}
\xi_k^{(\sigma)}(t, \psi_0) &= b_k^{(\sigma)} \Psi_k(t), \\
\xi_k(t, \psi_0) &= a_k \chi_k(t),
\end{align*}
\]

where the annihilation $(a_k, b_k^{(\sigma)})$ and creation $(a_k^\dagger, b_k^{(\sigma)}\dagger)$ operators comply with the algebra

\[
\begin{align*}
\left[ b_k^{(\sigma)}, b_k^{(\sigma')}\dagger \right] &= \delta^{\sigma \sigma'} \delta^{(3)}(\vec{k} - \vec{k}'), \\
\left[ a_k, a_k^\dagger \right] &= \delta^{(3)}(\vec{k} - \vec{k}'), \\
\left[ a_k^\dagger, a_k \right] &= 0.
\end{align*}
\]

Notice that we are considering $A_\mu(t, \vec{r}, \psi_0)$ as a $U(1)$ gauge field on the effective 4D metric (13).

Finally, it is very interesting to study the magnetic field fluctuations on the infrared (IR) sector (i.e., on scales very bigger than the horizon radius) in the physical frame: $\left( B^2_{\text{phys}} \right)_{|_{IR}} = \left( B^2_{B_i} \right)_{|_{IR}} = \left( \langle \epsilon^{ijk} \nabla_j A_k \rangle (\epsilon_{\mu \nu \lambda \sigma} \nabla^\lambda (A^\mu_{\text{phys}})) \right)_{|_{IR}}$ which are related to the expectation value of the magnetic field energy density $\langle \rho_B \rangle = \frac{1}{8\pi} \left( B^2_{B_i} \right)_{|_{IR}}$

\section*{IV. EXAMPLES}

In order to illustrate the formalism we can study two examples, which are interesting for the cosmological expansion of the early universe.

\subsection*{A. de Sitter expansion: $\Lambda = \Lambda_0$}

As a first example we shall consider the case where $\Lambda = \Lambda_0$. When we make $\psi_0 = \sqrt{\frac{3}{\Lambda_0}}$, this case give us a de Sitter inflationary expansion of the universe with tetra-velocities: $u^\alpha = (1, 0, 0, 0)$ for a comoving frame. Furthermore, the effective 4D line element is

\[
ds^2 = dt^2 - \frac{3}{\Lambda_0} \epsilon^2 \sqrt{\Lambda_0/3} d\tau^2,
\]
In this case the general solutions for the modes of photons $\bar{\Psi}_k(t) = e^{\frac{t}{2\sqrt{2}} \mathcal{H}(t)} \Psi_k(t)$ and the inflaton $\bar{\chi}_k(t) = e^{\frac{t}{2\sqrt{2}} \chi_k(t)} \chi_k(t)$, once normalized by the conditions:

$$\bar{\Psi}_k \left( \bar{\Psi}_k \right)^* - (\bar{\Psi}_k)^* \bar{\Psi}_k = i,$$  \hspace{1cm} (27) \\
$$\bar{\chi}_k \left( \bar{\chi}_k \right)^* - (\bar{\chi}_k)^* \bar{\chi}_k = i \sqrt{\frac{\Lambda_0}{3}}.$$  \hspace{1cm} (28) \\

result to be

$$\bar{\Psi}_k(t) = \frac{1}{2} \sqrt{\frac{\pi}{\Lambda_0/3 m_0}} \mathcal{H}(t)^{2} \left[ k e^{-\sqrt{2\pi} t} \right],$$  \hspace{1cm} (29) \\
$$\bar{\chi}_k(t) = \frac{i}{2} \sqrt{\pi \mathcal{H}(t)^{2}} \left[ k e^{-\sqrt{2\pi} t} \right].$$  \hspace{1cm} (30) \\

where $0 < m_0 < 3/2$ is some value of $m$. An estimation of the squared magnetic field fluctuations on the infrared sector in the physical and comoving frames, respectively, give us

$$\langle B^2_{(phys)} \rangle \bigg|_{IR} \simeq N \left( \frac{\Lambda_0}{3} \right)^2 e^{5-\sqrt{2}\pi} e^{-2\sqrt{2\pi} t},$$  \hspace{1cm} (31) \\
$$\langle B^2_{(com)} \rangle \bigg|_{IR} \simeq N \left( \frac{\Lambda_0}{3} \right)^2 e^{5-\sqrt{2}\pi} e^{2\sqrt{2\pi} t}.$$  \hspace{1cm} (32) \\

where $N$ and $\epsilon \ll 1$ are constants. The power spectrums of $\langle \phi^2 \rangle$ and $\langle B^2 \rangle$, go as

$$\mathcal{P}(\phi^2)(k) \sim k^{3 \left[ 1 - \sqrt{1 - \frac{2}{3}m_0^2} \right]},$$  \hspace{1cm} (33) \\
$$\mathcal{P}(B^2)(k) \sim k^{5 - \sqrt{2}\pi} \sim k^{0.42}.$$  \hspace{1cm} (34) \\

Note that $\mathcal{P}(\phi^2)(k)$ becomes nearly scale invariant for $m_0^2 \ll 1$. Furthermore, we can relate the parameter $m_0$ with the mass of the inflaton field $M$ and the Hubble parameter obtained in standard 4D theories of inflation: $m_0^2 = \frac{M^2}{H_0^2} \equiv \frac{3M^2}{\Lambda_0} \ll 1$. On the other hand we see in [21] that the spectrum of $\langle B^2 \rangle \big|_{IR}$ (and hence the expectation value for the energy density due to magnetic fields) on the infrared sector: $\langle \rho_B \rangle$, go as $\sim k^{0.42}$. This implies that magnetic fields should be more intense on smaller scales. This agrees with observation, because the observed strength of magnetic fields on galactic scales are bigger than whole of cosmological scales.

**B. Decaying cosmological parameter: $\Lambda = 3p^2/t^2$**

A more interesting example can be obtained making $\Lambda = 3p^2/t^2$ and $\psi_0 = \sqrt{\frac{\Lambda_0}{3}}$, where $\Lambda$ is the cosmological parameter when inflation starts. In this case the effective 4D line element is given by [10]. Since we require that $g_{CD} u^C u^D = 1$, on the comoving frame $u^x = u^y = u^z = 0$ for hypersurfaces $u^\psi = 0$, one obtains $u^t = \sqrt{\mathcal{F}} = \sqrt{\frac{\Lambda_0}{3\Lambda}} = \left( \frac{t}{t_0} \right)$. The normalized solutions for the time-dependent modes of the the inflaton field and the photons, are

$$\bar{\Psi}_k(t) = \left( \frac{t}{t_0} \right)^{\frac{1+6p}{2}} \Psi_k(t), \hspace{1cm} \bar{\Psi}_k(t) = \frac{i}{2} \sqrt{\frac{\pi}{m_0 p}} \mathcal{H}(t)^{2} \left[ k \left( \frac{t_0}{t} \right)^p \right],$$  \hspace{1cm} (35) \\
$$\bar{\chi}_k(t) = \left( \frac{t}{t_0} \right)^{3/2} \chi_k(t), \hspace{1cm} \bar{\chi}_k(t) = \frac{i}{2} \sqrt{\frac{\pi}{p}} \mathcal{H}(t)^{2} \left[ k \left( \frac{t_0}{t} \right)^p \right].$$  \hspace{1cm} (36)
The squared magnetic field fluctuations on the physical and comoving frames, are

\[
\langle B^2_{(\text{phys})} \rangle_{IR} \simeq \frac{K}{p^2} \left( \frac{\Lambda_0}{3} \right)^2 \epsilon^{5-\sqrt{21}} (21 p^2 - 1) \frac{2^{\sqrt{21}}}{t^0} \left( \frac{t}{t_0} \right)^{-2p},
\]

(37)

\[
\langle B^2_{(\text{com})} \rangle_{IR} \simeq \frac{K}{p^2} \left( \frac{\Lambda_0}{3} \right)^2 \epsilon^{5-\sqrt{21}} (21 p^2 - 1) \frac{2^{\sqrt{21}}}{t^0} \left( \frac{t}{t_0} \right)^{2p},
\]

(38)

where \( K \) is a constant of integration. The power spectrums of \( \langle \varphi^2 \rangle \) and \( \langle B^2 \rangle \), go as

\[
\mathcal{P}(\varphi^2)(k) \sim k^3 \left[ 1 - \sqrt{1 + \left( \frac{1}{p} - 4m^2 \right)} \right],
\]

(39)

\[
\mathcal{P}(B^2)(k) \sim k^{5-\sqrt{21}} \sim k^{0.42}.
\]

(40)

Note that \( \mathcal{P}(\varphi^2)(k) \) becomes nearly scale invariant for \( m_0 \simeq \frac{1}{2p} \ll 1 \). Furthermore, the parameter \( m_0 \) is related to the power of expansion \( p \) of the universe. This result agrees with whole that one expects in the sense that \( m_0 \) will be very small for a very accelerated universe (i.e., for \( p \gg 1 \)).

V. FINAL COMMENTS

In this work we have shown how large-scale magnetic fields with sufficiently large amplitude can be generated in the early universe from a 5D vacuum state. We have explored two examples, which are relevant for cosmology. The first one is the well known de Sitter expansion. The second describes an universe governed by a decaying cosmological parameter. In both cases we obtained the same power for the spectrum of \( \langle B^2 \rangle_{IR} \), because the origin of this power is geometrical and depends on the components of the Ricci tensor on the effective 4D hypersurface. In the examples here worked these components are the same: \( R^i_i = \Lambda_0 \). An important result here obtained is that the power of the spectrums for \( \langle B^2 \rangle_{IR} \) we found is positive. It suggests that more intense magnetic fields should be on smaller scales, which is in agreement with observation. Furthermore, in both cases \( \langle B^2_{(\text{phys})} \rangle_{IR} \sim a^{-2} \) (and not as \( a^{-4} \)), due to the superadiabatic amplification of the modes produced during inflation. Notice that the results obtained in this paper depends of the gauge we choose. A gauge invariant formalism will be studied in a future work. It is important to notice that the theory we have worked is not conformally invariant, but in a natural manner. The origin of this rupture is in the gravitational contribution (through \( g_{BC} \)) of the operator \( Q_{BC} = F_{BC} + \gamma g_{BC} \left( A^D_i \right) \).

Acknowledgments

The authors acknowledge CONICET and UNMdP (Argentina) for financial support.

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