1. INTRODUCTION

The short (millisecond) time variability of gamma-ray bursts is believed to arise in internal shocks, i.e., when faster moving ejecta from the explosion collides with slower moving material ejected at an earlier time (Narayan, Paczyński, & Piran 1992; Paczyński & Xu 1994; Rees & Mészáros 1994; Sari & Piran 1997). The optical identification and measurement of redshifts for five gamma-ray bursts (GRBs) have determined their distances and the amount of energy that would be radiated in an isotropic explosion (e.g., Metzger et al. 1997; Kulkarni et al. 1998; Kelson et al. 1999; Piran 1999; and references therein). In three of these cases (GRB 971214, GRB 980703, and GRB 990123), the total isotropic energy radiated is estimated to be in excess of $10^{53}$ ergs. For GRB 990123, the isotropic energy in the gamma-ray burst is estimated to be $3.4 \times 10^{54}$ ergs. However, the steepening of the falloff of the optical light curve, $\sim 2$ days after the explosion, suggests that the explosion was not isotropic, and the total radiated energy might only be $\sim 6 \times 10^{52}$ ergs (Kulkarni et al. 1999; Mészáros & Rees 1999). There is little evidence for beaming in the other two cases.

The energy radiated in photons in gamma-ray bursts is only a fraction of the total energy released in the explosion. Collisions of shells or ejecta from the central source, believed to produce the highly variable gamma-ray burst emission, converts but a small fraction of the kinetic energy of the ejecta into thermal energy which is shared among protons, electrons, and magnetic field. If the initial temperature of the fireball is larger than a few MeV, then a fraction of the fireball energy is lost to neutrinos. Thus, a significantly larger amount of energy than “observed” must be released in these explosions. The purpose of this Letter is to provide an estimate for the radiative efficiency of GRBs in the framework of the internal shock model (§ 2). Some aspects of the work presented here have been previously considered by Kobayashi et al. (1997) and Daigne & Mochkovitch (1998). The main points are summarized in § 3.

2. GAMMA-RAY BURST ENERGETICS

A fraction of the kinetic energy of ejecta in GRBs is converted into photons as a result of internal collision during the main burst. This efficiency factor is calculated below in § 2.1. Just after the explosion, when the adiabatic cooling is small and the temperature of the fireball is several MeV, neutrinos are copiously produced and carry away a fraction of the energy of the explosion. The fraction of energy lost to neutrinos is calculated in § 2.2.

2.1. Efficiency of Internal Shocks

The efficiency of conversion of the kinetic energy of ejecta to radiation via internal shocks has been considered by Kobayashi et al. (1997) and Daigne & Mochkovitch (1998). There are several differences between the calculation presented here and previous works. One is that we calculate synchrotron emission from forward and reverse shocks in colliding shells and Compton upscattering of photon energy, by solving appropriate equations for shock and radiation, to determine the observed fluence in the $10^{10}$ keV energy band. We also take into consideration that about one-third of the total thermal energy produced in colliding shells is taken up by electrons, and only this fraction is available to be radiated away. Finally, we treat in a consistent manner energy radiation in shell collisions when the fireball is optically thick to Thomson scattering. In this case, photons do not escape the expanding ejecta but instead deposit their energy back into shells and increase the kinetic energy of ejecta. Most of this kinetic energy is not converted back to thermal energy until some later time when interstellar material is shocked. The reason for this is that shell mergers reduce the relative Lorentz factor of remaining shells and their subsequent mergers produce less thermal energy. The optical depth is important for bursts of duration 10 s or less (hereafter referred to as short-duration bursts).

We model the central explosion as resulting in random ejection of discrete shells, each carrying a random amount of energy ($\epsilon_i$) and with a random Lorentz factor ($\gamma_i$). The baryonic mass of $i$th shell ($m_i$) is set by its energy ($\epsilon_i$) and $\gamma_i$; $m_i = \epsilon_i/(c^2\gamma_i)$. The time interval between the ejection of two consecutive shells is taken to be a random number with mean time interval such as to give the desired total burst duration. The Lorentz factor of shells is taken to be uniformly distributed between a minimum ($\gamma_{\text{min}} = 5$) and a maximum ($\gamma_{\text{max}}$) value. The energy conversion efficiency is more or less independent of the number of shells ejected in the explosion so long as the number of shells is greater than a few.
The energy and momentum absorbed by the shell determines the change to its bulk velocity and its expansion, which we include in our numerical simulation to determine the radiative efficiency of internal collisions. Also included in our calculation is the conversion of the thermal energy of protons and the magnetic field to bulk motion as a result of adiabatic expansion.

The radiative efficiency $\eta$ of a burst is defined as the total energy radiated in the $10^{-10} \text{ keV}$ energy band, during a time interval in which shell collisions take place, divided by the total energy released in the explosion. Figure 1 shows a plot of $\eta$ as a function of burst duration. The total energy in bursts in all of the cases shown in the figure was taken to be $10^{52} \text{ ergs}$, independent of the burst duration. The value of $\eta$ is found to be about 1% for long-duration bursts. The bolometric radiative efficiency of random internal shocks is found to be larger by a factor of about 4. The efficiency decreases with
decreasing duration (for a fixed $\gamma_{\text{max}}$). Internal shocks are very inefficient for short-duration bursts because of photon trapping, since a number of shell collisions occur when the shell radii are small and the fireball is optically thick. For instance, the radiative efficiency for bursts of 1 s duration is about 0.2% if $\gamma_{\text{max}} = 200$. The radiative efficiency for short-duration bursts can increase significantly if the Lorentz factor of ejecta is larger in shorter duration bursts (see Fig. 1). Choice of a different distribution function for the Lorentz factor of ejecta has little effect on the efficiency of long-duration bursts. However, the efficiency of short-duration bursts can increase significantly if the width of the distribution function is taken to be small so that shells collide at larger radii, enabling photons to escape freely; for instance, in the case in which $\gamma_{\text{min}} = 50$ and $\gamma_{\text{max}} = 200$, the radiative efficiency is nearly constant ($\eta \approx 0.006$) for bursts of duration 1 s and longer (see Fig. 1). The efficiency for short-duration bursts is also enhanced if they are less energetic than longer duration bursts, thereby requiring smaller baryonic loading.

2.2. Energy Loss Due to Neutrino Production

Some fraction of gamma-ray bursts display variability on a millisecond timescale, if not smaller. The energy of explosion in these cases is expected to be generated in a region of size about 100 km. If the total energy release in an explosion underlying a GRB is $E$ and it involves ejection of $N$ shells, each of which have an initial radius of $r_0$, then the mean initial temperature of shells is $T_0 = (3E/(4\pi N r_0^3))^{1/4} = 20.6$ MeV $E^{1/8} N^{-3/4}$, where $E$ is the radiation constant, $E_{\text{r1}}$ is energy in units of $10^{53}$ ergs, and $r_{100} = r_0/100$ km. We note that the energy of the explosion ($E$) is greater than the observed energy in the gamma-ray emission by a factor of at least 10 because of the inefficiency of photon production discussed in § 2.1. Moreover, the value of $E$ that should be used in calculating the temperature is the total isotropic energy of explosion and not the reduced energy due to the finite opening angle of jet, so long as the jet was produced in the initial explosion and not by some collimation effect of the surrounding medium subsequent to a spherical explosion. Thus, $E \approx 10^{53}$ ergs is a reasonable value for the five GRBs with known redshift distance.

Neutrinos produced by $e^- e^+$ annihilation and the decay of muons and pions result in a loss of a fraction of the energy of explosions. The energy-loss rate due to $e^- e^+$ annihilation is given by

$$\frac{dE}{dt} = -2n_e \sigma T \epsilon (4\pi r^2 n_e),$$

where $E_x = E/N$, $n_e$ is the number density of electrons, $\epsilon$ is the mean thermal energy of electrons, $4\pi r^2 n_e$ is the volume of the shell in its comoving frame when the shell has expanded to a radius $r$ (the shell thickness $r_1$ is very nearly constant in the initial acceleration phase), and $\sigma = 2 \times 10^{-41} (\epsilon / 1 \text{ MeV})^2 \text{ cm}^2$ is the effective cross section for $e^- e^+$ annihilation to produce neutrinos of all different flavors. Since $E_x \approx 12\pi r^2 n_e \epsilon (4\pi r^2 n_e)$, $n_e = 2.34 \times 10^{33} T_{10}^{-1}$ cm$^{-3}$ ($T_{10} = T/10$ MeV), and $\epsilon = 3.15kT$, we find

$$\frac{d\ln E}{dt} = -9.5 \times 10^3 \frac{T_{10}}{\gamma}.$$

Initially the Lorentz factor of shells ($\gamma$) increases linearly with their radius and the temperature declines as the inverse of the radius. Using these relations, we can integrate the above equation and find that

$$\ln \left[ \frac{E(2t_0)}{E(t_0)} \right] = -1.9 \times 10^4 \gamma \left( \frac{T_{10}}{10 \text{ MeV}} \right)^5,$$

where $t_0$ is the larger of $r_0/c$ and the time when the shell becomes optically thin to neutrinos; shells become optically thin to electron neutrinos when $T_0 \lesssim 10.2$ MeV. A neutrino propagating outward sees the mean electron energy and density decrease, and therefore the opacity for scattering in an expanding medium is smaller than a corresponding static shell. For $r_0 = 10^7$ cm and $T_0 = 7$ MeV, we find that 10% of the energy of the explosion is lost to neutrinos from $e^- e^+$ annihilation, and for $T_0 = 10$ MeV, 50% of the energy is lost.

We next calculate the fraction of energy carried away by neutrinos produced by the decay of muons and pions. For instance, we consider an unstable particle ($\mu^+$ or $\pi^+$) of mass $m_\mu$ that has a lifetime of $\tau_\mu$, number density $n_\mu$, and an amount of energy carried by neutrinos when it decays $\epsilon$. In the temperature range of interest to us, these particles are created by $e^{-} e^{+}$ interaction on a timescale short compared to their decay time, and so their number density is given by the thermal distribution, i.e.,

$$n_\mu = 10.5 T^{1/2} \frac{(m_\mu^e c^2/kT)}{kT} \text{ cm}^{-3}.$$  \(\text{1}\)

The rate of loss of energy of the explosion to escaping neutrinos produced by the decay of these particles is given by

$$\frac{dE}{dt} = -8 \pi r^2 n_\mu \epsilon \approx -\frac{E_{r_2}}{8 \tau_\mu kT} \left( \frac{m_\mu^e c^2}{kT} \right)^{3/2} \exp \left( -m_\mu^e c^2/kT \right).$$

This equation can be easily integrated to yield

$$\ln \left[ \frac{E(2t_0)}{E(t_0)} \right] = -\frac{2}{3} \frac{T_{10}}{\gamma} \left( \frac{m_\mu^e c^2}{kT_0} \right)^{1/2} \exp \left( -m_\mu^e c^2/kT_0 \right).$$

For the muons $m_\mu = 105.66$ MeV, $\tau_\mu = 2.2 \times 10^{-6}$ s, and $\epsilon_e \approx 70$ MeV. Thus the fraction of energy lost by the decay of $\mu^-$ for $T_0 = 10$ MeV and $T_0 = 3.3 \times 10^{-4}$ s is 0.5%, whereas at $T_0 = 15$ MeV, 10% of the energy of the fireball is lost to neutrinos from muon decay.

For pions $m_\pi = 139.6$ MeV, $\tau_\pi = 2.55 \times 10^{-8}$ s, and $\epsilon_e \approx 29$ MeV. The fraction of energy lost by the decay of $\pi^+$ if we take $T_0 = 10$ MeV and $T_0 = 3.3 \times 10^{-4}$ s is 2%, whereas at $T_0 = 15$ MeV, 50% of the energy of the explosion is lost to neutrinos from pion decay.

The energy of these pre-GRB neutrinos is about 10–30 MeV, and they are undetectable from a typical GRB source at $z \sim 1$.

3. SUMMARY AND DISCUSSION

We find that the efficiency for internal shocks to convert the energy of explosion to radiation in the 10–10$^4$ keV energy band is of order 1% if electrons are in equipartition with protons and magnetic field. The efficiency is smaller if the electron energy is less than the equipartition value as suggested by

$^{1}$The $\nu_e$'s produced in these decays find the shell to be optically thin so long as the shell temperature is less than about 15 MeV. For $T_0 \approx 15$ MeV, the $\nu_\mu$'s are trapped in the fireball and their distribution is thermal in equilibrium with $e^{-}$. In this case, roughly 50% of the fireball energy is lost to neutrinos.
analysis of afterglow emission. Energy loss due to neutrino production at initial times, when the fireball temperature is $\sim 10$ MeV for short-duration bursts, could be significant, further reducing the energy available for radiation by a factor of $\sim 2$. The bolometric radiative efficiency of random internal shocks is found to be a factor of about 4 larger. A recent work of Panaitescu, Spada, & Mészáros (1999) finds the radiative efficiency of internal shocks in the 50–300 keV band to be about 1% and is consistent with our result.

For GRB 971214, GRB 980703, and GRB 990123, the total isotropic energy radiated in the BATSE energy band has been estimated from their observed redshifts and fluences and found to be $3 \times 10^{53}$, $2 \times 10^{53}$, and $3.5 \times 10^{54}$ ergs, respectively. The flux in higher energy photons could increase the total energy budget by a factor of $\sim 2$. These three bursts are the most energetic of the five bursts for which redshifts (or lower limits to $z$) are known. These energies should of course be corrected for beaming and the efficiency for photon production.

It has been suggested that the energy for GRB 990123, $3.5 \times 10^{54}$ ergs for isotropic explosion, is reduced by a factor of about 50 due to finite beaming angle (Kulkarni et al. 1999; Mészáros & Rees 1999). However, the inefficiency of producing radiation raises the energy budget by a factor of about 100, so the energy in the explosion is more than $10^{54}$ ergs even if beaming is as large as suggested. For GRB 980703 (at $z = 0.966$), for which there is no evidence for beaming, the energy in the explosion is also of order $10^{54}$ ergs. So it appears that the total energy of explosion for the most energetic bursts is close to or possibly greater than $10^{54}$ ergs. This energy is greater than what one can realistically hope to extract from a neutron star–mass object.

The efficiency for gamma-ray production is significantly increased if photons during the main burst are produced in both internal and external shocks. However, since it is very difficult to get short time variability in external shocks (Sari & Piran 1997), only a small fraction of energy in highly variable bursts can arise in external shocks. The energy requirement is also reduced if shells ejected in explosions are highly inhomogeneous. This will be discussed in a future paper.

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REFERENCES

Daigne, F., & Mochkovitch, R. 1998, MNRAS, 296, 275
Kelson, D. D., Illingworth, G. D., Franz, M., Magee, D., & van Dokkum, P. G. 1999, IAU Circ. 7096
Kobayashi, S., Piran, T., & Sari, R. 1997, ApJ, 490, 92
Kulkarni, S. R., et al. 1998, Nature, 393, 35
——, 1999, preprint (astro-ph/9902272)
Mészáros, P., & Rees, M. J. 1999, preprint (astro-ph9902367)

Metzger, M. R., et al. 1997, Nature, 387, 879
Narayan, R., Paczyński, B., & Piran, T. 1992, ApJ, 395, L83
Paczyński, B., & Xu, G. 1994, ApJ, 427, 708
Panaitescu, A., Spada, M., & Mészáros, P. 1999, preprint (astro-ph/9905026)
Piran, T. 1999, Phys. Rep., in press
Rees, M. J., & Mészáros, P. 1994, ApJ, 430, L93
Sari, R., & Piran, T. 1997, MNRAS, 287, 110