Interaction of Dual N=1, D=10 Supergravity with Yang-Mills Matter Multiplet

N.A. Saulina $^{a,b}$, M.V. Terentiev $^a$, K.N. Zyablyuk $^{a,c}$

$^a$ Institute of Theoretical and Experimental Physics
$^b$ Moscow Physical Technical Institute
$^c$ Moscow State University

Abstract

The lagrangian of the N=1, D=10 dual supergravity interacting with the Yang-Mills matter multiplet is constructed starting immediately from the equations of motion obtained from the Bianchi Identities in the superspace approach. The difference is established in comparison with the Gates-Nishino lagrangian at the forth order level in fermionic fields.

1 Introduction

The dual version of ten-dimensional (D=10) simple (N=1) supergravity (DUAL SUGRA) was discussed in [1], [2]. (The complete lagrangian was written in [2]). Some arguments were presented in [3], that this theory may be considered as a low-energy limit of the genterotic superstring. In the same paper special nonlocal interactions at the superstring level were constructed to reproduce the DUAL SUGRA in the string theory. It was shown in [4], that DUAL SUGRA may be introduced naturally in the framework of a five-brane theory, which is apparently related by duality to

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the theory of superstring. The connection of the DUAL SUGRA with the superstring (and/or five-brane) approach is important for many reasons. In particular, the special nonlinear superstring corrections are necessary for compensation of anomalies.

Unfortunately, the five-brane theory has not yet studied properly. But in the superstring approach one encounters complicated nonlocal interactions in the derivation of DUAL SUGRA. So, it is not clear how to obtain the DUAL SUGRA anomaly-free lagrangian from the five-brane and/or superstring approach even in the tree approximation.

Nevertheless it is possible to construct anomaly-free DUAL SUGRA immediately using symmetry considerations only. One must use in addition some knowlege on the structure of the Green-Schwarz anomaly compensating terms which arise in the process of dual transformation from the USUAL SUGRA theory. This approach was accepted in [6] and it was shown finally in [7] that equations of motion (e.m.’s) of the resulting theory are much more simple than in the USUAL SUGRA (where e.m.’s has no closed form and can be presented only as an infinite series in terms of 3-form graviphoton superfield, see [8] and other references therein). Unfortunately the discussed approach is based on superspace methods and corresponds specifically to the mass-shell description, i.e. to the e.m.’s level.

The construction of the lagrangian is a separate problem. In general it is not clear a priori that this problem may be solved in the anomaly-free case, where a lot of nonlinear terms (tree-level superstring corrections) are present. The absence of the explicit e.m.’s makes it difficult to study this problem in the USUAL SUGRA case. Another situation is in the DUAL SUGRA, where one may hope to construct the lagrangian explicitely. That is our
final purpose in the process of studying of the DUAL SUGRA.

To solve this problem one must study first the simplest (anomaly-full) case of DUAL SUGRA (without nonlinear superstring corrections). The lagrangian of such a theory has been derived from the USUAL SUGRA lagrangian [9], [10] by an explicit dual transformation, see [2]. (This method can not be generalized to the anomaly-free case). Our purpose here is to derive this lagrangian immediately from the e.m.’s in the superspace approach, because we believe that namely this approach can be generalized to the more general anomaly-free case. The main idea of this derivation was formulated in the short paper by one of us [11], where the pure gravity sector was considered. In the present paper the straightforward generalization to the case of gravity interacting with matter is considered. We found out in the process of this study that some fermionic forth order terms are given incorrectly in [2].

In Sec. 2 the supercovariant e.m.’s for matter fields are presented, in Sec. 3 the lagrangian for matter sector is derived from these equations. In this section the lagrangian for gravity sector is also presented, which is taken from [11].

In Sec. 4 the e.m.’s for gravity sector (including matter contribution) is derived from the superspace aproach and the derivation of these equations from the lagrangian is also discussed. That provides the complete check of the procedure. In Sec. 5 the supersymmetry transformations are discussed.

We use the special field parametrization [4] (which is closely related to the parametrization in [12]), which greatly simplify calculations. In Sec. 6 the super-Weil transformation is discussed, connecting our fields to the set of fields from [2] and [11].
In Sec. 7 we discuss the scale-invariance, which will be important in analysing the general structure of the lagrangian with superstring/fivebrane corrections at the next stage of our work. This invariance is also helpful in establishing of general structure of Bianchi Identities (BI’s) in the superspace approach.

In the Appendix the supercovariant form of the e.m.’s is presented together with some important constraints for superfields.

Finally the additional comment is needed. The most of the studies in the superspace approach stop at the level of equations of motion. There are reasons for this. These e.m.’s are written usually in terms of torsion-full spin-connection and fields are defined in the superspace covariant notations. So, there is a long way from this level of description to the lagrangian level. But it is a technical problem. What is more important that the presence of additional constraints in the superspace approach complicates the transition from the e.m.’s to the lagrangian. With some parametrizations used in the literature (i.e. with some choice of the constraints) it is even not clear, that the lagrangian level may be consistently realized. Our purpose in this paper is to go along this way from the beginning to the end.

Our notations and conventions in general are the same as in [11]. (Some small differences will be noted below). These notations correspond to [7] up to the sign of curvature and spin-connection. The complete description of the notations and their connections to that from other papers see in [6].
2 Supercovariant Equations of Motion for Matter Fields

The derivation of the matter fields e.m.’s is the standard procedure in the superspace approach. (For example see [13], [14] and references therein). We are presenting here some basic formulas only to define our notations.

The starting point is the BI for the Yang-Mills field-strength superfield $\mathcal{F}_{AB}$:

$$D_{[A}\mathcal{F}_{BC]} + T_{[AB}^Q \mathcal{F}_{QC]} \equiv 0,$$

where supercovariant derivatives $D_A$ obey the commutative (anticommutative) relations:

$$(D_A D_B - (-1)^{ab} D_B D_A) V_C =$$

$$= - T_{AB}^Q D_Q V_C - \mathcal{R}_{ABCD}^D V_D - (\mathcal{F}_{AB} V_C - (-1)^{c(a+b)} V_C \mathcal{F}_{AB}),$$

where the supertorsion $T_{AB}$ is defined as in [7], but the supercurvature $\mathcal{R}_{ABCD}$ differs in sign in comparison with [7]; $\mathcal{F}_{AB}$ is in the algebra of the Yang-Mills internal symmetry group $G$: $\mathcal{F}_{AB} \equiv \mathcal{F}_{AB}^J X^J$, where $(X^J)_i^j$ are anti-hermitian matrices - generators of $G$. Our definition corresponds to the space-time components of the Yang-Mills field-strength in the form: $F_{mn} = \partial_m A_n - \partial_n A_m - [A_m, A_n]$, where $A_m$ is the space-time components of vector-potential. That corresponds to $F = dA + A^2$ for the field-strength 2-form. The same definition is accepted for the curvature 2-form: $dR = dw + w^2$, where $w$ is the spin-connection 1-form.
We use the same set of constraints as in [7], so we get the same solution for the torsion and curvature BI’s.

To find the mass-shell solution of BI (2.1) it is necessary to impose the additional constraint:

$$\mathcal{F}_{\alpha\beta} = 0$$  \hspace{1cm} (2.3)

Then, using the the standard procedure (cf. [14] and other references therein) one can derive the relations which follow from (2.1) and (2.3):

$$\mathcal{F}_{a\alpha} \equiv (\Gamma_a)_{\alpha\beta} \lambda^\beta,$$ \hspace{1cm} (2.4)

$$D_\alpha \lambda^\gamma = \frac{1}{4} \mathcal{F}_{ab} (\Gamma^{ab})_\alpha^\gamma$$ \hspace{1cm} (2.5)

$$D_\gamma \mathcal{F}^{ab} = 2(\Gamma^{[a})_{\gamma\beta} D^{b]} \lambda^\beta - T^{abc} (\Gamma_c)^{\gamma\beta} \lambda^\beta - \frac{1}{36} (\hat{T} \Gamma^{ab})_{\gamma\beta} \lambda^\beta$$ \hspace{1cm} (2.6)

where \(\lambda^a\) (which is a 16 IR of \(O(1.9)\)) must be interpreted as the gluino superfield.

Now, applying spinorial derivatives to eq. (2.5), then taking the symmetrical part in spinorial indices of the resulting expression and using (2.2), (2.6), one gets the following equations of motion:

$$\Gamma^a D_a \lambda + \frac{1}{12} T_{abc} \Gamma^{abc} \lambda = 0,$$ \hspace{1cm} (2.7)

$$D_a \mathcal{F}^{ab} + T_{ab} \Gamma^b \lambda + 2 \lambda \Gamma^b \lambda = 0.$$ \hspace{1cm} (2.8)

We do not write spinorial indices explicitely in the cases, where their position may be reconstructed unambiguously.
Taking the zero superfield-component of these equations, one gets immediately the e.m.’s for gluon and gluino fields in supercovariant notations. (In the following we make no difference for the relations between superfields and their zero components because in most of the cases it can not produce misunderstanding).

3 Lagrangian for Physical Matter Fields

The simple equations obtained in Sec. 2 are not suitable for construction of the lagrangian, because they are written in terms of superspace-covariant tangent-space components. (We are not able to write a lagrangian in terms of these components). As usual, to return to the space-time components one must use the special gauge for the superspace veilbein $E^A_M$ (cf. [15]):

$$E^A_M = \begin{pmatrix} e_m^a & \psi_m^\alpha \\ 0 & \delta_\mu^\alpha \end{pmatrix}, \quad (3.1)$$

where $\psi_m^\alpha$ is the gravitino field.

The supercovariant derivative $D_a \equiv E^a_M D_M$ is equal to:

$$D_a = e_a^m D_m - \psi_a^\beta D_\beta, \quad (3.2)$$

where $\psi_a = e_a^m \psi_m$ and the space-time component of the covariant derivative is:

$$D_m \lambda = \partial \lambda - \omega_m \lambda - [A_m, \lambda], \quad (3.2')$$

where $(\omega_m)^{\beta}_\gamma \equiv \frac{1}{4} \omega_m^{ab} \Gamma_{ab}^{\beta}_\gamma$ is the spin-connection which is in the algebra of $O(1.9)$.
Now we introduce the usual tangent-space components of physical fields instead of supercovariant quantities used in the superspace approach of refs. [7] and others. Namely:

\[ F_{ab} \equiv e^m_a e^n_b F_{mn}, \quad \omega_{cab} \equiv e^m_c \omega_{mab}, \quad (3.3) \]

\[ M_{abc} = \frac{1}{7!} \varepsilon_{abc}^{a_1...a_7} (e_{a_1}^{m_1} \cdots e_{a_7}^{m_7} N_{m_1...m_7}), \quad (3.4) \]

where \( N_{m_1...m_7} = 7 \partial_{[m_1} M_{m_2...m_7]}, \) and \( M_{m_1...m_6} \) is the 6-form graviphoton potential of DUAL SUGRA.

It is possible by the standard way, using the definition of supertorsion \( T_{MN}^A = D_M E_N^A - (-1)^{mn} D_N E_M^A, \) to find the relation between the torsion-full spin-connection in the eq.(3.3) and the usual spin-connection \( \omega_{cab}^{(0)} \) defined in terms of derivatives of \( e^a_m: \)

\[ \omega_{cab} = \omega_{cab}^{(0)}(e) + \frac{1}{2} T_{cab} + C_{cab}, \quad (3.5) \]

where:

\[ C_{cab} = \psi_a \Gamma_c \psi_b - \frac{3}{2} \psi_{[a} \Gamma_c \psi_{b]} \quad (3.5') \]

We need the special notation \( \nabla_m \) for the covariant derivative with the spin-connection \( \omega_m^{(0)} \) \( (\nabla_m e^a_n = 0). \) We define also \( \nabla_a \equiv e^m_a \nabla_m. \)

Now it is the straightforward procedure to connect the physical fields introduced before with \( \mathcal{F}_{ab} \) and other supercovariant fields from [7]:

\[ \mathcal{F}^{ab} = F^{ab} + 2 \psi^{[a} \Gamma^{b]} \lambda \quad (3.6) \]
Substituting obtained relations in the eqs.(2.7),(2.6) and taking into account the eq. (3.2), we get equations of motion for physical matter fields in the final form. For gluino:

\[ \hat{\nabla} \lambda - \frac{1}{24} \hat{M} \lambda + \frac{1}{48} (\psi^f \Gamma_{fabcd} \psi^d) \Gamma^{abc} \lambda + \frac{1}{4} \Gamma^a \hat{F} \psi_a + \]

\[ + \frac{1}{8} (\psi_a \Gamma_b \psi_c) \Gamma^{abc} \lambda - \frac{1}{2} (\psi_a \Gamma^a \lambda) \Gamma^b \psi_b + \frac{1}{2} (\psi_m \Gamma_a \lambda) \Gamma^a \psi^m = 0, \]  

(3.9)

For gluon:

\[ \nabla_b (F^{ba} - \lambda \Gamma^c \Gamma^{ba} \psi_c) + 2 \lambda \Gamma^a \lambda + \frac{1}{2} M^{abc} F_{bc} = 0, \]  

(3.10)

where

\[ \hat{\nabla} = \Gamma^a \nabla_a, \quad \hat{F} = F_{ab} \Gamma^{ab}, \quad \hat{M} = M_{abc} \Gamma^{abc} \]  

(3.10')

The matter-field lagrangian-density is reconstructed immediately from eqs.(3.9), (3.10):

\[ \mathcal{L}_Y^M = \frac{1}{g^2} Tr \left[ -\frac{1}{4} F_{ba} F^{ba} + \frac{1}{8 \cdot 6!} \varepsilon^{a_1 \ldots a_{10}} M_{a_1 \ldots a_6} F_{a_7 a_8} F_{a_9 a_{10}} + \right. \]

\[ + \lambda \hat{\nabla} \lambda - \frac{1}{24} \lambda \hat{M} \lambda + \frac{1}{2} \lambda \Gamma^a \hat{F} \psi_a \]

\[ + (\lambda \Gamma_b \psi_a) (\lambda \Gamma^a \psi_b) - \frac{1}{2} (\lambda \Gamma^b \psi_b)^2 - \frac{1}{2} (\lambda \Gamma_a \psi_b)^2 \]  

(3.11)
where $Tr$ is calculated in the adjoint representation of the group $G$ ($Tr AB = G_{JK} A^J B^K$, where $G_{JK}$ is a Killing tensor), $g$ is a coupling constant.

The total lagrangian is equal to (in all the cases here and in the following the term ”lagrangian” is used for the lagrangian density):

$$L_{tot} = L_{YM} + L_{GRAV}, \quad (3.12)$$

where the first term in the r.h.s. of (3.12) is the lagrangian (3.11), but the second term is the lagrangian for the pure gravity supermultiplet, obtained in [11]:

$$L_{GRAV} = \phi (R - \frac{1}{3} T^2) | + 2 \chi \Gamma^{ab} T_{ab} |, \quad (3.13)$$

where $\phi$ and $\chi_\alpha$ are dilaton and dilatino fields ($\alpha$ is spinorial index), $R$ is the supercovariant scalar curvature, $T^{a}_{ab}$ and $T_{abc}$ are supercovariant torsion components, $T^2 \equiv T_{abc} T^{abc}$, gravitational coupling constant is put equal to one.

This form of the lagrangian follows from the linearity of the superspace e.m.’s in terms of dilaton and dilatino fields (see Appendix).

It is not a direct procedure to obtain from (3.13) the lagrangian in terms of physical fields. (The presence of constraints at the on-shell level makes it difficult to relate the off-shell curvature in (3.13) with the corresponding on-shell quantity, defined in [11]). This problem was solved in [11] and the resulting lagrangian takes the form:

$$L_{GRAV} = \phi R - 2 \phi \psi_a \Gamma^{abc} \psi_{c;b} - 2 \phi \psi_a \psi^a \Gamma_b \psi^b + 4 \psi_a \Gamma^{ab} \chi_{;b} +$$
Some specific property of the graviphoton $M_{m_1...m_6}$-field equation of motion (namely the fact that this equation has the form of derivative, because only the field-strength appears in the lagrangian) is necessary to derive (3.14) from (3.13) [11], i.e. only partial information on the superspace e.m.’s is sufficient for the construction of the complete lagrangian. Afterwards, having (3.11), (3.12) and (3.14), one may write explicitly all the other equations of motion.

To check the consistency of the approach we take the following procedure in the next Section. We derive the explicit form of the superspace e.m.’s and then compare the result with the e.m.’s which follow from the lagrangian. We shall find that the lagrangian e.m.’s are, in general, complicated linear combinations of the superspace e.m.’s. Nevertheless there is the complete correspondence between them (see below).

The additional comment is necessary. The lagrangian (3.11) does not contain terms of order of $\lambda^4$. (The choice of variables corresponding to the canonical kinetic terms in the lagrangian does not change this conclusion, see Sec. 6). That contradicts to the result of [2]. (We find also the discrepancy with [2] in some other forth order terms in fermionic fields).
4 Gravity Multiplet Equations of Motion

The complete set of the superspace e.m.’s, as derived in [7], is presented in the Appendix. They have the universal form for DUAL and USUAL SUGRA. These equations contain the $A_{abc}$ -field, which, at the first sight, is not fixed completely in the DUAL SUGRA. Nevertheless, we believe that equations (A.11), (A.12) fix the $A_{abc}$-field unambiguously up to some numerical multiplicative factor. The simplest way to find the solution of these equations (it is the unique solution consistent with BI’s), is to consider BI’s for the graviphoton field $H_{ABC}$ in the USUAL SUGRA. Such a procedure was used in [7] to find the contribution of superstring corrections. Now we use it to find the contribution of matter degrees of freedom to the gravity multiplet e.m.’s.

In the presence of matter fields the $H$-superfield BI’s take the form:

$$D[A H_{BCD}] + \frac{3}{2} T_{[AB}^Q H_{QCD]} =$$

$$= -\frac{3}{4} c_Y Tr[\mathcal{F}_{[AB} \mathcal{F}_{CD]}] - \frac{3}{2} c_L R_{[AB}^e R_{CD]}^f_{ef} \quad (4.1)$$

Note, that $c_L = -\gamma \neq 0, \ c_Y = 0$ in [7]. Now we are considering the case $c_L = 0, \ c_Y \neq 0$.

The factor $c_Y$ is fixed in the USUAL SUGRA if the $H$-field normalization is fixed by the choice of constraints (4.2) (see below) but the $\mathcal{F}$-field normalization is fixed by the choice of kinetic terms in the lagrangian. The value of $c_Y$ also follows in the framework of the DUAL SUGRA (see below, eq. (4.11)). Due
to the fact, that $H_{abc}$ and $M_{abc}$ are connected with the same quantity, the torsion-component $T_{abc}$, one can easily relate the normalization of $H$ and $M$-fields. So one can easily establish the self-consistency of the $c_Y$-definition by two different procedures.

The constraints for the $H$-superfield may be self-consistently defined in terms of dilaton $\phi$ and dilatino field $\chi = D\phi$ in the form:

$$H_{\alpha\beta a} = \phi (\Gamma_a)_{\alpha\beta}, \quad (4.2)$$

Using (4.2), one can find the solution of (4.1) which is consistent with the solution of torsion BI’s in [7]. The result is:

$$A_{abc} = -\frac{c_Y}{96} Tr[\lambda \Gamma_{abc} \lambda], \quad (4.4)$$

$$H_{abc} = -(\Gamma_{bc} \chi)_a, \quad H_{abc} = -\phi T_{abc} - \frac{c_Y}{4} Tr[\lambda \Gamma_{abc} \lambda] \quad (4.5)$$

One may check that (4.4) is also the explicit solution of the $A$-field equations (A.11), (A.12) (see Appendix). Now one may forget the USUAL SUGRA, considering the described procedure as the helpful auxiliary method to find the $A_{abc}$-field explicitly, - not more.

Then it is a straightforward procedure to write the superspace equations of motion (A.6)-(A.10) in terms of physical fields entering in the lagrangian. The calculations and the results are rather cumbersome. We present here only the terms which come from the matter-fields contribution via the $A_{abc}$-tensor in the e.m.’s (A.6)-(A.10). (The pure gravity contribution was discussed in [11]). We get the equations:

for the gravitino:
\[
Q_a \equiv \phi T_{ab} \Gamma^b - \nabla_a \chi + \ldots + \\
+ \frac{C_Y}{96} \text{Tr}[(\lambda \Gamma_{bcd} \lambda) \Gamma^{bcd} \psi_a + 4 F_{cd} \Gamma_a^{cd} \lambda + 8 (\psi_c \Gamma_{d} \lambda) \Gamma_a^{cd} \lambda - \\
- 40 F_{ac} \Gamma^c \lambda - 80 (\psi_a \Gamma_{c} \lambda) \Gamma^c \lambda] = 0, \quad (4.6)
\]
(here and in the following the notation \ldots is used for the pure gravity contribution, nonlinear in fields),

for the dilatino:

\[
Q \equiv \hat{\nabla} \chi + \ldots - \\
- \frac{C_Y}{96} \text{Tr}[(\lambda \Gamma_{bcd} \lambda) \Gamma^a \Gamma^{bcd} \psi_a - 8 \hat{F} \lambda - 16 (\psi_a \Gamma_{b} \lambda) \Gamma^{ab} \lambda] = 0,
\]

(4.7)

for the dilaton:

\[
S \equiv \nabla_a \nabla^a \phi + \ldots + \\
+ \frac{C_Y}{96} \text{Tr}[(\lambda \Gamma_{abc} \lambda) (\psi_f \Gamma^{abc} \psi_f) - 8 (F^{ab} F_{ab}) - \\
- 32 (\psi_a \Gamma_{b} \lambda) F^{ab} - 32 (\psi_{[a} \Gamma_{b]} \lambda)^2 + \frac{4}{3} (\lambda \hat{T} \lambda)] = 0,
\]

(4.8)

for the graviton:

\[
S_{ab} \equiv \phi R_{ab} + \nabla_{(a} \nabla_{b)} \phi + \ldots + \\
+ \frac{C_Y}{96} \text{Tr}[(\lambda \Gamma_{cde} \lambda) (\psi_a \Gamma^{cde} \psi_b) + 4 \eta_{ab} (F^{cd} F_{cd}) + 16 \eta_{ab} (\psi_c \Gamma_{d} \lambda) F^{cd} + \\
+ 16 \eta_{ab} (\psi_{[c} \Gamma_{d]} \lambda)^2 - 48 (\nabla_{(a} \lambda) \Gamma_{b)} \lambda + 6 (\lambda \Gamma^{cd}_{(b} \lambda) T_{a)cd} - \\
- 12 (\lambda \Gamma^{cd}_{(b} \lambda) C_{a)cd} + 12 \psi_{(a} \hat{F} \Gamma_{b)} \lambda + 24 (\psi_{(a} \Gamma^{ij}_{b)} \lambda) (\psi_i \Gamma_{j} \lambda) - \\
- \frac{2}{3} \eta_{ab} (\lambda \hat{T} \lambda) + 48 F_{ac} F^c_{b} + 96 (\psi_{[a} \Gamma_{c]} \lambda) F^c_{b} + 96 (\psi_{[b} \Gamma_{c]} \lambda) F^c_{a} + \ldots
\]

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\[+120 (\psi_i \Gamma_{(a} \lambda) (\psi_b) \Gamma^i \lambda) - 72 (\psi^a \Gamma^j \lambda) (\psi_b \Gamma_j \lambda) - \\
-48 (\psi^c \Gamma_a \lambda) (\psi^c \Gamma_b \lambda)\] = 0, \hspace{1cm} (4.9)

for the graviphoton:

\[S_{abcd} \equiv \nabla_{[a} (\phi M_{bcd}]) + \ldots + \]
\[+ \frac{c_Y}{8} Tr[-6 F_{[a b} F_{c d]} - 24 (\psi_{[a} \Gamma_{b \lambda}) (\psi^c \Gamma_{d \lambda}) - \]
\[-24 (\psi_{[a} \Gamma_{b \lambda}) F_{c d]} + 4 \nabla_{[a} \lambda \Gamma_{bcd]} \lambda - \]
\[-6 \lambda \Gamma^j_{[a b \lambda} C_{c d]j} - \psi_{[a} \hat{F} \Gamma_{bcd]} \lambda - 2 (\psi_{[a} \Gamma^{ij} \Gamma_{bcd]} \lambda) (\psi_i \Gamma_j \lambda)\] = 0, \hspace{1cm} (4.10)

These equations may be derived independently by variation of the lagrangian (3.12). (matter-field contribution to them follows from (3.11)). The result of this variation is consistent with eq.’s (4.6)-(4.10) if:

\[c_Y = \frac{1}{g^2}. \hspace{1cm} (4.11)\]

But there is no direct correspondence between (4.6)-(4.10) and the equations obtained by variation of the lagrangian.

Note from the beginning that the dilaton eq.(4.8) immediately follows from (4.9) due to the constraint (A.4c) (one must multiply (4.9) by \(\eta^{ab}\) to get (4.8)); the dilatino eq. (4.7) follows from (4.6) due to the constraint (A.4b), \[7\] (one must multiply (4.6) by \(\Gamma^a\) to get (4.7)).

The variation of (3.12) with respect to the gravitino field \(\psi_m\) produces the equation:

\[Q_a + \Gamma_a Q = 0. \hspace{1cm} (4.12)\]
The variation of (3.12) with respect to the graviphoton field $M_{m_1...m_6}$ produces the equation:

$$S_{abcd} + 3 \psi_{[a} \Gamma_{bc} Q_{d]} = 0.$$  \hspace{1cm} (4.13)

The variation of (3.12) with respect to the graviton field $e^a_m$ produces the equation

$$S_{ab} + \eta_{ab} \left( \frac{1}{2} B - S \right) - 2 \psi_{(a} Q_{b)} - \frac{1}{2} \psi^c \Gamma_{ab} Q_c - \psi^c \Gamma_{c(a} Q_{b)} -$$

$$- \frac{1}{2} \left( \psi^c \Gamma_{cab} + 2 \psi_a \Gamma_b \right) Q - \eta_{ab} \psi_c \Gamma^c Q - \frac{1}{2} \eta_{ab} Tr(\lambda \Lambda) - \frac{1}{4} Tr(\lambda \Gamma_{ab} \Lambda) = 0.$$  \hspace{1cm} (4.14)

where $B \equiv -\phi (R - \frac{1}{3} T^2)$, but $\Lambda \equiv (\hat{\nabla} \lambda + \ldots) = 0$ is the l.h.s. of the gluino equation (3.9), $Q$, $Q_a$, $S$, $S_{ab}$, $S_{abcd}$ are defined by (4.6)-(4.10).

The direct variation of (3.12) with respect to the dilaton $\phi$ and the dilatino $\chi$-fields produces the constraints (A.4b), (A.4c) as it follows from (3.13). (Note that $\phi$ and $\chi$ does not enter into the matter part of the lagrangian (3.11)). So $B = 0$ in (4.14).

Calculating $\Gamma_a$ projection from (4.12) one immediately obtains $Q = 0$, and then $Q_a = 0$. So, $S_{abcd} = 0$ as it follows from (4.13). Contracting $a, b$ indices in (4.14) one obtains $S = 0$, and then $S_{ab} = 0$. So, all the equations (4.6)-(4.10) follow from (4.12)-(4.14).

This discussion demonstrates the complicated inter-connection between the lagrangian and the superspace e.m.’s. It is the price one must pay for the simplicity of the superspace mass-shell formulation.

The consideration of pure gravity sector (terms ... in (4.6)-(4.10)) leads to the same equations (4.12)-(4.14). (This calcu-
lation was done by one of us (K.N.Z) and provide a check of the procedure.

It is important that the same combinations (4.12)-(4.14) must follow from the variation of the lagrangian if the contribution of superstring corrections is taken into account according to [7] (if a lagrangian exists in this case) because, as it has been just shown, only consideration of matter-gravity interaction terms is sufficient for the derivation of (4.12)-(4.14). This observation must help the construction of the lagrangian from the superspace e.m.’s in the presence of superstring corrections.

5 Supersymmetry Transformations

The supersymmetry transformations for any physical field follows immediately from the super-gauge transformation for the corresponding superfield (cf. [13]). Our definitions are the following.

The super-gauge transformation is:

\[ \delta_Q(\epsilon) = \delta_{GCT}(\xi^N) + \delta_L(L_{ab}) + \delta_G, \]  

where \(\delta_{GCT}\) is a special superspace general coordinate transformation:

\[ \delta_{GCT}(\xi^N)V_M = -\xi^N \partial_N V_M - \partial_M \xi^N V_N, \]  

where \(\xi^N = (\epsilon^\nu, 0)\) is a parameter, \(V_M\) is any field with a world-index in the superspace;

\(\delta_L\) is a Lorentz-transformation:

\[ \delta_L(L_{ab})F = -(L_{ab} \hat{M}^{ab}) F, \]  

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where $L_{ab}$ are parameters, $\hat{M}^{cd}$ are Lorentz-group generators, $F$ is any field with a tangent-space index in the superspace. (Our definitions are: $\hat{M}^{cd} \chi^\alpha = \frac{1}{4} (\Gamma^{cd})^\alpha_\beta \chi^\beta$ and $\hat{M}^{cd} X_a = \delta_{[ab]}^c X^b$ for fields with spinorial and vector indices);

$\delta_G$ is a gauge transformation:

$$\delta_G(\Omega) A_m = -[\Omega, A_m] - \partial_m \Omega, \quad (5.4')$$

$$\delta_G(f) M_{n_1...n_6} = -6 \partial_{[n_1} f_{n_2...n_6]} \quad (5.4'')$$

where $\Omega$ and $f_{n_1...n_5}$ are gauge transformation parameters.

One can easily find (using the standard procedure [13]) all the parameters in (5.3), (5.4) from the condition, that only the superveibein transformation contains the derivative of $\epsilon$. We find:

$$L_{AB} = -\epsilon^\nu \omega_{\nu AB}, \quad \Omega = -\epsilon^\nu A\nu, \quad f_{n_1...n_5} = -\epsilon^\nu M_{\nu n_1...n_5}$$

Then we get for the super-veilbein:

$$\delta_Q(\epsilon) E^A_M = -D_M \xi^A - \epsilon^A T^A_\alpha M, \quad (5.5)$$

where $\xi^A = (\epsilon^\alpha, 0)$; for any gauge-covariant field:

$$\delta_Q(\epsilon) X = -\epsilon^\alpha D_\alpha X \quad (5.6)$$

and for field-potentials:

$$\delta_Q(\epsilon) A_m = -\epsilon^\alpha F_{\alpha m} \quad (5.7')$$

$$\delta_Q(\epsilon) M_{n_1...n_6} = -\epsilon^\alpha N_{\alpha n_1...n_6} \quad (5.7'')$$

We write here the final form of the supersymmetry transformation in terms of superfields. The result for zero components
(physical fields) follows immediately with the help of relations from sec.’s 2-4.

For matter multiplet:

\[ \delta_Q(\epsilon) \lambda = \frac{1}{4} \mathcal{F}_{ab} \Gamma^{ab} \epsilon \]  \hspace{1cm} (5.8a)
\[ \delta_Q(\epsilon) A_m = -\lambda \Gamma_m \epsilon, \]  \hspace{1cm} (5.8b)

(where as usual \( \Gamma_m \equiv e_m^a \Gamma_a \)); for gravity multiplet:

\[ \delta_Q(\epsilon) e^a_m = -\psi^a_m \Gamma^a \epsilon, \]  \hspace{1cm} (5.9a)
\[ \delta_Q(\epsilon) \psi^a_m = -D_m \epsilon - \frac{1}{72} \Gamma_m \hat{T} \epsilon, \]  \hspace{1cm} (5.9b)
\[ \delta_Q(\epsilon) \phi = \chi \epsilon, \]  \hspace{1cm} (5.9c)
\[ \delta_Q(\epsilon) \chi = \frac{1}{2} D_a \phi \Gamma^a \epsilon - \left( \frac{1}{36} \phi \hat{T} - \hat{A} \right) \epsilon, \]  \hspace{1cm} (5.9d)
\[ \delta_Q(\epsilon) M_{m_1...m_6} = 6 \psi_{[m_1} \Gamma_{m_2...m_6]} \epsilon \]  \hspace{1cm} (5.9e)

where \( \mathcal{F}^{ab} \) is defined in (3.6); \( \hat{T} = T_{abc} \Gamma^{abc} \), (the same for \( \hat{A} \)), \( T_{abc} \) is defined in (3.8), \( A_{abc} \) is defined in (4.4).

The additional terms should be included in the \( A_{abc} \)-field if superstring corrections are present, cf. [7]. It is the advantage of our parametrization, that matter degrees of freedom (as well as superstring corrections) ”penetrate” the gravity multiplet supersymmetry transformations only due to the \( A_{abc} \)-contribution as in (5.11).

The supersymmetry algebra for physical fields is closed up to equations of motion and gauge transformations. Namely:

\[ [\delta_Q(\epsilon_2), \delta_Q(\epsilon_1)] X = (\delta_{GCT}(\xi^m) + \delta_Q(\epsilon') + \\
+ \delta_L(L_{ab}) + \delta_G(\Omega_{YM}) + \delta_G(f_{n_1...n_5})) X + \text{(e.m.'s)}, \]  \hspace{1cm} (5.10)
where $X$ is any field from gravity or matter multiplet.

The transformation parameters in (5.10) are:

$$\xi^m = \epsilon_1 \Gamma^m \epsilon_2 .$$

$$\Omega_{YM} = -\xi^m A_m .$$

$$\Omega_{m_1,\ldots,m_5} = -\xi^n M_{m_1,\ldots,m_5,n} .$$

$$L_{ab} = -\xi^n \omega_{nab} + \frac{5}{12} \xi^c T_{abc} + \frac{1}{36} \epsilon_1 \Gamma_{ab}^{cde} \epsilon_2 T_{cde} .$$

$$\epsilon' = \xi^n \psi_n .$$

Eq. (5.10) takes place for any $A_{abc}$-field (not specifically for that, defined by eq.(4.4)). Only the representation (A.13) for the $A_{abc}$-superfield spinorial derivative is necessary for the derivation of (5.10).

6 Super-Weil transformations

To find the natural variables, where the lagrangian is more complicated, but all the kinetic terms have a canonical structure, the corresponding nonlinear transformation of the fields must be established. The most important part of this transformation was found in [11] by a direct study of a lagrangian structure. In the superspace approach it is a Super-Weil (SW) transformation [18] which relates the system of constraints from [3] (we define it as set I) with that from [12] (set II). Set I was used in [6]. This set produces the canonical lagrangian. Set II was used in [7] and in the present paper.

All quantities corresponding to set I are primed in the following to distinguish them from the same objects in the set II.
In all other respects we follow closely to the notations from [7], [6]. We find the SW-transformation in the form:

\[ E_a^{M'} = \exp(2\rho) (E_a^M + f_a^\gamma E_\gamma^M), \quad E_\beta^{M'} = \exp(\rho) E_\beta^M \]

\[ E_M^b' = \exp(-2\rho) E_M^b, \quad E_M^\beta' = \exp(-\rho) (E_M^\beta - E_M^b f_\beta^b) \]

\[ D_\alpha' = \exp(\rho) (D_\alpha + \frac{1}{2} f_{\alpha,cd} \hat{M}^{cd}) \]

\[ D_a' = \exp(2\rho) (D_a + f_a^\gamma D_\gamma + \frac{1}{2} f_{a,cd} \hat{M}^{cd}), \quad (6.1) \]

where \( \hat{M}^{cd} \) are \( O(1.9) \)-generators;

\[ \rho = -\frac{1}{16} \log \phi, \quad \rho_\beta \equiv D_\beta \rho = -(16\phi)^{-1 \chi_\beta}, \quad \rho_a \equiv D_a \rho \]

\[ f_a^\gamma = -2 \Gamma_a^{\gamma \beta} \rho_\beta, \quad f_\beta ab = -4 (\Gamma_{ab})^{\gamma} \rho_\gamma, \]

\[ f_{[a,b]c} = T_{abc} + \Sigma_{abc} + 4 \eta_{c[a \rho_b]}, \quad f^{a}_{\cdot,ab} = -36 \rho_b \quad (6.2) \]

where

\[ \Sigma_{abc} \equiv \rho \Gamma_{abc} \rho = (256\phi^2)^{-1} s_{abc}, \]

but \( s_{abc} \equiv \chi \Gamma_{abc} \chi \).

The relations (6.2) may be derived if one calculates the primed torsion-components:

\[ T_{BC}^{A'} \equiv (-1)^{b(m+c)} E_C^M E_B^{N'} T_{NM}^{A'} = \]

\[ = (-1)^{b(n+c)} E_C^{N'} D_B' E_N^{A'} - (-1)^{cn} E_B^{N'} D_C' E_N^{A'} \]

in terms of the unprimed torsion-components using (6.1). By this way one obtains the equations which may be solved immediately, because \( T_{AB}' \) and \( T_{AB} \) are known from the solution of corresponding BI's.
The same procedure may be applied to the \( N_{A_1...A_7}' \). (One must take into account the factor \( -1/2 \) due to the different normalization of the graviphoton field in the notations of \([7]\) and \([6]\)). Note that the SW transformation (6.1) does not affect world-space components, so:

\[
N'_{M_1...M_7} = -\frac{1}{2} N_{M_1...M_7}
\]

\[
\mathcal{F}'_{MN} = \mathcal{F}_{MN}
\]  

(6.3)

By this way one also finds the relation between primed and unprimed sets of physical fields. The result is:

\[
e^a_m = \exp\left(\frac{1}{6} \phi'\right) e'^a_m
\]

\[
\phi = \exp\left(-\frac{4}{3} \phi'\right), \quad \chi = -\frac{4}{3} \exp\left(-\frac{17}{12} \phi'\right) \chi'
\]

\[
\psi_m = \exp\left(\frac{1}{12} \phi'\right) \left(\psi'_m - \frac{1}{6} \Gamma'_m \chi'\right)
\]

\[
N_{abc} = -2 \exp\left(-\frac{7}{6} \phi'\right) L'_{abc} - \frac{7}{12} \exp\left(-\frac{1}{6} \phi'\right) s'_{abc}
\]

\[
A_{abc} = \frac{1}{3} \exp\left(-\frac{3}{2} \phi'\right) Z'_{abc},
\]

\[
F_{ab} = \exp\left(-\frac{\phi'}{3}\right) F'_{ab}, \quad \lambda = \exp\left(-\frac{\phi'}{4}\right) \lambda'
\]  

(6.4)

where \( L', Z', s' \) are defined in \([3]\) (these objects appear in \([6]\) without primes!), \( E'^a_m = \psi'^a_m \) and \( \Gamma'_m = e^a_m \Gamma_a \), \( \lambda' \) is defined according to (2.4) in terms of \( (\mathcal{F}^a_{\beta})' \).

To be complete we present also the kinetic part of the lagrangian \( \mathcal{L}'_{tot} \) (\( e\mathcal{L}'_{tot} = e' \mathcal{L}'_{tot} \)):
\[ \mathcal{L}'_{\text{tot}} = \frac{1}{g^2} \left( -\frac{1}{4} \exp(\phi') (F'_{ab})^2 + \exp(\phi') \chi' \nabla' \chi' + R' + 2 (\partial_a' \phi')^2 - \frac{1}{3} \exp(-2\phi') (M'_{abc})^2 - 2 \psi_a' \Gamma^{abc} \nabla'_b \psi'_c + 4 \chi' \nabla' \chi' \right) \] 

Note also relations, connecting matrix elements in the 16-component formalism used here with the corresponding quantities in the 32-component formalism:

\[
\psi \Gamma_{(2k+1)} \psi = i \bar{\Psi} \gamma_{(2k+1)} \Psi, \quad \text{(the same for lambda and } \Lambda) \\
\chi \Gamma_{(2k+1)} \chi = -i \bar{X} \gamma_{(2k+1)} X, \\
\psi \Gamma_{(2k)} \chi = i \bar{\Psi} \gamma_{(2k)} X, \quad \chi \Gamma_{(2k)} \psi = -i \bar{X} \gamma_{(2k+1)} \Psi, \quad (6.6)
\]

where \( \Psi, \Lambda, X, \gamma_n \) are the 32-component formalism analogs of \( \psi, \lambda, \chi, \Gamma_n \). (Note, that \( \Psi, \Lambda \) are 32-component spinors with positive, but \( X - \) with negative chirality). Eq.’s (6.4)-(6.6) provide the complete correspondence between our notations and that from other papers.

### 7 Scaling Transformation

The D=10 supergravity equations of motion are invariant under the scale transformation of the type \([19], [3]\):

\[ X_j \rightarrow \mu^{q_j} X_j \] 

where \( X_j \) is an arbitrary field, but \( q_j \) is a numerical factor, which has a specific value for each field, \( \mu \) is an arbitrary common factor. This invariance may be reproduced at the lagrangian
level if one transforms a lagrangian according to the general rule (7.1) with \( q = 3 \). (The transformation (7.1) does not touch the space-time coordinates).

It is important that this invariance also takes place when matter fields and tree-level superstring corrections are taken into account, i.e. equations of motions (A.6)-(A.11) and equations (4.6) - (4.10) are scale-invariant. (Note that corrections of higher order in the string-slope \( \alpha' \) -parameter as well as one-loop supergravity corrections break this invariance).

We present below the transformation rules for different fields (the numerical factors in the table are the values of \( q_j \) for each field):

| Field | \( q \) |
|-------|--------|
| \( \phi \) | -1 |
| \( e^a_m \) | 1/2 |
| \( D_a \) | -1/2 |
| \( D_\alpha \) | -1/4 |
| \( F_{ab} \) | -1 |
| \( T_{abc} \) | -1/2 |
| \( T^{\gamma}_{ab} \) | -3/4 |
| \( H_{abc} \) | -3/2 |
| \( \psi^\gamma_a \) | -1/4 |
| \( N_{abc} \) | -1/2 |
| \( \chi \) | -5/4 |
| \( A_{abc} \) | -3/2 |
| \( R_{ab}^{cd} \) | -1 |
| \( \lambda \) | -3/4 |
| \( e^{-1}L \) | -2 |

This scale invariance is extremely helpful in establishing of the lagrangian general structure and the structure of any possible intermediate expression. It is this invariance helps us to select in [7] the tree-level superstring/fivebrane corrections from all other possible superstring correction terms in the equations of motion. It is also the basis for use in [7] the simplest form of the \( N \)-field BI’s (with zero in the r.h.s), as opposed to the case of \( H \)-field BI’s in the usual supergravity, where Chern-Simons contributions enter in the r.h.s. (There is no possibility to introduce the Chern-Simons form into the r.h.s. of the \( N \)-field BI’s without breaking the (7.1) scale invariance, as opposed to the case of \( H \)-field BI’s). The corrections, related with the Green-
Schwarz anomaly compensating terms enter in the game only through the $A_{abc}$-field in the DUAL SUGRA and do not break the (7.1) scale invariance.

Appendix

We present here relations between the superfields (and their zero superspace components) in the DUAL SUGRA, which were derived in [7] and used in the text of the present paper.

The torsion and the graviphoton BI’s are used in the form:

$$D_{[A} T_{BC]} D + T_{[AB} Q T_{QC]} D - R_{[ABC]} D = 0. \quad (A.1)$$

$$D_{[A_1} N_{A_2\ldots A_8]} + \frac{7}{2} T_{[A_1 A_2} Q N_{Q A_3 \ldots A_8]} \equiv 0 \quad (A.2)$$

The nonzero superfield components are $T_{abc}$ (which is completely antisymmetric), $T_{ab}^{\beta}$ and:

$$T_{\alpha\beta}^{c} = \Gamma_{\alpha\beta}^{c}, \quad T_{\alpha\beta}^{\gamma} = \frac{1}{72} (\hat{T} \Gamma_{\alpha})_{\beta}^{\gamma},$$

$$N_{\alpha\beta a_1\ldots a_5} = -(\Gamma_{a_1\ldots a_5})_{\alpha\beta},$$

$$N_{abc} = T_{abc}, \quad (A.3)$$

where

$$N_{abc} \equiv \frac{1}{7!} \epsilon_{abc}^{b_1\ldots b_7} N_{b_1\ldots b_7} \quad \hat{T} \equiv T_{abc} \Gamma^{abc}.$$

All the super-curvature components are not equal to zero and may be derived in terms of torsion components and their spinorial derivatives. There are constraints:

$$D^{a} T_{abc} = 0, \quad (A.4a)$$

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\[ T_{ab} \Gamma^{ab} = 0, \quad (A.4b) \]
\[ \mathcal{R} - \frac{1}{3} T^2 = 0, \quad (A.4c) \]

where \( \mathcal{R} \) is a supercurvature scalar \((\mathcal{R} \equiv \mathcal{R}_{abcd} \eta^{ac} \eta^{bd}, \quad T^2 \equiv T_{abc} T^{abc})\). (There are a lot of additional relations, which are not interesting for our purposes here, see \[7\] for details).

The dilaton \( \phi \) and dilatino \( \chi_\alpha \equiv D_\alpha \phi \)- superfields are introduced independently. The \( A_{abc} \)-superfield (which is an arbitrary field up to the moment) appears for the first time in the most general expression of the dilatino-field spinorial derivative:

\[
D_\alpha \chi_\beta = -\frac{1}{2} \hat{D}_{\alpha \beta} \phi + \left( -\frac{1}{36} \phi T_{abc} + A_{abc} \right) \Gamma^{abc}_{\alpha \beta}, \quad (A.5)
\]

Now we are ready to present the complete set of e.m.’s for the independent superfields. (For our present purposes it is sufficient to consider only zero superspace components of these equations):

**Gravitino equation of motion:**

\[
Q_a \equiv \phi L_a - D_a \chi - \frac{1}{36} \Gamma_a \hat{T} \chi - \frac{1}{24} \hat{T} \Gamma_a \chi + \frac{1}{42} \Gamma_a \Gamma^{ijk} D A_{ijk} + \frac{1}{7} \Gamma^{ijk} \Gamma_a D A_{ijk} = 0, \quad (A.6)
\]

**Dilatino equation of motion:**

\[
Q \equiv \hat{D} \chi + \frac{1}{9} \hat{T} \chi + \frac{1}{3} \Gamma^{ijk} D A_{ijk} = 0. \quad (A.7)
\]

**Dilaton equation of motion:**

\[
S \equiv D^2 \phi + \frac{1}{18} \phi T^2 - 2 T A - \frac{1}{24} D \Gamma^{ijk} D A_{ijk} = 0. \quad (A.8)
\]

**Graviton equation of motion:**

\[
S_{ab} \equiv \phi \mathcal{R}_{ab} - L_{(a} \Gamma_{b)} \chi - \frac{1}{36} \phi \eta_{ab} T^2 + D_{(a} D_{b)} \phi -
\]
graviphoton equation of motion:

\[-2T(aA_b) + \frac{3}{28}D\Gamma^{ij}_{k(a}DA_{b)ij} - \frac{5}{336}\eta_{ab}D\Gamma^{ij,k}DA_{ijk} = 0. \quad (A.9)\]

The following notations are introduced in (A.6)-(A.10):

\[T_A = T_{ijk}A^{ijk}, \quad (TA)_{ab} = T_{aij}A^{ij}_b, \quad L_a = T_{ab}\Gamma^b \]

\[(TA)_{abcd} = T_{abj}A^{cdj}, \quad (T\epsilon A)_{abcd} = T^{ijk}\epsilon_{ijklabcdmn}sA^{mn}. \]

There are two additional equations for the \(A_{abc}\)-superfield. The first one follows from the self-consistency of eq.(A.6) (cf. [12], [6], [17]):

\[D\Gamma^{ij}_{[a}DA_{b]ij} + 56D^jA_{jab} - \frac{64}{3}(TA)_{[ab]} = 0. \quad (A.11)\]

The second one means, that 1200 IR contribution to the \(A\)-field spinorial derivative is equal to zero:

\[(DA_{abc})^{(1200)} = 0, \quad (A.12)\]

This condition may be derived immediately from (A.5) [17]. Note, that the most general solution of (A.12) takes the form:

\[DA_{abc} = \Gamma_{abc}^dX_{de}. \quad (A.13)\]

where \(X^\gamma_{ab}\) is an arbitrary function which is 16 + 144 + 560 IR of O(1.9).
Using (A.13) one may may get rid of spinorial derivatives in
the equations of motion and consider them as equations for zero
superfield components. The explicit expression of $X_{ab}$-superfield
in terms of physical fields may be derived using (2.5) and (4.4)
(for matter sector contribution) and using eq. (3.19) from [7]
(for superstring corrections contribution).

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