Two–loop heavy Higgs correction to Higgs decay into vector bosons

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Abstract

The leading $m_H$ radiative correction to the Higgs decay width into a pair of weak vector bosons is calculated at the two–loop level, using the equivalence theorem in Landau gauge. The result indicates the breakdown of perturbation theory if the Higgs boson is heavier than $\sim 930$ GeV, in spite of the smallness of the one–loop radiative correction.
1 Introduction

After the recent discovery of the top quark at the Tevatron, the mechanism of spontaneous electroweak symmetry breaking undoubtedly remains the most obscure part of the standard model. The screening theorem \([1, 2]\), which states that the leading contributions in \(m_H\) cancel in the radiative corrections to low energy processes, results in the present loose limits on the Higgs mass derived from radiative corrections.

A Higgs mass of the order of 1 TeV is definitely not excluded by the present data. A recent analysis of the LEP results even points in the direction of a heavy Higgs \([3]\). However, other analyses give somewhat different results \([4]\). The uncertainty is apparently too large to reach a conclusive estimate of the Higgs mass.

A large Higgs mass means strong interactions in the spontaneous symmetry breaking sector, and would challenge the theory because of the breakdown of the perturbative approach. For a not too heavy Higgs, one may try to resum the asymptotic perturbative series by means of Padé approximants or by using various nonlinear sequence transformations such as the Levin’s sequence transformation. Such techniques proved useful on simple quantum mechanical examples \([5]\), but strictly speaking there is no proof that they would converge in the case of radiative corrections in the standard model.

At the same time, strong couplings in the spontaneous symmetry breaking sector may result in a rich, nonperturbative spectrum of new phenomena at the TeV energy scale. In the limit of infinite Higgs mass, the Higgs sector reduces formally to the Lagrangian of the nonlinear sigma model, with the role of the chiral symmetry played by the custodial symmetry. In the low energy limit one can use chiral perturbation theory to derive anomalous self-interactions of the electroweak gauge bosons \([6]\). At the TeV energy scale, a nonperturbative spectrum of resonances may appear by analogy with pion physics \([7]\), and restore partial wave unitarity in longitudinal vector boson scattering. The BESS model was originally proposed as a possible description of such bound states of the electroweak symmetry breaking sector \([8]\).

One can estimate the validity range of perturbation theory from unitarity violation of tree level amplitudes \([9, 10]\), and from the size of one–loop radiative corrections \([11, 12]\). A perhaps more explicit criterion to estimate the range beyond which the asymptotic series stops converging at all requests knowledge of radiative corrections at two–loop order. The value of each term in the perturbative expansion of an amplitude depends on the renormalization scheme (see for instance ref. \([13]\)), and therefore the range
of validity of the perturbative approach is scheme dependent. However, for a given scheme, at the point where the two–loop corrections are as large as the one–loop ones, the perturbative series shows no sign of convergence, and there is no warranty that the tree or the one–loop level amplitudes calculated in that scheme are reasonable approximations.

Calculating two–loop radiative corrections in the electroweak theory is not an easy task. However, the problem becomes considerably simpler if the external momenta can be neglected. It was shown by van der Bij and Veltman that any two–loop Feynman diagram evaluated at vanishing external momenta can be expressed in terms of the derivatives of the so–called master diagram, for which they found an analytical formula in terms of Spence functions [14]. This made it possible to evaluate two–loop heavy Higgs effects on low energy observables such as the $\rho$ parameter [14], the masses of the vector bosons [15], and the trilinear selfcouplings of the vector bosons [16]. As expected, these effects are rather small because of the screening of the heavy sector, and perturbation theory only breaks down for a Higgs particle as heavy as 3–4 TeV.

To calculate two–loop corrections to processes involving the symmetry breaking scalars, one has to evaluate two–loop diagrams with finite external momenta. This is a more difficult problem, and calculations of physical quantities have not been performed until recently, in spite of the considerable effort which was devoted to solving massive two–loop integrals. The reason is that in general the massive two–loop scalar diagrams with arbitrary masses and external momenta cannot be expressed in terms of known and easy to evaluate functions, like polylogarithms. For instance, the two–loop selfenergy diagram with three propagators was shown to be related to a certain generalization of the hypergeometric series, the Lauricella function [17].

A number of techniques were proposed to deal with some two–loop diagrams. Without exhausting the list, let us merely mention a few. Analytical results exist for certain diagrams evaluated at special values of the masses and of the external momenta [18, 19]. Several integral representations were proposed for some diagrams [20, 22, 21, 23], as well as Monte–Carlo integration over the Feynman parameters [28, 29]. Momentum expansions were worked out too [24, 27, 20], and conformal transformations combined with Padé approximants or Levin’s sequence transformation were used to extend their convergence domain [27].

This paper uses a recently proposed method to deal with massive two–loop diagrams in a systematic way [30]. It can be used, at least in principle,
to evaluate any massive two–loop scalar integral with controllable accuracy, and is suitable for implementation in a computer program to generate and calculate automatically the Feynman graphs relevant for a given physical process. The same method was used to calculate the Higgs selfenergy at two–loop level and to extract the leading corrections to the location of the Higgs resonance as seen in fermion scattering [30]. This gives a perturbative bound on the Higgs mass of $\sim 1.2$ TeV. The two–loop heavy Higgs corrections to the Higgs fermionic width give a somewhat lower perturbative bound of $\sim 1.1$ TeV [31, 39].

This paper presents the two–loop corrections to the $H \to Z^0Z^0$ and $H \to W^+W^-$ decays in the heavy Higgs limit. More precisely, only the leading $m_H$ corrections will be retained, which grow like $m_H^4$ at two–loop level. As expected, these processes allow one to derive a stronger perturbative bound on the mass of the Higgs boson. The two–loop correction becomes as large as the one–loop one for $m_H \sim 930$ GeV, indicating that, at least in the OMS renormalization scheme, the perturbation theory is not reliable anymore.

## 2 Lagrangian and renormalization

Since one is only interested in radiative corrections at leading order in the Higgs mass, the natural choice is to use the equivalence theorem and the Landau gauge.

The equivalence theorem [32]—[35] relates the Green functions with external longitudinal vector bosons $V_L^i$ to the Green functions with the corresponding Goldstone bosons $\phi$ replacing the vector boson legs:

$$A(V_L^{i_1}, V_L^{i_2}, \ldots, V_L^{i_n}) = (iC)^n A(\phi^{i_1}, \phi^{i_2}, \ldots, \phi^{i_n}) + O(m_W^2 \sqrt{s}).$$

Such relations are a consequence of the Slavnov–Taylor identities of the theory. The coefficient $C$ with which the amplitude has to be multiplied for each external longitudinal vector boson replaced by a Goldstone boson is gauge dependent and is in general not unity beyond tree level [35].

The calculation is simpler if performed in Landau gauge. In this gauge the coefficient $C$ in eq. 1 is one at leading order in $m_H$ [35], and the diagrammatics is considerably simplified. There are no Goldstone boson–vector boson mixing terms, and diagrams containing fermions, vector bosons or Fadeev–Popov ghosts need not be taken into account since they are non–leading. The only diagrams which give leading contributions in $m_H$ are those which contain only the symmetry breaking scalars.
The scalar sector of the standard model reads:

\[ \mathcal{L} = \frac{1}{2} (\partial_\mu H_0) (\partial^\mu H_0) + \frac{1}{2} (\partial_\mu z_0) (\partial^\mu z_0) + (\partial_\mu w^+_0)(\partial^\mu w^-_0) - g^2 \frac{m^2_{H_0}}{m^2_{W_0}} \left[ w^+_0 w^-_0 + \frac{1}{2} z^2_0 + \frac{1}{2} H^2_0 + \frac{2m_{W_0}}{g} H_0 + \frac{4 \delta t}{g^2 m^2_{H_0}} \right]^2, \]  

where \( H_0, \ w^\pm_0 \) and \( z_0 \) denote the bare Higgs and Goldstone fields. The gauge coupling constant \( g \) can be defined by using the muon decay as \( g^2 = 4 \sqrt{2} m^2_{W} G_F, \) with \( G_F = 1.16637 \cdot 10^{-5} \text{ GeV}^{-2}, \) and \( m_{W} = 80.22 \text{ GeV}. \) The energy scale where the gauge coupling constant is defined is actually irrelevant within this context because \( g \) is not renormalized at leading order in \( m_H. \) \( \delta t \) is the tadpole counterterm which will be fixed by the condition that the vacuum expectation value of the Higgs field, \( v, \) should not receive quantum corrections, and is related to the selfenergy of the Goldstone bosons at zero momentum transfer [36].

To renormalize the theory, one splits the bare Lagrangian of eq. 2 into the renormalized Lagrangian and counterterms:

\[ H_0 = Z^{1/2}_H H, \]
\[ z_0 = Z^{1/2}_G z, \]
\[ w_0 = Z^{1/2}_G w, \]
\[ m^2_{H_0} = m^2_H - \delta m^2_H, \]
\[ m^2_{W_0} = m^2_W - \delta m^2_W, \]  

(3)

We use an on–shell renormalization scheme with field renormalization. In this renormalization scheme, the counterterms can be determined from the following renormalization conditions:

\[ \Sigma_{HH}(k^2 = m^2_H) + i \delta m^2_H - i \delta t + i \delta m^2_H \delta Z_H - i \delta t \delta Z_H = 0 \]
\[ \frac{\partial}{\partial k^2} \Sigma_{HH}(k^2 = m^2_H) + i \delta Z_H = 0 \]
\[ \Sigma_{w^+w^-}(k^2 = 0) - i \delta t - i \delta t \delta Z_G = 0 \]
\[ \frac{\partial}{\partial k^2} \Sigma_{w^+w^-}(k^2 = 0) + i \delta Z_G = 0 \]
\[ \Sigma_{W^+W^-}(k^2 = 0) + i \delta m^2_W = 0, \]  

(4)
where $\hat{\Sigma}$ contain the loop and loop–counterterm selfenergy diagrams, but not the pure counterterm diagrams. $\hat{\Sigma}_{W_1 W_2} (k^2)$ is the coefficient of the $-g_{\mu\nu}$ piece of the vector boson selfenergy. Because of the Ward identity $\delta m_W^2 = -m_W^2 \delta Z_G$, it’s not really necessary to consider the gauge sector to calculate the mass counterterm of the vector bosons, as done in the last line of eqns. 4, but this was done nevertheless because it provides a useful check on the calculation. Another check which was done is to compute the Higgs tadpole diagrams and to check that they cancel upon the tadpole counterterm derived from the Goldstone boson selfenergy at zero momentum transfer.

At one–loop level, one needs to evaluate the $H-H$, $w-w$ and $W-W$ selfenergies whose topologies are shown in fig. 1, to find the following counterterms at order $g^2 m_H^2/m_W^2$:

\[
\delta t^{\text{(1-loop)}} = g^2 \frac{m_H^2}{m_W^2} \left( \frac{m_H^2}{4\pi\mu^2} \right)^{\epsilon/2} \left( \frac{m_H^2}{16\pi^2} \left\{ -\frac{3}{4} + \frac{3}{8} - \frac{3\gamma}{8} \right\} + \epsilon \left\{ \frac{3}{2} - \frac{3\gamma}{8} - \frac{3\sqrt{3}\pi}{8} + \frac{3\sqrt{3}\pi}{8} \right\} + \frac{\pi^2}{16} \right) \]
\[
\delta m_H^{2\text{(1-loop)}} = g^2 \frac{m_H^2}{m_W^2} \left( \frac{m_H^2}{4\pi\mu^2} \right)^{\epsilon/2} \left( \frac{m_H^2}{16\pi^2} \left\{ -\frac{3}{4} + \frac{3}{8} - \frac{3\gamma}{8} \right\} + \epsilon \left\{ \frac{3}{2} - \frac{3\gamma}{8} - \frac{3\sqrt{3}\pi}{8} + \frac{3\sqrt{3}\pi}{8} \right\} + \frac{\pi^2}{16} \right) \]
\[
\delta m_W^{2\text{(1-loop)}} = g^2 \frac{m_H^2}{m_W^2} \left( \frac{m_H^2}{4\pi\mu^2} \right)^{\epsilon/2} \left( \frac{m_H^2}{16\pi^2} \left\{ -\frac{3}{4} + \frac{3}{8} - \frac{3\gamma}{8} \right\} + \epsilon \left\{ \frac{3}{2} - \frac{3\gamma}{8} - \frac{3\sqrt{3}\pi}{8} + \frac{3\sqrt{3}\pi}{8} \right\} + \frac{\pi^2}{16} \right) \]
\[
\delta Z_H^{\text{(1-loop)}} = g^2 \frac{m_H^2}{m_W^2} \left( \frac{m_H^2}{4\pi\mu^2} \right)^{\epsilon/2} \left( \frac{m_H^2}{16\pi^2} \left\{ -\frac{3}{4} + \frac{3}{8} - \frac{3\gamma}{8} \right\} + \epsilon \left\{ \frac{3}{2} - \frac{3\gamma}{8} - \frac{3\sqrt{3}\pi}{8} + \frac{3\sqrt{3}\pi}{8} \right\} + \frac{\pi^2}{16} \right) \]
\[
\delta Z_G^{\text{(1-loop)}} = g^2 \frac{m_H^2}{m_W^2} \left( \frac{m_H^2}{4\pi\mu^2} \right)^{\epsilon/2} \left( \frac{m_H^2}{16\pi^2} \left\{ -\frac{3}{4} + \frac{3}{8} - \frac{3\gamma}{8} \right\} + \epsilon \left\{ \frac{3}{2} - \frac{3\gamma}{8} - \frac{3\sqrt{3}\pi}{8} + \frac{3\sqrt{3}\pi}{8} \right\} + \frac{\pi^2}{16} \right) \].
\]

In these expressions the space–time dimension is $n = 4 + \epsilon$, and $Cl$ is the Clausen function, $Cl(x) = \sum_{n=1}^{\infty} \sin (nx)/n^2$. Numerically, $Cl(5\pi/3) = 1.0149416064\ldots$. The one–loop counterterms are needed at order $\epsilon$ because
these terms combine with the poles of one–loop scalar integrals to give finite contributions at two–loop order.

By evaluating the two-loop H–H, w–w, and W–W selfenergies, one finds the following $O((g^2 m_W^2)^2)$ counterterms \[30, 31\]:

\[
\delta t^{(2\text{-loop})} = (g^2 m_H^2 m_W^2) \left( \frac{m_H^2}{4\pi\mu^2} \right)^\epsilon \frac{m_H^2}{(16\pi^2)^2} \left[ \frac{45}{16\epsilon^2} + \frac{1}{\epsilon} \left( -\frac{33}{8} \right) \\
+ \frac{45\gamma}{16} + \frac{9\sqrt{3}\pi}{16} + \frac{609}{128} - \frac{33\gamma}{8} + \frac{45\gamma^2}{32} - \frac{45\sqrt{3}\pi}{64} \\
+ \frac{9\sqrt{3}\gamma\pi}{32} - \frac{3\pi^2}{32} - \frac{21\sqrt{3}}{32} C(\frac{\pi}{3}) + \frac{9\sqrt{3}\pi \log(3)}{32} \right]
\]

\[
\delta m_W^2(2\text{-loop}) = -\left( g^2 m_H^2 m_W^2 \right) \left( \frac{m_W^2}{4\pi\mu^2} \right)^\epsilon \frac{m_W^2}{(16\pi^2)^2} \left[ \frac{3}{32\epsilon} - \frac{1}{128} + \frac{3}{32\gamma} - \frac{\pi^2}{192} + \frac{3\sqrt{3}\pi}{64} - \frac{3\sqrt{3}}{16} C(\frac{\pi}{3}) \right]
\]

\[
\delta m_H^2(2\text{-loop}) = \text{Re}\left\{ (g^2 m_H^2 m_W^2) \left( \frac{m_H^2}{4\pi\mu^2} \right)^\epsilon \frac{m_H^2}{(16\pi^2)^2} \left[ -\frac{9}{\epsilon^2} \\
+ \frac{3}{32\epsilon} \left( 169 - 96\gamma - 24\sqrt{3}\pi \right) \\
- \left( 4.785031 \pm 4.2 \cdot 10^{-5} \right) - i \left( 0.412438 \pm 1.6 \cdot 10^{-5} \right) \right] \right\}
\]

\[
\delta Z_G^{(2\text{-loop})} = \left( g^2 m_H^2 m_W^2 \right) \left( \frac{m_H^2}{4\pi\mu^2} \right)^\epsilon \frac{1}{(16\pi^2)^2} \left[ \frac{3}{32\epsilon} - \frac{1}{128} + \frac{3}{32\gamma} - \frac{\pi^2}{192} + \frac{3\sqrt{3}\pi}{64} - \frac{3\sqrt{3}}{16} C(\frac{\pi}{3}) \right]
\]

\[
\delta Z_H^{(2\text{-loop})} = \text{Re}\left\{ (g^2 m_H^2 m_W^2) \left( \frac{m_H^2}{4\pi\mu^2} \right)^\epsilon \frac{1}{(16\pi^2)^2} \left[ \frac{3}{32\epsilon} - \frac{1}{128} + \frac{3}{32\gamma} - \frac{\pi^2}{192} + \frac{3\sqrt{3}\pi}{64} - \frac{3\sqrt{3}}{16} C(\frac{\pi}{3}) \right] \right\} + \epsilon R(\sqrt{3}) \left[ \frac{1}{32\epsilon} \left( 169 - 96\gamma - 24\sqrt{3}\pi \right) \\
- \left( 6.6296 \pm 2.5 \cdot 10^{-4} \right) - i \left( 1.00233 \pm 2.5 \cdot 10^{-4} \right) \right] \right\}
\]

These counterterms agree with an independent calculation of the Higgs and Goldstone selfenergies by P.N. Maher, L. Durand, and K. Rieselmann \[38\].

3 The calculation

The partial decay widths of the Higgs boson into a pair of vector bosons are given at tree level by:
\[
\Gamma(H \to W^+ W^-) = \frac{g^2 m_H^3}{64 \pi m_W} \left( 1 - 4 \frac{m_W^2}{m_H^2} \right)^{1/2} \times \\
\left[ 1 - 4 \frac{m_W^2}{m_H^2} + 12 \frac{m_W^4}{m_H^4} \right] \\
\Gamma(H \to Z^0 Z^0) = \frac{g^2 m_H^3}{128 \pi m_W} \left( 1 - 4 \frac{m_Z^2}{m_H^2} \right)^{1/2} \times \\
\left[ 1 - 4 \frac{m_Z^2}{m_H^2} + 12 \frac{m_Z^4}{m_H^4} \right] 
\] (7)

The one-loop radiative corrections at leading order in \(m_H\) can be calculated from the diagrams shown in fig. 3. The resulting \(HW^+ W^-\) coupling, including the order \(g^2 m_H^2/m_W^2\) corrections, is:

\[
-i g \frac{m_H^2}{2 m_W} \left\{ 1 + \frac{g^2 m_H^2}{16 \pi^2 m_W^2} \left[ \frac{19}{16} + \frac{5 \pi^2}{48} - \frac{3 \sqrt{3} \pi}{8} \right] \\
+ i \pi \left( \log \frac{2}{4} - \frac{5}{8} \right) \right\}, 
\] (8)

and the partial widths of eq. 7 correspondingly get a correction factor

\[
1 + \frac{g^2 m_H^2}{16 \pi^2 m_W^2} \left( \frac{19}{8} + \frac{5 \pi^2}{24} - \frac{3 \sqrt{3} \pi}{4} \right). 
\] (9)

The real part of the correction of eq. 8 and the corresponding correction to the widths of eq. 9 agree with the results of ref. [12]. Note that at two-loop level one needs the imaginary part of the one-loop radiative correction of eq. 8 as well because it gives a correction to the widths of order \((g^2 m_H^2/m_W^2)^2\).

One problem related to the use of the Landau gauge is the presence of massless particles in the theory. Some Feynman diagrams may display mass singularities, and there is the problem of the arbitrariness of integrals of the type \(\int d^np / (1/p^4)\) in the framework of dimensional regularization. Such problems do not appear in this calculation at one-loop level, since the only place where one encounters a \(\int d^np / (1/p^4)\) type integral is the \(k_\mu k_\nu\) piece of the vector boson propagator. They are however present at
two–loop level, and one needs a regularization procedure to deal with these problems in a consistent way. The approach adopted in this paper is to work in a nearly–Landau gauge, that is, to keep a small gauge parameter $\xi$ during the calculation, and to let $\xi \to 0$ in the final results. This amounts to giving the Goldstone bosons a small mass $\sqrt{\xi} m_W$. This procedure is consistent with discarding the diagrams containing fermions, gauge bosons and Fadeev–Popov ghosts on the internal lines, since they do not give rise to contributions of $\mathcal{O}(g^2 m_H^2/m_W^2)$ in the limit $\xi \to 0$. In this way, an additional check on the calculation is present, namely the cancellation of the poles and logarithms of the gauge parameter in the final result. This cancellation involves both the analytical and the numerical parts of the calculation. Another possibility, advocated in ref. [38], is to calculate the singular diagrams with vanishing Goldstone boson masses but at off–shell momentum, and to check that their sum remains finite when the external momentum is put on–shell.

Let us now turn to the actual two–loop calculation. The main task is to calculate the two–loop proper diagrams corresponding to the $Hw^+w^-$ vertex. The topologies which are involved are shown in fig. 4.

Considering the large number of Feynman diagrams which need to be calculated and the lengthy expressions which are involved at intermediary steps, the whole calculation was done by computer. The algorithm is essentially the same which was used to derive the two–loop counter terms given in the previous section.

As a first step, a computer program generates all relevant Feynman diagrams. This was done by giving by hand the possible topologies of the proper vertex diagrams, which are shown in fig. 4. For each given topology, the program then substitutes for each internal line all possible propagators, that is, $H-H$, $z-z$, $w^+-w^-$, and $w^--w^+$. This way many diagrams are generated which contain nonexistent vertices. The program then compares the vertices of the diagrams generated with a complete list of the vertices of the theory, discards the spurious diagrams, and substitutes the actual expressions of the vertices in the correct diagrams. The combinatorial factors are automatically correct, provided one divides each topology by its symmetry factor. If a certain topology has $m$ sets of equivalent lines or nodes which contain $n_1, n_2, \ldots, n_m$ elements, then its symmetry factor is $n_1! n_2! \ldots n_m!$.

In the next step some algebra is necessary to bring the resulting Feynman diagrams into a standard form. After doing all the partial fractioning which is possible, the program decides which diagrams can be calculated analytically and which ones need numerical integration. The two–loop diagrams
evaluated at vanishing external momentum can be expressed analytically in
terms of Spence functions, and in this case their mass expansion is needed
in order to retain only the $O((g^2 m_H^2/m_W^2)^2)$ terms in the final expression.
In the case of the two–loop diagrams evaluated at finite external momenta,
the propagators with the same loop momentum are combined by means of
Feynman parameters, and the resulting scalar integrals are calculated in
terms of two basic functions, $g$ and $f$, whose definitions are given in the
Appendix.

Some care is needed when introducing Feynman parameters in trian-
gular diagrams. It is useful to parametrize in the same way the diagrams
with a similar structure in order to avoid the unnecessary proliferation of
expressions resulting from introducing Feynman parameters. There are two
types of triangular diagrams which appear in this calculation, and they can
be parametrized in the following way:

\[
\frac{1}{(p^2 - M^2)^{\alpha_1}(p^2 - m^2)^{\alpha_2}} = \frac{\Gamma(\alpha_1 + \alpha_2)}{\Gamma(\alpha_1)\Gamma(\alpha_2)}\frac{\Gamma(\alpha_3)}{\Gamma(\alpha_3)}
\times \int_0^1 dx \int_0^1 dy \frac{x(x-y)^{\alpha_1-1}(x(1-y))^{\alpha_2-1}(1-x)^{\alpha_3-1}}{[(p + k_1)^2 - m_1^2 + i\eta]^{\alpha_1+\alpha_2+\alpha_3}},
\]

where $k_1$ and $k_2$ are the momenta of the external Goldstone bosons, $M \equiv m_H$ and $m \equiv \sqrt{\xi} m_W$ are the masses of the Higgs and Goldstone bosons, and

\[
\begin{align*}
\tilde{k}_1 &= -xy k_1 + x(1-y) k_2 \\
\tilde{k}_2 &= -xy k_1 + (1-x) k_2 \\
m_1^2 &= x[1 - xy(1-y)] M^2 + (1-x) m^2 \\
m_2^2 &= x[1 - y(2-x)] M^2 + [1 - x(1-y)] m^2.
\end{align*}
\]

The diagrams containing the second structure have at least one two-
particle cut, and correspondingly an imaginary part. After introducing
Feynman parameters, this translates into the presence of poles which one has
to avoid by a suitable choice of the integration path. With respect to the numerical integration over the Feynman parameters $x$ and $y$, the parametrization given in eq. 11 has the advantage that the solution of the equation $m_2 = 0$ has a simple structure in the integration domain $(0, 1) \times (0, 1)$, and therefore it is easy to find an integration path to avoid this singularity.

Also related to this parametrization is the problem of using the Landau gauge to calculate the diagrams of fig. 5. Such diagrams taken separately are not well defined for physical momenta because of an endpoint singularity, and it is only their sum which is finite. More precisely, these diagrams have two two-particle cuts, and after introducing two Feynman parameters, one obtains an expression of the type in eq. 11. The singularities of the integrand in the $xy$ plane lie on a curve given by $m_2 = 0$, which intersects the boundaries of the integration domain $(0, 1) \times (0, 1)$. These singularities can be avoided easily by deforming the integration paths in a convenient way inside the square $(0, 1) \times (0, 1)$, but not on its frontiers, and therefore the integral is logarithmically divergent. One way to regularize this divergency is to calculate these diagrams slightly off-shell, for instance by keeping a small but finite $\eta$ in eq. 11, and to take the limit $\eta \rightarrow 0$ in the sum of these three diagrams. The problem is that also the Landau gauge limit must be taken at the end of the calculation, and this must be handled with care because these two limits do not commute. This is due to the presence of pieces which are nonleading in $m_H$ and ought to vanish in Landau gauge, but which display also the endpoint logarithmic singularity. Therefore, the correct way to take the limit is first to set $\xi \rightarrow 0$, and then to go on-shell.

A simple way to get rid of this problem is to introduce explicitly in the diagram of fig. 5 c) the terms $\xi m_W^2 \delta Z_G w^+ w^-$ and $1/2 \xi m_W^2 \delta Z_G z z$, which in fact exist in the Lagrangian of eq. 2 when one is not exactly in Landau gauge, but which only generate contributions which vanish when $\xi \rightarrow 0$. Taking these terms into account in the diagram 5 c) ensures the cancellation of the nonleading contributions which have the endpoint singularity, after which the order in which one takes the limits $\xi \rightarrow 0$ and $\eta \rightarrow 0$ becomes irrelevant. Since the endpoint divergency is only logarithmic, no large numerical cancellations appear. Moreover, the cancellations among the diagrams already occur at the level of the integrands, before the numerical integration is carried out, because the diagrams were parametrized in the same way. The result is thus numerically stable.

Some checks on the algebraic part were also included in the program. Where possible, the analytical cancellation of the poles and logarithms of the gauge parameter was checked. Some subsets of diagrams must be free
of ultraviolet divergencies, in agreement with Bogoliubov’s proof of renormalizability. This is the case with the combinations $c + d + r$, $k + l + s$, and $g + h + i + j + o$, where the notations of topologies are defined according to fig. 4. Note that in order to check this, one needs first to set the one-loop field renormalization counterterms to zero.

The algorithm described was encoded in a FORM [37] program. It takes approximately seven hours to generate all Feynman diagrams, and to perform the necessary algebra and the checks on a NeXT computer. In the end, one obtains an analytical part and a number of numerical integrals over Feynman parameters of $g$ functions, which were encoded in FORTRAN. Their evaluation took approximately 10 hours on an Apollo 9000/720 workstation.

In the end, one obtains the following result for the $Hw^+w^-$ coupling, including the order $(g^2 m_H^2/m_W^2)^2$ radiative corrections:

$$
-i \frac{g m_H^2}{2 m_W} \left\{ 1 + \frac{g^2 m_H^2}{16 \pi^2 m_W^2} \left[ \frac{19}{16} + \frac{5 \pi^2}{48} - \frac{3 \sqrt{3} \pi}{8} + i \pi \left( \frac{\log 2}{4} - \frac{5}{8} \right) \right] + \left( \frac{g^2 m_H^2}{16 \pi^2 m_W^2} \right)^2 \left[ -0.53673 \pm 4.1 \cdot 10^{-4} \right] - i (0.32811 \pm 3.1 \cdot 10^{-4}) \right\}. \tag{13}
$$

Correspondingly, the partial widths of eq. 7 get the following correction factor:

$$
1 + \frac{g^2}{16 \pi^2} \frac{m_H^2}{m_W^2} \left( \frac{19}{8} + \frac{5 \pi^2}{24} - \frac{3 \sqrt{3} \pi}{4} \right) + \left( \frac{g^2 m_H^2}{16 \pi^2 m_W^2} \right)^2 \left( 0.97103 \pm 8.2 \cdot 10^{-4} \right). \tag{14}
$$

This correction factor is shown in fig. 6 as a function of $m_H$. The two-loop correction has the same sign as the one-loop correction, and for $m_H \approx 930$ GeV it becomes as large as the latter. This is an indication for the validity range of perturbation theory. This is a rather surprising result, taking into account that the one-loop radiative correction is quite small for such a Higgs mass, at the level of $\sim 13\%$. 

11
The partial decay widths corresponding to the main decay channels of the Higgs boson, including the two–loop $\mathcal{O}((g^2 m_H^2/m_W^2)^2)$ radiative corrections, are given in table 1. The $t\bar{t}$ channel, which was calculated with similar methods in ref. [31], is also given. It should be noted that by multiplying the tree level decay rates in eqns. 7 by the correction factor of eq. 14 some subleading terms are also generated. They start with $\mathcal{O}(g^2)$ terms in the one–loop correction, and with $\mathcal{O}(g^4 m_H^2/m_W^2)$ terms at two–loop. Such contributions are of course incomplete, but they were not explicitly subtracted from the numerical results given in table 1 because they are formally of the same order as the theoretical uncertainty due to the full subleading contributions they are part of, and also numerically negligible. A similar discussion holds for the $t\bar{t}$ channel as well.

### 4 Conclusions

The decays $H \to W^+W^-$ and $H \to Z^0Z^0$ were calculated at two–loop level in the limit of large Higgs mass. The calculation was performed in Landau gauge and by using the equivalence theorem, in order to obtain the leading $m_H$ contributions.

The two–loop radiative corrections have the same sign as the one–loop ones, and thus result in an enhancement of the Higgs width. The two–loop corrections become as large as the one–loop ones for $m_H = 930$ GeV. For this value of the Higgs mass, the sum of one–loop and two–loop radiative corrections is as large as 26% of the tree level widths. This is indicative for
the point beyond which the perturbative series stops converging at all in this renormalization scheme, and calculations performed by means of Feynman diagrams become unreliable.

This result is rather surprising, considering that for \( m_H = 930 \) GeV the one-loop corrections are quite small, at 13\% level. They only become substantial for a Higgs as heavy as 1.3 TeV \[12\]. Considering that most of the existing calculations in the electroweak theory were done in the OMS renormalization scheme, this raises the question of the validity range of the calculations of other processes involving the spontaneous symmetry breaking sector, such as the \( WW \to WW \) scattering which is of interest in view of searches for the Higgs boson at future hadron colliders.

Acknowledgement

The author gratefully acknowledges interesting discussions with prof. Jochum van der Bij.

Appendix

Here we give some details related to the use of the techniques of ref. \[30\] to evaluate the scalar integrals needed for this calculation.

The following two basic functions were introduced in ref. \[30\]:

\[
g(m_1, m_2, m_3; k^2) = \int_0^1 dx \left[ Sp\left(\frac{1}{1-y_1}\right) + Sp\left(\frac{1}{1-y_2}\right) + y_1 \log \frac{y_1}{y_1-1} + y_2 \log \frac{y_2}{y_2-1}\right], \tag{15}
\]

\[
f(m_1, m_2, m_3; k^2) = \int_0^1 dx \left[ \frac{1-\mu^2}{2\kappa^2} - \frac{1}{2} y_1 \log \frac{y_1}{y_1-1} - \frac{1}{2} y_2 \log \frac{y_2}{y_2-1}\right], \tag{16}
\]

where

\[
y_{1,2} = \frac{1 + \kappa^2 - \mu^2 \pm \sqrt{\Delta}}{2\kappa^2}
\]

\[
\Delta = (1 + \kappa^2 - \mu^2)^2 + 4\kappa^2 \mu^2 - 4i\kappa^2 \eta, \tag{17}
\]

and
\[
\mu^2 = \frac{ax + b(1-x)}{x(1-x)}
\]
\[
a = \frac{m_2^2}{m_1^2}, \quad b = \frac{m_3^2}{m_1^2}, \quad \kappa^2 = \frac{k^2}{m_1^2}.
\] (18)

No attempt was done to further integrate these functions analytically, since this cannot be done in terms of known and easy to calculate functions, such as polylogarithms. It is presumably possible to relate them to the Lauricella functions \[17\], but it is not clear that this would lead to an efficient way to calculate them. Instead, a FORTRAN routine was written to integrate numerically the \(g\) and \(f\) functions, as well as the necessary derivatives of \(g\), to the desired accuracy. This can be done easily by using an adaptative deterministic integration algorithm.

Some tricks were used to perform the integration in an efficient way. First, one extracts the singularities at the ends of the integration path, which are of logarithmic type, by a convenient change of variables, such as \(t = \sqrt{x}\). Then, the program chooses the appropriate integration path in the complex plane of the integration variable \(x\), in order to avoid the eventual singularities of the integrand. The aim is twofold. First, on such a path the integrand has a small variance and is smooth, so the integral can be calculated to high accuracy by using a small number of points. Second, by carrying out the integration on such a path, one avoids automatically the numerical instabilities due to large cancellations which occur in the computation of the integrand near its branching points. To compute the suitable integration path, one needs to know that the integrands of the \(g\) and \(f\) functions given in eq. 15 and 16 have two branching points in the plane of the complex Feynman parameter \(x\). They lie on the real axis when the functions are calculated above the physical threshold \(-k^2 > (m_1+m_2+m_3)^2\), and their location is given by:

\[
x_{1,2} = \frac{1}{2\mu_1^2} [-a + b + \mu_1^2 \pm \sqrt{(a - b - \mu_1^2)^2 - 4b\mu_1^2}]
\]
\[
\mu_{1,2}^2 = 1 - \kappa^2 \mp 2\sqrt{-\kappa^2}.
\] (19)

The functions \(g\) and \(f\) are the finite parts of the following scalar integrals:
\begin{align}
\mathcal{G}(m_1, m_2, m_3; k^2) & \equiv \\
\int d^npd^nq & \frac{1}{(p^2 + m_1^2)(q^2 + m_2^2)(p + q)^2 + m_3^2} = \\
\pi^4 \left\{ \frac{2}{\epsilon^2} + \frac{1}{\epsilon} \left[ -1 + 2\gamma + 2\log(\pi m_1^2) \right] + \frac{1}{4} + \frac{\pi^2}{12} \\
+ \frac{1}{4} \left[ -1 + 2\gamma + 2\log(\pi m_1^2) \right]^2 - 1 + g(m_1, m_2, m_3; k^2) \right\}, \\
(20)
\end{align}

\begin{align}
\mathcal{F}(m_1, m_2, m_3; k^2) & \equiv \\
- \int d^npd^nq & \frac{(p + q).k}{(p^2 + m_1^2)(q + k)^2 + m_3^2} = \\
k^2 \pi^4 \left\{ -\frac{1}{2\epsilon} + \frac{9}{8} - \frac{1}{2} \left[ \gamma + \log(\pi m_1^2) \right] + f(m_1, m_2, m_3; k^2) \right\}, \\
(21)
\end{align}

Let us introduce the following notation:

\begin{align}
\mathcal{G}(m_1, \alpha_1; m_2, \alpha_2; m_3, \alpha_3; k^2) & = \\
\int d^npd^nq & \frac{1}{(p^2 + m_1^2)^{\alpha_1}(q^2 + m_2^2)^{\alpha_2}((r + k)^2 + m_3^2)^{\alpha_3}}. \\
(22)
\end{align}

All \( \mathcal{G} \) scalar integrals can be obtained from \( \mathcal{G} \) and \( \mathcal{F} \) with the help of the following relations:

\begin{align}
\mathcal{G}(m_1, \alpha_1 + 1; m_2, \alpha_2; m_3, \alpha_3; k^2) & = \\
- \frac{1}{\alpha_1} \frac{\partial}{\partial m_1^2} & \mathcal{G}(m_1, \alpha_1; m_2, \alpha_2; m_3, \alpha_3; k^2), \\
\mathcal{G}(m_1, 1; m_2, 1; m_3, 1; k^2) & = \\
\frac{1}{3 - n} & \left\{ m_1^2 \mathcal{G}(m_1, m_2, m_3; k^2) + m_2^2 \mathcal{G}(m_2, m_1, m_3; k^2) \\
+ m_3^2 \mathcal{G}(m_3, m_1, m_2; k^2) + \mathcal{F}(m_1, m_2, m_3; k^2) \right\}, \\
(23)
\end{align}

We further notice that any two–loop scalar integral is either a \( \mathcal{G} \) integral, or it can be written as an integral of a certain \( \mathcal{G} \) after combining all propagators with the same loop momentum (\( p, q \) or \( p + q \)) by introducing Feynman parameters. Where further numerical integrations are needed, the numerical techniques are similar to those used for the computation of the \( g \) and \( f \)
functions. The main trick is to perform the integration over the remaining Feynman parameters on a complex path to avoid the eventual singularities of the integrand. The speed of the integration increases dramatically if one uses an optimized integration path, along which the integrand is smooth enough. The path was defined by means of spline functions.

Some scalar integrals need to be evaluated at vanishing external momentum. In this case \( F \) vanishes, and the function \( g \) can be integrated analytically in terms of Spence functions [14, 30]:

\[
g(m_1, m_2, m_3; 0) = 1 - \frac{1}{2} \log a \log b - \frac{a + b - 1}{\sqrt{\Delta'}} \left[ Sp\left( -\frac{u_2}{v_1} \right) + Sp\left( -\frac{v_2}{u_1} \right) \right] + \frac{1}{4} \log^2 \frac{u_2}{v_1} + \frac{1}{4} \log^2 \frac{v_2}{u_1} + \frac{1}{4} \log \frac{u_1}{v_1} - \frac{1}{4} \log \frac{u_2}{v_2} + \frac{\pi^2}{6} \right], \quad (24)
\]

where

\[
u_{1,2} = \frac{1}{2} \left( 1 + b - a \pm \sqrt{\Delta'} \right)
\]
\[
v_{1,2} = \frac{1}{2} \left( 1 - b + a \pm \sqrt{\Delta'} \right)
\]
\[
\Delta' = 1 - 2(a + b) + (a - b)^2. \quad (25)
\]

Only two masses appear in this calculation: \( m_H \), the mass of the Higgs boson, and \( \sqrt{\xi} m_W \), the mass of the Goldstone modes, with the Landau gauge limit taken in the end of the calculation. Therefore one needs some mass expansions of the \( g(m_1, m_2, m_3; 0) \) function.

The necessary expansions are given in the following, sometimes with unnecessary precision:

\[
\begin{align*}
j(M, M, m) &= x(-1 + \frac{\log x}{2}) + x^2 \frac{-5 + 3 \log x}{36} + O(x^3) \quad (26) \\
j(M, m, m) &= \frac{\pi^2}{6} + x(-2 + 2 \log x) + x^2 \left( -\frac{3}{2} + \frac{\pi^2}{3} \right) + 3 \log x + \log^2 x) + O(x^3) \quad (27) \\
j(m, M, m) &= -\frac{\pi^2}{6} + \frac{\log^2 x}{2} + x(2 - \frac{\pi^2}{3} - 2 \log x - \log^2 x) +
\end{align*}
\]
\[ j(m, M, M) = -2 + \log x - \frac{\log^2 x}{2} + x \frac{13 - 6 \log x}{18} + \]
\[ + x^2 \frac{26 - 15 \log x}{300} + O(x^3) \]  
\[ j^{[1,0,0]}(M, M, m) = \frac{1}{M^2} \left[ 1 + x \left( \frac{7}{18} - \frac{\log x}{3} \right) + O(x^2) \right] \]  
\[ j^{[1,0,0]}(M, m, m) = \frac{1}{M^2} \left[ -2x \log x + O(x^2) \right] \]  
\[ j^{[1,0,0]}(m, M, m) = \frac{1}{M^2} \left[ \frac{\log x}{x} - 2 \log x - \frac{2}{3} \pi^2 + 12 \log x + 3 \log^2 x + O(x^2) \right] \]  
\[ j^{[1,0,0]}(m, M, M) = \frac{1}{M^2} \left[ \frac{1 - \log x}{x} + \frac{7 - 6 \log x}{18} + \right. \]
\[ + x \frac{37 - 30 \log x}{300} + O(x^2) \]  
\[ j^{[0,1,0]}(M, M, m) = \frac{1}{M^2} \left[ -1 + x \frac{2 - 3 \log x}{18} + O(x^2) \right] \]  
\[ j^{[0,1,0]}(M, m, M) = \frac{1}{M^2} \left[ -1 + \log x \right. \]
\[ + x \frac{-7 + 6 \log x}{36} + O(x^2) \]  
\[ j^{[0,1,0]}(m, m, M) = \frac{1}{M^2} \left[ \log x + x \left( \frac{\pi^2}{3} + 4 \log x + \right. \]
\[ + \log^2 x \right) + O(x^2) \]  
\[ j^{[0,1,0]}(m, m, M) = \frac{1}{M^2} \left[ -\frac{\pi^2}{3} - 2 \log x - \log^2 x + x \left( 4 - 4 \frac{\pi^2}{3} - \right. \]
\[ - 12 \log x - 4 \log^2 x \right) + O(x^2) \]  
\[ j^{[0,1,0]}(m, M, M) = \frac{1}{M^2} \left[ \log x + x \left( \frac{\pi^2}{3} + 7 \log x + \right. \]
\[ + \log^2 x \right) + O(x^2) \]  
\[ j^{[0,1,0]}(m, M, M) = \frac{1}{M^2} \left[ -1 + \log x \right. \]
\[ + x \frac{-7 + 6 \log x}{36} + O(x^2) \]  

Here, \( x = \frac{m^2}{M^2} \), and the following notations were introduced:

\[ j(m_1, m_2, m_3) = g(m_1, m_2, m_3; 0) - 1 \]
\[
    j^{[\alpha,\beta,\gamma]}(m_1, m_2, m_3) = \frac{\partial^\alpha}{\partial m_1^{2\alpha}} \frac{\partial^\beta}{\partial m_2^{2\beta}} \frac{\partial^\gamma}{\partial m_3^{2\gamma}} j(m_1, m_2, m_3),
\]
and similar for the derivatives of \(f\).

One also needs the following relations:

\[
    j^{[0,1,0]}(M, M, M) = \frac{4\sqrt{3}}{3} M Cl(\frac{\pi}{3}) - 2 j^{[1,0,0]}(M, M, M) \tag{40}
\]

\[
    j(M, M, M) = -\frac{2\sqrt{3}}{3} Cl(\frac{\pi}{3}) \tag{41}
\]

\[
    f(M, M, M; -M^2) = -\frac{3}{2} \tag{42}
\]

\[
    f(M, m, m; -M^2) = -\frac{1}{2} + O(x) \tag{43}
\]

\[
    f^{[0,0,1]}(M, m, m; -M^2) = -\frac{1}{2} \frac{1}{M^2} + O(x) \tag{44}
\]

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Figure captions

*Fig.1* The topologies of the one–loop selfenergy diagrams.

*Fig.2* The topologies of the two–loop selfenergy diagrams.

*Fig.3* The topologies of the one–loop proper vertex diagrams.

*Fig.4* The topologies of the two–loop proper vertex diagrams.

*Fig.5* Triangular diagrams which display endpoint singularities. The solid line denotes the Higgs, and the dashed one the Goldstone bosons.

*Fig.6* The radiative corrections to the partial decay width of the Higgs to vector bosons in the one–loop (solid line) and the two–loop (dashed line) approximations as a function of the mass of the Higgs boson.