Cogging torque reduction based on segmented skewing magnetic poles with different combinations of pole-arc coefficients in surface-mounted permanent magnet synchronous motors

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Abstract
The calculation and reduction of cogging torque are fundamental to the evaluation and optimisation of motor performance. Considering that the accuracy of the cogging torque calculated by analytical method (AM) is low, an accurate calculation model of the effective air-gap length considering the actual stator slot structure and suitable for any motor is proposed, the cogging torque of 6p36s surface-mounted permanent magnet synchronous motor (SPMSM) are calculated accurately and verified by finite element method (FEM). Furthermore, the segmented skewing magnetic poles with different combinations of pole-arc coefficients are designed to weaken cogging torque, and the optimal combination of pole-arc coefficients and tilting angle of the adjacent magnetic poles are summarised by AM and particle swarm optimisation (PSO) algorithm. Finally, the cogging torques of six SPMSMs with different slot-pole combinations including 6p36s, 4p36s, 6p9s, 8p9s, 8p12s and 10p12s are analysed, and the results indicate that the cogging torque can be greatly weakened based on segmented skewing magnetic poles with different combinations of pole-arc coefficients, and other performances of the motors are basically not affected.

1 | INTRODUCTION

Due to the high efficiency and power density, permanent magnet synchronous motors (PMSMs) are widely used in electric vehicles, aerospace and other fields. The cogging torque will cause torque ripple, vibration and noise, which directly affects the performance of the motor. Therefore, the accurate calculation and weakening of cogging torque are especially critical in the design stage of the motors.

There are two common calculation methods for cogging torque, scilicet the finite element method (FEM) and the analytical method (AM). The effects of magnetic leakage, saturation and complex slot structure on cogging torque can be considered by FEM, and the calculation result is accurate. But the computing time is long, especially when studying the influence of structural parameters on cogging torque. In References [1,2], a novel and efficient FEM is proposed to calculate the cogging torque of surface-mounted permanent magnet synchronous motors (SPMSM), the computing time to solve the cogging torque is significantly reduced. In References [3,4], two novel asymmetric pole coaxial magnetic gears (CMGs) are proposed; the permanent magnets on the inner rotor of the CMG adopt Halbach arrays with unequal magnetic poles and eccentric magnet structure, and the air gap flux density and output torque are improved, which is verified by the FEM. However, the change laws of cogging torque with structural parameters are difficult to be summarised by FEM, which is not conducive to the weakening of cogging torque.

The AM is widely used in the calculation of cogging torque. In Reference [5], an AM is developed for predicting the magnetic field distribution and the cogging torque of linear rotary permanent magnet actuator (LRPMA); the magnetic field curvature factor is introduced, but the influence of stator slotting is neglected. Afterwards, the subdomain method is...
presented to calculate cogging torque with different pole-arc coefficients [6], magnet imperfections [7] and rotor eccentricity [7,8] of SPMSMs. Obviously, the air-gap permeance affected by the actual stator slot structure is difficult to be accurately calculated. Overcoming the shortcomings of the AM and the FEM in the calculation of the air-gap magnetic field, a fast and accurate calculation method is proposed, and the electromagnetic force waves with auxiliary slots are analysed [9]. However, the accurate calculation method of air-gap permeance in Reference [9] is only applicable to SPMSMs. In this paper, an accurate calculation model of the air-gap permeance (effective air-gap length) suitable for any motor is proposed.

Lots of weakening methods of cogging torque in SPMSMs have been studied. From the perspective of optimising the stator structure parameters, the reduction methods of cogging torque based on the eccentric structures of stator teeth [10], unequal teeth widths [11], shifting elementary-cogging unit [12], shifting the slot-openings [13] and inserting auxiliary slots [14] are presented, and other performances of the motors are basically not affected. Optimising the structure parameters of PMs is another effective technique to reduce cogging torque in SPMSMs, such as using different magnet widths [15], different pole-arc coefficients [16], eccentric magnetic poles shapes [17] and sinusoidal magnetic pole shapes [18] in the axial direction. For the surface-mounted permanent magnet synchronous motor (SPMSSM) with rotor step skewing structure, the cogging torque calculated by synthesised 2-D FEA is slightly larger than that calculated by 3-D FEA but still has high accuracy [19]. For the flux-switching permanent magnet (FSPM) with doubly salient structure, this chamfering method is very effective for weakening cogging torque [20–22]. Adopting the segmented skewing rotor structure with different combinations of pole-arc coefficients is very effective in reducing the cogging torque of SPMSMs, but relevant research is few. When electrical machine is running with rated load or overload, the local saturation and cross-magnetisation will inevitably occur in the electrical machine, and the operating point of the stator and rotor silicon steel sheets will change, which makes the cogging torque of the motor slightly different from that under no-load conditions [23,24]. In References [25–27], the cogging torques under load are analysed by frozen permeability method. The results show that the cogging torques under load are slightly larger than that under no-load. In recent years, some optimisation algorithms have been applied to the optimisation of cogging torque and other motor performances [28–30]. However, blindly adopting optimisation algorithms and unreasonable range of optimisation parameters often make the optimisation results complicated, and it is difficult to get universal conclusions that are suitable for different slot-pole combinations motors.

In this paper, an accurate calculation model of effective air-gap length is proposed, and the air-gap flux density and cogging torque are calculated based on the proposed accurate calculation model and verified by FEM. Furthermore, the weakening measure of cogging torque based on segmented skewing magnetic poles with different combinations of pole-arc coefficients is analysed, and the optimal combination of pole-arc coefficients and tilting angle of the adjacent magnetic poles in SPMSMs with different slot-pole combinations are summarised.

2 | CALCULATION AND ANALYSIS OF COGGING TORQUE

2.1 | Accurate calculation method of cogging torque

The relative position of stator and rotor of the SPMSM is shown in Figure 1. The position of \( \theta = 0^\circ \) is set at the midline of a specified PM. The angle between the centre line of a stator tooth and the centre line of a specified PM is defined as \( \alpha \). Ignoring the magnetic potential drop on the stator core, the distribution of the air-gap flux density along the circumference can be expressed as

\[
B_{ag}^\alpha(\theta, \alpha) = \mu_0 \frac{F(\theta)}{b_c + \delta(\theta, \alpha)}
\]  

(1)

where \( F(\theta), b_c \) and \( \delta(\theta, \alpha) \) are the distribution of the air-gap magnetomotive force (MMF), PM thickness and the effective air-gap length along the circumference, respectively. The magnetic co-energy stored in the air gap can be described as

\[
W_{ag} = \frac{1}{2\mu_0} \left[ B_{ag}^\alpha(\theta, \alpha) dV \right] = \frac{\mu_0}{2} \left[ F^2(\theta) \left( \frac{1}{b_c + \delta(\theta, \alpha)} \right)^2 dV \right]
\]  

(2)

Therefore, the cogging torque can be calculated by

\[
T_{cog} = -\frac{\partial W_{ag}}{\partial \alpha}
\]  

(3)

For the common tile-shaped PMs in the SPMSMs with radial magnetisation, the air-gap MMF generated by the PMs along the circumference is shown in Figure 2. The Fourier expansion of \( F^2(\theta) \) on the interval \([\pi/2p, \pi/2p]\) can be expressed as

![FIGURE 1 Analysis model of the surface-mounted permanent magnet synchronous motor (SPMSSM)](image-url)
The Fourier expansion of \( \left[ \frac{1}{\mu_0 + \delta(\theta, \alpha)} \right]^2 \) on the interval \([-\pi/\pi, \pi/\pi, \pi/\pi]\) can be expressed as

\[
\left[ \frac{1}{h + \delta(\theta, \alpha)} \right]^2 = G_0 + \sum_{k=1}^{\infty} G_k \cos k(\theta + \alpha) \tag{7}
\]

where \(G_0\) and \(G_k\) are the Fourier coefficients, \(z\) is the number of stator slots.

Substituting Equations (7) and (4) into Equation (2), the expression of the cogging torque can be simplified as

\[
T_{\text{cog}} = \frac{\mu_0 \pi L_s}{4} \left( R_2^2 - R_1^2 \right) \sum_{k=1}^{\infty} kG_k F_n \sin k\alpha \tag{8}
\]

where \(L_s\) is the axial length of the armature, \(R_1\) and \(R_2\) are the outer radius of the rotor core and the inner radius of the stator, \(k, n\) are positive integers and satisfy \(n = kz/2p\), the minimum value of \(k\) is \(2p/\text{GCD}(2p, z)\).

### 2.2 Verification of calculation method

The main parameters and finite element model of the 6-pole/36-slot (6p36s) SPMSM are shown in Table 1 and Figure 4, respectively. The structure parameters of the stator slot are shown in Figure 5. The effective air-gap length within a tooth pitch of the 6p36s SPMSM can be accurately calculated by the proposed accurate calculation model, as shown in Figure 6.

Substituting the calculation result of the effective air-gap length and the relevant parameters of PM into Equation (1), the air-gap flux density of the motor can be obtained. The prototype test platform of the 6p36s SPMSM is established, as shown in Figure 7. A thin wire is attached to the surface of any one tooth, and the induced electromotive force \(E(\alpha)\) in the wire can be easily measured. The \(E(\alpha)\) in the wire is shown in Figure 8 when the prototype is dragged by the prime mover to run stably at 1000 r/min. The air-gap flux density that changes with time at the centre line of the stator tooth can be obtained by the following equation [9].

\[
B(\alpha) = \frac{E(\alpha)}{\varepsilon L_\ell a}\tag{9}
\]

### Table 1 The main structure parameters of the 6p36s surface-mounted permanent magnet synchronous motor (SPMSM)

| Parameters | Value | Parameters | Value |
|------------|-------|------------|-------|
| Rated power/kW | 7.5 | Rated speed/(r/min) | 750 |
| Length of armature/mm | 98 | Number of poles | 6 |
| Outer diameter of stator/mm | 260 | Number of slots | 36 |
| Inner diameter of stator/mm | 180 | Magnet thickness/mm | 3.5 |
| Outer diameter of rotor/mm | 178 | Pole-arc coefficient | 0.833 |
structure, the detailed components of the effective air-gap length and the air-gap permeance can be obtained accurately by the proposed accurate calculation model. In addition, the calculation model of the effective air-gap length only needs to use the magnetostatic solver of Maxwell, which greatly reduces the calculation time.

Further, the Fourier coefficients $G_0$ and $G_k$ in Equation (7) can be obtained, as shown in Table 2. Substituting the Fourier coefficients $G_k$ and corresponding $F_n$ into Equation (8), the cogging torque within a tooth pitch can be calculated, as shown in Figure 10. The cogging torques obtained by the accurate calculation method and the FEM are compared in Figure 11. It can be seen that the calculation result of cogging torque has high accuracy.

According to Maxwell’s tensor method, the radial electromagnetic force density with no load of the SPMSM can be calculated by the following equation [9]

$$p_r = \frac{B_{mg}^2(\theta, t)}{2\mu_0} = \frac{\mu_0}{2} \left[ \frac{F(\theta, t)}{B_c + \delta(\theta, \alpha)} \right]^2$$

(10)

The main low-order radial electromagnetic force density of the 6p36s SPMSM with no load is compared in Figure 12. Ignoring the influence of the high-order harmonic components of the MMF and air-gap permeance, the low-order radial electromagnetic force density calculated by the proposed accurate calculation model is smaller, but still has high accuracy.

3 | REDUCTION OF COGGING TORQUE BASED ON SEGMENTED MAGNETIC POLES

3.1 | Analysis of cogging torques under segmented magnetic poles with different combinations of pole-arc coefficients

The rotor structures with entire magnetic poles and segmented magnetic poles are shown in Figure 13. Compared with the conventional rotor structures with entire magnetic poles, segmented magnetic poles with different pole-arc coefficients $\alpha_{p1}$ and $\alpha_{p2}$ are staggered in the axial direction, and the arrangement of the magnetic poles of N-pole and S-pole is
opposite. Selecting appropriate combination of pole-arc coefficients can effectively weaken the $F_n$, which has an effect on cogging torque, so as to reduce the cogging torque and improve the performance of the motor.

The Fourier expansion of $F_2(\theta)$ generated by the first-segment magnetic pole is

$$F_2(\theta) = F_{s0} + \sum_{n=1}^{\infty} F_{sn} \cos n\theta$$  \hspace{1cm} (11)

where

$$F_{s0} = \frac{F^2}{2} (\alpha_{p1} + \alpha_{p2})$$

$$F_{sn} = \frac{2F^2}{n\pi} \left[ \sin \frac{n\pi\alpha_{p1}}{2} + (-1)^n \sin \frac{n\pi\alpha_{p2}}{2} \right]$$  \hspace{1cm} (12)

Therefore, the cogging torque generated by the first-segment PM is

$$T_{\text{cog1}} = \frac{\mu_0\pi L_i}{4b} (R^2 - R_i^2) \sum_{k=1}^{\infty} kG_kF_{sn} \sin k\alpha$$

$$= C \sum_{k=1}^{\infty} \frac{2kF^2}{b} \frac{n\pi}{n\pi} \left[ \sin \frac{n\pi\alpha_{p1}}{2} + (-1)^n \sin \frac{n\pi\alpha_{p2}}{2} \right] \sin k\alpha$$  \hspace{1cm} (13)

where $k, n$ are positive integers and satisfy $n = kz/p$, $b$ is the number of axial segments of the magnetic pole, and $C = \frac{\mu_0\pi L_i}{4b} (R^2 - R_i^2)$. When $b$ is an even number, the total cogging torque of $b$ magnetic poles can be expressed as

$$T_{\text{cog}} = \begin{cases} 0 & n \text{ is an odd number} \\
\frac{4kF^2}{n\pi} \frac{n\pi}{4} \frac{n\pi}{4} (\alpha_{p1} + \alpha_{p2}) & n \text{ is an even number} \\
C \sum_{k=1}^{\infty} \frac{n\pi}{4} \left( \alpha_{p1} - \alpha_{p2} \right) \sin k\alpha \end{cases}$$  \hspace{1cm} (14)

It can be seen from Equation (14) that only the even components of $F_{sn}$ have effect on the cogging torque when the number of axial segments $b$ is an even number. The lower the order of $F_{sn}$, the greater the effect. Therefore, the cogging torque can be weakened by weakening the lower-order even components of $F_{sn}$. Ensure that the magnetic flux of each magnetic pole is basically unchanged to make the effect of changing the magnetic pole structure on other electromagnetic performance small; $\alpha_{p1}$ and $\alpha_{p2}$ satisfy $\alpha_{p1} + \alpha_{p2} = 2\alpha_p$, where $\alpha_p$ is the pole-arc coefficient of the entire magnetic pole before optimisation. Consequently, when $b$ is an even number, the combinations of $\alpha_{p1}$ and $\alpha_{p2}$ should satisfy

**TABLE 2** The Fourier coefficients $G_0$ and $G_k$

| Parameters | Value       |
|------------|-------------|
| $G_0$      | $4.350 \times 10^4$ |
| $G_1$      | $7.185 \times 10^3$ |
| $G_2$      | $-6.657 \times 10^3$ |
| $G_3$      | $7.291 \times 10^3$ |
| $G_4$      | $-4.013 \times 10^3$ |
| $G_5$      | $2.141 \times 10^3$ |
| $G_6$      | $1.155 \times 10^3$ |
| $G_7$      | $-1.231 \times 10^3$ |
| $G_8$      | $1.399 \times 10^3$ |
| $G_9$      | $-1.039 \times 10^3$ |

**FIGURE 9** The components of the air-gap flux density

**FIGURE 10** The calculation result of the cogging torques

**FIGURE 11** The Comparison of the cogging torques obtained by the accurate calculation method and the finite element method (FEM)

![Graphs and tables from the document]
the magnetic flux of $N$-pole and $S$-pole cannot be kept the same when the number of axial segments is an odd number. Consequently, the number of axial segments should be an even number, and the combinations of pole-arc coefficients should be determined according to Equation (15).

The combination of pole-arc coefficients in six SPMSMs with different slot-pole combinations including 6p36s, 4p36s, 6p9s, 8p9s, 8p12s and 10p12s are determined, as shown in Table 3. The cogging torques of 6p36s SPMSM with the combination of pole-arc coefficients of $(0.883, 0.717)$ calculated by superimposed 2-D FEM and 3-D FEM are compared in Figure 14. The amplitude of cogging torque calculated by superimposed 2-D FEM is slightly larger, but still has high accuracy. Therefore, the superimposed 2-D FEM is used to analyse the cogging torque in the subsequent analysis to shorten the calculation time. When the number of segments of magnetic poles is four, the cogging torques with different combinations of pole-arc coefficients are compared by superimposed 2-D FEM in Figure 15. For the 6p36s SPMSM, the components of $F_{c12}$, $F_{c24}$, and $F_{c36}$ are weakened by adopting the combinations of pole-arc coefficients of $(0.883, 0.717)$, $(0.842, 0.758)$ and $(0.828, 0.772)$, respectively. It can be seen that the combinations of pole-arc coefficients determined according to the above analysis can effectively weaken the cogging torque. The lower the order of weakened $F_m$, the better the cogging torque weakening result. Similarly, for the 4p36s, 6p9s, 8p9s, 8p12s and 10p12s PMSM, the optimal combinations of pole-arc coefficients are $(0.856, 0.744)$, $(0.967, 0.633)$, $(0.856, 0.744)$, $(0.883, 0.717)$ and $(0.967, 0.633)$, respectively. Therefore, the determination of the optimal combination of pole-arc coefficients should be based on the principle of weakening the lowest order component of $F_m$ that has the greatest impact on cogging torque.

3.2 Analysis of cogging torque under segmented skewing magnetic poles

The rotor structures with entire magnetic poles and segmented skewing magnetic poles are shown in Figure 16. For the rotor structure with segmented skewing magnetic poles, the MMFs of the PMs have phase difference along the axial direction. For the segmented skewing magnetic poles with axial segment

\[
\begin{align*}
|a_{p1} + a_{p2} = 2a_p \\
\cos \frac{n\pi}{4}(a_{p1} - a_{p2}) = 0 \text{ } n \text{ is an even number} \\
\end{align*}
\]

when $b$ is an odd number, the total cogging torque can be expressed as

\[
T_{cog} = C \frac{4kF^2}{\pi} \sum_{k=1}^{n} G_k \sin \left( \frac{n\pi}{4} \right) (a_{p1} - a_{p2}) \cos \left( \frac{n\pi}{4} \right) (a_{p1} + a_{p2}) \sin kza \quad n \text{ is an odd number}
\]

\[
C \frac{4kF^2}{\pi} \sum_{k=1}^{n} G_k \sin \left( \frac{n\pi}{4} \right) (a_{p1} + a_{p2}) \cos \left( \frac{n\pi}{4} \right) (a_{p1} - a_{p2}) \sin kza \quad n \text{ is an even number}
\]

\[
(15)
\]

It can be seen that the cogging torque produced by the even components of $F_{m}$ is consistent with the results when the number of axial segments is an even number. The amplitudes of the cogging torque generated by the odd components of $F_{m}$ are relatively small. The more the number of segments, the smaller the influence of odd components of $F_{m}$. In addition,

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{Parameters} & (a_{p1}, a_{p2}) & 6p36s & 4p36s & 6p9s & 8p9s & 8p12s & 10p12s \\
\hline
(\alpha_p = 0.8) & (0.883, 0.717), (0.842, 0.758), (0.828, 0.772) & (0.856, 0.744), (0.828, 0.772), (0.819, 0.781) & (0.967, 0.633), (0.883, 0.717), (0.856, 0.744) & (0.856, 0.744), (0.828, 0.772), (0.819, 0.781) & (0.883, 0.717), (0.842, 0.758), (0.828, 0.772) & (0.967, 0.633), (0.883, 0.717), (0.856, 0.744) \\
\hline
\end{array}
\]

Abbreviation: SPMSMs, surface-mounted permanent magnet synchronous motors.
number $b$ and tilting angle $\gamma$ between two adjacent magnetic poles, the cogging torque generated by $i$-th magnetic pole is

$$ T_{cog} = \frac{\mu_0 \pi z L_a}{4b} \left( R_2^2 - R_1^2 \right) \sum_{k=1}^{\infty} k F_n \sin k z (\alpha + (i-1)\gamma) $$

(17)

The total cogging torque can be expressed as

$$ T_{cog} = \begin{cases} 
C \sum_{k=1}^{\infty} k G_n F_n \sin \frac{k z y}{2} \sin k z (\alpha + (b-1)\gamma) \\
C \sum_{k=1}^{\infty} k G_n F_n \sin k z a 
\end{cases} $$

(18)

$$ k z y \neq m 360^\circ (m = 1, 2, 3...) $$

$$ k z y = m 360^\circ (m = 1, 2, 3...) $$

where $k$, $n$ are positive integers and satisfy $n = k z / 2p$. In theory, selecting appropriate $b$ and $\gamma$ to make $\sin k z y = 0$ on the premise of $\sin k z y \neq 0$ can eliminate the cogging torque. Generally, the tilting angle $(b-1)\gamma$ does not exceed one pitch.

**FIGURE 14** The cogging torques of 6p36s surface-mounted permanent magnet synchronous motor (SPMSM) with the combination of pole-arc coefficients of $(0.883, 0.717)$ calculated by superimposed 2-D FEM and 3-D FEM

**FIGURE 15** The comparisons of the cogging torques of different slot-pole combinations surface-mounted permanent magnet synchronous motors (SPMSMs) with different combinations of pole-arc coefficients

**FIGURE 16** The rotor structures with (a) entire magnetic poles and (b) segmented skewing magnetic poles.
If the minimum value of $k$ satisfies $\sin \frac{k\pi}{z} = 0$, there is always $\sin \frac{bk\gamma}{2} = 0$ for any value of $\langle k \rangle$. Therefore, the determinations of $b$ and $\gamma$ should satisfy

$$b\gamma = \frac{360^\circ}{k_{\text{min}}z} \quad (k_{\text{min}}z \neq 360^\circ) \quad (19)$$

The cogging torques of six SPMSMs with different axial segment number are compared by superimposed 2-D FEM in Figure 17. It can be seen that the segmented skewing magnetic poles with axial segment number $b$ and tilting angle $\gamma$ determined by Equation (19) can effectively weaken the cogging torque. The greater the segment number, the better the reduction effect of the cogging torque. The change of cogging torque amplitudes of 6p36s SPMSM with the axial segment number is shown in Figure 18. When the segment number of magnetic poles exceeds four, the reduction of the amplitudes of the cogging torque is very small with the increase of the segment number, thus it is not

$$T_{\text{cog1}} = \frac{C}{4} \sum_{k=1}^{\infty} \frac{2kF^2}{n\pi} G_k \left[ \sin \frac{n\pi\alpha_{p1}}{2} + (-1)^n \sin \frac{n\pi(2\alpha_p - \alpha_{p1})}{2} \right] \sin k\alpha$$

$$T_{\text{cog2}} = \frac{C}{4} \sum_{k=1}^{\infty} \frac{2kF^2}{n\pi} G_k \left[ \sin \frac{n\pi(2\alpha_p - \alpha_{p1})}{2} + (-1)^n \sin \frac{n\pi\alpha_{p1}}{2} \right] \sin k(\alpha + \gamma)$$

$$T_{\text{cog3}} = \frac{C}{4} \sum_{k=1}^{\infty} \frac{2kF^2}{n\pi} G_k \left[ \sin \frac{n\pi\alpha_{p1}}{2} + (-1)^n \sin \frac{n\pi(2\alpha_p - \alpha_{p1})}{2} \right] \sin k(\alpha + 2\gamma)$$

$$T_{\text{cog4}} = \frac{C}{4} \sum_{k=1}^{\infty} \frac{2kF^2}{n\pi} G_k \left[ \sin \frac{n\pi(2\alpha_p - \alpha_{p1})}{2} + (-1)^n \sin \frac{n\pi\alpha_{p1}}{2} \right] \sin k(\alpha + 3\gamma)$$

FIGURE 17 The cogging torques of six surface-mounted permanent magnet synchronous motors (SPMSMs) with different axial segment number.
necessar y to further increase the segment number of magnetic poles, so as not to make the rotor structure more complicated.

### 3.3 Analysis of cogging torque under segmented skewing magnetic poles with different combinations of pole-arc coefficients

Combining the structures of segmented skewing magnetic poles and segmented magnetic poles with different combination of pole-arc coefficients, the segmented skewing magnetic pole structure with different combinations of pole-arc coefficients is designed in Figure 19. Considering the previous analysis results comprehensively, the magnetic pole with four segments is a reasonable design.

The cogging torques generated by the four magnetic poles are

The total cogging torque can be expressed as

\[
T_{\text{cog}}(\alpha) = T_{\text{cog1}} + T_{\text{cog2}} + T_{\text{cog3}} + T_{\text{cog4}}
\]  

In this section, the particle swarm optimisation (PSO) algorithm is applied to search the optimal combination of pole-arc coefficients and tilting angle of magnetic poles. The key steps are as follows:

I. Initialisation

Randomly initialise each particle in entire population.

II. Calculation of fitness values

Bring each particle after initialisation into the cogging torque expression to get the fitness value of each particle.

III. Optimal solution update

Update the individual optimal solution and the global optimal solution in the particle swarm.

IV. Speed and position update

Each particle needs to continuously update its position according to its own optimal position, global optimal position and current speed, which can be expressed as

\[
v_{k+1} = \omega v_k + c_1 r_1 (P_{\text{best}} - x_k) + c_2 r_2 (G_{\text{best}} - x_k)
\]

\[
x_{k+1} = x_k + v_{k+1}
\]

where \(x_k\) and \(v_k\) respectively represent the position and speed of the current particle, \(P_{\text{best}}\) represents the optimal position of a single particle, \(G_{\text{best}}\) represents the global optimal position of the entire particle group, \(c_1\) and \(c_2\) are the acceleration

---

**TABLE 4** The pole coefficient \(\alpha_p\) and tilting angle \(\gamma\) of six SPMSMs with different slot-pole combinations

| Parameters | \(\alpha_p\) | PSO algorithm (15) | PSO algorithm (19) |
|------------|-------------|--------------------|--------------------|
| 6p36s (\(\alpha_p = 0.8\)) | 0.7171 | 0.717 | 2.4763 | 2.5 |
| 4p36s (\(\alpha_p = 0.8\)) | 0.7422 | 0.744 | 2.4581 | 2.5 |
| 6p9s (\(\alpha_p = 0.8\)) | 0.6332 | 0.633 | 9.9225 | 10 |
| 8p9s (\(\alpha_p = 0.8\)) | 0.7438 | 0.744 | 2.4532 | 2.5 |
| 10p12s (\(\alpha_p = 0.8\)) | 0.7152 | 0.717 | 1.4654 | 1.5 |
| 8p12s (\(\alpha_p = 0.8\)) | 0.6314 | 0.633 | 7.4655 | 7.5 |

Abbreviation: SPMSMs, surface-mounted permanent magnet synchronous motors.
constants, \( r_1 \) and \( r_2 \) are random numbers between 0 and 1, \( w \) is the inertia coefficient and can be expressed as

\[
  w = \frac{(C_L - C_C)}{C_L} (w_{\text{max}} - w_{\text{min}}) + w_{\text{min}}
\]

(24)

where \( w_{\text{max}} \) and \( w_{\text{min}} \) represent the maximum and minimum values of \( w \), \( C_C \) represents the current iteration steps, and \( C_L \) represents the maximum iteration steps.

V. Termination condition

If the termination condition is met, the calculation is ended and the optimisation result is output. Otherwise, the algorithm skips to step III and continues to update the optimal values for new iterative calculations.

The pseudocode of PSO algorithm is shown in Algorithm 1. The objective function and associated constraints are

\[
  \min T_a(\alpha_p, \gamma) = \min \left\{ \max T_{\text{cog}}(\alpha) \right\}
\]

\[
  \begin{cases}
    0.6 \leq \alpha_p \leq 0.8 \\
    0 \leq \gamma \leq \frac{360}{z}
  \end{cases}
\]

(25)

The parameters in the PSO algorithm are as follows: \( c_1 = c_2 = 2 \), \( w_{\text{max}} = 0.9 \), \( w_{\text{min}} = 0.4 \). The pole-arc coefficient \( \alpha_p \) and tilting angle \( \gamma \) of six SPMSMs with different slot-pole combinations are calculated in Table 4. It can be seen that the optimal combination of pole-arc coefficients obtained by the PSO algorithm are basically consistent with that

Algorithm 1 The pseudocode of particle swarm optimisation (PSO) algorithm

// Description: Find the minimum values of the objective function
// Parameters: \( N \) is the group size

procedure PSO
  for each particle \( k \n\)
  Initialize speed \( v_k \) and position \( x_k \) for particle \( k \n\)
  Evaluate particle \( k \) and set \( P_{\text{best}} = x_k \n\)
  end for

  \( G_{\text{best}} = \min \{ P_{\text{best}} \} \)

  while not stop
    for \( k = 1 \) to \( N \n\)
      Update the speed and position of particle \( k \n\)
      Evaluate particle \( k \n\)
      if \( \text{fit}(x_k) < \text{fit}(P_{\text{best}}) \n\)
        \( P_{\text{best}} = x_k \)
      end if
      if \( \text{fit}(P_{\text{best}}) < \text{fit}(G_{\text{best}}) \n\)
        \( G_{\text{best}} = P_{\text{best}} \)
      end if
    end for
  end while

  Print \( G_{\text{best}} \)
end procedure
determined by Equation (15) with a small error, and the determined tilt angles also basically satisfy \( \sin \frac{2\pi}{k} = 0 \). Consequently, the optimal combination of pole-arc coefficients and tilting angle of the adjacent magnetic poles can be determined by

\[
\begin{align*}
\alpha_{p1} + \alpha_{p2} &= 2\alpha_p \\
\cos \frac{n\pi}{4} (\alpha_{p1} - \alpha_{p2}) &= 0 \quad n \text{ is an even number} \\
\gamma &= \frac{m360^\circ}{k_{\min}z_f} \quad (m \text{ is an integer from } 1 \text{ to } k_{\min}, \\
&\quad k_{\min}z_f \neq m360^\circ)
\end{align*}
\]

Further, the cogging torques of six SPMSMs with different slot-pole combinations under different magnetic poles structures are compared in Figure 20. It can be seen that all three magnetic pole structures can weaken the cogging torques to varying degrees, and the weakening effects of cogging torques with segmented skewing magnetic poles are better than that under segmented magnetic poles with different combinations of pole-arc coefficients. Combining the two magnetic pole structures, the cogging torques can be greatly reduced.

The no-load back electromotive forces (EMFs) and their fundamental amplitudes of six SPMSMs with different slot-pole combinations under different magnetic poles structures are analysed by FEM, as shown in Figure 21. It can be seen that the fundamental amplitudes of back EMFs are slightly reduced when the segmented skewing magnetic poles with different combination of pole-arc coefficients are adopted, but the normal operation of the motors can still be guaranteed.

The output torques and torque ripples of six SPMSMs running at rated speed are compared in Figure 22 and
Table 5, respectively. It can be seen that the average values of output torques are basically unchanged, and the torque ripples are greatly reduced due to the weakening of the cogging torques. The efficiencies $\eta$ of six SPMSMs with different magnetic poles structures are shown in Table 6. $P_1$ and $P_2$ are the input power and output power of the motor, respectively. The efficiencies of six SPMSMs under the two magnetic poles structures basically remain unchanged. Therefore, the cogging torques can be effectively reduced by adopting the segmented skewing magnetic poles with different combinations of pole-arc coefficients under the precursor that ensures the normal operation of the motors.

4 | CONCLUSION

Considering that the effects of the complex slot structure, an accurate calculation model of the effective air-gap length suitable for any motor is proposed, the air-gap flux density and cogging torque of 6p36s SPMSM is calculated and verified by FEM. The results indicate that the effective air-gap length and the resulting cogging torque of the motor can be quickly and accurately calculated by the proposed accurate calculation model.

Furthermore, the cogging torques under segmented magnetic poles with different combination of pole-arc coefficients, segmented skewing magnetic poles and composite
magnetic poles structure are derived; six different slot-pole combinations SPMSMs including 6p36s, 4p36s, 6p9s, 8p9s, 8p12s and 10p12s are analysed by superimposed 2-D FEM; and the amplitude of cogging torque calculated by superimposed 2-D FEM is slightly larger, but still has high accuracy. For different slot-pole combinations SPMSMs, the optimal combination of pole-arc coefficients and tilting angle of adjacent magnetic poles obtained by the PSO algorithm are basically consistent with that determined by Reference (26), which effectively verify the universality of the conclusion. Therefore, the cogging torques can be effectively reduced by adopting the segmented skewing magnetic poles with different combinations of pole-arc coefficients, and other performances of the motors are basically not affected. Normally, the axial segment number of magnetic poles of four is a reasonable choice.

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**REFERENCES**

1. Ho, S.L., Chen, N., Fu, W.N.: An optimal design method for the minimisation of cogging torques of a permanent magnet motor using FEM and genetic algorithm. IEEE Trans. Appl. Super. 20(3), 861–864 (2010)

2. Chen, N., Ho, S.L., Fu, W.N.: Optimization of permanent magnet surface shapes of electric motors for minimisation of cogging torque using FEM. IEEE Trans. Mag. 46(6), 2478–2481 (2010)

3. Jing, L.B., Huang, Z.X., Chen, J.L., Qu, R.H.: An asymmetric Pole coaxial magnetic gear with unequal Halbach arrays and spoke structure. IEEE Trans. Appl. Super. 30(4), 1–5 (2020)

4. Jing, L.B., et al.: Design, analysis and realization of a hybrid-excited magnetic gear during overload. IEEE Trans. Ind. Appl., 1–8 (2020)

5. Jin, P., et al.: Analytical magnetic field analysis and prediction of cogging force and torque of a linear and rotary permanent magnet actuator. IEEE Trans. Mag. 47(10), 3004–3007 (2011)

6. Shin, K., et al.: Analytical calculation and experimental verification of cogging torque and optimal point in permanent magnet synchronous motors. IEEE Trans. Mag. 53(6), 1–4 (2017)

7. Qian, H., et al.: Analytical solution for cogging torque in surface-mounted permanent-magnet motors with magnet imperfections and rotor eccentricity. IEEE Trans. Mag. 50(8), 1–15 (2014)

8. Li, Y., et al.: Superposition method for cogging torque prediction in permanent magnet machines with rotor eccentricity. IEEE Trans. Mag. 52(6), 1–10 (2016)

9. Xing, Z.Z, Wang, X.H., Zhao, W.L.: Research on weakening measure of radial electromagnetic force waves in permanent magnet synchronous motors by inserting auxiliary slots. IET Elect. Power Appl. 14(8), 1381–1395 (2020)

10. Wang, K., et al.: Cogging torque reduction by eccentric structure of teeth in external rotor permanent magnet synchronous motors. IET Elect. Power Appl. 13(1), 57–63 (2019)

11. Petrov, I., et al.: Unequal teeth widths for torque ripple reduction in permanent magnet synchronous machines with fractional-slot non-overlapping windings. IEEE Trans. Mag. 51(2), 1–9 (2015)

12. Gao, J., et al.: Cogging torque reduction by elementary-cogging-unit shift for permanent magnet machines. IEEE Trans. Mag. 53(1), 1–5 (2017)

13. Liu, T., et al.: Cogging torque reduction by slot-opening shift for permanent magnet machines. IEEE Trans. Mag. 49(7), 4028–4031 (2013)

14. Xia, C., et al.: Cogging torque modelling and analysing for surface-mounted permanent magnet machines with auxiliary slots. IEEE Trans. Mag. 49(9), 5112–5123 (2013)

15. Wang, D., Wang, X., Jung, S.: Cogging torque minimisation and torque ripple suppression in surface-mounted permanent magnet synchronous machines using different magnet widths. IEEE Trans. Mag. 49(5), 2295–2298 (2013)

16. Feng, J., Shuhua, F., Ho, S.: Distribution characteristic and combined optimization of maximum cogging torque of surface-mounted permanent-magnet machines. IEEE Trans. Mag. 54(3), 1–5 (2018)

17. Pang, Y., Zhu, Z.Q., Feng, Z.J.: Cogging torque in cost-effective surface-mounted permanent-magnet machines. IEEE Trans. Mag. 47(9), 2269–2276 (2011)

18. Shah, S.A., Lipo, T.A., Kwon, B.: Modelling of novel permanent magnet pole shape SPM motor for reducing torque pulsation. IEEE Trans. Mag. 48(11), 4626–4629 (2012)

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**TABLE 6** The efficiencies of six SPMSMs with different slot-pole combinations

| Parameters | Entire structure | Optimised structure |
|------------|------------------|---------------------|
|            | Average values/N·m | Ripples/% | Average values/N·m | Ripples/% |
| 6p36s      | 71.06             | 33.83     | 70.57              | 9.61      |
| 4p36s      | 47.13             | 37.54     | 47.04              | 15.45     |
| 6p9s       | 51.93             | 35.14     | 50.20              | 18.73     |
| 8p9s       | 50.66             | 24.21     | 49.78              | 5.77      |
| 10p12s     | 87.31             | 21.32     | 86.23              | 7.08      |
| 8p12s      | 68.89             | 34.44     | 66.32              | 18.27     |

Abbreviation: SPMSMs, surface-mounted permanent magnet synchronous motors.

**TABLE 5** The torque ripples of six SPMSMs with different slot-pole combinations

| Parameters | Entire structure | Optimised structure |
|------------|------------------|---------------------|
|            | Average values/N·m | Ripples/% | Average values/N·m | Ripples/% |
| 6p36s      | 2.36              | 1.12     | 2.34              | 0.98      |
| 4p36s      | 2.47              | 1.23     | 2.45              | 1.05      |
| 6p9s       | 2.58              | 1.34     | 2.56              | 1.16      |
| 8p9s       | 2.69              | 1.45     | 2.67              | 1.28      |
| 10p12s     | 2.81              | 1.56     | 2.79              | 1.40      |
| 8p12s      | 2.92              | 1.67     | 2.90              | 1.52      |
19. Fei, W., Zhu, Z.Q.: Comparison of cogging torque reduction in permanent magnet brushless machines by conventional and herringbone skewing techniques. IEEE Trans. Energy Conv. 28(3), 664–674 (2013)
20. Zhu, X., Hua, W., Wu, Z.: Cogging torque minimisation in FSPM machines by right-angle-based tooth chamfering technique. IET Elect. Power Appl. 12(5), 627–634 (2018)
21. Xiu, J., Wang, S., Xiu, Y.: Reducing cogging torque of 6/4 pole FSPM machine by optimising parameters of chamfering and flange rotor pole shape without skewing teeth. IET Elect. Power Appl. 13(3), 277–284 (2019)
22. Jin, M.J., Wang, Y., Shen, J.X.: Cogging torque suppression in a permanent magnet flux-switching integrated-starter-generator. IET Elect. Power Appl. 4(8), 647–656 (2010)
23. Abbaszadeh, K., Alam, F.R.: On-load field component separation in surface-mounted permanent magnet motors using an improved conformal mapping method. IEEE Trans. Mag. 52(2), 1–12 (2016)
24. Shin, K.H., Choi, J.Y., Cho, H.W.: Characteristic analysis of interior permanent-magnet synchronous machine with fractional-slot concentrated winding considering nonlinear magnetic saturation. IEEE Trans. Appl. Super. 26(4), 1–4 (2016)
25. Wang, W.Y., et al.: A calculation method for the on-load cogging torque of permanent magnet synchronous machine. IEEE Access. 7, 106316–106326 (2019)
26. Wu, D., Zhu, Z.Q.: Design tradeoff between cogging torque and torque ripple in fractional slot surface-mounted permanent magnet machines. IEEE Trans. Mag. 51(11), 1–4 (2015)
27. Chu, W.Q., Zhu, Z.Q.: Reduction of on-load torque ripples in permanent magnet synchronous machines by improved skewing. IEEE Trans. Mag. 49(7), 3822–3825 (2013)
28. Xue, Z.Q., et al.: Analytical prediction and optimization of cogging torque in surface-mounted permanent magnet machines with modified particle swarm optimization. IEEE Trans. Ind. Elect. 64(12), 9795–9805 (2017)
29. Min, S.G., Bramerforger, G., Sarlioglu, B.: Analytical modelling and optimization for electromagnetic performances of fractional-slot PM brushless machines. IEEE Trans. Ind. Elect. 65(5), 4017–4027 (2018)
30. Ashabani, M., Mohamed, A.R.I.: Multiobjective shape optimization of segmented Pole permanent-magnet synchronous machines with improved torque characteristics. IEEE Trans. Mag. 47(4), 795–804 (2011)

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