A note on black-hole entropy, area spectrum, and evaporation

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Abstract – A model based on the topology change of a quantum manifold is used to explain the origin of the black-hole thermodynamics. We show that this model can explain the origin of the black-hole entropy, and why black-hole obey a generalized second law of thermodynamics. The method we use to do this is to analyze the selection rules for black-hole area transitions in the evaporation process driven by topology change. A discrete spectrum, which become increasingly spaced as the black hole approaches the Planck scale, is obtained for the black-hole area. As a striking consequence, as the lines of the area spectrum become more and more spaced, the black-hole radiation becomes less and less entropic. This result points to the possibility that some information about the black-hole initial quantum state can be recovered as it evaporates.

R. R. Landim has dedicated this paper to the memory of his wife Isabel Mara.

Introduction. – It is not an exaggeration to say that one of the most exciting predictions of general relativity is that there may exist black holes. This is mainly due to the belief that black holes may play a major role in our attempts to shed some light on the quantum nature of the spacetime such as the role played by atoms in the early development of quantum mechanics. Black-hole developments in the last forty years have shown that black holes have thermodynamics properties like entropy and temperature, and as a consequence of the instability of the vacuum in strong gravitational fields, they are sources of quantum radiation [1–3]. String theory and loop quantum gravity, lately, showed that the origin of the black-hole thermodynamics must be related with the quantum structure of the spacetime, bringing together the developments in black-hole physics and the improvement of our understanding on the nature of the spacetime in quantum gravity regime [4,5].

However, the understanding of black-hole thermodynamics in the semiclassical and, furthermore, in quantum regime has been a very difficult, and it is still an unsolved problem. To explain the situation, we know that, in statistical physics, entropy counts the number of accessible microstates that a system can occupy, where all states are presumed to occur with equal probability. Black holes, on the other hand, can be completely characterized by only three externally observable classical parameters: mass, electric charge, and angular momentum. All other information about the matter which formed a black hole “disappears” behind its event horizon, and, therefore, the nature of these microstates is obscure. Thus, the origin of the black-hole entropy is not clear. Furthermore, in order to justify the name “entropy”, one must to explain also why the sum of the entropy of the black hole and the entropy of its vicinity is a non-decreasing function of time. In other words, why black holes obey the so-called “Generalized Second Law of thermodynamics (GSL)”.

The situation becomes even worse if we consider black-hole evaporation. Since black holes evaporate, one could expect, from black-hole radiation, any information about the state which collapsed into the black hole. However, Hawking showed, through semiclassical arguments, that black-hole radiation is thermal, and therefore does not carry any information about its initial state. In this situation, the matter that formed the black hole, which initially was in a pure state has evolved into a mixed state. This fact brings us a contradiction with quantum mechanics,
where a pure state can only evolve into another pure state because of the unitarity of the evolution operator [1,2,6,7]. In this context, a new phenomenon arises as one way to solve the drawbacks between black-hole physics and quantum mechanics. This phenomenon is related with quantum gravity, and consists in a topology change of the spacetime, where a new topologically disconnected region arises inside the black hole, and information is preserved, and stored there. This scenario can be produced by the gravitational collapse, which would lead to a region of Planckian densities and curvature where quantum gravitational effects become important. Topology change must occur inside the black-hole horizon, in a way that, it is entirely invisible to observers outside the horizon, which see the usual Hawking evaporation of the hole. In this situation, a complete specification of the state of the (now topologically nontrivial) universe requires a wave function which has a component on the new topologically disconnected region too. In this way, observers without access to this new region, have incomplete information about the universe as a whole.

Earlier work on topology change and its relation to black-hole information includes an unpublished preprint by Dyson [8], papers by Strominger [9], Polchinski and Strominger [10], Jacobson [11,12], Easson and Brandenberger [13], Hsu [14], and Silva [15]. Also of interest is the work of Ashtekar and Bojowald [16,17], in which quantization of the classical singularity region of a black hole allows evolution of a large spacetime. The authors of [8,9,11–13, 15–17] all state that new universe creation might alleviate the black-hole information problem. For related work on dynamical mechanisms by which black-hole formation might lead to new universe creation, see [18–28]. For some early discussions of spacetime topology change in string theory, see [29–31].

In this paper, we will address the black-hole thermodynamics in the context of topology change, as conceived for some classes of quantum spaces, called fuzzy spheres. We argue that a model based on the topology change of fuzzy manifolds can be used to shed some light on the origin of the black-hole entropy, including why the black-hole evaporation process obeys the GSL. To do this, we discuss the selection rules for the black-hole area transitions in the evaporation process. Moreover, we discuss the possibility that some information about the black-hole initial state could be recovered by an observer in our universe, where we can perform measures.

The paper is organized as follows. In the second section, we address the relation between fuzzy spaces topology change and black-hole thermodynamics. In the third section, we obtain the selection rules for the black-hole area transitions in the evaporation process. In the fourth section, we investigate the obedience to GSL by the black-hole evaporation in the fuzzy topology change approach. The fifth section is devoted to conclusions. In this work we shall consider all fundamental constants ($c, \hbar, k_B, G$) equal to unity.

**Fuzzy spaces topology change and black-hole thermodynamics.** – Recently it was suggested by one of us that it is possible to realize a topology change process to black holes without break unitarity or cluster decomposition [15]. The basic idea of this proposal is to see the black-hole event horizon as a fuzzy sphere $S_F^2$ [32], taking into account some quantum symmetry properties related with a Hopf algebra structure.

Fuzzy spheres consist in one of the simplest examples of non-commutative spaces, and appear as vacuum solutions in Euclidean gravity [33,34]. It is obtained when we quantize the usual sphere $S^2$ replacing the commutative algebra of functions on this manifold by the non-commutative algebra of matrices. To do this one quantizes the coordinates $x^\mu$ ($\mu = 1, 2, 3$) on $S^2$, performing the transformation

$$x^\mu \to \hat{x}^\mu = k \hat{J}^\mu,$$

(1)

with

$$\hat{J}^\mu \hat{J}_{\mu} = r^2 1,$$

(2)

where $\hat{J}^\mu$ form the $n$-dimensional irreducible representation of the $SU(2)$ algebra, whereas $r$ is the fuzzy-sphere radius.

In this way, we have that the coordinates on the fuzzy sphere $S_F^2$ satisfy the following commutation relations:

$$[\hat{x}^\mu, \hat{x}^\nu] = i \lambda r^{-1} \epsilon^{\mu\nu\alpha} \hat{x}_\alpha,$$

(3)

where $\lambda$ has a dimension of (length)$^2$, and plays here a role analogous to that played by Planck’s constant in quantum mechanics. The fact that the coordinates $\hat{x}^\mu$ do not commute anymore implies that the points on the sphere are “smeared out”, and we have to substitute the idea of points for the idea of elementary (Planck) cells. The commutative limit is given by $\lambda \to 0$.

The use of fuzzy spheres in the context of black-hole physics is mostly motivated by Bekenstein’s limit [3], which says that the black-hole entropy is finite and proportional to the event horizon area. If the entropy is to be finite it is then necessary that there be a finite number of degrees of freedom associated with the event horizon area. It has driven the general belief that the event horizon area should be quantized [35]. In this context, fuzzy spheres offer a very concrete and interesting method to implement the event horizon quantization due to its non-commutative geometry [32]. Moreover, since fuzzy spheres, are obtained from quantization of a compact space, they are described by finite dimensional matrices, in a way that the number of independent states defined on the fuzzy sphere is limited, and the entropy associated with these states is finite, in agreement with the Bekenstein’s limit [15,33,36–38].

Another important feature of fuzzy spheres is their close relationship with Hopf algebras, which allow us to define a linear operation (the coproduct of a Hopf algebra) on $S_F^2$ and compose two fuzzy spheres preserving the algebraic properties intact. This operation produces a
topology change process where a fuzzy sphere splits into two others [39], and can be used as a good mathematical model to black-hole topology change [15].

To describe the fuzzy-sphere topology change, we have that under the quantization procedure (1), functions defined on $S^2$ are replaced by matrices on $S^2_F$ [32]. In this way, let a matrix $\hat{M}$ describe a wave function on $S^2_F$, the Hopf coproduct $\Delta$: $S^2_F(j) \to S^2_F(K) \otimes S^2_F(L)$ acts on $\hat{M}$ as

$$\Delta_{(K,L)}(\hat{M}) = \sum_{\mu_1,\mu_2,m_1,m_2} C_{K,L,J,\mu_1,\mu_2} C_{K,L,J,m_1,m_2} \times M_{\mu_1,\mu_2,m_1,m_2} e^{i\mu_1 m_1}(K) \otimes e^{i\mu_2 m_2}(L),$$

(4)

where $C$'s are the Clebsh-Gordan coefficients and $e^{i\mu m}$'s are the basis for a matrix space defined on the fuzzy sphere [39].

The coproduct $\Delta$ has the following interesting properties:

$$\Delta(M^\dag) = \Delta(M)^\dag,$$

(5)

$$\Delta_{KL}(MN) = \Delta_{KL}(M)\Delta_{KL}(N),$$

(6)

$$\text{Tr}(\Delta(M)) = \text{Tr}(M),$$

(7)

$$\Delta(M), \Delta(N) = (M,N).$$

In this way, it preserves the Hermitian conjugation, the matrix product, the matrix trace, and the inner product. The last is defined as

$$(M,N) = \frac{1}{2J+1} \text{Tr}M^\dag N,$$

(8)

where $2J+1$ is the dimension of the fuzzy-sphere Hilbert space. One can show that the properties above assure that (4) is a unitary process, and preserves the algebraic properties of the operators defined on the fuzzy sphere [15, 39], as well as cluster decomposition (locality) [15].

The meaning of eq. (4) is that a wave function $\hat{M} \in S^2_F(J)$ splits into a superposition of wave functions on $S^2_F(K) \otimes S^2_F(L)$. In this way, the information in $\hat{M}$ is divided between two regions of the spacetime, i.e., the two fuzzy spheres with spins $K$ and $L$, respectively.

The fuzzy-spheres features that come from their relationship with quantum groups support the basic assumptions of this article, which are introduced as follows:

i) If one uses the fuzzy-sphere Hilbert space as the ones of the black hole, the maximum of information about the black hole that an outside observer can obtain would be encoded in wave functions defined on the fuzzy-sphere Hilbert space.

ii) We shall find out, through the Hopf coproduct $\Delta$, a topology change process for the black hole. In this process the information about the black-hole initial state, will be divided into two spacetime regions. One of them is a fuzzy sphere with spin $K$, which we shall consider as the original world and name it “the main world”. The other one is a fuzzy sphere with spin $L$ which we shall name “the baby world”.

iii) Since the baby world arises in the black-hole interior, an observer in the main world cannot access the degrees of freedom there. In this way, from his standpoint, the black hole will appear to evolve from a pure to a mixed state described by a density matrix $\hat{\rho}$. It enables us to define an entropy, measured by the observer in the main world, associated to the black-hole horizon.

The assumptions above are our basic assumptions in this article, in a way that the topology change of a fuzzy sphere can be used to model the process leading the Hawking phenomenon. From this assumption, in the following sections we will discuss, the relationship between the fuzzy-sphere topology change and black-hole evaporation. It is worth mentioning again that the topology change approach had, traditionally, been claimed to break unitarity and cluster deposition (locality) [14,40]. However, the model above can describe the evaporation of a black hole through a topology change process in a way that unitarity and locality can be conserved. This is a consequence of the algebraic properties of the coproduct $\Delta$, and a detailed discussion on this can be find in ref. [15].

Fuzzy spaces topology change and black-hole evaporation process. – In this section we shall analyze how the topology change process drives the black-hole evaporation. To do this we will investigate how the fuzzy topology change drives the black-hole area transitions. At first, let us analyze the simplest transition $J \to J - 1/2$. We shall admit that the selection rules for the black-hole area transitions are the ones for the topology change. These rules are obtained from eq. (4), when one traces over the degrees of freedom in the baby universe. In this way, the splitting process (4) for a matrix $\hat{M} = | J,J,m \rangle \langle J,m' |$ with $L = 1/2$, and $K = J - 1/2$ is given by

$$\Delta(| J,J,m \rangle \langle J,m' |) =$$

$$\frac{\sqrt{(K+mK+1)(K+m'K+1)}}{2K+1} | K,m-1/2 \rangle \langle K,m'-1/2 |$$

$$+ \frac{\sqrt{(K-mK+1)(K-m'K+1)}}{2K+1} | K,m+1/2 \rangle \langle K,m'+1/2 |.$$

From the equation above, the probability amplitude for a one-step transition $J \to J - 1/2$ is given by

$$a_{J \to J-1/2} = \left( \frac{2J+1}{2J} \right).$$

(9)

Now, the probability amplitude for a black-hole $n$-steps transition can be obtained from eq. (9). In ref. [15], it has been shown that the splitting process (4) obeys cluster decomposition (locality). Then, we have that different steps $J \to J - 1/2$, in the black-hole evaporation, are independent events. In this way, the probability amplitude of $n$ steps occurring in the black-hole evaporation process is given by the product of the probability amplitudes of each of these steps occurring by itself. Then, taking the
product of the probability amplitudes (9) for subsequent steps in the black-hole evaporation, we have
\[ a_{J_n} = \left( \frac{2J + 1}{2J - n + 1} \right). \] (10)

In order to analyze the black-hole area transitions, we will introduce a canonical ensemble in which our system (the black hole) can occupy different area microstates. The idea of using these types of ensembles goes back to Krasnov [41–43] and is, somehow, a necessity in the loop quantum gravity formalism as the count of states is naturally done by using the horizon area instead of the black-hole mass [5,44,45]. In this framework, the probability amplitude for the black hole to evaporate is given by
\[ a_{J_n} = e^{-\beta A_{J_n}}, \] (11)
where \( \beta \) is a temperature-like parameter dual to the black-hole area [46–48].

We will identify the probability amplitude (10) with (11) in a way that the value of the black-hole area in the \( J \)-state will be written as
\[ A_J = \beta^{-1} \frac{\hbar^2}{8\pi} \ln(2J + 1). \] (12)

Moreover, we can write the density matrix describing the black-hole quantum states as
\[ \hat{\rho} = (1/Z) \sum_{J=0}^{\infty} e^{-\beta A_J} | J \rangle \langle J | = (1/Z) e^{-\beta \hat{A}}, \] (13)
where \( Z = \text{Tr} e^{-\beta \hat{A}} \) is the partition function. The matrix \( \hat{\rho} \) in eq. (13) satisfies the Bloch equation
\[ i \frac{\partial \hat{\rho}}{\partial \Theta} = -\frac{\hat{A}}{8\pi} \hat{\rho}, \] (14)
where we have replaced \( \beta \) by \(-i\Theta/8\pi\).

Equation (14) will govern the transitions between black-hole area states. It must be used when working in the Euclidean confirmation of the black hole, supplementing the Wheeler-DeWitt equation, where \( \Theta \) plays the role of a sort of “dimensionless internal time” associated with the horizon [46–48]. We have still \( \Theta = i\Theta_E \), where \( \Theta_E \) is the Euclidian angle. The regularity of the Euclidean manifold at the horizon imposes a fixed Euclidian angle given by \( \Theta_E = 2\pi \). In this way, at the horizon \( \beta = 1/4 \).

From eq. (12), and the results above, the entropy \( S = -\text{Tr}(\ln \hat{\rho}) = \ln(2J + 1) \), associated for an outside observer to black hole is given by
\[ S = \frac{A}{4}, \] (15)
which corresponds to the Bekenstein-Hawking formula.

The logarithmic dependence of the black-hole area spectrum on \( J \), in expression (12), tells us that the decrease in the horizon area is continuous at large values of \( J \), and discrete to small values of \( J \), when the black-hole approaches the Planck scale. The black-hole area spectrum is showed in fig. 1 above.

In the next section we will see how the selection rules, inherited from the topology change process, will bring essential consequences to the way in which entropy is emitted by black holes. We have that the Hawking radiation is known semiclassically to be continuous. However, the Hawking quanta of energy are not able to hover at a fixed distance from the horizon since the geometry of the horizon has to fluctuate, once gravitational effects are included. Thus, one suspects a modification of the description of the black-hole emission process occurs at the final stages of black-hole evaporation, where its area spectrum becomes discrete.

**Entropy emitted during the evaporation process.**

The entropy of a system measures one’s lack of information about its actual internal configuration. Suppose that everything we know about the internal configuration of the system is that it may be found in any of a number of states, with probability \( p_n \) for the \( n \)-th state. Then the entropy associated with the system is given by Shannon’s well-known relation \( S = -\sum p_n \ln p_n \) [49–52].

The probability for a black hole to emit a specific quantum should be given by expression (10), in which we must still include a gray-body factor \( \Gamma \) (representing a scattering of the quantum off the spacetime curvature surrounding the black hole). Thus, the probability \( p_n \) to the black hole going \( n \) steps down in the area ladder represented in fig. 1 is proportional to \( \Gamma(n) e^{-\frac{J_n}{4\hbar^2}} \). Moreover, the discrete area spectrum (12) implies a discrete line emission from a quantum black hole. For a Schwarzschild black hole, the radiation emitted will be at frequencies given by
\[ \omega_{J_n} = \frac{\hbar}{2\pi} \left( \sqrt{\ln(2J + 1)} - \sqrt{\ln(2J + 1 - n)} \right). \]

To gain some insight into the physical problem, we shall consider a simple toy model suggested by Hod [53,54]. To begin with, it is well known that, for a massless field, \( \Gamma(M\omega) \) approaches 0 in the low-frequency limit, and has a high-frequency limit of 1. A rough approximation of this effect can be achieved by introducing a low-frequency
From the graphic for the black hole decreases as the area spacing increases.

The ratio of entropy emission rate from the quantum black-hole to the rate of black-hole entropy decrease.

cutoff at some \( \omega = \omega_c \) [55]. That is, \( \Gamma(\omega) = 0 \) for \( \omega < \omega_c \), and \( \Gamma(\omega) = 1 \) otherwise, where \( \omega = M\omega \) [56-58].

The ratio \( R = |\dot{S}_{rad}/\dot{S}_BH| \) of entropy emission rate from the quantum black hole to the rate of black-hole entropy decrease is given by

\[
R = \left| \frac{\sum_{n=1}^{N_s} \sum_{J=1}^{2J} C T(n) e^{-\frac{\delta A_{Jn}}{4}} \ln \left[ C T(n) e^{-\frac{\delta A_{Jn}}{4}} \right]}{\sum_{n=1}^{N_s} \sum_{J=1}^{2J} C T(n) e^{-\frac{\delta A_{Jn}}{4}} \left( \frac{\delta A_{Jn}}{4} \right)} \right|, \tag{16}
\]

where \( C \) is a normalization factor, defined by the normalization condition:

\[
\sum_{i=1}^{N_s} \sum_{n=1}^{2J} C T(n) e^{-\frac{\delta A_{Jn}}{4}} = 1. \tag{17}
\]

For the effective number of particle species emitted \( (N_s) \), we will take into account the various massless modes emitted. We will consider

\[
N_s = \begin{cases} 
2J + 1, & \text{for } 2J + 1 < 112, \\
112, & \text{for } 2J + 1 \geq 112.
\end{cases} \]

In this way \( N_s \) is upper limited by the number of modes of massless particles in nature which make the dominant contribution to the black-hole spectrum (the 1/2, 3/2, 5/2 neutrino modes, the 1 and 2 photon modes, and the 2 and 3 graviton modes [53,54,56–58]), and by the size of the fuzzy-sphere Hilbert space.

In fig. 2, we have plotted down \( R \) taking \( \omega_c \approx 0.2 \) (the location of the peak in the total power spectrum [56–58]). With this frequency cutoff, the minimal non-null value to the quantum number \( J \), in order to have \( \Gamma \neq 0 \), is \( J = 6.0 \).

In this point, the black hole must evaporate completely. From the graphic for \( R \), we have that the non-unitary evolution of the black-hole geometry in the main world, due to the topology change process, imposes obedience to a “second law of thermodynamics” for the black-hole evolution process, since \( R \) is ever larger than (or equal to) unity. The value of \( R \) approaches the value of 1.3 at the large-\( J \) limit in agreement with known semiclassical results [55].

It is important to notice that the entropy emitted from the black hole decreases as the area spacing increases.

The entropy of the radiation should be maximal in the semiclassical limit where the black hole can be in any area state, and the various transitions have almost the same probabilities. On the other hand, in the quantum limit, only special values are allowed to the black-hole area, and then only special transitions are allowed. In this way, the entropy of the radiation emitted by the black-hole becomes smaller. The striking consequence of this is the possibility that, since the black-hole radiation becomes less and less entropic as the evaporation process takes place, some information about the black-hole initial state could leak out from its interior and be accessible to an observer in our universe, where we can perform measures. The possibility of information leakage from a black hole with a discrete area spectrum has been already pointed out by Hod [53,54].

**Remarks and conclusions.** – In this work we have argued that a model based on the topology change of a quantum manifold can be used to shed some light on the problem of the origin of black-hole entropy: it is generated because of the non-unitary evolutions of the geometry of the main world. To do this, we have put the process of emission of quanta of radiation by black hole in connection with topology changes in the quantum manifold which is assumed to describe the horizon (the fuzzy sphere).

From the topology change model we have obtained the selection rules for the black-hole area transitions. In this way, we were able to derive an expression for the probability amplitudes of black-hole transitions. From them, an understanding of the Bekenstein-Hawking formula for black-hole entropy is provided. We have that the topology change approach gives us a relation of states to points that brings together the black-hole entropy and our standard concept of entropy as the logarithm of the number of microstates.

Through the study of the black-hole evaporation process, an area spectrum, which is continuous in the semiclassical limit, and becomes discrete as the black-hole approaches the Planck scale has been obtained. In order to investigate the influences of the area spectrum shape on the black-hole emission, we have calculated the ratio \( R \) between the rates of entropy emission and black-hole entropy decrease. At first, we have that \( R \) is found to be larger than 1, showing that the considered mechanism is able to produce a generalized second law of thermodynamics. Then, we have that \( R \) approaches 1 as the black hole shrinks to the Planck scale, and the area spectrum becomes discrete. This points to a possible information leak out from the black hole, since its radiation becomes less and less entropic as the black-hole evaporates, and could alleviate the information problem for an observer outside the black hole. Since the possible information leakage would occur more strongly in the quantum gravity limit, it would not require radical modifications in the laws of physics above the Planck scale. The task of finding
an appropriate quantum mechanism for information leakage remains.

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