Quantum Anonymous Veto protocol

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Anonymous Veto (AV) and Dining cryptographers (DC) are two basic primitives for the cryptographic problems that can hide the identity of the sender(s) of classical information. They can be achieved by classical methods and the security is based on computational hardness or requires pairwise shared private keys. In this regard, we present a secure quantum protocol for both DC and AV problems by exploiting the GHZ correlation. We first solve a generalized version of the DC problem with the help of multiparty GHZ state. This allows us to provide a secure quantum protocol for the AV problem. Security of both the protocols rely on some novel and fundamental features of the GHZ correlation known as GHZ paradox.

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I. INTRODUCTION

In the classical world, where any physical transmission can be traced to its origin, it seems impossible to setup a secure way for message transmission without revealing its senders’ identity. Dining cryptographers (DC) problem \cite{1} introduced by Chaum is one of the primary attempts in this context. In a DC problem, three cryptographers are curious to find out whether their agency NSA (U.S. National Security Agency) or one of them pays for the dinner. At the same time they respect each other’s right to make an anonymous payment. A generalized version of the DC problem called DC-net where one of the member from an agency publicizes a secret message (anonymously) such that no passive adversary can learn which member publicizes the message \cite{1}. To setup and unconditionally secure DC-net requires pairwise shared (secure) keys and an authenticated broadcast channel. Since, the security of DC-net relies on the generation of secure key between pairs of members so it is not unconditionally secure\textsuperscript{1} if members are not allowed to pre-share bilateral private keys. Another major flaw of DC problem is that, it is vulnerable against multiple payments. It shows zero pay i.e. no transmission of message if even number (0, 2, . . .) of members pay for the dinner and detects payment if an odd (1, 3, . . .) number of members pay for the dinner. There is another loophole in DC problems called collusion loophole where some of the participants may cooperate among them to trace the person who pays. There are some works where partially resolve the multiple payment case and the collusion problem but non of them provides an unconditional secure solution of the problem \cite{2, 4}.

Another variant of DC problem known as Anonymous Veto (AV) problem \cite{3}. Here a group of jury members, who need to take an unanimous decision, but at the same time want their individual decisions to remain secret i.e. without ever disclosing the identity of possible vetoing member(s). This could be very important in many aspects of human societies. Security of the classical solution of this problem is also based on the computational hardness like other classical cryptographic protocols or put restrictions on the number of dishonest players \cite{6}. In this context, Boykin \cite{7} provided a quantum protocol to send classical information anonymously by distributing pairwise shared EPR pair among players. In 2005, Christandl and Wehner \cite{8} proved that the protocol presented by Boykin is not perfectly secure since it does not satisfy the traceless property and they provided an alternative quantum scheme of the DC-type problem with the traceless feature. In this regard, we present secure quantum protocol for both DC and AV problems with the help of multi-qubit GHZ correlation and GHZ paradox \cite{9}.

We start with a brief description of the GHZ paradox, which will allow us to present a quantum protocol for the three-party DC problem with a detection of multiple payments. This three party DC protocol is quite similar to the protocol presented in \cite{8}. We then extend the protocol into n-party DC problem without any detection of multiple payments. By exploiting this generalized version of DC problem we demonstrate a quantum protocol for the AV problem.

II. GHZ PARADOX

In 1989, Greenberger, Horne and Zeilinger (GHZ) \cite{9} provided a way to show a direct contradiction of quantum mechanics with local realism without using any statistical

\[|\psi^{+}\rangle := \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)\]
inequality. Consider a three qubit maximally entangled\footnote{this entanglement is maximal in the sense that it gives the maximum violation of Bell’s inequality for a given set of observables}.

$$|\Psi\rangle = \frac{|000\rangle - |111\rangle}{\sqrt{2}}$$ (1)

known as GHZ states. This GHZ state satisfies the following four constraints

$$\sigma_x \otimes \sigma_x \otimes \sigma_x |\Psi\rangle = (-1)|\Psi\rangle,$$

$$\sigma_y \otimes \sigma_y \otimes \sigma_y |\Psi\rangle = (+1)|\Psi\rangle,$$

$$\sigma_x \otimes \sigma_y \otimes \sigma_y |\Psi\rangle = (-1)|\Psi\rangle,$$

$$\sigma_y \otimes \sigma_y \otimes \sigma_y |\Psi\rangle = (+1)|\Psi\rangle,$$ (2)

where, $\sigma_x, \sigma_y, \sigma_z$ are the Pauli matrices. One should note that,

$$(\sigma_x \otimes \sigma_y \otimes \sigma_y)(\sigma_y \otimes \sigma_x \otimes \sigma_z)(\sigma_y \otimes \sigma_y \otimes \sigma_x) = \sigma_x \otimes \sigma_x \otimes \sigma_z.$$ (3)

If local-realistic (LR) value assignment to the spin observables is possible for the above quantum state $|\Psi\rangle$, then the left hand side of (3) would produce $+1$ for the given quantum state $|\Psi\rangle$ whereas the right hand side would produce $-1$. This is a contradiction. Hence, the correlation of $|\Psi\rangle$ cannot be described by any LR theory \cite{footnote:1}. Similarly, for another three qubit GHZ state

$$|\Psi^\perp\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}}$$ (4)

we have,

$$\sigma_x \otimes \sigma_x \otimes \sigma_x |\Psi^\perp\rangle = (+1)|\Psi^\perp\rangle,$$

$$\sigma_x \otimes \sigma_y \otimes \sigma_y |\Psi^\perp\rangle = (-1)|\Psi^\perp\rangle,$$

$$\sigma_y \otimes \sigma_x \otimes \sigma_y |\Psi^\perp\rangle = (-1)|\Psi^\perp\rangle,$$

$$\sigma_y \otimes \sigma_y \otimes \sigma_x |\Psi^\perp\rangle = (+1)|\Psi^\perp\rangle.$$ (5)

Like the previous case, one can easily show that the above four constraints cannot be satisfied simultaneously by any LR theory.

III. QUANTUM DINING CRYPTOGRAPHERS (QDC) PROTOCOL

Imagine that three cryptographers Alice, Bob and Charlie want to play the Dining Cryptographers (DC) problem. To do that they first share a number (say $L$) of copies of the GHZ state $|\Psi\rangle$ given in (1), one qubit each from each copy. Here a copy of the states corresponds to a run of the protocol. Onward we use both the notations copy of the state or run of the protocol simultaneously. After receiving all the qubits from $L$ copies of GHZ states they randomly select some runs (say $L_1$) and check whether the selected states satisfy the GHZ paradox or not. If yes, rest of the shared states (say, $L_2 = L - L_1$) are genuine copies of GHZ state $|\Psi\rangle$. The detail of the genuineness check of GHZ state is discussed later. After conformation of genuineness of the states the protocol goes as follows:

**Protocol: QDC(3)**

- **Resource:** Shared $L$ copies of $|\Psi\rangle$ and verify its genuineness.
- **S1.** Each member performs $\sigma_x$ on his qubits if want to pay the dinner otherwise does noting.
- **S2.** Performs $\sigma_x$ on qubit associated to a selected run to distinguish between the cases (i) even and (ii) odd no. of multiple payments.
- **S3.** Distinguish ‘no pay’ vs. ‘double pay’ in case (i) and ‘single pay’ vs. ‘triple pay’ in case (ii).

S1: Performing local unitary operation to encode payment: Alice performs local unitary operation $\sigma_x$ on each of her qubits from $L_2$ if she wishes to pay the dinner. Otherwise, she does nothing. Bob and Charlie follow the same.

(i): If an even number (*i.e.* zero/two) of members pay the bill (*i.e.*, apply $\sigma_x$) then the states of all the members of $L_2$ remain in the same GHZ state (11) as

$$I \otimes I \otimes I |\Psi\rangle = |\Psi\rangle; \quad I \otimes \sigma_x \otimes I |\Psi\rangle = |\Psi\rangle; \quad \sigma_x \otimes I \otimes I |\Psi\rangle = |\Psi\rangle.$$ (6)

Here, first equation represents the situation when no member willing to pay the dinner. Whereas, last three cases represent the situation when two of the members willing to pay for the dinner.

(ii): If an odd number (*i.e.* one/three) of members want to pay the dinner (*i.e.*, apply $\sigma_x$) then the states of the members of $L_2$ would become copies of $|\Psi^\perp\rangle$ given in (4).

$$\sigma_x \otimes I \otimes I |\Psi\rangle = |\Psi^\perp\rangle; \quad I \otimes \sigma_x \otimes I |\Psi\rangle = |\Psi^\perp\rangle; \quad \sigma_x \otimes \sigma_x \otimes I |\Psi\rangle = |\Psi^\perp\rangle,$$

where the first three equations represent the situation when only one of the member pays the dinner and the last equation represents the case when all the three members pay for the dinner. In the next step of the protocol members distinguish two cases (i) and (ii) without disclosing payer(s) identity.

S2: Distinguishing case (i) and case (ii): To distinguish case (i) and case (ii), members randomly select one of the run (say $r_1$) from $L_2$. Now the task is to identify the state ($|\Psi\rangle$ or $|\Psi^\perp\rangle$) corresponding to the run $r_1$. Thus, the distinguishability task between case (i) and case (ii) reduces to the problem of distinguishability of two orthogonal three-qubit GHZ states $|\Psi\rangle$ and $|\Psi^\perp\rangle$. To do this, each member measures his/her qubit corresponding to the run
r₁ in σₓ basis and after measurement declares the measurement outcome (+1 or −1). If the product of announced results of three local measurement is −1 then the desire state is |Ψ⟩ i.e., case (i) happens. On the other hand, if the product is +1 then the state is |Ψática⟩ i.e., case (ii) happens. This is quite easy to observe since, σₓ ⊗ σₓ ⊗ σₓ |Ψ⟩ = (−1)|Ψ⟩ and σₓ ⊗ σₓ ⊗ σₓ |Ψática⟩ = (+1)|Ψática⟩.

Now the task is to distinguish between subcases ‘zero pay’ vs. ‘double pay’ for case (i) and ‘single pay’ vs. ‘triple pay’ for case (ii).

S3: Distinguishing between subcases: Members randomly selects one of the run (say, r₂) from L₂ \ {r₁}. If case (i) occurs in the previous step then each of the member measures his qubit associated to the run r₂ in σᵧ basis if (s)he pays for the dinner (i.e. applied σₓ in step S1.) otherwise, measures the qubit in σₓ basis (i.e., not paying the dinner). After measurement each member announces his/her measurement result. If the product of the local measurements is −1 then no member has paid for the dinner (i.e., zero pay) and if the product is +1 then two members have paid for the dinner (i.e., double pay). The first case follows from the top equation of 2 whereas, the second case follows from the last three equations of 2. If zero pay occurs NSA will pay the dinner and if double pay occurs the payment will be cancelled.

In case (ii) each of the member measures his qubit for the run r₂ in σₓ basis if (s)he pays for the dinner (i.e. applied σₓ in step S1.) otherwise, measures the qubit in σᵧ basis (i.e., not paying the dinner) and announces the measurement result. If the product of the local measurements is +1 then all the three members have paid for the dinner (i.e., triple pay) and if the product is −1 then only one member has paid for the dinned (i.e., single pay). The first case follows from the first equation of 3 whereas, the second case follows from the last three equations of 3. If a single pay occurs payment will be accepted otherwise payment will be cancelled.

One may argue that for step S2. and step S3. two copy of the states, one copy for each step, are sufficient. Therefore, it is enough if the list L₂ contains just two runs. This is true only if all the members honestly follow the entire protocol i.e., perform local measurements (consistently) whenever asked according to their action and declares the true outcome for each such measurement. If they act dishonestly, they do it solely to trace payer(s) only, and not to create any confusion regarding payment. The member who get chance to announce his results last (i.e., at the end of all other members’ announcement) in both the steps S2. and S3. enjoys some advantage. (S)He may change the case by just sending a flipped result of her/his measurement outcome. To deal with this problem members choose a list of runs instead of just one run in both steps S2. and S3. and the ordering of the announcement of the result is random for each such selected run in both steps S1. and S2.. Therefore, no members gets advantage by delaying her/his announcement and if someone flips the result then that will lead to an inconsistent conclusion and subsequently, they abort the protocol and starts a new one with a fresh set of resources.

A. Security analysis of QDC protocol

Since the payer(s) information in step S1. encoded inside the phase of the GHZ state and due to the party symmetry of the state no quantum operation can reveal the identity of the payer(s). So the local operations for distinguishing the cases never disclose any information about the payer(s) identity. Step S2. only disclose the information whether the total number of payers are odd or even. This information in no way harm the purpose, rather it helps to detect multiple payments. In step S3. members only reveal their individual measurement result and not the choice of measurement to identify the ‘no pay’ in case of (i) and a ‘single pay’ in case of (ii). By knowing measurement result one cannot predict the measurement choice as that would immediately imply a violation of causality principle. If two of the members co-operate to each other then they can certainly predict the measurement choice and hence the action of the third party by knowing the measurement result. But this is quite obvious, since the anonymity exists only among a set of possible performers, and if the set is singleton, its member is always traceable i.e., no protocol can keep the singleton members set untraceable. In our QDC protocol if the payment accepted i.e. a ‘single pay’ happens then no non-payer have any information about the payer. But, in case of rejection of payment the identity of payers may be disclosed in two cases (i) if the ‘double payment’ occurs then the non-payer knows that the other two members are the payers and (ii) if the ‘triple pay’ happens then every body know that all the three members are payers. These two cases can be avoided if we assume that the dinner is paid only once like the original DC problem and then the protocol will end at step S2. as step S3. is unnecessary. Since, step S2. is sufficient to distinguish between the ‘zero pay’ and a ‘single pay’ as other two multiple cases will never occur. Based on the assumption that multiple payments will never occur, one can easily generalize our QDC protocol for n (≥ 3) number of members.

B. Generalized QDC protocol

Let a group of n cryptographers are sitting for dinner at a restaurant and they want to find out whether their agency NSA or one of them pays for the dinner, while respecting each other’s right to make a payment anonymously. To implement the protocol n-cryptographers
share a copy of the generalized n-qubit GHZ state
\[ |\Psi_n⟩ = \frac{1}{\sqrt{2}} [(000 \ldots 0) - |111 \ldots 1⟩]. \] (6)

The above GHZ state has the following correlation:
\[ \sigma^t_z |\Psi_n⟩ = \frac{1}{\sqrt{2}} [(000 \ldots 0) - (-1)^t|111 \ldots 1⟩] \]
\[ = \frac{1}{\sqrt{2}} [(000 \ldots 0) - |111 \ldots 1⟩] = |\Psi_n⟩ \]
(\text{if } t \text{ is even}),
\[ = \frac{1}{\sqrt{2}} [(000 \ldots 0) + |111 \ldots 1⟩] = |\Psi^+_n⟩ \]
(\text{if } t \text{ is odd}), (7)

where \( \sigma^t_\xi \) denotes that in \( t \)-number of places \( \sigma_\xi \) acts and
in rest of the places \( 2 \times 2 \) identity matrices \( \mathbb{I} \) acts where \( t = 0, 1, \ldots, n \).

**S1**: Similar to the step S1. of the previous protocol. Each member applies \( \sigma_\xi \) on his local qubit if he pays for the dinner otherwise does nothing. (i) If one of the member pays for the dinner the desire state become \( |\Psi^+_n⟩ \) which is orthogonal to the state \( |\Psi_n⟩ \). (ii) If no members pay for the dinner then the state remains same. Now the task is to distinguish case (i) and case (ii).

**S2**: To distinguish case (i) and (ii) each of the members measures his qubit in \( \sigma_\xi \) basis and after measurement announces the measurement result. If the product of all the local results is \(+1\) then the desire state is \( |\Psi^+_n⟩ \), i.e., one of the member paying for the dinner (case (i)). If the product is \(-1\) then NSA pays for the dinner i.e., case (ii).

The security of the protocol is guaranteed from the fact that the payer information encoded inside the phase of the GHZ state like in previous QDC protocol. By exploiting generalized QDC protocol we now provide an secure quantum protocol for the AV problem for odd number of parties. Then we extend it for the even number of parties.

**IV. QUANTUM ANONYMOUS VETO (QAV) PROTOCOL**

Imagine a jury with \( n \) (odd) no. of members, who need to take an unanimous decision, but at the same time want their individual decisions to remain secret. The generalized GHZ state \( |\Psi_n⟩ \) given in (6) would allow them to achieve this. The quantum AV protocol starts with sharing \( R \) number (say \( R \geq 2 \)) of copies of \( |\Psi_n⟩ \) between jury members. Each member gets one qubit from each of the copy of \( |\Psi_n⟩ \).

| Protocol: QAV (odd-\( n \)) |
|---------------------------|
| **Resource**: Shared \( L \) copies of \( |\Psi_n⟩ \) and verify its genuineness. |
| **S1**: Each member performs \( \sigma_\xi \) on his qubits if he wants to vote in ‘against’ otherwise, does noting. |
| **S2**: Performs \( \sigma_\xi \) on qubit associated to a selected run to distinguish between the cases (i) even (including zero) and (ii) odd no. of ‘against’ votes. |
| **S3**: Distinguish between the cases (a) unanimity in favor vs. (b) an even (excluding zero) no. of ‘against’ votes in case (i). |

**S1**: After receiving all the associate qubits each member performs the unitary operation \( \sigma_\xi \) if he is ‘against’ the decision and does nothing if he is ‘in favor’ on all his qubits from \( R \). (i) If an even number of members (including zero) ‘against’ the decision all the desire copies of \( |\Psi_n⟩ \) remain same. (ii) Otherwise, all of them reduce to the state \( |\Psi^+_n⟩ \).

**S2**: Jury members select one of the copy (say, \( r \)) of the state to distinguish between the cases (i) and (ii). Each member performs a \( \sigma_\xi \) measurement on his qubit associated with the selected state and announces the measurement outcome. They now calculate the product of all local outcomes. Product \(-1\) implies case (i) and \(+1\) implies case (ii). Unanimity in favor of the decision happens only if no members (i.e. zero members) voted against. Since, the case (ii) represents the case where at least one of the member voted against so it does not required any further analysis. But, case (i) represents the unanimity ‘in favor’ of the decision and (b) an even number (2, 4, \ldots) of members voted against the decision.

**S3**: To distinguish between subcases (a) and (b) each member performs (again) on all his \( R_1 = R \setminus \{r\} \) qubits the unitary operation \( \sigma_\xi(1) = \left( \begin{array}{cc} 1 & 0 \\ 0 & \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \end{array} \right) \) if he is ‘against’ the decision and if he is ‘in favor’ does nothing. (i) If the number (including zero) of members against the decision is even multiple of 2 (i.e., multiple of four) then all the states of the selected copies will remain in \( |\Psi_n⟩ \). (ii) Otherwise, (i.e., the number of members against the decision is odd multiple of 2) all of the selected copies reduce to the state \( |\Psi^+_n⟩ \).  

1.: Like step S2”, jury members select (randomly) one of the copy (say, \( s_1 \)) of the state from \( R_1 \) copies to distinguish case (i) and (ii). Each member performs a \( \sigma_\xi \) measurement on his qubit associated with the selected state \( s_1 \) and announces the measurement outcome. They
now calculate the product of all local outcomes. Product −1 implies case (i) and +1 implies case (ii). Since, the case (ii) represents the case where an odd multiple of 2 no. of members voted against, so it does not require any farther analysis. But, case (i) represents (1a) the unanimity in favor of the decision and (1b) multiple of four (i.e., 4, 8, 12, 16, 20, . . .) no. of members voted against the decision.

2.: To distinguish between subcases (1a) and (1b) each member performs the following unitary operation \( \sigma_z(2) = \begin{pmatrix} 1 & 0 \\ 0 & \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \end{pmatrix} \) on rest of his \( R_2 = R_1 \setminus \{ s_1 \} \) qubits if he is ‘against’ the decision and if he is ‘in favor’ does nothing. (i) If an even multiple of 4 no. (i.e., multiple of eight) of members (including zero) against the decision all the desire copies of \( | \Psi_n \rangle \) remain same. (ii) Otherwise, (i.e., an odd multiple of 4 no. of members excluding zero against the decision) all of them reduce to the copies of \( | \Psi_{n'} \rangle \).

3.: Repeat step 1. to distinguish case (i) and (ii). Jury members randomly select one state (say \( s_2 \)) from \( R_2 \) and each member performs a \( \sigma_x \) measurement on his qubit associated with the selected state \( s_2 \) and announces the measurement outcome. If the product of local results is −1 then it is case (i) and if the product +1 then it is case (ii). Like previous here also, (ii) represents the case where an odd multiple of 4 no. (excluding zero) of members voted against so it does not require any farther analysis. But, case (i) represents (2a) the unanimity in favor of the decision and (2b) multiple of 8 (i.e., 8, 16, 24, . . .) no. of members voted against the decision.

4.: To distinguish subcases (2a) and (2b) repeat step 2. with unitary operation \( \sigma_x(3) = \begin{pmatrix} 1 & 0 \\ 0 & \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \end{pmatrix} \) on rest of the runs \( R_3 = R_2 \setminus \{ s_2 \} \). (i) If an even multiple of 8 (i.e., multiple of sixteen) no. of members (including zero) voted against the decision all the selected copies will remain in \( | \Psi_n \rangle \). (ii) Otherwise, (i.e., an odd multiple of 8 no. of members excluding zero against the decision) all of the copies reduce to the state \( | \Psi_{n'} \rangle \).

5.: Jury members repeat these steps. In general, to distinguish the case of even multiple of \( 2^t \) no. of members (including zero) favouring the decision and the odd multiple of \( 2^t \) no. of members against the decision, the required unitary operation will be \( \sigma_z(t) = \begin{pmatrix} 1 & 0 \\ 0 & \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \end{pmatrix} \). Since, the total number of jury members are finite so after a finite number of (repeated) steps they can detect whether there is any unanimity ‘in favor’ of the decision.

Note that in the entire protocol the identity of the member giving veto(es) is not revealed. The thing that is revealed is the information regarding the number of vetoes. Here also the security is guaranteed from the genuineness of the GHZ states.

If the number of jury members \( n (> 2) \) is even then members share copies of \( | \Psi_{n+1} \rangle \) with first jury member holds two qubits from each copy. Here, except the first jury member all the other members follow the similar protocol as described in case of odd no. of members. In each run, the first member treats the first qubit (from the pair of qubits he receives at each run) as earlier i.e., performs operation/measurement according to his choice of decision. Whereas, on the second qubit he always performs the operations related to ‘in favor’ case. Obviously, this arrangement does not provide any advantage to the first member and hence does not effect the objectivity of the protocol.

V. GENUINENESS CHECK OF GHZ STATE

Security of all the above mentioned protocols is solemnly depend on the genuineness of the corresponding GHZ state. Since, one can construct a secure even \((n−1)\)-parties QDC(QAV) protocol from an odd \( n\)-parties QDC protocol so here we describe only the genuineness check of GHZ states for odd \( n \). To check the genuineness of \( n\)-odd-qubit GHZ state each player \( j (j = 1, 2, \ldots, n) \) randomly selects some runs \( R_j \) (i.e. copies of the shared \( n\)-qubit GHZ state) and for each \( r \in R_j \) he again randomly chooses an operator \( O_{r,j} \in \{ O_{r,1}, \ldots, O_{r,n} \} \) where

\[
O_0 = \sigma_x^1 \otimes \cdots \otimes \sigma_x^{i-1} \otimes \sigma^j_x \otimes \sigma_x^{i+1} \otimes \sigma_x^{i+1} \otimes \cdots \otimes \sigma_x^n
\]

and

\[
O_i = \sigma_x^1 \otimes \cdots \otimes \sigma_x^{i-1} \otimes \sigma_y^{j} \otimes \sigma_y^{i+1} \otimes \sigma_x^{i+1} \otimes \cdots \otimes \sigma_x^n,
\]

for \( i = 1, 2, \ldots, n \) with the convention \( n + 1 \equiv 1 \). The upper indices on Pauli matrices represent the identity of the party. Now player \( j \) asks player \( t(t = 1, 2, \ldots, n) \) to measure his qubit (associated with the run \( r \)) in the basis that present in the \( t\)-th place of the operator \( O_{r,j} \) and send the measurement outcome. Player \( j \) checks whether or not the product of all the local measurement results (including his won measurement result) is equal to the eigenvalue \( \lambda_{r,j} \) of the eigenvalue equation

\[
O_i | \Psi_n \rangle = \lambda_i | \Psi_n \rangle,
\]

where \( \lambda_0 = (−1)^n \) and \( \lambda_i = +1 \) for \( i = 1, 2, \ldots, n \). The above relations provide a GHZ like contradiction with LR-theory for an \( n\)-qubit system when \( n \) is odd. By
employing relations given in [S] one can construct the following LR inequality for \( n \)- (odd) two level system.

\[
\mathcal{O} = \left| \sum_{i=1}^{n} \mathcal{O}_i - \mathcal{O}_0 \right| \leq (n - 1).
\] (9)

The two extreme eigenvalues of the operator \( \sum_{i=1}^{n} \mathcal{O}_i - \mathcal{O}_0 \) are \( \pm(n + 1) \) and the corresponding eigenstates are \( |\Psi^+_n\rangle \) and \( |\Psi^-_n\rangle \) respectively. Therefore, only \( |\Psi^+_n\rangle \) and \( |\Psi^-_n\rangle \) give us the maximum algebraic value of \( \mathcal{O} \) i.e., \( \mathcal{O} = n + 1 \), hence violets the inequality (9) maximally. Hence, for odd \( n \) the relations given in [S] uniquely determines the correlation of \( |\Psi^+_n\rangle \). Therefore, if the product of the local measurement results associated to the observable \( \mathcal{O}_j \) is equal to the eigenvalue \( \lambda_j \), then the correlation is a genuine \( n \)-qubit GHZ correlation.

**VI. CONCLUSION**

In conclusion, we have presented secure quantum protocols for both the Dining Cryptographers (DC) problem and the Anonymous Veto (AV) problem. The security of these protocols are based on GHZ paradox and the properties of the GHZ correlation. In our DC protocol multiple payments can be detected if there is any. Whereas, no classical protocol has this luxury with an unconditional security proof. We then generalize the DC problem for \( n \) number of members based on the assumption that no multiple payments would occur. By exploiting this generalized DC problem we have shown that the multi-qubit GHZ state allow us to find a simple solution for the Anonymous Veto problem.

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