A Global Fit to
Extended Oblique Parameters

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Abstract

The $STU$ formalism of Peskin and Takeuchi is an elegant method for encoding the measurable effects of new physics which couples to light fermions dominantly through its effects on electroweak boson propagation. However, this formalism cannot handle the case where the scale of new physics is not much larger than the weak scale. In this case three new parameters ($V, W$ and $X$) are required. We perform a global fit to precision electroweak data for these six parameters. Our results differ from what is found for just $STU$. In particular we find that the preference for $S < 0$ is not maintained.

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1. Introduction

As we impatiently await our first glimpse of physics beyond the standard model, an important task is to develop methods for parametrizing measurable effects of new physics. This activity constitutes the vital link between experiment and the actual calculation of the effects of specific underlying models. One such parametrization is the \textit{STU} treatment of Peskin and Takeuchi [1], the end product of which is a set of expressions for electroweak observables, consisting of a standard model prediction corrected by some linear combination of the parameters $S$, $T$ and $U$. The \textit{STU} formalism is applicable to the case of new physics which contributes to light-fermion scattering dominantly through changes to the propagation of the electroweak gauge bosons, and so is independent of the light-fermion generation. These contributions are sometimes called ‘oblique’ corrections. A wide variety of models can be parametrized in this type of analysis, such as technicolor models, multi-Higgs models, models with extra generations, and the like.

The \textit{STU} parametrization suffices when the scale of new physics $M$ is large enough to justify approximating the new-physics contributions to gauge-boson self energies at linear order in $q^2/M^2$. As is shown in [2], however, the formalism breaks down when this approximation of linearity fails, even if the dominant corrections are still of the oblique form. Ref. [2] extends the \textit{STU} formalism to the case of general oblique corrections, and shows that only three new parameters, $V$, $W$ and $X$, are required to parametrize present data. The necessity for only six parameters in all comes as something of a surprise, but is a consequence of the present limitation of precision electroweak measurements to momentum transfers $q^2 \approx 0$ and $q^2 = M^2_Z$ or $M^2_W$.

The main situation for which the complete \textit{STUVWX} formalism is pertinent is where the scale, $M$, of new physics is not large in comparison with the weak scale. Comparatively light scalars and fermions arise in a great many models, and can be potentially quite numerous in some of them, such as in SUSY models for example. Since these parameters are defined with an explicit factor of the electromagnetic fine-structure constant, $\alpha$, and since the gauge bosons couple universally with strength of order $g = e/s_w$, the contribution of any one of these light particles to the parameters $S$ through $X$ is expected to be of order $1/4\pi s_w^2 \approx 0.3$. The contribution of one or two electroweak multiplets of such states can be expected to therefore contribute an amount that is comparable to 1.

The complete parametrization in terms of $S - X$ can also be required even if the underlying scale happens to be large: $M \sim 1$ TeV. In this case the more complete parametrization is necessary if the observables are to be studied with a precision which is sensitive to
corrections\(^1\). One instance where this accuracy is required arises in the study of the loop-induced low-energy bounds on anomalous electroweak boson self couplings [4].

In this paper we perform a global fit to the current range of precision electroweak measurements using this extended set of parameters. We do so with two motivations in mind. First, we wish to determine the size of the constraints that are implied by a joint fit for the parameters \(S\) through \(X\). This permits the extraction of quantitative bounds on specific types of new physics that do not satisfy the assumptions of the \(STU\) parametrization, and indicates what kinds of models can be usefully constrained in this way. Given their conventional normalization, we find the parameters to be bounded to be \(O(1)\).

Our second motivation is to see how the inclusion of \(V\), \(W\) and \(X\) alters the previously-obtained bounds that have been obtained for \(S\), \(T\) and \(U\) [1]. In this case global fits tended to favour central values for the parameter \(S\) that were negative, with \(S = 1\) being excluded to the 2\(\sigma\) level. This conclusion was particularly interesting considering that many models of the underlying physics at scale \(M\), such as technicolour models, predict positive values for \(S\) and \(T\) [5]. Our more general fit finds that the preference for negative \(S\) no longer holds. In a joint fit for all six parameters we find that the 2\(\sigma\) allowed range for \(S\) becomes \(-4.3 < S < 2.5\).

2. Expressions for Observables in Terms of \(S\) through \(X\)

In so far as it is sufficient to encode new physics effects in gauge-boson self energies only, one can express electroweak observables as the usual SM prediction plus some linear combination involving new physics self energies \(\delta \Pi(q^2)_{ab}\), where \(a, b = W, Z,\) and \(\gamma\). In this case, to the extent that precision observables only probe \(q^2 \approx 0\) and \(q^2 = M_Z^2\) and \(M_W^2\), it turns out that one can express all corrections to electroweak observables in terms of six linearly independent combinations of the various \(\delta \Pi\)’s. The contributions to linear order in \(q^2\) may be parametrized by the three parameters \(S\), \(T\) and \(U\), defined by

\[
\frac{\alpha S}{4s_w^2c_w^2} = \left[ \frac{\delta \Pi_{ZZ}(M_Z^2) - \delta \Pi_{ZZ}(0)}{M_Z^2} \right] \frac{(c_w^2 - s_w^2)}{s_w c_w} \delta \Pi'_{Z\gamma}(0) - \delta \Pi'_{\gamma\gamma}(0),
\]

(1)

\[
\alpha T = \frac{\delta \Pi_{WW}(0)}{M_W^2} - \frac{\delta \Pi_{ZZ}(0)}{M_Z^2},
\]

(2)

\[
\frac{\alpha U}{4s_w^2} = \left[ \frac{\delta \Pi_{WW}(M_W^2) - \delta \Pi_{WW}(0)}{M_W^2} \right] - c_w^2 \left[ \frac{\delta \Pi_{ZZ}(M_Z^2) - \delta \Pi_{ZZ}(0)}{M_Z^2} \right] \\
- s_w^2 \delta \Pi'_{\gamma\gamma}(0) - 2s_w c_w \delta \Pi'_{Z\gamma}(0).
\]

(3)

\(^1\) The subdominant terms in the \(q^2\) expansion have been examined in Ref. [3].
where the prime (′) denotes differentiation with respect to \( q^2 \). The remaining contributions may be subsumed into the new parameters \( V, W \) and \( X \), defined as

\[
\alpha V = \delta \Pi'_{ZZ}(M_Z^2) - \left[ \frac{\delta \Pi_{ZZ}(M_Z^2) - \delta \Pi_{ZZ}(0)}{M_Z^2} \right],
\]

\[
\alpha W = \delta \Pi'_{WW}(M_W^2) - \left[ \frac{\delta \Pi_{WW}(M_W^2) - \delta \Pi_{WW}(0)}{M_W^2} \right],
\]

\[
\alpha X = -s_w c_w \left[ \frac{\delta \Pi_{Z\gamma}(M_Z^2) - \delta \Pi'_{Z\gamma}(0)}{M_Z^2} \right].
\]

Manifestly, these expressions would vanish if \( \delta \Pi_{ab}(q^2) \) were simply a linear function of \( q^2 \), in which the parametrization of new physics effects could be achieved adequately with the set \( STU \).

It is shown in Refs. [1] and [2] how to calculate the dependence of electroweak observables on the variables \( S \) through \( X \). In this analysis, as is commonly done, we take as numerical inputs the following three observables: \( \alpha \) as measured in low-energy scattering experiments, \( G_F \) as measured in muon decay, and \( M_Z \). These observables are chosen because they are the most precisely measured. With this choice, the parameters \( U \) appears only in the observables \( M_W \) and \( \Gamma_W \), and \( W \) only appears in \( \Gamma_W \).

In observables defined at \( q^2 \approx 0 \), only the usual parameters \( S \) and \( T \) contribute. For example, the effective value of the weak mixing angle, \((s_w^2)_{eff}\), as measured in various low-energy asymmetries, (such as atomic parity violation, the low-energy neutral current scattering ratio \( R = \sigma(\nu_{\mu}e)/\sigma(\bar{\nu}_{\mu}e), \text{etc.} \)) is given by

\[
(s_w^2)_{eff}(q^2=0) = (s_w^2)_{SM} + \frac{\alpha S}{4(c_w^2 - s_w^2)} - \frac{s_w^2 c_w}{c_w^2 - s_w^2} \alpha T,
\]

and the relative strength of the low-energy neutral- and charged-current interactions is given by

\[
\rho = \rho_{SM}(e, G_F, M_Z) (1 + \alpha T).
\]

As for measurements at the \( Z \) resonance, the effective weak mixing angle is given by

\[
(s_w^2)_{eff}(q^2 = M_Z^2) = (s_w^2)_{SM}(q^2 = M_Z^2) + \frac{\alpha}{4(c_w^2 - s_w^2)} S - \frac{c_w^2 s_w}{(c_w^2 - s_w^2)} \alpha T + \alpha X,
\]

\[\text{By contrast, a different choice of inputs – such as } M_Z, M_W \text{ and } \alpha \text{ for instance – would lead to } U\text{-dependence throughout all the neutral current observables.}\]
and an example of a correction to $Z$-decay is

$$\Gamma(Z \to \bar{\nu}\nu) = \Gamma_{SM}(Z \to \bar{\nu}\nu) (1 + \alpha T + \alpha V). \quad (10)$$

We thus see that $V$ describes a contribution to the overall normalization of the strength of the neutral-current interaction, while $X$ acts to shift the effective value of $(s^2_w)_{\text{eff}}$ measured at the $Z$ pole.

The $W$ boson mass and width are

$$M_W^2 = (M_{SM}^2(e, s_w, m_Z)) \left[ 1 - \frac{\alpha S}{2(c_w^2 - s_w^2)} + \frac{c_w^2 \alpha T}{(c_w^2 - s_w^2)} + \frac{\alpha U}{4s_w^2} \right], \quad (11)$$

$$\Gamma(W \to \text{all}) = \Gamma_{SM}(W \to \text{all}) \left[ 1 - \frac{\alpha S}{2(c_w^2 - s_w^2)} - \frac{s_w^2 \alpha}{(c_w^2 - s_w^2)} T + \frac{\alpha}{4s_w^2} U + \alpha W \right]. \quad (12)$$

As advertised, the parameter $W$ turns out to appear only in the expression for $\Gamma_w$.

A comprehensive list of expressions for the electroweak observables that we include in our analysis is given in Table I. These expressions consist of a radiatively corrected standard model prediction plus a linear combination of the six parameters $S$, $T$, $U$, $V$, $W$ and $X$. $\Gamma_z$ and $\Gamma_{b\bar{b}}$ are the total width and partial width into $b\bar{b}$; $A_{FB}(f)$ is the forward-backward asymmetry for $e^+e^- \to f\bar{f}$; $A_{pol}(\tau)$, or $P_\tau$, is the polarization asymmetry defined by $A_{pol}(\tau) = (\sigma_R - \sigma_L) / (\sigma_R + \sigma_L)$, where $\sigma_{L,R}$ is the cross section for a correspondingly polarized $\tau$ lepton; $A_e(P_\tau)$ is the joint forward-backward/left-right asymmetry as normalized in Ref. [6]; and $A_{L,R}$ is the polarization asymmetry which has been measured by the SLD collaboration at SLC [7]. The low-energy observables $g_L^2$ and $g_R^2$ are measured in deep inelastic $\nu N$ scattering, $g_V^e$ and $g_A^e$ are measured in $\nu e \to \nu e$ scattering, and $Q_w(Cs)$ is the weak charge measured in atomic parity violation in cesium.

There are several features in Table I worth pointing out. First, as has already been mentioned, due to the choice of numerical inputs ($\alpha$, $G_F$, $M_Z$), only the two parameters $S$ and $T$ contribute to the observables for which $q^2 \sim 0$; the parameter $U$ appears only in $M_w$ and $\Gamma_w$. The limit on $U$ comes principally from the $M_w$ measurement, since $\Gamma_w$ is at present comparatively poorly measured. For the same reason, the parameter $W$ is weakly bounded, since it contributes only to $\Gamma_w$. In addition to $S$ and $T$, observables on the $Z^0$ resonance are also sensitive to $V$ and $X$, which are expressly defined at $q^2 = M_Z^2$. Observables that are not explicitly given in Table I can be obtained using the given expressions. In particular the parameter $R$ is defined as $R = \Gamma_{\text{had}} / \Gamma_{l\bar{l}}$, and $\sigma_p^h = 12\pi \Gamma_{ee} \Gamma_{\text{had}} / M_Z^2 \Gamma_Z^2$ is the hadronic cross section at the $Z$-pole.
Expressions for Observables

\begin{align*}
\Gamma_Z &= (\Gamma_Z)_SM - 0.00961S + 0.0263T + 0.0194V - 0.0207X \text{ (GeV)} \\
\Gamma_{bb} &= (\Gamma_{bb})_SM - 0.00171S + 0.00416T + 0.00295V - 0.00369X \text{ (GeV)} \\
\Gamma_{ll^-} &= (\Gamma_{ll^-})_SM - 0.000192S + 0.000790T + 0.000653V - 0.000416X \text{ (GeV)} \\
\Gamma_{had} &= (\Gamma_{had})_SM - 0.00901S + 0.02007 + 0.0136V - 0.0195X \text{ (GeV)} \\
A_{FB}(\mu) &= (A_{FB}(\mu))_SM - 0.00677S + 0.00479T - 0.0146X \\
A_{pol}(\tau) &= (A_{pol}(\tau))_SM - 0.0284S + 0.0201T - 0.0613X \\
A_e(P_\tau) &= (A_e(P_\tau))_SM - 0.0284S + 0.0201T - 0.0613X \\
A_{FB}(b) &= (A_{FB}(b))_SM - 0.0188S + 0.0131T - 0.0406X \\
A_{FB}(c) &= (A_{FB}(c))_SM - 0.0147S + 0.0104T - 0.03175X \\
A_{LR} &= (A_{LR})_SM - 0.0284S + 0.0201T - 0.0613X \\
M_w^2 &= (M_w^2)_SM (1 - 0.00723S + 0.0111T + 0.00849U) \\
\Gamma_w &= (\Gamma_w)_SM (1 - 0.00723S - 0.00333T + 0.00849U + 0.00781W) \\
g_2^Z &= (g_2^Z)_SM - 0.00269S + 0.00663T \\
g_2^R &= (g_2^R)_SM + 0.000937S - 0.000192T \\
g_v^e(ve \rightarrow ve) &= (g_v^e)_SM + 0.00723S - 0.00541T \\
g_A^e(ve \rightarrow ve) &= (g_A^e)_SM - 0.00395T \\
Q_w^{133}(Cs) &= Q_w(Cs)_SM - 0.795S - 0.0116T
\end{align*}

| TABLE I |

Summary of the dependence of electroweak observables on \( S, T, U, V, W \) and \( X \). In preparing this table we used the numerical values \( \alpha(M_Z^2) = 1/128 \) and \( s_w^2 = 0.23 \).

3. Numerical Fit of \( STUVWX \)

We now determine the phenomenological constraints on \( STUVWX \) by performing a global fit to the precision data. The experimental values and standard model predictions of the observables used in our fit are given in Table II. The standard model predictions are taken from Ref. [8] and have been calculated using the values \( m_t = 150 \text{ GeV} \) and \( M_H = 300 \text{ GeV} \). The LEP observables in Table II were chosen because they are closest to what is actually measured, and are relatively weakly correlated. In our analysis we include the combined LEP values for the correlations [9].

In Table III are displayed the results of the fit. In the second column are shown the results of individual fits, obtained by setting all but one parameter to zero. The third column is a fit of \( STU \), with \( VWX \) set to zero. Finally, in column four, we give the results
| Quantity             | Experimental Value          | Standard Model Prediction |
|----------------------|-----------------------------|--------------------------|
| $M_Z$ (GeV)          | $91.187 \pm 0.007$ [10]     | input                    |
| $\Gamma_Z$ (GeV)     | $2.488 \pm 0.007$ [10]      | $2.490[\pm 0.006]$       |
| $R = \Gamma_{had}/\Gamma_{\ell\ell}$ | $20.830 \pm 0.056$ [10]   | $20.78[\pm 0.07]$       |
| $\sigma_p^h$ (nb) | $41.45 \pm 0.17$ [10]    | $41.42[\pm 0.06]$       |
| $\Gamma_{bb}$ (MeV)  | $383 \pm 6$ [10]          | $375.9[\pm 1.3]$        |
| $A_{FB}(\mu)$        | $0.0165 \pm 0.0021$ [10]  | $0.0141$                 |
| $A_{pol}(\tau)$      | $0.142 \pm 0.017$ [10]    | $0.137$                  |
| $A_e(P_\tau)$        | $0.130 \pm 0.025$ [10]    | $0.137$                  |
| $A_{FB}(b)$          | $0.0984 \pm 0.0086$ [10]  | $0.096$                  |
| $A_{FB}(c)$          | $0.090 \pm 0.019$ [10]    | $0.068$                  |
| $A_{LR}$             | $0.100 \pm 0.044$ [7]     | $0.137$                  |
| $M_W$ (GeV)          | $79.91 \pm 0.39$ [11]      | $80.18$                  |
| $M_W/M_Z$            | $0.8798 \pm 0.0028$ [12]  | $0.8793$                 |
| $\Gamma_W$ (GeV)     | $2.12 \pm 0.11$ [13]       | $2.082$                  |
| $g_L^2$              | $0.3003 \pm 0.0039$ [6]    | $0.3021$                 |
| $g_R^2$              | $0.0323 \pm 0.0033$ [6]    | $0.0302$                 |
| $g_A^c$              | $-0.508 \pm 0.015$ [6]     | $-0.506$                 |
| $g_V^c$              | $-0.035 \pm 0.017$ [6]     | $-0.037$                 |
| $Q_W(C_S)$            | $-71.04 \pm 1.58 [0.88]$ [14] | $-73.20$                |

**TABLE II**

Experimental values for electroweak observables included in global fit. The $Z$-pole measurements are the preliminary 1992 LEP results taken from Ref. [10]. The couplings extracted from neutrino scattering data are the current world averages taken from Ref. [6]. The values for standard model predictions are taken from Ref. [8] and have been calculated using $m_t=150$ GeV and $M_H=300$ GeV. We have not shown the errors in the standard model predictions associated with theoretical uncertainties in radiative corrections or with the uncertainty regarding the measurement of $M_Z$, since these errors are in general overwhelmed by experimental errors. The exception is the error due to uncertainty in $\alpha_s$, shown in square brackets. We include this error in quadrature in our fits. The error in square brackets for $Q_W(C_S)$ reflects the theoretical uncertainty regarding atomic wavefunctions [15] and is also included in quadrature with the experimental error.
| Parameter | Individual Fit | STU Fit | STUVWX Fit |
|-----------|----------------|---------|------------|
| $S$       | $-0.19 \pm 0.20$ | $-0.48 \pm 0.40$ | $-0.93 \pm 1.7$ |
| $T$       | $0.06 \pm 0.19$  | $-0.32 \pm 0.40$ | $-0.67 \pm 0.92$ |
| $U$       | $-0.12 \pm 0.62$ | $-0.12 \pm 0.69$ | $-0.6 \pm 1.1$ |
| $V$       | $-0.09 \pm 0.45$ | —       | $0.47 \pm 1.0$ |
| $W$       | $2.3 \pm 6.8$    | —       | $1.2 \pm 7.0$ |
| $X$       | $-0.10 \pm 0.10$ | —       | $0.10 \pm 0.58$ |

**TABLE III**

Global fits of $STUVWX$ to precision electroweak data. The second column contains the results of individual fits, obtained by setting all but one parameter to zero. The third column is a fit of $STU$ setting $VWX$ equal to zero, and the final column allows all parameters to vary simultaneously. We have shown the 1σ errors.

for the fit in which all six parameters were allowed to vary simultaneously.

The most important observation concerning these results is that all of the parameters are consistent with zero. In other words there is no evidence for physics outside the standard model. The second observation is that including $VWX$ in our fits weakens the constraints on $STU$. This can be seen graphically in Fig. 1 where we have plotted the 68% and 90% C.L. contours for $S$ and $T$. We show the results for the case in which the parameters $VWX$ have been set to zero as well as that in which they have been allowed to vary. Notice in particular that while the entire 1−σ allowed range for $S$ in the $STU$ fit satisfies $S < 0$, this is not true for the fit with all six parameters.

4. Conclusions

We have performed a global fit for the complete set of six oblique correction parameters, $S$ through $X$. This fit extends the results of previous fits for $S$, $T$ and $U$ to a much wider class of models for the underlying physics, including in particular new light particles which need not be much heavier that the weak scale. We find that these parameters are bounded by the data to be $\lesssim 1$, corresponding to an $O(1\%)$ correction to the weak-boson vacuum polarizations, $\delta \Pi(q^2)$. Such bounds are sensitive enough to constrain many models for new physics near the weak scale, much as did the original $STU$ analysis for technicolour models at the TeV scale.
We have also compared our joint fit of the six parameters $S$ through $X$ to a three-parameter fit involving only $S$, $T$, and $U$ (with $V=W=X=0$). Not surprisingly, we find in the general case that the allowed ranges for $S$, $T$, and $U$ are relaxed. In particular, the preference found in earlier fits for negative values for $S$ – which had been uncomfortable for many underlying models – are no longer present for the six-parameter fit.

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Figure Captions

• Figure 1: Constraints on $S$ and $T$ from a global fit of precision electroweak measurements. The solid line represents the 68% C.L. setting $VWX$ to zero, the dashed line represents the 90% C.L. setting $VWX$ to zero, the dotted line represents the 68% C.L. allowing $VWX$ to vary, and the dot-dashed line represents the 90% C.L. allowing $VWX$ to vary.
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