Exploration of the behavior of understanding mathematical concepts of junior high school students

I Gunawan\textsuperscript{1,2,*}, D Darhim\textsuperscript{1} and K Kusnandi\textsuperscript{1}

\textsuperscript{1}Department of Mathematics Education, Universitas Pendidikan Indonesia, Jl. Dr. Setiabudi No. 229, Bandung 40154, Indonesia, Indonesia
\textsuperscript{2}Department of Mathematics Education, Universitas Langlangbuana, Jl Karapitan No. 116, Bandung, Indonesia

*igun1978@student.upi.edu

Abstract. This study aims to explore the behaviour of understanding mathematical concepts of junior high school students. Six participants were taken from grade VIII students with high, medium and low ability. The method used in this research is qualitative with grounded theory approach. Five questions were given to the participants to measure students’ understanding of mathematical concepts. Then students were interviewed by questions referring to behavioral aspects. All the interview processes were documented in the video. The video was transcribed and then combined with the test result to be classified into predefined rubric. Based on the analysis of video and test results, three categories of students' behaviour of understanding mathematical concepts were obtained: “instrumentalist”, “semi relationalist” and “relationalist”. Also found the consistency of response performed by students in each category.

1. Introduction

One of the competencies that students must acquire after learning mathematics is an understanding of mathematical concepts. The concept is an abstract idea that is used to understand a fact by expressing the similarities and differences of it. In mathematics, for example, circles, numbers, and others are a concept. [1]. Conceptual understanding of mathematics is comprehension of mathematics concepts, operations and relationships, for example students construct it by linking with another [2]. The ability to understand concepts is essential because students who understand the concept can (1) look at the characteristics of it more deeply, (2) view information specifically in fast situations, and (3) represent a case and view the status using mental model [3]. Understanding of a concept can also be determined measure the knowledge of concepts and categorised in instrumental and relational [4], while the power of a student's understanding correlate with how much they can relate the ideas, facts and procedures of a concept [5]. Even the development of students' knowledge is stated that the event of conceptual occurs through a dynamic, recursive process and stratified. The stratified such as; (1) Primitive knowing, (2) Image making, (3) Image having, (4) Proficient noticing, (5) Formalizing, (6) Observing, (7) Structuring, and (8) Inventing [6]. Based on the opinions of these scholars, one student is said to understand the concept in mathematics if the student; First, can represent it in a different perspective. Second, be able to use the procedure skilfully, and fifth can associate with a concept.
Many researchers have conducted research related to the understanding of the concept. However, there is no study about the mathematics conceptual understanding behaviour even though behaviour is an indicator of learning outcomes. Behavior is essentially observable physical activity for example a pigeon pecks a disk, a student says reason from his answer, a student raises his hand, and so forth [7].

The sign of learning outcomes is the change of attitude. Behaviour referred to in this study is the response to stimuli given to students. Mathematics conceptual understanding behaviour is a response to the stimulus provided in the form of a written test related to its indicators. We can measure behavior through observation. In this study, researcher asked several research questions: (1) What behaviors are shown by six students in understanding mathematical concepts?, (2) Are the students consistently perform the behaviors?

2. Method

2.1. Method

The method used in this research is qualitative with grounded theory approach. A grounded theory is a research methodology qualitative theory that emphasizes the discovery of theories from empirical observation data in the field with inductive method, a grounded theory approach was selected because of the lack of knowledge regarding the specific factor and factor relationships that comprise the process of physical activity behavioral [8].

2.2. Participant

The purpose of this research is to examine the mathematics conceptual understanding behaviour of six junior high school students. Researchers will look at aspects of knowledge ownership, control, confidence, and affective. The results of the test and interview analysis will produce these aspects. Students will be categorized based on a predefined rubric. For example, if the students satisfy eight point’s aspects or 80% in the group of relational behaviour, then the students tend to behave relationalist in understanding the concept.

2.3. Instrument

The instruments are in the form of test questions and rubric of concept understanding behaviour. The test consists of 5 items to measure the ability to understand the concept of mathematics. The first question is directed to expose students’ ability in presenting concepts from different points of view. The second question is to reveal students’ ability to translate concepts into verbal and written languages or vice versa. The third issue to reveal the ability of students in predicting a pattern. Fourth question to reveal students 'ability in using skilled procedures, and the fifth question to uncover students' ability in relating a concept to another concept.

The behaviour of conceptual understanding is through a rubric prepared on the theories advanced by previous researchers. As reveals the categories of instrumental and relational concepts [4]. Nickerson showing the characteristics of people who understand the concept [3]. Pirie and Kieren views the depths of people who master a concept [5]. Hiebert and Carpenter argue that understanding is an action, processes, and outcomes of the work [9]. Sierpinska sees people who understand the characteristics of having faith, being comfortable, forgetting what they have understood, being able to communicate to others and have confidence [10]. Duffin and Simpson see the development of conceptual understanding [6]. Of all the theories made a rubric to express the behaviour of students who understand the concept of mathematics.

Six students were given tests to measure understanding of the broad concept and circumference of the circle. Then combine test results and learn math to create categorisation in selecting the six students. Selected students were interviewed and recorded using video; then the results were transcribed. The interview guide refers to the behavioural aspects of the following table 1.
Table 1. The rubric of mathematics conceptual understanding behavioral.

| Factor | Indicator | Instrumentalist | Semi relationalist | Relationalist |
|--------|-----------|-----------------|--------------------|---------------|
| Knowledge ownership | Previous knowledge | Not utilising prior knowledge | There is an attempt to utilise prior knowledge | Make use of prior knowledge |
| Represents concepts in different view | Cannot represent concepts from a different view | There is an error in representing a concept from a different point of view | Can represent a concept correctly from a different point of view |
| Translating concepts in the verbal language into writing and vice versa | Cannot translate a concept into verbal or written language | There is an error in translating a concept into verbal or written language | Can translate a concept into verbal and written language |
| Predicts trend patterns | Cannot predict the trend of a particular pattern | There is an error in predicting certain pattern trends | Can predict certain pattern trends |
| Use the procedure skilfully | Cannot use the procedure skilfully | There is a mistake in using the procedure | Can use the procedure skilfully |
| Linking one concept with another concept | Unable to associate a concept with another concept | There is an error in associating a concept with another concept | Can link a concept with another concept |
| Control | Metacognitive thinking | Metacognitive thinking does not appear, either in written or verbal communication | The metacognitive thinking appears verbally | Metacognitive thinking is evident in written and verbal responses |
| Belief | Belief in the concept he understands | Unsure of his conceptual understanding | Can solve confidently some of the problems related to the concept he understands | Sure can solve the problem related to the concept he understands |
| Affective | Confidence | Not confident | A little confident | Confidence |
| Fast and precise in showing his understanding | Slow in showing their understanding through answers to questions | Showing doubt in his or her understanding through answers to questions | Fast and precise in showing his understanding through answers to questions |

Researchers will look through the rubric whether the behaviour of students in understanding the concept of mathematics in accordance with the rubric. Then the researchers describe based on students’ perceptions.

3. Results and discussion
Researchers categorise the six students based on the results of concept comprehension skills test on the subject of a circle and its area, and combine it with the score of mathematical ability of the report cards. The categories are high, medium, and low. Here are the categorisation table 2.

Table 2. Participants group.

| KM and PKM Category | Student |
|---------------------|---------|
| High                | M Rafi F (S1), Trisa (S2) |
| Medium              | Laely (S3), Miftah (S4) |
| Low                 | Puspita (S5), Safa (S6) |
The results of the analysis of written tests and interviews, obtained: Students S1 are categorized on relationalist behaviour. The evidence can see from the knowledge ownership aspect. He can use his knowledge to solve problems, be able to present a concept correctly from a different point of view, able to translate an idea in verbal or written language and to predict pattern trends, to use procedures skillfully, and to relate concepts to one concept the other correctly. In another aspect, he was able to show a clear metacognitive way of thinking, both written and verbal. Seen from the element of belief, he shows his view in the concept he has understood. From the affective aspect, he reveals high confidence and quickly responded to questions related to understanding his idea.

He answers all written questions using the knowledge he already has. It shows the characteristic of someone who understands the concept as expressed [5] regarding the development of conceptual understanding at the level of first development, i.e. knowing. Written answers from students S1 can be seen at figure 1.

![Figure 1. Student answer representation for problem 1.](image)

Student S1 has been able to utilise previous knowledge related to the concept of the square area combined with the broad idea of a circle so that he can solve the problem correctly. Besides that he has been able to link one concept with another concept, able to arrange the procedure well, also can interpret the shaded area in the form of its language that is, the shaded area is subtracting the square area by the width of one circle. Here’s an excerpt of the interview:

P: “Explain how to answer number one?”
R: “The area of the shading is the area of the square minus the circle area, i.e. 784 minus 616 equals 168.”
P: “What is the square area?”
R: “Square area equal to s times s, side times side.”
P: “What is the area of the circle?”
R: “In general the formula is pi r squared.”
P: “Are you sure of that answer?”
R: “Sure”

The control aspect is visible from work, i.e. no traces of scratch as a sign of doubt. While confident and affective looks when answering questions in the interview. The student responded quickly, loudly, and correctly. The S1 student also can use the procedure skillfully. It appears in his written answer. He first looks for a square area, then searches the area of the circle, then looks for the shaded area. In reply
to the last question in an interview about his belief, he responded with confidence and said he was convinced.

After analysing the behavior of Students S2 and S3, seen understanding the concept of semi-relasionalis categorised. On their written test answers sheets, there are still wrong answers. Once confirmed through interviews, and motivated by some questions from concepts they have learned before, they respond immediately and can answer the questions correctly. Here’s an example of an answer to question number 5 can be seen at figure 2.

![Figure 2. Student control aspect representation.](image)

The answer sheet shows the effort of the S2 students to take advantage of the previous concept, namely the circumference of the circle. But still mistaken in utilizing it. Then he is still wrong in representing or viewing the concepts. He does not calculate the whole circumference of the circle when calculating the boundary of the shaded area. When calculating the entire circumference, there is an error in estimating the diameter. However, after being confirmed and directed, he deftly can improve his way of thinking and answer back until the answer becomes true. It shows metacognitive look verbally. The S2 student still looks hesitant for some concept he understands. This can be seen from the following interview,

P: “About number five?”
T: “About this number five there is a debate,”
P: “Why?”
T: “Because it is a circumference, this one is in or not (while showing the whole circle)”
P: “If I want to travel from here back here again (while pointing at the shaded area) is it okay?”
T: (Trini moving pencil shows the way around the shaded area, but the whole circle is skipping)
P: “I try to circle the shaded area here ... through this one or not?” (while showing the entire ring)
T: “No, uh. no need, uh but it could be.”
P: “yes,”
T: “oh ... yes, it counts,”
P: “so how to calculate this one ?,” (show the shaded area)
T: “Think first, so if to calculate the shaded circumference this adds this one and this one round the circle” (while showing the picture)
P: “how to find the circumference of that circle.”
T: “phi times diameter.”
P: “How long is the width?”
T: “er .. possibly wrong, I think the diameter is 28,”
P: “Why 28?”
T: “I think this circle is the same, its radius is 14, then the diameter of the whole circle is 28, but it seems different.”

From the interview, it appears that there is still doubt in understanding the concept of the circumference of the circle associated with other. However, once confirmed, the student S2 can immediately change his understanding quickly and can answer the problem with the correct one. S3 also have experience that behaviour.

Students S4, S5, and S6 show the instrumentalist behavioural categories. They cannot use prior knowledge to solve problems, cannot represent concepts from different perspectives even though they have a clue but still cannot reproduce the answer. In the process, the procedure does not look good. For problems that require a connection with other concepts, they cannot answer at all. They cannot predict patterns or its trends. In his work did not appear a scratch, but the answer given was wrong. When asked seem to have no confidence and slow in responding to questions. The results of their written test can be seen at figure 3.

![Figure 3. Student cannot use prior knowledge to solve problems.](image)

From the written answer does not appear the procedure, does not seem the use of prior knowledge. The answer is no scratch, but the result is wrong. In addition to the conclusion of written tests, we can also note from the following interview footage,

P: “What's the question on number 4?”
Pit: “the shaded area.”
P: “What is the name of the shaded region?”
Pit: “segment.”
P: “still remember its formula?”
Pit: “I forgot.”
P: “if the formula of the sector?”
Pit: “60 degrees per 360 degrees times? ... wide circle”
P: “What is the formula for the triangular area?”
Pit: “90 times b squared.”
P: “No, what kind of triangle?”
P: “Where’s the right angle?”
Pit: (silence, does not answer)
P: “What triangles do you know?”
Pit: “same side, equilateral,”
P: “If this is what triangle? Equilateral?”
Pit: “I am not sure,”

From the interview, it appears that S5 students cannot relate the concept of a triangle with a circle. He was unsure of the answers he raised and responded to questions very slowly. Thus he fulfils the aspects of instrumentalist behaviour. S4 and S6 students also have instrumentalist behaviour.

Based on the results of the analysis, it was found that students with high mathematical abilities and high understanding mathematical concepts, categorized had relationalist behavior. These findings are supported by the fulfillment of indicators on each aspect of understanding mathematical concept behavior. Then there is degradation both in cognitive abilities and in the fulfillment of behavioral indicators so that semirelationalist and instrumentalist categories emerge. It is undeniable that learning outcomes are a behavior, it is in line with the opinion [11] that learning is a change in behavior that can be observed as a result of communication between stimulus and response.

Students who are categorized have relationalist behavior, tend to be able to utilize prior knowledge this is in line with the findings [12], then can view the problem from a variety of different points of view in line with the opinion [13] related to divergent thinking. Indicators can translate in verbal language into written language supported by opinions [14]. While in the aspect of control, students who are relationalist behaves are able to think metacognitively both verbally and in writing this is in line with opinions [15]. The belief aspect is fulfilled when he has confidence in what he believes in line with the results of the study [16] related to the impact of beliefs on student behavior. And the apective aspect, students who relationalist behavior have high self-confidence and are quick and precise in responding to questions, this is supported by the results of the study [14]. Whereas semirelationalist and instrumentalist categories can be distinguished from the level of each aspect that appears in student behavior.

4. Conclusion
As a result of this study, there are three categories of behavioural understanding of the concept of junior high school students in Bandung based on aspects of behaviour. The three categories of behaviour are an instrumentalist, semi-relationalist, and relationalist. These categories correspond to aspects of knowledge, control, confidence, and affirmation ownership. Each category has its aspects according to the categorization of the understanding behaviour of mathematical concepts. There is consistency between one student and another. In each group, they show the same attitude.

Acknowledgment
This article is part of the dissertation, so thank to Mathematics teacher from Junior high school 7 Bandung.

References
[1] Barr H, Graham J, Hunter P, Keown P, and McGee J 1997 A position paper: Social studies in the New Zealand school curriculum (Hamilton: University of Waikato)
[2] Kilpatrick J, Swafford J, and Findell B 2001 Adding it up: Helping children learn mathematics (Washington DC: National Academy Press)
[3] Nickerson R S 1985 Understanding understanding American Journal of Education 93 2, 201-239
[4] Skemp 1976 Relational Understanding and Instrumental Understanding Mathematics Teaching 77 20–26
[5] Pirie S and Kieren T 1994 Growth in mathematical understanding: How can we characterize it and how can we represent it? *Educational Studies in Mathematics* 26 165-190

[6] Duffin J M and Simpson A P 2000 A Search for understanding *Journal of Mathematical Behavior* 18 4 415-427

[7] Bergner R M 2011 What is behavior? And so What? *New Ideas in Psychology* 29

[8] Strauss A and Corbin J 1998 *Basics of qualitative research, techniques and procedures for developing grounded theory* (Thousand Oaks CA: Sage)

[9] Hiebert J and Carpenter T P 1992 Learning and Teaching with Understanding in Grouws D A *Handbook of Research on Mathematics Teaching and Learning* (New York: Macmillan) pp 65-97

[10] Sierpinska A 1994 *Understanding in Mathematics* (London: Falmer)

[11] Aliakbari F et al. 2015 Learning Theories Application in Nursing Education *Journal of Education and Health Promotion* 4

[12] Silver E A and Smith J P 1980 Think of a Related Problem in S Krulik and R E Reys *Problem Solving in School Mathematics 1980 yearbook* (Reston, VA: National Council of Teachers of Mathematics) pp146–156

[13] Izzati N 2010 *Meningkatkan Kemampuan Berpikir Matematis pada Tingkat Koneksi dan Analisis Siswa MTs Negeri Melalui Pembelajaran Kolaboratif MURDER* (Tesis SPs UPI Bandung)

[14] Opolot C 2014 *Improving Communication Skills in Science and Mathematics Education for Quality Student Outcomes* (Makerere University: Uganda)

[15] Muir, T 2008 I’m not Very Good at Solving Problems”; An exploration of student’s problem solving behaviours *The Journal of Mathematical Behavior* 27 228-241

[16] Sokolov M 2017 Student Beliefs about Mathematics and Their Effect on Academic Performance *OCMA 37th Annual Conference 2017*