Determination of blur kernel for HR-pQCT with bilevel optimization

B.Sixou¹, Y.Li¹, F.Peyrin¹
¹CREATIS, INSA-Lyon; Inserm, U1044; CNRS UMR 5220; Université Lyon 1; Université de Lyon; 69621, Lyon, France
E-mail: bruno.sixou@insa-lyon.fr

Abstract. In this work, we determine the blur kernel of HR-pQCT scanner with bilevel optimization. The method is compared with an optimization of the kernel based on the \( L_2 \) distance followed by regularization with Total Variation. An improvement is obtained for the reconstructed image.

1. Introduction
X-ray CT techniques are well adapted to image bone structure. However, imaging bone micro architecture \textit{in vivo} remains challenging because it is difficult to achieve a high spatial resolution (order of 100\( \mu \)m) with a low X-ray dose. High Resolution peripheral Quantitative CT (HR-pQCT) providing images with spatial resolution of about 100-150\( \mu \)m have been developed recently[1]. However, the segmentation and the extraction of bone micro-structure parameters remains an issue since the size of the structures is of the same order than the spatial resolution. To improve the spatial resolution, super resolution techniques may be applied. However, the HR-pQCT blurring kernel involved in the image formation process is unknown and has to be determined. The aim of this work is to investigate the degradation kernel of HR-pQCT for given high resolution and low resolution images. We consider the image acquired from parallel-beam synchroton micro-CT as the high resolution image[2], and the image generated by HR-pQCT as the low resolution image. In a previous work, we have used \( L_2 \) norm and mutual information minimization without any regularization. The kernels obtained with these approaches are then used for a classical Total Variation regularization with real data to solve the super-resolution problem. [3, 4, 5].

We determine here the blur kernel of HR-pQCT with a bilevel optimization method. Bilevel approaches for learning variational models have been much developed recently in image processing and inverse problems[6, 7, 8, 9]. For example, bilevel optimization for finite dimensional Markov random field models is presented in [10, 11] A bilevel approach for blind deconvolution is discussed in [6]. In this work, the bilevel approach is used and formulated in a discrete setting. The regularization functional includes a non-differentiable TV term but a smoothed approximation is considered for numerical computation. This paper is organized as follows. After the introduction, the HR-pQCT inverse problem considered is summarized and it is formulated as a bilevel optimization problem. The method to solve the optimization problem based on the adjoint method is presented. Numerical results are displayed before concluding remarks.
2. Bilevel framework for the HR-pQCT imaging problem

2.1. The image problem formulation

The reconstruction of a 3D image with an improved resolution from a single low-resolution image can be expressed as follows. We assume the low and high resolution images are defined on a bounded region Ω. The low-resolution image is obtained from a high-resolution image with blur, down sampling and noise. The forward problem can thus be written as:

\[ g = U(h * f) + \epsilon \]  

where \( g \in \mathbb{R}^N \) denotes the \( N \)-voxels 3D low-resolution noisy image, and \( f \in \mathbb{R}^{N'} \) denotes an \( N' = N \times p^3 \)-voxels high-resolution image with super-resolution factor \( p = 2 \) in each dimension, \( \epsilon \) is the additive noise component, \( U \) is the undersampling operator and \( h \) the convolution kernel accounting for the blurring of the images. The down sampling operator is described as \( g[i,j,k] = f[2i−1,2j−1,2k−1] \) for all indexes \( i, j, k \). Other voxels of \( f \) are discarded during the sampling process. In this work, we do not try to optimize this operator \( U \).

2.2. Bilevel optimization formulation

In this work, we investigate a smoothed bilevel optimization approach. We focus on the following upper-level optimization problem:

\[ \min_h \tilde{J}(h) = \| f(h) - f_{\text{true}} \|^2 \]  

where \( f_{\text{true}} \) is the ground truth image obtained with high resolution CT. The image \( f(h) \) is defined as the solution of the smoothed lower level variational problem:

\[ f(h) = \arg \min_f \| U(h * f) - g \|^2 + \alpha \phi_\gamma(Df) \]  

for a regularization parameter \( \alpha > 0 \). In this formula, \( D \) denotes the discrete gradient and \( \phi_\gamma \) is a smoothing function approximating the \( L_1 \) norm.

We assume that the lower level optimization problem admits an unique solution.

With this regularization approach of the \( L_1 \) norm, the first order optimality condition can be easily derived \[7\]. In the following, we consider the \( C^2(\mathbb{R}, \mathbb{R}) \) Huber smoothing function:

\[ \phi_\gamma(t) = \begin{cases} \frac{3\gamma}{8}t^2 + \frac{3\gamma}{8}|t|^2 - \frac{1}{8\gamma^2}|t|^4 & |t| < \gamma \\ \frac{3\gamma}{2} |t|^2 & |t| \geq \gamma \end{cases} \]  

We have also

\[ \phi'_\gamma(t) = \begin{cases} \frac{-t^3}{2\gamma^3} + \frac{3t}{2\gamma} |t| < \gamma \\ \frac{3}{2\gamma} sgn(t) & |t| \geq \gamma \end{cases} \]  

\[ \phi''_\gamma(t) = \begin{cases} \frac{-3t^2}{2\gamma^3} + \frac{3}{2\gamma} |t| < \gamma \\ 0 & |t| \geq \gamma \end{cases} \]  

For a vector \( a = (a_1, ..., a_N)^t \), \( \phi_\gamma(a) = (\phi_\gamma(a_1), ..., \phi_\gamma(a_N))^t \in \mathbb{R}^N \)

2.3. Adjoint method.

The necessary and sufficient optimality condition can be written with \( E : \mathbb{R}^{N'} \times \mathbb{R}^{N'} \rightarrow \mathbb{R}^{N'} \)[7]:

\[ E(f, h) = h_\ast h * f - h_\ast U^t * g + \alpha D^t \phi'_\gamma(Df) = 0 \]
where \( h_\text{a} \) is the kernel defined by \( h_\text{a}(x) = h(-x) \) for \( x \in \Omega \). We will assume that the kernel is symmetric and \( h_\text{a}(x) = h(-x) = h(x) \) for \( x \in \Omega \).

In the following, we will admit that the constraint operator \( E \) is differentiable with smooth derivatives. Let \( v = (f, h) \), the first derivative of the mapping \( f \) in the direction \( \delta h \) is:

\[
 f'(h)\delta h = -E_f^{-1}(v)E_h(v)\delta h \tag{8}
\]

where \( E_f \) and \( E_h \) denote the derivatives of \( E \) with respect to \( f \) and \( h \) respectively. By the chain rule, we obtain the gradient of \( \hat{J} \) at a control \( h \):

\[
 \nabla \hat{J}(h) = J_h(h) - E^*_h(v)E_f^*(v)J_f(h) \tag{9}
\]

where \( E^*_h \) and \( E_f^* \) denotes the adjoints of the operators.

In the framework of the adjoint method, we introduce the variable \( p \) defined as

\[
 p = -E_f^*(v)J_f(v) \tag{10}
\]

and the gradient can be rewritten:

\[
 \nabla \hat{J}^k(h) = J_h(h) + E^*_h(v)p \tag{11}
\]

For every \( v = (f, h) \in \mathbb{R}^{N'} \times \mathbb{R}^{N'} \), \( E_f(f, h) : \mathbb{R}^{N'} \rightarrow \mathbb{R}^{N'} \) is given by:

\[
 E_f(f, h)w = h_\text{a} * h * w + \alpha D^t \phi^\alpha(Df) \otimes Dw \tag{12}
\]

where \( a \otimes b = (a_1 b_1, \ldots, a_n b_n) \). Let \( H \) the linear operator associated to the convolution by the operator \( h \). The function \( \phi^\alpha \) is positive and thus, assuming that \( \text{Ker} H = \{0\} \), we obtain that \( E_f \) is an homeomorphism.

With \( J_f = f - f_{\text{true}} \), \( p \) is the solution of the adjoint equation:

\[
 E_f^*(f, h)p = h_\text{a} * h * p + \alpha D^t \phi^\alpha(Df) \otimes Dp = -(f - f_{\text{true}}) \tag{13}
\]

We have also to compute the Fréchet derivative of the constraint with respect to \( h \) and its adjoint. The operator \( E_h(f, h) : \mathbb{R}^{N'} \rightarrow \mathbb{R}^{N'} \), is given by

\[
 E_h(\delta h) = \delta h_\text{a} * (h * f - g) + h_\text{a} * \delta h * f \tag{14}
\]

Its adjoint \( E_h(f, h)^* : \mathbb{R}^{N'} \rightarrow \mathbb{R}^{N'} \) is given by

\[
 E_h(f, h)^*u = u * (h * f - g)_\text{a} + f_\text{a} * h * u \tag{15}
\]

The evaluation of the gradient \( \nabla \hat{J}^k(h_k) \) at the point \( h_k \) can be summarized with the following algorithm:

1) Solve the low level optimality equation to obtain \( f_k \) from the current approximation \( (h_k) \) with Eq.7.
2) Solve the backward adjoint equation to obtain the adjoint variable \( p_k \), with Eq.13.
3) Return the gradient with respect to \( h \) with Eq.11.
3. Numerical simulation

3.1. Implementation details

In order to optimize the blurring kernel, we use an entire database of pairs of 3D ground truth and corresponding degraded images of trabecular bone[12]. The imaging data were collected for a custom-built bone structure phantom. The phantom was comprised of cadaveric distal tibia sections (1cm thick) embedded in an 8 cm diameter cylinder comprised of polymethylmethacrylate (PMMA) and polyethylene resin. The phantom was scanned using typical in vivo protocol settings by HR-PQCT (XtremeCt, Scanco Medical AG), which generates reconstruction with 82 μm isotropic voxels [1]. The reference scan data was obtained from parallel-beam synchrotron micro-CT [2] images which provided high resolution reconstructions, with 24 μm isotropic voxels.

After registration with B-spline interpolation, the voxel size was equal to 41 μm. In this work, we consider that images that are perfectly aligned to each others such that only a deconvolution and upscaling is necessary. The voxel size of the low and high resolution images are 82 μm and 41 μm respectively. The size of the HR-pQCT μCT images is 420 × 510 × 110, and 840 × 1020 × 220 respectively. For the training step two tibia samples are used in which 50 crops of size 100 × 100 × 100 and 200 × 200 × 200 are cut from the images. At the end of the training process, one test crop is selected. Some sections of the high and low resolution images for the test crop are displayed in Figure 1.

The second approach is detailed in [3]. The optimization of the kernel is performed with the minimization of the square of the $L^2$ norm $\|U(h \ast f) - g\|^2$. This is followed by a TV super resolution restoration with ADMM (Alternate Direction of Multipliers Method) with optimized parameters. This second method is similar to the traditional blind deconvolution techniques with an unknown convolution kernel [13]. The Otsu segmentation method is applied to calculate DICE, BV/TV (bone volume/total volume) for the two approaches in order to evaluate the quality of the reconstruction. The DICE is defined as $\frac{|f_b \cap f_b^*|}{|f_b^*|}$, where $f_b$ and $f_b^*$ are binary images of $f$ and $f^*$ respectively. $|f_b \cap f_b^*|$ counts the number of elements where the two binary images overlap. The BV/TV is the percentage of the bone volume (BV) with respect to total volume (TV) in the image. BV is simply calculated by counting the number of 1 in the segmented image.

3.2. Results and discussion

The reconstructed images with the bilevel approach is displayed in Figure 2, together with the estimated kernel. At the end of the training process, the relative norm change between the initial guess $h_0$ and the estimated kernel $\hat{h}$, $\frac{\|\hat{h} - h_0\|}{\|h_0\|}$ is 0.08. Table 1 summarizes the DICE, BV/TV for the two methods. As explained above, the DICE is a measure of the distance between the binary high resolution image and the binary reconstructed image. The bilevel method improves this error measure. For the high resolution image, the BV/TV value is 0.25. This structural parameter is also improved with the bilevel approach.

Promising results are obtained with the bilevel optimization scheme and the DICE and BV/TV are slightly improved. It should be noted that the regularization parameter is fixed during the algorithm and that it could be also optimized in a more general framework.

4. Conclusion

In this paper, we have used a bilevel approach to determine the kernel of HR-pQCT. The upper optimization problem is the $L_2$ norm distance between the image obtained with a smoothed TV regularization and the ground truth. This method is compared with a $L_2$ data term norm minimization followed by a TV regularization on bone micro-structure images. Promising results are obtained with the bilevel optimization. In future studies, the optimization of the
Figure 1: (a) High resolution image. (b) Low resolution image.

Figure 2: (a) Reconstructed image with bilevel approach. (b) Kernel obtained with bilevel optimization

Table 1: DICE, BV/TV for the two approaches. For the high resolution $\mu$CT image, the BV/TV is 0.25.

| kernel                  | DICE | BV/TV |
|-------------------------|------|-------|
| Bilevel optimization    | 0.78 | 0.32  |
| $L_2 + TV$              | 0.74 | 0.4   |

A regularization parameter will be inserted in the bilevel optimization method with a more general functional.
References

[1] S.Boutroy, M.L.Bouxsein, F.Munoz, P.Delmas In vivo assessment of trabecular bone microarchitecture by high-resolution peripheral quantitative computed tomography, Inverse Problems and Imaging, 90 (2005) 6508-6515.

[2] M.Salome, F.Peyrin, P.Cloetens, C.Odet, A.M.Laval-Jeantet, J.Baruchel and Per Spanne A synchrotron radiation microtomography system for the analysis of trabecular bone samples, Medical Physics, 26 (1999) 2194-2204.

[3] Y.Li, B.Sixou and F.Peyrin Estimation of the blurring kernel in experimental HR-pQCT images based on mutual information, European Signal Processing Conference, Eusipco, (2017) 2086-2090.

[4] A.Toma, L.Denis, B.Sixou, J.B.Pialat and F.Peyrin Total variation super-resolution for 3D trabecular bone micro-structure segmentation, European Signal Processing Conference, Eusipco, (2014) 2220-2224.

[5] A.Toma, B.Sixou and F.Peyrin Iterative choice of the optimal regularization parameter in TV image restoration, Inverse Problems and Imaging, 9 (2014) 860-867.

[6] M.Hintermuller and T.Wu Bi-level optimization for calibrating point spread functions in blind deconvolution, Inverse Problems in Imaging, vol.9, 1139-1169, 2015.

[7] K.Kunisch and T.Pock “A bi-level optimization approach for parameter learning in variational models”, SIAM J.Imaging Sci., vol.6, 938-983, 2013.

[8] J.C.De Los, Reyes, C.B.Schonlieb, T.Valkonen “The structure of optimal parameters for image restoration problems”, Journal of Mathematical Analysis and Applications, vol.434, 464-500, 2016.

[9] Y.Chen, T.Pock and H.Bischof “Learning of $l_1$ based analysis and synthesis sparsity priors using bi-level optimization”, Workshop on Analysis Operator Learning vs.Dictionary learning, NIPS 2012.

[10] Y.Chen, R.Ranftl and T.Pock Insights into analysis operator learning: from patch-based sparse models to higher-order MRFs, IEEE Transactions on Image Processing, 23 (2014) 1060-1072.

[11] S.Roth and M.J.Black Fields of experts: a framework for learning image priors, Computer Vision and Pattern Recognition, 2 (2005) 860-867.

[12] A.J.Burghardt, J.B.Pialat, G.J.Kazakia, S.Boutroy, K.Engelke, J.M.Patsch, A.Valentinitsch, D.Liu, E.Szabo, C.E.Bogado Multicenter precision of cortical and trabecular bone quality measures assessed by high-resolution peripheral quantitative computed tomography, Journal of Bone and Mineral Research, 28 (2013) 524-536.

[13] M.Burger and O.Scherzer Regularization methods for blind deconvolution and blind source separation problems, Math. Control Signal Syst., 14 (2001) 358-383.