Heavy monopole potential in gluodynamics

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We discuss predictions for the interaction energy of the fundamental monopoles in gluodynamics introduced via the ’t Hooft loop. At short distances, the heavy monopole potential is calculable from first principles. At larger distances, we apply the Abelian dominance models. We discuss the measurements which would be crucial to distinguish between various models. Non-zero temperatures are also considered. Our predictions are in qualitative agreement with the existing lattice data. We discuss further measurements which would be crucial to check the model.

1. HEAVY MONOPOLE

The fundamental monopoles can be introduced via the ’t Hooft loop \cite{1} on the lattice and correspond to point-like objects in the continuum limit. On the lattice the monopoles are identified with the endpoints of the Dirac strings which in turn are defined as piercing negative plaquettes. In more detail, consider the standard Wilson action of $SU(2)$ lattice gauge theory:

$$S_{\text{lat}}(U) = -\beta \sum_p \frac{1}{2} \text{Tr} U_p .$$

Then the ’t Hooft loop is formulated (see, e.g. \cite{2,3} and references therein) in terms of a modified action $S(\beta, -\beta)$:

$$S(\beta, -\beta) = -\beta \sum_{p \in \Sigma_j} \frac{1}{2} \text{Tr} U_p + \beta \sum_{p \in \Sigma_j} \frac{1}{2} \text{Tr} U_p ,$$

where $\Sigma_j$ is a manifold which is dual to a surface spanned on the monopole world-line $j$. Introducing the corresponding partition function, $Z(\beta, -\beta)$ and considering a time-like planar rectangular $T \times R$, $T \gg R$ contour $j$ one can define

$$V_{m\bar{m}}(R) \equiv -\frac{1}{T} \ln \frac{Z(\beta, -\beta)}{Z(\beta, \beta)} .$$

Since the external monopoles become point-like particles in the continuum limit the potential $V_{m\bar{m}}(R)$ is the same fundamental quantity as, say, the heavy-quark potential $V_{Q\bar{Q}}$ related to the expectation value of the Wilson loop. By analogy, we will call the quantity $V_{m\bar{m}}(R)$ the heavy monopole potential. First direct measurements of $V_{m\bar{m}}(R)$ on the lattice were reported \cite{2,3}. Motivated by these measurements we review the theoretical predictions \cite{4-6} on the heavy monopole potential.

2. POTENTIAL AT $T = 0$

Consider first the potential $V_{m\bar{m}}(R)$ at short distances. Then the interaction between the monopoles is Coulomb-like (see \cite{4,5}),

$$V_{m\bar{m}}^{\text{Coul}}(R) = -\left( \frac{2\pi}{g(R)} \right)^2 \frac{1}{4\pi R} = -\frac{\pi}{g^2(R)} \frac{1}{R} ,$$

where $g(R)$ is the running coupling of the gluodynamics. This equation makes manifest that monopoles in gluodynamics unify Abelian and non-Abelian features. Namely, the overall coefficient, $\pi/g^2$ is the same as in the Abelian theory with fundamental electric charge $e = g$ while the running of the coupling $g^2$ is governed by the non-Abelian interactions.
As far as the physics of short distances is concerned, the next step is to consider power corrections to \( \mathcal{O}(\mu^2) \). Theoretically, the prediction is that there is no linear in \( R \) term at small \( R \):

\[
V_{mn}(R) \approx V_{mn}^{\text{Coul}}(R) + a_0 \Lambda_{QCD} + a_2 \Lambda_{QCD}^3 R^2 + \ldots,
\]

where \( a_{0,2} \) are constants sensitive to the physics in the infrared. One can show that the absence of the linear in \( R \) term at short distances follows from general principles and eventually is related to the fact that the monopoles are not confined.

It is a common point that at large distances \( V_{mn}(R) \) is of the Yukawa type,

\[
V_{mn}(R) = - C \cdot \frac{e^{-\mu R}}{R},
\]

see, e.g., [2, 3]. At a closer look, however, there is a variety of model dependent predictions for the parameters \( C \) and \( \mu \).

Historically, the first prediction for \( \mu \) was obtained in [4]. Namely, it was shown that to any order in the strong coupling expansion the mass \( \mu \) coincides with the mass of \( 0^{++} \) glueball, \( \mu = m_G \).

As for the constant \( C \), there is no reason for it to be the same for the Yukawa and Coulomb like potentials (i.e., \( C \neq \pi/g^2 \)).

A different prediction arises within effective field theories with monopole condensation. Note that the monopoles which condense are of course not the fundamental monopoles which are introduced via the ’t Hooft loop as external probes. To describe the condensation one introduces a new (effective) field \( \phi \) interacting minimally with the dual gluon, \( B \), (for review and references see, e.g., [5, 6]). Within this model,

\[
\mu = m_V, \quad C = \pi/g^2,
\]

where \( m_V \) is the mass of the vector field \( B \) acquired through the Higgs mechanism.

It is worth emphasizing that \( m_V \) can well be below the lightest glueball mass, \( m_V < m_G \). Indeed, because of the presence of the Dirac string there is no dispersion representation for the ’t Hooft loop granted. Observation of \( m_V < m_G \) via the lattice measurements would be an amusing demonstration of existence of the Euclidean “masses” which have no direct counterpart in the Minkowski space.

### 3. Potential \( T \neq 0 \)

The screening mechanism at high temperatures is the Debye screening. Using the fact that classical limit of the state created by ’t Hooft loop is an Abelian monopole pair [7] and utilizing the Abelian dominance conjecture, one can estimate the mass \( \mu(T) \) as \( \mu^2 = m_D^2 = \frac{\pi^2 \rho}{\varepsilon_3^2} \), where \( \rho \) is the density of the Abelian monopoles and \( \varepsilon_3(T) \) is the three-dimensional coupling constant corresponding to the dimensionally reduced model. This formula is valid at asymptotically high temperatures which ensure that the monopoles form low density gas and the dimensional reduction works well. Below we used the results obtained in the Maximal Abelian projection [8].

To estimate the temperature dependence of \( m_D \) we use the numerical results of Ref. [9], where the density of Abelian monopoles was obtained,

\[
\rho = 2^{-7}(1 \pm 0.02) \varepsilon_0^2.
\]

At the tree level one can express 3D coupling constant \( \varepsilon_3 \) in terms of 4D Yang–Mills coupling \( g \), \( \varepsilon_3^2(T) = g^2(\Lambda, T) T \), where \( g(\Lambda, T) \) is the running coupling calculated at the scale \( T \) and \( \Lambda \) is a dimensional constant which can be determined from lattice simulations.

At present, the lattice measurements of the \( \Lambda \) parameter are ambiguous and depend on the quantity which is used to determine it. We use two ”extreme” values: \( \Lambda = 0.262(18) T_c \) obtained in Ref. [10] and \( \Lambda = 0.076(13) T_c \).

### 4. Comparison with Data

Here we summarize briefly the comparison of the lattice measurements with predictions above.

**Short distances.** The Coulomb-like potential [3] is confirmed in the numerical simulations in the classical approximation [2]. There is no running of \( g^2 \) on this level of course. As for the full quantum simulations the normalization [3] of the potential at short distances is confirmed within a factor of about 2, Ref. [11]. As for the power corrections, all the data so far [3] are fitted smoothly with a Yukawa potential [12].

**Screening mass at \( T = 0 \).** In Figure 1 the dual gluon mass [13], \( m_V \approx 1 GeV \), and the mass of the lightest \( 0^{++} \) glueball [14] are shown by cross and star respectively. Existing data [3, 15] seem to
be not accurate enough to distinguish between the glueball exchange and monopole condensation models. Moreover, no checks of the vector-particle exchange have been made.

**Temperature dependence of the screening mass.**

In Figure 1 we have summarized the existing data on temperature dependence of the screening mass together with our predictions, Section 3. The direct measurement of the screening mass \[ \frac{m}{T} \] is shown by the diamonds and squares, respectively. The theoretically expected temperature dependence of the screening mass (described in the previous Section) is denoted by the shaded region. Although numerically the screening mass seems to be systematically higher than is predicted, the prediction based on the Abelian dominance hypothesis is rather close to the numerical results within the errors.

Figure 1. Temperature dependence of the screening mass in the heavy monopole potential \[ \frac{m}{T} \].

The triangles in Figure 1 denote the results of Ref. \[ \text{[8]} \] obtained in the Maximal Abelian projection using an effective Abelian monopole action. At small temperatures these results are quantitatively in agreement with Refs. \[ \text{[2,3]} \] while at higher temperatures the screening mass obtained in Ref. \[ \text{[8]} \] falls essentially lower. As is explained in Ref. \[ \text{[8]} \], the prediction of Ref. \[ \text{[8]} \] is justified at very low and very high temperatures only.

To summarize, the heavy monopole potential at small distances is of the Coulomb type with a known overall normalization. The large–distance potential is of the Yukawa type. According to the Abelian dominance model, the screening mass is the same as entering the effective Ginzburg-Landau Lagrangian relevant to the confinement. The temperature dependence of the screening mass can also be predicted. The existing lattice data do not contradict the predictions but allow only for a qualitative tests of the model. Further measurements are desirable for quantitative tests.

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