Theoretical Analysis of In-Band Full-Duplex Radios With Parallel Hammerstein Self-Interference Cancellers

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Abstract—In-band full-duplex (IBFD) communication systems utilize self-interference cancellation to mitigate high-power self-interference caused by simultaneous transmission and reception at the same frequency in the digital baseband domain. Self-interference is distorted by transceiver nonlinearity. Thus, the IBFD literature includes reports of nonlinear self-interference cancellers developed to achieve better cancellation performance. However, there are no detailed theoretical studies analyzing the performance of nonlinear cancellers in IBFD systems. In this work, we develop a theoretical analysis technique for IBFD systems using parallel Hammerstein self-interference cancellers. The nonlinear characteristics of the system are expanded by a generalized Fourier series using orthonormal Laguerre polynomials. Then, the canceller’s performance and the system’s symbol error rate (SER) are analyzed using the obtained Fourier coefficients. The analytical results are compared with simulation results, demonstrating good correlation in a wide range of situations, from extremely nonlinear cases to good linear cases. Additionally, we show that the SER of the IBFD system is reduced by moderately nonlinearizing rather than linearizing the amplifier.

Index Terms—Full-duplex radio, self-interference, digital cancellation, amplifier nonlinearity, theoretical analysis.

I. INTRODUCTION

A S DEMAND for wireless communications increases unabated, achieving efficient frequency utilization is an ongoing challenge. In-band full-duplex (IBFD) [2]–[6], in which the same frequency band is used for simultaneous transmission and reception, definitely will cause a paradigm shift in wireless communication. However, IBFD has yet to be implemented in traditional wireless systems, primarily because of high-power self-interference caused by the signal transmitted by the terminal. In the IBFD literature, self-interference cancellation has been achieved using TX/RX isolation [7]–[10], analog domain cancellation [11]–[13], and digital domain cancellation [14]–[19]. Digital cancellation is the final step in a series of cancellation processes. Thus, the performance of an IBFD system is determined by the performance of the digital canceller.

Additionally, transceiver nonlinearity is one of the main sources of performance degradation in full-duplex systems. This nonlinearity is caused by the non-ideality of analog circuits such as amplifiers, I/Q mixers, analog-to-digital (A/D) and digital-to-analog (D/A) converters. Since self-interference is affected by these nonlinearities, the self-interference canceller needs to eliminate the distorted self-interference signal. Thus, we need a mechanism for cancelling nonlinear self-interference to successfully implement IBFD.

The parallel Hammerstein canceller is one of most well-studied nonlinear cancellers in the IBFD literature. This type of canceller was initially developed to deal with amplifier nonlinearity [4], [20], but versions have subsequently been developed to deal with IQ imbalance [14], [18], [21] and crosstalk in multiple-input and multiple-output (MIMO) systems [22]. These papers [4], [14], [18], [20]–[22] have analyzed the performance of such Hammerstein cancellers using computer simulation. However, the literature does not contain any detailed theoretical analyses, or comparisons between simulation and theoretical results.

A. IIP-Based Distortion Analysis

In the conventional radio-frequency (RF) engineering literature, the distortion from an RF component is usually calculated from the input intercept point (IIP). The power of the \( n \)-th order distortion from an RF component can be estimated as

\[
D_n = \frac{P_{\text{out}}}{(\text{IIP}_n/P_{\text{in}})^{n-1}},
\]

where \( P_{\text{in}} \) and \( P_{\text{out}} \) are the input and output power of the RF component, respectively, and \( \text{IIP}_n \) is the \( n \)-th order IIP of the component. Some papers [20], [21], [23] have calculated the power of distortions caused in a transceiver using the IIP-based method.

Although this method is suitable for simple estimations of distortion power, it cannot be used in applications that require detailed analysis. Figure 1 shows the power growth of
power is computed by simulation. In the OFDM case, each component power is the input to a Rapp model [24] with AM-AM characteristic of linear and nonlinear components with a two-tone signal and Fig. 1. The linear and nonlinear component powers of two-tone and OFDM KOMATSU et al. contributions and peak-to-average-power ratio (PAPR) of these behaviors are not represented in (1).

Additionally, the results of the two-tone test are not useful for analyzing OFDM systems because the probability distributions and peak-to-average-power ratio (PAPR) of these two signals are completely different. In Fig. 1, the third-order component of the OFDM signal is about 10 dB or larger than that of the two-tone signal since the OFDM signal is more susceptible to nonlinear distortion due to its higher PAPR. As in the examples we have shown so far, the IIP-base analysis cannot be used for applications that require detailed analysis of current wireless systems.

B. Contributions

State-of-the-art studies of the theoretical analysis on in-band full-duplex radios [20], [21], [23] have calculated the power of distortions using IIP-based method. These studies are very useful and very important, e.g., for the design of full-duplex terminals and digital cancellers, as they allow easy estimation of distortion power using the IIP. However, as written in Section I-A, IIP-based method is not an exact analysis and involves rough approximations. Thus, these studies only analyze power level of distortions, and these techniques cannot calculate the performance of in-band full-duplex communications such as symbol error rate (SER) or bit error rate (BER). In this paper, we use a generalized Fourier series expansion with orthonormal Laguerre polynomials to analyze the performance of in-band full-duplex radios with parallel Hammerstein cancellers. While amplifiers are modeled with a few parameters of IIP in the papers [20], [21], [23], we model them as arbitrary functions, which increases the degree of freedom of analysis. Arbitrary functions used here are mathematically infinite dimensional vectors, and our method has infinite dimensional degrees of freedom for modeling of memoryless amplifiers. The core of our analysis is based on studies of OFDM systems in half-duplex communication [25]–[29]. We extend them to include the analysis of nonlinear self-interference cancellers and in-band full-duplex systems that have nonlinear amplifiers in their transmitters and receivers. The proposed method can be used to analyze both the performance of parallel Hammerstein cancellers and the SER of in-band full-duplex systems. In addition, we show that the theoretical and simulation results match well each other.

C. Organization and Notations

The rest of this paper is organized as follows: In Section II, we provide a summary of papers [25]–[29] on nonlinear analysis, and present additional useful theorems for analysis of parallel Hammerstein cancellers. In Section III, we present our detailed analysis method for in-band full-duplex transceivers with parallel Hammerstein cancellers, which is the main contribution of this paper. In Section IV, results obtained using this analysis approach are compared with results from equivalent baseband simulations, and discussed in detail. Finally, Section V concludes the paper.

In this paper, the complex Gaussian distribution with mean $\mu$ and variance $\sigma^2$ is denoted $\mathcal{CN}(\mu, \sigma^2)$, and the exponential distribution with mean $\rho^2$ is denoted $\text{Exp}(\rho^2)$, i.e., $|x|^2 \sim \text{Exp}(\rho^2)$ when $x \sim \mathcal{CN}(0, \sigma^2)$. The expected value of a random variable $R$ is written as $E[R] = \mathbb{E}$. The binary operator $*$ denotes convolution. For a matrix $M$, the transpose of $M$ is denoted by $M^T$, and the inverse of $M$ is denoted by $M^{-1}$. $I_n$ denotes an $n \times n$ identity matrix, and $0_{m \times n}$ denotes an $m \times n$ zero matrix. We write the Fourier transform for a signal $s(t)$ as $\mathcal{F}[s(t)]$. A tilde above a variable (e.g., $\tilde{x}$) indicates it to be a coefficient of a generalized Fourier series expansion with orthonormal Laguerre polynomials, as defined in Section II. In contrast, a dot above a variable (e.g., $\dot{a}$) indicates it to be a coefficient of a power series expansion. The variables used in this paper are listed in TABLE I.

II. THEORETICAL BACKGROUND AND THEOREMS

In this section, we describe a generalized Fourier series expansion and some related theorems that form the core of our theoretical analysis. In Section II-A, we provide a summary of papers [25]–[29] relevant to the analysis of the nonlinearities shown in Fig. 2. In Section II-B, we describe some useful theorems for analyzing nonlinear self-interference cancellers using the signal model of Fig. 3.
TABLE I

| Variable | Defined by | Mean |
|----------|------------|------|
| \(c_n(t)\) | Fig. 4, Eq. (44) | Regenerated self-interference signal by the canceller on the \(n\)-th terminal |
| \(\bar{c}_p(\tau), \bar{C}_p(f)\) | Eq. (27) | Response related to \(\psi_p(x)\) of the nonlinear canceller related on Fig. 3 |
| \(c_p(\tau)\) | Eq. (25) | Response related to \(x|x|^{p-1}\) of the nonlinear canceller on Fig. 3 |
| \(d_n(t)\) | Eq. (15), Eq. (17) | Distorted output signal from \(\alpha(x(t))\) |
| \(D_{ij}(f)\) | Eq. (52) | Power spectral density of the distorted signal from the \(i\)-th terminal to the \(j\)-th terminal |
| \(g_n\) | Fig. 4 | Gain of the VGA on the \(n\)-th terminal |
| \(h_{p}(\tau), \bar{H}_{p}(f)\) | Eq. (21) | Response related to \(\psi_p(x)\) of the parallel Hammerstein model on Fig. 3 |
| \(h_{\psi}(\tau)\) | Eq. (31) | Response related to \(x|x|^{p-1}\) of the parallel Hammerstein model on Fig. 3 |
| \(h_{ij}(\tau), H_{ij}(f)\) | Eq. (33), Eq. (35) | Channel response from the \(i\)-th terminal to the \(j\)-th terminal observed by baseband signal processors |
| \(T_{in}\) | Eq. (47) | Mean power of the self-interference before digital cancellation on the \(n\)-th terminal |
| \(T_{in}^R\) | Eq. (48) | Mean power of the residual self-interference after digital cancellation on the \(n\)-th terminal |
| \(T_{in}^Q\) | Eq. (53) | Power spectral density of the residual self-interference after digital cancellation on the \(n\)-th terminal |
| \(T_{in}^R(f)\) | Eq. (59) | Mean power spectral density of the residual self-interference after digital cancellation on the \(n\)-th terminal |
| \(l_{m,i}\) | Eq. (4) | The \(i\)-th coefficient of the power series expansion of \(\psi_{2m+1}(x)\) |
| \(P\) | Eq. (3) | Generalized Laguerre polynomial |
| \(N_{NL,n}\) | Eq. (39) | Mean power of \(x_{NL,n}(t)\) |
| \(N_{thermal}\) | Eq. (49) | Power of the thermal noise |
| \(N_{thermal}(f)\) | Eq. (49) | Power spectral density of the thermal noise |
| \(N_{out,n}\) | Eq. (49) | Mean power of the total noise \(x_{out,n}(t)\) on the \(n\)-th terminal |
| \(N_{out,n}(f)\) | Eq. (54) | Power spectral density of the total noise \(x_{out,n}(t)\) on the \(n\)-th terminal |
| \(P\) | Eq. (64) | Order of the nonlinear canceller |
| \(R_{ii}(\cdot)\) | Eq. (61) | Cross-correlation function or auto-correlation function |
| \(SER_n\) | Eq. (61) | Mean symbol error rate (SER) on the \(n\)-th terminal |
| \(SICR_n\) | Eq. (66) | Performance of the self-interference canceller on the \(n\)-th terminal |
| \(SIDNR_n(f)\) | Eq. (50) | Signal to interference, distortion, and noise ratio (SIDNR) on the \(n\)-th terminal |
| \(U_{in}(f)\) | Eq. (53) | Power spectral density of the useful signal from the i-th terminal to the j-th terminal |
| \(U_{in}(f)\) | Eq. (60) | Mean power spectral density of the useful signal from the i-th terminal to the j-th terminal |
| \(x_{in}(t)\) | Fig. 4, Eq. (45) | Residual signal after self-interference cancellation on the \(n\)-th terminal |
| \(y_{in}(t)\) | Fig. 4, Eq. (35) | Received baseband signal on the \(n\)-th terminal |
| \(z_n(t)\) | Eq. (33) | Thermal noise on the \(n\)-th terminal |
| \(z_{NL,n}(t)\) | Eq. (37) | Distorted signal from the LNA on the \(n\)-th terminal |
| \(z_{out,n}(t)\) | Eq. (43) | Total noise: thermal noise plus distorted signal from the LNA on the \(n\)-th terminal |
| \(\alpha(x)\) | Fig. 2 | Nonlinear transfer function on Fig. 2 |
| \(\alpha_p\) | Eq. (9) | The \(p\)-th coefficient of the generalized Fourier series expansion of \(\alpha(x)\) |
| \(\alpha_n(x)\) | Fig. 4 | Transfer function of the PA on the \(n\)-th terminal |
| \(\alpha_{m,n}(p)\) | Eq. (42) | The \(p\)-th coefficient of the generalized Fourier series expansion of the \(n\)-th transmitter’s transfer function \(\alpha_n(g_{in})\) |
| \(\beta_p\) | Eq. (4) | Transfer function of the LNA on the \(n\)-th terminal |
| \(\beta_{NL,n}\) | Eq. (38) | Linear gain of the LNA on the \(n\)-th terminal |
| \(\lambda_{SIR,n}(f)\) | Eq. (50) | Inverse of the signal to distortion ratio (SIR) on the \(n\)-th terminal |
| \(\lambda_{SNR,n}(f)\) | Eq. (57) | Inverse of the signal to interference plus noise ratio (SNR) on the \(n\)-th terminal |
| \(\rho_n\) | Eq. (40) | Transmission power from the \(n\)-th terminal |
| \(\rho_p\) | Eq. (40) | Gain of the response \(H_p(f)\) |
| \(\rho_{ij}(\tau)\) | Eq. (36) | Channel gain of \(h_{ij}(\tau)\) |
| \(\psi_p(x)\) | Eq. (2) | Orthonormal polynomial for \(CN(0, 1)\) |
| \(\psi_p(f)^2\) | Eq. (12), Eq. (64) | Power spectral density of \(\psi_p(x)\) |

where \(L_m^1(z)\) is a generalized Laguerre polynomial defined as

\[
L_m^1(z) = \sum_{n=0}^{m} (-1)^n \frac{(m+1)(m+2)\cdots(m+n)}{n!} z^n,
\]

and the coefficients \(l_{m,i}\) can be expressed as

\[
l_{m,i} = (-1)^{i+m} \frac{i!}{\sqrt{m+1}} \frac{m+1}{i+1}.
\]

Equation (2) is orthonormal with the following inner product:

\[
\langle \psi_p(x), \psi_q(x) \rangle = \mathbb{E} [\psi_p(x) \psi_q^*(x)] = \frac{1}{\pi} \int_C \psi_p(x) \psi_q^*(x) e^{-|x|^2} dx = \begin{cases} 1, & (p = q) \\ 0, & (p \neq q) \end{cases}
\]

where

\[
\int_C f(x) dx = \int_0^{2\pi} d\theta \int_0^\infty f(r e^{i\theta}) r dr,
\]
For an arbitrary function \( f(x) \), also, the cross-correlation of \( \psi_p(x(t)) \) has orthogonality as

\[
R_{\psi_p \psi_q} (\tau) = \begin{cases} 
R_{xx}(\tau) |R_{xx}(\tau)|^{p-1}, & (p = q) \\
0, & (p \neq q)
\end{cases} \quad (7)
\]

where \( R_{xx}(\tau) \) is the cross-correlation function of \( x(t) \) [27]. This means that the orthogonality of polynomials of different orders are all uncorrelated. Then, for the signal model in Fig. 2, we define the generalized Fourier series expansion of nonlinear amplifier \( \alpha(x) \) with orthonormal Laguerre polynomials as follows:

**Definition 1 (Generalized Fourier Series Expansion):** If an amplifier whose transfer function \( \alpha(x) = \alpha(|x|) |x|^p \) has only AM-AM and AM-PM nonlinearity, and \( \mathbb{E} \left[ |\alpha(x)|^2 \right] < \infty \) is satisfied by \( x \sim \mathcal{CN}(0, 1) \), the transfer function can be expanded with the orthonormal Laguerre polynomials as

\[
\alpha(x) = \sum_{p=1,3,\ldots}^{\infty} \tilde{\alpha}_p \psi_p(x), \quad (8)
\]

where \( \tilde{\alpha}_p \) is the \( p \)-th Fourier coefficient, given by

\[
\tilde{\alpha}_p = \frac{1}{\pi} \int_{0}^{\infty} \alpha(x) \psi_p(x) e^{-|x|^2} \, dx = \int_{0}^{\infty} \alpha(\tau) \psi_p(\tau) \cdot 2re^{-r^2} \, dr. \quad (9)
\]

The series expansion of (8) is referred to as the generalized Fourier series expansion with orthonormal Laguerre polynomials.

The generalized Fourier series expansion has suitable properties for analyzing nonlinear characteristics with a \( \mathcal{CN}(0, 1) \)-distributed input signal. In [25], [27], the following theorem is introduced and proved.

**Theorem 1:** For the signal model in Fig. 2, the autocorrelation function of the output signal \( \alpha(x(t)) \) is given by

\[
R_{\alpha\alpha} (\tau) = \sum_{p=1,3,\ldots}^{\infty} |\tilde{\alpha}_p|^2 R_{xx}(\tau) |R_{xx}(\tau)|^{p-1}, \quad (10)
\]

where \( R_{xx}(\tau) \) is the autocorrelation function of the input signal \( x(t) \). Also, the power spectral density (PSD) of the output signal \( \alpha(x(t)) \) is given by

\[
|A(f)|^2 = \sum_{p=1,3,\ldots}^{\infty} |\tilde{\alpha}_p|^2 |\Psi_p(f)|^2, \quad (11)
\]

where \( |\Psi_p(f)|^2 \) is defined as

\[
|\Psi_{2m+1}(f)|^2 = \mathcal{F} \left[ R_{xx}(\tau) |R_{xx}(\tau)|^{2m} \right] = |X(f)|^2 |X(f)|^2 \cdots |X(f)|^2 \star \cdots \star |X(-f)|^2\quad \text{m-times convolution}
\]

\[
\cdots |X(-f)|^2 \star |X(-f)|^2\quad \text{m-times convolution}
\]

and \( |X(f)|^2 = \mathcal{F} [R_{xx}(\tau)] \) is the PSD of \( x(t) \).

**Proof:** See the paper [27, Eq. (23) and (24)] with \( \sigma_x^2 = 1 \).

We can then use **Theorem 1** to prove Parseval’s theorem and Bussgang’s theorem as follows:

**Corollary 1 (Parseval’s Theorem):** In the signal model of Fig. 2, the expected output power of \( \alpha(x) \) is given by

\[
\mathbb{E} \left[ |\alpha(x)|^2 \right] = \sum_{p=1,3,\ldots}^{\infty} |\tilde{\alpha}_p|^2. \quad (13)
\]

**Proof:** Substituting \( \tau = 0 \) in (10), we get the following equation:

\[
\mathbb{E} \left[ |\alpha(x(t))|^2 \right] = \sum_{p=1,3,\ldots}^{\infty} |\tilde{\alpha}_p|^2 \mathbb{E} \left[ |x(t)|^2 \right]^p. \quad (14)
\]

Then, we obtain (13) since \( \mathbb{E} \left[ |x(t)|^2 \right] = 1 \).

**Corollary 2 (Bussgang’s Theorem):** In the signal model of Fig. 2, we can express the output signal from a nonlinear transfer function \( \alpha(x) \) with an input signal \( x(t) \sim \mathcal{CN}(0, 1) \) as

\[
\alpha(x(t)) = \tilde{\alpha}_1 x(t) + d_{\alpha}(t), \quad (15)
\]

where \( d_{\alpha}(t) \) is a distorted signal uncorrelated with \( x(t) \). Also, the power of the output distorted signal \( d_{\alpha}(t) \) can be written as

\[
\mathbb{E} \left[ |d_{\alpha}(t)|^2 \right] = \sum_{p=3,5,\ldots}^{\infty} |\tilde{\alpha}_p|^2 = \mathbb{E} \left[ |\alpha(x(t))|^2 \right] - |\tilde{\alpha}_1|^2. \quad (16)
\]

**Proof:** By comparing (8) and (15), we can see that the distorted signal \( d_{\alpha}(t) \) is described by

\[
d_{\alpha}(t) = \sum_{p=3,5,\ldots}^{\infty} \tilde{\alpha}_p \psi_p(x(t)). \quad (17)
\]

Then, the cross-correlation of \( x(t) \) and \( d_{\alpha}(t) \) can be written as

\[
R_{xd_{\alpha}} (\tau) = \sum_{p=3,5,\ldots}^{\infty} \tilde{\alpha}_p R_{xx}(\tau), \quad (18)
\]
where $R_{\psi \psi}(\tau)$ is the cross-correlation of $x(t)$ and $\psi_p(x(t))$. Since $\psi_1(x(t)) = x(t)$, Eq. (18) can be rewritten as

$$R_{xa_0}(\tau) = \sum_{p=3,5,\ldots}^{\infty} \bar{\alpha}_p R_{\psi_1 \psi_p}(\tau) = 0 \quad (19)$$

due to the orthogonality of (7). Thus, $a_0(t)$ is uncorrelated with $x(t)$. In addition, from Theorem 1, the power of the distorted signal can be written as

$$E \left[ |d_0(t)|^2 \right] = \sum_{p=3,5,\ldots}^{\infty} |\bar{\alpha}_p|^2 = \left( \sum_{p=1,3,\ldots}^{\infty} |\bar{\alpha}_p|^2 \right) - |\bar{\alpha}_1|^2$$

$$= E \left[ |\alpha(x(t))|^2 \right] - |\bar{\alpha}_1|^2. \quad (20)$$

B. Theorems Related to the Parallel Hammerstein Canceller

Theorem 1, Theorem 1, and Theorem 2 are useful for evaluating the output signal of the nonlinear transmitter. However, these theorems are inadequate for analyzing the received self-interference since they do not consider a channel’s impulse response. In Fig. 3, we assume that the input signal $x(t)$ is distributed in $CN(0, 1)$. The following theorem can address the parallel Hammerstein model with orthonormal polynomials for Fig. 3.

Theorem 2: For self-interference signal model of Fig. 3, it is assumed that the signal $y(t)$ can be expressed as

$$y(t) = \sum_{p=1,3,\ldots}^{\infty} \tilde{h}_p(\tau) \ast \psi_p(x(t)), \quad (21)$$

where $\tilde{h}_p(\tau)$ is an impulse response corresponding to the orthonormal Laguerre polynomials $\psi_p(x)$, and their frequency response is $\tilde{H}_p(f) = \mathcal{F} \left[ \tilde{h}_p(\tau) \right]$. Then, the following expressions hold:

$$|Y(f)|^2 = \sum_{p=1,3,\ldots}^{\infty} |\tilde{H}_p(f)|^2 |\Psi_p(f)|^2, \quad (22)$$

$$E \left[ |Y(f)|^2 \right] = \sum_{p=1,3,\ldots}^{\infty} \rho_p^2 |\Psi_p(f)|^2, \quad (23)$$

$$E \left[ |y(t)|^2 \right] = \sum_{p=1,3,\ldots}^{\infty} \rho_p^2, \quad (24)$$

where $Y(f) = \mathcal{F} [y(t)]$ is the frequency-domain representation of the output signal $y(t)$, and $\rho_p^2 = E \left[ |\tilde{H}_p(f)|^2 \right]$.

Proof: See Appendix A.

To perform a theoretical analysis of nonlinear cancellers within a functional analysis framework, we must first define them in a manageable form. Thus, we define the parallel Hammerstein canceller as follows:

Definition 2 (parallel Hammerstein canceller): For the signal model of Fig. 3, the output signal of the parallel Hammerstein canceller composed of up to $P$-order polynomials is defined as

$$c(t) = \sum_{p=1,3,\ldots}^{P} \tilde{c}_p(\tau) \ast x(t) |x(t)|^{p-1}, \quad (25)$$

where $\tilde{h}_p(\tau)$ are impulse responses that minimize the following residual self-interference power:

$$E \left[ |y(t) - c(t)|^2 \right]. \quad (26)$$

Then, we can prove the existence of impulse responses $\tilde{c}_p(\tau)$ to satisfy (27) with some trivial manipulations.

$$c(t) = \sum_{p=1,3,\ldots}^{P} \tilde{c}_p(\tau) \ast \psi_p(x(t)) \quad (27)$$

The impulse responses $\tilde{c}_p(\tau)$ and $\tilde{c}_p(\tau)$ have the following relationships:

$$\begin{bmatrix} c_1(\tau) \\ c_3(\tau) \\ \vdots \\ c_P(\tau) \end{bmatrix} = \begin{bmatrix} l_{0,0} & l_{1,0} & l_{2,0} & \ldots & l_{m,0} \\ 0 & l_{1,1} & l_{2,1} & \ldots & l_{m,1} \\ 0 & 0 & l_{2,2} & \ldots & l_{m,2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & l_{m,m} \end{bmatrix} \begin{bmatrix} \bar{c}_1(\tau) \\ \bar{c}_3(\tau) \\ \vdots \\ \bar{c}_P(\tau) \end{bmatrix}$$

$$\Leftrightarrow \tilde{c}_p(\tau) = L_{m+1}^{-1} \tilde{c}_p(\tau), \quad (28)$$

where $m = (P - 1)/2$. In other words, there is a method for completely transforming (25) and (27), and Eq. (27) also minimizes (26). Additionally, we can provide the following noteworthy theorem by taking advantage of the useful properties of (27) and orthonormal polynomials.

Theorem 3: In the signal model of Fig. 3, the impulse responses $\tilde{c}_p(\tau)$ corresponding to the generalized Fourier expansion of the parallel Hammerstein canceller (27) are given by

$$\tilde{c}_p(\tau) = \tilde{h}_p(\tau). \quad (29)$$

Proof: Equation (26) can be transformed into the following expression:

$$E \left[ |y(t) - c(t)|^2 \right]$$

$$= \sum_{p=1,3,\ldots}^{P} \int_{-\infty}^{\infty} E \left[ \left| \tilde{H}_p(f) - \tilde{C}_p(f) \right|^2 \right] |\Psi_p(f)|^2 \, df$$

$$+ \sum_{p=P+1,3,\ldots}^{\infty} \int_{-\infty}^{\infty} E \left[ \left| \tilde{H}_p(f) \right|^2 \right] |\Psi_p(f)|^2 \, df$$

$$+ E \left[ |\tilde{z}(t)|^2 \right], \quad (30)$$

where $\tilde{H}_p(f)$ and $\tilde{C}_p(f)$ are the frequency response of $\tilde{h}_p(\tau)$ and $\tilde{c}_p(\tau)$, respectively. When $\tilde{H}_p(f) = \tilde{C}_p(f)$ is satisfied, the above expression is minimized. Thus, (29) can be derived.

Theorem 3 states that if the received self-interference is approximated in the form of (21), we can analyze the theoretical characteristics of the parallel Hammerstein canceller. Additionally, we can analyze the characteristic of the residual self-interference signal, denoted $\tilde{w}(t) = y(t) - c(t)$. Although we have described the time-domain Hammerstein canceller in Theorem 3, a similar relationship, $\tilde{H}_p(f) = \tilde{C}_p(f)$, can be established for the frequency-domain Hammerstein.
canceller [18], [30] due to the one-to-one correspondence between the frequency response and the impulse response. Note that even if the received signal is expanded to a Hammerstein model of $x|x|^{p−1}$ and impulse responses $\hat{h}_p(\tau)$ as

$$y(t) = \sum_{p=1,3,...}^{\infty} \hat{h}_p(\tau) \ast x(t)|x(t)|^{p−1} + z(t), \quad (31)$$

the $p$-th impulse response of the Hammerstein canceller $\hat{c}_p(\tau)$ is not represented by $\hat{h}_p(\tau)$, i.e.,

$$\hat{c}_p(\tau) \neq \hat{h}_p(\tau). \quad (32)$$

We provide the proof for the above inequality in Appendix B. In contrast, (29) is an identity, and it is always holds true for an arbitrary transfer function. Hence we use the generalized Fourier series expansion for the theoretical analysis of nonlinear self-interference cancellers. Of course, the model of (31) can be analyzed by transforming it into the model of (21) using the matrix $L_{m+1}$. Equation (81) of Appendix B is a formula of the transformation between the two models.

III. THEORETICAL ANALYSIS OF FULL-DUPLEX RADIOS WITH PARALLEL HAMMERSTEIN CANCELLER

In this section, we describe a theoretical analysis technique for full-duplex transceivers and nonlinear self-interference cancellers. First, we show the analytical model and apply equation deformations to make the analysis easier. Then, we show how the performance of the canceller and the symbol error rate can be analyzed.

A. Analytical Model

In preparation for the analysis, we describe the analytical model summarized in Fig. 4. Only one terminal is depicted, but we assume that there are two terminals, terminal#1 and terminal#2. The transmitted baseband signal is the OFDM signal with many subcarriers, and we can assume that its envelope amplitude and power have a Rayleigh distribution with $\sigma_x^2 = 0.5$ and an exponential distribution Exp(1), respectively.

In the transmitter of terminal#1, the transmitted signal is distorted by the power amplifier (PA), and the signal received by the receiver antenna can be described as

$$y_{\text{ANT},1}(t) = h_{\text{SL},1}(\tau) \ast a_1(g_1 x_1(t)) + h_{21}(\tau) \ast a_2(g_2 x_2(t)) + z_1(t), \quad (33)$$

where $h_{\text{SL},1}(\tau)$ is the impulse response between the TX and RX antennas, $h_{21}(\tau)$ is the channel impulse response between terminal#1 and terminal#2, and $z_1(t)$ is the thermal noise. Also, $x_1(t)$ and $x_2(t)$ are the signal transmitted from terminal#1 and terminal#2, respectively. The nonlinear functions $a_1(\cdot)$ and $a_2(\cdot)$ are the AM-AM and AM-PM transfer functions of both terminals’ PAs.

Generally, in full duplex, an RF canceller is used to prevent saturation of the receiver LNA or A/D converter. The residual self-interference, which is the input signal to the LNA, can be expressed as

$$y_{\text{RFSIC},1}(t) = (h_{\text{SL},1}(\tau) - h_{\text{CIR},1}(\tau)) \ast a_1(g_1 x_1(t)) + h_{21}(\tau) \ast a_2(g_2 x_2(t)) + z_1(t), \quad (34)$$

where $h_{\text{CIR},1}(\tau)$ is the impulse response of the RF canceller. Then, the nonlinear distorted signal from the LNA can be described as

$$y_1(t) = \beta_1(h_{11}(\tau) \ast a_1(g_1 x_1(t)) + h_{21}(\tau) \ast a_2(g_2 x_2(t)) + z_1(t)), \quad (35)$$

where $h_{11}(\tau) = h_{\text{SL},1}(\tau) - h_{\text{CIR},1}(\tau)$, and the nonlinear function $\beta_1(\cdot)$ is the AM-AM and AM-PM transfer function of the LNA. In this paper, we assume that the self-interference channel after RF cancellation $h_{11}(\tau)$ and the channel between terminals $h_{21}(\tau)$ are Rayleigh fading channels, and the mean power gain of $h_{11}(\tau)$ and $h_{21}(\tau)$ are $\rho_{11}$ and $\rho_{21}$, respectively. In other words, $\rho_{ij}$ can be written as

$$\rho_{ij}^2 = \mathbb{E}\left[|H_{ij}(f)|^2\right], \quad (36)$$

where $H_{ij}(f)$ is the frequency response of $h_{ij}(\tau)$, and $H_{ij}(f)$ is distributed on CN(0, $\rho_{ij}^2$). To obtain the coefficients of the nonlinear canceller using Theorem 3, we need to transform (35) to the form of (21). Thus, we derive (37) from (35) using Bussgang’s theorem.

$$y_1(t) = \tilde{\beta}_{1,1} h_{11}(\tau) \ast a_1(g_1 x_1(t)) + \tilde{\beta}_{1,1} h_{21}(\tau) \ast a_2(g_2 x_2(t)) + \tilde{\beta}_{1,1} z_1(t) + z_{\text{NL},1}(t), \quad (37)$$

where $\tilde{\beta}_{1,1}$ is the linear gain of the LNA, and $z_{\text{NL},1}(t)$ is the nonlinear distortion caused by the LNA. For simplicity, we make the following assumptions:

- If there is no self-interference signal, the LNA does not saturate and operates as a linear amplifier.
- The power of the self-interference is much larger than the received desired signal and noise.

From above assumptions, we can formulate the following equations:

$$\tilde{\beta}_{1,1} = \frac{1}{\rho_{11} \nu_1} \int_0^\infty \beta_1(\rho_{11} \nu_1 r) r \cdot 2r e^{-r^2} dr, \quad (38)$$

$$\mathbb{E}_{\text{NL},1} = \mathbb{E}\left[|z_{\text{NL},1}(t)|^2\right] = \int_0^\infty |\beta_1(\rho_{11} \nu_1 r)|^2 r \cdot 2r e^{-r^2} dr - \tilde{\beta}_{1,1} \rho_{11} \nu_1^2, \quad (39)$$

Fig. 4. Analysis model of the full-duplex transceiver.
where $\rho_{11}^2$ is the mean power gain of $h_{11}(\tau)$, $\mathcal{N}_{NL,1}$ is the mean power of $z_{NL,1}(t)$, and $\nu_t^2$ is the mean transmission power of terminal#1, expressed as

$$
\nu_t^2 = \int_0^\infty |\alpha_1(g_1 r)|^2 \cdot 2 e^{-r^2} dr.
$$

(40)

Applying the generalized Fourier series expansion with the orthonormal Laguerre polynomials to (37), we obtain

$$
y_1(t) = \beta_{1,1} h_{11}(\tau) \ast \sum_{p=1,3,...}^\infty \tilde{\alpha}_{1,p} \psi_p(x_1(t))
$$

$$+ \beta_{1,1} h_{21}(\tau) \ast \sum_{p=1,3,...}^\infty \tilde{\alpha}_{2,p} \psi_p(x_2(t))
$$

$$+ z_{tot,1}(t),
$$

(41)

where

$$
\tilde{\alpha}_{n,p} = \int_0^\infty \alpha_n(g_n r) \psi_p(r) \cdot 2 e^{-r^2} dr,
$$

(42)

$$
z_{tot,1}(t) = \beta_{1,1} z_1(t) + z_{NL,1}(t).
$$

(43)

From Theorem 3, the regenerated self-interference signal of the nonlinear canceller composed of up to $P$-order polynomials can be written as

$$
c_1(t) = \beta_{1,1} h_{11}(\tau) \ast \sum_{p=1,3,...}^P \tilde{\alpha}_{1,p} \psi_p(x_1(t)).
$$

(44)

Thus, the residual signal after self-interference cancellation can be expressed as

$$
w_1(t) = y_1(t) - c_1(t)
$$

$$= \beta_{1,1} h_{11}(\tau) \ast \sum_{p=1,3,...}^\infty \tilde{\alpha}_{1,p} \psi_p(x_1(t))
$$

$$+ \beta_{1,1} h_{21}(\tau) \ast \sum_{p=1,3,...}^\infty \tilde{\alpha}_{2,p} \psi_p(x_2(t)) + z_{tot,1}(t).
$$

(45)

B. Cancellation Performance

The following self-interference cancellation ratio (SICR), which indicates the performance of a digital canceller, can be expressed as

$$
\text{SICR}_1 = \frac{\mathcal{T}_{11} + \mathcal{N}_{tot,1}}{\mathcal{T}_{11}^R + \mathcal{N}_{tot,1}},
$$

(46)

where $\mathcal{T}_{11}$ is the mean power of the self-interference before digital cancellation, and $\mathcal{T}_{11}^R$ is the mean power of the residual self-interference after digital cancellation. From Theorem 2, $\mathcal{T}_{11}$ and $\mathcal{T}_{11}^R$ can be written as

$$
\mathcal{T}_{11} = \mathbb{E} \left[ |\beta_{1,1} h_{11}(\tau) \ast \alpha_1(g_1 x(t))|^2 \right] = |\beta_{1,1}|^2 \rho_{11}^2 \nu_t^2.
$$

$$
\mathcal{T}_{11}^R = \mathbb{E} \left[ |\beta_{1,1} h_{11}(\tau) \ast \sum_{p=P+2,P+4,...}^\infty \tilde{\alpha}_{1,p} \psi_p(x_1(t))|^2 \right]
$$

(47)

C. Symbol Error Rate

To analyze the symbol error rate (SER), we need to derive statistical properties for the signal to interference, distortion, and noise ratio (SIDNR). The SIDNR on terminal#1 is described by

$$
\text{SIDNR}_1(f) = \frac{U_{21}(f)}{D_{21}(f) + I_{11}^R(f) + \mathcal{N}_{tot,1}(f)},
$$

(50)

where $U_{21}(f)$ and $D_{21}(f)$ are the powers of the useful and distortion signals of the received desired signal, and $I_{11}^R(f)$ is the residual self-interference power after digital cancellation. $U_{21}(f)$, $D_{21}(f)$, and $I_{11}^R(f)$ can be written as

$$
U_{21}(f) = |\beta_{1,1}|^2 |H_{21}(f)|^2 |\tilde{\alpha}_{2,1}|^2 |X(f)|^2,
$$

(51)

$$
D_{21}(f) = |\beta_{1,1}|^2 |H_{21}(f)|^2 \sum_{p=3,5,...}^\infty |\tilde{\alpha}_{2,p}|^2 |\Psi_p(f)|^2,
$$

(52)

$$
I_{11}^R(f) = |\beta_{1,1}|^2 |H_{11}(f)|^2 \sum_{p=P+2,P+4,...}^\infty |\tilde{\alpha}_{1,p}|^2 |\Psi_p(f)|^2.
$$

(53)

The total noise contains the nonlinear distortion caused by the LNA, and its PSD $\mathcal{N}_{tot,1}(f)$ is not flat in the band $-\frac{1}{2} < f < \frac{1}{2}$. In the proposed analysis, we approximate the PSD of the total noise as

$$
\mathcal{N}_{tot,1}(f) = |\beta_{1,1}|^2 \mathcal{N}_{thermal}(f) + \mathcal{N}_{NL,1} |\Psi_3(f)|^2,
$$

(54)

where $\mathcal{N}_{thermal}(f)$ is the PSD of the thermal noise. Thus, SIDNR$1(f)$ can be expressed as

$$
\text{SIDNR}_1(f) = \frac{1}{\lambda_{\text{sdr,1}}(f) + \lambda_{\text{sdr,1}}(f)},
$$

(55)

where

$$
\lambda_{\text{sdr,1}}(f) = \frac{D_{21}(f)}{U_{21}(f)} = \frac{1}{|\tilde{\alpha}_{2,1}|^2} \sum_{p=3,5,...}^\infty |\tilde{\alpha}_{2,p}|^2 |\Psi_p(f)|^2,
$$

(56)

$$
\lambda_{\text{sdr,1}}(f) = \frac{I_{11}^R(f) + \mathcal{N}_{tot,1}(f)}{U_{21}(f)}.
$$

(57)

Since (56) does not depend on the channels’ frequency responses, Eq. (56) is not a random variable and can have a fixed value. Thus, if the probability density function (PDF) of (57) is derived, we can formulate the PDF of the SIDNR and
analyze the SER of the full-duplex system. From Appendix C, the PDF of (57) can be written as
\[ p_{\text{ser},1}(x; f) = \frac{N_{\text{tot},1}(f)(xU_{21} + \overline{T}^{R}_{11}(f)) + xU_{21}T^{H}_{11}(f)}{x(xU_{21} + \overline{T}^{R}_{11}(f))^2} \times \exp\left(-\frac{N_{\text{tot},1}(f)}{xU_{21}}\right), \quad (58) \]

where
\[ \overline{T}^{R}_{11}(f) = \left| \tilde{\alpha}_{1,1}\rho_{11} \right|^2 \sum_{p=P+2, P+4, \ldots}^{\infty} |\tilde{\alpha}_{1,p}|^2 |\Psi_p(f)|^2, \quad (59) \]
\[ \overline{U}_{21} = \left| \tilde{\alpha}_{1,1}\rho_{21}\tilde{\alpha}_{2,1} \right|^2. \quad (60) \]

Therefore, the average SER can be analyzed by averaging the random variable in the band \(-\frac{1}{2} < f < \frac{1}{2}\) and can be expressed by the following integral:
\[ \text{SER}_1 = \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_0^{\infty} P_s \left( \frac{1}{x + \lambda_{\text{ser},1}(f)} \right) p_{\text{ser},1}(x; f) \, dx \, df, \quad (61) \]

where \(P_s(\gamma)\) is the symbol error probability of the subcarrier modulation. If we use \(M\)-ary QAM for each subcarrier, \(P_s(\gamma)\) can be written as [31]
\[ P_s(\gamma) = 1 - \left[ 1 - \left( 1 - \frac{1}{\sqrt{M}} \right) \text{erfc} \left( \sqrt{\frac{3\gamma}{2(M-1)}} \right) \right]^2. \quad (62) \]

D. Analysis Summary

In summary, we can analyze the performance of full-duplex communication using the following procedure:

1) Determine the following parameters:
   - Nonlinear transfer function of amplifiers: \(\alpha_1(x), \alpha_2(x), \beta_1(x), \text{ and } \beta_2(x)\)
   - The gain of the VGA \(g_n\)
   - The input back-off:
     \[ \text{IBO}_n \text{ (dB)} = 20 \log_{10} \frac{A_{\text{sat},n}}{g_nG_n}, \quad (63) \]
   - Where \(A_{\text{sat},n}\) and \(G_n\) are the output saturation level and linear gain of \(\alpha_n(x)\), respectively.
   - Sum of propagation and RF cancellation (dB) = \(-20 \log_{10} \rho_{nn}\)
   - The propagation gain between terminals: \(\rho_{21}\) and \(\rho_{12}\)
   - Thermal noise level, including the noise figure of the LNA: \(N_{\text{thermal}}\)
   - The nonlinear order of the canceller \(P\)

2) Compute the transmission power of the \(n\)-th terminal using (40).

3) Compute the Fourier coefficients \(\tilde{\alpha}_{n,p}\) of the \(n\)-th terminal's transmitter using (42).

4) Compute the linear gain \(\tilde{\beta}_{n,1}\) and nonlinear distortion power \(N_{\text{NL,n}}\) of the \(n\)-th terminal's LNA with (38) and (39), respectively.

5) Analyze the cancellation performance of a digital canceller SICR with (46)-(49).

6) Analyze the SER with (54), (56), and (58)–(61), where the PSD of the orthonormal Laguerre polynomial can be written as [27]
\[ |\Psi_p(f)|^2 = \sum_{k=0}^{p} \frac{(-1)^k}{(p-k)!} \left( \frac{p}{k} \right) \left( |f| - k + \frac{p}{2} \right)^{p-1}, \quad (64) \]

where
\[ (u)_+^p = \begin{cases} u^p, & (u > 0), \\ 0, & (u \leq 0). \end{cases} \quad (65) \]

In the above procedure, we use numerical integration formulas because some integrals cannot be evaluated with closed-form expressions. The source code for an example implementation of the proposed analysis is available from the author's GitHub repository.1

IV. EXAMPLES AND VERIFICATION

In this section, we provide some results from the proposed analysis, and some simulation results for verifying the proposed analysis. TABLE II and TABLE III list the simulation parameters. In the simulations, the self-interference channel \(h_{nn}(\tau)\), comprising the wireless channel and the RF self-interference canceller on the \(n\)-th terminal, was modeled

\[ \text{https://github.com/k3kaimu/theory_of_nonlin_canceller} \]

---

1. TABLE I

| Parameter | Value |
|-----------|-------|
| Constellation | 16QAM |
| FFT size | 64 |
| Active subcarriers | 32 |
| Cyclic prefix size | 16 |
| Bandwidth | 20 MHz |

| TABLE II

OFDM MODULATION SPECIFICATIONS

| Parameter | Value |
|-----------|-------|
| Constellation | 16QAM |
| FFT size | 64 |
| Active subcarriers | 32 |
| Cyclic prefix size | 16 |
| Bandwidth | 20 MHz |

| TABLE III

SIMULATION AND ANALYSIS SPECIFICATIONS

| Parameter | Value |
|-----------|-------|
| Oversampling rate | 8 |
| Sampling rate | 20 MHz × 8 = 160 MHz |
| SI channel after RF-SIC | Rayleigh fading |
| # of taps of SI channel | 64 taps |
| Desired channel | Rayleigh fading |
| # of taps of desired channel | 64 taps |
| Transmission power | 23 dBm |
| PA Gain | 30 dB |
| PA output saturation level | 30 dBm |
| PA smoothness factor | 3 |
| LNA noise figure | 4 dB |
| LNA Gain | 20 dB |
| LNA output saturation level | 14 dBm |
| LNA smoothness factor | 3 |
| # of taps of canceller | 64 taps |
| # of training symbols of canceller | 200 OFDM symbols |
| Trials | 10000 |
| # of transmission bits | 10^6 bits on each trial |
as a quasi-static Rayleigh fading channel with a constant
impulse response in a given simulation trial, with different
impulse responses used in different simulation trials. We
assumed there to be two full-duplex terminals, with identical
nonlinear characteristics and parameters. Thus, in the simu-
lation, the SICR$_a$ and SER$_a$ of the two terminals were
identical. The nonlinearities of the PAs and LNAs were imple-
mented using the Rapp model [24], which is often used to simu-
late the baseband behaviors of class AB solid-state amplifiers. The
AM-AM conversion of the Rapp model can be expressed as
\begin{equation}
\text{Rapp}(x; G, A_{\text{sat}}, s) = \frac{Gx}{1 + (\frac{|Gx|}{A_{\text{sat}}})^{2s}}, \quad (66)
\end{equation}
where $G$ is the linear gain of the amplifier, $A_{\text{sat}}$ is the
output saturation level, and $s$ is the smoothness factor of the
Rapp model. The larger the smoothness factor $s$, the stronger
the linearity of the Rapp model. When the smoothness factor
$s$ is infinity, the Rapp model becomes an ideally predistorted
amplifier, represented by the following AM-AM conversion:
\begin{equation}
\text{IdealPA}(x; G, A_{\text{sat}}) = \text{Rapp}(x; G, A_{\text{sat}}, \infty) = \begin{cases} 
Gx, & (Gx) \leq A_{\text{sat}}, \\
A_{\text{sat}} \frac{x}{G}, & (Gx) > A_{\text{sat}}.
\end{cases} \quad (67)
\end{equation}
We also implemented time-domain parallel Hammerstein
cancellers [4], [20] in the simulator. The regenerated
self-interference signal by the cancellers can be written as
\begin{equation}
c[n] = \sum_{m=0}^{M-1} \sum_{p=1}^{P} c_p[m] x[n-m] |x[n-m]|^{p-1}, \quad (68)
\end{equation}
where $c_p,q[m]$ is estimated by the least squares algorithm
with 200 OFDM symbols to achieve best performance. In
this section, we provide results from the Rapp model. For
reference, we also provide results from the Saleh model [32],
including both AM-AM and AM-PM nonlinearities in Appen-
dix D and results in the presence of IQ imbalance and phase
noise in Appendix E.

A. Cancellation Performance

Figure 5 shows the cancellation performance of the parallel
Hammerstein canceller with different value of order $P$ under
various conditions of propagation and RF domain cancellation.
We can confirm that the theoretical results and simulation
results match well for nonlinear cancellers, demonstrating the
accuracy of the proposed analysis method.

A more detailed discussion of Fig. 5 is as follows: The
self-interference and noise ratio, INR, is defined as
\begin{equation}
\text{INR}_1 = \frac{\mathbb{E}[|h_{11}(\tau) * \alpha_1(g_1 x_1(t))|^2]}{\mathbb{E}[|z_1(t)|^2]} = \rho_{11}^2 \frac{\nu_1^2}{\mathcal{N}_{\text{thermal}}}, \quad (69)
\end{equation}
where the numerator indicates the power of the self-interference on the first terminal. The upper bound of the cancellation performance SICR$_1$ can be expressed by using INR$_1$ as
\begin{equation}
\text{SICR}_1 \leq \frac{T_{11} + \mathcal{N}_{\text{thermal}}}{\mathcal{N}_{\text{thermal}}} \leq \text{INR}_1 + 1. \quad (70)
\end{equation}
Cancellers achieve the above upper bound when they have been trained with a sufficient number of OFDM symbols, and the LNA is not saturated by self-interference. In Fig. 5, we can see that the performance of the canceller with $P = 7$ reaches to the upper bound when $\rho_{11}^2 < -60$ dB. In contrast, when the order of the canceller $P$ is less than seven, the performance cannot reach the upper bound when $\rho_{11}^2 < -60$ dB because the residual nonlinear self-interference distorted by the PA becomes larger than the thermal noise level. In Fig. 5, we can also see that the cancellation performance is significantly degraded by saturation of the LNA when $\rho_{11}^2 > -55$ dB even if $P = 7$. The parallel Hammerstein canceller cannot estimate and regenerate the nonlinear self-interference caused by the LNA; to achieve higher performance, we need to consider the nonlinearity of the LNA [19], [33]–[39].

Figure 6 shows the cancellation performance of the parallel
Hammerstein canceller with various smoothness factors for
both terminals’ PAs. Firstly, we can confirm good correlation between the theoretical and simulation results. Furthermore, the linear canceller (i.e., \( P = 1 \)) achieves the best cancellation performance when the PAs are ideally predistorted. The higher smoothness factor improves amplifier linearity. Thus, amplifier linearization is an effective technique for a full-duplex system with linear canceller. More interestingly, the nonlinear cancellers such as \( P = 3, 5, \) and 7 achieve maximum cancellation performance when the PAs show moderate nonlinearity. This means that the linearization degrades the cancellation performance of the nonlinear cancellers. We provide a detailed discussion of this phenomenon in Section IV-C.

### B. Symbol Error Rate

Figure 7 shows the average SER of the full-duplex system under various back-offs for both terminals’ PAs, i.e., under various transmit powers. Similar to the results of the cancellation performance shown in Fig. 5 and Fig. 6, the theoretical and simulation results match well in Fig. 7. When the input back-off is less than 10 dB, the SER degrades rapidly because the distortion introduced by the PAs increases. When the input back-off is greater than 15 dB, the SER also degrades slowly because the power of the desired signal decreases. Thus, there is an optimum back-off value, which depends on the nonlinear canceller’s order \( P \).

Figure 8 shows the average SER of the full-duplex system with various PA smoothness factors. Again, the theoretical and simulation results show good correlation. The full-duplex system with a linear canceller such as \( P = 1 \) achieves the minimum SER when the PAs are ideally predistorted. In contrast, when we use nonlinear cancellers such as \( P = 3, 5, \) and 7, the full-duplex system achieves the minimum SER for the smoothness factor \( 1 < s < 2 \). As in Fig. 6, these results indicate that amplifier linearization is not the best approach for a full-duplex system with the nonlinear cancellers. Although this result deviates from the common knowledge of half-duplex systems, it is confirmed by the theoretical analysis and simulation results shown in Fig. 8 for a full-duplex system with nonlinear self-interference canceller. These results are very interesting; in the following section we will clearly show why such results are obtained.

### C. How Does a Nonlinear Amplifier Perform Better Than a Linearized Amplifier?

From Fig. 6 and Fig. 8, we can confirm that the full-duplex terminal with a nonlinear canceller does not achieve the best performance using ideally linearized amplifiers for the transmitters. We can explain these results in terms of the residual self-interference power \( T^R_{11} \) defined as (48) and rewritten as

\[
T^R_{11} = |\beta_{1,1}|^2 \rho_{11}^2 \left( |\tilde{\alpha}_{1,1}|^2 + |\tilde{\alpha}_{1,5}|^2 + \cdots \right). \tag{71}
\]

To achieve high cancellation performance and low SER, it is important to reduce the residual self-interference \( T^R_{11} \). For the linear canceller, \( P = 1 \), reducing the sum of all nonlinear distortions \( |\tilde{\alpha}_{1,3}|^2 + |\tilde{\alpha}_{1,5}|^2 + \cdots \) leads to a reduction in \( T^R_{11} \). Thus, the linearization reducing the total distortion power is effective for the full-duplex system with a linear canceller. However, for nonlinear cancellers such as \( P = 3, 5, 7, \ldots \), reducing the sum of all nonlinear distortions \( |\tilde{\alpha}_{1,3}|^2 + |\tilde{\alpha}_{1,5}|^2 + \cdots \) does not lead to a reduction in \( T^R_{11} \). This is because when the total power of distortions is reduced, the nonlinear component, with a higher order than \( P \), does not necessarily become smaller. The third-order distortion has the greatest power among the nonlinear distortions. If the total distortion power can be reduced, the linearization will try to reduce the third-order distortion even if it increases the fifth- or seventh-order distortion power. Therefore, linearization does not reduce the residual self-interference when we use nonlinear cancellers. In summary, linearization is not the best approach for full-duplex systems with nonlinear cancellers. In addition, if we want to achieve higher performance in full-duplex communications, we should design predistorters such that the SER is small.
proposed analysis technique can be used to guide the design of such predistorters.

V. CONCLUSION

This paper first presented some useful theorems for analysis of parallel Hammerstein cancellers and residual self-interference in full-duplex systems. Using these theorems, we demonstrated that the performance of a full-duplex system with parallel Hammerstein cancellers can be expressed by coefficients of the generalized Fourier series expansion for nonlinear amplifiers. We compared results from the proposed analysis with simulation results in terms of self-interference cancellation performance and SER, and confirmed that they match well. In addition, discussion of the results revealed that amplifier linearization is not the best approach in full-duplex systems with the nonlinear cancellers. In Appendix E, we analyzed the power of residual self-interference under the effect of IQ imbalance.

APPENDIX A

PROOF OF THEOREM 2

In this appendix, we provide the proof of Theorem 2. The autocorrelation function of \( y(t) \) is expressed as

\[
R_{yy}(\tau) = \mathbb{E}_t [y(t)y^*(t + \tau)] = \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} R^{(p,q)}_{yy}(\tau),
\]

where \( \mathbb{E}_t [\cdot] \) denotes the expected value for time \( t \), and \( R^{(p,q)}_{yy}(\tau) \) is defined as

\[
R^{(p,q)}_{yy}(\tau) = \mathbb{E}_t \left[ \left( \int_0^\infty \tilde{h}_p(\tau_1)\psi_p(x(t - \tau_1))d\tau_1 \right) \times \left( \int_0^\infty \tilde{h}_q^*(\tau_2)\psi_q^*(x(t + \tau - \tau_2))d\tau_2 \right) \right]
\]

\[
= \int_0^\infty \int_0^\infty \tilde{h}_p(\tau_1)\tilde{h}_q(\tau_2)\mathbb{E}_x [\psi_p(x(t + \tau - \tau_2))]d\tau_1d\tau_2,
\]

where \( \mathbb{E}_x [\cdot] \) is the cross-correlation function of \( \psi_p(x(t)) \) and \( \psi_q(x(t)) \). From (7), \( \mathbb{E}_x [\psi_p(x(t))\psi_q(x(t))] = 0 \) when \( p \neq q \). Thus, the frequency domain representation of \( R^{(p,q)}_{yy}(\tau) \) can be expressed as

\[
\mathcal{F} \left[ R^{(p,q)}_{yy}(\tau) \right] = \begin{cases} \left| \tilde{H}_p(f) \right|^2 \left| \Psi_p(f) \right|^2, & (p = q) \\ 0, & (p \neq q) \end{cases}
\]

where \( \tilde{H}_p(f) \) is the frequency response of \( \tilde{h}_p(\tau) \). Then, the PSD of \( y(t) \) can be written as

\[
|Y(f)|^2 = \sum_{p=1}^{\infty} \left| \tilde{H}_p(f) \right|^2 \left| \Psi_p(f) \right|^2.
\]

By taking the expected values of both sides of the above equation, Eq. (23) can be derived. Also, the expected power of \( y(t) \) can be expressed as

\[
\mathbb{E} \left[ |y(t)|^2 \right] = \int_{-\infty}^{\infty} \mathbb{E} \left[ |Y(f)|^2 \right] df
\]

\[
= \sum_{p=1}^{\infty} \int_{-\infty}^{\infty} \rho_p^2 \left| \Psi_p(f) \right|^2 df
\]

\[
= \sum_{p=1}^{\infty} \rho_p^2 \mathbb{E} \left[ |\psi_p(x(t))|^2 \right] = \sum_{p=1}^{\infty} \rho_p^2. \quad (77)
\]

Thus, Eq. (24) can be derived.

APPENDIX B

CONDITION OF \( \dot{c}_p(\tau) = \dot{\tilde{h}}_p(\tau) \)

In this appendix, we prove the following theorem:

Theorem 4: It is assumed that the received self-interference is expressed by a \( Q \)-order Hammerstein model as

\[
y(t) = \sum_{p=1}^{Q} \tilde{h}_p(\tau) \ast x(t) |x(t)|^{p-1} + z(t), \quad (78)
\]

where \( \tilde{h}_p(\tau) \) is an impulse response corresponding to \( x(t) |x(t)|^{p-1} \), and \( z(t) \) is a signal uncorrelated with the transmitted signal \( x(t) \). Then, the impulse responses \( \dot{c}_p(\tau) \) of the parallel Hammerstein canceller in (25) composed of up to \( P \)-order power series are given by the following identity

\[
\dot{c}_p(\tau) = \dot{\tilde{h}}_p(\tau), \quad (79)
\]

when \( P \geq Q \). In contrast, when \( P < Q \), \( \dot{c}_p(\tau) \neq \dot{\tilde{h}}_p(\tau) \). In other words, we cannot theoretically analyze the residual nonlinear self-interference by using the non-orthogonal polynomial expansion.

Proof: The received signal \( y(t) \) can be expressed as the following orthonormal Laguerre polynomial expansion:

\[
y(t) = \sum_{p=1}^{Q} \tilde{h}_p(\tau) \ast \psi_p(x(t)) + z(t), \quad (80)
\]

where \( \tilde{h}_p(\tau) \) is an impulse response corresponding to \( \psi_p(x(t)) \). Similar to (28), we can write the relationship between \( \tilde{h}_p(\tau) \) and \( \tilde{h}_p(\tau) \) as

\[
\tilde{h}(\tau) = L_{n+1}^{-1} \tilde{h}(\tau), \quad (81)
\]

where \( n = (Q - 1)/2 \), and

\[
\tilde{h}(\tau) = \begin{bmatrix} \tilde{h}_1(\tau) & \tilde{h}_3(\tau) & \cdots & \tilde{h}_Q(\tau) \end{bmatrix}^T, \quad (82)
\]

\[
\tilde{h}(\tau) = \begin{bmatrix} \tilde{h}_1(\tau) & \tilde{h}_3(\tau) & \cdots & \tilde{h}_Q(\tau) \end{bmatrix}^T. \quad (83)
\]

From Theorem 3, when \( P \geq Q \), \( \dot{c}(\tau) \) is given by

\[
\dot{c}(\tau) = \begin{bmatrix} I_{n+1} \\ 0_{(m-n) \times (n+1)} \end{bmatrix} \tilde{h}(\tau), \quad (84)
\]

where \( m = (P - 1)/2 \). Using (28) and (81), \( \dot{c}(\tau) \) is given by

\[
\dot{c}(\tau) = L_{m+1} \begin{bmatrix} I_{n+1} \\ 0_{(m-n) \times (n+1)} \end{bmatrix} L_{n+1}^{-1} \tilde{h}(\tau) = \begin{bmatrix} \dot{\tilde{h}}(\tau) \end{bmatrix} 0_{m-n}. \quad (85)
\]

Thus, \( \dot{c}_p(\tau) = \dot{\tilde{h}}_p(\tau) \) is derived: when the order of the self-interference model \( Q \) is less than the order of the canceler \( P \) (i.e., \( P \geq Q \)), all self-interference of (78) can be removed. However, the theoretical analysis of the case where all self-interference is removed remains easy. Also, our main focus is
the case where there is a residual nonlinear self-interference, i.e., \( P < Q \). When \( P < Q \), \( \dot{c}(\tau) \) is given by

\[
\dot{c}(\tau) = L_{m+1} [ I_{m+1} \ 0_{(m+1) \times (n-m)} ] L_{n+1}^{-1} \dot{h}(\tau)
\]

where \( \dot{h}_{i,j} = (L_{n+1}^{-1})_{i,j} \). Then, we derive \( \dot{c}_p(\tau) \neq \dot{h}_p(\tau) \) when \( P < Q \). The result shows that the analysis of residual self-interference is very difficult when using the non-orthogonal polynomial expansion.

\[\square\]

**APPENDIX C**

**Probability Density Function of (57)**

In this appendix, we derive the PDF of (57). We assume that \( |H_{11}(f)|^2 \) and \( |H_{21}(f)|^2 \) have exponential distributions \( \text{Exp}(\rho_{11}^2) \) and \( \text{Exp}(\rho_{21}^2) \), respectively. Thus, \( I_{11}^R(f) \) and \( U_{21}(f) \) are distributed exponentially, and expected values for these random variables in \(-\frac{1}{2} < f < \frac{1}{2}\) can be expressed as

\begin{equation}
T_{11}^R(f) = E[I_{11}^R(f)] = |\bar{\beta}_{1,1}\rho_{11}|^2 \sum_{p=P+2}^{P+4} |\bar{\alpha}_{1,p}|^2 |\Psi_p(f)|^2,
\end{equation}

\begin{equation}
U_{21} = E[U_{21}(f)] = |\bar{\beta}_{1,1}\rho_{21}\bar{\alpha}_{2,1}|^2.
\end{equation}

Thus, the probability distribution function of \( I_{11}^R(f) \) and \( U_{21}(f) \) can be described as

\begin{equation}
p_{I_{11}^R}(x; f) = \frac{1}{T_{11}^R(f)} \exp\left(-\frac{x}{T_{11}^R(f)}\right),
\end{equation}

\begin{equation}
p_{U_{21}}(x) = \frac{1}{U_{21}} \exp\left(-\frac{x}{U_{21}}\right).
\end{equation}

Then, the cumulative distribution function of (57) in \(-\frac{1}{2} < f < \frac{1}{2}\) can be written as

\[
\text{Prob}\{\lambda_{\text{sinr},1}(f) < x\} = \int_{\lambda_{\text{sinr},1}}^{\infty} \int_0^x p_{I_{11}^R}(I; f) p_{U_{21}}(U) dU dI = \exp\left(-\frac{N_{\text{tot},1}}{xU_{21}}\right) \left(1 - \frac{1}{U_{21}} \left(\frac{x}{T_{11}^R(f)} + \frac{1}{U_{21}}\right)^{-1}\right),
\]

where \( p_{I_{11}^R}(x; f) \) and \( p_{U_{21}}(x) \) are PDFs of the exponential distributions of \( I_{11}^R(f) \) and \( U_{21}(f) \), respectively. In (91), the domain of the two-dimensional integration is as shown in Fig. 9. Thus, the PDF of (57) can be described as

\begin{equation}

\frac{d}{dx} \text{Prob}\{\lambda_{\text{sinr},1}(f) < x\} = \frac{N_{\text{tot},1}(xU_{21} + T_{11}^R(f)) + xU_{21} T_{11}^R(f)}{(xU_{21} + T_{11}^R(f))^2} \exp\left(-\frac{N_{\text{tot},1}}{xU_{21}}\right).
\end{equation}

**APPENDIX D**

**Analysis Examples on Saleh Model**

To prove that the proposed technique can analyze AM-PM characteristics as well as AM-AM characteristics, we show additional analysis results using the Saleh model [32]. The transfer function of the Saleh model can be expressed as

\begin{equation}
f(x) = \frac{A_1x}{1 + B_1|x|^2} \exp\left(j \frac{A_2}{1 + B_2|x|^2}\right),
\end{equation}

where \( A_1, A_2, B_1, \) and \( B_2 \) are parameters that characterize the nonlinearity of the Saleh model. When the linear gain, output saturation level, and phase displacement at the saturation point are \( G, A_{\text{sat}}, \) and \( \Phi_{\text{sat}} \), respectively, the parameters of the Saleh model can be expressed as

\begin{equation}
A_1 = G, A_2 = \frac{2\Phi_{\text{sat}}G^2}{4A_{\text{sat}}^2}, B_1 = B_2 = \frac{G^2}{4A_{\text{sat}}^2}.
\end{equation}

In this paper, we use \( \Phi_{\text{sat}} = \pi/6 \), and \( G \) and \( A_{\text{sat}} \), as with the Rapp model. Figure 10 shows the average symbol error rate of the full-duplex system which has Saleh-modeled amplifiers with various back-offs. The simulation and analysis parameters of Fig. 10 are the same as those in Fig. 7 except for the nonlinear transfer function. As with the results of the Rapp model, which does not have an AM-PM characteristic, we can also confirm that the theoretical results and simulation results match well even if the transfer functions have an AM-PM characteristic.

**APPENDIX E**

**Discussions About IQ Imbalance and Phase Noise**

The main focus of the proposed analysis is the effect of AM-AM and AM-PM characteristic on the performance of parallel Hammerstein cancellers. However, the effects of IQ imbalance and phase noise are also important research subjects...
Fig. 10. The average symbol error rate (SER) of the two full-duplex terminals with different values of input-back-offs of both terminals’ PAs when $\rho_{11}^2 = 50$ dB and $\rho_{22}^2 = 70$ dB. The nonlinear amplifiers $\alpha_n(x)$ and $\beta_n(x)$ are modeled using the Saleh model with $\Phi_{sat} = \pi/6$.

for full-duplex radios. In this appendix, we provide simulation results under IQ imbalance and phase noise and compare them with analysis results. The simulation considers IQ mixers with image rejection rate (IRR) of 25 dB at the transmitter and receiver, and a local oscillator that generates phase noise of about $-90$ dBc/Hz at 10 kHz offset. These parameters are set based on NI 5791R transceiver [40] and previous literature on in-band full-duplex communication [21], [41]. The other parameters are the same as in Section IV, but the number of simulation trials is 1001. Also, in Appendix E-B, we provide a modification of the proposed analysis to analyze the performance of FD radios when the self-interference caused by IQ imbalance cannot be cancelled sufficiently.

A. Comparison of Analysis and Simulation Results in the Presence of IQ Imbalance and Phase Noise

Figure 11 shows the cancellation performance of the parallel Hammerstein canceller with various smoothness factors for both terminals’ PAs as in Fig. 6. In contrast to Fig. 6, the IQ imbalance and phase noise are taken into account on the simulation results, and simulated cancellers can estimate and regenerate self-interference signal affected by IQ imbalance. The regenerated self-interference signal by the cancellers can be written as

$$c[n] = \sum_{m=0}^{M_f-1} \sum_{p=1}^{P} \sum_{q=0}^{P} c_{p,q}[m] (x[n-m])^q (x^*[n-m])^{p-q}$$

instead of Eq. (68), where $c_{p,q}[m]$ is estimated by the least squares algorithm. In addition, the analysis results are shown exactly the same as Fig. 6. For the canceller with $P = 1$ to 5, we can see from Fig. 11 that the simulation results are in good agreement with the results of the proposed theoretical analysis, as in Fig. 6. Even for the canceller with $P = 7$, the difference between the simulation and theoretical results is only a few dB. This shows that the most serious problem in cancellers that support IQ imbalance is the nonlinearity from amplifiers, and that IQ imbalance and phase noise have only a small effect on the canceller performance. As for phase noise, in in-band full-duplex, the transmitter and receiver can share a local oscillator, so the phase noise generated by the transmitter is reduced by the receiver. Therefore, in this situation, our analysis method is still valid even in the presence of IQ imbalance and phase noise.

B. Modification of the Proposed Analysis for IQ Imbalance

Figure 12 shows the cancellation performance of the parallel Hammerstein canceller with various smoothness factors for both terminals’ PAs as in Fig. 11. Unlike Fig. 6, IQ imbalance and phase noise are also taken into account in the simulation. In contrast to Fig. 11, the simulated cancellers cannot estimate and regenerate self-interference signal affected by IQ imbalance. On the theoretical analysis, the residual self-interference power is calculated by Eq. (96).
that cannot be removed. Since our proposed method does not take into account the effect of IQ imbalance, the performance of cancellers cannot be analyzed well if no modification is made to the proposed method. To address this issue, we use the following residual interference power:

$$\sum_{p=1,3,}\left|g_{p,1,1}\right|^2 \rho_{1,1}^2 \left(1 + \frac{\alpha}{\gamma}\right) - \sum_{p=1,3,}\left|g_{p,1,1}\right|^2.$$  \hspace{1cm} (96)

instead of Eq. (48) where IRR is the image rejection rate of IQ mixers. The coefficient $\gamma$ means that IQ imbalance components that are difficult to remove occur in the transmitter and receiver, respectively. Figure 12 shows that the performance of the canceller can be evaluated accurately by Eq. (96) instead of Eq. (48).

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