A NEW MEASURE OF THE CLUSTERING OF QSO HEAVY-ELEMENT ABSORPTION-LINE SYSTEMS

JEAN M. QUASHNOCK
Department of Astronomy and Astrophysics, University of Chicago, Chicago, IL 60637; jmq@oddjob.uchicago.edu

AND

MICHAEL L. STEIN
Department of Statistics, University of Chicago, Chicago, IL 60637; stein@galton.uchicago.edu

Received 1998 August 21; accepted 1998 November 30

ABSTRACT

We examine the line-of-sight clustering of QSO heavy-element absorption-line systems, using a new measure of clustering, called the reduced second moment measure, $K(r)$, that directly measures the mean overdensity of absorbers on scales $\leq r$. This measure—while closely related to other second-order measures such as the correlation function or the power spectrum—has a number of distinct statistical properties that make possible a continuous exploration of clustering as a function of scale. From a sample of 352 $\text{C} \text{IV}$ absorbers with median redshift $\langle z \rangle = 2.2$, drawn from the spectra of 274 QSOs, we find that the absorbers are strongly clustered on scales from $1-20 \, h^{-1} \text{Mpc}$. Furthermore, there appears to be a sharp break at $20 \, h^{-1} \text{Mpc}$, with significant clustering on scales up to $100 \, h^{-1} \text{Mpc}$ in excess of that which would be expected from a smooth transition to homogeneity. There is no evidence of clustering on scales greater than $100 \, h^{-1} \text{Mpc}$. These results suggest that strong $\text{C} \text{IV}$ absorbers along a line of sight are indicators of clusters and possibly superclusters, a relationship that is supported by recent observations of “Lyman break” galaxies.

Subject headings: cosmology; observations — intergalactic medium — large-scale structure of universe — methods: statistical — quasars: absorption lines

1. INTRODUCTION

In a previous series of investigations (Vanden Berk et al. 1996; Quashnock, Vanden Berk, & York 1996; Quashnock & Vanden Berk 1998), the clustering properties of $\text{C} \text{IV}$ and $\text{Mg} \text{II}$ absorbers have been investigated, using an extensive catalog of heavy-element absorption-line systems drawn from the literature. These authors used a line-of-sight correlation function analysis and found evidence for strong (and evolving) power-law clustering on comoving scales of $1-16 \, h^{-1} \text{Mpc}$ of a form that is consistent with that found for galaxies and clusters at low redshift, and of amplitude such that absorbers are correlated on scales of clusters of galaxies. Furthermore, there also appears to be superclustering on scales of $50-100 \, h^{-1} \text{Mpc}$ (Quashnock et al. 1996), suggesting that these absorbers are biased tracers of the higher density regions of space and that agglomerations of strong absorbers along a line of sight are indicators of clusters and superclusters.

This relationship is supported by recent observations of so-called “Lyman break” galaxies (Steidel et al. 1998) that were found to be concentrated in coherent structures of size $\sim 10 \, h^{-1} \text{Mpc}$. These structures were found to contain metal-line systems. Also, the amplitude of the correlation function of these Lyman break galaxies at $z = 3.04$ ($r_0 = 2.1 \pm 0.7 \, h^{-1} \text{Mpc}$; Giavalisco et al. 1998) is consistent with that found for $\text{C} \text{IV}$ absorbers ($r_0 = 2.2 \, h^{-1} \text{Mpc}$; Quashnock & Vanden Berk 1998). While the exact relationship between high-redshift galaxies and heavy-element absorbers is unclear, it does appear that these systems are tracing the richer agglomerations of the clustering network, perhaps one that is similar to that found in detailed three-dimensional numerical investigations of the distribution of the richest Ly$\alpha$ absorbers (see, e.g., Zhang et al. 1998).

Thus, it is of great interest to measure and characterize the clustering of the absorbers, over as broad a range in scale as possible and with special attention given to the largest scales, using the best statistical tools that are at hand. Quashnock et al. (1996) were unable to relate the superclustering found on very large ($\sim 100 \, h^{-1} \text{Mpc}$) scales with the power-law clustering found later on smaller scales (Quashnock & Vanden Berk 1998), nor locate the approximate scale dividing these two regimes, because they used a two-point correlation function analysis requiring bins too large ($25 \, h^{-1} \text{Mpc}$) for this purpose.

Here, we examine the line-of-sight clustering of QSO heavy-element absorption-line systems, using a new measure of clustering, called the reduced second moment measure, $K(r)$, that directly measures the mean overdensity of absorbers on scales $\leq r$. This measure—while closely related to other second-order measures such as the correlation function or the power spectrum—has a number of distinct statistical properties that make possible a continuous exploration of clustering as a function of scale. It has been well-studied by statisticians (Ripley 1988; Baddeley 1998) and recently astrophysicists (Martinez et al. 1998), and several estimators have been developed for it.

The absorber catalog, with a total of over 2200 absorbers listed over 500 QSOs, permits exploration of clustering over a large range in scale (from about 1 to over $100 \, h^{-1} \text{Mpc}$) and redshift ($z$ from about 1 to 4). Ultimately, we are interested in a three-dimensional description of the absorber distribution; nevertheless, much of the useful information about this distribution lies in the one-dimensional distribution of the absorption-line systems along the lines of sight to QSOs (see, e.g., Crotts et al. 1985). The large number of such lines of sight makes it possible to make some inferences.
about three-dimensional clustering from one-dimensional statistical measures.

The outline of the paper is as follows: In § 2 we define the reduced second moment measure, present the estimator we have used for it, and discuss its statistical properties. In § 3 we present our results for the reduced second moment measure, using a large sample of C IV absorbers with median redshift $\langle z \rangle = 2.2$. In § 4 we discuss the implications of these results on our picture of absorber clustering.

2. THE REDUCED SECOND MOMENT MEASURE

Here we assume that the clustering of absorbers is stationary (does not depend on time) and homogeneous (does not depend on direction or location). The first assumption is likely not to be strictly true, since growth of the correlation with decreasing redshift has been detected (Quashnock & Vanden Berk 1998). Thus, our results here are averages for our sample, which has a characteristic redshift given by the median $\langle z \rangle = 2.2$. We follow the usual convention and take the Hubble constant, $H_0$, to be 100 km s$^{-1}$ Mpc$^{-1}$ and take $q_0 = 0.5$ and $\Lambda = 0$.

2.1. Definitions

Consider the process of absorber locations along some line of sight, and let $N$ be the mean number of absorbers per unit comoving length. Define the reduced second moment measure, $K(r)$, as the conditional expectation, or average—given that there is an absorber at $x_i$—of the number of absorbers (other than the one at itself), $N(x_i, r)$, that are within a comoving distance $r$ of $x_i$, normalized by $N$:

$$K(r) = \frac{1}{N} \mathbb{E}[N(x_i, r)|\text{absorber at } x_i].$$

(1)

Because of our assumption of homogeneity, the expected number of absorbers in equation (1) does not depend on $x_i$. With $q_0 = 0.5$ and $\Lambda = 0$, the comoving distance $r$ between two absorbers at redshifts $z_i$ and $z_j$ is $r = (2c/H_0)/\sqrt{1+z_i-1/1+z_j}$.

In terms of the two-point correlation function $\xi(r)$ (Peebles 1980, 1993), the reduced second moment measure is given by

$$K(r) = 2 \int_0^r du \left[ 1 + \xi(u) \right].$$

(2)

If no correlations are present, then $K(r) = 2r$. Simply put, in this case the number of surrounding absorbers within distance $r$ of $x_i$ would not depend on the fact that there is an absorber at $x_i$ and would simply be equal to $2rN$. (The factor 2 arises because we consider distinct absorbers within a distance $r$ of $x_i$ and in the interval $(x_i - r, x_i + r)$ around any given absorber.) The quantity $K(r)/2r \equiv 1 + \rho(r)$ is then a measure of the relative mean density of absorbers around other absorbers, averaged over scales less than $r$. The relative mean overdensity, $\rho(r)$, can be written in terms of the power spectrum, $P(k)$, the Fourier transform of the correlation function $\xi(r)$, or equivalently, in terms of the dimensionless power per logarithmic wavenumber, $\Delta^2(k) \equiv k^3 P(k)/2\pi^2$:

$$\rho(r) = \int_0^\infty \frac{dk}{k} \Delta^2(k) \frac{\text{Si}(kr)}{kr}.$$  

(3)

Here $\text{Si}(z) \equiv \int_0^z dt \sin(t)/t$ is the sine-integral.

Thus, the reduced second moment measure, $K(r)$, is closely related to other second-order measures such as the correlation function or the power spectrum, and it directly measures the mean overdensity of absorbers on scales less than $r$. However, it has a number of distinct and desirable statistical properties, which we examine below in § 2.3.

2.2. Estimating $K(r)$

Let $T_i$ be the comoving length of the $i$th line of sight, i.e., the section of the $i$th QSO spectrum that has been effectively searched for absorbers. In Figure 1, we show the cumulative distribution of the comoving lengths of 274 QSO lines of sight (over an approximate redshift range $1.2 < z < 3.2$) in the Vanden Berk et al. catalog. Almost all of the lengths are shorter than $400 h^{-1}$ Mpc, but the median length $\langle T \rangle = 350 h^{-1}$ Mpc, meaning that there is information on the clustering of the absorbers on scales of $100 h^{-1}$ Mpc or more.

Let $n_i$ be the number of absorbers found in the $i$th line of sight at positions $x_{i1}, \ldots, x_{in}$. If there are a total of $n$ lines of sight, then the total comoving length and number of absorbers are $T = \sum_{i=1}^n T_i$ and $n = \sum_{i=1}^n n_i$, respectively. An estimate for the mean number of absorbers per unit comoving length is $N = n/T$.

From equation (1), a natural estimate of the reduced second moment measure, $\hat{K}(r)$, is

$$\hat{K}(r) = \frac{T}{n(n-1)} \sum_{i=1}^n \sum_{j \neq i} \theta(r - |x_{ij} - x_{ij}|),$$

(4)

where $\theta(x)$ is the Heaviside step function. This estimate sums over pairs of absorbers that are on the same line of sight and within distance $r$ of each other.

However, this estimator is biased low, because neighboring absorbers that lie outside the line of sight cannot be counted. One way to remove the bias due to edge effects is to use the rigid motion corrected estimator (Miles 1974; Osher & Stoyan 1981), which corrects for these edge effects by weighting the summand in equation (4) by a factor $f(|x_{ij} - x_{ij}|)$, which depends on the separation $|x_{ij} - x_{ij}|$ relative to the lengths $T_i$ of the lines of sight.\(^2\) This factor is the probability, given that there is a first absorber somewhere on some line of sight, that a second absorber of fixed separation from the first would also be contained within the same line if sight. We find that this probability is $f(|x_{ij} - x_{ij}|) = \sum_{k=1}^{m-1} (T_i - |x_{ij} - x_{ij}|) + T_i$ (where the “+” superscript indicates that a summand is included in the sum only if it is positive), so that the edge-corrected estimator we use is

$$\hat{K}(r) = \frac{T}{n(n-1)} \sum_{i=1}^n \sum_{j \neq i} \theta(r - |x_{ij} - x_{ij}|) f(|x_{ij} - x_{ij}|).$$

(5)

2.3. Statistical Properties

This estimator has the following statistical property:

$$\mathbb{E}[n(n-1)\hat{K}(r)/T^2] = N^{-2}K(r)$$

exactly under any homogeneous and isotropic model for the absorbers. Furthermore, while $\hat{K}(r)$ is not an exactly unbiased estimator for $K(r)$, it is a consistent estimator for $K(r)$ in the sense that $\hat{K}(r)$ tends to $K(r)$ in probability as $n$ increases.

\(^2\) Other estimators, which may have lower variance, have been found by Stein (1993), but we defer a discussion and treatment of these to a later work.
Let us contrast this estimator with the quantity $m_{aa}(r)$ used in Quashnock et al. (1996) to measure clustering. For an interval $\Delta r = (r_1, r_2)$, $\xi_{aa}(\Delta r)$ is the number of pairs of absorbers whose separation is in the interval $\Delta r$ divided by the number of pairs that would be expected in $\Delta r$ if the $n$ absorbers were randomly distributed, minus 1. This statistic has the desirable property that it tends to 0 as $m$ increases if $\xi(r)$ is identically 0 on $\Delta r$. Furthermore, positive values of $\xi_{aa}(\Delta r)$ indicate clustering over the range of distances in $\Delta r$. However, it does not provide an appropriate estimate of $\frac{d}{dr}$. In particular, $\frac{\xi_{aa}(\Delta r)}{r_2 - r_1}$ does not tend to $\int_{r_1}^{r_2} \xi(r) \, dr$ as $m$ increases. For example, if all lines of sight were of equal length $T_1$, it is possible to show that as $m \to \infty$,

$$
\xi_{aa}(\Delta r) \to \frac{\int_{T_1}^{2T_1} \xi(u) \, du}{T_1(r_2 - r_1) - (r_2^2 - r_1^2)/2} \quad \text{in probability.} \tag{6}
$$

The fact that this limit generally depends on $T_1$ is undesirable for purposes of obtaining a quantitative assessment of the clustering of absorbers. When $\xi$ is nearly constant on $\Delta r$, the limit in equation (6) is approximately $\xi[(r_1 + r_2)/2]$, as one would hope. Unfortunately, the moderate size of this data set requires the use of rather wide bins, and Quashnock et al. (1996) use values of $r_2 - r_1$ of $25 \, h^{-1}$ Mpc and greater. Using the relationship between $\xi$ and $K$ in equation (2), we can easily obtain a consistent estimator of $\int_{r_1}^{r_2} \xi(r) \, dr$ as $m$ increases. Specifically, $[\hat{K}(r_2) - \hat{K}(r_1)]/2$ converges in probability to $\int_{r_1}^{r_2} \xi(r) \, dr$ as $m$ increases.

By examining $K(r)$ as a function of $r$, we can make a continuous exploration of clustering as a function of scale, without the binning required when using a correlation function analysis. In particular, this permits a more detailed examination of the relationship between small-scale clustering (Quashnock & Vanden Berk 1998) and the super-clustering found by Quashnock et al. (1996). The reduced second moment measure estimator (eq. [5]) is easy to compute and has well-understood statistical properties.

3. RESULTS

We have used equation (5) to estimate the reduced second moment measure, $\hat{K}(r)$, for 274 QSO lines of sight, obtained from the Vanden Berk et al. catalog. A total of 352 C IV absorbers have been selected from this heterogeneous catalog, using selection criteria (Quashnock et al. 1996; Quashnock & Vanden Berk 1998) designed to obtain as homogeneous a data set as possible. We refer the reader to these papers for a detailed description of the selection criteria.

In Figure 2, we show our results for the quantity $\frac{\hat{K}(r)}{2r} \equiv 1 + \beta(r)$ (solid line) for this sample. This quantity has expectation value unity, if there is no clustering of absorbers along lines of sight (see eq. [2]). We have constructed 1000 data sets of 352 absorbers uniformly distributed along the 274 QSO lines of sight, these lines having the same distribution of comoving lengths as in our actual data sample (see Fig. 1). The 95% region of variation of $\frac{\hat{K}(r)}{2r}$
for these 1000 simulated data sets, about the expectation value of unity, is also shown in Figure 2 (dashed lines). Our estimated value of \( \hat{K}(r)/2r \) for the C IV absorber data set is much greater than the upper limit of this band, for values of \( r \) between 1 and 20 \( h^{-1} \) Mpc. For example, for \( r = 10 \ h^{-1} \) Mpc our estimate is more than 12 \( \sigma \) above unity, meaning that a simulated data set with uniformly distributed absorbers would essentially never have a \( \hat{\rho} \) as large as is measured. Thus, C IV absorbers cluster significantly on these scales.

We compare these results for \( \hat{K}(r)/2r \) with those of Quashnock & Vanden Berk (1998)—who found that the correlation function of C IV absorbers on scales of 1–16 \( h^{-1} \) Mpc is consistent with a power law of the form \( \xi(r) = (r_0/r)^\gamma \), with \( r_0 = 3.4 \ h^{-1} \) Mpc and \( \gamma = 1.75 \)—by substituting this form of the correlation function into equation (2). In Figure 2 (light line), we show the value of \( K(r)/2r \) if absorbers have this power-law correlation function. This form of clustering appears to describe the estimated reduced second moment measure \( \hat{K}(r) \) reasonably well, out to scales \( r \sim 20 \ h^{-1} \) Mpc; afterward, there appears to be a break in the form of \( \hat{K}(r)/2r \).

We have investigated the significance of this excess by examining the quantity \( [\hat{K}(r) - \hat{K}(20)]/[2(r - 20)] = [\int_{20}^{r} \xi(u) \ du]/(r - 20) \equiv \Delta_{20}(r) \), shown in Figure 3a (solid line) for the same sample of C IV absorbers as in Figure 2. From equation (2), \( \Delta_{20}(r) \) also has expectation value of unity, if the correlation function is zero on scales greater than 20 \( h^{-1} \) Mpc. From Figure 3, it appears that \( \Delta_{20}(r) \) is greater than unity on scales \( r \gtrsim 30 \ h^{-1} \) Mpc. We have estimated the error in the estimate of \( \Delta_{20}(r) \) by a bootstrap resampling method in which we randomly pick 274 QSO lines of sight from the actual data sample, with replacement, i.e., allowing for the same line of sight to be picked multiple times (see Efron & Tibshirani 1993 or Davison & Hinkley 1997 for a review of bootstrap methods for estimating errors). This method ensures that the distribution of lengths of the resampled data sets is the same as that of the actual sample (Fig. 1). In Figure 3a (dashed lines), we also show the bootstrap-estimated 95% pointwise (i.e., for each value of \( r \)) confidence region for \( \Delta_{20}(r) \). While there is some uncertainty in the estimate of this quantity, it does appear that there is significant excess clustering on scales \( r \gtrsim 30 \ h^{-1} \) Mpc. For example, when resampling data sets by the bootstrap method, \( \Delta_{20}(50 \ h^{-1} \) Mpc) is greater than unity 99.998% of the time.

The bootstrap procedure for obtaining confidence intervals we have employed here has the desirable property that its validity does not require any special assumptions about
the nature of the absorber distribution along a line of sight. More specifically, because it uses lines of sight as the sampling unit in the resampling scheme, it only requires that the location of absorbers on different lines of sight are independent. Since the majority of lines of sight are not within 100 \(h^{-1}\) Mpc of any other line of sight, this independence assumption is reasonable. By using a resampling in which groups of lines of sight are resampled rather than individual lines, we believe it should be possible to detect if this independence assumption is appropriate. We plan to investigate this possibility and other refinements of the bootstrapping procedure in future work.

The procedures used by Quashnock & Vanden Berk (1998) and Quashnock et al. (1996) also assume that absorber locations on different lines of sight are independent. In addition, they both make use of further approximations about the absorber location process within a line of sight. Quashnock & Vanden Berk (1998) obtain approximate confidence intervals for the line-of-sight correlation function up to distances of 16 \(h^{-1}\) Mpc by assuming that every pair of points whose separation is in the interval \(\Delta r\) is independent of every other such pair. This assumption may be a good approximation when \(r_2 - r_1\) is small, although simulations in Stoyan, Bertram, & Wendrock (1993) suggest that such an assumption may often lead to overoptimistic confidence intervals. On larger scales, for which Quashnock et al. (1996) have used fairly wide bins (greater than 25 \(h^{-1}\) Mpc wide), assuming independence between pairs with distance in \(\Delta r\) may be problematic. There, the confidence intervals are based on assuming that one can ignore correlations beyond second-order in absorber locations along a line of sight. While we have no evidence that such an assumption is wrong, the bootstrapping procedure we employ is valid whether or not this assumption is reasonable.

We have also searched for clustering on scales greater than 50, 100, and 150 \(h^{-1}\) Mpc, by examining the quantities \(\Delta_{50}(r), \Delta_{100}(r),\) and \(\Delta_{150}(r),\) shown (solid lines) in Figures 3b, 3c, and 3d. Again we also show the bootstrap-estimated 95% confidence region for each quantity (dashed lines). We find that \(\Delta_{50}(r)\) is significantly greater than unity, for scales \(r > 50 h^{-1}\) Mpc, meaning that there is significant clustering on those scales. Namely, when resampling data sets by the bootstrap method, \(\Delta_{50}(100 h^{-1}\) Mpc\) is greater than unity 99.91\% of the time. However, \(\Delta_{100}(r)\) is statistically consistent with unity for all \(r > 150 h^{-1}\) Mpc, and \(\Delta_{150}(r)\) is consistent with unity everywhere except (marginally) for \(r \sim 200 h^{-1}\) Mpc. This supports the conclusion of Quashnock et al. (1996) that at present there is no significant evidence for clustering of absorbers on scales greater than 100 \(h^{-1}\) Mpc.

4. DISCUSSION

We have demonstrated that the line-of-sight clustering of QSO heavy-element absorption-line systems can be examined using a new measure of clustering, called the reduced second moment measure, \(K(r)\), that directly measures the mean overdensity of absorbers on scales \(\lesssim r\). By estimating \(K(r)\), we find that the absorbers are strongly clustered on scales from 1 to 20 \(h^{-1}\) Mpc, in a manner that is consistent with a power-law correlation function of the form found by Quashnock & Vanden Berk (1998). The form and amplitude of this clustering strongly suggests that the absorbers are tracing the large-scale structure seen in the distribution of galaxies and clusters.
However, because we have only examined the clustering of absorbers in one dimension, along the line of sight, there remains the possibility that some or all of the excess clumping is due to velocity effects, i.e., groups of component absorbers spread out in redshift due to velocity dispersion. (Note that at redshift (z) = 2.2, 1 h⁻¹ Mpc corresponds to velocity differences Δv = 180 km s⁻¹ in the rest frame: The flattening in 1 + ρ(r) seen near 1 h⁻¹ Mpc in Figure 2 may be due to velocity dispersion, as well as limited spectral resolution.) This has been argued by Crotts, Burles, & Tytler (1997), who explore the spatial clustering of C IV systems along adjacent lines of sight and claim that it is significantly weaker than clustering along a line of sight. Quashnock & Vanden Berk (1998) have shown that, whether due to peculiar motions inside clusters, or to actual spatial clustering on megaparsec scales, that the scale, the form, and the amplitude of the clustering are all indicative of an association of strong absorbers with clusters. Such an association is also supported by observations of “Lyman break” galaxies (see §1).

In Figure 2, there is a sharp break in the form of K(r) at 20 h⁻¹ Mpc. It thus appears that (for q = 0.5) this is the scale marking the boundary of power-law clustering on smaller scales. Using the reduced second moment measure has permitted an approximate determination of this break. From Figures 3a–3d there is evidence for clustering on scales of up to 100 h⁻¹ Mpc—but not on larger scales—in excess of that which would be expected from a smooth transition to homogeneity.

One possible interpretation of this excess is that it is due to superclustering on scales of 50–100 h⁻¹ Mpc (Heisler, Hogan, & White 1989; Dinshaw & Impey 1996; Williger et al. 1996; Quashnock et al. 1996 and references therein), much like what is seen locally in the distribution of galaxies: If true, this means that these absorbers are biased tracers of the higher density regions of space and that agglomerations of strong absorbers along a line of sight are indicators of clusters and superclusters.

However, Richards et al. (1998) have recently claimed that there may be evidence in the data catalogs that the number of C IV absorbers along the line of sight depends on the intrinsic properties of the QSO. These authors argue that there may be a significant contamination of true intervening systems along the line of sight by absorbers that are actually associated with the QSO, and that such a contamination may extend to relative velocities as great as 75000 km s⁻¹ from the QSO. In this work, we have adopted the standard cutoff and excluded absorbers that are closer than 5000 km s⁻¹ to the QSO (Foltz et al. 1988; this corresponds to comoving distances of about 30 h⁻¹ Mpc in this work).

It is possible that such a contamination is present in the large-scale excess ρ(r) in Figure 2. A more detailed analysis of this possible effect will require an indicator capable of distinguishing, at least statistically, associated absorption-line systems from true intervening ones. While the exact nature of this large-scale excess is still uncertain, its existence on scales of 20 h⁻¹ Mpc has been unambiguously revealed by the present analysis.

This work was supported in part by NASA grant NAG 5-4406 and NSF grant DMS 97-09696 (J. M. Q.), and by NSF grant DMS 95-04470 (M. L. S.).

REFERENCES

Baddeley, A. 1998, in Stochastic Geometry: Likelihood and Computation, ed. O. E. Barndorff-Nielsen, W. S. Kendall, & M. N. M. van Lieshout (London: Chapman & Hall), chap. 2
Crotts, A. P. S., Burles, S., & Tytler, D. 1997, ApJ, 489, L7
Crotts, A. P. S., Melott, A. L., York, D. G., & Fry, J. N. 1985, Phys. Lett. B, 155B, 251
Davison, A. C., & Hinckley, D. V. 1997, Bootstrap Methods and their Application (Cambridge: Cambridge Univ. Press)
Dinshaw, N., & Impey, C. D. 1996, ApJ, 458, 73
Efron, B., & Tibshirani, R. J. 1993, An Introduction to the Bootstrap (New York: Chapman & Hall)
Foltz, C. B., Chaffee, F. H., Jr., Weymann, R. J., & Anderson, S. F. 1988, in QSO Absorption Lines: Probing the Universe, Proc. QSO Absorption Line Meeting, ed. J. C. Blades, D. A. Turnshek, & C. A. Norman (Cambridge: Cambridge Univ. Press), 53
Giavalisco, M., Steidel, C. C., Adelberger, K. L., Dickinson, M. E., Pettini, M., & Kellogg. M. 1998, ApJ, 503, 543
Heisler, J., Hogan, C. J., & White, S. D. M. 1989, ApJ, 347, 52
Martinez, V. J., Pons-Bordera, M.-J., Moyede, R. A., & Graham, M. J. 1998, MNRAS, 298, 1212
Miles, R. E. 1974, in Stochastic Geometry, ed. E. F. Harding & D. G. Kendall (Chichester: Wiley), 228
Osher, J., & Stoyan, D. 1981, Biom. J., 23, 523
Peckers, P. J. E. 1980, The Large-Scale Structure of the Universe (Princeton: Princeton Univ. Press)
Quashnock, J. M., & Vanden Berk, D. E. 1998, ApJ, 500, 28
Quashnock, J. M., Vanden Berk, D. E., & York, D. G. 1996, ApJ, 472, L69
Richards, G. T., York, D. G., Yanny, B., Kollgaard, R. I., Laurent-Muehleisen, S. A., & Vanden Berk, D. E. 1999, ApJ, 513, in press
Ripley, B. D. 1981, Statistical Inference for Spatial Processes (Cambridge: Cambridge Univ. Press)
Steidel, C. C., Adelberger, K. L., Dickenson, M., Giavalisco, M., Pettini, M., & Kellogg. M. 1998, ApJ, 492, 428
Stein, M. L. 1993, Biometrika, 80, 443
Stoyan, D., Bertram, U., & Vendrock, H. 1993, Ann. Inst. Statist. Math., 45, 211
Vanden Berk, D. E., Quashnock, J. M., York, D. G., & Yanny, B. 1996, ApJ, 469, 78
Williger, G. M., Hazard, C., Baldwin, J. A., & McMahon, R. G. 1996, ApJS, 104, 145
York, D. G., Yanny, B., Crotts, A., Carilli, C., Garrison, E., & Matheson, L. 1991, MNRAS, 250, 24
Zhang, Y., Meiksin, A., Anninos, P., & Norman, M. L. 1998, ApJ, 495, 63