Collins-Soper Equation for the Energy Evolution of Transverse-Momentum and Spin Dependent Parton Distributions

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Abstract

The hadron-energy evolution (Collins and Soper) equation for all the leading-twist transverse-momentum and spin dependent parton distributions is derived in the impact parameter space. Based on this equation, we present a resummation formulas for the spin dependent structure functions of the semi-inclusive deep inelastic scattering.
I. INTRODUCTION

Semi-inclusive processes at low transverse momentum have attracted considerable interest in recent years. These processes can provide much information on aspects of perturbative and non-perturbative quantum chromodynamics (QCD), and on the internal structure of the nucleon in particular. Theoretical study of these processes began with the classical work of Collins and Soper in which a nearly back-to-back hadron pair is produced in $e^+e^-$ collision [1]. Applications to the Drell-Yan process and the semi-inclusive deep-inelastic scattering (SIDIS) were made lately in [2, 3, 4]. A factorization theorem for the process was established [1], involving a new class of non-perturbative hadronic observables depending on the transverse-momentum of hadrons and/or partons: the transverse-momentum dependent (TMD) fragmentation functions and parton distributions. Based on the recent development of the gauge invariant definition of the TMD parton distributions [5, 6], the proof of the factorization has been extended to the SIDIS and Drell-Yan processes at low transverse momentum (one the order of $\Lambda_{QCD}$) [7, 8], where the TMD parton distributions and fragmentation functions as important ingredients were emphasized.

On the other hand, at low transverse momentum, $P_\perp \ll Q$, where $Q$ represents some hard scale (e.g. the invariant mass of the virtual photon in SIDIS), the standard perturbative QCD (pQCD) calculations generate large logarithms $\alpha_s^n \ln^{2n} Q^2/P_\perp^2$ in perturbation series. These large logarithms must be resummed to make predictions reliable [2, 9, 10]. According to the factorization theorem for the semi-inclusive processes, these large logarithms can be attributed to the hadron-energy dependence of the TMD parton distributions and fragmentation functions, which contains double logarithms [1, 7]. The Collins-Soper equation is precisely the equation which governs the energy dependence of the TMD parton distributions and the fragmentation functions [1]. The factorization proof and an extensive study for the energy evolution equation for the unpolarized quark distribution have been performed in [1]. The large logarithms mentioned above can be resummed by solving this evolution equation [1, 2]. In this paper, we will follow these studies to analyze the evolution equations for the spin-dependent quark distributions. We will show that the original Collins-Soper evolution equation also applies to the spin-dependent distributions. With these evolution equations, we can perform the large logarithmic resummations for the spin-dependent structure functions and asymmetries in the semi-inclusive DIS [7, 8]. Another important point of our calculations is that we use Feynman gauge, and the TMD parton distributions in this gauge are defined in such a way to guarantee the gauge invariance [5, 6]. In [1], a specified gauge (axial gauge) was used, while in [11] the energy evolution for the Sudakov form factor was calculated in Feynman gauge.

The rest of this paper is organized as follows. In Sec.II, we briefly review the factorization of the energy evolution equation for the TMD quark distribution following [1, 11], and point out that this factorization works also for the spin-dependent distributions. The soft and hard parts in the factorization are defined. In Sec.III, the evolution equation for the unpolarized quark distribution at one-loop order is rederived. In Sec.IV, we show the results for all the leading-twist TMD quark distributions. In Sec.V, with these evolution equations, we show how to resum the large logarithms. We conclude in Sec.VI.
II. FACTORIZATION OF THE ENERGY DERIVATIVE FOR THE TMD QUARK DISTRIBUTIONS

In the non-singular gauge (e.g. Feynman gauge), the TMD quark distributions can be defined through the following matrix: \[^{[2]}^{[11]}^{[12]},\]

\[
\mathcal{M}(x, k_\perp, \mu, x\zeta, \rho) = p^+ \int \frac{d\xi^-}{2\pi} e^{-i\xi^-p^+} \int \frac{d^2b_\perp}{(2\pi)^2} e^{ib_\perp \cdot \vec{k}_\perp} \times \langle PS | \bar{\psi}_q(\xi^-, 0, \vec{b}_\perp) \mathcal{L}_v^\dagger(\infty; \xi^-, 0, \vec{b}_\perp) \mathcal{L}_v(\infty; 0) \psi_q(0) | PS \rangle, \tag{1}
\]

where \(\psi_q\) is the quark field with the Dirac- and color indices implicit. The parent hadron has a momentum \(P^\mu\) along the \(z\)-direction, and is polarized with a spin vector \(S^\mu\). In the following, we use the light-cone coordinates \(k^\pm = (k^0 \pm k^3)/\sqrt{2}\), and write any four-vector \(k^\mu\) in the form of \((k^-, \vec{k}) = (k^-, k^+, k^\perp)\), where \(k^\perp\) represents two perpendicular components \((k^x, k^y)\). The light-like vector \(p\) is chosen to be \(p^\mu = \Lambda(0, 1, 0)\), and \(v^\mu\) is a time-like dimensionless \((v^2 > 0)\) four-vector with zero transverse components \(v^-, v^+, 0\). We choose \(v^- \gg v^+\) so that \(v^\mu\) is very close to the light-like vector \(n^\mu = (1, 0, 0)/2\Lambda\). The variable \(\zeta^2\) is essentially the energy of the hadron, \(\zeta^2 = (2P \cdot v^2)/v^2 = 2(P^+)^2v^-/v^+\). \(\mathcal{L}_v\) is a gauge link along \(v^\mu\),

\[
\mathcal{L}_v(\infty; \zeta) = \exp \left( -ig \int_0^\infty d\lambda v \cdot A(\lambda v + \xi) \right). \tag{2}
\]

Here a non-light-like gauge link is introduced to regulate the light-cone singularity associated with the plus component of the gluon momentum \(l^+ \to 0\) \[^{[1]}^{[1]}^{[1]}\]. We avoid the use of the singular gauge (e.g. the light-cone gauge), because in such a gauge it is well known that the gauge potential does not vanish at infinity, and therefore a gauge link at infinity will be needed to ensure gauge invariance \[^{[6]}\].

In the above definition, we have divided by a soft factor which is defined as \[^{[7]}\];

\[
S(\vec{b}_\perp, \mu^2, \rho) = \frac{1}{N_c} \langle 0 | \mathcal{L}_{v\dagger}(\vec{b}_\perp, -\infty) \mathcal{L}_{vj}(\infty; \vec{b}_\perp) \mathcal{L}_{vjk}(\infty; 0) \mathcal{L}_{ekl}(0; -\infty) | 0 \rangle, \tag{3}
\]

where \(\vec{v}\) is another off-light-cone vector with \(\vec{v}^+ \gg \vec{v}^-\) and \(\vec{v} \cdot \vec{b} = 0\). The parameter \(\rho\) is defined as \(\rho^2 = v^- \vec{v}^+ / v^+ \vec{v}^+\). This soft factor will also appear in the factorization theorem for the semi-inclusive processes at low \(P_\perp\) \[^{[7]}\].

The above definition of the matrix \(\mathcal{M}\) is for deep inelastic scattering. For the Drell-Yan process, we need a different form where the gauge links are along the backward direction to \(-\infty\) \[^{[5]}^{[5]}^{[6]}\], and the soft factor has to be modified as well \[^{[8]}\]. In the following discussions, we will restrict ourselves to those for SIDIS, although the same can be done for the distributions in the Drell-Yan process.

The leading-twist expansion of the matrix \(\mathcal{M}\) contains eight distributions \[^{[13]}^{[14]}\],

\[
\mathcal{M} = \frac{1}{2} \left[ q(x, k_\perp) \cdot \vec{p} + \frac{1}{M} \delta q(x, k_\perp) \sigma^{\mu\nu} k_\mu p_\nu + \Delta q_L(x, k_\perp) \gamma_5 \cdot \vec{p} \right] + \frac{1}{M} \Delta q_T(x, k_\perp) \gamma_5 \cdot \vec{k}_\perp \tag{4}
\]

\[
+ \frac{1}{M^2} \delta q_T(x, k_\perp) i\sigma_{\mu\nu} \gamma_5 p^\mu \left( \vec{S}_\perp - \frac{1}{2} \vec{S}_\perp \right) + \frac{1}{M^2} \delta q_T(x, k_\perp) \epsilon^{\mu\nu\alpha\beta} \gamma_\mu p_\alpha k_\beta S_\beta, \nonumber
\]

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where $M$ is the nucleon mass. We have omitted the arguments $\zeta$, $\mu$ and $\rho$ in the parton distributions on the right-hand side. The polarization vector $S^\mu$ has been decomposed into a longitudinal component $S^\mu_\parallel$ and a transverse one $S^\mu_\perp$, and $\lambda$ is the helicity. The notations for the distributions follow Ref. [15].

The energy evolution of the TMD parton distribution is governed by the Collins-Soper equation [1],

$$
\zeta \frac{\partial}{\partial \zeta} f(x,b,\mu,x\zeta,\rho) = (K(b,\mu,\rho) + G(x\zeta,\mu,\rho)) f(x,b,\mu,x\zeta,\rho),
$$

(5)

where $K$ and $G$ depend on soft and hard physics, respectively. In the following we will show that this equation is in fact valid for all the leading-twist quark distributions in the impact parameter space. It is important to note that the above factorization of soft and hard parts for the energy derivative is true only for the leading power (1/\zeta) contribution, and any higher power corrections have been neglected [1, 11]. So, the evolution equations like Eq. (5) and here after are only valid in the leading power of 1/\zeta. Another point we want to point out is the $\rho$ dependence in $K$ and $G$, which was absent in [1, 11]. The $\rho$ dependence comes from our definition of $K$ in Eq. (13) below to avoid possible light-cone singularity in higher loop calculations. However, the $\rho$ dependence of $K$ and $G$ cancels out, and energy derivative of the TMD parton distribution does not depend on $\rho$, (see detailed discussion below).

Since there is no energy dependence in the soft factor $S$, the only source for energy dependence comes from the numerator in the matrix $M$ in Eq. (1). We call the numerator the un-subtracted TMD parton distribution $Q(x,k_\perp,x\zeta)$ as in [7, 8]. So, the Collins-Soper equation for any un-subtracted distribution reads

$$
\zeta \frac{\partial}{\partial \zeta} \mathcal{F}(x,b,\mu,x\zeta) = (K(b,\mu,\rho) + G(x\zeta,\mu,\rho)) \mathcal{F}(x,b,\mu,x\zeta).
$$

(6)

where $K$ and $G$ depend on soft and hard physics, respectively, as we shall explain. Since any un-subtracted distribution does not depend on the soft part, then there is no $\rho$ dependence in $\mathcal{F}$. This means that sum $(K(b,\mu,\rho) + G(x\zeta,\mu,\rho))$ is $\rho$-independent while each term could be $\rho$-dependent.

The different TMD quark distributions can be obtained from Eq. (4) by making appropriate spin projections. For example, the unpolarized quark distribution is related to,

$$
Q(x,k_\perp,\mu,x\zeta) = \frac{1}{2} \int \frac{d\xi d^2\vec{b}_\perp}{(2\pi)^3} e^{-ix\xi p^+ + i\vec{b}_\perp \cdot \vec{k}_\perp} \left\langle P \left| \bar{\psi}_q(\xi^-,\vec{b}_\perp) \mathcal{L}_\nu^\gamma + \mathcal{L}_v \psi_q(0) \right| P \right\rangle.
$$

(7)

Since $\zeta^2 = (2v \cdot P)^2/v^2$, using the chain rule, the derivative on $\zeta$ can be related to the derivative on $v$,

$$
\zeta \frac{\partial}{\partial \zeta} = \delta v^\alpha \frac{\partial}{\partial v^\alpha},
$$

(8)

where $\delta v$ is another dimensionless vector: $\delta v^- = v^-$, $\delta v^+ = -v^+$, and $\delta v_\perp = 0$. So that, we have $\delta v^2 = -v^2 < 0$ and $\delta v \cdot v = 0$. From Eq. (7), we see that the only dependence of $v$ comes from the gauge link $\mathcal{L}_v$. So, we have the following differential equation for the
\[
\begin{align*}
\frac{i}{v \cdot k + i \epsilon} (-ig) v^\mu &= g \frac{v^\mu}{v \cdot k + i \epsilon} \\
\frac{v \cdot k \delta v^\mu - \delta v \cdot k v^\mu}{(v \cdot k + i \epsilon)^2}
\end{align*}
\]

FIG. 1: Feynman rules for the eikonal vertex in the TMD parton distribution (a) and its derivative (b).

un-subtracted TMD parton distribution,

\[
\zeta \frac{\partial}{\partial \zeta} Q(x, k_\bot, v) = \delta v^\alpha \frac{\partial}{\partial v^\alpha} Q(x, k_\bot, v)
\]

\[
= \frac{1}{2} \int \frac{d\xi^2 d^2 \xi_\perp}{(2\pi)^3} e^{-ix \xi} p^+ + i \xi_\perp \cdot k_\perp
\]

\[
\times \left\{ \left\langle P \mid \overline{\psi}_q(\xi) \mathcal{L}_v^\dagger \gamma^+ (-ig) \int_0^\infty d\lambda [\delta v \cdot A(\lambda v) + \lambda \delta v \cdot \partial A(\lambda v) \cdot v] \mathcal{L}_v \psi_q(0) \mid P \right\rangle
\right.
\]

\[
+ \left. \left\langle P \mid \overline{\psi}_q(\xi) \mathcal{L}_v^\dagger (ig) \int_0^\infty d\lambda [\delta v \cdot A(\lambda v + \xi) + \lambda \delta v \cdot \partial A(\lambda v + \xi) \cdot v] \gamma^+ \mathcal{L}_v \psi_q(0) \mid P \right\rangle \right\} .
\]  

The relevant Feynman rules for the eikonal vertex from the gauge link contribution are shown in Fig. 1 for the TMD parton distributions (a), and the derivative \( \zeta \frac{\partial}{\partial \zeta} \) (b).

Similar to the structure functions for the semi-inclusive DIS discussed in [7, 8], the derivative \( \zeta \frac{\partial}{\partial \zeta} \) on the un-subtracted distribution \( Q \) receives contributions from three different regions of the gluon momentum: soft, hard, and collinear contributions, respectively.

First, let us examine the collinear contribution to the derivative. From the above, the special vertex reads,

\[
g \frac{v \cdot k \delta v^\mu - \delta v \cdot k v^\mu}{(v \cdot k + i \epsilon)^2}.
\]  

If the gluon momentum \( k \) is in the collinear region, i.e., \( k \propto P \), we have \( k^+ \sim Q \), \( k^- \sim \lambda Q \), \( k_\perp \sim \sqrt{\lambda} Q \), where \( \lambda \) is a small parameter. For such momentum, the above vertex will lead to

\[
\sim g \frac{k^+ v^-}{(v \cdot k + i \epsilon)^2} (\delta v - v)^\mu,
\]

and their contribution will be suppressed if we contract the above vertex with the collinear momentum in the jet part of the TMD parton distributions.

Thus, the derivative \( \zeta \frac{\partial}{\partial \zeta} \) on the parton distribution will receive contributions only from the soft and hard regions of the gluon momentum. A detailed analysis of these contributions leads to a factorization of the derivative of the TMD distribution as illustrated in the second line in Fig. 2 [1, 11]. This can be represented by the following differential equation,

\[
\zeta \frac{\partial}{\partial \zeta} Q(x, k_\perp, x\zeta) = \int [K + G] \otimes Q(x, k_\perp, x\zeta) ,
\]  

5
where the soft part is called $K$, and hard part $G$; and the label $\otimes$ means the momentum space convolution. Again, we emphasize that the above factorization is only valid for the leading power contribution. After applying a Fourier transformation to the coordinate space, the above differential equation reads,

$$
\zeta \frac{\partial}{\partial \zeta} Q(x, b, x\zeta) = [K(b, \mu, \rho) + G(x\zeta, \mu, \rho)] \times Q(x, b, x\zeta),
$$

where the convolution in momentum space becomes products in the impact parameter $b$-space. The soft-part $K$ depends on $b$, while the hard part $G$ depends on hard scale $\zeta$; and both of them depend on the renormalization scale $\mu$ and $\rho$, but the sum does not. The above equation is valid for any value of $b$. If $b$ is small as $1/b \gg \Lambda_{QCD}$, we can further have a factorization for the TMD parton distribution which depends on the integrated parton distributions, and then we can get another form for the evolution equation with an extra term $[1]$. 

The soft part can be calculated from the Feynman diagrams by using the Grammer-Yennie approximation $[10]$. In $[7]$, we demonstrated how to get the soft contribution in the TMD parton distribution. Here, we follow the same procedure, and define the soft part for the derivative of the TMD quark distributions by the following matrix element,

$$
K(b, \mu, \rho) = \left\langle 0 \left| \mathcal{L}_v^i(\vec{b}_\perp, -\infty) \mathcal{L}_v^i(\infty, \vec{b}_\perp) \left\{ (-ig) \int_0^\infty d\lambda [\delta v \cdot A(\lambda v) + \lambda \delta v \cdot \partial A(\lambda v) \cdot v] \\
+ (ig) \int_0^\infty d\lambda [\delta \bar{v} \cdot A(\lambda v + b_\perp) + \lambda \delta \bar{v} \cdot \partial A(\lambda v + b_\perp) \cdot v] \right\} \mathcal{L}_v(\infty, 0) \mathcal{L}_{\bar{v}}(0, -\infty) \right| 0 \right\rangle,
$$

where we have introduced the off-light-cone vector $\bar{v}$ defined above to regulate possible light-cone singularities $[7]$. Notice that this definition is different from that in $[11]$ where a light-cone vector $p = (0, 1, 0_\perp)$ was used instead of our off-light-cone vector $\bar{v}$. Introducing $\bar{v}$ leads to the $\rho$ dependence of $K$. Since the leading order (one-loop) calculation has no light-cone singularity, there is no difference between using $\bar{v}$ and $p$ at this order, and the result
does not depend on $\rho$. However, at higher-loop, there might be light-cone singularity associated with the gluon momentum $l^+ \to 0$. We note that so far there is no explicit calculation of the soft factor $K$ beyond one-loop order in non-abelian gauge theory (a QED result has been given in [11]). It is not clear at present that we can take the light-cone limit for higher loop calculations. So, it is necessary to include the off-light-cone vector $\tilde{v}$ in the formal definition. If there is no light-cone singularity, we can take the light-cone limit ($\tilde{v} \to p$ and $\rho \to \infty$). Like in the factorization of SIDIS structure function, $\rho$ is just a parameter which separates the hard and soft physics, and it does not affect any prediction power in the resummation formalism. Normally, we should take $\rho \gg 1$, while in practice we can choose $\rho$ between 3 and 10 if there is any $\rho$ dependence in $K$ to avoid the large logarithms associate with $\rho$.

Comparing Eq. (13) with Eq. (3), we find that the soft factor in the derivative can be related to that in the parton distribution,

$$K(b, \mu, \rho) = \frac{1}{S(b, \mu, \rho)} \delta v^\alpha \frac{\partial}{\partial v^\alpha} S(b, \mu, \rho) = \frac{1}{S(b, \mu, \rho)} \rho \frac{\partial}{\partial \rho} S(b, \mu, \rho).$$

(14)

Our one-loop result below verifies this relation. Since the distribution is $\rho$-independent, the $\rho$ dependence in $K$ must cancel the $\rho$ dependence in $G(x\zeta, \mu, \rho)$.

The renormalization scale dependence of the soft-factor $K$ is determined by the cusp anomalous dimension [17],

$$\mu \frac{d}{d\mu} K = -\gamma_K,$$

(15)

which is a series in $\alpha_s$ and free of infrared singularities. The hard part $G$ can be calculated through a systematic subtraction, as will be illustrated by the one-loop example in the next section. From the definition, it is obvious that the soft factor is spin-independent.

The above analysis of the factorization of energy derivative and the definitions of the hard and soft parts can be extended to the spin-dependent TMD quark distributions, because they all come from the same matrix $\mathcal{M}$ Eq. (1) with different spin projections, and the arguments supporting the factorization can be generalized to all the leading-twist distributions. We will explicitly show this in more detail in the following one-loop calculations, and give general argument for all orders.

III. RE-DERIVATION OF COLLINS-SOPER EQUATION FOR UNPOLARIZED QUARK DISTRIBUTION

In this section, we demonstrate how to calculate, in the present formulation, the Collins-Soper evolution kernels $K$ and $G$ for the unpolarized quark distribution $Q$ at one-loop order. In next section, we will discuss the spin dependence and present the evolution equations for all the leading-twist TMD quark distributions.

We first calculate the leading contribution to the soft part $K$, linearly proportional to the strong coupling constant $\alpha_s$. The relevant diagrams are shown in Fig. 3. Fig. 3(a) vanishes because $\delta v \cdot v = 0$, and 3(c) vanishes for the same reason and for being sub-leading order in $1/\zeta^2$. The contribution from Fig. 3(b) is

$$K(b, \mu, \rho)|_{\text{Fig. 3(b)}} = -\frac{\alpha_s C_F}{\pi} \ln \frac{\mu^2}{\Lambda^2},$$

(16)
where we have included a factor of two to account for the two vertex correction diagrams. \( \lambda \) is the gluon mass, introduced to regulate the infrared singularity for individual diagrams. However, the sum of all contributions is free of the infrared divergence. Fig. 3(d), together with its mirror diagram, contributes

\[
K(b, \mu, \rho)_{\text{fig.3(d)}} = \frac{\alpha_s C_F}{\pi} \left[ \ln \frac{4}{b^2 \lambda^2} - 2 \gamma_E \right].
\]  

(17)

Summing up, we get,

\[
K(b, \mu, \rho) = -\frac{\alpha_s C_F}{\pi} \left[ \ln \frac{\mu^2 b^2}{4} + 2 \gamma_E \right],
\]  

(18)

which agrees with the previous calculations [1, 11, 18]. Moreover, up to first order it has no dependence on \( \rho \), which means that we can take the light-cone limit for \( \tilde{v} \) (i.e., let \( \tilde{v} = p \)) at this order. We remark here that higher order calculation of \( K(b, \mu, \rho) \) may show explicit \( \rho \) dependence. Comparing this result with the soft factor \( S(b, \mu, \rho) \) at one-loop order calculated in [7], Eq. (14) is clearly satisfied. The one-loop cusp anomalous dimension is,

\[
\mu \frac{\partial}{\partial \mu} K(b, \mu) = -\gamma_K = -2 \frac{\alpha_s C_F}{\pi},
\]  

(19)

which is well known. As we stated in the previous section, the soft part is spin independent, hence the above result for \( K \) is the same for all the leading order TMD quark distributions.

We now calculate the complete one-loop contribution to the right-hand side of the Collins-Soper equation, from which we will subtract the above soft contribution to get the hard part \( G \). All the one-loop diagrams are shown in Fig. 4. The contributions from Fig. 4(a) and (c) vanish because of the same reason as that for Fig. 3(a) and (c).
The contribution from Fig. 4(b) in momentum space reads,
\[ \frac{\partial}{\partial \ln \zeta} Q(x, k_\perp, x\zeta)_{\text{fig.4(b)}} = \frac{\alpha_s C_F}{\pi} \left( 1 - \ln \frac{x^2 \zeta^2}{\lambda^2} \right) Q(x, k_\perp, x\zeta), \tag{20} \]
which is the same as that in the impact parameter \( b \) space. The explicit dependence on \( \zeta \) and \( \lambda \) indicates the presence of both hard and soft contributions. Subtracting the soft contribution in Eq. (16), we get the hard part \( G \) as,
\[ G(x\zeta, \mu) = \frac{\alpha_s C_F}{\pi} \left( 1 - \ln \frac{x^2 \zeta^2}{\mu^2} \right), \tag{21} \]
which depends on the hard scale \( \zeta \) and the renormalization scale \( \mu \).

Fig. 4(d) is dominated by the contribution from the soft gluon momentum region: \( q^+ \ll k^+ \), where \( q \) is the gluon momentum and \( k \) is the quark momentum. After the soft-gluon approximation, we get
\[ \frac{\partial}{\partial \ln \zeta} Q(x, k_\perp, x\zeta)_{\text{fig.4(d)}} = \frac{\alpha_s C_F}{2\pi^2} \int \frac{d^2 q_\perp}{q_\perp^2 + \lambda^2} Q(x, \vec{k}_\perp - \vec{q}_\perp, x\zeta). \tag{22} \]
Fourier-transforming to the \( b \) space, we have
\[ \frac{\partial}{\partial \ln \zeta} Q(x, b, x\zeta)_{\text{fig.4(d)}} = \frac{\alpha_s C_F}{\pi} \left[ \ln \frac{4}{b^2 \lambda^2} - 2\gamma_E \right] Q(x, b, x\zeta), \tag{23} \]
which can be reproduced by the soft factor \( K \) from Eq. (17) (Fig. 3(d)).

The above results show that the factorization is valid at one-loop order with the sum of \( K \) and \( G \) reads,
\[ K(b, \mu) + G(x\zeta, \mu) = -\frac{\alpha_s C_F}{\pi} \ln \frac{x^2 \zeta^2 b^2}{4} e^{2\gamma_E - 1}. \tag{24} \]
This result agrees also with that in [7], where the distribution itself was calculated to one-loop order.

IV. COLLINS-SOPER EVOLUTION EQUATIONS FOR SPIN-DEPENDENT TMD DISTRIBUTIONS

In this section, we study the energy evolution of the spin-dependent TMD distributions. Except for the unpolarized \( q(x, k_\perp) \), all other leading-twist TMD quark distributions depend on the polarization of either the initial hadron or the probing quark. The polarized distributions can be obtained from spin projections of the matrix \( \mathcal{M} \) Eq. (11). There are three leading-twist projections,
\[ \gamma^+ : \, q(x, k_\perp), \, q_T(x, k_\perp); \]
\[ \gamma^+ \gamma^5 : \, \Delta q_L(x, k_\perp), \, \Delta q_T(x, k_\perp); \]
\[ \gamma^+ \gamma^i \gamma^5 : \, \delta q_T(x, k_\perp), \, \delta q_L(x, k_\perp), \, \delta q(x, k_\perp), \, \delta q_T(x, k_\perp), \tag{25} \]
corresponding to the unpolarized, longitudinally-polarized, and transversely-polarized quark distributions, respectively. Moreover, different distributions may have different \( k_\perp \) dependence. For example, so-called \( k_\perp \)-even (under the exchange \( k_\perp \to -k_\perp \)) quark distributions,
\( q(x, k_\perp) \), \( \Delta q_L(x, k_\perp) \), and \( \delta q_T(x, k_\perp) \), correspond to the unpolarized, helicity, and transversity distributions, respectively. These distributions survive after integrating over \( k_\perp \). The other five distributions are associated with the \( k_\perp \)-odd structures, and vanish when \( k_\perp \) are integrated over.

In the following, we use the Sivers function as an example to demonstrate the calculation of the energy evolution kernel. The results for the other distributions can be obtained similarly. The Sivers function results through \( \gamma^+ \) projection from the matrix \( M \) Eq. (1),

\[
Q_T(x, k_\perp, \mu, x\zeta) = \frac{1}{2\epsilon^{ij} S_i k_j} \int \frac{d\xi - d\vec{b} \overline{e}}{(2\pi)^3} e^{-i\xi \cdot \vec{k} - i\vec{b} \cdot \vec{q}} \nonumber
\]

\[
\times \left\langle PS_{\perp} \left| \overline{\psi}_q(\xi^-, \vec{b}_\perp) \mathcal{L}_i^\dagger \gamma^+ \mathcal{L}_v \psi_q(0) \right| PS_{\perp} \right\rangle \bigg|_{\text{spin dependent part}},
\]

where the explicit transverse momentum and spin dependence has been included. The Feynman diagrams for the energy derivative are the same as those for the unpolarized distribution discussed in last section; and Figs. 4(a) and (c) vanish as before. The contribution from Fig. 4(d) reads,

\[
\frac{\partial}{\partial \ln \xi} \frac{\epsilon^{ij} S_i k_j Q_T(x, k_\perp, x\zeta)}{\text{fig. 4(d)}} = \frac{\alpha_s C_F}{2\pi^2} \int \frac{d^2 q_\perp}{q_\perp^2 + \lambda^2} \epsilon^{ij} S_i (k - q) j Q_T(x, \vec{k}_\perp - \vec{q}_\perp, x\zeta),
\]

where again we have made the soft approximation. To find the above result, we have applied the \( \gamma^+ \) projection to the quark matrix \( M \). Moreover, because the Sivers function is spin-dependent and associated with a \( k_\perp \)-odd structure, only such structure is isolated and kept. Fourier transforming to the impact parameter space, we get,

\[
\frac{\partial}{\partial \ln \zeta} \hat{Q}_T(x, b, x\zeta)\big|_{\text{fig. 4(d)}} = \frac{\alpha_s C_F}{\pi} \left[ \ln \frac{4}{b^2 \lambda^2} - 2\gamma_E \right] \partial_i^b Q_T(x, b, x\zeta),
\]

where \( \partial_i^b = \partial/\partial b_i \) is a derivative on the Sivers function. Apart from the explicit derivative, the above contribution is the same as that for the unpolarized distribution in the previous section.

The contribution from Fig. 4(b) is also the same as that for the unpolarized quark distribution,

\[
\frac{\partial}{\partial \ln \zeta} Q_T(x, k_\perp, x\zeta)\big|_{\text{fig. 4(b)}} = \frac{\alpha_s C_F}{\pi} \left( 1 - \ln \frac{x^2 \zeta^2}{\lambda^2} \right) Q_T(x, k_\perp, x\zeta),
\]

and similar equation holds in the \( b \)-space. Combining the above results, we have the entire energy evolution of the Sivers function at one-loop order,

\[
\zeta \frac{\partial}{\partial \zeta} \hat{g}_T(x, b, \mu, x\zeta, \rho) = (K(b, \mu, \rho) + G(x\zeta, \mu, \rho)) \partial_i^b g_T(x, b, \mu, x\zeta, \rho),
\]

where \( K \) and \( G \) are the same as those for the unpolarized distribution.

The above analysis can be repeated for all other leading-twist quark distributions, and the one-loop evolution kernels are found again to be the same as those for the unpolarized quark distribution. There are two important features supporting the above finding. First, the
leading-twist projection matrices $\gamma^+, \gamma^+\gamma^5$, and $\gamma^+\gamma^i\gamma^5$ lead to an identical trace. Second, there is no mixing between the different leading-twist distributions. This latter property, however, does not hold for the evolution of the higher-twist distributions, a topic beyond the scope of this paper.

In fact, we argue that, to all orders in perturbation theory, the same evolution kernels determine the energy evolution of all the leading-twist distributions. First of all, from the factorization of the energy derivative discussed in the Section II, only soft and hard regions contribute to the evolution. For the soft part, there is no spin dependence, as is clear from its definition Eq. (13). Therefore, for any leading-twist distribution, the soft part of the evolution kernel is the same. Second, the contribution from the hard part is also spin-independent, because the hard contribution is calculable in perturbative QCD and the perturbative processes for massless quarks conserve helicity. Any spin projection will lead to the same Dirac algebra if there is no mixing between different distributions.

To summarize, the Collins-Soper evolution kernel has no spin dependence for the leading-twist TMD quark distributions. For $k_\perp$-even distributions, the following evolution equation,

$$ \zeta \frac{\partial}{\partial \zeta} f(x, b, \mu, x\zeta, \rho) = (K(b, \mu, \rho) + G(x\zeta, \mu, \rho)) f(x, b, \mu, x\zeta, \rho) , \quad (31) $$

holds for $f = q, \Delta q_L$, and $\delta q_T$. For $k_\perp$-odd quark distributions, we have

$$ \zeta \frac{\partial}{\partial \zeta} \partial^i_b f(x, b, \mu, x\zeta, \rho) = (K(b, \mu, \rho) + G(x\zeta, \mu, \rho)) \partial^i_b f(x, b, \mu, x\zeta, \rho) , \quad (32) $$

which works for $f = q_T, \Delta q_T, \delta q$, and $\delta q_L$. Finally, for $\delta q'_T$,

$$ \zeta \frac{\partial}{\partial \zeta} \left( \partial^i_b \partial^j_b - \delta^{ij} \partial^2_b / 2 \right) \delta q'_T(x, b, \mu, x\zeta, \rho) $$

$$ = (K(b, \mu, \rho) + G(x\zeta, \mu, \rho)) \left( \partial^i_b \partial^j_b - \delta^{ij} \partial^2_b / 2 \right) \delta q'_T(x, b, \mu, x\zeta, \rho) . \quad (33) $$

As a final remark, we would like to point out that the energy evolution equations for the TMD quark distributions in the Drell-Yan process will be the same as the above for the DIS process. This is because we have the universality for the parton distributions between the two processes [5, 6].

V. RESUMMATION FOR THE STRUCTURE FUNCTIONS IN POLARIZED SEMI-INCLUSIVE DIS

In the physical cross sections with two widely separated scales, say, $P_\perp$ and $Q$, there are large double logarithms of the type $\alpha_s^n \ln^2 Q^2 / P_\perp^2$ as well as sub-leading ones. To have a reliable theoretical prediction, one has to re-sum these contributions. In this section, we perform the resummation for the large logarithms in polarized semi-inclusive DIS by solving the Collins-Soper evolution equations obtained in the last section.

In [3, 4, 5], we have obtained the factorization formulas for the various structure functions. In
the impact parameter $b$ space, they can be expressed as 

\[
\begin{align*}
\tilde{F}_{UU}^{(1)}(b) &= q(x_B, z_h b) \hat{q}(z_h, b) S^+(b) H_{UU}^{(1)}(Q^2) , \\
\tilde{F}_{LL}^{(1)}(b) &= \Delta q_L(x_B, z_h b) \hat{q}(z_h, b) S^+(b) H_{LL}(Q^2) , \\
\tilde{F}_{UT}^{(1)}(b) &= -\frac{1}{M z_h} \partial_b^i \left[ (\partial_b^i q_{T}(x_B, z_h b)) \hat{q}(z_h, b) S^+(b) H_{UT}^{(1)}(Q^2) \right] , \\
\tilde{F}_{UT}^{(2)}(b) &= -\frac{1}{M_h} \partial_b^i \left[ \delta q_{T}(x_B, z_h b) (\partial_b^i \delta \hat{q}(z_h, b)) S^+(b) H_{UT}^{(2)}(Q^2) \right] , \\
\tilde{F}_{LT}(b) &= -\frac{1}{M z_h} \partial_b^i \left[ (\partial_b^i \Delta q_{T}(x_B, z_h b)) \hat{q}(z_h, b) S^+(b) H_{LT}(Q^2) \right] , \\
\tilde{F}_{UL}^{(2)}(b) &= \frac{\partial_b^i \partial_b^j - \partial_b^2 \delta^{ij}}{M M_h z_h} \left[ (\partial_b^i \delta q(x_B, z_h b)) (\partial_b^j \delta \hat{q}(z_h, b)) S^+(b) H_{UL}^{(2)}(Q^2) \right] , \\
\tilde{F}_{UL}^{(3)}(b) &= \frac{-4\delta_b^i \partial_b^j \partial_b^k + 2\delta^{ik} \partial_b^2 \delta^{ij}}{M^2 M_h z_h^2} \left[ (\partial_b^i \delta \hat{q}(z_h, b)) S^+(b) H_{UL}^{(3)}(Q^2) \right] , \\
\end{align*}
\]

(34) 

where we followed the notations used in [8], and the parton distributions and fragmentation functions are calculated at the energy scale $x_B^2 \zeta^2 = \hat{\zeta}^2 / z_h^2 = \rho Q^2$. $\tilde{F}(b)$ are the Fourier transformation of the structure functions in the impact parameter space. For the unpolarized structure function, we define

\[
\tilde{F}_{UU}^{(1)}(b) = \int d^2 P_{h \perp} \tilde{F}_{UU}^{(1)}(P_{h \perp}) e^{i \vec{P}_{h \perp} \cdot \vec{b} \perp} ,
\]

(35) 

and similarly for $F_{LL}$. Others denote transverse-momentum-weighted Fourier transformations, for example,

\[
\tilde{F}_{UT}^{(1)}(b) = \int d^2 P_{h \perp} |\vec{P}_{h \perp}| \tilde{F}_{UT}^{(1)}(P_{h \perp}) e^{i \vec{P}_{h \perp} \cdot \vec{b} \perp} ,
\]

(36) 

and similarly for $\tilde{F}_{UT}^{(2)}$ and $\tilde{F}_{LT}$. For $\tilde{F}_{UU}$ and $F_{UL}$, we define

\[
\tilde{F}_{UU}^{(2)}(b) = \int d^2 P_{h \perp} |\vec{P}_{h \perp}|^2 \tilde{F}_{UU}^{(2)}(P_{h \perp}) e^{i \vec{P}_{h \perp} \cdot \vec{b} \perp} .
\]

(37) 

For $\tilde{F}_{UT}^{(3)}$, we define

\[
\tilde{F}_{UT}^{(3)}(b) = \int d^2 P_{h \perp} |\vec{P}_{h \perp}|^3 \tilde{F}_{UT}^{(3)}(P_{h \perp}) e^{i \vec{P}_{h \perp} \cdot \vec{b} \perp} .
\]

(38) 

In [7], the large logarithms have been re-summed for the unpolarized structure function by solving the relevant Collins-Soper equation for the TMD quark distribution and fragmentation function. From Eq. (34) and the results of the previous section concerning the Collins-Soper evolution for the polarized quark distributions, we conclude that the polarized structure functions have the same evolution equation as the unpolarized case. In the following, we take the structure function $F_{UT}^{(1)}$, depending on the transversely-polarized nucleon spin, as an example to demonstrate the re-summation procedure.
Rewriting the structure function $F_{UT}^{(1)}$ as,

$$
\tilde{F}_{UT}^{(1)}(b, Q^2) = \frac{-1}{M z_h} \partial_b \mathcal{F}_{UT}(b, Q^2) ,
$$

where

$$
\mathcal{F}_{UT}(b, Q^2) = \left( \partial_b q_T(x_B, z_h b) \right) \hat{q}(z_h, b) S^+(b) H_{UT}^{(1)}(Q^2) .
$$

Since the Sivers function $q_T$ and the unpolarized fragmentation function $\hat{q}$ obey the same Collins-Soper evolution equation as that for the unpolarized quark distribution, we have the following derivative equation respect to $Q^2$ \cite{2, 7},

$$
Q^2 \frac{\partial}{\partial Q^2} \mathcal{F}_{UT}(x_B, z_h, b, Q^2) = \left[ K(b \mu, g(\mu), \rho) + G_{UT}'(Q/\mu, g(\mu), \rho) \right] \mathcal{F}_{UT}(x_B, z_h, b, Q^2) ,
$$

where $K$ is the same as before, and $G_{UT}'$ contains additional contribution from the hard part. The sum of $K$ and $G_{UT}'$ has no dependence on $\rho$ although they separately might have. The solution to this differential equation has the following form \cite{2},

$$
\mathcal{F}_{UT}(x_B, z_h, b, Q^2) = \mathcal{F}_{UT}(x_B, z_h, b, \mu_1^2/C_2^2) e^{-S(Q^2, b, C_2)} ,
$$

where the distribution and fragmentation function are evaluated at $x_B^2 \zeta^2 = \hat{\zeta}^2/\hat{z}_h^2 = \rho \mu_1^2/C_2^2$. The Sudakov suppression form factor reads,

$$
S(Q^2, \mu_L^2, b, C_2) = \int_{\mu_L}^{C_2Q} \frac{d\bar{\mu}}{\bar{\mu}} \left[ \ln \left( \frac{C_2 Q^2}{\bar{\mu}^2} \right) A(b \mu_L, \bar{\mu}, \rho) + B(C_2, b \mu_L, \bar{\mu}, \rho) \right] .
$$

Here $C_2$ is a parameter in the order of 1, and $\mu_L$ is a low-energy, but still perturbative, scale. The functions $A$ and $B$ have perturbative expansions in $\alpha_s$, $A = \sum_n A^{(n)}(\alpha_s/\pi)^n$ and $B = \sum_n B^{(n)}(\alpha_s/\pi)^n$. They are defined as

$$
A(b \mu_L, \bar{\mu}, \rho) = \gamma_K(\bar{\mu}) + \beta \frac{\partial}{\partial g} K(b \mu_L, g(\bar{\mu}), \rho) ,
$$

$$
B(C_2, b \mu_L, \bar{\mu}, \rho) = -2K(b \mu_L, g(\bar{\mu}), \rho) - 2G_{UT}'(1/C_2, g(\bar{\mu}), \rho) .
$$

The $A$-function is the same as that for the unpolarized case \cite{2, 7} since it comes only from the spin-independent soft part. On the other hand, the $B$-function contains contribution from the unknown hard part $H_{UT}^{(1)}$, and hence it could be different from that of the unpolarized one.

Substituting the above results into Eq. \eqref{39}, we will get \cite{7}

$$
\tilde{F}_{UT}^{(1)}(b, Q^2) = \frac{-1}{M z_h} \partial_b \left[ \left( \partial_b q_T(x_B, z_h b) \right) \hat{q}(z_h, b) S^+(b) H_{UT}^{(1)}(\mu_L^2/C_2^2) e^{-S(Q^2, \mu_L^2, b)} \right] .
$$

From the above derivations, we confirmed that the $\rho$ dependence in the soft factor $K$ does not affect the resummation of the large logarithms. This is because the resummation concerns the logarithms of the form $\ln^2 Q^2 b^2$ in impact parameter space, while $\rho$ is just a parameter separating the hard and soft physics. In addition, there is no $\rho$ dependence in the Sudakov suppression form factor $S$. 

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The re-summation for all the other structure functions can be done in a similar way. With these re-summation formulas, one can further study the $Q^2$ dependence of the $P_\perp$ spectrum of the polarized cross section and asymmetries in the semi-inclusive DIS \[21\].

If $Q^2$ is not too large, for example, at the order of tens GeV$^2$, it is legitimate to re-sum only the leading double logarithms (DL) \[2, 9, 10\]. In this approximation, we only need to take into account the first term in the expansion of $A$ function, neglecting the contribution from $B$. Since $A^{(1)} = 4/3$ from the cusp anomalous dimension $\gamma_K$, the Sudakov suppression factor reduces to

\[
S^{(\text{DL})}(Q^2, \mu^2_L, C_2) = \frac{4}{3} \int_{\mu_L}^Q \frac{d\bar{\mu}}{\bar{\mu}} \ln \left( \frac{C_2 Q^2}{\bar{\mu}^2} \right),
\]

in the DL approximation. It only depends on $Q^2$ and $\mu^2_L$, but not on $b$. Moreover, since the cusp anomalous dimension is the same for all the leading-twist TMD quark distribution, the Sudakov suppression factor will be the same for all the leading-twist polarized structure functions in \(34\). One of the consequences is that one can predict the $P_\perp$ spectrum at higher $Q^2$ for semi-inclusive hadron production in DIS from the result at lower $Q^2$. For example, in the DL approximation, the unpolarized structure function $F_{UU}$ and the single transversely polarized structure function $F_{UT}^{(1)}$ have the following dependence on $Q^2$,

\[
F_{UU}(x_B, z_h, P_{h\perp}, Q^2) = F_{UU}(x_B, z_h, P_{h\perp}, \mu^2_L) e^{-S^{(\text{DL})}(Q^2, \mu^2_L)}
\]

\[
F_{UT}^{(1)}(x_B, z_h, P_{h\perp}, Q^2) = F_{UT}^{(1)}(x_B, z_h, P_{h\perp}, \mu^2_L) e^{-S^{(\text{DL})}(Q^2, \mu^2_L)}.
\]

The polarization asymmetry (the ratio of these two structure functions) as a function of $P_{h\perp}$ will remain the same for different $Q^2$ at fixed $x_B$ and $z_h$. This constancy in $Q^2$ has been seen from the comparison of the HERMES data at HERA with that of CLAS at JLab on various spin asymmetries, with the average $Q^2$ varying by a factor of three \[22\]. The above analysis applies to all the leading-twist polarized structure functions and polarization asymmetries. It will be interesting to test this prediction based on DL approximation with future DIS experiments at different $Q^2$.

We notice that the above formalism also applies for the Drell-Yan process, which have plenty data at low transverse momentum and not very high $Q^2$ \[23\]. It will be useful to compare the above DL approximation prediction with these experimental data, and gain insight for the transverse momentum dependence for the TMD quark distributions. We will carry this out in a future publication. This approach is different from what has been done so far in the literature \[24 25 26 27 28\], where the predictions solely depend on the integrated parton distributions and the Collins-Soper-Sterman formalism \[2\], and one needs nonperturbative parametrization for the large $b$ behavior of the distributions and the evolution as well. In our approach, the TMD parton distributions are important nonperturbative ingredients.

If $Q^2$ is very large (e.g., for $W$ and $Z$ bosons production), the above approximation breaks down. One has to take into account sub-leading logarithmic contributions, perhaps up to $A^{(2)}$ and $B^{(2)}$ in the expansion of the functions $A$ and $B$ \[2, 24 27 26 27 28 29\].

VI. CONCLUSIONS

In this paper, we have studied the Collins-Soper energy evolution equation for all the leading-twist TMD quark distributions. The evolution equation has contributions from both
the hard and soft regions of the gluon momentum. Since both parts are independent of the quark helicity, the evolution kernel is spin independent. Based on the evolution equation, we can perform re-summation for the large double logarithms in the polarized structure functions.

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