Understanding Terrorist Organizations with a Dynamic Model

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Terrorist organizations change over time because of processes such as recruitment and training as well as counter terrorism measures, but the effects of these processes have mostly been studied qualitatively and reductively. Seeking a more quantitative and integrated understanding, the author constructed a simple dynamic model where equations describe how these processes change an organization’s membership. Analysis of the model yields a number of intuitive as well as novel findings. Most importantly, it becomes possible to predict whether counter terrorism measures would be sufficient to defeat the organization. Furthermore, the author can prove in general that an organization would collapse if its strength and its pool of foot soldiers decline simultaneously. In contrast, a simultaneous decline in its strength and its pool of leaders is often insufficient and short-termed. These results and others like them demonstrate the great potential of dynamic models for informing terrorism scholarship and counter terrorism policy making.

1. Introduction
The goal of this article is to study terrorist organizations using a dynamic model. Generally speaking, in such a model a phenomenon is represented as a set of equations that describe it in simplified terms. The equations represent how the phenomenon changes in time or space, and cast empirically based knowledge in precise mathematical language. Once the model is constructed, it can be studied using powerful mathematical techniques to yield predictions, observations, and insights that are difficult or impossible to collect empirically (Aris, 2005; Ellner and Guckenheimer, 2006). For example, a dynamic model could be constructed for the various militant groups operating in Iraq and then used to predict their strength a year in the future. Moreover, given the model, it would be possible to evaluate the efficacy of various counterinsurgency policies.

Mathematical models can help fill a large methodological void in terrorism research: the lack of model systems. Whereas biologists studying pathogens can do experiments in vitro, there are no such model systems in terrorism research, except for mathematical models. In this sense, the method developed in this article provides an in vitro form of terrorism, which can be investigated in ways not possible in its in vivo kind. Like all model
systems, mathematical models are imperfect because they rely on large simplifications of the underlying political phenomena, and one can rightfully ask whether their predictions would be sufficiently accurate. Fortunately, complex phenomena in fields like biology have been studied very successfully with this mathematical technique (Ellner and Guckenheimer, 2006). Therefore, even phenomena as complex as found in terrorism research may, in some cases, be productively studied using mathematical models and indeed, previous models have brought considerable insights.

The rest of the article describes a simple model of a terrorist organization. The model is new in its focus, methodology, and audience: It focuses on a single terrorist organization and models its processes of recruitment, its internal dynamics as well as the impact of counterterrorism measures on it. As to methodology, with a few exceptions (Chamberlain, 2007; Feichtinger et al., 2001; Stauffer and Sahimi, 2006; Udwadia, Leitmann, and Lambertini, 2006), the powerful mathematical technique of differential equations has not been applied extensively in terrorism research. Finally, the article is specifically written to an audience of non-mathematicians: the main body of the article uses non-technical language to explain the terminology and to describe the equations and assumptions used in the model, whereas the technical analysis is offered in the Appendix.

The model described herein was built following two design principles. First, it was desired to have a model of broad applicability across organizations and conflicts. Indeed, the model is so general that it can be applied to insurgencies or even to some non-terrorist organizations. As will be shown, despite this generality it makes non-trivial observations: more importantly, it specifies sufficient conditions for victory over the organization (see subsection 4.2). Second, it was desired to build a simple model so as to facilitate interpretation, analysis, and further development. It was hoped that the model would establish a methodological prototype that could be easily extended and modified to fit specific cases.

The organization of the article is as follows. Section 2 describes the model—its variables, parameters, and relations between them. Section 3 graphically illustrates the model’s predictions about terrorist organizations. Sections 4 and 5 discuss the insights gleaned from the model, and the implications to counterterrorism policies. The conclusions are in Section 6. Finally, all of the technical arguments are gathered in the Appendix.

2. A Mathematical Model

There are many ways of describing a terrorist organization, such as its ideology or political platform, its operational patterns, or its methods of recruitment. Here it is considered from the “human resources” point of view. Namely, the study is interested in examining how the numbers of “leaders” and “foot soldiers” in the organization change with time. The former includes experienced managers, weapon experts, financiers, and even politicians and religious leaders who help the organization, whereas the latter are the more numerous rank-and-file. These two quantities arguably give the most important information about the strength of the organization. The precise characteristics of the two groups and their relative numbers would depend on the organization under consideration. Nevertheless, this distinction remains relevant even in the very decentralized organizations like the post-Afghanistan Al Qaeda movement, because one can identify the “leaders” as the experienced terrorists, as compared to the new recruits (Hoffman, 2003; Sageman, 2004). The subdivision between those two groups is also important in practice because policymakers must often compare policies that target one of the two groups (Wolf, 1989; Ganor, 2005, ch. 5): while the leaders represent more valuable targets, they are also often harder to reach. Section 5 compares the policy alternatives.
Therefore, a terrorist organization will be represented as two time-varying quantities, $L$ and $F$, corresponding to the number of leaders and foot soldiers, respectively. Also, $L$ and $F$ determine the overall “strength” $S$ of the organization. However, because leaders possess valuable skills and experience, they contribute more to the strength than an equivalent number of foot soldiers. Hence, strength $S$ is taken to be a weighted sum of the two variables, with more weight ($m > 1$) given to leaders:

$$S = mL + F$$

The article now identifies a set of processes that the author believes to be the processes governing changes in the numbers of leaders and foot soldiers. These processes constitute the mathematical model. Although some of these processes are self-evident, others could benefit from quantitative comparison with data. The latter task is non-trivial given the scarcity of time-series data on membership in terrorist organizations and hence it is left out for future work.

The histories of Al Qaeda as well as other terrorist organizations (e.g., Laqueur, 2001; Harmon, 2000; Hoffman, 2006) suggest that the pool of terrorist leaders and experts grows primarily when foot soldiers acquire battle experience or receive training (internally, or in terrorist-supporting states; Jongman and Schmid, 2005). Consequently, the pool of leaders ($L$) is provisioned with new leaders at a rate proportional to the number of foot soldiers ($F$). This process is called “promotion” and its parameter is labeled the parameter of proportionality process $p$. This growth is opposed by internal personnel loss due to demotivation, fatigue, and desertion, as well as in-fighting and splintering (Horgan, 2005, ch. 6). This phenomenon is modeled as a loss of a fraction ($d$) of the pool of leaders per unit time. An additional important influence on the organization are the counterterrorism (CT) measures targeted specifically at the leadership, including arrests, assassinations, as well as efforts to disrupt communications and to force the leaders into long-term inactivity. Such measures could be modeled as the removal of a certain number ($b$) of people per unit time from the pool of leaders. CT is modeled as a constant rate of removal rather than as a quantity that depends on the size of the organization because the goal is to see how a fixed resource allocation toward CT would impact the organization. Presumably, the human resources and funds available to fight the given terrorist organization lead, on average, to the capture or elimination of a fixed number of operatives. In sum, it is assumed that on average, every interval of time, the pool of leaders is nourished through promotion, and drained because of internal losses and CT (see Appendix, Equation (1).)

The dynamics of the pool of foot soldiers ($F$) are somewhat similar to the dynamics of leaders. Like in the case of leaders, some internal losses are expected. This is modeled as the removal of a fraction ($d$) of the pool of operatives per unit time where for simplicity the rate $d$ is the same as the rate for leaders (the complex case is discussed in subsection 5.2). Much like in the earlier case of leaders, counterterrorist measures are assumed to remove a fixed number ($k$) of foot soldiers per unit time. Finally, and most importantly, one needs to consider how and why new recruits join a terrorist organization. Arguably, in many organizations growth in the ranks is in proportion to the strength of the organization, for multiple reasons: Strength determines the ability to carry out successful operations, which increase interest in the organization and its mission. Moreover, strength gives the organization the manpower to publicize its attacks, as well as to man and fund recruitment activities. By assuming that recruitment is proportional to strength, one captures the often-seen cycle where successful attacks lead to greater recruitment, which lead to greater strength
and more attacks. Overall, the pool of foot soldiers is nourished through recruitment, and
drained because of internal losses and CT (see Appendix, Equation (2)).

The numerical values of all of the aforementioned parameters \( p, d, b, r, m, k \) are
dependent on the particular organization under consideration, and likely vary somewhat
with time. Fortunately, it is possible to draw many general conclusions from the model
without knowing the parameter values, and this article shall do so shortly. Finally, it should
be noted that counterterrorism need not be restricted to the parameters \( b, k \) (removal of
leaders and foot soldiers, respectively), and measures such as public advocacy, attacks on
terrorist bases, disruption of communication, and others can weaken the organization by
reducing its capabilities as expressed through the other parameters.

In the above description, it was assumed that counterterrorism measures are parameters
that can be changed without effecting recruitment. This is a significant simplification
because in practice it may be difficult to respond to terrorist attacks without engendering
a backlash that actually promotes recruitment (see e.g., Hanson and Schmidt, 2007).
Nevertheless, the advantages of this simplification outweigh the disadvantages: First, it is
clear that any model that would consider such an effect would be much more complicated
than the current model. Namely, it would be nonlinear and consequently much harder to
analyze or use. Second, the current model can be easily extended to incorporate such an
effect if it desired. Third, the strength of this effect is difficult to describe in general because
it depends extensively on factors such as the specific CT measures being used, the terrorist
actions, and the political environment. Indeed, Udwadia, Leitmann, and Lambertini (2006),
who incorporated this effect, constructed their model based on observations of a specific
context within the current conflict in Iraq.

The model includes additional implicit assumptions. First, it assumes a state of stable
gradual change, such that the effect of one terrorist or counterterrorist operation is smoothed.
This should be acceptable in all cases where the terrorist organization is not very small
and thus changes are not very stochastic. Second, the model assumes that an organization’s
growth is constrained only by the available manpower, and factors such as money or
weapons do not impose an independent constraint. Third, it is assumed that the growth in
foot soldiers is not constrained by the availability of potential recruits—and it is probably
true in the case of Al Qaeda because willing recruits are plentiful (for the case of England,
see Manningham-Buller, 2006). This point is discussed further in subsection 4.3.

3. Analysis of the Model

Having written down the governing equations, the task of studying a terrorist organization
is reduced to the standard problem of studying solutions to the equations. Because the
equations indicate rates of change in time, the solutions would be two functions, \( L(t) \) and
\( F(t) \), giving the number of leaders and foot soldiers, respectively, at each time. Suppose
that currently (time 0) the organization has a certain number of leaders and foot soldiers,
\( L_0 \) and \( F_0 \), respectively, and is subject to certain CT measures, quantified by \( b \) and \( k \).
One would want to see whether the CT measures are adequate to defeat the organization.
Mathematically, this corresponds to the question of whether at some future time both \( L \) and
\( F \) would reach zero. Intuitively, one expects that the organization would be eliminated if it is
incapable of recovering from the losses inflicted on it by CT. In turn, this would depend on its
current capabilities as well as the parameters \( p, d, r, m \), which characterize the organization.

Mathematical analysis of the model (see the Appendix) shows that most terrorist
organizations\(^4\) evolve in time like the organizations whose “orbits” are displayed in
Figure 1. (a) Typical solution curves of the equations coded by ultimate fate: thin dark gray for successfully neutralized organizations, thick light gray for those remaining operational and growing. The parameters were set to representative values, but as was said earlier, all realistic organizations are qualitatively similar and resemble these. (b) “Vector field” of $L$ and $F$. At each value of $L$, $F$ the direction and length of the arrow give the rate of change in $L$ and $F$.

Figure 1(a,b). Figure 1(a) plots eight different organizations with different starting conditions. Another perspective can be seen in Figure 1(b), which graphically illustrates the dynamical equations via arrows: the direction of each arrow and its length indicates how an organization at the tail of the arrow would be changing and at what rate. By picking any starting location $(L_0, F_0)$ and connecting the arrows, it is possible to visually predict the evolution into the future. Another illustration is found in Figure 2, which shows how two example organizations change with time.

Figure 2. Evolution of strength, leaders, and foot soldiers ($S$, $L$, $F$, respectively) in two terrorist organizations as a function of time. In (a), due to CT, $F$ falls initially but eventually the organization recovers through promotion. In (b), $L$ and $S$ fall initially but eventually the organization recovers through recruitment. The vertical axis has been rescaled by dividing each quantity by the maximum it attains during the time evolution. This makes it possible to represent all three quantities on the same plot. The units of time are unspecified because they do not affect the analysis.
In general, it is found that the dynamics of the organization are dependent on the position of the organization with respect to a threshold line, which can be termed the “sink line”: an organization will be neutralized if and only if its capabilities are below the sink line. In other words, the current CT measures are sufficient if and only if the organization lies below that threshold (solid thick line in Figure 3). The threshold is impassable: an organization above it will grow, and one below it is sure to collapse. This threshold is also

![Figure 3](image)

**Figure 3.** Plot of the sink (solid thick) and trend lines (thin dashed). The two lines intersect at a “saddle point.”

![Figure 4](image)

**Figure 4.** The effects of the parameters $b$ and $k$ on the dynamical system, (a) and (b), respectively, as seen through the effect on the sink line. In each case, as the CT measures are increased, the sink line moves up confining below it additional terrorist organizations.
Figure 5. The effects of the parameters $p$ (a), $r$ (b), and $d$ (c) on the dynamical system as seen through the effect on the sink line. When $p$ or $r$ are increased the organizations are able to grow faster, causing the sink line to move down, making the existing CT measures no longer sufficient to neutralize some terrorist organizations. In contrast, when $d$ is increased, the sink line moves up because the organization is forced to replace more internal losses to survive.

very sharp: two organizations may lie close to the line, but the one above it would grow, whereas the one below it would shrink even if the differences in initial capabilities are small. In addition to the sink line, the model also predicts that all successful organizations would tend toward a particular trajectory. This “trend line” (a dashed black line on Figure 3) is discussed further in subsection 4.1.

Suppose now that the model predicts that the given organization is expected to grow further despite the current CT measures, and therefore increased CT measures would be needed to defeat it. To see the effect of additional CT measures, one needs to examine how the dynamical system changes in response to increases in the values of the parameters, in particular, the parameters $b$ and $k$, which express the CT measures directed at leaders and foot soldiers, respectively (Figure 4). It is also possible to affect the fate of the organization by influencing the values of other parameters affecting its evolution, such as recruitment and promotion (Figure 5). In general, to bring the terrorist organization under control, it is necessary to change the parameters individually or simultaneously so that the organization’s current state $(L, F)$, is trapped under the sink line. An interesting finding in this domain is that both $b$ and $k$ are equivalent in the sense that both shift the sink link up in parallel (Figure 4).

4. Discussion

4.1. Nascent Terrorist Organizations

Recall that the sink line (Figure 3) distinguishes two classes of terrorist organizations—those destined to be neutralized and those that will continue growing indefinitely. Within the latter group, another distinction is introduced by the trend line—a distinction with significance to counterterrorism (CT) efforts: organizations lying to the left of it have different initial growth patterns compared to those lying to the right (Figure 1). The former start with a large base of foot soldiers and a relatively small core of leaders. In these organizations, $F$ may initially decline because of CT, but the emergence of competent leaders would then start organizational growth (e.g., Figure 2(a)). In contrast, the latter type of organizations start with a large pool of leaders but comparatively few recruits. CT could decimate their leadership, but they would develop a wide pool of foot soldiers, recover, and grow (e.g., Figure 2(b)). Thus, all successful terrorist organizations may be classified as either “p-types” (to the left of the trend line) or “r-types” (to the right of the trend line) in
reference to the parameters $p$ of promotion and $r$ of recruitment. In p-type organizations, early growth occurs mainly through promotion of their foot soldiers to leaders, whereas in the latter mainly through recruitment of new foot soldiers.

This classification could be applied to many actual organizations. For example, popular insurgencies are clearly p-type, whereas Al Qaeda’s history since the late 1990s closely follows the profile of an r-type: Al Qaeda may be said to have evolved through three stages: First, a core of followers moved with bin Laden to Afghanistan. They were well trained but the organization had few followers in the wider world (for a history see Wright, 2006). Then the attacks on the leadership in the fall of 2001 reduced the organization’s presence in Afghanistan, leaving its operatives outside the country with few leaders. Finally, the organization cultivated a wide international network of foot soldiers, but they were ill-trained as compared to their predecessors. This description closely matches the profiles in Figure 1 where r-type organizations start from a small, well-trained core, move toward a smaller ratio of leaders to foot soldiers, and then grow through recruitment.

As was noted, nascent organizations tend toward the trend line, regardless of how they started (Figure 1). The slope of this line is $\frac{r+\sqrt{r^2+4rpm}}{2p}$, and this number is the long-term ratio between the number of foot soldiers and the number of leaders. Notice that this formula implies that ratio is dependent on just the parameters of growth — $r$, $m$, $p$ — and does not depend on either $d$ or the CT measures $k$, $b$. This ratio is generally not found in failing organizations, but is predicted to be ubiquitous in successful organizations. It may be possible to estimate it by capturing a division of an organization, and then, it can help calculate the model’s parameters. However, it is important to note that $L$ includes not just commanding officers, but also any individuals with substantially superior skills and experience. The existence of the ratio is a prediction of the model, and if the other parameters are known, it could be compared to empirical findings.

### 4.2. Conditions for Victory

Recall that the model indicates that all terrorist organizations belong to one of three classes: r-types, p-types, and organizations that will be defeated. Each class exhibits characteristic changes in its leaders, foot soldiers, and strength ($L$, $F$, and $S$, respectively) over time. This makes it possible to determine whether any given organization belongs to the third class, that is, to predict whether it would be defeated.

One finding is that if a terrorist organization weakens, that is, shows a decline in its strength $S$, it does not follow that it would be defeated. Indeed, in some r-type organizations it is possible to observe a temporary weakening of the organization and yet unless counter-terrorism (CT) measures are increased, the organization would recover and grow out of control (see Figure 2(b)). Even a decline in the leadership is not by itself sufficient to guarantee victory. The underlying reason for this effect is out-of-control growth in $F$, which would ultimately create a new generation of terrorist leaders. Similarly, it is possible for an organization to experience a decline in its pool of foot soldiers, and yet recover. The aforementioned cases indicate that it is easy during a CT campaign to incorrectly perceive apparent progress in reducing the organization as a sign of imminent victory. Fortunately, under the model it is possible to identify reliable conditions for victory over the organization:

1. For a p-type organization, it is impossible to have a decline in strength $S$. If such a decline starts happening, the organization would be defeated.
2. For an r-type organization, it is impossible to have a decline in foot soldiers $F$. If such a decline starts happening, the organization would be defeated.

Consequently:

A terrorist organization would collapse if counter-terrorist measures produce both:

(1) a decline in its strength $S$ and (2) a decline in its foot soldiers $F$.

To apply the theorem to an organization of an unknown type, one needs merely to estimate whether the organization’s pool of foot soldiers and strength is declining. The latter could be found indirectly by looking at the quantity and quality of terrorist operations. It is not necessary to know the model’s parameters or changes in the pool of leaders—the latter could even be increasing. Furthermore, while it may take some time to determine whether $S$ and $F$ are indeed declining, this time could be much shorter compared to the lifetime of the organization. Therefore, the theorem suggests the following two-step counter-terrorism campaign:

1. Estimate the scale of CT measures necessary to produce a decline in $S$ and $F$ and implement them.
2. Measure the effect on $S$ and $F$. If they both declined, then sustain the scale of operations (i.e. do not reduce $b$, $k$); Otherwise an increase in CT measures would be necessary.

The theorem and findings above give sufficient conditions for victory but they do not characterize the only possible victory scenario. For example, it is possible for an organization to see an increase in its pool of foot soldiers $F$ yet ultimately collapse: these are organizations that lie to the right of the trend line and just slightly under the sink line. More generally, it should be remembered that to prove the theorem it was necessary to use a simplified model of a terrorist organization, as described in section 2. Nevertheless, it is likely that some form of the theorem would remain valid in the more complicated models because its truth stems from the precursor-product relationship between foot soldiers and leaders—a relationship that does not depend on the model.

### 4.3. Stable Equilibria

Recall that the model does not have a stable equilibrium (Figure 3). Yet, in many practical cases, terrorist organizations seem to reach a stable equilibrium in terms of their structure and capabilities. It is plausible that such stability is the result of a dynamic balance between the growing terrorist organization and increasing CT measures directed against it. Indeed, rather than staying constant, numbers like $b$, $k$, CT may actually grow when the organization presents more of a threat. Aside from CT, stability may be the result of organizations reaching an external limit on their growth—a limit imposed by constrains such as funding, training facilities, or availability of recruits. The case of funding could be modeled by assuming that the growth of the organization slows as the organization approaches a maximum point, $(L_{\text{max}}, F_{\text{max}})$. Alternatively, it is quite possible and consistent with the model that there would be a perception of stasis because the organization is changing only slowly.

### 5. Counterterrorism Strategies

Recall that the general counterterrorism (CT) strategy in this model is based on the location of the sink line, which should be placed above the terrorist organization (in Figure 1). To implement this strategy, it is necessary first to calculate the models parameters for
a given organization \((p, r, m, d)\), and second, to determine the efficacy of the current counterterrorism measures \((b, k)\). Then, it remains “just” to find the most efficient way of changing those parameters so as to move the sink line into the desired location. The article now considers several strategic options.

### 5.1. Targeting the Leaders

An important “counter terror dilemma” (Ganor, 2005) is whom to target primarily—the leaders or the foot soldiers. Foot soldiers are an inviting target: not only do they do the vital grunt work of terrorism, they also form the pool of potential leaders, and thus their elimination does quiet but important damage to the future of the organization. Moreover, subsection 4.1 showed that while an organization can recover from a decline in both its strength and leadership pool, it cannot recover from a decline in both its strength and foot soldiers pool. That finding does not say that attacking leaders is unlikely to bring victory—indeed, they form an important part of the organization’s overall strength, but it does suggest that a sustained campaign against an organization is more likely to be successful when it includes an offensive against its low-level personnel.

Notwithstanding those arguments, the neutralization of a terrorist leader may be more profitable because the leader is more valuable to the organization than a foot soldier, and his or her loss would inevitably result in command and control difficulties that may even disrupt terrorist attacks. This intuition is confirmed by detailed calculations: for realistic parameter values, an increase in \(b\) gives a greater rise in the sink line above its baseline than a comparable rise in \(k\). Perhaps surprisingly, this finding holds true even in “bottom-heavy” organizations, that is, in organizations where the ratio of \(F\) to \(L\) is high, possibly like present day Al Qaeda. In any case, policy prescriptions must be applied with consideration of counterterrorist capabilities and policy costs. Thus, while on paper attacking leaders is more productive, capturing the rank-and-file could be more feasible.

It is often argued that counterterrorist policies have considerable side effects. For instance, there is evidence that targeted assassinations of leaders have led terrorist organizations to escalate, in what has been called the “boomerang effect” (Crenshaw, 1996, 125). Fortunately, the model suggests that the policymaker has useful substitutes, with possibly fewer policy side effects. As Figure 5 shows, making recruitment \((r)\) lower has an effect similar to increasing \(k\). Likewise, decreasing the rate of promotion to leadership \((p)\) can substitute for increasing \(b\). This agrees with intuition: for example, in the case of the foot soldiers, growth can be contained either actively through, for example arrests or proactively by slowing the recruitment of new operatives (through, e.g., attacks on recruitment facilities or advocacy).

### 5.2. Encouraging Desertion

Fatigue and attrition of personnel have been empirically found to be an important effect in the evolution of terrorist organizations. In interviews with captured or retired terrorists, they often complained about the psychological stress of their past work, its moral contradictions, and the isolation from relatives and friends (Horgan, 2005, ch. 6). This is part of the reason why terrorist organizations cannot remain inactive (as in a cease-fire) for very long without experiencing irreparable loss of personnel due to loss of motivation, and many organizations even resort to coercion against desertion. Therefore, encouraging operatives to leave through advocacy or amnesties may be an effective counter-terrorist strategy.
Figure 6. The effects of $d_L$ (a) and $d_F$ (b) on the dynamical system, as seen through the effect on the sink line. As the desertion rates increase, the sink line moves up and its slope changes, thus trapping additional terrorist organizations.

The model introduced here brings theoretical insight into this phenomenon. One prediction of the model is that even if such desertion exceeds recruitment (i.e., $d > r$) the organization would still sustain itself as long as it has a sufficiently large rate of promotion ($p$) or leaders of sufficiently high caliber ($m$). However, if $d$ is even greater, namely, exceeds $d = \frac{1}{2}(r + r \sqrt{1 + \frac{mp}{r}})$, then the organization would be destroyed regardless of starting conditions, or counter-terrorism efforts ($b, k$).

Organizations with lower $d$ are, of course, also effected by desertion. Earlier, Figure 5 showed how increasing $d$ raises up the sink line. To see the phenomenon in more detail, the author replaced $d$ by two (not necessarily equal) parameters $d_L$ and $d_F$ for the desertion of $L$ and $F$, respectively. The two parameters change the slope of the sink line: increasing $d_L$ flattens it, whereas increasing $d_F$ makes it more steep (Figure 6). Therefore, increasing $d_L$ could be a particularly effective strategy against nascent r-type organizations, whereas increasing $d_F$ could be effective against the nascent p-types.

6. Conclusions

Much of the benefit of mathematical models is due to their ability to elucidate the logical implications of empirical knowledge that was used to construct the model. Thus, whereas the empirical facts used to construct the models should be uncontroversial, their conclusions should offer new insights. The model proposed here is a very simplified description of real terrorist organizations. Despite its simplicity, it leads to many plausible predictions and policy recommendations. Indeed, the simplicity of the model is crucial to making the model useful. More detailed models of this kind could provide unparalleled insights into counterterrorism policies and the dynamics of terrorism.

Notes

1. For examples of dynamic models: Allanach et al. (2004), Chamberlain (2007), Farley (2007), Feichtinger et al. (2001), Johnson et al. (2006), Stauffer and Sahimi (2006), Udwadia, Leitmann, and Lambertini (2006); rational choice models: Anderton and Carter (2005), Sandler, Tschirhart, and
Cauley (1983), Sandler (2003), Wintrobe (2006); agent-based models: MacKerrow (2003); Tsvetovat and Carley (2007).

2. A minor assumption in our model is that once a foot soldier is promoted a new foot soldier is recruited as a replacement. It is shown in the appendix that if in some organizations such recruitment is not automatic, then the current model is still valid for these organizations as long as \( p < r \). In any case the drain due to promotion is marginal because foot soldiers are far more numerous than leaders even in relatively “top heavy” organizations. This model is similar to structured population models in biology, where the foot soldiers are the “larvae” and the leaders are the “adults.” However, an interesting difference is that whereas larvae growth is a function of the adult population alone, in a terrorist organization the pool of foot soldiers contributes to its own growth.

3. The simplest approach to estimating them would be to estimate the number and leaders and foot soldiers at some point in time, and then find the parameter values by doing least-squares fitting of the model to the data on the terrorist attacks, where the terrorist attacks are considered to be a proxy of strength. However, this approach has some limitations.

4. That is, those with realistically low rates of desertion: \( d < \frac{1}{2}(r + r\sqrt{1 + \frac{mp}{r}}) \). A higher rate of desertion \( d \) always causes the organization to collapse and is not as interesting from a policy perspective (see subsection 5.2 for a discussion of desertion).

5. See the Appendix for proof.

6. It would be a straightforward task to modify the model to incorporate such a control-theoretic interaction, but the task is more properly the subject of a follow-up study.

7. Realistic: all organizations where the rate of recruitment and promotion are non-vanishing, the rate of internal loss (\( d \)) is not too large, that is, \( d < \frac{1}{2}(r + r\sqrt{1 + \frac{mp}{r}}) \).

8. Of course, the dynamical system is unrealistic once either \( F \) or \( L \) fall through zero. However, by the logic of the model, once \( F \) reaches zero, the organization is doomed because it lacks a pool of foot soldiers from which to rebuild inevitable losses in its leaders.

9. The degenerate case of \( \lambda = 0 \) has probability zero, and is not discussed.

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Appendix

The original differential equations are:

\[
\frac{dL}{dt} = pF - dL - b
\]

\[
\frac{dF}{dt} = r(mL + F) - dF - k
\]

If one wished to incorporate the drain of the foot soldiers due to promotion \((-pF)\) in Equation (2), then one could adjust the original parameters by the transformation \(r \rightarrow r - p\) and \(m \rightarrow \frac{rm}{r-p}\). However, this would affect some of the analysis below, because for \(r < p\) it would not longer be the case that \(r > 0\), even though \(rm > 0\) would still hold true. Alternatively, one could change the internal losses parameter for foot soldiers: \(dF \rightarrow dF + p\) and break the condition \(dF \neq dL\).

The linearity of the system of differential equations makes it possible to analyze the solutions in great detail by purely analytic means. The fixed point is at:

\[
L^* = \frac{kp - b(r - d)}{d(r - d) + rmp} \quad F^* = \frac{kd + rmb}{d(r - d) + rmp}
\]
The eigenvalues at the fixed point are

$$\lambda_{1,2} = \frac{r - 2d \pm \sqrt{(r - 2d)^2 + 4(rmp + d(r - d))}}{2}$$  \hspace{1cm} (4)

From Equation (4), the fixed point is a saddle when \( rmp + d(r - d) > 0 \), that is, \( \frac{r - \sqrt{r^2 + 4rmp}}{2} < d < \frac{r + \sqrt{r^2 + 4rmp}}{2} \) (physically, the lower bound on \( d \) is 0). The saddle becomes a sink if \( r < 2d \) and \( rmp + d(r - d) < 0 \). By Equation (3), this automatically gives \( F_\ast < 0 \), that is the organization is destroyed. It is impossible to obtain either a source because it requires \( r - 2d > 0 \) and \( rmp + d(r - d) < 0 \), but the latter implies \( d > r \), and so \( r - 2d > 0 \) is impossible; or any type of spiral because \( (r - 2d)^2 + 4(rmp + d(r - d)) < 0 \) is algebraically impossible. It is also interesting to find the eigenvectors because they give the directions of the sink and trend lines:

$$e_{1,2} = \left( \begin{array}{c} \frac{2p}{r \pm \sqrt{r^2 + 4rmp}} \\ \frac{r \pm \sqrt{r^2 + 4rmp}}{2p} \end{array} \right)$$  \hspace{1cm} (5)

It can be seen that the slope of \( e_2 \), which is also the slope of the sink line—the stable manifold—is negative. Therefore, it can be concluded that the stable manifold encloses, together with the axes, the region of neutralized organizations. Concurrently, the slope of \( e_1 \)—the trend line, that is, the unstable manifold—is positive. Thus, the top half of the stable separatrix would point away from the axes, and gives the growth trend of all non-neutralized organizations (\( \frac{\Delta F}{\Delta L} = \frac{r + \sqrt{r^2 + 4rmp}}{2p} \)).

**Proof of the Theorem**

Recall that the article wished to show that a terrorist organization that experiences both a decline in its strength and a decline in the number of its foot soldiers will be destroyed. The proof rests on two claims: First, a p-type organization cannot experience a decline in strength, and second, an r-type organization cannot experience a decrease in \( F \) (for a graphic illustration see Figure 7). Thus, both a decline in strength and a decline in the number of foot soldiers cannot both occur in a r-type organization nor can they both occur in a p-type organization. Hence, such a situation can only occur in the region of defeated organizations.

As to the first claim, note that slope of the sink line is always greater than the slope of the iso-strength lines (\( = -m \)). By Equation (5) the slope is \( \frac{r - \sqrt{r^2 + 4rmp}}{2p} = -m \frac{2}{1 + \sqrt{1 + 4\frac{d}{r}}} > -m \). Therefore, the flow down the sink line has \( \frac{dS}{dt} > 0 \). (Down is the left-to-right flow in the figure.) Now, note that in a p-type organization, the flow must experience an even greater increase in strength. Let \( A \) be the matrix of the dynamical system about the equilibrium point and let the state of the terrorist organization be \( (L, F) = d_1e_1 + d_2e_2 \) where \( e_1, e_2 \) are the distinct eigenvectors corresponding to the eigenvalues \( \lambda_1, \lambda_2 \). Consideration of the directions of the vectors (Equation (5)) shows that for a p-type organization, \( d_1 > 0 \) and \( d_2 < 0 \). The direction of flow is therefore \( d_1\lambda_1e_1 + d_2\lambda_2e_2 \). Notice that \( \lambda_1 > 0, \lambda_2 < 0 \), and so the flow has a positive component \( = d_1\lambda_1 \) in the \( e_1 \) direction (i.e., up the trend line). Because the flow along \( e_1 \) experiences an increase in both \( L \) and \( F \), it must experience an increase in strength. Consequently, a p-type organization must have \( \frac{dS}{dt} > 0 \), which is even more positive than the flow along the sink line (where \( d_1 = 0 \)). Thus, \( \frac{dS}{dt} > 0 \) for p-types.
As to the second claim, note that r-type organizations have \( d_1 > 0 \) and \( d_2 > 0 \). Moreover, in an r-type organization, the flow \( d_1 \lambda_1 e_1 + d_2 \lambda_2 e_2 \) has \( \frac{dF}{dt} \) greater than for the flow up the right side of the sink line (right-to-left in the figure): the reason is that \( e_1 \) points in the direction of increasing \( F \) and while in an r-type \( d_1 > 0 \), along the sink line \( d_1 = 0 \). The flow up the sink line has \( \frac{dF}{dt} > 0 \), and so \( \frac{dF}{dt} > 0 \) in an r-type organization. In sum, \( \frac{dS}{dt} < 0 \) simultaneously with \( \frac{dF}{dt} < 0 \) can only occur in the region \( d_1 < 0 \)—the region of defeated organizations. QED.