Radiative neutron capture rate of $^{19}\text{N}(n,\gamma)^{20}\text{N}$

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Abstract. Low and medium mass neutron rich nuclei have been thought of as important constituents in the final abundance patterns resulting from the r-process reaction network calculations. However, a direct study of these nuclei is extremely difficult and indirect methods like Coulomb dissociation are used for their investigation. We calculate the reaction rate of the $^{19}\text{N}(n,\gamma)^{20}\text{N}$ radiative capture reaction using Coulomb dissociation of the medium mass exotic nucleus $^{20}\text{N}$, under the aegis of finite range distorted wave Born approximation theory and compare our results with those from the experiment. The reaction rate, when included in the full reaction network, is thought to be crucial to the final abundance of Fluorine isotopes.

1. Introduction

Prodigious advancements in experimental facilities all over the world have now enabled the study of nuclear systems away from the valley of stability. They are interesting because their characteristics are peculiar when compared to their more famous, stable siblings and are thus, called exotic. Breaking of the conventional shell gaps of the spherical shell model picture, presence of deformation and/or halo constitution are some of their properties that help them survive despite their usually small life times. Nevertheless, small life times and unstable natures do not mean that these nuclei are not significant. In fact, they are the pathways to elemental formation during nucleosynthesis processes in stars [1]. An example is the r-process, which is responsible for the production of heavy to medium mass neutron rich nuclei during highly cataclysmic events [2]. However, it is the low mass neutron excess nuclei that provide seeds for the nuclei that initiate the r-process. These low mass neutron rich nuclei, when included in reaction network calculations, are known to significantly affect the final r-process abundances of stellar nuclei [3, 4].

Since these nuclei are extremely short-lived and radioactive, studies about their genesis and structure using direct reactions are tedious as they are exceedingly difficult to produce in a laboratory. Therefore, indirect techniques, which are a combination of theory and experiment, are used to produce and examine their reaction and structural properties using inverse kinematics. The cross-sections so obtained are then used to calculate their reaction rates [5, 6]. Recently, the method of Coulomb dissociation (CD) was applied to an experimental study of $^{20}\text{N}$ [7], where a secondary radioactive ion beam of this nucleus was produced by bombarding...
an Ar beam on a Be target and segregated using time-of-flight measurements. The secondary 20N beam was then made to impinge on a 208Pb target and dissociate into 19N and a neutron. Inverse kinematics was then applied to study the radiative neutron capture reaction 10N(n,γ)20N. This reaction is crucial as the rate so obtained was found to significantly influence the abundance of Fluorine isotopes when provided as an input to the full reaction network calculations of the elemental abundance estimation [7].

In view of this measurement, we present here our calculations and results for the 19N(n,γ)20N radiative capture reaction using the same method of Coulomb dissociation but under the framework of the fully quantum mechanical finite range distorted wave Born approximation (FRDWBA) theory. CD, realised from the perspective of FRDWBA, has been successfully applied to a variety of reactions in the past and is one of the leading formalisms for indirect approaches in nuclear astrophysics [8, 9]. In particular, it has been quite adequate when applied to 34Na, 19C and 37Mg to determine that the r-process isotopic flow is towards the neutron drip line [10, 11, 12]. In fact, reaction rates obtained via Coulomb dissociation studies have seen contribution not only from the ground state, but also from the excited states of both the projectile and its core [13]. In case of 20N, it has been suggested that the contribution to the reaction rate from the excited states of the core is as vital as that from the ground state [7]. However, it will be interesting to see the range of the temperature for their contribution.

In the next section, we present the Formalism while in the following segment, we discuss and compare our results obtained via FRDWBA with those from the experimental observations. The last section highlights the conclusions.

2. Theory and Formalism

We contemplate the breakup of a two-body projectile a (20N) in the dynamic Coulomb field of a target t (208Pb) into fragments b (a 19N core) and c (a neutron), as: a + t → b + c + t. Since we consider the elastic Coulomb breakup, the target remains in its ground state.

The triple differential cross section for the above reaction, in terms of the relative coordinates is given by

$$\frac{d^3\sigma}{dE_{bc}d\Omega_{bc}d\Omega_{at}} = \frac{2\pi}{h\nu_{at}} \frac{\mu_{bc}\mu_{at}p_{bc}p_{at}}{h^6} \sum_{\ell m} \frac{1}{[2\ell + 1]} |\beta_{\ell m}|^2,$$  

with $\nu_{at}$ being the projectile-target relative velocity in the entrance channel while $E_{bc}$ is the core-valence neutron relative energy in the final channel. $p_{bc}$ and $p_{at}$ are the suitable linear momenta, $\mu_{bc}$ and $\mu_{at}$ are the reduced masses and $\Omega_{bc}$ and $\Omega_{at}$ are the solid angles corresponding to the $b-c$ and $a-t$ systems, respectively. $\beta_{\ell m}$ is the reduced amplitude in post form FRDWBA, given by

$$\beta_{\ell m} = \left< \chi_b^{(-)}(q_{bc}, r_1) e^{i\delta_{q_c} r_1} | \chi_a^{(+)}(q_{ac}, r_1) \right> \left< e^{i(\gamma_{q_c} - \alpha K) r_1} | V_{bc}(r_1) | \phi_{a\ell m}(r_1) \right>. $$

In Eq. (2), the $r_i$'s ($i = j, i, c$) are the position vectors appropriate for the three-body Jacobi system, details about which can be found in Ref. [14]. The first term containing the Coulomb distorted waves $\chi^{(\pm)}$ describes the dynamics of the reaction and can further be expressed analytically in terms of the Bremsstrahlung integral [15], while the second term comprising of the projectile bound state wave function $\phi_{a\ell m}(r_1)$, of any angular momentum $\ell$ (with projection $m$), manifests the structure part. $V_{bc}$ is the interaction between $b$ and $c$ in the initial channel whose expression can be extended to include deformation effects in a projectile nucleus, if desired [16]. K represents an effective local momentum appropriate to the core-target relative system and $\mathbf{q}_i$'s ($i = a, b, c$) are the Jacobi wave vectors of the respective particles. For more details on these quantities one is referred to Refs. [14, 16, 17].
Once the relative energy spectrum \(\frac{d\sigma}{dE_{bc}}\) is obtained from Eq. (1) by integrating over the appropriate solid angles, it can be used to obtain the photodisintegration cross section \(\sigma_{\gamma,n}^{\pi\lambda}\) through,

\[
\sigma_{\gamma,n}^{\pi\lambda} = \frac{E_{\gamma}}{n_{\pi\lambda}} \frac{d\sigma}{dE_{bc}}.
\]

(3)

Eq. (3) is valid provided the reaction \(a + \gamma \rightarrow b + c\) is dominated by a single multipolarity \((\pi\lambda, \text{ were } \pi \text{ is the electric or magnetic type and } \lambda \text{ is the multipolarity})\). For a given \(Q\)-value of the reaction, \(E_{\gamma} = E_{bc} + Q\) is the photon energy and \(n_{\pi\lambda}\) is the equivalent virtual photon number which depends upon the \(a - t\) system [18].

Then the radiative capture cross section \(\sigma_{n,\gamma}\) may be computed by invoking the principle of detailed balance,

\[
\sigma_{n,\gamma} = \frac{2(2j_a + 1)}{(2j_b + 1)(2j_c + 1)} \left(\frac{k_{\gamma}}{k_{bc}}\right)^2 \sigma_{\gamma,n}^{\pi\lambda},
\]

(4)

where \(j_a, j_b\) and \(j_c\) are the spins of particles \(a, b\) and \(c\), respectively. \(k_{\gamma}\) and \(k_{bc}\) are the wave numbers of the photon and that of relative motion between \(b\) and \(c\), respectively.

Subsequently, the non-resonant reaction rate per mole \(N_A \langle \sigma v \rangle_{nr}\) \((N_A \text{ being the Avogadro’s constant})\) can be calculated from the neutron capture cross section \(\sigma_{n,\gamma}(E_{bc})\) as [19]:

\[
N_A \langle \sigma v \rangle_{nr} = N_A \sqrt{\frac{8}{(k_B T)^3 \mu_{bc}}} \int_0^\infty \sigma_{n,\gamma}(E_{bc}) E_{bc} \exp\left(-\frac{E_{bc}}{k_B T}\right) dE_{bc},
\]

(5)

where \(k_B\) is the Boltzmann constant and \(T\) is the temperature in Kelvin (K) (usually taken in \(T_9\) terms, where \(T_9 = 10^9\) K). It has been shown that the major contribution to the reaction rate comes from a very small range of energies and hence, the whole integration range in the above equation need not be considered [10, 13].

3. Results and discussion

In this section, we discuss the theoretical elastic Coulomb breakup of \(^{20}\text{N}\) to form \(^{19}\text{N}\) and a neutron when bombarded on a \(^{208}\text{Pb}\) target at 458 MeV/u, i.e., \(^{20}\text{N} + ^{208}\text{Pb} \rightarrow ^{19}\text{N} + n + ^{208}\text{Pb}\). The beam energy is taken to be in consonance with the experimental conditions when considering the forward boost applied to the secondary ion beam [7]. Suitable measurement circumstances, like high beam energies make the study of low relative energy outgoing fragments easier, which can give information regarding astrophysically vital reactions in a few keV to a few hundreds of keV range. The \(^{20}\text{N}\) nucleus, containing 13 neutrons to the 7 protons, is a highly exotic species with a neutron separation energy, \(S_n\), of 2.16 MeV. Its valence neutron couples to a \(^{19}\text{N}\) core and rests in the \(d_{5/2}\) orbital giving it a ground state (g.s.) spin of \(2^-\). We also consider the case when the core captures the neutron from its first excited state (f.e.s.), which for \(^{19}\text{N}\) is at 1.143 MeV having a total angular momentum of \(3/2\). This increases the neutron separation energy for the \(^{20}\text{N}\) to 3.303 MeV.

The triple differential cross-section for the reaction was calculated for the given forward reaction and was integrated over appropriate angles to extract the relative energy spectrum. Since the reaction is dipole dominated [7], the photodissociation cross-section ensuing from Eq. (3) was used to compute the radiative neutron capture cross-section for the inverse \(^{19}\text{N} + n \rightarrow ^{20}\text{N} + \gamma\) reaction. For population of the excited states relative to the ground state and their subsequent rate we used Eqs. (11) and (12) of Ref. [7]. The reaction rates for the radiative capture reaction \(^{19}\text{N}(n,\gamma)^{20}\text{N}\), so obtained were then compared with those from the previous
Table 1. Reaction rates for the $^{19}\text{N}(n,\gamma)^{20}\text{N}$ from the ground state (g.s.) and the first excited state (f.e.s.) of $^{19}\text{N}$ calculated from the CD of $^{20}\text{N}$. All the rates are in [cm$^3$mol$^{-1}$s$^{-1}$]. The statistical estimates presented are taken from Rauscher et al. [20]. Experimental results taken from Ref. [7].

| $T_0$ (K) | FRDWBA (g.s.) | Experiment (g.s.) [7] | FRDWBA (f.e.s.) | Experiment (f.e.s.) [7] | Rauscher et al. [20] |
|-----------|---------------|----------------------|-----------------|-------------------------|----------------------|
| 0.02      | 13.45         | 105.6                | -               | -                       | 15                   |
| 0.1       | 418.1         | 160.3                | -               | -                       | 158                  |
| 0.5       | 753.2         | 1646.3               | -               | -                       | 804                  |
| 0.62      | 768.2         | 1611.1               | -               | -                       | 980                  |
| 1.0       | 793.6         | 1339                 | -               | -                       | 1701                 |
| 2.0       | 816.6         | 877.6                | 9.2             | -                       | 3058                 |
| 3.0       | 904.4         | 697.4                | 85.7            | -                       | 4491                 |
| 4.0       | 992.2         | 664.8                | 249.4           | 155.6                   | 6076                 |
| 5.0       | 1108.1        | 648.8                | 452.3           | 288.5                   | 7645                 |
| 6.0       | 1255.6        | 636.9                | 648.3           | 443.5                   | 9278                 |
| 7.0       | 1438.2        | 612.7                | 814.3           | 600.1                   | 10638                |
| 8.0       | 1659.1        | 600.9                | 943.7           | 780.4                   | 12181                |
| 9.0       | 1921.5        | 595.2                | 1038.9          | 928.3                   | 13814                |
| 9.9       | 2195.5        | 624.6                | 1099.3          | 1094.9                  | 15217                |

theoretical studies and the available experimetnal results [20, 7]. The values for these rates are presented in Table 1. The rates less that 1 cm$^3$mol$^{-1}$s$^{-1}$ were not considered as relevant as they were orders of magnitude lower than others.

From the tabular values, it is evident that the excited state contribution is negligible for temperatures lower than $T_0 = 3$. In fact, at the equilibrium temperature of $T_0 = 0.62$ [3], only the ground sate is relevant for the capture reaction under scrutiny. The results from the FRDWBA match fairly well with the experimental results within their limits of root mean square statistical uncertainties [7]. Both results show that the percentage g.s. contribution decreases with increase in temperature. However, the direct capture results [20] show no such behaviour and are therefore, less reliable for network calculations. An important difference is the negative percentage decrease observed in the experimentally extracted rates, whereas the FRDWBA results show no such slump. They continue to increase although the percent increase decreases as the temperature rises. The percentage decrease in the increase of the ground state contribution can be attributed to the fact that excited state contribution increases, but as averred, only for higher temperatures. The overall rate tends to saturate, as at higher temperatures, other processes like $\alpha$-capture begin to interfere and dominate [3]. It will interesting to further analyse these rates from both the experimental and theoretical view points, especially when the rate for $^{20}\text{N}(n,\gamma)^{21}\text{N}$ is also considered for the abundance of Flourine isotopes via reaction network calculations, which is seen to change by as much as 10 % when these more precise calculations of rates are taken into account.

4. Conclusions
We have used the method of Coulomb dissociation via the finite-range distorted-wave Born approximation theory and investigated the theoretical elastic Coulomb breakup of $^{20}\text{N}$ on $^{208}\text{Pb}$ at 458 MeV/u beam energy to give off a $^{19}\text{N}$ core and a valence neutron. Around this beam energy ($\sim$ a few hundred MeV/u), the final channel fragments emanate with higher velocities.
and are usually easier to detect. Although relativistic effects start to come into the picture, but they are neglected due to the lack of a better relativistic theory [21]. We then used the principle of detailed balance to study the reverse capture reaction $^{19}\text{N}(n,\gamma)^{20}\text{N}$ and calculate its reaction rate in the relevant stellar temperature range. Our results are in good agreement with the experimental values and boost the hypothesis that the contribution of the core excited states to the radiative neutron capture reaction rates is significant only at higher temperatures. Further analysis is required both experimentally and theoretically to conclusively establish the role of core and/or projectile excited state contribution to reaction rates and subsequently, towards the understanding of nucleosynthesis.

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References

[1] Wallerstein G., et al. 1997 Rev. Mod. Phys. 69-4, 995.
[2] Kajino T., Aoki W., Balantekin A. B., Diehl R., Famiano M. A., Mathews G. J. 2019 ArXiv e-prints, arXiv: 1906.05002v1 [astro-ph.HE].
[3] Terasawa M., Sumiyoshi K., Kajino T., Mathews G. J., and Tanihata I. 2001 Astrophys. J. 562, 470.
[4] Sasaqui T., Kajino T., Mathews G. J., Otsuki K., and Nakamura T. 2005 Astrophys J. 634, 1173.
[5] Baur G., Bertulani C. A., and Rebel H. 1986 Nucl. Phys. A 458, 188.
[6] Bertulani C. A. 1999 J. Phys. G: Nucl. Part. Phys. 25, 1959.
[7] Röder M., et al. 2016 Phys. Rev. C 93, 065807.
[8] Tribble R. E., Bertulani C. A., Cognata M. La, Mukhamedzhonov A. M., and Spitaleri C. 2014 Rep. Prog. Phys. 77, 106901.
[9] Casal J., Rodríguez-Gallardo M., Arias J. M., and Gómez-Camacho J. 2016 Phys. Rev. C 93, 041602(R).
[10] Singh G., Shubhchintak, and Chatterjee R. 2017 Phys. Rev. C 95, 065806.
[11] Shubhchintak, Chatterjee R., and Shyam R. 2017 Phys. Rev. C 92, 025804.
[12] Dan M., Singh G., Shubhchintak, and Chatterjee R. 2019 Phys. Rev. C 99, 035801.
[13] Neelam, Shubhchintak, and Chatterjee R. 2015 Phys. Rev. C 92, 044615.
[14] Singh G., Shubhchintak, and Chatterjee R. 2016 Phys. Rev. C 94, 024606.
[15] Nordsieck A. 1954 Phys. Rev. 93, 785.
[16] Shubhchintak, and Chatterjee R. 2014 Nucl. Phys A 922, 99.
[17] Chatterjee R., and Shyam R. 2018 Prog. Part. Nucl. Phys. 103, 67.
[18] Bertulani C. A., and Baur G. 1988 Phys. Rep. 163, 299.
[19] Rolfs C. E., and Rodney W. S. 1988 Cauldrons in the Cosmos (University of Chicago Press, Chicago).
[20] Rauscher T., Applegate J. H., Cowan J. J., Thielemann F.-K., and Wiescher M. 1994 Astrophys J. 429, 499.
[21] Chatterjee R., Shyam R., Tsushima K., and Thomas A. W. 2013 Nucl. Phys A 913, 116.