The local plasticity characterizing the area around the crack tip affects the elastic strain field area due to the boundary interaction between the two zones. Its effects on the SIF and on the Paris’ Law are known, hence it must be considered in the crack growth monitoring and the fatigue limit estimation. Among the different approaches developed, those focusing their attention on the interaction between the elastic stress field and the plastic area seems to be more promising in the evaluation of the crack growth behaviour. In this work, an experimental approach was adopted to identify the plastic zone around the crack. In particular the Thermal Signal Analysis was employed and the thermal footprint was compared with shape and size of the plastic zone predicted through the application of Digital Image Correlation (DIC) in combination with two theoretical models: Westergaard’s model for the stress distribution around the crack tip and the more recent Christopher–James–Patterson (CJP) model.

Keywords: thermal signal analysis, plastic area, crack tip, DIC, Titanium.

1. Introduction
In fracture mechanics, the correct assessment of the strain and stress fields around the tip of a growing crack is an important aspect which can lead to a better understanding of the crack growth driving forces and thus to an improved damage tolerance design approach.

It has been shown how the local plasticity characterizing the area around the crack tip can affect the elastic strain field area due to the boundary interaction between the two zones [1]; in this contest, the characterization of the shape and size of the plastic zone around the crack tip has a key role in the description of the fracture behavior of materials.

Despite the availability of theoretical models mainly based on purely elastic behaviors allows to predict the plastic area, and thus the stress distribution, by means of FEM analysis, the need for experimental feedback remains for the validation of those models.

In several researches experimental techniques where employed in order to characterize the fracture behavior of materials and components [2-14]; among them DIC and thermographic techniques showed great potential having the important feature of being full-field.
Both the techniques are widely used in fracture mechanics for the identification of stress/strain distribution and SIF [2-7]. In recent works these techniques have also been employed for the estimation of dissipated energy [8-11] and the plastic zone shape and size [11-13]. In previous works of some of the authors, an experimental study showed the improved capability of prediction of a crack tip stress field model that incorporate the influence on the elastic stress field of any stresses induced by the plastically deformed area [13].

The objective of this work is to identify the plastic zone around the crack tip by means full-field experimental methods such as DIC and Thermal Signal Analysis. In particular the thermal footprint was compared with shape and size of the plastic zone predicted through the application of DIC in combination with two theoretical models: Westergaard’s model for the stress distribution around the crack tip and the more recent Christopher–James–Patterson (CJP) model.

2. Theory

2.1. Crack tip elastic stress models

In this work two theoretical models for the elastic stress distribution around the crack tip have been employed. The coefficients defining the models have been evaluated by applying an Over Deterministic Method in combination with DIC experimental displacement. The first is Westergaard’s model and assume material with a purely elastic behavior. The second is he CJP model which consider the boundary stresses acting at its interface with the plastic zone.

2.1.1. Westergaard’s model

Considering a polar coordinates system with its centre at the crack tip, the Westergaard’s equations which describe the stress field are the following [14]:

\[
\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) - \frac{K_{II}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \left( 2 + \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \right) + \sigma_0x 
\]

\[
\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) + \frac{K_{II}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} 
\]

\[
\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \sin \frac{\theta}{2} \sin \frac{3\theta}{2} + \frac{K_{II}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) 
\]

Where K_I and K_{II} represents the respectively the mode I and II stress intensity factors and σ the T-stress. Introducing the constitutive law, the same relation can be written in terms of horizontal and vertical displacement:

\[
u = \frac{K_I}{2G} \sqrt{\frac{r}{2\pi}} \cos \frac{\theta}{2} \left( \kappa - 1 - 2\sin^2 \frac{\theta}{2} \right) + \frac{K_{II}}{2G} \sqrt{\frac{r}{2\pi}} \sin \frac{\theta}{2} \left( \kappa + 1 + 2\cos^2 \frac{\theta}{2} \right) + \sigma_0x (\kappa + 1) \cos \theta 
\]

\[
u = \frac{K_I}{2G} \sqrt{\frac{r}{2\pi}} \sin \frac{\theta}{2} \left( \kappa - 1 - 2\cos^2 \frac{\theta}{2} \right) + \frac{K_{II}}{2G} \sqrt{\frac{r}{2\pi}} \cos \frac{\theta}{2} \left( \kappa + 1 + 2\sin^2 \frac{\theta}{2} \right) + \sigma_0x (\kappa - 3) \sin \theta 
\]

with G = \frac{E}{2(1+\nu)} shear modulus ratio, E the Young modulus, ν Poisson’ modulus and \( \kappa = \frac{3-\nu}{1+\nu} \) for plane stress and \( \kappa = 3 - 4\nu \) for.
• $K_c$, the opening mode SIF, which characterize the forces perpendicular to the plane of the crack (mode I);
• $K_t$, the shear SIF, which characterises shear stress in the plane of the crack induced by the compatibility at the interface between the plastic area and the surrounding elastic field;
• $K_r$, the retardation SIF, which characterizes the forces applied in the plane of the crack and counteracting the crack growth.

The equations can be written in terms of stress in a polar coordinate system as follow:

$$
\sigma_x = -\frac{1}{2}(A - 4B + 8F)r^{-\frac{1}{2}}\cos \frac{\theta}{2} - \frac{1}{2}Br^{-\frac{1}{2}}\cos \frac{5\theta}{2} - C
- \frac{1}{2}Fr^{-\frac{1}{2}}\left[\ln(r)\left(\cos \frac{5\theta}{2} + 3\cos \frac{\theta}{2}\right) + \theta\left(\sin \frac{5\theta}{2} + 3\sin \frac{\theta}{2}\right)\right] + O\left(r^{-\frac{3}{2}}\right)
$$

(6)

$$
\sigma_y = \frac{1}{2}(A - 4B + 8F)r^{-\frac{1}{2}}\cos \frac{\theta}{2} + \frac{1}{2}Br^{-\frac{1}{2}}\cos \frac{5\theta}{2} + H
+ \frac{1}{2}Fr^{-\frac{1}{2}}\left[\ln(r)\left(\cos \frac{5\theta}{2} - 5\cos \frac{\theta}{2}\right) + \theta\left(\sin \frac{5\theta}{2} - 5\sin \frac{\theta}{2}\right)\right] + O\left(r^{-\frac{3}{2}}\right)
$$

(7)

$$
\tau_{xy} = -\frac{1}{2}r^{-\frac{1}{2}}(A\sin \frac{\theta}{2} + B\sin \frac{5\theta}{2}) - Fr^{-\frac{1}{2}}\sin \frac{\theta}{2}\left[\ln(r)\cos \frac{3\theta}{2} + \theta\sin \frac{5\theta}{2}\right] + O\left(r^{-\frac{3}{2}}\right)
$$

(8)

And in terms of displacements as:

$$
2G(u + iv) = \kappa \left[-2(B + 2F)z^2 + 4Fz^2 - 2Fz^2 \ln(z) - \frac{C - H}{4}z\right]
- z\left[-(B + 2F)z^2 + \frac{z}{2} \ln(z) - \frac{C - H}{4}\right]
- \left[Az^2 + Dz^2 \ln(z) - 2Dz^2 + \frac{C + H}{2}z\right]
$$

(9)

To guarantee an appropriate asymptotic behaviour of the stress along the crack flank, the assumption $D + F = 0$ must be made. Therefore, crack tip displacement fields are defined from the five coefficients: $A, B, C, F$ and $H$. The coefficients are linked to three different stress intensity factors and the $T$-stress components through the following relations:

$$
K_F = \lim_{r \to 0} \left[\sqrt{2\pi r} \left(\sigma_y + 2Fr^{-\frac{1}{2}}\ln r\right)\right] = \frac{\pi}{\sqrt{2}}(A - 3B - 8F)
$$

(10)

$$
K_s = \lim_{r \to 0} \left[\sqrt{2\pi r} \tau_{xy}\right] = \frac{\pi}{\sqrt{2}}(A + B)
$$

(11)

$$
K_R = \lim_{r \to 0} \left[\sqrt{2\pi r} \sigma_x\right] = -(2\pi)^{\frac{3}{2}}F
$$

(12)

$$
T_x = -C
$$

(13)

$$
T_y = -H
$$

(14)

2.2. Thermal Signal Analysis

The thermal signal analysis is performed by acquiring the thermal response of components and samples undergoing dynamical loads.
The application of a sinusoidal load with a sufficiently high frequency, make the heat transfer negligible and the Fourier harmonic components can be experimentally measured performing a lock-in analysis of the signal.

For a homogenous and isotropic material, the calibrated thermographic signal $T$ during a fatigue test, can be written in the time domain as [13]:

$$ T(t) = T_0 + at + T_1 \sin(\omega t + \Phi_1) + T_2 \sin(2\omega t + \Phi_2) $$

(15)

The first and the second term in eq.15 represent respectively the mean temperature $T_0$ and its linear increase, defined through the coefficients $a$ which depends on the physic characteristics and geometry of the component. This term is affected by the heat transfer (conduction in the specimen or convection and radiance of the environment).

The third term in eq.15 represents the temperature variations induced by the thermoelastic effect; it varies at the same angular frequency as the load; and it is characterized by the amplitude $T_1$ and the phase $\Phi_1$. The latter represent the delay between the load and the thermal response and under adiabatic conditions it is constant throughout the component. If the adiabatic condition is no longer respected, $\Phi_1$ varies; this could be the case of high stress gradients, which lead to conduction effects [17, 18] or heat generation due to local plasticity [19, 20].

The fourth term in eq.15 represents the result of two effects both occurring at the twice of the loading frequency: the thermoelastic response due to the second order effect [21, 22] and the temperature variation due to dissipation [23]; it is defined through the amplitude $T_2$ and the phase $\Phi_2$.

When local plasticization effects occur, the thermoelastic component of the amplitude $T_2$ is at least one order of magnitude smaller than the second and therefore it can be neglected, in this case, the measured $T_2$ can be considered only related to the energy dissipation due to plasticization.

The capability of these parameters in fracture mechanics behavior assessment was demonstrated by several authors [24,13]. In particular, the amplitude $T_2$ allows the evaluation of the plastic and crack closure areas while the phase shifts $\Phi_1$ and $\Phi_2$ are used to localize the crack tip.

If the influence of the temperature on mechanical and thermo-physical characteristics of the material is negligible, the temperature variation induced by the thermoelastic effect [25] is related to the sum of principle stresses $\Delta \sigma$.

$$ S \cdot A = \Delta \sigma $$

(16)

Where $S$ is the amplitude of the first Fourier harmonic of the uncalibrated signal detected by the IR detector while $A$ is the calibration constant and it can be evaluated by means of three different approaches [27-28].

From the combination of eq.16 and Westergaard’s model for elastic stress field surrounding the plastic enclave at the crack tip, Stanley et al.[4] demonstrated the following important relation for the mode I loading:

$$ y = \left( \frac{3\sqrt{\pi} \Delta K_i^2}{4\pi A^2} \right) \frac{1}{S_{\text{max}}^2} $$

(17)

Where $y$ represents the vertical distance of a line parallel to the crack line and $S_{\text{max}}$ is the maximum signal in that line, which occurs at $\theta=60^\circ$.

The Stanley method for the evaluation of the $\Delta K_i$ is based on the linear relation between $y$ and $1/S_{\text{max}}^2$. In fact, once the constant $A$ is known, $\Delta K_i$ can be obtained from the gradient of a graph of $y$ versus $1/S_{\text{max}}^2$, as shown in Figure.1.
Figure 1. Application of the Stanley’s method for the evaluation of $\Delta K_f$. The experimental maximum signals $S_{\text{max}}$ along the lines parallel to the crack line are collected from the selected area. $\Delta K_f$ can be obtained from the gradient of the graph of $y$ versus $1/S_{\text{max}}^2$.

3. Experimental campaign

3.1. Samples material and geometry

The activity was carried out using five Compact-Tensile specimens extracted from a 1 mm thick sheet of pure Titanium grade 2. The samples geometry is reported in Figure 2.

All the samples were prepared by painting both faces. On the side used for the DIC work a speckle pattern has been realized spraying black painting on a white background. The other face used for the Thermal acquisitions was painted with a black mat spray in order to enhance the surface emissivity (Figure 2).

Figure 2. Samples geometry. The speckle pattern (a) was painted on the side used for DIC work while the other face used for the Thermal acquisitions was painted with a black mat spray (b). In (c) the dimensions are in mm.

One dog-bone sample extracted from the same sheet was also employed to perform the Thermoelastic Stress calibration in order to evaluate the constant $A$.

In Table 1 the mechanical properties of this commercially pure titanium alloy are given.
Table 1. Mechanical characteristics for the commercially pure Titanium employed in this work.

| Mechanical Characteristics | Young’s modulus [GPa] | Yield stress [MPa] | Ultimate stress [MPa] | Elongation [%] | Poisson’s ratio |
|-----------------------------|-----------------------|--------------------|-----------------------|----------------|-----------------|
| Pure Titanium Grade 2       | 105                   | 390                | 448                   | 20             | 0.33            |

3.2. Test procedure and data acquisition

The tests were carried out by using a loading frame MTS model 370 with a 25 kN of capacity (Figure 3). According to ASTM E 647, the test procedure involved constant-force-amplitude load, fixed $R$ ratio and fixed loading frequency. All the samples were tested with the same maximum load (750 N) and frequency (17 Hz) but with different $R$ ratio. Table 2 shows the test parameter adopted for each sample.

Table 2. Load characteristics for the five samples.

| Loading conditions | specimen | $P_{\text{min}}$ [N] | $P_{\text{max}}$ [N] | $P_{\text{med}}$ [N] | $\Delta P$ [N] | $R$ | $f$ [Hz] |
|--------------------|----------|----------------------|----------------------|---------------------|----------------|-----|-----------|
| CT1                | 75       | 750                  | 412.5                | 337.5               | 0.1            | 17  |           |
| CT2                | 150      | 750                  | 450                  | 300                 | 0.2            |     |           |
| CT3                | 225      | 750                  | 487                  | 262.5               | 0.3            | 17  |           |
| CT4                | 300      | 750                  | 525                  | 225                 | 0.4            | 17  |           |
| CT5                | 375      | 750                  | 562.5                | 187.5               | 0.5            | 17  |           |

Figure 3. Experimental set-up.

DIC data were acquired by using a Marlin F146B CCD camera 1280X960 fitted with a CF zoom lens 13-130mm and a 9.7 μm/pixel ratio. The images were captured statically; after a defined number of cycles the fatigue cycling was paused and a stepwise loading was applied. Three images were captured at the three different load level 0, $P_{\text{min}}$ and $P_{\text{max}}$. 
Thermal data were acquired by means of a FLIR X6581 cooled IR camera with a window of 640X356 pixels and a μm/pixel ratio of about 30.2. Sequences of 1500 were recorded with a sampling frequency of 200Hz and an integration time of 2287.4 μs.

4. Methods of data processing
The data processing workflow is showed in Figure 4. DIC data were employed to provide Von Mises equivalent stress maps both with the direct method and the indirect method based on the application of theoretical analytical predictions. Thermal data were employed to provide $T_1$, $T_2$, $\Phi_1$ and $\Phi_2$ maps.

4.1. DIC data processing
DIC data were analyzed with the software Ncorr which provided the maps of vertical and horizontal displacements $u$ and $v$.

4.1.1. Direct method. The direct method for the estimation of the stress field and thus the determination of the plastic zone shape and size, consisted in the following steps:

- Assessment of the vertical and horizontal displacements $u$ and $v$ (Ncorr). The determination of the plastic zone was carried out considering the unloaded and the maximum load conditions respectively as reference and current images, while for the evaluation of the $\Delta K_I$, the minimum and the maximum loading conditions were compared.
- Determination of the strain fields at the crack tip by differentiation of the displacement fields. In this regard, the Green–Lagrange strain tensor was employed in order to consider second-order terms.
- Determination of the stress fields using Hooke’s law.
- Calculation of the equivalent stress map by applying Von-Mises’ criterion (Figure 5).
- Individuation of the plastic zone size and shape by connecting all points where the yield criterion is met, that is, where the equivalent stress is equal to the yield stress.

Figure 4. Activity workflow.
4.1.2. Indirect method. The plastic zone shape and size evaluated with the direct method was compared with the theoretical analytical predictions provided by the two models described in section 2.1 (Westergaard and CJP). In this paper the multi-point over-deterministic method [17] was implemented in order to determine from analysis of the experimental displacements the coefficients which describe the two crack tip stress models.

**Figure 5.** Von Mises’ Equivalent stress maps obtained for the CT5 from the combination of DIC data with the Westergaard’s model (a) and the CJP model (b) and from the direct experimental displacements derivation (c). The plastic zone shape and size were identified putting the Yield strength as limit. In (d) the plastic zone profiles predicted by the theoretical models are overlapped to the binarized experimental map.
The application of the indirect method was carried out as follow:

- Assessment of the vertical and horizontal displacements $u$ and $v$ (Ncorr)
- Definition of an annular mesh (Figure 6). The selection of the inner and outer radii was made in such a way to avoid including plastic deformation and to be in the singularity-dominated zone. For the same reason, the mesh does not encompass the crack flanks.
- Iterative selection of the crack tip position and fitting of experimental displacements with the models equation. The solution (in terms of stress intensity factor) was found using the best fit identified by the lowest residuals.
- Prediction of the stress fields using the theoretical models
- Calculation of the equivalent stress map by applying Von-Mises’ criterion (Figure 7)
- Individuation of the plastic zone size and shape by connecting all points where the yield criterion is met, that is, where the equivalent stress is equal to the yield stress.

4.2. Thermal Signal processing

The thermal sequences acquired were processed by using the software IRTA®. For the evaluation of the heat dissipation thermal footprint (Figure 7), the procedure for the data processing involved the following steps:

- Lock-in analysis and pixel by pixel evaluation of the calibrated Fourier harmonic components: $T_1$ and $T_2$ and their phases $\Phi_1$ and $\Phi_2$
- Pixel by pixel subtraction of the mean value evaluated in an area not affected by the singularity of the crack
- Application of a Gaussian 2-D smoothing in order to obtain noise reduction.
- Flipping and resizing of the image in order to compare the map with the DIC images.

The evaluation of $\Delta K_I$ by means of the Stanley method required the following steps:

- Lock-in analysis and pixel by pixel evaluation of the uncalibrated first Fourier harmonic signal $S$
- Extraction of the analysis area around the maximum value of the signal $S$
Application of the Stanley method (as described in section 2.2) and evaluation of \( \Delta K_I \). The constant \( A \) was experimentally evaluated following the classical calibration procedure [27] on a dog-bone sample.

Figure 7. Data acquisition and processing workflow to obtain the map of amplitude and phase of the first and second Fourier harmonic of the thermal signal.

5. Results and discussion

5.1. Plastic zone from DIC direct and indirect method and thermal maps comparison

The plastic zone shapes and sizes identified with the direct and indirect methods have been compared with the thermal maps obtained from the thermal signal processing.

The thermal parameters affected by the presence of localized plasticization are \( \Phi_1 \) and \( T_2 \). Variations in the second harmonic phase are also related to dissipative phenomena due to plasticization, but it is hard to identify the boundary of the area due to the high noise level (Figure 7).

Figure 8 shows the superposition of the plastic zone boundaries on the phase maps for two different load ratios \((R=0.1 \text{ and } R=0.5)\) and a comparable crack length \((4.1 \text{ mm and } 3.8 \text{ mm respectively})\). Among the three boundaries (DIC direct method, Westergaard and CJP), the one that best approximates the thermal parameter footprint is the one predicted by the CJP model.

In Figure 8 it is shown the \( \Phi_1 \) profiles along the crack line; in the same plot the Von Mises’ equivalent stress profiles are also reported. Between the two tests \((R=0.1 \text{ and } R=0.5)\) there is no significative difference in \( \Phi_1 \) and in both cases the crack tip determinate by theoretical models fitting is closer to the minimum phase signal rather than the sign inversion.

Figure 9 shows the same maps and profile plots for the thermal parameter \( T_2 \). In this case the lower load ratio determines a higher and less noisy signal (being \( T_2 \) proportional to \( \Delta P \)) but in both cases the area interested from an above average increase in \( T_2 \) falls within the three predictions and also in this case it is better approximated by the CJP model.

In any case, due to the influence of some test parameters (such as \( R \) on \( T_2 \) and frequency on \( \Phi_1 \)), it is not possible to establish a single threshold for the thermal parameters that identify the plastic zone.
Figure 8. Phase of the first Fourier Harmonic for (a) CT1 ($R=0.1$) at the crack length of 4.1 mm and (c) CT5 ($R=0.5$) at the crack length of 3.8 mm. The plastic zone profiles predicted by direct experimental displacements derivation and the theoretical models are overlapped. The $\Phi_1$ profiles along the crack line and the Von Mises’ equivalent stress profiles are also reported (b and d).
Figure 9. Amplitude of the second Fourier Harmonic for (a) CT1 (R=0.1) at the crack length of 4.1 mm and (c) CT5 (R=0.5) at the crack length of 3.8 mm. The plastic zone profiles predicted by direct experimental displacements derivation and the theoretical models are overlapped. The $T_2$ profiles along the crack line and the Von Mises’ equivalent stress profiles are also reported (b and d).

5.2. $\Delta K_I$ from and Stanley’s method comparison
Table 3 reports the results obtained from the evaluation of the $\Delta K_I$ by using the DIC indirect method with the two theoretical models (Westergaard and CJP) and applying the Stanley’s method (as described in section 4.2).
In Table 3 are also reported the $\Delta K_I$ nominal values, evaluated as prescribed in ASTM E 647 [30].
The % errors evaluated with respect to this nominal value are shown in Table 4.
In all tests the CJP model provides a $\Delta K_I$ value closer to the nominal one, with an error ranging from 1.5 to 19.6 %.
This result could be expected since, contrary to the CJP model, both Stanley and Westergaard do not consider the boundary interactions between the elastic stress field and plastic zone surrounding the crack tip.
Furthermore, the Stanley's method does not consider the presence second order effects which, in metals such as Titanium, are not negligible.
Table 3. $\Delta K_f$ obtained by using the DIC indirect method with the two theoretical models (Westergaard and CJP) and applying the Stanley’s method. The nominal value evaluated as prescribed in ASTM E 647 is also reported.

| $R$ | $Nc_{TSA}$ | $Nc_{DIC}$ | crack length [mm] | $\Delta K_f$ [MPa\(\cdot\)m\(^{1/2}\)] | $\Delta K_{\text{West}}$ [MPa\(\cdot\)m\(^{1/2}\)] | $\Delta K_{\text{St}}$ [MPa\(\cdot\)m\(^{1/2}\)] | $\Delta K_{\text{ASME}}$ [MPa\(\cdot\)m\(^{1/2}\)] |
|-----|-------------|-------------|------------------|-----------------|-----------------|-----------------|-----------------|
| 0.1 | 20900       | 21500       | 5.0              | 34.5            | 33.2            | 34.5            | 42.9            |
| 0.2 | 23400       | 24000       | 5.0              | 39.7            | 30.3            | 36.8            | 40.3            |
| 0.3 | 25200       | 26000       | 5.2              | 33.1            | 28.6            | 24.9            | 36.1            |
| 0.4 | 47400       | 48000       | 4.9              | 31.7            | 23.5            | 20.6            | 29.9            |
| 0.5 | 84200       | 85000       | 5.0              | 26.1            | 23.1            | 22.4            | 25.5            |

Table 4. Percentage error in the determination of the $\Delta K_f$ with respect to the nominal value evaluated as prescribed in ASTM E 647.

| $R$ | $Nc_{TSA}$ | $Nc_{DIC}$ | crack length [mm] | CJP % er | Wester. % er | Stan. % er |
|-----|-------------|-------------|------------------|----------|--------------|------------|
| 0.1 | 20900       | 21500       | 5.0              | 19.6     | 22.6         | 19.6       |
| 0.2 | 23400       | 24000       | 5.0              | 1.5      | 24.8         | 8.7        |
| 0.3 | 25200       | 26000       | 5.2              | 8.3      | 20.8         | 31.0       |
| 0.4 | 47400       | 48000       | 4.9              | 6.0      | 21.4         | 31.1       |
| 0.5 | 84200       | 85000       | 5.0              | 2.4      | 9.4          | 12.2       |

6. Conclusion and future work

In this work two experimental full-field techniques were employed to assess the shape and size of the plasticized area around the tip of a growing crack. The thermal footprint obtained from the Thermal Signal Analysis was compared with shape and size of the plastic zone predicted through the application of DIC both by using the direct derivation of strains/stress field and in combination with two theoretical models: Westergaard’s and CJP. The study led the following results:

- The maps of the thermal parameters show an agreement with the results obtained from the DIC analysis in combination with the theoretical models; however, effects that vary with the test parameters (such as $R$ and the loading frequency) are not negligible and make it difficult to establish a limit for the identification of the plastic zone.
- The theoretical model which showed a better agreement both with the plastic zone predicted by using the direct method and the thermal footprint of both the considered parameters is the CJP model. It is confirmed that the model gives an improved description of the elastic stress field surrounding the crack enclave.
- Furthermore, the CJP model proved, for each $R$, to provide a $\Delta K$ for the opening mode closer to what was calculated with the ASTM standard.
- The Stanley’s method for the evaluation of $\Delta K_f$ resulted in values affected by an error of up to 31% compared to the ASTM standard. This result should consider that the method does not incorporate the boundary interactions between the elastic stress field and plastic zone (since it is based on Westergaard’s equations) and does not consider the presence of second order effects which, in metals such as Titanium and Aluminium, are not negligible.

The present research work continues with the aim of identifying all the effects that affect the thermal signal characterizing the plastic area around the crack. In particular, the study is continuing with:
• An analysis at different test frequencies aimed at isolating secondary effects on the thermal footprint
• A FEM modelling of all the heat sources affecting the system, including the crack closure phenomenon, aimed to validate experimental data and to develop a procedure for the quantitative assessment of the plastic zone shape and size based on thermal parameters.
• The development of a new method for the evaluation of $\Delta K_I$ that takes into account the presence of T-stress and the effects of the second order (when not negligible), based on the Thermoelastic Stress Analysis general model and Williams’ equations.

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