Humans experience small fluctuations in their gait when walking on uneven terrain. The fluctuations deviate from the steady, energy-minimizing pattern for level walking and have no obvious organization. But humans often look ahead when they walk, and could potentially plan anticipatory fluctuations for the terrain. Such planning is only sensible if it serves some an objective purpose, such as maintaining constant speed or reducing energy expenditure, that is also attainable within finite planning capacity. Here, we show that humans do plan and perform optimal control strategies on uneven terrain. Rather than maintaining constant speed, they make purposeful, anticipatory speed adjustments that are consistent with minimizing energy expenditure. A simple optimal control model predicts economical speed fluctuations that agree well with experiments with humans (N = 12) walking on seven different terrain profiles (correlated with model ρ = 0.55 ± 0.11 P < 0.05 all terrains). Participants made repeatable speed fluctuations starting about six to eight steps ahead of each terrain feature (up to ±7.5 cm height difference each step, up to 16 consecutive features). Nearer features matter more, because energy is dissipated with each succeeding step's collision with ground, preventing momentum from persisting indefinitely. A finite horizon of continuous look-ahead and motor working space thus suffice to practically optimize for any length of terrain. Humans reason about walking in the near future to plan complex optimal control sequences.

Walking over irregular terrain presents considerable control challenges compared to level ground (Fig. 1A). Whereas level walking is periodic and largely explained by minimizing energy expenditure (1–3), uneven terrain disrupts periodicity and costs considerably more energy (4, 5). Steps onto ground of uneven height perturb forward walking and cause walking speed to fluctuate (Fig. 1B), in a pattern dictated by a combination of the terrain and the human compensatory control strategy (Fig. 1C). Any number of control strategies are possible and could be determined by criteria such as to maintain constant speed or minimize energy expenditure. But such control is also potentially complex, because the optimal trajectory could depend on the entire terrain profile, which must therefore be anticipated and planned for at once. It is unknown whether humans anticipate and plan for uneven steps, whether for energy economy or other criteria, and how they manage the complexity. Walking on uneven terrain may thus be indicative of human capacity to anticipate and plan complex control tasks.

Many environmental challenges to walking are anticipated ahead of time. Humans obviously look far ahead to navigate and to plan an overall route. They also look more closely—only a few steps ahead—to step over an obstacle (6) or to select a foothold (7, 8). But in addition to kinematics, walking also has significant inertial dynamics (9) and thus momentum. For example, birds appear to use dynamics and leg posture to adjust for a change in ground height while walking (10). Similarly, humans appear to compensate for an upward step such as a sidewalk curb by gradually speeding up a few steps ahead, losing momentum while ascending the curb, and finally gradually regaining speed a few steps after (11). A nearly opposite one applies to stepping down a curb, and the compensations of birds and humans seem compatible with energy minimization. It appears that a single uneven step is anticipated ahead of time, and that the compensation occurs over several surrounding steps.

A sequence of multiple uneven steps may require more complex compensation (Fig. 1A). The momentum of one step carries forth into succeeding ones, and thus the compensation for one isolated uneven step might be incompatible with its neighbors. It may therefore be important to anticipate and plan a trajectory for many steps at once (Fig. 1B), but for computational complexity that typically increases exponentially with the number of steps, termed a “curse of dimensionality” (12, 13). There is thus need for either a means to plan optimal trajectories despite high dimensionality, or a heuristic and practical but suboptimal compensation strategy.

Significance

Humans obviously look ahead as they walk, but it is unknown what adjustments they make for uneven terrain and for what objective. We show that humans purposefully (albeit subconsciously) plan and adjust their forward speed ahead of upcoming terrain, as predicted by minimization of energy expenditure. This research shows how the dynamics, energetics, and control of walking are predictable. Humans have greater capacity to anticipate and reason about the body’s interactions with the world than previously understood. They use this capacity to plan complex control sequences that save them energy.
The optimization approach has been applied to a variety of human motions. Upper extremity reaching motions have long been thought to minimize kinematic objectives (19), and more recently energy expenditure (20, 21). Similarly, walking routes have been proposed to minimize kinematic objectives such as speed–curvature trade-offs (22), and empirical energy objectives to predict curved walking paths through doorways (23). There is potential to mechanistically predict the dynamics of walking, but most studies to date have been concerned only with steady, periodic gait (24). The challenge of walking on uneven terrain suggests the need to integrate mechanics, energetics, and planning, while somehow avoiding or mitigating high complexity.

The purpose of the present study was to test whether humans optimize their compensations for uneven terrain consisting of a complex pattern of unequal height steps. We used optimization of the pendulum-like walking model to predict speed fluctuation trajectories for a variety of uneven terrain profiles (Fig. 1B). We also considered whether a finite planning horizon can yield similar predictions to a full horizon, but with less complexity. We then performed a human subjects experiment to test whether the optimal compensations could predict human responses. If humans do favor energy economy as modeled by step-to-step transition dynamics, and the optimization is feasible and predictive for multiple steps, then the model should reasonably predict human responses. This will reveal whether humans favor energy economy when walking on uneven terrain, and whether and how far they plan into the future.

**Results**

**Model Predictions.** The simple walking model demonstrates compensatory control strategies for walking on uneven terrain. The optimization criteria predict specific trajectories of walking speed, time duration, and energy cost. Here, the proposed *Minimum energy objective* is contrasted with two sample alternatives. These criteria yield distinct, terrain-specific predictions, to be tested experimentally. Finally, the model predicts how planning horizon affects walking motions, to contrast full vs. finite horizon planning.

Three alternative control strategies are examined: reactive control, tight regulation, and the proposed minimum energy objective. The task is to traverse multiple steps of uneven height in nominal time (Fig. 2A; nominal defined as steady level walking), using a model of pendulum-like leg dynamics (Fig. 2B), where momentum is controlled by modulating active push-off (PO) onto each uneven step (Fig. 2C, PO). Reactive control does not anticipate the upcoming terrain, and instead compensates for disturbances only after it encounters the first uneven step, with the minimum work achievable from that point on (therefore still anticipating subsequent steps just as minimum energy). Tight regulation is a high gain compensation that minimizes fluctuations in step times, and thus maintains nearly constant speed for each step regardless of energetic cost. Minimum energy plans an optimal strategy for the entire (full horizon) terrain sequence in advance, to minimize the push-off work for step-to-step transitions. As demonstration, these strategies are applied to a sample uneven terrain profile that gently slopes up and then down over several steps (termed Pyramid, Fig. 2D).

The model predicts that all the three strategies can traverse the terrain in nominal overall speed and time (zero cumulative time gain), but at different costs. Reactive control and tight regulation, respectively, perform 12.1% and 9.2% more PO work than nominal level gait (0.718 \( MgL \) overall work; \( MgL \) is the gravitational potential energy of the model’s COM, for body mass \( M \) and leg length \( L \)). The minimum energy strategy is most economical, performing about 6.2% more work (0.044 \( MgL \) than nominal). Its speed profile starts several steps before and ends several steps after the uneven terrain. It achieves economy by pushing off harder and speeding up several steps ahead, in anticipation of a loss of speed while ascending the upward steps. The optimal descent is nearly the reverse in time, regaining speed on the downward steps, and then slowing down again afterward. The overall strategy gains time in the first half and then loses it in the second (Fig. 2, time gain). This saves work compared to any other alternative and requires 80% less work than the gravitational potential energy of
Walking over uneven terrain. (A) Task is to traverse successive uneven steps of height \( h_i \), in same overall time and speed as nominal level walking. Step index \( i = 0 \) is first step deviating from level ground. (B) Simple walking model with pendulum-like legs takes even or (C) uneven steps, punctuated by step-to-step transition consisting of active, impulsive PO by trailing leg just prior to leading leg's dissipative CO with ground. (D) Model predictions for a terrain of several steps up and then down (termed Pyramid), for three hypothetical strategies: reactive control that optimally adjusts the walking pattern with no anticipation; tight regulation that controls against step time changes; and minimum energy, which looks ahead to a full horizon of uneven steps and plans anticipatory, optimal control. Strategies shown in terms of (top to bottom rows) speed fluctuations vs. time, cumulative time gain relative to nominal walking, and PO work vs. time. All strategies take same overall time, but minimum energy required the least work. Simulations are shown as discrete steps (dot symbols), with speed \( V \), sampled at mid-stance instant \( t = 0 \) aligned by vertical dashed line. In experiment, human subjects walked over similar Pyramid terrain in a walkway (30 m long), with step height increments of 7.5 cm. Model parameters: \( a = 0.44 \), nominal mid-stance speed \( V = 0.44 \).

the Pyramid itself (0.225 \( MgL \)). It is thus possible to ascend steps with less work than would be needed to lift the body against gravity alone. This demonstrates the advantage of planning strategically for an entire terrain sequence at once.

Using the minimum-energy strategy, the model also predicts distinct compensation strategies for each of the seven experimental terrain profiles (Fig. 3), in comparison to level nominal walking (Control). The terrains consist of sequences of discrete ground-height changes, each up to 0.075 \( L \). The profiles have between one and sixteen such height changes and are termed Up-step (U), Down-step (D), Pyramid (P, see also Fig. 2), Up-Down (UD), Down and Up-Down (D&UD), Complex 1 (C1), and Complex 2 (C2). The task is to traverse the terrain and regain nominal speed and time, with compensations starting no earlier than six steps before the first uneven step and ending no later than six steps after. Given a full planning horizon, the optimal strategy for Up-step (Down-step) is a tri-phasic speed fluctuation pattern, as described previously (16). The Pyramid strategy (repeated from Fig. 2) is accompanied by parameter variations (Fig. 3, Parameters inset) for three different speeds and four different step lengths (shorter, nominal, and longer fixed lengths, and lengths increasing with speed according to the human preferred relationship, as detailed in Materials and Methods). The speed profiles vary in overall amplitude and time for these cases, but all retain a similar scalable shape in terms of discrete steps. The remaining terrains also yield deterministic, scalable, and distinct optimal profiles, whose fluctuation patterns are to be tested against human.

The minimum-energy strategies (Fig. 3) constitute the main testable predictions for humans, who regardless of self-selected speed and step length are expected to produce speed fluctuation patterns similar to model. The predictions extend beyond the section of uneven terrain alone and include anticipatory speed changes before the first uneven step and recovery beyond the final uneven step. There are, of course, many other models and optimization objectives possible, but each would be expected to produce distinct predictions for various terrain profiles, making the present model falsifiable.

The model also predicts compensation strategies for shorter, finite planning horizons (Fig. 4). In contrast to the previous full horizon, here the optimization plans based only on horizons of \( m \) upcoming steps at time. Only the first step of the \( m \)-step plan is executed, because the horizon is continually updated and a new compensation strategy is planned, based on this finite receding horizon. The planner is aware of the ultimate goal of regaining nominal time, but not of the distant terrain until it enters the horizon. Very short horizons yield speed fluctuations and PO work trajectories (Fig. 4 A and B, respectively) very different from full
Fig. 4. Effect of finite planning horizon for model walking over Pyramid (P) terrain with minimum energy. (A) Compensation strategy in terms of optimal speed fluctuations vs. time, for finite horizon lengths $m$ ranging 2 to 14 (lighter to darker colored lines), compared to full horizon optimum (gray lines). (B) PO work trajectories vs. time for traversing terrain in nominal time, for a range of horizon lengths. (C) Work cost vs. horizon length $m$. Cost is defined as average PO work per step to traverse the terrain, compared to full horizon (Fig. 3). (D) Correlation of finite horizon strategies with full horizon. For all plots, the longer the horizon, the greater the resemblance to planning over a full horizon (defined as 21 steps here). As few as seven to eight steps are sufficient to gain most of the economy and performance of a full horizon. Optimization minimizes PO work to traverse terrain while conserving overall speed. Conditions are equivalent to human walking at 1.5 m/s with 7.5 cm increments in step height on Pyramid terrain.

Experimental Results

Human subjects walked with a unique speed fluctuation profile on each terrain (Fig. 5). Even though each walked at their own self-selected overall speed, all had similar speed fluctuation patterns (Fig. 5, compare individual vs. overall average). The patterns compared reasonably well with model predictions. We first report a basic summary of the control condition of level walking, which serves as a basis of comparison for the uneven terrains (see SI Appendix for details). In three sets of terrains (Materials and Methods), self-selected control speeds were 1.38 ± 0.10 m/s, 1.52 ± 0.13 m/s and 1.51 ± 0.12 m/s (mean ± SD across subjects), respectively. Each person walked consistently, with average speed varying about 2.4 to 4.8% c.v. (coefficient of variation) across their control trials, even though there was also a significant recovery component, summarized by the average speeds of the two steps after uneven terrain (all terrains except D&UD and Up terrains). The control speeds on all uneven terrains regardless of complexity. The overall speed and walking duration differences within each set of terrain conditions including corresponding level control were typically less than 6% and none were statistically significant (repeated-measures ANOVA; see SI Appendix for details). Individual walking speeds were also fairly consistent across trials of a specific terrain, varying by 2 to 3% (c.v.). A time loss or difference would be expected if there were no compensation (Fig. 2). The observed conservation of the overall walking duration and walking speed, regardless of the terrain, indicates that humans compensated for all terrains. The approximate time conservation was in accordance with experimental instructions, and despite no feedback having been provided regarding speed or duration at any point in the experiment.

Humans approximately conserved overall walking speed and duration on uneven terrain. Subjects walked at similar overall speeds on all uneven terrains regardless of complexity. The overall speed and walking duration differences within each set of terrain conditions including corresponding level control were typically less than 6% and none were statistically significant (repeated-measures ANOVA; see SI Appendix for details). Individual walking speeds were also fairly consistent across trials of a specific terrain, varying by 2 to 3% (c.v.). A time loss or difference would be expected if there were no compensation (Fig. 2). The observed conservation of the overall walking duration and walking speed, regardless of the terrain, indicates that humans compensated for all terrains. The approximate time conservation was in accordance with experimental instructions, and despite no feedback having been provided regarding speed or duration at any point in the experiment.

Humans produced a repeatable speed fluctuation pattern for each uneven terrain. Each terrain yielded a specific speed fluctuation profile vs. time (Fig. 5). Two basic quantifications are the overall speed variability for a terrain (rms variability) and the maximum speed fluctuations within that terrain. The variability was greater than corresponding control (all $P < 0.05$; paired $t$ tests), ranging about 2 to 5% (c.v.). The maximum fluctuations differed in magnitude and location for different terrains, for example +5.68% at $i = -1$ for Up-step terrain, and +4.4 at $i = 2$ for Down-step. All were greater (all $P < 0.05$; paired $t$ tests), by an average factor of 7.6, than those in the Control conditions. These fluctuations do not support the tight regulation hypotheses, for which very little deviations were expected. (See SI Appendix for more quantification.)

The compensations included anticipatory and recovery components. Anticipation was summarized as a tendency to speed up before a first upward step (U, UD, P, C1, C2 terrains), and to slow down before a first downward step (D and D&UD terrains). For initial upward steps, the preceding step ($i = -1$) appeared to speed up 3.74 to 5.70% relative to nominal, and the step preceding initial downward steps to slow down about 2.05%. Moreover, the two preceding steps (average $v_l$ for $i = -2$, -1) for uneven terrains tended to exhibit anticipatory speed up or slow down consistent with prediction (all terrains except D&UD and Up terrain, respectively D&UD $P < 0.007$; paired $t$ tests with Bonferroni correction). Similarly, there was also a significant recovery component, summarized by the average speeds of the two steps after uneven terrain (all $P < 0.007$; paired $t$ tests with Bonferroni correction). The anticipatory adjustments do not support the reactive compensation and tight regulation hypotheses, and the recovery adjustments do not support tight regulation.

The individual speed responses were similar across subjects. Quantitatively, the correlation between individual speed fluctuations and (across-subject) averages was significant and positive for all terrains. The correlations $\rho$ ranged 0.66 ± 0.13 to 0.85 ± 0.11 (for Complex 2 terrain and Up terrain, respectively; mean ± SD). Some control trials also had a small amount of correlation (0.46 ± 0.33, 0.32 ± 0.32, and 0.29 ± 0.40, respectively, for the three sets), but with far smaller maximum speed fluctuations generally about one-sixth those for uneven terrain. Incidentally, the compensation strategies predicted to be nearly opposite to each other were negatively correlated in data. For example, responses for Up- and Down-steps were negatively correlated with average Down- and Up-steps, respectively ($-0.36 \pm 0.17$ and $-0.27 \pm 0.29$), and similarly for Up-Down and Down-and-Up-Down (0.52 ± 0.083 and $-0.53 \pm 0.15$, respectively). The similarity between individuals across all terrains is indicative of systematically repeatable responses.
Humans walking speed fluctuations were consistent with minimum-energy predictions. The compensation strategies agreed reasonably well with model predictions for minimizing energy expenditure over a full horizon (Fig. 6). Visual inspection reveals an overall resemblance in speed profiles between human and minimum energy model (Fig. 6A). This is quantified by the correlation between experimental fluctuations for a terrain and corresponding model prediction (13 to 28 steps per terrain for each subject across seven terrains, at least 11 subjects per terrain). Zero correlation would indicate a nonpredictive model or random human strategies. Instead, the experimental correlation coefficients ranged 0.35 to 0.67 across the seven terrains (Figs. 6 and 7A and B), all statistically significant (P-values by terrain: U 1.5e-12, D 1.7e-21, UD 1.3e-22, P 5.7e-21, C1 1.5e-10, C2 6.2e-18). The lowest correlations were for the two Complex terrains (0.35 ± 0.10 for C1 and 0.46 ± 0.09 for C2, mean ± 95% CI) and higher correlations for the shorter and simpler terrains. The average correlation across terrains was 0.55 ± 0.11 SD. Log likelihood ratios for optimal model compared to random models were at least 33 ± 14 (mean ± SD across randomly shuffled models; see SI Appendix for details). This was equivalent to a range of 2.6 to 6.1 bits per step of predictive information. (A model predicting only the direction of speed change would have 1 bit/step of information.) The minimum Bayes factor was 4.9e14, for U terrain; it is the factor by which a prior odds ratio (proposed model to random model) is adjusted to yield a posterior odds ratio. The correlations and highly significant P-values suggest that the model is predictive, and the log likelihood ratios suggest that the model adds substantial predictive information.

There were some steps and some terrains where human and model did not agree well. By visual inspection of Up- and Down-steps (Fig. 6), humans usually spent an extra step (about \( i = 2 \)) at greatly deviated speed compared to the model. On Down-and-Up-Down, humans were slower than model on step \( i = 4 \). And in the middle of both Complex terrains, human speed fluctuations did not vary as much as model (e.g. \( i = 8 \)); they were similar to applying a low-pass filter to model responses. Such differences emphasize that the general agreement between human and model did not apply to every step of every terrain.

A finite planning horizon predicts most human compensations. The human speed fluctuations could also be predicted reasonably well with a finite planning horizon. This was quantified by the correlation between human responses and model predictions...
for a range of \( m \)-step horizons (Fig. 7C). For all terrains, short planning horizons had almost no predictive ability, and longer horizons generally did better. The overall correlation, averaged across terrains (Fig. 7C, gray line), resembled a saturating exponential, with a nearly monotonic increase with horizon length \( m \), maximized with the full horizon. The saturation is consistent with model dynamics (Fig. 4), for which the immediate upcoming step is most important, and succeeding steps exponentially less so. This makes it highly advantageous to look at least a few steps ahead, but with diminishing returns for farther lookahead. The observed advantage of longer planning horizons was terrain specific, in that several (U, UD, D&UD, C1) had peak correlation with a model planning for as few as 5 to 8 steps and others (D, P, C2) only for a full horizon. As a basic summary, the average correlation was within 92 to 93% of the saturation value with horizons of six to eight steps.

![Fig. 6. Comparison of model and human walking speed fluctuations vs. time, for all terrains. Each terrain (defined in Fig. 3) is shown with paired model and human (Top and Bottom, respectively; human \( N \geq 11 \)) trajectories, along with the correlation coefficient \( \rho \) between them (±95% CI; * indicates statistical significance, \( P < 0.05 \)). Model predictions are from full-horizon, minimum-energy model (Fig. 3), in terms of discrete body speed at each step, for comparison with human average trajectory of body speed (shaded area represents ±1 SD from Fig. 5). Each data point for model and human is body speed at a footfall, defined as stride length divided by stride time ending at that footfall. The first uneven step is indicated by vertical dashed line and overall walking speed by horizontal solid line. Model trajectories are plotted in dimensionless speed and time, equivalent to units of \( g^{0.5}L^{0.5} = 3.13 \, \text{m/s} \) and \( g^{-0.5}L^{0.5} = 0.32 \, \text{s} \), respectively, using gravitational acceleration \( g \) and human leg length \( L = 1 \, \text{m} \).](image)

![Fig. 7. Correlation between human and model based on full and finite horizon predictions. (A) Human vs. model speed fluctuations for each terrain (all subjects, \( N \geq 11 \); terrains defined in Fig. 3), using full-horizon, minimum-energy prediction. Consecutive steps (lines; same data as Fig. 5) by each person on a terrain (filled symbols for individual steps) are plotted against model based on full horizon prediction. Speed fluctuation trajectories have been demeaned and scaled per subject for visualization purposes (correlation measures are independent of scale). Human speed fluctuations are expected to be positively correlated with model. (B) Overall human–model correlation \( \rho \), based on full horizon optimization (all significantly nonzero, \( P < 0.05 \); error bars denote 95% CI); mean across terrains is \( \rho = 0.55 \) (dotted line). (C) Correlation between human and finite horizon models vs. horizon length, for all terrains (one line per terrain; mean across terrains in gray). A horizon length of about six to eight steps is sufficient to predict human responses on most terrains.)
planning horizon of at least such length could thus be regarded as sufficient to predict most human responses.

Discussion

We had sought to determine whether humans compensate for uneven terrain by planning and adjusting their forward momentum. The human data showed systematic speed fluctuations that were consistent across individuals, and specific to each of the terrain patterns, including complex patterns of consecutive uneven steps. The speed fluctuations included an anticipatory component prior to actually contacting the uneven terrain and showed patterns that were consistent with the minimum-energy optimization model. That optimization becomes increasingly complex for more uneven steps. However, we found it sufficient to consider only a finite horizon of upcoming steps to yield near-optimal economy. We interpret these results to suggest that humans optimally plan and control for uneven terrain, in a manner that is bounded in planning complexity.

The human speed fluctuation patterns were not mere noise. They might superficially seem to have little organization (e.g., Fig. 5 complex terrain C1), but quantification shows that the patterns were repeatable across subjects, unique to each terrain, and predictable by model. Part of the repeatability may be explained by inverted pendulum dynamics, for example downward steps should gain (and upward steps lose) forward speed and momentum. But dynamics alone do not explain how walking durations could be conserved across different terrains. Without any compensatory control for terrain variations (i.e., using nominal PO for all steps), overall walking speeds would have been reduced (16). With compensatory but purely reactive control, it would be possible to regain lost time (Fig. 2, reactive compensation), but not before losing it. Another possibility would have been to tightly regulate speed to nearly constant with each uneven step (Fig. 2, tight regulation). But the significant and repeatable speed fluctuations observed do not support these possibilities. Instead, there was clear anticipatory compensation, consistent with the model. Subjects made systematic adjustments before the first uneven step, speeding up significantly before a first upward step and slowing down before a first downward step (compare D and D&UD against other terrains). They also systematically recovered beyond the final uneven step (compare D and D&UD against other terrains), as predicted by model. Subjects approximately conserved speed and duration, and in part by adjusting their speed ahead of time. Humans appear plan ahead for multiple steps, perhaps extending several steps before and after a segment of uneven terrain.

Such planning appears to be performed over a multistep horizon. A simple indication of lookahead is that walking speed started deviating from steady state ahead of the first uneven step (Fig. 5). A better indicator is the predictability of human responses with the model’s horizon length \( m \) (Fig. 7C). Longer planning horizons generally allowed the model to better predict human (Fig. 7C, gray line) with a gradually saturating behavior. The advantage of planning ahead is explained by simple governing principles, namely the step-to-step transitions of pendulum-like walking, which dissipate energy with each ground CO. About 70% of the forward momentum persists from one step to the next (16), and less than 1% of the forward kinetic energy persists beyond seven steps (25). A momentary perturbation, whether from the ground or by the person, thus has consequences for succeeding steps, making it generally optimal to plan a sequence of steps at once. Limited persistence also suggests no advantage to planning infinitely into the future. Much of the predictive ability of the finite horizon model (Fig. 7C) comes from the first few steps of lookahead, and six to eight steps of lookahead appear sufficient to explain over 90% of the observed human responses.

This does not, however, mean that humans literally optimize for each individual step. It might be practical to reduce complexity by reasoning about chunks of steps, for example treating Pyramid terrain (Fig. 4) as simply one chunk with a gentle upward followed by downward undulation rather than twelve individual steps (Fig. 7C, P terrain). It may also be more important for the human to attend to overall, low-frequency ups and downs rather than the detailed variations. This may explain why human responses on complex terrains seemed to be low-pass filtered versions of model (Results). It is quite possible that the long lookahead observed here is better represented as a smaller number of chunks or low-frequency step combinations.

The proposed criterion for this planning is energy economy. Much of the natural world imposes unsteady and uneven conditions, making it important to economize not only for the steady, level case, but also for when energy costs are highest. We used a mechanistic and quantitative model to predict how economy may be achieved, as governed step-to-step transitions (16). These dynamics describe how speed is lost with an upward step and gained by a downward one, and how PO may be modulated to change speed and affect the CO loss. Step-to-step transitions have previously been tested against humans during steady walking, where parameters such as step length or width (15) are readily manipulated and the associated energy cost tested. Such experiments do not apply to unsteady conditions, and so here we experimentally manipulated only the terrain and computationally predicted the compensatory trajectories. And through testing on seven different terrains, there is low probability that a random model could predict human responses well by chance.

Our results suggest that humans reason about energetics, dynamics, and timing for locomotion. Energetics refers to the ability to judge the upcoming terrain profile with respect to the hypothesized criterion of energy minimization. Although humans are understood to prioritize economy for steady, level walking (26), some form of energetic prediction is at least as important for selecting from the many options available for uneven terrain. Dynamics refers to the translation of that criterion into an appropriate sequence of control actions. Humans seem to reason about the momentum lost or gained by a change in ground height and how active PO and other control can influence that change. Just as the ability to catch a ball suggests reasoning about the ball’s dynamics in flight, the ability to conserve time and energy on uneven terrain implies reasoning about the body’s own dynamics, described here by a model of pendulum dynamics and the step-to-step transition. Timing refers to an ability to form an expectation of the time to traverse a given distance (Fig. 1) and to use that to guide the dynamic control actions with minimum energy expenditure. We treat the overall reasoning as tantamount to a central nervous system internal model (27) of walking that enables planning for economy.

The present study is concerned with a type of control that is intermediary between lower- and higher-level concerns. At the lower level, the central nervous system performs real-time control with relatively fast feedback (within tens of milliseconds) from somatosensory and other inputs, much of it mediated at the level of the spinal cord (28). At a higher level, humans can consciously plan many seconds or minutes into the future, for example the best route to the supermarket. The anticipatory, dynamic planning considered here is intermediate, with an apparent temporal update rate on the order of a walking step, or about half a second. Spatially, it integrates higher-level terrain awareness with lower-level walking
control and appears to work subconsciously, in that subjects exhibit little cognitive awareness of what they are planning or how their momentum varies on uneven terrain. The planning observed here is reminiscent of upper-extremity reaching movements, which appear mediated by internal dynamical models represented in the cerebellum and motor cortex (29, 30). Internal models may contribute to state estimation and control during walking (31). Other internal models with short-term storage or working space for anticipatory adjustments may also be employed for dynamic planning of locomotion.

The proposed planning horizon may seem longer than where people usually look. Humans allocate most of their gaze two to four steps ahead on rocky, rough terrain (7, 32). Such information is especially important for foot placement, which impacts balance (31, 33, 34) and energy expenditure (35). The nearest steps are considered critical for not only humans (6, 36) and robots (37), but also our simple models (16, 33). But on less challenging terrain, humans walk faster, look ahead farther (32), and expend less energy (4, 38). The present terrain is in that category, consisting of broad and flat surfaces that placed little demand for foot placement and perhaps visual attention. Our interpretation is that humans may also look ahead a variety of distances, including far ahead to determine and anticipate a path (36, 39), and intermediate distances (say, at least six to eight steps ahead) to plan forward momentum. Such lookahead might require only a brief glance yet still serve valuable purpose.

The present optimization approach is related to studies of humanoid robot control and simulation. Real-time control for many-legged robots is performed with MPC over a short horizon, computed repeatedly within a timing loop on the order of a millisecond. This acts as a feedback control that achieves stability and near-optimal performance with manageable computations (18, 40, 41), for robots with many more degrees of freedom and more complex contact conditions than modeled here. We employed a slower, per-step timing loop and a longer multistep planning horizon, because our focus was on economically managing momentum. However, MPC control is generally applicable over a variety of time scales and horizons. Although we designed the control explicitly, it could also be adapted via reinforcement learning techniques, which can learn quite robust control over uneven terrain, typically also with a finite horizon terrain profile obtained through vision and mapped directly to control commands (42, 43). Thus, humans could potentially embed the optimization within a vision-to-control mapping, rather than continually optimize for each step. An interesting observation is that robustness may be achieved with simple rewards such as to make forward progress, without need for explicit stability criteria, because successful progress implies stability. Our present model for human momentum planning assumes the existence of an executive goal set at higher levels and a lower-level control for fast feedback. This planner could potentially achieve both economy and stability by looking several steps ahead to direct the upcoming step-to-step transition. At present, finite horizon optimization or learning is a highly viable approach, for both understanding humans and controlling robots.

This study uses mechanistic principles to predict transient walking on uneven terrain. There is ample evidence that humans direct their gaze (7, 8) and start taking actions (44) only a few steps ahead, but with few operational principles for determining the actions. Our model is mechanistic in that it produces a walking gait governed entirely by physical first principles (e.g., inverted pendulum and the step-to-step transition). There were no free parameters or opportunities to fit model to data. We are unaware of any other models that are similarly principled and can predict compensation for uneven terrain. Perhaps, the closest analogy is an empirical energetic cost model (23) that predicts dynamic paths for economically taking turning paths such as through doorways (on level ground). We consider our mechanistic model to be largely compatible with such empirical costs, and suspect that a single, first-principle model might therefore explain both uneven and turning walking trajectories.

There are a number of limitations to this work. In terms of experimentation, we found the model to be significantly but only modestly predictive of humans (e.g., correlation 0.66 on Down terrain, Fig. 6). This was due in part to the noisiness of human walking, as individuals did not behave identically to each other (correlation 0.85 on Up terrain), let alone to any model. It may be better to apply larger terrain disturbances to exceed the noise floor, but we intentionally examined relatively small disturbances where the inverted pendulum model of walking is most applicable. Another limitation is that our terrain disturbances consisted of discrete and flat steps, whereas actual uneven ground is more continuous and requires additional decisions regarding foot placement and orientation. We also experimentally tested the optimization hypothesis through speed trajectories, but did not test actual mechanical work or metabolic energetics. The task was too long to measure ground reaction forces to estimate work over many steps and too short to allow for the steady-state measurement of oxygen consumption needed to estimate energetics. These remain avenues for further testing of model and its alternatives.

Other limitations were from the simplicity of the dynamic walking model. The model applies inverted pendulum dynamics as a fundamental feature of walking. Here, we observed some cases where the model’s rapid speed fluctuations were not matched by human (Fig. 6; see central step of Pyramid, several steps of Complex 1 and 2). We interpret this low-pass filter effect as the human stance leg behaving less like an inverted pendulum, allowing the COM to deviate somewhat from a pendular arc. Additional model features such as a previously hypothesized energetic cost for rapid force production (45) could potentially explain some of the mismatches observed here. But even if humans approximate an inverted pendulum on flat and slightly uneven ground, that is certainly not the case for larger terrain disturbances or for stair steps, where it is necessary to use the knees. When ascending large steps, the leading stance knee flexes and extends substantially and can perform substantial positive work for climbing. And when descending large steps, the trailing leg does not behave like a (noninverted) pendulum, perhaps to actively dissipate gravitational potential energy. Once in swing, it might also be actively flexed to allow the swing foot to clear the step. Dynamic walking models have been devised to include knee joints with actuation (46, 47), legs that telescope (48), and actuators with elasticity (49). Such models have potential to capture more aspects of human walking, especially larger terrain disturbances than examined here.

There are, of course, many other degrees of freedom present in human. For example, a feature missing here is actuation of the swing leg, which humans might use to modulate step length, for example to line up with an uneven step or ascend or descend it more economically. Our model takes fixed step lengths at fixed cost, where the control of step length may also exact a cost (24). But transient step adjustments deviate from steady-state step lengths (50, 51), at a cost yet to be modeled. Other relevant degrees of freedom that could be included in a dynamic walking model include the ankles (52) and lateral body motions (33, 53, 54), although with less effect on economy than pendulum-like step-to-step transitions. It is, however, not straightforward to devise appropriate optimization objectives to predict behavior for many degrees of freedom. We adopted a highly simplified model in part to avoid fitting to data, in favor of first principles.
We showed that humans plan and control their gait to economically locomote over uneven terrain. They anticipate upcoming steps and can adjust their speed and momentum to conserve energy and time. These actions resemble a dynamic optimization procedure, which has potential for impractically high dimensionality. Fortunately, walking momentum does not persist for long, and so it appears sufficient to plan dynamics for as little as six to eight steps into the future. Economy has long been established as a governing principle for level, steady walking, and our findings suggest that it similarly applies to unsteady and uneven walking of arbitrary distance.

Materials and Methods

We performed an experiment to test whether humans optimally compensate for uneven terrain. Predictions were obtained from a simple optimal control model of walking, described in detail previously (11, 16). Here, we briefly summarize the model, followed by a description of the present study’s experiment. Model code and experimental data are available in open access repositories (55, 56).

Model Dynamics. The optimal control model determines a compensation strategy for traversing a sequence of uneven-height steps at minimum energetic cost while conserving travel time (Fig. 2). The task is to walk down a sidewalk interrupted by uneven terrain, starting and ending from steady walking on flat ground. The model’s only energetic cost is from positive mechanical work needed to power walking. The key feature determining the optimal strategy is that work required to redirect the body COM velocity between pendulum-like steps. This work is performed by pushing off with the trailing leg each step, and the sequence of such POs comprises the decision variables for optimal control. Conservation of traversal time is a constraint that the total time be equal to that for steady walking on flat ground alone. The overall time and the walking dynamics governing the momentum of each step are expressed as constraints in the optimal control problem.

The model and optimization were described previously (11) and are briefly summarized here and in SI Appendix. The dynamic walking model treats the stance leg as a simple inverted pendulum (16, 24) supporting a point-mass pelvis (Fig. 2B) and the swing leg as a simple pendulum of infinitesimal mass. Forward speed $v$ is sampled at the mid-instance stance of each step, when the inverted pendulum is vertical. An impulse (PO, Fig. 2C) is applied along the trailing leg just before the leading leg’s impulsive CO with ground, with uneven step height $b_i$. It is optimal to purposefully modulate PO with each step, causing walking speed to fluctuate.

Nominal model parameters were selected to correspond to typical human walking. The nominal gait was for a person with leg length $L$ of 1 m walking at 1.5 m/s, step length of 0.79 m, and step time of 0.53 s. Here, we constrained the model to take steps of fixed length and examined alternative step lengths in parameter sensitivity studies. These included shorter and longer steps (0.59 and 0.96 m, respectively), as well as preferred step lengths increasing with speed according to the preferred human relationship, approximately $v^{0.42}$ (24, 57). The nominal parameter values were $a = 0.41$, PO 0.0342 $M^2$, step time $1.665 g^{-0.11} L^{0.54}$, precollision speed $0.601 g^{0.06} L^{0.51}$, and mid-stance speed $0.44 g^{0.16} L^{0.55}$. Most step heights were in increments of $b = 0.075 L$, equivalent to about 7.5 cm for a human. Previous parameter studies with this model (16, 25) have shown similar fluctuation patterns across a wide range of parameter values.

The energetics of the present model only include PO work. This is motivated by the observation that the human COM moves like an inverted pendulum each step, accompanied by an exchange of kinetic with gravitational potential energy (58). Although a pendulum conserves energy, the step-to-step transition requires mechanical work. It predicts how humans perform increasing positive mechanical work, and with no loss of overall speed compared to steady-level waking. The sequence consists of $N$ POs $w_i$ where the control is exerted for each step $i$. The problem starts and ends with nominal walking at speed $V$ and takes the same amount of overall time as $N$ steps of nominal walking with nominal step time $T$, despite a middle interval of uneven steps. Also serving as a constraint are the model dynamics, including pendulum-like walking punctuated by the step-to-step transition. The optimization may be formulated as:

\[
\text{Minimize total work:} \sum_{i=1}^{N} w_i
\]
\[\text{Subject to:}\]
\[v_0 = V, v_N = V\]
\[\text{and nominal total time: } \sum_{i=0}^{N-1} \tau_i = T \cdot N\]
\[f(v_{i+1}, v_i, \tau_i, \delta_i, u_i) = 0\]

The walking dynamics describe how the forward speed of the next step $v_{i+1}$ and the intervening step duration $\tau_i$ (SI Appendix) are related to the current step’s speed and PO ($v_i$ and $u_i$). The steps are indexed such that the zero index $i = 0$ is the first uneven step, $i_0$ is the level, nominal step at the beginning, and $i_N$ the nominal step at the end. Any number of uneven steps occur in the middle, padded on both sides by several level (but not necessarily nominal) steps to allow the model to anticipate and recover from the disturbance. We chose a padding of six steps, for example, the longest uneven interval was sixteen steps long (Complex 1 and 2), padded on each side to yield $N = 28$. The six-step advance padding is sufficient to gain most of the economic advantage of planning ahead, which keeps increasing but negligibly so for more steps (16). Paddling after the last uneven step serves a different purpose, which is to define an objective terminal goal of regaining nominal walking and timing on level terrain.

The optimization was performed over horizons of various lengths. A full horizon refers to optimizing all $N$ steps until arriving back to nominal walking, yielding a full control trajectory. A finite horizon refers to repeatedly optimizing with knowledge of only $m$ upcoming steps, and nothing beyond that horizon. The objective is to regain nominal timing by the $m$th step (or sooner if the $N$th step overall occurs first), to keep track of nominal timing, the optimization is informed of the cumulative time gain or loss thus far relative to nominal. The finite horizon yields $m$ commands (which are suboptimal with respect of full horizon) and executes only the first one. The optimization is then repeated anew each step, starting from the end of the previous step and still intending to meet the original nominal timing, but over a new $m$-step horizon. This receding horizon is generally suboptimal but can approach the full horizon’s optimality with sufficient $m$.

The primary prediction of interest was speed fluctuation patterns. This refers to the waveform shape for $v_i$, as opposed to absolute waveform amplitude. The point-mass model is not expected to accurately predict speed amplitudes, which scale slightly with leg length (or relative terrain amplitude) and self-selected walking speed (16). However, the fluctuation patterns are remain quite similar regardless of parameter values such as average speed, body mass, and leg length (11, 16). The optimization hypothesis is therefore to be tested through scale-invariant correlation of speed fluctuations (Fig. 3).

Alternative hypotheses: Reactive compensation and tight regulation. We considered two alternative ways to compensate for uneven terrain while conserving time. One, termed reactive compensation, refers to an optimal control that does not act until the terrain has been encountered. It then reactively optimizes a control sequence that will regain time and conserve energy. Even though it is optimal, it is not anticipatory of the first uneven step and is thus suboptimal compared to a control that acts ahead of time. The second alternative, tight regulation, refers to a feedback controller that maintains constant step time for each step, despite disturbances. It adjusts each PO to ensure constant time, and thus need not plan ahead. This type of control is simple to perform but is suboptimal because it does not take advantage of past or future information. A prediction from reactive compensation is that there should be no anticipatory speed fluctuations before the uneven terrain. A prediction from
tight regulation is that there should be very small speed fluctuations across uneven terrain.

**Experiment.** We measured speed fluctuations as healthy adult subjects walked on each of seven different terrain profiles plus level control, for a total of eight conditions. The profiles (Fig. 3) consisted of an integer number (one to sixteen) of evenly spaced steps, each deviating in height by a small integer multiple (between −3 and 3) of 2.54 cm. The eight profiles were labeled with the following names: Control, Up-step (U), Down-step (D), Up-Down (UD), Down-and-Up-Down (D&DUD), Pyramid (P), Complex 1 (C1), and Complex 2 (C2). Each was assembled from layers of polystyrene insulation foam, providing a flat surface for each footfall. Each profile occurred about halfway down a level walkway, with subjects initially walking with a steady, level gait, and also ending with a similar steady, level gait after the terrain. There were two groups of subjects walking in three sets of trials. The first group \( N=12; \) seven males, five females, all under 30 y age) performed trials of Control, U, D, UD, D&DUD. The second group \( N=11; \) seven males, four females, all under 30 y age) performed trials of Control, P in one set, and Control, C1, C2 in another set. A separate Control was collected for each set of terrains (called n1, n2, n3), and consisted of level ground in the same walkway, but on the side of the uneven steps. The entire walkway was about 30 to 40 m long, but data were only analyzed for a middle portion encompassing the terrain plus six steps (about 3.5 m) before and after. The six-step padding was to allow walking speed to deviate from nominal both before and after the uneven terrain, as has been observed in both model and human (11, 16). The full study protocol was approved by the institutional review boards (IRB HUM00020554 at University of Michigan and REB21-1497 at University of Calgary) and prior to the experiment, subjects gave their written informed consent according to these institutional review board procedures.

In all conditions, subjects walked at self-selected, comfortable speed. The main instruction was to walk from a start line and past a finish line “in about the same time” throughout the experiment, to avoid large variations in overall speed across conditions. This instruction was intended to provide only broad context, because the hypotheses were not dependent on any particular speed, and so subjects received no feedback about their timing. There was a brief pause between trials, in which subjects turned around and stood briefly before starting the next trial, in opposite direction. Except for opposite pairing, the conditions were conducted in random order, with at least four to seven trials per condition in each experimental set (all chosen randomly), also interleaved with occasional Control trials inserted at random. Prior to data collection, subjects walked several times on the terrains to gain familiarity. For the first set of terrains (U, D, UD, D&DUD, ordered randomly), there was a visual cue on the floor, a paper sticker placed approximately 5 m from the first uneven step. This was intended to help subjects line up their steps, but we observed that subjects paid little attention to the cue, and it was therefore eliminated for the three remaining terrains (P, C1, and C2).

Walking speed and timing was measured inertially, using inertial measurement units fixed atop the instep of each foot. We used a drift correction algorithm described previously (61), to yield forward body speed each stride, defined as the stride length divided by stride duration for each foot. A trajectory of discrete walking speeds was thus measured for each trial, and subject’s trials within a condition were averaged at the discrete step numbers (see SI Appendix for details). For comparison to human (Fig. 6), model predictions were converted from mid-stance speeds (Fig. 3) to body speeds.

We compared each subject’s average speed trajectory against all subjects and against model, for each terrain condition. Intersubject consistency was assessed by comparing each subject’s trajectory against the average across subjects with Pearson’s correlation coefficient \( r \), yielding one correlation per subject and terrain. Humans were also compared against model trajectory using correlation, between all subject’s trajectories for a terrain (13 to 28 steps per subject) and model’s, yielding one correlation coefficient per terrain. Correlations are independent of scale and thus test the model’s predicted fluctuation patterns (16) without regard to individual amplitudes, absolute walking speed, body mass, or body size, and without any fitting of model parameters to data. A truly predictive model should be positively correlated with human responses, whereas a random model (null hypothesis) would be expected to have zero correlation. Statistical significance of the human vs. model correlation coefficients was tested with a threshold of \( P < 0.05 \). We also performed a few additional tests of hypothesized planning criteria based on speed data. For the No compensation hypothesis, we expected that average speeds would be lower (or walking durations higher) on uneven than level terrain. For the tight speed regulation hypothesis, we expected that speeds would fluctuate very little, similar to level. And for the minimum energy hypothesis, we expected that speeds would fluctuate significantly, including before and after the uneven terrain (termied anticipatory and recovery compensations, defined as two steps preceding or following terrain).

The effect of horizon length was also assessed by computing optimal predictions for \( m \) ranging up to full horizon, again correlating against empirical human responses. If humans plan with a finite horizon, they may exhibit peak correlation with a finite model, and if fully optimally with the full horizon model. We used this same approach to also test for sensitivity to shorter (three-step) padding before and after terrain. We found shorter padding to yield negligible differences, with the same correlation patterns and no more than 0.05 difference in correlation coefficients compared to nominal six-step padding.

**Data, Materials, and Software Availability.** Data, code data have been deposited in Github (https://github.com/kuo-lab/uneventerrainexperiment, https://github.com/kuo-lab/simplelocomotionmodel) (55, 56).

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