Unified description of hadron yield ratios from dynamical core–corona initialization

Yuuka Kanakubo,¹*, Yasuki Tachibana,²,1‡ and Tetsufumi Hirano1†

¹Department of Physics, Sophia University, Tokyo 102-8554, Japan
²Department of Physics and Astronomy, Wayne State University, Detroit, MI 48201, USA

(Dated: March 2, 2020)

We develop the dynamical core–corona initialization framework as a phenomenological description of the formation of quark gluon plasma (QGP) fluids in high-energy nuclear collisions. Using this framework, we investigate the fraction of the fluidized energy to the total energy and strange hadron yield ratios as functions of multiplicity, and we scrutinize the multiplicity scaling of hadron yield ratios recently reported by the ALICE Collaboration. Our results strongly indicate that the QGP fluids are partly formed even at the averaged multiplicity for nonsingle diffractive p+p events.

PACS numbers: 25.75.-q, 12.38.Mh, 25.75.Ld, 24.10.Nz

I. INTRODUCTION

Properties of the quark gluon plasma (QGP) have been investigated through high-energy nuclear collision experiments at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory and at the Large Hadron Collider (LHC) at CERN. The experimental results have brought us a great deal of information about properties of the QGP. The collective behavior of the QGP is well described by relativistic hydrodynamics [1–5], in which the system is postulated to keep local thermal equilibrium during expansion. Since dissipative effects on expansion are quite small regardless of remarkably large expansion rate, the QGP exhibits near-perfect fluidity, which has led to the concept of strongly coupled/interacting QGP (sQGP) [2–4].

Comparisons of the experimental results from heavy-ion collisions with those from small colliding systems, such as proton-proton and proton-nucleus collisions, have been made to see how small the QGP droplets can be. Small colliding systems had been believed to provide us with reference data for a long time since the size of the system is postulated to keep local thermal equilibrium during expansion. Since dissipative effects on expansion are quite small regardless of remarkably large expansion rate, the QGP exhibits near-perfect fluidity, which has led to the concept of strongly coupled/interacting QGP (sQGP) [2–4].

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Besides the azimuthal anisotropy, the strangeness enhancement in small colliding systems was proposed a long time ago to be a signal of the QGP formation in high-energy nuclear collisions [38, 39]. Initial colliding nuclei do not contain the strange quark as the valence one, yields of strange hadrons would be sensitive to details of reaction dynamics. If the production rate of strange quarks becomes sufficiently high in thermalized systems, chemical equilibrium of strange quarks besides up and down quarks is expected to be reached within the time scale of nuclear collisions. In fact, most of the yield ratios of hadrons including multi-strange baryons have been well reproduced by statistical models [40], which is recognized to be one of the strong signals of the QGP formation in heavy-ion collisions at RHIC and LHC energies. Therefore the strangeness enhancement in high-multiplicity proton-proton and proton-nucleus collisions reported by the ALICE Collaboration [37] strongly indicates the QGP formation even in such a small colliding system.

This continuous increase of the strange hadron yield ratio can be interpreted as follows. Final hadrons originate from two different sources having different hadron production processes. The values of yield ratios in low-multiplicity events are described by results from string fragmentation, in which the string tension (κ ≈ 1 GeV/fm) controls the yield ratios of hadrons. On the other hand, the values of yield ratios in high-multiplicity events are explained by the statistical hadronization approach where the matter is assumed to be in chemical equilibrium. In this case, chemical freezeout temperature (Tch ≈ 160 MeV) plays an essential role in describing yield ratios of hadrons. Thus, the dominant process of final hadron production is supposed to change grad-
ually from string fragmentation at the low-multiplicity limit to statistical hadronization at the high-multiplicity limit. At intermediate multiplicity, final hadron production would be a superposition of these two contributions.

The "core–corona" picture has been adopted to demonstrate the above idea [41–46]. At the first contact of two nuclei, the number of collisions per nucleon should be different depending on where they are located in the transverse plane being perpendicular to the collision axis. High-density regions just after the initial collision in which nucleons in colliding nuclei suffer a lot of collisions are called "core" and assumed to be sources of the QGP medium in local thermal and chemical equilibrium. Low-density regions in which nucleons suffer only a few collisions are referred to as "corona" and supposed to be likely to create hadrons via string fragmentation without formation of bulk medium. Since the "core" regions become dominant production mechanism with increasing multiplicity, the continuous change from string fragmentation to statistical hadronization with increasing multiplicity could naturally be explained by the "core–corona" picture.

In this paper, we introduce the core–corona picture into the dynamical initialization framework to obtain the unified description of hadron yield ratios from small colliding systems to heavy-ion collisions. The dynamical initialization framework [47–50] describes the dynamics of gradual forming of the QGP fluids phenomenologically. In this framework, the initial condition of the QGP fluids is obtained via energy-momentum deposition from the partons, strings, or fields generated just after the collision. As an extension of the dynamical initialization description for generation of QGP fluids from initially produced partons [47–50], we introduced the dependence on initial parton densities in the dynamical initialization [50], which we call the "dynamical core–corona initialization (DCCI) model" in this paper. Given the various definitions of the core and the corona in the literature [41–46], we here define them explicitly as follows: The core represents the fluids under local thermal and chemical equilibrium, while the corona represents the system composed of nonequilibrated partons traversing the fluids or the vacuum. In the DCCI model, the continuous separation between the core and the corona is attributed to the spatial density of initially produced partons. Thus, it should be emphasized that we do not introduce the threshold to separate the core from the corona explicitly, unlike the model in Refs. [45, 46]. In the DCCI model, the nonequilibrated partons can coexist with the fluids at some space-time point.

This paper is organized as follows: In Sec. II, we give an overview of the dynamical initialization framework and its extension to the DCCI model. In Sec. III, we first investigate how much the energy is fluidized at midrapidity when we reproduce the multiplicity dependence of particle ratio from experimental data using the DCCI model. Then we extract the fraction of fluidized energy as a function of multiplicity from experimental data in both small and large colliding systems at LHC energies. After that we discuss a universal behavior of the hadron yield ratios as functions of multiplicity. Finally Sec. IV is devoted to summary of this paper.

Throughout this paper, we use the natural units, $\hbar = c = k_B = 1$, and the Minkowski metric, $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$.

II. MODEL

Before going into detailed explanations, we briefly give an outline of the DCCI model employed in this paper. Initial partons are obtained with the Monte Carlo event generator Pythia [51] and start traveling in the vacuum at their formation time. QGP fluids are dynamically generated via energy-momentum deposition from these partons by solving the relativistic hydrodynamic equation with source terms. Here the four-momentum deposition per unit time is parametrized to be proportional to the initially produced parton density so that we take into account the core–corona picture: Partons in regions with larger density of surrounding partons are more likely to deposit their four-momentum and generate QGP fluids, while those in dilute regions tend to traverse, keeping their four-momentum that they have just after the collision of nuclei. In this way, separation of initially produced partons into the QGP fluids (the core) and the non-equilibrated partons (the corona) is settled as a consequence of dynamical four-momentum deposition of each parton. After the dynamical initialization with the core–corona picture, we continue hydrodynamic evolution for fluids until their temperature drops to the decoupling temperature. Fluids are particlized into hadrons at the decoupling hypersurface via the Cooper–Frye formula [52]. On the other hand, surviving partons are hadronized under string fragmentation in Pythia. Therefore, the final hadron yield in this model is a summation of the contribution from particlization of the medium under local thermal and chemical equilibrium and that from string fragmentation.

A. Hydrodynamic equations with source terms

In conventional hydrodynamic simulations, the equation of continuity for energy and momentum,

$$\partial_\mu T^{\mu\nu}(x) = 0,$$

is solved, where $T^{\mu\nu}$ is the energy-momentum tensor of the fluid assuming that energy-momentum conservation is satisfied within the fluid. However, during the
formation of the locally equilibrated medium in high-energy nuclear collisions, there exist incoming energy and momentum for the formation of the medium from non-equilibrated subsystems. In our DCCI model, the initially produced partons deposit their energy and momentum for the formation of the medium fluid and the energy-momentum conservation is imposed for the total system which is a composite one of the medium fluids and the traversing partons,

\[ \partial_\mu T^\mu_\text{tot}(x) = \partial_\mu (T^\mu_\text{fluid}(x) + T^\mu_\text{parton}(x)) = 0. \] (2)

Here we assume that energy and momentum deposited by partons are equilibrated instantaneously. Equation (2) can be written in the form of the relativistic hydrodynamic equation with source term,

\[ \partial_\mu T^\mu_\text{fluid}(x) = J^\nu(x), \] (3)

if the source term is defined as

\[ J^\nu = -\partial_\mu T^\mu_\text{parton}. \] (4)

The hydrodynamic equation with source term (3) is also commonly used in the simulations to discuss the physics of jet quenching and its effects on the medium responses [53–58]. If we assume ideal fluids, the energy-momentum tensor of QGP fluids is decomposed as

\[ T^\mu_\text{fluid} = (e + P)u^\mu u^\nu - Pg^{\mu\nu}. \] (5)

Here, \( e, P \), and \( u^\mu \) are energy density, hydrostatic pressure, and velocity of the fluid, respectively. In order to close the equations of motion, we need the equation of state (EoS) which has a form of hydrostatic pressure as a function of energy density, \( P = P(e) \). In this study, we adopt the EoS from (2+1)-flavor lattice QCD calculations [59]. In other words, \( u, d, s \) quarks (and their anti-quarks) and gluons are under thermal and chemical equilibrium in the QGP fluids.

In this study, we do not solve the equation for the conservation of charges, such as baryon number, strangeness and electric charges, since matter generated at RHIC and LHC energies is supposed to be almost baryon free, strangeness neutral, and charge neutral around midrapidity.

We also note that, although Eqs. (3) and (5) are denoted in Cartesian coordinates to avoid complex notations, the actual calculation is performed in Milne coordinates. In Milne coordinates, the time axis is represented with proper time \( \tau = \sqrt{t^2 - z^2} \). The other spatial coordinates are \( x \) and \( y \), which are the transverse coordinates perpendicular to the collision axis, and \( \eta_s = (1/2) \ln[(t + z)/(t - z)] \), which is space-time rapidity.

B. Dynamical initialization of QGP fluids

In this study, the phase space distribution of initially produced partons is assumed to be

\[ f_\text{parton}(x, p; t)d^3xd^3p \]
\[ = \sum_i G(x - x_i(t))\delta^{(3)}(p - p_i(t))d^3xd^3p, \] (6)

\[ G(x - x_i(t))d^3x = \frac{1}{\sqrt{(2\pi\sigma^2)^3}} e^{-\frac{(x - x_i(t))^2}{2\sigma^2}} d^3x. \] (7)

Here \( \sigma \) is a width of Gaussian function in coordinate space. The Gaussian is introduced to give the scale of the region which is supposed to be involved in the interaction by a parton in the model. The trajectory of a parton is assumed to be eikonal, and rapidity of a parton is constant during the dynamical initialization, i.e., \( \eta_i = \eta_{s,i} = \text{constant} \) where index \( i \) represents the \( i \)th parton. Under this assumption, the position of the \( i \)th parton is defined as

\[ x_i(t) = \frac{p_i}{p^0_i}(t - t_{\text{form},i}) + x_{\text{ini},i}, \] (8)

where \( (p^0_i, p_i) \) is the four-momentum of the \( i \)th parton. Here, \( x_{\text{ini},i} \) is the position at the formation time of the \( i \)th parton, \( t_{\text{form},i} \). Under these assumptions, we derive the explicit form of Eq. (4). By putting the phase space distribution in Eq. (6) into the kinetic definition of the energy-momentum tensor, the source term (4) becomes

\[ J^\mu(x) = -\partial_\mu T^\mu_\text{parton} \]
\[ = -\sum_i \int_0^{t_{\text{form},i}} \frac{d^3p}{d^3p} \partial_\mu f_\text{parton}(x, p; t) \]
\[ = -\sum_i \frac{dp^0_i}{dt} G(x - x_i(t)). \] (9)

For details of this derivation, see Appendix A.

We call \( dp^0_i/dt \) in Eq. (9) the “fluidization rate”, which is the rate of four-momentum deposition for the \( i \)th parton. Since the index \( i \) represents each parton produced in each event, the summation is taken for all the initial partons in an event.

We assume that QGP fluids are generated by the initial partons during the proper time period from the formation time \( \tau = \tau_0 \) (0.1 fm) to initial time of fluids \( \tau = \tau_0 \) (0.6 fm). In this paper, the common constant value of \( \tau_0 \) is assumed for all partons, \( \tau_{\text{form},i} = \sqrt{t_{\text{form},i}^2 - \tau_0^2} \). We start with the vanishing energy-momentum tensor of QGP fluids, \( T^\mu_\text{fluid}(\tau = \tau_0) = 0 \), and solve Eq. (3) from \( \tau = \tau_0 \) to \( \tau = \tau_0 \). Here we note that energy-momentum conservation is satisfied among the QGP fluids and the traversing partons during the dynamical initialization.

As we mentioned in the previous subsection, our actual simulations are performed in (3+1)-dimensional Milne
coordinates. Thus the explicit form of the Gaussian distribution function in Eq. (9) is replaced as
\[ G(\mathbf{x} - \mathbf{x}_i(t)) d^3x \rightarrow \frac{1}{2\pi\sigma_{\perp}^2} \exp \left[ -\frac{(\mathbf{x}_{\perp} - \mathbf{x}_{\perp,i}(\tau))^2}{2\sigma_{\perp}^2} \right] \times \frac{1}{\sqrt{2\pi^2\sigma_{\eta}^2}} \exp \left[ -\frac{(\eta_{s,i}(\tau))^2}{2\sigma_{\eta}^2} \right] \tau d\eta_{s} d^2x_{\perp}, \tag{10} \]

where \( \mathbf{x}_{\perp,i}(\tau) \) and \( \eta_{s,i}(\tau) \) are the transverse coordinates and the space-time rapidity of the \( i \)-th parton. In this study, we adopt \( \sigma_{\perp} = 0.5 \text{ fm} \) and \( \sigma_{\eta} = 0.5 \) for transverse and longitudinal widths of the Gaussian function, respectively.

C. Energy-momentum deposition rate with the core–corona picture

We introduce the core–corona picture as an extension of the dynamical initialization framework [50]. To introduce the core–corona picture, we parametrize the fluidization rate as
\[ \frac{dp^\mu_{T,i}}{dt}(t) = -a_0 \rho_i(\mathbf{x}_i(t)) \frac{p^\mu_{T,i}}{p^\mu_{T,i}} - p^\mu_{T,i}(t), \tag{11} \]

where \( p^\mu_{T,i} \) and \( p^\mu_{T,i} \) are the transverse momentum and the four-momentum of the \( i \)-th parton, respectively. Here, \( \rho_i \) is the spatial density of partons surrounding the \( i \)-th parton. It is defined as
\[ \rho_i(\mathbf{x}_i(t)) d^3x = \rho_i(\mathbf{x} = \mathbf{x}_i(t)) d^3x \]
\[ = \sum_{j \neq i} G(\mathbf{x} - \mathbf{x}_j(t)) d^3x \bigg|_{\mathbf{x} = \mathbf{x}_i(t)}. \tag{12} \]

The dimensionless factor \( a_0 \) is a free parameter to control the intensity of energy-momentum deposition. The Gaussian function \( G \) has the same form as the one introduced in Eq. (6) with the form of Eq. (10) in Milne coordinates.

The parton density \( \rho_i \) introduced in Eq. (11) is the key factor to capture the feature of the core–corona picture. In high parton density regions, a large fraction of the energy and momentum of the parton is deposited to create the QGP fluids through the source term in Eq. (3). On the other hand, fluids are not likely to be generated in low parton density regions. It should be emphasized here that the criterion for the formation of the medium fluid is not governed by a certain threshold for the parton density but by the configuration of initial partons and their dynamics. In this framework, medium fluids and nonequilibrated partons can even coexist at the same time.

The factor \( p_{T,i}^{-2} \) in the fluidization rate accounts for the tendency that lower \( p_T \) partons are more likely to deposit their energy and momentum to form the fluids. We should note that the power of \(-2\) comes from consideration of dimension in Eq. (11). Although one may use more complicated forms of the fluidization rate to capture the equilibration processes, we leave this consideration for our future work.

As we discuss in the next subsection, the event generator PYTHIA, which we employ for generation of initial partons, provides us with the color flows of partons to form strings. During the dynamical core–corona initialization, we trace color flows of the partons in every time step and calculate the invariant mass of each string. As the partons lose their energy and momentum, the invariant mass of strings becomes smaller and, eventually, some strings cannot undergo string fragmentation due to the lack of their invariant masses. To avoid such a situation, we assign the threshold of invariant mass for each string. If the invariant mass becomes below the threshold to undergo the string fragmentation in PYTHIA, we put all the energy and momentum of partons in that string into fluids. In this paper, we use the threshold of invariant mass as \( m_{\text{th}} = m_1 + m_2 + 1.0 \) in units of GeV for strings which have a quark/antidiquark and antiquark/diquark at the each end point. Here \( m_1 \) and \( m_2 \) are the constituent masses of the leading quark/antidiquark and antiquark/diquark.

D. Generation of initial partons

In our DCCI framework, the event generator PYTHIA 8.230 [51] is used to simulate the initial parton production. In PYTHIA, we switch on the option to obtain partonic vertices (PartonVertex:setVertex=on) and switch off the hadronization (HadronLevel:all=off) to obtain the phase space distribution of partons at \( \tau = \tau_0 \). PYTHIA provides us with the vertices for parton production through multi parton interactions, final state radiations, and initial state radiations [60] which can be used in Eq. (8). Since the Milne coordinates are employed in the actual simulations, we take the transverse coordinates of the \( i \)-th parton, \( \mathbf{x}_{\perp,i} \), directly from PYTHIA and assume \( \eta_{\text{ini},i} = \eta_{\text{ini},i} \), where \( \eta_{\text{ini},i} \) is rapidity of a parton generated with PYTHIA. Here, it should be noted that this version of PYTHIA handles heavy-ion reactions at high-energies using the Angantyr model [61, 62].

E. Particulation of QGP fluids

The dynamics of the medium after \( \tau = \tau_0 \) is treated in the same way as that in conventional hydrodynamic simulations. In this paper, we do not consider the energy and momentum loss of traversing partons due to the parton-medium interaction after \( \tau = \tau_0 \) for simplicity, and we solve Eq. (3) without the source term until the maximum temperature of fluids becomes lower than the fixed decoupling temperature \( T_{\text{dec}} = 160 \text{ MeV} \).
effects of medium response on anisotropic flow in the dynamical initialization can be found in Ref. [47].

We use the Cooper–Frye formula [52] to obtain spectra of hadrons emitted directly from the decoupling hypersurface. The yields from the medium fluids are obtained by integrating the spectra of each hadron species \(i\)

\[
N_i = \frac{g_i}{(2\pi)^3} \int \frac{d^3 p}{p^0} \int_\Sigma \exp \left[ \frac{p^\mu u_\mu(x)}{T_{\text{dec}}} \mp_{\text{BF}} \right] \frac{d^4 \sigma_{\mu}(x)}{d^3 p},
\]

(13)

where \(g_i\) is the degeneracy, \(\mp_{\text{BF}}\) corresponds to Bose or Fermi distributions, \(\Sigma\) is the decoupling hypersurface at \(T = T_{\text{dec}}\), and \(d\sigma_{\mu}\) is the normal vector of its hypersurface. Since we assume baryon-free matter in this paper, the chemical potential of baryon number does not appear in Eq. (13). Here we should note that some fluid elements are already lower than the decoupling temperature at \(\tau = \tau_0\). For such fluid elements, we assume that the partonization is performed at the initial time of fluid \(\tau_0\) with the corresponding temperature.

For the consideration of feed down from resonance decays, we correct the direct yields based on statistical model calculations [40]. We estimate the ratio of the total yields to the contribution from directly produced hadrons \(c_i\) from Fig. 2 of Ref. [40], and multiply the ratio \(c_i\) with the direct yield obtained from Eq. (13). Here we use the ratio factors, \(c_\pi = 3.2, c_\rho = 3.0, c_\Lambda = 4.7, c_\Xi = 1.7,\) and \(c_\phi = 1.0\), to obtain the total yields of these hadrons.

F. String fragmentation of traversing partons

We assume that the partons surviving after the dynamical core–corona initialization form color singlet strings and are hadronized under the string fragmentation. We push back the surviving partons into Pythia with their energy and momentum at \(\tau = \tau_0\) (note that no parton energy loss happens after \(\tau_0\) in this paper) and perform hadronization with the option \texttt{forceHadronLevel()} to get the final hadronic spectra. Here we correct the energy of parton to be mass-on-shell using the momentum at \(\tau = \tau_0\) and rest mass for the execution of the string fragmentation, since most partons are mass-off-shell due to the four-momentum deposition of Eq. (11). We checked that the violation of energy conservation due to this correction is small enough to be ignored. It should also be noted that we switch off weak decays of strange baryons which are stable against strong decays (except \(\Sigma^0 \rightarrow \Lambda + \gamma\)) in Pythia.

III. RESULTS

The parton density distribution \(\rho_i\) in the fluidization rate per parton (11) governs the separation of the core and the corona. In the dynamical core–corona initialization, partons traversing the high-density region tend to deposit their four-momentum and to generate the fluids (the core). In contrast, partons traversing the low-density region tend to stay as surviving partons (the corona). As a result of the dynamical core–corona initialization, the final hadron yields contain the contributions from those two components. The fraction of each contribution is sensitive to distribution of the parton density from event to event.

In the following, the multiplicity \(\langle dN_{ch}/dy \rangle\) describes the number of charged particles per unit pseudorapidity in \(|y| < 0.5\) calculated with default settings in Pythia. To obtain the hadron yield ratios, we first calculate final hadron yields from the core and the corona separately and the final hadron yield in each multiplicity class is the sum of them,

\[
\left\langle \frac{dN_i}{dy} \right\rangle = \left\langle \frac{dN_i}{dy} \right\rangle_{\text{core}} + \left\langle \frac{dN_i}{dy} \right\rangle_{\text{corona}},
\]

(14)

The angle bracket means the event average in a multiplicity class. Although the final yield should be obtained as a sum of two contributions from the core and the corona on an event-by-event basis, the number of events for hydrodynamic simulations is smaller than that for parton generation and hadronization in Pythia in order to reduce the computational cost. In the results of the hadron yield ratios, error bars represent statistical ones of yields from string fragmentation in Pythia, while the shaded band represents statistical errors from the number of events in hydrodynamic simulations. Since central collisions are more weighted and the corresponding weight factor is provided for each event in the heavy-ion mode in Pythia, we consider it in our statistical analysis. We also note that the hadron yields from the core are calculated via Eq. (13) at \(y = 0\) assuming approximate boost invariance. On the other hand, the hadron yields from the corona are obtained by counting the hadrons in \(|y| < 2.0\) to gain sufficient statistics.

A. Parameter \(a_0\) dependence on particle ratio

The free parameter \(a_0\) in Eq. (11) controls intensity of the fluidization of partons besides the \(\rho_i\). First we check the \(a_0\) dependence on the particle ratio. We perform simulations of Pb+Pb collisions at \(\sqrt{s_{NN}} = 2.76\) TeV with \(a_0 = 10, 20,\) and 100 in the DCCI model.

Figure 1 shows the hadron yield ratio of cascades (\(\Xi^- + \Xi^+\)) to charged pions (\(\pi^+ + \pi^-\)) as a function of multiplicity, i.e., the event-averaged charged hadron pseudorapidity density \(\langle dN_{ch}/dy \rangle\) at midrapidity \(|y| < 0.5\). The hadron yield ratios in Pb+Pb collisions at

2 It should be noted that it is better to see the increasing behavior of the yield ratio of omega baryons since it shows more rapid enhancement. However, we do not calculate them since it is statistically difficult to obtain enough yield of omega baryons within our framework.
The parameter $a_0$ controls the amount of four-momentum deposition from initial partons from Eq. (11). The results shown in Fig. 1 demonstrate this expectation.

We also note that the hadron yield ratios from both the fluids and the string fragmentation do not seem to depend much on the multiplicity since the hadron yield ratios from the core are determined from the decoupling temperature $T_{\text{dec}}$ while the ones from the corona are determined from the string tension $\kappa$.

For a more quantitative view of how the parameter $a_0$ affects the hadron yield ratio as a function of multiplicity, we fit the results shown in Fig. 1 using the function

$$f(x) = (F - S)\frac{x^n}{x^n + k^n} + S,$$

where the valuable $x$ denotes multiplicity. Here we use $F = 0.0055$ and $S = 0.0018$ for the hadron yield ratio of cascades to charged pions purely from fluids and that from string fragmentation, respectively, assuming that the ratio from those components is constant as a function of multiplicity.\(^3\) The function in Eq. (15) captures the behavior of the multiplicity dependence of hadron yield ratios,

$$\lim_{x \to \infty} f(x) = F, \quad \lim_{x \to 0^+} f(x) = S.$$

This fitting function has two fitting parameters, $k$ and $n$. Using this function, we quantify the multiplicity, $x_{\text{sat}}$, at which the hadron yield ratio reaches 90% of the difference between the values obtained from the fluids and the one from string fragmentation, namely, $f(x_{\text{sat}}) = S + 0.9(F - S)$. Once the results are fitted, $x_{\text{sat}} = k \sqrt{9.0}$, which is referred as saturation multiplicity, is obtained. For details of the fittings, see Appendix B.

Figure 2 shows the $a_0$ dependence of saturation multiplicity. As expected from the results shown in Fig. 1, the saturation multiplicity exhibits a clear monotonic decrease as a function of $a_0$. We also simulate $p+p$ collisions at $\sqrt{s} = 7$ TeV and $p+Pb$ collisions at $\sqrt{s_{NN}} = 5.02$ TeV in addition to Pb+Pb collisions with $a_0 = 100$ in the DCCI model and perform the function fittings of these results simultaneously. Even if we include results from $p+p$ and $p+Pb$ collisions in the fitting, the resultant saturation multiplicity is almost identical with the one only from Pb+Pb collisions with the same $a_0$ value. We also apply the global fitting for the ALICE experimental data in $p+p$, $p+Pb$, and $Pb+Pb$ collisions [37, 64, 65] and estimate its saturation multiplicity as $\langle dN_{\text{ch}}/d\eta \rangle \approx 31$, shown as a solid line.

The value of the parameter $a_0$ which is most likely to reproduce the experimental data can be extracted by fitting the results obtained in the DCCI model in Fig. 2. Regarding the $a_0$ dependence of saturation multiplicity

\(^3\) This function is motivated by the Hill equation which characterizes ligand bindings in biochemical reactions [63].
as a power function, we obtain the optimized function

\[ f(a_0) = a_0^{-1.8} \times 10^6, \]

with reduced chi-square \( \chi^2 \approx 1.2. \) This optimized function is shown as a dashed line in Fig. 2. The value of \( a_0 \) at the intersection point of the solid and dashed lines in Fig. 2 enables us to reproduce the experimental data most reasonably within the DCCI model. We estimate this value to be \( a_0 \approx 368. \) Here, to obtain the optimized value for \( a_0 \), we use the central value of the plots from the fitting for the experimental data in Fig. 2.

### B. Fraction of the fluidized energy

We simulate \( p+p \), \( p+Pb \), and \( Pb+Pb \) collisions with \( a_0 = 368 \) in the DCCI model to demonstrate how much energy and momentum just after the collisions are turned into the fluids. We define the fraction of fluidized energy as

\[
R = \frac{dE_{\text{fluid}}/d\eta_s}{dE_{\text{tot}}/d\eta_s} \bigg|_{\eta_s=0},
\]

which is the ratio of the energy density turned into the fluids to the total energy density at midrapidity (\( \eta_s = 0 \)). The fluidized energy can be obtained by integrating the time component of source terms from \( \tau = \tau_{00} \) (formation time) to \( \tau = \tau_0 \) (hydrodynamic initial time) in the transverse plane as

\[
\frac{dE_{\text{fluid}}}{d\eta_s} \bigg|_{\eta_s=0} = \int_{\tau_{00}}^{\tau_0} d\tau \int d^2x_\perp J (\tau, x_\perp, \eta_s = 0). \]

The total energy density \( dE_{\text{tot}}/d\eta_s \) is calculated by taking the sum of all the initial partons’ energy at \( \tau = \tau_{00} \).

In Fig. 3, the fraction of fluidized energy \( R \) in \( p+p \), \( p+Pb \), and \( Pb+Pb \) collisions is shown as a function of multiplicity. The results exhibit the multiplicity scaling regardless of system sizes or collision energies. \( R \) increases monotonically and saturates around \( \langle dN_{\text{ch}}/d\eta \rangle_{|\eta|<0.5} \approx 20 \). Note that hydrodynamic simulations are performed in the center-of-mass frame while multiplicities are calculated in the laboratory frame. Therefore there is a rapidity shift \( \Delta \eta_s = 0.47 \) in \( p+Pb \) collisions at \( \sqrt{s_{NN}} = 5.02 \) TeV between those frames. The most remarkable thing is that the fraction of fluidized energy \( R \) is finite even at the averaged multiplicity for nonsingle diffractive \( p+p \) events, \( \langle dN_{\text{ch}}/d\eta \rangle = 5.74 \pm 0.15 \) [66], which is generally considered to be too small to generate the QGP. This means that, within our model calculations, the contribution from the fluids to final hadron yields is crucial to reproduce the yield ratio of cascades to pions in those multiplicity events. Therefore this would be a strong indication of the partial QGP generation even in nonsingle diffractive \( p+p \) collisions at

FIG. 2. (Color Online) Parameter \( a_0 \) dependence of saturation multiplicity in \( \text{Pb+Pb} \) collisions (circles) is compared with the result from simultaneous fitting for \( p+p \), \( p+Pb \), and \( \text{Pb+Pb} \) collisions (cross) and that from fitting for the experimental data (horizontal line). The dashed line shows the fitting function for the saturation multiplicities (circles) fitted by using a power function.

FIG. 3. (Color online) Fraction of the fluidized energy to the total energy at \( \eta_s = 0 \) as a function of multiplicity at midrapidity, \( \langle dN_{\text{ch}}/d\eta \rangle \) (\( |\eta| < 0.5 \)), in \( p+p \) (diamonds) collisions at \( \sqrt{s} = 7 \) TeV, \( p+Pb \) (triangles) collisions at \( \sqrt{s_{NN}} = 5.02 \) TeV, and \( \text{Pb+Pb} \) (squares) collisions at \( \sqrt{s_{NN}} = 2.76 \) TeV.
\( \sqrt{s} = 7 \text{ TeV} \).

We should note that the fraction of fluidized energy \( R \) could be overestimated quantitatively in the DCCI model. Suppose some hadronic strings are stretched between two surviving partons (possibly between a quark/antiquark and a diquark/antidiquark) each of which is in the projectile and in the target regions. Although these strings do not contain partons near midrapidity, they can decay into hadrons in the midrapidity region. We find \( \approx 10\% \) of the yield at midrapidity originates from these strings even when all partons are fluidized in the midrapidity region, \textit{i.e.}, \( R \approx 1 \). It is expected that such strings would melt when they penetrate the deconfined matter and do not contribute to the final hadron yield at midrapidity. This issue is to be resolved in future work.

C. Hadron yield ratios

We show the various hadron yield ratios in \( p+p \), \( p+Pb \), and \( Pb+Pb \) collisions as functions of multiplicity and compare them with ALICE data. In Fig. 4, the yield ratios as functions of multiplicity are shown for \( (a) \) cascades with \( |S| = 2 \), \( (b) \) lambda with \( |S| = 1 \), \( (c) \) phi mesons with \( |S| = 0 \), and \( (d) \) protons and antiprotons. In the DCCI model, we use \( a_0 = 368 \) extracted in Sec. III A. Since the parameter \( a_0 \) is fitted for experimental data of cascades, we reproduce the behavior of them well, as shown in Fig. 4(a). In addition, we also reproduce experimental data for lambda and phi mesons reasonably well. From Fig. 3, it is clear that the continuous change of hadron yield ratios results from competition between contributions from the core and the corona. Thus, continuous change of the dominant particle production mechanism from the corona to the core enables the model to describe the experimental data. These results also indicate that the DCCI model describes the different tendencies of strangeness enhancement among strange baryons which have different strangeness quantum numbers.

It is noted that the increasing behavior of the phi meson yield ratios would not be described by the canonical suppression model [69] since phi mesons are hadrons with hidden strangeness. According to the fact that the models based on the core–corona picture, including our model, show good agreement with the experimental data [46, 50, 72], the core–corona picture turns out to be a more essential description of the multiplicity dependence of hadron yield ratios compared to the canonical suppression model.

Regarding proton and antiproton yield ratios, our results show continuous increase with multiplicity, which is the same behavior as the other hadron yield ratios from the DCCI model. However, the experimental data exhibit the opposite tendency: The data decrease with multiplicity and deviate gradually from our results. The value of the ratio of protons and antiprotons to charged pions from the DCCI model is \( \approx 0.05-0.06 \) in a few lowest-multiplicity bins, which is almost the same value as the one that the corona gives. On the other hand, the ratio is \( \approx 0.07 \) in the highest-multiplicity bin in this model, which is almost consistent with the value obtained from the core. Thus the difference between our results and the experimental data in high-multiplicity events indicates a mechanism missing in the present model. Since this deviation can be explained from baryon-antibaryon annihilation during hadronic evolution [46, 70], we might resolve this issue by combining our model with hadronic cascade models in the late stage.

Motivated by the fact that hadron yield ratios scale with multiplicity regardless of system size or collision energy in the ALICE data [37], we study whether or not the DCCI model gives the same tendency. To see the system size independence at LHC energies, the hadron yield ratios in Xe+Xe collisions at \( \sqrt{s_{NN}} = 5.44 \text{ TeV} \) are compared with the ones in \( p+p \), \( p+Pb \), and \( Pb+Pb \) collisions in Fig. 5. The results in \( Xe+Xe \) collisions trace the ones from the other collision systems, which exhibit the same behavior as ones reported in Ref. [73].

Figures 6(a) and 6(b) show collision energy (in)dependence of hadron yield ratios as functions of multiplicity in nuclear and \( p+p \) collisions, respectively. The results in \( Au+Au \) collisions at \( \sqrt{s_{NN}} = 200 \text{ GeV} \) are compared with the ones in \( Pb+Pb \) collisions at \( \sqrt{s_{NN}} = 2.76 \text{ TeV} \) in Fig. 6(a). On the other hand, a comparison between \( p+p \) collisions at \( \sqrt{s} = 7 \) and 100 TeV is made in Fig. 6(b). It is noted that this extremely large collision energy of \( \sqrt{s} = 100 \text{ TeV} \) is the same as the one planned at the Future Circular Collider (FCC) experiment. The tendency is the same as that of the other results again: The hadron yield ratios exhibit scaling behavior as functions of multiplicity and there is almost no collision energy dependence regardless of a gap of collision energy of one or two orders of magnitude for each comparison.

These results demonstrate absence of size and energy dependence for hadron yield ratios as functions of multiplicity. Therefore it can be said that the multiplicity is one of the keys to control the fraction of energy converted into the QGP fluids in high-energy nuclear collisions.

IV. Summary

In this study, we analyzed multiplicity dependence of the hadron yield ratios in various collision systems and in a wide range of collision energy, based on the dynamical core–corona initialization framework, and studied how much in fraction the equilibrated matter is formed in \( p+p \), \( p+Pb \), and \( Pb+Pb \) collisions.

Assuming that fluids are generated via four-momentum deposition of the initially produced partons, we described the initial stage of forming the QGP fluids with hydrodynamic equations with source terms. We formulated the source term considering the spatial geometry of the initial partons under the concept of the
FIG. 4. (Color online) Hadron yield ratios of (a) cascades ($\Xi^-$ and $\Xi^+$), (b) lambdas ($\Lambda$ and $\bar{\Lambda}$), (c) phi mesons ($\phi$), and (d) protons ($p$ and $\bar{p}$) to charged pions ($\pi^-$ and $\pi^+$) as functions of multiplicity at midrapidity, $\langle dN_{ch}/d\eta\rangle$, in p+p (diamonds), p+Pb (triangles) and Pb+Pb (squares) collisions at LHC energies. The center-of-mass collision energies per nucleon pair are $\sqrt{s_{NN}} = 7$ TeV (p+p), 5.02 TeV (p+Pb), and 2.76 TeV (Pb+Pb). Results from the DCCI model (closed symbols) are compared with the ALICE data (open symbols) [37, 65, 67–71].
core–corona picture. In this picture, partons in the high-density region are likely to be components of the QGP fluids, while those in the low-density region tend to survive as partons traversing the QGP fluids or vacuum. Using the dynamical core–corona initialization framework, we performed numerical simulations of nucleon–nucleon, nucleon–nucleus, and nucleus–nucleus collisions at RHIC, LHC, and FCC energies. First we generated initial partons with the extracted best value to describe the experimental data within our model. Using the extracted $a_9$, we performed simulations of $p+p$, $p+Pb$, and $Pb+Pb$ collisions at LHC energies and quantified the fraction of the fluidized energy in various colliding systems and in a wide range of collision energy. Except for protons, our model calculations showed reasonable agreement with the ALICE experimental data. We also analyzed the multiplicity dependence of yield ratios of cascades, lamdas, phi mesons, and protons with the extracted $a_9$ in various noninclusive diffractive $p+p$ events there exists hadron production from the chemically equilibrated matter to reproduce the monotonically increasing behavior of multiplicity dependence of hadron yield ratios reported by the ALICE Collaboration. We described the continuous increase of hadron yield ratios observed in the experimental data as a result of competition between the core and the corona. Although our results describe the experimental data reasonably, we admit that their detailed characteristics are not reproduced. Nevertheless, there is still room to introduce additional dynamics such as hadronic rescatterings and viscosity. We leave implementation of them as our future work. In the dynamical core–corona initialization model, system size and collision energy dependences of the hadron yield ratio were analyzed by performing simulations of $p+p$, $Xe+Xe$, and $Au+Au$ collisions at $\sqrt{s_{NN}} = 100$ TeV, 5.44 TeV, and 4.4 TeV.

FIG. 5. (Color Online) Hadron yield ratios as functions of multiplicity in $Xe+Xe$ collisions at $\sqrt{s_{NN}} = 5.44$ TeV (closed circles) are compared with the ones in $p+p$ (open diamonds), $p+Pb$ (open triangles), and $Pb+Pb$ (open squares) collisions at LHC energies.
FIG. 6. (Color Online) (a) Hadron yield ratio as a function of multiplicity in heavy-ion collisions at RHIC and LHC energies. Results in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV (open symbols) are compared with the ones in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV (closed symbols). (b) Hadron yield ratio as a function of multiplicity in p+p collisions at LHC and FCC energies. Results at $\sqrt{s} = 7$ TeV (open symbols) are compared with the ones at $\sqrt{s} = 100$ TeV (closed symbols).

200 GeV, respectively. We studied whether there is a system size dependence of the hadron yield ratio as a function of multiplicity by comparing the results of Xe+Xe collisions with the ones of other collision systems at LHC energies. We also checked whether the collision energy dependence appears when comparing results at RHIC energy with the ones at LHC energy, and results in p+p collisions at FCC energy with the ones at LHC energy. Since both results show clear scaling with multiplicity, there are no system size and collision energy dependences within the dynamical core–corona initialization model.

In this work, we focused on the bulk property of the system created in high-energy nuclear collisions and saw that the dynamical core–corona initialization model reasonably describes the multiplicity dependence of strange hadron yield ratios from small to large colliding systems. On the other hand, it is still not revealed to what extent collectivity observed in small colliding systems originates from the collective hydrodynamic flow of the QGP fluids. Flow observables in small colliding systems have been discussed frequently in the context of QGP formation theoretically and experimentally. We point out here that the bulk properties such as hadron yield ratios also should be explained as well as the flow observables within the same framework. Conventionally viscous hydrodynamic models, which were successful in description of flow observables in high-energy heavy-ion collisions, have been applied also to small colliding systems. However, even if these hydrodynamic models reproduce flow data in small colliding systems, they might not reproduce the bulk properties, in particular, hadron yield ratios. In the light of our analysis and interpretation of hadron yield ratios data, there should exist a certain contribution from nonequilibrated systems in the final hadron production which would dilute collective flow signals to some extent. Therefore in our future work we plan to study to what extent hydrodynamic flow signals generated by the core would be affected by the corona as a nonequilibrated system within the dynamical core–corona initialization model. In order to perform sophisticated analysis, we will introduce viscosities in fluids and hadronic rescatterings in the late stage in the dynamical core–corona
initialization model.

It is also worth studying the effect of dynamical core–corona initialization on transverse dynamics in high-energy nuclear collisions. We anticipate that high \( p_T \) partons behave as the corona and traverse the core after the dynamical core–corona initialization. Thus we separate hard from soft components naturally and dynamically. This would be a starting point to investigate the modification of jet structure in a hydro-based full event generator. These will be discussed in our future publications.

ACKNOWLEDGMENTS

The work by Y.T. is supported in part by a special grant from the Office of the Vice President of Research at Wayne State University and in part by the National Science Foundation (NSF) within the framework of the JETSCAPE Collaboration under Grant No. ACI-1550300. The work by T.H. is partly supported by JSPS KAKENHI Grant No. JP17H02900.

Appendix A: Derivation of fluidization rate

The fluidization rate is obtained by substituting Eq. (6) into the kinetic definition of energy-momentum tensor in Eq. (4). The right hand side of Eq. (4) without minus sign becomes

\[
\partial_\mu T^\mu_\text{parton} = \sum_i \int d^3p \frac{p^\mu p^\nu}{p^0} \partial_\mu G(x - x_i(t)) \delta^{(3)}(p - p_i(t)) \\
= \sum_i \int d^3p \frac{p^\mu p^\nu}{p^0} \left[ \partial_\mu G(x - x_i(t)) \right] \delta^{(3)}(p - p_i(t)) \\
+ G(x - x_i(t)) \left( \partial_\mu \delta^{(3)}(p - p_i(t)) \right).
\]

(A1)

The first term in Eq. (A1) vanishes as

\[
\sum_i \int d^3p \frac{p^\mu p^\nu}{p^0} \cdot \nabla \left( \frac{1}{\sqrt{(2\pi\sigma^2)^3}} e^{-\frac{(x-x_i(t))^2}{2\sigma^2}} \right) \delta^{(3)}(p - p_i(t)) \\
= \sum_i \int d^3p \frac{p^\mu p^\nu}{p^0} \cdot \nabla \left( \frac{1}{\sqrt{(2\pi\sigma^2)^3}} e^{-\frac{(x-x_i(t))^2}{2\sigma^2}} \right) \delta^{(3)}(p - p_i(t)) \\
= 0.
\]

Here we use \( p_i^0 \frac{dx_i}{dt} = p_i \). On the other hand, the second term in Eq. (A1) is

\[
\sum_i \int d^3p \frac{p^\mu p^\nu}{p^0} \frac{\partial}{\partial t} \delta^{(3)}(p - p_i(t)) \\
= \sum_i \int d^3p \frac{p^\mu p^\nu}{p^0} \frac{\partial}{\partial t} \left( \frac{1}{(2\pi\sigma^2)^{3/2}} e^{-\frac{(x-x_i(t))^2}{2\sigma^2}} \right) \delta^{(3)}(p - p_i(t)) \\
= - \frac{1}{(2\pi)^3} \sum_i G(x - x_i) \\
\times \int d^3p \frac{p^\mu p^\nu}{p^0} \frac{\partial}{\partial t} \left( \frac{1}{(2\pi\sigma^2)^{3/2}} e^{-\frac{(x-x_i(t))^2}{2\sigma^2}} \right) \delta^{(3)}(p - p_i(t)) \\
= \sum_i G(x - x_i) \int d^3p \frac{\partial p^\mu}{\partial t} \frac{\partial p^\nu}{\partial p^\mu} \delta^{(3)}(p - p_i) \\
= \sum_i G(x - x_i) \frac{dp^\mu}{dt} \frac{\partial p^\nu}{\partial p^\mu} \delta^{(3)}(p - p_i) \\
= \sum_i G(x - x_i) \frac{dp^\mu}{dt} \delta^{(3)}(p - p_i).
\]

Thus the source term (4) is derived as

\[
J^\mu(x) = \sum_i \frac{dp^\mu_i}{dt} G(x - x_i(t)), \quad \text{(A2)}
\]

by assuming the phase space distribution as Eq. (6).

Appendix B: Reduced chi-square values of function fitting

Table I shows the reduced chi-square values of function fitting for yield ratios of cascades to pions as functions of multiplicity with various \( a_0 \) parameters shown in Fig. 1. Since the statistical errors from hydrodynamic simulations are relatively small, only the statistical errors due to string fragmentation are considered in the chi-square fitting. We also show the reduced chi-square value from function fitting for the yield ratio of cascades to pions in ALICE experimental data [37].


| $a_0$ | Collision system | $k$ | $n$ | Reduced $\chi^2$ |
|-------|------------------|-----|-----|-----------------|
| 10    | Pb+Pb            | 577.785 | 0.656 | 35.8272         |
| 20    | Pb+Pb            | 43.195  | 0.461 | 3.61187         |
| 50    | Pb+Pb            | 3.03    | 0.324 | 2.87592         |
| 100   | Pb+Pb            | 4.985   | 0.459 | 2.37556         |
| 100   | p+p, p+Pb, and Pb+Pb | 8.595   | 0.529 | 11.2227         |
| 125   | Pb+Pb            | 6.745   | 0.68  | 4.92507         |
| 150   | Pb+Pb            | 1.16    | 0.445 | 21.4127         |
| 1000  | Pb+Pb            | 0.995   | 0.267 | 4.87633         |
| N/A   | p+p, p+Pb, and Pb+Pb | 6.685   | 1.434 | 0.971169        |

TABLE I. Reduced chi-square values in function fittings for yield ratios of cascades to pions as functions of multiplicity for various $a_0$ parameters in Fig. 1 and for ALICE data [37] in the bottom line.

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