A novel robust fixed-time fault-tolerant tracking control of uncertain robot manipulators

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Abstract
This paper presents a novel robust fixed-time fault-tolerant control for global fixed-time tracking of uncertain robot manipulators with actuator effectiveness faults. With the sufficient consideration of the effects on uncertain dynamics, external disturbances and actuator effectiveness faults to the trajectory tracking performance, a singularity-free robust fault-tolerant control with an auxiliary vector is constructed for the fixed-time tracking control of uncertain robot manipulators. Lyapunov stability theory is employed to prove the global fixed-time stability ensuring that both the position and velocity tracking errors converge globally to the origin within a fixed time. The appealing advantages of the proposed control are as follows: (i) it is easy to implement with the global robust fixed-time fault-tolerant tracking control for uncertain robot manipulators featuring with faster convergence rate and higher steady-state tracking precision; (ii) the settling time is independent of the initial states of closed-loop system and can be calculated in advance for robot manipulators with uncertain dynamics, external disturbances and actuator faults. Extensive simulations on a two-DOFs robot are presented to demonstrate the effectiveness and improved performances of the proposed approach.

1 | INTRODUCTION

Tracking control of robot manipulators with a high reliability requirement on accuracy, stability, and safety has been a critical issue in both academic and industrial applications [1]. With the sufficient consideration of uncertain dynamics, external disturbances and actuator effectiveness faults, it is still a challenge to develop a simple robust fixed-time tracking control with an improved tracking performance and transient respond for uncertain robot manipulators in the research community [2].

Since robot manipulators are a typical mechanical interconnected system, several actuator faults such as low input voltage and larger load, which causes the actuators to lose its partial effectiveness of actuators, have been affecting the tracking precision of robot system. To improve the tracking performance, transient respond and reliability of robot, thus fault-tolerant control (FTC) schemes [3] have been developed in these robot manipulators. Generally, they can be divided into two categories, that is, passive FTC (PFTC) and active FTC (AFTC). For AFTC, it is worth noting that the control input depends on the feedback of fault estimation obtained from a fault diagnosis (FD) observer [4, 5]. Since the requirement of an additional observer, nevertheless, these approaches featuring with high computational complexity and fault feedback time delay may decrease the tracking performance and increase the computational complexity even become unstable. In contrast, for PFTC, no requirement of any feedback from fault observer has been encountered in the design of robust control for both normal and fault operation [6]. As a result, the PFTC has faster speed to compensate the actuator fault than AFTC. However, since the fault effects imposed on the PFTC system are heavier than that of the active approaches. Consequently, it is necessary to develop a robust fault-tolerant control with higher robustness for robot manipulators in the presence of uncertain dynamics, external disturbances and partial loss of actuator effectiveness faults.

With this purpose, several approaches have been developed to improve the tracking performance of robot manipulators...
in the presence of uncertain dynamics, external disturbances, and partial loss of actuator effectiveness faults. In the initial approaches, PID control [7], intelligent and learning controls [8, 9], optimal controls [10], robust controls [11, 12], etc., have been developed. Among them, robust controls show a higher robustness and disturbance and/or fault rejection capability. Furthermore, sliding mode controls (SMC) are a well-known robust control technique since its strong robustness against uncertainties and disturbances [13-15]. This great feature of SMCs have been applied in the formulation of FTC systems [16, 17]. A novel nonsingular fast terminal sliding mode control based on adaptive backstepping technique is developed in [18] for the fault-tolerant control of robot manipulators. By utilising the backstepping control technique, furthermore, the fault-tolerant controls [19, 20] have been proposed for a class of nonlinear systems, which can obtain the improved tracking performance with finite-time convergence. Nevertheless, the settling time of these mentioned approaches is related to the initial states of the closed-loop systems, which cannot be used in a non-linear system with time-constraints tracking control.

One minor drawback of finite-time controls is that it has a slower convergence than exponential stable systems if the system states are far away from the equilibrium point. The reason behind this is that the settling time of these finite-time controls depends on the initial states of closed-loop systems. As a result, the closed-loop system under the different initial states has different convergence performance. In light of above analysis, a strong finite-time stable system named as fixed-time control has been developed in [21, 22]. In comparison with conventional finite-time control systems, the fixed-time control guarantees that the settling time independently of initial states is uniformly bounded by a fixed time and calculated in advance [23]. Recognising these advantages, Tian et al. [24] proposes a continuous output feedback control scheme for the fixed-time stabilization of the double integrator system; while Zhang et al. [25] developed a prescribed fixed-time tracking control for robot manipulators by utilising a disturbance observer. Both of these approaches [24, 25] use the bi-limit homogeneous technique to obtain fixed-time stable controller and observer designs. Nevertheless, the settling time of these fixed-time controls [24, 25] may not be given as an exact time constant in advance since it uses the bi-limit homogeneous technique to obtain the fixed-time stable control. Based on seminal work, Zuo [26] proposes a TSMC for fixed-time stabilization of double integrators and applies for consensus tracking of second-order multi-agent systems. This fixed-time control is later extended to a class of non-linear second-order systems in the form of double integrators with matched uncertainties and perturbation [27]. By utilising a non-linear function, an approximate fixed-time sliding mode control is proposed in [28], which guarantees the system states converge to an arbitrarily small region centred at the origin. Upon analysing the existing robust fault tolerant controls, there still has one or more of following drawbacks that limit its performance in real applications [24]: (i) it does not provide a fixed time convergence; (ii) although it possesses a good transient response in normal operation, it is worse at tackling the fast variation effects of the disturbance and/or faults; and (iii) its control input has high computational complexity. Consequently, it is more desirable to develop a simple robust fixed-time fault-tolerant tracking control featuring with simplicity and robustness subject to uncertain dynamics, external disturbances and actuator effectiveness faults.

Motivated by the above analysis, this paper visits the robust fixed-time fault-tolerant tracking of robot manipulators with uncertain dynamics, external disturbances and partial loss of actuator effectiveness faults. Inspired by the works in [18, 27], a robust fixed-time fault-tolerant tracking control (RFTC) is proposed. The contributions of our paper are as follows: (i) the proposed RFTC is constructed without using the acceleration of joints or the assumption that the lumped uncertainty involving the acceleration of joints are bounded by a constant, which not only overcomes the algebraic loop problem [29] but also obtains the fixed-time fault-tolerant tracking of uncertain robot manipulators; (ii) in comparison with the existing finite-time tracking controls [13, 14], the convergence time of the proposed RFTC is independent of the initial states of robotic system and can be calculated in advance; (iii) compared with the fixed-time tracking controls [24, 26], the proposed RFTC considers the effects of actuator faults on tracking performance, and also has more higher steady-state tracking precision and simpler control structure in both position and velocity trajectory tracking for robot manipulators with uncertainties, external disturbances and actuator faults; (iv) in comparison with the fixed-time fault-tolerant control [25], the proposed RFTC did not use the bi-limit homogeneous technique to prove the fixed-time stability, thus its settling time can be calculated from controller parameters in advance. Lyapunov stability theory is employed to prove the global fixed-time stability ensuring that both the position and velocity tracking errors converge globally to the origin within a fixed time. Simulation comparisons have been performed for uncertain robot manipulators in the presence of uncertain dynamics, external disturbances and actuator effectiveness faults. The simulation results demonstrate that the proposed controller gains the improved tracking performance including faster transient and higher steady-state tracking precision in both position and velocity trajectory tracking.

The reminder of this paper is organised as follows. In Section 2, Some preliminaries including the model and properties of robot manipulators and fixed-time stability of dynamical systems are introduced. The controller design and stability analysis are presented in Section 3. In Section 4, numerical comparisons are performed. Finally, a conclusion is included in Section 5.

2 PRELIMINARIES

2.1 Robot manipulator model and properties

The n-joints rigid manipulators are described as [30]

\[ M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \Gamma \tau + d, \]  (1)
where $q, \dot{q}, \ddot{q} \in \mathbb{R}^a$ denote the vectors of position, velocity and acceleration, respectively, $M(q) \in \mathbb{R}^{a \times a}$ is the symmetric positive definite inertia matrix, $C(q, \dot{q}) \in \mathbb{R}^{a \times a}$ stands for the centrifugal-Coriolis matrix, $g(q) \in \mathbb{R}^a$ denotes the vector of gravitational torque, $\delta \in \mathbb{R}^a$ denotes the bounded external disturbances and is upper bounded by $\|d\| \leq d_m$ with a known constant $d_m$, $\tau \in \mathbb{R}^a$ is the control input, $\Gamma = \text{diag}(\gamma_i(t)), i = 1, \ldots, n$ denotes the actuator health condition with $\gamma_0 \leq \gamma_i(t) \leq 1$, and $\gamma_0 \in (0, 1]$ stands for a known positive constant.

**Remark 1.** Such an actuator fault formulation described by system (1) can be found in many previous results, such as [31–33]. Since the faults of control circuit or servo system was often happened in the application of robot manipulators, partial actuator loss of effectiveness is a kind of common fault of robot systems, which always affects the tracking precision and even stability of closed-loop system. Thus, the assumption of actuator fault for system (1) is reasonable and a question worth studying.

In order to facilitate the following analysis, the proposed robust fixed-time fault-tolerant tracking control for uncertain robot manipulators will be accomplished on the following fundamental facts [13, 30].

The subsequent development is based on the assumption that $q$ and $\dot{q}$ are available, and the desired trajectory $q_d \in \mathbb{R}^a$ be $C^2$ for the robotic system. Additionally, the following assumption will be exploited [13].

**Assumption 1.** The model parameters can be described as [13–15]

\[
M(q) = M_0(q) + \Delta M(q) \\
C(q, \dot{q}) = C_0(q, \dot{q}) + \Delta C(q, \dot{q}) \\
g(q) = g_0(q) + \Delta g(q),
\]

where $M_0(q), C_0(q, \dot{q})$ and $g_0(q)$ denote the nominal parts, and $\Delta M(q), \Delta C(q, \dot{q})$ and $\Delta g(q)$ stand for the uncertain parts.

Without loss of generality, it is assumed that the norms of desired vectors are upper bounded by the following positive constants

\[
\|q_d\| \leq P_q, \quad \|\dot{q}_d\| \leq P_q, \quad \|\ddot{q}_d\| \leq P_q
\]

where $q_d, \dot{q}_d, \ddot{q}_d \in \mathbb{R}^a$ are the vectors of desired position, velocity and acceleration, respectively, and $P_q, P_\dot{q}$ and $P_{\ddot{q}}$ are some known positive constants.

To facilitate the following design and analysis, we define the vector $\text{Sgn}(\xi) \in \mathbb{R}^a$ and $\text{Sig}^\alpha(\xi) \in \mathbb{R}^a$ as follows:

\[
\text{Sgn}(\xi) = [\text{sign}(\xi_1), \ldots, \text{sign}(\xi_a)] \\
\text{Sig}^\alpha(\xi) = [\text{sign}^\alpha(\xi_1), \ldots, \text{sign}^\alpha(\xi_a)]^T.
\]

**Definition 1 (Fixed-time stability [26]).** Consider the system $\dot{x} = g(t, x)$ with $x(0) = x_0$, where $g: \mathbb{R}^+ \times \mathbb{R}^a \rightarrow \mathbb{R}^a$ is a non-linear function with system states $x \in \mathbb{R}^a$ which can be discontinuous. The origin of system $\dot{x} = g(t, x)$ is said to be globally fixed-time stable if the settling time function $T$ is globally bounded, that is, there exists a fixed constant $T_{\text{max}} \in \mathbb{R}^+$ such that $T \leq T_{\text{max}}$ and $x(t) = 0$ for all $t \geq T$ and $x_0 \in \mathbb{R}^a$.

**Lemma 1.** Consider a scalar system [26]

\[
\dot{y} = -\eta|y|^\mu \text{sign}(y) - \mu|y|^{\mu-1} \text{sign}(y), \quad y(0) = y_0,
\]

where $r_1$ and $r_2$ are two known positive constants satisfied $r_1 > 1$ and $0 < r_2 < 1$, and $\eta > 0$ and $\mu > 0$ denote some positive constants which depend on controller parameters. Then, the equilibrium point of system (7) is fixed-time stable, and the settling time $T$ is derived as

\[
T < T_{\text{max}} = \frac{\Delta}{\eta (r_1 - 1)} + \frac{1}{\mu(1 - r_2)}. \tag{8}
\]

**Lemma 2.** For $\alpha \in \mathbb{R}^+$, $x \in \mathbb{R}$, the following inequalities hold [14]

\[
\frac{d}{dt} |x|^{\alpha+1} = (\alpha + 1)|x|^{\alpha} \dot{x} \text{ sign}(x) \\
\frac{d}{dt} (|x|^{\alpha+1} \text{ sign}(x)) = (\alpha + 1)|x|^\alpha \dot{x}, \tag{9}
\]

where $\text{sign}(\cdot)$ denotes the standard signum function.

## 3 | CONTROL DESIGN AND STABILITY ANALYSIS

To facilitate the subsequent control design and stability analysis, we begin with the open-loop error system development aimed to obtain an upper bound of the lumped
uncertainty that does not involve the joint acceleration. There-
after, the control formulation is presented including the fixed-
time fault-tolerant control law along with fixed-time stability
analysis.

3.1 Open-loop system development

Based on Assumption 1, the system (1) can be rewritten as

$$M_0(q)\ddot{q} + C_0(q, \dot{q})\dot{q} + g_0(q) = \Gamma \tau + \rho,$$

where the lumped uncertainty $\rho \in \mathbb{R}^n$ is defined as

$$\rho = -\Delta M(q)\ddot{q} - \Delta C(q, \dot{q})\dot{q} - \Delta g(q) + d.$$  

In light of (1) and (2), it follows that

$$\Delta M(q)\ddot{q} = E(\Gamma \tau - C(q, \dot{q})\dot{q} - g(q) + d),$$

where $E \in \mathbb{R}^{n \times n}$ defined by [29]

$$E = I_n - M_0(q)M^{-1}(q),$$

with $I_n$ denoting the $n \times n$ identity matrix.

Observed by the work [29], once $M_0(q)$ is chosen as

$$M_0 = \frac{2}{\gamma_1 + \gamma_2} I_n,$$  

where $\gamma_1$ and $\gamma_2$ are two known positive constants given by

$$\gamma_1 \leq \|M^{-1}(q)\| \leq \gamma_2$$

then $E$ is upper bounded by [29]

$$\|E\| \leq \sigma$$

with $\sigma$ standing for a known positive constant given by

$$\sigma = \frac{\gamma_2 - \gamma_1}{\gamma_1 + \gamma_2}.$$  

Note that $M_0(q)$ is written as $M_0$ in the subsequent development owing to the $M_0(q)$ defined by (14) is defined as a constant matrix. By virtue of Assumption 1, the lumped uncertainty $\rho$ given by (11) is upper bounded by [15]

$$\|\rho\| \leq a_0 + a_1\|\dot{q}\|^2 + \sigma\|\tau\|,$$

where $a_i$, $i = 0, 1$ denote two positive constants that depend on the robotic system, and $\sigma$ is defined by (17).

The proof of (18) can be found in Appendix.

3.2 Control formulation

Upon substituting (6) into (10), the error closed-dynamic equation for $\dot{e}$ takes

$$M_0\ddot{e} = \Gamma \tau + \rho + \eta,$$

where $M_0$ is a constant matrix given by (14), the lumped uncertainty $\rho \in \mathbb{R}^n$ is defined by (11) and the nominal part $\eta \in \mathbb{R}^n$ is described as follows:

$$\eta = -C_0(q, \dot{q})\dot{q} - g_0(q) - M_0\ddot{q}.$$  

To facilitate the subsequent control design, we defined an auxiliary vector as follows:

$$\chi = \dot{e} + \lambda e,$$

where $\lambda \in \mathbb{R}^+$ denotes a given constant, and $\dot{e}$ and $e$ are given by (6).

For system (19) and the auxiliary vector (21), then, a robust fixed-time fault-tolerant tracking control (RFTC) is defined as

$$\tau = -\eta + \tau_0 + \tau_1 + \tau_2,$$

where $\eta \in \mathbb{R}^n$ is defined by (20) and

$$\tau_0 = -\lambda M_0\ddot{e} - M_0\text{Sgn}(\chi)e^T e - M_0\text{Sgn}(\chi)\sum_{i=1}^n |\chi_i| + \frac{1}{2} |\xi_i|^2,$$

$$\tau_1 = -\frac{\gamma_1 + \gamma_2}{2} M_0\text{Sgn}(\chi)\alpha_1;$$

FIGURE 1 Block diagram of the proposed RFTC control strategy
with \( \chi \) is given by (21) and

\[
w = \frac{1}{\gamma_0 - \sigma} \left( a_0 + a_1 \| \hat{\eta} \| \right)^2 + \sigma \| \tau_0 + \tau_2 - \eta \|.
\]

where \( \alpha > 0, \beta > 0, p > 1 \) and \( r > 1 \) are some positive constants, \( \gamma_1, \gamma_2, \sigma, a_0 \) and \( a_1 \) are defined by (15), (17) and (18), respectively, and \( \gamma_0 \) is defined by (1) satisfied \( \gamma_0 > \sigma \).

Remark 3. Observed by (22)–(26), the formulation of control components \( \tau_1 \) and \( \tau_2 \) does not include its upper bounds \( \| \tau_1 \| \) and \( \| \tau_2 \| \), respectively. As a result, the proposed RFTC can overcomes the algebraic loop problem [29] completely. Compared with the existing robust controls, the uncertain dynamics, external disturbances and partial loss of actuator effectiveness faults will be considered adequately in the formulation of the proposed RFTC given by (22)–(26), which also preserve a simple control structure and appropriate controller gains to implement the trajectory tracking of robot manipulators.

Remark 4. Observed by (15) and (17), for most robot manipulators systems, we can also obtain a small enough \( \sigma \) because of the derivation of \( \gamma_1 \) and \( \gamma_2 \) comes from the fact \( \gamma_1 \leq \| M^{-1}(\hat{q}) \| \leq \gamma_2 \). On the other hand, the parameters \( \sigma \) can be modified as \( \sigma = 1 - 2 \gamma_1 / (\gamma_1 + \gamma_2) \) from (17), which can also prove that \( \sigma \) is small enough since \( \gamma_1 \gg \gamma_2 \) can always be established on robot manipulators. Accordingly, the assumption \( \gamma_0 > \sigma \) is realistic and reasonable observed from (15).

Remark 5. Observed by (22)–(26), the nominal parts \( \eta \) of robot manipulators have been used in the formulation of the proposed approach. Moreover, the control term \( \eta \) given by (20) can also be modified as \( \eta = -M_0 \hat{q}_1 \), which implies that no prior knowledge of robot manipulators have been used in the formulation of the proposed approach observed from (22)–(26). As a result, the proposed RFTC can be further converted to a fixed-time robust tracking controller in which no prior knowledge of robot manipulators are used in the formulation of the proposed approach.

After substituting (22) into (19), we have

\[
M_0 \dot{\eta} = \tau_0 + \tau_1 + \Gamma \tau_2 - (I_a - \Gamma)(\tau_0 + \tau_1 - \eta) + \rho.
\]

3.3 Stability analysis

For system (27), we are in a position to state the following result.

Theorem 1. Given the uncertain robot manipulators (1), the proposed RFTC given by (22)–(26) ensures that the position and velocity tracking errors converge globally to the origin within a fixed time \( T \leq T_{\text{max}} \Delta \equiv T_r \). The settling time \( T_r \) is derived as

\[
T_r = \frac{r}{\alpha} + \frac{r}{\beta(p - 1)},
\]

where \( \alpha, \beta, r, p \) are given by (23).

Proof. For system (27), the positive definite Lyapunov function candidate is proposed as follows:

\[
V = \sum_{i=1}^{n} \left( |\chi_i| + \frac{1}{2} |\xi_i| \right)^2.
\]

Differentiating \( V \) with respect to time along the trajectory of dynamics Equation (27), it follows that

\[
V' = r \left( \sum_{i=1}^{n} \left( |\chi_i| + \frac{1}{2} |\xi_i| \right)^{r-1} \right) \left( \chi_i \hat{\chi}_i + \xi_i \hat{\xi}_i \right)
\]

Upon substituting (27) into (30) yields

\[
V' \leq r \left( \sum_{i=1}^{n} \left( |\chi_i| + \frac{1}{2} |\xi_i| \right)^{r-1} \right) \left( \left( M_0^{-1}(\tau_0 + \tau_1 + \Gamma \tau_2) \right)^T S_{\gamma}(\chi) + \lambda \hat{\xi}^T S_{\gamma}(\hat{\chi}) + \hat{\xi}^T \hat{\xi} + \| \rho \| + \| I_a - \Gamma \| (\| \tau_1 \| + \| \tau_2 - \eta \|) \right).
\]

where \( \| I_a - \Gamma \| \leq (1 - \gamma_0), \Gamma \geq \gamma_0 I_a, \| M_0^{-1} \| = (\gamma_1 + \gamma_2) / 2 \) and \( \| \tau_1 \| = \| \tau_2 \| \) are involved from (1), (14) and (24).

Then, substituting (23) and (24) into (31), we have

\[
V' \leq \left( \sum_{i=1}^{n} \left( |\chi_i| + \frac{1}{2} |\xi_i| \right)^{r-1} \right) \left( \gamma_1 + \gamma_2 \right) \frac{\alpha}{r} \left( a_0 + a_1 \| \hat{\eta} \| \right)^2 + \sigma \| \tau_1 \| \frac{\beta}{r} \left( \sum_{i=1}^{n} \left( |\chi_i| + \frac{1}{2} |\xi_i| \right)^{r-1} \right)^{r-1}.
\]

where the fact \( \| M_0^{-1} \| = (\gamma_1 + \gamma_2) / 2 \) is used from (14).

By utilising (22), (26) and the upper bound of \( \rho \) given by (18), it follows that

\[
-\gamma_0 w + \| \rho \| = -\gamma_0 w + \left( a_0 + a_1 \| \hat{\eta} \| \right)^2 + \sigma \| \tau_1 \| \leq -\gamma_0 w + \sigma \| \tau_1 \| + \left( a_0 + a_1 \| \hat{\eta} \| \right)^2 + \sigma \| \tau_0 + \tau_2 - \eta \|
\]

\[
= -\gamma_0 w - \sigma w + \sigma \| \tau_1 \| + \left( a_0 + a_1 \| \hat{\eta} \| \right)^2 + \sigma \| \tau_0 + \tau_2 - \eta \|
\]

\[
= -\sigma w + \sigma \| \tau_1 \|
\]

\[
= 0,
\]
where the fact $\|\tau\| \leq \|\tau_0 + \tau_2 - \eta\| + \|\tau_1\|$ and $\|\tau_1\| = \omega$ with $\omega > 0$ are involved from (22), (24) and (26), respectively.

Upon substituting (29) and (33) into (32), we have

$$V' \leq -\alpha \left( \sum_{i=1}^{n} \left( |x_i| + \frac{1}{2} |\dot{x}_i|^2 \right) \right)_{r-1}^p - \beta \left( \sum_{i=1}^{n} \left( |x_i| + \frac{1}{2} |\dot{x}_i|^2 \right) \right)_{r-1}^p$$

$$= -\alpha \left[ \left( \sum_{i=1}^{n} \left( |x_i| + \frac{1}{2} |\dot{x}_i|^2 \right) \right) \right]_{r-1}^p + \beta \left[ \left( \sum_{i=1}^{n} \left( |x_i| + \frac{1}{2} |\dot{x}_i|^2 \right) \right) \right]_{r-1}^p$$

$$= -\alpha^{1/p} - \beta^{1/p},$$

where $\alpha$ and $\beta$ denote known constants.

**Remark 6.** The proposed RFTC does not refer to model parameters in the control law formulation and would gain global robust fixed-time fault-tolerant tracking of robot manipulators in the presence of uncertain dynamics, bounded external disturbances and partial loss of actuator effectiveness faults. Compared with the existing robust finite-time stable controllers, the settling time of the proposed approach is independent of the initial states and can be calculated in advance. As a result, observed from the section "Stability analysis", the proposed RFTC has higher steady-state tracking precision due to the position and velocity tracking errors $e$ and $\dot{e}$ converge globally to the origin within a fixed time $T_r$ defined by (28).

This completes the proof. \qed

**Remark 7.** Different from the work [25], the proposed approach removes the assumption that the acceleration has an upper bound. Since the work [25] uses bi-limit homogeneous technique to prove the fixed-time stability in sliding phase of sliding mode control (see (9) of [25]); moreover, it cannot provide an exact convergence time in advance. In other words, the fixed settling time of the work [25] cannot be calculated from the controller parameters. In contrast, the settling time of the proposed approach has been obtained from (28) in advance. Accordingly, the proposed RFTC offers an alternative approach for improving the tracking performance of robot manipulators subject to uncertain dynamics, external disturbances and actuator faults.

**Remark 8.** The robust fixed-time fault-tolerant control (22)-(26) proposed in this paper has some components that contains a signum function, which may result in chattering. The chattering situation not only decreases the tracking performance but also even damages the actuator of robot manipulators. The chattering will be eliminated by replacing the signum function to the following function [35]

$$\text{sign}(\chi_i) = \frac{\exp(\kappa \chi_i) - 1}{\exp(\kappa \chi_i) + 1},$$

where $\kappa$ denotes a known constant.

### 4 SIMULATION COMPARISONS

Consider the dynamics of two-DOFs robot are given by [13]

$$M(q) = \begin{bmatrix} H_1 + 2H_2\cos(q_2) & H_3 + H_5\cos(q_2) \\ H_5 + H_3\cos(q_2) & H_4 \end{bmatrix},$$

$$C(q, \dot{q}) = \begin{bmatrix} -H_2\sin(q_2)\dot{q}_1 - 2H_2\sin(q_2)\dot{q}_2 \\ 0 \end{bmatrix},$$

$$g(q) = \begin{bmatrix} H_5\cos(q_1) + H_6\cos(q_1 + q_2) \\ H_5\cos(q_1 + q_2) \end{bmatrix}^T,$$

with

$$H_1 = (m_1 + m_2) r_1^2 + m_2 r_2^2 + J_1, \quad H_2 = m_2 r_1 r_2$$

$$H_3 = m_2 r_2^2, \quad H_5 = H_4 + J_2$$

$$H_6 = (m_1 + m_2) r_1 \dot{g}_1, \quad H_6 = m_2 r_2 \dot{g}_1.$$

The parameters of robot manipulators are summarised as follows: $m_1 = 5.0 \text{ kg}$, $m_1 = 0.5 \text{ kg}$, $m_2 = 1.5 \text{ kg}$, $r_1 = 1.0 \text{ m}$, $r_2 = 0.8 \text{ m}$ and $\dot{g}_1 = 9.8 \text{ m/s}^2$. The nominal value of $m_1$ and $m_2$ are $\hat{m}_1 = 0.4 \text{ kg}$ and $\hat{m}_2 = 1.2 \text{ kg}$. In order to show the robustness of the proposed RFTC, the external disturbances are assumed as follows:

$$d = \left[ 2 \sin(t) + 0.5 \sin(200\pi t), \cos(2t) + 0.5 \sin(200\pi t) \right]^T.$$

The desired trajectories $q_d = [q_{d1}, q_{d2}]^T \text{ (rad)}$ are presented as

$$q_{d1} = 1.25 - 7/5 \exp(-t) + 7/20 \exp(-4t)$$

$$q_{d2} = 1.25 + \exp(-t) - 1/4 \exp(-4t).$$

The sampling period is 1 ms. The initial conditions are

$$q(0) = [1.0, 1.5]^T, \quad \dot{q}(0) = [0, 0]^T.$$

In this section, we verify the effectiveness of the proposed RFTC in the following two aspects: (i) upon the sufficient consideration of uncertain dynamics, external disturbances and actuator effectiveness faults, we have focused on the improved convergence property of the proposed RFTC in both position and velocity trajectory tracking; (ii) it is emphasised on the advantage of the proposed RFTC in convergence time, which means the settling time is independent of the initial conditions and only related to the controller parameters.
4.1 Tracking performance with actuator effectiveness faults

In order to show the improved tracking performance, the uncertain dynamics (1), external disturbances (1) and the following actuator effectiveness faults will be considered in the following simulation comparisons. The actuator health condition (1) will be given as

\[ \Gamma= \begin{cases} I_{2\times 2}, & t < 8 \text{ s} \\ \text{diag}\{0.7 + 0.01 \sin(10t), 0.65\}, & t \geq 8 \text{ s} \end{cases} \]  

where the first and second actuators are assumed to be lost up to its 31% and 35% effectiveness from the time 8 s, respectively.

By considering the uncertain dynamics (2) and partial loss of actuator effectiveness faults (43), accordingly, we have involved the tracking performance of the proposed RFTC in fault operations compared with finite-time integral backstepping control (FIBC) [20], computed torque controller (CTC) [36], PID controller [37], PID-based SMC (PID-SMC) [38], and non-singular terminal SMC (NTSMC) [39, 40]. Upon these seminal works, they are designed as follows [18].

For FIBC controller [20], it is given as

\[ \tau_{\text{fibc}} = \tau_N + \tau_J \]

\[ \tau_N = M_0 (\dot{\phi}_1(x_1) + \varphi_2(z) - x_1 + \ddot{\eta}_d) + C_0(q, \dot{q})\dot{q} + g_0(q) \]

\[ \tau_J = -M_0 \text{diag}\{\text{sign}(\zeta_i)\} \Omega \]

where \( x_1 = \varepsilon, x_2 = \ddot{\varepsilon}, z = x_2 - \varphi_1(x_1), \Omega = [\dot{\phi}_1 \ldots, \dot{\phi}_n] \) with \( \dot{\phi}_i \) is the estimate of \( \rho_i, \varphi_1(x_1) = [\phi_1(x_{11}) \ldots, \phi_1(x_{1n})]^T \) denotes the time derivative of \( \varphi_1(x_1) \), and

\[ \dot{\phi}_i = \begin{cases} 0, & \text{if } |\zeta_i| \leq \varepsilon \\ \nu_i |\zeta_i|, & \text{if } |\zeta_i| > \varepsilon \end{cases} \]

\[ \varphi_1(x_1) = -K_1x_1 - K_2 \varphi_2(x_1) \]

\[ \varphi_2(z) = -K_2z - K_3 \varphi_1^2(z) - K_4 \varphi_2^2 - K_5 \varphi_2(z) \]

\[ \dot{\varphi}_1(x_{1i}) = \begin{cases} -k_{11}x_{2i} - k_{1i} |x_{1i}|^{\alpha-1} x_{2i}, & |x_{1i}| \geq \gamma & \text{and } x_{2i} \neq 0 \\ -k_{11}x_{2i} - k_{1i} |\Delta_i|^\alpha x_{2i}, & |x_{1i}| < \gamma & \text{and } x_{2i} \neq 0 \\ 0, & x_{2i} = 0 \end{cases} \]

where \( \varphi = z, \varphi^2 \) is defined by (5) with \( 0 < \alpha < 1, K_i \in \mathbb{R}^{n \times n}, i = 1, \ldots, 6 \) stand for the diagonal positive matrices, \( k_1 \) and \( k_4 \) denote the 4th components of the matrices \( K_1 \) and \( K_4 \), respectively, and \( \nu, \varepsilon, \gamma \) and \( \Delta \) stand for some small positive constants.

For the CTC [36] and PID [37] controllers, they can be designed as

\[ \tau_{\text{ctc}} = -M_0(k_2\varepsilon + k_2\dot{e} - C_0(q, \dot{q})\dot{q} - g_0(q) - \ddot{\eta}_d), \]

where \( k_2, K_2 \) and \( K_3 \) denote the proportional, differential and integral gain matrix, respectively.

Then, the PID-SMC sliding surface and controller [38] are given as

\[ S_{\text{pid}} = k_1e + k_1\dot{e} + k_1\int_0^t e(\zeta)d\zeta, \]

\[ \tau_{\text{pid}} = \tau_{eq} + \tau_J, \]

where \( k_1, K_1 \) and \( K_2 \) are given by (50), Sgn(\( \cdot \)) is defined by (4), \( \Delta \) denotes the upper bound of the lumped uncertainty and actuator faults with \( ||\Delta|| \leq \Delta \) and \( \varsigma \) denotes arbitrarily small positive constant.

While the NFTSM controller and sliding surface [39, 40] are designed as

\[ S_{\text{nft}} = e + k_1 \varphi^2(e) + k_2 \varphi^q(e), \]

\[ \tau_{\text{nft}} = \tau_{eq} + \tau_J, \]

where \( k_1 = \text{diag}[k_{11}, \ldots, k_{1n}], i = 2, \text{are for diagonal constant matrices}, \rho \text{ and } q \text{ are two odd constants satisfying } 1 < \rho < 2, \lambda > 1 \text{ denotes a positive constant, } \text{Sgn}(\cdot) \text{ and } \varphi^q(\cdot) \text{ are defined by (4) and (5), respectively, and } \Delta \text{ and } \varsigma \text{ are defined by (54).} \]

For above controllers, by using (36) and the above given system parameters, the lower and upper bounds of the inverse inertial matrix \( M(q) \) defined by (15) are given by \( \gamma_1 = 0.09 \) and \( \gamma_2 = 0.2 \), and hence \( M_0 = 6.89I_n \) and \( \sigma = 0.38 \) given by (14) and (17) in [29]. Furthermore, \( C_0(q, \dot{q}) \) and \( g_0(q) \) are chosen by replacing \( m_1 \) and \( m_2 \) of (37)–(39) with the nominal ones \( \hat{m}_1 \) and \( \hat{m}_2 \). The selected parameters of these controllers are reported in Table 1.

First, we complete the simulation comparisons with the typical robust controllers (FIBC (44–48), CTC (49) and PID (50)). Figure 2 depicts the position tracking of the proposed RFTC. Figures 3 and 4 show the position and velocity tracking.
The controller parameters selection

| Controllers | Parameters |
|-------------|------------|
| FIBC        | $K_1 = K_2 = K_3 = \text{diag}[3, 3], K_4 = K_5 = K_6 = \text{diag}[1, 1]$ |
|             | $\alpha = 0.8, \rho_0 = [2; 2], \sigma_0 = [0; 0], \varepsilon = \zeta = 0.05, \gamma_1 = 0.01$ |
| CTC         | $K_7 = \text{diag}[150, 250], K_8 = \text{diag}[10, 15]$ |
| PID         | $K_p = 200, K_Q = 100, K_d = 30$ |
| PID-SMC     | $K_p = 50, K_Q = 100, K_d = 50, \Delta = 20, \zeta = 0.1$ |
| NFTSMC      | $k_1 = 10, k_2 = 5, \lambda = 1.4, \rho = 9, q = 7, \Delta = 20, \zeta = 0.1$ |
| FTC         | $\lambda = 5, \alpha = 1.2, \beta = 1.1, r = 1.2, y_1 = 0.09, y_2 = 0.2$ |
|             | $\rho = 1.3, \gamma_0 = 0.65, a_0 = 12, \phi_1 = 2.8, \sigma = 0.38$ |

In addition to $\lambda, \alpha, \beta, r$ and $p$ given by (59), the parameters of the proposed FTC given by (22)–(26) have the same values as the above comparison depicted by Figures 2–8 in Table 1. Figures 9 and 10 show the position tracking errors and control torque of the FTC with six different cases, respectively. Observed by the cases P1 and P2 of Figure 9, the transient convergence performance depends mainly on the parameter $\lambda$. As a result, the convergence time of the proposed FTC can be decreased with the increased $\lambda$, but the control inputs will be increased with the larger $\lambda$. Moreover, the objective of the cases P3, P4, P5 and P6 is to verify how does $\alpha, \beta, r$ and $p$ affect the system tracking performance. Obviously, the increased $\alpha, \beta, r$ and $p$ cannot significantly improve the tracking performance as shown in Figures 9 and 10. However, these increased parameters may enlarge the control torque input from Figure 10. Based on the above discussion and analysis, accordingly, we can conclude that the proposed FTC selects the parameters on a trade-off between the tracking performance and control inputs.

Finally, to further quantize the improved performance of the proposed FTC and also for an easier comparisons, the position and velocity tracking precisions, and its efforts on the control torque are compared after 2 s at the beginning of simulation.

Sample text: 

errors of the FIBC, CTC, PID and the proposed FTC with its zoomed plots, respectively; while Figure 5 depicts the control inputs. Without considering the actuator faults of uncertain robot manipulators, the proposed FTC shown in Figures 3 and 4 at the range 0 to 8 s represents a better tracking performance including faster transient rate and higher steady-state precision than the FIBC, PID and CTC. Obviously, the CTC shows worse robustness than the FIBC, PID and the proposed FTC especially in the presence of uncertain dynamics and external disturbance. Furthermore, the uncertain robot manipulators will be affected by the actuator faults after the time 8 s from Figures 3 and 4, while the FIBC, CTC and PID provide worse tracking performance. No matter whether the system is affected by the uncertain dynamics, external disturbance and/or actuator effectiveness faults, consequently, we can conclude that the proposed FTC always provides faster transient rate and higher steady-state precision in both position and velocity trajectory tracking. Moreover, these superior tracking performances of the proposed FTC have been obtained without using the excessive control input observed by Figure 5.

Secondly, we have accomplished the simulation comparisons with the typical robust fault-tolerant controllers (PID-SMC and NFTSMC given by (51)–(58)). Figures 6 and 7 show the position and velocity tracking errors of the PID-SMC, NFTSMC and the proposed FTC with its zoomed plots, respectively; while Figure 8 depicts their control inputs. Similarly, the proposed FTC without considering the actuator faults has a better tracking performance including faster transient rate and higher steady-state precision than the PID-SMC and NFTSMC from Figures 6 and 7 before the time 8 s. Since the integral component of PID are crucially significant, the PID-SMC shows a better robustness than the NFTSMC especially in the presence of uncertain dynamics and external disturbance. Furthermore, the uncertain robot manipulators have been affected by the actuator faults after the time 8 s from Figures 6 and 7, then the NFTSMC provides worse tracking performance in both position and velocity trajectory tracking; especially the velocity tracking performance of the NFTSMC tends to deteriorate immediately when the faults occur. Consequently, we have obtained the conclusion from Figures 6–8 as follows: (i) the proposed FTC provides an improved transient and steady-state tracking performance than the PID-SMC and NFTSMC no matter what form interferences the robot manipulators receive; (ii) both the position and velocity tracking performances have been enhanced by the proposed FTC.

After that, in order to show the effects of the controller parameters on the tracking performance, the following simulation comparisons have been accomplished with different six groups parameters as shown in Figures 9 and 10. These different controller parameters are as follows:

\[
\begin{align*}
\text{P1:} & \quad \lambda = \text{diag}[5, 5], \alpha = 1.2, \beta = 1.1, r = 1.2, p = 1.3 \\
\text{P2:} & \quad \lambda = \text{diag}[1, 1], \alpha = 1.2, \beta = 1.1, r = 1.2, p = 1.3 \\
\text{P3:} & \quad \lambda = \text{diag}[5, 5], \alpha = 1.2, \beta = 1.1, r = 3, p = 1.3 \\
\text{P4:} & \quad \lambda = \text{diag}[5, 5], \alpha = 1.2, \beta = 1.1, r = 1.2, p = 3 \\
\text{P5:} & \quad \lambda = \text{diag}[5, 5], \alpha = 3, \beta = 1.1, r = 1.2, p = 1.3 \\
\text{P6:} & \quad \lambda = \text{diag}[5, 5], \alpha = 1.2, \beta = 3, r = 1.2, p = 1.3
\end{align*}
\]

(59)

Auxiliary figure: 

**Figure 2** Joint positions of FTC.
where \( N \) denotes the total number of samples, \( e(k) \), \( \dot{e}(k) \) and \( \tau(k) \) stand for the position and velocity tracking errors and the control input at the \( k \)–th sampling instant, respectively. The comparisons of three performance indexes are summarised in Table 2.

As shown in Table 2, the proposed RFTC without using an excessive control torque input can obtain the minimal position and velocity tracking errors than the CTC, PID, PID-SMC and NFTSMC. The comparison results of Table 2 are to further verify the improved transient and steady-state tracking performance of the proposed RFTC in both position and velocity trajectory tracking. In particular, the proposed RFTC gains more smaller velocity tracking errors than others (See \( E_v \) of Table 2), which provides more higher tracking precision for uncertain robot manipulators.

### 4.2 Tracking performance with different initial conditions

Another advantage of the proposed RFTC is that its settling time is independent of the initial conditions of closed-loop sys-

**TABLE 2**  Comparison of control performance

| Controller | \( E_p \)     | \( E_v \)    | \( E_\tau \)  |
|------------|---------------|--------------|---------------|
| FIBC       | \( 4.2 \times 10^{-2} \) | \( 0.2 \times 10^{-2} \) | 20.86         |
| CTC        | \( 4.96 \times 10^{-2} \) | \( 0.47 \times 10^{-2} \) | 54.83         |
| PID        | \( 1.89 \times 10^{-2} \) | \( 2.35 \times 10^{-2} \) | 23.91         |
| RFTC       | \( 3.2 \times 10^{-4} \) | \( 2.2 \times 10^{-4} \) | 20.46         |
| PID-SMC    | \( 7.54 \times 10^{-4} \) | \( 21 \times 10^{-4} \) | 26.21         |
| NFTSMC     | \( 1.99 \times 10^{-2} \) | \( 0.71 \times 10^{-2} \) | 22.42         |
**Figure 5** Input torques of RFTC, FIBC, CTC and PID

**Figure 6** Position tracking errors of RFTC, PID-SMC and NFTSMC

**Figure 7** Velocity tracking errors of RFTC, PID-SMC and NFTSMC
Consequently, in this part we focused on the effects of initial states and controller parameters on the position and velocity tracking performance. The different initial states are defined as

\[
\begin{align*}
\text{Case 1:} & \quad \mathbf{q}(0) = \begin{bmatrix} 1.5, 1.8 \end{bmatrix}^T \\
\text{Case 2:} & \quad \mathbf{q}(0) = \begin{bmatrix} 1.0, 1.5 \end{bmatrix}^T \\
\text{Case 3:} & \quad \mathbf{q}(0) = \begin{bmatrix} -0.5, 0.5 \end{bmatrix}^T \\
\text{Case 4:} & \quad \mathbf{q}(0) = \begin{bmatrix} -1.0, -1.5 \end{bmatrix}^T \\
\text{Case 5:} & \quad \mathbf{q}(0) = \begin{bmatrix} -2.0, -2.5 \end{bmatrix}^T.
\end{align*}
\]

By utilising different initial states to the proposed RFTC, Figures 11 and 12 depict the position and velocity tracking errors with their zoomed plots, respectively, where the same controller parameters and different initial states are adopted in Table 1 and (62), respectively. According to Theorem 1 and (28), the position and velocity tracking errors always arrive at the origin within a fixed time \( T_r = 3.28 \text{ s} \). Based on this seminal analysis, we have concluded from these simulation
results from Figures 11 and 12 that the position and velocity tracking errors can arrive at the range \((-2, 2) \times 10^{-4}\) (rad) and \((-5, 5) \times 10^{-4}\) (rad/s) within a fixed time \(T_r = 3.28\) s, respectively. The simulation results of Figures 11 and 12 are to further verify Theorem 1 in which the tracking performance of the proposed RFTC depends mainly on the controller parameters instead of the initial conditions. The simulation comparisons of Figures 11 and 12 will be further verified the effectiveness of the proposed RFTC for uncertain robot manipulators in both position and velocity trajectory tracking.

Remark 9. Observed by Figures 11 and 12, the periodic steady-state tracking error comes mainly from the discrete nature of simulations. The sampling frequency can be added to further improve the tracking precision. However, the computation complexity will be increased with the increased sampling frequency. Accordingly, the selection of sampling period is a trade-off between the tracking precision and the computation complexity.

Upon the basis of the above simulation comparisons A and B, no matter the systemic states start from any positions in state space, the proposed RFTC always achieves the improved tracking performance such as faster transient and higher steady-state precision in both position and velocity trajectory tracking with a fixed time \(T_r\).

5 | CONCLUSION

In this paper, a novel robust fixed-time fault-tolerant control has been developed for global fixed-time tracking for robot manipulators subject to uncertainties, disturbances and actuator faults. Fixed-time stability analysis of the tracking system has been accomplished on Lyapunov theory. Numerical simulations demonstrate the enhanced tracking performance of the proposed approach in comparison with the traditional robust controls and the fault-tolerant controls with sliding mode in both position and velocity tracking. As a result, the proposed approach gets higher tracking precision with a fixed time than other robust controllers. Meanwhile, the developed approach provides higher robustness subject to uncertain dynamics, external disturbances and actuator faults. Future efforts will focus on finding a robust fixed-time tracking control with actuator constraints and continuity for robot manipulators with uncertain dynamics, external disturbances and actuator faults.
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APPENDIX A
For the convenience of the proof of (18), the following properties will be exploited [30]:
Property 1. The matrices $M(q)$ and $C(q, \dot{q})$ and the vector $g(q)$ are upper bounded by \[30\]
\[
\|M(q)\| \leq M_M, \quad \|C(q, \dot{q})\| \leq C_M \|\dot{q}\|, \quad \|g(q)\| \leq G_M,
\] (63)
where $M_M$, $C_M$ and $G_M$ are some known positive constants.

Property 2. The matrices $\Delta M(q)$ and $\Delta C(q, \dot{q})$ and the vector $\Delta g(q)$ are upper bounded by \[30\]
\[
\|\Delta M(q)\| \leq M_m, \quad \|\Delta C(q, \dot{q})\| \leq C_m \|\dot{q}\|, \quad \|\Delta g(q)\| \leq G_m,
\] (64)
where $M_m$, $C_m$ and $G_m$ are some known positive constants.

From (11), accordingly, the upper bound of the lumped uncertainty $\rho \in \mathbb{R}^n$ is derived as
\[
\|\rho\| \leq \|\Delta M(q)\| \|\dot{q}\| + \|\Delta C(q, \dot{q})\| \|\dot{q}\| + \|\Delta g(q)\| + \|d\|,
\] (65)
with the upper bound of $\Delta M(q)\dot{q}$ given by (12) is
\[
\|\Delta M(q)\dot{q}\| \leq \|E\| (\|\Gamma\| + \|C(q, \dot{q})\| \|\dot{q}\| + \|g(q)\| + \|d\|)
= \sigma \|\tau\| + \sigma C_M \|\dot{q}\|^2 + \sigma G_M + \sigma d_m,
\] (66)
where Property 1 and (16) are used.

Substituting (64) and (66) into (65), it follows that
\[
\|\rho\| \leq \sigma \|\tau\| + \sigma C_M \|\dot{q}\|^2 + \sigma G_M + \sigma d_m + \sigma G_m + d_m
= a_0 + a_1 \|\dot{q}\|^2 + \sigma \|\tau\|,
\] (67)
where
\[
a_0 = \sigma C_M + \sigma d_m + G_m + d_m, \quad a_1 = \sigma G_M + \sigma C_M.
\] (68)

This completes the proof.