A Measurement Method for Wide-frequency Harmonic Signal in Power Grid Based on Window Interpolation FFT and RLS-Adaline Neural Network

Jiawei Peng¹, Shuguo Pan¹*, Wenxiu Wang², Min Zhang³,⁴, Jian Shen³,⁴, Wang Gao¹ and Chenglin Xia³,⁴

¹ School of Instrument Science and Engineering, Southeast University, Nanjing, Jiangsu, 210096, China
² School of Electronic and Information Engineering, Jinling Institute of Technology, Nanjing, Jiangsu, 211169, China
³ Nari Technology Co., Ltd, Nanjing, Jiangsu, 211106, China
⁴ State Key Laboratory of Smart Grid Protection and Control, Nanjing, Jiangsu, 211106, China

*Corresponding author’s e-mail: psg@seu.edu.cn

Abstract. With the increase of non-linear loads in power grid and the development of power electronics, the growing serious harmonic pollution has put forward higher requirements for the accuracy and real-time performance of the measurement method. For this problem, this paper proposes a wide-frequency harmonic signal measurement method based on window interpolation FFT and adaptive neural network. After the window function is selected, the frequency of harmonic is estimated by double-spectrum-line interpolation algorithm based on three-term third derivative Nuttall window, and then the smoothed frequencies are input into RLS-Adaline neural network to estimate the amplitude and phase of each harmonic. The simulation results show that the proposed method has higher measurement accuracy and stability under few data conditions and low signal-to-noise ratios than conventional window interpolation algorithms. High accuracy measurement of harmonics with short time window in the range of 0~2.5kHz is effectively realized.

1. Introduction

With the rapid development of renewable energy and smart grids in recent years, the power grid has shown the development trend of complex interconnection and power electronic[1-2]. Various types of nonlinear loads and power electronic equipment introduce complex harmonic signals including high-order harmonics into the grid. This has led to the complexity of power system maintenance and device design, making grid operation increasingly difficult and risky[3].

Real-time and accurate analysis and measurement of harmonics are important for the safe and stable operation of power systems. The IEC 61000 series of standards and technical reports on electromagnetic compatibility and high-voltage power systems published by the International Electrotechnical Commission (IEC) provide analysis and limitation for harmonics within the 50th order[4]. However, most of the existing measurement techniques and devices are aimed at monitoring and analysing grid signals in the low and medium frequency ranges, making it difficult to meet the requirement of wide-
frequency signals measurements with short time window in the context of power electronic. The Fast Fourier Transform (FFT), which has the advantages of short computing time, is most widely used in harmonic analysis. Although scholars have proposed applying window functions[5], spectral line interpolation correction[6] and multiple modulation zoom spectrum analysis (ZoomFFT) [7] to deal with the fence effect and spectrum leakage of this kind of algorithm, the measurement accuracy is low when there are short time windows and fluctuations in fundamental frequency. The research on various modern spectrum estimation algorithms applied to grid signal analysis has been continuously developed. Among them, the algorithms based on Multiple Signal Classification (MUSIC) [8-9] and Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) [10] have the properties of super-resolution, but neither is suitable for signal measurement in wide frequency range and the accuracy is susceptible to noise. Machine learning methods have the characteristics of strong adaptivity and high accuracy. Compared with methods such as radial basis function (RBF) neural network and support vector machine (SVM) [11-12], adaptive linear (Adaline) neural network does not require sample training[13-15]. Nevertheless, the method requires accurate signal frequencies to ensure high estimation accuracy, and frequency deviations will result in no convergence. The iterative algorithm of Adaline neural network is an important factor affecting its performance. Although the commonly used LMS algorithm has the advantages of simple structure and easy implementation, its convergence rate is slow and there is a contradiction between it and the residual error. While the convergence rate of RLS algorithm is an order of magnitude higher than that of LMS algorithm, it can also ensure high convergence accuracy[16].

To this end, after the selection of the window function, this paper combines window interpolation FFT with RLS-Adaline neural network to realize high accuracy estimation of harmonics’ parameters in the range of wide frequency. The effectiveness of the method in this paper is verified by simulation. In this way, the basis is provided for the realization of real-time and accurate analysis of power electronic grid.

2. Window interpolation FFT and frequency smoothing

2.1. Window function selection

As different window functions have different characteristics and interpolation correction processes, the windows are compared and selected. The time domain expression of cosine window function is:

\[ w(n) = \sum_{h=0}^{H-1} (-1)^h a_h \cos(2\pi nh/N), n = 0,1,\ldots, N-1 \]  

(1)

where \( H \) is the number of terms of the window function and there is \( \sum_{h=0}^{H-1} a_h = 1, \sum_{h=0}^{H-1} (-1)^h a_h = 0 \). The main-lobe widths and side-lobe characteristics of common cosine windows are shown in Table 1[17-18].

| Windows                     | Main-lobe width | Sidelobe peak (dB) | Asymptotic decay (dB/oct) |
|-----------------------------|----------------|--------------------|---------------------------|
| Hanning                     | 8\pi/N         | -31.5              | 18                        |
| Hamming                     | 8\pi/N         | -43                | 6                         |
| Blackman                    | 12\pi/N        | -58                | 18                        |
| Three-term first derivative Nuttall | 12\pi/N    | -64                | 18                        |
| Three-term third derivative Nuttall | 12\pi/N  | -47                | 30                        |
| Four-term third derivative Nuttall  | 16\pi/N  | -83                | 30                        |
Four-term fifth derivative Nuttall 
16\pi/N  -61  42

Five term Rife-Vincent(I) 
20\pi/N  -74.5  30

Fourier transform is difficult to realize synchronous sampling. However, in the case of asynchronous sampling and applying short time windows, there will be more serious spectrum leakage and fence effects leading to inaccurate parameter estimation. When the Adaline network is used for signal parameter estimation, it needs to be based on exact harmonic number and frequencies to ensure high estimation accuracy, while deviations of frequency will lead to a significant drop in estimation accuracy. Therefore, the focus should be on selecting a window function with excellent side-lobe characteristic and a narrow main lobe, so as to realize the suppression of spectrum leakage and high frequency resolution. Since there is a contradiction between the main-lobe width and side-lobe characteristic, this paper intends to select a window with no more than four coefficients from Table 1 to process the signal.

2.2. Double-spectrum-line interpolation algorithm
A proper window can suppress spectrum leakage, but the fence effect requires spectrum line interpolation algorithms to reduce its impact. The interpolation algorithm of the proposed method is only used to estimate the frequency of harmonics. Compared with multi-spectrum-line interpolation, the double-spectrum-line interpolation algorithm with a small quantity of computation can fully meet the accuracy requirement.

The expression of a grid signal containing $M$ harmonic components is:

$$x(t) = \sum_{i=1}^{M} A_i \sin(2\pi f_i t + \phi_i)$$

where $f_0$ is fundamental frequency, $k_i$ is the order of harmonics, $A_i$ and $\phi_i$ are the amplitude and phase of the $k_i$th harmonic.

A cosine window is multiplied with the sequence obtained by sampling $x(t)$ at a sampling frequency $f_s$:

$$y(n) = x(n)w(n), n = 0, 1, ..., N-1$$

The result of the DFT of $y(n)$ in digital frequency form is:

$$Y(\lambda) = Y(e^{j\lambda})_{\lambda=2\pi N} = A_m W(\lambda - \lambda_m)e^{j((-N-1)\pi)(\lambda - \lambda_m)/N + \phi_m}$$

where $\lambda = Nf_0/2\pi$, $\lambda_m = Nf_m/2\pi = Nf_s/f_s$, $W(\lambda) = \sum_{h=0}^{H-1} a_h [W_r(\lambda - h) + W_r(\lambda + h)]/2$, $a_h$ is the coefficient of cosine window, $W_r(\lambda) = \sin(\lambda\pi)/\sin(\lambda\pi/N)$ is the amplitude spectrum of rectangular window in digital frequency form.

At this time, the digital frequency position corresponding to $f_m$ is:

$$\lambda_m = k_m + \delta_m$$

where $k_m$ is a sampling point, $\delta_m$ is the interpolation coefficient and $\delta_m \in [0, 1)$. In the case of asynchronous sampling, $\delta_m$ is not equal to 0, indicating that there is a deviation between the amplitude spectrum $X(k_m)$ at $k_m$ and the real frequency $f_m$.

The amplitude ratio of two adjacent spectral lines near the frequency point is $\beta_m$. Then Substituting equations (4) and (5) into $\beta_m$, we obtain
where \( c_i \) and \( b_{2i} \) are polynomial functions of \( a_m \) [19].

Among the cosine windows, only the Hanning, the three-term third derivative Nuttall and the Four-term fifth derivative Nuttall have \( i icb i = 0, i \neq 0 \). Thus, explicit formulas for \( \delta_m \) with respect to \( \beta_m \) are obtained to correct the result of spectrum analysis.

The correction formula for frequency is:

\[
f_w = \frac{(k_w + \delta_m) f_s}{N}
\]
2.3. Frequency smoothing

Since the interpolation algorithm corrects each spectrum peak individually, the correction accuracy varies at different frequency points. In addition, the fence effect is more significant when applying short time windows, further resulting in lower correction precision.

For the purpose of solving this problem, this paper adopts frequency smoothing to reduce the frequency estimation error. Firstly, according to the property that harmonics are all integer multiples of the fundamental frequency, the fundamental frequency corresponding to each harmonic obtained by window interpolation FFT is calculated. Then, after removing the outliers (i.e. the maximum and minimum values), the remaining fundamental frequencies are averaged. Finally, the smoothed fundamental frequencies are used to calculate the harmonic frequencies in turn, thereby reducing the frequency estimation error.

3. RLS-Adaline neural network

![Diagram of RLS-Adaline neural network.](image)

Adaline neural network, consisting of adaptive linear neurons, is a signal parameter estimation method suitable for few data conditions. The basic structure of the network is shown in the figure 2, where \( z(n) = [z_{1n}, z_{2n}, z_{3n}, \ldots, z_{kn}]^T \) is the input vector, \( w(n) = [w_{on}, w_{1n}, w_{2n}, \ldots, w_{kn}]^T \) is the weight vector, \( \hat{s}(n) = w^T(n)z(n) \) is the output of the network, \( s(n) \) is the desired output, i.e. the signal sampling sequence, and \( e(n) = s(n) - \hat{s}(n) \) is the posterior error.

Least mean square (LMS) method or least squares (LS) method is generally used to iterate the network weight vector, thereby allowing the error function to gradually converge and reach a specified threshold. At this point, the output of Adaline neural network can effectively track the sampling sequence.

When using the network for harmonic parameter estimation, the sum-and-difference expansion of the sampling sequence of equation (2) is:

\[
x(n) = \sum_{i=1}^{M} A_i \sin(k_i \omega_n + \phi_i) + d_0 = \sum_{i=1}^{M} [a_i \cos(k_i \omega_n) + b_i \sin(k_i \omega_n)] + d_0
\]

where \( \omega_0 = \frac{2\pi f_b}{M} \), \( a_i = A_i \sin \phi_i \), \( b_i = A_i \cos \phi_i \).

It is obvious that the amplitude and phase of each harmonic can be obtained from \( a_i \) and \( b_i \). At this time, Adaline neural network is set to:

\[
z(n) = [z_{on}, z_{2n}, z_{3n}, \ldots, z_{kn}]^T
\]

\[
= [1, \cos(\omega_n), \sin(\omega_n), \cdots, \cos(k_M \omega_n), \sin(k_M \omega_n)]^T
\]

\[
w(n) = [w_{on}, w_{1n}, w_{2n}, \cdots, w_{kn}]^T
\]

\[
= [d_0, a_1, b_1, \cdots, a_M, b_M]^T
\]
Recursive Least Squares (RLS), which has a faster convergence rate than LMS, is a fast algorithm of least squares. When RLS is used as the learning algorithm for the network, a cost function based on exponential weighting is used:

$$J(n) = \sum_{i=0}^{n} \lambda^{n-i} |e(n)|^2$$  \hspace{1cm} (12)

where $\lambda$ is the forgetting factor and $\lambda \to 1^-$, which has the effect of forgetting the past errors and thus affects the dynamic tracking performance and convergence accuracy of the RLS method.

The process of weight updating of RLS-Adaline neural network is as follows[22].

$$k(n) = \frac{P(n)z(n)}{\lambda + z'(n)P(n)z(n)}$$  \hspace{1cm} (13)

$$w(n+1) = w(n) + k(n)e(n)$$  \hspace{1cm} (14)

$$P(n+1) = \frac{P(n) - k(n)z^T(n)P(n)}{\lambda}$$  \hspace{1cm} (15)

where $k(n)$ is the Kalman gain vector, $e(n) = s(n) - w'(n-1)x(n)$ is the a priori error, $P(n)$ is the inverse correlation matrix and $P(0) = \delta^{-1}I_{2M+1}$, $\delta$ is the regularization parameter related to SNR and $0<\delta=1$, and $I_{2M+1}$ is the unit matrix.

Finally, when the error is less than the specified threshold or the algorithm reaches the maximum number of iterations, the learning is completed. The amplitude and phase of each harmonic can be calculated from the weight vector:

$$A_i = \sqrt{a_i^2 + b_i^2}$$  \hspace{1cm} (16)

$$\varphi_i = \arctan\left(\frac{a_i}{b_i}\right)$$  \hspace{1cm} (17)

4. Simulation results

According to the characteristics of the actual grid signal, the simulation parameters of a wide-frequency harmonic signal (SNR = 40 dB) set in this paper are shown in Table 2.

| Signal component | Order of harmonic | Frequency /Hz | Amplitude /V | Phase/° |
|------------------|-------------------|---------------|--------------|--------|
| 1                | 1                 | 50.12         | 100          | 30     |
| 2                | 3                 | 150.36        | 16           | 20     |
| 3                | 5                 | 250.60        | 8.5          | -30    |
| 4                | 7                 | 350.84        | 4.5          | 120    |
| 5                | 9                 | 451.08        | 5.6          | 80     |
| 6                | 21                | 1052.52       | 3.2          | -120   |
| 7                | 38                | 1904.56       | 1.7          | 90     |
| 8                | 49                | 2455.88       | 2.1          | 150    |

All tests are performed with sampling frequency $f_s = 6.4kHz$ and the number of sampling points $N = 2 \times f_s / 50 = 256$ (i.e., about 2 fundamental periods), which is shorter than the time window for spectrum analysis specified in the IEC standard (10 fundamental periods). In RLS-Adaline neural network, the forgetting factor is $\lambda = 0.9995$ and the regularization parameter is $\delta = 0.0036$. In order to avoid contingency, the results are averaged after repeating the simulation test 2000 times in this paper. The signal estimation results are shown in Figure 3, where the error bar is the standard deviation ($\pm \sigma$) of the relative error of each parameter.
Figure 3. Estimation results and relative errors of the measurement method in this paper (left) and interpolation FFT based on three-term third derivative Nuttall window (right).

The simulation results show that the proposed method effectively combines the advantages of window interpolation FFT and RLS-Adaline neural network algorithm, and realizes the parameter estimation of harmonics under the conditions of low signal-to-noise ratio, short time window and fundamental frequency offset.

In terms of frequency estimation, thanks to frequency smoothing, the average frequency estimation error of interpolation algorithm is reduced from 0.068% to 0.023%, resulting in a more accurate
harmonic frequency input to RLS-Adaline neural network. With respect to amplitude estimation, the accuracy of both methods is positively correlated with amplitude due to the susceptibility of low amplitude harmonics to background noise. The error of RLS-Adaline neural network is less than 3.1%, and the average error is 27.9% lower compared with the interpolation FFT based on three-term third derivative Nuttall window. With regard to phase estimation, RLS-Adaline neural network has an estimation error of less than 3% except for the 38th harmonic with the smallest amplitude. Compared with the interpolation algorithm, the error fluctuation of the method is smaller and the average error is reduced by 55.6%. In summary, the measurement method proposed in this paper has high accuracy and good stability, with significant improvements in frequency and phase estimation accuracy.

5. Conclusions
Aiming at the demand of high accuracy harmonic measurement under the development trend of wide-frequency grid, this paper proposes a measurement method for harmonic that can be applied to short time windows. After selecting and determining the window function based on interpolation process and frequency estimation accuracy, the harmonic frequencies obtained by double-spectrum-line interpolation algorithm based on three-term third derivative Nuttall window are smoothed. Subsequently, they are input to RLS-Adaline neural network for amplitude and phase estimation. The proposed method is simulated and compared with the conventional window interpolation FFT algorithm. The results show that, on the one hand, the method successfully realizes high accuracy measurement of wide frequency harmonic signals. In the case of low signal-to-noise ratio and fundamental frequency offset, the parameters of each harmonic including low amplitude high-order harmonic are estimated stably and effectively. On the other hand, the method effectively combines the advantages of both algorithms, and the accuracy is better than that of the conventional double-spectrum-line interpolation algorithm. On the other hand, the advantages of the both algorithms are effectively combined, and the accuracy is better than that of the conventional double-spectrum-line interpolation algorithm. The method is suitable for real-time tracking measurements of grid harmonic signals on the hardware platform.

It should be noted that the accuracy of the method proposed in this paper is adequate for most harmonic measurement. However, when there are inter-harmonics with frequencies close to the harmonic frequencies in the signal, it is necessary to improve the frequency resolution by increasing the time window length or adopting other frequency estimation algorithms.

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