Phase Space Factor for Two-Body Decay if One Product is a Stable Tachyon

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Abstract

We calculate the phase space factor for a two-body decay in which one of the products is a tachyon. Two threshold conditions, a lower and an upper one, are derived in terms of the masses of the particles and the speed of a preferred frame. Implicit in the derivation is a consistently formulated quantum field theory of tachyons in which spontaneous Lorentz symmetry breaking occurs. The result is to be contrasted with a parallel calculation by Hughes and Stephenson, which, however, implicitly adheres to strict Lorentz invariance of the underlying quantum field theory and produces the conclusion that there is no threshold for this process.

1 Introduction

We deal with a long-standing obstacle to the computation of the two-body decay rate involving a tachyon as a product: the claim by [14] that there is no threshold for the phase space factor $R_2$ when a particle of (regular) mass $M$ decays into a regular particle of mass $m_1$ and a tachyon of mass parameter $m_2$. Related to this is the observation that the formula obtained in [14] for $R_2$ (in which 2 is a tachyon) is not obtained by the simple replacement $m_2^2 \rightarrow -m_2^2$ into the usual formula for $R_2$ (in which 2 is a regular particle).

In this letter, we first point out that a scalar quantum field theory of tachyons cannot lead to a sensible description of particles if the usual Lorentz invariance axiom is insisted upon.
Rather, we assume that an underlying cutoff exists in the one-particle energy-momentum spectrum associated with the QFT. This property may be characterized by stating that a preferred (inertial) frame exists in which all the energies of both particle and anti-particle are positive or zero. Lorentz transformation from the preferred frame then determines the 4-momenta in the energy-momentum spectrum in any other (inertial) frame; therefore, in any frame, boosted with respect to the preferred one, the energy components of certain 4-momenta are allowed to be negative. Clearly, this situation can be viewed as one in which spontaneous symmetry breaking of the Lorentz group has occurred. We may also refer the reader to [8] for an alternative formulation using an unconventional synchronization scheme, which also yields a preferred frame in the QFT.

We claim that with this cut-off in the one-particle spectrum, a lower threshold depending on the speed of the preferred frame $\beta$ is introduced into the two-body decay scenario. Furthermore, when this $\beta$ is fixed, and the tachyonic mass parameter $m_2$ is allowed to approach 0, while $M, m_1$ are held fixed, the limit of $R_2$ obtained is the same as it would be if $m_2$ were a regular mass allowed to approach 0. Furthermore, with $\beta$ fixed and $m_2$ small enough, the formula for $R_2$ (tachyonic particle 2) is indeed obtained from $R_2$ (regular particle 2) by the replacement $m_2^2 \rightarrow -m_2^2$.

By arguing that an underlying QFT involving Lorentz symmetry breaking should be used in calculating the phase space factors, and presumably could be used to make sense of any other QFT type calculation, we dispute the conclusion of [14], namely that “there is strong circumstantial evidence against the proposal that at least one neutrino is a tachyon.” In effect, therefore, the present letter provides indirect support for the tachyonic neutrino hypothesis of Chodos, Hauser, and Kostelecký [5], whose ideas the authors of [14] were clearly attempting to refute.

A brief outline of the contents of this letter is as follows: In Section 2 we summarize the basic elements which we deem essential in a tachyonic quantum field theory. We describe
briefly why we consider the cutoff property to be necessary in the theory, and also offer hints as to how we expect it to be sufficient for (a new notion of) causality and for renormalization. Section 3 reviews the results of calculating the values of the energies of the product particles, as well as the common magnitude of their momenta, first for the case of regular (massive) product particles, then for the case that one is tachyonic. In Section 4 we (re-)derive the single threshold condition for the regular mass case, and the lower and upper threshold conditions for the tachyonic case. Section 5 similarly covers the calculation for the two-body phase space factor for the two cases (regular and tachyonic). Section 6 justifies the claims made in the introduction concerning the (good) behaviour of the phase space factor in the tachyonic case when the tachyonic mass parameter goes to zero. In the conclusions section (Section 7), we attempt to round out the list of clarifications and modifications that would need to be applied to the parts of [14] dealing with two-body decay involving a tachyon. In addition, we work out some of the details of this approach for pion decay, and its reverse process, within the context of assuming that the muon neutrino is tachyonic.

2 Rudiments of tachyonic quantum field theory

In the following we briefly explain the reasons why a quantum field theory of tachyons is expected to require a preferred frame.

To start with, we choose units in which $c = \hbar = 1$ and the metric $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$. Here, $\Box = \eta^{\mu\nu}\partial_\mu \partial_\nu$. We assume that, when constructing a reasonable quantum field theory of tachyons, a choice of space-like hyperplane (i.e., a 3-dimensional subspace of 4-dimensional Minkowski space) through the origin in energy-momentum space, labelled by $(E, \mathbf{p})$, must be made to separate “positive” and “negative” energy modes. We note that, for the tachyonic case, these labels reduce to their usual meaning in the preferred frame, but are mere labels for any inertial frame Lorentz boosted with respect to that frame. (Perhaps a better pair of terms for these would be “upper” and “lower”. ) This distinction is easy to make in the usual
case, since the mode solutions of, say, the Klein-Gordon equation $\Box \phi + m^2 \phi = 0$

$$e^{-i(Et - p \cdot x)}$$

have four-momenta $(E, p)$ which lie on the hyperboloid of two sheets $E^2 - p^2 = m^2$, with

$E > 0$ or $E < 0$. (Here $E = 0$ is included if $m = 0$.) In the case of $m^2 > 0$, the upper sheet is
the natural choice to make to define positive energy modes. One may regard any space-like
hyperplane through the origin as separating the upper and lower mass hyperboloids.

In the case of $m = 0$, we delete the point $(E, p) = 0$ from the cone $E^2 - p^2 = 0$, and once
again any space-like hyperplane through the origin separates the upper and lower cones.

However, for the tachyonic case, the Klein-Gordon equation becomes $\Box \phi - m^2 \phi = 0$, with
a tachyonic mass parameter $m^2 > 0$, and the modes of the form (1) lie on the hyperboloid
of one sheet, $E^2 - p^2 = -m^2$. Thus, the choice of which modes are “positive” and which are
“negative” (or “upper” and “lower”) energy depends upon the choice of space-like hyperplane
through the origin used to separate them. Hence, Lorentz symmetry is spontaneously broken
at this point in the construction.

The alternative to breaking Lorentz symmetry would be to treat all of the modes (resp.
none of them) as “positive energy” modes, and in any frame the appearance of arbitrarily
large negative particle energies (resp. large negative anti-particle energies) would lead to
an unsuitable theory from the QFT point of view. The problem is that the singularities of
the two-point function and Feynman propagator (the Green’s functions), which arise from
a theory which is strictly Lorentz invariant (in the usual sense), would not be appropri-
ate for constructing a renormalizable theory when reasonable interactions are introduced.
Specifically, the wave front set of the theory would not satisfy a restriction (called elsewhere
the “wave front set spectral condition” \[16\] [17], or the “microlocal spectrum condition” \[1\])
which, if satisfied, would ostensibly allow renormalization to proceed in a straightforward
way \[3\].
Insisting upon strict Lorentz invariance in the QFT would also introduce the possibility, however remote, of constructing causality-violating devices. In other words, suitably well-equipped experimentalists would be enabled to construct a device consisting of a relay at a space-like separation from their own lab, by which they could send a message backwards in time to themselves. In progress (15) is a more comprehensive discussion of these points, and a more detailed description of a model satisfying the above symmetry breaking requirement, which allows the wave front set condition on the two-point function to be satisfied, renormalizability to be incorporated, and strong causality violations to be circumvented. The model would also exclude exponentially growing or decaying modes and so would avoid any possibility of producing unstable observables. Furthermore, it would exhibit a natural mechanism (i.e., at the basic QFT level) for maximal parity breaking of the neutrino.

Now suppose a “cut” is made in the spectrum along $E' = 0$ in a certain frame $O' = T$, so that positive and negative energy modes are chosen to have $E' > 0$ and $E' < 0$ respectively. Frame $T$ is denoted the tachyon frame. In a lab frame $O$, we find (according to the usual transformation law for a relativistic boost) that a set of energies $E'$ labelled “positive” in the old frame $T$ now has $E < 0$ in the lab frame $O$. Furthermore, another set of modes labelled “positive” in $T$ now no longer exists with those energies in $O$. From this one might argue that symmetry breaking still does not eliminate the “unphysical” negative energy modes from the theory. However, there exists an argument to rebut this: in $O$ the negative energy particles are understood to be moving backward in time and can ultimately be “re-interpreted” as positive energy anti-particle states moving forward in time. Hence the usual notion of instability, in which particles which move forward in time with negative energy can be created out of the vacuum (such as may be encountered in situations in many body physics), is side-stepped here.

For the purposes of clarity, we shall not invoke the “re-interpretation principle” in $O$, e.g., to ensure that the energies are always positive, but rather we shall consider the
“vacuum state” as defining a particular designation of “upper” (some of which are negative) and “lower” (some of which are positive) energies in $O$. Perhaps there is some merit in viewing the surface $E' = 0$ in $O'$ as defining the Fermi level of a half-filled Dirac sea of particles in $O$, via a Lorentz transformation. (Note that this picture would be especially appropriate for the spin-$\frac{1}{2}$ neutrino.)

Also note that we shall assume the usual connection between spin and statistics, as do [7]. Contrast Feinberg’s assumption of fermionic statistics for the spin-0 case of tachyons [12]. We do not find that this assumption fits with the others we make here, since in the limit in which the tachyonic mass parameter goes to zero, the wrong connection of spin with statistics is obtained for a massless (scalar) theory.

3 Dynamical variables for two-body decay

The calculation for the decay rate of a particle of rest mass $M$ into two regular particles of rest masses $m_1$ and $m_2$, has been reviewed by [14] and is a standard calculation in a first course in elementary particle physics [13]. We list the results here for convenience. The magnitude of the three momentum squared is

$$p^2 \equiv p_1^2 = p_2^2 = \frac{(M^2 - m_1^2 - m_2^2)^2 - 4m_1^2m_2^2}{4M^2}$$

in the rest frame of $M$, while the energies of the resulting particles 1 and 2 are

$$E_1 = \frac{M^2 + m_1^2 - m_2^2}{2M}, \quad \text{and} \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M},$$

respectively. Eq.(2), simply by requiring $p^2$ to be positive, suggests the threshold inequality:

$$M \geq m_1 + m_2.$$
should read

\[ p^2 \geq m_2^2 , \]  

(6)

Here we have changed notation slightly from that of [14]: in the tachyonic case, instead of allowing \( m_2^2 < 0 \) we replace \( m_2^2 \) by \(-m_2^2\); i.e., we change the sign in front of \( m_2^2 \) wherever this power of \( m_2 \) appears. E.g., \( E_2^2 - p_2^2 = m_2^2 < 0 \) becomes \( E_2^2 - p_2^2 = -m_2^2 < 0 \). Then \( m_2 \) is always understood to be a positive real parameter. Note that in Eq.(10) of [14] the inequality goes the wrong way even with their choice of sign convention for \( m_2^2 \). However the above formula Eq.(6) follows easily from the formula for \( p^2 \) when 2 is tachyonic:

\[
\begin{align*}
p^2 &= \frac{(M^2 - m_1^2 + m_2^2)^2 + 4m_1^2m_2^2}{4M^2} \\
&= \frac{(M^2 - m_1^2 - m_2^2)^2 + 4M^2m_2^2}{4M^2} \geq m_2^2 .
\end{align*}
\]

(7)

(8)

It appears the wrong inequality in Eq.(10) of [14] is just a typo, so we have no quibble with it once it is corrected as in our formula Eq.(6). In fact, it agrees with the notion that there should be no exponentially growing modes (i.e., imaginary energy modes) in the QFT.

Note that when particle 2 becomes tachyonic, Eqs.(3) and (4) become (by making the replacement \( m_2^2 \rightarrow -m_2^2 \))

\[
\begin{align*}
E_1 &= \frac{M^2 + m_1^2 + m_2^2}{2M} , \quad \text{and} \\
E_2 &= \frac{M^2 - m_1^2 - m_2^2}{2M} .
\end{align*}
\]

(9)

(10)

4 Threshold condition for tachyonic case

We now take into account the dependence of the lower bound of the tachyonic particle’s energy \( E_2 \) on the preferred frame. We orient the axes so that the preferred frame is moving in the \(-z\) direction, at speed \( \beta \). The condition that the lower bound of the particle’s energy \( E'_2 \) is 0 in the preferred frame \( (E'_2 \geq 0) \) leads to the following condition in the rest frame of \( M \):

\[ \gamma(E_2 + \beta p \cos \theta) \geq 0 , \]

(11)
where $\theta$ is the angle of particle 2’s momentum $\mathbf{p}$ from the $z$ axis. Because of the mass shell condition $p = \sqrt{E_2^2 + m_2^2}$, this is equivalent to

$$E_2 \geq -\frac{m_2 \beta \cos \theta}{\sqrt{1 - \beta^2 \cos^2 \theta}} .$$

(12)

Combining Eqs. (10) and (12) we then obtain

$$\frac{M^2 - m_1^2 - m_2^2}{2M} \geq -\frac{m_2 \beta \cos \theta}{\sqrt{1 - \beta^2 \cos^2 \theta}} .$$

(13)

This inequality determines a range of values of $\theta$, which is possibly empty. These are the allowed directions of the momentum of the tachyon for which the decay can proceed. To find these, we express the formula $\frac{\cos \theta}{\sqrt{1 - \beta^2 \cos^2 \theta}}$ as $\gamma g(x)$, where $x = \cos \theta$, and note that

$$g(x) = \frac{x}{\gamma \sqrt{1 - \beta^2 x^2}}$$

(14)

ranges between $-1$ and $1$ as $x$ varies from $-1$ to $1$. Thus, in order for the reaction to proceed for at least one direction, we must have

$$E_2 = \frac{M^2 - m_1^2 - m_2^2}{2M} \geq -m_2 \beta \gamma .$$

(15)

This is the lower threshold condition for the case that particle 2 is a tachyon, and the preferred frame’s speed is $\beta$ relative to the rest frame of $M$. This condition may also be written as

$$(M + m_2 \beta \gamma)^2 \geq m_1^2 + m_2^2 \gamma^2 , \quad \text{or}$$

$$M \geq \sqrt{m_1^2 + m_2^2 \gamma^2} - m_2 \beta \gamma .$$

(16)

(17)

The range of directions for which decay is possible would then be

$$1 \geq x \geq -h \left( \frac{M^2 - m_1^2 - m_2^2}{2Mm_2 \beta \gamma} \right) ,$$

(18)

where $h$ is the inverse function of $g$. Namely,

$$h(y) = \frac{\gamma y}{\sqrt{1 + \beta^2 \gamma^2 y^2}} = \frac{y}{\sqrt{1 - \beta^2 (1 - y^2)}} .$$

(19)
Thus

$$0 \leq \theta \leq \arccos \left[ -h \left( \frac{M^2 - m_1^2 - m_2^2}{2Mm_2\beta} \right) \right]. \quad (20)$$

Note that, given any positive values for $M, m_1$ and $m_2$, one can find a sufficiently large $\beta$ such that the lower threshold condition Eq.(15) holds. Compare this with the idea, within the context of the tachyonic neutrino hypothesis, of proton decay in a sufficiently boosted frame [7, 6], which has been used as a way to explain the bend in the knee of the primary cosmic ray spectrum [9, 10, 11].

5 Phase space factor for tachyonic case

The phase space factor relevant for two-body decay for the case of regular masses $m_1$ and $m_2$ (following the conventions of [14], who ignore factors of $2\pi$) is

$$R_2 = \int \int \frac{d^3p_1}{2E_1} \frac{d^3p_2}{2E_2} \delta(M - p_1^0 - p_2^0) \delta^{(3)}(p_1 + p_2)$$

$$= \frac{\pi p}{M}, \quad (21)$$

$$= \frac{\pi p}{M}, \quad (22)$$

where $p$ is as in Eq.(2). (Note that in Eq.(11) of [14] the step function factors $\theta(p_1^0), \theta(p_2^0)$ are implied in the product of delta functions $\delta(p_1^2 - m_1^2)\delta(p_2^2 - m_2^2)$.) More explanatory details can be found in Ch. 6 of [13].

In order to perform the analogous calculation in the case where particle 2 is a tachyon, we write the two-body phase space factor as

$$R_2 = \int \int d^4 p_1 \delta(p_1^2 - m_1^2) \theta(p_1^0) d^4 p_2 \delta(p_2^2 + m_2^2) \theta(p_2^0 + \beta p_2 x_2) \delta^{(4)}(P - p_1 - p_2). \quad (23)$$

Here $P = (M, 0)$ and the second theta function incorporates the cutoff in the energy-momentum spectrum of the tachyon, with the preferred frame assumed to be moving in the $-z$ direction at speed $\beta$ as in Section 4. It is expeditious to evaluate the quadruple integral with respect to the $p_2^\mu$ variables by first integrating over $p_2 = |p_2|$, the magnitude
of the spatial momentum of 2. (The quadruple integral with respect to the $p_1^\mu$ variables is integrated first over $p_1^0$, as usual.) The phase space factor then becomes

$$R_2 = \pi p M \left\{ \int_{-1}^{1} dx_2 \theta(E_2 + \beta p_2 x_2) \right\},$$

(25)

where $x_2 = \cos \theta_2$ and $\phi_2, \theta_2$ are the azimuthal and polar angles for the spatial momentum of 2, respectively.

Now one easily sees that if the lower threshold condition Eq. (15) is not satisfied, then the theta function in Eq. (25) is always 0, giving $R_2 = 0$. Otherwise, the integral in Eq. (25) would be a fixed factor between 0 and 2 depending only on $M, m_1, m_2$ and $\beta$. If the condition

$$E_2 \geq \beta p_2 \iff E_2 \geq \beta \gamma m_2$$

(26)

holds, then clearly the theta function is always 1 on the range of integration, the factor in braces in Eq. (25) is 2, and

$$R_2 = \frac{\pi p}{M}.$$  

(27)

If $-\beta \gamma m_2 \leq E_2 \leq \beta \gamma m_2$ then the theta function in Eq. (25) is 1 when $1 \geq x_2 \geq -\frac{E_2}{\beta p_2}$, the factor in braces is $\{1 + \frac{E_2}{\beta p_2}\}$, and

$$R_2 = \frac{\pi}{2M} \left( p_2 + \frac{E_2}{\beta} \right).$$  

(28)

Note throughout that $p_2 = p$ and $E_2$ are evaluated according to Eqs. (8) and (10) respectively. The upper threshold condition Eq. (26) is equivalently expressed as

$$|M - \beta \gamma m_2| \geq \sqrt{m_1^2 + \gamma^2 m_2^2}.$$  

(29)

6 $R_2$ as $m_2$ approaches 0

Our intuition tells us that when $m_2 \to 0$ the phase space factors $R_2$ for the regular and tachyonic cases must converge to the same thing, namely the phase space factor $R_2$ for a massless particle 2. Let us see how this is borne out in direct calculation.
The lower threshold condition for the massless case $m_2 = 0$ is $M \geq m_1$, and a repetition of the derivation as for the massive case gives us the formulae

\[
p = p_1 = p_2 = \frac{M^2 - m_1^2}{2M},
\]

\[
E_1 = \frac{M^2 + m_1^2}{2M},
\]

\[
E_2 = \frac{M^2 - m_1^2}{2M},
\]

which are clearly the limits of Eqs. (2), (3) and (4) as $m_2 \to 0$. Furthermore the phase space factor $R_2$ for $m_2 = 0$ evaluates easily to $R_2 = \frac{\pi p}{M} = \frac{\pi (M^2 - m_1^2)}{2M^2}$, which is the massive case in the limit $m_2 \to 0$.

In order to check that the tachyonic $R_2$ approaches the above limit as $m_2 \to 0$, we need to verify that in this limit, the factor in braces in Eq. (25) approaches 2. If $M = m_1$, then $R_2$ goes to zero as $m_2$ goes to zero (same as the case of massless $m_2$, where $M = m_1$), thus the factor in braces is irrelevant here. It remains to consider the case $M > m_1$. Here, $E_2 > 0$ for small enough $m_2$, and we can take $m_2$ even smaller, if necessary, so that $E_2 \geq \beta \gamma m_2$, which, as we have seen, forces the factor in braces to be 2, and $R_2$ to be $\frac{\pi p}{M}$, as required.

As a further observation here, we note that in this last scenario, with $E_2 > 0$, and $m_2 > 0$ tachyonic and small enough so that $R_2 = \frac{\pi p}{M}$ (i.e., the factor in braces is 2), the expressions for regular and tachyonic particle 2 can be obtained from each other by the replacement $m_2^2 \leftrightarrow -m_2^2$. This justifies the assertions made about $R_2$ (as $m_2 \to 0$) in the introduction.

7 Conclusions

In light of these results, we note some modifications and clarifications that would appear to be necessary in the attempt made by [14] to evaluate the two-body phase space factor. First, with the lower cutoff in the energy-momentum spectrum of the tachyon, there would now be a (lower) threshold condition, giving an inequality between $M, m_1, m_2$ and $\beta$ which specifies under what circumstances the decay may proceed at all, Eq. (15).
Secondly, the phase space factor $R_2$ (always positive or zero) is, in the tachyonic case, no larger than the expression obtained by replacing $m_2^2$ by $-m_2^2$ in the phase space factor for regular $m_2$. When the upper threshold condition Eq. (26) is satisfied, $R_2$ would be exactly equal to the expression obtained by this replacement. The expression Eq. (25) thus supercedes the formula Eq.(13) in [14]. Note that the latter formula must also be considered suspect because it gives the wrong limit as $m_2 \to 0$.

Thirdly, note that, according to Eq.(15), if $M^2 \geq m_1^2 + m_2^2$, then the lower threshold condition would be satisfied for any $\beta$ (always positive). Applying this to the case of pion decay, assuming a tachyonic muon (anti-) neutrino

$$\pi^- \to \mu^- + \bar{\nu}_\mu,$$  \hspace{1cm} (33)

this restriction on $m_2 = m_{\nu_\mu}$ translates into

$$m_2 \leq \sqrt{M^2 - m_1^2} = \sqrt{139.57^2 - 105.66^2} = 91.19 \text{ MeV}.$$ \hspace{1cm} (34)

For the simple reason that any violation of this inequality would surely have shown up in pion decay measurements by now, it seems safe to assume it is satisfied. Then the lower threshold condition is satisfied for any speed $\beta$ of the preferred frame, and pion decay cannot be prevented in any frame.

In the paper [14], the authors consider the inverse process (when the muon neutrino is tachyonic)

$$\mu^- \to \pi^- + \nu_\mu.$$ \hspace{1cm} (35)

Here, the authors suggest that this decay is always allowed (because they have no lower restriction on the energies of the tachyon). In the present approach there is a restriction: the lower threshold condition Eq. (15). Applying it to this case (interchanging $M$ and $m_1$ from the last case), the condition is

$$\frac{m_2}{2M} + \left(\frac{m_2^2 - M^2}{2M}\right) \frac{1}{m_2} \leq \beta \gamma.$$ \hspace{1cm} (36)
The minimum value of the LHS is obtained for \( m_2 = \sqrt{m_1^2 - M^2} = 91.19 \text{ MeV} \), and is \( \frac{\sqrt{m_1^2 - M^2}}{M} = 0.863 \). Thus \( \beta \) must be at least 0.653 in order to allow this decay at all, and that minimum value is achieved for the noticeably large (tachyonic) neutrino mass parameter 91.19 MeV. Taking the results of pion decay measurements at face value, the “worst” tachyonic value so far obtained [1] is \( m_2 \approx 0.4 \text{ MeV} \). We plug in this value as an example of how large \( \beta \) must be to permit any “reverse” decay to occur: the LHS of Eq. (36) is 98.38, which implies a \( \beta \) of over 0.9999. Thus, it would seem that an unreasonably high \( \beta \) would have to exist in order to allow the reverse decay.

A fourth clarification: in the paragraph before their Eq.(12), the authors of [14] claim that, while for regular particles the restriction \( p^0 \geq 0 \) must be made to restrict to the positive energy mass shell, the analogous restriction for tachyons is \( p \geq m_2 \) (using our notation and conventions). We would postulate that the additional restriction \( p^0 + \beta p \cos \theta \geq 0 \) must also be made here, and should be considered the analogue of the restriction to the positive part of the mass shell. The condition \( p \geq m_2 \), which restricts the magnitude of the three momentum of the tachyon, is a very reasonable additional assumption that must be made to rule out imaginary energy modes (which lead to observables growing exponentially in time). Indeed in the two-body decay, we have seen that it follows from the dynamics of the situation under consideration (e.g., conservation of energy and momentum).

In view of the reasonable results we have obtained here for a two-body decay (lower) threshold condition and the phase space factor \( R_2 \), we argue that no (egregiously) “unphysical consequences” are looming here, and suggest that similar sense can be made of the calculations attempted in other parts of [14], according to a QFT incorporating the cut-off in the energy-momentum spectrum of the tachyon. The prospect of such a suitable theory, which was not considered at all in [14], would thus at least partly undermine their claim (as stated in their abstract) that “there is strong circumstantial evidence against the proposal that at least one neutrino is a tachyon.” Indeed, one may view their “unphysi-
cal consequences” as arising from a faulty starting assumption about the underlying QFT, which need not be made (namely that there is no cut-off such as what we have used here). Hence we conclude that the results of the present letter at least partly support the tachyonic neutrino hypothesis initially made in [5].

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