Light scattering by radiation fields: The optical medium analogy

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received 27 February 2013; accepted in final form 5 April 2013
published online 6 May 2013

PACS 04.20.Cv – Classical general relativity: Fundamental problems and general formalism
PACS 04.20.Jb – Exact solutions

Abstract – The optical medium analogy of a radiation field generated by either an exact gravitational plane wave or an exact electromagnetic wave in the framework of general relativity is developed. The equivalent medium of the associated background field is inhomogeneous and anisotropic in the former case, whereas it is inhomogeneous but isotropic in the latter. The features of light scattering are investigated by assuming the interaction region to be sandwiched between two flat spacetime regions, where light rays propagate along straight lines. Standard tools of ordinary wave optics are used to study the deflection of photon paths due to the interaction with the radiation fields, allowing for a comparison between the optical properties of the equivalent media associated with the different background fields.

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Introduction. – An electromagnetic field in any gravitational background, associated with the metric $ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$ and Cartesian-like coordinates $(t, x, y, z)$, can be thought of as propagating in flat spacetime but in the presence of a medium whose properties are determined by conformally invariant quantities constructed from the metric tensor. In fact, the covariant electromagnetic field $F_{\mu\nu}$ and its rescaled contravariant counterpart $\sqrt{-g} F^{\mu\nu}$ can be decomposed as $F_{\mu\nu} \rightarrow (E, B)$ and $\sqrt{-g} F^{\mu\nu} \rightarrow (-D, H)$ to yield the usual Maxwell’s equations in a medium [1–4]:

$$\nabla \cdot B = 0, \quad \nabla \times E = -\partial_t B,
$$

$$\nabla \cdot D = 4\pi \rho, \quad \nabla \times H = \partial_t D + 4\pi J,$$

(1)

where $J_{\mu} \rightarrow (\rho, J)$ satisfies the conservation law

$$\partial_t \rho + \nabla \cdot J = 0.$$

(2)

To these equations the constitutive relations

$$D_a = \epsilon_{ab} E_b - (M \times H)_a, \quad B_a = \mu_{ab} H_b + (M \times E)_a$$

(3)

must be added, where

$$\epsilon_{ab} = \mu_{ab} = -\sqrt{-g} \frac{g^{ab}}{g_{tt}}$$

(4)

play the role of electric and magnetic permeability tensors and

$$M_a = -\frac{g_{ta}}{g_{tt}}$$

(5)

is a vector field associated with rotations of the reference frame. Within this framework, the description of electromagnetic fields in a curved spacetime is equivalently accomplished by solving Maxwell’s equations in a flat spacetime but in the presence of a medium, whose properties are fully specified by the associated constitutive relations. Such a material medium is in general anisotropic and has no birefringence, due to the proportionality between polarization tensors. Notice that the conformal invariance of Maxwell’s equations is reflected, in the present formulation, by the independence of the dielectric tensors $\epsilon_{ab}$ and $\mu_{ab}$ from a conformal factor in

1We use here geometrical units so that $c = 1 = G$ and $+++-$ for the Lorentzian signature of the metric tensor. Greek indices run from 0 to 4 and Latin ones from 1 to 3.
the metric components, a property also shared by the “spatial” vector $\mathbf{M}$.

Following [4], if one introduces the complex vectors
\begin{align}
\mathbf{F} &= \mathbf{E} + i\mathbf{H}, & \mathbf{S} &= \mathbf{D} + i\mathbf{B},
\end{align}
the constitutive relations read
\begin{align}
S_a &= \epsilon_{ab} F_b + i(M \times F)_a.
\end{align}
Furthermore, it is possible to write the electromagnetic equations (1) in a form which is particularly suitable for the discussion of wave phenomena, i.e.,
\begin{align}
\frac{1}{i} \nabla \times \mathbf{F} = \partial_t \mathbf{S} + 4\pi \mathbf{J}, & \quad \nabla \cdot \mathbf{S} = 4\pi \rho.
\end{align}
Plane waves satisfy the above equations in a small spacetime region, where the metric tensor can be always assumed to vary in space and time only slightly with respect to the wavelength and the period of the wave, respectively. Therefore, one can apply th\emdash the standard tools of ordinary wave optics to study the optical properties of the equivalent medium. For instance, in the absence of currents, one may look for solutions of the form
\begin{align}
\mathbf{E} &= E_0 e^{i(n\mathbf{k}\cdot \mathbf{x} - \omega t)}, & E_0 &= \text{const}, \quad |\mathbf{k}| = \omega,
\end{align}
where $n$ is an effective refraction index. Similar expressions hold for $\mathbf{B}$, $\mathbf{H}$ and $\mathbf{D}$. Maxwell’s equations (1) then imply
\begin{align}
n \times \mathbf{E} &= \mathbf{B}, & n \times \mathbf{H} &= -\mathbf{D}
\end{align}
and $\mathbf{n} \cdot \mathbf{B} = 0 = \mathbf{n} \cdot \mathbf{D}$, where $\mathbf{n} = (n/\omega)\mathbf{k} = ne$ and $e = k/\omega$ is the spatial unit vector of the photon direction.

A substitution of the constitutive relations (3) into eqs. (10) leads to
\begin{align}
[\epsilon_{ik} + \epsilon_{ijr} \epsilon_{srk}(\mathbf{n} - \mathbf{M})(\mathbf{n} - \mathbf{M})_a(e^{-1})_{ka}] E_b = 0,
\end{align}
where $\epsilon_{ijk}$ denotes the Levi-Civita alternating symbol. One finds a similar equation for $H_b$. The existence of eigen-solutions for $\mathbf{E}$ implies the generalized Fresnel equation
\begin{align}
n^2\epsilon_{ee} - 2n\epsilon_{aM} + \epsilon_{MM} - \text{det}(\epsilon) = 0,
\end{align}
where the compact notation $X_{ab} A^a B^b = X_{ab}$ has been introduced for contraction of a generic matrix $X_{ab}$ with vectors $A_a$ and $B_b$. The above equation gives the relation between the effective refraction index of the medium and the components of the polarization tensors and the direction of propagation of the electromagnetic wave [2]. The solution for the refraction index is thus given by
\begin{align}
n = \frac{1}{\epsilon_{ee}} \left[ \epsilon_{eM} + \sqrt{\text{det}(\epsilon)[\epsilon_{ee} - (\epsilon^{-1})_{ee}]} \right],
\end{align}
where $c = \mathbf{M} \times e$ and $\text{det}(\epsilon) = \epsilon_1 \epsilon_2 \epsilon_3$, in a coordinate system in which electric and magnetic permeability tensors are diagonal, i.e., $\epsilon_{ab} = \text{diag}[\epsilon_1, \epsilon_2, \epsilon_3] = \mu_{ab}$. In the special case $M_a = 0$, the above expression simplifies to
\begin{align}
n = \sqrt{\text{det}(\epsilon) / \epsilon_{ee}}.
\end{align}

In the literature the “optical medium analogy” has been widely used to study both the propagation of electromagnetic waves in a gravitational field and scattering processes by either static or rotating black holes as well as by some cosmological solution, e.g., the Gödel universe [3–6]. Mashhoon and Grishchuk [7] have also considered some weak gravitational field background associated with a gravitational plane wave. In the present work we are interested in investigating the scattering of light by a radiation field given by either an exact gravitational plane wave or an exact electromagnetic wave. We assume that the interaction region is sandwiched between two flat spacetime regions. A similar situation has been considered in ref. [8], where the scattering of massive particles was studied instead. We first develop the equivalent medium analogy of the associated background fields, by exploring the optical properties of the corresponding effective material media. Using standard tools of ordinary wave optics we then study the deflection of photon paths due to the interaction with the radiation field, allowing for a comparison between the optical properties of the equivalent medium associated with the different background fields.

Radiation scattering from a sandwiched gravitational wave. – The background of a radiation field (either a gravitational plane wave or an electromagnetic wave) propagating along the $z$-axis of an adapted coordinate system $(t, x, y, z)$ is described by the line element [9]
\begin{align}
ds^2 = -dt^2 + dx^2 + F(t - z)^2 dz^2 + G^2(t - z)^2 dy^2
&= -2du dv + F(u)^2 dx^2 + G^2(u) du^2.
\end{align}
where the null coordinates $u = (t - z)/\sqrt{2}$ and $v = (t + z)/\sqrt{2}$ are related to the Cartesian ones in a standard way.

In this section we consider the (vacuum) case of a gravitational plane wave with a single polarization state ($+$ state), corresponding to metric functions
\begin{align}
F(u) = \cos b_{(gw)} u, & \quad G(u) = \cosh b_{(gw)} u,\end{align}
where the background quantity $b_{(gw)}$ is related to the frequency of the wave by $b_{(gw)} = \sqrt{2\omega_{(gw)}}$.

Let the plane gravitational wave be sandwiched between two Minkowskian regions $u \in (-\infty, 0) \cup (u_1, \infty)$. The coordinate horizon of the metric (15) at $b_{(gw)} u = \pi_2$, where it is degenerate, is avoided by restricting the coordinate $u$ to the interval $[0, u_1]$ with $b_{(gw)} u_1 < \pi_2$. The matching conditions at the boundaries of the sandwich impose restrictions on the metric functions $F$ and $G$ before and after the passage of the wave where the spacetime is Minkowskian.
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A possible choice to extend the metric to all values of \( u \) is the following [10]:

\[
F(u) = \begin{cases} 
  1, & u < 0, \\
  \cos b_{gw} u, & 0 \leq u \leq u_1, \\
  \alpha + \beta u, & u \geq u_1,
\end{cases} \quad (I)
\]

\[
G(u) = \begin{cases} 
  1, & u < 0, \\
  \cos b_{gw} u, & 0 \leq u \leq u_1, \\
  \gamma + \delta u, & u \geq u_1,
\end{cases} \quad (I)
\]

where labels I, II and III refer to in-zone \((u \leq 0)\), wave-zone \((0 < u < u_1)\) and out-zone \((u \geq u_1)\), respectively. The constants \(\alpha, \beta, \gamma\) and \(\delta\) can be found by requiring \(C^1\) regularity conditions for the metric functions at the boundaries \(u = 0\) and \(u = u_1\) of the sandwich, namely

\[
\alpha = \cos b_{gw} u_1 + b_{gw} u_1 \sin b_{gw} u_1, \\
\beta = -b_{gw} \sin b_{gw} u_1, \\
\gamma = \cos b_{gw} u_1 - b_{gw} u_1 \sin b_{gw} u_1, \\
\delta = b_{gw} \sinh b_{gw} u_1.
\] (18)

![Diagram](image.png)

Fig. 1: The null coordinate relationships in the \(t-z\) plane (orthogonal to the plane-wave fronts aligned with the \(x-y\) planes) for a sandwiched radiation field between the two null hypersurfaces \(u = 0\) and \(u = u_1 > 0\).

The “optical medium analogy”. Working out the electromagnetic analogy, we find for the electric and magnetic permeability tensors (4) of the corresponding equivalent medium in the wave-zone the following expressions:

\[
\epsilon_{ab} = \mu_{ab} = \text{diag}[\epsilon_1, \epsilon_2, \epsilon_3] = \text{diag} \begin{bmatrix} G & F & FG \end{bmatrix},
\] (19)

while the rotation vector \(M_a = 0\) vanishes identically, so that the medium is linear and nongyrotropic. In the weak field limit \(b_{gw} u \ll 1\) we have

\[
\epsilon_1 = \frac{\cosh b_{gw} u}{\cosh b_{gw} u} \sim 1 + b_{gw} u^2, \\
\epsilon_2 = \frac{1}{\epsilon_1} \sim 1 - b_{gw} u^2, \\
\epsilon_3 = \cosh b_{gw} u \cos b_{gw} u \sim 1,
\] (20)

up to the second order. The general definition (14) of the refraction index then gives

\[
n = \frac{FG}{\sqrt{G^2 \epsilon_1^2 + F^2 \epsilon_2^2 + F^2 G^2 \epsilon_3^2}}
\] (21)

The anisotropic properties of the medium are evident considering photons traveling in the three coordinate directions. For instance, for photons along the \(x\)-axis is \(e = \partial_x\) so that \(n = n_x = F\), whereas for photons along the \(y\)-axis is \(n = n_y = G\) and for photons along the \(z\)-axis is \(n = n_z = 1\). In fact, starting from the vanishing of the line element \(ds^2 = 0\) for photons, we have

\[
dt^2 = F(u)^2 dx^2 + G(u)^2 dy^2 + dz^2 \\
= \epsilon_2 \epsilon_3 dx^2 + \epsilon_1 \epsilon_3 dy^2 + dz^2,
\] (22)

so that the travel coordinate time of light can be expressed in terms of an associated optical path

\[
dt = \sqrt{n_x^2 dx^2 + n_y^2 dy^2 + n_z^2 dz^2},
\] (23)

whereas the identification

\[
n_x = \sqrt{\epsilon_2 \epsilon_3}, \quad n_y = \sqrt{\epsilon_1 \epsilon_3}, \quad n_z = 1,
\] (24)

characterizes the anisotropic properties of the medium. Explicitly, in the wave region, we find

\[
n_x = F(u) < 1, \quad n_y = G(u) > 1,
\] (25)

and hence it follows that the \(x\) and \(y\) axes are naturally defined as the superluminal and subluminal direction of light propagation, respectively. It is worth noticing that such a definition holds only for coordinate components of the velocity of light in the chosen coordinate system.

In the in-zone we have simply

\[
\epsilon_1 = \epsilon_2 = \epsilon_3 = 1,
\] (26)

so that \(n = n_x = n_y = n_z = 1\), whereas in the out-zone it is

\[
\epsilon_1 = \gamma + \delta u = \frac{1}{\epsilon_2}, \quad \epsilon_3 = (\gamma + \delta u)(\alpha + \beta u).
\] (27)

In the latter case (out-zone) it is always possible to perform a coordinate transformation \((t, x, y, z) \rightarrow (T, X, Y, Z)\) that will convert the line element to the standard Minkowskian form

\[
U = u, \quad X = F(u)x, \quad Y = G(u)y, \\
V = v + \frac{1}{2} F(u)F'(u)x^2 + \frac{1}{2} G(u)G'(u)y^2, \\
T = \frac{1}{\sqrt{2}} (V + U), \quad Z = \frac{1}{\sqrt{2}} (V - U).
\] (28)
In the new Cartesian coordinates, the electromagnetic waves propagate along the null geodesics of the spacetime [11]. Therefore, one can study how an incoming light beam propagating along a straight line in the in-zone is deflected when passing through the scattering region before emerging in the out-zone, where it moves again along a straight line but with different direction.

In the in-zone, the constant photon 4-momentum \( P_I \) can be parametrized in terms of the conserved specific momenta \( p_x, p_y \) and \( p_y \) associated with the three Killing vectors \( \partial_u, \partial_x, \partial_y \) as

\[
P_I = -p_v \left( \partial_u + \frac{p_x^2}{2p_v} \partial_x \right) + p_\perp = -\frac{1}{\sqrt{2}} p_v \left( 1 + \frac{p_x^2}{2p_v^2} \right) \partial_t + p_\perp
\]

\[
-\frac{p_v}{\sqrt{2}} \left( -1 + \frac{p_x^2}{2p_v^2} \right) \partial_z,
\]

where \( p_\perp = p_x \partial_x + p_y \partial_y \) (with \( p_x^2 + p_y^2 \) using the flat spacetime notation for convenience) and \( p_v < 0 \) for \( P_I \) to be future-pointing.

In the wave-zone we have instead

\[
P_{II} = -p_v \partial_u - \frac{1}{2p_v} \left( \frac{p_x^2}{F(u)^2} + \frac{p_y^2}{G(u)^2} \right) \partial_t
\]

\[
+ \frac{p_x}{F(u)^2} \partial_x + \frac{p_y}{G(u)^2} \partial_y,
\]

\[
= -\frac{p_v}{\sqrt{2}} \left[ 1 + \frac{1}{2p_v^2} \left( \frac{p_x^2}{F(u)^2} + \frac{p_y^2}{G(u)^2} \right) \right] \partial_t
\]

\[
- \frac{p_v}{\sqrt{2}} \left[ -1 + \frac{1}{2p_v^2} \left( \frac{p_x^2}{F(u)^2} + \frac{p_y^2}{G(u)^2} \right) \right] \partial_z
\]

\[
+ \frac{p_x}{F(u)^2} \partial_x + \frac{p_y}{G(u)^2} \partial_y,
\]

Finally, in the out-zone the constant photon 4-momentum in \((U, V, X, Y)\) coordinates is given by

\[
P_{III} = -Q_V \left( \partial_u + \frac{Q_x}{2Q_V} \partial_X \right) + Q_\perp,
\]

where \( Q_\perp = Q_X \partial_X + Q_V \partial_V \) (with \( Q_x^2 = Q_x^2 + Q_y^2 \)), and the constants \( Q_v, Q_X, Q_V \) are determined by imposing the matching conditions at the boundary II–III where \( u = u_1 \).

Thus one finds

\[
Q_V = p_v,
\]

\[
Q_X = p_x \sin(b_{(gw)}u_1)b_{(gw)}x_0 + p_x \cos(b_{(gw)}u_1),
\]

\[
Q_Y = -p_v \sinh(b_{(gw)}u_1)b_{(gw)}y_0 + p_y \cosh(b_{(gw)}u_1),
\]

where \( x_0 \) and \( y_0 \) denote the initial values of the coordinates \( x \) and \( y \) along the directions transverse to the wave propagation. Recalling that at the boundary \( u = u_1 \) (i.e., the most relevant surface to study light scattering in terms of refraction phenomena) we have

\[
F(u_1) = \cos(b_{(gw)}u_1) = n_x|u_1|
\]

\[
G(u_1) = \cosh(b_{(gw)}u_1) = n_y|u_1|,
\]

and

\[
F'(u_1) = -b_{(gw)} \sin(b_{(gw)}u_1) = n_x'|u_1|
\]

\[
G'(u_1) = b_{(gw)} \sinh(b_{(gw)}u_1) = n_y'|u_1|,
\]

(a prime denoting differentiation with respect to \( u \)), the transverse momentum in the out-zone can be written as

\[
Q_\perp = A p_\perp - p_v A' X_0,
\]

where

\[
A = \begin{pmatrix} n_x & 0 \\ 0 & n_y \end{pmatrix}, \quad A' = \begin{pmatrix} n_x' & 0 \\ 0 & n_y' \end{pmatrix}
\]

and \( X_0 = (x_0, y_0) \). The symbol \(|u_1|\) has been omitted for simplicity.

If the photon impacts the wave region (in the transverse plane) at the origin of the coordinates, i.e., \( x_0 = 0 = y_0 \), eq. (35) implies

\[
Q_x = n_x p_x, \quad Q_y = n_y p_y,
\]

expressing along each axis the Snell law of refraction of ordinary optics. Since \( n_x \neq n_y \), the relation (35) for \( X_0 = 0 \) can be represented by an ellipse in the space of momenta, as shown in fig. 2. One can also evaluate the angle between the directions of \( p_\perp \) and \( Q_\perp \), i.e., the scattering angle \( \alpha_s \), defined by

\[
\cos \alpha_s = \frac{p_\perp \cdot Q_\perp}{|p_\perp| |Q_\perp|} = \frac{n_x p_x^2 + n_y p_y^2}{\sqrt{p_x^2 + p_y^2} \sqrt{n_x^2 p_x^2 + n_y^2 p_y^2}}
\]

If \( p_x = 0 \) (or \( p_y = 0 \)), one simply finds \( \alpha_s = 0 \). Introducing the polar representation of transverse momenta \( p_x = p_\perp \cos \psi, p_y = p_\perp \sin \psi \), the previous expression becomes

\[
\cos \alpha_s = \frac{n_x \cos^2 \psi + n_y \sin^2 \psi}{\sqrt{n_x^2 \cos^2 \psi + n_y^2 \sin^2 \psi}},
\]

which is invariant for \( \psi \rightarrow \pi - \psi \). Its behavior as a function of \( \psi \) is shown in fig. 3. It turns out that the scattering angle \( \alpha_s \) remains close to zero in the whole range of allowed transverse momenta.

Vice versa, if the incident photon is purely longitudinal, i.e., \( p_x = 0 = p_y \), eq. (35) reduces to \( Q_\perp = -p_v A' X_0 \). In this case the photon travels along the same direction as the gravitational wave, a fact that makes trivial the whole problem.
Fig. 2: (Colour on-line) Gravitational wave case. In the space of transverse momenta, the different refraction indices \( n_x \) and \( n_y \) on the x and y axes imply that an initial circular distribution of momenta for incident photons becomes an ellipse after the refraction process. The plot shows two cases: \( p_\perp = 1 \) (external circle and ellipse) and \( p_\perp = 1/2 \) (internal circle and ellipse). The (generic) line intersecting the circle and the ellipse (in both cases) identifies corresponding pairs \((Q_x, Q_y)\) related by eq. (35) with choice of \( x_0 = 0 \) and using \( b_{\text{gw}} u_1 = \pi/4 \).

Fig. 3: Gravitational wave case. The angle \( \alpha \), given by eq. (38) and written as a function of the angle \( \psi \) of the polar representation of transverse momenta in the case \( b_{\text{gw}} u_1 = \pi/4 \).

**Radiation scattering from a sandwiched electromagnetic wave.** — Let us now consider the similar situation in which an electromagnetic plane wave propagates along the positive z-axis and is sandwiched between two flat spacetime regions, as shown in fig. 2 for the gravitational wave. The corresponding conformally flat line element is given by eq. (15) with functions [12]

\[
F = \cos(b_{\text{em}} u) = G,
\]

(40)
differing from the corresponding gravitational wave case only by a trigonometric rather than hyperbolic cosine appearing in \( G \), so that the above analysis is easily repeated. The present case corresponds to a nonvacuum spacetime which is a solution of the Einstein equations with energy momentum tensor

\[
T = \phi_0 K \otimes K, \quad K = b_{\text{em}} \partial_u,
\]

(41)
where \( \phi_0 = 1/4\pi \) and the background quantity \( b_{\text{em}} \) is related to the frequency of the wave by \( b_{\text{em}} = \sqrt{2}\omega_{\text{em}} \).

As in the previous section, the coordinate horizon of the metric at \( b_{\text{em}} u = \pi/2 \) is avoided by restricting the coordinate \( u \) to the interval \([0, u_1]\) with \( u_1 < \pi/(2b_{\text{em}}) \).
The matching conditions at the two null hypersurface boundaries now imply

\[
\alpha = \cos(b_{\text{em}} u_1) + b_{\text{em}} u_1 \sin(b_{\text{em}} u_1) = \gamma,
\]

\[
\beta = -b_{\text{em}} \sin(b_{\text{em}} u_1) = \delta.
\]

(42)
The previous analysis can be repeated straightforwardly, now implying in the wave region

\[
n_x = n_y = \cos b_{\text{em}} u \equiv n,
\]

(43)
so that \( n < 1 \). The relation between in- and out-momenta is now given by

\[
Q_V = p_v, \quad Q_X = p_x \sin(b_{\text{em}} u_1) b_{\text{em}} x_0 + p_x \cos(b_{\text{em}} u_1), \quad Q_Y = p_y \sin(b_{\text{em}} u_1) b_{\text{em}} y_0 + p_y \cos(b_{\text{em}} u_1).
\]

(44)
Recalling that at the boundary

\[
F(u_1) = \cos(b_{\text{em}} u_1) = n|_{u_1}, \quad F'(u_1) = -b_{\text{em}} \sin(b_{\text{em}} u_1) = n'|_{u_1},
\]

(45)
the emerging transverse momentum can be written in the same form as in eq. (35) with \( A = n I \) and \( A' = n'| I \), \( I \) being the identity matrix, so that

\[
Q_\perp = n p_\perp - p_x n' X_0.
\]

(46)
For $X_0 = 0$ this relation implies

\[ Q_x = np_x, \quad Q_y = np_y, \tag{47} \]

which can be represented by a circle in the momentum space, as shown in fig. 4. If $X_0 = 0$ (for simplicity) the scattering angle is always $\alpha_s = 0$, showing the difference between scattering by a gravitational and an electromagnetic wave.

**Concluding remarks.** – The optical medium analogy of a radiation field sandwiched between two flat regions has been analyzed for the two cases where the radiation field is that of an exact gravitational plane wave or that of an exact electromagnetic wave in a general relativistic context. In particular we have studied the typical scattering problem of a light beam by such a radiation field, elucidating the main features of the process in terms of associated optical properties of the equivalent optical active medium. The most relevant physics involves the plane transverse to the direction of propagation of the wave: here the equivalent medium of the gravitational wave is inhomogeneous and anisotropic, whereas that of an electromagnetic wave is inhomogeneous but isotropic. In both cases the medium is active in the sense that the refraction index depends on time. Nevertheless, one can treat the scattering process as a series of multiple refractions through media with nearly constant refraction indices all the way up to the boundary of the interaction region, so that we are allowed to adopt the standard tools of ordinary optics.

Moreover, there exist directions in which the effective refraction index of the medium is less than 1 (so that the coordinate components of the velocity of light become greater than 1). This realizes an interesting and novel point of view for looking at the photon scattering by electromagnetic as well as gravitational waves in the exact theory of general relativity, which may also have a counterpart in experiments. For instance, by means of nonlinear optical materials or “metamaterials” available nowadays [13,14], one can arrange for an analogue material of the above-mentioned radiation field spacetimes and compare the geometrization of physical interactions with experimental data. In an experimental scenario, however, we are aware that a number of difficulties arises. For example, in order to mimic the transient event of a passing gravitational wave one would need to consider a moving optical medium. This can be remedied by constructing a medium that is susceptible to nonlinear effects, such as an optical activation induced by an ultrashort, intense laser pulse. Then the optical medium itself remains at rest but its effective properties change as a propagating front when the medium is activated by the laser pulse. This method has been used previously to create fiber-optical analogs of a black hole’s event horizon in laboratory [14].

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