Investigation of Sensors & Actuators based on Hankel Singular Values

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Abstract. The work presented here proposes a method to rank the efficiency of the actuators and sensors for the attenuation of excited structural modes. Aeroelastic wing designed with multiple control surfaces and equipped with sensors at different locations is used as a test case. A unifying theme to this work is the application of Hankel Singular Values (HSV), an analytical approach applied to the experimental data conducted during the Ground Vibrational (GVT) and Wind Tunnel (WT) tests. Modal information gathered from experimental results is used to define actuator/sensor relationships, once the mode of interest is decided then the sensor and actuator location is determined to effectively cater the structural vibrations, it paves the way for the design of specific performance metrics. The research will add redundancy to the available set of instrumentation by paving the way for the dedicated use of control surface for the stable flight mechanics or flight dynamics instability. The results presented here are supported by Power Spectral Dens (PSD’s) and band pass filters.

1. Introduction

Multi-surface control is used for static and dynamic aeroelastic tailoring of the wing where, these surfaces can be used to alter the camber of the wing by redistributing the aerodynamic load. One beneficial feature of the multi-surface control wing is that it allows the freedom of multitasking for various purposes like flutter suppression, load attenuation or flight mechanics, these possibilities make multi surface wing an attractive choice for aeroelastic wing. Multiple control surfaces offer many ways in which these surfaces can be instructed i.e. All the surfaces can be commanded at once (one surface) or different surfaces can be made to work independently of each other. Constrained optimization is helpful in maximizing the effectiveness of the of a control surface for the given mode [1].

The availability of the multi surface analysis provide redundancy and helps to rank the suitability of the control surface for any structural mode attenuation and to reduce the control system design from multi-input multi-output system to single-input single-output system. Advanced wings that employ multiple control surfaces are used to enhance their aeroelastic response under the projects of NASA Active Flexible Wing (A.F.W.) and Active Aeroelastic Wing [2, 3, 4]. Optimized placement of sensors with Identical Location of Acceleration and Forces (ILAF) technique enables to avoid spill over into other modes by focusing on modes of interest [5]. Work is presented for the optimal location of the sensors and actuators in the preliminary design phase, it is cost effective as well as energy efficient solution to the attenuation of the structural modes. The work relies on the novel usage of Hankel
Singular Values (HSV) which define the relation between inputs and outputs of the system in the time domain for lightly damped structures [6].

The work presented here also uses the HSV to rank the outputs and inputs for the associated excited structural modes and their corresponding damping. The aeroelastic wing has four control surfaces, two control surfaces are located at the leading edge and are named as Leading edge Outboard (LEO), Leading Edge Inboard (LEI). Similarly, two control surfaces are present at the trailing edge and are termed as Trailing Edge Outboard (TEO) and Trialing Edge Inboard (TEI). The accelerometers attached by following the ILAF approach provides the structural accelerations.

The following section II presents the analytical approach to the problem formulation, followed by the HSV analysis of experimental results in the Section III.

2. HSV Analytical Modelling

The analytical evaluation of the performances of each control surface is implemented by keeping in view the worst-case scenario, where one or more control surfaces is not functional and not getting the sufficient performance from the control surface. It can be simply inferred as quantifying the suppression of vibrations with respect to the energy/cost related to the actuation of the surface.

A unifying theme to this research is the usage of Hankel singular values (HSV) for quantitative attenuation analysis of actuator and sensor combination. This scheme is originally presented for optimized placement of actuator/sensor in the preliminary design phase. As the wing is already manufactured with predetermined location of instrumentation, HSV is used experimentally for integrated design perspective for this research. For lightly damped structure, its modal properties can be used to obtain HSV for modal coupling at preliminary design phase, given by the expression:

\[
\sigma_{yu}^2 = \frac{[b^T_i b_i][c^T_i c_i]}{(4\zeta_i\omega_i)^2}
\]  

(1)

With ‘b,’ as the input matrix and ‘c,’ as output matrix of the i-th mode for the state space model. The numerator provides the coupling of actuator and sensor for the respective mode. Denominator defines the time constant of the state in terms of damping ratio ‘\(\zeta\)’ and excited frequency ‘\(\omega\)’ of the i-th excited mode as:

\[
\tau_i = \frac{1}{(\zeta_i\omega_i)}
\]

(2)

As for the presented case the data is preferred to be analyzed in the frequency domain. Also, the task on hand is to compare the efficient attenuation which includes the comparison of structural vibrations for active and inactive control system along with taking into consideration the optimized performance of the actuator. So, the task on hand is now to find the suitable combination for modal attenuation, the equation is transformed to find the effective coupling of actuators and sensors to damp out the first bending and first torsion modes under control law which minimizes the variance of actuator movement:

\[
\sigma_{yu}^2 \leq \frac{(b_{on}^T)(c_{on}^T/c_{off})(c_{on}^T)}{(4\zeta_i\omega_i)^2}
\]  

(3)

Where ‘b,’ and ‘c,’ represents the variance extracted from actuator inputs and structural outputs, subscripts ‘on’ and ‘off’ represents the readings taken when the control system is active or inactive. For the predetermined actuator sensor locations, the optimum coupling path of actuator signal and sensor output between each mode \(N_m\) is given as:

\[
\sigma_{yu} = \text{diag}(\sigma_{yu1}, \ldots, \sigma_{yun}, \ldots, \sigma_{yunm})
\]

(4)
[7], suggested to improve the performance of control, by coupling each mode with performance-disturbance path as:

$$\sigma_{zw} = \text{diag}(\sigma_{zw1}, \ldots, \sigma_{ZwP}, \ldots, \sigma_{zwN})$$

In the design metric form, it is given as:

$$J_{qp} = \sum_{i=1}^{N_0} \frac{\sigma_{yui}^2}{\sigma_{yui}^2}$$

The essence of this task is to select the best couple of q-th sensor and p-th actuator for increase performance and efficiency. $$\sigma_{yui}^2$$ defines the square of all HSV values for actuator/sensor coupling. A similar approach has been adopted to minimize the variance of the outputs: showed that performance ‘z’ can be controlled by introducing the weighting function in the method with systems performance. Design metric can be utilized to switch between the robustness and performance based cost functions.

3. HSV Experimental Analysis

An experimental setup was prepared to equip the wing with four PCB monoaxial accelerometers with bandwidth ranges from 0.5 Hz to 3 kHz. Two accelerometers are located near the wing tip on either side of the elastic axis and two accelerometers are located on the midspan of the wing. The following figure 1, shows the sensor/actuator schematic of the wing with symbolic representation of sensors.

![Figure 1. Instrumentation of the wing](image)

The four control surfaces are equipped with the electric motors Portescap mod 17N78-210E for driving these control surfaces (LEI, LEO, TEI and TEO). are selected by fulfilling the criteria set by physical design constraints such as weight and maximum allowable size of the motors [18]. Each motor is also equipped with encoder. Thanks to planetary gears in shaft/line torque transmission, chief concern of torque and gear reduction are met. The signals from the sensors are filtered by KEMO antialiasing filters while the connectivity between the host and the target system is defined by the Real Time Application Interface (RTAI), the system provided the real time data processing as well as data was stored to be processed at the later stage.
The following figures shows the process of experimentally extracting variance and Hankel singular values for already assigned actuator/sensor locations. The PSD’s from each sensor configuration are shown in the Figure 2 to extract the variance of sensor readings in the bending mode, the signals are filtered with 6th order equiripple filter so that the signal contains only one mode.

![Figure 2. Extraction of Bending Mode – Control Off](image)

Same procedure is followed to extract the variance for the bending signals when the control system is active (On) as shown in figure 3. Figure 4 shows the PSD of the attenuated and filtered modes under active control. The effect of active control system is evident by observing the reduction in structural vibrations.
Figure 4, shows the PSD of actuator movement for active control during the attenuation various modes of bending mode. PSD of actuator movement in bending mode is realized also through equiripple filter. The extracted PSD is used to calculate the variance of the actuator movement in the bending mode. The Hankel singular Values can be calculated to show the effect of active control under optimized actuation movement for all the control surface working under ILAF concept. The values are analytically calculated by the help of the formula described in equation (6). It shows that most effective surface for bending mode is trailing edge outboard and most suitable surface for torsion mode is trailing edge inboard. Table 1 is mentioned below to signify the importance of each actuator and significance of actuator/sensor combination for cost function is, it can be readily seen that outboard trailing edge is more effective in bending mode participating frequencies while inboard trailing edge is more effective in torsional mode attenuation. Moreover, it is also noted the deficiency of leading edge strip i.e. LEO and LEI, as compared to trailing edge strip i.e. TEO and TEI, to cater the turbulent loads produced by the airbrake.
Figure 4. Actuator Movement during Bending Mode

Table 1. Hankel singular values and effective performance factor

| Sensor Location / Control surface | Bending Mode HSV | Torsion Mode HSV | Bending Mode Effective Perf. | Torsion Mode Effective Perf. |
|----------------------------------|-----------------|-----------------|-----------------------------|-----------------------------|
| LEI / LEI                        | ×               | 0.118           | 4.711                       | 1.665                       |
| LEO / LEO                        | 0.108           | 0.634           | 2.661                       | 0.105                       |
| TEI / TEI                        | 0.360           | 1.760           | 10.91                       | 4.650                       |
| TEO / TEO                        | 2.032           | ×               | 14.94                       | ×                           |

Effective performance factor, *Error! Reference source not found.* can be extracted by normalizing the attenuation with the power consumption of each actuator during active control scheme.

Table 2. Effective performance of the sensors/actuators

| Sensor Location / Control surface | Bending Mode | Torsion Mode |
|----------------------------------|--------------|--------------|
| Midspan LEI / LEI                | 4.711        | 1.665        |
| Wingtip LEO / LEO                | 2.661        | 0.105        |
| Midspan TEI / TEI                | 10.91        | 4.650        |
| Wingtip TEO / TEO                | 14.94        | ×            |
Conclusions
Aspect of suitable actuator/sensor configuration for modal attenuation is dealt in this work. Frequency response, power consumption, effectiveness and Hankel singular values are used experimentally to rank the performance of actuators in specific mode of interest. Trailing edge outboard outperformed other control surfaces for bending mode attenuation, while contrary to this result trailing edge inboard is the most suitable surface for torsion mode attenuation.

The presented research could pave the way for dedicated control surface selection for flight controls and flight dynamics and the add the redundancy based on optimized performance of the actuators and sensors.

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