The grain parameters determination based on elements of the elasticity theory

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Abstract. The grain of agricultural crops during the machine process undergoes loads exceeding the permissible ones, as a result of which, under the influence of the mechanisms' units, the grain material becomes damaged. The paper is devoted to search of the solution for the problem of reducing the grain mechanical damages which appears when the seeds pass through the machining. Using buckwheat as an example, the author considers the application of methods of the theory of elasticity in order to assess the magnitudes of stresses arising in caryopses under their force loading. In this regard, it is recommended to represent the grain model of this crop in the form of a cylindrical trihedron. The text of the work contains the experimentally proven correspondence of the volume of the proposed model to the real one, depending on the humidity conditions, the value of the elastic modulus is determined. The considered method for assessing the magnitudes of stresses at a known value of the compressive force and the shape of the seed makes it possible to take into account the design parameters when designing agricultural machines.

1. Introduction

The grain injury is the one of the most important reasons for the decline in its sowing and commercial qualities. More often it occurs during harvest and post-harvest processing. According to Ng H. F. and Tarasenko A. P. [1, 2], the number of grains with mechanical damages in a batch can be 60 percent or more. Such values exceed the accepted quality standards of grain material, which significantly affects its further use. For grain intended for sowing, there is a decrease in field germination and yield, while for food grain its technological properties deteriorate. In addition, mechanically damaged grains are less stable in storage. However, the complete elimination of grain damage during mechanized processing is not possible, in this regard, the search of the methods to reduce the amount of mechanical damages by the improvement of the technological process and technical means of its implementation is an urgent task.

2. Problem Statement

Using the static interaction of the buckwheat grains with the units and parts of agricultural machines, it is possible to obtain the necessary understanding of the dynamic force loading. When interacting bodies come into contact, the pressure is transmitted over small areas, and the body itself at this moment is in a volumetric stress state. The values of stresses and deformations can be determined by the methods of the theory of elasticity [3, 4].
Let’s consider the technological process of mechanized processing using buckwheat as an example. In contact with the working bodies of machines and mechanisms, buckwheat grains receive mechanical damage. The main reason in this case is the violation of the strength conditions [3, 6, 5].

\[ \sigma_d = k_d \sigma_s \leq \sigma_b, \quad (1) \]

where \( \sigma_d \) and \( \sigma_s \) – equivalent static and dynamic stresses in the grain, Pa;
\( \sigma_b \) – compressive strength, Pa;
\( k_d \) – dynamic coefficient, the value of which varies depending on the speed of collision of the grain with the working body of the machine and on the magnitude of the static deformation.

3. Research Questions

With the known value of the arising load and the shape of the grain, it is possible to estimate the magnitude of the mechanical stresses arising in the grain. Earlier in works [2, 7, 8], as the example of grains of barley, wheat, corn, peas, soybeans, the assessment of contact stresses based on various mathematical relationships was already considered. But even with the same amount of force interaction with the working elements of machines, the stresses in the seeds of these crops are different from each other, and given analytical dependences are not applicable for buckwheat grain, due to the difference in its geometric shape.

4. Purpose of the Study

Considering all of the above, we define the purpose of this article to develop a model of buckwheat grain and, on the basis of the model under consideration we will reveal the influence of the geometric and mechanical parameters of the grain on the stresses arising in it.

5. Research Methods

We represent a buckwheat grain in the form of a cylindrical trihedron in this example. The proposed figure, in our opinion, is the closest to the real shape of the buckwheat grain, which makes it possible to increase the accuracy of assessing the values of the arising stresses for their subsequent comparison with the limiting ones.

The surface of the described shape is formed as a result of the intersection of three cylinders with the axes \( O_1O_1, O_2O_2, O_3O_3 \) and radii equal \( R \). The faces of the cylindrical trihedron are bounded by three elliptical edges \( PAQ, PBQ, PCQ \), and the figure has only two peaks \( P \) and \( Q \). When the plane \( f \) is drawn perpendicularly through the middle of the diagonal \( PQ \) – point \( O \), the result of the intersection will be a flat figure in the form of an equilateral triangle \( ABC \) with median intersection point \( O \). Points \( O_1, O_2, O_3 \) are intersections of the cylinders axes in the plane \( f \).

Thus, if you know the geometric dimensions of the grain (in millimeters): length \( l \), thickness \( h \) - you can determine the radius of curvature \( R_i \) and the location of the axes of the cylindrical faces \( a \) using the following expressions:

\[ R_i = \frac{4h^2 + 9l^2}{24h}, \quad (2) \]
\[ a = \frac{9l^2 - 4h^2}{24h}. \quad (3) \]
Relation (3) reflects the relationship between the grain width $b$ and the thickness $h$:

$$ b = \frac{2h}{\sqrt{3}} = 1,1547 \, h. $$ (4)

The volume $V$ of the figure in question can be determined by the formula:

$$ V = 3\sqrt{3} \left[ R_1^2 l + a^2 l^2 - \frac{l^3}{12} - a l \left( R_1^2 - \frac{l^2}{4} - 2aR_1^2 \cdot \arcsin \frac{l}{2R_1} \right) \right]. $$ (5)

Let’s consider, as an example, a grain of buckwheat variety "Chishminskaya". A photo of the grain is shown in figure 2 in three projections and perspective view.

![Figure 2. Buckwheat grain: a - frontal view; b - top view; c - side view; d - axonometric projection](image)

Supposing that, in the general case, the contact points of the working bodies of agricultural machines can be reduced to a plane or cylinder. Figure 3 shows the most probable options for static contacts of proposed model and alleged forms of working units under the influence of compressive forces $P$. Grain injuries usually occur during its fall from a great height, for example, when unloading a vehicle onto the concrete floor of an inlet pit of a production line, as shown in figure 3, or in contact with the cylindrical edges of the machine details, as shown in figures 4, 5. The result of the interaction of bodies is the formation in the place of their contact of an elliptical area with the semiaxes $c$ and $t$. Thus, for known values of $c$ and $t$ according to formula (5), the value of the highest compressive stress is:

$$ \sigma_{\text{max}} = \frac{3P}{2\pi ct}, $$ (6)

where $P$ – compressive force value, N.

The most dangerous point is located on the line of action of the force at a certain depth from the interaction area, which depends on the ratio of the semiaxes of the contact ellipse $c/t$.

Using the method indicated in [3, 4], it is possible to determine the value of $c$ and $t$ in the generalized case of contact between two contacting bodies. If you follow this method, you first need to find the values of the main radii of curvature $\rho_1$, $\rho_2$, $\rho_3$, $\rho'$ and angle $\varphi$ which is between the principal planes of curvature. Then, using the formula (6), determine the value of the auxiliary angle $\psi$. 

![Figure 1. Trihedron – the intersection of three cylinders](image)
\[
\psi = \arccos \left( \frac{1}{\rho_1} - \frac{1}{\rho'_1} \right)^2 + \left( \frac{1}{\rho_2} - \frac{1}{\rho'_2} \right)^2 + 2 \left( \frac{1}{\rho_1} - \frac{1}{\rho'_1} \right) \left( \frac{1}{\rho_2} - \frac{1}{\rho'_2} \right) \cos 2\phi \right) \frac{1}{\rho_1 + \frac{1}{\rho_1} + \frac{1}{\rho_2} + \frac{1}{\rho'_2}} \tag{7}
\]

According to the tables presented in [3, 6] with a known \(\psi\) coefficient values \(m\) and \(n\) are interpolated which are used in expression (7) to determine the semiaxes of the tangency ellipse.

\[
c = m \cdot \sqrt{\frac{3\pi}{4} \frac{P}{\rho_1 + \frac{1}{\rho_1} + \frac{1}{\rho_2} + \frac{1}{\rho'_2}}} \left( \frac{1 - \mu_1^2}{E_1} + \frac{1 - \mu_2^2}{E_2} \right) \tag{8}
\]

\[
t = n \cdot \sqrt{\frac{3\pi}{4} \frac{P}{\rho_1 + \frac{1}{\rho_1} + \frac{1}{\rho_2} + \frac{1}{\rho'_2}}} \left( \frac{1 - \mu_1^2}{E_1} \right) \tag{9}
\]

where \(\mu_1\) and \(\mu_2\) – Poisson's ratios of the bodies in contact, for the approximation we take \(\mu_1=\mu_2=0.3;\)

\(E_1\) and \(E_2\) – modules of elasticity of interacting bodies, according to (Pisarenko, 1988), for steel \(E_2=2 \times 10^8\)MPa.

Since the value of the elastic modulus of grain is much less than steel \(E_1<<E_2\), then we can neglect by the value of \(\frac{1 - \mu_1^2}{E_1}\). As the result the formulas (7) are represented like:

\[
c = m \cdot \sqrt{\frac{3\pi}{4} \frac{P}{\rho_1 + \frac{1}{\rho_1} + \frac{1}{\rho_2} + \frac{1}{\rho'_2}}} \left( \frac{1 - \mu_2^2}{E_2} \right) \tag{8'}
\]

\[
t = n \cdot \sqrt{\frac{3\pi}{4} \frac{P}{\rho_1 + \frac{1}{\rho_1} + \frac{1}{\rho_2} + \frac{1}{\rho'_2}}} \left( \frac{1 - \mu_1^2}{E_1} \right) \tag{9'}
\]

Making the necessary calculations in case of contact of the cylindrical trihedron with the plane \(\omega\) in figure 3, we have: \(\rho_1 = R_1; \rho'_1 = \infty; \rho_2 = \infty; \rho'_2 = \infty; \varphi = 90^0\).

When a cylinder of radius \(R_1\) and a plane is pressed against each other by forces \(P\), the half-width of the rectangular contact area \(t\) is:

\[
t = \frac{4PR_1}{\pi \cdot b} \left( \frac{1 - \mu_1^2}{E_1} + \frac{1 - \mu_2^2}{E_2} \right), \tag{10}
\]

where \(E_1\) and \(E_2\) – moduli of elasticity of the grains and the material of the working detail, Pa; \(\mu_1\) and \(\mu_2\) – transverse deformation coefficients of grains and the working detail material.

\(b\) – grain width, m.

Thus, since the load from the action of the forces \(P\) is distributed uniformly over the width of the grain \(b\), the greatest stress acting at the points of the axis of the contact area can be determined:

\[
\sigma_{\text{max}} = \sqrt{\frac{P}{bR_1} \left( \frac{1 - \mu_1^2}{E_1} + \frac{1 - \mu_2^2}{E_2} \right)} \tag{11}
\]

With parallel axes of the cylinders during the contact of the cylindrical trihedron with the cylindrical edge of the working detail as shown in figure 4 \(\rho_1 = R_1; \rho'_1 = \infty; \rho_2 = R_2; \rho'_2 = \infty; \varphi = 0^0\):

\[
t = \frac{P}{\pi \cdot b} \frac{R_1R_2}{R_1 + R_2} \left( \frac{1 - \mu_1^2}{E_1} + \frac{1 - \mu_2^2}{E_2} \right), \tag{12}
\]

The highest voltage value:
\[
\sigma_{\text{max}} = \frac{P}{b} \frac{R_1 + R_2}{R_1 R_2} \sqrt{\pi \cdot \left( \frac{1 - \mu_1^2}{E_1} + \frac{1 - \mu_2^2}{E_2} \right)}.
\]  

(13)

If the axes are perpendicular to the cylinder during contact trihedron cylindrical with a cylindrical edge of the working detail in figure 5 \( \rho_1 = R_1; \rho_1' = \infty; \rho_2 = R_2; \rho_2' = \infty; \varphi = 90^0 \), and the value \( \psi \):

\[
\psi = \arccos \left( \frac{1}{\sqrt{R_1^2 + R_2^2} \cdot \frac{2}{R_1 R_2}} \right).
\]  

(14)

Using expressions (5), (7), (8), it is possible to determine the values of the semiaxes of the contact area and the value of the greatest compressive stress.

Nevertheless, the value of the elastic modulus \( E_1 \) is subject to experimental determination for the grain.

**Figure 3.** The contact of a cylindrical trihedron with a plane

**Figure 4.** Contact of a cylindrical trihedron with a cylindrical edge of the working detail, the axes are parallel
"Chishminskaya" at humidity value of force measuring vessel filled with water, repeating five times. According to the table 1, the difference of the values of the ellipse semiaxes are grains with straight faces. The value of expression (3) will be for grain of the "Chishminskaya" grain volume \( V_l \) – width \( b \), thickness \( t \), measured grain volume \( V_2 \). And the varieties: 1 – Chishminskaya, 2 – Shatilovskaya 5, 3 – Idel.

The parameters of the buckwheat grain: weight of 1000 grains \( m \), sizes of grain (Length \( l \), width \( b \), thickness \( c \), radius of curvature of the face \( R_i \), calculated grain volume \( V_l \), measured grain volume \( V_2 \). and by force loading, as shown in table 2.

| Var. | \( m \) (g) | \( l \) | \( b \) | \( c \) | \( R_i \) | \( V_l \) \( \text{(mm}^3 \) | \( V_2 \) \( \text{(mm}^3 \) | Deviation |
|------|----------|------|------|------|-------|-----------|-----------|---------|
| 1    | 27.64 ±0.41 | 6.34±0.13 | 4.2±0.09 | 3.55±0.05 | 4.84 | 7.56 | 40.2 | 25.56±2.6 | 10.09 |
| 2    | 28.16±0.51  | 6.64±0.15 | 4.48±0.08 | 3.71±0.06 | 5.08 | 7.62 | 60.7 | 29.23±3.4 | 14.64 |
| 3    | 31.16±0.51  | 7.30±0.15 | 5.02±0.10 | 3.95±0.06 | 5.72 | 7.12 | 58.1 | 36.35±2.9 | 12.52 |

Basing on the data in the table, it can be concluded that the bulk of buckwheat grain consignments are grains with straight faces. The value of expression (3) will be for grain of the "Chishminskaya" variety \( \frac{b}{h} = 4.2 \) \( \frac{3.55}{1.1831} \), with the deviation 2.46%; for grain of the "Shatilovskaya 5" variety \( \frac{b}{h} = 4.48 \) \( \frac{3.71}{1.2075} \), with the deviation 4.57%; for grain of the "Idel" variety \( \frac{b}{h} = 5.02 \) \( \frac{3.95}{1.2709} \), with the deviation 9.53%.

The volume of a caryopsis is measured by immersing the total number of grains of each variety in a measuring vessel filled with water, repeating five times. According to the table 1, the difference of the actual and estimated volume of grains, depending on the grade, is about 10 - 15%.

The method [8], which is based on the compression of pre-prepared grains of different moisture content between the indicator tip and the screw head of a micrometer of a special tool, was used to determine the \( E_i \) of buckwheat. Loading and unloading of grain is performed using an adjustable power source with a constant speed of 4 N/s to the value of force \( P \).

By an imprint on colored paper, located between the grain and the tip, using a measuring magnifier, the values of the ellipse semiaxes \( c \) and \( t \) of the contact area were found during static interaction at the value of force \( P \) just before the moment of grain destruction.

The values of the sizes \( c \) and \( t \) were experimentally established for buckwheat variety "Chishminskaya" at humidity \( W \) and by force loading, as shown in table 2.

| ellipses dimension (mm) | 2\(c\) | 2\(t\) | \(S_c\) \(\text{(mm}^3 \) | \(P_{\text{lim}}\) (N) | \(P_{\max}\) (MPa) | \(E_i\) (MPa) |
|------------------------|------|------|----------------|--------|-----------|--------|
| 10.00±0.68             | 32.28±2.06 | 2.89±0.19 | 0.89±0.17 | 2.05±0.48 | 15.75±1.57 | 107.62±7.11 |
| 15.68±0.28             | 38.21±2.27 | 3.94±0.33 | 1.62±0.12 | 5.05±0.80 | 7.56±0.71 | 38.07±3.02 |
| 20.75±0.87             | 30.51±2.79 | 4.08±0.23 | 2.34±0.21 | 7.51±0.85 | 4.06±0.42 | 15.48±1.22 |
| 24.00±0.10             | 16.72±1.74 | 5.16±0.37 | 3.09±0.27 | 12.60±1.80 | 1.33±0.12 | 4.66±0.50 |
| 29.92±0.52             | 11.28±1.28 | 5.89±0.43 | 3.32±0.25 | 15.42±3.93 | 0.73±0.06 | 2.00±0.21 |
| 35.41±0.74             | 7.57±1.04 | 6.03±0.46 | 3.32±0.29 | 15.65±1.44 | 0.48±0.05 | 1.28±0.13 |

6. Findings
At the end of the standard harvesting technology with a visual control of the shape and measurements of overall dimensions, average grain samples were taken from three batches of different grades, 500 pieces of each type [8 - 10]. Table 1 summarizes the specified information.
According to the formula (14), the values of the elastic modulus are determined depending on the moisture content:

\[ E_i = \frac{4PR}{\pi \cdot b \cdot f^2} \cdot \left(1 - \mu_i^2\right) \].

(15)

Calculated values of the elastic modulus of the grain \( E_1 \), contact area \( S_K = \pi ct \) and the greatest stress \( \sigma_{\text{max}} \), are also presented in table 2.

Figure 6 displays graphical dependencies \( S_k = f(W) \), \( \sigma_{\text{max}} = f(W) \), \( E_i = f(W) \).

![Graphical dependencies](image)

Figure 6. Dependency graphics \( S_k = f(W) \), \( \sigma_{\text{max}} = f(W) \), \( E_i = f(W) \).

The curves obtained experimentally at a relative humidity \( W \) from 8 to 35% are approximated using the least squares method by the expressions:

\[ S_k = -0.0094W^2 + 1.0191W + 7.8835 \]

\[ \sigma_{\text{max}} = 66.996e^{-0.1459W} \]

\[ E_i = 671.45e^{-0.1722W} \].

(16)

7. Conclusion

This paper tells, using buckwheat as an example, a grain model in the form of a cylindrical trihedron. This form of presentation is the closest to the real one in terms of volume by 85-90%, and the deviation of the actual overall dimensions from the calculated ones is no more than 10%.

It was found that the value of the area of the contact ellipse arising during grain compression increases with increasing moisture content, while the values of the highest compressive stress and elastic modulus have an inverse relationship. Also the value of the approximation reliability coefficient is rather high, \( \eta^2 > 0.95 \).
The stresses arising during mechanical contact with machine assemblies have values that depend on the surface shape of interacting bodies, their elastic modules, transverse deformation coefficients, material moisture, and also on the value of the rate of impact of grain with the machines details.

According to the research results, the following recommendations can be proposed to reduce mechanical injury to grain during its mechanized processing:

– processing should be carried out in a zone of elastic deformation (at a moisture content of 15–20%);
– to use machine details made of materials with a significantly low modulus of elasticity compared to steel;
– to change the operating mode of the machine in order to reduce the impact speed of the contacting bodies;
– to prevent sharpening of the edges of parts interacting with grain, to conduct periodic monitoring of their condition.

The considered method for assessing the magnitudes of stresses with a known value of the compressive force and the shape of the grain makes it possible to take into account the design parameters when designing agricultural machines.

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