Kaon Photoproduction Near Threshold in Six Isospin Channels Revisited

T. Mart
Departemen Fisika, FMIPA, Universitas Indonesia, Depok 16424, Indonesia
E-mail: terry.mart@sci.ui.ac.id

Abstract. In this talk I review the progress achieved in the investigation of kaon photoproduction on the nucleon near the production threshold. This investigation has been performed by using the so-called isobar model, which makes use of Feynman diagrams for the background terms and the Breit-Wigner multipoles for the resonance terms. The future prospect as well as application of this model in the hadronic sector are also briefly discussed.

1. INTRODUCTION
It has been well known that the high threshold energy of kaon photoproduction on the nucleon is the origin of the complexity of phenomenological models, that attempt to explain this process. In fact, these high energies not only increase the level of complexity of the models, but also allow for the strangeness degree of freedom to enter the calculations. We note that for $K^+\Lambda$ photoproduction, for which the experimental data are abundant, at the production threshold ($W_{\text{thr.}} = 1609$ MeV ), a number of established nucleon resonances already contribute to the process. They are the $S_{11}(1650)$, $D_{15}(1675)$, and $F_{15}(1680)$ states. Theoretically, below the production threshold the well known Roper resonance $P_{11}(1440)$, as well as the $D_{13}(1520)$ and $S_{11}(1535)$ states, have been excited before they could create the explicit $s\bar{s}$ pair in the form of a kaon and a hyperon. Moreover, the presence of strangeness degree of freedom discussed above dictates the use of SU(3) symmetry formulation, rather than SU(2) one, as in the case of pion production. Obviously, formulating the theoretical explanation of this process could become a daunting task.

On the experimental side significant progresses have been achieved in the last two decades. As a result, abundant experimental data are available from threshold up to $W = 2.8$ GeV. We note that in this energy range there are 21 nucleon resonances listed in the new Particle Data Book [1]. For the $K\Sigma$ photoproduction there also exist 20 $\Delta$ resonances, that must be also taken into account in the model. Including all these excited states in the phenomenological model is obviously very difficult, because their hadronic and electromagnetic coupling constants are hardly known. In principle, it is possible to fit all these unknown parameters. However, in view of the presently available data accuracy, too many possible solutions can be obtained.

Therefore, except by limiting the energy of interest, there exist no plausible reason to limit the number of resonances that should be put into the model. In other words, performing the
investigation near the production threshold seems to be the only way out of this problem. There exist 6 isospin channels for kaon photoproduction of the nucleon (proton and neutron). Their threshold energies are listed in Table 1. From Table 1 it is clear that the $K^+\Lambda$ channel has the lowest production threshold. Therefore, we have started our investigation with this channel [2]. By exploiting the SU(3) symmetry and appropriate information from the Particle Data Book [1] we have extended this isobar model to include both photo- and electroproduction of the $K^0\Lambda$ [3]. This channel is especially of interest, because we can investigate the effect of the electromagnetic form factor of neutral kaon $K^0$ on the corresponding observables. Finally, we have investigated the four $K\Sigma$ channels listed in Table 1 by using a similar technique, although their threshold energies $E_{\text{thr}}^{\gamma}$ are almost 150 MeV above those of the $K\Lambda$ channels [4]. A new isobar model with nucleon resonances having spins up to 5/2, that works beyond the present energy of interest has been just published [5].

2. FORMALISM
A complete explanation of the formalism used in this work is given in Refs. [2, 3, 4, 5]. However, to facilitate the reader we will briefly present this formalism in this Section.

The scattering amplitude obtained from Fig. 1 can be decomposed in terms of four gauge and Lorentz invariant matrices $M_j$ as [2]

$$ M_{\vec{r}} = \bar{u}(p_Y) \sum_{j=1}^{4} A_j(s,t,k^2) M_j u(p_N), $$

(1)

Table 1. Threshold energies of the $K\Lambda$ and $K\Sigma$ photoproductions on the nucleon in terms of the photon laboratory energy ($E_{\gamma}^{\text{thr}}$) and the total c.m. energy ($W^{\text{thr}}$).

| No. | Isospin Channel     | $E_{\gamma}^{\text{thr}}$ (MeV) | $W^{\text{thr}}$ (MeV) |
|-----|---------------------|-----------------|-----------------|
| 1   | $\gamma + p \rightarrow K^+ + \Lambda$ | 911             | 1609             |
| 2   | $\gamma + n \rightarrow K^0 + \Lambda$ | 915             | 1613             |
| 3   | $\gamma + p \rightarrow K^+ + \Sigma^0$ | 1046            | 1686             |
| 4   | $\gamma + p \rightarrow K^0 + \Sigma^+$ | 1048            | 1687             |
| 5   | $\gamma + n \rightarrow K^+ + \Sigma^-$ | 1052            | 1691             |
| 6   | $\gamma + n \rightarrow K^0 + \Sigma^0$ | 1051            | 1690             |
where $s$ and $t$ are the Mandelstam variables,

$$s = (k + p_N)^2, \quad t = (k - q_K)^2, \quad u = (k - p_Y)^2,$$

(2)

whereas the gauge and Lorentz invariant matrices $M_j$ are given by

$$M_1 = \frac{1}{2} \gamma_5 (\not{q} - \not{k} q),$$

(3)

$$M_2 = \gamma_5 [(2q_K - k) \cdot \epsilon P \cdot k - (2q_K - k) \cdot k P \cdot \epsilon],$$

(4)

$$M_3 = \frac{\gamma_5}{2} (k q_K - q_K \cdot \epsilon k),$$

(5)

$$M_4 = i \epsilon_{\mu\nu\rho\sigma} \gamma_{\mu} q_K \epsilon_{\rho} k^\sigma,$$

(6)

where we have defined $\epsilon$ as the photon polarization, $P = \frac{1}{2} (p_N + p_Y)$, and $\epsilon_{\mu\nu\rho\sigma}$ is the four dimensional Levi-Civita tensor with $\epsilon_{0123} = +1$.

For the background amplitude there are two possible couplings in the $KYN$ hadronic vertex (see Fig. 1). In the pseudoscalar (PS) scheme we have to replace the vertex factor in the pseudoscalar (PS) theory, i.e., $\gamma_5 g_{KYN}^{PS}$ with $\gamma_5 g_{KYN}^{PV}$, where the PV coupling $g_{KYN}^{PV}$ is related to the PS one through $g_{KYN}^{PV} = g_{KYN}/(m_N + m_Y)$ [6]. In this paper we only present the result with PS coupling, since in this case the agreement with experimental data is satisfactory. A complete list of the form functions $A_j$ can be found in Ref. [2] for $KA$ channels and Ref. [4] for $K\Sigma$ channels.

For the resonance amplitude the electric and magnetic multipoles ($E^{\ell \pm}$ and $M^{\ell \pm}$) of a state with the mass $M_R$, width $\Gamma$, and angular momentum $\ell$ can be parametrized to the Breit-Wigner form [7, 8]

$$A_R^{\ell \pm}(W) = \bar{A}_{\ell \pm} e_{KYN} f_{KR}(W) \frac{\Gamma_{tot}(W) M_R f_{KR}(W)}{M_R^2 - W^2 - i M_R \Gamma_{tot}(W)} e^{i \phi}.\quad (7)$$

The Breit-Wigner factor $f_{KR}$ reads

$$f_{KR}(W) = \left[ \frac{1}{(2j + 1)\pi} \frac{k_W m_N \Gamma_{KYN}}{|q| W \Gamma_{tot}} \right]^{1/2}, \quad k_W = \frac{W^2 - m_N^2}{2W},\quad (8)$$

and the partial width $\Gamma_{KYN}$ is defined as

$$\Gamma_{KYN} = \beta_K \Gamma_R \left( \frac{|q|}{q_R} \right)^{2\ell + 1} \left( \frac{X^2 + q_R^2}{X^2 + q^2} \right) \frac{W_R}{W}.\quad (9)$$

Furthermore, the $\gamma NR$ vertex is given by

$$f_{\gamma R} = \left( \frac{k_W}{k_R} \right)^{2\ell + 1} \left( \frac{X^2 + k^2_R}{X^2 + k_W^2} \right) e^{\ell}.\quad (10)$$

A comprehensive explanation of all variables and factors in Eqs. (7) - (10) can be found in Ref. [9].

To combine the background and resonance amplitudes we can write Eq. (1) in terms of the CGLN amplitudes $F_i$ [10],

$$M_\bar{R} = \bar{u}(p_Y) \sum_{j=1}^{4} A_j(s, t, k^2) M_j u(p_N) = \chi_1 F \chi_1,\quad (11)$$
where
\[
F = \sigma \cdot \epsilon F_1 - i \sigma \cdot \hat{q} \sigma \cdot (k \times \epsilon) F_2 + \sigma \cdot \hat{k} \hat{q} \cdot \epsilon F_3 + \sigma \cdot \hat{q} \hat{q} \cdot \epsilon F_4,
\] (12)
where the relation between \(F_i\) and the form functions \(A_j\) can be found, e.g., in Refs. [6, 11]. As discussed in the next Section, in the case of the \(K\Lambda\) photoproduction near threshold, the resonance terms contribute only to \(F_1\), i.e.,
\[
F_1 = \bar{E}_{0+},
\] (13)
whereas for the \(K\Sigma\) photoproduction the resonance terms contribute to all CGLN amplitudes,
\[
F_1 = E_{2-} + 3M_{2-} + 3(E_{1+} + M_{1+}) \cos \theta,
\] (14)
\[
F_2 = 2M_{1+} + M_{1-} + 6M_{2-} \cos \theta,
\] (15)
\[
F_3 = 3(E_{1+} - M_{1+}),
\] (16)
\[
F_4 = -3(M_{2-} + E_{2-}),
\] (17)
since four resonances may contribute near the threshold, due to the higher threshold energies of \(K\Sigma\) channels as compared to those of \(K\Lambda\) ones. From the combined CGLN amplitudes \(F_i\) we can calculate the desired observables, i.e., the cross section or polarization observables [11], which can be compared or fitted to the relevant experimental data.

3. RESULTS AND DISCUSSION

3.1. Proton Channels
As listed in Table 1 there are three proton channels possible in kaon photoproduction, i.e., the \(K^+\Lambda, K^+\Sigma^0,\) and \(K^0\Sigma^+\) channels. For the the \(K^+\Lambda\) channel, by limiting the energy of interest up to 50 MeV above the production threshold there is only one nucleon resonance, which can contribute to the process, i.e., the \(S_{11}(1650)\) state. Consequently, from Eq. (7) only the resonant electric multipole \(E_{0+}\) amplitude exists in the scattering amplitude, i.e.,
\[
E_{0+}(W) = \bar{E}_{0+} e^{i\phi} \frac{f_{\gamma R}(W) \Gamma_{tot}(W)m_R f_{KR}(W)}{m_R^2 - W^2 - im_R \Gamma_{tot}(W)} e^{i\phi}.
\] (18)
Therefore, all information required for the resonance term can be obtained from Particle Data Book [1]. The unknown coupling constants in the background terms are extracted by fitting to experimental data. By limiting the energy up to 50 MeV above the threshold one has 139 data points for the \(K^+\Lambda\) channel. The value of \(\chi^2/N\) is close to unity, which indicates a good agreement with experimental data is achieved.

Comparison between the calculated differential cross section from the multipole model described above and experimental data for the \(K^+\Lambda\) channel is shown in Fig. 2, where the prediction of Kaon-Maid [16] is also shown for comparison. Obviously, the agreement with experimental data is found to be better for the multipole model, in spite of the fact that the experimental error bars are relatively large in most cases.

It is important to note that the cross section shown in Fig. 2 is almost flat close to the threshold, where we can see that both models are consistent, whereas as soon as the energy increases the cross section decreases at both forward and backward angles. This phenomenon indicates the effect of both \(t\)- and \(u\)-channel in the new model, which is negligible at energy close to the threshold. This is in contrast to the prediction of Kaon-Maid, which shows a slightly forward peaking behavior, especially at higher energies.

In the case of the \(K^+\Sigma^0\) production it is obvious from Fig. 3 that both models are consistent. The fact that the new model provides a better agreement with experimental data is understood.
Figure 2. Angular distribution of the kaon photoproduction differential cross section of the $\gamma + p \to K^+ + \Lambda$ channel calculated from fitting the multipole model (solid red lines) [2] and Kaon-Maid (dashed black lines) [16]. Experimental data are obtained from the CLAS collaboration (solid squares [12] and open squares [13]), LEPS collaboration (solid triangles [14]), and Crystal Ball collaboration (open circles [15]).

Figure 3. As in Fig. 2, but for the $\gamma + p \to K^+ + \Sigma^+$ channel. Experimental data are obtained from the CLAS collaboration (solid squares [17], solid circles [12] and solid triangles [18]), and SAPHIR collaboration (open circles [19]).

as the result of fitting process, whereas the result obtained from Kaon-Maid shown Fig. 3 is merely a prediction.

The $K^0\Sigma^+$ production exhibits an interesting phenomenon, its cross section reaches the minimum point at $\theta \approx 90^\circ$, which is in contrast to both $K^+\Lambda$ and $K^+\Sigma^0$ ones. Presumably this originates from the lack of the $t$-channel in the $K^0\Sigma^+$ production, because the $K^0$ intermediate state cannot interact with a real photon, together with the weak $\Sigma^+$ contribution in the $u$-channel. Nevertheless, unlike in both $K^+\Lambda$ and $K^+\Sigma^0$ channels, the $K^0\Sigma^+$ channel has only very limited experimental data. Therefore, measuring the observables in this isospin channel would be an important agenda for the future experimental study.
3.2. Neutron Channels, the $K^0$ Electroproduction

Although beyond the present interest and discussion, it is important to note that it is relatively straightforward to extend the available $K^+\Lambda$ channel to include the $K^0\Lambda$ channel. This can be performed by using the isospin symmetry to relate the coupling constants in the background terms of both channels and using the information from the PDG for the neutron couplings in the resonance terms [3]. In addition, by starting with the $K^+\Sigma^0$ and $K^0\Sigma^+$ channels we can predict the observables of $K^0\Sigma^0$ and $K^+\Sigma^-$ channels, both are available with the neutron target.

The prediction of the observables in $K^0\Lambda$ channel is important in view of the experimental measurement of the $K^0$ photoproduction on the deuteron performed by the Tohoku group [21]. Besides that extending this model to the finite $Q^2$ region, i.e., the $K^0\Lambda$ electroproduction, is also of interest because by using this process we can measure the effect of the $K^0$ charge form factor. Note that, unlike the neutral pion, the neutral kaon has a form factor because the difference between the strange and non-strange quark masses in the $K^0$ creates a non-uniform charge distribution. Therefore, although its total charge is zero, the $K^0$ has an electric form factor. This could lead to a sensitive test of phenomenological models that attempt to describe
Figure 6. (Left panel) Change of the $\chi^2$ in the fit of the $\gamma + p \to K^+ + \Lambda$ channel due to the inclusion of the $P_{11}$ resonance with the mass $M_{N^*}$ scanned from 1620 to 1730 MeV (step 10 MeV) and for $\Gamma_{\text{tot.}}$ scanned from 1 to 10 MeV (step 1 MeV). The largest $\chi^2$ changes are obtained for $M_{N^*} = 1.65$ GeV [23]. (Right panel) Differential cross section as a function of the total c.m. energy for the isobar model corresponding to the left panel (solid line) and Kaon-Maid (dashed line). New data from Crystal Ball collaboration are shown by solid black circles [15].

the nonperturbative QCD.

Figure 5 displays the predicted differential cross sections for neutral kaon electroproduction on a neutron as functions of kaon angle, virtual photon momentum squared, and total c.m. energy. Obviously, for obtaining a sizable effect of the neutral kaon form factor in this channel, Fig. 5 recommends the experimental measurement with kinematics $Q^2 \approx 0.5$ GeV$^2$ at forward angles ($\theta \approx 0^\circ$), and in the higher energy region.

3.3. Searching for the Narrow Resonance

The existence of the non-strange partner of pentaquark, the $J^{\pi} = 1^+$ narrow state, has become an interesting discussion recently [22, 23, 24, 25, 26]. The possibility of its existence has been investigated in the $\pi N$ [22], $\eta N$ [27], and $K^+ \Lambda$ [23] channels, since it has been predicted that this narrow state has significant decay widths to these three channels [28]. For this purpose, a precise prediction of the model is necessary, since the signal for this narrow resonance could be very small.

To investigate this resonance we can include a new resonance in the model and investigate the change in the $\chi^2$ by scanning the resonance mass. Since the width of the resonance is also unknown, it is also required to scan the resonance width. The result of this scanning is shown in left panel of Fig. 6 [23], where the mass is varied between 1.62 and 1.74 GeV, while the total width is varied between 1 and 10 MeV. It is obvious from this figure that there is significant decrease in $\chi^2$ for the resonance mass of 1650 MeV. This conclusion does not change if we used a different isobar model or change the $K^+ \Lambda$ branching fraction.

There is also possibility that the resonance could have different spin and parity. However, as shown in Ref. [23], the $J^P = 1^+$ state is more preferred than any other spin-parity states.

The new data from the Crystal Ball collaboration [15] indicates an interesting phenomenon. It has been shown that by using the resonance width obtained in the last $\eta$ photoproduction
Figure 7. Total cross sections of kaon photoproduction for the six available isospin channels as a function of the total c.m. energy, relative to their threshold energies [2, 3]. Notation for the model calculations and experimental data is as in Fig. 2.

experiment, the resonance mass is found to be consistent with the recent PDG estimate [29]. Furthermore, the inclusion of the new Crystal Ball data as shown in the right panel Fig. 6 can further constrain both the mass and the width of this narrow resonance [30]. Clearly, this result might help to further limit the uncertainty of this resonance.

3.4. Future Prospect
Measurements of the $K^0$ photoproduction on the neutron are currently in progress. At the Research Center for ELeCtron PHoton Science (ELPH), Tohoku University, Japan, $K^0$ photoproduction is measured with the photon energy from threshold up to 1.2 GeV, which corresponds to $W = 1.77$ GeV or 160 MeV above threshold [31]. At MAMI, Mainz, Germany, this experiment is also carried out by using the Crystal Ball and TAPS detectors, where in this case the total c.m. energy can reach 1.90 GeV [32]. Clearly, an extension of our model to the higher energy region is strongly recommended.

Prediction of the Kaon-Maid for total cross sections of kaon photoproduction in six isospin channels with total c.m. energy around 100 MeV above thresholds are depicted in Fig. 7, where the calculated and predicted cross section from Refs. [3, 4] are also shown for comparison. However, for the neutron channels increasing the total c.m. energy by 100 MeV above the threshold would become a daunting task, since as shown in Refs. [3, 4] the uncertainties of the cross section increases significantly. Furthermore, the number of involved resonances would also increases, whereas, on the other hand, most of the resonance couplings listed by PDG have relative error bars more than 100%. In view of this fact, we recommend the prediction of the complete model, i.e., the model that is valid from threshold up to $W \approx 2.8$ GeV.
ACKNOWLEDGMENTS

This work has been partly supported by the Research-Cluster-Grant-Program of the University of Indonesia, under contract No. 1862/UN.R12/HKP.05.00/2015.

References
[1] Olive K A et al. 2014 Chin. Phys. C 38 090001
[2] Mart T 2010 Phys. Rev. C 82 025209
[3] Mart T 2011 Phys. Rev. C 83 048203
[4] Mart T 2014 Phys. Rev. C 90 065202
[5] Mart T, Clymton S and Arifi A J 2015 Phys. Rev. D 92 094019
[6] Deo B B and Bisoi A K 1974 Phys. Rev. D 9 288
[7] Drechsel D, Hanstein O, Kamalov S S and Tiator L 1999 Nucl. Phys. A 645 145
[8] Tiator I, Drechsel D, Kamalov S S, Giannini M M, Santopinto E and Vassallo A 2004 Eur. Phys. J. A 19 55
[9] Mart T and Sulaksono A 2006 Phys. Rev. C 74 055203
[10] Chew G F, Goldberger M L, Low F E, and Nambu Y, 1957 Phys. Rev. 106 1345
[11] Knöcklein G, Drechsel D, and Tiator L 1995 Z. Phys. A 352 327
[12] Bradford R et al. 2006 Phys. Rev. C 73 035202
[13] McCracken M E et al. 2010 Phys. Rev. C 81 025201
[14] Sumihama M et al. 2006 Phys. Rev. C 73 035214
[15] Jude T C et al. 2014 Phys. Lett. B 735 112
[16] Mart T and Bennhold C 1999 Phys. Rev. C 61 012201
[17] Dey B et al. 2010 Phys. Rev. C 82 025202
[18] McNabb J W C et al. 2004 Phys. Rev. C 69 042201
[19] Glander K H et al. 2004 Eur. Phys. J. A 19 251
[20] Lawall R et al. 2005 Eur. Phys. J. A 24 275
[21] Tsukada K et al. 2008 Phys. Rev. C 78 014001
[22] Arndt R A, Azimov Ya I, Polyakov M V, Strakovsky I I, and Workman R L 2004 Phys. Rev. C 69 035208
[23] Mart T 2011 Phys. Rev. D 83 094015
[24] Mart T 2013 Few Body Syst. 54 311
[25] Mart T 2011 AIP Conf. Proc. 1454 19
[26] Mart T 2013 PoS Hadron 2013 144
[27] Kuznetsov V et al. 2007 Phys. Lett. B 647 23
[28] Diakonov D, Petrov V, and Polyakov M 1997 Z. Phys. A 359 305
[29] Mart T 2013 Phys. Rev. D 88 057501
[30] Mart T in preparation
[31] Tsuchikawa Y 2015 Proc. of the 10th Int. Workshop on the Physics of Excited Nucleons (NSTAR2015), May 2015, Osaka, Japan (to be published).
[32] Werthmueller D 2015 Proc. of the 10th Int. Workshop on the Physics of Excited Nucleons (NSTAR2015), May 2015, Osaka, Japan (to be published).