Power Flow Analysis Based on the Secant Method in the Environment of the Offshore Wind Farm

Chiwei Huang, Ruixiong Lu, and Weitai Hsu*

Department of Electrical Engineering, Zhaoqing University, 526061, China
Email:wthsu1@gmail.com

Abstract. China's current exploration of the development for the offshore wind resources is three times than for the onshore wind resources. The development and utilization of offshore wind resources have a broad prospect. In this study, a set of simplified asynchronous generators equivalent to a steady-state model applied in bus test systems is established according to the characteristics of offshore wind turbines. The calculation of the Newton-Raphson method in power flow analysis is relatively large to avoid the occurrence of a Jacobian matrix and reduce the calculation time. In this particular use of the idea of the secant method, the differential equation into the algebraic equation significantly reduced the amount of calculation but also overcame the Newton-Raphson method of sensitivity to the initial value problems. The solution of the equation is simplified, and the purpose of solving the equation is achieved more accurately, while the computation time is similar to the PQ decoupling analysis method.

1. Introduction

Power flow calculation is mathematically a method for solving multivariate nonlinear equations. It is the most critical and necessary calculation in power system analysis. It is the basis for power system operation, planning, safety, reliability, and optimization. It mainly refers to a class of algorithms that give a given input of active and reactive power, line parameters, and network structure in the power system to calculate the distribution of power flow and node voltage. Since the application of computers to power systems in the 20th century, many power flow methods have emerged, and the most classic of which is the Newton-Raphson method. However, with the increase in the complexity of the power system, the Newton-Raphson method is no longer to meet the requirement of calculation. Although it has a second-order convergence characteristic, its sensitivity to initial values is high. Therefore, in this paper, the secant method is applied to the power flow analysis for a network with wind turbines. Through the simplification and calculation correction of the power flow equation, the power flow problem can solve quickly and accurately [1, 2].

Since the 1990s, the installed capacity of wind power in the world has grown at an average rate of 22% per year, and the growth rate in the past five years has been 35% to 50%. Among the various types of power generation, the growth rate of wind power ranked first [3]. However, wind power is a fluctuating, intermittent power source. The stable operation of local power grids is affected by the large-scale grid connection of wind power. With the increasing scarcity of fossil energy, rising prices, and the deterioration of the global environment, wind power has been increasingly valued by various countries. More and more countries have applied wind power technology to large power grids, which makes the calculation of power flow more and more complicated. In this paper, the secant method is used to calculate the power flow of a grid-connected offshore wind farm. That is, the differential equation is transformed into an algebraic equation, and the step of calculating the Jacobian matrix in
the Newton-Raphson method is omitted, thereby significantly reducing the iteration time and the number of iterations, and improving the calculation speed. Finally, this paper is used as a five-bus system to prove the correctness and effectiveness of the secant method.

2. Mathematical model of offshore wind turbine
At present, there are many types of wind turbines, which can be classified according to their structure, operation mode, and control principle. Due to the variability and instability of offshore wind speeds, in this paper, a continuously variable-speed wind turbine is used in the system. This type of wind turbine is available for a wide range of rates above or below the synchronous speed. The most critical generator model is the asynchronous generator. Generally, large offshore wind farms use dozens or hundreds of variable-speed wind turbines. When calculating the power flow of the grid-connected offshore wind farm, the steady-state mathematical model of the wind turbine is considered and extended to the system's equations for a simultaneous solution.

Figure 1 is shown the power transfer relationship and an equivalent circuit diagram of the asynchronous generator. Wind energy is converted into mechanical power $P_\Omega$ on the rotor of the generator by the wind turbine blades. In the equivalent circuit, corresponding to the electric power on the variable resistor $r_2(1-s)/s$ on the rotor circuit, where $s$ refers to the slip coefficient of the generator. The electric power $P_e$ injected into the grid is obtained by subtracting the rotor copper loss $P_{cu2}$, the core loss $P_{Fe}$, and the stator copper loss $P_{cu1}$. Among them, $x_m$ is the excitation reactance, $r_m$ is the excitation resistance, $r_1$ is the stator resistance, $x_1$ is the stator reactance, $x_2$ is the rotor reactance, and $r_2$ is the rotor resistance [4].

![Figure 1. Power relationship of asynchronous generator.](image1)

Because $x_m \gg x_1$ in figure 1, the stator resistance and the power loss of the core can be ignored. Therefore, the excitation branch is moved to the circuit head, as shown in figure 2. The simplified Γ equivalent model of the asynchronous generator is obtained [4, 5].

![Figure 2. Simplified equivalent circuit diagram of asynchronous generator.](image2)
According to [6], the equation of the simplified circuit shown in Figure 2 is as follows:

\[
\frac{z}{s} = U = U \times \frac{-U}{r_2 / s + j(x_1 + x_2)} = -\frac{U^2 r_2 s}{r_2^2 + s^2 (x_1 + x_2)} - j\frac{U^2 r_2}{r_2^2 / s^2 + (x_1 + x_2)^2}
\]  

(1)

The real part in equation (1) is the active power \( P \) from the wind turbine.

\[
P = \frac{U^2 r_2 s}{r_2^2 + s^2 (x_1 + x_2)}
\]  

(2)

The imaginary part in equation (1) is the reactive power \( Q \) from the wind turbine.

\[
Q = \frac{U^2 r_2 / s + jU^2 (x_1 + x_2)}{r_2^2 / s^2 + (x_1 + x_2)^2}
\]  

(3)

The conventional power flow analysis is divided the system bus into three categories: load buses, named PQ node, regulated buses, called PV node, and slack bus, named Vθ node. Offshore wind turbines absorb reactive power while generating active power. The active power output depends on the wind speed. The absorbed reactive power is closely related to the terminal voltage, the generated active power, and the slip coefficient, so it cannot be simply treated as a constant-power PQ node. Moreover, wind turbines cannot merely belong to a specific type of node. When solving the power system problem that includes offshore wind farms, the characteristics of offshore wind turbines must be considered: asynchronous generators do not have excitation regulation devices and voltage regulation capabilities, so they cannot be used like conventional synchronous generators treat it as a PV node with a constant voltage amplitude. When solving the power flow problem, the characteristics of the offshore wind turbine are necessary to be considered. Asynchronous generators do not have excitation regulation devices and voltage regulation capabilities, so regarded as PV nodes with constant voltage amplitude like conventional synchronous generators is not applicable. Due to the above reasons, power flow calculations involving wind farms are usually divided into two parts: traditional power flow calculation and the calculation of the asynchronous generator of the internal circuit. The Newton-Raphson method is mostly used in solving the power flow problem. However, considering the calculation time and complexity of the Newton-Raphson method, the secant method is used in this paper for the power flow solution.

3. Optimization of reactive power compensation and LCL inverter

Let a function \( f(x) \) be continuous on the closed interval \([a, b]\), have finite derivatives on the open range \((a, b)\), and \( f'(x), f''(x) \) exist and be invariant, then the function satisfies the conditions of the secant method [7].

The secant method is the method for solving the root of a nonlinear equation and belongs to a point-by-point linearization method. It is an improvement based on the Newton-Raphson method. The basic idea is to use a straight line and a function to intersect at two points in the interval \([a, b]\). This line is called the secant line of the function. In the range \([a, b]\), the slope of the tangent line is used to replace the slope of the tangent line of the objective function, and the abscissa of the intersection of the secant line and the horizontal axis is used as an approximation to the root of the objective function [7]. The curve and secant of the function \( f(x) \) intersect at two points \( h \) and \( k \) in \([a, b]\). The intersection of the secant and abscissa is \( X_{k+1} \), as shown in figure 3.
The general Newton-Raphson method is expressed as [8]:

\[ J(x^{(k)}) = \frac{\partial f(x^{(k)})}{\partial x(k)} \]

\[ \Delta x^{(k)} = -J^{-1}(x^{(k)}) f(x^{(k)}) \]

\[ x(k+1) = x^{(k)} + \Delta x^{(k)} \]  \hspace{1cm} (4)

\( J(x(k)) \) is a Jacobian matrix. The process is to avoid forming a new Jacobian matrix with each iteration, and the secant method transforms the differential equation into an algebraic equation through the idea of difference quotient:

\[ J(x^{(k)}) = \frac{\partial f(x^{(k)})}{\partial x^{(k)}} = \frac{f(x^{(k)}) - f(x^{(k-1)})}{x^{(k)} - x^{(k-1)}} \]  \hspace{1cm} (5)

Substituting the equation (5) into the equation (4), the secant method iterative equation is as follows:

\[ x^{(k+1)} = x^{(k)} - \frac{f(x^{(k)})}{f(x^{(k)}) - f(x^{(k-1)})} (x^{(k)} - x^{(k-1)}) \]  \hspace{1cm} (6)

The intersection of the Newton-Raphson and abscissa is \( X_{k+1} \), as shown in figure 4.

Figure 3. Secant method.

Figure 4. Newton-Raphson method.
From Figures 3 and 4, one can see that after one iteration, the secant method is closer to the value of $f(x)=0$ (that is, the actual value), and the Newton-Raphson method is still far from the real value. Therefore, for solving the solution of the equation $f(x)=0$, the secant method can be completed in a relatively small number of iterations. Based on the equation (3), the secant method does not generate a matrix similar to Newton's Jacobian matrix $J(x(k))$ when iterating. The calculation speed of the secant method is faster than the Newton-Raphson method. Thus, the secant method has more considerable advantages in solving complex power flow problems.

It is pointed out in [1] that the Newton-Raphson method has second-order convergence in the case of a single root. With good initial value, the Newton-Raphson method can quickly iterate and converge to the solution of the equation, but in each iteration, in addition to calculating the value of $f(x(k))$, $J(x(k))$ and $f'(x(k))$ if $f'(x(k))$ is more complicated. The workload of calculating $f'(x(k))$ may be significant, especially when $|f'(x(k))|$ is tiny, considerable rounding errors have occurred. The calculation speed is significantly reduced, and the appropriate initial value is not easy to estimate. All the separate Newton-Raphson methods are not suitable for the fast power flow solution. The convergence order of the secant method is 1.618 [9]. In theory, under the same iteration conditions, the number of iterations of the secant method is less than that of the Newton-Raphson method. However, the disadvantage is that the secant method must provide two edible initial approximate roots $X_0$, $X_1$.

4. Power flow calculation for offshore wind farm

The reactive power consumed by the power system varies with the operating conditions of the system, so the power factor of offshore wind farms varies under different operating modes. The motive power of the offshore wind turbine is uncontrollable. The power output for the offshore wind turbine is depended on the wind speed. The offshore wind turbine itself does not have an excitation adjustment device and is not able to adjust the voltage. In this case, it cannot merely wind power nodes belong to PV nodes or PQ nodes. If the Newton-Raphson method is applied to the power flow calculation, the partial derivative of the reactive power is increased by the offshore wind farm node in the Jacobian matrix. It increases voltage [6]. In this paper, the secant method is applied to skip this step. According to [10], the power flow equation in the polar coordinate system is expressed as:

$$\begin{align*}
\Delta P_i &= P_i^{op} - U_i \sum_{j=i} U_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0, \quad i = 1, 2, \ldots, n \\
\Delta Q_i &= Q_i^{op} - U_i \sum_{j=i} U_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0, \quad i = 1, 2, \ldots, r
\end{align*}$$

(7)

Simplification of the iterative equation and correction of the iterative direction are as following:

$$\begin{align*}
\Delta P_i^{(k)} &= P_i^{(k)} (\theta_i^{(k)}, \ldots, \theta_i^{(k)}, \ldots, V_i^{(k)}, \ldots, V_i^{(k)}, \ldots) \\
\Delta Q_i^{(k)} &= Q_i^{(k)} (\theta_i^{(k)}, \ldots, \theta_i^{(k)}, \ldots, V_i^{(k)}, \ldots, V_i^{(k)}, \ldots)
\end{align*}$$

(8)

$$\begin{align*}
\Delta P_i^{(k)} &= P_i^{(k)} (\theta_i^{(k+1)}, \ldots, \theta_i^{(k+1)}, \ldots, V_i^{(k)}, \ldots, V_i^{(k)}, \ldots) \\
\Delta Q_i^{(k)} &= Q_i^{(k)} (\theta_i^{(k+1)}, \ldots, \theta_i^{(k+1)}, \ldots, V_i^{(k+1)}, \ldots, V_i^{(k+1)}, \ldots)
\end{align*}$$

(9)

The following is the calculation procedure of the power flow analysis, including offshore wind farms:
1) The wind speed of the offshore wind farm is given.
2) The initial voltage of the node for the offshore wind farm is given, including the amplitude and phase.
3) The original data to form the node admittance matrix is entered.
4) The active and reactive power of the wind turbine by equations (2) and (3) is determined under the given wind speed.
5) The value of each iteration is found according to equation (6).
6) The correction equation is found according to equation (7), and modification of the voltage and phase for each node.
If it converges, the calculation is finished, otherwise, use the revised node voltage as the initial value, and then go to step (4) for the next iteration.

5. Simulation and discussion
This example is used the five-bus system in [11], and the wind power model in [12] is served as the offshore wind turbine. The system diagram is shown in figure 5, and the Simulink simulation diagram is shown in figure 6.

![Figure 5. System diagram.](image)

![Figure 6. Simulink simulation diagram.](image)
This article is used 575V/35kV for the wind turbine terminal voltage. The electricity generated by the offshore wind turbine is first collected by the line and then sent to the system through the main transformer of the wind farm booster station. The wind farm capacity is 6×1500kW; the wind turbine is the same model, the cut-in wind speed, cut-out wind speed and rated wind speed of the wind turbine are the same, and the rated voltage is 575V. The stator impedance of the offshore equivalent wind turbine is 0.00706+j0.171; the rotor reactance is 0.005+j0.156, the excitation reactance is j2.9 (select SB=100MVA), cosψ=0.89 and the system voltage is 1.0p.u. This example is to verify the difference between the secant method and Newton-Raphson method, so the effect of the change in the wind speed on the system is ignored, that is, the wind speed is set to a fixed value. Moreover, the effect of parallel capacitor compensation at the offshore wind turbine outlet is also ignored; that is, the capacitor has not been put in for the time being.

According to the above algorithms, the following is the comparison of the simulation times and results of the Newton-Raphson method in Table 1 and the secant method in Table 2 after the simulation.

| Table 1. Iteration times and results for the Newton-Raphson method. |
|----------------------|----------------------|----------------------|----------------------|----------------------|
| Times               | Node (Voltage in p.u.) | 1       | 2       | 3       | 4       | 5       |
| 1                   | 0.5750                | 0.6181  | 1.0950  | 0.9720  | 1.0503  |
| 2                   | 0.5750                | 0.4167  | 1.0510  | 0.7135  | 1.0500  |
| 3                   | 0.5750                | 0.3237  | 1.0223  | 0.4640  | 1.0500  |
| 4                   | 0.5750                | 0.4044  | 1.2082  | 3.4913  | 1.0500  |
| 5                   | 0.5750                | 0.3538  | 1.1269  | 2.0134  | 1.0500  |
| 6                   | 0.5750                | 0.3206  | 1.0648  | 1.1806  | 1.0620  |
| 7                   | 0.5750                | 0.3016  | 1.0353  | 0.7413  | 1.0511  |
| 8                   | 0.5750                | 0.2863  | 1.0128  | 0.4356  | 1.0502  |
| 9                   | 0.5750                | 0.3258  | 1.0697  | 1.2719  | 1.0501  |
| 10                  | 0.5750                | 0.3041  | 1.0392  | 0.7971  | 1.0503  |
| 11                  | 0.5750                | 0.2889  | 1.0169  | 0.4859  | 1.0501  |

| Table 2. Iteration times and results for the secant method. |
|----------------------|----------------------|----------------------|----------------------|----------------------|
| Times               | Node (Voltage in p.u.) | 1       | 2       | 3       | 4       | 5       |
| 1                   | 0.7875                | 0.6597  | 1.1071  | 0.9835  | 1.0850  |
| 2                   | 0.7875                | 0.4979  | 1.0533  | 0.7270  | 1.0506  |
| 3                   | 0.7875                | 0.4427  | 1.0297  | 0.5052  | 1.0500  |
| 4                   | 0.7875                | 0.3289  | 0.8035  | 3.2745  | 1.0500  |
| 5                   | 0.7875                | 0.3748  | 0.9467  | 1.5696  | 1.0997  |
| 6                   | 0.7875                | 0.4023  | 0.9716  | 0.6069  | 1.0590  |
| 7                   | 0.7875                | 0.4180  | 0.9969  | 0.2753  | 1.0503  |
| 8                   | 0.7875                | 0.4282  | 1.0166  | 0.4008  | 1.0500  |
| 9                   | 0.7875                | 0.4420  | 1.0433  | 0.7784  | 1.0501  |
| 10                  | 0.7875                | 0.4323  | 1.0247  | 0.5054  | 1.0501  |

Because the secant method and the Newton-Raphson method take different initial values, the simulation results are also diverse, but the overall is similar. It can be seen from the table that the number of iterations of the secant method is less than the Newton-Raphson method and because the relationship of the Jacobian matrix is not calculated. The secant method (0.0052s) takes significantly less time than the Newton-Raphson method (0.026s). Although the number of iterations of the secant
method is only one less than the Newton-Raphson method in the above example, this is only the five-bus system. If the secant method is used on other more massive systems, the number of iterations is significantly distinctive.

6. Conclusion
As the capacity of offshore wind turbines and the scale of wind farms continue to increase, it inevitably has an impact on the planning, operation, and design of power systems. The grid-connected process of a large number of wind turbines is brought a series of problems to the planning, design, and operation of the power system. Therefore, it is necessary to perform power flow analysis of power systems containing offshore wind farms to verify the impact of wind power on the network. The calculation of the power flow of the grid-connected offshore wind system is the basis for studying the influence of offshore wind power on the net. Based on the simplified steady-state model of the asynchronous generator, the secant method is proposed to solve the power flow problem with offshore wind farms. The influence of the initial value on the convergence of the iterative process is avoided by addressing the power flow equation using the secant method. It makes the iterative process develop towards the real value, which is effectively improved the calculation speed. This paper also verifies by examples that the secant method has fewer iterations and time than the Newton-Raphson method in power flow calculation. The feasibility, accuracy, and timeliness of the secant method to solve the power flow problem are verified, which is conducive to quickly and accurately explaining the power flow distribution.

7. References
[1] J.H. Shi, Summary of optimization power flow algorithm (in Chinese), Chinese Science and Technology Information. 1 (2016) 59-61.
[2] Z. Zhao, F.Y. He, Solving power flow equations with secant method (in Chinese), Journal of China Institute of Water Resources and Hydropower Research. 16 (2018) 156-160.
[3] P.F. Shi, Progress and trends of wind power generation, Electric Power. 35 (2002) 86-90.
[4] Y.H. Lei, Studies on wind farm integration into power system, Automation of Electric Power Systems. 27 (2003) 84-89.
[5] H.C. Wang, S.X. Zhou, Z.X. Lu, J.L. Wu, A joint iteration method for load flow calculation of power system contacting unified wind farm and its application, Power System Technology. 29 (2005) 259-262.
[6] W. Zhang, L. Yu, Y.J. Liu, Study on calculation of power flow of grid connected offshore wind farm, Renewable Energy Resources. 28 (2010) 36-38.
[7] M.J.P. Nijmeijer, A method to accelerate the convergence of the secant algorithm, Advances in Numerical Analysis. 2014 (2014) 1–14.
[8] Z. Zhao, F.Y. He, Solving power flow equations with secant method, Journal of China Institute of Water Resources and Hydropower Research. 16 (2018) 156-160.
[9] J.H. Sheng, Fundamentals of Numerical Analysis, 2nd Ed., Tonji University Press, Shanghai, 2004, pp.187-189.
[10] B.M. Zhang, et al., Advanced Power Network Analysis, 2nd Ed, Tsinghua University Press, Beijing, 2007, pp.181-184.
[11] X.P. Meng, Y. Gao, Power System Analysis, 2nd Ed, Higher Education Press, Beijing, 2010, pp. 352-354.
[12] Q. Yu, N. Cao, MATLAB / Simulink Power System Modelling and Simulation, 2nd Ed, China Machine Press, Beijing, 2017, pp. 215-221.