0+ and 1+ heavy-light exotic mesons at N2LO in the chiral limit

R.M Albuquerque, F. Fanomezana, S. Narison, A. Rabemananjara, D. Rabetiarivony, G. Randriamanatrika

Faculty of Technology, Rio de Janeiro State University (FAT, UERJ), Brazil
Institute of High Energy Physics of Madagascar (iHEP-MAD), University of Antananarivo, Madagascar
Laboratoire Univers et Particules (LUPM), CNRS-IN2P3 & Université de Montpellier II, Case 070, Place Eugène Bataillon, 34095 - Montpellier Cedex 05, France.

Abstract

We use QCD spectral sum rules (QSSR) and the factorization properties of molecule and four-quark currents to estimate the masses and couplings of the 0+ and 1+ molecules and four-quark at N2LO of PT QCD. We include in the OPE the contributions of non-perturbative condensates up to dimension-six. Within the Laplace sum rules approach (LSR) and in the MS-scheme, we summarize our results in Table 2, which agree within the errors with some of the observed XZ-like molecules or/and four-quark. Couplings of these states to the currents are also extracted. Our results are improvements of the LO ones in the existing literature.

Keywords: QCD Spectral Sum Rules, molecule and four-quark states, heavy quarkonia.

1. Introduction

The recent discovery of the Zc(3900) 1++ by Belle [1] and BESIII [2] has motivated different theoretical analysis [3]. However, all of the previous analysis like e.g. in [4,5] from QCD Spectral Sum Rules (QSSR) [7,10] have been done at LO of PT QCD. In this paper, we are going to use QSSR to evaluate the masses and couplings of some 0+ and 1+ molecules at N2LO in the PT series and compare the results with those obtained at lowest order and with experiments. This work is a part of the original papers in [13] and also in [14,15].

2. QCD analysis of the ones in molecule states

\begin{itemize}
  \item Currents and two-point functions
  
  The currents \( J_{\text{mol}}^{(\mu)} \) used for these molecules states are given by:

  - For 0+:
    \[
    \bar{M} M : (\bar{q}_s Q)(\bar{Q}_s q) \\
    \tilde{M}^{\dagger} M^* : (\bar{q}_s \gamma_\mu Q)(\bar{Q}_s \gamma_\mu q) \\
    \bar{M}_0^0 M_0 : (\bar{q}_s Q)(\bar{Q}_s q) \\
    \tilde{M}_1 M_1 : (\bar{q}_s \gamma_\mu \gamma_5 Q)(\bar{Q}_s \gamma_\mu \gamma_5 q) 
    \]

  - For 1+:
    \[
    \bar{M}^* M : \frac{1}{\sqrt{2}}[(\bar{Q}_s q)(\bar{q}_s Q) - (\bar{q}_s Q)(\bar{Q}_s q)] \\
    \tilde{M}_0^0 M_1 : \frac{1}{\sqrt{2}}[(\bar{Q}_s q)(\bar{q}_s \gamma_5 Q) + (\bar{q}_s Q)(\bar{Q}_s \gamma_5 q)] 
    \]

 \end{itemize}

where \( q, Q \) represent respectively light and heavy quarks. The associated two-point correlation function is:

\[
\Pi_{\text{mol}}^{\mu\nu}(q) = i \int d^4x e^{iq\cdot x} \langle 0 | T J_{\text{mol}}^{\mu}(x) J_{\text{mol}}^{\nu\dagger}(0) | 0 \rangle \\
= -(q^2 g^{\mu\nu} - q^\mu q^\nu) \Pi_{\text{mol}}^{(0)}(q^2) + q^\mu q^\nu \Pi_{\text{mol}}^{(0)}(q^2),
\]

\[ (3) \]
where Π_{mol} and Π_{mol}^{(0)} are associated to the spin 1 and 0 molecule states. Parametrizing the spectral function by one resonance plus a QCD continuum, the lowest resonance mass M_H and coupling f_H normalized as:

\[ \langle 0|J^\mu|[H] = f_H M_H^4 e^{\mu}, \]

(4)
can be extracted by using the Laplace sum rules (LSR) \([7][12]\):

\[ M_H^2 = \int_{M_0}^{\infty} dt \frac{t e^{-\tau t}}{\pi} \text{Im} \Pi_{mol}(t) \]

(5)
and

\[ f_H^2 = \int_{M_0}^{\infty} dt \frac{t e^{-\tau t}}{\pi} \text{Im} \Pi_{mol}(t) \]

(6)

where M_0 is the heavy quark on-shell mass, τ the LSR parameter, t_\tau the continuum threshold and Im\Pi_{mol}(t) is the QCD expression of the molecular spectral function.

**The QCD two-point function at N2LO**

To derive the results at N2LO, we assume factorization and then use the fact that the two-point function of a molecular state can be written as a convolution of the spectral functions associated to quark bilinear currents. We have \([16][17]\):

\[ \frac{1}{\pi} \text{Im} \Pi_{mol}^{(0)(1)}(t) = \theta(t - 4M_0^2) \left( \frac{1}{4\pi} \right)^2 \int_{M_0}^{\infty} dt_1 \int_{M_0}^{\infty} dt_2 \frac{t_1^{\tau^2} - t_2^{\tau^2}}{t_1 t_2} \times ... \]

(7)

For spin 0:

\[ ... = \lambda^{1/2} \left[ \left( \frac{t_1}{t} + \frac{t_2}{t} - 1 \right)^2 \frac{1}{\pi} \text{Im} \Pi^{(0)(1)}(t_1) \frac{1}{\pi} \text{Im} \Pi^{(0)(0)}(t_2) \right] \]

or

\[ ... = \lambda^{1/2} \left[ \frac{1}{\pi} \text{Im} \Pi^{(0)(1)}(t_1) \frac{1}{\pi} \text{Im} \Pi^{(0)(0)}(t_2) \right] \]

For spin 1:

\[ ... = \lambda^{1/2} \left[ \left( \frac{t_1}{t} + \frac{t_2}{t} - 1 \right)^2 + \frac{8t_1 t_2}{t^2} \right] \times \frac{1}{\pi} \text{Im} \Pi^{(0)(0)}(t_1) \frac{1}{\pi} \text{Im} \Pi^{(1)(1)}(t_2) \]

with the phase space factor:

\[ \lambda = \left( 1 - \frac{\sqrt{t_1} - \sqrt{t_2}}{t} \right) \left( 1 - \frac{\sqrt{t_1} + \sqrt{t_2}}{t} \right) \].

(8)

Im\Pi^{(1)(1)}(t) and Im\Pi^{(0)(0)}(t) are respectively the spectral functions associated to the (axial)vector and to the (pseudo)scalar bilinear currents. The QCD expression of the spectral functions for bilinear currents are already known up to order α_s^2 and including non-perturbative condensates up to dimension 6. It can be found in \([18][21]\) for the on-shell mass M_Q. We shall use the relation between the on-shell M_Q and the running mass \(\overline{m}_Q(\mu)\) to transform the spectral function into the \(\overline{MS}\)-scheme \([22][23]\):

\[ M_Q = \overline{m}_Q(\mu) \left[ 1 + \frac{4}{3} \Delta_s + (16.2163 - 1.041n_l)\alpha_s^2 \right] \]

\[ + \ln \left( \frac{\mu}{M_Q} \right)^2 \left( a_s + (8.8472 - 0.361n_s)\alpha_s^2 \right) \]

\[ + \ln^2 \left( \frac{\mu}{M_Q} \right) \left( 1.7917 - 0.0833n_t \right)\alpha_s^2 \],

(9)

where \(n_l = n_f - 1\) is the number of light flavours and \(\alpha_s(\mu) = \alpha_s(\mu)/\pi\) at the scale \(\mu\).

**QCD parameters**

The PT QCD parameters which appear in this analysis are α_s, the charm and bottom quark masses m_c,b (the light quark masses have been neglected). We also consider non-perturbative condensates which are the quark condensate \(\langle \bar{q}q \rangle\), the two-gluon condensate \(\langle g^2 G^2 \rangle\), the mixed condensate \(\langle g\bar{q}Gq \rangle\), the four-quark condensate \(\rho\langle \bar{q}q\rangle^2\), the three-gluon condensate \(\langle g^3 G^3 \rangle\), and the two-quark multiply two-gluon condensate \(\rho\alpha_s\langle \bar{q}q\rangle\langle g^2 G^2 \rangle\) where \(\rho\) indicates the deviation from the four-quark vacuum saturation. Their values are given in Table 1 and more recently in \([24]\).

Table 1: QCD input parameters\(\big)\) the original errors for \((\alpha_s G^2)\), \((\langle g^2 G^2 \rangle)\) and \(\rho\langle \bar{q}q\rangle^2\) have been multiplied by about a factor 3 for a conservative estimate of the errors (see also the text).

| Parameters | Values |
|-----------|--------|
| α_s(M_z) | 0.325(8) |
| \(\hat{m}_s(\Mu)\) | (0.114 ± 0.006) GeV |
| \(\bar{m}_c(\Mu)\) | 1261(12) MeV |
| \(\bar{m}_b(\Mu)\) | 4177(11) MeV |
| \(\bar{m}_q(\Mu)\) | average 25(15) |
| \(\bar{m}_b(\Mu)\) | 253(6) MeV |
| \(\langle \bar{q}q \rangle\) | (0.8 ± 0.2) GeV^2 |
| \(\langle \alpha_s G^2 \rangle\) | (7 ± 3) × 10^{-2} GeV^4 |
| \(\langle g^2 G^2 \rangle\) | (8.2 ± 2.0) GeV^2 × \(\langle \alpha_s G^2 \rangle\) |
| \(\rho\alpha_s\langle \bar{q}q\rangle^2\) | (5.8 ± 1.8) × 10^{-4} GeV^6 |

**3. Mass of the \(\bar{D}D(0^+)\) molecule state**

**τ and \(t_\tau\) stabilities**

We study the behavior of the mass in term of LSR variable τ at different values of \(t_\tau\) as shown in Fig. 1. We consider as a final and conservative result the one corresponding to the beginning of the τ stability for \(t_\tau=23\)
$\tau \simeq 0.25 \text{ GeV}^{-2}$ until the one where $t_c$ stability is reached for $t_c \simeq 32 \text{ GeV}^2$ and $\tau \simeq 0.35 \text{ GeV}^{-2}$.

**Convergence of the PT series**

According to these analysis, we can notice that the $\tau$-stability begins at $t_c = 23 \text{ GeV}^2$ and the $t_c$-stability is reached from $t_c = 32 \text{ GeV}^2$. Using these two extremal values of $t_c$, we study in Fig. 2 the convergence of the PT series for a given value of $\mu = 4.5 \text{ GeV}$. We observe that from LO to NLO the mass increases by about +1% while from NLO to N2LO, it only increases by +0.1%. This result indicates a good convergence of PT series which validates the LO result obtained in the literature when the running quark mass is used.

**$\mu$-stability**

We improve our previous results by using different values of $\mu$ (Fig. 3). Using the fact that the final result must be independent of the arbitrary parameter $\mu$, we consider as an optimal result the one at the inflexion point for $\mu \simeq (4.0 - 4.5) \text{ GeV}$:

$$M_{DD} = 3898(36) \text{ MeV},$$

where the second error comes from the localisation of the inflexion point, QCD condensates and higher dimension contributions.

4. Coupling of $DD(0^+)$ molecule state

We can do the same analysis to derive the decay constant $f_H$ defined in Eq. (4). Noting that the bilinear pseudoscalar heavy-light current acquires an anomalous dimension, then the decay constant runs as:

$$f_H^{(1)}(\mu) = f_H^{(1)}(-\beta_1 \alpha_s \mu^{2\beta_1}) / r_m,$$

$$f_H^{(2)}(\mu) = f_H^{(2)}(-\beta_1 \alpha_s \mu^{2\beta_1}) / r_m,$$

where $f_H$ is a scale invariant coupling; $\beta_1 = (1/2)(11 - 2n_f/3)$ is the first coefficient of the QCD $\beta$-function for $n_f$ flavors and $\alpha_s \equiv \alpha_s/\pi$. The QCD corrections numerically read:

$$r_m(n_f = 4) = 1 + 1.014 \alpha_s + 1.389 \alpha_s^2,$$

$$r_m(n_f = 5) = 1 + 1.176 \alpha_s + 1.501 \alpha_s^2.$$  

Taking the Laplace transform of the correlator, this definition will lead us to the expression of the running coupling in Eq. (6). We show in Fig. 4 the $\tau$-behaviour of the running coupling $f_{DD}(\mu)$ for two extremal values of $t_c$ where $\tau$ and $t_c$ stabilities are reached. These values are the same as in the mass determination. One can see

in this figure that the $\alpha_s$ corrections to the LO term of PT series are still small though bigger than in the case of the mass determination from the ratio of sum rules. It is about +5% from LO to NLO and +2% from NLO.
Figure 5: $\mu$-behavior of $\hat{f}_{\bar{D}D}$ at N2LO

to N2LO. In the Fig. 5, we show the $\mu$ behaviour of the invariant coupling $\hat{f}_{\bar{D}D}$. Taking the optimal result at the minimum for $\mu \approx 4.5$ GeV, we obtain in units of MeV:

$$\hat{f}_{\bar{D}D} = (62 \pm 6) \text{ keV} \implies f_{\bar{D}D}(\mu) = (170 \pm 15) \text{ keV},$$

(13)

5. $0^+$ and $1^+$ four-quark states

We can do the same analysis for the case of four-quark states. The interpolating currents used are:

$$0^+: \epsilon_{abc} \epsilon_{dec} \left[ (\bar{q}_e C \gamma_5 Q_d) (\bar{q}_d C \gamma_5 Q_e) + k (\bar{q}_d C Q_e) (\bar{q}_e C \gamma_5 Q_d) \right]$$

$$1^+: \epsilon_{abc} \epsilon_{dec} \left[ (\bar{q}_e C \gamma_5 Q_d) (\bar{q}_d C \gamma_\mu \gamma_5 Q_e) + k (\bar{q}_d C Q_e) (\bar{q}_e C \gamma_\mu \gamma_5 Q_d) \right],$$

where $Q \equiv c$ (respectively $b$) in the charm (resp. bottom) channel and $q \equiv u, d$. $k$ is the mixing of the two operators. We use $k = 0$ as shown in [4, 53].

The behavior of the curves of masses and couplings are very similar to the molecules ones. Considering all the possible currents and channels configurations, we have in Table 2 the results for $0^+$ and $1^+$ molecule and four-quark states.

6. Conclusions

We have presented improved predictions of QSSR for the masses and couplings of the $0^+$ and $1^+$ molecule and four-quark states at N2LO of PT series and including up to dimension six non-perturbative condensates. We can see a good convergence of the PT series after including higher correction. This good convergence confirms the veracity of our results. The results are improvements of all the precedent works about the masses of exotic hadrons obtained at LO. Our analysis has been done within stability criteria with respect to the LSR variable $\tau$, the QCD continuum threshold $t_c$ and the subtraction constant $\mu$ which have provided successful predictions in different hadronic channels. The optimal values of the masses and couplings have been extracted at the same value of these parameters where the stability appears as an extremum and/or inflection points. The ill-defined heavy quark mass definition used at LO is not enough to have results. The effects are often large for the coupling. The masses of $\bar{D}D$ and $\bar{D}'D'$ are also around the $Z_c(3900)$ $0^{++}$ state. We do not include higher dimension condensates contributions in our estimate but only consider them as a source of the errors. One can conclude that $Z_c(3900)$ can be well described with an almost pure $D^*D$ molecule. One can notice that the masses of the $J^P = 1^+, 0^+$ states are most of them below the corresponding $DD, BB$-like thresholds and are compatible with some of the observed XYZ masses suggesting that these states can be interpreted as almost pure molecules or/and four-quark states.
Table 2: 0° and 1° molecules and four-quark masses, invariant and running couplings from LSR within stability criteria from LO to N2LO of PT.

| Channels | $\xi_0$ [keV] | LO | NLO | N2LO | LO | NLO | N2LO |
|----------|----------------|-----|------|-------|-----|------|-------|
| $b\bar{b}$ | | | | | | | | |
| $D_0^*D_0$ | | | | | | | | |
| $D_1^0D_1^-$ | | | | | | | | |
| $S_c$ | 62 | 67 | 70(7) | 173 | 184 | 191(20) | 3902 | 3901 | 3898(54) |
| $BB$ | 4.0 | 4.4 | 5(1) | 14.4 | 15.6 | 17(4) | 1605 | 1605 | 1595(58) |
| $B^*B^-$ | | | | | | | | | |
| $B^*_0B^*_-$ | 2.1 | 2.3 | 4(1) | 7.7 | 11.3 | 14(4) | 10653 | 10649 | 10648(113) |
| $B^*_1B^*_1$ | | | | | | | | | |
| $S_0$ | 4.6 | 5.0 | 5.3(1.1) | 16 | 17 | 19(4) | 10562 | 10553 | 1054(109) |
| $D^*D$ | 87 | 93 | 97(10) | 146 | 154 | 161(17) | 3901 | 3901 | 390(62) |
| $D^*_0D_0^-$ | | | | | | | | | |
| $B^*_0B^*_1$ | 4 | 6 | 7(1) | 8 | 11 | 14(2) | 10670 | 10670 | 10682(132) |
| $A_c$ | 8.7 | 9.5 | 10(2) | 16 | 18 | 19(5) | 10730 | 10701 | 10680(172) |

References

[1] Z. Q. Liu et al. [Belle Collaboration], arXiv:1303.0121 [hep-ex].
[2] M. Albikim et al. [BESIII Collaboration], arXiv:1303.5949 [hep-ex].
[3] For reviews, see e.g.: F. S. Navarra, M. Nielsen, S. H. Lee, Phys. Rep. 497 (2010) 4; S. L. Zhu, Int. J. Mod. Phys. E17 (2008); E.Swanson, Phys. Rep. 429 (2006) 243; N. Brambilla et al., Eur. Phys. J. C71 (2011) 1534; A. Ali, J. S Lange, Sheldon Stone, Prog. Part. Nucl. Phys. 97 (2017) 123.
[4] R.M. Albuquerque, F. Fanomezana, S. Narison, A. Rabemananjara, Phys. Lett. B715 (2012) 129.
[5] F. Fanomezana, S. Narison and A. Rabemananjara, Nucl. Part. Phys. Proc. 258-259, 152 (2015).
[6] F Fanomezana, S Narison and A Rabemananjara, Nucl. Part. Phys. Proc. 258-259, 156 (2015).
[7] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B147 (1979) 385, 448.
[8] P. Bell and R.A. Bortzmann, Nucl. Phys. B177, (1981) 218; Nucl. Phys. B187, (1981) 285.
[9] S. Narison and E. de Rafael, Phys. Lett. B522, (2001) 266.
[10] For a review, see e.g.: S. Narison, QCD spectral sum rules, World Sci. Lect. Notes Phys. 26 (1989) 1.
[11] For a review, see e.g.: S. Narison, Phys. Rept. 44 (1982) 263; S. Narison, Acta Phys. Pol. B 26 (1995) 687.
[12] R. Albuquerque, S Narison, A. Rabemananjara and D. Ratibetairony, Int. J. Mod. Phys. A 31 (2016) no.36, 1650196.
[13] R. Albuquerque, S. Narison, A. Rabemananjara and D. Ratibetairony, Int. J. Mod. Phys. A 31 (2016) no.17, 1600503.
[14] R. Albuquerque et al. Nucl. Part. Phys. Proc. 282-284 (2017) 83.
[15] A. Pich and E. de Rafael, Phys. Lett. B158 (1985) 477.
[16] A. Narison and E.Vivarov, Phys. Lett. B327 (1994) 341.
[17] S. Narison, Phys. Lett. B718 (2013) 1321.
[18] S. Narison, arXiv:1404.6642 [hep-ph].
[19] S. Narison, Phys. Lett. B624 (2005) 232.
[20] R. M. Albuquerque, F. Fanomezana, S. Narison and A. Rabemananjara, Nucl. Phys. B (Proc. Suppl.) 234, 158 (2013).