Prediction of Deflection of Shear-Critical RC Beams Using Compatibility-Aided Truss Model

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Abstract: This study proposes a method for predicting the deflection of shear-critical reinforced concrete (RC) beams. Shear deterioration of shear-critical RC beams occurs before flexural yielding. After shear deterioration occurs in the shear-critical RC beams, the deflection caused by shear is greater than the flexural deflection obtained from the elastic bending theory. To reasonably predict the deflection of shear-critical RC beams, it is necessary to evaluate deflections due to shear as well as flexure. In this study, the deflections produced by flexure and shear were calculated and superposed to evaluate the deflection of shear-critical RC beams. The method recommended by ACI 318-19 was employed to calculate the flexural deflection, and a compatibility-aided truss model able to calculate the shear stress and shear deformation at each load stage was used to consider the shear deflection. A comparison of the experimental and analytical results showed that the proposed analytical method can effectively predict the deflection of shear-critical RC beams.

Keywords: deflection; shear; flexure; truss model; RC beams

1. Introduction

A lot of existing structures worldwide remain vulnerable to shear stress due to the lack of shear analysis and design technologies. Although the shear design technology itself is advanced, there are many cases in which the shear capacity of the structure is made insufficient by errors in structural design and construction or problems in management. In recent times, existing structures tend to be repaired and rehabilitated to save raw materials and reduce carbon emissions and energy consumption. The effectiveness of such repair and rehabilitation depends on an accurate evaluation of the stress and strain states of damaged structures.

The deflection of flexure-critical RC beams has been widely investigated. Branson [1] proposed a deflection analysis method that reflects the decrease in the stiffness of RC beams due to flexural cracks using the effective moment of inertia. Subsequently, many other researchers have conducted studies to revise Branson’s formula [2–7]. Among them, Bischoff et al. [8,9] reported that Branson’s equation revealed a substantial deviation in the steel ratio amounting to 1% or less and proposed a new equation for the effective moment of inertia. In the ACI building code, Bischoff’s equation had replaced Branson’s equation in 2019 [10,11].

Flexure-critical RC beams have been designed to exhibit sufficient ductility by flexural yielding of the tension reinforcement. However, in shear-critical RC beams with low shear strength, shear deterioration occurs before flexural yielding. Shear deterioration causes an increase in shear deformation that cannot be evaluated by using the elastic bending theory. Therefore, to analyze the deformation of shear-critical RC beams, a shear analytical model that can predict both the shear strength and shear deformation of RC beams is required.

A theoretical approach to the shear analysis of RC members uses the 45-degree truss model proposed by Ritter and Mörsch as a basis and continues to develop steadily. Unlike bending, shear is not easy to analyze because it is largely related to the tensile properties...
of concrete. Most shear analysis theories, including the 45-degree truss model, cannot predict shear deformation because they only consider the stress equilibrium condition. The first shear analytical model to predict the shear deformation of RC members was the compression field theory (CFT) developed by Mitchell and Collins [12] based on the tension field theory proposed by Wagner. The CFT can predict the shear stress and shear strain of RC elements by considering the strain compatibility conditions in addition to the stress equilibrium conditions. Subsequently, Vecchio and Collins [13] developed a modified compression field theory (MCFT) that reflects the tensile properties of concrete, significantly improving the accuracy of the shear analysis.

A truss model that predicts shear stress and shear deformation by considering the strain compatibility conditions as well as the stress equilibrium conditions is called a ‘compatibility-aided truss model’. This model can be divided into rotating and fixed angle truss models, depending on whether the crack angle is fixed. The rotating angle-softening truss model (RA-STM) developed by Hsu [14] is included in the rotating angle truss model together with the MCFT. The fixed angle-softening truss model (FA-STM) proposed by Pang and Hsu [15] is the first compatibility-aided truss model considering a fixed crack angle, which assumes that the directions of principal compressive stress and fixed cracks are the same.

The difference between directions of the principal compressive stress and fixed cracks increases as the load increases when the difference in the bidirectional steel ratio is generally substantial, such as in RC beams. Lee et al. [16] proposed a transformation angle truss model (TATM) to solve this problem. The TATM can predict the shear stress and shear strain of RC beams at each load stage. This study proposes a method to calculate the deflection of shear-critical RC beams using the compatibility-aided truss model, TATM, and bending theory.

2. Calculating Method of Deflection of Shear-Critical RC Beams

Figure 1 shows an RC beam subjected to a concentrated load. The load acting on the beam is transferred from the loading point to the support by flexure and shear, and the total deformation of the beam is represented by the sum of the deformations due to flexure and shear. Deflection, which is a type of deformation, can be expressed by the following equation:

$$\Delta_{\text{tot}} = \Delta_f + \Delta_s$$

(1)

![Figure 1. Flexural and shear deflections of a RC beam.](image)

After the shear-critical RC beam reaches the peak load, as shown in Figure 1, the deflection increases, but the load decreases. This post-peak behavior is the result of a sudden collapse in the shear-resistance mechanism of the shear-critical RC beam. In this step, shear deformation is dominant rather than flexural one. Therefore, it is noted that
after the peak load, the principle of superposition in Equation (1) cannot be applied, and only the shear deflection should be considered.

The flexural deflection, \( \Delta_f \), in Equation (1) can be calculated from the curvature of the section; it shows a different behavior from the elastic theory due to cracking, which is a characteristic of concrete structures. The flexural deflection of cracked RC beams can be easily obtained using the following Bischoff’s equation recommended by ACI 318-19 [11].

\[
I_e = \frac{I_{cr}}{1 - \left(\frac{2}{3} \frac{M_c}{M_a}\right)^2 \left(1 - \frac{l_a}{l_g}\right)}
\]

(2)

Equation (2) can be used to calculate the deflection of RC beams using the elastic deflection equation. For 3- and 4-point loading, the deflection of RC beams can be obtained from the following formula:

\[
\Delta_f = \frac{M_a}{24E_cI_e} (3l^2 - 4a^2)
\]

(3)

Figure 1 shows the behavior of typical flexure- and shear-critical RC beams. As shown in Figure 1, unlike flexure-critical RC beams, shear-critical RC beams exhibit nonlinear behavior after shear deterioration, resulting in greater deflection for the same load than flexure-critical RC beams. Equation (3) is an elastic deflection equation that does not consider shear deflection. Therefore, as shown in Figure 1, when deflection is additionally increased owing to shear deterioration, a shear analytical model that can reasonably consider shear deformation is required. In this study, the average shear strain of an RC beam was calculated using the TATM developed by the author [16]. The theoretical background and calculation methods of the TATM are described in detail in Section 3. As shown in Figure 1, the average shear strain \( \gamma_{lt} \) obtained using the TATM is used to obtain the shear deflection from the relationship with the shear span as follows:

\[
\Delta_s = a \gamma_{lt}
\]

(4)

3. Shear Analytical Model for Calculating Shear Deflection

This section briefly describes the theoretical background of a shear analytical model, TATM. A detailed description for the TATM is in reference [16]. The TATM can calculate the shear stress and shear strain for each load stage of RC beams subjected to shear by using the stress equilibrium conditions, strain compatibility conditions, and material constitutive laws. In this paper, the calculation method of TATM is improved more effectively using Mohr’s circle.

3.1. Governing Equations

As shown in Figure 2, it can be assumed that a shear critical element exists in the web of an RC beam, and the stress states can be expressed as shown in Figure 3. Based on the assumption that reinforcing bars transmit only axial force, the stress equilibrium conditions can be derived by considering the equilibrium between the external and internal forces shown in Table 1. Furthermore, using the transformation formula for strain under the assumption that the strains of the concrete and rebar are the same, the strain compatibility equations of the shear critical RC element can be obtained, as shown in Table 1.
In a beam without an axial force, the initial shear crack occurs in a 45-degree direction. As the load increases after the initial crack occurs, the principal compressive stress direction, which was the same as the initial crack direction, rotates in the l-direction with more reinforcement. In particular, since the difference between the steel ratio in the l- and t-directions is generally large for RC beams, the actual experimental results can be overestimated if the rotations of the principal stress and strain are not considered. The TATM calculates the stress and strain in the m-n coordinate system from the principal stress and strain as follows using coordinate transformations, as shown in Table 1. The constitutive laws of materials used in shear analytical model TATM are listed in Table 2.
Table 2. Constitutive laws of materials used in shear analytical model.

| Materials          | Formulas                                                                 | Behavior Characteristics | Ref. |
|--------------------|---------------------------------------------------------------------------|--------------------------|------|
| Concrete in compression | $\sigma_c^e = v_f f' \left[ 2 \left( \frac{f_c}{f'} \right) - \left( \frac{f_c}{f'} \right)^2 \right]$ | $\nu = \frac{1}{1 + 2\nu v_c} \leq 1.0$ | [13,17] |
| Concrete in tension | $\sigma_1^c = E_c \epsilon_1$ for $\epsilon_1 \leq \epsilon_{cr}$, $\sigma_1^c = \sigma_{cr}'$ for $\epsilon_1 > \epsilon_{cr}$, $E_c = \frac{f_c}{\epsilon_0}$, $\sigma_{cr}' = 0.33 \sqrt{f_c} (\text{MPa})$ |                                                                 | [13] |
| Concrete in shear   | $\tau_{mn}^c = 3.8\sqrt{\frac{f'_c}{f_s}} \frac{\gamma_{mn}}{\epsilon_1 + \gamma_{mn}}$ | $w = s_c \epsilon_n$, $\delta = s_c \gamma_{mn}$ | [18] |
| Steel bar           | $f_t = E_s \epsilon_t \leq f_y$, $f_t = E_s \epsilon_t \leq f_y - \frac{\epsilon_t}{\rho_1}$ |                                                                 | [16] |

3.2. Calculation Procedure

Figure 4 shows the calculation procedure for the proposed analytical shear model. Solutions that satisfied all the equations were obtained while the principal compressive strain of the concrete gradually increased from a low value. The steel ratio $\rho_1$ in the l-direction was calculated using Equation (4); the tension steel ratio was applied in the first step. Then, solutions satisfying all equations were obtained by assuming $\epsilon_1$ and $\gamma_{mn}$. From Figure 5, which is represented by Mohr’s strain circle, the transformation angle $\beta$ can be obtained as follows.

$$\sin 2\beta = \frac{\gamma_{mn}}{\epsilon_1 - \epsilon_2}$$  (5)
The calculation procedure provided in this study is more reasonable than in previous study [16] because the angle $\beta$ here can be calculated from strains obtained at the same calculation step.

4. Verification of Proposed Method

4.1. Details of RC Beam Specimens Tested in Previous Study

Figure 6 and Table 3 show the details of the six RC beams tested in a previous study [16]. All specimens were simply supported and received a concentrated load at the mid-span of the specimens. All the specimens failed in shear before the tensile reinforcing bars yielded. The shear span-to-depth ratios of the S1- and S2-series specimens varied from 3.0 to 4.0 and 2.5 to 3.5, respectively. As shown in Figure 6, the shear reinforcement was
divided into the S1-series, wider than the effective depth d, and the S2 series, narrower than d, in order to reflect construction and design errors.

Figure 6. Details of typical specimens (unit: mm): (a) S1 series (S1-3.5); (b) S2 series (S2-3.5).

Table 3. Details of specimens and experimental and analytical results.

| Specimens | Experimental Details and Results | Analytical Results |
|-----------|---------------------------------|--------------------|
|           | $f'_c$ (MPa) | $a/d$ | $\rho_{lt}$ (%) | $f_{ty}$ (MPa) | $\rho_t$ | $\eta$ | $P_{u,exp.}$ (kN) | $\Delta_{u,exp.}$ (mm) | $\Delta_{u,exp.}$ | $P_{u,exp.}$ |
| S1-3.0    | 42.2 | 3.0 | 3.60 | 402.0 | 0.215 | 0.05 | 372.5 | 5.86 | 0.39 | 1.23 | 0.96 |
| S1-3.5    | 42.2 | 3.5 | 3.60 | 402.0 | 0.215 | 0.05 | 327.5 | 8.08 | 0.41 | 1.03 | 0.98 |
| S1-4.0    | 42.2 | 4.0 | 3.60 | 436.3 | 0.215 | 0.05 | 284.3 | 8.15 | 0.60 | 1.39 | 1.11 |
| S2-2.5    | 37.0 | 2.5 | 3.29 | 402.0 | 0.324 | 0.09 | 325.4 | 3.55 | 0.42 | 1.58 | 1.03 |
| S2-3.0    | 37.0 | 3.0 | 3.29 | 402.0 | 0.324 | 0.09 | 280.3 | 5.56 | 0.42 | 1.21 | 1.07 |
| S3-3.5    | 37.0 | 3.5 | 3.29 | 402.0 | 0.324 | 0.09 | 271.5 | 8.72 | 0.40 | 0.95 | 0.99 |

Mean 0.44 1.23 1.02
COV 17.9% 18.7% 5.9%

4.2. Verification of Proposed Method

Figure 7 shows the experimental results of S1-3.0 failed in shear. The figure on the left shows the load–deflection curve, and the one on the right shows the strain measured by the strain gauge attached to the tension reinforcement at the mid-span of the specimen. Since all the specimens shared similar characteristics to those shown in Figure 7, this study describes only the experimental results of S1-3.0, treating it as the representative sample. During the early loading stage, the first flexural crack occurred at the mid-span of the specimen. As shown in Figure 7, this initial flexural crack had a significant effect on the behavior of the tension reinforcement. The load–deflection curve of the specimen displayed a linear behavior after flexural cracking until severe shear deterioration occurred; the behavior of the tension reinforcement was also linear. As shown in Figures 1 and 7, unlike flexure-critical RC beams, the stiffness of shear-critical RC beams decreased and deflection increased after shear deterioration occurs. On the other hand, the strain of the tension reinforcement of S1-3.0 continued linearly up to the peak load, regardless of the shear deterioration. This means that the flexural deformation up to the peak load of the shear-critical RC beams can be calculated without being affected by the shear deformation.
That is, the calculation of flexural deflection using the effective moment of inertia assumed in Section 2 is valid.

As shown in Table 3, the proposed method predicted the shear strength of the six specimens with an average of 1.02 and a coefficient of variation (COV) of 5.9%. The deflection at peak load was evaluated somewhat conservatively as approximately 1.2 times. On the other hand, the analytical results obtained using Equation (3), which represents the flexural deflection relationship up to the peak load was predicted using the proposed shear analytical model.

As shown in Table 3, the proposed method predicted the shear strength of the six specimens with an average of 1.02 and a coefficient of variation (COV) of 5.9%. The deflection at peak load was evaluated somewhat conservatively as approximately 1.2 times. On the other hand, the analytical results obtained using Equation (3), which represents the flexural deflection, greatly underestimated the experimental results with an average of 0.44. This is because shear deflection was not considered in Equation (3).

Figure 8 shows the experimental and analytical results for the load–deflection relationships of the six specimens. In the figure, the solid line indicates the experimental result, and the circular symbol indicates the calculated result by the proposed analytical method. As shown in Figure 8, the predicted behavior was similar to the experimental one. Since the proposed shear model could be analyzed after the occurrence of shear cracks, the analytical results, marked below 140 kN in Figure 8, were calculated using only the flexural deflection represented in Equation (3). The analytical results marked at 140 kN or higher is the result calculated by Equation (1).

To examine the accuracy of the proposed method for the load–deflection relationship after shear cracking, Table 4 compares the analytical and experimental results at the same load. It is noted that since the deflections at the same load are compared, Table 4 deals with arbitrary loads lower than the experimental peak load of each specimen. As shown in Table 4, when only flexural deflection was considered, the experimental result was greatly underestimated at 0.65, whereas the proposed method considering both flexural and shear deflections predicted the experimental results more reasonably with an average of 1.12 and a COV of 13.2%. It is noted that time-dependent deflection due to creep and shrinkage was not considered in this study. As a result of comparing the experimental results with the analysis results, the proposed method can be used to calculate the deflection of shear-critical RC beams.
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Figure 8. Prediction results for load-deflection relationship of shear-critical RC beams: (a) S1-3.0; (b) S1-3.5; (c) S1-4.0; (d) S2-2.5; (e) S2-3.0; (f) S2-3.5.
Table 4. Prediction results of deflection of specimens for each load step.

| Specimens | Load (kN) | Experimental ∆exp. (mm) | Analytical ∆f (mm) | Ana./Exp. ∆pro. (mm) | ∆f/∆exp. | ∆pro./∆exp. |
|-----------|-----------|-------------------------|-------------------|----------------------|----------|-------------|
| S1-3.0    | 148.6     | 1.34                    | 0.94              | 1.16                 | 0.70     | 0.87        |
|           | 193.3     | 1.81                    | 1.24              | 2.01                 | 0.69     | 1.11        |
|           | 240.4     | 2.54                    | 1.55              | 3.04                 | 0.61     | 1.20        |
|           | 284.5     | 3.51                    | 1.84              | 4.26                 | 0.53     | 1.21        |
|           | 323.4     | 4.26                    | 2.10              | 5.67                 | 0.49     | 1.33        |
| S1-3.5    | 147.8     | 2.13                    | 1.50              | 1.77                 | 0.71     | 0.83        |
|           | 192.2     | 2.88                    | 1.97              | 2.93                 | 0.68     | 1.02        |
|           | 238.8     | 4.05                    | 2.46              | 4.39                 | 0.61     | 1.08        |
|           | 268.1     | 4.93                    | 2.76              | 5.54                 | 0.56     | 1.12        |
|           | 295.2     | 5.83                    | 3.05              | 6.85                 | 0.52     | 1.18        |
| S1-4.0    | 147.4     | 2.68                    | 2.25              | 2.57                 | 0.84     | 0.96        |
|           | 176.2     | 3.28                    | 2.70              | 3.53                 | 0.82     | 1.08        |
|           | 207.2     | 4.23                    | 3.18              | 4.68                 | 0.75     | 1.11        |
|           | 266.8     | 6.58                    | 4.11              | 7.54                 | 0.62     | 1.15        |
| S2-2.5    | 139.4     | 0.88                    | 0.61              | 0.75                 | 0.69     | 0.85        |
|           | 185.2     | 1.36                    | 0.82              | 1.48                 | 0.60     | 1.09        |
|           | 233.2     | 2.02                    | 1.04              | 2.40                 | 0.51     | 1.19        |
|           | 276.4     | 2.62                    | 1.24              | 3.47                 | 0.47     | 1.33        |
|           | 304.2     | 3.06                    | 1.36              | 4.36                 | 0.45     | 1.43        |
| S2-3.0    | 139.0     | 1.42                    | 1.06              | 1.23                 | 0.75     | 0.87        |
|           | 172.1     | 1.92                    | 1.32              | 1.98                 | 0.69     | 1.03        |
|           | 208.2     | 2.8                     | 1.61              | 2.92                 | 0.57     | 1.04        |
|           | 242.5     | 3.52                    | 1.88              | 4.03                 | 0.53     | 1.14        |
|           | 273.4     | 4.39                    | 2.12              | 5.30                 | 0.48     | 1.21        |
| S2-3.5    | 138.4     | 1.85                    | 1.68              | 1.90                 | 0.91     | 1.03        |
|           | 171.1     | 2.48                    | 2.09              | 2.94                 | 0.84     | 1.19        |
|           | 206.6     | 3.48                    | 2.54              | 4.27                 | 0.73     | 1.23        |
|           | 229.3     | 4.16                    | 2.82              | 5.33                 | 0.68     | 1.28        |
|           | 250.5     | 5.02                    | 3.09              | 6.54                 | 0.61     | 1.30        |
| Total     |           |                         |                   |                      | Mean 0.65 | COV 18.7%   |

5. Conclusions

A method to predict the deflection of shear-critical RC beams was developed. Existing methods for calculating the deflection of RC beams may considerably underestimate the deflection values, as the prediction of shear deformation of shear-critical RC beams is essential for reasonably calculating the deflection. In this study, a compatibility-aided truss model was used to predict shear deformation. The calculated results were compared with the experimental results and verified. The following conclusions can be drawn:

1. The flexural deflection analytically obtained using the effective moment of inertia substantially underestimated the experimental results for the deflection at peak load with an average of 0.44. This is because shear deflection increases as the shear deterioration of shear-critical RC beams occurs. However, the bending theory does not consider this phenomenon. On the other hand, the proposed analytical method predicted an average of 1.02 for the peak load and an average of 1.23 for the deflection at peak load. Although the deflection prediction using the proposed method gave somewhat conservative results, it was shown to be reliable and reasonable in terms of applicability.
(2) The experimental results showed that the strain of the tension reinforcing bars behaved almost linearly until the peak load, even if the shear-critical RC beams experienced shear deterioration. The superposition of flexural and shear deflections is a reasonable for the analysis of this result. In contrast, while the shear deformation increased rapidly after the peak load, the strain of the tension reinforcing bars no longer increased. Therefore, after reaching the peak load in the shear-critical RC beams, flexural and shear deflections cannot be superposed, and only the shear deflection obtained from the shear analytical model can be used.

(3) The load–deflection curves of shear-critical RC beams obtained using the proposed method are similar to those obtained from experimental results. By analyzing the deflection of the specimens in various load stages by using the proposed method, the experimental results were predicted to be relatively reasonably with an average of 1.12 and a COV of 13.2%. In contrast, when only the flexural deflection based on the bending theory was considered, the experimental result was considerably underestimated with an average of 0.65.

**Funding:** This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (No. 2021R1A2C2008923).

**Institutional Review Board Statement:** Not applicable.

**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** Not applicable.

**Conflicts of Interest:** The author declares no conflict of interest.

**Notations**

- $a$: shear spans of RC beam
- $A_{lt}$: cross-sectional area of tension reinforcement
- $b$: beam width of RC beam
- $d$: effective depth of RC beam
- $d_{es}$: effective shear depth of RC beam, taken as $0.9d$
- $E_c$: elastic modulus of concrete
- $E_s$: elastic modulus of steel bar
- $f'_c$: compressive strength of concrete
- $f_{l,l}$, $f_{t,t}$: stresses of reinforcement in the $l$- and $t$-directions, respectively
- $f_{y,l}, f_{y,t}$: yield strengths of longitudinal and transverse steel bars
- $I_{cr}$: moment of inertia of cracked section transformed to concrete
- $I_e$: effective moment of inertia
- $I_g$: moment of inertia of gross concrete section about centroidal axis
- $jd$: lever arm, taken as $0.9d$
- $l$: clear span of RC beam
- $M_a$: moment at which the deflection is calculated
- $M_{cr}$: cracking moment
- $P_u$: peak load of RC beam
- $s_c$: crack spacing
- $V$: shear force
- $w$: crack width
- $\alpha$: initial angle of cracks due to external loads
- $\beta$: angle between the $l$- and $m$- directions
- $\gamma_{mn}$: shear strain in the $m$-$n$ coordinate system
- $\gamma_{ll}$: average shear strain of RC beam
- $\delta$: crack slip
- $\Delta_{exp}$: deflection measured in experiment
- $\Delta_f$: flexural deflection of RC beam calculated using Equations (2) and (3)
- $\Delta_{pro}$: deflection of RC beam calculated using proposed method
\[ \Delta_s \] deflection of RC beam due to shear
\[ \Delta_{t\text{ot}} \] total deflection of RC beam
\[ \Delta_a \] deflection at peak load of RC beam
\[ \Delta_{a,exp} \] deflection at peak load of RC beam measured in experiment
\[ \Delta_{a,f} \] flexural deflection of RC beam at peak load calculated using Equations (2) and (3)
\[ \varepsilon_0 \] strain at peak stress in the stress-strain relationship of concrete
\[ \varepsilon_{c1}, \varepsilon_{c2} \] principal tensile and compressive strains of concrete, respectively
\[ \varepsilon_{crr} \] cracking strain of concrete, taken as 0.00008
\[ \varepsilon_{e1}, \varepsilon_{e2} \] strains in the l- and t-directions, respectively
\[ \varepsilon_{m,n} \] strains in the m- and n-directions, respectively
\[ \eta \] parameter defined as \( (\rho_{1f} f_y) / (\rho_{1l} f_y) \)
\[ \nu \] softening coefficient
\[ \rho_{1l} \] reinforcement ratios in the l- and t-directions, respectively
\[ \rho_{1f} \] total steel ratio in the l-direction, taken as tension reinforcement ratio in this study
\[ \rho_{1l} \] steel ratio resisting bending moment in the l-direction
\[ \sigma_{t1}, \sigma_{c2} \] principal tensile and compressive stresses of concrete, respectively
\[ \sigma_{cr} \] cracking strength of concrete
\[ \sigma_{e1}, \sigma_{e2} \] applied normal stresses acting in the l-t coordinate system, respectively
\[ \sigma_{e1}^{t}, \sigma_{e2}^{c} \] concrete shear resistances in the l-t coordinate system, respectively
\[ \sigma_{m}^{t}, \sigma_{n}^{c} \] concrete normal stresses in the m-n coordinate system, respectively
\[ \tau_{1l}, \tau_{1t} \] applied shear stress and concrete shear stress in the l-t coordinate system, respectively, and
\[ \tau_{c}^{t} \] concrete shear stress in the m-n coordinate system

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