Surrounding Vehicles Motion Prediction for Risk Assessment and Motion Planning of Autonomous Vehicle in Highway Scenarios

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ABSTRACT Safety is the cornerstone of autonomous driving vehicles. For autonomously controlled vehicles driving safely in complex and dynamic traffic scenarios, it is essential to precisely predict the evolution of the current traffic situation in the near future and make an accurate situational risk assessment. The precise motion prediction of surrounding vehicles is an essential prerequisite for risk assessment and motion planning of autonomous vehicles. In this paper, we propose a risk assessment and motion planning method for autonomously controlled vehicles based on motion prediction of surrounding vehicles. Firstly, surrounding vehicles’ trajectories are predicted based on fusing constant turn rate and acceleration-based motion prediction model and maneuver-based motion prediction model with interactive multiple models. Then, considering both the probability of collision event and collision severity, the collision risk assessment between autonomously controlled vehicle and surrounding vehicles is conducted with a collision risk index. After that, the motion planning of the autonomously controlled vehicle is formulated as a multi-objectives and multi-constraints optimization problem with a model predictive control framework. Finally, the proposed method is applied to several traffic scenarios to validate its feasibility and effectiveness.

INDEX TERMS Autonomous vehicles, motion prediction, risk assessment, motion planning, model predictive control.

I. INTRODUCTION Safety is the eternal theme of automotive technology. The frequent road traffic accidents have led to a higher demand for automotive safety. Since autonomously controlled vehicles (ACVs) have great potential in improving road safety, researchers have drawn much attention to the research and development of autonomous driving systems [1]. Motion planning is one of the core technologies for ACV. With environmental perception information provided by multi-sensors, the motion planning module of ACV generates a safe, stable, comfortable, and feasible reference trajectory for the trajectory tracking module. The traffic scenario around ACV may involve dynamic and uncertain traffic participants. Meanwhile, the ACV must satisfy kinematic constraints and kinetic constraints. All these make motion planning of ACV complicated.

Trajectory planning methods have been used in mobile robots and ACV for several decades. According to environmental modeling and searching strategies, trajectory planning methods can be classified into graph-based methods, sampling-based methods and optimization-based methods [2]. Graph-based methods separate the free space with graphs, such as the visibility graph [3] and the cell decomposition [4]. An optimal path is found with searching algorithms such as Dijkstra [5] and Anytime D* [1]. During the 2007 DRRPA Urban Challenge, the team named BOSS segmented the state space with state lattice consisting of position, heading angle and velocity, and Anytime D* was employed for searching the sub-optimal path in unstructured environments [1]. However, the completeness and optimality of graph-based methods are related to the size of state lattice. The complexity for finding the optimal path depends on
the dimension of state space, the number of obstacles and
the size of the state space. The graph-based methods are
deterministic, while the sampling-based methods discrete
the configuration space by sampling randomly. The rapidly-
exploring-random-tree (RRT) is a typical sampling-based
method [6]. MIT made a few extensions of standard RRT and
applied it to uncertain and dynamic scenarios in the DARPA
Urban Challenge [7]. Since collision detection is required
during tree extension each time, the complexity of RRT is
extremely high in crowded obstacle areas. Optimization-
based trajectory planning methods formulate the holonomic
and non-holonomic constraints as equalities and inequalities,
respectively. The optimal trajectory is found by solving a
constrained optimization problem. However, there may exist
several local extremums. A local and continuous trajectory
planning with only a single global optimum was proposed
for driving Bertha-Benz-Memorial-Route autonomously [8].
Static and dynamic obstacle constraints are represented
as polygons. A left-right decision and geometric processing
were incorporated for modifying the original polygons,
which resulted in the solution converging to a single, global
optimum.

The motion planning of ACV is essentially a multi-
objectives and multi-constraints optimization problem.
Safety is one of the basic demands of ACV. Risk assess-
ment is used for evaluating the safety of ACV. Researchers
typically construct risk indicators for risk assessment based
on motion prediction of ego vehicle and surround vehicles
(SVs) [9]–[11]. Consequently, risk assessment usually
involves two essential parts, which refers to motion prediction
of SVs and the construction of a reasonable risk indicator.

According to levels of information abstraction, motion
prediction of vehicles are divided into physics-based,
maneuver-based and interaction-aware motion models [12].
In physics-based motion models, vehicles are viewed as
dynamic entities controlled by physical laws. Kinematic
models or kinetic models are used for motion prediction
of vehicles in the future. The typical physics-based motion
models includes Constant Velocity (CV) Model, Constant
Acceleration (CA) Model, Constant Turn Rate and Velocity
(CTRV) Model, Constant Turn Rate and Acceleration
(CTRA) Model, Constant Steering Angle and Velocity Model
(CSAV), Constant Steering Angle and Acceleration (CSAA)
Model [12]. Schubert et al. made a comprehensive com-
parison of numerous physics-based motion models [13].
Although physics-based motion models have good real-
time efficiency, they are insufficient to describe changes in
vehicle motion due to abrupt maneuvering behaviors or envi-
ronmental factors. In maneuver-based motion models, vehi-
cles’ motion on the road network can be viewed as several
independent maneuvers. The vehicles’ trajectories are pre-
dicted based on recognized maneuvers. Typical maneuver
recognition methods include Support Vector Machine [14],
Hidden Markov Model [15] and Dynamic Bayesian Net-
work [16], et al. However, maneuver-based motion models
do not take into account the interaction among traffic
participants. In interaction-aware motion models, the motion
of one vehicle is influenced by the other traffic participants,
which makes them the complete models for the motion pre-
diction of vehicles. Li et al. proposed an interaction-aware
motion model for surrounding vehicles, which modeled the
interaction among vehicles with a performance function
penalizing the possible collisions [17].

The existing risk indicators mainly include time-
based, distance-based, deceleration-based, probability-based
and field theory-based measures [18], such as Time-
To-Collision (TTC) [19], Time-Headway (TH) [20], Time-
To-React (TTR) [21], Predicted Minimum Distance [22],
Required Deceleration [23], et al. However, in time-
based, distance-based and deceleration-based risk indicators,
the motion of ego vehicle and surrounding vehicles are
commonly described with CV or CA models. Although they
have good real-time performance, they are only suitable for
single-lane risk assessment in rear-end collision scenarios.
Xu et al. proposed an integrated risk assessment indicator
for multi-lane traffic scenarios by synthesizing the TTC, TH
and original constructed Time-To-Front (TTF) [24]. Interactive
multiple models (IMMs) is used for motion prediction of
surrounding vehicles. Hruschka et al. used maneuver-based
models for predicting the motion of SVs, and evaluated the
collision risk with a combination of collision probability
and collision severity [25]. In literature [26], the risk is
defined as the expected cost related to a future critical event.
Future event probability and damage probability are used to
determine risk indicator for motion planning of ego vehicle.

Due to low accuracy in the long prediction horizon,
physics-based motion models are not suitable for risk assess-
ment in complicated traffic scenarios. Maneuver-based and
interaction-aware motion models mainly depend on machine
learning methods. A great deal of data is needed for model
training. Besides, the trained model may be constrained to
road topology similar to those in the training process. Mean-
while, real-time efficiency cannot be satisfied due to high
computation complexity. In this paper, a risk assessment and
motion planning method based on motion prediction of SVs
is proposed, as illustrated in Fig. 1. The motion of surround-
ing vehicles is predicted by integrating the CTRA model
and a simplified maneuver model with IMM. Specifically,
the adopted maneuver model recognizes maneuvers by the
spatial-temporal relationship between the vehicle’s historical
trajectory and road geometry shape, which is different from
traditional maneuver recognition methods relying on a large
amount of training data. Then, the collision risk indicator
is used for risk assessment between ACV and SVs, which
synthesizes collision probability and collision severity. The
proposed risk indicator is applied to the motion planning of
ACV in highway scenarios.

The remainder of this work is structured as follows.
Section II briefly presents an introduction to the definition of
coordinates. In Section III, the motion prediction of surround-
ing vehicles is stated. The construction of the risk indicator
is explained in Section IV. In Section V, the motion planning of
ACV is formulated as a multi-objective and multi-constraints problem with model predictive control (MPC) framework. The proposed method is applied to several traffic scenarios to validate its feasibility and effectiveness in Section VI. Finally, Section VII gives the conclusions and future work.

II. THE DEFINITION OF COORDINATES

This section introduces several coordinates used for motion prediction of SVs and motion planning of ACV. As shown in Fig. 2, there are global coordinates system, road coordinate system and ego vehicle coordinate system. The global coordinate system is denoted by superscript G. The origin point of the global coordinate system is fixed. Its x axis points to the east, and the y axis points to the north. The local road frame is a curvilinear coordinate system. The Frenet frame is adopted for the road coordinate system, which is labeled with superscript F. The origin points of the local road frame is fixed to the right road boundary. Its x axis coincides with the right road boundary. The ego vehicle coordinate system is distinguished with superscript E, whose origin point coincides with the center of mass of ACV. The direction of x axis in the ego vehicle coordinate system is the same as that of velocity for ACV. SVs and ACV are regarded as particles making a curvilinear motion in a two-dimensional plane. In Fig. 2, the $x^G_e$ and $x^G_o$ refer to the longitudinal position of ACV and SV in road Frenet frame. The $y^F_e$ and $y^F_o$ are the lateral position of ACV and SV, respectively. The $\psi^G_e$ and $\psi^G_o$ mean the heading angle of ACV and SV in the global coordinate system. The $\psi^F_e$ and $\psi^F_o$ represent the heading angle of ACV and SV in the road Frenet frame. The $v^G_e$ and $v^G_o$ are the velocity of ACV and SV.

III. MOTION PREDICTION FOR SURROUNDING VEHICLES

In this section, the motion prediction of SV is discussed based on the CTRA motion model. Then, taking into account the temporal-spatial relationship between SV’s historical trajectory and geometry shape of the road boundary, a simplified maneuver recognition model is proposed. Motion prediction of SV is conducted on the basis of the simplified maneuver recognition model. Finally, IMMs is used for predicting SV’s trajectory by integrating the CTRA motion model and the simplified maneuver recognition model.
A. MOTION PREDICTION OF SV BASED ON CTRA MOTION MODEL

Physics-based motion models view vehicles as dynamic entities governed by physical laws. Vehicle motion is described by correlations among motion states of vehicle. The most commonly used physics-based motion models are CV model, CA model, CTRV model, CTRA model. CV and CA models are one-dimensional, which assumes that the vehicle moves straightly with constant velocity or acceleration. CTRV and CTRA models are two-dimensional, which regards vehicle as a particle moving in a curvilinear line with constant yaw rate and velocity or constant yaw rate and acceleration. Obviously, two-dimensional models are much complete and more accurate in motion prediction. Consequently, the CTRA model is adopted for motion prediction of SVs in this work.

Let $x^G_{o_i}$ be a state vector for surrounding vehicle $i$ in the global coordinate system, as illustrated in Eq. (1). The present motion states information of SV can be obtained from active sensors equipped with ego vehicle or V2V between ego vehicle and SV.

$$x^G_{o_i} = \begin{pmatrix} x^G_{o_i} \ y^G_{o_i} \ \theta^G_{o_i} \ v^G_{o_i} \ a^G_{o_i} \ \omega^G_{o_i} \end{pmatrix}^T \tag{1}$$

where $(x^G_{o_i}, y^G_{o_i})$ represents the position in the global frame, $\theta^G_{o_i}$ is the heading angle, $v^G_{o_i}$ is the velocity, $a^G_{o_i}$ is the tangential acceleration, and $\omega^G_{o_i}$ is the yaw rate.

In the CTRA motion model, the yaw rate and the acceleration are constant in the future. The state transition process is illustrated in Eq. (2), as shown at the bottom of the page. The statistical characteristic for process noise and measurement noise are illustrated in Eq. (2c). In particular, we adopt Kalman Filtering for system states prediction in the future instead of estimation of current system states. Since there are no measurement information in the future, the measurement equation is omitted in this work, where $f(x^G_{o_i})$ is state transition function, $w^G_{a_k}$ is process noise vector, $w^G_{a_k}$ is process noise for acceleration, $w^G_{\Omega_{k}}$ is process noise for yaw rate.

Both $w^G_{a_k}$ and $w^G_{\Omega_{k}}$ are zero-mean Gaussian white noise. $Q^G_{\Omega_{k}}$ is the non-negative covariance matrix for the process noise, $\Delta t$ is the time interval of each step.

Since the transition equation involves sine items and cosine items, this process is nonlinear. Unscented Kalman Filter (UKF) is used for recursively estimating the state vector and the covariance matrix. UKF is a filter algorithm for approaching the posterior distribution of the nonlinear system based on Unscented Transformation (UT). In UT, several Sigma sampling points are chosen. These Sigma sampling points have the same mean and covariance of the system state distribution. The most important thing in UT is to determine the number, position and weight coefficients of Sigma sampling points. In this work, a symmetrical sampling strategy is adopted, as shown in Eq. (3). After the Sigma sampling points are obtained, the nonlinear transformation function $f(\cdot)$ is acted on them, as illustrated in Eq. 4. Finally, the state vector and covariance are calculated with Eq. (5). The prediction uncertainty of the state vector can be reflected in the covariance matrix.

$$\xi_{i,k} = \begin{cases} \hat{x}^G_{o_i,k}, i = 0 \\ \hat{x}^G_{o_i,k} + \left(\sqrt{(n+\lambda)P^G_{o_i,k}}\right)_i, i = 1, 2, \cdots, n \\ \hat{x}^G_{o_i,k} - \left(\sqrt{(n+\lambda)P^G_{o_i,k}}\right)_i, i = n + 1, n + 2, \cdots, 2n \end{cases} \tag{3a}$$

$$W^m_{i,k} = W^c_{i,k} = \begin{pmatrix} \lambda & \frac{n + \lambda}{2} & 0 \\ \frac{n + \lambda}{2} & \lambda \end{pmatrix} \tag{3b}$$

where $\xi_{i,k}$ is Sigma sampling point, $\hat{x}^G_{o_i,k}$ is estimated state vector in the last moment, $P^G_{o_i,k}$ is covariance matrix in the last moment, $\left(\sqrt{(n+\lambda)P^G_{o_i,k}}\right)_i$ is the $i$th row of the square root matrix for $(n + \lambda) P^G_{o_i,k}$. $W^m_{i,k}$ is the first-order weighting

$$x^G_{o_i,k+1} = f(x^G_{o_i,k}) + w^G_{a_k} \tag{2a}$$

$$w^G_{a_k} = \begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix}^T \tag{2b}$$

$$E\left(w^G_{o_i,k}\right) = 0, E\left(w^G_{o_i,k} \left(w^G_{o_i,j}\right)^T\right) = Q^G_{o_i,k} \delta _{ij} \tag{2c}$$

$$f(x^G_{o_i,k}) = \begin{pmatrix} x^G_{o_i,k} + \frac{v^G_{o_i,k} + a^G_{o_i,k} \Delta t}{2} \sin(\omega^G_{o_i,k} \Delta t + \theta^G_{o_i,k}) - \frac{v^G_{o_i,k} \sin \theta^G_{o_i,k}}{a^G_{o_i,k}} + \frac{\omega^G_{o_i,k} \Delta t}{2} \cos(\omega^G_{o_i,k} \Delta t + \theta^G_{o_i,k}) - \cos \theta^G_{o_i,k} + \Delta k \omega^G_{o_i,k} \Delta t + \theta^G_{o_i,k} \Delta k \right) \\ y^G_{o_i,k} + \frac{v^G_{o_i,k} \cos \theta^G_{o_i,k}}{a^G_{o_i,k}} - \frac{v^G_{o_i,k} + a^G_{o_i,k} \Delta t}{2} \cos(\omega^G_{o_i,k} \Delta t + \theta^G_{o_i,k}) + \frac{\omega^G_{o_i,k} \Delta t}{2} \sin(\omega^G_{o_i,k} \Delta t + \theta^G_{o_i,k}) - \sin \theta^G_{o_i,k} + \Delta k \omega^G_{o_i,k} \Delta t + \theta^G_{o_i,k} \Delta k \\ a^G_{o_i,k} \Delta t + v^G_{o_i,k} \\ \omega^G_{o_i,k} \Delta t + v^G_{o_i,k} \\ \Delta t + v^G_{o_i,k} \\ a^G_{o_i,k} \Delta t + \omega^G_{o_i,k} \Delta k \end{pmatrix} \tag{2d}$$
coefficient of sigma sampling point, \( W_{i,k}^F \) is the second-order weighting coefficient of sigma sampling point, \( \lambda \) is used to adjust the relative distance between sigma point and the state vector in the last moment.

\[
\mathbf{y}_{i,k} = \mathbf{f}(\xi_{i,k}) \quad \text{for} \quad i = 0, 1, \ldots, 2n
\]  

(4)

\[
\mathbf{x}_{0i,k+1|k} = \sum_{i=0}^{2n} W_{i,k}^m \mathbf{y}_{i,k}
\]  

(5a)

\[
\mathbf{P}_{0i,k+1|k}^G = \sum_{i=0}^{2n} W_{i,k}^c \left( \mathbf{y}_{i,k} - \hat{\mathbf{x}}_{0i,k+1|k}^G \right) \mathbf{y}_{i,k} - \hat{\mathbf{x}}_{0i,k+1|k}^G + Q_{0i,k}^G
\]  

(5b)

where \( \hat{\mathbf{x}}_{0i,k+1|k} \) is the estimated value of state vector, \( \mathbf{P}_{0i,k+1|k}^G \) is its covariance matrix, \( Q_{0i,k}^G \) is process noise matrix.

**B. MOTION PREDICTION OF SV BASED ON A SIMPLIFIED MANEUVER RECOGNITION MODEL**

In maneuver-based vehicle trajectory prediction models, vehicle motion on the road network is associated with particular maneuvers, such as lane-keeping, lane change and turns. Each maneuver can be represented by a cluster of vehicle trajectories. In literature [27], the quaternion-based rotationally invariant longest common subsequence was adopted to measure similarities between trajectories. The radial basis function classified these trajectories into several types. Vehicle motion in the near future was predicted by matching its historical trajectory with these samples in the database collected in advance. In literature [28], the previously observed motion patterns were combined with Gaussian Mixture Models to infer a joint probability distribution as a motion model in the future. In summary, these maneuver recognition model based vehicle trajectory prediction model can recognize maneuvers by the temporal-spatial relationship between the vehicle’s historical trajectory and road geometry shape, which is independent of training data. After that, vehicle motion is predicted based on the recognized maneuver and its typical motion pattern.

1) MANEUVER RECOGNITION OF SV

Let \( \mathbf{x}_{0i}^F \) be a state vector for SV in the local road frame, as illustrated in Eq. (6).

\[
\mathbf{x}_{0i}^F = \left( x_{0i}^F \ y_{0i}^F \ v_{0i,x}^F \ v_{0i,y}^F \ a_{0i,x}^F \ a_{0i,y}^F \right)^T
\]  

(6)

where \( x_{0i}^F \) is the arc length along the right road boundary, \( y_{0i}^F \) is the lateral position relative to the right road boundary, \( v_{0i,x}^F \) is the tangential velocity in road frame, \( v_{0i,y}^F \) is the normal velocity in road frame, \( a_{0i,x}^F \) and \( a_{0i,y}^F \) are the tangential acceleration and the normal acceleration in road frame respectively.

Since \( y_{0i}^F \) represents SV’s lateral position in the road frame, maneuvers such as lane-keeping, lane change left and lane change right can be distinguished by the time serial of lateral position \( y_{0i}^F \). Consequently, we recognize maneuver by utilizing the temporal-spatial relationship between SV’s past trajectory and road geometry shape. Taking into account the temporal sequence of SV’s past trajectory, an integrated lateral position indicator is constructed, as shown in Eq. (7).

\[
\mathbf{y}_{0i,k}^F = \frac{1}{N-1} \sum_{j=0}^{N-1} \alpha_j y_{0i,k-j}^F
\]  

(7a)

\[
\alpha_j = e^{-(N-1-j)\Delta t}, \quad j = 0, 1, \ldots, N-1
\]  

(7b)

where \( \mathbf{y}_{0i,k}^F \) is the weighted lateral position in road frame after integrating several past lateral positions; \( \alpha_j \) is weighting coefficient, which allocates more importance to the nearer points in past trajectory; \( N \) is the number of steps.

Let \( \omega_L \) be the width of the lane. The sequence \( \{y_{0i,k-(N-1-j)}^F\}, j = 0, 1, \ldots, N-1 \) represents the lateral position temporal serial for SV in the past few seconds. \( M_1 \) is used to count the number of increasing elements in the sequence \( \{y_{0i,k-(N-1-j)}^F\} \). \( M_2 \) is used to count the number of decreasing elements in the sequence \( \{y_{0i,k-(N-1-j)}^F\} \), as illustrated in Eq. (8). The initial values \( M_1 \) and \( M_2 \) are zeros. When SV makes a lane change left maneuver, its lateral position increases over time. The temporal sequence \( \{y_{0i,k-(N-1-j)}^F\} \) is approximately monotonically increasing. As a sequence, the value of \( M_1 \) is near to \( N \). Similarly, when SV is making a lane change left maneuver, its lateral position decreases over time. The temporal sequence \( \{y_{0i,k-(N-1-j)}^F\} \) is approximately monotonically decreasing. The value of \( M_2 \) approximates \( N \). The maneuver recognition of SV in the three-lane highway scenario is shown in Table 1 and Fig. 3. Only normal maneuvers such as lane change left, lane change right and lane-keeping are considered. Abnormal behaviors can be detected by comparing the maneuver recognition result with the recognized maneuver.

**TABLE 1. Maneuver recognition of SV in a three-lane highway scenario.**

| Discriminating conditions | Classification of maneuvers |
|---------------------------|----------------------------|
| \( M_1 \geq 0.8N, M_2 \leq 0.8N, \) \( 0 \leq \{y_{0i,k}^F\} \leq 1.5\omega_L \) | Lane change left from lane 1 to lane 2 |
| \( M_1 \leq 0.8N, M_2 \geq 0.8N, \) \( 0 \leq \{y_{0i,k}^F\} \leq 1.5\omega_L \) | Lane change right from lane 2 to lane 1 |
| \( M_1 \geq 0.8N, M_2 \leq 0.8N, \) \( 1.5\omega_L \leq \{y_{0i,k}^F\} \leq 3\omega_L \) | Lane change left from lane 2 to lane 3 |
| \( M_1 \leq 0.8N, M_2 \geq 0.8N, \) \( 1.5\omega_L \leq \{y_{0i,k}^F\} \leq 3\omega_L \) | Lane change right from lane 3 to lane 2 |
| \( M_1 \leq 0.8N, M_2 \leq 0.8N, \) \( 0 \leq \{y_{0i,k}^F\} \leq \omega_L \) | Lane-keeping in lane 1 |
| \( M_1 \geq 0.8N, M_2 \leq 0.8N, \) \( 2\omega_L \leq \{y_{0i,k}^F\} \leq 3\omega_L \) | Lane-keeping in lane 2 |
| \( M_1 \leq 0.8N, M_2 \leq 0.8N, \) \( 2\omega_L \leq \{y_{0i,k}^F\} \leq 3\omega_L \) | Lane-keeping in lane 3 |
The longitudinal and lateral motion of SV is illustrated in an Ornstein-Uhlenbeck process, which means that the lateral model is continuous-time model, the longitudinal motion of SV is described by the CA public highway scenarios are involved. In literature [29], vehicle trajectory prediction models for maneuvers, such as following road, following vehicle, target braking, lane change and turn, were established by taking into account vehicle and environmental evidence. Lienke et al. formulated lane-keeping and lane-change maneuvers as quartic polynomial and quartic polynomial, respectively. In this work, only lane-keeping, lane-change left and lane-change right maneuvers in public highway scenarios are involved.

In the lane-keeping maneuver-based motion prediction model, the longitudinal motion of SV is described by the CA model. The lateral motion is modeled as a continuous-time Ornstein-Uhlenbeck process, which means that the lateral position of SV converges to the middle of the current lane. The longitudinal and lateral motion of SV is illustrated in Eq. (9).

\[
\begin{bmatrix}
M_1 = M_1 + 1, y_{o_l,k,j+1}^F > y_{o_l,k-j}^F \\
M_2 = M_2 + 1, y_{o_l,k,j+1}^F < y_{o_l,k-j}^F,
\end{bmatrix}
\quad j = 0, 1, \ldots, N - 1
\quad (8)
\]

2) MOTION PREDICTION OF SV BASED ON MANEUVER RECOGNITION MODEL

Vehicle maneuvers in public traffic can be described by certain types of curves. In literature [29], vehicle trajectory prediction models for maneuvers, such as following road, following vehicle, target braking, lane change and turn, were established by taking into account vehicle and environmental evidence. Lienke et al. formulated lane-keeping and lane-change maneuvers as cubic curves [30]. In literature [31], the lateral and longitudinal position during lane-change and lane-keeping maneuvers were modeled as quartic polynomial and quartic polynomial, respectively. In this work, only lane-keeping, lane-change left and lane-change right maneuvers in public highway scenarios are involved.

In the lane-keeping maneuver-based motion prediction model, the longitudinal motion of SV is described by the CA model. The lateral motion is modeled as a continuous-time Ornstein-Uhlenbeck process, which means that the lateral position of SV converges to the middle of the current lane. The longitudinal and lateral motion of SV is illustrated in Eq. (9).

\[
\begin{bmatrix}
\Delta x^F_{o_l,k+1} \\
\Delta v^F_{o_l,k+1} \\
\Delta a^F_{o_l,k+1}
\end{bmatrix}
= \begin{bmatrix}
1 & \frac{(\Delta t)^2}{2} \\
0 & 1 \frac{\Delta t}{2} \\
0 & 0 1
\end{bmatrix}
\begin{bmatrix}
\Delta x^F_{o_l,k} \\
\Delta v^F_{o_l,k} \\
\Delta a^F_{o_l,k}
\end{bmatrix}
+ \begin{bmatrix}
\frac{(\Delta t)^2}{2} \\
\frac{\Delta t}{2} \\
1
\end{bmatrix}
\omega^F_{o_l},
\quad (9a)
\]

\[
y^F_{o_l,k+1} = e^{-\beta \cdot \Delta t} y^F_{o_l,k} + \left( 1 - e^{-\beta \cdot \Delta t} \right) y^F_{l_{o_l}} + \omega^F_{y^F_{o_l}},
\quad (9b)
\]

where \(\omega^F_{o_l}\) is process noise of longitudinal acceleration, \(\omega^F_{x^F_{o_l}} \sim N(0, \sigma^2_{e^F})\), \(\omega^F_{y^F_{o_l}}\) is process noise of lateral position, \(\omega^F_{y^F_{o_l}} \sim N(0, \sigma^2_{e^F} \cdot (1 - e^{-2\beta \cdot \Delta t}))\), \(\beta\) is decay factor, \(y^F_{l_{o_l}}\) is the coordinate value of the middle of the current lane occupied by SV.

In the lane-change maneuver-based motion prediction model, the motion of SV is modeled as a half-cycle sine curve. It starts in the middle of the current lane and ends at the middle of the target lane. Since lane-change left and lane-change right maneuvers are approximately symmetric. For simplicity, only motion prediction about lane-change left from lane 1 to lane 2 is introduced here, as illustrated in Eq. (10). The longitudinal motion of SV is expressed with CA model, which is the same as that of longitudinal motion in lane-keeping maneuvers. After the value of \(x^F_{o_l,k+1}\) is obtained from the CA model, we calculate the value of \(y^F_{o_l,k+1}\) with Eq. (10).

\[
y^F_{o_l,k+1} = -\frac{1}{2} \omega^F_{x^F_{o_l}} \sin \left( \frac{\pi l^*_{s} \Delta x^F_{o_l,k+1} + \pi}{2} \right) + \omega^F_{y^F_{o_l}},
\quad (10a)
\]

\[
\Delta x^F_{o_l,k+1} = x^F_{o_l,k+1} - (x^F_{o_l})^* \quad (10b)
\]

where \(l^*_{s}\) is the projection length along the \(X^F\) axis for the whole lane-change process, \((x^F_{o_l})^*\) is the coordinate value of when the lane-change process starts, unknown parameters \(l^*_{s}\) and \((x^F_{o_l})^*\) can be calculated by nonlinear least-squares estimate with SV’s historical trajectory in the near past.
C. MOTION PREDICTION OF SV BASED ON IMM

The physics-based motion prediction model is accurate only in a short prediction horizon, less than one second. The maneuver-based motion prediction model achieves more precise performance in the long term. Combining physics and maneuver-based models for vehicle trajectory prediction gets more accurate prediction results in the whole prediction horizon. Aris et al. proposed a hierarchical-structured method fusing an adaptive kinematic model with the lane-keeping maneuver model in a linear weighting way for vehicle trajectory prediction [32]. In literature [31], the cubic spline curve was selected as a weighting function for combing the CTRA motion model with lane-keeping and lane-change maneuver-based motion models. The maneuver was recognized by a weighted distance between the vehicle’s historical trajectory and the road centerline. Kim and Yi integrated a vehicle state filter, a road geometry filter and a path-following model for vehicle motion prediction, where the path-following model was adopted to generate the desired yaw rate in the future for the vehicle state filter [33]. Xie et al. proposed a vehicle trajectory prediction method using the CTRA motion model and the DBN-based maneuver recognition model [16].

For systems whose structure are unknown or parameters are variable, the hybrid-system-based multiple-models is a powerful adaptive estimation method. Multiple-models-based estimation usually involves the model set design, choosing filters, estimation fusion and filters re-initialization. How to re-initialize each filter in the model set is associated with the accuracy and time efficiency of the multiple-models method. IMMs is a typical fixed structure multiple-models method. Assuming the model transition obeys the Markov process, the performance of IMMs is the same as the second-order generalized pseudo-Bayesian algorithm and its time efficiency is comparable with the first-order generalized pseudo-Bayesian algorithm. In this work, we propose a vehicle trajectory prediction method by fusing the CTRA model and the simplified maneuver recognition-based motion model with IMMs, as illustrated in Fig. 4. IMMs is a classic sub-optimal algorithm for state estimate of the hybrid system [34]. When IMMs is used for state estimation, considering each sub-model in the model set may be an effective model at present, the initial condition of each sub-model is a composite of the prediction result of each filter at the previous moment. The overall state estimate vector and covariance are obtained by integrating all outputs from sub-models. There are two sub-models in this research. The one is the CTRA motion model. The other is a simplified maneuver recognition-based motion model. IMMs is recursive. It typically includes four steps: model-conditional re-initialization, model-conditional filtering, model probability update and estimate fusion. The details are the following.

Model-conditional re-initialization refers to re-calculating the input of each sub-model with both outputs from the last moment. Supposing the matching sub-model at the present time \( k \) is \( M_k^i \), and the matching sub-model in the next moment \( k + 1 \) is \( M_{k+1}^j \), the mixed probability is illustrated in Eq. (11).

\[
\mu_{k+1}^j \triangleq p(M_{k+1}^j | M_k^i) = \frac{1}{c} \pi_{ij} \mu_k^j
\]  

where \( \tilde{c}_j \) is the normalization factor, \( \mu_k^j \) is the prior probability for sub-model \( i \), \( \pi_{ij} \) is model transfer probability.

After model-conditional re-initialization, the state vector and covariance matrix for each sub-model are shown in Eq. (12).

\[
\begin{align*}
\hat{x}_{k+1}^j &= \sum_{i=1}^{2} \hat{x}_k^i \mu_{k+1}^j \\
\hat{P}_{k+1}^j &= \sum_{i=1}^{2} \left[ \hat{P}_k^i + \left( \hat{x}_k^i - \hat{x}_k^j \right) \cdot \left( \hat{x}_k^i - \hat{x}_k^j \right)^T \right] \mu_{k+1}^j
\end{align*}
\]  

Given inputs from model-conditional re-initialization, model-conditional filtering means predicting vehicle trajectory with each sub-model, as shown in Eq. (13).

\[
(\hat{x}_{k+1|k}^j, \hat{P}_{k+1|k}^j) = \text{Pred}(\hat{x}_k^j, \hat{P}_k^j)
\]  

where \( \hat{x}_{k+1|k}^j \) is the estimate state vector from the sub-model \( j \) in the next moment \( k+1 \), \( \hat{P}_{k+1|k}^j \) is the covariance matrix from the sub-model \( j \) in the next moment \( k+1 \).

Model probability update refers to calculating the posterior probability of each sub-model, as illustrated in Eq. (14).

\[
\begin{align*}
\mu_{k+1}^i \triangleq p(M_{k+1}^i | \hat{x}_{k+1}^j, \hat{P}_{k+1}^j) = \frac{1}{c} N_{k+1} \tilde{c}_j \\
N_{k+1}^j \triangleq \frac{1}{\hat{P}_{k+1|k}^j (\hat{x}_{k+1|k}^j, \hat{P}_{k+1|k}^j) (y_{k+1}^j) + \hat{P}_{k+1|k}^j (y_{k+1}^j)} \\
c = \sum_{i=1}^{2} N_{k+1}^i \tilde{c}_i
\end{align*}
\]
where $\mu_{j+1}^k$ is the posterior probability, $\Lambda_{j+1}^k$ is the possibility for sub-model matching with the hybrid system, $\hat{P}_{x,j+1|k}^k(x'_{j+1})$ is prediction variance in $x$ axis for sub-model, $\hat{P}_{y,j+1|k}^k(y'_{j+1})$ is prediction variance in $y$ axis for sub-model, is a normalization factor.

Estimate fusion means calculating the overall state vector and variance matrix with that of sub-models, as shown in Eq. (15).

\begin{align}
\hat{x}_{k+1} &= \sum_{i=1}^{2} x_{i+1|k}^i \mu_{i+1}^k \\
\hat{P}_{k+1} &= \sum_{i=1}^{2} \left[ \hat{P}_{i+1|k}^k + \left( \hat{x}_{i+1|k}^k - \hat{x}_{k+1}^k \right) \right] \mu_{i+1}^k
\end{align}

where $\hat{x}_{k+1}$ is the overall estimated state vector for the hybrid system, $\hat{P}_{k+1}$ is the overall estimated covariance for the hybrid system.

D. SIMULATION ANALYSIS OF MOTION PREDICTION METHODS

In order to verify the effectiveness and superiority of the presented IMM-based motion prediction methods, we compare it with two commonly used motion prediction models: CTRA and Lane Keeping Model (LKM). The simulation scenario is defined in Fig.5. The vehicle is moving in a straight road at a constant speed of 25 m/s. Firstly, the vehicle keeps in lane 1. Then, the vehicle makes a lane change left behavior from lane 1 to lane 2. The lane change process lasts about 6s. Finally, the vehicle keeps in lane 2. The yaw rate of the vehicle in the whole duration is illustrated in Fig. 6.

![Figure 5. Lane-keeping and lane change process in a straight road.](image)

We mainly focus on the prediction results for the lane change duration. The prediction horizon is 5s. The Root Mean Square Error (RMSE) is adopted as a metric for evaluating prediction accuracy. The prediction errors of three different approaches in the lane change process is shown in Table 2. Fig.7 depicts the prediction results for three different methods at 1s, 2s, 3s and 4s after lane change starts. As shown in Table 2 and Fig.7, the CTRA-based trajectory prediction method has a large prediction error in the whole lane change process. The trajectory prediction accuracy of the LKM model is low before the vehicle enters the target lane, as illustrated in Fig. 7.a, Fig. 7.b and Fig. 7.c. After the vehicle crosses the lane line, the prediction results of the LKM model is extremely close to that of IMM-based approaches, as depicted in Fig. 7.d. Specifically, the proposed IMM-based motion prediction method has an excellent prediction accuracy in the whole process.

![Figure 6. Vehicle yaw rate in lane keeping and lane change process.](image)

![Table 2. Prediction error (RMSE) of three methods in lane change process.](image)
motion prediction for surrounding vehicles, Kim and Kum proposed a metric for assessing the collision probability between ego vehicle and surrounding vehicle based on vehicle shape and TTC [11]. In literature [37], the relative kinetic energy density between the ego vehicle and the preceding vehicle was employed for collision risk assessment in multi-vehicle collision avoidance scenarios. In literature [38], the projected area of ego vehicle along the direction of relative velocity was used as a metric for measuring the severity of the collision event.

Taking into account the temporal-spatial relationship between ACV and SV, an innovative metric is proposed for collision risk assessment. The probability of a collision event is measured by the longitudinal distance, lateral distance and the time headway between ACV and SV. The severity of a collision event is evaluated by the kinetic energy of ACV. The detailed expressions are illustrated in Eq. (16).

$$\text{Risk}(x_F^e, x_F^{i}, t) \equiv P\left(x_F^e, x_F^{i}, t\right) \cdot S\left(x_F^e, t\right)$$

(16a)

$$P\left(x_F^e, x_F^{i}, t\right) = e^{-\frac{(y_F^e - y_F^{i})^2}{\alpha_1^2}} \cdot e^{-\frac{(x_F^e - x_F^{i})}{\alpha_2^2}} \cdot e^{-\frac{1}{\alpha_3^2}(\frac{x_F^e - v_F}{\alpha_4})}$$

(16b)

$$S(x_F^e, t) = e^{-\frac{2}{\alpha_1^2 \cdot \mu(v_F^e)^2}}$$

(16c)

where Risk\((x_F^e, x_F^{i}, t)\) is the collision risk between ACV and SV \(i\), \(P(x_F^e, x_F^{i}, t)\) is the probability of a collision event, \(S(x_F^e, t)\) is the severity of a collision event. \(x_F^e\) is the state vector of ACV in road frame, \(x_F^{i}\) is the state vector of SV \(i\) in road frame, \(m_e\) is mass of ACV, \(\alpha_1, \alpha_2, \alpha_3\) and \(\alpha_4\) are parameters for adjusting the corresponding weight in collision risk.

### B. EXAMPLE OF RISK ASSESSMENT FOR A REAR-END COLLISION AVOIDANCE SCENARIO WITH BRAKING

Fig. 8 shows the initial condition for a rear-end collision scenario. The initial speed of the ego vehicle is 25 m/s. There is a static vehicle in front of the ego vehicle. The initial distance between the ego vehicle and the static vehicle is 80 m. Fig. 9 illustrates the collision risk map for rear-end avoidance with different braking accelerations. When the ego vehicle brakes with a low deceleration, the collision risk is high, as illustrated in Fig. 9. When the ego vehicle brakes with a high deceleration, the collision risk is lower at the end of the prediction horizon.

### V. MOTION PLANNING OF ACV

#### A. BEHAVIOR PLANNING OF ACV

Given the global reference path consisting of a serial of road sections, behavior planning means generating
reasonable behaviors for guiding ACV along the global reference path based on the perception of other traffic participants, road conditions and traffic signs. Since the driving environment and optional behaviors can be represented as several sets, each behavior is indicated as a particular state in a finite state machine. In this paper, we model the behavior selection process in highway scenarios with the finite state machine, as illustrated in Fig. 10. The motion of SVs is predicted with methods introduced in section III. The motion prediction of ACV is constrained by an optimized control vector, which is presented in part B of section V in detail.

The normal behaviors in highway scenarios include lane keeping and lane change. Lane-keeping can be further divided into cruising, following and leading based on the relative position between ACV and SVs. When there is a SV approaching ACV from behind, the given behavior for ACV is cruising, as illustrated in Fig. 11.c. If there is no SV around ACV in its current lane, the behavior for ACV is leading, as indicated in Fig. 11.b. The reference lane and reference speed for ACV in the following behavior are illustrated in Eq. (17a).

When there is a SV in the front of ACV, ACV is assigned the following behavior, as shown in Fig. 11.a. The reference lane and reference speed for ACV in the following behavior are shown in Eq. (17b). If there is no SV in its current lane, the behavior for ACV is cruising, as illustrated in Fig. 11.c. The reference lane and reference speed for ACV are shown in Eq. (17c).

The reference lane and reference speed for ACV are shown in Eq. (17c).

\[
\begin{align*}
&v_{e,ref}^{G}(t) = v_{Lane_o}^{F}(t), \quad x_e^{F}(t) < x_o^{F}(t), \quad t \in [0, t_p] \\
&v_{e,ref}^{G}(t) = v_o^{G}(t), \quad x_e^{F}(t) > x_o^{F}(t), \quad t \in [0, t_p] \\
&v_{e,ref}^{G}(t) = v_{des}^{G}, \quad t \in [0, t_p]
\end{align*}
\]  

where \(v_{e,ref}^{G}(t)\) is reference speed, \(v_{e,ref}^{F}(t)\) is the middle position for reference lane, \(v_{Lane_o}^{F}(t)\) is the middle position for lane occupied by SV, \(v_{o}^{G}(t)\) is the speed of SV, \(v_{des}^{G}\) is the desired speed for ACV in highway scenario, \(t_p\) is the length of the prediction window.

If the reference speed in the adjacent lane approaches the desired speed more close than that of the current lane, a lane change behavior is assigned for ACV, as illustrated in Fig. 12. In this case, the setting of the reference lane and reference speed is shown in Eq. (18).

\[
\begin{align*}
&y_{e,ref}^{G}(t) = y_{Lane_o}^{F}(t) \\
&v_{e,ref}^{G}(t) = v_o^{G}(t), \\
&v_{des}^{G} < v_e^{G}(t) - v_{des}^{G}, \quad t \in [0, t_p]
\end{align*}
\]

\[B. TRAJECTORY PLANNING OF ACV BASED ON MPC\]

Due to its superiority in dealing with the constrained optimization problems, MPC has been widely used for trajectory planning and trajectory tracking of ACV [10], [39]–[44]. In this work, taking into account the motion prediction of SVs and ACV, a predictive trajectory planning framework is proposed for ACV. It incorporates internal and external constraints resulted from kinematic and kinetic constraints of ACV, traffic rules and safe distance between ACV and
SVs. Based on the MPC framework, the trajectory of ACV is planned in a predictive way in order to adapt to the dynamic traffic environments.

1) MOTION PREDICTION OF ACV

The ACV is regarded as a particle moving in a curvilinear way. The position of the particle coincides with the mass point of ACV. Let \( \mathbf{x}_e \) be the state vector representing motion of ACV, as illustrated in Eq. (19).

\[
\mathbf{x}_e = \begin{bmatrix} x^F_e & y^F_e & v^G_e & a^G_e & \psi^F_e & \psi^G_e \end{bmatrix}^T
\]  

where \( x^F_e \) is the longitudinal position of ACV in the road frame, \( y^F_e \) is the lateral position of ACV in the road frame, \( v^G_e \) is the velocity of ACV, \( a^G_e \) is the acceleration of ACV, \( \psi^F_e \) is the heading angle of ACV in the road frame, \( \psi^G_e \) is the yaw rate of ACV in the global coordinate system.

With the control input of the desired acceleration and desired yaw rate, the state transition process of ACV is shown in Eq. (20).

\[
\begin{align}
\dot{x}^F_e &= v^G_e \cos \psi^F_e - y^F_e \dot{\psi}^F_R \\
\dot{y}^F_e &= v^G_e \sin \psi^F_e \\
\dot{v}^G_e &= a^G_e \\
a^G_e &= \frac{1}{T_v} (a^G_{des} - a^G_e) \\
\dot{\psi}^F_e &= \dot{\psi}^G_e - \dot{\psi}^F_R \\
\dot{\psi}^G_e &= \frac{1}{T_\psi} (\dot{\psi}^G_{des} - \dot{\psi}^G_e)
\end{align}
\]

where \( a^G_{des} \) and \( \dot{\psi}^G_{des} \) are control inputs from the solution of MPC problem, \( a^G_e \) is desired acceleration, \( \dot{\psi}^G_e \) is desired yaw rate, \( \psi^F_R \) is the angle between \( x \) axis of road frame and \( x \) axis of the global coordinate system, \( \dot{\psi}^F_R \) is the turn rate of road frame in the global coordinate system, \( T_v \) and \( T_\psi \) are time constants when the lower dynamic control system of ACV is simplified as a first-order system [33].

The rotation of the road frame is illustrated in Eq. (21).

\[
\dot{\psi}^F_R = \dot{\psi}^G_R \kappa \left( x^F_e \right)
\]  

where \( \kappa(x^F_e) \) is the curvature of the right road boundary in \( x^F_e \), the road geometry shape is known in advance.

Substituting in Eq. (20a) and (20e) with Eq. (21), the two equations are reorganized, as illustrated in Eq. (22).

\[
\begin{align}
\dot{x}^F_e &= v^G_e \cos \psi^F_e - y^F_e \kappa \left( x^F_e \right) \\
\dot{\psi}^F_e &= \dot{\psi}^G_e - v^G_e \cos \psi^F_e \kappa \left( x^F_e \right) / (1 + y^F_e \kappa \left( x^F_e \right))
\end{align}
\]

2) CONSTRAINTS

For generating a safe, comfortable and feasible trajectory for ACV, it’s essential to take into account internal and external constraints, which involves kinematic and kinetic constraints of ACV, traffic rules and safe distance constraint between SVs and ACV.

Internal constraints include kinematic and kinetic constraints of ACV. Kinematic constraint means that the curvature of the ACV should be bounded, as illustrated in Eq. (23). Kinetic constraint means that ACV must satisfy the tire-road friction ellipse condition, as shown in Eq. (24).

\[
-\kappa_{\text{max}} \leq \frac{\dot{\psi}^G_e}{v^G_e} \leq \kappa_{\text{max}}
\]  

where \( \kappa_{\text{max}} \) is the maximum curvature.

\[
\left( \dot{\psi}^F_{e,x} \right)^2 + \left( \dot{\psi}^F_{e,y} \right)^2 \leq \left( \mu_H g - \zeta_g \right)
\]

where \( \mu_H \) and \( \zeta_g \) are the maximum friction coefficient and the gravitational acceleration, respectively.
where $\mu_H$ is the peak adhesion coefficient, $a^E_{x,i}$ is the tangential acceleration of ACV, $a^E_{y,i}$ is the normal acceleration of ACV, $f_{e,y}$ is a scale factor for normal acceleration, $\xi_k$ is a relaxation factor used to make ACV stay in the stable area.

External constraints refer to the influence of traffic rules and SVs. The speed of ACV in highway scenarios should be bounded, as illustrated in Eq. (25). Meanwhile, the ACV should stay within the drivable area, as shown in Eq. (26). ACV should keep a safe distance with SVs. The vehicle shape and position prediction uncertainty should also be considered, as illustrated in Eq. (27).

$$v_{\text{min}} \leq v^G_e \leq v_{\text{max}}$$  
Eq. (25)

where $v_{\text{max}}$ is the maximum speed, $v_{\text{min}}$ is the minimum speed.

$$y^F_{\text{min}} \leq y^F_e \leq y^F_{\text{max}}$$  
Eq. (26)

where $y^F_{\text{min}}$ represents the right road boundary, $y^F_{\text{max}}$ represents the left road boundary.

$$\left(\frac{x^F_e - x^F_{e,0}}{\Delta x^F_{e,0}}\right)^2 + \left(\frac{y^F_e - y^F_{e,0}}{\Delta y^F_{e,0}}\right)^2 \geq 1$$  
Eq. (27a)

$$\Delta x^F_{e,0} = \frac{1}{2}L_e + \frac{1}{2}L_{o,i} + 3\sigma^F_{o,x}$$  
Eq. (27b)

$$\Delta y^F_{e,0} = \frac{1}{2}W_e + \frac{1}{2}W_{o,i} + 3\sigma^F_{o,y}$$  
Eq. (27c)

where $\Delta x^F_{e,0}$ is the minimum safe distance in $x$ axis of road frame between ACV and SV $i$, $\Delta y^F_{e,0}$ is the minimum safe distance in $y$ axis of road frame between ACV and SV $i$. $L_e$ is the length of ACV, $W_e$ is the width of ACV, $L_{o,i}$ is the length of SV $i$, $W_{o,i}$ is the width of SV $i$, $\sigma^F_{o,x}$ and $\sigma^F_{o,y}$ are standard deviations in $x$ and $y$ axes of road frame for SV $i$ respectively, which represents the position prediction uncertainty.

3) MPC PROBLEM FORMULATION

On the basis of internal and external constraints, the trajectory planning of ACV is formulated as solving a constrained optimization problem, as illustrated in Eq. (28). The first item in the objective function means minimizing the reference path tracking error. The second item in objective function refers to minimizing energy consumption of control inputs. The third item in the objective function represents minimizing collision risk between ACV and SVs. The motion of ACV is subject to equality constraints and inequality constraints. The equality constraints are the state transfer process of ACV, as illustrated in Eq. (20). The inequality constraints indicate the influence of SVs and traffic rules on ACV, as shown in Eq. (23)-(27).

$$\begin{align*}
\min_{u_1, u_2, \ldots, u_N} & \sum_{k=1}^{N_p} \|y_k - r_k\|_P + \sum_{k=1}^{N_p} \|u_k\|_Q \\
& + \sum_{k=1}^{N_p} \sum_{i=1}^{N_q} \|\text{Risk}(x^F_{e,i,k}, x^F_{o,i,k}, u^F_i)\|_R^2
\end{align*}$$  
Eq. (28)

where $N_p$ is the number of control steps, $N_o$ is the number of surrounding vehicles, $\nu^F_{e,k}$ represents the reference lane, $v^F_{e,k}$ is the reference speed, $\nu^F_{e,k}$ and $v^F_{e,k}$ determine the reference path for ACV, which are given from the behavior generation module. $P$, $Q$, $R$ are weighted matrixes for tracking error, control energy consumption and collision risk respectively.

VI. RESULTS AND DISCUSSION

A. PARAMETERS DEFINITION

The proposed predictive trajectory planning method is applied to two scenarios to validate its effectiveness and feasibility, as illustrated in Fig. 13 and Fig. 22. The parameters for motion prediction of SVs with uncertainty are defined in Table 3. The parameters for MPC-based trajectory planning are shown in Table 4.

![FIGURE 13. The initial condition for scenario 1.](image)

### TABLE 3. Parameters for motion prediction of SVs with uncertainty.

| Symbol          | Parameters                                    | Values             | Unit       |
|-----------------|-----------------------------------------------|--------------------|------------|
| $\sigma_{a,x}$  | Process noise of acceleration in CTRA motion model | $N(0,0.05^2)$      | m/$s^2$   |
| $\sigma_{a,y}$  | Process noise of yaw rate in CTRA motion model  | $N(0,0.001^2)$     | rad/$s^1$ |
| $\sigma_{a,o}$  | Standard deviation of longitudinal acceleration in maneuver-based model | 0.05               | m/$s^2$   |
| $\sigma_{e,o}$  | Standard deviation of lateral position in maneuver-based model | 0.05               | m         |

B. SCENARIO 1 SIMULATION RESULTS ANALYSIS

As shown in Fig. 13, scenario 1 is a straight road three-lane scene. The initial state parameters of ACV and SVs are defined in Table 5. At the initial moment, SV1 is moving in
Table 4. Parameters for MPC-based trajectory planning.

| Symbol    | Parameters                          | Values | Unit |
|-----------|-------------------------------------|--------|------|
| $T_a$     | Time constant of acceleration       | 13.3   | s    |
| $T_v$     | Time constant of yaw rate           | 0.125  | s    |
| $\kappa_{\text{max}}$ | Maximal curvature                       | 0.125 | m⁻¹   |
| $\mu_{f}$ | Road adhesion coefficient           | 0.8    | /    |
| $g$       | Acceleration of gravity             | 9.8    | m·s⁻² |
| $\xi_f$   | Relaxation factor for tire-road friction ellipse | 2 | m·s⁻¹ |
| $\nu_{\text{max}}$ | Maximal speed                   | 35     | m·s⁻¹ |
| $\nu_{\text{min}}$ | Minimal Speed                   | 15     | m·s⁻¹ |
| $y_{\text{left}}$ | Left road boundary                       | 10.5   | m    |
| $y_{\text{right}}$ | Right road boundary                       | 0      | m    |
| $L_a$     | Length of ACV                        | 5.21   | m    |
| $W_a$     | Width of ACV                         | 2.04   | m    |
| $L_v$     | Length of SVs                        | 4.04   | m    |
| $W_v$     | Width of ACVs                        | 1.96   | m    |
| $y_{\text{lane1}}$ | Lateral position of lane 1                  | 1.75   | m    |
| $y_{\text{lane2}}$ | Lateral position of lane 2                  | 5.25   | m    |
| $y_{\text{lane3}}$ | Lateral position of lane 3                  | 8.75   | m    |
| $\nu_{\text{d}}$ | Desired speed                   | 30     | m·s⁻¹ |
| $P_a$     | Weight factor of speed              | 30     | /    |
| $P_v$     | Weight factor of lateral position    | 10     | /    |
| $Q_a$     | Weight factory of acceleration       | 5      | /    |
| $Q_v$     | Weight factor of yaw rate           | 5      | /    |
| $R$       | Weight factor of collision risk      | 10     | /    |
| $N_c$     | Number of control steps             | 50     | /    |
| $N_v$     | Number of surrounding vehicles       | 3      | /    |

Table 5. Initial state parameters for scenario 1.

| Vehicles | Symbol    | Parameters                          | Values | Unit |
|----------|-----------|-------------------------------------|--------|------|
| ACV      | $x_0^e$   | Initial longitudinal position of ACV | 50     | m    |
|          | $y_0^e$   | Initial lateral position of ACV     | 5.25   | m    |
|          | $\nu_0^e$ | Initial speed of ACV                | 30     | m·s⁻¹ |
|          | $x_0^v$   | Initial longitudinal position of SV1 | 100    | m    |
|          | $y_0^v$   | Initial lateral position of SV1     | 1.75   | m    |
|          | $\nu_0^v$ | Initial speed of SV1                | 28     | m·s⁻¹ |
|          | $x_0^s$   | Initial longitudinal position of SV2 | 150    | m    |
|          | $y_0^s$   | Initial lateral position of SV2     | 5.25   | m    |
|          | $\nu_0^s$ | Initial speed of SV2                | 30     | m·s⁻¹ |
|          | $a_0^v$   | Initial acceleration of SV2         | -2     | m·s⁻² |
|          | $x_0^v$   | Initial longitudinal position of SV3 | 0      | m    |
|          | $y_0^v$   | Initial lateral position of SV3     | 8.75   | m    |
|          | $\nu_0^v$ | Initial speed of SV3                | 35     | m·s⁻¹ |

From the trajectory planning of ACV in the whole process, we analyze the collision risk between ACV and SVs at three moments $t = 0$, $10$, $15$ under different control inputs of acceleration and yaw rate for ACV. The trajectories of SVs are predicted by IMM. The motion of ACV is determined by state transfer equation and control inputs, as illustrated in Eq. (20). The collision risk between ACV and SVs is calculated with Eq. (16). The length of the prediction time window is 5s.

At $t = 0s$, the ACV’s current position and the predicted trajectories of SVs are shown in Fig. 14.a. All SVs stay in their original lanes and make lane-keeping maneuvers. SV1 and SV3 are moving at a constant speed. SV2 slows down. Fig. 15 indicates the collision risk in the prediction horizon between ACV and SVs with a different control input of acceleration for ACV. When the acceleration control input is positive, the longitudinal distance between ACV and
SV2 decreases with time. As a consequence, the collision risk between ACV and SV2 increases with time under positive acceleration control input, as shown in Fig. 15. When the yaw rate control input is negative, ACV turns right towards lane 1. The lateral distance between ACV and SV1 decreases with time at first and then increase. That results in a peak value for collision risk between ACV and SV1 at a certain moment, as illustrated in Fig. 16.a. When the yaw rate control input is positive, ACV makes left turns. The distance lateral between ACV and SV3 firstly decreases and then increases. The collision risk between ACV and SV3 achieves high value with positive yaw rate input, as depicted in Fig. 16.c. No matter the yaw rate input is positive or negative, the lateral distance between ACV and SV2 increases with time under non-zeros yaw rate input. As a result, the collision risk between ACV and SV2 appears minimum values at the end of the prediction time horizon, as illustrated in Fig. 16.b.

At $t = 10s$, Fig. 14.b illustrates ACV’s current position and SVs’ predicted trajectories. ACV has made a right lane change from lane 2 to lane 1, moving behind SV1. SV1 and SV3 keep in their original lanes. SV2 brakes in lane 2. The longitudinal distance between ACV and SV1 decreases with positive acceleration control input for ACV. The collision risk between ACV and SV1 increases with time under positive acceleration inputs, as shown in Fig. 17. When there is a positive yaw rate input for ACV, the distance between...
ACV and SV2 or SV3 firstly decreases and then increases. The collision between ACV and SV2 or SV3 reaches maximum values with positive yaw rate inputs, as illustrated in Fig. 18.b and Fig. 18.c. When the yaw rate input is non-zero, ACV makes left or right turns. The lateral distance between ACV and SV1 increases. Therefore, the collision between ACV and SV1 decreases with time under non-zero yaw rate inputs, as depicted in Fig. 18.a.

At \( t = 15 \) s, ACV and SV1 are moving in lane 1, as shown in Fig. 14.c. The longitudinal distance between ACV and SV1 is smaller than that at \( t = 10 \) s. SV2 has become static. SV3 makes a lane-keeping maneuver in lane 3. When the acceleration control input is negative, ACV brakes in lane 1. The longitudinal distance between ACV and SV1 or SV3 increases with time. The collision risk between ACV and SV1 or SV3 reaches a maximum value at the beginning of the prediction horizon, as illustrated in Fig. 19.a and Fig. 19.c. When the acceleration for ACV is large enough, ACV will take overtake SV1 or SV3 in a short time, the longitudinal distance between ACV and SV1 or SV3 firstly decreases and then increases, which leads to a lower value at the end of the prediction window. No matter the acceleration control input is negative or positive, the longitudinal distance between ACV and SV2 increases with time. As a result, the collision risk between ACV and SV2 decreases with time, as shown in Fig. 19.b. The ACV turns left with a positive yaw rate.
input. The lateral between SV2 or SV3 achieves a minimum value at a certain moment, which makes the collision risk between ACV and SV2 or SV3 reaches the maximum, as depicted in Fig. 20.b and Fig. 20.c. The collision risk between ACV and SV1 decreases with time under non-zero yaw rate inputs, as illustrated in Fig. 20.a.

The planned trajectory for ACV in scenario 1 is depicted in Fig. 21. ACV moves from lane 2 to lane 1 at the beginning. Then, ACV follows SV1 in lane 1. After ACV has overtaken SV2 in the longitudinal direction, ACV moves back to lane 2. Finally, ACV is moving at a speed of 30 m·s⁻¹. Throughout the whole process, the lateral and longitudinal acceleration keeps inside the tire-road friction ellipse. The maximum lateral acceleration is 2.8 m·s⁻².

C. SCENARIO 2 SIMULATION RESULTS ANALYSIS

Scenario 2 is a three-lane curve road scene, as shown in Fig. 22. The initial condition for ACV and SVs is indicated in Table 6. SV1 and SV2 are moving in lane 1. SV1 is 100 m in front of ACV. At t = 4 s, SV1 makes a left lane change from lane 1 to lane 2. The lane change duration is about 4 s. SV2 is 100 m behind ACV, moving at a speed of 30 m·s⁻¹. SV3 is moving in lane 3 at a speed of 35 m·s⁻¹. The collision risk between ACV and SVs at t = 0 s, 5 s, 10 s under different acceleration or yaw rate control inputs is described as follows.

| Vehicles | Symbol | Parameters | Values | Unit |
|----------|--------|------------|--------|------|
| ACV      | $x^e$  | Initial longitudinal position of ACV | 200    | m    |
|          | $y^e$  | Initial lateral position of ACV     | 5.25   | m    |
|          | $v^e$  | Initial speed of ACV                 | 30     | m·s⁻¹|
| SV 1     | $x^e$  | Initial longitudinal position of SV1 | 300    | m    |
|          | $y^e$  | Initial lateral position of SV1      | 1.75   | m    |
|          | $v^e$  | Initial speed of SV1                 | 25     | m·s⁻¹|
| SV 2     | $x^e$  | Initial longitudinal position of SV2 | 100    | m    |
|          | $y^e$  | Initial lateral position of SV2      | 1.75   | m    |
|          | $v^e$  | Initial speed of SV2                 | 30     | m·s⁻¹|
| SV 3     | $x^e$  | Initial longitudinal position of SV3 | 0      | m    |
|          | $y^e$  | Initial lateral position of SV3      | 8.75   | m    |
|          | $v^e$  | Initial speed of SV3                 | 35     | m·s⁻¹|

The ACV’s current position and SVs’ predicted trajectories in the prediction horizon at t = 0 s are depicted in Fig. 23.a. SV1 and SV2 are moving in lane 1. SV1 is in front of SV2.
SV3 moves in lane 3 at a constant speed. When the acceleration control input for ACV is negative, the longitudinal distance between ACV and SV2 or SV3 decreases with time. The collision risk between ACV and SV2 or SV3 increases with time under negative acceleration inputs, as illustrated in Fig. 24. b and Fig. 24.c. Since positive acceleration leads to the decreasing longitudinal distance between ACV and SV1, the collision risk between ACV and SV1 increases with time under positive acceleration inputs, as shown in Fig. 24.a. The ACV turns right with negative yaw rate inputs. The distance between ACV and SV1 or SV2 will achieve a minimum at a certain moment, which leads to the collision risk between ACV and SV1 or SV2 reaches the maximum at that point, as depicted in Fig. 25.a and Fig. 25.b. Similarly, the collision risk between ACV and SV3 becomes high at a particular moment in the prediction horizon under positive yaw rate inputs.

At $t = 5s$, the ACV’s current position and SVs’ predicted trajectories in the prediction horizon are depicted in Fig. 23.b SV1 is moving from lane 1 to lane 2. SV2 and SV3 still keep in original lanes. Fig. 26 and Fig. 27 depict the collision risk with all SVs under different acceleration or yaw rate control inputs. In this case, the collision risk with SV3 is obviously smaller than that of SV1 and SV2. The collision risk with all SVs appears peak values at the beginning of the prediction horizon.

At $t = 10s$, ACV has made a right lane change from lane 2 to lane 1, moving in front of SV2, as depicted in Fig. 23.c. SV1 keeps in lane 2. SV2 is moving in lane 1. SV3 keeps in lane 3. The longitudinal distance between ACV and SV1 decreases with time under positive acceleration inputs, which results in the collision risk between ACV and SV1 increases with time, as depicted in Fig. 28.a. When the acceleration input is negative, the distance between ACV and
SV2 firstly decreases and then increases. The collision risk between ACV and SV2 achieves the minimum at the end of prediction under negative acceleration inputs, as illustrated in Fig. 28. b. Since the lateral distance between ACV and SV3 is larger than others, the collision risk between ACV and SV3 is relatively smaller, as depicted in Fig. 28.c. ACV makes...
left turns with positive yaw rate control input. The lateral distance between ACV and SV1 or SV3 firstly decreases and then increases, which leads to the collision risk between ACV and SV1 or SV3 reaches the maximum at a certain moment in the prediction horizon, as illustrated in Fig. 29.a and Fig. 29.c. When the yaw rate control input is non-zero, the lateral distance between ACV and SV2 increases with time. As a consequence, the collision risk between ACV and SV2 achieves the minimum at the end of the prediction window, as shown in Fig. 29.c.

The trajectory planning results for ACV in scenario 2 is depicted in Fig. 30. In the beginning, ACV keeps in lane 2. At $t = 4.3s$, ACV starts moving from lane 2 to lane 1 in order to avoid colliding with SV1 in the next few moments. After ACV ends its lane-change maneuver, ACV keeps in lane 1 at a speed of $30\, m\cdot s^{-1}$. In the whole process, the maximum lateral acceleration of ACV is $2.6\, m\cdot s^{-2}$, which satisfies the tire-road friction ellipse constraint.

**VII. CONCLUSION**

In this work, a predictive trajectory planning method for ACV is proposed based on the motion prediction of SVs. Firstly, the IMM is used for predicting the motion of SVs by integrating the CTRA motion model and the simplified maneuver-based motion model. Specifically, the maneuver is recognized through the temporal-spatial relevance between
the historical trajectory and road geometry shape, which is independent of training data. Then, a risk indicator is constructed for collision risk assessment, which involves the possibility of a collision event and the severity of a collision event. After that, taking account the internal and external constraints resulted from kinematic and kinetic characteristics of ACV, traffic rules and safe distance restrict between ACV and SVs, the trajectory planning of ACV is formulated as a constrained optimization problem based on the MPC framework. Finally, the proposed predictive trajectory planning method is applied to two scenarios to validate its effectiveness and feasibility. In future work, predictive trajectory planning methods should involve an interaction-aware motion prediction model for SVs.

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