The Man who Invented Descriptive Geometry

Gaspard Monge is known as the father of modern descriptive and differential geometry. In 1764, he was engaged to draw a detailed plan of a fortification in his hometown, which was seen by an officer at the École Royale du Génie de Mézières. This plan was a success and his techniques were marked as a military secret for a long period of time. In 1780, he was elected to the Academy of Science and participated in the work of the Commission for Weights and Measures, that was in charge of moving the system from imperial to metric. In 1794, Monge helped setting up the École Centrale des Travaux Publics (later École Polytechnique) where he was lecturing Descriptive Geometry. In 1798, Napoleon undertook a campaign in Egypt. The famous chemist Claude Louis Berthollet was asked to recruit prominent scientists. Among them were Fourier, Monge, Dolomieu and Malus. Institut d’Égypte was established by Napoleon and Monge was named as its first president. Monge passed away on July 28, 1818. His name is inscribed on the base of the Eiffel Tower and it is located on the third façade opposite the Military Academy. Besides descriptive geometry, he carried on many different researches in chemistry and physics.

Keywords: Gaspard Monge, Descriptive geometry, Napoléon Bonaparte, Claude Louis Berthollet, Egypt, Beaune, École Polytechnique

1. LIFE OF GASPARD MONGE

Gaspard Monge (Fig. 1) was born on May 9, 1746, in Beaune in France. He was the eldest child of Jacques Monge (1718-1775), a peddler and knife grinder, and Jeanne Rousseau (1711-1773). Even though the Monge family lived in hard economic conditions, Gaspard completed his primary and secondary education at the Oratorian College in Beaune [1-3].

At the age of sixteen (1762), Gaspard made a map of his hometown on his own initiative. Fascinated by his work, teachers sent him to Lyon to the Oratorian College de La Trinité, which was run by their priests. Under the pressure to become one of them, Monge returned to Beaune in 1764, where he was jobless until the moment he got the offer to work at the École Royale du Génie de Mézières (his drafts were seen by an officer who recognized his potentials). Since he was coming from a poor family, he could only be a draftsman, not a student. There were twenty students per year and only ten of them graduated annually as lieutenants in engineering. In his leisure time he studied math, which eventually helped him to do demanding calculations in the theory of fortification [1-4]. Monge drew up a map of a fortification by developing a new geometrical method for solving the problem of object’s graphical representation, so that no part of the fortress should be exposed to the direct fire of an enemy. This new method was based on representing a 3D object by orthogonal projection of its points on two mutually orthogonal planes [3]. This was the very beginning of descriptive geometry and his method was declared secret for more than fifteen years. For the first time, in 1794, he was allowed to teach it at the École Normale in Paris and among the pupils was J. L. Lagrange. Lagrange’s reaction to descriptive geometry was the following one ”Before hearing Monge I did not know that I knew descriptive geometry” [1-5].

Figure 1. Portrait of Gaspard Monge [29]

The École Royale de Génie du Mézières was founded in 1748 and it was considered to be the elite school for future military engineers. The department for
Mathematics was led by famous scientist Charles Bossut. In 1768, Bossut left the École, since he was elected to the Académie des Sciences and became professor of hydrodynamics at the Louvre. Monge was appointed to succeed Bossut as a lecturer in January 1769. He devoted his leisure time to solving mathematical spatial problems in the theory of partial differential equations, curvature of surfaces, calculus of variations, the equations of vibrating strings, etc. [1-4].

In January 1769, Monge wrote to Bossut about his work on the evolutes of curves of double curvature and he asked him to give an opinion about it. Bossut’s reaction was positive and Monge’s paper was published in June in the Journal Encyclopédique [2]. This paper present the results obtained on space curves in general, but many other important Monge’s discoveries as well [6]. The completed work was submitted to the Académie des Sciences in 1770 and read in front of its members in 1771 [4, 5].

In 1777, Gaspard married Marie-Cathérine Huart (1747-1846), a young widow, and the couple had three daughters [1, 2]. Since his wife brought in a smathy as a dowry, he became interested in metallurgy as well [2, 3]. Later, in 1786, he published, together with A. T. Vandermonde and C. L. Berthollet, a paper on iron characteristics in different metallic states [2, 3, 21].

In 1780, Monge was elected as adjoint géomètre to the Académie des Sciences replacing Vandermonde, who was elected as a associé. During the next four years, Monge was working at the École Royale de Génie du Mézières and at the Académie des Sciences as well. He taught hydrodynamics as Bossut’s substitute and participated in projects led by the Académie. Finally, in 1784, he left his job as a lecturer at the École Royale de Génie du Mézières and he was replaced by his substitute Claude-Joseph Ferry (1756-1845) [1-4].

The year 1789 was crucial in French history. The storming of Bastille on July 14, 1789 marked the beginning of the French Revolution, which came to an end in 1799, when Napoleon established the French Consulate [7]. A that time, Gaspard Monge was one of the leading scientists in Paris. He was a big supporter of the French Revolution, and as many of his colleagues he joined the Société de Luxembourg and eventually the Club des Jacobins [2, 4, 7].

The French King Louis XVI tried to leave the country on June 20, 1791, but he was captured at Varennes and brought back to Paris, to the Tuileries Palace. In 1792, France declared war on Prussia and Austria, since the newly elected Legislative Assembly believed that French émigrés were planning a counterrevolution. Meanwhile, a group of revolutionists led by the extremist Jacobins entered the royal residence by force and arrested the King. The Legislative Assembly was replaced by the National Convention, the monarchy was abolished and the French Republic was proclaimed. In January, 1793, the king was executed and nine months later, his wife as well. In June, M. Robespierre overtook the control of the National Convention and ordered many executions of suspected enemies of the revolution [7].

In 1792, Monge was offered the position of Minister of the Marine and as a Minister, he signed the act that condemned the King to death. Even though he tried to fulfill his obligations as a Minister, he failed. In April, 1793, Monge resigned and dedicated his time to work at the Académie des Sciences. Unfortunately, the Académie des Sciences was abolished in August, 1793, by the National Convention [1, 2, 4, 7].

Since the Académie des Sciences and most of the French educational institutions were closed, the education of young people was impossible (schools and convents were transformed into prisons). In attempts to reconstruct what had been destroyed, the Convention founded the École Centrale des Travaux Publques (later the École Polytechnique), the École Normale and the Institut National in 1795. Monge was appointed director of the École Polytechnique and member of the Section des Arts Mécaniques de la première class of the Institut in 1797 [1-4].

The year 1796 was significant for Monge. He received a letter from Napoleon, whom he had already met in 1792, while Napoleon was just an officer and Monge Minister of the Marine [1, 2, 4]. At that time (1796), Napoleon was a commander in chief of the army in Italy [1, 2, 4, 10]. Monge took part in the Italian campaign. He and general Louis-Alexandre Berthier, Napoleon’s Chief of Staff, were in charge of transferring the Treaty of Campo Formio to Paris in 1797 [2, 3, 8-10]. Monge, as appointed member of the Commission des Sciences et des Arts en Italie, was sent to Italy to select all valuable works of art, which were supposed to be given to France as part of the Italian contribution to the expenses of Napoleon’s campaign. Berthollet accompanied Monge on the trip to Italy and met Napoleon through Gaspard [1, 2, 3, 11]. Since Berthollet was born in Savoy, he spoke Italian as fluently as French. In one of his reports from Italy, he discussed means of restoring several paintings of Raphael. Monge wrote to his wife that 300 crates were needed to transport objects from Italy to France. One of these pieces of art was Mona Lisa [2, 4, 8-12].

Next year, 1798, Monge returned to Rome participating in setting up the republic of Rome. The same year, Napoleon asked Berthollet to organize a "Committee on the Arts and Sciences" to accompany the army on his Egyptian expedition (Fig. 2). Members of the expedition, which was led by Berthollet, were Gaspard Monge, Joseph Fourier, the zoologist Étienne St. Hilaire, Nicolas-Jacques Conté (inventor of the graphite pencil and the Conté crayon), the mineralogist Déodat de Dolomieu (whose name is given to the Dolomites in the Alps and to the mineral dolomite) and the physicist Étienne Malus (discoverer of the polarization of light by reflection) [1-4, 10, 11, 13, 23].

Malta and Alexandria were occupied very fast and the delta of Nile was quickly conquered. One month later, the French fleet was destroyed by Nelson’s fleet in the Battle of the Nile. Napoleon established the Institut d’Égypte and Monge was appointed first president. Napoleon named himself vice-president and the Institut had 12 members of the mathematics division (including Fourier and Malus). Berthollet succeeded Monge as the second president. In 1799, Napoleon decided to abandon his army and to return to France secretly, choosing only Monge and Berthollet to accompany him on the trip back [2, 3, 4, 10, 11, 13].
After the Egyptian campaign, Napoleon created the Legion d’Honneur in 1802. Lagrange, P. S. Laplace, Berthollet and Monge were proclaimed Grand Officers of the Legion of Honour, Counts and Senators for life. In 1803, Napoleon established a system, in which France was divided into 15 districts for legal appeals. Monge and Berthollet were two of only three scientists selected for these positions [1-4, 10, 11]. When Fourier returned to France from Egypt in 1801, he was appointed by Napoleon as prefect of the Department of Isere, in Grenoble. His administration was outstanding during a 12-year tenure, and Napoleon made him a Baron in 1808. Although Fourier had difficulty in surviving Napoleon’s downfall, he was finally appointed Director of the Statistical Bureau of the Seine and elected to the Académie des Sciences [1, 5, 10, 11, 15]. Laplace was named Minister of the Interior by Napoleon, but he was a poor administrator and was replaced by Napoleon after only six weeks in office. Laplace successfully shifted loyalty under numerous changes in regime. After the Bourbon restoration, Louis XVIII raised him to the peerage as the Marquis de Laplace [1, 5, 10, 11, 15, 16].

After his return to Paris, Monge resumed his duties as a director of the École Polytechnique. As a senator, he had less time to dedicate to scientific research, especially when he was elected president of the Senate in 1806. In 1809, he gave up his teaching career at the École Polytechnique, since his health began to deteriorate [2, 3, 4].

In 1812, Napoleon assembled his Grande Armée of about half a million soldiers, including men from Austria and Prussia who were forced to serve and march to Russia. Napoleon’s Russian campaign was a complete disaster, but in autumn Napoleon entered the deserted city of Moscow. While his army was withdrawing, soldiers from Austria and Prussia abandoned the Grande Armée. Eventually, Napoleon returned to Paris and in 1813 he had some military success [10, 14].

The allied armies (Austria, Prussia, the United Kingdom, Sweden, Spain, Portugal and many smaller German states) began to move against France, and Monge left. After the War of the Sixth Coalition (known as War of Liberation as well), in 1814, Napoleon abdicated and according to the Treaty of Fontainebleau, he was exiled to the island of Elba. Monge returned to Paris after Napoleon’s abdication trying to pick up his former life again. Napoleon escaped from Elba and by March 20, 1815 he returned to Paris and governed for a period of time which is known as the Hundred Days. Monge gave him his full support. After the terrible defeat at Waterloo, Monge continued to meet Napoleon regularly until he was exiled to the island of Saint Helena in July. Here the great emperor died in 1821 [2, 4, 10, 17].

Since Monge escaped from France after Napoleon’s exile, in 1816 he returned to Paris. A few days later he was expelled from the Institut de France and harassed politically. He passed away on July 28, 1818 and was buried in the Père Lachaise cemetery [1, 2].

The students of the École Polytechnique appreciated their professor profoundly and he was their idol (Monge had protected them from Napoleon’s domineering interference). The King prohibited them to attend Monge’s funeral. Berthollet held the speech at his funeral, and the following day students went to the cemetery to pay tribute to their master (Fig. 3) [1-4].

In 1898, when Gustav Eiffel built his famous tower, he decided to honor 72 distinguished French scientists by inscribing their names on the base of tower. There are eighteen names per side of the tower and Monge’s name is located on the third façade, opposite to the Military Academy [2, 3, 4]. Monge’s work was continued by his students. Among them are: Michel-Ange Lancret, Charles Dupin, Augustin-Louis Cauchy, Jean Nicolas Pierre Hachette, Jean-Victor Poncelet, Gaspard-Gustave de Coriolis, Barré de Saint-Venant, André-Marie Ampère [2, 6, 18, 19].

Michel-Ange Lancret (1774-1807) was among the first graduate students of Monge at the École Polytechnique. As a senator, he had less time to dedicate to scientific research, especially when he was elected president of the Senate in 1806. In 1809, he gave up his teaching career at the École Polytechnique, since his health began to deteriorate [2, 3, 4].

Charles Dupin (1784-1873) made his first discovery in 1801, under the guidance of Monge- cyclids of Dupin. In 1834, he became Minister of Marine. During the years 1836-1844 he was Vice-President of the Académie des Sciences and in 1852 he was appointed to
the Senate. Dupin’s theorem in differential geometry states that three families of orthogonal surfaces intersect in lines of curvature [19, 24].

Jean Nicolas Pierre Hachette (1769-1834) was Monge’s student and assistant. He edited Monge’s Géométrie Descriptive, which was published in 1799 [24].

2. CONTRIBUTION OF GASPARD MONGE

As previously mentioned, Gaspard Monge is considered to be the father of modern descriptive geometry. Orthographic projections (Fig. 4) are integrated in almost all 3D-CAD programs, and a detailed study of them is mandatory for a better understanding of 3D-modelling.

Monge worked on the synthesis of water, iron metallurgy, decomposition of CO₂, capillarity, mirages and the establishment of the unit of length as well. With Lavoisier, Laplace, Berthollet and Antoine François, he translated Richard Kirwan’s book about phlogiston and the constitution of acids into French [3].

2.1 Descriptive geometry

Monge begins his works with these sentences: "The objectives of descriptive geometry are two: first, to give an understanding of the methods representing objects which in nature have three dimensions on a two-dimensional surface. The second objective is to teach the way to determine the forms of objects and to deduce all the properties resulting from their respective representations." [15, 20, 22]

Figure 4. Diagram of projection in Monge's Géométrie Descriptive [22]

Descriptive Geometry is the method of representing 3D objects in two dimensions by using certain procedures so that all of their geometric properties can be studied and described correctly. With the development of computers and computer graphics, the approach to teaching Descriptive Geometry has significantly changed. [28, 33].

The first attempt to represent a 3D object in a 2D drawing is by means of some pictorial view, such as perspective, isometric, or oblique views [33]. In any such attempt, some information is necessarily lost, and the object is only partially or ambiguously represented [33]. Graphical methods were mostly used in engineering calculations, and some papers that have survived through time, gave us a glimpse into the development of these methods, especially in architecture and shipbuilding. Among these methods was the use of two coordinated drawings to present three-dimensional information, as mentioned by Vitruvius [33, 34].

Monge’s method is based on orthographic projections of an object on two or more planes while establishing relations between different projections. The two planes are the horizontal and a vertical one, and the line of intersection is the ground line [6, 15, 20, 22].

Monge considers two general types of surfaces. The first one is that which can be conceived of as being formed by bending a plane surface without stretching, crumpling or tearing it. This is a developable surface. The main problem is to determine the shape of the plane figure formed by the boundary of a piece of developable surface as the latter is rolled out into a plane. A circular cylinder could be an example- when it slits along an element, becomes a rectangular figure. In mathematics, a developable surface is a smooth surface with zero Gaussian curvature [22, 25].

The second type of surface is that which cannot be formed by bending a piece of a plane. Two figures of this type are the sphere and the ellipsoid. Since these figures are not developable surfaces, they cannot be rolled out so as to form a plane surface. Various map projections are based on projections of developable and non-developable surfaces [18, 22].

During the last few decades, great progress has been made within the field of Computer Vision, a topic with the main goal to endow a computer with a sense of vision [32]. It is based on the principles of Descriptive Geometry and it works on enabling computers to identify and process images in the same way that human vision does.

2.2 Monge’s theorem

Monge’s theorem (Fig. 5) states that for any three circles in a plane, none of which is completely inside one of the others, the intersection points of each of the three pairs of external tangent lines are collinear [26].

For any two circles in a plane, an external tangent is a line that is tangent to both circles but does not pass between them. There are two such external tangent lines for any two circles [37]. Each such pair has a unique intersection point in the extended Euclidean plane. Monge's theorem states that the three such points given by each circle are always in a straight line [26]. Monge’s theorem can be proved by using Desargues' theorem and Menelaus’ theorem [37]. The Monge’s theorem has found its application in the construction of arched worm gear drive pairs [26, 27].
During the Egypt campaign, Monge became fascinated by a phenomenon called mirage. While soldiers were marching across the desert from Alexandria to Cairo, they saw what seemed to be huge lakes in the distance. They could even see distant villages, that appeared as if they were on some island surrounded by a lake with an inverted image of the village reflecting off its surface. As they were approaching, the water was retreating until it finally disappeared totally [3, 4, 13, 23, 36].

This phenomenon was described in many reports and scientific papers starting from Diodorus Sicilus, a Greek historian. Monge was the first who tried to explain the mirage from an optical point of view. Monge wrote that the light rays were bent by the layer of superheated air just above the sand. Heat radiating from the desert’s surface warms the air immediately above it and reduces the air density. There is a boundary layer between the heated and the cooler layer, denser air above it acts as a lens, bending or refracting the light rays from the sky back upward toward the cooler layer. The bigger the temperature difference, the bigger the effect of the mirage will be [3, 13, 18, 35].

Today, mirages can be divided into three groups: "inferior", "superior" and "Fata morgana". An inferior mirage is called this way because it is located under the real object. In the desert, an inferior mirage may appear to be like in distance, as it was the case with Napoleon’s soldiers. A superior mirage is less common than the inferior one and it is quite common in polar regions. Since the warm air is above cold air, which is the opposite of the normal temperature gradient (temperature inversion), the image appears above the real object. "Fata morgana" is a kind of complex form of superior mirage. It can be seen at the sea or on land, as well as in polar regions and deserts. Often, Fata morgana mirages change rapidly and distort objects on which they are based, so that these objects become completely unrecognizable [36].
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**ЧОВЕК КОЈИ ЈЕ СТВОРИО НАЦРТНУ ГЕОМЕТРИЈУ**

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Гаспар Монж је познат као отац модерне нацртне и диференцијалне геометрије. Године 1764. ангажован је да изради детаљан нацрт утврђења у свом редном граду и његов рад је прематио један официр из војне школе École Royale du Génie de Mézières. Будући да је нацрт био јако добар, методе које је Монж користио чуване су као војна тајна дуги низ година. Године 1780. Монж је постао члан Академије наука и учествовао је у раду Комитета за тегове и мере, који је империјални мерни систем преводио у мерици. Гаспар је помагао у оснивању школе École Centrale des Travaux Publics (касије École Polytechnique) где је и предавао нацртну геометрију. Године 1798. Наполеон је кренуо у поход на Египат и замолио је чуvenог хемичара Клода Бертолеа да ретрутуje истакнуте научнике који би му се придружили у походу. Међу њима су били Фурије, Монж, Доломју и Малу. Наполеон је основао Египатски институт и Монж је био његов први директор. Гаспар Монж је преминуо 28. јула 1818. године у Паризу. Његово име је уређено у темељ Ајфеловог топља и то на месту тачно преко пула Војне академије. Поред нацртне геометрије, Монж се бавио хемијом и физиком.