Effect of an interstitial fluid on the dynamics of three-dimensional granular media

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The propagation of mechanical energy in granular materials has been intensively studied in recent years given the wide range of fields that have processes related to this phenomena, from geology to impact mitigation and protection of buildings and structures. In this article, we experimentally explore the effect of an interstitial fluid on the propagation of a mechanical pulse in a granular packing under controlled confinement pressure. The experimental results unravel the occurrence of a complex elasto-hydrodynamic mechanism at the scale of the contacts between wet particles; we describe our results using a proper effective medium theory including the presence of the fluid, that precisely match our observations without adjusting parameter. Finally, we study the effect of material weakening as a function of impulse amplitude and viscosity of the interstitial fluid; our observations corroborate previous results and demonstrate that the presence of a wetting fluid impedes such mechanism.

I. INTRODUCTION

The propagation of a mechanical impulsion in dry granular materials is a complex phenomena that covers the interest of several fields both in basic and applied research [1]. In a granular media, the complexity of the contact network [2, 3] and the non-linearity of the interaction force between grains determines the propagation of mechanical impulses. In three dimensional granular packing it is well known that a short mechanical impulse input will propagate a coherent ballistic pulse that travels directly from the source to the receiver and an incoherent wave, namely S-wave, that propagates following different paths across the contact network. Furthermore, the S-wave is highly sensitive to the contact network configuration and has been used to reveal the modifications of the internal properties [4]. In recent experiments it has been shown that it is possible to propagate both linear and non linear waves, depending on the confinement pressure applied to the packing and the amplitude of the propagating impulsion [5-6].

The dynamical behavior of wet granular media has been also intensively investigated in recent years due to the great number of industrial applications, from mining to food and pharmaceutical industries. In wet granular media, besides the complexities mentioned before, one must also consider the surface tension of the liquid bridges and the rheology of the fluid. Most of the literature available deals with the behavior of static wet granular media, sheared or vibrated media [7-10]. Despite the intensive effort made, there are still many open questions regarding the interaction of mechanical waves with the structure of a wet granular material. A wet granular media has a fluid phase that partially occupies the interstitial volume available between grains and their motion will be determine by the lubrication regime. For low Reynold’s number, if two grains are initially separated by a fluid layer, a minimum force will be needed to establish a mechanical contact between both grains. This threshold force is proportional to the fluid’s viscosity [11].

The non-linearity of the interaction between grains plays a very important role in the propagation of mechanical impulses in dry granular materials. The repulsive nonlinear interaction between two spheres with radius R and elastic modulus E relies on the Hertz contact force, \( F \propto E R^{3/2} \delta^{3/2} \) [12]; the later implies that the contact force increases rapidly with the deformation \( \delta \). The Hertzian interaction force vanishes when there is no mechanical contact between both spheres and, therefore there is no tensile force in the dry case. However, in a wet granular media there is a restitution force due to the liquid bridge between grains [13].

Weakening in granular materials was reported in recent papers by several authors [14-19] and it has been for proposed as a possible mechanism for triggering secondary earthquakes after the occurrence of a main seismic event. The propagation of the mechanical impulsion interacts with the grains in the material and mobilizes the grains with unconsolidated or weak contacts. This modifications in the contact network can trigger major faults and, thus lead to the emission of new quakes. Given that this mechanism involves only weak or unconsolidated contacts, it corresponds to a non-linear process.

This article is organized as follows: In Section II we present the experimental setup and protocols used, in Section III a set of typical data obtained is shown and the behavior of the impulsion is discussed in terms of the propagation velocity and frequency spectrum of the experimental results. In Section IV-A we show that the velocity of the propagating impulsion depends strongly on the viscosity of the fluid and in Section IV-B we show that the weakening effect disappears when the contacts are lubricated by a sufficiently viscous fluid. Finally, section IV summarizes the results and the observations.
presented in this article.

II. EXPERIMENTAL SETUP

As we presented in a previous article [5], the experimental setup (see Fig. 1) consists in a packing of approximately 1000 glass beads (density $\rho_0 = 2400 \text{ kg/m}^3$) of radius $R = 2.5 \text{ mm}$, that is confined inside an elastic cylinder of length $L = 15 \text{ cm}$ and diameter $\phi = 5 \text{ cm}$, formed from a thin latex sheet. The cylinder is hermetically sealed by clamping the latex sheet at each end between two plastic rings that were specially fabricated to allow the emergence of the accelerometer cable and a vacuum hose. A controlled static pressure $P_0$ is imposed on the packing by evacuating the interstitial air using a vacuum pump. This process ensures a constant hydro-static pressure of as high as $P_0 \approx 83 \text{ kPa}$ and a constant packing fraction of approximately 0.63. A single short impulse is initiated at one cap of the cylinder by the impact of a rigid pendulum head against a dynamic force sensor (PCB Piezotronics model 208C01); the amplitude of the impact is controlled through adjustment of the initial release angle of the pendulum. The impact head of the pendulum consists of a hexagonal brass head with a glass bead mounted on the side that impacts the force sensor; its total mass is 57 g, and the length of the pendulum is 21 cm. The impact head was designed to permit modifications to the material and geometry, thereby allowing control of the duration and form of the input excitation. At the opposite end of the cylinder, a miniature accelerometer (PCB Piezotronics model 352A24), located at the center of the cap, records the outgoing pulse. The mass and size of the accelerometer were chosen to be close to those of an individual grain of the medium.

The signals from the force sensor and accelerometer are conditioned and amplified by a signal conditioner (PCB Piezotronics model 482C). The outputs of the conditioner and the amplified signal from the pressure sensor (Honeywell 19C015PV5K with an INA114 low-noise amplifier) are acquired by a computer via a simultaneous sampling acquisition card at 500 kS/s.

We use a set of silicone oils from Sigma Aldrich with viscosities ranging from $\mu_{\text{min}} = 0.1 \text{ Pa.s}$ up to $\mu_{\text{max}} = 30 \text{ Pa.s}$ at standard temperature. For each experiment we add a fluid volume equivalent to the 37% of the inter-particle space at packing fraction $\phi = 0.63$, to the grains and we make sure that all grains are coated in oil. Then, the wet grains are poured into the elastic container and the sensors are placed as mentioned before. After each measurement, the grains are washed several times using alcohol and then dried in an electric oven.

III. RESULTS

Figures 2 and 3 show a set of typical signals obtained when a confinement pressure $P_0 = 3.2 \text{ kPa}$ and $P_0 = 83.0 \text{ kPa}$ is applied, respectively. In both plots, the left column shows the input impulse force amplitude, with duration of less than half of a millisecond. This short impulse has a band-width that extends up to 8 kHz, while the output signal barely surpasses 1 kHz.

From the experimental data at low confinement it is possible to observe that the amplitude of the impulse decreases systematically with the increase viscosity of the coating fluid as dissipation increases. In the absence of interstitial fluid, the propagated signal carries a high frequency component that vanishes with the addition of the fluid.

At high confinement, as observed previously the output signal contains a low frequency oscillation that remains in the packing after the passing of the impulse and decays exponentially with time.

From the spectra of the signals (not shown here) we can distinguish two main components in the output acceleration signal: the first is a high frequency oscillation of approximately 1 kHz and the second component is a low frequency oscillation, of the order of 100 Hz, it was related to the longitudinal resonance of the packing in our previous work.

A. Impulse velocity

We measure the propagation velocity of the impulsion by means of the time of flight of the signal. We register the times at which the input and output signals passes a threshold value (three times the average noise) at each corresponding sensor. Then, we use the time difference and the distance between the sensors to evaluate the propagation velocity $c$. In Fig. 4 we show the propagation velocity for both low (a) and high (b) confinements. In each plot we have measured the velocity for different viscosities.

For $P_0 = 3.2 \text{ kPa}$, we start with the dry contact case (circles in Fig. 4(a)) and find that it corresponds with previous experimental and theoretical results, i.e. $c$ scales as a power law of the impulse force amplitude as it is predicted from Hertzian contact [5, 20]. When we increase the fluid viscosity, the exponent of the power law fitting decreases to a near zero value and the propagation velocity remains independent of the input
force. We observe that the propagation velocity increases with \( \mu \), this feature was also observed by other authors before for both one and three dimensional granular materials [8, 9].

For high confinement, the propagation velocity does not depend on the force amplitude of the impulse, and as shown in Fig. 3(b) there is a weak dependence on the viscosity of the added fluid. The latter result implies that the fluid is still present in the contact area as the impulse propagates, as if it were partially evacuated from the contact region by the slow compression of the confinement applied.

In order to account for the effect of the viscous fluid over the propagation velocity, we make use of an elasto-hydrodynamic (EHD) contribution of the fluid to the contact dynamics during the impulse propagation.
We first determine the effective $P$-wave modulus $M$ of our samples, both in dry and wet cases, from the measurements of the speed of the pressure waves shown in Fig. 4.

\[ M = K + (4/3)G = \rho \phi c^2, \quad (1) \]

where $\rho = 2400$ kg/m$^3$ is the density of the particles and $\phi = 0.63$ is the compacity of the packing. According to the effective medium theory \[21\] (EMT), the effective bulk and shear moduli, $K$ and $G$, of a random packing of frictional spheres of radius $R$ are given by

\[ K = (2\phi Z)^{2/3} \kappa_n / (12\pi R_\phi), \quad (2) \]
\[ G = (2\phi Z)^{2/3} [\kappa_n + (3/4)\kappa_t] / (20\pi R_\phi), \]

where $\kappa_n$ and $\kappa_t$ are the normal and tangential contact stiffness between two spheres, respectively, and $R_\phi = R/2$ is the reduced radius of curvature. In the dry case, the frictional spheres interact via the Hertz-Mindlin potential \[21\] \[22\], such that

\[ \kappa_n = 2E_c R_\phi (3\pi p / E_0)^{1/3}, \quad \kappa_t = 8E_c R_\phi [(1 - \nu) / (2 - \nu)] (3\pi p / E_0)^{1/3}, \quad (3) \]

where $E_c = E/(2(1 - \nu^2))$ is the reduced elastic modulus, and $E = 69$ GPa and $\nu = 0.2$ are the Young modulus and the Poisson’s ratio of glass spheres. It thus turns out that, in the dry case, the effective $P$-wave modulus non-linearly depends on the local pressure $p$

\[ M_d = \alpha_d \times E_0(p / E_0)^{1/3}, \quad (4) \]

where $\alpha_d = 0.944$ in theory, assuming a coordination number $Z \approx 4$ \[21\]. The experimental results are presented in Fig. 5(a); at high confinement pressure, $p$ stands for the static pressure $p = p_0 \gg p_m$ where $p_m$ is the magnitude of the perturbation; at low confinement pressure, $p$ stands for the magnitude of the perturbation, $p = p_m = F / \pi R_0^2 / \phi^{2/3} \sim p_0$, where $R_0 = 2.5$ cm is the sample cross-sectional area and $\phi^{2/3}$ accounts for the cross-sectional filling fraction. Matching the data shown in Fig. 5(a) to the Eq. 4 provides an estimation for the pre-factor of Eq. 4 $\alpha_d = 1.027 \pm 20\%$, in fair agreement with the EMT model reminded in Eqs. 2 and 3. Interestingly, the data at low and high confinement pressure, i.e. in the nonlinear and in the linear regimes respectively, both rely on an unique trend, as already demonstrated in \[5\].

In the wet case, the presence of an interstitial fluid induces a noticeable and non-trivial augmentation of the speed of the pressure waves. As shown in Fig. 4 the wave speed increases with the fluid viscosity, in both low and high confinement limits. Owing to the confining pressure the fluid resides at the periphery of the dry region of elastic contacts between particles \[23\]. The spatial extent of the dry region is here determined by the radius of the contact disk, $a_d = \sqrt{R_0 R_\phi}$, where $R_\phi \sim R_c(p / E_0)^{2/3}$ is the overlap deformation between spheres. The extent of the dry contact is $a_d \approx 118 \mu m$ at $P_0 = 3.2$ kPa and $a_d \approx 349 \mu m$ at $P_0 = 83$ kPa. When a pressure wave propagates in the sample, the fluid is squeezed outward from the contact area: due to the geometrical singularity near the center of the contact region, the flow likely generates sufficiently large shear rate, shear stress and hydrodynamic pressure to deform the elastic solids, resulting in an elastic confinement of the fluid \[10\]. It stands to reason that the elastic-hydrodynamic response at the periphery of a contact depends on both the fluid and the solid features via a non-dimensional number, $\mu \omega / E$ \[24\] \[26\]. In first approximation, the effect of the peripheral fluid can be accounted as a friction-less contact in the tangential direction, owing to the lubrication by the fluid, and as an elasto-hydrodynamic response in the normal direction, according to Leroy \[25\] \[26\] and Villey \[10\].

\[ \kappa_t = 0 \quad \text{and} \quad \kappa_n = \beta_n \times E_c (\mu_\omega / E)^{1/3}, \quad (5) \]

where $\beta_n$ is a numerical constant. It is worth mentioning that the configuration studied by Leroy \[25\] \[26\] and Villey \[10\] relies on the same mechanisms but slightly differs from ours; in their case, a sphere interacts with an elastic substrate through a viscous fluid with finite thickness $D > 0$. When the fluid thickness is larger than a cutoff distance, $D \gg D_c$, the rupture matches the Reynolds lubrication force, $F = -6\pi \mu \nu R_c / D$ where $\nu$ is the relative velocity of approach. In this case, the hydrodynamic field extends over a distance $d_n = \sqrt{2R_c D}$ near the contact region. Here, the cutoff $D_c$ corresponds to the limit at which the hydrodynamic pressure reaches sufficiently high

\[ \text{FIG. 4: Impulsion velocity as a function of impact amplitude for different fluid viscosity for both low (a: 3.2 kPa) and high (b: 83 kPa) confinement pressures. Markers correspond to different interstitial fluid viscosity: } \circ: 10^{-4} \text{ Pa.s (air), } \triangle: 0.1 \text{ Pa.s, } \square: 1 \text{ Pa.s, } \downarrow: 10 \text{ Pa.s and } \phi: 30 \text{ Pa.s. The dashed lines correspond to the model given in Eq. 6, estimated at the each viscosity.} \]
to deform the elastic solids. When the spheres approach closer than $D_c$, but the fluid thickness and the extent of the hydrodynamic field saturate, $D \to D_c$ and $a_w \to \sqrt{2K_cD_c}$: the fluid is confined and the solids deform to accommodate their displacements. We estimate the typical extent to be $D_c \approx 92, 198, 426$ and 615 $\mu$m at $\mu = 0.1, 1, 10$ and 30 Pa s, respectively, and $f = \omega/2\pi \approx 1$ kHz. In the asymptotic limit of vanishing $D$, the mechanical impedance of the wet contact tends to the ansatz given in Eq. 5 and the pre-factor can be determined numerically at $\beta_0 \approx 1.163 \times (3\pi/4) \times (\sqrt{3} + j)^{25}$. As a consequence, the effective $P$-wave modulus given by Eqs. 2 and 5 becomes

$$M_w = \alpha_w \times E_i(\mu\omega/E_i)^{1/3},$$

with $\Re(\alpha_w) = 0.222$. It is worth mentioning that according to Eqs. 4 and 6, the precise knowledge of the characteristics of the spheres is not necessary; the ratio of wet to dry $P$-wave modulus only depends on the pressure, the viscosity and the frequency, $M_w/M_d \propto (\mu\omega/p)^{1/3}$, as clearly demonstrated in Fig. 5(b). Here, we use the fact that the typical frequency content of the transmitted waves lies in the range of $f = \omega/2\pi \approx 1$ kHz.

First, we observe that the data at low confinement pressure collapse on a master curve, with the predicted exponent, for the three highest viscosity: here, the spatial extent of the hydrodynamic field saturates, the size of the dry region, $a_w \gg a_d$, which can thus be safely neglected. A noticeable exception occurs at the lowest viscosity, for which the extents are of the same order of magnitude: here, we observe that the wet $P$-wave modulus saturates at the value of the dry case. The fit of the data at low confinement pressure, without considering the lowest viscosity, provides an experimental pre-factor $\alpha_w = 0.224 \pm 38\%$ in agreement with the model. In the high confinement pressure case, the extent of the dry region exceeds the extent of the hydrodynamic field, $a_d \geq a_w$, and cannot be neglected; we thus consider a correction contribution, $M_w = M_d + M_w'$, by considering that the fluid resides at the periphery of the dry contact region. Accounting for such a correction demonstrates that the features of the fluid (non-trivially) follows the same trend as in the low confinement limit, see Fig. 5(b). At this point, a fit of the differential $P$-wave modulus provides a pre-factor at $\alpha_w = 0.28 \pm 31\%$, see Fig. 5(c), again in fair agreement with the EMT model including and EHD mechanism.

### B. Material weakening

Material weakening has been proposed as a triggering mechanism for the emission of secondary pulses after the passing of a principal mechanical impulse. Several authors [17, 19, 27, 28] have linked the breaking of unconsolidated and weak contacts with the loss of material strength and dynamical change in form of the Coda of the propagating wave. We probe the weakening of the wet granular material by systematically increasing the amplitude of the mechanical impulsion for different fluid viscosities. We start by following the frequency of the lowest mode in the spectrum of the outgoing acceleration signal for a dry media (see Fig. 6), for both low and high confinement pressure. We observe that for high confinement the frequency of the mode ($f_0 \approx 100$ Hz), remains constant for all impulse amplitudes. On the other hand, in the low confinement case, the mode frequency decreases with increasing impulse amplitude and we find a 20% variation respect to the initial frequency. Next, we repeat the same procedure for different oil viscosities.

In Fig. 7, the evolution of the lowest mode for every fluid is shown as a function of the impulse amplitude, and it is possible to observe that the weakening mechanism remains in the low confinement case, for low viscosities only. This result is compatible with the weak contacts hypothesis, given that the number of consolidated contacts increases with confinement. The role of the fluid seems to point in the direction of the consolidation of contacts.

In order to estimate the origin of the force exerted by the interstitial fluid between the contacts, we consider two distinct phenomena that could increase the cohesion of grains: (i) Capillary bridges and (ii) Viscous drag force. For the first mechanism, we calculate that the capillary force between two grains is of the order $F_c \approx 1\mu N$, given the known surface tension and contact angle of silicone oils [29]. We now consider the viscous drag force experienced by a spherical particle of radius $R$ that moves in a viscous fluid. It should be noted that the grains in our experiments are coated in oil rather than immersed on it. By introducing a finite volume correction [11], it is possible to calculate the force exerted by the fluid on the grains as they propagate the impulsion. We use the maximum acceleration shown in Fig. 2 to calculate the total force over an individual grain $F_d \approx 2.4 mN$ and the mean velocity of the grain $v_{mean} = 15 mm/s$. By equating this force to the corrected Stokes drag force, we obtain a value of viscosity $\mu^* = 3.1 Pa s$, from which the grain will be blocked to move.

![FIG. 5: Color online](image-url)

(a) $P$-wave effective modulus in the dry case as a function of the pressure $p$; $p$ stands either for the static pressure $p_0$ at high confinement or for the magnitude of the perturbation in the low confinement limit. (b) Ratio of the wet to dry $P$-wave modulus as a function of $\mu\omega/p$. At high confinement pressure, the differential modulus $M_w' = M_w - M_d$ follows the same trend as the low pressure data. The inset (c) is the differential modulus as a function of $\mu\omega/E_i$. The markers are identical to Fig. 4 and the solid lines shows the prediction of Eqs. 4 and 6 the gray shades shows the incertitude from the fits.
Therefore, given that the drag force $F_d$ is three orders of magnitude superior to the capillary force $F_c$, we anticipate that for fluid viscosity superior to the obtained value of $\mu^*$ the material will not weaken, given that the unconsolidated contacts will be quickly blocked to move by the fluid’s viscous drag, as the results show in Fig. 7.

IV. CONCLUSIONS

The experimental evidence presented in this article shows that the interstitial fluid in a granular media significantly modifies the contact dynamics between particles and, consequently, the effective features of mechanical wave propagation in the long wavelength approximation. In both low and high confinement cases, we found that the fluid introduces a very complex elasto-hydrodynamic mechanism that increases the rigidity of the contacts, rendering a higher propagation velocity for the impulse, as compared with a dry case. We also demonstrated that the material weakens when submitted to strong perturbations and that this weakening effect is impeded by either the confining pressure or the presence of the viscous fluid. We discussed all our results in terms of an effective mean field theory, in addition to an exact description of the elasto-hydrodynamic interaction between spherical particles; we found quantitative agreement for the wave speed at intermediate frequency ranges and qualitative agreements for the dependence of the lowest mode frequency and the coordination number.

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