Gauge supergravity in $D = 2 + 2$

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Abstract

We present an action for chiral $N = (1, 0)$ supergravity in 2+2 dimensions. The fields of the theory are organized into an $OSp(1|4)$ connection supermatrix, and are given by the usual vierbein $V^a$, spin connection $\omega^{ab}$, and Majorana gravitino $\psi$. In analogy with a construction used for $D = 10 + 2$ gauge supergravity, the action is given by $\int STr(R^2 \Gamma)$, where $R$ is the $OSp(1|4)$ curvature supermatrix two-form, and $\Gamma$ a constant supermatrix containing $\gamma_5$. It is similar, but not identical to the MacDowell-Mansouri action for $D = 2 + 2$ supergravity. The constant supermatrix breaks $OSp(1|4)$ gauge invariance to a subalgebra $OSp(1|2) \oplus Sp(2)$, including a Majorana-Weyl supercharge. Thus half of the $OSp(1|4)$ gauge supersymmetry survives. The gauge fields are the selfdual part of $\omega^{ab}$ and the Weyl projection of $\psi$ for $OSp(1|2)$, and the antiselfdual part of $\omega^{ab}$ for $Sp(2)$. Supersymmetry transformations, being part of a gauge superalgebra, close off-shell. The selfduality condition on the spin connection can be consistently imposed, and the resulting “projected” action is $OSp(1|2)$ gauge invariant.
1 Introduction

In most approaches to supergravity, local supersymmetry appears as the “square root” of diffeomorphisms, and has a natural interpretation as coordinate transformation along Grassmann directions. In this framework supersymmetry is part of a superdiffeomorphism algebra in superspace.

In Chern-Simons supergravities, on the other hand, supersymmetry “lives” on the fiber of a gauge supergroup rather than on a (super) base space. It is part of a gauge superalgebra of transformations leaving the Chern-Simons action invariant, up to boundary terms.

These two conceptually different ways of interpreting supersymmetry are fused together in the group geometric approach (a.k.a. group manifold or rheonomic framework, see for ex. [1]). Recent advances in superintegration theory [2] have shown how this approach interpolates between superspace and component actions.

In this paper we work in the gauge supersymmetry paradigm, that has been explored since long ago [3–5] and has allowed the construction of Chern-Simons supergravities in odd dimensions [6,7,8]. Recently it has been used to construct chiral gauge supergravity in $D = 10 + 2$ dimensions [9]. The twelve dimensional action is written in terms of the $OSp(1|64)$ curvature supermatrix, but is invariant only under its $OSp(1|32) \oplus Sp(32)$ subalgebra. Supersymmetry is part of this superalgebra: it is generated by a Majorana-Weyl supercharge and closes off-shell. The constructive procedure relies on the existence of Majorana-Weyl fermions, and
can in principle be applied in all even dimensions with signatures \((s, t)\) satisfying \(s - t = 0 \pmod{8}\).

Here we apply it for the case \(s = 2, t = 2\) to find an action for \(D = 2 + 2\) chiral supergravity. This action is given by \(\int STr(R \wedge R \Gamma)\), where \(R\) is the \(OSp(1|4)\) curvature supermatrix two-form, and \(\Gamma\) is a constant supermatrix involving \(\gamma_5\). Due to the presence of \(\Gamma\), the action is not invariant under the full \(OSp(1|4)\) superalgebra, but only under a subalgebra \(OSp(1|2) \oplus Sp(2)\) that includes a Majorana-Weyl supercharge. Thus chiral \((1,0)\) supersymmetry survives. This is an important difference with the MacDowell-Mansouri action, for which gauge supersymmetry is completely broken. \(^1\)

Supergravity theories in \(D = 2 + 2\) dimensions have been considered by many authors in the past (for a very partial list of references see \([11, 12]\)). They are candidate backgrounds for the \(N = 2\) superstring \([13, 14, 15]\), and are related after dimensional reduction to integrable models in \(D = 2\) \([16, 17, 18]\). The actions were obtained in most cases by supersymmetrizing the self-dual Einstein-Palatini action with \((2,2)\) signature, supersymmetry invariance being of the “base space” type, as in usual supergravity in \(D = 3 + 1\). The version we propose here differs because supersymmetry is chiral \((1,0)\), and part of a gauge superalgebra, entailing automatic off-shell closure without need of auxiliary fields.

The paper is organized as follows. Section 2 recalls the definitions of \(OSp(1|4)\) connection and curvature, and their \(5 \times 5\) supermatrix representation. Section 3 deals with the chiral \(D = 2 + 2\) action, its invariances, field transformation laws, the explicit expression of the action in terms of component fields, field equations and selfduality condition. Section 4 contains some conclusions. Gamma matrix conventions in \(D = 2 + 2\) are summarized in the Appendix.

2 \(OSp(1|4)\) connection and curvature

This section and the next one closely parallel the analogous sections for \(D = 10 + 2\) supergravity of ref. \([9]\).

2.1 The algebra

The \(OSp(1|4)\) superalgebra is given by:

\[
[M_{ab}, M_{cd}] = \eta_{bc}M_{ad} + \eta_{ad}M_{bc} - \eta_{ac}M_{bd} - \eta_{bd}M_{ac} \quad (2.1)
\]

\[
[M_{ab}, P_c] = \eta_{bc}P_a - \eta_{ac}P_b \quad (2.2)
\]

\[
[P_a, P_b] = M_{ab} \quad (2.3)
\]

\[
\{\bar{Q}_\alpha, Q_\beta\} = -(C_{\gamma}^a)_{\alpha\beta}P_a + \frac{1}{2}(C_{\gamma}^{ab})_{\alpha\beta}M_{ab} \quad (2.4)
\]

\[
[M_{ab}, \bar{Q}_\beta] = \frac{1}{2} (\gamma_{ab})^\alpha_\beta \bar{Q}_\alpha \quad (2.5)
\]

\(^1\)It is “restored” in second order formalism, or by modifying the spin connection transformation law, see for ex. \([5, 10]\).
\([P_a, Q_\beta] = \frac{1}{2} (\gamma_a)^\beta_\alpha Q_\alpha\) \tag{2.6}\]

where \(M_{ab}\) and \(P_a\), dual to the one-forms \(\omega^{ab}\) (spin connection) and \(V^a\) (vierbein), generate the \(Sp(4) \approx SO(3, 2)\) bosonic subalgebra, and the supercharge \(Q_\alpha\) is dual to the Majorana gravitino \(\psi^\alpha\). Conventions on \(D = 2 + 2\) gamma matrices and charge conjugation \(C_{\alpha\beta}\) are given in the Appendix.

### 2.2 The 5 × 5 supermatrix representation

The above superalgebra can be realized by the 5 × 5 supermatrices:

\[
M_{ab} = \left( \frac{1}{2} \gamma_{ab}, 0 \right), \quad P_a = \left( \frac{1}{2} \gamma_a, 0 \right), \quad \bar{Q}_\alpha = Q^\beta C_{\beta\alpha} = \left( 0, \frac{\delta^\beta_\alpha}{C_{\sigma\alpha}} \right) \tag{2.7}\]

To verify the anticommutations (2.4), one needs the identity

\[
\delta^\rho_\alpha C_{\sigma\beta} + \delta^\rho_\beta C_{\sigma\alpha} = -\frac{1}{2} (C_{\gamma a})_{\alpha\beta} (\gamma^a)^\rho_\sigma + \frac{1}{4} (C_{\gamma ab})_{\alpha\beta} (\gamma^{ab})^\rho_\sigma \tag{2.8}\]
deducible from the Fierz identity (A.4) by factoring out the two spinor Majorana one-forms.

### 2.3 Connection and curvature

The 1-form \(OSp(1|4)\)-connection is given by

\[
\Omega = \frac{1}{2} \omega^{ab} M_{ab} + V^a P_a + Q_\alpha \psi^\alpha \tag{2.9}\]

In the 5 × 5 supermatrix representation:

\[
\Omega = \left( \begin{array}{cc} \Omega & \psi \\ -\bar{\psi} & 0 \end{array} \right), \quad \bar{\Omega} = \frac{1}{4} \omega^{ab} \gamma_{ab} + \frac{1}{2} V^a \gamma_a \tag{2.10}\]

The corresponding \(OSp(1|4)\) curvature two-form supermatrix is

\[
R = d\Omega - \Omega \wedge \bar{\Omega} \equiv \left( \begin{array}{cc} R & \Sigma \\ -\Sigma & 0 \end{array} \right) \tag{2.11}\]

where simple matrix algebra yields\(^2\)

\[
R = \frac{1}{4} R^{ab} \gamma_{ab} + \frac{1}{2} R^a \gamma_a \tag{2.12}\]

\[
\Sigma = d\psi - \frac{1}{4} \omega^{ab} \gamma_{ab} \psi - \frac{1}{2} V^a \gamma_a \psi \tag{2.13}\]

\[
\bar{\Sigma} = d\bar{\psi} - \frac{1}{4} \bar{\psi} \omega^{ab} \gamma_{ab} - \frac{1}{2} \bar{\psi} V^a \gamma_a \tag{2.14}\]

\[
R^{ab} \equiv d\omega^{ab} - \omega^{ac} \omega_{cb} - V^a V^b - \frac{1}{2} \bar{\psi} \gamma^{ab} \psi \tag{2.15}\]

\[
R^a \equiv dV^a - \omega^a_c V^c - \frac{1}{2} \bar{\psi} \gamma^a \psi \tag{2.16}\]

\(^2\)we omit wedge products between forms.
We have also used the Fierz identity for 1-form Majorana spinors in [A.4].

3 The $D = 2 + 2$, $N = 1$ (chiral) supergravity action

3.1 Action

The action is written in terms of the $OSp(1|4)$ curvature two-form $R$ as:

$$ S = -2 \int STr(RR\Gamma) $$ (3.1)

where $STr$ is the supertrace and $\Gamma$ is the constant matrix:

$$ \Gamma \equiv \begin{pmatrix} \gamma_5 & 0 \\ 0 & 1 \end{pmatrix} $$ (3.2)

All boldface quantities are $5 \times 5$ supermatrices.

3.2 Invariances

Under the $OSp(1|4)$ gauge transformations:

$$ \delta_\epsilon \Omega = d\epsilon - \Omega \epsilon + \epsilon \Omega \implies \delta_\epsilon R = -R\epsilon + \epsilon R $$ (3.3)

where $\epsilon$ is the $OSp(1|4)$ gauge parameter:

$$ \epsilon \equiv \begin{pmatrix} \frac{1}{4} \epsilon^{ab} \gamma_{ab} + \frac{1}{2} \epsilon^a \gamma_a & \epsilon \\ -\bar{\epsilon} & 0 \end{pmatrix} $$ (3.4)

the action (3.1) varies as

$$ \delta S = -2 \int STr(RR[\Gamma, \epsilon]) $$ (3.5)

Computing the commutator yields

$$ [\Gamma, \epsilon] = \begin{pmatrix} \epsilon^a \gamma_a \gamma_5 & (\gamma_5 - 1) \epsilon \\ \bar{\epsilon}(\gamma_5 - 1) & 0 \end{pmatrix} $$ (3.6)

Thus the action is invariant under gauge variations with $\epsilon^a = 0$ and $(\gamma_5 - 1)\epsilon = 0$ (which implies also $\bar{\epsilon}(\gamma_5 - 1) = 0$). These restrictions on the gauge parameters determine a subalgebra of $OSp(1|4)$, generated by $M_{ab}$ and $\bar{Q}_a P_+$, where $P_+ = \frac{1}{2}(1 + \gamma_5)$. These generators close on the $OSp(1|2) \oplus Sp(2)$ subalgebra:

$$ [M^\pm_{ab}, M^\pm_{cd}] = \eta_{bc} M^\pm_{ad} + \eta_{ad} M^\pm_{bc} - \eta_{ac} M^\pm_{bd} - \eta_{bd} M^\pm_{ac} $$ (3.7)

$$ \{\bar{Q}_a, \bar{Q}_b\} = \frac{1}{2} (C\gamma^a)_{ab} M^+_b $$ (3.8)

$$ [M^+_a, \bar{Q}^+_b] = \frac{1}{2} (\gamma^+_a)_{ab} \bar{Q}^+_b $$ (3.9)

$$ [M^-_a, M^+_b] = [M^-_a, \bar{Q}^+_b] = 0 $$ (3.10)
with
\[ \gamma_{ab}^+ = \frac{1 + \gamma_5}{2} \gamma_{ab} = \frac{1}{2}(\gamma_{ab} - \frac{1}{2} \epsilon_{abcd} \gamma^{cd}) \] (3.11)
and
\[ M_{ab}^+ = \frac{1}{2}(M_{ab} + \frac{1}{2} \epsilon_{abcd} M^{cd}), \quad Q^+_\alpha = Q_\alpha \frac{1 + \gamma_5}{2} \] (3.12)

The selfdual \( M_{ab}^+ \) and Weyl projected \( \bar{Q}^+_\alpha \) generate \( OSp(1|2) \), while the antiselfdual \( M_{ab}^- \) generates \( Sp(2) \).

### 3.3 \( OSp(1|2) \oplus Sp(2) \) transformation laws

Restricting the gauge parameter \( \epsilon \) to the \( OSp(1|2) \oplus Sp(2) \) subalgebra as described above, from (3.3) we deduce the transformation laws on the fields \( \omega^{ab} \), \( V^a \) and \( \psi \) leaving the action (3.1) invariant:

\[ \delta \omega_{ab}^+ = d\epsilon_{ab}^{+} + \omega^{ac}_{+} \epsilon_{+}^{cd} \eta_{cd} + \omega^{bc}_{+} \epsilon_{+}^{da} \eta_{cd} + \bar{\epsilon}_{+} \gamma_{ab}^{+} \psi_{+} \] (3.13)
\[ \delta \omega_{ab}^- = d\epsilon_{ab}^{-} - \omega^{ac}_{-} \epsilon_{-}^{cd} \eta_{cd} + \omega^{bc}_{-} \epsilon_{-}^{da} \eta_{cd} \] (3.14)
\[ \delta V^a = (\epsilon_{+}^{ab} + \epsilon_{-}^{ab}) V^c \eta_{bc} - \bar{\epsilon}_{+} \gamma_{a}^{+} \psi_{-} \] (3.15)
\[ \delta \psi_{+} = d\epsilon_{+} - \frac{1}{4} \omega^{ac}_{+} \gamma_{ab} \epsilon_{+} + \frac{1}{4} \epsilon_{+}^{ab} \gamma_{ab} \psi_{+} \] (3.16)
\[ \delta \psi_{-} = - \frac{1}{2} V^a \gamma_{a} \epsilon_{+} + \frac{1}{4} \epsilon_{-}^{ab} \gamma_{ab} \psi_{-} \] (3.17)

where \( \epsilon_{+} = P_{+}\epsilon \) is the Weyl projected supersymmetry parameter, and \( \epsilon_{+}^{ab}, \epsilon_{-}^{ab} \) are the selfdual and antiselfdual \( SO(2, 2) \approx Sp(2) \times Sp(2) \) parameters. Moreover \( \psi_{+} \) and \( \psi_{-} \) are respectively Weyl and anti-Weyl projections of the Majorana gravitino, i.e. \( \psi_{\pm} = P_{\pm}\psi \).

Thus we see that the \( OSp(1|2) \oplus Sp(2) \) gauge fields \( \omega^{ab}_{\pm}, \psi_{\pm} \) transform with the \( OSp(1|2) \oplus Sp(2) \) covariant derivative of the gauge parameters, whereas the “matter fields” \( V^a, \psi_{-} \) transform homogeneously. Note also that gauge and matter fields do not mix, separating into a gauge and a matter multiplet under \( OSp(1|2) \oplus Sp(2) \) transformations.

Finally, \( \omega^{ab}_{-} \) is inert under supersymmetry, This will be important for the consistency of the selfduality condition \( \omega^{ab}_{-} = 0 \), see Section 3.6.

### 3.4 The action in terms of component fields

Recalling that \( \int ST(rRR) \) is a topological term, we have:

\[ S = -2 \int ST(rRR\Gamma) = 4 \int ST(rRR\frac{1 - \Gamma}{2}) \] (3.18)

up to boundary terms. Carrying out the supertrace leads to:

\[ S = 4 \int Tr(RRP_{-}) + \Sigma P_{-}\Sigma \] (3.19)
with $R$ and $\Sigma$ as defined in Section 2.3, and $P_- = (1 - \gamma_5)/2$. After inserting the curvature definitions the action takes the form

$$S = \int \mathcal{R}^{ab} V^c V^d \varepsilon_{abcd} - 4 \tilde{\rho} + \gamma_a \psi - V^a - \frac{1}{2}(V^a V^b V^c V^d + \bar{\psi} \gamma^{ab} \psi V^c V^d) \varepsilon_{abcd} + \frac{1}{2} \mathcal{R}^{ab}_{\pm} \bar{\psi} \gamma^{cd} \psi \varepsilon_{abcd}$$

(3.20)

with

$$\mathcal{R}^{ab} \equiv d\omega^{ab} - \omega^a_c \omega^{cb}, \quad \rho \equiv d\psi - \frac{1}{4} \omega^{ab} \gamma_{ab} \psi, \quad \rho_+ \equiv P_+ \rho$$

(3.21)

We have dropped the topological term $R^{ab} - R^{cd} - \varepsilon^{abcd}$ (sum of Euler and Pontryagin forms), and used the identities

$$R^a R^a = -\mathcal{R}^{ab} V^a V^b - \bar{\rho} \gamma_a \psi + \frac{1}{2} \bar{\psi} \gamma_a \psi R^a + \text{total derivative}$$

(3.22)

$$\rho - \rho_- = -\frac{1}{4} \bar{\psi} \gamma_{ab} \psi R^{ab} + \text{total derivative}$$

(3.23)

$$\bar{\psi} \gamma^a \psi R^a = -2 \bar{\rho} \gamma^a \psi V^a + \text{total derivative}$$

(3.24)

$$R^a_{\pm} = \mp \frac{1}{2} \varepsilon^{ab}_{cd} R^{cd}$$

(3.25)

and the Bianchi identities

$$dR^a - \omega^a_b R^b = -\mathcal{R}^a_b V^b + \bar{\rho} \gamma^a \psi$$

(3.26)

$$d\rho - \frac{1}{4} \omega^{ab} \gamma_{ab} \rho = -\frac{1}{4} \mathcal{R}^{ab} \gamma_{ab} \psi$$

(3.27)

consequences of the definitions (2.16) and (3.21) of $R^a$ and $\rho$.

This action is similar to a MacDowell-Mansouri action in $D = 2 + 2$, or also to $R^2$-type actions previously considered in the literature concerning self-dual supergravity, but with the important difference that it is invariant under a gauge (chiral) supersymmetry, closing off-shell.

### 3.5 Equations of motion

The variational equations for the action (3.20) read:

$$R^{ab} V^c \varepsilon_{abcd} - 2 \Sigma_+ \gamma_{ab} \psi_+ = 0 \quad (\text{Einstein eq.s})$$

(3.28)

$$\gamma_a \Sigma_+ V^a - \gamma_a \psi_+ R^a = 0 \quad (\text{gravitino } \psi_+)$$

(3.29)

$$4 \gamma_a \Sigma_+ V^a + R^{ab}_{\pm} \gamma_{cd} \psi_\pm \varepsilon_{abcd} = 0 \quad (\text{gravitino } \psi_-)$$

(3.30)

$$R^a V^d \varepsilon_{abcd} + \frac{1}{2} \Sigma_- \gamma^{cd} \psi_- \varepsilon_{abcd} = 0 \quad (\text{torsion eq.})$$

(3.31)

These equations admit the vacuum solution $OSp(1|4)$ curvatures = 0.
3.6 Self-dual D=2+2 supergravity

We can impose the self-duality condition on the spin connection:

\[ \omega_-^{ab} \equiv \frac{1}{2} (\omega^{ab} + \frac{1}{2} \varepsilon^{ab}_{\ cd} \omega^{cd}) = 0 \]  

(3.32)

Recalling that

\[ R^{ab} = R^+_a + R^-_a \]  

(3.33)

with

\[ R^\pm_a = d\omega^\pm_a - \omega^\pm_a \omega^\pm_b \eta_{bd} \]  

(3.34)

we can implement the selfduality condition (3.32) in the action by discarding the \( R^-_a \) component of \( R^a_b \) in the first term of (3.20), and the \( R^a_- \) part of \( R^a_- \) in the last term. The resulting action is invariant under the transformations of \( \omega^a_\pm, V^a, \psi^+, \psi^- \) given in Section 3.3, with \( \varepsilon^{ab} = 0 \), i.e. under \( OSP(1|2) \) transformations whose gauge fields are \( \omega^a_+ \) and \( \psi_+ \). Indeed the condition \( \omega^a_- = 0 \) breaks \( OSP(1|2) \times Sp(2) \) to its first factor \( OSP(1|2) \).

Second order formalism is retrieved by solving the torsion equation of motion (3.31), which for \( \omega^a_- = 0 \) allows to express \( \omega^a_+ \) as a function of \( V^a, \psi^+ \) and \( \psi^- \).

4 Conclusions

We have presented a \( D = 2 + 2 \) supergravity action, made out of the fields contained in the \( OSP(1|4) \) connection. It is invariant only under a subalgebra \( OSP(1|2) \oplus Sp(2) \) of \( OSP(1|4) \). This closely resembles what happens for the Mac Dowell-Mansouri action in \( D = 3 + 1 \): there too the supergravity fields are organized in a \( OSP(1|4) \) connection, but the action itself is invariant only under the Lorentz subalgebra, whereas in the present paper also (1,0) supersymmetry survives, being part of the invariance subalgebra of the action.

A selfdual condition can be imposed on the spin connection, and breaks the \( OSP(1|2) \oplus Sp(2) \) invariance to the first factor \( OSP(1|2) \).

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A D=2+2 \( \Gamma \) matrices

Clifford algebra

\[ \{\gamma_a, \gamma_b\} = 2\eta_{ab}, \quad \eta_{ab} = (+1, +1, -1, -1), \quad \gamma_5 = \gamma_1 \gamma_2 \gamma_3 \gamma_4, \quad \gamma_5^2 = 1 \]  

(A.1)
The $D = 2 + 2$ charge conjugation matrix $C$ satisfies
\[
\gamma^T_a = -C\gamma_a C^{-1}, \quad C^T = -C, \quad [C, \gamma_5] = 0 \tag{A.2}
\]
so that $C\gamma_a$ and $C\gamma_{ab}$ are symmetric.

**Duality relation:**
\[
\gamma^{ab} = -\frac{1}{2}\epsilon^{abcd}\gamma_{cd}\gamma_5 \tag{A.3}
\]

**Fierz identity**

The following Fierz identity holds for Majorana spinor one-forms ($\bar{\psi} = \psi C$):
\[
\psi\bar{\psi} = \frac{1}{4}(\bar{\psi}\gamma^a\psi\gamma_a - \frac{1}{2}\bar{\psi}\gamma^{ab}\psi\gamma_{ab})\tag{A.4}
\]
(to prove it, just multiply both sides by $\gamma_c$ or $\gamma_{cd}$ and take the trace on spinor indices).

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