Quality Selection in Two-Sided Markets: A Constrained Price Discrimination Approach

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Abstract

Online platforms collect rich information about participants and then share some of this information back with them to improve market outcomes. In this paper we study the following information disclosure problem in two-sided markets: If a platform wants to maximize revenue, which sellers should the platform allow to participate, and how much of its available information about participating sellers’ quality should the platform share with buyers? We study this information disclosure problem in the context of two distinct two-sided market models: one in which the platform chooses prices and the sellers choose quantities (similar to ride-sharing), and one in which the sellers choose prices (similar to e-commerce). Our main results provide conditions under which simple information structures commonly observed in practice, such as banning certain sellers from the platform while not distinguishing between participating sellers, maximize the platform’s revenue. An important innovation in our analysis is to transform the platform’s information disclosure problem into a constrained price discrimination problem. We leverage this transformation to obtain our structural results.

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1 Introduction

Online platforms have an increasingly rich plethora of information available about market participants. These include rating systems, public and private written feedback, purchase behavior, among others. Using these sources, platforms have become increasingly sophisticated in classifying the quality of the sellers that participate in their platform (for example, see Tadelis (2016), Filippas et al. (2018), Donaker et al. (2019), and Garg and Johari (2019)). This information can be used both to increase the platform’s revenue, and to enhance the welfare of the platform’s participants. For example, cleaning services and ridesharing platforms remove low quality sellers from their platforms. Platforms can also boost the visibility of high quality sellers with certain badges, as is done by online marketplaces such as Amazon Marketplace and eBay. We refer broadly to such market design choices by platforms as \textit{quality selection}.

In this paper, we study quality selection in two-sided markets. In particular, we investigate which sellers a two-sided market platform should allow to participate in the platform, as well as the optimal amount of information about the participating sellers’ quality that the platform should share with buyers in order to maximize its own revenue. Our results characterize conditions under which simple information structures, such as just banning a portion of low quality suppliers or giving badges to high quality suppliers, emerge as optimal designs.

We introduce two different two-sided market models with heterogeneous buyers and heterogeneous sellers. Sellers are heterogeneous in their quality levels and buyers are heterogeneous in how they trade-off quality and price. In the first model, the platform chooses prices and the sellers choose quantities (e.g., how many hours to work). This setting is loosely motivated by labor platforms such as ride-sharing and cleaning services. In the second model, the sellers choose prices, and quantities are determined in equilibrium. This setting is motivated by online marketplaces such as Amazon Marketplace. In both models, quality selection by the platform involves deciding on an \textit{information structure}, that is, how much of the information it has about the sellers’ quality to share with buyers. The platform’s goal is to choose an information structure that maximizes the platform’s revenue. The information structure can consist of banning a certain portion of the sellers, and also richer structures that share more granular information with buyers about the quality of participating sellers.

The mapping from the information that the platform shares about the sellers’ quality to market outcomes is generally complicated. After the platform chooses an information structure, the buyers and the sellers take strategic actions. Market outcomes such as prices and offered qualities are determined by these strategic actions and the resulting equilibrium conditions, including market clearing: not only must the buyers’ incentive compatibility and individual rationality constraints be satisfied, (as in a standard price discrimination problem, e.g., Mussa and Rosen (1978)), but the total supply must also equal the total demand. One of our paper’s key observations is that the platform’s information disclosure problem transforms into a \textit{constrained price discrimination problem}. We show that every information structure induces a certain subset of price-expected quality pairs which we call a \textit{menu}, from which the buyers can choose. Optimization over feasible
menus yields a price discrimination problem.

Note that platforms can use the information they collect about the sellers’ quality to induce a menu in many different ways. For example, giving badges to high quality sellers can influence the prices such sellers charge, the quantities they sell, and their market entry decisions (Hui et al., 2018). Similarly, banning some low quality sellers can also influence the prices, the quantities sold, and the participating sellers’ quality.

We show that finding the optimal menu in the constrained price discrimination problem is equivalent to finding the optimal information structure. This equivalence proves to be beneficial for two reasons. First, deriving structural results in the constrained price discrimination problem (see Section 3) is simpler than solving for the optimal information structure in a two-sided market model directly. This is similar to the Bayesian persuasion literature where the sender’s optimization problem is usually reformulated in order to simplify the analysis (see Kamenica (2019) and Section 1.1). Second, the constrained price discrimination problem is general and can capture different market arrangements and different two-sided market models.

Using our results from the constrained price discrimination problem, we provide a broad set of conditions under which a simple information structure in which the platform bans a certain portion of low quality sellers and does not distinguish between participating sellers maximizes the platform’s revenue. This resembles a common practice in ride-sharing and cleaning services platforms (in these cases the participating suppliers’ review scores are typically so high that they do not reveal much information (Tadelis, 2016)). To obtain this result, we require two conditions. First, we require a regularity condition on the induced set of feasible menus in the constrained price discrimination problem; as we suggest later, this regularity condition is natural and likely to be satisfied in a wide range of market models. Given this regularity condition, our second requirement is an appropriate convexity condition on the demand; as we note, this condition reduces to the requirement that the demand elasticity is not too low. We also provide results involving only local demand elasticity that guide the market design decision of whether to share less information about sellers’ quality. We provide a simple example in Section 2 that illustrates the key features of our analysis.

We then apply the equivalence between the constrained price discrimination problem and the information disclosure problem in order to study the two different two-sided market models mentioned above. In both models, the platform’s decisions (the platform decides on an information structure and prices in the first model, and on an information structure in the second model) generate a game between buyers and sellers. Given the platform’s decisions there are four equilibrium requirements. First, the sellers choose their actions (prices or quantities) to maximize their profits. Second, the buyers choose whether to buy the product and if so, what (expected) quality to buy to maximize their utility. Third, given the information structure that the platform chooses, the buyers form beliefs about the sellers’ qualities that are consistent with Bayesian updating and with the sellers’ actions. Fourth, we require market clearing: the total supply equals the total demand.

We show that each equilibrium of the game induces a certain subset of price-quality pairs; each pair consists of a price, and the expected quality of sellers selling at that price. The platform’s goal
is to choose a menu that maximizes the platform’s revenue. Finding the set of equilibrium menus that the platform can choose from depends on the equilibrium outcomes of the game. Hence, this set is determined by the specific two-sided market model being studied and can be challenging to characterize. For our first model (in which the platform sets prices), we show that for every information structure there exists a strictly convex optimization problem whose unique solution yields the unique menu of induced price-quality pairs. For the second model, Bertrand competition between the sellers pins down the equilibrium prices, so we are able to explicitly provide the menu that each information structure induces. In each setting, we then leverage the analysis of the constrained price discrimination problem to characterize the platform’s optimal information disclosure, and in particular to find conditions under which the policy of banning low quality sellers, and not distinguishing between the remaining high quality sellers, is optimal.

The rest of the paper is organized as follows. Section 1.1 discusses related literature. In Section 2 we describe a simple example that captures the main features of our analysis. In Section 3 we study the general constrained price discrimination problem. In Section 4 we present the platform’s initial information and information structures. In Section 5 we present our first model where the platform chooses prices and the sellers choose quantities. In Section 6 we present our second model where the sellers choose prices and quantities are determined in equilibrium. In Section 7 we provide a summary, followed by an Appendix.

1.1 Related Literature

Our paper is related to several strands of literature. We discuss each of them separately below.

**Information design.** There is a vast recent literature on how different information disclosure policies influence the decisions of strategic agents and equilibrium outcomes in different settings. Applications include Bayesian persuasion (Aumann and Maschler (1966) and Kamenica and Gentzkow (2011)), dynamic contests (Bimpikis et al., 2019), matching markets (Ostrovsky and Schwarz, 2010), queuing theory (Lingenbrink and Iyer, 2019), games with common interests (Lehrer et al., 2010), transportation (Meigs et al., 2020), inventory systems (Kostami, 2019), ad-auctions (Varadaraja et al., 2018), exploration in recommendation systems (Papanastasiou et al. (2017) and Immorlica et al. (2019)), social networks (Candogan and Drakopoulos (2020) and Candogan (2019)), social services (Anunrojwong et al., 2020), the retail industry (Lingenbrink and Iyer (2018) and Drakopoulos et al. (2019)), warning policies (Alizamir et al., 2020), and many more (see Candogan (2020) for a recent review of information design in operations.)

In this paper we focus on the amount of information about the sellers’ quality that a two-sided market platform should share with buyers. Our information disclosure policy problem is different from the previous literature because the platform faces equilibrium constraints when informing buyers about the sellers’ quality; these constraints emerge because actual two-sided market outcomes are determined endogenously by buyers’ and sellers’ behavior, subsequent to the information disclosure choices of the platform. There are at least three salient characteristics of our setting. First, the platform does not have full information about the sellers’ quality. Second, buyers’ beliefs
about the sellers’ quality can depend on the sellers’ actions (in addition to the standard dependence of the buyers’ beliefs on the platform’s information disclosure policy). For example, if the sellers choose quantities (e.g., how many hours to work) these quantities influence the expected qualities.\footnote{Because the buyers’ beliefs are consistent with the sellers’ actions, our model also relates to the adverse selection literature (see Akerlof (1970)).} Third, the prices and the sellers’ expected qualities must form an equilibrium in the two-sided market (i.e., the total supply equals the total demand). Overall, these constraints significantly limit the platform’s feasible information structures, and therefore, the typical techniques used in the Bayesian persuasion literature cannot be applied.

Similar to the Bayesian persuasion literature, we reformulate the platform’s optimization problem in order to simplify the analysis. In the Bayesian persuasion literature, it can be shown that the platform’s (sender) payoffs are determined by the receivers’ posterior beliefs. The standard approach is to optimize over these posterior beliefs instead of over information structures. This approach leads, at least in some cases, to sharp characterizations of the optimal information disclosure policy (see, e.g., Aumann and Maschler (1966) and Kamenica and Gentzkow (2011)). In our setting, we can show that the platform’s payoffs are determined by the buyers’ (i.e., the receivers) equilibrium posterior quality means and by the equilibrium prices. Our approach is to optimize jointly over posterior means and prices, and thus, we transform the information disclosure problem to a price discrimination problem. This approach leads to sharp characterizations of the optimal information structure under certain conditions.

**Nonlinear pricing.** Nonlinear pricing schemes are widely studied in the economics and management science literature (see Wilson (1993) for a textbook treatment). The price discrimination problem that we consider in this paper is closest to the classical second-degree price discrimination problem (Mussa and Rosen, 1978) and (Maskin and Riley, 1984).

The problem that the platform solves in our setting differs from the previous literature on price discrimination in two major aspects. First, the costs for the platform from producing higher quality products are zero. This is because in the two-sided market models that we study, the costs of producing a higher quality product are incurred by the sellers and not by the platform. Hence, the platform’s revenue maximization problem transforms into a constrained price discrimination problem with no costs. Second, the platform cannot simply choose any subset of price-quality pairs (menus) that satisfies the incentive compatibility and individual rationality constraints. The set of menus from which the platform can choose is determined by the additional equilibrium requirements described in the introduction.

These differences significantly change the analysis and the platform’s optimal menu. First, a key part of our analysis is to incorporate equilibrium constraints into the price discrimination problem, introducing significant additional complexity. In addition, under the regularity assumption that the virtual valuation function is increasing, Mussa and Rosen (1978) show that the optimal menu assigns different qualities of the product to different types. In contrast, the results in our paper are drastically different: under certain regularity assumptions, the optimal menu assigns the same
quality of the product to different types.\footnote{Another difference from most of the previous literature is that in our model each menu is finite (i.e., there is a finite number of price-quality pairs), and thus the standard techniques used to analyze the price discrimination problems in the previous literature cannot be used. Bergemann et al. (2011) study a price discrimination problem with a finite menu in order to study a setting with limited information. However, because the platform’s costs are zero in our setting, we cannot use the Lloyd-Max optimality condition that Bergemann et al. (2011) employs.}

**Two-sided market platforms.** Recent papers study how platforms can use information and other related market design levers to improve market outcomes. In the context of matching markets, Arnosti et al. (2018) and Kanoria and Saban (2019) suggest different restrictions on the agents’ actions in order to mitigate inefficiencies that arise in those markets. Vellodi (2018) studies the role of design of rating systems in shaping industry dynamics. In Romanyuk and Smolin (2019) the platform designs what buyer information the sellers should observe before the platform decides to form a match.

The paper most closely related to ours is the contemporaneous work by Bimpikis et al. (2020) that studies the interaction between information disclosure and the quantity and quality of the sellers participating in the platform. Studying a dynamic game theoretic model, Bimpikis et al. (2020) focuses on how information design influences supply-side decisions, showing that information design can be a substitute to charging lower fees when solving the “cold start” problem. As in our paper, in the papers noted above the full disclosure policy is not necessarily optimal, and hiding information can increase the social welfare and/or the platform’s revenue.

## 2 A Simple Motivating Model

In this section we provide a simple model that illustrates many important features of our paper. While this model ignores important features of our more general model, it will be helpful to highlight important aspects of our analysis and main results.

Consider a platform where heterogeneous sellers and heterogeneous buyers interact. In our simple model of this section, there are two types of sellers: high quality sellers $q_H$ and low quality sellers $q_L$ with $q_H > q_L > 0$. The platform knows the sellers’ quality and considers two policies. Policy $B$ is to ban the low quality sellers and keep only the high quality sellers on the platform. Policy $K$ is to keep both low quality and high quality sellers on the platform and share the information about the sellers’ quality with the buyers.

The total supply of products by sellers whose quality level is $i = H, L$ is given by the function $S_i(p^j_i)$. When the platform chooses policy $j = B, K$, $p^j_i$ is the price of the product sold by sellers whose quality level is $i = H, L$. We assume that the total supply is increasing in the price. The total supply can also depend on the mass of sellers whose quality level is $i = H, L$ and on the sellers’ costs. In our two-sided market models the supply function will be micro-founded, but we abstract away from these details for now.

Buyers are heterogeneous in how much they value quality relative to price. A buyer with type $m$ that decides to purchase from a seller whose quality level is $i = H, L$ has a utility $mq_i - p^j_i$. 
We normalize the utility associated to not buying to zero. The distribution of the buyers’ types is described by a probability distribution function $F$. We assume that $F$ admits a density function $f$. The buyers choose to buy or not to buy the product from sellers whose quality level is $i = H, L$ in order maximize their own utility. The buyers’ decisions generate demand for quality $i = H, L$ sellers $D_i^K(p_L^K, p_H^K)$ when the platform chooses policy $K$, and demand for quality $H$ sellers $D_H^B(p_H^B)$ when the platform chooses policy $B$ (when the platform chooses option $B$, there is no demand for low quality sellers as they are banned).

The platform’s goal is to choose a policy that maximizes the total transaction value given that prices form an equilibrium. Equilibrium requires that the market must clear: that is, supply must equal demand. Note that if the platform charges commissions from each side of the market, maximizing the total transaction value is equivalent to maximizing the platform’s revenue. For this reason, we will refer to the platform’s objective as “revenue” or “total transaction value” interchangeably. If the platform chooses policy $B$, then the total transaction value is $p_H^B D_H^B(p_H^B)$ and the equilibrium requirement is $S_H(p_H^B) = D_H^B(p_H^B)$. If the platform chooses policy $K$, then the total transaction value is $p_H^K D_H^K(p_L^K, p_H^K) + p_L^K D_L^K(p_L^K, p_H^K)$ and the equilibrium requirements are

$$S_H(p_H^K) = D_H^K(p_L^K, p_H^K) \quad \text{and} \quad S_L(p_L^K) = D_L^K(p_L^K, p_H^K). \quad (1)$$

For simplicity, we assume that the prices that satisfy the equilibrium requirements are unique. That is, $(p_L^K, p_H^K)$ are the unique prices that solve the equations in (1) and $p_H^B$ is the unique price that solves $D_H^B(p_H^B) = S_H(p_H^B)$. In this case, the platform’s revenue maximization problem transforms into a constrained price discrimination problem. Choosing policy $B$ is equivalent to showing the buyers the price-quality pair $(q_H, p_H^B)$, while choosing policy $K$ is equivalent to showing the buyers the price-quality pairs $(q_H, p_H^K)$ and $(q_L, p_L^K)$. Hence, each policy is equivalent to a subset of price-quality pairs that we call a menu, and the platform’s goal is to choose the menu with the higher revenue.

In our simple model, the set of feasible menus (denoted by $C$) contains only two menus. We introduce our general model in Section 3, where we study a general price discrimination problem with a rich set of possible menus $C$, defined by a general constraint set. Furthermore, in the model we consider in this section, the sellers’ qualities are fixed and the prices are constrained by the equilibrium requirements. In the general two-sided market models we consider (see Sections 5 and 6), the expected qualities are also determined in equilibrium. Hence, the set of feasible menus $C$ in the corresponding price discrimination problem is determined by the specific two-sided market model that we study. When the model is complicated, characterizing the set $C$ can be challenging as it requires computation of the equilibria of the two-sided market model.

While the price discrimination problem in this example is simple, we later show that we can solve a general constrained price discrimination problem with similar arguments (see Section 3). We
analyze the price discrimination problem in two stages. In the first stage, we compare the revenue from policy $K$ (showing the price-quality pairs $(q_H, p^K_H)$ and $(q_L, p^K_L)$) to the revenue from the infeasible policy $I$: showing the price-quality pair $(q_H, p^K_H)$. Policy $I$ is generally infeasible because while the pair $(q_H, p^K_H)$ and $(q_L, p^K_L)$ clears the market, only showing $(q_H, p^K_H)$ will generally not do so: demand will be higher than supply.

Note that the equilibrium requirements imply that the price of the product sold by high quality sellers is higher than the price of the product sold by low quality sellers, i.e., $p^K_H > p^K_L$. Now, if the platform were to choose policy $I$ then fewer buyers would participate in the platform compared to policy $K$, but the participating buyers would pay the higher price $p^K_H$. Policy $I$ would be better than policy $K$ if and only if the revenue gains from the participating buyers that pay a higher price when choosing $I$ instead of $K$ outweigh the revenue losses from the mass of buyers that do not participate in the platform when choosing $I$ instead of $K$. This depends on the *elasticity of the density function* $\partial \ln f(m)/\partial \ln m$. Intuitively, when the density function’s elasticity is not too “low” the mass of buyers that the platform loses is not too “high”. We show in Theorem 1 a general version of the following: when the density function’s elasticity is bounded below by $-2$, policy $I$ yields more revenue than policy $K$ (see a detailed analysis of the elasticity condition in Section 3).

In the second stage of the analysis, we compare the revenue from policy $B$ to the revenue from (potentially infeasible) policy $I$. The equilibrium requirements imply that $p^B_H \geq p^K_H$. To see this, note that $D^B_H(p^K_H) \geq D^K_H(p^K_L, p^K_H) = S_H(p^K_H)$, i.e., the demand for high quality sellers in policy $B$ is greater than the demand for high quality sellers in policy $K$ when the price is $p^K_H$. This follows because for some buyers, buying from the high quality sellers yields a positive utility that is smaller than the utility from buying from the low quality sellers. Hence, in policy $B$, these buyers buy from the high quality sellers, while in policy $K$ they buy from the low quality sellers. Thus, the demand for high quality sellers under the price $p^K_H$ exceeds the supply. Because the supply is increasing and the demand is decreasing in the price, we must have $p^B_H \geq p^K_H$.

Before proceeding with the second stage of the analysis, we note that for some models it is the case that $p^B_H = p^K_H$, like in the Bertrand competition model that we study in Section 6. In this model, because supply is perfectly elastic prices drop down all the way to marginal cost independently of whether low quality sellers participate in the platform. In this case, this second stage of the analysis is not necessary.

Now, if the platform shows the buyers the menu $(q_H, p)$ only the buyers whose valuations satisfy $mq_H - p \geq 0$ buy the product from the high quality sellers. Thus, $pD^B_H(p) = p(1 - F(p/q_H))$. When the density function’s elasticity is bounded below by $-2$, the function $F(m)m$ is convex (see Section 3), and hence, the revenue function $R_H(p) := p (1 - F(p/q_H))$ is concave in the price $p$. Thus, as shown in Figure 1 below, policy $B$ yields more revenue than policy $I$ if the equilibrium price $p^B_H$ is lower than the *monopoly price* $p^M_H$, i.e., the unconstrained price that maximizes the platform’s revenue $p^M_H$ ignoring equilibrium conditions:

$$p^M_H = \arg\max_{p \geq 0} p \left(1 - F \left( \frac{p}{q_H} \right) \right).$$
Figure 1: The platform’s revenue as a function of the price.

Intuitively, the equilibrium price $p^H_B$ is lower than the price that maximizes the platform’s revenue $p^M_H$ if the total supply of high quality sellers is large enough. In particular, if the total supply of high quality sellers exceeds the total demand under the price $p^M_H$, then the equilibrium price $p^B_H$ must be lower than $p^M_H$ to ensure the market clears. In many two-sided markets, competition between platforms and between sellers, platform subsidies on the supply side, penetration pricing strategies, and other factors decrease equilibrium prices considerably. Hence, in our context it is natural to assume that the monopoly price is higher than or equal to the equilibrium price, i.e., $p^H_B \leq p^M_H$. In addition, if the equilibrium price was higher than the price that maximizes the platform’s revenue the platform could introduce balanced transfers for each side of the market, i.e., paying suppliers and charging buyers in order to decrease the equilibrium price.

In the general two-sided market models that we study in Sections 5 and 6, the qualities are also determined in equilibrium and the set of possible menus that the platform can choose from can be very large. We will call this set regular if it satisfies a general version of the conditions $p^M_H \geq p^B_H \geq p^K_H$ discussed above. That is, the set is regular if removing low quality sellers increases the equilibrium price for high quality sellers; and if, in addition, the monopoly price is higher than this equilibrium price. These conditions give rise to natural constraints on the equilibria that can arise in the two-sided market models that we study (see the discussion after Definition 1 in Section 3).

We conclude that when the elasticity of the density function is not too low, and the monopoly price is higher than the equilibrium price, then policy $B$ yields more revenue than policy $K$. That is, banning low quality sellers and keeping only the high quality sellers yields more revenue than keeping both low quality and high quality sellers on the platform and distinguishing them for buyers. In the next sections we study this and other structural results in the context of general two-sided market models and information structures.

3 A Constrained Price Discrimination Problem

In the simple model of the previous section, we observed that the platform’s problem of choosing how much information to share with the buyers about the sellers’ quality transforms into a price discrimination problem with constraints on the menu that can be chosen by the platform. In this section, we study a general constrained price discrimination problem; the simple model in the
previous section is a special case. In the price discrimination problem we consider, the platform chooses a subset of price-quality pairs, i.e., a menu, from a feasible space of possible menus (referred to as the constraint set). The constraint set restricts the possible choices of menus available to the platform.

In the two-sided market models that we study in Sections 5 and 6, the constraint set is determined by the endogenously-determined equilibrium in these markets: i.e., the price-quality pairs in the menu must form an equilibrium, in the sense that the prices and qualities agree with the buyers’ and sellers’ optimal actions, and supply equals demand. Different two-sided market models generate different constraint sets. In this section, we consider a general constraint set. The platform’s problem is to choose a subset of price-quality pairs (the menu) that belongs to the constraint set in order to maximize the total transaction value, while knowing only the distribution of valuations of possible buyers. As previewed in the simple model of the previous section, in Sections 5 and 6 we will show that the platform’s information disclosure problem in our two-sided market models transforms into the constrained price discrimination problem that we study in this section.

3.1 Preliminaries

In this subsection we collect together basic concepts needed for our subsequent development.

Menus. A menu $C$ is a finite set of price-quality pairs.

Constraint set. We denote by $\mathcal{C}$ the nonempty set of all possible menus from which the platform can choose. $\mathcal{C}$ is called a constraint set.

Buyers. We assume a continuum of buyers. Given a menu, the buyers choose whether to buy a unit of the product and if so, at which price-quality pair to buy it. Each buyer has a type that determines how much they value quality relative to price. The utility of a type $m$ buyer over price-quality combinations is $mq - p$. The type distribution is given by a continuous cumulative distribution function $F$. We assume that $F$ is supported on an interval $[a, b] \subseteq \mathbb{R}_+ := [0, \infty)$.\footnote{All the results in the paper can be extended to the case that the utility of a type $m$ buyer over price-quality combinations is $z(m)q - p$ for some strictly increasing function $z$. In this case we can define the distribution function $\bar{F} := F(z^{-1})$ and our results hold when the assumptions on $F$ are replaced by the same assumptions on $\bar{F}$.}

Platform optimization problem and optimal menus. Given the constraint set $\mathcal{C}$, the platform chooses a menu $C = \{(p_1, q_1), \ldots, (p_k, q_k)\} \in \mathcal{C}$ to maximize the total transaction value, subject to the standard incentive compatibility and individual rationality constraints.

In other words, the platform chooses a menu $C \in \mathcal{C}$ to maximize:

$$\pi(C) := \sum_{(p_i, q_i) \in C} p_i D_i(C),$$

where $D_i(C)$ is the total mass of buyers that choose the price-quality pair $(p_i, q_i)$ when the platform
chooses the menu \( C \in \mathcal{C} \). That is,

\[
D_i(C) := \int_a^b 1\{m;mq_i-p_i \geq 0\}(m)1\{m;mq_i-p_i=\max_{(p_i,q_i) \in C} mq_i-p_i\}(m)F(dm),
\]

where \( 1_A \) is the indicator function of the set \( A \). A menu \( C' \in \mathcal{C} \) is called \textit{optimal} if it maximizes the total transaction value, i.e., \( C' = \arg\max_{C \in \mathcal{C}} \pi(C) \).

\( k \)-\textit{separating menus}. Let \( \mathcal{C}_p = \{ C \in \mathcal{C} : D_i(C) > 0 \text{ for all } (p_i,q_i) \in C \} \) be the set that contains all the menus \( C \) such that the mass of buyers that choose the price-quality pair \( (p_i,q_i) \) is positive for every \( (p_i,q_i) \in C \). A menu \( C = \{(p_1,q_1),\ldots,(p_k,q_k)\} \in \mathcal{C}_p \) is said to be \( k \)-\textit{separating} for a positive integer \( k \) if \( C \) contains exactly \( k \) different price-quality pairs. That is, a \( k \)-separating menu \( C \) satisfies \( |C| = k \) where \( |C| \) is the number of price-quality pairs on the menu \( C \). We let \( \mathcal{C}_1 \subseteq \mathcal{C}_p \) be the set of all \( 1 \)-separating menus. For the rest of the section, we assume without loss of generality that prices are labeled so that \( p_1 \leq p_2 \leq \ldots \leq p_k \) for every \( k \)-separating menu \( C = \{(p_1,q_1),\ldots,(p_k,q_k)\} \).

### 3.2 Optimality of 1-Separating Menus

The main result of this section (Theorem 1) shows that under certain conditions, a 1-separating menu is optimal. Translating this to the two-sided market model, it means that the platform bans a portion of the sellers and provides no further information to buyers about the quality of the remaining sellers that participate in the platform.

Our theorem shows that this result holds under two key conditions on the model, each of which is related to conditions discussed in Section 2. The first is a regularity condition that will be satisfied by a wide range of two-sided market models, including those we consider in this paper. The second is the convexity of \( F(m)m \) which relates to demand elasticities. We now discuss each condition in turn.

\textbf{Regularity.} The first condition that we introduce is regularity. This condition imposes natural restrictions on the possible equilibria that can arise in the two-sided market models. As we discussed in Section 2, the constraint set in the price discrimination problem describes the set of equilibrium menus in the two-sided market models that we study in Sections 5 and 6. Hence, the condition on the constraint set that we describe next relates to the equilibrium properties of the two-sided market models.

\textbf{Definition 1} We say that the constraint set \( \mathcal{C} \) is regular if the following two conditions hold:

(i) If \( C = \{(p_1,q_1),\ldots,(p_k,q_k)\} \in \mathcal{C}_p \) then there exists a feasible 1-separating menu \( \{(p,q)\} \in \mathcal{C}_1 \) such that \( p \geq p_k \) and \( q \geq q_k \).

\( ^4 \)If there is a subset of price-quality pairs \( C' \) such that for some type \( m \) buyer we have \( mq_i-p_i \geq 0 \) and \( mq_i-p_i = \max_{C \in C} mq_i-p_i \) for all \( (p_i,q_i) \in C' \) then we assume that the buyer chooses the price-quality pair with the highest index. This assumption does not change our analysis because we assume that \( F \) does not have atoms.

\( ^5 \)Recall that we assume without loss of generality that \( p_1 \leq p_2 \leq \ldots \leq p_k \) for every menu \( C = \{(p_1,q_1),\ldots,(p_k,q_k)\} \).
(ii) Let \( \{(p, q)\} \in C_1 \) be such that \( p \geq p' \) for all \( \{(p', q')\} \in C_1 \). Then \( p \leq p^M(q) \).\(^6\)

Condition (i) in Definition 1 can be interpreted in the two-sided market models as follows: For a feasible menu (i.e., a menu that can arise in equilibrium), banning all sellers other than the highest quality sellers in that menu increases the equilibrium price and the equilibrium quality of those sellers. This is a natural condition in markets as decreasing the supply of low quality sellers increases the equilibrium price and quality. Condition (ii) in Definition 1 means that when the platform uses a 1-separating menu, the highest equilibrium price that can arise in the two-sided market model is lower than the monopoly price. As we discussed in Section 2, this is also a natural condition because market factors such as competition and supply subsidizing suggest that the equilibrium price is lower than the monopoly price. In the two-sided market models that we study, a sufficient condition that implies condition (ii) in Definition 1 is that the supply of high quality sellers is not very low. In this case, the equilibrium price is not very high and condition (ii) holds (see Section 5). The two conditions in Definition 1 generalize the regularity condition discussed in the simple model we presented in Section 2. We believe that regularity is a mild condition over two-sided market models; hence, we think of the demand elasticity condition that we introduce next as the primary determinant of the optimality of 1-separating menus.

**Convexity of \( F(m)m \).** The second condition that we require is the convexity of \( F(m)m \). If we suppose that \( F \) has a strictly positive and continuously differentiable density \( f \), then an elementary calculation shows that \( F(m)m \) is convex if and only if:

\[
\frac{\partial f(m)}{\partial m} m = f'(m)m = \frac{f'(m)m}{f(m)} \geq -2.
\]

In words, the *elasticity* of the density function must be bounded below by \(-2\). A number of distributions satisfy this condition, e.g., power law distributions \( F(m) = d + cm^k \) for some constants \( k > 0, c, d \); beta distributions \( f(m) = \Gamma(\alpha + \beta) \Gamma(\alpha) \Gamma(\beta) m^{\alpha-1} (1-m)^{\beta-1} \) with \( \beta \leq 1 \), where \( \Gamma \) is the gamma function); and Pareto distributions \( F(m) = 1 - \left( \frac{c}{m} \right)^{\alpha} \) on \([c, \infty)\), where \( c \geq 1 \) is a constant and \( \alpha \leq 1 \). It is also worth noting that the condition that \( F(m)m \) is convex is distinct from monotonicity of the so-called *virtual value function* \( r(m) := m - (1 - F(m))/f(m) \), a condition that plays a key role in the price discrimination literature.\(^7\)

To see the dependence on the density function’s elasticity, consider a simple price discrimination setting inspired by the example of Section 2. In particular, suppose that the platform has only two price-quality pairs available: \((p_L, q_L) = (1, 1.5)\) and \((p_H, q_H) = (2, 4)\), and the platform can either choose the 1-separating menu \( \{(p_H, q_H)\} \) consisting of high quality only, or the full (2-separating)

\(^6\)Recall that given some quality \( q \), the monopoly price ignoring equilibrium conditions, \( p^M(q) \) is given by

\[
p^M(q) = \inf \arg\max_{p \geq 0} \left( 1 - F \left( \frac{p}{q} \right) \right).
\]

\(^7\)See Mussa and Rosen (1978) and Maskin and Riley (1984), and more generally the mechanism design literature (e.g., Myerson (1981)), for use of the monotonicity of the virtual valuation function. Convexity of \( F(m)m \) can be shown to be equivalent to monotonicity of the *product* of the virtual valuation with the density, \( r(m)f(m) \).
menu \{ (p_L, q_L), (p_H, q_H) \} consisting of both qualities. In Figure 2 we demonstrate the consequences of different elasticities of \( f \). In the figures in the left column, the platform chooses the full menu, the black color represents the buyers that choose not to participate in the platform, the green color represents the buyers that choose \( L \), and the red color represents the buyers that choose \( H \). In the figures in the right column, the platform chooses the 1-separating high quality menu, the black color represents the buyers that choose to not participate in the platform, and the orange color represents the buyers that choose to buy the product.

The 1-separating high quality menu yields more revenue than the full menu if and only if the area between the points \( B \) and \( C \) times \( p_H \) is greater than or equal to the area between the points \( A \) and \( C \) times \( p_L \), that is, the revenue losses from losing the participation in the platform of buyers whose valuations are between 1.5 and 2 are smaller than the revenue gains from charging the participating buyers whose valuations are between 2 and 2.5 the higher price. Intuitively, when the elasticity is lower, this difference is higher. In other words, when the elasticity is lower, the full menu is more attractive because the platform loses too much revenue when choosing the 1-separating high quality menu instead.

![Figure 2: Density functions with low and high elasticities.](image)

**Main result.** We can now state our main result using the previous two conditions. The following theorem states that our constrained price discrimination problem admits an optimal solution that is 1-separating. All the proofs in the paper are deferred to the Appendix.

**Theorem 1** Suppose that \( F(m) \) is a strictly\(^8\) convex function on \([a,b]\) and that \( C \) is regular.

\(^8\)The assumption that \( F(m) \) is strictly convex implies that the monopoly price is unique. This assumption is for mathematical convenience and does not influence the result.
Assume that the set of all 1-separating menus \( \mathcal{C}_1 \) is a compact subset of \( \mathbb{R}^2 \). Then there is an optimal 1-separating menu. In addition, the optimal 1-separating menu \( \{(p,q)\} \) is maximal in \( \mathcal{C}_1 \): for every \( \{(p',q')\} \in \mathcal{C}_1 \) such that \((p',q') \neq (p,q)\) we have \( p > p' \) or \( q > q' \).

In the Appendix we also show that we can slightly weaken the regularity condition.

We note that if for every menu \( C = \{(p_1,q_1),\ldots,(p_k,q_k)\} \) that belongs to \( \mathcal{C} \), the 1-separating menu \( C' = \{p_k,q_k\} \) belongs to \( \mathcal{C} \) then the second condition in Definition 1 is not needed in order to prove the optimality of a 1-separating menu. The proof of this follows immediately from the proof of Theorem 1. The intuition for this result follows from the argument in Section 2 that shows that the second stage of the analysis of the example provided there is not needed when such menu \( C' \) belongs to \( \mathcal{C} \). As we discussed in Section 2, this is useful for the two-sided market model where sellers compete in a Bertrand competition (see Section 6). We use the next Corollary to prove the optimality of a 1-separating menu in that model.

**Corollary 1** Suppose that \( F(m)m \) is a convex function on \([a,b]\) and that for every menu \( C = \{(p_1,q_1),\ldots,(p_k,q_k)\} \in \mathcal{C} \) we have \( C' = \{p_k,q_k\} \in \mathcal{C} \). Assume that the set of all 1-separating menus \( \mathcal{C}_1 \in \mathcal{C} \) is compact. Then there is an optimal 1-separating menu.

Corollary 1 can be applied for some important constraint sets as the following example shows.

**Example 1** (i) In this example, the platform can choose any subset of price-quality pairs from a pre-fixed set of price-quality pairs. Suppose that there is a given set \( \mathcal{P} \) of \( R \) price-quality pairs, \( \mathcal{P} = \{(p_1,q_1),\ldots,(p_R,q_R)\} \). Then the constraint set is \( \mathcal{C}_\mathcal{P} = 2^\mathcal{P} \) where \( 2^\mathcal{X} \) is the set of all subsets of a set \( \mathcal{X} \).

(ii) In this example, the platform can choose any finite string \( (p_1,q_1),\ldots,(p_k,q_k) \) in \( \mathbb{R}^{2k} \) for \( k \leq N \) where \( N \geq 1 \), \( p_i \in [0,\bar{p}] \) and \( q_i \in [0,\bar{q}] \) for all \( 1 \leq i \leq k \). That is, the constraint set is given by \( \mathcal{C}_N = \{C : C \text{ is a } k \text{-separating menu for } k \leq N \text{ such that } (p,q) \in [0,\bar{p}] \times [0,\bar{q}] \text{ for all } (p,q) \in C\} \).

In the two-sided market model in Section 6, the constraint set that the platform faces is the same as the constraint set in Example 1 part (i) (see Theorem 3). The constraint set in Example 1 part (ii) is standard in the price discrimination literature (see for example Bergemann et al. (2011)).

We now discuss two additional results that expand on Theorem 1. First, the following corollary shows that for some menus \( C \in \mathcal{C} \), it is enough to show that the function \( F(m)m \) is convex on a subset of \([a,b]\) in order to prove that there exists a 1-separating menu that yields more total transaction value than the menu \( C \). Thus, the menu that maximizes the total transaction value can still be 1-separating for a distribution function that is convex on a subset of the distribution’s support.

For a \( k \)-separating menu \( C = \{(p_1,q_1),\ldots,(p_k,q_k)\} \in \mathcal{C}_\mathcal{P} \), let \( m_i(C) = (p_i - p_{i-1}) / (q_i - q_{i-1}) \) for \( i = 1,\ldots,k \) where \( p_0 = q_0 = 0 \). Corollary 2 follows immediately from the proof of Theorem 1.

\(^9\)In the two-sided market models that we study the constraint set is finite, and hence, \( \mathcal{C}_1 \) is compact.
Corollary 2 Let $C = \{(p_1, q_1), \ldots, (p_k, q_k)\} \in \mathcal{C}_p$ be a k-separating menu where $p_i < p_j$ if $i < j$. Suppose that $F(m)m$ is convex on $[m_1(C), m_k(C)]$ and that $C$ is regular. Then there exists a 1-separating menu $C^*$ that yields more revenue than $C$, i.e., $\pi(C) \leq \pi(C^*)$.

In addition, if $F(m)m$ is convex on $[m_1(C), m_k(C)]$ for every menu and $C_1$ is compact, then there is a 1-separating menu that maximizes the total transaction value.

We can also show that when the function $F(m)m$ is not convex, we can find a constraint set $C$ that satisfies the condition of Corollary 1 such that no 1-separating menu exists that maximizes the total transaction value. In particular, we can find a simple constraint set $C = 2^C$ where $C = \{(p_1, q_1), (p_2, q_2)\} \in \mathbb{R}$ (see Example 1 part (i)), for which a 1-separating menu is not optimal.

Proposition 1 Suppose that $F(m)m$ is not convex on $(a, b)$. Then there exists a menu $C = \{(p_1, q_1), (p_2, q_2)\}$ and a constraint set $C = 2^C$ such that the menu $C \in C$ maximizes the total transaction value and yields strictly more revenue than any 1-separating menu in $C$.

When $F(m)m$ is not convex on $(a, b)$, Proposition 1 shows that we can construct a constraint set where a 2-separating menu yields more total transaction value than any 1-separating menu. Similarly, when $F(m)m$ is not concave on $(a, b)$, we can construct a constraint set where a 1-separating menu yields more total transaction value than any 2-separating menu. Thus, in the case that $F(m)m$ is not convex or concave everywhere on $[a, b]$, a general characterization of the optimal menu for an arbitrary constraint set is not achievable. However, in the next subsection we derive some positive results that depend only on local convexity properties.

3.3 Local Results

In practice, because of operational considerations or other constraints, a platform might only consider a small number of options. For example, an e-commerce platform can introduce a new top rated sellers category or remove an existing category. In this section we show that our main result holds also locally. That is, the values of the density function’s elasticity on some local region remain the key condition when deciding which option will yield more total transaction value.

For simplicity, suppose that the platform considers only two menus $C = \{(p_1, q_1), \ldots, (p_n, q_n)\} \in \mathcal{C}_p$ and $C' = C \setminus \{(p_1, q_1)\}$ where $p_i < p_j, q_i < q_j$ if $i < j$. In our two sided-market model where sellers choose prices, the menu $C'$ is feasible and can be obtained from the menu $C$ by banning some low quality sellers (see Section 6). The platform does not seek to find the optimal menu across all menus but only to determine which menu yields more total transaction value: $C$ or $C'$. In Proposition 2 we show that the menu $C$ yields lower (higher) total transaction value than the menu $C'$ if the density function’s elasticity is bounded below (above) by $-2$ on the interval $A := [p_1/q_1, (p_2-p_1)/(q_2-q_1)]$.

Note that $C \in \mathcal{C}_p$ implies $m_i(C) < m_j(C)$ for $i < j$ and that $[m_1(C), m_k(C)] \subseteq [a, b]$ (see the proof of Theorem 1).
Proposition 2 Let \( C = \{(p_1, q_1), \ldots, (p_n, q_n)\} \in C_p \) and let \( C' = C \setminus \{(p_1, q_1)\} \). Assume without loss of generality that \( p_i < p_j \) whenever \( i < j \).

Then, \( \pi(C) \leq \pi(C') \) if \( F(m)m \) is convex on \([p_1/q_1, (p_2 - p_1)/(q_2 - q_1)]\) and \( \pi(C) \geq \pi(C') \) if \( F(m)m \) is concave on \([p_1/q_1, (p_2 - p_1)/(q_2 - q_1)]\).

We can obtain some intuition for the preceding result as follows. A type \( m \) buyer chooses the price-quality pair \((p_1, q_1)\) under the menu \( C \) if and only if \( m \in A \). Thus, in order to compare \( C \) and \( C' \), the density function’s elasticity must be bounded below or above \(-2\) on the set of buyers’ types that choose the price-quality pair \((p_1, q_1)\). Further, the elasticity of many standard density functions is decreasing. In such a case, we can check the density function’s elasticity at just one point to determine which menu yields more total transaction value: \( C \) or \( C' \) (see more details in Section 6).

In the Appendix we prove a general version of Proposition 2 (see Proposition 4). We compare any two menus \( C \) and \( C' \) such that \( C' \in 2^C \) where \( 2^C \) is the power set of \( C \). In the two-sided market model the menu \( C' \) can be obtained by removing some sellers from the platform (not necessarily the lowest quality sellers). We show that \( C' \) yields more (less) total transaction value than \( C \) under convexity (concavity) of \( F(m)m \) on a certain relevant local region.

A similar “local” analysis can be applied to the two-sided market model where sellers choose quantities (see Section 5) but this requires additional conditions on the set feasible menus. These conditions are similar to the regularity condition (see Definition 1).

4 Information Structures

Having described our constrained price discrimination problem, we are now in a position to describe how we apply that framework to design information disclosure policies in two-sided markets. We begin in this section by describing the information the platform has about the sellers’ quality levels and the set of information structures from which the platform can choose.

Seller quality. Let \( X \) be the set of possible sellers’ quality levels. We assume that \( X \) is the interval\(^{11}\) \([0, \overline{x}]\) for some \( \overline{x} > 0 \). We denote by \( B(X) \) the Borel sigma-algebra on \( X \) and by \( P(X) \) the space of all Borel probability measures on \( X \). The distribution of the sellers’ quality levels is described by a probability measure \( \phi \in P(X) \).

Platform’s information. The platform’s information is summarized by a finite (measurable) partition \( I_o = \{A_1, \ldots, A_t\} \) of \( X \). We assume that \( \phi(A_i) > 0 \) for all \( A_i \in I_o \). The platform has no information about the sellers’ quality levels if \( |I_o| = 1 \) where \( |I_o| \) is the number of elements in the partition \( I_o \).

Information structures. Given the platform’s information \( I_o \), the platform chooses an information structure to share with buyers. We now define an information structure.

**Definition 2** An information structure \( I \) is a family of disjoint sets such that every set in \( I \) is a union of sets in \( I_o \), i.e., \( B \in I \) implies \( \cup_i A_i = B \) for some sets \( A_i \in I_o \).

\(^{11}\)All our results can be easily generalized for the case that \( X \) is any compact set in \( \mathbb{R}_+^n \).
While the class of information structures we study is relatively simple, it provides enough richness for our analysis. An interesting direction for future work is to expand our analysis to other information structures. We now provide examples of information structures.

**Example 2** Suppose that \( X = [0, 1] \), \( I_0 = \{A_1, A_2, A_3, A_4\} \), \( A_j = [0.25(j-1), 0.25j] \), \( j = 1, \ldots, 4 \).

Two examples of information structures are the information structure \( I_1 = \{A_3, A_4\} \)

and the information structure \( I_2 = \{A_3 \cup A_4\} \)

In the information structure \( I_1 \), the sellers whose quality levels belong to the sets \( A_1 \) and \( A_2 \) are “banned” from the platform, and the sellers whose quality levels belong to the sets \( A_3 \) and \( A_4 \) can participate in the platform. The platform shares the information it has about the sellers whose quality levels belong to the sets \( A_3 \) and \( A_4 \), i.e., the buyers know that the quality level of a seller in the set \( A_4 \) is between 0.75 and 1, and the quality level of a seller in the set \( A_3 \) is between 0.5 and 0.75. In the information structure \( I_2 \), the sellers whose quality levels belong to the sets \( A_1 \) and \( A_2 \) are banned from the platform and the platform does not share the information it has about the other sellers. Hence, buyers cannot distinguish between sellers in \( A_3 \) and \( A_4 \).

Note that the platform’s information structure \( I = \{B_1, \ldots, B_n\} \) determines both which sellers are banned from the platform (in particular, sellers in \( X \setminus \cup_{B_i \in I} B_i \) are banned from the platform), as well as the amount of information that the platform shares with buyers regarding the sellers that participate in the platform.

Given an information structure \( I \), we define the measure space \( \Omega_I = (X, \sigma(I)) \) where \( \sigma(I) \) is the sigma-algebra generated by \( I \). Recall that a function \( p : (X, \sigma(I)) \to \mathbb{R} \) is \( \sigma(I) \) measurable if and only if \( p \) is constant on each element of \( I \), i.e., \( x_1, x_2 \in B \) and \( B \in I \) imply that \( p(x_1) = p(x_2) \) := \( p(B) \).

Given the platform’s initial information on the sellers’ quality levels \( I_0 \), we denote by \( \mathbb{I}(I_0) \) the set of all possible information structures.

**k-separating information structures.** We say that an information structure \( I \) is \( k \)-separating if \( I \) contains exactly \( k \) elements, i.e., \( |I| = k \). For example, the information structure \( I_1 \) described in Example 2 is 2-separating and the information structure \( I_2 \) is 1-separating.

\(^{12}\)Note that equilibrium conditions will be required to fully specify buyers’ beliefs on seller quality within each element of the information structure.
4.1 Remarks On The Assumptions

We now provide a few remarks on our assumptions.

**Exogenous quality.** In our two-sided market models we assume that sellers choose quantities or prices while their qualities are their types. In some platforms sellers can choose or improve their quality. In those cases, the sellers’ types can be their opportunity cost, investment cost, or another feature. In principle, we could incorporate this into our model and the transformation to a constrained price discrimination problem would still hold. However, the set of feasible menus (equilibrium menus) is determined by the specific two-sided market model we study and by the market arrangement. Hence, the set of feasible menus would be different and harder to characterize when sellers can also choose their quality.

**The platform’s initial information.** As we discussed in the introduction, platforms collect information about the sellers’ quality from many sources. In this paper we abstract away from the data collection process and assume that the platform has already collected some information about the sellers’ quality and classified the sellers’ quality (the partition $I_o$ represents this classification). We focus on how much of this information the platform should share with buyers to maximize its revenues. An interesting future research direction is to incorporate dynamic considerations that are related to learning, such as learning the sellers’ quality, into our framework.

**Information structures.** The information structures available to the platform in our model are more limited than the information structures available to the platform (sender) in the standard information design literature. For example, we do not allow the platform to use a mixed strategy (i.e., mix over sets in the platform’s initial information $I_o$). Allowing for mixed strategies would actually simplify our analysis as is typically the case in the information design literature. However, in our context of quality selection we think that platform’s pure strategies are more realistic. Also, because we assume that the platform’s information about the sellers’ quality is partial and is given by a finite partition, every information structure that the platform can choose as well as the set of possible information structures that the platform can choose from are finite. The analysis of the constrained price discrimination problem in Section 3 shows that our framework can be generalized to the case of uncountable information structures.

5 Two-Sided Market Model 1: Sellers Choose Quantities

In this section we consider a model in which the platform chooses the prices, and the sellers choose the quantities.

The platform chooses an information structure $I \in \mathcal{I}(I_o)$ and a $\sigma(I)$ measurable pricing function $p$. The measurability of the pricing function means that if the platform does not reveal any information about the quality of two sellers, i.e., the two sellers belong to the same set $B$ in the information structure $I$, then these sellers are given the same price under the platform’s pricing function. The measurability condition is natural because the buyers do not have any information on the sellers’ quality except the information provided by the platform, so any rational buyer will
not buy from a seller \( x \) whose price is higher than a seller \( y \) when \( x \) and \( y \) have the same expected quality.

With slight abuse of notation, for an information structure \( I = \{B_1, \ldots, B_n\} \), we denote a \( \sigma(I) \) measurable pricing function by \( p = (p(B_1), \ldots, p(B_n)) \) where \( p(B_i) \) is the price that every seller \( x \) in \( B_i \) charges. A pricing function \( p = (p(B_1), \ldots, p(B_n)) \) is said to be positive if \( p(B_i) > 0 \) for all \( B_i \in I \).

An information structure \( I = \{B_1, \ldots, B_n\} \) and a pricing function \( p \) generate a game between the sellers and the buyers. The platform’s decisions and the structure of the game are common knowledge at the start of the game. In the game, the sellers choose quantities, and the buyers choose whether to buy a product and if so, from which set of sellers \( B_i \in I \) to buy it. Each equilibrium of the game induces a certain revenue for the platform. The platform’s goal is to choose an information structure and prices that maximize the platform’s equilibrium revenue. We now describe the buyers’ and sellers’ decisions in detail.

5.1 Buyers

Buyers are heterogeneous in how much they value the quality of the product relative to its price; in particular, every buyer has a type in \([a, b] \subseteq \mathbb{R}_+ := [0, \infty)\), with buyers’ types distributed according to the probability distribution function \( F \) on \([a, b]\), with continuous probability density function \( f \). The buyers do not know the sellers’ quality levels, but they know the information structure \( I = \{B_1, \ldots, B_n\} \) and the pricing function \( p \) that the platform has chosen.

The buyers choose whether to buy a product and if so, from which set of sellers \( B_i \in I \) to buy it. A type \( m \in [a, b] \) buyer’s utility from buying a product from a type \( x \in B_i \) seller is given by

\[
Z(m, B_i, p(B_i)) = m \lambda_{B_i} - p(B_i).
\]

The probability measure \( \lambda_{B_i} \) describes the buyers’ beliefs about the quality levels of sellers in the set \( B_i \), and \( \lambda_{B_i} \) is the seller’s expected quality given the buyers’ beliefs \( \lambda_{B_i} \).\(^{14}\) In equilibrium, the buyers’ beliefs are consistent with the sellers’ quantity decisions and with Bayesian updating.

A type \( m \) buyer buys a product from a type \( x \in B_i \) seller if \( Z(m, B_i, p(B_i)) \geq 0 \) and \( Z(m, B_i, p(B_i)) = \max_{B_j \in I} Z(m, B_j, p(B)) \), and does not buy it otherwise.\(^{15}\) The total demand in the market for products sold by type \( x \in B_i \) sellers given the information structure \( I \) and the pricing function \( p \), \( D_I(B_i, p) \) is given by

\[
D_I(B_i, p) = \int_a^b 1_{\{Z(m, B_i, p(B_i)) \geq 0\}} 1_{\{Z(m, B_i, p(B_i)) = \max_{B_j \in I} Z(m, B_j, p(B))\}} F(dm).
\]

---

\(^{13}\)Here quantities can correspond, for example, to how many hours the sellers choose to work.

\(^{14}\)All of our results hold if a type \( m \in [a, b] \) buyer’s utility is given by \( Z(m, B_i, p(B_i)) = mv(\lambda_{B_i}) - p(B_i) \) for some function \( v : \mathcal{P}(X) \rightarrow \mathbb{R}_+ \) that is increasing with respect to stochastic dominance. For example, the function \( v \) can capture buyers’ risk aversion.

\(^{15}\)If there are multiple sets \( \{B_i\}_{i \in \mathcal{I}} \) such that for some type \( m \) buyer we have \( Z(m, B_i, p(B_i)) \geq 0 \) and \( Z(m, B_j, p(B_j)) = \max_{B_k \in I} Z(m, B_k, p(B)) \), then we break ties by assuming that the buyer chooses to buy from the set of sellers with the highest index, i.e., \( \max_{i \in \{i \mid B_i \in \mathcal{I}\}} i \).
5.2 Sellers

Given the information structure $I$ and the pricing function $p$, a type $x \in B_i \subseteq X$ seller’s utility is given by

$$U(x, h, p(B_i)) = hp(B_i) - k(x)h^{\alpha+1}/(\alpha + 1).$$

Each seller chooses a quantity $h \in \mathbb{R}_+$ in order to maximize their utility. For a type $x$ seller, the cost of producing $h$ units is given by $k(x)h^{\alpha+1}/(\alpha + 1)$. The seller’s cost function depends on their type and on the quantity that they sell. We assume that $k$ is measurable and is bounded below by a positive number. We also assume that the cost of producing $h$ units is strictly convex in the quantity, i.e., $\alpha > 0$. This cost structure is quite general and simplifies the characterization of the constraint set, i.e., the set of equilibrium menus (see Proposition 3 and Lemma 1 in the Appendix) but showing that the constraint set is regular can be done under more general cost structures.

Let $g(x, p(B_i)) = \arg\max_{h \in \mathbb{R}_+} U(x, h, p(B_i))$ be the quantity that a type $x \in B_i$ seller chooses when the pricing function is $p = (p(B_1), \ldots, p(B_n))$. Note that $g$ is single-valued because $U$ is strictly convex in $h$. Let

$$S_I(B_i, p(B_i)) = \int_{B_i} g(x, p(B_i)) \phi(dx)$$

be the total supply in the market of sellers with types $x \in B_i$.

5.3 Equilibrium

Given the information structure and the pricing function that the platform chooses, there are four equilibrium requirements. First, the sellers choose quantities in order to maximize their utility. Second, the buyers choose whether to buy a product and if so, from which set of sellers to buy it in order to maximize their own utility. Third, the buyers’ beliefs about the sellers’ quality are consistent with Bayesian updating and with the sellers’ actions. Fourth, demand equals supply for each set $B_i$ that belongs to the information structure. We now define an equilibrium formally.

**Definition 3** Given an information structure $I = \{B_1, \ldots, B_n\}$ and a positive pricing function $p = (p(B_1), \ldots, p(B_n))$, an equilibrium is given by the buyers’ demand $\{D_I(B_i, p)\}_{i=1}^n$, sellers’ supply $\{S_I(B_i, p(B_i))\}_{i=1}^n$, and buyers’ beliefs $\{\lambda_{B_i}\}_{i=1}^n$ that satisfy the following conditions:

(i) Sellers’ optimality: The sellers’ decisions are optimal. That is,

$$g(x, p(B_i)) = \arg\max_{h \in \mathbb{R}_+} U(x, h, p(B_i))$$

is the optimal quantity for each seller $x \in B_i \in I$.

(ii) Buyers’ optimality: The buyers’ decisions are optimal. That is, for each buyer $m \in [a, b]$ that buys from type $x \in B_i$ sellers, we have $Z(m, B_i, p(B_i)) \geq 0$ and $Z(m, B_i, p(B_i)) = \max_{B_i \in I} Z(m, B, p(B))$.

(iii) Rational expectations: $\lambda_{B_i}(A)$ is the probability that a buyer is matched to sellers whose
quality levels belong to the set $A$ given the sellers’ optimal decisions, i.e.,

$$\lambda_{B_i}(A) = \frac{\int_A g(x, p(B_i)) \phi(dx)}{\int_{B_i} g(x, p(B_i)) \phi(dx)}$$

(2)

for all $B_i \in I$ and for all measurable sets $A \subseteq B_i$.

(iv) Market clearing: For all $B_i \in I$ the total supply equals the total demand, i.e.,

$$S_I(B_i, p(B_i)) = D_I(B_i, p)$$

where $D_I(B_i, p)$ and $S_I(B_i, p(B_i))$ are defined in Sections 5.1 and 5.2 respectively.

The equilibrium requirements limit the platform’s ability to design the market. The buyers’ beliefs about the expected sellers’ quality depends on the sellers’ quantity decisions, which the platform cannot control. Thus, the platform’s ability to influence the buyers’ beliefs by choosing an information structure is constrained. Furthermore, the prices and the expected sellers’ qualities must form an equilibrium (i.e., supply equals demand) in each set of the information structure. This equilibrium requirement is in addition to the more standard requirement in the market design literature that the buyers’ and sellers’ decisions are optimal. Hence, the platform cannot implement every pair of an information structure and pricing function. This motivates the following definition.

**Definition 4** An information structure and pricing function pair $(I, p)$ is called implementable if there exists an equilibrium $(D, S, \lambda)$ under $(I, p)$ where $D = \{D_I(B_i, p)\}_{B_i \in I}$, $S = \{S(B_i, p(B_i))\}_{B_i \in I}$, and $\lambda = \{\lambda_{B_i}\}_{B_i \in I}$. We say that $(D, S, \lambda)$ implements $(I, p)$ if $(D, S, \lambda)$ is an equilibrium under $(I, p)$.

We denote by $\mathcal{W}_Q$ the set of all implementable pairs of an information structure and pricing function $(I, p)$. The platform’s goal is to choose an information structure $I = \{B_1, \ldots, B_n\}$ and a pricing function $p$ that maximize the total transaction value $\pi^Q$ given by

$$\pi^Q(I, p) := \sum_{B_i \in I} p(B_i) \min\{D_I(B_i, p), S_I(B_i, p(B_i))\}$$

under the constraint that $(I, p)$ is implementable. That is, the platform’s revenue maximization problem is given by $\max_{(I, p) \in \mathcal{W}_Q} \pi^Q(I, p)$.

5.4 Equivalence with Constrained Price Discrimination

The main motivation for studying the constrained price discrimination problem that we analyzed in Section 3 is that the platform’s revenue maximization problem described above trans-
forms into this constrained price discrimination problem. To see this, let \((I, p)\) be an information structure-pricing function pair where \(I = \{B_1, B_2, \ldots, B_n\}\) and \(p = (p(B_1), \ldots, p(B_n))\). Let \(D = \{D_I(B_i, p)\}_{B_i \in I}\), \(S = \{S_I(B_i, p(B_i))\}_{B_i \in I}\), and \(\lambda = \{\lambda_{B_i}\}_{B_i \in I}\) be an equilibrium under \((I, p)\). Then \((I, p)\) induces a subset of price-expected quality pairs \(C\). The menu \(C\) is given by \(C = \{(p(B_1), \mathbb{E}_{\lambda_{B_1}}(X)), \ldots, (p(B_n), \mathbb{E}_{\lambda_{B_n}}(X))\}\) where \(\mathbb{E}_{\lambda_{B_i}}(X)\) is the equilibrium expected quality of the sellers that belong to the set \(B_i\).

Denoting, \(q_i := \mathbb{E}_{\lambda_{B_i}}(X)\), the menu \(C\) yields the total transaction value

\[
\pi(C) := \sum_{(p_i, q_i) \in C} p_i D_i(C) \\
= \sum_{B_i \in I} p(B_i) D_I(B_i, p) \\
= \sum_{B_i \in I} p(B_i) \min\{D_I(B_i, p), S_I(B_i, p(B_i))\} \\
= \pi^Q(I, p).
\]

The first equality follows from the definition of \(\pi\) (see Section 3). The third equality follows from the fact that \((I, p)\) is implementable. We conclude that the implementable information structure-pricing function pair \((I, p)\) yields the same revenue as the menu \(C\) that it induces.

We denote by \(C^Q\) the set of all menus \(C\) that are induced by some implementable \((I, p) \in \mathcal{W}^Q\). With this notation, the platform’s revenue maximization problem is equivalent to the constrained price discrimination problem of choosing a menu \(C \in C^Q\) to maximize \(\sum p_i D_i(C)\) that we studied in Section 3. That is, we have \(\max_{(I, p) \in \mathcal{W}^Q} \pi^Q(I, p) = \max_{C \in C^Q} \pi(C)\).

An information structure is optimal if it induces a menu that maximizes the platform’s revenue. The next subsection studies optimal information structures in this model, leveraging the equivalence with the constrained price discrimination problem.

### 5.5 Results

In this section we present our main results regarding the two-sided market model where the sellers choose quantities and the platform choose prices.

Note that if \((I, p)\) induces the menu \(C\) and \(I\) is a \(k\)-separating information structure, then \(C\) is a \(k\)-separating menu. We let \(C^Q_k \subseteq C^Q\) be the set of \(k\)-separating menus. From the fact that the platform’s revenue maximization problem transforms into the constrained price discrimination problem, Theorem 1 implies that if \(C^Q\) is regular and \(F(m)m\) is convex, then the optimal information structure is 1-separating, i.e., the optimal information structure consists of one element. In this subsection, we establish certain natural conditions on the market model primitives that ensure regularity; these conditions then imply that if in addition \(mF(m)\) is convex, then a 1-separating information structure is optimal.

Let \(\varphi^Q : \mathcal{I}(I_o) \rightrightarrows C^Q\) be the set-valued mapping from the set \(\mathcal{I}(I_o)\) of all possible information
structures to the set of menus $C^Q$ such that $C \in \varphi^Q(I)$ if and only if $C$ is a menu that is induced by some implementable $(I, p)$. That is, $\varphi^Q(I)$ contains all the menus that can be induced when the platform uses the information structure $I$. We note that the mapping $\varphi^Q$ is generally complicated and there is no simple characterization of this mapping. However, we make substantial progress via the following proposition. In particular, it can be shown that associated to every information structure $I$ there is a strictly convex program over the space of pricing functions $p$, such that $(I, p)$ is implementable if and only if the solution to the program is $p$. Since every strictly convex program has at most one solution, this result also implies that the cardinality of $\varphi^Q(I)$ is at most one; in other words, there is no more than one menu $C$ such that $C \in \varphi^Q(I)$.

**Proposition 3** For every information structure $I \in \Pi(I_o)$, there exists a strictly convex program over pricing functions such that $(I, p)$ is implementable if and only if the solution to the program is $p$. Therefore, there is at most one menu $C$ such that $C \in \varphi^Q(I)$.

To construct the claimed convex program in the preceding proposition, for every information structure $I = \{B_1, \ldots, B_n\}$ we define an associated excess supply function. We show that the excess supply function satisfies the law of supply, i.e., the excess supply function is strictly monotone$^{18}$ on a convex and open set $P \subseteq \mathbb{R}^n$ such that if $p$ is an equilibrium price vector then $p \in P$. The excess supply function is the gradient of some function $\psi$. Thus, minimizing $\psi$ over $P$ is a strictly convex program that has a solution (minimizer) if and only if the solution is a zero of the excess supply function, i.e., an equilibrium price vector. The result is helpful because it introduces a tractable convex program that for a given information structure provides an implementable price vector as its solution.

In the remainder of this subsection, we establish conditions for regularity of the space of menus induced under $\varphi^Q$; these conditions are analogous to those discussed for the simple model in Section 2. First, note that in the Appendix we prove Lemma 1 that states that given an information structure, the sellers’ expected qualities do not depend on the prices as long as the prices are positive. This follows from the sellers’ cost functions which imply that the sellers’ optimal quantity decisions are homogeneous in the prices. We assume for the rest of the section that $\sum_{i=1}^n x_i y_i$ denotes the standard inner product between two vectors $x$ and $y$ in $\mathbb{R}^n$.  

$^{18}$A function $\zeta: P \rightarrow \mathbb{R}^n$ is strictly monotone on $P$ if for all $p = (p_1, \ldots, p_n)$ and $p' = (p'_1, \ldots, p'_n)$ that belong to $P$ and satisfy $p \neq p'$, we have

$\langle \zeta(p) - \zeta(p'), p - p' \rangle > 0$

where $\langle x, y \rangle := \sum_{i=1}^n x_i y_i$ denotes the standard inner product between two vectors $x$ and $y$ in $\mathbb{R}^n$.  

23
\{(p(B^H), \mathbb{E}_{\lambda_B}(X))\} \in \varphi^Q(\{B^H\}) \) and \{(p(B), \mathbb{E}_{\lambda_B}(X))\} \in \varphi^Q(\{B\}) imply \( p(B^H) \geq p(B) \) for every 1-separating information structure \( \{B\} \) such that \( \{B\} \in \{\{A_1\}, \ldots, \{A_l\}\} \).

Theorem 2 shows that if

\[
S_{\{B^H\}}(B^H, p^M(B^H)) \geq D_{\{B^H\}}(B^H, p^M(B^H))
\]

and \( F(m)m \) is strictly convex, then the optimal information structure is 1-separating. Inequality (3) says that under the information structure \( \{B^H\} \) and the price \( p^M(B^H) \), the supply exceeds the demand. This implies that under the information structure \( \{B^H\} \), the equilibrium price is lower than the optimal monopoly price that maximizes the platform’s revenue, similarly to the condition discussed in Section 2. Hence, inequality (3) implies condition (ii) of the regularity definition (see Definition 1) holds. In order to prove that the optimal information structure is 1-separating we show that condition (i) of the regularity definition also holds, and hence, the set of equilibrium menus \( C^Q \) is regular. As we discussed in Section 3, condition (i) means that removing low quality sellers increases the equilibrium price for high quality sellers. This is a natural condition in the context of two-sided market models. In the two-sided market model that we study in this Section we show that condition (i) holds without any further assumptions on the model’s primitives. Thus, under the mild condition that ensures that the supply of high quality sellers is not too low (inequality (3)), we can apply Theorem 1 to prove that the optimal information structure is 1-separating under the convexity of \( F(m)m \).

**Theorem 2** Assume that \( F(m)m \) is strictly convex on \([a, b]\). Assume that inequality (3) holds. Then,

(i) The set \( C^Q \) is regular.

(ii) There exists a 1-separating information structure \( I^* \) such that

\[
(I^*, p^*) = \arg\max_{(I, p) \in W^Q} \pi^Q(I, p).
\]

That is, there exists a 1-separating information structure \( I^* \) that maximizes the platform’s revenue.

(iii) The pair \((I^*, p^*)\) induces a menu that is maximal in \( C^Q_1 \) and \( B^* \in I_0 = \{A_1, \ldots, A_l\} \) where \( I^* = \{B^*\} \) is the optimal information structure.\(^{19}\)

Theorem 2 shows that there exists a unique equilibrium price \( p^{eq}(A_j) \) that the platform can induce when it chooses the 1-separating information structure \( I = \{A_j\} \), i.e., \( \varphi^Q(I) \) is single-valued when \( I \) is a 1-separating information structure. Further, under the natural condition that the equilibrium price is increasing in the sellers’ quality, i.e., \( p^{eq}(A_j) \leq p^{eq}(A_k) \) whenever \( \mathbb{E}_{\lambda_B}(X) < \mathbb{E}_{\lambda_B}(X) \), it is simple to show that there exists only one information structure-price pair \((\{A_j\}, p^{eq}(A_j))\) that induces a maximal menu in \( C^Q_1 \). Hence, in this case, Theorem 2 implies

\(^{19}\)Recall that a menu \( \{(p, q)\} \in C^Q_1 \) is maximal in \( C^Q_1 \) if for every menu \( \{(p', q')\} \in C^Q_1 \) such that \( (p', q') \neq (p, q) \) we have \( p > p' \) or \( q > q' \).
that the optimal 1-separating information structure is \( \{A_l\} \). That is, banning all sellers except the highest quality sellers is optimal for the platform.

Checking if inequality (3) holds is straightforward given the model’s primitives. The following example illustrates that inequality (3) holds if the sellers’ costs in \( B^H \) are low enough and/or the size of the supplier set \( B^H \) is large enough. We note that if we introduce transfers or subsidies for each side of the market then the platform can always charge buyers and pay sellers in a way that inequality (3) holds and the subsidies do not influence the platform’s revenue.

**Example 3** Suppose that \( F(m) \) is the uniform distribution on \([0, 1] \), i.e., \( F(m) = m \) on \([0, 1] \). Assume also that \( \alpha = 1 \). A direct calculation shows that \( p^M(B) = \frac{E_{\lambda^H}(X)}{2} \). Hence, inequality (3) holds if and only if

\[
1 - \frac{p^M(B^H)}{E_{\lambda^H}(X)} \leq p^M(B^H) \int_{B^H} k(x)^{-1}\phi(dx) \iff 1 \leq \int_{B^H} xk(x)^{-1}\phi(dx)
\]

where we use the fact that \( E_{\lambda^H}(X) \int_{B^H} k(x)^{-1}\phi(dx) = \int_{B^H} xk(x)^{-1}\phi(dx) \) (see Lemma 1 in the Appendix). Thus, the size of the set \( B^H \), the sellers’ qualities in \( B^H \), and the sellers’ costs in \( B^H \) determine whether inequality (3) holds. In order to determine the information structure \( \{B^H\} \) with the highest equilibrium price we can solve for the equilibrium price:

\[
1 - \frac{p^{eq}(B)}{E_{\lambda^H}(X)} = p^{eq}(B) \int_{B} k(x)^{-1}\phi(dx) \iff p^{eq}(B) = \frac{\int_{B} xk(x)^{-1}\phi(dx)}{\int_{B} k(x)^{-1}\phi(dx)(1 + \int_{B} xk(x)^{-1}\phi(dx))}
\]

and choose the set \( B \in \{A_1, \ldots, A_l\} \) with the highest equilibrium price.

When the support of \( F \) is unbounded it can be the case that inequality (3) trivially holds because the supply under the price that maximizes the platform’s revenue tends to infinity. For example, suppose that \( F \) has the Pareto distribution, i.e., \( F(m) = 1 - \frac{1}{m^\beta} \) on \([1, \infty) \). Then \( F(m)m \) is convex for \( \beta < 1 \). In this case, the support of \( F \) is unbounded so \( p^M \) is not necessarily well defined. Indeed, for every \( q > 0 \) we have

\[
\lim_{p \to \infty} p \left( 1 - F \left( \frac{p}{q} \right) \right) = \lim_{p \to \infty} p \left( \frac{q^\beta}{p^\beta} \right) = \infty.
\]

Thus, the price that maximizes the platform’s revenue tends to infinity which means that the supply under this price tends to infinity and inequality (3) trivially holds.

### 6 Two-Sided Market Model 2: Sellers Choose Prices

In this section we consider a model in which the sellers choose the prices and the quantities are determined in equilibrium.

The platform chooses an information structure \( I \in I_o \) (see Section 4). An information structure generates a game between buyers and sellers. In this game, sellers make entry decisions first. After
the entry decisions, in each set of sellers that belongs to the information structure, the participating
sellers engage in Bertrand competition. Buyers form beliefs about the sellers’ quality and choose
whether to buy a product and if so, from which set of sellers to buy it.

Each equilibrium of the game induces a certain revenue for the platform. The platform’s goal
is to choose the information structure that maximizes the platform’s equilibrium revenue. We now
describe the sellers’ and buyers’ decisions in detail.

6.1 Buyers

In this section we describe the buyers’ decisions. The buyers make their decisions after the sellers’
entry and pricing decisions have been made. We denote by \( H(B_i) \subseteq B_i \) the set of quality \( x \in B_i \)
sellers that participate in the platform and by \( p_x \) the price that a quality \( x \in \cup_{B_i \in I} H(B_i) \) seller
charges.

As in Section 5.1, the buyers’ heterogeneity is described by a type space \([a, b] \subset \mathbb{R}_+\), and buyers’
types are distributed according to a probability distribution function \( F \) on \([a, b]\). The buyers do
not know the sellers’ quality levels, but they know the information structure \( I = \{ B_1, \ldots, B_n \} \) that
the platform has chosen. Because the buyers do not have any information about the sellers’ quality
aside from the information structure \( I \), and there are no search costs or frictions, the buyers that
decide to buy a product from quality \( x \in B_i \) sellers buy it from the seller (or one of the sellers)
with the lowest price in \( B_i \).

The preceding requirement implies that sellers cannot use prices in order to signal quality. That
is, two sellers with quality levels \( x_1, x_2 \) such that \( x_1 \in B_i, x_2 \in B_i \) for some set \( B_i \) in the information
structure \( I \) cannot disclose information about their quality level to the buyers. Because the main
focus of this section is examining the platform’s quality selection decisions, we abstract away from
information that sellers can disclose to buyers. In particular, our model abstracts away from the
possibility that the sellers signal their quality through higher prices. This may be an interesting
avenue for future research.

Given the information structure \( I = \{ B_1, \ldots, B_n \} \) and the sets of sellers that participate in the
platform \( \{ H(B_i) \}_{B_i \in I} \), \( H(B_i) \subseteq B_i \), the buyers form beliefs \( \lambda_{B_i} \in \mathcal{P}(X) \) about the quality level
of type \( x \in B_i \) sellers.\(^{20}\) In equilibrium, the buyers’ beliefs are consistent with the sellers’ entry
decisions and with Bayesian updating. That is, \( \lambda_{B_i} \) describes the conditional distribution of \( \phi \) given
\( H(B_i) \), i.e., \( \lambda_{B_i}(A) = \phi(A|H(B_i)) \) where \( \phi(A|H(B_i)) := \frac{\phi(A \cap H(B_i))}{\phi(H(B_i))} \) for every (measurable) set \( A \)
and all \( B_i \in I \) such that \( \phi(H(B_i)) > 0 \).

We denote by \( p(B_i) = \inf_{x \in H(B_i)} p_x \) the lowest price among the sellers in the set \( B_i \). A type
\( m \in [a, b] \) buyer’s utility from buying a product from quality \( x \in B_i \) sellers is given by

\[
Z(m, B_i, p(B_i)) = m \mathbb{E}_{\lambda_{B_i}}(X) - p(B_i)
\]

\( \mathbb{E}_{\lambda_{B_i}}(X) \) is the sellers’ expected quality given the buyers’ beliefs \( \lambda_{B_i} \). A type \( m \) buyer buys a product

\(^{20}\)With slight abuse of notations we use similar notations to those of Section 5.1.
from a quality \( x \in B_i \) seller if \( Z(m, B_i, p(B_i)) \geq 0 \) and \( Z(m, B_i, p(B_i)) = \max_{B \in I} Z(m, B, p(B)) \), and does not buy a product otherwise.

The total demand in the market for products that are sold by type \( x \in B_i \) sellers \( D_I(B_i, p(B_1), \ldots, p(B_n)) \) who charge the lowest price in \( B_i \) is given by

\[
D_I(B_i, p(B_1), \ldots, p(B_n)) = \int_{a}^{b} 1\{Z(m, B_i, p(B_i)) \geq 0\} 1\{Z(m, B_i, p(B_i)) = \max_{B \in P} Z(m, B, p(B))\} F(dm).
\]

The total demand in the market for products that are sold by type \( x \in B_i \) sellers that do not charge the lowest price in \( B_i \) is zero.

### 6.2 Sellers

In this section we describe the sellers’ decisions. Sellers first choose whether to participate in the platform or not. In each set \( B_i \in I \) that belongs to the information structure, participating sellers price their products simultaneously and engage in price competition with other sellers whose quality levels belong to the set \( B_i \in I \). Because a buyer that decides to buy a product from a quality \( x \in B_i \) seller buys it from the seller (or one of the sellers) who charges the lowest price in the set \( B_i \), the price competition between the sellers resembles Bertrand competition.

A quality \( x \in B_i \subseteq X \) seller that participates in the platform sells a quantity given by \( h_I(B_i, H(B_i), p_x, p(B_1), \ldots, p(B_n)) \) units if the set of participating sellers is \( H(B_i) \), the price that \( x \) charges is \( p_x \in \mathbb{R}_+ \), and \( p(B_i) = \inf_{x \in H(B_i) \setminus \{x\}} p_x \) is the lowest price among the other sellers in the set \( H(B_i) \). We denote by \( M_I(B_i, p(B_1), \ldots, p(B_n)) \) the total mass of sellers whose quality levels belong to \( B_i \) and who charge the price \( p(B_i) \). The quantity allocation function \( h_I \) is determined in equilibrium and is given by

\[
h_I(B_i, H(B_i), p_x, \textbf{p}) = \begin{cases} 
\infty & \text{if } p_x < p(B_i), \quad D_I(B_i, \textbf{p}) > 0 \\
\frac{D_I(B_i, \textbf{p})}{M_I(B_i, \textbf{p})} & \text{if } p_x = p(B_i), \quad D_I(B_i, \textbf{p}) > 0 \\
0 & \text{if } p_x > p(B_i), \text{ or } D_I(B_i, \textbf{p}) = 0
\end{cases}
\]

where \( \textbf{p} := (p(B_1), \ldots, p(B_n)) \) and we define \( D_I(B_i, \textbf{p})/M_I(B_i, \textbf{p}) = \infty \) if \( M_I(B_i, \textbf{p}) = 0 \) and \( D_I(B_i, \textbf{p}) > 0 \). This quantity allocation resembles the quantity allocation in the standard Bertrand competition model with a continuum of sellers. In particular, when multiple active sellers’ charge the same lowest price within a set, the buyers’ demand splits evenly between those sellers.

A quality \( x \in B_i \subseteq X \) seller’s utility from participating in the platform is given by

\[
\overline{U}(x, H(B_i), p_x, p(B_1), \ldots, p(B_n)) = h_I(B_i, H(B_i), p_x, p(B_1), \ldots, p(B_n))(p_x - c(x)).
\]

We assume that the cost function \( c \) is positive and constant on each element of the partition \( I_o \), i.e., \( x_1, x_2 \in A_i \) and \( A_i \in I_o \) imply \( c(x_1) = c(x_2) = c(A_i) \). The assumption that the cost function \( c \) is constant on each element of the partition \( I_o \) means that the cost function is measurable with respect to the platform’s information, i.e., the platform knows the sellers’ costs but not the sellers’
quality levels. This assumption simplifies the analysis but is not essential to our results. We also assume that the cost function is increasing, i.e., \( c(A_i) < c(A_j) \) for \( i < j \). This assumption means that producing higher quality products costs more. A quality \( x \in X \) seller’s utility from not participating in the platform is normalized to 0.

6.3 Equilibrium

In this section we define the equilibrium concept that we use for the game described above. For simplicity, we focus on a symmetric equilibrium in the sense that for all \( B_i \in I \), all the sellers that participate in the platform charge the same price. With slight abuse of notation, we denote this price by \( p(B_i) \), i.e., \( p_x = p(B_i) \) for all \( x \in H(B_i) \), \( B_i \in I \).

**Definition 5** Given an information structure \( I = \{B_1, \ldots, B_n\} \), an equilibrium consists of a vector of positive prices \( p = (p(B_1), \ldots, p(B_n)) \in \mathbb{R}^{|I|} \), positive masses of sellers that participate in the platform \( \{M_I(B_i, p)\}_{B_i \in I} \), positive masses of demand \( \{D_I(B_i, p)\}_{B_i \in I} \), and buyers’ beliefs \( \lambda = (\lambda_{B_i})_{B_i \in I} \) such that

(i) Sellers’ optimality: The sellers’ decision are optimal. That is,

\[
p(B_i) = \arg\max_{p_x \in \mathbb{R}_+} U(x, H(B_i), p_x, p)
\]

is the price that seller \( x \in H(B_i) \) charges. In addition, seller \( x \in B_i \) enters the market, i.e., \( x \in H(B_i) \), if and only if \( U(x, H(B_i), p(B_i), p) \geq 0 \).

(ii) Buyers’ optimality: The buyers’ decisions are optimal. That is, for each buyer \( m \in [a, b] \) that buys from type \( x \in B_i \) sellers, we have \( Z(m, B_i, p(B_i)) \geq 0 \) and \( Z(m, B_i, p(B_i)) = \max_{B_i \in I} Z(m, B, p(B)) \).

(iii) Rational expectations: \( \lambda_{B_i}(A) \) is the probability that a buyer is matched to sellers whose quality levels belong to the set \( A \) given the sellers’ entry decisions, i.e.,

\[
\lambda_{B_i}(A) = \phi(A|H(B_i)) = \frac{\phi(A \cap H(B_i))}{\phi(H(B_i))}
\]

for every (measurable) set \( A \) and for all \( B_i \in I \).

(iv) Market clearing: For all \( B_i \in I \) we have

\[
M_I(B_i, p) h_I(B_i, H(B_i), p(B_i), p) = D_I(B_i, p),
\]

where \( M_I(B_i, p) = \phi(H(B_i)) \) is the mass of sellers in \( B_i \) that participate in the platform; \( D_I(B_i, p) \) and \( h_I(B_i, H(B_i), p(B_i), p) \) are defined in Sections 6.1 and 6.2, respectively.

We say that an information structure \( I \) is implementable if there exists an equilibrium \((p, D, M, \lambda)\) under \( I \) where \( D = \{D_I(B_i, p)\}_{B_i \in I} \), \( M = \{M(B_i, p)\}_{B_i \in I} \), and \( \lambda = \{\lambda_{B_i}\}_{B_i \in I} \). We denote by \( \mathcal{W}^P \) the set of all implementable information structures.
The platform’s goal is to choose an implementable information structure to maximize the total transaction value \( \pi^P \) given by

\[
\pi^P(I) := \sum_{B_i \in I} p(B_i) \min\{D_I(B_i, p), M_I(B_i, p) h_I(B_i, H(B_i), p(B_i), p)\}.
\]

### 6.4 Equivalence with Constrained Price Discrimination

As in Section 5.4, the platform’s revenue maximization problem described above transforms into the constrained price discrimination problem that we analyzed in Section 3. To see this, note that an implementable information structure \( I = \{B_1, B_2, \ldots, B_n\} \) and an associated equilibrium price vector \( p = (p(B_1), \ldots, p(B_n)) \) induce a menu \( C \) that is given by

\[
C = \{(p(B_1), E_{\lambda_{B_1}}(X)), \ldots, (p(B_n), E_{\lambda_{B_n}}(X))\}
\]

where \( E_{\lambda_{B_i}}(X) \) is the equilibrium expected quality of the sellers that belong to the set \( B_i \) and \( p = (p(B_1), \ldots, p(B_n)) \) is the vector of equilibrium prices. The implementable information structure \( I \) yields the same revenue as the menu \( C \) that it induces (see Section 5.4). We denote by \( C^P \) the set of all menus \( C \) that are induced by some implementable information structure \( I \in W^P \). With this notation, the platform’s revenue maximization problem is equivalent to the constrained price discrimination problem of choosing a menu \( C \in C^P \) to maximize \( \sum p_i D_i(C) \) that we studied in Section 3.

### 6.5 Results

In this section we present our main results regarding the two-sided market model in which the sellers choose the prices.

Let \( \varphi^P : I_o \rightharpoonup C^P \) be the set-valued mapping from the set \( I_o \) of all possible information structures to the set of menus \( C^P \) such that \( C \in \varphi^P(I) \) if and only if \( C \) is a menu that is induced by the information structure \( I \). As opposed to the two-sided market model that we study in Section 5, the mapping \( \varphi^P \) can be explicitly characterized in the current setting. This is because Bertrand competition pins down the equilibrium prices (to the lowest marginal costs within a set in the information structure).

For an information structure \( I = \{B_1, \ldots, B_n\} \) let \( L(I) = \{G_1, \ldots, G_n\} \) be an information structure such that \( G_j \in I_o \) for all \( G_j \in L(I) \) and \( G_j \) is the set with the lowest index among the blocks of \( B_j \), i.e., among the sets \( \{A_k\} \) such that \( B_j = \cup_k A_k \). For example, if \( B_1 = A_1 \cup A_2 \), then \( G_1 = A_1 \). We assume without loss of generality that \( c(G_1) < \ldots < c(G_n) \) for every information structure \( I \). The following theorem shows that for every implementable information structure \( I \) and for every set \( B_i \in I \), the equilibrium price for sellers in \( B_i \) equals \( c(G_i) \). This fact follows directly from our Bertrand competition assumption. Further, using this characterization of the equilibrium prices it follows directly that \( C^P \) satisfies the condition of Corollary 1.
Theorem 3 Let $I$ be any information structure. Suppose that $C \in \varphi^P(I)$.

(i) We have
\[
C = \{ (c(G_1), E_{\lambda G_1}(X)), \ldots, (c(G_n), E_{\lambda G_n}(X)) \}
\]
where $L(I) = \{ G_1, \ldots, G_n \}$ and $\lambda G_i(A) = \phi(A \cap G_i) / \phi(G_i)$ for every measurable set $A$.

(ii) We have \( \{ (c(G_n), E_{\lambda G_n}(X)) \} \in \varphi^P(\{ B_n \}) \).

(ii) Suppose that $I_o$ is implementable and $C_o \in \varphi^P(I_o)$. Then $C^P = 2^C o$.

The proof of the following Corollary follows immediately from Theorem 3 and Corollary 1.

Corollary 3 Assume that $F(m)m$ is convex on $[a, b]$. Then there exists a 1-separating information structure that maximizes the platform’s revenue.

Note that the only 1-separating information structure that induces a menu that is maximal in $C^P_1$ is \{ $A_l$ \}. Thus, when $I_o$ is implementable and the constraint set $C^P = 2^C o$ is regular (i.e., the equilibrium price is lower than the monopoly price under the information structure \{ $A_l$ \}), Theorem 1 implies that the optimal information structure is \{ $A_l$ \}. That is, the optimal information structure bans all sellers except the highest quality sellers.

As we discussed in Section 3.3, in practice, a platform might consider only a small number of options, e.g., removing the lowest quality sellers or keeping them. In order to determine whether banning these low quality sellers is beneficial, the platform needs to measure the density function’s elasticity only locally. If the density function’s elasticity is bounded below by $-2$ (i.e., $F(m)m$ is convex) on some local region that depends on the prices and qualities of the low quality sellers, then it is beneficial to ban these sellers. Conversely, if the density function’s elasticity is bounded above by $-2$ (i.e., $F(m)m$ is concave) on this local region, then it is beneficial to keep these sellers (see Corollary 4). For many distribution functions the density function’s elasticity is decreasing. In this case Corollary 4 implies that the platform needs to check the density function’s elasticity only at one point. For example, if at the highest point of the relevant interval (this point depends on the equilibrium prices and qualities) the density function’s elasticity is greater than $-2$, then it is greater than $-2$ over the relevant interval. In practice, the platform might be able to estimate this elasticity with price experimentation.

Corollary 4 Let $I = \{ B_1, \ldots, B_n \}$ be an implementable information structure.

Let $C = \{ (p(G_1), E_{\lambda G_1}(X)), \ldots, (p(G_n), E_{\lambda G_n}(X)) \} \in \varphi^P(I)$ where $L(I) = \{ G_1, \ldots, G_n \}$. Consider the (implementable) information structure $I' = \{ B_2, \ldots, B_n \}$. Then
\[
\pi^P(I) \leq \pi^P(I') \text{ if } F(m)m \text{ is convex on } \left[ \frac{p(G_1)}{E_{\lambda G_1}(X)}, \frac{p(G_2) - p(G_1)}{E_{\lambda G_2}(X) - E_{\lambda G_1}(X)} \right]
\]
\[
\pi^P(I) \geq \pi^P(I') \text{ if } F(m)m \text{ is concave on } \left[ \frac{p(G_1)}{E_{\lambda G_1}(X)}, \frac{p(G_2) - p(G_1)}{E_{\lambda G_2}(X) - E_{\lambda G_1}(X)} \right]
\]
We also show that when $F(m)m$ is concave and $I_o$ is implementable, the optimal information structure is $I_o$, i.e., the platform reveals all the information it has about the sellers’ quality. The proof of the following Corollary follows from Theorem 3 and Proposition 2.

**Corollary 5** Assume that $I_o$ is implementable. Let $C_o = \{(p(A_1), E_{\lambda A_1}(X)), \ldots, (p(A_l), E_{\lambda A_l}(X))\} \in \varphi^P(I_o)$. Suppose that $F(m)m$ is concave on 

$$\left[ \begin{array}{cc} p(A_1) & p(A_l) - p(A_{l-1}) \\ \frac{1}{E_{\lambda A_1}(X)} & \frac{1}{E_{\lambda A_l}(X)} - \frac{1}{E_{\lambda A_{l-1}}(X)} \end{array} \right].$$

Then the optimal information structure is $I_o$.

7 Conclusions

In this paper we study optimal information disclosure policies for online platforms. We introduce two distinct two-sided market models. In the first model the sellers choose quantities, and in the second model the sellers make entry and pricing decisions. A key element of our analysis is showing that the platform’s information disclosure problem transforms into a constrained price discrimination problem, where the constraints are given by the equilibrium requirements and depend on the specific two-sided market model being studied. We use this equivalence to provide conditions that are related to demand elasticities, under which a simple information structure where the platform removes a certain portion of low quality sellers and does not share any information about the other sellers is revenue-optimal for the platform.

There are some interesting potential extensions for future work. For example, in practice, the platform and the buyers learn the sellers’ quality as they make their decisions. One possible extension of our work would be to incorporate learning into our setting. Another direction for future work is to introduce competition between platforms. In many industries, fierce competition between platforms has a first order effect on the market design choices made by platforms.

Finally, a third interesting direction for future research is to incorporate search frictions in our setting. In some platforms (e.g., e-commerce platforms) search frictions play a significant role. In some of these platforms, because of rating inflation, the sellers’ star rating does not provide substantial information about the sellers’ quality (see, e.g., Tadelis (2016)). In this case, the menu observed in practice sometimes looks similar to a 2-separating menu: certified sellers, sellers that are not certified, and sellers that are banned. While the results in this paper show that a 1-separating menu is optimal under an appropriate condition on demand elasticity, we conjecture that extending our setting to incorporate search costs would change the optimal menu. In particular, in order to mitigate the impact of search, a 2-separating menu might be more attractive.
Appendix

A.1 Proofs of Section 3

We first introduce some definitions. A menu $C \in C_p$ is called price-$M$ if for all $(p, q) \in [0, \infty) \times [0, \infty)$ such that $C \cup \{p, q\} \in C_p$, we have $p \leq p'$ for some $(p', q') \in C$. In words, a menu $C$ is price-$M$ if it is not feasible to add a price-quality pair to $C$ with positive demand and a higher price than all the other prices in the menu $C$.

Step 4 in the proof of Theorem 1 shows that the optimal menu (if it exists) is price-$M$. This also shows that Theorem 1 holds under the following weaker version of the first condition of Definition 1 (the regularity condition): For every price-$M$ menu $C = \{(p_1, q_1), \ldots, (p_k, q_k)\}$ there exists a 1-separating menu $\{p, q\} \in C_1$ such that $p \geq p_k$ and $q \geq q_k$.

Recall that given some quality $q > 0$, the price that maximizes the platform’s revenue $p^M(q)$ is given by

$$p^M(q) = \arg\max_{p \geq 0} p \left(1 - F\left(\frac{p}{q}\right)\right).$$

Note that $p^M(q)$ is single-valued under the assumptions of Theorem 1. A 1-separating menu $\{(p, q)\}$ is maximal in $C_1$ if for every $\{(p', q')\} \in C_1$ such that $(p', q') \neq (p, q)$ we have $p > p'$ or $q > q'$.

**Proof of Theorem 1.** Let $C = \{(p_i, q_i)_{i=1}^n\} \in C$ be a menu such that $p_k \leq p_j$ for all $k < j$ and $n > 1$. We can assume$^{21}$ that the demand for each price-quality pair in $C$ has a positive mass. That is

$$D_i(C) = \int_a^b 1_{\{mq_i-p_i \geq 0\}} 1_{\{mq_i-p_i = \max_{i=1,\ldots,n}mq_i-p_i\}} F(dm) > 0$$

for all $1 \leq i \leq n$. Note that $D_i(C) > 0$ for all $1 \leq i \leq n$ implies that $q_k < q_j$ for all $k < j$.

**Step 1.** The total transaction value from the menu $C$ is given by

$$\pi(C) = \sum_{i=1}^n p_i \left(F(m_{i+1}) - F(m_i)\right)$$

where $m_{n+1} = b$ and the numbers $\{m_i\}_{i=2}^n$ satisfy $m_i \in [a, b]$ for all $2 \leq i \leq n$ and

$$m_i q_i - p_i = m_i q_{i-1} - p_{i-1}$$

where $q_0 = p_0 = 0$. The number $m_1$ satisfies $m_1 = \max\{a, p_1/q_1\}$.

**Proof of Step 1.** The proof of Step 1 is standard (see Maskin and Riley (1984)). We provide it here for completeness.

Because $q_n > q_j$ for all $1 \leq j \leq n - 1$, if for some $1 \leq j \leq n - 1$ and $m \in [a, b]$ we have

$$m(q_n - q_j) \geq p_n - p_j$$

$^{21}$If for some $(p_k, q_k)$ in $C$ we have $D_k(C) = 0$, then the menu $C \setminus \{(p_k, q_k)\}$ has the same total transaction value as the menu $C$. Thus, we can consider the menu $C \setminus \{(p_k, q_k)\}$ instead of the menu $C$. 

32
then
\[ m' (q_n - q_j) \geq p_n - p_j \]
for all \( m' \in [m, b] \). Thus, if for some \( m \in [a, b] \) we have
\[ \text{mq}_n - p_n \geq \max\{ \max_{1 \leq j \leq n-1} \text{mq}_j - p_j, 0 \} \]
then inequality (8) holds for all \( m' \in [m, b] \). In other words, if a type \( m \) chooses the price-quality pair \((p_n, q_n)\), then every type \( m' \) with \( m \leq m' \leq b \) chooses the price-quality pair \((p_n, q_n)\).

Let
\[ W_n := \{ m \in [a, b] : \text{mq}_n - p_n \geq \max\{ \max_{1 \leq j \leq n-1} \text{mq}_j - p_j, 0 \} \} \]
be the set of types that choose the price-quality pair \((p_n, q_n)\). Define \( m_n = \min W_n \). \( D_n(C) > 0 \) implies that the set \( W_n \) is not empty. From the fact that \( m \in W_n \) implies \( m' \in W_n \) for all \( m \leq m' \leq b \), \( W_n \) equals the interval \([m_n, b]\). Thus,
\[ D_n(C) = \int_a^b 1_{W_n}(m) F(dm) = F(b) - F(m_n) = F(m_{n+1}) - F(m_n) \]
where \( m_{n+1} := b \) so \( F(m_{n+1}) = 1 \).

Define \( m_i = \min W_i \) where we define the sets
\[ W_i := \{ m \in [a, m_{i+1}] : \text{mq}_i - p_i \geq \max\{ \sup_{1 \leq j \leq i-1} \text{mq}_j - p_j, 0 \} \} \]
for all \( 1 \leq i \leq n - 1 \). \( D_i(C) > 0 \) implies that \( W_i \) is not empty. Thus, \( m_i \) is well defined. From the same argument as the argument above, if a type \( m \in W_i \) chooses the price-quality pair \((p_i, q_i)\), then every type \( m' \) with \( m \leq m' \leq m_{i+1} \) chooses the price-quality pair \((p_i, q_i)\). Thus, \( W_i \) equals the interval \([m_i, m_{i+1}]\) and
\[ D_i(C) = \int_a^b 1_{W_i}(m) F(dm) = F(m_{i+1}) - F(m_i) > 0 \]
for all \( 1 \leq i \leq n \).

Note that \( W_1 = \{ m \in [a, m_2] : \text{mq}_1 - p_1 \geq 0 \} \). The continuity of the function \( \text{mq}_1 - p_1 \) implies that \( m_1 = \min W_1 \) satisfies \( m_1 = \max\{ a, p_1/q_1 \} \). Using continuity again and the definition of \( m_2 \) we conclude that \( m_2 q_2 - p_2 = m_2 q_1 - p_1 \). Similarly, \( m_i q_i - p_i = m_i q_{i-1} - p_{i-1} \) for all \( 2 \leq i \leq n \).

Thus, the total transaction value from the menu \( C \) is given by
\[ \pi(C) = \sum_{i=1}^n p_i D_i(C) = \sum_{i=1}^n p_i (F(m_{i+1}) - F(m_i)) \]
where \( m_{n+1} = b \) and the numbers \( \{m_i\}_{i=1}^n \) satisfy \( m_i \in [a, b] \) for all \( 1 \leq i \leq n \) and \( m_i q_i - p_i = m_i q_{i-1} - p_{i-1} \), \( q_0 = p_0 = 0 \).
Step 2. The function $f(x, y) = xF\left(\frac{z}{y}\right)$ is convex on $E = \{(x, y) : x/y \in [a, b], y > 0\}$.

Proof of Step 2. Recall that the perspective function $\bar{f}(x, y) = yg\left(\frac{x}{y}\right)$ is convex on $E$ whenever $g$ is convex on $[a, b]$. Suppose that $g(x) = F(x)x$. Then $g$ is convex on $[a, b]$ from the theorem’s assumption. Thus,

$$\bar{f}(x, y) = yg\left(\frac{x}{y}\right) = yF\left(\frac{x}{y}\right) = xF\left(\frac{x}{y}\right) = f(x, y)$$

is convex on $E$.

Step 3. Let $0 = d_0 < d_1 < \ldots < d_k$ and $0 = z_0 < \ldots < z_k$. Assume that $(z_i - z_{i-1}) / (d_i - d_{i-1}) \in [a, b]$ for all $1 \leq i \leq k$. Then

$$z_k F\left(\frac{z_k}{d_k}\right) \leq \sum_{i=1}^{k} (z_i - z_{i-1}) F\left(\frac{z_i - z_{i-1}}{d_i - d_{i-1}}\right).$$ (9)

Proof of Step 3. From Step 2 the function $f(x, y) = xF\left(\frac{z}{y}\right)$ is convex on $E$. From Jensen’s inequality we have

$$k^{-1} \sum_{i=1}^{k} x_i F\left(\frac{k^{-1} \sum_{i=1}^{k} x_i}{k^{-1} \sum_{i=1}^{k} y_i}\right) = f\left(\frac{k^{-1} \sum_{i=1}^{k} (x_i, y_i)}{k^{-1} \sum_{i=1}^{k} y_i}\right) \leq k^{-1} \sum_{i=1}^{k} f(x_i, y_i) = k^{-1} \sum_{i=1}^{k} x_i F\left(\frac{x_i}{y_i}\right)$$

for all $(x_1, \ldots, x_k)$ and $(y_1, \ldots, y_k)$ such that $(x_i, y_i) \in E$ for all $i = 1, \ldots, k$. Thus,

$$\sum_{i=1}^{k} x_i F\left(\frac{\sum_{i=1}^{k} x_i}{\sum_{i=1}^{k} y_i}\right) \leq \sum_{i=1}^{k} x_i F\left(\frac{x_i}{y_i}\right).$$

Let $z_i - z_{i-1} = x_i \geq 0$ and $d_i - d_{i-1} = y_i > 0$. Note that $\sum_{i=1}^{k} x_i = z_k$ and $\sum_{i=1}^{k} y_i = d_k$ to conclude that inequality (9) holds.

Step 4 The menu that maximizes the total transaction value is price-M.

Proof of Step 4. Assume that $C$ is not price-M. Then there exists a price-quality pair $(p_{n+1}, q_{n+1})$ such that $p_{n+1} > p_n$ and $C \cup \{p_{n+1}, q_{n+1}\}$ belongs to $C_p$, i.e., $D_i(C) > 0$ for all $1 \leq i \leq n + 1$. From Step 1, we have $m_i q_i - p_i = m_i q_{i-1} - p_{i-1}$ for all $i$ (recall that $q_0 = p_0 = 0$). This implies that

$$m_i = \frac{p_i - p_{i-1}}{q_i - q_{i-1}}.$$
for all $i$. We have

$$\pi(C \cup \{p_{n+1}, q_{n+1}\}) - \pi(C) = \sum_{i=1}^{n} p_i (F(m_{i+1}) - F(m_i)) + p_{n+1}(1 - F(m_{n+1}))$$

$$- \sum_{i=1}^{n-1} p_i (F(m_{i+1}) - F(m_i)) - p_n(1 - F(m_n))$$

$$= p_n \left( F \left( \frac{p_{n+1} - p_n}{q_{n+1} - q_n} \right) - F \left( \frac{p_n - p_{n-1}}{q_n - q_{n-1}} \right) \right) + p_{n+1} \left( 1 - F \left( \frac{p_{n+1} - p_n}{q_{n+1} - q_n} \right) \right)$$

$$- p_n \left( 1 - F \left( \frac{p_n - p_{n-1}}{q_n - q_{n-1}} \right) \right) > 0.$$ 

Thus, $C$ is not optimal. The inequality follows from the facts that $p_{n+1} > p_n$ and $D_{n+1} = 1 - F \left( \frac{p_{n+1} - p_n}{q_{n+1} - q_n} \right) > 0$. We conclude that the menu that maximizes the total transaction value (if it exists) is price-M.

**Step 5.** Let $C^* = \{(p_n, q_n)\}$. We have

$$\pi(C) \leq \pi(C^*).$$

**Proof of Step 5.** From Step 1 we have

$$\pi(C) = \sum_{i=1}^{n} p_i (F(m_{i+1}) - F(m_i))$$

$$= \sum_{i=1}^{n-1} p_i \left( F \left( \frac{p_{i+1} - p_i}{q_{i+1} - q_i} \right) - F \left( \frac{p_i - p_{i-1}}{q_i - q_{i-1}} \right) \right) + p_n \left( 1 - F \left( \frac{p_n - p_{n-1}}{q_n - q_{n-1}} \right) \right)$$

$$= p_n - \sum_{i=1}^{n} (p_i - p_{i-1}) F \left( \frac{p_i - p_{i-1}}{q_i - q_{i-1}} \right).$$

The first equality follows from Step 1. In the second equality we use the fact that $F(m_{n+1}) = F(b) = 1$.

Let $C^* = \{(p_n, q_n)\}$. Using Step 1 again we have

$$\pi(C^*) = p_n \left( 1 - F \left( \frac{p_n}{q_n} \right) \right)$$

Thus, we have $\pi(C) \leq \pi(C^*)$ if and only if

$$p_n F \left( \frac{p_n}{q_n} \right) \leq \sum_{i=1}^{n} (p_i - p_{i-1}) F \left( \frac{p_i - p_{i-1}}{q_i - q_{i-1}} \right). \quad (10)$$

From Step 1, $m_i = (p_i - p_{i-1}) / (q_i - q_{i-1}) \in [a, b]$ for all $1 \leq i \leq n$. Thus, from Step 3, inequality (10) holds. We conclude that $\pi(C) \leq \pi(C^*)$.

**Step 6.** We have $p^M(q) \geq p$ for every 1-separating menu $\{(p, q)\}$ that is maximal in $C_1$.  

35
Proof of Step 6. We first show that for any two 1-separating menus \((p, q)\) and \((p', q')\) we have \(p^M(q) \geq p^M(q')\) whenever \(q \geq q' > 0\).

Because \(F(m)m\) is strictly convex on \([a, b]\), \(p^M(q)\) is single-valued. In addition, we clearly have \(a \leq p^M(q)/q < b\). Hence, we have

\[
\max_{p \geq 0} p \left(1 - F\left(\frac{p}{q}\right)\right) = \max_{q \leq p} p \left(1 - F\left(\frac{p}{q}\right)\right).
\]

Assume in contradiction that \(p^M(q) < p^M(q')\) and \(q \geq q'\). Then \(p^M(q)/q < p^M(q')/q'\). The first order conditions for the optimality of \(p^M\) and the fact that the strict convexity of \(F(m)m\) on \([a, b]\) implies that the function \(F(m) + mf(m)\) is strictly increasing on \([a, b]\) yield

\[
0 \geq 1 - \left(F\left(\frac{p^M(q)}{q}\right) + \frac{p^M(q)}{q} f\left(\frac{p^M(q)}{q}\right)\right)
\]

\[
> 1 - \left(F\left(\frac{p^M(q')}{q'}\right) + \frac{p^M(q')}{q'} f\left(\frac{p^M(q')}{q'}\right)\right) = 0
\]

which is a contradiction. We conclude that \(p^M(q) \geq p^M(q')\) whenever \(q \geq q' > 0\).

Let \(\{(p^H, q^H)\} \in C_1\) be such that \(p^H \geq p'\) for all \(\{(p', q')\} \in C_1\), and let \(\{(p, q)\}\) be a maximal element in \(C_1\). From the definition of \(p^H\) we have \(p^H \geq p\). Because \(\{(p, q)\}\) is maximal in \(C_1\) we have \(q \geq q^H\). Thus, we have \(p^M(q) \geq p^M(q^H)\). Because \(C\) is regular we have \(p^M(q^H) \geq p^H\). We conclude that \(p \leq p^H \leq p^M(q^H) \leq p^M(q)\) which proves Step 6.

Step 7. There exists a 1-separating menu \(C' \in C\) such that \(\pi(C^*) \leq \pi(C')\) where \(C^* = \{(p_n, q_n)\}\).

Proof of Step 7. Because \(C\) is regular and \(C = \{(p_i, q_i)\}_{i=1}^n \in C_p\), there exists a 1-separating menu \(\{(p', q')\} \in C_1\) such that \(p' \geq p_n\) and \(q' \geq q_n\). We consider two cases.

Case 1. \(\{(p', q')\}\) is maximal in \(C_1\).

From Step 6 we have \(p' \leq p^M(q')\). We conclude that \(p_n \leq p' \leq p^M(q')\). The convexity of \(F(m)m\) on \([a, b]\) implies that \(p \left(1 - F\left(\frac{p}{q}\right)\right)\) is increasing in \(p\) on \([p_n, p^M(q')\]). Thus,

\[
p_n \left(1 - F\left(\frac{p_n}{q_n}\right)\right) \leq p_n \left(1 - F\left(\frac{p_n}{q'}\right)\right) \leq p' \left(1 - F\left(\frac{p'}{q'}\right)\right).
\]

Thus, the menu \(\{(p', q')\}\) yields more total transaction value than the menu \(\{(p_n, q_n)\}\).

Case 2. \(\{(p', q')\}\) is not maximal in \(C_1\).

In this case, because \(C_1\) is compact, there exists a menu \(\{(p, q)\} \in C_1\) such that \(p \geq p'\) and \(q \geq q'\), and \(\{(p, q)\}\) is maximal in \(C_1\). From Step 6 we have \(p \leq p^M(q)\).
Hence, we have \( p_n \leq p \leq p^M(q) \) which implies
\[
p_n \left( 1 - F \left( \frac{p_n}{q_n} \right) \right) \leq p_n \left( 1 - F \left( \frac{p_n}{q} \right) \right) \leq p \left( 1 - F \left( \frac{p}{q} \right) \right).
\]

That is, the menu \( \{(p, q)\} \in C_1 \) yields more total transaction value than the menu \( \{(p_n, q_n)\} \). This proves Step 7.

Step 5 and Step 7 prove that for any menu \( C = \{(p_i, q_i)_{i=1}^n\} \in C \) there exists a 1-separating menu \( C' \in C \) such that \( \pi(C) \leq \pi(C') \). Thus,
\[
\sup_{C \in C} \pi(C) \leq \max_{C \in C_1} \pi(C)
\]
which proves the Theorem. The maximum on the right side of the last inequality is attained because the distribution function \( F \) is continuous and \( C_1 \) is a compact set.

From Case 2 in Step 7, for every 1-separating menu \( C \) that is not maximal in \( C_1 \) there exists a 1-separating menu that is maximal in \( C_1 \) that yields more total transaction value than \( C \). We conclude that the optimal 1-separating menu is maximal in \( C_1 \). □

**Proof of Proposition 1.** Suppose that \( g(z) = F(z)z \) is not convex on \((a, b)\). Then there exist non-negative numbers \( z_1 \in (a, b), z_2 \in (a, b) \) and \( 0 < \lambda < 1 \) such that
\[
g(\lambda z_1 + (1 - \lambda) z_2) > \lambda g(z_1) + (1 - \lambda) g(z_2).
\]
Let \( k_1, k_2, d_1, d_2, \) and \( 0 < \theta < 1 \) be such that \( k_1 \geq 0, k_2 \geq 0, d_1 > 0, d_2 > 0, d_1 z_1 = k_1, d_2 z_2 = k_2, \) and \( \theta d_1 = \lambda (\theta d_1 + (1 - \theta) d_2) \).

Note that \( 1 - \lambda = (1 - \theta) d_2 / (\theta d_1 + (1 - \theta) d_2) \).

Denote \( d_\theta := \theta d_1 + (1 - \theta) d_2 \) and \( k_\theta := \theta k_1 + (1 - \theta) k_2 \). Note that
\[
\lambda z_1 + (1 - \lambda) z_2 = \frac{\theta d_1 k_1}{d_\theta} + \frac{(1 - \theta) d_2 k_2}{d_\theta} = \frac{k_\theta}{d_\theta}.
\]
We have
\[
\theta d_1 g \left( \frac{k_1}{d_1} \right) + (1 - \theta) d_2 g \left( \frac{k_2}{d_2} \right) = d_\theta \left( \frac{\theta d_1 g \left( \frac{k_1}{d_1} \right)}{d_\theta} + \frac{(1 - \theta) d_2 g \left( \frac{k_2}{d_2} \right)}{d_\theta} \right) < d_\theta g \left( \frac{k_\theta}{d_\theta} \right).
\]
We conclude that the function \( f(x, y) := yg \left( \frac{x}{y} \right) = x F \left( \frac{x}{y} \right) \) is not convex on \( E^* = \{(x, y) : x/y \in (a, b), y > 0 \} \).

Since \( f \) is continuous and not convex it is not midpoint convex.\(^{22}\)

Thus, there exists \((x_1, y_1) \in E^* \) and \((x_2, y_2) \in E^* \) such that
\[
f \left( \frac{(x_1, y_1)}{2} + \frac{(x_2, y_2)}{2} \right) > f \left( \frac{x_1, y_1}{2} \right) + f \left( \frac{x_2, y_2}{2} \right).
\]
(11)

\(^{22}\)Recall that the function \( f : E^* \to \mathbb{R} \) is midpoint convex if for all \( e_1, e_2 \in E^* \) we have \( f \left( (e_1 + e_2)/2 \right) \leq (f(e_1) + f(e_2))/2 \). A continuous midpoint convex function is convex. We conclude that \( f \) is not midpoint convex.
If \( x_1 = x_2 = 0 \) then the left-hand-side and the right-hand-side of the last inequality equal 0 which is a contradiction, so we have \( x_1 + x_2 > 0 \).

Assume in contradiction that \( \frac{x_2}{y_2} = \frac{x_1}{y_1} \). We have

\[
\frac{f \left( \frac{(x_1, y_1)}{2} + \frac{(x_2, y_2)}{2} \right)}{2} > \frac{f \left( x_1, y_1 \right)}{2} + \frac{f \left( x_2, y_2 \right)}{2}
\]

\[
\iff (x_1 + x_2) F \left( \frac{x_1 + x_2}{y_1 + y_2} \right) > x_1 F \left( \frac{x_1}{y_1} \right) + x_2 F \left( \frac{x_2}{y_2} \right)
\]

\[
\iff F \left( \frac{x_1 + x_2}{y_1 + y_2} \right) > F \left( \frac{x_1}{y_1} \right)
\]

\[
\iff \frac{x_1 + x_2}{y_1 + y_2} > \frac{x_1}{y_1} \iff \frac{x_2}{y_2} > \frac{x_1}{y_1},
\]

which is a contradiction. Thus, \( \frac{x_2}{y_2} \neq \frac{x_1}{y_1} \).

Assume without loss of generality that \( \frac{x_2}{y_2} > \frac{x_1}{y_1} \). Then \( x_2 > 0 \).

Let \( p_2 > p_1 \) and \( q_2 > q_1 \) be such that \( p_2 - p_1 = x_2 > 0 \), \( p_1 = x_1 \), \( q_2 - q_1 = y_2 \) and \( y_1 = q_1 \). Define the menus \( C = \{(p_1,q_1),(p_2,q_2)\} \), \( C^* = \{(p_1,q_1)\} \), and \( C^{**} = \{(p_2,q_2)\} \). Let \( C = \{C,C^*,C^{**}\} \).

We now show that \( D_1(C) > 0 \), \( D_2(C) > 0 \) and that \( C \) yields more total transaction value than the 1-separating menus \( C^* \) and \( C^{**} \).

Note that \( \frac{x_2}{y_2} > \frac{x_1}{y_1} \) implies

\[
m_2 = \frac{p_2 - p_1}{q_2 - q_1} > \frac{p_1}{q_1} = m_1
\]

where \( m_1 \) and \( m_2 \) are defined in Step 1 in the proof of Theorem 1.

Since \( F \) is supported on \([a,b]\), \( F \) is strictly increasing on \([a,b]\). Note that \( m_1 \) and \( m_2 \) belong to \((a,b)\) so \( m_2 > m_1 \) implies that \( F(m_2) > F(m_1) \). We have \( D_1(C) = F(m_2) - F(m_1) > 0 \). In addition, because \( m_2 = x_2/y_2 \) and \((x_2,y_2) \in E^* \) we have \( m_2 < b \), so \( D_2(C) = 1 - F(m_2) > 0 \).

Inequality (11) implies that

\[
p_2 F \left( \frac{p_2}{q_2} \right) > (p_2 - p_1) F \left( \frac{p_2 - p_1}{q_2 - q_1} \right) + p_1 F \left( \frac{p_1}{q_1} \right).
\]

Because \( D_1(C) > 0 \) and \( D_2(C) > 0 \), from Step 5 in the proof of Theorem 1, the last inequality implies \( \pi(C) > \pi(C^*) \) where \( C^{**} = \{(p_2,q_2)\} \).

The menu \( C^* = \{(p_1,q_1)\} \) does not maximize the total transaction value because

\[
\pi(C^{**}) = p_1 \left( 1 - F \left( \frac{p_1}{q_1} \right) \right) < p_2 \left( 1 - F \left( \frac{p_2 - p_1}{q_2 - q_1} \right) \right) + p_1 \left( F \left( \frac{p_2 - p_1}{q_2 - q_1} \right) - F \left( \frac{p_1}{q_1} \right) \right) = \pi(C)
\]

where the equalities follow from Step 1 in the proof of Theorem 1.

We conclude that the 2-separating menu \( C \) yields more total transaction value than the 1-separating menus \( C^* \) and \( C^{**} \). ■

Recall that for a menu \( C = \{(p_1,q_1), \ldots, (p_n,q_n)\} \in C_p \) we define \( m_i(C) = (p_i - p_{i-1}) / (q_i - q_{i-1}) \) for \( i = 1, \ldots, n \) where \( p_0 = q_0 = 0 \). We now prove the following general version of Proposition 2.
Proposition 4 Let $C = \{(p_1, q_1), \ldots, (p_n, q_n)\} \in C_p$ and let $C' = \{(p_{\mu(1)}, q_{\mu(1)}), \ldots, (p_{\mu(k)}, q_{\mu(k)})\} \in 2^C$. Assume without loss of generality that $p_i < p_j$ and $\mu(i) < \mu(j)$ whenever $i < j$. Define $\mu_0 = 0$.

Assume that $\mu(k) = n$. Then, $\pi(C) \leq \pi(C')$ if $F(m)m$ is convex on $[m_{\mu(j-1)+1}(C), m_{\mu(j)}(C)]$ for all $j = 1, \ldots, n$ such that $\mu(j) - \mu(j - 1) > 1$. Further, $\pi(C) \geq \pi(C')$ if $F(m)m$ is concave on $[m_{\mu(j-1)+1}(C), m_{\mu(j)}(C)]$ for all $j = 1, \ldots, n$ such that $\mu(j) - \mu(j - 1) > 1$.

Proof of Proposition 4. Clearly $C \in C_p$ implies $C' \in C_p$. From Step 1 in the proof of Theorem 1 and using the fact that $\mu(k) = n$ we have

$$
\pi(C) - \pi(C') = p_n - \sum_{i=1}^{n} (p_i - p_{i-1}) F \left( \frac{p_i - p_{i-1}}{q_i - q_{i-1}} \right)
- \left( p_{\mu(k)} - \sum_{i=1}^{k} (p_{\mu(i)} - p_{\mu(i-1)}) F \left( \frac{p_{\mu(i)} - p_{\mu(i-1)}}{q_{\mu(i)} - q_{\mu(i-1)}} \right) \right)
- \sum_{i=1}^{k} (p_{\mu(i)} - p_{\mu(i-1)}) F \left( \frac{p_{\mu(i)} - p_{\mu(i-1)}}{q_{\mu(i)} - q_{\mu(i-1)}} \right).
$$

Let $j$ be such that $\mu(j) - \mu(j - 1) = d > 1$ and assume that $F(m)m$ is convex on $[m_{\mu(j-1)+1}(C), m_{\mu(j)}(C)]$. From Step 2 in the proof of Theorem 1 the function $f(x, y) = x F \left( \frac{x}{y} \right)$ is convex on $E = \{(x, y) : x/y \in [m_{\mu(j-1)+1}(C), m_{\mu(j)}(C)], y > 0 \}$. Hence, using Jensen’s inequality with the points $(x, y) = (p_i - p_{i-1}, q_i - q_{i-1}) \in E$ for $i = \mu(j - 1) + 1, \ldots, \mu(j)$ yields

$$
\sum_{i=\mu(j-1)+1}^{\mu(j)} d^{-1} f(x_i, y_i) \geq f \left( \sum_{i=\mu(j-1)+1}^{\mu(j)} d^{-1} x_i, \sum_{i=\mu(j-1)+1}^{\mu(j)} y_i \right)
,$$

i.e.,

$$
\sum_{i=\mu(j-1)+1}^{\mu(j)} (p_i - p_{i-1}) F \left( \frac{p_i - p_{i-1}}{q_i - q_{i-1}} \right) \geq (p_{\mu(j)} - p_{\mu(j-1)}) F \left( \frac{p_{\mu(j)} - p_{\mu(j-1)}}{q_{\mu(j)} - q_{\mu(j-1)}} \right).
$$

Summing the last inequality over all $j$ such that $\mu(j) - \mu(j - 1) > 1$ shows that $\pi(C') \geq \pi(C)$. The case where $F(m)m$ is concave is proven by an analogous argument. ■

A.2 Proofs of Section 5

We first prove the following Lemma:

Lemma 1 Fix an information structure $I = \{B_1, B_2, \ldots, B_n\}$ in $I(I_o)$. Then, for every positive pricing function $p$ we have

$$
\mathbb{E}_{\lambda_{\alpha}}(X) = \frac{\int_{B_n} x(k(x))^{-1/\alpha} \phi(dx)}{\int_{B_n} (k(x))^{-1/\alpha} \phi(dx)}
$$

\(^{23}\)If $\mu(k) < n$ then $C'$ is not price-M. Hence, the menu $C' \cup \{p_n, q_n\}$ yields more total transaction value than $C'$ (see the proof of Theorem 1).
The probability measure $\lambda_{B_i}$ is given in Equation (2) in Section 5. That means the expected sellers’ qualities do not depend on the prices.

**Proof of Lemma 1.** Fix an information structure $I = \{B_1, B_2, \ldots, B_n\}$ in $\mathbb{I}(I_o)$.

Given a positive pricing function $p$, the optimal quantity of a seller $x$ in $B_i$, $g(x, p(B_i)) = \arg\max_{h \in \mathbb{R}_+} U(x, h, p(B_i))$ is given by

$$g(x, p(B_i)) = \left(\frac{p(B_i)}{k(x)}\right)^{1/\alpha}. \quad (12)$$

Hence, we have

$$\mathbb{E}_{\lambda_{B_i}}(X) = \int_{B_i} x \lambda_{B_i}(dx) = \frac{\int_{B_i} x g(x, p(B_i)) \phi(dx)}{\int_{B_i} g(x, p(B_i)) \phi(dx)} = \frac{\int_{B_i} x (k(x))^{-1/\alpha} \phi(dx)}{\int_{B_i} (k(x))^{-1/\alpha} \phi(dx)}.$$  

Thus, the expected sellers’ quality $\mathbb{E}_{\lambda_{B_i}}(X)$ does not depend on the prices when the pricing function is positive. ■

**Proof of Proposition 3.** For the rest of the proof except for Step 3, we fix an information structure $I = \{B_1, B_2, \ldots, B_n\}$ in $\mathbb{I}(I_o)$ and assume that $\mathbb{E}_{\lambda_{B_1}}(X) < \ldots < \mathbb{E}_{\lambda_{B_n}}(X)$ where the expected sellers’ quality $\mathbb{E}_{\lambda_{B_i}}(X)$ is given in Lemma 1.

Let $P$ be the set of all pricing functions such that the demand for each set $B_i \in I$, $D_i(B_i, p)$ is greater than 0, each price is greater than 0, and the prices are ordered according to an ascending order. That is,

$$P = \{ p \in \mathbb{R}_+^n : D_i(B_i, p) > 0 \text{ for all } i = 1, \ldots, n, 0 < p(B_1) < \ldots < p(B_n) \}.$$  

To simplify notation, for the rest of the proof we denote $p_i = p(B_i)$, $p'_i = p'(B_i)$, $s_i(p_i) = S_i(B_i, p(B_i))$, $\mathbb{E}_{\lambda_{B_i}}(X) = q_i$, and $d_i(p) = D_i(B_i, p)$. Note that $p \in P$ implies $0 < q_1 < \ldots < q_n$ (recall that Lemma 1 implies that the expected sellers’ quality $q_i$ does not depend on the prices).

Define the function $\psi : P \rightarrow \mathbb{R}$ by

$$\psi(p) = \sum_{i=1}^{n} \frac{p_i^{\alpha+1}}{\alpha} \int_{B_i} k(x)^{-1/\alpha} \phi(dx) - p_n + \sum_{i=0}^{n-1} F_2 \left( \frac{p_{i+1} - p_i}{q_{i+1} - q_i} \right)(q_{i+1} - q_i) \quad (13)$$

where $F_2(x) = \int_{a}^{x} F(m)dm$ is the antiderivative of $F$ and $q_0 = p_0 = 0$. Note that $p \in P$ implies that for every $1 \leq i \leq n - 1$ we have $a \leq (p_{i+1} - p_i)/(q_{i+1} - q_i) \leq b$ (see Step 1 in the proof of Theorem 1). Because the function $F$ is continuous, the fundamental theorem of calculus implies that the function $F_2$ is differentiable and $F'_2 = F$ Thus, $\psi$ is continuously differentiable.

Let $\nabla \psi$ be the gradient of $\psi$ and let $\nabla_i \psi$ be the $i$th element of the gradient. A direct calculation
shows that for \(1 \leq i \leq n - 1\) we have

\[
\nabla_i \psi(p) = p_i^{1/\alpha} \int_{B_i} k(x)^{1-1/\alpha} \phi(dx) - F'_1 \left( \frac{p_{i+1} - p_i}{q_{i+1} - q_i} \right) + F'_2 \left( \frac{p_i - p_{i-1}}{q_i - q_{i-1}} \right) \\
= p_i^{1/\alpha} \int_{B_i} k(x)^{1-1/\alpha} \phi(dx) - F \left( \frac{p_{i+1} - p_i}{q_{i+1} - q_i} \right) + F \left( \frac{p_i - p_{i-1}}{q_i - q_{i-1}} \right) \\
= s_i(p_i) - d_i(p).
\]

The last equality follows from Step 1 and Step 5 in the proof of Theorem 1, the fact that \(p \in P\), and Equation (12) (see the proof of Lemma 1). Similarly,

\[
\nabla_i \psi(p) = p_n^{1/\alpha} \int_{B_n} k(x)^{1-1/\alpha} \phi(dx) - 1 + F \left( \frac{p_n - p_{n-1}}{q_n - q_{n-1}} \right) = s_n(p_n) - d_n(p).
\]

Thus, the excess supply function is given by \(\nabla \psi(p) = (\nabla_1 \psi(p), \ldots, \nabla_n \psi(p))\) where \(\nabla_i \psi(p) = s_i(p_i) - d_i(p)\) for all \(i\) from 1 to \(n\). Note that \(\nabla \psi(p) = 0\) implies that \((I, p)\) is implementable.

Our goal is to prove that \((I, p)\) is implementable if and only if \(p\) is the unique minimizer of \(\psi\). To show that \(\psi\) has at most one minimizer we prove that \(\psi\) is strictly convex on the convex set \(P\).

We proceed with the following steps:

**Step 1.** The set \(P\) is bounded, convex and open in \(\mathbb{R}^n\).

**Proof of Step 1.** We first show that \(P\) is bounded. Let \(\overline{p} = q_n b\) and let \(p = (p_1, \ldots, p_n)\) be a vector such that \(p_i > \overline{p}\) for some \(1 \leq i \leq n\). Then

\[
mq_i - p_i \leq bq_n - p_i < bq_n - \overline{p}.
\]

Hence \(d_i(p) = 0\). That is, \(p\) does not belong to \(P\). We conclude that \((\overline{p}, \ldots, \overline{p})\) is an upper bound of \(P\) under the standard product order on \(\mathbb{R}^n\). Clearly, \(P\) is bounded from below. Hence, \(P\) is bounded.

We now show that \(P\) is a convex set in \(\mathbb{R}^n\). Let \(p, p' \in P\) and \(0 < \lambda < 1\).

We need to show that \(\lambda p + (1 - \lambda)p' \in P\). First note that

\[
0 < \lambda p_1 + (1 - \lambda)p'_1 < \ldots < \lambda p_n + (1 - \lambda)p'_n
\]

so we only need to show that \(d_i(\lambda p + (1 - \lambda)p') > 0\) for all \(i = 1, \ldots, n\). Let \(1 \leq i \leq n - 1\). Because \(d_i(p) > 0\) and \(d_i(p') > 0\) we have \(F(p_{i+1} - p_i) - F(p_{i-1} - p_i) > 0\) and \(F(p'_{i+1} - p'_i) - F(p'_{i-1} - p'_i) > 0\). Strict monotonicity of \(F\) on its support implies \(\frac{p_{i+1} - p_i}{q_{i+1} - q_i} > \frac{p_{i-1} - p_i}{q_{i-1} - q_i}\) and \(\frac{p'_{i+1} - p'_i}{q_{i+1} - q_i} > \frac{p'_{i-1} - p'_i}{q_{i-1} - q_i}\). Hence,

\[
\frac{\lambda p_i + (1 - \lambda)p'_{i+1} - (\lambda p_i + (1 - \lambda)p'_i)}{q_{i+1} - q_i} > \frac{\lambda p_i + (1 - \lambda)p'_i - (\lambda p_{i-1} + (1 - \lambda)p'_i)}{q_i - q_{i-1}}.
\]
Using again the strict monotonicity of $F$ we conclude that

$$F\left(\frac{(\lambda p_{i+1} + (1 - \lambda)p'_{i+1} - (\lambda p_{i+1} + (1 - \lambda)p'_i)}{q_{i+1} - q_i}\right) - F\left(\frac{(\lambda p_i + (1 - \lambda)p'_i - (\lambda p_{i-1} + (1 - \lambda)p'_i)}{q_i - q_{i-1}}\right) > 0.$$ 

That is, $d_i(\lambda p + (1 - \lambda)p') > 0$. Similarly we can show that $d_n(\lambda p + (1 - \lambda)p') > 0$. Thus, $P$ is a convex set.

Because $d_i(p)$ is continuous on $P$ for all $1 \leq i \leq n$, it is immediate that the set $P$ is an open set in $\mathbb{R}^n$.

**Step 2.** The function $\psi$ is strictly convex on $P$.

**Proof of Step 2.** We claim that $\nabla \psi$ is strictly monotone on $P$, i.e., for all $p = (p_1, \ldots, p_n)$ and $p' = (p'_1, \ldots, p'_n)$ that belong to $P$ and satisfy $p \neq p'$, we have

$$\langle \nabla \psi(p) - \nabla \psi(p'), p - p' \rangle > 0$$

where $\langle x, y \rangle := \sum_{i=1}^n x_i y_i$ denotes the standard inner product between two vectors $x$ and $y$ in $\mathbb{R}^n$. Because $P$ is a convex set it is well known that $\nabla \psi$ is strictly monotone on $P$ if and only if $\psi$ is strictly convex on $P$.

Let $p, p' \in P$ and assume that $p \neq p'$.

Because $g$ is strictly increasing in $p_i$, $k$ is a positive function, and $\phi(B_i) > 0$, the supply function $s_i(p_i) = p_i^{-\alpha} \int_{B_i} k(x)^{-1/\alpha} \phi(dx)$ is strictly increasing in the price $p_i$. Thus, $s_i(p_i) > s_i(p'_i)$ if and only if $p_i > p'_i$. Combining the last inequality with the fact that $p \neq p'$ implies

$$\sum_{i=1}^n (p_i - p'_i)(s_i(p_i) - s_i(p'_i)) > 0.$$

Let $p_0 = p'_0 = 0$. We have

$$\sum_{i=1}^n (p_i - p'_i)(d_i(p) - d_i(p')) = \sum_{i=1}^{n-1} (p_i - p'_i) \left( F\left(\frac{p_{i+1} - p_i}{q_{i+1} - q_i}\right) - F\left(\frac{p_i - p_{i-1}}{q_i - q_{i-1}}\right)\right) - \sum_{i=1}^{n-1} (p_i - p'_i) \left( F\left(\frac{p'_i - p_{i-1}}{q_i - q_{i-1}}\right) - F\left(\frac{p_i - p'_{i-1}}{q_i - q_{i-1}}\right)\right) + (p_n - p'_{n-1}) \left( F\left(\frac{p'_{n-1}}{q_{n-1} - q_n}\right) - F\left(\frac{p_n - p_{n-1}}{q_n - q_{n-1}}\right)\right) = \sum_{i=1}^n (p_i - p_{i-1} - (p'_i - p'_{i-1})) \left( F\left(\frac{p'_i - p'_{i-1}}{q_i - q_{i-1}}\right) - F\left(\frac{p_i - p_{i-1}}{q_i - q_{i-1}}\right)\right) \leq 0.$$
The last inequality follows from the monotonicity of \( F \). Thus,

\[
\langle \nabla \psi(p) - \nabla \psi(p'), p - p' \rangle = \sum_{i=1}^{n} (s_i(p_i) - d_i(p) - (s_i(p'_i) - d_i(p'))(p_i - p'_i)
= \sum_{i=1}^{n} (p_i - p'_i)(s_i(p_i) - s_i(p'_i)) - \sum_{i=1}^{n} (p_i - p'_i)(d_i(p) - d_i(p'))
> 0.
\]

We conclude that \( \nabla \psi \) is strictly monotone on the convex set \( P \). Hence, \( \psi \) is strictly convex on \( P \).

**Step 3.** \((I, p)\) is implementable if and only if \( p \) is the unique minimizer of \( \psi \).

**Proof of Step 3.** Suppose that \((I, p)\) is implementable where \( I = \{B_1, B_2, \ldots, B_n\} \) and \( p = (p(B_1), \ldots, p(B_n)) \). Let \( D = \{D_I(B_i, p)\}_{B_i \in I} \), \( S = \{S(B_i, p(B_i))\}_{B_i \in I} \), and \( \lambda = \{\lambda_{B_i}\}_{B_i \in I} \) be an equilibrium under \((I, p)\).

Because \((I, p)\) is implementable we have \( p(B_i) > 0 \) for all \( B_i \in I \) and

\[
D_I(B_i, p) = S_I(B_i, p(B_i)) = \int_{B_i} g(x, p(B_i)) \phi(dx) > 0
\]

where the last inequality follows because \( g \) is positive (see the proof of Lemma 1) and \( \phi(B_i) > 0 \). We can assume without loss of generality that \( \mathbb{E}_{\lambda_{B_1}}(X) < \ldots < \mathbb{E}_{\lambda_{B_n}}(X) \). To see this, note that if \( \mathbb{E}_{\lambda_{B_i}}(X) = \mathbb{E}_{\lambda_{B_j}}(X) \) for some \( i < j \) then \( \min\{D_I(B_i, p), D_I(B_j, p)\} = 0 \) which contradicts the implementability of \((I, p)\). Thus, relabeling if needed, we can assume \( \mathbb{E}_{\lambda_{B_i}}(X) < \mathbb{E}_{\lambda_{B_j}}(X) \) for all \( i < j \). This implies that \( p(B_i) < p(B_j) \) for all \( i < j \). Thus, \( p \) belongs to \( P \). Hence, \( \nabla \psi(p) = 0 \) for some \( p \in P \). Because \( \psi \) is strictly convex on the convex set \( P \), there is at most one \( p \in P \) such that \( \nabla \psi(p) = 0 \). We conclude that for every information structure \( I \in \mathcal{I}(I_o) \) there exists at most one pricing function \( p \) such that \((I, p)\) is implementable.

Furthermore, because the set \( P \) is an open set, we have \( \nabla \psi(p) = 0 \) if and only if \( p \) is the unique minimizer of the strictly convex function \( \psi \) on \( P \). We conclude that \((I, p)\) is implementable if and only if \( p \) is the unique minimizer of \( \psi \).

**Proof of Theorem 2.** We show that \( C^Q \) is regular. Then, Theorem 1 implies that the optimal menu is 1-separating, and hence, the optimal information structure consists of one set of sellers. We proceed with the following steps:

**Step 1.** Let \( \{B\} \) be a 1-separating information structure and let \( \{(p(B), \mathbb{E}_{\lambda_B}(X))\} \in \mathcal{V}^Q(\{B\}) \). Then for every \( p > 0 \) we have \( S_{\{B\}}(B, p) \geq D_{\{B\}}(B, p) \) if and only if \( p \geq p(B) \).

**Proof of Step 1.** Assume in contradiction that \( p(B) > p > 0 \) and \( S_{\{B\}}(B, p) \geq D_{\{B\}}(B, p) \). Recall that the sellers’ expected quality \( \mathbb{E}_{\lambda_B}(X) \) does not depend on the price (see Lemma 1). We
have

\[
1 - F \left( \frac{p}{\mathbb{E}_{\lambda B}(X)} \right) = D_{\{B\}}(B, p) \leq S_{\{B\}}(B, p)
\]

\[
= \int_B g(x, p) \phi(dx)
\]

\[
< \int_B g(x, p(B)) \phi(dx)
\]

\[
= 1 - F \left( \frac{p(B)}{\mathbb{E}_{\lambda B}(X)} \right)
\]

which is a contradiction to the fact that \( F \) is increasing. The strict inequality follows because \( g \) is strictly increasing in the price and \( \phi(B) > 0 \) (see the proof of Lemma 1). The last equality follows from the fact that \( \{(p(B), \mathbb{E}_{\lambda B}(X))\} \in \varphi^Q(\{B\}) \). This proves that \( S_{\{B\}}(B, p) \geq D_{\{B\}}(B, p) \) implies \( p \geq p(B) \). The other direction is proven in a similar manner.

**Step 2.** Suppose that \( (\{B\}, p(B)) \) induces a menu that is maximal in \( C.Q \). Then \( B \in I_o = \{A_1, \ldots, A_l\} \).

**Proof of Step 2.** Let \( I = \{B\} \) be a 1-separating information structure and assume that \( B \neq A_i \) for all \( A_i \in I_o \). Thus, \( B \) is a union of at least two elements of \( I_o \). Let \( k \) be highest index among these elements. Hence, \( \mathbb{E}_{\lambda A_j}(X) \leq \mathbb{E}_{\lambda A_k}(X) \) for all \( A_j \subseteq B, A_j \in I_o \). We have

\[
\mathbb{E}_{\lambda B}(X) = \frac{\int_B x(k(x))^{-1/\alpha} \phi(dx)}{\int_B (k(x))^{-1/\alpha} \phi(dx)}
\]

\[
= \frac{\sum_{A_i: A_i \subseteq B, A_i \in I_o} \int_{A_i} x(k(x))^{-1/\alpha} \phi(dx)}{\sum_{A_i: A_i \subseteq B, A_i \in I_o} \int_{A_i} (k(x))^{-1/\alpha} \phi(dx)}
\]

\[
\leq \frac{\int_{A_k} x(k(x))^{-1/\alpha} \phi(dx)}{\int_{A_k} (k(x))^{-1/\alpha} \phi(dx)}
\]

\[
= \mathbb{E}_{\lambda A_k}(X).
\]

The first and last equalities follow from Lemma 1. The inequality follows from the elementary inequality \( \sum_{i=1}^n x_i / \sum_{i=1}^n y_i \leq \max_{1 \leq i \leq n} x_i / y_i \) for positive numbers \( x_1, \ldots, x_n \) and \( y_1, \ldots, y_n \).

Assume that \( (I, p(B)) \) is implementable and that it induces the menu \( \{(p(B), \mathbb{E}_{\lambda B}(X))\} \). Then the arguments above imply \( \mathbb{E}_{\lambda B}(X) \leq \mathbb{E}_{\lambda A_k}(X) \).

We claim that \( p(B) < p(A_k) \) where \( p(A_k) \) is the (unique) equilibrium price under the information structure \( \{A_k\} \) (the existence of this equilibrium price follows from the arguments in Step 3). To
see this, note that

\[ S_{A_k}(B, p(B)) = \int_{A_k} \left( \frac{p(B)}{k(x)} \right)^{1/\alpha} \phi(dx) \]

\[ < \frac{p(B)}{k(x)} \phi(dx) \]

\[ = S_I(B, p(B)) = D_I(B, p(B)) \]

\[ = 1 - F \left( \frac{p(B)}{\mathbb{E}_{\lambda_k}(X)} \right) \]

\[ \leq 1 - F \left( \frac{p(B)}{\mathbb{E}_{\lambda_k}(X)} \right) \]

\[ = D_{A_k}(B, p(B)). \]

The first inequality follows from the facts that \( k \) is a positive function, \( B \supseteq A_k \), and \( \phi(B \setminus A_k) > 0 \). The second inequality follows from the fact that \( F \) is increasing. Hence, the demand exceeds the supply under the price \( p(B) \). From Step 1 we have \( p(B) < p(A_k) \). Thus, the information structure-price pair \((\{B\}, p(B))\) does not induce a menu that is maximal in \( C^Q_1 \).

**Step 3.** \( C^Q \) is regular.

**Proof of Step 3.** Let \((I, p)\) be implementable where \( I = \{B_1, B_2, \ldots, B_n\} \). Let

\[ C = \{(p(B_1), \mathbb{E}_{\lambda_{B_1}}(X)), \ldots, (p(B_n), \mathbb{E}_{\lambda_{B_n}}(X))\} \]

be the menu that is induced by \((I, p)\). Suppose that \((D, S, \lambda)\) implements \((I, p)\). We can assume that \( D(B_i, p) > 0 \) for all \( B_i \in I \) and \( 0 < p(B_1) < \ldots < p(B_n) \) (see the proof of Proposition 3). Note that \( D(B_i, p) > 0 \) for \( B_i \in I \) implies \( 0 < \mathbb{E}_{\lambda_{B_1}}(X) < \ldots < \mathbb{E}_{\lambda_{B_n}}(X) \).

Consider the 1-separating information structure \( I' = \{B_n\} \).

We claim that there exists a \( p^q(B_n) \geq p(B_n) \) such that \((I', p^q(B_n))\) is implementable and \((I', p^q(B_n))\) induces the menu \( \{(p^q(B_n), \mathbb{E}_{\lambda_{B_n}}(X))\} \).

From Step 1 in the proof of Theorem 1, we have \( D_{I'}(B_n, p(B_n)) = 1 - F \left( \frac{p(B_n)}{\mathbb{E}_{\lambda_{B_n}}(X)} \right) \). Note that there exists a \( \bar{p} > p(B_n) \) such that \( D_{I'}(B_n, \bar{p}) = 0 \) (for example we can choose \( \bar{p} = \mathbb{E}_{\lambda_{B_n}}(X)b \)).

Define the excess demand function \( \tau : [p(B_n), \bar{p}] \rightarrow \mathbb{R} \) by \( \tau(\cdot) = D_{I'}(B_n, \cdot) - S_{I'}(B_n, \cdot) \). From the definition of \( \bar{p} \) we have \( \tau(\bar{p}) < 0 \).

Note that

\[ \tau(p(B_n)) = D_{I'}(B_n, p(B_n)) - S_{I'}(B_n, p(B_n)) \]

\[ = D_{I'}(B_n, p(B_n)) - S_I(B_n, p(B_n)) \]

\[ \geq D_I(B_n, p) - S_I(B_n, p(B_n)) = 0 \]

The first equality follows from the definition of \( \tau \). The second equality follows from the fact that
\[ S_I(B_n, p(B_n)) = S_I(B_n, p(B_n)) = \int_{B_n} g(x, p(B_n)) \phi(dx), \] i.e., seller \( x \)'s optimal quantity decision does not change when the information structure changes. The inequality follows from the definition of the demand function. The last equality follows from the fact that \((I, p)\) is implementable.

Because the distribution function \( F \) and the optimal quantity function \( g \) are continuous in the price, the excess demand function \( \tau \) is continuous on \([p(B_n), \overline{p}]\). Thus, from the intermediate value theorem, there exists a \( p^\theta(B_n) \) in \([p(B_n), \overline{p}]\) such that \( \tau(p^\theta(B_n)) = 0 \). We conclude that \((I', p^\theta(B_n))\) is implementable and that \( p^\theta(B_n) \geq p(B_n) \). Thus, the menu \( \{p^\theta(B_n), \mathbb{E}_{\lambda B_n}(X)\} \) is a 1-separating menu that belongs to \( C^1 \) and condition (i) of Definition 1 holds.

Condition (ii) of Definition 1 immediately follows from using Step 2 to conclude that \( B^H \in I_o \) and applying Step 1 to the information structure \( \{B^H\} \). Thus, \( C^Q \) is regular.

Theorem 1 implies that the optimal 1-separating menu is maximal. Combining this with Step 2 imply that the optimal 1-separating information structure-price pair induces a menu that is maximal in \( C^Q \) and \( B^* \in I_o = \{A_1, \ldots, A_l\} \) where \( I^* := \{B^*\} \) is the optimal information structure. This concludes the proof of the Theorem. \( \blacksquare \)

### A.3 Proofs of Section 6

**Proof of Theorem 3.** Let \( I = \{B_1, \ldots, B_n\} \) be an information structure and let \( L(I) = \{G_1, \ldots, G_n\} \).

(i) Suppose that \( C \in \varphi^P(I) \). Let \( p = (p(B_1), \ldots, p(B_n)) \) be the equilibrium price vector that is associated with the menu \( C \). We claim that \( p(B_i) = c(G_i) \).

If \( p(B_i) < c(G_i) \) then for every seller \( x \in B_i \) we have \( U(x, H(B_i), p(B_i), p) < 0 \) so the mass of sellers that participate in the platform equals to 0 which contradicts the implementability of \( I \). If \( p(B_i) > c(G_i) \) then the sellers’ pricing decisions are not optimal. Sellers in \( G_i \subseteq B_i \) can decrease their price and increase their utility. Thus, \( I \) is not implementable. We conclude that \( p(B_i) = c(G_i) \) for all \( B_i \in I \).

Let \( B_i \in I \). Because \( c(A_i) < c(A_j) \) whenever \( i < j \) we have \( U(x, H(B_i), p(B_i), p) < 0 \) for sellers \( x \in B_i \setminus G_i \) under the equilibrium price vector \( p = (c(G_1), \ldots, c(G_n)) \). Thus, sellers in \( B_i \setminus G_i \) do not participate in the platform and only the sellers in \( G_i \subseteq B_i \) participate in the platform. This completes the proof of part (i).

(ii) First note that \( D_{\{B_n\}}(B_n, c(G_n)) \geq D_I(B_n, (c(G_1), \ldots, c(G_n))) > 0 \) (see the proof of Theorem 2). Furthermore, under the price \( c(G_n) \), it is optimal for all the sellers in \( G_n \subseteq B_n \) to participate in the platform and for all the sellers in \( B_n \setminus G_n \) to not participate in the platform. So \( \mathbb{E}_{\lambda G_n}(X) \) is the sellers’ expected quality given the sellers’ optimal entry decisions and the price \( c(G_n) \). Also, it is easy to see that the price \( c(G_n) \) maximizes the participating sellers’ utility. From the quantity allocation function \( h_I \) it follows immediately that the market clearing condition is satisfied. We conclude that \( \{(c(G_n), \mathbb{E}_{\lambda G_n}(X))\} \in \varphi^P\{\{B_n\}\} \).

(iii) From part (i) we have \( C_o = \{(c(A_1), \mathbb{E}_{\lambda A_1}(X)), \ldots, (c(A_l), \mathbb{E}_{\lambda A_l}(X))\} \). Let \( C \in \varphi^P(I) \). Then part (i) implies that \( C = \{(c(G_1), \mathbb{E}_{\lambda G_1}(X)), \ldots, (c(G_n), \mathbb{E}_{\lambda G_n}(X))\} \). Thus \( C \in 2^{C_o} \). We conclude that \( C^P \subseteq 2^{C_o} \). Now consider a menu \( C' = \{(c(A_{\mu_1}), \mathbb{E}_{\lambda A_{\mu_1}}(X)), \ldots, (c(A_{\mu_j}), \mathbb{E}_{\lambda A_{\mu_j}}(X))\} \in \)
$2^{C_o}$ for sum increasing numbers $\{\mu_k\}_{k=1}^j$. Consider the information structure $I' = \{A_{\mu_1}, \ldots, A_{\mu_j}\}$. Because $I_o$ is implementable we have $D_{I'}(A_{\mu_i}, (c(A_{\mu_1}), \ldots, c(A_{\mu_j}))) \geq D_{I_o}(A_{\mu_i}, (c(A_1), \ldots, c(A_l))) > 0$ for all $A_{\mu_i} \in I'$. An analogous argument to the argument in part (ii) shows that $I'$ is implementable and $C' \in \varphi^P(I')$. That is, $2^{C_o} \subseteq C^P$. We conclude that $2^{C_o} = C^P$ which proves part (iii).

References

Akerlof, G. A. (1970): “The market for ‘lemons’: Quality uncertainty and the market mechanism,” *The Quarterly Journal of Economics*, 84, 488–500.

Alizamir, S., F. de Véricourt, and S. Wang (2020): “Warning against recurring risks: An information design approach,” *Management Science*.

Anunrojwong, J., K. Iyer, and V. Manshadi (2020): “Information design for congested social services: Optimal need-based persuasion,” *arXiv preprint arXiv:2005.07253*.

Arnosti, N., R. Johari, and Y. Kanoria (2018): “Managing congestion in matching markets,” *Working paper*.

Aumann, R. J. and M. Maschler (1966): “Game theoretic aspects of gradual disarmament,” *Report of the US Arms Control and Disarmament Agency*, 80, 1–55.

Bergemann, D., J. Shen, Y. Xu, and E. M. Yeh (2011): “Mechanism Design with limited information: the case of nonlinear pricing,” in *International Conference on Game Theory for Networks*, Springer, 1–10.

Bimpikis, K., S. Ehsani, and M. Mostagir (2019): “Designing dynamic contests,” *Operations Research*, 67, 339–356.

Bimpikis, K., Y. Papanastasiou, and W. Zhang (2020): “Information Disclosure in Online Platforms: Optimizing for Supply,” *Working paper*.

Candogan, O. (2019): “Persuasion in Networks: Public Signals and k-Cores,” *Working paper*.

——— (2020): “Information Design in Operations,” *Available at SSRN*.

Candogan, O. and K. Drakopoulos (2020): “Optimal signaling of content accuracy: Engagement vs. misinformation,” *Operations Research*, 68, 497–515.

Donaker, G., H. Kim, M. Luca, M. Weber, S. House Rich, G. Duhon, R. Berman, S. Melumad, C. Humphrey, R. Meyer, et al. (2019): “Designing Better Online Review Systems.” *Harvard Business Review. Nov/Dec2019*, 97, 3.

Drakopoulos, K., S. Jain, and R. S. Randhawa (2019): “Persuading customers to buy early: The value of personalized information provisioning,” *Working paper*.

Filippas, A., J. J. Horton, and J. Golden (2018): “Reputation inflation,” in *Proceedings of the 2018 ACM Conference on Economics and Computation*, 483–484.

Garg, N. and R. Johari (2019): “Designing Informative Rating Systems for Online Platforms: Evidence from Two Experiments,” *Working paper*.

Hui, X., M. Saeedi, G. Spagnolo, and S. Tadelis (2018): “Certification, reputation and
entry: An empirical analysis,” Working paper.

IMMORLICA, N., J. MAO, A. SLIVKINS, AND Z. S. WU (2019): “Bayesian Exploration with Heterogeneous Agents,” in The World Wide Web Conference, ACM, 751–761.

KAMENICA, E. (2019): “Bayesian persuasion and information design,” Annual Review of Economics, 11, 249–272.

KAMENICA, E. AND M. GENTZKOW (2011): “Bayesian persuasion,” American Economic Review, 101, 2590–2615.

KANORIA, Y. AND D. SABAN (2019): “Facilitating the Search for Partners on Matching Platforms: Restricting Agents’ Actions,” Working paper.

KOSTAMI, V. (2019): “Price and leadtime disclosure strategies in inventory systems,” Available at SSRN 3431895.

LEHRER, E., D. ROSENBERG, AND E. SHMAYA (2010): “Signaling and mediation in games with common interests,” Games and Economic Behavior, 68, 670–682.

LINGENBRINK, D. AND K. IYER (2018): “Signaling in online retail: Efficacy of public signals,” Working paper.

——— (2019): “Optimal signaling mechanisms in unobservable queues,” Operations research, 67, 1397–1416.

MASKIN, E. AND J. RILEY (1984): “Monopoly with incomplete information,” The RAND Journal of Economics, 15, 171–196.

MEIGS, E., F. PARISE, A. OZDAGLAR, AND D. ACEMOGLU (2020): “Optimal dynamic information provision in traffic routing,” arXiv preprint arXiv:2001.03232.

MUSSA, M. AND S. ROSEN (1978): “Monopoly and product quality,” Journal of Economic theory, 18, 301–317.

MYERSON, R. B. (1981): “Optimal auction design,” Mathematics of Operations Research, 6, 58–73.

OSTROVSKY, M. AND M. SCHWARZ (2010): “Information disclosure and unraveling in matching markets,” American Economic Journal: Microeconomics, 2, 34–63.

PAPANASTASIOU, Y., K. BIMPIKIS, AND N. SAVVA (2017): “Crowdsourcing exploration,” Management Science, 64, 1727–1746.

ROMANYUK, G. AND A. SMOLIN (2019): “Cream skimming and information design in matching markets,” American Economic Journal: Microeconomics, 11, 250–76.

Tadelis, S. (2016): “Reputation and feedback systems in online platform markets,” Annual Review of Economics, 8, 321–340.

VARADARAJA, A. B., K. BHAWALKAR, AND H. XU (2018): “Targeting and Signaling in Ad Auctions,” Working paper.

VELLIDI, N. (2018): “Ratings Design and Barriers to Entry,” Working paper.

WILSON, R. B. (1993): Nonlinear pricing, Oxford University Press on Demand.