Experimental and numerical analysis of turbulent velocity fluctuations in a Francis turbine draft tube in part load operation

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Abstract. The part load operation of turbines with non-regulated runner blades is often associated with the occurrence of a rotating corkscrew-shaped vortex rope in the draft tube. Due to its negative influence on power plants, this phenomenon is an important factor in the design process of a water turbine. Investigations using CFD aim to predict the draft tube flow, especially regarding the efficiency and the losses of the turbine. As there are still discrepancies between CFD simulations and experimentally gained data, a research project on this subject was initiated, including extensive measurements with 2D-Laser-Doppler-Anemometry. CFD simulations were carried out using high-resolution hybrid RANS-LES turbulence models. In this paper, the interrelation of the turbulent velocity fluctuations and the periodic vortex rope movement is being examined. Therefore, the measured data is provided time averaged and phase resolved. The specialties of the swirling flow influenced by the periodic vortex concerning the turbulence are illustrated on two planes, one downstream the runner in the cone and one at the lowest position of the draft tube. The results show good conformity with the CFD data including periodic part of the velocity fluctuations. The dependence of the velocity field on the rotating vortex rope is significantly higher in the cone than at the end of the draft tube bend.

1. Introduction

Hydraulic turbines in power plants are increasingly operated in off-design states due to the growing rate of fluctuating energy supply. In part load operation, a rotating vortex rope can develop in the draft tube, moving with a frequency of about 20% - 40% of the runner-frequency. This periodic flow phenomenon can lead to pressure fluctuations, power fluctuations, efficiency drop and damage to material [1]. Thus, the exact knowledge about the flow-processes in the draft tube is of great importance in the design of turbines and the operation of existing power plants.

Especially in part load operation, the complex behavior of the swirling flow is a challenge for CFD simulations and the currently usual RANS turbulence models. The velocity fluctuations are an important criterion to assess the quality of numerical data as it demands high accuracy on turbulence modelling and computational grid. For validation of CFD simulations conducted with such high effort, measurement data with high accuracy and extent are required, too. In literature, many investigations
concerning part load flow can be found, offering a wide range of experimental data using different measurement techniques. Examples include the FLINDT project [2], [3] and the Francis-99 workshop [4]. Based on this data, several numerical studies were carried out, focusing on different aspects of the complex flow.

This work looks at the periodic influence to the draft tube flow in its entirety, regarding turbulent velocity fluctuations [5]. Therefore velocity measurements using 2D-Laser-Doppler-Anemometry (LDA) were carried out on planes along the turbine draft tube [6]. This non-invasive measurement method is well suited for the acquisition of velocity fluctuations and flow instabilities.

First, the experimental and numerical setup is described. The parameters used to investigate the flow are then introduced, regarding the specifics of the measurement technique. After presenting the time averaged data, the phase resolved results of the measurements are analyzed and compared to the CFD results.

2. Experimental Setup

For the investigation, a model-sized Francis turbine with a runner outlet diameter of \( D = 372 \) mm has been installed in the test rig of the Institute of Fluid Mechanics and Hydraulic Machinery (IHS). The turbine is schematically shown in figure 1. The global coordinate system is located in the runner axis at the guide vanes mid elevation. The elbow-shaped draft tube is connected to a downstream tank. A pier is installed in the diffuser, dividing it into two equally sized channels.

Along the draft tube, acrylic glass inserts are installed to gain optical access to the flow. In this paper, two measurement planes are presented. One plane is located horizontally in the cone at 0.63 \( D \) downstream the runner. The diameter of the plane is 1.16 \( D \). It is measured by means of 32 radial lines. The second plane is located in the diffuser, at the lowest point of the draft tube, right before the pier. It consists of a vertically oriented Cartesian grid. The measurement technique provides two-dimensional data perpendicular to the laser beam. Thus in the cone, the axial component in \( z \)-direction and the tangential component in \( \varphi \)-direction can be obtained using the coordinate system in figure 1. In the diffuser, the components in \( x \)- and \( y \)-directions are measured. Here, a right-handed local coordinate system \([x^*y^*z^*]\) is used, being located on the lower wall in the middle of the plane. The vortex rope position is found by means of a trigger system. Therefore, a pressure sensor is installed between the runner and the LDA measurement plane. For a detailed description of the test stand and the measurement process see [7].

![Figure 1. Overview of the Francis model turbine with the right-handed coordinate system. The measurement positions are marked with blue dotted lines.](image)

The part load operating point is at about 70 % discharge of the best efficiency point. It was chosen with respect to obtain a fully developed vortex rope. The rotating frequency of the corkscrew vortex rope is 0.30 \( f_r \) related to the runner frequency with a relatively stable behavior.
3. CFD simulations

The CFD simulations were carried out at the IHS with the proposition to achieve the highest possible accuracy in predicting the draft tube flow and are explicitly described in [8] and [9]. The complete turbine from spiral case to draft tube as well as tail water tank was included in the numerical model. Different mesh sizes up to 300 million knots were compared to eliminate the error caused by the mesh as far as possible. Additionally, the RANS-SST turbulence model was compared to a hybrid SAS-SST model approach. To perform those extensive simulations, high parallelization is necessary.

For the comparison of simulation and measurement, only the version with the largest mesh and the SAS-SST turbulence model is used. The turbulence provided by the CFD simulations consists of a modelled and a numerically resolved part. Data presented here use the combined value of both, modeled and the resolved part.

4. Definition of the investigated Data

The velocity field is related to the vortex rope position by means of phase resolving. A triple decomposition according to [10] and [11] is performed, resulting in the following definition for the velocity field:

\[ \nu = \nu' + \bar{\nu} + \tilde{\nu} \]  (1)

Here, the instantaneous velocity is divided in a chaotic and hence turbulent fluctuation \( \nu' \), a periodic fluctuation \( \bar{\nu} \) and the overall mean value \( \tilde{\nu} \). Besides the velocity vectors, the velocity fluctuations are used to describe and validate the flow. Therefore, the variance is calculated. With the definitions made in equation 1, the variance at a specific geometric location is defined as:

\[ \sigma^2_{xx} = \frac{1}{N-1} \sum_{i=1}^{N} (\nu_{x,i} - \bar{\nu}_{x,i})^2 \]  (2)

This value contains both the turbulent and the periodic part \( \nu_{x,i} \) and \( \bar{\nu}_{x,i} \). The periodic influence can be eliminated to obtain the turbulent energy in one spatial direction. Thus, the phase averaged variance \( \tilde{\sigma}^2_{xx} \) corresponding to turbulence is independent of the known periodic fluctuations:

\[ \tilde{\sigma}^2_{xx} = \frac{1}{N-1} \sum_{i=1}^{N} (\nu_{x,i} - \bar{\nu}_{x} - \bar{\nu}_{x,i})^2 \]  (3)

The mean values are calculated by the arithmetic time average of all measured counts in one point. To synchronize the data to the vortex rope position, an angle \( \alpha_{VR} \) is introduced, indicating the circumferential position. According to the coordinate system given in figure 1, at the angle \( \alpha_{VR} = 0^\circ \) the vortex core is located on the positive x-axis. In the cone plane the vortex moves in positive \( \phi \)-direction. One vortex rope revolution is divided into 180 equidistant bins with an increment of \( \Delta \alpha_{VR} = 2^\circ \). Within one bin, the periodic velocity fluctuation caused by the vortex rope is low, including an uncertainty caused by the limited discretization.

This method to subdivide the circumference in a number of increments is used to assess the instantaneous phase resolved variance \( \sigma^2_{xx,\alpha_{VR}} \) at one spatial point, excluding the periodic part of the fluctuations. It is calculated by using the velocity data of one single bin and therefore associated to a specific vortex rope angle \( \alpha_{VR} \):

\[ \sigma^2_{xx,\alpha_{VR}} = f(\alpha_{VR}) \]  (4)

Afterwards, the mean value of the bin variance \( \sigma^2_{xx,\alpha_{VR}} \) over a complete vortex rope rotation is calculated. By this means the phase averaged variance \( \tilde{\sigma}^2_{xx} \) in one spatial point introduced in equation 3 is obtained:

\[ \tilde{\sigma}^2_{xx} = \frac{1}{360} \sum_{\alpha_{VR}=0}^{360} \sigma^2_{xx,\alpha_{VR}} \]  (5)

All values are provided in a normalized form. As a reference, either the maximum length scale of the measurement plane \( l_{ref} \) or the mean velocity \( v_{ref} = Q/A_{ref} \) are used.

5. Results

The results are being presented on measurement lines as well as on whole planes. In the cone, the data are interpolated to illustrate a continuous plane and to avoid gaps near the cone wall caused by the
distance between measurement lines. Therefore the velocity is mapped on a uniform grid using a 2\textsuperscript{nd} order method. In the diffuser, the data can be plotted without further modification.

5.1. Time averaged results
The time averaged velocities in the cone are shown in figure 2. In axial direction, data are presented as a contour plot, giving an overview of the simulated and measured velocity field. Measurement data are represented by solid lines, CFD data by dotted lines. The profile is characteristic for part load flow with the main flow being located at the outer rim near the cone wall. A marginal asymmetry can be observed. The comparison shows good conformity along the whole plane.

The circumferential velocity is plotted on the x-axis exemplary. Close to the runner axis, an N-shaped profile is visible in the measurement data, which is caused by the vortex rope rotation. In the simulation, this characteristic shape is not present. At larger radii, the values match relatively well.

Figure 2. Time averaged velocities of the measurement and CFD data in the cone plane.

Figure 3 shows the time averaged velocity fluctuations on the x-axis in the cone. The vortex rope moves on a radius of about $r/r_{ref} = 0.45$. This leads to a periodically rotating flow field inside this area. Therefore, the tangential variance increases significantly towards the middle of the cone, reaching values far above the other areas and far beyond the axial variance. This effect is primarily related to the moving cork screw vortex and hardly assigned to the turbulent velocity fluctuations.

Figure 3. Time averaged velocity fluctuations according to eq. 2 of the measurement and CFD data in the cone.
In both, axial and tangential variance, measurement and CFD data show good agreement in the quantitative comparison. However, deviations can be found in the axial variance along the entire diameter. For \( r/r_{ref} > 0.5 \), the profile of the measurement data shows a more uniform distribution. The high peak close to the cone wall in the simulation is caused by the boundary layer, which is not included in the measurements.

In the diffuser, a difference between the right and the left side of the plane can be found, see figure 4. The time averaged velocities are plotted in the main flow direction \( x^* \) as contour lines. The discharge is significantly shifted to the right side of the diffuser. On the left side, a large area of values less than 20 % of the maximum velocity is present. This profile is a result of the flow influenced by the vortex rope. The comparison of the simulated and the measured data shows slight differences only, the overall profile appears fairly similar. At \( y^* = 0 \), the velocity contour shows a S-shaped profile, which matches quite well between the measurement and the simulation.

![Figure 4](attachment://figure4.png)

**Figure 4.** Time averaged velocities in mean flow direction of the measurement and CFD data in the diffuser plane.

The fluctuations in the diffuser plane are shown in figure 5 as variance in the mean flow direction on seven vertical lines. In general, the gradient between the upper and the lower side is relatively small. On the right side of the diffuser, towards negative \( y^* \)-values, the fluctuations in the values of the simulation and the measurement are higher.

![Figure 5](attachment://figure5.png)

**Figure 5.** Time averaged velocity fluctuations according to eq. 2 in mean flow direction on seven lines in the diffuser plane.

For the comparison of the measurements and the CFD data, the difference value (\( \sigma_{xx,meas}^2 - \sigma_{xx,sim}^2 \)) is calculated and presented as a contour plot in figure 6. Here, on the left diffuser side only little differences are visible. On the right side, two areas with high variation stand out. At \( y^* \approx -0.1 \) the measured value is significantly higher than the simulation, whereas at \( y^* \approx -0.32 \) the simulation data lie above the measurement data.

![Figure 6](attachment://figure6.png)
5.2. Phase resolved results

By phase resolving the velocity field, it can be accessed at a certain vortex rope position. Figure 7 shows the axial velocities of the CFD simulations and the measurements. The results are presented as contour plots with the same scale. The vortex rope is exemplary shown at four circumferential positions: At $\alpha_{VR} = 0^\circ$, the vortex rope cuts the positive x-axis, at $\alpha_{VR} = 90^\circ$ it cuts the positive y-axis.

![Figure 7. Phase resolved velocity field in axial direction in the cone plane at four vortex rope positions $\alpha_{VR} = 0^\circ, \alpha_{VR} = 30^\circ, \alpha_{VR} = 60^\circ$ and $\alpha_{VR} = 90^\circ$.](image)
Near the vortex rope position, a high velocity gradient occurs between the vortex core at $r/r_{rel} = 0.45$ and the cone wall. Here, the highest velocity values can be found in a small area close to the wall precluding laser measurements. For the most part, the velocities show a good conformity for values $v_z/v_{ref} < -0.7$. Within this border, an area with small gradients and two backflow zones are visible, in figure 7 identified by red contour lines. On the side of the cone facing the vortex rope, the backflow zone is significantly bigger in the CFD simulation. On the opposite side, the zone is relatively large in the measurements, while the CFD data show no backflow at all. The velocity field is very sensitive to the operating point, which is found to show minor deviations between measurement and CFD. This presumably leads to the differences within the low-velocity zone.

The phase resolved velocity fluctuations are presented on the measurement line $y = 0$ corresponding to the x-axis, see figure 8. The variance in circumferential and axial direction is given at the vortex rope positions $\alpha_{VR} = 0^\circ$ and $\alpha_{VR} = 90^\circ$. For the measurements, the bin variance $\sigma_{VR}^2$ and the phase averaged variance $\tilde{\sigma}^2$ are plotted, cf. equations 4 and 5. The simulated data are given as instantaneous values at the current vortex rope position, corresponding to the bin variance.

![Figure 8](image_url)

Figure 8. Phase resolved velocity fluctuations in axial and tangential direction on the x-axis measurement line. The bin variance $\sigma_{VR}^2$ according to eq. 4 is presented for the measurement (red circles) and the simulation (green line); the phase averaged variance $\tilde{\sigma}^2$ according to eq. 5 is presented for the measurement (blue crosses). The vortex rope positions $\alpha_{VR} = 0^\circ$ and $\alpha_{VR} = 90^\circ$ are shown exemplary.
The phase-dependant values of the bin variance $\sigma_{V_R}^2$ shows a gain near the vortex core ($\alpha_{VR} = 0^\circ$), as expected. The measured radial position of the peak accords with the CFD simulations quite well. However, the maximum value at the vortex core is higher in the measured data for the axial and the tangential variance. Along the remaining line, the comparison shows relatively good conformity.

At $\alpha_{VR} = 90^\circ$, the variance profile is relatively symmetric. The axial measurement data has a small peak in the cone centre, which is absent in the simulation. This effect can be seen along the complete vortex revolution and therefore in the phase averaged variance $\tilde{\sigma}^2$. The simulated data is constantly below the measurement in both the axial and the tangential deviation.

Compared to the mean value plotted in figure 3, no significant increase is visible in the cone centre for the phase averaged tangential variance $\tilde{\sigma}_{\phi\phi}^2$. Thus, it can be concluded that the phase averaged variance correlates to the actual turbulent fluctuation, excluding the periodic part.

![Figure 9](image)

Figure 9. Phase resolved velocity fields in main flow direction. The vortex rope positions $\alpha_{VR} = 0^\circ$, $\alpha_{VR} = 30^\circ$, $\alpha_{VR} = 60^\circ$ and $\alpha_{VR} = 90^\circ$ are shown exemplary.

Similar to figure 7, four vortex rope positions from $\alpha_{VR} = 0^\circ$ to $\alpha_{VR} = 90^\circ$ are presented for the diffuser, see figure 9. The flow field is more irregular in the diffuser than in the cone. Consequently, the periodicity of the velocity is not as distinct as in the cone. The overall appearance of the contour
profile does not vary significantly. Most noticeable is the area of low discharge on the left side of the pier. Here, at $y^* \approx 0.03$, a backflow zone appears, which is not visible in the time average in figure 4. At $\alpha_{VR} = 30^\circ$ the backflow is only present in the simulation. For $\alpha_{VR} = 60^\circ$ and $\alpha_{VR} = 90^\circ$ it also occurs in the measurement data. In this area with low velocity values, the difference between measurement and CFD is higher than in the planes presented previously.

Figure 10 shows the normalized variance in x-direction, depending on the position of the vortex rope. In relation to the velocity values, the velocity fluctuations show relatively few deviations over a vortex rope rotation. On line $y^* = 0.26$, the fluctuation in simulation and measurement remains at a value of about 0.1. The largest values occur on the opposite side, at $y^* = -0.26$. Here in particular, the simulation shows a large dependence on the vortex rope position, which is not visible in the measurement to this extent. Regarding the time averaged data in figure 5, the phase resolved values do not differ as in the cone. This leads to the assumption, that within the diffuser, the periodic part of the velocity fluctuations is relatively low in comparison to the cone region.

Figure 10. Phase resolved velocity fluctuations according to eq. 4 in mean flow direction on seven lines across the diffuser plane for the vortex rope positions $\alpha_{VR} = 0^\circ$, $\alpha_{VR} = 90^\circ$, $\alpha_{VR} = 180^\circ$ and $\alpha_{VR} = 270^\circ$. 
6. Conclusion
The results of CFD simulations and LDA measurements of the draft tube flow with a rotating vortex rope have been compared. The time-dependent velocity has been decomposed into the mean, periodic and fluctuating part for both simulated and measured data. By dividing the vortex rope rotation into 180 bins the turbulent fluctuations can be presented for certain vortex rope positions. This method allows very detailed validation of CFD results considering unsteady effects in the draft tube. Especially the turbulent fluctuation of the flow is a sensitive parameter under the periodic influence of a rotating vortex rope. Therefore, the comparisons made under this basic prerequisite can be considered relatively accurate.

In the cone, the periodic part is dominant in both, velocity field and velocity fluctuations. Downstream, in the diffuser, the dependence on the periodic rotation is less visible in the velocity fluctuations. Hence, the turbulence is predominant in the calculated variance.

With the highly precise measurements presented in this paper a database has been created, that makes detailed analysis of the fluid mechanics in part load operation possible and is used to validate numerical methods. The comparison shows that the dominating structures in the flow field are resolved by the CFD and therefore accord very well to the measured data. In general, a good match between measurement and simulation can be stated which is sufficient for most present technical questions. Regarding the phase resolved velocity in main flow direction, there is still potential to optimize the numerical simulation approach, in particular for the vortex core region.

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