EFFICIENCY, RTS, AND MARGINAL RETURNS FROM SALARY ON THE PERFORMANCE OF THE NBA PLAYERS: A PARALLEL DEA NETWORK WITH SHARED INPUTS

SAEED ASSANI
Department of Mathematics, Nanjing University of Aeronautics and Astronautics
Nanjing 210016, China

MUHAMMAD SALMAN MANSOOR*
School of Management, University of Science and Technology of China
Hefei 230026, China

FAISAL ASGHAR
School of Management, University of Science and Technology of China
Hefei 230026, China
Government College University Faisalabad
Punjab, Pakistan

YONGJUN LI AND FENG YANG
School of Management, University of Science and Technology of China
Hefei 230026, China

(Communicated by Ruhul Sarker)

ABSTRACT. National Basketball Association (NBA) is one of the popular sports leagues worldwide and is also a business source that generates enormous financial resources. Generally, the salary of sports players is associated with their performance in the field. However, the NBA players’ performance in the game is related to specific technical features in the offensive and defensive activities. This paper aims to measure the impact of NBA players’ salary on their efficiency levels using a big data set of eleven seasons (2604 players from 2005 to 2016) by considering the players’ performance in offensive and defensive activities. First, we propose models to measure players’ overall, offensive, and defensive efficiencies based on a non-homogeneous parallel data envelopment analysis (DEA) network. Then, we introduce input-output oriented network models to estimate the marginal returns from salary on the outcomes of both offensive and defensive activities. Results indicated that all players’ average overall efficiency is low (63.5%), with 17 efficient players. The offensive efficiency is 12.8% higher than the defensive efficiency. When the impact of salary on offensive (defensive) activity is considered, about 73% (47%) of the players’ observations indicate increasing marginal returns, respectively.

2020 Mathematics Subject Classification. Primary: 90B10, 90B30, 90C05, 90C90; Secondary: 90C08.

Key words and phrases. Data envelopment analysis (DEA), parallel processes, returns to scale (RTS), marginal returns, NBA games, salary.

* Corresponding author: Muhammad Salman Mansoor.
1. **Introduction.** In professional team sports, the distribution of players’ salaries varies across teams. Although some teams pay each player with similar rates, other teams concentrate their payroll on a small star-players group. [15] reported that, if the players’ average salary is high, players’ efficiency would also be increased, resulting in more winning games or matches. In view of financial constraints, such as the salary cap, [20] stated that the factors affecting the National Basketball Association (NBA) owners to pay players are of great importance. Given a fixed payroll budget, it remains unclear which salary structure leads to better team performance.

The salary of sports players is associated with their performance in the field. However, the performance of players in the game is related to specific technical features. For example, the performance of NBA players is affected by their performance in the game’s offensive and defensive activities. Some players are voted to be in the all-stars list based on their high offensive or defensive stats. Therefore, there is a need to simultaneously evaluate NBA players’ performance considering their offensive and defensive activities. This measurement allows the managers to know their players’ efficiency level and identify the inefficiency place for each player. This leads to this paper’s first contribution: to measure the performance of a big data set of NBA players, 2604 players played from 2005 to 2016, using a parallel network of offensive and defensive processes. For the inefficient players, we propose a model to project them on the efficient frontier using the information of the most productive scale size (MPSS).

The association between the salary of NBA players and their offensive and defensive performance should be highlighted so that teams’ managers can know when to increase or decrease players’ salaries based on their performance in both offensive and defensive activities. In the literature, most studies that discussed NBA players’ salaries were based on the players’ overall performance. In other words, there are no available studies on the impact of salary on offensive and defensive activities. For this purpose, we introduce a new model associated with returns to scale estimation to measure the marginal returns from salary on the outcomes of offensive and defensive activities. This will help the managers and decision makers if there is a need to increase or decrease players’ salaries.

In summary, we introduce a framework to deal with a big data set of players using network data envelopment analysis (DEA) methodology. First, we consider the data of 2604 NBA players who played in the period from the season 2005-2006 to 2015-2016. Each player is treated as a decision-making unit (DMU), and his offensive and defensive activities are taken into account in a non-homogeneous parallel network. We measure players’ overall technical efficiency based on the variable returns to scale (VRS) concept and decompose it into the offensive and defensive efficiencies. A new model to estimate the marginal returns of salary on the outcomes of both offensive and defensive processes is introduced.

The rest of this paper is organized as follows. The next section gives background research on the topic. In Section 3, we give our developed models to evaluate the overall efficiency and decompose it into the offensive and defensive efficiencies. Also, we introduce a model to measure the marginal returns from salary on the outcomes of both offensive and defensive activities. Section 4 presents the data sources and results on the efficiency of the NBA players (2005-2016) and the association with salary. The conclusions are presented in Section 5.
2. Background.

2.1. DEA methodology. DEA is a mathematical approach based on linear programming used to evaluate the relative efficiency of a set of homogeneous DMUs with the same input and output measures; the amount may vary from one DMU to another [6]. The group of efficient DMUs identifies the efficient frontier. Moreover, DEA gives the required information to move the inefficient DMUs to the efficient frontier [3]. DEA has been applied in various areas and applications such as sports [17, 25], research and development (R&D) [1], banks [10], and hospitals [18].

Conventional DEA treats the DMU as a black box that converts the consumed inputs into final outputs. When a DMU is evaluated as inefficient, it is hard to identify the place of inefficiency. Therefore, network DEA is proposed to open the black box and take the interrelations within the evaluated DMU into account. The overall efficiency of the evaluated DMU can be decomposed into the efficiencies of the internal processes. This decomposition allows the decision makers to identify the inefficiency place in the evaluated DMU so that the necessary improvements can be made [3, 11]. [12] reviewed the network DEA structures and classified them into series [13], parallel [23], mixed structure [2], hierarchic [26], and dynamic [19].

2.2. Homogeneous parallel DEA network. Consider a homogeneous standard parallel system in which \(q\) processes are connected in parallel to form a network system (see Figure 1). Let \(X_{ij}\) and \(Y_{rj}\) are the \(i\)th input and \(r\)th output of DMU\(j\) (\(j = 1, \ldots, n\)), respectively. The index set for the input is \(I = (1, 2, \ldots, m)\), and \(O = (1, 2, \ldots, s)\) is the index set for the outputs. Process \(p\) consumes inputs \(X_{ij}^{(p)}, i \in I^{(p)}\), to produce outputs \(Y_{rj}^{(p)}, r \in O^{(p)}\), where \(I^{(p)} \subset I\) and \(O^{(p)} \subset O\) are the index sets for the inputs and outputs of process \(p\). The \(i\)th input of the system of DMU\(j\) is the sum of the \(i\)th input for all processes, i.e., \(\sum_{p=1}^{q} X_{ij}^{(p)} = X_{ij}\). This is also applied to outputs; that is, \(\sum_{p=1}^{q} Y_{rj}^{(p)} = Y_{rj}\).

[11] introduced DEA models based on constant returns to scale concept to measure the overall efficiency of the parallel network and decompose it into the internal processes’ efficiencies. In the following, we adapt the CCR models in [11] to the variable returns to scale assumption with output orientation as follows:

\[
T_k = \min \sum_{i=1}^{m} v_i X_{ik} + u_{ok}
\]

\[
\text{s.t.} \quad \sum_{r=1}^{s} u_r Y_{rk} = 1,
\]

\[
\sum_{r=1}^{s} u_r Y_{rk} - \left( \sum_{i=1}^{m} v_i X_{ik} + u_{ok} \right) \leq 0,
\]

\[
\sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \leq 0, \quad j = 1, 2, \ldots, n, \quad j \neq k,
\]

\[
\sum_{r \in O^{(p)}} u_r Y_{rj}^{(p)} - \left( \sum_{i \in I^{(p)}} v_i X_{ij}^{(p)} + u_{ok}^{(p)} \right) \leq 0, \quad p = 1, \ldots, q.
\]

\[
\sum_{r \in O^{(p)}} u_r Y_{rj}^{(p)} - \sum_{i \in I^{(p)}} v_i X_{ij}^{(p)} \leq 0, \quad p = 1, \ldots, q, \quad j = 1, \ldots, n, \quad j \neq k,
\]
Figure 1. Classical parallel structure

\[ u_{ok} = \sum_{p=1}^{q} u_{ok}^{(p)}. \]

\[ u_{ok}, u_{ok}^{(p)} \in \mathbb{R}, u_r, v_i \geq \varepsilon, r = 1, \ldots, s, i = 1, \ldots, m. \]

The second and third constraints in Model (1) correspond to the system, and the fourth and fifth constraints correspond to the internal processes. The sixth constraint is added so that the system constraints are the sum of the \( q \) processes constraints. At optimality, the BCC efficiencies of the parallel system and the internal processes are given as follows:

\[ T_k = \sum_{i=1}^{m} v_i^* X_{ik} + u_{ok}^* \over \sum_{r=1}^{s} u_r^* Y_{rk} = \sum_{i=1}^{m} v_i^* X_{ik} + u_{ok}^*. \]

\[ T_k^{(p)} = \left( \sum_{i \in I(p)} v_i^* X_{ik}^{(p)} + u_{ok}^{(p)*} \right) \over \sum_{r \in O(p)} u_r^* Y_{rk}^{(p)}, p = 1, \ldots, q. \]

If we assume the weight of process \( p \) as the aggregate outputs produced by process \( p \) in that produced by all \( q \) processes, \( \omega^{(p)} = \sum_{r \in O(p)} u_r^* Y_{rk}^{(p)} / \sum_{r=1}^{s} u_r^* Y_{rk} \), then we have:

\[ \sum_{p=1}^{q} \omega^{(p)} T_k^{(p)} = \sum_{p=1}^{q} \left( \left( \sum_{r \in O(p)} u_r^* Y_{rk}^{(p)} \right) \left( \sum_{i \in I(p)} v_i^* X_{ik}^{(p)} + u_{ok}^{(p)*} \right) \right) \over \sum_{r \in O(p)} u_r^* Y_{rk}^{(p)} \]

\[ = \sum_{p=1}^{q} \left( \sum_{i \in I(p)} v_i^* X_{ik}^{(p)} + u_{ok}^{(p)*} \right) \over \sum_{r=1}^{s} u_r^* Y_{rk} = T_k. \]
In the case of existing multiple solutions in Model (1), the decomposition mentioned above would not be unique, and thereby the comparison of the efficiency among all DMUs will lack a common basis. [14] proposed an approach to find the set of multipliers that produces the largest $T_k(p)$ (suppose process $p$ is considered the most important) while maintaining the overall efficiency score at $T_k$ calculated from Model (1).

3. Performance evaluation models for NBA players.

3.1. Overview. The NBA is one of the popular sports leagues worldwide and is also a business source that generates enormous financial resources. An organized basketball game includes two teams comprising five players per team. A match involves the two teams, and the team which scores more points is the winner. Many statistics about the game activities are available (i.e., when players score a goal to secure points, when a player saves the ball from a basket, the efforts of different positions while playing the game, the number of minutes played when the total time for each game is 48 minutes). NBA games are broadcasted in 215 countries in 41 languages during the 2008–2009 seasons, suggesting that the NBA has reached most parts of the world.

The average pay across all NBA Players is nearly 5,497,859 USD in 2017. The NBA remains the best-paid sports league by the year 2017; therefore, the NBA’s supremacy remains a trend (Global Sports Salaries Survey [GSSS] 2017). The players are pursued and hired after exhibiting their skills in one or more aspects of the game stats, such as points, rebounds, steals, turnovers, blocks, and assists. However, they may have skills in other areas of game statistics as well. Hence the appropriate player selection for ownership has always been a very arduous, planned, and data-driven process [20].

3.2. The proposed non-homogeneous parallel network of NBA player activities. In this paper, we propose a parallel network to reflect the performance activities of the NBA players. Each player is considered a DMU, and this player contributes to the offensive and defensive activities in the match. These two activities are considered two processes connected in parallel. The two activities’ outputs are not in the same types; that is, the two activities are non-homogeneous. For example, giving an assist is an output in the offensive activity, but it is not so in the defensive activity. Also, obtaining a block is an output in the defensive activity, and it is not so in the offensive activity. The proposed network of NBA players consumes two inputs: the number of minutes played and the player’s salary, and the outputs are the achievements in both offensive and defensive activities (see Figure 2).

The selection of inputs and outputs of the proposed network is based on the critical viewpoints of the basketball coaches and has strong research evidence. We considered the offensive and defensive activities as two processes because the NBA player presents different playing tactics concerning both activities and can play better on one side than on the other side. Several articles focused on both offensive and defensive in DEA [22, 9, 5]. By considering these two activities, the results help the managers and coaches decide what game strategy their best practicing player can use, resulting in-game improvements that lead to winning; this can be realized using offensive and defensive activities [21]. A player’s value can be improved if he can prevent the opponent from gaining another scoring chance
Figure 2. The non-homogeneous parallel network of NBA player activities

by grabbing or snatching defensive rebounds, which provides additional scoring chances for his team resulting from his offensive rebounds [20]. Previous studies have reported that assists, field goal, steals, blocks, and rebounds are significant variables in determining NBA players’ efficiency [17, 22, 8, 16, 7, 24]. Field goal stats only consider the points obtained from 2-pints and 3-points and does not consider the free throws; therefore, we kept the free throws variable in the analysis. The winning teams have achieved better shooting efficiency and a higher number of defensive rebounds [9]. Furthermore, [5] revealed that free throws effectiveness is the most significant factor. The effective offensive play has led to teams’ success in the NBA, allowing them to improve their offensive efficiency [21] significantly.

Consistent with the prior literature on basketball performance evaluation [17, 22, 8, 16, 7, 24], we incorporated two inputs to the parallel network of NBA player’s activities, and two sets of outputs are produced as follows:

- **Input variables:** minutes played (MP) and salary (SLR).
- **Output variables:** the classification of output variables for each process using a non-homogeneous parallel network structure is as follows:
  - Offensive process: offensive rebounds (ORB), field goals (FG), free throws (FT), assists (AST).
  - Defensive process: defensive rebounds (DRB), steals (STL), blocks (BLK).

Each player is denoted as $DMU_j$ $(j = 1, \ldots, n)$, in which the process $p$ $(p = 1, 2)$ consumes inputs $(X^1_{ij})$ and $(X^2_{ij})$ $(i = 1, 2)$ to generate two sets of outputs $(Y^1_{rj}, r = 1, 2, 3, 4)$ and $(Y^2_{lj}, l = 1, 2, 3)$, respectively.

Let a parameter $\alpha$ ($0 < \alpha < 1$) denote the proportion of inputs dedicated to the offensive process. Then, the overall inputs $(X_{ij}, i = 1, 2)$ are divided into two parts for the two processes as follows:

$$X^1_{ij} = \alpha X_{ij} \text{ and } X^2_{ij} = (1 - \alpha) X_{ij}, \quad i = 1, 2, \quad j = 1, \ldots, n,$$

where $\alpha$ is a parameter. If $\alpha = 0$, it means that the defensive side consumes all inputs; on the contrary, $\alpha = 1$ means that all inputs $X_j$ are consumed by the offensive side. Given that the participating players considered in this paper have played for at least one minute in the match, inputs on each side cannot be zero.
In other words, each participating player has consumed some of the overall inputs to obtain the nonzero outputs on each side. Thus, the parameter of $\alpha$ is in an interval range of $(0, 1)$. We develop the \cite{11}'s BCC model to be consistent with a non-homogeneous parallel network of NBA players activities under the output orientation as follows:

**Model (2):**

\[
T_k = \min u_{ok} X_{ik} + u_{ok} \\
\text{s.t. } \sum_{r=1}^{4} u_r Y_{ik} - \alpha \sum_{i=1}^{2} v_i X_{ik} + u_{ok} \leq 0,
\]

Offensive activity:

\[
\sum_{r=1}^{4} u_r Y_{ij} - \left( \alpha \sum_{i=1}^{2} v_i X_{ij} \right) \leq 0, \quad j = 1, \ldots, n, \ j \neq k,
\]

Defensive activity:

\[
\sum_{l=1}^{3} u_l^2 Y_{ik} - \left( (1-\alpha) \sum_{i=1}^{2} v_i X_{ik} + u_{ok} \right) \leq 0,
\]

Overall system:

\[
\sum_{r=1}^{4} u_r Y_{ik} + \sum_{l=1}^{3} u_l^2 Y_{ik} - \left( \sum_{i=1}^{2} v_i X_{ik} + u_{ok} \right) \leq 0,
\]

\[
\sum_{r=1}^{4} u_r Y_{ij} + \sum_{l=1}^{3} u_l^2 Y_{ij} - \sum_{i=1}^{2} v_i X_{ij} \leq 0, \quad j = 1, \ldots, n, \ j \neq k,
\]

\[
u = \omega_{ok}^1 + \omega_{ok}^2,
\]

where $u_{ok}, u_{ok}^1 \in \mathbb{R}, p = 1, 2, v_i, u_i, u_i^1 \geq c, \ i = 1, 2, r = 1, \ldots, 4, l = 1, 2, 3$.

In Model (2), $v_i, u_i, u_i^1$ and $u_i^2$ are the multipliers attached to the inputs, the outputs of process 1, and the outputs of process 2, respectively. The variable $u_{ok}$ can be used as an index for the returns to scale information. If $u_{ok}$ is zero, positive, or negative, then constant returns to scale (CRS), increasing returns to scale (IRTS), or decreasing returns to scale (DRTS), respectively, are derived.

At optimality, the objective function of Model (2) can be written as follows:

\[
T_k = \frac{v_i X_{ik} + u_{ok}}{\sum_{r=1}^{4} u_r Y_{ik} + \sum_{l=1}^{3} u_l^2 Y_{ik}} = \frac{\sum_{r=1}^{4} u_r Y_{ik}}{\sum_{r=1}^{4} u_r Y_{ik} + \sum_{l=1}^{3} u_l^2 Y_{ik}} \times \frac{v_i X_{ik} + u_{ok}}{\sum_{r=1}^{4} u_r Y_{ik}}
\]

\[+ \frac{\sum_{l=1}^{3} u_l^2 Y_{ik}}{\sum_{r=1}^{4} u_r Y_{ik} + \sum_{l=1}^{3} u_l^2 Y_{ik}} \times \frac{v_i X_{ik} + u_{ok}}{\sum_{r=1}^{4} u_r Y_{ik}} = \omega_1^k \times T_k^1 + \omega_2^k \times T_k^2,
\]

where $\omega_1^k$ and $\omega_2^k$ are the importance of process 1 and process 2, respectively, and the aggregate performance measure $T_k$ is a convex combination of the two process measures for $DMU_k$. 
At the beginning of each game, the referee throws the basketball up in the air, and one player from each team attempts to hit the ball to their teammates, which is called a ‘jump ball.’ It means that every team tries to get the jump ball to start the offensive activity. Based on this fact and following [14]’s approach, we measure the efficiency of the offensive activity, $T_k^1$, before the defensive activity, $T_k^2$, while maintaining the overall efficiency score at $T_k$ calculated from Model (2) as follows:

**Model (3):**

$$T_k^1 = \min v_i X_{ik}^1 + u_{ok}^1$$

subject to $\sum_{r=1}^{4} u_{ik}^1 Y_{ik}^1 = 1$,

Offensive activity:

$$\sum_{r=1}^{4} u_{ik}^1 Y_{ik}^1 - (\alpha \sum_{i=1}^{2} v_i X_{ik} + u_{ok}^1) \leq 0,$$

$$\sum_{r=1}^{4} u_{ij}^1 Y_{ij}^1 - (\alpha \sum_{i=1}^{2} v_i X_{ij}) \leq 0, \quad j = 1, \ldots, n, \quad j \neq k,$$

Defensive activity:

$$\sum_{l=1}^{3} u_{lk}^2 Y_{lk}^2 - (1-\alpha) \sum_{i=1}^{2} v_i X_{lk} + u_{ok}^2 \leq 0,$$

$$\sum_{l=1}^{3} u_{lj}^2 Y_{lj}^2 - (1-\alpha) \sum_{i=1}^{2} v_i X_{lj} \leq 0, \quad j = 1, \ldots, n, \quad j \neq k,$$

Overall system:

$$T_k^* = \left( \sum_{r=1}^{4} u_{rk}^1 Y_{rk}^1 + \sum_{l=1}^{3} u_{lk}^2 Y_{lk}^2 \right) - \left( \sum_{i=1}^{2} v_i X_{ik} + u_{ok} \right) = 0, \quad j = 1, \ldots, n,$$

$$u_{ok} \geq \frac{2}{p} u_{ok}^{(p)},$$

$u_{o}, u_{o}^{(p)} \in \mathbb{R}, \quad p = 1, 2, v_i, u_{r}, u_{l}^2 \geq \varepsilon, \quad i = 1, 2, r = 1, \ldots, 4, l = 1, 2, 3.$

As shown in Model (3), the evaluated player’s overall efficiency, calculated via Model (2), stays unchanged. Similarly, we obtain the efficiency of defensive activity.

**Corollary 1.** An NBA player’s overall efficiency is the weighted average of the efficiencies of offensive and defensive activities.

**Corollary 2.** A player is considered overall efficient if and only if he is efficient in the offensive and defensive activities.
3.3. Marginal returns from salary on the outcomes of NBA player’s activities. The dual envelopment form of Model (2) is given as follows:

**Model (4):**

$$\max \emptyset$$

s.t. \( \sum_{p=1}^{2} \left( \sum_{j=1}^{n} \lambda_{j}^{p} X_{ij}^{p} \right) \leq X_{ik}, \ i = 1, 2, \)

$$\sum_{p=1}^{2} \left( \sum_{j=1}^{n} \lambda_{j}^{p} Y_{rj}^{p} \right) \geq \emptyset \left( Y_{rk}^{1} + Y_{ik}^{2} \right), \ r = 1, \ldots, 4, l = 1, 2, 3,$$

$$\sum_{j=1}^{n} \lambda_{j}^{p} = 1, \ p = 1, 2,$$

$$\lambda_{j}^{p} \geq 0, j = 1, \ldots, n, p = 1, 2, \emptyset: \text{unrestricted}.$$

Based on Model (4), we establish the following input-output oriented model:

**Model (5):**

$$\max \emptyset / \theta$$

s.t. \( \sum_{p=1}^{2} \left( \sum_{j=1}^{n} \lambda_{j}^{p} X_{ij}^{p} \right) \leq \theta X_{ik}, \ i = 1, 2, \)

$$\sum_{p=1}^{2} \left( \sum_{j=1}^{n} \lambda_{j}^{p} Y_{rj}^{p} \right) \geq \emptyset \left( Y_{rk}^{1} + Y_{ik}^{2} \right), \ r = 1, \ldots, 4, l = 1, 2, 3,$$

$$\sum_{j=1}^{n} \lambda_{j}^{p} = 1, \ p = 1, 2,$$

$$\lambda_{j}^{p} \geq 0, j = 1, \ldots, n, p = 1, 2, \emptyset \geq 1, \theta \leq 1.$$

In Model (5), \( \theta \) and \( \emptyset \) are the distance measures representing the contraction and expansion factors applied to the aggregated inputs and outputs, respectively, in the evaluated player. This model maximizes the parallel system inputs and outputs mix’s productivity average by considering the system’s internal processes. In other words, this model can measure the best economic scale, the most productive scale size (MPSS).

**Corollary 3.** A player is evaluated as the most productive scale size if the objective function value of Model (5) equals unity.

Model (5) not only measures the best economic scale of players, but it helps the inefficient players to move to the efficient frontier.

**Corollary 4.** A player \((X_{k}, Y_{rk}^{1}, Y_{ik}^{2})\) must be overall efficient (frontier point) in both offensive and defensive activities using this projection \((\theta^{*} X_{k}, \emptyset^{*} Y_{rk}^{1}, \emptyset^{*} Y_{ik}^{2})\) where \(\theta^{*}\) and \(\emptyset^{*}\) are the optimal solutions of Model (5).

In the following, we study the impact of salary on the outcomes of offensive and defensive activities. Consider the impact of salary on the outcomes of offensive activity. Suppose a player (DMU\(_{k}\)) is efficient or is moved onto the efficient frontier by Corollary 4. Let \(\partial\) be a change in salary and \(\gamma\) the resulted change in outcomes of offensive activity. Consider the following problem:

**Model (6):**

$$\max \gamma / \partial$$
Based on the optimal solution of the Model (6), the marginal returns from salary on the outcomes of the offensive activity of NBA players can be estimated as follows:

- If $\gamma^*/\partial^*>1$, then the marginal returns from salary are increasing. This indicates that managers should consider increasing their players’ salaries further.
- If $\gamma^*/\partial^*=1$, the marginal returns from salary are constant. This indicates that the current level of salary is appropriate.
- If $\gamma^*/\partial^*<1$, the marginal returns from salary are decreasing. This indicates that the salary of players should be reduced.

Similarly, we can obtain the marginal returns from salary on the outcomes of the defensive activity.

It is worthy of keeping in mind that the marginal returns and returns to scale are different concepts. In Model (6), the marginal returns are calculated by scaling up only the salary and keeping the other inputs (minutes played) the same. In contrast, the returns to scale derived from the proposed network BCC Model (2) are obtained when all inputs (salary and minutes played) are scaled up.

4. Efficiency evaluation and marginal returns from salary.

4.1. Data collection. Data of 4481 players over 11 regular seasons from 2005 to 2016 are taken from [http://www.stat-nba.com](http://www.stat-nba.com), which is a widely accessible website. The players who played less than 25% of the regular season matches (about less than 1000 minutes played) are excluded to have reliable results. Accordingly, 2604 players are considered and entered the analysis. The salary data are collected from Celtics Hub ([http://www.celtichub.com](http://www.celtichub.com)) and Patricia Bender repository ([http://www.eskimo.com/~pbender/](http://www.eskimo.com/~pbender/)), which are the most accurate sources of salary data and have been widely used in various studies on the NBA. However, in other places, the data figures provided by these sources are different. We considered the highest quoted salary in our work. In the dataset, all players have at least one salary figure quoted. To obtain the maximum/relevant results, the last known salary is used. Besides, if a player’s salary is not available, that player is excluded.
Table 1. Summary of inputs and outputs descriptive statistics of NBA players

| Variables            | Mean  | S.D.   | Min   | Max   |
|----------------------|-------|--------|-------|-------|
| **Inputs**           |       |        |       |       |
| Minutes played       | 1923  | 573    | 1000  | 3424  |
| Salary               | 6224124 | 5107788 | 160244 | 30453805 |
| **Outputs**          |       |        |       |       |
| **Offensive activity** |      |        |       |       |
| Assists              | 180   | 145    | 6     | 925   |
| Offensive rebounds    | 85    | 65     | 5     | 440   |
| Field goals          | 308   | 146    | 33    | 978   |
| Free throws          | 153   | 110    | 9     | 756   |
| **Defensive activity** |     |        |       |       |
| Defensive rebounds   | 248   | 131    | 39    | 882   |
| Steals               | 60    | 31     | 7     | 217   |
| Blocks               | 38    | 37     | 1     | 285   |

from the data [15]. The dataset is available on an external repository [4] (Mendeley Data, http://dx.doi.org/10.17632/fm86gnkw6x.1).

The following Table 1 summarizes the statistical descriptive of inputs and outputs for the NBA players. Using 11 years of data, the overall efficiency and the process efficiencies can be evaluated for each DMU separately; each year has a distinct frontier. However, the calculated efficiency of players over 11 years may not be comparable. For this, all the DMUs have been placed in one bundle to evaluate all players under one DEA frontier.

4.2. Efficiencies and RTS of the NBA players. This section presents the proposed non-homogeneous parallel DEA network results for all the players (N=2604). For each year, the overall, offensive, and defensive efficiencies for each player are calculated from Models (2) and (3). Then, the average values of these evaluated efficiencies are obtained for each year, as reported in Table 2.

The average of the overall efficiencies of the eleven seasons is (63.49%). The offensive efficiency is 67.49%, which is about 12.8% greater than the defensive efficiency (59.83%). The overall efficiency average reached the highest rate (65.37%) in the season 2015-2016 as well as the offensive activity (70.33%), while the defensive activity obtained the highest efficiency (62.81%) in the seasons 2005-2006.

Based on the proposed network models, 41 and 27 players are found to be efficient in the offensive and defensive activities, respectively. Only 17 players are found to be overall efficient (Kobe Bryant 05–06, Tim Duncan 11–12, Kevin Durant 09–10, James Harden 15–16, Blake Griffin 10–11, Dwyane Wade 08–09, Dwight Howard 07–08,10–11, Zach Randolph 09–10, Chris Paul 07–08,08–09, Rudy Gobert 14–15, Russell Westbrook 15–16, Marcus Camby 07–09, Andre Drummond 13–14,14–15,15–16). The upper scripts stand for the seasons. It is noteworthy that these players are evaluated as efficient in both offensive and defensive activities, which is consistent with Corollaries 1 and 2. For the other inefficient players, efficiency decomposition is applied to explore the inefficiency activity. There are 24 efficient players in offensive activity and inefficient in defensive activity, such as Pau Gasol 13–14 and Kevin Garnett 13–14. In contrast, there are ten efficient players in defensive activity
Table 2. Efficiency evaluation and RTS of NBA players from 2005-2016

| Season | Network BCC Models (2) and (3) | RTS |
|--------|-------------------------------|-----|
|        | Overall | Offensive | Defensive | IRTS | DRTS |
| 05-06  | 0.6440  | 0.6610    | 0.6281    | 54.8% | 45.2% |
| 06-07  | 0.6387  | 0.6584    | 0.6168    | 52.3% | 47.7% |
| 07-08  | 0.6415  | 0.6620    | 0.6020    | 52.8% | 47.2% |
| 08-09  | 0.6413  | 0.6738    | 0.5957    | 53.1% | 46.9% |
| 09-10  | 0.6402  | 0.6850    | 0.5916    | 53.4% | 46.6% |
| 10-11  | 0.6277  | 0.6724    | 0.5768    | 55.9% | 44.1% |
| 11-12  | 0.6134  | 0.6690    | 0.5741    | 54.6% | 45.4% |
| 12-13  | 0.6148  | 0.6570    | 0.5764    | 55.6% | 44.4% |
| 13-14  | 0.6254  | 0.6846    | 0.5864    | 53.5% | 46.5% |
| 14-15  | 0.6430  | 0.6969    | 0.6068    | 66.7% | 33.3% |
| 15-16  | 0.6537  | 0.7033    | 0.6265    | 64.3% | 35.7% |
| Average| 0.6349  | 0.6749    | 0.5983    | 58.6% | 41.4% |

and inefficient in offensive activity, such as Dirk Nowitzki \(^{09-10}\) and James Harden \(^{13-14}\).

Based on the proposed network Model (2), the RTS information of NBA players over the eleven seasons are reported in the last two columns of Table 2. We found that 58.6% of players had IRTS, and 41.4% had DRTS indicating that the managers should further increase the inputs (salary and minutes played).

By applying Model (5), the MPSS values of NBA players are calculated. Among the 17 efficient players, only five players are evaluated as MPSS; Kobe Bryant \(^{05-06}\), Kevin Durant \(^{09-10}\), Dwight Howard \(^{10-11}\), Chris Paul \(^{08-09}\), Andre Drummond \(^{15-16}\).

Model (5) can also determine the projections of inefficient players on the efficient frontier. Table 3 reports the original and projected inputs and outputs for LeBron James \(^{15-16}\) from Model (5) where \(\theta^* = 0.9527\) and \(\emptyset^* = 1.0601\).

Model (5) shows that the projection (the enhanced level) of salary for LeBron James \(^{15-16}\) should be \(21883995\). The original value of the salary is \(22970500\). This result indicates that reducing 4.7% of minutes played and salary would have efficiently generated better offensive and defensive outcomes. The optimal values of \(\lambda_j^p (p = 1, 2)\) in Model (5) indicate the benchmark players in the offensive and defensive activities, i.e., Kevin Durant \(^{09-10}\) (\(\lambda = 0.7412\)) and Kobe Bryant \(^{05-06}\) (\(\lambda = 0.2588\)) are used as referent players for LeBron James \(^{15-16}\) in the offensive process, while Andre Drummond \(^{15-16}\) (\(\lambda = 0.6834\)) and Chris Paul \(^{07-08}\) (\(\lambda = 0.3166\)) are used as referent players for LeBron James \(^{15-16}\) in the defensive process.

Figure 3 shows the overall efficiency values’ tendencies with the gradual change of inputs allocation (\(\alpha\)), calculated by Model (2), for LeBron James in different seasons. We notice that the efficiency tendency curve in season 2006-2007 is flat at first till (\(\alpha = 0.30\)), then is rising with the increase of (\(\alpha\)). In season 2008-2009, the efficiency tendency curve is almost flat for all inputs allocation values (\(\alpha\)). In season 2012-2013, the efficiency tendency curve is also flat at first but till (\(\alpha = 0.80\)), then is rising with the increase of (\(\alpha\)). Finally, in season 2015-2016, the efficiency tendency curve is flat at first till (\(\alpha = 0.70\)), then is rising sharply with the increase of (\(\alpha\)). In other words, in season 2006-2007, LeBron James needed to share at least 30% of minutes played and salary amount to the offensive activity to get a higher
Table 3. Original and efficient inputs and outputs for LeBron James

|          | MP   | SLR      | AST | ORB | FG  | FT  | DRB | STL | BLK |
|----------|------|----------|-----|-----|-----|-----|-----|-----|-----|
| Original | 2709 | 22970500 | 514 | 111 | 737 | 359 | 454 | 104 | 49  |
| Efficient| 2581 | 21883995 | 545 | 118 | 781 | 381 | 481 | 110 | 52  |

Referent players for offensive process
- Kevin Durant\(^{09-10}\) (\(\lambda = 0.7412\)),
- Kobe Bryant\(^{05-06}\) (\(\lambda = 0.2588\))

Referent players for defensive process
- Andre Drummond\(^{15-16}\) (\(\lambda = 0.6834\)),
- Chris Paul\(^{07-08}\) (\(\lambda = 0.3166\))

rate of overall efficiency, while he needed to share 80% and 70% to the offensive activity in the seasons 2012-2013 and 2015-2016, respectively.

4.3. Efficiencies and salary of NBA players. To explore the association between players' salary and their performance in the overall, offensive, and defensive activities, we divided the efficiency levels of players' activities and calculated the salary average for each efficiency level (see Figure 4).

We notice that defensive activity had a higher salary rate than offensive activity when the efficiency level is less than 80%. However, when the efficiency level is higher than 80%, the offensive activity had a significantly higher salary rate.

4.4. Marginal returns from salary on the outcomes of offensive and defensive activities. Using Model (6), Table 4 reports the marginal returns from salary on NBA players' offensive and defensive activities. For the overall set of players (2604 players), when the impact of salary on offensive activity is considered, 72.80% of the players' observations indicate increasing marginal returns. In contrast, when the effect of salary on defensive activity is considered, 47.14% of the players' observations indicate increasing marginal returns.
As shown in Table 4, most NBA players have increasing marginal returns in offensive activity. This further indicates that the managers should consider increasing the players' salaries based on their offensive outcomes.

5. Conclusions and implications. Using DEA, this paper evaluated NBA players' performance considering their offensive and defensive activities in the match in a non-homogeneous parallel network with shared inputs. The impact of the players' salary on the outcomes of the offensive and defensive activities is determined. The results showed that the overall efficiency of NBA players is low with higher offensive efficiency. When the impact of salary on offensive activity is considered, 72.8% of the players’ observations indicate increasing marginal returns.
The obtained results can help managers and coaches develop guidelines and suggestions for the sports industry concerning the player’s efficiency. Furthermore, this paper allows the managers to know the impact of the player’s salary on his performance on the field so that future increasing or decreasing salary may be considered.

Our methodology can measure NBA players’ overall efficiency and decompose it into the non-homogeneous parallel offensive and defensive processes. Such decomposition can also provide a guide for performance enhancement. Thus, we obtained more comprehensive and practical performance evaluation results. With the rapidly developing sports industry, this practice should theoretically influence NBA team management decisions to determine player salary. As stated earlier, this may also redirect the NBA management’s focus on using non-scoring metrics in developing formulas and models that can lead to better and more realistic approaches to rewarding players.

As future works, this work can be conducted at the NBA teams’ level to determine the marginal returns from each team’s salary package on the team performance. Also, players’ salaries can be predicted using multivariate regression analysis in one step. In the second step, the obtained parameters can be integrated with the proposed parallel DEA model of offensive and defensive activities to get players’ estimated efficiency.

Acknowledgments. The authors are grateful for the valuable comments from the respected editor and reviewers which enriched the quality of this paper. This research was financially supported by the National Natural Science Foundation of China (Grant Nos. 72071192, 71671172, 71631006, 71991464, 71991460).

REFERENCES

[1] S. Assani, J. Jiang, A. Assani and F. Yang, Scale efficiency of China’s regional R & D value chain: A double frontier network DEA approach, *Journal of Industrial & Management Optimization*, **17** (2021), 1357–1382.

[2] S. Assani, J. Jiang, A. Assani and F. Yang, Most productive scale size of China’s regional R&D value chain: A mixed structure network, preprint, arXiv:1910.03805.

[3] S. Assani, J. Jiang, R. Cao and F. Yang, Most productive scale size decomposition for multi-stage systems in data envelopment analysis, *Computers and Industrial Engineering*, **120** (2018), 279–287.

[4] S. Assani and M. S. Mansoor, Salary, offensive, and defensive stats of 2604 NBA players over 11 seasons (2005-2016), *Mendeley Data*, V1 (2020).

[5] J. E. Boscá, V. Liern, A. Martínez and R. Sala, Increasing offensive or defensive efficiency? An analysis of Italian and Spanish football, *Omega*, **37** (2009), 63–78.

[6] A. Charnes, W. W. Cooper and E. Rhodes, Measuring the efficiency of decision making units, *European Journal of Operational Research*, **2** (1978), 429–444.

[7] Y. Chen, Y. Gong and X. Li, Evaluating NBA player performance using bounded integer data envelopment analysis, *INFOR: Information Systems and Operational Research*, **55** (2017), 38–51.

[8] W. W. Cooper, J. L. Ruiz and I. Sirvent, Selecting non-zero weights to evaluate effectiveness of basketball players with DEA, *European Journal of Operational Research*, **195** (2009), 563–574.

[9] Ó. Gutiérrez and J. L. Ruiz, Data envelopment analysis and cross-efficiency evaluation in the management of sports teams: The assessment of game performance of players in the Spanish handball league, *Journal of Sport Management*, **27** (2013), 217–229.

[10] C.-K. Hu, F.-B. Liu, H.-M. Chen and C.-F. Hu, Network data envelopment analysis with fuzzy non-discretionary factors, *Journal of Industrial & Management Optimization*.

[11] C. Kao, Efficiency decomposition for general multi-stage systems in data envelopment analysis, *European Journal of Operational Research*, **232** (2014), 117–124.
[12] C. Kao, Network data envelopment analysis: A review, *European Journal of Operational Research*, 239 (2014), 1–16.
[13] C. Kao and S.-T. Liu, Cross efficiency measurement and decomposition in two basic network systems, *Omega*, 83 (2019), 70–79.
[14] C. Kao and S.-N. Hwang, Decomposition of technical and scale efficiencies in two-stage production systems, *European Journal of Operational Research*, 211 (2011), 515–519.
[15] H. Katayama and H. Nuch, A game-level analysis of salary dispersion and team performance in the national basketball association, *Applied Economics*, 43 (2011), 1193–1207.
[16] B. L. Lee and A. C. Worthington, A note on the ‘Linsanity’ of measuring the relative efficiency of National Basketball association guards, *Applied Economics*, 45 (2013), 4193–4202.
[17] Y. Li, L. Wang and F. Li, A data-driven prediction approach for sports team performance and its application to National Basketball Association, *Omega*, 98 (2021), 102–123.
[18] Y. Li, X. Lei and A. Morton, Performance evaluation of nonhomogeneous hospitals: the case of Hong Kong hospitals, *Health Care Management Science*, 22 (2019), 215–228.
[19] Y. Li, X. Shi, A. Emrouznejad and L. Liang, Environmental performance evaluation of Chinese industrial systems: A network SBM approach, *Journal of the Operational Research Society*, 69 (2018), 825–839.
[20] R. Lyons, E. N. Jackson and A. Livingston, Determinants of NBA player salaries, *The Sport Journal*, 18 (2015).
[21] K. Mikolajec, A. Maszczyk and T. Zajac, Game indicators determining sports performance in the NBA, *Journal of Human Kinetics*, 37 (2013), 145–151.
[22] P. Moreno and S. Lozano, A network DEA assessment of team efficiency in the NBA, *Annals of Operations Research*, 214 (2014), 99–124.
[23] A. Stefaniec, K. Hosseini, J. Xie and Y. Li, Sustainability assessment of inland transportation in China: A triple bottom line-based network DEA approach, *Transportation Research Part D: Transport and Environment*, 80 (2020), 102258.
[24] G. Villa and S. Lozano, Dynamic network DEA approach to basketball games efficiency, *Journal of the Operational Research Society*, 69 (2018), 1738–1750.
[25] M. Yang, Y. Wei, L. Liang, J. Ding and X. Wang, Performance evaluation of NBA teams: A non-homogeneous DEA approach, *Journal of the Operational Research Society*, (2020), 1–12.
[26] L. Zhang and K. Chen, Hierarchical network systems: An application to high-technology industry in China, *Omega*, 82 (2019), 118–131.

Received August 2020; revised December 2020.

E-mail address: saeedassani@nuaa.edu.cn
E-mail address: mansoor@mail.ustc.edu.cn
E-mail address: faisal_beaconite@hotmail.com
E-mail address: lionli@ustc.edu.cn
E-mail address: fengyang@ustc.edu.cn