Chapter 13
Blue Stragglers in Globular Clusters: Observations, Statistics and Physics

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13.1 Straw-Man Models for Blue Straggler Formation

In order to gain some intuition, let us start by considering the two simplest distinct formation channels for blue stragglers in globular clusters. First, blue stragglers may form in the same way in clusters as they do in the Galactic field, i.e. via mass transfer or coalescence in binary systems. In this case, we may expect the number of blue stragglers in any given cluster ($N_{\text{BSS}}$) to scale with the number of binary stars in the cluster ($N_{\text{bin}}$),

$$N_{\text{BSS}} \propto N_{\text{bin}} \propto f_{\text{bin}} M_{\text{tot}},$$  \hspace{1cm} (13.1)

where $f_{\text{bin}}$ is the fraction of binaries among the cluster members and $M_{\text{tot}}$ is the total mass of the cluster. In reality, $f_{\text{bin}}$ should really be the fraction of close binaries (since only these can be the progenitors of blue stragglers), but let us assume for the moment that these two quantities track each other, so that we can ignore this subtlety.

The second possibility is that blue stragglers in globular clusters form primarily via dynamical encounters. Here, the simplest possibility is that the most important encounters are direct collisions between two single stars. In this case, the number of blue stragglers should scale with the 1+1 collision rate ($\Gamma_{\text{coll},1+1}$), which is determined by the conditions in the dense cluster core via

$$N_{\text{BSS}} \propto \Gamma_{\text{coll},1+1} \propto R_c^3 n_c^2 \sigma_c^{-1},$$  \hspace{1cm} (13.2)

Here, $R_c$ is the core radius of the cluster, $n_c$ is the stellar density in the core, and $\sigma_c$ is the core velocity dispersion (which is a measure of the characteristic speed at which stars move in the core).
These scaling relations are clearly vast oversimplifications. Perhaps most obviously, both channels may produce significant number of blue stragglers in globular clusters. However, more importantly, even the very distinction between binary and dynamical formation channels is something of a false dichotomy. After all, the close binaries that are the progenitors of blue stragglers in the binary evolution channel may themselves have been formed or hardened by previous dynamical encounters (e.g. (11)). Similarly, it is not at all obvious that the total rate of stellar collisions should be dominated by encounters between single stars. In fact, Leigh & Sills (21) show that the rate of dynamical encounters is dominated by binaries (or even triples) even in environments with only modest binary (or triple) fractions (Figure 13.1). The probability of an actual stellar collision occurring in such encounters is discussed by (16). Thus, in reality, the binary channel may involve dynamical encounters, and the dynamical channel may involve binaries.

Fig. 13.1 The “phase diagram” of dynamical encounters. For any stellar population described by a particular combination of binary fraction and triple fraction, it is possible to determine which type of dynamical encounter will dominate. The plot above shows the regions of parameter space dominated by the various different encounters, assuming a particular set of characteristic binary and triple parameters. Reproduced from Figure 1 of Leigh & Sills (21). *An analytic technique for constraining the dynamical origins of multiple star systems containing merger products*, MNRAS, 410, 2370.
Does this mean that our simple straw-man models are useless? Not at all. First, the most extreme cases one can envision within the two channels are basically distinct. If blue straggler formation is dominated by mass transfer in or coalescence of primordial binaries that have not been affected (much) by dynamical encounters, \( N_{BSS} \) will scale with \( N_{bin} \) and not with \( \Gamma_{coll,1+1} \). Conversely, if the dominant channel are really single-single encounters, then \( N_{BSS} \) will scale with \( \Gamma_{coll,1+1} \) and not with \( N_{bin} \). Second, and perhaps more importantly, we might expect the basic scaling relations to be valuable even if binaries are affected by encounters and collisions involve binaries. This is particularly easy to see for the binary channel. Here, the relationship \( N_{BSS} \propto N_{bin} \) should presumably hold regardless of how the relevant binaries were formed (so long as our definition of \( N_{bin} \) does, in fact, refer to the “relevant” binary population, which may be a significant challenge in practice). For example, suppose that most of the close binaries that evolve into blue stragglers have been previously hardened in three-body encounters. In this case, we would expect \( N_{BSS} \propto N_{bin} \propto \Gamma_{1+2} \), where \( \Gamma_{1+2} \) is the 1+2 encounter rate. Similarly, if blue stragglers are predominately formed by direct collisions occurring during 1+2 encounters, we would expect \( N_{BSS} \propto P_{coll,1+2} \propto P_{coll,1+2} \Gamma_{1+2} \propto f_{bin} (a_{bin}/R_{*}) \Gamma_{coll,1+1} \), where \( P_{coll,1+2} \) is the probability of a physical collision occurring during a 1+2 encounter, \( a_{bin} \) is the characteristic binary separation and \( R_{*} \) is the characteristic stellar radius; see [17] for expressions linking the various encounter rates.

These specific examples show that, at the most basic level, the straw-man relations should remain roughly valid, even if reality is more complex than the limiting cases they formally represent. If blue straggler formation mostly involves binaries, we expect a scaling with \( N_{bin} \); if it mostly involves encounters, we expect a scaling with \( \Gamma_{coll,1+1} \). If binaries and dynamics work in tandem, \( N_{bin} \) and \( \Gamma_{coll,1+1} \) will simply be less distinct quantities.

Two final, technical points are worth noting here. First, \( N_{bin} \) and \( \Gamma_{coll,1+1} \) will always be statistically correlated, even if binaries are primordial and encounters dominated by single stars. After all, there are both more encounters and more binaries in an environment containing more stars. Davies, Piotto & De Angeli [3] show that this effect induces a scaling of \( \Gamma_{coll,1+1} \propto M_{tot}^{3/2} \), and \( N_{bin} \propto f_{bin} M_{tot} \). This needs to be kept in mind when comparing \( N_{BSS} \) to either of these quantities.

Second, two different conventions are sometimes used in statistical studies of blue stragglers. The first and simplest is to use raw numbers, \( N_{BSS} \), corrected (if necessary) for partial coverage of the relevant cluster. The second is to use blue straggler frequencies, \( N_{BSS}/N_{ref} \), where \( N_{ref} \) refers to the number of some reference population (e.g. horizontal branch stars) in the same field of view. It is important to understand that this difference matters. In particular, the straw-man scalings we have derived above hold only for blue straggler numbers. In the collision scenario, blue straggler frequencies should scale with the specific encounter rate, \( \Gamma_{1+1}/M_{tot} \). In the binary scenario, blue straggler frequencies should scale simply with the binary fraction, \( f_{bin} \).
13.2 All Theory is Grey: Binary Coalescence and Dynamical Encounters in Practice

It is interesting to ask at this point whether there is any empirical evidence that the physical mechanisms we are invoking in our two basic blue straggler formation channels actually occur in nature. Let us first consider the binary channel. Mass transfer is, of course, a well-established process in many close binary systems, including X-ray binaries (in which the accretor is a neutron star or black hole), cataclysmic variables (in which the accretor is a white dwarf) and Algols (in which the accretor is a main sequence star). But is there also evidence that full coalescence can occur?

As it turns out, there is. It has been known for quite a long time that some binary system, and in particular the eclipsing W UMa stars, are contact binaries, in which both binary components overfill their respective Roche lobes. In many such systems, the predicted time scale for full coalescence is much shorter than a Hubble time. W UMa binaries are therefore obvious progenitor candidates for apparently single blue stragglers in the Galactic field. In fact, quite a few blue stragglers in globular clusters are known to be (in) W UMa binaries.

![Figure 13.2](image)

**Fig. 13.2** Left: The long-term OGLE light curve of “Nova” Sco 2008 = V1309 Sco. Right: The phase-folded pre-eruption light curves of V1309 Sco for the 2002-2006 OGLE observing seasons. Reproduced from Figure 1 and 3 of Tylenda et al. [38], *V1309 Scorpii: Merger of a Contact Binary*, A&A, 528, A114.

However, we can actually do even better. In 2008, astronomers in Japan and China discovered an apparent nova in the constellation Scorpius [27]. Follow-up observations [22] quickly revealed that Nova Sco 2008 (aka V1309 Sco) was quite an unusual transient and probably related to the rare class of “red novae” (like V838 Mon). By a huge stroke of luck, the nova happened to lie in the footprint of the OGLE microlensing survey [39]. The pre- (and post-)eruption OGLE data of
V1309 Sco is astonishing \cite{38}. Not only does it provide an exquisite light curve of what turns out to be a \(\approx 6\) mag eruption, but it also reveals that the system was a \(P_{\text{orb}} \approx 1.4\) d W UMa contact binary prior to the outburst (Figure 13.2)! In fact, the OGLE data is good enough to provide several accurate measurements of the orbital period in the lead-up to the eruption (Figure 13.3). These measurements show that \(P_{\text{orb}}\) decreased significantly in just the \(\approx 6\) years leading up to the outburst. The implication is that Nova Sco 2008 represents a binary coalescence event caught in real time!

What about dynamical encounters or direct stellar collisions in globular clusters? No such event has been unambiguously observed in real time to date. This is not surprising given the low frequency and short duration of such events. There is nevertheless very strong empirical evidence that some exotic stellar populations in globular clusters are preferentially formed in dynamical encounters.

It has been known since the 1970s that bright low-mass X-ray binaries are over-abundant by a factor of \(\approx 100\) in globular clusters, relative to the Galactic field. This was quickly ascribed to the availability of unique dynamical formation channels in globular clusters, such as 1+1 tidal captures \cite{12,2,6}. However, observational confirmation of this idea required much more powerful X-ray telescopes and took nearly another three decades. The first convincing empirical case was made by Pooley et al. \cite{31}, who showed that the number of moderately bright X-ray sources in a given
cluster (which are dominated by a variety of accreting compact binaries) exhibits a clear scaling with the predicted dynamical collision rate in the cluster (Figure 13.4).

![Graph showing the number of X-ray sources detected in globular clusters versus the normalised collision rate of the cluster.](image)

**Fig. 13.4** The number of X-ray sources detected in globular cluster above $L_x \simeq 4 \times 10^{30}$ ergs$^{-1}$ versus the normalised collision rate of the cluster. Reproduced by permission of the AAS from Fig 2 of Pooley et al. [31], *Dynamical Formation of Close Binaries in Globular Clusters*, ApJL, 591, L141.

Pooley & Hut [30] later showed that, in high-collision-rate clusters, this scaling holds independently for both low-mass X-ray binaries and cataclysmic variables. However, there is also tentative evidence that, in low-collision-rate clusters, the number of these sources may instead scale with cluster mass. This suggests that the evolution of (primordial?) binaries may be sufficient to produce the few X-ray sources observed in such clusters. Our simple straw-man models thus do a rather good job in accounting for the observed number of X-ray binaries across the full range of Galactic globular clusters.
13.3 The Search for the Smoking Gun Correlation I: The Near Constancy of Blue Straggler Numbers

Let us return to blue stragglers. The first reasonably complete catalogue of blue stragglers in Galactic globular clusters was constructed and analysed by Piotto et al. [28]. This catalogue was based on an HST/WFPC2 survey that provided V and I colour-magnitude diagrams for 74 clusters [29]. Blue stragglers could be reliably selected in 56 of these clusters, yielding a total sample of nearly 3000 stars.

The results obtained by Piotto et al. [28] were surprising. Most importantly, they found no correlation between the frequency of blue stragglers and the cluster collision rate. Moreover, they found a weak anti-correlation between blue straggler frequency and cluster luminosity (i.e. total mass). One potentially confounding issue in their analysis is that, it is somewhat unnatural to correlate blue straggler frequencies against collision rate and total mass. As emphasised in Section 13.1, it is the number of blue stragglers that should scale linearly with these quantities in our straw-man models, not their frequency.

As it turns out, however, this issue is not the main cause of the unexpected results. Indeed, the same data base was re-analysed and interpreted by Davies et al. [3], who showed that blue straggler numbers also do not correlate significantly with collision rate (Figure 13.5 top panel). They also argued that blue straggler numbers are largely independent of total cluster mass/luminosity, although an inspection of their figure suggests that there may be a mild, positive, but sub-linear correlation between these quantities (Figure 13.5 bottom panel).

The number of horizontal branch stars does scale linearly with cluster mass or luminosity, as one would expect for a “normal” stellar population (Figure 13.5 top panel). More interestingly, the number of horizontal branch stars actually also correlates with the collision rate (Figure 13.5 bottom panel). This correlation is no doubt induced by the intrinsic correlation between cluster mass and collision rate (Section 13.1). Indeed, the scaling between horizontal branch numbers and collision rate is broadly in line with the relation we would expect in this case, i.e. \( N_{\text{HGB}} \propto \Gamma_{\text{coll}}^{2/3} \). But this only highlights the central mystery: apparently blue stragglers exhibit a weaker correlation with collision rate than horizontal branch stars, a population that is certainly not produced in dynamical encounters.

13.4 Do Clusters Deplete their Reservoir of Binary Blue Straggler Progenitors?

Davies et al. [3] suggested an interesting interpretation for the near constancy of blue straggler numbers (Figure 13.5). Their idea invokes a combination of binaries and dynamical encounters. Specifically, they consider a binary mass transfer scenario in which blue stragglers are formed when the primary star leaves the main sequence, expands and fills its Roche lobe. This initiates mass transfer onto the sec-
Fig. 13.5  *Top panel:* The observed number of blue stragglers and horizontal branch stars as a function of stellar collision rate. *Bottom panel:* The estimated number of blue stragglers and horizontal branch stars as a function of the absolute magnitude of the cluster. Reproduced from Fig 1 of Davies et al. [3], *Blue Straggler Production in Globular Clusters*, MNRAS, 348, 129.

Secondary, which can then be converted into a blue straggler. Davies et al. note that, in dense globular clusters, each binary is likely to undergo many encounters with single stars. In each such encounter, the most likely outcome is the ejection of the least massive star, so these encounters strongly affect the mass distribution of the binary population. This, in turn, affects the ability of this population to form blue stragglers, since only systems with primaries close to the turn-off mass are viable progenitors.
The central argument put forward by Davies et al. [3] is that, in high collision rate clusters\(^1\), relatively massive stars near the cluster turn-off mass are likely to have exchanged into binaries well before the present day. Such clusters may therefore have used up their blue straggler binaries by the present day and may thus now be deficient in blue stragglers derived from the binary channel. On the other hand, blue stragglers formed via direct stellar collisions should be more numerous in these clusters. Davies et al. therefore suggest that these two effects broadly cancel. This would imply that binary-derived blue stragglers dominate in low-collision-rate clusters, while collisional blue stragglers dominate in high-collision-rate clusters, even though the absolute numbers in both types of clusters are more or less the same.

In support of this argument, Davies et al. carried out a simple simulation. In this, a set of initial (“primordial”) binaries was created, in which the mass of each binary component is drawn independently from a simple initial mass function [4, 5]. Each binary was then subjected to a series of exchange encounters with single stars whose masses are also drawn from the same IMF. In each encounter, the least massive of the three stars involved was ejected, and the remaining two assumed to remain as a binary system. After each encounter, a system was labelled as a blue straggler if the mass of its primary satisfied

\[0.8 \, M_\odot < M_1 < 0.816 \, M_\odot.\]

This corresponds to the range of turn-off masses over the last 1 Gyr – a typical blue straggler life time – for a typical Galactic globular cluster.

In order to gain some insight, we have repeated their simulation. The black histogram in Figure 13.6 shows how the fraction of blue stragglers among the simulated binaries, \(f_{pbs}\), depends on the number of encounters a binary has undergone, \(N_{enc}\). In agreement with their results, we see that \(f_{pbs}\) initially increases as the number of encounters goes up, but then peaks at \(N_{enc} = 6\) and declines monotonically towards larger \(N_{enc}\). The expected number of encounters in the highest collision rate clusters is larger than six over the cluster lifetime, so these results appear to suggest that binary-derived blue stragglers will indeed be rare in such clusters today.

There are, however, problems with this scenario. For example, one would still expect a scaling of blue straggler numbers with \(\Gamma_{coll,1+1}\) at least for high-collision-rate clusters (which is not really observed). Moreover, since exchange encounters are required to produce even binary-derived blue stragglers in this model, we might actually still expect a scaling with \(\Gamma_{coll,1+1}\) for these objects (via an intrinsic scaling with \(I_{1+2}\); see Section 13.1).

However, the most fundamental problem with the simulation is that it assumes that all stellar masses are available for all encounters. In reality, each successive encounter represents a later time in the evolution of the cluster, so stars above the main sequence turn-off corresponding to this time will already have evolved off the main sequence. In order to test if this effect matters, we have repeated the simulation once more, but this time with a rough model for stellar evolution.

In this new calculation, we first estimate the typical time interval between encounters for a given set of representative binary and cluster parameters. This provides the time step for the simulation. We then evolve the binary population forward

\(^{1}\) Strictly speaking, we are talking here about clusters with high specific collision rates, i.e. clusters in which each binary undergoes many encounters.
The fraction of “blue stragglers” among a simulated binary population versus the number of encounters with single stars the binary has undergone. Blue stragglers are defined as binaries with primaries near the present-day turn-off mass. The black line corresponds to the case where the masses of the initial binary components, as well as the masses of the single stars the binary encounters, are drawn from an unrestricted initial mass function. This is the case considered by Davies et al. [3]. The red line corresponds to the case where, at each encounter, all stellar masses are replaced with a suitable compact object mass if they exceed the appropriate turn-off mass at the time of the encounter (see text for details). Note that while the black line declines fairly quickly towards large $N_{\text{enc}}$, the red line does not.

by allowing each binary to encounter a single star at each time step. At this point, we first check if the primary has turned off the main sequence since the last time step. If so, we assume that mass transfer has already started and that no further exchange encounters will take place. If not, we once again ask if an exchange encounter will happen. However, we now also first check if the single star the binary has encountered has turned off the main sequence. If so, we replace its mass with that of the relevant compact object\footnote{If $M_i > 18 \, M_\odot$, we assume the star has turned into a black hole, so that $M_f = 10 \, M_\odot$; if $7 \, M_\odot < M_i < 18 \, M_\odot$, we assume the star has turned in a neutron star, so that $M_f = 1.4 \, M_\odot$; finally, if $M_{\text{ms}}(t) < M_i < 7 \, M_\odot$, we assume the star has become a white dwarf, so that $M_f = 0.5 \, M_\odot$. These mass ranges, as well as the main sequence lifetimes, are estimated using SSE [10].}. Once the present day is reached, we calculate the fraction of blue stragglers in the same way as before (but excluding systems with white dwarf secondaries). We then carry out the same calculation for a wide range of assumed cluster densities (and hence collision rates), with each density corresponding to a different number of encounters between the birth of the cluster and the present day.

The results of this modified simulation are shown by the red line in Figure\ref{fig:13.6}. With our simplistic treatment of stellar evolution included, $f_{\text{psb}}$ now rises fairly
quickly up to $N_{\text{enc}} \simeq 10$ and then stays nearly constant out to at least $N_{\text{enc}} \simeq 60$. The absence of a sharp decline towards high $N_{\text{enc}}$ is actually easy to understand. In the original simulation, where all stellar masses are available in all encounters, the overall binary population quickly becomes dominated by systems with main sequence primaries more massive than 0.816 M$_\odot$. It is the increasing dominance of these systems that fundamentally causes the decline in $f_{\text{pbs}}$ towards larger $N_{\text{enc}}$. But this is of course unphysical, since such systems should not exist at the present day. In the revised simulation, $f_{\text{pbs}}$ stays high because the most massive stars in the cluster (ignoring the extremely rare neutron stars and black holes) are now always stars with masses near the turn-off mass.

Davies et al. [3] emphasised that their simulation ignored several important physical effects, and this is still true of our revised simulation as well. Some of these effects are discussed in more detail in Chapter 9. Nevertheless, the results in Figure 13.6 suggest that the depletion of blue straggler binaries in high collision rate clusters may not offer as natural an explanation for the observed blue straggler numbers as previously envisaged.

### 13.5 The Search for the Smoking Gun Correlation II: The Core Mass Correlation

One obvious explanation for the lack of convincing correlations between global blue straggler numbers and cluster parameters is that both the binary and the collisional channels contribute. In particular, it seems plausible that each channel may dominate in different regions within a cluster, with collisions perhaps dominating in the dense core, and binary evolution dominating in the periphery. More generally, it seems safe to assume that if collisions/dynamics dominates blue straggler production anywhere, it will be in cluster cores. So is it possible that a cleaner picture may emerge if we focus specifically on blue stragglers found in the cores of their parent clusters?

We investigated this idea in [13], building on a new blue straggler catalogue constructed by Leigh, Sills & Knigge [18, 19]. This catalogue was still based on the WFPC2 data set of Piotto et al. [29], but included only systems found in the cluster core by a consistent photometric selection algorithm. Our hope and expectation was that the number of core blue stragglers would show a strong correlation with cluster collision rate. However, Figure 13.7 shows that we were wrong. More in desperation than expectation we then decided to also have a look at the binary hypothesis. Since no comprehensive set of empirically derived core binary fractions, $f_{\text{bin,core}}$, were available in 2009, our only option was to use the total core mass, $M_{\text{core}}$, as a proxy for the number of binaries in the core. This is reasonable so long as core binary fractions do not vary dramatically between clusters.

Much to our surprise, plotting $N_{\text{BSS,core}}$ vs $M_{\text{core}}$ immediately revealed a clear correlation (Figure 13.8). This would seem to suggest that stragglers are preferentially formed via the binary channel, even in dense cluster cores. However, the observed
scaling is clearly sub-linear, and a fit to the data suggests \( N_{\text{BSS,core}} \propto M_{\text{core}}^{0.4} \). Can this be accommodated within a simple binary scenario?

The simplest way to accomplish this is to remember that the intrinsic scaling should be with the number of binaries in the core, not just the core mass, i.e. \( N_{\text{BSS,core}} \propto N_{\text{bin,core}} \propto f_{\text{bin,core}} M_{\text{core}} \) in the binary picture. Thus the observed scaling could be trivially understood if the core binary fractions themselves scale with core mass as \( f_{\text{bin,core}} \propto M_{\text{core}}^{-0.6} \).

As already noted above, there was no definitive set of core binary fractions available to test this prediction in 2009. However, Sollima et al. [36, 37] had derived empirical binary fraction for a small sample of low-density clusters and had already shown that these correlated positively with the blue straggler frequencies in these clusters. Also, Milone et al. [26] had just obtained preliminary estimates of the core binary fractions in a larger sample of clusters, based on the HST/ACS survey of Galactic globular clusters [33] and found a clear anti-correlation between core binary fractions and total cluster mass. Even though neither set of core binary fractions were suitable for combining directly with the blue straggler data based used in [13], they permitted preliminary tests for a correlation with core mass. These tests
suggested that $f_{\text{bin,core}} \propto M_{\text{core}}^{-0.35}$, not too far from the expected relation, albeit with considerable scatter.

In Section 13.7 we will consider whether more recent, higher quality observations confirm, refute or modify these results. However, in 2009, our conclusion was that blue stragglers seem to be derived mainly from binary systems, even in dense cluster cores.

### 13.6 Alternative Constraints on Formation Channels

Let us accept for the moment that the scaling of blue straggler numbers with cluster parameters tends to favour a binary formation channel. Are there other strands of evidence that would challenge this idea?

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3 We were careful not to rule out the possibility that the relevant binary population may be affected by dynamical encounters. However, we also noted that the absence of a scaling with collision rates seemed hard to understand in any scenario involving lots of dynamical encounters (see Section 13.1).
As discussed elsewhere in this book, it is extremely difficult to confidently assign a specific formation mechanism to a particular blue straggler. The only convincing cases are the Carbon/Oxygen-depleted blue stragglers, which were initially discovered by Ferraro et al. [9] in 47 Tuc. This chemical anomaly is an expected consequence of mass transfer, since this process can dredge up CNO-processed material from the stellar interior. By contrast, no unique spectroscopic signature for dynamically/collisionally formed blue stragglers is known.

Nevertheless, there are at least two other types of observations that may shed light on blue straggler formation in globular clusters. They are (i) the radial distribution of blue stragglers in a given cluster, and (ii) the discovery of a double blue straggler sequence in M30 (and perhaps other clusters). Both of these observations are discussed in much more detail elsewhere in this book, so here we will merely ask whether (or to what extent) they conflict with the idea that most blue stragglers derive from binaries, rather than from dynamical encounters.

### 13.6.1 Radial Distributions

In most globular clusters, the dependence of blue straggler frequency on radius is bimodal (Figure 13.9; e.g. [8, 32, 40, 14, 15]. These distributions have been modelled quite successfully by Mapelli et al. [23, 24]. Their simulations follow the motion of blue stragglers in a static cluster potential, assuming that collisional blue stragglers form only within the core, while binary-derived blue stragglers all start their lives outside the core. The ratio of collisional to binary blue stragglers is a free parameter of the model.

Mapelli et al. [23, 24] obtained the best fits to the observed distributions with both channels contributing a comparable number of blue stragglers to the total population. Moreover, they also found that collisional blue stragglers completely dominate the population in the cluster core, while binary blue stragglers are dominant in the cluster halo, beyond the minimum in the radial distribution. Yet this conclusion seems incompatible with the lack of a correlation between core blue straggler numbers and cluster collision rates.

It is useful to take a step back at this point and consider the physics that produces the bimodal blue straggler distributions. As discussed by Mapelli et al. [23, 24] and described in more detail elsewhere in this book, the key dynamical process is dynamical friction. Since blue stragglers are relatively massive, they tend to sink towards the cluster core. The time scale on which this happens, $t_{df}$, changes as a function of radius. We can therefore define a critical radius, $R_{min} \simeq R(t_{dc} = t_{gc})$, where $t_{gc}$ is the lifetime of the cluster. Binary blue stragglers born well inside $R_{min}$ have had plenty of time to sink to the core, while those born well outside this radius have barely moved from their original location. The minimum in the blue straggler distribution therefore corresponds roughly to $R_{min}$.

These considerations highlight an important point: a blue straggler population containing only binary blue stragglers should still produce a bimodal radial distri-
Fig. 13.9 The radial distribution of blue straggler frequencies in the globular clusters 47 Tuc and NGC 6752. Blue straggler frequencies are defined here as the number of blue stragglers normalised to the number of horizontal branch stars. The filled blue point and solid blue lines are the observational data, while the open red circles and red dashed lines are the best-fitting models of Mapelli et al. [24]. The dotted lines mark the location of the minimum in the radial distributions. See text for details. Figure adapted from Figure 2 of Mapelli et al. [24], The Radial Distribution of Blue Straggler Stars and the Nature of their Progenitors, MNRAS, 373, 361.

bution. So why are collisional blue stragglers needed at all in the simulations? The answer is that not enough binary blue stragglers were seeded inside $R_{\text{min}}$. But this is just an assumption. If the birth distribution of binary blue stragglers is allowed to be centrally peaked, a population consisting exclusively of such systems may be able to match the data as well (see [24]).

It is perhaps also worth emphasising here that the scenario preferred by Mapelli et al. [23, 24] is very different from that suggested by Davies et al. [3]. Both scenarios do favour a combination of binary-derived and collisional blue stragglers. However, in Davies et al. model, different formation channels dominate in different clusters, whereas in Mapelli et al.’s model, different channels operate in different locations within a given cluster. In any case, the key point for our purposes here is that a bimodal radial distribution does not necessarily require distinct formation channels for the core and halo blue straggler populations.

13.6.2 Double Blue Straggler Sequences

One other recent discovery is highly relevant to the question of blue straggler formation channels. Ferraro et al. [7] showed that colour magnitude diagram of the globular cluster M30 appears to contain two distinct blue straggler sequences (Fig
ure 5.7 in Chapter 5). Their interpretation of this observation is that objects on the blue sequence were formed via collisions, while those on the red sequence are derived from binaries.

Why should there be such a clean separation between these two types of systems in M30? The idea put forward by Ferraro et al. [7] is that, in M30, the collisional blue stragglers all formed recently in a short burst, most likely when the cluster underwent core collapse. All of these objects therefore share the same evolutionary state, so that all of them line up on a well-defined main sequence. By contrast, the red sequence lies roughly 0.75 mag above the extension of the cluster zero-age main sequence, as expected for a population of roughly equal-mass binaries. If this idea is correct, then both sequences could be present in many/most clusters, but would usually overlap too much to be noticeable as distinct entities.

The double blue straggler sequence in M30 is almost certainly an important clue, and Ferraro et al. present new data elsewhere in this book (see Chap. 5) that appear to show a similar double sequence in another cluster. If confirmed, it would be nice if each of the two sequences really does correspond to a distinct formation channel, even if this may make it harder to understand other results, such as the core mass correlation. However, there is at least one surprising aspect to the double sequence in M30. As noted by Ferraro et al. [7], their blue straggler sample for this cluster contains 3 W UMa binaries and two other variables that are likely binaries. However, these are not all located on the red (binary) sequence. Rather, one W UMa and one other binary are located nicely on the collisional sequence. Perhaps this simply means that these two binaries were produced in (or affected by) dynamical encounters, while the others are mostly primordial. Nevertheless, if each sequence corresponds cleanly to a particular formation channel, it does seem surprising that the known binaries should be split nearly evenly between them.

**Fig. 13.10** The fraction of binaries with mass ratios $q > 0.5$ in the cluster core versus the blue straggler frequency in the core. The two red points near the bottom right of the plot correspond to post-core-collapse clusters. Figure reproduced from Figure B5 of Milone et al. 55, *The ACS Survey of Galactic Globular Clusters XII: Photometric Binaries along the Main Sequence*, A&A, 540, A16.
13.7 The Search for the Smoking Gun Correlation III: Once More, With Binary Fractions...

Binaries are key to the study of cluster dynamics. In fact, the late dynamical evolution of globular clusters is thought to be driven by binary systems (e.g. (11)). It was therefore a major breakthrough when Milone et al. [25] presented photometric estimates of binary fractions for 59 clusters, based on the HST/ACS survey of Galactic globular clusters already mentioned in Section 13.5.

Three trends discovered by Milone et al. are of immediate relevance to the blue straggler formation problem. First, core binary fractions correlate only weakly with $\Gamma_{\text{coll},1+1}$. Second, they anti-correlate more strongly with total cluster luminosity (and hence mass). Third, they correlate very strongly with blue straggler frequency (Figure 13.10). All of these trends are quite promising for the idea that binaries dominate blue straggler production, as suggested in (13).

However, the availability of binary fractions makes it possible to test the key formation scenarios much more directly. For example, we can now compare the number of core blue stragglers directly to the number of binaries in the core, rather than just to the total core mass. Similarly, we can now directly estimate 1+2 and 2+2 encounter rates ($\Gamma_{1+2}$ and $\Gamma_{2+2}$; see Section 13.1). If blue straggler production is dominated by encounters involving binaries (e.g. exchange encounters; see Section 13.4), blue straggler numbers should correlate strongly with $\Gamma_{1+2}$ or $\Gamma_{2+2}$.

We carried out these tests in [17]. For this purpose, we combined the core blue straggler numbers derived by Leigh et al. [20] with the core binary fractions obtained by Milone et al. [25]. These data sets are ideally matched, since both are based on the HST/ACS survey of Galactic globular clusters [33].

Let us first look at the results for dynamical formation scenarios. Figure 13.11 shows plots of $N_{\text{BSS,core}}$ against each of $\Gamma_{\text{coll},1+1}$ (top left panel), $\Gamma_{1+2}$ (top right panel) and $\Gamma_{2+2}$ (bottom panel). None of the encounter rates correlate cleanly with the number of blue stragglers in cluster core.

Now let us look at the binary evolution scenario. The top left panel in Figure 13.12 shows that the ACS data confirms the existence of a strong correlation between $N_{\text{BSS,core}}$ and core mass, with $N_{\text{BSS}} \propto M_{\text{core}}^{0.4}$ (also see [20]). The top right panel in Figure 13.12 shows that the data also confirm the prediction of a strong anti-correlation between core binary fraction and $M_{\text{core}}$. A power-law fit to this relation gives roughly $f_{\text{bin,core}} \propto M_{\text{core}}^{-0.6}$, a little shallower than predicted, but not too far from the expected $M_{\text{core}}^{-0.6}$ dependence. So far, so promising. However, the bottom panel in Figure 13.12 shows what happens when we directly compare $N_{\text{BSS,core}}$ to $N_{\text{bin,core}} \propto f_{\text{bin,core}} M_{\text{core}}$. Instead of improving on the correlation with core mass alone, the addition of empirical binary fractions actually degrades it!

This is a surprising result. One possibility is that it simply means that all of our straw-man models are too simplistic after all (although, as noted in Section 13.1).

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4 As already noted by Leigh et al. [20], the correlation between $N_{\text{BSS,core}}$ and $\Gamma_{\text{coll},1+1}$ is formally significant, but probably induced by the intrinsic correlation between $\Gamma_{\text{coll},1+1}$ and $M_{\text{core}}$ (see Section 13.1).
it seems quite hard to avoid all of the expected correlations, even in more complex formation scenarios). However, before we accept that we need “new physics”, we should check if we can somehow reconcile one of the existing models with our new findings.

Since the binary evolution model predicts at least the observed correlation with core mass (and the lack of a correlation with collision rate), let us imagine that all blue stragglers are exclusively formed from (primordial) binaries. In this case, $N_{\text{BSS, core}} \propto N_{\text{bin, core}} \propto f_{\text{bin, core}} M_{\text{core}}$, with just some modest intrinsic scatter. We already know empirically that $f_{\text{bin, core}}$ anti-correlates quite strongly with $M_{\text{core}}$. But now suppose that the intrinsic anti-correlation is even stronger than the observed one, i.e. that the scatter in the middle panel of Figure [13.12] is mostly due to observational errors on $f_{\text{bin, core}}$, rather than any intrinsic dispersion. In this limit, $M_{\text{core}}$ actually becomes a better predictor of the true core binary fractions than the observationally estimated values. The number of binaries in the core – and hence the
number of blue stragglers – will then also be predicted more accurately by $M_{\text{core}}$ alone than by the empirically estimated combination of $f_{\text{bin,core}}M_{\text{core}}$.

We have carried out some simple simulations to test and illustrate this idea. In these simulations, we create mock data sets of similar size and dynamic range as the real data and assume that the number of blue stragglers scales perfectly and linearly with the number of binaries, i.e. $N_{\text{BSS,core}} \propto N_{\text{bin,core}}$. We also assume that $f_{\text{bin,core}} \propto M_{\text{core}}^{0.6}$, with only a slight intrinsic dispersion, $\sigma_{\text{int}}$. Finally, we assume that our observational estimates of $f_{\text{bin,core}}$ are subject to an observational uncertainty of $\sigma_{\text{obs}}$, which we vary in the range $0.1\sigma_{\text{int}} \leq \sigma_{\text{obs}} \leq 10.0\sigma_{\text{int}}$. We then analyse each mock data set to estimate the correlation coefficients of $N_{\text{BSS,core}}$ against the “observationally estimated” $M_{\text{core}}$ and $N_{\text{bin,core}} = f_{\text{bin,core}}M_{\text{core}}$. We also fit the latter correlation with a power law and estimate the power law index.
The effect of observational uncertainties in binary fractions on correlations between blue straggler numbers and cluster parameters. The data shown in both panels are derived from simulations designed to roughly mimic the data shown in Figure 13.12 (see text for details). The ordinate in both panels is the ratio of the assumed observational uncertainty, $\sigma_{\text{obs}}$, to the assumed intrinsic dispersion in the relation between $f_{\text{bin},\text{core}}$ and $M_{\text{core}}$. Top panel: The ratio of the correlation coefficients between $N_{\text{BSS,core}}$ and $N_{\text{bin,core}}$, on the one hand, and $N_{\text{BSS,core}}$ and $M_{\text{core}}$, on the other; note that the correlation with $N_{\text{bin,core}}$ will only seem stronger than that with $M_{\text{core}}$ if $\sigma_{\text{obs}} << \sigma_{\text{int}}$. Bottom panel: The inferred power law index of the $N_{\text{BSS,core}}$ versus $N_{\text{bin,core}}$ correlation; note that the correct value of unity is only obtained if $\sigma_{\text{obs}} << \sigma_{\text{int}}$. Figure reproduced from Figures 9 of Leigh et al. [17]: The Origins of Blue Straggler and Binarity in Globular Clusters, MNRAS, 428, 897.
Figure 13.13 shows how the ratio of the estimated correlation coefficients, and also the inferred power law index of the $N_{\text{BSS,core}}$ vs $N_{\text{bin,core}}$ relation, depend on the ratio of $\sigma_{\text{obs}}/\sigma_{\text{int}}$. As expected, when $\sigma_{\text{obs}} < < \sigma_{\text{int}}$, the “empirical” core binary fractions add value. In this case, the correlation coefficient between $N_{\text{BSS,core}}$ and $N_{\text{bin,core}}$ is larger than that between $N_{\text{BSS,core}}$ and $M_{\text{core}}$. Also, the inferred power law index of the $N_{\text{BSS,core}}$ vs $N_{\text{bin,core}}$ relation is unity, i.e. we correctly infer that the intrinsic relation is linear. However, when $\sigma_{\text{obs}} \gg \sigma_{\text{int}}$, the empirical binary fractions only serve to degrade the underlying signal. In this limit, the correlation of $N_{\text{BSS}}$ with $M_{\text{core}}$ is stronger than that with the estimated $N_{\text{bin,core}}$, and the relationship between $N_{\text{BSS,core}}$ and $N_{\text{bin,core}}$ is incorrectly inferred to be sub-linear. So the binary evolution might still be consistent with the observations, but only if (core) binary fractions correlate extremely cleanly with cluster (core) masses.

13.8 Summary & Outlook

What are the key points to take away from our look at blue straggler statistics? On the observational front, we have seen that (i) blue straggler numbers do not correlate with dynamical encounter rates; (ii) they do correlate strongly with cluster (core) masses; (iii) empirically estimated core binary numbers (obtained by combining core masses with photometrically determined core binary fractions) correlate less strongly with blue straggler numbers than core masses alone.

The first point would seem to argue against a dynamical origin for most blue stragglers in globular clusters. Yet presumably encounters and collisions must happen in such dense environments at roughly the predicted rates. So can the efficient production of blue stragglers via this channel actually be avoided? Sills et al. [34] followed the evolution of a simulated stellar collision products. One of their key findings was that, if left to their own devices, such objects tend to exceed the breakup velocity and can be completely disrupted if unbound mass shells are successively removed from the surface. In their words: “either blue stragglers are not created through physical off-axis collisions or some mechanism(s) can remove angular momentum from the star on short timescales” [34]. Thus perhaps stellar collisions occur, but do not produce blue stragglers.

On the other hand, recent dynamical simulations of globular clusters suggest that, even if blue stragglers are produced predominantly by (mainly binary-mediated) collisions, their numbers may scale only weakly with $\Gamma_{\text{coll,1+1}}$ [1] [35]. The reason for this is not immediately apparent, however, and it is also not clear if a strong scaling with $\Gamma_{1+2}$ could be avoided as well.

The second point – the core mass correlation – can be explained most naturally in the context of a binary scenario for blue straggler formation. However, the third point – the poor correlation obtained when core masses are combined with empirically estimated binary fractions – seems at first sight inconsistent with a binary

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5 We should note, however, that Chatterjee et al. [1] and Sills et al. [35] argue that the dynamically-formed blue stragglers in their simulations also produce a core mass correlation.
scenario. We have seen that this discrepancy can be resolved if core binary fractions are extremely tightly coupled to core masses. If this idea is correct, it would have significant implications for our understanding of cluster dynamics, well beyond the realm of blue stragglers.

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