Dissipation-driven superconductor-insulator transition in linear arrays of Josephson junctions capacitively coupled to metallic films

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We study the low-temperature properties of linear Josephson-junction arrays capacitively coupled to a proximate two-dimensional diffusive metal. Using bosonization techniques, we derive an effective model for the array and obtain its critical properties and phases at $T = 0$ using a renormalization group analysis and a variational approach. While static screening effects given by the presence of the metal can be absorbed in a renormalization of the parameters of the array, backscattering originated in the dynamically screened Coulomb interaction produces a non-trivial stabilization of the insulating groundstate and can drive a superconductor-insulator transition. We study the consequences for the transport properties in the low-temperature regime. In particular, we calculate the resistivity as a function of the temperature and the parameters of the array, and obtain clear signatures of a superconductor-insulator transition that could be observed in experiments.

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I. INTRODUCTION

Low-dimensional superconductors are systems displaying a surprisingly complex and rich physics, allowing the study of paradigmatic phenomena in condensed matter physics, like quantum phase transitions and quantum critical behavior, electronic localization, Coulomb blockade, etc. In particular, an intriguing superconductor-insulator phase transition (SIT) was observed experimentally in superconducting films, wires, and in ultrasmall-capacitance Josephson junction arrays (JJAs) in two- and one-dimensional systems, giving rise to an intense theoretical activity. In this transition, as one of the parameters is varied (e.g., the normal-state resistance of the film, the thickness of the wire, the Josephson coupling $E_J$ in the array, etc.) the groundstate of the system changes from superconducting to insulator.

In one-dimensional (1D) superconductors, one particular kind of excitation, the so-called quantum-phase slip (QPS) processes, have been recently the focus of an intense research. The interest is based both on the putative role of QPS in the SIT in 1D, as well as for their potential uses in novel qubit architectures, a fact that has stimulated recent interesting experimental research in 1D JJJAs. A phase-slip is a discrete process occurring in a 1D superconductor, in which the amplitude of the order parameter vanishes temporarily at a particular point, allowing the phase of the order parameter to change abruptly in units of $2\pi$. In particular, a QPS is a phase-slip excitation originated in macroscopic quantum tunneling of the phase of the order parameter.

On the other hand, since the seminal works by Caldeira and Leggett, it has been known that dissipation in macroscopic quantum systems plays a central role. For instance, in a two-dimensional JJA capacitively coupled to a proximate two-dimensional electron gas (2DEG), Rimberg et al. observed a tunable-SIT upon variation of the backgate voltage $V_g$ applied to the 2DEG. In that work, it was shown that $V_g$ has the effect of tuning the sheet resistance $R_{\square}$ in the 2DEG through the modulation of its electronic density, a fact that in turn modifies the electromagnetic environment of the JJA. It was argued later by Wagenblast et al. that due to the incomplete screening of the Coulomb interaction provided by the 2DEG, the 2D plasma mode in the array is overdamped and the charging energy in the junction $E_C$ is renormalized to higher values, producing a SIT whenever the ratio $E_J/E_C \sim 1$, with $E_C$ the renormalized $E_C$. While this scenario is reasonable in a 2D geometry, in a 1DJJA capacitively coupled to a 2DEG, the screening provided by the metal is typically very efficient, and a significant damping of the 1D propagating plasma mode is not expected. This leads to the naive conclusion that in the 1DJJA/2DEG geometry, a dissipation-driven SIT should not occur. However, in a closely related Luttinger-liquid system placed in proximity to a metallic plane, a dissipation-driven quantum phase transition was predicted to occur. This transition is driven by backscattering events originated in the Luttinger liquid under the effect of the dynamically screened Coulomb interaction. It is therefore interesting to study to what extent the same dissipative processes will affect the dynamics of QPS in 1D superconductors in proximity of a
diffusive metallic plane. Indeed, the question of dissipation in 1D superconductors is an active area of research, and theoretical predictions point towards the important role of intrinsic and/or extrinsic dissipation mechanisms in determining their $T = 0$ phase diagram.

In this article we explore the possibility of a dissipation-driven SIT in a 1DJJA capacitively coupled to a proximate diffusive 2DEG. We concentrate in particular on the low-temperature phase diagram and on the transport properties of the array. Using a bosonization approach, we derive the dissipative effective action from a microscopic Hamiltonian, and we elucidate the role of dissipation in the SIT. One important conclusion in our work is that for weak dissipation, the transition occurs always between a superconducting and an insulating phase, in contrast to other works predicting quadrupolar and normal phases. We believe this is a consequence of a different kind of dissipation in the model. We find that, except in the experimentally challenging situation in which the Cooper-pair density in the superconducting islands is not commensurate, the SIT is always of the Berezinskii-Kosterlitz-Thouless (BKT) type and is originated in the unbinding of QPS/anti-QPS pairs. Dissipation stabilizes the insulating groundstate through the introduction of friction in the dynamics of the 1D superfluid density, a fact that could be observed experimentally in the dc-resistivity of the 1DJJA. Specifically, we predict a resistivity of the form $\rho(T) \sim A_1 T^{\nu_1} + A_2 T^{\nu_2}$ in the superconducting phase, and $\rho(T) \sim \eta T e^{|\Delta|/\Delta}$, in the insulating phase, with $\Delta$ the insulating gap and $\eta$ the dissipation parameter.

The paper is organized as follows. In Section II we derive the effective model for a 1DJJA coupled to a 2DEG, in Section III we derive the $T = 0$ phase-diagram as a function of the parameters of the model, Section IV is devoted to the study of the experimental consequences of our results, and finally in Section V we present a summary and our conclusions.

## II. MODEL

We start the analysis by considering an ideally isolated JJA, with length $L \to \infty$. To simplify the analysis, we neglect in the following the fermionic degrees of freedom forming the Cooper-pairs at a microscopic level. This “boson-only” approximation is believed to describe correctly the critical properties of a JJA at temperatures $T \ll T_c$, with $T_c$ the superconducting critical temperature in the bulk of the superconducting island. The usual description of the isolated, infinite 1DJJA is given in terms of the quantum phase model:

$$H_{\text{JJA}} = \frac{1}{2} \sum_{i,j} (n_i - \bar{n}) v_{ij} (n_j - \bar{n}) + \sum_{\langle ij \rangle} E_J (1 - \cos \theta_i - \theta_j).$$

(1)

The dynamical variables of this model are the number of Cooper pairs $n_i$ and the phase of the superconducting order parameter $\theta_i$ at every site $i$ in the array. These variables obey the usual phase-number commutation relation in the BCS groundstate, i.e., $[\theta_i, n_j] = i \delta_{ij}$. The first term in Eq. (1) represents the charging energy, with $v_{ij}$ the unscreened Coulomb interaction [cf. Eq. (3)] between the excess charges at sites $i$ and $j$, and $\bar{n}$ corresponds to an average charge imposed, e.g., by external gate voltages. The second term is the Josephson energy contribution, parametrized by $E_J$. In the following we use the convention $\hbar = k_B = 1$.

The critical properties of model Eq. (1) are more conveniently studied using a field-theoretical approach, valid for fluctuations of wavelengths much larger that the lattice parameter of the array $a$. We therefore introduce the coarse-grained superfluid density $\delta \rho(x)$, defined as $\delta \rho(x_i) = (n_i - \bar{n})/a$, and we expand the Josephson term as $E_J \cos (\theta_i - \theta_j) \simeq E_J a^2 (\nabla \theta(x_i))^2$. At low temperatures, the continuum limit of Hamiltonian Eq. (1) reads

$$H_{\text{JJA}} = \frac{1}{2} \int dxdx' \delta \rho(x) v(x - x', 0) \delta \rho(x') + \frac{1}{2} E_J a \int dx (\nabla \theta(x))^2.$$  

(2)

Here the 1D superfluid interacts via the bare Coulomb potential, which we define for convenience as

$$v(r, z) = \frac{e^2}{\epsilon \sqrt{r^2 + z^2 + a^2}},$$

(3)

where $r = |r|$ and $z$ are, respectively, the distance in the $xy$-plane and along the azimuthal direction between two point-charges [cf. Eq. (10)]. Here the lattice parameter $a$ acts as the short-distance regularization of the interaction and $\epsilon_r$ is the permittivity of the insulating medium surrounding the islands. Note that in Eq. (2) we do not assume an a priori short-ranged, screened interaction as is usually done when dealing with JJAs. This will result as a natural consequence of the interaction with the 2DEG (see below). One problem of this field-theoretical approach is that Mott-instabilities (crucial when the superfluid density is commensurate with the lattice) are lost in Eq. (2) after taking the continuum limit. One way to cure this problem is to introduce a phenomenological term $H_1 = -\int dx V_1(x) \rho(x)$, where the effective superfluid density $\rho(x)$ [cf. Eq. (10)] couples to the phenomenological potential $V_1(x)$, having the same periodicity of the lattice.

The electrons in the 2DEG are described by the Hamiltonian

$$H_{\text{2DEG}} = \int d^2r \sum_{\sigma} \left[ -\frac{1}{2m} \eta_{\sigma}^\dagger \nabla^2 \eta_{\sigma} + V_{\text{imp}} \eta_{\sigma}^\dagger \eta_{\sigma} \right] + \frac{1}{2} \int d^2r d^2r' \delta \rho_{\text{2DEG}}(r) v(r - r', 0) \delta \rho_{\text{2DEG}}(r'),$$

(4)

where the fermionic field-operator $\eta_{\sigma}^\dagger \equiv \eta_{\sigma}^\dagger (r)$ creates an electron in the 2DEG with spin projection $\sigma$ at spatial position $r \equiv (x, y)$, and $V_{\text{imp}} \equiv V_{\text{imp}}(r)$ represents a weak static impurity potential which provides
a finite resistivity and dissipation in the metal. In terms of $\eta_x^i (r)$, $\eta_x^\pm (r)$, the density-operator $\rho_{2D}$ (r) in the 2DEG writes $\rho_{2D} (r) \equiv \sum_x \eta_x^i (r) \eta_x^\pm (r)$, and $\delta \rho_{2D} (r) \equiv \rho_{2D} (r) - \rho_{0,2D}$, with $\rho_{0,2D}$ the average density in the metal.

Finally, the interaction between the 1DJJA and the 2DEG placed at a distance $d$ (cf. Fig. 1) is described by the Hamiltonian

$$H_{\text{int}} = \int d^2r dx' \delta \rho (x') v (x' - r, d) \delta \rho_{2D} (r).$$

Our goal in this Section is to derive an effective model for the 1DJJA capacitively coupled to the 2DEG. To that end we introduce the partition function of the system:

$$Z = \int \mathcal{D} [\rho, \theta] \mathcal{D} [\eta, \eta] e^{-S},$$

where $S$ is the Euclidean action of the problem

$$S = S_{1\text{JA}} + S_{2D} + S_{\text{int}},$$

where

$$S_{1\text{JA}} = \int_0^\beta d\tau \int dx i \partial_\tau (x, \tau) \rho (x, \tau) + \int_0^\beta d\tau H_{1\text{JA}} (\tau),$$

$$S_{2D} = \int_0^\beta d\tau \left[ \int d^2r \bar{\eta} (r, \tau) (\partial_\tau - \mu_{2D}) \eta (r, \tau) + H_{2D} (\tau) \right],$$

$$S_{\text{int}} = \int_0^\beta d\tau H_{\text{int}} (\tau).$$

Here $\mu_{2D} = k_F^2/2m - eV_g$ is the effective chemical potential in the metal, with $k_F = |k_F|$ the Fermi wavevector, and $V_g$ the gate voltage applied to the 2DEG, which allows to change the value of $\rho_{0,2D}$, and therefore, the sheet-resistance $R_{\square}$.

The first step in the derivation of an effective model for the array is to integrate out the fermionic degrees of freedom $\bar{\eta} (r, \tau), \eta (r, \tau)$ in the 2DEG. Assuming that the term $S_{\text{int}}$ can be treated perturbatively (we check the consistency of this assumption later), the integration of the fermionic degrees of freedom in the metal yields

$$S_{\text{eff}} \simeq S_{1\text{JA}} - \frac{1}{2} \int d\tau d\tau' \int dx dx' \delta \rho (x, \tau) \times v_{\text{scr}} (x - x', \tau - \tau') \delta \rho (x', \tau').$$

We do not provide the details of this derivation here, and we refer the interested reader to Refs. 23 and 26. In Eq. (7) we have introduced the 1D effective screening potential $v_{\text{scr}} (x - x', \tau - \tau')$, which encodes all the screening effects provided by the 2DEG. This quantity writes more conveniently in Fourier representation

$$v_{\text{scr}} (k, \omega_m) \equiv \frac{1}{L} \sum_{k_{\perp}} \frac{\left| v_{2D} (k, \omega_m) \right|^2}{1 + v_{2D} (k, 0)} \chi_{0,2D} (k, \omega_m),$$

where $\omega_m = 2\pi m/\beta$ are the bosonic Matsubara frequencies, and $k = (k, k_\perp)$ is the wavevector in 2D, where we have made explicit the component $k_\perp$ in the 2DEG, perpendicular to the 1DJJA. The quantity $v_{2D} (k, d) = \left( 2\pi e^2/\epsilon_\parallel \right) \exp (-|k| \sqrt{d^2 + a^2})/|k|$ is the 2D Fourier transform of the Coulomb potential Eq. (8). We assume that the length of the array is $L < \xi_{\text{loc}}$, with $\xi_{\text{loc}}$ the Anderson localization length in the 2DEG, a condition well fulfilled in practice. In that case, the density-density response function in the 2DEG, averaged over disorder configurations, writes $\chi_{0,2D} (k, \omega_m) = 2N_{2D}^2 Dk^2/(Dk^2 + |\omega_m|)$, where $D$ and $N_{2D}$ are, respectively, the diffusion constant and the density of states (at the Fermi energy) per spin projection.

We can now define the total effective retarded interaction

$$v_{\text{eff}} (k, \omega_m) = v_{1D} (k, 0) - v_{\text{scr}} (k, \omega_m),$$

where $v_{1D} (k, 0) = 2e^2K_0 (|k| a)/\epsilon_0$ is the Fourier transform of Eq. (3) in 1D, and $K_0 (x)$ is the zeroth-order modified Bessel function. Physically, the effective potential $v_{\text{eff}} (k, \omega_m)$ describes the interaction among charges in the array, both via the direct intrawire Coulomb interaction, as well as indirectly via the coupling to the diffusive modes in the 2DEG, which corresponds to the retarded interaction $v_{\text{scr}} (k, \omega_m)$ Eq. (8).

We now introduce a more convenient representation of the superfluid density in the 1DJJA. To motivate our approach, we first note that in the absence of Josephson coupling (i.e., $E_J = 0$ in Eq. (4)), the Cooper-pair occupation number $n_i$ is a good quantum number in each island, fixed by $\bar{n}$ via the application of an external gate-voltage. Increasing $E_J$ will evidently introduce fluctuations in $n_i$ due to the transfer of Cooper-pairs between neighboring islands, and $n_i$ is no longer a good quantum number. However, we expect that in the experimentally interesting regime $E_J/E_0 \sim 1$, where $E_0$ is the characteristic charging energy in the island, fluctuations $\Delta n_i = n_i - \bar{n}$ will be of order $\Delta n_i \simeq 1$, and that all other charging states such that $|\Delta n_i| > 1$ will be energetically forbidden. We therefore truncate those states from our description and focus on charge-fluctuations of $\Delta n_i = \pm 1$. In terms of a continuous field $\phi (x)$, which is slowly varying on the scale of $a$, the superfluid density in this effective model can be more conveniently written as

$$\rho (x) = \left[ \rho_0 - \frac{1}{\pi} \nabla \phi (x) \right] \sum_k e^{i2\pi k_\parallel a} e^{-\phi (x)},$$

where the parameter $\rho_0$ is defined as $\rho_0 \equiv 1/a$ in the commensurate case. Note that $\rho_0$ is an effective parameter of our model, and cannot be interpreted as the total physical density, in contrast to truly 1D systems. Only the
fluctuations $\delta \rho (x) \equiv \rho (x) - \rho_0$ have a physical meaning in our model.

In order to obey the phase-number commutation relations in the BCS-groundstate[23] note that the field $\phi (x)$ must verify the new commutation relation

$$[\theta (x), \nabla \phi (x')] = i \pi \delta (x - x') . \quad (11)$$

The contribution in squared brackets in Eq. (10) describe long-wavelength density fluctuations around the average value $\rho_0$, while in the last term, each contribution describes low-energy density fluctuations of momentum $k \sim 2p\rho_0$, where $p$ is an integer. When replaced into the effective action Eq. (7) we obtain the following effective model

$$S_{\text{eff}} [\phi] = S_0 [\phi] + S_1 [\phi] + S_2 [\phi] , \quad (12)$$

where the contribution $S_0$ corresponds to a Luttinger liquid mode[22]

$$S_0 [\phi] = \frac{1}{2\pi \beta L} \sum_{k, \omega_m} \left[ \frac{\omega_m^2}{uK} + \frac{uK^2}{K} + \frac{\eta |\omega_m| |k|}{2\pi c} \right] |\phi (k, \omega_m)|^2 . \quad (13)$$

resulting from the slow fluctuations of the density $\delta \rho (x) \sim - \nabla \phi (x) / \pi$ and from the hydrodynamic (i.e., $(k, \omega_m) \to 0$) sector of $v_{\text{eff}} (k, \omega_m)$. $u$ and $K$ are respectively the velocity of the 1D plasmon and the interaction Luttinger parameter

$$K = \pi \sqrt{\frac{E_J}{E_0}} , \quad (14)$$

$$u = a \sqrt{E_J E_0} , \quad (15)$$

where $E_0 \equiv e^2 / 2C_0$ is the charging energy with respect to the ground, with $C_0 = \epsilon \tau a / 4 \ln (2d/a)$ the effective ground capacitance of the Josephson junction. In our treatment, due to the screening provided by the 2DEG, the static effective potential $v_{\text{eff}} (k, 0)$ is effectively short-ranged for distances $x \gg d$, and therefore the Luttinger parameter $K$ is a constant[22] in terms of the capacitance matrix $C_{ij}$ of the 1DJJA, this amounts to neglecting the interjunction capacitance $C$, since this contribution, although relevant for density fluctuations of momentum $k \sim a^{-1}$, drops off in the long-wavelength sector $k \to 0$ (i.e. the interaction is screened in a length $L_{\text{act}} \sim a \sqrt{C / C_0}$[23]. In the present context, the Luttinger parameter $K$ physically represents the competition between coherence and charging effects in the array [cf. Eq. (14)]. Therefore, a large parameter $K$ favors superconducting correlations, while a small value of $K$ tends to destroy superconductivity due to strong charging effects[22].

The dissipative parameter $\eta$ is defined as

$$\eta \equiv \frac{c}{\epsilon_8 \pi} \frac{R_{\square}}{R_Q} , \quad (16)$$

where $R_{\square}$ is the sheet-resistance of the 2D film and $c$ is a numerical constant of order $c \sim \mathcal{O} (1)$. Eq. (13) with a non-vanishing $\eta$ describes a 1D plasmon-mode with a finite lifetime $\Gamma \sim |k| / \eta$[24]. Physically, a broadening of the 1D plasma mode occurs due to coupling to the diffusive modes in the 2DEG. The term $\sim \eta |\omega_m| |k|$ in Eq. (13) is the result of combining the leading contribution in powers of $|\omega_m| / D K_{\text{TF}} |k|$ (with $K_{\text{TF}}$ the Thomas-Fermi momentum in the 2DEG) in the expansion of the retarded potential $v_{\text{eff}} (k, \omega_m)$, and the long-wavelength fluctuations of the density $\sim (\nabla \phi)^2$, which contributes a term $\sim k^2 |\phi (k, \omega_m)|^2$ in Fourier representation. Note that since the scaling dimension of the term $\sim |k| |\omega_m|$ is 2, the critical properties of the system are not modified. Moreover, for a metallic plane [cf. Ref. (10)] with $R_{\square} \sim 0.1 R_Q$, $\eta \simeq 10^{-2} \ll 1$ and it can be effectively ignored, allowing us to write

$$S_0 [\phi] \simeq \frac{1}{2\pi \beta L} \sum_{k, \omega_m} \left[ \frac{uK^2}{2\pi c} + \frac{uK}{K} \right] |\phi (k, \omega_m)|^2 . \quad (17)$$

This assumption greatly simplifies the analysis, since the action $S_0$ recovers Lorentz-invariance in space-time.

The next term $S_1$ in Eq. (12) originates in the phenomenological potential $V_1 (x)$, which has the same periodicity of the array. Therefore, it can be decomposed in Fourier components as $V_1 (x) = \sum_n V_n \cos (Q n x)$, with $Q = 2\pi / a$. In general, all terms other than $p = n = 0$ in Eq. (10) are rapidly oscillating and vanish under the integral sign. However, if $Q n = 2\pi p_0$, or equivalently $p_0 a = n / p$ (i.e., the average density of bosons is commensurate with the lattice), then the term $\sim \int dx V_1 (x) \rho (x)$ yields a term $e^{-i (Q n - 2\pi p_0) x} = 1$ which is not oscillating, and, in addition to the term $p = n = 0$, we have the additional term

$$S_1 [\phi] = - \lambda \frac{\alpha}{a \tau_0} \int dx d\tau \cos (2\phi (x, \tau)) , \quad (18)$$

where we have only kept the most important commensurability ($p = 1$), and where we have defined the dimensionless parameter $\lambda$

$$\lambda \equiv V_1 \tau_0 . \quad (19)$$

and the short-time cutoff $\tau_0 \equiv a / u$. The term $V_0$ can be reabsorbed in a redefinition of the chemical potential of the externally imposed charge $\overline{n}$, so in the following we will not consider it. Physically, the dimensionless parameter $\lambda$ is related to the QPS rate in the Josephson junction by $\Gamma_{\text{QPS}} = \lambda / \tau_0$. Estimated experimental values for $\Gamma_{\text{QPS}}$ are in the order of $\sim 1$ GHz[22], which yields $\lambda \simeq 0.06$.

The final term in Eq. (12) comes from the dissipative part of $v_{\text{eff}} (k, \omega_m)$. Due to the strongly oscillating factors $e^{-i 2\pi p p_0 x}$ in Eq. (10), it results in the local dissipative term

$$S_2 [\phi] = - \frac{\eta}{a} \int d\tau d\tau' \sum_{p > 0} \frac{1}{p} \cos 2\phi (x, \tau) - \phi (x, \tau') \right) (\tau - \tau')^2 , \quad (20)$$
This contribution is consistent with that of Ref. [26] obtained in the context of Luttinger liquids capacitively coupled to diffusive metals. In spite of the small magnitude of $\eta$, we will show that this contribution has important consequences for the critical properties of the 1DJJA, in contrast to the term proportional to $\eta$ in Eq. [13].

In the following we study the critical properties and phases of the model obtained in Eq. [12].

### III. PHASE DIAGRAM

#### A. Weak-coupling renormalization group analysis

We first focus on the phases of the 1DJJA at $T = 0$. To that end, we perform a weak-coupling renormalization group (RG) analysis of the model Eq. [12], assuming that $S_1$ and $S_2$ in Eqs. [18] and [20], respectively, are weak perturbations to the Luttinger liquid $S_0$ in Eq. [17]. Since the action $S_0$ is Lorentz-invariant in space and imaginary time, we adopt an RG procedure that rescales homogeneously space and time. As usual, we assume that the original theory is defined up to a certain momentum cutoff $\Lambda (l) = \Lambda_0 e^{-l}$ (with $\Lambda_0 \sim a^{-1}$), and we study how the action $S_0$ is renormalized upon integration of high-energy modes in a window between $\Lambda (l) / s < |q| < \Lambda (l)$, with $s = e^{d l}$, where we have employed the compact notation $q \equiv \{ k, -\frac{m}{a} \}$ and $x \equiv \{ x, u \tau \}$.

We obtain the perturbative RG-flow equations of the model by performing a one-loop correction in $S_2$ and a two-loop correction in $S_1$, and requiring that the term $S_0$ is invariant upon scaling. We obtain the RG-flow equations

\[
\frac{dK (l)}{dl} = \left[ -2\pi \eta (l) - (2\pi)^2 K (l) \lambda^2 (l) C \right] K^2 (l),
\]

\[
\frac{du (l)}{dl} = -2\pi \eta (l) u (l) K (l),
\]

\[
\frac{d\lambda (l)}{dl} = \left[ 2 - K (l) \right] \lambda (l),
\]

\[
\frac{d\eta (l)}{dl} = \left[ 1 - 2K (l) \right] \eta (l),
\]

where the numerical constant $C$ is of order unity.

Note that both $S_1$ and $S_2$ tend to destroy superconducting correlations in the Luttinger liquid phase, a fact that is reflected in Eq. [21], where the Luttinger parameter $K (l)$ is renormalized to smaller values, meaning that charging effects are enhanced. This can be interpreted as an effective increase of the charging energy $E_0$ in Eq. [14]. In addition, since $S_2$ is the only term that breaks the Lorentz invariance of the theory, note that the plasmon velocity $u (l)$ is proportional only to $\eta (l)$, and is independent of $\lambda (l)$.

When $K (l) < 2$, the perturbative parameter $\lambda (l)$ flows to strong-coupling [cf. Eq. [23]], and the perturbative RG procedure is no longer valid. In the limit $\eta \to 0$ we recover the usual Mott-transition of the BKT-type described by the sine-Gordon model, and below the critical value $K_c = 2$, the 1DJJA is in the insulating phase. Using Eq. [14], this means that in absence of dissipation, the SIT occurs for $E_I / E_0 = (2/\pi)^2$ [22]. Note that our situation corresponds strictly to the case when the superfluid density in the 1DJJA is commensurate to the lattice, and is in clear distinction to the non-commensurate situation (i.e., $\lambda = 0$), where dissipation (i.e., the term $S_2$) becomes relevant for $K (l) < 1/2$, inducing a different kind of non-superconducting groundstate [26].

In the present case, the scaling dimension of the dissipative parameter $\eta (l)$ is always smaller than that of $\lambda (l)$, which means that for $K (l) \approx 2$, $S_1$ is a stronger perturbation as compared to $S_2$. Therefore, one would expect the nature of the non-superconducting groundstate to be determined essentially by $S_1$. However, based on this fact, one could naively conclude that the term $S_2$ is unimportant near the SIT, a conclusion we prove incorrect. In fact, a more detailed analysis reveals the importance of the term $S_2$ near the SIT. Physically, the coupling to the diffusive degrees of freedom in the 2DEG quenches charge-fluctuations in the 1DJJA, resulting in an enhanced effective charging energy $E_0^s$. This phenomenon is more precisely described by the RG-flow equation for $K (l)$ [cf. Eq. [21]], where $K (l)$ is renormalized to lower values by $\eta (l)$. Indeed, near the SIT, a small increase in the initial value $\eta_0 \equiv \eta (l = 0)$ (i.e., an increase in $R_0$) can effectively control the RG-flow of $K (l)$ and therefore, that of $\lambda (l)$, inducing the SIT. We illustrate this point in Fig 2, where the schematic phase diagram obtained by integration of the RG-flow Eqs. [21]–[24], with initial parameter $\lambda_0 \equiv \lambda (l = 0) = 0.01$. Note the stabilization

![Figure 2: Schematic phase diagram of the 1DJJA in the $K−\eta$ plane, obtained from the integration of the RG-flow Eqs. [21]–[24], with the initial parameter $\lambda_0 \equiv \lambda (l = 0) = 0.01$. An increase of $R_0$ in the 2DEG, and consequently, of the dissipative parameter $\eta$, can induce a SIT. Note that, in absence of dissipation, the critical value $K_c = K_c (\lambda_0) \simeq 2.1$ is slightly shifted with respect to the value $K_c (\lambda_0 \to 0) \to 2$.](image)
of the insulating groundstate due to Ohmic dissipation induced by the coupling to the 2DEG.

In a first approximation, this effect is similar to the dissipation-driven SIT observed in 2D JJAs capacitively coupled to a diffusive 2DEG. However, important differences appear with respect to the 2D case. In that case, it was argued that dissipation produced a renormalization of the effective parameters $E_J$ and $E_C$ of the array due to the incomplete screening of the Coulomb interaction in a certain frequency-regime. Physically, the slow diffusive response of the 2DEG cannot follow the faster dynamics of the 2D plasma mode, and cannot screen it efficiently. However, in the 1D geometry the 1D plasmon is effectively very well screened by the 2DEG, and it could be naively concluded that no dissipation-driven SIT should be observed. However, this screening effect is compensated by the presence of strong backscattering occurring in 1D [i.e., action $S_2$, Eq. (27)], and originated in the retarded interaction $v_{\text{eff}}(x,\tau)$. The net result is that the dissipation-driven SIT is restored in 1D.

Although one expects the nature of the non-superconducting groundstate to be of the Mott-insulating type, by analogy with the well-known results for the sine-Gordon model, strictly speaking we cannot extrapolate the results in this Section to the strong-coupling situation, and a different method is needed in that regime.

### B. Self-consistent harmonic approximation

To gain more insight into the phase in which the parameter $\lambda(l)$ flows to strong-coupling, in this Section we make use of the variational self-consistent harmonic approximation. This method consist in finding the optimal propagator $g_{\text{tr}}^{-1}(q)$ of a Gaussian trial action of the 1DJJJA

$$S_{\text{tr}} = \frac{1}{2\beta L} \sum q g_{\text{tr}}^{-1}(q) |\phi_q|^2,$$  

(25)  

where $\phi_q$ is the Fourier transform of $\phi(x,\tau)$. Here we have introduced the compact notation $q = (k, -\omega_m/u)$. The idea is to minimize the compact notation $\phi(x,\tau)$, and we obtain the following estimate for the gap increase

$$\Delta = 4\lambda \left( \frac{\zeta K \pi + 2\sqrt{K\pi \Delta}}{4} \right)^{K/2},$$  

(31)  

obtained replacing the solution Eq. (29) back into Eq. (28). Starting from the self-consistent solution of Eqs. (30) and (31) for $\Delta$ in absence of dissipation (i.e., $\eta = 0$), we can study the regime $\eta \ll \lambda \ll 1$ perturbatively in $\eta$, and we obtain the following estimate for the gap increase.
due to dissipative effects

\[ \delta \Delta \simeq 2\pi^2 \eta K \Delta^2 / \lambda. \] (32)

This result is consistent with the fact that dissipation in the density (i.e., field \( \phi \)) quenches charge-fluctuations and therefore favors an insulating groundstate.

In Fig. 3 we show numerical results for \( \Delta \) and \( \zeta \) as a function of \( K \) for the values \( \lambda = 0.05 \) and \( \eta = 0.01 \). Note the sharp increase of both \( \Delta \) and \( \zeta \) for \( K < 2 \). This result is consistent with the RG-analysis, which predict the breakdown of the Luttinger liquid phase for \( K < 2 \) in the weak-coupling regime. Within the SCHA, the physics of the strong-coupling fixed point is encoded in non-vanishing values of \( \zeta \) and \( \Delta \), providing a complementary description to the RG-analysis.

**IV. TRANSPORT PROPERTIES**

In this section we concentrate on the dc-conductivity of the 1DJJA, a quantity of central interest in experiments.\(^{[11]}\) We first focus on the current-density \( j(x) \). Since the field \( \nabla \theta(x) / \pi \) is the momentum of Cooper-pairs [cf. Eq. (2)], the usual minimal coupling procedure \( \nabla \theta(x) / \pi \rightarrow [\nabla \theta(x) - 2eA(x)] / \pi \) (with \( e \) the electron charge and \( A \) the vector potential) in Hamiltonian Eq. (2) allows to obtain the current as \( j(x) \equiv -\delta H_{JJA} / \delta A(x) \). In our problem, it explicitly reads\(^{[23]}\)

\[ j(x) = uK \left( \frac{2e}{\pi} \right) [\nabla \theta(x) - 2eA(x)]. \] (33)

The conductivity along the wire is obtained from the Kubo formul\(^{[23,13]}\)

\[ \sigma(\omega) \equiv \chi^{R}_{jj}(0,\omega) / i(\omega + i\delta), \] (34)

where \( \chi^{R}_{jj}(k,\omega) \equiv \lim_{\omega_{m} \rightarrow \omega + i\delta} \chi_{jj}(q) \) is the retarded current-current correlation function and \( \chi_{jj}(q) \equiv \langle j_{-}^{\ast}(q) j_{-}(q) \rangle = \delta^{2} \ln Z / \delta A(q) / \delta A^{\ast}(q) \rangle_{A=0} \) is the current-current correlation function obtained in the linear-response regime. It is convenient to express this correlator as \( \chi_{jj}(q) = \chi^{d}_{jj} + \chi^{p}_{jj}(q) \), where \( \chi^{d}_{jj} \equiv -(2e)^{2} uK / \pi \) is the diamagnetic contribution and

\[ \chi^{p}_{jj}(q) \equiv \left( \frac{2e}{\pi} \right)^{2} (uK)^{2} k^{2} \langle \theta(q) \theta(-q) \rangle, \] (35)

is the paramagnetic term\(^{[33]}\). In absence of current-decaying mechanisms [i.e., \( \lambda = \eta = 0 \) in Eq. (6)], the conductivity writes

\[ \sigma_{0}(\omega) \equiv \langle (2e)^{2} / \hbar \rangle uK \left[ \delta(\omega) + i p \left( \frac{1}{\pi \omega} \right) \right], \]

where we have restored the Planck constant and where we have used that \( \chi^{p}_{jj}(q) \rightarrow 0 \) in the limit \( k = 0 \).\(^{[20]}\) Note that the real part of \( \sigma_{0}(\omega) \) consists of a Drude-peak at \( \omega = 0 \), as expected for a superconductor. This result can be understood from the fact that the total charge current \( J_{c} = \int dx j(x) \) is a conserved quantity in absence of QPS and dissipation processes, i.e., it commutes with the hamiltonian \( H_{JJA} \).

The effect of a finite \( \eta \) in the Gaussian sector of the theory [cf. Eq. (13)] has been studied in Ref. \(^{[25]}\) and produces a broadening of the plasmon peak, whose width \( \Gamma \) vanishes as \( \Gamma \sim |k| \). Consequently, only taking into account this effect, a well-defined Drude-peak in \( \sigma(\omega) \) for \( \omega = 0 \) is recovered, and the system should behave as a perfect conductor.

Let us now study the effects of the terms \( S_{1} \) and \( S_{2} \). When \( \lambda \) and \( \eta \) are irrelevant perturbations (in the RG sense), their effects on the conductivity can be studied within the theoretical framework of the memory function formalism.\(^{[33]}\) In this approach, the central assumption is that the Kubo formula for the conductivity Eq. (34) can be recasted as\(^{[20]}\)

\[ \sigma(\omega, T) = \frac{i (2e)^{2}}{\pi \hbar} \frac{uK}{\omega + M(\omega, T)}, \] (36)

where \( M(\omega, T) \) (i.e., the memory function) is a meromorphic function depending on the terms in the Hamiltonian responsible for degrading the current, and hence producing a finite resistivity. Current-decay originated in QPS and in the coupling to the dissipative modes in the 2DEG induce finite resistivity in the 1DJJA for all temperatures \( T < T_{c} \). In particular for temperatures \( T \ll T_{c} \), and perturbatively in \( \lambda \) and \( \eta \), we obtain

\[ \varrho(T) = \frac{\hbar}{a (2e)^{2}} \left[ A_{1} T^{2K-3} + A_{2} T^{2K} \right] \] (37)

where

\[ A_{1} \equiv \lambda^{2} 4 \pi^{3} \left[ \cos \left( \frac{\pi K}{2} \right) B \left( \frac{K}{2} - 1, K \right) \right]^{2} \left( \frac{2 \pi a}{u} \right)^{2K-3}, \] (38)

\[ A_{2} \equiv \eta 32 \pi^{3} \cos [(1 + K) \pi] B [1 + K, -1 - 2 K] \left( \frac{2 \pi a}{u} \right)^{2K}, \] (39)

where the function \( B(x, y) \) is defined as \( B(x, y) \equiv \Gamma(x) \Gamma(y) / \Gamma(x + y) \), and \( \Gamma(x) \) is the standard Euler’s Gamma function.\(^{[80]}\) The term \( \sim T^{2K-3} \) in Eq. (37) is the contribution due to QPS processes, consistent with former theoretical predictions.\(^{[31-41]}\) The second term \( \sim T^{2K} \) originates in backscattering effects induced by dissipation, and is consistent with the behavior predicted by Cazalilla \textit{et al}.\(^{[26]}\) This last effect can be interpreted as a frictional drag produced by the diffusive modes in the 2DEG.\(^{[85]}\) Note that at lowest order in \( \lambda \) and \( \eta \), the two
contributions add up independently, indicating that for temperatures $T^* < T \ll T_c$, where $T^* = \sqrt{A_1/A_2}/2\pi \tau_a$, the resistivity in the 1DJJA is dominated by frictional drag, while for $T < T^* \ll T_c$ the effect of QPS takes over.

The non-trivial effects due to the renormalization of the bare couplings can be taken into account integrating the RG-flow Eqs. (21)-(24), and injecting them in the above Eqs. (37), (38) and (39). We integrate the RG-flow up to the scale given by the temperature $a(l) = a(0) e^l = u(l)/2\pi T$, and we use formula Eq. (37) with the parameters of the model calculated at the scale $a(l)$. This allows to obtain $\varrho(T(l))$ vs $T(l)$.

In Fig. 4 we show the resistivity $\varrho(T)$ of the 1DJJA, calculated for different values of the parameter $K$ and using the estimations for the bare parameters $\lambda_0 = 0.01$ and $\eta_0 = 0.01$. The results are normalized to a “high-temperature” resistivity $\varrho(T_0)$, where $T_0 \simeq a(0)/u = \tau_0$, represents a high-temperature cutoff in the theory (e.g., $T_c$).

![Figure 4: Dc-resistivity $\varrho(T)$ of the 1DJJA, normalized to a “high-temperature” value $\varrho(T_0)$, as a function of $T/T_0$, calculated for the parameters $\lambda_0 = 0.01$ and $\eta_0 = 0.01$, and for different values of $K = \pi \sqrt{E_1/E_0}$. A low-temperature upturn of $\varrho(T)$ signals the formation of the insulating phase.](image)

Note that for the values $K = 2.5$ and $K = 2.3$, the resistivity shows a monotonically decreasing behavior, indicating a superconducting groundstate and consistent with the RG-analysis of Sec. II.A. We also note a small kink around $T^* \approx 0.4 T_0$, signalling the aforementioned crossover from dissipation-dominated to QPS-dominated resistivity. For $K = 2.1$, the resistivity first decreases and then shows a low-temperature upturn, indicating that the array is near the quantum critical point $K_c$. Finally, for lower values of $K$, the insulating behavior in the 1DJJA is clear. Since both the integration of the RG-flow equations and the calculation of the memory-function formulas are perturbative in $\lambda$ and $\eta$, the calculation of the resistivity must be stopped whenever $\lambda(l)$ or $\eta(l)$ become of order unity.

In Fig. 5 we show the resistivity as a function of $T/T_0$, calculated for fixed $K = 2.3$ and $\lambda = 0.01$, and for different values of parameter $\eta$. We see that for $\eta = 0$ (i.e., $R_{\square} = 0$ in the 2DEG), the array shows superconducting behavior, and the resistivity due to QPS processes is well described by the predicted power-law $\varrho(T) \sim T^{2K-3}$, with $K = K(l \rightarrow \infty) \approx 2.2$ the renormalized value predicted by Eq. (23). Upon increasing the parameter $\eta$, the resistivity of the array increases, developing the aforementioned kink, but most importantly, the low-temperature resistivity develops an upturn, indicating a dissipation-driven phase transition to the insulating phase.

![Figure 5: Resistivity of the 1DJJA (in units of $h/e^2 a$) as a function of $T/T_0$, calculated for parameters $K = 2.3$ and $\lambda = 0.01$ and for different values of $\eta$. Although in absence of dissipation, the array is in the superconducting phase, a dissipation-driven SIT occurs upon increasing $\eta$, consistent with the results in Fig. 4. The curve $\varrho(T) \sim T^{2K-3}$ is shown for comparison.](image)

More insight into the insulating phase can be obtained using the Luther-Emery reification formalism for $K = 1$, in absence of dissipation (i.e., $\eta = 0$) an exact solution is obtained in terms of non-interacting fermions, with a gap $\Delta \equiv \pi a \lambda$ in their spectrum of excitations. Using the Kubo formula, one obtains the following expression for the dc-conductivity at low temperatures $T \ll \Delta$:

$$\sigma(\omega) \approx \frac{e^2}{h} u \frac{2\pi T}{\Delta} e^{-\Delta/T} \delta(\omega).$$

This contribution arises from the excited quasiparticles above the gap $\Delta$, which have an exponential population at low enough temperatures. This infinite conductivity occurs because in absence of dissipation, excited quasiparticles are infinitely long-lived. Using the memory-function approach for the reformation problem in the regime $\eta \ll \lambda, T \ll \Delta$, we find the analytical result.
\[ \sigma (\omega = 0) = c_2 \frac{e^2}{h} \frac{1}{\eta} \tilde{\Delta}^2 T e^{-\tilde{\Delta}/T}, \]

where \( c_2 \) is a numerical coefficient \( c_2 \approx O(1) \). As expected, dissipation introduced a finite lifetime in the quasiparticles, and a finite resistivity is obtained at \( \omega = 0 \).

V. SUMMARY AND CONCLUSIONS

We have investigated the properties of a linear JJA capacitively coupled to a diffusive 2DEG placed in close proximity. Using a bosonization approach, we have derived an effective model for the 1DJJA, and have obtained its critical properties and phases at \( T = 0 \). Our main result is the possibility to observe a SIT tuned by the parameter \( \eta \sim R_{c2}/R_Q \) [cf. Eq. (10)]. This setup could be used to investigate the superconductor-insulator transition in a 1DJJA under better controlled experimental conditions as compared to other setups used in the past.\[23,24\] Our work could shed some light on the understanding of other 1D superconducting systems showing a similar behavior, such as ultra-thin superconducting wires built by molecular templating\[25,26\] or by e-beam lithography\[27,28\] techniques.

We have shown that besides the more or less trivial static screening effect, the presence of a 2DEG induces dissipative effects in the quantum dynamics of the 1DJJA due to backscattering processes induced by the dynamically screened Coulomb interaction, and explicitly depend on the sheet-resistance \( R_{c2} \) of the 2DEG. These dynamical effects play an important role in the quantum phase diagram of the 1DJJA. This situation is different from previous approaches in higher dimensions.\[29,30\] Indeed, in 1D the plasmon mode is almost statically screened\[23\], and this would lead to the naive conclusion that dynamical effects are not important. However, a more careful analysis shows that backscattering originated in the dynamically screened Coulomb potential has the effect of restoring the SIT. In our system, these dynamical effects have important consequences for the critical properties of the array, and should be possible to observe them in dc-transport measurements. Physically, the coupling to diffusive modes in the metal induces charging effects which are local in space (i.e., of the order of the lattice parameter \( a \) of the 1DJJA) but which are non-local in time (i.e., Ohmic dissipation effects), and tend to quench charge fluctuations, rendering superconductivity weaker.

By the means of a weak-coupling RG-analysis and a variational approach, we predict a SIT driven by the presence of dissipation in the 2DEG. This SIT is of the BKT-type and mediated by unbinding of QPS/anti QPS pairs, like in the dissipationless case.\[23\] Near the critical line the effects of QPS are stronger than those originated in dissipation and results in a SIT. This scenario is corroborated by a subsequent variational analysis of action Eq. (12), which suggests the formation of a gap \( \Delta \) in the spectrum of excitations of the 1DJJA [cf. Eq. (29)].

Our results suggest that dissipation renormalizes the QPS-rate to higher values and the ratio \( \sqrt{E_J/E_0} \) to lower values [cf. Eqs. (21)–(24)], rendering superconductivity in the 1DJJA weaker. Eventually, an increase of \( R_{c2} \) (and therefore of \( \eta \), in view of Eq. (16)), could drive the system into the insulating phase, as can be seen in Figs. 2 and 5. This phenomenon is different to the case studied by Cazalilla et al., where QPS processes were absent, and it was dissipation itself that drove the quantum phase transition for the critical value \( K_c = 1/2 \).\[23\]

We have also studied the consequences on the temperature-dependent dc-resistivity of the array \( \varrho(T) \). We have shown that a non-vanishing \( R_{c2} \) induces a rich behavior of \( \varrho(T) \). In particular in the superconducting phase, where the 1DJJA is in the Luttinger liquid universality class, and the effects of QPS and dissipation are perturbative, the resistivity of the array \( \varrho(T) \) follows a power-law behavior \( \varrho(T) = A_1 T^{\nu_1} + A_2 T^{\nu_2} \), with exponents \( \nu_1 = 2K - 3 \) and \( \nu_2 = 2K \) [cf. Eq. (37)] generated by QPS and dissipation, respectively. Therefore, the results of this paper could be relevant in the interpretation of experimental results of transport through superconducting circuits subject to dissipative effects. In the insulating phase, the low-temperature dc-resistivity is expected to show thermally-activated behavior.\[31,32\] In particular for \( K = 1 \), the resulting model can be studied analytically with a refermionization approach, and results in a resistivity \( \varrho(T) \sim \eta T e^{\Delta/\nu_1 T} \). Quite importantly, note in this expression that the resistivity depends also implicitly on \( \eta \) via a renormalization of the gap \( \eta \).

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1 G. Schönh and A. D. Zaikin, Physics Reports 198, 237 (1990), ISSN 0370-1573.
2 R. Fazio and H. van der Zant, Physics Reports 355, 235 (2001).
3 D. B. Haviland, Y. Liu, and A. M. Goldman, Phys. Rev. Lett. 62, 2180 (1989).
