Model of multilevel formation of normal cracks in reinforced concrete elements and structures

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Abstract. This article proposes a model of multilevel formation of normal cracks. It is based on the most important patterns of adhesion of reinforcement with concrete. The occurrence of the first normal cracks of the first level is calculated by the limiting deformations of stretched concrete, which are fixed by the extreme criterion of its strength. The distances between the cracks are determined by the equilibrium of the maximum forces in the stretched concrete and the forces of active adhesion in the area between the cracks. It is proposed to calculate the force of active coupling by the nonlinear function of the average stresses of this coupling, justified by the power of expression. It is substantiated why for bending reinforced concrete elements the distances between normal cracks of new and previous levels will always differ from each other. In the developed model, the width of the most dangerous normal crack opening is calculated from the positions of successive accumulation of reinforcement and concrete mutual displacements.

1. Introduction

The issues of reinforced concrete elements and structures fracture resistance have always been a priority in the theory of concrete and reinforced concrete. When designing these elements, there was always a need to calculate the basic parameters that can accurately characterize their stress-strain state in the operational stage.

In the power models, the actual process of crack formation and development could not be reproduced even close. This is due to the fact that they are based on idealized rectangular stress plots in compressed concrete without involving into the calculations the hypothesis of flat sections.

The introduction of deformation models into the modern practice of designing reinforced concrete elements and structures has laid some possibilities for reproduction of real processes of multistage formation and crack opening due to the use of real material deformation diagrams and the hypothesis of flat sections. However, in most cases these capabilities are extremely difficult to implement because of the intrinsic static uncertainty of the cross-section of reinforced concrete elements and structures.

Even more opportunities for effective modeling of the real process of sequential multilevel formation and the opening of normal cracks are given by the deformation-force model of resistance of reinforced concrete elements and structures by force influences. These possibilities are due, first of all, to the use of an additional analytical dependence of the generalized state diagram of the concrete elements “moment-curvature”.

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2. Analysis of recent research and publications

The issues of reinforced concrete elements and structures fracture resistance are devoted to many works of domestic and foreign researchers [1]. However, studies that would reproduce real processes of gradual multilevel crack formation are currently very limited. They are united by the fact that they are all accompanied by direct modeling of the coupling of the reinforcement with concrete. The interaction of reinforcement with concrete is built on the basic provisions of mechanics of deformed solid or fracture mechanics.

In the framework of the mechanics of a deformed solid, the multilevel crack formation is modeled by:

• diagrams of displacement of reinforcement with respect to stretched concrete in the areas between adjacent cracks [2, 3];
• average stresses of reinforcement with concrete adhesion in the block between cracks [4].

It is difficult to reproduce the process of coupling reinforcement with concrete by means of diagrams of their mutual displacement. The results of numerous studies show that the appearance of these diagrams in the process of reinforced concrete elements and structures deformation is constantly changing [5-7]. Therefore, integrating them directly to determine clutch effort becomes virtually impossible.

At the same time, the average stresses of reinforcement with concrete adhesion make it possible to simulate the processes of multilevel crack formation in the simplest way. However, the implementation of this method in studies [4] raises a number of serious reservations about:

• linearity of the connection between the average adhesion stresses of the reinforcement with concrete and the boundary normal stresses in the reinforcing rod, regardless of the level of reinforced concrete element deformation;
• the validity of the determination in the concrete element of the first crack of the first level by the extreme criterion $dM_w/d\varepsilon = 0$;
• the lawfulness of using expression $dM/d\varepsilon = 0$ as an extreme criterion in general.

The modeling of multilevel crack formation within the framework of fracture mechanics is carried out by means of a "double console" element in section with a crack [8, 9] or in strictly numerical ways [10, 11].

The model of the "double-console" element in section with the crack remains quite complex today, and even controversial in some issues. In particular, the authors of [8, 9] explain the reduction of stretched concrete deformation in the cracks zone by the appearance of compressive stresses in this zone. Is it not the cause of such a phenomenon of deformation of the elastic aftereffect, which is manifested with the gradual weakening of the reinforcement with concrete adhesion as new cracks form. In addition, for this model, the careful correlation of the stress-strain state of the reinforced concrete element with the patterns of reinforcement to the concrete adhesion also remains quite problematic.

Numerical methods [10, 11] are iterative, and therefore implemented only programatically using the finite element method. Their engineering reach is often lost, since it completely or partially eliminates the physical essence of not only the processes of adhesion of reinforcement with concrete, but also the stages of crack formation in the reinforced concrete element.

3. The purpose and objectives of research

These studies are aimed at developing a generalized model of sequential multilevel formation of normal cracks in reinforced concrete elements and structures. This model is proposed to be the basis of the engineering method of calculating the step formation and cracking at any stage of reinforced concrete elements deformation. At the same time the solution has two main tasks:

• the developed model must remain engineering-focused;
• it must be equally successful in both numerical and analytical (express) calculations of the fracture toughness of reinforced concrete elements and structures.
4. Results and discussion
First of all, it should be noted that the developed model of consistent multilevel formation of normal cracks is an integral part of the generalized deformation-force model of reinforced concrete elements and structures resistance by force influences. Therefore, it is also based on the basic relations of the mechanics of deformed solid (MDS), which are expressed by the generally recognized system of equations:

\[
M = f(e_c, e_{ct}, e_s), \quad N = f(e_c, e_{ct}, e_s)
\]

\[
1/r = f(e_c, e_{ct}, e_s)
\]

with the addition of its analytical dependence of the state diagram of the element \( M - 1/r \) and the function of boundary deformation of compressed concrete \( e_{cu} = f(e_{si}, \rho_{li}, x_{si}, m_h, m_b) \) [12].

Under such conditions, the first normal cracks of the first level in the reinforced concrete element under the ultimate deformations of the tensile concrete \( e_{cu} \), which are fixed by the extreme criterion of its strength \( dN_{ct} / de_{ct} = 0 \) or bearing strength \( dM_{ct} / d(1/r) = 0 \) or are assumed equal, occur. The application \( dM / de = 0 \) of the criterion for determining the moment of occurrence of cracks [4] is incorrect, since in this case the maximum bearing capacity loses only the stretched concrete, and not the element as a whole. In addition, expression cannot be considered a criterion at all. In terms of physical content, the Fermat criterion characterizes the rigidity \( E_c \) for centrally-loaded concrete elements, \( EA \) - for centrally-loaded reinforced concrete elements, \( EI \) - for bending and off-center elements), at which these elements lose their bearing capacity.

As for the distances between the cracks, they must be determined subject to the equilibrium of maximum forces in the stretched concrete \( N_{ct,cr} = f(e_{cu}) \) and the forces of active adhesion in the area between the cracks \( N_{bd,cr} \). The stated force of active coupling of reinforced concrete reinforcement is proposed to be calculated by the following power-law expression for the nonlinear function of the average stresses of this coupling, substantiated in [13, 14]:

\[
\tau_{lim} = \eta_1 \cdot \eta_2 \cdot f_{ck} \cdot (\sigma_{si} / \sigma_y)^{1-1/\eta_s}
\]

where \( \eta_1 \) - coefficient taking into account the profile of reinforcement [1] by the Rem criterion; \( \eta_2 \) - coefficient taking into account the influence of the reinforcement diameter [15, 16]; \( f_{ck} \) - characteristic value of concrete tensile strength; \( \sigma_{si} \) - stresses in the armature in section with a crack of the appropriate level; \( \sigma_y \) - limit stresses in reinforcement \( (\sigma_y = f_{yk}) \), \( 1/\eta_s \) - clutch intensity parameter (for periodic profile reinforcement \( \eta_s = \eta_1 \), for smooth profile reinforcement \( \eta_s = 6 \cdot \eta_1 \)).

Then, taking into account the above, the distance between two adjacent cracks of the first level at the level of the gravity center of the stretched reinforcement, for the stresses in it \( \sigma_{s1} = e_{cu} \cdot E_s \), should be calculated by the expression:

\[
s_{r1} = \frac{\emptyset_s}{4 \cdot \eta_1 \cdot \eta_2 \cdot f_{ck} \cdot (e_{cu} \cdot E_s / f_{yk})^{1-1/\eta_s} \cdot N_{ct,cr}}{A_s}
\]

For a centrally stretched element (Figure 1), the distance between said cracks will be inversely proportional to its reinforcement coefficient \( \rho_{1,t} = A_s / N_{ct,cr} \):

\[
s_{r1} = \frac{\emptyset_s}{4 \cdot \eta_1 \cdot \eta_2 \cdot (e_{cu} \cdot E_s / f_{yk})^{1-1/\eta_s} \cdot \rho_{1,t}}
\]
Given that the distance between the second-level cracks will be halved \( s_{r2} = \frac{s_{r1}}{2} \), the stresses at which these cracks begin to form can be determined by the following dependence:

\[
\sigma_{s2} = f_{yk} \cdot \left( \frac{\varnothing}{2 \cdot \eta_1 \cdot \eta_2 \cdot s_{r1} \cdot \rho_{l,t}} \right)^{\eta_s} \frac{\eta_s}{\eta_{s-1}} \tag{5}
\]

In bending elements (Figure 2), the distances between adjacent cracks of the first level should also be calculated by the expression (3), but the maximum efforts in the concrete of the stretched zone \( N_{ct,cr} = f(e_{ctu}) \) must be determined using the system of equations (1). Since the normal stresses in the reinforcement within the sections between the cracks are variable, the average adhesion stresses on these sections by expression (2) will also be different. This means that the distances between the normal cracks of the new and previous levels (Figure 2) will also always differ from each other. Based on the equality of adhesion efforts [14] on both sides of the probable second-level crack, the distance to it from the first-level cracks (Figure 2) will be determined by the following expressions:

\[
s_{r2,1} = \frac{s_{r1,1} \cdot \tau_{bm2,2}}{\tau_{bm2,1} + \tau_{bm2,2}}, \quad s_{r2,2} = \frac{s_{r1,1} \cdot \tau_{bm2,1}}{\tau_{bm2,1} + \tau_{bm2,2}} \tag{6}
\]

The distances between the cracks of all previous and subsequent levels are similarly determined. For most reinforced concrete elements, considering 2-3 levels of crack formation is sufficient.

The opening of normal cracks is calculated according to the hypothesis of Thomas from the standpoint of consistent accumulation of mutual displacements of reinforcement and concrete in the areas of active clutch located on both sides of the crack [14]. It is almost impossible to perform a direct integration of the mutual displacements dependence of the reinforcement and concrete \( e_r(z) \) since it cannot be described by a single continuous function.

Therefore, it is proposed to calculate the total opening width of the most dangerous normal crack using the simplified formula:
\[ w_k = s_{r1,1} \cdot (e_{sm,1} + e_{sm,2} + \Delta e_{sm,2} - e_{ctm}) - s_{r2,1} \cdot (e_{sm,2} + \Delta e_{sm,2} - e_{ctm}) \]  \hspace{1cm} (7)  

where \( e_{sm,1} \) is the increase in the average relative deformation of the tensile reinforcement on the section between the cracks from the beginning of loading to the moment of the first level crack formation; \( e_{sm,2} \) also from the second (next) crack appearance of the first level until the crack formation of the second level; \( \Delta e_{sm,2} \) - also from the appearance of the second level crack until the moment of reaching the operational load; \( e_{ctm} \) is average values of boundary deformations of stretched concrete in the areas between cracks. The average deformations of the reinforcement in the areas between the cracks should be determined directly in the diagram of the element state \( M = 1/r \) when using the flat hypothesis \( e_{sm,i} = f(1/r) \). The average deformations of the stretched concrete at the respective site can be calculated according to \([14]\) or not taken into account at all.

For the 8 reinforced concrete beams \([17]\), the results of experimental studies of the normal crack width opening were compared with their theoretical values, calculated by three different methods. The results of the comparison (see table 1) show that the authors developed a model of multilevel crack formation that allows to more accurately calculate the of normal cracks width opening in comparison with other methods and models.

**Figure 2.** a) scheme of level formation of cracks, b) corresponding plots of adhesion stresses and c) moments in the bending element.
Table 1. Comparison of theoretical and experimental values of width crack opening in reinforced concrete beams [17] by methods.

| Code beams | Correlation $w_{k,calc} / w_{m,exp}$ |
|------------|-------------------------------------|
|            | DSTU [15]  | EC-2 [16]  | Authors (7) |
| S3-1-1     | 1.23       | 1.21       | 1.13        |
| S3-1-2     | 1.86       | 1.89       | 1.40        |
| S3-2-1     | 1.32       | 1.35       | 1.15        |
| S3-2-2     | 1.46       | 1.41       | 1.21        |
| S3-2-3     | 1.38       | 1.41       | 1.19        |
| S3-2-6     | 1.61       | 1.59       | 1.32        |
| S3-2-9     | 1.55       | 1.56       | 1.25        |
| S3-2-10    | 1.49       | 1.45       | 1.22        |

5. Conclusions
Thus, as a result of the above studies, a model of level formation and opening of normal cracks was developed, which:

- is an integral part of the generalized model of reinforced concrete elements deformation;
- preserves the physical nature and takes into account the most important regularities of the processes of reinforcement with concrete adhesion;
- it is universal because it remains acceptable for any reinforced concrete elements that undergo both homogeneous and heterogeneous deformation;
- based on the express methods of calculating the formation and opening of normal cracks;
- it is generalized because it relates the levels of crack formation in reinforced concrete elements with the basic parameters of their stress-strain state (curvature, bending moments, stresses in the reinforcement and its deformations, etc.);
- remains engineering comprehensible and suitable for both software and engineering implementation in practical calculations.

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