New explicit and exact traveling waves solutions to the modified complex Ginzburg Landau equation

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Abstract
This paper applies function transformation method to obtain under certain conditions bright, dark, kink and W-shaped dark solitons waves solutions to the modified Complex Ginzburg Landau Equation. The relevant aspect that is encountered using the applied method is the ability to lead to many types of interesting soliton solutions to the model. These new obtained solutions can be useful in many applications such as communication, medicine, hydrodynamic, thermodynamic just to name a few and can allow to explain physical phenomena observed in fundamental sciences and engineering.

Keywords Function transformation method · Solitons solutions · Modified Complex Ginzburg Landau equation

1 Introduction

Solitary waves propagation in nonlinear media received great attention these recent years because of their extensive applications in the field of ultra-fast science (Konar and Biswas 2005; Arshad et al. 2017; Konar et al. 1999; Crutcher et al. 2006; Kevrekidis and Frantzeskakis 2016; Mandal and Chowdhury 2007; Biswas 2012), there has been quite a bit of focus on the study of this equation with Kerr and power laws of nonlinearity. One of the most important aspects of this equation is in its integrability. Previously some authors have addressed the integrability of this modified Complex Ginzburg Landau equation in Kerr and power law nonlinearity. Also, the semi-inverse variational principle method was applied to carry out the integration of this equation. The latter were known through the pioneers works refs. Drazin and Johnson (1996), Hereman (2009), like wave that can propagate over a long distance without undergoing modification and also maintains its origins. However, in various works solitary waves terms and

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solitons are generally related in the definition through properties such as soliton is a solitary waves having permanent form, localized, interact highly with other solitons and can retain its oneness.

More precisely, the important remark which has pointed out the interest of solitary waves theory in many fields is that the nonlinear and dispersion terms of nonlinear evolution equations can balance to form solitons solutions. Thus, there is a category of solitary waves in literature namely bell-shape, periodic solutions, kinks wave, peaked solitary waves, solitons exhibited cups and so on. Furthermore, the most commonly used names are the bright, dark and singular solitons Crutcher et al. (2006), Kevrekidis and Frantzeskakis (2016), Mandal and Chowdhury (2007), Biswas (2012), Malomed et al. (1997), Drazin and Johnson (1996), Hereman (2009), by using the ansatz method, Houria Triki et al. obtained such kind of solitons.

Several studies are made by researchers in order to exhibit how much solitons and other fractional solutions of nonlinear evolution equations are important to bring efficient solution to the problem of communication, medicine, hydrodynamic by avoiding signal loose (Konar and Biswas 2005; Arshad et al. 2017; Konar et al. 1999; Crutcher et al. 2006; Kevrekidis and Frantzeskakis 2016; Mandal and Chowdhury 2007; Biswas 2012; Malomed et al. 1997; Osman et al. 2018), such study has been made Dépélair et al. (2021), by Djennadi et al. (2021), and Arqub (2017, 2016). So, in the case of nonlinear optical fibers, optical amplifiers have to be employed to compensate this loss. For example, soliton propagating in a monomode optical fibers have been used for future bits in long move communications and other fiber optic founded ultra fast pulse inspection devices. Indeed, high bit rate data relocation over a distance of thousands kilometers using optical solitons and smash erbium doped fiber amplifiers appeared to be technologically viable in close prospectives. In addition, it has been used a periodic fluctuation of the core diameter to master soliton parameters. Otherwise, it has been demonstrated that solitons can propagate over 14,000 km using an amplifying signals technique in an optical fiber. In addition, the use of the soliton in the field of chemistry has ensured the electrical conduction of conductive plastics, while acoustic soliton have contributed to minimize waves shock when trains entered tunnels. More recently, it has been emphasized the behavior of the new soliton solution to the longitudinal wave equation in a magneto-electro-elastic circular rod by applying the extended trial equation method. We also noted that solitons appeared on the surface during nonlinear distortions in DNA functions, where nonlinear interactions can give rise to stable excitations. Stable excitation (solitons) play an important role in the key processes of respiration, replication and transcription of DNA (Konar and Biswas 2005; Arshad et al. 2017; Konar et al. 1999; Crutcher et al. 2006; Kevrekidis and Frantzeskakis 2016; Mandal and Chowdhury 2007; Biswas 2012; Malomed et al. 1997; Osman et al. 2018; Hirota 1973; Majid et al. 2020; Rezazadeh 2018; Yomba and Kofane 2003; Hong 2007; Mohamadou et al. 2005; Shamsldeen et al. 2017; Zhang and Dai 2005; Alphonse et al. 2019; Osman et al. 2019; Gangwar et al. 2007; Yan 1996; Seadawy and Jalil 2018; Ehab et al. 2016; Seadawy and El-Rashidy 2018; Seadawy and Alamri 2018; Abdullah et al. 2017; Seadawy 2017; Seadawy and El-Rashidy 2016; Seadawy 2017; Asghar et al. 2017; Savaissou et al. 2020a, b, c, d; Souleymanou et al. 2019a, b, 2011, 2012; Mustafa et al. 2018a; Aliyu et al. 2019, 2019a; Abdullahi et al. 2019b; Arqub 2019; Arqub and Hasan 2018; Arqub 2019, 2018; Arqub and Nabil 2019).

Among these studies of soliton-like solutions which propagated by conserving its velocity and shaped, several studies have been conducted on the modified Complex Ginzburg Landau Equation (CGLE) (Rezazadeh 2018; Yomba and Kofane 2003; Hong 2007; Mohamadou et al. 2005; Osman et al. 2019), where methods such as the $G'/G$ method, exp-function method,
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Riccati equation method, simplest equation method, Fan’s F-expansion method, Lie symmetry analysis, collective variable approach, and many others are used.

CGLE is a complex nonlinear equation which depicts generically the dynamics of waveling, spatially prolonged systems close to the start of oscillations. This equation describes also a variety of physical phenomena in plasmas, optical wave guides and fibers, Bose Einstein condensation, phase transitions, bimolecule dynamics, open flow motions, spatially extended non equilibrium systems Rezazadeh (2018), Yomba and Kofane (2003), Hong (2007), Mohamadou et al. (2005). We cite some examples such as Osman et al. who study Complex wave structures for abundant solutions related to the CGLE model Osman et al. (2019), this equation for Kerr law nonlinearity was studied using the Hirota bilinear method. The modulational stability of this equation was also addressed Mohamadou et al. (2005). In this work, some wave structures as solutions to the CGL equation have been reported. Mohamadou et al. have conducted study on Pattern selection and modulation instability in one dimensional modified CGL equation Mohamadou et al. (2005), they have highlighted that modulation unstable pattern is selected and propagated into an initially unstable motionless state in the system.

In the present work, we will investigate some solitons solutions such as bright, dark, kink and W-shaped dark, and other periodic structures for the modified Complex Ginzburg Landau equation. To reach this aim, the present paper is structured as follows: in Sect. 2, we exhibit the model; in Sect. 3, solutions are given to the model by applying function transformation method; in Sect. 4, we summarize the work by giving some important remarks and graphical representations.

2 Skeleton of the model

The complex field $\psi(x, t)$ of 1-D modified CGL equation is represented as follows Rezazadeh (2018), Mohamadou et al. (2005), Osman et al. (2019)

$$i\psi_t + \alpha \psi_{xx} + \lambda |\psi|^2 \psi = c \frac{\psi \psi_x^*}{\psi^*} + d \nabla^2 (\sqrt{\psi \psi^*}) \sqrt{\psi / \psi^*} + i \gamma \psi,$$  \hspace{1cm} (1)

where the dependent variable $\psi(x, t)$ represents the wave profile that arises in various physical systems, including nonlinear optics, plasma physics and others. The independent variables are $x$ and $t$ that represents the spatial and temporal variables respectively, $\alpha$ and $\lambda$ represents the coefficients of dispersion and nonlinearity. The operator $\nabla^2 \equiv \frac{\partial^2}{\partial x^2}$ and $c$, $d$ and $\gamma$ are real valued constants.

Meanwhile, the expression $d \nabla^2 (\sqrt{\psi \psi^*}) \sqrt{\psi / \psi^*}$ in the right side of equation (1) can be rewritten as

$$d \nabla^2 (\sqrt{\psi \psi^*}) \sqrt{\psi / \psi^*} = d \left[ \frac{1}{2} \frac{\partial^2 (\psi \psi^*)}{\partial x^2} \psi \psi^* - \frac{1}{4} \left( \frac{\partial (\psi \psi^*)}{\partial x} \right)^2 \frac{1}{\psi \psi^*} \right].$$  \hspace{1cm} (2)

Employing Eq. 2 into Eq. 1, the CGL equation can be explicitly rewritten as follows

$$i\psi_t + \alpha \psi_{xx} + \lambda |\psi|^2 \psi = c \frac{\psi \psi_x^*}{\psi^*} + d \left[ \frac{1}{2} \frac{\partial^2 (\psi \psi^*)}{\partial x^2} \psi \psi^* - \frac{1}{4} \left( \frac{\partial (\psi \psi^*)}{\partial x} \right)^2 \right] \frac{1}{\psi \psi^*} + i \gamma \psi.$$  \hspace{1cm} (3)

Next, we employ the transformation hypothesis to build solitary waves to the CGL equation and discussed the obtained miscellaneous soliton solutions. So, we set...
\[ \psi(x, t) = \Psi(\xi) \exp[i\theta(x, t)], \quad \theta(x, t) = \kappa x - \omega t + \theta_0, \quad \xi = x - vt, \] (4)

where \( \Psi(\xi) \) is a real function considered as the amplitude of the wave, \( \theta(x, t) \) is the phase of the wave, \( \kappa \) is the wave number, \( \omega \) is the frequency of the soliton, \( \theta_0 \) is the phase constant, \( \xi \) is the traveling coordinate and \( v \) is given in terms of the group velocity of the wave packet as \( v = \frac{1}{v} \) in an optical fiber setting.

Inserting Eq. 4 into Eq. 1 gives

\[ (a - d)\Psi_{\xi\xi\xi} - \frac{c \Psi^2}{\Psi} + \lambda \Psi^3 + (\omega - ak^2 - ck^2)\Psi = 0, \] (5)

and

\[ (2ak - \nu)\Psi_\xi - \gamma \Psi = 0. \] (6)

The latter denotes real and imaginary parts respectively. From Eq. 6, we obtain

\[ \Psi_\xi = \frac{\gamma \Psi}{2ak - \nu}. \] (7)

Inserting Equation Eq. 7 into equation Eq. 5 yields to

\[ \Psi_{\xi\xi} + \frac{\lambda}{\alpha - d} \Psi^3 + \frac{1}{\alpha - d} (\omega - ak^2 - ck^2 - \frac{cy^2}{(2ak - \nu)^2})\Psi = 0. \] (8)

The integration Eq. 8 with respect to \( \xi \) gives the following expression

\[ \Psi^2_\xi = \frac{\lambda}{2(d - \alpha)} \Psi^4 + \frac{1}{\alpha - d} (\omega - ak^2 - ck^2 - \frac{cy^2}{(2ak - \nu)^2})\Psi^2 + K, \] (9)

where \( K \) is an integration constant. After setting the following constants as \( c_1 = \frac{1}{d - a} (\omega - ak^2 - ck^2 - \frac{cy^2}{(2ak - \nu)^2}) \), and \( c_2 = \frac{\lambda}{2(d - a)} \).

Eq. 9 takes the following expression

\[ \Psi^2_\xi = c_2 \Psi^4 + c_1 \Psi^2 + K. \] (10)

It is important to emphasize that nonlinear ordinary equation (NODE) represents by Eq. 8 which depends only on \( \xi \), can be solved by employing auxiliary equation methods, rational function method or ansatz methods. But to point out analytical solutions in this work, we apply the function transformation method Seadawy (2012). Recently, in ref. Ayaz (2003), the two-dimensional differential transform method have been employed to facilitate computational difficulties. The used of this method usually imposed the boundary condition like the decomposition method. The important matter is that this method facilitated the investigation of the solitary waves without computer codes.

Employing mathematical transformation Eq. 10 takes the form of

\[ \frac{d\Psi}{\sqrt{\frac{1}{d - a} (\omega - ak^2 - ck^2 - \frac{cy^2}{(2ak - \nu)^2})\Psi^2 + \frac{\lambda}{2(d - a)} \Psi^4 + K}} = d\xi. \] (11)
3 Exact soliton solutions

According to the different values of constant of integration $K$ and employing the same transformation in ref. Seadawy (2012), equation Eq. 11 has different explicit and exact analytic solutions as follows.

**Case(I)** For localized solutions, we will consider $K = 0$ and $\frac{\lambda}{2(d-a)} > 0$ in Eq. 11. Therewith, the following solitary wave solution for the ordinary differential Eq. 1 can be obtained as

$$\Psi(\xi) = \frac{4\sqrt{Y} \exp(\sqrt{Y}(\xi - \xi_0))}{4\Gamma \exp(2\sqrt{Y}(\xi + \xi_0)) - 1}, \quad (12)$$

and

$$\Psi(\xi) = \frac{4\sqrt{Y} \exp(\sqrt{Y}(\xi - \xi_0))}{4\Gamma \exp(2\sqrt{Y}(\xi_0) - \exp(2\sqrt{Y}\xi))}. \quad (13)$$

With $\xi_0$ a second constant of integration. However, $\sqrt{1 - \frac{1}{d-a}(\omega - ak^2 - ck^2 - \frac{\gamma^2}{(2ak-c)^2})}$ and $\Gamma = \frac{\lambda}{2(d-a)}$. Then, the solution of the modified CGL equation Eq. 1 can stay in the following form

$$\psi_{1,1}(x, t) = \frac{4\sqrt{Y} \exp(\sqrt{Y}(x - vt + \xi_0))}{4\Gamma \exp(2\sqrt{Y}(x - vt + \xi_0)) - 1} \exp(i(\kappa x - \omega t + \theta_0)), \quad (14)$$

and

$$\psi_{1,2}(x, t) = \frac{4\sqrt{Y} \exp(\sqrt{Y}(x - vt + \xi_0))}{4\Gamma \exp(2\sqrt{Y}\xi_0) - \exp(2\sqrt{Y}(x - vt))} \exp(i(\kappa x - \omega t + \theta_0)). \quad (15)$$

The constraint relation is $\frac{1}{d-a}(\omega - ak^2 - ck^2 - \frac{\gamma^2}{(2ak-c)^2}) > 0$.

The validity of Eq. 14 arises on $4\Gamma \exp(2\sqrt{Y}(x - vt + \xi_0)) - 1 \neq 0$ and the same time the validity of Eq. 15 is given by $4\Gamma \exp(2\sqrt{Y}(x - vt)) - \exp(2\sqrt{Y}(x - vt)) \neq 0$.

**Case(II)**

Under certain conditions on parametric constants, with $\xi_0 = 0$ contrary to the Case (I), we obtain the following two types of solutions of Eq. (1):

(a) If $K = 0$, $\xi_0 = 0$, $\frac{1}{d-a}(\omega - ak^2 - ck^2 - \frac{\gamma^2}{(2ak-c)^2}) > 0$ and $\frac{\lambda}{2(d-a)} < 0$, the following bell-shaped solitary wave solution of equation Eq. 10) can be obtained

$$\Psi(\xi) = \frac{Y}{\Gamma \sech(\sqrt{Y}\xi)}. \quad (16)$$

Hence, the first bright soliton solutions a of Eq. 1 is

$$\psi_{2,1}(x, t) = \frac{Y}{\Gamma \sech(\sqrt{Y}(x - vt)) \exp(i(\kappa x - \omega t + \theta_0))}. \quad (17)$$

(b) If $K = 0$, $\xi_0 = 0$, $\frac{1}{d-a}(\omega - ak^2 - ck^2 - \frac{\gamma^2}{(2ak-c)^2}) < 0$ and $\frac{\lambda}{2(d-a)} < 0$ in equation Eq. 11, the following triangular solution of equation Eq. 10 can be obtained
\[ \Psi(\xi) = \frac{-Y}{\Gamma} \operatorname{sech}(\sqrt{-Y} \xi). \] (18)

Then, Eq. 1 has the following solution
\[ \psi_{2,2}(x, t) = \frac{-Y}{\Gamma} \operatorname{sech}(\sqrt{-Y}(x - vt)) \exp(i(\kappa x - \omega t + \theta_0)). \] (19)

**Case (III)** (a) If \( K = \frac{-1}{\lambda(d-a)}(\omega - ak^2 - ck^2 - \frac{cy^2}{(2ak-\omega)^2}), \) \( \xi_0 = 0, \) while \( \frac{1}{d-a}(\omega - ak^2 - ck^2 - \frac{cy^2}{(2ak-\omega)^2}) < 0 \) and \( \frac{\lambda}{2(d-a)} < 0, \) the following kink-shaped solitary wave solution of Eq. 10 can be found
\[ \Psi(\xi) = \frac{-Y}{\Gamma} \tanh(\sqrt{-Y} \xi). \] (20)

Then, equation (1) has the following solution
\[ \psi_{3,1}(x, t) = \frac{-Y}{\Gamma} \operatorname{tanh}(\sqrt{-Y}(x - vt)) \exp(i(\kappa x - \omega t + \theta_0)). \] (21)

However, the constraint relation of the obtained Eq. 19 is \( Y < 0 \) and the constraint relation of Eq. 21 is \( Y > 0. \)

(b) If \( K = \frac{-1}{\lambda(d-a)}(\omega - ak^2 - ck^2 - \frac{cy^2}{(2ak-\omega)^2}) \) and \( \xi_0 = 0, \) while \( \frac{1}{d-a}(\omega - ak^2 - ck^2 - \frac{cy^2}{(2ak-\omega)^2}) > 0 \) and \( \frac{\lambda}{2(d-a)} < 0, \) the following triangular solution of Eq. 10 is unearthed
\[ \Psi(\xi) = \frac{Y}{\Gamma} \tan(\sqrt{Y} \xi). \] (22)

Then, the solution of equation Eq. 1 becomes
\[ \psi_{3,2}(x, t) = \frac{Y}{\Gamma} \tan(\sqrt{Y}(x - vt)) \exp(i(\kappa x - \omega t + \theta_0)). \] (23)

**Case (IV)** Under certain conditions on the parametric constants, equation (1) admits three jacobian elliptic function solutions as follows:

(a) If \( K = \frac{Y(1-m^2)}{\Gamma(2m^2-1)} \) with \( \frac{1}{d-a}(\omega - ak^2 - ck^2 - \frac{cy^2}{(2ak-\omega)^2}) > 0. \) While \( \frac{\lambda}{2(d-a)} > 0, \) the following solution for equation Eq. 10 can be obtained
\[ \Psi(\xi) = \sqrt{\frac{-Ym^2}{\Gamma(2m^2-1)}} \operatorname{cn}(\sqrt{\frac{Y}{2m^2-1}} \xi). \] (24)

Then, equation Eq. 1 has the following solution
\[ \psi_{4,1}(x, t) = \sqrt{\frac{-Ym^2}{\Gamma(2m^2-1)}} \operatorname{cn}(\sqrt{\frac{Y}{2m^2-1}}(x - vt)) \exp(i(\kappa x - \omega t + \theta_0)). \] (25)

(b) If \( K = \frac{Y(1-m^2)}{\Gamma(2m^2-1)} \) with \( \frac{1}{d-a}(\omega - ak^2 - ck^2 - \frac{cy^2}{(2ak-\omega)^2}) > 0 \) and \( \frac{\lambda}{2(d-a)} > 0, \) the following solution for equation Eq. 10 can be found
Thereafter, Eq. 1 has the following solution

\[
\psi_{4,2}(x,t) = \sqrt{\frac{-\gamma}{\Gamma(2 - m^2)}} \frac{d}{dn}\left(\sqrt{\frac{\gamma}{2 - m^2}}(x - vt)\exp(i(\kappa x - \omega t + \theta_0))\right).
\]  \hspace{1cm} (27)

(c) If \(K = \frac{\gamma m^2}{\Gamma(m^2 + 1)^2}, \frac{1}{d-a}(\omega - ak^2 - ck^2 - \frac{c^2}{(2ak-o)^2}) < 0\) and \(\frac{\lambda}{2(d-a)} < 0\), the following solution for Eq. 10 gives

\[
\Psi(\xi) = \sqrt{\frac{-\gamma m^2}{\Gamma(m^2 + 1)^2}} \frac{d}{dn}\left(\sqrt{\frac{-\gamma}{m^2 + 1}}(x - vt)\exp(i(\kappa x - \omega t + \theta_0))\right).
\]  \hspace{1cm} (28)

Then, Eq. 1 turns to

\[
\psi_{4,3}(x,t) = \sqrt{\frac{-\gamma m^2}{\Gamma(m^2 + 1)^2}} \frac{d}{dn}\left(\sqrt{\frac{-\gamma}{m^2 + 1}}(x - vt)\exp(i(\kappa x - \omega t + \theta_0))\right).
\]  \hspace{1cm} (29)

The following restrictive condition in the above solutions is

\[
\frac{1}{d-a}(\omega - ak^2 - ck^2 - \frac{c^2}{(2ak-o)^2}) > 0.
\]

In the same time Eq. 25 is invalid for \(m = \pm \frac{\sqrt{2}}{2}\) and Eq. 27 is valid for \(m \neq \pm \sqrt{2}\) along with \(\Gamma > 0\).

The obtained above results of the 1D-complex Ginzburg Landau are recovered by using the direct integration (function transformation). These solutions are rational function solutions, bright soliton, dark soliton and jacobian elliptic function solutions. Compared to refs. (Mohamadou et al. 2005; Shamseldeen et al. 2017; Zhang and Dai 2005), it is unearthed some new results (i.e. Eqs. 14, 15, 17 and 19). Besides, using an adequate parameters of the model, some additional results are obtained such as dark and an additional jacobian elliptic function solutions compared to Shamseldeen et al. (2017), Zhang and Dai (2005). These results are very important in diverse field such as optic fibers, ocean engineering, fluid mechanics, biology and so on Zhang and Dai (2005), Alphonse et al. (2019), Osman et al. (2019), Gangwar et al. (2007). Between others, it should be emphasized that, without using computer codes, the obtained results are comparatively look the same with those investigated by adopting several integration technics in refs. (Seadawy and Jalil 2018; Ehab et al. 2016; Seadawy and El-Rashidy 2018; Seadawy and Alamri 2018; Abdullah et al. 2017; Seadawy 2017; Seadawy and El-Rashidy 2016; Seadawy 2017; Asghar et al. 2017) just to list a few.

Furthermore, by considering zero value the integration constant, we yield to Eqs. 14, 15 which are rational solutions. At the same time, to set the integration constant to zero \((K = 0)\) and taking \((\xi_0 = 0)\), it is gained bright solitons. Moreover, taking now the value of integration constant different to zero and assuming \((\xi_0 = 0)\), we unearth dark soliton. The constraint relation on the dark soliton reads \(\frac{1}{d-a}(\omega - ak^2 - ck^2 - \frac{c^2}{(2ak-o)^2}) < 0\). While the jacobian elliptic function solutions are given under the constraint relation \(\frac{1}{d-a}(\omega - ak^2 - ck^2 - \frac{c^2}{(2ak-o)^2}) > 0\) and \(\frac{\lambda}{2(d-a)} > 0\). The obtained self-localized waves of the CGLE can describe the dynamic of the solitary waves in various field of science and engineering.
Figure 1 depicts the spatiotemporal evolution of $|\psi_{1,1}|^2$ of the solution Eq. 14 for $d = 1.000012$, $\alpha = 1$, $k = 0.000125$, $c = 0.0001045$, $\lambda = -0.047$, $\xi_0 = 0.0012$, $v = 0.000015$ and $d = 0.00012$, $\alpha = 1$, $k = 0.000125$, $c = 1.01045$, $\lambda = -0.047$, $\xi_0 = 0.0012$, $v = 0.000015$ while $-150 \leq x \leq 150$ respectively.

Figure 2 shows the bright soliton of $|\psi_{2,1}|^2$ of the solution Eq. 17 for $d = 0.02$, $\alpha = 1$, $k = 0.000125$, $c = 10.045$, $\lambda = 0.47$, $v = 0.25$ at $t = 0$, $t = 5$, $t = 10$, $t = 15$, $t = 20$ and $-150 \leq x \leq 150$ respectively.

Figure 1 depicts the spatiotemporal evolution of $|\psi_{1,1}|^2$ of Eq. 14, and Fig. 2 is the bright soliton of $|\psi_{2,1}|^2$. However, Fig. 3 shows the spatiotemporal evolution of the bright soliton $|\psi_{2,2}|^2$ of Eq. 18 at different values of the soliton speed. Furthermore, in Fig. 4 the W-shaped dark (a) and the corresponding dark soliton (b) of $|\psi_{3,2}|^2$ of Eq. 23 are stressed.

The obtained bright and dark solitons solutions can be useful in communication system via the optical fibers (Savaissou et al. 2020a, b, c, d; Souleymanou et al. 2019a, b, 2011, 2012). These results will certainly encourage the search for solutions to nonlinear differential...
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4 Conclusions

In this paper, we investigate exact soliton solutions to the modified Complex Ginzburg Landau equation through function transformation method. The first step consists to use the transformation hypothesis to obtain the nonlinear ordinary differential equation (NODE). To unearth exact traveling waves solutions to the NODE, we employ the function transformation method which does not use mathematics codes. From there, considering certain conditions on the modified CGL equation parameters, it is revealed bright, dark, kink, W-shaped dark and rational solutions. These results have been illustrated over the 3-D graphical representations by the help of MATLAB software. It is also important to emphasize that the obtained NODE could be handle by other methods.

More recently, the modified CGL model has been investigated in refs. Yomba and Kofane (2003), Hong (2007), Mohamadou et al. (2005), Osman et al. (2019). They have unearthed pulses, front, kink soliton and periodic solutions by adopting Painlevé analysis and Hirota bilinear methods Hong (2007), Mohamadou et al. (2005). The steady state of
these results have been also set out in ref. Hong (2007). In this paper, the obtained results are new compared to Rezazadeh (2018), Yomba and Kofane (2003), Hong (2007), Mohamadou et al. (2005), Gangwar et al. (2007), Yan (1996), Seadawy and Jalil (2018), Ehab et al. (2016), Seadawy and El-Rashidy (2018), Seadawy and Alamri (2018), Abdullah et al. (2017), Seadawy (2017), Seadawy and El-Rashidy (2016) and will certainly have some important effects in many applications in the field of solitary waves such as communication (optical fiber), medicine (fibroscopy), hydrodynamic (rogue waves), thermodynamic (heat) and will extensively be helpful to explain some phenomena of ultra fast science. Next, others methods will be applied in order to emphasize and show out relevant interpretations and related applications.

Declarations

Conflict of interest The authors declare that they have no competing interests.

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