The critical current density in type-II superconducting bulk materials

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Abstract

The critical current density $J_c$ in type-II conventional and high-$T_c$ superconducting bulk materials is investigated based on the quantum theory for the vortex dynamics. It is shown that for a constant magnetic field, the critical current density $J_c$ decreases weakly with increasing temperature when $T < T_{dp}$ (depinning temperature); when $T_{dp} < T < T_f$ (boundary fluctuation temperature), $J_c$ is power-law-decaying, and when $T > T_f$, $J_c$ decays exponentially; while for a constant temperature, $J_c$ first decreases, then increases after reaching a maximum, and finally decreases again as the magnetic field increases. These results are in agreement with the experiments.

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1. INTRODUCTION

Vortex dynamics in type-II conventional and high-$T_c$ superconductors, especially the critical current density $J_c$ has been investigated intensively [1-46], after Anderson [2] first pointed out that the critical current density was reached when the Lorentz force on vortex lattice was balanced by the pinning force [3-8] due to inhomogeneities [7-10] in the specimen [11, 12]. It is understood that the quenched disorder always destroys the long-range order of the vortex lattice, after which only short-range order, the vortex bundle, remains [2, 14, 17, 18, 22, 23].

In this paper we investigate the critical current density $J_c$ for type-II superconducting bulk materials based on the quantum theory of vortex dynamics we [23] have developed. By applying this theory, we have calculated the eigenmodes of the Hamiltonian for the vortex bundles, taken into account the quantum, random and thermal averages of the square of the fluctuations of the deformation and free displacement operators, the critical current density is studied through the balance of the Lorentz force and collective pinning force of the vortex bundle.

It is shown that the critical current density $J_c$ decreases weakly with increasing temperature for temperature $T$ less than the depinning temperature $T_{dp}$; while $J_c$ is power-law-decaying when the range of temperature is between the depinning temperature $T_{dp}$ and boundary fluctuation temperature $T_f$, and $J_c$ decays exponentially for temperature $T$ greater than the boundary fluctuation temperature $T_f$ for a constant magnetic field. On the other hand, $J_c$ first decreases, then increases after reaching a maximum, and finally decreases again as the magnetic field increases for a constant temperature.

The rest of this paper is organized as follows. In the next section, a mathematical description of the model is presented. In section 3, the critical current density due to the collective pinning is investigated for a constant magnetic field as well as for a constant temperature. Several general and important issues about our theory are discussed in section 4. Finally, the concluding remarks are given in section 5.

2. MATHEMATICAL DESCRIPTION OF THE MODEL

Let us consider a type-II high-$T_c$ or conventional superconductor. The full Hamiltonian of the fluctuation for the flux line lattice (FLL) in the $z$-direction is given by [16-18, 22, 23]

$$H = H_f + H_R$$  \hspace{1cm} (1)

where $H_f = H_{\text{kin}} + H_c$ represents the Hamiltonian for the free modes [16-18, 22, 23], with $H_{\text{kin}}$ the kinetic energy [16-18, 22, 23]
\[ H_{\text{el}} = \frac{1}{2\rho} \sum_{\mu} \frac{1}{\mu} P_{\mu}(\vec{K})P_{\mu}(\vec{K}) \]  

(2)

\[ H_r \text{ the elastic energy [16-18, 22-24],} \]

\[ H_r = \frac{1}{2} \sum_{\mu \nu} C_{\mu \nu} K_{\mu \nu} S_{\mu}(\vec{K})S_{\nu}(\vec{K}) + \frac{1}{2} \sum_{\mu \nu} (C_{\mu \nu} K_{\mu \nu}^2 + C_{\mu \nu}^2) S_{\mu}(\vec{K})S_{\nu}(\vec{K}) \]  

(3)

and \( H_r \) represents the random Hamiltonian, given as [16-18, 22, 23],

\[ H_r = \sum_{\mu \nu} \hat{f}_{\mu \nu}(\vec{K})S_{\mu}(\vec{K}) \]  

(4)

where \((\mu, \nu)=(x, y), \rho \) is the effective mass density of the flux line [25], \( K_{\mu \nu}^2 = K_{\mu x}^2 + K_{\mu y}^2 \).

\( P_{\mu \nu}(\vec{K}), S_{\mu}(\vec{K}) \) are the Fourier transformations of the momentum and displacement operators, and \( C_{\mu \nu}, C_{\mu \nu} \) are temperature- and \( \vec{K} \)-dependent bulk modulus, compression modulus, tilt modulus and shear modulus, respectively [7, 8, 26-31]. \( \hat{f}_{\mu \nu}(\vec{K}) \) is the Fourier transformation of the collective pinning force \( \hat{f}_r(r) = -\nabla V_r(r) \), with \( V_r(r) \) the random potential energy of the collective pinning [32-35], which is the sum of the contributions of defects within a distance \( \xi \) of the vortex core position \( r \), where \( \xi \) is the temperature-dependent coherence length. The correlation functions of the collective pinning random force are assumed to be the short-range correlation [23],

\[ \langle \langle f_{\mu \nu}(k) f_{\mu \nu}^*(k') \rangle \rangle \equiv \beta(T, B) \delta(k - k') \]  

(5)

with \( \langle \langle \rangle \rangle \) are the quantum, thermal, and random averages, and \( \beta(T, B) \) is the temperature- and magnetic field-dependent correlation strength. It is the quantum correlation strength \( \beta^Q(T, B) \) in quantum limit, while in the classical limit it is the classical correlation strength \( \beta^C(T, B) \).

The free modes Hamiltonian \( H_f \) can be diagonalized [16-18, 22, 23] as follows:

\[ H_f = \sum_{\nu \mu} [N_{\nu \mu} + \frac{1}{2} \hbar \omega_{\nu \mu} \delta(k - k')] \]  

(6)

\[ N_{\nu \mu} = \alpha_{\nu \mu}^\dagger \alpha_{\nu \mu} \]  

(7)

with \( \alpha_{\nu \mu}^\dagger, \alpha_{\nu \mu} \) are the creation and annihilation operators for the corresponding eigenmodes as

\[ \alpha_{\nu \mu}^\dagger = \frac{1}{\sqrt{2\hbar}} \left[ \frac{-i}{\sqrt{\rho \omega_{\nu \mu}}} P_{\nu}(\vec{K}) + \sqrt{\rho \omega_{\nu \mu}} S_{\nu}(\vec{K}) \right] \]  

(8)
\[ \alpha_{k\mu} = \frac{1}{\sqrt{2\hbar}} \frac{i}{\sqrt{\rho_0 k_{\mu}}} P_\mu(-K) + \sqrt{\rho_0 k_{\mu}} S_\mu(K) \]  

(9)

where \( \mu = 1 \) presents the component parallel to the \( \vec{K}_\perp \) direction, while \( \mu = 2 \) is perpendicular to the \( \vec{K}_\perp \) direction, and the corresponding eigenmodes spectrum are given by \( [16-18, 22, 23] \)

\[ \omega_{k1} = \frac{1}{\rho} (C_1 K_\perp^2 + C_{44} K_\perp^2) \]  

(10)

\[ \omega_{k2} = \frac{1}{\rho} (C_{66} K_\perp^2 + C_{44} K_\perp^2) \]  

(11)

The equation of motion for the displacement operator \( S_\mu(K) \) can be obtained from Eq. (1) as

\[ \rho \ddot{S}_\mu(K) + C_\mu(K \cdot \dot{S}(K)) K_\mu + (C_{66} K_\perp^2 + C_{44} K_\perp^2) S_\mu(K) + f_\mu(K) = 0 \]  

(12)

Then the solution to Eq. (12) is expressed as

\[ S_\mu(K) = S'^\mu(K) + S'^R_\mu(K) \]  

(13)

where \( S'^R_\mu(K) \) is the random displacement operator owing to the collective pinning of the random function \( \tilde{f}_R(K) \), and \( S'^\mu(K) \) denotes the free displacement operator which is the displacement operator of the free modes. They are obtained as

\[ S'^R_\mu(K) = \left[ (\vec{K} \cdot \tilde{f}(K)) \frac{K_\mu}{K_\perp} \right] \frac{1}{C_1 K_\perp^2 + C_{44} K_\perp^2} + \left[ f_\mu(K) \right] \frac{K_\mu}{K_\perp} \frac{1}{C_{66} K_\perp^2 + C_{44} K_\perp^2} \]  

(14)

and

\[ S'^\mu(K) = \frac{\hbar}{2 \rho_0 k_{\mu}} \alpha_{k\mu} + \alpha^*_{k\mu} \]  

(15)

respectively.

For type-II superconducting bulk materials, the corresponding random and free displacement correlation functions can be obtained as

\[ \langle \langle S^R(K) - S^R(0) \rangle \langle S^R(K) - S^R(0) \rangle \rangle_{\text{th}} = \frac{d^3k}{(2\pi)^3} 2 (1 - \cos(\vec{K}_\perp \cdot \vec{R}_L + K_z L)) \frac{\beta(T, B)}{(C_{66} K_\perp^2 + C_{44} K_\perp^2)^2} \]  

= \frac{\beta(T, B)}{2 \pi^2 C_{66} \sqrt{C_{44} C_{66}}} \left[ R_z^2 + \frac{a_0^2 L^2}{\lambda^2} \right] \]  

(16)
\[ << (\hat{S}^f)^2 >>_{th} = \frac{k_B T}{\pi^2 C_{\alpha_{1}C_{\alpha_{6}}}} \cdot \frac{1}{\xi^2} \]  

(17)

respectively, once again, \( << >>_{th} \) are the quantum, random and thermal averages, \( \beta(T,B) \) is the temperature- and magnetic field-dependent correlation strength, \( \bar{\rho} = (\bar{\rho}_{\perp}, \bar{L} \vec{c}_z) \), with \( \bar{\rho}_{\perp} \) and \( L \) are the transversal and longitudinal sizes of the vortex bundle, \( \lambda \) is the temperature-dependent penetration depth, the lattice constant \( a_0 = (2\Phi_0 / B\sqrt{3})^{1/2} \), \( \Phi_0 \) is the unit flux, \( B \) is the applied magnetic field, \( \xi_0 \) is the coherence length at zero temperature and \( k_B \) is the Boltzmann constant. In deriving the above equations, we have taken into account the fact that \( C_{\alpha_{1}} >> C_{\alpha_{6}} \), performed the average over \( \phi \), which is the angle between \( \bar{K}_{\perp} \) and \( \bar{R} \), and applied the cutoff values for small \( k \) as \( k_y = 2(R_{\perp}^2 + a_0^2L^2 / \lambda^2)^{1/2} \) and for large \( k \) as \( k_L = 1 / \xi_0 \).

3. CRITICAL CURRENT DENSITY \( J_c \)

In this section we would like to investigate the critical current density \( J_c \) in type-II superconducting bulk materials by considering the balance of the Lorentz force and collective pinning force, \( J_c \) can be obtained as [22, 23]

\[ J_c = \frac{1}{B} \frac{\beta(T,B)}{\pi R_{\perp}^2 L} \]  

(18)

where \( B \) is the applied magnetic field, \( R_{\perp} \) and \( L \) are the transverse and longitudinal sizes of the vortex bundles, which are determined by the relation

\[ << (\hat{S}^R(R) - \hat{S}^R(0))^2 >>_{th} = \eta^2 \]  

(19)

where \( \eta \) stands for the collective pinning force range. Let us define the following characteristic temperatures [22] for the upcoming calculations and discussions of the critical current density \( J_c \): the division temperature \( T_D \), where for \( T < T_D \) only the quantum statistics is applicable, while in the case of \( T > T_D \) the classical statistics can also be used. The depinning temperature \( T_{dp} \), which is defined by the condition

\[ << (\hat{S}^f)^2 >>_{th} = \xi^2 \]  

(20)

the boundary fluctuation temperature \( T_f \), which is determined by the relation

\[ << (\hat{S}^f)^2 >>_{th} = \chi a_0^2 \]  

(21)
with $\chi$ is a dimensionless constant. The collective pinning force range $\eta$ can now be identified for different temperature regimes as

$$\eta = \bar{\xi}, \quad \text{for} \quad T < T_{dp}$$

$$= \sqrt{\langle (\bar{S}/\chi)^2 \rangle_{\eta}}, \quad \text{for} \quad T > T_{dp} \quad (22)$$

where the quantum, random and thermal averages of the free displacement correlation function $\langle (\bar{S}/\chi)^2 \rangle_{\eta}$ is given by Eq. (17). The critical current density $J_c$ for type-II bulk materials can now be obtained as follows.

First for temperature $T < T_D$, inserting Eqs. (19) and (22) into (18), the critical current density $J_c$ can be evaluated as

$$J_c = \sqrt{\frac{\alpha_0}{\lambda}} \frac{\beta^Q(T,B)^2}{2B\sqrt{2\pi(\pi\bar{\xi})^3}C_{44}^2(C_{66}C_{44})^{3/4}} \quad (23)$$

where $\beta^Q(T,B)$ is the temperature-and magnetic field-dependent quantum correlation strength that we have discussed earlier in section 2.

For temperature in the interval $T_D < T < T_{dp}$, we have

$$J_c = \sqrt{\frac{\alpha_0}{\lambda}} \frac{\beta^C(T,B)^2}{2B\sqrt{2\pi(\pi\bar{\xi})^3}C_{44}^2(C_{66}C_{44})^{3/4}} \quad (24)$$

and

$$J_c = \sqrt{\frac{\alpha_0}{\lambda}} \left[ \frac{\beta^C(T,B)^2}{B\sqrt{\pi(2k_B T C_{66})^{3/2}}} \right] \quad (25)$$

for $T_{dp} < T < T_f$. Finally, for $T > T_f$, the charge-density-wave type [36-38] pinning regime, we arrive at

$$J_c = \frac{\beta^C(T,B)^{1/2}}{B[\pi R_j^2 L_f \exp\left(\frac{(3\sqrt{2})k_BT}{\alpha_0\pi C_{44}C_{66}}\frac{1}{\bar{\xi}_0}\right)]^{1/2}} \quad (26)$$

where $\beta^C(T,B)$ is the temperature- and magnetic field-dependent classical correlation strength, and $R_j$ and $L_f$ are the transverse and longitudinal bundle sizes at temperature $T = T_f$, respectively.
The above calculated results show that, for \( T < T_D \), \( J_c \) depends on temperature weakly through the parameters of quantum correlation strength \( \beta^q(T, B) \), coherence length \( \xi \), penetration depth \( \lambda \), shear modulus \( C_m \) and tilt modulus \( C_d \) implicitly, for \( T_D < T < T_{dp} \), \( J_c \) decreases weakly with increasing temperature through the parameters of classical correlation strength \( \beta^c(T, B) \), coherence length \( \xi \), penetration depth \( \lambda \), shear modulus \( C_m \) and tilt modulus \( C_d \) implicitly. For \( T_{dp} < T < T_f \), \( J_c \) decreases in the form of power law, while for \( T > T_f \), \( J_c \) decays exponentially with increasing temperature for a constant magnetic field, when \( T_{dp} < T < T_f \) (boundary fluctuation temperature). \( J_c \) is power-law-decaying, while for \( T > T_f \), \( J_c \) decays exponentially for a constant magnetic field; on the other hand, \( J_c \) first decreases, then increases after reaching a maximum, and finally decreases again as the magnetic field increases for a constant temperature.

3.1. Critical Current Density for Constant Applied Magnetic Field

First we would like to investigate the critical current density \( J_c \) for constant magnetic field. It is understood that \( \beta^c(T, B) \) decreases with increasing temperature for a constant magnetic field \( B \) due to the reduction of condensation energy. As we have mentioned above the calculations shown that, for \( T < T_D \) the depinning temperature, \( J_c \) decreases weakly for both quantum and classical limits, for \( T_D < T < T_f \), \( J_c \) decreases in the form of power law, finally for \( T > T_f \), \( J_c \) decays exponentially with increasing temperature. It is interesting to make numerical estimates of the characteristic temperatures \( T_D \), \( T_{dp} \), \( T_f \) and the critical current density \( J_c \) as a function of \( T/T_c \) for a constant applied magnetic field. We obtained \( T_D \approx 10^2 K \), \( T_{dp} \approx 18.23 K \), \( T_f \approx 72 K \), \( J_c (10^{-4}) \approx 2.2 \times 10^9 A/m^2 \), \( J_c (0.192) \approx 2 \times 10^9 A/m^2 \), \( J_c (0.4) \approx 1.41 \times 10^9 A/m^2 \), \( J_c (0.758) \approx 3.163 \times 10^9 A/m^2 \), \( J_c (0.9) \approx 10^9 A/m^2 \), and \( J_c (1) \approx 0 \). These values are in agreement with the experimental results for YBa\(_2\)Cu\(_3\)O\(_{6-\delta}\) superconducting bulk material [40]. In obtaining the above results the following approximate data have been employed [12]: \( B = 1T \), \( \xi_0 = 1 \times 10^{-9} m \), \( \lambda_0 = 2 \times 10^{-7} m \), \( T_c = 95 K \), \( \chi = 1.8 \times 10^{-3} \), \( \beta^c(T_{dp}) \approx 18.23; B = 1 \) \( \approx 2.429 \times 10^{-3} N^2/m^3 \), \( \beta^c(T = 38; B = 1) \approx 1.013 \times 10^{-2} N^2/m^3 \), \( \beta^c(T_f = 72; B = 1) \approx 5.06 \times 10^{-3} N^2/m^3 \), \( \beta^c(T = 85.5; B = 1) \approx 6.183 \times 10^{-5} N^2/m^3 \), \( \beta^c(T_D = 10^2; B = 1) \approx 8.124 \times 10^{-3} N^2/m^3 \).

3.2. Critical Current Density for Constant Temperature

In this subsection we would like to study the critical current density \( J_c \) for constant temperature. It is understood that when the applied magnetic field is above the irreversible line or depinning line, the pins become ineffective, and the vortices can move freely, in other words, \( J_c = 0 \), for \( B \geq B_{irr} \), where \( B_{irr} \) is the irreversible field [12]. However, for \( B < B_{irr} \), \( \beta^c(T, B) \) increases with increasing magnetic field, after reaching a maximum value, then decreases again for constant temperature due to the competition between the reduction of condensation energy and the increase of pinning. The numerical calculations of critical current density \( J_c \) as function of \( B/B_{irr} \) are also obtained as follows: \( J_c (0.01) \approx 5.012 \times 10^9 A/m^2 \), \( J_c (0.1) \approx 3.17 \times 10^9 A/m^2 \), \( J_c (0.2) \approx 7.95 \times 10^9 A/m^2 \).
$J_c(0.3) = 10^9 \text{A/m}^2$, $J_c(0.4) = 8.92 \times 10^6 \text{A/m}^2$, $J_c(0.5) = 7.08 \times 10^9 \text{A/m}^2$, $J_c(0.7) = 2.51 \times 10^8 \text{A/m}^2$, $J_c(0.8) = 7.08 \times 10^7 \text{A/m}^2$, $J_c(1) = 0$. These values are in agreement with the experimental results for YBa$_2$Cu$_3$O$_{y+\delta}$ superconducting bulk material [40]. In obtaining the above results the following approximate data have been employed [12]: $T_c = 95 \text{K}$, $\chi = 1.8 \times 10^{-3}$, $\beta^c(T=72;B=0.1) = 7.106 \times 10^{-4} \text{N}^2/\text{m}^3$, $\beta^c(T=72;B=1) = 5.06 \times 10^{-3} \text{N}^2/\text{m}^3$, $\beta^c(T=72;B=2) = 2.082 \times 10^{-2} \text{N}^2/\text{m}^3$.

4. DISCUSSION

In this section we would like to make several general and important points about our calculations. First of all, the critical current density $J_c$ in type-II superconducting bulk materials for constant magnetic field as well as for constant temperature has been calculated based on the quantum theory of vortex dynamics we [23] have developed.

Secondly, the model of our calculations is very general it can be used for conventional and high-$T_c$ superconductors. Although their mechanisms and the method of pairing are different, these only affect the structure of the vortex lattice and are a minute effect in our calculations.

Thirdly, the coherence length of the superconductor plays a very important role, since it sets the smallest length scale in our theory that can be seen from the large-$k$ cutoff in the $k$-integration.

Finally, it is worthwhile to know that we only discuss the systems that are in thermodynamic equilibrium with the environment any time dependent behavior shall not be discussed.

5. CONCLUSION

The quantum, thermal and random fluctuations of the free displacement as well as the random displacement operator of the flux line lattice and critical current density $J_c$, as a function of temperature as well as applied magnetic field in type-II conventional and high-$T_c$ superconducting bulk materials are investigated based on the quantum model for vortex dynamics that we have developed in both quantum and classical regimes. It is shown that for a constant magnetic field, the critical current density $J_c$ decreases weakly with increasing temperature when $T < T_{dp}$ (depinning temperature); when $T_{dp} < T < T_f$ (boundary fluctuation temperature), $J_c$ is power-law-decaying, and when $T > T_f$, $J_c$ decays exponentially; while for a constant temperature, $J_c$ first decreases, then increases after reaching a maximum, and finally decreases again as the magnetic field increases. These results are in agreement with the experiments.
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