Two-photon exchange and elastic scattering of longitudinally polarized electron on polarized deuteron

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Structure functions and polarization observables in elastic scattering of longitudinally polarized electron on polarized deuteron are considered within approximation of one-photon + two-photon exchange. It is shown that contribution of two-photon exchange in the generalized structure function $A$ is of order of few percent, while in the generalized structure function $B$ it is of order of 10–20%. We have found that components $T^{20}$ and $T^{21}$ of tensor analyzing power are mainly determined by one-photon exchange, but $T^{22}$ is mainly determined by interference between one-photon exchange and two-photon exchange. We have also considered polarization observables $T^{11}$, $C^{21}$ and $C^{22}$ which are proportional to imaginary part of the reaction amplitude and vanish in the framework of one-photon exchange.

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I. INTRODUCTION

Over recent years, the interest to study of polarization observables in electron scattering on hadron systems (nucleons, pions, lightest nuclei) has been raised. This interest is based on the significant progress in experimental technique, as well as on the fact that polarization measurements give an opportunity to get more information about structure of the hadron systems, than study of unpolarized cross section. For example, polarization data obtained in resent years together with unpolarized cross sections make it possible to extract important information about effects beyond Born approximation in elastic electron-proton scattering (see, e.g., [1] and references therein).

In case of electron-deuteron scattering polarization measurements play an important role already in one-photon exchange (OPE) approximation. Indeed, the deuteron, due to its spin structure, has three electromagnetic form factors (the charge, $G_C$, quadrupole, $G_Q$, and magnetic, $G_M$, form factors), which are real functions of one variable, $Q^2$. The Rosenbluth separation allows to perform and obtain two structure functions of the deuteron: $A(Q^2)$ (a combination of $G_C^2$, $G_Q^2$ and $G_M^2$) and $B(Q^2)$ (proportional to $G_M^2$). To separate all three form factors we need to measure an additional observable. Usually this is $t_{20}$ component of tensor polarization of the deuteron.

Similarly to elastic electron-nucleon scattering, two-photon exchange (TPE) is one of the most important effects beyond Born approximation in elastic $ed$-scattering [2, 3].

Following Ref. [2], Feynman diagrams for TPE diagrams in $ed$ scattering fall into two types: diagrams, where both intermediate photons interact with the same nucleon, (type I) and diagrams, where photons interact with different nucleons, (type II). In Ref. [2] diagrams of type II were calculated using the simplest gaussian wave function of the deuteron. In turn, in Refs. [3, 4] some effects connected with diagrams of type I were examined in the framework of effective Lagrangian approach [6, 7]. Both types of diagrams were calculated simultaneously in Ref. [5] within semi-relativistic approximation with the deuteron wave functions for “realistic” NN potentials.

The aim of the present paper is to study differential cross section and polarization observables in the elastic scattering of longitudinally polarized ultra-relativistic electron on polarized deuteron in the framework of OPE+TPE approximation.

The paper is organized as follows. In Sec. I we discuss density matrix for spin-1 particle and define polarization observables in elastic scattering of longitudinally polarized electron on polarized deuteron. In Sec. II the polarization observables are calculated in the framework of OPE+TPE approximation. Numerical results and discussion are given in Sec. III.
rected along transfer momentum $\vec{q}$ and $y$-axis is directed along $\vec{k} \times \vec{k}'$, where $k_\mu$ and $k'_\mu$ are momenta of incoming and outgoing electrons.

In this case, according to Madison Convention [8], the density matrix of a spin-1 particle (the deuteron) is given by the relation

$$\rho = \frac{1}{3} \sum_{kq} t^*_{kq} \tau_{kq},$$

where $t_{kq}$ are polarization parameters of the deuteron and $\tau_{kq}$ are spherical tensors. The later are expressed through the deuteron spin matrices $\vec{S}$

$$S_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_y = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix},$$

$$S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$ (3)

From (2) and hermiticity of the spin operator we immediately get

$$\tau^\dagger_{kq} = (-1)^q \tau_{k-q}$$

and the hermiticity condition for the density matrix yields for $t_{kq}$

$$t^*_{kq} = (-1)^q t_{k-q}.$$ (5)

After that we come to explicit expression of the deuteron density matrix

$$\rho = \frac{1}{3} \left( \begin{pmatrix} 1 & \sqrt{2} t_{10} + \frac{1}{\sqrt{2}} t_{20} & \sqrt{3} t_{1-1} + t_{2-1} \\ -\sqrt{2} (t_{11} + t_{21}) & 1 - \sqrt{2} t_{20} & \sqrt{3} t_{1-1} - t_{2-1} \\ \sqrt{3} t_{22} & -\sqrt{2} (t_{11} - t_{21}) & 1 - \frac{1}{\sqrt{2}} t_{10} + \frac{1}{\sqrt{2}} t_{20} \end{pmatrix} \right).$$ (6)
The cross section for elastic scattering of electron with helicity sign $h$ on polarized deuteron is given by
\[
\frac{d\sigma(h)}{d\Omega} = \frac{d\sigma_0}{d\Omega} \left( 1 + \sum_{k=1}^{2} \sum_{q=-k}^{k} t_{kq} T_{kq}^h \right), \tag{7}
\]
where $\frac{d\sigma_0}{d\Omega}$ is cross section for unpolarized particles and $t_{kq}$ and $T_{kq}^h$ are polarization tensor of the incoming deuteron and analyzing power of the reaction, respectively.

The two conditions allow us to write
\[
T_{kq}^h = \frac{\text{Tr}[M^h T_{kq} M^h]}{\frac{1}{2} \sum_h \text{Tr}[M^h M^h^\dagger]}, \tag{8}
\]
where $M^h$ is scattering amplitude. Similarly to polarization tensor, $T_{kq}^h$ obeys the hermiticity condition
\[
T_{kq}^h = (-1)^q T_{-k-q}^h. \tag{9}
\]
Furthermore, parity conservation implies
\[
T_{kq}^h = (-1)^{k+q} T_{-k-q}^{-h}. \tag{10}
\]
These two conditions allow us to write
\[
\begin{align*}
T_{10}^h &= h C_{10}, \\
T_{11}^h &= i T_{11} + h C_{11}^L, \quad T_{1-1}^h = i T_{11} - h C_{11}^L, \\
T_{20}^h &= T_{20}, \\
T_{21}^h &= T_{21} + i h C_{21}^L, \quad T_{2-1}^h = -T_{21} + i h C_{21}^L, \\
T_{22}^h &= T_{22} + i h C_{22}^L, \quad T_{2-2}^h = T_{22} - i h C_{22}^L,
\end{align*} \tag{11}
\]
where $T_{kq}$ and $C_{kq}^L$ are purely real. We call $T_{kq}$ and $C_{kq}^L$ analyzing power and correlation parameter, respectively. It should be noted that OPE amplitude is real and $T_{11}, C_{21}^L$ and $C_{22}^L$ are nonvanishing only beyond OPE approximation.

Substituting (11) in (7) and using relation (8) we get the differential cross section for elastic scattering of electron with longitudinal polarization $p$ on polarized deuteron
\[
\frac{d\sigma}{d\Omega} = \frac{d\sigma_0}{d\Omega} \left[ 1 + 2 \Im \text{mt}_{11} T_{11} + t_{20} T_{20} + 2 \Re t_{21} T_{21} + 2 \Re t_{22} T_{22} + p \left( t_{10} C_{10}^L + 2 \Re t_{11} C_{11}^L + 2 \Re t_{21} C_{21}^L + 2 \Re t_{22} C_{22}^L \right) \right], \tag{12}
\]

III. POLARIZATION OBSERVABLES IN THE SECOND ORDER PERTURBATION CALCULATIONS

It follows from $P$ and $T$ invariance that elastic scattering amplitude of a spin $\frac{1}{2}$-particle (electron) on a spin-1 particle (deuteron) is determined by 12 invariant amplitudes. Putting the electron mass to zero reduce the number of invariant amplitudes (form factors) to 6 (see, e.g., [10]).

All calculations will be done in the Breit frame. In this frame it is useful, instead of usual amplitude $M$, to introduce reduced amplitude $T_{\lambda,\lambda',h}$
\[
M = \frac{16\pi\alpha}{Q^2} E_e E_d T_{\lambda,\lambda',h}, \tag{13}
\]
where $\alpha \approx 1/137$ is the fine-structure constant, $\lambda, \lambda'$ are spin projections of the incoming and outgoing deuteron and $E_e$ and $E_d$ are energies of the electron and deuteron in the Breit frame. For the reduced amplitude we will adopt the following parametrization [10]
\[
T_{\lambda,\lambda',h} = \begin{pmatrix}
G_{11} \cos \frac{\theta}{2} & -\sqrt{\frac{2}{3}} G_{10}^h & G_{11}^h \\
\sqrt{\frac{2}{3}} G_{10}^h & G_{00} \cos \frac{\theta}{2} & -\sqrt{\frac{2}{3}} G_{10}^h \\
G_{11}^{-h} & \sqrt{\frac{2}{3}} G_{10}^h & G_{11} \cos \frac{\theta}{2} \\
\end{pmatrix}, \tag{14}
\]

FIG. 3: The TPE correction to $T_{20}$ at $Q^2 = 2$ and 3 GeV$^2$ (solid and dashed respectively). The bold and thin lines are for calculations with CD-Bonn and Paris NN-potentials. $T_{20}^{\text{OPE}} \approx 0$ at $Q^2 = 1$ GeV$^2$.

FIG. 4: The TPE correction to $T_{21}$ at $Q^2 = 1$ and 3 GeV$^2$ (solid and dot-dashed lines, respectively). The bold and thin lines are for calculations with CD-Bonn and Paris NN-potentials. $T_{21}^{\text{OPE}} \approx 0$ at $Q^2 \approx 2$ GeV$^2$ and relative correction becomes infinite.
FIG. 5: $T_{22}$ at $\theta_{\text{lab}} = 70^\circ$. The dashed line is for OPE calculations, the solid and dash-doted lines are for OPE+TPE calculated with CD-Bonn and Paris NN-potentials, respectively. Data are from [16] and [17] (circles and boxes, respectively).

FIG. 6: $T_{11}$ at $Q^2 = 1, 2$ and 3 GeV$^2$ (solid, dashed and dot-dashed lines, respectively). The bold and thin lines are for calculations with CD-Bonn and Paris NN-potentials.

where $\theta$ is scattering angle in the Breit frame, $\eta = Q^2/(4M^2)$, $M$ is the deuteron mass and

$$G_{10}^h = f_1 + h \sin \frac{\theta}{2} f_2, \quad G_{1-1}^h = f_3 + h \sin \frac{\theta}{2} f_4.$$  \hspace{1cm} (15)

The form factors $G_{11}, G_{00}, f_1, ..., f_4$ are complex functions of the two independent kinematical variables, for example, $Q^2$ and the commonly used polarization parameter

$$\epsilon = \frac{\cos \frac{\theta}{2}}{1 + \sin \frac{\theta}{2}}.$$  \hspace{1cm}

Instead of the the form factors $G_{11}, G_{00}, f_1, ..., f_4$ the following linear combinations are introduced

$$G_C = \frac{1}{3} (G_{11} + 2G_{00}), \quad G_Q = \frac{1}{2\eta} (G_{00} - G_{11}), \quad G_M = \frac{f_1 + \sin^2 \frac{\theta}{2} f_2}{1 + \sin^2 \frac{\theta}{2}}, \quad g_1 = \frac{f_1 - f_2}{1 + \sin^2 \frac{\theta}{2}}, \quad g_2 = f_3, \quad g_3 = f_4.$$  \hspace{1cm} (16)

We call $G_C(Q^2, \epsilon), G_Q(Q^2, \epsilon)$ and $G_M(Q^2, \epsilon)$ the generalized electric, quadrupole and magnetic form factors. In zeroth order in $\alpha$ the generalized electric, quadrupole and magnetic form factors are reduced to the usual electric, quadrupole and magnetic form factors, $G_C(Q^2), G_Q(Q^2)$ and $G_M(Q^2)$, while the form factors $g_1(Q^2, \epsilon), g_2(Q^2, \epsilon)$ and $g_3(Q^2, \epsilon)$ are of order $\alpha$ and vanish in the Born approximation.

Substituting (14)-(16) in Eq. (8) we arrive at the following expressions for analyzing powers

$$T_{20} = \frac{-\eta}{3\sqrt{2S}} \left[ S \left( \Re(G_C G_Q) + \frac{\eta}{3} |G_M|^2 \right) \right] + (1 + 2\tan^2 \frac{\theta}{4}) |G_M|^2,$$

$$T_{21} = -\frac{\eta}{3\cos^2 \frac{\theta}{2} S} \left[ 2\cos \frac{\theta}{2} \eta \Re(G_M G_Q) + G_M \Re(\sin^2 \frac{\theta}{2} g_3 + g_2) + 2\sin^2 \frac{\theta}{2} \cos \frac{\theta}{2} \Re(g_1 G_Q) \right],$$

$$T_{22} = \frac{1}{2\sqrt{3}\cos^2 \frac{\theta}{2} S} \left[ -\cos^2 \frac{\theta}{2} \eta |G_M|^2 - 4\sin^2 \frac{\theta}{2} \eta G_M \Re g_1 + 4\cos \frac{2\theta}{3}(G_C - \frac{2}{3} \eta G_Q) \Re g_2 \right],$$

and polarization correlations

$$C_{21} = \sqrt{\frac{\eta}{3\cos^2 \frac{\theta}{2} S}} \times \left[ -\eta \sin \theta (3m (G_M^* G_Q) + G_Q 3m g_1) + \sin \frac{\theta}{2} G_M 3m (g_2 + g_3) \right],$$

$$C_{22} = -\frac{\eta}{\sqrt{3}\cos^2 \frac{\theta}{2} S} \times \left[ \sin \theta (G_C - \frac{2}{3} \eta G_Q) 3m g_3 + \sin \frac{\theta}{2} (\sin^2 \frac{\theta}{2} + 1) \eta G_M 3m g_1 \right],$$

$$C_{10} = \sqrt{\frac{2\eta}{3\tan \frac{\theta}{2}}} \left[ -|G_M|^2 + \cos^2 \frac{\theta}{2} G_M \Re g_1 \right],$$

$$C_{11} = \sqrt{\frac{\eta}{3\cos^2 \frac{\theta}{2} S}} \times \left[ \sin \theta (\Re(G_M^* (3G_C + \eta G_Q)) - (3G_C + \eta G_Q) \Re g_1) + 3\sin \frac{\theta}{2} G_M \Re (g_2 + g_3) \right],$$

where we neglected terms of order $\alpha^2$ and thus $G_K G_L = G_K G_L + \delta G_K G_L + \delta G_K \delta G_L, (K, L) = (C, Q, M), G_K = G_K + \delta G_K, \delta G_L = G_L + \delta G_L$.

$$S = A + B\tan^2 \frac{\theta}{4},$$  \hspace{1cm} (19)
with
\[ \mathcal{A}(Q^2, \theta) = |g_C|^2 + \frac{8}{3} \eta |g_Q|^2 + \frac{2}{3} \eta |g_M|^2, \]
\[ \mathcal{B}(Q^2, \theta) = \frac{2}{3}(1 + \eta)|g_M|^2 \]
are generalized structure functions and \( \tan^2 \theta_{\text{lab}}/2 = (1 + \eta)^{-1} \tan^2 \theta/2 \).

\section{IV. Numerical Results and Conclusions}

In previous section the generalized structure functions \( \mathcal{A}(Q^2, \theta) \) and \( \mathcal{B}(Q^2, \theta) \) and polarization observables were expressed by generalized form factors. Now we will calculate behavior of all this quantities.

In numerical calculations of the TPE amplitudes we used semi-relativistic approach of Ref. [5] with the deuteron wave function for CD-Bonn [11] and Paris [12] NN-potentials. The deuteron form factors \( G_C(Q^2) \), \( G_Q(Q^2) \) and \( G_M(Q^2) \) were taken from parametrization of Ref. [13] with parameters taken from fit of Ref. [14].

We find that TPE contribution is of order of few percent (see Fig. 1) in the generalized structure function \( \mathcal{A} \), while in the generalized structure function \( \mathcal{B} \) it is more significant. For example, at \( \theta = 180^\circ \) and \( Q^2 > 2.5 \text{ GeV}^2 \) TPE effect are estimated to be 10 to 20% in \( \mathcal{B} \) (Fig. 2).

One also sees, that value of TPE correction is strongly dependent on the deuteron wave function.

The \( \epsilon \) dependence of relative TPE correction to \( T_{20} \) and \( T_{21} \) at few values of \( Q^2 \) are shown in Figs. 3 and 4. One can conclude that TPE is not significant in \( T_{20} \). In \( T_{21} \) the role of TPE effects increases with \( Q^2 \). For example, at \( Q^2 = 3 \text{ GeV}^2 \) the TPE correction comes up to 20%. In \( T_{22} \) the interference between OPE and TPE amplitudes becomes a dominant contribution, Fig. 5. The latter conclusion is obvious, because \( G_C/G_M \sim 10 \) and the term \( \frac{4}{3} \cos^2 \theta G_C \) \( \Re g_{92} \) ranks over the term \( \cos^2 \theta G_M^2 \) in the expression for \( T_{22} \) (see Eqs. 17).

Our estimations for the observables, which vanish in the OPE approximation (\( T_{11} \), \( C_{21} \) and \( C_{22} \)), are displayed in Figs. 6 and 7.

We have also found that TPE makes only slight changes (not more than few percent) in \( C_{10} \) and \( C_{11} \) correlations, Fig. 8.

In conclusion, we have considered general structure of
differential cross section for elastic scattering of longitudinally polarized electron on polarized deuteron.

The generalized structure functions $A$ and $B$ and polarization observables in this reaction are calculated in the framework of the one-photon + two-photon exchange. We find that TPE contribution in the generalized structure function $A$ is less than experimental errors (about few percent) and about 10-20% in the generalized structure function $B$ at $Q^2 > 2.5$ GeV$^2$.

While in $T_{20}$ TPE correction is minor, interference between OPE and TPE becomes dominant contribution in $T_{22}$ at $Q^2 > 0.5$ GeV$^2$. This means that $T_{22}$ may be a good object for experimental study of two-photon exchange in $ed$-scattering. We have also calculated polarization observables $T_{11}$, $C_{21}$ and $C_{22}$ which are proportional to the imaginary part of the reaction amplitude and vanish in the framework of one-photon exchange.

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