Charmonium spectral functions in $N_f = 2$ QCD at high temperature

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Charmonium systems in two-flavour QCD at non-zero temperature are studied at, and above the deconfining transition. Using anisotropic lattices the MEM approach is used to extract spectral functions for these channels. By carefully varying some of the irrelevant parameters in the MEM procedure, we confirm that our systematic effects are under control. The $\eta_c$ and $J/\psi$ states are found to persist at temperatures $\gtrsim 1.3T_c$ in agreement with our previous dynamical studies.
1. Introduction

The recent experimental evidence of the quark-gluon plasma phase of QCD in RHIC [1] provides experimentalists and theorists with a formidable challenge: How is the physics of this new phase uncovered from the products of the fireball when these products have necessarily condensed back into the confined phase?

To answer this question it is crucial to know whether hadronic states in fact persist in the quark-gluon plasma phase. Current understanding of the plasma phase suggests that while it is not confining, it is still strongly interacting, suggesting that hadronic states can indeed survive above the de-confining temperature, \( T_c \) [2].

This study investigates mesonic states at and above \( T_c \) via their spectral functions, \( \rho(\omega, p) \).\( \rho \) can be defined from Euclidean hadronic 2–point function, \( G(t, p) \), as

\[
G(t, p) = \int_0^\infty \rho(\omega, p) K(t, \omega) \frac{d\omega}{2\pi}
\]

where the kernel, \( K(t, \omega) \), at temperature, \( T \), is given by

\[
K(t, \omega) = \frac{\cosh[\omega(t - 1/(2T))] - \omega/(2T)}{\sinh[\omega/(2T)]}.
\]

Spectral functions contain information on the stability of hadronic states, transport coefficients and dilepton production rates, and thus are fundamental physical quantities of the system. They can be derived from lattice calculations of hadronic correlators via the Maximum Entropy Method (MEM)[3]. However, the success of this approach relies on the quality of the input data, in particular on having estimates of \( G(t) \) over a large time range. Using isotropic lattices, this requirement runs at odds with finite temperature studies which restrict the temporal extent to be small. We overcome this conflict by using anisotropic lattices, to ensure that the number of time points, \( N_t \), is large, while maintaining a non-zero temperature, \( T = 1/(a_s N_t) \) (where \( a_s \) is the temporal lattice spacing which we take much smaller than the spatial spacing, \( a_s \)). Using this approach, we have performed dynamical studies of charmonium states at temperatures \( T_c \lesssim T \lesssim 2T_c \) and are able to confirm that they are still bound even for temperatures above \( T_c \). This work is a continuation of that presented in [4].

2. Lattice Parameters

Because our lattices used dynamical fermions, tuning the two anisotropy parameters in the gauge and quark actions is highly non-trivial. This is because there is feedback from the fermions to the gluons which is not present in the quenched anisotropic case. Details of this procedure for the choice of light quark mass and gauge coupling used in this work can be found in [5]. The ensembles studied are more highly tuned than in our previous work presented in [4] and have a renormalised anisotropy for both quarks and gluons very close to 6.

A highly improved anisotropic gauge action was used with Wilson + Hamber-Wu fermions with stout links. The details of this action can be found in [5]. Table 1 lists the parameters used in the simulations. As can be seen, three temperatures were studied: \( T/T_c \sim 1, 1.3, 2 \) and two spatial
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| Light quarks | $M_{\pi}/M_{\rho}$ | $\sim 0.5$ |
|--------------|---------------------|------------|
| Heavy quarks | $a m_c$             | 0.080, 0.092 |
| (Renormalised) Anisotropy | $\xi$ | 6 |
| Lattice spacings | $a_t$ | $\sim 0.025$ fm |
| Spatial Volume | $N^3_s$ | 8$^3$ ((&$12^3$)) |
| Temporal Extent | $N_t$ | 16 $\longleftrightarrow$ $\sim 2T_c$
| | | 24 $\longleftrightarrow$ $\sim 1.3T_c$
| | | 32 $\longleftrightarrow$ $\sim T_c$
| Statistics | $N_{cfg}$ | $\sim 500$ |

Table 1: The lattice parameters used.

volumes were simulated enabling a finite volume analysis. Two heavy quark masses were studied, with the $a m_c = 0.080$ being closer to the experimental charm quark.

The deconfining transition was found between $N_t = 34$ and 33 by studying the Polyakov line on the $N^3_s = 12^3$ spatial volume. (No discontinuity in the Polyakov line was found in the $N^3_s = 8^3$ case.)

Before embarking on the full simulation, it is worth studying the systematic effects induced by the lattice discretisation. Figure 1 shows the pseudoscalar and vector spectral functions for the free theory for both the continuum and lattice. (See [6] for details of how this calculation was performed.) In the lattice case, the anisotropy of 6 was chosen and two temporal extents, $N_t = 24$ and 32 are shown. As can be seen, the lattice functions reproduce the continuum ones for small and moderate energies, but there is an unphysical cusp at $a_t \omega \sim 0.7$ which is independent of $N_t$ and channel. This leads us to conclude that lattice spectral functions above this $a_t \omega$ value are likely to be contaminated by systematic effects. Note also that the cusp for the run parameters in [4] occurs at the smaller value of $a_t \omega \sim 0.6$, and hence the results presented here are closer to the continuum ones over a slightly larger energy range.

3. Results

Since we have correlation functions calculated for a number of different parameters, we can study how the spectral functions vary with:

- energy resolution for the MEM
- starting time for the MEM analysis
- spatial volume
- heavy quark mass
- channel (i.e. pseudoscalar, vector, axial, scalar)
- temperature
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**Figure 1:** The spectral function for the vector and pseudoscalar mesons in free field theory for the parameter values in this work. The continuum results and those from the lattice with $N_t = 24$ and 32 are shown.

**Figure 2:** LEFT: The spectral function with various energy resolutions, i.e. the number of discretised $\omega$ points, $N_\omega$, showing the insensitivity of the spectral function to this quantity. The scalar channel with $am_c = 0.092$ on the $8^3 \times 24$ lattice was used. RIGHT: The spectral function with various starting times, $t_{\text{start}}$ for the MEM analysis. This shows the insensitivity of the spectral function to $t_{\text{start}}$ within the range $2 \leq t_{\text{start}} \leq 4$. (The same channel as the left panel was used.)

The hope is that the spectral function determined from MEM will not vary with the first three items since they are unphysical, and this is what we find. The left panel in figure 2 shows the total lack of effect when the energy resolution of the MEM is varied from $n_\omega = 100$ through to 1000.

In the right panel of figure 2 the effect of varying the start time, $t_{\text{start}}$, of the MEM is studied. Note that the spectral functions for $t_{\text{start}} = 2, 3$ or 4 are all compatible, i.e. there is a range of sensible $t_{\text{start}}$ values which produce consistent results. However, for $t_{\text{start}} = 1$ and $t_{\text{start}} \geq 5$ the spectral functions do vary. This can be understood since there are contact terms which enter the $t_{\text{start}} = 1$ case, and if $t_{\text{start}} \geq 5$ is used, there is little signal left for the MEM to latch onto ($N_t = 24$ in this case).
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Finite volume effects are studied in left panel of figure 3. As can be seen, there is little variation in the spectral function as $N_s$ is changed, confirming that our results are not dominated by finite volume systematics.

Now that these lattice systematic effects are studied and have been shown not to contaminate our spectral functions, we turn to varying the physical parameters. The right panel of figure 3 shows the effect of varying the charm quark mass from $a m_c = 0.08$ to 0.092. The ground state peak is slightly higher in the latter case as expected, but otherwise the spectral function shows little dependency on $m_c$.

Finally we consider varying the temperature. Figures 4 show the spectral functions for the three temperatures studied for the four mesonic channels. ($N_s = 8$ is used throughout.) As can be seen there is clear evidence of a bound state in the pseudoscalar and vector channels which survives above $T_c$ and melts between $1.3 T_c$ and $T_c$. At $2T_c$ there is evidence of non-zero spectral weight at small energies. In the case of the axial channel, the state seems to melt by around $\sim 1.3 T_c$, and for the scalar channel the situation is less clear.

4. Conclusions

This work continues that first presented in [4] where anisotropic dynamical lattices were used together with the MEM technique to determine charmonium spectral functions at non-zero temperatures. Improvements over the work described in [4] were that the anisotropy parameters are now better tuned, a larger volume considered with more than one heavy quark mass simulated.

We have confirmed the spectral functions from MEM to check that they are stable for sensible variations in the MEM parameters (energy discretisation, starting value of the time interval, volume).

Our results confirm our earlier work [4] and previous quenched studies [7]: the pseudoscalar ($\eta_c$) and vector ($J/\psi$) charmonium states melt between $1.3 T_c$ and $2T_c$, while the scalar ($\chi_{c0}$) and axialvector ($\chi_{c1}$) states are less certain, but seem to melt by $1.3 T_c$. 
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![Graphs showing meson spectral functions at various temperatures with $am_c = 0.08$ and $N_t = 8$.]

Figure 4: The meson spectral function at various temperatures (with $am_c = 0.08$ and $N_t = 8$).

Future work will include varying the default model, inclusion of non-zero momenta and studies of light hadrons.

References

[1] I. Arsene et al. [BRAHMS Collaboration], Nucl. Phys. A 757 (2005) 1 nucl-ex/0410020; B. B. Back et al., *ibid.* 28 nucl-ex/0410022; J. Adams et al. [STAR Collaboration], *ibid.* 102 nucl-ex/0501009; K. Adcox et al. [PHENIX Collaboration], *ibid.* 184 nucl-ex/0410003.

[2] M. Gyulassy and L. McLerran, Nucl. Phys. A 750 (2005) 30 nucl-th/0405013; E. V. Shuryak, *ibid.* 64 hep-ph/0405066.

[3] M. Asakawa, T. Hatsuda and Y. Nakahara, Prog. Part. Nucl. Phys. 46 (2001) 459 hep-lat/0011040.

[4] G. Aarts, C. R. Allton, R. Morrin, A. O. Cais, M. B. Oktay, M. J. Peardon and J. I. Skullerud, PoS LAT2005 (2006) 176 hep-lat/0509115; Nucl.Phys.Proc.Suppl. 153 (2006) 296 hep-lat/0511028; hep-lat/0608009.

[5] R. Morrin, A. O. Cais, M. Peardon, S.M. Ryan, J.-I. Skullerud Phys.Rev. D74 (2006) 014505 hep-lat/0604021.

[6] G. Aarts and J. M. Martinez Resco, Nucl. Phys. B 726 (2005) 93 hep-lat/0507004.
Charmonium spectral functions in $N_f = 2$ QCD at high temperature

[7] M. Asakawa and T. Hatsuda, Phys. Rev. Lett. 92, 012001 (2004) hep-lat/0308034; S. Datta, F. Karsch, P. Petreczky and I. Wetzorke, Phys. Rev. D 69, 094507 (2004) hep-lat/0312037; T. Umeda, K. Nomura and H. Matsufuru, Eur. Phys. J. C 39 S1, 9 (2005) hep-lat/0211003.