Research on the shear of reinforced concrete T beams based on the MCFT calculation method

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Abstract. The MCFT on shear theoretical calculation method in reinforced concrete beams is expressed by the mathematical relationship based on the theory of MCFT, and the theoretical calculation formula is simplified. Finally the results with shear theoretical calculation method of simplified MCFT can be well fit with the test datas. That shows the simplified theoretical calculation method of shear is reasonable, thus another simple method for shear calculation of the reinforced concrete T beams is provided.

1. MCFT
The main clue on the development of the theory to calculate the shearing of reinforced concrete members is expanded along the truss model. In 1899 and 1902, the papers were independently published by the Swiss engineer Ritter and German engineer Mörsch, in which the shear failure truss models of reinforced concrete were put forward[1], in the classical truss model, the angle of inclined cracks was assumed to be 45 degrees, but in most cases the inclined crack angle was less than 45 degrees, so the model was conservative in the shear bearing capacity calculation. Based on the early days of truss model, the variable angle truss model used for calculating the shear and torsion bearing capacity of reinforced concrete and prestressed concrete beams was put forward by the European and American researchers, then developed into a plastic truss model.

Since 1980s, the MCFT that the tensile stress beared by the cracking concrete was put forward by professor Vecchio and professor Collins at the University of Toronto, a complete description of mechanical behavior on reinforced concrete elements in conditions of the plane stress was given, and the mechanical performance of reinforced concrete elements was revealed with a new experimental method[2]. But in accordance with the MCFT, considering the tensile stress of concrete, even without transverse reinforcement that has also a certain shear capacity in reinforced concrete members, and considering cracking tensile stress of concrete that can improve the stiffness of components and reduce the strain of concrete, larger shear can be beared before the concrete destruction. The axial tensile strength design value \( f_t \) of concrete is considered in the shear calculation formula of concrete on inclined section in the” Code for Design of Concrete Structures”[3], that is consistent with MCFT theory, but in the” Code for Design of Highway Reinforced Concrete and Prestressed Concrete Bridges and Culverts structures” the compressive strength standard value \( f_{cu,k} \) of concrete is used in calculation[4], that seemed to be conflict with the MCFT, but the contribution of the shear strength in compressive area concrete after crack propagation is considered in T beams. The major contribution of the MCFT in constitutive model is introduced to stress-strain curve of concrete under the pull-compressive stress state, and the basic concept of compression softening of concrete is also put
forward. The MCFT is based on solid mechanics including the equilibrium condition, deformation coordination conditions and stress-strain relationship of concrete and steel\[^5\]. Wang Jingquan studied the shear unified calculation method of reinforced concrete beams with web reinforcement and without stirrups\[^6\], Deng Zongcai studied the shear of the ultra high performance steel fiber reinforced concrete column based on the MCFT\[^7\], Zhang Facheng studied the shear of the steel fiber self-compacting concrete beams\[^8\], Zhu Weiqing studied the shear of the steel ultra high strength concrete columns\[^9\], Wei Weiwei studied the shear calculation of the reinforced concrete flexural member with web reinforcement based on the MCFT\[^10\]. The calculation model of the reinforced concrete beams is set up and simplified based on the MCFT, finally the rationality of model is verified through comparing the experimental data with the theoretical calculation results.

2. The shear performance analysis method of T beams

2.1. The shear performance analysis method under the pure shear state

When only beared shear stress, for reinforced concrete beams the shear bearing capacity of the cross section can be obtained with the iterative approach through basic equations. The crack width \(w\) and the average principal tensile stress \(f_1\) are calculated with the tensile strain \(\varepsilon_1\) of the given cross section through the assumptive principal compressive strain angle \(\theta\) and the average stress of stirrup \(f_{sz}\). In normal situation, the vertical compressive stress \(f_z=0\) is considered, and the shear stress of cross section is solved according to the Eq.(1), then when the pure shear state the shear stress of cross section uniformly is distributed along the cross section, this moment the shear \(V\) solved by the Eq.(2). Then the principal compressive stress of oblique compression bars is solved, and the compressive strain is determined whether meet the requirements, if not the angle \(\theta\) will be assumed again. The actual stress is solved based on the constitutive relationship of steel, then the actual stress of reinforcement and the hypothesis stress is judged whether they are consistent, if not the iterative method will be used, the calculated stress of stirrups will be recurrently calculated as assumed the initial value, until to be meet the requirements. \(N\) calculated by the Eq. (3) is judged whether it is equal to the initial value, if they are equal the next main tensile strain \(\varepsilon_1\) is analysed, if not the angle \(\theta\) is returned to adjust until to be meet the requirements. The shear strain calculated by the Eq.(4) is used to get the relationship curve for shear stress and shear strain under the pure shear state, in turn the limiting shear stress of the cross section is calculated\[^11\].

\[
\begin{align*}
 f_z &= f_1 + p_{sz}f_{sz} - vtan\theta \\
 v &= \frac{p}{bwv} \\
 N &= Vcot\theta + f_1bwv + A_{sx}f_{sx} \\
 \gamma &= \frac{2(\varepsilon_2 - \varepsilon_z)}{tan\theta}
\end{align*}
\]

2.2. The shear performance analysis method under the combined curved scissors

Normally the shear zone of beams is all under the action of bending moment and shear force, the pull of the longitudinal tensile reinforcement and the pressure of the compression concrete will be increased by the bending moment, then the shear bearing capacity be impacted. The stress of concrete and steel are used to balance the external force roleed on the concrete members, the average stress is determined by the average strain in constitutive relation. For compression concrete, the constitutive relation is\[^12\]:

\[
\begin{align*}
 f_{cz} &= f_{cz\max} \cdot \left[2\left(\frac{\varepsilon_2}{\varepsilon_c}\right) - \left(\frac{\varepsilon_2}{\varepsilon_c}\right)^2\right] \\
 f_{cz\max} &= \frac{f_{c2max}}{0.8-0.34\varepsilon_i/e_c} \leq 1.0
\end{align*}
\]

The stress and strain states of the double cross section stratification analysis method and the single section stratification analysis method and the simplified analysis method are separately described in Fig.1, from which the longitudinal strain of the cross section only assumed in the double cross section.
method is meet to the flat section assumption and in accordance with the actual stress state, while the distribution regular of the shear stress along the section height is assumed again in the single section method, in the simplified analysis method the main compressive stress angle of fiber in the shear stress distribution area is increased again that is consistent with assumption [13].

The double cross section stratification analysis method and the single section stratification analysis method and the simplified analysis method are proved by literature [13] that can be better predict the shear capacity of the test beams. The assumption of shear stress distributed uniformly and the main compressive stress angle fixed along the section height is feasible to analyze the shear capacity of the reinforced concrete beams.

3. Simplified model of the MCFT

The analysis method based on the MCFT can be better analyzed to the shear bearing capacity, but it is more complex to analyze the shear capacity of the reinforced concrete T beams with the MCFT, so the theoretical model of the MCFT should be simplified and the simplified calculation formula should be given.
Fig. 2. Shear model based on the MCFT

The bending moment of the critical shear plane is undertaken by the internal force couple composed of the compression concrete force \( C \) with the steel tensile \( T \), the shear \( V \) is assumed to undertake commonly by the stirrup vertical force \( V_s \) crossed the diagonal crack and the vertical component of the shear stress \( v_a \) passed along the fracture plane. The shear calculation formula of the reinforced concrete beams based on the MCFT is as follows:

\[
V = f_1 b_w d_v \cot \theta + \rho_s f_s b_w d_v \cot \theta
\]  

(7)

In the formula, \( s \) is the stirrup spacing, the first item \( f_1 b_w d_v \cot \theta \) is the contribution of the concrete, the second item \( \rho_s f_s b_w d_v \cot \theta \) is the contribution of the stirrup. The shear \( v \) undertaken by the beams can be expressed as:

\[
v = \frac{V}{b_w d_v} = f_1 \cot \theta + \rho_s f_s \cot \theta = \beta f'_c + \rho_s f_s \cot \theta
\]  

(8)

Among them: \( \beta \) simplified as the the factors of the strain and geometry is the parameter related to the shear stress \( v_a \) passed fracture plane, the Eq. (9) is the recommended formula on \( \beta, \theta \) given as the main compressive stress angle is simplified to calculate as shown in Eq. (10)\(^{[11]}\):

\[
\beta = \frac{0.4}{1 + 1500 \varepsilon} \cdot \frac{1300}{1000 + S_{xe}}
\]  

(9)

\[
\theta = (29 + 7000 \varepsilon)(0.88 + \frac{S_{xe}}{2500}) \leq 75
\]  

(10)

Where \( \beta f'_c \) is the shear of concrete, \( \rho_s f_s \cot \theta \) is the shear of stirrup, \( b_w d_v \) is the effective shear area, \( f'_c \) is the compressive strength design values of concrete, \( \rho_s \) is the ratio of stirrups, \( f_s \) is the stress of stirrups, \( f_s \) is the yield stress \( f_{sY} \) in the limit state, \( \varepsilon \) is the longitudinal strain of shear unit, \( S_{xe} \) is the fracture spacing.

When the simplified analysis method is used to solve the shear capacity of T beams, \( \beta \) and \( \theta \) will be solved, but that need to be known the longitudinal strain of the cross section. Collins etc. advice the longitudinal strain of the shear unit of the web with abdominal muscle to be half of the strain \( \varepsilon_x \) of the longitudinal tensile reinforcement under the joint action of the bending, shear and axial force\(^{[14]}\).
The calculation mode of the longitudinal reinforcement strain $\varepsilon_x$ produced by the bending, shear and axial force is shown in Fig. 3. The calculation formula of the the longitudinal reinforcement $\varepsilon_x$ is following:

$$\varepsilon_x = \varepsilon_{x,M} + \varepsilon_{x,V} + \varepsilon_{x,N} = \frac{M_u/d_v + 0.5(V \cot \theta + N)}{E_s A_s}$$  \hspace{1cm} (11)

where $M, V, N$ is the bending, shear and axial force of cross section; $\theta$ is the main compressive stress angle; $d_v$ is the arm length of bending moment, it’s value is the distance between longitudinal reinforcement; $E_s$ is the elastic modulus of longitudinal reinforcement; $A_s$ is the area of longitudinal reinforcement.

So the calculation formula of the the longitudinal strain $\varepsilon_x$ of the shear unit of the web with abdominal muscle is shown:

$$\varepsilon = \frac{\varepsilon_x}{2} = \frac{M_u/d_v + 0.5(V \cot \theta + N)}{E_s A_s}$$  \hspace{1cm} (12)

4. Experiment and calculation analysis

Three prestressed concrete T beams are designed, the geometry and reinforcement of test beams are as shown in Fig. 4, the beams length is 6.2 meters, the calculation span is 6 meters, the longitudinal reinforcement is used with 3ф12, the longitudinal reinforcement ratio $\rho_x$ is 0.539, the stirrup is used with Ф6@200 (the pure bending section is 200, the rest is encrypted), the thickness of the concrete cover is 15mm, the concrete grade is 50MPa. The values of deflection and strain in the cross section of 1/4 span and midspan and 3/4 span are measured under the loads of each level when multi-stage loading. The development and change of cracks are observed after beam cracking, the parameters of crack width under different load are shown in Table 1[15].

![Fig 4. Cross section and reinforcement of T beam](image-url)
By the known conditions, the transverse reinforcement $\rho_s f_s = 0.587$ MPa, the strain of longitudinal reinforcement initially is assumed to correspond to the value of maximum strain, namely $\varepsilon_x = 1.0 \times 10^{-3}$. From Tab.1 the parameter of crack width $S_{xe} = 0.28$mm is selected when the transverse reinforcement yielded, and plugged in Eq. (9), Eq. (10), then $\beta = 0.297$, $\theta = 28.6^\circ$ are obtained, the shear strength of beams is:

$$v = v_c + v_s = 1.43 + 1.22 = 2.65 \text{MPa}$$

Before the longitudinal reinforcement yielded, from $f_x = \rho_x f_{sx} + f_1 - v_c \cot \theta$ follows:

$$f_{sx} = \frac{f_x - f_1 + v_c \cot \theta}{\rho_x}$$

While $f_{sx} = E_s \varepsilon_x$,

$$\varepsilon_x = \frac{f_{sx}}{E_s} = \frac{f_x - f_1 + v_c \cot \theta}{E_s \rho_x}$$

Where the axis pressure $f_x = 0$, $f_1 = v_c / \cot \theta$, so:

$$\varepsilon_x = \frac{f_{sx}}{E_s} = \frac{v_c \cot \theta - v_c / \cot \theta}{E_s \rho_x}$$

In this case,

$$\varepsilon_x = \frac{2.65 \times \cot 28.6^\circ - 1.43 / \cot 28.6^\circ}{200000 \times 0.539} = 1.6 \times 10^{-5}$$

$\varepsilon_x = 1.0 \times 10^{-3} \neq 1.6 \times 10^{-5}$, $\varepsilon_x$ must be assumed and calculated again, after four times iteration loop computations, finally converged to a value $\varepsilon_x = 7.1 \times 10^{-5}$, the total shear stress for the damage is:

$$v = \frac{v}{b_w d_y} = f_1 \cot \theta + \rho_s f_s \cot \theta = \beta \sqrt{f_c} + \rho_s f_s \cot \theta = 3.74 \text{MPa}$$

The shear capacity of T beams solved by the simplified analysis method of MCFT is in good agreement with the measured value of shear bearing capacity. From Fig.5, after increased the number of stirrups, the shear stiffness and the shear capacity are increased too, but the strain of longitudinal reinforcement is no longer grew with the increase of load, the ductility of the member is reduced, that is verified the rigidity of the component can be improved and the strain of concrete is reduced by the tensile stress of cracking concrete in the MCFT, the concrete can be undertaken larger shear before destruction.
5. Conclusion
In this paper the shear calculation method of the MCFT is simplified, the complex degree of computational work is reduced, the results solved by the simplified analysis method can be in good agreement with the test results, that shows it is feasible to calculate the shear of reinforced concrete T beams. At the same time it is also proved that the rigidity of the component can be improved and the strain of concrete is reduced when the load is increased and the crack is expanded, the concrete can be undertaken larger shear before destruction. Because the test results of three only beams are compared, number of specimens is less, validation test of datas is not fully, and in test analysis the influence of the prestressed degree and the flange width of T beams on shear are not considered in the test analysis, therefore it is worth studying furtherly.

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