Estimating the Ages of Open Star Clusters from Properties of Their Extended Tidal Tails

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Abstract

The most accurate current methods for determining the ages of open star clusters, stellar associations, and stellar streams are based on isochrone fitting or the lithium depletion boundary. We propose another method for dating these objects based on the morphology of their extended tidal tails, which have been recently discovered around several open star clusters. Assuming that the tidal tails originate from the stars released from the cluster during early gas expulsion, or that they form in the same star-forming region as the cluster (i.e., being coeval with the cluster), we derive the analytical formula for the tilt angle $\beta$ between the long axis of the tidal tail and the orbital direction for clusters or streams on circular trajectories. Since at a given Galactocentric radius $\beta$ is only a function of age $t$ regardless of the initial properties of the cluster, we estimate the cluster age by inverting the analytical formula $\beta = \beta(t)$. We illustrate the method on a sample of 12 objects, which we compiled from the literature, and we find a reasonable agreement with previous dating methods in $\approx70\%$ of the cases. This can probably be improved by taking into account the eccentricity of the orbits and by revisiting the dating methods based on stellar evolution. The proposed morphological method is suitable for relatively young clusters (age $\lesssim300$ Myr), where it provides a relative age error of the order of $10\% - 20\%$ for an error in the observed tilt angle of $5^\circ$.

Unified Astronomy Thesaurus concepts: Stellar ages (1581); Open star clusters (1160); Stellar associations (1582); Stellar kinematics (1608)

1. Introduction

Together with stellar mass and chemical composition, stellar age is one of the most important properties characterizing a star, and methods of its determination are of great interest. The knowledge of the stellar age is important, for example, for testing theories of stellar evolution, for reconstructing the star formation history of the Galaxy and the relationship between enrichment and kinematics in the Galaxy, and for estimating the lifetimes of some circumstellar structures (e.g., debris disks).

Two most accurate methods for determining the ages of open star clusters are isochrone fitting and the lithium depletion boundary. However, each method suffers from limitations. Isochrone fitting is sensitive to not fully understood processes in stellar evolution such as convective overshooting, rotational mixing, internal gravity waves, and diffusion (Meynet et al. 2009; Soderblom 2010). Moreover, stellar rotation prolongs stellar lifetimes and increases the luminosity for a given stellar mass (Meynet & Maeder 2000). Blue stragglers, if present, add another bias (Brandt & Huang 2015), as they appear as stars of a younger age. These uncertainties in stellar evolution introduce an error in the age determination of the order of tens of a percent (Meynet et al. 2009). Systematic errors (e.g., omission of the most luminous stars) can easily lead to much larger errors. For example, Meingast et al. (2019) overestimate the age of the Pisces—Eridanus stream by a factor of 8 as later corrected by Curtis et al. (2019).

The other dating method, based on the lithium depletion boundary, is considered to be more accurate (but see Jeffries et al. 2017; Bouvier et al. 2018, for complications due to rotation and uncertainties in stellar radii). This method can be applied only for the most nearby clusters because of the very low luminosity of the relevant objects. Up to date, only a dozen clusters have ages determined by this method. In general, the lithium depletion boundary method provides older ages (by $\approx50\%$) than isochrone fitting (e.g., Soderblom 2010; Binks et al. 2021). The difference between these two dating methods is not restricted to the youngest clusters. For example, Stauffer et al. (1998) estimate the age of the Pleiades based on the lithium depletion boundary to be $\approx125$ Myr, while isochrone fitting provides the age from 70 to 100 Myr.

Another reason for dating clusters is that they can be used as anchors for other (and generally less accurate) dating methods, for example, gyrochronology, chromospheric emission, lithium abundance, and coronal X-ray emission (e.g., Skumanich 1972; Soderblom et al. 1991; Lachaume et al. 1999; Barnes 2007; Mamajek & Hillenbrand 2008), which are instrumental in estimating the ages of field stars. Unlike isochrone fitting and the lithium depletion boundary, these methods lack a theoretical background, and they need to be calibrated empirically on coeval stellar systems such as star clusters or associations.

In addition to the dating methods described above, which are based on stellar evolution, there are also attempts to provide age estimates from stellar kinematics or cluster dynamics. Kinematical methods usually backtrace a population of stars until they assume the smallest volume, which is taken as the time of formation, or they estimate the onset of expansion from the current size and expansion velocity. Kinematical estimates are usually less accurate than those using stellar evolution, and they can be used only for the youngest objects (age $\lesssim20$ Myr).
because of gravitational interactions within the cluster and the external field of the Galaxy, which bends the stellar trajectories (Blauw 1991; Brown et al. 1997). More recently, Crundall et al. (2019) suggest a more sophisticated tool taking into account the external Galactic field and the complicated morphology of star-forming regions, extending the usability of the kinematic method up to \(\approx 100\) Myr for objects with a sufficiently low velocity dispersion. Dynamical methods compare numerical models of star clusters at a different degree of dynamical evolution with observations. They compare the radial mass distribution (Buchholz & Schmidt-Kaler 1980), or the ratio of the number of stars in two different mass bins (Kroupa 1995). Dynamical methods are rarely used.

In this work, we present another method, which utilizes stellar kinematics but in a very different way than the previous methods. Instead of tracing individual stars backward or forward in time, we investigate the shape of the expanded tidal structure, which is formed from an expanding group of stars. The expanding stellar structure is supposed to be formed of stars that escape from a star cluster as the result of gas expulsion (e.g., Lada et al. 1984; Geyer & Burkert 2001; Kroupa et al. 2001), which terminates its embedded phase. The remnant of the star cluster that survives this event is accompanied by these stars. The existence of the extended stellar structures was predicted theoretically from the results of N-body simulations (Kroupa et al. 2001) and further explored by Dinnbier & Kroupa (2020a), before similar structures surrounding star clusters were observed (Meingast et al. 2021) thanks to the unprecedented sensitivity of the Gaia mission (Gaia Collaboration et al. 2016, 2018). At least some of these structures are coeval with the star cluster (Bouma et al. 2021), which supports the view of their common origin. In an alternative scenario where the extended structures were never bound to the clusters, but probably formed close to them and are coeval with them, this dating method can be applied as well.

We introduce the new method for age determination in Section 2, and then we apply it to the open star clusters that are surrounded by known extended structures in Section 3. In addition, we analyze two gravitationally unbound stellar streams. The method is discussed in Section 4. We conclude in Section 5.

### 2. The Time Evolution of the Tilt and the Size of the Tidal Tail

We assume that star clusters form with a relatively low star formation efficiency (SFE \(\approx\) stellar mass/stellar and gaseous mass within the star-forming volume; Lada & Lada 2003; Megeath et al. 2016; Banerjee & Kroupa 2018) and that they expel the non-star-forming gas on a timescale that is short in comparison to the cluster crossing time. These conditions unbind a substantial fraction of stars from the cluster (typically more than 60%; Lada et al. 1984; Goodwin 1997; Kroupa et al. 2001; Baumgardt & Kroupa 2007), which expand and form an extended tidal structure surrounding the cluster (Dinnbier & Kroupa 2020a, 2020b, hereafter Papers I and II, respectively). We refer to this tidal structure as *tidal tail I*. The post-gas expulsion cluster revirializes (Banerjee & Kroupa 2013) and loses stars gradually owing to encounters between stars, producing the classical S-shaped tidal tail (Chunnak & Rastorguev 2006; Küpper et al. 2008, 2010), which is referred to as *tidal tail II*. On the timescale of several hundreds of Myr, tail I contains substantially more stars and is more extended than tail II (Dinnbier & Kroupa 2020b). The tidal tails of type I and II have been recently explored using observational data (e.g., Pang et al. 2021).

The method takes advantage of the following kinematic property of stars. Stars that escape from the cluster through gas expulsion have the Galactic orbital velocity either slightly larger or smaller than the orbital velocity of the cluster. The stars with larger orbital velocity have the guiding centers of their orbits outside the orbit of the cluster (which is at Galactocentric radius \(R_0\)), while the stars with smaller orbital velocity have the guiding centers inside the orbit of the cluster. The former stars trail behind the cluster, while the latter overtake it. Consequently, the volume occupied by the escaping stars gradually stretches from a sphere to an elongated spheroid, and the direction \(\beta\) (see its meaning in Figure 1) of the long axis of the spheroid gets more aligned with the direction of the cluster orbit with time. Since all the stars escaped almost at the same time and they follow epicyclic motions with the same epicyclic frequency \(\kappa\) to a good approximation, they reach their birth Galactocentric radius \(R_0\) at the same time at \(2\pi n/\kappa\), where \(n\) is a positive integer, but the stars are stretched along the azimuthal direction. This means that the tidal tail is aligned with the direction of motion each time \(2\pi n/\kappa\), and it tilts relative to this direction in between these time events. Thus, the age of the tail can be estimated by inverting the theoretical time dependence of the tail tilt \(\beta = \beta(t)\).

The width of the stellar structure depends on the characteristic speed of escaping stars, which is a function of the embedded cluster mass and the phase of the tail oscillation (for example, the thickness of the idealized tail drops to zero at \(2\pi n/\kappa\)). The width of the tail at the known age can constrain the initial mass of the cluster. In this section, we derive the time dependence of two quantities: the tilt (Section 3.1) and width (Section 3.2) of the tidal tail.

The present semianalytic study is an extension of the analysis of Paper I; it uses the same assumptions, and it applies only to tail I. The first assumption is that star clusters release many stars during a time window whose duration is short in comparison to the epicyclic timescale \(2\pi/\kappa\) (which is \(\approx 168\) Myr at the solar circle; Allen & Santillán 1991). The physical process that is responsible for unbinding the stars is assumed to be gas expulsion of the residual gas from the newly formed open star clusters; however, the solution is general, and it applies to any isotropically expanding stellar system in an external tidal field as long as the initial size of the stellar system is significantly smaller than the Galactocentric distance. The more general examples include a shock caused by an encounter between a star cluster (not necessarily young) and a molecular cloud, or a dissolution of a population of sparse clusters as their natal clouds are disrupted, forming a gravitationally unbound stellar stream.

The second assumption is that the speeds of escaping stars \(v_{e,1}\) are approximately equal to or larger than the speed \(v_{e,2}\) corresponding to the difference between the maximum and minimum of the gravitational potential around the tidal radius \(r_t\). This condition means that stars of typical velocity \(v_{e,1}\) can overcome the Jacobi potential at any direction without significant change to \(v_{e,1}\) and thus escape the cluster with comparable probability in any direction. As shown in Paper I (Sections 2.1 and 4.7 there), this condition is fulfilled for practically all embedded star clusters currently forming in the Galaxy.
We adopt the usual coordinate system, where the x-axis points in the direction of the Galactic anticenter, and the y-axis points in the direction of the Galactic rotation. The star cluster is on a circular orbit, so it is at rest at the origin of the coordinate axes. In this idealized model, we assume that all stars escape the cluster with the same velocity $v_{e,1}$. The components of the velocity vector $v_x$, $v_y$, where $v_{e,1}^2 = v_x^2 + v_y^2$, are parallel to the coordinates $(x, y)$, which lie in the plane of the Galaxy. The vertical motion is neglected.

The star escapes the cluster at an angle $\alpha$, whose meaning is shown in Figure 1 of Paper I, and then moves along an epicycle of a guiding center at Galactocentric radius $R_g$ and of semimajor axis $Y$ and semiminor axis $X$. The guiding center itself moves in the azimuthal direction at velocity $v_y$ relative to the cluster. The relative position of the star to the cluster at time $t$ (since its escape) is given by (Equation (12) in Paper I)

$$
x(\alpha, t) = X(\alpha)\cos(\alpha)\sin(\kappa t) - X(\alpha)\sin(\alpha)(1 - \cos(\kappa t)),$$
$$y(\alpha, t) = \gamma X(\alpha)\cos(\alpha)(\cos(\kappa t) - 1) - \gamma X(\alpha)\sin(\alpha)\left\{\sin(\kappa t) + \frac{t}{\gamma}\left(\frac{\kappa^2}{2\omega} - 2\omega\right)\right\},$$

where $\omega$ and $\kappa$ are the orbital and epicyclic frequency, respectively, and $\gamma = 2\omega/\kappa$ (Binney & Tremaine 2008).

The value of the epicyclic semiminor axis $X$ depends not only on the velocity $v_{e,1}$ but also on the direction of escape $\alpha$. From Equations (9) and (10) of Paper I, it follows that

$$X(\alpha) = \frac{i}{\kappa}\frac{k_{e,1}}{\sqrt{\gamma^2 \cos^2(\alpha) + \sin^2(\alpha)}}.$$

### 2.1. The Tilt of the Tidal Tail

At time $t$, stars that escape at velocity $v_{e,1}$ are located at the curve given by Equation (1) with $X$ substituted from Equation (2). Figure 1 shows the shape of tail I formed by stars escaping at $v_{e,1} = 2$ km s$^{-1}$ at seven time events for the frequencies $\omega = 8.381 \times 10^{-16}$ s$^{-1}$ and $\kappa = 1.185 \times 10^{-15}$ s$^{-1}$, which are expected at the solar circle (at Galactocentric radius $R_0 = 8.5$ kpc) for the Galaxy model by Allen & Santillan (1991).

At the age of 40 Myr, the tail points almost toward the Galactic center (red contour in Figure 1), and with progressing time the tail aligns with the direction of the orbit around the Galaxy (at 160 Myr; blue contour). Later, the tail is again tilted with respect to its orbit (240 Myr; yellow contour), and these oscillations continue with time.

At a given time $t$, the tail is parameterized by the angle $\alpha$. Thus, a quantitative description of the tail tilt can be obtained by finding the value of $\alpha$ for which the distance from the cluster $r = \sqrt{x^2 + y^2}$ is the largest. The value of $\alpha_{\text{ext}}$, which extremalizes the distance on the tail, can be found by setting $\partial r/\partial \alpha = 0$, which results in

$$\frac{(x^2 + y^2)^2}{(\gamma^2 - 1)}(\sin(\alpha_{\text{ext}})\cos(\alpha_{\text{ext}})) - \frac{x}{\gamma^2}(\sin(\alpha_{\text{ext}})\sin(\kappa t) + \cos(\alpha_{\text{ext}})(1 - \cos(\kappa t)))$$
$$+ \frac{y}{\gamma}(\sin(\alpha_{\text{ext}})(1 - \cos(\kappa t)))$$
$$- \cos(\alpha_{\text{ext}})\left\{\sin(\kappa t) + \frac{t}{\gamma}\left(\frac{\kappa^2}{2\omega} - 2\omega\right)\right\} = 0,$$

where the dimensionless quantities $\bar{x}$ and $\bar{y}$ are defined as $\bar{x} = x/X$ and $\bar{y} = y/X$, respectively.

In the interval $\alpha \in (-\pi/2, \pi/2)$, Equation (3) has typically two solutions: one corresponding to the maximum distance $r$ (at angle $\alpha_{\text{ext}}^+$), and the other corresponding to the minimum distance $r$ (at angle $\alpha_{\text{ext}}^-$). The tilt of the tail is the angle $\beta$ at which the tip of the tail is seen from the cluster relative to the positive y-axis (Figure 1), i.e.,

$$\tan(\beta) = -\frac{x(\alpha_{\text{ext}}^+(t), t)}{y(\alpha_{\text{ext}}^+(t), t)}.$$

At a given $t$, the angles $\alpha_{\text{ext}}^+$ and $\alpha_{\text{ext}}^-$ depend only on the Galactic frequencies $\omega$ and $\kappa$, and they are independent of any property of the cluster, such as $v_{e,1}$. Thus, the tidal structure occupied by stars escaping the cluster at different velocity $v_{e,1}$ forms concentric curves with respect to the origin; a group of stars escaping at velocity $k \times$ larger reaches a $k \times$ larger distance from the cluster in any direction in the x-y plane (Equation (2)), but the tidal structure is concentric with the same tilt for a
group of stars of any $v_{e,I}$. The time evolution of the angle $\beta$ is shown in Figure 2.

To check our calculations, we compared the value of $\beta$ obtained from Equations (3) and (4) with NBODY6 models of initial stellar mass $M_{\text{ecl}} = 1400 M_\odot$ and $M_{\text{ecl}} = 4400 M_\odot$ (dots). The time dependence of the theoretical tail tilt angle $\beta$ (according to Equations (3) and (4)). The value of $\beta$ at a given age is independent of the velocity of escaping stars $v_{e,I}$ or any other cluster property. Inset: comparison of the theoretical solution (black line) with $N$-body models of initial stellar mass $M_{\text{ecl}} = 1400 M_\odot$ and $M_{\text{ecl}} = 4400 M_\odot$. (dots).

Figure 2.

Although the dating method uses stellar kinematics, because it is based on Equation (1), the only important observational quantity is the tail tilt angle $\beta$. This might make the method simpler to use than standard kinematic methods, because it does not require stellar velocities for the age determination, velocities being used only to identify the tidal tail. For this reason, we will refer to this method in the following as the morphological method (and corresponding ages as morphological ages). The standard methods of cluster age determination, which are based on stellar evolution (e.g., isochrone fitting, lithium depletion boundary, and gyrochronology), are referred to as stellar evolutionary methods (and corresponding ages as stellar evolutionary ages). Another advantage of the present method is its insensitivity to completeness because $\beta$ can be estimated only from a fraction of the stars in the tail.

2.2. The Width and Length of the Tidal Tail

The width $b$ of the tail is attained at the solution to Equation (3) that corresponds to the minimum distance, i.e., at

$$b(t) = 2 \sqrt{x^2(a_{\text{ext}}^{-}(t), t) + y^2(a_{\text{ext}}^{-}(t), t)}.$$  

(5)

At a given age, $b(t)$ is a linear function of $\bar{v}_{e,I}$ (Equation (2)). The time dependence of $b$ for $\bar{v}_{e,I} = 1$, $2$, and $5$ km s$^{-1}$ is shown in the top panel of Figure 3. We note that although the shape of tail I resembles an ellipse, the curve is not an exact ellipse, because Equation (1) does not represent a parametric equation of an ellipse.

Likewise, the tail reaches its longest distance $a$ from the origin for stars escaping at the angle $\alpha_{\text{ext}}$.

$$a(t) = 2 \sqrt{x^2(\alpha_{\text{ext}}^{+}(t), t) + y^2(\alpha_{\text{ext}}^{+}(t), t)}.$$  

(6)
The tail length obtained by this way for the three values of $\theta_{\phi}$ is shown in the middle panel of Figure 3. The bottom panel of the figure plots the time evolution of the aspect ratio of the tail, $b(t)/a(t)$, which shows that the tail gets more elongated with time.

3. Application to the Star Clusters with Known Extended Tails and Unbound Stellar Streams

3.1. Age Determination

We illustrate the proposed age determination method on data that we compiled from the existing literature. The examples below serve only as a consistency check of the method, and they are not meant for improvement of the current age estimates of these clusters and streams because the angle $\beta$ was estimated only by eye from $x$-$y$ maps of the tidal structures, and we assume that the clusters or streams orbit the Galaxy on exactly circular trajectories (see Section 4.1 for more details).

We took as the basis for our cluster sample the $x$-$y$ maps from Meingast et al. (2021, their Figure A.2). In addition to these structures, which still surround gravitationally bound clusters, we include two tidal streams in our sample: the Psc−Eri and $\mu$ Tau streams. For Messier 39 and NGC 2516, we give the morphological age estimates during the second tail oscillation, as these are in a better agreement with the stellar evolutionary ages. The ratio between the mean and standard deviation as agreement with the stellar evolutionary ages. The stellar evolutionary age and mass estimates are adopted from Meingast et al. (2021) for the open clusters (above the horizontal line) and from Curtis et al. (2019), Ratzenböck et al. (2020), and Gagné et al. (2020) for the Psc−Eri and $\mu$ Tau streams.

We exclude the Pleiades from further analysis because there has been no prominent tidal tail found around the Pleiades so far, and the putative tail found in the data of Meingast et al. (2021) points in the direction toward the Galactic anticenter (i.e., $\beta < 0$), for which no age solution exists. From the 11 objects left, six (Platais 9, NGC 2516, NGC 2451A, Blanco 1, $\mu$ Tau, and $\alpha$ Per) have the morphological age range either in complete agreement with or differing at maximum by 10% from the most probable stellar evolutionary age. A notable example is NGC 2516, which agrees with its stellar evolutionary age during its second tail oscillation (not the first oscillation), which might constrain its age to the interval from $\approx 200$ to 260 Myr.

Three other clusters (NGC 2547, Messier 39, and IC 2602) differ more from their stellar evolutionary ages, but the difference is not huge (Figure 4); in the case of NGC 2547, $t_{\text{ev}}$ differs from the lower morphological estimate $t_{\text{mph,min}}$ by only 7 Myr (see Table 1 for details); for Messier 39, $t_{\text{ev}} = 310$ Myr differs by a factor of 1.3 from $t_{\text{mph,max}} = 245$ Myr; and the morphological age estimate of IC 2602 lies within the interval allowed by the stellar evolutionary age.

Table 1 lists the measured tilt angle $\beta$ and the estimated age range $(t_{\text{mph,min}}, t_{\text{mph,max}})$ obtained by inverting the function $\beta = \beta(t)$ from Equation (4) as illustrated in the top panel of Figure 4. We estimate that the angle $\beta$ is measured with an uncertainty of $\Delta \beta = \pm 5^\circ$ for each object, and the uncertainty is propagated to the age uncertainty. The red and blue points represent, respectively, the age estimate for the first (up to 168 Myr) and second (between 168 and 336 Myr) oscillation of the tail, and they illustrate that the age determination is degenerated for objects with $\beta - \Delta \beta < 18^\circ$ (Platais 9, NGC 2516, Messier 39, IC 2391, and Blanco 1). Because of the degeneracy, it is useful to have a priori age knowledge for objects with $\beta - \Delta \beta < 18^\circ$ so that the age can be searched for during either the first or second oscillation. The red and blue bars in the bottom panel of Figure 4 represent the estimated morphological age interval $(t_{\text{mph,min}}, t_{\text{mph,max}})$ during the first and second tail oscillation, respectively. The range of age determination from various stellar evolutionary signposts is indicated by the gray bars, and the most probable age (according to Meingast et al. 2021) is indicated by the black bars.

Table 1: Morphological Age and Mass Estimates for 10 Open Star Clusters and Two Tidal Streams (Psc−Eri Stream and $\mu$ Tau)

| Object Name | $t_{\text{ev,min}}$ (Myr) | $t_{\text{ev,max}}$ (Myr) | $t_{\text{ev}}$ (Myr) | $M_{\text{cl,obs}}$ ($M_\odot$) | $\beta$ (deg) | $t_{\text{mph,min}}$ (Myr) | $t_{\text{mph,max}}$ (Myr) | $M_{\text{cl,mph}}$ ($M_\odot$) |
|-------------|---------------------------|---------------------------|----------------------|-----------------------------|--------------|---------------------------|---------------------------|-----------------------------|
| Platais 9   | 78                        | 347                       | 100                  | 285                         | 15           | 103                       | 126                       | 20                          |
| Messier 39  | 279                       | 1023                      | 310                  | 325                         | 22           | 215                       | 245                       | 20                          |
| $\alpha$ Per| 35                        | 110                       | 87                   | 1030                        | 31           | 75                        | 92                        | 60                          |
| NGC 2451A  | 32                        | 148                       | 44                   | 425                         | 58           | 36                        | 50                        | 20                          |
| IC 2602    | 30                        | 100                       | 35                   | 400                         | 51           | 46                        | 60                        | 20                          |
| NGC 2547   | 27                        | 78                        | 27                   | 590                         | 80           | 46                        | 72                        | 870                         |
| Blanco 1   | 63                        | 209                       | 94                   | 365                         | 22           | 91                        | 110                       | 5                           |
| IC 2391    | 26                        | 81                        | 36                   | 445                         | 21           | 93                        | 113                       | 50                          |
| NGC 2516   | 63                        | 299                       | 251                  | 2550                        | 17           | 205                       | 265                       | 30                          |
| Pleiades   | 86                        | 176                       | 86                   | 850                         | $-32$        | $\cdots$                  | $\cdots$                  | $\cdots$                    |
| Psc−Eri stream | 120                       | 120                       | 120                  | $\geq 2000$                 | 39           | 62                        | 78                        | 200                         |
| $\mu$ Tau  | 55                        | 69                        | 62                   | $\approx 250$               | 51           | 45                        | 60                        | 1200                        |

Note. The minimum, maximum, and the most probable stellar evolutionary age estimate is denoted by $t_{\text{ev,min}}$, $t_{\text{ev,max}}$, and $t_{\text{ev}}$, respectively. The measured angle of the tail tilt and the minimum and maximum morphological age are denoted by $\beta$, $t_{\text{mph,min}}$ and $t_{\text{mph,max}}$, respectively, and the observed and expected mass are denoted by $M_{\text{cl,obs}}$ and $M_{\text{cl,mph}}$, respectively. For Messier 39 and NGC 2516, we give the morphological age estimates during the second tail oscillation, as these are in a better agreement with the stellar evolutionary ages. The stellar evolutionary age and mass estimates are adopted from Meingast et al. (2021) for the open clusters (above the horizontal line) and from Curtis et al. (2019), Ratzenböck et al. (2020), and Gagné et al. (2020) for the Psc−Eri and $\mu$ Tau streams.

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5 The estimate is based on the assumption that the tidal tail is divided into $k_{nseg}$ equal azimuthal segments with the origin at the cluster, and that the number of stars $n_{seg}$ in each segment is distributed according to the Poisson distribution. Requiring the ratio between the mean and standard deviation as $\langle n_{seg} \rangle/\sigma_{n_{seg}} \geq 2$, we obtain $\langle n_{seg} \rangle \geq 4$. Further assuming that each tidal tail contains typically at least 300 stars (Meingast et al. 2021, their Figure 12), one obtains $k_{nseg} = 300/\langle n_{seg} \rangle = 75$ azimuthal segments, i.e., each segment spans $4.8^\circ$, which we identify with $\Delta \beta$. This is an order-of-magnitude estimate, as the signal-to-noise ratio of 2 and the number of stars in the tail of $\geq 300$ might be too optimistic. On the other hand, the estimate is provided for a nonelongated tidal tail; an elongated tidal tail has more stars in the segments along its longer axis, which increases the signal-to-noise ratio.

6 Using gyrochronology, Bouma et al. (2021) find a slightly younger age of $\approx 150$ Myr.
Equations (2) and (5)), and from this quantity $\sigma$ because $\sigma \propto v_{e,1}$, and finally from $\sigma$ we estimate the initial cluster mass. Because we expect the volume density of tail I to be the highest (and the least contaminated) along its short axis, we estimate the tail extent (i.e., its width) in this direction.

We take the typical velocity of tail I stars as $v_{e,1} = 4 \, \text{km s}^{-1}$ for $M_{\text{ecl}} = 4400 \, M_\odot$ (Table 1 in Paper II), and from the arguments above it follows that

$$v_{e,1} \, \text{km s}^{-1} = 5.7 \left( \frac{M_{\text{ecl}}}{10^4 \, M_\odot} \right)^{7/16}.$$  

(7)

The tail width $b$ expressed from Equation (5) is shown by lines as a function of $\beta$ in Figure 5. Each of the panels corresponds to the same age interval when the tail tilt evolves monotonically: the left panel is for the first tail oscillation ($\beta$ decreases), the middle panel for the first part of the second oscillation ($\beta$ increases), and the right panel for the second part of the second oscillation ($\beta$ decreases). For comparison, we plot there the tail width for $N$-body models C03G13 and C10G13 (blue and cyan crosses, respectively) of Paper II, which is calculated as the standard deviation of the stellar distances in the tail direction (i.e., excluding the stars in the cluster) in the direction of the tail semiminor axis. The $N$-body models, which have cluster masses of 1400 and 4400 $M_\odot$, fit at the expected positions in the $\beta - b$ plane between the theoretical curves for 1250, 2500, and 5000 $M_\odot$ for most of the time.

The tail widths for the observed clusters and streams are plotted by black squares in Figure 5. To provide an order-of-magnitude estimate, we neglect the uncertainty in the measurements of tail widths, which is probably dominated by incompleteness as discussed below. For simplicity, we show them during the first oscillation only. For the tail width at given $t$, we obtain the velocity $\bar{v}_{e,1}$ from Equation (5) and the estimated initial mass $M_{\text{ecl, mph}}$ from Equation (7). However, the mass obtained in this way is substantially smaller (often by a factor of 10; Table 1) for most of the objects than the observed mass $M_{\text{ecl, obs}}$ of the cluster and tail combined. In other words, when the expected tail width is calculated from the observed cluster mass $M_{\text{ecl, obs}}$, it is substantially (by a factor of 3) larger than its observed value (yellow symbols in Figure 5) for most of the objects. Possible reasons for the discrepancy are discussed in Section 4.4.

4. Discussion

4.1. Comparison to Stellar Evolutionary Age Estimates

The analysis of Section 3.1 provides age estimates that are in very good agreement with stellar evolutionary age estimates in $\approx 55\%$ of the cases, in reasonable agreement in $\approx 25\%$ of cases, and in tension in $\approx 20\%$ of cases. The agreement for the majority of cases gives some support to the morphological method. The discrepancy in the age for $\approx 20\%$ of cases (the Psc–Eri stream and IC 2391) indicates that one of the methods is incorrect. It is possible that it is either one of the stellar evolutionary methods or the morphological method. For example, the main-sequence fitting method for the Psc–Eri stream provided an age of $\approx 1$ Gyr (Meingast et al. 2019), which was later found to be younger by a factor of 8 (Curtis et al. 2019) because of incompleteness in the former data. An excellent match with the morphological age for both objects could be obtained by changing the stellar evolutionary age by a factor of two, which is substantially less

Figure 4. The morphological age determination for the star clusters with known extended tail structures from Meingast et al. (2021), and for the Psc–Eri and $\mu$ Tauri streams (Meingast et al. 2019; Gagné et al. 2020). The top panel shows the ages that correspond to the observed values of $\beta$. The degeneracy of age for a given $\beta$ is illustrated by the dot color: the age corresponding to the first and second tail oscillation is shown by the red and blue dots, respectively. The typical uncertainty in measuring $\beta$ is assumed to be $5^\circ$ (error bars). Bottom panel: comparison between the morphological cluster age determination from the tail tilt (red and blue bars) and the age estimates based on stellar evolution (gray bars; see Table 1 for details). The most probable cluster age $t_{\text{ecl}}$ is indicated by the black vertical bars.

3.2. Estimate of the Initial Embedded Mass

The stars forming tidal tail I escape the cluster at a typical velocity $v_{e,1}$, which is proportional to the initial velocity dispersion $\sigma$ in the embedded cluster. The velocity dispersion is related to the initial cluster mass $M_{\text{cl}}$ and virial radius $R_V$ by $\sigma^2 = GM_{\text{cl}}/(2R_V)$ (Aarseth 2003). For a cluster represented by the Plummer model of scale radius $a_0$, for which $R_V = 16a_0^2/(3\pi)$, the velocity dispersion reads $\sigma^2 = 3\pi GM_{\text{cl}}/(32a_0^3)$. Assuming that the cluster length scale $a_0$ depends only on the cluster mass (Marks & Kroupa 2012), the velocity dispersion can be expressed only as a function of the initial (embedded) mass in stars, $M_{\text{cl}}$. Thus, from the extent of the tidal tail at a given age, we can determine $v_{e,1}$ (from objects have $t_{\text{ecl}}$ substantially different from the morphological age estimate (Psc–Eri stream and IC 2391).
The area enclosing clusters of mass from 312 to $10^4 M_\odot$ by the yellow squares. The majority of the observed tails are too narrow for their observed mass.

A given clusters, respectively. The simulated point of the age nearest to the selected ages and streams of Table 1 are indicated by the black squares. The expected tail width for the same clusters, but calculated from their observed mass evolves monotonically; $\beta$ decreases from 0 to 168 Myr (left panel), increases from 168 to 228.7 Myr (middle panel), and decreases from 228.7 to 336.1 Myr (right panel). For a given $\beta$, the tail width increases monotonically with the cluster mass. The blue and cyan crosses represent NBODY6 simulations of the $M_{\text{ecl}} = 1400$ and $4400 M_\odot$ clusters, respectively. The simulated point of the age nearest to the selected ages (black dots) is shown by the large cross. The widths of tails of the observed clusters and streams of Table 1 are indicated by the black squares. The expected tail width for the same clusters, but calculated from their observed mass $M_{\text{ecl,obs}}$, is indicated by the yellow squares. The majority of the observed tails are too narrow for their observed mass.

than what was the discrepancy for the Psc–Eri stream in the example above.

The morphological method can be further improved by taking into account the noncircularity of the orbit and the possible influence of the Galactic bar. The degree of noncircularity (eccentricity) can be obtained from the proper motion of the cluster or stream and then utilized for a correction of the relationship between the morphological age and angle $\beta$ for the given eccentricity. It is possible that including the effect of the orbital eccentricity would also improve the agreement between the stellar evolutionary and morphological age estimates.

Giant molecular clouds passing through a tidal tail can stir asymmetry in the tail (Jerabkova et al. 2021), which was indicated in the tail (in this case tail II) of the Hyades (Röser et al. 2019). According to the results of Jerabkova et al. (2021), giant molecular clouds might distort also tidal tail I so that its tilt $\beta$ would no longer be a clear function of the age of the cluster or stream.

Another mechanism that is likely to distort the shape of tail I is gas expulsion occurring off the cluster center (e.g., caused by a massive star, which was sent to the outskirts of the still-embedded cluster by an encounter near the center). This would accelerate stars in one direction at a larger velocity than in the opposite direction, breaking the assumption of isotropy. We intend to explore some of these possibilities in a follow-up paper.

4.2. Applications and Limitations

Morphological age determinations can be applied both to tidal tails enveloping gravitationally bound clusters from which the tails presumably originated and to stellar streams, which contain no gravitationally bound remnant cluster. Moreover, it is not necessary that the stellar stream originated from a gravitationally bound object. For example, consider a star-forming region with sparse star formation occurring only in small groups or clusters, which completely dissolve and release all their stars to the field during or soon after the star-forming cloud is disrupted. These stars will expand in random directions from the location of their birth clusters and will be subjected to the Galactic tidal field. At a given time $t$, stars released from each cluster will occupy an area approximately bounded by the contour of Figure 1 corresponding to $t$ and scaled by the typical escape velocity $v_{\text{esc}}$ from each individual cluster.

Whether the morphological dating provides a robust estimate depends mainly on the volume occupied by the star-forming region. Since the majority of star formation in the Galaxy occurs in filaments (André et al. 2014), the stars that formed in the filaments occupy a filamentary configuration long after the star-forming region became inactive with all gas removed (Jerabkova et al. 2019). Stellar relic filaments reach sizes up to several hundreds of parsecs (Jerabkova et al. 2019; Kounkel & Covey 2019; Beccari et al. 2020; Wang et al. 2021) and often span large distances between gravitationally bound clusters (Beccari et al. 2020). In this case, parts of relic filaments might be confused with tidal tails, which probably possess the largest limitation for the morphological dating method.

On the other hand, many star-forming regions occupy smaller volumes (e.g., the maximum extent of the Taurus-Auriga star-forming region is $\approx 60$ pc; Galli et al. 2019) than the volume of the tail already at a young age ($\approx 100$ pc; Figure 1), and the tidal tails from the clusters will be superimposed one over another, overlapping and probably being indistinguishable from each other. Nevertheless, the tilt $\beta$ of the tail will be comparable for all the clusters (the age spread of clusters forming within the same star-forming region is $\lesssim 25$ Myr; e.g., Kawamura et al. 2009; Jeffries et al. 2011; Dobbs & Pringle 2013, which is much less than $2\pi/\kappa \approx 170$ Myr), so the morphological dating method can be applied to initially unbound configurations as well.
In order to estimate the limitations due to relic filaments, we consider that a typical relic filament is of length 100 pc and that the filament forms a star cluster at one of its extremities. The cluster releases stars to tail I at typical speeds of $\bar{v}_{\text{e},1} \approx 2\,\text{km s}^{-1}$. These stars will overtake the relic filament at an age of $\approx 30\text{–}50\,\text{Myr}$, when both the filament and the tidal tail will be clearly discernible from one another. This time is also near the upper limit on the observed age of relic filaments (Jerabkova et al. 2019; Beccari et al. 2020), as well as the age when the elongation of the tail I becomes less spherical so that its tilt can be measured (see the red contour in Figure 1). At the same time, the density of the relic filament decreases with time as the filament expands radially, lowering its contamination of tail I. Thus, if the cluster forms as a part of a large filamentary structure, its morphological age can be obtained after it is older than $\approx 40\,\text{Myr}$.

Another complication in determining the morphological age are the stars that evaporate from the cluster and form tail II. These stars occupy a very similar area in the position–velocity space as stars from tail I so that both tails would be probably indistinguishable in the Gaia data. This impacts in particular older clusters because stars forming tail II evaporate from clusters at an approximately time-independent rate (Aarseth 2003; Baumgardt & Makino 2003; Chumak & Rastorguev 2006). Also, stars in tail II move at lower speeds than stars in tail I (Paper II), so tail II is not as close to clumps to the cluster and at an elevated stellar density. Unlike tail I, tail II is S shaped in the vicinity of the cluster (Chumak & Rastorguev 2006; Küpper et al. 2008), which would spuriously increase $\beta$, thereby underestimating the proper cluster age.

In order to suppress contamination by tail II, we suggest that for measuring $\beta$, only sufficiently distant stars from the cluster are taken into account. Since the typical speeds in tail II are relatively low ($\bar{v}_{\text{e},II} \approx 1.4\,\text{km s}^{-1}$ for a rather massive cluster of $4400\,M_\odot$ and lower for lower-mass clusters; see Table 1 of Paper II), the analysis should ignore stars located closer than $\approx \bar{v}_{\text{e},II} \times t$ to the cluster, where $t$ is the prior estimate of the cluster age.

How many objects do we expect to be accessible for the method given the astrometry of the Gaia DR2 release? The velocity cut of $2\,\text{km s}^{-1}$ can be provided for A0 stars closer than 1 kpc (Gaia Collaboration et al. 2016). At this distance, the position error $\approx 10\,\text{pc}$, which should be sufficient for the detection of tail I. Porras et al. (2003) find 16 very young star clusters having more than 100 stars within the circle of radius 1 kpc centered at the Sun. For this estimate, we require that only more numerous clusters (having more than 800 stars) can produce tails of discernible tilt. Assuming that the Galaxy forms embedded clusters according to an embedded cluster mass function in the form of $\frac{dn_{\text{ecl}}(M_{\text{ecl}})}{dM_{\text{ecl}}} \propto M_{\text{ecl}}^{-1.2}$ (e.g., Lada & Lada 2003; de la Fuente Marcos & de la Fuente Marcos 2004), spanning up to $M_{\text{ecl}} \approx 10^4\,M_\odot$ (Johnson et al. 2017), 5/8 of the star clusters with more than 100 stars contain more than 800 stars. Further assuming that the catalog of Porras et al. (2003) is complete for clusters younger than 5 Myr and that the morphological method can be used for clusters in the age interval from 40 to 340 Myr (see Section 4.3), we obtain $5/8 \times 16 \times (340\text{–}40)/5 \approx 550$ objects (i.e., clusters or streams) accessible to the method within 1 kpc from the Sun. Even though this is the likely upper estimate (because many of the clusters form within the same star-forming region so that their tails cannot be distinguished from each other and the upper mass of clusters that form within the solar circle is probably lower than $10^4\,M_\odot$ according to Pfannm-Altenburg & Kroupa 2008), we estimate that there might be more than 100 objects in the Gaia DR2 data accessible for this dating method.

4.3. The Range of Cluster Ages Available for the Morphological Dating Method and Its Accuracy

The morphological age determination method is suited for younger clusters, where $\beta$ sensitively depends on the age. This is mainly because the function $\beta = \beta(t)$ is not an injective function; there is one $t$ for $\beta > 18^2$, three possible values for $t$ for $\beta \in (10^2.5, 18^2)$, five possible values of $t$ for $\beta \in (7^2.4, 10^2.5)$, and the degeneracy quickly increases for $\beta \leq 7^2.4$ (Figure 2). Any systematic error in the determination of $\beta$ (e.g., from the cluster having an eccentric orbit or confusion between tail I and tail II stars) results in a large uncertainty in age $t$ for $\beta$ sufficiently small. Taking $\Delta \beta = 5^\circ$ as an estimate of the uncertainty in $\beta$ determination, this method is useless for $t > 336\,\text{Myr}$ because $\beta$ changes by less than $\approx 2\Delta \beta$ since this age. On the other hand, $\beta$ changes rapidly for younger clusters (it decreases from $\approx 90^\circ$ to the smallest angle of uniquely determined time of $18^2$ in 105 Myr), offering a very sensitive tool for clusters in this age range. Likewise, $\beta$ is sensitive to $t$ also during the second tail oscillation (between 168 and 336 Myr). The knowledge of a prior estimate of the tail age (e.g., from a stellar evolutionary method) would be useful because it would constrain whether the tail is in its first or second oscillation, and then the inversion $t = \beta(\beta)$ could be done in the appropriate time interval, which would provide the morphological estimate for the age. We show an example of using the prior estimate for the tail age in Section 3.1 (for NGC 2516; see also Figure 4).

The form of $\beta = \beta(t)$ and the assumed error of $\Delta \beta = 5^\circ$ can be used for estimating the age of clusters younger than $\approx 340\,\text{Myr}$. On the other hand, the method appears to be less accurate or problematic for clusters younger than $\approx 40\,\text{Myr}$ because of the possible presence of relic filaments and the initially spherical expansion of tail I (see Section 4.2 for details).

The assumed uncertainty in $\beta$ of $\pm 5^\circ$ propagates to an age uncertainty of $\approx 12\,\text{Myr}$ during the first tail oscillation (i.e., at ages younger than 168 Myr), and to an age uncertainty of $\approx 30\,\text{Myr}$ during the second tail oscillation (i.e., between 168 and 336 Myr). This uncertainty, which is $\approx 10\%$ to 20%, is comparable to the most accurate stellar evolutionary dating methods, i.e., isochrone fitting and lithium depletion boundary (e.g., Meynet et al. 2009; Soderblom 2010; Jeffries et al. 2013; Martin et al. 2018; Binks et al. 2021), but the morphological method is much easier to apply to a particular cluster or stream, as it does not need stellar evolutionary models or dedicated spectroscopic observations. However, the role of possible systematic errors (e.g., noncircular orbits or contamination) remains to be clarified.

4.4. Estimate of the Initial Cluster Mass

Although the morphological age estimate is in agreement with other dating methods for the majority of the clusters in our sample, the estimate of the initial mass (Section 3.2) provides substantially lower cluster masses than observed in most of the cases. The discrepancy in the initial cluster masses (or
equivalently the tail widths) might point to incompleteness in the observational data, or possibly contamination with stars of tail II, which is most conspicuous close to the cluster. The estimate of cluster mass is based on more assumptions (e.g., on the initial cluster radius and the cluster’s virial state) than the estimate for the age, which might amplify the uncertainty. For the order-of-magnitude estimate in this work, we inspected the available data by eye only. It is possible that more sophisticated tools (e.g., a Bayesian analysis of the probability that a star belongs to tail I) might lead to a substantial improvement of the present estimate.

Alternatively, the same outcome could be caused if the velocity $\vec{v}_b$ is lower than assumed, for example, if the clusters form with the SFE being larger than 1/3, or if the gas expulsion timescale is adiabatic, or if the majority of the extended structure originates from stars that were never gravitationally bound to the cluster, but which formed nearby to the cluster in the same star-forming region.

5. Summary

We propose a new method (morphological method) for dating open star clusters with extended tidal tails and stellar streams based on the tilt angle $\beta$ of their extended tidal structure measured from the direction of their orbit around the Galaxy. The tidal tail, which is coeval with the cluster, forms at an early age either from stars released owing to expulsion of non-star-forming gas from the cluster or from stars formed in the same star-forming region in the vicinity of the cluster. Classical tidal tails forming by gradual evaporation of stars from the cluster cannot be used by this method.

We show analytically that the tilt angle $\beta$ for objects (i.e., clusters or streams) at a given Galactocentric distance is only a function of the object age $t$ and not a function of any other property of the object such as its initial mass or radius. The age can be found by inverting the theoretical dependence $\beta = \beta(t)$ for observed $\beta$ (top panel of Figure 4). The method is suitable for younger objects ($40 \text{ Myr} \lesssim t \lesssim 350 \text{ Myr}$), where we estimate the accuracy to be 10% to 20% of the age of the object (for an error in $\beta$ of 5$^\circ$), which is comparable to the errors of the currently most accurate stellar evolutionary dating methods, which utilize isochrone fitting or the lithium depletion boundary. The morphological method does not necessarily aim at exceeding the accuracy of the stellar evolutionary methods, but at providing an estimate, which is completely independent of models of stellar evolution.

The main advantage of this method is its ease of use and its independence from stellar evolutionary models and stellar velocity measurements (apart from the velocity cut to detect the tidal structure). The present derivation applies to clusters or streams on circular orbits only, but at providing an estimate, which is completely independent of models of stellar evolution.

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