CP-odd $WWZ$ couplings induced by vector-like quarks

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Abstract

A minimal extension of the standard model includes extra quarks with charges $2/3$ and/or $-1/3$, whose left-handed and right-handed components are both SU(2) singlets. This model predicts new interactions of flavor-changing neutral current at the tree level, which also violate CP invariance. We study CP-odd anomalous couplings for the gauge bosons $W$, $W$, and $Z$ induced by the new interactions at the one-loop level. These couplings become nonnegligible only if both an up-type and a down-type extra quarks are incorporated. Their form factors are estimated to be maximally of order $10^{-5}$. Such magnitudes are larger than those predicted in the standard model, though smaller than those in certain other models.

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1 Introduction

Experiments at LEP2 and planned $e^+e^-$ linear colliders can directly probe gauge-boson self-interactions, an aspect of the standard model (SM) characteristic of its non-Abelian nature. Their precise measurements serve a detailed examination of the SM, whose predictions have been well studied including quantum corrections. If some deviations from the SM predictions are found, the SM will have to be extended. For this task, studying peculiar features of various models is indispensable. Theoretical analyses therefore have been made on the gauge-boson self-interactions in the extensions of the SM [1, 2]. In particular, CP-even couplings for the gauge bosons $W$, $W$, and $Z$ have been studied in the two-Higgs-doublet model [3], the model with Majorana neutrinos [4], the supersymmetric model [5], and so on. The CP-odd $WWZ$ couplings have also recently been discussed in the supersymmetric model [6].

One of minimal extensions of the SM is the vector-like quark model (VQM). This model includes extra quarks with charges $2/3$ and/or $-1/3$, whose left-handed components, as well as right-handed ones, are singlets under SU(2). Although many features of the SM are not significantly modified, new sources of CP violation and flavor-changing neutral current (FCNC) are incorporated. Therefore, their effects on the $K$-meson and $B$-meson systems have been studied extensively [7]. It has been also argued [8] that baryon asymmetry of the universe could be attributed to these new sources of CP violation.

In this paper, we study the effects of the VQM on CP-odd couplings for the $WWZ$ vertex. This model predicts that the $Z$ boson couples to quarks of different generations, causing interactions of FCNC at the tree level. For a pair of light ordinary quarks, these interactions should be suppressed from experimental results. However, the $t$ quark and the extra up-type quark $U$ could have a sizable coupling with the $Z$ boson. Since these interactions of FCNC also violate CP invariance, non-negligible CP-odd couplings for the $WWZ$ vertex may be induced at the one-loop level. On the other hand, the SM does not contain the CP-odd $WWZ$ couplings at the tree level nor the one-loop level. The supersymmetric model predicts them at the one-loop level [6]. The CP-odd $WWZ$ couplings at the one-loop level could become a window for physics beyond the SM. It will be shown that form factors for the couplings in the VQM can be evaluated without making many assumptions on mixings among quarks. The form factors are nonnegligible at the one-loop level, though the possible maximal magnitudes are smaller than those in the supersymmetric model.
This paper is organized as follows. In sect. 2 we briefly summarize the model. In sect. 3 the CP-odd form factors are obtained and numerical analyses are performed. Summary is contained in sect. 4.

2 Model

The quark sector is enlarged to have extra quarks whose transformation properties are given by (3, 1, 2/3) or (3, 1, −1/3) for the SU(3)×SU(2)×U(1) gauge symmetry. Both the left-handed and right-handed components have the same properties. For definiteness, we assume the particle contents with one up-type and one down-type extra quarks. The quark masses are generated by Yukawa couplings and bare mass terms. The mass matrices are given by 4×4 matrices, which are denoted by $M^u$ and $M^d$ respectively for up-type and down-type quarks. The mass eigenstates are obtained by diagonalizing the mass matrices as

$$A^u_L M^u A^u_R = \text{diag}(m_{u1}, m_{u2}, m_{u3}, m_{u4}),$$

$$A^d_L M^d A^d_R = \text{diag}(m_{d1}, m_{d2}, m_{d3}, m_{d4}),$$

where $A^u_L$, $A^u_R$, $A^d_L$, and $A^d_R$ are unitary matrices. We express the mass eigenstates by $u^a$ and $d^a$ ($a = 1 - 4$), $a$ being the generation index, which may be also called as $(u, c, t, U)$ and $(d, s, b, D)$.

The interaction Lagrangian for the quarks with the $W$ boson is given by

$$\mathcal{L} = \frac{g}{\sqrt{2}} u^a V_{ab} \gamma^\mu \frac{1 - \gamma^5}{2} d^b W_\mu^\dagger + \text{h.c.}. \quad (3)$$

Here the 4×4 matrix $V$ stands for an extended Cabibbo-Kobayashi-Maskawa matrix, which is defined by

$$V_{ab} = \sum_{i=1}^{3} (A^u_L)_{ai} (A^d_L)_{ib}. \quad (4)$$

Note that $V$ is not unitary. The interaction Lagrangian for the quarks with the $Z$ boson is given by

$$\mathcal{L} = -\frac{g}{\cos \theta_W} u^a \gamma^\mu \left( F^u_L \frac{1 - \gamma^5}{2} + F^u_R \frac{1 + \gamma^5}{2} \right) u^b Z_\mu$$

$$-\frac{g}{\cos \theta_W} d^a \gamma^\mu \left( F^d_L \frac{1 - \gamma^5}{2} + F^d_R \frac{1 + \gamma^5}{2} \right) d^b Z_\mu, \quad (5)$$

$$F^u_L = \frac{1}{2} V V^\dagger - \frac{2}{3} \sin^2 \theta_W, \quad F^u_R = -\frac{2}{3} \sin^2 \theta_W,$$

$$F^d_L = -\frac{1}{2} V V^\dagger + \frac{1}{3} \sin^2 \theta_W, \quad F^d_R = \frac{1}{3} \sin^2 \theta_W.$$
Since $V$ is not a unitary matrix, off-diagonal elements of $F^u_L$ and $F^d_L$ become non-vanishing, leading to FCNC at the tree level. The Lagrangian in Eq. (5), as well as that in Eq. (3), can induce CP violation.

3 Form factors

For the pair production of $W$ bosons in $e^+e^-$ annihilation, the trilinear gauge-boson interaction for $W$, $W$, and $Z$ is generally expressed as$$
\mathcal{L}_{\text{eff}} = g \cos \theta_W \Gamma^{\mu\nu\lambda} W^\dagger_{\mu} W_{\nu} Z_{\lambda},
$$

$$
\Gamma^{\mu\nu\lambda} = f_1 (p - \bar{p})^{\lambda} q^{\mu} q^{\nu} + f_2 \frac{1}{M_W^2} (p - \bar{p})^{\lambda} q^{\nu} q^{\mu} + f_3 (q^{\mu} g^{\lambda\nu} - q^{\nu} g^{\lambda\mu})
+ i f_4 (q^{\mu} g^{\lambda\nu} + q^{\nu} g^{\lambda\mu}) + i f_5 \varepsilon^{\mu\nu\lambda\rho} (p - \bar{p})_{\rho} + f_6 \varepsilon^{\mu\nu\lambda\rho} q_{\rho}
+ f_7 \frac{1}{M_W^2} (p - \bar{p})^{\lambda} \varepsilon^{\mu\nu\rho\sigma} q_{\rho} (p - \bar{p})_{\sigma},
$$

where $p$ and $\bar{p}$ denote the outward momenta of the gauge bosons $W^-$ and $W^+$, respectively, and $q$ the inward momentum of $Z$. The couplings with the form factors $f_1$, $f_2$, $f_3$, and $f_5$ are $CP$-even, while those with $f_4$, $f_6$, and $f_7$ are $CP$-odd. Both in the SM and in the VQM, only the form factors $f_1$ and $f_3$ have non-vanishing values at the tree level.

The $CP$-odd form factors receive contributions from the one-loop diagrams in which up-type quarks or down-type quarks couple to the $Z$ boson as shown in Fig. 1.

We obtain the form factors arising from the diagram in Fig. 1(a) as

$$
f_4 = \frac{-g^2}{64\pi^2 \cos^2 \theta_W} \sum_{a=1}^{4} \sum_{b=1}^{4} \sum_{c=1}^{4} \text{Im} \left[ V_{ac} V_{bc}^*(VV^\dagger)_{ba} \right] I_4(m_{ua}, m_{ub}, m_{dc}),
$$

$$
f_6 = \frac{-g^2}{64\pi^2 \cos^2 \theta_W} \sum_{a=1}^{4} \sum_{b=1}^{4} \sum_{c=1}^{4} \text{Im} \left[ V_{ac} V_{bc}^*(VV^\dagger)_{ba} \right] I_6(m_{ua}, m_{ub}, m_{dc}),
$$

$$
f_7 = 0,
$$

where the functions $I_4$ and $I_6$ are defined by

$$
I_4(m_{ua}, m_{ub}, m_{dc}) = \int \int_D \frac{M_W^2(1 - x - y)^2(x - y) + (m_{ua}^2 - m_{ub}^2)xy}{-M_W^2(1 - x - y)(x + y) - q^2 xy + m_{ua}^2 x + m_{ub}^2 y + m_{dc}^2(1 - x - y) - i\varepsilon},
$$

$$
I_6(m_{ua}, m_{ub}, m_{dc}) = \int \int_D \frac{M_W^2(1 - x - y)^2(x - y) + (m_{ua}^2 - m_{ub}^2)xy}{-M_W^2(1 - x - y)(x + y) - q^2 xy + m_{ua}^2 x + m_{ub}^2 y + m_{dc}^2(1 - x - y) - i\varepsilon}.
$$
The values of $S_4$ in Fig. 3 the $\sqrt{m}$ boson. We can safely neglect mass differences among the light quark $s$. Then, taking the imaginary parts do not vary much with $m$ and (iv) represent $\text{Re}(m)$. The domain $D$ for integration is given by

$$x \geq 0, \quad y \geq 0, \quad x + y \leq 1. \quad (12)$$

The contributions of the diagram in Fig. 1(b) can be obtained similarly. However, these contributions are negligible compared to those from Fig. 1(a) as shown later.

The form factors $f_4$ and $f_6$ in Eqs. (9) and (10) are expressed more simply by taking approximations for the quark masses. Since the quarks of the first two generations and the $b$ quark are much lighter than the $W$ boson, the integrands of $I_4$ and $I_6$ in Eqs. (11) and (12) are determined almost only by the $W$-boson mass $M_W$, the heavy quark masses $m_t, m_U, m_D$, and the momentum-squared $q^2$ of the $Z$ boson. We can safely neglect mass differences among the light quarks. Then, taking $m_u = m_c$ and $m_d = m_s = m_b$, the form factors are written by

$$f_i = \frac{g^2}{32\pi^2 \cos^2 \theta_W} C S_i \quad (i = 4, 6), \quad (13)$$

$$C = \text{Im} \left[ V_{34} V_{44}^* (V V^*)_{43} \right],$$

$$S_i = I_i(m_u, m_t, m_d) - I_i(m_u, m_U, m_d) + I_i(m_t, m_U, m_d)$$

$$- I_i(m_u, m_t, m_D) + I_i(m_u, m_U, m_D) - I_i(m_t, m_U, m_D).$$

Here, $C$ depends on mixing parameters for quarks, while $S_4$ and $S_6$ depend on mass parameters $m_U, m_D$ and an experimental parameter $q^2$. It is seen that $S_4$ and $S_6$ vanish if an equality $m_u = m_t$ is assumed. Correspondingly, the contributions to $f_4$ and $f_6$ from the diagram in Fig. 1(b) become negligible, since an equality $m_d = m_b$ holds to a good approximation.

Numerical analyses for the form factors are now in order. We first consider $S_4$ and $S_6$. The integrals in Eqs. (11) and (12) are precisely evaluated by a numerical method [3]. In Figs. 2(a) and 2(b) the absolute values for the real and imaginary parts of $S_4$ and $S_6$ are shown as functions of $m_U$ for $m_U \geq 200$ GeV, taking $\sqrt{q^2} = 200$ GeV and $m_D = 200$ GeV (Fig. 2(a)), 500 GeV (Fig. 2(b)). Curves (i), (ii), (iii), and (iv) represent $\text{Re}(S_4), \text{Im}(S_4), \text{Re}(S_6)$, and $\text{Im}(S_6)$, respectively. In wide ranges the imaginary parts do not vary much with $m_U$ and $m_D$, being $\text{Im}(S_4) \approx -0.3$ and $\text{Im}(S_6) \approx 0.9$. The real parts are in the ranges $|\text{Re}(S_4)| \lesssim 0.2$ and $|\text{Re}(S_6)| \lesssim 0.4$. The values of $S_4$ and $S_6$ are not much dependent on $m_D$ for $m_D \gtrsim 400$ GeV. In Fig. 3 the $\sqrt{q^2}$-dependencies of $S_4$ and $S_6$ are shown for $\sqrt{q^2} \geq 180$ GeV, taking
Next we consider the magnitude of $C$. It is seen from Eqs. (3) and (5) that $V^{34}$, $V^{∗44}$, and $(V V^∗)_{43}$ are related to the couplings of $tD_W$, $UD_W$, and $UtZ$, respectively. Experimental results available at present have not yet given much information on their values. However, $C$ is expressed in terms of the unitary matrices $A_u^u$ and $A_d^d$ as

$$C = -\text{Im} \left[ (A_u^u)_{43} (A^u_{L})_{34} (A^d_{L})_{44} (A_u^u)_{44} - (A_u^u)_{43} (A^u_{L})_{34} (A^d_{L})_{44} \right]$$

Since the mass of the $D$ quark is considered to be significantly larger than those of the $d$ and $s$ quarks, we may neglect mixings between the $D$ quark and these light quarks, taking $(A_d^d)_{4i} = (A^d_{L})_{4i} = 0$ ($i = 1, 2$). Then, $C$ is given by

$$C = -|(A_d^d)_{43}|^2 \text{Im} \left[ (A_u^u)_{43} (A^u_{L})_{33} (A^d_{L})_{34} (A_u^u)_{44} \right].$$

(15)

We can see that $C$ is proportional to $(A_d^d)_{34}$. If there is no mixing between the $D$ quark and the ordinary down-type quarks, the form factors $f_4$ and $f_6$ vanish. In order to have nonnegligible CP-odd couplings, the extra down-type quark $D$, as well as $U$, should exist and be mixed with other down-type quarks. The magnitude of $C$ is estimated to be at most of order 0.1, since $C$ contains the product of four different elements of a unitary matrix in addition to $|(A_d^d)_{34}|^2$. If the mixings of the quarks for the third and fourth generations are not suppressed, $|C|$ becomes maximal. However, the mixing for the down-type quarks would be suppressed, since the mass difference between the $b$ quark and the $D$ quark is still large. Therefore, we take the allowed range of $C$ for $|C| < 0.01$ as a conservative constraint.

The form factors are written from Eq. (13) as

$$f_i = 1.7 \times 10^{-3} CS_i \quad (i = 4, 6).$$

(16)

Taking into account the constraints on $C$ and $S_i$, we make an estimate

$$|f_4|, |f_6| < 1 \times 10^{-5}.$$

(17)

Possible maximal values of this result are larger than the predictions in the SM, though smaller by two order of magnitude than those in the supersymmetric model [6], where the CP-odd form factors are also induced at the one-loop level. Assuming a maximal value, a total of more than $10^{10}$ pairs of $W$ bosons would be necessary to detect the form factors. It seems to be difficult to achieve such a number of events in near-future experiments.
4 Summary

We have discussed CP-odd couplings for the WWZ vertex within the framework of the VQM. These couplings could be sizably induced through the one-loop diagram in which the Z boson couples to the up-type quarks. Both up-type and down-type extra quarks are necessary to have nonnegligible form factors. Their possible maximal magnitudes have been estimated without assuming a detailed structure for the quark mixings, giving at most of order $10^{-5}$. These magnitudes are larger than the predictions by the SM but smaller than those by the supersymmetric model. The VQM does not yield CP-odd WWZ couplings which can be detected experimentally in the near future.

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Figure 1: Possible one-loop diagrams which induce $CP$-odd couplings for the $WWZ$ vertex.
Fig. 2(a)
Figure 2: The absolute values of the real and imaginary parts of $S_4$ and $S_6$ as functions of $m_U$ at $\sqrt{q^2} = 200$ GeV. Four curves (i)–(iv) correspond to Re($S_4$), Im($S_4$), Re($S_6$), Im($S_6$). (a) $m_D = 200$ GeV, (b) $m_D = 500$ GeV.
Figure 3: The values of the real and imaginary parts of $S_4$ and $S_6$ as functions of $\sqrt{q^2}$ for $m_U = 300$ GeV and $m_D = 300$ GeV. (i) Re($S_4$), (ii) Im($S_4$), (iii) Re($S_6$), (iv) Im($S_6$).