Constraints on the mass of Majorana neutrinos from Cosmology

M. Agostini,1, 2 G. Benato,3 S. Dell’Oro,4, 5 S. Pirro,3 and F. Vissani3, 6

1Department of Physics and Astronomy, University College London, Gower Street, London WC1E 6BT, UK
2Physik-Department, Technische Universität München, 85748 Garching, Germany
3INFN, Laboratori Nazionali del Gran Sasso, 67100 Assergi, L’Aquila, Italy
4INFN Sezione di Milano-Bicocca, 20126 Milano, Italy
5University of Milano-Bicocca, 20126 Milano, Italy
6Gran Sasso Science Institute, 67100 L’Aquila, Italy

(Dated: December 29, 2020)

We discuss the impact of the cosmological measurements towards possible predictions of the Majorana mass of the neutrinos, the same parameter probed by neutrinoless double-beta decay experiments. Using only this empirical information, we quantify the probability that a search for neutrinoless double-beta decay will have to see or not a signal and introduce a new graphical representation that might be of interest for the experimental community.

a. Introduction The idea that neutrinos are described by a theory that is symmetric between particles and antiparticles, namely the Majoran theory of fermions [1], has gained credibility over the years. The tiny neutrino masses, proven by the observation of neutrino oscillations, are consistent with the assumption that there is new physics at ultra-high mass-scales [2–5] and is fully compatible with the gauge structure of the Standard Model [6]. A direct way to test Majorana’s theory is to search for neutrinoless double-β decay (0νββ) [7].

Given the importance of 0νββ, it is of paramount importance to build quantitative projections on the Majorana mass-matrix determining the decay rate. In the absence of a conclusive and convincing theory of fermion masses, these predictions should be as much as possible driven by the data and independent by the model. This approach was proposed long ago in Ref. [8], immediately identifying its limitations and initiating their analysis. In fact, the Majorana neutrino mass depends on a set of parameters whose uncertainty can strongly limit our capability of making accurate projections. These parameters are: (1) the mixing angles and mass splittings of neutrino oscillations; (2) the type of neutrino mass spectrum; (3) the value of the lightest neutrino mass; (4) the characteristic phases of Majorana mass.

The huge experimental effort of the last decades led to unprecedented accuracies on the measurement of neutrino properties. Current-generation 0νββ experiments have reached sensitivities to Majorana masses at the level of 100 meV and next-generation searches is foreseen to probe the tens-of-meV region [9, 10]. Precise measurements of the neutrino oscillations have virtually eliminated the uncertainties on the mixing angles and mass splittings relevant for a prediction on the Majorana mass, and global fits are now indicating a mild preference for the normal ordering of the neutrino mass spectrum [11, 12]. At the same time, an increasing number of cosmological measurements is converging to constrain the mass of the lightest neutrino.

In this letter, we revisit and update the discussion on the expectations on the neutrino Majorana mass and their impact on 0νββ searches. We propose a new point of view, in which we minimize the amount of assumptions on the unknown parameters – e.g. the Majorana phases – which would otherwise require a prior affecting the results (see e.g. Refs. [13–16]). We set our discussion within the theoretical framework of 0νββ mediated by the exchange of ordinary light neutrinos endowed with Majorana masses. In fact, the small masses of the three known neutrinos represent today the only unequivocal experimental indication for the existence of physics beyond the Standard Model, which is consistent with the overall picture described above. We will also assume that neutrino masses follow the normal ordering. The projections for 0νββ assuming inverted ordering are not as interesting because next-generation 0νββ experiments will explore the entire parameter space allowed by this model, fully covering any possible projection for mββ.

b. Information from neutrino oscillations The relevant parameter for the 0νββ transition is the absolute value of the ee-entry of the neutrino mass matrix, conventionally indicated with mββ. This parameter can be expressed in terms of the real neutrino masses mββ, where i = 1, 2, 3, and of the complex mixing matrix elements Uei, namely mββ = |∑i |Uei|2mββ|. While the real parts of the matrix elements are constrained by the unitary relation ∑i |Uei| = 1, the complex parts are completely unconstrained. Given the absolute lack of information on the complex parts, in this work we will base our inferences on the two extreme scenarios in which mββ assumes the maximum and minimum allowed value [8]:

\begin{equation}
\begin{aligned}
\mbox{m}_{\beta\beta}^{\text{max}} &= \sum_{i=1}^{3} |U_{ei}| \mbox{m}_{i}, \\
\mbox{m}_{\beta\beta}^{\text{min}} &= \max \left\{ 2|U_{ei}| \mbox{m}_{i} - \mbox{m}_{\beta\beta}^{\text{max}}, 0 \right\}, \quad i = 1, 2, 3.
\end{aligned}
\end{equation}

The squared-mass differences \((\mbox{m}_{ei}^{2} - \mbox{m}_{i}^{2})\) and the parameters \(|U_{ei}|\) have been precisely measured by oscillation experiments and can be considered as fixed. Thus, Eqs. (1a) and (1b) are a function of only one parameter which is the lightest neutrino mass (i.e. \(m_{1}\) assuming normal ordering). The corresponding true value of \(m_{\beta\beta}\) realized in Nature must hence satisfy the condition

\begin{equation}
\begin{aligned}
\mbox{m}_{\beta\beta}^{\text{max}} &= \sum_{i=1}^{3} |U_{ei}| \mbox{m}_{i}, \\
\mbox{m}_{\beta\beta}^{\text{min}} &= \max \left\{ 2|U_{ei}| \mbox{m}_{i} - \mbox{m}_{\beta\beta}^{\text{max}}, 0 \right\}, \quad i = 1, 2, 3.
\end{aligned}
\end{equation}
The sensitivity is more accurately defined as the median upper limit that can be set at a certain confidence level. The actual definition is however not relevant to the purpose of this work as we will not consider concrete experiments.
The most recent analysis by the Planck Collaboration provides a limit at 95% confidence interval on $\Sigma$ of 120 meV, which has been derived using BAO data and assuming $\Sigma > 0$, under the approximation that all three neutrinos have the same mass [17]. An analysis of the same dataset, but with an improved description of the three massive neutrinos imposing the mass differences as fixed by the oscillation experiments, gives a bound on $\Sigma$ of 152 meV at 95% confidence level (C. L.) [11]. These two results are in good agreement. The results obtained by Planck is in good approximation a Gaussian probability distribution with central value at $\Sigma = 0$ and width $\delta \Sigma \simeq 61$ meV. If we now restrict this probability distribution to the physical region $\Sigma \geq \Sigma_{\text{min}}$, we obtain an upper limit on $\Sigma$ of 146 meV. The $\chi^2$ from Ref. [11] is also quasi-parabolic and can be approximated – i.e., we get a consistent limit – by setting (refer to Eq. (2))

$$\Sigma = 48.9 \text{ meV} / \delta \Sigma = 52.6 \text{ meV}. \quad (5)$$

Similar results have been obtained already a few years ago by including the Lyman-$\alpha$ in an analysis similar to those mentioned above. The Authors of Ref. [18] obtained a limit $\Sigma < 140$ meV at 95% C. L. by using Lyman-$\alpha$ data from BOSS and XQ-100, from an almost-Gaussian likelihood that can be approximated by [16]

$$\Sigma = 41.3 \text{ meV} / \delta \Sigma = 49.7 \text{ meV}. \quad (6)$$

More recently, it has been shown in Ref. [19] that the inclusion of the new data from Planck [17] could lead to a tighter bound of $\Sigma < 122$ meV at 95% C. L. (still in the approximation of degenerate neutrino masses).\footnote{Unfortunately, neither the likelihood nor the $\chi^2$ are given in the reference and we cannot include it in our analysis.}

In the near future, we expect important progress from cosmology, with the possibility of extracting more precise results. These could lead to an uncertainty as small as $\delta \Sigma = 12$ meV [20, 21].

The limits extracted by cosmological data appear to be consistent over a large set of assumptions. The use of different hypotheses could make the same bound tighter (e.g., using a smaller value for $H_0$ [11, 17, 19]), or looser (e.g., assuming primordial fluctuations to be distributed with a ‘running spectral index’ [17, 19]), however, the most comprehensive results that are currently available lead to similar conclusions on the sum of the neutrino mass. In particular, in both Refs. [11] and [18], the value of $\Sigma$ which minimizes the $\chi^2$ is compatible with $\Sigma_{\text{min}}$.

**d. Implications for $m_{\beta\beta}$** We have discussed how $m_{\beta\beta}$ and $\Sigma$ are connected to the value of the lightest neutrino mass $m_1$. We will now exploit this connection to convert existing information from cosmology into projected discovery probabilities for future $0\nu\beta\beta$ experiments. This can be done by taking a probability distribution on $\Sigma$ and converting it into a probability distribution for $m_{\beta\beta}$. The cumulative distribution function (CDF) of $m_{\beta\beta}$ can hence be directly interpreted as the probability of observing a signal as a function of the experimental sensitivity.

The connection between $\Sigma$ and $m_{\beta\beta}$ is however not univocally defined. Even considering the oscillation parameters to be perfectly known, we still have no information on the Majorana phases and do not want to introduce unnecessary assumptions on their values. For this purpose, we have been focusing on the most extreme scenarios, i.e., those in which $m_{\beta\beta}$ is equal to the maximum or minimum allowed value. In these two scenarios, the relationship between $\Sigma$ and $m_{\beta\beta}$ becomes univocal and a probability distribution on $\Sigma$ can be converted into one on $m_{\beta\beta}^{\text{max}}$ or $m_{\beta\beta}^{\text{min}}$. Projections for models predicting intermediate values of $m_{\beta\beta}$ can be obtained by interpolating between the results for our two reference scenarios.

To illustrate the procedure in details, let us consider the likelihood described in Eq. (2) with the parameter values set according to Eq. (6)\footnote{Incidentally, this allows a direct comparison with the our previous results reported in Ref. [16].} and a particular $m_{\beta\beta}^*$ value. We are interested in the two values $m_{\beta\beta}^{\text{min}}(m_{\beta\beta}^*)$ and $m_{\beta\beta}^{\text{max}}(m_{\beta\beta}^*)$ (black bullets in Fig. 1). The discovery probability can hence be calculated using two values of the CDFs in Eq. (3):

$$F_{\Sigma}^{\text{max}} = F_{\Sigma} \left( \Sigma \left( m_{\beta\beta}^{\text{max}}(m_{\beta\beta}^*) \right) \right) \quad (7a)$$
$$F_{\Sigma}^{\text{min}} = F_{\Sigma} \left( \Sigma \left( m_{\beta\beta}^{\text{min}}(m_{\beta\beta}^*) \right) \right). \quad (7b)$$

which satisfy the condition $0 \leq F_{\Sigma}^{\text{min}} < F_{\Sigma}^{\text{max}} < 1$ since the CDF is a monotonically non-decreasing function and $0 \leq m_{\beta\beta}^{\text{min}} < m_{\beta\beta}^{\text{max}}$.

The results are shown in Fig. 2 for $m_{\beta\beta}$ sensitivity spanning from about 1 to 50 meV. The solid black line shows the discovery probability for the most unfavorable scenario in which $m_{\beta\beta}$ is at its minimal value. The black dashed line shows the probability for the most favorable scenario, in which $m_{\beta\beta}$ is equal to its maximum allowed value. The discovery probability monotonically increases by lowering the $m_{\beta\beta}$ value to which an experiment is sensitive. Assuming normal ordering, future experiments with sensitivities of the order of 10 meV will achieve a discovery power between 20 and 80%. Searches able to

| $m_{\beta\beta}$ (Fig. 1) | inaccess. exploration | observation |
|--------------------------|----------------------|-------------|
| 5.0 meV (left)           | $F_{\Sigma}^{\text{min}} - F_{\Sigma}^{\text{max}}$ | 1 - $F_{\Sigma}^{\text{max}}$ |
| 2.5 meV (center)         | 0                    | 1 - $F_{\Sigma}^{\text{max}}$ |
| 0.5 meV (right)          | 0                    | $F_{\Sigma}^{\text{min}} - F_{\Sigma}^{\text{max}}$ | 1 - ($F_{\Sigma}^{\text{min}} - F_{\Sigma}^{\text{max}}$) |
reach 5 meV could reach a discovery power between 50 and 100%.

Fig. 2 can be interpreted also in terms of probabilities of the three possible outcomes of an experiment that have been discussed before: i.e. inaccessibility, observation and exploration. The sum of the probability of these three outcomes is 1 for any value of $m_{\beta\beta}$, as the three outcomes are complementary and mutually exclusive. Their probabilities can be expressed in terms of the CDF as reported in Table I. These probabilities can also be directly read from Fig. 2, by looking at the width of the red, white and green bands, respectively. The smaller the value of $m_{\beta\beta}$ is, the larger the green band becomes, signaling an increasing probability of observing a signal even assuming a signal even in the worst case scenario (red, inaccessibility), and when observing a signal depends on the value of the Majorana phases (white, exploration).

The discovery probabilities shown in Fig. 2 are also reported numerically in Table II. These values have been computed for a Gaussian probability distribution of $\Sigma$ with the centroid and sigma value of Eq. (5). The discovery probabilities change by just a few percents when we fix the centroid and sigma to the other values mentioned earlier in this manuscript. We have also estimated the impact of the functional form of the probability distribution of $\Sigma$. Adopting other reasonable shapes – e.g., a decreasing exponential function – can change the discovery probabilities by up to 10% at the $m_{\beta\beta}$ value of interest for the next-generation experiments, but it does not alter the overall features of our analysis. These considerations confirm that our results and conclusions are very robust and valid regardless of which cosmological analysis is used to extract information on $\Sigma$.

e. Summary

We investigated the impact of cosmological measurements on the possible values of the parameter $m_{\beta\beta}$ and introduced a new procedure (and graphical representation) that presents the advantages of relying on purely empirical facts, i.e. the cosmological measurements, and of not depending on theoretical assumptions, i.e. priors. Our approach is motivated both by the lack of a precise theoretical prediction for the Majorana mass of the neutrino and by the mounting evidence that precision cosmology studies have significant implications on the absolute values of neutrino masses, and hence on the possible values of $m_{\beta\beta}$. In the assumption that the exchange of light Majorana neutrinos dominates the rate of the $0\nu\beta\beta$ transition, the results of this work reinforce the view that future-generation experiments will have a high discovery power even assuming a normal neutrino mass ordering. However, even larger multi-tonne detectors might be needed to find a signal in the less favorable scenarios.

ACKNOWLEDGMENTS

We thank Jason Detwiler, Simone Marcocci and Javier Menéndez for valuable discussions. This work has been...
supported by the Science and Technology Facilities Council (grant number ST/T004169/1), by the Deutsche Forschungsgemeinschaft (SFB No. 1258); and by the research grant number 2017W4HA7S “NAT-NET: Neutrino and Astroparticle Theory Network” under the program PRIN 2017 funded by the Italian Ministero dell’Università e della Ricerca (MIUR).

[1] E. Majorana, Nuovo Cim. 14, 171 (1937).
[2] P. Minkowski, Phys. Lett. B 67, 421 (1977).
[3] T. Yanagida, Proc. Workshop Baryon Number of the Universe and Unified Theories, Tsukuba, Japan, February 1979, 95 (1979).
[4] M. Gell-Mann, P. Ramond, and R. Slansky, Proceedings of the Supergravity Workshop, Stony Brook, New York, USA, September 1979, 315 (1979).
[5] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).
[6] S. Weinberg, Phys. Rev. Lett. 43, 1566 (1979).
[7] W. H. Furry, Phys. Rev. 56, 1184 (1939).
[8] F. Vissani, J. High Energy Phys. 06, 022 (1999).
[9] S. Dell’Oro, S. Marcocci, M. Viel, and F. Vissani, Adv. High Energy Phys. 2016, 2162659 (2016).
[10] A. Pocar et al., (2020), [Discussion presented at Snowmass Workshop].
[11] F. Capozzi, E. Di Valentino, E. Lisi, A. Marrone, A. Melchiorri, and A. Palazzo, Phys. Rev. D 101, 116013 (2020).
[12] I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, T. Schwetz, and A. Zhou, J. High Energy Phys. 09, 178 (2020).
[13] G. Benato, Eur. Phys. J. C 75, 563 (2015).
[14] A. Caldwell, A. Merle, O. Schulz, and M. Totzauer, Phys. Rev. D 96, 073001 (2017).
[15] M. Agostini, G. Benato, and J. Detwiler, Phys. Rev. D 96, 053001 (2017).
[16] S. Dell’Oro, S. Marcocci, and F. Vissani, Phys. Rev. D 100, 073003 (2019).
[17] N. Aghanim et al. (Planck Collaboration), (2018), [Accepted for publication in Astron. Astrophys.], arXiv:1807.06209 [astro-ph.CO].
[18] C. Yèche, N. Palanque-Delabrouille, J. Baur, and H. du Mas des Bourboux, J. Cosmol. Astropart. Phys. 06, 047 (2017).
[19] N. Palanque-Delabrouille, C. Yèche, N. Schöneberg, J. Lesgourgues, M. Walther, S. Chabanier, and E. Arnegaud, J. Cosmol. Astropart. Phys. 04, 038 (2020).
[20] K. Abazajian et al., (2019), arXiv:1907.04473 [astro-ph.IM].
[21] M. A. Alvarez, S. Ferraro, J. C. Hill, R. Hložek, and M. Ilape, (2020), arXiv:2006.06594 [astro-ph.CO].