Optical limiters transmit low-level radiation while blocking electromagnetic pulses with excessively high energy (energy limiters) or with excessively high peak intensity (power limiters). A typical optical limiter absorbs most of the high-level radiation which can cause its destruction via overheating. Here we introduce the novel concept of a reflective energy limiter which blocks electromagnetic pulses with excessively high total energy by reflecting them back to space, rather than absorbing them. The idea is to use a defect layer with temperature dependent loss tangent embedded in a low-loss photonic structure. The low energy pulses with central frequency close to that of the localized defect mode will pass through. But if the cumulative energy carried by the pulse exceeds certain level, the entire photonic structure reflects the incident light (and does not absorb it!) for a broad frequency window. The underlying physical mechanism is based on self-regulated impedance mismatch which increases dramatically with the cumulative energy carried by the pulse.

I. INTRODUCTION

The protection of photosensitive optical components from high incident radiation has applications to areas as diverse as microwave and optical communications to optical sensing [1][9]. As a result, a considerable research effort has focused on developing novel protection schemes and materials that provide control of high-level optical and microwave radiation and prevent damages of optical sensors (including the human eye) and microwave antennas [4][9]. Optical limiters constitute an important class of such protection devices. They are supposed to transmit low-level radiation, while blocking light pulses with high level of radiation. A typical passive optical limiter absorbs most of the high-level radiation, which can cause its destruction via overheating. The most common set-up of a passive optical limiter consists of a single protective layer with complex permittivity \( \epsilon = \epsilon' + i\epsilon'' \), where the imaginary part \( \epsilon'' \) increases sharply with the radiation level. For low-level radiation, the absorption is negligible, and the protective layer is transparent. An increase in the radiation level results in an increase in \( \epsilon'' \), which renders the protective layer opaque. As a consequence, most of the high-level radiation will be absorbed by the limiter, which can cause its overheating and destruction. It turns out that if the same protective layer is incorporated into a certain photonic layered structure, the entire multilayer can become highly reflective for high-level radiation, while remaining transmissive at certain frequencies if the radiation level is low. Such a reflective limiter can be immune to overheating and destruction by high-level laser radiation, which is our main objective.

The physical reasons for the sharp increase in \( \epsilon'' \) with the radiation level can be different. For instance, it can be photoconductivity, heating, two-photon absorption, or any combination of the these mechanisms. In our previous publication [10] we considered the particular case of a strong non-linear dependence of \( \epsilon'' \) of the protective layer on light intensity. This can be attributed, for instance, to a two-photon absorption. We showed that incorporation of such a non-linear layer in a properly designed low-loss layered structure makes the entire assembly act as a reflective power limiter. In this paper, we consider a more practical particular case where the increase in \( \epsilon'' \) is due to heating of the protective layer. We show that, depending on the pulse duration as compared to the thermal relaxation time, the properly design layered structure incorporating such a protective layer can act as a reflective energy limiter, or as a reflective power limiter. Specifically, for short pulses, such a layered structure acts as an energy limiter, reflecting light pulses carrying excessively high energy. By comparison, for sufficiently long pulses, the same structure will act as a power limiter. In either case, most of the incident radiation will be reflected back to space, even though a stand-alone protective layer would act as an absorptive optical limiter.

The proposed architecture consists of a (protective) defect layer embedded in a low-loss Bragg grating. In contrast to the reflective power limiter introduced in [10], the defect layer does not have to be nonlinear, but it must display strong temperature dependence \( \epsilon''(T) \) of the imaginary part of its permittivity. If the total energy carried by the pulse is low, \( \epsilon''(T) \) remains small enough to support a localized mode and the resonant transmittance associated with this mode. If, on the other hand, the energy carried by the pulse exceeds certain level, the defect layer becomes lossy enough to suppress the localized mode, along with the resonant transmittance. The entire stack turns highly reflective, which is consistent with our goal. We refer to this limiter as a reflective energy limiter in order to distinguish it from the nonlinear reflective power limiter introduced in [10]. Finally, if the pulse duration significantly exceeds the thermal relaxation time of the defect layer, the entire layered structure will again
act as a reflective power limiter with the cut-off light intensity determined by the thermal relaxation time of the defect layer – not by the nonlinearity in $\epsilon''$, as was the case in [10].

The organization of the paper is as follows. In Sec. [II] a conceptual design for the reflective energy limiter is presented, along with the mathematical formalism used in our calculations. In Sec. [III] we analyze the role of thermal conductivity. The latter plays an important role if the pulse duration is comparable or exceeds the thermal relaxation time of the defect layer.

![Fig. 1](image-url) *(Color online)* A schematics of a reflective energy limiter. Two identical lossless Bragg reflectors are placed on the left and right of a lossy layer (green). The value of $\epsilon''$ in the defect layer is an increasing function of temperature. (a) Field distribution at the frequency of resonance transmission for an incident pulse with low energy – the field amplitude at the location of the defect layer is exponentially higher than that of the incident wave. (b) Transmittance vs. light wavelength for low incident light energy. (c) Field distribution at the frequency of maximum transmittance for an incident pulse with high energy – the amplitude of the suppressed localized mode is lower than that of the incident wave. (d) Transmittance vs. wavelength for an incident pulse with high energy.

## II. PHYSICAL STRUCTURE AND MATHEMATICAL MODEL

We consider two identical lossless Bragg reflectors consisting of two alternating layers. Each mirror consists of forty layers which are placed at $-L \leq z \leq 0$ and $d \leq z \leq L + d$. For the sake of the discussion we assume that the layers consist of Al$_2$O$_3$ and SiO$_2$ with corresponding permittivities $\epsilon_1 = 3.08$ and $\epsilon_2 = 2.1$. These values are typical for these materials at wavelengths $\lambda \sim 1 \mu m$. The width of layers is assumed to be $d_1 = 151 nm$ and $d_2 \approx 183 nm$ respectively. At $0 \leq z \leq d$ we introduce a defect lossy layer with complex permittivity $\epsilon_d = \epsilon_d' + i\epsilon_d''$. We further assume that the imaginary part of the permittivity of the defect layer depends on the temperature $T$ i.e. $\epsilon_d'' = \epsilon_d''(T)$. For simplicity, we assume linear dependence i.e. $\epsilon_d''(T) = c_1 + c_2 T$ where $c_1$, $c_2$ are some characteristic constants of the defect. Below we assume that $c_1 = 12.11$ (which is a typical value for, say GaAs, at near infrared), $c_1 = 10^{-5}$ and $c_2 = 1$ while the width of the defect layer is taken to be $d = 151 nm$.

The transmittance $T$, reflectance $R$ and absorption $A$ of our set up, and the field profile at any frequency can be calculated via the transfer matrix approach. Specifically, a monochromatic electric field of frequency $\omega$ satisfies the Helmholtz equation:

$$\frac{\partial^2 E(z)}{\partial z^2} + \frac{\omega^2}{c^2} \epsilon(z) E(z) = 0 \quad . \quad (1)$$

At each layer inside the grating, Eq. [1] admits the solution $E^{(j)} = E^{(j)}_f \exp(in_j k z) + E^{(j)}_r \exp(-in_j k z)$, where $n_j = \sqrt{\epsilon_j}$ is the refraction index of the $j$-th layer and $k$ is the wave vector $k = \omega / n_0 c$ ($c$ is the speed of light in the vacuum and $n_0$ is the refractive index of air). Imposing continuity of the field and its derivative at each layer interface, as well as taking into consideration the free propagation in each layer, we get the following iteration relation

$$\begin{pmatrix} E^{(j)}_f \\ E^{(j)}_r \end{pmatrix} = M^{(j)} \begin{pmatrix} E^{(j-1)}_f \\ E^{(j-1)}_r \end{pmatrix} ; M^{(j)} = P_R^{(j)} Q^{(j)} K^{(j)} P_L^{(j)}$$

where

$$Q^{(j)} = \begin{pmatrix} e^{i n_j d_j} & 0 \\ 0 & e^{-i n_j d_j} \end{pmatrix} \quad , \quad K^{(j)} = \begin{pmatrix} n_j + n_j - 1 & n_j - n_j + 1 \\ n_j - n_j - 1 & n_j + n_j - 1 \end{pmatrix} \quad , \quad P_R^{(j)} = \begin{pmatrix} e^{i n_j z} & 0 \\ 0 & e^{-i n_j z} \end{pmatrix} \quad , \quad P_L^{(j)} = \begin{pmatrix} e^{-i n_{j-1} (z-d_j)} & 0 \\ 0 & e^{i n_{j-1} (z-d_j)} \end{pmatrix} \quad (3)$$

At the same time the field outside the layered structured can be written as $E^{(0)}_f(z) = E^{(f)} f \exp(ikz) + E^{(+)}_b \exp(-ikz)$ for $z < -L$ and $E^{(+)}_b(z) = E^{(f)}_f \exp(ikz) + E^{(+)}_b \exp(-ikz)$ for $z > L + d$. The amplitudes of forward and backward propagating waves on the left $z < -L$ and right $z > L + d$ domains are related via the total transfer matrix $\mathcal{M} = I_R^{(2N+2)} K^{(2N+2)} I_L \mathcal{M}^{(j)}$ (where $N$ is the number of layers on each grating and $n_{2N+2} = n_0$):

$$\begin{pmatrix} E^{(+)}_f \\ E^{(+)}_b \end{pmatrix} = \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{21} & \mathcal{M}_{22} \end{pmatrix} \begin{pmatrix} E^{(-)}_f \\ E^{(-)}_b \end{pmatrix} \quad (4)$$

The transmission and reflection coefficients and the field profile, say for a left incident wave, can be obtained by iterating backwards Eqs. [4] together with the boundary conditions $E^{(+)}_b = 0$ and $E^{(+)}_f = 1$ (due to the lineararity of the equations, one can always impose a value for
the outgoing field and calculate via a backward iteration of the transfer matrices the input field to which corresponds (12). Specifically we have $\mathcal{T} \equiv |E^+_f/E^+_r|^2; \mathcal{R} \equiv |E^-_f/E^-_r|^2$. These can be expressed in terms of the transfer matrix elements as $\mathcal{T} = \frac{1}{M_{22}^2}; \mathcal{R} = \frac{M_{23}^2}{M_{22}^2}$. The absorption coefficient $A$ can then be evaluated in terms of transmittances and reflectances as $A \equiv 1 - \mathcal{T} - \mathcal{R}$.

### III. THEORETICAL ANALYSIS

In the case that the permittivity of the defect layer is replaced by $\epsilon_d = \epsilon_1$, the whole structure is periodic and displays a typical dispersion relation consisting of transparent frequency windows (bands) where light is transmitted with near-unity transmittance alternated with frequency windows (gaps) where the incident light is experiencing almost complete reflection.

When the defect is included in the middle of the grating, for zero temperature $T = 0$ corresponding to permittivity $\epsilon_d \approx \epsilon'_d$, the layered structure supports a localized resonant defect mode (see Fig. 1a) with a frequency lying in a photonic band gap of the Bragg grating (see Figs. 1d). For the specific set up that we consider here, we find that a resonant mode is located in the middle of the gap at wavelength $\lambda_0 \approx 1060\,nm$. This defect mode is localized in the vicinity of the defect layer and decays exponentially away from the defect (see Fig. 1b). In the vicinity of the localized mode frequency $\omega$, the entire layered structure displays a strong resonant transmission due to the excitation of the localized mode (see Fig. 1e). In other words, the transmittance is $\mathcal{T} \approx 1$ while the reflectance and the absorption in the absence of any losses are $\mathcal{R} \approx 0$ and $\mathcal{A} \approx 0$ respectively. This picture is still applicable even in the presence of small (but non-zero) dissipative permittivity $\epsilon'_d \neq 0$ (see Fig. 1f).

An alternative expression for the absorption coefficient $A$ can be given in terms of the permittivity and the field intensity $|E(z)|^2$ inside the defect layer. The resulting expression is derived by subtracting the product of Eq. (1) with $E^*(z)$ from its complex conjugate form and then integrating the outcome over the interval $-L \leq z \leq L$.

\[
\left(\frac{E^*}{dz} - E \frac{dE^*}{dz}\right)_{z=L} - \frac{2ik^2}{L} \int_{-L}^{L} \mathcal{I}m\epsilon(z)|E(z)|^2\,dz = 0.
\]

Substituting in Eq. (5), the expressions of the electric field at $z = -L$ and $z = L$ respectively we get

\[
A \equiv 1 - \mathcal{T} - \mathcal{R} = \frac{k}{|E^+_f|^2} \int_{-L}^{L} dz |E(z)|^2 \mathcal{I}m\epsilon(z).
\]

Furthermore, we assume that $\mathcal{I}m\epsilon(z)$ is zero everywhere inside the layered structure apart from the interval $0 \leq z \leq d$ where the defect layer is placed. In this interval it takes a uniform value $\mathcal{I}m\epsilon(0 \leq z \leq d) = \epsilon''_d(T)$.

These simplifications allow us to express the absorption coefficient of Eq. (6) in the following form

\[
\mathcal{A}(T) = \rho(T) \omega \epsilon''_d(T)
\]

where $\rho(T) = \mathcal{I}_d/|E^+|^2$ is the ratio of the integral of light intensity $\mathcal{I}_d = \int_0^d \,dz |E(z)|^2$ at the lossy layer and the incident light intensity. It is obvious from Eq. (7) that $\mathcal{A}(T)$ depends on both the dissipative part of the permittivity and the value of the electric field inside the defect layer. Although the former increases monotonically with the temperature $T$ and thus with the duration time of the incident pulse, this is not true for $\rho(T)$. The latter, which is a unique function of the permittivity, remains approximately constant up to some value of $\epsilon''_d$ above which it decreases, leading eventually to a total decrease of the absorption coefficient together with a simultaneous increase of the reflectivity of the structure. This is related to the fact that the increase of $\epsilon''_d$ spoils the resonant localized mode (see Fig. 1g) which is responsible for high transmittance. Specifically, when the losses due to $\epsilon''_d$ overruns the losses due to leakage from the boundaries of the structure, the resonant mode cease to exist (see Fig. 1h) and the structure becomes reflective, i.e. $\mathcal{R} \approx 1$, and $\mathcal{T} \approx 0$, see Fig. 1i. As a consequence we have that $\mathcal{A} = 1 - \mathcal{T} - \mathcal{R} \approx 0$ and the system does not absorb the high incident energy of the incoming light source but rather reflects it back in space.

In fact, the non-monotonic shape of the envelope of the scattering field in Fig. 1i is a direct consequence of the fact that the structure becomes reflective $\mathcal{R} \approx 1; \mathcal{T} \approx 0$. One has to realize that in the case that both Bragg gratings on the left and right of the defect layer are finite, the field inside each half-space is written as a linear combination of two evanescent contributions with exponentially decreasing and exponentially increasing amplitudes. Their relative weight is determined by the boundary conditions $E(z = L) = E_0^+(-L)$ and $E(L) = E_0^+(L) = E_0^-\sqrt{T}$ at the two outer interfaces of the layered structure. In the case of reflective structures these boundary conditions lead to the relation $E(-L) = E_f^+ \sim O(1)$ and $E(L) \approx 0$. It can be shown rigorously that in this case, the field on the left half-space of the structure is dominated originally by the exponentially decaying component while after some turning point $z_0$ the exponentially increasing component becomes dominant up to the defect layer. After that the field decays exponentially as in the resonant case. Similar scattering field profiles have been found in cases of active (gain) defects [13].

One can use a simple qualitative argument that allows to estimate the condition under which $\mathcal{A}(T)$ continues to increase. As we discuss previously, we assume that the electromagnetic energy losses occur in the lossy defect layer. The dissipated power can be estimated from Eq. (7) to be $\dot{Q} \propto \mathcal{A} \cdot |E^+_f|^2 = \omega \epsilon''_d \mathcal{I}_d$. Due to the energy conservation, the rate of energy dissipation cannot
exceed the energy supply provided by the incident wave. The latter is \( S_{\text{in}} \propto c \cdot |E_f|^2 \). Taking this constraint into account we get the following upper limit on the field intensity at the defect layer location

\[
\frac{c}{\omega \epsilon''(T) d} |E_f|^2 \geq |E_d|^2 \tag{8}
\]

Above we have made the additional approximation that \( I_d \sim |E_d|^2 \cdot d \), where \( E_d \) is a typical value of the field inside the defect layer.

Next we recall that a resonant mode with a frequency \( \omega \) inside the band-gap has a Bloch wave number which is imaginary \( k = ik'' \). The electric field inside the layered structure, can be expressed as a pair of evanescent modes, one of which is decaying with the distance \( z \) and another one which is growing i.e. \( E(z) = E_f \exp(-k''z) + E_b \exp(k''z) \). To the left of the defect \( (-L < z < 0) \), the electric field is dominated by the rising evanescent mode \( E(z) \approx E_b \exp(k''z) \) while to the right of the defect \( (0 < z < L) \), the dominant contribution is provided by the decaying mode \( E(z) \approx E_f \exp(-k''z) \) \([14] \).

The field \( E_d \) at the location of the defect layer is provided by the rising evanescent mode evaluated at \( z = 0 \) i.e. \( E_d \sim E_b \). Therefore, the value of this evanescent mode at the left stack boundary at \( z = -L \) is

\[
E(-L) \propto E_d \exp(-k''L) \tag{9}
\]

Comparing (8) and (9) we can conclude that if

\[
\frac{c}{\omega \epsilon''(T)d} \exp(-2k''L) \ll 1 \tag{10}
\]

then the amplitude of rising evanescent mode \( E(z = -L) \) at the left stack boundary is much less than amplitude of the incident wave

\[
|E(z = -L)|^2 \ll |E_f|^2. \tag{11}
\]

The latter condition Eq. (11), implies that the energy density inside the left grating is much smaller than the energy density of the incident wave, hence, only a small portion of the incident light energy \( S_I \propto c \left| E_f \right|^2 \) will cross the stack boundary at \( z = -L \). In other word, the condition Eq. (10) automatically implies high reflectivity at the stack interface. The condition Eq. (10) for high stack reflectivity (and hence low transmittance and absorption) will always be satisfied if the loss tangent \( \epsilon''(T) \) of the defect layer is large enough and/or if the number of layers in the Bragg grating is large enough.

Next, we want to quantify the above arguments. To this end, we calculate explicitly the transport characteristics of our grating structure for an incident laser pulse. Although the analysis can be generalized for any incident pulse shape, in our numerical simulations below, we have assumed for simplicity that the incident laser pulse has

\[
\frac{\epsilon_d'(t)}{t} \ll \frac{1}{t} \tag{8}
\]

\[
\frac{c}{\omega \epsilon''(T)d} \exp(-2k''L) \ll 1 \tag{10}
\]

\[
|E(z = -L)|^2 \ll |E_f|^2. \tag{11}
\]
which expresses the temporal behavior of permittivity \( \epsilon''_d \) as a function of the pulse duration \( t_f \). Notice that for train pulses the pulse duration \( t_f \) is directly analogous of the total incident energy \( U_f \). We will therefore alternate, in our presentation below, the dependence of \( \epsilon''_d, T, R, A \) from the pulse duration with the (more natural parameter for an energy limiter) total incident energy of the pulse.

Originally \( \epsilon''_d \) is essentially unaffected by the incident energy and the same is true for the resonance mechanism (via the defect mode) that is responsible for high transmittance in the absence of losses. In this domain \( T \approx 1, \ R \approx 0 \) while there is a slow increase of the absorption \( A \), as it can be seen from Fig. 2 (solid lines). Once the incident energy (pulse duration time) exceeds some critical value, there is a rather abrupt increase in \( \epsilon''_d \) which results to the destruction of the resonance mode. Subsequently, the incident energy does not resonate into the structure, leading to a decaying absorption \( A \approx 0 \), while the same is true for the transmittance \( T \approx 0 \). At the same time, there is a noticeable growth of the reflectance which becomes approximately equal to unity \( R \approx 1 \). For comparison we also plot at the same figure the results of the stand-alone layer. We find that for large incident energies (pulse durations \( t_f \)) the absorption \( A(t) \) is higher by more than two orders of magnitude as compared to the case of reflective energy limiter.

We have also performed the same analysis for the case where the thermal conductance \( \kappa \) is different from zero. In Fig. 3 we report the results of the numerical integration of Eq. (15) in the presence of thermal conductivity. For long pulse duration we find a steady state behavior of the transport characteristics of the reflective energy limiter. The physical nature of the steady-state regime is quite obvious. It corresponds to the situation when the heat released in the defect layer is completely carried away by thermal conductivity. At this point, the temperature of the defect layer stabilizes and the time derivative \( d\Phi(t)/dt \) in Eqs. (14,15) vanishes. The latter condition determines the steady-state values of the defect layer temperature as a function of the incident light amplitude. In this limiting case our structure acts as a power limiter. For comparison, the results of the stand-alone layer are also reported in this figure. We find that in the steady-state regime our structure performs superbly resulting in absorption values which are more than two orders of magnitude smaller than the ones achieved by the stand-alone lossy layer.

IV. CONCLUSIONS

At infrared and optical frequencies, the reflectivity of known uniform materials is well below 90%, especially so when the incident light intensity is dangerously high. So, if we want to build a highly reflective optical limiter, we have to rely on photonic structures which would support some kind of low-intensity resonant transmission via slow or localized modes at photonic band-gap frequencies. If
the incident light intensity increases, the respective localized mode must disappear, and the entire photonic structure will behave as a simple Bragg reflector. Here we considered the so-called "dissipative" mechanism of the localized mode suppression. At first glance, it seems counterintuitive, because the high reflectivity and low absorption are caused by the increase in the loss tangent of the defect layer in Fig. 1. A qualitative explanation for such a phenomenon is that the large value of $\epsilon''$ in the defect layer results in decoupling of the left and the right Bragg reflectors in Fig. 1. Of course, there might be other ways to suppress resonant transmittance when the incident light intensity, or the total energy of the pulse, grow dangerously high. Still, the presented "dissipative" mechanism seems simple and practical.

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