Direct Detection of Non-Chiral Dark Matter

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Abstract

Direct detection experiments rule out fermion dark matter that is a chiral representation of the electroweak gauge group. Non-chiral real, complex and singlet representations, however, provide viable fermion dark matter candidates. Although any one of these candidates will be virtually impossible to detect at the LHC, it is shown that they may be detected at future planned direct detection experiments. For the real case, an irreducible radiative coupling to quarks may allow a detection. The complex case in general has an experimentally ruled out tree-level coupling to quarks via Z-boson exchange. However, in the case of two SU(2)_L doublets, a higher dimensional coupling to the Higgs can suppress this coupling, and a remaining irreducible radiative coupling may allow a detection. Singlet dark matter could be detected through a coupling to quarks via Higgs exchange. Since all non-chiral dark matter can have a coupling to the Higgs, at least some of its mass can be obtained from electroweak symmetry breaking, and this mass is a useful characterization of its direct detection cross-section.

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1 Introduction

The evidence for the existence of non-baryonic dark matter is overwhelming. Within the concordance ΛCDM cosmological model, the required dark matter relic density is now known to remarkable accuracy [1]. The nature of the dark matter particles within this model, however, is unknown.

There is a possibility that new physics associated with electroweak symmetry breaking (EWSB) might contain a dark matter candidate with the correct relic density. This is because weakly interacting massive particles (WIMPs) can have the observed dark matter relic density through thermal freeze-out if their mass is on the order of the electroweak (EW) scale. In addition, it is possible to stabilize WIMPs by including a symmetry that forbids their decay into other particles. This allows them to be good dark matter candidates.

The preferred mass of WIMPs suggests the possibility that they may be produced and detected at the upcoming Large Hadron Collider (LHC) at CERN. Two other types of experiments attempting to detect dark matter are indirect and direct detection experiments. While the indirect detection experiments look for the particles that are produced from annihilating dark matter, the direct detection experiments attempt to infer the presence of dark matter particles as they scatter off nuclei within detectors by looking for the resulting nuclear recoil.

The rationale for the direct detection experiments is that the dark matter lies in a halo which encompasses our Milky Way galaxy. As the earth and sun rotate around the galactic center, detectors on the earth move through the halo and intersect the path of dark matter particles, which are expected to scatter off the nuclei inside the detectors. Since the local dark matter density is not known better than to within a factor of two, there is some uncertainty in the expected scattering rate [2]. Depending on the experimental setup, the nuclear recoil from the scattering would produce ionization, phonons or scintillation, any of which can be observed. Examples of direct dark matter detection experiments include
CDMS, DAMA, NaIAD, PICASSO, ZEPLIN, EDELWEISS, CRESST, XENON and Warp [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13].

The dark matter scattering off nuclei within a detector can proceed via two fundamentally different types of interactions. There is, on the one hand, a spin-independent, or coherent, interaction between the dark matter and the nucleons. In this case the contribution of each nucleon to the total scattering cross-section interferes constructively across the nucleus. Scattering off nuclei is therefore enhanced roughly by a factor of $A^2$ in the cross-section, where $A$ is the number of nucleons in the nucleus. This large enhancement factor is absent for the other type of interaction, which is spin-dependent, and couples the dark matter spin to the spin of the nuclei. The large enhancement factor is also the main reason that much tighter constraints (a factor of about $10^5 - 10^6$) exist on the SI cross-section, normalized to cross-section per nucleon, than on the SD cross-section.

In this paper, fermion dark matter transforming under the EW gauge group $SU(2)_L \times U(1)_Y$ will be added to the standard model (SM), and the observational consequences at a direct detection experiment will be discussed. In particular, chiral and non-chiral (real and complex) representations of $SU(2)_L \times U(1)_Y$ will be considered in §2 and §3, respectively, and the focus will be on spin-independent interactions for the reasons discussed in the previous paragraph. §4 discusses how the direct detection cross-section may be characterized in terms of the fraction of the dark matter mass that is obtained through EWSB. This characterization is particularly useful for EW singlet dark matter. The conclusions are presented in §5.

The results of this paper are summarized in Figure 3. Shown are the current experimental upper bounds on the spin-independent cross-section for WIMP scattering off nucleons from XENON10 (solid line) [14], the projected upper bounds for SuperCDMS 2-ST at Soudan (blue dashed line), SuperCDMS 25kg / 7-ST at Snolab (green dashed line), XENON1T (magenta dashed line) and SuperCDMS Phase C (red dashed line) [15, 16, 17]. The cross-sections for chiral and non-chiral dark matter are shown, in addition to the Higgs contribution to the direct detection cross-section for a variety of parameter choices.

## 2 Chiral Electroweak Dark Matter

Chiral EW matter is forbidden to have an explicit mass term in the Lagrangian since such a mass term is not gauge invariant. It instead has a Yukawa coupling to the Standard Model Higgs field and gains all its mass from EWSB through the Higgs mechanism. Chiral EW dark matter particles are thus Dirac fermions.

EW precision measurements put tight constraints on additional chiral matter. For example, an additional doublet of colorless heavy fermions gives a contribution of $1/6\pi$ to the electroweak $S$-parameter, which is about $1.8\sigma$ away from its measured central value. An additional degenerate generation is disfavored even more strongly at the 99.95% confidence level [18].

Although EW precision measurements still allow room for chiral EW dark
matter, direct detection experiments rule it out as a viable dark matter candidate. The reason is that it has a vector coupling to the $Z$-boson and can therefore scatter coherently off the nuclei inside the detector via a tree-level $Z$-boson exchange. The resulting cross-section is large enough that such dark matter particles would already have been seen [19].

In general, the cross-section per nucleon for dark matter scattering coherently off nuclei via the exchange of a $Z$-boson is given by

$$\sigma \simeq \frac{G_F^2 m_N^2}{2\pi} \left[ (1 - 4 \sin^2 \theta_W) Z - (A - Z) \right] \frac{1}{A^2} \mathbf{Y}^2. \quad (1)$$

Here, $G_F$ is the Fermi coupling constant, $m_N$ is the reduced mass of the dark matter mass ($m_\chi$) and nucleon mass ($m_N$), $A$ ($Z$) is the mass (atomic) number of the nucleus, $\theta_W$ is the weak mixing angle, and $\mathbf{Y} \equiv \frac{1}{2}(Y_L + Y_R)$, where $Y_L$ and $Y_R$ are the hypercharge of the left- and right-handed components of the dark matter particle [19]. The convention chosen here is $Q = T_3 + \frac{1}{2}Y$, where $Q$ is the electric charge, $T_3$ is the third component of the isospin, and $Y$ is the hypercharge of the particle. The term proportional to $Z$ in the square brackets is for the dark matter scattering off the protons inside the nucleus. It is suppressed since $1 - 4 \sin^2 \theta_W$ is very small. The term proportional to $A - Z$ is for the dark matter scattering off the neutrons inside the nucleus, and it dominates. The factor of $1/A^2$ normalizes the cross-section to a cross-section per nucleon.

Chiral EW dark matter has $Y_R = Y_L \pm 1$, i.e. $\mathbf{Y} = Y_L \pm \frac{1}{2}$. For the CDMS experiment, for example, which uses Germanium ($^{73}_{32}$Ge), the scattering cross-section per nucleon then becomes

$$\sigma \gtrsim 5 \times 10^{-40} \text{ cm}^2, \quad (2)$$

for $\mathbf{Y} \geq \frac{1}{2}$. This result is roughly independent of the mass of the dark matter, at least for a large enough dark matter mass. A Dirac neutrino [1] saturates the lower bound as it has $Y_L = 1$ and $Y_R = 0$, and thus $\mathbf{Y} = \frac{1}{2}$. For $m_\chi$ above roughly 10 GeV, the cross-section is larger than current bounds, see Figure 3 and such chiral EW dark matter is therefore ruled out as a viable dark matter candidate. Note that for $m_\chi$ less than about 10 GeV (and down to about 2 eV, at which point the dark matter ceases to be “cold”), the direct detection cross-section is not larger than the experimental bound. However, since these particles couple to the $Z$-boson, the $Z$ could have decayed into them. The precise CERN LEP measurement of the invisible decay of the $Z$-boson rules out this possibility.

### 3 Non-Chiral Dark Matter

Non-chiral, or vector, matter is different from chiral matter in that an explicit mass term in the Lagrangian is allowed. Even though, a priori, there is nothing...
that protects this explicit mass term from being large, its size can nevertheless naturally be on the order of the EW scale. This may happen if, for example, the underlying high-scale theory has a global chiral symmetry that is spontaneously broken at the EW scale, but that forbids an explicit mass term at higher scales.

Non-chiral matter is not subject to the same tight constraints from EW precision measurements as is chiral matter. This is because there is no renormalizable coupling to the Higgs field. Although there is a higher dimensional (non-renormalizable) coupling to the Higgs, this does not cause any conflict with EW precision measurements. Instead, this coupling implies that non-chiral matter gains some small fraction of its mass from EWSB. It will be seen that the fraction of the dark matter particle's mass that comes from EWSB is useful characterization of the dark matter's direct detection cross-section. This will be discussed further in §4.

Stability and electric neutrality are basic requirements of any dark matter particle. Since massive non-chiral representations are allowed to carry conserved quantum numbers, which prohibits their mixing with Standard Model fermions, the lightest state of such an additional representation can indeed be stable. Moreover, such representations contain both new neutral and new charged particles. The charged particles are several hundred MeV heavier than the neutral particles due to EWSB. Intuitively one can understand the mass difference as arising from different one-loop corrections to the masses and wave-functions: the charged components receive corrections from both virtual photons and Z-bosons in the loop, whereas the neutral components receive corrections only from virtual Z-bosons [20]. This means that the lightest state of an additional massive non-chiral representation can also be expected to be neutral.

It is useful to divide non-chiral representations up further into real and complex representations. Each of these will now be discussed by focusing on an explicit example.

### 3.1 Real representations of $SU(2)_L \times U(1)_Y$

If the dark matter particle is part of a real representation of $SU(2)_L \times U(1)_Y$, then its hypercharge, $Y$, must be zero. Since the charge, $Q$, of the dark matter must be zero, this also implies $T_3 = Q - \frac{1}{2}Y = 0$. The dark matter particle, now a Majorana fermion, therefore does not couple to the Z-boson, and there is no coherent tree-level scattering off nuclei. This makes it “safe” from the current experimental bounds.

As an example, consider the dark matter to be part of an $SU(2)_L$ triplet with zero hypercharge,

$$L = \begin{pmatrix} L^+ \\ L^0 \\ L^- \end{pmatrix}. \quad (3)$$

Here the neutral component $L^0$ is a possible dark matter candidate. The explicit
mass term in the Lagrangian is given by

\[ \mathcal{L} \supset -\frac{m}{2} (2L^+L^- + L^0L^0). \]  

(4)

The non-renormalizable operator that, after EWSB, splits the mass of the neutral components from the mass of the charged components by several hundred MeV is given by

\[ \mathcal{L} \supset \epsilon^{abc} L^a L^b H^\dagger T^c H, \]

(5)

where the \( T^a, a = 1, 2, 3, \) are the \( SU(2)_L \) generators, and \( H \) is the Standard Model Higgs field.

The interactions of \( L^0 \) with the Standard Model gauge bosons and the charged fields \( L^\pm \) are given by

\[ gW^+_\mu (-L^+\bar{\sigma}^\mu L^0 + L^0\bar{\sigma}^\mu L^-) + gW^-_\mu (-L^0\bar{\sigma}^\mu L^+ + L^-\bar{\sigma}^\mu L^0). \]

(6)

Two-component spinor notation for the dark matter is employed throughout this paper, while four-component Dirac notation will be used below for the quark fields (in equation (6), \( \bar{\sigma}^\mu = (I_2, \vec{\sigma}) \) and \( \bar{\sigma}^\mu = (I_2, -\vec{\sigma}) \), where \( \vec{\sigma} \) are the usual Pauli matrices).

Note the absence of any coupling of the neutral component \( L^0 \) to the \( Z \)-boson. This means there is no tree-level scattering for \( L^0 \) off nuclei, making this a viable dark matter candidate. There is, however, an irreducible one-loop coupling to nucleons, which will be discussed in \( \S 3.3 \).

The particle \( L^0 \) behaves like a wino-like lightest supersymmetric particle (LSP) found in the Minimal Supersymmetric Standard Model (MSSM). Assuming that \( L^0 \) makes up all of the dark matter in the universe, it may be shown that it must have a mass of about

\[ m_{L^0} \approx 2 \text{ TeV} \]

(7)

to give the correct dark matter relic density. This mass was estimated from Figure 4 in [21]. Non-perturbative electroweak corrections to the dark matter annihilation cross-section as included in [22] require the dark matter to have a mass of about 2.7 TeV to obtain the correct relic density.

It is interesting to note that if \( L^0 \) makes up most of the dark matter component in the universe, it will most likely be very difficult to detect at the LHC. Although a detailed collider study is beyond the scope of this paper, the following comments are meant to give an indication of this difficulty. Since the \( L^{\pm,0} \) are heavy and weakly interacting, their production cross-sections are small. They may be very roughly estimated to be on the order of \( 10^{-5} - 10^{-4} \) pb, as may be extrapolated from Figure 2 in [23], which shows the production cross-section for the related wino-like neutralinos and charginos in the MSSM. Moreover, the charged states \( L^\pm \) are split from the neutral state \( L^0 \) only by a small amount, so that even though they produce ionizing charged tracks, they do so only within the inner portion of the detector, before they each decay into the neutral state by emitting a soft pion [20]. The missing energy from the two neutral particles escaping the detector
balances, so that there is not much visible missing energy. At the LHC it is very
difficult to trigger on this, and such dark matter particles will thus be extremely
difficult to detect at the LHC. It is possible but unlikely that a detailed collider
study will change this conclusion.

3.2 Complex representation of $SU(2)_L \times U(1)_Y$

If the dark matter particle is part of a complex representation of $SU(2)_L \times U(1)_Y$, then its hypercharge is nonzero. Since the charge of the dark matter must be zero, $T_3 = -\frac{1}{2}Y$. The dark matter particle, now a Dirac fermion, therefore couples to the $Z$-boson at tree-level. In the notation of equation (1), $Y_L = Y_R \equiv Y$, and the cross-section per nucleon for scattering off nuclei is given by

$$\sigma \simeq \frac{G_F^2 m_N^2}{2\pi} \frac{1}{A^2} \left[ (1 - 4 \sin^2 \theta_W)Z - (A - Z) \right]^2 Y^2. \quad (8)$$

For the CDMS experiment, using Germanium, the scattering cross-section per
nucleon then becomes

$$\sigma \simeq 2 \times 10^{-39} \ Y^2 \ \text{cm}^2,$$  

where $Y = -1$. The explicit
mass term in the Lagrangian is given by

$$\mathcal{L} \supset -m L_1 L_2,$$  

where the $SU(2)_L$ indices are contracted as $\epsilon_{\alpha\beta} L_1^\alpha L_2^\beta$. The neutral components of each doublet together form a neutral Dirac fermion.

There is an accidental $U(1)_{L_1 L_2}$ symmetry under which $L_1$ and $L_2$ transform
opposite to each other. This symmetry requires the neutral components to be part of a Dirac fermion, and thus allows the tree-level scattering off nuclei via $Z$-boson exchange. An operator which violates this symmetry can, however, split the Dirac state into a pseudo-Dirac state, which consists of two Majorana fermions that have a tiny mass splitting. This splitting can substantially suppress the tree-level scattering.
The non-renormalizable operator that, after EWSB, splits the mass of the neutral components from the mass of the charged components by several hundred MeV is given by
\[ L \supset L_2 T^a L_1 H^\dagger T^a H, \] (12)
where the \( T^a \) are the \( SU(2) \) generators \([20]\). This operator, however, only affects the splitting of the charged states from the neutral states. Since it does not violate the \( U(1)_{L_1 L_2} \) symmetry, it does not affect the neutral Dirac state, whose scattering off nuclei remains unchanged.

However, a non-renormalizable operator that does violate the \( U(1)_{L_1 L_2} \) symmetry is given by
\[ L \supset -\frac{c}{M} (L_1 H)(L_1 H) + h.c. - \frac{c^*}{M} (L_2 H^c)(L_2 H^c) + h.c., \] (13)
where brackets indicate that the \( SU(2)_L \) indices are contracted, \( H^c = i\sigma_2 H^* \), and \( H \) has been assigned hypercharge \( Y = -1 \). The scale \( M \) is some high mass scale at which this operator is generated, and \( c \) is an \( O(1) \) coefficient. Note that in writing down this term, the discrete symmetry \( L_1 \leftrightarrow (L_2)^c \) was assumed, so that the coefficients are the same up to complex conjugation (removing this assumption leaves unchanged the main conclusion, namely that the neutral Dirac state will be split). This operator only exists for dark matter that has hypercharge \( |Y| = 1 \).

Once the Higgs field obtains a vacuum expectation value, \( v \), and EW symmetry has been broken, the neutral components get an additional contribution to the mass, which can be written as \( \delta = \frac{c}{M} v^2 \). \( M \) will have to be large enough to ensure \( |\delta| \ll m \). Including corrections up to \( O\left(\frac{\text{Im} \delta}{m}\right) \) or \( O\left(\frac{\text{Re} \delta}{m}\right) \), the mass term may be written as
\[ -\frac{1}{2} \begin{pmatrix} L_1^0 & L_2^0 \end{pmatrix} \begin{pmatrix} \delta & m \\
-m & \delta^* \end{pmatrix} \begin{pmatrix} L_1^0 \\
L_2^0 \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} \chi_2 & \chi_1 \end{pmatrix} \begin{pmatrix} m + \text{Re} \delta & 0 \\
0 & m - \text{Re} \delta \end{pmatrix} \begin{pmatrix} \chi_2 \\
\chi_1 \end{pmatrix}, \] (14)
where the neutral mass eigenstates are given by
\[ \chi_1 \simeq \frac{i}{\sqrt{2}} \left(-1 + \frac{1}{2} \frac{\text{Im} \delta}{m}\right) L_1^0 + \left(1 + \frac{1}{2} \frac{\text{Im} \delta}{m}\right) L_2^0 \] (15)
and
\[ \chi_2 \simeq \frac{1}{\sqrt{2}} \left(1 + \frac{1}{2} \frac{\text{Im} \delta}{m}\right) L_1^0 + \left(1 - \frac{1}{2} \frac{\text{Im} \delta}{m}\right) L_2^0 \] (16)
These are the two Majorana fermions that make up the pseudo-Dirac state. Ignoring higher order corrections, the mass eigenstates may also be written as
\[ \chi_1 \simeq \frac{i}{\sqrt{2}} (-L_1^0 + L_2^0), \quad m_1 = m - \text{Re} \delta \] (17)
and
\[ \chi_2 \simeq \frac{1}{\sqrt{2}} (L_1^0 + L_2^0), \quad m_2 = m + \text{Re} \delta. \] (18)
Here $\chi_1$, the lighter of the two Majorana particles, is the dark matter particle. It behaves like a higgsino-like LSP found in the MSSM. Assuming that $\chi_1$ makes up all of the dark matter in the universe, it must have a mass of about

$$m_{\chi_1} \simeq 1 \text{ TeV},$$

(19)
to give the correct dark matter relic density. This mass was estimated from Figure 4 in [21]. Non-perturbative electroweak corrections are negligible as discussed in [22].

At lowest order, the couplings among the neutral fields, $\chi_1$ and $\chi_2$, and the charged fields, $L_1^-$ and $L_2^+$, are given by

$$L_1^i(i\bar{\sigma}^\mu \partial_\mu)L_1 + L_2^i(i\bar{\sigma}^\mu \partial_\mu)L_2 + gW_\mu^+ \left[\frac{1}{2}(\chi_2^\dagger - i\chi_1^\dagger)\bar{\sigma}^\mu L_1^\dagger\right]$$

$$-\frac{1}{2}L_2^+\bar{\sigma}^\mu(\chi_2 - i\chi_1) + gW_\mu^- \left[-\frac{1}{2}(\chi_2^\dagger + i\chi_1^\dagger)\bar{\sigma}^\mu L_2^\dagger + \frac{1}{2}L_1^\dagger\bar{\sigma}^\mu(\chi_2 + i\chi_1)\right]$$

$$+ \frac{g}{\cos\theta_W} Z_\mu \left[L_1^\dagger\bar{\sigma}^\mu(-\frac{1}{2} + \sin^2\theta_W)L_1^- + L_2^\dagger\bar{\sigma}^\mu(\frac{1}{2} - \sin^2\theta_W)L_2^+\right]$$

$$+ i\frac{g}{2}(\chi_2^\dagger\bar{\sigma}^\mu\chi_1 - \chi_1^\dagger\bar{\sigma}^\mu\chi_2)\right] + eA_\mu \left[ - L_1^\dagger\bar{\sigma}^\mu L_1^- + L_2^\dagger\bar{\sigma}^\mu L_2^+ \right].$$

(20)

Including the next higher order correction, the coupling of the dark matter to the $Z$-boson becomes

$$\frac{g}{2\cos\theta_W} Z_\mu \left[i(\chi_2^\dagger\bar{\sigma}^\mu\chi_1 - \chi_1^\dagger\bar{\sigma}^\mu\chi_2) + \frac{\text{Im}\delta}{m}(\chi_2^\dagger\bar{\sigma}^\mu\chi_2 - \chi_1^\dagger\bar{\sigma}^\mu\chi_1)\right].$$

(21)

Equations (20) and (21) show that the only coupling $\chi_1$ has to itself at tree-level is suppressed by a factor of $\frac{\text{Im}\delta}{m}$. The dominant coupling of $\chi_1$ is to $\chi_2$, and it is possible for $\chi_1$ to scatter inelastically off nucleons via $Z$-boson exchange ($\chi_1 \to \chi_2$). This inelastic scattering will be kinematically inaccessible if the mass splitting between $\chi_1$ and $\chi_2$ ($\sim 2\text{ Re}(\delta)$) is large enough. Since the typical recoil energies of the nuclei in the detector are expected to be on the order of a few 10’s of keV, a splitting of a few 10’s of keV is required in order to forbid the inelastic scattering via $Z$-boson exchange [2, 30]. This means that $\frac{\text{Im}\delta}{m}$ can be as small as $\sim 10^{-7} - 10^{-8}$, so that the cross-section for the scattering of $\chi_1$ to $\chi_1$ off nuclei is suppressed by a factor of $\left(\frac{\text{Im}\delta}{m}\right)^2 \sim 10^{-14} - 10^{-16}$, which ensures it lies well below the current experimental bound. Note also that this requires the scale of the new physics which generates the operator that breaks the $U(1)_{L_1L_2}$ symmetry to be roughly $M \lesssim 10^8 - 10^9$ GeV.

\^2The question of whether the scattering is kinematically allowed or not depends critically on the mass of the nuclei in the detector. It is thus possible to carefully choose $\delta$ in such a way that scattering will take place in a heavier target such as NaI used by DAMA, but not in a lighter target such as Ge used by CDMS. The possibility of using this to explain the DAMA signal, in the absence of a signal by CDMS and others, was discussed in [28, 29]. (The fact that the dark matter in the halo would follow a Maxwell-Boltzmann distribution of velocities complicates, but does not invalidate, the statements just made.)
For appropriate values of the mass splitting the dark matter can therefore not scatter off the nuclei at tree-level. This makes it “safe” from current experimental bounds. There is, however, again an irreducible one-loop coupling to nucleons, which will be discussed in §3.3.

It should be noted that $\chi_1$ will most likely be extremely difficult to detect at the LHC. The reasoning is similar to that mentioned at the end of §3.1 for the case of the $SU(2)_L$ triplet with zero hypercharge. The LHC production cross-section of $\chi_1$ here is only marginally larger (since it is less massive), about $10^{-4} - 10^{-3}$ pb. This was estimated from Figure 2 in [31], which shows the production cross-section for the related higgsino-like neutralinos and charginos in the MSSM. Moreover, the direct production of this type of dark matter and the associated charged particles will again only give rise to signals that are very difficult to trigger on at the LHC. Their associated production with jets, for example, has a cross-section that is too small to be visible above background events (see [32], which looked at collider signatures for a higgsino-like lightest supersymmetric particle). The non-chiral dark matter proposed in this paper thus seems to be extremely difficult to detect at the LHC. Although a detailed LHC collider study is beyond the scope of this paper, it seems unlikely that it would change this conclusion.

### 3.3 Direct detection of non-chiral dark matter

The previous two subsections considered non-chiral dark matter that is either a real or a complex representation of $SU(2)_L \times U(1)_Y$. For real representations, there is no tree-level coupling between the dark matter and the nuclei. For complex representations, the tree-level coupling is completely negligible, if the Dirac state has been appropriately split into a pseudo-Dirac state. Although the absence of any tree-level coupling allows non-chiral dark matter particles to be consistent with current experimental limits, there is an irreducible one-loop coupling which is large enough for it to be detectable in future direct detection experiments. These irreducible one-loop couplings are given in Figure 11.

For real representations, the one-loop diagrams involve the $W$-bosons, but not the $Z$-boson. As an explicit example, consider the $SU(2)_L$ triplet with zero hypercharge ($L^0$). Its couplings to the $W$-bosons and to the additional charged states ($L^{\pm}$) are given in equation (6). The effective Lagrangian for the coherent interaction between the dark matter and the quarks is

$$4 \alpha_2^2 \pi \sum_q \left[ \frac{1}{8} I_L^W (m_W / m_{L^0}) \frac{1}{m_W m_h^2} (L^0 L^0 + L^{0i} \bar{L}^{0i}) m_q \bar{q}q + \frac{1}{12} \bar{L}^0 i D^\mu \sigma^\nu L^{0i} \bar{L}^{0i} i D^\mu \sigma^\nu L^0 \right] \times$$

$$\bar{q}(\gamma_\mu i D_\nu + \gamma_\nu i D_\mu - \frac{1}{2} g_{\mu\nu} i \mathcal{D}) q.$$

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3For indirect dark matter detection rates and for prospects of detecting the associated charged particles among the ultra-high energy cosmic rays see [22].
Fig. 1: Feynman diagrams for the irreducible one-loop couplings between non-chiral dark matter and quarks. For real representations of $SU(2)_L \times U(1)_Y$, there are only $W$-bosons within the loop. For complex representations of $SU(2)_L \times U(1)_Y$, there are both $W$- and $Z$-bosons in the loop. The symbol $h$ denotes the Standard Model Higgs boson, $\chi$ denotes the dark matter particle, and $q$ refers to quarks. There is also a cross-diagram for the diagram on the left which needs to be included.

This result\textsuperscript{4} was obtained by assuming that the momentum carried by the quarks in the Feynman diagram on the left in Figure 1 is small but non-zero; in the Feynman diagram on the right the momentum of the quarks was set to zero, and therefore no momentum was assumed to flow through the Higgs propagator. The functions $f^W_I$ and $f^W_{II}$ are given by

$$f^W_I(x) = \frac{1}{3\pi} \left( \frac{12 - 12x^2 + 2x^4}{\sqrt{4 - x^2}} \arctan \left( \frac{1}{x} \sqrt{4 - x^2} \right) + 2x + (4x - x^3) \ln x^2 \right)$$

(23)

$$f^W_{II}(x) = \frac{1}{4\pi} \left( \frac{16 + 12x^2 - 12x^4 + 2x^6}{\sqrt{4 - x^2}} \arctan \left( \frac{1}{x} \sqrt{4 - x^2} \right) - 5x + 2x^3 + (4x^3 - x^5) \ln x^2 \right).$$

(24)

These functions have been normalized to equal one in the limit $x \to 0$. This is a useful normalization since here $x \approx m_W/m_{L^0} \ll 1$.

For higher dimensional representations, there is an additional factor in equation (22). For an $n$-tuplet of $SU(2)_L$ with zero hypercharge this additional factor is given by $(n^2 - 1)/8$.

For complex representations, the one-loop diagrams involve the $W$- and $Z$-bosons. As an explicit example, consider the dark matter candidate from two

\textsuperscript{4}The result for the one-loop computation agrees on-shell with [33] for $m_W/m_{L^0} \to 0$, although here the operator $\left( \frac{1}{2} L^0 L^0 + \frac{1}{4} L^{0\dagger} L^{0\dagger}) \right) \bar{q} \bar{D} q$ is found to vanish, and the coefficient of the twist-two operator is a factor of two larger than in [33]. The results of this paper do not agree off-shell with [34], who considered wino-like and higgsino-like lightest supersymmetric particles in the MSSM. Since the result agrees on-shell with [33], the final cross-sections calculated in this paper are also very similar in magnitude. (It is more difficult to numerically compare the cross-sections with those of [34] since their’s is dependent on various MSSM parameters.)
SU(2)_L doublets of opposite hypercharge (χ_1). Its couplings to the W- and Z-bosons, to the additional charged states L_1^- and L_2^+, and to the slightly heavier neutral state χ_2 are given in equation (20). The effective coherent interaction between the dark matter and the quarks due to W-bosons in the loop is given by equation (22) by replacing L^0 with χ_1 and by including a factor of 1/4 which multiplies the whole equation. The effective Lagrangian for the coherent interaction between the dark matter and the quarks due to a Z-boson in the loop is given by

\[
\frac{\alpha^2 \pi}{\cos^4 \theta_W} \sum_q \left[ \left( \frac{1}{16} f^Z_I(m_Z/m_{χ_1}) \left( \frac{c_V^q}{m_Z^2} - \left( \frac{c_A^q}{m_Z^2} \right)^2 \right) + \frac{1}{16} f^Z_{II}(m_Z/m_{χ_1}) \frac{1}{m_Z m_h^2} \right) \times \left( \chi_1 \chi_1^* + \chi_1^\dagger \chi_1^\dagger \right) m_q \bar{q}q \\
+ \frac{1}{24} f^Z_{III}(m_Z/m_{χ_1}) \left( \frac{c_V^q}{m_Z^2} + \left( \frac{c_A^q}{m_Z^2} \right)^2 \right) \left( \chi_1 i D^\mu \sigma^\nu \chi_1^\dagger + \chi_1^\dagger i D^\mu \bar{\sigma}^\nu \chi_1 \right) \times \bar{q} \left( \gamma_\mu \bar{D} \nu + \gamma_\nu \bar{D} \mu - \frac{1}{2} g_{\mu \nu} \bar{D} \right) q \right] \tag{25}
\]

where the functions f^Z_I, f^Z_{II} and f^Z_{III} are given by

\[
f^Z_I(x) = \frac{1}{\pi} \left( \frac{4 - 2x^2 + x^4}{\sqrt{4 - x^2}} \right) \arctan \left( \frac{1}{x} \sqrt{4 - x^2} \right) + x - \frac{1}{2} x^3 \ln x^2 \tag{26}
\]

\[
f^Z_{II}(x) = \frac{1}{\pi} \left( \frac{4 + 4x^2 - 2x^4}{\sqrt{4 - x^2}} \right) \arctan \left( \frac{1}{x} \sqrt{4 - x^2} \right) - 2x + x^3 \ln x^2 \tag{27}
\]

\[
f^Z_{III}(x) = \frac{1}{8\pi} \left( \frac{32 + 16x^2 - 32x^4 + 8x^6}{\sqrt{4 - x^2}} \right) \arctan \left( \frac{1}{x} \sqrt{4 - x^2} \right) - 4x + x^3 + (8x^3 - 4x^5) \ln x^2 \right) \tag{28}
\]

These functions have also been normalized to equal one in the limit x → 0. This is again a useful normalization since here x ≡ m_Z/m_{χ_1} ≪ 1. On the quark line the coupling of the Z boson to the quarks is given by −\frac{g}{\cos \theta_W} \gamma^\mu \frac{1}{2} (c_V^q - c_A^q \gamma^5), where c_V^q = T^q_q - 2 \sin^2 \theta_W Q_q, c_A^q = T^q_q, Q_q is the quark charge, and T^q_q = +\frac{1}{2} (-\frac{1}{2}) for up (down)-type quarks.

For higher dimensional complex representations, there are additional factors in the W-bosons contribution in equation (22) and in the Z-boson contribution in equation (25). For an n-tuplet of SU(2) with n = Y + 1, there is an additional factor of (n^2 - (1 - Y)^2)/16 multiplying equation (22). However, if n > Y + 1, then there are more charged states that the dark matter particle can couple to, comparing the result found here with the on-shell result given in the published version of [33] for m_Z/m_{χ_0} → 0 the following discrepancy is found: the Higgs contribution is a factor of 3 smaller here, and (on-shell) the factor of 3(c_V^q)^2 appearing in [33] for the box-diagram is here found to be (c_A^q)^2.
and the additional factor multiplying equation (22) is given by \((n^2 - (1 + Y^2))/8\). For an \(n\)-tuplet of hypercharge \(Y\), the factor that needs to multiply equation (25) is given by \(Y^2\).

The effective coupling between dark matter and the quarks involves several operators at a scale of order \(m_Z\) (which is the value of the dominant momentum in the loops of the diagrams in Figure 1). These operators are the scalar operator \(m_q\bar{q}q\), the trace operator \(\bar{q}i\partial q\) (which was found to vanish, but there is nothing that in principle forces it to vanish), and the traceless twist-two operator \(\frac{1}{2}\bar{q}(\gamma_\mu iD_\nu + \gamma_\nu iD_\mu - \frac{1}{2}g_\mu_\nu \partial)q\). The traceless twist-two operator and trace operator are part of the quark energy momentum tensor given by \(\bar{q}\gamma(\mu i\partial_\nu)q\).

The nucleon matrix element of the scalar operator \(m_q\bar{q}q\) for light quarks is

\[
\langle N|m_q\bar{q}q|N\rangle = f_{T_u}^N m_{N\bar{N}N},
\]

where on the right hand side of the equation \(N\) denotes a nucleon, and

\[
f_{T_u}^p \simeq 0.020 \pm 0.004, \quad f_{T_d}^p \simeq 0.026 \pm 0.005, \quad f_{T_s}^p \simeq 0.118 \pm 0.062
\]

\[
f_{T_u}^n \simeq 0.014 \pm 0.003, \quad f_{T_d}^n \simeq 0.036 \pm 0.008, \quad f_{T_s}^n \simeq 0.118 \pm 0.062.
\]

The main contribution comes from the strange quark content of the nucleon, which also has the largest uncertainty. Heavy quarks, \(Q\), also contribute to the mass of the nucleon. This can be derived by making use of the anomaly relating the heavy quarks to the gluons [2],

\[
\langle N|m_Q\bar{Q}Q|N\rangle = \langle N \rangle - \frac{\alpha_s}{12\pi} G_\mu G^{a\mu}|N\rangle = \frac{2}{27} f_{T_G}^N m_{N\bar{N}N},
\]

where,

\[
\frac{2}{27} f_{T_G}^N = \frac{2}{27} \left(1 - \sum_{u,d,s} f_{T_s}^N\right) \simeq 0.062.
\]

Although the results in this paper suggest that the trace operator \(\bar{q}i\partial q\) vanishes, and its nucleon matrix element is therefore not needed, one may estimate it as follows: The nucleon matrix element for light quarks may be estimated as

\[
\langle N|\bar{q}i\partial q|N\rangle = \langle N|m_q\bar{q}q|N\rangle.
\]

An accurate determination of the nucleon matrix element for the trace operator with heavy quarks involves the calculation of higher loop diagrams as shown in Figure 2 and is beyond the scope of this paper (see for example [36]). Instead, as a crude approximation, equation (33) may also be used for the heavy quarks \(Q\) together with (31).

The twist-two quark operator is given by

\[
\mathcal{O}_q^{(2)\mu\nu} = \frac{1}{2}\bar{q}(\gamma_\mu iD_\nu + \gamma_\nu iD_\mu - \frac{1}{2}g_\mu_\nu \partial)q.
\]

A linear combination of scale-dependent twist-two quark operators,

\[
\sum_q \lambda_q [\mathcal{O}_q^{(2)}] \frac{m_Z^2}{m_Z^2},
\]

12
is generated at the scale $m_Z$, with coefficients $\lambda_q$ that may be read from equations (22) and (25). For the $W$-contribution to the scattering amplitude, the coefficients are the same for all quarks $q$, but for the $Z$-contribution they differ for up- and down-type quarks. Under QCD rescaling, the twist-two quark operator mixes with the twist-two gluon operator. This may be taken into account by rewriting equation (35) as a linear combination of operators that rescale multiplicatively [37]. One of these operators is the QCD energy-momentum tensor $T^{\mu\nu}$

$$T^{\mu\nu} = \sum_q O_q^{(2)\mu\nu} + O_G^{(2)\mu\nu},$$

where $O_G^{(2)}$ is the twist-2 gluon operator given by

$$O_G^{(2)\mu\nu} = G^{a\rho\mu} G^{a\rho\nu} - \frac{1}{4} g^{\mu\nu} G^{a\rho} G^{a\rho}. \quad (37)$$

Another operator that may be rescaled multiplicatively is

$$O_-^{\mu\nu} = \frac{16}{3} \sum_q O_q^{(2)\mu\nu} - n_f O_G^{(2)\mu\nu}, \quad (38)$$

where $n_f$ is the number of active quark flavors ($n_f = 5$ at the scale $m_Z$). In the case of the $W$-contribution, for which all $\lambda_q$ are the same, equation (35) can be rewritten in terms of the operators (36) and (38). For the $Z$-contribution, however, $\lambda_q$ differs for up- and down-type quarks, so that other operators that rescale multiplicatively are required. These are flavor non-singlet combinations of the individual quark operators $O_{qi}^{(2)} - O_{qj}^{(2)}$ that do not mix with the gluon operator since the gluon contributions cancel out.
The linear combination of twist-two quark operators \( \mathcal{O}_{\mu
u} \) can thus be rewritten in terms of operators whose QCD rescaling is simple. The operators may then be rescaled down to low scales, so that \( \mathcal{O}_{\mu
u} \) may be written in terms of operators that are evaluated at low scales. The energy-momentum tensor \( T_{\mu\nu} \) has zero anomalous dimension, whereas \( \mathcal{O}_{\mu
u} \) and the flavor non-singlet combinations \( \mathcal{O}_{q_i}^{(2)} - \mathcal{O}_{q_j}^{(2)} \) have positive anomalous dimension given by \( \frac{16}{3\alpha_s/\pi} \left( \frac{16}{3} + n_f \right) \) and \( \frac{16\alpha_s/\pi}{3} \), respectively. This means that running to the infrared, \( T_{\mu\nu} \) does not get renormalized whereas the other operators, \( \mathcal{O}_{\mu\nu} \) and \( \mathcal{O}_{q_i}^{(2)} - \mathcal{O}_{q_j}^{(2)} \), both decrease. The dominant contribution at low scales to the linear combination of twist-two quark operators generated at \( m_Z \) is thus from the quark energy momentum tensor, whose contribution is known exactly. The other contributions are subdominant, and may be estimated from the parton distribution functions (PDFs); helpful for this is [38]. The expression for (35), written in terms of the operators evaluated at a lower scale, will not be reproduced here. However, it was checked that for a lower scale equal to 1 GeV, the subdominant contributions that require knowledge of the PDFs amount to only about 17% in the case of the \( W \)-contribution and 14% in the case of the \( Z \)-contribution (care was taken to decrease the active number of quark flavors from five to four at the scale of the bottom quark mass and from four to three at the scale of the charm quark mass). This shows that the nucleon matrix element of the twist-two quark operator can be estimated reliably.

The nucleon matrix element of the twist-two quark operators may be evaluated by using the expression [2]

\[
\langle N(p)|\mathcal{O}_{q}^{(2)}_{\mu\nu}|N(p)\rangle = \frac{1}{m_N}\left( p^\mu p^\nu - \frac{1}{4}m_N^2 g^{\mu\nu} \right)
\times \int_0^1 dx \, x \left( q(x, \mu^2) + \bar{q}(x, \mu^2) \right), \tag{39}
\]

where \( p^\mu \) denotes the momentum of the nucleon, and zero momentum transfer was assumed. The PDF \( q(x, \mu^2) \) (or \( \bar{q}(x, \mu^2) \)) gives the probability density of finding the quark \( q \) (or anti-quark \( \bar{q} \)) in the nucleon with momentum fraction \( x \). The integral denotes the second moment of the PDF, and one may define

\[
q(2, \mu^2) = \int_0^1 dx \, x \, q(x, \mu^2). \tag{40}
\]

The PDFs depends on the scale \( \mu \) at which the twist-two operator was generated, so that here \( \mu = m_Z \). Using the website [38] and the results from the CTEQ group (CTEQ6M) [39], the second moment of the PDFs may be determined directly at this scale (equivalently, \( q(2, \mu^2) \) may be determined at \( \mu = 1 \) GeV if the linear combination of twist-two quark operators is first rescaled down to 1 GeV). The second moment of the PDFs for the proton for \( \mu = m_Z \) are given by

\[
u(2) \simeq 0.221, \quad \bar{u}(2) \simeq 0.034,
\]
\[
d(2) \simeq 0.115, \quad \bar{d}(2) \simeq 0.039,
\]
\[
s(2) \simeq 0.026, \quad \bar{s}(2) \simeq 0.026, \tag{41}
\]
\[ c(2) \simeq 0.019, \quad \bar{c}(2) \simeq 0.019, \]
\[ b(2) \simeq 0.012, \quad \bar{b}(2) \simeq 0.012, \]
\[ G(2) \simeq 0.47. \]

\( G(2) \) is the PDF of the gluon, which is not needed here. For the neutron, the values of \( u(2) \) and \( \bar{u}(2) \) are interchanged with \( d(2) \) and \( \bar{d}(2) \), respectively.

The nucleon matrix elements discussed above may now be used to write the spin-independent effective Lagrangian for non-chiral dark matter scattering off nucleons as
\[
\mathcal{L}_{\text{eff}, \, N} \simeq \mathcal{C} \, m_N \left( \frac{1}{2} \chi \chi^\dagger + \frac{1}{2} \chi^\dagger \chi \right) \bar{N} N, \tag{42}
\]
where \( \mathcal{C} \) is determined from equations \((22)\) and \((25)\) and using the nucleon matrix elements. The cross-section for the non-chiral dark matter particle to scatter off nuclei (normalized to a single nucleon) is then
\[
\sigma_\chi N = \frac{1}{\pi} \mu_{\chi N}^2 m_N^2 \mathcal{C}^2, \tag{43}
\]
where \( \mu_{\chi N}^2 \) is the reduced mass of the nucleon and the dark matter. The cross-section for a dark matter particle from an \( SU(2)_L \) triplet with \( Y = 0 \) is roughly the same when scattering off a proton or a neutron, and the average is given by
\[
\sigma_{\chi N}^{L_0} \simeq 1.9 \times 10^{-45} \text{ cm}^2. \tag{44}
\]
The cross-section for a dark matter particle from two \( SU(2)_L \) doublets with opposite hypercharge \( Y = \pm 1 \), after splitting the Dirac state into a pseudo-Dirac state, is also roughly the same when scattering off a proton or a neutron, and the average is given by
\[
\sigma_{\chi N}^{L_1} \simeq 2.1 \times 10^{-46} \text{ cm}^2. \tag{45}
\]
A Higgs mass of \( m_h = 120 \text{ GeV} \) was assumed. For higher dimensional representations there are additional factors which increase the cross-section, as discussed below equations \((22)\) and \((25)\). For example, a quintuplet of \( SU(2)_L \) with \( Y = 0 \) has a cross-section that is larger by a factor of 9 than the triplet cross-section, i.e.
\( \sigma \simeq 3.9 \times 10^{-44} \text{ cm}^2 \).

Figure 3 shows the results for the cross-section and how they compare to current experimental exclusion bounds, as well as projected future bounds. The current upper bound on the direct detection cross-section is roughly two to three orders of magnitude higher than the calculated cross-sections in \((44)\) and \((45)\), respectively. Interestingly, XENON1T will get close to the required sensitivity to see an \( SU(2)_L \) triplet with zero hypercharge and should be able to detect an \( SU(2)_L \) quintuplet with zero hypercharge, while SuperCDMS 25kg / 7-ST at Snolab may not quite be able to detect the triplet, but will get close to detecting the quintuplet. Experiments planned for well into the future, such as the proposed SuperCDMS “Phase C” \([16, 17]\), should be able to also probe the required parameter space for the case of the two \( SU(2)_L \) doublets with opposite hypercharge.
Fig. 3: A comparison of the results presented in this paper with current and projected experimental bounds for the cross-section of dark matter scattering off a nucleon. Shown are the current experimental upper bounds from XENON10 (solid black line) [14], and the projected upper bounds for SuperCDMS 2-ST at Soudan (blue dashed line), SuperCDMS 25kg / 7-ST at Snolab (green dashed line), XENON1T (magenta dashed line) and SuperCDMS Phase C (red dashed line) [15, 16, 17]. The dashed black horizontal line is the theoretical lower bound on the cross-section for chiral electroweak dark matter scattering coherently off nuclei via the exchange of a $Z$-boson, see §2. The black dot (•) is the predicted cross-section for a 1 TeV non-chiral dark-matter particle part of two $SU(2)_{L}$ doublets with opposite hypercharge (a complex representation of $SU(2)_{L}$), assuming its coupling to the $Z$-boson is forbidden by splitting the Dirac state into a pseudo-Dirac state; see §3.2. Without the latter assumption, the cross-section is given by the open circle (○) and would be ruled out. The black square (■) is the predicted cross-section for a 2 TeV non-chiral dark matter particle part of an $SU(2)_{L}$ triplet with zero hypercharge (a real representation of $SU(2)_{L}$), see §3.1. Dark matter from higher order real or complex representations has a larger direct-detection cross-section than those represented by the black square or by the black dot, respectively, see §3. The dotted diagonal lines represent the Higgs contribution to dark matter scattering off nucleons for a range of magnitudes of the Higgs to dark matter coupling. This coupling also determines what fraction, $f_{\text{ewsb}} \equiv m_{\chi}/m_{\chi}$, of the dark matter mass comes from electroweak symmetry breaking, and the lines shown are for various $f_{\text{ewsb}}$. The experimental results shown in this figure were obtained through [40].
4 Higgs contribution to the Direct Detection Cross-Section and Singlet Dark Matter

In this section, singlet dark matter will be discussed, and a useful characterization of its direct detection cross-section will be given. Dark matter that is a singlet under $SU(2)_L \times U(1)_Y$ does not have any irreducible couplings to quarks, unlike the non-chiral dark matter discussed in §3. It will be assumed that the singlet dark matter does not couple to the Higgs at the renormalizable level and does not obtain a mass spontaneously. Rather, the singlet will be allowed in the Lagrangian to have an explicit mass term which is not associated with the EW scale. Although there is no renormalizable coupling between the singlet and the SM, no symmetries forbid the existence of a non-renormalizable interaction generated by new physics beyond the SM at some high scale. The gauge invariant operator coupling the dark matter $\chi$ to the Higgs is an infinite sum of higher-dimensional operators,

$$L_{h\chi\chi} = \frac{c_1}{\Lambda_1} \chi H^\dagger H + \frac{c_2}{\Lambda_2} \lambda \chi (H^\dagger H)^2 + \ldots + \frac{c_n}{\Lambda_{n-1}^2} \lambda \chi (H^\dagger H)^n + \ldots, \quad (46)$$

where one Higgs field is replaced by the physical Higgs boson $h/\sqrt{2}$, and all others acquire a vacuum expectation value of $v/\sqrt{2} \approx 174$ GeV. The $c_n$ are dimensionless coefficients and the $\Lambda_n$ are the scales at which the higher dimensional operators are generated by new physics.

The Higgs-dark-matter coupling (46) is allowed more generally for any non-chiral dark matter, whether it is a singlet or forms a non-trivial representation of the EW gauge group. For singlet dark matter, the coupling (46) is generated at a scale $\Lambda_1$ by new physics. For non-chiral dark matter with non-trivial EW quantum numbers, the coupling is already generated at the EW scale by integrating out the $W$-bosons (and, for complex representations, also the $Z$-boson), as shown in Figure 1 in §3.

The existence of this Higgs to dark matter coupling also implies the existence of additional contributions to the dark matter mass when all of the Higgs fields in (46) acquire a vacuum expectation value. This means that non-chiral dark matter obtains at least some of its mass from EWSB. Denoting the dark matter mass by $m_\chi$ and the mass that is not associated with EWSB by $m_0$, gives the relation

$$m_\chi = m_0 + m_{\text{EWSB}}, \quad (47)$$

where $m_{\text{EWSB}} \approx \frac{v^2}{2\Lambda_1} + \ldots$ is the mass gained from EWSB.

The mass obtained by the dark matter from EWSB is a useful characterization of the Higgs contribution to the direct detection cross-section. The latter is given by (see also [11], [42])

$$\sigma_N^\chi \approx \frac{g^2}{4\pi m_W^2 m_h^2} \mu_{\chi N}^2 m_N^2 \left( \sum_q f_{T_q}^N \right)^2 g_{h\chi\chi}, \quad (48)$$

where $g_{h\chi\chi} \approx c_1 v/2\Lambda_1 \approx m_{\text{EWSB}}/v$ is the Higgs to dark matter coupling, and $f_{T_q}^N$ may be taken from equations (29) – (32).
120 GeV gives
\[ \sigma_N^\chi \simeq 8 \times 10^{-47} \mu^2_{\chi N} m_{\text{ewsb}}^2, \]  
(49)
or,
\[ \sigma_N^\chi \simeq 8 \times 10^{-47} \mu^2_{\chi N} m_{\chi}^2 f_{\text{ewsb}}, \]  
(50)
where
\[ f_{\text{ewsb}} \equiv \frac{m_{\text{ewsb}}}{m_{\chi}} \]  
(51)
is the dark matter mass fraction obtained from EWSB. The cross-section is seen to be directly proportional to the square of this fraction.

The various dotted lines in Figure 3 show the cross-section for \( f_{\text{ewsb}} = 1, 10^{-1}, 10^{-2}, 10^{-3}, \) and \( 10^{-4} \), as well as the current experimental bounds. (Constraints on \( m_{\chi} \) and \( f_{\text{ewsb}} \) from the known dark matter relic density are not included in the present discussion, but see for example [41], [42]). These lines represent the Higgs contribution to the direct detection cross-section. Modulo destructive interference with other contributions, they represent the lower bounds of the direct detection cross-section also for non-chiral dark matter that is not an EW singlet.\(^6\)

If the dark matter is associated with new physics at the EW scale, the fraction \( f_{\text{ewsb}} \) should not be too small. The current bound has ruled out dark matter with a mass heavier than about 1 TeV and that obtains more than 10% of its mass from EWSB. SuperCDMS “Phase C” would be able to rule out dark matter with a mass heavier than about 1 TeV and that obtains more than about 0.1% of its mass from EWSB. This means that, assuming \( c_1 \sim \mathcal{O}(1) \), SuperCDMS “Phase C” would probe a scale of \( \Lambda_1 \sim \mathcal{O}(30 \text{ TeV}) \). As the direct detection experiments probe ever smaller values of \( f_{\text{ewsb}} \), the absence of any direct detection signal would make relevant the question of whether one should abandon the idea that dark matter is associated with new physics at the EW scale.

5 Conclusions

Fermion dark matter transforming under the electroweak gauge group \( SU(2)_L \times U(1)_Y \) was added to the standard model, and the observational consequences at a direct detection experiment were discussed. Figure 3 summarizes the results.

Chiral electroweak dark matter is well known to be not a viable dark matter candidate, as it has a spin-independent coupling to nuclei via the \( Z \)-boson, which gives a cross-section that is ruled out by two to three orders of magnitude.

Non-chiral dark matter from real representations of \( SU(2)_L \times U(1)_Y \) has an irreducible one-loop spin-independent coupling to nuclei. The triplet has a mass of about 2 TeV and a cross-section that is about two order of magnitude below current experimental bounds. A future experiment with a very large sensitivity, such as

\(^6\)Note that for the \( SU(2)_L \) triplet with zero hypercharge (\( \text{§3.1} \)), the Higgs contribution to the direct detection cross-section is about \( 8.7 \times 10^{-47} \text{cm}^2 \), so that \( f_{\text{ewsb}} \simeq 5.5 \times 10^{-4} \) and \( m_{\text{ewsb}} \simeq 1.1 \text{ GeV} \). For the \( SU(2)_L \) doublets with opposite hypercharge (\( \text{§3.2} \)), the Higgs contribution amounts to about \( 1.6 \times 10^{-47} \text{cm}^2 \), so that \( f_{\text{ewsb}} \simeq 4.5 \times 10^{-4} \) and \( m_{\text{ewsb}} \simeq 0.5 \text{ GeV} \).
the proposed XENON1T, is required to probe the relevant region of parameter space. Higher order representations have a larger cross-section which makes it easier to detect them.

Non-chiral dark matter from complex representation of $SU(2)_L \times U(1)_Y$ have a tree-level coupling to nuclei via $Z$-boson exchange, which would rule it out unless this tree-level coupling can be suppressed somehow. For two $SU(2)_L$ doublets with opposite hypercharge the tree-level coupling can be suppressed by a dimension five operator that couples the Higgs to the dark matter particle and is able to split the neutral Dirac state into a pseudo-Dirac state. The remaining irreducible one-loop coupling allows such a dark matter particle to be detected at a very sensitive future planned direct detection experiment such as SuperCDMS “Phase C”. Its mass is required to be about 1 TeV to reproduce the observed dark matter relic density.

Although a detailed LHC collider study was not done in this paper, non-chiral dark matter particles are most likely extremely difficult to detect at the LHC. The reason is that not many of them will be produced since they are not only required to be heavy to reproduce the observed relic density, but they are also weakly interacting. This is in addition to the fact that they would not even provide a signal that can easily be triggered on.

Non-chiral dark matter has a coherent coupling to the standard model fermions through the Higgs field. The existence of this coupling to the Higgs also means that at least some of its mass is obtained from electroweak symmetry breaking. Non-chiral dark matter from non-trivial representations of the electroweak gauge group does indeed gain a small fraction, about $10^{-3}$, of its mass from electroweak symmetry breaking. For dark matter that is a singlet under the electroweak gauge group, a non-renormalizable coupling to the Higgs could allow it to be detected at a direct detection experiment (the singlet’s dominant coupling to the Higgs was assumed to be through a dimension five operator). A useful characterization of the direct detection cross-section is given by the fraction of mass that the dark matter particle obtains through electroweak symmetry breaking, the amplitude being directly proportional to this fraction. The current experimental bound has ruled out dark matter with a mass heavier than about 1 TeV and that obtains more than 10% of its mass from EWSB. SuperCDMS “Phase C” would be able to rule out dark matter with a mass heavier than about 1 TeV and that obtains more than 0.1% of its mass from EWSB. As the direct detection experiments probe ever more of the available parameter space, the absence of any direct detection signal would at some point make relevant the question of whether one should abandon the idea that dark matter is associated with new physics at the EW scale.

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