Variations on leptogenesis

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Abstract

We study variations of the standard leptogenesis scenario that can arise if an additional mass scale related to the breaking of some new symmetry (as for example a flavor or the B-L symmetry) is present below the mass $M_{N_1}$ of the lightest right-handed Majorana neutrino. Our scheme is inspired by $U(1)$ models of flavor à la Froggatt-Nielsen, and involves new vectorlike heavy fields $F$. We show that depending on the specific hierarchy between $M_{N_1}$ and the mass scale of the fields $F$, qualitatively different realizations of leptogenesis can emerge. We compute the $CP$ asymmetries in $N_1$ decays in all the relevant cases, and we conclude that in most situations leptogenesis could be viable at scales much lower than in the standard scenario.

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1 Introduction

Baryogenesis through leptogenesis represents an attractive mechanism to explain the observed matter-antimatter asymmetry of the Universe [1,2]. Once the Standard Model (SM) is extended by including the see-saw mechanism in order to explain the strong suppression of the neutrino mass scale [3], the possibility of generating a cosmic lepton asymmetry via the lepton number and
CP violating out-of-equilibrium decays of the seesaw singlet neutrinos arises as a natural possibility. Partial conversion of the lepton asymmetry into a baryon asymmetry then proceeds by means of electroweak sphaleron interactions [4] that are non-perturbative SM processes. Qualitatively, it is almost unavoidable that a model that includes the seesaw mechanism will predict a certain amount of matter-antimatter asymmetry surviving until the present epoch, and then the question of whether leptogenesis is able to explain the puzzle of the baryon asymmetry of the Universe is essentially a quantitative one. In recent years, quantitative analysis of the standard leptogenesis scenario have become more and more sophisticated, taking into account many subtle but significant ingredients, such as various washout effects [5,6,7], thermal corrections to particle masses and CP violating asymmetries [8], spectator processes [9], flavor effects [10,11,12,13,14,15,16] and the possible effects of the heaviest right handed Majorana neutrinos $N_{2,3}$ [10,17,18,19] (for reviews of the most recent results see [20]).

One assumption that is common to all these studies is that between the scale of the breaking of lepton number and the electroweak breaking scale, there are no additional sources of new physics that could affect the mechanism of leptogenesis. This assumption is certainly justified both in terms of simplicity and also because it allows for a certain level of predictivity, that is mainly due to the fact that the same couplings that determine the CP asymmetries and the out-of-equilibrium conditions in the decays of the heavy Majorana singlets are also responsible for the seesaw masses of the light neutrinos. In particular, in the standard seesaw model, successful leptogenesis implies a lower bound on the values of $M_{N_1}$ that, even in the most favorable case of dominant $N_1$ initial abundance [8], puts a direct test of leptogenesis out of the reach of any foreseeable experiment.$^1$

In this paper we explore the implications of the presence below $M_{N_1}$, of an additional energy scale related to the breaking of a new symmetry (that could be for example a flavor symmetry). As we will see, in some cases a large enhancement of the $CP$ asymmetry in $N_1$ decays is easily obtained, while at the same time the scale of leptogenesis can be lowered by several orders of magnitude without conflicting with other conditions. In section 2 the main features of the model are outlined. In section 3, after reviewing the main results of standard leptogenesis, we highlight the most important new features that stem from the presence of the extra mass scale. In section 4 we present the conclusions. In the appendix we consider a minimal scenario where one light neutrino remains massless, and we derive a simple analytical expression for the ratio between the new mass scale and $M_{N_1}$.

$^1$ Only in the case of resonant leptogenesis [6], scales much lower than the limit $M_{N_1} \gtrsim 10^7$ GeV [8] seem to be possible.
2 The model

We assume that at some large scale close to the leptogenesis scale a horizontal $U(1)_X$ symmetry forbids direct couplings between the lepton doublets $l$ and the heavy Majorana neutrinos $N$. Light neutrino masses can arise because of the presence of heavy vectorlike fields $F, \bar{F}$ that are singlets with respect to $SU(2)_L \times U(1)_Y$ and charged under the additional $U(1)_X$ factor, and that couple to both the Majorana fields $N$ and to the lepton doublets $l$. We use Greek indices $\alpha, \beta = 1, 2 \ldots$ to denote the heavy Majorana neutrinos $N_\alpha$, Latin indices $a, b = 1, 2 \ldots$ to denote the heavy vectorlike fields $F_a, \bar{F}_a$ and Latin indices $i, j = 1, 2, 3$ to denote the SM lepton doublets $l_i$. The following Lagrangian gives a simple realization of this scheme:

$$
-\mathcal{L} = \frac{1}{2} N_\alpha M_{N_\alpha} N_\alpha + \bar{F}_a M_{F_a} F_a + \left( h_{ia} \bar{l}_i \gamma^5 {l_i} F_a \Phi + \lambda_{\alpha a} \bar{N}_\alpha F_a S + \text{h.c.} \right),
$$

where $\Phi$ is the $SU(2)_L$ Higgs doublet and $S$ is a complex scalar that is a singlet under $SU(2)_L \times U(1)_Y$ and charged under $U(1)_X$. The scalar $S$ is responsible for breaking the additional symmetry through a vacuum expectation value (vev) $\sigma \equiv \langle S \rangle$ that we assume to be somewhat smaller than the mass scale of the vectorlike fields $\sigma \lesssim M_F$. Note that the assignments $L(l_i, F_a) = +1$, $L(N_\alpha, \bar{F}_a) = -1$, $L(S, \Phi) = 0$ imply that all the interaction terms in eq. (1) conserve lepton number that, as in the standard seesaw, is broken (by two units) only by the $N_\alpha$ Majorana mass term. A simple $U(1)_X$ charge assignment yielding the Lagrangian (1) is for example $X(l_i, F_a) = +1$, $X(S, \Phi) = 0$. Note that if for all the SM fields the charges $X$ were proportional to $B - L$, the $U(1)_X$ symmetry would leave unaffected all the charged leptons and quarks Yukawa couplings. In contrast, if the set of $X$ charges for the quarks and leptons is not a trivial one, $U(1)_X$ could play a role as (part of) a flavor symmetry of the kind proposed long ago by Froggatt and Nielsen [21]. However, for the present discussion the issue if $U(1)_X$ also contributes to determine the charged fermion mass pattern is to a large extent irrelevant, and accordingly we will not necessarily adopt all the assumptions that realize the naturalness conditions of flavor models based on Abelian symmetries, and in particular:

- We do not constrain the couplings in the Lagrangian eq. (1) to be all of the same size and of order unity.
- We do not need to specify any precise value for the ratio $\sigma/M_F$.
- We do not constrain from below by means of FCNC considerations the scale of $U(1)_X$ breaking (this is consistent if e.g. we take $X = B - L$).
After $U(1)_X$ and electroweak symmetry breaking, the light neutrino mass matrix arising from (1) reads

$$-M_{ij} = \begin{bmatrix} h^* \sigma M_F^2 \lambda^T \frac{v^2 M_N^2}{M_F} h \end{bmatrix}_{ij} = \begin{bmatrix} \tilde{\lambda}^* \frac{v^2 M_N^2}{M_F} \tilde{\lambda} \end{bmatrix}_{ij}$$

(2)

where for convenience we have introduced the effective seesaw couplings

$$\tilde{\lambda}_{ai} = \left( \frac{\lambda}{M_F} h^i \right)_{ai}.$$  

(3)

That is, with respect to the standard seesaw mechanism the light neutrino masses have an additional suppression factor of the ratio $\sigma^2/M_F^2$ and are of fourth order in the fundamental couplings ($\lambda$ and $h$).

An important point to note is that in order to ensure that two light neutrinos are massive, as is required by oscillation neutrino data, a minimum field content of two right-handed neutrinos $N_\alpha$ and two vectorlike fields $(F_a, \bar{F}_a)$ is needed. A straightforward analysis then shows that even in this minimal scheme, both the matrices of the $h$ and $\lambda$ coupling constants contain physical complex phases that can be relevant for leptogenesis.

To summarize, in this model besides the electroweak breaking scale $v$ we have the following new mass scales:

- the mass scale of the heavy vectorlike fields, $M_F$.
- the lepton number breaking scale, $M_N$.
- the horizontal symmetry breaking scale, $\sigma$.

The consequences of the different hierarchies amongst these new scales is studied in detail in section 3.

### 3 The different possibilities

In this section we present the detailed results for the $CP$ asymmetry in $N_1$ decays for the different cases as are determined by the hierarchy between the relevant scales $M_{N_1}$, $M_F$ and $\sigma$, and we will explore qualitatively the implications of the different possibilities. As already said, we take $N_1$ to be the lightest one of the Majorana neutrinos, and $F_1$ to be the lightest of the vectorlike fields. For the different mass ratios we adopt the following notation:

$$z_\alpha = \frac{M_{N_\alpha}}{M_{N_1}^2}, \quad \omega_a = \frac{M_{F_a}^2}{M_{F_1}^2}, \quad r_a = \frac{M_{N_\alpha}}{M_{F_a}}.$$  

(4)
The $CP$-asymmetry in the decay of the heavy Majorana neutrinos $N_1$ is defined in the usual way as

$$\epsilon_{N_1} = \frac{\Gamma_{N_1} - \bar{\Gamma}_{N_1}}{\Gamma_{N_1} + \bar{\Gamma}_{N_1}},$$

(5)

where $\Gamma_{N_1}$ and $\bar{\Gamma}_{N_1}$ represent respectively the partial decay rates of $N_1$ into particles with lepton number $L = +1$ and antiparticles with lepton number $L = -1$ (regardless of the fact that they are $l$ or $F$ states). A non-vanishing numerator in (5) can arise from the interference between tree-level and loop-amplitudes, and the total decay rate in the denominator can be approximated with the tree level result. When the ratio (5) is phase-space independent, as is the case in two-body decays, the $CP$-asymmetry can be simply calculated in terms of products of the tree-level $M_0$ and loop $M_1$ amplitudes. However, when there is a dependence on the phase space as is the case for three-body decays discussed in section 3.2.1, the full decay widths have to be taken into account.

In standard leptogenesis models the lepton number breaking scale is constrained by the out-of-equilibrium condition on the decay rate of $N_1$, and as the scale of lepton number violation is lowered, in order to satisfy this condition the size of the $N_1$ Yukawa couplings must be accordingly reduced. If we further require that the neutrino oscillation data are accounted for just by the (type 1) seesaw mass matrix, and we forbid any additional source for the light neutrino masses, then it can be shown that a large suppression of the $CP$ asymmetry in $N_1$ decays is unavoidable. Then the requirement that the final lepton asymmetry is large enough to account for the baryon asymmetry of the Universe, implies a lower bound on the mass of $N_1$ that is several orders of magnitude larger than the electroweak breaking scale.

As we will discuss below, in some cases the presence of a new scale and of a new set of couplings associated with it can allow to satisfy the out-of-equilibrium condition and to account for the light neutrino mass scale, without necessarily implying any particular suppression of the $CP$ asymmetries, even when $M_{N_1}$ is lowered down to the TeV scale. This ‘decoupling’ of the size of the $CP$ asymmetry from the decay width $\Gamma_{N_1}$ and from the scale of the light neutrino masses is rendered possible by the fact that, while the latter two quantities are mainly controlled by the $\lambda$ parameters that couple the heavy vectorlike fields $F$ to the right handed neutrino $N_1$, the $CP$ asymmetry is essentially determined by the couplings $h$ between the fermions $F$ and the lepton doublets $l$.

3.1 The standard leptogenesis case: $M_F, \sigma \gg M_N$

When all the scales related to the $U(1)_X$ symmetry are larger than the Majorana neutrino masses $(M_F, \sigma > M_N)$ there are no major differences from the
standard Fukugita-Yanagida leptogenesis model \[1\]. After integrating out the $F$ fields one obtains the standard seesaw Lagrangian containing the effective operators $\tilde{\lambda}_{i\alpha}N_\alpha l_i \Phi$ with the seesaw couplings $\tilde{\lambda}_{i\alpha}$ given in eq. (3). The right handed neutrino $N_1$ decays predominantly via 2-body channels as shown in fig. 1. This yields the standard results that for convenience we recall here. The total decay width is $\Gamma_{N_1} = (M_{N_1}/16\pi) (\tilde{\lambda}\tilde{\lambda}^\dagger)_{11}$ and the sum of the vertex and self-energy contributions to the $CP$-asymmetry for $N_1$ decays into the flavor $l_j$ reads \[22\]

$$\epsilon_{N_1 \rightarrow l_j} = \frac{1}{8\pi(\tilde{\lambda}\tilde{\lambda}^\dagger)_{11}} \sum_{\beta \neq 1} \text{Im} \left\{ \tilde{\lambda}_{\beta j} \tilde{\lambda}^*_{1j} \left[ (\tilde{\lambda}\tilde{\lambda}^\dagger)_{\beta 1} \tilde{F}_1(z_{\beta}) + (\tilde{\lambda}\tilde{\lambda}^\dagger)_{1\beta} \tilde{F}_2(z_{\beta}) \right] \right\}, \quad (6)$$

where

$$\tilde{F}_1(z) = \frac{\sqrt{z}}{1-z} + \sqrt{z} \left( 1 - (1+z) \ln \frac{1+z}{z} \right), \quad \tilde{F}_2(z) = \frac{1}{1-z}. \quad (7)$$

At leading order in $1/z_\beta$ and after summing over all leptons $l_j$, eq. (6) yields for the total asymmetry:

$$\epsilon_{N_1} = \frac{3}{16\pi(\tilde{\lambda}\tilde{\lambda}^\dagger)_{11}} \sum_{\beta} \text{Im} \left\{ \frac{1}{\sqrt{z_\beta}} (\tilde{\lambda}\tilde{\lambda}^\dagger)_{\beta 1}^2 \right\}. \quad (8)$$

where the sum over the heavy neutrinos has been extended to include also $N_1$ since for $\beta = 1$ the corresponding combination of couplings is real.

In the hierarchical case $M_{N_1} \ll M_{N_{2,3}}$ the size of the total asymmetry in (8) is bounded by the Davidson-Ibarra limit \[23\]

$$|\epsilon_{N_1}| \leq \frac{3}{16\pi} \frac{M_{N_1}}{v^2} (m_{\nu_3} - m_{\nu_1}) \lesssim \frac{3}{16\pi} \frac{M_{N_1}}{v^2} \frac{\Delta m^2_{\text{atm}}}{2m_{\nu_3}}, \quad (9)$$

where $m_{\nu_i}$ (with $m_{\nu_1} < m_{\nu_2} < m_{\nu_3}$) are the light neutrinos mass eigenstates and $\Delta m^2_{\text{atm}} \sim 2.5 \times 10^{-3} \text{eV}^2$ is the atmospheric neutrino mass difference. It is now easy to see that (9) implies a lower limit on $M_{N_1}$. The amount of $B$ asymmetry that can be generated from $N_1$ dynamics can be written as:

$$\frac{n_B}{s} = -\kappa_s \epsilon_{N_1} \eta \quad (10)$$

where $\kappa_s \approx 1.3 \times 10^{-3}$ accounts for the dilution of the asymmetry due to the increase of the Universe entropy from the time the asymmetry is generated with respect to the present time, $\eta$ (that can range between 0 and 1, with typical values $10^{-1} - 10^{-2}$) is the efficiency factor that accounts for the amount of $L$ asymmetry that can survive the washout process. WMAP data on the cosmic background anisotropy \[24\] and considerations of big bang nucleosynthesis \[25\] yield the experimental value $n_B/s \approx (8.7 \pm .4) \times 10^{-11}$, and
therefore, assuming that $\epsilon_{N_1}$ is the main source of the $B - L$ asymmetry [19], eqs. (9) and (10) yield:

$$M_{N_1} \gtrsim 10^9 \frac{m_{\nu_3}}{\eta \sqrt{\Delta m^2_{\text{atm}}}} \text{GeV}. \quad (11)$$

This limit can be somewhat relaxed depending on the specific initial conditions [8] or when flavor effects are included [12,15] but the main point remains, and that is that the value of $M_{N_1}$ should be well above the electroweak scale.

3.2 Variations on leptogenesis: $\sigma < M_{N_1}$

Interesting new possibilities arise when the $U(1)_X$ symmetry breaking scale is lower than the leptogenesis scale, that is $\sigma < M_{N_1}$. In this case $N_1$ cannot decay directly into the light lepton doublets via the two body channel. In this regime, we can distinguish three cases:

1) $\sigma < M_{N_1}$ but all the masses $M_{F_a}$ are larger than $M_{N_1}$: then $N_1$ can decay to $l_j$ only via the three body channel $N_1 \to S \Phi l_j$ depicted in figure 2.

2) $M_{N_1}$ is larger than all the scales related to the $U(1)_X$ symmetry ($M_F, \sigma$): then $N_1$ will decay via two body channels to $F_a$ and $\bar{F}_a$, (see figure 3). The heavy fermions $F$ will then transfer part of the asymmetry to the light leptons via lepton number conserving processes (decays and scatterings).

3) If $M_N$ arises from the same source than $M_F$ (as for example from the vev of a singlet) then some of the heavy fermions (for example $F_1$) could be lighter than $N_1$ while the others can be heavier. Then $N_1$ will decay dominantly into $F_1, \bar{F}_1$ via the two body channel in figure 3. However, a new diagram contributing to the $CP$ asymmetry is present in this case. This diagram is interesting, since it yields the possibility of decoupling the lifetime of $N_1$ from the size of the $CP$ violating asymmetry $\epsilon_{N_1 \to F_1}$.

In the next sections we will analyze in some detail these different possibilities.
3.2.1 Case 1: $M_F > M_{N_1}$

In the case when all the vectorlike fermions $F$ are heavier than $N_1$ but the horizontal $U(1)_X$ is still a good symmetry at $T \lesssim M_{N_1}$, there is only one diagram contributing to the $CP$ asymmetry, that is the wave function type of diagram depicted in figure 2(b). This is because on the one hand the loop correction to the $N_1 FS$ vertex with $N_\beta (\beta \neq 1)$ and $F$ internal lines does not develop an imaginary part, and on the other hand there is no correction at one loop to the vertex $Fl\Phi$ (the first correction arises only at two-loops). This is due to the fact that the standard vertex correction depicted in figure 1(b) is of the Majorana type (with an inverted flow of fermion number) while in the present case both $F$ and $l$ are Dirac fermions.

Unlike the standard case and the next two cases discussed in sections 3.2.2 and 3.2.3, for three body final states the squared amplitudes and interference terms are not phase-space independent. Our results for the total decay width and $CP$-asymmetries given below correspond to the leading terms in the mass ratios $r_{a,a',b} < 1$. For the $N_1$ total decay width we obtain

$$\Gamma_{N_1} = \frac{M_{N_1}}{192 \pi^3} \sum_{a,a'} r_{a'} r_a \left( h^\dagger h \right)_{a'a} (\lambda_{1a'}^* \lambda_{1a}) r_{a} r_{a'} \left( M_{N_1} \frac{1}{\sigma} \right)^2 (\tilde{\lambda}\tilde{\lambda})_{11}.$$  \hspace{1cm} (12)

The interference between diagrams 2(a) and 2(b), summed over all the leptons $l_m$ running in the loop, yields the $CP$-asymmetry:

$$\epsilon_{N_1} = \frac{3}{128\pi} \sum_{a,a'} \text{Im} \left[ \left( h^\dagger h \right)_{a'a} (h^\dagger h)_{a'a} (\lambda_{1a'}^* \lambda_{1a}) r_{a} r_{a'} \right] \frac{r_{a} r_{a'}}{r_{a} r_{a'}}$$

$$= \frac{3}{128\pi} \text{Im} \left[ \tilde{\lambda} h r^2 h^\dagger \tilde{\lambda}^\dagger \right]_{11} = 0.$$  \hspace{1cm} (13)

When written in terms of the effective couplings $\tilde{\lambda}$ as in the second line in eq. (13), the vanishing of the $CP$ asymmetry at leading order in the mass ratios $r_{a,a',b} < 1$ is apparent. However, this is true also beyond the leading order, and is related to the fact that the loop in diagram 2(b) does not involve lepton number violation. Mathematically, one can see from the first equality
in eq. (13) that the vanishing of the \( CP \)-asymmetry follows from the fact that by exchanging the two indices \( a \leftrightarrow a' \) the combination of couplings within square brackets goes into its complex conjugate, and thus the sum over \( a \) and \( a' \) yields a real term for any value of the index \( b \). Since the full phase-space function (that in eq. (13) has been replaced with the leading term \( \sim r_ar'_a \)) is symmetric under the exchange \( r_a \leftrightarrow r'_a \) this holds also for the complete result. The fact that there is no source term for the lepton asymmetry implies that in the present case leptogenesis can occur only through the effects of lepton flavor dynamics combined with the violation of lepton number that is provided by the washout processes. The single lepton-flavor \( CP \)-asymmetries are indeed non-vanishing:

\[
\epsilon_{N_1 \to l_j} = \frac{3}{128\pi} \frac{\sum_{a,a'} \text{Im} \left[ (h^\dagger h)_{a'a'} h_{ja}^* \lambda_{1a'} \lambda^*_{1a} \right] r_{a'} r_a r^2_b}{\sum_{a,a'} \left( h^\dagger h \right)_{a'a} \lambda_{1a'} \lambda^*_{1a} r_{a'} r_a} = \frac{3}{128\pi} \frac{\sum_i \text{Im} \left[ (hr^2h^\dagger)_{ij} \tilde{\lambda}_{1i} \tilde{\lambda}^*_{1j} \right]}{(\tilde{\lambda} \tilde{\lambda}^*)_{11}} \neq 0. \tag{14}
\]

and will be a source of non vanishing asymmetry-densities in the different flavors. These asymmetries will then suffer washouts processes (like \( s \) and \( t \) channels scatterings \( l_j \Phi \leftrightarrow S^*N_1, SN_1 \leftrightarrow \overline{l}_j \Phi \)) that in general are characterized by different rates for the different flavors. As is discussed e.g. in ref. [13], under these conditions a net lepton asymmetry can result even if \( \epsilon_{N_1} = 0 \).

A quick inspection of equation (14) shows that if the \( \tilde{\lambda} \) couplings are all of the same order of magnitude, the \( CP \) asymmetry \( \epsilon_{N_1 \to l_j} \) is roughly proportional to \( |hr^2h^\dagger| \). That is, the dominant contribution to the \( CP \) asymmetry is determined by the \( h \) couplings that are different from the effective couplings \( \tilde{\lambda} \) appearing in the neutrino mass matrix, and in particular larger by a factor of \( (\lambda \sigma/M_F)^{-1} \) (see eq. (3)). If we further assume the hierarchy \( hr > \lambda \) between the fundamental couplings then, independently of the particular value of \( M_{N_1} \), leptogenesis will always occur in a regime when the \( F_{a} \) interactions with the light leptons determine a ‘flavor’ basis \( \ell_a = h_{ia} l_i/\sqrt{(hh^\dagger)_{aa}} \). This ensures that the requirement that flavor dynamics participates in the generation of a lepton asymmetry is satisfied.

We will now address the following two interesting points: 1) In the present case is leptogenesis still compatible with a reasonable scale for the light neutrino masses? 2) Is the leptogenesis scale still bounded from below, or can it be lowered to values that are experimentally accessible?

The out-of-equilibrium condition necessary to ensure that a macroscopic lep-
ton asymmetry can be generated reads

$$\Gamma_{N_1} \lesssim \xi \cdot H(M_{N_1}),$$

(15)

where the out-of-equilibrium parameter $\xi$ can normally lie in the range $\xi \sim 0.1 - 10$ (but values as large as $10^2$ are possible) and the Hubble parameter at decay time is $H(M_{N_1}) \simeq 1.66\sqrt{g_*}M_{N_1}^2/M_P$ with $g_* = 106.75$ the number of relativistic degrees of freedom at $T \sim M_{N_1}$ and $M_P$ the Planck mass. Using (12) this yields the condition

$$(\bar{\lambda}\lambda^\dagger)_{11} \lesssim 10^5 \xi \left(\frac{\sigma}{M_{N_1}}\right)^2 \frac{M_{N_1}}{M_P}.\quad (16)$$

From equation (2) we have

$$\sum_i m_{\nu_i} = \frac{v^2}{M_{N_1}} \text{Tr} \left(\bar{\lambda}^T z^{-1/2} \lambda\right)_{11} \approx \frac{v^2}{M_{N_1}} (\bar{\lambda}\lambda^T)_{11},\quad (17)$$

where in the second relation we have assumed that $M_{N_1}$ dominates the seesaw matrix. From (16) and (17) we obtain the order-of-magnitude relation

$$\sum_i m_{\nu_i} \approx 0.3 \xi \left(\frac{\sigma}{M_{N_1}}\right)^2 \text{eV}.\quad (18)$$

This ensures that if the ratio $\sigma/M_{N_1}$ is not exceedingly small, the out-of-equilibrium condition can be satisfied for the correct scale of neutrino masses. When the value $\xi \cdot (\sigma/M_{N_1})^2 \sim 10^{-1}$ suggested by the previous equation is inserted into eq. (16) we obtain $(\bar{\lambda}\lambda^\dagger)_{11} \lesssim 10^{-12} (M_{N_1}/1 \text{TeV})$, that is the $\bar{\lambda}$ couplings should be of the order of the electron Yukawa coupling when $M_{N_1}$ is at the TeV scale. On the other hand this limit does not constrain the size of the asymmetry in eq. (14), and for $|h r^2 h^\dagger| \gtrsim 10^{-3}$ the CP asymmetry could be sufficiently large for successful leptogenesis. For such a low leptogenesis scale, direct production of the $F$ states could be possible in collider experiments e.g. via off-shell $X$ boson exchange ($M_X \sim g_X \sigma < M_F$). However, $F \to N_1$ decays (if kinematically accessible) would be strongly suppressed by the small values of $\lambda$, and thus a direct detection of the Majorana neutrinos would be a rather difficult task.

3.2.2 Case 2: $M_F < M_{N_1}$

In this case $N_1$ decays proceed through the diagram in figure 3(a). A lepton asymmetry is first generated in the $F$ states, and is transferred in part to the light leptons through the $L$ conserving interactions controlled by the couplings $h$. If we assume that the $h$-interactions are in equilibrium at $T \sim M_{N_1}$ (as is the case if the couplings $h$ are larger than the couplings $\lambda_{1a}$), then the reduction
of the asymmetry in the $F$ states implied by chemical equilibrium with the light leptons $l$ also implies a reduction in the rates of the washout processes, with a corresponding enhancement of the efficiency $\eta$. This would favor the survival of a sizeable asymmetry. Neglecting in first approximation phase space suppressions from final state masses, the total decay width for this case reads:

$$\Gamma_{N_1} = \frac{M_{N_1}}{32\pi} (\lambda\lambda^\dagger)_{11}. \quad (19)$$

The $CP$ asymmetry is determined by the interference between diagram (a) and the loop diagrams (b) and (c) in figure 3. Note that even if these diagrams are the same as in the standard case, the result for the asymmetry is different, since in contrast to the lepton doublets the vectorlike fields $F$ do not couple chirally to the Majorana neutrinos. From the interference of diagrams (a) and (b) in figure 3 we obtain:

$$\epsilon^{(a+b)}_{N_1 \to F_a} = \frac{1}{8\pi(\lambda\lambda^\dagger)_{11}} \sum_{\beta \neq 1} \text{Im} \left\{ (\lambda\lambda^\dagger)_{\beta 1} \lambda_{\beta a} \lambda_{1 a}^* \right\} F_1(z_\beta) \quad (20)$$

where $F_1$ given by

$$F_1(z) = -2z - \sqrt{z} + \left[ z(1 + 2z) + \sqrt{z}(1 + z) \right] \log \left( \frac{1 + z}{z} \right) \xrightarrow{z \to \infty} \frac{1}{2\sqrt{z}} + \frac{1}{6z} + \ldots$$

As regards the self-energy type of diagrams, like in the standard case besides the diagram depicted in figure 3(c) there is another diagram ($c'$) of the Majorana type with opposite fermion flux in the loop. The contribution from the interference of these diagrams with diagram 3(a) is:

$$\epsilon^{(a+c+c')}_{N_1 \to F_a} = \frac{-1}{8\pi(\lambda\lambda^\dagger)_{11}} \sum_{\beta \neq 1} \text{Im} \left\{ \left[ (\lambda\lambda^\dagger)_{\beta 1} + (\lambda\lambda^\dagger)_{1 \beta} \right] \lambda_{\beta a} \lambda_{1 a}^* \right\} F_2(z_\beta) \quad (21)$$

with

$$F_2(z) = \frac{1 + \sqrt{z}}{1 - z} \xrightarrow{z \to \infty} -\frac{1}{\sqrt{z}} - \frac{1}{z} + \ldots \quad (22)$$

Adding the two contributions (20) and (21), summing over all the possible final states $F_a$, and using the fact that $\text{Im} \left\{ (\lambda\lambda^\dagger)_{1 \beta} (\lambda\lambda^\dagger)_{\beta 1} \right\} = 0$ to include in
the leading order result also the sum over $\beta = 1$, we obtain:

$$
\epsilon_{N_1} = \frac{3}{16\pi(\lambda\lambda^\dagger)_{11}} \sum_{\beta} \Im \left\{ \frac{1}{\sqrt{2}_\beta} (\lambda\lambda^\dagger)_{\beta 1}^2 \right\}.
$$

(23)

Even if the two functions $F_1$ and $F_2$ are not the same as in the standard case with chiral leptons couplings [22], once the effective couplings $\tilde{\lambda}$ are replaced with the fundamental $\lambda$’s, at leading order eq. (23) coincides with the standard expression eq. (8). However, the fact that the couplings in eq. (23) are not the seesaw couplings $\tilde{\lambda}$ is rather important, because the fundamental $\lambda$’s do not directly determine the light neutrinos masses. This implies that for this case the bound of eq. (9) does not hold. As regards the consistency among different phenomenological requirements, an order of magnitude estimate suggests that there is some tension between the out-of-equilibrium condition (15), that by means of eq. (19) yields $(\lambda\lambda^\dagger)_{11} \lesssim 10^3 \xi M_{N_1}/M_P$, and the light neutrino mass scale. In fact the latter can get too much suppression from the small values of $\tilde{\lambda}$ that are reduced by the factor $h\sigma/M_F$ with respect to the $\lambda$ that determine the decay rate. This yields the rough estimate $\sum m_{\nu_i} \sim O(\xi 10^{-5})$ eV. While it is always possible to assume a large hierarchy between the combination of couplings that control the out-of-equilibrium condition and the couplings that determine the light neutrino masses, the fair conclusion is that in the case under discussion successful leptogenesis can be ensured only by means of a careful choice of the relevant parameters.

### 3.2.3 Case 3 ($M_{F_{2,3}} > M_{N_1} > M_{F_1}$)

This case corresponds to the situation when the value of $M_{N_1}$ lies in between different values of $M_{F_a}$. For definiteness we assume $M_{F_1} < M_{N_1} < M_{F_b}$ with $b \neq 1$. Neglecting the contributions from three body decays $N_1 \rightarrow S l_i \Phi$ (suppressed by $O(hh^*)$ and by phase space factors) the $N_1$ decay rate reads

$$
\Gamma_{N_1} = \frac{M_{N_1}}{96\pi} |\lambda_{11}|^2.
$$

(24)

For the $CP$ asymmetry, besides the diagrams in figure 3(a) the new type of diagram depicted in figure 4(d) also contributes. This contribution is qualitatively different from the previous ones in figure 3 since it involves the couplings $h_{ia}$ of the light leptons $l_i$ to the vectorlike fermions $F_a$. It is easy to see that in the case when the $h$ couplings dominate over the $\lambda$, diagram 4(d) gives the leading contribution to the $CP$ asymmetry. The interference between diagrams 3(a) and 4(d) yields:

$$
\epsilon_{N_1 \rightarrow F_1}^{(a+d)} = \frac{1}{8\pi|\lambda_{11}|^2} \sum_{\beta \neq 1} \Im \left[ (h^\dagger h)_{1b} \lambda_{11} \lambda_{1b}^* \right] F_2(\omega_b),
$$

(25)
where the function $F_2$ is given in eq. (22). After approximating $F_2(\omega) \sim -1/\sqrt{\omega}$ the sum can be extended over all $b$ since $\text{Im} \left((hh^\dagger)_{11}\lambda_{11}\lambda_{11}^*\right) = 0$. In this case, while the constraint from the out-of-equilibrium condition is only slightly relaxed with respect to the previous case ($|\lambda_{11}|^2 \lesssim 10^4 \xi M_{N_1}/M_P$), it involves only the coupling $\lambda_{11}$. It is then conceivable that some mechanism could suppress just this particular entry, without affecting the light neutrino mass scale (e.g. a texture zero in the matrix of the $\lambda$'s lifted by some higher order effect). As in the case discussed in section 3.2.1, small values of $\lambda$ do not necessarily imply that the $CP$ asymmetry is small, since the contribution in eq. (25) depends in a crucial way on the size of the couplings $h$, and the factor of $|\lambda_{11}|^2$ in the denominator can also enhance $\epsilon_{N_1 \rightarrow F_1}$. Also in this case, the scale of leptogenesis could be much lower than the bound in eq. (11) without being in conflict with other conditions. For example, by taking $|\lambda_{11}|$ of the order of the electron Yukawa coupling $\sim 10^{-6}$ the out-of-equilibrium condition can be satisfied for values of $M_{N_1}$ as low as $\sim 1$ TeV. By assuming that the $\lambda$ couplings different from $\lambda_{11}$ are at least of the order of the $\mu$ Yukawa $\sim 10^{-4}$, and taking for the seesaw suppression factor $(h \sigma/M_F) \sim 10^{-2}$, we also obtain a reasonable value for the light neutrino mass scale. At the same time, the contribution to the $CP$ asymmetry in eq. (25) can remain as large as $O(h^2)$.

4 Conclusions

In this paper we have explored the consequences for leptogenesis of adding to the seesaw model a new scale related to the breaking of an additional $U(1)_X$ symmetry. Our framework is inspired by $U(1)$ models for flavor à la Froggatt-Nielsen, but it differs from the usual schemes firstly in its simplicity, and secondly because we do not impose particular conditions for the values of the fundamental couplings, nor on the ratio between the symmetry breaking scale and the masses of the heavy vectorlike fermions. As a consequence, while the model represents an interesting playground to study variations of the standard leptogenesis scenario, it does not pretend to account also for the pattern of fermion masses and mixings. (This could still be achieved in more complicated schemes in which $U(1)_X$ appears as a component of the full flavor symmetry.) We have found that in all the cases in which leptogenesis occurs while the
\( U(1)_X \) symmetry is still unbroken, the expressions for the \( CP \) asymmetries in the decays of the lightest Majorana neutrino \( \epsilon_{N_1} \) differ from the standard case. The most interesting situations occur when the \( N_1 \) lifetime and the \( CP \) asymmetry \( \epsilon_{N_1} \) are controlled by two different sets of couplings, and are thus unrelated. In these cases successful leptogenesis can be achieved even at a scale as low as a few TeV, without conflicting with the requirement of \( N_1 \) out-of-equilibrium decays, and ensuring at the same time a reasonable value for the scale of the light neutrino masses.

**Appendix A**

In this section we present some analytic results for the minimal 2+2 model with only two right-handed neutrinos \( N_{1,2} \) and two pairs of vectorlike fermions \( F_{1,2} \) and \( \bar{F}_{1,2} \). This is the minimal field content that ensures that two light neutrinos are massive, as is required by neutrino oscillations data. Since in this case one neutrino is exactly massless, the neutrino mass squared differences measured in oscillations experiments completely determine the light neutrinos masses, thus allowing for a certain level of predictivity. For a normal hierarchy spectrum we have \( m_{\nu_2} = \sqrt{\Delta m^2_{\text{sol}}} \approx 9 \cdot 10^{-3} \text{eV} \) and \( m_{\nu_3} = \sqrt{\Delta m^2_{\text{atm}}} + \Delta m^2_{\text{sol}} \approx 5 \cdot 10^{-2} \text{eV} \).

Focusing on the case discussed in section 3.2.1, that is defined by the condition \( M_F > M_N \), the \( 2 \times 3 \) matrix of the effective couplings \( \tilde{\lambda} \) can be written as [26]

\[
\tilde{\lambda} = \frac{1}{v} M_N^{1/2} \text{R} \, m^{1/2}_{\nu} U^\dagger,
\]

where \( \text{R} \) is a complex \( 2 \times 3 \) matrix satisfying \( \text{R} \, \text{R}^T = I \) and \( \text{R}^T \text{R} = \text{diag}(0, 1, 1) \) [12], \( U = U_D \cdot \text{diag}(1, e^{-i\theta/2}, 1) \) with \( U_D \) the leptonic mixing matrix, and \( M_N \) and \( m_{\nu} \) are respectively the matrices of the heavy and light neutrinos mass eigenvalues. Substituting (26) in the combination of couplings that determines the decay rate, eq. (12), we obtain

\[
(\tilde{\lambda} \tilde{\lambda}^\dagger)_{11} = \frac{M_{N_1}}{v^2} (m_{\nu_2} |R_{12}|^2 + m_{\nu_3} |R_{13}|^2).
\]

Then eq. (16) implies the following constraint on the ratio of the \( U(1)_X \) breaking scale and the lightest Majorana mass \( M_{N_1} \):

\[
\xi \left( \frac{\sigma}{M_{N_1}} \right)^2 \gtrsim 10^{-5} \frac{M_P \sqrt{\Delta m^2_{\text{atm}}}}{v^2} \left[ |R_{13}|^2 + \sqrt{\frac{\Delta m^2_{\text{sol}}}{\Delta m^2_{\text{atm}}}} |R_{12}|^2 \right] \gtrsim 0.2 \left( |R_{13}|^2 + 0.17 |R_{12}|^2 \right).
\]

This suggests that a mild hierarchy between \( \sigma \) and \( M_{N_1} \) is certainly possible and, as was implicit in our scheme, allows for the possibility that the three
scales $M_F$, $M_{N_1}$ and $\sigma$ can actually lie within a few orders of magnitude.

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