Searching For New Physics With $B \rightarrow K\pi$ Decays

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We propose a method to quantify the Standard Model uncertainty in $B \rightarrow K\pi$ decays using the experimental data, assuming that power counting provides a reasonable estimate of the subleading terms in the $1/m_b$ expansion. Using this method, we show that present $B \rightarrow K\pi$ data are compatible with the Standard Model. We analyze the pattern of subleading terms required to reproduce the $B \rightarrow K\pi$ data and argue that anomalously large subleading terms are not needed. Finally, we find that $S_{K\pi 0}$ is fairly insensitive to hadronic uncertainties and obtain the Standard Model estimate $S_{K\pi 0} = 0.74 \pm 0.04$.

A decade of physics studies at the $B$ factories produced the impressive set of results on $B \rightarrow K\pi$ decays summarized in Table I. As data became more and more accurate, phenomenological analyses based on flavour symmetries and/or hadronic models were not able to fully reproduce the data. This led several authors to introduce the $K\pi$ puzzle in its different incarnations [1,2]. In particular, the difference $\Delta A_{CP} = A_{CP}(K^+\pi^0) - A_{CP}(K^+\pi^-)$ has recently received considerable attention, following the new measurement $\Delta A_{CP} = 0.164 \pm 0.037$ published by the Belle collaboration [3]. It has been argued that $\Delta A_{CP}$ could be a hint of New Physics (NP), but alternative explanations within the Standard Model (SM) have also been considered.

To understand whether $B \rightarrow K\pi$ decays are really puzzling, possibly calling for NP, one has to control the SM expectations for the $B \rightarrow K\pi$ amplitudes with a level of accuracy dictated by the size of the potential NP contributions. Thanks to the progress of theory in the last few years, we know that two-body non-leptonic $B$ decay amplitudes are factorizable in the infinite $b$-quark mass limit, i.e. computable in terms of a reduced set of universal non-perturbative parameters [7,8,9]. However, the accuracy of the predictions obtained with factorization is limited by the uncertainties on the non-perturbative parameters on the one hand and by the uncalculable subleading terms in the $1/m_b$ expansion on the other. The latter problem is particulary severe for $B \rightarrow K\pi$ decays where some power-suppressed terms are doubly Cabibbo-enhanced with respect to factorizable terms [10]. Indeed factorization typically predicts too small $B \rightarrow K\pi$ branching ratios, albeit with large uncertainties. The introduction of subleading terms, certainly present at the physical value of the $b$ quark mass, produces large effects in branching ratios and CP asymmetries, leading to a substantial model dependence of the SM predictions. Given this situation, NP contributions to $B \rightarrow K\pi$ amplitudes could be easily misidentified.

In this paper, we suggest a method to estimate the SM uncertainty given the experimental data, assuming that subleading terms are at most of order $1/m_b$. 1 This procedure provides a solid starting point for NP searches. Clearly, we are not sensitive to the presence of NP contributions of the same size as the subleading corrections to factorization.

We now describe our method in detail. We start with a general parametrization of the $B \rightarrow K\pi$ amplitudes derived from the one in ref. [12]. The decay amplitudes are given by:

$$A(B^+ \rightarrow K^0\pi^+) = -V_{ts}V_{tb}^* P + V_{us}V_{ub}^* A,$$
$$A(B^+ \rightarrow K^+\pi^0) = \frac{1}{\sqrt{2}}(V_{ts}V_{tb}^* (P + \Delta P_1 + \Delta P_2) - V_{us}V_{ub}^* (E_1 + E_2 + A)),$$
$$A(B^0 \rightarrow K^+\pi^-) = V_{ts}V_{tb}^* (P + \Delta P_1) - V_{us}V_{ub}^* E_1.$$

1 An early attempt at this method was presented in ref. [11].

| Decay Mode | HFAG average | global fit | fit prediction |
|------------|--------------|------------|----------------|
| $10^6 \text{BR}(K^+\pi^-)$ | 19.4 $\pm$ 0.6 | 19.5 $\pm$ 0.5 | 19.7 $\pm$ 1.0 |
| $10^6 \text{BR}(K^+\pi^0)$ | 12.9 $\pm$ 0.6 | 12.7 $\pm$ 0.5 | 12.4 $\pm$ 0.7 |
| $10^6 \text{BR}(K^0\pi^+)$ | 23.1 $\pm$ 1.0 | 23.8 $\pm$ 0.8 | 24.9 $\pm$ 1.2 |
| $10^6 \text{BR}(K^0\pi^0)$ | 9.8 $\pm$ 0.6 | 9.3 $\pm$ 0.4 | 8.7 $\pm$ 0.6 |
| $A_{CP}(K^+\pi^-)$ [%] | $-9.8 \pm 1.2$ | $-9.5 \pm 1.2$ | $3.9 \pm 6.8$ |
| $A_{CP}(K^+\pi^0)$ [%] | 5.0 $\pm$ 2.5 | 3.6 $\pm$ 2.4 | $-6.2 \pm 6.0$ |
| $A_{CP}(K^0\pi^+)$ [%] | 0.9 $\pm$ 2.5 | 1.8 $\pm$ 2.1 | 6.2 $\pm$ 4.5 |
| $C(K_{S}\pi^0)$ | 0.01 $\pm$ 0.10 | 0.09 $\pm$ 0.03 | 0.10 $\pm$ 0.03 |
| $S(K_{S}\pi^0)$ | 0.57 $\pm$ 0.17 | 0.73 $\pm$ 0.04 | 0.74 $\pm$ 0.04 |
| $\Delta A_{CP}$ [%] | 14.8 $\pm$ 2.8 | 13.1 $\pm$ 2.6 | 1.7 $\pm$ 6.1 |

TABLE I: Experimental inputs and fit results for $B \rightarrow K\pi$. For each observable, we report experimental results (BR and $A_{CP}$) taken from HFAG [6], the results of the fit using all the constraints (third column) and the prediction obtained using all constraints except the considered observable (fourth column). For $\Delta A_{CP}$, the prediction is obtained by removing both $A_{CP}(K^+\pi^0)$ and $A_{CP}(K^+\pi^-)$ from the fit.
FIG. 2: P.d.f. obtained from the global fit for $\Delta A_{CP}$ (left) and for $S(K_S\pi^0)$ (right).

$$A(B^0 \to K^0\pi^0) = -\frac{1}{\sqrt{2}}(V_{ts}V_{tb}^*(P - \Delta P_2) + V_{us}V_{ub}^*E_2).$$ (1)

In terms of the parameters of ref. [12], our parameters read

$$E_1 = E_1(s, q, q; B, K, \pi) - P^G_{1}\text{GIM}(s, q; B, K, \pi),$$
$$E_2 = E_2(q, q; B, \pi, K) + P^G_{1}\text{GIM}(s, q; B, K, \pi),$$
$$A = A_1(s, q; B, K, \pi) - P^E_{1}\text{GIM}(s, q; B, K, \pi),$$
$$P = P_1(s, d; B, K, \pi),$$
$$\Delta P_1 = P_1(s, u; B, K, \pi) - P_1(s, d; B, K, \pi),$$
$$\Delta P_2 = P_2(s, u; B, \pi, K) - P_2(s, d; B, \pi, K).$$ (2)

With respect to the most general parametrization, we have neglected isospin breaking in the hadronic matrix elements of the effective weak Hamiltonian, yet fully retaining the effects of the electroweak penguins (EWP). This assumption reduces the number of independent parameters and removes the dependence on meson charges in the arguments of the parameters on the r.h.s. of eqs. [2], where $q$ denotes the light quarks.

Our procedure is to fit the hadronic parameters to the experimental data, taking into account the hierarchy between leading and subleading terms in the $1/m_b$ expansion by imposing an upper bound to subleading corrections. Only the correction to the dominant penguin amplitude is well determined by the fit. The information on the subdominant terms is limited, while their presence contributes to the theoretical uncertainty. The theoretical error on the predicted observables is thus determined by the allowed range for the subleading parameters. While quantifying this range is somewhat arbitrary, extreme situations in which the leading and subleading terms are comparable would imply a failure of the infinite mass limit. Of course, one has to be careful about possible parametric or dynamical enhancements which could invalidate the power counting. Chirally-enhanced terms in $B \to K\pi$ amplitudes are well-known examples of terms that are formally subleading but numerically of $O(1)$. We have therefore included them in the leading factorized amplitudes. We now quantify the allowed ranges we use for subleading corrections. To this aim, we write each parameter as follows:

$$E_1 = E_1^F + Fr(E_1),$$
$$E_2 = E_2^F + Fr(E_2)e^{i\delta(E_2)},$$
$$A = A^F + Fr(A)e^{i\delta(A)},$$
$$P = P^F + Fr(P)e^{i\delta(P)},$$
$$\Delta P_1 = \Delta P_1^F + Fr_{\text{em}}(\Delta P_1)e^{i\delta(\Delta P_1)},$$
$$\Delta P_2 = \Delta P_2^F + Fr_{\text{em}}(\Delta P_2)e^{i\delta(\Delta P_2)},$$ (3)

where the factorized amplitudes in the limit $m_b \to \infty$ are

$$E_1^F = A_{\pi K}(\alpha_1 - \alpha_4^* + \alpha_4 - \alpha_{4,EW}^* - \alpha_{4,EW}^*),$$
$$E_2^F = A_{K\pi}(\alpha_2 + \frac{3}{2}(\alpha_{3,EW} - \alpha_{3,EW}^*)) + A_{\pi K}(\alpha_4^* - \alpha_4 - \frac{1}{2}(\alpha_{4,EW} - \alpha_{4,EW}^*)),$$
$$A^F = A_{\pi K}(\alpha_4^* + \alpha_4 - \alpha_{4,EW}^* - \alpha_{4,EW}^*),$$
$$P^F = A_{\pi K}(\alpha_4^* - \alpha_4 - \frac{1}{2}(\alpha_{4,EW} - \alpha_{4,EW}^*)),$$
$$\Delta P_1^F = -A_{\pi K}\frac{3}{2}\alpha_{4,EW}^*,$$
$$\Delta P_2^F = -A_{K\pi}\frac{3}{2}\alpha_{3,EW}^*,$$ (4)

in terms of the parameters $\alpha$ defined in eq. (31) of ref. [13]. We note that we have discarded non-factorizable contributions to the chirally enhanced terms. Furthermore,

$$A_{\pi K} = G_F/\sqrt{2}m_B^2f_KF_\pi(0),$$
$$A_{K\pi} = G_F/\sqrt{2}m_B^2f_KF_K(0).$$ (5)
FIG. 3: 1D and 2D p.d.f.’s obtained from the global fit for the parameters $r(E_2)$, $\delta(E_2)$, $r(P)$, $\delta(P)$, and $r(A)$, $\delta(A)$ defined in eqs. (3).

The coefficient $F$ in eqs. (3) sets the normalization of subleading corrections and is equal to $A_{rK}$ computed using the central value of the form factor. The phase convention is chosen such that the power correction to $E_1$ is real.

The subleading terms in units of $F$ are given by $r(X) = [0, 0.5]$ for $X = \{E_1, E_2, A, \Delta P_1, \Delta P_2\}$. Since $r(P)$ is very well determined by the fit, for computational efficiency we used $r(P) = [0, 0.2]$. For the sake of comparison, Ref. [13] quotes a value of $0.09^{+0.32}_{-0.09}$ for the contribution to $r(P)$ from penguin annihilation, compatible with the range we use. All strong phases vary in the range $[-\pi, \pi]$.

Using the ranges above for the hadronic parameters and the input parameters reported in Table II, we perform a fit to the data in Table II using the method described in ref. [11]. Flat priors are used for the hadronic parameters. Two sets of results are summarized in Table II. On one hand, when using all the experimental information as input we test the consistency of the SM description of the decay amplitudes in a global fit. On the other hand, by removing one of the inputs from the fit
we obtain a prediction of the corresponding experimental observable, using all the other inputs to constrain the hadronic parameters.

Two main results are obtained from the global fit: i) the BR values are well reproduced, and they are fairly insensitive to the $1/m_b$ contributions, but for the CKM-enhanced charming penguin $P$. ii) The values of the $\Delta \mathcal{C}_P$ are well reproduced, thanks to the $1/m_b$ contributions. In particular, the presence of $\Delta P_2$ ($E_2 + A$) in the CKM-enhanced (CKM-suppressed) part of the $B^+ \to K^+\pi^0$ amplitude (see Eq. 1) allows to obtain simultaneously a positive value of $\mathcal{A}_{CP}(K^+\pi^0)$ and a negative value of $\mathcal{A}_{CP}(K^+\pi^-)$. This is shown in the left plot of Fig. 1, where the output distribution of $\Delta \mathcal{A}_{CP}$ is fully consistent with the experimental world average $\Delta \mathcal{A}_{CP} = 0.148 \pm 0.028$.

The results for the hadronic parameters are shown in Figs. 2 and 3. Both the charming penguin parameters $r(P)$ and $\delta(P)$ are well determined, in agreement with the old results of ref. 10. In particular, $r(P)$ is found to be of $\mathcal{O}(1/m_b)$, as expected from the power expansion in QCD factorization. Small values of $r(A)$ are favoured, although values as large as 0.5 are not excluded. However, a large $r(A)$ requires a $\delta(A)$ small and negative. The corrections to the parameter $E_2$, on the other hand, are pushed towards the upper half of the allowed range, namely $0.3 \div 0.5$, showing a preference for a large correction to the color-suppressed emission amplitude [2, 17, 18]. However, we have checked that the p.d.f. for $r(E_2)$ falls for values larger than 0.6 (although there are other allowed regions for $r(E_2) \gg 1$, see below). No information on the other parameters can be extracted from the fit, but for a slight modulation of the phases in the region of absolute values close to the upper bound.

### TABLE II: Input values used in the analysis. Form factors are taken from lattice QCD calculations [14]. CKM parameters have been taken from ref. [13]. Wave function parameters can be found in Table I of ref. [13].

| Parameter | Value |
|-----------|-------|
| $f_s$ | 0.1307 GeV |
| $F_{B\to\pi}^B$ | 0.27 ± 0.08 |
| $\tau_{B_0}$ | 1.546 · 10$^{-12}$ ps |
| $m_B$ | 5.2794 GeV |
| $m_{\pi}$ | 0.14 GeV |
| $\lambda$ | 0.2258 ± 0.0014 |
| $\tilde{\rho}$ | 0.154 ± 0.022 |
| $\tilde{\rho}$ | 0.154 ± 0.022 |
| $\delta$ | 0.342 ± 0.014 |
| $f_K$ | 0.1598 GeV |
| $F_{B\to\pi}^B$ | 1.20 ± 0.10 |
| $\tau_{B_+}$ | 1.674 · 10$^{-12}$ ps |
| $f_B$ | 0.189 ± 0.027 GeV |

FIG. 4: 1D and 2D p.d.f.’s obtained from the global fit for the parameters $r(\Delta P_1)$, $\delta(\Delta P_1)$ and $r(\Delta P_2)$, $\delta(\Delta P_2)$ defined in eqs. [3].
We have checked that the result for $(\Delta P_1 + \Delta P_2)/(E_1 + E_2)$ is in agreement with the prediction of ref. [19] (obtained in the SU(3) limit neglecting left-right electroweak penguins). To quantify this statement, we define, following ref. [20], the SU(3) breaking ratio of matrix elements

$$r = \frac{\langle K\pi | I = 3/2 | Q-B \rangle}{\langle K\pi | I = 3/2 | Q+B \rangle}.$$  

In factorization, this ratio is tiny due to the fact that $f_K F_{B\to\pi} \sim f_K F_{B\to K}$, so that $r \sim \frac{|f_K F_{B\to\pi} - f_K F_{B\to K}|}{f_K F_{B\to\pi} + f_K F_{B\to K}} \sim O(10^{-2})$. However, this cancellation is not related to SU(3) (in fact, it also holds for $B \to K^*\pi$, where the SU(3) argument does not apply). More generally, one expects $|r| \lesssim 20\%$. In Fig. 5 we present the value of $r$ obtained from our global fit, yielding $|r| = 0.20 \pm 0.08$. The fit is fully compatible with the general expectations on SU(3) breaking. The factorization predictions are also compatible with the fit result, although the fit prefers larger values of SU(3) breaking.

Going back to the parameters on the r.h.s. of Eq. (3), we can conclude that $P_{1,2}^{\text{SM}}$ is not the dominant source of power corrections in $E_1$, $E_2$ and $A$ as this would imply definite correlations among $E_1$, $E_2$ and $A$ which are not observed.

Another mechanism for reproducing the $K\pi$ data proposed in the literature [18, 21, 22] is a NP contribution enhancing the EWP amplitudes with a new CP-violating phase. While we do not include NP phases in our analysis, we checked that removing subleading corrections to emissions and annihilations and allowing $r|\Delta P_2|$ to violate the $1/m_W$ power counting, it is not possible to reproduce the $K\pi$ data.

The predictions for the BR, obtained by removing them one by one from the fit, show that the observed values can be easily explained, all the values being in the $\pm 1\sigma$ range, the error on the prediction being comparable to the experimental one. On the other hand, the error on the predictions for $A_{CP}$ is much larger than the experimental precision (up to a factor six for $A_{CP}(K^+\pi^-)$). Within these large uncertainties, the predictions are in agreement with the experimental values at the $1$–$2\sigma$ level, as shown in Fig. 6.

The choice of the upper limit for the subleading terms used in our fit clearly dictates the theoretical error associated to the fit predictions. For example, raising the upper limit from 0.5 to 1 the error on the fit prediction for $\Delta A_{CP}$ increases from 0.06 to 0.09. On the other hand, the results of the global fit are fairly independent of this choice provided that the upper limit is large enough, as shown in Fig. 7. In fact, our point is that a good fit of the experimental data can be obtained for subleading terms compatible with power counting. Once a good fit is obtained, the dependence on the upper bound becomes

FIG. 5: P.d.f. obtained from the global fit for Im($r$) vs. Re($r$).

FIG. 6: Compatibility plots for $A_{CP}(K^+\pi^0)$ (upper) and $A_{CP}(K^+\pi^-)$ (lower). The cross denotes the experimental values. The colour code indicates the level of compatibility with the SM prediction.
negligible. On the other hand too small values of the upper limit would result in a worse agreement between the theory and the data, showing that the factorization formulae need to be completed with non-perturbative $1/m_b$ corrections to give a good description of the data.

Removing both $S_{K_{S}\pi^0}^{\text{exp}}$ and $C_{K_{S}\pi^0}^{\text{exp}}$ from the fit, an interesting prediction can still be obtained for the parameters of the $B^0 \to K_S \pi^0$ time-dependent CP asymmetry. We get $C_{K_{S}\pi^0} = 0.10 \pm 0.04$, in good agreement with the experimental measurement, and $S_{K_{S}\pi^0} = 0.74 \pm 0.04$, which is compatible with the experimental world average at the $\sim 1\sigma$ level. In Fig. S we show the selected region on the $S_{K_{S}\pi^0} - C_{K_{S}\pi^0}$ plane, compared to the experimental determination. Both the prediction and the measurement are limited by the experimental precision, since the other $K \pi$ data are a crucial ingredient in our fit. For example, reducing all experimental errors by a factor of two, the error on the fit prediction for $S_{K_{S}\pi^0}$ decreases to 0.03, while the error on $C_{K_{S}\pi^0}$ decreases to 0.02. It is then mandatory to improve the experimental information. Considering the difficulties related to the study of $B^0 \to K^0 \pi^0$ in the crowded environment of LHC, SuperB appears as the ideal facility to accomplish this task.

Recently, ref. [24] pointed out a correlation between $S_{K_{S}\pi^0}$ and $C_{K_{S}\pi^0}$. Using the experimental value of $C_{K_{S}\pi^0}$ they obtained $S_{K_{S}\pi^0} = 0.96^{+0.01+0.00+0.00}_{-0.07-0.10-0.06}$. Similar results were found in ref. [25]. Both papers make the following assumptions: $\Delta I = 3/2$ amplitude fixed from $\pi\pi$ data using $SU(3)$ symmetry (neglecting also left-right electroweak penguins). Under these assumptions, they solve for the amplitudes $A_{00} = A(B^0 \to K^0 \pi^0)$, $A_{+-} = A(B^0 \to K^+ \pi^-)$ and the CP-conjugate ones $\bar{A}_{00}, \bar{A}_{+-}$, up to a four-fold ambiguity. This ambiguity can be lifted using phenomenological arguments partly based on $SU(3)$ and involving charged $B \to K \pi$ modes, further neglecting annihilations. Both papers find a large value of $\phi_{00} = \arg(A_{00} \bar{A}_{00}) \sim 42^\circ$ leading to a prediction for $S_{K_{S}\pi^0}$ close to one. We have repeated the analysis and were able to reproduce the results of refs. [24, 25]. In addition, we computed the values of the relevant hadronic parameters (in our notation: $E_{1,2}$ and $P$) corresponding to the four solutions for the amplitudes $A_{00}, A_{+-}, \bar{A}_{00}, \bar{A}_{+-}$. In particular, neglecting annihilations and $\Delta I = 1/2$ EWP, the solution with $\phi_{00} \sim 42^\circ$ has a value of $P$ giving a $\text{BR}(B^+ \to K^0 \pi^0) \sim 18 \times 10^{-6}$, incompatible with the measured value $(23.1 \pm 1.0) \times 10^{-6}$. In any case, one gets a huge value of $|E_2/E_1|$; using the input of ref. [25], we find $E_2/E_1 = 1.96^{+176}_{-176}$. Clearly this value is not compatible with factorization and would imply a breakdown of the heavy quark expansion. In our fit, by limiting the range of the power corrections, we discarded this possibility. In fact, we have shown that a good agreement with the experimental data is possible without introducing huge corrections to factorization. Another recent analysis, presented in ref. [26], obtained
a good agreement with experimental data, fixing the ratio of EWP to current-current operator matrix elements using QCD factorization and fitting all other matrix elements. The range found in ref. 24 for $|E_2/E_1| = [0.52, 3]$ can possibly be compatible with both our findings and the results of refs. 24, 25. Indeed it may overlap with
can possibly be compatible with both our findings and
in the lower range allowed for $|E_2/E_1|$ but also with those of refs. 24, 25 in the upper range where
$|E_2/E_1|$ violates the $1/m_s$ power counting.

In this letter, we presented a data-driven method to estimate the hadronic uncertainties in $B \to \pi \pi$ amplitudes compatible with the $1/m_s$ expansion. This is a basic requirement to meaningfully look for NP in these channels. We found that $K \pi$ data can be accounted for by the SM, including direct CP violation. CP violating asymmetries are predicted with a large uncertainty, except for $S_{K_{2\pi}}$ and $C_{K_{2\pi}}$, where the theoretical error is much smaller than the experimental one. Thus, these asymmetries are a better place to look for NP than direct CP violation in the other $B \to K \pi$ decay modes, where possible NP contributions are obscured by hadronic uncertainties.

Note added
During the completion of this work, we were informed that similar results have been obtained by M. Duraisamy and A. Kagan in an ongoing analysis of power corrections to $B \to PP, PV,$ and $VV$ decays. Earlier results by the same group can be found in ref. [27].

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