Are spin junction transistors suitable for signal processing?

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Abstract

A number of spintronic junction transistors, that exploit the spin degree of freedom of an electron in addition to the charge degree of freedom, have been proposed to provide simultaneous non-volatile storage and signal processing functionality. Here, we show that some of these transistors unfortunately may not have sufficient voltage and current gains for signal processing. This is primarily because of a large output ac conductance and poor isolation between input and output. The latter also hinders unidirectional propagation of logic signal from the input of a logic gate to the output. Other versions of these transistors appear to have better gain and isolation, but not better than those of a conventional transistor. Therefore, these devices may not improve state-of-the-art signal processing capability, although they may provide additional functionality by offering non-volatile storage. They may also have niche applications in non-linear circuits.

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A number of spin analogs of conventional transistors have been proposed recently with a view to providing both signal processing and non-volatile storage functions with the same device. In this letter, we examine whether these devices meet the stringent requirements for signal processing.

“Analog” signal processing usually requires devices to have both large current and voltage gain for power amplification. “Digital” logic devices, on the other hand, have other critical requirements [1], three of which are that (i) the device must have a large voltage gain to regenerate logic levels at signal nodes, (ii) a large current gain for adequate fan out, and (iii) no feedback from the output terminal to the input terminal (a property known as “isolation between input and output”) so that logic signal can propagate unidirectionally from the input to the output. A conventional transistor has all these attributes and therefore has become the workhorse of analog and digital (as well as “mixed signal”) circuits. Spin transistors need to have the same attributes to be useful.

There are two basic types of transistors: the field effect type (FET) and the bipolar junction type (BJT). In this letter, we will focus on spin analogs of the latter, since we had already examined the device potentials of spintronic FETs earlier [2]. Two varieties of spin-BJTs have been proposed: (i) unipolar spin junction transistor (USJT) that mimics a conventional BJT [3], and (ii) bipolar spin junction transistor (BSJT) whose only difference with a conventional BJT is that the base is ferromagnetic and has a non-zero equilibrium spin polarization [4, 5, 6].

We consider first the USJT of ref. [3]. This device consists of three layers of spin polarized material which act as emitter, base and collector. In the emitter and collector layers, spin of one kind (say, “downspin”) is majority, while in the base, spin of the other kind (“upspin”) is majority. Since all three layers can have the same charge polarity, the device is “unipolar”, but operationally it mimics a conventional BJT. Ref. [3] derived the expressions for the collector current \( I_C \) and emitter current \( I_E \) as functions of the emitter-base bias \( V_{EB} \) and collector-base bias \( V_{CB} \): 

\[
I_C = -\frac{qJ_0}{\sinh(W/L)} \left[ \left(e^{-qV_{EB}/kT} - 1\right) - \left(e^{-qV_{CB}/kT} - 1\right) \cosh(W/L) \right] - qJ_0 \left[e^{qV_{CB}/kT} - 1\right] 
\]

\[
I_E = -\frac{qJ_0}{\sinh(W/L)} \left[ \left(e^{-qV_{EB}/kT} - 1\right) \cosh(W/L) - \left(e^{-qV_{CB}/kT} - 1\right) \right] + qJ_0 \left[e^{qV_{EB}/kT} - 1\right] 
\]

where \( qJ_0 \) is the constant saturation current, \( W \) is the base width and \( L \) is the minority spin diffusion length (assumed same in all layers) [7]. In the active mode of operation, \( V_{EB} < 0 \) and \( V_{CB} > 0 \).

In Fig. 1(a), we show the universal low-frequency small-signal equivalent circuit of a BJT [8], which applies to both the USJT and BSJT. First, we will derive expressions for two small-signal parameters – the transconductance \( g_m \) and the output conductance \( g_o \) – in the “active mode”. They are given by [9]:

\[
g_m = \left. \frac{\partial I_C}{\partial V_{EB}} \right|_{V_{BC}=constant} = \frac{q^2J_0}{kT \sinh(W/L)} e^{q|V_{EB}|/kT} 
\]

\[
g_o = \left. \frac{\partial I_C}{\partial V} \right|_{V_{CB}=constant} = -\frac{\partial I_C}{\partial V} \left|_{V_{EB}=constant} \right. = \frac{q^2J_0}{kT} \left[ e^{q|V_{CB}|/kT} - \coth(V/W) e^{-q|V_{CB}|/kT} \right] 
\]
Figure 1: (a) Universal low-frequency small-signal equivalent circuit of a BJT which applies to both USJT and BSJT. The small signal conductances are shown. (b) A biased transistor.
\[ g_o \approx \frac{q^2 J_0}{kT} e^{q |V_{CB}| / kT}. \]

The output conductance \( g_o \) can be quite large. If we assume realistic values, e.g. \( qJ_0 = 1 \text{ fA} \), and \( |V_{CB}| = 1 \text{ V} \), then \( g_o \approx 9000 \text{ S} \), which is extremely large. The situation can be improved slightly by having a larger spin splitting in the collector than in the emitter. In that case, the exponent in the last term of Equation (1) will be modified as \( V_{CB} \rightarrow V_{CB} - \Delta_c + \Delta_b \), where \( \Delta_c \) and \( \Delta_b \) are the spin splittings in the collector and base respectively. Accordingly, 

\[ g_o \approx \frac{q^2 J_0}{(kT)} e^{q |V_{CB}| - \Delta_c + \Delta_b / kT}. \]

Realistically, spin splitting energies in known materials hardly exceed \( kT \) at room temperature. Therefore, if we assume that \( \Delta_c - \Delta_b = 25 \text{ mV} \), then \( g_o = 3330 \text{ S} \), which is \( \approx 3 \) times smaller, but still very large. Of course, we can improve the situation by choosing smaller values of \( |V_{EB}| \) and \( |V_{CB}| \), but this is not advisable. The voltage levels must be several times larger than the thermal voltage \( kT / q \) to preserve good noise margin. Currently, power supply voltages are several volts and there is no movement towards making them millivolts.

Using the above results, we find that the small signal voltage gain in the common-emitter or common-base configuration (the voltage gain in common-collector configuration is always less than unity) is \[ a_v = \frac{|g_m/g_o| = \frac{\exp[q |V_{EB}| - q |V_{CB}| + \Delta_c - \Delta_b / kT]}{\sinh(W/L)}}. \]

If we assume the previous values for the junction voltages, and \( W/L = 0.001 \), then \( a_v = 0.017 \), which is far less than unity.

One obvious way to make the voltage gain larger than unity, is to make \( |V_{EB}| > |V_{CB}| \). But this poses a problem. When the transistor is in the “off” state and \( I_C, I_E \approx 0 \), the emitter-base forward bias voltage \( |V_{EB}| \) must be approximately 0 (see Equations (1) and (2)). Therefore, if \( |V_{EB}| > |V_{CB}| \), then \( |V_{CB}| \approx 0 \). Consequently, \( V_{CE} = V_{CB} + V_{BE} \approx 0 \). But now, consider the biased transistor circuit shown in Fig. 1(b). Kirchoff’s voltage law dictates that \( V_{CC} = I_C R_C + I_E R_E + V_{CE} \). Since near the “off” state, \( I_C, I_E \) and \( V_{CE} \) are all approximately zero, then \( V_{CC} \approx 0 \) as well! Therefore, we have an inconsistency. The transistor can never be switched off unless the power supply voltage is vanishingly small. We can of course designate the “off” state to be one with non-zero collector and emitter currents, but this leads to unacceptable standby power dissipation. On the other hand, if we work with a vanishingly small power supply voltage, then we operate with unacceptable noise margin and furthermore, the logic levels (“on” and “off” states) are not well separated, leading to unacceptable bit error rates. Therefore, we cannot operate under the condition that \( |V_{EB}| > |V_{CB}| \). In other words, we are constrained to operate with a small voltage gain, which is good for neither digital, nor analog circuitry.

Next, we calculate the small-signal feedback conductance \( g_\mu \) which is defined as \( g_\mu = \partial I_B / \partial V_{CB} \) [9] evaluated at a constant value of \( V_{EB} \), where \( I_B = I_E - I_C \). As is evident from Fig. 1(a), the physical significance of \( g_\mu \) is that it is a measure of the isolation between the output and input of the transistor since this conductor connects the output to the input. Ideally, \( g_\mu = 0 \), which implies perfect isolation. However, after some algebra, we can show that in the active mode the output conductance \( g_o \) can be quite large.
For a normal BJT, this quantity is approximately $q^2 J_0/(kT)e^{-q|V_{CB}|/kT}$. Therefore the $g_\mu$ for a USJT is about $e^{2q|V_{CB}|-\Delta_c+\Delta_b}/kT$ times larger than for a normal BJT. Again, if we assume that $|V_{CB}| = 1$ V and $\Delta_c - \Delta_b = 25$ mV, we find that the $g_\mu$ for a USJT is more than $10^{34}$ times worse than that for a conventional BJT, resulting in that much poorer isolation between the input and the output of the transistor. Isolation is an extremely important issue since there must exist a unilateral cause-effect relationship between the input and the output of a logic device. Without sufficient isolation between the input and output, it is impossible to ensure this relationship. To understand this issue, consider Fig. 1(a) if $g_\mu = 0$. Then there will be no direct connection between the input and output. In that case, the output voltage ($-g_m v_{be}/g_o$) directly depends on the input voltage $v_{be}$, but the input voltage $v_{be}$ is an independent variable that does not depend on the output voltage. Therefore, the input controls the output, and not the other way around. However, if $g_\mu \neq 0$, then there is a feedback from the output to the input which impairs the unilateral cause-effect relationship between the input and output. In fact, a non-zero $g_\mu$ makes the input voltage clearly depend on the output voltage. This does not bode well for logic applications.

Next, we consider the short circuit current gain of a USJT. This is given by [8]

$$a_i = \frac{\beta_0 (1 - g_\mu/g_m)}{1 + \beta_0 g_\mu/g_m}$$

(6)

where $\beta_0 = I_C/I_B$. If $g_\mu = 0$, then $a_i = \beta_0$, which can be very large. But if $g_\mu \neq 0$, then using Equations (3)-(6), we find that

$$a_i \approx \frac{\beta_0 (1 - 1/a_v)}{1 + \beta_0/a_v}.$$  

(7)

Since $a_v$ is small, the short circuit current gain is degraded from $\beta_0$, and will be small. In fact, if $a_v << 1$, then $a_i \approx -1$. This degradation is a consequence of a non-zero $g_\mu$.

We conclude that a USJT is not competitive with conventional BJTs for mainstream analog or digital signal processing applications since it has much larger output and feedback conductances. Of course, that does not preclude other niche applications for the USJT.

Now, we consider the BJST device of ref. [7, 4]. Since this device is very similar to a conventional BJT, with the only difference being that the base is ferromagnetic, we will be able to directly compare it with a conventional BJT. Using the current expressions derived in ref. [10] for an npn transistor, we get

$$I_C = qA D_{nb} \frac{1}{L_{nb} \sinh(W/L_{nb})} n_{be} [e^{qV_{EB}/kT} - 1]$$

$$-qA D_{nb} \frac{1}{L_{nb}} \coth(W/L_{nb}) n_{bc} [e^{qV_{CB}/kT} - 1]$$

$$-qA D_{pc} \frac{1}{L_{pc}} \coth(W_c/L_{pc}) p_{oc} [e^{qV_{CB}/kT} - 1]$$

(8)

$$I_E = qA D_{nb} \frac{1}{L_{nb}} \coth(W/L_{nb}) n_{be} [e^{qV_{EB}/kT} - 1]$$

$$-qA D_{nb} \frac{1}{L_{nb} \sinh(W/L_{nb})} n_{bc} [e^{qV_{CB}/kT} - 1]$$

$$+ qA D_{pc} \frac{1}{L_{pc}} \coth(W_c/L_{pc}) [e^{qV_{CB}/kT} - 1].$$

(9)
where $A$ is the cross-sectional area of the transistor, $W_c$ is the width of the collector, $W_e$ is the width of the emitter, $D_{nb}$ ($L_{nb}$) is the minority carrier diffusion constant (length) for electrons in the base, $D_{pc}$ ($L_{pc}$) is the minority carrier diffusion constant (length) for holes in the collector, $D_{pe}$ ($L_{pe}$) is the minority carrier diffusion constant (length) for holes in the emitter, $n_{be} = (n_i^2/N_{AB})(1 + \alpha_e \alpha_{ob})/\sqrt{1 - \alpha_{ob}^2}$, $n_{bc} = (n_i^2/N_{AB})(1 + \alpha_e \alpha_{ob})/\sqrt{1 - \alpha_{ob}^2}$, $p_{oc} = (n_i^2/N_{DC})$, $p_{oe} = (n_i^2/N_{DE})$, $n_i$ is the intrinsic carrier concentration in the material, $N_{AB}$ is the acceptor dopant concentration in the base, $N_{DC}$ is the donor dopant concentration in the collector, $N_{DE}$ is the donor dopant concentration in the emitter, $\alpha_e$ and $\alpha_c$ are the non-equilibrium spin polarizations in the emitter and collector, $\alpha_{ob} (= tanh(\Delta/kT))$ is the equilibrium spin polarization in the base, and $2\Delta$ is the magnitude of energy splitting between the majority and minority spin in the base.

As before, we calculate the small signal parameters in the active mode:

\[
\begin{align*}
g_m &= \frac{\partial I_C}{\partial V_{EB}} |_{V_{BC}=constant} = \frac{q^2 A D_{nb}}{kT L_{nb} \sinh(W/L_{nb}) \sqrt{1 - \alpha_{ob}^2}} e^{qV_{EB}/kT} \\
\frac{D_{nb}}{L_{nb}} &- \frac{n_i^2}{N_{AB}} 1 + \frac{\alpha_c \alpha_{ob}}{\sqrt{1 - \alpha_{ob}^2}} e^{qV_{EB}/kT} \\
\frac{D_{pe}}{L_{pe}} &- \frac{2D_{pc}}{L_{pc} N_{DC}} 1 + \frac{\alpha_c \alpha_{ob}}{\sqrt{1 - \alpha_{ob}^2}} e^{qV_{EB}/kT} \\
\frac{D_{pe}}{L_{pe}} &- \frac{2D_{pc}}{L_{pc} N_{DC}} 1 + \frac{\alpha_c \alpha_{ob}}{\sqrt{1 - \alpha_{ob}^2}} e^{qV_{EB}/kT} \\
\end{align*}
\]

In deriving the expression for $g_m$, we have used the fact that $I_B = I_E - I_C$.

If we assume that the equilibrium spin polarization in the base is larger than 76% so that $\Delta > kT$, and additionally if we assume that the the base and collector dopings are about equal, as well as the base and collector widths are about equal, then the above expressions can be approximated as:

\[
\begin{align*}
g_m &\approx \frac{q^2 A}{2kT} \left[ \frac{D_{nb}}{L_{nb}} \frac{1}{\sinh(W/L_{nb}) \sqrt{1 - \alpha_{ob}^2}} (1 + \alpha_e \tanh(\Delta/kT)) \right] e^{q(V_{EB}+\Delta)/kT} \\
g_o &\approx \frac{q^2 A}{2kT} \left[ \frac{D_{nb}}{L_{nb}} \coth \left( \frac{W}{L_{nb}} \right) \frac{n_i^2}{N_{AB}} (1 + \alpha_e \tanh(\Delta/kT)) \right] e^{q(V_{CB}+\Delta)/kT} \\
g_\mu &\approx \frac{q^2 A}{2kT} \left[ \frac{D_{nb}}{L_{nb}} \frac{W}{2L_{nb} N_{AB}} (1 + \alpha_e \tanh(\Delta/kT)) \right] e^{q(V_{CB}+\Delta)/kT} \\
\end{align*}
\]

We can now compare the parameters of a BSJT with those of a conventional BJT:

\[
\frac{g_m(BSJT)}{g_m(BJT)} = (1 + \alpha_e \tanh(\Delta/kT)) e^{\Delta/kT} \\
\frac{g_o(BSJT)}{g_o(BJT)} = (1 + \alpha_e \tanh(\Delta/kT)) e^{\Delta/kT} \\
\frac{g_\mu(BSJT)}{g_\mu(BJT)} = (1 + \alpha_e \tanh(\Delta/kT)) e^{\Delta/kT} \\
a_v(BSJT) = \frac{(1 + \alpha_e \tanh(\Delta/kT))/\sqrt{1 + \alpha_e \tanh(\Delta/kT)}}{1 + \alpha_e \tanh(\Delta/kT)} \approx 1
\]
\[
\frac{a_i(\text{BSJT})}{a_i(\text{BJT})} = 1
\] (12)

where we have used Equation (6) to evaluate the ratio of the current gains.

The above comparison shows that there is no significant advantage or disadvantage of a BSJT compared to a conventional BJT as far as current and voltage gains are concerned. However, there is a drawback in terms of having a larger feedback conductance \(g_\mu\) which degrades isolation between input and output of the device, and this impairs logic functionality. The degradation becomes progressively worse with increasing \(\Delta\) or increasing spin polarization in the base. Because of this, it is unlikely that a BSJT will replace a BJT in digital signal processing applications.

In conclusion, we have found that spin analogs of bipolar junction transistors do not offer an advantage over their conventional counterparts in mainstream signal processing applications. This is consistent with our earlier finding regarding spin analogs of field effect transistors [2]. We stress that neither the proponents of Spin-BJT, nor the proponents of Spin-FETs claimed explicitly that their devices have an advantage over conventional transistors in signal processing. The small signal analysis in this Letter confirms that there is indeed no such advantage. However, spin transistors do have some special features that are absent in their conventional counterparts. They can store information via magnetism and perform non-conventional tasks such as spin filtering [5], magnetic field sensing [11], etc. The current gains of BSJT's depend on the degree of spin polarization in the base, which can be altered with an external magnetic field using the Zeeman effect. This feature can be exploited to realize mixers/modulators and other non-linear circuits. For example, if the base current is a sinusoid with angular frequency \(\omega_1\) and the external magnetic field is sinusoidal with angular frequency \(\omega_2\), then the collector current will have frequencies \(\omega_1 \pm \omega_2\). Therefore, it appears that the role of spin transistors is not in mainstream digital and analog applications, but perhaps in unusual applications where their unique features make them particularly suitable entities.

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