KNOTS IN RIEMANNIAN MANIFOLDS

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Abstract. In this paper we study submanifold with nonpositive extrinsic curvature in a positively curved manifold. Among other things we prove that, if \( K \subset (S^n, g) \) is a totally geodesic submanifold in a Riemannian sphere with positive sectional curvature where \( n \geq 5 \), then \( K \) is homeomorphic to \( S^{n-2} \) and the fundamental group of the knot complement \( \pi_1(S^n-K) \cong \mathbb{Z} \).

1. Introduction

In [Re] the author constructed nontrivial torus knots in \( S^3 \) which are totally geodesic with respect to some Riemannian metric with positive curvature. Inspired by this work, it is interesting to ask the following

Problem 1: Let \( (S^n, g) \) be a Riemannian sphere with positive sectional curvature and let \( i : K \rightarrow (S^n, g) \) be a codimension 2 totally geodesic submanifold. Could \( i(K) \) be a nontrivial knot if \( n \geq 2 \)?

The problem emerges naturally in the study of transformation group theory acting on manifolds. The famous Smith conjecture asserts that if a cyclic group acting on \( S^3 \) has 1-dimensional fixed point set, then the fixed point set must be an unknot. The proof of the conjecture was finally given in 1979 depended on several major advances in 3-manifold theory, in particular the work of William Thurston on hyperbolic structures on 3-manifolds, results by William Meeks and Shing-Tung Yau on minimal surfaces in 3-manifolds, and work by Hyman Bass on finitely generated subgroups of \( GL(2, \mathbb{C}) \) (cf. [MB]). The well-known Thurston’s conjecture for 3-orbifold, proved in [BLP], implies that any such an action is topologically conjugate to a linear action. Clearly, every fixed point component of a linear action is a trivial knot and so the latter assertion implies the Smith conjecture.

2000 Mathematics Subject Classification. Primary 53C42; Secondary 53C22.

Key words and phrases. fundamental group, positive curvature, extrinsic curvature, totally geodesic.

Fuquan Fang was supported by NSF Grant of China #10671097 and the Capital Normal University.

Sérgio Mendonça was supported with a fellowship from CNPq, Brazil.
However, it is known for a long period that the higher dimensional \((\geq 5)\) analog of Smith’s conjecture is not true (cf. [Go]). It is interesting to ask the Riemannian geometric version of the higher dimensional Smith conjecture:

**Problem 2:** If \(\mathbb{Z}_p\) acts isometrically on a positively curved Riemannian sphere \((S^n, g)\) with a codimension 2 fixed point set \(K\), where \(n \geq 5\), then \(K\) is a unknot.

It is easy to see that the above conjecture is equivalent to claim that the action is topologically conjugate to a linear action. Therefore, a possibly higher dimensional analog of Thurston’s elliptic orbifold conjecture is the following

*If a Lie group \(G\) acts isometrically on a positively curved Riemannian sphere \((S^n, g)\), then the \(G\)-action is topologically conjugate to a linear action.*

To attack the above question the first measure is the fundamental group of the knot complement. In dimension 3 it is well-known that a knot is trivial if and only if its complement has infinite cyclic fundamental group. The main result in this paper implies that the higher dimensional Smith conjecture is true at the fundamental group level, namely, the knot complement \(S^n - K\) has infinite cyclic fundamental group.

The above problems have their analogs in algebraic geometry. It is an important theme to study the complement of an algebraic variety in \(\mathbb{P}^n\). The main tool in the studies is the well-known Zariski connectedness theorem.

Recently, Wilking [Wi] obtained a connectedness theorem for totally geodesic submanifolds in a positively curved closed manifold. In [FMR] the authors generalized Wilking’s Theorem and the Frankel Theorems (cf. [Fr2]) to a unified connectedness principle (compare [La], [FL], [FM]) which applies also to the case of non totally geodesic submanifolds, e.g., submanifolds with nonpositive extrinsic curvature[^1]. The principle may be considered as the Riemannian geometric counterparts of the connectedness principle in algebraic geometry, except the following Zariski type connectedness conjecture:

[^1]: An isometric immersion \(f : N \to M\) has nonpositive *extrinsic curvature* if the sectional curvatures \(K(X, Y)\) of \(N\) do not exceed the corresponding sectional curvatures \(\bar{K}(X, Y)\) in \(M\).
Conjecture 1.1. Let $M$ be an $m$-dimensional Riemannian manifold of positive sectional curvature. If $N, H$ are totally geodesic closed submanifolds of dimensions, respectively, $n$ and $h$, which intersect transversely, then the homomorphism

$$j_* : \pi_i(N - (N \cap H)) \to \pi_i(M - H)$$

induced by inclusion is an isomorphism for $i \leq 2n - m$ and an epimorphism for $i = 2n - m + 1$.

We remark that, by the transversality theorem, $N \cap H \subset N$ is a submanifold of codimension $m - h$, and the inclusion induces an isomorphism $\pi_i(N - N \cap H) \cong \pi_i(N)$ if $i \leq m - h - 2$ and an epimorphism $\pi_i(N - N \cap H) \twoheadrightarrow \pi_i(N)$ if $i = m - h - 1$.

As a byproduct of this paper we prove the above conjecture at the level of fundamental group. By the above remark and Frankel’s theorem [Fr2] it is new only if $h = m - 2$.

Now let us state our main results:

**Theorem 1.2.** Let $(S^n, g)$ be a Riemannian sphere with positive sectional curvature. If $K \subset (S^n, g)$ is a closed embedded submanifold of codimension 2 with nonpositive extrinsic curvature, then $K$ is homeomorphic to $S^{n-2}$ and the fundamental group $\pi_1(S^n - K) \cong \mathbb{Z}$, provided one of the following conditions holds:

- (1.2.1) $n \geq 14$;
- (1.2.2) $n \geq 5$ and $K$ is totally geodesic.

It is possible to classify a higher dimensional knot $K \subset S^n$ such that $\pi_1(S^n - K) \cong \mathbb{Z}$ in certain range, e.g., a theorem of Levine [Lv] asserts that 3-knots in $S^5$ are determined up to isotopy by the $S$-equivalence classes of their Seifert matrices. Following Levine, we call such a knot a simple knot.

**Corollary 1.3.** Let $K \subset (S^5, g)$ be a totally geodesic 3-manifold in a Riemannian 5-sphere with positive sectional curvature. Then $K$ is a simple knot.

The following theorem particularly verifies the first nontrivial case of Conjecture 1.1:

**Theorem 1.4.** Let $M$ be an $m$-dimensional closed Riemannian manifold of positive sectional curvature. Let $H \subset M$ be a codimension 2 submanifold of non-positive extrinsic curvature, and let $N \subset M$ be a submanifold of dimension $n$ which in general position with $H$ (i.e., intersects $H$ transversely) with non-positive extrinsic curvature, then the
homomorphism
\[ j_+: \pi_1(N - (N \cap H)) \to \pi_1(M - H) \]
induced by the inclusion is surjective, provided one of the following conditions holds:
\begin{align*}
(1.4.1) \ & n \geq \frac{3m}{4} \text{ and } m \geq 9; \\
(1.4.2) \ & n \geq \frac{1}{2} m \text{ and } m \geq 5, \text{ provided } N, H \text{ are both totally geodesic.}
\end{align*}

For a Riemannian manifold \( M \) and a given integer \( 1 \leq k \leq m - 1 \), we say that \( M \) has positive \( k \)-Ricci curvature if
\[ \sum_{i=1}^{k} K(v, e_i) > 0, \]
where \( v \in T_pM \) is any unit tangent vector, \( K(v, e_i) \) is the sectional curvature associated to the plane generated by \( v \) and \( e_i \), and \( v, e_1, \ldots, e_k \) are orthonormal vectors. This definition is due to Rovenski (\[Ro\]). A slightly different definition of positivity of \( k \)-Ricci curvature was given previously by Wu (\[Wu\]). Observe that \( M \) has positive sectional curvature when \( k = 1 \) and positive Ricci curvature when \( k = m - 1 \).

**Remark 1.5:** The same conclusion in the above Theorem 1.4 holds true if we replace the inequalities by either of the following conditions:
\begin{align*}
(1.5.1) \ & n \geq \frac{3m+k-1}{4} \text{ and } m \geq k + 8; \\
(1.5.2) \ & n \geq \frac{m+k-1}{2} \text{ and } m \geq k + 4, \text{ provided both } N, H \text{ are totally geodesic.}
\end{align*}

**Remark 1.6:** The proof to Theorem 1.2 implies easily the following consequence: Let \( M \) be a closed \( m \)-dimensional Riemannian manifold and \( H \) a simply connected codimension 2 submanifold with nonpositive extrinsic curvature. Then \( \pi_1(M - H) \) is cyclic if one of the following conditions holds:
\begin{align*}
(1.6.1) \ & M \text{ has positive sectional curvature, } m \geq 8; \\
(1.6.2) \ & M \text{ has positive } k \text{-Ricci curvature, } m \geq k + 7.
\end{align*}

2. **Two key lemmas**

Let \( M \) be a closed \( m \)-dimensional Riemannian manifold, and let \( H \subset M \) be a closed submanifold. Following \[FMR\], the asymptotic index of \( H \subset M \) is defined by \( \nu_H = \min_{x \in H} \nu(x) \), where \( \nu(x) \) is the maximal dimension of a subspace of \( T_xH \) on which the second fundamental form vanishes (cf. p 188 of \[Fl\]). Clearly, \( H \) is totally geodesic if and only if \( \nu_H = \dim(H) \).

In this section we will prove two key lemmas using variation theory in the presence of positive curvature.
Lemma 2.1. Let $M$ be a closed $m$-dimensional Riemannian manifold of positive $k$-Ricci curvature and let $H$ be an embedded submanifold of $M$. Let $V$ be the $\epsilon$-tubular open neighborhood of $H$ for some small $\epsilon > 0$, with closure $\bar{V}$ and boundary $\partial \bar{V}$. Let $\gamma : [0,1] \to M$ be a geodesic such that
\begin{align}
\gamma(t) &\in M - \bar{V} \text{ for } t \in (0,1) \text{ and } \gamma(0), \gamma(1) \in \partial \bar{V}; \\
\gamma'(0), \gamma'(1) &\perp \partial \bar{V}.
\end{align}

If $\nu_H \geq \frac{m+k-1}{2}$, then there exists a smooth variation $\gamma_s$ of $\gamma = \gamma_0$ with $\gamma_s(0), \gamma_s(1) \in \partial \bar{V}$ and $\gamma_s(t) \in M - \bar{V}$ for all $t \in (0,1)$ and all $s$, such that the length $L(\gamma_s) < L(\gamma)$ if $s \neq 0$.

Proof. For $\epsilon$ sufficiently small $\partial \bar{V}$ is contained in the $2\epsilon$-tubular neighborhood $U$ of $H$. We recall that, for all $x \in U$, the gradient of the distance function $\rho$ from $H$ satisfies $\nabla \rho(x) = \sigma'(\rho(x))$, where $\sigma : [0,\rho(x)] \to U$ is a minimal geodesic from $H$ to $x$ with $\sigma'(0) \perp H$ and $|\sigma'(\rho(x))| = 1$. Since $\gamma'(0), \gamma'(1) \perp \partial \bar{V}$ and $\partial \bar{V} = \rho^{-1}(\{\epsilon\})$, we have that $\gamma'(0) = \lambda_1 \nabla \rho(\gamma(0))$ and $\gamma'(1) = \lambda_2 \nabla \rho(\gamma(1))$. Thus the geodesic $\gamma$ can be extended to a geodesic $\tilde{\gamma} : [-\epsilon,1+\epsilon] \to M$ with $p = \tilde{\gamma}(-\epsilon) \in H, q = \tilde{\gamma}(1+\epsilon) \in H$ and $\tilde{\gamma}'(-\epsilon), \tilde{\gamma}'(1+\epsilon) \perp H$.

The parallel transport along $\tilde{\gamma}$ defines an isometric linear injective map $\tilde{P} : T_p H \to T_q M$. Consider $\nu_H$-dimensional linear subspaces $W \subset T_p H$ and $\tilde{W} \subset T_q H$ such that the second fundamental form $\alpha$ vanishes on $W$ and $\tilde{W}$. Since $\nu_H \geq \frac{m+k-1}{2}$ and $(P(W) + \tilde{W}) \subset \{\tilde{\gamma}'(1+\epsilon)\}^\perp$ we have
\[
\dim(P(W) \cap \tilde{W}) = \dim(P(W)) + \dim(\tilde{W}) - \dim(P(W) + \tilde{W}) \\
\geq m + k - 1 - \dim(P(W) + \tilde{W}) \geq m + k - 1 - (m - 1) = k,
\]
hence there exist orthonormal parallel vector fields $e_1(t), \ldots, e_k(t)$ along $\tilde{\gamma}$ with $e_i(-\epsilon) \in W$ and $e_i(1+\epsilon) \in \tilde{W}$ for $i = 1, \ldots, k$.

For each $i$ consider a variation $\gamma_s^i$ of $\gamma$ with $\gamma_s^i(-\epsilon), \gamma_s^i(1+\epsilon) \in H$ and $\frac{\partial \gamma_s^i(t)}{\partial s} = e_i(t)$ and let $L_i(s)$ be the length of the curve $\gamma_s^i$. The positivity of the $k$-Ricci curvature, the fact that $\alpha$ vanishes on $W$ and $\tilde{W}$, and the second variation formula imply together that there exists $i$ such that $L_i^\prime(0) < 0$. Thus we get a variation $\gamma_s^i$, denoted by $\gamma_s$, so that
\[
\tilde{\gamma}_0 = \gamma, \tilde{\gamma}_s(-\epsilon), \tilde{\gamma}_s(1+\epsilon) \in H \text{ and } L(\tilde{\gamma}_s) < L(\gamma) \text{ if } s \neq 0.
\]

Obviously, there exist $-\epsilon < t_s < u_s < 1 + \epsilon$ such that $\tilde{\gamma}_s(t_s), \tilde{\gamma}_s(u_s) \in \partial \bar{V}$ and the image of the restriction $\tilde{\gamma}_s|_{(t_s,u_s)}$ is contained in $M - \bar{V}$. By the transversality of the intersection between $\gamma$ and $\partial \bar{V}$ we obtain that $t_s, u_s$ depend smoothly on $s$. For $s \neq 0$ we have:
\[
2\epsilon + L(\gamma) = L(\tilde{\gamma}) > L(\tilde{\gamma}_s) = L(\tilde{\gamma}_s|_{(-\epsilon,t_s)}) + L(\tilde{\gamma}_s|_{[t_s,u_s]}) + L(\tilde{\gamma}_s|_{[u_s,1+\epsilon]})
\]
Thus we have $L(\gamma) > L(\tilde{\gamma}_s|_{[t_s, u_s]})$. We define $\gamma_s = \tilde{\gamma}_s|_{[t_s, u_s]} \circ \sigma_s$ for some smooth change of parameters $\sigma_s : [0, 1] \rightarrow [t_s, u_s]$. The desired result follows.

Lemma 2.2. Let $H \subset M$ and $V$ be as in Lemma 2.1. If $\nu_H \geq \frac{m+k-1}{2}$, then $\pi_1(M - V, \partial V) = 0$.

Proof. It suffices to prove that, for any curve $\gamma : (I, \partial I) \rightarrow (M - V, \partial V)$, there is a homotopy $\tau_s : (I, \partial I) \rightarrow (M - V, \partial V)$ for all $s \in [0, 1]$ with $\tau_0 = \gamma$ and $\tau_1(I) \subset \partial V$, where $I = [0, 1]$.

We argue by contradiction. Assume there is a nonempty set $S$ consisting of the continuous curves $\gamma$ as above which can not homotopy to curves in $\partial V$. Let $a$ denote the infimum of the length $L(\gamma)$ for $\gamma \in S$.

Let $U$ be a $\delta$-tubular neighborhood of $\partial V$ so that $U$ strongly deformation retracts to $\partial V$ for some small $\delta > 0$. Note that $a \geq 2\delta > 0$. Consider a sequence of curves $\sigma_\ell \in S$ satisfying $L(\sigma_i) \rightarrow a$. We may assume further that $\sigma_\ell$ is parameterized proportionally to the arc length. Since $L(\sigma_\ell)$ is uniformly bounded, the curves $\sigma_\ell$ form an equicontinuous sequence. By the Ascoli-Arzela Theorem, passing to a subsequence, if necessary, we may assume that $\sigma_\ell$ converges uniformly to a continuous curve $\sigma : [0, 1] \rightarrow M - V$ with length $L(\sigma) \leq a$.

We claim that $\sigma \in S$. In fact, we may easily find normal open $r_i$-balls $B_1, \ldots, B_j$ with $r_i < \delta$ for all $i$ and a partition $t_0 = 0 < t_1 < \cdots < t_j = 1$ such that $\sigma([t_{i-1}, t_i]) \subset B_i$, for all $i$. For sufficiently large $\ell$, there exists a homotopy $\sigma^u$ between $\sigma_\ell$ and $\sigma$ with $\sigma^u([0, 1]) \subset \Omega = (M - V) \cup U$. Since $U$ strongly deformation retracts to $\partial V$, hence $\sigma$ is homotopic to $\sigma_\ell$, and so $\sigma \in S$. As a consequence, $L(\sigma) = a$.

By the minimality of $L(\sigma)$ and the first variation formula we know that $\sigma$ is a geodesic satisfying that $\sigma'(0), \sigma'(1) \perp \partial V$. Hence, by using Lemma 2.1 we obtain a homotopy $\tau$ of $\sigma$ with length $L(\tau) < L(\sigma) = a$. A contradiction. This completes the proof.

Remark 2.3: By the proofs of Lemmas 2.1 and 2.2 we know that, if $H \subset M^m$ is closed orientable hypersurface with orientable normal bundle where $M^m$ possesses positive sectional curvature, then $H$ separates $M^m$, provided either of the following conditions holds:

(2.3.1) $\nu_H \geq m/2$;
(2.3.2) $H$ has nonpositive extrinsic curvature and $m \geq 4$.

3. Proofs of Theorems 1.2 and 1.4

In the proof we need the following two results from Theorems 0.7, 0.8 and Theorem C in [FMR].
Theorem 3.1 (FMR). Let $M$ be an $m$-dimensional closed Riemannian manifold of positive $k$-Ricci curvature, and $N, H$ closed embedded submanifolds of $M$ with asymptotic indices $\nu_N, \nu_H$ respectively. If $N$ and $H$ intersect transversely, then the following natural homomorphisms

$$i_1 : \pi_i(N, N \cap H) \mapsto \pi_i(M, H), \quad i_2 : \pi_i(H, N \cap H) \mapsto \pi_i(M, N)$$

are isomorphisms for $i \leq \nu_N + \nu_H - m - k + 1$ and are surjections for $i = \nu_N + \nu_H - m - k + 2$.

Theorem 3.2 (FMR). Let $M$ be an $m$-dimensional closed Riemannian manifold of positive $k$-Ricci curvature, and let $N$ be a closed embedded submanifold. Then the inclusion $i : N \mapsto M$ is $(2\nu_N - m - k + 2)$-connected.

Lemma 3.3. Let $M$ be an $m$-dimensional closed Riemannian manifold of positive $k$-Ricci curvature. Let $H \subset M$ be a codimension 2 submanifold with asymptotic index $\nu_H$, and let $N \subset M$ be a submanifold of dimension $n$ with asymptotic index $\nu_N$, which in general position with $H$ (i.e., intersects transversely), then the homomorphism

$$j_* : \pi_1(N - (N \cap H)) \mapsto \pi_1(M - H)$$

induced by the inclusion is surjective, provided $\nu_N, \nu_H \geq \frac{m+k-1}{2}$ and

$$\nu_N + \nu_H \geq m + k.$$

Proof. By Theorem 3.2 we know that $\pi_1(M, N) = 0$. By Theorem 3.1 we get further that $\pi_1(H, N \cap H) = 0$.

Let $V$ be the open $\varepsilon$-tubular neighborhood of $H$ defined in Lemma 2.1. Since $\nu_H \geq \frac{m+k-1}{2}$, by Lemma 2.2 it follows that

$$\pi_1(\partial \bar{V}) \mapsto \pi_1(M - V)$$

is surjective. Observe that $N \cap \partial \bar{V}$ is contained in $N - (N \cap H)$, since $H$ and $N$ are in general position. In fact, $N \cap \partial \bar{V}$ is diffeomorphic to the normal sphere bundle of $N \cap H$ in $N$. Therefore, it suffices to show that the inclusion induces an epimorphism, $\pi_1(N \cap \partial \bar{V}) \mapsto \pi_1(\partial \bar{V})$, because the inclusion factors through $N - (N \cap H) \mapsto M - V$.

Note that $\partial \bar{V}$ is an $S^1$-bundle over $H$, whose pullback on $N \cap H$ is isomorphic to the normal circle bundle of $N \cap H$ in $N$, by the transversality of $N$ and $H$. Since $\pi_1(H, N \cap H) = 0$, by comparing the exact sequences for the circle bundles $S^1 \mapsto \partial \bar{V} \cap N \mapsto N \cap H$ (resp. $S^1 \mapsto \partial \bar{V} \mapsto H$) we know that $\pi_1(N \cap \partial \bar{V}) \mapsto \pi_1(\partial \bar{V})$ is surjective. The desired result follows. \qed

Proof of Theorem 1.2. By Florit’s Theorem in [Fl], $\nu_K \geq n - 4$ if $K$ has nonpositive extrinsic curvature. Thus, $K \mapsto S^n$ is $(n - 7)$-connected, by Theorem 3.2. This implies that the homology group $H_i(K; \mathbb{Z}) = 0$.
for all $0 < i \leq n - 8$. By Poincaré duality we know that $H_i(K; \mathbb{Z}) = 0$ for all $0 < i < n - 2$, provided $n - 8 \geq \frac{1}{2}(n - 2)$, i.e., $n \geq 14$. As a consequence $K$ is a homotopy sphere, and so homeomorphic to $S^{n-2}$ by Smale’s theorem. On the other hand, by Lemma 2.2 we know that $\pi_1(\partial \bar{V}) \to \pi_1(S^n - K)$ is surjective. Note that $\partial \bar{V}$ is a circle bundle over $K$, and so $\pi_1(\partial \bar{V}) \cong \mathbb{Z}$. This shows that $\pi_1(S^n - K)$ is abelian, and so $\pi_1(S^n - K) \cong H_1(S^n - K) \cong H^{n-1}(S^n, K) \cong \mathbb{Z}$.

If $K$ is totally geodesic, we may use Wilking’s theorem (cf. [Wi]) instead in the above argument, to show that $K$ is homeomorphic to $S^{n-2}$ if $n \geq 5$. The rest of the proof is the same as above. This proves the theorem.

Proof of Theorem 1.4. We apply Lemma 3.3 with $k = 1$. By Florit’s theorem ([Fl]) the asymptotic index $\nu_H \geq m - 4$, and $\nu_N \geq 2n - m$. It is easy to see that (1.4.1) implies the conditions of Lemma 3.3.

If $N, H$ are both totally geodesic we have $\nu_N = \text{dim}(N)$ (resp. $\nu_H = \text{dim}(H)$). Thus the assumption (1.4.2) implies the desired result by using Lemma 3.3.

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