Meso-damaged VIB analysis for whole failure process of rock

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Abstract. Based on the VIB model and the similarity of VIB two parameters cohesive force law at micro scale and the rock’s stress-strain curve at macro scale, a density evolving function $D(\theta)$ about the elements softened and damaged at micro scale is proposed. The fracture informations of rock material are incorporated into the constitutive equations which in incorporating the density revolving function $D(\theta)$, two kinds state of the elements—air element and solid element are proposed to describe the process of micro element. According to the consistence hypothesis between AE accumulations and the damaged elements, the AE simulation of rock fracture process is realized. The reasonableness of meso-damaged VIB analysis for whole failure process of rock is proved by a rock’s uniaxial compression test.

1. Introduction

Rock destruction is the process of beginning, expanding and penetrating cracks on different scales, which has important theoretical and practical significance for the study of rock destruction, which can help people to understand many important engineering phenomena clearly, and then make predictions and take some effective protective measures. [1] Laboratory tests and in-situ tests are used to understand the mechanism of different kinds rock failure under different kinds of stress, and abundant results are acquired. With the appearance of computers, an important method which using numerical simulation to reappear the mechanic behavior of rock under environments and loads, which is used to study the mechanic behavior of rocks, involving the complex mechanic mechanism of failure process et al. According to the different continuum hypothesis of numerical simulation, the numerical methods using to study the properties of rock mechanics can be categorized into two kinds: one kind is continuum mechanic model. Based on the realization heterogeneity of rock, Tang Chunan et al [2-3] presented numerical simulation model of rock failure(RFPA$^2$D, RFPA$^3$D), and used widely; Pan Pengzhi et al [4] has created 3D elasto-plastic cellular automation model which can simulate the evoluting process of rock failure and has exploited EPCA$^3$D software. The second kind is discrete element model, Xing Jibo et al [5-6] used three dimension beam-particle model to simulate the fracture process of anisotropic rock materials; Jiao Yuyong ea at [7] used discontinuous deformation analysis(DDA) to simulate the extension of rock cracks; Liu Ling et al [8] used the grain flow(PFC) software to simulate the time effects of fracture extension of marble rock.

H.J.Gao et al [9] and P.Klein et al [10] presented virtual internal bond model (VIB) when simulating the cracks extension of solid materials. Being a multi-scale model, which makes it possible that adjusts and controls the solid material’s macro mechanic behaviors at micro scale. The model considers solid materials constituting discrete solid particles, and a virtual internal bond connects two particles each other, based on a special cohesive force law, the VIB model can simulate the fracture of solid materials needing no failure criteria, the material’s constitutive equation at macro scale is derived.
by the interaction between the particles. As the VIB model has the virtues of both continuum mechanic model and the discrete element model, the prospect of this type of approach in numerical simulation of fracture seems to be highly promising.

Based on VIB model and the simulating of VIB two parameters cohesive-force law at the micro scale and the rock’s stress-strain curve at macro scale, a density evolving function \( D(0^+) \) about the elements softened and damaged at micro scale is proposed, Ke Changren and Ge Xiurun [11-13] successfully used it to simulate the whole process curve and the failure process of rock material. According to the anterior works, this paper explains the meso-damaged VIB analysis for whole failure process of rock basing on the laboratory test of rocks, and bases on the hypothesis of consistence between accumulation of AE and damaged elements: \( N \propto D \), the AE numerical simulation of rock failure has been done. The reasonableness of meso-damaged VIB analysis for whole failure process of rock is proved by a rock’s uniaxial compression test.

2. The theory of meso damaged vib model

The following hypothesis have been done at the works of meso-damaged VIB analysis for whole failure process of rock:

(1) Rock materials are viewed as hyperelastic materials, which satisfy thermodynamical equilibrium equation, existing a strain energy density function \( \phi(e_0) \), which defines the material tangent modulus \( C_{ijmn} \).

(2) The meso-damaged elements of rock material are decided by local tension strain, when a element’s tension strain somewhere equal or even exceed the material’s strain corresponding top stress of macro stress-strain, the meso element damages automatically.

(3) The damaged elements are deleted from the model, meaning the meso crack advances a step forward, the elements else share the loads again, then a element somewhere damages , when all the damaged elements interpenetrate mutually and form a line or a plane, constituting a macro fracture zone , the sample failure.

2.1. Theory of vib model

Consider a homogeneous, isotropic, hyperelastic solid which has a microstructure consisting of internal cohesive bonds between a network of randomly distributed material particle (Figure 1).

![Figure 1. Virtual internal bond model.](image)

We assume that each bond can be described by a potential energy function \( U(l) \) where \( l \) denotes the bond length. According to Cauchy-Born rule [9], the bond length after deformation may be described as

\[
l = l_0 \sqrt{1 + 2 \xi_l E_{ij} \xi_l},
\]

(1)

Where \( \xi_l \) denotes the bond orientation and \( l_0 \) the length of bond when unstretched; \( E_{ij} \) denotes the Green-Lagrange strain tensor.

We assume that each bond can be described by a potential energy function \( U(l) \). The strain energy per unit undeformed volume in such a random network of bonds can be obtained as

\[
\Phi(E_{ij}) = (U(l)) = (U(l_0 \sqrt{1 + 2 \xi_l E_{ij} \xi_l}))
\]

(2)

Each bond is characterized by a position in the spherical coordinate system \((l_0, \theta, \phi)\). For conciseness, we have adopted the notation:
As a weighted average with respect to a bond density function \(D(l_0, \theta, \phi)\), where \(D(l_0, \theta, \phi)\sin \partial l_0, d\partial \phi\) is the number of bonds per unit volume in the undeformed solid with bond length between \(l_0\) and \(l_0 + dl_0\) and bond orientation between \((\theta + d\theta, \phi + d\phi)\).

According to superelastic theory [9], Piola-Kitchhoff stress tensor and the material tangent modulus may be written as

\[
S_{ij} = \frac{\partial \sigma}{\partial E_{ij}} = \left\langle \frac{U'(l)}{l} \right\rangle_{\theta, \phi} D(\theta) d\theta
\]

(4)

\[
C_{ijkl} = \frac{\partial^2 \sigma}{\partial E_{ij} \partial E_{kl}} = \left\langle \frac{U''(l)}{l^2} \right\rangle_{\theta, \phi} D(\theta) d\theta
\]

(5)

Paper [11] derived the material tangent modulus in case of small deformation, obtained the elastic tensor of two dimensional plane as

\[
C_{ijmn} = \frac{8E}{3\pi(1-\mu)^2} \int_0^{\pi} \sigma_{\theta\theta} \sigma_{\phi\phi} D(\theta) d\theta
\]

(6)

For case of plane stress:

\[
C_{ijmn} = \frac{8E}{\pi(1+\mu)(1-2\mu)} \int_0^{\pi} \sigma_{\theta\theta} \sigma_{\phi\phi} D(\theta) d\theta
\]

(7)

As the material tangent modulus \(C_{ijmn}\) is decided mainly by VIB density function \(D(\theta)\), seen in elastic constitutive equation:

\[
\sigma_{ij} = C_{ijmn} \varepsilon_{mn}
\]

(8)

The VIB density function \(D(\theta)\) decides the rock material’s mechanic properties.

2.2. Vib density function

Paper [2] derived the rock’s apparent average stress in case of one dimensional as:

\[
\sigma = Ec(1-D)
\]

(9)

Where \(\sigma\) is the Cauchy stress, \(E\) denotes elastic modulus of undamaged material, \(D\) denotes damage parameter, it means the ratio of micro crack in material elements under uniaxial compression.

If initial damage \(D_0 = 0\), seen in Weibull statistical distribution function.

\[
D = \int_0^\infty \phi(x)dx = \int_0^{\infty} \left(\frac{m}{\xi_0}\right)^{m+1} e^{-\frac{x^m}{\xi_0}} dx = 1 - e^{-\frac{\xi_0}{\xi_1}}^m
\]

(10)

This is the rock damage parameter denoted by rock’s meso elements, it reflects the damage accumulation as a whole. The function \(D\) can be seen in figure 2.

Figure 2.Strain damage parameter D’s curve of D-strain.

It can be seen from equation (9) and (10)

\[
\sigma = Ec\cdot e^{-\frac{\xi_0}{\xi_1}}^m
\]

(11)

This is the stress-strain equation of rock’s uniaxial compression when the elements’s strength satisfy Weibull distribution.

Comparing equation (9) and (10), the VIB density function is hypothesized as the following under uniaxial compression
$$D(\theta) = e^{-\left[\epsilon/\epsilon_0\right]}$$

(12)

Where $\epsilon$ denotes rock material’s deformation; $\epsilon_0$ denotes the deformation when the stress reaching the highest; $\epsilon_1$, $\epsilon_2$ denote the model parameter, which are used to adjust the shape of stress-strain curve.

2.3 Description of heterogeneity

We not only hypothes the distribution of mechanic properties of elements after discreted are statistical, but also use Weibull distribution function to describe, that is

$$F(\alpha) = \frac{m}{\alpha_0} \left(\frac{\alpha}{\alpha_0}\right)^{m-1} \exp\left[-\left(\frac{\alpha}{\alpha_0}\right)^m\right]$$

(13)

Where $\alpha$ denotes rock element’s mechanic properties parameters (strength, elastic modulus et al); $\alpha_0$ denotes mean of element’s mechanic properties; $m$ denotes shape parameter of distribution function, it’s physical meaning reflects rock material’s isotropy, and defined as rock material’s homogeneity factor; $F(\alpha)$ denotes statistical distribution density of rock element’s mechanic properties parameter $\alpha$.

Equation (13) reflect meso mechanic properties’ heterogeneity distribution of rock element. The higher of $m$, the mechanic properties of the element locate on a narrow span, the more homogeneous of rock’s properties; when isotropy parameter $m$ degrees, the distribution span of element’s mechanic properties will be wider, it means rock material’s properties becoming heterogeneous, which can be seen in figure (3).

3. Numerical simulation of the model

In order to verify the adaptability of the model to rock material, the theory is used to simulate the fracture process of rock which loaded under uniaxial compression, the datas come from the lab tests at Institute of Rock and Soil Mechanics, Chinese Academy of Sciences, using MTS815.03 electronic servo rock mechanic machine, the sample’s size and boundary condition are seen in figure 4.

Figure 3. Distribution curve of elements’ mechanic properties with different $m$.

Figure 4. Sample size and boundary conditions.

The material’s parameter are shown at table 1. In the model, the dimension of sample is 50mm × 100mm, the plane strain triangular elements are used, the elements number is 900, nodes number is 486. The displacement control method is used during the loading progress, each step is $\Delta s = 0.002mm$, the whole steps is 600.
Table 1. Rock material’s parameters and model parameters.

| Sample number | Elastic modulus $E$(GPa) | Poisson’s ratio $\mu$ | Top stress $\sigma_{\text{max}}$(MPa) | Top strain $\varepsilon_{\text{max}}$/10$^{-3}$ | Element number $NE$ | $D_{\text{max}}$ ($\times10^3$) | $c_1$ | $c_2$ |
|---------------|-------------------------|----------------------|-------------------------------|------------------------|------------------|-----------------|-----|-----|
| 2-1           | 4.06                    | 0.33                 | 30.06                         | 8.20                   | 900              | 1.5             | 4.0 | 8.0 |

3.1. The influences of model parameters on stress-strain curve

In the equation (12) of VIB density evolving equation, two parameters need to be decided, that is $c_1$ and $c_2$, which decide the rock sample’s stress-strain curve’s shape, as seen in Figure 5: $c_1$ decides the sample’s top strength, the higher the $c_1$ is, the smaller the sample’s top strength is; $c_2$ decides the sample’s mechanic character of the curve which located after the top. The higher the $c_2$ is, the steeper the curve after top strength is; the smaller the $c_2$ is, the more even the curve after top strength is. Then the suitable parameters can be selected to reappear rock sample’s stress-strain curve.

Figure 5. Influences of stress-strain curve with model parameters.

3.2. The influences of heterogeneity

The stress-strain curves with different isotropy parameters are shown in figure 6. It can be seen that with the increase of $m$, the higher pot strength of sample, and deeper the curve after top strength curve: when isotropy parameter is small, the rock sample’s top strength is low, the curve after top strength is even, which means the more isotropic the rock is, the higher the material’s macro strength. It may be explained at meso scale as following: when the isotropic parameter is bigger, the elements strength are distributed evenly, all the elements share the loads together, the sample’s bearing capacity is larger. Once local elements damage, the elements along which damage soon, the progress is so rapid that the curve after top strength is deep.

Figure 6. Rock sample’s stress-strain curve with different $m$.

The failure modes with different isotropic parameters are shown in figure 7. It can be seen that the smaller the isotropic parameter is, the damaged elements appear at several places, and appear dispersed modes, which result in the lower bearing capacity of sample at macro scale; the bigger the isotropic parameter is, the damaged elements appear at a concentrical place, and extend near the place, at last all
the damaged elements integrate a main crack, declaring the rock sample failures, the higher bearing capacity of sample at macro scale. Which works well both at meso scale and at macro scale in figure 6 and figure 7.

![Figure 7. Failure modes with different $m$.](image)

### 3.3. Stresses-load step and ae curve

Estimated intuitively, not counting each element’s AE frequency and energy, and supposing the damage of each element has contribution to AE, a conclusion may be drawn that there is a direct proportional relationship between the damaged ratio in rock sample and the rock’s AE. Base on this idea, Tang Chunan [2] proposed a hypothesis that the AE accumulations and the damaged elements work consistently.

$$N \propto D$$

(14)

Where $N$ denotes the rock’s AE, $D$ denotes rock’s damaged parameter. A relationship between AE accumulations and rock’s constitutive equation under one dimensional condition is built in theory.

According to Equation (14), $D$ denotes the damaged amounts per volume or area, so the ratio of AE is direct proportional to the rock damaged area. In the paper, there are two kinds of states for each element: damaged or not damaged. One element damaged, which can be viewed as a meso crack, so no matter how big the element is, it represents one meso crack, the AE equals one. If the grids are divided not evenly, the AE amounts of the elements which have different size are not comparable, obviously this is not the case. So the grids are divided evenly in the paper.

Figure 8 shows the stress-load step curve and AE curve under uniaxial compression with different isotropy. It can be seen that AE counts distribute mainly near the top stress of stress-load step curve on matter how many the $m$ is. But the smaller the $m$ is, the AE counts distribute disperse, the character of macro failure appears brittle; the bigger the $m$ is, the AE counts distribute mainly near to top stress of stress-load step curve, the higher the strength is, the character of macro failure appears ductile.

![Figure 8. Rock sample’s stress-loadstep and AE curve with different $m$.](image)

### 4. Conclusion

Based on the VIB model, heterogeneity is added to it, AE accounts simulation planned. A VIB density evoluting function $D(\theta)$ based on meso element damaged is proposed by comparing the micro VIB two parameter force law with the macro stress-strain curve, then needing no fracture criteria during the simulation of rock failure, a system that simulating rock’s failure process based on meso damaged VIB model is work out in the end.

For the correct selection of parameters $c_1$ and $c_2$ of VIB density function $\rho(\theta)$, the rock’s
stress-strain curves are got. For the realization of rock’s heterogeneity, a function of Weibull is used to describe the rock elements’ elastic modulus etc., the influences of parameters $c_1$ and $c_2$ to macro stress-strain curve are analyzed, also those of $m$ to macro stress-strain curve, failure modes and AE. The numerical simulation system is used to redraw rock’s stress-strain curve, and the higher the $m$ is, the sample showing brittle failure, the AE accounts locate mainly near the top of stress-load step curve.

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