On the architecture of spacetime geometry

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Abstract
We propose entanglement entropy as a probe of the architecture of spacetime in quantum gravity. We argue that the leading contribution to this entropy satisfies an area law for any sufficiently large region in a smooth spacetime, which, in fact, is given by the Bekenstein–Hawking formula. This conjecture is supported by various lines of evidence from perturbative quantum gravity, simplified models of induced gravity, the AdS/CFT correspondence and loop quantum gravity, as well as Jacobson’s ‘thermodynamic’ perspective of gravity.

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(Some figures may appear in colour only in the online journal)

Introduction
One of the most remarkable discoveries in fundamental physics was the realization that black hole horizons carry entropy [1]. This entropy is manifest in the spacetime geometry, as expressed by the Bekenstein–Hawking formula:

\[ S_{\text{BH}} = \frac{\mathcal{A}}{4G} , \]

where \( \mathcal{A} \) is the area of the horizon\(^1\). This celebrated formula draws an unexpected connection between spacetime geometry, thermodynamics, quantum theory and gravity. In fact, interest in this result has long been sparked by the possibility that it provides a window into the nature of quantum gravity.

In terms of the Planck length \( l_{\text{P}} = \frac{\hbar}{G m c} \), the geometric entropy (1) of the horizon is simply the horizon area measured in units of the Planck scale:

\(^1\) We adopt units \( \hbar = 1, c = 1, G = 1 \).
Above, $d$ denotes the spacetime dimension. This result applies for black hole solutions of Einstein’s equations and it was also extended to any Killing horizon \cite{2}, including de Sitter \cite{3} and Rindler \cite{4} horizons. In equation (2), we have indicated the possibility of contributions subleading to the area term, e.g. as would arise from higher curvature interactions in the gravitational theory \cite{5}.

In this paper, we propose that equation (2) in fact has much wider applicability with the following general conjecture. In a theory of quantum gravity, any states describing a smooth spacetime geometry manifest the following property: for any sufficiently large region, the entanglement entropy between the degrees of freedom describing the given region with those describing its complement is finite and to leading order, and takes the form given in equation (2).

An implicit assumption here is that the usual Einstein–Hilbert action (as well as, possibly, a cosmological constant term) appears as the leading contribution to the low energy effective gravitational action. The appearance of this geometric entropy (2) in such a general context emphasizes that in quantum gravity, the description of any macroscopic spacetime (even flat Minkowski space) entails a state with a great deal of nontrivial structure in terms of the microscopic degrees of freedom of the theory. Further with this proposal, the area law (2) for entanglement entropy serves as a characteristic signature for the emergence of a semiclassical metric in a theory of quantum gravity. The remainder of the paper will then present various lines of evidence that support our conjecture. We begin with some discussion of the relevant background material.

**Entanglement entropy**

Entanglement entropy has emerged as a useful measure of entanglement which can be used to characterize the correlations in a quantum system \cite{6}. Given a subsystem $A$ described by the reduced density matrix $\rho_A$, the entanglement entropy is simply the corresponding von Neumann entropy: $S_{EE} = -\text{tr} [\rho_A \log \rho_A]$. In the context of quantum field theory (QFT), when one considers the entanglement between a region and its complement, \cite{2} one finds that $S_{EE}$ is UV divergent because of short range correlations in the vicinity of the ‘entangling surface’ $\Sigma$ separating the two regions. If the calculation is regulated with a short distance cut-off $\delta$, the leading contributions for a QFT in $d$ spacetime dimensions generically take the form

$$S_{EE} = c_0 L^{d-2} \frac{\delta^{d-2}}{\delta^{d-2}} + c_2 L^{d-4} \frac{\delta^{d-4}}{\delta^{d-4}} + \cdots,$$

where $L$ is some (macroscopic) length scale characterizing the geometry of $\Sigma$. In fact, a closer examination shows that each of these terms has a precise geometric interpretation involving an integration of various factors over the boundary $\Sigma$ \cite{7}. For example, the leading term yields the famous ‘area law’ result with $S_{EE} \approx c_0 A_\Sigma / \delta^{d-2}$ \cite{8}. Unfortunately the dimensionless coefficients $c_k$ appearing in these power law divergent terms above are sensitive to the details of the UV regulator.

Entanglement entropy has also been discussed in the context of the AdS/CFT correspondence and more broadly, of gauge/gravity duality \cite{9}. There, the entanglement entropy in the $(d-1)$-dimensional boundary theory between a spatial region $A$ and its complement is

\footnote{Here and throughout the following, these are spatial regions on a fixed Cauchy surface.}
calculated by extremizing the following expression

\[ S(A) = \frac{2\pi}{\ell_p^{d-2}} \text{ext} [\mathcal{A}(\nu)] \]  

over \((d - 2)\)-dimensional surfaces \(\nu\) in the bulk spacetime which are homologous to the boundary region \(A\). In particular then, the boundary of \(\nu\) matches the ‘entangling surface’ \(\Sigma = \partial A\) in the boundary geometry. Implicitly, equation (4) assumes that the bulk theory is well approximated by classical Einstein gravity. Hence this prescription implicitly assigns a gravitational entropy given by equation (2) to these extremal surfaces in the bulk spacetime. Since, in general, the surfaces \(\nu\) do not coincide with an event horizon, this is a clear indication that equation (2) has a wider applicability. Further, we emphasize that the standard AdS/CFT dictionary equates entropies in the boundary QFT and bulk gravity theory. In fact, recent work \[10\] seems to indicate that entanglement entropy in the boundary theory is associated with entanglement entropy in the bulk. Let us also add that there have been previous speculations that evaluating equation (2) on other surfaces in the bulk geometry may yield additional entropic measures of entanglement in the boundary theory \[11, 12\]. Finally, recently boundary observables have been found which yield the Bekenstein–Hawking entropy (2) of arbitrary surfaces in the bulk spacetime \[13\]

\[ S_{\text{BH}} = S_{\text{EE}}. \]

Some initial proposals \[8\] to consider what is now called entanglement entropy stemmed from attempts to understand the entropy of black holes. The observation is that the horizon plays the role of an entangling surface separating the degrees of freedom between the interior and exterior of the black hole. Hence it was suggested that quantum correlations between these two regions might account for the black hole entropy. Recall that in evaluating the entanglement entropy for a QFT, the leading term obeys the desired area law:

\[ S_{\text{EE}} = c_0 \frac{A}{\ell_p^{d-2}} + \ldots. \]  

Of course, this is suggestive. In particular, it is natural to think that quantum gravity should regulate the entanglement entropy with \(\delta^{d-2} \sim G\), however, the precise mechanism is unclear, e.g. \[14\]. Further, as noted above, the dimensionless coefficient \(c_0\) is sensitive to the precise choice of a regulator.

A resolution of this puzzle was proposed by Susskind and Uglum in \[15\]. Their suggestion was that this area law contribution of ‘low energy’ degrees of freedom renormalizes the bare area term: \(S_0 = \mathcal{A}/4G_0\) where \(G_0\) is the ‘bare’ Newton’s constant, in the sense of perturbative QFT. That is, the renormalization of the Einstein term in the effective action coming from integrating out quantum fields also yields a power law divergence

\[ \Delta \left( \frac{1}{G} \right) = \frac{\tilde{c}_0}{\delta^{d-2}}. \]  

While \(\tilde{c}_0\) is regulator dependent, given a particular choice, the proposal \[15\] is that the renormalization of \(G\) exactly matches the area law contribution in the entanglement entropy, i.e. \(c_0 = \tilde{c}_0/4\). Hence, for the entanglement entropy across a black hole horizon, this term is precisely \(S_{\text{EE}} = \frac{c_0}{4} A_H\). While there was a great deal of work done to confirm this idea in explicit examples \[16\], unfortunately there were cases where it appeared that the desired matching was not achieved. Much of this confusion was resolved in \[17\] with an effective field theory perspective and the proposal of \[15\] was confirmed for any theories with spin 0,
1/2, 1 or 3/2 fields to all orders in the perturbation expansion of the corresponding relativistic QFT in any static curved spacetime background with a Killing horizon in any dimension. Unfortunately, there still remain open issues, e.g. in extending these arguments to the graviton itself or to nonminimally coupled scalars [18, 19]. Setting these issues aside, we note that these results [17] confirm that the area term in $S_{EE}$ can be seen as renormalizing $S_{BH}$ for a Rindler horizon in flat space.

However, even if $S_{EE}$ renormalizes the Bekenstein–Hawking entropy, the situation is still widely regarded as unsatisfactory. The horizon entropy is given by

$$S_{BH} = S_0 + S_{EE} = \frac{A}{4G_0} + \frac{A}{4} \left( \frac{1}{G} \right) + \cdots,$$

which leaves the question of how to interpret the bare term in the above contribution. One suggestion was to consider models of ‘induced gravity’ [20] with $1/G_0 = 0$. Hence the horizon entropy, as well as the entire gravity action, is generated by quantum fluctuations of some other degrees of freedom [21]. We would observe that most modern approaches to quantum gravity take essentially this perspective. While a fundamental description of the theory involves some microscopic degrees of freedom, graviton excitations, the low energy Einstein action and the spacetime geometry itself emerge as some collective phenomena. With this outlook, it is natural then to view the metric as an effective macroscopic variable and then the ‘off-shell’ method [15, 22] to calculating horizon entropy is precisely a calculation of the entanglement entropy in terms of these macroscopic variables.

We note that, in low-energy processes, the change $\Delta S_{EE}$ reproduces the Bekenstein–Hawking formula [23],

$$\Delta S_{EE} = \frac{\Delta A}{4G},$$

with $G$ being the low-energy Newton’s constant and $\Delta A$ the change in area of the event horizon. The variation $\Delta S_{EE}$ is finite and insensitive to the UV physics because a low-energy process affects only the IR part of the entropy. Moreover it is universal because of the universal coupling of the gravitational field to the energy–momentum tensor. This result from perturbative quantum gravity provides further support to the idea that $S_{BH} = S_{EE}$.

Of course, one would like to confirm $S_{BH} = S_{EE}$ in explicit microscopic models. The AdS/CFT correspondence provides one such example where the entropy of an eternal AdS black hole is interpreted as the entanglement entropy between the boundary CFT and its thermofield double [24]. In fact, this interpretation extends generally to asymptotically AdS spacetimes with a Killing horizon, e.g. [25, 26]. Similarly the idea that $S_{BH} = S_{EE}$ arises in calculations in loop quantum gravity and spin foams, where the entropy of the horizon at the leading order is given by the entanglement entropy of spin-network links crossing the entangling surface [27]. Further, simplified models of induced gravity also illustrate this point [28, 29].

**Entanglement Hamiltonians**

In this section, we work within the framework of QFT in curved spacetime, and we generalize the usual computation of the entanglement entropy for black hole horizons to general regions. However, as in the previous section, our cut-off is far removed from the Planck scale, i.e. $\delta \ll \ell_P$, and so these calculations do not address the fully quantum regime of gravity.

Recall that the first step in calculating $S_{EE}$ for some region $A$ was to determine the density matrix $\rho_A$. Here we note that an essential role played by $\rho_A$ is to encode the correlators of
operators whose support lies in $A$. In fact, by causality, $\rho_A$ describes such physics throughout the causal development $\mathcal{D}$ of $A$.\(^3\) Now since the reduced density matrix is both Hermitian and positive semidefinite, it can be expressed as

$$
\rho_A = e^{-H_A}
$$

(9)

for some Hermitian operator $H_A$. In the literature on axiomatic QFT, $H_A$ is known as the ‘modular Hamiltonian’ [30] while the same operator is referred to as the ‘entanglement Hamiltonian’ in the condensed matter literature [31]. We note that $H_A$ and the unitary operator $U(s) = \rho^{s\overline{s}} = e^{-iH_A}\overline{s}$ play an important role in axiomatic approaches to establish various formal properties of $\rho_A$ and the algebra of operators on $\mathcal{D}$. However, we must emphasize that generically $H_A$ is not a local operator and $U(s)$ does not generate a local (geometric) flow on $\mathcal{D}$. For example, if we begin with a local operator defined at a point, $O = \phi(x)$, then generally the operator $O(s) = U(s)O U^\dagger(s)$ becomes an operator with support over an extended region within $\mathcal{D}$. To be explicit, we might expect $H_A$ to have the schematic form

$$
H_A = \int d^{d-1}x \rho_1^{\mu\nu}(x)T_{\mu\nu} + \\
\quad + \int d^{d-1}x \int d^{d-1}y \rho_2^{\mu\nu;\rho\sigma}(x, y)T_{\mu\rho}T_{\nu\sigma} + \cdots.
$$

(10)

Furthermore, in general, we should expect that other operators beyond the stress tensor can also appear in this expansion. Of course, there are special cases where the modular flow and the entanglement Hamiltonian are in fact local. A well-known example is given by the Minkowski vacuum state restricted to the Rindler wedge $L$. That is, taking the entangling surface to be the line $\Sigma = \{t = 0, x = 0\}$ and the region $A$, the half-space $x > 0$ (and $t = 0$), then the corresponding causal development $\mathcal{D} \equiv L$ is a wedge of Minkowski space. In this case for any QFT, the entanglement Hamiltonian is just

$$
H_A = 2\pi K + c' = -2\pi \int_{x>0} d^{d-2}y \ dx \left[ xT_{00} \right] + c'
$$

(11)

where $K$ is the boost generator in the $x$ direction and $c'$ is a constant introduced to ensure the unit trace of the density matrix. In this case, the operator $U(s)$ translates operators along the boost orbits within $L$ and equation (9) describes a thermal state with respect to this notion of ‘time’ translations [32, 33].

Now given this formalism, let us turn to the question of calculating the entanglement entropy of some general region. We wish to frame the discussion in the context of a general curved background spacetime in which the curvatures are slowly varying. Within this background, we choose some smooth Cauchy surface and on this surface a smooth entangling surface $\Sigma$. We imagine that we are evaluating the entanglement entropy for a collection of quantum fields in this framework. The QFT will be provided with some regulator that introduces a short distance cut-off $\delta$. The latter is much smaller than any geometric scales $L_{\text{geom}}$ that arise in defining the background, the Cauchy surface and the entangling surface, but still much large than the Planck scale, i.e. $\ell_p \ll \delta \ll L_{\text{geom}}$. Now let us zoom in on a small spacetime region $\Gamma$ of size $L_{\text{reg}} \ll L_{\text{geom}}$ near the entangling surface $\Sigma$, as illustrated in figure 1. We imagine that the problem is such that we can choose $\delta \ll L_{\text{reg}} \ll L_{\text{geom}}$. Hence within $\Gamma$, the spacetime looks like flat space, the portion of the entangling surface looks simply like a straight line and the light sheets defining $\partial \mathcal{D}$ are flat surfaces extending from this line like Rindler horizons.

\(^3\) The causal development of $A$ is the set of all points $p$ for which all causal curves through $p$ necessarily intersect $A$. 5
Now given the general setting in which we have framed our discussion, it would be difficult to consider details of the vacuum state for the QFT without further information. In fact, it may well not be possible to define a unique vacuum state in general. However, rather than focussing on such a precise state, we instead consider general states with the property that correlators in these states reproduce the standard UV singularities of the Minkowski vacuum. For free fields, this is essentially the definition of Hadamard states, and for interacting fields, this Hadamard-like property can be seen as a defining characteristic of the relevant states [34]. For any such states, this ensures that within the region $\Gamma$, any correlators have the same form as in Minkowski space up to small corrections. Hence, if we are interested in short-distance correlators in the vicinity of the entangling surface, we know that the relevant part of the density matrix simplifies to have the same structure as in flat space. In particular, if we express $\rho_A$ in terms of the entanglement Hamiltonian as in equation (9), the latter will still generally have a complicated nonlocal form (10), however, we will find $[H_A, O(x)] = 2\pi \{K, O(x)\} + O\left(\frac{L_{\text{reg}}}{L_{\text{geom}}}\right)$ if we focus on some operator $O(x)$ near the entangling surface. That is, the leading contributions in $H_A$ encoding the short-distance correlations near the entangling surface take precisely the form of the Rindler Hamiltonian (11). Let us add that one can see the structure of equation (11) emerging from the full entanglement Hamiltonian (10) in certain explicit examples, i.e. $\chi^\nu \rightarrow -2\pi \times \delta^\nu \delta^\nu$. For example, this behaviour arises with the $H_A$ constructed in [26] for a CFT within a spherical entangling surface and also in the situation where the entangling surface corresponds to a fixed point of some background Killing vector $\zeta^\nu$ where $H_A = \int d\Sigma^\nu T_{\mu\nu} \zeta^\nu$, e.g. see [35].

We note that this reasoning produces agreement with the usual intuition about the local character of UV physics. As stressed above, the full entanglement Hamiltonian will generally be a nonlocal operator but here we have argued that the leading UV sensitive contribution to $S_{\text{reg}}$ is controlled by a local term in $H_A$. The same intuition suggests that the other UV divergent contributions to $S_{\text{reg}}$ should also be governed by local terms in $H_A$.

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**Figure 1.** Consider a region $A$ on a Cauchy slice of some smooth background geometry. The density matrix $\rho_A$ controls the physics throughout the causal domain $\mathcal{D}$. This geometry varies only on some large distance scales $L_{\text{geom}}$. We consider a spacetime region $\Gamma$ of size $L_{\text{reg}} \ll L_{\text{geom}}$ near the entangling surface $\Sigma$. Within this region, the spacetime looks like flat space and the light sheets defining $\partial \mathcal{D}$ look like Rindler horizons.
conclusion has two interesting implications. First, we know that the ‘density’ of the entanglement entropy for a Rindler horizon yields a constant leading divergence, i.e. \( s \approx s_0 \). While the constant \( s_0 \) depends on our precise choice of regulator, in the current framework the regulator is fixed and so each element of the entangling surface makes precisely the same contribution. Hence integrating over the entangling surface, we will find the leading singularity will be a contribution of the form \( S_{EE} \approx s_0 \frac{\delta}{2 \pi} + \cdots \). That is, for any calculation of \( S_{EE} \) within the general context set out above, we will find that the leading contribution to the entanglement entropy is an area law term. Furthermore, at this point, we may invoke the results of [17] as applied to Rindler horizons to further relate the constant \( s_0 \) to the renormalization of the effective Newton’s constant in this theory. Hence we arrive at the conclusion that the leading contribution to the entanglement entropy takes the form

\[
S_{EE} \approx \frac{\mathcal{A}}{4} \Delta \left( \frac{1}{G} \right) + \cdots .
\]

That is, for any general region in a smooth background spacetime, as described above, the entanglement entropy of the low energy fields has precisely the necessary form to renormalize the expression given in equation (2). At the very least, this is a highly nontrivial consistency check of the proposal made at the outset of this paper. That is, in order for equation (2) to be consistent with the coupling of the perturbative quantum fields to gravity, these low energy degrees of freedom must contribute to the entanglement entropy precisely as in equation (12) for general regions.

As in the discussion of black hole entropy, we are left here with the question of how to interpret the bare area term in \( S_{EE} \), which our proposal requires to be present. Here, we would advocate that applying the ‘off-shell’ method to calculate the Rindler entropy within each patch indicates that this bare term must be present as the entanglement entropy of the microscopic gravity degrees of freedom. There will also be many higher order corrections, e.g. corresponding to integrals of both intrinsic and extrinsic curvatures over the entangling surface. We expect that these will also be associated to the renormalization of various gravitational couplings [36]. Let us note here that this renormalization is an essential ingredient in the recent holographic entanglement entropy calculations of [10].

Of course, as before, it is much more satisfying to explicitly realize the desired result with calculations within a given microscopic model. Here, the Randall–Sundrum II braneworld [37] provides an interesting example as a model of induced gravity [38]. In particular, it has been observed that in this framework, using holographic prescription for entanglement entropy [9], black hole entropy corresponds to entanglement entropy [28]. However, it is also straightforward to show that the area term in equation (2) appears for any sufficiently large region, irrespective of whether or not the entangling surface corresponds to an event horizon [28, 36]. Similar results were noted in [29] for other simple induced gravity models using heat kernel techniques.

At this point, let us add that the preceding discussion can be combined with Jacobson’s ‘thermodynamic’ perspective of gravity [14, 39] to provide further support for our conjecture. Here one begins by assuming that the entanglement entropy \( S_{EE} = \alpha \mathcal{A} \) is finite for any Rindler horizon and is then lead to conclude that the spacetime metric must be dynamical and governed by the Einstein equations with Newton’s constant \( G = 1/4\pi \). However, reversing the order of the arguments [39], one can conclude that if the spacetime geometry is determined by the Einstein equations, then the entanglement entropy must be finite and given by equation (2). Further, if this reasoning could be applied to ‘local’ Rindler horizons, the same result (2) should apply for the entanglement entropy of general regions [14]. Now our discussion above gives a precise meaning to the intuitive idea of a ‘local’ Rindler horizon.
That is, in small patches near a general entangling surface, the entanglement Hamiltonian encoding the local physics takes precisely the Rindler form (11). Hence with this understanding, one successfully concludes that the leading contribution to the entanglement entropy of any general region will take precisely the form of equation (2).

To close this section, let us remark that a key assumption above is that we should restrict our attention to Hadamard(-like) states. These are the natural set of low-energy states to consider in the context of QFT in curved spacetime. However, one expects that this Hadamard-like property only emerges as an effective characteristic of low-energy semiclassical states in the context of a full theory of quantum gravity. The corrections to the UV correlations probed here should be suppressed by $\delta/L_\text{P} \ll 1$, however, we may expect that the structure of the short-distance correlations will be strongly modified as we approach the Planck scale. Hence we turn to loop quantum gravity in the next section to find support that our conjecture survives these modifications.

**Spin foam models**

In ‘loop quantum gravity’, a smooth macroscopic geometry is expected to emerge from a description of space and spacetime which is discrete at a fundamental level [40]. There has been recent progress in the understanding of black hole entropy in this context [27, 41] and so it is natural to ask whether these models give some evidence for our conjecture that general regions of macroscopic spacetimes carry an entanglement entropy given by equation (2).

Consider a cellular decomposition of a three-dimensional manifold, for instance, a triangulation. A spin-network graph with a node in each cell and a link connecting nodes in neighbouring cells is said to be dual to this triangulation. Lorentz-group representations label the links of the graph and determine a quantum geometry of the triangulation. Generally such states are highly entangled [42]. In particular, we consider the vacuum state defined using the covariant spinfoam dynamics, which has the properties that it is invariant under local Lorentz transformations and time translations. Now, even neglecting interactions between different links, the state has entanglement associated to the endpoints of each link. In the cellular picture, the quantum geometries of two nearby cells in the three-dimensional manifold are entangled.

Now we consider a three-dimensional region $A$ in the manifold. On the boundary $\Sigma$ of the region, the cellular decomposition induces a tessellation in two-dimensional cells. In the dual picture these are links $l$ crossing the surface $\Sigma$. Exactly as discussed above, the relevant part of the reduced density matrix $\rho_A$ can be written in the form (9) with the entanglement Hamiltonian

$$H_A = 2\pi \sum_l K_l + \log Z.$$  

The sum is over the links $l$ that cross the entangling surface $\Sigma$ and $K_l$ is the Hermitian generator of boosts in the unitary representation of the Lorentz group associated to the link. This expression has the same form as equation (11) for the QFT case. The term $\log Z$ provides the normalization of the density matrix $\rho_A = e^{-H_A}$, i.e. this term provides the constant $c'$ in equation (11). The entanglement entropy is now easily computed:

$$S_{EE} = -\text{Tr} \left( \rho_A \log \rho_A \right) = 2\pi \text{Tr} \left( \sum_l K_l \rho_A \right) + \log Z.$$  

(14)
The simplicity constraint on representations of the Lorentz group allows us to express the first term as the area $\mathcal{A}_L$ of the entangling surface [27]. The second term is proportional to the number $\mathcal{N}$ of links crossing $\Sigma$, so that we have

$$S_{EE} = \frac{\mathcal{A}_L}{4G_0} + \mu(\gamma)\mathcal{N},$$

(15)

where $\mu$ is a chemical potential that depends on the Immirzi parameter $\gamma$ [41]. The entanglement entropy is finite because the theory has no degrees of freedom below the scale $\ell_{\text{QG}} = (8\pi G_0)^{1/2}$, the physical cut-off scale in loop quantum gravity. As the area $\mathcal{A}_L$ is proportional to $\mathcal{N}$, the second term can be understood as a finite renormalization of $G_0$ and be reabsorbed in the first term in the same way as described in equation (7), thus providing further evidence for our conjecture.

**Discussion**

We have proposed that the Bekenstein–Hawking formula has a much wider applicability than previously considered. In fact, our conjecture is that equation (2) corresponds to the leading contribution to the entanglement entropy for any sufficiently large region in a theory of quantum gravity. Evidence for this conjecture was presented from four directions.

(i) In the AdS/CFT correspondence, the well-tested prescription for holographic entanglement entropy [9] clearly assigns an entropy to large classes of surfaces which are unrelated to horizons, with equation (2) as the leading term.

(ii) In examining quantum fields in curved spacetime, for any large region, the leading contribution to the entanglement entropy is an area term and the coefficient of this term matches precisely the renormalization of Newton’s constant in equation (2). Further, applying the ‘off-shell’ method to calculate the Rindler entropy locally along the entangling surface suggests the presence of a bare term $\mathcal{A}/4G_0$, as well.

(iii) A spin-off of the preceding discussion was that the part of the entanglement Hamiltonian encoding physics near the entangling surface takes the Rindler form (11). This insight combines with Jacobson’s ‘thermodynamic’ perspective of gravity [14, 39] to conclude that the leading contribution to the entanglement entropy of general regions takes precisely the form of equation (2).

(iv) In simplified models of induced gravity, the leading term to the entanglement entropy for large regions is finite and takes precisely the form given by equation (2) [28, 29, 36].

(v) Our preliminary investigations of spin foam models indicate that general regions will carry a finite entanglement entropy, again with the leading term described by equation (2). We feel that combining these results provides strong evidence for our conjecture as a general result. Our proposal provides a glimpse into the complexity of macroscopic spacetimes as described by quantum gravity and illustrates the role of quantum entanglement as a key mechanism underlying the emergence of spacetime geometry in any such theory.

Our proposal demands that quantum gravity effects two essential features for entanglement entropy. First, it ‘regulates’ entanglement entropies for general regions. This might be seen as another realization of the general lore that quantum gravity contains fewer states than QFT. The second property is that this regulator yields a simple universal result, i.e. equation (2). This property would seem to rely on the universal couplings of the effective Einstein theory emerging at low energies [23]. We expect this universality is a unique feature.
of the entanglement entropy. For example, the Rényi entropies \[43, 44\], which would provide another measure of the entanglement between regions, should also exhibit an area law behaviour at leading order. However, the precise coefficient of the area term would likely depend on the microscopic details of the underlying quantum gravity theory.

In quantum many-body systems, the entanglement entropy typically satisfies an area law \[45\]. However, this is not the behaviour of generic states in the full Hilbert space \[46\]. Hence it seems that the locality of the underlying Hamiltonian restricts the entanglement of the microscopic constituents in the low energy states of these systems. This feature was central to the recent development of tensor network techniques to better understand the nature of quantum matter \[47\]. Drawing an analogy here, we expect that generic states in the full Hilbert space of quantum gravity will not correspond to anything resembling a smooth spacetime. Rather states describing smooth macroscopic spacetimes require a certain structure for the short-range entanglement such that we get the area law behaviour \(2\) as conjectured here. Given that tensor network techniques are beginning to be applied to the description of quantum gravity with discrete models \[48\], it would be interesting to understand the role of the current discussion and the above lessons from condensed matter theory in this approach.

Of course, implicitly, we are assuming here that a definition of entanglement entropy extends beyond the semiclassical regime to the full theory of quantum gravity. A sketch of how such a definition might be formulated is as follows. Consider the Hilbert space of geometries prior to the imposition of the gravitational constraints. In this larger Hilbert space, one has a notion of local degrees of freedom and of regions of space and therefore entanglement entropy can be defined as usual. However, this construction then induces a notion of entanglement entropy for the subset of physical states that satisfy the gravitational constraints, i.e. the Wheeler–deWitt equation.\(^5\) Further, we expect that not all such physical states will satisfy the area law of equation \(2\). Hence this discussion suggests that an area law entanglement can be regarded as a signature of physical states that approximate smooth spacetimes in quantum gravity. In this way then, equation \(2\) provides us with a glimpse into the quantum architecture of macroscopic spacetime geometries.

In closing, we consider some future directions: clearly, it is of interest to further develop the calculations for the spin foam models. Of course, it would also be interesting to identify analogous calculations in the context of string theory, as well as to better understand our proposal in a holographic framework. Moreover, it would be useful to identify if this entanglement entropy has an operational meaning. Certainly, when applied to black hole horizons, there is an interpretation in terms of thermodynamic entropy of the black hole, where it should also correspond to a counting of states. Here we might note that low-energy perturbations of the entanglement entropy seem to admit such a thermodynamic interpretation \[23, 49\].

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\(^5\) This is what happens, for instance, in loop quantum gravity where the spin foam dynamics projects states on solutions of the Wheeler–deWitt equation.
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