LinEasyBO: Scalable Bayesian Optimization Approach for Analog Circuit Synthesis via One-Dimensional Subspaces

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Abstract—A large body of literature has proved that the Bayesian optimization framework is especially efficient and effective in analog circuit synthesis. However, most of the previous research works only focus on designing informative surrogate models or efficient acquisition functions. Even if searching for the global optimum over the acquisition function surface is itself a difficult task, it has been largely ignored. In this paper, we propose a fast and robust Bayesian optimization approach via one-dimensional subspaces for analog circuit synthesis. By solely focusing on optimizing one-dimensional subspaces at each iteration, we greatly reduce the computational overhead of the Bayesian optimization framework while safely maximizing the acquisition function. By combining the benefits of different dimension selection strategies, we adaptively balancing between searching globally and locally. By leveraging the batch Bayesian optimization framework, we further accelerate the optimization procedure by making full use of the hardware resources. Experimental results quantitatively show that our proposed algorithm can accelerate the optimization procedure by up to $9 \times$ and $38 \times$ compared to LP-EI and REMBOpBO respectively when the batch size is 15.

I. INTRODUCTION

As the development of the integrated circuit (IC) technology, the circuit scale increases dramatically and the unwanted parasitic effect complicates the circuit design process significantly. The increasing demands on speed, reliability, low power consumption and short time-to-market further aggravate the difficulty of designing circuit manually, especially for the analog circuit [1]. Sophisticated analog circuit design automation tools are in great need. The analog circuit design procedure generally can be divided into two stages: topology selection and device sizing. In this paper, we mainly focus on the device sizing problem.

The analog circuit device sizing problem typically can be formulated as a noisy, multimodal, expensive and derivative-free black-box function. A large body of literature has been published on designing an efficient optimization algorithm to solve this problem [1]–[7]. The traditional optimization algorithms for analog circuit device sizing problem can be divided into two groups: the model-based and simulation-based approaches. The model-based approaches try to approximate the circuit performance and provide a cheap-to-evaluate model for space exploration. Instead of searching for the global optimum with expensive circuit simulator, the constructed model can greatly reduce time consumption on observation, thus, accelerate the optimization procedure. One popular model-based approach is the geometric programming [8]–[10]. However, there is no theoretical guarantee that the constructed model is accurate over the whole design space. Therefore, the obtained global optimum may deviate from the real global optimum.

Instead of constructing a surrogate model to capture the behavior pattern of the circuit performance, the simulation-based approaches simply search the state space by mimicking the physical or biological phenomenon. By treating the circuit performance as a black-box function, the simulation-based approaches guide the search with a selection engine and invoke the circuit simulator on the fly. There are several popular simulation-based approaches, including simulated annealing (SA) [11], [12], multiple starting point (MSP) algorithm [6], [13], evolutionary algorithm [14], [15] and particle swarm optimization (PSO) algorithm [15], [16]. The limitation of the simulation-based approaches is their relatively low convergence rate.

Inspired from above, the Bayesian optimization framework has been proposed to pave a way out of the dilemma by combining the benefits of both model-based and simulation-based methods [17], [18]. Generally, there are two key elements in the Bayesian optimization framework: the statistical surrogate model and the acquisition function. Injected with our prior beliefs about the latent function, the surrogate model provides both the predictive mean and uncertainty estimation for each location based on the collected data points. By leveraging the posterior distribution provided by the surrogate model, the acquisition function works as a utility function to measure the potential of each location over the design space and observe the most promising data points in an iterative and greedy manner. After a limited number of iterations, the global optimum can be reached with a theoretical guarantee [19], [20]. Compared to the model-based methods that only build the model once and searches for the global optimum offline, the Bayesian optimization approach incrementally updates the surrogate model and iteratively invokes the circuit simulator on the fly. Compared to the simulation-based methods that solely explores the state space with a heuristically designed selection engine, the Bayesian optimization approach makes full use of the previously collected information and facilitates the searching procedure more economically. Prior researches have demonstrated the efficiency and effectiveness of the Bayesian optimization framework for analog circuit synthesis [2], [3], [5], [7], [21].

Although lots of efforts have spent on designing an informative surrogate model and efficient acquisition function [3], [5], [7], [19], [22]–[26], the acquisition function optimization procedure itself is always assumed to be relatively easy and largely ignored. However, the global optimum can only be theoretically guaranteed for the Bayesian optimization framework when it successfully finds the global optimum of the acquisition function at each iteration. The efficiency of the Bayesian optimization framework can be significantly hurt if less promising data points are accumulated across iterations. In other words, the acquisition function optimization procedure is important both theoretically and practically. With the increase of the dimensionality, this problem only deteriorates and further sabotages the efficiency of the Bayesian optimization framework. In this paper, we propose a fast and robust Bayesian optimization approach via one-dimensional subspaces [27] for analog circuit synthesis. Unlike the traditional Bayesian optimization methods that optimize the acquisition function over the entire design space, we solely focus on one dimension at a time. In this way, our proposed algorithm makes solid progress at each iteration and ensures information gain regardless of the dimensionality. We also leverage the carefully designed batch Bayesian optimization framework to further increase sampling efficiency. Experimental results quantitatively demonstrate that our proposed algorithm can accelerate the optimization procedure by up to $9 \times$ and $38 \times$ compared to LP-EI [28] and REMBOpBO [29] respectively while obtaining the same optimization results when the batch size is 15.

The rest of the paper is organized as follows. In §II, we formulate the device sizing problem and briefly review the knowledge background of our proposed algorithm. In §III, we analytically highlight the challenges of the acquisition function optimization procedure and our proposed method of handling these problems. In §IV, we conduct experiments on two real-world analog circuits.

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and quantitatively demonstrate the efficiency and effectiveness of our proposed algorithm. Finally, we conclude the paper in §V.

II. BACKGROUND

In this section, we present both the problem formulation of device sizing (§II-A) and a brief review of the Bayesian optimization framework (§II-B).

A. Problem Formulation

Given a topology selection, the analog circuit device sizing problem can be formulated into a black-box function optimization problem, which can be expressed as follow:

\[ \text{minimize } f(x). \]  

By searching the \( d \)-dimensional design space, our target is to find an optimal circuit design \( x \) that minimizes the Figure of Merit (FOM) \( f(x) \) over the design space \( X \). For each design \( x \), we regard the simulation results generated by circuit simulators including commercial HSPICE and Spectre software as the ground-truth circuit performances \( f(x) \).

B. Review of the Bayesian Optimization Framework

There are two basic elements in the Bayesian optimization framework: the surrogate model and the acquisition function [17]. The surrogate model captures our prior beliefs about the latent function and provides an informative posterior distribution for each location over the design space. The posterior distribution means both the predictive mean and uncertainty estimation are provided. The predictive mean describes the behavior pattern of the latent function predicted by the surrogate model. The uncertainty estimation shows how much confidence the model has on its predictive mean. In other words, the surrogate model not only tells us what it believes but also how much we can trust its beliefs. Compared to deterministic models that only provide predictive mean and guide the search in a greedy manner, the stochastic statistical surrogate model employed by the Bayesian optimization framework can better facilitate the searching process and help to make more rational decisions.

One commonly used surrogate model is the Gaussian process regression (GPR) model. Assume we have an initial dataset \( D = \{ X, y \} \), where \( X = \{ x_1, \ldots, x_N \} \), \( y = \{ y_1, \ldots, y_N \} \), and \( N \) is the size of the dataset. By injecting our prior assumption about the latent function with mean function \( m(\cdot) \) and kernel function \( k(\cdot, \cdot) \), the posterior distribution for each location \( x \) can be expressed as [30]:

\[
\begin{align*}
\mu(x) &= m(x) + k(x_1, x)K^{-1}(y - m)
\sigma^2(x) &= k(x, x) - k(x, X)K^{-1}k(X, x),
\end{align*}
\]

where \( m = \{ m(x_1), \ldots, m(x_N) \}^T \), \( K = (k(x_1, X), \ldots, k(x_N, X))^T \), and \( \sigma^2_n \) is the noise variance. In this paper, we fix the mean function as \( m(x) = 0 \) and the kernel function as the square exponential (SE) covariance function.

By taking both the predictive mean and uncertainty estimation into consideration, the acquisition function works as a utility function to prioritize each location over the design space. By favoring the potential area with high uncertainty estimation, the acquisition function explores the unknown region to avoid getting stuck in the local optimum. By selecting locations with high probability to be optimal, the acquisition function exploits the collected information to avoid too much sampling around the less promising area. In other words, the acquisition function trades off between searching globally (exploration) and locally (exploitation). Popular acquisition functions include the probability of improvement (PI) [31], expected improvement (EI) [32], lower confidence bound (LCB) [19], Thompson sampling (TS) [33] and entropy search (ES) [34]. A portfolio of several acquisition functions is also possible [25], [35].

The Bayesian optimization framework starts the optimization procedure by first randomly sampling an initial dataset. By embedding our prior beliefs about the unknown objective function, the surrogate model builds upon the collected data points and provides the posterior distribution for each location over the design space. By taking both exploration and exploitation into consideration, the acquisition function helps to select the query points and traverse the design space efficiently. At each iteration, selected data points are observed to incrementally update the dataset. With a limited number of iterations, the Bayesian optimization framework can theoretically obtain the global optimum. The corresponding framework of the Bayesian optimization algorithm is presented in Algorithm 1.

### Algorithm 1 Bayesian Optimization Framework

Randomly sample an initial dataset \( D_0 = \{ X, y \} \)

for \( t = 1 \) to \( T \) do

1. Build a Gaussian process regression model with \( D_{t-1} \)
2. Select \( x_t \) that optimizes the acquisition function
3. Observe the selected data \( y_t = f(x_t) \)
4. Update the dataset \( D_t = \{ D_{t-1}, \{ x_t, y_t \} \} \)

end for

return The best \( f(x) \) recorded during optimization

III. PROPOSED APPROACH

In this section, we first highlight the fundamental challenges of the acquisition function optimization procedure and how it affects the efficiency of the Bayesian optimization framework (§III-A). Then, we explicitly describe our propose algorithm to handle this problem (§III-B and §III-C). Finally, we summarize the proposed method (§III-D).

A. Challenges of Acquisition Function Optimization

A large body of literature has been published to improve the efficiency of the Bayesian optimization framework. However, most of the previous researches mainly focus on designing a more informative surrogate model or an efficient acquisition function. The inner optimization procedure (a.k.a the acquisition function optimization procedure) has been assumed to relatively easy and often neglected.

From a theoretical point of view, the global optimum is only granted with a limited number of iterations for the Bayesian optimization framework, if the global optimum of the acquisition function is obtained at each iteration. In other words, the success of the Bayesian optimization framework crucially relies on successfully optimizing the acquisition function. The less promising data points accumulated across iterations can violate the theoretical justification of the Bayesian optimization framework. Therefore, it is theoretically important to optimize the acquisition function optimally.

From a practical point of view, the efficiency of the acquisition function optimization procedure suffers greatly from the curse of dimensionality. With the increase of the dimensionality, the state space for exploration increases exponentially. To search for the global optimum globally and make full coverage of the design space, the required number of acquisition function estimations can be prohibitively large. Take the grid search as an example, let us assume that we evenly select \( M \) data points for acquisition function estimation at each dimension to make full coverage of the design space. The expected number of data points for acquisition function estimation is \( M^d \) when the design space is \( d \)-dimensional. Considering that the computational complexity of the training and prediction procedure for GPR model is \( O(N^3) \) and \( O(N^2) \) respectively, the computational complexity for acquisition function optimization is \( O(N^2M^d) \).
Therefore, even if a large body of literature claims that the theoretical computational bottleneck of the Bayesian optimization framework is the model training process, the acquisition function optimization procedure easily dominates the computational overhead in practice. Although other algorithms have also been studied to facilitate the acquisition function optimization procedure including MSP algorithm [7], stochastic gradient ascent algorithm [36], evolutionary strategy [27], DIRECT [38], etc. This problem is only alleviated not solved.

In summary, the acquisition function optimization procedure is theoretically important and practically difficult. We tackle this problem by proposing a scalable Bayesian optimization framework in this paper.

B. Optimization in One-Dimensional Subspaces

Although optimizing the acquisition function over the whole design space is practically expensive or even intractable, searching for the global optimum of the acquisition function over the one-dimensional subspace at a time is relatively easy [39]. Instead of optimizing the acquisition function all over the design space, we constrain the searching space to a one-dimensional subspace \( \mathcal{L}_i = \{x_i + \beta_i z : \beta \in \mathbb{R}\} \cap X \) that contains the previously sampled best point \( \hat{x}_i \) [27], where \( \mathcal{L}_i \) represents the \( i \)-th one-dimensional subspace and \( l_i \) denotes the \( i \)-th direction. In this way, the global optimum of the acquisition function can be effectively obtained by grid search. Without surrendering the efficiency of the acquisition function optimization procedure, our proposed algorithm makes solid progress and ensures information gain at each iteration.

With the increase of the dimensionality, the traditional way of handling the acquisition function optimization procedure tends to follow the same behavior pattern as random search. Our proposed method instead makes solid and safe progress and maximizes information gain in the selected one-dimensional subspace, thus, improves the sampling efficiency.

C. Effective Dimension Selection

According to the above, another interesting question arises: how to select the effective dimension for optimization?

**Random Selection:** The intuitive and naive way of dimension selection is random selection. By randomly selecting the dimension for optimization, the design space at each one-dimensional subspace can be equally searched, thus, globally explored. However, this dimension selection policy greatly neglects the characteristics of different optimization problems and tends to evenly sample around the low and high potential subspaces.

**Descent Direction Estimation:** Another dimension selection policy is to calculate the descent direction heuristically based on the Thompson sampling [33]. By evaluating the utility of the best data point \( \hat{x}_n \) and the \( d \) data points around it with a small deviation \( \hat{x}_n + \delta l_i \) in each direction, we can calculate the gradient \( \hat{g} \) at each dimension and focus on the most crucial one. However, this dimension selection strategy tends to constantly sample around several most effective subspaces and easily get stuck in the local optimum.

In this paper, we propose a mixture of selection strategy to combine the benefits of random selection and descent direction estimation. With probability \( p \), we randomly select the dimension for optimization. With probability \( 1 - p \), we select the dimension with descent direction estimation based on the Thompson sampling. In this way, parameter \( p \) helps to balance between searching globally or locally. And we fix \( p \) as 0.8 in this paper.

D. Summary

Instead of optimizing the acquisition function over the whole design space, we propose to focus on a single dimension at a time to ensure information gain at each iteration. By combining the random selection strategy and the descent direction estimation policy, we adaptively avoid sampling too much in the low potential subspaces while globally exploring the design space. To fully utilize the hardware resources and further accelerate the optimization procedure, we also leverage the asynchronous batch Bayesian optimization framework EasyBO [40] to enable parallelism. By randomly selecting the exploration parameter, we achieve a delicate exploration-exploitation tradeoff and maximize the information gain per data point. The heuristically observed data points are employed to naturally penalize around the previously selected data points that are still under observation. The overall framework of our proposed algorithm LinEasyBO is presented in Algorithm 2.

**Algorithm 2 LinEasyBO Framework**

Randomly sample an initial dataset \( D_0 = \{X, y\} \), and the number of works \( B \) to evaluate the design in parallel

**for** \( t = 1 \) to \( T \) **do**

Wait for a worker to finish

Query the observed data \( D_t = \{D_{t-1}, \{x_t, y_t\}\} \)

Build a Gaussian process regression model with \( D_t \cup \{X, \hat{y}\} \), where \( \hat{X} = \{\hat{x}_1, \ldots, \hat{x}_{B-1}\} \) is the previously selected data points remaining to be observed, and \( \hat{y} = \{\mu(\hat{x}_1), \ldots, \mu(\hat{x}_{B-1})\} \) is the predictive mean of the \( \hat{X} \) that works as the heuristically observed data points

Get the current best design \( \hat{x}_n = \text{argmax}_x f(x) \)

Select the dimension \( l_i \) for optimization (§III-C)

Select \( x_t \) that optimizes the acquisition function (§III-B)

Observe the selected data with the idle worker \( y_t = f(x_t) \)

**end for**

**return** The best \( f(x) \) recorded during optimization

IV. EXPERIMENTAL RESULTS

In this section, we conduct experiments on two real-world analog circuits: the charge pump circuit (§IV-A) and the class-E power amplifier circuit (§IV-B). To demonstrate the efficiency of our proposed method in both sequential and batch mode, we compared the performances of LinEasyBO with several state-of-the-art algorithms in terms of both optimization results and wall-clock time. To distinguish between the sequential and batch mode, we label each algorithm with its corresponding batch size. In general, the tested algorithms can be divided into the following four categories.

C1 Random search, a perfect algorithm to measure the difficulty of optimization problems and the efficiency of the optimization algorithms [41].

C2 Sequential Bayesian optimization algorithms that work as baselines.

C3 Sequential Bayesian optimization algorithms that select the query points in one-dimensional subspaces including our proposed algorithm.

C4 Bayesian optimization algorithms that work in the batch mode.

For the sequential Bayesian optimization algorithms that work as baselines (C2), we run experiments on three algorithms: (1) EI [32], an improvement-based acquisition function that helps to explore the design space. (2) LCB [19], an optimistic policy that searches the design space in sequence. (3) EasyBO [40], an efficient Bayesian optimization algorithm that selects the exploration parameter randomly.

To demonstrate the effectiveness of optimizing the inner loop in the one-dimensional subspaces (C3), we run experiments on three alternative algorithms: (1) Line-EI, an improvement-based policy that optimizes the acquisition function in one-dimensional subspaces. (2) Line-LCB, an optimistic strategy that searches the next query points in line. (3) LinEasyBO, which is our proposed algorithm in sequential mode.
Finally, we comprehensively instantiate C4 with experiments on four algorithms: (1) LP-EI [28], an improvement-based strategy that maintains diversity in the batch with Lipschitz constant. (2) LP-LCB [28], an optimistic policy that introduces a local repulsive term to reduce redundantly sampling around the same area. (3) REMBOpBO [29], an efficient Bayesian optimization framework that handles the high-dimensional problems by projecting the design variables into a high effective subspace. (4) LinEasyBO, which is our proposed asynchronous algorithm in batch mode.

To ensure a fair comparison and reduce implementation bias, we implement the Gaussian process regression model with GPyOpt [28] library for all test algorithms. For REMBOpBO, we follow the experimental setting of [29] and optimize the acquisition function with NLopt [32] library. For the rest of the algorithms, we implement the inner optimization procedure with Scipy [43] library. For each algorithm, we limit the maximum number of circuit simulation as 350 and set the size of the initial dataset as 20. We also run each algorithm 20 times to reduce random fluctuations. All circuit performances are generated with the commercial HSPICE circuit simulator. And all experiments are conducted on a Linux workstation with two Intel Xeon CPUs and 128GB memory.

A. Charge Pump

The schematic of the charge pump circuit is presented in Figure 1. Implemented in an SMIC 40nm process, there are a total of 36 design variables, including lengths and widths of the transistors, the resistance of the resistors and the capacitance of the capacitors. The target of design is to ensure the current of M1 and M2 stay close to 40μA across 18 PVT corners [6]. The design specification of the charge pump is as follows:

\[
\begin{align*}
\text{minimize} & \quad 0.3 \times \text{diff} + 0.5 \times \text{deviation}, \\
\text{where} & \\
\text{diff}_1 &= \max_{PV T}(I_{M1,\text{max}} - I_{M1,\text{avg}}), \\
\text{diff}_2 &= \max_{PV T}(I_{M1,\text{avg}} - I_{M1,\text{min}}), \\
\text{diff}_3 &= \max_{PV T}(I_{M2,\text{max}} - I_{M2,\text{avg}}), \\
\text{diff}_4 &= \max_{PV T}(I_{M2,\text{avg}} - I_{M2,\text{min}}), \\
\text{diff} &= \sum_{i=1}^{4} \text{diff}_i, \\
\text{deviation} &= \max_{PV T}(I_{M1,\text{avg}} - 40\mu A) + \max_{PV T}(I_{M2,\text{avg}} - 40\mu A).
\end{align*}
\]

And the experimental results are presented in Table I.

Let us first focus on the sequential mode. It is worth notice that the performances of EI, LCB and EasyBO significantly outperform their traditional counterparts. This clearly suggests that searching the design space in one-dimensional subspaces is beneficial regardless of the acquisition function. This also emphasizes that the efficiency of the Bayesian optimization framework can be largely impacted by the inner optimization procedure. By solely focusing on one dimension at a time, space exploration efficiency can be greatly improved.

Now let us move on to analyzing the optimization results in batch mode. Compared to LP-EI, LP-LCB, and REMBOpBO, LinEasyBO consistently demonstrates better performances in terms of the optimization results and time consumption on average regardless of the batch size. This clearly shows the efficiency and effectiveness of our proposed algorithm. The performances of LP-EI, LP-LCB, REMBOpBO, and LinEasyBO deteriorate with the increase of the batch size. This implies that the design specification of the charge pump circuit is rather sensitive to the exploration parameter of the acquisition function. In other words, even if there is only a small change in the exploration-exploitation tradeoff of the acquisition function, the final optimization results can fluctuate greatly for this circuit. The fact that LinEasyBO constantly outperforms EI, LCB and EasyBO in both sequential and batch mode again emphasizes the importance of the inner optimization procedure. The fact that REMBOpBO is always outperformed to a large extent by LP-EI, LP-LCB, and LinEasyBO suggests that the low effective dimensionality presumption doesn’t hold for the charge pump circuit. The strong assumption that a low-

| Algo              | Best  | Worst | Mean  | Std   | Time  |
|-------------------|-------|-------|-------|-------|-------|
| Random Search     | 21.16 | 46.39 | 30.49 | 7.52  | -     |
| EI                | 3.42  | 6.47  | 4.09  | 0.35  | 7h45m32s |
| LCB               | 3.45  | 27.92 | 5.22  | 0.22  | 6h34m11s |
| EasyBO            | 5.09  | 7.43  | 6.15  | 0.69  | 7h57m51s |
| Line-EI           | 3.11  | 4.40  | 3.37  | 0.29  | 7h56m32s |
| Line-LCB          | 3.17  | 7.37  | 3.39  | 0.17  | 7h53m1s  |
| LinEasyBO         | 3.16  | 4.06  | 3.39  | 0.22  | 7h59m59s |
| LP-EI-5           | 3.49  | 5.13  | 4.06  | 0.39  | 2h10m34s |
| LP-LCB-5          | 3.63  | 5.08  | 4.17  | 0.36  | 1h48m6s  |
| REMBOpBO-5        | 9.86  | 32.75 | 14.70 | 4.87  | 2h21m48s |
| LinEasyBO-5       | 3.18  | 4.11  | 3.44  | 0.25  | 1h45m58s |
| LP-EI-10          | 3.77  | 5.63  | 4.59  | 0.45  | 1h35m38s |
| LP-LCB-10         | 3.53  | 4.89  | 4.24  | 0.30  | 53m52s   |
| REMBOpBO-10       | 8.76  | 27.68 | 17.51 | 5.12  | 1h43m2s  |
| LinEasyBO-10      | 3.32  | 4.71  | 3.71  | 0.34  | 49m58s   |
| LP-EI-15          | 4.02  | 5.85  | 5.02  | 0.49  | 1h22m23s |
| LP-LCB-15         | 3.60  | 4.93  | 4.29  | 0.33  | 42m31s   |
| REMBOpBO-15       | 11.35 | 32.12 | 18.31 | 5.62  | 1h28m2s  |
| LinEasyBO-15      | 3.23  | 4.90  | 3.95  | 0.41  | 33m33s   |

Fig. 1. The schematic of the charge pump circuit, which is reproduced from [6].

Fig. 2. The optimization results with respect to the wall clock time for the charge pump circuit, when the batch size is 15.
dimensional effective subspace exists for the optimization problem
can greatly limit the application of REMBOpBO, while LinEasyBO
is not affected by the characteristic of the problem and can be
widely and safely used without much consideration of the application
scenario. As is shown in Figure 2, compared with LP-LCB, LP-EI,
and REMBOpBO, LinEasyBO reduces 56.1%, 84.9%, and 96.6% of
the time consumption respectively when the batch size is 15, while
achieving the same optimization results. This translates to 2×, 6×,
and 29× optimization procedure acceleration respectively.

B. Class-E Power Amplifier

The schematic of the class-E power amplifier circuit is presented
in Figure 3. Implemented in a TSMC 180nm process, there are a
total of 12 design variables. The target of design for this circuit is
to maximize the power-added efficiency (PAE) and the output power
(Pout) simultaneously [25]. The corresponding design specification
is as follows:

\[
\begin{align*}
\text{maximize} & \quad 3 \times \text{PAE} + \text{Pout}.
\end{align*}
\]

And the optimization results are shown in Table II.

As usual, we start with looking at the sequential mode. Again,
the Bayesian optimization algorithms with our proposed acquisition
function optimizer consistently outperform their traditional coun-
terparts. This again demonstrates the efficiency and effectiveness
of searching the design space one dimension at a time and the
importance of successfully optimizing the acquisition function at
each iteration. Another interesting and unexpected phenomenon is
that random search algorithm obtains better optimization results than
EI and comparable performance with LCB. This strongly stresses
the fact that a less efficient inner optimization strategy can guide
the search in the wrong direction and greatly hurt the performance of
the optimization algorithm. By only focusing on a single dimension and
optimizing the acquisition function optimally at each iteration, the
Bayesian optimization algorithm can iteratively make solid progress
and search the state space in a safe and efficient manner. Our proposed
acquisition function optimization method can be easy-to-use and
effective regardless of the acquisition function and beneficial across
different dimensionality.

Let’s now analyze the optimization results in batch mode. One
interesting phenomenon is that REMBOpBO is constantly outper-
formed by random search. The assumption that only a small group
of design variables have a great impact on the circuit performance
can significantly hurt the performance of REMBOpBO when the
presumption is not met. This makes REMBOpBO a risky choice
when we have no information at hand about the black-box optimization
problem. The fact that LinEasyBO consistently outperforms LP-
EI, LP-LCB, and REMBOpBO with less time consumption again
demonstrates the efficiency and effectiveness of our proposed algo-
rithm. By focusing on a single dimension at a time, the acquisition
function optimization procedure can help to make solid and safe
decisions and maximize the information gain in the long run. The
comparable optimization results of LinEasyBO across different batch
size show the robustness of our proposed algorithm. As is presented
in Figure 4, LinEasyBO reduces the simulation time (accelerates the
optimization procedure) by 88.8% (9×), 91.7% (11×), and 97.4%
(38×) compared to LP-EI, LP-LCB, and REMBOpBO respectively
when the batch size is 15, while obtaining the same optimization results.

V. CONCLUSION

In this paper, we proposed a fast and robust Bayesian optimiza-
tion framework for analog circuit synthesis via one-dimensional
subspaces. Instead of searching the global optimum for acquisition
function over the whole design space, our proposed method focuses
on one dimension at a time. We also introduce an effective dimension
selection strategy to balance searching globally and locally. In this
way, our proposed algorithm guarantees information gain at each
step and searches the design space safely and efficiently. To further
accelerate the optimization procedure, we leverage the batch Bayesian
optimization framework to fully utilize the computational resources.
Compared to the traditional acquisition function optimization pro-
cedure, our method not only reduces the computational overhead

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**Table II**

| Algo      | Best | Worst | Mean | Std | Time  |
|-----------|------|-------|------|-----|-------|
| Random Search | 4.16 | 3.34 | 3.70 | 0.21 | -     |
| EI        | 4.36 | 1.60 | 3.68 | 0.52 | 12h22m8s |
| LCB       | 4.12 | 3.42 | 3.79 | 0.21 | 9h10m31s |
| EasyBO    | 4.51 | 3.69 | 4.01 | 0.15 | 6h49m46s |
| Line-EI   | 4.83 | 2.71 | 4.29 | 0.49 | 6h44m35s |
| Line-LCB  | 4.81 | 2.82 | 4.03 | 0.52 | 6h46m46s |
| LinEasyBO | 5.44 | 3.35 | 4.34 | 0.58 | 6h17m15s |

**Fig. 4.** The optimization results on average with respect to the wall-clock time for the class-E power amplifier circuit, when the batch size is 15.
for the Bayesian optimization framework but also increases the information gain per observation. We experimentally compare the performance of our proposed algorithm with several state-of-the-art optimization algorithms. Experimental results quantitatively show that our method accelerates the optimization procedure by up to 9× and 38× compared to LP-EI and REMBOpBO when the batch size is 15.

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