Cycles in $G$-orbits in $G^C$-flag manifolds

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Abstract

There is a natural duality between orbits $\gamma$ of a real form $G$ of a complex semisimple group $G^C$ on a homogeneous rational manifold $Z = G^C/P$ and those $\kappa$ of the complexification $K^C$ of any of its maximal compact subgroups $K$: $(\gamma, \kappa)$ is a dual pair if $\gamma \cap \kappa$ is a $K$-orbit. The cycle space $C(\gamma)$ is defined to be the connected component containing the identity of the interior of $\{g : g(\kappa) \cap \gamma \text{ is non-empty and compact}\}$. Using methods which were recently developed for the case of open $G$-orbits, geometric properties of cycles are proved, and it is shown that $C(\gamma)$ is contained in a domain defined by incidence geometry. In the non-Hermitian case this is a key ingredient for proving that $C(\gamma)$ is a certain explicitly computable universal domain.

0 Introduction and Notation

Let $G$ be a non-compact semi-simple Lie group without compact factors which is embedded in its complexification $G^C$ and let $Z = G^C/Q$ be a $G^C$-flag manifold, i.e., a compact, homogeneous, algebraic $G^C$-manifold. Denote by $K$ a maximal compact subgroup of $G$; in particular $G/K$ is a negatively curved Riemannian symmetric space. In the sequel we will assume that $G$ is simple. The necessary adjustments for the semi-simple case are straight-forward.

Let $Orb_G(Z)$ (resp. $Orb_G(K^C)$) denote the set of $G$-orbits (resp. $K^C$-orbits) in $Z$. It is known that these sets are finite ([W1]). If $\kappa \in Orb_G(K^C)$ and $\gamma \in Orb_G(Z)$, then $(\kappa, \gamma)$ is said to be a dual pair if $\kappa \cap \gamma$ is non-empty and compact.

If $\gamma$ is an open $G$-orbit, then $\kappa$ being dual to $\gamma$ is equivalent to $\kappa \subset \gamma$. In ([W1]) it is shown that every open $G$-orbit contains a unique compact $K^C$-orbit, i.e., duality at the level of open $G$-orbits. This is extended in ([M], see also [BL] and [MUV]) to the case of all orbits: For every $\gamma \in Orb_G(Z)$ there exists a unique $\kappa \in Orb_G(K^C)$ such that $(\gamma, \kappa)$ is a dual pair and vice versa. Furthermore, if $(\gamma, \kappa)$ is a dual pair then the intersection $\kappa \cap \gamma$ is transversal at each of its points and consists of exactly one $K$-orbit.

To motivate the notion of a cycle in this case, let us begin with the case of an open $G$-orbit $D$. The dual orbit $\kappa$ defines a point in the cycle

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space $C^i(D)$, where $q := \text{dim}\, \kappa$. The connected component $\Omega_W(D)$ of \{\(g \in G^C : g(\kappa) \subset D\)\} can be regarded as a family of cycles by the procedure of associating $g$ to the cycle $g(\kappa)$. Clearly $\Omega_W(D)$ is invariant by the action of $K^C$ on $G^C$ on the right and therefore we often regard it as being in the affine homogenous space $\Omega := G^C/K^C$.

The cycle space $C(\gamma)$ associated to a lower-dimensional orbit is defined analogously, at least when one has duality in mind: $C(\gamma)$ is the connected component containing the identity of the interior of the set

\[ \{g \in G^C : g(\kappa) \cap \gamma \text{ is non-empty and compact}\}. \]

Since the intersection $\kappa \cap \gamma$ is transversal, it is clear that $C(\gamma)$ is non-empty.

Here, in a result that we state at the outset, it is shown that this is in fact a reasonable set. The following is proved in §2.

**Proposition 1.** If $g \in C(\gamma)$, then $g(\ell(\kappa)) \cap \ell(\gamma) := M_g$ is a compact subset of $g(\kappa) \cap \gamma$. This intersection is transversal at each of its points and the manifold $M_g$ is homotopic in $\gamma$ to $M = M_e$. If $g \in bd(C(\gamma))$, then either $g(\kappa) \cap \gamma$ is non-compact or empty. In particular, if \{\(g_n\)\} is a divergent sequence in $C(\gamma)$, then there exists $z_n \in g(\kappa) \cap \gamma$ with \{\(z_n\)\} divergent in $\gamma$.

In the sequel we take this to be the meaning of the cycle space $C(\gamma)$ and regard the manifolds $M_g$ as cycles. Just as in the case of open orbits, we often regard $C(\gamma)$ as a $G$-invariant domain in $\Omega$.

Our main result is the complete characterization of the cycle spaces $C(\gamma)$ in the case where $G$ is non-Hermitian, where our contribution is in the case of the non-open orbits. For a given group $G$ these cycle spaces are all naturally biholomorphically isomorphic to a fixed domain which, when realized in $\Omega$, is denoted by $\Omega_{AG}$ (see Thm. 9).

This domain was discussed in (C) in the context of differential geometry. If $G \times_K \mathfrak{p}$ is the tangent bundle of the Riemannian symmetric space $M = G/K$, then $\Omega_{AG}$ is identified by the polar coordinates map $P : TM \rightarrow \Omega$, $[(g, \xi)] \mapsto g\exp(i\xi)$, with the maximal neighborhood of the 0-section on which $P$ is a local diffeomorphism. It was considered in (AG) from the point of view of neighborhoods of $M$ in $\Omega$ on which $G$ acts properly. It is known to be a Stein domain and its complex geometry is closely related to the Riemannian geometry of the symmetric space $M$ (BHH).

In terms of roots, if $\mathfrak{g} = \mathfrak{k} + \mathfrak{a} + \mathfrak{n}$ is an Iwasawa-decomposition, $\Phi$ is a system of roots on $\mathfrak{a}$ and $\omega_{AG}$ is the connected component containing $0 \in \mathfrak{a}$ of the set which is obtained by removing from $\mathfrak{a}$ the union affine hyperplanes

\[ \bigcup_{\alpha \in \Phi} \{\xi \in \mathfrak{a} : \alpha(\xi) = \frac{\pi}{2}\}. \]

Then $\Omega_{AG} = G\exp(i\omega_{AG}).x_0$, where $x_0 \in G^C/K^C$ is the base point.

Using a certain triality and related incidence varieties (HW1) along with $G$-invariant theoretic properties and the Kobayashi hyperbolicity of $\Omega_{AG}$
(H, FH), it has been recently shown that, with a few well-known Hermitian exceptions where $\Omega_W(D)$ is the associated bounded symmetric domain, the cycle domain $\Omega_W(D)$ of an open orbit is naturally identifiable with $\Omega_{AG}$.

Our work here makes use of the methods and results of these papers along with a result of (GM) which, together with knowledge of the intersection of the cycle domains for the open orbits in $G^C/B$, implies the inclusion $\Omega_{AG} \subset C(\gamma)$ for all $\gamma \in Orb_Z(G)$.

For a survey of these and other basic properties of $\Omega_{AG}$ see (HW1).

1 Basic triality

Here $B$ denotes a Borel subgroup of $G^C$ which contains the factor $AN$ of an Iwasawa decomposition $G = KAN$. As usual the closure $S$ of a $B$-orbit $O$ in $Z$ is referred to as a Schubert variety. Let $Y := S \setminus O$.

Since the set of Schubert varieties $S$ generates the homology of $Z$, for every $\kappa \in Orb_Z(K^C)$ the set

$$S_\kappa := \{ S \in S : S \cap \text{cl}(\kappa) \neq \emptyset \}$$

is non-empty. The following is a slightly refined version of Thm. 3.1 of (HW1).

**Theorem 2.** Let $(\kappa, \gamma)$ be a dual pair and $S \in S_\kappa$. Then

1. $S \cap \text{cl}(\kappa) \subset \kappa \cap \gamma$
2. The intersection $S \cap \gamma$ is open in $S$ and consists of finitely many $AN$-orbits each of which is open in $S$ and closed in $\gamma$.
3. If $z_0 \in S \cap \gamma$, then $\Sigma := (AN).z_0$ intersects $\kappa$ in exactly one point and that intersection is transversal in $Z$.

The only refinement is that, instead of $\Sigma \cap \kappa$ being finite, we now show that it consists of just one point. This is implicit in (HW1) (see §5 in that paper). Let us repeat the relevant details.

Let $\alpha : K.z_0 \times (AN).z_0 \rightarrow G.z_0$ be defined by multiplication, $(k, an) \mapsto kan$. As was shown in (HW1) $\alpha$ is a diffeomorphism onto a number of components of $G.z_0$. However, since the orbit $M := K.z_0$ is a strong deformation retract of $\gamma$, it follows that $G.z_0 / K.z_0$ is connected and consequently $\alpha$ is surjective. Thus $|\Sigma \cap \kappa| = 1$ is proved just as in Cor. 5.2 of (HW1). □

Given $\gamma, \kappa$, an Iwasawa-decomposition $G = KAN$, a Borel subgroup $B$ of $G^C$ which contains $AN$, a $B$ Schubert variety $S \in S_\kappa$ and an intersection point $z_0 \in \kappa \cap S$ as above, we refer to $\Sigma = AN.z_0$ as the associated Schubert slice. Note that $G$-conjugation yields a $G_p$-invariant family of Schubert slices $\Sigma$ at every point $p \in \gamma$.

It should also be noted that in the case that $\gamma = \gamma_{\ell}$ is closed, $\Sigma$ is just the associated $B$-fixed point.
2 Cycle transversality

Our main goal here is to prove Prop. 1. The first step is to show that if $g(\kappa) \cap \gamma$ is compact, the intersection of the variety $g(c_\ell(\kappa))$ with every Schubert slice $\Sigma$ has optimal transversality properties.

**Lemma 3.** For all $g \in G^C$ with $g(\kappa) \cap \gamma$ compact the number of points in the intersection $g(\kappa) \cap \Sigma$ is bounded by the intersection number $S.c_\ell(\kappa)$.

**Proof.** Since $\Sigma$ can be regarded as a domain in $O \cong \mathbb{C}^n$ and $g(\kappa) \cap \gamma$ is compact, it follows from the maximum principle that $g(\kappa) \cap \Sigma$ is finite and of course it is then bounded by the intersection number $S.c_\ell(\kappa)$. 

It should be underlined that, since $|g(\kappa) \cap \Sigma|$ is finite, it is semi-continuous in the sense that it can only increase as $g$ moves away from $g_0$.

Let

$$I := \{ g \in G^C : |g(\kappa) \cap \Sigma| = 1 \text{ for all } \Sigma \}$$

Here for all $\Sigma$ means for all choices of the maximal compact group $K$ and all Iwasawa factors $AN$, i.e., all Schubert slices which arise by $G$-conjugation of those $\Sigma$ which are connected components of $S \cap \gamma$ for a fixed $S \in S_n$.

We now consider the open set $I \cap C(\gamma)$.

**Lemma 4.** If $g \in \text{bd}(C(\gamma) \cap I)$, then either $g(\kappa) \cap \gamma$ is empty or non-compact. In particular, $C(\gamma) \subset I$.

**Proof.** We may assume that $g \notin I$, because if $g \in I$ and $g(\kappa) \cap \gamma$ is compact, then both conditions hold in an open neighborhood of $g$ and consequently $g \in C(\gamma)$.

We assume that $g(\kappa) \cap \gamma$ is compact and reach a contradiction. For this let $M$ be the limiting set of the sequence of manifolds $M_n := g_n(\kappa) \cap \gamma$.

Since $C(\gamma)$ is by definition connected and $M_n$ is connected, it follows that $M$ is a connected closed set.

Since $g(\kappa) \cap \gamma$ is compact, it follows that $M \cap \gamma$ is compact and is therefore closed in $M$. On the other hand $M \cap \text{bd}(\gamma)$ is closed in $M$ and consequently $M \cap \gamma$ is also open in $M$. By the semicontinuity of the intersection number and the fact that $g \notin I$, $M \cap \Sigma_0 = \emptyset$ for some Schubert slice $\Sigma_0$. In particular, since $|M_n \cap \Sigma_0| = 1$ for all $n$, $M \notin \gamma$. Since $M$ is connected, it follows that $M \cap \gamma = \emptyset$.

But $g(\kappa) \cap \gamma$ is non-empty. So there exists $p \in \gamma$ and an open neighborhood $U(p)$ so that $g_n(\kappa) \cap U = \emptyset$, but $p \in g(\kappa)$. On the other hand, if $\Sigma$ is a Schubert slice at $p$, and $p$ were isolated in $g(\kappa) \cap \Sigma$, it would follow that $g_n(\kappa) \cap U \cap \Sigma$ is non-empty for $n$ sufficiently large. 

**Proof of Proposition** Observe that if $g \in C(\gamma)$ and $g(c_\ell(\kappa)) \cap \gamma$ contained an additional point $p \in \text{bd}(\gamma)$ or in $\text{bd}(\kappa)$, then for $h \in C(\gamma)$ chosen appropriately, in particular small, we would find some $\Sigma_0$ which
contains \( h(p) \) as well as another point in \( M_{hg} \). This is contrary to the fact that \( C(\gamma) \subset I \) and that this intersection contains exactly one point.

The transversality of the intersection and the properties of \( M_{g} \) are also immediate consequences of \( C(\gamma) \) being connected and contained in \( I \).

If \( g \in \text{bd}(C(\gamma)) \), then, again since \( C(\gamma) \subset I \), by Lemma \( 4 \) \( g(\kappa) \cap \gamma \) is either empty or non-compact. \( \square \)

3 Description of the cycle spaces

3.1 Lifting cycle spaces

The above transversality results are only useful in the case when \( \gamma \) is not closed. Here we prove a Lifting Lemma which will be used to understand the cycle space \( C(\gamma_{cl}) \) of the closed orbit. For this let \( Z = G^c/P \) as usual, let \( \tilde{Z} = G^{c}/\tilde{P} \) be defined by a parabolic group \( \tilde{P} \) which is contained in \( P \) and \( \pi: \tilde{Z} \to Z \) is the natural projection.

Proposition 5. If \( (\gamma, \kappa) \) is a dual pair of orbits in \( Z \) with \( z_0 \in \gamma \cap \kappa \) and \( \tilde{z}_0 \) is in a closed \( G_{z_0} \)-orbit in the fiber \( F := \pi^{-1}(z_0) \), then \( \tilde{\gamma} := G\cdot z_0 \) and \( \tilde{\kappa} := \tilde{K}\cdot \tilde{z}_0 \) define a dual pair \( (\tilde{\gamma}, \tilde{\kappa}) \) in \( \tilde{Z} \). Furthermore, the mapping \( \pi|\tilde{\gamma}: \tilde{\gamma} \to \gamma \) is proper.

Proof. Since \( K_{z_0} \) is a maximal compact subgroup of \( G_{z_0} \), it acts transitively on the compact orbit \( G_{z_0}\cdot \tilde{z}_0 \). Consequently \( \tilde{\gamma} \cap F = K\cdot \tilde{z}_0 \) and, since \( \gamma \cap \kappa = K\cdot z_0 \), it follows that \( \tilde{\gamma} \cap \tilde{\kappa} = K\cdot \tilde{z}_0 \).

The properness of \( \pi|\tilde{\gamma} \) follows immediately from the fact that it is a homogeneous fibration with compact fiber. \( \square \)

In the following result we maintain the above notation.

Proposition 6. If \( \gamma \) is not closed, then \( C(\gamma) \subset C(\tilde{\gamma}) \)

Proof. Let \( \{g_n\} \) be a sequence in \( C(\tilde{\gamma}) \) which converges to \( g \in \text{bd}(C(\tilde{\gamma})) \).

By Prop. 4 there is a sequence \( \tilde{z}_n \) in \( g_n(\tilde{\kappa}) \cap \tilde{\gamma} \) which diverges in \( \tilde{\gamma} \). Since \( \pi|\tilde{\gamma} \) is proper, the corresponding sequence \( z_n \) in \( C(\gamma) \) is also divergent.

Now either \( g_n \notin C(\gamma) \) infinitely often, in which case \( g \notin C(\gamma) \) or we may assume that \( \{g_n\} \subset C(\gamma) \). In the latter case, since \( z_n \in g_n(\kappa) \cap \gamma \) is divergent in \( \gamma \) it likewise follows that \( g \notin C(\gamma) \).

Hence \( \text{bd}(C(\tilde{\gamma})) \cap C(\gamma) = \emptyset \) and the desired result follows. \( \square \)

Finally, let \( \gamma \) be such that \( \text{cl}(\gamma) = \gamma \cup \gamma_{cl} \) and \( \tilde{\gamma} \) be as above. If \( g_n \in C(\gamma) \) converges to \( g \in \text{bd}(C(\gamma)) \), then, again by Prop. 4, it follows that \( g(\text{cl}(\kappa)) \cap \gamma_{cl} \neq \emptyset \). Consequently, \( C(\gamma_{cl}) \subset C(\gamma) \).

Thus, in the notation above, we have the following consequence.

Corollary 7. \( C(\gamma_{cl}) \subset C(\gamma) \subset C(\tilde{\gamma}) \)
3.2 Proof of the main theorem

Let us first consider the case where $\gamma$ is not closed and let $S \in S_\kappa$ as above. Since $\kappa \cap S \cap \gamma$ already realizes the intersection number $S.c(\kappa)$, it follows that $g(c(\kappa)) \cap Y = \emptyset$ for all $g \in C(\gamma)$.

From now on we regard the homogeneous space $\Omega = G^C / K^C$ as a family of $q$-dimensional cycles, $q := \dim C(\kappa)$, via the identification of $g \in C(\gamma)$ with the cycle $g(c(\kappa))$.

Since $Y := S \setminus O$ can be given the structure of a very ample divisor in $S$, it follows that incidence variety of all $q$-dimensional cycles $C$ with $C \cap Y \neq \emptyset$ contains a complex hypersurface which pulls back to a complex hypersurface $H \subset \Omega$ ([BK],[BM]).

By the method of ([BK]), for $f \in \Gamma(S,\mathcal{O}(\ast Y))$ an appropriately chosen meromorphic function with poles along $Y$, the trace-transform $Tr(f)$ has a non-empty polar set $P$ which is the union of a certain number of components of $H$. Apriori it is possible that there are cycles $g \in C(\gamma)$ with $c(g(\kappa)) \cap Y \neq \emptyset$, but with $|c(g(\kappa)) \cap \Sigma| = S.c(g(\kappa))$. These would necessarily have positive-dimensional intersection with $S$ and the non-discrete components would lie in $S \setminus \Sigma$.

However, this phenomenon occurs on a lower-dimensional subvariety in $\Omega$, and such cycles are limits of generic cycles which intersect $S$ only in points of $\Sigma$ and of course at only finitely many points which are bounded away from $Y$. Thus the value of $Tr(f)$ at $g(c(g(\kappa)))$ is finite and therefore such cycles are not in $P$. As a consequence $P \cap C(\gamma) = \emptyset$ and we redefine $H$ to be $P$ in the sequel.

Proposition 8. If $\gamma$ is not closed, then there exists a Borel subgroup $B$ which contains a factor $AN$ of an Iwasawa-decomposition $G = KAN$ and a $B$-invariant hypersurface $H$ in $\Omega$ so that $\Omega_{AG} \subset C(\gamma) \subset \Omega_H$.

Proof. From the characterization of cycle spaces of the open $G$-orbits ([HW],[FH]) it follows in particular that the intersection of all such cycle spaces for the open orbits in $G^C / B$ is $\Omega_{AG}$ (This intersection property also follows in most cases from the results in [GM]). By Prop. 8.1 of ([GM]) it then follows that

$$\Omega_{AG} \subset C(\gamma).$$

In the case of a non-closed orbit we have the $B$-invariant hypersurface $H$ in the complement of $C(\gamma)$ and, since $C(\gamma)$ is $G$-invariant and contains the base point in $\Omega_{AG}$ it follows that it is contained in the connected component of

$$\Omega \setminus \left( \bigcup_{k \in K} k(H) \right)$$

which by definition is $\Omega_H$. \hfill \Box

Theorem 9. If $G$ is not of Hermitian type, then

$$C(\gamma) = \Omega_{AG}$$

for all $\gamma \in Orb_Z(G)$. 
Proof. Since $G$ is non-Hermitian, $\Omega_H$ is Kobayashi hyperbolic (see \cite{H}, \cite{FH}) and the main theorem of \cite{FH} can therefore be applied: A $G$-invariant, Stein, Kobayashi hyperbolic domain containing $\Omega_{AG}$ is $\Omega_{AG}$ itself. Thus $\Omega_{AG} = \Omega_H$ and the result for non-closed orbits follows from Prop. 8.

By Cor. 7 it then follows that $C(\gamma_{c,\ell}) \subset \Omega_{AG}$. But, if $\tilde{Z} = G^C/B$, it is clearly the case that $C(\gamma_{c,\ell}) \subset C(\gamma_{c,\ell})$ and it is known that $C(\gamma_{c,\ell}) = \Omega_{AG}$ (\cite{B}, \cite{H}, see also \cite{FH}). Thus, $C(\gamma_{c,\ell}) \supset \Omega_{AG}$ and the result is proved for closed orbits as well.

It should be remarked that we actually only use the Kobayashi hyperbolicity of $\Omega_H$ and that one might expect the following result in the Hermitian case: If $\Omega_H$ is not Kobayashi hyperbolic, then $C(\gamma)$ can be identified with the bounded Hermitian symmetric space associated with $G$.

Note added in proof. With a few added details which we note here, we also handle the Hermitian case. For this assume that $G$ is of Hermitian type and let $P$ be a complex subgroup of $G^C$ which properly contains $K^C$. Then $X := G^C/P$ is a compact Hermitian symmetric space. There are only two choices for $P$ and the (open) $G$-orbit of the neutral point in $X$ is the bounded Hermitian symmetric domain $B$ or its conjugate $\bar{B}$.

**Proposition 10.** If $G$ is of Hermitian type, then either

$$C(\gamma) = \Omega_{AG} = B \times \bar{B}$$

or the base dual cycle $c\ell(\kappa)$ is $P$-invariant and $C(\gamma)$ is either $B$ or $\bar{B}$, depending on the choice of sign.

Proof. In the Hermitian case it is known that $\Omega_{AG}$ agrees with $B \times \bar{B}$ in its natural embedding in $G^C/K^C$ (\cite{BHH}).

Let $\pi : G^C/K^C \to G^C/P =: X$, where $P$ is one of the two choices mentioned above, and let $B$ be an Iwasawa-Borel subgroup of $G^C$ which is being used for the incidence geometry. If the $B$-invariant hypersurface $H$ in $G^C/K^C$ of \cite{X} is not a lift $H = \pi^{-1}(H_0)$, then $\Omega_H$ is Kobayashi hyperbolic and therefore $\Omega_H = \Omega_{AG}$ (\cite{FH}). Thus, just as in the non-Hermitian case, the desired result follows from Prop. 8.

In order to complete the proof, we assume that $H$ is a lift, but that $c\ell(\kappa)$ is not $P$-invariant, and reach a contradiction.

Let $x_0 \in \gamma$ be a base point with $\kappa = K^C.x_0$. Since $\kappa$ is not $P$-invariant, $c\ell(P.x_0)$ contains $c\ell(\kappa)$ as a proper algebraic subvariety. Now the intersection $\kappa \cap \gamma$ is transversal in $Z$. Thus every component of $P.x_0 \cap \mathcal{O}$ is positive-dimensional. Since $\mathcal{O} = \mathbb{C}^{m(\mathcal{O})}$ is affine, every such component has at least one point of $Y$ in its closure.

Thus for $B$ an arbitrarily small neighborhood of the identity in $G^C$ there exists $p \in P$ and $g \in B$ with $g.p.x_0 \in Y$, and consequently $g.p.K^C \in \Omega_H$.

But $H$ was assumed to be a lift. This is equivalent to $\Omega_H$ being a lift, i.e., at the group level $\Omega_H$ is invariant under the action of $P$ defined by right-multiplication. Hence $g.K^C$ is also in $\Omega_H$. However, $g$ can be chosen
arbitrarily near the identity, contrary to $C(\gamma)$ being an open neighborhood of the neutral point in $G^C/K^C$.

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