Numerical simulation of air ionization processes in plasma-actuator electric field

P A Semenev*1,2, P D Toktaliev*1,3 and S I Martynenko1,3

1P. I. Baranov Central Institute of Aviation Motors, Moscow, Russia
2Bauman Moscow State Technical University, Moscow, Russia
3Institute of Problems of Chemical Physics of Russian Academy of Sciences

E-mail: semenyov@gmail.com

Abstract. Mathematical model based on drift-diffusion approximation of hydrodynamic method of plasma description with two-temperature approximation was carried out for air ionization case in order to model process of interaction between dielectric barrier discharge plasma actuator and gas flow. Electron-induced ionization-recombination mechanism was used. Air was considered as mixture of molecular oxygen and nitrogen in proportion at normal conditions. Process kinetics was described by electron-induced reactions with molecular nitrogen and oxygen, ions and electron components. Equations system consisted of transport equations for concentrations of all mixture components, electron temperature and equation for electric potential was solved. Extended Scharfetter-Gummel scheme for large quantity of transport equations was developed. It was shown, that main influence on charge distribution has electron temperature. This model could be used for volume and surface charge density estimations necessary for volume force calculation in the vicinity of electrodes.

1. Introduction
Process of interaction between dielectric barrier discharge (DBD) plasma actuator and gas flow is a complex multiphysical problem, which involves coupled disciplines: gas dynamics, turbulence, heat transfer, plasma dynamics and ion kinetics.

An approach for numerical simulation DBD-actuator influence on the flow based on empirical estimation of source terms in Navier-Stokes equations and interaction area size from experimental flow velocity fields was considered and carried out in previous works [1]. Empirical estimations was used for setting constant value source terms regions (so-called “active volumes”) within the framework of this approach. Main advantages of this approach are simplicity of setting a piecewise constant volume force and heat source fields and, hence, obtaining a quick estimations of possible influence on the flow. Disadvantages consist in the following: approach does not reveal physical features of the process; it does not allow connect values of the source terms with DBD electrical parameters and requires reliable empirical estimations.

More complicated mathematical model allows calculating continuous electrical potential ($\phi$) and volume charge density fields ($\rho$) in computational domain (hence, to calculate continuous volume force field by averaging and multiplying these two fields). It was carried out by means of Dual-Potential Method (DPM) based on reduced Maxwell equations [2]. In addition, it is become possible to obtain connection between calculated fields ($\rho$, $\phi$) and electrodes voltage and alternating current frequency – parameters of DBD actuator. More complex mathematical model allows describing
physical features of ionization process in the vicinity of actuator. Nevertheless, model has few empirical constants in boundary conditions used for potential and charge density equations solution. Both approaches do not consider ionization mechanism, i.e. mechanism of low temperature plasma formation nearby electrodes. For this purpose it was suggested a consecutive problem solution with the following steps: ionization modelling, calculating source terms fields (e.g. by means of DPM) and integration of these terms fields in Navier-Stokes equations numerical simulation (e.g. using RANS method).

2. Description and application of mathematical model
An approach based on drift-diffusion approximation of hydrodynamic method of plasma description with two-temperature approximation was carried out for volume and surface charge formation analysis for air ionization case. This approximation is widely used in electron and holes transport processes simulation in semiconductors [3-5]. An assumption is that velocity and energy relax time scales in charged particles system considerably lesser than typical time scale of the problem. Main part of this description is ionization-recombination mechanisms. Overview of this mechanism with commentaries is set out below; more detailed picture of kinetic processes taking place in air in the presence of electric field is presented in [6]. Air is considered as mixture of molecular oxygen and nitrogen in proportion at normal conditions. At first computation step ionization reactions rates was assumed constant values, however, analysis of calculation results showed that this simplification do not allow obtaining physical picture of charge distribution. Therefore, in further calculations electron and gas temperatures dependence for reactions rates approximation was used. Kinetic mechanism also includes electron-induced ionization reactions with molecular nitrogen [7] taking place at initial stages (all expressions in SI units)

\[
N_2 + e \rightarrow N_2^+ + 2e, \quad k = 3.855 \cdot 10^{-14} \exp(-17.04/T_e),
\]
\[
N_2 + e \rightarrow 2N + e, \quad k = 2.686 \cdot 10^{-14} \exp(-13.42/T_e),
\]

here \(T_e\) – electron temperature, \(k\) – reaction rate constant, besides initial stages molecular nitrogen ionization reactions, excitation of nitrogen inner vibrational degrees of freedom occurs, however, this phenomenon was disregarded. Nitrogen recombination reactions are:

\[
N_2^+ + e + M(N_2) \rightarrow N_2 + M(N_2), \quad k = 6 \cdot 10^{-33} \left(\frac{300}{T_e}\right)^{4.5},
\]
\[
N_2^+ + e \rightarrow N + N, \quad k = 2.8 \cdot 10^{-13} \left(\frac{300}{T_e}\right)^{0.5},
\]

here letter \(M\) stands for third-body. Electron-induced ionization reactions with molecular oxygen [8] (9-step model) taking place at initial stages are:

\[
e + O_2 \rightarrow 2O + e, \quad k = 4.2 \cdot 10^{-15} \exp(-5.6/T_e) \text{ (dissociation)},
\]
\[
e + O_2 \rightarrow O^- + O, \quad k = 8.672 \cdot 10^{-17} \exp(-4.4/T_e) \text{ (dissociative electron attachment)},
\]
\[
O_2^- + O^- \rightarrow O + O_2 + O, \quad k = 1.5 \cdot 10^{-13} \left(\frac{300}{T_e}\right)^{0.5} \text{ (recombination 1)},
\]
\[
O^+ + O^- \rightarrow 2O, \quad k = 2.5 \cdot 10^{-13} \left(\frac{300}{T_e}\right)^{0.5} \text{ (recombination 2)},
\]
\[
e + O^- \rightarrow O + 2e, \quad k = 1.96 \cdot 10^{-15} \exp(-5.5/T_e) \text{ (electron detachment)},
\]
\[
e + O_2 \rightarrow O^- + O_2^- + e, \quad k = 6.71 \cdot 10^{-17} T_e^{0.5} \exp(-17.0/T_e) \text{ (ion pair formation)},
\]
\[
e + O_2 \rightarrow O + O^+ + 2e, \quad k = 5.3 \cdot 10^{-16} T_e^{0.9} \exp(-20.0/T_e) \text{ (dissociative ionization)},
\]
\[ e + O \rightarrow O^+ + 2e, \quad k = 8.6 \cdot 10^{-15} r^6 \exp(-13.6 / T_e) \] (ionization 1).

In the general case, surface reactions of bimolecular ion and atom recombination of oxygen at dielectric surface should be added to mentioned reactions. Recombination effects on formation of surface charge, however, consideration of these processes requires additional model complication, and therefore it was not taking into account to a first approximation. Formation of surface charge was modeled by special boundary conditions. Reactions rate analysis showed that nitrogen ions formation proceeds considerably less intensive than oxygen ions, these ions forms neutral atoms at short time. Mechanism for oxygen differs, at low electron temperatures formed oxygen ions recombines quickly, but when electron temperature increase, ion formation start to go in a few different ways and local ion concentration increase occurs. Concentrations of neutral particles, ions and electrons were defined as \( n_i \), with the assumption of practically immobile heavy ions system of transport equations for concentration (9 unknown quantities: 3 nitrogen-related concentrations, 6 oxygen-related concentrations). Typical transport equations for electron, ion and neutral components are of the form:

\[
\frac{\partial n_i}{\partial t} + \nabla \left( n_i \mu_i \nabla \varphi - D_i \nabla n_i \right) = n_i \left( k_{i1} n_{j1} - k_{j1} n_{i1} + (k_{i1} - k_{j1}) n_{j1} + k_0 n_{j1} + k_{j1} n_{i0} \right),
\]

\[
\frac{\partial n_{ji}}{\partial t} + \nabla \left( n_{ji} \mu_{ji} \nabla \varphi - D_{ji} \nabla n_{ji} \right) = k_{ji} n_{j1} n_{i1} - k_{ji} n_{ji} n_{i1},
\]

\[
\frac{\partial n_{j1}}{\partial t} + \nabla \left( n_{j1} \mu_{j1} \nabla \varphi - D_{j1} \nabla n_{j1} \right) = -k_{j1} n_{j1} n_{i1} - k_{j1} n_{j1} n_{i2} + k_{j1} n_{j1} n_{i0}.
\]

All equations are transport equations with the individual convective velocity and source terms at right-hand part, which are modelling formation of mixture components; \( k_1, \ldots, k_{12} \) - reaction rate constants, numeration corresponds to order of mentioning. Charge particle mobility \( \mu_i \) and diffusion coefficient \( D_i \) were used in this calculation. These transport coefficients could be derived directly from velocity distribution function. For simplification purposes, transport coefficients were obtained using semiempirical formulas. Considering ion and electron mass ratio, mobility and diffusion of electrons have more significant influence on this calculation, than other ones. Therefore, ion transport coefficients were defined as constant values field intensity-independent. Charge mobility coefficients \( \mu_i \) were calculated using Langèvin formula. Connection of ion diffusion coefficient with mobility were obtained from Einstein formula, neutral component diffusion coefficients were defined as:

\[
D = 1/3 \cdot \langle v \rangle (\lambda) = 1/3 \cdot (8RT / \pi M)^{0.5} \left( kT / \pi d^2 \right)
\]

here \( \langle v \rangle \) – mean velocity, \( \langle \lambda \rangle \) – mean free path, \( M \) – molar mass, \( k \) – Boltzmann constant, \( p \) - pressure, \( R \) - universal gas constant. Dependence of \( D_e \) for electrons is:

\[
D_e = \mu_e T_e.
\]

Equation for electrostatic field in a form of Gauss law was used in addition to mentioned equations system:

\[
\nabla \left( \varepsilon \nabla \varphi \right) = e n_0 \left( n_e + n_{o} - n_{o2} - n_{o3} - n_{o4} \right),
\]

where \( \varepsilon = 10 \varepsilon_0 \) for dielectric volume, \( e \) – elementary charge, \( \varphi \)– scalar electric potential. Equation for electron temperature is of the form:

\[
\frac{3}{2} \frac{\partial n_e T_e}{\partial t} + \nabla \left( \frac{5}{2} \Gamma_e T_e - \frac{5}{2} D_e n_e \nabla T_e \right) = \frac{e}{k} \Gamma_e \cdot \nabla \varphi + S,
\]
\[ \Gamma_e = \mu_e n_e \nabla \varphi - D_e \nabla n_e. \]

Thus, solvable system consisted of concentration, electrostatic field and electron temperature equations. Initial mixture temperature was defined as constant value in computational domain and was equaled 300 K. In general case analogous equations should correspond to each mixture component. Boundary conditions for neutral components concentrations were identical for all computational gas domain boundaries and were of the form:

\[ \frac{\partial n}{\partial t} = 0. \tag{1} \]

Electron temperature and concentration have similar boundary conditions for all air domain boundaries. Considering negligible secondary emission of electron component boundary condition for \( n_e \) on walls and electrodes is of the form:

\[ \frac{\partial n}{\partial n} = 0.25 n_e \left( \frac{8kT_e}{\pi m_e} \right)^{0.5}. \tag{2} \]

Electron temperature boundary condition on walls and electrodes is:

\[ \frac{\partial T_e}{\partial n} = \frac{1}{5n_e D_e} T_e \frac{\partial n_e}{\partial n}. \tag{3} \]

Electric potential boundary condition were defined as: potential \( \varphi=0 \) V at grounded electrode and \( \varphi=1500\sin(10000\pi t) \) V at electrode exposed to air. Surface charge accumulation on dielectric-air boundary and related electric potential boundary condition are of the form:

\[ \frac{\partial \rho_s}{\partial t} = -e \frac{\partial n_e}{\partial n}, \tag{4} \]

\[ \varepsilon_r \frac{\partial \varphi}{\partial n_e} - \frac{\partial \varphi}{\partial n_e} = -\rho_s, \tag{5} \]

where \( n_e, n_i \) - gas and dielectric normal.

Continuity equations for each mixture component, expressions for \( \Gamma_m, \Gamma_U \) fluxes and electron temperature flux \( U = n_e T_e \) equation are of the form:

\[ \frac{\partial n_m}{\partial t} = S_m - \nabla \Gamma_m \tag{6} \]

\[ \Gamma_m = n_m \mu_m \nabla \varphi - D_m \nabla n_m \tag{7} \]

\[ \frac{3}{2} \frac{\partial U}{\partial t} + \nabla \cdot \Gamma_U = \frac{e}{k} \varepsilon_r \cdot \nabla \varphi + S \tag{8} \]

\[ \Gamma_U = \frac{5}{2} \Gamma_e T_e - \frac{5}{2} D_e n_e \nabla T_e = \frac{5}{2} \mu_e U \nabla \varphi - \frac{5}{2} D_e \nabla U \tag{9} \]

where in (3) it is assumed that \( S = P_{\text{ext}} - P_{\text{coll}} = P_{\text{ext}} - n_e n_{m,\text{me}} \sum_j K_j e_j, P_{\text{ext}} \) - source term, which takes collisionless electron component heat into account; conditions under consideration (pressure \( P \approx 1 \) atm and voltage change frequency \( f \approx 1-10 \) kHz) corresponds to \( P_{\text{ext}} \approx 0, \sum_j K_j e_j \) - total specific electron
component energy loss during ionization processes, excitation of internal degrees of freedom, elastic collisions; it is also assumed that this term is neglected with respect to Joule heat $P_{\text{coul}} \ll \frac{1}{k} \mathbf{\Gamma} \cdot \nabla \varphi$.

Transient problem (1)-(4) solution is considered in rectangular area $G: \{(x, y)|0 \leq x \leq L_x, 0 \leq y \leq L_y, 0 \leq t \leq T\}$, with meshes $\omega_{hi} = \{(x_i, y_j)|x_i = h_x, i = 0...IM, y_j = h_y, j = 0...JM, \omega_{hi} = \{t_n = n\tau, n = 0...N\}$ and $G: \omega_{hi} = \omega_{hi} \times \omega_{hi}$. Components of $\mathbf{\Gamma}_m$, $\mathbf{\Gamma}_U$ are approximated on $\omega_{hi}$ with a constant steps $h_x = L_x/IM$, $h_y = L_y/JM$, $\tau = T/N$, piecewise constant distribution of $\mathbf{\Gamma}_m$ is assumed:

$$\begin{align*}
(\mathbf{\Gamma}_m)_{x,i+1/2} &= n_m \mu_m (\nabla \varphi)_{x,i+1/2} - D_m (\nabla n_m)_{x,i+1/2} \\
(\mathbf{\Gamma}_m)_{y,j+1/2} &= n_m \mu_m (\nabla \varphi)_{y,j+1/2} - D_m (\nabla n_m)_{y,j+1/2}
\end{align*}
$$

(10) (11)

According to the Scharfetter-Gummel scheme [9] expression for $(\mathbf{\Gamma}_m)_{x,i+1/2}$ could be obtained by multiplying both sides of equation (5) by $e^{-\mu_m(\nabla \varphi)_{x,i+1/2} \frac{x-x_i}{h_x}}$ and integrating over the interval $(x_i, x_i+h)$:

$$\int_{x_i}^{x_i+h} (\mathbf{\Gamma}_m)_{x,i+1/2} e^{-\mu_m(\nabla \varphi)_{x,i+1/2} \frac{x-x_i}{h_x}} \frac{dx}{h_x} = \int_{x_i}^{x_i+h} \frac{\partial}{\partial x} \left( D_m n_m e^{-\mu_m(\nabla \varphi)_{x,i+1/2} \frac{x-x_i}{h_x}} \right) dx$$

$$= \left. \mu_m (\nabla \varphi)_{x,i+1/2} = \frac{n_{i+1} e^{-\mu_m(\nabla \varphi)_{x,i+1/2} \frac{h_x}{D_m}} - n_i}{e^{-\mu_m(\nabla \varphi)_{x,i+1/2} \frac{h_x}{D_m}} - 1} \right|$$

$$= \frac{D_m}{h_x} \left( B \left( \frac{x-x_i}{h_x} \right) \frac{h_x}{D_m} n_i - B \left( \frac{x-x_i}{h_x} \right) \frac{h_x}{D_m} n_{i+1} \right)$$

$$= \frac{D_m}{h_x} \left( B_{i+1/2} n_i - B_{i+1/2} n_{i+1} \right)$$

(12)

where $B_{i+1/2} = B\left( \frac{x-x_i}{h_x} \right) = \frac{\frac{x-x_i}{h_x}}{e^{\frac{x-x_i}{h_x}} - 1}$ - Bernoulli function. Second flux vectors projections expressions could be obtained similarly. Hence difference analogue of flux vectors $\mathbf{\Gamma}_m$, $\mathbf{\Gamma}_U$ divergence could be represented in (6),(8) as

$$\nabla \cdot \mathbf{\Gamma}_m = \frac{1}{h_x} \left( (\mathbf{\Gamma}_m)_{x,i+1/2} - (\mathbf{\Gamma}_m)_{x,i-1/2} \right) + \frac{1}{h_y} \left( (\mathbf{\Gamma}_m)_{y,j+1/2} - (\mathbf{\Gamma}_m)_{y,j-1/2} \right)$$

(13)

Known limitations of Scharfetter-Gummel scheme are connected with stability and could be overcome by insertion of additional fictitious nodes for gradient $\varphi$ calculation and solution recalculation as it is shown in [9]; or by Newton method application [4]. Mainly these limitations are connected with $E$ and $n_m \mu_m \nabla \varphi$ terms from (1)-(2). Hence there is no need in additional fictitious nodes at conditions under consideration with ion mobility independent from $E$ and at moderate electric field intensity $E$ magnitudes. Time-step limitations in (1) are connected with fastest process in the problem which is polarization of bound charges in dielectric, typical time scale of this process at conditions under consideration is $\Delta t = m_v_e \nu_{e\text{opt}}/(e\nu_{e\text{opt}}) \sim 10^{-15}$-$10^{-13}$s, here $\nu_{e\text{opt}}$ frequency of ion-electron collisions. Thus, use of explicit time schemes requires sufficiently small time-steps [10-12], implicit schemes are more preferable in spite of greater number of arithmetical operations per step.
The simplest alternate direction implicit scheme for (1)-(4) (subscripts $m$, $U$ are used in members with different ones only):

$$\alpha_{m,U} \frac{n_{i,j}^{n+1/2} - n_{i,j}^n}{0.5\tau} = \frac{1}{h_x} \left( \Gamma_{x,i-l/2}^{n+1/2} - \Gamma_{x,i+l/2}^{n+1/2} \right) + \frac{1}{h_y} \left( \Gamma_{y,j-l/2}^{n+1} - \Gamma_{y,j+l/2}^{n+1} \right) + \left( S_{m,U} \right)_{i,j}^n, \quad (14)$$

$$\alpha_{m,U} \frac{n_{i,j}^{n+1} - n_{i,j}^{n+1/2}}{0.5\tau} = \frac{1}{h_x} \left( \Gamma_{x,i-l/2}^{n+1/2} - \Gamma_{x,i+l/2}^{n+1/2} \right) + \frac{1}{h_y} \left( \Gamma_{y,j-l/2}^{n+1} - \Gamma_{y,j+l/2}^{n+1} \right) + \left( S_{m,U} \right)^{n+1/2}_{i,j}, \quad (15)$$

here $\alpha_m=1$, $\alpha_U=3/2$, $S_m = \sum_i k_i n_i$, $S_U = e \frac{\Gamma_x \cdot \nabla \varphi}{k}$. Solution of (14)-(15) after (10)-(13) substitution could be achieved through sequences of tridiagonal matrix algorithm scalar (or, considering all components and electron temperature, vector with variable $X = (n_1, ..., n_n, U)^T$) implementations.

$$B^-_{x,j-l/2} X_{i-1,j}^{n+1/2} - \left( B^+_{x,j-l/2} + B^+_{x,j+l/2} + \alpha_{m,U} \frac{h_x^2}{D \tau} \right) X_{i,j}^{n+1/2} + B^+_{x,j+l/2} X_{i+1,j}^{n+1/2} = -F^n_{i,j}, \quad (16)$$

$$F^n_{i,j} = \frac{h_x^2}{h_y^2} \left( B^-_{y,j-l/2} X_{i,j-1}^n - B^+_{y,j-l/2} X_{i,j}^n - B^+_{y,j+l/2} X_{i,j}^n + B^+_{y,j+l/2} X_{i,j+1}^n \right) + \frac{h_x^2}{D} \left( S_{m,U} \right)^n_{i,j} + \beta_{m,U} \frac{h_x^2}{D \tau} X_{i,j}^n, \quad (17)$$

$$F^n_{i,j} = \frac{h_x^2}{h_y^2} \left( B^-_{y,j-l/2} X_{i,j-1}^n - B^+_{y,j-l/2} X_{i,j}^n - B^+_{y,j+l/2} X_{i,j}^n + B^+_{y,j+l/2} X_{i,j+1}^n \right) + \frac{h_x^2}{D} \left( S_{m,U} \right)^{n+1/2}_{i,j} + \beta_{m,U} \frac{h_x^2}{D \tau} X_{i,j}^{n+1/2},$$

here $\alpha_m=\beta_m=1$, $\alpha_U=2/5$, $\beta_U=3/5$. Thus, (16) – (17) supplemented with boundary and initial conditions (1)-(5) allow obtaining all mixture components concentrations at next time-step.

3. Simulation results

Computational domain with a simple electrodes geometry is presented on figure 1. For charge density estimation purposes, it is sufficient to consider only electrodes vicinity in a form of rectangle. Typical length scales are: dielectric layer thickness – 1 mm, computational domain height – 10 mm. A problem was considered in two-dimensional unsteady statement for simplification. Time step was equal $10^{-8}$ s.
Figure 2. Electric potential field $\varphi$ (V) at time $t=0.003$ s.

Electric potential field at time $t=0.003$ s is presented on figure 2. Potential distribution practically has no discontinuity on media interface, small equipotential surfaces distortion could be observed above the buried electrode only. Equipotential surface tangent lines along the other part of the surface have no discontinuity. For the moment of time under consideration volume concentrations of ions have small values, e.g. for $O^{2+}$ ion maximum particle number per volume unit is $\max(n_{O^2+})=100$. Therefore, electric potential field does not change significantly in computational domain. Mean value of surface charge density above the buried electrode equals $\rho_s=10^{-6}$ C/m$^2$.

Electron temperature field at time $t=0.003$s is presented on figure 3. Maximum value of $T_e$ does not exceed $0.35$ eV, estimation for this value at given potential difference equals $T_e \sim 1eV$, therefore, system does not reach the quasi-stationary state at this time moment. Nevertheless, it could be concluded that $T_e$ increase area located between the electrodes – area of high electron-induced reactions intensity at high $T_e$. Using fields from figures 3 and 4 and ions concentration fields, different data, e.g., surface density value and its distribution along the surface above buried electrode could be obtained. These data could be used as a boundary conditions for higher level models, e.g., DPM.

4. Conclusions
Mathematical model based on drift-diffusion approximation of hydrodynamic method of plasma description with two-temperature approximation was carried out for air ionization case. It was shown that main influence on charge distribution has electron temperature (typical computational value for plasma actuator induced ionization is $\sim 1eV$). This model could be used for volume and surface charge density estimations necessary for volume force calculation in the vicinity of electrodes and boundary conditions setting for simulation of interaction between the DBD-actuator and gas flow.

Acknowledgments
Research has been supported by the RFBR grant No. 18-08-00271.
References

[1] Semenev P A and Pudovikov D E 2015 Dielectric barrier discharge numerical simulation and analysis of its flow control application effectiveness. 6th European conference for aeronautics and space sciences (EUCASS), 29 June – 3 July 2015, Krakow, Poland

[2] Semenev P A, Pudovikov D E and Toktaliev P D 2016 Numerical simulation of interaction process between dielectric barrier discharge and duct flow. VII European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS), 5–10 June 2016, Crete Island, Greece

[3] Stewart R A, Vitello P and Graves D B 1994 Two-dimensional fluid model of high density inductively coupled plasma sources J. Vacuum Sci. Technol. B 12

[4] Lymberopoulos D P and Economou D J 1993 Fluid simulations of glow discharges: Effect of metastable atoms in argon J. Appl. Phys. 73 3668

[5] Selberherr S 1984 Analysis and Simulation of Semiconductor Devices (Springer-Verlag)

[6] Kossyi I A, Kostinsky A Yu, Matveyev A A and Silakov V P 1992 Plasma Sources Sci. Technol. 1 207

[7] Bose D, Govindan T R and Meyyappan M 1999 A Continuum Model for the Inductively Coupled Plasma Reactor in Semiconductor Processing J. Electrochem. Soc. 146 (7) 2705–2711

[8] Chung T H, Yoon H J and Seo D C 1999 Global model and scaling laws for inductively coupled oxygen discharge plasmas. J. of Applied Physics 86 (7) 3536–3542

[9] Kulikovski A A 1995 A more accurate Sharfetter-Gummel Algorithm of electron transport for semiconductor and gas discharge simulation Journal of Computational Physics 119 149–155

[10] Barnes M S, Colter T J and Elta M E 1987 Large-signal domain modeling of low-pressure rf glow discharges Journal of Applied Physics 61 (1) 81–89

[11] Colella P, Dorr M R and Wakey D D 1999 A Conservative Finite Difference Method for the Numerical Solution of Plasma Fluid Equations Journal of Computational Physics 149 168–193

[12] Roy S, Singh K P and Gaitonde D V 2007 Air Plasma Actuators for Effective Flow Control 45th AIAA Aerospace Sciences Meeting and Exhibit, Reno, Nevada