Identification of kinematic errors, position and rotation of industrial robot used as the redundant measurement system

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Abstract. The issue of determining industrial robot’s kinematic errors does seem to be already fully addressed. However, due to the recent influx of new technological possibilities, it was decided to verify and apply new methods of determining both robot’s and its mounted equipment’s errors. It is possible for industrial robot with integrated scanning head to become a reliable, automatic system; one that could allow direct coordinate measuring on assembly line even in places that are both difficult to access and dangerous for a worker. However, determination of kinetic errors as well as measuring errors is going to be a start to improving accuracy of the device and modelling errors’ matrix. Due to better software correction, the matrix is going to allow installation of cheaper components in devices.

1. Introduction

Using industrial robots for more than just one task is more and more common in industry. Switching between a gripper and a scanning probe allows a significant cost reduction as expenditures required for measuring devices can be limited. Furthermore, the production time is reduced. There are two reasons for that: first, inter-operational breaks become shorter; second, since there is no need for checking dimensions in separate quality laboratory, in which component needs to be disassembled, moved, measured and reassembled.

Accuracy of an industrial robot can be improved by its calibration. The development of robot’s mathematical model is crucial for this process. During modelling it is necessary to take into consideration existing kinematic and non-kinematic errors as they affect the accuracy of the robot. In static robot calibration the effects of robot motion accuracy are neglected. The opposite is the dynamic robot calibration in which the impact of motion is taken into account. Static calibration is widely presented in professional literature [1,2,3], where either the proper relationship between the shift of the actual points and the signals from the encoder were determined or robot’s kinematic parameters were determined without taking into account elasticity or slackness in the connections. These two sources of inaccuracy are, however, taken into consideration for the calibration in[4] where both the kinematic parameters and connections between systems are identified when spring rate and links’ gravitation are taken into account.

Taking everything into consideration, it was decided to model errors of coordinate measuring system. Said system consists of the industrial robot with mounted measuring head.

The study was carried out in two stages. Those stages consisted of testing firstly the robot itself and then the entire system with the measuring head already mounted. This approach allows the application of two-step system correction: firstly, the robot itself, and secondly, the measuring head. The access to the robot was provided by ASTOR, located in Cracow.

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2. Modelling errors in robot without equipped scanning probe

In order to determine measuring robot’s errors, its actual geometrical elements were determined. The device’s errors were determined without the head, to prevent it from causing interference. To achieve that, LaserTracer was used. A retroreflector was mounted at the very end of the robot. Laser was used as a length standard.

Coordinates x,y,z of a measuring tip, Euler angles O,A,T (which were then calculated to direction cosine i,j,k) and readings from encoder’s angular measuring systems θ₁,..,θₙ (so-called configuration coordinates), were the information that was acquired while measuring point was gathered. In order to achieve that manufacturer’s software was used.

A matrix description of transformations in i-dimensional spaces was used as a next step. The transformation of the "i" system to the "i-1" system describes the product of the matrix

\[ A_{i-1} = A_{x_1,A_i}A_{x_2,A_i}A_{x_3,A_i}A_{x_4,A_i} \]

(2)

The position of the gripper or stylus in relation to the support system has been determined using formula (3):

\[ p_i = T_{i,p_i} \]

(3), where \( p_i = [x_i, y_i, z_i] \)

(4) \( T_{i,i} = A_{1}A_{2}A_{3}....A_{i} \)

(5) \( p_{i-1} - \) the position of the element’s point „i” in local reference system (4), \( T_{i,i-1} - \) matrix informing about transformation of the „i” system in relation to the support system. It is obtained by multiplying \( A_{i} \) matrix (4).

The \( T_{i,i} \) matrix (5) informs about the position vector of stylus’ \( p_{i} \) (6) and about the orientation \( B_{i} \) (7).

\[ T_{i} = \begin{bmatrix} B_{i}p_{i} \\
0 \quad 0 \quad 1 \end{bmatrix} \]  

(6), where \( B_{i} = [i_{ix}, i_{iy}, i_{iz}, j_{ix}, j_{iy}, j_{iz}, k_{ix}, k_{iy}, k_{iz}] \) - vector’s coordinates that define the rotation of individual elements.

System of equations with 24 unknowns was created since four DH parameters are needed to describe the rotation pair. After the data was obtained from both the actual measuring device and the standard, it was added to the system of equations created using the formula (9):

\[ D_{i} = \sqrt{(x_{i_1} - x_{i_2})^2 + (y_{i_1} - y_{i_2})^2 + (z_{i_1} - z_{i_2})^2} \]  

(9)

where \( D_{i} \) - the difference of the distance of two measuring lengths obtained from LaserTracer, \( x_{i_1}, x_{i_2}, y_{i_1}, y_{i_2}, z_{i_1}, z_{i_2} \) - formula for coordinates \( X,Y,Z \) of standard’s points 1 and 2, determined using forward kinematics and with configuration coordinates \( θ_{1},..,θ_{n} \).

During measuring, the standard was placed in several positions in the entire measuring space: three in each sector of measuring space, dividing it every 120° - one in front of the device and two diagonally. The entire procedure was repeated on the level below device’s mounting. As a result, the actual parameters of the device were obtained. Those were: \( L \)-the distance from the \( z \) axis to the \( z_{i+1} \) axis, measured on the \( x \) axis; \( α \) - the angle between the \( z_{i} \) and \( z_{i+1} \) axes, measured around the \( x \) axis; \( β \) - the distance between \( x_{i} \) and \( x_{i+1} \) axes, measured around the \( z \) axis; \( θ \) - angular displacement on the angle between \( x_{i} \) and \( x_{i+1} \) and \( θ \) angles, measured around \( z \), characterizing the length of the elements, torsion, angular displacements and eccentric of individual elements. The results are shown in Table 1.

![Figure 1](image1.png)  
**Figure 1** Rotation around any axis.  
**Figure 2** Orientation error’s definition

| Lp | Shift error along x from z to z+1 | Shift error along z from x to x+1 | α-angular displacement between z and z+1 | θ-angular displacement between x and x+1 |
|----|----------------------------------|----------------------------------|----------------------------------------|----------------------------------------|
| 1  | 0.0002                           | 0.0006                           | 90.0005°                               | 0.0001°                               |
| 2  | 0.0001                           | 0.0000                           | 90.0005°                               | 0.0000°                               |
| 3  | 0.0004                           | -0.0002                          | 89.9999°                               | 0.0002°                               |
| 4  | 0.0000                           | 0.0003                           | 0.0001°                                | 0.0000°                                |
| 5  | 0.0006                           | 0.0001                           | 0.0001°                                | 0.0001°                                |
| 6  | -0.0004                          | -0.0001                          | -0.0002°                               | 0.0001°                                |
2.1. Orientation error and the representation of rotation

Determining the position error is done using the formula (10):

\[ \overrightarrow{p_p} = \overrightarrow{p_d} - \overrightarrow{p} \] (10)

where the vector \( \overrightarrow{p_p} \) is the difference between the set vector \( \overrightarrow{p_d} \), and \( \overrightarrow{p} \) indicated by the machine.

A formula for converting rotation errors (i.e., the difference between the set and actual rotation) is needed to determine the orientation error. Generally, three vectors: n, o, a, are needed to determine rotation. The set orientation is denoted by \( R_d \), and the actual by \( R \). In our case it is possible to obtain only \( \overrightarrow{n} \) vector using the device (IR). \( \overrightarrow{n} \) is consistent with the position of the stylus. Determining the orientation error is done using the formula (11):

\[ \overrightarrow{e} = \overrightarrow{k} \sin \theta \] (11),

\[ \overrightarrow{e} = \text{Rot} (k, \theta) = R_d R^T \] (12)

Figure 2 shows how the orientation is determined. It also depicts the coordinate system, as well as vectors \( \overrightarrow{n} \) and \( \overrightarrow{n_d} \), unit vector \( k \) and angle of rotation \( \theta \). It was assumed that vectors \( \overrightarrow{n_d} \) and \( \overrightarrow{n} \) have their beginning at O. After that, it was necessary to definite the error using an orientation matrices \( R_d \) and \( R \). Since \( k \) is unit vector that defines a rotation axis and \( \theta \) angle is an equipollent angle of rotation between \( n \) o a \( a \) \( d \) \( d \) systems (meaning \( \overrightarrow{n} \) and \( \overrightarrow{n_d} \) vectors), the orientation error can be calculated [6] using formula (12). The formula explains which transformations are needed to achieve the overlap of \( R \) with \( R_d \) system as well as \( \overrightarrow{n} \) and \( \overrightarrow{n_d} \) vectors. Defining set vector as \( \overrightarrow{n} = [n_x \ n_y \ n_z] \) and the current vector from the machine as \( \overrightarrow{n_m} = [n_{xm} \ n_{ym} \ n_{zm}] \), after the multiplication the matrix is obtained. After doing calculations and transformation, according to [5] the formula for the orientation’s error is presented using rotation’s pseudovector (13):

\[ \overrightarrow{e} = \overrightarrow{k} \sin \theta = \begin{bmatrix} k_x \sin \theta \\ k_y \sin \theta \\ k_z \sin \theta \end{bmatrix} = \begin{bmatrix} n_{xm}n_{ym} - n_{xm}n_{zm} \\ \frac{1}{2} n_{ym}n_{ym} - n_{xm}n_{zm} \\ \frac{1}{2} n_{zm}n_{zm} - n_{xm}n_{ym} \end{bmatrix} = \begin{bmatrix} n_xn_{ym} - n_yn_{xm} \\ n_yn_{ym} - n_zn_{xm} \\ n_zn_{zm} - n_xn_{ym} \end{bmatrix} \] (13)

which is the sought relation to the orientation error. It is worth noting, that the angle constraints imply the conditions on the scalar product: \( n^T n_m \geq 0 \). If the vector \( \overrightarrow{n} \) overlaps \( \overrightarrow{n_m} \) vector, the orientation error equals 0 [8].

3. Determining position and orientation errors

After corrections were made to the forward kinematics it was possible to determine position and orientation errors. To achieve that, coordinates of 20 single points were collected in the measuring space – together with the direction of the last link and the readings from the encoders. Instead of using the i, j, k vectors (like in coordinate measuring arms), the direction is presented using Euler angles. After making the correction, the maximum position error is 0.0059 mm, while the average is 0.0010 mm, as shown in Diagram 1.

The orientation error was insignificant. The results are presented in Diagram 2. For the robot with mounted measuring head another 20 coordinates were collected in the measuring space and then corrections were made. The parameters are shown in Table 2, with shift included.

The maximum position error for the robot with mounted measuring head is 0.0305 mm, while the average is 0.0168 mm.

The orientation error is a rotation pseudovector of the robot’s head. This error has increased in relation to the results for the robot without the mounted head. It is caused by the transfer of the system from the end of the robot’s segment to the measuring probe, as seen in Diagram 4.

4. Conclusions

As a result of after conducted tests industrial robot’s kinematic errors were determined. After that, position and orientation errors were defined for both the robot with and without the mounted contact
Both errors significantly increased after the touch-trigger head was mounted. It was decided to model the head’s errors using neural networks. The head’s modelling results are shown in [Mathematical errors model of touch-trigger probe installed on the industrial robot] on this conference.

**Table 2** Parameters of 6-axis Kawasaki industrial robot with measuring head mounted (with corrections).

| Lp | 1- shift along x from z to z+1 | λ- shift along z from x to x+1 | 0 – reading from encoders | α angles between z and z+1 |
|----|---------------------------------|---------------------------------|---------------------------|---------------------------|
| 1  | 100.0002                        | 0.0006                          | Var±90.0005               | 90.0001°                  |
| 2  | 649.9999                        | 0.0000                          | Var±90.0000°              | 180                       |
| 3  | 0.0004                          | -0.0002                         | Var±89.9999°              | 90.0002°                  |
| 4  | 0.0004                          | 700.0003                        | Var ±0.0001°              | 270, 0000°                |
| 5  | 0.0006                          | 0.0001                          | Var                         | 90, 0001°                 |
| 6  | 0.1904                          | 239.0601                        | Var±0.0002°               | 0.0001°                  |

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