Pathologies in Lovelock AdS black branes and AdS/CFT

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Abstract

We study pathologies in AdS black branes in Lovelock theory. More precisely, we examine the conditions that AdS black branes have a naked singularity, ghost instability and dynamical instability. From the point of view of the AdS/CFT correspondence, pathologies in AdS black branes indicate pathologies in the corresponding CFT. Hence, in Lovelock theory, we need to be careful when we apply AdS/CFT to various phenomena such as the shear viscosity to entropy ratio in strongly coupled quantum field theory.

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1. Introduction

It is well known that AdS black branes play a central role in the application of the AdS/CFT correspondence to various phenomena such as condensed matter physics and fluid dynamics [1–3]. Remarkably, the AdS/CFT correspondence holds in any dimensions. In higher dimensions, however, a natural theory of gravity is not general relativity but Lovelock theory [4, 5]. Thus, it is natural to consider the AdS/CFT correspondence in the context of Lovelock theory.

The AdS/CFT correspondence in Lovelock theory has already been discussed in the context of the shear viscosity to entropy ratio. It is conjectured that the shear viscosity to entropy ratio \( \eta/s \) is larger than \( 1/4\pi \), which is called the Kovtun–Son–Starinets (KSS) bound [6]. Recently, in the case that the dual gravitational theory is Lovelock theory, this ratio has been calculated as

\[
\frac{\eta}{s} = \frac{1 - 2\alpha_2}{4\pi},
\]

where \( \alpha_2 \) is an appropriately normalized second-order Lovelock coefficient [7]. It seems that the KSS bound is violated for a positive \( \alpha_2 \). However, when we take into account pathologies in AdS black branes, \( \alpha_2 \) must be somewhat restricted.

Indeed, there are several works on the causality violation of AdS black branes in conjunction with the KSS bound [8–10]. Clearly, it is important to clarify pathologies in AdS black branes.

In this paper, we consider pathologies in AdS black brane solutions in Lovelock theory. First, we explain our method using analytically tractable cases in five and six dimensions.
other dimensions, we have to resort to numerical analysis. As typical examples, we numerically study pathologies in AdS black branes in 10 and 11 dimensions. In recent work, this issue has been investigated in [11] based on the master equations derived by us [12]. They used near horizon analysis in the most part of their work and alluded to the importance of the bulk geometry based on numerical results. However, they have never given general conditions for the occurrence of pathologies. In this paper, we explicitly present the conditions for the occurrence of pathologies. Using the conditions, we will give a detailed analysis of pathologies in Lovelock AdS black branes and discuss its implications in AdS/CFT.

The organization of this paper is as follows. In section 2, we review Lovelock theory and explain a graphical method for constructing black brane solutions. In section 3, we clarify the conditions for avoiding a naked singularity, ghost instability and dynamical instability. In section 4, we analytically study pathologies in AdS black branes in five and six dimensions. In section 5, we numerically examine pathologies in AdS black branes in 10 and 11 dimensions. In section 6, based on the analytical and numerical results, we discuss implications of our findings in the AdS/CFT correspondence. Section 7 is devoted to conclusion.

2. Lovelock AdS black branes

In this section, we review Lovelock theory and introduce a graphical method for constructing AdS black brane solutions.

The most general divergence free symmetric tensor constructed out of a metric and its first and second derivatives was obtained by Lovelock [4]. The corresponding Lagrangian can be constructed from $m$-th order Lovelock terms

$$L_m = \frac{1}{2m} g^{\lambda_1 \sigma_1 \cdots \lambda_m \sigma_m} R_{\lambda_1 \sigma_1 \cdots \lambda_m \sigma_m} R_{\mu_1 \sigma_1 \cdots \mu_m \sigma_m} \cdots$$

(1)

where $R_{\lambda \sigma \rho \kappa}$ is the Riemann tensor in $D$ dimensions and $\delta_{\mu_1 \mu_2 \cdots \mu_p}$ is the generalized totally antisymmetric Kronecker delta defined by

$$\delta_{\mu_1 \mu_2 \cdots \mu_p} = \det \left( \begin{array}{cccc}
\tilde{g}^{\mu_1 \nu_1} & \tilde{g}^{\mu_1 \nu_2} & \cdots & \tilde{g}^{\mu_1 \nu_p} \\
\tilde{g}^{\mu_2 \nu_1} & \tilde{g}^{\mu_2 \nu_2} & \cdots & \tilde{g}^{\mu_2 \nu_p} \\
\cdots & \cdots & \cdots & \cdots \\
\tilde{g}^{\mu_p \nu_1} & \tilde{g}^{\mu_p \nu_2} & \cdots & \tilde{g}^{\mu_p \nu_p}
\end{array} \right).$$

By construction, the Lovelock terms vanish for $2m > D$. It is also known that the Lovelock term with $2m = D$ is a topological term. Thus, Lovelock Lagrangian in $D$ dimensions is defined by

$$L = \sum_{m=0}^{k} c_m L_m,$$

(2)

where we defined the maximum order $k \equiv [(D - 1)/2]$ and $c_m$ are arbitrary constants. Here, $[z]$ represents a maximum integer satisfying $[z] \leq z$. Taking variation of the Lagrangian with respect to the metric, we can derive Lovelock equations

$$0 = \sum_{m=0}^{k} c_m g^{\lambda_1 \sigma_1 \cdots \lambda_m \sigma_m} R_{\lambda_1 \sigma_1 \cdots \lambda_m \sigma_m} R_{\mu_1 \sigma_1 \cdots \mu_m \sigma_m} \cdots$$

(3)

Hereafter, we set $c_0 = (D - 1)(D - 2)\lambda$, $c_1 = 1$ and $c_m = \alpha_m \left\{ m! \prod_{p=1}^{m-2} (D - 2 - p) \right\} (m \geq 2)$ for convenience. Note that the coefficients $\alpha_m$ are dimensionless.
It is well known that there exist static exact solutions of the Lovelock equations (3) [13–15]. Let us consider the following metric:

\[ ds^2 = r^2 \psi(r) \, dt^2 + \frac{\, dr^2}{-r^2 \psi(r)} + r^2 \delta_{ij} \, dx^i \, dx^j. \]  

(4)

We assume that \( \psi(r) \) is negative outside the horizon. Substituting this metric ansatz into equation (3), we can obtain an algebraic equation for \( \psi(r) \):

\[ W[\psi] \equiv \sum_{m=2}^{k} \left( \frac{\alpha_m}{m} \psi^m \right) + \psi + 1 = \frac{\mu}{r^{D-1}}, \]  

(5)

where \( \mu \) is a constant of integration which is related to the ADM mass and we assume that it is positive. Note that we fixed the scale by setting \( \lambda = 1 \) in equation (5).

Now, we explain how to construct solutions using a graphical method [16, 17]. Apparently, \( W \) must be positive. In general, there are many branches. In figure 1, we depicted \( y = W[\psi] \) and \( y = \mu/r^{D-1} \), with \( r \) being fixed in the \( \psi-y \) plane. The intersection of the curve and the line determines \( \psi \), once \( r \) is given. By varying \( r \), we obtain the solution of equation (5). Taking a look at the metric (4), we see that the horizon corresponds to \( \psi = 0 \). Hence, a black brane corresponds to the branch containing \( \psi = 0 \). Next, consider the asymptotic infinity \( r \to \infty \) or \( y \to \mu/r^{D-1} \to 0 \), the function \( \psi(r) \) in figure 1 approaches \( \psi_a \), which is the largest negative root of \( W[\psi] = 0 \). Thus, the curve between \( \psi = \psi_a \) and \( \psi = 0 \) defines a black brane solution.

3. Pathologies

In this section, we list the pathologies in Lovelock AdS black branes. In particular, we reveal the conditions for the occurrence of pathologies.

3.1. Naked singularity

In the graphical method, it is easy to find singularities. Let us recall the Kretschmann invariant which is calculated as

\[ R_{\mu\nu\rho\lambda} R^{\mu\nu\rho\lambda} = (\partial_{\alpha}(r^2 \psi))^2 + 2(D-2) \frac{(\partial_{\alpha}(r^2 \psi))^2}{r^2} + 2(D-2)(D-3) \psi^2. \]  

(6)
If this invariant diverges, there exist singularities. This occurs at $r = 0$ and the point where $\partial_r \psi$ diverges in fact. $\partial_r \psi$ diverges when $W[\psi]$ becomes an extremal value because of a relation $\partial_r \psi = -(D-1)W[\psi]/(\partial_\psi W)$ obtained from (5). Since $\partial_\psi W|_{\psi=0} = 1 > 0$, if $W[\psi]$ is a monotonically increasing function in the region $[\psi_a, 0]$, there is no naked singularity. Figure 2(a) corresponds to this case. However, as in figure 2(b), if $W[\psi]$ has an extremal point between $\psi_a$ and 0, there exists a naked singularity. Note that the shape of $W[\psi]$ depends only on the Lovelock coefficients $\alpha_m$, so whether a branch has a naked singularity or not is determined by these constants. Since we want to avoid a naked singularity, we have to exclude the solutions which have extrema between $\psi = \psi_a$ and $\psi = 0$. Note that there may be exotic cases for which $\psi_a$ does not exist. These solutions should be excluded because they necessarily have a naked singularity.

3.2. Ghost instability

In Lovelock theory, the sign in front of the kinetic term in the action could be negative, namely a ghost instability could occur. In the previous paper [12], we have shown that there exists a ghost instability, when

$$K[\psi] \equiv (D - 3)(\partial_\psi W)^2 - (D - 1)W \partial_\psi^2 W$$

becomes negative. Hence, we need to check the sign of $K[\psi]$ to check if a black brane has a ghost instability or not.

3.3. Dynamical instability

As we have shown in [12], the function $W(\psi)$ determines if a dynamical instability of Lovelock black branes occurs. Using the symmetry of the planar part of the metric, we can classify metric perturbations into the scalar, vector and tensor sectors. In the absence of a ghost instability, we can prove that there is no dynamical instability in the vector sector [12].

There exists a dynamical instability for the tensor sector, when

$$L[\psi] \equiv (D - 3)(D - 4)(\partial_\psi W)^2 - (D - 1)(6 + 2D - 10)\partial_\psi W \partial_\psi^2 W \partial_\psi^3 W$$

$$+ (D - 1)^2 W^2 \{\partial_\psi W \partial_\psi^3 W - (\partial_\psi^2 W)^2\}$$

is negative [12, 16]. Similarly, there exists a dynamical instability for the scalar sector, when

$$M[\psi] \equiv (D - 2)(D - 3)(\partial_\psi W)^2 - 3(D - 2)(D - 1)\partial_\psi^2 W (\partial_\psi W)^2$$

$$+ (D - 1)^2 W^2 \{3(\partial_\psi^2 W)^2 - \partial_\psi W \partial_\psi^3 W\}$$
is negative [12]. In both cases, the square of the effective speed of sound becomes negative. This kind of instability is found in the cosmological context for the first time [18].

In order to find a dynamical instability, what we have to check is the sign of $L[\psi]$ and $M[\psi]$ in the region $\psi_a < \psi < 0$. Note that these functions and $\psi_a$ are independent of $\mu$; hence, whether a dynamical instability exists or not depends only on the Lovelock coefficients $\alpha_m$.

4. Pathology inspection: analytic results

In this section, we analytically investigate pathologies in five and six dimensions for which we have $k = 2$. In these cases, $W$ is given by

$$W[\psi] = \frac{\alpha_2}{2} \psi^2 + \psi + 1, \quad \alpha_2 \neq 0.$$  \hspace{1cm} (10)

This can be written as

$$W = \frac{\alpha_2}{2} \left( \psi + \frac{1}{\alpha_2} \right)^2 + 1 - \frac{1}{2\alpha_2,$$  \hspace{1cm} (11)

from which we see that there is no solution for $W = 0$ if $\alpha_2 > 1/2$. Since there is an extremum in the region $\psi < 0$, we have a naked singularity in these cases. Thus, only the range $\alpha_2 \leq 1/2$ is allowed.

For $\alpha_2 \leq 1/2$, there are solutions for $W = 0$:

$$\psi_W = -1 \pm \sqrt{1 - 2\alpha_2}.$$  \hspace{1cm} (12)

We have defined $\psi_a$ as the largest negative root. In the cases $0 < \alpha_2 \leq 1/2$, the largest negative root should be

$$\psi_a = -1 + \frac{\sqrt{1 - 2\alpha_2}}{\alpha_2}.$$  \hspace{1cm} (13)

In the cases $\alpha_2 < 0$, that becomes

$$\psi_a = \frac{-1 + \sqrt{1 - 2\alpha_2}}{\alpha_2} = 1 - \frac{\sqrt{1 + 2|\alpha_2|}}{|\alpha_2|}.$$  \hspace{1cm} (14)

In any case, the plus sign in (12) corresponds to $\psi_a$. Thus, we do not have any naked singularity as long as $\alpha_2 \leq 1/2$.

Next, let us see if we have ghosts. With this aim, we need to look at the sign of $K[\psi]$. In five or six dimensions, we obtain

$$K = \begin{cases} 
2(1 - 2\alpha_2) & \text{(for } D = 5) \\
(\alpha_2 \psi + 1)^2/2 + 5(1 - 2\alpha_2)/2 & \text{(for } D = 6). 
\end{cases}$$  \hspace{1cm} (15)

Apparently, the condition for no naked singularity $\alpha_2 \leq 1/2$ guarantees $K \geq 0$. Hence, we do not have ghosts as long as we do not have naked singularities in six dimensions. In five dimensions, for $\alpha_2 = 1/2$, $K$ vanishes, which is singular. Hence, we have the condition $\alpha_2 < 1/2$ in five dimensions.

From now on, we check the stability of black branes in the tensor and scalar sectors. The analysis depends on the dimensions. Hence, we discuss the stability in five and six dimensions, separately.
4.1. Five dimensions

First, we discuss the stability in the tensor sector by looking at the sign of $L$. From the definition (8), $L$ can be calculated as

$$L = 2(1 - 2\alpha_2)\left\{3\alpha_2^2\psi^2 + 6\alpha_2\psi + (1 + 4\alpha_2)\right\}.$$  \hfill (16)

We see that there exists an instability for $\alpha_2 < -1/4$ because $L[0] < 0$ in this range. Then, we consider the remaining range $-1/4 \leq \alpha_2 < 1/2$. In this case, $L = 0$ has solutions

$$\psi_{L\pm} = \frac{-1 \pm \sqrt{1 - 2\alpha_2}}{\alpha_2}.$$  \hfill (17)

Comparing these solutions with $\psi_a$, we see $\psi_a > \psi_{L\pm}$ in the range $0 < \alpha_2 < 1/2$ and $\psi_a < \psi_{L\pm}$ in the range $-1/4 < \alpha_2 < 0$. Thus, in the range $[\psi_0, 0]$, we always have $L > 0$ for $0 < \alpha_2 < 1/2$ and $-1/4 < \alpha_2 < 0$.

Next, we study the stability in the scalar sector. From the definition (9), $M[\psi]$ is given by

$$M[\psi] = 6(1 - 2\alpha_2)\left(\alpha_2 \psi^2 - 2\alpha_2\psi + (1 - 4\alpha_2)\right).$$  \hfill (18)

For $1/4 < \alpha_2 < 1/2$, we have $M[0] < 0$ and there is an instability. Thus, we need to check the range $\alpha_2 \leq 1/4$ in the following. It is easy to see that $M = 0$ has solutions

$$\psi_{M\pm} = \frac{-1 \pm \sqrt{\alpha_2} - 2\alpha_2}{\alpha_2}.$$  \hfill (19)

Comparing these solutions with $\psi_a$, we find $\psi_- < \psi_a < 0 < \psi_+$ for $0 < \alpha_2 \leq 1/4$ and $\psi_+ < \psi_a < 0 < \psi_-$ for $\alpha_2 < 0$. Therefore, in the range $[\psi_0, 0]$, we conclude $M \geq 0$ for $0 < \alpha_2 < 1/4$ and $\alpha_2 < 0$.

Combining the above results, we found that black branes in five dimensions have no pathology for the range $-1/4 \leq \alpha_2 \leq 1/4$, where the trivial case $\alpha_2 = 0$ is included.

4.2. Six dimensions

Now, we investigate pathologies in AdS black branes in six dimensions. First, we check the stability in the tensor sector. In six dimensions, $L[\psi]$ becomes

$$L[\psi] = -\frac{\alpha_2^2}{4}\psi^4 - \alpha_2^3\psi^3 + (11 - 25\alpha_2)\alpha_2^2\psi^2 + 24\alpha_2\psi - 50\alpha_2^2\psi + (6 - 25\alpha_2^2).$$  \hfill (20)

From this expression, we can conclude $L[0] < 0$ for $\alpha_2 < -\sqrt{\frac{2}{5}}$. $\frac{\sqrt{2}}{5} < \alpha_2 \leq \frac{1}{2}$. Hence, in order not to have an instability in the tensor sector, we have to choose a parameter in the range $-\sqrt{6}/5 \leq \alpha_2 \leq \sqrt{6}/5$, where $\alpha_2 = 0$ is a trivial case. Note that the equation $L = 0$ has four solutions

$$\psi_{L\pm\pm} = -\frac{1}{\alpha_2} \pm \frac{\sqrt{\alpha_2} - 2\alpha_2}{\alpha_2} (\sqrt{15} \pm \sqrt{10}).$$  \hfill (21)

Here, $\psi_{L\pm\pm}$ distinguish four possible solutions. Comparing these solutions with $\psi_a$, we find

$$\psi_{L++} < \psi_{L--} < \psi_{L+-} < \psi_a < 0 < \psi_{L-+}$$  \hfill (22)

for $0 < \alpha_2 < \sqrt{6}/5$, and we have

$$\psi_{L--+} < \psi_a < 0 < \psi_{L++} < \psi_{L--} < \psi_{L-+}$$  \hfill (23)

for $-\sqrt{6}/5 < \alpha_2 < 0$. Thus, we see $L > 0$ in the range $[\psi_a, 0]$.

Remarkably, $M[\psi]$ can be written as

$$M[\psi] = \frac{1}{4}( -4 + 10\alpha_2 + 2\alpha_2\psi + \alpha_2^2\psi^2)^2.$$  \hfill (24)

Hence, there exists no instability in the scalar sector.

To conclude, there exists no pathology in AdS black branes in six dimensions as long as we take a parameter in the range $-\sqrt{6}/5 \leq \alpha_2 \leq \sqrt{6}/5$.  

5. Pathology inspection: numerical approach

Now, we are in a position to examine the pathologies in AdS black branes numerically. In this paper, we consider 10 and 11 dimensions because the analysis and the results in other dimensions are similar. Our strategy is very simple. For each coefficient $\alpha_m$, we search for $\psi_a$ and check the sign of $\partial_\psi W$, $K[\psi]$, $L[\psi]$ and $M[\psi]$ in the region $\psi_a < \psi < 0$. The mesh size of this calculation is $\Delta \alpha_m = 0.05$. We have checked that our numerical method can reproduce the analytical results in five and six dimensions.

5.1. Ten dimensions

In ten dimensions, the Lovelock black holes can be characterized by the functional

$$W[\psi] = \frac{\alpha_4}{4} \psi^4 + \frac{\alpha_3}{3} \psi^3 + \frac{\alpha_2}{2} \psi^2 + \psi + 1.$$  \hspace{1cm} (25)

Substituting this expression into $\partial_\psi W[\psi]$, (7)–(9), we can find the forbidden region in three-dimensional parameter space $[\alpha_2, \alpha_3, \alpha_4]$. In figure 3, we plot forbidden regions due to various reasons in the $\alpha_2$–$\alpha_3$ plane with $\alpha_4 = -1.5, 0, 0.5$, respectively. Interestingly, there is a region where both scalar and tensor sector instabilities exist. The shaded region represents an allowed region.

In figure 3, when $\alpha_4 = 0$, the border between the allowed region and the unstable region due to the scalar sector instability can be obtained from the condition $M[0] = 0$ as

$$\alpha_3 = \frac{3}{2} \alpha_2^2 - \frac{3(D-2)}{2(D-1)} \alpha_2 + \frac{(D-2)(D-3)}{2(D-1)^2}.$$  \hspace{1cm} (26)

Similarly, the border between the allowed region and the unstable region due to the tensor sector instability can be determined by the condition $L[0] = 0$ as

$$\alpha_3 = \frac{\alpha_2^2}{2} + \frac{D-6}{2(D-1)} \alpha_2 - \frac{(D-3)(D-4)}{2(D-1)^2}.$$  \hspace{1cm} (27)

These analytical results are in good agreement with our numerical results. Thus, we see that these dynamical instabilities occur near the horizon because $\psi = 0$ corresponds to the horizon. These results are consistent with those obtained in [11]. Note that $M[0]$ and $L[0]$ are determined by $\alpha_2$ and $\alpha_3$ and so these borders are independent of $\alpha_4$ if instabilities occur near the horizon. However, comparing three figures in figure 3, the region where black holes are unstable under scalar perturbations for $\alpha_4 = -1.5$ is very different from that for $\alpha_4 = 0$ and 0.5. This suggests that these instabilities occur away from the horizon. Therefore, $\alpha_4$ affects the behavior of $M[\psi]$ and changes the allowed region in the $\alpha_2$–$\alpha_3$ plane. Indeed, this fact can be understood more easily from figure 4. In figure 4, we plot forbidden regions in the $\alpha_2$–$\alpha_4$ plane with $\alpha_3 = -0.2, 0$ and 0.5, respectively. In these figures, we see vertical stripes for negative $\alpha_4$. For example, in figure 4 with $\alpha_3 = 0$, there are three vertical lines: $\alpha_2 \simeq -1.0$, $\alpha_2 \simeq 0.5$ and $\alpha_2 \simeq 0.75$. These lines can be obtained from $L[0] = 0$ as

$$\alpha_2 = -\frac{D-6}{2(D-1)} \pm \sqrt{2 \alpha_3 + \frac{5D^2 - 40D + 84}{4(D-1)^2}}.$$  \hspace{1cm} (28)

and from $K[0] = 0$ as

$$\alpha_2 = \frac{D-3}{D-1}.$$  \hspace{1cm} (29)

This agreement between the analytical and numerical results is remarkable. However, when $\alpha_4$ becomes large, the stripe structure collapses. This suggests that instabilities do not originate from the near horizon geometry. It turned out that $\alpha_4$ is a relevant parameter for AdS black branes.
Figure 3. We plot the allowed and forbidden regions in the $\alpha_2-\alpha_3$ plane with $\alpha_4 = -1.5$, 0 and 0.5, respectively. The shaded region represents an allowed region.
Figure 4. We plot the allowed and forbidden regions in the $\alpha_2 - \alpha_4$ plane with $\alpha_3 = -0.2$, 0 and 0.5, respectively. The shaded region represents an allowed region.
Figure 5. We plot the allowed and forbidden regions in the $\alpha_2 - \alpha_5$, $\alpha_3 - \alpha_5$ and $\alpha_4 - \alpha_5$ planes, respectively. The shaded region represents an allowed region. In these figures, we set other Lovelock coefficients to be 0.
5.2. Eleven dimensions

In 11 dimensions, the key functional is given by

$$W[\psi] = \frac{\alpha_5}{5} \psi^5 + \frac{\alpha_4}{4} \psi^4 + \frac{\alpha_3}{3} \psi^3 + \frac{\alpha_2}{2} \psi^2 + \psi + 1.$$  \hspace{1cm} (30)

Again, substituting this expression into $\partial_\psi W[\psi]$, (7)–(9), we can find the forbidden region in four-dimensional parameter space $(\alpha_2, \alpha_3, \alpha_4, \alpha_5)$. Of course, it is a formidable task to visualize such a higher dimensional space. Hence, we look at several sections in the parameter space. In figure 5, we plot forbidden regions in the $\alpha_2$--$\alpha_5$ plane with $(\alpha_2, \alpha_3) = (0, 0)$, $\alpha_3$--$\alpha_5$ plane with $(\alpha_2, \alpha_4) = (0, 0)$ and $\alpha_4$--$\alpha_5$ plane with $(\alpha_2, \alpha_3) = (0, 0)$, respectively.

From these figures, we see that $\alpha_5$ affects the allowed ranges of $\alpha_2$, $\alpha_3$ and $\alpha_4$. In particular, in the case of the $\alpha_3$--$\alpha_5$ and $\alpha_4$--$\alpha_5$ planes, the allowed region is finite. It indicates that AdS black branes in Lovelock theory are pathological in most cases.

6. Implications in AdS/CFT

Let us discuss the implications of our results in the AdS/CFT correspondence.

With the master equation in [16], the shear viscosity to entropy ratio $\eta/s$ has been calculated as

$$\frac{\eta}{s} = \frac{1}{4\pi} \left( 1 - \frac{D - 1}{D - 3} \alpha_2 \right)$$

through AdS/CFT correspondence [7]. Note that this depends only on $\alpha_2$. Hence, it seems that $\alpha_3$, $\alpha_4$ and $\alpha_5$ do not affect this value. However, as our numerical calculations have shown, $\alpha_3$, $\alpha_4$ and $\alpha_5$ affect the allowed region of $\alpha_2$. This fact was also noted in [11]. In five and six dimensions, the KSS bounds are lowered to $\eta/s = 1/8\pi$ and $\eta/s \simeq 0.59/4\pi$, respectively. Interestingly, in ten dimensions, we see from figure 3 that a positive $\alpha_2$ is not allowed for any $\alpha_3$ if $\alpha_5 = -1.5$. This means that the bound of $\eta/s$ must be larger than $1/4\pi$ if $\alpha_4 = -1.5$. As another example, we can take $\alpha_4 = 0$ and $\alpha_3 = 0.5$, then the maximal value of $\alpha_2$ is about $-0.1$. While if we take $\alpha_4 = 0.5$, the maximal $\alpha_2$ becomes 0.15 at which $\alpha_3 = 0.2$. Thus, it turned out that the KSS-like bound is sensitive to Lovelock coefficients.

It is also possible to apply our results to holographic superconductors [19]. There the universality for the ratio between the frequency-dependent conductivity and the critical temperature is found [20]. In Gauss–Bonnet theory, it is pointed out that this universality in holographic superconductors is violated for large $\alpha_2$ [21]. However, it is probable that this violation is due to the pathologies discussed in this paper. It would be interesting to extend holographic superconductors to Lovelock theory to clarify this point.

7. Conclusion

We have discussed the pathologies in AdS black branes in Lovelock theory, analytically in 5 and 6 dimensions, and numerically in 10 and 11 dimensions. We obtained the general conditions for the Lovelock coefficients $\alpha_m$ that these black branes have naked singularity, ghost instability and dynamical instability. It turned out that the dynamical instability could occur away from the horizon in contrast to a naive expectation. Thus, $\alpha_4$ and $\alpha_5$ also control the allowed region of $\alpha_2$ and consequently change the lower limit of $\eta/s$. We have also pointed out that the pathologies we have found could affect the interpretation of higher dimensional holographic superconductors.

In this paper, we did not consider the causality violation discussed in [8–11]. It is easy to take into account the causality violation based on the master equations [12]. Then, we could
further restrict the allowed region for $\alpha_2$. It is also straightforward to extend our analysis to other dimensions using the master equations [12].

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