TIC 454140642: A Compact, Coplanar, Quadruple-lined Quadruple Star System Consisting of Two Eclipsing Binaries

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Abstract

We report the discovery of a compact, coplanar, quadruple-lined, eclipsing quadruple star system from Transiting Exoplanet Survey Satellite data, TIC 454140642, also known as TYC 0074-01254-1. The target was first detected in Sector 5 with a 30-minute cadence in full-frame images and then observed in Sector 32 with a 2-minute cadence. The light curve exhibits two sets of primary and secondary eclipses with periods of $P_A = 13,624$ days (binary A) and $P_B = 10,393$ days (binary B). Analysis of archival and follow-up data shows clear eclipse-timing variations and divergent radial velocities, indicating dynamical interactions between the two binaries and confirming that they form a gravitationally bound quadruple system with a 2 + 2 hierarchy. The Aa+Ab binary, Ba+Bb binary, and A-B system are aligned with respect to each other within a fraction of a degree: the respective mutual orbital inclinations are $0\,^\circ.25$ (A versus B), $0\,^\circ.37$ (A versus A-B), and $0\,^\circ.47$ (B versus A-B). The A-B system has an orbital period of 432 days—the second shortest of the confirmed quadruple systems—and an orbital eccentricity of 0.3.

Unified Astronomy Thesaurus concepts: Eclipsing binary stars (444); Transit photometry (1709); Multiple stars (1081); Astronomy data analysis (1858); Astronomy data modeling (1859)

1. Introduction

A few percent of F- and G-type stars are members of stellar quadruples, and the fraction likely increases with stellar mass (e.g., Raghavan et al. 2010; De Rosa et al. 2014; Toonen et al. 2016; Moe & Di Stefano 2017; Tokovinin 2017). These systems are ideal laboratories to study stellar formation and evolution in a dynamically complex environment where the constituent binary stars can be close enough to interact with each other. These interactions can occur multiple times over the lifetime of the system, involve, e.g., Lidov–Kozai oscillations (Kozai 1962; Lidov 1962; Pejcha et al. 2013), mass transfer, tidal and dynamical interactions, mergers (Perets & Fabrycky 2009; Hamers et al. 2021), or the formation of tight binaries (e.g., Naoz & Fabrycky 2014, and references therein). Quadruple systems are important participants in stellar evolution as a potential source for Type I supernovae (SNe; Fang et al. 2018), as well as interesting outcomes such as common-envelope evolution and interactions with circumbinary disks.
Detection and detailed characterization of a bona fide quadruple-star system can be challenging. For example, it is not uncommon for close eclipsing binary (EB) stars to exhibit deviations from strict periodicity, and recent studies show that at least ~7% of Kepler’s eclipsing binary stars show such deviations within the 4 yr long observing window of the Kepler prime mission (e.g., Rappaport et al. 2013; Orosz 2015; Borkovits et al. 2016). Such eclipse-timing variations (ETVs) can be indicative of dynamical interactions between the eclipsing binary and additional star(s) on wider orbit(s), and provide an important detection method for triple, quadruple, and higher-order stellar systems, as well as for circumbinary planets (e.g., Borkovits et al. 2016; Welsh & Orosz 2018). Other pathways for the detection of quadruple (and higher-order multiples) are visual and spectroscopic observations (e.g., Raghavan et al. 2010; Tokovinin 2017; Pribulla et al. 2006), but these methods generally do not guarantee that the potentially additional star(s) are indeed gravitationally bound to the known binary.

A target star exhibiting multiple sets of eclipses, ETVs, and per-EB divergent radial velocities provides a unique opportunity not only to confirm the multiplicity of the system, but also to measure the orbital and physical parameters of its constituents with exquisite precision (e.g., Carter et al. 2011; Borkovits et al. 2018, 2020a, 2021). We note that because the outer orbital period of quadruple systems is much longer than the periods of the inner binaries (for otherwise, the system would become unstable), detecting such features requires continuous and long-baseline observations of the target. A combination of space-based photometric surveys, extensive ground-based survey photometry, and dedicated spectroscopy follow-up is ideally suited to provide such observations. This was demonstrated by the highly successful Kepler mission, which detected a number of EB systems exhibiting additional features indicative of either circumbinary planets or triple and higher-order stellar systems (e.g., Kirk et al. 2016; Borkovits et al. 2016; Welsh & Orosz 2018).

The Transiting Exoplanet Survey Satellite (TESS, Ricker et al. 2015) is equally capable of discovering multiply eclipsing stellar systems, as already demonstrated by Borkovits et al. (2020b, 2021) for stellar triples and quadruples and by Powell et al. (2021) for sextuples. The target presented here, TIC 454140642, exhibits the typical 2 + 2 hierarchy for quadruple systems, illustrated in Figure 1, and joins the small number of such confirmed, well-characterized systems—VW LMi (Pribulla et al. 2008), V994 Her (Zasche & Uhlafi 2016), V482 Per (Torres et al. 2017), EPIC 220204960 (Rappaport et al. 2017), EPIC 219217635 (Borkovits et al. 2018), CzeV1731 (Zasche et al. 2020), TIC 278956474 (Rowden et al. 2020), and BG Ind (Borkovits et al. 2021).

We note that while quadruple star systems have lower occurrence rates than triple systems, the larger outer orbit that is necessary for stability in the latter systems is generally well beyond the 27-day TESS observation time in a single sector. Specifically, the long outer orbital period substantially reduces the probability that tertiary eclipses or occultations of a stellar triple will be identified in one sector of TESS data. While such triples can (and do) produce multiple tertiary eclipses or occultations during one conjunction of the outer orbit in a single sector of TESS data (e.g., Borkovits et al. 2020b), detecting multiple such conjunctions in one sector of data is highly unlikely. In turn, this makes determining the outer period of such triples—and thus the overall configuration of the respective system—challenging. In contrast, quadruple systems do not need to be at a conjunction of the outer orbit to reveal their nature, and eclipses/occultations of their constituent binaries can (and do) easily occur in a single sector of TESS data.

Here we present the discovery of the quadruple star TIC 454140642, which exhibits four sets of eclipses as well as prominent, dynamically induced apsidal motion, and short-term four-body perturbations. At the time of writing, TIC 454140642 is the closest to coplanarity of the quadruple systems mentioned above, and has the second-shortest outer period (after VW LMi). This paper is organized as follows. Section 2 outlines the detection of the target in TESS data. In Sections 3 and 4 we describe the analysis of the available archival data and our follow-up observations, respectively. In Section 5 we present our comprehensive spectro-photodynamical analysis of the system parameters. We discuss the properties of the system in Section 6 and draw our conclusions in Section 7.

2. Detection

Our detection methodology is described in detail by Powell et al. (2021). Briefly, we found hundreds of thousands of eclipsing binaries using a neural network classifier on the TESS full-frame image light curves. Through manual examination of the light curves identified by the neural network as EBs, we found many that showed eclipses belonging to multiple periods in the same light curve. Through photocenter analysis, we determined that the large majority of these light curves are consistent with a superposition of two EBs originating from unrelated target stars within the same TESS aperture. A fraction of these, however, demonstrated an on-target photocenter for all detected eclipses, suggesting that they originate from the same stellar system.

TIC 454140642, a previously unknown EB, was one of the systems that we found early in our examination of TESS light curves. The nature of the system was initially quite challenging to decipher because all the eclipses are similar in depth and duration, in addition to the fact that we only had one sector of data with each eclipse present twice in the light curve. Preliminary analysis indicated that there are two sets of primary and secondary eclipses. The TESS eleanor light curve (Feinstein et al. 2019) for Sectors 5 and 32 is shown in Figure 2, highlighting the four sets of eclipses; there are no visible out-of-eclipse variations. In accordance with the
designation system developed by the IAU, these binaries are referred to as A (the brighter component with a period of 13.62 days, and components Aa and Ab) and B (the fainter component, with a period of 10.39 days, and components Ba and Bb). The contamination due to nearby sources is low according to the TESS Input Catalog (0.001), and photocenter analysis confirmed that both sets of eclipses originate from the target (see Figure 3), indicating a potentially quadruple system. This motivated us to submit the target for 2-minute cadence observations in Sector 32 (TESS DDT program 020 PI: Kostov), and also coordinate follow-up efforts with the TESS follow-up observing program (TFOP). The parameters of the system are listed in Table 1.

### 3. Photometric Data

The archival data on TIC 454140642 from the WASP and ASAS-SN surveys are extensive (Pollacco et al. 2006; Shappee et al. 2014; Kochanek et al. 2017). Altogether, they cover more than 4000 days. The time span of the available photometric data is shown in Figure 4, which includes the TESS data as well as our follow-up photometry.
clear apsidal motion (see Figure 6) and indicates dynamical interactions between the two binaries.

### 4. Follow-up Observations

#### 4.1. Spectroscopy

TIC 454140642 was monitored spectroscopically with the Tillinghast Reflector Echelle Spectrograph (TRES; Szenti-gyorgyi & Furész 2007; Furész 2008) mounted on the 1.5 m Tillinghast reflector at the Fred L. Whipple Observatory on Mount Hopkins (AZ). TRES is a bench-mounted, fiber-fed instrument with a resolving power of $R \approx 44,000$, and a wavelength coverage of 3900–9100 Å in 51 orders. In total, we gathered 26 spectra between September of 2020 and February of 2021, with signal-to-noise ratios in the region of the MgI b triplet (~5185 Å) ranging from 28 to 60 per resolution element of 6.8 km s$^{-1}$. Exposure times ranged between 450 and 2400 s. The spectra were extracted and reduced as per Buchhave et al. (2010), with wavelength solutions derived from bracketing thorium-argon lamp exposures.

The very first spectrum of TIC 454140642 showed four sets of lines, which we expected should represent the two components of two double-lined binaries in the system (see Figure 7). Radial velocities were measured using QUADCOR, a four-dimensional cross-correlation algorithm introduced by Torres et al. (2007), which uses four separate and possibly different templates matched to each star. In this case, however, the stars turn out to be quite similar to each other (see below), so we adopted the same template for the four components. We used a synthetic spectrum taken from a large library of calculated spectra based on model atmospheres by R. L. Kurucz (see Nordström et al. 1994; Latham et al. 2002), covering the region of the MgI b triplet. The template parameters were $T_{\text{eff}} = 6250$ K, $\log g = 4.5$, $[\text{m/H}] = 0.0$, and no rotational broadening. The choice of zero rotational broadening was based on experiments with a range of values for $v \sin i$, which indicated that the broadening for all four stars is smaller than our spectral resolution. In addition to the velocities, we used QUADCOR to measure the flux ratios of the components at the mean wavelength of our observations.

While approximate orbital periods were known from the photometry for the two binaries undergoing eclipses, at first it was not trivial to identify which set of lines corresponded to which component of which binary. In several of the spectra, the lines were heavily blended, so the velocity determinations were not always possible for all four stars at this early stage, and were guided by the location of the peaks in plots of the 1D cross-correlation functions. The process was further complicated by the degeneracy between the flux ratios and the velocity separations when the lines are blended, which made the velocities imprecise. Based on the more reliable flux ratios from the relatively few spectra with well-separated lines, it was eventually realized that one star was brighter than the other three, another was the faintest, and the remaining two had similar brightness. Gathering the velocities for the brighter and fainter components and sorting out the others based on the preliminary photometric ephemerides, we obtained initial orbital solutions for the two binaries based on the 18 observations through mid 2020 December. These fits showed considerable scatter, far in excess of the precision expected for the measurements. The primary and secondary residuals for the 13-day binary showed an obvious upward drift, and those of

#### Table 1

Stellar Parameters for TIC 454140642

| Parameter | Value | Error | Source |
|-----------|-------|-------|--------|
| T (mag)   | 9.8549| 0.0097| 1      |
| B (mag)   | 10.945| 0.111 | 4      |
| V (mag)   | 10.409| 0.008 | 1      |
| Gaia (mag)| 10.2224| 0.0028| 2      |
| $G_{\text{BP}}$ (mag)| 10.4997| 0.001265| 2 |
| $G_{\text{RP}}$ (mag)| 9.80662| 0.001475| 2 |
| $B_{\text{F}}$ (mag)| 10.769| 0.058 | 3      |
| $V_{\text{F}}$ (mag)| 10.329| 0.063 | 3      |
| $g'$ (mag) | 10.613| 0.098 | 4      |
| $r'$ (mag) | 10.229| 0.041 | 4      |
| $i'$ (mag) | 10.123| 0.021 | 4      |
| $J$ (mag) | 9.349| 0.027 | 5      |
| $H$ (mag) | 9.116| 0.026 | 5      |
| $K$ (mag) | 9.022| 0.019 | 5      |
| $W1$ (mag)| 9.010| 0.023 | 6      |
| $W2$ (mag)| 9.036| 0.020 | 6      |
| $W3$ (mag)| 9.046| 0.032 | 6      |
| $W4$ (mag)| 8.431| 0.028 | 6      |
| FUV (mag) | 18.381| 0.195 | 7      |
| NUV (mag)| 12.771| 0.007 | 7      |
| Contamination | 0.0014|       |        |
| $T_{\text{eff}}$ (K) | 6434| 30 | This work |
| $T_{\text{eff}}$ (K) | 6303| 30 | This work |
| $T_{\text{eff}}$ (K) | 6303| 30 | This work |
| $T_{\text{eff}}$ (K) | 6188| 35 | This work |
| $[\text{Fe/H}]$ | −0.039| 0.022 | This work |
| Age (Gyr) | 1.945| 0.27 | This work |

Sources: (1) TIC-8 (Stassun et al. 2018), (2) Gaia EDR3 (Gaia Collaboration et al. 2016, 2021), (3) Tycho-2 catalog (Hog et al. 2000), (4) APASS DR9 (Henden et al. 2015), (5) 2MASS All-Sky Catalog of Point Sources (Skrutskie et al. 2006), (6) AllWISE catalog (Cutri et al. 2012), (7) GALEX-DR5 (GR5) (Bianchi et al. 2011).

The ASAS-SN data of TIC 454140642 provide coverage of about 2000 days and overlap with TESS data. The target was observed by TESS with both the 85 and the 200 mm lens, although the signal-to-noise ratio of the former is poor, and we therefore only used the latter for our analysis. All four sets of eclipses are clearly recovered in the ASAS-SN and TESS data, as shown in Figure 5. A box least-squares analysis (BLS, Kovács et al. 2002) analysis of the two data sets provides similar orbital periods for the two binaries, as indicated in the figure captions and listed in Table 2. We note that the periods listed in the table are derived from the photometry alone; the final orbital periods are derived from the comprehensive spectro-photodynamical model described in Section 5.1. Altogether, the available photometry phase-folded on the periods listed in Table 2 shows
Figure 4. Photometry of the target from ASAS-SN (blue symbols for both g-filter and V-filter), FRAM (green symbols), TESS (red symbols), TFOP (magenta symbols), and WASP 200 mm lenses (cyan symbols), highlighting the covered baseline.

Figure 5. Folds of the ASAS-SN (top) and WASP (bottom) archival data for the two binaries about their respective orbital periods.
the 10-day binary a drift in the opposite direction (see Figure 8). This was the first sure sign that the two binaries are gravitationally bound to each other, and in hindsight, it

| Binary     | A          | B          |
|------------|------------|------------|
| ASAS — SN  | 13.623978  | 10.392850  |
| Period [days] | 5914.2859  | 5920.1721  |
| T0 [BJD—2,450,000] | 8440.8394  | 8445.5638  |
| TESS       |            |            |
| Period [days] | 13.62395   | 10.39335   |
| T0 [BJD—2,450,000] | 4694.1464  | 4693.7639  |
| WASP       |            |            |
| Period [days] | 13.623978  | 10.392850  |
| T0 [BJD—2,450,000] | 8454.4688  | 8445.5610  |
| Global Fitted Periods | 13.6239   | 10.3928   |
| Global Fitted T0 [BJD—2,450,000] | 8454.4688  | 8445.5610  |
explains our initial difficulty in identifying the components of the two binaries.

Solving for this linear drift separately in each of the double-lined orbits improved the velocities significantly and allowed us to determine approximate binary elements. In addition to the period ($P$), the elements for each binary are the velocity semi-amplitudes of the components ($K$), the orbital eccentricity ($e$) and argument of periapsis of the secondary ($\omega$), the common center-of-mass velocity of the primary and secondary ($\gamma$), a reference time of the periapsis passage ($\tau$), and a radial acceleration term ($\dot{\gamma}$) for the binary center of mass. The orbital periods were better determined from the TESS photometry as well as the WASP and ASAS-SN archival data (see Section 3), and were held fixed. These elements enabled us to better predict the velocities for the four components at epochs of heavy line blending, which were then used as a starting point for a refined analysis of those 18 observations and subsequent ones with QUADCOR. We report our final velocities for all epochs in Table 3. Typical uncertainties for stars Aa, Ab, Ba, and Bb are 0.66, 0.73, 0.95, and 1.01 km s$^{-1}$, respectively, and are based on the rms scatter of the residuals from the orbits described above. The spectroscopic flux ratios we obtained for stars Ab, Ba, and Bb relative to the primary of the 13-day binary (Aa) are 0.85, 0.81, and 0.67, respectively, at the mean wavelength of our observations ($\sim$5185 Å).

The first-cut binary parameters determined from this analysis are given in Table 4 and served to set initial values for the global photodynamical analysis described in Section 5. The masses of the four stars in the quadruple system average 1.15 $M_\odot$ (assuming $\sin i \approx 1$, as both binaries are eclipsing), and none of the stars has a mass that differs from this average by more than 0.05 $M_\odot$. Given that the total masses of the two binaries are rather similar, we note also that the straight average masses of the four stars in the quadruple system average 0.66, 0.73, 0.95, and 1.01 km s$^{-1}$ for stars Aa, Ab, Ba, and Bb, respectively.

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### Table 3

| Observation Date | RV Aa (km s$^{-1}$) | RV Ab (km s$^{-1}$) | RV Ba (km s$^{-1}$) | RV Bb (km s$^{-1}$) |
|------------------|---------------------|---------------------|---------------------|---------------------|
| BJD = 2,400,000  |                     |                     |                     |                     |
| 59097.9754       | $-56.29$            | $-49.83$            | $+121.71$           | $-8.53$            |
| 59110.9829       | $-49.11$            | $+50.30$            | $+44.69$            | $+64.00$           |
| 59117.0083       | $+35.20$            | $-36.63$            | $+86.98$            | $+16.78$           |
| 59118.9106       | $+63.97$            | $-64.35$            | $+115.66$           | $-13.81$           |
| 59120.9626       | $+35.06$            | $-32.31$            | $+59.15$            | $+43.65$           |
| 59130.9687       | $+49.18$            | $-39.79$            | $+70.83$            | $+24.54$           |
| 59131.9596       | $+66.35$            | $-56.74$            | $+33.01$            | $+61.01$           |
| 59164.8201       | $-24.29$            | $+57.10$            | $-19.90$            | $+95.43$           |
| 59165.8458       | $-34.06$            | $+70.49$            | $-23.97$            | $+95.35$           |
| 59170.8216       | $+36.46$            | $+0.21$             | $+97.46$            | $-32.57$           |
| 59172.8547       | $+78.94$            | $-43.53$            | $+44.59$            | $+22.68$           |
| 59175.9399       | $+40.22$            | $-0.75$             | $-29.14$            | $+96.57$           |
| 59178.7227       | $-22.22$            | $+65.17$            | $+43.06$            | $+20.25$           |
| 59180.7983       | $-31.47$            | $+77.27$            | $+94.78$            | $-36.39$           |
| 59186.8346       | $+84.48$            | $-42.34$            | $-27.90$            | $+88.42$           |
| 59188.8468       | $+61.05$            | $-16.05$            | $+31.70$            | $+24.74$           |
| 59195.8602       | $-11.14$            | $+63.39$            | $-29.43$            | $+85.94$           |
| 59197.8081       | $+36.24$            | $+15.42$            | $-19.08$            | $+73.06$           |
| 59199.7751       | $+82.82$            | $-32.63$            | $+47.12$            | $-4.03$            |
| 59208.7251       | $-19.33$            | $+76.92$            | $-7.22$             | $+55.46$           |
| 59216.7606       | $+50.26$            | $+7.48$             | $-35.74$            | $+82.01$           |
| 59233.6625       | $-15.88$            | $+85.25$            | $+76.54$            | $-42.23$           |
| 59246.6163       | $-4.70$             | $+76.49$            | $-11.52$            | $+44.92$           |
| 59253.6860       | $+84.15$            | $-12.72$            | $+78.43$            | $-50.96$           |
| 59269.6177       | $+93.88$            | $-20.79$            | $-46.47$            | $+76.01$           |
| 59273.6654       | $+1.48$             | $+77.41$            | $+68.18$            | $-44.34$           |

Note: Radial velocity measurements were taken with TRES on the 1.5 m reflector at the Fred Lawrence Whipple Observatory (FLWO) in Arizona. See text for details. The uncertainties on the individual measurements were determined empirically from the rms scatter in the residuals to the fitted model and are 0.66, 0.73, 0.95, and 1.01 km s$^{-1}$ for stars Aa, Ab, Ba, and Bb, respectively.
find the outer orbital separation, $a_{\text{out}}$, and phase, $\phi_{\text{out}}$, at the time of the RV observations, where $\phi$ is the angle from inferior conjunction of binary A,

$$\sqrt{G(M_A + M_B)/a} \sin \phi = \gamma_A - \gamma_B$$  \hspace{1cm} (1)

$$G(M_A + M_B)/a^2 \cos \phi = \gamma_A - \gamma_B.$$  \hspace{1cm} (2)

If we solve for $a_{\text{out}}$ and $\phi_{\text{out}}$ and then calculate the outer period, $P_{\text{out}}$, we find

$$a_{\text{out}} = 389 \pm 20 \, R_\odot$$

$$\phi = -38.8 \pm 5^\circ$$

$$P_{\text{out}} \simeq 419 \pm 19 \text{ d},$$

where the uncertainties are due only to those associated with the measurements used in the calculations, and not to the actual eccentric nature of the outer orbit. These approximations for the outer orbit served to guide the full photodynamical modeling described below.

### 4.2. Photometric Measurements

#### 4.2.1. TESS Follow-up Observing Program

We acquired ground-based time-series follow-up photometry of TIC 454140642 as part of the TESS Follow-up Observing Program (TFOP).\footnote{https://tess.mit.edu/followup} We used the TESS Transit Finder, which is a customized version of the Tapir software package (Jensen 2013), to schedule our eclipse observations. The photometric data were extracted using AstroImageJ (Collins et al. 2017). Observations were acquired using the Las Cumbres Observatory network (LCO; Brown et al. 2013) and the Hazelwood Private Observatory (Churchill, Victoria, Australia), as described in Table 5.

#### 4.2.2. Other Follow-up Ground-based Photometry

In addition to the observations described above, we also used other photometric data for confirmation of the ETVs predicted from TESS data. These observations of eclipses of both pairs were obtained using the following instruments:

1. FRAM telescopes: 30 cm telescope as part of the Pierre Auger Observatory in Argentina (The Pierre Auger

### Table 4

| Fitted Binary Parameters\(^a\) | Aa | Ab | Ba | Bb |
|--------------------------------|----|----|----|----|
| Period [days]\(^b\) | 13.6239 | 13.6239 | 10.3928 | 10.3928 |
| K [km s\(^{-1}\)] | 57.89 ± 0.41 | 60.79 ± 0.40 | 62.58 ± 0.51 | 65.05 ± 0.50 |
| $e$ | 0.075 ± 0.005 | 0.075 ± 0.005 | 0.022 ± 0.004 | 0.022 ± 0.004 |
| $\omega$ [deg] | 354.9 ± 5 | 174.9 ± 5 | 1.9 ± 16 | 181.9 ± 16 |
| $\gamma$ [km s\(^{-1}\)] | 9159.9 ± 0.2 | 9159.9 ± 0.2 | 9149.9 ± 0.5 | 9149.9 ± 0.5 |
| $\gamma$ [cm s\(^{-2}\)] | +11.34 ± 0.22 | +11.34 ± 0.22 | +41.17 ± 0.25 | +41.17 ± 0.25 |
| Mass [$M_\odot$]\(^d\) | 1.208 ± 0.018 | 1.151 ± 0.018 | 1.141 ± 0.019 | 1.098 ± 0.018 |

Notes.

\(^a\) Derived from the RV measurements alone.

\(^b\) Fixed at the values given in Table 2.

\(^c\) BJD—2,450,000.

\(^d\) Derived under the assumption that $\sin i \approx 1$.

### Table 5

| Telescope | Location | Date [UTC] | Filter | Aperture radius [arcsec] | Coverage |
|-----------|----------|------------|--------|--------------------------|----------|
| LCO-SAAO 0.4 m | South Africa | 2020-07-18 | $z$-short | 5.7 | no event, ephemeris updated |
| LCO-McD 0.4 m | Texas, U.S. | 2020-09-08 | $z$-short | 6.8 | no event, ephemeris updated |
| LCO-Hal 0.4 m | Haleakala, U.S. | 2020-10-20 | $z$-short | 4.0 | ingress |
| LCO-SSO 0.4 m | Siding Spring | 2021-01-10 | $z$-short | 5.1 | egress |
| Hazelwood 0.32 m | Australia | 2021-01-10 | $i'$ | 5.0 | mid-eclipse |
| LCO-SAAO 0.4 m | South Africa | 2021-01-20 | $z$-short | 4.0 | ingress |

Note. The $z$-short filter is the same as the Pan-STARRS Z-short passband, i.e., $\lambda_{\text{center}} = 870 \text{ nm}$, $\lambda_{\text{width}} = 104 \text{ nm}$. The $i'$ filter is the same as the SDSS $i'$ passband, i.e., $\lambda_{\text{center}} = 755 \text{ nm}$, $\lambda_{\text{width}} = 129 \text{ nm}$.
Collaboration et al. 2021), and 25 cm telescope in La Palma as part of the Cerenkov Telescope Array (CTA; Ebr et al. 2019).

2. Ondřejov Observatory, Czech Republic, 65 cm reflecting telescope, using a G2-3200 CCD camera.
3. Danish 1.54 m telescope located on La Silla in Chile, and using standard V photometric filter.
4. Small 34 mm refractor by R.U. - private observatory in Jilové u Prahy, Czech Republic, using a G2-0402 CCD camera.

4.2.3. Speckle Imaging

The target star was placed on the Southern Astrophysical Research Telescope (SOAR) speckle program in 2020 December. It was observed on 2021 February 27. The star was unresolved, and no companions with $\Delta I < 1$ mag and having a separation greater than $0.00048$ were detected. The achieved detection limits are $\Delta I < 2.9$ mag at $0.015$ and $\Delta I < 4.4$ mag at $1^\circ$. The details of these observations will be published jointly with other SOAR speckle results as part of the series of papers represented by Tokovinin et al. (2020).

The A-B pair, with an estimated semimajor axis of 5 mas, is not expected to be resolved at SOAR. However, the speckle imaging demonstrated the absence of other nearby faint stars that could otherwise be missed by the photometry or spectroscopy. To the best of our knowledge, then, TIC 454140642 is an isolated quadruple system.

5. Analysis of System Parameters

5.1. Combined Spectro-photodynamical Analysis

We used the LIGHTCURVEFACTORY software package (Borkovits et al. 2019, 2020a) to carry out a simultaneous, joint analysis of the light curves, ETV, RV curves and multi-passband spectral energy distribution (SED) data of the system. We used three separate light curves as follows:

(i) TESS Sectors 5 and 32 light curves. In the case of the Sector 32 data, we used the 2-minute-cadence data set to extract the ETV data; however, for the joint analysis, we binned these data into 1800 s, resulting an effective exposure length of 1425 s, to obtain the same sampling as was available for Sector 5 data. Furthermore, we excluded the flat out-of-eclipse light-curve sections and kept only the narrow $\pm 0.03$-phase-domain regions around each eclipse.

(ii) Similarly clipped regions of archival WASP data. After the removal of outlier data points, these data were averaged into 1800 s bins for the analysis.

(iii) The ground-based eclipse observations of our follow-up photometric campaign. These data were also binned at 1800 s.

Additionally, we used the four ETV curves (primary and secondary ETV data for both binaries; see Tables 6 and 7) and the first 18 epochs of the four TRES RV curves (see Table 3). Furthermore, the observed passband magnitudes tabulated in Table 1 were also used for the SED analysis.

The modeling runs proceeded as follows. LIGHTCURVEFACTORY calculates the positions and velocities of each star at any requested instant via the numerical integration of the orbital motion. Then, taking into account the radii, temperature, and other atmospheric properties of each star and their relative geometry with respect to the others and to the observer, the code predicts the net light curve of the whole system, enabling any kind of mutual (or multiple) eclipses. Note that in the present analysis, stellar radii and temperature were constrained through the stellar masses and the metallicity and age of the system with the use of built-in PARSEC stellar isochrones (Bressan et al. 2012), as was described in detail in Borkovits et al. (2020a). Furthermore, the same PARSEC tables are used to generate theoretical passband magnitudes for the SED fitting part of the analysis. To solve the inverse problem, LIGHTCURVEFACTORY employs a Markov chain Monte Carlo (MCMC)-based parameter search, implementing the generic Metropolis-Hastings algorithm (see, e.g., Ford 2005).

In most of the MCMC runs, we adjusted the following 20 parameters:

(i) Four orbital elements for binary A, i.e., the eccentricity and argument of periastron in the combinations of $(e \sin \omega)_A$ and $(e \cos \omega)_A$, its inclination, $i_A$, and the longitude of the ascending node $\Omega_A$ relative to the node of binary B.

51 Similar to our previous works, for the SED analysis, we used a minimum uncertainty of 0.03 mag for most of the observed passband magnitudes in order to avoid the outsized contribution of the extremely precise Gaia magnitudes and also to counterbalance the uncertainties inherent in our interpolation method during the calculations of theoretical passband magnitudes that are part of the fitting process. The two exceptions are the WISE W4 and GALEX FUV magnitudes, for which the uncertainties were inflated to 0.3 mag.
(ii) Three orbital elements for binary B, i.e., the same as above, with the exception of $\Omega_B$, for which the value was kept at zero.

(iii) Six orbital parameters for the outer orbit, i.e., its anomalistic period, $P_{AB}$, eccentricity, and argument of periastron, inclination, periastron passage time, $\tau_{AB}$, and the longitude of the ascending node relative to the node of binary B.

(iv) Four mass-related parameters, i.e., the masses of the two primaries, $m_{A,B}$, and the (inner) mass ratios of the two binaries, $q_{A,B}$.

(v) Three global parameters, i.e., the (logarithmic) age of the system, its metallicity, $[\text{m/H}]$, and the interstellar reddening $E(B-V)$.

Moreover, the following parameters were internally constrained:

(i) Four remaining orbital parameters of the inner orbits, i.e., their periods ($P_{A,B}$) and inferior conjunction times ($T_{\text{inv}}^{\text{AB}}$) at epoch $t_0$, which were constrained through the ETV curves (see Borkovits et al. 2019).

(ii) The systemic RV of the entire quadruple system ($\gamma$) was obtained at each step by minimizing the $\chi^2_{\text{RV}}$ value a posteriori.

(iii) Eight fundamental stellar parameters, i.e., the radii and temperature of the four stars, which were interpolated from the PARSEC grids with the use of $\{\text{mass, metallicity, age}\}$ triplets in the manner described by Borkovits et al. (2020a).

(iv) The (photometric) posterior distance of the system was also obtained through an a posteriori minimization of $\chi^2_{\text{SED}}$.

(v) Finally, the (logarithmic) limb-darkening coefficients were calculated at each trial step internally from the precomputed passband-dependent tables downloaded from the Phoebe 1.0 Legacy page. These tables were originally used in previous versions of the PHOEBE software (Prša & Zwitter 2005).

Table 8 lists the median values of the stellar and orbital parameters of the quadruple systems that are being either adjusted, internally constrained, or derived from the MCMC posteriors, together with their 1σ statistical uncertainties. The RV curves, light curves, ETV curves, and SED model of the lowest $\chi^2_{\text{global}}$ solution are plotted in Figures 8–11.

In addition to the usual orbital elements tabulated in the first section of Table 8, we list the mutual (i.e., relative to each other) inclinations of the three orbital planes as well ($i_{\text{inv},A-B}$, $i_{\text{inv},A-B}-i_{\text{inv},B}$). We also provide the angular orbital elements in the dynamical frame of reference, i.e., the inclinations of the orbits relative to the invariable plane of the quadruple system ($i_{\text{dyn}}$), and the longitude of the pericenter ($\omega_{\text{dyn}} = \Omega_{\text{dyn}} + \omega_{\text{dyn}}$) of each orbit defined as the sum of the dynamical longitude of the node and the dynamical argument of pericenter. We note that due to the almost perfect coplanarity of the entire system, the intersections of the three orbits with the invariable plane (and therefore all the three pairs of $\Omega_{\text{dyn}}$ and $\omega_{\text{dyn}}$) become ill defined; however, their sums, i.e., $\omega_{\text{dyn}} = \Omega_{\text{dyn}} + \omega_{\text{dyn}}$, remain well defined, and we tabulate the latter quantities in Table 8. The relevance of these dynamical orbital elements lies in the fact that they, along with the mutual inclinations, appear directly in the analytic perturbation theories of multiple star systems (see, e.g., Borkovits et al. 2015, for further discussions). Furthermore, in this section we also tabulate some parameters related to apsidal motion that are discussed in Section 6.3. Finally, for completeness, we also provide the inclination and nodal angle of the invariable plane ($i_{\text{inv}}$ and $\Omega_{\text{inv}}$, respectively).

We note that the values listed in Table 8 represent the (instantaneous) osculating orbital elements at the (arbitrarily chosen) epoch $t_0 = 2458437.5$. Therefore the respective periods listed in the first row of the table should not be used for eclipse-timing prediction for future follow-up observations. This is illustrated in Figure 12, where we show the various relevant periods for the system. The osculating orbital elements (gray lines in the figure) are calculated at each instant from the Cartesian (Jacobian) coordinates and velocities of each orbit as if they represented pure, unperturbed, Keplerian two-body motions. In other words, these orbital elements describe the (osculating) two-body orbit on which the motion would continue if the perturbing forces vanished at that instant. The true motion occurs as the envelope of the osculating orbits. Naturally, in the presence of time-varying external forces such

\begin{table}[h]
\centering
\begin{tabular}{ccccccccc}
\hline
BID & Cycle & std. dev. & BID & Cycle & std. dev. & BID & Cycle & std. dev. \\
\text{–2,400,000} & no. & (d) & \text{–2,400,000} & no. & (d) & \text{–2,400,000} & no. & (d) \\
\hline
54766.52662 & –354.0 & 0.00173 & 55873.51828 & –247.5 & 0.00075 & 59162.70763$^a$ & 69.0 & 0.00016 \\
54813.46435 & –349.5 & 0.00057 & 56268.45172 & –209.5 & 0.00050 & 59178.45726 & 70.5 & 0.00038 \\
54818.50060 & –349.0 & 0.00025 & 56595.68135 & –178.0 & 0.00023 & 59183.49229 & 71.0 & 0.00004 \\
55109.43785 & –321.0 & 0.00372 & 56647.61573 & –173.0 & 0.00111 & 59188.84938 & 71.5 & 0.00033 \\
55153.59379 & –318.5 & 0.00024 & 56663.38911 & –171.5 & 0.00024 & 59193.88343 & 72.0 & 0.00004 \\
55156.38361 & –316.5 & 0.00040 & 58440.54817 & –0.5 & 0.00040 & 59199.24128 & 72.5 & 0.00026 \\
55161.43771 & –316.0 & 0.00077 & 58445.56571 & 0.0 & 0.00007 & 59214.66803$^a$ & 74.0 & 0.00024 \\
55208.38458 & –311.5 & 0.00161 & 58455.95559 & 1.0 & 0.00006 & 59230.41337$^a$ & 75.5 & 0.00033 \\
55483.58635 & –285.0 & 0.00042 & 58461.32839 & 1.5 & 0.00012 & 59235.45419$^a$ & 76.0 & 0.00051 \\
\hline
\end{tabular}
\caption{Times of Minima of TIC 454140642 B}
\end{table}

\textit{Note.} Integer and half-integer cycle numbers refer to primary and secondary eclipses, respectively. Eclipses between cycles $–354.0$ and $–171.5$ were observed in the WASP project. Times of minima from cycles $–0.5$ to $1.5$ and $70.5$ to $72.5$ are determined from the TESS measurements. Times of minima labeled with $^a$ are obtained from ground-based follow-up observations.

$^{a}$ http://phoebe-project.org/1.0/download

\textsuperscript{33} The angular distance of the intersection of the given orbital plane and the invariable plane—the dynamical nodal line—from the intersection of the plane of the sky and the invariable plane, measured along the invariable plane.

\textsuperscript{34} The angular distance of the pericenter from the dynamical ascending node along the orbital plane.
Table 8
Median Values of the Parameters from the Double EB Simultaneous Light Curve, 2 x SB2 RV, Double ETV, Joint SED, and PARSEC Evolutionary Track Solution from LIGHTCURVEFACTORY

| Orbital Elements<sup>a</sup> | A | B | A-B |
|-----------------------------|---|---|-----|
| $P_\text{c}$ [days]         | 13.61594 ± 0.00016 | 10.39063 ± 0.00008 | 0.00055 ± 0.00008 |
| semimajor axis $[R_\odot]$  | 31.87 ± 0.02 | 26.27 ± 0.01 | 2000 ± 1000 |
| $i$ [deg]                   | 87.55±0.06 | 87.52±0.08 | 0.040 ± 0.004 |
| $e$                         | 0.07449 ± 0.000021 | 0.00700 ± 0.000012 | 0.000013 |
| $\omega$ [deg]              | 161.21 ± 0.16 | 190.89 ± 1.39 | 1.28 ± 1.16 |
| $e^\text{e}$ [BJD]          | 2458 437.0411 ± 0.0002 | 2458 443.3691 ± 0.0002 | 2.38 ± 0.28 |
| $\Omega$ [deg]              | −0.13 ± 0.20 | 0.00 ± 0.01 | 0.03 ± 0.03 |
| $\omega_\text{m}A_\text{...}$ [deg] | 0.0 | 0.25±0.11 | 0.26 ± 0.22 |
| $\omega_\text{m}B_\text{...}$ [deg] | 0.25±0.11 | 0.00 ± 0.01 | 0.24 ± 0.23 |
| $\omega_{\text{d}B_\text{...}}$ [deg] | 11.04±1.38 | 161.21±0.55 | 150.17 ± 0.56 |
| $i_{\text{f}}$ [deg]        | 0.31±0.16 | 0.41±0.21 | 0.09 ± 0.04 |
| $i_{\text{f}}^\text{e}$ [deg] | 87.67±0.40 | −0.13±0.24 |

| RV Curve Related Parameters | | | |
|-----------------------------|---|---|-----|
| mass ratio $[q = m_{\text{sec}}/m_{\text{ps}}]$ | 0.959±0.008 | 0.964±0.005 | 0.963±0.009 |
| $K_{\text{ps}}$ [km s<sup>-1</sup>] | 58.11±0.24 | 62.80±0.24 | 24.25±0.13 |
| $K_{\text{sec}}$ [km s<sup>-1</sup>] | 60.62±0.25 | 65.14±0.21 | 25.20±0.21 |
| $\gamma$ [km s<sup>-1</sup>] | ... | ... | 25.97±0.07 |

| Apsidal Motion Related Parameters<sup>b</sup> | | | |
|-----------------------------|---|---|-----|
| $\psi_{\text{m}A_\text{...}}$ [year] | 70 | 97 | 472 |
| $\psi_{\text{m}B_\text{...}}$ [year] | 87.7±1.0 | 109±1.0 | 480±3.0 |
| $\Delta\psi_{\text{A}}$ [arcsec/cycle] | 556.7±1.0 | 388±0.4 | 3194±12 |
| $\Delta\psi_{\text{B}}$ [arcsec/cycle] | 611±0.04 | 709±0.04 | 1.06±0.01 |
| $\Delta\psi_{\text{BA}}$ [arcsec/cycle] | 0.075±0.004 | 0.130±0.005 | ... |

| Stellar Parameters | Aa | Ab | Ba | Bb |
|-------------------|----|----|----|----|
| Relative Quantities | | | | |
| fractional radius $[R/a]$ | 0.039±0.0009 | 0.0365±0.0004 | 0.0442±0.0007 | 0.0415±0.0005 |
| fractional flux [in TESS band] | 0.305±0.009 | 0.274±0.009 | 0.247±0.009 | 0.204±0.008 |
| fractional flux [in SWASP band] | 0.311±0.0010 | 0.245±0.009 | 0.246±0.009 | 0.200±0.009 |
| fractional flux [in $R_\odot$ band] | 0.307±0.002 | 0.245±0.009 | 0.246±0.009 | 0.202±0.009 |

| Physical Quantities | | | | |
|---------------------|---|---|---|---|
| $m$ [$M_\odot$]     | 1.195±0.012 | 1.146±0.013 | 1.146±0.013 | 1.105±0.010 |
| $R^4$ [$R_\odot$]   | 1.255±0.018 | 1.161±0.014 | 1.161±0.014 | 1.091±0.013 |
| $T_{\text{eff}}$ [K] | 6434±10 | 6303±10 | 6303±10 | 6188±36 |
| $L_{\text{bol}}$ [$L_\odot$] | 2.420±0.006 | 1.917±0.010 | 1.917±0.010 | 1.570±0.007 |
| $M_{\text{bol}}$ | 3.81±0.006 | 4.06±0.010 | 4.06±0.010 | 4.28±0.006 |
| $M_{\text{bol}}^2$ | 3.81±0.006 | 4.07±0.010 | 4.07±0.010 | 4.29±0.006 |
| log $g^*$ [dex]       | 4.316±0.010 | 4.365±0.009 | 4.367±0.010 | 4.405±0.011 |

| Global Quantities | | | | |
|------------------|---|---|---|---|
| log(age)$^b$ [dex] | ... | ... | 9.289±0.045 |
| $[M/H]^b$ [dex] | −0.039±0.020 | −0.039±0.020 | 0.012±0.000 |
| $E(B-V)$ [mag] | 2.54±0.000 | 2.54±0.000 | ... |
| distance [pc] | 357±3 |

Note.
<sup>a</sup> Instantaneous, osculating orbital elements at epoch $t_0 = 2458437.5$; (b) time of periastron passage; (c) mutual (relative) inclination; (d) longitude of pericenter ($\omega_{\text{d}B_\text{...}}$) and inclination ($i_{\text{f}}^\text{e}$) with respect to the dynamical (relative) reference frame (see text for details); (e) inclination ($i_{\text{m}}$) and node ($\Omega_{\text{m}}$) of the invariant plane to the sky; (f) see Section 6.3 for a detailed discussion of the tabulated apsidal motion parameters; (g) interpolated from the PARSEC isochrones;
as those due to the gravitational perturbation from the second binary,\(^{35}\) the osculating elements vary continuously, with a characteristic period that is half of the orbital period of the given binary. Furthermore, if the outer orbit happens to be eccentric (as in the present case), the perturbing forces—as well as the amplitudes of the variations of the osculating elements—naturally increase around the periastron passage of the outer orbit. As a result, the value of a given osculating element depends strongly on the inner and outer orbital phase of the sampling. This is demonstrated in Figure 12 by the green curve, which connects the anomalistic periods that were sampled during all the consecutive inner periastron passages of the given binary and, similarly, by the magenta curve, which shows the osculating anomalistic periods during each primary eclipse.\(^{36}\) However, none of the periods discussed thus far can be observed. The periods that can be observed are, e.g., the time between two consecutive primary (or secondary) eclipses, or in case of the RV observations, the repeating time of the RV curve. As both the eclipses and the RV curve are periodic according to the true longitude-like quantity \(u = v + \omega\), where \(v\) denotes the true anomaly and \(\omega\) stands for the argument of periastron in the observational frame of reference,\(^{37}\) the measurable quantity is the \(2\pi\) increment of \(u\), instead of \(v\), i.e., some kind of averaged sidereal (or eclipsing) period, instead of an averaged anomalistic period. In Figure 12 we therefore plot the time intervals between two consecutive primary (red) and secondary (blue) eclipses. One should be aware, however, that these observable eclipsing or sidereal periods are systematically longer than the one-sidereal-period average of the instantaneous osculating anomalistic periods (brown symbols in the figure). The reason is that in a triple (and quadruple) system, the clock of the inner binary is slowed because on average, the perturbing forces from the outer bodies act as if the central mass of the binary were reduced. Finally, the black curve in Figure 12 represents the average value of the eclipsing period, which is practically the average of the red and blue curves and can be determined most easily from the ETV curves as it is simply the linear term of the ETV ephemeris (as listed in the last row of Table 2 and in Figure 9 as well).

5.2. PHOEBE Analysis

For completeness, we also analyzed the TESS data via a classical light-curve modeling approach using the Wilson-Devinney method (Wilson & Devinney 1971) as implemented in the PHOEBE code (Prša & Zwitter 2005). Both binaries were analyzed independently, using several simplifications for the analysis. Both orbital periods were kept fixed, and the mass ratio in each binary was set to the corresponding value given in Table 8. Likewise, the temperature of the primary in both binaries was taken from Table 8. See Table 9 for the results of the PHOEBE fit. These agree well with those derived with the more sophisticated approach using the Lightkurvefactory code. Note, however, that the uncertainties associated with these PHOEBE results are statistical only and have often been found to be underestimated.

\(^{35}\) We note that tidal forces may (and in several case should) be considered as well.

\(^{36}\) We note that the analytical forms of these curves are described and discussed by Borkovits et al. (2015).

\(^{37}\) For the difference between the observational and dynamical argument of periastron see, e.g., Borkovits et al. (2015).
6. Discussion

6.1. Origin

This hierarchical system has several distinctive features pointing to its origin, namely (i) nearly equal masses of all four stars, (ii) a compact outer orbit, and (iii) a near-perfect coplanarity. It is well known that close binaries cannot form directly with their present-day small separations owing to the so-called opacity limit to fragmentation on the order of 10 au (e.g., Larson 1972). Accretion of gas onto a nascent binary shrinks its orbit and at the same time increases the mass ratio because the secondary star accretes more; this mechanism can explain the statistics of the close-binary population, including inner subsystems in hierarchies (Tokovinin & Moe 2020). Preferential growth of the secondary component produces a subset of binaries with almost equal masses (twins), which is more prominent at short periods. A compact quadruple such as TIC 454140642 can be explained by the accretion-induced migration of both inner and outer subsystems, assuming that an initially wide and low-mass seed quadruple gained most of its mass by accretion. The seed quadruple could be produced, for example, through an encounter of two protostars A and B, each surrounded by a gas envelope. An encounter leads to a burst of accretion onto each star that produced the inner subsystems by disk fragmentation. However, other formation channels are also feasible. The important factor is a substantial mass growth of this progenitor quadruple by accretion that eventually created four stars with similar masses and shortened both inner and outer orbits. Dissipative interaction of the orbits with gas also led to the coplanarity of the entire system.

This formation scenario of compact 2 + 2 quadruples makes a strong prediction of near-equal masses and orbit alignment. Indeed, other known compact quadruples share these properties with TIC 45414062. The closest such system, VW LMi (11029 +3025, HD 95660), has an outer period of 355 days and inner periods of 0.478 and 7.93 days (Pribulla et al. 2020). The masses of the stars in the closest eclipsing pair are not equal, but this could be a result of the later mass transfer; the other 7.9-day subsystem is a twin with $q = 0.98$, and the outer mass ratio is 0.93; all orbits are coplanar. Another compact and doubly eclipsing quadruple V994 Her (=HD 170314) has an outer period of 2.9 yr and inner periods of 2.1 and 1.4 days (Zasche & Uhlar 2016). The inner mass ratios are 0.86 and 0.95, and the outer mass ratio is 0.67. There is another, more distant star in this system. Finally, the doubly eclipsing quadruple OGLE BLG-ECL-145467, found by Zasche et al. (2019), has an outer period of 4.2 yr, with inner periods of 3.3 and 4.9 days, and the masses of all four stars are comparable.

6.2. Gaia EDR3 Astrometry

The Gaia EDR3 astrometric excess noise for TIC 454140642 is 0.144 mas at a significance of $\sigma = 24.476$; the number of...
The Astrophysical Journal, 917:93 (18pp), 2021 August 20

Kostov et al.

Table 9

Calculated Parameters from PHOEBE

| Subsystem | Derived Physical Quantities |
|-----------|----------------------------|
| pair A    |                            |
| $i$ [deg] | 87.74 ± 0.08               |
| $e$       | 0.076 ± 0.003              |
| $\omega$ [deg] | 158.1 ± 1.9             |
| $q$ [=m$_{sec}$/m$_{pri}$] | 0.959 (fixed)          |
| pair B    |                            |

| Subsystem | Derived Physical Quantities |
|-----------|----------------------------|
| pair A    |                            |
| $i$ [deg] | 87.40 ± 0.17               |
| $e$       | 0.027 ± 0.002              |
| $\omega$ [deg] | 187.9 ± 5.4             |
| $q$ [=m$_{sec}$/m$_{pri}$] | 0.964 (fixed)          |

Figure 13. The folded RV curve of binary A after the removal of the contribution of the outer orbit. Red and orange lines represent the fold of primary and secondary RV curves for the forthcoming 9.5 yr. The consequence of the rapid, dynamically forced apsidal motion is clearly manifested in the varying shape of the curves. Black points represent the measured RVs (see in Table 3), while the blue lines represent the folded model curves during the interval of the observations. As one can see, the half-year duration of the available observations is insufficient to detect the apsidal motion, but it will certainly be detectable within a few years.

Figure 14. To-scale plot of the orbital motions in the quadruple TIC 454140642. The observer is in the $x-\gamma$ plane and viewing the system from a distant position along the negative $\gamma$-axis (i.e., the red arrow is the view direction).

good observations is 257 (Gaia EDR3, Gaia Collaboration et al. 2016, 2021). The physical displacement corresponding to the excess noise is $\Delta a_{\text{measured}} = 0.052$ au (Equation (3), Belokurov et al. 2020). Taking into account the mass and luminosity ratios between binary A and B (0.94 and 0.8, respectively) as well as the eccentricity and argument of periastron of the AB system, the expected astrometric wobble is $\Delta a_{\text{expected}} = 0.053$ au (Equation (5), Belokurov et al. 2020)—practically identical to $\Delta a_{\text{measured}}$. In addition, using the methodology of Stassun & Torres (2021) and the measured renormalized unit weight error (RUWE) of 1.299, the photocenter semimajor axis is 0.27 mas, which at the measured parallax of 2.786 mas corresponds to $\approx 0.1$ au—comparable to $\Delta a_{\text{measured}}$. Altogether, this indicates that Gaia has seen the astrometric wobble of the AB system.

6.3. Long-term Dynamical Stability and Orbital Evolution

The orbital elements listed in Table 8 represent a snapshot of the system at the reference epoch. Due to the strong dynamical interactions between the two binary stars, these orbital elements, in accordance with the perturbation theory of hierarchical stellar systems (see, e.g., Harrington 1968) vary over time with at least three characteristic timescales: (i) the inner periods, (ii) the outer period, and (iii) the ratio of $P_{\text{out}}^2/P_{\text{in}}$ (see, e.g., Figures 9, 12). The variations on the latter two timescales can nicely be confirmed on the ETV curves (Figure 9), where in addition to the strictly $P_{\text{out}}$-period cyclic variations (not to be confused with the much smaller amplitude Rømer delay, shown also in Figure 9), the dynamically forced apsidal motion is also manifested.

To quantify the properties of the apsidal motions for all of the three orbits, we calculated theoretical apsidal motion rates, $\Delta \omega$ (i.e., the variation of the argument of periastron, $\omega$, during one revolution of the given binary). For this approximate calculation, we used hierarchical three-body models. So, for example, to calculate the dynamical apsidal motion rate of binary A, we replaced the other binary (B) by a single outer body having mass $m_B$ and vice versa to compute the dynamical apsidal motion of binary B. In addition to the dynamically
forced apsidal motion, we also calculated the general relativistic and tidal contributions. For the two inner orbits we strictly followed the formulae and method described in Borkovits et al. (2015), Appendix C. For the outer orbit we considered the tidal contribution to be negligible, and furthermore, in the case of the dynamical term, we simply calculated the algebraic sum of the contributions from the two separate “triple systems” (formed by stars Aa, Ab, and B and Ba, Bb, and A). We list these apsidal advance rates in the “apsidal motion parameters” part of Table 8.

As one can see, the dynamical apsidal advance rate ($\Delta \omega_3$; where “3b” stands for “third body”) is higher by 4–6 orders of magnitude.
Figure 17. Upper panel: Evolution of the impact parameters between each star in each binary with respect to the two stars of the other binary (AaBa for the impact parameter between Aa and Ba, AaBb for the impact parameter between Aa and Bb, etc.) over the course of 5 million days. Lower panel: Same as above, but zoomed in on a 300-day interval to highlight two conjunctions.

The Astrophysical Journal, 917:93 (18pp), 2021 August 20
Kostov et al.

magnitude than the relativistic and tidal rates ($\Delta\omega_{\text{GR}}, \Delta\omega_{\text{tide}}$, respectively) for all the three orbits. Therefore these latter contributions are certainly negligible. Assuming that these apsidal motion rates remain nearly constant (which is a plausible assumption for such a nearly coplanar system), we estimated approximate theoretical apsidal motion periods for the three orbits. These periods were found to be $U_{\text{theo}} = 87 \pm 1, 109 \pm 1, \text{and } 480 \pm 1 \text{ yr}$ for binary A, B, and AB, respectively.

In order to check the plausibility of our assumptions, we determined the “true” apsidal motion periods of the three orbits. For this we numerically integrated the best-fit four-body model for $\sim 3000 \text{ yr}$ and then determined the average apsidal motion periods for all three orbits. We obtained $U_{\text{num}} = 70, 97, \text{and } 472 \text{ yr}$, respectively. Therefore we can conclude that our simple, theoretical three-body model slightly overestimated the true periods (i.e., in other words, slightly underestimated the amplitude of the perturbations); however, it resulted in a correct estimation of their magnitude.

Finally, we note that the rapid apsidal motion of binary A, in addition to the readily visible divergence of the primary and secondary ETV curves, will also be detectable within a few years through the variation of the shape of the RV curve of binary A. This is illustrated visually in Figure 13. (Note that despite the similarly rapid apsidal advance of binary B, due to its almost circular orbit, the same effect will remain below the uncertainties of the currently available RV data).

We also investigated the short-term dynamical stability of the system. To confirm this, we integrated the four-body equations of motion for 5 million days (corresponding to $\approx 10,000$ outer periods), using the parameters from Table 8. We used the high-order IAS15 integrator with adaptive time steps (Rein & Spiegel 2015), part of the REBOUND N-body package (Rein & Liu 2012). We provide a REBOUND jupyter notebook demonstrating the initialization and orbital element computation at https://github.com/vbkostov/TIC_454140642.38

The orbital configuration of the system is shown in Figure 14. The evolution of some of the orbital elements over the course of 100,000 days is shown in Figure 15, highlighting the periodic extrema corresponding to the periastron passages of the outer orbit. As shown in Figure 16, there are no indications of chaotic motion, and the system remains stable for the duration of this integration. The semimajor axis of binary A ($a_A$) oscillates between 31.8 and 32 $R_\odot$, and the eccentricity ($e_A$) varies between 0.066 and 0.082; the semimajor axis of binary B ($a_B$) oscillates between 26 and 26.2 $R_\odot$, and the eccentricity ($e_B$) varies between 0.023 and 0.030. The semimajor axis of binary AB ($a_{AB}$) oscillates between 400.6 and 402.5 $R_\odot$, and the eccentricity ($e_{AB}$) varies between 0.322 and 0.328. The orbital inclinations vary between 87.48 and 87.85 (binary A), 87°52 and 87°81 (binary B), and 87°64 and 87°69 (binary AB).

Despite the near edge-on orientation of the system, there are no mutual eclipses or occultations between the constituents stars of binary A and binary B. This is highlighted in Figure 17, where we show the evolution of the respective impact parameters between the four stars. While there are periodic precession-induced variations in the corresponding minimum impact parameters, the latter never reach below $\approx 4$.

7. Summary

We have presented the discovery of the quadruple-lined, doubly eclipsing quadruple system TIC 454140642. The target was observed by TESS in Sector 5 (with a 30-minute cadence)

38 The orbital elements for binaries A and B represent the corresponding osculating two-body orbits, while the elements for A-B represent the orbit of the centers of mass of binaries A and B around one another.
and again in Sector 32 (with a 10-minute cadence) and exhibited four sets of eclipses associated with two binary stars (A and B). Additional spectroscopic and photometric measurements reveal divergent radial velocities, eclipse-time variations, and apsidal motion for the two binaries, confirming that they form a gravitationally bound quadruple system with a 2 + 2 hierarchy. The two binary stars have orbital periods of $P_A = 13.62$ days and $P_B = 10.39$ days, respectively, and slightly eccentric orbits ($e_A = 0.07$, $e_B = 0.03$). The four stars are very similar in terms of mass (individual masses between 1.105 and 1.195 $M_\odot$), size (radii between 1.09 and 1.26 $R_\odot$), and effective temperature (between 6188 and 6434 K). The quadruple has an orbital period of $P_{AB} = 432$ days and an eccentricity of $e_{AB} = 0.32$. The entire system is essentially coplanar—the orbits of the two binaries and the wide orbit are aligned to within a fraction of a degree. TIC 454140642 is the newest addition to the small family of coplanar, multiply eclipsing quadruple systems.

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This work makes use of observations from the LCOGT network.

Facilities: Gaia, MAST, TESS, WASP, ASAS-SN, NCCS, FRAM-Augur, FRAM-CTA, PEST, TRES, SOAR, LCOGT.

Software: Astrocuted (Brasseur et al. 2019), AstrolmageJ (Collins et al. 2017), Astropy (Astropy Collaboration et al. 2013, 2018), Eleanor (Feinstein et al. 2019), IPython (Pérez & Granger 2007), Keras (Chollet et al. 2015), Keras-vis (Kotikalapudi 2017) Lightcurvefactory (Borkovits et al. 2013; Rappaport et al. 2017; Borkovits et al. 2018), Lightkurve (Lightkurve Collaboration et al. 2018), Matplotlib (Hunter 2007), Mpi4py (Dalci et al. 2008), NumPy (Harris et al. 2020), Pandas (McKinney 2010), PHOEBE (Prsa et al. 2011), SciPy (Virtanen et al. 2020), Tapir (Jensen 2013), Tensorflow (Abadi et al. 2015), Tess-point (Burke et al. 2020).

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The Astrophysical Journal, 917:93 (18pp), 2021 August 20
Kostov et al.
