Blind image separation based on exponentiated transmuted Weibull distribution

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Abstract—In recent years the processing of blind image separation has been investigated. As a result, a number of feature extraction algorithms for direct application of such image structures have been developed. For example, separation of mixed fingerprints found in any crime scene, in which a mixture of two or more fingerprints may be obtained, for identification, we have to separate them. In this paper, we have proposed a new technique for separating a multiple mixed images based on exponentiated transmuted Weibull distribution. To adaptively estimate the parameters of such score functions, an efficient method based on maximum likelihood and genetic algorithm will be used. We also calculate the accuracy of this proposed distribution and compare the algorithmic performance using the efficient approach with other previous generalized distributions. We find from the numerical results that the proposed distribution has flexibility and an efficient result.

Keywords—Blind image separation, Exponentiated transmuted Weibull distribution, Maximum likelihood, Genetic algorithm, Source separation, FastICA.

I. INTRODUCTION

Recently the blind source separation (BSS) has more attention because it can be considered as an advanced image/signal processing technique and has many applications such as: speech sound, image, communication, and biomedicine [1–4]. BSS aims to recover source (images/signals) from a mixture with little known information. There are many BSS algorithms that have been discussed from various viewpoints, including principle component analysis (PCA) [9], maximum likelihood [7], mutual information minimization [6], tensors [8], non-Gaussianity [5], and neural networks [10–12]. Regarding to BSS, the separation and optimization methods play the most important roles. Separation step is used as the measurement of separability and optimization step is used to get the optimum solution for the objective function which we get from separation mechanism. Using generalized distributions usually gives good results of blind separation due to the variant properties of its sub-models. In the independent component analysis (ICA) framework, accurately estimates the statistical model of the sources is still an open and challenging problem [2]. Practical BSS scenarios employ difficult source distributions and even situations where many sources with variant probability density functions (pdf) mixed together. Towards this direction, many parametric density models have been made available in recent literature. For examples of such models, the generalized Gaussian density
(GGD) [13], the generalized gamma density (GGD) [14], and even combinations and generalizations such as super and generalized Gaussian mixture model (GMM) [15], the Pearson family of distributions [16], the generalized alfa-beta distribution (AB-divergences) [17] and even the so-called extended generalized lambda distribution (EGLD) [18] which is an extended parameterizations of the aforementioned generalized lambda distribution (GLD) and generalized beta distribution (GBD) models [19]. In this paper, we have presented the exponentiated transmuted Weibull distribution (ETWD) which is a generalization of the Weibull distribution. We have evaluated the accuracy of our proposed ETWD and compare the algorithmic performance using many different previous distributions. The numerical results, shows that the ETWD give a good results comparing with many different cases. The rest of this paper is organized as follows: In section 2, we present the BSS model. In section 3, we will discuss the ETWD. In section 4, we will use maximum likelihood to estimate the parameters of ETWD based on genetic algorithm. Finally, we will present the computational efficient performance of our proposed technique.

II. BLIND SOURCE SEPARATION (BSS) MODEL

Let \( S(t) = [s_1(t), s_2(t), \ldots, s_N(t)]^T \) denote independent source image vector that comes from \( N \) image sources. We can get observed mixtures

\[
X(t) = [x_1(t), x_2(t), \ldots, x_K(t)]^T (N = K) \text{ under the circumstances of instantaneous linear mixture.}
\]

\[
X(t) = AS(t), \tag{1}
\]

where \( A \) is a \( N \times N \) mixing matrix. The task of the BSS algorithm is to recover the sources from mixtures \( x(t) \) by using

\[
U(t) = WX(t), \tag{2}
\]

where \( W \) is a \( N \times N \) separation matrix and \( U(t) = [u_1(t), u_2(t), \ldots, u_N(t)]^T \) is the estimate of \( N \) sources.

Often sources are assumed to be zero-mean and unit-variance signals with at most one having a Gaussian distribution. To solve the problem of source estimation the un-mixing matrix \( W \) must be determined. In general, the majority of BSS approaches perform ICA, by essentially optimizing the negative log-likelihood (objective) function with respect to the un-mixing matrix \( W \) such that

\[
L(u, W) = \sum_{l=1}^{N} [\log p_{ul}(u_l) - \log|\det(W)|], \tag{3}
\]

where \( E[.] \) represents the expectation operator and \( p_{ul}(u_l) \) is the model for the marginal pdf of \( u_l \), for all \( l = 1, 2, \ldots, N \). In effect, when correctly hypothesizing upon the distribution of the sources, the maximum likelihood (ML) principle leads to estimating functions, which in fact are the score functions of the sources.
\[ \varphi_l(u_l) = - \frac{d}{du_l} \log p_{ul}(u_l) \quad (4) \]

In principle, the separation criterion in (3) can be optimized by any suitable ICA algorithm where contrasts are utilized (see; e.g., [2]). The FastICA [3], based on

\[ W_{k+1} = W_k + D(E[\varphi(u)u^T] - \text{diag}(E[\varphi_l(u_l)u_l]))W_k, \quad (5) \]

where, as defined in [4],

\[ D = \text{diag} \left( \frac{1}{E[\varphi_l(u_l)u_l] - E[\varphi_l'(u_l)]} \right), \quad (6) \]

where \( \varphi(t) = [\varphi_1(u_1), \varphi_2(u_2), ..., \varphi_n(u_n)]^T \), valid for all \( l = 1, 2, ..., n \).

In the following section, we propose ETWD for image modeling.

III. EXPONENTIATED TRANSMUTED WEIBULL DISTRIBUTION (ETWD)

Following [20] ETWD is a new generalization of the two parameters Weibull distribution. The pdf of ETWD is defined as:

\[ f(x) = \frac{\beta \alpha^{\beta-1}}{\alpha} e^{-\left(\frac{x}{\alpha}\right)^\beta} \left[ 1 - \lambda + 2\lambda e^{-\left(\frac{x}{\alpha}\right)^\beta} \right] \times \left[ 1 + (\lambda - 1)e^{-\left(\frac{x}{\alpha}\right)^\beta} - \lambda e^{-2\left(\frac{x}{\alpha}\right)^\beta} \right]^{-1} \quad (7) \]

cumulative distribution function of ETWD is given by:

\[ F(x) = \left\{ 1 + (\lambda - 1)e^{-\left(\frac{x}{\alpha}\right)^\beta} - \lambda e^{-2\left(\frac{x}{\alpha}\right)^\beta} \right\} x \geq 0, \quad (8) \]

where \( \alpha, \beta > 0 \), and \( |\lambda| \leq 1 \) are the scale, shape and transmuted parameters, respectively. It is clear that the ETWD is very flexible. This is so since there are many several other distributions that can be considered as special cases of ETW, by selecting the appropriate values of the parameters. These special cases include eleven distributions as shown in Table (I). In Figure (1-4) there are several distributions generated from ETWD by changing the parameters.

IV. ESTIMATION OF THE PARAMETERS

To estimate the parameters of ETWD, the maximum likelihood is used.

Let \( X_1, X_2, ..., X_n \) be a sample of size \( N \) from an ETWD.

Then the log-likelihood function (\( \mathcal{L} \)) is given by:

\[ \mathcal{L} = \log \ell = \log \left( \prod_{i=1}^{n} \frac{\beta \alpha^{\beta-1}}{\alpha} e^{-\left(\frac{x_i}{\alpha}\right)^\beta} \left[ 1 - \lambda + 2\lambda e^{-\left(\frac{x_i}{\alpha}\right)^\beta} \right] \times \left[ 1 + (\lambda - 1)e^{-\left(\frac{x_i}{\alpha}\right)^\beta} - \lambda e^{-2\left(\frac{x_i}{\alpha}\right)^\beta} \right]^{-1} \right) \quad (9) \]
Therefore, maximum likelihood estimation of $\alpha$, $\beta$, $\lambda$, and $\nu$ are derived from the derivatives of $\mathcal{L}$. They should satisfy the following equations:

$$\frac{\partial \mathcal{L}}{\partial \alpha} = 0, \quad \frac{\partial \mathcal{L}}{\partial \lambda} = 0, \quad \frac{\partial \mathcal{L}}{\partial \beta} = 0, \quad \frac{\partial \mathcal{L}}{\partial \nu} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \alpha} = -\frac{n\beta}{\alpha} + \sum_{i=1}^{n} \left( \frac{X_i}{\alpha} \right)^{\beta-1} + \sum_{i=1}^{n} \frac{2\lambda e^{-\left(\frac{X_i}{\alpha}\right)^{\beta}}}{2\lambda e^{-\left(\frac{X_i}{\alpha}\right)^{\beta}}} \left( \frac{X_i}{\alpha^2} \right) \log\left( \frac{X_i}{\alpha} \right) + (\nu - 1) \sum_{i=1}^{n} \frac{(\lambda - 1)e^{-\left(\frac{X_i}{\alpha}\right)^{\beta}}}{(\lambda - 1)e^{-\left(\frac{X_i}{\alpha}\right)^{\beta}}} \log\left( \frac{X_i}{\alpha} \right) - \lambda + 1$$

$$\frac{\partial \mathcal{L}}{\partial \beta} = \sum_{i=1}^{n} \log(X_i) - n \log \alpha - \sum_{i=1}^{n} \left( \frac{X_i}{\alpha} \right)^{\beta} \log \left( \frac{X_i}{\alpha} \right) + \sum_{i=1}^{n} \left( \frac{X_i}{\alpha} \right)^{\beta} \log \left( \frac{X_i}{\alpha} \right) + (\nu - 1) \sum_{i=1}^{n} \frac{(\lambda - 1)e^{-\left(\frac{X_i}{\alpha}\right)^{\beta}}}{(\lambda - 1)e^{-\left(\frac{X_i}{\alpha}\right)^{\beta}}} \log\left( \frac{X_i}{\alpha} \right) - \lambda + 1$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \sum_{i=1}^{n} \frac{2e^{-\left(\frac{X_i}{\alpha}\right)^{\beta}}}{2\lambda e^{-\left(\frac{X_i}{\alpha}\right)^{\beta}}} - 1 + (\nu - 1) \sum_{i=1}^{n} \frac{e^{-\left(\frac{X_i}{\alpha}\right)^{\beta}} - e^{-\left(\frac{X_i}{\alpha}\right)^{\beta}}}{(\lambda - 1)e^{-\left(\frac{X_i}{\alpha}\right)^{\beta}}} - \lambda + 1$$

$$\frac{\partial \mathcal{L}}{\partial \nu} = \sum_{i=1}^{n} \log \left( (\lambda - 1)e^{-\left(\frac{X_i}{\alpha}\right)^{\beta}} - \lambda e^{-\left(\frac{X_i}{\alpha}\right)^{\beta}} + 1 \right) + \frac{n}{\nu}$$

To estimate the value of parameters, the system of equations (10-13) must be solved. However, it is difficult to solve this system so, the genetic algorithm (GA) [21-22] will be used as an alternative numerical method to estimate the parameters. The appeal of the GA optimization technique lies in the fact that it can minimize the negative of the log-likelihood objective function in (3), essentially without depending on any derivative information.
Table I
The ETWD sub-models, shows the specific values of the parameters used to generate the above mentioned eleven special cases, Where $\alpha > 0, \beta > 0, \nu > 0, |\lambda| \leq 1$

| $\beta$ | $\nu$ | $\lambda$ | Distribution       |
|--------|-------|-----------|--------------------|
| 2      |       |           | Exponentiated transmuted Rayleigh (ETR) |
| 1      | 1     | 0         | Transmuted exponential (TE) |
| 1      | 0     | 0         | Exponentiated exponential (EE) |
| 2      | 0     | 0         | Exponentiated Weibull (EW) |
| 1      | 0     | 0         | Exponentiated Rayleigh (ER) |
| 0      | 1     | 0         | Weibull (W) |
| 2      | 0     | 0         | Rayleigh (R) |
| 0      | 0     | 1         | Exponential (E) |

Figure 1. The ETWD with fixed $\alpha=3$. 
Figure 2. The ETWD with fixed $\beta=2$.

Figure 3. The ETWD with fixed $\lambda=0.5$.

Figure 4. The ETWD with fixed $\nu=2$. 
V. NUMERICAL RESULTS

Numerical experiments have shown that the GA method can converge to an acceptably accurate solution with substantially fewer function evaluations. We have generated random number from ETWD with parameters $\alpha$, $\beta$, $\nu$ and $\lambda$. By performing GA, we obtain best estimation of parameters as in table (II).

Applications of ETWD for BSS

We resolve to FastICA algorithm for blind signal separation (BSS). This algorithm depends on the estimated parameters and an un-mixing matrix $W$ which estimated by FastICA algorithm. By substituting (7) into (4) for the source estimates $u_l$, $l = 1, 2, \ldots, n$, it quickly becomes clear that the proposed score function inherits a generalized parametric structure, which can be attributed to the highly flexible ETWD parent model. So, a simple calculus yields the flexible BSS score function

$$
\phi_l(u_l) = -\frac{\partial}{\partial u_l} \log \frac{\nu \beta}{\alpha} \left( \frac{x_i}{\alpha} \right)^{\beta-1} e^{-\frac{x_i}{(\alpha^\beta)}} \left[ 1 - \lambda + 2\lambda e^{-\frac{x_i}{(\alpha^\beta)}} \right] \times \left[ 1 + (\lambda - 1)e^{-\frac{x_i}{(\alpha^\beta)}} - \lambda e^{-2\frac{x_i}{(\alpha^\beta)}} \right]^{\nu-1}
$$

In principle $\phi_l(u_l|\theta)$ is capable of modeling a large number of signals as well as various other types of challenging heavy- and light-tailed distributions. Experiments were done to investigate the performance of our method through three applications (two in source separation and one in image denoising) when impulsive noise is presented. In all experiments, the performance of our method is compared with generalized gamma [14], tanh, skew, pow3 [23], and Gauss [15]. Our performance is measured by the peak-signal-to-noise ratio (PSNR), defined as:

$$
PSNR = 20 \log_{10} \left( \frac{255}{MSE} \right)
$$

| Parameter estimation by using GA |
|---|---|---|---|---|---|---|
| $\lambda$ | $\nu$ | $\alpha$ | $\beta$ | $\lambda$ | $\nu$ | $\alpha$ |
| X1 | 0.5 | 2 | 3 | 4 | 0.59 | 1.86 | 2.97 | 4.11 | 0.02 |
| X2 | 1 | 2.5 | 5.2 | 6.8 | 1.16 | 2.42 | 5.27 | 6.80 | 0.06 |
| X3 | 3 | 5.7 | 1.9 | 8.2 | 2.98 | 5.63 | 1.98 | 8.12 | 0.006 |

Example 1

We have run the algorithm using natural images taken from [24]. We selected 4 noise-free natural images with 512x512 pixels. Further, to reduce the dimension of input image data, the data set X is centered and whitened by principal component analysis (PCA) method. Then, using the updating rules of $W$ defined in (5), the objective function given in (14) is minimized. Where Figure (5-6) show the original, mixed and separated images by Gauss,
pow3, skew, tanh, generalized gamma, and ETWD algorithms. Also, Table (III) illustrates the performance of these algorithms. From this table and Figure (5-6), the ETWD is higher performance than other algorithms.

| Distribution | First Image | Second Image | Third Image | Forth Image | Elapsed time (in seconds) |
|--------------|-------------|--------------|-------------|-------------|--------------------------|
|              | MSE | PSNR | MSE | PSNR | MSE | PSNR | MSE | PSNR | MSE | PSNR | MSE | PSNR |
| Gauss        | 0.1176 | 57.4255 | 0.2972 | 53.4009 | 0.1773 | 55.6426 | 0.1314 | 56.9444 | 8.757703 |
| Pow3         | 0.1375 | 56.7477 | 0.2130 | 54.8477 | 0.1736 | 55.7363 | 0.1259 | 57.1320 | 24.921161 |
| Skew         | 0.0044 | 71.7366 | 0.0177 | 65.6481 | 0.2340 | 54.4378 | 0.2193 | 54.7209 | 5.788523 |
| Tanh         | 0.1179 | 57.4172 | 0.1647 | 55.9628 | 0.1810 | 55.5538 | 0.0741 | 59.4309 | 6.852007 |
| Generalized Gamma | 0.1341 | 56.8571 | 0.2659 | 53.8840 | 0.1865 | 55.4237 | 0.1305 | 56.9746 | 4.333974 |
| ETWD         | 0.0011 | 77.6298 | 0.0159 | 66.1132 | 0.0026 | 73.9429 | 0.0015 | 76.2714 | 4.285013 |

**Example 2**

In this example, we illustrate the performance of our algorithm to denoise medical images taken from [25]. Where Figure (7-12) show the original images, noised images, and denoised images by different algorithms. After applying algorithms of Gauss, pow3, skew, tanh, generalized gamma and, our algorithm ETWD, the results are illustrated in Figure (7-12), also Table (IV) illustrates the performance of these algorithms. From table (IV) and Figure (7-12), the ETWD is higher performance than other algorithms.

| Distribution / PSNR | First Image (Medical) | Second Image (Medical) | Elapsed time (in seconds) |
|---------------------|-----------------------|------------------------|--------------------------|
|                     | MSE | PSNR | MSE | PSNR | MSE | PSNR | MSE | PSNR | MSE | PSNR |
| Gauss               | 0.0092 | 68.4753 | 0.0077 | 69.2751 | 1.724821 |
| Pow3                | 0.0077 | 69.2780 | 0.0093 | 68.4489 | 1.646659 |
| Skew                | 0.0077 | 69.2797 | 0.0093 | 68.4383 | 1.611382 |
| Tanh                | 0.0076 | 69.2967 | 0.0093 | 68.4483 | 1.729392 |
| Generalized gamma   | 0.0058 | 70.5134 | 0.0061 | 70.2859 | 1.578206 |
| ETWD                | 0.0050 | 71.1162 | 0.0039 | 72.1719 | 1.646362 |
Figure 5. A original images, B mixed images, C Gauss separated images, and D pow3 separated images.
Figure 6. E skew separated images, F tanh separated images, G generalized gamma separated images, and H ETWD separated images.
Figure 7. Medical image denoising using Gauss filter: A, D are the source images, B, E are the noised images, C, F are the denoised images.

Figure 8. Medical image denoising using pow3 filter: A, D are the source images, B, E are the noised images, C, F are the denoised images.
Figure 9. Medical image denoising using Skew filter: A, D are the source images, B, E are the noised images, C, F are the denoised images.

Figure 10. Medical image denoising using tanh filter: A, D are the source images, B, E are the noised images, C, F are the denoised images.
Figure 11. Medical image denoising using generalized gamma filter: A, D are the source images, B, E are the noised images, C, F are the denoised images.

Figure 12. Medical image denoising using ETWD filter: A, D are the source images, B, E are the noised images, C, F are the denoised images.
VI. Conclusion

In this paper, we introduced a new technique for blind image separation and image denoising based on exponentiated transmuted Weibull distribution. Our proposed technique outperforms existing solutions in terms of separation quality and computational cost. When the GA is used to estimate the parameters of ETWD and it gives small error. Also the results of ETWD are better than other algorithms.

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