Abstract

The inherent structure of the power grid deeply influences its secure functioning and stability. As power grids move towards becoming ‘smarter’ with increased demand response and decentralized control, the topological aspects of the grid have become even more important. Understanding the topology can thus lead to better strategies to control the smart grid as well as enable rapid identification and prevention of risks such as blackouts. This paper focuses on modeling and analyzing basic similarities in network structure of large power grids across America and Europe. A simple but accurate model of the power grid can help generate real-sized test cases to evaluate new grid technologies as well as aid in tractable theoretical analysis of the grid structure.

I. INTRODUCTION

The power grid is a complex interactive system that affects all the other sectors of the economy. Making the grid ‘smart’ has added further complexity to the system by creating distributed decision making in real time through demand response, real-time pricing and flexible demand. This leads to increased control in the hands of end consumers, reduced pricing and more careful supervision of the grid against failures like cascading blackouts and intentional attacks. To understand the usefulness of new technologies and their impact on the smart grid, they need to be implemented on test cases of the scale of large power grids together with the IEEE standard test bus systems. These standard IEEE bus systems and their data can be found at [15]. The test cases used to model the large power grids should follow the general trends of the topology present in massive networks around the world.

Modeling and dynamics of the power grid are growing fields of research. A good understanding of the underlying degree distribution (in addition to other physical network characteristics) of
the grid is of critical value in understanding and controlling it. There has been significant work in the past in connecting the performance of the power grid with its topology represented by the adjacency matrix of the grid or the degree distribution. Albert et al. [11] and others [2] have measured the robustness of the grid against random or directed attacks and established the importance of physical network of the power grid in determining its vulnerability. Similarly, research in other networks, including social networks [5], the Internet [4] and population models for spread of diseases [10] and effective vaccination [11].

In this context, there is a body of work on the degree distribution of the underlying physical network associated with large power grids. Though there has been some work reporting power-law/scale free distributions, a considerable body of work reports an exponential degree distribution or, in the least, an exponential tail. [13] reports the presence of exponential degree distribution while considering power plants, substations, and 115 - 765 kV power lines of the North American power grid. The exponential degree distribution is also observed for the western power grid as reported in [14]. Hines et al. in [16] also suggest an exponential degree distribution in The Eastern, Western and Texas Interconnects and reject the power-law degree distribution for these grids as well as for the 300 bus IEEE test case. A very clear exponential tail distribution has also been reported by [6] for the NYISO. Similarly, [8], [7], [9] discuss an exponential degree distribution associated with power grids in Europe - namely in Italy, UK, Ireland, Portugal as well as for the entire connected component of Europe obtained from the Union for the Co-ordination of Transmission Electricity, UCTE. Although power grids in different countries possess different average degrees per node due to differing geographical terrain and shape, an exponential degree distribution seems widespread across regions and large grids. There is existing work on developing generative models for the exponential degree distribution in the power grid. [12] introduces a well-known small-world model. The small-world model has nodes which get connected to neighbors in a ring lattice and links are rewired between nodes with some probability $p$. [6] develops a recent model called RT-nested-Smallworld based on the Watts and Strogatz small-world model. Here nodes in a ring lattice are connected to neighbors with some probability and rewiring happens by a Markov chain. Though the model in [6] compares well against the real world NYISO power grid, in some part due to several parameters which can be tuned to obtain a good fit, this model yields limited generative intuition for the physical network. In particular, such a model presumes a fixed (static) network configuration. However,
the power grid can be dynamic (particularly in the developing world), where new nodes are added over time. A generative model that encapsulates the temporal aspect of network growth is thus desirable. Growth of networks demonstrating a power-law degree distribution like the internet and airline network can be explained on the basis of the Barabasi-Albert model [5]. In a network with exponential degree distribution, however, the Barabasi-Albert model’s principle of ‘the rich gets richer’ does not apply. A modified generative model based on the Barabasi-Albert model for power-law distributed networks can be found in [19]. In this model, link formation has two components. New nodes enter the network and either form links to any node in the network randomly or form links to existing nodes with a preference given to nodes with higher degree. Here too the spatial distribution of nodes is not taken into account and the rationale for joining nodes with higher degree cannot be seen in a power network. Thus a generative model motivated by physical characteristics that sheds light on the structure of power grids in general, and their exponential distribution in particular, is yet absent. Our paper is dedicated to bridging this gap by providing a meaningful yet simple generative framework for understanding the network associated with power grids. Our approach differs from others in that we study the temporal evolution of the network in our generative model rather than restricting ourselves to explaining the final degree distribution alone.

It needs to be mentioned, though, that such modeling techniques do not lead to the exact networks with all associated characteristics seen in the real world. However, our simplified model does provide an accurate and intuitive approximation behind the common structures in different power grids with the added benefit of mathematical tractability of graph processes and measures. More information on the relative advantages and approximations associated with network modeling techniques can be found at [3].

The rest of this paper is organized as follows. The next section presents a description of the generative model for the exponential degree distribution for power grids. The degree distribution is analyzed and verified through numerical simulations in Section [III]. We provide empirical degree distributions of power grids in America and Europe and fit them with degree distributions produced by our generative model in Section [IV]. The diameter calculations for our generative model for different sizes of the power-grid are discussed in Section [V]. It is followed by betweenness calculations in Section [VI]. The performance of our model in propagation of infection and comparison with results of infection propagation in real power grid networks is presented...
II. Generative Model for Exponential Degree Distribution

A brief note on notation: we use $P(.)$ to denote the probability of an event, and $E(.)$ to denote expectation. $\delta(.)$ represents the dirac delta function, $\overline{\alpha} = 1 - \alpha$ and $I(.)$ the indicator function.

As mentioned in the introduction, power grids are found to possess an exponential degree distribution. Our goal is to explain it using a temporally-evolving generative model based on the framework of 2-D spatial Poisson point process theory. In this model, nodes are randomly placed in space according to a Poisson point process $P_\lambda$ with density $\lambda$. To capture the network’s temporal evolution, edges/links between nodes are formed when new nodes are introduced into the system. The new node thus connects to $K$ of its nearest pre-existing nodes in the network where $K$ is a random variable. This model is inspired by evolution of power grids over time where transmission lines/links are formed to connect newly established buses/nodes to the existing buses/links in the grid. The evolutionary aspect of the model also increases the possibility of creation of longer edges in the initial stages of the network compared to other models like the random geometric model.

We explain $P(K > 1) > 0$ as a technique of introducing robustness through multiple connections within the physical network. The selection of ‘nearest neighbors’ instead of arbitrary nodes is motivated by the objective of minimizing the cost of establishing these linkages. Thus, in our generative model, each new node forms $K$-redundant minimum-cost connections with the existing network. Next, we show that such an organic model of network growth is indeed reasonable, as it leads to an appropriate exponential distribution for the resulting network.

III. Analysis of the Degree Distribution of the Generative Model

Our generative model assumes creation/birth of a new node at each discrete time step and formation of $K$ new links from the new node to its $K$ closest neighbors. We represent the geographical area covered by the grid with a disk $R$ of known radius $r$ and generate nodes in it according to a Poisson point process of density $\lambda$. The nodes are numbered in an increasing order with the first-born node numbered as 1. If $K$ is taken to be a constant, then we form a complete network with the first $K + 1$ nodes. A new node born after the first $K + 1$ nodes connects to existing node $a$ present in $R$ if it lies within the region of influence of the node $a$. In Section VII, finally, concluding remarks are summarized in Section VIII.
Here ‘region of influence’ refers to the area $R_{at}$ within $R$ such that for every point $p$ within $R_{at}$, node $a$ is amongst the $K$ nearest neighbors of $p$ at time $t$. The evolution of the network and its degree distribution thus follows from the evolution of a $K^{th}$ order Voronoi-region defined by the Poisson point process of density $\lambda$ over $R$. If $K$ is a constant at 1, every new node forms a link to the single nearest neighbor and the network structure evolves according to the basic Voronoi-region evolution of a 2-D Poisson point process as shown in Figure 1. The dots in Figure 1 represent the nodes which already exist and probability of formation of an edge to a particular node is directly proportional to the area in its Voronoi-region.

Analysis of the evolution of higher-order Voronoi-regions is non-trivial and does not lead to closed-form results. To overcome that, we use a mean-field framework for analyzing the degree distribution of the network given by our generative model. The main principle behind a mean-field formulation is to replace the dynamics of a single point with the average dynamics of the entire system at each time step. A good reference for mean-field techniques can be found in [23].

We study the evolution of the network in discrete time steps. Let $N(m, t)$ represent the average
number of nodes of degree \( m \) at time \( t \) with \( K = k \) (constant) being the number of new links formed at each step. We discuss the case with variable \( K \) later. At time \( t + 1 \), the average number of nodes of each degree \( m \) increases if the new node coming in at time \( t + 1 \) forms links with nodes of degree \( m - 1 \); and decreases if links are formed with existing nodes of degree \( m \). For a single node, the probability of an increase in its degree at time \( t + 1 \) is proportional to its region of influence times \( k \). Note that at every time \( t \), the expected values of the volumes of the \( k^{th} \) order Voronoi-regions of the existing nodes depend only on their positions (which are random) and not on the order in which the nodes entered the network. Thus, at each time step, all present nodes have the same expected volume of ‘region of influence’ and the relative area covered by regions of influence of nodes with a particular degree \( m \) is given by the fraction of nodes with degree \( m \). This implies that the probability of the average number of nodes of a particular degree \( m \) increasing at a time step is proportional to the fraction of nodes of degree \( m - 1 \) and \( m \) at the previous time step. Due to creation of \( k \) edges, the new node gets a degree of \( k \). Mathematically,

\[
N(m, t + 1) - N(m, t) = \frac{N(m - 1, t)k}{t} - \frac{N(m, t)k}{t} + \delta(m = k) \tag{1}
\]

For \( m = k \), we have

\[
N(k, t + 1) - N(k, t) = -\frac{N(k, t)k}{t} + 1.
\]

For large \( t \), using Lemma 4.1.1. in [24], this becomes

\[
\frac{N(k, t)}{t} = \frac{1}{1 + k} \tag{2}
\]

For \( m \neq k \),

\[
N(m, t + 1) = N(m, t)(1 - \frac{k}{t}) + \frac{N(m - 1, t)k}{t} \tag{3}
\]

For large \( t \), again using Lemma 4.1.2 in [24], we obtain

\[
\frac{N(m, t)}{t} = \lim_{t \to \infty} \frac{N(m-1, t)k}{t} + k \tag{4}
\]

\[
\Rightarrow \frac{N(m, t)}{t} = \left( \frac{k}{1 + k} \right)^{(m-k)} \frac{1}{1 + k}. \tag{5}
\]
Therefore, the average fraction of nodes $n(d)$ with degree less than $d$ can be obtained from

$$n(d) = \frac{1}{1+k} + \frac{k}{1+k(1+k)} + \ldots + \frac{1}{1+k} \left(\frac{k}{1+k}\right)^{(d-k-1)}$$

$$\Rightarrow n(d) = 1 - \left(\frac{1+k}{k}\right)^{(k-d)}$$
$$\Rightarrow n(d) = 1 - e^{-\frac{d-k}{\mu_K}}$$

(6)

Next, we analyze the random variable $K$ in greater depth. Let $K$ have a support of 2, $k_1$ and $k_2$ with probability $\alpha$ and $\bar{\alpha}$ respectively. Here, $K$ has a mean of $\mu_K = \alpha k_1 + \bar{\alpha} k_2$ where $\bar{\alpha} = 1 - \alpha$. Note that $\mu_K$ represents the average number of edges formed at each time step. The change in $N(m, t+1)$ at time $t+1$ is governed by the following equation:

$$N(m, t+1) - N(m, t) = \mu_K \frac{N(m-1, t)}{t} - \mu_K \frac{N(m, t)}{t} + \alpha \delta(m = k_1) + \bar{\alpha} \delta(m = k_2)$$
(7)

For $m = k_1$, we have

$$N(k_1, t+1) - N(k_1, t) = \frac{-N(k_1, t) \mu_K}{t} + \alpha.$$

For large $t$ and using (2), this becomes

$$\frac{N(k_1, t)}{t} = \frac{\alpha}{1 + \mu_K}$$
(8)

For $k_1 < m < k_2$, at large $t$, using (4), we get

$$\frac{N(m, t)}{t} = \lim_{t \to \infty} \frac{N(m-1, t)}{t} \mu_K \frac{1}{1 + \mu_K} = \left(\frac{\mu_K}{1 + \mu_K}\right)^{(m-k_1)} \frac{\alpha}{1 + \mu_K}$$
(9)

Similarly, for $m = k_2$, at large $t$,

$$\frac{N(k_2, t)}{t} = \left(\frac{\mu_K}{1 + \mu_K}\right)^{(k_2-k_1)} \frac{\alpha}{1 + \mu_K} + \frac{\bar{\alpha}}{1 + \mu_K}$$
(10)

Finally, for $m > k_2$, for large $t$, using the same method we get:

$$\frac{N(m, t)}{t} = \left(\frac{\mu_K}{1 + \mu_K}\right)^{(m-k_1)} \frac{\alpha}{1 + \mu_K} + \frac{\bar{\alpha}}{1 + \mu_K}$$
(11)

Using equations (8), (9), (10) and (11), the fraction of nodes $n(d)$ with degree less than $d$ is given by

$$n(d) = \alpha (1 - e^{-\frac{d-k_1}{\mu_K}}) I(d > k_1) + \bar{\alpha} (1 - e^{-\frac{d-k_2}{\mu_K}}) I(d > k_2)$$
(12)
Continuing in the same way, if $K$ takes values $k_i$ with probability $\alpha_i$ respectively for some range of $i$, the p.d.f. of the degree distribution for some degree $d$ is given by:

$$pdf(d) = \sum_i \frac{\alpha_i}{\mu_K} e^{-\frac{d-k_i}{\mu_K}} \mathbb{1}(d \geq k_i)$$

(13)

The network created by our generative model, thus, has a degree distribution given by a combination of exponential distributions. Figure 2 shows the degree distribution for two networks given by our generative model with variable $K$ taking 3 possible values (3, 4 and 5). We see that the degree distribution in both cases is well approximated by a combination of exponential distributions as given by equation (13). We show later that this is in excellent agreement with the degree distribution observed in several real power grids.

![Fig. 2. Fitting sum of exponential p.d.f.s from equation (13) to the degree distribution for variable $K$ in radius 20](image)

The validity of using a mean-field model to represent the network growth can be established using the Azuma-Hoeffding’s inequality [24]. Let $Z(m, t)$ be the random variable representing the number of nodes of a given degree $m$ at time $t$. $Z(m, t)$ depends on the locations of the
incoming nodes which are given by \( y_1, y_2 \) etc. We obtain a martingale \( Z_i(m, t) \) as follows:

\[
Z_i(m, t) = E(Z(m, t) | y_1, y_2, ..., y_i)
\]

where \( N(m, t) = Z_0(m, t) = E(Z(m, t)) \) and \( Z(m, t) = Z_t(m, t) \). We have \(|Z_i(m, t) - Z_{i-1}(m, t)| \leq 2k\) as the number of nodes of a particular degree cannot change by more than \( 2k \) at each time step where \( k \) is the maximum value that \( K \) can take. Using the Azuma-Hoeffding inequality, we have

\[
P(Z(m, t) - N(m, t) \geq x) \leq e^{-\frac{x^2}{8k^2 t}}
\]

\[
\Rightarrow P(Z(m, t)/t - N(m, t)/t \geq x) \to 0 \text{ as } t \to \infty
\]

**IV. Fitting Degree Distribution of Power Grids**

We look at three different power grid networks in this section. We show that the degree distribution of each considered network is well approximated by a sum of exponentials. This is exactly the degree distribution of simulated networks given by our generative model as shown in the previous section.

First we look at the Western United States electrical power grid, which has 4941 nodes and 6594 edges. The network data for this grid is freely available [12]. We choose \( K \) to be a random variable, as given by equation (13) to match this grid’s degree distribution, obtaining a good fit as shown in Figure 3.

We also choose \( K \) to be a random variable when fitting the power grid data in Texas which is controlled by the Electric Reliability Council of Texas (ERCOT). The degree distribution has a clear exponential tail with a “kink” in the beginning. We find that our approach works well, as seen in Figure 4.

Finally, we look at the power grid of Union for Coordination of Transmission of Electricity (UCTE). [20] includes a good and well-cited approximate model of the grid using publicly available data having 1254 buses and 378 generators. The data can be found online at [21]. We use the admittance matrix to get the adjacency matrix of the network and fit it using a combination of exponential p.d.f.s for the degree distribution and observe a good fit in Figure 5.
Our generative model, thus provides degree distributions comparable to those seen in power grids. The model also successfully fits the “kinks” in the beginning of the degree distributions seen in all the considered power grids here.

V. DIAMETER OF THE GENERATIVE MODEL

In this section, we analyze the diameters of power grid networks and compare it with diameters given by our generative model networks. The diameter of a graph is the length of the longest route between any two nodes in the graph. The presence or absence of a short diameter in a power grid indicates the existence of hubs (nodes/buses with high degree) and influences the probability of fragmentation of the network following a directed or random attack by an adversary [13], [27]. In general, due to different topological constraints, power grids vary in their connectivity parameters. Thus, the scaling of the diameter of the network graph with the number of nodes is hard to quantify. Using the data given in [16], [6], we compare power grids with similar average nodal degree and observe that the average diameter of the network scales as the logarithm of the network size (number of nodes). Figure 6 illustrates this finding. We, now, simulate the diameter of networks given by our generative model for different values of $N$ (number of nodes) and $K$.
(number of connections per incoming node). The results given in Figure 7 show a logarithmic scaling between the diameter and the network size, which is similar to observations in power grids.

In the next section we compare another important graph feature, betweenness centrality, between real power grids and networks given by our generative model.

VI. BETWEENNESS CENTRALITY OF THE GENERATIVE MODEL

Betweenness centrality for a node $i$ (denoted by $l_i$) or an edge $e_{im}$ (denoted by $l_{im}$) in the network measures the number of shortest paths in the network which pass through that node or edge respectively [8]. For an undirected graph with $N$ nodes, we have

\[
l_i = \sum_{j,k \neq i} \frac{n_{jk}(i)}{n_{jk}}
\]

\[
l_{im} = \sum_{j,k \neq i,m} \frac{n_{jk}(im)}{n_{jk}}
\]
Here $n_{jk}$ is the total number of shortest paths between nodes $j$ and $k$ while $n_{jk}(i)$ and $n_{jk}(im)$ are the number of shortest paths between $j$ and $k$ which include node $i$ and edge $e_{im}$ respectively.

A node/edge with higher value of betweenness can be considered more important or relevant to the robustness of the network. In particular, attacking nodes with higher betweenness rather than random nodes or nodes with higher degree can make blackouts worse in a power grid [8], [25]. [18] mentions the positive correlation of betweenness of an edge and the product of the degree of the nodes connected by the edge in the network. This is considered a measure of the load carried by the edge and used in the analysis of cascades in the power grid [17].

We plot the p.d.f.s of node and edge betweenness centralities ($L$) for networks given by our generative model for variable $K$ in Figures 8 and 9 respectively. We observe that our generative model induces centrality characteristics comparable to that of real networks demonstrated in literature [27].

To compare the results, we also calculate the betweenness centrality of the power grid networks available with us. We begin with the UCTE power grid and calculate its node betweenness. Figure
average diameter for power grid graphs with similar average degree per node (between 2.67 and 2.73)

Fig. 6. Diameter of the network v/s size of the power grid

Fig. 7. Diameter of the network v/s size of the network $N$
Node betweenness for
\( k = 3 (p = .5) \),
\( k = 4 (p = .3), k = 5 (p = .2) \)

\( \lambda \) of Poisson distribution = .8
\( k \) = number of links per new node
\( p \) = probability of selecting \( k \)

Fig. 8. p.d.f. of node betweenness centrality for variable \( K \)

Edge betweenness
for \( k = 3 (p = .5) \),
\( k = 4 (p = .3), k = 5 (p = .2) \)

\( \lambda \) of Poisson distribution = .8
\( k \) = number of links per new node
\( p \) = probability of selecting \( k \)

Fig. 9. p.d.f. of edge betweenness centrality for variable \( K \)
Fig. 10. p.d.f. of node betweenness centrality for UCTE

[1] shows the result, which matches results obtained from our generative model. The ERCOT power grid network is then used to determine edge betweenness centrality in Figure [10]. This plot again compares well with the edge betweenness plot for our generative model.

We now focus on the performance of our generative model in modeling dynamics of network processes in power grids. In the following section, we look specifically at one such process: infection propagation.

VII. Vulnerability of the Generative Model

Random and intentional attacks by disruptive agents pose a serious threat to the secure operation of the grid and delivery of electricity. In extreme cases, even a small disturbance inserted in the system from outside can propagate to the rest of the network and cause a cascade and subsequent breakdown in the network. We consider two models of failure propagation in the power grid and analyze its vulnerability to them. The two models considered are the SIS (Susceptible-Infected-Susceptible) model and the SIR (Susceptible-Infected-Removed) model.

In the SIS model [28], each node exists either in the susceptible (S) state or infected (I) state. In
Every time step, an infected node returns to the susceptible state with probability $\delta$ independently of other nodes. A susceptible node, on the other hand, can get the infection from its infected neighbors through shared edges. The infection is spread from every infected neighbor to the susceptible node independently with probability $\beta$. Here, $\delta$ and $\beta$ are parameters of the infection.

The SIR model [28] has nodes in one of three possible states: susceptible (S), infected (I) and removed (R). A node in the susceptible state can get infected from any of its infected neighbors. Like the SIS model, each infected neighbor can spread the infection to susceptible node through the shared edge with probability $\beta$. Each infected node itself gets resistance and enters the removed state with probability $\gamma$. Eventually, in a SIR model the infection dies out with nodes being either in the removed state or the susceptible state.

A good review of the spread of infection under different network conditions and its mathematical formulation can be found in [29] and [30]. These infection propagation models, originally meant for studying diseases also help in analyzing the propagation of viruses and gossip in cellular and social networks. In the power grid, the same models have been used to study the
propagation of frequency oscillations and disturbances [31]. It is well known that vulnerability of the network to infection propagation depends on the network structure. We provide here simulations of infection propagation on real power grids and compare them with simulations on similar sized models developed through our generative model. Figures 12 and 13 show the propagation of infection through the SIS model in the ERCOT, Western American and UCTE Power Grid network respectively. In every figure, the dynamics of infection propagation on a similar-sized network formed by our generative model is also plotted. We observe that infection propagation on the generative model has excellent fit with the infection propagation on the real network, which justifies the accuracy of our generative model. Finally, Figures 14 and 15 show the propagation of SIR infection on the ERCOT, Western American Power Grid respectively and on similar sized networks generated by our generative model. The similarity in the simulations on real and generated networks proves a good potential of our generative model to help create test cases to study policies to improve the stability in real power grids.
Fig. 13. SIS infection propagation in the Western USA Power Grid

Fig. 14. SIR infection propagation in the ERCOT Power Grid
In this paper, we provide a generative model for the physical network representing power grids. The model provides a simple and intuitive explanation for the growth of the network based on Poisson Point Process. Through analysis and simulations, we show that the degree distribution of the generated network is given by a combination of exponentials and is in agreement with the degree distributions seen in power grids across America and Europe. We further demonstrate the efficacy of the developed model by favorably comparing graph diameter and betweenness of generated networks with real power grid networks. Finally, we simulate the dynamics of SIR and SIS infection propagation on networks created by our generative model and show that the dynamics match with those obtained by simulations on similar sized real power grid networks. An intuitive model for the evolution of power grids has several advantages in developing a theoretical understanding of the structure of the grid. Incorporating additional features of the power grid into our generative model without sacrificing mathematical tractability of the model is challenging. This is the focus of our future research in this domain.

Fig. 15. SIR infection propagation in the Western USA Power Grid
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