Measurement and shaping of biphoton spectral wavefunctions

N. Tischler\textsuperscript{1,2}, A. Büse\textsuperscript{1,2}, L. G. Helt\textsuperscript{1,3}, M. L. Juan\textsuperscript{1,2}, N. Piro\textsuperscript{4}, J. Ghosh\textsuperscript{5}, M. J. Steel\textsuperscript{1,3}, G. Molina-Terriza\textsuperscript{1,2}

\textsuperscript{1} MQ Photonics, 4 Quantum Tech, Department of Physics and Astronomy, Macquarie University, 2109, NSW, Australia
\textsuperscript{2} Center of Excellence for Engineered Quantum Systems, Macquarie University, 2109, NSW, Australia
\textsuperscript{3} Center of Excellence for Ultrahigh bandwidth Devices for Optical Systems, Macquarie University, 2109, NSW, Australia
\textsuperscript{4} École Polytechnique Fédérale de Lausanne (EPFL), CH-1015 Lausanne, Switzerland and
\textsuperscript{5} Department of Physics, Indian Institute of Technology Delhi, 110016, New Delhi, India

(Dated: March 31, 2015)

In this work we present a simple method to reconstruct the complex spectral wavefunction of a biphoton, and hence gain complete information about the spectral and temporal properties of a photon pair. The technique, which relies on quantum interference, is applicable to biphoton states produced with a monochromatic pump when a shift of the pump frequency produces a shift in the relative frequencies contributing to the biphoton. We demonstrate an example of such a situation in type-II parametric down-conversion (SPDC) allowing arbitrary paraxial spatial pump and detection modes. Moreover, our test cases demonstrate the possibility to shape the spectral wavefunction. This is achieved by choosing the spatial mode of the pump and of the detection modes, and takes advantage of spatiotemporal correlations.

With the ability to exhibit nonclassical properties such as entanglement, photon pairs (or biphotons) are of fundamental interest in quantum optics and constitute a useful resource. Many proposals exploiting this resource require knowledge of the biphoton quantum state. The complex spectral wavefunction, which contains both amplitude and phase information, provides complete knowledge of the spectrottemporal state of a photon pair, including the ability to calculate all observables related to this degree of freedom and predict interference phenomena. Unsurprisingly then, the reconstruction of the full complex spectral wavefunction has received a lot of attention. Several approaches have been pursued over the last years, but each proposal entails experimental challenges. Interferometric methods require a high level of stability\textsuperscript{[13]}. Other methods rely on nonlinear optical effects which are inherently inefficient at the low intensity levels typical of quantum light sources, requiring very large nonlinearities or high powers\textsuperscript{[14,15]}.

In the pioneering work of Hong, Ou and Mandel (HOM)\textsuperscript{[7]}, the coherence length and time delay between two photons was measured using quantum interference on a beam-splitter, circumventing the need for optical nonlinearities. In fact, the interference phenomenon has since then proven useful in a variety of applications, including quantum teleportation\textsuperscript{[8]}, quantum gates\textsuperscript{[9,10]}, linear optics quantum computation\textsuperscript{[11]}, Bell-state analysers\textsuperscript{[12]}, and the measurement of the group velocity of light\textsuperscript{[13]}, as well as of dispersion\textsuperscript{[14]}. Extensions of the HOM approach also enable the full reconstruction of complex spectral wavefunctions: Chen and co-workers\textsuperscript{[15]} rely on the time resolution of the detectors to directly measure the delay distributions, and therefore their method is applicable to very narrow-band biphotons. In contrast, Douce et al.\textsuperscript{[16]} propose a scheme to measure the biphoton Wigner function using HOM interference by adding shifts of the biphoton frequencies. Yet, a practical implementation of such shifts is not particularly simple or efficient\textsuperscript{[17]}.

In this letter, we propose and implement a variation of the scheme in Ref.\textsuperscript{[16]} that relies on the ability to effectively shift the relative frequency of the biphoton state in the generation process. As an example, our method allows us to measure the complex spectral wavefunction for type-II SPDC with a monochromatic pump in an arbitrary paraxial spatial mode, after projection of the down-converted photons into a likewise arbitrary paraxial spatial mode. The assumption of a monochromatic pump beam means that the frequencies of signal and idler photons are perfectly anticorrelated, hence reducing the problem to the determination of a complex-valued function of one variable. Our scheme is an extension of a conventional HOM type set-up by tuning either the temperature of the nonlinear crystal or the pump frequency, so that a quantum interference coincidence pattern is recorded as a function of path length difference and crystal temperature or pump frequency. We show that both pump frequency tuning and crystal temperature control produce the same effect: a frequency displacement of the wavefunction. Other systems, such as four-wave mixing in atomic species can be similarly controlled by tuning the frequencies of the pumps\textsuperscript{[15]}. We show that multivariable quantum interference patterns can in fact be used to reconstruct the complex spectral mode function $\Phi (\Omega)$, which determines the wavefunction $|\Psi\rangle = \int d\Omega \, \Phi(\Omega) \hat{a}^\dagger_{s} (\frac{\omega_s}{2} + \Omega) \hat{a}^\dagger_{i} (\frac{\omega_i}{2} - \Omega) |0\rangle$. Here $\hat{a}^\dagger_{m} (\omega)$ is the creation operator for a photon with frequency $\omega$ and polarisation as indicated by the subscript, and $\omega_p$ is the pump frequency.

We also demonstrate that in our chosen experimental implementation, the spectral wavefunction can be influenced through the choice of the spatial detection modes, owing to spatiotemporal correlations. This leads
to nontrivial complex spectra with marked differences to the standard sinc function \[15\], making their characterisation worthwhile. The ability to shape the spectral wavefunction is important for quantum information and communication applications, and has already been pursued for single photons and photon pairs using other approaches \[6, 19\].

The experimental set-up used in the implementation of our reconstruction scheme is shown in Fig. 1. Photon pairs are generated by pumping a periodically poled Potassium Titanyl Phosphate (ppKTP) crystal with a focused Gaussian beam in a collinear, type-II down-conversion configuration. They are then separated with a polarising beam splitter (PBS). As in a typical HOM experiment, the path length between the two arms differs by \(\Delta S\), the temperature-controlled nonlinear crystal (15 mm, ppKTP). The down-converted light is collimated by a lens after the crystal. The pump beam is discarded by a polarising beam-splitter, respectively. In addition, we have defined the transmission and reflection amplitudes of the HOM beam splitter, \(T, r\). Note that the wavefunction appears in the form of

\[
\Phi (\Omega; \omega_p) \equiv \Phi (\Omega; \omega_p) \Phi^* (-\Omega; \omega_p),
\]

which we refer to as the symmetrised wavefunction. Since \(F(\Omega, \omega_p)\) is Hermitian w.r.t. \(\Omega\), \(f(\Delta S, T, \omega_p)\) is real and from the coincidence rates (recall Eq. (1)):

\[
f(\Delta S, T, \omega_p) = \left( \frac{1}{2\pi^2} (t^4 + r^4 - R_{\text{coinc}}(\Delta S, T, \omega_p)) \right).
\]

We then obtain \(F(\Omega, T, \omega_p)\) by taking the Fourier transform of \(f(\Delta S, T, \omega_p)\) with respect to \(\Delta S\):

\[
F(\Omega, T, \omega_p) = \frac{1}{c^2 T} \int d\Delta S f(\Delta S, T, \omega_p) \exp (-i2\Omega \Delta S/c).
\]

Because the symmetrisation is not isomorphic, Eq. (4) can in general not be inverted to retrieve the wavefunction from the usual HOM dip. Nevertheless, we now show that the reconstruction is still possible by performing a temperature or a pump frequency sweep. To see this, we perform a multivariate Taylor expansion to leading orders of the wavevector \(\mathbf{q}\)-components for each of pump, signal, and idler (indicated by subscript \(m\)), about the values at which perfect phase matching takes place: at frequencies \(\omega_m = \omega_{0m}\), crystal temperature \(T = T_0\), and transverse wavevector \(\mathbf{q}_m = \mathbf{0}\). From the Taylor series approximation, symmetrised wavefunctions at different temperatures can be related by shifting the frequencies, while keeping the temperature fixed \[20\]:

\[
\Phi (\Omega; T_0 + \Delta T, \omega_p + \Delta \omega_p) \Phi^* (-\Omega; T_0 + \Delta T, \omega_{0p} + \Delta \omega_p) \\
\approx \Phi (\Omega + \Delta T c_l + \Delta \omega_p c_{wp}; T_0, \omega_p) \\
\times \Phi^* (-\Omega + \Delta T c_l + \Delta \omega_p c_{wp}; T_0, \omega_{0p}),
\]

The coincidence count rates can be modelled as

\[
R_{\text{coinc}}(\Delta S, T, \omega_p) \propto t^4 + r^4 - 2\sqrt{2} Re \{ f(\Delta S, T, \omega_p) \},
\]

where \(\Delta S = (S_p - S_s)\) is the difference between signal and idler path lengths, \(T\) is the crystal temperature, \(\omega_p\) the pump frequency, and \(t\) and \(r\) are the moduli of the transmission and reflection amplitudes of the HOM beam splitter, respectively. In addition, we have defined

\[
f(\Delta S, T, \omega_p) \equiv \int d\Omega \Phi (\Omega; T, \omega_p) \Phi^* (-\Omega; T, \omega_p) \times \exp (i\Delta S 2\Omega/c),
\]

where \(c\) is the speed of light in vacuum. The conventional HOM dip is a slice of such a surface \(R_{\text{coinc}}(\Delta S, T, \omega_p)\) along the \(\Delta S\) direction, keeping the crystal temperature and pump frequency fixed. The coincidence counts thus involve our complex wavefunction of interest,

\[
\Phi (\Omega; T, \omega_p) \equiv \int d\mathbf{q}_s d\mathbf{q}_i \Phi_{\text{full}}(\mathbf{q}_s, \mathbf{q}_i; \Omega; T, \omega_p) \\
\times G^*_s(\mathbf{q}_s) G^*_i(\mathbf{q}_i),
\]

where \(\Phi_{\text{full}}(\mathbf{q}_s, \mathbf{q}_i; \Omega; T, \omega_p)\) is the wavefunction before projection into the spatial modes \(G_s(\mathbf{q}_s)\) and \(G_i(\mathbf{q}_i)\), with \(\mathbf{q}\) being the transverse momenta \[20\].

FIG. 1. Experimental set-up: An 8 mW, monochromatic 404.25 nm pump beam is focused into the temperature-controlled nonlinear crystal (15 mm, ppKTP). The down-converted light is collimated by a lens after the crystal. The pump beam is discarded by a longpass filter. The photon pairs (s and i denoting signal and idler, respectively) are separated by a polarising beam splitter (PBS). A set of waveplates is used to maximize interference. The path length between the two arms differs by a controllable amount \(\Delta S\), before coupling into single-mode fibres. Alternatively, we can select different higher order modes with diffractive elements, prior to the fiber coupling. Finally, the photons pass through a fibre beam splitter (BS), and coincidences (&) are detected across two avalanche photodiodes (APDs), one for each fibre beam splitter output arm.

We measure the coincidence count rate as a joint function of path length difference and crystal temperature. Equivalently, it is possible to vary the pump laser frequency instead of the crystal temperature. We consider both cases in our theoretical analysis.
where we have defined

\[ c_t \equiv -\frac{X_T}{(\partial k_p / \partial T - \partial k_s / \partial T - \partial k_i / \partial T + \frac{2\pi}{\Lambda} \partial \Lambda)}, \]  

\[ X_T \equiv \left( \frac{\partial k_p}{\partial T} - \frac{\partial k_s}{\partial T} - \frac{\partial k_i}{\partial T} + \frac{2\pi}{\Lambda} \right); \]  

\[ c_{\omega p} \equiv -\frac{X_{\omega}}{(\partial k_p / \partial \omega - \partial k_s / \partial \omega - \partial k_i / \partial \omega)}; \]  

\[ X_{\omega} \equiv \left( \frac{\partial k_p}{\partial \omega} - \frac{\partial k_s}{2\partial \omega} - \frac{\partial k_i}{2\partial \omega} \right). \]

Here, \( k_m \) are the wavenumbers in the crystal (pump, signal, and idler indicated by subscripts), \( \omega_m \) the frequencies, and \( \Lambda \) the poling period of the crystal. All derivatives are evaluated at the reference temperature \( T_0 \) and frequencies \( \omega_m = \omega_{m0}, m \in \{p, s, i\} \). Given this, the complex mode function can be obtained as a slice through \( F(\Omega, \Delta T, \Delta \omega_p) \):

\[
\Phi(2c_t\Delta T + 2c_{\omega p}\Delta \omega_p; T_0, \omega_{p0}) = F^*\left(-c_t\Delta T - c_{\omega p}\Delta \omega_p, T_0 + \Delta T, \omega_{p0} + \Delta \omega_p\right) / |F(0, T_0, \omega_{p0})|.
\]

As previously mentioned, either temperature or pump frequency may be swept in the experiment. However, for an implementation of the experiment it is sufficient to scan one, while keeping the other variable fixed. Fig. 2 illustrates the data analysis process.

FIG. 2. Steps to determine \( \Phi(\Omega) \) from \( R_{\text{coinc}}(\Delta S, \Delta T) \). Starting with \( R_{\text{coinc}} \), the real coincidence counts, we perform elementary operations to obtain another real function \( f \) (Eq. (5)). Taking a Fourier transform of \( f \) w.r.t. \( \Delta S \), we get to the complex function \( F \) (by Eq. (6)). The desired wavefunction is obtained by taking an appropriate slice of \( F \) (by Eq. (12)).

In Fig. 3 we demonstrate the use of our reconstruction method, with theory and experimental results for three test cases. The test cases all use a Gaussian pump beam with a beam waist of 4.3 \( \mu m \), but differ in the detection modes. These are, each with a beam waist of 9.6 \( \mu m \), (a) Gaussians with the crystal centered, (b) Gaussians with the crystal displaced by 3 mm along the propagation direction, and (c) the Laguerre Gaussian modes (azimuthal index, radial index) = (1,0), (-1,0) with the crystal centered.

We first show the results of our theoretical analysis, illustrated in the left panel of Fig. 3. From our model of the nonlinear process and detection, we obtain the expected spectral wavefunction directly (black solid line) \( 20 \). We then calculate the expected quantum interference pattern, \( R_{\text{coinc}} \), by Eqs. (1) and (2), based on which the spectral wavefunction is reconstructed using Eqs. (5), (6) and (12) (red dotted line). Next we calculate the spectral and time delay distributions both from the original and the reconstructed wavefunction, shown in black solid and red dotted lines, respectively. For the experimental results, we measure the quantum interference pattern and reconstruct the wavefunction using Eqs. (5), (6) and (12), from which the time delay distribution is obtained (red dotted line).

The theoretical analysis allows us to compare the original and the reconstructed wavefunction, showing a good agreement in all cases. However, the quantum interference pattern and thus also the reconstruction are insensitive to even terms in the frequency dependence of the crystal’s refractive index, which have an impact on the wavefunction through the propagation of the down-converted photons to the end of the crystal. This causes an error in the reconstructed phase. For any particular implementation, the error is limited and depends on the optical properties and length of the crystal, as well as the spectral bandwidth. The deviation in phase cannot be seen easily in Fig. 3 because it is comparatively small, but there is a difference which is quadratic with \( \Omega \) and reaches up to 0.46 radians for the plotted section of frequencies. This, in turn, has a visible impact on the time delay distribution in (c), where a small deviation between reconstructed and calculated distributions is evident. We note that the discussed lack of sensitivity of the reconstruction method does not mean that it is only sensitive to odd functions of the phase, as the phase imparted through the spatial projection can be arbitrary and is recovered by our method.

The experimentally measured coincidences allow us to determine the complex spectral wavefunctions and time delay distributions of our experimental photon pairs. Interestingly, they also allow us to identify small imperfections in the experiment. For example, a slight off-centering of the crystal in cases (a) and (c) results in an asymmetry of the coincidence count map and an increased slope of the phase and mean time delay. In (c), we attribute differences with the theoretically predicted wavefunction to the fact that the radial profile of the theoretical detection mode is slightly different from the one in the experimental implementation.

A comparison of the three rows in Fig. 3 shows significant differences between the test cases. The Gaussian detection with the crystal centred (a) yields a quantum interference pattern that is symmetric w.r.t.
A considerable advantage of the method lies in its simplicity, both in the experimental implementation and in the data analysis. Indeed, our technique is not faced with challenging stabilisations typical of interferometric measurements \cite{3}, or the need for high pump powers incurred by measurements that rely on nonlinear optical effects \cite{5}. As an extension, following the results in Ref. \cite{10}, our two dimensional coincidence counts would provide the Wigner function in the case of a mixed state, and could still yield useful information for the case of a pulsed pump. We note that our technique has an applicability that is restricted to biphotons which can be manipulated in the appropriate manner through the generation process. Moreover, post-emission spectral manipulation will lead to a faulty reconstruction. A further limitation is that the reconstruction is unsuccessful at recovering the limited part of the phase that arises from the even order frequency dependence of the crystal’s refractive index.

Lastly, the freedom to choose spatial pump and detection modes offers some interesting possibilities. We have characterised the spectrotemporal properties of the biphoton, after projection into a spatial mode. Contrary to the intuitive idea that spatial degrees of freedom should not play a role, our results show that the choice of detection modes can have a pronounced effect on the spectrotemporal properties, in particular

\[ \Delta S, \text{ about a value that depends on the time delay acquired when signal and idler photons propagate through half the length of the nonlinear birefringent crystal.} \]

The phase of the wavefunction has a primarily linear trend due to this time delay. The spectrum is quite narrow, and the time delay distribution has a symmetric peak centred at the time delay acquired by propagation through half of the crystal. When the crystal is displaced (b), the quantum interference pattern becomes asymmetric, the phase changes, and the time delay distribution shifts \cite{21}. Using the Laguerre Gaussian detection mode with the crystal centered (c) changes the structure of the quantum interference pattern markedly, even transforming the dip into a peak. The phase of the wavefunction is similar to the Gaussian case, but the spectrum has a side lobe. Interestingly, the time delay distribution has a dip at the approximate axis of symmetry, which means that the probability of photons arriving with their mean time delay is suppressed.

In summary, we have proposed and demonstrated a method to reconstruct the complex spectral wavefunction of a biphoton, using HOM interference for type-II SPDC. The essence of our method lies in the fact that a change in temperature or pump frequency is approximately equivalent to a shift of the frequency for the symmetrised mode function that determines the quantum interference coincidence counts.

FIG. 3. Demonstration of the reconstruction method on three test cases. Theoretical (left) and experimental (right panel) results using as detection modes (a) Gaussians with the crystal centered, (b) Gaussians with the crystal displaced by 3 mm along the propagation direction, and (c) a pair of Laguerre Gaussian modes (1 0), (-1 0) with the crystal centered. We show, in both cases, (from left to right) the coincidence counts, the complex spectral wavefunctions (amplitude in arb. units), the spectral distributions (arb. units), and the time delay distributions (arb. units). For experiments, we omit the spectral distribution. The red dotted lines are the reconstruction results, while the black solid lines within the theory section are based on the simulated wavefunction. Our experimental conditions are simulated using \( T_0 = 58 \, ^\circ\text{C} \), so that \( c_r = -4.8698 \times 10^{11} \, (^\circ\text{C} \, \text{s})^{-1} \).
due to spatiotemporal correlations in the biphoton wavefunction [21–24]. Our method works for arbitrary paraxial pump and projection modes, so it is possible to influence the detected wavefunction by adjusting the modes.

This work was funded by the Australian Research Council’s Centres of Excellence for Engineered Quantum Systems (EQuS), and for Ultrahigh bandwidth Devices for Optical Systems (CUDOS). G.M.-T. also holds an Australian Research Council Future Fellowship.

[1] C. Ren and H. F. Hofmann, Physical Review A 84, 032108 (2011)
[2] C. Ren and H. F. Hofmann, Physical Review A 86, 043823 (2012)
[3] F. A. Beduini, J. A. Zieliska, V. G. Lucivero, Y. A. de Icaza Astiz, and M. W. Mitchell, Physical Review Letters 113, 183602 (2014)
[4] O. Jedrkiewicz, J.-L. Blanchet, E. Brambilla, P. Di Trapani, and A. Gatti, Physical Review Letters 108, 253904 (2012)
[5] K. O'Donnell and A. URen, Physical Review Letters 103, 123602 (2009)
[6] A. Pe'er, B. Dayan, A. Friesem, and Y. Silberberg, Physical Review Letters 94, 073601 (2005)
[7] C. K. Hong, Z. Y. Ou, and L. Mandel, Physical Review Letters 59, 2044 (1987)
[8] D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, Nature 390, 575 (1997)
[9] H. Hofmann and S. Takeuchi, Physical Review A 66, 024308 (2002)
[10] T. Ralph, N. Langford, T. Bell, and A. White, Physical Review A 65, 062324 (2002)
[11] E. Knill, R. Laflamme, and G. J. Milburn, Nature 409, 46 (2001)
[12] M. Michler, K. Mattle, H. Weinfurter, and A. Zeilinger, Physical Review A 53, R1209 (1996)
[13] A. Steinberg, P. Kwiat, and R. Chiao, Physical Review Letters 68, 2421 (1992)
[14] R. Okamoto, S. Takeuchi, and K. Sasaki, Physical Review A 74, 011801 (2006)
[15] P. Chen, C. Shu, X. Guo, M. Loy, and S. Du, Physical Review Letters 114, 010401 (2015)
[16] T. Douce, A. Eckstein, S. P. Walborn, A. Z. Khoury, S. Ducci, A. Keller, T. Coudreau, and P. Milman, Scientific reports 3, 3530 (2013)
[17] S. Preble, L. Cao, A. Elshaari, A. Aboketaf, and D. Adams, Applied Physics Letters 101, 171110 (2012)
[18] A. Fedrizzi, T. Herbst, M. Aspelmeyer, M. Barbieri, T. Jennewein, and A. Zeilinger, New Journal of Physics 11, 103052 (2009)
[19] D. Kielpinski, J. F. Corney, and H. M. Wiseman, Physical Review Letters 106, 130501 (2011)
[20] Details can be found in the supplementary information.
[21] A. Bøse, N. Tischler, M. J. Juan, and G. Molina-Terriza, arXiv:1502.02773, accepted in Journal of Optics (2015).
[22] E. Brambilla, L. Caspani, L. A. Lugliati, and A. Gatti, Physical Review A 82, 013835 (2010)
[23] C. I. Osorio, A. Valencia, and J. P. Torres, New Journal of Physics 10, 113012 (2008)
[24] A. Gatti, E. Brambilla, L. Caspani, O. Jedrkiewicz, and L. Lugliati, Physical Review Letters 102, 223601 (2009)