Data Programming: Creating Large Training Sets, Quickly

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Abstract

Large labeled training sets are the critical building blocks of supervised learning methods and are key enablers of deep learning techniques. For some applications, creating labeled training sets is the most time-consuming and expensive part of applying machine learning. We therefore propose a paradigm for the programmatic creation of training sets called data programming in which users provide a set of labeling functions, which are programs that heuristically label large subsets of data points, albeit noisily. By viewing these labeling functions as implicitly describing a generative model for this noise, we show that we can recover the parameters of this model to “denoise” the training set. Then, we show how to modify a discriminative loss function to make it noise-aware. We demonstrate our method over a range of discriminative models including logistic regression and LSTMs. We establish theoretically that we can recover the parameters of these generative models in a handful of settings. Experimentally, on the 2014 TAC-KBP relation extraction challenge, we show that data programming would have obtained a winning score, and also show that applying data programming to an LSTM model leads to a TAC-KBP score almost 6 F1 points over a supervised LSTM baseline (and into second place in the competition). Additionally, in initial user studies we observed that data programming may be an easier way to create machine learning models for non-experts.

1 Introduction

Many of the major machine learning breakthroughs of the last decade have been catalyzed by the release of a new labeled training dataset. Supervised learning approaches that use such datasets have increasingly become key building blocks of applications throughout science and industry. This trend has also been fueled by the recent empirical success of automated feature generation approaches, notably deep learning methods such as long short term memory (LSTM) networks, which ameliorate the burden of feature engineering given large enough labeled training sets. For many real-world applications, however, large hand-labeled training sets do not exist, and are prohibitively expensive to create due to requirements that labelers be experts in the application domain. Furthermore, applications’ needs often change, necessitating new or modified training sets.

To help reduce the cost of training set creation, we propose data programming, a paradigm for the programmatic creation of training datasets. Data programming extends the idea of distant supervision, in which an external knowledge base is mapped onto an input dataset to generate training examples. In data programming, users provide a set of heuristic labeling functions, which are user-defined programs that each provide a label for some subset of the data, and collectively generate a large but noisy training set. These labeling functions can be more general than distant supervision mappings—they can use external knowledge bases (as in distant supervision), model an individual annotator’s labels (as in crowdsourcing), or leverage a combination of domain-specific patterns and dictionaries—and thus may have widely varying error rates, may overlap, and may conflict on certain data points. To address this, we model the labeling functions as a generative process, which lets us automatically denoise the resulting training set by learning the accuracies of the labeling functions along with their correlation structure. In turn, we use this model of the training set to optimize a stochastic version of the loss function of the discriminative model that we desire to train. We show that, given certain conditions on the labeling functions, our method achieves the same asymptotic scaling as supervised learning methods,
but that our scaling depends on the amount of unlabeled data—using only a fixed number of labeling functions, which is small relative to the training set size.

Data programming is in part motivated by the challenges that users faced when applying prior programmatic supervision approaches, and is intended to be a new software engineering paradigm for the creation and management of training sets. For example, consider the scenario when two labeling functions of differing quality and scope overlap and possibly conflict on certain training examples; in prior approaches the user would have to decide which one to use, or how to somehow integrate the signal from both. In data programming, we accomplish this automatically by learning a model of the training set that includes both labeling functions. Additionally, users are often aware of, or able to induce, dependencies between their labeling functions. In data programming, users can provide a dependency graph to indicate, for example, that two labeling functions are similar, or that one “fixes” or “reinforces” another. We describe cases in which we can learn the strength of these dependencies, and for which our generalization is again asymptotically identical to the supervised case.

One further motivation for our method is driven by the observation that users often struggle with selecting features for their models, which is a traditional development bottleneck given fixed-size training sets. However, initial feedback from users suggests that writing labeling functions in the framework of data programming may be easier. While the impact of a feature on end performance is dependent on the training set and on statistical characteristics of the model, a labeling function has a simple and intuitive optimality criterion: that it labels data correctly. Motivated by this, we explore whether we can flip the traditional machine learning development process on its head, having users instead focus on generating training sets large enough to support automatically-generated features.

Summary of Contributions and Outline  Our first contribution is the data programming framework, in which users can implicitly describe a rich generative model for a training set in a more flexible and general way than in previous approaches. In Section 3 we first explore a simple model in which labeling functions are conditionally independent. We show here that under certain conditions, the sample complexity is nearly the same as in the labeled case. In Section 4 we extend our results to more sophisticated data programming models, generalizing related results in crowdsourcing [15]. In Section 5 we validate our approach experimentally on large real-world text relation extraction tasks in genomics, pharmacogenomics and news domains, where we show an average 2.34 point F1 score improvement over a baseline programmatic supervision approach—including what would have been a new competition-winning score for the 2014 TAC-KBP Slot Filling competition. Using LSTM-generated features, we would have placed second in this competition, achieving a 5.98 point F1 score gain over a state-of-the-art LSTM baseline trained on hand-labeled data [30]. Additionally, we describe promising feedback from a usability study with a group of bioinformatics users.

2 Related Work

Our work builds on many previous approaches in machine learning. Distant supervision is one preceding paradigm for programmatic creation training sets. The canonical example is relation extraction from text, wherein a knowledge base of known relations is heuristically mapped to label a set of mentions in an input corpus as ground truth examples [7][20]. Basic extensions group these mapped examples by the particular textual pattern w that they occur with, and cast the problem as a multiple instance learning one [13][23]. Other extensions actually model the accuracy of this pattern w using a discriminative feature-based model [25], or generative models such as hierarchical topic models [1][24][29]. Like our approach, these latter methods model a generative process of training set creation, however in a proscribed way that is not based on user input as in our approach. There is also a wealth of examples where additional heuristic patterns used to label training data are collected from unlabeled data [6] or directly from users [19][27], in a similar manner to our approach, but without a framework to deal with the fact that said labels are explicitly noisy.

Crowdsourcing is widely used for various machine learning tasks [11][16]. Of particular relevance to our problem setting is the theoretical question of how to model the accuracy of various experts without ground truth available, classically raised in the context of crowdsourcing [9]. More recent results provide formal guarantees even in the absence of labeled data using various approaches [3][8][14][15][22][31]. Our model can capture the model described in crowdsourcing, and can be equivalent in the independent case (Sec 5). However, in addition to generalizing beyond getting inputs solely from human annotators, we also model user-supplied dependencies between the “labelers” in our model, which is not natural within the context of crowdsourcing. Additionally, while crowdsourcing results focus on
In many applications, we would like to use machine learning, but we face the following challenges: (i) hand-labeled training data is not available, and is prohibitively expensive to obtain in sufficient quantities as it requires expensive domain experts; (ii) related external knowledge bases are either unavailable or insufficiently specific, precluding a traditional distant supervision or co-training approach; (iii) application specifications are in flux, changing the model we ultimately wish to learn.

In such a setting, we would like a simple, scalable and adaptable approach for supervising a model applicable to our problem. More specifically, we would ideally like our approach to achieve \( \epsilon \) expected loss with high probability, given \( O(1) \) inputs of some sort from a domain-expert user, rather than the traditional \( O(\epsilon^{-2}) \) hand-labeled training examples required by most supervised methods (where \( O \) notation hides logarithmic factors). To this end, we propose data programming, a paradigm for the programmatic creation of training sets, which enables domain-experts to more rapidly train machine learning systems and has the potential for this type of scaling of expected loss. In data programming, rather than manually labeling each example, users instead describe the processes by which these points could be labeled by providing a set of heuristic rules called labeling functions.

In the remainder of this paper, we focus on a binary classification task in which we have a distribution \( \pi \) over object and class pairs \((x, y) \in \mathcal{X} \times \{-1, 1\}\), and we are concerned with minimizing the logistic loss under a linear model given some features,

\[
l(w) = \mathbb{E}_{(x,y) \sim \pi} \left[ \log(1 + \exp(-w^T f(x)y)) \right],
\]

where without loss of generality, we assume that \( \|f(x)\| \leq 1 \). Then, a labeling function \( \lambda : \mathcal{X} \mapsto \{-1, 0, 1\} \) is a user-defined function that encodes some domain heuristic, which provides a (non-zero) label for some subset of the objects. As part of a data programming specification, a user provides some \( m \) labeling functions, which we denote in vectorized form as \( \lambda : \mathcal{X} \mapsto \{-1, 0, 1\}^m \).
Example 3.1. To gain intuition about labeling functions, we describe a simple text relation extraction example. In Figure 1 we consider the task of classifying co-occurring gene and disease mentions as either expressing a causal relation or not. For example, given the sentence “Gene A causes disease B”, the object $x = (A, B)$ has true class $y = 1$.

To construct a training set, the user writes three labeling functions (Figure 1a). In $\lambda_1$, an external structured knowledge base is used to label a few objects with relatively high accuracy, and is equivalent to a traditional distant supervision rule (see Sec. 2). $\lambda_2$ uses a purely heuristic approach to label a much larger number of examples with lower accuracy. Finally, $\lambda_3$ is a “hybrid” labeling function, which leverages both an external knowledge base and a heuristic filter.

A labeling function need not have perfect accuracy or recall; rather, it represents a pattern that the user wishes to impart to their model and that is easier to encode as a labeling function than as a hand-labeled examples. As illustrated in Ex. 3.1, labeling function can be based on external knowledge bases, libraries or ontologies, could be purely a heuristic pattern, or some hybrid of these types; we see evidence for the existence of such diversity in our experiments (Section 5). The use of labeling functions is also strictly more general than manual annotations, as a manual annotation can always be directly encoded by a labeling function. Importantly, labeling functions can overlap, conflict, and even have dependencies which users can provide as part of the data programming specification (see Section 4); our approach provides a simple framework for these inputs.

Independent Labeling Functions We first describe a model in which the labeling functions label independently, given the true label class. Under this model, each labeling function $\lambda_i$ has some probability $\beta_i$ of labeling an object and some probability $\alpha_i$ of labeling the object correctly; for simplicity we also assume here that each class has probability 0.5. This model has distribution

$$
\mu_{\alpha,\beta}(\lambda, y) = \begin{cases} 
1 & \text{if } \beta_i \alpha_i 1_{[\Lambda_i = y]} + \beta_i (1 - \alpha_i) 1_{[\Lambda_i = -y]} + (1 - \beta_i) 1_{[\Lambda_i = 0]} = 1 \\
0 & \text{otherwise}
\end{cases}
$$

(1)

where $\Lambda \in \{-1, 0, 1\}^m$ contains the labels output by the labeling functions, and $Y \in \{-1, 1\}$ is the predicted class. If we allow the parameters $\alpha \in \mathbb{R}^m$ and $\beta \in \mathbb{R}^m$ to vary, (1) specifies a family of generative models. In order to expose the scaling of the expected loss as the size of the unlabeled dataset changes, we will assume here that $0.3 \leq \beta_i \leq 0.5$ and $0.8 \leq \alpha_i \leq 0.9$. We note that while these arbitrary constraints can be changed, they are roughly consistent with our applied experience, where users tend to write high-accuracy and high-coverage labeling functions.

Our first goal will be to learn which parameters $(\alpha, \beta)$ are most consistent with our observations—our unlabeled training set—using maximum likelihood estimation. To do this for a particular training set $S \subset \mathcal{X}$, we will solve the problem

$$
(\hat{\alpha}, \hat{\beta}) = \operatorname{arg \ max}_{\alpha, \beta} \sum_{x \in S} \log P(\alpha, \beta; \mu_{\alpha, \beta})(\Lambda = \lambda(x)).
$$

(2)

In other words, we are maximizing the probability that the observed labels produced on our training examples occur under the generative model in (1). In our experiments, we use stochastic gradient descent to solve this problem; since this is a standard technique, we defer its analysis to the appendix.

Noise-Aware Empirical Loss Given that our parameter learning phase has successfully found some $\hat{\alpha}$ and $\hat{\beta}$ that accurately describe the training set, we can now proceed to estimate the parameter $w$ which minimizes the expected risk of a linear model over our feature mapping $f$, given $\hat{\alpha}, \hat{\beta}$. To do so, we define the noise-aware empirical risk $L_{\hat{\alpha}, \hat{\beta}}$ with regularization parameter $\rho$, and compute the noise-aware empirical risk minimizer

$$
\hat{w} = \operatorname{arg \ min}_{w} L_{\hat{\alpha}, \hat{\beta}}(w; S) = \operatorname{arg \ min}_{w} \frac{1}{|S|} \sum_{x \in S} E(\alpha, \beta; \mu_{\alpha, \beta}) \left[ \log \left( 1 + e^{-w^T f(x)^Y} \right) \right] \Lambda = \lambda(x) + \rho \|w\|^2
$$

(3)

This is a logistic regression problem, so it can be solved using stochastic gradient descent as well.

We can in fact prove that stochastic gradient descent running on (2) and (3) is guaranteed to produce accurate estimates, under conditions which we describe now. First, the problem distribution $\pi$ needs to be accurately modeled by some distribution $\mu$ in the family that we are trying to learn. That is, for some $\alpha^*$ and $\beta^*$,

$$
\forall \Lambda \in \{-1, 0, 1\}^m, Y \in \{-1, 1\}, \pi_{(\Lambda, y) - \pi^*}(\lambda(x) = \Lambda, y = Y) = \mu_{\alpha^*, \beta^*}(\Lambda, Y).
$$

(4)
Suppose that we run data programming, solving the problems in (2) and (3) using stochastic gradient descent to produce \( \hat{\alpha}, \hat{\beta} \) and \( \hat{\nu} \). Suppose further that our setup satisfies the conditions (4), (5), and (6), and suppose that \( m \geq 2000 \). Then for any \( \epsilon > 0 \), if the number of labeling functions \( m \) and the size of the input dataset \( S \) are large enough that

\[
|S| \geq \frac{356}{\epsilon^2} \log \left( \frac{m}{3\epsilon} \right)
\]

then our expected parameter error and generalization risk can be bounded by

\[
E \left[ \| \hat{\alpha}^* - \alpha^* \|^2 \right] \leq m \epsilon^2 \quad \text{and} \quad E \left[ \| \hat{\beta} - \beta^* \|^2 \right] \leq m \epsilon^2 \quad \text{and} \quad E \left[ l(\hat{\nu}) - \min_w l(w) \right] \leq \chi + \frac{\epsilon}{27 \rho}.
\]

We select \( m \geq 2000 \) to simplify the statement of the theorem and give the reader a feel for how \( \epsilon \) scales with respect to \( |S| \). The full theorem with scaling in each parameter (and for arbitrary \( m \)) is presented in the appendix.

This result establishes that to achieve both expected loss and parameter estimate error \( \epsilon \), it suffices to have only \( m = O(1) \) labeling functions and \( |S| = \tilde{O}(\epsilon^{-2}) \) training examples, which is the same asymptotic scaling exhibited by methods that use labeled data. This means that data programming achieves the same learning rate as methods that use labeled data, while requiring asymptotically less work from its users, who need to specify \( O(1) \) labeling functions rather than manually label \( \tilde{O}(\epsilon^{-2}) \) examples. In contrast, in the crowdsourcing setting, the number of workers \( m \) tends to infinity while here it is constant while the dataset grows. These results provide some explanation of why our experimental results suggest that a small number of rules with a large unlabeled training set can be effective at even complex natural language processing tasks.

### 4 Handling Dependencies

In our experience with data programming, we have found that users often write labeling functions that have clear dependencies among them. As more labeling functions are added as the system is developed, an implicit dependency structure arises naturally amongst the labeling functions: modeling these dependencies can in some cases improve accuracy. We describe a method by which the user can specify this dependency knowledge as a dependency graph, and show how the system can use it to produce better parameter estimates.

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**Figure 2:** Examples of labeling function dependency predicates.
Label Function Dependency Graph  To support the injection of dependency information into the model, we augment the data programming specification with a label function dependency graph, \( G \in \mathcal{D} \times \{1, \ldots, m\} \times \{1, \ldots, m\} \), which is an undirected graph over the labeling functions, each of the edges of which is associated with a dependency type from a class of dependencies \( \mathcal{D} \) appropriate to the domain. From our experience with practitioners, we identified four commonly-occurring types of dependencies as illustrative examples: similar, fixing, reinforcing, and exclusive (see Figure 2).

For example, suppose that we have two functions \( \lambda_1 \) and \( \lambda_2 \), and \( \lambda_2 \) typically labels only when (i) \( \lambda_1 \) also labels, (ii) \( \lambda_1 \) and \( \lambda_2 \) disagree in their labeling, and (iii) \( \lambda_2 \) is actually correct. We call this a fixing dependency, since \( \lambda_2 \) fixes mistakes made by \( \lambda_1 \). If \( \lambda_1 \) and \( \lambda_2 \) were to typically agree rather than disagree, this would be a reinforcing dependency, since \( \lambda_2 \) reinforces the truth proclaimed by \( \lambda_1 \).

Modeling Dependencies  The presence of dependency information means that we can no longer model our labels using the simple Bayesian network in (1). Instead, we model our distribution as a factor graph. This standard technique lets us describe the family of generative distributions in terms of a known factor function \( h : [-1, 0, 1]^m \times [-1, 1] \mapsto [-1, 0, 1]^M \) (in which each entry \( h_i \) represents a factor), and an unknown parameter \( \theta \in \mathbb{R}^M \) as

$$
\mu_\theta(\Lambda, Y) = Z_\theta^{-1} \exp(\theta^T h(\Lambda, Y)),
$$

where \( Z_\theta \) is the partition function which ensures that \( \mu \) is a distribution. Next, we will describe how we define \( h \) using information from the dependency graph.

To construct \( h \), we will start with some base factors, which we inherit from (1), and then augment them with additional factors representing dependencies. For all \( i \in \{1, \ldots, m\} \), we let

$$
h_0(\Lambda, Y) = Y, \quad h_i(\Lambda, Y) = \Lambda_i Y, \quad h_{m+i}(\Lambda, Y) = \Lambda_i Y, \quad h_{2m+i}(\Lambda, Y) = \Lambda_i Y, \quad h_{3m+i}(\Lambda, Y) = \Lambda_i Y.
$$

These factors alone are sufficient to describe any distribution for which the labels are mutually independent, given the class: this includes the independent family in (1).

We now proceed by adding additional factors to \( h \), which model the dependencies encoded in \( G \). For each dependency edge \((d, i, j)\), we add one or more factors to \( h \) as follows. For a near-duplicate dependency on \((i, j)\), we add a single factor \( h_j(\Lambda, Y) = 1[\Lambda_i = \Lambda_j] \), which increases our prior probability that the labels will agree. For a fixing dependency, we add two factors, \( h_j(\Lambda, Y) = -1[\Lambda_i = 0 \land \Lambda_j \neq 0] \) and \( h_{i+1}(\Lambda, Y) = 1[\Lambda_i = -Y \land \Lambda_j = Y] \), which encode the idea that \( \Lambda_j \) labels only when \( \Lambda_i \) does, and that \( \Lambda_j \) fixes errors made by \( \Lambda_i \). The factors for a reinforcing dependency are the same, except that \( h_{i+1}(\Lambda, Y) = 1[\Lambda_i = Y \land \Lambda_j = Y] \). Finally, for an exclusive dependency, we have a single factor \( h_j(\Lambda, Y) = -1[\Lambda_i \neq 0 \land \Lambda_j \neq 0] \).

Learning with Dependencies  We can again solve a maximum likelihood problem like (2) to learn the parameter \( \hat{\theta} \). Using the results, we can continue on to find the noise-aware empirical loss minimizer by solving the problem in (3).

In order to solve these problems in the dependent case, we typically invoke stochastic gradient descent, using Gibbs sampling to sample from the distributions used in the gradient update. Under conditions similar to those in Section 3, we can again provide a bound the accuracy of these results. We define these conditions now. First, there must be some set \( \Theta \subset \mathbb{R}^M \) that we know our parameter lies in. This is analogous to the assumptions on \( \alpha \) and \( \beta \) we made in Section 3 and we can state the following analog of (3):

$$
\exists \theta^* \in \Theta \text{ s.t. } \forall(\Lambda, Y) \in \{-1, 0, 1\}^m \times \{-1, 1\}, \ P_{(\theta^*)}^\mathcal{D} \left( \lambda(x) = \Lambda, y = Y \right) = \mu_\theta(\Lambda, Y).
$$

Second, for any \( \theta \in \Theta \), it must be possible to accurately learn \( \theta \) from full (i.e. labeled) samples of \( \mu_\theta \). More specifically, there exists an unbiased estimator \( \hat{\theta}(T) \) that is a function of some dataset \( T \) of independent samples from \( \mu_\theta \) such that, for some \( c > 0 \) and for all \( \theta \in \Theta \),

$$
\text{Cov} \left( \hat{\theta}(T) \right) \preceq (2c |T|)^{-1} I.
$$

Third, for any two feasible models \( \theta_1 \) and \( \theta_2 \in \Theta \),

$$
\mathbf{E}_{\theta_1 \theta_2} \left[ \mathbf{Var}_{\theta_1 \theta_2} \left( Y_2 | \Lambda_1 = \Lambda_2 \right) \right] \leq c M^{-1}.
$$

That is, we’ll usually be reasonably sure in our guess for the value of \( Y \), even if we guess using distribution \( \mu_{\theta_1} \) while the the labeling functions were actually sampled from (the possibly totally different) \( \mu_{\theta_2} \). We can now prove the following result about the accuracy of our estimates.
Table 1: Precision/Recall/F1 scores using data programming, as compared to distant supervision ITR approach, with both hand-tuned and LSTM-generated features.

| Features | Method | KBP (News) | Genomics | Pharmacogenomics |
|----------|--------|------------|----------|-----------------|
|          |        | Prec. | Rec. | F1 | Prec. | Rec. | F1 | Prec. | Rec. | F1 |
| Hand-tuned | ITR | 51.15 | 26.72 | 35.10 | 83.76 | 41.67 | 55.65 | 68.16 | 49.32 | 57.23 |
|           | DP   | 50.52 | **29.21** | **37.02** | **83.90** | 43.43 | 57.24 | **68.36** | 54.80 | **60.83** |
| LSTM      | ITR   | 37.68 | 28.81 | 32.66 | 69.07 | 50.76 | 58.52 | 32.35 | 43.84 | 37.23 |
|           | DP   | 47.47 | 27.88 | 35.78 | 75.48 | 48.48 | 58.99 | 37.63 | 47.95 | 42.17 |

Theorem 2. Suppose that we run stochastic gradient descent to produce $\hat{\theta}$ and $\hat{w}$, and that our setup satisfies the conditions (5)-(9). Then for any $\epsilon > 0$, if the input dataset $S$ is large enough that

$$|S| \geq \frac{2}{c^2\epsilon^2} \log \left( \frac{2 ||\theta_0 - \theta^*||^2}{\epsilon} \right),$$

then our expected parameter error and generalization risk can be bounded by

$$E \left[ \|\hat{\theta} - \theta^*\|^2 \right] \leq M\epsilon^2$$

$$E \left[ l(\hat{w}) - \min_w l(w) \right] \leq \chi + \frac{c\epsilon}{2\rho}.$$

As in the independent case, this shows that we need only $|S| = \tilde{O}(\epsilon^{-2})$ unlabeled training examples to achieve error $O(\epsilon)$, which is the same asymptotic scaling as supervised learning methods. This suggests that while we pay a computational penalty for richer dependency structures, we are no less statistically efficient. In the appendix, we provide more details, including an explicit description of the algorithm and the step size used to achieve this result.

5 Experiments

We seek to experimentally validate three claims about our approach. Our first claim is that data programming can be an effective paradigm for building high quality machine learning systems, and we test this across three real-world text relation extraction applications. Our second claim is that data programming can be used successfully in conjunction with automatic feature generation methods, such as LSTM models. Finally, our third claim is that data programming is an intuitive and productive framework for application domain-expert users, and we report on our initial experiences with a group of bioinformatics researchers.

Relation Mention Extraction Tasks  In the relation mention extraction task, our objects are relation mention candidates $x = (e_1, e_2)$, which are pairs of entity mentions $e_1,e_2$ in unstructured text, and our goal is to learn a model that classifies each candidate as either a true textual assertion of the relation $R(e_1,e_2)$ or not. We examine a news application from the 2014 TAC-KBP Slot Filling challenge\(^2\) where we extract relations between real-world entities from articles; a clinical genomics application, where we extract causal relations between genetic mutations and phenotypes from the scientific literature; and a pharmacogenomics application where we extract interactions between genes, also from the scientific literature.

For each application, we or our collaborators originally built a system where a ground truth training set was programmatically generated by ordering the labeling functions as a sequence of if-then-return statements, and for each candidate, taking the first label emitted by this script as the training label. We refer to this as the if-then-return (ITR) approach, and note that it often required significant domain expert development time to tune (weeks or more). For this set of experiments, we then used the same labeling function sets within the framework of data programming. In Table 1, we see that we achieve consistent improvements: on average by 2.34 points in F1 score, including what would have been a winning score on the 2014 TAC-KBP challenge\(^2\).

We observed these performance gains across applications with very different labeling function sets. We describe the labeling function summary statistics—coverage is the percentage of objects that had at least one label, overlap is the percentage of objects with more than one label, and conflict is the percentage of objects with conflicting labels—and see in Table 2 that even in scenarios where $m$ is small, and conflict and overlap is relatively less common, we still realize performance gains.

\(^2\)http://www.nist.gov/tac/2014/KBP/
Additionally, on a disease mention extraction task (see Usability Study), which was written from scratch within the data programming paradigm, we allowed developers to supply dependencies of the basic types outlined in Sec. 4 and report a 2.3 point F1 score boost from incorporating this dependency information, which we believe illustrates the potential of further pursuing this approach of providing dependency structure.

Automatically-generated Features  We additionally compare both hand-tuned and automatically-generated features, where the latter are learned via an LSTM recurrent neural network (RNN) [12]. Conventional wisdom states that deep learning methods such as RNNs are prone to overfitting, thus rendering them ineffective over distantly-supervised training sets. In our experiments, however, we find that training them with the data programming may be effective, reporting a 9.79 point boost to precision and a 3.12 point F1 score improvement on the benchmark 2014 TAC-KBP (News) relation extraction task, over the baseline if-then-return approach. Additionally for comparison, our approach is a 5.98 point F1 score improvement over a state-of-the-art LSTM approach applied to the TAC-KBP task which was trained on hand-labeled data [30].

Usability Study  One of our hopes is that a non-ML-expert user will be more productive iterating on labeling functions than on features. To test this, we arranged a hackathon involving a handful of bioinformatics researchers. Their goal was to build a disease tagging system, a common and important challenge in the bioinformatics domain [10]. However, the hackathon did not have access to a labeled training set, and no feature engineering was performed. The entire effort was restricted to iterative labeling function development and the setup of candidates to be classified. In under eight hours, they had a dataset that was within 10 points of F1 of the supervised baseline; the gap was mainly due to recall issue in the candidate extraction phase. This suggests data programming may be a promising way to build high quality extractors, quickly.

6 Conclusion and Future Work

We introduced data programming, a new approach to generating large labeled training sets. We demonstrate that our approach can be used with automatic feature generation techniques to achieve high quality results. For some relation extraction tasks, we provided anecdotal evidence that our methods may be easier for domain expert users to build on. We hope to explore the limits of our approach on more sophisticated machine learning tasks, notably in imaging and structured prediction.

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A General Theoretical Results

In this section, we will state the full form of the theoretical results we alluded to in the body of the paper. First, we restate, in long form, our setup and assumptions.

We assume that, for some function \( h : [-1, 0, 1]^m \times [-1, 1] \mapsto [-1, 0, 1]^M \) of sufficient statistics, we are concerned with learning distributions, over the set \( \Omega = [-1, 0, 1]^m \times [-1, 1] \), of the form

\[
\pi_\theta(\Lambda, Y) = \frac{1}{Z_\theta} \exp(\theta^T h(\Lambda, Y)),
\]

where \( \theta \in \mathbb{R}^M \) is a parameter, and \( Z_\theta \) is the partition function that makes this a distribution. We assume that we are given, i.e. can derive from the data programming specification, some set \( \Theta \) of feasible parameters. This set must have the following two properties.

First, for any \( \theta \in \Theta \), learning the parameter \( \theta \) from (full) samples from \( \pi_\theta \) is possible, at least in some sense. More specifically, there exists an unbiased estimator \( \hat{\theta} \) that is a function of some number \( D \) samples from \( \pi_\theta \) (and is unbiased for all \( \theta \in \Theta \)) such that, for all \( \theta \in \Theta \) and for some \( c > 0 \),

\[
\text{Cov}(\hat{\theta}) \leq \frac{I}{2cD}.
\]

Second, for any \( \theta_1, \theta_2 \in \Theta \),

\[
\mathbb{E}_{(x,y) \sim \pi_\theta} \left[ \text{Var}_{(x,y) \sim \pi_\theta}(y_1 | \Lambda_1 = \Lambda_2) \right] \leq \frac{c}{M}.
\]

That is, we’ll always be reasonably certain in our guess for the value of \( y \), even if we are totally wrong about the true parameter \( \theta \).

On the other hand, we are also concerned with a distribution \( \pi^* \) which ranges over the set \( \mathcal{X} \times [-1, 1] \), and represents the distribution of training and test examples we are using to learn. These objects are associated with a labeling function \( \lambda : \mathcal{X} \mapsto [-1, 0, 1]^m \) and a feature function \( f : \mathcal{X} \mapsto \mathbb{R}^n \). We make three assumptions about this distribution. First, we assume that, given \((x, y) \sim \pi^*\), the class label \( y \) is independent of the features \( f(x) \) given the labels \( \lambda(x) \). That is,

\[
(x, y) \sim \pi^* \Rightarrow y \perp f(x) | \lambda(x).
\]

Second, we assume that we can describe the relationship between \( \lambda(x) \) and \( y \) in terms of our family in (10) above. That is, for some parameter \( \theta^* \in \Theta \),

\[
\mathbb{P}_{(x,y) \sim \pi^*}(\lambda(x) = \Lambda, y = Y) = \pi_{\theta^*}(\Lambda, Y).
\]

Third, we assume that the features themselves are bounded; for all \( x \in \mathcal{X} \),

\[
\|f(x)\| \leq 1.
\]

Our goal is twofold. First, we want to recover some estimate \( \hat{\theta} \) of the true parameter \( \theta^* \). Second, we want to produce a parameter \( \hat{w} \) that minimizes the regularized logistic loss

\[
l(w) = \mathbb{E}_{(x,y) \sim \pi^*} \left[ \log(1 + \exp(-w^T f(x)y)) \right] + \rho \|w\|^2.
\]

We actually accomplish this by minimizing a noise-aware loss function, given our recovered parameter \( \hat{\theta} \),

\[
l_p(w) = \mathbb{E}_{(\hat{x}, y) \sim \pi^*} \left[ \mathbb{E}_{(\Lambda, y) \sim \pi_\theta}(\log(1 + \exp(-w^T f(\hat{x})y)) | \Lambda = \lambda(\hat{x})) \right] + \rho \|w\|^2.
\]

In fact we can’t even minimize this; rather, we will be minimizing the empirical noise-aware loss function, which is only this in expectation. Since the analysis of logistic regression is not itself interesting, we assume that we are able to run some algorithm that produces an estimate \( \hat{w} \) which satisfies, for some \( \chi > 0 \),

\[
\mathbb{E} \left[ l_p(\hat{w}) - \min_w l_p(w) \bigg| \hat{\theta} \right] \leq \chi.
\]

The algorithm chosen can be anything, but in practice, we use stochastic gradient descent.

We learn \( \hat{\theta} \) and \( \hat{w} \) by running the following algorithm.

Under these assumptions, we are able to prove the following theorem about the behavior of Algorithm[1].
Algorithm 1 Data Programming

Require: Step size $\eta$, dataset $S \subset X$, and initial parameter $\theta_0 \in \Theta$.

$\theta \rightarrow \theta_0$

for all $x \in S$ do

Independently sample $(\Lambda, Y)$ from $\pi_{\theta_0}$, and $(\bar{\Lambda}, \bar{Y})$ from $\pi_{\theta_0}$ conditioned on $\Lambda = \lambda(x)$.

$\theta \leftarrow \theta + \eta(h(\Lambda, Y) - h(\bar{\Lambda}, \bar{Y}))$.

$\theta = P_{\Theta}(\theta)$ \quad \text{Here, $P_{\Theta}$ denotes orthogonal projection onto $\Theta$.}

end for

Compute $\hat{w}$ using the algorithm described in (15)

return $(\theta, \hat{w})$.

Theorem A.1. Suppose that we run Algorithm 1 on a data programming specification that satisfies conditions (11), (12), (13), (14), (15), and (16). Suppose further that, for some parameter $\epsilon > 0$, we use step size $\eta = \frac{c \epsilon^2}{4}$ and our dataset is of a size that satisfies $|S| = \frac{2}{c^2 \epsilon^2} \log \left( \frac{2 \|\theta_0 - \theta^*\|^2}{\epsilon} \right)$.

Then, we can bound the expected parameter error with

$E \left[ \|\theta - \theta^*\|^2 \right] \leq \epsilon^2 M$

and the expected risk with

$E \left[ l(\hat{w}) - \min_w l(w) \right] \leq \chi + \frac{c \epsilon}{2 \rho}$.

This theorem’s conclusions and assumptions can readily be seen to be identical to those of Theorem 2 in the main body of the paper, except that they apply to the slightly more general case of arbitrary $h$, rather than $h$ of the explicit form described in the body. Therefore, in order to prove Theorem 2 it suffices to prove Theorem A.1 which we will do in Section C.

B Theoretical Results for Independent Model

For the independent model, we can obtain a more specific version of Theorem A.1. In the independent model, the variables are, as before, $\Lambda \in \{-1, 0, 1\}^m$ and $Y \in \{-1, 1\}$. The sufficient statistics are $\Lambda_i Y$ and $\Lambda_i^2$.

To produce results that make intuitive sense, we also define the alternate parameterization

$P_{\pi}(\Lambda_i|Y) = \begin{cases} \beta_i \frac{1 + \gamma_i}{2} & \Lambda_i = Y \\ (1 - \beta_i) & \Lambda_i = 0 \\ \beta_i \frac{1 - \gamma_i}{2} & \Lambda_i = -Y \end{cases}$

In comparison to the parameters used in the body of the paper, we have

$\alpha_i = \frac{1 + \gamma_i}{2}$.

Now, we are concerned with models that are feasible. For a model to be feasible (i.e. for $\theta \in \Theta$), we require that it satisfy, for some constants $\gamma_{\text{min}} > 0$, $\gamma_{\text{max}} > 0$, and $\beta_{\text{min}}$,

$\gamma_{\text{min}} \leq \gamma_i \leq \gamma_{\text{max}} \quad \beta_{\text{min}} \leq \beta_i \leq \frac{1}{2}$. For $0 \leq \beta \leq 1$ and $-1 \leq \gamma \leq 1$.

For this model, we can prove the following corollary to Theorem A.1.
Corollary B.1. Suppose that we run Algorithm 1 on an independent data programming specification that satisfies conditions (13), (14), (15), and (16). Furthermore, assume that the number of labeling functions we use satisfies

\[
m \geq \frac{9.34 \operatorname{artanh}(\gamma_{\text{max}})}{\gamma_{\text{min}}^2} \log \left( \frac{24m \beta_{\text{min}}}{\beta_{\text{min}}} \right).
\]

Suppose further that, for some parameter \(\epsilon > 0\), we use step size

\[
\eta = \frac{\beta_{\text{min}} \epsilon^2}{16}
\]

and our dataset is of a size that satisfies

\[
|S| = \frac{32}{\beta_{\text{min}}^2 \epsilon^2} \log \left( \frac{2 ||\theta_0 - \theta^*||^2}{\epsilon} \right).
\]

Then, we can bound the expected parameter error with

\[
E \left[ \|\hat{\theta} - \theta^*\|^2 \right] \leq \epsilon^2 M
\]

and the expected risk with

\[
E \left[ l(\hat{w}) - \min_w l(w) \right] \leq \chi + \frac{\beta_{\text{min}} \epsilon}{8 \rho}.
\]

We can see that if, as stated in the body of the paper, \(\beta_i \geq 0.3\) and \(0.8 \leq \alpha_i \leq 0.9\) (which is equivalent to \(0.6 \leq \gamma_i \leq 0.8\)), then

\[
2000 \geq 1896.13 = \frac{9.34 \operatorname{artanh}(0.8)}{0.3 \cdot 0.6^3} \log \left( \frac{24 \cdot 2000}{0.3} \right).
\]

This means that, as stated in the paper, \(m = 2000\) is sufficient for this corollary to hold with

\[
|S| = \frac{32}{0.3^2 \cdot \epsilon^2} \log \left( \frac{2m \operatorname{artanh}(0.8) - \operatorname{artanh}(0.6)^2}{\epsilon} \right) = \frac{356}{\epsilon^2} \log \left( \frac{m}{3 \epsilon} \right).
\]

Thus, proving Corollary B.1 is sufficient to prove Theorem A.1 from the body of the paper. We will prove Corollary B.1 in Section E.

C Proof of Theorem A.1

First, we state some lemmas that will be useful in the proof to come.

Lemma D.1. Given a family of maximum-entropy distributions

\[
\pi_\theta(x) = \frac{1}{Z_\theta} \exp(\theta^T h(x)),
\]

for some function of sufficient statistics \(h : \Omega \mapsto \mathbb{R}^M\), if we let \(J : \mathbb{R}^M \mapsto \mathbb{R}\) be the maximum log-likelihood objective for some event \(A \subseteq \Omega\),

\[
J(\theta) = \log P_{x \sim \pi_\theta} (x \in A),
\]

then its gradient is

\[
\nabla J(\theta) = E_{x \sim \pi_\theta} [h(x) | x \in A] - E_{x \sim \pi_\theta} [h(x)]
\]

and its Hessian is

\[
\nabla^2 J(\theta) = \text{Cov}_{x \sim \pi_\theta} (h(x) | x \in A) - \text{Cov}_{x \sim \pi_\theta} (h(x)).
\]
Lemma D.2. Suppose that we are looking at a distribution from a data programming label model. That is, our maximum-entropy distribution can now be written in terms of two variables, the labeling function values \( \lambda \in \{-1, 0, 1\} \) and the class \( y \in \{-1, 1\} \), as
\[
\pi_\theta(\lambda, y) = \frac{1}{Z_\theta} \exp(\theta^T h(\lambda, y)),
\]
where we assume without loss of generality that for some \( M, h(\lambda, y) \in \mathbb{R}^M \) and \( \|h(\lambda, y)\|_\infty \leq 1 \). If we let \( J : \mathbb{R}^M \rightarrow \mathbb{R} \) be the maximum expected log-likelihood objective, under another distribution \( \pi^* \), for the event associated with the observed labeling function values \( \lambda \),
\[
J(\theta) = \mathbb{E}_{(x, y^*) \sim \pi^*} \left[ \log P_{(\lambda, y) \sim \pi_\theta}(\lambda = \lambda^*) \right],
\]
then its Hessian can be bounded with
\[
\nabla^2 J(\theta) \leq M \mathbb{E}_{(x, y^*) \sim \pi^*} \left[ \text{Var}_{(\lambda, y) \sim \pi_\theta}(y|\lambda = \lambda^*) \right] - I(\theta),
\]
where \( I(\theta) \) is the Fisher information.

Lemma D.3. Suppose that we are looking at a data programming distribution, as described in the text of Lemma [D.2].

Suppose further that we are concerned with some feasible set of parameters \( \Theta \subset \mathbb{R}^M \), such that the any model with parameters in this space satisfies the following two conditions.

First, for any \( \theta \in \Theta \), learning the parameter \( \theta \) from (full) samples from \( \pi_\theta \) is possible, at least in some sense. More specifically, there exists an unbiased estimator \( \hat{\theta} \) that is a function of some number \( D \) samples from \( \pi_\theta \) (and is unbiased for all \( \theta \in \Theta \)) such that, for all \( \theta \in \Theta \) and for some \( c > 0 \),
\[
\text{Cov}(\hat{\theta}) \leq \frac{I}{2cD}.
\]

Second, for any \( \theta, \theta^* \in \Theta \),
\[
\mathbb{E}_{(x, y^*) \sim \pi^*} \left[ \text{Var}_{(\lambda, y) \sim \pi_\theta}(y|\lambda = \lambda^*) \right] \leq \frac{c}{M}.
\]

That is, we’ll always be reasonably certain in our guess for the value of \( y \), even if we are totally wrong about the true parameter \( \theta^* \).

Under these conditions, the function \( J \) is strongly concave on \( \Theta \) with parameter of strong convexity \( c \).

Lemma D.4. Suppose that we are looking at a data programming maximum likelihood estimation problem, as described in the text of Lemma [D.2].

Suppose further that the objective function \( J \) is strongly concave with parameter \( c > 0 \).

If we run stochastic gradient descent on objective \( J \), using unbiased samples from a true distribution \( \pi_{\theta^*} \), where \( \theta^* \in \Theta \), then if we use step size
\[
\eta = \frac{c \epsilon^2}{4}
\]
and run (using a fresh sample at each iteration) for \( T \) steps, where
\[
T = \frac{2}{c^2 \epsilon^2} \log \left( \frac{2 \|\theta_0 - \theta^*\|^2}{\epsilon} \right)
\]
then we can bound the expected parameter estimation error with
\[
\mathbb{E} \left[ \|\hat{\theta} - \theta^*\|^2 \right] \leq c^2 M.
\]

Lemma D.5. Assume in our model that, without loss of generality, \( \|f(x)\| \leq 1 \) for all \( x \), and that in our true model \( \pi^* \), the class \( y \) is independent of the features \( f(x) \) given the labels \( \lambda(x) \).

Suppose that we now want to solve the expected loss minimization problem wherein we minimize the objective
\[
l(w) = \mathbb{E}_{(x, y) \sim \pi^*} \left[ \log(1 + \exp(-w^T f(x) y)) \right] + \rho \|w\|^2.
\]

We actually accomplish this by minimizing our noise-aware loss function, given our chosen parameter \( \hat{\theta} \),
\[
l_\theta(w) = \mathbb{E}_{(x, y) \sim \pi^*} \left[ \mathbb{E}_{(\lambda, Y) \sim \pi_\theta} \left[ \log(1 + \exp(-w^T f(\tilde{x}) Y)) | \Lambda = \lambda(\tilde{x}) \right] \right] + \rho \|w\|^2.
\]
In fact we can’t even minimize this; rather, we will be minimizing the empirical noise-aware loss function, which is only this in expectation. Suppose that doing so produces an estimate \( \hat{w} \) which satisfies, for some \( \chi > 0 \),

\[
\mathbb{E} \left[ l(\hat{w}) - \min_w l(w) \right] \leq \chi.
\]

(Here, the expectation is taken with respect to only the random variable \( \hat{w} \).) Then, we can bound the expected risk with

\[
\mathbb{E} \left[ l(\hat{w}) - \min_w l(w) \right] \leq \chi + \frac{c\epsilon}{2\rho}.
\]

Now, we restate and prove our main theorem.

**Theorem A.1.** Suppose that we run Algorithm 1 on a data programming specification that satisfies conditions (11), (12), (13), (14), (15), and (16). Suppose further that, for some parameter \( \epsilon > 0 \), we use step size \( \eta = \frac{c\epsilon^2}{4} \) and our dataset is of a size that satisfies

\[
|S| = \frac{2}{c^2\epsilon^2} \log \left( \frac{2 \|	heta_0 - \theta^*\|^2}{\epsilon} \right).
\]

Then, we can bound the expected parameter error with

\[
\mathbb{E} \left[ \|\hat{\theta} - \theta^*\|^2 \right] \leq \epsilon^2 M
\]

and the expected risk with

\[
\mathbb{E} \left[ l(\hat{w}) - \min_w l(w) \right] \leq \chi + \frac{c\epsilon}{2\rho}.
\]

**Proof.** The bounds on the expected parameter estimation error follow directly from Lemma D.4, and the remainder of the theorem follows directly from Lemma D.5. \( \square \)

### D Proofs of Lemmas

**Lemma D.1.** Given a family of maximum-entropy distributions

\[
\pi_{\rho}(x) = \frac{1}{Z_{\rho}} \exp(\theta^T h(x)),
\]

for some function of sufficient statistics \( h : \Omega \mapsto \mathbb{R}^M \), if we let \( J : \mathbb{R}^M \mapsto \mathbb{R} \) be the maximum log-likelihood objective for some event \( A \subseteq \Omega \),

\[
J(\theta) = \log \mathbf{P}_{\pi_\rho}(x \in A),
\]

then its gradient is

\[
\nabla J(\theta) = \mathbf{E}_{\pi_\rho}[h(x)|x \in A] - \mathbf{E}_{\pi_\rho}[h(x)]
\]

and its Hessian is

\[
\nabla^2 J(\theta) = \mathbf{Cov}_{\pi_\rho}(h(x)|x \in A) - \mathbf{Cov}_{\pi_\rho}(h(x)).
\]

**Proof.** For the gradient,

\[
\nabla J(\theta) = \nabla \log \mathbf{P}_{\pi_\rho}(A)
\]

\[
= \nabla \log \left( \frac{\sum_{x \in A} \exp(\theta^T h(x))}{\sum_{x \in \Omega} \exp(\theta^T h(x))} \right)
\]

\[
= \nabla \log \left( \sum_{x \in A} \exp(\theta^T h(x)) \right) - \nabla \log \left( \sum_{x \in \Omega} \exp(\theta^T h(x)) \right)
\]

\[
= \frac{\sum_{x \in A} h(x) \exp(\theta^T h(x))}{\sum_{x \in \Omega} \exp(\theta^T h(x))} - \frac{\sum_{x \in \Omega} h(x) \exp(\theta^T h(x))}{\sum_{x \in \Omega} \exp(\theta^T h(x))}
\]

\[
= \mathbf{E}_{\pi_\rho}[h(x)|x \in A] - \mathbf{E}_{\pi_\rho}[h(x)].
\]
And for the Hessian,
\[
\nabla^2 J(\theta) = \nabla \frac{\sum_{x \in A} h(x) \exp(\theta^T h(x))}{\sum_{x \in A} \exp(\theta^T h(x))} - \nabla \frac{\sum_{x \in \Omega} h(x) \exp(\theta^T h(x))}{\sum_{x \in \Omega} \exp(\theta^T h(x))}
\]
= \frac{\sum_{x \in A} h(x)(h(x)^T \exp(\theta^T h(x)) - \left(\sum_{x \in A} h(x) \exp(\theta^T h(x))\right) \left(\sum_{x \in \Omega} h(x) \exp(\theta^T h(x))\right)^T}{\sum_{x \in A} \exp(\theta^T h(x))}
- \frac{\left(\sum_{x \in A} h(x) \exp(\theta^T h(x))\right) \left(\sum_{x \in \Omega} h(x) \exp(\theta^T h(x))\right)^T}{\left(\sum_{x \in \Omega} \exp(\theta^T h(x))\right)^2}
= E_{x \sim P} \left[h(x) (h(x)^T \mathbf{1} \in A) - E_{x \sim P} [h(x) | x \in A] E_{x \sim P} [h(x) | x \in A]^T\right]
- \left(E_{x \sim P} [h(x) | x \in A] - \left(E_{x \sim P} [h(x) | x \in A]\right)^T\right)
= \text{Cov}_{x \sim P} (h(x) | x \in A) - \text{Cov}_{x \sim P} (h(x)).
\]

Lemma D.2. Suppose that we are looking at a distribution from a data programming label model. That is, our maximum-entropy distribution can now be written in terms of two variables, the labeling function values \( \lambda \in \{-1, 0, 1\} \) and the class \( y \in \{-1, 1\} \), as
\[
\pi_\theta(\lambda, y) = \frac{1}{Z_\theta} \exp(\theta^T h(\lambda, y)),
\]
where we assume without loss of generality that for some \( M, h(\lambda, y) \in \mathbb{R}^M \) and \( \|h(\lambda, y)\|_\infty \leq 1 \). If we let \( J : \mathbb{R}^M \rightarrow \mathbb{R} \) be the maximum expected log-likelihood objective, under another distribution \( \pi \), for the event associated with the observed labeling function values \( \lambda \),
\[
J(\lambda) = E_{x \sim P} \left[\log P(\lambda, y \mid \lambda = \lambda')\right],
\]
then its Hessian can be bounded with
\[
\nabla^2 J(\lambda) \leq M I(\lambda') \text{Cov}_{(\lambda, y') \sim \pi} (y | \lambda = \lambda') - I(\theta),
\]
where \( I(\theta) \) is the Fisher information.

Proof. From the result of Lemma [D.1], we have that
\[
\nabla^2 J(\lambda) = E_{(\lambda', y') \sim \pi} \left[\text{Cov}_{(\lambda, y) \sim \pi} (h(\lambda, y) | \lambda = \lambda')\right] - \text{Cov}_{(\lambda, y) \sim \pi} (h(\lambda, y)).
\]
We start by defining \( h_0(\lambda) \) and \( h_1(\lambda) \) such that
\[
h(\lambda, y) = h(\lambda, 1) \frac{1 + y}{2} + h(\lambda, -1) \frac{1 - y}{2} = \frac{h(\lambda, 1) + h(\lambda, -1)}{2} + \frac{y h(\lambda, 1) - h(\lambda, -1)}{2} = h_0(\lambda) + y h_1(\lambda).
\]
This allows us to reduce (17) to
\[
\nabla^2 J(\lambda) = E_{(\lambda', y') \sim \pi} \left[h_1(\lambda') h(\lambda')^T \text{Var}_{(\lambda, y) \sim \pi} (y | \lambda = \lambda')\right] - \text{Cov}_{(\lambda, y) \sim \pi} (h(\lambda, y)).
\]
On the other hand, the Fisher information of this model at \( \theta \) is
\[
I(\theta) = E \left[\nabla_\theta \log \pi_\theta(x) \right]^2
= E \left[\nabla_\theta \log \left(\frac{\exp(\theta^T h(x))}{\sum_{z \in \Omega} \exp(\theta^T h(z))}\right)\right]^2
= E \left[\nabla_\theta \log \left(\frac{\exp(\theta^T h(x))}{\sum_{z \in \Omega} \exp(\theta^T h(z))}\right) - \nabla_\theta \log \left(\sum_{z \in \Omega} \exp(\theta^T h(z))\right)\right]^2
= E \left[h(x) - \frac{\sum_{z \in \Omega} h(z) \exp(\theta^T h(z))}{\sum_{z \in \Omega} \exp(\theta^T h(z))}\right]^2
= E \left[(h(x) - E[h(z)])^2\right]
= \text{Cov}(h(x)).
\]
That is, we’ll always be reasonably certain in our guess for the value of $y$, even if we are totally wrong about the true

This implies that $J$. Therefore, for all $\theta$.

This is the desired result.

**Lemma D.3.** Suppose that we are looking at a data programming distribution, as described in the text of Lemma D.2. Suppose further that we are concerned with some feasible set of parameters $\Theta \subset \mathbb{R}^M$, such that the any model with parameters in this space satisfies the following two conditions.

First, for any $\theta \in \Theta$, learning the parameter $\theta$ from (full) samples from $\pi_0$ is possible, at least in some sense. More specifically, there exists an unbiased estimator $\hat{\theta}$ that is a function of some number $D$ samples from $\pi_0$ (and is unbiased for all $\theta \in \Theta$) such that, for all $\theta \in \Theta$ and for some $c > 0$,

$$\text{Cov} \left( \hat{\theta} \right) \leq \frac{1}{2cD}.$$  

Second, for any $\theta, \theta' \in \Theta$,

$$\mathbb{E}_{(\lambda', y') \sim \pi} \left[ \text{Var}_{(\lambda, y) \sim \pi_0} (y|\lambda = \lambda') \right] \leq \frac{c}{M}.$$  

That is, we’ll always be reasonably certain in our guess for the value of $y$, even if we are totally wrong about the true parameter $\theta'$.

Under these conditions, the function $J$ is strongly concave on $\Theta$ with parameter of strong convexity $c$.

**Proof.** From the Cramér-Rao bound, we know in general that the variance of any unbiased estimator is bounded by the reciprocal of the Fisher information

$$\text{Cov} \left( \hat{\theta} \right) \geq (I(\theta))^{-1}. $$

Since for the estimator described in the lemma statement, we have $D$ independent samples from the distribution, it follows that the Fisher information of this experiment is $D$ times the Fisher information of a single sample. Combining this with the bound in the lemma statement on the covariance, we get

$$\frac{1}{2cD} \geq \text{Cov} \left( \hat{\theta} \right) \geq (DI(\theta))^{-1}. $$

It follows that $I(\theta) \geq 2cI$.

On the other hand, also from the lemma statement, we can conclude that

$$M\mathbb{E}_{(\lambda', y') \sim \pi} \left[ \text{Var}_{(\lambda, y) \sim \pi_0} (y|\lambda = \lambda') \right] \leq cI.$$  

Therefore, for all $\theta \in \Theta$,

$$\nabla^2 J(\theta) \leq M\mathbb{E}_{(\lambda', y') \sim \pi} \left[ \text{Var}_{(\lambda, y) \sim \pi_0} (y|\lambda = \lambda') \right] - I(\theta) \leq -cI.$$  

This implies that $J$ is strongly concave over $\Theta$, with constant $c$, as desired. \hfill \square

**Lemma D.4.** Suppose that we are looking at a data programming maximum likelihood estimation problem, as described in the text of Lemma D.2. Suppose further that the objective function $J$ is strongly concave with parameter $c > 0$.

If we run stochastic gradient descent on objective $J$, using unbiased samples from a true distribution $\pi_0$, where $\theta^* \in \Theta$, then if we use step size

$$\eta = \frac{ce^2}{4}$$

would be approximately $\mathbb{E}_{(\lambda', y') \sim \pi} \left[ \text{Var}_{(\lambda, y) \sim \pi_0} (y|\lambda = \lambda') \right] - cI$. This implies that $J$ is strongly concave over $\Theta$, with constant $c$, as desired. \hfill \square
and run (using a fresh sample at each iteration) for \( T \) steps, where

\[
T = \frac{2}{c^2 \epsilon^2} \log \left( \frac{2 \| \theta_0 - \theta^* \|^2}{\epsilon} \right)
\]

then we can bound the expected parameter estimation error with

\[
E \left[ \| \hat{\theta} - \theta^* \|^2 \right] \leq c^2 M.
\]

Proof. First, we note that, in the proof to follow, we can ignore the projection onto the feasible set \( \Theta \), since this projection always takes us closer to the optimum \( \theta^* \).

If we track the expected distance to the optimum \( \theta^* \), then at the next timestep,

\[
\| \theta_{t+1} - \theta^* \|^2 = \| \theta_t - \theta^* \|^2 + 2 \gamma (\theta_t - \theta^*) \nabla \hat{J}(\theta_t) + \gamma^2 \| \nabla \hat{J}(\theta_t) \|^2.
\]

Since we can write our stochastic samples in the form

\[
\nabla \hat{J}(\theta_t) = h(\lambda_t, y_t) - h(\bar{\lambda}_t, \bar{y}_t),
\]

for some samples \( \lambda_t, y_t, \bar{\lambda}_t, \) and \( \bar{y}_t \), we can conclude that

\[
\| \nabla \hat{J}(\theta_t) \|^2 \leq M \| \nabla \hat{J}(\theta_t) \|_\infty \leq 4M.
\]

Therefore, taking the expected value conditioned on the filtration,

\[
E \left[ \| \theta_{t+1} - \theta^* \|^2 | F_t \right] = \| \theta_t - \theta^* \|^2 + 2\gamma (\theta_t - \theta^*) \nabla \hat{J}(\theta_t) + 4\gamma^2 M.
\]

Since \( J \) is strongly concave,

\[
(\theta_t - \theta^*) \nabla J(\theta_t) \leq -\gamma \| \theta_t - \theta^* \|^2;
\]

and so,

\[
E \left[ \| \theta_{t+1} - \theta^* \|^2 | F_t \right] \leq (1 - 2\gamma c) \| \theta_t - \theta^* \|^2 + 4\gamma^2 M.
\]

If we take the full expectation and subtract the fixed point from both sides,

\[
E \left[ \| \theta_{t+1} - \theta^* \|^2 \right] - \frac{2\gamma M}{c} \leq (1 - 2\gamma c) E \left[ \| \theta_t - \theta^* \|^2 \right] + 4\gamma^2 M - \frac{2\gamma M}{c} = (1 - 2\gamma c) \left( E \left[ \| \theta_t - \theta^* \|^2 \right] - \frac{2\gamma M}{c} \right).
\]

Therefore,

\[
E \left[ \| \theta_t - \theta^* \|^2 \right] - \frac{2\gamma M}{c} \leq (1 - 2\gamma c) \left( \| \theta_0 - \theta^* \|^2 - \frac{2\gamma M}{c} \right),
\]

and so

\[
E \left[ \| \theta_t - \theta^* \|^2 \right] \leq \exp(-2\gamma c) \| \theta_0 - \theta^* \|^2 + \frac{2\gamma M}{c}.
\]

In order to ensure that

\[
E \left[ \| \theta_t - \theta^* \|^2 \right] \leq \epsilon^2,
\]

it therefore suffices to pick

\[
\gamma = \frac{c \epsilon^2}{4M}
\]

and

\[
t = \frac{2M}{c^2 \epsilon^2} \log \left( \frac{2 \| \theta_0 - \theta^* \|^2}{\epsilon} \right).
\]

Substituting \( \epsilon^2 \to \epsilon^2 M \) produces the desired result. \( \square \)
Lemma D.5. Assume in our model that, without loss of generality, $\|f(x)\| \leq 1$ for all $x$, and that in our true model $\pi^*$, the class $y$ is independent of the features $f(x)$ given the labels $\lambda(x)$.

Suppose that we now want to solve the expected loss minimization problem wherein we minimize the objective

$$l(w) = E_{(x,y) \sim \pi^*} \left[ \log(1 + \exp(-w^T f(x)y)) \right] + \rho \|w\|^2.$$  

We actually accomplish this by minimizing our noise-aware loss function, given our chosen parameter $\hat{\theta}$,

$$l_\theta(w) = E_{(x,y) \sim \pi} \left[ \log(1 + \exp(-w^T f(x)y))\|\Lambda = \lambda(x)\| + \rho \|w\|^2. $$

In fact we can’t even minimize this; rather, we will be minimizing the empirical noise-aware loss function, which is only this in expectation. Suppose that doing so produces an estimate $\hat{\theta}$ which satisfies, for some $\chi > 0$,

$$E \left[ l_\theta(\hat{\theta}) - \min_w l_\theta(w) \right] \leq \chi.$$  

(Here, the expectation is taken with respect to only the random variable $\hat{\theta}$.) Then, we can bound the expected risk with

$$E \left[ l(\hat{\theta}) - \min_w l(w) \right] \leq \chi + \frac{c \epsilon}{2^\rho}.$$ 

Proof. (To simplify the symbols in this proof, we freely use $\theta$ when we mean $\hat{\theta}$.)

The loss function we want to minimize is, in expectation,

$$l(w) = E_{(x,y) \sim \pi^*} \left[ \log(1 + \exp(-w^T f(x)y)) \right] + \rho \|w\|^2.$$  

By the law of total expectation,

$$l(w) = E_{(x,y) \sim \pi} \left[ E_{(x,y) \sim \pi^*} \left[ \log(1 + \exp(-w^T f(x)y)) \right] \right] + \rho \|w\|^2,$$

and by our conditional independence assumption,

$$l(w) = E_{(x,y) \sim \pi} \left[ \log(1 + \exp(-w^T f(x)y))\|\lambda = \lambda(x)\| \right] + \rho \|w\|^2.$$  

Since we know from our assumptions that, for the optimum parameter $\theta^*$,

$$P_{(x,y) \sim \pi^*} (\lambda) = \Lambda, y = Y = P_{(x,y) \sim \pi} (\lambda = \Lambda, y = Y),$$

we can rewrite this as

$$l(w) = E_{(x,y) \sim \pi^*} \left[ E_{(x,y) \sim \pi^*} \left[ \log(1 + \exp(-w^T f(x)y))\|\lambda = \lambda(x)\| \right] \right] + \rho \|w\|^2.$$  

On the other hand, if we are minimizing the model we got from the previous step, we will be actually minimizing

$$l_\theta(w) = E_{(x,y) \sim \pi} \left[ \log(1 + \exp(-w^T f(x)y))\|\lambda = \lambda(x)\| \right] + \rho \|w\|^2.$$  

We can reduce this further by noticing that

$$E_{(x,y) \sim \pi^*} \left[ \log(1 + \exp(-w^T f(x)y))\|\lambda = \lambda(x)\| \right]$$

$$= E_{(x,y) \sim \pi} \left[ \log(1 + \exp(-w^T f(x)y)) \right] + \log(1 + \exp(w^T f(x)))$$

$$= \log(1 + \exp(-w^T f(x))) + \log(1 + \exp(w^T f(x)))$$

$$+ \log(1 + \exp(-w^T f(x))) - \log(1 + \exp(w^T f(x))) E_{(x,y) \sim \pi^*} \left[ \Lambda = \lambda(x) \right]$$

$$= \frac{\log(1 + \exp(-w^T f(x))) + \log(1 + \exp(w^T f(x)))}{2} - \frac{w^T f(x) E_{(x,y) \sim \pi^*} \left[ \Lambda = \lambda(x) \right]}{2}.$$  

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It follows that the difference between the loss functions will be
\[ |l(w) - l_\theta(w)| = \left| \mathbf{E}_{(\bar{x},\bar{y})} \left[ \frac{w^T f(\bar{x})}{2} \left( \mathbf{E}_{(\bar{x},\bar{y})} [Y|\Lambda = \lambda(x)] - \mathbf{E}_{(\bar{x},\bar{y})} [Y|\Lambda = \lambda(\bar{x})] \right) \right] \right|. \]

Now, we can compute that
\[
\nabla_\theta \mathbf{E}_{(\bar{x},\bar{y})} [Y|\Lambda = \lambda] = \nabla_\theta \left[ \exp(\theta^T h(\lambda, 1)) - \exp(\theta^T h(\lambda, -1)) \right] \\
= \nabla_\theta \left[ \exp(\theta^T h(\lambda)) - \exp(-\theta^T h(\lambda)) \right] \\
= \nabla_\theta \tanh(\theta^T h(\lambda)) \\
= h_1(\lambda) \left( 1 - \tanh^2(\theta^T h(\lambda)) \right) \\
= h_1(\lambda) \mathbf{Var}_{(\bar{x},\bar{y})} (Y|\Lambda = \lambda).
\]

It follows by the mean value theorem that for some \( \psi \), a linear combination of \( \theta \) and \( \theta^* \),
\[ |l(w) - l_\theta(w)| = \left| \mathbf{E}_{(\bar{x},\bar{y})} \left[ \frac{w^T f(\bar{x})}{2} (\theta - \theta^*)^T h_1(\lambda) \mathbf{Var}_{(\bar{x},\bar{y})} (Y|\Lambda = \lambda) \right] \right|. \]
Since \( \Theta \) is convex, clearly \( \psi \in \Theta \). From our assumption on the bound of the variance, we can conclude that
\[ \mathbf{E}_{(\bar{x},\bar{y})} \left[ \mathbf{Var}_{(\bar{x},\bar{y})} (Y|\Lambda = \lambda) \right] \leq \frac{c}{M}. \]
By the Cauchy-Schwarz inequality,
\[ |l(w) - l_\theta(w)| \leq \frac{1}{2} \left[ \mathbf{E}_{(\bar{x},\bar{y})} \left[ |w| \|f(\bar{x})\| \|\theta - \theta^*\| \|h_1(\lambda)\| \mathbf{Var}_{(\bar{x},\bar{y})} (Y|\Lambda = \lambda) \right] \right] \leq \frac{\|w\| \|\theta - \theta^*\| \sqrt{M}}{2} \frac{c}{M} = \frac{c \sup_{\theta} \|\theta - \theta^*\|}{2 \sqrt{M}}. \]
Now, for any \( w \) that could conceivably be a solution, it must be the case that
\[ \|w\| \leq \frac{1}{2\rho}, \]
since otherwise the regularization term would be too large. Therefore, for any possible solution \( w \),
\[ |l(w) - l_\theta(w)| \leq \frac{c \sup_{\theta} \|\theta - \theta^*\|}{4\rho \sqrt{M}}. \]
Now, we apply the assumption that we are able to solve the empirical problem, producing an estimate \( \hat{w} \) that satisfies
\[ \mathbf{E} [l_\theta(\hat{w}) - l_\theta(w^*_\theta)] \leq \chi, \]
where \( w^*_\theta \) is the true solution to
\[ w^*_\theta = \arg \min_w l_\theta(w). \]
Therefore,
\[
E \left[ l(\hat{w}) - l(w^*) \right] = E \left[ l_\theta(\hat{w}) - l_\theta(w^*_\theta) + l_\theta(w^*_\theta) - l_\theta(\hat{w}) + l(\hat{w}) - l(w^*) \right] \\
\leq \chi + E \left[ l_\theta(w^*) - l_\theta(\hat{w}) + l(\hat{w}) - l(w^*) \right] \\
\leq \chi + E \left[ |l_\theta(w^*) - l(\hat{w})| + |l_\theta(\hat{w}) - l(\hat{w})| \right]
\leq \chi + E \left[ c \|	heta - \theta^*\| \right] \\
= \chi + \frac{c}{2 \rho \sqrt{M}} E \left[ \|	heta - \theta^*\| \right] \\
\leq \chi + \frac{c}{2 \rho \sqrt{M}} E \left[ \|	heta - \theta^*\| \right].
\]

We can now bound this using the result of Lemma D.4, which results in
\[
E \left[ l(\hat{w}) - l(w^*) \right] \leq \chi + \frac{c}{2 \rho \sqrt{M}} \sqrt{M} \epsilon^2 \\
= \chi + \frac{c \epsilon}{2 \rho}.
\]
This is the desired result. \(\square\)

**E  Proofs of Results for the Independent Model**

To restate, in the independent model, the variables are, as before, \(\Lambda \in \{-1, 0, 1\}^m\) and \(Y \in \{-1, 1\}\). The sufficient statistics are \(\Lambda_Y\) and \(\Lambda^2_Y\). That is, for expanded parameter \(\theta = (\psi, \phi)\),
\[
\pi_\theta(\Lambda, Y) = \frac{1}{Z} \exp(\psi^T \Lambda Y + \phi^T \Lambda^2).
\]

This can be combined with the simple assumption that \(P(Y) = \frac{1}{2}\) to complete a whole distribution. Using this, we can prove the following simple result about the moments of the sufficient statistics.

**Lemma E.1.** The expected values and covariances of the sufficient statistics are, for all \(i \neq j\),
\[
E[\Lambda_i Y] = \beta_i \gamma_i, \\
E[\Lambda_i^2] = \beta_i, \\
\text{Var}(\Lambda_i Y) = \beta_i - \beta_i^2 \gamma_i^2, \\
\text{Var}(\Lambda_i^2) = \beta_i - \beta_i^2, \\
\text{Cov}(\Lambda_i Y, \Lambda_j Y) = 0, \\
\text{Cov}(\Lambda_i^2, \Lambda_j^2) = 0, \\
\text{Cov}(\Lambda_i Y, \Lambda_j^2) = 0.
\]

We also prove the following basic lemma that relates \(\psi_i\) to \(\gamma_i\).

**Lemma E.2.** It holds that
\[\gamma_i = \tanh(\psi_i).\]

We also make the following claim about feasible models.

**Lemma E.3.** For any feasible model, it will be the case that, for any other feasible parameter vector \(\hat{\psi}\),
\[
P\left(\hat{\psi}^T \Lambda Y \leq \frac{m}{2} \gamma_{\min}(\gamma \beta)_{\min}\right) \leq \exp\left(-\frac{m(\gamma \beta)_{\min} \gamma^2_{\min}}{9.34 \arctanh(\gamma_{\max})}\right).
\]
We can also prove the following simple result about the conditional covariances

**Lemma E.4.** The covariances of the sufficient statistics, conditioned on $\Lambda$, are for all $i \neq j$,

\[
\text{Cov} \left( \Lambda_i Y, \Lambda_j Y \mid \Lambda \right) = \Lambda_i \Lambda_j \text{sech}^2 (\phi^T \Lambda)
\]

\[
\text{Cov} \left( \Lambda_i^2, \Lambda_j^2 \mid \Lambda \right) = 0.
\]

We can combine these two results to bound the expected variance of these conditional statistics.

**Lemma E.5.** If $\theta$ and $\theta^*$ are two feasible models, then for any $u$,

\[
E_{\theta^*} [\text{Var}_{\theta} (Y \mid \Lambda)] \leq 3 \exp \left( -\frac{m \beta_{\min}^2 \gamma_{\min}^2}{8 \text{artanh}(\gamma_{\max})} \right).
\]

We can now proceed to restate and prove the main corollary of Theorem A.1 that applies in the independent case.

**Corollary B.1.** Suppose that we run Algorithm 1 on an independent data programming specification that satisfies conditions (13), (14), (15), and (16). Furthermore, assume that the number of labeling functions we use satisfies

\[
m \geq 9.34 \frac{\text{artanh}(\gamma_{\max})}{(\gamma \beta_{\min} \gamma_{\min})} \log \left( \frac{24m \beta_{\min}}{\epsilon} \right).
\]

Suppose further that, for some parameter $\epsilon > 0$, we use step size

\[
\eta = \frac{\beta_{\min} \epsilon^2}{16}
\]

and our dataset is of a size that satisfies

\[
|S| = \frac{32}{\beta_{\min}^2 \epsilon^2} \log \left( \frac{2 \|\theta_0 - \theta^*\|^2}{\epsilon} \right).
\]

Then, we can bound the expected parameter error with

\[
E \left[ \|\hat{\theta} - \theta^*\|^2 \right] \leq \epsilon^2 M
\]

and the expected risk with

\[
E \left[ l(\hat{w}) - \min_w l(w) \right] \leq \chi + \frac{\beta_{\min} \epsilon}{8 \rho}.
\]

**Proof.** In order to apply Theorem A.1, we have to verify all its conditions hold in the independent case.

First, we notice that (11) is used only to bound the covariance of the sufficient statistics. From Lemma E.1, we know that these can be bounded by $\beta_{\min}^2 \gamma_{\min}^2 \geq \frac{\beta_{\min}}{2}$. It follows that we can choose

\[
c = \frac{\beta_{\min}}{4},
\]

and we can consider (11) satisfied, for the purposes of applying the theorem.

Second, to verify (12), we can use Lemma E.5. For this to work, we need

\[
3 \exp \left( -\frac{m \gamma_{\min} \gamma_{\max}^2}{9.34 \text{artanh}(\gamma_{\max})} \right) \leq \frac{c}{M} = \frac{\beta_{\min}}{8m}.
\]

This happens whenever the number of labeling functions satisfies

\[
m \geq 9.34 \frac{\text{artanh}(\gamma_{\max})}{(\gamma \beta_{\min} \gamma_{\min})} \log \left( \frac{24m}{\beta_{\min}} \right).
\]

The remaining assumptions, (13), (14), (15), and (16), are satisfied directly by the assumptions of this corollary. So, we can apply Theorem A.1 which produces the desired result. □
F Proofs of Independent Model Lemmas

Lemma E.1. The expected values and covariances of the sufficient statistics are, for all $i \neq j$,

\[
E[\Lambda_Y] = \beta_i \gamma_i \\
E[\Lambda_i^2] = \beta_i \\
\text{Var}(\Lambda_Y) = \beta_i - \beta_i^2 \gamma_i^2 \\
\text{Var}(\Lambda_i^2) = \beta_i - \beta_i^2 \\
\text{Cov}(\Lambda_Y, \Lambda_i Y) = 0 \\
\text{Cov}(\Lambda_i^2, \Lambda_i Y) = 0 \\
\text{Cov}(\Lambda_i Y, \Lambda_i^2) = 0.
\]

Proof. We prove each of the statements in turn. For the first statement,

\[
E[\Lambda_Y] = P(\Lambda_Y = Y) - P(\Lambda_Y = -Y) = \beta_i \frac{1 + \gamma_i}{2} - \beta_i \frac{1 - \gamma_i}{2} = \beta_i \gamma_i.
\]

For the second statement,

\[
E[\Lambda_i^2] = P(\Lambda = Y) + P(\Lambda = -Y) = \beta_i \frac{1 + \gamma_i}{2} + \beta_i \frac{1 - \gamma_i}{2} = \beta_i.
\]

For the remaining statements, we derive the second moments; converting these to an expression of the covariance is trivial. For the third statement,

\[
E[(\Lambda_Y)^2] = E[\Lambda_i^2 Y^2] = E[\Lambda_i^2] = \beta_i.
\]

For the fourth statement,

\[
E[(\Lambda_i^2)^2] = E[\Lambda_i^4] = E[\Lambda_i^2] = \beta_i.
\]

For subsequent statements, we first derive that

\[
E[\Lambda_i Y|Y] = \beta_i \frac{1 + \gamma_i}{2} - \beta_i \frac{1 - \gamma_i}{2} = \beta_i \gamma_i.
\]

and

\[
E[\Lambda_i^2|Y] = \beta_i \frac{1 + \gamma_i}{2} + \beta_i \frac{1 - \gamma_i}{2} = \beta_i.
\]

Now, for the fifth statement,

\[
E[(\Lambda_Y)(\Lambda_i Y)] = E[E[\Lambda_Y|Y]E[\Lambda_i Y|Y]] = \beta_i \gamma_i \beta_j.
\]

For the sixth statement,

\[
E[(\Lambda_i^2)(\Lambda_j^2)] = E[E[\Lambda_i^2|Y]E[\Lambda_j^2|Y]] = \beta_i \beta_j.
\]

Finally, for the seventh statement,

\[
E[(\Lambda_Y)(\Lambda_i^2)] = E[E[\Lambda_Y|Y]E[\Lambda_i^2|Y]] = \beta_i \gamma_i \beta_j.
\]

This completes the proof. □

Lemma E.2. It holds that

\[
\gamma_i = \tanh(\psi_i).
\]
Proof. From the definitions,
\[ \beta_i = \frac{\exp(\psi_i + \phi_i) + \exp(-\psi_i + \phi_i)}{\exp(\psi_i + \phi_i) + \exp(-\psi_i + \phi_i) + 1} \]
and
\[ \beta_i \gamma_i = \frac{\exp(\psi_i + \phi_i) - \exp(-\psi_i + \phi_i)}{\exp(\psi_i + \phi_i) + \exp(-\psi_i + \phi_i) + 1}. \]
Therefore,
\[ \gamma_i = \frac{\exp(\psi_i + \phi_i) - \exp(-\psi_i + \phi_i)}{\exp(\psi_i + \phi_i) + \exp(-\psi_i + \phi_i)} = \tanh(\psi_i), \]
which is the desired result. \(\square\)

Lemma E.3. For any feasible model, it will be the case that, for any other feasible parameter vector \(\hat{\psi}\),
\[ P\left( \hat{\psi}^T \Lambda Y \leq \frac{m}{2} \gamma_{\min} (\gamma \beta)_{\min} \right) \leq \exp\left( -\frac{m (\gamma \beta)_{\min} \gamma_{\min}^2}{9.34 \text{ artanh}(\gamma_{\max})} \right). \]

Proof. We start by noticing that
\[ \hat{\psi}^T \Lambda Y = \sum_{i=1}^{m} \hat{\psi}_i \Lambda_i Y. \]
Since in this model, all the \(\Lambda_i Y\) are independent of each other, we can bound this sum using a concentration bound. First, we note that
\[ |\hat{\psi}_i \Lambda_i Y| \leq \hat{\psi}_i. \]
Second, we note that
\[ E[\hat{\psi}_i \Lambda_i Y] = \hat{\psi}_i \beta_i \gamma_i \]
and
\[ \text{Var}(\hat{\psi}_i \Lambda_i Y) = \hat{\psi}_i^2 \left( \beta_i - \beta_i^2 \gamma_i^2 \right) \]
but
\[ |\hat{\psi}_i \Lambda_i Y| \leq \hat{\psi}_i \leq \text{artanh}(\gamma_{\max}) \triangleq \hat{\psi}_{\max} \]
because, for feasible models, by definition
\[ \gamma_{\min} \leq \text{artanh}(\gamma_{\min}) \leq \hat{\psi}_i \leq \text{artanh}(\gamma_{\max}). \]
Therefore, applying Bernstein’s inequality gives us, for any \(t\),
\[ P\left( \sum_{i=1}^{m} \hat{\psi}_i \Lambda_i Y - \sum_{i=1}^{m} \hat{\psi}_i \beta_i \gamma_i \leq -t \right) \leq \exp\left( -\frac{3t^2}{6 \sum_{i=1}^{m} \hat{\psi}_i^2 \gamma_i \beta_i \gamma_i + 2 \hat{\psi}_{\max} t} \right). \]
It follows that, if we let
\[ t = \frac{1}{2} \sum_{i=1}^{m} \hat{\psi}_i \beta_i \gamma_i, \]
then we get

\[
P \left( \sum_{i=1}^{m} \hat{\psi}_i \Lambda_i Y - \sum_{i=1}^{m} \hat{\psi}_i \beta_i \gamma_i \leq -t \right) \leq \exp \left( - \frac{3 \left( \frac{1}{2} \sum_{i=1}^{m} \hat{\psi}_i \beta_i \gamma_i \right)^2}{24 \sum_{i=1}^{m} \hat{\psi}_i^2 \beta_i \gamma_i + 4 \hat{\psi}_{\max} \sum_{i=1}^{m} \hat{\psi}_i \beta_i \gamma_i} \right) \]

\[
\leq \exp \left( - \frac{3 \left( \sum_{i=1}^{m} \hat{\psi}_i \beta_i \gamma_i \right)^2}{24 \sum_{i=1}^{m} \hat{\psi}_i^2 \beta_i \gamma_i + 4 \hat{\psi}_{\max} \sum_{i=1}^{m} \hat{\psi}_i \beta_i \gamma_i} \right) \]

\[
\leq \exp \left( - \frac{3 \gamma_{\min} \left( \sum_{i=1}^{m} \hat{\psi}_i \beta_i \gamma_i \right)}{24 \hat{\psi}_{\max} \sum_{i=1}^{m} \hat{\psi}_i \beta_i \gamma_i + 4 \hat{\psi}_{\max} \sum_{i=1}^{m} \hat{\psi}_i \beta_i \gamma_i} \right) \]

\[
\leq \exp \left( - \frac{3 \gamma_{\min} \left( \sum_{i=1}^{m} \hat{\psi}_i \beta_i \gamma_i \right)}{28 \hat{\psi}_{\max}} \right) \]

\[
\leq \exp \left( - \frac{m \gamma_{\min}^2 (\gamma \beta)_{\min}}{9.34 \hat{\psi}_{\max}} \right).
\]

This is the desired expression. \(\square\)

**Lemma E.4.** The covariances of the sufficient statistics, conditioned on \(\Lambda\), are for all \(i \neq j\),

\[
\text{Cov} \left( \Lambda_i Y, \Lambda_j Y | \Lambda \right) = \Lambda_i \Lambda_j \text{sech}^2(\psi^T \Lambda)
\]

\[
\text{Cov} \left( \Lambda_i^2, \Lambda_j^2 | \Lambda \right) = 0.
\]

**Proof.** The second result is obvious, so it suffices to prove only the first result. Clearly,

\[
\text{Cov} \left( \Lambda_i Y, \Lambda_j Y | \Lambda \right) = \Lambda_i \Lambda_j \text{Var} \left( Y | \Lambda \right) = \Lambda_i \Lambda_j \left( 1 - \mathbb{E} \left[ Y | \Lambda \right]^2 \right).
\]

Plugging into the distribution formula lets us conclude that

\[
\mathbb{E} \left[ Y | \Lambda \right] = \frac{\exp(\psi^T \Lambda + \phi^T \Lambda^2) - \exp(-\psi^T \Lambda + \phi^T \Lambda^2)}{\exp(\psi^T \Lambda + \phi^T \Lambda^2) + \exp(-\psi^T \Lambda + \phi^T \Lambda^2)} = \tanh^2(\psi^T \Lambda),
\]

and so

\[
\text{Cov} \left( \Lambda_i Y, \Lambda_j Y | \Lambda \right) = \Lambda_i \Lambda_j \left( 1 - \tanh^2(\psi^T \Lambda) \right) = \Lambda_i \Lambda_j \text{sech}^2(\psi^T \Lambda),
\]

which is the desired result. \(\square\)

**Lemma E.5.** If \(\theta\) and \(\theta^*\) are two feasible models, then for any \(u\),

\[
\mathbb{E}_{\theta^*} \left[ \text{Var}_{\theta}(Y|\Lambda) \right] \leq 3 \exp \left( - \frac{m \beta^2_{\min} \gamma^2_{\min}}{8 \arctanh(\gamma_{\max})} \right).
\]

**Proof.** First, we note that, by the result of Lemma E.4

\[
\text{Var}_{\theta}(Y|\Lambda) = \text{sech}^2(\psi^T \Lambda).
\]

Therefore,

\[
\mathbb{E}_{\theta^*} \left[ \text{Var}_{\theta}(Y|\Lambda) \right] = \mathbb{E}_{\theta^*} \left[ \text{sech}^2(\psi^T \Lambda) \right].
\]
Applying Lemma E.3, we can bound this with
\[
\mathbb{E}_{\theta} \left[ \text{Var}_\theta (u^T \Lambda Y | \Lambda) \right] \leq \left( \text{sech}^2 \left( \frac{m}{2} \gamma \beta_{\text{min}} \gamma_{\text{min}}^2 \right) + \exp \left( - \frac{m \gamma \beta_{\text{min}} \gamma_{\text{min}}^2}{9.34 \text{ artanh} (\gamma_{\text{max}})} \right) \right)
\]
\[
\leq \left( 2 \exp \left( - \frac{m}{2} \gamma \beta_{\text{min}} \gamma_{\text{min}}^2 \right) + \exp \left( - \frac{m \gamma \beta_{\text{min}} \gamma_{\text{min}}^2}{9.34 \text{ artanh} (\gamma_{\text{max}})} \right) \right)
\]
\[
\leq 3 \exp \left( - \frac{m \gamma \beta_{\text{min}} \gamma_{\text{min}}^2}{9.34 \text{ artanh} (\gamma_{\text{max}})} \right)
\]

This is the desired expression. □

G Additional Experimental Details

G.1 Synthetic Experiments

In Fig. 3(a-b), we ran synthetic experiments with labeling functions having constant coverage $\beta = 0.1$, and accuracy drawn from $\alpha \sim \text{Uniform} (\mu_\alpha - 0.25, \mu_\alpha + 0.25)$ where $\mu_\alpha = 0.75$ in the above plots. In both cases we used 1000 normally-drawn features having mean correlation with the true label class of 0.5.

In this case we compare data programming (DP-Pipelined) against two baselines. First, we compare against an if-then-return setup where the ordering is optimal (ITR-Oracle). Second, we compare against simple majority vote (MV).

In Fig. 3(c), we show an experiment where we add dependent labeling functions to a set of $m_{\text{ind}} = 50$ independent labeling functions, and either provided this dependency structure (LDM-Aware) or did not (Independent). In this case, the independent labeling functions had the same configurations as in (a-b), and the dependent labeling functions corresponded to “fixes” or “reinforces”-type dependent labeling functions.