The influence of the process-induced deformed state on straightness of prestressing strands after manufacture

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Abstract. Based on the study of elastic-plastic deformation, the expressions of longitudinal force $N$, bending $M$, and torque $T$ moments in wire-cross sections subject to the process parameters of rope torsion are obtained: wire tensioning efforts, untwisting and preformation.

A technique for determining the technological internal force factors (TIFF) in the rope cross-sections: the longitudinal forces $N_i(N_i)$; bending $M_i(N_i;N_b;M_b)$, $M_i(N_i;M_b;M_b)$ and torque $M_i(N_i;N_b;M_b)$ moments. Being guided by the theorem on unloading and FEM, the external deformed state of the rope unloading from TIFF is studied. The expression of the rope unloading deformations vector $\boldsymbol{\varepsilon} = [\varepsilon, \theta, \chi, \zeta]$ as a function of the TIFF vector using the global stiffness matrix of 4×4 rope cross-section corresponding to the elastic deformation is obtained.

Based on the bending deformations, an expression was obtained, the parameter $h$ ($h$ is the segment arrow at the chord length of 1 m), which, according to the regulatory standard, characterizes the degree of the rope straightness after manufacture. The connection between the parameters of torsion (tension, untwisting, and preformation) and the parameter $h$ of straightness is established.

It is determined that the unstraightness is directly affected only by the deviation from the uniformity of tension. The developed theory allows to determine the permissible deviation from the non-uniformity of tension with the permissible value of the parameter $h$ of straightness.

For example, a prestressing strand of a structure 1+6, having a diameter of 6.9 mm is considered to be of high quality according to the standard, if $h \leq 25 \text{mm}$. This value $h$ corresponds to the deviation of the tension of $0,8 \kappa$ one of the 6 wires of the layer. With an increase in deviation of tension, $h$ increases almost rectilinearly to $3 \kappa$. With further increase, a loop is formed – the loss of stability.

The obtained theory of the external deformed state of unloading provides further opportunity to study the internal deformed-stress state of the rope wires due to process-induced forces, which opens the prospect of even a deeper study of operational strength and durability of ropes.

1. Introduction

Prestressing strands are subject to the requirements of some strength and deformation characteristics [1, 2]. Numerical values of the characteristics vary from the state immediately after manufacture to a stable operating state.

Characteristics immediately after manufacture depend on the torsional tension (up to 10% of the breaking force of the wire [3]) and parameters of their curvature and cabling [4]:

$$b = \frac{\sin^2 \alpha}{r};$$  \hspace{1cm} (1)

$$t = \frac{\sin \alpha (k_0 + \cos \alpha)}{r},$$  \hspace{1cm} (2)
where $\alpha$ and $r$ – torsion angle and radius;
$\pm \kappa$ – cabling parameter ("+" twisting, "−" untwisting), provided by the ratio of the angular velocities of the coil and the rotor.

![Figure 1. – Tense state.](image)

In each wire, the internal forces of elasticity (Figure 1) are reduced to the longitudinal force $N_i$, bending $M_b$ and $M_t$ torque moments, which, being normalized to the axes of the rope cross-section, express its process-related internal force factors (TIFF). Their expressions given in [5–7] are specified and simplified in this work. TIFF provide a basis for determining the characteristics that are set by the standards [1–2].

On the basis of refined TIFF when using FEM to investigate the residual deformation after unloading from the action of TIFF and their relationship with the parameter that characterizes the straightness of the rope after manufacture.

2. **Presentation of the core material**

Figure 1 shows the stress state of the wires and the rope TIFF. Their expressions are obtained subject to the action of external loads

\[
N_i^T = \sum_{i=1}^{n} N_i \cos \alpha_i;
\]

\[
M_i^T = \sum_{i=1}^{n} (M_b \sin \alpha_i + M_t \cos \alpha_i + N_i r_i \sin \alpha_i);
\]

\[
M_i^b = \sum_{i=1}^{n} (M_b \cos \alpha_i - M_t \sin \alpha_i + N_i r_i \cos \alpha_i \sin \varphi_i);
\]

\[
M_i^t = \sum_{i=1}^{n} (- M_b \cos \alpha_i + M_t \sin \alpha_i - N_i r_i \cos \alpha_i),
\]

where $N_i^T$ is a longitudinal force in the cable cross-section;
\( M^T_s \) and \( M^T_x, M^T_z \) – torque and bending moment in the cable cross-section;
\( N_i \) – longitudinal force in the i-th wire caused by torsional tension;
\( M_{bi} \) and \( M_{ti} \) – bending and torque moments in the i-th wire cross-section, which emerge due to torsion of the wire line.

The central (core) wire takes only longitudinal force \( N_i \), caused by torsional tension.

In the elastic state
\[
M^y_b = EJ \tilde{b}^y; \\
M^y_i = GJ_0 \tilde{t}^y,
\]
where \( \tilde{b}^y \) and \( \tilde{t}^y \) – torsional deformations of bending and twisting in the stage of elastic deformation;
\( EJ = E\pi\delta^4 / 64 \); \( GJ_0 = G\pi\delta^4 / 32 \) – cross-sectional bending and twisting rigidity of the wire with diameter \( \delta \).

Deformations \( \tilde{b}^y \) and \( \tilde{t}^y \) are determined by expressions similar to formulas (1), (2):
\[
\tilde{b}^y = \frac{\sin \alpha^y}{r}; \\
\tilde{t}^y = \frac{\sin \alpha^y (\kappa \cos \alpha^y)}{r},
\]
where \( \alpha^y \) – angles in edge elastic state during torsion.

The edge elastic state of the wire cross-section for the specified process deformation is determined by reaching the yield parameters \( \sigma_f \) and \( \varepsilon_f \) in boundary points of the wire cross-section:
through the stress intensity
\[
\sigma_i = \sqrt{\sigma^2 + 3\tau^2} = \sigma_f (1 - \kappa);
\]
through strain intensity
\[
\varepsilon_i = \sqrt{\varepsilon^2 + \frac{3}{2}\gamma^2} = \varepsilon_f (1 - \kappa),
\]
where \( \kappa \leq 0.1 \) – the wires torsional tension factor;
\( \sigma_f \) and \( \varepsilon_f \) stress and deformation of yield limit in accordance with linearly schematized diagram of wire stretching (Figure 2, a). In Figure 2, b a diagram and formulas (11) i (12), are represented in relative terms to obtain generalized study results.
\[
\bar{\sigma}_i = \sqrt{\bar{\sigma}^2 + \bar{\tau}^2} = 1 - \kappa; \\
\bar{\varepsilon}_i = \sqrt{\bar{\varepsilon}^2 + \bar{\gamma}^2} = 1 - \kappa,
\]
where \( \bar{\sigma}_i = \sigma_i / \sigma_f \); \( \bar{\sigma} = \sigma / \sigma_f \); \( \bar{\tau} = \tau / \tau_f \); \( \bar{\varepsilon}_i = \varepsilon_i / \varepsilon_f \); \( \bar{\varepsilon} = \varepsilon / \varepsilon_f \); \( \bar{\gamma} = \gamma / \gamma_f \).

Parameters of torsional deformation for the edge elastic state
\[
\tilde{\xi}^y = \frac{\gamma_{max}}{\bar{b}^y} = \frac{\tilde{y}^y}{\bar{b}^y},
\]
Similarly for the final state
\[ \xi = \frac{\bar{t}}{b}. \]  

(16)

\[ \xi_y = \frac{\sin \alpha^y \cos \alpha^y}{\gamma} \frac{l}{\sin^2 \alpha^y} = \frac{\gamma}{\alpha^y}. \]  

(17)

In the presence of untwisting-twisting

\[ \xi_y = \frac{k_x + \cos \alpha^y}{\sin \alpha^y}. \]  

(18)

With respect to the deformation parameter \( \xi_y \) (17), (18), the strain intensity in the edge elastic state of the wire cross-section is expressed as follows

\[ \varepsilon_i = \varepsilon_y \sqrt{1 + \frac{1}{3} \xi^2} = \varepsilon_y (1 - \kappa). \]  

(19)

In relative terms

\[ \bar{\varepsilon}_i = \bar{\varepsilon} \sqrt{1 + \frac{1}{3} \xi^2} = 1 - \kappa. \]  

(20)

On the basis of (20) we obtain the curvature \( \hat{b}^y \), at which the wire will be in the edge elastic state due to bending with torsion during pre-stretching

\[ \hat{b}^y = \frac{2\sqrt{3} \varepsilon_y (1 - \kappa)}{\delta \sqrt{3 + \xi^2}}. \]  

(21)

Based on (21), determining of the torsion angles that correspond to the edge elastic state of the wire cross-section, is performed by successive approximations based on the equation

\[ \sin^2 \alpha^y = \frac{2\sqrt{3} \varepsilon_y (1 - \kappa)}{\delta \sqrt{3 + \xi^2}}. \]  

(22)

Table 1, on the basis of (22), shows the calculated values \( \alpha^y ; \hat{b}^y ; \bar{\xi}_i ; \bar{\xi}_f \) for the wires of prestressing strand of diameter \( d = 6.9 \text{ mm} \); \( \delta = 2.5 \text{ mm} \); \( \delta_2 = 2.2 \text{ mm} \); \( \alpha = 9 \); \( r = 2.35 \text{ mm} \); \( E = 2 \times 10^5 \text{ kN/mm}^2 \); \( \sigma_f = 1550 \text{ kN/mm}^2 \); the torsional tension factor \( \kappa = 0.1 \).
As can be seen from Table 1, in the absence of untwisting \((\kappa_0 = 0)\), the value in the edge elastic state has torque. At \(\kappa_0 = -\cos \alpha\) the wires are deformed only by bending with pretension.

**Table 1. Parameters of the edge elastic state**

| \(\kappa_0\) \(^a\) | \(\alpha\), degr | \(\tilde{b}^y \times 10^4 \text{ mm}^4\) | \(\xi^y\) | \(\tilde{M}_b^y\) | \(\tilde{M}_t^y\) |
|-----------------|----------------|----------------|--------|--------------|--------------|
| 0               | 1.46           | 2.796          | 39.23  | 0.0397       | 0.8991       |
| -1              | 1.65           | 3.511          | 34.71  | 0.0483       | 0.9987       |
| \(-\cos \alpha\) | 7.00           | 63.364         | -0.0611| 0.8994       | -0.0371      |
|                 | 7.38           | 70.405         | -0.0644| 0.9993       | -0.0372      |

\(^a\) For each \(\kappa_0\) the upper lines at \(\kappa = 0.1\); lower ones at \(\kappa = 0\).

The contour of the boundary elastic zone of the wire cross-section is escribed by a circle

\[
\left( \tilde{M}_b^y \right)^2 + \left( \tilde{M}_t^y \right)^2 = (1 - \kappa)^2 ,
\]

where bending (7) torque moments (8) of the edge elastic state are in the relative dimensionless form:

\[
\tilde{M}_b^y = \frac{EJ \tilde{b}^y}{\sigma_y W_b} = \frac{\sqrt{3} (1 - \kappa)}{\sqrt{3 + \xi_y^2}} ;
\]

\[
\tilde{M}_t^y = \frac{\sqrt{3} GJ_0 \tilde{t}^y}{\sigma_y W_0} = \frac{\xi_y (1 - \kappa)}{\sqrt{3 + \xi_y^2}} .
\]

The formation of the spiral shape of the wire ends in the stage of elastic-plastic deformation by bending with torsion during the pretension associated with the torsional tension of the wires.

It is known [8] that in the case of joint bending and torsion in the elastic-plastic stage, the hypothesis of flat sections and straight radii is unacceptable because in the process of deformation the boundary of the elastic region does not remain a circle (23). No exact solutions of such a problem in the theory of plasticity have been obtained up to date. Therefore, we will follow the path of an approximated solution.

With a completely plastic (flexible) cross-section, which is possible with very large deformation and using the model of a perfectly plastic rod \((\tilde{E}_y = 0)\), the limit curve of dependence that connects the bending and torque moments, without longitudinal force, i.e., for \(\kappa = 0\) is close to the ellipse [8].

In this case, that is, when bending with torsion after pretension \((\kappa \neq 0)\), we assume that the boundary curve with acceptable accuracy can also be taken as escribed by the ellipse of such an equation

\[
\frac{\left( \tilde{M}_b^y \right)^2}{\tilde{M}_b^2} + \frac{\left( \tilde{M}_t^y \right)^2}{\tilde{M}_t^2} = (1 - \kappa)^2 .
\]

The numerators of equation (26) represent the expressions of bending \(\tilde{M}_b^y\) and torque \(\tilde{M}_t^y\) moments corresponding to very large deformations \(\tilde{e}_{i, \text{max}}\); \(\tilde{e}_{\text{max}}\); \(\tilde{\gamma}_{\text{max}}\), at joint bending and torsion of ideally plas-
tic rod. In [8], and earlier by other authors, such expressions were obtained in solving the problem with \( k=0 \) by integrating through complete elliptic integrals.

Denominators in (26) \( \hat{M}_b = 16/3\pi \) and \( \hat{M}_r = 4/3 \) – the limit values of bending and torque in the case of separate bending and torsion of a perfectly plastic rod.

For any values of deformations in the interval \((1 \div \infty)\) we approximately take the relationship between bending and torque moments that are also described by the equations of ellipses, which occupy intermediate positions between the circle (23) and the ellipse (26). In this case, for the limit deformation intensities (20)

\[
\bar{\varepsilon}_{\text{max}} = \bar{\varepsilon}_{\text{max}} \sqrt{1 + \frac{1}{3} \varepsilon^2}
\]

the equations of the ellipses have the form of

\[
\left( \frac{M_b}{\hat{M}_b} \right)^2 + \left( \frac{M_r}{\hat{M}_r} \right)^2 = (1 - \kappa)^2,
\]

where \( \hat{M}_b \) – is the bending moment under the action of bending only, i.e., when the intensity of deformation in the boundary points of the rod cross-section \( \bar{\varepsilon}_{\text{max}} = \bar{\varepsilon}_{\text{max}} \); \( \hat{M}_r \) – torque under the action of torsion only, i.e., when \( \bar{\varepsilon}_{\text{max}} = \bar{\gamma}_{\text{max}} \).

The studies performed indicate that in the range of values inherent in the torsional deformations of the wires, the load parameters can be assumed equal to the deformation parameters, i.e. \( \lambda_b = \bar{t}/\bar{b} = \xi \); \( \lambda_r = \bar{b}/\bar{t} = 1/\xi \).

Then, taking into account these parameters on the basis of equation (27) we obtain the expressions of bending and torque moments in the circular cross-section of the rod during torsional bending after pretension in the following form

\[
\hat{M}_b = \frac{M_b M_r \sqrt{1 - \kappa}}{\sqrt{b^2 M_b^2 + M_r^2}};
\]

\[
\hat{M}_r = \frac{M_b M_r \sqrt{1 - \kappa}}{\sqrt{t^2 M_b^2 + M_r^2}},
\]

where \( \hat{M}_b = M_b/\sigma_T W_b \) and \( \hat{M}_r = M_r/(\sigma_T / \sqrt{3}) W_0 \) – expressions of bending and torque moments in the separate action.

In computations according to formulas (3–6) \( \hat{M}_b \) and \( \hat{M}_r \) are conveniently used in the form of functions of technological deformations of wire torsion: \( \hat{M}_b = f\left( \bar{\varepsilon}_{\text{max}} ; \bar{E}_T \right) ; \hat{M}_r = f\left( \bar{\gamma}_{\text{max}} ; \bar{G}_T \right) \),

where

\[
\bar{\varepsilon}_{\text{max}} = \frac{\sin^2 \alpha \cdot \delta}{\varepsilon_r r} \cdot \frac{\bar{t} \delta}{2} ; \quad \bar{\gamma}_{\text{max}} = \frac{\sin \alpha (\kappa_r + \cos \alpha)}{2\sqrt{3}\varepsilon_r} \cdot \frac{\bar{t} \delta}{2}. \quad (30)
\]

In terms of physical content, the bending moment is expressed by an integral equation

\[
M_b = \int_A \sigma y dA.
\]

\[\text{(31)}\]
Having assumed the deformation from torsional bending as a variable integration, expression (31) takes the form

\[
M_b = \frac{4r^3}{\sin^6 \alpha} e^\max \int_0^{e^\max} \sigma e \sqrt{e^2 - e^2} \, de .
\]  

(32)

In relative terms

\[
\bar{M}_b = \frac{M_b}{\sigma \gamma W_b} = \frac{128 r^3 \gamma^3}{\pi \delta^3 \sin^6 \alpha} \int_0^{e^\max} \sigma e \sqrt{e^2 - e^2} \, d e .
\]  

(33)

Stress in the cross-section for relative form in the zones:
elastic \( \bar{\sigma} = \bar{e} \); elastic-plastic

\[
\bar{\sigma} = 1 - \bar{E}_T + \bar{e} \bar{E}_T .
\]  

(34)

Taking into account (34)

\[
\bar{M}_b = \frac{128 r^3 \gamma^3}{\pi \delta^3 \sin^6 \alpha} \int_0^1 \bar{e}^2 \sqrt{\bar{e}^2 - \bar{e}^2} \, d \bar{e} + \int_1^{e^\max} (1 - \bar{E}_T + \bar{e} \bar{E}_T) \bar{e} \sqrt{\bar{e}^2 - \bar{e}^2} \, d \bar{e} .
\]  

After integration and transformations, the bending moment \( \bar{M}_b \) takes the following form

\[
\bar{M}_b = \frac{4}{\pi \gamma^3} \left[ (1 - \bar{E}_T) \left( e^\max \sqrt{e^2 - 1} + \left( e^2 - 1 \right)^{3/2} \right) \right] .
\]  

(35)

Torque \( \bar{M}_t \) in physical terms is expressed by an integral equation

\[
M_t = \int \tau \rho dA = 2\pi \int_0^\rho \tau \gamma^2 d\rho ,
\]  

(36)

where the current radius \( \rho \) and its differentiation operator

\[
\rho = 0.5\delta - \frac{\gamma}{\gamma^\max} ; \ d\rho = \frac{0.5\delta}{\gamma^\max} d\gamma .
\]  

(37)

Having applied (37) in (36), we obtained

\[
M_t = \frac{\pi \delta^3}{4\gamma^3} \int_0^{\gamma^\max} \tau \gamma^2 d\gamma .
\]  

(38)

In relative terms

\[
\bar{M}_t = \frac{M_t}{\tau \gamma W_0} = \frac{4}{\gamma^\max} \int_0^{\gamma^\max} \bar{\gamma} \gamma^2 d\bar{\gamma} ,
\]  

(39)

Tangential stresses in the zones:
elastic \( \bar{\tau} = \bar{\gamma} \);
elastic-plastic zone

\[
\bar{\tau} = 1 - \bar{G}_T + \bar{\gamma} \bar{G}_T .
\]  

(40)

Taking into account (40)
\[ M_t = \frac{4}{\gamma_{\text{max}}} \left[ \gamma_0^3 \int_0^{\gamma_{\text{max}}} \gamma^3 d\gamma + \int_1^{\gamma_{\text{max}}} (1 - \bar{G}_r + \bar{G} \gamma_r) \gamma^3 d\gamma \right]. \]  

\[(41)\]

After integration and transformation
\[ M_t = \frac{4}{\gamma_{\text{max}}} \left[ \frac{1}{4} \left( \frac{1 - \bar{G}_r}{\gamma_{\text{max}}^3} - 1 \right) + \bar{G}_r \frac{1}{4} \left( \gamma_{\text{max}}^4 - 1 \right) \right]. \]

\[(42)\]

For the formulas of TIFF (3) - (6) bending and torques moments \( M_b \) and \( M_t \) shall be represented in absolute terms:
\[ M_b = \frac{\bar{M}_b \gamma_{\text{max}}^{1 - \kappa}}{\sqrt{\lambda_1^2 M_b^2 + \lambda_2^2 M_t^2}} \frac{32}{\pi \delta^3} \sigma_r \quad M_t = \frac{\bar{M}_b \gamma_{\text{max}}^{1 - \kappa}}{\sqrt{\lambda_1^2 M_b^2 + \lambda_2^2 M_t^2}} \frac{16 \sqrt{3}}{\pi \delta^3} \sigma_r. \]

\[(43)\]

The whole set of rope unloading efforts is represented by the vector
\[ F = \begin{bmatrix} N_x^T \\ M_x^T \\ M_y^T \\ M_z^T \end{bmatrix}, \]

\[(44)\]

whose components are computed using formulas (3)–(6). Deformation of the rope under the action of the force vector (44) on the basis of the unloading theorem [9] gives grounds to consider it elastic. For its analytical description we apply FEM in a similar way to [10]:
\[ |G| \cdot |DK| = F. \]

\[(45)\]

Here the stiffness matrix
\[ |G| = \begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ G_{21} & G_{22} & G_{23} & G_{24} \\ G_{31} & G_{32} & G_{33} & G_{34} \\ G_{41} & G_{42} & G_{43} & G_{44} \end{bmatrix} = \sum_i \Lambda_i |K_i| \cdot |GII| \cdot |K_i|^T, \]

\[(46)\]

where \( G_{11}; G_{22} \) and \( G_{33}; G_{44} \) – is longitudinal, torsional and bending stiffnesses of the rope; \( G_{12} = G_{21}; ...; G_{34} = G_{43} \) – stiffness of influence.

Rope unloading deformations vector based on (45) and (46)
\[ |DK| = \begin{bmatrix} \varepsilon \\ \theta \\ \chi \\ \zeta \end{bmatrix} = \frac{1}{|D|} \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \begin{bmatrix} N_x^T \\ M_x^T \\ M_y^T \\ M_z^T \end{bmatrix}. \]

\[(47)\]

Deformations of rope based on (47):
\[ \varepsilon = (N_x^T A_{11} + M_x^T A_{12} + M_y^T A_{13} + M_z^T A_{14}) / |D|; \]

\[(48)\]

\[ \theta = (N_x^T A_{21} + M_x^T A_{22} + M_y^T A_{23} + M_z^T A_{24}) / |D|; \]

\[(49)\]
\[ \chi = \left( N_x^T A_{13} + M_y^T A_{23} + M_z^T A_{33} + M_x^T A_{34} \right) / |D|, \]  
(50)

\[ \zeta = \left( N_x^T A_{24} + M_y^T A_{34} + M_z^T A_{34} + M_x^T A_{34} \right) / |D|. \]  
(51)

**Figure 3** – Segment arrow.

The degree of the rope linearity after manufacturing after standard [1] is controlled by the segment arrow (Figure 3)

\[ h = r - \sqrt{r^2 - \frac{a^2}{4}}, \]  
(52)

which under \( a = 1 \) m standard [1] shall not exceed 25 mm,

where radius \( r \) is determined based on (50) and (51): \( r = 1/\sqrt{\chi^2 + \zeta^2} \).

Thus, theoretically obtained deformations of the rope associated with the unloading due to TIFF. They include bending deformations \( \chi \) and \( \zeta \), which determine the radius of curvature and the parameter \( h \) of the rope linearity control after manufacture. Numerical studies based on the obtained theory indicate that the most important technological argument of the ropes linearity after manufacture is the uniformity of wires tension during torsion. This confirms the urgent requirement in [2] for the level and uniformity of wire tension in the manufacture of the quality ropes.

By way of example in Figure 4 and in Table 2 for prestressing strand 1+6 with diameter 6.9 mm (Figure 5) shows the dependence of the parameter of straightness \( h \), associated with the deviation of the tension of the wire №2 (Figure 5) from the accepted level determined by the coefficient \( \kappa = 0,1 \). Figure 4 shows that with uniform tension of all wires \( h = 0 \). When the wire No.2 tension deviates from the total tension by \( \kappa = 0,1 \) more than 20%, the arrow \( h \) exceeds GOST [1] permissible value \([h] = 25 \) mm.

**Figure 4.** Dependence \( h \) on tension. **Figure 5.** The rope cross-section.
By way of example in Fig 4 and in Table 2 for prestressing strand 1+6 with diameter 6.9 mm (Figure 5) the dependence of the linearity parameter $h$, associated with the deviation $\kappa_0$ of the wire No.2 tension (Figure 5) from the accepted level determined by the coefficient $\kappa = 0.1$. Figure 4 shows that with uniform tension of all wires $h = 0$. When the wire No.2 tension deviates from the total tension by $\kappa = 0.1$ more than 20%, the arrow $h$ exceeds GOST [1] permissible value of 25 mm. Unstraightness occurs both in the case of undertensioning $\kappa_2 < \kappa = 0.1$, and as a result of excessive tensioning in relation to the set level.

As can be seen from Table 2 in the first three states of torsion, the parameter of unstraightness $h = 0$. After manufacture and unloading due to TIFF, the rope will be shortened and untwisted in proportion to the deformations $\varepsilon$ and $\theta$ but will be straight. At the same time, untwisting produces $\kappa_0 = -\cos \alpha$ and $\kappa_0 = -1$ virtually the same positive effects. In the absence of a untwisting ($\kappa_0 = 0$), as practical experience shows, the untwisting of the rope increases sharply after the manufacture. The impact of preformation should be further investigated.

3. Conclusions and recommendations

Improved method for determining the longitudinal force $N_t$, bending $M_b$ and torque $M_c$ moments in the ropes with the general process parameters of torsion and the method for determining the integrated internal forces in the cross-sections of the rope TIFF: longitudinal force of $N_t^y(N_t)$, torque $M_b^y(N_t;M_b;M_c)$ and bending moments $M_c^y(N_t;M_b;M_c)$ and $M_t^y(N_t;M_b;M_c)$. Algorithm of computation $M_b$ and $M_c$; initially $\tilde{M}_b$ and $\tilde{M}_c$ in a relative dimensionless form under the condition of their individual action directly due to torsional deformations of bending and twisting of wires; further ex-

| No. | $\frac{N_t}{H}$ | $M_b^y$ | $M_c^y$ | $M_t^y$ | $\varepsilon \cdot 10^4$ | $\theta \cdot 10^4$ | $\chi \cdot 10^4$ | $\zeta \cdot 10^4$ | $h$ |
|-----|----------------|----------|----------|----------|---------------------|----------------|----------------|----------------|-----|
| 1   | 4.252          | 1.907    | ~ 0      | ~ 0      | 1.640               | 1.988          | ~ 0            | ~ 0           | 0   |
| 2   | 4.252          | 1.907    | ~ 0      | ~ 0      | 1.640               | 1.988          | ~ 0            | ~ 0           | 0   |
| 3   | 4.252          | 17.486   | ~ 0      | ~ 0      | 5.100               | 10.33          | ~ 0            | ~ 0           | 0   |
| 4   | 4.136          | 951      | ~ 0      | 401      | 1.405               | 1.440          | -0.165         | 1.99          | 25  |
| 5   | 3.961          | 856      | ~ 0      | 1.003    | 1.359               | 1.346          | -0.421         | 4.99          | 64  |
| 6   | 4.543          | 1.174    | ~ 0      | -1003    | 1.517               | 1.670          | 0.396          | -4.99         | 64  |
| 7   | 3.903          | 824      | ~ 0      | 803      | 1.342               | 1.313          | -0.334         | 4.04          | 51  |
| 8   | 4.252          | 1015     | ~ 0      | 2007     | 1.440               | 1.509          | -0.819         | 9.97          | 134 |

Using formula (52) $a > r$: $h > r = 490, \text{MM}$: a loop is formed
pressions $\bar{M}_b^T$ and $\bar{M}_b^b$ under the condition of joint action during the previous tension in the form of $k$ coefficient of the torsional tension of the wires. Numerical values $\bar{M}_b^T$ and $\bar{M}_b^b$ for $k = 0$ in the interval $\alpha$ $(8 - 25^\circ)$ virtually coincide with the solution [9] obtained by another way (integration through complete elliptic integrals).

Using the unloading theorem, a technique for studying the external deformed state of the rope unloading due to TIFF was developed based on FEM. The expression of the rope unloading deformations vector $[DK] = [\epsilon, 0, \chi, \zeta]^T$ as a function of the $F = (N^T; M^T_v; M^T_b; M^T_b)$ TIFF vector using the global stiffness matrix of $4 \times 4$ rope cross-section corresponding to the elastic deformation is obtained. On the basis of bending deformations $\chi$ and $\zeta$ the expression of $h$ parameter is obtained. In accordance with the regulatory standard, parameter $h$ characterizes straightness of a rope after manufacturing. The connection between the parameters of torsion (tension, untwisting, and preformation) and the parameter $h$ of straightness is established. The unstraightness is directly affected only by the deviation from the uniformity of tension. The developed theory allows to establish the admissible deviation from non-uniformity of tension with admissible straightness value of parameter $h$ ($h$ is an arrow of a segment for the chord length of 1 m). For example, a prestressing strand of 1+6 structure, 6.9 mm in diameter according to the regulatory standard is considered to be of high quality if $h \leq 25.5 \text{mm}$. This value $h$ corresponds to the tension deviation $\Delta \kappa = 0.8$ of one of the 6 wires of the layer. With increasing deviation in tension $h$ increases almost rectilinearly to $\Delta \kappa = 3$. With further increase, a loop is $\Delta \kappa$ formed – the loss of stability.

The developed theory of the external deformed state of unloading provides further opportunity to study the internal deformed-stress state of the rope wires due to process-induced forces, which opens the prospect of even a deeper study of operational strength and durability of ropes.

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