Comparative analysis of accuracy and efficiency of different contact algorithms

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Abstract. In order to solve contact problems, ANSYS provides a variety of contact algorithms. However, there is no basis for how to select the contact algorithm when solving the contact problem in hydraulic structure engineering. Therefore, this paper introduces the basic concepts of contact algorithms briefly, such as penalty method, lagrange method, augmented lagrange method and L&P method, which are commonly used in ANSYS. Then, different examples are used to compare the four contact algorithms in terms of accuracy, convergence and efficiency. Finally, considering the accuracy, convergence and efficiency, it is suggested to adopt L&P method in hydraulic structure contact analysis.

1. Introduction

Contact problems widely exist in civil engineering, hydraulic engineering, mechanical engineering and other engineering fields, such as pile-foundation contact, arch dam transverse joint and gear mesh. It is the main research content of contact problem to determine the size of contact area and stress distribution on the contact surface, so as to find out the stress and deformation of contact body. The difficulty of the contact problem is that the frictional contact condition is a unilateral inequality constraint, which makes the fonctionelle of the system nonlinear and non-smooth, and makes the convergence and accuracy of the algorithm difficult to be guaranteed[1].

With the rise and development of numerical methods, there are many methods to solve contact problems, such as the the finite element method[2], the distance potential discrete element method[3], the boundary element method[4] and the scaled boundary the finite element method[5,6]. As the most effective numerical method to solve complex engineering problems, the finite element method is also a main method to solve contact problems[7-9].

ANSYS[10] is a commercial large-scale analysis software based on the finite element method. It integrates structure field, fluid field, electric field, magnetic field and sound field analysis into one. ANSYS is widely used in aerospace, mechanical manufacturing, civil engineering, shipbuilding, biomedical, water conservancy and many other fields. ANSYS is powerful and easy to operate. It has become the most popular finite element analysis software in the world. ANSYS software mainly includes three parts: pre-processing module, analysis and calculation module and post-processing module. Analysis and calculation module of ANSYS provides various contact algorithms, for instance, lagrange method, penalty method, augmented lagrange method and L&P method for users to choose.
freely. However, how to choose the appropriate algorithm becomes a problem when solving different engineering problems.

Therefore, this paper will introduce the basic concepts of different contact algorithms in ANSYS briefly, then compare and analyze the accuracy and efficiency of different contact algorithms in ANSYS through running several typical examples, and finally select a reasonable contact algorithm for the simulation of contact problem of arch dam contraction joint.

2. Contact method in ANSYS

2.1. Lagrange method

The lagrange method introduces an additional functionelle of the contact constraint deterministic solution condition by multiplying the lagrange $\lambda$ by the penetration-free condition[10]

$$\Pi_c = ig$$

(1)

Where $g$ is the contact clearance including the tangential clearance.

$\lambda$ contacts with the original problem, which does not contain the constraint, of total potential energy $\Pi(U)$ to form a modified functionelle, transforms the original problem into the unconditional extreme value

$$\min \Pi'(U, \lambda) = \Pi(U) + \Pi_c(U, \lambda)$$

(2)

Lagrange method can meet the constraints of contact accurately, however, because of the introduction of lagrange multiplier, each contact will add an unknown, which increased the degree of freedom of equation, and expanded the scale of the solution of the system, and the coefficient matrix for solving the equation is therefore no longer positive definite, the value of each multiplier item on the diagonal turn to zero. Appropriate methods must be adopted to ensure the convergence and stability of the equation, therefore, a penalty term was added to revise functionelle equation (2),

$$\Pi_c = -\lambda^2 E_p^{-1}/2$$

(3)

Where $E_p$ is the penalty factor, $E_p > 0$. equation (2) was rewrite as

$$\min \Pi'(U, \lambda) = \Pi(U) + \Pi_c(E_p)$$

(4)

Compared with equation (2), since $\lambda$ was added in the quadratic term, the main diagonal elements corresponding to $\lambda$ in the computed finite element scheme is no longer zero after the variation of the longitude, so the global stiffness matrix is no longer singular. When the penalty factor $E_p \rightarrow \infty$, the solution of equation (4) will converge to the solution of equation (2).

2.2. Penalty method

Penalty method is an additional functionelle using penalty function to introduce the condition of contact constraint

$$\Pi_p = ag^2$$

(5)

Where $a$ is the penalty parameter; $g$ is the contact clearance including tangential clearance. Contact with $\Pi(U)$ of the original problem, which does not contain the constraint of total potential energy to form a modified functionelle, therefore constraint conditional extreme value problems translate into the unconditional extreme value problems

$$\min \Pi'(U) = \Pi(U) + \Pi_p(U)$$

(6)

The greatest advantage of the penalty method is that the degree of freedom of the system is invariable when the contact condition is introduced, the original storage and calculation amount of the computer are not increased, and the coefficient matrix of the solution equation is positive definite automatically. The disadvantage of the penalty method is that the constraints can only be satisfied approximately. Theoretically speaking, increase the penalty parameters can improve the precision of
calculation, but increase the penalty parameter too much can easily lead to excessive equation morbid, and may lead to relative motion between objects in contact each other produce reverse by mistake, so that the solution of the process is not stable, and it often requires many times of penalty parameter selection in a complex practical problems[11].

2.3. Augmented lagrange method
Augmented lagrange method was formed by combining lagrange method and penalty method. Based on equation (2), the augmented lagrange method introduces additional fonctionelle

\[ \Pi_p = \frac{1}{2}g^2 / 2 \]  

(7)

the modified potential energy fonctionelle was constructed

\[ \Pi' = \Pi + \Pi_c + \Pi_p \]  

(8)

By applying

\[ \lambda_{k+1} = \lambda_k + ag \]  

(9)

To calculate the update of the lagrange without increasing the degree of freedom of the system. With the introduction of penalty factor with higher power \( \Pi_p \), the diagonal dominance of the equation matrix is increased, which can improve the convergence of the solution. Compared with the lagrange method, the augmented lagrange method has the same advantages as the penalty method. Compared with penalty method, augmented lagrange method usually has better adaptability, also, it is insensitive to the size of contact stiffness[11].

2.4. L&P method
L&P method is a method combining lagrange method and penalty method. lagrange method is used as contact constraint condition on contact normals direction, and tangential contact stiffness penalty method is used as contact constraint condition on friction plane. This method is only applicable to adhesive contact conditions that allow a small amount of slippage. The L&P method requires the setting of flutter control parameters TOLN, FTOL and the maximum allowable elastic slip parameters SLTOL. Based on tolerance, current normal contact force and friction coefficient, tangential contact stiffness FKS can be obtained automatically. In some cases, FKS can also be overridden by defining a scaling factor (positive input) or an absolute value (negative input)[10]. L&P method is similar to lagrange method in that it is easy to cause oscillation in the case of many iterations. If the value of SLTOL is too large and the value of FKS is too small, excessive elastic slip will occur. If the value of SLTOL is too small or the value of FKS is too large, the model may not converge.

3. Numerical examples

3.1. The contact problem of cylinder and rigid foundation
In order to compare the accuracy of four contact algorithms in ANSYS, the contact problem between cylinder and rigid foundation is selected for comparative analysis. The model is shown in Figure 1, the radius of cylindrical section \( R=8m \), elastic modulus \( E=1000Pa \), Poisson's ratio \( \nu=0.3 \), and external load \( P=240N \), 4 nodes PLANE182 plane elements was used to discrete the contact body, the total number of nodes is 198598. Contact constraints were imposed by the augmented lagrange method, penalty method, L&P method and lagrange method respectively, and FKN=10, FTOLN=0.1. The point-surface contact model was adopted for the contact surface.

In order to compare the errors of different contact algorithms, the root-mean-square error is calculated for each method. The root-mean-square error formula is as follows
error \left(\frac{\sum_{i=1}^{n}(S_i - S'_i)^2}{\sum_{i=1}^{n}(S_i)^2}\right)^{\frac{1}{2}} \times 100\% \hspace{1cm} (10)

Where $S_i$ is the analytic solution of the contact stress at the $i^{th}$ contact point, $S'_i$ is the contact stress at the $i^{th}$ contact point, and $n$ is the total number of contact points.

As shown in Figure 2, the distribution of contact stress obtained by the four contact algorithms is roughly the same as the trend of analytic solution, but the contact stress shows strong jumping phenomenon at the end point.

As shown in Table 1, under the same calculation conditions, compared with the other three methods, the contact stress error calculated by the augmented lagrange method is the smallest, but the error of the calculation results of the four algorithms does not differ much.

### Table 1. Root-mean-square errors of the four contact methods

| Method                  | Augmented lagrange | Penalty   | Lagrange&Penalty | Lagrange |
|-------------------------|--------------------|-----------|------------------|----------|
| Root-mean-square error  | 8.26%              | 10.30%    | 9.56%            | 9.56%    |

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#### 3.2. Three-dimensional curved beam contact problem

In order to compare the calculation time and convergence of the four methods, two 3D curved beam contact problems as shown in Figure 3 are selected for analysis. Concentrated loads are applied along the top of beam 1 along X, Y and Z, with values of -100 kN. The dotted area in Figure 3 is the contact surface, and the initial distance between the two beams is 0. The material properties of the two curved beams are the same: elastic modulus was set to 30 GPa, Poisson's ratio was set to 0.3 and friction coefficient was set to 0.5, 20-node SOLID95 solid element was used. 7 calculation model with 162, 282, 850, 5346, 10242, 20034, 41316 nodes, each of them was used for comparing of convergence of the calculation results. Contact constraints were imposed by the augmented lagrange method, penalty method, L&P method and lagrange method respectively. The contact surface adopted the surface-surface contact model, with FKN=1 and FTOLN=0.1.
In order to analyze the influence of contact tolerance FTOLN on the convergence of different contact algorithms, a calculation model with the number of nodes of 20034 was selected. Set FKN=1 and changing FTOLN. The results are shown in Table 2.

Table 2. Convergence comparison of different contact algorithms with FTOLN

| FTOLN | Method      | 1.0E-09 | 2.0E-09 | 3.0E-09 | 4.0E-09 | 5.0E-09 | 6.0E-09 |
|-------|-------------|---------|---------|---------|---------|---------|---------|
|       | Augmented lagrange | NO      | NO      | NO      | NO      | NO      | YES     |
|       | Penalty     | NO      | NO      | NO      | NO      | NO      | YES     |
|       | Lagrange&Penalty | NO      | NO      | YES     | YES     | YES     | YES     |
|       | Lagrange   | NO      | NO      | YES     | YES     | YES     | YES     |

It can be seen from Table 2 that, the convergence of L&P method and lagrange method is least affected by the setting of parameter FTOLN, and is superior to the augmented lagrange method and penalty method in terms of stability.

In order to compare the variation of the calculation results of different contact algorithms with the computational grid density, FKN=1, FTOLN=0.1, and other models with unchanged parameters were used for trial calculation. Take the midpoint of two long sides of curved beam to be point A and point B, and take the midpoint of the connecting line of point A and point B to be point E to compare the contact stress of these three points under different grid density. Figure 4 show the change of contact stress of points A, B and E with the increase of the number of nodes.

(a) Node A
(b) Node B
(c) Node E

Figure 4. The relationship between the contact stress and the number of nodes
As shown in Figures 4, the calculation results of the L&P method and lagrange method converge basically when the node number reach 5346, and the convergence speed is better than that of the augmented lagrange method and the penalty method. The results of the lagrange method and the L&P method are in good agreement, and the contact force obtained by the lagrange method and the L&P method is often larger than the contact force obtained by the augmented lagrange method and the penalty method.

3.3. Contact problem of arch dam contraction joint

In order to compare the application of these four contact algorithms in practical engineering, an example of an arch dam with a height of 240 m and eight transverse cracks is taken for analysis. In the calculation and analysis, considering the action of dead weight and hydrostatic pressure, the elastic modulus of the dam body is 21 GPa, Poisson's ratio was set to 0.167, density was set to 2400, friction coefficient was set to 0.5. In the analysis, the 20-node SOLID186 element was used for discretization, with a total number of node been 7,117. Contact constraints were imposed by the augmented lagrange method, penalty method, L&P method and lagrange method respectively. The contact surface adopted the face-to-face contact model, with FKN=1, FTOLN=0.1.

Figure 5. The computing mesh of Arch dam

![Figure 5. The computing mesh of Arch dam](image)

Figure 6. Total contact stress under the constrain of different contact algorithms

![Figure 6. Total contact stress under the constrain of different contact algorithms](image)

Figure 6 shows the contact stress distribution at the contact surface of arch dam transverse joints with different contact algorithms. It can be seen from Figure 6, under the same calculation conditions, the contact stress distribution obtained by different contact algorithms are approximately the same.

As shown in Figure 7, when the contact parameters are set in the same way, the calculation time required for arch dam models with different contact algorithms is calculated. The calculation speed of
these four methods ranges from slow to fast, in the following order: lagrange method, penalty method, augmented lagrange method, and L&P method.

Figure 7. Calculation time of different contact algorithms

4. Conclusion
By using several numerical examples to compare the contact algorithms in ANSYS, such as lagrange method, penalty method, augmented lagrange method and L&P method, the following conclusions are drawn:

- For frictionless contact problem: under the same mesh density and parameter setting, the distribution of contact stress obtained by the four contact algorithms is roughly the same as the trend of analytical solution, but the contact stress jumps strongly at the end points. The error of contact stress obtained by the augmented lagrange method is the smallest, but the error of the four algorithms have little difference from each other.

- 3D curved beam example shows that the convergence of L&P method and lagrange method is least affected by FTOLN, and the stability of L&P method is better than that of augmented lagrange method and penalty method. The convergence speed of L&P and lagrange contact algorithm is faster than augmented lagrange and penalty contact algorithm.

- Arch dam example shows that L&P method is more efficient than lagrange method, penalty method and augmented lagrange method under the same contact parameters.

- Considering the precision, convergence and efficiency, L&P method is suggested to be used in hydraulic structure contact analysis.

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