Generation of axially splitted ultra-long multiple optical needles/optical tubes using generalized cylindrical vector Bessel Gaussian beam phase modulated by annular Walsh function filter

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Abstract
Based on vector diffraction theory and annular Walsh function filter, axially splitted ultra-long multiple optical needles/optical tubes of electric as well as magnetic field distributions are numerically generated by generalized cylindrical vector Bessel Gaussian beam phase modulated by annular Walsh function filter. This multiple optical structures can be conveniently tailored by properly tuning the initial phase of the Cylindrical vector beam (CVB) and Walsh order of the annular Walsh function filter. Furthermore, the effect of energy flux density (Poynting vector) as well as Spin angular momentum (SAM) distribution in the focal region are theoretically analyzed. We expect that such an axially splitted focal systems could be applicable in optical trapping and manipulation of multiple particles, optical super resolution microscopy etc.

Keywords Annular Walsh function filter · Generalized CVB · Bessel Gaussian beam · SAM · Energy flux density

1 Introduction
Axially splitted sub wavelength multiple focal spots/holes with extended depth are useful for many potential application such as optical super resolution microscopy, optical trapping & manipulation of multiple particles, high density optical data storage etc. (Bingen et al. 2011; Rong et al. 2015; Yehoshua et al. 2015; Liang et al. 2020; Fuxi and Yang 2015; Li et al. 2015a; Ren et al. 2014). Recently, two extreme case of generalized cylindrical vector (radial & azimuthal) beams with different types of amplitude and phase filters are used to produce the axially splitted multiple focal spots/holes. For example radially polarized beam
can generates multiple focal spots by used different types of pupil plane filters (Yu and Zhan 2015; Wang et al. 2017; Prabakaran et al. 2015a). On the other hand, an azimuthally polarized beam can generates multiple focal holes in the focal plane by using different types of pupil filters (Guo et al. 2013; Yu et al. 2018; Chandrasekaran et al. 2016a; Lalithambigai et al. 2015). Except some trivial cases, synthesis & optimization of most of these pupil filters are more complicated as it suffers diffraction problem in general (Prabakaran et al. 2015b; Gould et al. 2012). It is also observed that the practical implementation of some continuous varying phase filters are get tedious because they have more number of ladder step approximation. To overcome the diffraction problems by using a set of orthogonal base function which can compute from decomposition of the pupil function. On the other hand annular Walsh function filters from annular Walsh functions overcomes the prespecified problem, as it composed of orthogonal functions as base function with finite number of discrete phase values (Hao et al. 2017). Order of the annular Walsh filters based on three transmission values such as zero amplitude (i.e., an obstruction), unity amplitude with zero phase, unity amplitude with pi phase & size of the annular region provides an additional degree of freedom to tailoring the focal structure(Walsh 1923). Hazra et al. tailored the image resolution of microscopy by annular Walsh filter (Mukherjee and Hazra 2013). P. Mukherjee et al. used annular filter as a pupil filter to delivered complex far-field amplitude distribution (Hao et al. 2017). The self-similarity nature of radial, annular, azimuthal and polar Walsh filters are analyzed in detailed by Hazra et al. (Hazra 2007; Mukherjee and Hazra 2014a, 2014b, 2014c). Recently, our group numerically generate multiple optical hole segments through tightly focused pure azimuthally polarized beam phase modulated with appropriate Walsh function filter (Thiruarul et al. 2020). So that, generation of axial multiple focal structures in the focal plane using annular Walsh filters as a pupil filter is simple unique and cost effective alternative for the prespecified methods. The distribution of Poynting vector and spin angular momentum (SAM) have great interest in the field of optical trapping & manipulation (Litvin et al. 2011; Sztul and Alfano 2008; Bergman, et al. 2008; Tang and Cohen 2010; Hendry et al. 2010; Bliokh and Nori 2011; Shi et al. 2020; Angelsky et al. 2012; Pan et al. 2019; Allen et al. 2003; Yao and Padgett 2011). Poynting vector describes the energy in the focal region. In optical trapping & manipulation process, the SAM related to spinning of the trapped particles to it’s own axis. There are so many articles demonstrate the distribution &redistribution of energy flow in the tightly focused region (Andrews and Babiker 2013; Man et al. 2018a, 2018b; Gaffar and Boruah 2015; Jiao et al. 2012; Yuan et al. 2011; Zhang and Ding 2009; Wu et al. 2014; Gao et al. 2017; Richards and Wolf 1959 ; Kotlyar et al. 2019; Stafeev et al. 2019; Stafeev and Kotlyar 2019). In recent years, the interaction of SAM with different type of polarizations are exploited (Zhao et al. 2007; Monteiro et al. 2009; Prajapati 2021; Han et al. 2018; Shu et al. 2011; Zhu et al. 2015; Bomzon et al. 2006; Chen and She 2008; Zhang et al. 2018; Bliokh et al. 2017). In more recently, Man et al. analytically demonstrate the angular momentum properties of hybrid cylindrical vortex vector beam in tightly focused optical system (Meng et al. 2019; Man et al. 2020). In this paper, we numerically discuss the generation of axially splitted multiple focal spots/holes through tightly focused generalized CVB with phase modulated by annular Walsh function filter. Furthermore, we numerically analyze the distribution of Poynting vector as well as SAM in the focal plane.
2 Theory

2.1 Generalized CVB

The states of polarization (SoP) of the generalized cylindrical vector beam is given by Youngworth and Brown (2000)

\[ E_0 = A_0[\cos(\phi + \phi_0)e_x + \sin(\phi + \phi_0)e_y] \]

Here \( A_0 \) is a constant, \( \phi \) is the azimuthal angle, \( \phi_0 \) is the initial phase, \( e_x \) and \( e_y \) are unit vectors along \( x \) and \( y \) axis, respectively. The SoP only depends on the azimuthal angle \( \phi(0 \leq \phi_0 \leq 2\pi) \) with their corresponding horizontal and vertical components are always in phase. Figure 1a-c depicts the three kind of CVB with \( \phi_0 = 0^\circ, 45^\circ, 90^\circ \). The cylindrical phase \( \phi_0 = 0^\circ \) and \( 90^\circ \), generates two extreme case of CVB’s such as radially and azimuthally polarized beams as described in Fig. 1d and f, respectively. The beam that posses cylindrical angle between \( \phi_0 = 0^\circ \) and \( 90^\circ \) are called generalized CVB.

2.2 Annular Walsh function

Walsh functions are a complete set of orthogonal functions over a given finite domain in which they take on values +1 or −1, except at a finite number of zero crossings within the domain. Number of zero crossings within the pre-specified domain is the order of

![Fig. 1 a–c Polarization distribution of CVB with \( \phi_0 = 0^\circ, 45^\circ, 90^\circ \). d–f polarization distribution with corresponding field intensity](image-url)
the Walsh function (Hao et al. 2017). Alike the Walsh function, annular Walsh function $\Psi_k^\delta(\theta)$ of index $k \geq 0$ & $\theta$ over an annular region with $\delta$ and 1 as inner and outer radii is represented in Fig. 2. In Fig. 2, black color represents the central annular region, blue and yellow colors represent the phase transition value $+\pi$ and $-\pi$, respectively. The orthogonality conditions implies to the annular Walsh functions over the interval $(\delta, 1)$ is (Walsh 1923)

$$\int_{\delta}^{1} \Psi_k^\delta(\theta)\Psi_m^\delta(\theta)\theta d\theta = \frac{1 - \delta^2}{2}\sigma_{mn}$$

(1)

where

$$\sigma_{km} = \begin{cases} 0, & k \neq m \\ 1, & k = m \end{cases}$$

(2)

$\sigma_{km}$ is the Kronecker delta.

The Walsh order $k$ is expressed as

Fig. 2 a and b Annular Walsh function $\Psi_k^\delta(\theta)$ in $(0, r)$ space and along $\theta$ for central annular obscuration $\delta = 0.3$, respectively.
\[
K_m = \sum_{n=0}^{v-1} K_m 2^n
\]

\[K_m\] are the bits, 0 or 1 of the binary numerical for \(k\), and \((2^v)\) is the power of 2 that just exceeds \(k\), for all \(0 < \theta < 1\), \(\Psi^\delta_k(\theta)\) is defined as

\[
\Psi^\delta_k(\theta) = \prod_{m=0}^{v-1} \text{sgn}\left\{ \cos \left[ K_m 2^m \pi \frac{(\theta^2 - \delta^2)}{(1 - \delta^2)} \right] \right\}
\]

where

\[
\text{sgn}(y) = \begin{cases} +1, & y > 0 \\ 0, & y = 0 \\ -1, & y < 0 \end{cases}
\]

The locations of the points of zero crossings for members of the set of functions \(\Psi^\delta_k(\theta)\), \(k=0,1,\ldots,(M-1)\) are given by

\[
\theta_i = \sqrt{\frac{(M-i)\delta^2 + i}{M}} \times \alpha
\]

The inner and outer angle of the filter is \(\theta_0 = \delta\) and \(\theta_M = \alpha\). \(\alpha = \arcsin(\text{NA})\). We note that the set of \((M-1)\) phase transiting (or zero crossing) locations, \(\theta_i, i = 1, 2, \ldots, (M-1)\) consist of all phase transiting locations required for specifying domains of this particular set of Walsh functions. And also an individual domain of this set of Walsh functions will have the same number of phase transition as its order.

In tight focusing condition, if the annular Walsh function presence in the input pupil, the pupil function \(f(\theta)\) is

\[
f(\theta) = \begin{cases} 0, & 0 \leq \theta < \delta \\ \Psi^\delta_k(\theta), & \delta \leq \theta < \alpha \end{cases}
\]

\(f(\theta)\) is binary (value either 0 or \(\pm \pi\)) only in the case of zero order annular Walsh function \(\Phi^\delta_0(\theta)\), for all other orders \(f(\theta)\) is ternary with \((0, +\pi, -\pi)\).

### 2.3 Tight focusing of CVB with annular Walsh function filter

A schematic representation for tight focusing is shown in Fig. 3. Here the generalized cylindrical vector Bessel Gaussian beam is phase modulated with a higher order annular Walsh function filter and focused through a high NA lens to produce multiple axial focal structures. Based on Richards and Wolf vector diffraction theory, the electric and magnetic energy densities near focal region can be derived in cylindrical coordinates \((r, \phi, z)\) as (Richards and Wolf 1959 Youngworth and Brown 2000)
Here $\alpha = \arcsin (\text{NA}/n)$ is maximum tangential angle of the high NA lens. $\text{NA}$ represents the numerical aperture of the objective lens. $n$ denotes the refractive index of the surrounding medium. $k = 2\pi/\lambda$ is the wave number in free space and $f$ is the focal distance.

The simplified forms of electric and magnetic fields are obtained by integrating along the azimuthal direction in Eqs. (8) and (9) as

$$\begin{align*}
\begin{bmatrix}
E_r(r, \psi, z) \\
E_\phi(r, \psi, z) \\
E_z(r, \psi, z)
\end{bmatrix} &= \frac{-ikf}{2\pi} \int_0^\alpha \int_0^{2\pi} \sin \theta \left( \sqrt{\cos \theta} \right) P(\theta) f(\theta) \\
& \times \exp[i(kz \cos \theta + r \sin \theta \cos(\phi - \psi))] \\
& \times \begin{bmatrix}
-\sin \varphi_0 \sin \phi + \cos \varphi_0 \cos \theta \cos \phi \\
\sin \varphi_0 \cos \phi + \cos \varphi_0 \cos \theta \sin \phi \\
-\cos \varphi_0 \sin \theta
\end{bmatrix} d\phi d\theta
\end{align*}$$

(8)

$$\begin{align*}
\begin{bmatrix}
H_r(r, \psi, z) \\
H_\phi(r, \psi, z) \\
H_z(r, \psi, z)
\end{bmatrix} &= \frac{-ikf}{2\pi} \int_0^\alpha \int_0^{2\pi} \sin \theta \left( \sqrt{\cos \theta} \right) P(\theta) f(\theta) \\
& \times \exp[i(kz \cos \theta + r \sin \theta \cos(\phi - \psi))] \\
& \times \begin{bmatrix}
-\cos \varphi_0 \sin \phi - \sin \varphi_0 \cos \theta \cos \phi \\
\cos \varphi_0 \cos \phi - \sin \varphi_0 \cos \theta \sin \phi \\
-\sin \varphi_0 \sin \theta
\end{bmatrix} d\phi d\theta
\end{align*}$$

(9)

Here $\alpha = \arcsin (\text{NA}/n)$ is maximum tangential angle of the high NA lens. $\text{NA}$ represents the numerical aperture of the objective lens. $n$ denotes the refractive index of the surrounding medium. $k = 2\pi/\lambda$ is the wave number in free space and $f$ is the focal distance.

The simplified forms of electric and magnetic fields are obtained by integrating along the azimuthal direction in Eqs. (8) and (9) as

$$\begin{align*}
\begin{bmatrix}
E_r(r, \psi, z) \\
E_\phi(r, \psi, z) \\
E_z(r, \psi, z)
\end{bmatrix} &= \frac{-ikf}{2} \int_0^\alpha P_e(\theta) f(\theta) \sqrt{\cos \theta} \sin \theta \exp(ikz \sin \theta) d\theta
\end{align*}$$

(10)

$$\begin{align*}
\begin{bmatrix}
H_r(r, \psi, z) \\
H_\phi(r, \psi, z) \\
H_z(r, \psi, z)
\end{bmatrix} &= \frac{-ikf}{2} \int_0^\alpha P_m(\theta) f(\theta) \sqrt{\cos \theta} \sin \theta \exp(ikz \sin \theta) d\theta
\end{align*}$$

(11)
where the polarization vector $P_e$ and $P_m$ are

$$P_e = \frac{m e^{i m \varphi}}{2} \left[ +i[J_{m+1}(\zeta) - J_{m-1}(\zeta)] \cos \theta \cos \varphi_0 - [J_{m+1}(\zeta) + J_{m-1}(\zeta)] \sin \varphi_0 \right]$$

$$-2J_m(\zeta) \sin \theta \cos \varphi_0$$

(12)

$$P_m = \frac{m e^{i m \varphi}}{2} \left[ -i[J_{m+1}(\zeta) - J_{m-1}(\zeta)] \cos \theta \sin \varphi_0 - [J_{m+1}(\zeta) + J_{m-1}(\zeta)] \cos \varphi_0 \right]$$

$$-2J_m(\zeta) \sin \theta \sin \varphi_0$$

(13)

Here $\zeta = kr \sin \theta$ and $J_m(\zeta)$ is the $m$th-order Bessel function of the first kind.

$P(\theta)$ is the relative amplitude of the input Bessel Gaussian beam at the entrance pupil and is given by Richards and Wolf (1959) Youngworth and Brown (2000)

$$P(\theta) = \exp \left[ -\beta_0^2 \left( \frac{\sin(\theta)}{\sin \alpha} \right)^2 \right] J_1 \left( 2\beta_0 \frac{\sin(\theta)}{\sin \alpha} \right)$$

(14)

Here $\beta_0$ is a beam parameter that indicates the ratio between pupil diameters to beam diameter. $J_1(x)$ is the first order Bessel function. If the annular Walsh function filter is placed at the pupil plane, the pupil function $P(\theta)$ is replacing by $P(\theta) f(\theta)$.

The Spin-angular momentum (SAM) density or an arbitrary time-harmonic beam is defined as (Meng et al. 2019)

$$S = \frac{\text{Im}[\varepsilon_0(E^* \times E) + \mu_0(H^* \times H)]}{4\omega}$$

(15)

where $\omega$ is the angular frequency of the laser beam. $*$ denotes the complex conjugate and $\text{Im}[x]$ denotes the imaginary value of the $x$. $\varepsilon_0$ and $\mu_0$ are the vacuum permittivity and permeability. The interaction between magnetic field and particle is weaker than the interaction between electric field and particle. So that, the SAM in terms of electric field is given by

$$S_r = \frac{\varepsilon_0}{4\omega} \text{Im}(E^*_r E_z - E_r E^*_z)$$

$$S_\phi = \frac{\varepsilon_0}{4\omega} \text{Im}(E^*_\phi E_z - E_\phi E^*_z)$$

$$S_z = \frac{\varepsilon_0}{4\omega} \text{Im}(E^*_r E_\phi - E_r E^*_\phi)$$

(16)

In terms of the electric and magnetic fields, the time averaged Poynting vector $P$ in cylindrical coordinates is given by Andrews and Babiker (2013)

$$P_r = \left( \frac{c}{8\pi} \right) \text{Re} \left( E_\phi M_z^* - E_z M_\phi^* \right)$$

$$P_\phi = \left( \frac{c}{8\pi} \right) \text{Re} \left( E_z M_r^* - E_r M_z^* \right)$$

$$P_z = \left( \frac{c}{8\pi} \right) \text{Re} \left( E_\phi M_r^* - E_\phi M_r^* \right)$$

(17)
where \( c \) denotes velocity of light and the asterisk means the complex conjugation. \( \text{Re}\{x\} \) denotes the real value of \( x \). The other values used for numerical computation are \( \lambda = 1 \), \( n = 1 \), \( k = 2\pi/\lambda \), \( NA = 0.85 \) and \( \alpha = \arcsin(NA) \).

### 3 Result and discussion

Figure 4 shows the electric field distribution for three kinds of CVBs in the focal plane. Keeping the cylindrical phase angle \( \rho_o \) as 0° with Walsh order \( k = 2 \) and annular obstruction \( \delta \) as 0.9 is found to generate an ultra long optical needle as depicted in Fig. 4a and the their corresponding axial and radial intensity distributions are mentioned in Fig. 4d and g, respectively. From Fig. 4d & g, the generated optical needle is found to possess FWHM (Full Width Half Maximum) of \(~0.37\lambda\) and DOF (Depth of Focus) as \(~14.47\lambda\). Tuning the cylindrical angle from 0° to 45°, the focal structure can be broadened in radial direction as shown in Fig. 4b, e, h. From Fig. 4h, the total electric field component is found to be dominated by peak centered \( E_r \) component as well as annular shaped \( E_{\phi} \) component. Further increasing \( \rho_o \) as 90°, an ultra long optical tube is generated with FWHM of \(~0.37\lambda\) and DOF of \(~14.47\lambda\) and are depicted in Fig. 4c, f, i. The total electric field is observed to be dominated only by \( E_{\phi} \) component (from Fig. 4i). We concluded from above result, one can

**Fig. 4** Calculated electric energy density in the focal region for initial phase \( \rho_o = 0°, 45°, 90° \) with Walsh order \( k = 2 \), annular obstruction \( \delta = 0.9 \) with \( \beta = 1.2 \). a-c, d-f, g-i describes the 2D surface distribution of \(|E|^2\) in \( r-z \) plane and their corresponding axial and radial intensity distributions, respectively.
generates both ultra long optical needle as well as optical tube for special case of generalized CVBs such as radial ($\phi_o=0^\circ$) and azimuthal ($\phi_o=90^\circ$) beams. Moreover, the FWHM and DOF achieved here is much almost same as the our previously proposed methods. (Seethalakshmi et al. 2019; Senthilkumar et al. 2019; Chandrasekaran et al. 2016b) These focal structures found potential applications in optical recording, optical trapping, optical microscopy etc. (Bingen et al. 2011; Rong et al. 2015; Yehoshua et al. 2015; Liang et al. 2020; Fuxi and Yang 2015; Li et al. 2015a; Ren et al. 2014).

Figure 5 shows the magnetic field distribution in the focal plane. The parameters are same as in Fig. 4. For radially polarized beam ($\phi_o=0^\circ$), generated an ultra long magnetic tube, dominated only by $M_\phi$ component as depicted in Fig. 5a, d, g. For initial angle $\phi_o$ as $45^\circ$, the magnetic field distribution is same as like as the electric field distribution as shown in Fig. 5b, e, h. For an azimuthally polarized beam ($\phi_o=90^\circ$), the magnetic field turns to an ultra long magnetic needle, dominated by only $M_z$ component as shown in Fig. 5c, f, i. The FWHM and DOF values of generated magnetic field distributions are almost same as it the electric field distribution. The generated ultra long magnetic needle and tube have application in magnetic recording, magnetic resonance microscopy etc. (Nie et al. 2017; He et al. 2020).

Figure 6 depicts same as Fig. 4, but for $k$ as 25. It is noted that for higher Walsh order ($k=25$), one can generates axially splitted, co-axially equidistant self similar
multiple optical needles and optical tubes in the geometrical plane. Focusing the radially polarized CVB ($\phi_o = 0^\circ, 45^\circ, 90^\circ$) with higher Walsh order ($k = 25$) can generate an optical segment with six self-similar optical needles in the range of $-25\lambda$ to $25\lambda$ along the optical axis. Each optical needle possesses FWHM of $\sim 0.34\lambda$, DOF of $\sim 4.36\lambda$, and axial separation between them measured as $\sim 4.4\lambda$. Further increasing $\phi_o$ as $45^\circ$, can generate an optical segment which contains six radial flattop profiled optical needles with FWHM of $\sim 0.55\lambda$ and DOF as $\sim 4.36\lambda$ and are shown in Fig. 6b, c, h. On the other hand, an azimuthally polarized CVB ($\phi_o = 90^\circ$) generated an optical segment with six self-similar, axially split optical tubes in the focal region as shown in Fig. 6c, f, i. The values such as FWHM and DOF are similar to those of optical needles in the optical segment for radially polarized beams. These multiple optical needles and optical tubes have potential applications in multiple optical trapping and recording, multiple optical manipulation, high-resolution microscopy etc. (Bingen et al. 2011; Rong et al. 2015; Yehoshua et al. 2015; Liang et al. 2020; Fuxi and Yang 2015; Li et al. 2015a; Ren et al. 2014).

On the other hand, higher Walsh order ($k = 25$) for magnetic fields, can generate magnetic tube segments as well as magnetic needle segments for radially and azimuthally polarized CVBs, respectively. The number of focal structures in the segment and size of them are the same as in Fig. 6. This multiple magnetic needles and magnetic tubes can increase the performance of magnetic recording, magnetic resonance microscopy etc. (Nie et al. 2017; He et al. 2020) (Fig. 7).
Overall, the initial phase angle of the generalized CVB can change the focal pattern from focal spot to focal hole (in electric field distribution) or focal hole to focal spot (in the magnetic field distribution). Similarly, increasing the Walsh order ($k > 2$) with high annular value can split the focal structures in the axial plane with long focal depth. The overall values of FWHM and DOF are summarized in Table 1.

Figure 8a shows that the Poynting vector distribution for cylindrical angle $\phi_0 = 0^\circ$ appeared as a set of discrete spots with field arrows shows the Poynting vector converges above and below the axial line. From Fig. 8d, the total Poynting vector distribution is found

![Image](image_url)

**Table 1** FWHM & DOF values for generated focal structures

| Focal Structures                          | Walsh order $k$ | Annular Obstruction $\delta$ | Beam parameter $\beta$ | FWHM($\lambda$) | DOF($\lambda$) |
|-------------------------------------------|-----------------|-------------------------------|------------------------|-----------------|----------------|
| Electric($\phi_0 = 0^\circ$) & magnetic ($\phi_0 = 90^\circ$) focal spot | 2               | 0.9                           | 1.2                    | 0.37            | 14.47          |
| Electric($\phi_0 = 90^\circ$) & magnetic ($\phi_0 = 0^\circ$) focal hole | 2               | 0.9                           | 1.2                    | 0.37            | 14.47          |
| Electric($\phi_0 = 0^\circ$) & magnetic ($\phi_0 = 90^\circ$) multiple spots | 25              | 0.9                           | 1.2                    | 0.34            | 4.36           |
| Electric($\phi_0 = 90^\circ$) & magnetic ($\phi_0 = 0^\circ$) multiple holes | 25              | 0.9                           | 1.2                    | 0.34            | 4.36           |
to be dominated by strong forward longitudinal component. From Fig. 8g & j, the total SAM for cylindrical angle $\phi_0 = 0^\circ$ is transverse and diverges radially from the axial line. The total SAM is only dominated by $S_{\phi}$ component is shown in Fig. 8j. Figure 8b, e & h, k shows the energy flux density and SAM density for cylindrical angle $\phi_0 = 45^\circ$. It is observed that the energy flow appears as a set of discrete spots that diverge above and below the axial line but converges to the edge of the each spot as depicted in Fig. 8b & e. From Fig. 8e, the total energy flow is observed to be dominated only by $S_z$ component. From Fig. 8h & k, the SAM is transverse and converges towards the centre of the axial line.
The total SAM density is contain both $S_r$ and $S_\phi$ components and dominated by $S_r$ component is shown in Fig. 8k. Further increase the cylindrical angle $\phi_0 = 90^\circ$, the total Poynting vector distribution is diverge above and below the axial line but not converge to the centre of each spot as it in the previous case and are shown in Fig. 8c & f. To compare the Fig. 8h & k with Fig. 8i & l, the SAM is totally diverge from the centre of the axial line at the same time total SAM only dominated by $S_r$ component. From the above results, we noted that the change of cylindrical angle $\phi_0$ can greatly affect the energy flow distribution and SAM distribution at the focal region. These findings are useful in multiple optical trapping and manipulation.

4 Conclusion

Application of annular Walsh function filter for the generation of axially splitted ultra-long multiple optical needles/optical tubes of electric as well as magnetic field distributions are numerically studied based on the vector diffraction theory. Results shows lower Walsh order one can generates single axially extended focal patterns of spot, flattop profile, hole corresponding to initial phase angle of the generalized cylindrical vector Bessel Gaussian beam. On the other hand, for higher Walsh order with appropriate initial cylindrical angle one can generates axially splitted self-similar multiple ultra long optical structures. The distribution properties of Poynting vector as well as SAM densities in the focal region are theoretically demonstrated and are found to be changes with the changed of cylindrical angle $\phi_0$. We conclude that, such an axially splitted focal systems could be applicable in optical trapping and manipulation of multiple particles, optical microscopy etc.

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Declarations

Conflict of interest The authors declare no conflicts of interest.

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