Light pseudoscalar $\eta$ and $H \to \eta\eta$ decay in the simplest little Higgs mode

Kingman Cheung

Department of Physics, National Tsing Hua University, Hsinchu, Taiwan, R.O.C.
The National Center for Theoretical Sciences, Hsinchu, Taiwan
Email: cheung@phys.nthu.edu.tw

Jeonghyeon Song

Department of Physics, Konkuk University, Seoul 143-701, Korea
Email: jhsong@konkuk.ac.kr

Abstract: The SU(3) simplest little Higgs model in its original framework without the so-called $\mu$ term inevitably involves a massless pseudoscalar boson $\eta$, which is problematic for $b$-physics and cosmological axion limit. With the $\mu$ term introduced by hand, the $\eta$ boson acquires mass $m_\eta \sim \mu$, which can be lighter than half the Higgs boson mass in a large portion of the parameter space. In addition, the introduced $\mu$ term generates sizable coupling of $H-\eta-\eta$. The Higgs boson can dominantly decay into a pair of $\eta$'s especially when $m_H$ below the $WW$ threshold. Another new decay channel of $H \to Z\eta$ can be dominant or compatible with $H \to W^+W^-$ for $m_H$ above the $Z\eta$ threshold. We show that the LEP bound on the Higgs boson mass is loosened to some extent due to this new $H \to \eta\eta$ decay channel as well as the reduced coupling of $H-Z-Z$. The Higgs boson mass bound falls to about 110 GeV for $f = 3 - 4$ TeV. Since the $\eta$ boson decays mainly into a $b\bar{b}$ pair, $H \to \eta\eta \to 4b$ and $H \to Z\eta \to Zb\bar{b}$ open up other interesting search channels in the pursuit of the Higgs boson in the future experiments. We discuss on these issues.

Keywords: Little Higgs, Collider Phenomenology, Higgs boson decay.
1. Introduction

The Higgs boson is the last ingredient of the standard model (SM) to be probed at experiments. Precision measurements of the electroweak parameters with logarithmic dependence on the Higgs boson mass give indirect but tantalizing limit on $m_H$ to be less than 186 GeV at the 95% confidence level (C.L.)\textsuperscript{[1]}. Direct search by the four LEP collaborations, ALEPH, DELPHI, L3 and OPAL, resulted in no significant data. A lower bound on the Higgs boson mass is established to be 114.4 GeV at the 95% C.L.\textsuperscript{[2]}, which is applicable to the SM and its extensions that preserve the nature of the SM Higgs boson, \textit{e.g.}, minimal supersymmetric SM (MSSM) in most parameter space.

In some other extensions, however, if the nature of the light Higgs boson is drastically modified, the limit from direct search at LEP becomes weaker. Phenomenologically, evading the LEP data is possible when the Higgs boson coupling $g_{ZZH}$ with the $Z$ boson is reduced and/or the Higgs boson decays into non-SM light particles. In the CP-conserving MSSM, for example, the lower bound on $m_H$ can be in the vicinity of 93 GeV at the 95\% C.L.\textsuperscript{[3]}. If we further allow CP violation the result becomes more dramatic that no absolute limits can be set for the Higgs boson mass\textsuperscript{[4]}. Since the Higgs mass bound has far-reaching implications on the Higgs search at the LHC, the examination of the LEP bound on $m_H$ in other new models is of great significance.

Recently, little Higgs models have drawn a lot of interests as they can solve the little hierarchy problem between the electroweak scale and the 10 TeV cut-off scale $\Lambda$\textsuperscript{[5]}. A relatively light Higgs boson mass compared to $\Lambda \sim 10$ TeV can be explained if the Higgs boson is a pseudo-Nambu-Goldstone boson (pNGB) of an enlarged global symmetry. Quadratically divergent Higgs boson mass at one-loop level, through the gauge, Yukawa,
and self-couplings of the Higgs boson, is prohibited by the collective symmetry breaking mechanism. According to the global symmetry breaking pattern, there are various models with the little Higgs mechanism \[9\]. Detailed studies have been also made, such as their implications on electroweak precisions data (EWPD) \[7\] and phenomenologies at high energy colliders \[8\].

Considering the possibility of evading the LEP data on the Higgs mass, the simplest little Higgs model \[4\] is attractive as it accommodates a light pseudoscalar boson \(\eta\), which the Higgs boson can dominantly decay into. The model is based on \([SU(3) \times U(1)_X]^2\) global symmetry with its diagonal subgroup \(SU(3) \times U(1)_X\) gauged. The vacuum expectation value (VEV) of two \(SU(3)-\)triplet scalar fields, \(\langle \Phi_{1,2} \rangle = (0, 0, f_{1,2})^T\), spontaneously breaks both the global symmetry and the gauge symmetry. Here \(f_{1,2}\) are at the TeV scale. Uneaten pNGB’s consist of a \(SU(2)_L\) doublet \(h\) and a pseudoscalar \(\eta\). Loops of gauge bosons and fermions generate the Coleman-Weinberg (CW) potential \(V_{CW}\) which contains the terms such as \(h^\dagger h\) and \((h^\dagger h)^2\): The Higgs boson mass and its self-coupling are radiatively generated. However the CW potential with non-trivial operators of \(|\Phi_1^* \Phi_2|\) does not have the dependence of \(\eta\) which is only a phase of sigma fields \(\Phi_{1,2}\) \[10, 11\]. This \(\eta\) becomes massless, which is problematic for \(\eta\) production in rare \(K\) and \(B\) decays, \(\bar{B}\)-\(B\) mixing, and \(Y \to \eta \gamma\), as well as for the cosmological axion limit.

One of the simplest remedies was suggested by introducing a \(-\mu^2 (\Phi_1^* \Phi_2 + h.c.)\) term into the scalar potential by hand. Even though this breaks the global \(SU(3)\) symmetry and thus damages the little Higgs mechanism, its contribution to the Higgs boson mass is numerically insignificant. This \(\mu\) determines the scale of \(\eta\) mass. By requiring negative Higgs mass-squared parameter for electroweak symmetry breaking (EWSB), we show that the \(\mu\) (and thus \(m_\eta\)) is of the order of 10 GeV. Thus, we have light pseudoscalar particles. In addition, the \(\mu\) term also generates the \(\lambda' hh \eta^2\) term in the CW potential. As the \(h\) field develops the VEV \(v\), the \(h^\dagger h \eta^2\) term emerges with the strength proportional to \(v \mu^2 / f^2\), with \(f = \sqrt{f_1^2 + f_2^2}\) at the TeV scale. The Higgs boson can then decay into two \(\eta\) bosons. Furthermore, this light \(\eta\) opens a new decay channel of \(H \to Z \eta\). Indeed, these two new decay channels can be dominant, as shall be shown later.

Another issue which we make a thorough investigation into is the condition for successful electroweak symmetry breaking (EWSB). The model with the \(\mu\) term is determined by four parameters: \(f\), \(\tan \beta = f_2 / f_1\), \(x_\lambda\), and \(\mu\). Here \(x_\lambda\) is the ratio of two Yukawa couplings in the third generation quark sector. The radiatively generated Higgs VEV \(v\) is also determined by these four parameters: The SM EWSB condition \(v = 246\) GeV fixes one parameter, \(e.g., \tan \beta\). For \(x_\lambda \in [1, 15]\), \(\mu \sim \mathcal{O}(10)\) GeV, and \(f = 2 - 4\) TeV, the \(v = 246\) GeV condition limits \(\tan \beta\) around 10. This large \(\tan \beta\) reduces the effective \(g_{ZZH}\) coupling in this model. With smaller \(g_{ZZH}\) and \(B(H \to b\bar{b})\) than in the SM, the LEP Higgs boson mass bound based on the limit \((g_{ZZH} / g_{ZZH}^{SM})^2 B(H \to b\bar{b})\) can be reduced \[4\]. Yet there was a general search by the DELPHI collaboration \[12\] in the channel \(e^+ e^- \to ZH \to Z(AA) \to Z + 4b\). The \(\eta\) boson in the present model is similar to the \(A\) boson. We shall apply the limit obtained in the DELPHI analysis to the present model, which shall be shown entirely unconstrained.

The organization of the paper is as follows. In the next section, we highlight the essence
of the original SU(3) simplest little Higgs model, in particular the Higgs sector. We will show that the original model can accommodate proper EWSB as well as the Higgs mass $\sim 100$ GeV. After explicit demonstration of no $\eta$ dependence on the scalar potential, we will discuss the problem of the massless pseudoscalar $\eta$. In Sec. 3, we introduce the $\mu$ term and discuss the EWSB implication as well as the mass spectra of the Higgs boson and $\eta$. In Sec. 4, we calculate the branching ratio $H \to \eta\eta$ and discuss its impact on the Higgs boson mass bound. We discuss further possibilities to investigate this scenario and then conclude in Sec. 5.

2. SU(3) simplest group model without the $\mu$ term

The SU(3) simplest little Higgs model is based on $[SU(3) \times U(1)_X]^2$ global symmetry with its diagonal subgroup $SU(3) \times U(1)_X$ gauged. The pNGB multiplet is parameterized by two complex SU(3) triplet scalar fields $\Phi_{1,2}$:

$$\Phi_1 = e^{i t_\beta \Theta} \Phi_1^{(0)}, \quad \Phi_2 = e^{-i \Theta / t_\beta} \Phi_2^{(0)},$$

(2.1)

where $t_\beta \equiv \tan \beta$ and

$$\Theta = \frac{1}{f} \left[ \begin{pmatrix} 0 & 0 & h \\ 0 & 0 & h^\dagger \\ h^\dagger & 0 & 0 \end{pmatrix} + \frac{\eta}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right] \equiv \frac{1}{f} \mathbb{H} + \frac{\eta}{\sqrt{2} f} \mathbb{I}_3.$$

(2.2)

The kinetic term for $\Phi_{1,2}$ is

$$\mathcal{L}_\Phi = \sum_{i=1,2} \left| \partial_\mu + ig A_\mu^a T^a - \frac{ig_x}{3} B_\mu \right| \Phi_i^2,$$

(2.3)

where $T^a$ are the SU(3) generators while $A_\mu^a$ and $B_\mu$ are the SU(3) and U(1) gauge fields, respectively. Two gauge couplings of $g$ and $g_x$ are fixed by the SM gauge couplings such that SU(3) gauge coupling $g$ is just the SM SU(2)$_L$ gauge coupling and $g_x = g' / \sqrt{1 - t_W^2 / 3}$.

Each of the SM fermionic doublets is promoted to a SU(3) triplet. Focusing on the third generation quarks, we introduce a 3 representation of SU(3), $\chi_L = (t_L, b_L, i U_L)^T$, as well as two weak-singlet quarks, $U_{R1}$ and $U_{R2}$. The Yukawa interaction is

$$\mathcal{L} = i \lambda_1 U_{R1}^\dagger \Phi_1^\dagger \chi_L + i \lambda_2 U_{R2}^\dagger \Phi_2^\dagger \chi_L + h.c.,$$

(2.4)

where the complex number $i$’s guarantee positive masses for fermions. According to the SU(3) representation of the first two generation quarks and all generation leptons, there are two versions for fermion embedding. This variation in model building is possible since light quarks and leptons make very little contributions to the radiative Higgs mass. The first fermion embedding is called “universal” embedding, where all three generations have identical quantum numbers. The other is the “anomaly-free” embedding where anomaly-cancellation is required for easier UV completion: The third generation quarks and all leptons are put into 3 representations of SU(3), while the first two generation quarks into
3. Yukawa couplings for light quarks and leptons in both embedding cases are referred to Ref. [11].

When \(\Phi_1\) and \(\Phi_2\) develop the aligned VEV of

\[
\langle \Phi_1 \rangle = \Phi_1^{(0)} = (0, 0, f \cos \beta)^T, \quad \langle \Phi_2 \rangle = \Phi_2^{(0)} = (0, 0, f \sin \beta)^T, \tag{2.5}
\]
two kinds of symmetry breaking occur. First, the global symmetry is spontaneously broken into its subgroup of \([\text{SU}(2) \times \text{U}(1)]^2\), giving rise to ten Nambu-Goldstone bosons. Second, the gauge symmetry \(\text{SU}(3) \times \text{U}(1)_X\) is broken into the SM \(\text{SU}(2)_L \times \text{U}(1)_Y\), as five Nambu-Goldstone bosons are eaten. Five new gauge bosons and one heavy top-like quark \(T\) appear with heavy mass of order \(f \sim \text{TeV}\). The heavy gauge bosons include a \(Z'\) gauge boson (a linear combination of \(A^8_\mu\) and \(B^x_\mu\)) and a complex SU(2) doublet \((Y^0, X^-)\) with masses of

\[
M_{Z'} = \sqrt{\frac{2}{3 - t_W^2}} g f, \quad M_{X^\pm} = M_Y = \frac{g f}{\sqrt{2}}. \tag{2.6}
\]

The new heavy \(T\) quark mass is

\[
M_T = \sqrt{2} \frac{t_\beta^2 + x_\lambda^2}{(1 + t_\beta^2)x_\lambda} \frac{m_t}{v} f, \tag{2.7}
\]
where \(x_\lambda = \lambda_1/\lambda_2\).

Brief comments on the EWPD constraint on \(f\) are in order here. According to Ref. [9], the anomaly-free model is less constrained. The strongest bound comes from atomic parity violation with \(f > 1.7\ \text{TeV}\) at the 95\% C.L. A more recent analysis in Ref. [14] gives a stronger bound of \(f > 4.5\ \text{TeV}\) at 99\% C.L. Main contribution comes from an oblique parameter \(\hat{S}\) due to the \(Z'\) gauge boson. They applied the approximation for \(Z'\) that is eliminated by solving its equation of motion. Considering both analyses, we take \(f = 2 - 4\ \text{TeV}\) as reasonable choices.

The gauge and Yukawa interactions of the Higgs boson explicitly break the \(\text{SU}(3)\) global symmetry, generating the Higgs mass at loop level. In the CW potential up to dimension four operators, only the \(|\Phi_1^\dagger \Phi_2|^2\) term leads to non-trivial result for the pNGB’s. A remarkable observation is that this \(|\Phi_1^\dagger \Phi_2|^2\) term does not have any dependence on \(\eta\)\[15\]. This can be easily seen by the expansion of, \(e.g., \Phi_1\) as

\[
\Phi_1 = \exp \left(\frac{i t_\beta \eta}{\sqrt{2} f}\right) \exp \left(\frac{i t_\beta \mathbb{H}}{f}\right) \Phi_1^{(0)}, \tag{2.8}
\]
which we have used the Baker-Hausdorff formula with \([\mathbb{H}, \mathbb{I}_3] = 0\). This compact form is very useful when calculating the \(\Phi_1^\dagger \Phi_2\):

\[
\Phi_1^\dagger \Phi_2 = f^2 s_\beta c_\beta e^{-i\left(t_\beta + h_0\right)} \frac{2}{\sqrt{2} f} \cos \left(\frac{h_0}{f c_\beta s_\beta}\right). \tag{2.9}
\]

The \(|\Phi_1^\dagger \Phi_2|^2\) term or the CW potential has no dependence on \(\eta\). Thus, the pseudoscalar \(\eta\) remains massless in the original model.
On the contrary, the Higgs boson mass is radiatively generated with one-loop logarithmic divergence and two-loop quadratic divergence. The troublesome one-loop quadratic divergence is eliminated by the little Higgs mechanism. The CW potential is

$$V_{CW} = -m_0^2 h^\dagger h + \lambda_0 (h^\dagger h)^2, \tag{2.10}$$

where

$$m_0^2 = \frac{3}{8\pi^2} \left[ \frac{\lambda t M_T^2}{M_T^2} \ln \frac{\Lambda^2}{M_T^2} - \frac{g^2}{4} M_X^2 \ln \frac{\Lambda^2}{M_X^2} - \frac{g^2}{8} (1 + t_W^2) M_Z^2 \ln \frac{\Lambda^2}{M_Z^2} \right], \tag{2.11}$$

$$\lambda_0 = \frac{1}{3s\beta c\beta} \frac{m_0^2}{f^2} + \frac{3}{16\pi^2} \left[ \lambda t^4 \ln \frac{M_T^2}{m_t^2} - \frac{g^4}{8} \ln \frac{M_X^2}{m_W^2} - \frac{g^4}{16} (1 + t_W^2)^2 \ln \frac{M_Z^2}{m_Z^2} \right]. \tag{2.12}$$

Here $\lambda_t = \sqrt{2} m_t/v$ and $\Lambda \simeq 4\pi f$. The negative mass-squared term for the Higgs doublet in Eq. (2.10) generates the VEV for the Higgs boson as $\langle h \rangle = v_0/\sqrt{2}$, which then triggers the EWSB and generates the Higgs boson mass $m_{H0}$, given by

$$v_0^2 = \frac{m_0^2}{\lambda_0}, \quad m_{H0}^2 = 2m_0^2. \tag{2.13}$$

This CW potential alone has been considered insufficient to explain the EWSB, due to excessively large soft mass-squared $m_0^2$. If $f = 2$ TeV and $x_\lambda = t_\beta = 2$, for example, $m_0 \simeq 710$ GeV and thus $m_H \simeq 1$ TeV. In addition, the quartic coupling $\lambda_0$ is also small since it is generated by logarithmically divergent diagrams, not by quadratically divergent ones. In the ordinary parameter space of $t_\beta$ and $x_\lambda$ of the order of one, the $v_{CW} \simeq 246$ GeV condition cannot be satisfied. However, this flaw in the original model without the $\mu$ term is not as serious as usually considered in the literatures. If we extend the parameter space allowing $x_\lambda$ and $t_\beta$ up to $\simeq 10$, the $v_0 \simeq 246$ GeV condition can be met easily. Reducing $m_0^2$ in Eq. (2.11) is possible if the heavy $T$ mass decreases. As discussed in Ref. [18], the heavy $T$ mass is minimized when $t_\beta = x_\lambda$ and $t_\beta$ increases. Larger $t_\beta$ can help to satisfy $v_0 \simeq 246$ GeV. In addition, large $t_\beta$ suppresses the new contributions to the EWPD [18].

When we require that the radiatively generated Higgs VEV be equal to the SM Higgs VEV, what is the SM Higgs VEV in this model is an important question. A definite way is to require that the SM Higgs VEV $v$ should explain the observed SM $W$ gauge boson mass. In this model, the $W$ gauge boson mass is modified into

$$m_W = \frac{gv}{2} \left[ 1 - \frac{v^2}{12f^2} \frac{t_\beta^4 - t_\beta^2 + 1}{t_\beta^2} + O \left( \frac{v^4}{f^4} \right) \right]. \tag{2.14}$$

The Higgs boson VEV explaining $m_W$, which we denote by $v_W$, is

$$v = v_0 \left[ 1 + \frac{v_0^2}{12f^2} \frac{t_\beta^4 - t_\beta^2 + 1}{t_\beta^2} + O \left( \frac{v^4}{f^4} \right) \right] \equiv v_W, \tag{2.15}$$

where $v_0 = 2m_W/g = 246.26$ GeV. With the observed $m_W$, the $v_W$ in this model depends on $t_\beta$ and $f$. 

- 5 -
Figure 1: Allowed parameter space of \((x_\lambda, \tan \beta)\) for \(\mu = 0, 30\) GeV by valid electroweak symmetry breaking. The red and blue (or thin) lines are the contours of \(m^2 = 0\) and \(v = v_W\) for \(\mu = 0\), respectively. The black and green (or thick) lines satisfies \(m^2 = 0\) and \(v = v_W\) for \(\mu = 30\) GeV, respectively.

In Fig. 1, we present the contours of \(m_0^2 = 0\) and \(v_0 = v_W\) (lines for \(\mu = 0\)). In the upper right corner, \(m_0^2\) becomes negative such that the EWSB is not possible. This is because too large \(t_\beta\) and thus too small \(M_T\) makes \(m_0^2\) negative. The \(\lambda_0 < 0\) region is contained in the excluded region by \(m_0^2 = 0\). Thin lines are for \(\mu = 0\) case: We do have considerably large parameter space, particularly around \(t_\beta \simeq 10\), to explain appropriate EWSB.

Apparently the EWSB condition does not really need the extra \(\mu\) term if we can take large \(t_\beta\) around 10. The most serious problem is the presence of massless pseudoscalar \(\eta\). Any term in the CW potential, proportional to \(|\Phi_1^\dagger \Phi_2|^n\) or \(|\Phi_1^\dagger \Phi_2|^n\), cannot accommodate the \(\eta\) dependence. Even though lower bounds on CP-odd scalar masses from the \(b\)-physics signal \([19]\) and cosmology \([20]\) are not very stringent, any pseudoscalar particle should be massive: The \(\eta\) mass can be as low as \(O(100)\) MeV from the \(b\)-physics signal such as rare \(K\), \(B\) and radiative \(\Upsilon\) decays with the \(\eta\) in the final state, \(B_s \rightarrow \mu^+ \mu^-\) and \(B-B\) mixing; the cosmological bound is also weak but finite, as low as 10 MeV. We should, therefore, extend the model to cure this massless pseudoscalar problem.

3. SU(3) model with the \(\mu\) term

The simplest solution to the massless \(\eta\) problem as well as generically large \(m_0^2\) problem is to introduce a new term of \(-\mu^2(\Phi_1^\dagger \Phi_2 + h.c.)\) into the scalar potential by hand \([9, 16, 17]\). Unfortunately, this explicitly breaks the global SU(3) symmetry. The little Higgs mechanism is lost as the Higgs loop generates the one-loop quadratically divergent corrections to the Higgs mass. Since this correction is numerically insignificant, we adopt this extension.
Since the new term can be written as
\[-\mu^2(\Phi^\dagger \Phi_1^2 + h.c.) = -2\mu^2 f^2 s_\beta c_\beta \cos \left( \frac{\eta}{\sqrt{2} s_\beta c_\beta f} \right) \cos \left( \frac{\sqrt{h^\dagger h}}{f c_\beta s_\beta} \right), \] (3.1)
the scalar potential becomes
\[V = -m^2 h^\dagger h + \lambda (h^\dagger h)^2 - \frac{1}{2} m_\eta^2 \eta^2 + \lambda' h^\dagger h \eta^2 + \cdots, \] (3.2)
where
\[m^2 = m_0^2 - \frac{\mu^2}{s_\beta c_\beta}, \quad \lambda = \lambda_0 - \frac{\mu^2}{12 s_\beta c_\beta}, \quad \lambda' = -\frac{\mu^2}{4 f^2 s_\beta c_\beta} \] (3.3)
The Higgs VEV \(v\), the Higgs mass \(m_H\), and \(\eta\) mass \(m_\eta\) are then
\[v^2 = \frac{m^2}{\lambda}, \quad m_H^2 = 2m^2, \quad m_\eta^2 = \frac{\mu^2}{s_\beta c_\beta} \cos \left( \frac{v}{\sqrt{2} f s_\beta c_\beta} \right). \] (3.4)

The CW potential as well as the masses of new heavy particles depend on the following four parameters:
\[f, \quad x_\lambda, \quad t_\beta, \quad \mu. \] (3.5)
As before, the \(v = v_W\) condition removes one parameter. In Fig. 1, we present the contours of \(v = v_W\) for \(\mu = 30\text{ GeV}\) and \(f = 2\text{ TeV}\). Increasing \(\mu\) reduces the allowed value of \(t_\beta\) by \(\sim 10\%\).

![Figure 1](image1.png)

**Figure 2**: Allowed parameter space of \((t_\beta, \mu)\) for \(x_\lambda = 3, 6, 10\) by requiring positive Higgs mass-squared parameter \(m^2\). We consider \(f = 2\text{ TeV}\) and \(f = 4\text{ TeV}\). Upper right corner is excluded since \(m^2 < 0\).

Unfortunately there is no prior information even about the scale of \(\mu\). Nevertheless upper bound on \(\mu\) can be imposed since \(\mu\) contributes negatively to the Higgs mass-squared parameter \(m^2\). If \(m^2\) becomes negative due to too large \(\mu\), the EWSB cannot occur. In Fig. 2, we present the allowed parameter space of \((t_\beta, \mu)\) for \(x_\lambda = 3, 6, 10\) and \(f = 2, 4\text{ TeV}\) by requiring \(m^2 > 0\). The upper right corner where \(m^2 < 0\) is excluded due to the EWSB.
condition. Since the \( v = v_W \) condition prefers \( t_\beta \simeq 10 \) as in Fig. 1, the scale of \( \mu \) is about \( \mathcal{O}(10) \) GeV.

With the constraint of \( v_{CW} = v \), two parameters of \( x_\lambda \) and \( \mu \) determine the masses of the Higgs boson and \( \eta \) at a given \( f \). In Fig. 3, we plot, as a function of \( x_\lambda \), the \( m_H \) (solid lines) and \( m_\eta \) (dashed line) for \( \mu = 0, 10, 30 \) GeV and \( f = 2, 4 \) TeV. Note that \( m_\eta = 0 \) for \( \mu = 0 \). For non-zero \( \mu \), the \( \eta \) mass is around \( \sim \mathcal{O}(10) \) GeV. And the Higgs boson mass is generically around \( \sim 100 \) GeV.

![Figure 3: The masses of the Higgs boson (solid line) and \( \eta \) (dashed line) as a function of \( x_\lambda \) for \( f = 2 \) TeV and \( f = 4 \) TeV. The value of \( t_\beta \) is determined by the \( v_{CW} = v \) condition.](image)

In addition, we find some other interesting features. First, both \( m_H \) and \( m_\eta \) attain a minimum with a given \( f \), which occurs when \( \mu = 0 \). This minimum of the Higgs boson mass is close to the LEP bound of 114.4 GeV, and decreases as \( f \) increases. For example, \( m_H^{(\text{min})} = 114.5 \) GeV for \( f = 2 \) TeV, and \( m_H^{(\text{min})} = 88.9 \) GeV for \( f = 3 \) TeV.
Investigation of the LEP bound on the Higgs boson mass is of great significant in this model. Second, $\mu$ increases both $m_H$ and $m_\eta$. Since $m_\eta \propto \mu$ as in Eq. (2.13), increasing $m_\eta$ with $\mu$ is easy to understand. However $m_H$ has negative contribution from increasing $\mu$ as in Eq. (3.3): Increasing $m_H$ with $\mu$ seems strange. This behavior is due to the $t_\beta$ value determined by the $v = v_W$ condition. With high $\mu$, the $t_\beta$ value for $v = v_W$ is reduced as in Fig. 3. Smaller $t_\beta$ raises the $M_T$, and thus also raises its radiative contribution to the Higgs boson mass.

Another important point is that $m_\eta$ can be quite light. In principle, $m_\eta$ can be as light as the current $b$ physics and/or cosmological bounds allow. In this paper, however, we adopt the generic mass scale for $\eta$, around $\mathcal{O}(10)$ GeV. This light pseudoscalar particle can have a significant implication on the phenomenology of the Higgs boson. The $\lambda h^* h \eta^2$ term in the scalar potential of Eq. (3.2) leads to the coupling of $H-\eta-\eta$: If $\eta$ boson is light enough, the Higgs boson can decay into a pair of $\eta$ and the Higgs discovery strategy should be reexamined. In Fig. 4, we present, with $f = 2, 4$ TeV, the parameter space of $(\mu, x_\lambda)$ where $2m_\eta < m_H$ (to the left-hand side of the contours). If $\mu$ is too large, $H \rightarrow \eta\eta$ decay is kinematically prohibited unless $x_\lambda$ is smaller than a certain value.

![Figure 4: The contours of $m_H = 2m_\eta$ in the parameter space $(\mu, x_\lambda)$ for $f = 2, 4$ TeV. To the left-hand (right-hand) side of the contour, $2m_\eta < (>) m_H$](image)

4. $H \rightarrow \eta\eta$ Decay and LEP implications

4.1 Branching ratios

In this model, major decay modes of the Higgs boson are SM-like ones with the partial
decay rates as
\[ \Gamma(H \to f \bar{f}) = \frac{N_C g^2 m_f^2}{32 \pi m_W^2} (1 - x_f)^{3/2} m_H, \quad \text{for } f = t, b, c, \tau, \quad (4.1) \]
\[ \Gamma(H \to W^+W^-) = \frac{g^2 m_W^2}{64 \pi m_W^2} \sqrt{1 - x_W} \left(1 - \frac{3}{4} x_W^2\right), \]
\[ \Gamma(H \to ZZ) = \frac{g^2 m_H^2}{128 \pi m_Z^2} \sqrt{1 - x_Z} \left(1 - \frac{3}{4} x_Z^2\right), \]
where \( x_i = 4m_i^2/m_Z^2, N_c = 3(1) \) for \( f \) being a quark (lepton). New decay channels are
\[ \Gamma(H \to \eta \eta) = \frac{\lambda^2 v^2}{8 \pi m_H^2} \sqrt{1 - x_\eta} = \frac{m_\eta^4}{8 \pi v^2 m_H} \sqrt{1 - x_\eta}, \quad (4.2) \]
\[ \Gamma(H \to Z \eta) = \frac{m_Z^3}{32 \pi f^2} \left( t_\eta - \frac{1}{t_\eta} \right) \chi^{3/2} \left(1, \frac{m_Z^2}{m_H^2}, \frac{m_\eta^2}{m_H^2}\right), \]
where \( \lambda(1, x, y) = (1 - x - y)^2 - 4xy \). The last decay mode was mentioned in Ref. \[17\], which could be dominant and phenomenologically quite interesting.

**Figure 5:** Contours of \( B(H \to \eta \eta) = 0.3, 0.5, 0.7 \) in the parameter space \((x_\lambda, \mu)\) for \( f = 2, 4 \) TeV.

Search strategy of the Higgs boson depends sensitively on its branching ratios (BR): In the SM, the major decay mode for \( m_H < 2m_W \) is into \( b \bar{b} \) while that for \( m_H \gtrsim 2m_W \) is into \( W^+W^- \). In this model, there are two new decay modes for the Higgs boson, \( H \to \eta \eta \) and \( H \to Z \eta \). In Fig. 3, we present the contours of \( B(H \to \eta \eta) = 0.3, 0.5, 0.7 \) in the parameter space \((x_\lambda, \mu)\) for \( f = 2, 4 \) TeV. Quite sizable portions of the parameter space can accommodate dominant decay of \( H \to \eta \eta \). For \( f = 2 \) TeV, \( B(H \to \eta \eta) > 0.5 \) requires \( x_{\lambda} \in [6, 14] \) and \( \mu \in [16, 30] \) GeV. A smaller \( \mu \) increases the 2-body phase-space factor since \( \mu \) is proportional to the produced \( \eta \) mass, while it reduces the \( H-\eta-\eta \) coupling. The optimal \( \mu \) for large \( B(H \to \eta \eta) \) is around 20 GeV. The size of parameter space for \( f = 4 \) TeV is relatively smaller with \( x_{\lambda} \in [5.6, 6.6] \) and \( \mu \in [10, 22] \) GeV. In this case, the optimal \( \mu \) is also around 20 GeV.

Figure 8 shows the same contours for \( B(H \to Z \eta) \), which depend quite sensitively on \( f \). For \( f = 2 \) TeV, sizable parameter space of \( x_{\lambda} \gtrsim 6 \) and \( \mu \lesssim 10 \) GeV can allow dominant
The decay of $H \to Z\eta$. When $f = 4$ TeV, only a small region around $x_\lambda \simeq 6$ and $\mu \lesssim 15$ GeV can accommodate dominant $H \to Z\eta$. This is mainly due to the $\eta$ mass. As can be seen in Fig. 3, $\eta$ for $f = 4$ TeV is relatively heavier than that for $f = 2$ TeV.

In order to see the $m_H$ dependence on each branching ratio, we present the branching ratios as a function of $m_H$ for $f = 2, 4$ TeV in Fig. 7. We fix $\mu = 20$ GeV for both $f = 2, 4$ TeV while vary $x_\lambda$ to generate various $m_H$. Different distribution of BRs for $f = 2$ TeV from that for $f = 4$ TeV is mainly due to the Higgs mass range. In the $f = 2$ TeV case, $Z\eta$ mode is solely dominant for $m_H$ from the $Z\eta$ threshold to $2m_W$. Even for $m_H > 2m_W$ $B(H \to Z\eta)$ is almost the same as $B(H \to W^+W^-)$. In the $f = 4$ TeV case, the $H \to \eta\eta$ is dominant for $140 \lesssim m_H \lesssim 160$ GeV, but the $H \to b\bar{b}$ becomes dominant if $m_H$ is below about 140 GeV. For $m_H$ above $WW$ threshold, $H \to WW$ is the leading decay mode, but not as dominant as in the SM because of the presence of the $Z\eta$ mode. The second important decay mode is into $Z\eta$, which is very different from a SM-like Higgs boson [17].

Brief comments on the decay of $\eta$ is in order here. If $m_\eta < 2m_W$, the decay pattern is very similar to that of the SM Higgs boson with the main decay mode into a SM fermion pair via the coupling $c(m_f/f)i\bar{f}\gamma_5f$, where $c \sim O(t_\beta)$ and $m_f$ is the mass of the fermion. Although this coupling is suppressed by $1/f$, the decay is still prompt in collider experiments for $f \sim O(\text{TeV})$. Therefore, the light $\eta$ boson mainly decays into a $b\bar{b}$ pair [17] if kinematically allowed. This characteristic feature of $\eta$ decay is useful to probe $\eta$ at high energy colliders.

4.2 LEP bound on $m_H$

Due to the presence of dominant decay of $H \to \eta\eta$, one may expect that the LEP bound on the Higgs mass can be loosened to some extent. The four LEP collaborations [2] searched for the Higgs boson via

$$e^+e^- \to ZH \to (l^+l^-, q\bar{q}, \nu\bar{\nu}) + b\bar{b}. \quad (4.3)$$

Here the main decay mode of the SM Higgs boson into $b\bar{b}$ dominates the width of the Higgs boson, with a branching fraction about 90% for most of the mass range and down to
Figure 7: Branching ratios of the Higgs boson in the simplest little Higgs model with the $\mu$ term as a function of $m_H$ for $f = 2$ TeV and $f = 4$ TeV. We fix $\mu = 20$ GeV but vary $x_\lambda$.

about 74% at $m_H = 115$ GeV. There is also a search using a minor mode of $H \rightarrow \tau^+\tau^-$. Nevertheless, the combined limit is almost the same as that using just the $b\bar{b}$ mode. The mass bound on the SM Higgs boson is 114.4 GeV [4]. For model-independent limits the LEP collaborations presented the upper bound on \([g_{ZZH}/g_{ZZH}^{SM}]^2 \times B(H \rightarrow b\bar{b})\) at the 95% C.L., as shown by the rugged curve in Fig. 8.

In the simplest little Higgs scenario with the $\mu$ term, one anticipates that the LEP bound on $m_H$ would be reduced, because of (i) sizable decay rate of $H \rightarrow \eta\eta$ such that $B(H \rightarrow b\bar{b})$ is substantially reduced as shown in Fig. 7, and (ii) the reduced coupling $g_{ZZH}$ in the simplest little Higgs model, especially when $t_\beta$ is large. In this model, the $g_{ZZH}$
deviates from the SM value by
\[ \frac{g_{ZZH}}{g_{ZZH}^{SM}} = \left[ 1 - \frac{v_0^2}{4f^2} \left( \frac{1}{t_s^2} - 1 + \frac{1}{t_s^2} + (1 - t_s^2)^2 \right) \right]. \tag{4.4} \]

In Fig. 8, we present the prediction of \( [g_{ZZH}/g_{ZZH}^{SM}]^2 \times B(H \to b\bar{b}) \) for \( f = 2, 3, 4 \) TeV, and compare to the 95% C.L. upper limit obtained by the LEP collaborations. We found the best value of \( \mu = 14 \) (15) GeV for \( f = 3 \) (4) TeV such that the prediction of \( [g_{ZZH}/g_{ZZH}^{SM}]^2 \times B(H \to b\bar{b}) \) for \( f = 2, 3, 4 \) TeV is the smallest. The \( f = 2 \) TeV case is safe because the minimum value of \( m_H \) predicted is already above 114 GeV. For \( f = 3, 4 \) TeV, however, the Higgs boson mass bound is restricted by the data as follows:

\[ m_H > 109 \text{ GeV} \quad \text{for} \quad f = 3 \text{ TeV}, \tag{4.5} \]
\[ m_H > 111 \text{ GeV} \quad \text{for} \quad f = 4 \text{ TeV}. \]

![Figure 8: Upper bound on \( [g_{ZZH}/g_{ZZH}^{SM}]^2 \times B(H \to b\bar{b}) \) established by the LEP collaborations, and the corresponding values in the simplest little Higgs model that we are considering.](image)

### 4.3 DELPHI limit on \( C_{Z(AA \to 4b)}^2 \)

The DELPHI collaboration [12] has searched for the process \( e^+e^- \to ZH \to Z(AA) \to Z + 4b \) for \( m_H > 2m_A \). Here \( A \) is a CP-odd scalar particle, for which \( \eta \) is a good candidate. The DELPHI collaboration parameterized the cross section by

\[ \sigma_{(AA)Z \to 4b+jets} = \sigma_{HZ}^{SM} \times B(Z \to \text{hadrons}) \times C_{Z(AA \to 4b)}^2, \tag{4.6} \]
where
\[
C_{Z(\AA \rightarrow 4b)}^2 = \left( \frac{g_{ZZH}}{g_{ZZH}^{SM}} \right)^2 \times B(H \rightarrow AA) \times B(A \rightarrow b\bar{b})^2. \tag{4.7}
\]

As no convincing evidence for a signal was found, the upper bound on \(C_{Z(\AA \rightarrow 4b)}^2\) was presented \[12\].

![Figure 9: Upper bound on \(C_{Z(\AA \rightarrow 4b)}^2\) by the DELPHI collaboration, and the values of \(C_{Z(\AA \rightarrow 4b)}^2\) in our model for \(f = 2, 3, 4\) TeV.](image)

We show the values of \(C_{Z(\AA \rightarrow 4b)}^2\) predicted in our model for \(f = 2, 3, 4\) TeV in Fig. 9. Here we fix \(\mu = 20, 14, 15\) GeV for \(f = 2, 3, 4\) TeV, respectively. We also show the upper bounds on \(C_{Z(\AA \rightarrow 4b)}^2\) for various combinations of \(m_H\) and \(m_A\) obtained by the DELPHI collaboration.\(^1\) For all three cases the \(C_{Z(\AA \rightarrow 4b)}^2\) values in this model are much smaller than the experimental upper bound. The DELPHI searches do not constrain the model at all. For \(f = 2\) TeV case, it is because the Higgs boson mass is already above the lower bound of 114.4 GeV. For \(f = 3, 4\) TeV, smaller \(m_H\) can evade the DELPHI search since \(g_{ZZH}\) decreases substantially for large \(t_\beta\) and \(H \rightarrow b\bar{b}\) is still dominant for \(m_H < 100\) GeV as discussed before. The kinks in the curves are due to the onset of the \(Z\eta\) mode when \(m_H > m_Z + m_\eta\).

5. Conclusions

Little Higgs models provide a very interesting perspective on answering the little hierarchy problem. As attributing the lightness of Higgs boson to its being a pseudo Nambu-

\(^1\)The mass ranges of the DELPHI data are \(12\) GeV < \(m_A\) < \(55\) GeV and \(2 m_A < m_H < 110\) GeV.
Goldstone boson, the collective symmetry breaking mechanism removes the quadratically divergent radiative-corrections to the Higgs mass at one-loop level. As a perfect type of “simple group” models, the SU(3) simplest little Higgs model has drawn a lot of interests due to its lowest fine-tuning associated to electroweak symmetry breaking 21. In the original framework, this simplest model cannot avoid the presence of massless pseudoscalar particle $\eta$. Cosmological lower bound on the axion mass requires to extend the model. One of the simplest choices is to add the so-called $\mu$ term in the scalar potential by hand. Then $\eta$ acquires a mass of order $\mu$, and the $H-\eta-\eta$ coupling is also generated of the order of $v\mu^2/f^2$. In order to accommodate the EWSB, this $\mu$ has a natural scale of a few ten GeVs, which leads to relatively light $\eta$. It is possible to allow a substantial branching ratio for the $H \to \eta \eta$ decay. In addition, the $H-Z-\eta$ coupling, which is present in the original model without the $\mu$ term, leads to $H \to Z \eta$ decay.

We found that the $H \to \eta \eta$ decay can be dominant for $m_H$ below the $WW$ threshold for $\mu \simeq 15-20$ GeV, while $H \to Z \eta$ dominant if $140$ GeV $\lesssim m_H \lesssim 2m_W$. For $m_H$ even above $2m_W$, the $H \to Z \eta$ decay can be as important as $H \to W^+W^-$. We have investigated the LEP bound on $[g_{ZZH}/g_{ZZH}]^2 B(H \to b\bar{b})$ in the search for the SM Higgs boson. In the $f = 2$ TeV case, the model restricts $m_H$ above the LEP bound. For the $f = 3$ ($4$) TeV cases, a lowering in the Higgs boson mass bound occurs: $m_H > 109$ ($111$) GeV, respectively. This is the main result of our work.

A few comments are in order here.

• This new and dominant decay channel can lead to important implications on the LEP search for the neutral Higgs boson. The DELPHI collaboration examined, in extended models, the process of $e^+e^- \to HQ \to (AA)Z \to (b\bar{b}b\bar{b})Z$, and presented the upper bound on $[g_{ZZH}/g_{ZZH}]^2 B(H \to \eta\eta)B(\eta \to b\bar{b})^2$. Our models with $f = 2,3,4$ TeV are not constrained by this bound.

• Further probes of the scenario are possible at LEP, at the Tevatron, and at the LHC. The LEP collaborations can investigate the scenario by searching for $e^+e^- \to ZH \to Z(\eta\eta) \to Z(4b,2b2\tau,4\tau)$, where $Z \to \ell^+\ell^-, \nu\bar{\nu}, q\bar{q}$. This mode may suffer from the fact that the coupling $g_{ZZH}$ is reduced relative to the SM one because of the little Higgs corrections. At the Tevatron, similar channels such as $p\bar{p} \to WH, ZH \to W/Z + (4b, 2b2\tau, 4\tau)$ can be searched for. At the LHC, the two-photon decay mode of the intermediate Higgs boson will suffer because of the dominance of $H \to \eta\eta$ mode in that mass range. Thus, the branching ratio into $\gamma\gamma$ reduces. On the other hand, $gg \to H \to \eta\eta \to 4b, 2b2\tau, 4\tau$ open, which may be interesting modes to search for the Higgs boson. However, a detailed study is needed to establish the feasibility.

• The $Z\eta$ decay mode of the Higgs boson is very unique in this simplest little Higgs model. In fact it dominates for $140$ GeV $< m_H < 2m_W$. Even for $2m_W < m_H$
the $Z\eta$ mode is as important as $WW$ mode. It is very different from a SM-like Higgs boson, which usually has the $ZZ$ mode in the second place. Since the $ZZ \rightarrow \ell^+\ell^-\ell^+\ell^-$ is the golden mode for Higgs discovery, the emergence of the $Z\eta$ mode will affect the Higgs detection significantly. Careful studies of $Z\eta$ mode is therefore important for Higgs searches.

- Another possibility to probe the $\eta$ is the direct production of the $\eta$ boson in $gg$ fusion or the associated production with a heavy quark pair. Although the production is suppressed by $1/f$ in the coupling of the $\eta$ to the SM fermion pair, this remains as an interesting possibility because the coupling to the heavy top quark is not suppressed.

We end here with an emphasis that $4b, 2b2\tau, 4\tau$ modes should be seriously searched for in the pursuit of the Higgs boson, which we have clearly demonstrated that it is possible in the simplest little Higgs models for $H \rightarrow \eta\eta$ and $H \rightarrow Z\eta$ to be dominant.

Acknowledgments

We thank the Physics division of the KIAS for hospitality during the initial stage of the work. K.C. also thanks K.S. Cheng and the Centre of Theoretical and Computational Physics at the University of Hong Kong for hospitality. And we would like to express our special gratitude to Alex G. Dias, for correcting our mistakes. We also appreciate the valuable comment from Juergen Reuter. The work of JS is supported by KRF under grant No. R04-2004-000-10164-0. The work of KC is supported by the National Science Council of Taiwan under grant no. 95-2112-M-007-001- and by the National Center for Theoretical Sciences.

References

[1] [ALEPH Collaboration], arXiv:hep-ex/0511027.
[2] R. Barate et al. [LEP Working Group for Higgs boson searches], Phys. Lett. B 565, 61 (2003).
[3] S. Schael et al. [ALEPH Collaboration], Eur. Phys. J. C 47, 547 (2006).
[4] P. Bechtle [LEP Collaboration], hep-ex/0602046.
[5] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Lett. B 513, 232 (2001); N. Arkani-Hamed, A. G. Cohen, E. Katz, A. E. Nelson, T. Gregoire and J. G. Wacker, JHEP 0208, 021 (2002); N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Lett. B 513, 232 (2001); M. Perelstein, arXiv:hep-ph/0512128.
[6] N. Arkani-Hamed, A. G. Cohen, E. Katz, A. E. Nelson, T. Gregoire and J. G. Wacker, JHEP 0208, 021 (2002); N. Arkani-Hamed, A. G. Cohen, E. Katz and A. E. Nelson, JHEP 0207, 034 (2002); T. Han, H. E. Logan, B. McElrath and L. T. Wang, Phys. Rev. D 67, 095004 (2003); I. Low, W. Skiba and D. Smith, Phys. Rev. D 66, 072001 (2002); S. Chang and J. G. Wacker, Phys. Rev. D 69, 035003 (2004); W. Skiba and J. Terning, Phys. Rev. D 68, 075001 (2003); S. Chang, JHEP 0312, 057 (2003); E. Katz, J. y. Lee, A. E. Nelson and D. G. E. Walker, JHEP 0510, 088 (2005); I. Low, JHEP 0410, 067 (2004); H. C. Cheng and I. Low, JHEP 0408, 061 (2004).
[7] C. Csaki, J. Hubisz, G. D. Kribs, P. Meade and J. Terning, Phys. Rev. D 67, 115002 (2003); J. L. Hewett, F. J. Petriello and T. G. Rizzo, JHEP 0310, 062 (2003); M. C. Chen and S. Dawson, Phys. Rev. D 70, 015003 (2004); Z. Han and W. Skiba, Phys. Rev. D 72, 035005 (2005), W. Kilian and J. Reuter, Phys. Rev. D 70, 015004 (2004).

[8] G. Burdman, M. Perelstein and A. Pierce, Phys. Rev. Lett. 90, 241802 (2003) [Erratum-ibid. 92, 049903 (2004)]; M. Perelstein, M. E. Peskin and A. Pierce, Phys. Rev. D 69, 075002 (2004); J. Hubisz and P. Meade, Phys. Rev. D 71, 035016 (2005); S. C. Park and J. Song, Phys. Rev. D 69, 115010 (2004).

[9] M. Schmaltz, JHEP 0408, 056 (2004).

[10] S. R. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).

[11] T. Han, H. E. Logan and L. T. Wang, JHEP 0601, 099 (2006).

[12] J. Abdallah et al. [DELPHI Collaboration], Eur. Phys. J. C 38, 1 (2004).

[13] O. C. W. Kong, arXiv:hep-ph/0307250; O. C. W. Kong, J. Korean Phys. Soc. 45, S404 (2004).

[14] G. Marandella, C. Schappacher and A. Strumia, Phys. Rev. D 72, 035014 (2005).

[15] Private communication from Alex Dias.

[16] D. E. Kaplan and M. Schmaltz, JHEP 0310, 039 (2003).

[17] W. Kilian, D. Rainwater and J. Reuter, Phys. Rev. D 71, 015008 (2005).

[18] K. Cheung, C. S. Kim, K. Y. Lee and J. Song, arXiv:hep-ph/0608259.

[19] G. Hiller, Phys. Rev. D 70, 034018 (2004) [arXiv:hep-ph/0404220].

[20] W. M. Yao et al. [Particle Data Group], J. Phys. G 33 (2006) 1.

[21] J. A. Casas, J. R. Espinosa and I. Hidalgo, JHEP 0503, 038 (2005).
Branching ratios

\[ m_H \text{ (GeV)} \]

- WW
- ZZ
- \( t \bar{t} \)
- \( \eta \eta \)
- b\overline{b}
- \( \tau \tau \)
- gg

\[ f = 2 \text{ TeV} \]
\[ \mu = 14 \text{ GeV} \]
Branching ratios

$\eta \eta$

$WW$

$ZZ$

$gg$

$\mu = 15$ GeV

$f = 4$ TeV

$m_H$ (GeV)