Characterizing Low-Quality-Factor Dissipative Superconducting Resonators

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Characterizing superconducting microwave resonators with highly dissipative elements is a technical challenge, but a requirement for implementing and understanding the operation of hybrid quantum devices involving dissipative elements, e.g. for thermal engineering and detection. We present experiments on λ/4 superconducting niobium coplanar waveguide (CPW) resonators, shunted at the antinode by a dissipative copper microstrip via aluminium leads, yielding a quality factor unresolvable from the typical microwave environment. By measuring the transmission both above and below this transition, we are able to isolate the resonance. We then experimentally verify this method with copper microstrips of increasing thicknesses, from 50 nm to 150 nm, and measure quality factors in the range of 10 ~ 67 in a consistent way.

Superconducting microwave resonators are the cornerstone of much current state of the art superconducting quantum technologies. Their intrinsic electromagnetic properties routinely enable high quality factors, typically over 10000, allowing for extremely sensitive measurements, for example, the multiplexed readout of the dispersive shift of a weakly coupled quantum bit, a fundamentally important tool for quantum information, or photon absorption of a microwave kinetic inductance detectors (MKIDs), a tool widely used in astronomy. On the other hand, dissipative elements added to superconducting microwave circuits are finding increasing applications in the fields of circuit quantum environment engineering, quantum information and circuit quantum thermodynamics.

In superconducting circuits, normal metal elements can be easily integrated into the existing chip architecture, and provide channels for intentional decoherence, such as for quantum bit initialization for precise temperature control (electronic heating and cooling), ultra sensitive calorimetry. Notably, these devices are in use as high-speed thermometers and microwave photon sources.

Experimentally characterizing such dissipative resonators is challenging by conventional single tone spectroscopy, as the dissipation results in a broad and shallow resonance, often unresolvable from the electronic noise of a cryogenic high electron mobility transistor (HEMT) amplifier, and the frequency dependent variations of transmission in the experimental setup. In this report, we present a novel method for this task utilizing an intermediate superconductor with a lower critical temperature, enabling us to isolate the resonance by performing characterization at differing bath temperatures, with measured quality factors as low as 10.

The method presented is demonstrated here using a structure consisting of two λ/4 niobium CPW resonators, both inductively coupled to a common CPW transmission line, shown in Fig. 1(a), used for multiplexed readout. On the left is a fully superconducting λ/4 CPW resonator to act as a reference. On the right resonator, the voltage node is shunted by an aluminium-copper-aluminium constriction, forming a Nb/Al/Cu/Nb heterostructure, with the copper acting as the dissipative normal metal shown in the micrograph in Fig. 1(b).

Samples are fabricated on a 330 μm thick c-plane sapphire substrate using a process described in Ref. 14. The sapphire surface is initially cleaned with an argon ion plasma milling before a 200 nm thick niobium film is deposited by DC magnetron sputtering. The coplanar waveguide resonator patterns are written by electron-beam lithography (EBL), and transferred to the niobium film using an SF6 + O2 reactive ion etching process. As the niobium layer is thick, subsequently comparatively thin evaporated aluminium and copper films are fragile and can become discontinuous at the intersection. Therefore, during the EBL exposure, the dose at the interface is incrementally changed, forming a ramp when etched to remove the discontinuity and increase the surface area. The Al/Cu/Al shunts are written by electron-beam lithography onto a bilayer resist and grown using double-angle deposition in an electron-beam evaporator, with galvanic contact between lithographic layers facilitated by an in-situ argon ion plasma milling process removing native oxides on the niobium. At the beginning of deposition, 10 nm of is evaporated to improve adhesion to the sapphire surface, followed by copper (of variable thickness: 50 nm, 100 nm, and 150 nm for different devices). Finally, two 110 nm thick contacts to the niobium CPW are deposited. After processing, the substrates are diced with a diamond-coated resin blade, wire-bonded to the sample stage assembly described in Ref. 21. Finally, the sample is loaded into a cryogen-free dilution refrigerator with a base temperature of 10 mK.

All spectroscopic measurements are performed using a vector network analyzer (VNA) at room temperature,
The signal leaving the sample is then passed through two circuit elements at different temperatures to the input of the dilution refrigerator, as shown in Fig. 1(c), is given near this resonance by $Z = -i\pi Z_\infty (f/f_r - f_r/f)/4$, where $Z_\infty = 50 \, \Omega$ is the characteristic impedance of an infinite transmission line. Power injected to the shunt at the half-power points $f = f_r \pm \Delta f/2$ can be written as $P \approx V^2/R[1 + (\pi Z_\infty/4R)^2(\Delta f/f_r)^2]] = V^2/2R$, where $\Delta f$ is the width of the resonance peak. By definition, the internal quality factor $Q$ of the resistively shunted resonator can be written as

$$Q = \frac{\pi Z_\infty}{4R}.$$  

To measure the low $Q$, the $Q_e$ is designed in the strongly coupled regime ($Q_e \ll Q$) in order to maximize the measurable signal. Due to the low overall quality factor, the depth of the notch in $S_{21}$ is of order 1 dB, which typically cannot be resolved within the background of the microwave setup. To isolate the resonator, the method presented here is based on measuring the spectrum above and below the critical temperature of aluminium $T_{c,Al}$. The lower quality factor $Q$ of the resonator above $T_{c,Al}$ suppresses the resonance totally for all practical purposes. Under these high temperature conditions, the transmittance can be regarded as a reference measurement for the background. The transmission through the CPW line is shown in Fig. 2(b), with the red and blue traces measured at different temperatures $T_H$ and $T_L$, above and below the critical temperature of the aluminium $T_{c,Al}$, respectively, yet staying sufficiently below the critical temperature $T_{c,Nb}$ of the niobium transmission line. Here, $T_{c,Al}$ and $T_{c,Nb}$ can be determined by the resistance-temperature characteristic of a co-process sample shown in the inset of Fig. 2.
FIG. 2. (a) Transmittance spectra of the device with 100 nm thick copper shunt. The red and blue curves are measured at $T_H = 2$ K and $T_L = 10$ mK respectively, as indicated with arrows in the inset. Here, the $T_L$ and $T_H$ satisfy $T_k < T_{c,Al} < T_H < T_{c,Nb}$. The inset shows resistance-temperature characteristics of a nominally identical co-process dissipative element, demonstrating three plateaus corresponding to the superconducting transitions of aluminium, and niobium, respectively. The yellow region demonstrates the frequency range of the resistively shunted superconducting resonator. We present in the right inset of (b) the quality factor of $S_{21}(T_H)/S_{21}(T_L)$ and the phase in the left inset. Within the yellow highlighted frequency region, we see the resonance of the dissipative resonator. Here, we extracted $f_r$ and $Q$ as 6.71 GHz and 45 corresponding to the green dashed fitting line.

(a). A narrow band of $S_{21}(T_L)$ shows in the right inset of Fig. 2(b) emphasising the narrow bandwidth of the standard reference resonator. Here, $f_0$ is extracted as 7.246 GHz with a corresponding 5 MHz shift with temperature, as the kinetic inductance increases with decreasing temperature. The dissipative resonator is not clearly visible in either trace. We observe, however, that as expected, due to lower operating temperature and thus more ideal superconducting characteristics of the CPW transmission line, the blue trace is consistently higher than the red trace, except in the region highlighted in yellow, centered around the design frequency of the dissipative resonator. By taking the ratio of the two $S_{21}$ measured at $T_L$ and $T_H$, one can isolate the resonance of the dissipative resonator. This ratio is shown in Fig. 2(b). Fitting a standard notched resonator model to the trace, the parameters $f_r$, $Q$, and $Q$ can be extracted, with data for different samples (with variable thickness of copper) presented in Table I.

In order to investigate how the $Q$ of the resistively shunted superconducting resonator depends on its resistance $R$, the co-process samples with identical shunt element (see the red dashed frame in Fig. 1(a)) are measured in current biased four probe configuration at 50 mK bath temperature. A measured IV curve is shown for the 150 nm thick copper shunt in Fig. 3(a). By ramping the bias current up through the shunt, we observe first the superconducting energy gap and then two resistive branches at higher bias. The superconducting state exists by virtue of the proximity effect in the SNS Josephson junction induced by the Al/Cu/Al structure, and the shunt switches to the first dissipative branch at current $I_{sw1} = 5.6$ nA with normal state resistance $R = 0.87 \Omega$. The normal resistance $R$ due to the copper wire and the imperfect contact between the aluminium and the copper layers determines the quality factor of the resonator in the microwave measurement at $T_L$. Continually increasing the current, the resistance increases at $I_{sw2} = 43.3$ nA with resistance $R_\approx = 4.83 \Omega$ when the current exceeds the critical current of aluminium. $R_\approx$ consists of $R$, the normal state resistance of aluminium wire and the Al/Nb contact. All the measured parameters $I_{sw1}$, $I_{sw2}$, $R$, and $Q$ extracted from IV measurement are given in Table II.

The presence of supercurrent in SNS junctions is due to the well-known proximity effect. Based on the ratio of the Thouless energy $\epsilon_c = \hbar D/L^2$ to the superconducting gap of aluminium $\Delta$, the junction is in the long junction regime, when $\epsilon_c/\Delta \ll 1$ and the zero temperature $e R I_c$ is found to be proportional to $\epsilon_c$ in this limit. Here $L$ is the length of the junction, $D = v_F \ell_c/3$ is the diffusion constant of the N metal, $v_F$ is the Fermi velocity, $I_c$ is the critical current of the junction, and $\ell_c$ is the elastic mean free path of electrons. In Table II $e R I_{sw1}$ is smaller than 5 $\mu eV$ indicating long junction limit.

The N lead between S superconductors of the resonator is an ideal element to localise heat. The small volume of normal metal enhances the temperature of the element at

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**TABLE I. Parameters of resistively shunted superconducting resonators extracted based on the measured $S_{21}$.**

| Thickness | 50 nm | 100 nm | 150 nm | 100 nm |
|-----------|-------|--------|--------|--------|
| sample    | A-1   | B-1    | B-2    | B-3    | C-1    | D-1    | D-2    |
| $f_r$ (GHz) | 6.54  | 6.74   | 6.69   | 6.70   | 6.53   | 6.67   | 6.67   |
| $Q_c$     | 333   | 339    | 200    | 253    | 323    | 151    | 455    |
| $Q$       | 10.3  | 45.3   | 53.2   | 44.7   | 66.3   | 20.4   | 26.5   |

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**TABLE II. DC properties of Nb/Al/Cu/Al/Nb junctions.**

| Thickness | 50 nm | 100 nm | 150 nm | 100 nm |
|-----------|-------|--------|--------|--------|
| sample    | A-2   | A-3    | B-4    | B-5    | C-2    | C-3    | D-3    | D-4    |
| $I_{sw1}$ ($\mu A$) | 0.24  | 0.30   | 2.05   | 1.83   | 5.60   | 2.59   | 0.74   | 0.93   |
| $I_{sw2}$ ($\mu A$) | 19.60 | 22.80  | 39.00  | 40.00  | 43.30  | 42.00  | 21.00  | 21.05  |
| $R$ ($\Omega$)      | 5.30  | 4.30   | 1.70   | 1.56   | 0.87   | 1.28   | 3.25   | 2.84   |
| $R_\approx$ ($\Omega$) | 21.49 | 16.03  | 8.16   | 7.94   | 4.83   | 5.19   | 19.38  | 19.52  |
fixed transferred power, whereas the S forms a Cooper-pair while injecting an electron from N and reflects a hole known as Andreev reflection that drops the temperature at the interface and localises the heat efficiently in the N element as the coupling between the copper heat bath and the resonator.

In Fig. 3 (c), we plot the measured $Q$ from Table I versus the reciprocal $R$ from Table I and the expected $Q$, as blue dots, and orange solid line, respectively. The expected slope by Eq. 2 is shown with $Z_{\text{in}}$ set to 50 $\Omega$. For a more general analysis of the internal quality factor, a model that takes into account non-vanishing quality factor above the aluminium superconducting transition temperature can be used to simulate the measurement technique by inserting $R_s$ into Eq. 2 to obtain $Q(T_H)$. $Q(T_L)$ and $Q(T_H)$ are independent dissipation channels with $1/Q_1 = 1/Q + 1/Q_c$, which are then inserted into Eq. 1 to obtain the general two-temperature model, $S_21(T_L)/S_21(T_H)$. Here $Q(T_L)$ is extracted from the model and plotted as green hollow circles. The slopes of extracted $Q$ versus $1/R$ from the two models differ by less than 12%, which places the values within the uncertainty of the fitting algorithm. This validates the vanishing quality factor assumption, particularly in the low $Q$ limit. This, however, is only approximately true since the normal aluminium sections have finite resistance. According to Table II, $R_s$ decreases while the thickness of the copper increases. The lower $R_s$ in the thicker samples yields a broader resonance when aluminium undergoes the transition to normal state, which explains the difference of extracted $Q$ for the thicker Cu films. However, the measured $Q$ is approximately double the expected value. This can be explained by the following: as shown in Table II the supercurrent $I_{sw1}$ increases with the thickness of the copper, and hence the dissipation is suppressed because the current passes through the lossless channel in the low-frequency regime ($\hbar \omega \ll \epsilon_c^{(2)}$). Additionally, a geometric inductance from the narrow, impedance mismatched geometry of the shunt element can be explicitly observed from the reduced resonance frequency of around 6.6 GHz in comparison to the designed frequency of 7.2 GHz, which also contributes to a parallel lossless channel.

In conclusion, we have presented a novel technique for isolating the resonance of low-quality dissipative superconducting resonators based on the transition temperature of an intermediate superconductor with a lower energy gap as compared to that of the superconducting resonator. We have verified this method by characterizing a series of resistively shunted $\lambda/4$ superconducting CPW resonators. From microwave measurements, we have extracted extremely low-quality factors of 10–67 for resonators with different resistances of copper shunt element. The technique here can be directly applied in understanding and characterizing superconducting devices, such as those being used in circuit quantum electrodynamics and thermodynamics experiments.

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