LOW-ENERGY PHOTON PRODUCTION
IN NEUTRINO NEUTRAL-CURRENT INTERACTIONS

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ABSTRACT

The search for $\nu_\mu \to \nu_e$ oscillations by the MiniBooNE Collaboration at Fermilab has revealed a low-energy signal which could be due either to electrons produced by $\nu_e$ or photons produced by the interaction of the weak neutral current on the target nucleus. One contribution to the latter is a Wess-Zumino-Witten anomaly leading to a term in the Lagrangian proportional to $\epsilon^{\mu\nu\kappa\lambda}Z_\mu\omega_\nu F_{\kappa\lambda}$. This term is normalized with the help of the known rates for the processes $f_1 \to \rho\gamma$ and $\tau \to \nu_\tau a_1$. A rate of about 1/4 of that employed in several previous estimates is obtained. As the anomaly term had already been found to play a subdominant role in photon production (e.g., in comparison with $\Delta$ excitation and decay), the present estimate reduces its strength even further.

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I INTRODUCTION

The search for $\nu_\mu \to \nu_e$ oscillations by the MiniBooNE Collaboration at Fermilab has revealed a low-energy signal which could be due either to electrons produced by $\nu_e$ or photons produced by the coherent interaction of the weak neutral current on the target nucleus. One proposed source of the latter is an interaction between the $Z$, $\omega$ meson, and photon due to a Wess-Zumino-Witten anomaly, whose strength has been calculated in Refs. [9,11]. It was concluded in Ref. [12,13] that the anomaly contribution was not enough to account for a photon signal. Although neutral-current nucleon excitation followed by photon emission was originally suggested as a source of the signal, it was found insufficient as well in Refs. [14,19]. A comprehensive review of neutrino-induced quasi-elastic scattering and single photon production is given in a workshop summary [20].

The imminent operation of the MicroBooNE Experiment at Fermilab [21] will be able to distinguish final-state photons from electrons. Hence it is timely to present an independent estimate of the strength of the anomaly-mediated interaction. In this paper we perform such an estimate based on dominance of the $Z^-\omega-\gamma$ interaction by the $a_1$ pole in the neutral current. The $a_1$ decay constant is obtained from the observed rate for $\tau \to a_1\nu_\tau$, while the

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Figure 1: Processes sensitive to WZW anomalies: (a) Coherent reaction $\nu A \rightarrow \nu \gamma A$ on a nucleus of atomic number $A$; (b) Induced $f_1 \rightarrow \gamma \rho$ decay; (3) Coherent $K_L \rightarrow K_S$ regeneration on a nucleus of atomic number $A$.

$a_1$–$\omega$–$\gamma$ coupling is obtained from the observed decay rate for $f_1 \rightarrow \gamma \rho$, which involves a coupling constant identical to $g_{a_1\omega\gamma}$ if $f_1$ contains only nonstrange quarks. This interaction was overlooked in an otherwise successful description of light meson radiative decays based on the quark model [22, 23].

In Sec. II we review the consequence for the MiniBooNE experiment of the assumed $Z$–$\omega$–$\gamma$ interaction. We then (Sec. III) derive the consequence of assuming $a_1$ dominance of the weak neutral current. The $a_1$ decay constant which arises in this derivation is evaluated with the help of the rate for $\tau \rightarrow a_1 \nu_\tau$ in Sec. IV, while the decay rate for $f_1 \rightarrow \gamma \rho$ is employed to evaluate the $a_1$–$\omega$–$\gamma$ coupling in Sec. V. Section VI contains predictions for the rates for $a_1^0 \rightarrow \omega \gamma$ and $a_1 \rightarrow \rho \gamma$ (all charge states). We sum up in Sec. VII.

II NEW INTERACTION AND MINIBOONE

The MiniBooNE experiment [1] at Fermilab was conceived to check a signal for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation observed at the LSND detector [24] operating at Los Alamos National Laboratory. The oscillation signature would be the appearance of electrons. Initially signals were restricted to an electromagnetic energy deposit greater than 475 MeV. An excess of events below this cutoff was observed, attributable either to electrons or to photons.

A possible source of photons in this experiment was identified by J. A. Harvey, C. T. Hill, and R. J. Hill [4]. A Wess-Zumino-Witten anomaly [6,7] gives rise to a term

$$\delta L = \frac{N_c e g_\omega g_2}{48\pi^2 \cos \theta_W} \epsilon_{\mu\nu\rho\sigma} \omega^\mu Z^\nu F^{\rho\sigma}.$$  

(1)

Here $N_c = 3$ is the number of quark colors, $e = \sqrt{4\pi\alpha} = 0.3028$ is the proton charge, $g_\omega$ is a coupling constant of the $\omega$ meson to baryon number whose value needs to be specified, $g_2$ is the electroweak SU(2) coupling constant, $\theta_W$ is the electroweak mixing angle, and $F^{\rho\sigma}$ is the photon field-strength tensor. The contribution of this term to the coherent process $\nu A \rightarrow \nu \gamma A$, where $A$ denotes a nucleus of atomic number $A$, is illustrated in Fig. 1(a). The induced decay $f_1 \rightarrow \gamma \rho$ is shown in Fig. 1(b), while a related WZW anomaly [6,7] leads to a $K\bar{K}\omega$ coupling responsible for $K_L \rightarrow K_S$ coherent regeneration [Fig. 1(c)] [25,26].
Table I: Coupling constants of $f\bar{f}$ to the $Z$. Here $x_W \equiv \sin^2 \theta_W$.

| $f\bar{f}$ pair | $a_L$ | $a_R$ |
|-----------------|-------|-------|
| $\nu\bar{\nu}$ | $\frac{1}{4}$ | 0 |
| $\ell\bar{\ell}$ | $\frac{1}{2}\left(-\frac{1}{2} + x_W\right)$ | $\frac{1}{2}x_W$ |
| $u\bar{u}$ | $\frac{1}{2}\left(\frac{1}{2} - \frac{2}{3}x_W\right)$ | $\frac{1}{2}\left(-\frac{2}{3}x_W\right)$ |
| $d\bar{d}$ | $\frac{1}{2}\left(-\frac{1}{2} + \frac{1}{3}x_W\right)$ | $\frac{1}{2}\left(\frac{1}{3}x_W\right)$ |

The term (1) leads to a cross section per nucleon $N$ in the zero-recoil limit [2, 4]:

$$\sigma(\nu N \rightarrow \nu\gamma N) = \frac{\alpha g_\omega^4 G_F^2}{48\pi^6 m_\omega^4} E_\nu^6 = 2.6 \times 10^{-41} E_\nu^6 (g_\omega/10.0)^4 \text{cm}^2 .$$

(2)

in the limit of $E_\nu$ below a few hundred MeV. The $E_\nu^6$ behavior can only be valid below threshold for nucleon excitation; contributions of Compton-like scattering and $\Delta$ excitation are more important in this regime [12, 13]. (For $E_\nu \approx 20$ GeV, upper limits on neutral-current photon production contradict (2) [27].) For production on a nucleus of atomic number $A$, the right-hand side of Eq. (2) is to be multiplied by a factor less than $A$ [12].

Ref. [4] quoted considerable uncertainty in the value of $g_\omega$ but noted that the nominal value $g_\omega = 10$ agreed roughly with the MiniBooNE signal interpreted as photons. This was found [12, 13] to be an overestimate, as a result of form factor and recoil effects, with $\Delta$ excitation and decay providing a dominant photon source, but the nominal value $g_\omega = 10$ was retained. Corrections for efficiency [17, 19] further reduced the expected signal. We shall use $a_1$ dominance of the weak neutral current to find an independent estimate of $g_\omega$, finding a value close to 7. This strengthens the conclusions of Refs. [9–15, 17–19] that if the MiniBooNE signal is indeed found to be photons, the WZW anomaly is unlikely to be their dominant source.

### III $a_1$ DOMINANCE OF NEUTRAL CURRENT

In the contribution (1) to the Lagrangian, the weak neutral current carried by the $Z$ may be viewed as dominated by the $a_1(1260)$ meson. The matrix element between $Z$ and $a_1$ may be written as $m_{a_1}f_{a_1}$, where [28] $m_{a_1} = 1230 \pm 40$ MeV and the neutral $a_1$ decay constant is to be determined. The interaction between a $Z$ and a fermion-antifermion pair $f\bar{f}$ is

$$\mathcal{L}_{ZF} = \frac{g_2}{\cos \theta_W} f \bar{\gamma}_\mu Z^\mu [(1 - \gamma_5) a_L + (1 + \gamma_5) a_R] f ,$$

(3)

where the coupling constants $a_L$ and $a_R$ are listed in Table I.

The axial vector coupling is then

$$\mathcal{L}_{Zf\bar{f}}^{\text{axial}} = \frac{g_2}{\cos \theta_W} f \bar{\gamma}_m u \gamma_5 Z^\mu (a_R - a_L) f ,$$

(4)

where

$$(a_R - a_L)(u\bar{u}) = -\frac{1}{4} , \quad (a_R - a_L)(d\bar{d}) = \frac{1}{4} .$$

(5)
The quark content of $a_1$ is $(d\bar{d} - u\bar{u})/\sqrt{2}$, so the $Z-a_1^0$ coupling may be written as

$$g_{Za_1^0} = \frac{g_2 f_a m_a}{2\sqrt{2} \cos \theta_W}.$$  \hspace{1cm} (6)

Assuming $a_1$ dominance of the weak neutral current, Eq. (1) then may be written as

$$\delta \mathcal{L} = \frac{f_a}{2\sqrt{2} m_a} \epsilon_{\mu\nu\rho\sigma} \omega^\mu Z^\nu F^\rho\sigma g_{a\omega\gamma}.$$  \hspace{1cm} (7)

Equating coefficients of equal terms in Eqs. (1) and (7) and taking $N_c = 3$, we find

$$g_{a\omega\gamma} = \frac{e g_\omega}{4\sqrt{2} \pi^2} \frac{m_a}{f_a}.$$  \hspace{1cm} (8)

We shall evaluate $f_a$ in the next Section.

IV EVALUATION OF $f_{a_1}$

The decays $\tau^- \rightarrow \pi^- \nu_\tau$ and $\tau^- \rightarrow \rho^- \nu_\tau$ are described by simple expressions involving the pion and rho decay constants, respectively (see, e.g., Ref. [29]). The corresponding expression for the decay $\tau^- \rightarrow a_1^- \nu_\tau$, in terms of the decay constant $f_a$ linking the $Z$ and the neutral $a_1$, is

$$\Gamma(\tau^- \rightarrow a_1^- \nu_\tau) = \frac{G_F^2 m_\tau f_a^2}{8\pi} |V_{ud}|^2 \left( 1 + \frac{2m_a^2}{m_\tau^2} \right) \left( 1 - \frac{m_a^2}{m_\tau^2} \right)^2.$$  \hspace{1cm} (9)

The Particle data Group does not give a branching fraction for this decay. However, assuming that the quoted branching fractions [28]

$$B(\tau^- \rightarrow \pi^- \pi^0 \pi^0 \nu_\tau) = (9.30 \pm 0.11)\%,$$  
$$B(\tau^- \rightarrow \pi^- \pi^+ \pi^- \nu_\tau) = (8.99 \pm 0.06)\%$$  \hspace{1cm} (10)

are dominated by the $a_1$, one obtains $B(\tau^- \rightarrow a_1^- \nu_\tau) = (18.29 \pm 0.13)\%$. Using the values $G_F = 1.16638 \times 10^{-5}$ GeV$^{-2}$, $m_\tau = 1776.82 \pm 0.16$ MeV, $|V_{ud}| = 0.97425 \pm 0.00022$, $m_a = 1230 \pm 40$ MeV, and $\tau_\tau = 290.3 \pm 0.5$ fs quoted in [28], we find a decay constant $f_a = (164.6 \pm 7.3)$ MeV, where the error is dominated by uncertainty in $m_a$. In our convention $f_a$ refers to the neutral current, whereas most authors quote a value which in our notation would be $\sqrt{2} f_a$. Our value is consistent with several others obtained theoretically or extracted from data [30][31]. The resulting decay constant may now be used in Eq. (8) to obtain the result $g_{a\omega\gamma} = (0.0405 \pm 0.0005)g_\omega$.

V EVALUATION OF $g_{a\omega\gamma}$ USING $f_1 \rightarrow \gamma\rho$ RATE

The coupling constants $g_{a\omega\gamma}$ and $g_{f\rho\gamma}$ both involve the isovector photon coupling to an isovector and isosinglet, and are equal by U(3) symmetry as long as $f_1$ contains no strange quarks: $g_{f\rho\gamma} = g_{a\omega\gamma}$. The rate for $f_1 \rightarrow \gamma\rho$ is given (see also Appendix C of Ref. [11]) by

$$\Gamma(f_1 \rightarrow \gamma\rho) = \frac{g_{f\rho\gamma}^2 E_\gamma^3}{3\pi m_\rho^2} \left( 1 + \frac{m_\rho^2}{m_f^2} \right) = (26.6 \pm 0.7) g_\omega^2 \text{ keV},$$  \hspace{1cm} (11)
where we have used \( m_\rho = (775.25 \pm 0.25) \text{ MeV} \), \( m_f = (1281.9 \pm 0.5) \text{ MeV} \), and \( E_\gamma = (m_f^2 - m_\rho^2)/(2m_f) = (406.5 \pm 0.4) \text{ MeV} \). The first and second terms in large parentheses correspond to longitudinal and transverse \( \rho \) polarizations, respectively, so longitudinal \( \rho \) polarization is dominant [22,33], in contradiction to the result found in Ref. [32].

The experimental partial width for \( f_1 \to \gamma \rho \) is the product of the total \( f_1 \) width and the corresponding branching fraction [28]:

\[
\Gamma(f_1 \to \gamma \rho) = \Gamma(f_1)B(f_1 \to \rho \gamma) = (24.2 \pm 1.1) \text{ MeV}(0.055 \pm 0.013) = (1.33 \pm 0.32) \text{ MeV}.
\]

(12)

When combined with Eq. (11) this yields \( g_\omega = 7.07 \pm 0.86 \), where the error is dominated by the experimental error in Eq. (12). This value is lower than the nominal one of 10 taken in [4,12], and leads to an anomaly contribution only about 1/4 of that previously estimated, thanks to the quartic power of \( g_\omega \) in Eq. (2).

The systematic errors that we are able to identify tend to decrease \( g_\omega \) by a modest amount. We have taken \( B(\tau \to a_1 \nu_\tau) \) to be as large as possible when ascribing all the \( 3\pi \nu_\tau \) decays to \( a_1 \). If \( B(\tau \to a_1^{-} \nu_\tau) \) is smaller, \( f_a \) is smaller, the coefficient of \( g_\omega \) is larger in Eq. (8), so \( g_\omega \) is smaller. We have also assumed the anomaly to fully account for the \( f_1 \to \rho \gamma \) decay rate, whereas a small quark model contribution of 150 keV was predicted in Ref. [22]. It is not clear whether the decay amplitude for longitudinal \( \rho \) production predicted in Ref. [22] should be added coherently to that predicted here.

VI RATES FOR \( a_1^0 \to \gamma \omega \) AND \( a_1 \to \gamma \rho \)

The decay \( f_1 \to \gamma \rho \) is related by \( U(3) \) symmetry to the decays \( a_1 \to \gamma \omega \) and \( a_1 \to \gamma \rho \):

\[
g_{\omega \gamma}^2 = 9g_{\rho \gamma}^2 = 0.082 \pm 0.020, \tag{13}
\]

where we have used the experimental value [12] in the expression (11) for the \( f_1 \to \gamma \rho \) rate. The corresponding formulae for the \( a_1 \) radiative decay widths are

\[
\Gamma(a_1^0 \to \gamma \omega) = \frac{g_{\omega \gamma}^2 E_\gamma^3}{3\pi m_\omega^2} \left( 1 + \frac{m_\omega^2}{m_\rho^2} \right), \tag{14}
\]

\[
\Gamma(a_1 \to \gamma \rho) = \frac{g_{\rho \gamma}^2 E_\gamma^3}{3\pi m_\rho^2} \left( 1 + \frac{m_\rho^2}{m_\omega^2} \right), \tag{15}
\]

where the photon energies, predicted rates, and range of predicted branching fractions (using \( \Gamma_{\text{tot}}(a_1) = 250 \) to 600 MeV [28]) are shown in Table II. We have used \( m_\omega = (782.65 \pm 0.12) \text{ MeV} \). The expression (15) holds for all \( a_1 \) charge states.

The branching fractions in Table II are quite small because of the large \( a_1 \) total width. Nevertheless, it may be possible to see the decay \( a_1^- \to \gamma \rho^- \) in the final state of \( \tau^\to a_1^- \nu_\tau \). The subprocess \( a_1^0 \to \gamma \omega \) may be observable through coherent photoproduction of \( a_1^0 \) on a heavy nucleus \( a = A: \gamma A \to a_1^0 A \to \pi^+\pi^-\pi^0 A \), proceeding via \( \omega \) exchange.

VII SUMMARY

A neutral-current interaction based on the \( Z^-\omega^-\gamma \) interaction depicted in Fig. 1(a) is predicted [4] on the basis of a Wess-Zumino-Witten anomaly [6,7]. This interaction is calibrated with the help of the decay \( \tau \to a_1 \nu \). It leads to a prediction for the low-energy signal.
Table II: Photon energies, predicted decay rates, and range of predicted branching fractions for the decays $a_1 \rightarrow \gamma\omega$ and $a_1 \rightarrow \gamma\rho$.

| Final state $f$ | $E_\gamma$ (MeV) | $\Gamma_f$ (MeV) | $B_f$ ($\times 10^{-3}$) |
|-----------------|-----------------|-----------------|------------------------|
| $\gamma\omega$  | 366             | $0.98 \pm 0.24$ | (1.2–4.9) $\times 10^{-3}$ |
| $\gamma\rho$    | 371             | $115 \pm 28$   | (1.4–5.7) $\times 10^{-4}$ |

in the MiniBooNE experiment [1], if interpreted in terms of photons rather than electrons or positrons, which is about a fourth of that previously estimated, which already was below the needed magnitude. (One proposed source of photons is the decay of a quasi-sterile neutrino with mass between 40 and 80 MeV [34, 35].)

For the future one looks forward to tests of the predicted rates for $a_1 \rightarrow \gamma\omega$ and $a_1 \rightarrow \gamma\rho$, to a more precise estimate of $f_{\alpha}$, and to an experimental distinction between electrons or positrons and photons in the final state studied by MiniBooNE. The MicroBooNE experiment [21], soon to begin operation at Fermilab, should resolve the question.

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