An Approach to Energy Efficiency in a Multi-Hop Network Control System through a Trade-Off between $\mathcal{H}_\infty$ Norm and Global Number of Transmissions

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Abstract – The present work proposes a new approach to energy efficiency for filtering of a system interconnected by a Multi-Hop network. The minimization of the energy cost per time unit is obtained through limiting the number of packets retransmission in the Hop-by-Hop mechanism. We explore a trade-off between system performance, measured by estimation error $\mathcal{H}_\infty$ norm and energy consumption. The proposal is validated using a theoretical model for energy consumption of MICA2 transceivers.

keywords: MJLS, Hop-by-Hop, Energy-Efficiency, NCS

1 Introduction

The classical design of a control application immersed in a digital network requires high service quality, ensuring a full-reliable communication and null or low delay. The information transported through a network has an operational cost and a probability of failure, we usually minimize the probability of failure by increasing the number of transmitted packets. This approach increases power consumption and delay per package. Using semi-reliable communication in control system implies in the performance degradation which sometimes can lead to instability.

Another application that requires a full-reliable communication is the image transmission, where the packet loss can cause useful information loss in an image [1]. The image transmission in particular wireless technologies as Low Rate Wireless Personal Area Network (LR-WPANs) can be complex where image reconstruction does not admit information loss, causing high energy consumption for the network. Wireless sensor network (WSN), a type of LP-WPAN, implemented with vision modules is useful for a wide range of applications. A new field of study is developed to find ways to minimize the implementation cost in image transmission by using semi-reliable communication reconstructing the lost packets using techniques like data selectivity, packet correlation, and others [2, 3]. The use of a semi reliable network instead of a full reliable one has a direct impact in the energy cost due to the decrease in the expected number of transmissions and receptions per packet.

The reference [4] proposes an approach similar to the problem with image transmission in LR-WPAN. This approach allows us to use a Multi-Hop network in filtering applications by decreasing the dynamic system performance in order to minimize the expected number of global transmissions. This techniques propose a trade off between the average number of transmissions and the estimation error $\mathcal{H}_\infty$ norm. In the present work we propose a new approach for energy efficiency of a Network Control System (NCS). We use the theoretical energy consumption model for the MICA2 transceivers [5] and apply it in the network using a Hop by Hop transport scheme limiting the maximum number of retransmissions per link. Figure 1 represents a control scheme in a Multi-Hop network as described by [6].

2 Preliminary

2.1 Markovian Jump Linear System (MJLS)

Consider the following discrete-time system,

$$
\begin{align*}
G: \quad & x(k+1) = A(\theta_k)x(k) + J(\theta_k)w(k), \\
& y(k) = C_y(\theta_k)x(k) + E_y(\theta_k)w(k), \\
& z(k) = C_z(\theta_k)x(k) + E_z(\theta_k)w(k),
\end{align*}
$$

(1)

where $x(k) \in \mathbb{R}^n$ is the state vector and $w(k) \in \mathbb{R}^m$ is the external disturbance. The vector $z(k) \in \mathbb{R}^r$ contains the signals to be estimated by the filter through the measured outputs $y(k) \in \mathbb{R}^q$. The random variable
θ_k ∈ K = \{1, 2, \ldots, N_n\} assumes its values according to a Markov Chain with probability matrix P = [p_{ij}], where p_{ij} = P(θ_{k+1} = j|θ_k = i). Clearly, p_{ij} are always positive and the sum of the elements in a row of P is equal to one. To simplify the notation, whenever θ_k = i, we write A(θ_k) = A_i, J(θ_k) = J_i and so forth.

An important assumption for filter and control design for MJLS is the availability of the Markov mode θ_k = i ∈ K to the controller or filter. If the mode is available we say the design is mode-dependent, or mode-independent otherwise. An intermediate approach considers partial availability, that is the mode itself is not available but only the cluster U_ℓ it belongs to. For more details refer to [7]. A particular case of the Markov chain is the generalized Bernoulli jump chain where p_{ij} = p_j for all i, j ∈ K, that is the probability matrix has identical rows. In the following sections we will consider such assumption.

**Optimum H_∞ filter design for a MJLS**

Among the markovian filters described in [8], we use a particular filter, the mode-independent one, to minimize estimation error H_∞ norm. A Luenberger filter that depends on the cluster availability of the Markov mode θ_k ∈ U_ℓ is given by

\[
O: \begin{align*}
x_f(k + 1) &= A_ℓx_f(k) + B_ℓ(y(k) - C_ℓy(k)), \\
z_f(k) &= C_ℓx_f(k) + D_ℓ(y(k) - C_ℓy(k)).
\end{align*}
\]  

(2)

Note that, we are assuming that A_i = A_ℓ, C_{yi} = C_ℓy and C_{zi} = C_ℓz for all i ∈ U_ℓ, that is, the plant matrices do not vary within a given cluster. Notice that mode-independent design is clearly a particular case for cluster design with only one cluster containing all Markov chain modes.

**Theorem 1** There exists a filter of the form (2) such that \|G_o\|_∞ < γ if and only if there exist symmetric matrices H_i, X, and the matrices F_ℓ, K_ℓ with compatible dimensions that satisfy the LMIs

\[
\begin{bmatrix}
H_i & \cdot & \cdot & \cdot \\
0 & \gamma I & \cdot & \cdot \\
X A_ℓ + F_ℓC_ℓ & X J_ℓ + F_ℓE_ℓ & X & \cdot \\
C_ℓz - K_ℓC_ℓ & E_ℓz - K_ℓE_ℓ & 0 & 1
\end{bmatrix} > 0,
\]

(3)

\[
\sum_{j=1}^{N_n} p_j H_j - X < 0,
\]

(4)

for all i ∈ U_ℓ ∩ K and ℓ ∈ L. The filter gains are given by, B_ℓ = -X^{-1}F_ℓ, D_ℓ = K_ℓ. The proof of this theorem can be found in [9].

**2.2 Hop by Hop transport model**

The Hop-by-Hop is a mechanism that distributes transport control along the Source-Sink path commonly used in wireless networks, where acknowledgement is transmitted locally between each hop. The Hop-by-Hop model and notation can be seen in [10, 11]. The probability of successful hop-by-hop transmissions P_S is determined according to,

\[
P_S = [1 - (1 - p)^L]^N.
\]

(5)
where $N$ is the number of hops, $L$ is the maximum allowed number of transmissions, $p$ is the link data delivery probability. This probability is directly related to the Markov chain governing the MJLS presented in the previous section.

2.2.1 Fundamental Concepts for Energy-Consumption

A network is composed by nodes, each one has a transceptor unit which can work in two modes: a transmission mode ($TX$) and a reception mode ($RX$). It is possible to define the energy consumption: $E_{SW}$, $E_{TX}(Lp, P_{out})$ and $E_{RX}(Lp)$, which are the energies used to switch between modes, to transmit for power $P_{out}$ and to receive a packet, respectively. Both $TX$ and $RX$ energy consumptions depend on the packet length $Lp$. Figure 2 corresponds to the schematized energy consumption per transceptor [12]. In control system applications, the packets with the measurements are transmitted in constant intervals determined by the sampling rate. The energy consumed by time per unit in a multi-hop network corresponds to the mathematical expectation of the energy consumed per package $E$ divided by the time between samples $T_s$. The theoretical models of energy consumption may consider more elements, which can be seen in [12, 2].

3 Numeric example

The plant used in the [4] is a rotary inverted pendulum [13], in which the joint angle is measured and transmitted every 50[ms] and we used a zero-order holder with a sampling frequency of 50[ms] to obtain a discrete-time model. The project is to design a state observer filter in semi-reliable communication using the Hop by Hop scheme and limiting the maximum allowed retransmissions per packet $L$. We consider a mode-independent filter, that is, $N_c = 1$. Figure 3a is a diagram of the plant and Figure 3b shows the $H_{\infty}$ norm degradation with respect to packet loss rate.

3.1 Network Model

The measured signal $y(k)$ is transmitted in a multi-hop network with 10 router units, whose implemented transport mechanism is the Hop-by-Hop. The energy consumption model is based in the wireless units which operates at 3V and have the following current consumption: the consumption per transmitted byte is 20[mA]

![Diagram of the plant and Norm](image)

(a) Plant

(b) Norm
for a power of 0[dBm]; for each received byte we spend 15[mA]; the system also has a fixed current consumption for each transition between modes, 15[mA/per] switch, the duration of those transitions are $250 \times 10^{-6}[s]$. A complete explanation about those values can be found in [12]. The average energy cost for a network with 10 routers was obtained by a Monte Carlo simulation with $1 \times 10^6$ iterations per packet.

### 3.2 Analysis Metrics

Through a Monte Carlo simulation we obtained the average energy consumed per unit time for preset values of $L$, $p$ and $N$. In order to better understand the results we created two new metrics, the first one is the rate $\Upsilon_H$ between the norm obtained with packet loss and the one obtained with the classic filtering without loss and the other is the ratio $\Upsilon_E$ between the average energy consumption with a given retransmissions limit and the average without any limit defined by the equation $\Upsilon_E = \frac{E[E|L<\infty]}{E[E|L\rightarrow\infty]}$.

### 3.3 Preliminary Results

This section displays the results in the same way as [4]. For our model of energy consumption, the $H_\infty$ norm converges faster to the classical than the energy consumption per time unit. The packet length used was 25 bytes contemplating the header. Figure 3 plots the metrics obtained for four values of $p$ in function of $L$. Through simulations it was possible to observe that, using the energy model proposed in this work, the energy consumption behavior has a direct correlation with the number of transmissions due the cost $E_{SW}$ in function of the number of transitions where the units found in reception mode and if the unit needs to transmit it switches to transmission mode and then go back to reception mode. Due the results of Monte Carlo simulation we obtained the average number of receptions have $p$ times the number of transmissions, where $pE(TX) \approx E(RX)$.

![Graphs showing the metrics for different values of p](image)

Figure 3: Variation the metric by $L$. 

4
4 Conclusion

Limiting the number $L$ in a multi hop network using the Hop by Hop scheme has a direct impact in the decrease of the average energy consumption per packet. The numerical results indicate that the percentage decrease of the average energy consumption is greater than the decrease in performance, as indicated by the $H_\infty$ norm. Like the image transmission problem is willing to lose quality in our case the the immunity to disturb ($H_\infty$) is possible to increase the network life spam. Limit the number of transmission can be useful for the network lowering the congestion and consequently the average delay.

For future work we propose to obtain the complete characterization of the stochastic process which describes the number of $TX$ and $RX$ of the transport model in a Hop by Hop network, in order to use those values in a more complete model of energy consumption. One of the most important perspectives is to formalize the control problem in a Networked Control System, which we term as “intelligent”, where there is cooperation between the controller and the network. This scheme intends to obtain the value of $L$ that fulfills the performance criteria given by $H_\infty$ norm and the life spam, congestion and others. Such a “intelligent” Networked Control System would have different controllers for different network status.

References

[1] C. Duran-Faundez and V. Lecuire, “Error resilient image communication with chaotic pixel interleaving for wireless camera sensors,” in Proceedings of the Workshop on Real-world Wireless Sensor Networks, ser. REALWSN 08, 2008, pp. 21–25.

[2] D. G. Costa and L. A. Guedes, “A discrete wavelet transform (dwt)-based energy-efficient selective retransmission mechanism for wireless image sensor networks,” Journal of Sensor and Actuator Networks, vol. 1, no. 1, pp. 3–35, 2012.

[3] C. Duran-Faundez, V. Lecuire, and F. Lepage, “Tiny block-size coding for energy-efficient image compression and communication in wireless camera sensor networks,” Signal Processing: Image Communication, vol. 26, no. 8, pp. 466–481, 2011.

[4] J. M. Palma, d. L. Carvalho, A. Gonçalves, C. Galarza, and A. Oliveira, “Application of control theory markov systems to minimize the number of transmissions in a multi-hop network,” in Proceedings of the Asia-Pacific Conference on Computer Aided System Engineering (APCASE) conference. Ecuador: IEEE, Jul. 2015, pp. 296–301.

[5] C. T. Inc, “The Smart Sensor Company,” [http://moog-crossboe.com](http://moog-crossboe.com), 2015, [Online; accessed 1-July-2015].

[6] F.-Y. Wang and D. Liu, Networked Control Systems: Theory and Applications, 1st ed. Springer Publishing Company, Incorporated, 2008.

[7] J. B. Do Val, J. C. Geromel, and A. P. GonçAlves, “The $h_2$-control for jump linear systems: cluster observations of the markov state,” Automatica, vol. 38, no. 2, pp. 343–349, 2002.

[8] A. P. d. C. Gonçalves, “Controle dinâmico de saída para sistemas discretos com saltos markovianos,” Ph.D. dissertation, Universidade Estadual de Campinas (UNICAMP). Faculdade de Engenharia Elétrica e de Computação, 2009.

[9] A. Fioravanti, A. Gonçalves, and J. Geromel, “Optimal and mode-independent filters for generalised bernoulli jump systems,” International Journal of Systems Science, vol. 46, no. 3, pp. 405–417, 2015.

[10] S. Heimlicher, P. Nuggehalli, and M. May, “End-to-end vs. hop-by-hop transport,” SIGMETRICS Perform. Eval. Rev., vol. 35, no. 3, Dec. 2007.

[11] S. Heimlicher, M. Karaliopoulos, H. Levy, and M. May, “End-to-end vs. hop-by-hop transport under intermittent connectivity,” p. 20, 2007.

[12] V. Lecuire, C. Duran-Faundez, and N. Krommenacker, “Energy-efficient image transmission in sensor networks,” International Journal of Sensor Networks, vol. 4, no. 1-2, pp. 37–47, 2008.

[13] A. M. de Oliveira, “Análise e controle de um sistema mecanico com dados transmitidos através da rede,” in Monografia. Unicamp, 2015.