ISGUR-WISE FORM FACTORS OF HEAVY BARYONS WITHIN A LIGHT-FRONT CONSTITUENT QUARK MODEL

F. Cardarelli and S. Simula

Istituto Nazionale di Fisica Nucleare, Sezione Sanità,
Viale Regina Elena 299, I-00161 Roma, Italy

Abstract

The space-like elastic form factors of baryons containing a heavy quark are investigated within a light-front constituent quark model in the limit of infinite heavy-quark mass, adopting a gaussian-like ansätz for the three-quark wave function. The results obtained for the Isgur-Wise form factors corresponding both to a spin-0 and a spin-1 light spectator pair are presented. It is found that the Isgur-Wise functions depend strongly on the baryon structure, being sharply different in case of diquark-like or collinear-type configurations in the three-quark system. It is also shown that the relativistic effects lead to a saturation property of the form factors as a function of the baryon size. Our results are compared with those of different models as well as with recent predictions from QCD sum rules and lattice QCD simulations; the latter ones seem to suggest the dominance of collinear-type configurations, in which the heavy-quark is sitting close to the center-of-mass of the light quark pair.

PACS numbers: 12.38.Lg; 12.39.Ki; 13.40.Gp; 14.20.Mr

Keywords: heavy baryons; relativistic quark model; electroweak form factors.
The investigation of heavy-hadron decays can provide relevant information on fundamental parameters of the Standard Model, like the Cabibbo-Kobayashi-Maskawa mixing angles and the quark masses. As it is well known, in case of hadrons containing a single heavy quark ($Q$), the complexity of the theoretical analysis is largely simplified by the Heavy Quark Symmetry (HQS) \cite{ref1, ref2, ref3}, which is a spin-flavour symmetry shared by (but not manifest in) the QCD Lagrangian when the limit of infinite heavy-quark mass ($m_Q \to \infty$) is considered. The HQS, which is only softly broken by terms of order $\Lambda_{QCD}/m_Q$, allows to derive several model-independent relations among hadronic properties; in particular, it requires that, when $m_Q \to \infty$, all the non-perturbative strong physics relevant for exclusive electroweak processes is contained in few universal form factors, known as the Isgur-Wise (IW) functions. The latter however cannot be predicted by the HQS, for the full knowledge of the non-perturbative structure of heavy hadrons is required. One has therefore to resort to models in order to make quantitative predictions, provided the HQS relations are checked in any particular model for internal consistency.

The aim of this letter is to investigate the IW form factors of heavy baryons using a relativistic constituent (CQ) quark model formulated on the light-front (LF). Our CQ model (see Refs. \cite{ref1, ref3}), which properly incorporates boost effects and the relativistic composition of CQ spins, has been already applied to the evaluation of the IW form factor in case of the ground-state of heavy-light mesons. It has been shown \cite{ref3} that in a wide range of values of the recoil ($\omega - 1$), where $\omega \equiv v \cdot v'$ is the product of the initial ($v$) and final ($v'$) hadron four-velocities, the calculated IW function $\xi^{(IW)}(\omega)$ exhibits a moderate dependence upon the choice of the heavy-meson wave function; moreover, the slope of the IW form factor at the zero-recoil point, $\rho^2 \equiv -[d\xi^{(IW)}(\omega)/d\omega]_{\omega=1}$, has been found to be remarkably increased by relativistic effects and quantitatively close to the value $\rho^2 \approx 1$ \cite{ref3} (cf. also Ref. \cite{ref4}). In this letter, the baryon IW form factors are obtained from the electromagnetic (e.m.) elastic form factors of heavy baryons, corresponding both to a spin-0 and a spin-1 light spectator pair, calculated for space-like values of the squared four-momentum transfer $q^2 \equiv q \cdot q < 0$. Though we limit ourselves to a gaussian-like ansatz for the baryon wave function, an interesting feature of the IW form factors emerges: indeed, it will be shown that the baryon IW functions are sharply different in case of diquark-like or collinear-type configurations in the three-quark system. In particular, the slope of the IW form factors at the zero-recoil point is remarkably smaller for collinear-type configurations, in which the heavy quark is sitting close to the center-of-mass of the two light spectator quarks. It is also found that the relativistic effects lead to a saturation property of the IW form factors as a function of the baryon size.

To begin with, let us consider a $Q(qq')$ baryon composed by a heavy quark $Q$ with constituent mass $m_Q$ and a partner light-quark pair $(qq')$ with constituent masses $m_q$ and $m_{q'}$. Within the LF formalism (cf., e.g., Refs. \cite{ref1, ref3}) the hadron wave functions are eigenvectors of a mass operator, e.g. $\mathcal{M} = M_0 + \mathcal{V}$, and of the non-interacting angular momentum operators $j^2$ and $j_n$, where the vector $\hat{n} = (0,0,1)$ defines the spin quantization axis. The operator $M_0$ is the free mass and the interaction term $\mathcal{V}$ is a Poincaré invariant. For baryons one has $M_0^2 = \sum_{i=Q,q,q'}(k_{i\perp}^2 + m_i^2)/\xi_i$, where $\xi_i = p_i^+ / P^+$ and $k_{i\perp} = \vec{p}_{i\perp} - \vec{P}_{\perp}$ are the intrinsic LF variables. The subscript $\perp$ indicates the projection perpendicular to the spin quantization axis and the plus component of a four-vector $p \equiv (p^0, \vec{p})$ is given by $p^+ = p^0 + \hat{n} \cdot \vec{p}$; finally, $\vec{P} \equiv (P^+, \vec{P}_\perp) = \vec{p}_Q + \vec{p}_q + \vec{p}_{q'}$ is the LF baryon momentum and $\vec{p}_i$ the quark one. In terms of
the longitudinal momentum \( k_{in} \), related to the variable \( \xi_i \) by 
\[
\xi_i = \frac{1}{2} \left[ \xi_i M_0 - (k_{i\perp}^2 + m_i^2) / \xi_i M_0 \right],
\]
the free mass operator acquires a familiar form, viz.
\[
M_0 = \sum_{i=Q,q,q'} \sqrt{m_i^2 + \left| \vec{k}_i \right|^2} \equiv \sum_{i=Q,q,q'} E_i
\]
with \( \vec{k}_i \equiv (\vec{k}_{i\perp}, k_{in}) \). Disregarding for simplicity the colour and flavour degrees of freedom and limiting ourselves to \( S \)-wave baryons, the LF wave function can be written as
\[
\langle \{\xi, \vec{k}_{i\perp} ; \nu_i\} | \Phi_{S_{qq'}}^{J\mu} \rangle = \sqrt{\frac{E_Q E_q E_{q'}}{M_0 \xi_Q \xi_q \xi_{q'}}} \sum_{\nu'_i} \langle \nu_i | R(\{\vec{k}_i; m_i\}) | \nu'_i \rangle \langle \{\vec{k}_i; \nu'_i\} | \chi_{S_{qq'}}^{J\mu} \rangle
\]
where \( J \) and \( \mu \) are the total angular momentum and its projection; \( S_{qq'} \) is the spin of the light quark pair \((qq')\); the curly braces \( \{ \) mean a list of indices corresponding to \( i = Q, q, q' \); \( \nu' \) is the third component of the \( CQ \) spin; \( R(\{\vec{k}_i; m_i\}) \equiv \Pi_{j=Q,q,q'} R_M(\vec{k}_j, m_j) \) with \( R_M(\vec{k}_j, m_j) \) being the (generalized) Melosh rotation. The canonical heavy-baryon wave function \( \langle \{\vec{k}_i; \nu'_i\} | \chi_{S_{qq'}}^{J\mu} \rangle \) is given by
\[
\langle \{\vec{k}_i; \nu'_i\} | \chi_{S_{qq'}}^{J\mu} \rangle = w(Qqq') (\vec{p}, \vec{k}) \cdot \Phi_{S_{qq'}}^{J\mu} (\nu'_i)
\]
where \( \vec{p} = [(m_q + m_q)\vec{k}_Q - m_Q(\vec{k}_q + \vec{k}_{q'})]/(m_Q + m_q + m_{q'}) = \vec{k}_Q = -\vec{k}_q - \vec{k}_{q'} \) and \( \vec{k} = (m_q \vec{k}_q - m_q \vec{k}_{q'})/(m_q + m_{q'}) \) are the Jacobi coordinates for the \( Q(qq') \) system; \( w(Qqq') \) is the \( S \)-wave radial wave function and the spin function \( \Phi_{S_{qq'}}^{J\mu} (\nu'_i) \) is defined as
\[
\Phi_{S_{qq'}}^{J\mu} (\nu'_i) = \sum_{\sigma} \left( \frac{1}{2} \nu'_Q S_{qq'} \sigma | J\mu \rangle \langle \frac{1}{2} \nu'_Q | \frac{1}{2} S_{qq'} \sigma \right)
\]
For a fixed value of \( S_{qq'} = 0, 1 \) the LF wave functions \( \{2\} \) with \( J = S_{qq'} \pm 1/2 \) belong to multiplets of the \( HQS \) (see Ref. [1](c)), namely to singlets for \( S_{qq'} = 0 \) (\( J = 1/2 \)) and doublets for \( S_{qq'} = 1 \) (\( J = 1/2, 3/2 \)). Finally, the wave function \( \{3\} \) is normalized as: \( \sum_{\nu_i} \int \{d\vec{k}_i\} \delta(\vec{k}_Q + \vec{k}_q + \vec{k}_{q'}) \left| \langle \{\vec{k}_i, \nu_i\} | \chi_{S_{qq'}}^{J\mu} \rangle \right|^2 = 1 \), which implies \( \int d\vec{p} d\vec{k} \left| w(Qqq')(\vec{p}, \vec{k}) \right|^2 = 1 \). In what follows we will limit ourselves to a gaussian ansatz for \( w(Qqq')(\vec{p}, \vec{k}) \), viz.
\[
w(Qqq')(\vec{p}, \vec{k}) = \left( \frac{1}{\pi \alpha_p} \right)^{3/2} e^{-|\vec{p}|^2/2 \alpha_p^2} \left( \frac{1}{\pi \alpha_k} \right)^{3/2} e^{-|\vec{k}|^2/2 \alpha_k^2}
\]
where \( \alpha_p \) and \( \alpha_k \) are the harmonic oscillator parameters governing the average value of the internal momenta, namely \( \langle |\vec{p}|^2 \rangle = \frac{3}{2} \alpha_p^2 \) and \( \langle |\vec{k}|^2 \rangle = \frac{3}{2} \alpha_k^2 \).

After the description of the structure of our LF three-quark wave function, we now briefly remind (see Ref. [1](c)) that in the heavy-quark limit all the relevant electroweak baryon form factors reduce to one \( IW \) function, \( \xi^{(0)}(\omega) \), in case of the singlets with \( S_{qq'} = 0 \) and to two \( IW \) form factors, \( \xi^{(1)}(\omega) \) and \( \zeta^{(1)}(\omega) \), in case of the doublets with \( S_{qq'} = 1 \). In this letter the
IW form factors are calculated through the matrix elements of the e.m. vector current $\bar{Q}\gamma^\rho Q$ between $J = 1/2$ baryons, viz.

$$V^\rho_{\mu\mu}(S_{qq'}) \equiv \langle \Psi_{S_{qq'}}^{1/2\mu'} | \bar{Q}\gamma^\rho Q | \Psi_{S_{qq'}}^{1/2\mu} \rangle = F_1^{(S_{qq'})}(\omega) \bar{u}(P', \mu') \gamma^\rho u(P, \mu) +$$

$$+ \left[ F_2^{(S_{qq'})}(\omega) \bar{u} + F_3^{(S_{qq'})}(\omega) \bar{u} \gamma^\rho \right] \bar{u}(P', \mu') u(P, \mu)$$

(6)

where $u(P, \mu)$ is a Dirac spinor (normalized as $\bar{u}u = 1$), $P = Mv$ and $P' = P + q = Mv'$, with $M$ being the heavy-baryon mass. Since we are considering an elastic process, the four-momentum transfer squared is given by $q^2 = 2M^2 \cdot (1 - \omega)$ and has space-like values ($q^2 \leq 0$). A formula similar to Eq. (6) can be written for the matrix elements of the axial-vector current $\bar{Q}\gamma^\rho \gamma_5 Q$, namely

$$A^\rho_{\mu\mu}(S_{qq'}) \equiv \langle \Psi_{S_{qq'}}^{1/2\mu'} | \bar{Q}\gamma^\rho \gamma_5 Q | \Psi_{S_{qq'}}^{1/2\mu} \rangle = G_1^{(S_{qq'})}(\omega) \bar{u}(P', \mu') \gamma^\rho \gamma_5 u(P, \mu) +$$

$$+ \left[ G_2^{(S_{qq'})}(\omega) \bar{u} + G_3^{(S_{qq'})}(\omega) \bar{u} \gamma^\rho \right] \bar{u}(P', \mu') \gamma_5 u(P, \mu)$$

(7)

In the heavy-quark limit one has

$$\lim_{m_Q \to \infty} F_1^{(0)}(\omega) = \lim_{m_Q \to \infty} G_1^{(0)}(\omega) = \xi^{(0)}(\omega)$$

(8)

$$\lim_{m_Q \to \infty} F_2^{(0)}(\omega) = \lim_{m_Q \to \infty} F_3^{(0)}(\omega) = 0$$

(9)

$$\lim_{m_Q \to \infty} G_2^{(0)}(\omega) = \lim_{m_Q \to \infty} G_3^{(0)}(\omega) = 0$$

(10)

$$\lim_{m_Q \to \infty} F_1^{(1)}(\omega) = \lim_{m_Q \to \infty} G_1^{(1)}(\omega) = -\frac{1}{6} \left[ 2\omega \xi^{(1)}(\omega) + (\omega - 1) \zeta^{(1)}(\omega) \right]$$

(11)

$$\lim_{m_Q \to \infty} F_2^{(1)}(\omega) = \lim_{m_Q \to \infty} F_3^{(1)}(\omega) = \frac{2}{3} \xi^{(1)}(\omega)$$

(12)

$$\lim_{m_Q \to \infty} G_2^{(1)}(\omega) = -\lim_{m_Q \to \infty} G_3^{(1)}(\omega) = \frac{2}{3} \left[ \xi^{(1)}(\omega) + \zeta^{(1)}(\omega) \right]$$

(13)

where the IW form factors $\xi^{(0)}(\omega)$ and $\xi^{(1)}(\omega)$ must satisfy the model-independent normalization $\xi^{(0)}(1) = \xi^{(1)}(1) = 1$ at the zero-recoil point $\omega = 1$, whereas the normalization $\zeta^{(1)}(1)$ is not known a priori. In what follows we will choose a reference frame where $q^+ \equiv q^0 + \bar{n} \cdot \bar{q} = 0$, which allows to suppress the contribution arising from the so-called Z-graph (pair creation from the vacuum) at any value of the heavy-quark mass.

All the hadronic form factors corresponding to a conserved current can always be expressed in terms of the matrix elements of the plus component of the current. As a matter of fact, adopting a reference frame where $q^+ = 0$, from Eq. (6) one easily gets

$$F_1^{(S_{qq'})}(\omega) = \frac{1}{2} \text{Tr} \left\{ \frac{\mathcal{V}^+(S_{qq'})}{2P^+} \right\} + \frac{M}{\sqrt{-q^2}} \text{Tr} \left\{ \frac{\mathcal{V}^+(S_{qq'})i\sigma_2}{2P^+} \right\}$$

(14)

$$F_2^{(S_{qq'})}(\omega) = F_3^{(S_{qq'})}(\omega) = -\frac{M}{2\sqrt{-q^2}} \text{Tr} \left\{ \frac{\mathcal{V}^+(S_{qq'})i\sigma_2}{2P^+} \right\}$$

(15)
On the contrary, from Eq. (14) one has

\begin{align}
G_1^{(S_{qq'})}(\omega) &= \frac{1}{2} \text{Tr} \left\{ \frac{A^+ (S_{qq'}) \sigma_3}{2 P^+} \right\} \\
G_2^{(S_{qq'})}(\omega) + G_3^{(S_{qq'})}(\omega) &= -\frac{M}{\sqrt{-q^2}} \text{Tr} \left\{ \frac{A^+ (S_{qq'}) \sigma_1}{2 P^+} \right\} \\
G_3^{(S_{qq'})}(\omega) - G_2^{(S_{qq'})}(\omega) &= \frac{2M^2}{-q^2} \left[ \text{Tr} \left\{ \frac{A^+ (S_{qq'}) \sigma_3}{2 P^+} \right\} - \frac{P^+}{M} \text{Tr} \left\{ \frac{A_1 (S_{qq'}) \sigma_1}{2 P^+} \right\} \right]
\end{align}

with $P^+ = \sqrt{M^2 - q^2/4}$. Therefore, the evaluation of the form factors $G_2^{(S_{qq'})}(\omega)$ and $G_3^{(S_{qq'})}(\omega)$ requires also the use of the matrix element of the transverse component of the (non-conserved) axial current.

We have evaluated the right-hand side of Eqs. (14-18) using the three-quark wave function given by Eqs. (2-4) adopting the gaussian ansatz (5). The numerical integrations, involving six-dimensional integrals, have been performed through a well-established Monte Carlo procedure \[7\]. The heavy-quark limit ($m_Q \to \infty$) is obtained by increasing the value of the heavy-quark mass until full convergence of the calculated form factors is reached. We have found that all the HQS relations (8-13) are fulfilled at any value of $\omega$, except Eq. (13) at the zero-recoil point $\omega = 1$, because of the occurrence of a divergence at $q^2 = 0$ in the right-hand side of Eq. (18). The understanding and elimination of such a divergence is outside the scope of the present letter and will be addressed in a future work \[8\]. Here, we will limit ourselves to note that within the LF formalism the transverse components of the current can contain the so-called instantaneous propagation terms, which are instead absent in the plus component. Thus, the IW form factors $\xi^{(0)}(\omega)$, $\xi^{(1)}(\omega)$ and $\zeta^{(1)}(\omega)$ will be determined only via the matrix elements of the plus component of the (conserved) vector current, viz.

\begin{align}
\xi^{(0)}(\omega) &= \frac{1}{2} \lim_{m_Q \to \infty} \text{Tr} \left\{ \frac{V^+ (0)}{2P^+} \right\} \\
\xi^{(1)}(\omega) &= -\frac{3}{4\omega \sqrt{2(\omega - 1)}} \lim_{m_Q \to \infty} \text{Tr} \left\{ \frac{V^+ (1) i \sigma_2}{2P^+} \right\} \\
\zeta^{(1)}(\omega) &= -\frac{3}{\omega - 1} \lim_{m_Q \to \infty} \left[ \text{Tr} \left\{ \frac{V^+ (1)}{2P^+} \right\} + \frac{3}{2\sqrt{2(\omega - 1)}} \text{Tr} \left\{ \frac{V^+ (1) i \sigma_2}{2P^+} \right\} \right]
\end{align}

The explicit expressions of the traces appearing in Eqs. (19-21) will be reported elsewhere \[8\]. Here, we point out that the absolute normalizations $\xi^{(0)}(1) = \xi^{(1)}(1) = 1$ are recovered in the right-hand side of Eqs. (19-21) thanks to the normalization of the canonical wave function (3) and to the $SU(2)$ Clebsch-Gordan algebra; moreover, for the same reasons the right-hand side of Eq. (21) behaves regularly when $\omega \to 1$.

In our calculations, besides the masses $m_q$ and $m_{q'}$ of the spectator quarks, there are two more parameters, $\alpha_p$ and $\alpha_k$, appearing in the radial wave function (5). Instead of them we now
introduce two other parameters, which are combinations of $\alpha_p$ and $\alpha_k$, inspired by the fact that in the heavy-quark limit ($m_Q \to \infty$) the canonical wave function $\xi(0)$ with the gaussian ansatz (3) implies a root mean square radius of the baryon equal to $\sqrt{\langle r^2 \rangle_B} = \sqrt{\frac{3}{2}} \alpha^{-1}$, where

$$\alpha \equiv \frac{\alpha_p}{\sqrt{2 + \frac{1+\eta^2}{(1+\eta)^2} \beta^2}}$$

$$\beta \equiv \frac{\alpha_p}{\alpha_k} \left( \frac{\langle \vec{p}^2 \rangle}{\langle \vec{k}^2 \rangle} \right)$$

with $\eta \equiv m_q/m_{q'}$ being the light $CQ$ mass ratio. Thus, the parameter $\alpha$ (Eq. (22)) governs the (canonical) baryon size, whereas $\beta$ (Eq. (23)) is the ratio of the average internal momenta. Let us first focus on $\alpha$ and consider only the form factor $\xi(0)(\omega)$ for sake of simplicity. In the non-relativistic limit the charge radius of $\xi(0)(\omega)$, $\rho(0)^2 \equiv -[d\xi(0)(\omega)/d\omega]_{\omega=1}$, is proportional to the square of the (canonical) baryon size $\langle r^2 \rangle_B$, so that when $\alpha \to \infty$ one should have $\rho(0)^2 \to 0$. However, a well-known feature of the relativistic effects is the delocalization of the position of the light particles, triggered when the baryon size becomes smaller than $\sqrt{\lambda_q^2 + \lambda_{q'}^2}$, where $\lambda_q(q') \equiv 1/m_q(q')$ is the $CQ$ Compton wavelength. Therefore, we expect the slope $\rho(0)^2$ to be a monotonically decreasing function of $\alpha$, and moreover, when $\alpha \gtrsim \alpha_{sat}$ with

$$\alpha_{sat} \equiv \sqrt{\frac{3}{2}} \frac{m_q m_{q'}}{\sqrt{m_q^2 + m_{q'}^2}},$$

the slope $\rho(0)^2$ should saturate and become independent of the baryon size, being dominated by relativistic effects. These expectations, including the dependence of $\alpha_{sat}$ (Eq. (24)) upon the spectator $CQ$ masses, are fully confirmed by explicit calculations of the slope $\rho(0)^2$ in the $(u, d, s, c)$ spectator sector. In Fig. 1 we have reported the results obtained for the form factor $\xi(0)(\omega)$ for various values of $\alpha$ at fixed $\beta$. It can clearly be seen that in a wide range of values of $\omega$, including the range of interest for the $b \to c$ transition, the form factor itself saturates for $\alpha \gtrsim \alpha_{sat}$. Similar results hold as well at any value of $\beta$ and for the spin-1 form factors $\xi(1)(\omega)$ and $\zeta(1)(\omega)$, so that for $\alpha \gtrsim \alpha_{sat}$ the baryon $IW$ form factors becomes almost independent of the parameter $\alpha$, i.e. of the baryon size. Since the (flavour-independent) confinement scale yields $\alpha \sim 0.4 \text{GeV}$ (cf. Ref. [3]), from Eq. (24) it follows that in case of spectator $CQ$ masses in the $(u, d, s)$ sector the baryon $IW$ form factors are expected to be almost independent of the baryon size and therefore to be in our gaussian model function of $\beta$ and $\eta$ only.

As far as the parameter $\beta$ is concerned, we have sketched in Fig. 2 two types of three-quark spatial configurations corresponding to the limiting cases $\beta \ll 1$ (Fig. 2(a)) and $\beta \gg 1$ (Fig. 2(b)). In the heavy-quark limit ($m_Q \to \infty$) the internal momentum $\vec{p}$ describes the motion of the center-of-mass of the spectator pair with respect to the heavy-quark sitting on the center-of-mass of the baryon, i.e. the Jacobian momentum $\vec{p} \equiv [\vec{k}]$ is conjugate to the spatial variable $\vec{R}_{Q(qq')} \equiv \vec{r}_Q - (m_q \vec{r}_q + m_{q'} \vec{r}_{q'})/(m_q + m_{q'}) \cdot [\vec{r}_q - \vec{r}_{q'}]$, where $\vec{r}_j$ is the position of the quark

\footnote{However, the saturation value of the slope $\rho(0)^2$ still depends on the particular functional form adopted for the radial wave function $w(Qqq')(\vec{p}, \vec{k})$.}
Since $\beta = \langle |\vec{R}_q - \vec{r}_q|\rangle / \langle |\vec{R}_{Q(q\bar{q})}|\rangle$, the range of values $\beta \lesssim 1$ is characterized by values of $|\vec{R}_{Q(q\bar{q})}|$ larger than the average relative distance $|\vec{r}_q - \vec{r}_q'|$ between the light spectator quarks; this kind of configuration resembles a diquark-like cluster (see Fig. 2(a) in the limiting case $\beta << 1$). On the contrary, when $\beta > 1$, one has on average $|\vec{R}_{Q(q\bar{q})}| < |\vec{r}_q - \vec{r}_q'|$, resembling now a collinear-type configuration, where the heavy-quark is sitting close to the center-of-mass of the spectator $CQ$ pair (see Fig. 2(b) in the limiting case $\beta >> 1$). In Fig. 3 the $IW$ form factors $\xi^{(0)}(\omega)$, $\xi^{(1)}(\omega)$ and $\zeta^{(1)}(\omega)$ are shown for various values of the parameter $\beta$ in the saturation region $\alpha > \alpha_{sat}$. It can be seen that the $IW$ form factors depend strongly on the baryon structure, being sharply different in case of diquark-like ($\beta \lesssim 1$) or collinear-type ($\beta > 1$) configurations in the three-quark system. This result is at variance with what happens in the heavy-light meson case, where the sensitivity of the $IW$ form factor to the choice of the meson wave function is moderate. Therefore, in case of heavy-light baryons the investigation of the $IW$ functions appears to be very appealing in order to shed light on the non-perturbative aspects of $QCD$. The dynamical determination of the parameter $\beta$, which should depend upon the spin of the spectator quark pair, is clearly called for and, in this respect, calculations based on baryon wave functions derived from spectroscopic $CQ$ models are in progress.

In Fig. 3(a) our predictions for $\xi^{(0)}(\omega)$ are compared with existing results from the diquark model of Ref. [3], the MIT bag model of Ref. [11], the Skyrme model of Ref. [11], the $QCD$ sum rule technique of Ref. [12] and with recent lattice $QCD$ simulations [13]. It can be seen that, in qualitative agreement with our results, the diquark model predicts the steepest $IW$ function $\xi^{(0)}(\omega)$, while both the lattice points and the $QCD$ sum rule results indicate a much softer form factor, suggesting the dominance of collinear-type configurations in the structure of heavy-light baryons.

Finally, in Fig. 4 we have reported the results obtained for the slopes $\rho^{(i)}_2 \equiv -[d\xi^{(i)}(\omega)/d\omega]|_{\omega=1}$ ($i = 0, 1$) at the zero-recoil point as a function of the parameter $\beta$ in the saturation region $\alpha > \alpha_{sat}$. The slope $\rho^{(1)}_2$ results to be systematically larger than $\rho^{(0)}_2$ and both of them are monotonically decreasing function in a wide range of values of $\beta$. In the limit $\beta \rightarrow \infty$ the slope $\rho^{(1)}_2$ approaches the value $\simeq 2.1$, while $\rho^{(0)}_2$ monotonically decreases up to $\simeq 1.1$, i.e. it becomes very close to the value of the slope $\rho^2$ of the $IW$ form factor for heavy-light mesons [3]. The values of $\rho^{(0)}_2$; which can be found in literature, are spread in a quite large range, namely: $\rho^{(0)}_2 = 3.7$ [4], 2.4 [14], 1.3 [11], 1.15 [12] and $1.2^{+0.8}_{-1.1}$ [13].

In conclusion, the space-like elastic form factors of baryons containing a heavy quark have been investigated within the light-front constituent quark model of Refs. [4, 5] in the limit of infinite heavy-quark mass, adopting a gaussian-like ansatz for the three-quark wave function. It has been found that the Isgur-Wise functions, corresponding both to a spin-0 and a spin-1 light spectator pair, depend strongly on the baryon structure, being sharply different in case of diquark-like or collinear-type configurations in the three-quark system. Moreover, it has been shown that relativistic effects lead to a saturation property of the calculated Isgur-Wise form factors as a function of the baryon size. Finally, our predictions have been compared with those of different models as well as with recent predictions from $QCD$ sum rules and lattice $QCD$ simulations; the latter ones seem to suggest the dominance of collinear-type configurations, in

\[ \eta = m_q/m_{q'}. \]
which the heavy-quark is sitting close to the center-of-mass of the light quark pair.

References

[1] (a) N. Isgur and M.B. Wise: Phys. Lett. B232 (1989) 113; (b) ib. B237 (1990) 527; (c) Nucl. Phys. B348 (1991) 276.

[2] D. Politzer and M.B. Wise: Phys. Lett. 208B (1988) 504. M.B. Voloshin and M.A. Shifman: Sov. J. Nucl. Phys. 47 (1988) 511. H. Georgi: Phys. Lett. 240B (1990) 447.

[3] For a review see, e.g., M. Neubert: Phys. Rep. 245 (1994) 259, and references therein quoted.

[4] F. Cardarelli et al: Phys. Lett. B332 (1994) 1; Phys. Lett. B349 (1995) 393; Phys. Lett. B357 (1995) 1; Phys. Lett. B359 (1995) 1; Few-Body Syst. Suppl. 8 (1995) 345; Few-Body Syst. Suppl. 9 (1995) 267; Phys. Rev. D53 (1996) 6682; Phys. Lett. B371 (1996) 7; Phys. Lett. B397 (1997) 13; e-print archive nucl-th/9612063.

[5] S. Simula: Phys. Lett. B373 (1996) 193. I.L. Grach et al.: Phys. Lett. B385 (1996) 317; Nucl. Phys. B502 (1997) 227 (e-print archive hep-ph/9603369); Nucl. Phys. B (Proc. Suppl.) 55A (1997) 84.

[6] F.E. Close and A. Wambach: Nucl. Phys. B412 (1994) 169; Phys. Lett. B349 (1995) 207. See also A. Le Yaouanc et al.: Phys. Lett. B386 (1996) 304 and e-print archive hep-ph/9705324.

[7] G.P. Lepage: J. Comp. Phys. 27 (1978) 192.

[8] F. Cardarelli and S. Simula: to be published.

[9] X.-H. Guo and P. Kroll: Z. Phys. C59 (1993) 567. See also X.-H. Guo and T. Muta: phys. Rev. D54 (1996) 4629.

[10] M. Sadzikowski and K. Zalewski: Z. Phys. C59 (1993) 677.

[11] E. Jenkins, A. Manohar and M.B. Wise: Nucl. Phys. B396 (1996) 38.

[12] A.G. Grozin and O.I. Yakovlev: Phys. Lett. B291 (1992) 441. O.I. Yakovlev: in Proc. of the III German-Russian Workshop on Heavy Quark Physics, Dubna(Russia), May 20-22, 1996, e-print archive hep-ph/9608348.

[13] UKQCD collaboration, K.C. Bowler et al.: e-print archive hep-lat/9709028.
Figure 1. The form factor $\xi(0)(\omega)$ (Eq. (19)), evaluated using the LF wave function (2) and adopting the gaussian ansatz (5) for the radial function, as a function of the recoil $\omega$ for various values of the parameter $\alpha$ (Eq. (22)). The dot-dashed, dotted, dashed, long-dashed and solid lines correspond to $\alpha = 0.1, 0.2, 0.3, 0.4$ and $1.0$ GeV, respectively. The light CQ masses are $m_q = m_{q'} = 0.220$ GeV and the parameter $\beta$ (Eq. (23)) is fixed at the value $\beta = 2$.

Figure 2. Diquark-like (a) and collinear-type (b) three-quark configurations, corresponding to the limiting cases $\beta << 1$ and $\beta >> 1$, respectively (see text). The full dot represents the heavy-quark, while a spectator CQ mass ratio $\eta = m_q/m_{q'} = 1$ is understood.

Figure 3. The IW form factors $\xi(0)(\omega)$ (a), $\xi(1)(\omega)$ (b) and $\zeta(1)(\omega)$ (c), given by Eqs. (19)-(21) calculated in our LF CQ model adopting the gaussian ansatz (5), as a function of the recoil $\omega$ for various values of the parameter $\beta$ (Eq. (23)). The dot-dashed, dotted, dashed, long dashed and solid lines correspond to $\beta = 0.5, 1.0, 2.0, 3.0$ and $10.0$, respectively. The value of $\alpha$ (Eq. (22)) is always in the saturation region $\alpha > \alpha_{sat}$ (see Eq. (24)), while the value of the spectator CQ mass ratio is $\eta = m_q/m_{q'} = 1$. In (a) the results of the diquark model of Ref. [9] (with $\varepsilon = 0.6$ GeV and $b = 1.77$ GeV$^{-1}$), the MIT bag model of Ref. [10], the Skyrme model of Ref. [11] and the QCD sum rule of Ref. [12] (corresponding to $\rho_{10}^2 = 1.15$) are reported by the full triangles, dots, diamonds and squares, respectively. The open dots and squares correspond to recent lattice QCD simulations [13] of axial and vector form factors, respectively. In (b) the long dashed and solid lines are indistinguishable. In (c) the error bars on the solid line indicate the uncertainty related to the Monte-Carlo integration procedure in the calculation of the form factor $\zeta(1)(\omega)$ (in case of $\xi(0)(\omega)$ and $\xi(1)(\omega)$ the uncertainty is smaller by one order of magnitude).

Figure 4. The slopes $\rho_i^2 \equiv -[d\xi^{(i)}(\omega)/d\omega]_{\omega=1}$ ($i = 0, 1$) at the zero-recoil point, predicted by our LF CQ model using the gaussian ansatz (5), as a function of the parameter $\beta$ (Eq. (23)). The solid and dashed line correspond to $i = 0$ and 1, respectively. The value of the parameter $\alpha$ (Eq. (22)) is always in the saturation region $\alpha > \alpha_{sat}$ (see Eq. (24)), while the value of the spectator CQ mass ratio is $\eta = m_q/m_{q'} = 1$. 
Figure 1
Figure 2
Figure 3(a)
Figure 3(b)
Figure 3(c)
Figure 4