We analyze the response of a detector with a uniform acceleration $\alpha$ in $\kappa$–Minkowski spacetime using the first order perturbation theory. The monopole detector is coupled to a massless complex scalar field in such a way that it is sensitive to the Lorentz violation due to the noncommutativity of spacetime present in the $\kappa$–deformation. The response function deviates from the thermal distribution of Unruh temperature at the order of $1/\kappa$ and vanishes exponentially as the proper time of the detector exceeds a certain critical time, a logarithmic function of $\kappa$. This suggests that the Unruh temperature becomes not only fuzzy but also eventually decreases to zero in this model.

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I. INTRODUCTION

In the last decade, there has been a great interest in attempting to explain the cosmic observational data as a quantum gravitational effect. As a theoretical framework to study the quantum gravity effects phenomenologically, several field theories on noncommutative spacetimes [1, 2, 3, 4, 5, 6, 7, 8] have been studied.

In this direction of research, one common aspect is the introduction of deformed symmetries and results in Lorentz symmetry breaking. This is reformulated in noncommutative spacetimes where a dimensional parameter is introduced, related to the Planck mass. This dimensional parameter is expected to suppress the Lorentz symmetry breaking in the commutative limit. However, careful estimates suggest that there may exist a strong fine-tuning problem in noncommutative spacetime approach at a one-loop level [9]. In addition, unitarity of the theory is in question in noncommutative spacetime field theories [10, 11]. This consideration, however, does not decrease the interest in noncommutative spacetime field theories. On the contrary, one needs to find a good candidate for a realistic model.

The $\kappa$-Minkowski spacetime introduces a dimensionful deformation parameter $\kappa$, whose natural choice is to put $\kappa = M_P$, the Planck mass [12, 13, 14]. The $\kappa$–Minkowski spacetime respects rotational symmetry and appears to be a good candidate to study the quantum gravity effect. Scalar field theory has been studied by introducing the differential structure in $\kappa$–Minkowski space [12, 13]. The $\kappa$–deformation is extended to the curved space with $\kappa$–Robertson-Walker metric and is applied to cosmic microwave background radiation in [16].

On the other hand, it is an interesting question how the quantum gravity effect changes the structures of vacua and particle in curved spacetime. Unruh [15, 16] calculated the response of a particle detector moving with a uniform acceleration, under the assumption that the state of field is initially in its vacuum state, i.e., the Minkowski vacuum and the field interacts only with the detector. It is shown that the detector responds as if it would have remained un-accelerated but immersed in a heat bath at temperature $\alpha/2\pi$, the Unruh temperature. This is called the Unruh effect and has been studied further in [19]. The idea has been extended to include back-reaction problem and generalized to realistic detectors [20, 21].

The Unruh effect is interesting since it gives an analogy of the Hawking radiation in blackhole spacetime due to the thermal form of the transition probability in the presence of event horizon. In this point of view, the accelerating frame (Rindler spacetime) is regarded as a simplest toy model simulating the radiation effect of a blackhole. Since
it is not easy to calculate the field theory in curved-noncommutative spacetime containing a blackhole directly, one may instead, seek for the effect on an accelerating frame first.

In the present paper, we study the response function of a uniformly accelerating monopole particle detector which interacts with massless complex scalar field in $\kappa$–Minkowski spacetime. Since the primary effect of $\kappa$–deformation is the Lorentz symmetry breaking, we design a particle detector model so that it is sensitive to this symmetry violation.

In Sec. II, we briefly summarize the complex scalar field theory in $\kappa$–Minkowski spacetime using the differential structure given in [12]. We derive the Feynman propagator from the action and propose Wightman functions. In Sec. III, we consider a uniformly accelerating detector interacting with massless complex scalar field and calculate the transition amplitude of the detector. It is found that the Lorentz symmetry violation effect appears to the response function at $O(1/\kappa)$. We summarize the results in Sec. IV.

II. SCALAR FIELD THEORY IN $\kappa$-MINKOWSKI SPACETIME

In this section, we briefly introduce the $\kappa$–deformation of the Minkowski spacetime and construct the scalar field theory. Some details can be found in Ref. [12]. Here we use the signature of spacetime metric, $(+,−,−,−)$.

A. $\kappa$–Minkowski spacetime

The time and space coordinates are not commuting in $\kappa$–Minkowski spacetime but satisfy the commutation relations

$$[\hat{x}^0, \hat{x}^i] = \frac{i}{\kappa} \hat{x}^i, \quad [\hat{x}^i, \hat{x}^j] = 0, \quad i, j = 1, 2, 3. \quad (1)$$

$\kappa$ is a positive parameter which represents the deformation of the space-time. Its Hopf algebra $H_x$ is described by the co-product

$$\Delta(\hat{x}^\mu) = \hat{x}^\mu \otimes 1 + 1 \otimes \hat{x}^\mu.$$  

The $\kappa$-deformed Poincaré algebra is constructed using commuting four-momenta $[p^\mu, p'^\nu] = 0$ and the dual Hopf algebra $H_p$. The co-product of four momenta is given as

$$\Delta(p^0) = p^0 \otimes 1 + 1 \otimes p^0, \quad \Delta(p^i) = p^i \otimes e^{-p^0/\kappa} + 1 \otimes p^i. \quad (2)$$

Exponential operator $e^{-ip\hat{x}}$ is the basic ingredient which transforms the theory in space-time coordinates to the theory in momentum space. Here $\hat{x} = (\hat{x}^0, \mathbf{x})$, $p = (p^0, \mathbf{p})$ and $p \cdot \hat{x} \equiv p^0 \hat{x}^0 - \mathbf{p} \cdot \mathbf{x}$. Its ordering is defined as

$$e^{-ip\hat{x}} := e^{-ip^0\hat{x}^0} e^{i\mathbf{p} \cdot \mathbf{x}}.$$  

Multiplication of two ordered exponentials follows from Eqs. (1) and (3):

$$e^{-ip\hat{x}} e^{-iq\hat{x}} := e^{i\tilde{p} \cdot \mathbf{x}} e^{-i\tilde{q} \cdot \mathbf{x}}.$$  

which is indicated in the four momentum addition rule described by the coproduct (2).

To find the differential calculus one differentiates Eq. (1) and finds

$$[\tau^0, \hat{x}^i] + [\hat{x}^0, \tau^i] = \frac{i}{\kappa} \tau^i, \quad [\tau^i, \hat{x}^j] + [\hat{x}^j, \tau^i] = 0,$$

where $\tau^\mu = d\hat{x}^\mu$ is the exterior derivatives along the four dimensional space-time direction. One may choose the commutation relations

$$[\tau^0, \hat{x}^i] = \frac{i}{\kappa} \tau^i, \quad [\tau^i, \hat{x}^0] = 0, \quad (5)$$

and fix the unique differential structure through Jacobi’s identities:

$$[\tau^\mu, \hat{x}^\nu] = \frac{1}{4} \eta^{\mu\nu} \tau^5 - \frac{i}{\kappa} \eta^{\mu\nu} \tau^0 + \frac{i}{\kappa} \eta^{0\nu} \tau^\mu, \quad [\tau^5, \hat{x}^\mu] = -\frac{4}{\kappa^2} \tau^\mu. \quad (6)$$
This demonstrates that the differential calculus is not closed in 4-dimensions but needs a new exterior derivative along the fifth direction $\tau^5$. One may identify $\tau^5$ as

$$\tau^5 = [\tau^\mu, x_\mu] + \frac{3i}{\kappa} \tau^0.$$ 

The covariance property under the action of $\kappa$–deformed Poincaré group was established in Ref. [12, 14] and a five-dimensional bi-covariant differential calculus was considered in Ref. [12].

Using $\tau^A$ with $A = 0, 1, 2, 3, 5$, one writes the derivative of the exponential function as

$$d : e^{-ip\hat{x}} : = -i : \chi_A(p)e^{-ip\hat{x}} : \tau^A$$

where

$$\chi_0(p) = \kappa \left( \sinh \frac{p_0}{\kappa} + \frac{\vec{p}^2}{2\kappa^2} e^{p_0/\kappa} \right)$$

$$\chi_i(p) = p_i e^{p_0/\kappa}$$

$$\chi_5(p) = -\frac{i}{8} M_\kappa^2(p).$$

$M_\kappa^2(p)$ is the first Casimir operator of the algebra [29],

$$M_\kappa^2(p) = \left( 2\kappa \sinh \frac{p_0}{2\kappa} \right)^2 - \vec{p}^2 e^{p_0/\kappa},$$

which is invariant under the deformed Poincaré transformations. As $\kappa \to \infty$, this reduces to the commutative relation, $M_\infty^2(p) = p_0^2 - \vec{p}^2 = m^2$. For finite $\kappa$, the on-shell three-momentum is bounded from above by $\vec{p}^2 \leq \kappa^2$ [24] since

$$\frac{p_0^2}{\kappa^2} = 1 - \left( 2 + \frac{M_\kappa^2}{\kappa^2} \right) e^{-p_0/\kappa} + e^{-2p_0/\kappa}$$

and $M_\kappa^2$ is a non-negative constant. $p_0$ goes to infinity when the momentum reaches the maximal value $\kappa$. In addition, defining 5-dimensional momentum

$$\mathcal{P}_A = (\chi_0, \vec{\chi}, \kappa + 4i\chi_5/\kappa),$$

we realize that the momentum space constitutes a 4-dimensional de Sitter spacetime:

$$(\mathcal{P}_0)^2 - (\mathcal{P}_i)^2 - (\mathcal{P}_5)^2 = -\kappa^2.$$

### B. Scalar field representation

Scalar field in $\kappa$-Minkowski spacetime is represented using the $\kappa$-deformed Fourier transformation. Suppose $\hat{\Phi}(p)$ is a classical function in commuting four-momentum space. The scalar field on the $\kappa$-Minkowski spacetime is then defined as

$$\Phi(\hat{x}) = \int_p \hat{\Phi}(p) : e^{-ip\hat{x}} :$$

where $\int_p$ denotes $\int \frac{d^4p}{(2\pi)^4}$. This scalar field on the non-commuting coordinates smoothly reduces to the commuting case if $\kappa \to \infty$. The definition also allows us to transform the integration over non-commuting coordinates unambiguously into the ones with commuting momentum space integration if one uses the delta-function relation

$$\int e^{-ip\hat{x}} = \int \frac{d^4p}{(2\pi)^4} \delta^4(p) = \int \delta^4(p),$$

where $\int \delta^4(p)$ denotes $\int \frac{d^4\dot{x}}{2\pi^2}$. For example, the multiplication of fields integrated over $\dot{x}$ is expressed as

$$\int \Phi^2(\dot{x}) = \int \hat{\Phi}(p) \hat{\Phi}(-p^0, -p\epsilon^0/\kappa).$$
One may obtain a conjugate field if one defines the conjugation of ordered exponential as
\[
\left( e^{-ip\hat{x}} \right)^\dagger = e^{-ip\hat{x}\ast} e^{ip\hat{x}} = e^{-i(e^{po/\kappa})p\cdot \hat{x} + ip^0\hat{x}^0}.
\] (12)

With this, we have
\[
\left( \Phi^\dagger(\hat{x}) \right)^\dagger = \Phi(\hat{x}) , \quad \left( \Phi_1(\hat{x})\Phi_2(\hat{x}) \right)^\dagger = \Phi_2^\dagger(\hat{x})\Phi_1^\dagger(\hat{x}),
\] (13)

and the conjugate scalar field is given as
\[
\Phi^\dagger(\hat{x}) = \int_p \Phi(p)^\dagger \left( e^{-ip\hat{x}} \right)^\dagger = \int_p \Phi_c(p) : e^{-ip\hat{x}} :
\]
\[
\Phi_c(p) = e^{3p_0/\kappa}\Phi(-p_0, -e^{p_0/\kappa}p).
\] (14)

\(\Phi^\dagger(p)\) represents the ordinary complex conjugate of \(\Phi(p)\) for classical field and hermitian conjugate for quantum field. Likewise in Eq. (11), one may write the multiplication of fields as
\[
\int_{\hat{x}} \Phi_1(\hat{x})\Phi_2(\hat{x}) = \int_p \Phi_1^\dagger(p)\Phi_2(p)
\]
\[
\int_{\hat{x}} \left( \Phi_1(\hat{x})\Phi_2(\hat{x}) \right)^\dagger = \int_p \Phi_2^\dagger(\hat{x})\Phi_1^\dagger(\hat{x}) = \int_p \Phi_2^\dagger(-pe^{p_0/\kappa}, -p_0)\Phi_1^\dagger(p).
\] (15)

If the scalar field \(\Phi(\hat{x})\) is real, we need \(\Phi(\hat{x})^\dagger = \Phi(\hat{x})\). This, in turn gives \(\Phi_c(p) = \Phi(p)\). (Our definition differs from the one in Ref. [12]. There, \(\Phi_c(p)\) is replaced by \(\Phi_\dagger(p)\).)

The partial derivative of field is defined from the partial derivative of the exponential functions in Eq. (7),
\[
\hat{\partial}_\mu : e^{-ip\hat{x}} : = -i \chi_\mu(p) : e^{-ip\hat{x}} :.
\] (16)

The adjoint derivative \(\hat{\partial}^\dagger\) is obtained from the relation
\[
\int_{\hat{x}} \Phi_1(\hat{x})\hat{\partial}_\mu\Phi_2(\hat{x}) = \int_{\hat{x}} \left( \hat{\partial}_\mu^\dagger\Phi_1(\hat{x}) \right)\Phi_2(\hat{x})
\]
which leads to
\[
\hat{\partial}_\mu^\dagger : e^{-ip\hat{x}} : = -i \chi_\mu^\dagger(p) : e^{-ip\hat{x}} :.
\] (17)

\[
\chi_\mu^\dagger(p) = \chi_\mu(-p^0, -pe^{p_0/\kappa}).
\]

This results in a useful relation
\[
\left( \hat{\partial}_\mu\Phi(\hat{x}) \right)^\dagger = -\hat{\partial}_\mu^\dagger\Phi^\dagger(\hat{x}).
\] (18)

C. Free field action of a complex field

The free field action of a complex scalar field in \(\kappa\)–Minkowski spacetime can be written in analogy with the commutative one as
\[
S = \int_{\hat{x}} \left[ (\hat{\partial}_\mu^\dagger\Phi^\dagger(\hat{x}))(\hat{\partial}^\mu\Phi(\hat{x}) - m^2\Phi^\dagger(\hat{x})\Phi(\hat{x})) \right].
\] (19)

Here the fifth direction is omitted and evaluated in 4-dimensions only. This action has not SO(4, 1) invariance in [12] but the deformed Poincaré symmetry is respected. (See Eq. (20) below). This action can be written in momentum space representation by using the Fourier transform (9),
\[
S = \int_p \Phi^\dagger(p) \Delta_{\Phi}^{-1}(p) \Phi(p)
\]
\[
\Delta_{\Phi}^{-1}(p) = \left[ M_{\kappa}^2(p) \left( 1 + \frac{M_{\kappa}^2(p)}{4\kappa^2} \right) - m^2 + i\epsilon \right],
\] (20)
where $\epsilon$ is the usual small number prescription for the Feynman propagator. This action shows that the non-commutative spacetime modifies the propagator for the free fields defined in the commutative coordinate space

$$\phi(x) = \int_p e^{-ip \cdot x} \Phi(p).$$  \hfill (21)

The number of poles in $\Delta_F$ are, however, infinitely many on the complex momentum plane. Explicit pole positions are given as $\omega_n^\pm = \omega^\pm + in\kappa\pi$ with $n$ an arbitrary integer;

$$\omega^\pm = -\kappa \ln \left( \sqrt{1 + \frac{m^2 - i\epsilon}{\kappa^2}} + \sqrt{1 + \frac{m^2 + p^2 - i\epsilon}{\kappa^2}} \right).$$  \hfill (22)

For the massless case, $\omega^\pm$ reduces to

$$\omega^\pm = -\kappa \ln (1 \pm \frac{|p| - i\epsilon}{\kappa}).$$  \hfill (23)

$\omega^\pm$ represent the two stable on-shell spectra and due to the restriction $|p|^2 < \kappa^2$, $\omega^+$ is positive and $\omega^-$ is negative. The existence of the two stable spectra suggests that one may define the Minkowski vacuum $|0_M\rangle$ which reduces to commutative vacuum when $\kappa \to \infty$. Then the time-ordered product is given as

$$G_F(x, y) = \langle 0_M | T \phi(x) \phi^\dagger(y) | 0_M \rangle = \int_p e^{-ip \cdot (y-x)} i\Delta_F(p) = G_F(x - y),$$  \hfill (24)

while

$$\langle 0_M | T \phi(x) \phi(y) | 0_M \rangle = \langle 0_M | T \phi^\dagger(x) \phi^\dagger(y) | 0_M \rangle = 0.$$  

The asymmetry of the pole positions of the Feynman propagator in Eq. (20), $\omega^+ + \omega^- = -\kappa \ln (1 - p^2/\kappa^2) \neq 0$, demonstrates that the Lorentz symmetry is broken at the order of $1/\kappa$. It is natural to define $\omega^+$ as the particle spectrum and $\omega^-$ as the anti-particle one. In this case, the vacuum $|0_M\rangle$ does not respect the particle and antiparticle symmetry.

In addition, it is noted that when $\Delta x^0 > 0$, $G_F(\Delta x)$ creates not only the excitation with $\omega^+$ but also unstable ones with $\omega_n^+$ and $\omega_n^-$ with $n$ negative integers. Thus we may define particle spectra as $\omega^+$, $\omega_n^+$ and $\omega_n^-$ with $n < 0$. Similarly, when $\Delta x^0 < 0$, $G_F(\Delta x)$ creates the stable excitation with $\omega^-$ as well as unstable ones with $\omega_n^+$ and $\omega_n^-$ with $n$ positive integers, which indicates that $\omega^-$, $\omega_n^+$ and $\omega_n^-$ with $n > 0$ represent anti-particle spectra.

The positive Wightman function $W_+(x, y)$ is defined as

$$W_+(x, y) = \langle 0_M | \phi(x) \phi^\dagger(y) | 0_M \rangle.\tag{25}$$

$W_+(x, y)$ measures the amplitude to create particles including the unstable ones and is defined to be the Feynman propagator when $\Delta x^0 > 0$. Thus it can be represented as

$$W_+(x, y) = W_+(x - y) = \sum_{\omega = \omega^+, \omega_n^- < 0} \int \frac{d^3p}{(2\pi)^3 2|p|} \frac{e^{-i\omega(x_0 - y_0) + ip \cdot (x - y)}}{1 + M_n^2(\omega, p)/(2\kappa^2)}.\tag{26}$$

This result can be written formally as

$$W_+(\Delta x) = \int_{p^+} e^{-ip \cdot \Delta x} 2\pi \delta \left( M_n^2 \left( 1 + \frac{M_n^2}{4\kappa^2} \right) - m^2 \right).$$

Here $\int_{p^+}$ indicates that the integral over $p^0$ includes not only the real mass-shell position $\omega^+$ but also the complex ones, $\omega_n^+$ and $\omega_n^-$ with $n < 0$. The delta function integration is evaluated using the property

$$\frac{\partial M_n^2}{\partial p_0} \bigg|_{\omega^\pm} = \left[ \kappa \left( 1 - \frac{p^2}{\kappa^2} \right) e^{p_0/\kappa} - \kappa e^{p_0/\kappa} \right]_{\omega^\pm} = \pm 2|p|.$$  \hfill (27)

Likewise, the negative Wightman function $W_-(x, y)$ is defined as

$$W_-(x, y) = \langle 0_M | \phi^\dagger(y) \phi(x) | 0_M \rangle = W_-(x - y) = \sum_{\omega = \omega^-, \omega_n^+ > 0} \int \frac{d^3p}{(2\pi)^3 2|p|} \frac{e^{-i\omega(x_0 - y_0) + ip \cdot (x - y)}}{1 + M_n^2(\omega, p)/(2\kappa^2)}.\tag{28}$$
The explicit form of $W_{\pm}(\Delta x)$ for massless case is needed in the next section. When $\kappa|\Delta x^0| \gg 1$, the complex mass-shell contributions decay exponentially, representing the unstable excitations. As a result, among the infinitely many contributions to $W_{\pm}(\Delta x)$, the main contribution comes from the $\omega^+ \pm$ part. After the angular integrations and rescaling $|p|$ by $\kappa$, we have

$$
W_{\pm}(\Delta x) = \frac{\kappa}{8i\pi^2|\Delta x|}(g_{\pm}(\Delta x) - g_{\pm}(\Delta x)),
$$

$$
g_{\pm}(\Delta x) = \int_0^1 dz \, e^{ik[\log(1+z) - \Delta x_0 \mp |z|\Delta x]}.
$$

Using the result in Appendix A, we have, to the order of $1/\kappa$,

$$
W_{\pm}(\Delta x) = -\frac{1}{4\pi^2} \left\{ \xi - \frac{i\Delta x_0}{\kappa} \left( \frac{3(\Delta x_0)^2 + (\Delta x)^2}{\xi} \right) \right\}^{-1} + O(\kappa^{-2}),
$$

(30)

where $\xi = (\Delta x_0)^2 - (\Delta x)^2$. For $W_{-}(\Delta x)$, we have

$$
W_{-}(\Delta x) = \frac{\kappa}{8i\pi^2|\Delta x|}(h_{-}(\Delta x) - h_{-}(\Delta x))
$$

(31)

$$
h_{-}(\Delta x) = \int_0^1 dz \, e^{ik[\log(1+z) - \Delta x_0 \mp |z|\Delta x]}.
$$

To the order of $1/\kappa$, we have

$$
W_{-}(\Delta x) = -\frac{1}{4\pi^2} \left\{ \xi - \frac{i\Delta x_0}{\kappa} \left( \frac{3(\Delta x_0)^2 + (\Delta x)^2}{\xi} \right) \right\}^{-1} + O(\kappa^{-2}).
$$

(32)

Here we assume that $\kappa\Delta x_0 \gg 1$ and $\xi \gg 1$, even though $W_{\pm}(\Delta x)$ holds for $\xi \sim 0$ in the commutative limit. In addition, it is noted that the Lorentz symmetry breaking is reflected in this result since $W_{-}(\Delta x) \neq W_{+}(\Delta x)$ at the order of $1/\kappa$.

### III. PARTICLE DETECTOR INTERACTING WITH A MASSLESS SCALAR FIELD IN $\kappa$–MINKOWSKI SPACE

Suppose a particle detector is moving along a world line $X^\mu(\tau)$, where $\tau$ is the detector’s proper time. Unruh [17] calculated the response of the particle detector moving with a uniform acceleration: If the detector interacts with a free massless field, and the system initially lies in the Minkowski vacuum state, then the detector responds as if it were immersed in a heat bath in an un-accelerated frame whose temperature turns out to be $\text{acceleration}/2\pi$, the Unruh temperature. This effect is called the Unruh effect.

In the Minkowski spacetime, we will let the detector interact with a massless complex scalar field through the detector’s monopole moments, $\mathcal{M}(\tau)$ and $\mathcal{M}^\dagger(\tau)$. The interaction is written as

$$
S_I = c \int d\tau \int_x \delta^4(x - X(\tau)) \left( \mathcal{M}^\dagger(\tau) \phi(x) + \phi^\dagger(x) \mathcal{M}(\tau) \right)
$$

(33)

$$
= \int d\tau \left( \mathcal{M}^\dagger(\tau) \phi(X(\tau)) + \phi^\dagger(X(\tau)) \mathcal{M}(\tau) \right)
$$

where $c$ is a small coupling constant.

In the $\kappa$-Minkowski spacetime, the complex scalar field is affected by the non-commutative nature of the spacetime and its action is written as Eq. (20) with vanishing mass. For the detector part, on the other hand, we assume that it experiences the time evolution under the ordinary quantum mechanics while moving along the commutative world line $X^\mu(\tau)$.

The detector can be coupled to the field in various ways. To find the possibility we consider a $\kappa$-deformed delta function

$$
\delta^{(4)}(\hat{x} - X(\tau)) = \int_p e^{-ip(\hat{x} - X(\tau))} = \int_p e^{ipX(\tau)} e^{-ip\hat{x}}.
$$

(34)
Thus we need

\[ M(0) = \int e^{i p \cdot X(\tau)} \Phi(\hat{x}) \Phi(\hat{x}) \]

This delta function gives the property

\[ \int \Phi^{\dagger}(\hat{x}) \delta^{(4)}(\hat{x} - X(\tau)) = \int \Phi^{\dagger}(p) e^{i p \cdot X(\tau)} = \Phi^{\dagger}(X(\tau)). \]  

However, the $\kappa$-deformed delta function is not self-conjugate, since

\[ \delta^{(4)}(\hat{x} - X(\tau)) = \int e^{i p \cdot X(\tau)} \left( e^{-i p \cdot \hat{x}} : \right) = \int e^{i p_{0}/\kappa e^{i p_{0}/\kappa} X(\tau) - i e^{p_{0}/\kappa} \hat{x} X(\tau)} : e^{-i p \cdot \hat{x}} : \neq \delta(\hat{x} - X(\tau)). \]

Thus we need

\[ \Phi(X(\tau)) = \int \delta^{(4)}(\hat{x} - X(\tau)) \Phi(\hat{x}). \]  

Thanks to this property of the delta function, we may rewrite the interaction Eq. (33) for the interaction in the $\kappa$-Minkowski spacetime as

\[ S_I = c \int d\tau \left[ M^{\dagger}(\tau) \int \Phi^{\dagger}(p) e^{-i p \cdot X(\tau)} + M(\tau) \int \Phi^{\dagger}(\hat{x}) \delta^{(4)}(\hat{x} - X(\tau)) \right]. \]  

This choice is the simplest one, in the sense that complicated coproduct of the $\kappa$-Minkowski spacetime does not appear in the interaction term. There exist other choices of interaction, which may give more complicated non-commutative effect but this possibility is not considered in the present paper.

Suppose the detector lies initially in its ground state $|E_{0}\rangle$ of a quantum mechanical hamiltonian $H_{0}$. As the detector moves along a trajectory, it will in general find itself undergoing a transition from the initial to an excited state $|E\rangle$ with $E > E_{0}$. In addition, the field might be affected by the detector since the field is coupled to the detector and there will be a field contribution to the transition.

We assume for simplicity that the field initially is in the ground state $|0_M\rangle$, the vacuum state with respect to the Minkowski spacetime and finally makes a transition to an excited state $|\phi\rangle$, which may be assumed to be a one particle state. Then we expect the whole transition amplitude of the system $A_{f_{\tau-i}}$ to be evaluated by the first order perturbation theory:

\[ A_{f_{\tau-i}} = i c(E, \phi) \int_{-\infty}^{\tau_{0}} d\tau \left[ M^{\dagger}(\tau) \int \Phi^{\dagger}(p) e^{-i p \cdot X(\tau)} + M(\tau) \int \Phi^{\dagger}(\hat{x}) \delta^{(4)}(\hat{x} - X(\tau)) \right] |0_M, E_{0}\rangle \]  

where $\tau_{0}$ is the time when the detector and the field reach the final state $|\phi, E\rangle$. Using the time evolution of the monopole moment operator $M(\tau)$

\[ M(\tau) = e^{i H_{0} \tau} M(0) e^{-i H_{0} \tau}, \quad M^{\dagger}(\tau) = e^{i H_{0} \tau} M^{\dagger}(0) e^{-i H_{0} \tau}, \]  

$A_{f_{\tau-i}}$ may factorize to give

\[ A_{f_{\tau-i}} = \int_{-\infty}^{\tau_{0}} d\tau e^{i(E-E_{0})\tau} \int P e^{-i p \cdot X(\tau)} \langle \phi | \Phi(p) | 0_M \rangle \]

where $\gamma = 1/\sqrt{1 - v^{2}/c^{2}}$. Then the integral in $[40]$ with $\tau_{0} \to \infty$ becomes

\[ A_{f_{\tau-i}} = i c(E, M(0)) \int_{-\infty}^{\tau_{0}} d\tau 2 \pi \delta \left( E - E_{0} - \gamma (p_{0} - v \cdot \mathbf{v}) \right) e^{i p \cdot X} \langle \phi | \Phi(p) | 0_M \rangle \]

The expectation values $\langle \phi | \Phi(p) | 0_M \rangle$ ($\langle \phi | \Phi^{\dagger}(p) | 0_M \rangle$) is nonzero only for the case of $p_{0} < 0$ ($p_{0} > 0$) as in the commutative case. In addition, since $E > E_{0}$ and $|p_{0}| \geq p \cdot \mathbf{v}$, the delta functions make $A_{f_{\tau-i}}$ vanish, which reflects the fact that the $\kappa$-Minkowski spacetime has the time-translational invariance.
To explore the case when the detector follows a uniformly accelerating path with acceleration $\alpha$, whose coordinates are chosen as
\[X^0(\tau) = \alpha^{-1} \sinh \alpha \tau, \quad X^1(\tau) = \alpha^{-1} \cosh \alpha \tau, \quad X^2(\tau) = 0 = X^3(\tau), \tag{41}\]
we consider the transition probability,
\[|A_{f+1}|^2 = c^2 \sum_{E} \left( M_+(E, E_0) F_+(E - E_0) + M_-(E, E_0) F_-(E - E_0) \right), \tag{42}\]
where we sum over intermediate states using the completeness of the basis \{ | \phi \rangle \} and
\[M_\pm(E, E_0) = \frac{1}{2} \left( |\langle E | M^\dagger(0) | E_0 \rangle|^2 \pm |\langle E | M(0) | E_0 \rangle|^2 \right). \]
Note that $M_+(E, E_0)$ is always larger than $|M_-(E, E_0)|$. $F_\pm$ is the part due to the response of the field and is given by
\[F_\pm(E) = \int_{-\infty}^{\tau_0} d\tau f_\pm(X(\tau), X(\tau')), \]
where
\[f_\pm(X(\tau), X(\tau')) = \langle 0_M | \left[ \Phi(X(\tau)) \Phi^\dagger(X(\tau')) \mp \Phi^\dagger(X(\tau)) \Phi(X(\tau')) \right] | 0_M \rangle \]
\[= W_+(X(\tau) - X(\tau')) \mp W_-(X(\tau') - X(\tau)) = f_\pm(X(\tau) - X(\tau')).\]
In the Lorentz invariant systems such as in the commutative field theory, $F_-(E)$ vanishes since $W_+(\Delta X) = W_-(\Delta X)$. In our case, $F_-(E)$ does appear at the order of $1/\kappa$.

Excitation rate $R(\tau_0, E)$ is the rate of the transition probability,
\[R(\tau_0, E) = \frac{d|A_{f+1}|^2}{d\tau_0} = c^2 \sum_{E} \left( M_+(E, E_0) S_+(\tau_0, E) + M_-(E, E_0) S_-(\tau_0, E) \right), \tag{43}\]
where $S_\pm(\tau_0, E)$ is called the response function \textsuperscript{26} representing the Unruh effect:
\[S_\pm(\tau_0, E) = \frac{dF_\pm(E)}{d\tau_0} = \int_{-\infty}^{\tau_0} d\tau e^{iE(\tau - \tau_0)} f_\pm(X(\tau_0), X(\tau)) + \int_{-\infty}^{\tau_0} d\tau e^{-iE(\tau - \tau_0)} f_\pm(X(\tau), X(\tau_0)). \tag{44}\]
Shifting $\tau$ by $\tau_0$ we have
\[S_\pm(\tau_0, E) = \int_{-\infty}^{0} d\tau e^{iE\tau} f_\pm(X(\tau_0), X(\tau + \tau_0)) + \int_{-\infty}^{0} d\tau e^{-iE\tau} f_\pm(X(\tau + \tau_0), X(\tau_0)) \tag{45}\]
\[= \int_{-\infty}^{\infty} d\tau e^{-iE\tau} f_\pm(X(\tau_0 - |\tau|/2 + \tau/2), X(\tau_0 - |\tau|/2 - \tau/2)). \]
For the massless case, $f_\pm(\Delta X)$ is given by
\[f_+(\Delta X) = -\frac{1}{2\pi^2 \xi} + O(\kappa^{-2}), \tag{46}\]
\[f_-(\Delta X) = -\frac{i}{2\pi^2 \kappa} \frac{\Delta X^0 \left( 3(\Delta X^0)^2 + (\Delta X)^2 \right)}{\xi^3} + O(\kappa^{-2}). \tag{47}\]
Along the uniformly accelerating path in Eq. \textsuperscript{411}, $\Delta X = X(\tau) - X(\tau')$ is parameterized as as
\[\Delta X^0 = \frac{2}{\alpha} \sinh \left( \frac{\alpha(\tau - \tau')}{2} \right) \cosh \left( \frac{\alpha(\tau + \tau')}{2} \right), \tag{48}\]
\[\Delta X^1 = \frac{2}{\alpha} \sinh \left( \frac{\alpha(\tau - \tau')}{2} \right) \sinh \left( \frac{\alpha(\tau + \tau')}{2} \right), \]
\[\xi = (\Delta X^0)^2 - (\Delta X^1)^2 = \frac{4}{\alpha^2} \sinh^2 \left( \frac{\alpha(\tau - \tau')}{2} \right),\]
and we have
\[
\begin{align*}
 f_+ (\Delta X) &= -\frac{\alpha^2}{8\pi^2} \left\{ \frac{1}{\sinh \frac{\alpha (\tau - \tau')}{2}} \right\} + O \left( \frac{1}{\kappa^2} \right), \\
f_- (\Delta X) &= \frac{i \alpha^3}{8\pi^2 \kappa} \left\{ \frac{4 \cosh^3 \frac{\alpha (\tau + \tau')}{2} - \cosh \frac{\alpha (\tau + \tau')}{2}}{\sinh^3 \frac{\alpha (\tau - \tau')}{2}} \right\} + O \left( \frac{1}{\kappa^2} \right).
\end{align*}
\] (49)

Thus \( f_\pm \) in Eq. (45) is given explicitly in terms of the proper-time parametrization
\[
\begin{align*}
 f_+ (X((\tau_0 - |\tau|/2 + \tau/2), X(\tau_0 - |\tau|/2 - \tau/2)) &= -\frac{\alpha^2}{8\pi^2} \left\{ \frac{1}{2 \sinh \frac{\alpha \tau}{2}} \right\} + O(\kappa^{-2}), \\
f_- (X((\tau_0 - |\tau|/2 + \tau/2), X(\tau_0 - |\tau|/2 - \tau/2)) &= -\frac{i \alpha^3}{8\pi^2 \kappa} \left\{ \frac{\cosh 3\alpha \tau_0 \cosh \frac{3\alpha \tau}{2} + 2 \cosh \alpha \tau_0 \cosh \frac{\alpha \tau}{2}}{\sinh^{3} \frac{\alpha \tau}{2}} \right\}
\end{align*}
\]
\[\epsilon(t) \frac{\sinh 3\alpha \tau_0 (2 \cosh \alpha \tau + 1) + 2 \sinh \alpha \tau_0}{\sinh^{2} \frac{\alpha \tau}{2}} \right\} + O(\kappa^{-2}).
\]

FIG. 1: The distribution functions \( s_{BE}(2E/\alpha) \) and \( s(2E/\alpha) \).

We evaluate the response function using contour integration. To do this we need to detour the contour so that we exclude the singularity around \( \tau = 0 \). Using the contour integration result given in Appendix, we finally have
\[
\begin{align*}
 S_+ (\tau_0, E) &= \frac{\alpha}{2\pi} s_{BE}(2E/\alpha) + O \left( \frac{1}{\kappa^2} \right), \\
 S_- (\tau_0, E) &= -\frac{\alpha}{2\pi} \kappa \left\{ \left[ \frac{\alpha}{2E} \left( 9 \cosh(3\alpha \tau_0) + 2 \cosh \alpha \tau_0 \right) - \frac{2E}{\alpha} \left( \cosh(3\alpha \tau_0) + 2 \cosh(\alpha \tau_0) \right) \right] s_{BE}(2E/\alpha) \right. \\
&\left. + \frac{1}{\pi} \left[ 3 \sinh(3\alpha \tau_0) + 2 \sinh(\alpha \tau_0) \right] s(2E/\alpha) \right\} + O \left( \frac{1}{\kappa^2} \right),
\end{align*}
\] (50)

where
\[
\begin{align*}
 s_{BE}(\zeta) &= \frac{\zeta}{e^{\pi \zeta} - 1}, \\
 s(\zeta) &= \int_0^\infty \frac{dk}{k} \left( \frac{\zeta |k-1|}{e^{\pi \zeta |k-1|} - 1} - \frac{\zeta |k+1|}{e^{\pi \zeta |k+1|} - 1} \right).
\] (52)

The distribution function \( s_{BE}(2E/\alpha) \) is the Bose-Einstein one, finite at \( E = 0 \) and decays exponentially for large \( E \). On the other hand, \( s(2E/\alpha) \) shows no definite statistics but fuzziness, vanishes at \( E = 0 \), and decreases slowly as \( s(2E/\alpha) \sim \alpha/(4\pi^2 E) \) for large \( E \). The behaviors of \( s_{BE} \) and \( s \) are plotted in Fig. 1.

It is to be noted that \( S_+ \) only reproduces the commutative result and presents no \( O(1/\kappa) \) correction. Lorentz symmetry breaking appears in \( S_- \) at \( O(1/\kappa) \). Here the correction term shows the preferred-frame effect, the dependency of the detector time \( \tau_0 \).
Since the transition probability in Eq. (43) also depends on how the detector couples to the complex scalar field, one may tune the detector without modifying the field theory of the massless complex scalar field. Thus if one tune the detector so that $M$ is hermitian ($M = M^\dagger$), then the detector does not see the Lorentz violation at $O(1/\kappa)$. Since $M_+(E, E_0) \geq |M_-(E, E_0)|$ and $M_-$ is sensitive to the violation of the Lorentz symmetry, one may consider the detector with $M_+(E, E_0) = M_-(E, E_0)$, maximally sensitive to the violation of Lorentz symmetry.

In this maximal case, the response function is given by

$$S_{\text{max}}(E, \tau_0) = S_+(E, \tau_0) + S_-(E, \tau_0).$$

(53)

An explicit example of the response function with the restrictions given in Eqs. (55) and (56) below is plotted in Fig. 2. As seen in the figure, the response function deviates slightly from the Bose-Einstein result.

FIG. 2: Plot of the response function $S_{\text{max}}(\tau_0, E)$. $\alpha$ is set as $5 \cdot 10^{-5} \kappa$. The shaded curve represents the commutative case, the Bose-Einstein result. The solid curves denote the response function at time $\alpha \tau_0 = 0$, $\alpha \tau_0 = 1$, $\alpha \tau_0 = 2$, respectively, from top to bottom. At each time scale, the curves are shown for the valid energy range of the relevant perturbation result.

The Lorentz symmetry breaking results in $\tau_0$ dependence and the excitation rate depends on the proper-time. At $\tau_0 \sim 0$, $S_-$ is given by $s_{BE}$ term only since $\sinh(\alpha \tau_0) = 0$. Since the $O(1/\kappa)$ correction term must be smaller than the $O(\kappa^0)$ term, we have the range of validity of the results (50) and (51),

$$\frac{\alpha^2}{\kappa} \ll E \ll \kappa.$$ (54)

The maximal response function becomes

$$S_{\text{max}}(\tau_0 = 0, E) \simeq \frac{E}{\pi} e^{2\pi E/\alpha} \left[\frac{1}{1 - \frac{\alpha}{2\kappa} \left(\frac{11\alpha}{2E} - \frac{6E}{\alpha}\right)}\right].$$

At $\alpha \tau_0 \sim O(1)$, the infrared part of $S_-$ is dominated by $s_{BE}$ term and the ultraviolet part is dominated by $s$ term. For $1 \ll \alpha \tau_0 < \alpha \tau_c = (\ln(2\kappa/(3\alpha)))/3$, the proper time contribution becomes large and thus the range of validity is limited as

$$\frac{9\alpha}{2\kappa} e^{3\alpha \tau_0} \ll \frac{2E}{\alpha} \ll \frac{2\kappa}{\alpha} e^{-3\alpha \tau_0}.$$ (55)

(At $\tau_0 = \tau_c$, the left and right-hand side become equal and the range is empty). In the UV part of the spectrum, another restriction appears by comparing the behavior of $s(2E/\alpha)$ and $s_{BE}(2E/\alpha)$:

$$\left(\frac{\alpha}{2E}\right)^2 e^{2\pi E/\alpha} \ll \frac{4\pi^3 \kappa}{3\alpha} e^{-3\alpha \tau_0}.$$ (56)

When $\tau_0 \gg \tau_c$, we cannot use the result of $W_{\pm}$ in Eqs. (50) and (52). This is because after this time, if $\tau \sim \tau_0 \rightarrow \infty$, the four vector $X(\tau) - X(\tau_0)$ is not time-like but becomes almost light-like. Therefore, the series expansion used in Appendix is not valid anymore. In the limit, $|X(\tau) - X(\tau_0)| \rightarrow 0$ with $X(\tau_0) \rightarrow \infty$, the coefficient $(a - b)$ in the
integral Eq. \(\text{[A1]}\) in Appendix vanishes. Therefore, the integration becomes a Gaussian-like integral proportional to \(\int_0^\infty d\mu e^{-\kappa X \mu^2/2}\). After explicit evaluation, the Wightman function \(W_+(X(\tau), X(\tau_0))\) can be shown to be

\[
W_+(X(\tau), X(\tau_0)) \propto \sqrt{\kappa} |X(\tau) - X(\tau_0)|^{-3/2} = \sqrt{\kappa} \left[ \frac{2}{\alpha} \sinh \frac{\alpha(\tau - \tau_0)}{2} \sinh \frac{\alpha(\tau + \tau_0)}{2} \right]^{-3/2}.
\]

(57)

In addition, for time-like interval of \(|X(\tau) - X(\tau_0)|\) when \(|\tau_0 - \tau| \to \infty\), \(W_+(X(\tau), X(\tau_0))\) is roughly of the form, \(\frac{\alpha^2}{8\pi^2} \sinh^{-2}(\alpha(\tau - \tau_0)/2)\). Using these result, one may write the long-time behavior of the response function as

\[
S(\tau_0, E) \propto e^{-3\alpha \tau_0/2} \quad \text{when} \quad \tau_0 \gg \tau_c.
\]

(58)

This implies that the uniformly accelerating detector is not excited by the interaction of the scalar field in \(\kappa\)-Minkowski vacuum when \(\tau_0 \gg \tau_c\). This result can be understood as follows: The Wightman function measures the correlation of the field between the two points. If the detector is uniformly accelerating, and if two points are connected by almost light-like path, the energy difference between two points become very large even for the case of small separation (or momentum difference), because of the dispersion relation \([\text{S}]\) between the momentum and energy. The difference of the oscillation frequency between the two points may be very large even if the momentum difference of the two is small. Therefore, the correlation between the two points decays very fast, due to the fast oscillation (difference) of the energy eigenfunction.

This indicates that the \(\kappa\)-deformation completely alters the vacuum structure of uniformly accelerating observers. In the simplest model in \(\kappa\)-Minkowski spacetime, the (fuzzy) thermal spectrum of the Minkowski vacuum is seen to a uniformly accelerating observer for a long but finite duration of proper time, \(\tau_c \sim (\ln(\kappa/\alpha))\alpha\). After this time, the response function exponentially decays to zero.

### IV. SUMMARY

We have studied a uniformly accelerating particle detector, interacting with massless complex scalar field in \(\kappa\)-Minkowski spacetime. Introducing the simplest interaction of the detector with the scalar field we calculated the response function using the first order perturbation theory. The correction term depends on the proper time of the detector as a result of the Lorentz symmetry breaking effect.

One may devise a maximally sensitive detector whose response function gradually deviates from the Bose-Einstein result as the proper time increases. This deviation of the response function is much enhanced when the proper time approaches to a critical time \(\tau_c \sim (\ln\kappa/\alpha)/\alpha\). This critical time is of the order of the logarithm of the Plank mass after which the temperature of the Minkowski vacuum state seen by the observer decreases to zero.

It may have some implication to blackhole radiation problem. However small the spacetime noncommutativity is, it can nullify the thermal spectrum of the blackhole eventually. This implies that we need to examine the blackhole radiation problem carefully if there is a spacetime non-commutativity with \(\kappa\)-deformation. Note, however, that the present result is not a conclusive one since we have used the first order perturbation theory in obtaining the response function. To have better understanding on this feature one must incorporate higher order perturbations or try non-perturbative calculations. In addition, one may has to incorporate the coordinates of the accelerating observer having the symmetry consistent with the spacetime non-commutativity, \([\hat{t}, \hat{x}^i] = i\hat{x}^i/\kappa\). This will be considered in future publications.

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Appendix A: Integration

The integrations relevant for the calculation of the response function are

\[ I(a, b) = \int_0^1 dz \ e^{i(\log(1-z)a + zb)} \]  
\[ J(a, b) = \int_0^1 dz \ e^{i(\log(1+z)a + zb)}. \]

Putting \( \log(1 - z) = -t \), we can write \( I(a, b) \) as

\[ I(a, b) = \int_0^\infty dt \ e^{-t-i(ta-b(1-e^{-t}))}. \]

The integration has the main contribution around \( t \sim 0 \). Thus we may expand \( (1-e^{-t}) \) around \( t = 0 \), if \( a - b \neq 0 \):

\[ I(a, b) = \int_0^\infty dt \ e^{-t-it(a-b)} \times e^{ib(-\frac{t^2}{2} \cdots)} \]
\[ = \frac{1}{1 + i(a-b)} - \frac{ib}{(1 + i(a-b))^3} + \cdots. \]

If \( a \) and \( b \) are order of \( \kappa \), we may rescale \( a \) and \( b \) to \( \kappa a \) and \( \kappa b \), and expand the result in \( 1/\kappa \) as,

\[ I(\kappa a, \kappa b) = -\frac{i}{\kappa(a-b)} + \frac{a}{\kappa^2(a-b)^3} + \cdots. \]

Likewise for \( J(a, b) \) we have

\[ J(a, b) = \int_0^1 dz \ e^{i((z^{\frac{3}{2}} \cdots) a + zb)} \]
\[ = \frac{e^{i(a+b)} - 1}{i(a+b)} - \frac{a}{2(a+b)} \frac{e^{i(a+b)}}{a+b^2} + \frac{a(e^{i(a+b)} - 1)}{(a+b)^3} + \cdots. \]

Rescaling \( a \) and \( b \) to \( \kappa a \) and \( \kappa b \), we have

\[ J(\kappa a, \kappa b) = -\frac{i}{\kappa(a+b)} - \frac{a}{\kappa^2(a+b)^3} + \cdots. \]

Finally, we list some integral formulas needed in Sec. III:

\[ \int_{-\infty}^{\infty} d\tau \ \frac{e^{-i\zeta \tau}}{\sinh^2(\tau - i\epsilon)} = -2\pi \zeta \sum_{n=1}^{\infty} e^{-i\zeta \tau_n} = -\frac{2\pi \zeta}{e^{\pi \zeta} - 1}, \]
\[ \int_{-\infty}^{\infty} d\tau \ \frac{e^{-i\zeta \cosh(k \tau)}}{\sinh^2(\tau - i\epsilon)} = -\pi i((k^2 - 1) - \zeta^2) \sum_{n=1}^{\infty} e^{-i\zeta \tau_n} = -\frac{\pi i((k^2 - 1) - \zeta^2)}{e^{\pi \zeta} - 1}, \]
\[ \int_{-\infty}^{\infty} d\tau \ \frac{e^{-i\zeta \varepsilon(\tau)}}{\sinh^2(\tau)} = \frac{1}{2\pi i} \int_{-\infty}^{\infty} dk \int_{-\infty}^{\infty} d\tau \ \frac{e^{-i\zeta \tau}}{\sinh^2(\tau)} \left( \frac{e^{ik \tau} - e^{-ik \tau}}{k - i\epsilon} \right) \]
\[ = i \int_{-\infty}^{\infty} dk \left( \frac{|k - \zeta|}{e^{\pi|k-\zeta|} - 1} - \frac{|k + \zeta|}{e^{\pi|k+\zeta|} - 1} \right) \]
\[ = 2i \int_{0}^{\infty} dk \left( \frac{|k - \zeta|}{e^{\pi|k-\zeta|} - 1} - \frac{|k + \zeta|}{e^{\pi|k+\zeta|} - 1} \right). \]
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