Steady State Drifting Controller for Rear-Wheel Independent Driving Electric Vehicles

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Abstract. The use of independent driving electric vehicles for extreme sports is a matter to consider. This paper presents a steady-state drift controller for rear-wheel independent driving electric vehicles. The controller is based on linear quadratic regulator technology. A drift equilibrium condition calculator is designed based on the longitudinal force pre-distribution of rear tires and the Pacejka tire model. The results are used as the system reference input. Finally, the CarSim-MATLAB/Simulink co-simulation is carried out to verify the controller.

1. Introduction

Drifting is a kind of extreme driving mastered by expert drivers and often appears in off-road rally. The driver controls the throttle and the steering wheel to make the driving wheels of the vehicle reach the limit of adhesion, so that the vehicle can slide sideways while maintaining the stability of the vehicle. Drift has three significant characteristics: large side slip angle, counter-steering, and rear wheel saturation[1].

Many previous researches on vehicle drifting have focused on centralized driving vehicles. R. Chaichaowarat et al. designed a drift control strategy for rear-wheel-drive (RWD) vehicles using a modified tire model[2]. A kind of drift control system based on single-track model is designed. The control inputs are the steering angle and driving torque to control the vehicle speed and stabilize the yaw rate and the side slip angle. Because the system is underactuated, the control system is very complicated [3], and some studies have used a four-wheel models for controller design [4].

In recent years, many scholars have studied the control technologies for independent driving electric vehicles. For example, Direct Yaw Moment Control (DYC) and Active Steering, the stable driving space is defined in a small range to avoid the tire to reach the adhesion limit. The control algorithm limits the side slip angle within a small range (eg.6°) to maintain vehicle stability [5-6].

In this paper, considering that the lateral force and the longitudinal force of the vehicle are coupled under the extreme condition, a drift controller for the rear wheel independently driving electric vehicle is designed. Firstly, based on the pre-distribution of longitudinal force, the drift balance value is calculated, and the result is taken as the reference value. The outputs of the controller are the steering angle and independent rear wheel drive torques. Three state quantities are tracked and avoid the system underactuation. The steering angle is obtained through vehicle model inversion [7]. Finally, the effectiveness of the control strategy was verified by CarSim-MATLAB/Simulink co-simulation.
2. Vehicle Model

2.1. Vehicle states and Dynamics
In this section, a four-wheel vehicle dynamic model, including the chassis longitudinal, sideslip and yaw motion as well as the rotations of wheels, is introduced as displayed in Figure 1. The definition of the symbols in the model is displayed in Table 1.

The equations of the vehicle state are as follows:

\[
\dot{\beta} = \frac{1}{m v_x} \left( (F_{fx} + F_{frx}) \sin \delta + (F_{fly} + F_{fry}) \cos \delta + F_{rly} + F_{rry} \right) - \gamma \tag{1}
\]

\[
v_x = \frac{1}{m} \left[ (F_{fx} + F_{frx}) \cos \delta - (F_{fly} + F_{fry}) \sin \delta + F_{rly} + F_{rry} \right] + v_x \beta \gamma \tag{2}
\]

\[
\dot{\gamma} = \frac{1}{l_z} \left[ (F_{frx} - F_{fx}) \cos \delta \frac{d}{2} + (F_{rrx} - F_{rrx}) \frac{d}{2} + l_f (F_{frx} + F_{fx}) \sin \delta \right]
\]

\[
+ l_r (F_{fly} + F_{fry}) \cos \delta - l_r (F_{rly} + F_{rry}) + (d_f (F_{fly} - F_{fry}) \sin \delta) / 2 \right] \tag{3}
\]

\[
I_w \omega_{ij} = T_{ij} - F_{ijx} R, \quad i = f, r, j = l, r \tag{4}
\]

![Vehicle plane motion model](image)

**Figure 1. Vehicle plane motion model**

| Symbol | Definitions |
|--------|-------------|
| CG     | vehicle mass center |
| $F_{fij}$ | longitudinal force of the tire |
| $F_{rjy}$ | lateral force of the tire |
| $l_w$ | rotational inertia of the wheel |
| R | wheel radius |
| $T_{ij}$ | motor torque on the wheel |
| $\omega_{ij}$ | wheel rotational speed |
| $v_x$ | longitudinal velocity |
| r | yaw rate |
| $\beta$ | sideslip angle |
| $\delta$ | steering wheel angle |

2.2. Front tire for Modeling
The Pacejka magic model is used to model front tire. It is a sophisticated and reliable empirical tire model, and is widely used. The magic formula for the lateral force of the front tire is as follows:

\[
F_y(\alpha) = D \sin(C \tan^{-1}(B(1 - E)\alpha + E \tan^{-1}(B\alpha))) \tag{5}
\]
Where $B$ is the stiffness factor, $C$ is the shape factor, $D$ is the peak factor, and $E$ is the curvature factor.

To simplify the calculation process, we rewrite the expression.

$$F_y(\alpha) = D \sin(C \tan^{-1}(B\alpha)) \quad (6)$$

2.3. Rear tire for Modeling

When the vehicle is drifting, the rear tire force reaches the adhesion limit. All the friction that can be generated by the tire is used. The lateral force and the longitudinal force are coupled.

$$F_{rjy} = \pm \sqrt{(\mu F_{rjx})^2 - F_{rjx}^2}, \quad j = l, r \quad (7)$$

Where $\mu$ is the coefficient of friction, $F_{rjx}$ is the rear tire load force.

3. Equilibrium Analysis

In this section, the equilibrium of the vehicle is analyzed based on the pre-distribution of the longitudinal force of the rear tire, and also designs a equilibrium condition calculator. It is helpful to understand the state and property of drifting equilibrium, to study the method of achieving stable drifting, and to understand the relationship between input and vehicle state.

3.1. Longitudinal Force Pre-distribution

Load transfer has a great influence on drift motion. The greater the vertical load of the tire, the larger the friction circle of the tire, and the greater the tire force available. Based on the vertical load, the longitudinal force of the tire is pre-distributed, and the static load, longitudinal and transverse acceleration of the vehicle are taken into account. After the new vertical load of the tire is calculated, the longitudinal force of the left and right wheels is pre-distributed. Because the vehicle is driven independently by the rear wheels, the load transfer of the front wheels is not considered.

$$F_{rlx} = \frac{mg_f}{2(l_f + l_r)} + \frac{ma_x}{2(l_f + l_r)} - \frac{mha_y}{2d}, F_{rrz} = \frac{mg_f}{2(l_f + l_r)} + \frac{ma_x}{2(l_f + l_r)} + \frac{mha_y}{2d} \quad (8)$$

Where $g$ is the gravitational acceleration, $h$ is the height of the center of mass, $a_x$ is the Longitudinal acceleration, and $a_y$ is the lateral acceleration.

The left and right lateral forces are distributed as follows:

$$\frac{F_{rlx}}{F_{rrx}} = \frac{F_{rlz}}{F_{rrz}} \quad (9)$$

3.2. Equilibrium Condition Calculator Design

For the nonlinear vehicle model, the state space $z = f(z, u)$, with $z$ is the state vector and $u$ is the input vector, the equilibrium point $(z^{eq}, u^{eq})$ makes the differential of the state vector equal to zero: $f(z^{eq}, u^{eq}) = 0$. To complete the equilibrium analysis, we set the longitudinal velocity to a fixed value, and grid the steering angle. The front longitudinal tire forces are ignored because no torque is applied to the front wheel and the two front wheel forces are combined to form the front axle forces.

$$F_{fjy} + F_{frjy} = F_{fjy} \quad (10)$$

The front lateral tire force and rear lateral tire force follow the tire models given in Eq.(6-7), described in the previous section. Use the Matlab nonlinear solver to solve algebraic equations given in Eq. (9-13) finally. Calculate other equilibrium values of steering angle in the range -20deg ~ 15 deg.

$$0 = \frac{1}{mv_x^{eq}}(F_{fjy}^{eq} \cos \delta^{eq} + F_{frjy}^{eq} + F_{rrjy}^{eq}) - \nu^{eq} \quad (11)$$

$$0 = \frac{1}{I_z} \left( F_{frjy}^{eq} - F_{rjy}^{eq} \frac{d}{2} + l_f F_{fjy}^{eq} \cos \delta^{eq} - l_r (F_{frjy}^{eq} + F_{rrjy}^{eq}) \right) \quad (12)$$
\[ 0 = \frac{1}{m} \left( F_{rrx}^{eq} + F_{rlx}^{eq} - F_{fy}^{eq} \sin \delta^{eq} \right) + v_{x}^{eq} \beta^{eq} \gamma^{eq} \]  

\hfill (13)

The right-hand drifting is ignored because it works the same way as left-hand drifting except in the opposite direction. Figure 2 shows the equilibrium values under the normal steering and the left-hand drifting conditions. Figure 2(a) shows that the sideslip angle is within a very small range for normal steering (black line). When the vehicle is drifting, the sideslip angle can become very large (red line). Figure 2(b) shows that the yaw rate increases with the increase of the steering angle under conventional steering conditions. The Yaw rate can be maintained at a large positive value when drifting to the left.

During the drift, the vehicle has a large range of motion and more load transfer. Figure 2(c) shows the difference of the load on the two rear wheels during the drifting process. According to the rule that the larger the load, the larger the pre-distributed longitudinal force, figure 2(d) shows that the longitudinal force of the right tire is larger than that of the left tire under the condition of the left drift.

4. Controller Design

This section describes a drift controller using the linear quadratic regulator (LQR) method. In the design of the controller, the system is linearized at the equilibrium position, which is derived from the equilibrium calculation in the previous section. \( z^{eq}(\beta^{eq}, r^{eq}, v^{eq}) \) is the equilibrium state variable of the system, \( u^{eq}(F_{yy}^{eq}, F_{rlx}^{eq}, F_{rrx}^{eq}) \) is the equilibrium input.

\[ \Delta \dot{z} = A \Delta z + B \Delta u \]  

\hfill (14)

Where \( \Delta z = z - z^{eq} \) and \( \Delta u = u - u^{eq} \).

Jacobian matrix A and B are given as follows:
The LQR controller minimizes following quadratic cost function.

$$J = \int_{t=0}^{\infty} (\Delta z^T Q \Delta z + \Delta z^T R \Delta u) \, dt$$  \hspace{1cm} (16)

The Q and R matrices are positive definite matrices.

Compute LQR control policy.

$$u = u_{eq} + \Delta u$$  \hspace{1cm} (17a)

$$\Delta u = -R^{-1} B^{-1} P \Delta u = -k \Delta z$$  \hspace{1cm} (17b)

The matrix P can be obtained by solving the following equation.

$$A^T P + P A - P B R^{-1} B^T P + Q = 0$$

Finally, the controller input $F_{fy}$ can not be directly used as a standard input command. We use the inverse vehicle model to invert the front side force into the steering angle [7]. The structure of the controller is shown in Figure 3.

![Diagram](image)

**Figure 3. Control system proposed**

5. Simulation Results

We imitate the drift-starting operation of the real-life driver: starting at a certain speed, stepping on the accelerator and reversing the steering wheel, designed an open-loop operation. When the direction of the front wheel corner is detected to have changed (from positive to negative), the closed loop control (LQR) is activated. Vehicle parameters are selected according to class C car in carsim, except $m=1600$kg.
Figure 4. Comparison of simulation state results with the reference

Figure 5. Comparison of simulation input with the reference

Figure 4 shows the vehicle states during the simulation. The dark lines show the output of the simulation and the red lines indicate the control reference. When $t < 1.5s$, the vehicle is under open loop control, when $t > 1.5s$, the closed-loop controller is activated. After 5 seconds the vehicle reaches the calculated reference state $(-0.24, 0.36, 19.7)$. Because the vehicle is a complex non-linear system, the calculated equilibrium value is slightly different from the actual value.

Figure 5(a) shows the value of the steering angle. The early steering angle is positive (+) and then quickly turns in the opposite direction (-) to achieve the target stable value. Immediately after activating the closed-loop Controller, a peak appears in the steering angle, and soon stabilizes. This is because we do not use specific numerical values to design open-loop control, we rely on closed-loop control to make the car tend to the target state. The larger the load, the larger the driving torque, Figure 5(b) shows the drive torque of the two independent wheels controlled by the controller. The drive torque of the right wheel is about 930N, the stable driving Torque of the left wheel is about 480N.

6. Conclusion

In this paper, a drift controller for rear wheel independent drive electric vehicle is proposed to explore independent drive vehicle for extreme sports. It make use of the different wheel drive torque to coordinate the yaw dynamics. The designed steady-state calculator can be used as an reference for the analysis of equilibrium. Simulation results verify the effectiveness of the control strategy.

References

[1] M. Abdulrahim. On the dynamics of automobile drifting. In Proceedings of SAE World Congress 2006, April 2006. SAE Technical Paper 2006-01-1019.

[2] R. Chaichaowarat and W. Wannasuphoprasit, “Dynamics and simulation of RWD vehicle drifting at steady state using BNP-MNC tire model,” SAE Technical Paper, No.2013-01-0001, 2013.

[3] Rami Y. Hindiyeh and J. Christian Gerdes. “A Controller Framework for Autonomous Drifting: Design, Stability, and Experimental Validation”. In: Journal of Dynamic Systems, Measurement, and Control 136.5 (July 2014), p. 051015.
[4] Efstathios Velenis, Diomidis Katzourakis, Emilio Frazzoli, Panagiotis Tsiotras, and Riender Happee. “Steady-state drifting stabilization of RWD vehicles”. In: Control Engineering Practice 19.11 (2011), pp. 1363–1376.

[5] Russell P. Osborn, Taehyun Shim. Independent control of all-wheel-drive torque distribution[J]. Vehicle System Dynamics, 2006, 44(7):529-546.

[6] Sakai S I, Sado H, Hori Y. Dynamic driving/braking force distribution in electric vehicles with independently driven four wheels[J]. Electrical Engineering in Japan, 2002, 138(1):79–89.

[7] Goh J Y, Goel T, Gerdes J C. A Controller for Automated Drifting Along Complex Trajectories[J]. AVEC, 2018.

[8] Rami Yusef Hindiyeh. Dynamics and Control of Drifting in Automobiles. PhD thesis. Stanford University, 2013