Transport in Quasi One-Dimensional Systems

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Abstract. The interplay of Umklapp scattering from a periodic potential and other scattering processes determine the conductivity of (quasi) one-dimensional metals. We show that the transport at finite temperature is qualitatively and quantitatively strongly influenced by a number of approximate conservation laws. Typically, not the strongest but the second strongest scattering mechanism determines the dc-conductivity. We discuss the optical conductivity both of strongly anisotropic, quasi one-dimensional Fermi liquids and of Luttinger liquids.

Thanks to the miracles of bosonization, conformal field theory, Bethe ansatz and renormalization group, the theoretical description of one-dimensional systems is one of the best developed areas in the theory of strongly correlated systems [1]. The enormous theoretical advance is contrasted by failure to understand certain experimental quasi one-dimensional systems like the Bechgaard salts even on a qualitative level [2]. In recent years, carbon nanotubes have proven to be an almost ideal one-dimensional system where theoretical concepts can be tested in some detail.

Astonishingly, the theory of finite temperature transport in clean one-dimensional (1d) systems is not developed very far. The technical reason for this is that in a pure Luttinger-liquid the conductivity is infinite and \( \sigma \) is therefore a singular function of irrelevant operators and naive perturbation theory can be problematic and misleading. In Ref. [3], we argued that main features of the optical conductivity \( \sigma(\omega, T) \) can be understood from the analysis of certain approximate conservation laws and proposed to calculate \( \sigma \) from a hydrodynamic theory of the corresponding slowly decaying modes. The main result is, that in many experimentally relevant situations, the strongest scattering process alone cannot lead to a relaxation of the current due to some conservation law. As a consequence, temperature dependence of the dc-conductivity is often determined by the second strongest scattering processes. This point of view has been tested numerically in [4]. We will first discuss in some detail the role of approximately conserved quantities for a quasi one-dimensional Fermi liquid, afterwards we will shortly review the results for Umklapp scattering in a Luttinger liquid and discuss how the results generalize for other scattering mechanisms.
1 Transport in an Anisotropic Fermi Liquid

Before we investigate the transport properties of a Luttinger liquid, we study the role of Umklapp scattering in an anisotropic Fermi liquid. The arguments given here will be published in a more extended version in Ref. [5].

![Fermi surface of an anisotropic metal. Both “forward” (I) and “Umklapp” (II) scattering processes do not lead to an decay of the pseudo momentum $\tilde{P}_x$ as long as the momenta are within the shaded area. The scattering event III conserves momentum but leads to a decay of $\tilde{P}_x$. It is exponentially suppressed even at moderate temperatures as it involves quasi-particles far away from the Fermi surface.](image)

1.1 Pseudo Momentum Conservation Close to Half Filling

We consider an anisotropic metal close to half filling with a clearly defined most-conducting axis in $x$-direction. It is assumed that two well defined Fermi sheets perpendicular to this axis exist (see Fig. 1). The curvature of those sheets is not required to be extremely small, for our argument it is sufficient it is less than e.g. a quarter of the width of the Brillouin zone (i.e. within the shaded area of Fig. 1). We consider a rather arbitrary lattice, assuming only the existence of a translation vector $a_1$ of the underlying lattice in $x$-direction.

Umklapp processes lead to a decay of any macroscopic momentum. For example, close to $1/2$ filling a low-energy Umklapp processes (type II in Fig. 1) with a momentum transfer $G_x$ relaxes the momentum in $x$ direction, where $G$ is a reciprocal lattice vector. However, a pseudo momentum $\tilde{P}_x$ can be defined, which is conserved by two-particle scattering processes close to the Fermi surface as we will show in the following

$$\tilde{P}_x = \sum_{k,\sigma} \delta k_x c_{k,\sigma}^\dagger c_{k,\sigma}, \quad \text{with} \quad \delta k_x = k_x - \frac{G_1}{4} \text{sign}(k_x).$$

(1)

The pseudo momentum $\delta k_x$ is measured with respect to the line $k_x = \pm \frac{G_1}{4}$ (dashed line in Fig. 1). Here, we concentrate on systems close to half filling – for a quasi 1d system close to a different commensurate filling, other pseudo-momenta are more relevant.

To check to what extent $\tilde{P}_x$ is conserved, we calculate the commutator of $\tilde{P}_x$ with a generic 2-particle scattering term:
\[ H_2 = \sum_{1.\ BZ} c_{k_1}^\dagger c_{k_2}^\dagger c_{k_1'}^\dagger c_{k_2'}^\dagger V_{k_1,k_2,k_1',k_2'} \sum_{G_n} \delta(k_1 + k_2 - k_1' - k_2' - G_n) \]

\[
[\tilde{P}_x, H_2] = \sum_{1.\ BZ} c_{k_1}^\dagger c_{k_2}^\dagger c_{k_1'}^\dagger c_{k_2'}^\dagger V_{k_1,k_2,k_1',k_2'} (\delta k_{1x} + \delta k_{2x} - \delta k_{1'x} - \delta k_{2'x}) \times \sum_{G_n} \delta(k_1 + k_2 - k_1' - k_2' - G_n) \]

(2)

where the \( G_n \) are reciprocal lattice vectors. It is easy to check, that all those terms on the right-hand side of (2) vanish if all four momenta are in the shaded region of Fig. 1, i.e. \(|\delta k_{1x}|, |\delta k_{2x}|, |\delta k_{1'x}|, \delta k_{2'x}| < \frac{G_{1x}}{4}\). For “forward scattering” processes of type I in Fig. 1, \( k_{1x} + k_{2x} - k_{1'x} - k_{2'x} = 0 \), both momentum \( P_x \) and pseudo momentum \( \tilde{P}_x \) are conserved as for all 4 momenta have the same sign. While “Umklapp” processes of type II pick up a lattice momentum \( G_x \), this is exactly compensated by the fact that two electrons are moving from the right to the left Fermi surface due to the term sign \([k_x]G_{1x}/4\) in the definition of \( \delta k_x \) and \( \tilde{P}_x \) is again conserved. The pseudo momentum can only decay by high-energy processes far from the Fermi surface, e.g. III in Fig. 1. In a two-particle scattering event (e.g. in 2nd order perturbation theory) such a scattering process is exponentially suppressed even at moderate temperatures.

In high orders of perturbation theory, however, low-energy contribution can result from these high-energy processes due to virtual excitations. Or to put it in the language of renormalization group: \( N \)-particle interactions are generated. Will they relax \( \tilde{P}_x \)? From the definition of the pseudo momentum it is clear that \( \tilde{P}_x \) is conserved modulo \( G_{1x}/2 \). Therefore any \( N \)-particle scattering event will change \( \tilde{P}_x \) by a multiple of \( G_{1x}/2 \). Accordingly, a relaxation of \( \tilde{P}_x \) is not possible if all 2 \( N \) pseudo momenta \( \delta k_{ix} \) involved in the scattering process are smaller than \( \frac{G_{1x}}{4} \). At low \( T \), a decay of \( \tilde{P}_x \) by \( N \)-particle collision can only happen for

\[
N > \frac{G_{1x}/4}{\max |\delta k_{F_x}|} \]

(3)

where \( \max |\delta k_{F_x}| \) is the maximal distance of the Fermi surface from the plane \( k_x = \pm G_{1x}/4 \) (dashed line in Fig. 1). At sufficiently high temperatures, the broadening of the Fermi-surface and the thermal excitation of states with higher energy will favor decay channels of the pseudo momentum with smaller \( N \) (this effect can crudely be described by adding \( T/v_F \) to \( \max |\delta k_{F_x}| \) in (3)).

The temperature dependence of the decay-rate \( \Gamma_{\tilde{P}_x} \) of \( \tilde{P}_x \) at low \( T \) in the Fermi liquid regime is determined by the usual phase-space arguments: a particle of energy \( \omega \sim T \) decays in \( 2N - 1 \) particle and hole excitation, one of the energies is fixed by energy conservation, the remaining \( 2N - 2 \) energies each have a phase-space of order \( \omega \) and therefore

\[
\Gamma_{\tilde{P}_x} \propto T^{2N-2} \]

(4)
where the integer $N$ is the smallest value consistent with (3). The prefactor in (4) depends in a rather delicate way on the strength and range of the interaction, the screening and the band-curvature. Note, that a local $N$-particle interaction will give no contribution for $N > 2$ due to the Pauli-principle. Therefore the scattering rate is strongly suppressed for weakly coupled chains with well-screened interactions. Furthermore, additional logarithmic temperature dependences of the scattering vertices are expected even in the Fermi-liquid regime as it is well known from Fermi liquid theory.

We want to stress that the analysis given above is valid for interactions of arbitrary strength as long as a Fermi liquid description is possible (For strong interaction, one should, however, consider the pseudo momentum of quasi-particles which slightly differs from the pseudo momentum of the bare electrons, defined above).

### 1.2 Pseudo Momentum and Conductivity

In the preceding section we have established that the pseudo momentum $\tilde{P}_x$ will decay very slowly in a quasi one-dimensional metal. How does this influence the optical conductivity $\sigma(\omega)$? We will argue in the following, that the pseudo momentum conservation leads to well defined peak in the optical conductivity at zero frequency and we will show how its weight can be calculated reliably. Furthermore, the $T$-dependence of the dc-conductivity is primarily given by the decay rate of $\tilde{P}_x$ at sufficiently low temperatures.

We will first try to derive our results using rather simple hand-waving arguments which will be substantiated by some rigorous results derived many years ago [6,7]. We consider the following Gedankenexperiment: at time $t = 0$ we prepare a state with a finite current $\langle J_x(t = 0) \rangle > 0$. As the current is not conserved, it will decay rather fast by two-particle collisions ($\Gamma_{J_x} \propto T^2$ at low $T$). Typically, the initial state with finite current will also have a finite pseudo momentum $\langle \tilde{P}_x(t = 0) \rangle$ which will decay much slower than the current; $\Gamma_{\tilde{P}_x} \ll \Gamma_{J_x}$. The important point is now to realize that any state with finite pseudo momentum will typically carry a finite current $\langle J_x \rangle = J(\langle \tilde{P}_x \rangle)$. Accordingly, a finite fraction of $J_x$ will not decay with the fast rate $\Gamma_{J_x}$ but with the much smaller rate $\Gamma_{\tilde{P}_x}$ as is shown schematically in Fig. 2.

How large is the fraction of the current which decays slowly? To answer this question, we have to consider two separate questions: first, how large is $\langle \tilde{P}_x(t = 0) \rangle$, and second, how much current does a state with finite $\tilde{P}_x$ carry? At $t = 0$ one obtains in linear response theory the ratio of the expectation values of the operators $\langle \tilde{P}_x(t = 0) \rangle / \langle J_x(t = 0) \rangle = \chi_{\tilde{P}_x J_x} / \chi_{J_x J_x}$ in terms of the corresponding susceptibilities (defined as usual). To answer the second question, we consider a situation where a field conjugated to $\tilde{P}_x$ is applied and find in analogy to the argument above $\langle J_x(t \rightarrow \infty) \rangle / \langle \tilde{P}_x(t \rightarrow \infty) \rangle = \chi_{J_x \tilde{P}_x} / \chi_{\tilde{P}_x \tilde{P}_x}$. Therefore, we expect (we will discuss later under what condition this statement is rigorously true) that the low-energy peak in the optical
Fig. 2. In a Gedankenexperiment, at time $t = 0$ a state with current $\langle J_x \rangle$ is prepared. $J_x$ decays rapidly while the pseudo-momentum $\tilde{P}_x$ is approximately conserved and has a much lower decay rate $\Gamma_{\tilde{P}_x}$. In the conductivity $\sigma(\omega)$ the approximate conservation of $\tilde{P}_x$ leads to a low-frequency peak in the optical conductivity characterized by the decay-rate $\Gamma_{\tilde{P}_x}$, here assuming that $N^*$ is the smallest integer consistent with (3): a scattering process involving $N^*$ particles can relax $\tilde{P}_x$. The width is given by $\Gamma_{\tilde{P}_x}$, the weight by (6).

conductivity carries a fraction

$$\frac{D}{D_0} = \frac{\chi_{J_x,\tilde{P}_x}^2}{\chi_{\tilde{P}_x,\tilde{P}_x}\chi_{J_x,\tilde{P}_x}}$$

of the total weight $\pi\chi_{J_x,\tilde{P}_x} = 2\pi D_0 = \pi \frac{na^2}{m}$ with $\frac{a}{m} = \sum_{\kappa \sigma} \frac{\partial^2}{\partial k^2} \langle c_{\kappa \sigma}^\dagger c_{\kappa \sigma} \rangle$. Note, that due to the Cauchy-Schwartz inequality the relative weight $D/D_0$ is always smaller than 1 as it has to be. For lower and lower temperature the peak in $\sigma(\omega)$ gets sharper. According to (6), the total weight of the peak is

$$D = \frac{1}{2} \frac{\chi_{J_x,\tilde{P}_x}^2}{\chi_{\tilde{P}_x,\tilde{P}_x}}.$$ 

$D$ should not be confused with the zero-temperature Drude weight as we have not included frequencies of the order of the short-time decay rate $\Gamma_{J_x} \propto T^2$ in the definition of its weight.

1.3 Rigorous Results

It is obvious that the weight of the low-frequency peak in $\sigma(\omega)$ (shaded area in Fig. 2) can rigorously be extracted only in the limit $\Gamma_{\tilde{P}_x}/\Gamma_{J_x} \to 0$, i.e. in a situation where $\tilde{P}_x$ is exactly conserved and $D$ is really a finite-temperature Drude weight. In this situation, Mazur [6] has derived long ago an exact
inequality for certain correlation functions which in our context reads

\[
D \geq \frac{1}{2} \sum_{m=1}^{M} \frac{\chi_{Q_n, Q_m}^2}{\chi_{Q_n, Q_m}}
\]

It is valid if all \(Q_n\) are conserved and \(\chi_{Q_n, Q_m} = 0\) for \(n \neq m\). The importance of Mazur’s inequality has been recently emphasized by Zotos et al. \cite{Zotos} (see below).

Furthermore, Suzuki \cite{Suzuki} showed that the inequality in (7) can be replaced by an equality if the sum includes all conservation laws! If therefore \(\tilde{P}_x\) is the only (approximately) conserved quantity in the system with a finite overlap to the current \(\chi_{J, Q} \neq 0\) (i.e. if \(\tilde{P}_x\) is the slowest “current-like” mode in the system and \(\Gamma_{\tilde{P}_x}\) the smallest decay rate) then (6) is exact as was tested numerically for a simple model in \cite{Rosch}.

1.4 Low-Frequency Weight in Fermi Liquid Theory

\(\chi_{\tilde{P}_x, \tilde{P}_x}\) can easily be calculated at low \(T\) within Fermi liquid theory following standard text books \cite{Luttinger}. The result will in general depend on the details of the momentum dependence of the effective interactions and the band-structure.

We assume a quasi 1D system with a Fermi velocity \(v_F = k_F/m^*\) parallel to the most conducting axis and – for simplicity – completely local interactions characterized by two Fermi liquid parameter \(F_{++}\) and \(F_{+-}\) in the spin-singlet channel to describe the interactions of two density excitations \(\delta n_k\) on the same Fermi sheet or on different sheets, respectively. With \(F_m = F_{++} - F_{+-}\), the relative weight of the low frequency peak in the optical conductivity for low \(T\) is given by

\[
\frac{D}{D_0} \approx \frac{m}{m^*} \left( \frac{\langle (\delta k_x)^2 \rangle_{\text{FS}}}{\langle \delta k_x^2 \rangle_{\text{FS}}} - \frac{F_m}{1 + F_m} \right)^{-1}.
\]  

(8)

\(\langle \ldots \rangle_{\text{FS}}\) is defined as an average over the Fermi sheet, for example \(\langle \delta k_x^2 \rangle_{\text{FS}} = \int \int dk_ydk_z (k_x^2 - k_x^2) / (\int \int dk_ydk_z)\), where \(k_x = k_x(k_y, k_z)\) is the x-component of the Fermi momentum on the right Fermi sheet. Note that due to Luttinger’s theorem \(\Delta n = 2 \langle \delta k_x \rangle_{\text{FS}} / (a_y a_z \pi)\), where \(\Delta n\) is the deviation of the electron-density from half filling.

If the interactions are sufficiently weak so that no phase transition is induced, the low-frequency weight \(D\) vanishes close to half filling with

\[
\frac{D}{D_0} \approx \frac{m}{m^*} \left( \frac{\epsilon_F^*}{\epsilon_F^*} \right)^2 \left( \frac{\Delta n}{n} \right)^2.
\]

(9)

where \(\epsilon_F^* = k_Fv_F\) is the renormalized Fermi energy. We expect that the low-frequency weight \(D\) decreases with increasing temperature, mainly due to the thermal broadening of \(\langle (\delta k_x)^2 \rangle_{\text{FS}}\). Leading finite-\(T\) corrections to (8) or (9) are of order \((T/\epsilon_F^*)^2\).
2 Luttinger Liquids

The physics of approximate conservation laws and their influence on transport discussed in the previous section is even more important for exactly one-dimensional systems.

Surprisingly, little is known about the conductivity \( \sigma(T) \) of an ideal one-dimensional wire in the presence of Umklapp scattering induced by a periodic potential. For a long time, there has not even an agreement whether \( \sigma \) is finite or infinite at \( T > 0 \) for generic systems [9,10]. For example, T. Giamarchi [9,10] found a finite conductivity for \( T > 0 \) within a certain perturbation theory. However, using a Luther-Emery transformation of the dominant Umklapp process, he concluded that the conductivity is actually infinite [9]. Furthermore, Castella, Zotos and others [11,12,13] were able to calculate the Drude weight for a number of exactly solvable, integrable lattice models and they also found an infinite conductivity for \( T > 0 \). From our discussion in 1.2 this is not too surprising as any integrable model is characterized by an infinite number of conservation laws – this point of view has been emphasized by Zotos et al. [13], who proposed to use (7) to obtain an estimate for the Drude weight in these models. On general grounds, however, one would expect a finite conductivity in any generic lattice model.

Most of the above discussed discrepancies can be resolved from an analysis of the approximate conservation laws of the appropriate low-energy theories as has been realized recently by the author and N. Andrei [3]. Within a certain hydrodynamic theory of the approximately conserved quantities, the main characteristics of the optical conductivity can be calculated reliably.

2.1 Pseudo Momenta

The topology of the Fermi surface of a 1d metal determines its low-energy excitations. Two well defined Fermi-points exist at momenta \( k = \pm k_F \), allowing us to define left and right moving excitations, to be described by \( \Psi_{L/R,\sigma=\uparrow} \) (see Fig. 3). We shall include in the fields momentum modes extending to the edge of the Brillouin zone, usually omitted in treatments that concentrate on physics very close to the Fermi-surface.

![Fig. 3. In d = 1, a metal is characterized by two Fermi points and the fermionic excitations can be separated in left- and right-movers. Certain Umklapp processes (here for n = 3) can scatter electrons from the left to the right Fermi surface and vice versa.](image-url)
The Hamiltonian, including high energy processes, is

\[
H = H_{LL} + H_{irr} + \sum_{n,m} H_{U,n,m}.
\]

(10)

\(H_{LL}\) is the well-known Luttinger liquid Hamiltonian capturing the low energy behavior \[1\],

\[
H_{LL} = v_F \int (\Psi_{L,\sigma}^\dagger i \partial_x \Psi_{L,\sigma} - \Psi_{R,\sigma}^\dagger i \partial_x \Psi_{R,\sigma}) + g \int \rho(x)^2 = \frac{1}{2} \int \frac{dx}{2\pi} \sum_{\nu=\sigma,\rho} v_{\nu} \left[ K_{\nu} (\partial_x \theta_{\nu})^2 + \frac{1}{K_{\nu}} (\partial_x \phi_{\nu})^2 \right]
\]

\(v_F\) is the Fermi velocity, \(g > 0\) measures the strength of interactions, \(\rho = \rho_L + \rho_R\) is the sum of the left and right moving electron densities. In the second line we wrote the bosonized \[1\] version of the Hamiltonian. Here \(v_\sigma, v_\rho\) are the spin and charge velocities, and the interactions determine the Luttinger parameters \(K_\nu\) with \(v_\nu K_\nu = v_F, v_\rho/K_\rho = v_F + g/\pi, v_\sigma/K_\sigma = v_F - g/\pi\).

The high energy processes are captured in the subsequent terms which are formally irrelevant at low energies (we consider only systems away from a Mott transition, i.e. away from half filling). Some of them, however, determine the low-frequency behavior of the conductivity at any finite \(T\), since they induce the decay of the conserved modes of \(H_{LL}\) (they are “dangerously irrelevant”). We classify these irrelevant terms with the help of two operators which will play the central role in our discussion. The first one is the translation operator \(P_T\) of the right- and left-moving fields, the second one, \(J_0 = N_R - N_L\), is the difference of the number of right- and left-moving electrons, and is up to \(v_F\), the charge current of \(H_{LL}\):

\[
P_T = \sum_\sigma \int dx \left[ \Psi_{R,\sigma}^\dagger (-i \partial_x) \Psi_{R,\sigma} + \Psi_{L,\sigma}^\dagger (-i \partial_x) \Psi_{L,\sigma} \right]
\]

(11)

\[
J_0 = N_R - N_L = \sum_\sigma \int dx \left[ \Psi_{R,\sigma}^\dagger \Psi_{R,\sigma} - \Psi_{L,\sigma}^\dagger \Psi_{L,\sigma} \right]
\]

(12)

The linear combination \(P_0 = P_T + k_F J_0\) can be identified with the total momentum of the full Hamiltonian \(H\).

We proceed to the classification of the formally irrelevant terms in the Hamiltonian. This classification allows us to select all those terms (actually few in number) that determine the current dynamics. \(H_{irr}\) includes all terms in \(H - H_{LL}\) which commute with both \(P_T\) and \(J_0\), such as corrections due to the finite band curvature, due to finite-range interactions and similar terms. We will not need their explicit form.

The Umklapp terms \(H_{U,n,m}^{n,m} (n, m = 0, 1, \ldots)\) convert \(n\) right-movers to left-movers (and vice versa) picking up lattice momentum \(m2\pi/a = mG\), and do not commute with either \(P_T\) or \(J_0\) (see Fig. 3). Leading terms are of the
form,

\[ H_{0,m}^U \approx g_{0,m}^U e^{i\Delta k_{0,m}} x (\rho_L + \rho_R)^2 + h.c. \]  

\[ H_{1,m}^U \approx g_{1,m}^U \sum_\sigma e^{i\Delta k_{1,m} x} \psi_{R\sigma}^\dagger \psi_{L\sigma} \rho_{-\sigma} + h.c. \]  

\[ H_{2,m}^U \approx g_{2,m}^U \int e^{i\Delta k_{2,m} x} \psi_{R\uparrow}^\dagger \psi_{R\downarrow}^\dagger \psi_{L\downarrow} \psi_{L\uparrow} + h.c. \]

with momentum transfer \( \Delta k_{n,m} = n2k_F - mG \).

The important step is to realize that certain pseudo momenta, \( \tilde{P}_{nm} \), defined as linear combinations of \( J_0 \) and \( P_T \):

\[ \tilde{P}_{nm} = \frac{\Delta k_{nm}}{2n} J_0 + P_T \]

are approximately conserved. For \( m = 0 \), \( \tilde{P}_{n0} \) is nothing but the usual momentum, for \( n = 2, m = 1 \) we recover the pseudo momentum discussed in section 1.1.

If the Hamiltonian includes only a single type of Umklapp process \( H_{nm}^U \), then the pseudo momentum is exactly conserved

\[ \left[ H_{LL} + H_{IR} + H_{nm}, \tilde{P}_{nm} \right] = 0. \]

even in the presence of band-curvature and similar terms. Therefore we can expect an infinite conductivity in such a model. At least two independent Umklapp terms are required to lead to a complete decay of the current and a finite conductivity (an exception are exactly commensurate systems, see [3]).

In any generic lattice model, certainly all types of Umklapp processes are present or are “generated” in the language of renormalization group. It is clear that in such a situation, the strongest Umklapp process will not determine the \( T \) dependence of the dc-conductivity as the associated conservation law prohibits the decay of the current. Instead, the second strongest Umklapp determines the decay rate of the current as we have argued in section 1.2. As the second strongest Umklapp process is typically far from the Fermi surface, the conductivity can be very large. For lowest temperatures, we find in [3] close to a commensurate filling \( n \approx M/N \), where \( M \) and \( N \) are integers with

\[ \sigma(n = \frac{M}{N} + \Delta n, T) \sim \max \left[ (\Delta n)^2 e^{\beta vG/N}, T^{-N^2K} \right] \]  

while \( \sigma(T) \sim \exp[-(T_0/T)^{2/3}] \) at typical incommensurate fillings. Details and omitted prefactors can be found in Ref. [3]. The calculation of the conductivity in Ref. [3] is based on the memory matrix formalism [15]. For a certain classical model of charge excitations of a weakly doped Mott insulator these effects have been calculated numerically in [4]. These numerical calculations have served as a test for the analytical methods used in [3].
2.2 Competition of Scattering Processes and Role of Integrability

If one analyses various scattering processes in (quasi) one-dimensional materials, one realizes that many of them can be characterized by approximate conservation laws. We argued above, that Umklapp scattering $H^U_{n,m}$ conserves pseudo-momentum but also scattering from low-energy phonons or slowly varying potential fluctuations are characterized by well-known conserved quantities as is shown in Table 1. The consequences can be that e.g. in a situation with very strong Umklapp scattering the $T$ dependence of the conductivity is determined by phonons, while in a regime where phonons dominate, the power-laws associated to Umklapp processes show up in $\sigma(T)$.

Table 1. Scattering processes and the associated conservation laws

| scattering mechanism                          | conserved quantity |
|-----------------------------------------------|--------------------|
| Umklapp $H^U_{n,m}$                           | pseudo-momentum $\tilde{P}_{n,m}$ |
| acoustic phonons (1D or 3D)                   | $N_R - N_L$        |
| long-range disorder(forward scattering)      | $N_R - N_L$        |
| short-range impurities                        | no conservation    |

This analysis applies both to one- and quasi one-dimensional materials independent of whether the system is better described by a Luttinger liquid, a Fermi liquid or something else (however, the $T$ dependence of $\sigma$ will be very different).

Generically, the low-energy behavior of exactly one-dimensional systems is characterized by an integrable fixed-point Hamiltonian with an infinite number of conservation laws. It is therefore important to analyze whether these more complicated conservation laws influence measurable quantities like the conductivity for generic models. For incommensurate systems it is easy to check that at low temperature it is sufficient to keep track of the pseudo-momentum conservation as the decay rate of the relevant $P_{nm}$ is exponentially small in temperature while other more complicated quantities decay with algebraically small rates. The situation is more complicated for an exactly commensurate filling as $\chi_{\tilde{\Delta}P_{n,m}}$ vanishes with exponential precision if $\Delta k_{nm} = 0 \ [3,5]$. In this situation one has to analyze in more detail the influence of other approximate conservation laws. Preliminary results suggest that for 1d Mott insulators with a gap $\Delta$, and for temperatures $\Delta < T < \epsilon_F$, more complicated conservation laws of the associated fixed point Hamiltonian (the sine-Gordon model in this case) are indeed important. This implies, that the optical conductivity will develop a well defined low-frequency peak as sketched in Fig. 2 associated to slowly decaying modes which have their origin in the structure of conservation laws of the sine-Gordon model.
3 Conclusions

In this paper, we have discussed various approximate conservation laws which determine the low-frequency conductivity of clean (quasi) one-dimensional materials. For a large class of situations, the dominant scattering process leads not to a decay of the current due to the presence of a slowly decaying mode. In this situation the second strongest scattering process determines the dc-conductivity. The most important signature of this type of physics is a well-defined low frequency peak in the optical conductivity as is shown schematically in Fig. 2. The weight of such a peak can be calculated from (6). In experiments on Bechgaard salts [2], indeed a well defined low-frequency peak with a small 1% weight have been found. Presently, it is however not clear whether one of the conservation laws discussed in this paper is at the origin of this feature.

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References

1. J. Sólyom: The Fermi gas model of one-dimensional conductors, Adv. Phys. 28, 201–303 (1979); V.J. Emery in Highly Conducting One-Dimensional Solids, eds. J. Devreese et al. (Plenum, New York, 1979), p. 247
2. V. Vescoli et al.: Dimensionality-driven insulator-to-metal transition in the Bechgaard salts, Science, 281, 1188 (1998); A. Schwartz et al.: On-chain electrodynamics of metallic (TMTSF)₂X salts: Observation of Tomonaga-Luttinger liquid response, Phys. Rev. B 58, 1261 (1998)
3. A. Rosch and N. Andrei: Conductivity of a clean one-dimensional wire, Phys. Rev. Lett. 85, 1092–1096 (2000)
4. M. Garst and A. Rosch: Transport in a classical model of an one-dimensional Mott insulator: Influence of conservation laws, preprint. [cond-mat/0102109]
5. A. Rosch and N. Andrei, to be published
6. P. Mazur: Non-ergodicity of phase functions in certain systems, Physica 43, 533–545 (1969)
7. M. Suzuki: Ergodicity, constants of motion and bounds for susceptibilities, Physica 51, 277–289 (1971)
8. D. Pines and P. Nozières, The Theory of Quantum Liquids: Volume 1, Benjamin (New York 1966)
9. T. Giamarchi: Umklapp process and resistivity in one-dimensional fermion systems, Phys. Rev. B 44, 2905–2913 (1991)
10. T. Giamarchi and A.J. Millis: Conductivity of a Luttinger liquid, Phys. Rev. B 46, 9325–9331 (1992)
11. S. Fujimoto and N. Kawakami: Exact Drude weight for the one-dimensional Hubbard model at finite temperatures J. Phys. A 31, 465–474 (1998)
12. X. Zotos: Finite temperature Drude weight of the one-dimensional spin-1/2 Heisenberg model, Phys. Rev. Lett. 82, 1764 (1998); H. Castella, X. Zotos, and P. Prelovšek: Integrability and ideal conductance at finite temperature, Phys. Rev. Lett. 74, 972 (1995)
13. X. Zotos, F. Naef, and P. Prelovšek: Transport and conservation laws, Phys. Rev. B 55, 11029 (1997)
14. S. Kirchner et al., Phys. Rev. B 59, 1825 (1999); S. Sachdev and K. Damle, Phys. Rev. Lett. 78, 943 (1997); V.V. Ponomarenko and N. Nagaosa, Phys. Rev. Lett. 79, 1714 (1997); A.A. Odintsov, Y. Tokura, S. Tarucha, Phys. Rev. B 56, 12729 (1997); M. Mori, M. Ogata, H. Fukuyama, J. Phys. Soc. J. 66, 3363 (1997). K. Le Hur, cond-mat/0001439
15. D. Forster, Hydrodynamic Fluctuations, Broken Symmetry, and Correlation Functions, (Benjamin, Massachusetts, 1975)