Hybrid voltage stability and security assessment using synchrophasors with consideration of generator Q-limits

Syed Muhammad Hur Rizvi1, Pratim Kundu1, Anurag K. Srivastava1
1Department of Electrical Engineering and Computer Science, Washington State University, Pullman, USA
E-mail: syed.rizvi@wsu.edu

Abstract: Increased power demands, push for economics and limited investment in grid infrastructure have led utilities to operate power systems closer to their stability limits. Voltage instability may trigger cascade tripping, wide-area voltage collapse, and power blackouts. Real-time voltage stability monitoring possible with deployment of phasor measurement units (PMUs) is essential to take proactive control actions and minimise the impact on the system. This study presents a novel online algorithm for (a) hybrid perturbation analysis based voltage stability monitoring (HPVSM), (b) including Q-limit in voltage stability index, and (c) real-time security analysis using voltage stability index. HPVSM-based voltage stability index is computed using the data obtained from linear state estimator and PMU measurements. Typically measurement-based schemes ignore the impact of generator Q-limits and security analysis is not feasible. The proposed HPVSM-based index considers the impact of generator Q-limits violations by anticipating the critical generators using real-time PMU measurements. Contingencies are ranked using the proposed voltage stability index for security analysis. Results simulated for the 9-bus WECC, IEEE 14, 57, and 118 bus systems highlight the superiority of the proposed method in real-time voltage stability and security analysis.

1 Introduction

Power system voltage stability assessment is performed to identify impending instability condition [1] using knowledge of the existing power system state. Traditionally voltage stability has been analysed using continuation power flow (CPF) [2–4]. CPF approaches because of their computational requirements are suitable for planning studies rather than real-time monitoring. Phasor measurement units (PMUs) provide the opportunity for real-time stability monitoring schemes [5].

Machine learning-based approaches have also been used for synchrophasor-assisted voltage stability assessment. The decision tree-based approach has been presented in [6]. In [7], Zheng et al. proposed regression tree-based real-time voltage stability assessment where robustness against measurement errors and topology variation is also analysed. An artificial neural network-based approach has also been used for voltage stability assessment [8–10]. A major issue with the machine learning-based approach is the lack of actual labelled training data leading to overly optimistic results and dynamic nature of the power system operation with multiple operating points.

Local voltage and current phasors are used for obtaining the classic Thevenin equivalent for voltage stability assessment [11–13]. The ratio of the load and Thevenin impedance is used as an index for real-time voltage stability monitoring. As the system approaches instability, load impedance and Thevenin impedance become almost equal in magnitude. Thevenin parameters are obtained by solving an over-determined system of equations using a window of real-time PMU measurements on the concerned bus. The Thevenin-based voltage stability assessment concept has been extended to wide-area voltage stability monitoring in [14, 15]. Coupled single port theory has also been used for wide-area voltage stability monitoring [16–19]. One problem associated with measurement-based techniques based on Thevenin equivalent is convergence issues because of very little change in operating conditions during normal operation [20].

The inability of the traditional measurement-based approaches to consider the impact of generator reactive power limits has been a hindrance in efficient voltage stability monitoring. Recent work focused on a possible extension of measurement-based stability analysis techniques to consider the impact of reactive power limits [21] using the concept of CPF predictor, but this approach does not consider architecture for real-time analysis. A correction factor is introduced in [22] to predict the impact of Q-limits on Thevenin-based voltage stability index (VSI), but this work does not consider the mathematical formulation to estimate the impact of Q-limits. Impact of Q-limits on security analysis is not considered in [21, 22]. Impact of Q-limits on the coupled single port-based approach is presented in [23]. A hybrid approach proposed in [24] considers...
security analysis, but the impact of \(Q\)-limits on the index is not considered. A measurement-based approach presented in [25] considers the impact of \(n-1\) contingencies on power limits of individual tie-lines without considering the impact of \(Q\)-limits. There is a need for a real-time voltage stability assessment method, which can perform monitoring with consideration of reactive power limits and provides operators a feature to analyse what-if contingency scenarios. Moreover, the impact of estimation issues on assessment performance (e.g. traditional Thevenin-based approaches) should be minimised.

This study introduces a hybrid perturbation analysis based voltage stability monitoring (HPVSM) approach. The proposed approach is considered hybrid because it utilises real-time PMU measurements and system network models. A network model is used by linear state estimator (LSE) to provide the best estimate of wide-area PMU measurements. Linear state estimation implementation considered by the authors of [26, 27] can enhance the quality of signals used for the proposed application.

The novelty of the proposed approach lies in the consideration of \(Q\)-limits in the VSI and online security assessment to suggest proactive control. Weighted VSI (WVSI) is introduced in this work, which combines the two indices VSI and VShu. VSI is the traditional index, which at any point considers only the system's network limit, which is computed using the most recent state of the power system. VSI proactively considers the impact of diminishing reactive power reserves by considering the \(Q\)-limits. The weights used to combine these two indices come from the reactive power reserve status of the generators. The idea behind WVSI is to consider both the impact of transmission network limits and system reactive power limits in one index. The main contributions in this work are summarised as follows:

- Consideration of reactive power limits in the computation of the voltage instability index using the proposed hybrid approach.
- A new index is proposed that combines the two causes of voltage instability, i.e. transmission network limit and reactive power limit in one index.
- Online security analysis for contingency screening and ranking with consideration of reactive power limits is proposed.
- Validation with multiple test cases.

2 Background on measurement based and hybrid voltage stability assessment

Measurement-based voltage stability approaches have attracted lots of attention due to the increased deployment of phasor measurement units in the modern power grid. The concept of assessing voltage stability using synchrophasor data was first realised in the Thevenin-based VSI. Despite the apparent advantages of measurement-based stability, the implementation issues became apparent mainly because of estimation problems. These issues highlight the need to move towards hybrid voltage stability assessment approaches where the assessment utilises both measurement data and power system model information. Hybrid schemes have the potential to be more robust to estimation issues and are especially attractive for long-term but real-time voltage stability assessment.

2.1 Thevenin-based VSI

The use of local phasor measurements for construction of the Thevenin equivalent at the concerned bus is proposed in [11]. Least squares estimation is performed on a window of phasor data for the determination of Thevenin parameters (Thevenin impedance and voltage). It is assumed that system conditions remain almost the same during the window

\[
Z_{th} = r + jv, \quad E_{th} = u + jw
\]

Here, \(Z_{th}\) is the Thevenin impedance, \(r\) and \(v\) are the real and imaginary parts of \(Z_{th}\), \(E_{th}\) represents the Thevenin voltage, and \(u\) and \(w\) are its real and imaginary parts. Using the complex voltage and current measurements at the load bus, an over-determined set of equations can be set up [11] for finding the Thevenin parameters. The VSI is computed as follows:

\[
VSI = \frac{|PV_{th}|}{|PV_{load}|}, \quad Z_{load} = \frac{V_{load}}{T_{load}}. \tag{1}
\]

Despite its apparent simplicity and ease of interpretation, this approach suffers from estimation issues. Accurate estimation of Thevenin parameters is not guaranteed if the change in measurement set is not enough. Moreover, the assumption that system conditions remain the same during the estimation window can lead to inaccurate results.

2.2 Hybrid VSI computation

2.2.1 Concept of hybrid assessment: The hybrid approach uses PMU measurements along with measurements or model information from LSE or Supervisory Control and Data Acquisition (SCADA). In [28, 29], a hybrid approach is proposed that relies on PMU measurements along with network topology information from SCADA to compute VSI in an online and iteration free manner. The main advantage of such a hybrid approach as compared to the PMU measurement-based Thevenin approach in Section 2.1 is not relying on a time series of PMU measurements to compute VSI. The voltage stability assessment approach proposed in [28, 29] performs perturbation analysis using the most recent available state from PMU measurements and network topology information.

2.2.2 Perturbation analysis and computation of VSI: Perturbation for computation of VSI is modelled by changing the real and reactive power injections as per the load change direction. Applying perturbation on the Jacobian, fictitious measurements are obtained as follows:

\[
\frac{\Delta \delta}{\Delta Y} = \begin{bmatrix} J_{pq} & J_{pv} \\ J_{qf} & J_{pf} \end{bmatrix} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}
\]

\[
\frac{\delta^*}{|V^*|} = \begin{bmatrix} \Delta \delta + \delta \\ \Delta V + |V| \end{bmatrix}
\]

Using fictitious measurements, Thevenin impedance is computed as follows:

\[
Z_{th} = \frac{V^f}{I^f}, \quad I^f = Y_{bus}V^f. \tag{4}
\]

Here, measurement \(V^f\) corresponds to the complex fictitious measurement obtained from perturbation analysis and \(V\) corresponds to the complex voltage measurement obtained from linear state estimation results. Once the value of \(Z_{th}\) has been computed, the VSI is obtained using (1).

3 Hybrid perturbation analysis based voltage stability monitoring (HPVSM)

In this study, a hybrid approach has been introduced with the ability to accommodate \(Q\)-limits and perform online security analysis. The approach is considered hybrid because the analysis is carried out based on the results of the LSE, PMU measurements from generators and network topology information. The LSE takes the updated network topology information and PMU data from all buses as input and provides filtered measurements that are used for voltage stability assessment using the proposed approach. If all the buses do not have PMUs, the LSE is required for the reduced set of buses considering the optimal location of PMUs with complete observability [30]. The reactive power reserve is calculated using the knowledge of reactive power generation (from PMUs) and pre-specified value of reactive power generation limits. The proposed method predicts the critical generators and evaluates the impact of \(Q\)-limits on voltage stability. The method computes a WVSI, where
weights are computed using PMU data and the reactive power reserve information. The security analysis feature in the proposed hybrid approach is used for online contingency screening. The ability to perform online security analysis can enhance the awareness of the operator and improve system resilience.

3.1 Consideration of Q-limits

Voltage stability problems often stem from insufficient availability of reactive power. Thevenin-based VSI inherently does not have the ability to anticipate $Q$-limits and leads to an optimistic estimation of the margin from collapse. In this section, a new method for the anticipation of $Q$-limits is considered, which is able to predict the critical generators, and then accommodate the impact directly in the computed VSI.

3.1.1 Prediction of the critical generators: To consider the impact of generator reactive power limits, the reactive power reserve of each generator in the system is modelled as a quadratic function of total reactive power load consumption similar to [31], where reactive power loss in the network is modelled. It is assumed that current reactive power generation and reactive power consumption at all the load buses are available from PMU measurements. Let $RPR_i$ be the reactive power reserve of the $i$th generator. $Q_T$ be the total reactive power consumption in the system. $Q_T$ is considered equal to the sum of reactive power consumption at all the load buses. The $i$th generator reactive power reserve is modelled as follows:

$$RPR_i = a_i Q_i^2 + b_i Q_i + c_i$$  \hspace{1cm} (5)

To identify the parameters, a weighted least squares algorithm can be used on the window of synchrophasor data. In a simplistic scenario where the window has four measurements over-determined system of equations for least-squares estimation would take the following form:

$$\begin{bmatrix}
RPR(n) \\
RPR(n+1) \\
RPR(n+2) \\
RPR(n+3)
\end{bmatrix} = \begin{bmatrix}
Q_T(n) \\
Q_T(n+1) \\
Q_T(n+2) \\
Q_T(n+3)
\end{bmatrix} \begin{bmatrix}1 \\
a \\
b \\
c
\end{bmatrix}$$  \hspace{1cm} (6)

Parameters corresponding to all generators can be evaluated in the same way.

Once the parameters have been estimated the critical value of the total load at which $RPR_i$ would become zero can be computed. This can be done by evaluating the roots of $RPR_i$ equation. The realistic roots can be selected to make a list of critical generators expected to hit the $Q$-limits

$$0 = a_i Q_i^2 + b_i Q_i + c_i$$  \hspace{1cm} (7)

$$Q_{cr,i} = \frac{-b_i \pm \sqrt{b_i^2 - 4ac_i}}{2a_i}$$  \hspace{1cm} (8)

$Q_{cr,i}$ represents the predicted value of total reactive power load at which the $i$th generator would exhaust its reactive power reserve ($RPR_i$). Once values of $Q_{cr,i}$ corresponding to each reactive power resource have been computed, a list of most critical generators is identified, including ones having values close to $Q_T$. The threshold defined in Algorithm 1 is used to populate the list of critical generators. During thorough analysis, it was observed that rather than having a fixed threshold for populating the list of critical generators, the threshold should be dependent on the total system reactive power load. This approach makes the computed index less conservative. Time taken to compute $Q_{cr,i}$ for all generators of the IEEE 57 bus system is found to be 0.009206 s on Intel Core i-7 laptop computer when 30 measurements are used to set-up the over-determined system.

3.1.2 Evaluating the impact of $Q$-limits: In the proposed hybrid approach, we propose perturbation analysis by modification of the power system Jacobian. The power system Jacobian at the current operating point is given as

$$J = \begin{bmatrix}
J_{p_0} & J_{p_0} \\
J_{q_0} & J_{q_0}
\end{bmatrix}$$  \hspace{1cm} (9)

Let us assume that the generator at bus $x$ is predicted to hit the $Q$-limit next. Once it has been identified the Jacobian can be modified considering bus $x$ as a PQ bus instead of PV bus

$$J = \begin{bmatrix}
J_{p_0} & J_{p_0} & J_{p_0} \\
J_{q_0} & J_{q_0} & J_{q_0}
\end{bmatrix} \begin{bmatrix}
J_{q_0} & J_{q_0} & J_{q_0} \\
J_{q_0} & J_{q_0} & J_{q_0}
\end{bmatrix}$$  \hspace{1cm} (10)

Here, $v_i$ is the newly added variable, which is the voltage of generator $x$ and $q_i$ is the reactive power injection at bus $x$. The size of the power flow Jacobian is increased by 1.

3.1.3 Weighted VSI (WVSI): A WVSI is introduced here. The main purpose is to combine the information received from VSI computed at the current operating point and the anticipated VSI with violation of $Q$-limits

$$\frac{\Delta \delta}{\Delta V} = \begin{bmatrix}
J_{p_0} & J_{p_0} & J_{p_0} \\
J_{q_0} & J_{q_0} & J_{q_0}
\end{bmatrix}^{-1} \begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix}$$  \hspace{1cm} (11)

$$V_i = \begin{bmatrix}
\Delta V + \Delta V^{SE} \\
\Delta V + V^{SE}
\end{bmatrix}$$  \hspace{1cm} (12)

$$V_i = \begin{bmatrix}
\Delta V \\
\Delta V + V^{SE}
\end{bmatrix}$$  \hspace{1cm} (13)

$$Z_n = \frac{V^SE - V^{SE}}{I^SE - I}$$  \hspace{1cm} (14)

$$VSI[i] = \frac{\sum_{i \in \text{List} RPR} \|RPR[i] \|}{\sum_{i \in \text{List} Q_{cr}} \|Q_{cr,i} \|}$$  \hspace{1cm} (15)

$$w_1 = \frac{\sum_{i \in \text{List} RPR} \|RPR[i] \|}{\sum_{i \in \text{List} Q_{cr}} \|Q_{cr,i} \|}$$  \hspace{1cm} (16)

Measurements with superscript $SE$ represent measurements filtered through LSE. The WVSI allows alarm processing by taking into account both current network load-ability constraints and the status of reactive power reserves. Here the ‘List’ refers to a list of the identified critical generators using Algorithm 1. VSI represents the index computed without considering the impact of generator $Q$-limits, and $w_1$ is the corresponding weight. WVSI is the index computed assuming critical generator buses as PQ buses and $w_2$ is the corresponding weight.

3.1.4 Algorithm: Evaluation of the impact of $Q$-limits on the computed VSI has two steps. The first step is the prediction of critical generators expected to hit the $Q$-limits and the second step is estimating the impact on the computed VSI at the current operating point.

Algorithm 1 (Evaluating the impact of $Q$-limits):

1: Estimate the model parameters for reactive power reserve of each generator as a quadratic function of load.
2: Estimate the roots of quadratic equation (7) to compute \( Q_{cr,i} \) for every generator. Realistic roots are selected corresponding to \( Q_{cr,i} > Q_i \). Moreover, generators having imaginary roots are eliminated from the updated list.

3: Formulate a list of critical generators. The criterion adopted in this study is to first select the generator having minimum predicted critical total reactive power load \( (\Delta Q_i^m) \) and then select the generators having \( Q_{cr,i} \) in the following range to form the critical list:

\[
[i \in \text{List}, \text{if } Q_{cr,i} \in \left(Q_{cr,min}^n + \text{th} \times Q_i \right)]
\]

The threshold adopted in this study is 0.01.

4: Convert the shortlisted generators to PQ mode and remodel the Jacobian matrix at the current operating point.

5: Use the modified Jacobian to evaluate the impact of nearest \( Q \)-limits in the form of WVSI.

The concept is demonstrated for WECC 9 bus system. The VSI for the current operating point and anticipated index with consideration of reactive power limits is shown for bus 7 as the load is incremented until non-convergence of power flow. It can be seen in Fig. 1 that VSI jumps suddenly from 0.7 to more than 0.9 when the system becomes unstable. This sudden jump comes due to reactive power reserve exhaustion of the second generator. Such a sudden jump can catch the operator off-guard if relying on the VSI. WVSI, on the other hand, anticipates the critical generators and estimates the impact of reactive power reserve exhaustion in advance, thereby providing the operator an early alert and more time to devise preventive control actions.

3.2 Security analysis using the proposed approach

This section introduces a hybrid approach to online security analysis. Traditional measurement-based voltage stability approaches cannot analyse the impact of contingencies on the computed index in an online manner. Security analysis involves two steps for a given contingency; the first step is the prediction of post contingency state and critical generators using the PMU measurements and network topology information. The second step involves the computation of WVSI. The analysed contingency is then classified as critical or non-critical. For the consideration of the \( N-1 \) contingencies piecewise linear method proposed in [32] is adopted for prediction of post-contingent states and critical generators.

3.2.1 Piecewise linear estimation: Piecewise linear and linear estimation of post-contingency states were considered in [32]. Piecewise linear estimation is superior because of its ability to inherently consider the impact of \( Q \)-limits, which resulted in better estimation of voltage magnitudes and reactive powers of the generators. This subsection briefly reviews the concept behind the piecewise linear estimation.

Contingency to be analysed is parameterised in terms of \( K \), which varies from 0 to 1. Starting from iteration 1 \( (i = 1) \) and piecewise linear parameter \( K = 0 \), perturbation analysis is performed at each step to predict post-contingency states as \( K \) varies from 0 to 1. Reactive power change at each unconstrained generator bus in terms of minimum \( (\Delta Q_i^m) \) and maximum \( (\Delta Q_i^j) \) reactive power generation allowed, at the \( i \)th step is given as

\[
\Delta Q_i^m = Q_i^m - Q_i^{(0)} \geq 0
\]

\[
\Delta Q_i^j = Q_i^j - Q_i^{(0)} \leq 0
\]

Voltage deviation \( (\Delta V_i^j) \) at each constrained bus for the \( i \)th is given as

\[
\Delta V_i^j = V_i^j - V_i^{(0)}
\]

Critical constrained and unconstrained generator buses are identified. \( P_i \) and \( N_i \) represent critical unconstrained buses in terms of maximum and minimum reactive powers, respectively. These lists are updated as \( K \) varies from 0 to 1. \( P_i \) and \( N_i \) represent critically constrained generator buses in terms of maximum and minimum reactive powers, respectively. Once the list is finalised, step-change in piecewise parameter \( \Delta K \) is identified as

\[
\Delta K^0 = \min \left\{ \min_{j \in P_i} \Delta Q_i^j, \min_{j \in N_i} \Delta Q_i^j, \min_{j \in P_i} \Delta V_i^j, \min_{j \in N_i} \Delta V_i^j \right\}
\]

Gradients \( \frac{\Delta Q_i^j}{\Delta K} \) and \( \frac{\Delta V_i^j}{\Delta K} \) are computed using mathematical formulation in [32]. Once the value \( \Delta K^0 \) has been computed, new values of \( Q_i^m \) and \( V_i^j \) are computed using linear extrapolation for \( \Delta K^0 \). This is continued until \( K \) approaches 1, which is the point that gives post contingency states.

3.2.2 Contingency screening: In this work, a list of contingencies is screened and critical contingencies are singled out. The first step is the prediction of post contingency states. The second step is the computation of post contingency voltage stability indices. The maximum VSI of the system is used as a screening index. The list of critical generators \( (P_i \) and \( N_i \)) corresponding to the final state of piecewise linear estimation is used for computation of WVSI for a selected contingency. Once an approximate post contingency state is known using piecewise linear estimation, perturbation analysis on the modified Jacobian according to a specific contingency is used to compute the post-contingency WVSI. Contingency screening and ranking are performed using post-contingency WVSI. From the list of contingencies, contingencies having maximum WVSI are considered as the most important contingencies. If any of the screened contingencies have threshold greater than a specified value like 0.75 then that contingency is considered as a critical contingency. Most Thevenin-based approaches use threshold in the range 0.7–0.9 for alarm processing. Fig. 2 shows the architecture of the proposed approach. In Fig. 2, the proposed approach relies on LSE to provide the best approximate of the power system state using wide-area PMU measurements and network topology. LSE can mitigate the impact of noise and anomalous data to give the best approximation of the most recent state, which is used for both voltage stability and security analysis. Table 1 compares the proposed approach with Thevenin and previously proposed hybrid approach for WVSI computation. Fig. 3 describes the architecture for the proposed security analysis approach.

The advantage associated with this approach is that it is not prone to ill-conditioning issues associated with real-time least square-based approaches. Moreover, the proposed approach also avoids the assumption that system condition remains the same during the estimation process. The update rate of the index using the proposed approach depends on the frequency of LSE results.
Since voltage stability is a slowly varying dynamic phenomenon, such a hybrid approach is fast enough for timely long-term voltage stability assessment.

4 Case studies and simulation results

This section presents case studies for voltage security analysis using the proposed method. IEEE 14 bus, 57 bus, and 118 bus systems have been studied. Section 4.1 provides the basis for the application of the considered hybrid voltage stability analysis method by validation from CPF. In Section 4.2, all the benchmark systems are studied with regard to the anticipation of $Q$-limits. $N - 1$ contingency screening results are presented in Section 4.3.

4.1 Continuation power flow-based validation

In this subsection, the effectiveness of the hybrid index computation method is validated using CPF for different IEEE benchmark systems. Repeated power flow results are used to compute the index at each point as the load is varied from the base case until power flow fails to converge. Continuation power flow results are then used for validation by comparison of the PV curve and WVSI for the most critical bus. Continuation of power-flow implementation in MATPOWER [33] is used for validation.

4.1.1 IEEE 14 bus system: For IEEE 14 Bus system, the load was increased from the base case until power flow failed to converge. Corresponding to each converged solution VSI was computed using the proposed method. Bus 9 was found to be the most critical bus from the observed values of voltage stability indices just before collapse as shown in Fig. 4.

Continuation power flow was used to validate the results. Fig. 5a shows the PV curve of bus 9 along with the index values for critical buses. The proposed method was able to identify the voltage instability. Buses 14 and 12 were also found to be critical with a threshold of 0.9 for the IEEE 14 bus system.

4.1.2 IEEE 57 bus system: Bus 32 was observed to be the most critical bus from the observed values of voltage stability indices just before collapse. Bus 25 is also identified as critical with a threshold of 0.9. Fig. 6 shows voltage stability indices of buses having index $> 0.75$ just before collapse. Continuation power flow was then used to validate the results. Fig. 5b shows the PV curve of bus 32 and the index values for identified critical buses with a threshold of 0.9.

4.1.3 IEEE 118 bus system: Similarly, for the IEEE 118 bus system, the load was increased from the base case until power flow failed to converge. Corresponding to each converged solution, VSI was computed using the proposed method. Bus 14 was observed to be the most critical bus from the observed values of voltage stability indices just before collapse.

Continuation power flow was used to validate the results. Fig. 5c shows the PV curve of bus 14 and the index values for the identified critical buses. The proposed method was able to identify the voltage instability. Buses 7, 14, 20, 29, 47, and 60 were also critical if a threshold of 0.9 was selected for the IEEE 118 bus system. Fig. 7 shows voltage stability indices of buses having an index $> 0.8$ just before collapse.
4.2 Consideration of reactive power limits

This subsection demonstrates the impact of reactive power limit consideration. First, the generator with a minimum predicted critical total reactive power load ($Q_{\text{cr},i}^{\min}$) is selected in the list. Then list is updated to include generators having $Q_{\text{cr},i}$ within the range $[Q_{\text{cr},i}^{\min}, 0.01Q_T + Q_{\text{cr},i}^{\min}]$. Here, $Q_{\text{cr},i}$ is the predicted critical value of the total load corresponding to the $i$th generator. Once the list comprising the critical generators is populated, Algorithm 1 is used to accommodate reactive power limits in the index.

4.2.1 IEEE 14 bus system: IEEE 14 bus system was studied using the proposed algorithm for identifying the critical generators and predicting the impact on the VSI. Near the base case loading ($k = 1.04$), generator 2 was the most critical generator followed by generator 8. Generator 2 is pro-actively considered in PQ mode to compute VSI and WVSI in accordance with (18). At the loading level of 1.22, generator 2 is already exhausted and generators 6 and 8 are now used to compute VSI and WVSI according to (18) resulting in a jump in the index. At the loading level of 1.32 generator 8 is exhausted and generator 3 is identified as a critical generator to compute VSI and WVSI resulting in another jump in the index value. Fig. 8a shows the plot of WVSI and VSI for the most critical bus in the system. Fig. 9 shows the variation of generator reactive power reserves as a function of $Q_T$. Table 2 shows the generators having minimum reactive power reserves at various loading levels and Table 3 shows the quadratic parameters for a generator at bus 2 at different loading levels.

4.2.2 IEEE 57 bus system: At $k = 1.015$, generators at buses 9 and 12 were predicted as critical generators. The generator at bus 9 was predicted to hit the $Q$-limits first closely followed by the generator on bus 12. This information matched the observed actual reactive power reserves from repeated power flows. At the loading of $k = 1.3$, the generators at buses 3 and 6 were predicted as critical generators. The generators at buses 9 and 12 were already at the $Q$-limits. After $k = 1.72$, the power flow failed to converge. At this loading, generators at buses 8 and 2 were the critical generators. Generator at bus 8 was predicted to hit the $Q$-limits first closely followed by the generator at bus 2. Fig. 8b shows the impact of $Q$-limits on bus 32 of the IEEE 57 bus system. Fig. 10 shows the variation of generator reactive power reserves as a function of $Q_T$. Table 4 shows the generators having minimum reactive power reserves at various loading levels and Table 5 shows the quadratic parameters for the generator at bus 9 at different loading levels.

4.2.3 IEEE 118 bus system: The proposed approach was correctly able to identify the generators to hit the $Q$-limits and thus include their impact when anticipating VSI. Preparing a list instead of predicting just one critical generator makes the process more robust when several generators could hit the $Q$-limits at almost the same system load like $k = 2.0$ for the IEEE 118 bus system when both generators at buses 104 and 105 were predicted to hit the $Q$-limits. Looking at actual power flow results, the generator at bus...
105 hits the Q-limit just before the generator at bus 104. For IEEE 118 bus system, imaginary roots of (7) are found for some non-critical generators whose RPR stays almost constant throughout as the load is varied from the base-case until power flow fails to converge. Fig. 8c shows the impact of generator Q-limits on the most critical bus in the IEEE-118 bus system for this load increase scenario. Table 6 shows the generators having minimum reactive power reserves at various loading levels and Table 7 shows the quadratic parameters for the generator at bus 12 at different loading levels.

Fig. 8 Anticipating impact of Q-limits for IEEE benchmark systems
(a) IEEE 14 bus System—bus 9, (b) IEEE 57 bus system—bus 32, (c) IEEE 118 bus system—bus 14

105 hits the Q-limit just before the generator at bus 104. For IEEE 118 bus system, imaginary roots of (7) are found for some non-critical generators whose RPR stays almost constant throughout as the load is varied from the base-case until power flow fails to converge. Fig. 8c shows the impact of generator Q-limits on the most critical bus in the IEEE-118 bus system for this load increase scenario. Table 6 shows the generators having minimum reactive power reserves at various loading levels and Table 7 shows the quadratic parameters for the generator at bus 12 at different loading levels.

### 4.3 Security analysis

The proposed approach for security analysis is demonstrated here for IEEE 14, IEEE 57 bus, and IEEE 118 bus systems. The first step for security analysis is the prediction of contingency states using the piecewise linear method. Once the post contingency states are known, perturbation analysis is performed to compute the VSI. The criticality of contingency is decided based on the computed index.

#### 4.3.1 Classification of contingencies

In this subsection, the ability of the proposed scheme to classify contingencies accurately as critical or non-critical is demonstrated. Contingency classification capability of the proposed scheme is validated by...
forming a confusion matrix where actual labels come from CPF and predicted labels come from the proposed approach. For the WVSI-based approach, a threshold of 0.75 is used to classify critical and non-critical contingencies for IEEE 57 and IEEE 14 bus systems. $\Delta \lambda$ is the margin from the nose point in terms of per unit apparent power. Contingencies having $\Delta \lambda > 0.75$ are considered critical and contingencies having $\Delta \lambda < 0.75$ are non-critical. Table 9 presents the classification performance metrics using the proposed approach. Tables 10–12 demonstrate the examples of security analysis for IEEE 14, IEEE 57, and IEEE 118 bus systems, respectively.

### 4.3.2 Ranking of contingencies:

To validate the ability of the proposed approach, a threshold of 0.9 is chosen based on simulation experiments. Offline studies can be used to compute the threshold for different network configurations. Table 9 presents the classification performance of the proposed approach. Table 8 presents the confusion matrix to demonstrate the classification performance of the proposed approach. Table 8 shows that out of 459 non-critical contingencies, the proposed approach is correctly able to identify 65 of them as critical. Similarly, 446 out of 459 non-critical contingencies are identified correctly as non-critical. Table 9 presents the classification performance metrics using the proposed approach. Tables 10–12 demonstrate the examples of security analysis for IEEE 14, IEEE 57, and IEEE 118 bus systems, respectively.
status and reactive power reserve status in a single index. Security

tables, the operator can be better prepared to identify and deal
problems in the system. Knowledge of critical generators can be

determined and used to compute a WVS. WSVI considers the present voltage stability
status and reactive power reserve status in a single index. Security
analysis to identify critical contingencies from a given list is
performed by using the developed index. The proposed approach
provide timely alerts to the operator about the voltage stability
problems in the system. Knowledge of critical generators can be

utilised to plan effective control actions. As the deployment of
PMUs in power system increase, linear state estimation is likely
to become a key application for efficient power system monitoring.
The proposed approach can be deployed at control centres to
perform wide-area monitoring using PMU-based LSEs. The
proposed approach can also work with SCADA-based state
estimation but at the slow reporting rate of VSI. The developed
approach has been validated for multiple test systems and shows
superior performance compared to measurement-based approach
(e.g. estimation-based Thevenin's approach). Future work will
include benchmarking with CPF and implementation with real
utility data.

6 Acknowledgments

The authors would like to acknowledge the support from the
Fulbright scholarship to conduct this work. We would also like to
thank Dr Saugata Biswas for preliminary work.

7 References

[1] Taylor, W.C.: ‘Power system voltage stability’ (EPRI Power System
Engineering, USA, 1994)
[2] Ajjarapu, V., Christy, C.: ‘The continuation power flow: a tool for steady state voltage
stability analysis’, 1992, 7, (1), pp. 416–422
[3] Li, S., Chiang, H.: ‘Nonlinear predictors and hybrid corrector for fast
continuation power flow’, IET Gener. Transm. Distrib., 2008, 2, (3), pp. 341–
354
[4] Xu, P., Wang, X., Ajjarapu, V.: ‘Continuation power flow with adaptive stepszie control via convergence monitor’, IET Gener. Transm. Distrib., 2012,
6, (7), pp. 673–679
[5] De, L., Centeno, V., Thorp, J.S., et al.: ‘Synchronized phasor measurement applications in power systems’, IEEE Trans. Smart Grid, 2010, 1, (1), pp. 20–
27
[6] Diao, R., Sun, K., Vittal, V., et al.: ‘Decision tree-based online voltage security assessment using PMU measurements’, IEEE Trans. Power Syst.,
2009, 24, (2), pp. 832–839
[7] Zheng, C., Malbas, V., Kezunovic, M.: ‘Regression tree for stability margin prediction using synchrophasor measurements’, IEEE Trans. Power Syst.,
2013, 28, (2), pp. 1978–1987
[8] Zhou, D.Q., Annakkage, U.D., Rajapakse, A.D.: ‘Online monitoring of voltage stability margin using an artificial neural network’, IEEE Trans.
Power Syst., 2010, 25, (3), pp. 1566–1574
[9] Nakawiro, W., Erlich, I.: ‘Online voltage stability monitoring using artificial neural network’, 2008 Third Int. Conf. on Electric Utility Deregulation and Restructuring and
Reforming Technologies, Nanjing, People’s Republic of China, 2008, pp. 941–947
[10] Shah, H., Verma, K.: ‘PMU-ANN based approach for real-time voltage stability monitoring’, 2014 IEEE 6th Int. Conf. on Power Systems (ICPS),
New Delhi, India, 2016, pp. 1–5
[11] Vu, K., Begovic, M.M., Novosel, D., et al.: ‘Use of local measurements to estimate voltage-stability margins’, IEEE Trans. Power Syst., 1999, 14, (3), pp. 1029–1035
[12] Corsi, S., Taranto, G.N.: ‘A real-time voltage instability identification algorithm based on local phasor measurements’, IEEE Trans. Power Syst.,
2008, 23, (3), pp. 1271–1279
[13] Li, W., Wang, Y., Chen, T.: ‘Investigation on the Thevenin equivalent parameters for online estimation of maximum power transfer limits’, IET
Gener. Transm. Distrib., 2010, 4, (10), pp. 1180–1187
[14] Milosevic, B.D., Begovic, M.: ‘Voltage stability protection and control using a wide area network of phasor measurements’, IEEE Power Eng. Rev.,
2002, 22, (9), pp. 57–57
[15] Zima, M., Larsson, M., Korba, P., et al.: ‘Design aspects for wide-area monitoring and control systems’, Proc. IEEE, 2005, 93, (3), pp. 980–996
[16] Wang, Y., Pordanjani, I.R., Li, W., et al.: ‘Voltage stability monitoring based on the concept of coupled single-port circuit’, IEEE Trans. Power Syst.,
2011, 26, (4), pp. 2154–2163
[17] Liu, J., Chu, C.: ‘Wide-area measurement-based voltage stability indicators by modified coupled single-port models’, IEEE Trans. Power Syst.,
2014, 29, (2), pp. 756–764
[18] Liu, J.-H., Chu, C.-C.: ‘PMU measurement-based voltage stability indicators by modified multi-port equivalent models’, 2013 IEEE Power Energy Society
General Meeting, Vancouver, BC, Canada, 2013, pp. 1–5
[19] Yu, J., Li, W., Ajjarapu, V., et al.: ‘Identification and location of long-term voltage instability based on branch equivalent’, IET Gener. Transm. Distrib.,
2014, 8, (1), pp. 46–54
[20] Mesgarnejad, H., Shahrtash, S.M.: ‘Power system voltage stability assessment applying phasor measurement units’, 2010 The 2nd Int. Conf. on Electric Utility Deregulation and
Automation Engineering (ICCAE), Singapore, Singapore, 2010, vol. 5, pp. 13–19
[21] Matavalam, A.R.R., Ajjarapu, V.: ‘Sensitivity based Thevenin index with systematic inclusion of reactive power limits’, IEEE Trans. Power Syst.,
2017, 32, (1), pp. 932–942
[22] Dalali, M., Kazemi Karegar, H.: ‘Voltage instability prediction based on reactive power reserve of generating units and zone selection’, IET Gener. Transm. Distrib., 2019, 13, (8), pp. 1432–1440

Table 12 Example IEEE 118 bus system security analysis

| Contingency | Loading = 1.0 | Loading = 1.4 |
|-------------|--------------|--------------|
| WVSStat | Status | WVSStat | Status |
| Br 69-70 | 0.668 | NC | 0.9802 | C |
| Br 24-70 | 0.631 | NC | 0.7467 | NC |
| Br 70-71 | 0.6284 | NC | 0.78 | NC |
| Br 69-75 | 0.6489 | NC | 0.9751 | C |
| Br 68-81 | 0.6435 | NC | 0.9897 | C |
| Br 75-77 | 0.603 | NC | 0.788 | NC |
| Br 74-75 | 0.5938 | NC | 0.75 | NC |
| Br 79-80 | 0.603 | NC | 0.812 | NC |

Table 13 Ranking performance: Wilcoxon signed rank test

| Case | N | Loading multiplier | Confidence interval | p-value | Confidence, % |
|------|---|--------------------|---------------------|---------|---------------|
| IEEE 14 | 10 | 1 | [−3.5, 8] | 0.203 | 95 |
| IEEE 14 | 1.3 | [−2, 3] | 0.9463 | 95 |
| IEEE 57 | 10 | 1 | [−5, 7.5] | 0.5703 | 95 |
| IEEE 57 | 1.3 | [−1, 5.5] | 0.1152 | 95 |
| IEEE 118 | 10 | 1 | [−4, 6.5] | 0.3594 | 95 |
| IEEE 118 | 1.4 | [−3, 4] | 0.8457 | 95 |

Table 14 Ranking performance: t-test

| System | Loading | p-value | Null-hypothesis |
|--------|---------|---------|-----------------|
| IEEE 14 | 1.0 | 0.2256 | valid |
| IEEE 14 | 1.3 | 0.8899 | valid |
| IEEE 57 | 1.0 | 0.4881 | valid |
| IEEE 57 | 1.3 | 0.1796 | valid |
| IEEE 118 | 1.0 | 0.2791 | valid |
| IEEE 118 | 1.4 | 0.7912 | valid |

Table 15 Example IEEE 118 bus system security analysis

| Contingency | Loading = 1.0 | Loading = 1.4 |
|-------------|--------------|--------------|
| WVSStat | Status | WVSStat | Status |
| Br 69-70 | 0.668 | NC | 0.9802 | C |
| Br 24-70 | 0.631 | NC | 0.7467 | NC |
| Br 70-71 | 0.6284 | NC | 0.78 | NC |
| Br 69-75 | 0.6489 | NC | 0.9751 | C |
| Br 68-81 | 0.6435 | NC | 0.9897 | C |
| Br 75-77 | 0.603 | NC | 0.788 | NC |
| Br 74-75 | 0.5938 | NC | 0.75 | NC |
| Br 79-80 | 0.603 | NC | 0.812 | NC |
[23] Su, H.Y., Liu, C.W.: ‘Estimating the voltage stability margin using PMU measurements’, *IEEE Trans. Power Syst.*, 2015, 31, (4), pp. 3221–3229

[24] Yuan, H., Li, F.: ‘Hybrid voltage stability assessment (VSA) for n – 1 contingency’, *Electr. Power Syst. Res.*, 2015, 122, pp. 65–75

[25] Hu, F., Sun, K., Shi, D., et al.: ‘Measurement-based voltage stability assessment for load areas addressing n – 1 contingencies’, *IET Gener. Trans. Distrib.*, 2017, 11, (15), pp. 3731–3738

[26] Yang, T., Sun, H., Bose, A.: ‘Transition to a two-level linear state estimator—part I: architecture’, *IEEE Trans. Power Syst.*, 2011, 26, (1), pp. 46–53

[27] Yang, T., Sun, H., Bose, A.: ‘Transition to a two-level linear state estimator—part II: algorithm’, *IEEE Trans. Power Syst.*, 2011, 26, (1), pp. 54–62

[28] Biswas, S.S., Srivastava, A.K.: ‘Voltage stability monitoring in power systems’, Google Patents, 2018, US Patent 9,876,352

[29] Biswas, S.S., Tushar, , Srivastava, A.K.: ‘Performance analysis of a new synchrophasor based real time voltage stability monitoring (RT-VSM) tool’. 2014 North American Power Symp. (NAPS), Pullman, WA, USA, 2014, pp. 1–6

[30] Wu, T., Chung, C.Y., Kamwa, I.: ‘A fast state estimator for systems including limited number of PMUs’, *IEEE Trans. Power Syst.*, 2017, 32, (6), pp. 4329–4339

[31] Karki, M.J.: ‘Methods for online voltage stability monitoring’ (Digital Repository@Iowa State University, USA, 2009)

[32] Ruiz, P.A., Sauer, P.W.: ‘Voltage and reactive power estimation for contingency analysis using sensitivities’, *IEEE Trans. Power Syst.*, 2007, 22, (2), pp. 639–647

[33] Zimmerman, R.D., Murillo-Sánchez, C.E., Thomas, R.J.: ‘Matpower: steady-state operations, planning, and analysis tools for power systems research and education’, *IEEE Trans. Power Syst.*, 2011, 26, (1), pp. 12–19