Banks of filters for implementation of DMWT of an image

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We describe some possible options for implementation of the Discrete multiwavelet transform (DMWT) of an image by using filter banks. DMWT can be implemented by two channel bank of vector filters which are made by cross-connected scalar filters. The properties of DGHM, CL, BiHermite and SA4 multiwavelets are here analyzed, and compression analysis for output normalization of DMWT is performed. A procedure is design of equivalent replacing of 2 channel multifilters bank by 4 channel bank of single scalar filters. Finally, we deal with a possible reduction and combinations of subbands and suggest their use.

Keywords: multiwavelet, bank of filter, multifilter, DGHM, SA4, CL

1 Introduction

The discrete wavelet transform (DWT) and discrete multiwavelet transform (DMWT) became very popular in the last few decades. The transforms are used in a wide spectrum of applications, for example, we can mention compression [1-3], classification [4], multifocal and multi-exposure images fusion [5,6], medical image denoising [7], human emotion detection [8], image steganography [9,10] and many others. One of drawbacks of the classical DWT is that during the signal or image analysis it uses only one scale and one wavelet function. DMWT eliminates this drawback. DMWT works at least with 2 scale and 2 wavelet functions in contrast with the classical DWT [11,12]. The number of used functions is given by the parameter multiplicity \( r \). The most common value of multiplicity for DMWT used in practice \( r = 2 \) and therefore we consider only this form of DMWT. The processor of DMWT can be realized by the bank of multifilters but this implementation is relatively difficult. We analyze a simplification of this DMWT implementation. DMWT implemented by multifilters bank which are preceded by prefilter. The matrix impulse responses of DMWT multifilters for 4 different multiwavelets is discussed and the connection of multifilters implemented by the cross-connected scalar filters are described. The method of cross-connected filter impulse response creation from multifilters bank is described in detail and the transfer functions of filters for all considered multiwavelets are analyzed as well. Chapter 4 focuses on the analysis of output normality and compression properties of DMWT with considered multiwavelets. The cross-connected scalar filter implementation of the 2 channel multifilters bank (2CH) needs 4 scalar filters for each subband. Thus, 8 scalar filters are needed for LP (Lowpass) and HP (Highpass) band of MDWT. In addition the input signal has to be divided by pre-filtering [13] into 2 components which are brought to input of LP and HP filters [14]. Hence, we analyze the implementation of an equivalent 4 channel (4CH) bank of scalar single filters in chapter 5. The main benefit of this implementation is that on the outputs we get directly all 4 subbands of DMWT and in addition there is no need for the input image prefiltering. As a result, DMWT is realized only by 4 scalar single filters which means a significant simplification of its implementation. In the last chapter the possible methods of reduction and combination of DMWT subbands are analyzed. In this article some possible approaches and possibilities of subband combination for practical exploitation are described.

2 DMWT of image implemented by multifilters bank

Regardless of the used multiwavelets, a multiwavelet transform is implemented by banks of multifilters and multidecimators. The block diagram of one level multiwavelet transform with multiplicity \( r = 2 \) is in Fig. 1, showing one decomposition stage having 2 inputs and 4 outputs. The multiplicity of outputs allows the input image analysis by multiple scaling and wavelet functions. Conventionally DMWT is performed as a convolution of the multifilter impulse response and the input vector signal. For DMWT with \( r = 2 \) the low pass (LP) transfer function is given as a matrix \( G_0 (z) = \left[ G_{l,m}^{(0)} (z) \right]_{2 \times 2} \) \( l, m \in (1, 2) \) and the high pass (HP) transfer function is described by matrix \( G_1 (z) = \left[ G_{l,m}^{(1)} (z) \right]_{2 \times 2} \) \( l, m \in (1, 2) \).

The input are vector signals obtained by a prefiltering [1, 2] described by (1).

\[
\begin{bmatrix}
    c_{0,1} (n) \\
    c_{0,2} (n)
\end{bmatrix} = \begin{bmatrix} f (2n) \\
    f (2n + 1) \end{bmatrix} \tag{1}
\]
The 2D DMWT of an image after the 2 level decomposition is shown.

In Fig. 4(b) DMWT of the Lena image after 2 levels is arranged as shown in Fig. 4(a). For a better illustration the 2D DMWT of the Lena image after 2 levels is shown in Fig. 4. After the second level of the transformation 28 subimages are obtained and they are arranged as shown in Fig. 4(a). For a better illustration in Fig. 4(b) DMWT of the Lena image after 2 levels of decomposition is shown.
3 Banks of multifilters

The concerned multiwavelets are DGHM [4], CL created by Chui and Lian [5], symmetric-antisymmetric SA4 [6,7] and BiHermite [8] therefore the article deals only with DMWT using these multiwavelets. Below equations (2),(3) define the impulse responses of two multifilters for DGHM.

\[ c_n(n) \rightarrow G_{n0}(e^{j\omega n}) \rightarrow y_{n0}(n) \]

\[ c_n(n) \rightarrow G_{n1}(e^{j\omega n}) \rightarrow y_{n1}(n) \]

Fig. 5. Cross-connected scalar filters for realization of LP multifilter \( G_{0}(z) \)

For implementation of DMWT with multiwavelet CL the matrix impulse responses of multifilters are defined by (4),(5) and the impulse responses for SA4 multiwavelet by (6),(7), while the multiwavelet BiHermite is defined by (8),(9).

Multifilters \( G_0(z) \) and \( G_1(z) \) shown in Fig. 1 can be realized by the cross-connected scalar filters [14, 21]. Figure 5 shows such connection for an LP multifilter and an HP multifilter can be created in the same way.

The impulse responses of the individual scalar filters of DGHM LP multifilter are defined by (10)-(13) and HP filters are defined by (14-17).

The transfer functions of these scalar filters are shown in Fig. 6. It can be seen from plots in Fig. 6(a) that LP multifilter is created by scalar filters which have properties of both LP and HP filter. The transfer function of HP multifilter in Fig. 6(a) shows similar attributes. It can be seen from Fig. 6(a) that scalar filters with the impulse responses \( g_{1,1}(n) \) a \( g_{2,1}(n) \) are LP and the scalar filter \( g_{2,2}(n) \) is HP. The scalar filter \( g_{1,2}(n) \) is a band-
pass (BP) filter with constant attenuation in the whole frequency band.

Plots in Fig. 6(a) show that the scalar filters \( g_{1,1}^0(n) \) and \( g_{1,2}^1(n) \) are LP, the filter \( g_{2,2}^0(n) \) is HP and \( g_{2,2}^1(n) \) is BP. The transfer functions of scalar filters in Fig. 6 are crossed circa in the middle point \( \omega = \pi/2 \) of a frequency band. The impulse responses for another multiwavelets are obtained in the same way. The impulse response for multiwavelet CL are defined by (18-25) and the transfer functions of these filters are shown in Fig. 7.

In contrast with the multifiers for implementation of DGHM the character of scalar filters for CL corresponds to the character of a particular multifier. LP multifier with transfers in Fig. 7(a) is created by the LP scalar filters \( g_{1,1}^0(n), g_{2,2}^0(n) \) and the BP scalar filters \( g_{1,2}^0(n), g_{2,1}^0(n) \). The HP multifier, Fig. 7(b), is created by the HP scalar filters \( g_{1,2}^1(n), g_{2,1}^1(n) \) and the BP scalar filters \( g_{1,1}^1(n), g_{2,2}^1(n) \).

Impulse responses for BiHermite multiwavelet are defined by the next equations (26-33) and its transfer functions are shown in Fig. 8.

LP multifier is created by scalar filters with LP and BP character as it can be seen from Fig. 8(a). For this multifier LP scalar filter is dominant with the im-

\[
\begin{align*}
G_{1,1}^0(z) &\rightarrow g_{1,1}^0(n) = \left\{ \begin{array}{c} 3/10, 3/10 \\ 0, 0 \end{array} \right. \\
G_{1,2}^0(z) &\rightarrow g_{1,2}^0(n) = \left\{ \begin{array}{c} 2\sqrt{2}/5 \\ 0, 0 \end{array} \right. \\
G_{2,1}^0(z) &\rightarrow g_{2,1}^0(n) = \left\{ \begin{array}{c} \sqrt{2}/10, \sqrt{2}/10, -\sqrt{2}/10 \\ 0, 0 \end{array} \right. \\
G_{2,2}^0(z) &\rightarrow g_{2,2}^0(n) = \left\{ \begin{array}{c} \sqrt{2}/2, 0, -2\sqrt{2}/20 \\ 0, 0 \end{array} \right. \\
G_{1,1}^1(z) &\rightarrow g_{1,1}^1(n) = \left\{ \begin{array}{c} \sqrt{2}/10, \sqrt{2}/10, -\sqrt{2}/10 \\ 0, 0 \end{array} \right. \\
G_{1,2}^1(z) &\rightarrow g_{1,2}^1(n) = \left\{ \begin{array}{c} 3/20, 1/20, -3/20 \\ 0, 0 \end{array} \right. \\
G_{2,1}^1(z) &\rightarrow g_{2,1}^1(n) = \left\{ \begin{array}{c} 1/20, 9/20, 1/20 \\ 0, 0 \end{array} \right. \\
G_{2,2}^1(z) &\rightarrow g_{2,2}^1(n) = \left\{ \begin{array}{c} 3\sqrt{2}/2, 0, 3\sqrt{2}/20 \\ 0, 0 \end{array} \right. \\
\end{align*}
\]

\[
\begin{align*}
g_{1,1}^0(n) & = \left\{ \begin{array}{c} 1/2\sqrt{2}, -1/2\sqrt{2}, 1/2\sqrt{2} \\ 0, 0 \end{array} \right. \\
g_{1,2}^0(n) & = \left\{ \begin{array}{c} 1/2\sqrt{2}, 0, 1/2\sqrt{2} \\ 0, 0 \end{array} \right. \\
g_{1,2}^1(n) & = \left\{ \begin{array}{c} 1/4\sqrt{2}, 0, 1/4\sqrt{2} \\ 0, 0 \end{array} \right. \\
g_{2,1}^0(n) & = \left\{ \begin{array}{c} 1/4\sqrt{2}, -1/4\sqrt{2}, 1/4\sqrt{2} \\ 0, 0 \end{array} \right. \\
g_{2,2}^0(n) & = \left\{ \begin{array}{c} 1/2\sqrt{2}, 0, 1/2\sqrt{2} \\ 0, 0 \end{array} \right. \\
\end{align*}
\]

Fig. 8. Transfer functions of individual scalar filters of cross-connected (a) – LP, and (b) – HP multifier for BiHermite multiwavelet

Fig. 9. Transfer functions of individual scalar filters of cross-connected (a) – LP, and (b) – HP multifier for SA4 multiwavelet
4 Analysis of compression properties of DMWT

The compression properties of individual multiwavelets are evaluated in this part of the article. The evaluation is done on the base of the ratio of energy stored in subimages and energy of an input image. This comparison can be done only if the output of DMWT is normalized. Thus, the energy of the image after the transform $E_{\text{DMWT}}$ and the energy before the transform $E_{\text{IMG}}$ have to be equal.

$$\frac{E_{\text{DMWT}}}{E_{\text{IMG}}} = 1$$

This ratio for each multi-wavelet was verified on 72 test images [22] which were transformed and subsequently the energy ratios were computed. The results are listed in Tab. 1.

Table 1. Statistical evaluation of energy ratio between original and transformed image for DGHM, CL, BiHermite and SA4 multiwavelet

| Multiwavelet | Mean of ratio $E_{\text{DMWT}}/E_{\text{IMG}}$ | Standard deviation |
|--------------|---------------------------------------------|--------------------|
| DGHM         | 0.2503                                      | 0.0011             |
| CL           | 0.9992                                      | 0.0098             |
| BiHermite    | 0.1714                                      | 0.0020             |
| SA4          | 1.0003                                      | 0.0039             |

It is clear from Tab. 1 that output of DMWT with CL and SA4 multiwavelet are normalized but it is necessary to adjust multifilters for DGHM and BiHermite multiwavelets. For DGHM it is sufficient to multiply each multifilter with a constant $\frac{\sqrt{2}}{\sqrt{\pi}}$. Equations (43 and 44) define the normalized impulse responses. By the same process it was concluded that the first row of multifilters for multiwavelet BiHermite has to be multiplied by a constant of $\frac{\sqrt{2}}{\sqrt{\pi}}$. The corrected BiHermite multifilter impulse responses are defined by equations (45 and 46).

The mean values of the defined energy ratio (42) for DGHM and BiHermite multiwavelets are listed in Tab. 2.

Table 2. Statistical evaluation of energy ratio between original and transformed image for normalized multiwavelet DGHM and BiHermite

| Multiwavelet | Mean of ratio $E_{\text{DMWT}}/E_{\text{IMG}}$ | Standard deviation |
|--------------|---------------------------------------------|--------------------|
| DGHM         | 1.0000                                      | 0.0008             |
| BiHermite    | 1.0055                                      | 0.0177             |

DMWT with each multiwavelet for the Lena image with the subband energy distribution is shown in Fig. 10. The transformed images are shown in their absolute value for a better illustration.

On the base of the energy distribution shown in Fig. 10 it can be concluded that the energy of image is concentrated in the LP band ($L_1 L_1, L_1 L_2, L_2 L_1$ and $L_2 L_2$ subimages). The transform with the DGHM multiwavelet

$$g_{1,1}^1(n) = \left\{ k, k\vartheta^2, k\vartheta^2, k \right\}$$  (34)
$$g_{1,2}^1(n) = \left\{ k\vartheta, k\vartheta, -k\vartheta, -k\vartheta \right\}$$  (35)
$$g_{2,1}^1(n) = \left\{ k\vartheta, k\vartheta, -k\vartheta, -k\vartheta \right\}$$  (36)
$$g_{2,2}^1(n) = \left\{ k, k\vartheta^2, k\vartheta^2, k \right\}$$  (37)
$$g_{1,1}^2(n) = \left\{ k, -k\vartheta^2, k\vartheta^2, -k \right\}$$  (38)
$$g_{1,2}^2(n) = \left\{ -k\vartheta, k\vartheta, k\vartheta, -k\vartheta \right\}$$  (39)
$$g_{2,1}^2(n) = \left\{ k\vartheta, -k\vartheta, -k\vartheta, k\vartheta \right\}$$  (40)
$$g_{2,2}^2(n) = \left\{ k, k\vartheta^2, -k\vartheta^2, k \right\}$$  (41)

where, $\vartheta = 4 + \sqrt{15}$ and $k = \frac{1}{(\vartheta^2 + 1)\sqrt{2}}$.
\begin{align}
    g_0(0) &= \sqrt{2} \begin{bmatrix}
        \frac{3}{10} & 2\sqrt{2} / \sqrt{5} \\
        -\frac{1}{\sqrt{2}} & -\frac{3}{20}
    \end{bmatrix} \\
    g_0(1) &= \sqrt{2} \begin{bmatrix}
        \frac{3}{10} & 0 \\
        9\sqrt{2} / 40 & 1
    \end{bmatrix} \\
    g_0(2) &= \sqrt{2} \begin{bmatrix}
        0 & 0 \\
        9\sqrt{2} / 40 & -\frac{3}{20}
    \end{bmatrix} \\
    g_0(3) &= \sqrt{2} \begin{bmatrix}
        0 & 0 \\
        -\frac{\sqrt{2}}{40} & 0
    \end{bmatrix} \\
    g_1(0) &= \sqrt{2} \begin{bmatrix}
        -\sqrt{2} / 40 & -\frac{3}{20} \\
        1 & -3\sqrt{2} / 20
    \end{bmatrix} \\
    g_1(1) &= \sqrt{2} \begin{bmatrix}
        9\sqrt{2} / 40 & -\frac{1}{2} \\
        -\frac{9}{20} & 0
    \end{bmatrix} \\
    g_1(2) &= \sqrt{2} \begin{bmatrix}
        9\sqrt{2} / 40 & -\frac{3}{20} \\
        -\frac{9}{20} & 3\sqrt{2} / 20
    \end{bmatrix} \\
    g_1(3) &= \sqrt{2} \begin{bmatrix}
        -\frac{\sqrt{2}}{40} & 0 \\
        -\frac{9}{20} & 0
    \end{bmatrix} \\
\end{align}

\begin{align}
    g_0(0) &= \sqrt{2} \begin{bmatrix}
        \frac{27\sqrt{2}}{5} & -\frac{27\sqrt{2}}{4} \\
        -\frac{1}{8\sqrt{2}} & 1
    \end{bmatrix} \\
    g_0(1) &= \sqrt{2} \begin{bmatrix}
        \frac{27\sqrt{2}}{5} & 0 \\
        -\frac{1}{2\sqrt{2}} & 1
    \end{bmatrix} \\
    g_0(2) &= \sqrt{2} \begin{bmatrix}
        \frac{27\sqrt{2}}{5} & \frac{27\sqrt{2}}{8} \\
        \frac{1}{8\sqrt{2}} & 1
    \end{bmatrix} \\
    g_0(3) &= \sqrt{2} \begin{bmatrix}
        \frac{27\sqrt{2}}{5} & -\frac{27\sqrt{2}}{16} \\
        -\frac{3}{8\sqrt{2}} & \frac{1}{4\sqrt{2}}
    \end{bmatrix} \\
    g_1(0) &= \sqrt{2} \begin{bmatrix}
        \frac{27\sqrt{2}}{5} & -\frac{27\sqrt{2}}{16} \\
        -\frac{3}{8\sqrt{2}} & \frac{1}{4\sqrt{2}}
    \end{bmatrix} \\
    g_1(1) &= \sqrt{2} \begin{bmatrix}
        0 & \frac{27\sqrt{2}}{5} \\
        \frac{1}{2\sqrt{2}} & 0
    \end{bmatrix} \\
    g_1(2) &= \sqrt{2} \begin{bmatrix}
        \frac{27\sqrt{2}}{5} & -\frac{27\sqrt{2}}{16} \\
        -\frac{3}{8\sqrt{2}} & \frac{1}{4\sqrt{2}}
    \end{bmatrix} \\
    g_1(3) &= \sqrt{2} \begin{bmatrix}
        -\frac{\sqrt{2}}{40} & 0 \\
        -\frac{9}{20} & 0
    \end{bmatrix} \\
\end{align}

\begin{center}
\begin{tabular}{cccc}
45.59 & 19.82 & 1.99 & 0.11 \\
19.78 & 8.61 & 0.88 & 0.08 \\
1.93 & 0.86 & 0.11 & 0.04 \\
0.07 & 0.06 & 0.03 & 0.05 \\
\end{tabular}
\end{center}
\begin{center}
\begin{tabular}{cccc}
24.95 & 20.58 & 4.32 & 0.09 \\
20.62 & 17.01 & 3.55 & 0.06 \\
4.29 & 3.53 & 0.79 & 0.05 \\
0.06 & 0.04 & 0.04 & 0.02 \\
\end{tabular}
\end{center}

\begin{center}
\begin{tabular}{cccc}
45.09 & 18.73 & 3.27 & 0.05 \\
18.74 & 7.78 & 1.34 & 0.02 \\
3.10 & 1.27 & 0.51 & 0.03 \\
0.03 & 0.01 & 0.03 & 0.00 \\
\end{tabular}
\end{center}
\begin{center}
\begin{tabular}{cccc}
24.70 & 24.78 & 0.09 & 0.13 \\
24.73 & 24.82 & 0.10 & 0.15 \\
0.05 & 0.06 & 0.03 & 0.05 \\
0.08 & 0.01 & 0.05 & 0.07 \\
\end{tabular}
\end{center}

\textbf{Fig. 10.} Image transformed by DMWT and its subimage energy distribution in % for normalized multiwavelet (a) – DGHM, (b) – CL, (c) – BiHermite, and (d) – SA4.

It can be seen that SA4 in the other subimages concentrates less than 1% of the total energy. Other multiwavelets left much more energy in HP subimages. The

concentrates 93.8% of the energy in the LP subimages, for CL it is 83.16%, for BiHermite 90.34% and the highest compression of 99.03% is achieved by SA4 multiwavelet.
Table 3. Mean energy values of LP band for DMWT with DGHM, CL, BiHermite and SA4 multiwavelet

| Multiwavelet | L-subband | Mean(%) | Std(%) |
|--------------|-----------|---------|--------|
| DGHM         | $L_1L_1$  | 44.7085 | 2.0528 |
|              | $L_1L_2$  | 19.4907 | 0.6716 |
|              | $L_2L_1$  | 19.5289 | 0.6652 |
|              | $L_2L_2$  | 8.5517  | 0.1689 |
|              | $L_{sum}$ | 92.2772 | 3.4387 |
| CL           | $L_1L_1$  | 24.5937 | 0.9901 |
|              | $L_1L_2$  | 20.2894 | 0.7375 |
|              | $L_2L_1$  | 20.2659 | 0.7570 |
|              | $L_2L_2$  | 16.7556 | 0.8067 |
|              | $L_{sum}$ | 81.9048 | 2.9194 |
| BiHermite    | $L_1L_1$  | 43.9182 | 2.4780 |
|              | $L_1L_2$  | 18.2835 | 1.0079 |
|              | $L_2L_1$  | 18.2765 | 1.0120 |
|              | $L_2L_2$  | 7.6170  | 0.4976 |
|              | $L_{sum}$ | 88.0946 | 4.7957 |
| SA4          | $L_1L_1$  | 24.1023 | 1.3032 |
|              | $L_1L_2$  | 24.3110 | 1.0408 |
|              | $L_2L_1$  | 24.3228 | 0.9990 |
|              | $L_2L_2$  | 24.5620 | 0.9140 |
|              | $L_{sum}$ | 97.2976 | 4.0311 |

Comparison of the mean value energy stored in LP subbands is shown in Tab. 3.

These values were obtained on the base of transform of 72 different images.

Table 3 shows that DMWT with SA4 multiwavelet achieved the highest energy compression into LP subimages. In accordance with the previous experiments shown in Fig. 10 it can be concluded that the energy for SA4 is distributed evenly into the L subband, contrary to this other multiwavelets concentrate the energy into $L_1L_1$ subimage. On the base of standard deviation values it can be concluded that DMWT with BiHermite multiwavelet is the most sensitive to the image content and on the other hand CL multiwavelet is less sensitive to image content changes.

5 Banks of single filters

Banks of multifilters used for implementation of DMWT are created by the cross-connected scalar filters. These 2 channel (2CH) banks implement LP (approximation) and HP (detail) subbands. The realization of DMWT in this way is not very advantageous. It needs 8 scalar filters which are in addition preceded by multifilters. These pre-filters perform change of the input signal from scalar form to its vector form. An equivalent replacement of 2CH multifilters bank by four channel (4CH) single filter bank [23] shown in Fig. 11 was analyzed.

Figure 11 shows that 4CH single filter bank implementation of DMWT does not need prefiltering as it was seen in the case of 2CH multifilters bank. The input signal is brought directly on the input of this equivalent 4CH bank. This signal is subsequently filtered by scalar filters in 4 parallel branches. After decimation by factor 4 all sub-bands of DMWT are directly obtained on the outputs of these branches. Scalar filters with the transfer functions $G_0(z)$ and $G_1(z)$ implement LP subimages and on the other hand filters with the transfer functions $G_2(z)$ and $G_3(z)$ implement HP subbands. The transfer functions are created from scalar filters of 2CH multifilter bank by using (47-50).

In (49 and 50) a substitution of $z \rightarrow z^2$ is used. This substitution is caused by the order change of filtering and decimation [24]. The mathematical process of replacement of 2CH multifilter bank by 4CH single filters bank for DGHM multiwavelet is shown in the next text.

$$
\mathbf{G}_0(z) = \begin{bmatrix} G_{0,1}(z) \\ G_{0,2}(z) \\ G_{1,1}(z) \\ G_{1,2}(z) \end{bmatrix}
$$

$$
\mathbf{G}_1(z) = \begin{bmatrix} G_{0,1}(z) \\ G_{0,2}(z) \\ G_{1,1}(z) \\ G_{1,2}(z) \end{bmatrix}
$$

$$
\begin{bmatrix} G_0(z) \\ G_1(z) \end{bmatrix} = \mathbf{G}_0(z^2) \left[ \begin{array}{c} 1 \\ z \end{array} \right]
$$

$$
\begin{bmatrix} G_2(z) \\ G_3(z) \end{bmatrix} = \mathbf{G}_1(z^2) \left[ \begin{array}{c} 1 \\ z \end{array} \right]
$$

In (49 and 50) a substitution of $z \rightarrow z^2$ is used. This substitution is caused by the order change of filtering and decimation [24]. The mathematical process of replacement of 2CH multifilter bank by 4CH single filters bank for DGHM multiwavelet is shown in the next text.
The impulse responses for these transfer functions are

\[
\begin{bmatrix}
g_0(n) \\
g_1(n)
\end{bmatrix} = \sqrt{2} \begin{bmatrix}
\frac{3\pi}{4} & 0 & 0 & 0 \\
\frac{3\pi}{4} & 0 & 0 & 0
\end{bmatrix}
\]

(54)

The impulse responses in (54) implement LP filters. For HP transfer functions we can write (55). Thus, the impulse responses for HP subband are defined by (56).

\[
\begin{bmatrix}
G_2(z) \\
G_3(z)
\end{bmatrix} = \sqrt{2} \begin{bmatrix}
\frac{3\pi}{4} & 0 & 0 & 0 \\
\frac{3\pi}{4} & 0 & 0 & 0
\end{bmatrix}
\]

(55)

\[
\begin{bmatrix}
g_2(n) \\
g_3(n)
\end{bmatrix} = \sqrt{2} \begin{bmatrix}
\frac{3\pi}{4} & 0 & 0 & 0 \\
\frac{3\pi}{4} & 0 & 0 & 0
\end{bmatrix}
\]

(56)

The impulse responses in (54) implement LP filters. For HP transfer functions we can write (55). Thus, the impulse responses for HP subband are defined by (56).

For impulse responses in (54) and (56) we can display the transfer functions shown in Fig. 12(a) and therefore it can be concluded that each of these scalar filters show amplification in a different part of the frequency spectrum. The filter with the transfer \( G_0(z) \) covers the first quarter of the frequency band. In the second and third part the frequency band is covered by the transfer \( G_3(z) \) and \( G_1(z) \). The last part of the band is covered by filter with the transfer \( G_2(z) \). Filters behave as attenuation in the other parts of the frequency band. The filters \( G_0(z) \) and \( G_1(z) \) implement a scale function, which presents an approximation of the input signal. It is clear from Fig. 12(a), that these filters attenuate the input signal for frequencies \( \omega \in (0, \pi/4) \) as it can be seen from Fig. 12(a).

The impulse responses and transfer functions for other multiwavelets can be obtained in the same way as for DGHM multiwavelet. To be completely, the impulse responses for CL, BiHermite and SA4 multiwavelets are mentioned in the next part of the article.

The impulse responses of 4CH bank of single filter for DMWT with CL multiwavelets are defined by (57-60) and transfer functions are in Fig. 12(b). These functions show similar properties as it was observed for DGHM multiwavelet. Each filter shows amplification for different quarter of the frequency band. Thus, each frequency band of the transformed signal is created by frequencies of the particular quarter and only in small contribution by frequencies of the other bands. The LP filters \( G_0(z) \) and \( G_1(z) \) present more likely a band stop for frequencies \( \omega \in (\pi/4, 3\pi/4) \) and on the other hand, the HP filters \( G_2(z) \) and \( G_3(z) \) present a band pass in this subband. The filter \( G_2(z) \) allows frequencies for \( \omega \in (\pi/4, \pi/2) \) and
The filter $G_3(z)$ allows frequencies for $\omega \in (\pi/2, 3\pi/4)$. The impulse response of 4CH bank of single filters for DMWT with BiHermite multiwavelet are given by (61-64) and the transfer functions are in Fig. 12(c).

For impulse responses in (54 and 56) we can display the transfer functions shown in Fig. 12(a) and therefore it can be concluded that each of these scalar filters show amplification in a different part of the frequency spectrum. The filter with the transfer $G_0(z)$ covers the first quarter of the frequency band. In the second and third part the frequency band is covered by the transfer $G_3(z)$ and $G_1(z)$ . The last part of the band is covered by filter with the transfer $G_2(z)$. Filters behave as attenuation in the other parts of the frequency band. The filters $G_0(z)$ and $G_1(z)$ implement a scale function, which presents an approximation of the input signal. It is clear from Fig. 12(a), that these filters attenuate the input signal for frequencies $\omega \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{4}, \pi\right)$ and on the other hand the filters $G_2(z)$ and $G_3(z)$ implement wavelet functions which represent details of the input image. These filters attenuate the input signal for $\omega \in \left(0, \frac{\pi}{4}\right)$ as it can be seen from Fig. 12(a).

The impulse responses and transfer functions for other multiwavelets can be obtained in the same way as for DGHM multiwavelet. To be completely, the impulse responses for CL, BiHermite and SA4 multiwavelets are mentioned in the next part of the article. The impulse responses of 4CH bank of single filter for DMWT with CL multiwavelets are defined by (57-60).

The filters described by impulse responses (57-60) have transfer functions depicted in Fig. 12(h). These functions show similar properties as it was observed for DGHM multiwavelet. Each filter shows amplification for different quarter of the frequency band. Thus, each frequency band of the transformed signal is created by frequencies of the quarter and only in small contribution by frequencies of the other bands. The LP filters $G_0(z)$ and $G_1(z)$ present more likely a band stop for frequencies $\omega \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$ and on the other hand, the HP filters $G_2(z)$ and $G_3(z)$ present a band pass in this subband. The filter $G_2(z)$ allows frequencies for $\omega \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ and the filter $G_3(z)$ allows frequencies for $\omega \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$. The impulse response of 4CH bank of single filters for DMWT with BiHermite multiwavelet are given by the next equations (61-64) and the transfer functions are shown in Fig. 12(c).

For the transfer function in Fig. 12(c) it is possible to conclude that the filters $G_0(z)$ and $G_1(z)$ are band stop for $\omega \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$. The filter with the transfer $\omega \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ allows frequencies for $\omega \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$ and the filter $G_3(z)$ appears as an attenuation in the whole frequency band. The value of this attenuation is lowest for $\omega \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. The impulse responses for 4CH bank of single filters for DMWT with SA4 multiwavelet are defined by equations (65-68) and their transfer function are shown in Fig. 12(d).

The same as for CL and BiHermite we can conclude that the SA4 multiwavelet filters $G_0(z)$ a $G_1(z)$ implement approximation subband of DMWT and they are of band pass character for $\omega \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$. On the other hand, this particular frequency interval is allowed by the filters $G_2(z)$ and $G_3(z)$ which implement detail subband of DMWT. Contrary to the other multiwavelets SA4 multiwavelet shows a significant symmetry of the transfer functions.

5.1 Experimental equivalency evaluation of 2CH and 4CH banks for considered multiwavelets

The experiment was performed with 72 images for confirmation of 2CH multfilter and 4CH single filter bank equivalency. These images were transformed by both approaches for all 4 considered multiwavelets. In Tab. 4 mean values of SNR and standard deviation in dB are listed. The value of SNR is computed by (69). Where $I_{MDWT_{2CH}}$ represents an image transformed by 2CH bank and $I_{MDWT_{4CH}}$ represents an image transformed by 4CH bank.

\begin{align*}
g_0(n) &= \frac{1}{\sqrt{2}} \left[ \begin{array}{cccc} 1 & 1 & 0 & 1 \\ -1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \end{array} \right] \quad (57) \\
g_1(n) &= \frac{1}{\sqrt{2}} \left[ \begin{array}{cccc} 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ \end{array} \right] \quad (58) \\
g_2(n) &= \frac{1}{\sqrt{2}} \left[ \begin{array}{cccc} -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ \end{array} \right] \quad (59) \\
g_3(n) &= \frac{1}{\sqrt{2}} \left[ \begin{array}{cccc} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \end{array} \right] \quad (60) \\
g_0(n) &= k \left[ \vartheta \quad 1 \quad \vartheta^2 \quad -\vartheta \quad \vartheta^2 \quad -\vartheta \quad 1 \right] \quad (65) \\
g_1(n) &= k \left[ 1 \quad \vartheta \quad \vartheta^2 \quad \vartheta \quad 1 \quad -\vartheta \right] \quad (66) \\
g_2(n) &= k \left[ -\vartheta \quad 1 \quad -\vartheta^2 \quad \vartheta \quad \vartheta^2 \quad -\vartheta \quad -1 \right] \quad (67) \\
g_3(n) &= k \left[ 1 \quad \vartheta \quad -\vartheta^2 \quad -\vartheta \quad -\vartheta \quad -\vartheta \quad -1 \right] \quad (68)
\end{align*}
$$SNR = 10 \log \left( \frac{\sum_M \sum_N I_{DMWT,2k}^2}{\sum_M \sum_N (I_{DMWT,2k} - I_{DMWT,4k})^2} \right)$$

Table 4. Statistical evaluation of SNR between DMWT implemented by 2CH multifilter bank and 4CH single filter bank

| Multiwavelet | Mean value of SNR (dB) | Standard deviation (dB) |
|--------------|------------------------|-------------------------|
| DGHM         | 316.0696               | 0.3129                  |
| CL           | 318.3322               | 1.5975                  |
| SA4          | 315.7196               | 0.8287                  |
| BiHermite    | 317.2854               | 0.8531                  |

On the base of Tab. 4 it is possible to conclude that the implementations of DMWT performed by 2CH multifilter bank and 4CH single filter bank are equivalent.

6 Possible reduction and combinations of subbands

From the previous chapters it is clear that DWMT produces 4 times more subimages than classical DWT. For purposes of the further processing it is possible to modify this subimage representation in to its reduced form.

The three described modifications of the subimage combinations will be called option 1, 2 and 3.

6.1 All subband reduction (option 1)

This approach reduces all DMWT subbands. From Fig. 13 it can be seen that the approximation coefficients of 4CH DMWT are combined into one channel with the output sequence $u(n)$. The detail coefficients are combined into the sequence $v(n)$ in the same way. By a separate application of this connection on the rows and columns of the image the reduced transformed image is obtained. This transformed image compared to the regular DMWT image has 4 times less subimages as it is possible to observe from Fig. 13(b).

This subimage reduction is useful when it is necessary to compare DMWT with the classical DWT. For example, this option is advantageous for the compression properties comparison. The result of DMWT shown in Fig. 13(b) is very similar to the output of the classical DWT but it needs to be mentioned that the transformed image in Fig. 13(b) is still result of DMWT only with combined subimages. Because the number and arrangement of subimages are comparable to DWT, this option can be used for modification of standard JPEG2000 where the classical DWT with the scalar wavelet (9/7) is used. There is no need for a deep modification of the standard. For modification of JPEG2000 it is sufficient to replace DWT with MDWT with the subband reduction according to option 1. The other blocks of JPEG2000 stay unchanged after this modification. During decoding by the receiver the subbands are easily expanded into the original DMWT shape. The compression properties of DMWT with all the considered multiwavelets and the
classical DWT (9/7) are experimentally evaluated in Tab. 5, where mean values and theirs standard deviations of the subband energy distribution are listed. Experiment was performed with 72 input images. The best results were obtained by DMWT with SA4 multiwavelet, where the energy compression into LL subband is the highest, as it can be seen from Tab. 5. Classical DWT with the scalar wavelet (9/7) achieved worse results.

6.2 Approximation subband reduction (option 2)

This modification reduces only number of approximation subbands. The reduction is performed in the same way as in the option 1 but number of detail subbands stays unchanged. The resulting subband combination for DMWT of Lena is shown in Fig. 14(b).

This option can be used for implementation of the second level image transform. DMWT performed by 4CH

Fig. 13. (a) – block diagram of 4CH equivalent single filter bank with all subband reduction, and (b)– image transformed by DMWT with this reduction

Fig. 14. (a) – block diagram of 4CH equivalent bank with approximation subband reduction, and (b)– DMWT of image with reduction of approximation subbands

Fig. 15. Block diagram of two level DMWT implemented by 4CH equivalent single filters bank
single filter bank is applied on the input signal only in one channel in contrast with 2CH multifilter implementation were the second stage is connected directly on two LP outputs (Fig. 3). For 4CH implementation these LP outputs have to be joined as Fig. 15(a) shows. The block diagram of two level DMWT implemented by 4CH single filter bank is shown in Fig. 15. The Lena image transformed by DMWT with 4CH single filter bank is shown in Fig. 16.

### Table 5. Statistical evaluation of DMWT and classical DWT energy compression properties

| Transform   | Subbands | Mean(%) | Std(%) |
|-------------|----------|---------|--------|
| DGHM        | LL       | 92.277  | 3.4387 |
|             | LH       | 3.6001  | 1.3771 |
|             | HL       | 3.6675  | 1.5912 |
|             | HH       | 0.4559  | 0.7421 |
| CL          | LL       | 81.905  | 2.9194 |
|             | LH       | 8.4310  | 1.0440 |
|             | HL       | 8.4727  | 1.2217 |
|             | HH       | 1.1920  | 0.9159 |
| BiHermite   | LL       | 88.095  | 4.7957 |
|             | LH       | 5.3734  | 1.7404 |
|             | HL       | 5.5066  | 2.2439 |
|             | HH       | 1.0251  | 1.3475 |
| SA4         | LL       | 97.298  | 4.0311 |
|             | LH       | 1.0849  | 1.5497 |
|             | HL       | 1.1515  | 1.8500 |
|             | HH       | 0.4656  | 0.9883 |
| Wavelet transform (9,7) | LL       | 97.201  | 2.1722 |
|             | LH       | 1.4052  | 1.0400 |
|             | HL       | 1.2384  | 0.9422 |
|             | HH       | 0.1559  | 0.4309 |

### 6.3 Detail subband reduction (option 3)

For application of DMWT focused on analysis of approximation subband it is possible to leave the approximation subband unchanged but detail subbands can be reduced. The block diagram of this reduction is shown in Fig. 17(a) and DMWT of the Lena image with detail subimages reduced in this way is shown in Fig. 17(b).

Besides the previously mentioned modifications it is possible perform a reduction only of selected detail subbands. DMWT can be used not only for compression but for other applications as well. One of these applications is classification of textures and objects. In these cases it is often necessary to analyze detail coefficients of the horizontal and vertical detail subimages. The diagonal subimages contain a noise at higher level than the mentioned detail subimages. In this case the subbands $L_1H_1$, $L_1H_2$, $L_2H_1$, $L_2H_2$, $H_1L_1$, $H_1L_2$, $H_2L_1$, and $H_2L_2$ are analyzed and the subbands $H_1H_1$, $H_1H_2$, $H_2H_1$ and $H_2H_2$ are reduced into one $HH$ subband. The result of this reduction is shown in Fig. 18.
7 Conclusion

The implementations of image DMWT were analyzed in this article. DMWT with multiplicity \( r = 2 \) implemented by multifilters bank, where impulse responses are defined as matrices of the size \( 2 \times 2 \), was discussed in introduction of the article. The implementation of these multifilters by the cross-connected scalar filters was shown in the next part of the article.

By this connection it is possible to perform DMWT for arbitrary multiwavelet functions. The main drawback of this implementation is the need of an input signal prefiltering which splits this signal into 2 inputs of multifilter. The fact that for implementation of DMWT the LP and HP band 8 scalar filters are needed is another drawback. Transfer functions were analyzed in detail for all these scalar filters. The normality of DMWT outputs for the considered multiwavelets was verified on the base of experimental results. For the multiwavelets without normalized outputs the normalization constants were specified and impulse responses of the multifilters were corrected. The compression properties of several multiwavelets were evaluated. From the results it can by concluded that the best energy compression into LP subbands is achieved by DMWT with the SA4 multiwavelet. In the next part of this article the equivalent single filter bank implementation of DMWT was analyzed. The detail mathematical procedure of creation of the single filter impulse responses from the cross-connected scalar filter was described. These single filter bank impulse responses were derived for all the considered multiwavelets. Experimentally was confirmed that the proposed 4CH single filter bank solves the previously mentioned drawbacks of the cross-connected scalar filter bank implementation of DMWT. The transformed image is obtained directly from input image which in this case is not preprocessed by prefilters. The 4 parallel connected filters provide directly 4 outputs belonging to each subband of DMWT outputs. The equivalency of 4CH single filter bank and 2CH multifilter bank was experimentally confirmed as well. The attention in the final part of the article was focused on possible ways of subband combination. There were shown options which can be useful for different MDWT practical applications. The reduction of all subbands provides the output which structurally matches with the output of the classical DWT. This subband combination allows to replace the classical DWT by DMWT in standard JPEG 2000. The experimental results show that DMWT with SA4 multiwavelet achieves a better energy compression into LP subband than the classical DWT with wavelet (9/7). By the reduction of only LP band it is possible to get input for the second level DWMT implemented by 4CH single filter bank. The other mentioned methods of detail subimages combination can be useful in the field of image analysis, object and textures classification.

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