COBE and SUSY

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Abstract
We show that supersymmetry automatically leads to density fluctuations
\(\Delta T/T \sim 6 \times 10^{-6}\) in agreement with the recent COBE measurement.

1. Introduction

The cosmic microwave background radiation (CMBR) is of great interest to cosmologists because it is thought to faithfully reflect conditions in the early universe when electrons recombined with nuclei to form neutral atoms [1]. After this event, which happened when the universe was about 100,000 years old, photons stopped scattering off matter and simply redshifted to the 2.735K blackbody spectrum observed today. Perhaps the most interesting feature of the CMBR is its very small degree of temperature fluctuations \(\Delta T/T \sim 6 \times 10^{-6}\), which has recently been measured for the first time by the COBE satellite [2]. This measurement has given strong support to the idea that the CMBR fluctuations \(\Delta T/T\) derive from the same source as the matter fluctuations \(\delta \rho/\rho \sim 10^{-4}\) which were necessary to seed galaxy formation. In the most popular theories that incorporate this concept [3], both \(\Delta T/T\) and \(\delta \rho/\rho\) are produced by perturbations of a scalar quantum field \(\phi\) during a primordial inflationary period. Typical scenarios using cold dark matter predict that \(\delta \rho/\rho \sim (10 \sim 100)\Delta T/T\) [1], which is consistent with COBE’s measurement.

If the Lagrangian responsible for inflation contains the scalar field \(\phi\) in the generic form \(\lambda \phi^4\), then a given value of \(\delta \rho/\rho\) requires a value of \(\lambda = v_2 (\pi \lambda s)^2 N^{-3} (\delta \rho/\rho)^2\), where \(N\) is the number of e-foldings before the end of inflation that the relevant perturbations “leave the horizon” of the inflationary universe. For \(\delta \rho/\rho = 10^{-4}\) and \(N = 60\) (the number required to insure the flatness and homogeneity of the universe today) this demands a value of \(\lambda \sim 10^{-13}\); in realistic models [4,1] a lower \(N\) is usually relevant, but still a \(\lambda \sim 10^{-11} \sim 10^{-12}\) is required. The main difficulty in implementing inflationary models has always been the need to motivate this extremely small dimensionless parameter \(\lambda \sim 10^{-11} \sim 10^{-12}\). In non-supersymmetric theories, this...
has been achieved by unnatural fine-tuning of parameters and/or the introduction of ad hoc, non-renormalizable potentials for observable or hidden-sector scalar fields [5].

We show in this paper that all realistic supersymmetric theories (i.e., those containing an electron) naturally predict this magnitude of density fluctuations. This is because the supersymmetric scalar potential automatically includes a "quasi-flat" $\lambda \phi^4$ direction with

$$\lambda = (\lambda^e)^2 = \frac{g_2^2 m_e}{m_W} (1 + \langle h^0 \rangle^2 / \langle h^0 \rangle^2)^2 \sim 10^{-11},$$

where $\lambda^e$ is the electron Yukawa coupling. It is worth emphasizing that this parameter naturally arises from the supersymmetric relationship $\lambda = (\lambda^e)^2$ between the quartic scalar potential and the superpotential, which contains the somewhat small electron Yukawa coupling $\lambda^e \sim 10^{-5}$. Moreover, this small quartic scalar coupling is multiplicatively renormalized, and thus remains small to all orders in perturbation theory.

[Another related useful feature of supersymmetry is the likely presence [6] of stable, weakly-interacting massive particles (WIMPs), which provide highly plausible cold dark matter candidates [7]. Such dark matter is essential for the initial density fluctuations described above to begin to seed galaxy formation prior to recombination, thereby giving these fluctuations sufficient time to grow [1].]

In this paper we argue that the primordial density fluctuations arose during a period of chaotic inflation [8], which seems virtually inescapable in supersymmetric theories due to the large number of scalar fields (squarks, sleptons and higgses) present in such models. At timescales $\sim M_P^{-1}$ characteristic of the early universe, the uncertainty principle makes it highly unlikely that all of these fields $\phi_i$ would be localized initially within a distance $\langle \phi_i \rangle < M_P$ of the origin. This makes it almost inevitable that one would obtain through chaotic inflation the $> 60$ e-foldings of expansion needed for a viable inflationary scenario.

We show explicitly how this chaotic inflation leads to density fluctuations $\delta \rho / \rho \sim 10^{-4}$ ($\Delta T / T \sim 10^{-5} - 10^{-6}$) consistent with COBE, and under plausible assumptions to a baryon asymmetry $n_B / s \sim 10^{-10}$.

2. The SUSY Scalar Potential

In order to identify the $\lambda \phi^4$ direction whose $\lambda$ is described by Eq. (1), we recall that the scalar potential $V$ of a supersymmetric Lagrangian is composed of both D-terms and F-terms, with

$$D = \sum_a |g_a| \sum_{kl} \phi^*_k T_a^{kl} \phi_l |^2 \text{ and } F = \sum_k |\partial W / \partial \phi_k|^2,$$

where $\phi_k$ ranges over all scalar fields in the theory, the $T^a$ are the symmetry generators, and $W$ is the superpotential. For definiteness, we will use the superpotential of the minimal supersymmetric Standard Model (MSSM) [9]:

$$W = \lambda_1^i \epsilon_{rs} Q_{i\alpha r} d^c_{\alpha s} h_s + \lambda_2^j \epsilon_{rs} Q_{i\alpha r} u_{\alpha j}^c h_s + \lambda_3^j \epsilon_{rs} L_{ir} l^c_i h_s,$$
\[i, j = \text{family indices; } r, s = SU(2); \alpha = SU(3)\], since at least these terms will be present in the superpotential of any other realistic supersymmetric theory (e.g., SUSY GUTS, etc.). For convenience, we have chosen a basis in which both \(\lambda_1\) and \(\lambda_3\) are diagonal; in particular we have \(\lambda_1^3 = \lambda^3\).

The coefficients of the D-terms (i.e., the \(g_a\)) are all presumably \(O(1)\). It is the F-terms which convert the products of possibly-small Yukawa couplings \(\lambda_{ij}^k\) in \(W\) into coefficients of quartic terms. A simple one-parameter example of a quasi-flat direction is

\[\tilde{e} = \sqrt{2a}; \quad \tilde{e}^c = \tilde{e}_\alpha = \tilde{d}_\mu = \tilde{\nu}_\mu = a,\]

where \(a\) is any complex number and all other fields are zero. (Tildes denote scalar partners of Standard Model quarks and leptons.) This direction is D-flat, and it is also F-flat except for the extremely small term \(|\partial W/\partial h_1|^2 = |\lambda^e\tilde{e}\tilde{e}^c|^2 = |\lambda^e|^2|a|^4\). A two-parameter generalization of this direction is

\[\tilde{e} = \sqrt{|a|^2 + |b|^2} e^{i\phi}; \quad \tilde{e}^c = \tilde{\nu}_\mu = a; \quad \tilde{e}_\alpha = \tilde{d}_\mu = b.\]

There are many other such quasi-flat directions, making it all the more likely that at least one such direction will have initial conditions \(|\phi| \gg \text{few} \times M_P\), generating the required \(> 60\) e-foldings of quasi-flat chaotic inflation.

3. Generation of Density Perturbations

Following the principles of chaotic inflation, we expect that all scalar fields at the Planck time \(t = M_P^{-1}\) will have values \(O(M_P)\). For a random set of initial conditions \(\phi_i^I\), those directions in field space corresponding to F and D non-flat directions evolve quickly in a \(\lambda\phi^4\)-type potential:

\[
\ddot{\phi}_i + 3H \dot{\phi}_i = -\frac{\partial V}{\partial \phi_i}, \quad H^2 = \frac{8\pi\rho}{3M_P^2}.
\]

The directions in field space with the steepest slopes evolve first, moving in toward the origin or into nearby valleys if the potential \(V(\phi)\) contains flat directions. This initial phase of evolution in a generic \(\lambda\phi^4\) potential contributes \(N \sim \pi (\phi^f/M_P)^2\) e-foldings of inflation [8], where \(\phi^f\) characterizes the magnitude of the largest initial field v.e.v.’s. [For this discussion, we assume that the scalar fields \(\phi_i\) evolve according to their classical equations of motion [Eq. (6)]. Although we expect quantum fluctuations \(\langle \phi_i^2 \rangle\) to play an important role during the initial phase of chaotic inflation, our semi-classical approximation (6) becomes a good approximation by the onset of quasi-flat inflation.]

The last of the F and D non-flat directions to evolve is the quasi-flat direction \(\Phi_{QF}\) with its quartic self-coupling \(\lambda = |\lambda_{33}^3|^2 \sim 10^{-11}\) [10]. With an initial characteristic size \(\Phi_{QF} \gtrsim 4M_P\) for any one of the many quasi-flat directions, this direction would produce \(N_{QF} \sim \pi (\Phi_{QF}/M_P)^2 \gtrsim 50\) e-foldings of inflation during the final quasi-flat
period. Standard theories [1] predict that the density perturbations which influenced galactic structure formation on distance scales between 1–1000 Mpc had their origin approximately 45-50 e-foldings before the end of inflation. This falls within the above quasi-flat inflationary period and therefore guarantees a value of $\delta \rho / \rho \sim 10^{-4}$ in accord with observation.

4. Automatic Baryogenesis

Most quasi-flat directions in the scalar potential belong to families characterized by several parameters, where the potential $V$ may be independent of one or more of these parameters. We see this in the following nine-parameter quasi-flat direction (where radicals are understood to be multiplied by arbitrary phases):

\begin{align*}
\begin{array}{c|ccccccc}
\text{Gen} & \hat{Q}_1 & \hat{Q}_2 & \bar{U} & \bar{D} & \bar{L}_1 & \bar{L}_2 & \bar{L}^c \\
1 & a_\alpha & - & b_\tau & c_\tau & - & d & e \\
2 & \sqrt{|d|^2 + |h|^2} & -|c|^2 - |a|^2_\alpha & - & f_\tau & g_\tau & - & h \\
3 & - & \sqrt{|m|^2 - |c|^2} & \sqrt{|e|^2 + |c|^2} & \sqrt{+|g|^2 - |m|^2} & \sqrt{+|c|^2 + |g|^2} & m & - \\
\end{array}
\end{align*}

In this example the potential $V$ is again given simply by $|\lambda e de|^2$. As $e$ and/or $d$ fall toward zero, the other parameters generally remain at high values [11]. In other words the quasi-flat direction rolls down into an eight-parameter valley at a large distance from the origin.

A useful feature of this eight-parameter valley is that it gives non-zero expectation value to the following baryon number-violating operator:

$$O_{BX} = \left( \sum_{\text{gen}} \hat{Q} \bar{D}^* \right) \left( \sum_{\text{gen}} \bar{L} \bar{D}^* \right)$$

(8)

(\text{the antisymmetric } SU(2) \text{ and } SU(3) \text{ indices have been suppressed}). Such flat directions have been analyzed in detail by Affleck and Dine [12], who showed that they give rise to a baryon asymmetry $n_B/s \sim 1$ under the low-temperature conditions likely to prevail after inflation [13]. (A requirement of their analysis was the lifting of the degeneracy of the flat direction through an $m^2 \phi^2$ potential induced by soft-supersymmetry breaking.) Such baryogenesis is particularly efficient when the fields $\Phi_{AD}$ in $O_{BX}$ have values in the region $M_{GUT} < \Phi_{AD} < M_P$. Affleck and Dine argued that such v.e.v.’s could be generated along a flat direction by starting at the origin and then being driven up by quantum fluctuations in the early universe. In our model, these high v.e.v.’s are the natural endpoint of evolution from the quasi-flat direction (7), providing (we feel) a much better motivated lead-in to the elegant Affleck-Dine baryogenesis scenario.
In the most successful and plausible supersymmetric GUT, Flipped $SU(5)$ [14], the GUT symmetry is broken by the v.e.v. of a weakly-coupled field $\Phi$, which corresponds to a flat direction in the scalar potential. We have shown elsewhere [4] that the slow decay of the oscillations of this weakly-coupled field about the GUT minimum in a soft supersymmetry-breaking $m^2\Phi^2$ potential generates an entropy density $s$ that is sufficient to dilute Affleck and Dine’s problematic $n_B/s \sim 1$ down to today’s observed $n_B/s \approx 10^{-10}$. Thus automatic baryogenesis can be a natural side benefit of the above mechanism for generating density perturbations.

5. Conclusion

We have demonstrated that realistic supersymmetric theories contain many quasi-flat $\lambda\phi^4$ directions with $\lambda = (\lambda^e)^2 \sim 10^{-11}$ which naturally lead to density perturbations $\delta \rho/\rho \sim 10^{-4}$ consistent with the recent COBE measurement $\Delta T/T \sim 6 \times 10^{-6}$. In addition, we have shown that many of these quasi-flat directions flow directly into Affleck-Dine-type flat valleys at a distance $\sim M_P$ from the origin. In a supersymmetric GUT this will lead to Affleck-Dine baryosynthesis resulting in $n_B/s \sim 1$. If this GUT is broken by a v.e.v. along a flat direction [as in Flipped $SU(5)$], this large baryon asymmetry is naturally diluted by the decaying oscillations of this weakly-coupled flat direction.

We conclude that supersymmetric theories automatically explain a number of cosmological features, including $\delta \rho/\rho \sim 10^{-4}$, that they have not yet been credited for.

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10. The duration of $\lambda \phi^4$ chaotic inflation is proportional to $\lambda^{-1/2}$; e.g., the “electron slope inflation” should last longer than the “muon slope inflation” by a factor $\lambda^\mu / \lambda^e = m_\mu / m_e \sim 200$. Calculations show that $(\lambda^e)^2 \Phi^4_{QF}$ inflation lasts $\sim 10^6 M_P^{-1}$ [4].

11. If only one of the parameters $d$ and $e$ falls to zero (for example, $e$), then we effectively have a case of $V = \frac{1}{2} m^2 \phi^2$ chaotic inflation, with $m^2 = 2 \lambda d^2 \sim 10^{11} M_P^2$. The density perturbations will then be [8] $\delta \rho / \rho = 8 / (3\sqrt{3}\pi) (m/M_P) N \sim 50 \times 10^{-5.5} \sim 10^{-4}$, similar to above. (Here $N \sim 50$ is the number of e-foldings before the end of inflation when the perturbations leave the horizon.)

12. I. Affleck and M. Dine, *Nucl. Phys.* B249 (1985) 361.

13. The sample flat direction given in their paper was equivalent to our quasi-flat direction (7) with all parameters equal to zero except $a = h$ and $b = g$.

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