Muon $g - 2$ and Minimal Supersymmetric Standard Model

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Abstract

The supersymmetric contribution to the muon $g - 2$ is studied in light of the finalization of the LEP electroweak precision data. The recent precise measurement of the muon $g - 2$ of E821 experiment is explained well by the relatively light chargino and sleptons. We find that such the MSSM parameter space is also favored from the electroweak precision data, in which the fit to the data is better than that of the SM ($\Delta\chi^2_{\text{min}} \sim -2$), if the lighter chargino has a mixed higgsino-wino character ($\mu/M_2 \sim 1$). The models with light gauginos ($\mu/M_2 > 10$) or light higgsinos ($\mu/M_2 < 0.1$) also show the better fit over the SM, but the improvement is marginal as compared to the case of the mixed higgsino-wino case ($\mu/M_2 \sim 1$).
1 Introduction

Although looking for signatures of supersymmetry (SUSY) at high energy experiments is one of the most important tasks of particle physics, no evidence of the supersymmetry has been found in the collider experiments at the energy frontier. However, we may find important constraints on, or an indication of, the supersymmetric models from precise measurements of the electroweak experiments which are sensitive to the interactions of superparticles. A representative candidate is the electroweak precision measurements at LEP1 and SLC [1]. The enormous data of the electroweak measurements at LEP1 have been analyzed after its completion in 1995. The final combination of the results from four collaborations – ALEPH, DELPHI, L3 and OPAL – has been available on the Z-line shape and the leptonic asymmetry data [2]. The constraints on the parameter space of the Minimal Supersymmetric Standard Model (MSSM) has been studied comprehensively in Ref. [3] using the 1998 data, and its update confronting the finalization of the LEP1 analysis is given elsewhere [4].

The muon $g-2$ experiment is another candidate to find the indirect signal of physics beyond the Standard Model (SM). The precise measurement of the muon $g-2$ has been achieved at BNL [5] where the experimental uncertainty has been reduced by about factor 3 from the previous measurement [6]. The current world average of the muon $g-2$ is then given by

$$a_{\mu}^{\text{exp.}} = 11,659,203(15) \times 10^{-10}. \quad (1.1)$$

On the other hand, the theoretical prediction of $a_{\mu}$ is composed of: (1) QED corrections, (2) Electroweak (gauge bosons and Higgs boson in the SM) corrections, (3) Hadronic vacuum polarization effects and (4) Hadronic light-by-light scattering effects. We summarize individual contributions from (1) to (4) in Table 1. Summing up the theoretical estimations in Table 1, the SM prediction is given by

$$a_{\mu}^{\text{SM}} = 11,659,159.6(6.7) \times 10^{-10}. \quad (1.2)$$

The comparison of the data with the SM prediction is

$$a_{\mu}^{\text{exp.}} - a_{\mu}^{\text{SM}} = 43(16) \times 10^{-10}, \quad (1.3)$$

where the experimental and theoretical errors are added in quadrature. As seen in Table 1, the theoretical uncertainty is dominated by the hadronic vacuum polarization effect. Although consensus among experts should yet to emerge on the
Table 1: Summary of the theoretical estimation of $a_\mu$.

|                      | $\times 10^{-10}$ | references                       |
|----------------------|-------------------|----------------------------------|
| QED                  | $O(\alpha^5)$     | 11 658 470.56 (0.29)            |
| Electroweak          | $O(\alpha^2)$     | 15.1 (0.4)                      |
| Hadronic (Vacuum Pol.) | $O(\alpha^2)$     | 692.4 (6.2)                     |
| Hadronic (Light by light) | $O(\alpha^3)$     | -10.0 (0.6)                     |

magnitude and the error of the SM prediction \cite{13, 14}, we adopt the estimate of eq. (1.3) as a distinct possibility. The purpose of this talk is to examine if this possible inconsistency of the data and the SM can be understood naturally in the context of minimal supersymmetric SM (MSSM), when taking account of the electroweak precision data.

2 Muon $g - 2$ in the MSSM

In the MSSM, there are two contributions to the muon $g - 2$. One is the chargino $(\tilde{\chi}_j, j = 1, 2)$ and muon-sneutrino ($\tilde{\nu}_\mu$) propagation in the intermediate states, and the other is the neutralino ($\tilde{\chi}_0^j, j = 1 \sim 4$) and smuon ($\tilde{\mu}_i, i = 1, 2$) propagation. The Feynman diagrams which correspond to these contributions are shown in Fig. 4. The chargino-sneutrino contribution can be expressed as

$$a_\mu(\tilde{\chi}^-) = \frac{1}{8\pi^2} \frac{m_\mu}{m_{\tilde{\nu}_\mu}} \sum_{j=1}^2 \left\{ \frac{m_\mu}{m_{\tilde{\nu}_\mu}} G_1 \left( \frac{m^2_{\tilde{\chi}_j}}{m^2_{\tilde{\nu}_\mu}} \right) \left( \left| \tilde{\chi}_j \tilde{\nu}_\mu \right| ^2 + \left| \tilde{\chi}_j \tilde{\nu}_\mu \right| ^2 \right) \right\} + \frac{m_{\tilde{\nu}_\mu}}{m_{\tilde{\nu}_\mu}} G_3 \left( \frac{m^2_{\tilde{\chi}_j}}{m^2_{\tilde{\nu}_\mu}} \right) \text{Re} \left[ \left( \frac{g_{L}}{g_{R}} \right) \left( \frac{g_{L}}{g_{R}} \right)^* \right] \right\},$$

$$G_1(x) = \frac{1}{12(x - 1)^4} \left[ (x - 1)(x^2 - 5x - 2) + 6x \ln x \right],$$

$$G_3(x) = \frac{1}{2(x - 1)^3} \left[ (x - 1)(x - 3) + 2 \ln x \right],$$

while the neutralino-smuon contribution as

$$a_\mu(\tilde{\chi}_0^0) = \frac{1}{8\pi^2} \sum_{i=1}^4 \frac{m_\mu}{m_{\tilde{\mu}_i}} \sum_{j=1}^2 \left\{ \frac{m_\mu}{m_{\tilde{\mu}_i}} G_2 \left( \frac{m^2_{\tilde{\chi}_0^j}}{m^2_{\tilde{\mu}_i}} \right) \left( \left| \tilde{\chi}_0^j \tilde{\mu}_i \right| ^2 + \left| \tilde{\chi}_0^j \tilde{\mu}_i \right| ^2 \right) \right\} \right\} + \frac{m_{\tilde{\mu}_i}}{m_{\tilde{\mu}_i}} \text{Re} \left[ \left( \frac{g_{L}}{g_{R}} \right) \left( \frac{g_{L}}{g_{R}} \right)^* \right] \right\}.$$
Figure 1: Feynman diagrams for the muon $g - 2$ in the MSSM: The chargino-sneutrino exchange (left) and the neutralino-smuon exchange.

$$+rac{m_{\tilde{\chi}^0}}{m_{\tilde{\mu}_i}} G_4 \left( \frac{m_{\tilde{\chi}^0}}{m_{\tilde{\mu}_i}} \right) \text{Re} \left[ \left( \tilde{g}^{\tilde{\chi}^0_{\mu} \mu_i} \right)^* g^{\tilde{\chi}^0_{\mu} \mu_i} \right] \right], (2.2a)$$

$$G_2(x) = \frac{1}{12(x - 1)^4} \left[ (x - 1)(2x^2 + 5x - 1) - 6x^2 \ln x \right], \hspace{1cm} (2.2b)$$

$$G_4(x) = \frac{1}{2(x - 1)^3} \left[ (x - 1)(x + 1) - 2x \ln x \right]. \hspace{1cm} (2.2c)$$

The coupling constants in eqs. (2.1) and (2.2) follows the notation of Refs. [3, 15]

$$L = \sum_{\alpha = L,R} g_\alpha^{F_1 F_2 S} F_1 P_\alpha F_2 S, \hspace{1cm} (2.3)$$

where $F_1$ and $F_2$ are four-component fermion fields, $S$ denotes a scalar field, and

$$P_L = \frac{1 - \gamma_5}{2}, \quad P_R = \frac{1 + \gamma_5}{2}. \hspace{1cm} (2.4)$$

The number of parameters for the muon $g - 2$ in the MSSM is seven: the left- and right-handed smuon masses ($m_{\tilde{\mu}_L}$ and $m_{\tilde{\mu}_R}$), the ratio of vacuum expectation values of two Higgs doublets ($\tan \beta$), the scalar tri-linear coupling for the smuon ($A_\mu$), the higgsino mass ($\mu$) and the SU(2)$_L$ and U(1)$_Y$ gaugino masses ($M_2$ and $M_1$). In practice, we set $M_1 = \frac{5}{3} \tan \theta_W M_2$ and $A_\mu = 0$ for simplicity. Furthermore, we fix the lighter chargino mass $m_{\tilde{\chi}^-_1}$ by 100 GeV which corresponds to the lower mass bound from the direct search experiments, because the fit to the electroweak precision measurements may be slightly improved over that in the SM when the chargino is relatively light ($\sim 100$ GeV), due to its contribution to the oblique parameters (see, Sec.3). Then, the set of independent parameters which we use in the analysis is: $m_{\tilde{\mu}_L}$, $m_{\tilde{\mu}_R}$, $\tan \beta$ and the ratio $\mu/M_2$. We show the possible maximum contributions to the muon $g - 2$ in the MSSM in Fig.3. In the figure, we study for $\tan \beta = 3$ and 50, $m_{\tilde{\mu}_R} = 100, 300, 500$ GeV and $\mu/M_2 = 0.1 \sim 10.$
Figure 2: Possible maximal contribution to the muon $g - 2$ in the MSSM for $\tan \beta = 50$ (left) and 3 (right). The three lines correspond to $m_{\tilde{\mu}_R} = 100$ GeV (solid), 300 GeV (dotted) and 500 GeV (dashed).

We find that the $a_\mu$ parameter increases as $\tan \beta$ increases, while the right-handed smuon mass dependence diminishes for $m_{\tilde{\mu}_R} > 300$ GeV [16, 17, 18]. Furthermore, the MSSM contribution to $a_\mu$ is most efficient when, and the sign of the discrepancy between the data and the SM prediction in eq. (1.3) favors positive $\mu/M_2$. This may be understood intuitively from the diagram of Fig. 3, where the chirality flip occurs at the internal fermion line. In the chargino-neutrino exchanging diagram, the chirality flip occurs at the internal fermion line. We can tell from the diagram of Fig. 3 that the relevant MSSM contribution to the muon $g - 2$ is proportional to the product $M_2 \tan \beta$. In the wino or higgsino limit, the contribution is suppressed because one of the two charginos is heavy. The chirality flip due to $\tilde{\mu}_L - \tilde{\mu}_R$ mixing contributes negligibly to $a_\mu$ even at $\tan \beta = 50$ [17]. In the following analysis, we therefore ignore the small $\tilde{\mu}_L - \tilde{\mu}_R$ mixing effects.

3 Electroweak precision measurements and the MSSM

The electroweak precision measurements consist of 17 $Z$-pole observables from LEP1 and SLC, and the $W$-boson mass from Tevatron and LEP2. The supersymmetric particles affect these observables radiatively through the oblique corrections.
Figure 3: Feynman diagram for the muon $g - 2$ which is mediated by a sneutrino and charginos. The photon line should be attached to the charged particle lines.

which are parametrized by $S_Z, T_Z, m_W$, and the $Z f f$ vertex corrections $g^f_L$, where $f$ stands for the quark/lepton species and $\lambda = L$ or $R$ stands for their chirality. The parameters $S_Z$ and $T_Z$ [3] are related to the $S$- and $T$-parameters [19, 20] as:

$$\Delta S_Z = S_Z - 0.955 = \Delta S + \Delta R - 0.064 x_\alpha + 0.67 \frac{\Delta \delta G}{\alpha},$$

$$\Delta T_Z = T_Z - 2.65 = \Delta T + 1.49 \Delta R - \frac{\Delta \delta G}{\alpha},$$

where $\Delta S_Z$ and $\Delta T_Z$ measure the shifts from the reference SM prediction point, $(S_Z, T_Z) = (0.955, 2.65)$ at $m_t = 175$ GeV, $m_{H_{SM}} = 100$ GeV, $\alpha_s(m_Z) = 0.118$ and $1/\alpha(m^2_Z) = 128.90$. The $R$-parameter, which accounts for the difference between $T$ and $T_Z$, represents the running effect of the $Z$-boson propagator corrections between $q^2 = m^2_Z$ and $q^2 = 0$ [3]. The parameter $x_\alpha \equiv (1/\alpha(m^2_Z) - 128.90)/0.09$ allows us to take account of improvements in the hadronic uncertainty of the QED coupling $\alpha(m^2_Z)$. $\Delta \delta G$ denotes new physics contribution to the muon lifetime which has to be included in the oblique parameters because the Fermi coupling $G_F$ is used as an input in our formalism [3, 20]. The third oblique parameter $\Delta m_W = m_W - 80.402(\text{GeV})$ is given as a function of $\Delta S, \Delta T, \Delta U, x_\alpha$ and $\Delta \delta G$ [3].

The explicit formulae of the oblique parameters and the vertex corrections $\Delta g^f_L$ in the MSSM can be found in Ref. [3].

We study constraints on the oblique parameters from the electroweak data. In addition to the three oblique parameters, the $Zb_Lb_L$ vertex correction, $\Delta g^f_L$, is included as a free parameter in our fit because non-trivial top-quark-mass dependence appears only in the $Zb_Lb_L$ vertex among all the non-oblique radiative corrections in the SM. By using all the electroweak data [1] and the constraint $\alpha_s(m_Z) = 0.119 \pm 0.002$ [21] on the QCD coupling constant, we find from a five-parameter fit ($\Delta S_Z, \Delta T_Z, \Delta m_W, \Delta g^f_L, \alpha_s(m_Z)$) the following constraints on the
oblique parameters:
\[
\begin{align*}
\Delta S_Z - 25.1 \Delta g_L^b &= 0.002 \pm 0.104 \\
\Delta T_Z - 45.9 \Delta g_L^b &= -0.041 \pm 0.125 \\
\rho &= 0.88,
\end{align*}
\]
(3.2)

\[
\Delta m_W (\text{GeV}) = 0.032 \pm 0.037,
\]
for \(\Delta g_L^b = -0.00037 \pm 0.00073\). The \(\chi^2\) minimum of the fit is \(\chi^2_{\text{min}} = 22.6\) for the degree-of-freedom (d.o.f.) \(19 - 5 = 14\).

Through the expression (3.1a) of \(\Delta S_Z\), the QED coupling \(\alpha(m_Z^2)\) affects theoretical predictions for the electroweak observables. The LEP electroweak working group has adopted the new estimate \(\alpha(m_Z^2) = 128.936 \pm 0.046\), \(x_\alpha = 0.4 \pm 0.51\),

(3.3)

which takes into account the new \(e^+e^-\) annihilation results from BEPC \[23]. Using the central value of \(\alpha(m_Z^2)\) in eq. (3.3), \(x_\alpha = 0.4\), the SM best fit is found given at \((m_t(\text{GeV}), m_{H_{\text{SM}}}(\text{GeV}), \alpha_s(m_Z)) = (175.1, 116.0, 118.1)\). The \(\chi^2\) minimum is \(\chi^2_{\text{min}} = 24.4\) for the d.o.f. \(20 - 3 = 17\). At the SM best fit point, the oblique parameters are given by \((\Delta S_Z - 25.1 \Delta g_L^b, \Delta T_Z - 45.9 \Delta g_L^b, \Delta m_W) = (-0.010, -0.020, -0.009)\), which shows an excellent agreement with the data. Although the SM fit is already good, the further improvement of the fit may be found if new physics gives slightly positive \(\Delta m_W\). On the other hand, new physics contribution which gives large negative \(\Delta S_Z\) and positive \(\Delta T_Z\) is disfavored from the data.

The supersymmetric contributions to the oblique parameters have been studied in Ref. \[3\] in detail. In the MSSM, the oblique corrections are given as a sum of the individual contributions of (i) squarks, (ii) sleptons, (iii) Higgs bosons and the (iv) ino-particles (charginos and neutralinos). Squarks always give \(\Delta S_Z \sim 0\) and \(\Delta T_Z > 0\) while sleptons give \(\Delta S_Z \ll 0\) and \(\Delta T_Z > 0\). Both of them give \(\Delta m_W > 0\) which is favored from the data but the improvement is more than compensated by the disfavored contributions to \(\Delta S_Z\) and \(\Delta T_Z\). The contributions from the MSSM Higgs bosons are similar to that of the SM Higgs boson whose mass is around that of the lightest CP-even Higgs boson, as long as the CP-odd Higgs mass is not too small; \(m_A \gtrsim 300\) GeV \[3\]. We find no improvement of the fit through the oblique corrections in the Higgs sector.

The ino-particles give \(\Delta T_Z < 0\), owing to the large negative contribution to the \(R\)-parameter when there is a light chargino of mass \(\sim 100\) GeV \[3\]. They also make \(\Delta S_Z\) negative when the light chargino is either gaugino-like or higgsino-like \[3\].
However, we find that both $\Delta S_Z$ and $\Delta T_Z$ can remain small in the presence of a light chargino, if the ratio of the higgsino mass $\mu$ and the SU(2)$_L$ gaugino mass $M_2$ is order unity \[4\]. Let us recall that $S_Z$ is the sum of $S$- and $R$-parameters, while $T_Z$ is the sum of $T$- and $R$-parameters. The $S$- and $T$-parameters are associated with the SU(2)$_L \times U(1)_Y$ gauge symmetry breaking while the $R$-parameter is negative as long as a light chargino of mass $\sim 100 \text{ GeV}$ exists. The contributions of the ino-particles to the $S$- and $T$-parameters are essentially zero when the lighter chargino is almost pure wino or pure higgsino, whereas they both become positive when their mixing is large because the mixing occurs through the gauge symmetry breaking. As a consequence, the negative $R$ contributions from a light chargino can be compensated by the positive $S$ and $T$ contributions to the parameters $S_Z$ and $T_Z$, if the light-chargino has a mixed character. The parameter $\Delta m_W$ is increased by the light ino-sector contribution when $\mu/M_2 \sim 1$, and hence the fit is slightly improved. This is largely because of the positive $T$ contribution due to the symmetry breaking. The overall fit, therefore, can be improved in the MSSM if a light chargino with the mixed wino-higgsino character ($|\mu/M_2| \sim 1$) exists and all sfermions are heavy. We find no sensitivity to the sign of the ratio $\mu/M_2$ in the fit to the electroweak data.

Now we examine the effects of vertex and box corrections. Since we find that the light chargino can improve the fit slightly through the oblique corrections, we set the chargino mass to be $m_{\tilde{\chi}_1^\pm} = 100 \text{ GeV}$, as a representative number in our analysis\[4\]. Our task is to look for the possibility of further improving the fit through the vertex and box corrections when squarks and sleptons are also light. We find that sizable $Zff$ vertex corrections via the loop diagrams mediated by the left-handed squarks or the Higgs bosons make the fit worse always \[4\]. On the other hand, the fit is found to be improved slightly by the slepton contributions to the $Z\ell\ell$ vertices ($\Delta g_\ell^\ell$) and the muon lifetime ($\Delta \delta_G$), when the left-handed slepton mass is around $200 \sim 500 \text{ GeV}$. We show the total $\chi^2$ as a function of the left-handed smuon mass $m_{\tilde{\mu}_L}$ in Fig. 4. The $\tan \beta$ dependence is shown for $\tan \beta = 50$ (a) and $\tan \beta = 3$ (b), and the character of the $100 \text{ GeV}$ lighter chargino is shown by $\mu/M_2 = 1.0$ (solid), 0.1 (dotted) and 10 (dashed). For simplicity, we assume that the universality of the slepton mass parameters in the flavor space. We find no sensitivity to the right-handed slepton mass, and $m_{\tilde{\mu}_R}$ is fixed at $100 \text{ GeV}$. The masses of all the squarks and the CP-odd Higgs-boson mass are set at $1 \text{ TeV}\[1\]. The

\[1\] Our results are not significantly altered as long as the mass of the lighter chargino is smaller than about $150 \text{ GeV}$. 

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Figure 4: Total $\chi^2$ in the MSSM as a function of the left-handed smuon mass $m_{\tilde{\mu}_L}$ for $\tan \beta = 50$ (a) and 3 (b). Three lines are corresponding to $\mu/M_2 = 1.0$ (solid), 0.1 (dotted) and 10 (dashed), respectively. The right-handed smuon mass $m_{\tilde{\mu}_R}$ and the lighter chargino mass $m_{\tilde{\chi}^+_1}$ are fixed at 100 GeV. The flavor universality of the slepton masses is assumed. The masses of squarks and extra Higgs bosons are set at 1 TeV. The SM best fit is shown by the dotted horizontal line. The dot-hashed horizontal line is the SM fit with $m_{H_{SM}} = 128$ GeV (left) and $m_{H_{SM}} = 116$ GeV (right) which is the lightest Higgs mass predicted in the MSSM.

The improvement of the fit is maximum at around $m_{\tilde{\mu}_L} \simeq 300$ GeV for $\tan \beta = 50$ (a) and $\simeq 200$ GeV for $\tan \beta = 3$ (b) for $\mu/M_2 = 1.0$, where the total $\chi^2$ value is smaller than those of the decoupling limits, which are shown by the dot-dashed horizontal lines, by about 1.4 and 1.9, respectively.

The origin of the improvement at those points is found to come from the vertex corrections to the hadronic peak cross section on the $Z$-pole, which more than compensate the disfavored negative contributions to the oblique parameter $S_Z$ from the light left-handed sleptons [3]. The hadronic peak cross section $\sigma^0_h$ is given by

$$\sigma^0_h = \frac{12\pi \Gamma_e \Gamma_h}{m_Z^2 \Gamma_Z^2}, \quad (3.4)$$

and is almost independent of the oblique corrections [3, 24]. The final LEP1 data of $\sigma^0_h$ is larger than the SM best-fit value by about 2-$\sigma$. Since the squarks and Higgs bosons are taken to be heavy, the leptonic partial decay widths ($\Gamma_e$ or $\Gamma_{\nu_L}$) in $\Gamma_Z$ are the quantities which are affected significantly by the vertex corrections. The supersymmetric contribution to the leptonic partial decay widths is given by the sleptons and the ino-particles, which constructively interferes with the SM
prediction. The fit to the $\sigma_h^0$ data, therefore, improves if the sleptons and ino- particles are both light. The overall fit is found to improve when the left-handed slepton mass is around $200 \sim 500$ GeV, as shown in Fig. 4. If the slepton mass is too light ($m_{\tilde{\mu}_L} \lesssim 200$ GeV for $\tan \beta = 50$, $m_{\tilde{\mu}_L} < 150$ GeV for $\tan \beta = 3$), the total $\chi^2$ increases rapidly because of disfavored contributions to the $S_Z$-parameter and also from the muon lifetime ($\Delta\bar{G}_G$). Since, in the large $m_{\tilde{\mu}_L}$ limit, only the oblique corrections from the light chargino and neutralinos remain, the difference of $\chi^2$ between the value at its minimum and that at $m_{\tilde{\mu}_L} \sim 3$ TeV represents the improvement of the fit due to non-oblique corrections. Among the three cases of $\mu/M_2$ in Fig. 4, only the $\mu/M_2 = 1.0$ case shows a slight improvement of the fit via the non-oblique corrections. This is because the relatively light heavier chargino ($m_{\tilde{\chi}^\pm_1} \simeq 220$ GeV for $\mu/M_2 = 1$) contributes to $\Gamma_\ell$ or $\Gamma_\nu_\ell$ but has no other significant effects elsewhere. The improvement of the fit persists for $\mu/M_2 \sim 0.5$ or 2, but the smallest $\chi^2$ is found at $\mu/M_2 = 1.0$. Although we have shown results for $\mu/M_2 > 0$, we found that the electroweak data are insensitive to the sign of $\mu/M_2$.

4 Constraints on the MSSM parameters from electroweak precision data and the muon $g - 2$

We study constraints on the MSSM parameter space from the muon $g - 2$ data, in the light of the electroweak precision measurements. In Fig. 4, we show constraints on the left-handed smuon mass $m_{\tilde{\mu}_L}$ and $\tan \beta$ from the experimental data of $a_\mu$, eq. (1.3), for $m_{\tilde{\chi}^\pm_1} = 100$ GeV and $\mu/M_2 = 0.1$ (a), 1.0 (b) and 10 (c). The regions enclosed by solid and dashed lines are found for the right-handed smuon mass $m_{\tilde{\mu}_R} = 100$ GeV and 500 GeV, respectively. In the region of $m_{\tilde{\mu}_L}$ smaller than the vertical dotted lines, the MSSM fit to the electroweak data is worse than the SM fit ($\chi^2_{\text{min}}[\text{MSSM}] > \chi^2_{\text{min}}[\text{SM}]$). Fig. 3 (b) tells us that if the lighter chargino state has comparable amounts of the wino and higgsino components ($\mu/M_2 = 1.0$), which is favored from the electroweak data, relatively low values of $\tan \beta$ is allowed: $4 \lesssim \tan \beta \lesssim 8$ for $m_{\tilde{\mu}_L} \approx 110$ GeV. This is the region favored by the electroweak data in Fig. 4. On the other hand, if it is mainly higgsino ($\mu/M_2 = 0.1$) or wino ($\mu/M_2 = 10$), low values of $\tan \beta$ is excluded: $\tan \beta \gtrsim 15$ for $m_{\tilde{\mu}_L} \approx 200$ GeV, where the MSSM fit to the electroweak data is comparable to the SM. In all cases of $\mu/M_2$ in the figure, the right-handed smuon tends to make the bound on $\tan \beta$
Figure 5: 1-σ allowed region of the \( (m_{\tilde{\mu}_L}, \tan \beta) \) plane from the experimental data of muon \( g - 2 \) (1.3) for \( \mu/M_2 = 0.1 \) (a), 1.0 (b) and 10 (c). The lighter chargino mass \( m_{\tilde{\chi}_1^-} \) is set at 100 GeV. The region enclosed by the solid lines and dashed lines are obtained for the right-handed smuon mass \( m_{\tilde{\mu}_R} = 100 \text{ GeV} \) and 500 GeV, respectively. In the region of \( m_{\tilde{\mu}_L} \) smaller than the vertical line, the MSSM fit to the electroweak data is worse than the SM \( (\Delta \chi^2 \equiv \chi^2_{\text{min}} \text{[MSSM]} - \chi^2_{\text{min}} \text{[SM]} > 0) \).

The constraints on \( (M, \tan \beta) \) are given in Fig. 6(a), for three representative cases of \( \mu/M_2 = 0.1, 1.0 \) and 10. Because the muon \( g - 2 \) decreases if any of the three charged superparticles is heavier than the common value \( M \), we can regard the allowed range as an upper mass limit of charged superparticles. We find that, if the lighter chargino is either wino- or higgsino-like, i.e., \( \mu/M_2 \gg 1 \) or \( \mu/M_2 \ll 1 \), either the lighter chargino or smuons should be discovered by a lepton collider at \( \sqrt{s} = 400 \text{ GeV} \) for any \( \tan \beta \leq 50 \). On the other hand, if no superparticle
Figure 6: Constraints from the muon $g - 2$ data \cite{lepton} on $(M, \tan \beta)$ (a) and on $(\mu/M_2, \tan \beta)$ (b). The mass parameter $M$ is defined as $M \equiv m_{\tilde{\chi}_1^\pm} = m_{\tilde{\mu}_L} = m_{\tilde{\mu}_R}$. The solid, dot-dashed and dashed lines are corresponding to: (a) $\mu/M_2 = 0.1, 1.0$ and 10, and (b) $M = 100$ GeV, 200 GeV and 400 GeV, respectively.

is found at a 500 GeV lepton collider, then the chargino should have the mixed character and $\tan \beta$ should be bigger than 15. In Fig. 6(b) we show the constraint on $(\mu/M_2, \tan \beta)$ from the $a_\mu$ data \cite{lep}. We can clearly see from this figure that the lowest value of $\tan \beta$ is allowed at $\mu/M_2 = 1$.

5 Summary

We have studied the supersymmetric contributions to the muon $g - 2$ in the light of its recent measurement at BNL and the finalization of the LEP electroweak data. Although the SM fit to the electroweak data is good, slightly better fit in the MSSM is found when relatively light left-handed sleptons with mass $\sim 200$ GeV and a light chargino of mass $\sim 100$ GeV and of mixed wino-higgsino character ($\mu/M_2 \sim 1$) exist. The improvement is achieved via the light chargino contribution to the oblique parameters and also via the ino-slepton contribution to $\sigma^h$. The improvement of the fit disappears rapidly if the light chargino is higgsino- or wino-like, or if the light chargino mass is heavier ($\gtrsim 200$ GeV), or the sleptons are too light ($\lesssim 180$ GeV for $\tan \beta = 50$ and $\lesssim 120$ GeV for $\tan \beta = 3$). We find that the supersymmetric contribution to the muon $g - 2$ is most efficient for $\mu/M_2 \sim 1$. If $\tan \beta \lesssim 10$, the MSSM parameter space which is favored from the electroweak
data is also favored from the muon $g - 2$ data. The wino- or higgsino-dominant chargino contributes significantly to the muon $g - 2$ only for large $\tan \beta$ ($\gtrsim 15$), although it does not improve the fit to the electroweak data. The impact on the search for the superparticles at future colliders from the precise measurement of the muon $g - 2$ is also discussed. The present 1-$\sigma$ constraint (1.3) from the muon $g - 2$ measurement implies that either a chargino or charged sleptons are within the discovery limit of a 500 GeV lepton collider for any $\tan \beta(< 50)$ if the lighter chargino is dominantly wino ($\mu/M^2_2 \gtrsim 3$) or dominantly higgsino ($0 < \mu/M^2_2 \lesssim 0.3$).

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