Analysis of Investment Policy in Belarus

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Abstract

The optimal planning trajectory is analysed on the basis of the growth model with effectiveness. The saving per capital value has to be rather high initially with smooth decrement in the future years.

I. INTRODUCTION

For the development of the long-term economic strategy and for realization of appropriate investment policy one needs the theoretical model of economic growth. All developed countries pay a considerable attention to researching of such theoretical models, as well as to developing of corresponding instrumental means that are important for calculation of the concrete prognoses and programs [1]. The practical experience, accumulated by many countries, demonstrates that the most effective tool for the development of strategic directions of an economic policy on a long-run period are the special economic-mathematical models of small dimension (macromodels). Such models are elaborated on the base of the theory of economic growth [2].

In traditional macromodels the principal attention is payed to the estimation of future dynamics of the investments that determine trajectories of the economic growth. However, economic growth depends not only on scales of investment resources, enclosed in economics: these trajectories are also determined by a number of the so-called quality factors [3]. Moreover, the final economic growth results depend basically on these factors.
The orientation of economy to the extensive growth by escalating of wide area investments only cannot ensure an achievement of the useful final results without paying attention to the quality factors. The history of development of many countries, including the former USSR, shows the possibility of economical development by the principle of production for the sake of production. Economical system absorbs huge investment resources and augments volumetric parameters. Nevertheless this type of investment policy is not able to raise essentially standards of living.

The limited prospects of the economic development on the base of only extensive factors are demonstrated in the Solow model [4]. The estimation of influence of technical progress is made on the basis of the Solow model with technical progress [2]. This model accounts for the contribution of technical progress in the simplest way. It is based on the rather relative concept of autonomous progress advance. The intensity of the autonomous progress effect on production growth is completely determined by the time factor. A quantitative estimations of such effect are received on the basis of the production functions method. The parameters for this method are calculated by means of econometric processing of dynamic rows that define change of production volumes. A row of successive numbering of time periods (years, quarters or other periods) is used as a dynamic row, that corresponds to the change of technical progress.

In essence another approach to define the quality factors contribution to the growth of production volumes has been realized in growth model with effectiveness. This approach does not use insecure parameters of rather relative econometric models. It is based on a direct estimation of the change of an economic system material capabilities that are indispensable for the realization of social purposes. The growth model with effectiveness is most adequate for simulation of economic growth in the present economical situation for the Republic of Belarus. The main purpose of this work is to formulate the optimal planning problem for the growth model with effectiveness and solution of this problem: such solution is especially important for researches of economic growth problems, because it answers the main question that appears when choosing the investment policy. The essence of this question consists in
choosing between maintenance of current demand (consumption) and maintenance of the future demand (capital investment). The solution of the optimal planning problem allows to point such investment policy, at which the economic system works at best. The formulation of the depends on the purposes that the economic system has. In this work one takes the most realistic version of such purpose - maximization of the welfare integral which is consumption per man during the modeled period of time.

The article is organized as follows: section II deals with the concept of production effectiveness. Growth model with effectiveness is described in section III. In section IV one considers optimal planning problem formulation. Maximum principle is applied in section V. Solution of the optimal planning problem is analyzed in section VI. Policy options are considered in section VII. Main achievements of this work are briefly concluded in section VIII.

II. CONCEPT OF PRODUCTION EFFECTIVENESS

The growth model with effectiveness uses modern approach to predict the economic growth trajectory and define the quality factors contribution in increasing of production volumes. This approach does not rest on accident-sensitive parameters of rather conditional econometric models, but it is grounded on the direct estimation of those changes of material capabilities of an economical system that are necessary for realization of the chosen purposes. On the basis of such estimation it is expedient to draw a conclusion about the advance, reached in economics: one admits the existence of "advance" only in the case, when the capabilities for implementation of the social and economic purposes are extended. At the same time, from the standpoint of the target approach, opening capabilities scales shows the degree of the obtained advances.

Within the framework of such an approach there is a more successful name for the identification of the advance, reached in economics. This name is "increase of production effectiveness" (or, accordingly, its reduction, if the capabilities for the implementation of
the social and economic purposes are reduced). This term is used in this work to reflect all quality factors influence on economic growth. Thus, the primary task of this chapter is to design a criteria index that should characterize change of production effectiveness on the macrolevel. Such an index should be used as one of the most important variables of the dynamic macromodel, which reflects the relationship of this macromodel with other macroeconomic indexes that describe the intensity of production and use of resources, as far as accumulation and non-productive consumption. Within the framework of the dynamical macromodel of economical growth, the submodel of the production effectiveness has been developed from the standpoint of the target approach [5]. The economical system under consideration is supposed to be closed in the sense that the total amount of the needs is fulfilled at the expense of the production only. The system does not move its production into other countries in the debt or gratuitously (that does not eliminate barter with an environment on the equivalent basis for the coordination of commodity pattern of production with consumption pattern).

To define an effectiveness criterion of production it is necessary to reveal the certain economical form of the operational outcome of the economical system and resources, used for its achievement. For this purpose one needs to define not only spatial - organizational, but also temporal boundaries of this system in order to reveal and to agree final output indexes with the initial input factors involved in the process of production.

If we abstract from the natural and external economical factors and consider the effectiveness of an economical system, without taking into account its temporal boundaries, then the labour force is the only used resource, and the amount of created material benefits and services intended only for the non-productive consumption is the outcome of the system operation. In this case it would be possible to define the effectiveness index on the basis of the simple comparison of the total final consumption and used labour force. It is always necessary to associate an estimation of effectiveness with the particular time frame, therefore time also should play a role of the factor that limits the frameworks of the economical system. The inputs and outputs of the system reflect not the connection with the environment.
The evolution of the system depends on the past and defines the future evolution of the system. The relations of the resources reproduction costs to their volumes are the relevant characteristics of the reproduction process. For example the consumption per man $\psi$ reflects the level of workers material benefits. Any change of this index substantially determines the dynamics of all indexes of the population welfare. The relation of gross investment $S$ to a volume of accumulated productive capital $K$

$$\omega = \frac{S}{K}$$  \hspace{1cm} (1)

predetermines the rate of capital growth in a decisive measure. The estimations $\psi$ and $\omega$ are connected with the indexes of labour productivity $p$ and capital per man $r$ as follows

$$p = \psi + \omega \cdot r.$$  \hspace{1cm} (2)

This ratio is a consequence of the balance identity $Y = C + S$, where $Y$ - output, $C$ - consumption, which originates from the assumption about closure of the considered economical system (see fig.1).

To derive the equation (2) from the balance identity, one has to divide it by the labour force volume $L$ and take into consideration that

$$\frac{S}{L} = \frac{S}{K} \cdot \frac{K}{L} = \omega \cdot r.$$  \hspace{1cm} (3)

Relative resource estimations $\psi$ and $\omega$ can be accepted as objective functions that describe the implementation degree of two main purposes of the reproductive process. These purposes form one general purpose of an economical system. Here we denote it as "integral purpose". It implies the maximization of satisfaction of current and future needs of society. The index of effectiveness should serve as the objective function that permits to estimate quantitatively the integral purpose implementation.

For the concrete definition of the integral purpose and the integral index of effectiveness it is necessary to distribute the priorities between two introduced primary purposes. This distribution leads to their coordination and resolving of the contradiction between the primary
purposes. Such priorities are reflected quantitatively at distribution of the gross internal product to parts intended for reproduction of two kinds of manufacturing resources. The purposes of the society define the proportions of such distribution and concretize the effectiveness index formula that should serve as the integral object function of the reproduction process.

Below all variables are counted for the given year. Here we assume that during t-th year the usage of the manpower quantity $L_t$ and capital of the volume $K_t$ makes the gross internal product of the volume $Y_t$. The corresponding values of the relative indexes of labour productivity and capital per man are peer $p_t$ and $r_t$, respectively. The indexes $p_t$ and $r_t$ are the most important quantitative characteristics of the process of reproduction as generalized technology on the macrolevel. Though their values don’t determine base-line values of the object functions $\psi_t$ and $\omega_t$ uniquely, but they limit the area of their possible values. In accordance with the general limiting condition the above indicated main balance identity reads

$$p_t = \psi_t + \omega_t \cdot r_t.$$  \hspace{1cm} (4)

The particular values of estimations $\psi_t$ and $\omega_t$ can be derived from the area, restricted by the equality (4). This area depends on the distribution of the made gross internal product, which arrests the norm of accumulation $\delta_t$. The last is defined as the fraction of the gross accumulation in the total amount of the gross internal product

$$\delta_t = \frac{S_t}{Y_t}.$$  \hspace{1cm} (5)

At the given norm of accumulation and the certain labour productivity level as well as capital per man, the values of object functions are determined uniquely according to the formulas

$$\psi_t = (1 - \delta_t) \cdot p_t,$$

$$\omega_t = \frac{\delta_t \cdot p_t}{r_t}.$$  \hspace{1cm} (6)
Their values are the starting point for the analysis of the effectiveness dynamics.

Here we consider the values of variables for the next year \( t + 1 \). The capital per man increases to the value of \( r_{t+1} \). The labour productivity for the given capital per man reaches the value \( p_{t+1} \). The dynamics of the reproduction process effectiveness characterizes the change of the implementation capabilities of two main purposes that have to be reflected in the change of area of acceptable values of the object functions \( \omega \) and \( \psi \). In order to estimate the degree of the indicated change quantitatively one needs to compare labour productivity in current year with the value \( \psi_t + \omega_t \cdot r_{t+1} \), which describes that minimal production volume per man. This allows to keep the values of the object functions \( \omega \) and \( \psi \) at the level of the basic year.

\[
p_{t+1} = \psi_t + \omega_t \cdot r_{t+1}. \tag{7}
\]

The last corresponds to the equal effectiveness of production in current and basic years. It means that the values of both object functions can be saved at a basic level and both can not be increased at once. Note that actual values of estimations \( \omega \) and \( \psi \) in one year \( t + 1 \) can differ from \( \psi_t \) and \( \omega_t \), but one of them will be more basic, and another will be less.

Assume the labour productivity level in one year \( t + 1 \) surpasses the bound

\[
p_{t+1} > \psi_t + \omega_t \cdot r_{t+1}.
\]

Therefore in this year there appears the possibility for simultaneous increase of values of two object functions in contrast to the year \( t \). As mentioned before these object functions describe the level of implementation of the main purposes of the reproduction process. It gives the ground to suppose that in one year \( t + 1 \) the value of the integral object function is augmented and, therefore, the level of efficiency is raised. Moreover, the value of a difference \( p_{t+1} - \psi_t - \omega_t \cdot r_{t+1} \) allows to judge that there appeared capability to increase the values of object functions \( \omega \). Therefore this difference can serve as the characteristic of the effectiveness increase degree.

The actual increment of labour productivity \( \Delta p_t = p_{t+1} - p_t \) can be decomposed in two parts.
\[ \Delta_1 = \psi_t + \omega_t \cdot r_{t+1} - p_t, \tag{8} \]
\[ \Delta_2 = p_{t+1} - \psi - \omega \cdot r_{t+1}. \tag{9} \]

The value $\Delta_1$ represents the increment of the labour productivity, at which the level of effectiveness remains invariable. It can also be interpreted as the increment of the productivity reached at the expense of the extensive increase of the capital per man (at the basic level of the effectiveness). Another part of the increment of labour productivity $\Delta_2$ quantitatively characterizes the additional capabilities of implementation of the main purposes of production appeared in one year $t+1$. We start from the reason that the growth of the effectiveness is the only source of increase of such capabilities. Therefore it is possible to consider the value $\Delta_2$ as the increment of the productivity reached at the expense of the increase of efficiency. The ratio of the increment $\Delta_2$ to the base-line value of the labour productivity

\[ \frac{\Delta_2}{p_t} = \frac{p_{t+1} - \psi - \omega \cdot r_{t+1}}{p_t}. \tag{10} \]

represents the rate of the productivity increment at the expense of the effectiveness increase. This ratio can be identified with the rate of the increment of the effectiveness of production.

In equation (10) we replace $p_{t+1}$ with $p_t + \Delta p_t$ and $r_{t+1}$ with $r_t + \Delta r_t$ and simplify it using the balance identity (2). The rate of the increment of effectiveness depends on the pure increments of the labour productivity $\Delta p_t$ and capital per man $\Delta r_t$

\[ \frac{\Delta_2}{p_t} = \frac{p_{t+1} - \psi - \omega \cdot r_{t+1}}{p_t}. \tag{11} \]

The equation (11) can be rewritten using the absolute increases of the corresponding indexes to the rates of their change. For this purpose one substitutes the intensity of reproduction of the capital from the formula (6) into equation (11):

\[ i_\varphi = \frac{1}{p_t} \cdot \left( \Delta p_t - \frac{\delta_t \cdot p_t}{r_t} \cdot \Delta r_t \right) = \frac{\Delta p_t}{p_t} - \frac{\delta_t \cdot \Delta r_t}{r_t}. \]
Here we introduce the notations for rates of increment of the corresponding indexes \( i_p = \frac{\Delta p}{p_t} \) and \( i_r = \frac{\Delta r}{r_t} \). Then the estimation of dynamics of a production effectiveness takes the form

\[
i_\varphi = i_p - \delta \cdot i_r
\]

(12)

Thus, the conducted analysis results in a conclusion that the reference point for the society that seeks to achieve the social and economic purposes, can’t serve the increase of labour productivity, which traditionally was considered to be the main criteria index in our economics. The obtained two equivalent dynamical formulas of the index of a production effectiveness (11) and (12) demonstrate that for the estimation of outcomes of the economical development it is necessary to correct the growth of labour productivity allowing for the change of the capital per man. The bigger part of the made gross internal product is routed to the reproduction of the productive capital, the more is the deviation of the dynamics of the criteria index of effectiveness from dynamics of labour productivity. The last can serve as the main reference point of control only in that case, when the capital per man remains invariable. In this work we consider the usage of the effectiveness index as the preferred reference point of control.

### III. GROWTH MODEL WITH EFFECTIVENESS

The following six main macroeconomic indexes have been selected as variables of the Growth model with effectiveness \( r \) - capital per man, \( p \) - output per man, \( \psi \) - consumption per man, \( \omega \) - saving per capital, \( \lambda \) - sum of amortization rate and population rate, \( i_\varphi \) - the rate of economics effectiveness.

For simplification of model and its analysis the following preconditions are adopted: the number of workers in economics changes with constant rate of increment \( n \), (so that the dynamics of employment can be recorded with the help of the function \( L = L_0 \cdot e^{nt} \)). The amortization rate \( \beta \) is also invariable. Besides the differential form of notation is used. This form is more convenient for solution and analysis and widely used in the literature on
economic-mathematical modeling [5]. For conversion to the differential form one supposes, that the values of the considered economical indexes are continuously differentiable functions of time. The increments of indexes per unit of time, selected as a step for the analysis, are substituted by derivative from functions that describe their dynamics.

The main equation of the model is based on the definition of the effectiveness index. The formula (11), derived earlier for the estimation of dynamics of effectiveness and introduced in the incremental form, can be converted to the following equivalent equation in the differential form

\[ \dot{p}(t) = \dot{i}_\varphi(t) \cdot p(t) + \omega(t) \cdot \dot{r}(t). \]  \hspace{1cm} (13)

The differential equation (13) is included in the developed macroeconomic model and plays the role of production function, which links indexes of capital per man and labour productivity. Besides, the model includes the main balance identity (2)

\[ p(t) = \psi(t) + \omega(t) \cdot r(t). \]  \hspace{1cm} (14)

The ratio (6) can be rewritten in differential form as follows

\[ \omega(t) \cdot r(t) = \delta(t) \cdot p(t). \]  \hspace{1cm} (15)

The differential equation for \( r \) comes from the equation of capital dinamics

\[ dK = S(t) - \beta \cdot K(t). \]  \hspace{1cm} (16)

To pass from the increment of capital to the index of the increment of capital per man, it is necessary to differentiate the formula for capital per man

\[ dr = \frac{d\left(\frac{K(t)}{L(t)}\right)}{L(t)^2} \]
\[ = \frac{dK \cdot L(t) - K(t) \cdot dL}{L(t)^2} \]
\[ = \frac{dK}{L(t)} - \frac{K(t)}{L(t)} \cdot \frac{dL}{L(t)}. \]  \hspace{1cm} (17)
Here we introduce the population rate \( n = \frac{dL(t)}{L(t)} \). The last assumption allows to rewrite equation (17) in the following form:

\[
dr = \frac{S(t) - \beta \cdot K(t)}{L(t)} - n \cdot r(t) = \frac{S(t)}{L(t)} - (\beta + n) \cdot r(t).
\]

Now in (18) instead of \( \frac{S(t)}{L(t)} \) we put equivalent expression from (3), and for the sum of two constants \( n + \beta \) we enter new identification \( \lambda = n + \beta \). Then it is possible to enter into the model the equation, that reflects the correlation between capital per man and saving per capital.

\[
\dot{r}(t) = (\omega(t) - \lambda) \cdot r(t).
\]

Equations (13) - (15) and (19) form the set of four equations, which describes the correlations between six abovelisted main macroeconomic indexes.

**IV. OPTIMAL PLANNING PROBLEM FORMULATION**

It is nessesary to emphasize that this model is represented by six variables and four equations. Therefore, the obtained set of equations (13)-(15),(19) is incomplete. In order to close the set of equations one introduces control variables that describe investment policy. It is possible to do by assuming \( i_\varphi = \text{const} \) and \( \omega(t) \) is a control variable. Then one can formulate optimal planning problem. First, it is necessary to construct a target functional. The task of the central planning establishment is to select a feasible trajectory \( \omega(t) \) that is the optimum for achievement of some economic target. The economic target of central planning organ should be based on the standards of living, estimated by the consumption level. In particular, it is presumed that the central planning organ has an utility function, which determines utility \( U \) at any moment of time as a function of consumption per man \( U = U[\psi(t)] \). We assume that the utility function is doubly differentiable and that the marginal utility is positive and non-increasing function. Therefore, the utility function is
concave and monotonically increasing function. The utility function determines utility in a certain year. However, the problem of the central planning organ is to select the whole trajectory of consumption per man. For this purpose it is necessary to compare indexes of utility that correspond to different years. Suppose, that utilities in different years do not depend on each other. The utility in any year does not depend on either consumption or utility in any other year directly. Utilities for different years are supposed to be additive.

This assumption is based on the fact, that the proximate consumption is more important, than distant. We suppose that the norm of discounting \( d \) is constant and positive. The bigger norm of discounting testifies the greater preference of the utilities closest in time. Suspecting the exponential behavior of the discounting, we receive the value of utility in year \( t \) equal to \( e^{-d(t-t_0)}U[\psi(t)] \). During the indicated time period from \( t_0 \) to \( t_1 \) the welfare \( W \), corresponding to the pathway of consumption per man \( \psi(t) \), is determined by integrating of all instantaneous utilities over the whole interval. The time planning horizon \( t_1 \) can be finite or infinite. In case this time is finite, it is necessary to set the minimally acceptable value of capital per man in final year to ensure the possibility of consumption outside the given horizon of time. Here we try to avoid difficulties, connected with the definition of the minimal value of capital per man in a final moment of time. Consider, that \( t_1 \) is indefinite, so the control trajectory is selected for all times in the future.

However, in this case the welfare integral can miss. The convergence of an integral is guaranteed, if the following conditions are satisfied: the initial value of capital per man is less than the maximal accessible level \( \tilde{k} \) and the norm of discounting is positive. In this case \( c(t) \leq f(\tilde{k}) \) and

\[
\int_{t_0}^{\infty} e^{-d(t-t_0)}U(\psi(t))dt \leq \int_{t_0}^{\infty} e^{-d(t-t_0)}U(f(\tilde{k}))dt = \frac{U(f(\tilde{k}))}{d}.
\]  

So the integral of welfare is bounded above.

We choose \( r(t) \) and \( p(t) \) as state variables. We express consumption per man through control and state variables from (14). Then we receive optimal planning problem for model with effectiveness

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\[ W = \int_0^\infty e^{-d \cdot t} U(p - \omega \cdot r) dt \rightarrow \max_{\omega(t)} \]
\[ \dot{r} = (\omega - \lambda) r, \]
\[ \dot{p} = \pi_\varphi + \omega(\omega - \lambda) \cdot r, \]
\[ r(0) = r_0, \]
\[ p(0) = p_0. \]  

(21)

V. APPLYING OF THE MAXIMUM PRINCIPLE

In this section we derive the equation for the saving per capital trajectory. When solving optimal planning problems with the help of the Maximum principle, for each coordinate of state variables vector \( x \) a costate variable is used. The Hamiltonian function is \( H(x, y, u, t) = I(x, u, t) + yf(x, u, t) \), where \( I(x, u, t) \) stands for the integrand of a target functional, \( f(x, u, t) \) for the vector of motion equations right parts, \( u \) for control. We find functions \( u(t), x(t), y(t) \) that satisfy the following conditions

\[ \max_u H(x, u, y, t), t_0 \leq t, \]
\[ \dot{x} = \frac{\partial H}{\partial y}, x(t_0) = x_0, \]
\[ \dot{y} = -\frac{\partial H}{\partial x}, \]
\[ H = e^{-d(t-t_0)} U(p - \omega \cdot r) + y_r(\omega - \lambda)r + y_p [\pi_\varphi + \omega(\omega - \lambda)r]. \]  

(25)

where \( y_r, y_p \) are costate variables. Finally, by using of Eqs. (22) - (24) we can write the problem to be solved in the following form

\[ 0 = -e^{-d \cdot t} U'(p - \omega r) + y_r + 2y_p\omega - y_p\lambda, \]
\[ \dot{r} = (\omega - \lambda) r, \]
\[ \dot{p} = \pi_\varphi + \omega(\omega - \lambda)r, \]
\[ \dot{y}_r = \omega \cdot e^{-d \cdot t} U'(p - \omega \cdot r) - y_r(\omega - \lambda) - y_p\omega(\omega - \lambda), \]
\[ \dot{y}_p = -e^{-d \cdot t} U'(p - \omega \cdot r) - y_p\pi_\varphi. \]  

(26) (27) (28) (29) (30)
The system of five equations with five unknown functions is then obtained. Four equations of this system are differential, one is algebraic. Solution of this system of equations is equivalent to the solution of the problem (21).

The set of equations (26) - (30) has the solution for any kind of the utility function. However in this work we perform the analysis of the elementary case $U(z) = z$. In this case Eqs. (29) and (30) for costate variables $y_r$ and $y_p$ do not depend on state ones $r$ and $p$. In order to find an optimal savings trajectory $\omega(t)$ we have to solve only three equations (26), (29), and (30). After that one can find state variables $r$ and $p$. As a first step we obtain from Eq. (26) an explicit formula for $\omega(t)$ written as a function of costate variables $y_r$ and $y_p$:

$$\omega = \frac{\lambda}{2} + \frac{\chi - y_r}{2y_p},$$  \hspace{1cm} (31)

where $\chi = exp(-dt)$ is an exponential function. Note, that the solution of eq.(30) for $y_p$ can be written in the analytical form

$$y_p(t) = y_p(0)e^{-\mu t} + \frac{1}{\mu - d}(e^{-\mu t} - e^{-d t}),$$  \hspace{1cm} (32)

where $\mu = i\varphi$. Having differentiated the left and right parts of eq. (31), one arrives at

$$\dot{\omega} = -\frac{1}{2} \left( \frac{\dot{y}_r - \dot{\chi}}{y_p} \right) y_p - \frac{1}{2y_p} (y_r - \chi) \dot{y}_p.$$  \hspace{1cm} (33)

By rewriting Eq. (31) in the following form

$$\frac{y_r - \chi}{y_p} = \lambda - 2\omega.$$  \hspace{1cm} (34)

and substituting it in the right part of Eq. (33), we find

$$\dot{\omega} = -\frac{1}{2} \left( \frac{\dot{y}_r - \dot{\chi}}{y_p} \right) + \frac{1}{2} \frac{\dot{y}_p}{y_p}(\lambda - 2\omega).$$  \hspace{1cm} (35)

Then we replace $\dot{y}_r$ and $\dot{y}_p$ in the right part of eq. (29) and (30) respectively with the intermediate result

$$\dot{\omega} = -\frac{1}{2} \left( \frac{\omega \chi - y_r(\omega - \lambda) - y_p \omega (\omega - \lambda) - \chi}{y_p} \right) + \frac{1}{2} \left( \frac{-\chi - y_p \mu}{y_p} \right) (\lambda - 2\omega).$$ \hspace{1cm} (36)
Finally, we substitute the ratio of costate variables
\[
\frac{y_r}{y_p} = \lambda - 2\omega + \frac{\chi}{y_p}.
\] (37)

found from Eq. (34) in the equation (36) and obtain the following differential equation for saving per capital
\[
\dot{\omega} = -\frac{1}{2}\omega^2 + (\mu + \lambda + \frac{\chi}{y_p})\omega - \frac{\lambda^2}{2} - \frac{(2\lambda + d)\chi}{2y_p} - \frac{\mu\lambda}{2}.
\] (38)

Found equation together with Eq. (32) is equivalent to the optimal planning problem (21). Solution of this equation gives an optimal savings trajectory \(\omega(t)\).

VI. OPTIMAL SAVINGS TRAJECTORY

The differential linear equation (38) is Riccati equation which can be solved by standard methods. For the parameters values \(\mu = 0.015, d = 0.2\) (corresponding to decreasing significance of consumption in e times for 5 years), \(\lambda = 0.02\), and the initial value \(\omega(0) = 0.05\), the solution of the Riccati equation (38) has been found numerically by the Runge-Kutta method. The obtained solution is shown in figure 2.

The equation (38) enables analytical consideration in the asymptotic limit when \(\exp(-(d - \mu)t) \to 0\), i.e. when \(d > \mu\) and \(t \gg 1/(d - \mu)\). In this case we can neglect those terms in Eq. (38) proportional to \(\chi/y_p\) with a result
\[
\dot{\omega} = -\frac{3}{2}\omega^2 + \mu\omega - \frac{\lambda(\lambda + \mu)}{2} = -\frac{1}{2}(\omega - \omega_-)(\omega - \omega_+),
\] (39)

where \(\omega_\pm = (\lambda + \mu) \pm \sqrt{\mu(\lambda + \mu)}\). Note, that this equation is known as equation of interacting masses. Having integrated it we obtain an asymptotic optimal trajectory
\[
\omega(t) = \frac{\omega_- \exp(-t\sqrt{\mu(\lambda + \mu)}) + c\omega_+}{\exp(-t\sqrt{\mu(\lambda + \mu)}) + c}
\] (40)

where the constant \(c\) is defined by the values \(\omega_\pm\) and the value of \(\omega(t_\infty) = \omega_\infty\) taken as “initial” value for the asymptotic regime of evolution.
\[ c = (\omega_\infty - \omega_-)(\omega_+ - \omega_\infty). \]

As it follows from obtained solution an asymptotic trajectory will be slightly increased if \( \omega_- < \omega_\infty < \omega_+ \), being restricted by the value of \( \omega_+ = (\lambda + \mu) + \sqrt{\mu(\lambda + \mu)} \). An asymptotic behavior of state variable \( r \) is obtained from Eq. (27) by taking into account Eq. (40)

\[ r = \tilde{r} \exp \left( (\mu + \sqrt{\mu(\lambda + \mu)})t \right) \left( \exp \left( (\sqrt{\mu(\lambda + \mu)})t \right) + c \right)^2, \]

where \( \tilde{r} \) is a rate-fixing constant.

**VII. POLICY OPTIONS**

There are two main applications of this analysis: recommendations to the government for investment policy development and prediction of a long-run period macroeconomic system development.

To define recommended investment volume in the current year, we need to obtain the stock of capital statistical data for this year and to find the optimal saving per capital value using the optimal planning problem solution. Then we can evaluate the investment volume as a product of \( K \) and \( \omega \). The next year we change the optimal planning problem parameters to improve the accuracy of calculation. The procedure of recommended investment volume evaluation remains invariable. Thus this method allows the optimization of the investment policy during any period of time.

Variables from economic growth models play important role in models of other important fields of economic life too. A financial programming method combines these models. The optimal planning problem for the model with effectiveness can serve as a part of financial programming systems [7]. The solution of the optimal planning problem may be useful for evaluating financial programming system parameters.

To predict long-run period macroeconomic system development we put optimal saving per capital trajectory into the growth model with effectiveness and then calculate all macroeconomic variables from this model. Table 1 shows predictions based on the Republic
of Belarus economic parameters. Optimal saving per capital trajectory for this model is the result of the optimal planning problem solution from this work.

Table 1 shows that this variant of macroeconomic strategy allows the extension of productivity potential by increasing the capital per man by 72.4 % and the output per man by 55.5 %. The total growth of consumption per man is 51.6 %.

VIII. CONCLUSIONS

The main result of this article is the optimal investment trajectory for Belarus for the period of 2000-2020. The number of additional results may be of use for the investment policy development in other countries and other periods. The nonlinear system of equations allows to analyze dependencies between trajectories of main macroeconomic variables. The asymptotic solution shows long-run perspectives of macroeconomic system. Equation from this work may be used in financial programming systems.

A possible improvement of the present model is connected with the splitting of capital in two parts - private industrial capital and public overhead capital. Besides, a more realistic utility function can be chosen for the model. The results of the improvements will be published elsewhere.

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FIGURES

FIG. 1. Schematic representation of the production process

FIG. 2. Numerical solution of the optimal planning differential equation. Parameters represent the economy of Belarus. Numerical solution shows that the saving per capital ratio has to be rather high initially with smooth decrement in future years. This is the main result of this work.
TABLES

TABLE I. Republic of Belarus economic development variant, when the effectiveness rate is 1.5%

| Year | Capital per man | Consumption per man | Output per man | Savings per capital |
|------|-----------------|---------------------|----------------|---------------------|
|      | value | rate % | value | rate % | value | rate % | value | rate % |
| 2000 | 780   |        | 111,0 |        | 150   |        | 0,050 |        |
| 2001 | 803,6 | 103,0  | 113,3 | 102,1  | 153,5 | 102,3  | 0,049 | 99,3   |
| 2002 | 827,7 | 106,1  | 115,6 | 104,1  | 157,0 | 104,6  | 0,048 | 98,6   |
| 2003 | 852,1 | 109,2  | 118,0 | 106,3  | 160,6 | 107,0  | 0,047 | 98,0   |
| 2004 | 877,1 | 112,4  | 120,4 | 108,5  | 164,2 | 109,5  | 0,046 | 97,4   |
| 2005 | 902,4 | 115,7  | 122,9 | 110,7  | 167,9 | 112,0  | 0,046 | 96,8   |
| 2006 | 928,3 | 119,0  | 125,5 | 113,0  | 171,7 | 114,5  | 0,045 | 96,2   |
| 2007 | 954,6 | 122,4  | 128,1 | 115,4  | 175,6 | 117,1  | 0,045 | 95,6   |
| 2008 | 981,4 | 125,8  | 130,8 | 117,8  | 179,5 | 119,7  | 0,044 | 95,1   |
| 2009 | 1008,7| 129,3  | 133,5 | 120,3  | 183,6 | 122,4  | 0,044 | 94,6   |
| 2010 | 1036,5| 132,9  | 136,3 | 122,8  | 187,6 | 125,1  | 0,043 | 94,1   |
| 2011 | 1064,8| 136,5  | 139,2 | 125,4  | 191,8 | 127,9  | 0,043 | 93,7   |
| 2012 | 1093,7| 140,2  | 142,1 | 128,0  | 196,1 | 130,7  | 0,042 | 93,2   |
| 2013 | 1123,0| 144,0  | 145,1 | 130,7  | 200,4 | 133,6  | 0,042 | 92,8   |
| 2014 | 1153,0| 147,8  | 148,2 | 133,5  | 204,8 | 136,6  | 0,041 | 92,4   |
| 2015 | 1183,5| 151,7  | 151,3 | 136,3  | 209,4 | 139,6  | 0,041 | 92,0   |
| 2016 | 1214,5| 155,7  | 154,6 | 139,2  | 214,0 | 142,6  | 0,040 | 91,6   |
| 2017 | 1246,1| 159,8  | 157,9 | 142,2  | 218,6 | 145,8  | 0,040 | 91,2   |
| 2018 | 1278,4| 163,9  | 161,2 | 145,3  | 223,4 | 149,0  | 0,040 | 90,9   |
| 2019 | 1311,2| 168,1  | 164,7 | 148,4  | 228,3 | 152,2  | 0,039 | 90,5   |
| 2020 | 1344,6| 172,4  | 168,3 | 151,6  | 233,3 | 155,5  | 0,039 | 90,2   |
Figure 1.

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Figure 2.

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