Adaptive Multi-User Clustering and Power Allocation for NOMA Systems with Imperfect SIC

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Abstract—Non-orthogonal multiple access (NOMA) is recognized as a promising radio access technique for the next generation wireless systems. We consider a practical downlink NOMA system with imperfect successive interference cancellation and derive bounds on power allocation factors for a NOMA cluster. We propose a minimum signal-to-interference-plus-noise ratio difference criterion between two successive NOMA users in a NOMA cluster to achieve higher rates than an equivalent orthogonal multiple access system. We then propose adaptive multi-user clustering and power allocation algorithms for downlink NOMA systems. Through extensive simulations, we show that the proposed algorithms achieve higher rates than the state-of-the-art algorithms.

Index Terms—Imperfect successive interference cancellation (SIC), non-orthogonal multiple access (NOMA), power allocation, spectral efficiency, user clustering.

I. INTRODUCTION

Non-orthogonal multiple access (NOMA) is one of the key technologies for fifth-generation (5G) and beyond 5G cellular networks [1]. In power-domain NOMA, the transmitter multiplexes the symbols intended for multiple users with varying power levels and transmits it in the same time-space-frequency resource. The capacity improvement with the multiplexing of users in NOMA is significantly dependent on the channel conditions of the users [2]. Hence, to achieve higher user data rates, the base station (BS) must optimally choose the number of users to be clustered together for NOMA and suitably allocates the available power to all the users in each cluster [3].

The practical NOMA systems have imperfections in successive interference cancellation (SIC) which significantly impact the network throughput performance [4]. The imperfections in the SIC strongly impact the higher channel gain user rates, and hence, impose conditions where NOMA pairing for certain users may not be beneficial for the overall system [2]. However, most of the existing works like [5], [6] discuss the user clustering algorithms considering two users in each cluster under a perfect SIC scenario. A few works in the literature have considered the imperfect SIC while proposing 2-user pairing algorithms for NOMA [2], [7]. Nevertheless, limited analysis has been performed towards user clustering and power allocation for a generalized number of users in each cluster in the presence of imperfections in SIC [8], [9].

To the best of our knowledge, this is the first work that considers the Minimum SINR Difference (MSD) criterion for user clustering and power allocation for the downlink NOMA systems with a generalized number of users in each cluster and with the imperfect SIC. Clustering schemes have been proposed in literature under the assumption that NOMA consistently outperforms orthogonal multiple access (OMA). However, it is not always the case in practice.

Motivated by the aforementioned details, we present the following key contributions in this letter.

- We propose bounds on the power allocation for multi-user NOMA clusters to achieve higher data rates than an equivalent OMA system in presence of imperfect SIC.
- We derive bounds on channel coefficients for user clustering in the presence of imperfect SIC.
- Using these derived bounds, we present multi-user clustering (MUC) and adaptive multi-user clustering (AMUC) algorithms for NOMA systems.
- We then present a novel power allocation procedure for all the users in a cluster.

In this letter, we have formulated an adaptive clustering criterion for multiple users in a NOMA cluster by comparing their NOMA rates with corresponding OMA rates. The performance of the proposed AMUC algorithm is always better than OMA even in the presence of imperfections in SIC and saturates with an increase in the number of users in a cluster but never degrades, unlike the conventional user-pairing algorithms.

II. SYSTEM MODEL

We consider a set of users $U = \{1, \ldots, N\}$, where $N$ is the number of users associated to the BS under consideration and $G \leq N$ users may be clustered as NOMA clusters as shown in Fig. 1. The downlink signal-to-interference-plus-noise ratio...
For the NOMA formulation, at any time instant, we consider $G$ users in a NOMA cluster with channel coefficients satisfying $|h_1|^2 > |h_2|^2 > \ldots > |h_G|^2$. Then, the SINR of user $i$ in the NOMA system is given by

$$\gamma_i^{\text{NOMA}} = \frac{\alpha_i P_t |h_i|^2}{\sigma^2 + I + \sum_{j=1, j \neq i}^{G-1} \alpha_j P_t |h_j|^2 + \beta \sum_{k=i+1}^{G} \alpha_k P_t |h_k|^2}.$$  \hspace{1cm} (3)

where $\alpha_i$ is the fraction of total power allocated to the user $i$ and $\beta \in [0, 1]$ is the imperfection in SIC. Note that $\beta = 0$ indicates a perfect SIC. The normalized data rate of the user $i$ in NOMA is formulated as

$$R_i^{\text{NOMA}} = \log_2(1 + \gamma_i^{\text{NOMA}}).$$  \hspace{1cm} (4)

Further, based on NOMA principle \[10\], the following conditions hold.

$$\alpha_G > \alpha_{G-1} > \ldots > \alpha_2 > \alpha_1$$  \hspace{1cm} (5)

$$\sum_{i=1}^{G} \alpha_i = 1.$$  \hspace{1cm} (6)

From (5), we conclude the following result

$$\sum_{j=i+1}^{G} \alpha_j > (G-i)\alpha_i.$$  \hspace{1cm} (7)

Next, we present the bounds on power allocation factors and channel coefficients for users in a NOMA cluster in the presence of imperfections in SIC.

### III. Computation of Bounds

In this section, we derive a lower bound on the power allocation factor and an upper bound on the imperfections in SIC for a NOMA cluster under consideration in terms of channel coefficients. Further, we formulate MSD criterion for a NOMA multi-user cluster to achieve higher NOMA user rates as compared to OMA.

#### A. Lower Bound on $\alpha_i$

We consider the NOMA rate of each individual user should be greater than the OMA rate ($R_i^{\text{OMA}} > R_i^{\text{NOMA}}$). Thus, from (2) and (4), we have

$$\log_2(1 + \gamma_i^{\text{NOMA}}) > \frac{1}{G} \log_2(1 + \gamma_i^{\text{OMA}}),$$

yielding

$$\gamma_i^{\text{NOMA}} > \frac{1 + \gamma_i^{\text{OMA}}}{G} - 1.$$  \hspace{1cm} (8)

Using $\gamma_i^{\text{NOMA}}$ from (3) in (8), we have

$$\frac{1 + \sum_{j=1, j \neq i}^{G-1} \alpha_j \gamma_j^{\text{OMA}} + \beta \sum_{k=i+1}^{G} \alpha_k \gamma_k^{\text{OMA}}}{\gamma_i^{\text{OMA}}} > \frac{1 + \gamma_i^{\text{OMA}}}{G} - 1.$$  \hspace{1cm} (9)

Solving it further, we obtain lower bound on $\alpha_i$ as follows:

$$\alpha_i > \left(1 + \left(1 + (\beta - 1) \sum_{j=1}^{G} \alpha_j\gamma_j^{\text{OMA}}\right)\chi_0(\gamma_i^{\text{OMA}})\right) \chi_0(\gamma_i^{\text{OMA}}).$$  \hspace{1cm} (10)

Using (10), we reformulate (9) as $\alpha_i > \delta_i$, where

$$\delta_i = \left(1 + \left(1 + (\beta - 1) \sum_{j=1}^{G} \alpha_j\gamma_j^{\text{OMA}}\right)\chi_0(\gamma_i^{\text{OMA}})\right) \chi_0(\gamma_i^{\text{OMA}}).$$  \hspace{1cm} (11)

Substituting (7) in (11), we get

$$\alpha_i > \sum_{j=1}^{G} \alpha_j > (G-i)\alpha_i.$$  \hspace{1cm} (12)

Thus, if $\alpha_i > \left(1 + (1 + (\beta - 1) \sum_{j=1}^{G} \alpha_j\gamma_j^{\text{OMA}}\right)\chi_0(\gamma_i^{\text{OMA}})$, then $\alpha_i > \delta_i$. Solving $\alpha_i$ further, we obtain

$$\alpha_i > \frac{(1 + \gamma_i^{\text{OMA}})\chi_0(\gamma_i^{\text{OMA}})}{1 - (\beta - 1) \gamma_i^{\text{OMA}} \chi_0(\gamma_i^{\text{OMA}})}.$$  \hspace{1cm} (13)

Note that if (13) is satisfied, then $R_i^{\text{NOMA}}$ is always greater than $R_i^{\text{OMA}}$. Further, (9) is a sufficient condition to achieve higher NOMA rates, whereas (13) is a much stricter bound as compared to (9). Since (13) is dependent only on $\gamma_i^{\text{OMA}}$ and $G$, we use (13) to define constraints for multi-user clustering.

#### B. Upper Bound on the Imperfect SIC Parameter $(\beta)$

From (5), $\alpha_i > \alpha_{i-1}$. Using the bounds in (13), we get

$$\frac{(1 + \gamma_i^{\text{OMA}})\chi_0(\gamma_i^{\text{OMA}})}{1 - (\beta - 1) \gamma_i^{\text{OMA}} \chi_0(\gamma_i^{\text{OMA}})} > \frac{(1 + \gamma_i^{\text{OMA}})\chi_0(\gamma_i^{\text{OMA}})}{1 - (\beta - 1) \gamma_i^{\text{OMA}} \chi_0(\gamma_i^{\text{OMA}})}.$$  \hspace{1cm} (14)

For ease of exposition, we define the following terms

$$D_i = \frac{(1 + \gamma_i^{\text{OMA}})\chi_0(\gamma_i^{\text{OMA}})}{(1 + \gamma_i^{\text{OMA}})\chi_0(\gamma_i^{\text{OMA}})},$$

$$E_i = (G - i)\gamma_i^{\text{OMA}} \chi_0(\gamma_i^{\text{OMA}}),$$

$$E_i - 1 = (G - i + 1)\gamma_i^{\text{OMA}} \chi_0(\gamma_i^{\text{OMA}}).$$
Substituting $D_i$, $E_i$ and $E_{i-1}$ in (14), we get

$$1 - (\beta - 1)E_{i-1} > D_i[1 - (\beta - 1)E_i],$$

(15)

$$\beta < \frac{1 - D_i}{E_{i-1} - D_iE_i} + 1 = \zeta_{i-1,i}. $$

(16)

Thus, for the NOMA rates to be higher than the OMA rates, the typical value of imperfections in SIC must satisfy the constraint in (16) for the multi-user clustering in NOMA under consideration.

C. MSD between Successive Users

In case of $G$ users clustered in NOMA system, we define the MSD between two users for achieving higher NOMA rates as follows. We apply positivity constraint $\beta > 0$ in (16), thus,

$$1 - D_i > D_iE_i - E_{i-1} \rightarrow E_{i-1} > D_iE_i + D_i - 1. $$

(17)

Substituting $E_{i-1}$ from (15) in (17), we get

$$(G - i + 1)\gamma_{i-1}^{OMA} \Delta_{i-1} > D_iE_i + D_i - 1,$$

$$\gamma_{i-1}^{OMA} > \frac{D_iE_i + D_i - 1}{(G - i + 1)\alpha_i^{OMA}}, $$

$$\zeta_{i-1}^{OMA} > \gamma_{i-1}^{OMA} - \gamma_i^{OMA}. $$

Thus, we define the MSD between users $i - 1$ and $i$ as follows

$$\Delta_{i-1,i}^{MSD} = \frac{D_iE_i + D_i - 1}{(G - i + 1)\alpha_i^{OMA}} - \gamma_i^{OMA}. $$

(18)

Note that for a cluster of $G$ users, we have $G - 1$ combinations of $\Delta_{i-1,i}^{MSD}$ and $\zeta_{i-1,i}$, values. In case (16) and (18) are satisfied for all these combinations, then all the G users can be clustered together to achieve NOMA rates higher than their OMA counterparts. Further, we next use the MSD formulated in (18) and the lower bound on power allocation formulated in (9) to propose MUC, AMUC and power allocation algorithms for NOMA systems, respectively.

IV. PROPOSED ALGORITHMS

In this section, we explain the procedure for the clustering of users. We propose MUC and AMUC algorithms to achieve higher NOMA user rates and compare their performance with a conventional near-far (NF) user pairing algorithm. Given a set of users in a cluster, we then present power allocation for each user.

A. Multi-User Clustering (MUC) Algorithm

We use the upper bound on $\beta$ (16) and the MSD criterion (18) to propose an MUC algorithm for a generalized number of users as follows. With $G$ users in a cluster, we first evaluate $\zeta_{i-1,i}$ as in (16) and $\Delta_{i-1,i}^{MSD}$ as in (18) for all the $G - 1$ combinations. We consider clustering these $G$ users in NOMA only when (16) and (18) are satisfied for all the $G - 1$ combinations. Else, all the $G$ users are designated as OMA users. This way, the individual rates of each user will never be less than that of the corresponding OMA rates. Since, we check the criteria in (16) and (18) for $G - 1$ times in each of $N/G$ clusters, the complexity of the proposed MUC is $O(2N/G \times (G - 1))$. Further, the probability of users not meeting MSD criterion increases with an increase in the number of users in a cluster and imperfections in SIC.
with large sized user clustering and higher values of $\beta$, the MUC algorithm will designate most of the users as OMA. To address this issue, next, we propose an AMUC algorithm.

B. Adaptive Multi-User Clustering (AMUC) Algorithm

In the AMUC algorithm, whenever the $G$ users in a cluster fail to meet $\zeta_{i-1}$ and $\Delta_{i-1}^{\text{MDD}}$, we split the cluster into two halves. For this new cluster of users, we again perform the criterion check as formulated in (16) and (18). If the criterion is met for all the combinations, we continue clustering those $G/2$ users in NOMA. Otherwise, we continue splitting this cluster again into two new halves. We continue this procedure until we end up with a single user. If the MSD criterion is not met for any user clustering, then that single user will be designated as OMA user. Further, while splitting a cluster into two halves, we follow the NF [11] user pairing procedure. Otherwise, the throughput gains will not be achieved. In Fig. 3, we have presented an example of splitting a 4-user cluster into two 2-user clusters. Note that the 2 users clustered together after the splitting process in Fig. 3 are exactly the same as they would have been in the case of 2-user clustering in Fig. 2.

The complexity of the proposed algorithm is calculated as follows. For each of $N/G$ clusters, the proposed algorithm initially checks (16) and (18) criterion for $(G-1)$ combinations. If the conditions are not satisfied, then it splits the cluster into two halves and checks the criteria again. This results into an additional $4 \times ((G/2) - 1)$ computations. This procedure is continued till only one user is left in a cluster. Thus, in a worst case scenario, the complexity of the proposed AMUC algorithm is $O\left(\left(2N/G\right)\left(G\log_2 G - \left(1 - G\right)/(1 - \log_2 G)\right)\right)$. Next, we propose the power allocation algorithm for a given set of clustered users.

C. Power Allocation

Based on the lower bound formulated in (9), we allocate the minimum power required for each user as follows

$$
\alpha_i = \left(1 + \left(1 - \sum_{j=i+1}^{G} \alpha_j^{\text{OMA}}\right)\chi_0(\gamma_i^{\text{OMA}})\right).
$$

We begin allocation with user $G$, as $\alpha_0$ is dependent only on $\gamma_0^{\text{OMA}}$. We then use this allocated power $\alpha_0$ and $\gamma_0^{\text{OMA}}$ to recursively compute the power allocation factor $\alpha_{G-1}$. Likewise, we continue allocation till $\alpha_1$. Further, when we allocate the minimum power required for each user based on (9), $\sum_{i=1}^{G} \alpha_i$ is less than 1. Hence, to maximize the achievable sum rates, we allocate the remaining power $(1 - \sum_{i=1}^{G} P_i)$ to the strong user.

We have presented a pseudo-code to implement the proposed MUC, AMUC, and power allocation in Algorithm 1. Next, we present the simulation results.

V. RESULTS AND DISCUSSION

For the evaluation of the proposed algorithms, we have considered Poisson point distributed BSs and users with densities $25$ BS/km$^2$ and $2000$ users/km$^2$, respectively. The simulation parameters and the path loss model considered for the evaluation are as per the urban cellular scenario presented in [12]. For each user, we calculate the received SINR from each BS and then associate the user to the BS from which it receives maximum SINR. We then randomly pick $N = 64$ users associated with each BS and perform evaluation for user clusterings with $G \in \{2, \ldots, 32\}$. For each $G$ value, we perform the proposed user clustering, MUC, AMUC, and power allocation as mentioned in Algorithm 1. We then perform Monte-Carlo simulations to calculate the cell spectral efficiency for each algorithm.

In Fig. 4 we plot cumulative distribution function (CDF) of cell spectral efficiency for varying number of users in each cluster and for perfect SIC ($\beta = 0$). The performance of the OMA system does not vary with number of users in each cluster. With MUC, NOMA achieves higher performance than OMA, but the performance degrades as we further increase the number of users in each cluster. This is because, with a larger number of users in a cluster, the constraint (18) fails more often, and all the users in the cluster will be designated as OMA. Thus, the performance degrades with an increase in user clustering, but it still outperforms the OMA. With AMUC, whenever the constraint (18) fails, the algorithm tries to cluster the users in a smaller size. Hence, the performance saturates but never degrades with an increase in the number of users in a cluster. A similar trend is observed in Fig. 5 where we have presented the variation of mean cell-spectral efficiency with
various clustering algorithms. Note that we have evaluated the conventional NF user pairing \([11]\). This algorithm always considers only two users in a cluster, and hence, the mean cell spectral efficiency is constant for any number of users in a cluster. As shown in Fig. 5 with large user clustering \((G > 8)\), the proposed AMUC algorithm performs \(37.2\%\), and \(8.3\%\) better than the OMA baseline, and the conventional NF algorithm \([11]\) in terms of mean cell spectral efficiency for \(\beta = 0\), respectively.

In Fig. 6 we plot CDF of cell spectral efficiency with 4 users in each cluster and varying imperfection in SIC. The performance of NOMA systems decreases with an increase in \(\beta\). Whenever the user channel coefficients do not satisfy the constraint in \([16]\) and \([18]\), the performance with NOMA systems is equivalent to OMA systems (here for 4-user clustering, bound on \(\beta\) is observed to be 0.0561). However, the CDF of cell spectral efficiency of NF algorithm decreases beyond OMA for higher values of imperfections in SIC.

In Fig. 7 we present the mean cell spectral efficiency with varying imperfection in SIC. The performance of OMA does not change with number of users in each cluster and varying \(\beta\). The mean cell spectral efficiency of NF-based algorithm decreases beyond OMA with increasing values of \(\beta\). In case of MUC, for increasing \(\beta\) values, the mean cell spectral efficiency decreases gradually and converges with OMA. Further, for smaller value of \(\beta\), the mean cell spectral efficiency decreases with increasing number of users per cluster. With larger user clustering, the performance of MUC will be close to OMA systems as the condition fails most of the time. However, for smaller \(\beta\), the performance of the proposed AMUC saturates with an increase in user clustering, but it does not degrade. For larger \(\beta\), when the channel coefficients do not satisfy the upper bound on \(\beta\) presented in \([16]\), the performance converges with OMA. In such scenarios, AMUC tries to avoid clustering, and hence, convergence with OMA. Thus, in the presence of imperfect SIC, NOMA has superior performance only when the user’s channel coefficients satisfy \([16]\) and \([18]\).

VI. CONCLUSION

We have derived bounds on power allocation, imperfect SIC, and channel coefficients for multi-user clustering in a downlink NOMA system. We have proposed adaptive multi-user clustering and power allocation algorithms for a generalized number of users in a cluster. We have derived conditions and shown that it is beneficial to not utilize NOMA for certain users to achieve higher user data rates. We have also shown that an increase in the number of users in a cluster does not always achieve higher NOMA rates. In future, we plan to implement and evaluate the proposed algorithms on hardware testbeds.

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