A Poincaré Covariant Current Operator for Low and High Energy Phenomena

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Abstract

Within front-form dynamics and in the Breit frame where initial and final three-momenta of the system are directed along the $z$ axis, Poincaré covariance constrains the current operator only through kinematical rotations around the $z$ axis. Therefore, in this frame the current can be taken in the one-body form. Applications to deep inelastic structure functions in an exactly solvable model and to the deuteron magnetic form factor are presented.
The electromagnetic (em) and weak current operators should properly commute with the Poincaré generators and satisfy Hermiticity. The em current should also satisfy parity and time reversal covariance, as well as continuity equation. For instance, the tensor $\langle P', \chi'|J^\mu(x)J^\nu(0)|P', \chi' \rangle$ in deep inelastic scattering will have correct transformation properties relative to the Poincaré group only if both the nucleon state $|P', \chi' \rangle$, with four-momentum $P'$ and internal wave function $\chi'$, and the operator $J^\mu(x)$ have correct transformation properties with respect to the same representation of the Poincaré group.

In ref. [1] we investigated within the front-form dynamics [2] the constraints imposed on the current operator $J^\mu(x)$ by extended Poincaré covariance (continuous + discrete transformations), Hermiticity and current conservation using a spectral decomposition of the current operator and we showed that Poincaré covariance of $J^\mu(x)$ can take place if $J^\mu(0)$ satisfies Lorentz covariance [1].

Let $\mathcal{H}$ be the space of states for an $N$ particle system and let $\Pi_i$ be the projector onto the subspace $\mathcal{H}_i \equiv \Pi_i \mathcal{H}$ corresponding to the mass $M_i$ and the spin $S_i$. Since $J^\mu(0) = \sum_{ij} \Pi_i J^\mu(0) \Pi_j$ the operator $J^\mu(0)$ is fully defined by the operators $J^\mu(P_i, P_j)$, acting in the internal space and corresponding to definite values of the masses: $J^\mu(P_i, P_j) \equiv \langle \vec{P}_\perp, P^+|\Pi_i J^\mu(0) \Pi_j|\vec{P}_\perp^+, P^+ \rangle$, with $\vec{P}_\perp \equiv (P_x, P_y)$ and $P^\pm = (P^0 \pm P^z)/\sqrt{2}$. In the front form rotations around the $z$ axis are kinematical, while the ones around $x$ and $y$ axes are dynamical. To take advantage of this fact, we use the Breit frame with the initial and final three-momenta of the system directed along the $z$ axis. In order to satisfy Poincaré covariance, the operator $j^\nu(K\vec{e}_z; M_i, M_j)$ (which is the current operator $J^\mu(K, K'; M_i, M_j)$ in this particular Breit frame, with $K\vec{e}_z = K_\perp$) has to be covariant with respect to rotations around the $z$ axis [1]

$$j^\mu(K\vec{e}_z; M_i, M_j) = L(u_z)_{\mu} D^{S_i}(u_z) j^\nu(K\vec{e}_z; M_i, M_j) D^{S_j}(u_z)^{-1} \tag{1}$$

where $D^{S}(u)$ is the rank $S$ representation of the rotation $u_z$ around the $z$ axis.

Therefore for a non-interacting system the continuous Lorentz transformations constrain the current $j^\mu(K\vec{e}_z; M_i, M_j)$ in the same way as in the interacting case. The same property holds for the covariance with respect to the reflection of the $y$ axis, $\mathcal{P}_y$, and to the product of parity and time reversal, $\theta$, which leave the light cone $x^+ = 0$ invariant and are kinematical. Hence the constraints imposed on the current for an interacting system by extended Lorentz covariance can be fulfilled by a current composed in our frame by the sum of one-body currents (i.e., by $J^\mu_{\text{free}}(0) = \sum_{n=1}^{N} j^\mu_{\text{free},n}$, where $N$ is the number of constituents).

The Hermiticity, $j^\mu(K\vec{e}_z; M_i, M_j)^* = j^\mu(-K; M_j, M_i)$, is satisfied if

$$j^\mu(K\vec{e}_z; M_i, M_j)^* = L[r_x(\pi)]_{\mu} D^{S_i}[r_x(\pi)] j^\mu(K\vec{e}_z; M_j, M_i) D^{S_j}[r_x(\pi)]^{-1} \tag{2}$$

where the symbol $^*$ means Hermitian conjugation in internal space and $r_x(\pi)$ represents a rotation by $\pi$ around the $x$ axis. Equation (2) is a non-trivial constraint when $M_i = M_j$ (i.e., for elastic form factors), because in this case the rhs and the lhs contain the same operator [1].
As a first application, we will study the deep inelastic scattering in an exactly solvable model. Because of Hermiticity, the hadronic tensor for a system of mass \( m \), ground state \( \chi_0 \) and initial momentum \( \vec{P} \), in the Breit reference frame where \( \vec{P}_{\perp} = \vec{q}_{\perp} = 0, P_z = -P_z' = K > 0 \), can be written as follows

\[
W^{\mu\nu} = \frac{1}{4\pi} \sum (2\pi)^4 \delta^{(4)}(P + q - P') \langle \chi_0 | j^\mu(K\vec{e}_z; m, M') | \chi' \rangle \cdot \langle \chi_0 | j^\nu(\vec{K}\vec{e}_z; m, M') | \chi' \rangle
\]

where the sum is taken over all final states \( |P', \chi'\rangle \) of mass \( M' \).

As we have seen, the free current fulfills the extended Lorentz covariance in our Breit frame. Furthermore in the calculation of the structure functions only three components of the current are needed and can be chosen unconstrained by the current conservation, while the fourth component can be determined through the continuity equation. Therefore the structure functions can be calculated by using the \( + \) and \( \perp \) components of the free current in our Breit frame, even in the case where the final state interaction is present (cf. [1], [3]).

Let us consider a system of two particles, each one of mass \( m_o \) and spin \( (1/2, \sigma_n) \), interacting through a relativistic harmonic oscillator potential, with ground state \( \chi_0(\vec{k}_{\perp}, \xi, \sigma_1, \sigma_2) \) \((\vec{\xi} = p^+/P^+)\), and mass eigenvalues \( M_o = 2\{m_o^2 + a^2[2(n_x + n_y + n_z) + 3]\}^{1/2} \). If the exact harmonic oscillator eigenstates are used, the hadronic tensor and then the structure functions become sums of \( \delta \) functions. In order to obtain continuous functions, one can consider average values, \( \overline{F}_{1(2)}(x, Q) \), of the structure functions over small intervals of \( x \), which resemble the finite experimental resolution. Taking exactly into account the interaction, both in the initial and in the final states, in the Bjorken limit \((Q^2 \to \infty, x = Q^2/(2Pq))\) the averaged structure functions yield exactly the parton model results [3]:

\[
x = \xi, \quad \overline{F}_1(x) = \overline{W}^{11} = \sum_{\sigma_1, \sigma_2} \int |\chi_0(\vec{k}_{\perp}, x, \sigma_1, \sigma_2)|^2 d\vec{k}_{\perp} / \{4(2\pi)^3 x (1 - x)\}
\]

\[
W^{+\nu} = 0, \quad \overline{F}_2(x, Q) = 2x\overline{F}_1(x, Q).
\]

As a second application, we calculate elastic deuteron form factors. Let \( \Pi \) be the projector onto the subspace of bound states \( |m, S, S_z\rangle \). The following current:

\[
j^\mu(K\vec{e}_z; m, m) = \frac{1}{2} \{ \mathcal{J}^\mu(K\vec{e}_z; m, m) + \mathcal{J}^\mu(-K\vec{e}_z; m, m)^* \}
\]

\[
\mathcal{J}^\mu(-K\vec{e}_z; m, m) = L[r_x(-\pi)]\exp(i\pi S_x)\mathcal{J}^\nu(K\vec{e}_z; m, m)\exp(-i\pi S_x).
\]

(with \( \mathcal{J}^\mu(K\vec{e}_z; m, m) = \langle 0, P^+ | \Pi J_{\text{free}}^\mu(0) | 0, P'^+ \rangle \)) is compatible with extended Lorentz covariance and Hermiticity, Eq. (3), and fulfills current conservation \([1]\). In the elastic case one has only \( 2S + 1 \) independent matrix elements for the em current defined in Eq. (3), corresponding to the \( 2S + 1 \) elastic form factors \([1]\). Therefore, the extraction of em form factors is no more plagued by the ambiguities which are present when the reference frame \( q^+ = 0 \) is used (see, e.g., [3]). Only three matrix elements
Figure 1. - The deuteron magnetic form factor $B(Q^2)$ obtained with the Gari-Krümpelmann nucleon form factors and different $N - N$ interactions: RSC (dotted line), Av14 (solid line), Av18 (dashed line), Paris (dot-dashed line). Experimental data are from Refs. [6a] (open dots) and [6b] (full dots).

$\langle m_d SS_z | j^\mu(K e_z; m_d, m_d) | m_d SS'_z \rangle$ are independent for the deuteron (e.g., $\langle m_d 10 | j^+ | m_d 10 \rangle$, $\langle m_d 11 | j^+ | m_d 11 \rangle$, $\langle m_d 11 | j_x | m_d 10 \rangle$), corresponding to the three em elastic form factors.

In Fig. 1 we report our result for the deuteron magnetic form factor

$$B(Q^2) = 2 \langle m_d 11 | j_x (K e_z; m_d, m_d) | m_d 10 \rangle^2 / (3m_d^2)$$

(6)
corresponding to different $N - N$ interactions and the Gari-Krümpelmann nucleon form factors. A reasonable agreement with the available experimental data is achieved. A more stringent comparison with the new TJNAF data will be possible in the near future. In Table 1 our results for the deuteron magnetic moment

$$\mu_d = \lim_{Q \to 0} 2^{1/2} m_p \langle m_d 11 | j_x (K e_z; m_d, m_d) | m_d 10 \rangle / (Q m_d)$$

(7)

are compared with the results of Ref. [3], obtained using the free current in the $q^+ = 0$ reference frame. While the $q^+ = 0$ approach points to a low $P_D$, our covariant approach prefers higher $P_D$ values.

Calculations for charge and quadrupole form factors are in progress.

References

[1] F. Lev, E. Pace, G. Salmè: Nucl. Phys. A, in press.
Table 1. - $\mu^{th}_d$ for different $D$-state percentages, $P_D$ ($\mu^{exp}_d = 0.8574$).

| Interaction | $P_D$ | $\mu_d$ (ref. [5]) | $\mu_d$ (this paper) |
|-------------|-------|---------------------|---------------------|
| RSC         | 6.47  | 0.8500              | 0.8611              |
| Av14        | 6.08  | 0.8516              | 0.8608              |
| Paris       | 5.77  | 0.8531              | 0.8632              |
| Av18        | 5.76  | 0.8635              |                     |

[2] P.A.M. Dirac: Rev. Mod. Phys. 21, 392 (1949).

[3] E. Pace, G. Salmè, F. Lev: Phys. Rev. C 58, 2655 (1998).

[4] I.L. Grach and L.A. Kondratyuk: Yad. Fiz. 39, 316 (1984).

[5] P.L. Chung, F. Coester, B.D. Keister and W.N. Polyzou: Phys. Rev. C37, 2000 (1988).

[6] a) S. Platchkov et al.: Nucl. Phys. A510, 740 (1990); b) P.E. Bosted et al.: Phys. Rev. C 42, 1 (1990).