Anomaly-driven decay of massive vector bosons

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Abstract

We inquire if the total decay rate of neutral vector mesons can contain relevant off-shell contributions from the triangle anomaly.

The answer seems to be affirmative for the case of J/Ψ, and as a byproduct we get estimates of the decay width for neutral pions.

A vector meson can not decay into two real photons. But that doesn’t mean that the decay do not exist, it inhabitates the virtual channels. When we rescale the pion anomalous decay into other pseudoscalars, we get an estimate of the γγ decay of these particles. But when we scale up to vector mesons, we can not get real decay. Instead, the virtual decay will be a contribution to the total decay amplitude. We wonder in what cases, if any, this contribution will be the dominant one.

We can ask the question in a reverse way: to scale down, so that a dominant role of the triangle in the massive vector will amount to an estimate of the anomalous decay of π^0 (and, from Γ^π^0, also of the constant f^π^0)

Let’s explore the experimental data. From PDG 2004

\[ \Gamma^\text{exp}_{\pi^0} = 7.8 \text{ eV (}+0.6, -0.5, S = 3) \]

or, from second order perturbation theory [3],

\[ \Gamma^\text{th}_{\pi^0} = 8.10 \text{ eV (} \pm 0.08) \]

The ρ, K^*, ω, φ have access to stronger decay mechanisms via OZI-allowed transitions, so the contribution from the triangle anomaly goes unnoticeable and we can not make use of them \(^1\). Only for massive quarkonia, J/Ψ and Υ, the transitions allowed by OZI rule are closed by the respective energy thresholds of DD and BB.

So our first candidate to try is J/Ψ. We get

\[ \left( \frac{m_{\pi^0}}{m_{J/\Psi}} \right)^3 \Gamma_{J/\Psi} = 7.5 \text{ eV (} \pm 0.3) \]

which is a fairly perfect quantity, practically exhausting the available width.

On the contrary for Υ, the decay rate is too low. Perhaps something is closing the access to the triangle. The decay fails by about one MacGregor’s

\(^1\)Still, we could make some use of decays of ρ to photon plus anything, or of other mesons to photon plus charged plus anything, but a very subtle analysis of the decay process is needed.
magnitude\footnote{Experimentally, three jets are observed. Does it decay via three gluons, as expected from OZI violation, or via some box diagram? Or direct quark decay? (the order of magnitude seems right for the later possibility)}\footnote{Note also that the equidistance with the electroweak decaying pseudoscalars is maintained approximately}. It can be a hint of the special role of the third generation; for the top quark it is unnatural to assume a zero mass, and via SU(2) -or isospin if you prefer- the protection against anomaly extends to the bottom quark.

And now, a surprise! For $Z^0$, we can claim that a 't Hooft principle seems to be working behind the curtains, because we get an almost excessively exact

\[(\frac{m_{\pi}}{m_{Z^0}})^3 \Gamma_{Z^0} = 8.1 \text{ eV } (\pm0.0)\]

From this last result, it should mean that we can get $f_{\pi^0}$ from only the electroweak parameters (plus $m_\pi$ in higher orders). Lets write it explicitly, for instance by joining formulae (1.3) and (8.91) of \cite{8}

\[\xi^2(\frac{\alpha}{\pi})^2 \frac{1}{64\pi} \frac{1}{f_\pi^2} \approx \frac{G_F}{\sqrt{2}\pi} \sum_{f} C_f(|V_f|^2 + |A_f|^2)\]

Jointly, the three results could be interpreted as hints of a deeper symmetry that affects to the $Z$ particle too and at the same time protects the top/bottom quarks from getting a zero mass. About how this zero mass could appear, let me to address you towards the figure in \cite{5} where it was suggested to drive the fine structure constant down to zero at the same time that the masses of the first generation\footnote{In an intermediate step of the process, the mass of the muon becomes degenerate with the pion, an amusing "reflected susy". The pion becomes stable, and then -if having isospin symmetry- it still can get a non null mass because of the infinite in $1/f_\pi$}. We also suspect that the second generation should get a null mass against the electroweak scale, from hints in \cite{9}.

In a humbler approach, let me note that the discrepancy between theory and calculation for hadronic OZI-forbidden rates has been traditionally problematic, see for instance section 4.5.3 of \cite{7}; thus the possibility of a intermediate virtual state could help to settle the issue.

\textbf{About the figures:} This work was inspired by the classification of decays reviewed in \cite{2}, where integer powers of the fine structure constant are used, without accounting the mass nor another kinematic. Most of this impressive ordering gets lost when we scale with a power of the mass, so our graphs are not so interesting, and they only show the peculiarity of the $\Upsilon$ particles below the $B\bar{B}$ threshold. To put some separation in the horizontal axis, I have preferred the Inverse ArcTan visualisation communicated to me by Yuri Danoyan some weeks ago, but taking as mass origin the $\eta'$ instead of the nucleon mass. Yuri calls this parametrisation "dodecahedron" because it produces some angles about $180 - 144 = 36$ and $180 - 108 = 72$ degrees, as well as submultiples.

\textbf{References}

\begin{itemize}
\item [1] Z. P. Li, F. E. Close, T. Barnes, \textit{Relativistic effects in gamma gamma decays of P-wave positronium and qq-bar systems}, \textit{Phys. Rev. D} 43, 2161-2170 (1991)
\end{itemize}
[2] Malcolm H. Mac Gregor, *Electron generation of leptons and hadrons with reciprocal alpha-quantized lifetimes and masses*, arxiv:hep-ph/0506033

[3] J.L. Goity, A.M. Bernstein, B.R. Holstein, *The decay π0 to gamma gamma to next to leading order in Chiral Perturbation Theory*, Phys. Rev. D 66, 076014 (2002) arxiv:hep-ph/0206007

[4] C. N. Yang, *Selection Rules for the Dematerialization of a Particle into Two Photons*, Phys. Rev. 77, 242-245 (1950)

[5] Alejandro Rivero, Andre Gsponer, *The strange formula of Dr. Koide*, arxiv:hep-ph/0505220

[6] Hans de Vries, Alejandro Rivero, *Evidence for radiative generation of lepton masses*, arxiv:hep-ph/0503104

[7] A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, “Hadron Transitions In The Quark Model,” Gordon And Breach (New York, 1988) 311p

[8] R. K. Ellis, W. J. Stirling and B. R. Webber, “QCD and collider physics,” Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. 8, 1 (1996).
Figure 1: Decay widths of PDG 2004 table of particles, scaled according the cube of their mass, as described in the text. Electroweak particles are shown in the next figure, for completeness. We use Danoyan’s parametrisation in the horizontal axis, and MacGregor’s in the vertical.
Figure 2: Decay widths for the electroweak decaying particles, scaled according to $M^3$ as in previous plot. Note that the pion in this plot is the charged one.