Photonic Quantum Computation by Chiral Quantum Networks

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Inspiried by the excellent control of single photons realized by the atom-photon-chiral couplings, we propose a novel potential photonic-quantum-computation scheme. The single-photon rotating and phase-shift operations, which can be controlled by another single photon, are realized by properly designed atom-photon-chiral couplings. The operations can be integrated into a chiral quantum network to realize photonic quantum computation. Based on the proposal, an algorithm to perform the machine learning tasks is developed, in which the essential nonlinearities come from the appropriately designed operations.

Quantum computing realizes the quantum algorithms based on the quantum mechanical phenomena, such as quantum superposition and quantum entanglement. Thanks to the quantum properties, quantum computing shows superiority in certain problems compared to classical computers \cite{1, 2}. Single photons are one of the most suitable carriers for quantum information because they transport at the speed of light and rarely interact with the environment. In photonic quantum computation, it is critical to realize the efficient photonic gates at the level of single quanta, which are based on simple manipulations and can be effectively integrated into a quantum circuit, especially for the two-qubit gate consisting of the operation on a single photon controlled by another single photon.

On the other hand, machine learning aims to investigate the algorithms to learn from the data and make predictions \cite{3}. Benefiting from increasingly powerful computer and algorithms, machine learning becomes a rapidly developing field in computer science and has successful applications in many fields, such as computer vision, pattern recognition, data mining, speech recognition, natural language processing, and so on. It is expected to improve machine learning by quantum computing \cite{4, 5}, which is known as quantum machine learning. Currently, it is a significant issue to realize quantum machine learning based on the quantum hardware platform \cite{6}.

Recently, the chiral coupling between the transversely tightly confined photons and the atoms with polarization-dependent dipole transitions has been realized in practice \cite{7, 8}. This is underpinned by the fact that the transversely tightly confined photons, which propagate towards opposite directions, gain different local polarizations. Significantly, atom-waveguide chiral couplings provide the exciting approaches to manipulate the single photons. Inspired by this, we propose to realize photonic quantum computation based on a new potential scheme, i.e. one-dimensional (1D) waveguides chirally coupled to atoms. The efficient photonic rotating, phase, controlled-rotating and controlled-phase gates operated at the single-photon level are realized. Each of the gates is constituted by one atom chirally coupled to waveguides. It is feasible to integrate the gates into a chiral quantum network \cite{9} by properly designing the scheme structure. Therefore, it is expected that the study opens an avenue for photonic quantum computation based on the chiral quantum network. Moreover, we realize machine learning based on the proposed photonic-quantum-computation scheme. To verify the learning ability, the performance of supervised learning tasks is numerically simulated based on gradient descent optimization. The study also shows an excellent platform for photonic quantum machine learning.

We first consider three types of atom-waveguide-chiral couplings as shown in Fig. 1(a), which constitute essential gates for gate-based photonic quantum computation.

Photonic single-qubit rotating gate: There is a Λ-type atom chirally coupled to a pair of waveguides, type a in Fig. 1(a). The atomic transition $|1\rangle \leftrightarrow |2\rangle$ is coupled to the right-moving single photons in the waveguide $m_0$ and waveguide $m_1$ with strength $g$. The atom-waveguide coupling strength relates to the atomic location. We assume that all the atom-waveguide strengths are frequency independent in our scheme. This is equivalent to Weisskopf-Wigner approximation. An external Laser is introduced to drive the transition $|2\rangle \leftrightarrow |3\rangle$. Initially, the atom is in its ground state $|1\rangle$. Due to the atom-waveguide interaction, the single photon moving in the waveguide $m_0$ ($m_1$) towards the atom is delivered into the waveguide $m_1$ ($m_0$) with certain probability. The transfer matrix corresponding to this process is \cite{10}

$$ R(g, \Omega, \delta, \Delta) = \begin{pmatrix} \Delta_k (\delta_h - i \Delta) \Omega^2 & \Delta_k \delta_h \Omega^2 & \Delta_k (\delta_h - i \Delta) \Omega^2 \\ \Delta_k \delta_h \Omega^2 & \Delta_k (\delta_h - i \Delta) \Omega^2 & \Delta_k (\delta_h - i \Delta) \Omega^2 \\ \Delta_k (\delta_h - i \Delta) \Omega^2 & \Delta_k \delta_h \Omega^2 & \Delta_k (\delta_h - i \Delta) \Omega^2 \end{pmatrix} $$

with $\Omega$ the Rabi frequency of the Laser, $\Gamma = g^2$, and $\Delta_k = \delta_h - \Delta$. The symbol $\delta_h$ ($\Delta$) denotes the detuning between the input photon (external Laser) and the corresponding atomic transition.

In this work, a qubit is represented by the pair of waveguides. The facts that the single photon with energy $E$ is in the waveguide $m_0$ and waveguide $m_1$ are described by the two degenerate orthogonal photonic
quantum states of the qubit, respectively. The atom-waveguide interaction plays the role of a rotation between the two states and hence acts as a rotating gate.

Photonic single-qubit phase gate: There is a \(\Lambda\)-type atom chirally coupled to the waveguide \(m_{0}\), type \(b\) in Fig. 1(a). It is the special case of type \(a\) when the atom is decoupled to the waveguide \(m_{1}\). The transfer matrix has the form

\[
P(g, \Omega, \delta, \Delta) = \begin{pmatrix}
\frac{\Delta \delta g - \Omega^2}{\Delta g (\delta + i\Omega)} & 0 \\
0 & 1
\end{pmatrix}.
\]

The single photon in the waveguide \(m_{0}\) gains a phase shift, with the phase tuned by the Laser.

Photonic two-qubit controlled-rotating and controlled-phase gates: There is a five-level atom chirally coupled to the waveguide \(m_{1}\), waveguide \(m_{0}'\) and waveguide \(m_{1}'\), type \(c\) in Fig. 1(a). The atomic transitions \(|1\rangle \leftrightarrow |2\rangle\) and \(|3\rangle \leftrightarrow |5\rangle\) are driven by the right-moving photon in the waveguide \(m_{1}\) with strengths \(g\) and \(g'\), respectively. The transition \(|2\rangle \leftrightarrow |3\rangle\) is driven by an external Laser. The transition \(|4\rangle \leftrightarrow |5\rangle\) is driven by the right-moving photon in the waveguide \(m_{0}'\) and waveguide \(m_{1}'\) with strength \(g''\). For simplicity, we assume that all the atom-waveguide coupling strengths are equal to \(\sqrt{P}\) in our scheme. The atom is initialized in the state \(|1\rangle\), and the \(m\)-th pair of waveguides contains a right-moving photon. We label the two situations that the photon is contained in waveguide \(m_{1}\) and in waveguide \(m_{0}\) by situation \(A\) and situation \(B\), respectively. In situation \(A\), the atom will absorb the photon and then reemit it, meanwhile making the transition \(|1\rangle \rightarrow |5\rangle\) with probability \(P_{\text{conv}}\) or maintaining in the state \(|1\rangle\). The former transition corresponds to the inelastic scattering in most cases because of energy conservation. This constitutes a tunable frequency converter with the conversion efficiency \(P_{\text{conv}}\). The frequency of reemitted photon and the conversion efficiency can be tuned by the external Laser. The quantum frequency converter has many critical applications for connecting the quantum systems with different frequencies. In this work, we assume \(\omega_{51} = \omega_{32}\), the input photon and the Laser resonantly drive the atom, and the Rabi frequency of the Laser is equal to \(\frac{\pi}{\Omega}\). The symbol \(\omega_{ij}\) denotes the transition frequency between the levels \(|i\rangle\) and \(|j\rangle\). In this case, the atom is determinably in the state \(|5\rangle\) after the elastic scattering. In addition, the reemitted photon gains a phase shift of \(\frac{\pi}{2}\). If another single photon is subsequently injected into the waveguide \(m_{0}'\) (\(m_{1}'\)), it drives the transition \(|4\rangle \leftrightarrow |5\rangle\). Then it is delivered into waveguide \(m_{1}'\) \((m_{0}'\)) with the transfer matrix \(\frac{\delta_{ij} \epsilon - \Omega^2}{\delta_{ij} \Gamma - \Omega^2} I + \frac{\Omega^2}{\delta_{ij} \Gamma - \Omega^2} \sigma_x\).

The operator \(I\) denotes the two dimensional identity matrix, \(\sigma_x\) denotes the Pauli operator, and \(\delta_{ij} \Gamma\) is the detuning between the single photon and the atomic transition \(|4\rangle \leftrightarrow |5\rangle\). In situation \(B\), the photon does not drive the atomic transition and hence the atom maintains its initial state \(|1\rangle\). If another single photon is subsequently injected into the waveguide \(m_{0}'\) \((m_{1}'\)) , it is decoupled to the atom. Therefore, type \(c\) constitutes a combination of two operations. One is the rotating operation on the photonic state of the \(m'\)-th pair of waveguides, which is controlled by the photonic state of the \(m\)-th pair of waveguides. The other is the \(\frac{\pi}{2}\)-phase-shift operation operated on the photon in the waveguide \(m_{1}\). Especially, when the transition \(|4\rangle \leftrightarrow |5\rangle\) is decoupled to either of the \(m'\)-th pair of waveguides, the controlled-rotation gate becomes the controlled-phase gate. In the following learning tasks, we consider the resonant case, i. e. \(\delta_{ij} \Gamma = 0\), and hence the controlled-rotation gate reduces to the controlled-NOT gate.

We proceed to develop an algorithm to perform the su-
pective learning tasks by integrating the realized operations. The system under consideration consists of \( N \) pairs of 1D waveguides chiral coupled to atoms, as shown in Fig. 1(b). Initially, \( Q \) single photons are sequentially injected into the waveguides \( q_0 \), with \( q = 1, Q \). After waveguide-atom interactions, the paths of the single photons in the \( q \)-pair of waveguides change between the waveguides \( q_0 \) and waveguides \( q_i \). A detector is introduced to measure the probabilities, labeled by \( P_{Q_i} \), that the single photons injected into the waveguide \( Q_0 \) are delivered into the waveguide \( Q_1 \).

The output function is defined by a linear transformation of \( P_{Q_1} \), i.e. \( g(x, \theta) = \omega P_{Q_1}(x, \theta) + b \), with \( \omega \) and \( b \) real numbers. For a given input set \( \{x_i\} \) with the teacher data \( \{f(x_i)\} \), the supervised learning aims to update \( \theta \) to make the teacher data close to the output function. Here \( \theta \) consists of the tunable parameters, i.e. the frequencies and Rabi frequencies of the adjustable external Lasers. We use quadratic cost function, \( L = \sum_i[g(x_i, \theta) - f(x_i)]^2 \), to measure the gap between the teacher data and the output function. Based on the gradient descent, the parameters are iterated as \( \theta_{n+1} = \theta_n - \nabla \theta L \) until the value of the cost function is small enough.

The scheme is decomposed into two parts. The input set \( \{x_i\} \) is encoded into the photonic state by the atom-waveguide interactions in part one. The measurement bases are altered by variable parameters in part two. The tensor product structure of the system provides the measurement with rich components due to the increased dimension of the system.

Part one contains \( L \) layers. In each layer, each pair of waveguides interacts with \( N \) \( \Lambda \)-type atoms in the manner of type a. Then the correlations between the single photons in nearest neighbor pairs of waveguides are achieved by the controlled operations of type c. The non-local operations play an important role. The \( \Lambda \)-type atom-transition frequencies are different and keep constant during the training. All the detunings between the external Lasers and corresponding atomic transitions are equal to the input data. The encoding and the tensor product structure of the system directly bring in the non-linearity, which improves the learning ability. When the dimension of the system is large enough, it is difficult to calculate by classical computer. For the quantum system, the outcomes are obtained by running the operations and performing the measurements. Quantum computing is expected to perform the efficient calculation when the dimension is large, which is one purpose for most quantum algorithms. In the following numerical simulations, each of the detunings between the input photons and corresponding atomic transitions is the rand number in the range \([-2\Gamma, 2\Gamma]\) produced by the computer and then keeps constant. The Rabi frequencies are updated during the iterations.

Part two contains \( L' \) layers. Each pair of waveguides in each layer interacts with two \( \Lambda \)-type atoms, and a subsequent five-level atom in the manner of type c. One of the \( \Lambda \)-type atom is coupled to the waveguides in the manner of type a. The other \( \Lambda \)-type atom is coupled to a waveguide in the manner of type b. We assume that both the \( \Lambda \)-type atoms are resonantly driven by the guided photons. In this case, by ignoring the global phases, the type a interaction acts as the single-qubit \( X \) rotation \( R_X(\alpha) \), and the type b interaction acts as the single-qubit \( Z \) rotation \( R_Z(\beta) \). The angles, which can be tuned by external Lasers during the training, satisfy

\[
\cos \alpha = \frac{\Omega^2 - \Delta^2}{\Omega^2 + \Delta^2}, \quad \sin \alpha = \frac{2\Omega^2 \Delta}{\Omega^2 + \Delta^2}, \quad \cos \beta = \frac{\Omega_1^2 - 2\Omega_1^2}{\Omega_1^2 + 2\Omega_1^2}, \quad \sin \beta = \frac{-2\Omega_1^2 \Delta_1}{\Omega_1^2 + \Omega_1^2},
\]

The analytical gradients of the measurement with respect to variational parameters is obtained based on the parameter-shift rules developed in Ref. \[23\]. To distinguish the parameters corresponding to different \( \Lambda \)-type atoms in our scheme, it is necessary to bring in the subscript \( j \), which consists of a set of indexes. For example, \( \Omega_{j_1,j_2,j_3,j_4} \) denotes the Rabi frequency of the Laser that drives the \( j_2 \)-th \( \Lambda \)-type atom, corresponding to the \( j_1 \)-th

FIG. 2: Numerically simulating the performance of regression tasks. The red dots denote the teachers. The green solid lines denote the output functions obtained from the optimized scheme. The parameters are \( L = 4, N = 3, Q = 4, L' = 8, \omega = 2, b = -0.5, \) and \( \Gamma \) is taken as a unit. The target functions are: (a) \( f_1(x) = \sin^2(x \cdot \pi) \), (b) \( f_2(x) = e^{x-1} \), (c) \( f_3(x) = x^4 \), (d) \( f_4(x) = e^{-x^2/0.01} \), (e) \( f_5(x) = \frac{1}{1+e^{-x}} \), (f) Rectified Linear Unit, (g) \( |\cos(x \cdot \pi)|e^{-(z+1)} \).
The performance of the regression tasks is shown in Fig. 2, which demonstrates the fitting of different functions against the inputs, are represented in (b), (d), and (f), respectively. The parameters are equal to the ones in Fig. 2, except for $N = 6$.

The performance of the classification tasks is shown in Fig. 3. When the training data represented by the red triangles and blue dots denotes the two different classes, respectively. Accordingly, the facts that $P_{Q_1}$ is less than 1/2 and that $P_{Q_2}$ is greater than 1/2 indicate two different classes, respectively. The encoding method is not unique. For example, $\Delta_{j_1,j_2,j_3,1} = x_{i,2} - (j_2 + (j_3 - 1) N \mod 2)$; $\Delta_{j_1,j_2,j_3,1} = x_{i,2} - (j_2 \mod 2)$, or $\Delta_{j_1,j_2,j_3,1} = x_{i,2} - (j_3 \mod 2)$. All the encoding methods can be generalized into the high-dimension-input case.

In summary, photonic quantum computation has been realized with a new platform based on the atom-photonic-chiral couplings. As an important application of quantum computation, an algorithm to perform learning tasks based on our scheme has been proposed. We propose four elementary single-photons operations by chiral couplings. More complex operations used in quantum computation can be achieved by integrating the proposed operations. The running of the scheme can be considered as the scattering of the injected single photons. Therefore, the photonic probability measured after the scattering is a stable value in principle, which does not change against the time. This implies the low operating requirements on the time dimension. In both the regression and classification tasks, we have not introduced extra nonlinearity. The nonlinear mapping during the encoding relies heavily on the atomic structure. Therefore, our work implies a new issue of the quantum feature mapping by rich atomic structures. The learning ability is verified by numerically simulating the performance on some typical simple nonlinear learning tasks with classical computer. The updated parameters during the training are the tunable parameters of external Lasers, while the other parameters remain unchanged. It is reasonable that the scheme would be generalized to a high-dimensional complex structure in practice, which is difficult for the classical calculations, for various tasks. The work paves the way for the implementation of photonic quantum computation, especially for quantum machine learning.

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