On the generic higher-derivative $\mathcal{N} = 2$, $d = 3$ gauge theory

F. S. Gama, M. Gomes, J. R. Nascimento, A. Yu. Petrov, and A. J. da Silva

1Departamento de Física, Universidade Federal da Paraíba
Caixa Postal 5008, 58051-970, João Pessoa, Paraíba, Brazil
2Departamento de Física Matemática, Universidade de São Paulo,
Caixa Postal 66318, 05314-970, São Paulo, SP, Brazil

Abstract

We formulate a generic $\mathcal{N} = 2$ three-dimensional superfield higher-derivative gauge theory coupled to the matter, which, in certain cases reduces to the $\mathcal{N} = 2$ three-dimensional scalar super-QED, or supersymmetric Maxwell-Chern-Simons or Chern-Simons theories with matter. For this theory, we explicitly calculate the one-loop effective potential.
I. INTRODUCTION

The idea of extending field theory models through the introduction of additive higher-derivative terms is very old. Initially, higher derivative models have been introduced in the gravity context, aiming to construct a renormalizable gravity theory [1]. This gave rise to a great amount of studies on the quantum properties of different gravity models (for a review on these studies, see [2]).

It is clear that the introduction of higher derivatives in a theory highly improves its renormalization properties possibly making it finite. Therefore it was natural that within the supersymmetry context, higher derivatives were first considered in a (super)gravity model [3]. For the $\mathcal{N} = 1$ supergravity, the quantum dynamics for its higher-derivative extension has been studied [4], and only recently, one-loop studies for more generic examples of higher-derivative four-dimensional supersymmetric theories have been carried out [5]. In the three-dimensional case, the effective action for higher-derivative supersymmetric field theories have been studied in [6] at the one-loop level, and some two-loop aspects of the higher-derivative $3D$ super-QED were considered in [7].

In this work, we extend our studies to a new class of theories – we present the higher-derivative extension of the three-dimensional theories with extended supersymmetry. It is known that the $\mathcal{N} = 2$ superfield formulation in three-dimensional space-time is very similar to the $\mathcal{N} = 1$ superfield formalism for the four-dimensional space-time [8]. Recently, studies of the $\mathcal{N} = 2$ supersymmetric theories within this formulation have been carried out in [9]. Within this paper, we study the one-loop effective action for the higher-derivative theories described in this formalism.

Throughout this paper, we are using the conventions of [8], constructed on the base of those ones used in [10]. They are explained in the Appendix.

The structure of the paper is as follows. In the section II we present the classical formulation of the higher-derivative $\mathcal{N} = 2$, $d = 3$ supersymmetric gauge theory, in the section III we carry out the one-loop calculations, and the section IV is a Summary where the results are discussed. In the Appendix, the main relations for the $\mathcal{N} = 2$, $d = 3$ supersymmetry algebra are given.
II. HIGHER-DERIVATIVE $\mathcal{N} = 2, d = 3$ GAUGE THEORY

In the pure gauge sector, let us start with the most general $\mathcal{N} = 2, d = 3$ abelian gauge action which will be used to find the one-loop Kählerian effective potential (KEP)

$$S_G = \frac{1}{2} \int d^3 x d^4 \theta V [f(\Box) \bar{D}^\alpha D_\alpha + h(\Box) D^\alpha \bar{D}^2 D_\alpha] V ,$$

(1)

where $f(\Box)$ and $h(\Box)$ are analytical functions of the d’Alembertian operator $\Box$. In particular, if (up to multiplicative constants) $f = m$, where $m$ is a constant with mass dimension, and $h = 0$ we have a Chern-Simons theory; if $f = 0$ and $h = 1$ we have a Maxwell theory, and if $f = m$ and $h = 1$ we have a Maxwell-Chern-Simons theory. If $f$ and/or $h$ involves higher degrees of $\Box$ we have a higher-derivative supersymmetric gauge theory.

The structure of the expression (1) deserves some justification. Firstly, we have ignored in (1) terms higher than quadratic in the gauge superfield $V(z)$ due to the fact that the KEP is, by definition, a function only of background matter superfields, and such terms necessarily contribute with background gauge superfields in one loop. Therefore, terms higher than quadratic in $V(z)$ do not contribute to one-loop KEP. Secondly, for simplicity, we are working with an abelian theory because the one-loop KEP for a non-Abelian theory is the same as for an Abelian one, up to the constant depending on the algebraic factor, again due to the fact that the self-coupling of the gauge superfield does not contribute to the one-loop KEP. Lastly, $S_G$ is invariant under the gauge transformation $\delta V = i(\bar{\Lambda} - \Lambda)$ because the operators $\bar{D}^\alpha D_\alpha$ and $D^\alpha \bar{D}^2 D_\alpha$ commute with $\Box$ and annihilate the superfields $\bar{\Lambda}$ and $\Lambda$ which satisfy the conditions $D_\alpha \bar{\Lambda} = 0$ and $\bar{D}_\alpha \Lambda = 0$. Moreover, the higher-derivative operator in (1) was chosen to be linear in $\bar{D}^\alpha D_\alpha$ and $D^\alpha \bar{D}^2 D_\alpha$ due to the identities:

$$\left(\bar{D}^\alpha D_\alpha\right)^n = \Box^{n-1} \bar{D}^\alpha D_\alpha , \ n = 2l - 1,$$

(2)

$$\left(\bar{D}^\alpha D_\alpha\right)^n = -\Box^{n-1} D^\alpha \bar{D}^2 D_\alpha , \ n = 2l,$$

(3)

$$\left(D^\alpha \bar{D}^2 D_\alpha\right)^n = (-1)^{n+1} \Box^{n-1} D^\alpha \bar{D}^2 D_\alpha , \ n = 1, 2, 3, \ldots ,$$

(4)

where $l = 1, 2, 3, \ldots$.

We can add to (1) the following gauge-fixing term:

$$S_{GF} = -\frac{1}{2\alpha} \int d^3 x d^4 \theta V \{ D^2, \bar{D}^2 \} V .$$

(5)

Of course, we could have used a gauge-fixing term more sophisticated involving higher
derivatives like in [6], however, we will use (5) for convenience. Besides, we know that 
\( \delta V = i(\bar{\Lambda} - \Lambda) \) is an Abelian symmetry, and therefore the ghosts completely decouple.

Now, let us consider the matter sector. We will make two assumptions in order to simplify
the model involving the matter superfields. First, we will demand that the matter action
does not contain terms with higher derivatives. Second, we will not consider self-couplings
involving only \( \Phi \) or \( \bar{\Phi} \) superfields. Having made these assumptions, the most generic matter
action is given by

\[
S_M = \int d^3xd^4\theta K(\Phi, \Phi),
\]

where \( K(\Phi, \Phi) \) is the tree-level KEP.

In order to couple (6) to the gauge superfield, the function \( K(\Phi, \Phi) \) must firstly be
invariant under the global transformation \( \delta \Phi = i\lambda \Phi \), and \( \delta \bar{\Phi} = -i\lambda \bar{\Phi} \). It follows that
\( K(\Phi, \Phi) \) must satisfy the constraint

\[
\Phi \frac{\partial K(\bar{\Phi}, \Phi)}{\partial \Phi} = \frac{\partial K(\bar{\Phi}, \Phi)}{\partial \bar{\Phi}} \Phi.
\]

In particular this constraint is satisfied if \( K(\bar{\Phi}, \Phi) \) is a function of \( \bar{\Phi}\Phi \).

Now, we can introduce the gauge superfield \( V \) in (6) to obtain

\[
S_M = \frac{1}{2} \int d^3xd^4\theta \left[ K(\bar{\Phi}e^{2gV}, \Phi) + K(\bar{\bar{\Phi}}, e^{2gV} \Phi) \right],
\]

which is invariant under local transformations \( \delta \Phi = i\Lambda(z)\Phi \), \( \delta \bar{\Phi} = -i\bar{\Lambda}(z)\bar{\Phi} \), and \( \delta V = i(\bar{\Lambda} - \Lambda) \).

Finally, the generic higher-derivative \( \mathcal N = 2, d = 3 \) gauge theory that we will study in
this work follows from (1), (5), and (8):

\[
S = \frac{1}{2} \int d^3xd^4\theta \left\{ V[f(\Box)\bar{D}^{\alpha}D_{\alpha} + h(\Box)\bar{D}^{\alpha}\bar{D}_{\alpha} - \frac{1}{\alpha}\{D^2, \bar{D}^2\}]V + K(\bar{\Phi}e^{2gV}, \Phi) + K(\bar{\Phi}, e^{2gV} \Phi) \right\}.
\]
By using this prescription, we get from (9) 

\[
S_2[\Phi, \Phi; \bar{\phi}, \phi, V] = \frac{1}{2} \int d^3xd^4\theta \{ V[f(\Box)\bar{D}^\alpha D_\alpha + h(\Box)D^\alpha \bar{D}^2 D_\alpha - \frac{1}{\alpha} \{D^2, \bar{D}^2\}]V \\
+ \frac{(2g)^2}{2} (K_{\Phi} + K_{\bar{\phi}} + K_{\Phi} \bar{\phi}) V^2 + 2g(K_{\Phi} + K_{\bar{\phi}} \bar{\phi}) \bar{\phi} V + 2K_{\Phi} \bar{\phi} \phi \}
\]

where the derivatives of the background superfields were omitted due to our interest only in the KEP [11]. By differentiating the constraint (7) we obtain new identities which can be used to simplify (10), then we get 

\[
S_2[\Phi, \Phi; \bar{\phi}, \phi, V] = S_q + S_{int},
\]

\[
S_q = \frac{1}{2} \int d^3xd^4\theta \{ V[f(\Box)\bar{D}^\alpha D_\alpha - \Box h(\Box)\Pi_{1/2} - \frac{1}{\alpha} \Box \Pi_0] V + 2K_{\Phi} \bar{\phi} \phi \}
\]

\[
S_{int} = \frac{1}{2} \int d^3xd^4\theta \{ (2g)^2 K_{\Phi} \Phi V^2 + 2(2g)K_{\Phi} \Phi \bar{\phi} V + 2(2g)K_{\Phi} \Phi \bar{\phi} \Phi V \}
\]

These properties can be used to deduce the identities (2-4). Moreover, we can use them to extract the propagators from \( S_q \). Thus, in momentum space, we obtain 

\[
\langle V(1)V(2) \rangle = \left[ X(p^2) D^\alpha D_\alpha + Y(p^2) \Pi_{1/2} - \frac{\alpha}{p^2} \Pi_0 \right] \delta_{12},
\]

\[
\langle \bar{\phi}(1)\phi(2) \rangle = \left( \frac{1}{K_{\Phi} p^2} \right) \delta_{12},
\]

where 

\[
X(p^2) = \frac{f(-p^2)}{p^2} \left[ \frac{h^2(-p^2) + f^2(-p^2)}{p^2} \right] \quad \text{and} \quad Y(p^2) = \frac{h(-p^2)}{p^2} \left[ \frac{h^2(-p^2) + f^2(-p^2)}{p^2} \right].
\]

These propagators will be used for the one-loop calculations.

III. ONE-LOOP CALCULATIONS

Let us start the calculations of the one-loop supergraphs contributing to the KEP. At the one-loop order, we will have two types of contributions. In the first, all diagrams involve
only the gauge superfield propagators $\langle V(1)V(2) \rangle$ in the internal lines connecting the vertices $(2g)^2 K_{\Phi \Phi} \bar{\Phi} \Phi V^2$. Such supergraphs exhibit structures given at Fig. 1.

We can compute all the contributions by noting that each supergraph above is formed by $n$ "subgraphs" like these ones given by Fig. 2.

The contribution of this fragment is

$$Q_{12} = [(2g)^2 K_{\Phi \Phi} \bar{\Phi} \Phi]_1 (X \bar{D}^\alpha D_\alpha + Y \Pi_{1/2} - \frac{\alpha}{p^2} \Pi_0)_1 \delta_{12}.$$  \hspace{1cm} (19)

It follows from the result above that the contribution of a supergraph formed by $n$ fragments is given by

$$I_n = \int d^3x \frac{1}{2n} \int d^4\theta_1 d^4\theta_2 \cdots d^4\theta_n \int \frac{d^3p}{(2\pi)^3} Q_{12} Q_{23} \cdots Q_{n-1,n} Q_{n,1}$$

\[= \int d^3x \frac{1}{2n} \int d^4\theta_1 d^4\theta_2 \cdots d^4\theta_n \int \frac{d^3p}{(2\pi)^3} [(2g)^2 K_{\Phi \Phi} \bar{\Phi} \Phi]_1 (X \bar{D}^\alpha D_\alpha + Y \Pi_{1/2} - \frac{\alpha}{p^2} \Pi_0)_1 \delta_{12} \]

\[\times [(2g)^2 K_{\Phi \Phi} \bar{\Phi} \Phi]_n (X \bar{D}^\alpha D_\alpha + Y \Pi_{1/2} - \frac{\alpha}{p^2} \Pi_0)_n \delta_{n,1}, \hspace{1cm} (20)\]

where $2n$ is a symmetry factor. Such a contribution takes into account the Taylor series expansion coefficients of the effective action, the usual symmetry factor of each supergraph,
and the number of topologically distinct supergraphs \[12\]. The external momenta must be taken to be zero in the calculation of the effective potential.

We can integrate by parts the expression \( I_n \) and discard terms involving covariant derivatives of \( \bar{\Phi} \) and \( \Phi \) to get

\[
I_n = \int d^3 x d^4 \theta \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2n} [ (2g)^2 K_{\Phi \Phi} \bar{\Phi} \Phi ] n ( X \bar{D}^\alpha D_\alpha + Y \Pi_{1/2} - \frac{\alpha}{p^2} \Pi_0 )^n \delta_{\theta \theta'} |_{\theta = \theta'} .
\]  \hspace{1cm} (21)

The effective action is given by the sum of all supergraphs \( I_n \)

\[
\Gamma_1^{(1)} = \sum_{n=1}^{\infty} I_n = \int d^3 x d^4 \theta \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2n} [ (2g)^2 K_{\Phi \Phi} \bar{\Phi} \Phi ] n ( X \bar{D}^\alpha D_\alpha + Y \Pi_{1/2} )^n \delta_{\theta \theta'} |_{\theta = \theta'}
\]

\[- \frac{2}{p^2} \left( - \frac{\alpha}{p^2} \right)^n ,
\]  \hspace{1cm} (22)

where we used \((14), (15)\), and the fact that \( \Pi_0 \delta_{\theta \theta'} |_{\theta = \theta'} = -2/p^2 \). Summing over all \( n \) we get

\[
\Gamma_1^{(1)} = \int d^3 x d^4 \theta \int \frac{d^3 p}{(2\pi)^3} \left\{ - \frac{1}{2} \ln \left[ 1 - (2g)^2 K_{\Phi \Phi} \bar{\Phi} \Phi (X \bar{D}^\alpha D_\alpha + Y \Pi_{1/2}) \right] \delta_{\theta \theta'} |_{\theta = \theta'}
\]

\[+ \frac{1}{p^2} \ln \left[ 1 + \frac{\alpha(2g)^2 K_{\Phi \Phi} \bar{\Phi} \Phi}{p^2} \right] \right\} .
\]  \hspace{1cm} (23)

The first logarithm term can be splitted in two parts, then

\[
\Gamma_1^{(1)} = \int d^3 x d^4 \theta \int \frac{d^3 p}{(2\pi)^3} \left\{ - \frac{1}{2} \ln \left[ 1 - \frac{(2g)^2 K_{\Phi \Phi} \bar{\Phi} \Phi X}{1 - (2g)^2 K_{\Phi \Phi} \bar{\Phi} \Phi Y} \bar{D}^\alpha D_\alpha \right] \delta_{\theta \theta'} |_{\theta = \theta'}
\]

\[- \frac{1}{2} \ln \left[ 1 - (2g)^2 K_{\Phi \Phi} \bar{\Phi} \Phi Y \Pi_{1/2} \right] \delta_{\theta \theta'} |_{\theta = \theta'} + \frac{1}{p^2} \ln \left[ 1 + \frac{\alpha(2g)^2 K_{\Phi \Phi} \bar{\Phi} \Phi}{p^2} \right] \right\} .
\]  \hspace{1cm} (24)

Finally, we expand in Taylor series the first two logarithms and use \((24), (14)\) and \( (15) \), and \( \Pi_{1/2} \delta_{\theta \theta'} |_{\theta = \theta'} = 2/p^2 \) to obtain

\[
\Gamma_1^{(1)} = \int d^3 x d^4 \theta \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2p^2} \left\{ - \frac{1}{2} \ln \left[ 1 + p^2 \left( \frac{(2g)^2 K_{\Phi \Phi} \bar{\Phi} \Phi X}{1 - (2g)^2 K_{\Phi \Phi} \bar{\Phi} \Phi Y} \right)^2 \right]
\]

\[- \ln \left[ 1 - (2g)^2 K_{\Phi \Phi} \bar{\Phi} \Phi Y \right] + \ln \left[ 1 + \frac{\alpha(2g)^2 K_{\Phi \Phi} \bar{\Phi} \Phi}{p^2} \right] \right\} .
\]  \hspace{1cm} (25)

Let us proceed the calculation of the second type of one-loop supergraphs, which involve the gauge and matter superfield propagators in the internal lines connecting the vertices \((2g) K_{\Phi \Phi} \bar{\Phi} V \) and \((2g) K_{\Phi \Phi} \bar{\Phi} V \phi \). Such supergraphs exhibit the structure shown in Fig. 3.

It is worth to point out that we can insert an arbitrary number of vertices \((2g)^2 K_{\Phi \Phi} \bar{\Phi} V^2 \) into the gauge propagators. Therefore, we should firstly introduce a "dressed" propagator.
In this propagator, the summation over all vertices $(2g)^2K_{\phi\bar{\phi}}V^2$ is performed (see Fig. 4). As a result, this dressed propagator is equal to

$$
\langle V(1)V(2) \rangle_D = \langle V(1)V(2) \rangle + \int d^4\theta_3 \langle V(1)V(3) \rangle (2g)^2K_{\phi\bar{\phi}}\tilde{\Phi}\Phi_3 \langle V(3)V(2) \rangle \\
+ \int d^4\theta_3 d^4\theta_4 \langle V(1)V(3) \rangle (2g)^2K_{\bar{\phi}\phi}\Phi\Phi_3 \langle V(3)V(4) \rangle (2g)^2K_{\bar{\phi}\phi}\Phi\Phi_4 \\
\times \langle V(4)V(2) \rangle + \ldots .
$$

(26)

By using (16) and integrating by parts, we arrive at

$$
\langle V(1)V(2) \rangle_D = \sum_{n=0}^{\infty} [(2g)^2K_{\phi\bar{\phi}}\Phi\Phi]^n [(X\bar{D}\alpha D_{\alpha} + Y\Pi_{1/2})]^{n+1} + \left( -\frac{\alpha}{p^2} \right)^{n+1} \Pi_0 ] \delta_{12} .
$$

(27)

As before, we can compute all the contributions by noting that each supergraph above (Fig. 3) is formed by $n$ subgraphs, like those depicted in Fig. 5 and Fig. 6. Since both subgraphs, Figs. 5 and 6, provide the same contribution, we just need to calculate the one in the Fig. 5. This subgraph yields the contribution ($\Pi_- \equiv -\bar{D}^2D^2/p^2$)

$$
R_{13} = \int d^4\theta_2 [(2g)^2K_{\phi\bar{\phi}}\Phi]_1 \left\{ \sum_{n=0}^{\infty} [(2g)^2K_{\phi\bar{\phi}}\Phi\Phi]^n [(X\bar{D}\alpha D_{\alpha} + Y\Pi_{1/2})]^{n+1} \\
+ \left( -\frac{\alpha}{p^2} \right)^{n+1} \Pi_0 ] \delta_{12} \right\} [(2g)^2K_{\phi\bar{\phi}}\Phi]_2 \left[ -\left( \frac{\Pi_-}{K_{\phi\bar{\phi}}} \right) \delta_{23} \right] \\
= -\sum_{n=0}^{\infty} [(2g)^2K_{\phi\bar{\phi}}\Phi\Phi]^n (2g)^2K_{\phi\bar{\phi}}\Phi\Phi \right\} [(2g)^2K_{\phi\bar{\phi}}\Phi]_1 \left( -\frac{\alpha}{p^2} \right)^{n+1} (\Pi_-) \delta_{13} .
$$

(28)
By summing up, we arrive at

$$R_{13} = \left( \frac{(2g)^2 \alpha K_{\Phi\Phi} \Phi \Phi}{p^2 + (2g)^2 \alpha K_{\Phi\Phi} \Phi \Phi} \right) \delta_{13}. \tag{29}$$

FIG. 5: A typical link in one-loop supergraphs in mixed sector.

FIG. 6: Another typical link in one-loop supergraphs in mixed sector.

It follows from the result above that the contribution of a supergraph formed by \( n \) subgraphs is given by

$$J_n = \int d^3 x \frac{1}{2n} \int d^4 \theta_1 d^4 \theta_3 \ldots d^4 \theta_{2n-1} \int \frac{d^3 p}{(2\pi)^3} R_{13} R_{35} \ldots R_{2n-3, 2n-1} R_{2n-1, 1}$$

$$= \int d^3 x \frac{1}{2n} \int d^4 \theta_1 d^4 \theta_3 d^4 \theta_5 \ldots d^4 \theta_{2n-1} \int \frac{d^3 p}{(2\pi)^3} \left[ \frac{(2g)^2 \alpha K_{\Phi\Phi} \Phi \Phi}{p^2 + (2g)^2 \alpha K_{\Phi\Phi} \Phi \Phi} \right] \delta_{13}$$

$$\times \left[ \frac{(2g)^2 \alpha K_{\Phi\Phi} \Phi \Phi}{p^2 + (2g)^2 \alpha K_{\Phi\Phi} \Phi \Phi} \right] \delta_{35} \ldots \left[ \frac{(2g)^2 \alpha K_{\Phi\Phi} \Phi \Phi}{p^2 + (2g)^2 \alpha K_{\Phi\Phi} \Phi \Phi} \right] \delta_{2n-1, 1}$$

$$= \int d^3 x d^4 \theta \frac{1}{2n} \int \frac{d^3 p}{(2\pi)^3} \left( \frac{(2g)^2 \alpha K_{\Phi\Phi} \Phi \Phi}{p^2 + (2g)^2 \alpha K_{\Phi\Phi} \Phi \Phi} \right)^n \Pi_\theta \delta_{\theta \theta^\prime} |_{\theta = \theta^\prime} . \tag{30}$$

By using \( \Pi_\theta \delta_{\theta \theta^\prime} |_{\theta = \theta^\prime} = -1/p^2 \), we get the effective action

$$\Gamma_2^{(1)} = 2 \sum_{n=0}^{\infty} J_n = -\int d^8 x d^4 \theta \frac{1}{p^2} \ln \left[ 1 + \frac{\alpha(2g)^2 K_{\Phi\Phi} \Phi \Phi}{p^2} \right]. \tag{31}$$
It is worth to point out that the contribution (31) cancels the dependence of (25) on the
gauge parameter $\alpha$. By summing (25) to (31) we obtain the total one-loop effective action
\[
\Gamma^{(1)}[\bar{\Phi}, \Phi] = \int d^3 x d^4 \theta \int \frac{d^3 p}{(2\pi)^3} \frac{1}{p^2} \left\{ -\frac{1}{2} \ln \left[ 1 + p^2 \left( \frac{(2g)^2 K_{\Phi \Phi} \bar{\Phi} \Phi X}{1 - (2g)^2 K_{\Phi \Phi} \bar{\Phi} \Phi Y} \right)^2 \right] - \ln \left[ 1 - (2g)^2 K_{\Phi \Phi} \bar{\Phi} \Phi Y \right] \right\} .
\] (32)

Finally, we arrive to the following result for the KEP (as usual, the corresponding effective action can be restored from the relation $\Gamma^{(1)} = \int d^3 x d^4 \theta K^{(1)}$):
\[
K^{(1)}(\bar{\Phi}, \Phi) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{p^2} \left\{ -\frac{1}{2} \ln \left[ 1 + p^2 \left( \frac{(2g)^2 K_{\Phi \Phi} \bar{\Phi} \Phi X}{1 - (2g)^2 K_{\Phi \Phi} \bar{\Phi} \Phi Y} \right)^2 \right] - \ln \left[ 1 - (2g)^2 K_{\Phi \Phi} \bar{\Phi} \Phi Y \right] \right\} ,
\] (33)

where, $X$ and $Y$ are given by (18). Moreover, notice that (33) is independent of the gauge parameter for any choice of $K(\bar{\Phi} e^{2gV} \Phi), f(-p^2)$, and $h(-p^2)$.

The result (33) is rather generic. Therefore, in order to proceed with the calculation and solve explicitly the integral above, we have to specify the operators $f(\Box)$ and $h(\Box)$ in (18). So, let us consider two characteristic examples where the final result is expressed in closed form and in terms of elementary functions.

As our first example, let us take $f(\Box) = \xi_f (-\Box)^n$ and $h(\Box) = 0$ in (18), where $\xi_f$ is a parameter with a nontrivial mass dimension $[\xi_f] = [M]^{-2n+1}$, $\xi_f > 0$, and $n$ is a non-negative integer. This choice corresponds to a higher-derivative Chern-Simons theory (see (1)). It follows from (33) that
\[
K^{(1)}_{HC\text{S}}(\bar{\Phi}, \Phi) = -\frac{1}{2} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{p^2} \ln \left[ 1 + \frac{1}{(p^2)^{2n+1}} \left( \frac{(2g)^2 K_{\Phi \Phi} \bar{\Phi} \Phi}{\xi_f} \right) \right] ,
\] (34)
whose solution is given by
\[
K^{(1)}_{HC\text{S}}(\bar{\Phi}, \Phi) = -\frac{1}{4\pi} \csc \left[ \frac{\pi}{2(2n+1)} \right] \left( \frac{(2g)^2 K_{\Phi \Phi} \bar{\Phi} \Phi}{\xi_f} \right)^{\frac{1}{2n+1}} .
\] (35)

The second example is $f(\Box) = 0$ and $h(\Box) = \xi_h (-\Box)^n$ in (18), where $[\xi_h] = [M]^{-2n}$, $\xi_h > 0$. This choice corresponds to a higher-derivative Maxwell theory. It follows from (33) that
\[
K^{(1)}_{HQ\text{ED}}(\bar{\Phi}, \Phi) = -\int \frac{d^3 p}{(2\pi)^3} \frac{1}{p^2} \ln \left[ 1 + \frac{1}{(p^2)^{n+1}} \left( \frac{(2g)^2 K_{\Phi \Phi} \bar{\Phi} \Phi}{\xi_h} \right) \right] ,
\] (36)
whose solution is given by
\[
K^{(1)}_{HQ\text{ED}}(\bar{\Phi}, \Phi) = -\frac{1}{2\pi} \csc \left[ \frac{\pi}{2(n+1)} \right] \left( \frac{(2g)^2 K_{\Phi \Phi} \bar{\Phi} \Phi}{\xi_h} \right)^{\frac{1}{2(n+1)}} .
\] (37)
We notice that the one-loop corrections for the KEPs, namely (35,37), are finite and do not need any renormalization. Moreover, these results are universal, valid for any form of the potential $K(\Phi^2 g V(\Phi))$. We also notice that the functional structure of (35) and (37) does not involve any logarithm-like dependence, which is usually found in four-dimensional theories. We observe that, up to constants, $K^{(1)}_{HQED}(\Phi,\Phi)$ given in (37) is the same as in the $\mathcal{N} = 1$ case derived in [6]. Additionally, in [6] it was shown that the one-loop KEP vanishes for the $\mathcal{N} = 1, d = 3$ QED coupled to non-self-interacting matter; we see from (37) that it is not the case for $\mathcal{N} = 2, d = 3$ QED coupled to non-self-interacting matter.

IV. SUMMARY

We have calculated the one-loop effective potential for the $\mathcal{N} = 2$ supersymmetric three-dimensional higher-derivative theories. Our calculation was based on a formalism allowing to maintain the $\mathcal{N} = 2$ supersymmetry at all steps of the calculations. Discussing the properties of the result, we should emphasize, first, its finiteness which is a natural consequence of the presence of higher derivatives, second, the similarity of its form to the results obtained earlier for the four-dimensional $\mathcal{N} = 1$ supersymmetric theories [5]. Also, contrarily to the case of the $\mathcal{N} = 1, d = 3$, supersymmetric QED [6], in our theory the one-loop kählerian effective potential does not vanish. However, the one-loop $\mathcal{N} = 1$ and $\mathcal{N} = 2$ kählerian effective potentials for the supersymmetric Chern-Simons theories in $d = 3$ display similar structures.

APPENDIX

Let us briefly describe the $\mathcal{N} = 2$ supersymmetry algebra used in this paper. Here we work within the three-dimensional Minkowski space, so, we choose the gamma matrices as $(\gamma^\mu)^{\alpha\beta} = (\sigma^2, i\sigma^1, i\sigma^3)$, which satisfy the Clifford algebra $\{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu}$, with $\eta^{\mu\nu} = \text{diag}(-1,1,1)$. We raise and lower spinor indices with the matrix $C_{\alpha\beta} = \sigma^2$, so that $C^{12} = -C_{12} = i$, and $\psi^\alpha = C^{\alpha\beta}\psi_{\beta}$, $\psi_{\beta} = \psi^\alpha C_{\alpha\beta}$, $\psi^2 = \frac{1}{2}C_{\beta\alpha}\psi^\alpha\psi^\beta$. It follows from $(\gamma^\mu)^{\alpha\beta} = C^{\gamma\alpha}(\gamma^\mu)_{\gamma\beta}$ and $(\gamma^\mu)^{\alpha\beta} = \eta_{\mu\nu}C^{\beta\lambda}(\gamma^\nu)^{\alpha\lambda}$ that

$$(\gamma^\mu)^{\alpha\beta} = \{-\hat{1}, -\sigma^3, \sigma^1\}, \quad (\gamma^\mu)^{\alpha\beta} = \{-\hat{1}, -\sigma^3, \sigma^1\}.$$  

(38)
From these equations, we get

$$\gamma_\mu \alpha \beta \gamma_\nu \gamma_\delta = 2 \delta_{\mu \nu}, \quad \gamma_\mu \alpha \beta \gamma_\delta = (\delta_\alpha \gamma_\delta \gamma_\beta + \delta_\alpha \delta_\beta \gamma_\gamma). \quad (39)$$

Therefore, we can use the gamma matrices to map the components of 3-vectors into $2 \times 2$ symmetric (hermitian) matrices by means of the definitions

$$V^{\alpha \beta} = \frac{1}{\sqrt{2}} \gamma_\mu \alpha \beta V^\mu, \quad V^\mu = \frac{1}{\sqrt{2}} \gamma_\mu \alpha \beta V^{\alpha \beta}; \quad (40)$$

$$\partial_{\alpha \beta} = (\gamma_\mu \alpha \beta \partial_\mu, \quad \partial_\mu = \frac{1}{2} (\gamma_\mu \alpha \beta \partial_{\alpha \beta}; \quad (41)$$

$$x^{\alpha \beta} = \frac{1}{2} (\gamma_\mu \alpha \beta x^\mu, \quad x^\mu = (\gamma_\mu)_{\alpha \beta} x^{\alpha \beta}. \quad (42)$$

The $N = 2, d = 3$ supersymmetry algebra is

$$\{Q_i^\alpha, Q_j^\beta\} = 2 \delta_{ij} P_{\alpha \beta} \quad (i, j = 1, 2), \quad (43)$$

where $P_{\alpha \beta} = i \partial_{\alpha \beta}$. However, it is convenient to go over to a complex representation by defining

$$Q_\alpha = \frac{1}{2} (Q_1^\alpha + iQ_2^\alpha), \quad \tilde{Q}_\alpha = \frac{1}{2} (Q_1^\alpha - iQ_2^\alpha); \quad (44)$$

which can be used to express the algebra as,

$$\{Q_\alpha, \tilde{Q}_\beta\} = P_{\alpha \beta}, \quad \{Q_\alpha, Q_\beta\} = 0, \quad \{\tilde{Q}_\alpha, \tilde{Q}_\beta\} = 0. \quad (45)$$

Notice that these conventions and definitions are the exact analogues of those ones used in [10]. In fact, the $N = 2, d = 3$ superspace can be parametrized by the coordinates $z^M = (x^{\alpha \beta}, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$, with $(\theta^\alpha)^* = \bar{\theta}^{\dot{\alpha}}$, and the explicit forms of the generators and covariant derivatives are given by

$$Q_\alpha = i (\partial_\alpha - \frac{1}{2} \bar{\theta}^{\dot{\beta}} i \partial_{\alpha \beta}), \quad \tilde{Q}_\alpha = i (\partial_\alpha - \frac{1}{2} \theta^{\beta} i \partial_{\alpha \beta}), \quad (46)$$

$$D_\alpha = \partial_\alpha + \frac{1}{2} \bar{\theta}^{\dot{\beta}} i \partial_{\alpha \beta}, \quad \tilde{D}_\alpha = \partial_\alpha + \frac{1}{2} \theta^{\beta} i \partial_{\alpha \beta}. \quad (47)$$

We note that despite the derivatives $D_\alpha$ and $\tilde{D}_\beta$ are independent, there is no chirality in this case since both types of the derivatives (and of the spinors) are transformed under the same (unique) spinor representation of the Lorentz group.
The (anti)commutation relations for the $D_\alpha$ and $\bar{D}_\alpha$ are rather similar to those ones for the four-dimensional supersymmetry [10]. Indeed, one has

\[
\{D_\alpha, \bar{D}_\beta\} = i\partial_{\alpha\beta}; \quad \{D_\alpha, D_\beta\} = \{\bar{D}_\alpha, \bar{D}_\beta\} = 0, \quad D_\alpha D^2 = \bar{D}_\alpha \bar{D}^2 = 0; \\
D^\alpha D_\beta = \delta^\alpha_\beta D^2; \quad \bar{D}^\alpha \bar{D}_\beta = \delta^\alpha_\beta \bar{D}^2; \quad [D^\alpha, \bar{D}^2] = i\partial^\alpha \bar{D}_\beta; \quad [\bar{D}^\alpha, D^2] = i\partial^\alpha D_\beta; \\
\bar{D}^2 D^2 \bar{D}^2 = \Box \bar{D}^2; \quad D^2 \bar{D}^2 D^2 = \Box D^2. 
\] (48)

These (anti)commutation relations can be used to prove the identities (2,4) and (14,15). Moreover, it is clear that the use of the derivatives satisfying these rules is no more difficult as the use of the standard supercovariant derivatives either in three- or in four-dimensional case.

Finally, all quantum calculations were carried out using a Wick-rotated metric $\eta^{\mu\nu} = \text{diag}(+1,1,1)$.

**Acknowledgments.** This work was partially supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq). A. Yu. P. has been supported by the CNPq project No. 303438-2012/6. The work by F. S. Gama has been supported by the CNPq process No. 141228/2011-3.

[1] K. Stelle, Phys. Rev. D16, 953 (1977).
[2] I. L. Buchbinder, S. D. Odintsov, I. L. Shapiro. Effective action in quantum gravity. IOP Publishing, Bristol and Philadelphia, 1992.
[3] I. L. Buchbinder, S. M. Kuzenko, Phys. Lett. B202, 233 (1988).
[4] I. L. Buchbinder, A. Yu. Petrov, Class. Quant. Grav. 13, 2081 (1996), hep-th/9511205; Class. Quant. Grav. 14, 21 (1997), hep-th/9607217.
[5] M. Gomes, J. R. Nascimento, A. Yu. Petrov, A. J. da Silva, Phys. Lett. B682, 229 (2009), arXiv: 0908.0900; F. S. Gama, M. Gomes, J. R. Nascimento, A. Yu. Petrov, A. J. da Silva, Phys. Rev. D84, 045001 (2011), arXiv: 1101.0724.
[6] F. S. Gama, J. R. Nascimento, A. Yu. Petrov, Phys. Rev. D88, 045021 (2013), arXiv: 1307.3190; Phys. Rev. D88, 065029 (2013), arXiv: 1308.5834.
[7] E. A. Gallegos, R. Baptista, “Two-loop finiteness of self-energies in higher-derivative SQED3”, arXiv: 1308.4923.
[8] N. J. Hitchin, A. Karlhede, U. Lindström, and M. Rocek, Commun. Math. Phys. 108, 535 (1987).

[9] I. L. Buchbinder, N. G. Pletnev, I. B. Samsonov, JHEP 1004, 124 (2010), arXiv: 1003.4806; JHEP 1101, 121 (2011), arXiv: 1010.4967; I. L. Buchbinder, E. A. Ivanov, I. B. Samsonov, B. M. Zupnik, JHEP 1201, 001 (2012), arXiv: 1111.4145; I. L. Buchbinder, B. S. Merzlikin, I. B. Samsonov, Nucl. Phys. B680, 87 (2012), arXiv: 1201.5579.

[10] S. J. Gates, M. T. Grisaru, M. Rocek, W. Siegel. Superspace or One Thousand and One Lessons in Supersymmetry. Benjamin/Cummings, 1983, hep-th/0108200.

[11] I. L. Buchbinder and S. M. Kuzenko, Ideas and Methods of Supersymmetry and Supergravity. IOP Publishing, Bristol and Philadelphia, 1998.

[12] B. Hatfield. Quantum Field Theory of Point Particles and String Theory. Westview Press, 1998.