Correlation Analysis of Elevation and Slope in Henan

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Abstract. Elevation and slope are the important indexes to describe the regional geography, which affect the local temperature, illumination conditions as well as soil water conservation capacity to a great extent, determine the ways and directions of regional land use. Based on the digital elevation model, the paper utilizes spatial analysis method to calculate the slope and the correlation coefficient of elevation and slope, and analyzes the spatial distribution characteristics of elevation and slope, studies their spatial relations in Henan province. The results show that: Henan is dominated by medium and low elevations, with the increase of longitude, the maximum elevations show three steps that gradually decrease, the mean and minimum values decrease. With the increase of latitude, the maximum elevations show the characteristics of the three steps range of low high medium, and the mean values show two connected parabolic characteristic. Henan is dominated by low slope, with the increase of longitude, the maximum and mean of slopes gradually decrease, and the local fluctuation is obviously higher than that of elevation. With the increase of latitude, the maximum slope gradually increases, the trend of mean in latitude is similar with the pattern of the elevation, but there are obvious differences in local changes. The correlation between slope and elevation is weak, the elevation and slope is negatively correlated in the region above 1870m. The slope change is more obvious in the regions with low and high elevation regions. With the increase of elevation, the mean and variance is increasing. The topographic relief is obvious in the region below 200m and above 1870m, respectively shows concave geomorphic feature and convex geomorphic feature.

Introduction

Elevation and slope are important indexes in describing the regional geography, which affect the local temperature, illumination conditions as well as the soil water conservation ability to a great extent, which also determine the ways and directions of regional land use [1,2]. In recent years, the researches about the impact of elevation and slope on regional economic development have mainly focused on the spatial characteristics of land use and population distribution based on elevation and slope, as well as their evolution characteristics, and achieve some valuable results [3,4]. In fact, although there is a certain correlation between elevation and slope in the region, there are some differences in the distribution characteristics of slope on different elevation zones. the correlation analysis of the elevation and slope in different elevation zones could help to understand more about the regional topography feature, provide a useful reference for effectively using of natural resources [5,6,7].

Based on the 30mx30m digital elevation model (DEM), the paper utilizes spatial analysis method to calculate the slope and the correlation coefficient of elevation and slope, and analyzes the spatial distribution characteristics of elevation and slope, studies their spatial relations in Henan province.

Data and Research Methods

Research Area Overview

Henan Province (31°23'N-36°22'N, 110°21'E-116°39'E) is located in the middle and eastern part of China, with the total area of 167 thousand km$^2$. The three sides as the north, west and south are
semi-circular surrounded by Taihang, Funiu, Tongbai and Dabie mountains. The central and east regions are Huanghuaihai alluvial plain. The data used in the study include 30m×30m DEM provided by Computer Network Information Center (http://www.gscloud.cn).

**Research Methods**

Terrain slope describes the degree of steepness of a surface unit, defined as the angle between the normal direction of the surface unit and the vertical direction. For the calculation of slope based on the digital elevation model, firstly for one cell on the surface, extracting the elevation values of 16 direct cells in the 60m neighborhood, and using the bi-cubic polynomial surface difference method to fit the region surface equation [8,9]

\[
z = f(x, y) = \sum_{i,j} a_{ij} x^i y^j.
\]

For arbitrary \( P(x_0, y_0) \) belongs to the cell, its tangent equation is

\[
f_{x_0}(x_0, y_0) (x-x_0) + f_{y_0}(x_0, y_0) (y-y_0) - (z-z_0) = 0,
\]

and corresponding normal equation is

\[
f_{x_0}^{-1}(x_0, y_0) (x-x_0) = f_{y_0}^{-1}(x_0, y_0) (y-y_0) = -(z-z_0),
\]

the normal direction is \( (f_{x_0}^{-1}(x_0, y_0), f_{y_0}^{-1}(x_0, y_0), -1) \), The cosine of the angle with the vertical direction is

\[
cos \alpha = -\sqrt{(f_{x_0}^2(x_0, y_0) + f_{y_0}^2(x_0, y_0) + 1)}
\]

the slope of the cell is

\[
\alpha = \arccos \sqrt{(f_{x_0}^2(x_0, y_0) + f_{y_0}^2(x_0, y_0) + 1)}, \alpha \in [0, 90^\circ].
\]

**Correlation Coefficient**

Correlation coefficient is used to reflect the correlation degree between variables, by the sum of multiplied the dispersions of two variables. Suppose \((x_i, y_i), i=1,2,3\ldots n\) is the observations of two-dimensional random variable \((X, Y)\), and \(\bar{x}, \bar{y}, D(X)\) and \(D(Y)\) respectively are the means and variances of \(X\) and \(Y\), the correlation coefficient of \(X\) and \(Y\) is defined as [10]

\[
\rho = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{D(X)D(Y)}}
\]

\(\rho\) belongs to \([0,1]\), the greater \(\rho\) means the large correlation, as \(\rho = 1\) means \(X\) and \(Y\) is linearly dependence, \(\rho = 0\) means \(X\) and \(Y\) is independence.

Figure 1. The distribution characteristics of elevation in Henan province.
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Distribution Characteristics of Elevation in Henan Province

Fig. 1a shows the elevation distribution in Henan Province. The elevation ranges from 23.2m to 2413.8m with an average elevation of 403m. The province is dominated by medium and low elevations, the area of 185-860m occupies 90% of the total area, and the area below 185m accounts for 0.6%, while the area above 1500m is only 1.23% (Fig. 1b).

From the west to the east, the elevation maximums on longitude lines show three steps as high-medium-low, respectively are high elevation region located in the west of 112°E with the average 850m, the middle elevation region in 112°E-114°E with the average 387m, and the low elevation region in the east of 114°E with the average 227m (Fig. 1c). The mean and minimum gradually decrease with the increase of longitude.

From south to north, the elevation maximums on latitude lines also present three steps as low-high-medium (Fig. 1d). The first step is located in the south of 33.5°N with the average 950m, the second step is in 33.5°N-36.3°N with the average 2010m, and the north of 34.3°N is the third step with the average 1506m. The means show two connected parabolic characteristics linked at 34.3°N; the mean in the southern part is smaller than that in the northern part. The minimums remain at 132m except for the north-south edge.

Distribution Characteristics of Slope in Henan Province

Using Eq.1-5 to calculate the slope in Henan province, the results are showed in Fig.2a, the slopes is between 0.33°-78.12° (except the water region about 689 km²), the average slope of the province is about 8.71°. The areas of the slope ≤2°, 2°~6°, 6°~15°, 15°~25° and > 25° respectively are 10.63%, 40.63%, 32.41%, 9.74% and 6.59% of the total area (Fig. 2b).

On the longitude line, the slope maximum in the west of 114.13°E fluctuates around 68.57°, and from 114.13°E to the eastern the slope maximum gradually decreases. The slope mean gradually decreases with the longitude increase, which is similar to the trend of elevation on the longitude line, but there are obvious regional differences in local fluctuation (Fig. 2c).

On the latitude line, the slope maximum gradually increases with the latitude increase, the fluctuation amplitude in the south of 34.37°N is obviously smaller than that in the north region. The trend of average value and elevation is similar on the latitude line, but the local variation has obvious difference (Fig. 2d).

Figure 2. The distribution characteristics of slope in Henan province.

Correlation Analysis of Elevation and Slope

The slope based on DEM represents the angle between the vertical direction and the normal direction of the surface fitting by the elevation of the adjacent 16 grid cells, therefore the contribution rates of the 16 elevation value should be the same, and the contribution rate of each cell should not be greater than 6.25%. Affine transformation the Eq.1 shows that the slope α is independent with the average elevation of the neighborhood. Therefore, the absolute value of the correlation coefficient between the elevation and the slope is greater than 6.25%, shows that the contribution rate of the cell is greater than that of the cells in its neighborhood, the slope changes in the neighborhood is quite larger, The local topographic relief is obvious, the topography is rather complicated. On the one hand,
if the correlation coefficient between elevation and slope is greater than 0, it means that there are more concave geomorphic features in the region; conversely, the convex geomorphic features occupy the main advantage, the greater the absolute value of the correlation coefficient the more obvious advantages. On the other hand, because of the continuity of geomorphic characteristics, if the convex (concave) geomorphic features occupy the main advantage in the neighborhood of a cell, then there must be a local sudden change concave (convex) in the larger neighborhood of the cell, and the geomorphic undulation is obvious in the local region where the cell is located and the terrain is relatively complicated [10-11].

In order to study the relationship between elevation and slope, according to the characteristics of elevation distribution in the province, this study divides the elevations into 25 classes at interval of 100m, and the $i^{th}$ class includes the elevation belongs to $[100(i-1), 100i]$ (Fig.3a). According to formula (5), by using the partition statistics and map algebra calculate the correlation coefficient of elevation and slope in each class, the results shown in Table 1.

| Class | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   | 11   | 12   | 13   |
|-------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| EM    | 0.84 | 1.94 | 2.33 | 3.43 | 4.48 | 5.48 | 6.49 | 7.48 | 8.49 | 9.49 | 10.50| 11.49| 12.49|
| ESD   | 16.71| 6.27 | 26.37| 28.59| 29.  | 28.72| 28.99| 28.74| 28.81| 28.94| 28.92| 28.74| 28.85|
| SM    | 25.75| 5.47 | 5.20 | 7.28 | 10.76| 12.63| 15.23| 16.93| 18.75| 20.48| 21.69| 22.27| 23.59|
| SSD   | 15.22| 3.76 | 3.47 | 5.40 | 7.44 | 8.68 | 9.56 | 10.08| 10.30| 10.28| 10.28| 10.37| 10.37|
| CC%   | -1.45| 8.16 | 9.88 | 17.78| 7.42 | 6.64 | 5.72 | 6.18 | 2.83 | 5.33 | 1.38 | 2.93 | 3.07 |

| Class | 14   | 15   | 16   | 17   | 18   | 19   | 20   | 21   | 22   | 23   | 24   | 25   |
|-------|------|------|------|------|------|------|------|------|------|------|------|------|
| EM    | 13.48| 14.48| 15.47| 16.46| 17.45| 18.46| 19.46| 20.45| 21.43| 22.42| 23.41| 24.42|
| ESD   | 28.81| 28.75| 28.65| 28.61| 28.65| 28.70| 28.59| 28.39| 27.99| 28.10| 28.10| 28.10|
| SM    | 24.57| 25.04| 25.02| 25.43| 26.  | 26.21| 26.48| 26.65| 27.52| 27.52| 26.56| 26.71|
| SSD   | 10.37| 10.44| 10.45| 10.45| 10.49| 10.56| 10.54| 10.58| 10.85| 10.89| 11.38| 11.46|
| CC%   | 2.82 | -0.02| 0.65 | 1.47 | 1.77 | -0.73| 0.80 | 2.57 | 2.82 | -2.89| 0.41 | -4.74|

Table 1. The distribution characteristics of elevation and slope in Henan province.

Along with the elevation rise from 23m to 200m, the average slope gradually decreases. As the elevation is in $[200,300m)$ the average value of the slope remains at about 5.33°. The average value of the slope increases rapidly from 5.20°to 10.08°, when the elevation rises from 300m to 800m. As the elevation above 800m, the average slope increases from 10.08° to 18.72° , with an increase rate less than that in 300-800m (Fig.3b).

There are significant differences in the variance of slope in different elevation classes. as the elevation rises from 23m to 200m, the variance of slope decreases from 15.22 to 3.75. the elevation rises from 200m to 300m, the variance of slope basically keeps at about 3.61. above 300m, the variance of slope gradually increases form 5.40 to 11.45.

As elevation increases, the correlation coefficient between elevation and slope gradually increases, reaching a maximum of 17.18% at the 400m, and then gradually decreasing to -4.70% of 2500m. As the elevations in $[23,112), [1490,1530), [1870,1948),[2250,2387],[2408,2414]$ the elevation and slope shows negative correlation, while in the other elevation zones shows positive correlation. In the negative correlation region the absolute value of the correlation coefficients are less than 6.25%, and
in the positive correlation region, only in the elevation zone of \([180,553)\) the correlation coefficient is more than 6.25%.

**Conclusion and Discussion**

Based on the digital elevation model, the paper utilizes spatial analysis method to calculate the slope and the correlation coefficient of elevation and slope, and analyzes the spatial distribution characteristics of elevation and slope, studies their spatial relations. The results show that:

1. Henan is dominated by medium and low elevations. With the increase of longitude, the maximums elevation show three steps that gradually decrease, the mean and minimum values decrease. With the increase of latitude, the maximums elevation show the characteristics of the three steps with the range of low high medium, and the mean values show two connected parabolic characteristic.

2. Henan is dominated by low slope, with the increase of longitude, the maximum and mean of slopes gradually decrease, and the local fluctuation is obviously higher than that of elevation. With the increase of latitude, the maximum slope gradually increases. The trend of mean in latitude is similar with the pattern of the elevation, but there are obvious differences in local changes.

3. The correlation between slope and elevation is weak, the elevation and slope are negatively correlated in the region above 1870m. The slope change is more obvious in the regions with low and high elevation. With the increase of elevation, the mean and variance is increasing. The topographic relief is obvious in the region below 200m and above 1870m, respectively show concave geomorphic feature. and convex geomorphic feature.

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