OPTICALLY DETECTED MAGNETOPHONON RESONANCES IN POLAR SEMICONDUCTORS

G.-Q. Hai
Instituto de Física de São Carlos, Universidade de São Paulo,
13560-970, São Carlos, SP, Brazil

F. M. Peeters
Departement Natuurkunde, Universiteit Antwerpen (UIA),
B-2610 Antwerpen, Belgium

Magnetophonon resonances are found for $\omega_c = \omega_{LO}/N$ with $N = 1, 2, 3 \ldots$ in the polaron cyclotron resonance (CR) linewidth and effective mass of bulk polar semiconductors. The CR mass and the linewidth are obtained from the full polaron magneto-optical absorption spectrum which are calculated using the memory function technique. The amplitude of the resonant peak in the linewidth can be described by an exponential law at low temperature.

I. INTRODUCTION

The magnetophonon resonance (MPR) effect occurs when two Landau levels are a phonon energy apart which leads to a resonant scattering due to emission or absorption of phonons. Since the pioneering work by Gurevich and Firsov [1], this effect has been extensively studied in bulk [2-4] as well as low-dimensional semiconductor systems [5-10]. The resonant character makes it a powerful spectroscopic tool. Magnetophonon resonances have been used to obtain information on band structure parameters, such as the effective mass and the energy levels, and on the electron-phonon interaction. The vast majority of work on MPR was done on the transport properties of semiconductors, usually the magnetoresistance, which inevitably involves a complicated averaging of scattering processes. The oscillations in the magnetoresistance are the results of a combination of scattering and broadening processes which can lead to a quite complicated dependence of the resonance amplitudes on doping, sample structure, carrier concentration, and temperature. However, the MPR can also be observed directly through a study of the electron cyclotron resonance (CR) linewidth and effective mass, i.e., the so called optically detected MPR (ODMPR). This was demonstrated by Barnes et al. [11] for a two-dimensional (2D) semiconductor system formed in GaAs/AlGaAs heterojunctions. The ODMPR allows one to make quantitative measurements of the scattering strength for specific Landau levels and yields direct information on the nature of the electron-phonon interaction in semiconductors.

In this work, we extend the theory for ODMPR to three-dimensional (3D) systems and present the first theoretical study of the magnetophonon resonances in the frequency-dependent conductivity in bulk polar semiconductors. Our calculations show strong oscillations of both the linewidth and the effective mass in a 3D system of GaAs which indicate that the ODMPR should also be observed experimentally in bulk polar semiconductors.

The present paper is organized as follows. In Sec. II, we present our theoretical formulation of the problem. The numerical results and discussions are given in Sec. III, and we summarize our results in Sec. IV.

II. THEORETICAL FRAMEWORK

Magnetophonon resonance is essentially a single-particle effect and, consequently, can be treated as a one-polaron problem. We consider a polar semiconductor in a uniform magnetic field $\mathbf{B}$ directed along the $z$-axis. The system under consideration can be described by the following Hamiltonian,

$$H = H_e + H_{ph} + H_{int}$$  \hspace{1cm} (1)

with

$$H_e = \frac{1}{2m_b}(\vec{p} + e\vec{A})^2,$$  \hspace{1cm} (2)

and
\[ H_{ph} = \sum_q \hbar \omega_q (a_q^\dagger a_q + \frac{1}{2}), \]  

(3)



where \( m_b \) is the bare electron effective mass, the vector potential \( \vec{A} = B/2(-y, x, 0) \) is chosen in the symmetrical Coulomb gauge, \( \vec{q} \) (\( \vec{r} \)) the momentum (position) operator of the electron, \( a_q^\dagger \) (\( a_q \)) the creation (annihilation) operator of an optical-phonon with wave vector \( \vec{q} \) and energy \( \hbar \omega_q \). The electron-phonon interaction Hamiltonian \( H_{int} \) is given by the Fröhlich interaction Hamiltonian

\[ H_{int} = \sum_q (V_{\vec{q}} a_q e^{iq \cdot \vec{r}} + V_{\vec{q}}^* a_q^\dagger e^{-iq \cdot \vec{r}}), \]  

(4)

where

\[ V_{\vec{q}} = -\frac{i\hbar \omega_{LO}}{2m_b \omega_{LO}} \left( \frac{1}{2\pi} \right)^{1/4} \sqrt{4\pi \alpha} \frac{V_{\vec{q}}}{V_{\vec{q}}^2}, \]  

(5)

and \( \alpha \) is the electron-LO-phonon coupling constant.

First, we calculate the optical aborption spectrum of the polaron in the presence of a magnetic field from which we are able to investigate the polaron CR spectrum and the corresponding MPR effects. For convenience we use units such that \( \hbar = m_b = \omega_{LO} = 1 \). Within linear response theory the frequency-dependent magneto-optical absorption spectrum for cyclotron resonance [12-14] is given by

\[ A(\omega) = \frac{1}{2} \frac{\text{Im} \Sigma(z)}{[\omega - \omega_c - \text{Re} \Sigma(z)]^2 + [\text{Im} \Sigma(z)]^2}, \]  

(6)

where \( \omega_c = eB/m_b \) is the unperturbed electron cyclotron frequency, \( \Sigma(z) \) is the so called memory function, and \( z = \omega + i\gamma \) where \( \gamma \) is a broadening parameter. Notice that \( \gamma \) is introduced semi-empirically to remove the divergence of the Landau level density of states. We take \( \gamma \) as a constant. For the magneto-optical absorption spectrum in the Faraday (active-mode) configuration, which corresponds to the cyclotron resonance experiments, the memory function is given by [13]

\[ \Sigma(z) = \frac{1}{2m_b} \sum_q q^2 |V_{\vec{q}}|^2 F_{\vec{q}}(z), \]  

(7)

with

\[ F_{\vec{q}}(z) = -\frac{2}{\pi} \int_0^\infty dt (1 - e^{izt}) \text{Im} \langle [b_{\vec{q}}(t), b_{\vec{q}}^\dagger(0)] \rangle, \]  

(8)

where \( b_{\vec{q}} = a_{\vec{q}} e^{iq \cdot \vec{r}} \), and the correlation function is given by

\[ \langle [b_{\vec{q}}(t), b_{\vec{q}}^\dagger(0)] \rangle = [1 + n(\omega_{LO})] e^{-i\omega_{LO}t} S^*(-\vec{q}, t) - n(\omega_{LO}) e^{-i\omega_{LO}t} S(\vec{q}, t), \]  

(9)

where

\[ n(\omega_{LO}) = \frac{1}{e^{\hbar \omega_{LO}} - 1}, \]  

(10)

is the number of LO-phonons and

\[ S(\vec{q}, t) = \langle e^{i\vec{q} \cdot \vec{r}(t)} e^{-i\vec{q} \cdot \vec{r}(0)} \rangle, \]  

(11)

is the space Fourier transform of the electron density-density correlation function. In Eq. \( \text{(10)} \) \( \beta = 1/k_B T \), where \( k_B \) is the Boltzmann constant. For a weak electron-LO-phonon coupling system, i.e. \( \alpha \ll 1 \), the density-density correlation function is calculated for a free electron in a magnetic field which is given by

\[ S(\vec{q}, t) = e^{q^2 D(t)} e^{-q^2 D_H(t)}, \]  

(12)

where
\[ D(t) = \frac{1}{2}(-it + t^2/\beta), \] (13)

and
\[ D_H(t) = \frac{1}{2\omega_c}[1 - e^{i\omega_c t} + 4n(\omega_c) \sin^2(\omega_c t/2)], \]

From the above equations, we obtain the memory function for \( \gamma = 0 \). The calculation proceeds along the lines of a similar calculation which was presented in Ref. [13]. The results for the memory function are

\[
\text{Re}\Sigma(\omega) = \frac{\alpha \sqrt{\beta}}{2\pi \omega} \frac{\omega_c \tanh(\beta \omega_c/2)}{\sinh(\beta/2)} \sum_{n,n' = 0}^{\infty} \frac{2 \cosh(\beta \omega_c/2)^{(n+n')}}{n!n!'!} \int_0^\infty \frac{dx}{x} E_{n+n'+1} \left( \frac{x^2}{\omega_c \tanh(\beta \omega_c/2)} \right) 
\times \left\{ \exp\left( \frac{\beta \omega_{nn'}}{2} \right) \left\{ 2D \left( \frac{\sqrt{\beta} x}{2} + \frac{\sqrt{\beta}}{2x} \omega_{nn'} \right) 
- D \left( \frac{\sqrt{\beta} x}{2} + \frac{\sqrt{\beta}}{2x} (\omega_{nn'} + \omega) \right) - D \left( \frac{\sqrt{\beta} x}{2} + \frac{\sqrt{\beta}}{2x} (\omega_{nn'} - \omega) \right) \right\} 
+ \exp\left( -\frac{\beta \omega_{nn'}}{2} \right) \left\{ 2D \left( \frac{\sqrt{\beta} x}{2} - \frac{\sqrt{\beta}}{2x} \omega_{nn'} \right) 
- D \left( \frac{\sqrt{\beta} x}{2} - \frac{\sqrt{\beta}}{2x} (\omega_{nn'} + \omega) \right) \right\}, \tag{14}
\]

and
\[
\text{Im}\Sigma(\omega) = -\frac{\alpha \sqrt{\beta}}{4\pi \omega} \frac{\omega_c \sinh(\beta \omega_c/2) \tanh(\beta \omega_c/2)}{\sinh(\beta/2)} \sum_{n,n' = 0}^{\infty} \frac{2 \cosh(\beta \omega_c/2)^{(n+n')}}{n!n!'!} \int_0^\infty \frac{dx}{x} E_{n+n'+1} \left( \frac{x^2}{\omega_c \tanh(\beta \omega_c/2)} \right) 
\times \left\{ \exp\left( -\frac{\beta x}{4} - \frac{\beta}{4x} (\omega_{nn'} - \omega) \right) + \exp\left( -\frac{\beta x}{4} + \frac{\beta}{4x} (\omega_{nn'} - \omega) \right) \right\}, \tag{15}
\]

where, \( \omega_{nn'} = 1 + (n-n')\omega_c \), \( D(x) \) is the Dawson integral function, and
\[
E_n = \int_0^\infty dt \frac{t^n e^{-t}}{t+x}. \tag{16}
\]

In the case of non zero broadening, i.e. \( \gamma \neq 0 \), the calculation is more tedious. We obtain the following results for the memory function,
\[
\text{Re}\Sigma(\omega) = -\frac{\alpha}{\sqrt{2\pi} (\omega^2 + \gamma^2)} \left[ \omega I_1(\omega) + \gamma I_2(\omega) \right], \tag{17}
\]

and
\[
\text{Im}\Sigma(\omega) = \frac{\alpha}{\sqrt{2\pi} (\omega^2 + \gamma^2)} \left[ \omega I_2(\omega) + \gamma I_1(\omega) \right], \tag{18}
\]

with
\[
I_1(\omega) = -\sqrt{\beta} \omega_c \frac{\omega_c \tanh(\beta \omega_c/2)}{\sinh(\beta/2)} \sum_{n,n' = 0}^{\infty} \frac{2 \cosh(\beta \omega_c/2)^{(n+n')}}{n!n!'!} \int_0^\infty \frac{dx}{x} E_{n+n'+1} \left( \frac{x^2}{\omega_c \tanh(\beta \omega_c/2)} \right) 
\times \left\{ \exp\left( \frac{\beta \omega_{nn'}}{2} \right) \left\{ 2D \left( \frac{\sqrt{\beta} x}{2} + \frac{\sqrt{\beta}}{2x} \omega_{nn'} \right) 
- D \left( \frac{\sqrt{\beta} x}{2} + \frac{\sqrt{\beta}}{2x} (\omega_{nn'} + \omega + i\gamma) \right) + D \left( \frac{\sqrt{\beta} x}{2} + \frac{\sqrt{\beta}}{2x} (\omega_{nn'} - \omega + i\gamma) \right) \right\} 
+ \exp\left( -\frac{\beta \omega_{nn'}}{2} \right) \left\{ 2D \left( \frac{\sqrt{\beta} x}{2} - \frac{\sqrt{\beta}}{2x} \omega_{nn'} \right) 
- D \left( \frac{\sqrt{\beta} x}{2} - \frac{\sqrt{\beta}}{2x} (\omega_{nn'} - \omega - i\gamma) \right) + D \left( \frac{\sqrt{\beta} x}{2} - \frac{\sqrt{\beta}}{2x} (\omega_{nn'} + \omega - i\gamma) \right) \right\} \right\}, \tag{19}
\]

\[ \]
and
\[ I_2(\omega) = -\frac{\sqrt{\pi} \beta \omega_c \tanh(\beta \omega_c/2)}{2\sqrt{2} \sinh(\beta/2)} \sum_{n,n'=0}^{\infty} \left[ \frac{2 \cosh(\beta \omega_c/2)}{n!} \right] \int_0^\infty \frac{dx}{n+n'+1} \left( \frac{x^2}{\omega_c \tanh(\beta \omega_c/2)} \right) \times \left( \exp \left( \frac{-\beta \omega_{nn'}}{2} \right) \right) \left[ \text{Re} W \left( \frac{\sqrt{\beta x}}{2} + \frac{\sqrt{\beta x}}{2} (\omega_{nn'} + \omega + i\gamma) \right) - \text{Re} W \left( \frac{\sqrt{\beta x}}{2} + \frac{\sqrt{\beta x}}{2} (\omega_{nn'} - \omega + i\gamma) \right) + \text{Re} W \left( \frac{\sqrt{\beta x}}{2} - \frac{\sqrt{\beta x}}{2} (\omega_{nn'} - \omega - i\gamma) \right) - \text{Re} W \left( \frac{\sqrt{\beta x}}{2} - \frac{\sqrt{\beta x}}{2} (\omega_{nn'} + \omega - i\gamma) \right) \right] \right], \] (20)

where \( W(z) = e^{-z^2} \text{erfc}(-iz) \) is the complex error function.

### III. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we present our numerical results on the magneto-optical absorption spectra and study the magneto-phonon resonant effects. As an example, we apply our theory to the semiconductor GaAs where \( \alpha = 0.07 \). First, we show numerical results for \( T = 77 \) K and level broadening parameter \( \gamma = 0 \). Due to the importance of the memory function in the absorption spectrum, we plot the real and imaginary parts of the memory function in Fig. 1(a) and 1(b), respectively, as a function of frequency at three different magnetic fields. We see that, as \( \omega = |\omega_{LO} - n\omega_c| \) \( (n = 0, 1, 2, \ldots) \), \( \text{Im} \Sigma(\omega) \) exhibits a jump while \( \text{Re} \Sigma(\omega) \) diverges logarithmically. The discontinuity of \( \text{Re} \Sigma(\omega) \) and the divergence in \( \text{Im} \Sigma(\omega) \) reflects the resonant coupling between the state \( E_0 + \omega_{LO} \) and Landau level \( E_n = (1/2 + n)\omega_c \). The stronger this coupling, the larger the discontinuity in \( \text{Re} \Sigma(\omega) \). Actually, the real part of the memory function \( \text{Re} \Sigma(\omega) \) is responsible for the shift in the observed CR energy which is due to the electron-phonon interaction. While, the imaginary part leads to a broadening of the spectrum which is a result of scattering. When \( \text{Im} \Sigma(\omega) \) is zero like in a 2D system, the absorption is a \( \delta \) function and its position is determined by the equation \( \omega^*_c - \omega_c - \text{Re} \Sigma(\omega^*_c) = 0 \). Fig. 1(b) shows that for a bulk system \( \text{Im} \Sigma(\omega) \) is non zero which reflects the 3D character of the electron states. The scattering which is mainly in the direction parallel to the magnetic field results in a finite \( \text{Im} \Sigma(\omega) \) and consequently a finite linewidth even for \( \gamma = 0 \). In Fig. 1(c), we show the corresponding magneto-optical absorption spectra. The position of the absorption peaks corresponds to the cyclotron resonance frequency \( \omega^*_c \) at which the cyclotron resonance occurs. We notice an asymmetric double peak structure around \( \omega = \omega_{LO}/2 \) for \( \omega_c = \omega_{LO}/2 \) (the solid curve) and the absorption becomes zero at \( \omega = \omega_c = \omega_{LO}/2 \). The zeros in the absorption spectrum are a consequence of the divergences in \( \text{Im} \Sigma(\omega) \) and which can be traced back to the divergent nature of the density of states. The double peak structure is a consequence of the magneto-phonon resonance which leads to an anticrossing behavior in the CR spectrum. When the CR frequency \( \omega_c \) deviates from \( \omega_{LO}/N \), this splitting becomes very weak and difficult to observe in the absorption spectrum. As we will see below, however, the magneto-phonon resonance will strongly affect the linewidth of the magneto-optical absorption and the CR mass. From the dash and the dotted curves, we notice that the absorption peak appears at a frequency \( \omega^*_c < \omega_c \) due to the polaron effect which shifts the cyclotron frequency to lower frequencies. The latter is often interpreted as an increase of the cyclotron mass, i.e. \( \omega^*_c = eB/m^*c \). In Fig. 2, we show the absorption spectrum around (a) \( \omega_c = \omega_{LO}/2 \) and (b) \( \omega_c = \omega_{LO}/3 \). The double peak structure disappears when \( \omega_c \) deviates from \( \omega_{LO}/N \). The absorption spectra also demonstrate clearly a nonlinear magnetic field dependence of the peak position and linewidth around \( \omega_c/N \).

Fig. 3 demonstrates the effect of the broadening parameter \( \gamma \) on the absorption spectrum. Notice that, with increasing \( \gamma \), i) the double peak structure disappears for \( \gamma > 0.01\omega_{LO} \), ii) the zero in the absorption spectrum disappears when \( \gamma > 0 \), and iii) the position of the absorption peak shifts to higher frequency. This indicates that the anticrossing behavior in the CR spectrum will be difficult to be observed experimentally at \( \omega_c = \omega_{LO}/2 \) due to broadening effects which are a consequence of scattering on e.g. impurities and acoustical phonons.

As soon as the polaron CR frequency \( \omega^*_c \) is determined from the position of the magneto-optical absorption peak, the CR mass of the polaron is obtained by
\[ m^*/m_b = \omega_c/\omega^*_c. \] (21)

The numerical results of the polaron CR mass and the FWHM (full width at half maximum) for \( \gamma = 0.05\omega_{LO} \) are plotted as a function of the unperturbed CR frequency at different temperatures in Figs. 4(a) and 4(b), respectively.
One observes that the polaron CR mass is an oscillating function of the magnetic field. Fig. 4(b) shows that the FWHM of the polaron magneto-optical absorption spectrum reaches a local maximum at $\omega_c = \omega_{LO}/N$ where the polaron mass has an inflection point. This result demonstrates the derivative-like relation between the polaron CR mass and the linewidth which are due to the fact that the real and imaginary part of the memory function are related to each other through a Kramers-Kronig relation. One finds that, for temperature $T < 100$ K, the resonance grows rapidly with increasing $T$. This effect can lead to a direct measurement of the optical-phonon scattering rate. We notice also an overall increase of the linewidth with temperature, but an overall decrease of the effective mass when $T > 80$ K. The resonant position is slightly larger than the non-interacting resonant condition $\omega_c = \omega_{LO}/N$ and is almost independent of temperature. A detailed analysis indicates that, at $N = 2$ and $3$, the peak position both in the FWHM and in the derivative of the CR mass is at 0.50$\omega_{LO}$ and 0.336$\omega_{LO}$, respectively. Experimentally this position determines the so called fundamental field $B_0 = m^*\omega_{LO}/e$ which is an important quantity to study the effective mass, nonparabolicity of the energy band, as well as the LO-phonon frequency. The linewidth is a direct measure of the lifetime of the state. Notice that the conventional MPR occurs in the resistivity, which is given by $\rho_{zz} = \text{Im}\Sigma(\omega = 0)$. But ODMPR is related to both the real and imaginary part of the memory function which occurs for $\omega \neq 0$ and is a dynamical MPR.

Fig. 5 shows the CR mass oscillation amplitude at $\omega_c/\omega_{LO} = 1/2$ and $1/3$ as a function of temperature. With increasing temperature, the number of phonons increases and consequently, the oscillation amplitude increases. On the other hand, the background electron-phonon scattering (coupling) increases which results in a suppression of the oscillation amplitude. Fig. 6 shows an activation plot of the amplitude of the resonant peak in the FWHM at $\omega_c/\omega_{LO} = 1/2$ and $1/3$ as a function of $T^{-1}$. We find that, for the resonance around $N = 2$, the linewidth can be described rather well by the exponential law $\exp(-\hbar\omega_{LO}/2kT)$ for $T < 240$ K, while that around $N = 3$ can be described by $\exp(-2\hbar\omega_{LO}/3kT)$ for $T < 140$ K. This exponential behavior can be understood as follows. MPR is proportional to the number of LO-phonons which are present and therefore should increase as $n(\omega_{LO})$. On the other hand, thermal broadening of the Landau levels, which is proportional to $n(\omega_c)$, will diminish the resonant structure in $\Delta\text{FWHM}$. Thus this contribution decreases the resonant character and consequently we expect that $\Delta\text{FWHM} \sim n(\omega_{LO})/n(\omega_c) \approx \exp(-\hbar(\omega_{LO} - \omega_c)/kBT)$ which agrees with the exponential laws found for $N = 2$ and $N = 3$.

IV. SUMMARY

We have extended the theory for ODMPR to three-dimensional (3D) systems and present the first detailed theoretical study of the magnetophonon resonance in the frequency-dependent conductivity of electrons in bulk GaAs. In comparison to the corresponding 2D systems, the theoretical obtained amplitudes for the oscillations of both the linewidth and the effective mass in a 3D system are for GaAs predicted to be about half of those in 2D. Therefore, we believe that ODMPR can also be observed experimentally in bulk polar semiconductors. Our numerical results indicate that the amplitude of the resonant peak in the FWHM can be described by an exponential law for not too large temperatures.

ACKNOWLEDGMENTS

This work was supported by FAPESP, CNPq (Brazil) and FWO, IUAP (Belgium).

[1] J. R. Barker, J. Phys. C 5, 1657 (1972).
[2] R. J. Nicholas, Prog. Quantum Electron. 10, 1 (1985); R. V. Parfen’ev, G. I. Kharus, I. M. Tsidil’kovskii, and S. S. Shalyt, Sov. Phys. -Usp. 17, 1 (1974).
[3] J. Van Royen, J. De Sitter, L.F. Lemmens, and J.T. Devreese, Physica B 89, 101 (1977); J. Van Royen, J. De Sitter, and J.T. Devreese, Phys. Rev. B 30, 7514 (1984); J.P. Vigneron, R. Evrard, and E. Kartheuser, Phys. Rev. B 18, 6930 (1978).
[4] D. Schneider, C. Brink, G. Irmer, and P. Verma, Physica B 256-258, 625 (1998); D. Schneider, K. Pricke, J. Schulz, G. Irmer, and M. Wenzel, in Proceedings of the 23rd International Conference on the Physics of Semiconductors, Eds. M. Scheffler and R. Zimmermann (World Scientific, Singapore, 1996), p. 221.
FIG. 1. (a) Re\(\Sigma(\omega)\) and (b) Im\(\Sigma(\omega)\) as a function of frequency \(\omega\) in GaAs at different magnetic fields \(\omega_c/\omega_{LO} = 0.3\) (dotted curves), 0.4 (dashed curves) and 0.5 (solid curves). The corresponding absorption spectrum is given in (c). The broadening parameter \(\gamma = 0\) and temperature \(T = 77\) K.

FIG. 2. The magneto-optical absorption spectrum around (a) \(\omega_c/\omega_{LO} = 1/2\) and (b) \(\omega_c/\omega_{LO} = 1/3\) for \(T = 77\) K and \(\gamma = 0\).

FIG. 3. The magneto-optical absorption spectra as a function of frequency \(\omega\) in GaAs for \(\gamma/\omega_{LO} = 0\) (solid curve), 0.001 (dash curve), 0.01 (dotted curve) and 0.1 (dash-dotted curve) at \(\omega_c/\omega_{LO} = 0.5\) and \(T = 77\) K.

FIG. 4. (a) Polaron CR mass and (b) FWHM (full linewidth at half-maximum) as a function of \(\omega_c\) at different temperatures from 60 K to 200 K with broadening \(\gamma/\omega_{LO} = 0.05\).

FIG. 5. The CR mass oscillation amplitude as a function of temperature at \(\omega_c/\omega_{LO} = 1/2\) (dots) and \(\omega_c/\omega_{LO} = 1/3\) (solid squares) with \(\gamma/\omega_{LO} = 0.05\).

FIG. 6. An activation plot of the amplitude of the resonant peak in the FWHM at \(\omega_c/\omega_{LO} = 1/2\) (circles) and \(\omega_c/\omega_{LO} = 1/3\) (triangles) as a function of \(T^{-1}\) for \(\gamma/\omega_{LO} = 0.05\). The solid line \(\propto \exp(-\hbar \omega_{LO}/2k_B T)\) and the dotted line \(\propto \exp(-2\hbar \omega_{LO}/3k_B T)\).
\[ \text{Re}\Sigma(\omega) \]

\[ \omega_c/\omega_{\text{LO}} = 0.3 \]

\[ \omega_c/\omega_{\text{LO}} = 0.4 \]

\[ \omega_c/\omega_{\text{LO}} = 0.5 \]

\[ T = 77 \text{ K} \]
Figure (b) shows the plot of \( \text{Im} \Sigma(\omega) \) vs \( \omega / \omega_{\text{LO}} \). The graph exhibits various peaks and troughs, indicating the behavior of the system at different frequencies relative to the LO frequency.
\[ A(\omega) \]

\[ \frac{\omega_c}{\omega_{LO}} = 0.46 \]

\[ \frac{\omega}{\omega_{LO}} = 0.48 \]

\[ \frac{\omega}{\omega_{LO}} = 0.5 \]

\[ \frac{\omega}{\omega_{LO}} = 0.52 \]

\[ \frac{\omega}{\omega_{LO}} = 0.54 \]
FWHM \( \omega / \omega_{LO} \)

\( \omega_c / \omega_{LO} \)

- 80
- 100
- 120
- 140
- 160
- 180
- 200

T = 60 K
\[ \frac{\Delta m^*}{m_b} \]

Graph showing the relationship between temperature (T) in Kelvin (K) and \( \frac{\Delta m^*}{m_b} \) for different values of N.

- N=2
- N=3
\[ \Delta \text{FWHM} (\omega_{LO}) \]

vs.

\[ \frac{1}{T} \text{ (K}^{-1}) \]

For different values of \( N \):

- \( N=2 \)
- \( N=3 \)

The graph shows a linear relationship between \( \Delta \text{FWHM} (\omega_{LO}) \) and \( \frac{1}{T} \text{ (K}^{-1}) \) for both \( N=2 \) and \( N=3 \) cases.