Un-differenced precise point positioning model using triple GNSS constellations

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Abstract: This paper introduces a dual-frequency precise point positioning (PPP) model, which combines the observations of three different global navigation satellite system (GNSS) constellations, namely GPS, Galileo, and BeiDou. A drawback of a single GNSS system such as GPS, however, is the availability of sufficient number of visible satellites in urban areas. Combining GNSS observations offers more visible satellites to users, which in turn is expected to enhance the satellite geometry and the overall positioning solution. However, combining several GNSS observables introduces additional biases, which require rigorous modeling, including the GNSS time offsets and hardware delays. In this paper, un-differenced ionosphere-free linear combination PPP model is developed. The additional biases of the GPS, Galileo, and BeiDou combination are accounted for through the introduction of a new unknown parameter, which is identified as the inter-system bias, in the PPP mathematical model. Natural Resources Canada’s GPSPace PPP software is modified to enable a combined GPS, Galileo, and BeiDou PPP solution and to handle the newly introduced biases. A total of four data-sets collected at four different IGS stations are processed to verify the developed PPP model. Precise satellite orbit and clock products from the International GNSS Service Multi-GNSS Experiment (IGS-MGEX) network are used to correct the GPS, Galileo, and BeiDou measurements. It is shown that the un-differenced GPS-only post-processed PPP solution indicates that the model is capable of obtaining a sub-decimeter-level accuracy. However, the solution takes about 20 min...
to converge to decimeter-level precision. The convergence time of the combined GNSS post-processed PPP solutions takes about 15 min to reach the decimeter-level precision, which represent a 25% improvement in comparison with the GPS-only post-processed PPP solution.

Subjects: Aerospace Engineering; Civil, Environmental and Geotechnical Engineering; Earth Sciences

Keywords: PPP; GPS; Galileo; BeiDou

1. Introduction
Precise point positioning (PPP) has proven to be capable of providing positioning accuracy at the sub-decimeter and decimeter levels in static and kinematic modes, respectively. PPP accuracy and convergence time are controlled by the ability to mitigate all potential error sources in the system. Several comprehensive studies have been published on the accuracy and convergence time of undifferenced combined GPS/Galileo PPP model (see, e.g. Afifi & El-Rabbany, 2015; Collins, Bisnath, Lahaye, & Héroux, 2010; Colombo, Sutter, & Evans, 2004; Ge, Gendt, Rothacher, Shi, & Liu, 2008; Hofmann-Wellenhof, Lichtenegger, & Wasle, 2008; Kouba & Héroux, 2001; Leick, 2004; Zumberge, Heflin, Jeffrey, Watkins, & Webb, 1997). PPP relies essentially on the availability and use of precise satellite products, namely orbital and clock corrections. At present, the Multi-global navigation satellite systems (GNSS) Experiment (MGEX) of the International GNSS Service (IGS) provides the precise satellite orbital and clock corrections for all the GNSS (Montenbruck et al., 2014).

Unfortunately, the use of a single constellation limits the number of visible satellites, especially in urban areas, which affects the PPP solution. Recently, a number of researchers showed that combining GPS and Galileo observations in PPP solution enhances the positioning convergence and precision in comparison with the GPS-only PPP solution (Afifi & El-Rabbany, 2016a; Melgard, Tegedor, de Jong, Lapucha, & Lachapelle, 2013). At present, the IGS-MEGX network provides the GNSS users with precise clock and orbit products to all currently available satellite systems (Montenbruck et al., 2014). This makes it possible to obtain a PPP solution by combining the observations of two or more GNSS constellations. This research focuses on combining the GPS, Galileo, and BeiDou observations in a PPP model.

Presently, there exist four operational GNSS. These include the US global positioning system (GPS), the Russian global navigation satellite system (GLONASS), the European Galileo system, and the Chinese BeiDou system. Combining the measurements of multiple systems can significantly improve the availability of a navigation solution, especially in urban areas (Afifi & EL-Rabbany, 2016b). GPS satellites transmit signals on three different frequencies, which are controlled by the GPS time frame (GPST). Currently, the GPS users can receive the modernized civil L2C and L5 signals. On the other hand, Galileo satellite constellation foresees 27 operational and three spare satellites positioned in three nearly circular medium earth orbits (MEO). Galileo system transmits six signals on different frequencies using the Galileo time system (GST). Unlike GLONASS satellite system, Galileo and GPS have partial frequency overlaps, which simplify the dual-system integration. In addition, GPS and Galileo operators have agreed to measure and broadcast a GPS to Galileo time offset (GGTO) parameter, in order to facilitate the interchangeable mode (Melgard et al., 2013). BeiDou navigation satellite system, being developed independently by China, is pacing steadily forward toward completing the constellation. China has indicated a plan to complete the second generation of Beidou satellite system by expanding the regional service into global coverage. Beidou system transmits three signals on different frequencies using the Beidou time frame (BDT). The Beidou-2 system is proposed to consist of 30 medium Earth orbiting satellites and five geostationary satellites (Beidou, 2015; ESA, 2015; Hofmann-Wellenhof et al., 2008; IAC, 2015).

In this paper, a triple GNSS constellation (GPS, Galileo, and BeiDou) PPP model is developed. Four combinations are considered in the PPP modeling namely; GPS/Galileo, GPS/BeiDou, Galileo/BeiDou, and GPS/Galileo/BeiDou. All the combined PPP models results are compared with the GPS-only PPP
model results. In the developed model GPS L1/L2, Galileo E1/E5a, and BeiDou B1/B2 signals are used in a dual-frequency ionosphere-free linear combination. Precise satellite corrections from the International GNSS Service multi-GNSS experiment (IGS-MEGX) network are used to account for GPS, Galileo and BeiDou satellite orbit and clock errors (Montenbruck et al., 2014). As these products are presently referenced to the GPS time and since we use mixed GNSS receivers that also use GPS time as a reference, the GGTO and the GPS to BeiDou time offset are canceled out in our model. The inter-system bias is treated as an additional unknown parameter. The hydrostatic component of the tropospheric zenith path delay is modeled through the Hopfield model, while the wet component is considered as an additional unknown parameter (Hofmann-Wellenhof et al., 2008; Hopfield, 1972). All remaining errors and biases are accounted for using existing models as shown in Kouba (2009).

The inter-system bias parameter was found to be essentially constant over the one-hour observation time span and was receiver dependent. The positioning results of the developed combined GPS/Galileo, GPS/BeiDou, Galileo/BeiDou, and GPS/Galileo/BeiDou PPP models showed a sub-decimeter accuracy level and 25% convergence time improvement in comparison with the GPS-only PPP results.

2. Un-differenced PPP models
PPP has been carried out using dual-frequency ionosphere-free linear combinations of carrier-phase and pseudorange GPS measurements. Equations (1)–(6) show the ionosphere free linear combination of GPS, Galileo, and BeiDou observations (Afifi & El-Rabbany, 2016a).

\[
P_G = \rho + c[dt_G - dt^*] + c[\alpha d_{p1} - \beta d_{p2}] + c[\alpha d_{p1} - \beta d_{p2}]^2 + T_G + \epsilon_{PG} \tag{1}
\]

\[
P_E = \rho + c[dt_E - GGTO - dt^*] + c[\alpha d_{E1} - \beta d_{E5a}] + c[\alpha d_{E1} - \beta d_{E5a}]^2 + T_E + \epsilon_{PE} \tag{2}
\]

\[
P_B = \rho + c[dt_B - GB - dt^*] + c[\alpha d_{B1} - \beta d_{B2}] + c[\alpha d_{B1} - \beta d_{B2}]^2 + T_B + \epsilon_{PB} \tag{3}
\]

\[
\Phi_G = \rho + c[dt_G - dt^*] + c[\alpha \delta_{L1} - \beta \delta_{L2}] + c[\alpha \delta_{L1} - \beta \delta_{L2}]^2 + T_G + N_G + \phi_{LG} + \phi_{\delta_{G}} + \epsilon_{\phi_G} \tag{4}
\]

\[
\Phi_E = \rho + c[dt_E - GGTO - dt^*] + c[\alpha \delta_{E1} - \beta \delta_{E5a}] + c[\alpha \delta_{E1} - \beta \delta_{E5a}]^2 + T_E + N_E + \phi_{LE} + \phi_{\delta_{E}} + \epsilon_{\phi_E} \tag{5}
\]

\[
\Phi_B = \rho + c[dt_B - GB - dt^*] + c[\alpha \delta_{B1} - \beta \delta_{B2}] + c[\alpha \delta_{B1} - \beta \delta_{B2}]^2 + T_B + N_B + \phi_{LB} + \phi_{\delta_{B}} + \epsilon_{\phi_B} \tag{6}
\]

where the subscripts G, E, and B refer to the GPS, Galileo, and BeiDou satellite systems, respectively; \(P_G, P_E, P_B\) are the ionosphere-free pseudoranges in meters for GPS, Galileo, and BeiDou systems, respectively; \(\Phi_G, \Phi_E, \Phi_B\) are the ionosphere-free carrier phase measurements in meters for GPS, Galileo, and BeiDou systems, respectively; \(GGTO\) is the GPS to Galileo time offset; GB is the GPS to BeiDou time offset; \(\rho\) is the true geometric range from receiver at reception time to satellite at transmission time in meter; \(dt, dt^*\) are the clock errors in seconds for the receiver at signal reception time and the satellite at signal transmission time, respectively; \(d_{p1}, d_{p2}, d_{E1}, d_{E5a}, d_{B1}, d_{B2}\) are frequency-dependent code hardware delays for the receiver at reception time in seconds; \(d_{p1}^s, d_{p2}^s, d_{E1}^s, d_{E5a}^s, d_{B1}^s, d_{B2}^s\) are frequency-dependent code hardware delays for the satellite at transmission time in seconds; \(\alpha, \beta, \delta_{L1}, \delta_{L2}, \delta_{E1}, \delta_{E5a}, \delta_{B1}, \delta_{B2}\) are frequency-dependent carrier-phase hardware delays for the receiver at reception time in seconds; \(\alpha, \beta, \delta_{E1}, \delta_{E5a}, \delta_{B1}, \delta_{B2}\) are frequency-dependent carrier-phase hardware delays for the satellite at transmission time in seconds; \(T\) is the tropospheric delay in meter; \(N_G, N_E, N_B\) are the ionosphere-free linear combinations of the ambiguity parameters for both GPS, Galileo, and BeiDou carrier-phase measurements in meters, respectively; \(\phi_{\alpha}, \phi_{\beta}, \phi_{\delta_{G}}, \phi_{\delta_{E}}, \phi_{\delta_{B}}\) are ionosphere-free linear combinations of frequency-dependent initial fractional phase biases in the receiver and satellite channels for both GPS, Galileo, and BeiDou in meters, respectively;
Precise orbit and satellite clock corrections of IGS-MGEX networks are produced for both GPS/Galileo observations and are referred to GPS time. IGS precise GPS satellite clock correction includes the effect of the ionosphere-free linear combination of the satellite hardware delays of L1/L2 (Y) code, while the Galileo counterpart includes the effect of the ionosphere-free linear combination of the satellite hardware delays of the Galileo E1/E5a pilot code. In addition, BeiDou satellite clock correction includes the effect of the ionosphere-free linear combination of the satellite hardware delays of B1/B2 code (Montenbruck et al., 2014). By applying the precise clock products for both GPS/Galileo/BeiDou observations, Equations (1)–(6) will take the following form:

\[ P_{G_x} = \rho_G + c[dt_{G} - dt_{E}^{prec}] + c[ad_{p1} - \beta d_{p2}]_r + T_G + \epsilon_{G_x} \]  
\[ P_{E_x} = \rho_E + c[dt_{E} - dt_{E}^{prec}] + c[ad_{E1} - \beta d_{E5a}]_r + T_E + \epsilon_{E_x} \]  
\[ P_{B_y} = \rho_B + c[dt_{B} - dt_{B}^{prec}] + c[ad_{B1} - \beta d_{B2}]_r + T_B + \epsilon_{B_y} \]  
\[ \Phi_{G_x} = \rho_G + cdt_{G} - c[dt_{E}^{prec} + [ad_{p1} - \beta d_{p2}]^1] + c[\alpha d_{L1} - \beta \delta_{L2}]_r - c[\alpha d_{L1} - \beta \delta_{L2}]^1 + T_G + N_{G_x} + \phi_{\alpha_{G_x}} + \phi_{\beta_{G_x}} + \epsilon_{G_x} \]  
\[ \Phi_{E_x} = \rho_E + cdt_{E} - c[dt_{E}^{prec} + [ad_{E1} - \beta d_{E5a}]^1] + c[\alpha d_{E1} - \beta \delta_{E5a}]_r - c[\alpha d_{E1} - \beta \delta_{E5a}]^1 + T_E + N_{E_x} + \phi_{\alpha_{E_x}} + \phi_{\beta_{E_x}} + \epsilon_{E_x} \]  
\[ \Phi_{B_y} = \rho_B + cdt_{B} - c[dt_{B}^{prec} + [ad_{B1} - \beta d_{B2}]^1] + c[\alpha d_{B1} - \beta \delta_{B2}]_r - c[\alpha d_{B1} - \beta \delta_{B2}]^1 + T_B + N_{B_y} + \phi_{\alpha_{B_y}} + \phi_{\beta_{B_y}} + \epsilon_{B_y} \]  

For simplicity, the receiver and satellite hardware delays will take the following forms:

\[ b_{r_x} = c[ad_{p1} - \beta d_{p2}]_r \quad b_{r_x}^s = c[ad_{p1} - \beta d_{p2}]^s \]  
\[ b_{e_x} = c[ad_{E1} - \beta d_{E5a}]_r \quad b_{e_x}^s = c[ad_{E1} - \beta d_{E5a}]^s \]
In the combined GPS/Galileo un-differenced PPP model, the GPS receiver clock error is lumped with the GPS receiver differential code biases. In order to maintain consistency in the estimation of a common receiver clock offset, this convention is used when combining the ionosphere-free linear combination of GPS L1/L2, Galileo E1/E5a, and BeiDou B1/B2 observations in PPP solution. This, however, introduces an additional bias in the Galileo ionosphere-free PPP mathematical model, which represents the difference in the receiver differential code biases of both systems. Such an additional bias is commonly known as the inter-system bias, which is referred to as ISB in this paper. In our PPP mode, the Hopfield tropospheric correction model along with the Vienna mapping function are used to account for the hydrostatic component of the tropospheric delay (Boehm & Schuh, 2004; Hopfield, 1972). Other corrections are also applied, including the effect of ocean loading (Bos & Scherneck, 2013; IERS, 2010), Earth tide (Koubou, 2009), carrier-phase windup (Leick, 2004; Wu, Wu, Hajj, Bertiger, & Lichten, 1993), Sagnac (Kaplan & Heagarty, 2006), relativity (Hofmann-Wellenhof et al., 2008), and satellite and receiver antenna phase-center variations (Dow, Neilan, & Rizos, 2009). The noise terms are modeled stochastically using an exponential model, as described in Afifi and El-Rabbany (2015). With the above consideration, the GPS/Galileo ionosphere-free linear combinations of both pseudorange and carrier phase can be written as:

\[ P_{G_{\phi}} = \rho_G + \delta t_{RG} - \delta t_{\text{prec}}^s + T_G + \epsilon_{PG_{\phi}} \]  

(16)

\[ P_{E_{\phi}} = \rho_E + \delta t_{RG} - \delta t_{\text{prec}}^s + \text{ISB}_{GE} + T_E + \epsilon_{PE_{\phi}} \]  

(17)

\[ \Phi_{G_{\phi}} = \rho_G + \delta t_{RG} - \delta t_{\text{prec}}^s + T_G + \tilde{N}_{G_{\phi}} + \epsilon_{\Phi_{G_{\phi}}} \]  

(18)

\[ \Phi_{E_{\phi}} = \rho_E + \delta t_{RG} - \delta t_{\text{prec}}^s + T_E + \tilde{N}_{E_{\phi}} + \text{ISB}_{GE} + \epsilon_{\Phi_{E_{\phi}}} \]  

(19)

where \( \delta t_{RG} \) represents the sum of the receiver clock error and receiver hardware delay \( \delta t_{RG} = \delta t_{\text{PREC}} + b_{i} \); ISB is the inter system bias as follows \( \text{ISB}_{GE} = b_{t} - b_{t} \); \( \tilde{N}_{G_{\phi}} \) and \( \tilde{N}_{E_{\phi}} \) are given by:

\[ \tilde{N}_{G_{\phi}} = N_{G_{\phi}} + b_{t_{g}} + b_{t_{e}} - b_{s_{\phi}} - b_{s_{e}} \]  

(20)

\[ \tilde{N}_{E_{\phi}} = N_{E_{\phi}} + b_{t_{e}} + b_{t_{g}} - b_{s_{\phi}} - b_{s_{e}} \]  

(21)

In case of combining GPS and BeiDou observations in a PPP model ionosphere-free linear combinations of both pseudorange and carrier phase can be written as:

\[ P_{G_{\phi}} = \rho_G + \tilde{\delta} t_{RG} - \delta t_{\text{prec}}^s + T_G + \epsilon_{PG_{\phi}} \]  

(22)

\[ P_{B_{\phi}} = \rho_B + \tilde{\delta} t_{RG} - \delta t_{\text{prec}}^s + \text{ISB}_{GB} + T_B + \epsilon_{PB_{\phi}} \]  

(23)

\[ \Phi_{G_{\phi}} = \rho_G + \tilde{\delta} t_{RG} - \delta t_{\text{prec}}^s + T_G + \tilde{N}_{G_{\phi}} + \epsilon_{\Phi_{G_{\phi}}} \]  

(24)
\[ \Phi_{B_y} = \rho_B + \Delta t_{rg} - \Delta t_{\text{prec}} + T_B + \tilde{N}_{B_y} + ISB_{GB} + \epsilon_{\Phi_{B_y}} \]  

(25)

where \( \Delta t_{rg} \) represents the sum of the receiver clock error and receiver hardware delay \( \Delta t_{\text{prec}} = c \Delta t_{rg} + b_s; ISB \) is the inter system bias as follows \( ISB_{GB} = b_{s_y} - b_s; \tilde{N}_{G_y} \) and \( \tilde{N}_{B_y} \) are given by:

\[ \tilde{N}_{G_y} = N_{G_y} + b_{s_y} + b_{s} - b_{p} - b_{p}^s \]  

(26)

\[ \tilde{N}_{B_y} = N_{B_y} + b_{s_y} + b_{s} - b_{p}^s - b_{B}^s \]  

(27)

In the combined Galileo and BeiDou un-differenced PPP model, the Galileo receiver clock error is lumped with the Galileo receiver differential code biases. In order to maintain consistency in the estimation of a common receiver clock offset, this convention is used when combining the ionosphere-free linear combination of Galileo E1/E5a and BeiDou B1/B2 observations. This, however, introduces an additional bias in the BeiDou ionosphere-free PPP mathematical model, which represents the difference in the receiver differential code biases of both systems. As a result, the Galileo and BeiDou combined PPP model ionosphere-free linear combinations of both pseudorange and carrier phase can be written as:

\[ P_{E_y} = \rho_{E} + \Delta t_{\text{IE}} - \Delta t_{\text{prec}} + T_E + \epsilon_{PE_y} \]  

(28)

\[ P_{B_y} = \rho_{B} + \Delta t_{\text{IE}} - \Delta t_{\text{prec}} + ISB_{EB} + T_B + \epsilon_{PB_y} \]  

(29)

\[ \Phi_{E_y} = \rho_{E} + \Delta t_{\text{IE}} - \Delta t_{\text{prec}} + T_E + \tilde{N}_{E_y} + \epsilon_{\Phi_{E_y}} \]  

(30)

\[ \Phi_{B_y} = \rho_{B} + \Delta t_{\text{IE}} - \Delta t_{\text{prec}} + T_B + \tilde{N}_{B_y} + ISB_{EB} + \epsilon_{\Phi_{B_y}} \]  

(31)

where \( \Delta t_{\text{IE}} \) represents the sum of the receiver clock error and receiver hardware delay \( \Delta t_{\text{prec}} = c \Delta t_{\text{IE}} + b_s; ISB \) is the inter system bias as follows \( ISB_{EB} = b_{s_y} - b_s; \tilde{N}_{E_y} \) and \( \tilde{N}_{B_y} \) are given by:

\[ \tilde{N}_{E_y} = N_{E_y} + b_{s_y} + b_{s} - b_{p} - b_{p}^s \]  

(32)

\[ \tilde{N}_{B_y} = N_{B_y} + b_{s_y} + b_{s} - b_{p}^s - b_{B}^s \]  

(33)

When using the combined GPS/Galileo or GPS/BeiDou or Galileo/BeiDou un-differenced PPP model, the ambiguity parameters lose their integer nature as they are contaminated by receiver and satellite hardware delays. It should be pointed out that the number of unknown parameters in the combined PPP model equals the number of visible satellites from any system plus six parameters, while the number of equations equals double the number of the visible satellites. This means that the redundancy equals \( n_B + n_G - 6 \). In other words, at least six mixed satellites are needed for the solution to exist. In comparison with the GPS-only un-differenced scenario, which requires a minimum of five satellites for the solution to exist, the addition of Galileo or BeiDou satellites increases the redundancy by \( n_G - 1 \). In other words, we need a minimum of two satellites from any GNSS system in order to contribute to the solution.

3. Least-squares estimation technique

Under the assumption that the observations are uncorrelated and the errors are normally distributed with zero mean, the covariance matrix of the un-differenced observations takes the form of a diagonal matrix. The elements along the diagonal line represent the variances of the code and carrier phase measurements. In our solution, we consider the ratio between the standard deviation of
the code and carrier-phase measurements to be 100. The general linearized form for the above observation equations around the initial (approximate) vector \( \mathbf{u}^0 \) and observables \( \mathbf{l} \) can be written in a compact form as:

\[
\mathbf{f}(\mathbf{u}, \mathbf{l}) \approx \mathbf{A} \Delta \mathbf{u} - \mathbf{w} - \mathbf{r} \approx 0
\]  

(34)

where \( \mathbf{u} \) is the vector of unknown parameters; \( \mathbf{A} \) is the design matrix, which includes the partial derivatives of the observation equations with respect to the unknown parameters \( \mathbf{u} \); \( \Delta \mathbf{u} \) is the unknown vector of corrections to the approximate parameters \( \mathbf{u}^0 \), i.e. \( \mathbf{u} = \mathbf{u}^0 + \Delta \mathbf{u} \); \( \mathbf{w} \) is the misclosure vector and \( \mathbf{r} \) is the vector of residuals. The sequential least-squares solution for the unknown parameters \( \Delta \mathbf{u}_i \) at an epoch \( i \) can be obtained from Vanicek and Krakiwsky (1986):

\[
\Delta \mathbf{u}_i = \Delta \mathbf{u}_{i-1} + \mathbf{M}^{-1}_i \mathbf{A}_i^T \left( \mathbf{C}_i + \mathbf{A}_i \mathbf{M}^{-1}_i \mathbf{A}_i^T \right)^{-1} \left[ \mathbf{w}_i - \mathbf{A}_i \Delta \mathbf{u}_{i-1} \right]
\]  

(35)

\[
\mathbf{M}^{-1}_i = \mathbf{M}^{-1}_{i-1} - \mathbf{M}^{-1}_{i-1} \mathbf{A}_i^T \left( \mathbf{C}_i + \mathbf{A}_i \mathbf{M}^{-1}_i \mathbf{A}_i^T \right)^{-1} \mathbf{A}_i \mathbf{M}^{-1}_{i-1}
\]  

(36)

\[
\mathbf{C}_{\Delta \mathbf{u}_i} = \mathbf{M}^{-1}_i - \mathbf{M}^{-1}_{i-1} \mathbf{A}_i^T \left( \mathbf{C}_i + \mathbf{A}_i \mathbf{M}^{-1}_i \mathbf{A}_i^T \right)^{-1} \mathbf{A}_i \mathbf{M}^{-1}_{i-1}
\]  

(37)

where \( \Delta \mathbf{u}_{i-1} \) is the least-squares solution for the estimated parameters at epoch \( i-1 \); \( \mathbf{M} \) is the matrix of the normal equations; \( \mathbf{C} \) and \( \mathbf{C}_{\Delta \mathbf{u}_i} \) are the covariance matrices of the observations and unknown parameters, respectively. It should be pointed out that the usual batch least-squares adjustment should be used in the first epoch, i.e. for \( i = 1 \). The batch solution for the estimated parameters and the inverse of the normal equation matrix are given, respectively, by Vanicek and Krakiwsky (1986):

\[
\Delta \mathbf{u}_1 = \left( \mathbf{C}_1^{-1} + \mathbf{A}_1^T \mathbf{A}_1 \right)^{-1} \mathbf{A}_1^T \mathbf{w}_1
\]  

(38)

\[
\mathbf{M}_1^{-1} = \mathbf{C}_1^{-1} + \mathbf{A}_1^T \mathbf{A}_1
\]  

(39)

where \( \mathbf{C}_1 \) is a priori covariance matrix for the approximate values of the unknown parameters.

In case of the combined GPS/Galileo PPP model, the design matrix \( \mathbf{A} \) and the vector of corrections to the unknown parameters \( \Delta \mathbf{x} \) take the following forms:

\[
\mathbf{A} = \begin{bmatrix}
\begin{pmatrix}
\frac{x-x^0}{\lambda_i}
\frac{y-y^0}{\lambda_i}
\frac{z-z^0}{\lambda_i}
\end{pmatrix}
\begin{pmatrix}
\frac{x-x^0}{\lambda_i}
\frac{y-y^0}{\lambda_i}
\frac{z-z^0}{\lambda_i}
\end{pmatrix}
\begin{pmatrix}
\frac{x-x^0}{\lambda_i}
\frac{y-y^0}{\lambda_i}
\frac{z-z^0}{\lambda_i}
\end{pmatrix}
\vdots
\vdots
\vdots
\begin{pmatrix}
\frac{x-x^0}{\lambda_i}
\frac{y-y^0}{\lambda_i}
\frac{z-z^0}{\lambda_i}
\end{pmatrix}
\begin{pmatrix}
\frac{x-x^0}{\lambda_i}
\frac{y-y^0}{\lambda_i}
\frac{z-z^0}{\lambda_i}
\end{pmatrix}
\end{bmatrix}
\begin{pmatrix}
1 & m^1_x & 0 & 0 & \ldots & 0 & 0
1 & m^1_y & 0 & 1 & \ldots & 0 & 0
1 & m^1_z & 0 & 0 & \ldots & 1 & 0
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots
1 & m^6_x & 0 & 0 & \ldots & 0 & 0
1 & m^6_y & 0 & 1 & \ldots & 0 & 0
1 & m^6_z & 1 & 0 & \ldots & 0 & 0
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots
1 & m^6_x & 0 & 0 & \ldots & 1 & 0
1 & m^6_y & 0 & 0 & \ldots & 0 & 0
1 & m^6_z & 0 & 0 & \ldots & 0 & 0
\end{pmatrix}\begin{pmatrix}
\Delta x \\
\Delta y \\
\Delta z \\
c \ dt_x \\
rd w \\
\text{ISB}_w \\
N_{S_E} \\
N_{E} \\
N_{S} \\
N_{E} \\
N_{E} \\
N_{E} \\
N_{E}
\end{pmatrix}
\end{bmatrix}
\]  

(40)

where \( n_i \) refers to the number of visible GPS satellites; \( n_i \) refers to the number of visible Galileo satellites; \( n = n_{GPS} + n_{Galileo} \) is the total number of the observed satellites for both GPS/Galileo systems; \( X^0, Y^0, Z^0 \) and \( z_0 \) are the approximate receiver coordinates; \( X^i, Y^i, Z^i, j = 1, 2, \ldots, n_{GPS} \) are the known GPS satellite
coordinates; \( X^k, Y^k, Z^k, k = 1, 2, \ldots, n_E \) are the known Galileo satellite coordinates; \( \rho_0 \) is the approximate receiver–satellite range. The unknown parameters in the above system are the corrections to the receiver coordinates, \( \Delta x, \Delta y, \) and \( \Delta z, \) the wet component of the tropospheric zenith path delay \( \zeta_{dp}, \) the inter-system bias \( ISB, \) and the non-integer ambiguity parameters \( \tilde{N}. \)

In case of the combined GPS/BeiDou PPP model, the design matrix \( A \) and the vector of corrections to the unknown parameters \( \Delta x \) take the following forms:

\[
A = \begin{bmatrix}
\begin{array}{ccccccc}
\frac{x_i^g - x_i^g}{\lambda} & \frac{y_i^g - y_i^g}{\lambda} & \frac{z_i^g - z_i^g}{\lambda} & 1 & m_{1i}^g & 0 & 0 & 0 & 0 & 0 \\
\frac{x_i^g - x_i^g}{\lambda} & \frac{y_i^g - y_i^g}{\lambda} & \frac{z_i^g - z_i^g}{\lambda} & 1 & m_{1i}^g & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{x_i^g - x_i^g}{\lambda} & \frac{y_i^g - y_i^g}{\lambda} & \frac{z_i^g - z_i^g}{\lambda} & 1 & m_{1i}^g & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{x_i^g - x_i^g}{\lambda} & \frac{y_i^g - y_i^g}{\lambda} & \frac{z_i^g - z_i^g}{\lambda} & 1 & m_{1i}^g & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\end{bmatrix}
\]

\[
\Delta x = \begin{bmatrix}
\Delta x \\ \Delta y \\ \Delta z \\
\end{bmatrix}
\]

where \( n_g \) refers to the number of visible GPS satellites; \( n_b \) refers to the number of visible BeiDou satellites; \( n = n_g + n_b \) is the total number of the observed satellites for both GPS/BeiDou systems; \( x_i^g, y_i^g, \) and \( z_i^g \) are the approximate receiver coordinates; \( X^k, Y^k, Z^k, k = 1, 2, \ldots, n_E \) are the known GPS satellite coordinates; \( X^k, Y^k, Z^k, k = 1, 2, \ldots, n_E \) are the known BeiDou satellite coordinates; \( \rho_0 \) is the approximate receiver–satellite range. The unknown parameters in the above system are the corrections to the receiver coordinates, \( \Delta x, \Delta y, \) and \( \Delta z, \) the wet component of the tropospheric zenith path delay \( \zeta_{dp}, \) the inter-system bias \( ISB, \) and the non-integer ambiguity parameters \( \tilde{N}. \)

In case of the combined Galileo/BeiDou PPP model, the design matrix \( A \) and the vector of corrections to the unknown parameters \( \Delta x \) take the following forms:

\[
A = \begin{bmatrix}
\begin{array}{ccccccc}
\frac{x_i^g - x_i^g}{\lambda} & \frac{y_i^g - y_i^g}{\lambda} & \frac{z_i^g - z_i^g}{\lambda} & 1 & m_{1i}^g & 0 & 0 & 0 & 0 & 0 \\
\frac{x_i^g - x_i^g}{\lambda} & \frac{y_i^g - y_i^g}{\lambda} & \frac{z_i^g - z_i^g}{\lambda} & 1 & m_{1i}^g & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{x_i^g - x_i^g}{\lambda} & \frac{y_i^g - y_i^g}{\lambda} & \frac{z_i^g - z_i^g}{\lambda} & 1 & m_{1i}^g & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{x_i^g - x_i^g}{\lambda} & \frac{y_i^g - y_i^g}{\lambda} & \frac{z_i^g - z_i^g}{\lambda} & 1 & m_{1i}^g & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\end{bmatrix}
\]

\[
\Delta x = \begin{bmatrix}
\Delta x \\ \Delta y \\ \Delta z \\
\end{bmatrix}
\]

where \( n_g \) refers to the number of visible Galileo satellites; \( n_b \) refers to the number of visible BeiDou satellites; \( n = n_g + n_b \) is the total number of the observed satellites for both Galileo/BeiDou systems; \( x_i^g, y_i^g, \) and \( z_i^g \) are the approximate receiver coordinates; \( X^k, Y^k, Z^k, j = 1, 2, \ldots, n_E \) are the known
Galileo satellite coordinates; $X^k, Y^k, Z^k, k = 1, 2, \ldots, n_B$ are the known BeiDou satellite coordinates; $\rho_0$ is the approximate receiver-satellite range. The unknown parameters in the above system are the corrections to the receiver coordinates, $\Delta x, \Delta y,$ and $\Delta z$, the wet component of the tropospheric zenith path delay $zpd_w$, the inter-system bias $ISB_{EB}$, and the non-integer ambiguity parameters $N$.

4. Results and discussion

To verify the developed combined PPP models, GPS, Galileo, and BeiDou observations at four globally distributed stations (Figure 1) were selected from the IGS tracking network (Dow et al., 2009). Those stations are occupied by GNSS receivers, which are capable of simultaneously tracking the GNSS constellations. Only one hour of observations with maximum possible number of Galileo and BeiDou satellites of each data-set is considered in our analysis. All data-sets have an interval of 30 s.

The positioning results for stations DLF1 are presented below. Similar results are obtained for the other stations. However, a summary of the convergence times and the three-dimensional PPP solution standard deviations are presented below for all stations. Natural Resources Canada’s GPSpace PPP software was modified to handle data from GPS, Galileo, and BeiDou systems, which enables a combined PPP solution as detailed above (Afifi & El-Rabbany, 2016b). In addition to the combined PPP model, we also obtained the solutions of the un-differenced ionosphere-free GPS-only which is...
used to assess the performance of the newly developed PPP model. Figure 2 summarizes the satellite availability during the analysis time (one hour) for each system at DLF1 station.

As shown in Figure 2, the GPS system offers eight visible satellites for one hour, however by adding the Galileo system the number of visible satellites will be 13 satellites. In case of combining GPS and BeiDou the number of visible satellites will be 14 satellites, however by combining the three satellite systems the number of visible satellites will be 19 satellites. Figure 3 shows the positioning results in

Figure 3. The positioning results of the GPS-only PPP model.

Figure 4. The positioning results of the GPS/Galileo PPP model.

Figure 5. The positioning results of the GPS/BeiDou PPP model.
the East, North, and Up directions, respectively, for the GPS-only PPP model. As can be seen, the undifferenced GPS-only PPP solution indicates that the model is capable of obtaining a sub-decimeter-level accuracy. However, the solution takes about 20 min to converge to decimeter-level accuracy.

Figure 4 shows the positioning results combined GPS/Galileo PPP model. As shown in Figure 4, the positioning results of the combined GPS/Galileo traditional PPP model have a convergence time of 15 min to reach the decimeter-level of accuracy.

Figure 5 shows the combined GPS/BeiDou PPP model positioning results. As shown in Figure 5, the convergence time of the combined GPS/BeiDou PPP model is similar to the combined GPS/Galileo PPP model which is 15 min to reach the decimeter level of accuracy.

Figure 6 shows the combined Galileo/BeiDou PPP model positioning results. Similar to the previous combined PPP models, the convergence time of the combined Galileo/BeiDou PPP model is 15 min to reach the decimeter level of accuracy.

Figure 7 shows the combined GPS/Galileo/BeiDou PPP model positioning results. As shown in Figure 7, the convergence time of the combined GPS/Galileo/BeiDou PPP model has a convergence time of 15 min to reach the decimeter level of accuracy.

Figure 8 summarizes the convergence times for all combined PPP models, which confirm the PPP solution consistency at all stations.
To further assess the performance of the various PPP models, the solution output is sampled every 10 min and the standard deviation of the computed station coordinates is calculated for each sample. Figure 9 shows the position standard deviations in the East, North, and Up directions, respectively. Examining the standard deviations of the combined PPP models is almost comparable to the GPS-only PPP model. As the number of epochs, and consequently the number of measurements, increases, the performance of the various models tends to be comparable.

5. Conclusions
This paper presented a PPP model, which combines GPS/Galileo, GPS/BeiDou, Galileo/BeiDou, and GPS/Galileo/BeiDou observations in the un-differenced mode. The developed PPP model accounts for the combined effects of the different GNSS time offsets and hardware delays through the introduction of a new unknown parameter, the inter-system bias, in the PPP mathematical model. Four combinations are considered in the PPP modeling namely; GPS/Galileo, GPS/BeiDou, Galileo/BeiDou, and GPS/Galileo/BeiDou. All the combined PPP models results are compared with the GPS-only PPP model results. In the developed model GPS L1/L2, Galileo E1/E5a, and BeiDou B1/B2 are used in a dual-frequency ionosphere-free linear combination. It has been shown that the positioning results of the GPS-only and GPS/Galileo/BeiDou PPP are comparable and are at the sub-decimeter-level accuracy. However, the convergence time of the combined PPP models improved by about 25% in comparison with the GPS-only PPP.
Acknowledgments

This research was partially supported by the Natural Sciences and Engineering Research Council (NSERC) of Canada. The authors would like to thank the International GNSS service-Multi-GNSS Experiment (IGS-MEGX) network.

Funding

The authors received no direct funding for this research.

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Citation information

Cite this article as: Un-differenced precise point positioning model using triple GNSS constellations, Akram Afifi & Ahmed El-Rabbany, Cogent Geoscience (2016), 2: 1223899.

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