Multivariate Discrimination in Quantum Target Detection

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We describe a simple multivariate technique of likelihood ratios for improved discrimination of signal and background in multi-dimensional quantum target detection. The technique combines two independent variables, time difference and summed energy, of a photon pair from the spontaneous parametric down-conversion source into an optimal discriminant. The discriminant performance was studied in experimental data and in Monte-Carlo modelling with clear improvement shown compared to previous techniques. As novel detectors become available, we expect this type of multivariate analysis to become increasingly important in multi-dimensional quantum optics.

Non-classical correlation is at the heart of a range of quantum-enhanced technologies. In quantum optics, correlated photon pairs are routinely produced using the workhorse spontaneous parametric down-conversion (SPDC) sources. These sources have been used to generate pairs of photons that are correlated in almost every imaginable degree of freedom (DOF), including time, polarization, position-momentum, orbital angular momentum (OAM), or frequency. The polarization degree of freedom naturally lends itself to quantum information theory, indeed, much of the seminal work in the field employed polarization-entangled photon pairs. However, polarization is by nature only two dimensional so each photon can carry only a single bit of information. Other DOFs are in principle unbounded offering high-dimensional encoding. But, while these high-dimensional states offer great promise, measuring them efficiently remains a significant challenge. Traditional avalanche single photon detectors offer excellent temporal resolution but are single-mode so require scanning techniques to measure continuous DOFs. Alternatively, single photon sensitive cameras can be employed but they suffer from low frame rate making continuous readout with good temporal resolution impossible.

The Tpx3Cam is an optical camera based on a technology originating in the high-energy physics that has been adapted for optical detection by bonding a fast readout chip to an optical sensor. The resulting spatial resolution is comparable to intensified CCD or EMCCD cameras, but with a so-called data-driven readout, only pixels in which the readout exceeds a threshold are read out allowing continuous operation and efficient time stamping with nanosecond resolution. By appending an image intensifier, the Tpx3Cam can be made to detect single photons, bringing a paradigm shift in quantum imaging devices.

In a recent paper, we exploited the multidimensional capabilities of the Tpx3Cam to enhance the sensitivity of quantum target detection. Pairs of photons were generated by SPDC with one photon from each pair (the ‘herald’) measured locally and the other (the ‘signal’) sent to a target, which is hidden in a large amount of background light. After interacting with the target, correlations between the scattered signal photons and the herald photons are measured. This technique provides improved background rejection compared to simply measuring the back-scattered signal because the signal and herald modes are perfectly correlated, whereas the background is uncorrelated. By using the Tpx3Cam as a two-photon spectrometer, it was possible to simultaneously measure frequency and time correlations, significantly improving the signal-to-background ratio (SBR) compared to measuring time-only correlations as used in previous work.

In our recent work, the multi-variable correlations were analysed in a simple fashion with a temporal ‘coincidence window’ used to isolate pairs of photons that arrive with the correct time separation and then, subsequently, a spectral cut selected appropriate frequency correlations. These ‘box cuts’ are not optimal. By applying a multivariate, or combined, discriminant to our previous data we are able to show improved performance compared to the box-cut analysis. While optimal discrimination is widely used in particle physics and other fields to our knowledge, this is the first time it has been applied to quantum optics. As the Tpx3Cam, and other readout-driven cameras, become more prevalent in quantum optics we expect this type of analysis to become increasingly important beyond quantum target detection. Furthermore, it is simple to extend this analysis to higher dimensions, for example, to analyse multi-variable hyper-entangled states.

Here we use one of the most straightforward multivariate techniques, likelihood ratio, to combine the time difference and photon energy into a single discriminant. It can be shown that this combination is optimal, which means that the resulting discriminating variable provides the best possi-
The discriminant can be written as a product of different distributions for signal and background. For independent variables, the discriminant can be written as a product of

$$Y = \frac{f_B(x_1, \ldots, x_n)}{f_S(x_1, \ldots, x_n)} = \prod_{i=1}^n \frac{f_B(x_i)}{f_S(x_i)} = \prod_{i=1}^n Y_i,$$

where $Y_i$ is the ratio of probability density functions for signal, $f_S$, and background, $f_B$. The above procedure is very simple and generalizes to any number of discriminating variables.

The approach described above requires knowledge of the signal and background distributions for the variables which are used to form the discriminant. These distributions are measured experimentally and modeled using Monte-Carlo (MC) simulations. The MC simulations allow us to test the discriminant and evaluate performance in different regimes that were not investigated experimentally (see supplementary information).

The experimental setup used for the measurements is shown schematically in Figure 1 and is described in detail elsewhere. Briefly, an SPDC source is employed to produce pairs of photons with wavelength centered around 810 nm. One of the photons (signal) is sent onto a target and subsequently collected with a small telescope, while the other (herald) photon is sent directly to the camera. Before entering the fast camera the two photons are dispersed spectroscopically with a diffractive grating. The target is obscured by broadband ‘jamming’ light from a halogen lamp introduced from behind the target.

The fast camera, Tpx3Cam, is based on a Timepix3 chip with 1.5 ns timing resolution coupled to an optical sensor. The data obtained from the camera consists of $x, y$ position of hit pixels, $\text{ToF}$ (Time of Arrival) and $\text{ToT}$ (Time over Threshold) of the signal. The latter specifies deposited energy within the pixel. In order to achieve single photon sensitivity, an intensifier is employed, converting single photons to flashes of light, which are registered by the camera. The Hi-QE Red intensifier from Photonis has a quantum efficiency of about 20% at 810 nm and employs P47 fast scintillator, which has timing performance compatible with nanosecond scale resolution. All 256 \times 256 pixels of the camera function independently with low dead-time and can be read out with a maximum total rate of about 10M photons per second.

The raw data is post-processed to identify ‘clusters’, collections of pixels each corresponding to a single photon, and to perform centroiding. The centroiding improves the signal resolution using a profile of the deposited energy in the cluster. We also apply a ToT-based correction to remove the time-walk effect in ToA and to further improve the time resolution. The post-processing steps are discussed in details elsewhere.

The inset of Figure 1 shows the measured data as a two-dimensional distribution of pixel occupancy of the camera data. The signal and herald modes after the diffractive grating appear as two horizontal stripes while the uniform background is mostly due to the intensifier dark counts and remaining stray light. In the spectrometer the photon wavelength has a linear relationship to the position along the stripe which can be derived by a simple calibration procedure. The downconversion process in the crystal requires conservation of energy and, therefore,

$$\frac{hc}{\lambda_p} = \frac{hc}{\lambda_h} + \frac{hc}{\lambda_o},$$

implying

$$\lambda_o = \frac{\lambda_h \lambda_p}{\lambda_h - \lambda_p},$$

where $\lambda_p$ is the wavelength of the pump photon from the laser, 405 nm, and $\lambda_{h(s)}$ is the wavelength of the herald (signal) photon. The spectral resolution is different for the herald and signal photons due to different style of multi-mode fibers used for their collection and is measured to be 1.6 and 3.2 pixels respectively for the herald and signal photons. The pump laser has a full-width half maximum linewidth of $\Delta \lambda_p = 0.6 \text{ nm}$.

To get the number of time coincidences for the photon pairs, we employed a previously used algorithm. The data are selected according to the regions of interest, the two stripes, and for every event in one stripe, an event with the smallest $\Delta t \equiv |\text{ToA}_1 - \text{ToA}_2|$ is found in the other stripe. The two-dimensional distribution of the sum energy and time difference of the photon pairs in the data is shown in Figure 2(a). The sum energy, expressed through the pump photon wavelength, can be described by a normal distribution of width 0.36 nm due to a combination of pump laser linewidth and spectrometer resolution. The time coincidence peak is also a normal distribution of width 7.55 ns due to the temporal resolution of the camera.

To study the signal and background separation using the likelihood ratio discriminant we developed a MC model corresponding to experimental conditions such as signal and background resolutions and rates, including various inefficiencies of the whole system. More details regarding the model and its matching to the dataset are described in the supplementary material.

Next we applied the aforementioned coincidence algorithm to find pairs of photons. It blindly processes the MC sample.
and determines which events are paired based on the closest ToA. We can then plot one-dimensional histograms of the time difference $\Delta T$ distribution and the sum energy, represented by pump energy $\lambda_p$, as plotted in Figure 3(a) and (b), respectively. The MC simulations are in good agreement with the measured data. Note that each photon’s origin was tagged in the MC to track them when forming photon pairs in the coincidence algorithm. This allows us to unambiguously find true signal coincidence events (brown stars in Figure 3), and identify different types of background events (green diamonds). This is a useful feature of the MC simulation which is unavailable in experimental data. Figure 3 also illustrates very well that, before any selections are made, the signal to background ratio (SBR) is very poor.

The same data can be plotted in the two-dimensional representation shown in Figure 2 for both data and MC. The bright spot in the centre of the plot is due to true coincidences between photons produced in a pair which are highly correlated in time and anti-correlated in wavelength. The background is due to uncorrelated events such as photon-background or background-background coincidences. It is easy to see that signal-to-background contrast is far higher in the 2D representation than in either of the 1D histograms. Indeed, Figures 2 and 3 are a good visual representation of the difference between the ‘box-cuts’ applied in our previous work, and the combined discriminant employed here. In the previous work, a region of interest was defined first in one degree of freedom (time), and then the other (energy) which is equivalent to selecting the peaks in the one dimensional histograms. Here, instead, the combined discriminant combines both variables when defining a region of interest effectively selecting an ellipse around the peak of the two-dimensional histogram. This idea is explored more rigorously below.

Above, we studied in detail two discriminating variables, one derived from the temporal measurements and the other one derived from the spectroscopic measurements. We emphasize that this information is available on the pair by pair basis and, therefore, can be combined individually for each registered pair. This is a novelty of this work where we employed the fast camera which is recording simultaneously the coordinate and temporal information for each photon.

The combined discriminant that we define is a function with two inputs: the photon pair time difference $\Delta T$ and wavelength of the reconstructed pump photon $\lambda_p$. To combine $\Delta T$ and $\lambda_p$, we start by defining background to signal ratios for these two variables, $Y_{\lambda_p}$ and $Y_{\Delta T}$, as

$$Y_{\lambda_p}(\lambda_p) = \frac{A(\lambda_p - \lambda_{b0}) + B}{2\pi \sigma_{\lambda_p}} \exp \left(-\frac{(\lambda_p - \lambda_{b0})^2}{2\sigma_{\lambda_p}^2}\right),$$

$$Y_{\Delta T}(\Delta T) = \frac{C\exp \left(-\frac{\Delta T^2}{2\sigma_{\Delta T}^2}\right)}{2\pi \sigma_{\Delta T}} \exp \left(-\frac{(\Delta T - \Delta T_{b0})^2}{2\sigma_{\Delta T}^2}\right).$$

Parameters of the functions: $A$, $B$, $C$, $N$, experimental resolutions $\sigma_{\lambda_p}$, $\sigma_{\Delta T}$, and offset values $\lambda_{b0}$, $\lambda_{p0}$, $\Delta T_{b0}$ used in (3) and (4) were obtained from the fits to the data. Combining the ratios $Y_{\lambda_p}$ and $Y_{\Delta T}$ according to (1),

$$Y(\lambda_p, \Delta T) \equiv Y(Y_{\lambda_p}, Y_{\Delta T}) = Y_{\lambda_p} \cdot Y_{\Delta T}$$

yields the two-dimensional likelihood ratio function $Y$ with the result shown in Figure 4 with a deep, well-defined minimum of the function in the center.

The selection criteria to discriminate the signal from background will be determined by $Y$-isolines. All events inside the area surrounded by an isoline would correspond to a dataset with optimum separation of signal and background. This selection, which can be chosen according to the experiment needs, would correspond to a certain value of signal efficiency $\eta$, defined as the fraction of selected signal counts $s$ and total signal $\Sigma$, and sample purity $p$ defined as

$$p \equiv \frac{s}{s+b} = \frac{SBR}{1+B},$$

FIG. 2: Two-dimensional distribution of spectroscopic and temporal variables for (a) data and (b) MC simulation.

FIG. 3: (a) Time difference $\Delta T$ distribution of photon pairs. Simulated data fit is obtained by Gaussian fit of signal-signal and exponential fit of all backgrounds. (b) Pump photon wavelength $\lambda_p$ distribution of the pairs. Simulated data fit is obtained by Gaussian fit of signal-herald and linear fit of all backgrounds.

FIG. 4: Two-dimensional likelihood ratio $Y$ for both time difference and pump photon wavelength.
where \( b \) is the selected background counts.

We tested the discriminating power of this newly obtained variable \( Y \) by comparing it to \( Y_{\lambda} \) and \( Y_{\text{AT}} \) performances on their own. We also analysed the MC data using simple box-cuts where the temporal cut was fixed at \( \pm 10 \) ns and the spectral cut width was varied, as in reference [19].

In Figure 5, the sample purity and signal efficiency are plotted for various selection parameters using different techniques: sum energy-only, time difference only, energy and time cuts, and combined discriminant. In each case, as the selected region becomes smaller, the sample purity increases as more background is eliminated. However, if the region becomes too small, then true coincidences are also rejected which reduces the signal efficiency. Therefore, there is a trade-off between efficiency and sample purity, which can be used to optimise different aspects of the data analysis. It is also clear that the performance is vastly superior for the multivariate techniques compared to the single-variable approaches, and that the combined discriminant outperforms the box-cut method. According to the MC simulations, for a constant signal efficiency \( \eta = 0.80 \) the sample purities achieved with different methods of combined discrimination are 0.63 and 0.56, improving the SBR by 26% for the optimal discriminant. For comparison, we also apply a similar analysis to the experimental data, shown with dotted lines in Figure 5. The MC/data agreement is within errors, with the same functional form.

In summary, we employed a novel fast camera, Tpx3Cam, in the context of quantum target detection and considered an optimal discriminant based on the likelihood ratios for two measured variables, energy and time. We achieved a 26% improvement of SBR for the same signal efficiency compared to the previously used selections.

We believe this multivariate approach is a promising venue to analyse quantum sensing protocols using correlated photon pairs and, in general, high-dimensional quantum states. Finally, another opportunity for the future work is to extend the multivariate analysis to determine distance to the target using all available information in the data.

See supplementary material for more information on the MC model and on the performance predictions for different resolutions and background rates.

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FIG. 5: Signal efficiency \( \eta_s \) (ratio of signal counts to the total number of signal events) plotted as a function of sample purity \( p \) for four cases: discriminants based on time difference, based on spectral information and based on the combined discriminant; and discriminant based on traditional box cuts, see the text. Solid lines are MC results, dotted lines are derived from the experimental data with errors shown as color bands.
