The Geometrical Meaning of Time -
Some Cosmological Implications

Asher Yahalom
Ariel University, Ariel 40700, Israel
E-mail: asya@ariel.ac.il

Abstract. It is stated in many text books that the any metric appearing in general relativity
should be locally Lorentzian i.e. of the type $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ this is usually presented
as an independent axiom of the theory, which cannot be deduced from other assumptions. The
meaning of this assertion is that a specific coordinate (the temporal coordinate) is given a unique
significance with respect to the other spatial coordinates. It was shown that the above assertion
is a consequence of requirement that the metric of empty space should be linearly stable and
need not be assumed. Some cosmological implications of the above result will be suggested.

1. Introduction
It is well known that our daily space-time is approximately of Lorentz (Minkowski) type that
is, it possesses the metric $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$. The above statement is taken as one of
the central assumptions of the theory of special relativity and has been supported by numerous
experiments. But why should it be so?

Furthermore, many textbooks [2] state that in the general theory of relativity any space-
time is locally of the type $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$, although it can not be presented so
globally due to the effect of matter. This is a part of the demands dictated by the well known
equivalence principle. The above principle is taken to be one of the assumptions of general
relativity other assumption such as diffeomorphism invariance, and the requirement that theory
reduce to Newtonian gravity in the proper regime lead to the Einstein equations:

$$G_{\mu\nu} = -\frac{8\pi G}{c^4}T_{\mu\nu}$$

in which $G_{\mu\nu}$ is the Einstein tensor, $T_{\mu\nu}$ is the stress-energy tensor, $G$ is the gravitational
constant and $c$ is the velocity of light.

The Principle of Equivalence rests on the equality of gravitational and inertial mass,
demonstrated by Galileo, Huygens, Newton, Bessel, and Eötvös. Einstein reflected that, as
a consequence, no external static homogeneous gravitational field could be detected in a freely
falling elevator, for the observers, their test bodies, and the elevator itself would respond to the
field with the same acceleration [2]. This means that the observer will experience himself as
free, not feeling the effect of any force at all. Mathematically speaking for the observer space
time is locally (but not globally) flat and Minkowskian.

It was shown by several authors that one need not assume that space-time is locally
Minkowskian based on an empirical (unexplained) fact, rather this can be justified theoretically.
Greensite [5] and Carlini & Greensite [6, 7] have studied the metric \( \eta_{\mu\nu} = \text{diag} (e^{i\theta}, -1, -1, -1) \) in which \( \theta \) the "Wick angle" was treated as a quantum field dynamical variable. They have shown that the real part of the quantum field effective potential is minimized for the Lorentzian metric \( \theta = 0 \) and for the same case the imaginary part of the quantum field effective potential is stationary. Furthermore they have calculated the fluctuations around this minimal value and have shown them to be of the order \( (l_p R)^3 \) in which \( l_p \) is the Planck length and \( R \) is the scale of the universe. Elizalde & collaborators [8] have shown that the same arguments apply to a five dimensional Kaluza-Klein universe of the type \( R^4 \times T^1 \).

Itin & Hehl [9] have deduced that space time must have a Lorentzian metric in order to support classical electric/magnetic reciprocity.

H. van Dam & Y. Jack Ng [10] have argued that in the absence of a Lorentzian metric one cannot obtain an appropriate finite representation of the relevant groups and hence the various quantum wave equations can not be written.

What is common to the above approaches is that additional theoretical structures & assumptions are needed in order to justify what appears to be a fundamental property of space-time. In previous works [1, 11, 12] it was claimed otherwise. It was shown that General relativistic equations and some "old fashioned" linear stability analysis will lead to a unique choice of the Lorentzian metric being the only one which is linearly stable for empty space. Other metrics are also allowed for empty space but are unstable and thus can exist in only a limited region of space-time. It should be mentioned that the choice of coordinates in the Fisher approach to physics can be also be justified using the stability approach [13]. The nonlinear stability question of the Lorentzian metric was settled by D. Christodoulou & S. Klainerman [14].

This paper assumes that space time must have four dimensions, it does not explain why this is so. For a possible explanation derived from string theory one can consult a paper by S. K. Rama [15]. The problem of the "arrow of time" that is time directionality was thought to be explained by Boltzman’s H theorem. However, this explanation is thought to be wrong by some researchers who argue that this theorem cannot provide the required result unless one assumes low entropy at the beginning of the universe, for a popular account see a book by S. Carroll [16]. We will not discuss time directionality here.

For non empty space-time the situation can be drastically different. The existence of the intuitive partition of 4 dimensional space into "spatial" space and "temporal" time, is a feature of an almost empty space-time. This does not contradict the fact that such a partition can not be demonstrated in general solutions of equation (1) such as the one discovered by Gödel [26]. But this problem is not a characteristic of exotic space-times rather it is a property of standard cosmological models.

Standard Cosmology has many fundamental problems those include the horizon problem, the flatness problem, the entropy problem and the monopole problem [17]. A possible solution to those problems were suggested by Alan Guth using his famous inflation theory [18]. Entropy problems which plagued the original inflation model has led to a new inflation model suggested by Linde [19] which solve the entropy problem but required fine tuning of parameters. The same criticism holds for chaotic inflation also suggest by Linde [20]. On 17 March 2014, astrophysicists of the BICEP2 collaboration announced the detection of inflationary gravitational waves in the B-mode power spectrum, which if confirmed, would provide clear experimental evidence for the theory of inflation [21]. However, on 19 June 2014, lowered confidence in confirming the findings was reported.

It is the opinion of the author of this paper that a basic flaw in common to all inflation models. All inflation models require to postulate one or more scalar fields which have no function, implication or purpose in nature except for their ad-hoc use in the inflation model. This is in sharp contradiction with the principle of Occam’s razor which demand that a minimum number
of assumptions will explain a maximum number of phenomena. Postulating a physical field for every phenomena does not serve the purpose of theoretical science. Moreover, it will be shown that a perfectly good explanation within the frame-work of standard Cosmology does exist for the horizon problem if one looks closely at the metric changes of the Friedman-Lemaitre-Robertson-Walker metric.

The plan of this paper is as follows: First we look at non empty space-times and see how generic solutions of the Einstein equations in the presence of matter change signature. Then we will discuss some possible implications of our results, in particular the conditions under which a particle can travel in speeds exceeding the speed of light. Finally some cosmological implications and conclusions would be given.

2. Mechanisms Of Signature Change

It was shown that among the possible flat space metrics only the Lorentzian metric is stable in empty space [1]. Nevertheless one may still inquire if a mechanism exists by which a metric change does occur\(^1\), can we create somehow a metric of the type \(g_{\mu\nu} = \text{diag} (+1, +1, -1, -1)\) in some region of space-time? The answer obviously has to do with the only reason a metric should change according to equation (1) and this is \(T_{\mu\nu}\). Looking at available solution of general relativity one finds that metric changes are quite common.

The Schwarzschild square interval (in terms of spherical coordinates \(t, r, \theta, \phi\)) is given by:

\[
c^2 d\tau^2 = (1 - \frac{r_s}{r})c^2 dt^2 - \frac{dr^2}{1 - \frac{r_s}{r}} - r^2(d\theta^2 + \sin^2 \theta d\phi^2)
\]

In which \(\tau\) is the proper time, and \(r_s\) is the Schwarzschild radius (in meters) of the massive body, which is related to its mass \(M\) by \(r_s = \frac{2GM}{c^2}\). It is obvious that while for \(r > r_s\) the metric is locally (up to scaling) \(g_{\mu\nu} = \text{diag} (+1, +1, -1, -1)\). For \(r < r_s\) the metric is locally (up to scaling) \(g_{\mu\nu} = \text{diag} (-1, +1, -1, -1)\). Hence the direction of temporal and (one) spatial axis is exchanged. Notice, however, that although the sign of the eigen-values did change we are still left with a Lorentzian metric.

Another example is the Friedman-Lemaitre-Robertson-Walker square interval which is well known in cosmological models:

\[
c^2 d\tau^2 = c^2 dt^2 - a(t)^2 \left( \frac{dr^2}{1 - \kappa r^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right)
\]

\(a(t)\) is known as the “scale factor” and \(\kappa\) may be taken to have units of length\(^{-2}\), in which case \(r\) has units of length and \(a(t)\) is unit less. \(\kappa\) is then the Gaussian curvature of the space at the time when \(a(t) = 1\). Hence for radial distances such that \(r < \frac{1}{\sqrt{\kappa}}\) the metric is locally (up to scaling) \(g_{\mu\nu} = \text{diag} (+1, -1, -1, -1)\) that is Lorentzian. However, for \(r > \frac{1}{\sqrt{\kappa}}\) the metric is locally (up to scaling) \(g_{\mu\nu} = \text{diag} (+1, +1, -1, -1)\). This means that a particle propagating in a radial direction will experience an Euclidean metric. Of course the critical radius \(\frac{1}{\sqrt{\kappa}}\) only exists for closed universes in which \(\kappa > 0\). Flat universes with \(\kappa = 0\) or open universes with \(\kappa < 0\) do not have such a critical radius.

One should notice that in the above cases a signature change is accompanied by a metric singularity while the signature changes considered by Eddington [4] involve zeros. However, metric singularities are not curvature singularities and can be removed by proper choice of coordinates.

\(^1\) In the sense that the eigen-values of the metric change signs.
3. Implications For Particle Trajectories

A particle travelling in a space-time with a constant metric can be described by the Action $\mathcal{A}$ and Lagrangian $L$:

$$\mathcal{A} = \int L d\tau, \quad L = \frac{1}{2}m u_\alpha u^\alpha + \frac{q}{c} u_\alpha A^\alpha$$

In the above $\tau$ is some parameter along the trajectory, $x_\alpha$ are the particle coordinates, $u_\alpha \equiv \frac{dx_\alpha}{d\tau}$, $m$ is the particle mass, $q$ is the particle charge and $A_\alpha$ are some functions of the particle coordinates (that transform as a four dimensional vector). Basic variational analysis leads to the following equations of motion:

$$m \frac{du^\alpha}{d\tau} = -\frac{q}{c} u^\beta (\partial_\beta A^\alpha - \partial^\alpha A_\beta)$$

It is customary to use as a parameter the length of the trajectory:

$$d\tau^2 = |\eta_{\alpha\beta} dx^\alpha dx^\beta|$$

The analysis of trajectories in different flat metrics is described in [22] and will not be repeated here due to space limitations.

The main results are as follows: a subluminal particle in a Lorentz space-time will remain subluminal for ever (as is well known). A superluminal particle in a Lorentz space-time will remain superluminal for ever. A particle in an Euclidean space-time can attain any velocity.

4. Possible Cosmological Implications

One obvious physical implication of the previous analysis is that a particle can be accelerated to a velocity close to the velocity $c$ in a Lorentz space, enter into an Euclidean space and be accelerated further in this region to velocities above the speed $c$ and emerge in a Lorentz space in which it will remain above the speed $c$ for ever unless it is decelerated in an Euclidean space again.

This certainly may happen to a particle which travels radially in a Friedman-Lemaître-Robertson-Walker metric passing outwards the critical radius of $r_c = \frac{1}{\sqrt{\kappa}}$ and then coming back at superluminal velocities.

But if such particles do exist how would their existence bear on existing physical and astrophysical problems?

An obvious implication has to do with the homogeneity (horizon) problem. According to an analysis given by Narlikar [17] a proper radius for a particle horizon of a sub luminal particle at the radiation dominated epoch was $R_L = 2ct$, taking into account temperatures of the early universe led him to conclude that this radius was of the order of magnitude of about 1 meter on present day scales whether in reality the cosmological micro wave background is homogeneous on a scale of $10^{26}$ meters. Notice, however, that superluminal particles are not restricted by the velocity of light and hence can bring a very young universe into thermal equilibrium. Of course a more popular mechanism for achieving this is inflation [24]. However, one should notice that a Higgs type fields do not give the correct density perturbation spectrum [24], hence one is forced to postulate a new field which is not a part of any particle model and thus is a possible but inelegant solution of the homogeneity problem. Alternatively one can speculate that homogeneity is achieved by ordinary matter which can become superluminal as the current analysis shows.

5. Conclusions

Mathematically speaking one of the main differences between time and space is encapsulated in the flat metric of our space-time which is locally of the Lorentzian type $\eta_{\mu \nu} = \text{diag}(1,-1,-1,-1)$. But this is an empirical fact or a mathematical postulate thus unexplained. One
can imagine also other flat metrics such as the Euclidian metric: \( \text{diag} (1,1,1,1) \). In Euclidian metrics there is no restriction on the speed of any moving body as the speed of light restricts the speed of propagation only in the presence of a Lorentzian metric. Why is our space-time Lorentzian and not Euclidean? The answer is that only the Lorentzian metric is stable [1] for an (almost) empty space. But space-time is not empty and thus the notion of time always progressing forward with the increase of entropy is probably just a consequence of the scales of reality that we are exposed to. In huge cosmological scale the Friedman-Lemaitre-Robertson-Walker universe loses its Lorentzian character. The horizon problem related to the homogeneity of cosmic microwave background can be solved if one takes into account the superluminal motion of particles for \( r > r_c \) and the same particles moving into the \( r < r_c \) Lorentzian domain. Another scenario that one needs to take into account are particles created such that their initial velocities are superluminal. Statistical physics of such particles is needed and hopefully will be developed in the future.

References
[1] Yahalom, A. (2008). The Geometrical Meaning of Time, Foundations of Physics, Volume 38, Number 6, Pages 489-497.
[2] Weinberg, S. (1972). Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity, John Wiley & Sons, Inc.
[3] Misner, C. W. Thorne, K.S. & Wheeler J.A.(1973). Gravitation W.H. Freeman & Company.
[4] Eddington, A. S. (1923). The mathematical theory of relativity, Cambridge University Press.
[5] Greensite, J. (1992). Los Alamos Archive gr-qc/9210008.
[6] Carlini, A. & Greensite, J. (1993). Los Alamos Archive gr-qc/9308012.
[7] Carlini, A. & Greensite, J. (1994). Phys. Rev. D, Volume 49, Number 2.
[8] Elizalde, E., Odintsov, S. D. & Romeo, A. (1994). Class. Quantum Grav. 11 L61-L67.
[9] Itin, Y. & Hehl, F. W. (2004). Los Alamos Archive gr-qc/0401016.
[10] van Dam, H. & Jack, Y. (2001). Los Alamos Archive hep-th/0108067.
[11] Yahalom, A. (2009). The Gravitational Origin of the Distinction between Space and Time, International Journal of Modern Physics D, Vol. 18, Issue: 14, pp. 2155-2158.
[12] Yahalom, A. (2011). Advances in Classical Field Theory, Chapter 6, Bentham eBooks eISBN: 978-1-60805-195-3.
[13] Yahalom, A. (2010). Gravity and the Complexity of Coordinates in Fisher Information, International Journal of Modern Physics D, Vol. 19, No. 14 1-5, World Scientific Publishing Company.
[14] Christodoulou, D. & Klainerman, S. (1989-1990). The global nonlinear stability of the Minkowski space, Seminaire Equations aux derives partielles (dit "Goulaouic-Schwartz"), Exp. No. 13, p. 29.
[15] Kalyana Rama, S. (2006). Los Alamos Archive hep-th/0610071 v2.
[16] Carroll, S. (2010). From Eternity to Here: The Quest for the Ultimate Theory of Time, A Plume Book.
[17] Narlikar, J. V. (1993). Introduction to Cosmology, Cambridge University Press.
[18] Guth, A. H. (1981). Inflationary universe: A possible solution to the horizon and flatness problems, Phys. Rev., D23, 347 (1981).
[19] Linde, A. (1982). A new inflationary universe scenario, Phys. Lett., B108, 389.
[20] Linde, A. (1983). Chaotic inflation, Phys. Lett., B129, 177.
[21] Ade, P. A. R. et al. (BICEP2 Collaboration) (2014). Detection of B-Mode Polarization at Degree Angular Scales by BICEP2, Phys. Rev. Lett. 112, 241101 Published 19 June 2014.
[22] Yahalom, A. (2013). Gravity and Faster than Light Particles, Journal of Modern Physics (JMP), Vol. 4 No. 10 PP. 1412-1416 DOI: 10.4236/jmp.2013.410169.
[23] Weinberg, S. (1995). The Quantum Theory of Fields, Cambridge University Press.
[24] Guth, A. H. (1995). Starting the universe: the Big Bang and cosmic inflation, p. 105 in Bubbles, voids and bumps in time: the new cosmology, Edited by James Cornell, Cambridge University Press.
[25] Rubin, V. C. (1995). "Weighting the universe: dark matter and missing mass" p. 73 in Bubbles, voids and bumps in time: the new cosmology, Edited by James Cornell, Cambridge University Press.
[26] Yourgrau, P. (2006). A World Without Time, Basic Books.