Understanding Catastrophic Forgetting and Remembering in Continual Learning with Optimal Relevance Mapping

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Abstract
Catastrophic forgetting in neural networks is a significant problem for continual learning. A majority of the current methods replay previous data during training, which violates the constraints of an ideal continual learning system. Additionally, current approaches that deal with forgetting ignore the problem of catastrophic remembering, i.e. the worsening ability to discriminate between data from different tasks. In our work, we introduce Relevance Mapping Networks (RMNs) which are inspired by the Optimal Overlap Hypothesis. The mappings reflect the relevance of the weights for the task at hand by assigning large weights to essential parameters. We show that RMNs learn an optimized representational overlap that overcomes the twin problem of catastrophic forgetting and remembering. Our approach achieves state-of-the-art performance across all common continual learning datasets, even significantly outperforming data replay methods while not violating the constraints for an ideal continual learning system. Moreover, RMNs retain the ability to detect data from new tasks in an unsupervised manner, thus proving their resilience against catastrophic remembering.

1. Introduction
Continual learning refers to a learning paradigm where different data and tasks are presented to the model in a sequential manner, akin to what humans usually encounter. But, unlike humans or animal learning, which is largely incremental and sequential in nature, artificial neural networks (ANNs) prefer learning in a more concurrent way and have been shown to forget catastrophically. The term catastrophic forgetting (CF) in neural networks is usually used to define the inability of ANNs to retain old information in the presence of new one.

Continual Learning (CL) in Neural Networks. The widely understood formulation of continual learning refers to a learning paradigm where ANNs are trained strictly sequentially on different data and tasks (Chen & Liu, 2018; Mundt et al., 2020). The important conditions of the training paradigm are:

1. Sequential Training i.e. for a single neural network $f$ with parameters $\theta$ trained at time $T$ with sequentially available data $D_1 \ldots N$, $T_1[f_\theta(D_1)] < T_2[f_\theta(D_2)] < \ldots < T_N[f_\theta(D_N)]$
2. No negative exemplars, examples or feedback i.e. future (or past) data samples cannot be provided to the network with the current data/task. $(D_1 \cap D_T) \cup \ldots \cup (D_{T-1} \cap D_T) \cup (D_T \cap D_{T+1}) \cup (D_T \cap D_{T+2}) \ldots \cup (D_T \cap D_{T+N}) = \emptyset$

Despite this formulation, since (Robins, 1993) showed the promise of memory replay methods in dealing with CF and the prevalence of cognitive/neuro science inspired theories regarding memory, it is of no surprise that rehearsal/replay buffer/generative replay methods dominate the current state of the art (SOTA) benchmarks (Titsias et al., 2020; Kemker & Kanan, 2017; Pan et al., 2021; Lee et al., 2020). However, they clearly violate the conditions of the CL paradigm. Additionally, some most prominent current CL methods (Serr et al., 2018; Lee et al., 2020; Guo et al., 2020; Yoo et al., 2020) change the ANN altogether by adding new convolutional/linear layers for each task (for e.g. using multi heads -different last linear layer for each task- has become a common practice) or using a mixture of ANNs which violates the above conditions as well since we are not training the same ANN $f_\theta$ anymore. In contrast, we aim to develop a learning paradigm that strictly obeys the formulation of CL without applying data replay or introducing new sets of additional ANN models or convolutional/linear layers during training and inference.

Catastrophic Forgetting is a direct implication of continual learning in ANNs and is largely considered a direct consequence of the overlap of distributed representations in the network. Most prior works deal with CF by either completely removing the representational overlap (French, 1991; Kirkpatrick et al., 2017) or more frequently, by replaying data from previous tasks. Data replay methods can deal
with CF but, in turn, lead to a reduced capability of the network to discriminate between old and new inputs (Sharkey & Sharkey, 1995b). This is referred to as Catastrophic Remembering (CR) (Refer to Section 3.1 for a detailed discussion) and has been shown to be a significant limitation of replay methods (Robins, 1993; Sharkey & Sharkey, 1995b).

The goal of this work is that we attempt to develop a method for continual learning for deep neural networks which can alleviate the twin problem of Catastrophic Forgetting and Catastrophic Remembering at the same time, without violating or relaxing the conditions of a strict continual learning framework.

Our proposed approach builds on the following Optimal Overlap Hypothesis: For a strictly continually trained deep neural network, catastrophic forgetting and remembering can be minimized, without additional memory or data, by learning optimal representational overlap, such that the representational overlap is reduced for unrelated tasks and increased for tasks that are similar.

More formally, for an ANN $f_{\Theta}(D)$ with parameter $\Theta$ and sequentially available data $D_i$ over number of tasks/data $i \in [1, T]$ instead of trying to enforce over-generalization which is to learn a superset parameter space which encompasses the sequential tasks $\{\theta_i | \cup \theta_1...T = \Theta \cap \cap \theta_1...T = \emptyset\}$ or complete separation of weight space $\{\theta_i | \cup \theta_i \subseteq \Theta\}$, we try and learn optimal overlaps amongst the sequentially learned parameter sets. That means $\{\forall i, j \in T \exists i \neq j | \theta_i \cap \theta_j = A \cup \theta_i = \Theta\}$ where $A \in [0, \theta_{i/j}]$.

Inspired by this hypothesis, we propose Relevance Mapping for continual learning to alleviate CF and CR. During the continual learning process, our method learns the neural network parameters and a task-based relevance mask on the hidden layer representation concurrently. The almost-binary relevance mask keeps a portion of the neural network weights static and hence is able to maintain the knowledge acquired from previous tasks, while the rest of the network adapts to the new task. Our experiments demonstrate that Relevance Mapping Networks outperform all related works by a wide margin on many popular continual learning benchmarks (Permuted MNIST, Split MNIST, Split Omniglot, Split CIFAR-100), hence alleviating catastrophic forgetting without relaxing or violating the conditions of a strict continual learning framework. Moreover, we demonstrate that Relevance Mapping Networks are able to detect new sequential tasks in an unsupervised manner with high accuracy, hence alleviating catastrophic remembering.

In summary, our contributions are:

- We introduce Relevance Mapping Networks which learns binary relevance mappings on the weights of the neural network concurrently to every task. We demonstrate that our model efficiently deal with the twin problem of catastrophic forgetting and remembering.

- Our method achieves SOTA results on all popular continual learning benchmarks without relaxing the conditions of a strict continual learning framework.

- We re-introduce the concept of Catastrophic Remembering for deep neural networks and show that our method is capable of dealing with the same (becoming the first modern methodology to elevate catastrophic forgetting and remembering concurrently).

2. Related Work

Continual Learning. Current continual learning mechanisms dealing with CF are broadly classified into regularization approaches, dynamic architecture, complementary learning systems and replay architectures (Parisi et al., 2018). Primarily based on the Stability-Plasticity Dilemma (Mermillod et al., 2013) concept, regularization approaches impose constraints on weight updates to alleviate catastrophic forgetting like Elastic Weight Consolidation (EWC) (Kirkpatrick et al., 2017) and Learning Without Forgetting (Li & Hoiem, 2018). These methods do not ordinarily violate the conditions of the CL framework but have been shown to suffer from brittleness due to representational drift (Titsias et al., 2020; Kemker et al., 2017) and thus are usually combined with other methods. Rehearsal/replay buffer methods, like (Titsias et al., 2020) which are the state-of-the-art methods, use a memory store of past observations to remember previous tasks in order to alleviate the brittleness problem. However, these are not representative of strict sequential learning insofar that they still require re-learning of old data to some extent and perform significantly worse the less samples are replayed, and they may struggle to represent uncertainty about unknown functions.

There are no known methods which deal with CR in continual learning framework, with our method being the first of its kind to be able to combat catastrophic forgetting and remembering in a strict continual learning framework.

Catastrophic Remembering refers to the tendency of artificial neural networks to abruptly lose the ability to discriminate between old and new data/task during sequential learning. It is an important problem and inherently attached to the problem of catastrophic forgetting. But, unlike the problem of catastrophic forgetting, which has a rich literature of research, catastrophic remembering has not been explored outside of minor discussions in early works (Sharkey & Sharkey, 1995a; Lewandowsky & Li, 1995; French, 1991). In this work, we discuss CR from a probabilistic perspective (Section 3.1) and demonstrate that related work suffers from
CR in our experiments in Section 5.2. Finally, we demonstrate that our proposed Relevance Masking Networks are much more resilient to catastrophic remembering.

**Similar Methods.** The idea of using soft-masking in networks (usually on non-linear activations) has been utilized before in novel ways for solving different problems. However, few of them, if any, ground these methods in some underlying concept (Optimal Overlap in our case) and often these methods include masks which are mutually exclusive, for example, for sparsity learning (Zhu & Gupta, 2017), joint learning (Mallya et al., 2018) where piggyback a pretrained network by using a non-differentiable mask thresholding function and value, etc. In contrast, we don’t require our models to be pretrained or be thresholded. In CL, the following methods appear to be closest to our Relevance Mapping Networks (RMNs):

(Serr et al., 2018) proposes hard attention (HAT), a task based attention mechanism which can be considered the most similar to our RMN. It differs from RMN due to following reasons- (i) They utilize task embeddings and a positive scaling parameter - and a gated product of these two is used to produce a non-binary mask - unlike our RMNs which don’t use either a task embedding or a scaling parameter and is necessarily binary. (ii) Unlike RMNs, the attention on the last layer in HAT is manually hard-coded for every task. (iii) A recursive cumulative attention mechanism is employed to deal with multiple non binary mask values over tasks in HAT. RMNs however have no need for such a mechanism. (iv) HAT cannot be used in an unsupervised CL setup or to deal with CR and has not been implemented with more complex network architectures like Residual Networks.

(Jung et al., 2020) uses proximal gradient descent algorithm to progressively freeze nodes in an ANN. (i) Unlike RMNs, this method employs selective regularization to signify node importance (which is calculated by lasso regularization). (ii) This method progressively uses up the parameter set of the ANN and it is unclear whether it can be used for an arbitrary large number of sequential tasks. (iii) This method is also unable to deal with unsupervised learning scenario or CR. (iv) This method uses a different classification layer for each task - relaxing the core constraints of the problem altogether.

(Aljundi et al., 2018) (i) calculates the parameter importance by calculating sensitivity of the squared l2 norm of the function output to their changes and then uses regularization (similar to (Kirkpatrick et al., 2017)) to enforce in sequential learning, unlike RMNs. (ii) The method enforces fixed synaptic importance between tasks irrespective to their similarity and unlike our work, doesn’t seem to be capable of working under Unsupervised Learning scenarios.

(Yoo et al., 2020) propose SNOW and (i) uses a unique channel pooling scheme to evaluate the channel relevance for each specific task which differs from RMN’s individual node relevance mapping strategy. (ii) Importantly, this work, unlike RMNs, employs a pre-trained model which is frozen source model which already overgeneralizes to the CL problem at hand and thus makes this method inapplicable for dealing with CR. (iii) It also doesn’t seem to be capable of handling unsupervised learning/testing scenarios.

**3. Strict Continual Learning and a**

**Catastrophic Forgetting Memory**

Continual learning has been an important topic since the onset of Machine Learning and the fact that ANNs are incapable of learning continually due to Catastrophic Forgetting has been a significant drawback. CF has been strongly identified with overlap of distributed representations (French, 1991).

Catastrophic forgetting from a probabilistic view. Intuitively, given a learnt initial set of parameters \( \theta_i \) for a neural network \( f \) and a task \( i \) with data \( D_i \), the network’s parameters get overwritten when it learns a new set of network parameters \( \theta_{i+1} \) from new data \( D_{i+1} \) for the \((i+1)_{th}\) task. To facilitate the conceptual understanding of CF, we consider continual learning from a probabilistic perspective, where optimizing the parameters \( \Theta \) of \( f \) is tantamount to finding their most probable values given some data \( D \mid \mathcal{D} \supseteq D_1, ..., D_n \) (Kirkpatrick et al., 2017). We can compute the conditional probability of the first task \( \mathcal{P}(\theta_1 \mid D_1) \) from the prior probability of the parameters \( \mathcal{P}(\theta_1) \) and the probability of the data \( \mathcal{P}(D_1 \mid \theta_1) \) by using Bayes’ rule. Hence, for the first task,

\[
\log \mathcal{P}(\theta_1 \mid D_1) = \log \mathcal{P}(D_1 \mid \theta_1) + \log \mathcal{P}(\theta_1) - \log \mathcal{P}(D_1). \quad (1)
\]

Note that the likelihood term \( \log \mathcal{P}(D_1 \mid \theta_1) \) simply represents the negative of the loss function for the problem at hand (Kirkpatrick et al., 2017). Additionally, the posterior term is usually intractable and only approximated for ANNs (Titias et al., 2020; Nguyen et al., 2017; Kirkpatrick et al., 2017) and we are just considering it here without change for analysis purposes only.

If we were to now train the same network for a second task, the posterior from (1) now becomes a prior for the new posterior. If no regularization or method is included to preserve the prior information, we’d optimize for the second task,

\[
\log \mathcal{P}(\theta_{1:2} \mid D_{1:2}) = \log \mathcal{P}(D_2 \mid \theta_2) + \log \mathcal{P}(\theta_1 \mid D_1) - \log \mathcal{P}(D_2) \quad (2)
\]

\[
= \log \mathcal{P}(D_2 \mid \theta_2) + \log \mathcal{P}(D_1 \mid \theta_1) + \log \mathcal{P}(\theta_1) - \log \mathcal{P}(D_1) - \log \mathcal{P}(D_2). \quad (3)
\]

If the likelihood term is not optimised over both \( \theta_1 \) and \( \theta_2 \), as would happen normally in an ordinary ANN training setup,
the prior information can be overwritten, leading to the condition commonly referred to as catastrophic forgetting.

**Overcoming catastrophic forgetting.** We can clearly see from Eq. 3, if we had access to the previous data $D_1$ or if both $\theta_1$ and $\theta_2$ were independent of one another, we could approximate a well optimized posterior. However, in a typical continual learning setting we do not have access to the previous data and cannot ordinarily make independence assumptions on the sequentially learnt parameters. However, this provides us a with a crucial conceptual understanding of how to deal with $CF$ and an insight towards understanding the mechanisms of current popular $CL$ methods, which aim to overcome either of the two mentioned restrictions. In particular, some recent works (Mallya et al., 2018; Serr et al., 2018; Jung et al., 2020) try to effectively separate the model parameters $\theta_i$ for different tasks, as initially proposed by (French, 1994). The most successful recent works (Pan et al., 2021; Titsias et al., 2020; Guo et al., 2020; Kemker & Kanan, 2017) involve data replay methods, which relax the previous data availability restriction and have been shown to be effective initially by (Robins, 1993). Despite its success in dealing with $CF$ to an extent, data replay drastically diminishes the discriminative ability of the ANN, which is referred to as Catastrophic Remembering (Robins, 1993). This usually happens when the ANN learns a more general function $f(\Theta)$ than necessary, generalizing not only to the individual tasks, but to the entire sequential set of tasks $\{\theta \mid \forall \theta_i \subseteq \Theta\}$ which we has been referred to as overgeneralization (Robins, 1993). The network can then experience a sense of extreme déjà vu (Sharkey & Sharkey, 1995b) and is unable to differ the old from new data.

### 3.1. Catastrophic Remembering

For a better understanding of $CR$ and why $CF$ alleviation aggravates it, we calculate the posterior after the $n_{th}$ task learnt continually using Eq. (3),

$$
\log \mathcal{P}(\theta_{1:n}|D_{1:n}) = \log \mathcal{P}(D_n|\theta_n) + \sum_{i=1}^{n-1} \log \mathcal{P}(D_i|\theta_i) + \log \mathcal{P}(\theta_i) - \mathcal{C}
$$

(4)

where $\mathcal{C}$ is a constant representing the sum of the normalization constants $\sum_{i=1}^{n} \log \mathcal{P}(D_i)$. As discussed earlier, the information from the previous tasks is passed to the next sequential task as a prior ($\sum_{i=1}^{n-1} \log \mathcal{P}(D_i|\theta_i)$). The problem of loss of discriminative ability arises when for an arbitrary large $n$ when the prior term far exceeds the currently optimized likelihood. For Eq. (4), that means

$$
\log \mathcal{P}(D_n|\theta_n) < \sum_{i=1}^{n-1} \log \mathcal{P}(D_i|\theta_i).
$$

(5)

In the context of data replay methods this intuitively means that if the number of data from the previous tasks $\{D_1, \ldots, D_{n-1}\}$ is far bigger than the data in the current task $D_n$, the contribution of the present likelihood to the posterior is negligible and no new features are learnt by the ANN to account for the new dataset/task. This, in turn, gives the model a sense of false familiarity with a new input and the model is no longer able to discriminate between old and new inputs. The above explanation, though not exhaustive, provides a initial understanding from a Bayesian viewpoint.

One may argue against the necessity of the discriminative property that $CR$ attacks in $ANNs$. While, it is true that just concentrating on generalization may allow us to ignore the problems of $CR$, but novel input and task detection are important problems in Artificial Intelligence, Computer Vision and Robotics. It can be necessary to detect new inputs to learn more robust features for the current data, e.g. a self driving network may need to identify whether it is familiar with a current set of input data. Additionally, Recognition and discrimination memory are important aspects of human memory and learning - concepts which artificial networks have been trying to replicate.

### Balancing Forgetting and Remembering

Having gained a basic understanding about $CF$ and $CR$, an astute reader realizes the crux of the problem that we are dealing with. Alleviating $CF$ appears to aggravate $CR$. While current literature focuses on alleviating $CF$, the problem of $CR$ does not receive much attention. One aim of this work is to shed light on the twin problem of catastrophic forgetting and remembering and to introduce a method which can balance alleviating both problems concurrently.

### 4. Relevance Mapping for Continual Learning

We introduce Relevance Mapping, which is a method inspired by the Optimal Overlap Hypothesis, that aims to learn an optimal representational overlap, such that unrelated tasks use different network parameters, while allowing similar tasks to have a representational overlap. Note that our method avoids data replay and instead aims to achieve independence between network weights that are used for different sequential tasks.

**Algorithmic implementation of Relevance Mapping.** To illustrate and motivate Relevance Mapping Networks (RMNs) using a simple example, we consider a multilayer perceptron (MLP) with two layers $f$ defined as

$$f(x) = \sigma(W_2\sigma(W_1x)),
$$

(6)

where $x \in \mathbb{R}^{d_1}$, $W_1 \in \mathbb{R}^{d_2 \times d_1}$, and $W_2 \in \mathbb{R}^{d_3 \times d_2}$, and $\sigma$ denotes a nonlinear activation function. We denote the set of weights as $W \equiv \{W_1, W_2\}$. Although it may depend on the dimensionality of the task, overparameterization occurs even in these simple MLP settings. For a sufficiently simple task, only a subset of the parameters in $W$
are often required (Frankle & Carbin, 2019). For example, if the optimization task has ground-truth outputs specified as \( f^*(x) = \sigma(W_1^T \sigma(W_2^T x)) \) for optimized weights \( \{W_1^*, W_2^*\} \), and \( \|W_1^T\|_0 + \|W_2^T\|_0 \ll d_3d_2 + d_2d_1 \) (i.e. the number of non-zero weights needed for the ground-truth function is much less than the number of total weight parameters) then only \( \|W_1^T\|_0 + \|W_2^T\|_0 \) weight parameters are necessary to be learned in network \( f \). In theory, if we could learn the importance or relevance of each weight node, we could apply a zero-mask to the non-essential parameters without pruning or modifying them and still successfully learn the ground-truth. A set of mappings can be denoted as \( M_p = \{M_{p1}, M_{p2}\} \), where \( M_{p1} \in \{0, 1\}^{d_2 \times d_1} \) and \( M_{p2} \in \{0, 1\}^{d_3 \times d_2} \), explicitly representing the neuron-to-neuron connections of the network. The initialized relevance mappings of an ANN can be approximated by a logit-normal distribution mixture which is rounded during inference.

\[
M_p \approx \prod_k \mathcal{L}_R(N(\mu_k, \sigma_k^2))
\]

where \( \mu, \sigma \) are the initializing distribution parameters and \( \mathcal{L}_R \) sigmoidal pseudo-round function:

\[
\mathcal{L}_R(x_k; \beta) = \frac{1}{1 + \exp(-\beta(x_k - 0.5))} \tag{7}
\]

This is done in order to make the mappings differentiable and the individual mixture components are jointly optimized for the task with the network parameters.

In theory, any network \( f \) with weight tensors \( W \) can have such corresponding sets of neuron connection representations \( M_{p1}, M_{p2}, \ldots, M_{pT} \) for \( T \) tasks/mappings, where each set \( M_{pi} \) activates a subnetwork mapping in \( f \) that could be used for various purposes for a task \( i \).

Note that \( \lim_{\beta \to \infty} (\mathcal{L}_R(x_k; \beta)) \) for \( x \in [0, 1] \) is equivalent to the rounding function. Here, \( \beta \) is a learnable, layer-wise parameter (i.e., in our implementation, there is one specific \( \beta \) for every layer of a given network) that controls the “tightness” of \( R \). To achieve an approximate neuron-connection representation, we define \( \hat{M}_p = \mathcal{L}_R(M_p; \beta) \) where \( \hat{M}_p \) is initialized from some distribution with support \([0, 1]\) (in experiments, we initialize \( \hat{M}_p \) with a clipped, skewed normal distribution).

In our presented work, we can think of the RMNs as replacing the weights of a network with the product of the weights and a binary relevance mixture. In this work, we introduce two algorithms, **Algorithm 1 and 2 (Supplementary Sec. 2)** which make use of Relevance Mapping. The former is used for traditional **Supervised CL experiments** which used to evaluate \( CF \) alleviation. The latter is used for the **Unsupervised scenario** (new task detection and unsupervised task inference) concerning evaluation of \( CR \) alleviation. Importantly, neither of the algorithms relax the conditions of a

\[\text{strict CL framework (Section 1).}^{1}\]

**Probabilistic interpretation of Relevance Mapping.** French introduced the method of **context-biasing** in (1994) which produces internal representations which are both well distributed and well separated to deal with \( CF \). **RMN** preserves a similar idea of distribution and separability without constraining for an explicit representation separation amongst posteriors learnt for the sequential tasks. The separation, in turn, is provided by the relevance mappings.

\[
\mathcal{P}(\theta_1, M_{p1}|D_1) \propto \mathcal{P}(D_1|\theta_{M_{p1}}) \mathcal{P}(\theta_{M_{p1}}) \tag{8}
\]

The 1st task of the **CL** problem presented in Eq. (8) is similar to Eq. (1) with relevance mappings introduced under the conditions of the algorithm presented. \( \theta_{M_{p1}} \) represents only a subset of \( \theta \) for which \( M_{p1} = 1 \). For learning the second task we optimize

\[
\mathcal{P}(\theta_{1:2}, M_{p2}|D_{1:2}) \propto \mathcal{P}(D_{2}|\theta_{M_{p2}}) \mathcal{P}(\theta_{1:2}) \tag{9}
\]

In Eq. (9), the second term on the right doesn’t contribute anything to the optimization over the second task due to the presence of independent relevance mappings which effectively disengages \( \theta_{M_{p1}} \) from further tampering and the next task receives a slightly constrained prior distribution that we can refer to as \( \theta_{2} \). The \( \theta_{M_{p1}} \) parameter set is however still available to the second task. Eq. (9) now becomes

\[
\mathcal{P}(\theta_{1:2}, M_{p2}|D_{1:2}) \propto \mathcal{P}(\theta_{2}) \tag{10}
\]

which is effectively now a problem of just jointly optimizing an ANN’s parameters \( (\Theta, M_{p2}) \) without any dependence on the previous task’s posterior. We have effectively decomposed the sequential task parameters. There are three scenarios that may occur w.r.t the optimised parameters i.e., \( k \) represents the individual elements (i) \( M_{p2} = M_{p1} \Rightarrow \theta_{M_{p2}} = \theta_{M_{p1}} \) (ii) \( M_{p2} = 1 \& M_{p1} = 0 \Rightarrow \theta_{M_{p2}} = 0 \) (iii) \( M_{p2} = 0 \& M_{p1} = 1 \). All of these scenarios can be handled by **RMNs** thanks to the \( O_2 \) hypothesis.

For \( n \) tasks, Eq. (10) becomes,

\[
\mathcal{P}(\Theta, M_{p}|D_{1:n}) \propto \prod_{i=1}^n \mathcal{P}(D_{i}|\theta_{M_{p_i}}) \mathcal{P}(\theta_i) \tag{11}
\]

Looking at (11) which is a basic Bayesian expression for a normal ANN, we can now understand that **O2 hypothesis** inspired **RMN** algorithm is capable of learning well separated and well distributed internal representations thanks to the **posterior decomposition** induced by our method. This takes care of the problem of **CF** and since the parameters of the model are jointly optimized over both the **RMN** parameters \( \Theta \) and the relevance mappings \( M_p \), the network cannot

\[\text{Refer to Supplementary for further method details}\]
Table 1. Results on sequential learning tasks for the Split-MNIST (S-MNIST), Permutated-MNIST (P-MNIST), Sequential Omniglot (S-Omniglot), Split Cifar-100(20 tasks) with Resnet18 (RES-CIFAR) and Split Cifar-100(5 tasks) (S-CIFAR100) tasks. Mean test accuracy results with standard deviation over five trials are shown where applicable.

| ALGORITHM | P-MNIST | S-MNIST | S-Omniglot | RES-CIFAR | S-CIFAR100 |
|-----------|---------|---------|------------|-----------|------------|
| VCL(NGuyen et al., 2017) † | 90 (200 pts/task) | 97 (40 pts/task) | 53.86 ± 2.3 | – | – |
| HAT(Serr et al., 2018) † | 91.6 | 99 (3 pts/character) | 5.5 ± 11.1 | 23.6 ± 8.8 | 59.2 ± 0.7 |
| RWALK(Chaudhry et al., 2018) † | – | 82.5 | 71.0 ± 5.6 | 70.1 | 58.1 ± 1.7 |
| AGS-CL(Jung et al., 2020) † | – | – | 82.8 ± 1.8 | 27.6 ± 3.6 | 64.1 ± 1.7 |
| FRCL(Titsias et al., 2020) † | 94.3 ± 0.2 (200 pts/task) | 97.8 ± 0.7 (40 pts/task) | 81.47 ± 1.6 (3 pts/character) | – | – |
| MEGA-II(Guo et al., 2020) † | 91.21 (256 pts/task) | – | – | 66.12 ± 1.94 ‡ | – |
| SNOW(Yoo et al., 2020) † | – | – | 82.8 ± 1.8 | – | – |
| FROMP(Pan et al., 2021) † | 94.9 ± 0.1 (40 pts/task) | 99.0 ± 0.1 (40 pts/task) | – | – | – |
| DLP(Smol et al., 2003) | 82 | 61.2 | – | – | – |
| EWC(Kirkpatrick et al., 2017) | 84 | 63.1 | 67.43 ± 4.7 ‡ | 42.67 ± 4.24 ‡ | 60.2 ± 1.1 ‡ |
| Si(Zenke et al., 2017) | – | 57.6 | 54.9 ± 16.2 | 45.49 ± 0.2 ‡ | 60.3 ± 1.3 ‡ |
| MAS(Aljundi et al., 2018) † | – | – | 81.4 ± 1.8 | 42 ± 1.9 | 61.5 ± 0.9 |
| RMN (Ours) | 97.727 ± 0.07 | 99.5 ± 0.2 | 85.33 ± 1.7 | 80.01 ± 0.9 | 70.02 ± 2.5 |

† SIMILAR METHODS(SECTION 2) ‡ USES PRETRAINED NETWORK ‡ USES DATA REPLAY BUFFER
‡ MULTIHEADED LAYER IMPLEMENTATION ** NOT TRAINED OVER ALL TASKS ^ ADDITIONAL MODEL IS USED

overgeneralize to a specific task given only Θ which, in turn takes care of CR.

The focus in RMNs is not to force a zero representational overlap or just generalize to all the sequential tasks altogether but rather to utilize the over-parameterization property of ANNs (Frankle & Carbin, 2019) and learn an optimal representational overlap for all tasks in the weight space - corroborating the Optimal Overlap Hypothesis. Therefore, there’s no constraint on the maps M_p to minimize the overlap with each other or a global loss function which takes in account the loss of individual tasks. The map M_p for each task helps define a subset of the final weight mapping of the ANNs. This subset may be disjoint or overlapping with other M_p defined weight subsets. Since, all the sequential tasks’ parameter mappings are subsets of the final weight mapping (with M_p defining the set relationship), we are able to alleviate both CF (the final mappings generalizes well for all the tasks) and CR (the M_p preserve the relationship between the global and individual parametric mappings).

5. Experiments

5.1. Supervised CL (Testing Catastrophic Forgetting)

We evaluate RMNs on supervised sequential learning tasks, which enables us to measure their ability to alleviate CF. In this setup, the network is given data for learning one task followed by another. The challenge lies in retaining the performance on previous tasks even as new tasks are learned, hence alleviating catastrophic forgetting. This experimental framework is commonly used in CL literature.

Setup. We use standard baseline architectures, including CNNs (LeCun et al.), Siamese Networks (Koch et al., 2015), and Residual Networks(He et al., 2015) and apply Relevance Mapping to them (denoted as CNN-RMN, Siamese-RMN, Resnet18-RMN, etc.). For task-wise classification, we let the classification output f(x; W, M_p,…, M_pT) ≜ argmax_{i∈{1,…,T}}{f(x; W, M_{pi})} so that no task-specific information is utilized at inference time. We also augment the loss function with L1-norm penalty on M_p masks and sum of overlap of M_p,…, M_pT) to reward sparsity and optimal separation of weight spaces, respectively. We adhere to the strict CL(Section 1) framework in RMN experiments.

Figure 1. Average accuracy results on CIFAR-100 (10 tasks)
Models. As is common in related work (Titsias et al., 2020; Kirkpatrick et al., 2017; Nguyen et al., 2017; Pan et al., 2021; Jung et al., 2020), we evaluate RMNs on five benchmarks: Permuted-MNIST (Kirkpatrick et al., 2017) (P-MNIST), Split-MNIST (S-MNIST), Sequential Omniglot (S-OMNIGLOT) (Schwarz et al., 2018), 10 task Split-Cifar100 (Zenke et al., 2017) (S-CIFAR100) and 20 task Split-Cifar100 (RES-CIFAR). To validate the efficacy of RMNs on complex architectures, the RES-CIFAR model is trained on a Resnet18. For 10 task S-Cifar100, we used 6 convolution layers followed by 2 fully connected layers (with ReLU activations). S-MNIST, P-MNIST and S-Omniglot architectures are same as in (Titsias et al., 2020).

Results and Discussion. As seen in Table 1, our RMNs set the new state of the art across all continual learning benchmarks presented, with improvements of 2.8% (P-MNIST), 0.5% (S-MNIST), 3.9% (S-Omniglot), 8.7% (S-Cifar100) and 13.9% (RES-CIFAR) over the previous SOTA. RMNs show their versatility in both simple (MLP) and complex (ResNet) architectures over both long (S-Omniglot, RES-CIFAR) and short continual learning, demonstrating their versatility. Figure 1 shows an example of the effectiveness of RMNs as compared to other methods when dealing with CF.

To keep comparisons fair amongst methods, Table 1 is divided into two parts by a 2-line separator. The above part includes all the methods which do not obey the conditions of a strict CL framework. Some compared methods (Nguyen et al., 2017; Titsias et al., 2020; Chaudhry et al., 2018; Guo et al., 2020; Pan et al., 2021) employ data replay buffers while others cannot work efficiently without one or more multi-headed layers (Nguyen et al., 2017; Chaudhry et al., 2018; Jung et al., 2020) or a version of it - for e.g. (Serr et al., 2018) use manual hard coding of layers per task. The lower part of Table 1 consists of methods which are implemented in a strict sequential learning setup. Unlike most of the compared methods, RMNs do not require any replay buffers, ensemble networks, meta networks, multi-headed layers or pretrained models and yet are able to outperform methods which do use such methods. We also compare RMNs with similar methods as mentioned in Section 2 and see that RMNs substantially outperform every one of them as seen in Table 1.

5.2. Unsupervised CL (Testing Catastrophic Remembering)

Measuring Catastrophic Remembering. For a good measure of CR, we need to evaluate how well a sequentially trained ANN discriminates between old and new data as well as how well does it discriminate between all the tasks/data after it has been trained on all of them. To that end, we propose two tests: (1) (Unsupervised) New Task/Data Detection and (2) Unsupervised Task Inference. In the new task detection setup, the ANN is given no supervision with respect to the new data or task and has to detect this change. The model’s performance (for e.g., the accuracy for each task in case of classification) is compared with the supervised continual learning version. In the second test, a trained model has to detect which specific task does the test input data belongs to. A point to be noted is that the preconditions of the sequential training paradigm mentioned in Section 1 are to be strictly observed. The tests are mutually inclusive in terms of representing effectiveness w.r.t CR alleviation i.e. a method should perform well on both tasks to be good candidate for fixing Catastrophic Remembering (while still being able to alleviate Catastrophic Forgetting).

5.2.1. New Task/Data Detection

Setup. The model is given no information about the tasks during training (and inference) time. The results are then compared with the fully supervised version (Table 1). The performance degradation from the supervised learning results allow us to evaluate how well the model can alleviate CR. Here, we assume that no information of task labels is given, including the number of disparate tasks.

RMN Methodology. In this case, we initialize \( f \) with only a single \( M_p \), i.e. only a single forward inference path can be learned at initialization, as seen in Line 2 of Algorithm 2 (Suppl. Sec. 2). We set the current task indicator as \( est_j = 0 \). Then, for each minibatch \( x \) encountered, we run a task-switch-detection (TSD) method, denoted as \( TSD(x) \) which returns a boolean value. If \( TSD(x) \) returns True, then \( est_j \) is incremented and another set of \( M_p \) is added to \( f \). We use a Relevance modified Welsh’s t-test on the KL divergence between prior and posterior distributions of the model to determine a task switch (Titsias et al., 2020; Hendrycks & Gimpel, 2016; Lee et al., 2018).

Results and Discussion. Few methods (Titsias et al., 2020; Lee et al., 2020; Pan et al., 2021) have tried to effectively deal with the harder problem of learning continually without task labels and none of these follow a strict CL framework. (Titsias et al., 2020) and (Pan et al., 2021) both employ a data replay buffer whereas (Lee et al., 2020) uses generative replay and a mixture of expert models (which leads to a large increase in computational and memory requirements).
The usual methodology that is followed in an Unsupervised CL learning setup involves boundary detection between current and new tasks. Table 2 shows the results of this setup amongst all the relevant methods. RMNs achieve the state of the art for Unsupervised CL Learning without the usage of data replay buffer, mixture of expert models or any kind of generative replay, unlike (Titsias et al., 2020; Pan et al., 2021; Lee et al., 2020). Ordinarily, these task detection methods employ statistical tests like Welch’s T-test over clean batches of data (the entire batch data belongs to either the current or the next task). This methodology fails when the incoming previous and the incoming batch are noisy with the incoming batch consisting of the new task data as well as old data. RMNs however can easily deal with this by filtering the incoming batch via final layer activations - only if RMNs have seen the data do they have high and confident activations before calculating the KL divergence test between the prior and posterior to detect the presence of a new set of data. However, none of the methods (Titsias et al., 2020; Lee et al., 2020) mentioned are capable of deploying over such a noisy data setup and are unable to learn continually. (Lee et al., 2020) does employ a Fuzzy Testing scenario for Split-MNIST in which there are transition phases between tasks where the amount of new data increases linearly in each batch. Comparison on the same experiment is presented in Table 3.

5.2.2. UNSUPERVISED TASK INFERENCE

Under this novel setup, the algorithm has to identify at inference time which task a data input belongs to amongst all the tasks it has learned. From a practical point of view, knowing which task in the sequential task list does the current inference data element belongs to, without human intervention opens up huge opportunities for automation and analysis.

Setup. After the ANN has been trained, the test data is randomized and provided to the model for inference without its task identity (something which would happen in real world CL scenario). The model identifies the task to which the data belongs to and then the test accuracy is calculated from the correctly identified task over the entire task set. For RMNs, as task $j$ is not given at inference time, thus $\max_k f(x, k; W)$ is returned, as seen in Algorithm 2 (Suppl. Sec. 2). Our experimental results show that for any ground-truth task label $j$, indeed the desired result is $f(x, j; W) \approx \max_k f(x, k; W)$, which allows for unsupervised CL inference, as the pathways of different tasks don’t overlap unless the tasks are the same.

### Table 3. Fuzzy Unsupervised Learning.

| ALGORITHM | S-MNIST |
|-----------|---------|
| CN-DPM    | 93.22 ± 0.07 |
| RMN (OURS)| 99.1 ± 0.5  |

![Figure 2. P-MNIST Randomized Unsupervised Task Inference](image)

**Results and Discussion.** Unfortunately, we couldn’t find any SOTA CL method which can be used for this experiment, or can be used with trivial modifications. It should be possible for (Lee et al., 2020) to possibly be able to extend the method to do unsupervised task inference. However since the method employs a mixture of expert models for every task as well as generative replay which in turn rapidly drives up computational and storage memory requirements for even small ANNs, it cannot be considered a strict CL setup or even a slightly relaxed version of the same. In Figure 2, we show how our algorithm is able to detect the right task - we see that the relevance-weight combination achieves the correct maximum activation in the final layer only when the correct relevance is used. We also display the percentage of correct activations for other relevance values even if they are not maximum activations. According to our knowledge, our method is the only known continual learning method under strict CL setup constraints capable of successfully accomplishing unsupervised task inference.

### 6. Conclusion

In this work, we study the twin problem of catastrophic forgetting and remembering in continual learning. To resolve them, we introduce Relevance Mapping for continual learning, which applies a relevance map on the parameters of a neural network that is learned concurrently to every task. In particular, Relevance Mapping learns an optimal overlap of network parameters between sequentially learned tasks, reducing the representational overlap for dissimilar tasks, while allowing for overlap in the network parameters for related tasks. We demonstrate that our model efficiently deals with catastrophic forgetting and remembering, and achieves SOTA performance across a wide range of popular benchmarks without relaxing the conditions of a strict continual learning framework.
References

Ahn, H., Cha, S., Lee, D., and Moon, T. Uncertainty-based Continual Learning with Adaptive Regularization. pp. 11.

Aljundi, R., Babiloni, F., Elhoseiny, M., Rohrbach, M., and Tuytelaars, T. Memory Aware Synapses: Learning What (not) to Forget. In Ferrari, V., Hebert, M., Sminchisescu, C., and Weiss, Y. (eds.), Computer Vision – ECCV 2018, volume 11207, pp. 144–161. Springer International Publishing, Cham, 2018. ISBN 978-3-030-01218-2 978-3-030-01219-9. doi: 10.1007/978-3-030-01219-9_9. URL http://link.springer.com/10.1007/978-3-030-01219-9_9. Series Title: Lecture Notes in Computer Science.

Ando, R. K. and Zhang, T. A framework for learning predictive structures from multiple tasks and unlabeled data. J. Mach. Learn. Res., 6:1817–1853, December 2005. ISSN 1532-4435.

Chaudhry, A., Dokania, P. K., Ajanthan, T., and Torr, P. H. S. Riemannian walk for incremental learning: Understanding forgetting and intransigence. In Proceedings of the European Conference on Computer Vision (ECCV), September 2018.

Chen, Z. and Liu, B. Lifelong machine learning. Synthesis Lectures on Artificial Intelligence and Machine Learning, 12(3):1–207, 2018.

Ebrahimi, S., Elhoseiny, M., Darrell, T., and Rohrbach, M. Uncertainty-guided Continual Learning with Bayesian Neural Networks. April 2020. URL https://iclr.cc/virtual_2020/poster_Hk1UCCVKDB.html.

Frankle, J. and Carbin, M. The lottery ticket hypothesis: Finding sparse, trainable neural networks, 2019.

French, R. Dynamically constraining connectionist networks to produce distributed, orthogonal representations to reduce catastrophic interference, 1994.

French, R. M. Using semi-distributed representations to overcome catastrophic forgetting in connectionist networks. 1991.

Guo, Y., Liu, M., Yang, T., and Rosing, T. Improved schemes for episodic memory-based lifelong learning, 2020.

He, K., Zhang, X., Ren, S., and Sun, J. Deep residual learning for image recognition, 2015.

Hendrycks, D. and Gimpel, K. A baseline for detecting misclassified and out-of-distribution examples in neural networks, 2016.

Jung, S., Ahn, H., Cha, S., and Moon, T. Continual learning with node-importance based adaptive group sparse regularization, 2020.

Kemker, R. and Kanan, C. Fearnet: Brain-inspired model for incremental learning. 11 2017.

Kemker, R., McClure, M., Abitino, A., Hayes, T., and Kanan, C. Measuring catastrophic forgetting in neural networks, 2017.

Kirkpatrick, J., Pascanu, R., Rabinowitz, N., Veness, J., Desjardins, G., Rusu, A. A., Milan, K., Quan, J., Ramalho, T., Grabska-Barwinska, A., et al. Overcoming catastrophic forgetting in neural networks. Proceedings of the national academy of sciences, 114(13):3521–3526, 2017.

Knoblauch, J., Husain, H., and Diebthe, T. Optimal Continual Learning has Perfect Memory and is NP-hard. arXiv:2006.05188 [cs, stat], June 2020. URL http://arxiv.org/abs/2006.05188. arXiv: 2006.05188.

Koch, G., Zemel, R., and Salakhutdinov, R. Siamese neural networks for one-shot image recognition. In ICML deep learning workshop, volume 2. Lille, 2015.

Krizhevsky, A. Learning multiple layers of features from tiny images. University of Toronto, 05 2012.

Kurle, R., Cseke, B., Klushyn, A., Smagt, P. v. d., and Günnemann, S. Continual Learning with Bayesian Neural Networks for Non-Stationary Data. April 2020. URL https://iclr.cc/virtual_2020/poster_SJlsFpVtDB.html.

Lake, B. M., Salakhutdinov, R., and Tenenbaum, J. B. Human-level concept learning through probabilistic program induction. Science, 350(6266):1332–1338, 2015. ISSN 0036-8075. doi: 10.1126/science.aab3050. URL https://science.sciencemag.org/content/350/6266/1332.

LECUN, Y. The mnist database of handwritten digits. http://yann.lecun.com/exdb/mnist/. URL https://ci.nii.ac.jp/naid/10027939599/en/.

LeCun, Y., Bengio, Y., et al. Convolutional networks for images, speech, and time series.

Lee, J., Hong, H. G., Joo, D., and Kim, J. Continual Learning With Extended Kronecker-Factored Approximate Curvature. In 2020 IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR), pp. 8998–9007, Seattle, WA, USA, June 2020. IEEE. ISBN 978-1-72817-168-5. doi: 10.1109/CVPR42600.2020.00902. URL https://ieeexplore.ieee.org/document/9157569/.
Lee, K., Lee, K., Lee, H., and Shin, J. A simple unified framework for detecting out-of-distribution samples and adversarial attacks, 2018.

Lewandowsky, S. and Li, S.-C. 10 - catastrophic interference in neural networks: Causes, solutions, and data. In Dempster, F. N., Brainerd, C. J., and Brainerd, C. J. (eds.), Interference and Inhibition in Cognition, pp. 329 – 361. Academic Press, San Diego, 1995. ISBN 978-0-12-208930-5. doi: https://doi.org/10.1016/B978-012208930-5/50011-8. URL http://www.sciencedirect.com/science/article/pii/B9780122089305500118.

Li, Z. and Hoiem, D. Learning without forgetting. IEEE Transactions on Pattern Analysis and Machine Intelligence, 40(12):2935–2947, Dec 2018. ISSN 1939-3539. doi: 10.1109/tpami.2017.2773081.

Mallya, A., Davis, D., and Lazebnik, S. Piggyback: Adapting a single network to multiple tasks by learning to mask weights. In Ferrari, V., Sminchisescu, C., Weiss, Y., and Hebert, M. (eds.), Computer Vision – ECCV 2018 - 15th European Conference, 2018, Proceedings, Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics), pp. 72–88. Springer-Verlag Berlin Heidelberg, January 2018. ISBN 9783030012243. doi: 10.1007/978-3-030-01225-0_5. 15th European Conference on Computer Vision, ECCV 2018 ; Conference date: 08-09-2018 Through 14-09-2018.

Mermillod, M., Bugaiska, A., and BONIN, P. The stability-plasticity dilemma: investigating the continuum from catastrophic forgetting to age-limited learning effects. Frontiers in Psychology, 4:504, 2013. ISSN 1664-1078. doi: 10.3389/fpsyg.2013.00504.

Mundt, M., Hong, Y. W., Plischch, I., and Ramesh, V. A wholistic view of continual learning with deep neural networks: Forgotten lessons and the bridge to active and open world learning, 2020.

Nguyen, C. V., Li, Y., Bui, T. D., and Turner, R. E. Variational continual learning, 2017.

Oswald, J. v., Henning, C., Sacramento, J., and Grewe, B. F. Continual learning with hypernetworks. September 2019. URL https://openreview.net/forum?id=SJgwNerKvB.

Pan, P., Swaroop, S., Immer, A., Eschenhagen, R., Turner, R. E., and Khan, M. E. Continual deep learning by functional regularisation of memorable past, 2021.

Parisi, G. I., Kemker, R., Part, J. L., Kanan, C., and Wermter, S. Continual lifelong learning with neural networks: A review, 2018.

Robins, A. Catastrophic forgetting in neural networks: the role of rehearsal mechanisms. In Proceedings 1993 The First New Zealand International Two-Stream Conference on Artificial Neural Networks and Expert Systems, pp. 65–68, Dunedin, New Zealand, 1993. IEEE Comput. Soc. Press. ISBN 978-0-8186-4260-9. doi: 10.1109/ANNES.1993.323080. URL http://ieeexplore.ieee.org/document/323080/.

Schwarz, J., Luketina, J., Czarnecki, W. M., Grabska-Barwinska, A., Teh, Y. W., Pascanu, R., and Hadsell, R. Progress & compress: A scalable framework for continual learning. arXiv preprint arXiv:1805.06370, 2018.

Serr, Suris, Miron, and Karatzoglou. Overcoming catastrophic forgetting with hard attention to the task. ArXiv, abs/1801.01423, 2018.

Sharkey, N. and Sharkey, A. An analysis of catastrophic interference. Connect. Sci., 7:301–330, 01 1995a.

Sharkey, N. E. and Sharkey, A. J. C. Backpropagation discrimination geometric analysis interference memory modelling neural nets. Connection Science, 7(3-4):301–330, 1995b.

Smola, A., Vishwanathan, V., and Eskin, E. Laplace propagation. 01 2003.

Srijith, P. K. and Shevade, S. Gaussian process multi-task learning using joint feature selection. In Calders, T., Esposito, F., Hüllermeier, E., and Meo, R. (eds.), Machine Learning and Knowledge Discovery in Databases, pp. 98–113, Berlin, Heidelberg, 2014. Springer Berlin Heidelberg. ISBN 978-3-662-44845-8.

Titsias, M. K., Schwarz, J., Matthews, A. G. d. G., Pascanu, R., and Teh, Y. W. Functional Regularisation for Continual Learning with Gaussian Processes. April 2020. URL https://iclr.cc/virtual_2020/poster_HkxCzeHFDB.html.

Yoo, C., Kang, B., and Cho, M. SNOW: Subscribing to Knowledge via Channel Pooling for Transfer & Lifelong Learning of Convolutional Neural Networks. April 2020. URL https://iclr.cc/virtual_2020/poster_rJxtgJBKDr.html.

Zenke, F., Poole, B., and Ganguli, S. Continual learning through synaptic intelligence, 2017.

Zhu, M. and Gupta, S. To prune, or not to prune: exploring the efficacy of pruning for model compression. arXiv preprint arXiv:1710.01878, 2017.
Supplementary

1. Concepts

1.1. Strict Continual Learning

In Section 1 of the main paper, the concept of Continual Learning is defined and it is noted that most of the current state of the art Continual Learning methods relax the constraints of a strict continually learning framework. In Table 1, we quote some of the major violations of a strict continual learning framework with reference to the state of the art methods compared in the main paper.

**Data Replay** refers to the usage of old or future task data in any way to train the neural network. Methods like (Guo et al., 2020; Kemker & Kanan, 2017; Nguyen et al., 2017; Titsias et al., 2020; Pan et al., 2021; Chaudhry et al., 2018), etc. all employ this tactic in unique ways to learn continually. It often involves saving old task data in memory modules and been originally inspired from (Robins, 1993) who was among the first researchers to show that data replay helps in alleviating catastrophic forgetting in artificial neural networks, albeit at an expense of relaxing the constraints of a strict continual learning setup.

**Multihead** usually refers to the usage of different last (usually linear) layer for each task. This has become particularly common in continual learning benchmarks with many methods (Chaudhry et al., 2018) dissuading the usage of single heads for a continually learning neural network. There are a few methods which also employ similar but unique methodology. For e.g. (Serr et al., 2018) uses a binary hard coded final layer per task.

**Pretrained** refers to using a pretrained model (usually trained on a more complex dataset like ImageNet (Deng et al., 2009)) for training on a simpler problem or dataset (e.g. CIFAR (Krizhevsky, 2012)). Having used data outside of the continual learning problem setup and since the model has now probably over-generalized to entire sequel data/task set, using a pretrained model relaxes the constraints of a strict continual learning.

**Generative Replay** ordinarily refers to the usage of generative modelling for the dataset or task at hand and using these generated samples for some form of data replay. The usage of additional generative model (even for simple classification models and tasks), saving old data points, etc. all violate the conditions of a strict continual setup.

**Multi-Models** refers to the usage of a neural model other than the original model to help with the continual learning problem. This can reflect in using meta networks, or in strategies similar to the one presented in (Yoo et al., 2020) where separate delta models are used per task to help the original neural network learn continually.

The aforementioned concepts are not mutually exclusive and clearly do not present an exhaustive list of methods employed to relax a strict continual learning framework, however they do provide us a reference among-st the compared SOTA methods to ascertain which method displays the best results with least amount of constraint relaxation.

Table 1 show that our method, RMNs, do not need to use any of the aforementioned methods to violate strict continual learning constraints and still produces the state of the art results in common Continual Learning benchmarks.

1.2. Catastrophic Remembering

A common solution for the problem of Catastrophic Forgetting is to overgeneralize to the entire set of sequential data/tasks. For example, methods which employ data replay, pre-training and knowledge distillation directly employ over-generalization for CF alleviation.

**Over-generalization** The concept of over-generalization (in the case of back propagated artificial neural networks) refers to the learning of parameter set by the neural network tries to or has already learned a much more general function than what is required.

A simple example to understand Catastrophic Remembering was provided in the work by French (1999) where a network has a task of reproducing an input as output. A new input is detected if output diverges by a large margin. If the network learns too well and learns the identity function, then it has overgeneralized and hence loses the ability to detect new input. This trivial example presents one aspect of Catastrophic Remembering. However, there is no guarantee that the loss of discrimination always leads to correct generalization - the network just becomes too familiar with the input irrespective of whether the output is correct.
2. Relevance Mapping Method

2.1. Algorithms

2.1.1. Supervised

In the supervised continual learning setup, task labels are available both during training and inference (though RMNs do not require task labels as such). This kind of experimental setup is currently the most common form of evaluation used for Continual Learning methods. A point to note that adding regularization to induce sparsity in RMNs is optional and is not required to obtain an optimally trained model (as shown in Section 2.3). Additionally, model weights are never pruned in RMN methodology. The pruning mentioned in Algorithm 1 and 2 refers to zeroing out of relevance mappings which have not tightened towards value of 1. The prune parameter $\mu$ may also refer to the combination of weight and $M_P$.

Algorithm 1 RMN Supervised Continual Learning

1: **Input:** data $x$, ground truth $y$ for $n$ tasks, prune parameter $\mu$, corresponding task labels $i$ paired with all $x$

2: **Given:** parameters $W$ & initilaized relevance mappings $M_P$

3: **for each task $i$ do**

4: $f(x_i; W, M_{P_i}) \Rightarrow \hat{y}_i = \sigma((W \odot M_{P_i}) \odot x_i)$

5: Compute Loss : $L(\hat{y}_i, y_i)$

6: Optional: Add Sparsity Loss : $L(\hat{y}_i, y_i) + (M_{P_i})_{ij}$

7: Backpropagate and optimize

8: Prune $M_P \leq \mu$ only.

9: Stabilize (fix) parameters in $f$ where $M_P = 1$

10: **end for**

11: **Inference:** For data $x$ and ground-truth task label $i$:

12: **Output:** $f(x, i; W)$

2.1.2. Unsupervised

For Unsupervised learning setup (which is used as a measure of Catastrophic Learning in our work), we introduce two sub tests - new task/data detection and unsupervised/randomized task inference.

In New Task/Data Detection, task label information is unavailable during both training and inference time and the model has to detect the new task in an unsupervised manner.

Algorithm 2 RMN Unsupervised Continual Learning

1: **Input:** data $x$, ground truth $y$, prune parameter $\mu$

2: **Given:** parameters $W$, $M_{Pest,j}$ with $est,j = 0$, Task Switch Detection Method $TSD$

3: **for each task $j$ do**

4: Filter input $x$ on $f(x; W, M_{Pest,j-1})$

5: $f(x; W, M_{Pest,j}) \Rightarrow \hat{y}$

6: Compute Loss : $L(\hat{y}, y)$

7: **if** $TSD(x)$ **is True**

8: $est,j + +$

9: Add $M_{Pest,j}$ to learn-able parameter list

10: $f(x; W, M_{Pest,j}) \Rightarrow \hat{y}$

11: Re-Compute Loss : $L(\hat{y}, y)$

12: **end if**

13: Backpropagate and optimize

14: Sample $x_g$ from standard Gaussian distribution with same shape as $x$

15: $f(x_g; W, M_{Pest,j}) \Rightarrow \hat{y}$

16: Compute Loss : $||\hat{y} - 0||^2$

17: Backpropagate and optimize

18: Prune $M_P \leq \mu$ only.

19: Stabilize (fix) parameters in $f$ where $M_P \approx 1$

20: **end for**

21: **Inference:** For data $x$:

22: **Output:** $\max_k f(x, k; W)$

Table 1. Common Methods used in Continual Learning which relax a strict CL framework

| Algorithm  | Data Replay | Multihead | Pretrained | Generative Replay | Multi-Models |
|------------|-------------|-----------|------------|-------------------|-------------|
| VCL        | ✔           | ✔         |           |                   |             |
| HAT        | ✔           | ✔         |           |                   |             |
| RWALK      | ✔           | ✔         |           |                   |             |
| AGS-CL     | ×           | ×         |           |                   |             |
| FRCL       | ✔           | ✔         |           |                   |             |
| MEGA-II    | ✔           | ×         |           |                   |             |
| SNOW       | ×           | ✔         |           |                   |             |
| FROMP      | ×           | ×         |           |                   |             |
| DLP        | ×           | ×         |           |                   |             |
| EWC        | ×           | ×         |           |                   |             |
| SI         | ×           | ×         |           |                   |             |
| MAS        | ×           | ×         |           |                   |             |
| RMN (Ours) | ×           | ×         |           |                   |             |

Exceptions exist
This usually involves task boundary detection during training which often involves statistical tests like Welch’s T-test or KL-Divergence on the model’s loss (Titsias et al., 2020; Pan et al., 2021). However, this setup often assumes clean sequential data over the entire training set as well as in a mini-batch. New Task/Data detection also involves learning in an unsupervised way when the incoming mini-batch is noisy i.e. a mix of old and new task data. The Fuzzy Unsupervised Learning experiment done on Sequential-MNIST follows the procedure laid out in (Lee et al., 2020).

The Randomized Unsupervised Task Inference experiment also capitalizes on the aforementioned weakness of the former test. Under this, a continually trained model has to identify the task ID during inference under circumstances where the inference input is task randomized. However, since RMNs form unique subnetworks in the original neural network, they are capable of trivially identifying the task ID as well as filtering out the noisy training data for new tasks.

2.2. \(\beta\) parameter

As presented in the main text, we used a sigmoidal pseudo-round function during training which is completely rounded during inference:

\[
\mathcal{L}_R(x_k; \beta) = \frac{1}{1 + \exp(-(\beta(x_k - 0.5)))}
\]

We also noted that \(\lim_{\beta \to \infty} (\mathcal{L}_R(x, \beta))\) for \(x \in [0, 1]\) is equivalent to the rounding function. Here, \(\beta\) is a learnable, layer-wise parameter (i.e., in our implementation, there is one specific \(\beta\) for every layer of a given network) that controls the “tightness” of \(\mathcal{L}_R\). We experimented with different values of \(\beta\) and noted that for an arbitrary high value of \(\beta\) (\(\geq 80\)), there’s not any visible difference in results and that the \(\beta\) value doesn’t require any tuning. If we tried and learn the \(\beta\) parameter instead of fixing its value, we noted that the \(\beta\) value tightened over time.

2.3. Sparsity Analysis

Sparsity \(S\) in a RMN \(f(W, M_P)\) is calculated according to the following formula:

\[
S = \frac{\prod_{i=1}^{t} \cap \forall M_P = 0}{\text{num}(W)}
\]

Figure 1 shows the model sparsity and usage for Resnet-18 model trained on Cifar-100 (Krizhevsky, 2012) dataset over 20 tasks. For this experiment, there are no loss functions, regularization or any method employed to constrain the model parameters or \(M_P\) for sparsity. If we do however constrain for sparsity using \(L_1\) or \(L_0\) regularization, we can observe much higher sparsity levels. We can observe that the model capacity usage even over time which can be explained due to subsequent tasks finding overlap amongst old task parameters.

2.4. Model Computational Complexity w.r.t number of Tasks

RMNs require only the learned weights of the continually learning network, though this is achieved through creating distinct sub-network mappings in each network. This does increase the number of parameters, but ultimately reduces the effective model size because all additional parameters can be converted to binary tensors. Thus, the memory complexity can be written as \(O(tk)\) where \(t\) is an integer and equal to the number of tasks and \(k\) is a constant. Thus, for a finite and constrained value of \(t\), the memory complexity of RMNs is \(O(1)\) i.e. constant. The value of \(k\) depends on the amount of overlap in our model as well as the method used to save binary parameters. For e.g. For the model in Table 2, the theoretical worst case scenario (a model with no overlap amongst the relevance mappings) results in 12kb of memory. Practically, as noticed in the RMN sparsity discussion, the model has not been observed to fully utilize its weights over the period of sequential tasks and unused parameters can be effectively removed post training. Additionally, we do not implement bias parameters in our RMN.

2.5. The Lottery Ticket Hypothesis and Relevance Mapping

A question arises as to whether the slight constraints introduced in the weight space by our algorithm worsen the per-
formance of the sequential tasks. The Lottery ticket Hypothesis introduced in a seminal work (Frankle & Carbin, 2019) states that - A randomly-initialized, dense neural network contains a subnetwork that is initialized such that—when trained in isolation—it can match the test accuracy of the original network after training for at most the same number of iterations. Additionally, our method doesn’t remove the previous tasks parameters and subsequent methods can choose to use their predecessors parameters optimally. Therefore, no performance drop is expected in our method and results from our experiments prove the same.

3. Experimental Details

The experimental implementation for most comparative methods mentioned in Table 1 have been taken from official implementations of (Serr et al., 2018), (Titsias et al., 2020), (Jung et al., 2020) and (Pan et al., 2021).

3.1. Architectural Details

In this section, we provide detailed descriptions of the architectures used for our experiments. We denote 2D convolutional layers as Conv2D, linear layers as Linear, Rectified Linear Unit as ReLU, and Batch Normalization as BN. For RMN versions of each layer with included Relevance Mapping $M$, we add an “M-” prefix, e.g. M-Conv is the RMN version of Conv.

For fair comparison purposes, for all architectures, we attempt to keep architectural representational capacity and module sequences as similar as possible to referenced methods.

| Table 2. Details of the Multi-layer Perceptron network used for the Permuted-MNIST and Split-MNIST tasks. This architecture is of equivalent representational capacity to the network used in (Titsias et al., 2020). |
|---|
| **INPUT:** $x \in \mathbb{R}^{784}$ |
| **M-LINEAR** (784) $\rightarrow$ 100 |
| **M-BN** (100) |
| **ReLU** |
| **M-LINEAR** (100) $\rightarrow$ 100 |
| **M-BN** (100) |
| **ReLU** |
| **M-LINEAR** (100) $\rightarrow$ 100 |
| **M-BN** (100) |
| **ReLU** |
| **M-LINEAR** (100) $\rightarrow$ 10 |
| **SOFTMAX** |

Table 3. Details of the convolutional network used for the Sequential Omniglot task. This architecture is of similar or equivalent representational capacity to the network used in (Titsias et al., 2020), cited as “Baseline” in their results sections.

| **INPUT:** $x \in \mathbb{R}^{1\times 105 \times 105}$ |
| **RESIZE** $\rightarrow x \in \mathbb{R}^{1\times 28 \times 28}$ |
| **M-CONV2D 1ch $\rightarrow$ 250ch** |
| **M-BN (250)** |
| **ReLU** |
| **MAXPOOLING (2 x 2), stride = 2** |
| **M-CONV2D 250ch $\rightarrow$ 250ch** |
| **M-BN (250)** |
| **ReLU** |
| **MAXPOOLING (2 x 2), stride = 2** |
| **M-CONV2D 250ch $\rightarrow$ 250ch** |
| **M-BN (250)** |
| **ReLU** |
| **MAXPOOLING (2 x 2), stride = 2** |
| **M-LINEAR $\rightarrow$ 60** |
| **SOFTMAX** |

Tablea 2 and 3 provide the main architecture details of the continual learning experiments dealing with Sequential MNIST, Permuted MNIST and Sequential Omniglot benchmarks.

For the sequential Cifar-100 (10 tasks) (Krizhevsky, 2012) benchmark, we follow the experimental and architectural details from (Jung et al., 2020). For the RMN, however, we do not use bias parameters and dropout layers. Table 4 shows the architecture details for the RMN used in the experiment.

For the RES-CIFAR experiment which uses a Resnet-18 (He et al., 2015) trained over Sequential Cifar-100 (20 tasks), we use the original Resnet-18 architecture for all experiments with modifications as required by a specific method. For the RMN-Resnet-18, we do not make use of bias parameters and dropout layers.

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1HAT (Serr et al., 2018) authors mentioned that there is no official implementation for Residual Networks

2AGS-CL (Jung et al., 2020) authors did not reply to our query concerning official Residual network implementation.
Table 4. Details of the convolutional network used for the Sequential Cifar 100 (10 tasks) task. This architecture is of similar or equivalent representational capacity to the network used in (Jung et al., 2020)

| Layer | Description |
|-------|-------------|
| INPUT | \( x \in \mathbb{R}^{3\times32\times32} \) |
| M-CONV2D | 3ch \( \rightarrow 32ch \) ReLU |
| M-CONV2D | 32ch \( \rightarrow 32ch \) ReLU |
| MAXPOOLING | (2 x 2), stride = 2 |
| M-CONV2D | 32ch \( \rightarrow 64ch \) ReLU |
| M-CONV2D | 64ch \( \rightarrow 64ch \) ReLU |
| MAXPOOLING | (2 x 2), stride = 2 |
| M-CONV2D | 64ch \( \rightarrow 128ch \) ReLU |
| M-CONV2D | 128ch \( \rightarrow 128ch \) ReLU |
| MAXPOOLING | (2 x 2), stride = 2 |
| M-LINEAR | \( \rightarrow 256 \) |
| M-LINEAR | \( \rightarrow 10 \) |

3.2. Hyperparameter Details

In this section, we provide detailed descriptions of training, optimization, and hyperparameter details.

In (Titsias et al., 2020) and (Jung et al., 2020), they select experiment with a range of hyperparameters, choose the values that return the highest validation accuracy and then retrain on the union of the train and validation set. When applicable, we select hyperparameter values similar or equivalent to those arrived at in (Titsias et al., 2020) for MNIST and Omniglot experiments and (Jung et al., 2020) for Cifar-100 experiments. For all continual learning tasks, we make use of the Adam optimizer and have separate learning rates for weights and \( M_p \) parameters. Subsets of weights are frozen via gradient masking as tasks increase, where \( \prod_{t}^{T} M_{p_{t}} = 1 \) is the mask applied to the weights at task \( T + 1 \).

For the Permuted-MNIST and Split-MNIST tasks, we use a 90-10 train-test split, 0.002 learning rate for all parameters, and batch size of 128. For all tasks, the network is trained for 250 epochs.

For the Sequential Omniglot task, we use an 80-20 train-test split, 0.0002 learning rate for all parameters, except \( M_p \) parameters, a learning rate of 0.0001 for \( M_p \) parameters, a batch size of 16. For the first task, the network is trained for 150 epochs, for subsequent tasks the network trained for 200 epochs.

In S-CIFAR-100 and RES-CIFAR, we train all comparative methods with mini-batch size of 256 for 100 epochs using Adam optimizer (Kingma & Ba, 2014) with initial learning rate 0.001 and decaying it by a factor of 3 if there is no improvement in the validation loss for 5 consecutive epochs, similarly as in (Jung et al., 2020; Serr et al., 2018).

For our method (RMNs), we keep the same mini batch size, training epochs and optimizer as mentioned in (Jung et al., 2020). For Split Cifar-100 (10 tasks) and RES-CIFAR (Split Cifar100-20 tasks with Resnet-18), the model weight parameters initial learning rate is .001 and for \( M_p \) the learning rate is .01. The prune parameter value is .05 and .01 respectively which is used to prune the relevance mappings. The pruning is done whenever the model’s task loss converges which varies from epoch 20 – 80 for different tasks.

4. Similar Methods

4.1. Differences with Relevance Mapping Method

(Serr et al., 2018) proposes hard attention (HAT), a task based attention mechanism which can be considered the most similar to our RMN.

It differs from RMN due to following reasons:

1. They utilize task embeddings and a positive scaling parameter - and a gated product of these two is used to produce a non-binary mask - unlike our RMNs which don’t use either a task embedding or a scaling parameter and is necessarily binary.

2. Unlike RMNs, the attention on the last layer in HAT is manually hard-coded for every task.

3. A recursive cumulative attention mechanism is employed to deal with multiple non binary mask values over tasks in HAT. RMNs however have no need for such a mechanism.

4. HAT cannot be used in an unsupervised CL setup or to deal with CR and has not been implemented with more complex network architectures like Residual Networks.

(Jung et al., 2020) uses proximal gradient descent algorithm to progressively freeze nodes in an ANN.

1. Unlike RMNs, this method employs selective regularization to signify node importance (which is calculated by lasso regularization).
2. This method progressively uses up the parameter set of the ANN and it is unclear whether it can be used for an arbitrary large number of sequential tasks.

3. This method employs two group sparsity-based penalties in order to regularize important nodes, however AMN do not require usage such kind of sparse based penalty.

4. This method is also unable to deal with unsupervised learning scenario or CR. (iv) This method uses a different classification layer for each task - relaxing the core constraints of the problem altogether.

(Aljundi et al., 2018)

1. Calculates the parameter importance by calculating sensitivity of the squared $l^2$ norm of the function output to their changes and then uses regularization (similar to (Kirkpatrick et al., 2017) to enforce in sequential learning, unlike RMNs.

2. The method enforces fixed synaptic importance between tasks irrespective to their similarity and unlike our work, doesn’t seem to be capable of working under Unsupervised Learning scenarios.

(Yoo et al., 2020) propose SNOW and

1. Uses a unique channel pooling scheme to evaluate the channel relevance for each specific task which differs from RMN’s individual node relevance mapping strategy.

2. Importantly, this work, unlike RMNs, employs a pre-trained model which is frozen source model which already overgeneralizes to the CL problem at hand and thus makes this method inapplicable for dealing with CR.

3. It also doesn’t seem to be capable of handling unsupervised learning/testing scenarios.

References

Aljundi, R., Babiloni, F., Elhoseiny, M., Rohrbach, M., and Tuytelaars, T. Memory Aware Synapses: Learning What (not) to Forget. In Ferrari, V., Hebert, M., Sminchisescu, C., and Weiss, Y. (eds.), Computer Vision – ECCV 2018, volume 11207, pp. 144–161. Springer International Publishing, Cham, 2018. ISBN 978-3-030-01218-2 978-3-030-01219-9. doi: 10.1007/978-3-030-01219-9_9. URL http://link.springer.com/10.1007/978-3-030-01219-9_9. Series Title: Lecture Notes in Computer Science.

Chaudhry, A., Dokania, P. K., Ajanthan, T., and Torr, P. H. S. Riemannian walk for incremental learning: Understanding forgetting and intransigence. In Proceedings of the European Conference on Computer Vision (ECCV), September 2018.

Deng, J., Dong, W., Socher, R., Li, L.-J., Li, K., and Fei-Fei, L. ImageNet: A Large-Scale Hierarchical Image Database. In CVPR09, 2009.

Frankle, J. and Carbin, M. The lottery ticket hypothesis: Finding sparse, trainable neural networks, 2019.

French, R. M. Catastrophic forgetting in connectionist networks. Trends in cognitive sciences, 3(4):128–135, 1999.

Guo, Y., Liu, M., Yang, T., and Rosing, T. Improved schemes for episodic memory-based lifelong learning, 2020.

He, K., Zhang, X., Ren, S., and Sun, J. Deep residual learning for image recognition, 2015.

Jung, S., Ahn, H., Cha, S., and Moon, T. Continual learning with node-importance based adaptive group sparse regularization, 2020.

Kemker, R. and Kanan, C. Fearnet: Brain-inspired model for incremental learning. 11 2017.

Kingma, D. P. and Ba, J. Adam: A method for stochastic optimization, 2014.

Kirkpatrick, J., Pascanu, R., Rabinowitz, N., Veness, J., Desjardins, G., Rusu, A. A., Milan, K., Quan, J., Ramalho, T., Grabska-Barwinska, A., et al. Overcoming catastrophic forgetting in neural networks. Proceedings of the national academy of sciences, 114(13):3521–3526, 2017.

Krizhevsky, A. Learning multiple layers of features from tiny images. University of Toronto, 05 2012.

Lee, J., Hong, H. G., Joo, D., and Kim, J. Continual Learning With Extended Kronecker-Factored Approximate Curvature. In 2020 IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR), pp.
Nguyen, C. V., Li, Y., Bui, T. D., and Turner, R. E. Variational continual learning, 2017.

Pan, P., Swaroop, S., Immer, A., Eschenhagen, R., Turner, R. E., and Khan, M. E. Continual deep learning by functional regularisation of memorable past, 2021.

Robins, A. Catastrophic forgetting in neural networks: the role of rehearsal mechanisms. In Proceedings 1993 The First New Zealand International Two-Stream Conference on Artificial Neural Networks and Expert Systems, pp. 65–68, Dunedin, New Zealand, 1993. IEEE Comput. Soc. Press. ISBN 978-0-8186-4260-9. doi: 10.1109/ANNES.1993.323080. URL http://ieeexplore.ieee.org/document/323080/.

Serr, Suris, Miron, and Karatzoglou. Overcoming catastrophic forgetting with hard attention to the task. ArXiv, abs/1801.01423, 2018.

Titsias, M. K., Schwarz, J., Matthews, A. G. d. G., Pascanu, R., and Teh, Y. W. Functional Regularisation for Continual Learning with Gaussian Processes. April 2020. URL https://iclr.cc/virtual_2020/poster_HkxCzeHFDB.html.

Yoo, C., Kang, B., and Cho, M. SNOW: Subscribing to Knowledge via Channel Pooling for Transfer & Life-long Learning of Convolutional Neural Networks. April 2020. URL https://iclr.cc/virtual_2020/poster_rJxtgJBKDr.html.