Multi-path entanglement of two photons

Alessandro Rossi,1∗ Giuseppe Vallone,2, 1, 3, Andrea Chiuri,1 Francesco De Martini,1, 4 and Paolo Mataloni1, 3

1 Dipartimento di Fisica, Sapienza Università di Roma, Roma, 00185 Italy
2 Centro Studi e Ricerche “Enrico Fermi”, Via Panisperna 89/A, Compendio del Viminale, Roma 00184, Italy
3 Consorzio Nazionale Interuniversitario per le Scienze Fisiche della Materia, Roma, 00185 Italy
4 Accademia Nazionale dei Lincei, via della Lungara 10, Roma 00165, Italy

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We present a novel optical device based on an integrated system of micro-lenses and single mode optical fibers. It allows to collect and direct into many modes two photons generated by spontaneous parametric down conversion. By this device multiqubit entangled states and/or multilevel qu-dit states of two photons, encoded in the longitudinal momentum degree of freedom, are created. The multi-path photon entanglement realized by this device is expected to find important applications in modern quantum information technology.

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Entangling two photons in a high-dimension Hilbert space allows the realization of important quantum information tasks. These deal with a complete analysis of Bell states [1, 2, 3] and novel protocols of superdense coding [4], the possibility to perform secure quantum cryptography [5, 6] and, in a fast, high-fidelity one-way quantum computation [7, 8, 9, 10, 11], besides the realization of novel quantum nonlocality tests [12, 13, 14, 15]. Indeed, a very large number of qubits are in principle available by this geometry. However, a successful realization of this idea strongly depends on the possibility to overcome the practical difficulties represented by independently collecting and manipulating the SPDC radiation belonging to a large number of k-modes.

By the device realized in this experiment, photon pairs travelling along a large number of k-modes are efficiently coupled into a bundle of single mode optical fibers by an integrated system of micro-lenses. Four pairs of correlated k-modes have been selected to generate two maximally entangled ququads, i.e. qu-dits with d = 4 (or equivalently a 4-qubit HE state) encoded in the longitudinal momentum of the photons:

$$\left| \Psi \right\rangle_k = \frac{1}{2} \sum_{j=1}^{4} e^{i\phi_j} \left| j \right\rangle_A \left| j \right\rangle_B$$

(1)

being \( \left| j \right\rangle_A \) (\( \left| j \right\rangle_B \)) the A (B) photon mode of the jth mode pair and \( \phi_j \) the corresponding phase. Figure 1(b) shows the annular section of the degenerate cone, with four pairs of correlated modes. It is divided in two half-rings, respectively corresponding to the Alice (A) and Bob (B) side.

This particular geometry allows the creation of multi-dimensional entangled states. However, it is worth to remember that using an increasing number of k-modes necessarily implies an exponential requirement of resources since this number doesn’t scale linearly with the number of qubits. In fact, \( 2^N \) k-modes per photon must be selected within the emission cone to encode \( N \) qubits in each photon. On the other hand, since a qu-dit, with \( d = 2^N \), is equivalent to \( N \) qubits, the number of modes scales linearly with the size \( d \) of the state.

We can re-label for convenience the modes \( \left| 1 \right\rangle_A, \cdots, \left| 4 \right\rangle_A \) belonging to the A side as \( \left| E, \ell \right\rangle_A, \left| I, \ell \right\rangle_A, \left| I, r \right\rangle_A \) and \( \left| E, r \right\rangle_A \), where \( \ell \) (r) and \( E \) (I) refer to the left (right) and external (internal) emission modes (see Figure 1(b)). They are respectively correlated to the

∗URL: http://quantumoptics.phys.uniroma1.it/
FIG. 1: a) Labelling of the correlated pairs of SPDC modes. Photon $A$ ($B$) can be collected with equal probability into one of the four modes $|1\rangle_A, |2\rangle_A, |3\rangle_A$ or $|4\rangle_A$ ($j = A, B$), which are relabelled as shown in b). b) Scheme needed for measurement of photon $A$. Here the left ($\ell$) and right ($r$) modes interfere on beam splitters $BS_1$ and $BS_1'$, while further interference occurs on $BS_2$ between the external ($E$) and internal ($I$) modes. An identical scheme is needed for photon $B$. c) Experimental setup: The GL-SMF integrated system injects photon pairs emitted by the crystal into the chained interferometer. Path of photon $A$ ($B$) is indicated by the continuous red (dashed blue) line. By this scheme $BS_1$ plays simultaneously the role of $BS_1$ and $BS_1'$ on side b) for both photons. The internal and external modes coming out of the right side of $BS_1$ interfere on the beam splitter $BS_2$. Photon $A$ and photon $B$ are separately detected by two single photon counting modules. Waveplates $\lambda/4$ and $\lambda/2$ enable polarization restoration of photons after SMF transmission. Inset: picture of the 8 GL-SMF system.

$B$ emission modes $|E, r\rangle_B, |I, r\rangle_B, |I, \ell\rangle_B$ and $|E, \ell\rangle_B$, labelled as $|1\rangle_B, \ldots, |4\rangle_B$ in Figure 1(b).

In the experiment (cf. Figure 1), a Type I $2\text{mm}$ thick $\beta$-barium-borate (BBO) crystal, cut at $\theta = 51.4^\circ$ deg and shielded by a vertically polarized cw single-longitudinal mode MBD-266 Coherent laser ($P = 100\text{mW}$, $\lambda_p = 266\text{nm}$), produces degenerate photon pairs ($\lambda_i = 532\text{nm}$), with horizontal polarization, along correlated directions belonging to the external surface of a cone. Then, a positive lens $L_p$ (focal length $f = 9.5\text{cm}$), located at a distance $f$ from the BBO, transforms the conical emission into a cylindrical one with transverse diameter $D = 12\text{mm}$.

The device collecting SPDC radiation into a set of single mode fibres consists of an integrated system of 8 graded-index lenses (GLs), regularly spaced along the circumference of the degenerate ring. Each GL (Grin-tech, mod. GT-LFRL-100-025-50-NC (532), length=2.381\text{mm}, diameter=1.0mm, numerical aperture=0.5) was glued to a pre-aligned single-mode fiber (SMF). We measured a coupling efficiency of almost 60% for each GL-SMF system. Each GL-SMF pair system, corresponding to two correlated $k$-modes, was then glued in a 8-hole mask, after preliminary optimization of its alignment (cf. inset of Figure 1(c)). In these conditions, for each GL-SMF pair, nearly $7 \cdot 10^6\text{coinc/s were measured over a bandwidth } \Delta \lambda = 4.5\text{nm, within a coincidence window of } 7\text{ns. This corresponds to a coincidences/singles ratio of almost 10\%}.

By this system, the probability that more than one photon pair excites the entire mode set within the time coincidence window is related to the number of accidental coincidences, hence it grows quadratically with the number of mode pairs, while the real coincidences increase linearly. This would contribute to reduce the signal to noise ratio if a larger number of mode pairs would be adopted.

The entanglement existing between the four mode pairs was characterized by the interferometric apparatus sketched in Figure 1(c). For any couple of mode pairs $|i\rangle_A |i\rangle_B$ and $|j\rangle_A |j\rangle_B$ ($i \neq j$) we can define a visibility

$$V_i^{(j)} = (C_{ii} + C_{jj} - C_{ij} - C_{ji})/(C_{ii} + C_{jj} + C_{ij} + C_{ji}),$$

where $C_{ab}$ is the number of coincidences measured between the mode $a$ of photon $A$ and the mode $b$ of photon $B$. Since almost no coincidence was detected between any pair of non-correlated modes (such as, for instance, $|1\rangle_A$ and $|2\rangle_B$) we have $V_i^{(j)} = 0.990 \pm 0.005$ for any couple of mode pairs. Then, any positive value of visibility measured in any superposition basis is sufficient to demonstrate the existence of path-entanglement for any couple of mode pairs. Similarly, in a 2-qubit entangled state, such as $\frac{1}{\sqrt{2}}(|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B)$, the entanglement witness $W = 1 - \sigma_z(A) \sigma_z(B) - \sigma_x(A) \sigma_x(B)$ reveals the existence of entanglement if the sum of visibilities in the standard basis ($|0\rangle$ and $|1\rangle$) and in the superposition basis ($|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$ is larger than $1$. Indistinguishability between the four modes on which each photon is emitted can be achieved by the sequence of beam splitters shown in Figure 1(d). In the actual chained interferometric setup used in the experiment (cf. Figure 1(c)), the $k$-modes of photons $A$ or $B$, corresponding each to a SMF, are matched on the up- and down-side of a common 50:50 beam splitter ($BS_1$), where temporal
mode matching is realized by fine adjustment of three optical paths by spatial delays $\Delta x_1$, $\Delta x_2$, $\Delta x_3$. Precisely, on the Alice’s side, left modes $|I, \ell\rangle_A$ and $|E, \ell\rangle_A$ interfere with the corresponding right modes $|I, r\rangle_A$ and $|E, r\rangle_A$. The same happens with the corresponding Bob’s side modes. The interference on $BS_1$ derives from spatial and temporal indistinguishability obtained by independent adjustment of delays $\Delta x_1$ and $\Delta x_3$, for the internal and external mode sets, respectively. This operation acts as a measurement on the $\ell - r$ qubit since the $BS_1$ output modes are respectively $\frac{1}{\sqrt{2}}(|\ell\rangle + e^{i\phi_{\ell}}|r\rangle)$ ($j = A, B$), regardless of the value of the $E-I$ qubit.

Interference between internal and external modes is the necessary step to test the existence of path-entanglement over the entire set of k-modes. This was performed by creating interference on the second beam splitter $BS_2$, i.e. by making indistinguishable the four events $|1\rangle_A|1\rangle_B$, $|2\rangle_A|2\rangle_B$, $|3\rangle_A|3\rangle_B$ and $|4\rangle_A|4\rangle_B$. Hence, interference can be observed between the output modes of $BS_1$, i.e. by spatial superposition of the $I$ and $E$ modes on $BS_2$. Let’s consider the input ports of the 50:50 beam splitter $BS_2$ in Figure 1c). There are two interfering modes for the Alice and two for the Bob side, coming one from the internal and the other from the external side of the first interferometer. Thus, temporal indistinguishability on $BS_2$ is obtained by varying path delay of the $I$ modes with respect to the $E$ modes. This can be achieved by simultaneously varying $\Delta x_2$ and $\Delta x_3$. Precisely, for any $\Delta x_2 = \delta L$ and $\Delta x_3 = 2\delta L$ (with $\delta L/c$ lower than then pump coherence time), we kept at the same time the interference of the external modes on $BS_1$ and varied the delay of external mode with respect to the internal modes on $BS_2$. In this way we were able to simultaneously achieve interference on both $BS_1$ and $BS_2$. Fine alignment of $BS_1$ and $BS_2$ allowed accurate phase tuning of the entangled state. In the experiment, phase stability could be achieved by mechanical stabilization of the interferometric setup and by thermal heating of the fiber system. All the measurements presented in this paper were performed without temperature stabilization of the fiber apparatus.

The interference patterns given in Figures 2a) and 2b), obtained by respectively varying $\Delta x_1$ and $\Delta x_3$, demonstrate the existence of path-entanglement (on $BS_1$), both for the internal and external modes. As already explained in [19], when the delay is simultaneously changed for two modes (as it happens by changing $\Delta x_1$ for the internal modes in Figure 2a)), the state phase is self-stabilized (in this case it is almost equal to $\pi$). This is evident in Figure 2a), where the small asymmetries come from small temperature fluctuations. On the other hand, by changing only one path, as done by varying $\Delta x_3$ in Figure 2c) (or $\Delta x_2$ in Figure 2c)), phase fluctuates randomly. For the same reason, the FWHM of the interference pattern of Figure 2b) (and 2c)) doubles the one of Figure 2a).

The two experimental results (with visibilities $V = 0.73 \pm 0.02$ and $V = 0.80 \pm 0.02$) demonstrate the en-
tanglement for the internal (2a) and external (2b) modes separately. The presence of a multi-path entanglement over the entire set of modes is demonstrated by the third interference pattern of Figure 2c) \((V = 0.80 \pm 0.02)\) occurring on \(BS_2\) between \(|I, i\rangle_A |I, r\rangle_B\) and \(|E, i\rangle_A |E, r\rangle_B\), and obtained by varying \(\Delta x_2\).

As a further demonstration of multi-path entanglement we compare in Figure 3 the coincidence oscillations obtained for different values of delays \(\Delta x_1, \Delta x_2\) and \(\Delta x_3\). In these measurements the phase was almost constant within each data acquisition (1 s) but varied randomly due to temperature fluctuations from acquisition to acquisition. The first two oscillations correspond to independent interference of the internal (2\(a\)) and external (2\(b\)) modes on \(BS_1\). The third oscillation corresponds to the coincidences measured in conditions of no temporal interference on \(BS_2\) (i.e. by setting \(\Delta x_1 = \Delta x_3 = 0\) and \(\Delta x_2 \neq 0\)). This must be compared with the fourth oscillation data, obtained when \(\Delta x_1 = \Delta x_2 = \Delta x_3 = 0\), that clearly shows the enhancement of signal due to the complete indistinguishability of the four pairs of modes caused by multi-path entanglement. The upper bound of coincidence counts (1130), compared to the maximum that can be obtained in absence of interference (660), corresponds to a total visibility \(V = 0.71 \pm 0.02\).

In conclusion, we presented in this paper the first experimental realization of a multi-path entanglement of two photons. The experiment was performed by collecting four pairs of correlated modes of the degenerate SPDC radiation emitted by a NL crystal cut for Type I phase matching. On this purpose we successfully realized an integrated system of eight graded index lenses coupled to a corresponding set of single mode optical fibers. We tested the entanglement generated over the entire SPDC conical emission of the crystal by using an especially developed chained multi-path interferometer.

The novel device presented in this paper can be extended to an even larger number of \(k\) modes by exploiting the continuum emission of the crystal and used together with an integrated optical circuit to realize miniaturized quantum optical networks of increasing complexity [23].

Besides multiqubit entanglement, this system can be successfully used in other quantum information processes. As an example, it can be adopted in the realization of a “quasi-deterministic” source of heralded single photons emitted along one half of the SPDC modes and triggered by the photons emitted along the other half of correlated modes [26]. The same device allows the efficient multiplexing transmission to many pairs of users and over long fiber distances of time-bin entanglement, which is not affected by thermal or mechanical fiber instabilities [27].

This device is a useful tool for an efficient generation and distribution of entanglement since it allows to maximize the emission of photon pairs for a given value of the pump power. Other configurations can be used in this purpose, for instance those based on a sequence of mirrors and beam splitters or several nonlinear crystal, but at the price of a strong reduction of SPDC coincidences.

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