Estimation of admixture of twelve quark bag state in $^4H$e nucleus

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The $p^4H$e elastic scattering at the energy range from 0.695 to 393 GeV is analyzed in the framework of the Glauber theory. The Glauber amplitudes were evaluated using isospin-averaged nucleon-nucleon amplitudes and the $^4H$e wave function as a superposition of the Gaussian functions. The values of the calculated differential cross sections usually exceed the experimental ones. In order to overcome the discrepancy, it is assumed following to the paper by L.G. Dakno and N.N. Nikolaev (Nucl. Phys. A436 (1985) 653) that the ground state wave function of $^4H$e has an admixture of a twelve quark bag. Neglecting all transition amplitudes, the proton - 12q bag scattering amplitude was chosen in a simple gaussian form. The inclusion of the 12q bag leads to decreasing the $p^4H$e differential cross section and to a shift of the dip position to a large values of $t$ what is needed for a successful description of the experimental data. While fitting the data it is found that the weight of the 12q bag state in the ground state of the $^4H$e nucleus is $\sim 10.5\%$, $\sigma_{total}^{p-12q} \sim 34$ mb, and the slope parameter of the $p - 12q$ bag elastic scattering is $\sim 23 (GeV/c)^{-2}$. Inelastic shadowing is not taken into account at the calculations.

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Introduction

The study of structure of light nuclei such as $^4$He, $^6$He, $^{11}$Li and so on is very popular now. There is a big progress in understanding the structure of light exotic nuclei $^6$He, $^{11}$Li, ... . The deuteron structure is a subject of continuous discussions. Only few hypotheses about the structure of $^4$He exist now. In paper by L.G. Dakno and N.N. Nikolaev [1] it was assumed and shown that 12% admixture of twelve quark bag configuration in the ground state wave function of the $^4$He nucleus allows one to understand the irregularities of proton -$^4$He elastic scattering at high energies. We believe that the hypothesis will permit to describe other reactions $d + ^4$He, $^4$He + $^4$He, $^4$He + C, ... etc. The matter is the carbon and oxygen nuclei are considered as strong clustering nuclei, consisting of $\alpha$ particles. So, the peculiarities of the $^4$He nucleus can manifest themselves in the structure of $^{12}$C and $^{16}$O nuclei.

To show these, one needs to calculate elastic and inelastic scattering of $^4$He on different nuclei. The Glauber diffraction theory [2] of multiple scattering processes has been generally accepted as a suitable framework for such calculations. But it was recognized many years ago that the model predictions have been far from being perfect even for the hadron-nucleus scattering process.

Many authors believe that it is due to inelastic screening, and many attempts have been made to take them into account [3, 4, 5, 6, 7, 8]. According to different calculations, the inelastic screening corrections to the total hadron-nucleus cross sections are at the level of 2–5%. It is not enough to describe the $p^4$He scattering. Inclusion of the corrections into calculations of the $p^4$He elastic scattering leads to a shift of the first diffraction minimum to low values of the momentum transfer, $t$. But a good description of the cross sections demands shift of the minimum to large values of $t$. In order to solve the problem at the first step of our study, we will omit the corrections.

The content of the paper is as follows: Sec. 1 describes calculation of the $^4$He form factor with different parametrizations of the ground state wave function. Sec. 2 gives calculations of the $p^4$He elastic scattering amplitude and differential cross section with these parametrizations. In Sec. 3 we include the twelve quark bag admixture and fit the parameters of the twelve quark bag using experimental data. In the last Sec. we summarize our results.

1 Form-factor of $^4$He

The main characteristic properly of a nucleus is a nuclear form-factor.

$$F(q) = \int e^{iqr_1} |\psi(r_1, \ldots, r_A)|^2 \prod_{i=1}^A dr_i,$$

where $\psi$ is the wave function of a nucleus in the ground state, $A$ – a mass number of the nucleus, $r_1$, $r_2$, ... – radius vectors of nuclear nucleons, $q$ - momentum transfers. It is very often assumed in Glauber calculations that the square module of $\psi$ can be represented as

$$|\psi(r_1, \ldots, r_A)|^2 = (2\pi)^3 \rho_c \delta \left( \sum_{i=1}^A r_i \right) \prod_{i=1}^A \varphi(r_i).$$
The δ-function is introduced in order to satisfy the obvious condition

\[
\left( \sum_{i=1}^{A} \vec{r}_i \right) = 0.
\]  

(3)

For the \(^4\text{He}\) nucleus in the paper [1] the following parametrizations of \(\varphi(\vec{r})\) were proposed:

(A) \(\varphi(\vec{r}) = \exp[-\vec{r}^2/R_1^2]\),

(B) \(\varphi(\vec{r}) = \exp[-\vec{r}^2/R_1^2] - D_1 \exp[-\vec{r}^2/R_2^2]\),

(C) \(\varphi(\vec{r}) = (\exp[-\vec{r}^2/2R_1^2] - D_1 \exp[-\vec{r}^2/2R_2^2])^2\),

(D) \(\varphi(\vec{r}) = \exp[-\vec{r}^2/R_1^2] + D_1 \exp[-\vec{r}^2/R_2^2] - (1 + D_1 - D_2) \exp[-\vec{r}^2/R_3^2]\).

The parameters are given in Table 1.

|   | \(R_1^2\) (GeV/c\(^{-2}\)) | \(R_2^2\) (GeV/c\(^{-2}\)) | \(R_3^2\) (GeV/c\(^{-2}\)) | \(D_1\) | \(D_2\) |
|---|-----------------|-----------------|-----------------|-------|-------|
| A | 51.01           |                 |                 |       |       |
| B | 48.07           | 3.67            |                 |   1.0 |       |
| C | 47.29           |                 | 1.6             |   1.6 |       |
| D | 62.06           | 19.0            | 10.13           | 3.79  | 0.31  |

We will use a general form for the function \(\varphi\) as

\[
\varphi(\vec{r}) = \sum_{i=1}^{N} C_i e^{\vec{r}^2/R_i^2}.
\]  

(4)

In Eq. (2) \(\rho_c\) is the normalization constant determined from the condition

\[
\int |\psi(\vec{r}_1, \ldots, \vec{r}_A)|^2 \prod_{i=1}^{A} d^3r_i = 1.
\]  

(5)

Substituting Eq. (2) in the normalization condition (3), we have

\[
\rho_c (2\pi)^3 \int \delta \left( \sum_{i=1}^{4} \vec{r}_i \right) \prod_{i=1}^{4} \varphi(\vec{r}_i) d^3r_i = 1.
\]  

(6)

Using the following representation of the δ function

\[
\delta \left( \sum_{i=1}^{4} \vec{r}_i \right) = \frac{1}{(2\pi)^3} \int d^3\alpha \ e^{i\vec{\alpha} \cdot (\sum_{i=1}^{4} \vec{r}_i)},
\]

the Eq. (6) can be re-written as

\[
\rho_c (2\pi)^3 \left( \frac{1}{2\pi} \right)^3 d^3\alpha \ e^{i\vec{\alpha} \cdot (\sum_{i=1}^{4} \vec{r}_i)} \prod_{i=1}^{4} \varphi(\vec{r}_i) d^3r_i = 1
\]  

(7)
Then
\[ \rho_c^{-1} = \int d^3\alpha \prod_{i=1}^{4} e^{i \vec{\alpha} \cdot \vec{r}_i} \varphi(\vec{r}_i) d^3 r_i = \int d^3\alpha \prod_{i=1}^{4} e^{i \vec{\alpha} \cdot \vec{r}_i} \left( \sum_{j=1}^{N} C_j e^{\vec{r}_i^2 / R_j^2} \right) d^3 r_i. \tag{8} \]

Integration with respect to \( r_i \) gives
\[ \rho_c^{-1} = \int d^3\alpha \prod_{i=1,j=1}^{4} \sum_{i=1}^{N} C_j \left( \pi R_j^2 \right)^{3/2} e^{-\alpha^2 R_j^2 / 4} = \int d^3\alpha \left( \sum_{j=1}^{N} C_j \left( \pi R_j^2 \right)^{3/2} e^{-\alpha^2 R_j^2 / 4} \right)^4 \]
\[ = \int d^3\alpha \sum_{i_1,i_2,i_3,i_4} C_{i_1} C_{i_2} C_{i_3} C_{i_4} \left( \prod_{s=1}^{4} \left( \pi R_{i_s}^2 \right)^{3/2} \right) e^{-\frac{\alpha^2}{4} \left( \sum_{j=1}^{4} R_{i_j}^2 \right)} \tag{9} \]
and final integration with respect to \( \alpha \) yields
\[ \rho_c^{-1} = \sum_{i_1,i_2,i_3,i_4} C_{i_1} C_{i_2} C_{i_3} C_{i_4} \left( \prod_{s=1}^{4} \left( \pi R_{i_s}^2 \right)^{3/2} \right) \left( \frac{4\pi}{\sum_{j=1}^{4} R_{i_j}^2} \right)^{3/2}. \tag{10} \]

The one-particle density function is determined as
\[ \rho(\vec{r}) = \int |\psi(\vec{r}, \vec{r}_2, \vec{r}_3, \vec{r}_4)|^2 d^3 r_2 d^3 r_3 d^3 r_4, \tag{11} \]
and can be calculated in an analogous way. The functions \( \rho(\vec{r}) \) corresponding to the parametrizations (A – D) of the wave function are shown in Fig. 1. All densities are close to each other at large values of \( r \), and they are different in the nucleus center. So, the parametrizations take various short range \( NN \) correlations into account.

![Figure 1: The one-particle density of the \(^4\text{He}\)](image-url)
Performing nearly the same calculations, we have the following expression for the form factor:

\[
F(\vec{q}) = \int e^{i\vec{q}\cdot\vec{r}_1} (2\pi)^3 \rho_c \delta \left( \sum_{i=1}^{4} \vec{r}_i \right) \prod_{i=1}^{4} \varphi(\vec{r}_i) d^3r_i
\]

\[
= (2\pi)^3 \rho_c \int \frac{d^3\alpha}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}_1} e^{i\vec{a}\cdot(\sum_{i=1}^{4} \vec{r}_i)} \prod_{i=1}^{4} \left( \sum_{j=1}^{N} C_j e^{-R_j^2/\tilde{R}_j^2} \right) d^3\alpha
\]

\[
= \rho_c \int d^3\alpha \left( \sum_{j=1}^{N} C_j \left( \pi R_j^2 \right)^{3/2} e^{\frac{\vec{q}^2}{4}(\vec{q}+\vec{a})^2} \right) \prod_{i=2}^{4} \left( \sum_{j=1}^{N} C_j \left( \pi R_j^2 \right)^{3/2} e^{-\frac{R_j^2}{4}} \right) d^3\alpha
\]

Integrating it with respect to \( r_i \), we obtain

\[
F(\vec{q}) = \rho_c \int d^3\alpha \left( \sum_{j=1}^{N} C_j \left( \pi R_j^2 \right)^{3/2} e^{\frac{\vec{q}^2}{4}(\vec{q}+\vec{a})^2} \right) \prod_{i=2}^{4} \left( \sum_{j=1}^{N} C_j \left( \pi R_j^2 \right)^{3/2} e^{-\frac{R_j^2}{4}} \right) d^3\alpha
\]

\[
= \rho_c \int d^3\alpha \left( \sum_{j=1}^{N} C_{i_1} C_{i_2} C_{i_3} C_{i_4} \prod_{j=1}^{4} \left( \pi R_j^2 \right)^{3/2} e^{\frac{\vec{q}^2}{4}(\vec{q}+\vec{a})^2} \right) \prod_{j=1}^{4} \left( \sum_{j=1}^{N} C_j \left( \pi R_j^2 \right)^{3/2} e^{-\frac{R_j^2}{4}} \right)
\]

\[
= \rho_c \sum_{i_1,i_2,i_3,i_4=1}^{N} C_{i_1} C_{i_2} C_{i_3} C_{i_4} \prod_{j=1}^{4} \left( \pi R_j^2 \right)^{3/2} \left( \frac{4\pi}{\sum_{j=1}^{4} R_j^2} \right)^{3/2} \exp \left( -\frac{R_j^2 q^2}{4} \right)
\]

The charge form factor, \( F_{ch}(\vec{q}) \), of the \(^4\text{He}\) is connected with \( F(\vec{q}) \),

\[
F_{ch}(\vec{q}) = F(\vec{q}) G_N(\vec{q}),
\]

where \( G_N(\vec{q}) \) is the nucleon charge form factor, \( G_N(\vec{q}) = G_p(\vec{q}) + G_n(\vec{q}) \). \( G_p \) is the proton form factor chosen in the dipole form, \( G_p(t) = (1 - t/0.71)^{-2} \). \( G_n \) is the neutron form factor, \( G_n(\vec{q}) = (1 + r_1^2 q^2)^{-2} - (1 + r_2^2 q^2)^{-2} \), where \( r_1^2 = 1.24 \text{ (GeV/c)}^{-2}, r_2^2 = 1.50 \text{ (GeV/c)}^{-2} \). \( t = -q^2 \) is the four momentum transfer in \( \text{(GeV/c)^2} \).

In Fig. 2 the charge form factor calculations at the different parametrizations \((B - D)\) are compared with the experimental data of R.F. Frosh et. al. \( [13] \). The charge form factor predicted by parametrization \( A \) is not presented because it does not reproduce the data at \( q^2 > 0.35 \text{ (GeV/c)^2} \). As seen, at small values of \( t \) all parametrizations give the same good description of the data. They are different only at large values of \( t \) due to the difference of the corresponding one-particles densities in the center of the nucleus (see Fig. 1). We consider parametrization \( D \) as the best one though it gives a dip position at a somewhat smaller value of \( t \) than it is needed for a perfect description of the data. We think that the inclusion of the twelve quark bag component of the ground state wave function will not change the results drastically (see consideration in Ref. [14]).
2 The differential elastic cross section

The Glauber amplitude for hadron-nucleus scattering has a form [2]:

\[ F_{1A}(\vec{q}) = \frac{ip}{2\pi} \int d^2b \ e^{i\vec{q}\cdot\vec{b}} \langle \psi_f | 1 - \prod_{j=1}^{A} \left( 1 - \gamma(\vec{b} - \vec{s}_j) \right) | \psi_i \rangle, \]  

(15)

where \( \vec{b} \) is the impact parameter, \( p \) is the momentum of the projectile hadron, \( \psi_i \) and \( \psi_f \) are initial and final states wave functions, respectively. \( \gamma \) is the NN elastic scattering amplitude in the impact parameter representation. The corresponding differential cross section is given as

\[ \frac{d\sigma}{d\Omega} = |F_{1A}|^2. \]  

(16)

In the case of the elastic \( p^4He \) scattering the amplitude \( F_{1A} \) given by Eq. (15) can be re-written as

\[ F_{1A}(\vec{q}) = \frac{ip}{2\pi} \rho_c \int d^2b \ d^3\alpha e^{i\vec{q}\cdot\vec{b}} \left[ 1 - \prod_{j=1}^{4} \left( 1 - \gamma(\vec{b} - \vec{s}_j) \right) \right] |\psi(\vec{r}_1, \ldots, \vec{r}_4)|^2 \prod_{j=1}^{4} d^2r_j. \]  

(17)

Substituting Eq. (2) in Eq. (17) gives

\[ F_{1A}(\vec{q}) = \frac{ip}{2\pi} \rho_c \int d^2b \ d^3\alpha e^{i\vec{q}\cdot\vec{b}} \left[ 1 - \prod_{j=1}^{4} \left( 1 - \gamma(\vec{b} - \vec{s}_j) \right) \right] e^{i\vec{\alpha}\cdot\sum_{j=1}^{4} \vec{r}_j} \prod_{j=1}^{4} \phi(\vec{r}_j) d^3r_j, \]  

(18)

where \( \vec{r} = \vec{s} + \vec{z}, \vec{z} \) is the component of the position vector \( \vec{r} \) in the direction along the projectile momentum \( \vec{p} \). We assume it is the direction of the z-axis. Taking into account...
we can write the amplitude as a sum of the multiple scattering terms. Every term can be calculated separately if \( \gamma \) is chosen as

\[
\gamma(\bar{b}) = \beta e^{-\bar{p}^2/2B_{NN}},
\]

where \( \beta = (\sigma_{NN}^\text{tot}(1 - i\alpha_{NN}) / (4\pi B_{NN}) \), \( \sigma_{NN}^\text{tot} \) is the NN total cross section, \( B_{NN} \) - the slope parameter of the NN differential elastic cross section at zero momentum transfer, \( \alpha_{NN} \) - the ratio of the real to imaginary parts of the NN elastic scattering amplitude at zero momentum transfer. Then, the first term will be

\[
F_{14}^{(1)} = \int d^2 b \ d^3 \alpha \left( \sum_{k_1=1}^4 \gamma(\bar{b} - \bar{s}_{k_1}) e^{i\bar{q} \cdot \bar{b}} \right) \left( \prod_{i=1}^4 C_i \ e^{i\bar{\alpha} \cdot \bar{r}_j} e^{-\bar{r}_j^2/R_{ij}^2} d^3 r_j \right)
\]

\[
= \int d^2 b \ d^3 \alpha e^{i\bar{q} \cdot \bar{b}} \left( \sum_{k_1=1}^4 \gamma(b - s_{k_1}) \right) \left( \sum_{i=1}^N C_i e^{i\bar{\alpha} \cdot \bar{r}_j} e^{-\bar{r}_j^2/R_{ij}^2} d^3 r_j \right)
\]

\[
\prod_{k_2,k_3,k_4} \left( \sum_{i=1}^N C_i e^{i\bar{\alpha} \cdot \bar{r}_{k_2} e^{-\bar{r}_{k_2}^2/R_{k_2}^2} d^3 r_{k_2}} \right)
\]

\[
= \sum_{k_1=1}^4 \int d^2 b \ d^3 \alpha \gamma(b - s_{k_1}) e^{i\bar{q} \cdot \bar{b}} \left( \sum_{i=1}^N C_i e^{i\bar{\alpha} \cdot \bar{r}_k} e^{-\bar{r}_k^2/R_{ik}^2} d^3 r_k \right)
\]

\[
\prod_{k_2,k_3,k_4} \left( \sum_{i=1}^N C_i e^{i\bar{\alpha} \cdot \bar{r}_{k_2} e^{-\bar{r}_{k_2}^2/R_{k_2}^2} d^3 r_{k_2}} \right)
\]
\[
\text{where } \beta = 4 = 4 \beta_k \left( \sum_{i=1}^{N} C_i e^{(b-s_k_1)^2} e^{i\alpha t_k_1} e^{-r_{k_1}^2/R_k^2} \right)
\]

\[
\prod_{k_2, k_3, k_4} \left( \sum_{i=1}^{N} C_i e^{i\alpha t_k_4} e^{-r_{k_4}^2/R_k^2} \right)
\]

\[
= 4 \sum_{k_1=1}^{N} \beta \int d^2 b \, d^3 \alpha e^{i\vec{q} \cdot \vec{b}} \left( \sum_{i=1}^{N} C_i e^{(b-s_k_1)^2} e^{i\alpha t_k_1} e^{-r_{k_1}^2/R_k^2} \right)
\]

\[
\prod_{k_2, k_3, k_4} \left( \sum_{i=1}^{N} C_i \left( \pi R_i^2 \right)^{3/2} e^{-\alpha^2 R_i^2/2} \right)
\]

\[
= 4 \sum_{k_1=1}^{N} \beta \int d^2 b \, d^3 \alpha e^{i\vec{q} \cdot \vec{b}} \left( \sum_{i=1}^{N} C_i \left( \pi R_i^2 \right)^{3/2} e^{-\alpha^2 R_i^2/2} \right) \frac{1}{\pi} \right)^2 \left( \sum_{i=1}^{N} C_i e^{-b^2/2B} e^{-\alpha^2 R_i^2/4} \left( \frac{2\pi BR_i^2}{2B + R_i^2} \right) \right)
\]

\[
\exp \left[ -\left( \frac{1}{R_i^2} + \frac{1}{2B} \right) \left( s_{k_1}^2 - \frac{1}{2} \left( \frac{b}{B} + i\alpha_2 \right) \left( \frac{2\pi BR_i^2}{2B + R_i^2} \right) \right) \right]
\]

\[
= 4 \beta \sum_{k_1=1}^{N} \left( \prod_{j=1}^{4} \left( \pi R_k^2 \right)^{3/2} \right) \left( \frac{2B}{2B + R_k^2} \right) \int d^2 b \, d^3 \alpha e^{i\vec{q} \cdot \vec{b}}
\]

\[
e^{-\frac{\alpha^2}{4} \sum_{j=1}^{4} R_k^2} e^{-\frac{3}{4} \left( \sum_{j=1}^{4} R_k^2 \right) e^{-b/B} e^{i\alpha t_j} \left( \frac{R_k^2}{R_k^2 + 2B} \right)} \frac{1}{\pi} \left( \frac{R_k^2}{R_k^2 + 2B} \right) e^{-\frac{\alpha^2}{4} \left( \sum_{j=1}^{4} R_k^2 \right)}
\]

\[
= 4 \beta \sum_{k_1, j, l=1}^{4} C_{k_1} C_{k_2} C_{k_3} C_{k_4} \left( \prod_{j=1}^{4} \left( \pi R_k^2 \right)^{3/2} \right) \left( \frac{2B}{2B + R_j^2} \right) \left( \frac{4\pi}{\sum_{j=1}^{4} R_k^2} \right) \left( \frac{4\pi}{M_1} \right) e^{-q^2/4M_1} \exp \left[ q^2 S_1^2/4M_1 H_1 \right]
\]

Where

\[
M_1 = \frac{1}{B} - \frac{1}{B} \frac{R_k^2}{R_k^2 + 2B} = \frac{1}{R_k^2 + 2B}, \quad S_1 = \frac{R_k^2}{R_k^2 + 2B}
\]

\[
F_{14}^{(1)} = 4 \beta \sum_{k_1, j=1}^{4} C_{k_1} C_{k_2} C_{k_3} C_{k_4} \left( \prod_{j=1}^{4} \left( \pi R_k^2 \right)^{3/2} \right) \left( \frac{2B}{2B + R_j^2} \right) \left( \frac{4\pi}{\sum_{j=1}^{4} R_k^2} \right) \frac{\pi}{M_1} \frac{4\pi}{H_1} e^{-q^2/4M_1} \exp \left[ q^2 S_1^2/4M_1 H_1 \right]
\]

(24)
The second term also will be,

\[
F_{14}^{(2)} = \int d^2 b \, d^3 \alpha \left( \sum_{k_1,k_2=1}^{4} \gamma(b - s_{k_1}) \gamma(b - s_{k_2}) \right) e^{i\vec{q} \cdot \vec{b}} \prod_{j=1}^{N} \sum_{i=1}^{4} C_j e^{i\vec{a}_i \cdot \vec{r}_j} e^{-r_{ij}^2/R_i^2} d^3 r_i \\
= \int d^2 b \, d^3 \alpha e^{i\vec{q} \cdot \vec{b}} \left( \sum_{k_1,k_2=1}^{4} \gamma(b - s_{k_1}) \gamma(b - s_{k_2}) \right) \prod_{k_1,k_2} \left( \sum_{i=1}^{N} C_i e^{i\vec{a}_i \cdot \vec{r}_{k_j}} e^{-r_{ij}^2/R_i^2} d^3 r_{k_j} \right) \\
= \sum_{k_1,k_2=1}^{4} \int d^2 b \, d^3 \alpha \gamma(b - s_{k_1}) \gamma(b - s_{k_2}) e^{i\vec{q} \cdot \vec{b}} \prod_{k_1,k_2} \left( \sum_{i=1}^{N} C_i e^{i\vec{a}_i \cdot \vec{r}_{k_j}} e^{-r_{ij}^2/R_i^2} d^3 r_{k_j} \right) \\
= \sum_{k_1,k_2=1}^{4} \beta^2 \int d^2 b \, d^3 \alpha e^{i\vec{q} \cdot \vec{b}} \prod_{k_1,k_2} \left( \sum_{i=1}^{N} C_i \gamma(b - s_{k_1}) e^{i\vec{a}_i \cdot \vec{r}_{k_j}} e^{-r_{ij}^2/R_i^2} d^3 r_{k_j} \right) \\
= \sum_{k_1,k_2=1}^{4} \beta^2 \int d^2 b \, d^3 \alpha e^{i\vec{q} \cdot \vec{b}} \left( \sum_{i=1}^{N} C_i \left( \frac{\pi R_i^2}{2} \right)^{3/2} e^{-\alpha^2 R_i^2/2} \right)^2 \\
\prod_{k_1,k_2} \left( \sum_{i=1}^{N} C_i e^{-\beta^2/2B} e^{-\alpha_i^2 R_i^2/4} \left( \pi R_i^2 \right)^{1/2} \left( \frac{2BR_i^2}{2B + R_i^2} \right) \right) \\
= \exp \left[ - \left( \frac{1}{R_i^2} + \frac{1}{2B} \right) \left( \sum_{i=1}^{N} C_i \left( \pi R_i^2 \right)^{3/2} e^{-\alpha^2 R_i^2/2} \right)^2 \right] \\
= \sum_{k_1,k_2}^{4} \beta^2 \int d^2 b \, d^3 \alpha e^{i\vec{q} \cdot \vec{b}} \left( \sum_{i=1}^{N} C_i \left( \pi R_i^2 \right)^{3/2} e^{-\alpha^2 R_i^2/2} \right)^2 \\
\prod_{k_1,k_2} \left( \sum_{i=1}^{N} C_i e^{-\beta^2/2B} e^{-\alpha_i^2 R_i^2/4} \left( \pi R_i^2 \right)^{1/2} \left( \frac{2BR_i^2}{2B + R_i^2} \right) \right) \\
= 6\beta^2 \sum_{k_1,k_2}^{4} C_{k_1} C_{k_2} C_{k_3} C_{k_4} \prod_{j=1}^{N} \left( \pi R_{k_j}^2 \right)^{3/2} \prod_{j=1}^{N} \left( \frac{2B}{2B + R_i^2} \right)
\begin{align*}
\int d^3 b \, d^3 \alpha e^{i\vec{q}\cdot\vec{b}} e^{-\frac{\alpha^2}{4} \sum_{j=1}^{4} R_{k_j}^2} e^{-\frac{\alpha^2}{4} \left( R_{k_3}^2 + R_{k_4}^2 \right)}
\cdot \left( \sum_{j=1}^{2} \frac{R_{k_j}^2}{R_{k_j}^2 + 2B} \right) e^{-\frac{B\alpha^2}{2} \left( \sum_{j=1}^{2} \frac{R_{k_j}^2}{R_{k_j}^2 + 2B} \right)} e^{-\frac{B\alpha^2}{2} \left( \sum_{j=1}^{2} \frac{R_{k_j}^2}{R_{k_j}^2 + 2B} \right)}
\cdot \exp \left[ -\frac{\pi R_{k_3}^2}{2} \right] = 6\beta^2 \sum_{k_{ij}, j=1}^{4} C_{k_1} C_{k_2} C_{k_3} C_{k_4} \left( 4 \prod_{j=1}^{4} \left( \pi R_{k_j}^2 \right)^{3/2} \right) \prod_{j=1}^{2} \left( \frac{2B}{2B + R_{j}^2} \right) \left( \frac{4\pi}{\sum_{j=1}^{4} R_{k_j}^2} \right) \frac{1}{2}
\int d^3 b \, d^3 \alpha e^{-B\alpha^2 S_2/2} e^{-\frac{\alpha^2}{4} \left( R_{k_3}^2 + R_{k_4}^2 \right)} e^{-b/B} e^{-\left( i(q + \vec{\alpha}_2 S_2) \cdot \vec{b} / M_2 + (i(q + \vec{\alpha}_2 S_2) / M_2)^2 \right)} 
\exp \left[ -M_2 \left( b^2 - i \left( \vec{q} + \vec{\alpha}_2 S_2 \right) \cdot \vec{b} / M_2 + (i(q + \vec{\alpha}_2 S_2) / M_2)^2 \right) \right],
\end{align*}

where

\begin{align*}
M_2 &= \frac{1}{B} - \frac{1}{2B} \sum_{j=1}^{2} \frac{R_{k_j}^2}{R_{k_j}^2 + 2B} = \sum_{j=1}^{2} \frac{1}{R_{k_j}^2 + 2B}, 
S_2 &= \sum_{j=1}^{2} \frac{R_{k_j}^2}{R_{k_j}^2 + 2B}. 
\end{align*}

By the same way the other terms will be

\begin{align*}
F_{14}^{(2)} &= 6\beta^2 \sum_{k_{ij}, j=1}^{4} C_{k_1} C_{k_2} C_{k_3} C_{k_4} \left( 4 \prod_{j=1}^{4} \left( \pi R_{k_j}^2 \right)^{3/2} \right) \prod_{j=1}^{2} \left( \frac{2B}{2B + R_{j}^2} \right) \left( \frac{4\pi}{\sum_{j=1}^{4} R_{k_j}^2} \right) \frac{1}{2}
\left( \frac{\pi}{M_2} \right) \left( \frac{4\pi}{H_2} \right) e^{-q^2/4M_2} \exp \left[ q^2 S_2^2/4M_2 H_2 \right],
\end{align*}

\begin{align*}
H_2 = R_{k_3}^2 + R_{k_4}^2 + 2BS_2 + \frac{S_2^2}{M_2}. 
\end{align*}

\begin{align*}
F_{14}^{(3)} &= 4\beta^3 \sum_{k_{ij}, j=1}^{4} C_{k_1} C_{k_2} C_{k_3} C_{k_4} \left( 4 \prod_{j=1}^{4} \left( \pi R_{k_j}^2 \right)^{3/2} \right) \prod_{j=1}^{2} \left( \frac{2B}{2B + R_{j}^2} \right) \left( \frac{4\pi}{\sum_{j=1}^{4} R_{k_j}^2} \right) \frac{1}{2}
\left( \frac{\pi}{M_3} \right) \left( \frac{4\pi}{H_3} \right) e^{-q^2/4M_3} \exp \left[ q^2 S_3^2/4M_3 H_3 \right],
\end{align*}

\begin{align*}
M_3 &= \frac{3}{2B} - \frac{1}{2B} \sum_{j=1}^{3} \frac{R_{k_j}^2}{R_{k_j}^2 + 2B} = \sum_{j=1}^{3} \frac{1}{R_{k_j}^2 + 2B}, 
S_3 &= \sum_{j=1}^{3} \frac{R_{k_j}^2}{R_{k_j}^2 + 2B}. 
\end{align*}

\begin{align*}
F_{14}^{(4)} &= \beta^4 \sum_{k_{ij}, j=1}^{4} C_{k_1} C_{k_2} C_{k_3} C_{k_4} \left( 4 \prod_{j=1}^{4} \left( \pi R_{k_j}^2 \right)^{3/2} \right) \prod_{j=1}^{2} \left( \frac{2B}{2B + R_{j}^2} \right) \left( \frac{4\pi}{\sum_{j=1}^{4} R_{k_j}^2} \right) \frac{1}{2}
\left( \frac{\pi}{M_4} \right) \left( \frac{4\pi}{H_4} \right) e^{-q^2/4M_4} \exp \left[ q^2 S_4^2/4M_4 H_4 \right],
\end{align*}

\begin{align*}
M_4 &= \frac{2}{B} - \frac{1}{2B} \sum_{j=1}^{4} \frac{R_{k_j}^2}{R_{k_j}^2 + 2B} = \sum_{j=1}^{4} \frac{1}{R_{k_j}^2 + 2B}, 
S_4 &= \sum_{j=1}^{4} \frac{R_{k_j}^2}{R_{k_j}^2 + 2B}. 
\end{align*}

\begin{align*}
H_4 = 2BS_4 + \frac{S_4^2}{M_4}. 
\end{align*}
In many experimental papers [11, 12, 13] the authors included the Coulomb scattering amplitude in a simple way in order to extract the nuclear total \( p^4He \) cross section,

\[
\frac{d\sigma}{dt} = |F_c e^{i\phi} + F_N|^2,
\]

where

\[
F_c(t) = \frac{4\alpha\sqrt{\pi}}{\beta t} G_p(t) G_{He}(t),
\]

\( \alpha = 1/137 \) is the fine structure constant, \( \beta = v/c \) is the proton velocity in the laboratory system, \( G_p(t), G_{He}(t) \) are the electromagnetic form factor of the proton and \( He \), respectively,

\[
G_{He}(t) = \exp \left[ \frac{r_{He}^2 t}{6} \right],
\]

\( r_{He} = r_e - r_p \), \( r_e = 1.67 \text{ fm}, \ r_p = 0.812 \text{ fm} \).

\( \phi = 2\alpha\beta^{-1} [\ln (B|t|) + 0.577...] \).

\( B = 29 (GeV/c)^{-2} \) and \( t = -q^2 \). \( F_n \) is the nuclear amplitude written at small \( t \) in the form

\[
F_n = \frac{\sigma_{tot}^{NN}}{4\hbar\sqrt{\pi}} (i + \alpha) e^{\frac{1}{2}Bt}.
\]

We follow the same way replacing \( F_N \) by the Glauber scattering amplitude.

The available experimental data on the \( p^4He \) elastic scattering have been presented by G.N. Velichko et. al. [11] at the energies of 0.695, 0.793, 0.89, 0.991 GeV; by A. Bujak et. al. [12] at the energies of 45, 97, 146, 200, 259, 301, 393 GeV, and by J.P. Burq et al. [13] at the energies of 100, 150, 250, 300 GeV. To calculate the Glauber amplitudes at these energies, it is needed to have the values of the nucleon-nucleon amplitude parameters \( \sigma_{tot}^{NN}, B_{NN} \) and \( \alpha_{NN} \). \( \sigma_{tot}^{NN} \) was estimated as an average of the neutron-proton total cross section, \( \sigma_{np}^{tot} \), and the proton-proton total cross section, \( \sigma_{pp}^{tot} \), which can be taken from the compilation of the experimental data [14].

More complicated situation is with \( B_{NN} \). There are only few experimental data, and it is not enough for all energies. Thus we have used another way to evaluate \( B_{NN} \) from the total and elastic \( NN \) cross sections. At chosen form of \( \gamma(\vec{b}) \) (see Eq. 22) the elastic \( NN \) cross section, \( \sigma_{NN}^{el} \), is given as

\[
\sigma_{NN}^{el} = \frac{1}{p^2} \left( \frac{p}{2\pi} \right)^2 \int \gamma(\vec{b}_1) e^{i\vec{q} \cdot \vec{b}_1} \gamma^*(\vec{b}_2) e^{-i\vec{q} \cdot \vec{b}_2} d^2b_1 d^2b_2 d^2q
\]

\[
= \frac{1}{(2\pi)^2} \int e^{i\vec{q} \cdot (\vec{b}_1 - \vec{b}_2)} \gamma(\vec{b}_1) \gamma^*(\vec{b}_2) d^2b_1 d^2b_2 d^2q
\]

\[
= \int \delta(\vec{b}_1 - \vec{b}_2) \gamma(\vec{b}_1) \gamma^*(\vec{b}_2) d^2b_1 d^2b_2 = \int |\gamma(\vec{b})|^2 d^2b
\]

\[
= \left( \frac{\sigma_{tot}^{NN}}{4\pi B_{NN}} \right)^2 (1 + \alpha_{NN}^2) \int e^{-\frac{q^2}{\pi B_{NN}}} d^2b
\]

\[
= \left( \frac{\sigma_{tot}^{NN}}{4\pi B_{NN}} \right)^2 (1 + \alpha_{NN}^2) \pi B_{NN} = \frac{(\sigma_{tot}^{NN})^2}{16\pi B_{NN}} (1 + \alpha_{NN}^2). \quad (34)
\]
Since $\alpha_{NN}$ is very small, we neglect it in our calculations. In this case $B_{NN}$ can be calculated as

$$B_{NN} = \frac{(\sigma_{NN}^{tot})^2}{16\pi\sigma_{el}^{el}_{NN}}. \quad (35)$$

$\sigma_{NN}^{el}$ was taken from the compilation of the experimental data [14] as an average of $pn$ and $pp$ cross sections.

The values of $\alpha_{NN}$ for all mentioned above energies were extracted from the compilation of the experiential data [15].

Table 2: The parameters used at the calculations of the Glauber amplitudes

| $E_{kin}$ GeV | $\sigma_{el}^{el}_{NN}$ mb | $\sigma_{pp}^{tot}$ mb | $\sigma_{pn}^{tot}$ mb | $\sigma_{NN}^{tot}$ mb | $B_{NN}$ (GeV/c)$^{-2}$ | $\alpha_{NN}$ |
|--------------|----------------|-----------------|----------------|----------------|-----------------|----------------|
| 0.695        | 24.2           | 42.4            | 38.38          | 40.39          | 4.069           | -0.205         |
| 0.795        | 22.5           | 46.8            | 38.56          | 42.68          | 4.134           | -0.1975        |
| 0.890        | 24.4           | 47.3            | 38.73          | 43.01          | 3.872           | -0.19          |
| 0.991        | 24.27          | 47.6            | 39.24          | 43.42          | 3.967           | -0.185         |
| 0.992        | 24.27          | 47.6            | 39.24          | 43.42          | 3.967           | -0.185         |
| 45           | 7.402          | 38.48           | 38.32          | 38.4           | 10.173          | -0.087         |
| 97           | 6.985          | 37.94           | 38.89          | 38.4           | 10.783          | -0.090         |
| 100          | 6.985          | 37.94           | 38.89          | 38.4           | 10.783          | 0.1            |
| 146          | 7.03           | 38.29           | 39.12          | 38.71          | 10.884          | -0.049         |
| 150          | 7.03           | 38.69           | 39.12          | 38.91          | 10.996          | 0.105          |
| 200          | 6.895          | 38.98           | 39.56          | 39.27          | 11.422          | -0.022         |
| 250          | 6.89           | 39.34           | 39.83          | 39.58          | 11.614          | 0.11           |
| 259          | 6.89           | 39.34           | 39.83          | 39.58          | 11.614          | 0.024          |
| 300          | 6.888          | 39.46           | 39.83          | 39.65          | 11.653          | 0.115          |
| 301          | 6.888          | 39.46           | 39.83          | 39.65          | 11.653          | 0.031          |
| 393          | 7.016          | 40.19           | 40.01          | 40.1           | 11.703          | 0.067          |

All the parameter values used for our calculation are presented in Table 2. Typical results of the calculations in comparison with experimental data [11, 12] are shown in Figs. 3, 4. As seen, the model calculations are above the experimental data. The first diffraction minimums are shifted to small $t$. We can confirm now that the model calculations can not reproduce the data with required accuracy. This pushed us to search for modification of the model.

3 The twelve quark bag admixture

Any nucleus consists of $3A$-quarks. In the ground state the quarks are forming clusters, bags and nucleons. Following [1] we assume that the $^4He$ wave function is given as

$$|q_1...q_{12}> = \alpha|NNNN> + \beta|12q>, \quad |\alpha|^2 + |\beta|^2 = 1, \quad (36)$$

where $|q_1...q_{12}>$ is the 12- quark bound state wave function of the $^4He$. $|12q>$ stands for a component left after projecting $|q_1...q_{12}>$ onto the four nucleons state. Since $|NNNN>$ vanishes in the central part of $^4He$, $|12q>$ must strongly peak in the central part of the
According to the assumption of Ref. [1] $NNNN|12q>=0$. Thus we neglect $12q−NNNN$ interference terms that vanish at $q = 0$ in the elastic $p^4He$ scattering amplitude and write

$$F_{14} = (1 − w_{12q}) F_{Gl} + w_{12q} F_{12q},$$

where $F_{Gl}$ is the Glauber amplitude of the $p−4N$ scattering, and $w_{12q}$ is the weight of the $12q$ bag quark state, $w_{12q} = |β|^2$. We take the nucleon - twelve quark bag scattering amplitude in a simple form,

$$F_{12q} = \frac{\sigma_{12q}}{2} e^{b_{12q}/2},$$

where $\sigma_{12q}$ is the $N−12q$ bag total cross section, and $b_{12q}$ is the slope parameter.

We found the parameters $\sigma_{12q}$, $b_{12q}$ and $w_{12q}$ fitting the experimental data [11, 12, 13]. The values are presented in Table 3. As one can see, the parameter uncertainty is very large at low energies ($E_{kin} < 1$ GeV). This means that at the energies one does not need to add anything to the Glauber amplitude. At higher energies the values become more stable excepting the results at 146 GeV.

| $E_{kin}$ (GeV) | $w_{12q}$ | $\sigma_{12q}$ (mb) | $b_{12q}$ (GeV/c)$^{-2}$ | $\chi^2$/NOF |
|----------------|-----------|---------------------|--------------------------|-------------|
| 0.695          | 4.1 ±66.5 | 126.0 ±169.7        | 32.6±100.5               | 72/65       |
| 0.795          | 9.8 ±1.8  | 117.0 ±10.7         | 40 ±228.9                | 45/81       |
| 0.890          | 10.6 ±174.4 | 169.4 ±464.4     | 37.4±164.2               | 56/95       |
| 45             | 8.34 ±0.92 | 32.62 ±10.23       | 20.38±3.90               | 557/127     |
| 97             | 9.23 ±1.31 | 30.83 ±13.37       | 20.46±5.54               | 146/84      |
| 146            | 13.51±0.42 | 65.57 ±2.54        | 32.46±0.52               | 222/84      |
| 200            | 10.51±1.83 | 28.86 ±17.24       | 21.62±7.33               | 285/84      |
| 259            | 9.72 ±2.91 | 29.10 ±30.04       | 22.11±12.69              | 264/86      |
| 301            | 11.08±1.28 | 25.94 ±11.78       | 21.68±5.46               | 173/86      |
| 393            | 10.80±1.59 | 25.73 ±14.95       | 21.14±6.96               | 118/85      |

We have excluded from the fit the data at the energies of 100, 150, 250, and 300 GeV [13]. The data are above the Glauber calculations. Thus at the fitting an unreasonable large weight of 12q-bag (> 50 %) and $\sigma_{12q}$ was obtained. We believe the data are not quite well normalized. To show this, we plot the data at close energies [12] on the same figures 3, 4.

As seen, there is a clear difference between the two groups of experimental data. Maybe, it is due to a normalization error. We do not know a reason of the error. However, one can see that the data by Ref. [13] are falling out from the whole set of the experimental data, and it is not possible to fit them correctly.

The figures show influence of the 12q bag admixture on the differential cross section. The inclusion of the admixture leads to decreasing the Glauber amplitude if $\sigma_{12q}$ is smaller than $\sigma_{pHe}$. In the region of the dip where the Glauber amplitude vanishes, $F_{12q}$ is positive and shifts the dip to a larger values of $t$. So, the hypothesis really allows one to solve the main part of the problem.
Figure 3: The $p^4He$ differential elastic cross sections. The point are the experimental data [12, 13]. The solid and dashed lines are our calculations with and without 12q admixture, respectively.

Clearly, inclusion of the inelastic screening into calculations will lead to decreasing the cross section in the region of small $t$, and to increasing in the region of the large $t$ values. To compensate these, one have to increase $w_{12q}$, $\sigma_{12q}$, and $b_{12q}$. From this point of view we can understand the results of Ref. [1]. According to the Fig. 10 of the Ref. [1], $\sigma_{12q} \sim 140 \text{ mb}$ what is near to the $p4He$ total cross section, the slope parameter of that $F_{12q}$ is larger than ours. As a result, $w_{12q} \sim 12 \%$. We have the average value of $w_{12q} \sim 10.5\%$. So, two values agree quite reasonable with each other. At the same time, our $\sigma_{12q}$ is too small.

Let us mark that our amplitude $F_{12q}$ is more simple than that of the Ref. [1]. It can be easily used for future calculations.
Conclusion

The 12q bag admixture to the ground state wave function of the $^4$He nucleus allows one to describe quite well the elastic $p^4He$ scattering. According to our estimations, the weight of the 12q bag is $\sim 10.5\%$, the proton - 12q bag total cross section is $\sim 34 \text{ mb}$, and the slope parameter of the $p - 12q$ bag elastic scattering is $\sim 23 \, (\text{GeV}/c)^{-2}$.

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References

[1] L.G. Dakno and N.N. Nikolaev, *Nucl. Phys.* A436 (1985) 653.

[2] R.J. Glauber, *Lectures in theoretical physics*. W.E. Brittin, L.G. Dunham (eds.). Vol. 1, p.315, New York: Interscience 1959; A.G. Sitenko, *Ukr. Fiz. Zh.* 4 (1959) 152.

[3] E.S. Abers et al., *Nuovo Cimento* 42A (1966) 365.

[4] V.N. Gribov, *Sov. Phys. JETP* 29 (1969) 483.

[5] J. Pumplin and M. Ross, *Phys. Rev. Lett.* 21 (1968) 1778.

[6] G. Alberi and L. Bertocchi, *Nuovo Cimento* 61A (1969) 201.

[7] D.R. Harrington, *Phys. Rev.* D1 (1970) 260.

[8] C. Quigg and L.L. Wang, *Phys. Lett.* 42 (1973) 314.

[9] R.P. Feynman, *Photon - Hadron Interactions* (1972) W.A.Benjamin, Inc Reading, Massachusetts.

[10] R.F. Frosch, J.S. McCarthy, R.E. Rand, and M.R. Yearian, *Phys. Rev.* 160 (1967) 874.

[11] G.N. Velichko et. al., *Yad. Fiz.* 42 (1985) 1325.

[12] A. Bujak et. al., *Phys. Rev.* D23 (1981) 1895; JINR Preprint E1-81-289 (1981).

[13] J.P. Burk et. al., *Nucl. Phys.* B187 (1981) 205.

[14] V. Flaminio, W.G. Moorhead, D.R.O. Morrison, N. Rivoire, *Compilation of cross–sections: p and p̅ induced reactions* CERN–HERA 84-01 (1984).

[15] O. Benary, L.R. Price, G. Alexander, *NN and ND interactions above 0.5 GeV/c – a compilation* UCRL-20000NN (1970).