Leading finite-size effects on some three-point
correlators in $TsT$-deformed $AdS_5 \times S^5$

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Abstract

We compute the leading finite-size effects on the normalized structure constants in semi-classical three-point correlation functions of two finite-size giant magnon string states and three different types of "light" states - primary scalar operators, dilaton operator with nonzero momentum and singlet scalar operators on higher string levels. This is done for the case of $TsT$-transformed, or $\gamma$-deformed, $AdS_5 \times S^5$ string theory background, dual to $\mathcal{N} = 1$ super Yang-Mills theory in four dimensions, arising as an exactly marginal deformation of $\mathcal{N} = 4$ super Yang-Mills.
1 Introduction

The AdS/CFT duality [1] between string theories on curved space-times with Anti-de Sitter subspaces and conformal field theories in different dimensions has been actively investigated in the last years. A lot of impressive progresses have been made in this field of research based mainly on the integrability structures discovered on both sides of the correspondence (for recent overview on the AdS/CFT duality, see [2]). The most studied example is the correspondence between type IIB string theory on AdS$_5 \times S^5$ target space and the $\mathcal{N} = 4$ super Yang-Mills theory (SYM) in four space-time dimensions. However, many other cases are also of interest, and have been investigated intensively.

Different classical string solutions play important role in checking and understanding the AdS/CFT correspondence [3]. To establish relations with the dual gauge theory, one has to take the semiclassical limit of large conserved charges [4]. A crucial example of such string solution is the so called ”giant magnon”, discovered by Hofman and Maldacena in $R_t \times S^2$ subspace of AdS$_5 \times S^5$ [5]. It gave a strong support for the conjectured all-loop $SU(2)$ spin chain, arising in the dual $\mathcal{N} = 4$ SYM, and made it possible to get a deeper insight in the AdS/CFT duality. Characteristic feature of this solution is that the string energy $E$ and the angular momentum $J_1$ go to infinity, but the difference $E - J_1$ remains finite and related to the momentum of the magnon excitations in the dual spin chain in $\mathcal{N} = 4$ SYM. This string configuration have been extended to the case of dyonic giant magnon, being solution for a string moving on $R_t \times S^3$ and having second nonzero angular momentum $J_2$ [6]. Further extension to $R_t \times S^5$ have been also worked out in [7]. It was also shown there that such type of string solutions can be obtained by reduction of the string dynamics to the Neumann-Rosochatius integrable system, by using a specific ansatz.

An interesting issue to solve is to find the finite-size effect, i.e. $J_1$ large, but finite, related to the wrapping interactions in the dual field theory [8]. For (dyonic) giant magnons living in AdS$_5 \times S^5$ this was done in [9, 10]. The corresponding string solutions, along with the (leading) finite-size corrections to their dispersion relations have been found.

Here, we are going to consider the leading finite-size effects on some three-point correlation functions in $\gamma$-deformed [11] or TsT-transformed [12] AdS$_5 \times S^5$ string theory background. To this end, we will need to use our knowledge about the properties of the finite-size (dyonic) giant magnon solutions on this target space. The corresponding information can be found in [13, 14].

In this paper we will be interested in the case of three-point correlators, when two of the ”heavy” string states are finite-size giant magnons, while the third state is a ”light” one[1]. We will consider three different types of ”light” states: primary scalar operators, dilaton operator with nonzero momentum and singlet scalar operators on higher string levels. The finite-size effects on such correlation functions in TsT-transformed AdS$_5 \times S^5$ have been

[1] The first papers in which three-point correlation functions of two ”heavy” operators and a ”light” operator have been computed are [15, 16].
found in [17, 18]. There, the normalized structure constants in these correlators are given in terms of several parameters and hypergeometric functions of two variables depending on them. On the other hand, it is important to know their dependence on the conserved string charges $J_1$, $J_2$ and the worldsheet momentum $p$, because namely these quantities are related to the corresponding operators in the dual $\mathcal{N} = 1$ SYM, and the momentum of the magnon excitations in the dual spin-chain. That is why, we are going to find this dependence here. Unfortunately, this can not be done exactly for the finite-size case due to the complicated dependence between the above mentioned parameters and $J_1$, $J_2$, $p$. Because of that, we will consider only the leading order finite-size effects on the three-point correlators of this type. Moreover, due to computational complications, we will restrict ourselves to the case of $J_2 = 0$.

This paper is organized as follows. In Sec. 2, we give a short review of the finite-size (dyonic) giant magnon’s solution on $\gamma$-deformed $AdS_5 \times S^5$. Also, we give the corresponding exact semiclassical results for the three-point correlators we are interested in, found in [17, 18]. Sec. 3 is devoted to the computation of the leading order finite-size effects on the three-point correlators given in Sec. 2 in terms of the conserved string angular momentum $J_1$ and the worldsheet momentum $p$. In Sec. 4 we conclude with some final remarks.

2 Finite-size giant magnons
on $TsT$-deformed $AdS_5 \times S^5$ and
some three-point correlators

2.1 Short review of the giant magnon solutions

Investigations on AdS/CFT duality for the cases with reduced or without supersymmetry is of obvious importance and interest. An example of such correspondence between gauge and string theory models with reduced supersymmetry is provided by an exactly marginal deformation of $\mathcal{N} = 4$ SYM and string theory on a $\beta$-deformed $AdS_5 \times S^5$ background suggested by Lunin and Maldacena in [11]. When $\beta \equiv \gamma$ is real, the deformed background can be obtained from $AdS_5 \times S^5$ by the so-called TsT transformation on $S^5$. It includes T-duality on one angle variable, a shift of another isometry variable, then a second T-duality on the first angle [12]. The $AdS_5$ part of the background is untouched, so the conformal invariance remains.

An essential property of the TsT transformation is that it preserves the classical integrability of string theory on $AdS_5 \times S^5$ [12]. The $\gamma$-dependence enters only through the twisted boundary conditions and the level-matching condition. The last one is modified since a closed string in the deformed background corresponds to an open string on $AdS_5 \times S^5$ in general.
The parameter $\tilde{\gamma}$, which appears in the string action, is related to the deformation parameter $\gamma$ as

$$\tilde{\gamma} = 2\pi T\gamma = \sqrt{\lambda} \gamma,$$

where $T$ is the string tension and $\lambda$ is the t’Hooft coupling.

The effect of introducing $\gamma$ on the field theory side of the duality is to modify the superpotential as follows

$$W \propto \text{tr} \left( e^{i\pi\gamma} \Phi_1 \Phi_2 \Phi_3 - e^{-i\pi\gamma} \Phi_1 \Phi_3 \Phi_2 \right).$$

This leads to reduction of the supersymmetry of the SYM theory from $N = 4$ to $N = 1$.

Since we are going to consider three-point correlation functions with two vertices corresponding to giant magnon states, we can restrict ourselves to the subspace $R_t \times S_3^\gamma$ of $AdS_5 \times S_5^\gamma$ background. Then one can show that by using the ansatz

$$t(\tau, \sigma) = \kappa \tau, \quad \theta(\tau, \sigma) = \omega_j \tau + f_j(\xi),$$

$$(2.1)$$

the string Lagrangian in conformal gauge, on the $\gamma$-deformed three-sphere $S_3^\gamma$, can be written as [19] (prime is used for $d/d\xi$)

$$\mathcal{L}_\gamma = \left( \alpha^2 - \beta^2 \right) \left[ \theta'^2 + G \sin^2 \theta \left( f'_1 - \frac{\beta \omega_1}{\alpha^2 - \beta^2} \right)^2 + G \cos^2 \theta \left( f'_2 - \frac{\beta \omega_2}{\alpha^2 - \beta^2} \right)^2 - \alpha^2 \left( \frac{\alpha^2 - \beta^2}{\alpha^2 - \beta^2} \right)^2 G \left( \omega_1^2 \sin^2 \theta + \omega_2^2 \cos^2 \theta \right) + 2\alpha \tilde{\gamma} G \sin^2 \theta \cos^2 \theta \frac{\omega_2 f'_1 - \omega_1 f'_2}{\alpha^2 - \beta^2} \right],$$

$$(2.2)$$

where

$$G = \frac{1}{1 + \tilde{\gamma}^2 \sin^2 \theta \cos^2 \theta}.$$

By using (2.2) and the Virasoro constraints, one can find the following first integrals of the string equations of motion

$$f'_1 = \frac{\Omega_1}{\alpha} \frac{1}{1 - v^2} \left[ \frac{vW - uK}{1 - \chi} - v(1 - \tilde{\gamma}K) - \tilde{\gamma}u\chi \right],$$

$$f'_2 = \frac{\Omega_1}{\alpha} \frac{1}{1 - v^2} \left[ \frac{Kw}{\chi} - uv(1 - \tilde{\gamma}K) - \tilde{\gamma}v^2W + \tilde{\gamma}(1 - \chi) \right],$$

$$\chi' = \frac{2\sqrt{1 - u^2}}{1 - v^2} \sqrt{(\chi_p - \chi)(\chi - \chi_m)(\chi - \chi_n)},$$

$$(2.3)$$

where

$$\chi = \cos^2 \theta, \quad v = -\beta/\alpha, \quad u = \frac{\Omega_2}{\Omega_1}, \quad W = \left( \frac{\kappa}{\Omega_1} \right)^2, \quad K = \frac{C_2}{\alpha \Omega_1},$$

$$\Omega_1 = \omega_1 \left( 1 + \tilde{\gamma} \frac{C_2}{\alpha \omega_1} \right), \quad \Omega_2 = \omega_2 \left( 1 - \tilde{\gamma} \frac{C_1}{\alpha \omega_2} \right), \quad C_1, C_2 = \text{constants.}$$
Also, the following equalities hold

\[
\chi_p + \chi_m + \chi_n = \frac{2 - (1 + v^2)W - u^2}{1 - u^2},
\]
\[
\chi_p\chi_m + \chi_p\chi_n + \chi_m\chi_n = \frac{1 - (1 + v^2)W + (vW - uK)^2 - K^2}{1 - u^2},
\]
\[
\chi_p\chi_m\chi_n = -\frac{K^2}{1 - u^2}.
\]

The case of dyonic finite-size giant magnons corresponds to

\[
0 < u < 1, \quad 0 < v < 1, \quad 0 < W < 1, \quad 0 < \chi_m < \chi < \chi_p < 1, \quad \chi_n < 0.
\]

The AdS$_5$ part of the giant magnon solution, in Euclidean Poincare coordinates, can be written as\(^2\) \((t = \sqrt{W}\tau, \ i\tau = \tau_e)\)

\[
z = \frac{1}{\cosh(\sqrt{W}\tau_e)} \quad x_{0e} = \tanh(\sqrt{W}\tau_e), \quad x_i = 0, \quad i = 1, 2, 3.
\]

Let us also write down the exact expressions for the conserved charges and the angular differences

\[
\mathcal{E} \equiv \frac{2\pi E}{\sqrt{\lambda}} = \frac{2(1-v^2)\sqrt{W}}{\sqrt{1-u^2}} \frac{K(1-\epsilon)}{\sqrt{\chi_p - \chi_n}},
\]
\[
\mathcal{J}_1 \equiv \frac{2\pi J_1}{\sqrt{\lambda}} = \frac{2}{\sqrt{1-u^2}} \left[ \frac{1 - \chi_n - v(vW - uK)}{\sqrt{\chi_p - \chi_n}} K(1-\epsilon) - \sqrt{\chi_p - \chi_n} E(1-\epsilon) \right],
\]
\[
\mathcal{J}_2 \equiv \frac{2\pi J_2}{\sqrt{\lambda}} = \frac{2}{\sqrt{1-u^2}} \left[ \frac{u\chi_n - vK}{\sqrt{\chi_p - \chi_n}} K(1-\epsilon) + u\sqrt{\chi_p - \chi_n} E(1-\epsilon) \right],
\]
\[
p_1 \equiv \Delta\phi_1 = \phi_1(L) - \phi_1(-L) = \frac{2}{\sqrt{1-u^2}} \left\{ \frac{vW - uK}{(1 - \chi_p)\sqrt{\chi_p - \chi_n}} \Pi \left( \frac{-\chi_p - \chi_m}{1 - \chi_p} |1 - \epsilon| - [v(1 - \tilde{\gamma}K) + \tilde{\gamma}u\chi_n] \frac{K(1-\epsilon)}{\sqrt{\chi_p - \chi_n}} \right) \right. \\
\left. -\tilde{\gamma}u\sqrt{\chi_p - \chi_n} E(1-\epsilon) \right\},
\]

\(^2\)Euclidean continuation of the time-like directions to \(t_e = it, x_{0e} = ix_0\), will allow the classical trajectories to approach the AdS$_5$ boundary \(z = 0\) when \(\tau_e \rightarrow \pm\infty\), and to compute the corresponding correlation functions.

\(^3\)We set \(\alpha = \Omega_1 = 1\) for simplicity.
\[ p_2 \equiv \Delta \phi_2 = \phi_2(L) - \phi_2(-L) = \frac{2}{\sqrt{1 - u^2}} \]

\[
\times \left\{ \frac{K}{\chi_p \sqrt{\chi_p - \chi_n}} \Pi \left( 1 - \frac{\chi_m}{\chi_p} | 1 - \epsilon \right) - \left[ u v + \tilde{\gamma} v (v W - u K) - \tilde{\gamma} (1 - \chi_n) \right] \frac{K(1 - \epsilon)}{\sqrt{\chi_p - \chi_n}} \right. \\
- \tilde{\gamma} \sqrt{\chi_p - \chi_n} E(1 - \epsilon) \right\},
\]

where the following notation has been introduced

\[ \epsilon = \frac{\chi_m - \chi_n}{\chi_p - \chi_n}. \] (2.10)

Here, \( E, J_{1,2} \) are the string energy and angular momenta. \( K(1 - \epsilon), E(1 - \epsilon) \) and \( \Pi \left( 1 - \frac{\chi_m}{\chi_p} | 1 - \epsilon \right) \) are the complete elliptic integrals of first, second and third kind. The parameter \( L \) appeared above is related to the size of the giant magnons. For finite-size giant magnons \( L \) is finite, while for infinite-size giant magnons \( L \rightarrow \infty \).

Let us also point out that for the \( \gamma \)-deformed case even the giant magnon with \( J_2 = 0 \) lives on \( S^3 \). It happens because that is the smallest consistent reduction due to the twisted boundary conditions \[13\].

The dyonic giant magnon dispersion relation, including the leading finite-size correction, can be written as \[14\]

\[ \mathcal{E} - J_1 = \sqrt{J_2^2 + 4 \sin^2(p/2)} - \frac{\sin^4(p/2)}{\sqrt{J_2^2 + 4 \sin^2(p/2)}} \cos(\Phi) \epsilon, \] (2.11)

where

\[ \epsilon = 16 \exp \left[ - \frac{2 \left( J_1 + \sqrt{J_2^2 + 4 \sin^2(p/2)} \right) \sqrt{J_2^2 + 4 \sin^2(p/2) \sin^2(p/2)}}{J_2^2 + 4 \sin^4(p/2)} \right]. \] (2.12)

The second term in (2.11) represents the leading finite-size effect on the energy-charge relation, which disappears for \( \epsilon \rightarrow 0 \), or equivalently \( J_1 \rightarrow \infty \). It is nonzero only when \( J_1 \) is finite. The \( \gamma \)-deformation effect is represented by \( \cos(\Phi) \).

In the next section, we will restrict our considerations to the case \( J_2 = 0 \). Then (2.11) simplifies to

\[ \mathcal{E} - J_1 = 2 \sin(p/2) - \frac{1}{2} \sin^3(p/2) \cos(\Phi) \epsilon, \] (2.13)

where

\[ \epsilon = 16 \exp \left[ - \frac{J_1}{\sin(p/2)} - 2 \right], \quad \Phi = 2 \pi \left( n_2 - \frac{\tilde{\gamma}}{2 \pi} J_1 \right), \quad n_2 \in \mathbb{Z}. \] (2.14)
2.2 Semiclassical three-point correlation functions

It is known that the correlation functions of any conformal field theory can be determined in principle in terms of the basic conformal data \( \{ \Delta_i, C_{ijk} \} \), where \( \Delta_i \) are the conformal dimensions defined by the two-point correlation functions

\[
\langle O^i(x_1) O^j(x_2) \rangle = \frac{C_{ij}}{|x_1 - x_2|^{2\Delta_i}}
\]

and \( C_{ijk} \) are the structure constants in the operator product expansion

\[
\langle O_i(x_1) O_j(x_2) O_k(x_3) \rangle = \frac{C_{ijk}}{|x_1 - x_2|^{\Delta_1 + \Delta_2 - \Delta_3} |x_1 - x_3|^{\Delta_1 + \Delta_3 - \Delta_2} |x_2 - x_3|^{\Delta_2 + \Delta_3 - \Delta_1}}.
\]

Therefore, the determination of the initial conformal data for a given conformal field theory is the most important step in the conformal bootstrap approach.

The three-point functions of two "heavy" operators and a "light" operator can be approximated by a supergravity vertex operator evaluated at the "heavy" classical string configuration \([20, 21]\):

\[
\langle V_H(x_1) V_H(x_2) V_L(x_3) \rangle = V_L(x_3)_{\text{classical}}.
\]

For \(|x_1| = |x_2| = 1, x_3 = 0\), the correlation function reduces to

\[
\langle V_H(x_1) V_H(x_2) V_L(0) \rangle = \frac{C_{123}}{|x_1 - x_2|^{2\Delta_H}}.
\]

Then, the normalized structure constants

\[
\mathcal{C} = \frac{C_{123}}{C_{12}}
\]

can be found from

\[
\mathcal{C} = c_\Delta V_L(0)_{\text{classical}},
\]

were \( c_\Delta \) is the normalized constant of the corresponding "light" vertex operator.

By now, investigations on the finite-size effects in the three-point correlators have been performed in \([22, 23, 19, 18, 24]\). This was done for the cases when the "heavy" string states are finite-size giant magnons, with one or two angular momenta\(^4\), and for three different "light" states:

1. Primary scalar operators: \( V_L = V_j^{pr} \)

\(^4\)See also \([23]\), where the finite-size correction to a three-point correlation function is found, when the "heavy" state is not giant magnon one.
2. Dilaton operator: $V_L = V^d_j$

3. Singlet scalar operators on higher string levels: $V_L = V^q$

According to [20], the corresponding (unintegrated) vertices are given by

$$V^{pr}_j = (Y_4 + Y_5)^{-\Delta_{pr}} (X_1 + i X_2)^j \left[ z^{-2} \left( \partial x_m \bar{\partial} x^n \partial z - \partial X_k \bar{\partial} X_k \right) \right],$$

where the scaling dimension is $\Delta_{pr} = j$. The corresponding operator in the dual gauge theory is $Tr(Z^j)$.

$$V^d_j = (Y_4 + Y_5)^{-\Delta_d} (X_1 + i X_2)^j \left[ z^{-2} \left( \partial x_m \bar{\partial} x^n + \partial z \bar{\partial} z \right) \right],$$

where now the scaling dimension $\Delta_d = 4 + j$ to the leading order in the large $\sqrt{\lambda}$ expansion. The corresponding operator in the dual gauge theory is proportional to $Tr(F^2_{\mu\nu} Z^j + \ldots)$, or for $j = 0$, just to the SYM Lagrangian.

$$V^q = (Y_4 + Y_5)^{-\Delta_q} (\partial X_k \bar{\partial} X_k)^q.$$  \hspace{1cm} (2.18)

This operator corresponds to a scalar string state at level $n = q - 1$, and to the leading order in $\frac{1}{\sqrt{\lambda}}$ expansion

$$\Delta_q = 2 \left( \sqrt{(q - 1) \sqrt{\lambda} + 1 - \frac{1}{2} q(q - 1) + 1} \right).$$  \hspace{1cm} (2.19)

The value $n = 1(q = 2)$ corresponds to a massive string state on the first exited level and the corresponding operator in the dual gauge theory is an operator contained within the Konishi multiplet. Higher values of $n$ label higher string levels.

In (2.16), (2.17), (2.18) we denoted with $Y$, $X$ the coordinates in AdS and sphere parts of the $AdS_5 \times S^5$ background.

$$Y_1 + i Y_2 = \sinh \rho \ \sin \eta \ e^{i \varphi_1},$$

$$Y_3 + i Y_4 = \sinh \rho \ \cos \eta \ e^{i \varphi_2},$$

$$Y_5 + i Y_0 = \cosh \rho \ e^{i t}.$$

The coordinates $Y$ are related to the Poincare coordinates by

$$Y_m = \frac{x_m}{z},$$

$$Y_4 = \frac{1}{2z} \left( x^m x_m + z^2 - 1 \right),$$

$$Y_5 = \frac{1}{2z} \left( x^m x_m + z^2 + 1 \right).$$

$^5Z$ is a complex scalar.
where $x^m x_m = -x_0^2 + x_i x_i$, with $m = 0, 1, 2, 3$ and $i = 1, 2, 3$.

The semiclassical results found in [17, 18] for the normalized structure constants (2.15), in the case of finite-size giant magnons on the $\gamma$-deformed $AdS_5 \times S^5_\gamma$ and the above three vertices, are given by

\[
C_{pr}^{\gamma} = \frac{\pi^{3/2} c_{j_p r}^{pr}}{c_{j_r}^{pr}} \frac{\Gamma \left( \frac{j_r + 1}{2} \right)}{\Gamma \left( \frac{j_r + 1}{2} \right)} \frac{(1 - v^2) x_p^{j_r/2}}{(1 - u^2) (\chi_p - \chi_n)} (2.20)
\]

\[
\left\{ \left[ \sqrt{W} \frac{j - 1}{j + 1} + \frac{1}{\sqrt{W(1 - v^2)}} \right] \left( 2 - (1 + v^2) W - 2\tilde{\gamma} K \right) \right\} \times F_1 \left( 1/2, 1/2, -j/2; 1; 1 - \epsilon, 1 - \frac{x_m}{x_p} \right) \\
\times F_1 \left( 1/2, 1/2, -1 - j/2; 1; 1 - \epsilon, 1 - \frac{x_m}{x_p} \right)
\]

\[
C_{d}^{\gamma} = 2\pi^{3/2} c_{4+j}^{d} \frac{\Gamma \left( \frac{j + 1}{2} \right)}{\Gamma \left( \frac{j + 1}{2} \right)} \frac{x_p^{j/2}}{\sqrt{W(1 - u^2)} (\chi_p - \chi_n)} (2.21)
\]

\[
\left\{ \left[ 1 - \tilde{\gamma} K - u (u + \tilde{\gamma} (vW - uK)) \right] \chi_p F_1 \left( 1/2, 1/2, -1 - j/2; 1; 1 - \epsilon, 1 - \frac{x_m}{x_p} \right) \\
- (1 - W - \tilde{\gamma} K) F_1 \left( 1/2, 1/2, -j/2; 1; 1 - \epsilon, 1 - \frac{x_m}{x_p} \right) \right\}
\]

\[
C_{q}^{\gamma} = c_{\Delta q} \pi^{3/2} \frac{\Gamma \left( \frac{\Delta q + 1}{2} \right)}{\Gamma \left( \frac{\Delta q + 1}{2} \right)} \frac{(-2)^q}{(1 - v^2)^q \sqrt{(1 - u^2)W(\chi_p - \chi_n)}} (2.22)
\]

\[
\sum_{k=0}^{q} \frac{q!}{k!(q - k)!} \left( -\frac{B}{A} \right)^k \chi_p^k F_1 \left( \frac{1}{2}, \frac{1}{2}, -k; 1; 1 - \epsilon, 1 - \frac{x_m}{x_p} \right)
\]

where

\[
A = 1 - \frac{1}{2} (1 + v^2) W - \tilde{\gamma} K, \quad B = 1 - \tilde{\gamma} K - u [u - \tilde{\gamma} (K u - v W)]
\]

and $F_1(a, b_1, b_2; c; z_1, z_2)$ is one of the hypergeometric functions of two variables ($AppellF_1$).
3 Leading order finite-size corrections

From now on, we restrict ourselves to the case $J_2 = 0$, $J_1$ large but finite, i.e. $J_1 \gg \sqrt{\lambda}$. This means that the problem reduces to consider the limit $\epsilon \to 0$, since $\epsilon = 0$ corresponds to the infinite-size case, i.e. $J_1 = \infty$ (see (2.14)). To this end, we introduce the expansions

$$
\chi_p = \chi_{p0} + (\chi_{p1} + \chi_{p2} \log(\epsilon)) \epsilon,
$$

$$
\chi_m = \chi_{m0} + (\chi_{m1} + \chi_{m2} \log(\epsilon)) \epsilon,
$$

$$
\chi_n = \chi_{n0} + (\chi_{n1} + \chi_{n2} \log(\epsilon)) \epsilon,
$$

$$
v = v_0 + (v_1 + v_2 \log(\epsilon)) \epsilon,
$$

$$
u = u_0 + (u_1 + u_2 \log(\epsilon)) \epsilon,
$$

$$
W = W_0 + (W_1 + W_2 \log(\epsilon)) \epsilon,
$$

$$
K = K_0 + (K_1 + K_2 \log(\epsilon)) \epsilon. \tag{3.1}
$$

To be able to reproduce the dispersion relation for the infinite-size giant magnons, we set

$$
\chi_{m0} = \chi_{n0} = K_0 = 0, \quad W_0 = 1. \tag{3.2}
$$

Also, it can be proved that if we keep the coefficients $\chi_{m2}$, $\chi_{n2}$, $W_2$ and $K_2$ nonzero, the known leading correction to the giant magnon energy-charge relation in (2.13) will be modified by a term proportional to $J_1^2$. That is why we choose

$$
\chi_{m2} = \chi_{n2} = W_2 = K_2 = 0. \tag{3.3}
$$

In addition, since we are considering for simplicity giant magnons with one angular momentum ($J_2 = 0$), we also set

$$
u_0 = 0, \tag{3.4}
$$

because the leading term in the $\epsilon$-expansion of $J_2$ is proportional to $u_0$. Thus, (3.1) simplifies to

$$
\chi_p = \chi_{p0} + (\chi_{p1} + \chi_{p2} \log(\epsilon)) \epsilon,
$$

$$\chi_m = \chi_{m1} \epsilon,
$$

$$\chi_n = \chi_{n1} \epsilon,
$$

$$v = v_0 + (v_1 + v_2 \log(\epsilon)) \epsilon,
$$

$$u = (u_1 + u_2 \log(\epsilon)) \epsilon,
$$

$$W = 1 + W_1 \epsilon,
$$

$$K = K_1 \epsilon. \tag{3.5}
$$
By replacing (3.5) in (2.4) and (2.10), one finds

\[ \chi_{p0} = 1 - v_0^2, \]
\[ \chi_{p1} = \frac{v_0}{1 - v_0^2} \left[ v_0 \sqrt{(1 - v_0^2)^4 - 4K_1^2(1 - v_0^2)} - 2(1 - v_0^2)v_1 \right], \]
\[ \chi_{p2} = -2v_0v_2, \]
\[ \chi_{m1} = \frac{(1 - v_0^2)^2 + \sqrt{(1 - v_0^2)^4 - 4K_1^2(1 - v_0^2)}}{2(1 - v_0^2)}, \]
\[ \chi_{n1} = -\frac{(1 - v_0^2)^2 - \sqrt{(1 - v_0^2)^4 - 4K_1^2(1 - v_0^2)}}{2(1 - v_0^2)}, \]
\[ W_1 = -\frac{\sqrt{(1 - v_0^2)^4 - 4K_1^2(1 - v_0^2)}}{1 - v_0^2}. \]

The expressions for the other parameters in (3.5) and (3.6) can be derived in the following way.

First, we impose the conditions \( J_2 = 0 \) and \( p_1 \) to be independent of \( \epsilon \). This leads to four equations with solution

\[ v_1 = \frac{v_0 \sqrt{(1 - v_0^2)^4 - 4K_1^2(1 - v_0^2)}}{4(1 - v_0^2)} (1 - \log 16), \]
\[ v_2 = \frac{v_0 \sqrt{(1 - v_0^2)^4 - 4K_1^2(1 - v_0^2)}}{4(1 - v_0^2)}, \]
\[ u_1 = \frac{K_1v_0 \log 4}{1 - v_0^2}, \]
\[ u_2 = -\frac{K_1v_0}{2(1 - v_0^2)}, \]

where

\[ v_0 = \cos \frac{p_1}{2}. \]

Second, expanding \( J_1 \) and \( p_2 = 2\pi n_2 \) \((n_2 \in \mathbb{Z})\) to the leading order in \( \epsilon \), we obtain (compare with (2.13))

\[ \epsilon = 16 \exp \left( -\frac{J_1}{\sin \frac{p_2}{2}} - 2 \right), \quad K_1 = \frac{1}{2} \sin^3 \frac{p_1}{2} \sin \Phi, \quad \Phi = 2\pi \left( n_2 - \frac{\gamma}{2\pi} J_1 \right). \]

Now, we are going to use the above results to find the leading order finite-size effects on the normalized structure constants in terms of \( J_1 \equiv J, p_1 \equiv p \) and \( \Phi \).

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\(^6\)This follows from the periodicity condition on \( \phi_2 \).
3.1 Giant magnons on $AdS_5 \times S^5_\gamma$ and primary scalar operators

As was pointed out in [24], where the undeformed case has been considered, $j = 1$ and $j = 2$ are special values. That is why we will find the corresponding normalized structure constants in Appendix A. Here, we will deal with $j \geq 3$, when we can use the following representation of $F_1(a, b_1, b_2; c; z_1, z_2)$ [26]:

$$F_1(a, b_1, b_2; c; z_1, z_2) = \sum_{k=0}^{\infty} \frac{(a)_k (b_2)_k}{(c)_k} 2F_1(a+k, b_1; c+k; z_1) \frac{z_2^k}{k!}.$$  \hfill (3.10)

For all cases we are going to consider, primary, dilaton and higher string level vertices, only $b_2$ is different, while the other parameters and arguments of $F_1$ are the same (see (2.20), (2.21), (2.22)):

$$a = \frac{1}{2}, \quad b_1 = \frac{1}{2}, \quad c = 1, \quad z_1 = 1 - \epsilon, \quad z_2 = 1 - \frac{\chi_m}{\chi_p}. \hfill (3.11)$$

Then, expending $2F_1\left(\frac{1}{2} + k, \frac{1}{2}; 1+k; 1 - \epsilon\right) (1 - \chi_m/\chi_p)^k$ around $\epsilon = 0$, one finds

$$2F_1\left(\frac{1}{2} + k, \frac{1}{2}; 1+k; 1 - \epsilon\right) \left(1 - \frac{\chi_m}{\chi_p}\right)^k \approx \frac{\Gamma(1+k)}{\sqrt{\pi} \Gamma(\frac{1}{2} + k)} \left\{ \log(4) - H_{k-\frac{1}{2}} - \frac{1}{4\chi_p0} \left[ 2\chi_p0 + (4k\chi_m1 - (1+2k)\chi_p0) \left(\log(4) - H_{k-\frac{1}{2}}\right) \right] \epsilon - \log(\epsilon) - \frac{\chi_p0 + 2k(\chi_p0 - 2\chi_m1)}{4\chi_p0} \epsilon \log(\epsilon) \right\}, \hfill (3.12)$$

where $H_z$ is defined as [26]

$$H_z = \psi(z + 1) + \gamma.$$

The replacement of (3.12) in (3.10), taking into account (3.11), gives

$$F_1\left(\frac{1}{2}, \frac{1}{2}, b_2; 1 - \epsilon, 1 - \frac{\chi_m}{\chi_p}\right) \approx C_0 + C_1 \epsilon + C_2 \epsilon \log(\epsilon) + C_3 \log(\epsilon), \hfill (3.13)$$
where

\[
C_0 = \frac{\Gamma(-b_2)}{\sqrt{\pi} \Gamma\left(\frac{1}{2} - b_2\right)} + \frac{\log(16)}{\pi} {}_1F_0(b_2, 1), \tag{3.14}
\]

\[
C_1 = \frac{1}{4\pi} \left\{ \frac{1}{\chi_{p0}} \left[ -\sqrt{\pi} \Gamma(-1 - b_2) \right] \left( \chi_{p0} + 2b_2 \chi_{m1} \right) + 8 \log(2) b_2 (\chi_{p0} - 2 \chi_{m1}) \right\},
\]

\[
C_2 = -\frac{1}{4\pi \chi_{p0}} \left[ \chi_{p0} \ {}_1F_0(b_2, 1) + 2b_2 (\chi_{p0} - 2 \chi_{m1}) \right] \right\},
\]

\[
C_3 = -\frac{1}{\pi} {}_1F_0(b_2, 1).
\]

Here, \( {}_1F_0(b_z) \) is one of the hypergeometric functions.

In the normalized structure constants (2.20), there are two hypergeometric functions \( F_1 \left( \frac{1}{2}, \frac{1}{2}, b_2; 1; 1 - \epsilon, 1 - \frac{x_0}{x_p} \right) \) with \( b_2 = -j/2 \) and \( b_2 = -1 - j/2 \). By using (3.13), (3.14) in (2.20) and expanding it about \( \epsilon = 0 \), we can write down the following approximate equality for \( j \geq 3 \)

\[
C_{j^\gamma}^{pr} \approx A_0 + A_1 \epsilon + A_2 \epsilon \log(\epsilon), \tag{3.15}
\]

where the coefficients are given by

\[
A_0 = c_j^{pr} \pi \frac{\Gamma(\frac{1}{2})^2}{\Gamma\left(\frac{1}{4} + \frac{j}{2}\right)\Gamma\left(\frac{3}{4} + \frac{j}{2}\right)} j \chi_{p0}^{(j-1)} (1 - v_0^2 - \chi_{p0}), \tag{3.16}
\]

\[
A_1 = c_j^{pr} \pi \frac{\Gamma(\frac{1}{2})^2}{4 \Gamma\left(\frac{1}{4} + \frac{j}{2}\right)\Gamma\left(\frac{3}{4} + \frac{j}{2}\right)} \chi_{p0}^{(j-3)} \left\{ 4(W_1 + \chi_{m1})\chi_{p0} - 2\chi_{p0}^2 - 2\chi_{m1}(1 - v_0^2 - \chi_{p0}) + \chi_{p0}(1 - v_0 + 8v_1 + 2v_0W_1) \right. \\
\left. - \chi_{p0}(1 - 2W_1) - 2(1 - v_0^2 + \chi_{p0})\chi_p \right\} j^3 \\
+ \left[ \chi_{p0} - 4W_1v_1\chi_{p0} + \chi_{m1}(1 - v_0^2 - \chi_{p0}) - v_0^2(\chi_{n1} + W_1\chi_{p0} - 3\chi_{p1}) - 3\chi_{p1} + \chi_{p0}(-\chi_{n1} + W_1(-1 + \chi_{p0} + \chi_{p1})) \right] j^3 \\
+ (1 - v_0^2 - \chi_{p0})\chi_p j^3 \\
+ \tilde{\gamma} \left[ 4K_1\chi_{p0} + (2\chi_{p0}(K_1 - 2(K_1 + v_0u_1)\chi_{p0})) j + (2\chi_{p0}(v_0u_1\chi_{p0} - K_1(1 - \chi_{p0}))) j^2 \right] \right\},
\]

\[
A_2 = -c_j^{pr} \pi \frac{\Gamma(\frac{1}{2})^2}{2 \Gamma\left(\frac{1}{4} + \frac{j}{2}\right)\Gamma\left(\frac{3}{4} + \frac{j}{2}\right)} j \chi_{p0}^{(j-3)} \left[ 4v_0v_2\chi_{p0} + (1 - v_0^2 + \chi_{p0})\chi_{p2} - (1 - v_0^2 - \chi_{p0})\chi_{p2} j - 2\tilde{\gamma}v_0u_2\chi_{p0}^2 \right].
\]

Now, our goal is to express (3.15) in terms of \( J, p, \) and \( \Phi \). To this end, we replace (3.6) - (3.9) in (3.16). This leads to the following final result for the normalized structure constants
Let us point out that (3.17) reduces exactly to the result found for the undeformed case in [24], when $\tilde{\gamma} = 0$, $\Phi = 0$. Moreover, it generalizes it for any $j \geq 3$. The cases $j = 1$ and $j = 2$ will be considered separately in Appendix A.

### 3.2 Giant magnons on $AdS_5 \times S^5$ and dilaton operator

Since the hypergeometric functions in (2.21) are the same as in (2.20) (only the coefficients in front of them are different), we can use (3.13), (3.14) for the case under consideration, with $b_2 = -j/2$ and $b_2 = -1 - j/2$. Thus, expanding (2.21) to the leading order in $\epsilon$, one finds ($j \geq 1$)

$$
C_{j\gamma}^{\pi} \approx C_{j\gamma}^{\pi} \frac{\pi}{4} \frac{\Gamma \left( \frac{j}{2} \right)}{\Gamma \left( \frac{1+j}{2} \right)} \frac{\Gamma \left( \frac{3+j}{2} \right)}{\Gamma \left( \frac{4+j}{2} \right)} \frac{\Gamma \left( \frac{5+j}{2} \right)}{\Gamma \left( \frac{6+j}{2} \right)} \sin^{1+j}(p/2) \left\{ j - \frac{1}{8} \left[ (4 - j(1 + 3j)(1 + \cos p) - j(1 + j)(1 + \cos p)) \csc(p/2) \right] J \cos \Phi \right. \\
- \left. \tilde{\gamma}(4 \sin(p/2) - j(1 + \cos p)J) \sin \Phi \right\} \epsilon.
$$

Taking into account (3.6) - (3.9) in (3.18) one finally derives

$$
C_{j\gamma}^{\pi} \approx C_{j\gamma}^{\pi} \frac{\pi}{4} \frac{\Gamma \left( \frac{j}{2} \right)}{\Gamma \left( \frac{1+j}{2} \right)} \frac{\Gamma \left( \frac{3+j}{2} \right)}{\Gamma \left( \frac{4+j}{2} \right)} \sin^{1+j}(p/2) \left\{ j - \frac{1}{8} \left[ (4 - j(1 + 3j)(1 + \cos p) - j(1 + j)(1 + \cos p)) \csc(p/2) \right] J \cos \Phi \right. \\
- \left. \tilde{\gamma}(4 \sin(p/2) - j(1 + \cos p)J) \sin \Phi \right\} \epsilon.
$$

### 3.3 Giant magnons on $AdS_5 \times S^5$ and singlet scalar operators on higher string levels

For this case, we were not able to obtain a general formula for the leading finite-size corrections to the three-point correlation functions in terms of $J$, $p$, and $\Phi$, for any $q \geq 1$. That is why, we are going to present here the results for $q = 1, ..., 5$ (string levels $n = 0, 1, 2, 3, 4$).
Let us first point out that the hypergeometric functions \( F_1 \left( \frac{1}{2}, \frac{1}{2}; -k; 1 - \epsilon, 1 - \frac{\chi_m}{\chi_p} \right) \) entering (2.22) can be expressed in terms of the complete elliptic integrals \( K(1 - \epsilon), E(1 - \epsilon) \) of first and second kind. For example,

\[
F_1 \left( \frac{1}{2}, \frac{1}{2}, 0; 1 - \epsilon, 1 - \frac{\chi_m}{\chi_p} \right) = \frac{2}{\pi} K(1 - \epsilon),
\]

\[
F_1 \left( \frac{1}{2}, \frac{1}{2}, -1; 1 - \epsilon, 1 - \frac{\chi_m}{\chi_p} \right) = \frac{2}{\pi} \left( \frac{\chi_m - \epsilon \chi_p}{1 - \epsilon} \right) \frac{K(1 - \epsilon)}{(1 - \epsilon) \chi_p},
\]

\[
F_1 \left( \frac{1}{2}, \frac{1}{2}, -2; 1 - \epsilon, 1 - \frac{\chi_m}{\chi_p} \right) = \frac{1}{3\pi(1 - \epsilon)^2 \chi_p^2} \left[ 2 \left( \left( 1 - \epsilon \right) \chi_m^2 - 4 \epsilon \chi_m \chi_p \right) - (1 - 3 \epsilon) \epsilon \chi_p^2 \right] K(1 - \epsilon) - 4 \left( \chi_m - \chi_p \right) \left( (2 - \epsilon) \chi_m + (1 - 2 \epsilon) \chi_p \right) E(1 - \epsilon).
\]

Then, (2.22) can be written in terms of hypergeometric functions of the type \( pF_q \) with argument \( 1 - \epsilon \). However, this is just much more complicated representation of the semiclassically exact result.

Here, we are interested in the case of small \( \epsilon \) (or, equivalently, large \( J \)) limit. So, we will expand everything in \( \epsilon \). Since the computations are similar to the previously considered cases, we will write down the final results only. They are given by the following approximate equalities:

\[
C_1 \approx c_{\Delta_1} \sqrt{\frac{\pi}{2}} \frac{\Gamma \left( \frac{\Delta_1}{2} \right)}{\Gamma \left( 1 + \frac{\Delta_1}{2} \right)} \sin(p/2) \left\{ 16 - 8J \csc(p/2) + \left[ 4 - (2 - \cos p + J^2 \cot^2(p/2)) \right] \right\},
\]

\[
+ J \left( 5 - \cos p \right) \csc(p/2) \cos \Phi + 8\tilde{\gamma} J \sin^2(p/2) \sin \Phi \right\} \epsilon,
\]

\[
C_2 \approx -c_{\Delta_2} \sqrt{\frac{\pi}{24}} \frac{\Gamma \left( \frac{\Delta_2}{2} \right)}{\Gamma \left( 1 + \frac{\Delta_2}{2} \right)} \left\{ 8(2 \sin(p/2) - 3J) + \left[ 12 \sin(p/2) \right] \right\},
\]

\[
+ (2(27 + 5 \cos p) \sin(p/2) - J(31 + 13 \cos p + 3J \left( 1 + \cos p \right) \csc(p/2))) \cos \Phi + 8\tilde{\gamma} \sin(p/2)(8 \sin(p/2) - J(7 + \cos p)) \sin \Phi \right\} \epsilon,
\]

\[
C_3 \approx c_{\Delta_3} \sqrt{\frac{\pi}{120}} \frac{\Gamma \left( \frac{\Delta_3}{2} \right)}{\Gamma \left( 1 + \frac{\Delta_3}{2} \right)} \left\{ 8(38 \sin(p/2) - 15J) + \left[ 60 \sin(p/2) \right] \right\},
\]

\[
+ (18(13 + 19 \cos p) \sin(p/2) - J(187 + 97 \cos p + 15J \left( 1 + \cos p \right) \csc(p/2))) \cos \Phi - 12\tilde{\gamma} \sin(p/2)(48 \sin(p/2) - J(23 - 7 \cos p)) \sin \Phi \right\} \epsilon,
\]
\begin{align*}
C_5^4 & \approx -c_{\Delta_4} \frac{\sqrt{\pi}}{840} \frac{\Gamma \left( \frac{\Delta_4}{2} \right)}{\Gamma \left( 1 + \frac{\Delta_4}{2} \right)} \left\{ 1264 \sin(p/2) - 840 \mathcal{J} + \left[ \sin(p/2) (420 \\
+ (4730 + 2054 \cos p - \mathcal{J} (1837 + 1207 \cos p) \csc(p/2) - 210 \mathcal{J}^2 \cot^2(p/2)) \cos \Phi \\
- 16 \tilde{\gamma} (424 \sin(p/2) - 3 \mathcal{J} (79 + 9 \cos p)) \sin \Phi) \right] \epsilon \right\}.
\end{align*}

\begin{align*}
C_5^5 & \approx c_{\Delta_5} \frac{\sqrt{\pi}}{2520} \frac{\Gamma \left( \frac{\Delta_5}{2} \right)}{\Gamma \left( 1 + \frac{\Delta_5}{2} \right)} \left\{ 8(902 \sin(p/2) - 315 \mathcal{J}) + \left[ 1260 \sin(p/2) \\
+ (2(6093 + 7667 \cos p) \sin(p/2) - \mathcal{J} (6343 + 4453 \cos p) \\
+ 315 \mathcal{J}(1 + \cos p) \csc(p/2))) \cos \Phi \\
- 20 \tilde{\gamma} \sin(p/2)(1376 \sin(p/2) - \mathcal{J} (523 - 107 \cos p)) \sin \Phi) \right] \epsilon \right\}.
\end{align*}

4 Concluding Remarks

In this article, we have derived the leading finite-size effects on the normalized structure constants in some semiclassical three-point correlation functions in $\text{AdS}_5 \times S^5$, dual to $\mathcal{N} = 1$ SYM theory in four dimensions, arising as an exactly marginal deformation of $\mathcal{N} = 4$ SYM, expressed in terms of the conserved string angular momentum $\mathcal{J}$, and the worldsheet momentum $p$, identified with the momentum $p$ of the magnon excitations in the dual spin-chain. More precisely, we found the leading finite-size effects on the structure constants in three-point correlators of two “heavy” giant magnon’s string states and the following three “light” states:

1. Primary scalar operators;
2. Dilaton operator with nonzero-momentum ($j \geq 1$);
3. Singlet scalar operators on higher string levels.

It would be interesting to investigate other cases for which the finite-size corrections to the giant magnon’s dispersion relations are known, like $\text{AdS}_4 \times \mathbb{C}P^3$, $\text{AdS}_4 \times \mathbb{C}P_\gamma^3$, $\text{AdS}_5 \times T^{1,1}$, or $\text{AdS}_5 \times T^{1,1}_\gamma$. 
A Giant magnons on $AdS_5 \times S^5_\gamma$

and primary scalar operators with $j = 1$ and $j = 2$

Let us start with the case $j = 1$. Expanding the coefficients in $C_{1\gamma}^{pr}$ according to (3.5), one can rewrite it in the following form

\[
C_{1\gamma}^{pr} \approx c_{1\gamma}^{pr} \frac{\pi^2}{2} \left\{ \frac{1}{\chi_{p0}} F_1 \left( 1/2, 1/2, -1/2; 1; 1 - \epsilon, 1 - \frac{\chi_{m1}}{\chi_{p0}} \epsilon \right) \left[ (1 - v_0^2) \chi_{n1} \epsilon \\
+ (2 - (4v_0v_1 + 4\bar{\gamma} K_1) \epsilon - v_0^2(2 + W_1 \epsilon)) \chi_{p0} - 4v_0v_2\chi_{p0} \epsilon \log(\epsilon) \right]
\right.
\]

\[
- F_1 \left( 1/2, 1/2, -3/2; 1; 1 - \epsilon, 1 - \frac{\chi_{m1}}{\chi_{p0}} \epsilon \right) \\
\times \left[ 4\chi_{p0} + 2(\chi_{n1} - (W_1 + 2\bar{\gamma}(K_1 + v_0u_1))\chi_{p0} + 2\chi_{p1}) \epsilon
+ 4(\chi_{p2} - \bar{\gamma}v_0u_2\chi_{p0}) \epsilon \log(\epsilon) \right]
\}

(A.1)

In order to represent $C_{1\gamma}^{pr}$ as a function of $\mathcal{J}$, $p$ and $\Phi$, one have to use (3.6) - (3.9) in (A.1). This leads to

\[
C_{1\gamma}^{pr} \approx -c_{1\gamma}^{pr} \frac{\pi^2}{4} \sin^2(p/2) \left\{ 8F_1 \left( 1/2, 1/2, -3/2; 1; 1 - \epsilon, 1 - \frac{1}{2} (1 + \cos \Phi) \epsilon \right)
\right.
\]

\[
- 4F_1 \left( 1/2, 1/2, -1/2; 1; 1 - \epsilon, 1 - \frac{1}{2} (1 + \cos \Phi) \epsilon \right)
\]

\[
+ \left[ F_1 \left( 1/2, 1/2, -1/2; 1; 1 - \epsilon, 1 - \frac{1}{2} (1 + \cos \Phi) \epsilon \right)
\right.
\]

\[
\times (1 - \cos \Phi (9 + 2 \cos p + \mathcal{J}(1 + \cos p) \csc(p/2)) + 4\bar{\gamma} \sin(p/2) \sin \Phi)
\]

\[
- F_1 \left( 1/2, 1/2, -3/2; 1; 1 - \epsilon, 1 - \frac{1}{2} (1 + \cos \Phi) \epsilon \right)
\]

\[
\times (2 - 2 \cos \Phi (5 + 2 \cos p + \mathcal{J}(1 + \cos p) \csc(p/2)))
\]

\[
+ \bar{\gamma} \left( \mathcal{J}(1 + \cos p) + 4 \sin(p/2) \sin \Phi \right) \epsilon \]

\}

(A.2)

For the undeformed case, when $\bar{\gamma} = 0$, $\Phi = 0$, (A.2) simplifies to

\[
C_{1\gamma}^{pr} \approx -c_{1\gamma}^{pr} \frac{\pi^2}{4} \sin(p/2) \left[ 3 \sin(p/2) + \sin(3p/2) + \mathcal{J}(1 + \cos p) \right] \epsilon^2.
\]

(A.3)

This is in accordance with the result $C_{1\gamma}^{pr} \approx 0$ found in [24], where only the leading order in $\epsilon$ was taken into account.
Now, let us consider the case $j = 2$, when (2.20) reduces to

$$C_{2\bar{\gamma}} = \frac{-8}{3} C^p C^r \frac{1}{\epsilon^2 (1 - u^2) W (\chi_p - \chi_m)} \left\{ \left[ 3 - (1 + 2v^2) W - 3\tilde{\gamma} K \right] (1 - \epsilon) \right\}^{\frac{1}{2}} \left( 1 - \epsilon \right)\right) (A.4)

\times \left[ (\chi_m - \chi_p) E (1 - \epsilon) - (\chi_m - \chi_p^2) K (1 - \epsilon) + (1 - u (u - \tilde{\gamma} K u - v W)) - \tilde{\gamma} K \right]

\times \left( 2(\chi_p - \chi_m) ((2 - \epsilon) \chi_m + (1 - 2\epsilon) \chi_p) E (1 - \epsilon) + ((3 - \epsilon) \chi_m^2 - 4\chi_m \chi_p \epsilon \right.

\left. - \chi_p^2 (1 - 3\epsilon) \epsilon) K (1 - \epsilon) \right) \right\}.\right)

Expanding (A.4) in $\epsilon$, and taking into account (3.6) - (3.9), one finds

$$C_{2\bar{\gamma}} \approx \frac{2}{3} C^p C^r \sin^2(p/2) \left[ 2J \cos \Phi - \tilde{\gamma} \left( 2 \sin(p/2) - J (1 + \cos p) \right) \sin(p/2) \sin \Phi \right] \epsilon.\right) (A.5)

Obviously, the result for the undeformed case [24] is properly reproduced by the above formula.

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