Strangeness enhancement and flow-like effects in $e^+e^-$ annihilation at high parton density

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Abstract

Strangeness enhancement and collective flow are considered signatures of the quark gluon plasma formation. These phenomena have been detected not only in relativistic heavy ion collisions but also in high energy, high multiplicity events of proton–proton and proton–nucleus (“small systems”) scatterings. Indeed, a universal behavior emerges by considering the parton density in the transverse plane as the dynamical quantity to specify the initial condition of the collisions. On the other hand, $e^+e^-$ annihilation data at LEP and lower energies indicate that there is no strangeness enhancement and no flow-like effect. We show that the parton density in the transverse plane generated in $e^+e^-$ annihilation at the available energy is too low to expect such effects. The event-by-event multiplicity where strangeness suppression and flow-like phenomenon could show up in $e^+e^-$ is evaluated.

1 Introduction

Recent experimental results in proton–proton ($pp$) and proton–nucleus ($pA$) collisions [1–9] support the conclusion that the system created in high energy, high multiplicity collisions with “small” initial settings ($pp$ and $pA$), is essentially the same as that one produced with “large” initial nucleus–nucleus ($AA$) configurations.

The ALICE collaboration reported [1] an enhanced production of multi-strange hadrons, previously observed in $PbPb$ collisions [10], in high energy, high multiplicity $pp$ events.

Moreover, the energy loss in $AA$ collisions was shown to scale in small and large systems [11] by considering a dynamical variable, previously introduced to predict the strangeness enhancement in $pp$ [12–15]. It corresponds to the initial entropy density of the collisions and takes into account the transverse size (and its fluctuations) of the initial configuration in high multiplicity events.

Another important similarity among $pp$, $pA$, and $AA$ collisions was identified in several measurements of long-range di-hadron azimuthal correlations [3,4,6,16] indicating universality in flow-like patterns.

More recently, the $e^+e^-$ annihilation LEP data have been reconsidered [17] to check if a flow-like behavior is generated with this initial, small, non hadronic, setting.

The answer is negative and confirmed at lower energy by the BELLE collaboration [18].

Furthermore, there is no strangeness enhancement in $e^+e^-$ annihilation. Figures 1, 2 show the strangeness suppression factor, $\gamma_s$, in the Statistical Hadronization Model (SHM) [19,20,38] as a function of the available energy ($\gamma_s \simeq 1$ means no strangeness suppression and $\gamma_s < 1$ suppressed strangeness production).

According to the universality point of view, strangeness enhancement and collective flow are both indications of the formation of an initial system with high entropy density, i.e. high parton number density in the transverse plane.

In this letter we show that at the available energies in $e^+e^-$ annihilation the parton density in the transverse plane is small and therefore the previous signatures should not arise here.

The event-by-event multiplicity and the corresponding energy where strangeness suppression and flow-like phenomenon could show up in $e^+e^-$ turns out to be quite large.

In the next section the universality in hadronic and nuclear collision will be recalled. Section 3 is devoted to evaluate the parton density in the transverse plane for $e^+e^-$ annihilation.
and to verify that the energy/multiplicity is actually too low to follow the universal trend, observed in the small and large hadronic and nuclear systems. Section 4 contains comments and conclusions.

2 Universality in hadronic and nuclear collisions

2.1 Strangeness enhancement

One of the most striking observations in high energy multi-hadron production is that both species abundances and transverse momentum spectra (provided effects of collective flow and gluon radiation are removed) follow the thermal pattern of an ideal hadron-resonance gas, with a universal temperature \( T \simeq 150 \pm 10 \text{ MeV} \) [19,20] (see Fig. 3).

More precisely, the relative yields of the different hadron species are well accounted for by an ideal gas of all hadrons and hadronic resonances with one well-known caveat: strangeness production is reduced with respect to the predicted rates. This suppression can, however, be taken into account by one further parameter, \( 0 < \gamma_s \leq 1 \). The predicted rate for a hadron species containing \( \nu_s = 1, 2, 3 \) strange quarks is then suppressed by the factor \( \gamma_s^{\nu_s} \) [21]. Figures 1, 4 are for \( \nu_s = 1 \).

The basic quantity for the resonance gas description is the grand-canonical partition function for an ideal gas at temperature \( T \) in a spatial volume \( V \)

\[
\ln Z(T) = V \sum_i d_i \frac{\gamma_i^{\nu_i}}{(2\pi)^3} \phi(m_i, T),
\]

with \( d_i \) specifying the degeneracy (spin, isospin) of species \( i \), and \( m_i \) its mass; the sum runs over all species. Here

\[
\phi(m_i, T) = \int d^3 p \exp \left\{ \frac{\sqrt{p^2 + m_i^2}}{T} \right\}
\]

\( \simeq \exp \left\{ \frac{-m_i}{T} \right\} \)

is the Boltzmann factor for species \( i \), so that the ratio of the production rates \( N_i \) and \( N_j \) for hadrons of species \( i \) and \( j \) is given by

\[
\frac{N_i}{N_j} = \frac{d_i \gamma_i^{\nu_i} \phi(m_i, T)}{d_j \gamma_j^{\nu_j} \phi(m_j, T)}.
\]

where \( \nu_i = 0, 1, 2, 3 \) specifies the number of strange quarks in species \( i \). We note that in the grand-canonical formulation the volume cancels out in the form for the relative abundances.

The Statistical Hadronization Model (SHM) is in agreement with the high energy data for large (nucleus–nucleus) and small (proton–proton and e\(^+\)e\(^-\)) initial settings with the same hadronization temperature as shown in Fig. 3 [20,22].

It was early proposed, on perturbative QCD basis, that the quark-gluon plasma formation would enhance strange particle production in nucleus–nucleus (AA) collisions [24,25]. Indeed, \( \gamma_s \simeq 1 \) well describes the high energy AA data, but \( \gamma_s < 1 \) for proton–proton scattering at energies less than those at the Large Hadron Collider (LHC). Figure 1 shows \( \gamma_s \) as a function of the collision energy.

More recently, the ALICE collaboration reported for \( pp \) collisions [1] the enhanced production of multi-strange hadrons, previously observed in \( PbPb \) collisions, in high energy, high multiplicity, proton–proton \( pp \) events. Indeed a universal behavior of strangeness production, suggested on theoretical grounds in Refs. [12–15,26,28], emerges by considering a specific dynamical variable corresponding to the parton density in the transverse plane of the collision,
which takes into account the transverse size (and its fluctuations) of the initial configuration in high multiplicity events.

Figure 4 from Ref. [26] clearly shows the universal pattern of $\gamma_s$, for different initial settings $pp$, $pA$, $AA$ at various energies, when plotted versus the initial entropy density $s_0$, which in the one-dimensional hydrodynamic formulation [27] is given by

$$s_0 \tau_0 \simeq \frac{1.5}{A_T} \frac{dN_{ch}^s}{dy} = \frac{1.5}{A_T} \frac{N_{part}^s}{2} \frac{dN_{ch}^s}{dy} \bigg|_{y=0},$$

with $x = pp$, $pA$, $AA$. Here $A_T$ is the transverse area, $(dN_{ch}^s/dy)_{y=0}$ denotes the number of produced charged secondaries, normalized to half the number of participants $N_{part}^s$, in reaction $x$. The transverse area $A_T$ is correlated with the multiplicity and it can be evaluated in $AA$ and $pA$ by Glauber Monte Carlo simulation [29]. The high multiplicity events in $pp$, are associated with fluctuation of the transverse area, which, in the Color Glass Condensate, are indeed fluctuations of the initial gluon field configurations [30–32]. In proton–proton, proton–nucleus and heavy ion collisions at high energies, high multiplicities, $\gamma_s \rightarrow 1$ and it becomes a universal function of $s_0 \tau_0$ as shown in Fig. 4.

Notice that the strangeness saturation, with $\gamma_s \gtrsim 0.95$, requires $s_0 \tau_0 \geq 6 \text{ fm}^{-2}$ (a more refined determination of this cut-off value, by a model reproducing the behavior in Fig. 4, is in progress.)

2.2 Elliptic flow and participant eccentricity

The scaling behavior in $pp$ and $AA$ of the ratio between the elliptic flow, $v_2$, and the participant eccentricity, $\epsilon_{part}$, has been shown in Ref. [26]. Indeed the plot of $v_2/\epsilon_{part}$ versus $x = dN_{ch}/S$, where $S$ is the transverse area associated with $\epsilon_{part}$, shows a universal trend starting from $x \geq 2.5 \text{ fm}^{-2}$, i.e. $s_0 \tau_0 \geq 3.8 \text{ fm}^{-2}$. Moreover the difference between the geometrical transverse area $A_T$ (i.e. the overlapping almond shape in $AA$ collisions) and $S$ is crucial to obtain the smooth interpolation among $pp$ and $AA$ data. The definition of $S$ becomes meaningless in $e^+e^-$, since it is related to the event by event fluctuations of the projectile/target constituents. Since $S < A_T$ one needs, in general, a value of $s_0$ larger than that one evaluated by geometrical criteria. In other words, a flow-like effect requires large enough parton density in the transverse area.

3 Transverse parton density in $e^+e^-$ annihilation

According to previous discussion, an universal behavior emerges if the parton density in the transverse plane is used
as the relevant dynamical variable to define the initial setting of the collisions and if it is large enough.

Let us now study this quantity in \( e^+e^- \) annihilation and of the saturation (if any) to \( \gamma_s \to 1 \) arises. To evaluate the effective parton density in the transverse plane for this particular non-hadronic setting, one has to know the multiplicity and the transverse area (which are not independent quantities).

The problem is a reliable evaluation of an effective transverse size for \( e^+e^- \). Indeed, in the energy range up to \( \approx 200 \) GeV, the multiplicity is known and similar to the nucleus–nucleus one (normalized to half the number of participants \( N_{\text{part}} \)): \( dN/dy \) at \( y = 0 \), with respect to thrust axis, is plotted in Fig. 5 versus the cms energy.

Let us recall, in a simplified way, the steps of the hadronization cascade of a primary quark or antiquark produced in \( e^+e^- \) annihilation.

The color field flux tube (string), initially created along the direction of the separating \( q \) and \( \bar{q} \), produces a further pair \( q_1, \bar{q}_1 \) and then provides an increasing of their longitudinal momentum. When \( q_1, \bar{q}_1 \) reaches the critical distances for the string breaking still another pair \( q_2, \bar{q}_2 \) is created and the state \( q_2, \bar{q}_1 \) is emitted as a hadron. The string multifragmentation produces the final multiplicity in Fig. 5.

The transverse size, \( R_T \), of a quark–antiquark string at center mass energy \( \sqrt{s} \) turns out to be [39,40]

\[
R_T^2 = \frac{2}{\pi \sigma} \sum_{k=0}^{N} \frac{1}{2k+1},
\]

where \( \sigma \) is the string tension and \( N = \sqrt{s}/2\sigma \). Moreover

\[
\sum_{k=0}^{N} \frac{1}{2k+1} = \frac{\gamma}{2} + \ln(2) + \frac{1}{2} \left[ \Psi(N + 3/2) \right]
\]

where \( \gamma \) is the Euler–Mascheroni constant and \( \Psi \) is the digamma function which, for large values of the argument, can be approximated as

\[
\Psi(x) \simeq \ln(x).
\]

Finally the transverse size can evaluated by

\[
R_T^2 = \frac{2}{\pi \sigma} \left[ \frac{\gamma}{2} + \ln[2 \left( N + \frac{3}{2} \right)^{1/2}] \right],
\]

which shows a weak, logarithmic, dependence on the energy. Therefore, a small uncertainties on \( R_T \) is expected since all the string transverse fluctuations (i.e. the excited states of the string) have been taken into account in Eqs. (5, 6, 8) and, moreover, according to lattice results [41], the uncertainty due to the fluctuations is about 10% [40].

The result is plotted in Fig. 6 and compared with the transverse size of a \( pp \) collision, evaluated by the Color Glass parameterization, which takes into account the event-by-event fluctuations of the initial gluon configuration [30–32], and by the phenomenological fit of the multiplicity

\[
\frac{dN}{dy}_{pp} = a_p + b_p \sqrt{s}^{0.22}
\]

with \( a_p = 0.04123 \) and \( b_p = 0.797 \) [42].

The similarity between the transverse size in \( e^+e^- \) and in \( pp \) should not be surprising since it is well known that the multiplicity in \( pp \) collisions is related with the multiplicity in \( e^+e^- \) annihilation if one takes into account the leading particle effect, i.e. the energy removed from the genuine hadronization cascade due to the leading particles [43,44].

The initial value of \( s_0\tau_0 \) in \( e^+e^- \) annihilation, can now be estimated by data in Fig. 5, fitted by \( (dN_{ch}/dy)e^+e^- = 0.3493 + 0.6837 \sqrt{s}^{0.3} \), and by \( R_T \) in Fig. 6. The energy range 14 – 186 GeV correspond to rather narrow interval of \( s_0\tau_0 \), 2 fm\(^{-2} \lesssim s_0\tau_0 \lesssim 3 \) fm\(^{-2} \) and the \( \gamma_s \) data in Fig. 2 can be plotted on the universal curve as a function of \( s_0 \) for comparison with \( pp \) and \( AA \) (Fig. 7).
The previous analysis clarifies that in $e^+e^−$ annihilation at the LEP or lower energies there is no chance of observing the enhancement of the strangeness production, that is $γ \gtrsim 0.95$, because the parton density in the transverse plane is too small: $s_0$ turns out to be $\lesssim 3$ but a value larger than 6 is required. The multiplicity one needs in $e^+e^−$ annihilation to obtain the same value of $s_0\tau_0$ determined for $pp$ and $Pb−Pb$ collisions is reported in Table 1, by extrapolating to very high energy the fit of the multiplicity at lower energy in Fig. 5. The corresponding center-mass energy depends on the power of $\sqrt{s}$ in the fit and with previous formula for $dN_{ch}/dy$ one gets $s_0\tau_0 = 5.2 \pm 0.8$ fm$^{-2}$, corresponding to $dN/dy \simeq 10 \pm 2$, at $\sqrt{s} \simeq 7^{+4}_{−3}$ TeV, a energy range that could be reached in the planned $e^+e^−$ colliders.

Similarly, the value $s_0\tau_0|_{e^+e^−}$ is too small for observing flow-like effect, although in this case a precise value is difficult to determine, due to the uncertainty in the estimate of the transverse area associated with the eccentricity.

Moreover, the question of collective behaviors in $e^+e^−$ collisions has been raised in Ref. [45] from the perspective of hydrodynamics and in Ref. [46] by a string-based model. The interesting aspect concerns the role of the initial geometry of the colliding system, because the long range correlations observed in ”small” ($pp$ and $pA$) settings suggest it is not the unique condition for collective effects. In $e^+e^−$ annihilation, at some very high energy, the parton density in the transverse plane should be large enough but the geometric pre-condition could not be fulfilled for a $v_2$ measurement. Therefore, the possible (un)detetection of elliptic flow in $e^+e^−$ should clarify the role of the two main ingredients (geometric and entropic) of collective dynamics.

In conclusion, one can expect to see in $e^+e^−$ annihilation the asymptotic behavior measured in high energy $pp$ and $AA$ collisions only at very much higher energies than so far available. There is a hierarchy in energy and multiplicity requirements to see the “collective” effects, starting from low energy in $AA$ to larger energy and multiplicity in $pp$ collisions and still much larger energies in $e^+e^−$ annihilation.

**Data Availability Statement** This manuscript has no associated data or the data will not be deposited. [Authors’ comment: The reported data are available in the corresponding quoted publications.]

### Table 1

| $\frac{1.5\ dN_{ch}}{A_{T}\ dy}$ | $\frac{dN_{ch}}{dy}|_{e^+e^-}$ | $\frac{dN_{ch}}{dy}|_{pp}$ | $\frac{dN_{ch}}{dy}|_{PbPb}$ | $PbPb$ Cent. |
|---------------------------------|---------------------------------|-----------------------------|-----------------------------|--------------|
| 20.1 ± 0.8                     | 58 ± 3                          | 100. ± 4                    | 1943. ± 56                  | 0–%          |
| 17.5 ± 1.1                     | 49 ± 4                          | 87. ± 5                     | 1587. ± 47                  | 5–10%        |
| 15.4 ± 0.9                     | 42 ± 3                          | 76. ± 4                     | 1180. ± 31                  | 10–20%       |
| 12.2 ± 0.6                     | 31 ± 2                          | 60.6 ± 3.1                  | 649. ± 13                   | 20–40%       |
| 8.3 ± 0.7                      | 19 ± 2                          | 41.2 ± 3.4                  | 251. ± 7                    | 40–60%       |
| 5.2 ± 0.8                      | 10 ± 2                          | 26. ± 4                     | 70.6 ± 3.4                  | 60–80%       |
| 3.1 ± 1.1                      | 5 ± 3                           | 12.4 ± 3                   | 17.5 ± 1.8                  | 80–90%       |

**Fig. 6** Transverse radius, $R_T$ in fm, in $e^+e^−$ (black) and $pp$ (red)

**Fig. 7** $γ_s$ versus $s_0$

4 Comments and conclusions

The previous analysis clarifies that in $e^+e^−$ annihilation at the LEP or lower energies there is no chance of observing the enhancement of the strangeness production, that is $γ \gtrsim 0.95$, because the parton density in the transverse plane is too small: $s_0$ turns out to be $\lesssim 3$ but a value larger than 6 is required.

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