Construction of simulation models of lunar observations

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Abstract. The aim of the present paper is to establish optimal parameters for simulation models of the lunar telescope. Modelling of the observations taken from the surface of the Moon is carried out on the basis of dynamic algorithms describing diurnal motion of stars and specifically designed software modules. The first steps in the development of simulation models were made when planning the ILOM (In-Situ Lunar Orientation Measurement) mission which implied the installation of an optical telescope on one of the lunar Poles. The main task of the project was to observe physical libration of the Moon directly from its surface in order to reveal subtle effects related to the characteristics of internal structure of our natural satellite, including the refinement of the Lava elasticity coefficients, tidal dissipation and dissipation at the core-mantle boundary parameters, core’s size, ellipticity, and its chemical composition.

The results obtained in this work allow for the development of lunar physical libration and dynamics theory in order to apply them in the future lunar observations taken with the automated optical telescope.

1. Introduction

The joint works by Kazan Federal University (KFU) and the National Astronomical Observatory of Japan (NAOJ) within In-Situ Lunar Orientation Measurement (ILOM) project started in 2003 [1–3]. The Japanese side performed the analytical and technical works on construction of the lunar telescope, while the Russian side developed theoretical support for further lunar observations such as development of an analytical theory of the Moon’s physical libration (PLM) and simulation of observations from the lunar surface in order to reveal qualitative opportunities and restrictions for future observations. It should be noted that the study of physical libration is one of the available sources of information on the lunar internal structure. This is why, at the modern stage of PLM conceptions’ development, its application in this direction is emphasized. The new geophysical data and highly accurate models of the Moon’s gravitational field generate considerable opportunities to refine both the numerical and analytical theories of PLM in terms of including the parameters characterizing the complex structure of the lunar body. And that will be used to create a space reference net.

2. The algorithm of the lunar observations reduction

The first stage of simulation includes:
- selection of stars covered by the telescope in the direction of the lunar Pole’s precessional motion;
- analysis of star tracks’ behavior during their observation;
- control over the sensitivity of determined selenographic coordinates of stars to the change of lunar dynamic model’s parameters and the lunar body’s elasticity parameters.

To solve these tasks, the analytical theory of physical libration of the solid Moon by Petrova [4] enhanced by inclusion of selenopotential’s harmonics of 4th order and additional terms of free libration, was used. Since the aim of the study is qualitative assessment of possibility of taking observations from the lunar surface, the accuracy of PLM theory, on the one hand, does not matter much, but on the other hand, the analytical theory of PLM is an important component during simulation.
During the simulation of the observing process, an “ideal” coordinate system was taken; it was assumed that the telescope was placed exactly on the dynamic Pole of the Moon, its vertical axis was directed strictly along the main moment of inertia \(C\) and axes of the objective were directed along the other two main moments of inertia \(A\) and \(B\).

In accordance with the telescope’s parameters, in the neighborhood of the Poles’ precessional motion the stars of magnitude more than 12 were chosen [5]. A software was developed both to select the stars from a number of star catalogues and transform ICRF stars’ coordinates into ecliptic ones \(\lambda_s, \beta_s\) (s was taken for the observation date with taking into account reducing amendments for the precession of celestial pole, aberration, and proper motion of stars). From the full list of stars, the ones visible through the telescope on certain date and observation period were chosen by their coordinates. Thus, for 13 sidereal months from January 1, 2013 for the Northern Pole 48 stars were selected. A fictive star with coordinates of the northern lunar Pole: \(\lambda_{PN} = \Omega + 90^0, \beta_{PN} = 90^0 - l\). Here, \(\Omega\) is average longitude of the lunar orbit’s ascending node on observation date.

The simulation process included 2 problems of lunar physical libration – direct and inverse ones. A system of the equations that link Cartesian selenographic \((x_S, y_S, z_S)\) or equatorial \((\alpha_S, \delta_S)\) coordinates of a star to its ecliptic \((\lambda_S, \beta_S)\) ones is based on rotation matrices:

\[
\begin{pmatrix}
    x_S \\
    y_S \\
    z_S
\end{pmatrix} = \begin{pmatrix}
    \cos \delta_S \cos \alpha_S \\
    \cos \delta_S \sin \alpha_S \\
    \sin \delta_S
\end{pmatrix} = \Pi_2(\varphi + 180^0) \times \Pi_X(-\Theta) \times \Pi_Z(\psi) \times \begin{pmatrix}
    \bar{X}_S = \cos \beta_S \cos \lambda_S \\
    \bar{Y}_S = \cos \beta_S \sin \lambda_S \\
    \bar{Z}_S = \sin \beta_S
\end{pmatrix}.
\]

When solving the direct problem using the PLM theory on the basis of a certain model of gravitational field (numerical values of Stokes coefficients \(S\) that are substituted into Poisson series), the calculation of left-hand side of the equation of the “observed” in Cartesian selenographic coordinates of stars at the chosen period of observation was carried out.

When solving the inverse problem, the Cartesian coordinates of stars, determined at the stage of the direct problem, were taken as “observed” coordinates of stars in order to determine the “unknown” angles of PLM. In equation these unknown ones are introduced in the expressions for rotation matrices and may be determined approximately. To solve the inverse problem, we used the gradient method. It was shown that the error of determining libration at tilt angle \(\rho(t)\) and node \(\sigma(t)\) did not exceed the errors in the star’s observed selenographic coordinates, while libration in longitude \(\tau(t)\) could not be determined from the polar lunar-based observations.

3. Modeling of stellar trajectories
The construction of 48 stars’ trajectories was performed when solving the direct problem [5]. In contrast to terrestrial diurnal star parallels, lunar ones represent spirals rather than closed circles. This occurs for 2 reasons:

1. The precession period of the lunar Pole (18.6 years) is much less than the period of the terrestrial one (26 000 years).
2. “Lunar days” last longer than terrestrial ones and have a duration of 23.7 terrestrial days. Here we have introduced the definition that 1 “lunar day” is equal to a sidereal month.

In work [6] authors demonstrated this effect in simulating ILOM observations for the first time. However, due attention was not given to the astrometrical features of star trajectories and the factors defining those features were not taken into consideration either. An important moment was found in the behavior of star tracks. The stars whose longitude at the time of observation appears to be less than the one of the lunar pole describe loops on the celestial sphere. This occurs due to the Pole’s reverse motion: initially, a star moves towards the Pole, the polar distance decreases, and when their longitudes become equal, the star goes in spiral from the center of a telescope. If the lunar rotation is described only by the Cassini laws, then the spirals have symmetrical forms. Inclusion of physical
liberation into the calculations leads to distortion of spirals’ circular forms as well as to the general shift from the center of the telescope placed in the point of average lunar pole.

During the simulation, the sensitivity of stars’ selenographic coordinates accuracy to small changes in the lunar internal structure was investigated. The analytical theory allows simulating coordinates dependence on both time and parameters of the lunar gravitational field as Poisson series. This enables to perform simple calculations for various sets of the dynamic parameters characterizing mass distribution in the lunar body. The divergences in simulated selenographic coordinates are caused by application of 4 different gravitational field models (dynamic ones):

1. The model based on lunar laser ranging (LLR) data [7].
2. GLGM-2 based on Doppler monitoring for “Clementine” satellite [8].
3. LP150Q based on Lunar Prospector mission [9].
4. SGM1OOh improved gravitational field model based on Doppler monitoring of SELENE satellite system [10].

Analysis of the star tracks built at the stage of direct problem for the dynamic models listed above shows:

1. Owing to physical libration, star tracks shift from the center of the telescope placed at the average Pole of lunar rotation; for January 2013 this shift occurred in the opposite direction from the Earth, circular form of spirals gets distorted.
2. Star tracks calculated for different dynamic models significantly split up. Even for the most modern models SGM100 and LP150 the difference in simulated stars’ coordinates exceed hundreds of milliseconds. This provides hope that future ILOM observations, whose accuracy is planned to be 1 mas, will allow refining parameters of the gravitational field.

4. Analysis the viscoelastic Moon model
The PLM theory by Petrova is constructed for rigid Moon. For deformable Moon in work [11] a semi-analytical expansion of the analytical model [12] was obtained on the basis of DE245 dynamic model. The calculations reconciling the solutions obtained in this paper and in [11] were carried out. As a result, trigonometric series (particular case of Poisson series) describing the model of viscoelastic Moon [13] were obtained. Analysis of this model was conducted within the inverse problem of PLM.

The solution algorithm is as follows:

1. In solving the direct problem, selenographic coordinates \( x_\text{obs}^\text{s}, y_\text{obs}^\text{s}, z_\text{obs}^\text{s} \) defined as observed ones are determined on the basis of modified theory of PLM for viscoelastic Moon. Those coordinates are made artificially noisy using a random number generator; the value of “noise” was \( \varepsilon = 1 \) ms.
2. When solving the reverse problem, the “observed” values of PLM angles \( \tau^\text{obs}(t), \rho^\text{obs}(t), l\sigma^\text{obs}(t) \) are calculated. Here, we should notice that for the inverse problem the values of angles \( \tau^c(t), \rho^c(t), l\sigma^c(t) \) calculated for the rigid model of PLM are substituted as an initial approximation.
3. Then “residual differences” are analyzed:

\[
\Delta \tau(t) = \tau^\text{obs}(t) - \tau^c(t), \Delta \rho(t) = \rho^\text{obs}(t) - \rho^c(t), \Delta l\sigma(t) = l\sigma^\text{obs}(t) - l\sigma^c(t).
\]

If the lunar body corresponded to the rigid-body model, then the values of the differences would be within observational errors. But the “observed” selenographic coordinates include the information on viscoelastic structure of the Moon. As a result, it follows that in \( \Delta \rho(t), \Delta l\sigma(t) \) “residual differences” there are both periodic variations of significant amplitudes and constant shifts for the values considerably exceeding the specified error \( \varepsilon = 1 \) ms.

5. Summary and conclusions
Residual differences for libration in longitude \( \Delta \tau(t) \), as it was suggested, remain at the level of zero and do not “sense” changes in the lunar body structure. The procedure of a fast Fourier transform was applied to the obtained spectrum of \( \Delta \rho \) and \( \Delta l\sigma \) residual differences. Analysis of a resulting frequency
spectrum [13] allowed revealing a few harmonics sensitive both to the elasticity and viscosity effects. Taking into account the elasticity effect causes shifts in a star’s position and correspondingly the libration angles calculated according to the position. There is also sensitivity to Love $k_2$ number. Viscous terms of the expansion are manifested as a phase shift in corresponding harmonics for rigid Moon. Owing to viscosity, there are a constant shift in $I\sigma$ ($-0.2619^{\circ}$) and additional incline ($0.0051^{\circ}$) in $\rho$; at the same time, the constant shift $0.0066^{\circ}$ in $\rho$ is explained by the Moon’s elasticity.

Thus, the harmonics revealed are found to be most sensitive to viscoelastic properties of the lunar body and may be used as indicators when refining elasticity and viscosity parameters of the Moon [14]. The presented studies were performed within the international cooperation and contributed much to the modern investigation of the lunar rotation and to space navigation [15]. The results of the present work open up new prospects for PLM theory development in order to apply it for processing the future observations taken with the lunar polar telescope [16]. The obtained results can be used to create the selenocentric coordinate system [17–21].

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