We present a model for tail wavelets, a phenomenon also known as “echo” in the literature. The tail wavelet may appear in signal reconnaissances in the merger of binary compact objects, including black holes and neutron stars. It follows main gravitational waves (GWs) emitted during the merger process. We study, among other generic properties, the pressure of the gravitational waves and their subsequent effects to the surrounding matters/dark matters. We demonstrate that the matters/dark matters surrounding the compact objects lead to the speculated tail wavelet following the main GW. In this scenario, the surrounding matter is first pushed away to some altitude by the main wave, and then splash down to the black hole region, and thereby excites the tail wavelet. We illustrate this idea in a simplified model, where numerical estimations are carried out concerning the specific distribution of the dark matter outside the black hole horizon and the threshold values in accordance with observations. We demonstrate that the tail wavelet can be a natural phenomenon in frame of general relativity, without invoking any modified gravities or quantum effects. Furthermore, in frame of this model one can investigate the distribution of dark matters surrounding the black holes through analysing the waveform of the tail wavelet from the forthcoming data with better resolution, especially those from the third generation GW detectors.

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I. INTRODUCTION

The observations of gravitational waves (GWs) from binary compact objects provide unprecedented opportunities for direct verification of general relativity in the strong field and high-speed regime. It symbolizes the beginning of an era of GW astronomy as a branch of observational astronomy with considerable precision. Regarding fundamental physics, we finally have the experimental observables at our disposal for testing whether a quantum theory of gravity is plausible. The GW is an incisive tool to study the structure of the black holes and other compact objects. In particular, it may unwrap the enigma about the quantum structure of the spacetime beyond the horizon. In regard to the quantum effect, it is believed that the spacetime is no longer a smooth manifold. It may possess a specific non-trivial structure when the quantum fluctuations become significant [1]. In fact, the structure of the black hole horizon has been suggested to solve the black hole information problem, for example, in terms of the fuzzball model [2] and the firewall model [3]. Such a speculated structure may be detected by the GWs emitted from the binary black holes [4]. In these models, the horizon is not a smooth sphere, but a shell with finite thickness. A wave can bounce back and forth between the inner wall and the outer wall of the shell multiple times. As a result, tail wavelets referred as “echoes” may appear after the main GWs have been emitted [5]. The waveform of the echo model suggested in [5] may be ruled out by analysis in [8–10]. However, the tail wavelet may be in a different form compared with [5]. One can deal with this problem from an opposite direction. Is there completely no signal in the data after the main wave? An detailed analysis in [11] demonstrates that signals appear for several GW events with a p-value of order 1% or sometimes significantly less. More approaches for signal reconnaissance of the tail wavelets are presented in [12]. In this context, it turns out to be somewhat a surprise when the tail wavelets are also observed in GW events of the binary neutron stars, besides those of binary black holes. While explanations are given in terms of the resulting black hole remnant [13], the observed events also imply the possibility that the tail wavelet may be related to specific generic properties of the binary system other than the near-horizon quantum structure [14]. This is because it would be quite challenging to interpret the appearance of tail wavelets in terms of the properties of the horizon if the latter might not exist in the first place.

Following the above train of thought, we proceed by discussing some of the generic properties of the compact objects, which might potentially cause the tail wavelets. Generally, a black hole/neutron star is surrounded by specific form of matter invisible to the accessible electromagnetic spectrum, whose density is much higher than that of the average value in the universe. We
will show later that the matter or dark matter distributed in the vicinity of the compact object can provide a mechanism for the observed tail wavelets. Further quantitative studies of the wavelets may be helpful to extract the distribution of the dark matter around the compact object. We note that in the literature there exist extensive studies on the distribution of dark matter around celestial bodies. Some pioneer works [15] have explored specific properties of the dark matter distributions in the halos of various mass scales. A remarkable feature of the distribution of the dark matter is its universality. Halos of different scales, namely, from the mass of the celestial body to that of a cluster of galaxies, all possess similar distribution [16]. To be specific, for the supermassive black holes in elliptical galaxies and barred/ordinary spiral galaxies, such as our Milky Way, the observed density profile reads [17]

\[ \rho \sim r^{-\gamma}, \quad \gamma \in (1.5, 2.5). \]  

(1)

For the stellar mass black holes, direct observation concerning the dark matter halo is still absent. Regarding the universality of the distribution of the dark matter halos, in the present study, we will assume that it also obeys the form of Eq. (1).

Our proposed scenario is described as follows. The passage of the main GWs from the binary system merger pushes up the matter to a certain altitude. The latter causes the infalling of the surrounding matter. To be specific, after the initial waves traverse through, the matter falls back towards the black hole horizon and subsequently produces the tail wavelets. The relevant physical quantities are the energy density, pressure, energy flux of the GW. For linear plane GWs [1] with frequency \( \omega \) and amplitude \( A \), the energy density \( \rho \) and pressure \( P \) are found to be,

\[ \rho = \frac{\omega^2}{32\pi}(A_+^2 + A_\times^2), \]  

(2)

and

\[ P = -\frac{\omega^2}{32\pi}(A_+^2 + A_\times^2), \]  

(3)

while the energy flux \( f \) in the direction of propagation reads,

\[ f = \frac{\omega^2}{32\pi}(A_+^2 + A_\times^2). \]  

(4)

Here \( P \) is negative since it is evaluated for a volume element in which the gravity ray is out-going. Also, the luminous velocity \( c \) has been set to \( c = 1 \) in the above equations. The above expressions are in coincidence with the Landau-Lifshitz pseudo-tensor for GWs. In a general case, the stress-energy tensor for the GWs can be defined as a second order perturbation of the Einstein tensor [18].
The general definition of GWs in the strong field region leads to some uncertainties, which depend on the specific choice of the background for the GWs in question. For example, the obtained waves possess different forms when one expands the metric about a Minkowski spacetime as compared to a Schwarzschild one. Nonetheless, the relation between power per area $S$ and the pressure $P$ satisfies

$$P = S/c,$$  \hspace{1cm} (5)

for massless fluctuations (gravitons). It is noted that Eq. (5) is valid for GWs in a general sense, independent of the background metric. As the total energy radiated from a binary object can be calculated, one thus obtains the corresponding pressure. In what follows, we estimate the order of magnitude of the pressure by considering the GW150914 event as an example. The system emits an amount of energy equivalent to 3 solar mass in about 0.1 seconds. The GW emission is “directional”. However, its directivity is much weaker than electromagnetic radiation, since the lowest order radiation is of the quadrupole. Thus, if one omits the GWs’ directivity, one may approximate the emission as isotropic and estimate the pressure of GW at the horizon by,

$$P = \frac{w}{4\pi r^2 c} \sim \frac{3 \times 10^{49}}{4\pi (62 \times 3 \times 10^5)^2 \times 3 \times 10^8} = 2.3 \times 10^{29} \text{Pa}.$$  \hspace{1cm} (6)

As for comparisons, the pressure at the center of the sun is about $2 \times 10^{16} \text{Pa}$, and the pressure at the center of a neutron star can reach $10^{33}$ to $10^{34} \text{Pa}$. So the estimated value of the GW pressure is found to be comparable to those of the interior of stellar systems.

At the region near horizon, the Newton gravity fails. We just make an estimate to clear which force, the pressure or attraction, dominates for an object around the black hole. Assuming an object whose mass is 1kg and area is 1 $m^2$. At the horizon, it is attracted by the black hole with force,

$$F = G \frac{m_1 m_2}{r^2} = 2 \times 10^{11} \text{N}.$$  \hspace{1cm} (7)

At the same time, the pressure it sensed is

$$F' = 2.3 \times 10^{29} \text{N},$$  \hspace{1cm} (8)

when the main wave passes. It is clear the the repulsive force is much lager than than attractive force for this object. Both the repulsive force and attractive force decrease with $r^{-2}$. Thus, at large distance, the ratio between the repulsive force and attractive force is exactly $F'/F$, if the wave is completely absorbed by the object.
Furthermore, we assume that the distributed matter can absorb a tiny fraction of GW. Meanwhile, the pressure also decreases as the GW gradually loses its energy. A rigorous treatment of the scattering between the graviton and matter demands a full-fledged theory of quantum gravity, which is still unknown to us. Tree level calculations may also be entirely plagued by loop corrections. Nevertheless, some preliminary studies on the scattering of gravitons have been carried out, see for example [19]. By very general arguments, similar to those for earlier approaches of quantum physics, one can estimate the total cross-section of a graviton scattered by a matter particle [20],

$$\sigma \sim 10^{-68} \text{m}^2.$$  \hfill (9)

Apparently, this cross-section is quite small. According to the known GW events, a typical event emits an amount of energy of several solar mass, and the characteristic frequency of the GW is about 100Hz. Thus for a typical GW event, the number of gravitons is approximately

$$N \sim \frac{M \Omega c^2}{h \nu} = 3 \times 10^{78}. \hfill (10)$$

Besides, the sunken region of the gravitational potential may concentrate a large number of dark matter particles, as studied in many previous works. Therefore, the total energy transferred from GW to the surrounding dark matter can be considerable. As a rigorous approach is not yet feasible, in the remainder of the present work, we carry out a phenomenological approach to investigate this interaction between GWs and dark matter.

The present paper is organized as follows. In the next section we develop a microscopic model to describe how the matter or dark matter is pushed away, and subsequently falls back, and eventually generates the tail wavelets. In section III, we study the waveform of the resultant wavelets numerically. Further discussions and concluding remarks are given in section IV.

II. GENERATION OF THE TAIL WAVELETS

From a macroscopic viewpoint, the rate of energy transfer from the GWs to the surrounding matter should be proportional to the energy flux of the wave as well as to the local density of the matter, where in principle, the absorption coefficient can be calculated by the corresponding microscopic theory. Let’s consider a spherical distribution of matter outside the horizon, for $r > r_h$. For a thin layer of matter distribution, the energy transferred, $dE$, should be proportional to the total energy of the GWs traversing the matter $H$, and the mass of the layer $dm$, namely,

$$dE(r) = \chi H(r) dm,$$  \hfill (11)
where $\chi$ is the absorption coefficient. As discussed in the preceding section, the magnitude of $\chi$ is mostly small but still appreciable for our present model. For a spherical shell with thickness $dr$, one has,

$$dm = 4\pi r^2 \rho_m dr.$$  \hspace{1cm} (12)

As the GWs sweeps across the surrounding matter, the total energy flux on a 2-surface decreases with increasing $r$, as a result of the energy loss while it is gradually transferred to the dark matter. Because of the non-locality of gravitational energies, it is difficult to locate the exact spacetime position of the source of the GWs, which is different from the electromagnetic waves. We do not consider the interior structure of a black hole. Therefore, the innermost surface we consider is the horizon. On the other hand, if the final product is a neutron star, the innermost surface in question is the surface of the star. By assuming the GW energy across the horizon to be $H_0$, we can easily evaluate the energy transferred to the mass $E(r)$, satisfying

$$H(r) = H_0 - E(r).$$ \hspace{1cm} (13)

By submitting (13) into (11) and integrating from $r_h$ to $r > r_h$, one finds,

$$E(r) = H_0(1 - e^{-\chi(m(r) - m(r_h))}).$$ \hspace{1cm} (14)

Given the precision of $1/c^2$, the potential in post-Newtonian approximation possesses the same form as the Newtonian one \cite{21}. Thus one may simply adopt the Newtonian potential in our calculations of the potential for the matter. A thin shell of matter at radius $r$ has the potential

$$d\Omega = GM \left( \frac{1}{r_h} - \frac{1}{r} \right) dm.$$ \hspace{1cm} (15)

Therefore, the total potential energy of matter for an initial matter distribution between $r_h$ and $r$ reads,

$$\Omega_i = GM \int_{r_h}^{r} \left( \frac{1}{r_h} - \frac{1}{r} \right) dm = 4\pi GM \int_{r_h}^{r} \left( \frac{1}{r_h} - \frac{1}{r} \right) \rho_r r^2 dr.$$ \hspace{1cm} (16)

After the main GWs traverse through the matter or dark matter, the potential energy of final distribution

$$\Omega_f = GM \int_{r_h}^{r} \left( \frac{1}{r_h} - \frac{1}{r} \right) dm = 4\pi GM \int_{r_h}^{r} \left( \frac{1}{r_h} - \frac{1}{r} \right) \rho_f r^2 dr.$$ \hspace{1cm} (17)

The energy difference between the initial and final configurations gives rise to the energy of subsequent wavelet tails. As a first approximation, we assume

$$\epsilon = \beta(\Omega_f - \Omega_i) = \beta H_0(1 - e^{-\chi(m(r) - m(r_h)))},$$ \hspace{1cm} (18)
where we have introduced $\beta$ as a coefficient measuring the efficiency of the tail wavelets excitation, one has obviously $\beta < 1$. In practice, as the matter or dark matter distributed is mostly in the vicinity of the compact object, only a small portion of the energy of the main GWs is absorbed. Subsequently, it gives rise to the generation of the tail wavelets where $\beta$ is small in magnitude.

Now we turn to study the time scale between the emission of the main GWs and the tail wavelets. This can be estimated by simply investigating the equation of motion of a free-falling particle from a given height. Here we suppose that the background spacetime to be Schwarzschild

$$ds^2 = - \left(1 - \frac{2GM}{c^2r}\right)dt^2 + \left(1 - \frac{2GM}{c^2r}\right)^{-1}dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2),$$

(19)

and the emission of the tail wavelets do not affect the background spacetime, and write down,

$$r^2 \dot{\phi} = 1/C_1,$$

(20)

$$(1 - 2GM/(c^2 r))c \dot{t} = C_2,$$

(21)

$$(1 - 2GM/(c^2 r))^{-1} \dot{r}^2 + r^2 \dot{\phi}^2 - (1 - 2GM/(c^2 r))c^2 \dot{t}^2 = -\eta_0.$$  

(22)

For simplicity, we only consider the radial motion by taking $\eta_0 = 0$. The results for the numerical solution of Eqs. (20-22) are presented in Fig. 1. There, we consider a particle infalls from an initial altitude of $R$ towards the horizon $r_h$.

To estimate the amplitude of the GW, one makes use of Eq. (2). The energy density is proportional to the square of the amplitude $L_0$, thus around the horizon, one has

$$\frac{\rho_{r_h}}{\rho_L} = \frac{L_0^2}{r_h^2}.$$  

(23)

If the energy loss of the GWs is insignificant, the energy density is inversely proportional to the square of the distance from the center, namely,

$$\frac{A_{r_h}}{A_L} = \frac{L_0}{r_h}.$$  

(24)

For the event GW150914, we have $r_h = 1.8 \times 10^5$ meters, $L_0 = 1.2 \times 10^{25}$ meters, and $A_L = 10^{-21}$. Then one obtains,

$$A_{r_h} = \frac{1}{15}.$$  

(25)

Hence it seems to be reasonable to consider that the amplitude of the initial main GWs to be $A_{r_h} \sim 0.1$ in the following analysis.
FIG. 1: The numerical solutions of the free-fall equation Eqs. (20-22) as an estimation of the time scale between the main GWs and the tail wavelets. The calculated results are presented regarding different initial conditions of $R = r(0)$, which have been rescaled into a same value in this figure. The location of the event horizon is depicted by the red solid line.

III. CHARACTERISTIC WAVEFORM OF THE TAIL WAVELETS

The waveform of the tail wavelet by generated by the infalling of the matter is similar to that of the ordinary quasinormal modes [24]. Therefore, in our model, the observed tail wavelets are also a manifestation of the black hole quasinormal modes. Its amplitude is considerably insignificant, due to the fact that it is triggered by only a tiny portion of the energy of the main GWs. However, there is a subtlety. Since the infalling process is continuous, initial disturbance period carries the information not only on the black hole itself but also about the matter distribution surrounding it. It is therefore expected that the waveform in initial disturbance period will be characteristically different from the ordinary quasinormal modes. In this section, we present the numerical calculations of the waveform associated with the gravitational perturbation of the metric owing to the infalling of the matter.

The radial master equation of the gravitational fluctuation around the black hole reads [25],

$$f(r) \frac{\partial}{\partial r} \left( f(r) \frac{\partial \Phi}{\partial r} \right) - \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} - V(r) \Phi = 0. \tag{26}$$

Here $V$ denotes the potential of the black hole. For the odd and even modes, $V$ reads,

$$V_o = f(r) \left( \frac{L(L+1)}{r^2} - \frac{3r_h}{r^3} \right), \tag{27}$$
\[ V_e = \frac{2}{r^3} f(r) \left( \frac{9r_\text{h}^3/8 + 9\lambda r_\text{h}^2 r/4 + 3\lambda^2 r_\text{h} r^2/2 + \lambda^2 (1 + \lambda)r^3}{(3r_\text{h}/2 + \lambda r)^2} \right), \] 

respectively. Here,

\[ \lambda = L(L + 1)/2 - 1, \]

where \( L \) is azimuthal quantum number, and

\[ f(r) = 1 - r_\text{h}/r. \] 

Based on the analysis in the previous sections, we take the following initial conditions. For convenience, we choose \( \eta = ct/r_\text{h} \) and consider the following different cases:

Case A, \( \Phi = 0 \) as \( \eta \in (0, 5) \).
Case B, $\Phi = \eta/250$, $\eta \in (0, 2.5)$; $\Phi = (\eta - 1)/150$, $\eta \in [2.5, 5)$.

Case C, $\Phi = 0.01\exp\left(-\frac{(\eta-2)^2}{2}\right)$, where $\eta \in (0, 5)$.

Case D, $\Phi = 0.01$ as $\eta = 0$

Figs. 2 and 3 demonstrate the waveforms of the odd and even fluctuations for all the four cases of initial conditions. For comparison purpose, we also show the ordinary quasinormal modes of a black hole. According to the present estimations, the magnitude of the tail wavelets is at most $\sim 10\%$ of the main GWs. Thus we take the initial perturbation to be $\Phi = 0.01$ for our calculations. One could observes that the waveforms generated by assumed forms of perturbations are different from those trigger by a pulse, the latter corresponds to the ordinary quasinormal mode which only carries the information of the black hole itself. We expect forthcoming data with better resolution for the waveform, especially those from the third generation GW detectors like the Einstein Telescope and Cosmic Explorer, may shed light on the feasibility of the present model.

IV. CONCLUDING REMARKS

According to our model, there might be tail wavelets occurring after the main GWs from the merger of binary systems. Such tail wavelets exist not only for the GW events of binary black holes but also for those of binary neutron stars. Therefore, it is speculated that the physical mechanism behind the phenomenon might not be related to the quantum structure of the black hole horizon but is rather associated with certain generic properties of the collapsing binary system. The main characteristic of our approach is that the explanation is given within the framework of Einstein’s general relativity, rather than originated from a modified theory of gravity or quantum effect.

Following this line of thought, in this work, we present a more natural and straightforward scenario for the generation of the tail wavelets. In our model, the cause of the phenomenon is attributed to the matter or dark matter surrounding the binary system. In particular, the dark matter distribution around a compact object has been studied in different scales for many years. We carefully investigate the pressure of the GWs and demonstrate that the GWs can perturb and even push away the matter distributed around the compact star. After the main wave traverses through, the matter falls back towards the black hole horizon, which in turn excites the tail wavelets. Based on our analysis, we assume four different initial conditions for the related gravitational perturbations and evaluate the corresponding waveforms.

We look forward to testing the proposed model against the forthcoming data from the GW detectors of the third generation. Further studies concerning specific dark matter distributions are
in progress.

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