Theoretical and experimental investigations on the dynamic and thermodynamic characteristics of the linear compressor for the pulse tube cryocooler

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Abstract: Theoretical and experimental investigations on the dynamic and thermodynamic characteristics of a linear compressor incorporating the thermodynamic characteristics of the inertance tube pulse tube cold finger have been made. Both the compressor and cold finger are assumed as a one-dimensional thermodynamic model. The governing equations of the thermodynamic characteristics of the working gas are summarized, and the effects of the cooling performance on the working gas in the compression space are discussed. Based on the analysis of the working gas, the governing equations of the dynamic and thermodynamic characteristics of the compressor are deduced, and then the principles of achieving the optimal performance of the compressor are discussed in detail. Systematic experimental investigations are conducted on a developed moving-coil linear compressor which drives a pulse tube cold finger, which indicate the general agreement with the simulated results, and thus verify the rationality of the theoretical model and analyses.

1. Introduction
The past three decades have witnessed a worldwide quest for space-qualified Stirling-type pulse tube cryocoolers (SPTC), and the ensuing successful applications in a wide variety of space missions since the first launch in the late 1990s have provided abundant evidences of the SPTC becoming a new generation enabling space regenerative cryocooler [1–3]. Most of the SPTCs used in space are driven by the Oxford-type moving-coil linear compressors because of the proven high efficiency, low EMI noise, enhanced manufacturability and reliability [4–6]. The inertance tube, due to its simple configuration, powerful phase-shifting ability, especially considering its complete passivity, has been widely employed as an effective phase-shifting approach in developing reliable and efficient SPTCs for space applications. The intention of this paper is to make systematic theoretical and experimental investigations on the dynamics and thermodynamics of the linear compressor, especially under the condition that it is coupled to the pulse tube cold finger with the inertance tube as the phase shifter.

The efficiency and capacity of the linear compressor is determined by its dynamic and thermodynamic characteristics, whose key parameters mainly include the input current, the operating frequency, the stroke, the forces on the piston, the dynamic pressure and the volume flow rate in the compression space, and also the phase angle between the dynamic pressure and the displacement. The dynamic and thermodynamic characteristics of the linear compressor have been investigated, in varying degrees, in several publications. For example, Marquardt et al. [4] described a set of
governing equations of the dynamic and thermodynamic characteristics used to design the various components in the Oxford-type linear compressor under given specified performance criteria. Since their study was focused on the linear compressor itself, it did not discuss the cases when the cold fingers were coupled. Koh et al. [6] studied the dynamic and thermodynamic characteristics of the linear compressor in the Stirling cryocooler, and indicated that the operating frequency and charge pressure had significant effect on the compressor performance. The work mainly studied the linear compressor itself, whereas the effect of the cold finger on the compressor was not discussed. Ko and Jeong [7] made the analysis about the SPTC in consideration of the dynamics of the linear compressor, and found that the dynamic behavior of the piston in the compressor was directly influenced by the SPTC’s load condition. Subsequently, Ko et al. [8] further made experimental investigations on the dynamic behavior of the linear compressor, and the results indicated that the longer inertance tube imposed a larger gas spring effect in the dynamic system of the compressor, and the characteristics and cooling performance of a SPTC were governed by the dynamic behavior of the piston as well as the cooler configuration. In the last two studies, the linear compressor and the cold finger were coupled together, and the dependence of the characteristics of the linear compressor on that of the cold finger was revealed. However, the two mentioned studies were mainly focused on the cold finger, whereas the systematic theoretical analyses of the linear compressor were not made.

As shown in the above literature survey, most of the previous studies, especially the theoretical investigations, only paid attention to the governing equations of the linear compressor. A few of the studies experimentally investigated the effects of the cold finger on the compressor. However, systematic analyses have not yet been carried out. For a typical SPTC, the compressor and the cold finger act on each other. Hence, in order to develop a high performance linear compressor for the SPTC, it is very important to investigate the dynamic and thermodynamic characteristics of the linear compressor in consideration of the characteristics of the cold finger, in which the latter mainly includes the cooling temperature, the cooling capacity and the frequency of the working gas, etc.

2. Analytic model of SPTC

Figure 1 shows the simplified model of the SPTC. In this model, the linear compressor consists of the piston and the compression space, which serves as the exit of the linear compressor and the inlet of the cold finger as well. Therefore, the compression space is the key component connecting the compressor and cold finger. To investigate the linear compressor, the influence of the cold finger on the thermodynamic characteristic of the gas in the compression space should be determined at first.

![Schematic of the simplified SPTC model](image)

2.1. Governing equation of the cold finger

In order to simplify the analyses, the working gas oscillating in the cooler can be assumed as a simple one-dimensional thermodynamic process. The SPTC driven by the linear compressor and with an inertance tube as the phase-shifter can be analogized as an electric circuit, as shown in Fig. 1. Each component in the SPTC can be considered as a complex impedance, which consists of the hydraulic capacity, hydraulic inductance and hydraulic resistance. Since the thermodynamic characteristics of the working gas in the compression space is the very object to be determined, the calculations of the characteristics should start from the reservoir.

Since the reservoir is much larger than other components, the volume flow rate and the amplitude of
the dynamic pressure in it can be ideally assumed as zero. As a result, the volume flow rate \( \bar{\Delta U}_0 \) and the amplitude of the dynamic pressure \( \bar{\Delta P}_0 \) at the inlet of the reservoir are given by [9]:

\[
\bar{\Delta U}_0 = \frac{i \omega f_{\text{IT}}}{\gamma P_{\text{m}}} \Delta \bar{P}_0
\]

where \( \omega, \gamma, V_I \) and \( P_m \) are angle frequency, ratio of specific heat, reservoir volume and charge pressure. \( \bar{\Delta U} \) and \( \bar{\Delta P} \) along the inertance tube can be expressed as [9–11]:

\[
\bar{\Delta U} = \int_{0}^{l_{\text{IT}}} \left( \frac{4 i \omega f_{\text{IT}}}{\pi D_{\text{IT}}^2} + \frac{64 f_{\text{IT}}}{\pi D_{\text{IT}}^2} \right) \rho \Delta \bar{U} \, dx
\]

\[
\bar{\Delta U} = \int_{0}^{l_{\text{IT}}} \frac{i \omega}{4 \gamma P_{\text{m}}} \Delta \bar{P} \, dx
\]

where \( D_{\text{IT}}, l_{\text{IT}}, f_{\text{IT}} \) and \( \rho \) are tube diameter, tube length, fanning friction factor and gas density.

For an idea pulse tube, the pressure can be regarded as identical along the axial direction. So the resistance and the inertance of the pulse tube can be neglected, while only the capacity exerts the effect on the working gas. Hence, the volume flow rate at the inlet of the pulse tube is given by [9]:

\[
\bar{\Delta U}_3 = \frac{i \omega f_{\text{IT}}}{\gamma P_{\text{m}}} \Delta \bar{P}_2
\]

The inertance of the regenerator can be neglected compared with the resistance owing to the large ratio between its length and diameter. Therefore, the impedance of the regenerator consists of the resistance and the capacity. Based on the continuity equation and the momentum equation of the working gas, the characteristics of the working gas in the regenerator can be expressed as [9, 11]:

\[
\bar{\Delta U}_3 - \bar{\Delta U}_4 = \int_{0}^{l_{\text{reg}}} \left( i \omega \frac{\pi D_{\text{h}}^2}{4 \gamma P_{\text{m}}} \Delta \bar{P}_3 + \frac{2 (V_r - T_a)}{\gamma \rho (V_r + T_a)} \Delta \bar{U}_3 \right) \, dx
\]

where \( \epsilon, l_{\text{reg}}, d_h \) and \( A_{\text{reg}} \) are porosity, regenerator length, hydraulic diameter, and cross area of regenerator, respectively. And \( a \) and \( b \) are coefficients of hydraulic resistance [10].

The SPTC shown in Fig. 1 contains three heat exchangers, which can be assumed isothermal. The inertances of them can be neglected. Therefore, the characteristics of the gas can be expressed as:

\[
\Delta \bar{P}_{\text{in}} - \Delta \bar{P}_{\text{out}} = \int_{0}^{l_{\text{HX}}} \frac{\eta_{\text{HX}}}{\gamma P_{\text{m}}} \Delta \bar{U}_3 \, dx
\]

\[
\Delta \bar{U}_{\text{in}} - \Delta \bar{U}_{\text{out}} = \int_{0}^{l_{\text{HX}}} \frac{i \omega}{\gamma P_{\text{m}}} \Delta \bar{P}_3 \, dx
\]

where \( \eta_{\text{HX}}, A_{\text{HX}} \) are resistance per unit length and valid cross area of the heat exchanger, respectively.

Based the above equations, the thermodynamic characteristics of the gas in the compression space can be determined accordingly. The resistance and inductance of the compression space can be neglected in front of its capacity owing to the ratio of its length to diameter. Therefore, the dynamic pressure at the surface of the piston equals the pressure at the inlet of the regenerator. The volume follow, and the angle \( \alpha \) between the pressure and the volume flow rate are given by:

\[
\Delta \bar{P} = \Delta \bar{P}_6
\]

\[
\Delta \bar{U} = \Delta \bar{U}_6 + i \omega \frac{V_{\text{com}}}{\gamma P_{\text{m}}} \Delta \bar{P}_6
\]

\[
\alpha = \pi/2 - \varphi = \arctan \left( \left| \text{Im}[\Delta \bar{P} / \Delta \bar{U}] \right| / \left| \text{Re}[\Delta \bar{P} / \Delta \bar{U}] \right| \right)
\]

where \( \varphi \) is the phase angle between the dynamic pressure and the displacement, and \( V_{\text{com}} \) is the volume of compression space. For a dual-opposed compressor, the volume flow rate is given by [12]:

\[
\Delta \bar{U} = 2 f X A_p
\]

where \( X \) is piston stroke, \( A_p \) is the cross area of piston. Therefore, the stroke can be determined.

2.2. Cooling capacity
If the regenerator does not produce any thermal loss, the PV power at the cold end of the regenerator is completely used to generate the cooling effect. But in the actual situations, the ineffectiveness loss of the regenerator exerts a great effect on the cooling performance, which should be considered in the evaluation of the cooling capacity. The time-averaged PV power at the cold end, the time-averaged ineffectiveness loss \( Q_{\text{loss}} \) in regenerator and the time-averaged cooling capacity are given by [7, 13]:

\[
Q_e = W_{\text{pv},4} - Q_{\text{loss}} = \frac{\Delta P_s \Delta U_s}{2} - \lambda C_p (\rho_s T_s \Delta U_s - \rho_t T_t \Delta U_t) - (KA_{\text{reg}} \mu_{\text{reg}})(T_s - T_t) \tag{13}
\]

where \( \lambda \) is the ineffectiveness, \( C_p \) is the specific heat, and \( K \) is the thermal conductivity.

### 3. Dynamic and thermodynamic characteristics of compressor

Since the analytic model of the cold finger has been established and the thermodynamic characteristics of the gas in the compression space can be determined by the above analyses, the dynamic and thermodynamic characteristics of the linear compressor can then be calculated accordingly. Because the governing equations cannot graphically reveal the characteristics of the linear compressor, the compressor will be investigated with a specific SPTC example as shown in Table 1. The SPTC will work at 80 K with 3.3 MPa of the charge pressure and 4 mm of the stroke.

| Table 1. The geometrical parameters of the SPTC |
|-----------------------------------------------|
| Linear compressor | Cold finger |
|-------------------|-------------|
| Piston diameter   | 20 mm       | Regenerator | Φ 16 × 67 mm |
| Moving mass       | 200 g       | Pulse tube  | Φ 10.5 × 92 mm |
| Magnet force coeff | 15 N/A      | Inertance tube I | Φ 2.7 × 2100 mm |
| Wire resistance   | 3 Ω         | Inertance tube II | Φ 3.3 × 1600 mm |
| Damping coefficient | 4.5 N·s/m  | Reservoir volume | 400 cm³ |
| Stiffness of flexure springs | 4500 N/m       |

### 3.1. Governing equation of the linear compressor

During operation, there are mainly six forces on the piston of the linear compressor: the linear motor force, the inertial force, the mechanical spring force, the gas force, the viscous dissipation, and the force caused by the dynamic pressure on the backside of the piston [4]. For most practical compressors, the volume of the gas on the backside of the piston is much larger than that of the compression space, which means the corresponding force is negligible compared with other forces. So the remaining five forces form the balance, as shown in Fig. 2. The characteristics of linear compressor are thus represented by Eq. (14) and Eq. (15) [4, 6]:

\[
BII \cos \theta + \omega^2 mX = k_m X + \Delta P A_p \cos \phi \tag{14}
\]

\[
BII \sin \theta = \omega c X + \Delta P A_p \sin \phi \tag{15}
\]

where \( B \) is the magnetic field, \( m \) is the moving mass, \( A_p \) is the piston cross sectional area, \( k_m \) is the axial mechanic springs stiffness, \( \omega \) is the angle frequency, and \( X \) is the stroke. The above parameters are defined artificially. The remaining four parameters \( I, \theta, \Delta P \) and \( \phi \) will vary with \( \omega \) and \( X \), respectively. Since \( \Delta P \) and \( \phi \) are heavily dependent on the thermodynamic characteristics of the cold finger, the left two parameters can then be determined accordingly.

Fig. 3 shows the variations of the simulated \( I \) and \( \Delta P \) with the operating frequency. The amplitude of the dynamic pressure increases quickly while the frequency changes from 5 Hz to 50 Hz. When the frequency is larger than 50 Hz, the influence of the frequency on the pressure becomes smaller. The pressure even decreases when the frequency varies from 50 Hz to 75 Hz. The variation of the input current is similar to that of \( \Delta P \). Fig. 4 shows the variations of the simulated \( \theta \) and \( \phi \) with the operating frequency. Both phase angles change sharply as the frequency increases. According to the simulated results, the frequency exerts great and complex influences on these four parameters, simultaneously. Therefore, when the cold finger remains unchanged, it is impossible to achieve the optimal frequency by changing one of above four parameters, while the other three ones unchanged.
3.2. Performances of the linear compressor

The input voltage of the linear compressor is determined by the magnet field, and the resistance, the inductance and the length of the wire in the coil. The governing equation of the voltage is given by [6]:

$$U = RI + L_e \frac{dI}{dt} + BL \frac{dX}{dt}$$

Therefore, the time-averaged input electric power of the compressor can be expressed as follows:

$$W_e = \frac{1}{T} \int_0^T U I dt = \frac{1}{2} I^2 R + \frac{1}{2} \omega X (\omega c X + \Delta P A_p \sin \varphi)$$

Fig. 5 shows the input electric power, Joule loss, and the cooling capacity varying with the operating frequency. Both electric power and Joule loss go up with the increase of the frequency, except when the frequency changes from 45 Hz to 65 Hz, which accords with the variations of $f$ and $\Delta P$. When the frequency changes from 20 Hz to 70 Hz, the Joule loss varies within a limited scope. When the frequency is higher than 70 Hz, the Joule loss increases quickly and starts to account for a larger percentage of the electric power, which will lead to the decrease of the compressor efficiency. However, the cooling capacity does not change according with the electric power. The maximum cooling capacity is achieved at 50 Hz with 7.0 W at 80 K. Hence, the cooling capacity is mainly determined by the thermodynamic characteristics of the working gas in the cold finger.

Fig.6 shows the simulated motor efficiency and relative Carnot efficiency versus the operating frequency. The optimal motor efficiency is about 90%, achieved at 65 Hz. However, the optimal relative Carnot efficiency is about 10.3%, achieved at 50 Hz. Therefore, the optimal frequency of the SPTC is mainly determined by the cold finger, but not by the compressor. As a result, the compressor must be optimized in consideration of the cold finger.

3.3. Effects of compressor geometrical parameters on motor efficiency
Fig. 7 shows the effect of the moving mass on the motor efficiency while the other parameters remain constant. The optimal frequency increases when the moving mass decreases from 300 g to 100 g. Hence, a larger moving mass can achieve a lower optimal frequency, according with the previous studies [6, 14]. However, in the simulation, it is found that the motor efficiency also decreases quickly with the increase of the moving mass, which has been ignored by most of the previous studies.

![Figure 6. Simulated motor efficiency and relative Carnot efficiency versus f](image)

![Figure 7. The effect of the moving mass on the motor efficiency](image)

Fig. 8 shows the effect of the magnet force coefficient $BL$ on the motor efficiency. Because the magnet field is difficult to be changed, the wire length is adjusted in this case, and the resistance also changes accordingly. As shown in Fig. 8, the motor efficiency is enhanced as $BL$ increases from 10 N•s/m to 20 N•s/m, and the optimal frequency remains constant. Therefore, increasing $BL$ is an effective method to achieve a well motor efficiency. However, a longer wire length will bring a larger moving mass, which should be considered in practical cases.

Fig. 9 shows the effect of the axial stiffness of the flexure springs on the motor efficiency. As the stiffness changes from 1000 N/m to 8000 N/m, the optimal motor efficiency only increases about 3%, while the optimal frequency keeps unchanged. Therefore, the axial stiffness of the flexure springs only has a slight influence on the motor efficiency.

![Figure 8. The effect of the magnet force coefficient on the motor efficiency](image)

![Figure 9. The effect of the axial stiffness of the flexure spring on the motor efficiency](image)

4. Experimental investigations
To verify the theoretical analyses, the experimental investigations are conducted on a linear compressor coupled with a coaxial pulse tube cold finger developed in authors’ laboratory. The SPTC is tested at 80 K with 70% of the stroke, and the frequency changes from 44 Hz to 52 Hz.

Fig. 10 shows the variations of the simulated and measured results with the operating frequency, including $\theta$, $\varphi$, $I$ and $\Delta P$. The simulated curves of the characteristics show the good agreement with the
corresponding experimental results. Corresponding with the theoretical analyses, the experimental results indicate that the operating frequency exerts great influences on the dynamic and thermodynamic characteristics of the linear compressor.

Figure 10. Simulated and measured values versus $f$: (a) $\theta$ and $\varphi$; (b) $I$ and $\Delta P$

Figs. 11 and 12 show the variations of the input electric power and the motor efficiency with the operating frequency, respectively. Same as $I$ and $\Delta P$, the simulated and experimental results also decrease when the frequency rises up. The experimental results are larger than the simulated values by about 7%. Since some losses are neglected in the theoretical analyses, the simulated values are better than the experimental results. The experimental efficiency increases linearly from 77.1% to 82.6%, and the simulated one changes from 77.9% to 83.2% with the increasing frequency. Both simulated and measured optimal efficiency are achieved at 52 Hz.

Figure 11. The electric power versus $f$  Figure 12. The motor efficiency versus $f$

Fig. 13 shows the variations of the cooling capacity with the operating frequency. The maximum capacities of the simulation and the experiment are 8.1 W and 7.0 W respectively, both of which are achieved at 47 Hz. Fig. 14 shows the variations of the relative Carnot efficiencies with the operating frequency. The simulated and measured optimal relative Carnot efficiencies are 18.5% and 15.8%, respectively, and both are achieved at 48 Hz. The results verify the theoretical analysis that the optimal efficiency of the SPTC is mainly determined by cold finger.

Figs. 10 to 14 show the good accordance between simulated and experimental results, which verifies the related theoretical analyses and the proposed simulation model.

5. Conclusions

This paper presents the theoretical and experimental investigations on the dynamic and thermodynamic characteristics of the Oxford-type moving-coil linear compressor in consideration of the thermodynamic characteristics of the inerance tube pulse tube cold finger.
An analytical model of the SPTC is established to calculate the thermodynamic characteristics of the working gas at any position of the SPTC, including the dynamic pressure, volume flow rate and the phase angle between them. This model is employed to determine effect of the pulse tube on the working gas in the compression space which connects the compressor and the cold finger.

Based on the analytical model, the dynamic and thermodynamic characteristics of the linear compressor are investigated and simulated with a specific case. The frequency exerts great and complex influence on $I, \theta, \Delta P$ and $\varphi$. To achieve the optimal frequency, the above-mentioned four parameters should be optimized simultaneously. The analyses also indicate that the cooling capacity is mainly determined by the thermodynamic characteristics of the working gas in the cold finger, and the optimal frequency of the SPTC is mainly determined by the cold finger, but not by the compressor. Therefore, the compressor must be optimized in consideration of the cold finger.

The experimental investigations are conducted on a linear compressor coupled with a coaxial pulse tube cold finger developed in the authors’ laboratory. The simulated results of the dynamic and thermodynamic characteristics indicate the fairly good agreement with the corresponding experimental results, which verify the rationality of the theoretical analyses.

![Figure 13. The cooling capacity versus $f$](image1)

![Figure 14. The relative Carnot efficiency versus $f$](image2)

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