On the Structure Functions of Mesons and Baryons in a Chiral Quark Model

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ABSTRACT. This research summarizes work done by myself (Nucl.Phys. A641 (1998)461), or in collaboration with R. M. Davidson (Phys.Lett. B348 (1995)163) and H. Weigel and L. Gamberg (Nucl.Phys. B560 (1999)xx).

I will discuss several topics related with the computation of structure functions in the quark model in general and its perturbative evolution. In particular, I address this topic in the Nambu–Jona–Lasinio model of hadrons, where the nucleon is constructed as a soliton. I show that the handling of the regularization procedure is crucial in order to obtain exact scaling in the Bjorken limit and fulfillment of sum rules. I also include some problems concerning the general validity of quark model calculations.

1. Reminder on Deep Inelastic Scattering

Deep inelastic scattering (DIS) provides some of the most convincing evidence for the quark sub–structure [1] of hadrons. The main object of study which is experimentally measured is the so-called hadronic tensor,

\[ W_{\mu\nu}(p, q; s) = \frac{1}{4\pi} \int d^4xe^{iq\cdot\xi} \langle p, s | [J^a_\mu(\xi), J^b_\nu(0)] | p, s \rangle. \tag{1.1} \]

where \( p, s \) are the hadron momentum and spin respectively, \( J^a_\mu(\xi) \) is a hadronic vector or axial current and \( q \) is the momentum transfer. Introducing the Lorentz invariants \( Q^2 = -q^2 \) and \( x = Q^2/2p \cdot q \), one has on the basis of gauge and relativistic invariance the following decomposition (omitting parity violating contributions)

\[ W_{\mu\nu}(p, q; s) = \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) M_N W_1(x, Q^2) \]
\[ + \left( p_\mu - q_\mu \frac{p \cdot q}{q^2} \right) \left( p_\nu - q_\nu \frac{p \cdot q}{q^2} \right) \frac{1}{M_N} W_2(x, Q^2) \]
\[ + i\epsilon_{\mu\nu\lambda\sigma} \frac{q^2 M_N}{p \cdot q} \left[ g_1(x, Q^2) + g_2(x, Q^2) \right] s^\sigma - \frac{q \cdot s}{q \cdot p} g_2(x, Q^2). \tag{1.2} \]

When a spin–zero hadron is considered, as e.g. the pion, the polarized structure functions \( g_1 \) and \( g_2 \) are ignored. Once the hadronic tensor is computed, the form factors are given by suitable projections. Finally the leading twist contributions to the structure functions are obtained from these form factors by assuming the Bjorken limit:

\[ Q^2 \to \infty \quad \text{with} \quad x = Q^2/2p \cdot q \quad \text{fixed}. \tag{1.3} \]

For the spin independent part the structure functions \( f_i \) are the linear combinations

\[ M_N W_1(x, Q^2) \xrightarrow{\text{Bj}} f_1(x) \quad \text{and} \quad \frac{p \cdot q}{M_N} W_2(x, Q^2) \xrightarrow{\text{Bj}} f_2(x) = 2xf_1(x). \tag{1.4} \]

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where the last identity is a direct consequence of the spin 1/2 nature of partons. The structure function is related to the quark and antiquark distribution functions, $q_i(x)$ and $\bar{q}_i(x)$ respectively by

$$f_1(x) = \frac{1}{2} \sum_{i=u,d,s} e_i^2 [q_i(x) + \bar{q}_i(x)],$$

(1.5)

with $e_i$ the quark electromagnetic charges. Actually, exact scaling is only true up to logarithms, $\log(Q^2/\Lambda^2)$, due to perturbative QCD radiative corrections. The most economic way of including them in a calculation is by means of the DGLAP equations which, using renormalization group arguments, relate in a linear fashion the structure functions at a given reference scale, say $Q_0^2$, to the scale of interest, $Q^2$,

$$f_i(x, Q^2) = U(Q^2, Q_0^2) f_i(x, Q_0^2).$$

(1.6)

Here, $U(Q^2, Q_0^2)$ is a linear matrix operator, fulfilling the properties $U(Q_1^2, Q_2^2)U(Q_2^2, Q_3^2) = U(Q_1^2, Q_3^2)$ and $U(Q^2, Q^2) = 1$. In principle, the former equation means that one only needs to know structure functions at a given scale, since $U$ generates structure functions at all other scales. So far $U$ can only be computed in perturbation theory. Due to asymptotic freedom, this means that in practice $U$ is reliable to relate high momentum scales, although nobody really knows what high momentum means in the present context; one is only left to practical convergence criteria between one loop and two loop calculations. The form of scaling violations predicted by QCD has been tested if one relates experimentally measured structure functions at different scales.

Expression (1.1) is not always convenient for practical calculations. In case the considered hadron represents the ground state with the specific quantum numbers, the hadronic tensor will be related to the forward virtual Compton amplitude

$$W^{ab}_{\mu\nu}(p, q; s) = \frac{1}{2\pi} \text{Im} T^{ab}_{\mu\nu}(p, q; s),$$

(1.7)

which is given as the matrix element of a time–ordered product of the currents

$$T^{ab}_{\mu\nu}(p, q; s) = i \int d^4 \xi e^{iq\cdot\xi} \langle p, s | T \left\{ J^{a}_\mu(\xi)J^{b\dagger}_\nu(0) \right\} | p, s \rangle$$

(1.8)

rather than their commutator. This time–ordered product has nicer properties than the commutator, since it can be built by functional differentiation with respect to external sources. Applying Cutkosky’s rules subsequently yields the absorptive part, $\text{Im} T^{ab}_{\mu\nu}$, which is to be identified with the hadronic tensor (1.7).

In the parton model, an analysis of the virtual Compton forward scattering amplitude leads, in the Bjorken limit, to the so called quark-target scattering formula. This formula is formally correct, but it may require some regularization, or introduction of form factors. It is obvious that not every choice of regularization or form factor is consistent with the original Compton amplitude one started with. This point is often disregarded in practical calculations.

The calculation of structure functions at a given scale remains a challenge within QCD, and one may recourse to quark models to compute them. In all considered models one is forced to the so-called valence quark approximation. This hypothesis is based on the observation that the momentum fraction carried by the valence quarks increases for decreasing momentum scales. So, there is a momentum scale, say $Q_0$, where the valence quarks carry all the momentum. In this picture, one starts with only valence quarks and gluons and sea quark distributions are generated as radiative corrections by means of DGLAP equations. Whether or not this assumption makes sense is analyzed.
below, but one should say that it is routinely applied in practical calculations. There is another assumption which is usually made, related to the fact that although models of hadrons deal with constituent quarks, perturbative QCD evolution is applicable to current quarks. In my opinion this point is at present not well understood, even in models which exhibit dynamical symmetry breaking, i.e. which allow for both kinds of quarks, and deserves further study.

2. The NJL model in the Pauli-Villars regularization

For practical calculations we adopt the Nambu–Jona–Lasinio (NJL) model \[7\] of quark flavor dynamics. This model offers a microscopic and non-perturbative description of dynamical chiral symmetry breaking for low energy non-perturbative hadron physics, and is rather successful from a phenomenological point of view. Mesons are described as quark-antiquark excitations of the vacuum in terms of poles of the corresponding Bethe-Salpeter amplitude. Baryons are described in the large \( N_C \) limit of the model as solitons. At a formal level the NJL quark distributions can be identified with those of the parton model. However, a major obstacle is that the bosonized NJL model contains quadratic divergences which have to be carefully regularized, to comply with the symmetries of the system. Actually, this point turns out to be crucial to ensure the fulfillment of sum rules, i.e. weighted integrals of structure functions in \( x \) space which yield low energy matrix elements.

In the NJL chiral soliton model the issue of systematically regularizing the nucleon structure functions is rather subtle and has been handled in several ways. In a first approach the contributions from the polarized vacuum are neglected \[10, 11\], since polarized vacuum contributions to static nucleon properties turn out to be numerically small, once the self-consistent soliton has been constructed. In other approaches the quark-target scattering formula has been regularized \[12, 13\], in a treatment which only works in the chiral limit since quadratic divergences do not appear. This connection to static properties for setting up the regularization description does not provide a definite answer for structure functions not having a sum rule; i.e., structure functions whose integral cannot be written as a matrix element of a local operator.

The NJL model is defined in terms of quark fields by the Lagrangian \[7\]

\[
\mathcal{L}_{\text{NJL}} = \bar{q} (i\not\partial - m_0) q + 2G_{\text{NJL}} \left\{ \left( \bar{q} \vec{\tau} q \right)^2 + \left( \bar{q} \vec{\gamma}^5 q \right)^2 \right\}
\]

(2.1)

with the quark interaction described by a chirally symmetric quartic potential. The current quark mass, \( m_0 \), parameterizes the small explicit breaking of chiral symmetry. Using functional techniques the quark fields can be integrated out in favor of auxiliary mesonic fields, \( M = S + iP \). According to the chirally symmetric interaction in the Lagrangian (2.1), \( S \) and \( P \) are scalar and pseudoscalar degrees of freedom, respectively. This results in the bosonized action \[8\]

\[
\mathcal{A}_{\text{NJL}} = -iN_C \text{Tr}_A \log \left\{ i\not\partial - m_0 - (S + i\gamma_5 P) \right\} + \frac{1}{4G} \int d^4x \text{tr} \left[ \mathcal{M} \mathcal{M}^\dagger \right].
\]

(2.2)

Here the ‘cut–off’ \( \Lambda \) indicates that the quadratically and logarithmically divergent quark loop requires regularization. In order to compute properties of hadrons from the action (2.2) a twofold procedure is in order. First, formal expressions for the symmetry currents have to be extracted. This is straightforwardly accomplished by adding external sources with suitable quantum numbers to the Dirac operator and taking the appropriate functional derivatives. Ignoring effects associated with the regularization, the currents would be as simple as \( \bar{q} \gamma_\mu (\gamma_5) t^a q \), with \( t^a \) being the appropriate flavor generator. Secondly, hadron states are constructed from the effective action which in turn allows one to calculate the relevant matrix elements of the symmetry currents. For the pion this will be a Bethe–Salpeter wave–function which is obtained by expanding the action (2.2) appropriately in the fields \( S \) and \( P \).
In the case of the nucleon we will determine a soliton configuration which after collective quantization carries nucleon quantum numbers \cite{14}.

We employ the Pauli–Villars regularization scheme since it is possible to formulate the bosonized NJL model \cite{7} completely in Minkowski space. This is particularly appropriate when applying Cutkosky’s rules in order to extract the hadronic tensor from the Compton amplitude. Also, let us remind that in ref.\cite{15}, scaling for the pion structure functions was accomplished in the Pauli-Villars regularization, and not in the proper–time regularization. In this context, it has been shown \cite{16} that in the Pauli-Villars regularization, unlike the more customary proper–time regularization where cuts in the complex plane appear \cite{17, 9}, dispersion relations are fulfilled. The Pauli–Villars regularization has been considered before in this context both for mesons and solitons and we refer to refs \cite{19, 20, 15} for more details and results in this regularization scheme. We will specifically follow ref \cite{15} because in that formulation a consistent treatment solely in Minkowski space is possible.

The regularized action of the bosonized NJL model is then given by\footnote{\textit{We denote traces of discrete indices by “tr” while “Tr” also contains the space–time integration.}}\footnote{\textit{In the case of two subtractions we need at least two cut-offs $\Lambda_1$ and $\Lambda_2$. In the limit $\Lambda_1 \to \Lambda_2 = \Lambda$, we have $\sum_i c_i f(A_i^2) = f(0) - f(A^2) + A^2 f'(A^2)$. For instance, $\sum_i c_i A_i^{2n} = (2n - 2)A^{2n}$.}}

$$A_{NJL} = A_R + A_I + \frac{1}{4G} \int d^4x \text{tr} \left[ S^2 + P^2 + 2m_0 S \right] (2.3)$$

$$A_R = -i \frac{N_C}{2} \sum_{i=0}^{2} c_i \text{Tr} \log \left[-DD_5 + \Lambda_i^2 - ie\right], \quad (2.4)$$

$$A_I = -i \frac{N_C}{2} \text{Tr} \log \left[-D (D_5)^{-1} - ie\right]. \quad (2.5)$$

The local term in eq (2.3) is the reminder of the quartic quark interaction of the NJL model. After having shifted the meson fields by an amount proportional to the current quark masses $m_0$ it also contains the explicit breaking of chiral symmetry. Furthermore we have retained the notion of real and imaginary parts of the action as it would come about in the Euclidean space formulation. This is also indicated by the Feynman boundary conditions. When disentangling these pieces, it is found that only the ‘real part’ $A_R$ is ultraviolet divergent. It is regularized within the Pauli–Villars scheme according to which the conditions\footnote{\textit{We denote traces of discrete indices by “tr” while “Tr” also contains the space–time integration.}}\footnote{\textit{In the case of two subtractions we need at least two cut-offs $\Lambda_1$ and $\Lambda_2$. In the limit $\Lambda_1 \to \Lambda_2 = \Lambda$, we have $\sum_i c_i f(A_i^2) = f(0) - f(A^2) + A^2 f'(A^2)$. For instance, $\sum_i c_i A_i^{2n} = (2n - 2)A^{2n}$.}}

$$c_0 = 1, \quad \Lambda_0 = 0, \quad \sum_{i=0}^{2} c_i = 0 \quad \text{and} \quad \sum_{i=0}^{2} c_i \Lambda_i^2 = 0 \quad (2.6)$$

hold. The ‘imaginary part’ $A_I$ is conditionally convergent, i.e. a principle value description must be imposed for the integration over the time coordinate. \textit{A priori} this does not imply that it should not be regularized. However, in order to correctly reproduce the axial anomaly we are constrained to leave it unregularized.

Essentially we have added and subtracted the (unphysical) $D_5$ model to the bosonized NJL model. Under regularization the sum, $\log (D) + \log (D_5)$ is then treated differently from the difference, $\log (D) - \log (D_5)$. In the case of the polarized nucleon structure functions ii turns out that this special choice of regularization nevertheless requires further specification.

\textit{2a. Vacuum and meson sectors}

We consider two different Dirac operators in the background of scalar ($S$) and pseudoscalar ($P$) fields \cite{15, 18}

$$iD = i\partial - (S + i\gamma_5 P) + \gamma + \gamma_5 =: iD^{(\sigma)} + i\gamma_5 \quad (2.7)$$
\[ i\mathbf{D}_5 = -i\mathbf{\hat{D}} - (S - i\gamma_5 P) - \mathbf{\hat{S}} + \mathbf{\hat{P}} = i\mathbf{D}_5^{(\pi)} - \mathbf{\hat{D}} - \mathbf{\hat{S}} + \mathbf{\hat{P}}. \] (2.8)

Here we have also introduced external vector \((v_\mu)\) and axial–vector \((a_\mu)\) fields. As noted above the functional derivate of the action with respect to these sources will provide the vector and axial–vector currents, respectively. For later use we have also defined Dirac operators, \(\mathbf{D}(\pi)\) and \(\mathbf{D}_5^{(\pi)}\), with these fields omitted. Of course, all fields appearing in eqs (2.7) and (2.8) are considered to be matrix fields in flavor space. It is worth noting that upon continuation to Euclidean space, \(\mathbf{D}_5\) transforms into the hermitian conjugate of \(\mathbf{D}\) [13].

In the vacuum sector the pseudoscalar fields vanish while the variation of the action with respect to the scalar field \(S\) yields the gap equation

\[
\frac{1}{2G} (m - m_0) = -4iN_C m \sum_{i=0}^{2} c_i \int \frac{d^4k}{(2\pi)^4} \left[ -k^2 + m^2 + \Lambda_i^2 - i\epsilon \right]^{-1}. \tag{2.9}
\]

This equation determines the vacuum expectation value of the scalar field \(\langle S \rangle = m\) which is referred to as the constituent quark mass. Its non–vanishing value signals the dynamical breaking of chiral symmetry. Next we expand the action to quadratic order in the pion field \(\mathbf{\tilde{\pi}}\). This field resides in the non–linear representation of the meson fields on the chiral circle

\[
\mathcal{M} = m U = m \exp \left( i\frac{g}{m} \mathbf{\tilde{\pi}} \cdot \mathbf{\tilde{\tau}} \right). \tag{2.10}
\]

This representation also defines the chiral field \(U\). The quark–pion coupling \(g\) will be specified shortly. Upon Fourier transforming to \(\mathbf{\tilde{\pi}}\) we find

\[
\mathcal{A}_{\text{Nijl}} = g^2 \int \frac{d^4q}{(2\pi)^4} \mathbf{\tilde{\pi}}(q) \cdot \mathbf{\tilde{\pi}}(-q) \left[ 2N_C q^2 \Pi(q^2) - \frac{1}{2G} \frac{m_0}{m} \right] + \mathcal{O}(\mathbf{\tilde{\pi}}^4), \tag{2.11}
\]

with the polarization function

\[
\Pi(q^2, x) = -i \sum_{i=0}^{2} c_i \int \frac{d^4k}{(2\pi)^4} \left[ -k^2 - x(1-x)q^2 + m^2 + \Lambda_i^2 - i\epsilon \right]^{-2} \quad \text{and} \quad \Pi(q^2) = \int_0^1 dx \Pi(q^2, x), \tag{2.12}
\]

parameterizing the quark loop. The on–shell condition for the pion relates its mass to the model parameters

\[
m_\pi^2 = \frac{1}{2G} \frac{m_0}{m} \frac{1}{2N_C \Pi(m^2_\pi)}. \tag{2.13}
\]

Requiring a unit residuum at the pion pole determines the quark–pion coupling

\[
\frac{1}{g^2} = 4N_C \left. \frac{d}{dq^2} \left[ q^2 \Pi(q^2) \right] \right|_{q^2=m^2_\pi}. \tag{2.14}
\]

The axial current is obtained from the linear coupling to the axial–vector source \(a_\mu\). Its matrix element between the vacuum and the properly normalized one–pion state provides the pion decay constant \(f_\pi\) as a function of the model parameters

\[
f_\pi = 4N_C mg \Pi(m^2_\pi). \tag{2.15}
\]
The empirical values $m_\pi = 138\text{MeV}$ and $f_\pi = 93\text{MeV}$ are used to determine the model parameters.

2b. Soliton sector

In order to describe a soliton configuration we consider static meson configurations. In that case it is suitable to introduce a Dirac Hamiltonian $h$ via

$$i\mathbf{D}(\pi) = \beta (i\partial_t - h) \quad \text{and} \quad i\mathbf{D}_5^{(\pi)} = (-i\partial_t - h)\beta .$$  \hspace{1cm} (2.16)

For a given meson configuration the Hamiltonian $h$ is diagonalized

$$h\Psi_\alpha = \epsilon_\alpha \Psi_\alpha$$  \hspace{1cm} (2.17)

yielding eigen–spinors $\Psi_\alpha$ and energy eigenvalues $\epsilon_\alpha$. In the unit baryon number sector the well–known hedgehog configuration minimizes the action for the meson fields. This configuration introduces the chiral angle $\Theta(r)$ via

$$h = \vec{\alpha} \cdot \vec{p} + \beta m \exp \left[ i\vec{r} \cdot \vec{\tau} \gamma_5 \Theta(r) \right].$$  \hspace{1cm} (2.18)

The eigenstates $|\alpha\rangle$ of this Dirac Hamiltonian are in particular characterized by their grand–spin quantum number $|21\rangle$. The grand–spin, $\vec{G}$ is the operator sum of total spin and isospin. Since $\vec{G}$ commutes with the Dirac Hamiltonian $h$ the state $\exp(i\pi G_2)|\alpha\rangle$ is also an eigenstate of $h$ with energy $\epsilon_\alpha$ and grand–spin $G_\alpha$. This rotational symmetry, which actually is a grand–spin reflection, will later be useful to simplify matrix elements of the quark wave–functions, $\Psi_\alpha$.

For unit baryon number configurations it turns out that one distinct level, $\Psi_{\text{val}}$, is strongly bound [9]. This level is referred to as the valence quark state. The total energy functional contains three pieces. The first one is due to the explicit occupation of the valence quark level to ensure unit baryon number. The second is the contribution of the polarized vacuum. It is extracted from the action (2.3) by considering an infinite time interval to discretize the eigenvalues of $\partial_t$. The sum over these eigenvalues then becomes a spectral integral [22] which can be computed using Cauchy’s theorem. Finally, there is the trivial part stemming from the local part of the action (2.2). Collecting these pieces we have [20, 9]

$$E_{\text{tot}}[\Theta] = \frac{N_C}{2} \left( 1 - \text{sign}(\epsilon_{\text{val}}) \right) \epsilon_{\text{val}} - \frac{N_C}{2} \sum_{i=0}^2 c_i \sum_\alpha \left\{ \sqrt{\epsilon_\alpha^2 + \Lambda_i^2} - \sqrt{\epsilon_{(0)}^2 + \Lambda_i^2} \right\}$$

$$+ m_\pi^2 f_\pi^2 \int d^3r \left( 1 - \cos(\Theta) \right).$$  \hspace{1cm} (2.19)

Here we have also subtracted the vacuum energy associated with the trivial meson field configuration and made use of the expressions obtained for $m_\pi$ and $f_\pi$ in the preceding subsection. The soliton is then obtained as the profile function $\Theta(r)$ which minimizes the total energy $E_{\text{tot}}$ self–consistently.

At this point we have constructed a state which has unit baryon number but neither good quantum numbers for spin and flavor. Such states are generated by canonically quantizing the time–dependent collective coordinates $A(t)$ which parameterize the spin–flavor orientation of the soliton. For a rigidly rotating soliton the Dirac operator becomes, after transforming to the flavor rotating frame [22],

$$i\mathbf{D}(\pi) = A\beta \left( i\partial_t - \vec{\tau} \cdot \vec{\Omega} - h \right) A^\dagger \quad \text{and} \quad i\mathbf{D}_5^{(\pi)} = A \left( -i\partial_t + \vec{\tau} \cdot \vec{\Omega} - h \right) \beta A^\dagger .$$  \hspace{1cm} (2.20)

Actual computations involve an expansion with respect to the angular velocities

$$A^\dagger \frac{d}{dt} A = \frac{i}{2} \vec{\tau} \cdot \vec{\Omega}.$$  \hspace{1cm} (2.21)
According to the quantization rules, the angular velocities are replaced by the spin operator

\[ \vec{\Omega} \rightarrow \frac{1}{\alpha^2} \vec{J}. \] 

(2.22)

The constant of proportionality is the moment of inertia \( \alpha^2 \) which is calculated as a functional of the soliton [22]. For the present purpose we remark that \( \alpha^2 \) is of the order 1/\( N_C \). Hence an expansion in \( \vec{\Omega} \) is equivalent to one in 1/\( N_C \). The nucleon wave–function becomes a (Wigner D) function of the collective coordinates. A useful relation in computing matrix elements of nucleon states is [14]

\[ \langle N|\frac{1}{2}\text{tr}(A_i^\dagger \tau_j A_j)|N\rangle = -\frac{4}{3}\langle N|I_i J_j|N\rangle. \] 

(2.23)

3. Bjorken limit and scaling

An important consequence of the Pauli-Villars regularization is given by the fact that in the Bjorken limit the model scales. A detailed analysis [23] shows that in this limit the contribution to the Compton scattering amplitude of the regularized Dirac determinant is given by a second functional differentiation of

\[ A^{(2,v)}_{\Lambda,R} = -\frac{iN_C}{4} \sum_{i=0}^2 c_i \text{Tr} \left\{ \left( -D^{(\pi)} D_5^{(\pi)} + \Lambda_i^2 \right)^{-1} \left[ Q^2 (\partial g)^{-1} D_5^{(\pi)} - D^{(\pi)} (\partial g)^{-1} D_5^{(\pi)} Q^2 \right] \right\} \] 

(3.1)

with respect to the vector sources. For the imaginary part of the action the expression analogous to (3.1) reads

\[ A^{(2,v)}_{\Lambda,1} = -\frac{iN_C}{4} \text{Tr} \left\{ \left( -D^{(\pi)} D_5^{(\pi)} \right)^{-1} \left[ Q^2 (\partial g)^{-1} D_5^{(\pi)} + D^{(\pi)} (\partial g)^{-1} D_5^{(\pi)} Q^2 \right] \right\}. \] 

(3.2)

In both cases, it is understood that the large photon momentum runs only through the operators in square brackets. With \( S_{\mu\nu\sigma\rho} = g_{\mu\rho} g_{\nu\sigma} + g_{\mu\nu} g_{\rho\sigma} - g_{\mu\nu} g_{\rho\sigma} \), the subscript ‘5’ means

\[ \gamma_\mu \gamma_\rho \gamma_\nu \gamma_5 \Rightarrow S_{\mu\nu\sigma\rho} \gamma^\sigma \gamma^5 \] 

while \( (\gamma_\mu \gamma_\rho \gamma_\nu)_{\mathbf{5}} = S_{\mu\nu\sigma\rho} \gamma^\sigma + i\epsilon_{\mu\nu\sigma\rho} \gamma^\sigma \gamma^5 \). 

(3.3)

The fact that the sum rules enforce this extension of the regularization scheme is not all surprising since the derivative operator \( i\partial \) fixes the Noether currents. Rather it is imposed and a consequence of the ‘sum rules’ of the model defined by \( D_5 \). The \( D_5 \) model, which is not physical, has been introduced as a device to allow for a regularization which maintains the anomaly structure of the underlying theory. Hence, further specification of this regularization prescription is demanded in order to formulate a fully consistent model. It should be stressed that this issue is not specific to the Pauli–Villars scheme but rather all schemes which do regularize the sum, \( \log(D) + \log(D_5) \) but not the difference, \( \log(D) - \log(D_5) \) will require the specification (3.3). Since only the polarized, \( i.e. \) spin dependent, structure functions are effected, this issue has not shown up when computing the pion structure functions in the Pauli–Villars scheme. In the unregularized case (\( \Lambda_i \equiv 0 \)) the contributions associated to the expansion of \( D_5 \) cancel in the sum (3.1) and (3.2) leaving

\[ A^{(2,v)} = \frac{iN_C}{2} \text{Tr} \left\{ \left( D^{(\pi)} \right)^{-1} \left[ Q^2 (\partial g)^{-1} \right] \right\}. \] 

(3.4)

\(^3\)When formally considering nucleon structure functions it will turn out that it is mostly sufficient to refer to the Dirac operators as defined in eqs (2.21) rather than to explicitly carry out the expansion in the angular velocities.
Finally, it should be mentioned that the property of scaling is not an automatic consequence of any regularization. For instance, in the Proper-Time regularization, there appear logarithmic corrections since the imaginary part of the one loop diagrams is not regularized.

4. The regularized structure functions

4a. Pion structure functions

Working through the expressions, one obtains after some calculation an explicit expression for the hadronic tensor. For the pion we have

\[
f_1(x) = \left( \frac{5}{9} \right) 4N_C g_0^2 \frac{d}{dq^2} \left[ q^2 \Pi(q^2, x) \right] \bigg|_{q^2=m_\pi^2}.
\]

which is trivially normalized. In the chiral limit \( m_\pi \to 0 \) we get

\[
f_1(x) = \left( \frac{5}{9} \right) \theta(x) \theta(1-x)
\]

This result is surprisingly simple for it does not seem to depend strongly on any dynamical assumptions, and it is equivalent to set the parton distributions, \( u_\pi(x) = \bar{d}_\pi(x) = 1 \) for \( \pi^+ \). In addition, if DGLAP evolution at LO \[15\] and NLO \[25\] is undertaken a very satisfactory description of valence quark distributions in the pion as extracted in ref.\[26\] is obtained. One should say however, that the gluon and sea quark distributions are too steep in the low \( x \) region and too flat in the high \( x \) region \[25\]. In ref.\[13\], parton distributions for the kaon are also computed.

4b. Nucleon structure functions

Here we will only discuss the contribution of the polarized vacuum to the nucleon structure functions on a formal and conceptual level, since up to now the final formulas have not been numerically evaluated. The contribution of the distinct valence level, which is not effected by the regularization, cf. eq \( (2.19) \), has previously been detailed \[4, 10\] and numerically evaluated. After some calculation, we obtain for the nucleon \[23\]

\[
W_{\mu\nu}(q) = -iM_N \frac{N_C}{4} \int \frac{d\omega}{2\pi} \sum_\alpha \int d^3\xi \int \frac{d\lambda}{2\pi} e^{iM_N x \lambda} \times \left\langle N \right| \left[ \bar{\Psi}_\alpha(\vec{\xi}) \gamma_\mu \Psi_\alpha(\vec{\xi}) e^{-i\lambda\omega} - \bar{\Psi}_\alpha(\vec{\xi}) \gamma_\mu \Psi_\alpha(\vec{\xi} - \vec{e}_3)e^{i\lambda\omega} \right] f_\alpha^+(\omega) \right|_{\text{pole}} \left| N \right\rangle,
\]

with the light-cone vector \( n = (1, 0, 0, 1) \) and the spectral functions

\[
f_\alpha^+(\omega) = \sum_{i=0}^{2} c_i \frac{\omega \pm \epsilon_\alpha}{\omega^2 - \epsilon^2_\alpha - \Lambda_i^2 + i\epsilon} \pm \frac{\omega \pm \epsilon_\alpha}{\omega^2 - \epsilon^2_\alpha + i\epsilon},
\]

8
and "pole" means computing the residue contribution of the poles of the spectral functions. The previous equations are similar to the decomposition into quark and anti–quark distributions. In the former expression Eq. (4.4), the sum is over the continuum Dirac spectrum. In the unregularized case, \( f^\alpha_\omega (\omega) = 0 \) while \( f_{\omega}^\alpha (\omega) \) pole \( = -4\pi i\delta (\omega - \epsilon_\alpha) \). Apparently the hadronic tensor then indeed becomes a sum of quark and anti–quark distributions. However, in the Pauli–Villars regularized scheme, we have additional contributions from quark and anti–quark distributions with dispersion relations which also contain the cut–offs, \( \Lambda_i \). Hence they differ from those dispersion relations naively expected from the solutions of the Dirac equation (2.17). From this expression structure functions can be directly obtained by contracting with appropriate projectors.

Note that within the Bjorken limit the Callan–Gross relation, \( f_2 (x) = 2x f_1 (x) \), is automatically fulfilled. The unpolarized structure function \( f_1 (x) \) then becomes

\[
\begin{align*}
 f_1 (x) & = -i M_N \frac{N_C}{2} \int \frac{d\omega}{2\pi} \sum_\alpha \int d^3 \xi \int \frac{d\lambda}{2\pi} e^{i M_N x \lambda} \left( \sum_{i=0}^{2} c_i \omega - \epsilon_\alpha - \Lambda_i^2 + i \epsilon \right)_\text{pole} \\
 & \times \langle N \bar{\Psi}_\alpha (\tilde{\xi}) Q_3 \Psi_\alpha (\tilde{\xi} + \lambda \hat{e}_3) e^{-i \omega \lambda} - \bar{\Psi}_\alpha (\tilde{\xi}) Q_3 \Psi_\alpha (\tilde{\xi} - \lambda \hat{e}_3) e^{i \omega \lambda} | N \rangle \\
 & = \frac{5}{36} M_N N_C \int \frac{d\omega}{2\pi} \sum_\alpha \int d\lambda \frac{e^{i M_N x \lambda}}{2\pi} \left( \sum_{i=0}^{2} c_i \omega - \epsilon_\alpha - \Lambda_i^2 + i \epsilon \right)_\text{pole} \\
 & \times \int d^3 \xi \left\{ \Psi_\alpha^\dagger (\tilde{\xi}) (1 - \alpha_3) \Psi_\alpha (\tilde{\xi} + \lambda \hat{e}_3) e^{-i \omega \lambda} - \Psi_\alpha^\dagger (\tilde{\xi}) (1 - \alpha_3) \Psi_\alpha (\tilde{\xi} - \lambda \hat{e}_3) e^{i \omega \lambda} \right\}
\end{align*}
\]  

(5.1)

to leading order in \( 1/N_C \). The structure function which enters the Gottfried sum rule [30] should not be regularized in contrast to previous studies. In ref [23] this structure function has been treated analogously to the one of neutrino nucleon scattering associated with the Adler sum rule. As discussed in ref [23] the latter indeed undergoes regularization. This example clearly exhibits that obtaining the formal expressions for structure functions from the defining action is unavoidable in cases when there is no relation to a static nucleon property. Similar expressions to leading order in \( 1/N_C \) for the polarized structure functions, \( g_1 (x) \) and \( g_2 (x) \) can also be found in ref. [23]. Since the soliton breaks translational invariance, these structure functions extend out of the interval \( 0 < x < 1 \). The issue of restoring the proper support has been addressed in ref. [21] by going to the infinite momentum frame.

Sum rules relate moments of the structure functions to static properties of hadrons and a consistently formulated model is required to satisfy these sum rules. Static properties are obtained by computing matrix elements of the symmetry currents. In ref. [23] it has been shown that the prescription (3.3) complies with the required sum rules for polarized and unpolarized structure functions.

5. Phase-Space considerations

The result, that the pion distribution function becomes unity in the chiral limit, seems to be a rather remarkable one, because it looks independent of any detailed dynamics. In addition, if a DGLAP evolution for the valence part is undertaken to leading order [15] and NLO [23] the agreement with experimental parameterizations is striking. The interesting aspect here is that within our calculation we are treating the pion as a composite \( q\bar{q} \) Goldstone boson. We will derive the same result by a rather crude method. The probability \( F (x) \) (normalized to unity) that a quark in a hadron with \( N \) constituents has a momentum fraction between \( x \) and \( x + dx \) is given by

\[
F (x) = \int dx_1 \ldots dx_N \delta \left( \sum_{i=1}^{N} x_i - 1 \right) f (x_1, \ldots, x_N) \frac{1}{N} \sum_{i=1}^{N} \delta (x - x_i)
\]

(5.1)
where \( f(x_1, \ldots, x_N) \) is the probability distribution of the N quarks in the hadron. If we make the choice \( f(x_1, \ldots, x_N) = 1 \), then we find

\[
F(x) = (N - 1)(1 - x)^{N-2}.
\]

(5.2)

We see that in the case \( N = 2 \), we have \( F_\pi(x) = 1 \), which was the result we obtained previously for the pion. It is of course very tempting to look at the nucleon case \( (N = 3) \), for which we get \( F_N(x) = 2(1 - x) \). After DGLAP evolution we get a reasonably good description of the valence part \( V(x) = [u(x) + d(x) - \bar{u}(x) - \bar{d}(x)]/3 \) (see figure), much better than what it is obtained in much more sophisticated calculations \([3]\). The agreement is not surprising if one looks at the low energy parameterizations of GRV 95' in ref. \([24]\), where one sees that as \( x \to 1 \) the valence contribution behaves as \( \sim (1 - x) \). Actually, after evolution one finds that for \( Q^2 = 4\text{GeV}^2 \) the distribution function behaves as \( F(x) \sim (1 - x)^{N-2+1.2} \) which only agrees very roughly with the counting rules found long ago \([2]\), \( F(x) \sim (1 - x)^{2M-1} \), with \( M = 2 \) for the nucleon. It is fair to say that the scale at which these counting rules were derived was never made clear, and that structure functions are not experimentally known for \( x > 0.75 \). It is also fair to say that the analysis above was already envisaged in the original paper of Bjorken and Paschos \([28]\) although to that time perturbative QCD evolution equations were not known, so they tried to build the full \( Q^2 \) dependent structure function by making a judicious choice of \( f(x_1, \ldots, x_N) \). Although, the results found in this section should be analyzed with care, it seems that the bulk of the valence distributions is understood in terms of the number of constituents only. Nevertheless, in both cases the gluon and sea distributions are too steep at low \( x \) and too flat at high \( x \). In addition, the \( f_2 \) is reasonably well described at LO and NLO at \( Q^2 = 5\text{GeV}^2 \) between \( 0.3 < x < 0.75 \).
6. Can low energy models be matched to perturbative QCD?

To finish the discussion let us analyze whether or not low energy models can be matched to perturbative QCD in a reasonable way. The traditional recipe for this kind of calculations is that after the low energy model structure functions are computed, a subsequent DGLAP evolution is undertaken. Since these models are thought not to provide a gluon or sea distribution, the initial condition consists of taking only the valence part, the radiative corrections due to gluons and sea quarks, are automatically generated by the solution of DGLAP equations. I have looked at this problem from a different point of view [29]. Namely, I have taken the parton distribution functions at very high energies, $Q^2 = m_b^2$, and I have evolved them downwards in energy, until some structure function becomes negative, in the interval $0 < x < 1$. This seems a natural place to stop evolution, since going below would violate unitarity. The relevant question to ask is whether or not around this point the gluon and the sea distributions are negligible. There is a subtlety in the procedure, since NLO DGLAP equations are often used in a form which is not manifestly “reversible”, i.e. the property $U(Q_1^2, Q_2^2)U(Q_2^2, Q_3^2) = U(Q_1^2, Q_3^2)$ is not fulfilled. After properly handling this point I have found that the unitarity limit takes place at the scale $Q_0 = 580 \text{MeV}$. At this scale one gets

\[ \langle x \rangle_{\text{val}} = 0.45, \quad \langle x \rangle_{\text{gluons}} = 0.36, \quad \langle x \rangle_{\text{sea}} = 0.19, \quad \alpha_s = 0.48 \]  

(6.1)

From the point of view of perturbation theory this scale is certainly more reliable than that required by the valence quark model point (defined as $\langle x \rangle_{\text{val}} = 1$, $\langle x \rangle_{\text{gluons}} + \langle x \rangle_{\text{sea}} = 0$ and which requires $\alpha_s = 1.80$), and it is still having a non-negligible gluon and antiquark distribution. Thus, there seems to be a gap between the low energy nucleon model and the perturbative QCD result. In perturbative QCD there exists a systematic expansion in powers of $\alpha_s$. In the quark model such a systematic expansion seems to be missing, and therefore ”improvements” of it are a bit arbitrary. What is needed is a better understanding on how to systematically improve the low energy model including gluons and antiquarks to be really able to work on the low energy side and hopefully fill in this gap.

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