Classical Strongly Coupled QGP:

VI. Structure Factors

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Abstract

We show that the classical and strongly coupled QGP (cQGP) is characterized by a multiple of structure factors that obey generalized Orstein-Zernicke equations. We use the canonical partition function and its associated density functional to derive analytical equations for the density and charge monopole structure factors for arbitrary values of \( \Gamma = V/K \), the ratio of the mean potential to Coulomb energy. The results are compared with SU(2) molecular dynamics simulations.

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I. INTRODUCTION

High temperature QCD is expected to asymptote a weakly coupled Coulomb plasma albeit with still strong infrared divergences. The latters cause its magnetic sector to be non-perturbative at all temperatures. At intermediate temperatures of relevance to heavy-ion collider experiments, the electric sector is believed to be strongly coupled.

Recently, Shuryak and Zahed [1] have suggested that certain aspects of the quark-gluon plasma in range of temperatures $(1 - 3) T_c$ can be understood by a stronger Coulomb interaction causing persistent correlations in singlet and colored channels. As a result the quark and gluon plasma is more a liquid than a gas at intermediate temperatures. A liquid plasma should exhibit shorter mean-free paths and stronger color dissipation, both of which are supported by the current experiments at RHIC [2].

To help understand transport and dissipation in the strongly coupled quark gluon plasma, a classical model of the colored plasma was suggested in [3]. The model consists of massive quarks and gluons interacting via classical colored Coulomb interactions. The color is assumed classical with all equations of motion following from Poisson brackets. For the SU(2) version both molecular dynamics simulations [3] and bulk thermodynamics were recently presented [4] including simulations of the energy loss of heavy quarks [5].

In this paper we follow up on our recent equilibrium analysis of the bulk thermodynamics [4] to the static structure factors. In section 2 we define the energy functional for the cQGP. In section 3 we derive generalized Ornstein-Zernicke equations for the pair correlation functions. In section 4 we show that the cQGP supports multiple structure factors that measure a variety of colored correlations. Each structure factor obeys a generalized Ornstein-Zernicke equation. In section 5 we introduce the Debye-Huckel-hole potential for the cQGP. In section 6, we use Debye charging process to derive analytical expressions for the lowest two structure factors in the cQGP for arbitrary $\Gamma$. In section 7, we construct numerically the lowest two structure factors using molecular dynamics simulations and compare them with our analytical results for values of $\Gamma$ in the liquid phase. Our conclusions are in section 8. Appendix A is added to streamline our conventions for the SU(2) color charges.
II. FREE ENERGY FUNCTIONAL

We consider the canonical partition function of a single species, either quarks and gluons, at finite temperature $1/\beta = T$ and in the presence of an external scalar source $\psi$

$$Z_N[\psi] = \frac{1}{N!} \int \prod_i \frac{dr_i dQ_i}{\lambda^3} \exp \left( \beta \int dr dQ n(r, Q) \psi(r, Q) \right) \times \exp \left( -\frac{g^2}{2} \int drr'dQdQ'n(r, Q) \frac{Q \cdot Q'}{|r - r'|} n(r', Q') \right)$$ (II.1)

The color charges are treated classically and we refer to [7, 8, 9] for further details regarding the nature of the measure. Here, we have defined

$$n(r, Q) = \sum_{i} \delta(r - r_i)\delta(Q - Q_i)$$ (II.2)

The generalization to many species is straightforward. The associated Coulomb parameter is

$$\Gamma = \frac{g^2 \beta C_2}{4\pi a_{WS}}$$ (II.3)

where $C_2$ is the quadratic Casimir ($= \sum_i Q_i^2/(N_c^2 - 1)$) and $a_{WS}$ is the Wigner-Seitz radius $4\pi a_{WS}^3/3 = 1/n$. For small $\Gamma$, Eq. (II.1) behaves as a screened but weakly coupled gas, while for intermediate values of $\Gamma$, Eq. (II.1) describes a liquid [3, 6]. At large values of $\Gamma$ Eq. (II.1) yields a solid as a ground state. From now on, the canonical charge of $g^2/4\pi$ will be set to 1 for simplicity, and will be restored in the final parameters by inspection.

The static correlations both in space and in phase space associated with Eq. (II.1) are involved and will be the subject of most of this paper. For that, we note that Eq. (II.1) yields the free energy generating functional

$$\mathcal{F}_N[\psi] = \frac{1}{\beta} \int dr dQ n^{(1)}(r, Q) \left( \ln (\lambda^3 n^{(1)}(r, Q)) - 1 \right) - \int dr \psi(r, Q) n^{(1)}(r, Q) + \mathcal{F}_c(n^{(1)}(r, Q)) + \frac{1}{2} \int drr'dQdQ'n^{(1)}(r, Q) \frac{Q \cdot Q'}{|r - r'|} n^{(1)}(r', Q')$$ (II.4)
Here we have set

\[ n^{(1)}(\vec{r}, \vec{Q}) = \langle n(\vec{r}, \vec{Q}) \rangle = \langle \sum_i \delta(\vec{r} - \vec{r}_i)\delta(\vec{Q} - \vec{Q}_i) \rangle \quad \text{(II.5)} \]

as the expectation value with the averaging carried using Eq. (II.1). The second contribution in Eq. (II.4) is the ideal classical contribution following from the measure in Eq. (II.1) using the asymptotic Stirling formulae. The third contribution is the excess free energy functional. \( \mathcal{F}_c \) is the connected free energy that sums up the second and higher cumulants of \( n(\vec{r}, \vec{Q}) \) from Eq. (II.1). We note that for zero scalar source \( \psi = 0 \),

\[ \mathcal{F}_N[0] = \mathcal{F}_{id} + \mathcal{F}_{ex} \quad \text{(II.6)} \]

where the first contribution is the classical ideal part and the second contribution the excess part.

### III. ORNSTEIN-ZERNICKE EQUATIONS

To quantify the static interactions between pairs of particles in (II.1) we define

\[ -\frac{1}{\beta} \frac{\delta^2 \mathcal{F}_N}{\delta \psi \delta \bar{\psi}} = \left\langle \sum_{i,j} \delta(\vec{r} - \vec{r}_i)\delta(\vec{r} - \vec{r}_j)\delta(\vec{Q} - \vec{Q}_i)\delta(\vec{Q} - \vec{Q}_j) \right\rangle \]

\[ = \left( n^{(1)}(\vec{r}, \vec{Q})n^{(1)}(\vec{r}', \vec{Q}')h(\vec{r} - \vec{r}', \vec{Q} \cdot \vec{Q}') + n^{(1)}(\vec{r}, \vec{Q})\delta(\vec{r} - \vec{r}')\delta(\vec{Q} - \vec{Q}') \right) \quad \text{(III.1)} \]

with \( h \) the pair correlation function for \( \psi = 0 \). The pair correlation function is invariant under space translation and color rotation. Generically

\[ h(\vec{r} - \vec{r}', \vec{Q} \cdot \vec{Q}') = \frac{1}{n^2} \left\langle \sum_{i \neq j} \delta(\vec{r} - \vec{r}_i)\delta(\vec{r} - \vec{r}_j)\delta(\vec{Q} - \vec{Q}_i)\delta(\vec{Q} - \vec{Q}_j) \right\rangle \quad \text{(III.2)} \]

The direct correlation function \( c_D \) follows from the excess free energy (II.6) through

\[ -\frac{1}{\beta} c_D(\vec{r} - \vec{r}', \vec{Q} \cdot \vec{Q}') = \frac{\delta^2 \mathcal{F}^{ex}}{\delta n^{(1)} \delta n^{(1)}} = \left( \frac{\vec{Q} \cdot \vec{Q}'}{|\vec{r} - \vec{r}'|} + \frac{\delta^2 \mathcal{F}_c}{\delta n^{(1)} \delta n^{(1)}} \right) \quad \text{(III.3)} \]
which plays the role of a correlated potential. \( c_D \) will be used below as a renormalized Coulomb potential in the liquid phase. It also obeys the identity

\[
\frac{\delta^2 F_N}{\delta n^{(1)} \delta n^{(1)}} = - \frac{\delta \psi}{\delta n^{(1)}} = \frac{1}{\beta} \left( \frac{1}{n} \delta(r - r') \delta(Q - Q') - c_D(r - r', Q \cdot Q') \right)
\]  

(III.4)

Using the chain rule,

\[
\int dr'' dQ'' \frac{\delta \psi (r, Q)}{\delta n^{(1)} (r'', Q'')} \frac{\delta n^{(1)} (r'', Q'')}{\delta \psi (r', Q')} = \delta(r - r') \delta(Q - Q')
\]  

(III.5)

we obtain

\[
h(r - r', Q \cdot Q') = c_D(r - r', Q \cdot Q') + n \int dr'' dQ'' h(r - r'', Q \cdot Q'') c_D(r'' - r', Q' \cdot Q'')
\]  

(III.6)

which is the Orstein-Zernicke equation that ties the pair correlation \( h \) to the direct correlation or the pair potential \( c_D \). For a uniform plasma (III.6) unfolds algebraically in momentum and color space using

\[
h_l(k) = c_{Dl}(k) + nh_l(k) c_{Dl}(k)
\]  

(III.8)

which holds for each partial waves. (III.8) are the generalized Orstein-Zernicke equations for each color partial wave of the SU(2) colored Coulomb plasma.

IV. STATIC STRUCTURE FACTORS

The statistical aspects of the colored charged particles are best captured by correlations in the phase space distributions. The static structure factor is defined as

\[
S_0(r - r', pp', Q \cdot Q') = \langle \delta f(rpQ) \delta f(r'p'Q') \rangle
\]  

(IV.1)
with formally

\[ f(rpQ) = \sum_i \delta(r - x_i) \delta(p - p_i) \delta(Q - Q_i) \]  
(IV.2)

and \( \delta f = f - \langle f \rangle \). The averaging in (IV.1) is carried using the canonical partition function (III.1). Color and translational invariance imply

\[ \langle f(rpQ) \rangle = nf_0(p) = n \left( \frac{\beta}{2\pi m} \right)^{3/2} e^{-\beta p^2/2m} \]  
(IV.3)

which is the Maxwellian distribution for massive constituent quarks or gluons. It is readily shown that

\[ S_0(r - r', pp', Q \cdot Q') = nf_0(p)\delta(r - r')\delta(p - p')\delta(Q - Q') + nf_0(p)f_0(p')h(r - r', Q \cdot Q') \]  
(IV.4)

The reduced static structure factor

\[ S_0(k, Q \cdot Q') = \frac{1}{n} \int dpdp' \int dk e^{ik \cdot (r - r')} S_0(r - r', pp', Q \cdot Q') \]  
(IV.5)

ties with the pair correlation function (III.2) through

\[ S_0(k, Q \cdot Q') = \delta(Q - Q') + nh(k, Q \cdot Q') \]  
(IV.6)

Its Legendre transform in the color charge reads

\[ S_0(k) = 1 + nh_l(k) \]  
(IV.7)

So the knowledge of the partial-wave structure factor \( S_l(k) \) yields the pair correlation \( c_{dl} \) through (III.7) and (IV.7)

\[ 1 = S_0(k)^{-1} + nc_{dl}(k) \]  
(IV.8)
We note that in configuration space the lth partial wave of the static structure factor is

$$S_{0l}(r - r') = \frac{1}{n} \int dp \int dp' \int dQ \ P_l(Q \cdot Q') S_0(r - r', pp', Q \cdot Q')$$  \hspace{1cm} (IV.9)

Using (III.2) and (IV.4) and enforcing space translational and color rotational invariance in the averaging process yields

$$S_{0l}(r) = \delta(r) + \frac{1}{N} \left\langle \sum_{i \neq j} \delta(r - r_{ij}) P_l(Q_i \cdot Q_j) \right\rangle$$  \hspace{1cm} (IV.10)

In particular, the two lowest static structure factors are the density structure factor

$$S_{00}(r) = \delta(r) + \frac{1}{N} \left\langle \sum_{i \neq j} \delta(r - r_{ij}) \right\rangle = \delta(r) + n h_0(r)$$  \hspace{1cm} (IV.11)

and the charge structure factor

$$S_{01}(r) = \delta(r) + \frac{1}{N} \left\langle \sum_{i \neq j} \delta(r - r_{ij}) Q_i \cdot Q_j \right\rangle$$  \hspace{1cm} (IV.12)

Higher structure factors are given by (IV.10) as they measure the various color correlation content of the SU(2) strongly coupled QGP. Below, we propose both an analytical and numerical derivation of the two lowest structure factors (IV.11) and (IV.12).

V. DEBYE-HUCKEL-HOLE POTENTIAL

To derive the static structure factors we will use the Debye charging procedure for a fixed color charge. For that, we need the Poisson-Boltzman equation for the 1-species SU(2) colored plasma in the presence of a colored test charge $q$.

$$\nabla^2 \phi(r, r', q) = -4\pi \left( q \delta(r - r') + \int dQ' Q' n(r, q) e^{-\beta Q' \cdot (\phi(r, r', q) - \Phi(r, q))} \right)$$  \hspace{1cm} (V.1)

with the fixed external density profile

$$n(r, q) = n + n(\Delta_0 + \Delta_1 \cdot q) \cos(k \cdot r)$$  \hspace{1cm} (V.2)
We note that (V.2) is a scalar under rigid and orthogonal color rotations $RQ$ if the external parameters $\Delta_l$ transform unitarily as $D(R)\Delta$ with $D(R)$ the Wigner rotation in the adjoint representation. This fixed density causes an imposed potential

$$\nabla^2 \Phi(r, q) = -4\pi q (n(r, q) - n) \quad (V.3)$$

which is used to normalize the Poisson-Boltzman equation in (V.1). We solve (V.1) in the linear approximation. For that we define the shifted potential $\delta \phi = \phi - \Phi$,

$$\left(\nabla^2 - \kappa_D^2 n(r, q)/n\right) \delta \phi(r, r', q) \approx -4\pi q \left( \delta(r - r') - (n(r, q) - n) \right) \quad (V.4)$$

with $\kappa_D^2 \equiv 4\pi\beta n C_2$ the squared Debye constant. (V.4) is the linearized Poisson-Boltzman or Debye-Huckel equation for the classical colored plasma. At short separations (V.4) is known to misrepresent the hole caused by the strong Coulomb correlations. To fix that we use the Debye-Huckel plus hole approximation[10]

$$\left(\nabla^2 - \kappa_D^2 n(r, q)/n\Theta\right) \delta \phi(r, r', q) \approx -4\pi q \left( \delta(r - r') - (n(r, q) - n) \right) \left( 1 - \Theta \right) \quad (V.5)$$

with $\Theta = \theta(|r - r'| - \sigma)$ the spherical hole insertion of radius $\sigma$. The mean-induced potential is

$$\Psi(r, q) = \lim_{r \to r'} \left( \delta \phi(r, r', q) - \frac{q}{|r - r'|} \right) \quad (V.6)$$

VI. DEBYE CHARGING PROCESS

To assess the static structure factors for the classical and strongly coupled colored plasma, we note that the excess free energy (II.6) can be readily rewritten in terms of the pair correlations

$$-\beta \mathcal{F}^{ex} = \frac{1}{2} \int dr dr' dQ dQ' n^{(1)}(r, Q)c_D(r - r', Q \cdot Q') n^{(1)}(r', Q') \quad (VI.1)$$
In Fourier (space) and Legendre (color) space (VI.1) reads

\[-\beta \mathcal{F}^{\text{ex}} = \frac{1}{2} \int d\mathbf{k} \, dQ \, dQ' \sum_{l,m} n_{lm}^{(1)}(\mathbf{k}, Q) c_{Dl}(\mathbf{k}) \, n_{lm}^{(1)}(-\mathbf{k}, Q')\]  

(VI.2)

with

\[n_{lm}^{(1)}(\mathbf{k}, Q) = \int d\mathbf{r} \, e^{-i\mathbf{k} \cdot \mathbf{r}} Y_{lm}(Q) n_{lm}^{(1)}(\mathbf{r}, Q)\]  

(VI.3)

and \(Y_{lm}^m\) a spherical harmonic for an SU(2) colored plasma. Using the partial wave form of the Orstein-Zernicke equations (IV.8), (VI.2) becomes

\[-\beta \mathcal{F}^{\text{ex}} = \frac{1}{2n} \int d\mathbf{k} \, dQ \, dQ' \sum_{l,m} n_{lm}^{(1)}(\mathbf{k}, Q) \left(1 - S_{0l}^{-1}(\mathbf{k})\right) n_{lm}^{(1)}(-\mathbf{k}, Q')\]  

(VI.4)

This shows that the quadratic change in the excess free energy caused by an external density profile \(n_{lm}(\mathbf{k})\) is directly proportional to the \(l\)th partial wave of the inverse of the static structure factor.

The external density profile (V.2) changes the color Coulomb potential locally through (V.4), thereby affecting the free energy. To assess the change in the latter we use the Debye charging procedure [11]. For that, we note that by dialing (V.2) the free energy shifts. The shift can be decomposed into three parts,

\[\mathcal{F} = \mathcal{F}_{\text{ideal}} + \mathcal{F}_{\text{imposed}} + \mathcal{F}_{\text{induced}}\]  

(VI.5)

The shift in the ideal part is set by the first term in (II.4) after inserting (V.2). The imposed contribution is

\[\mathcal{F}_{\text{imposed}} = \int d\mathbf{r} dQ (n(\mathbf{r}, \mathbf{q}) - n) \mathbf{q} \cdot \int_0^1 d\lambda \Phi(\mathbf{r}, \lambda \mathbf{q})\]  

(VI.6)

and follows from the imposed charge. Specifically,

\[\mathcal{F}_{\text{imposed}} = \frac{n^2}{2} q^2 (3\Delta_0^2 + |\Delta_1|^2) \int d\mathbf{r} d\mathbf{r}' \frac{\cos(\mathbf{k} \cdot \mathbf{r}) \cos(\mathbf{k} \cdot \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}\]  

(VI.7)
The induced free energy is

$$F_{\text{induced}} = \int d\mathbf{r} dQn(r, q) q \cdot \int_0^1 d\lambda \Psi(r, \lambda q)$$

(VI.8)

and follows from the induced but shifted screening potential (V.6).

FIG. 1: $S_{01}(q)$ for $\Gamma = 4, 8, 12, 16$ summed up to $l = 1$ (a) and $l = 2$ (b). See text.

The integrand can be obtained by solving (V.5) for $\delta \phi(\mathbf{r}, \mathbf{r}', \mathbf{q})$ with the help of the Green function,

$$\left[ \nabla^2 - \kappa_D^2 \Theta(|\mathbf{r} - \mathbf{r}'| - \sigma) \right] G(\mathbf{r}''', \mathbf{r} - \mathbf{r}') = -4\pi \delta^3(\mathbf{r}''' - (\mathbf{r} - \mathbf{r}'))$$

(VI.9)

The method has been developed in [12, 13, 14] for the one-component plasma and readily extends to our colored plasma. For that we evaluate the reduced free energy $f = \beta F/V$ to quadratic order in $\Delta^2$. By comparing the terms with (VI.4), we can extract $S_{00}^{-1}$ as the coefficient of $\Delta_0^2$ and $S_{01}^{-1}$ as the coefficient of $\Delta_1^2$. We find that both static structure factors are finite and identical in this approximation,
The three integral contributions are

\[ I_{-l}(z, y) = \int_0^z dw w^{-l} g_l(-w) j_l(y w) \]

\[ I_0^l(x, y) = \int_x^\infty dz z^{-l} g_l(z) j_l(y z) I_l^-(z, y) e^{2(x - z))} \]

\[ I_l^+(x, y) = \int_x^\infty dz z^{-l} g_l(z) j_l(y z) e^{2(x - z))} \]

(VI.11)

with \( g_l(z) = e^{z} z^{l+1} k_l(z) \). Here \( j_l(y z) \) is a spherical Bessel function, and \( k_l(z) \) a modified spherical Bessel function. The parameter \( w_\lambda \) is defined as

\[ w_\lambda = \left( 1 + \lambda^3 (3\Gamma)^2 \right)^{\frac{1}{2}} \]

(VI.12)

The static structure factors in (VI.10) involve summations over multiple partial waves. The sums are rapidly converging as we show in Fig. 1 with \( l = 1 \) retained (a) and \( l = 2 \) retained (b). Here \( q = k a_W S \) is a dimensionless wave-vector. To assess the accuracy of the analytical method developed above for the static correlation functions in the colored Coulomb plasma, we now carry numerical simulations for the same structure factors using molecular dynamics simulations.

VII. STRUCTURE FACTORS FROM MOLECULAR DYNAMICS

For an SU(2) plasma, the details of the molecular dynamics simulations can be found in [3]. Color motion is treated as a point coordinate on a 3-sphere with a fixed radius that
is equal to the quadratic Casimir for SU(2). Classical stability of the colored Coulomb gas at short distances is achieved by using a scalar core potential of the type

$$V_{\text{core}} = \frac{1}{n} \frac{1}{|r_i - r_j|^n} \quad \text{(VII. 1)}$$

with $n = 9$. The two-body interparticle colored potential is

$$V(r, Q \cdot Q') = \frac{g^2}{\lambda} \left[ \frac{1}{9} \left( \frac{\lambda}{r} \right)^9 + Q \cdot Q' \left( \frac{\lambda}{r} \right) \right] \quad \text{(VII. 2)}$$

with $\lambda$ setting the unit of length scale. At close packing the density is $n_{cp} = 1/\lambda^3$. We choose the unit of length $\lambda$ so that $n_{cp} = 1$. The unit of time is set by the inverse plasma frequency $\tau = \omega_p^{-1}$. In these units, the strength of the colored Coulomb potential is $\frac{1}{4\pi} \frac{1}{n\lambda^3}$. We have adopted the Verlet algorithm in [15] to integrate the equations of motion for a system composed of 108 particles. The particles are confined in a box and surrounded by images via periodic boundary conditions. The simulations are carried in a fixed volume $1/(n\lambda^3) = V/(N\lambda^3) = 2.72$ as in [16]. The Wigner-Seitz radius $a_{WS} = (4\pi n/3)^{-1/3}$ is 0.866. With these parameters, the interparticle interaction strength is set by the Coulomb constant $\Gamma$.

We first measure the particle radial distribution function $g(r) = h_0(r)$ as a function of $\Gamma$. $g(r)$ measures the probability of finding two particles between $r$ and $r + \Delta r$,

$$g(r) \equiv h_0(r) = \frac{1}{nN} \left\langle \sum_{i \neq j} N \delta(r - r_{ij}) \right\rangle \quad \text{(VII. 3)}$$

In Fig. 2 we show $g(r)$ versus $r$ for different Coulomb couplings $\Gamma = 2.2, 6.6, 12.8$. The larger $\Gamma$ the larger the size of the Coulomb hole surrounding each colored Coulomb particle. Also, the larger $\Gamma$, the higher the peak, the tighter the Coulomb packing.

The radial distribution function in Fig. 2 for the SU(2) colored Coulomb plasma appears overall similar to the one observed for the one component plasma (OCP). Although our colored particles attract for color antiparallel charges, they overall statistically repel due to the larger color repulsive orientations. The difference with the OCP is best seen by taking
FIG. 2: Radial distribution function for $\Gamma = 2.2$ (b), 6.6 (c) and 12.8 (d). See text.

the Fourier transform of (VII. 3) which is the $l = 0$ density structure factor $S_{00}(q)$

$$S_{00}(k) = \frac{1}{N} \langle |n_k|^2 \rangle$$  

(VII. 4)

with

$$n_k = \sum_{i=1}^{N} e^{ik \cdot r_i}$$  

(VII. 5)

Fig. 3 shows the behavior of $S_{00}(q)$ versus the dimensionless wave-vector $q = a_{WS} k$. The
nonvanishing of $S_{00}$ at the origin reflects on the coupling to the sound mode. In the static and long-wavelength approximation it is just

$$S_{00}(k) \approx \frac{k^2}{c_S^2 k^2} = \frac{1}{c_S^2}$$

(VII. 6)

with $c_S^2 = (\partial P/\partial \rho)_T$ the isothermal squared speed of sound, with $P$ the pressure and $\rho$ the mass density. Since $k$ is a multiple of $2\pi/L$ because of the finite cubic box $L \times L \times L$, only about a dozen points were accessible numerically. Since $L \approx N^{1/3}$, we need to increase the number of particles in the box to smoothen out the structure factor in momentum space.

In Fig. 3 we show the $l = 1$ or charge structure factor

$$S_{01}(k) = \frac{1}{N} \langle |\rho_k|^2 \rangle$$

(VII. 7)

with

$$\rho_k = \sum_{i=1}^N Q_i e^{ik \cdot r_i}$$

(VII. 8)

Unlike $S_{00}$ which correlates a pair of scalar densities, $S_{01}$ correlates a pair of charge densities. In the OCP plasma both correlators are identical. They are not in the SU(2) colored
Coulomb plasma. In the long wavelength approximation, the static density structure factor is saturated by the plasmon mode

\[ S_{01}(k) \approx \frac{k^2}{k_D^2} \]  

(VII. 9)

which is seen to vanish at zero momentum.

Our analytical result for \( S_{01}(k) \) in (VI.10) is in agreement with the molecular dynamics simulations for the charged correlator (VII. 7). Our analytical result for \( S_{00}(k) \) is identical with \( S_{01}(k) \). It differs from the molecular dynamics simulation results for small momenta.
since the sound mode drops out of the Debye-Huckel colored potential on which our charging
process was based. The contribution of the sound mode is additive at small momenta, and
drops out at large momentum due to damping through the shear viscosity.

VIII. CONCLUSIONS

The strongly coupled SU(2) QGP is characterized by a number of static correlators in
phase space with color treated as a classical 3-vector on $S^3$ with a radius fixed by the
second Casimir. Space translational invariance and color rotational invariance yields multiple
structure factors characterizing color correlations with color charges sourced by Legendre
polynomials. Each structure factor obeys a generalized Ornstein-Zernicke equation.

To evaluate analytically these multiple structure factors, we have made use of the Debye
charging process and the linearized Poisson-Boltzman equation in line with linear response
theory. We have derived explicit relations for the two lowest structure factors, ie $l = 0, 1$
which corresponds to the density and charge structure factors.

To check the validity of the linear response analysis, we have numerically extracted the
density and charge static structure factors using SU(2) molecular dynamics simulations.
Modulo the sound mode, both analytical structure factors compare favorably with the nu-
merical results. The current analysis extends to higher multipoles, ie $l = 2, 3, ...$ and gener-
alizes to higher color SU($N > 2$) groups.

The static structure factors play an important role in characterizing the correlations in
the colored SU(2) QGP at intermediate and large values of the coupling coupling $\Gamma$. They
also enter in the assessment of transport parameters at strong coupling. The results will be
presented elsewhere.

The current classical and strongly coupled SU(2) colored Coulomb plasma can be ex-
tended to several species to account for gluons, quarks and antiquarks [3]. The effects of
quantum mechanics being a renormalization of the constituent parameters such as the mass
and charge. It will be interesting to see whether a quantum phase space formulation of
QCD is achievable through the background field formulation in a way that allows for the
introduction of colored static structure factors.
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APPENDIX A: SU(2) COLOR CHARGES

The explicit representation of the classical color charges is \[8, 9\]

\[
Q^1 = \cos \phi_1 \sqrt{J^2 - \pi_1^2}, \quad Q^2 = \sin \phi_1 \sqrt{J^2 - \pi_1^2}, \quad Q^3 = \pi_1
\] (A.1)

with \(J^2\) the quadratic Casimir \(q_2 = \sum_{\alpha}^N \alpha^2 Q^\alpha Q^\alpha\). The measure in the SU(2) phase space can be set to

\[
dQ = c_R d\pi_1 d\phi_1 J dJ \delta(J^2 - q_2)
\] (A.2)

where \(c_R\) is a representation dependent constant. These SU(2) color charges satisfy

\[
\begin{align*}
\int dQ Q^\alpha &= 0 \\
\int dQ Q^\alpha Q^\beta &= C_2 \delta^{\alpha\beta}
\end{align*}
\] (A.3)

For fixed Casimir \(\sum_{\alpha} Q^\alpha Q^\alpha = (N_c^2 - 1)C_2\), we can chose the spherical representation for \(Q^\alpha\) for which the measure \(dQ\) reads

\[
dQ = \sin \theta d\theta d\phi
\] (A.5)

Equivalently,
\[ Q^1 = -\sqrt{\frac{2\pi}{3}} \left( Y^{-1}_{1}(\theta, \phi) - Y^{1}_{1}(\theta, \phi) \right) \]
\[ Q^2 = i \sqrt{\frac{2\pi}{3}} \left( Y^{-1}_{1}(\theta, \phi) + Y^{1}_{1}(\theta, \phi) \right) \]
\[ Q^3 = \sqrt{\frac{4\pi}{3}} Y^{0}_{1}(\theta, \phi) \]  
(A.6)

in terms of spherical harmonics. In the spherical representation, we have \( \int dQ = 4\pi \), \( \sum_{\alpha} Q^\alpha Q^\alpha = 1 \) and \( \int dQ Q \cdot Q = 4\pi \).