The interplay of observational errors through numerical simulations using the $1/V_{\text{max}}$ method for magnitude and proper-motion samples of local disk white dwarfs.

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Abstract. We examine the faint-end ($M_V > +14$) behavior of the disk white dwarf luminosity function using the $1/V_{\text{max}}$ method, but, for the first time, fully including the effects of realistic observational errors in the derived luminosity function. We find that observational errors, mostly in the bolometric corrections and trigonometric parallaxes, play a major role in obliterating (real or artificial) small scale fluctuations in the luminosity function. A better estimator of the true luminosity function seems to be the median over simulations, rather than the mean. When using the latter, an age of 10 Gyr or older can not be ruled out from the sample of Leggett, Ruiz, and Bergeron (1998).

1. Introduction

The classical method for determining the luminosity function (LF hereafter) of magnitude- and proper-motion selected samples is that proposed by Schmidt (1975). This method, called the $1/V_{\text{max}}$, stems from a generalization of a method proposed earlier by Schmidt (1968) for magnitude-limited samples. The method assumes that the LF does not change (“evolves”) as a function of distance from the observer, and that the sample is homogeneously distributed in space. The $1/V_{\text{max}}$ method computes the LF by “weighting” the contribution of each observed point by the equivalent volume where that particular object could have been observed under the pre-specified survey constraints. Several modifications have been proposed to the method in the case of magnitude-limited samples (Davis and Huchra 1982, Eales 1993), most notably that to be able to combine different samples coherently (Avni and Bahcall 1980). However, the basic scheme to determine the LF of magnitude and proper-motion selected samples has remained unchanged, and few and limited numerical simulations have been carried-out to explore the robustness and possible biases that the original method might have when dealing with complete but small, kinematically selected samples (Wood and Oswalt 1998).
Additionally, prompted by investigations of the LF of galaxies where, both, galaxy evolution and clustering (inhomogeneity) plays an important role, several new methods have been proposed to determine the LF of magnitude- and redshift-limited samples. These methods have been designed so as to be less sensitive than the $1/V_{\text{max}}$ to galaxy evolution and clustering. Some of these methods are parametric, some are not. For a review of the different methods, and their relative merit, the reader is referred to Willmer (1997). Unfortunately, these methods are difficult to generalize to proper-motion selected samples as they imply the simultaneous integration of the LF and the projected tangential velocity distribution function, and thus they are extremely model-dependent, unlike the $1/V_{\text{max}}$ which only uses kinematic information, without regard to the underlying velocity distribution. For example, the Step-Wise maximum-likelihood method (Efstathiou et al. 1988), which is one of the most robust non-parametric methods proposed so far (Willmer 1997) is difficult to generalize to proper-motion selected samples, because of the interlace between the LF and the velocity distribution function when predicting the behavior of samples selected simultaneously in magnitude and proper-motion.

Because the spatial density of white-dwarfs (WDs hereafter) is rather small (about $3.4 \times 10^{-3}$ stars/pc$^3$ down to $M_V \sim +16.75$), it is important to insure that the method being used to determine its LF is either free from biases, or that they can be at least reliably corrected. Also, it is important to understand the effects of the kinematic selection on the resulting LF. The purpose of this research has been precisely the elaboration of simulation tools that would allow us to answer some of these questions.

Finally, an interesting outcome of all this, is the fact that one might invert these simulations and use a well understood method. In this case, the determined LF can be used in the model to predict the kinematic distribution of the catalogue stars starting from a set of assumptions regarding the velocity dispersion of WDs. The model predictions are then compared to the observations in an iterative fashion so as to determine the best-fit velocity distributions that are compatible with the observed LF. The velocity dispersion of WDs is known only approximately, while its possible change as a function of luminosity, or cooling age, is completely unknown (Sion and Liebert 1977) because of the lack of large homogeneous samples, and the lack of interpretative models. A proper understanding of the kinematic characteristics of WDs in the solar neighborhood is critical to an understanding of the dynamical history of the local disk which is recorded in the WD luminosity cooling-clock.

2. Numerical simulations

The method proposed by Schmidt (1968, 1975) allows for a derivation of the LF for a sample of stars if we know their apparent magnitudes, parallaxes and (if used in the sample selection), proper motions. We also need to know the sample selection (or survey) limits. For more details of the method, the reader is referred to Schmidt’s papers.

Liebert, Dahn & Monet (1988, LDM88 thereafter) have presented trigonometric parallaxes, optical colors, and spectrophotometric data for intrinsically faint WDs in the context of a program to determine the faint end of the WD
Using the classical $1/V_{\text{max}}$ method, they derived a LF which indicated a downturn near $\log(L/L_\odot) \sim -4.4$, a stellar density of $3 \times 10^{-3}$ stars pc$^{-3}$, and a derived age for the disk in the range $7 - 10$ Gyr. More recently, Legget, Ruiz and Bergeron (1998, LRB98 thereafter) gathered new optical and near-IR data for the cool WDs in the LDM88 sample. Using stellar parameters derived from these data and more refined model atmospheres they re-derived the faint-end of the WD LF, also using the $1/V_{\text{max}}$ method. Comparing their LF with the (then) most recent cooling sequences, they derived a rather young age for the disk of $8 \pm 1.5$ Gyr. In both cases, the uncertainties on the LF was computed using the classical approach of assuming Poisson noise in the counts of every bin, without consideration of the actual observational errors for the quantities involved in the LF determination.

For our numerical simulations, it is assumed that observational errors represent the standard deviation, and that the true value follows a Gaussian distribution function. In every realization of the LF, a value with a mean and dispersion from input values is randomly drawn from a Gaussian distribution function. These values for the whole sample are then used to construct the LF for that particular simulation. Collective values, averaged over the simulations, are then output. In this way, it is possible to derive mean, median & quartiles for the LF over a given set of simulations, as well as any other indicator.

For the simulations, the value adopted for an observable is simply given by, e.g. for the bolometric correction:

$$b_{c,i,j} = b_{c,j} + \sigma_{b_{c,j}} \times G_i$$

where $b_{c,j}$ is the (mean) observed bolometric correction for star “j” in the sample, with “measurement error” $\sigma_{b_{c,j}}$, $G_i$ the Gaussian deviate for simulation “i”, and $b_{c,i,j}$ is the i-th simulation value for the bolometric correction of star j. The same is performed for proper-motion, apparent magnitude, and parallax (not distance!).

### 3. Results & Conclusions

Our simulations indicate that LRB98’s data, when properly accounting for observational errors, does not rule out a disk with an age as large as 10 Gyr (see Figure 1). This is good news because previous studies that find ages of 8 Gyr or younger using similar datasets are difficult to reconcile with an halo age of 15 Gyr (inferred from old globular clusters) given that Galactic formation and chemical evolution models suggest a delay of, at most, 3 Gyr between the onset of star formation in the halo and in the local disk (Wood and Oswalt 1998). Jimenez et al. (1998), using Hipparcos data, have found an upper limit for the age for the disk field population in the solar neighborhood of $11 \pm 1$ Gyr, which would be in agreement with our revised (older) age from the WD LF. Also, we find that current observational uncertainties and sample sizes do not allow us to establish the existence of small scale features in the WD LF which could be indicative of different episodes of star formation in the disk. This could only be alleviated by dramatically increasing the currently small samples.

We find, using the $< V/V_{\text{max}} >$ completeness criteria, that the LRB98 sample seems to be missing faint ($M_{\text{bol}} > +15.0$), large proper motion ($\mu >$...
2.0 arcsec yr$^{-1}$) stars, and that the sample is only complete for $\mu \leq 1.5 - 2.0$ arcsec yr$^{-1}$. However, we find that the precise luminosity break at the faint end of the WD LF is not extremely sensitive to the survey boundary and/or incompleteness effects.

Finally, we find that most of the current uncertainties in the observational WD LF come from uncertainties in bolometric corrections and in parallaxes, while photometry and proper-motions play a minor role. Therefore, refinements on theoretical models (such that $\sigma_{BC} \leq 0.05$ mag) and parallaxes (with $\sigma_\pi \leq 1$ mas), as well as larger samples ($N_{\text{samp}} \sim 200$, see Wood and Oswalt 1998), are urgently needed in order to produce a more refined luminosity function for white dwarfs.

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Figure 1. Bolometric LF for the LRB98 dataset. The solid squares with error bars indicate the LF using the Monte-Carlo mean LF on discrete 0.5 mag bin intervals, while the open squares reproduces the LF derived in the case of no errors, shifted by +0.04. The solid line shows the mean LF using a novel moving-box approach. The long-dashed line indicates the median over simulations LF from the very same simulation that generated the plotted mean LF. The dot-dashed and short-dashed lines show the latest theoretical WD LF published by Benvenuto and Althaus (1999), based on carbon-oxygen core WDs, for a 6 Gyr and a 10 Gyr disk respectively. The big difference between the mean and median LF at faint magnitudes indicates a highly skewed distribution of LF values. Surprisingly, we can also see that the median LF approaches very well the LF derived in the case of no errors. We also find that, when using the mean LF, we can not rule out a disk age of 10 Gyr given the present observational uncertainties.