Theoretical and Empirical Study on Risk Measurement Method Statistics and Portfolio Model

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Abstract: The fundamental purpose of securities investment is to obtain benefits. In order to diversify risks, many investors invest many kinds of securities simultaneously to achieve the maximum returns. Risk measurement methods and portfolio model have become the major issues faced with the financial sector. This paper analyses the calculating method and applicable condition of Markowitz risk remuneration model, Arrow-Pratt risk remuneration model and Jia & Dyer standard risk model. This paper gives the investment model optimized by mean entropy and portfolio model including transaction cost. Empirical analysis shows that the portfolio model is suitable for calculating the return and risk of portfolio. According to the model, investor’s different degree of risk avoidance will form different investment strategies.

1. Introduction

Securities investors, whether individuals or institutions, mainly aim to achieve the maximum expected returns. To this end, they can invest all their funds in one or a few securities with the highest returns in order to maximize their returns. However, the return and risk of investment go hand in hand and complement each other. High return must contain high risk and low risk must contain low return. Therefore, shrewd investors often choose a number of securities to match, rather than focusing on a certain kind of securities, in order to avoid the extreme situation of too high risk and too low return. Of course, securities are not random, but composed of purposeful selection, careful arrangement and scientific collocation. After its formation, it should also be monitored and adjusted regularly according to its effectiveness so as to enable it to achieve the predetermined objectives of investors. There are two different approaches and methods for portfolio management. One is the traditional theory of portfolio analysis, which focuses on determining the constraints of investors themselves. According to investors' demand for securities investment returns, this paper studies how to carry out portfolio from the aspects of current income and capital appreciation in order to meet investors' needs. We study how to portfolio securities from the perspectives of current income and capital appreciation in order to satisfy the purpose of investors, and most of their analysis belongs to qualitative analysis. The other is the modern portfolio theory, which studies how to reduce the risk and maximize the return from the perspective of the relationship between the expected return and risk of securities. Quantitative analysis method is widely used, which is more scientific and rigorous.

The securities market is the main market of the capital market. As the place of investment, the risk of the securities market has brought serious negative impact on the economy. Therefore, financial market risk identification and risk management has become an increasingly important and urgent topic for economists and financial experts around the world. The basis and core of risk management is the measurement of risk and the quantitative analysis and evaluation of risk. With the development of financial market and the scale, dynamics and complexity of financial transactions, the rapid development of financial theory and financial instruments has been promoted. It is impossible to avoid financial risks
completely. When financial risks have been aging and losses of financial institutions are inevitable, it is urgent to resolve financial risks. At the same time, effective risk management theories and methods are not only beneficial to the operation and development of banks, insurance companies, institutional investors and other financial institutions, but also provide objective standards for the management of regulatory authorities, which is conducive to standardizing the securities market, optimizing the allocation of resources, thus promoting the stable development of the economy. Therefore, it is of great theoretical and practical significance to carry out risk measurement and model building.

2. Risk measurement methods

2.1 Markowitz risk remuneration model
In 1952, Markowitz first proposed the method of measuring the risk of securities investment by the variance of return rate. He chose mean and variance as the basic measurement of return and risk, which is characterized by two-way understanding and measurement of risk. He regards the fluctuation of return around the mean as the manifestation of risk, measures the change of loss and the change of profit, and uses linear programming theory to analyse the return of investment, laying a foundation for applying mathematical method to determine the optimal portfolio investment. The theory makes the mean variance model the basis of theoretical research and practical application of portfolio model. Markowitz assumes that investors are risk averse, and rational investors always want to get the maximum expected return under the condition of known risk, while the minimum investment risk under the condition of known expected return. The portfolio with this nature is called efficient portfolio. Securities market is effective. The price of securities reflects the internal economic value of securities. Every investor has sufficient information to understand the expected return and standard deviation of each kind of securities. There is no transaction cost and tax. Investors are price recipients. Securities are infinitely separable. If necessary, they can buy some shares. In investment decision-making, investors only pay attention to the expected return and variance of the probability distribution of investment returns. The expected return reflects investor’s measurement of future return level, while the variance of return reflects investors’ estimation of risk. The investor's investment goal is to obtain as much profit as possible under certain risk conditions, or to reduce the risk as much as possible under certain yield conditions.

\[ R_m = E(w + z) - U^{-1}E[U(w + z)] \]

In the formulation, \( w \) is the initial wealth; \( z \) is the return on investment; \( U \) is the risk reward requirement of the owner for the revamped scheme.

2.2 Arrow-Pratt risk remuneration model
K. J. Arrow (1965) and J. W. Pratt (1964) respectively put forward the Arrow-Pratt measurement to measure consumer risk aversion tendency. Intuitively, the more concave the utility function, the stronger the risk aversion tendency of consumers. Therefore, the second derivative of utility function can be considered to measure the degree of risk aversion. However, utility functions that express the same preferences can be transformed into infinite numbers under affine transformation. Therefore, using second derivative to measure risk aversion tendency will change because of different utility functions representing the same preference, which obviously has problems. The solution to this problem is to standardize the measurement. We use the first derivative to remove the second derivative and get a reasonable measure. It is in this way that Arrow and Pratt give their risk aversion measures. Because the AP measure does not distinguish affine transformations of a class of functions, the transformations of both function \( f \) and AP measure are upper form without careful distinction. Therefore, although AP measures are not refined in describing the concavity of functions, it is this characteristic that makes AP measures extremely important in risk theory. Utility function is a quantity used to describe people's economic behaviour in a risky environment. People's attitude to risk determines their choice of utility function.

\[ R_m = \frac{1}{2}r(w)E[(z - \bar{z})^2] = \frac{1}{2}r(w)\sigma_z^2 \]
In the formula, $\sigma^2_Z$ is the securities return variance.

2.3 Jia & Dyer standard risk model

In recent years, some scholars have attempted to link risk measurement directly with investor's preference model and explore the general theory of risk measurement. The most representative is the standard risk measurement model proposed, and the previous risk measurement model is summarized as a special case. Jia & Dyer's standard risk measurement model directly links risk measurement with investor's preferences, which conforms to the expected utility theory, but it belongs to an explanatory theory. In the application, we must first determine the specific form of the investor utility function in order to select the corresponding risk measure, but this is actually not easy to achieve. In addition, the standard risk measure is based on the normal distribution factory, and has the constraint that risk is independent of the mean value, which is only satisfied in some utility functions. Different ways of investment can bring different risks, and there are many different kinds of risks. For example, financial risk can be divided into market risk, credit risk, liquidity risk, operational risk, legal risk, moral risk and systemic risk from different angles. According to the nature of risk and the different measures to deal with it, the risk of securities investment can be divided into systematic risk and non-systematic risk. Systematic risk is the possible change of investment returns caused by some global factors, including social, political and economic changes, which are reflected in market risk, interest rate risk, purchasing power risk and policy risk. Non-systemic risk refers to the risk that affects the securities of an industry or individual company. It is usually caused by a special factor and has no systematic and comprehensive relationship with the price of the whole stock market.

Assume $X^-$ is the expected return of security of $X$. We define the standard risk variable based on zero expected income benchmark $\bar{X} = (X - X^-)$. Then, the measure of $\bar{X}$ is:

$$R(\bar{X}) = E[U(X - X^-)]$$

In the formula, $U()$ is the utility function of V.N-M.

3. Portfolio model based on information entropy

3.1 Maximum entropy principle

There is a famous theorem about information entropy in information theory, which is the principle of maximum information entropy. In his paper *Information Theory and Statistical Mechanics*, Jaynes proposed a selection criterion. When reasoning based on partial information, such a set of probability distributions must be selected, which should have the maximum entropy and obey all known information. This is the only unbiased distribution we can make; using any other distribution is tantamount to making arbitrary assumptions about all the original information. The statistical inference rule established by Jaynes is called the maximum entropy principle. Although the criterion has a subjective aspect in nature, it is the most objective one. Because the estimated error is the smallest. Maximum Entropy Principle has been widely used as an accepted optimization standard. If for a stochastic process, no observable quantity can be obtained, that is to say, no observation data of statistical moment class can be obtained, then the relevant constraints in the problem will not exist. In addition, from the nature of the entropy function, it is known that under a certain number of possible results, the maximum value of the entropy risk function is obtained when the probability distribution $P$ is uniform distribution, and the value of the entropy function calculated by any other probability distribution will be less than this value. When there are statistical moment constraints in the formula, these constraints are involved. So the calculated probability distribution will deviate from the uniform distribution, resulting in the calculated value of the entropy function will be smaller than that of the uniform distribution. The more the number of moment constraints is. The more the probability distribution deviates from the uniform distribution. The smaller the calculated value of the entropy function is. From the perspective of information theory, this result is very reasonable. Because the more moment constraints there are. The more information you get, the less uncertainty you have about the problem you solve. Therefore, the probability distribution
based on the maximum entropy principle will be the nearest to the uniform distribution under the condition of obeying the statistical moment constraint.

3.2 Investment model optimized by mean entropy

Markowitz's mean-variance theory assumes that the return on investment obeys the normal distribution, and this method may fail if the return on investment does not obey the normal distribution. Statistics show that the return does not necessarily obey the normal distribution. Markowitz's mean-variance theory has two main shortcomings in measuring risk by variance. The variance represents both positive and negative deviations. For investors, they do not want real returns to be less than expected returns, but they do not refuse that real returns are higher than expected returns. To measure risk with variance, it is necessary to calculate the variance covariance matrix, when the amount of calculation is quite complex. In addition, securities investment is a dynamic process of investment selection, while Markowitz's mean-variance portfolio model is a static model, without considering the time factor. We build the mean-entropy portfolio model based on the connotation of entropy and the principle of maximum entropy. Maximum entropy is to find a distribution closest to the uniform distribution. Its advantage is that the risk of uncertainty expressed by entropy is more dynamic than that of variance. The method is relatively simple without considering the covariance matrix. This paper presents a new risk measurement model from the point of view of entropy. It makes some discussions on the risk measurement of portfolio theory. It is a new portfolio model under the condition of insufficient information and obeying known information. It is much easier to calculate the return of various securities in different time periods and the proportion of return to total return in each time period than to calculate the covariance matrix. The value-entropy optimization model not only obeys the above known information, but also achieves an unbiased estimation by maximizing the entropy function in the prediction of future securities market. This model is a portfolio model based on information entropy. It solves a linear programming problem. Compared with the mean-variance model, it does not need to calculate the complex covariance matrix.

3.3 Portfolio model including transaction cost

In the above model, the transaction cost in the process of portfolio investment is not considered. In fact, the transaction cost is a problem that cannot be ignored in investment management. In the process of portfolio investment, ignoring transaction costs will lead to inefficient portfolio investment. However, the investment model considering transaction costs is often more complex than the classical Markowitz model, because the calculation of variance or covariance, but also to deal with complex transaction functions. In order to reduce the workload, some authors proposed a method to simplify the investment model including transaction functions, which is linear programming. However, the simplification procedure is not simple enough to apply. Risk is the uncertainty of future earnings. Uncertainty includes randomness and fuzziness. At present, some authors have made some progress in the study of uncertainty by using fuzzy mathematics, which shows that fuzzy mathematics has its unique characteristics and advantages in the study of some issues in the securities market. Traditional mathematics uses precise and deterministic mathematical concepts to describe the objective world in order to establish a rigorous and perfect axiomatic system. However, due to the diversity and complexity of the objective world, many things are difficult to describe with precise and deterministic concepts, and the occurrence and development of certain events are random. Strictly speaking, the extension of these concepts is characterized by ambiguity or uncertainty. We call the concept of extension uncertainty a fuzzy concept. The ambiguous concept with unclear boundary is not caused by the fact that people's subjective knowledge cannot reach the objective reality, but an objective attribute of things, which is the result of the intermediate transition process between the differences of things. In order to describe the fuzziness of the characteristics of things, the theory of fuzzy sets and fuzzy mathematics have been developed. Since the established model is a multi-objective linear programming model, it is often impossible to achieve the optimal value of both objectives at the same time. At this time, we must take both into account. For investors, it is necessary to choose a compromise method to make each objective
relatively optimal. This process can be completed by using the method of fuzzy theory. Because of the complexity of the financial system and the unpredictability of the stock market, investors cannot give precise expectations of the two objectives of return and risk. Therefore, we can think that return and risk are two vague objectives of investors. Investors believe that returns are acceptable above a certain level, and the bigger the better, the lower the risk is acceptable, and the smaller the better.

3.4 Empirical study
In this section, we use the portfolio model established before to make an empirical analysis. We selected ten stocks traded on Shenzhen Stock Exchange. The data sampled in this paper are the daily closing price of stocks. The time span is from October 13, 2017 to October 12, 2018. Through the analysis, we can know that to get the final value of the model, we must know the distribution of securities returns. In view of the expected return and entropy risk of the stock, we adopt the following methods from the discrete point of view combined with the historical data of the stock: the first step is to collect the data for the daily return of the stock. The second step is to classify and summarize the returns calculated in the first step. According to the distribution of the returns, the sample points of the returns are divided into several groups. The third step is to get the frequency of the returns of stocks falling in the interval in the past year and the mean of the returns falling in the interval. The return of stocks falls in the interval. According to Bernoulli’s law of large number, the frequency of stock returns falling in the interval converges with probability. Thus, when enough data are collected, the probability of frequency estimation can be known. In addition, the average of the historical data of the return rate in the interval is used as the prediction of the future return rate of the stock in the interval; the fourth step is to calculate the risk value of each securities according to the definition of the risk of securities investment. According to the different degree of risk avoidance of investors, the corresponding investment strategy can be obtained by the model.

| $\lambda$ | $(x_1, x_2, \ldots, x_{10})$ | Profit | Entropy risk |
|---|---|---|---|
| 0 | (1,0,0,0,0) | -0.0052 | 0.1015 |
| 0.1 | (0,1,0,0,0) | -0.0024 | 0.0857 |
| 0.5 | (0,0,1,0,0) | -0.0091 | 0.0856 |
| 1 | (0,0,0,0,1) | -0.0075 | 0.0941 |

4. Conclusions
In the financial market, securities investors are most concerned about the return and risk of portfolio when making investment decisions. We draw the following conclusions from our analysis.
(1) In terms of risk measurement, Markowitz risk remuneration model, Arrow-Pratt risk remuneration model and Jia & Dyer standard risk model are the commonly used methods.
(2) This paper gives the investment model optimized by mean entropy and portfolio model including transaction cost. Empirical analysis shows that the portfolio model is suitable for calculating the return and risk of portfolio.
(3) Fuzzy model reflects investor’s subjective wishes. Different parameters will lead to different optimal investment strategies.

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