Simple formula for leading $SU(3)$ irreducible representation for nucleons in an oscillator shell

V.K.B. Kota$^a$

Physical Research Laboratory, Ahmedabad 380 009, India

Abstract

Applications of rotational $SU(3)$ symmetry in nuclei, using Elliott’s $SU(3)$ or pseudo-$SU(3)$ or proxy-$SU(3)$ model, often need just the lowest or leading $SU(3)$ irreducible representation (irrep) $(\lambda_H, \mu_H)$. For nucleons in an oscillator shell $\eta$, with $\mathcal{N} = (\eta + 1)(\eta + 2)/2$, we have the algebra $U(r\mathcal{N}) \supset [U(\mathcal{N} \supset SU(3)) \otimes SU(r)]$; $r = 2$ when there are only valence protons or neutrons and $r = 4$ for nucleons with isospin $T$. Presented in this paper is a simple general formula for the leading $SU(3)$ irrep $(\lambda_H, \mu_H)$ in any given irrep $\{f\}$ of $U(\mathcal{N})$. Results are provided for $(\lambda_H, \mu_H)$ irreps for $\eta$ values of interest in nuclei and for this for all allowed particle numbers. These results clearly show that prolate shape dominates over oblate shape in the shell model $SU(3)$ description.

$^a$ Phone:+917926314939, Fax:+917926314460

E-mail address: vkkota@prl.res.in (V.K.B. Kota)
I. INTRODUCTION

Elliott has recognized way back in 1958 that there is $SU(3)$ symmetry generating rotational spectra in nuclei within the spherical shell model (SM) picture and this is found to be good for light nuclei [1]. With the strong spin-orbit force breaking Elliott’s $SU(3)$ symmetry, Draayer et al recognized [2–5] that for heavier nuclei pseudo-$SU(3)$, based on pseudo spin and pseudo Nilsson orbits, will apply. More recently, proxy-$SU(3)$ scheme by Bonatsos et al [6–8] again brought $SU(3)$ symmetry into focus. In all these, the lowest or the leading $SU(3)$ irreducible representation (irrep), labeled in Elliott’s notation by $(\lambda_H, \mu_H)$, plays a significant role. For example, in light nuclei such as $^{20}$Ne the leading irrep (80) describes the ground $K^\pi = 0^+$ band, in $^{24}$Mg the leading irrep (84) describes the ground $0^+$ and the $2^+ \gamma$-band and so on [9, 10]. Similarly, using the pseudo-$SU(3)$ model with leading $SU(3)$ irreps for valence protons and neutrons respectively, low-lying rotational bands are described with examples from rare-earth and actinides nuclei [4]. This gave rise to integrity basis spectroscopy [9, 11]. More strikingly, leading $SU(3)$ irrep in the proxy-$SU(3)$ scheme is used to describe prolate shape dominance over oblate shape in nuclei with protons and neutrons occupying different oscillator shells and in some situations the same shell [7, 8].

With nucleons in a oscillator major shell $\eta$, we have $U(\mathcal{N}) \supset [U(\mathcal{N}) \supset SU(3) \supset SO(3)] \otimes SU(r)$; $\mathcal{N} = (\eta + 2)(\eta + 2)/2$ and $r = 2$ for identical nucleons and 4 for nucleons with isospin. Computer codes for determining the $SU(3)$ irreps contained in a $U(\mathcal{N})$ irrep are available [12–14] and obviously they give the leading $SU(3)$ irrep besides giving all other irreps with their multiplicities. However, due to their importance, it is clearly useful to have a formula for obtaining the leading irrep $(\lambda_H, \mu_H)$ rather than employing detailed computer codes. The purpose of the present note is to report a simple formula for $(\lambda_H, \mu_H)$ and provide tabulations for useful examples with $r = 2$ and $r = 4$.

In Section II formula for the leading $SU(3)$ irrep is given along with a tabulation for identical nucleons in oscillator shells $\eta = 2 − 6$. An application of these results is for nuclei with protons and neutrons in different shells. Section III gives results for proton-neutron systems and their applications. Finally, Section IV gives conclusions.
II. FORMULA FOR LEADING SU(3) IRREP: RESULTS FOR IDENTICAL NUCLEONS

Given an oscillator shell number $\eta$, we have $U((\eta + 1)(\eta + 2)/2) \supset SU(3)$. In the reminder of this paper we will use $\mathcal{N} = (\eta + 1)(\eta + 2)/2$. In nuclei we are interested in $\eta$ up to 6 or 7. Now, the problem we want to solve is to find the leading (or ground state) $SU(3)$ irrep $(\lambda_H, \mu_H)$ in the reduction of a irrep $\{f\}$ of $U(\mathcal{N})$. Note that $\{f\} = \{f_1, f_2, \ldots, f_N\}$ and $f_1 \geq f_2 \geq f_3 \geq \ldots \geq f_N \geq 0$. For example, for $m$ identical nucleons (protons or neutrons with spin 1/2 degree of freedom) in the oscillator shell $\eta$, $\{f\} = \{2p\}$ for $m$ even and total spin $S = 0$ with $p = m/2$. Similarly, $\{f\} = \{2p, 1\}$ for $m$ odd and $S = 1/2$ with $p = (m - 1)/2$; note that $m = \sum_i f_i$. For a given $\{f\}$, the irrep $(\lambda_H, \mu_H)$ is the one with largest value of $\epsilon = 2\lambda_H + \mu_H$ and for this $\epsilon$ with largest value of $\Lambda = \frac{\mu_H}{2}$ [15].

For a given $\eta$, the single particle orbits in $(n_z, n_x, n_y)$ representation are $(\eta - r, r - x, x)$ in the order with $r$ taking values 0 to $\eta$ and for each $r, x$ takes values 0 to $r$; note that $n_i$ gives number of oscillator quanta in $i$-th direction for a single particle. For example for $\eta = 3$, in order they are (300), (210), (201), (120), (111), (102), (030), (021), (012) and (003). Now, let us consider an irrep $\{f\}$ for $m$ particles. Putting $f_1$ number of particles in the first orbit, $f_2$ in the second orbit and so on with $f_N$ in the last orbit will give the total $N_z, N_x$ and $N_y$ (note that $N_i$ is the sum of single particle $n_i$ values). Then, we have $\lambda_H = N_z - N_x$ and $\mu_H = N_x - N_y$ [16]. This, we are able to convert into a simple formula that is valid for any $\eta$ and $\{f\}$. The final result is

$$
\{f\} \rightarrow (\lambda_H, \mu_H)_{SU(3)} \quad \text{with} \\
\lambda_H = \sum_{r=0}^{\eta} \sum_{x=0}^{r} (\eta - 2r + x) \times f_{1+x+\frac{r(r+1)}{2}} , \quad \mu_H = \sum_{r=0}^{\eta} \sum_{x=0}^{r} (r - 2x) \times f_{1+x+\frac{r(r+1)}{2}} .
$$

(1)

It is trivial to programme Eq. (1). Results obtained using this, for shells $\eta = 2, 3, 4, 5, 6$ for identical fermions with particle number $m = 2$ to $\mathcal{N}$ and spin $S = 0$ for even $m$ and $S = 1/2$ for odd $m$, are given in Table I. They can be extended easily to $\eta = 7, 8$ as may be needed for SHE nuclei and also for other spin $S$ values. In [7], detailed code given in [14] that generates the full $\{f\} \rightarrow (\lambda\mu)$ reductions is used to obtain $(\lambda_H, \mu_H)$ (see Table I of [7]).
III. APPLICATION TO HEAVY NUCLEI WITH VALENCE PROTONS AND NEUTRONS IN DIFFERENT SHELLS USING PROXY-$SU(3)$ SCHEME

Using the results for identical nucleons given in Table I, it is possible to infer prolate to oblate transition, in heavy nuclei with protons and neutrons in different shells, as number of valence nucleons in a given shell changing. This is indeed possible as nuclei will be prolate for $\lambda > \mu$ and oblate for $\lambda < \mu$ [17–19]. Here, we will use the $SU(3)$ symmetry extension to heavy nuclei by the proxy-$SU(3)$ model introduced in [6, 7]. Firstly, for identical nucleons in shells $\eta = 2, 3, 4, 5, 6$ prolate shape changes to oblate at $m = 8, 15, 23, 34$ and 47 respectively as seen from the $(\lambda_H, \mu_H)$ values shown in Table I for each $\eta$. Note that these $m$ values are higher than the mid-shell values (6, 10, 15, 21 and 28 respectively) and therefore prolate to oblate transition breaks particle-hole symmetry. For heavy nuclei with valence protons ($p$) and neutrons ($n$) occupying different shell say $\eta_p$ and $\eta_n$, the leading or ground state $SU(3)$ irrep will be $(\lambda_H^p + \lambda_H^n, \mu_H^p + \mu_H^n)$ where $(\lambda_H^p, \mu_H^p)$ and $(\lambda_H^n, \mu_H^n)$ are the leading $SU(3)$ irreps for protons and neutrons respectively and they can obtained from Table I. This is employed in the proxy-$SU(3)$ model and here for Ba to Pt isotopes with $88 \leq N \leq 120$, valence protons will be in 50-82 shell ($sdg$ or $\eta = 4$ in the proxy model) and neutrons in 82-126 shell ($fph$ or $\eta = 5$ in the proxy model). Then, it is easy to see that the oblate shape $(\lambda_H > \mu_H)$ appears for $^{190,192,194}$W, $^{192,194,196}$Os and $^{194,196,198}$Pt. From experiments there are strong indications that $^{190}$W, $^{192,194}$Os are oblate or more towards oblate shape [20–22]. Using the results in Table I, it is also possible to predict that some of the Dy, Er and HF isotopes (with neutron number $N$ between 88 and 120) also exhibit oblate shape. See [7, 8] for details of prolate-oblate transition within the proxy-$SU(3)$ model.

IV. LEADING $SU(3)$ IRREPS FOR PROTON-NEUTRON SYSTEMS WITH GOOD SPIN-ISOSPIN $SU(4)$ SYMMETRY

With protons and neutrons in the same oscillator shell $\eta$, we need to consider four column irreps of $U(\mathcal{N})$ and find $(\lambda_H, \mu_H)$ contained in these irreps. Firstly we have the algebra $U(4\mathcal{N}) \supset U(\mathcal{N}) \otimes SU(4)$; here we assume that the Wigner’s spin-isospin $SU(4)$ symmetry is good. Given the nucleon number $m$ and the isospin $T = |T_Z|$ (note that $T_Z = (m_p - m_n)/2$ where $m_p$ is number of valence protons and $m_n$ is number of valence neutrons with $m =$
an example. For this nucleus, ground state is oblate if we consider the valence nucleons to be
and it can be identified as follows. Firstly, \( \{ a \} \) will have, in Young tableaux notation, four boxes in
\( SU(4) \) irreps follow from \( U(4) \) irreps. For \( m \) even we have
\[
\{ F_1, F_2, F_3, F_4 \} = \left\{ \frac{m+2T}{4}, \frac{m+2T}{4}, \frac{m-2T}{4}, \frac{m-2T}{4} \right\} \quad \text{for} \quad \frac{m}{2} + T \quad \text{even},
\]
\[
\{ F_1, F_2, F_3, F_4 \} = \left\{ \frac{m+2T+2}{4}, \frac{m+2T-2}{4}, \frac{m-2T+2}{4}, \frac{m-2T-2}{4} \right\} \quad \text{for} \quad \frac{m}{2} + T \quad \text{odd}.
\]
The only exception is \( T = 0 \) for \( m = 4r + 2 \) type and then
\[
\{ F_1, F_2, F_3, F_4 \} = \left\{ \frac{m+2}{4}, \frac{m+2}{4}, \frac{m-2}{4}, \frac{m-2}{4} \right\} \quad \text{for} \quad \frac{m}{2} + T \quad \text{odd}.
\]
Similarly, for odd-\( m \) we have
\[
\{ F_1, F_2, F_3, F_4 \} = \left\{ \frac{m+2T+2}{4}, \frac{m+2T-2}{4}, \frac{m-2T+2}{4}, \frac{m-2T-2}{4} \right\} \quad \text{for} \quad \frac{m}{2} + T \quad \text{even}.
\]
Eqs. (2)-(4) are well known and given in many papers (but with different notations). See for example Refs. [23, 24]. Using Eqs. (2)-(4) it is easy to obtain the lowest \( U(4) \) irrep for a given \( m \) and \( |T_z| \). Note that, with \( \mathcal{N}' = (\eta + 1)(\eta + 2) \), values of \( T_Z \) for \( m \leq \mathcal{N}' \) are \( |T_Z| = m/2, m/2 - 1, \ldots, 0 \) or 1/2. For \( m > \mathcal{N}' \), we have \( |T_Z| = (2\mathcal{N}' - m)/2, (2\mathcal{N}' - m)/2 - 1, \ldots, 0 \) or 1/2. Now, our task is to find the \( U(\mathcal{N}) \) irrep \( \{ f \} \) that corresponds to a given \( \{ F_1, F_2, F_3, F_4 \} \). It is well known that \( \{ f \} \) must be conjugate of \( \{ F_1, F_2, F_3, F_4 \} \) and it can be identified as follows. Firstly, \( \{ f \} \) will be \( \{ 4^a 3^b 2^c 1^d \} \) type irrep. Then, it is easy to find that \( a = F_4, b = F_3 - F_4, c = F_2 - F_3 \) and \( d = F_1 - F_2 \). Therefore, \( \{ f \} \) will have, in Young tableaux notation, four boxes in \( a \) (= \( F_4 \)) number of rows, 3 boxes in \( b \) (= \( F_3 - F_4 \)) number of rows, two boxes in \( c \) (= \( F_2 - F_3 \)) number of rows and one box in \( d \) (= \( F_1 - F_2 \)) number of rows. With \( \{ f \} \) thus determined, we can find the leading \( SU(3) \) irrep in the four columned \( \{ f \} \) using Eq. (1). Results are given in Tables II-IV for \( \eta = 2, 3 \) and 4. These results will suffice as they are needed only for nuclei with valence nucleons in the same oscillator shell. Let us now consider some applications of these results.

Firstly, for \( N=Z \) and \( N=Z \pm 1 \) nuclei, it is easy to see that the leading or ground state
\( U(\mathcal{N}) \) irrep \( \{ f \} \) will be of \( \{ 4^r, p \} \) type with \( \{ 4^r \} \) for \( m = 4r \) (i.e. \( N=Z \) even-even), \( \{ 4^r, 2 \} \) for \( m = 4r + 2 \) (\( N=Z \) odd-odd) and \( \{ 4^r, 1 \} \) or \( \{ 4^r, 3 \} \) as appropriate for \( m = 4r + 1 \) or \( 4r + 3 \) (\( N=Z \pm 1 \)). The leading \( SU(3) \) irrep for these for \( \eta = 2, 3 \) and 4 shells are listed in Table II and they will suffice in practical applications. It is seen that the prolate to oblate transition here is for \( m = 16, 29 \) and 46 for \( \eta = 2, 3 \) and 4 shells respectively. Let us consider \( ^{72}\text{Kr} \) as an example. For this nucleus, ground state is oblate if we consider the valence nucleons to be
only in $^2p_{3/2}, \, ^1f_{5/2}$ and $^2p_{1/2}$ orbits with $^{56}$Ni core (then $\eta = 2$ will apply with pseudo $SU(3)$). However, if use the proxy-$SU(3)$ model and include the $^1g_{9/2}$ orbit (without the $\Omega = \pm 9/2$ states), then $\eta = 3$ shell will apply giving prolate shape. Experimental data for this nucleus indicates prolate with some oblate mixing; see for example [25, 26]. Let us also add that heavy N=Z nuclei (more so odd-odd N=Z nuclei) may not preserve $SU(3)$ symmetry and here $SO(N)$ structures from IBM-3 and IBM-4 models may be more appropriate [27, 28].

Turning to N $\neq$ Z nuclei but with valence protons and neutrons in the same shell, the leading $SU(3)$ irrep for these follow from Tables III and IV and their application is discussed in [7] with examples from Ba (Z=56) to Os (Z=78) isotopes. Results in Table III of [7] follow from Tables III and IV in the present paper. We will discuss in a future publication in detail the dominance of prolate over oblate shape that can be inferred from Tables I-IV and the experimental evidence for these.

V. CONCLUSIONS

Following the significance of leading $SU(3)$ irrep in Elliott’s $SU(3)$, pseudo-$SU(3)$ and proxy-$SU(3)$ models, we have presented a simple formula as given by Eq. (1) for the ($\lambda_H, \mu_H$) for nucleons in a given oscillator shell $\eta$. Tabulations (Tables I-IV) are provided for values of $\eta$ that are important for nuclei. They will cover all the situations with (i) valence protons and neutrons occupying different shells and (ii) valence protons and neutrons occupying the same shell as in N $\sim$ Z nuclei and in many other situations as given in Table III of [7]. Eq. (1) eliminates the need to use detailed computer codes for obtaining ($\lambda_H, \mu_H$). A brief discussion, based on the results in Tables I-IV, of prolate dominance over oblate shape is given with detailed discussion postponed to a separate paper.

It is useful to add that Eq. (1) is also useful in IBM-2, IBM-3 and IBM-4 models [25, 29]. In IBM-2 with $F$-spin we have $U(2N) \supset [U(N) \supset SU(3)] \otimes SU(2)$ and therefore $\{f\}$ of $U(N)$ with two rows are allowed. In IBM-3 with $U(3N) \supset [U(N) \supset SU(3)] \otimes SU_T(3)$, $\{f\}$ of $U(N)$ here can have maximum three rows (the other $SU(3)$ here generates isospin of the bosons). Similarly, in IBM-4 with $U(6N) \supset [U(N) \supset SU(3)] \otimes SU(6)$ we have $\{f\}$ of $U(N)$ with maximum six rows. Note that $\mathcal{N} = 6$ for $sdIBM$, 15 for $sdgIBM$ and so on. Applications of Eq. (1) for these will be discussed elsewhere.
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TABLE I. Ground state or leading SU(3) irrep $(\lambda_H, \mu_H)$ for a given number $m$ of identical particles in a oscillator shell $\eta$. Results are shown for $\eta = 2, 3, 4, 5$ and 6. For a given $m$ with $m \geq 4$, the $(\lambda_H, \mu_H)$ is given in the table as $(\lambda_H, \mu_H)^m$.

$\eta = 2$
(4, 0)$^2$, (4, 1)$^3$, (4, 2)$^4$, (5, 1)$^5$, (6, 0)$^6$, (4, 2)$^7$, (2, 4)$^8$, (1, 4)$^9$, (0, 4)$^{10}$, (0, 2)$^{11}$, (0, 0)$^{12}$

$\eta = 3$
(6, 0)$^2$, (7, 1)$^3$, (8, 2)$^4$, (10, 1)$^5$, (12, 0)$^6$, (11, 2)$^7$, (10, 4)$^8$, (10, 4)$^9$, (11, 2)$^{11}$, (12, 0)$^{12}$, (9, 3)$^{13}$, (6, 6)$^{14}$, (4, 7)$^{15}$, (2, 8)$^{16}$, (1, 7)$^{17}$, (0, 6)$^{18}$, (0, 3)$^{19}$, (0, 0)$^{20}$

$\eta = 4$
(8, 0)$^2$, (10, 1)$^2$, (12, 2)$^4$, (15, 1)$^5$, (18, 0)$^6$, (18, 2)$^7$, (18, 4)$^8$, (19, 4)$^9$, (20, 4)$^{10}$, (22, 2)$^{11}$, (24, 0)$^{12}$, (22, 3)$^{13}$, (20, 6)$^{14}$, (19, 7)$^{15}$, (18, 8)$^{16}$, (18, 7)$^{17}$, (18, 6)$^{18}$, (19, 3)$^{19}$, (20, 0)$^{20}$, (16, 4)$^{21}$, (12, 8)$^{22}$, (9, 10)$^{23}$, (6, 12)$^{24}$, (4, 12)$^{25}$, (2, 12)$^{26}$, (1, 10)$^{27}$, (0, 8)$^{28}$, (0, 4)$^{29}$, (0, 0)$^{30}$

$\eta = 5$
(10, 0)$^2$, (13, 1)$^3$, (16, 2)$^4$, (20, 1)$^5$, (24, 0)$^6$, (25, 2)$^7$, (26, 4)$^8$, (28, 4)$^9$, (30, 4)$^{10}$, (33, 2)$^{11}$, (36, 0)$^{12}$, (35, 3)$^{13}$, (34, 6)$^{14}$, (34, 7)$^{15}$, (35, 7)$^{16}$, (36, 6)$^{17}$, (36, 3)$^{18}$, (38, 3)$^{19}$, (40, 0)$^{20}$, (37, 4)$^{21}$, (34, 8)$^{22}$, (32, 10)$^{23}$, (30, 12)$^{24}$, (29, 12)$^{25}$, (28, 12)$^{26}$, (28, 10)$^{27}$, (28, 8)$^{28}$, (29, 4)$^{29}$, (30, 0)$^{30}$, (25, 5)$^{31}$, (20, 10)$^{32}$, (16, 13)$^{33}$, (12, 16)$^{34}$, (9, 17)$^{35}$, (6, 18)$^{36}$, (4, 17)$^{37}$, (2, 16)$^{38}$, (1, 13)$^{39}$, (0, 10)$^{40}$, (0, 5)$^{41}$, (0, 0)$^{42}$

$\eta = 6$
(12, 0)$^2$, (16, 1)$^3$, (20, 2)$^4$, (25, 1)$^5$, (30, 0)$^6$, (32, 2)$^7$, (34, 4)$^8$, (37, 4)$^9$, (40, 4)$^{10}$, (44, 2)$^{11}$, (48, 0)$^{12}$, (48, 3)$^{13}$, (48, 6)$^{14}$, (49, 7)$^{15}$, (50, 8)$^{16}$, (52, 7)$^{17}$, (54, 6)$^{18}$, (57, 3)$^{19}$, (60, 0)$^{20}$, (58, 4)$^{21}$, (56, 8)$^{22}$, (55, 10)$^{23}$, (54, 12)$^{24}$, (54, 12)$^{25}$, (55, 10)$^{26}$, (55, 10)$^{27}$, (56, 8)$^{28}$, (58, 4)$^{29}$, (60, 0)$^{30}$, (56, 5)$^{31}$, (52, 10)$^{32}$, (49, 13)$^{33}$, (46, 16)$^{34}$, (44, 17)$^{35}$, (42, 18)$^{36}$, (41, 17)$^{37}$, (40, 16)$^{38}$, (40, 13)$^{39}$, (40, 10)$^{40}$, (41, 5)$^{41}$, (42, 0)$^{42}$, (36, 6)$^{43}$, (30, 12)$^{44}$, (25, 16)$^{45}$, (20, 20)$^{46}$, (16, 22)$^{47}$, (12, 24)$^{48}$, (9, 24)$^{49}$, (6, 24)$^{50}$, (4, 22)$^{51}$, (2, 20)$^{52}$, (1, 16)$^{53}$, (0, 12)$^{54}$, (0, 6)$^{55}$, (0, 0)$^{56}$

Note that for $m = 2$ we have $\{f\} = \{2\}$ and $\{1^2\}$ and here complete solution for $\{f\} \rightarrow \lambda \mu$ is easy to derive. We have $\{f\} \rightarrow (\lambda \mu) = (2m, 0) \oplus (2m - 4, 2) \oplus (2m - 8, 4) \oplus \ldots$. Similarly, $\{1^2\} \rightarrow (2m - 2, 1) \oplus (2m - 6, 3) \oplus \ldots$
TABLE II. Leading SU(3) irrep \((\lambda_H, \mu_H)\) for a given number \(m\) of nucleons with \(\{f\} = \{4^r, p\}\) irrep for \(U((\eta + 1)(\eta + 2)/2)\) in a oscillator shell \(\eta\); \(m = 4r + p\). Note that uniquely: (i) for \(N=Z\) even-even nuclei lowest \(\{f\} = \{4^r\}\); (ii) for \(N=Z\) odd-odd nuclei, lowest \(\{f\} = \{4^r, 2\}\); (iii) for \(N=Z \pm 1\) nuclei, lowest \(\{f\} = \{4^r, 1\}\) or \(\{4^r, 3\}\). Results are shown for \(\eta = 2, 3,\) and 4. For a given \(m\), the \((\lambda_H, \mu_H)\) is given in the table as \((\lambda_H, \mu_H)^m\).

| \(\eta\) = 2 | \((4, 0)^2, (6, 0)^3, (8, 0)^4, (8, 1)^5, (8, 2)^6, (8, 3)^7, (8, 4)^8, (9, 3)^9, (10, 2)^{10}, \) |
|---|---|
| \((11, 1)^{11}, (12, 0)^{12}, (10, 2)^{13}, (8, 4)^{14}, (6, 6)^{15}, (4, 8)^{16}, (3, 8)^{17}, (2, 8)^{18}, (1, 8)^{19}, \) |
| \((0, 8)^{20}, (0, 6)^{21}, (0, 4)^{22}, (0, 2)^{23}, (0, 0)^{24}\) | |
| \(\eta\) = 3 | \((6, 0)^2, (9, 0)^3, (12, 0)^4, (13, 1)^5, (14, 2)^6, (15, 3)^7, (16, 4)^8, (18, 3)^9, (20, 2)^{10}, \) |
| \((22, 1)^{11}, (24, 0)^{12}, (23, 2)^{13}, (22, 4)^{14}, (21, 6)^{15}, (20, 8)^{16}, (20, 8)^{17}, (20, 8)^{18}, (20, 8)^{19}, \) |
| \((20, 8)^{20}, (21, 6)^{21}, (22, 4)^{22}, (23, 2)^{23}, (24, 0)^{24}, (21, 3)^{25}, (19, 6)^{26}, (15, 9)^{27}, (12, 12)^{28}, \) |
| \((10, 13)^{29}, (8, 14)^{30}, (6, 15)^{31}, (4, 16)^{32}, (3, 15)^{33}, (2, 14)^{34}, (1, 13)^{35}, (0, 12)^{36}, (0, 9)^{37}, \) |
| \((0, 6)^{38}, (0, 3)^{39}, (0, 0)^{40}\) | |
| \(\eta\) = 4 | \((8, 0)^2, (12, 0)^3, (16, 0)^4, (18, 1)^5, (20, 2)^6, (22, 3)^7, (24, 4)^8, (27, 3)^9, (30, 2)^{10}, \) |
| \((33, 1)^{11}, (36, 0)^{12}, (36, 2)^{13}, (36, 4)^{14}, (36, 6)^{15}, (36, 8)^{16}, (37, 8)^{17}, (38, 8)^{18}, (39, 8)^{19}, \) |
| \((40, 8)^{20}, (42, 6)^{21}, (44, 4)^{22}, (46, 2)^{23}, (48, 0)^{24}, (46, 3)^{25}, (44, 6)^{26}, (42, 9)^{27}, (40, 12)^{28}, \) |
| \((39, 13)^{29}, (38, 14)^{30}, (37, 15)^{31}, (36, 16)^{32}, (36, 15)^{33}, (36, 14)^{34}, (36, 13)^{35}, (36, 12)^{36}, (37, 9)^{37}, \) |
| \((38, 6)^{38}, (39, 3)^{39}, (40, 0)^{40}, (36, 4)^{41}, (32, 8)^{42}, (28, 12)^{43}, (24, 16)^{44}, (21, 18)^{45}, (18, 20)^{46}, \) |
| \((15, 22)^{47}, (12, 24)^{48}, (10, 24)^{49}, (8, 24)^{50}, (6, 24)^{51}, (4, 24)^{52}, (3, 22)^{53}, (2, 20)^{54}, (1, 18)^{55}, \) |
| \((0, 16)^{56}, (0, 12)^{57}, (0, 8)^{58}, (0, 4)^{59}, (0, 0)^{60}\) |
TABLE III. Leading SU(3) irrep \((\lambda_H, \mu_H)\) for a given number \(m\) of nucleons and isospin \(T = |T_z|\).

Results are shown for \(\eta = 2, 3\) and 4 with \(m\) even and \(m \geq 4\). The irreps are shown the table as 
\[(\lambda_H, \mu_H)_m^\eta\].

\(\eta = 2\)

| \(m\) | \(20\) | \(24\) | \(28\) | \(32\) | \(36\) | \(40\) |
|---|---|---|---|---|---|---|
| \(0\) | \(\{0, 0\}\) | \(\{0, 0\}\) | \(\{0, 0\}\) | \(\{0, 0\}\) | \(\{0, 0\}\) | \(\{0, 0\}\) |
| \(2\) | \(\{0, 2\}\) | \(\{0, 2\}\) | \(\{0, 2\}\) | \(\{0, 2\}\) | \(\{0, 2\}\) | \(\{0, 2\}\) |
| \(4\) | \(\{1, 0\}\) | \(\{1, 0\}\) | \(\{1, 0\}\) | \(\{1, 0\}\) | \(\{1, 0\}\) | \(\{1, 0\}\) |
| \(6\) | \(\{2, 0\}\) | \(\{2, 0\}\) | \(\{2, 0\}\) | \(\{2, 0\}\) | \(\{2, 0\}\) | \(\{2, 0\}\) |

\(\eta = 3\)

| \(m\) | \(16\) | \(20\) | \(24\) | \(28\) | \(32\) | \(36\) | \(40\) |
|---|---|---|---|---|---|---|---|
| \(0\) | \(\{0, 0\}\) | \(\{0, 0\}\) | \(\{0, 0\}\) | \(\{0, 0\}\) | \(\{0, 0\}\) | \(\{0, 0\}\) | \(\{0, 0\}\) |
| \(2\) | \(\{0, 2\}\) | \(\{0, 2\}\) | \(\{0, 2\}\) | \(\{0, 2\}\) | \(\{0, 2\}\) | \(\{0, 2\}\) | \(\{0, 2\}\) |
| \(4\) | \(\{1, 0\}\) | \(\{1, 0\}\) | \(\{1, 0\}\) | \(\{1, 0\}\) | \(\{1, 0\}\) | \(\{1, 0\}\) | \(\{1, 0\}\) |
| \(6\) | \(\{2, 0\}\) | \(\{2, 0\}\) | \(\{2, 0\}\) | \(\{2, 0\}\) | \(\{2, 0\}\) | \(\{2, 0\}\) | \(\{2, 0\}\) |

\(\eta = 4\)

| \(m\) | \(8\) | \(12\) | \(16\) | \(20\) | \(24\) | \(28\) | \(32\) |
|---|---|---|---|---|---|---|---|
| \(0\) | \(\{0, 0\}\) | \(\{0, 0\}\) | \(\{0, 0\}\) | \(\{0, 0\}\) | \(\{0, 0\}\) | \(\{0, 0\}\) | \(\{0, 0\}\) |
| \(2\) | \(\{0, 2\}\) | \(\{0, 2\}\) | \(\{0, 2\}\) | \(\{0, 2\}\) | \(\{0, 2\}\) | \(\{0, 2\}\) | \(\{0, 2\}\) |
| \(4\) | \(\{1, 0\}\) | \(\{1, 0\}\) | \(\{1, 0\}\) | \(\{1, 0\}\) | \(\{1, 0\}\) | \(\{1, 0\}\) | \(\{1, 0\}\) |
| \(6\) | \(\{2, 0\}\) | \(\{2, 0\}\) | \(\{2, 0\}\) | \(\{2, 0\}\) | \(\{2, 0\}\) | \(\{2, 0\}\) | \(\{2, 0\}\) |

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| \( \eta = 2 \) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (6, 2) | (3, 1) | (8, 5) | (1, 6) | (7, 3) | (4, 5) | (8, 3) | (1, 7) | (5, 6) | (3, 4) | (9, 2) | (6, 1) | (8, 7) | (4, 3) | (1, 9) | (5, 7) |
| (11, 2) | (8, 3) | (9, 1) | (19, 2) | (9, 1) | (19, 2) | (9, 1) | (19, 2) | (9, 1) | (19, 2) | (9, 1) | (19, 2) | (9, 1) | (19, 2) | (9, 1) | (19, 2) | (9, 1) | (19, 2) | (9, 1) | (19, 2) | (9, 1) | (19, 2) |
| (17, 5) | (15, 2) | (13, 6) | (11, 2) | (17, 5) | (15, 2) | (13, 6) | (11, 2) | (17, 5) | (15, 2) | (13, 6) | (11, 2) | (17, 5) | (15, 2) | (13, 6) | (11, 2) | (17, 5) | (15, 2) | (13, 6) | (11, 2) | (17, 5) | (15, 2) | (13, 6) |
| (10, 2) | (5, 3) | (10, 2) | (5, 3) | (10, 2) | (5, 3) | (10, 2) | (5, 3) | (10, 2) | (5, 3) | (10, 2) | (5, 3) | (10, 2) | (5, 3) | (10, 2) | (5, 3) | (10, 2) | (5, 3) | (10, 2) | (5, 3) | (10, 2) | (5, 3) |
| (11, 1) | (10, 2) | (9, 2) | (8, 3) | (7, 4) | (6, 5) | (5, 6) | (4, 7) | (3, 8) | (2, 9) | (1, 10) | (1, 10) | (1, 10) | (1, 10) | (1, 10) | (1, 10) | (1, 10) | (1, 10) | (1, 10) | (1, 10) | (1, 10) | (1, 10) |
| (3, 1) | (2, 1) | (1, 2) | (6, 5) | (5, 6) | (4, 7) | (3, 8) | (2, 9) | (1, 10) | (1, 10) | (1, 10) | (1, 10) | (1, 10) | (1, 10) | (1, 10) | (1, 10) | (1, 10) | (1, 10) | (1, 10) | (1, 10) | (1, 10) | (1, 10) |

**TABLE IV.** Same as Table III but for \( m \) odd. The leading \( SU(3) \) irreps are given in the table as \((\lambda_H, \mu_H)^{m,2T}\).