Buckyballs and gluon junction networks on the femtometer scale *

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Abstract

We explore the possibility that novel geometrical structures analogous to carbon Fullerenes may exist in Nature on the femtometer scale. The theory of strong interactions, Quantum Chromo Dynamics (QCD) predicts the existence of special topological gluon field configurations called baryon junctions and anti-junctions. Here we show that femto-scale structures, networks or closed (gluon field) cages, can be constructed in the theory of QCD as tiny cousins of familiar nano-scale structures such as carbonic Fullerenes $C_{60}$, $C_{70}$. The most symmetric polyhedra of QCD junctions (J-balls) are characterized by the “magic numbers” 8, 24, 48, and 120, and zero net baryon number. Tubes, prisms, tori and other topological structures can also be created. In addition, special configurations can be constructed that are odd under charge and parity conjugation (CP), although the QCD Lagrangian is CP even. We provide a semi-classical estimate for the expected mass range of QCD Buckyballs and discuss the possible conditions under which such novel topological excitations of the QCD vacuum may be produced in experiments of high energy physics.

Key words: Fullerenes, baryon number, QCD, junction, classical and semi-classical techniques, nonstandard multi-gluon states

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1 Introduction

The Buckminsterball is the nickname for the carbon molecule Buckminsterfullerene, C_{60}, a new form of carbon discovered in chemistry in 1985 by R. F. Curl, H. W. Kroto and R. E. Smalley[1]. The molecule was named after the geodesic dome, invented by the architect Buckminster Fuller, whose geometry approximates that of a truncated icosahedral (soccer-ball) shaped structure. The discovery of Buckyballs was followed by the discovery of a wide variety of other carbon molecules with interesting geometrical properties. Carbon tubes, helixes, tori, etc. opened the doorway to technology on the nanometer (10^{-9} m) scale. Carbon atoms can be arranged in novel geometric forms because the carbonic bonds can arrange into 3 way junction structures as illustrated in Fig. 1. Nanostructures have also been constructed using 3 and 4 way DNA junctions by Seaman et al. [2]. The field of nano-technology is developing rapidly using an assortment of molecular junctions as the chemical “lego” building blocks.

In nuclear/particle physics, where the distance scales are femtometers (10^{-15} m), the existence of special three-way QCD junctions (topological gluon field configurations) was predicted a long time ago[3]. Lattice QCD calculations were able to confirm the existence of such junctions only recently[4]. Data on baryon stopping and strangeness production in experiments with high energy heavy ion collisions from CERN SPS and BNL RHIC accelerators are also in agreement with model calculation assuming that QCD junctions carry the conserved baryon charge[5–8]. In this Letter, we explore what types of femtometer scale structures can be constructed from QCD using junctions and anti-junctions as a nuclear scale “lego set”. Our preliminary results were presented at a Symposium on multiparticle production in high energy physics [9].

According to QCD, hadrons are composite bound state configurations built up from the fundamental quark and gluon fields. Quarks, \( \Psi_{i,f}(x) \), carry color, \( i = 1, ..., N_c \), and flavor, \( f = u, d, s, c, b, t \) quantum numbers. Gluons, \( A_\mu^a(x) \), are the vector gauge bosons intermediating the color, \( a = 1, ..., N_c^2 - 1 \), interactions between the quarks and gluons. The form of the interaction is fixed by the principle of gauge invariance under the non-Abelian color \( SU(N_c) \) Lie group. The \( N_c = 1 \) limit is Quantum Electrodynamics (QED). Gauge invariance of composite operators can only be achieved with the help of open string operators, called Wilson lines [10], that keep track of the phase along an arbitrary path, \( \Gamma \), in space-time. In QED, \( U(\Gamma) = \exp [ie \int_\Gamma dx^\mu A_\mu(x)] \) is the well known Aharonov-Bohm phase [11] accumulated by an electron moving along a path \( \Gamma \) in an external electromagnetic field, \( A_\mu(x) \). In QCD, \( N_c = 3 \) and \( U(\Gamma) \) is a matrix defined by a path ordered exponential with dimension corresponding to that of the representation of the generators, \( T_a \), of the Lie algebra.
Closed Wilson loops, $\text{Tr}U(\Gamma xx)$, correspond to color singlet glueball configurations in QCD, while open “strings”, $\overline{\Psi}_{i, f_1}(x)U^{i_{123}}(\Gamma_{xy})\Psi_{i, f_2}(y)$, terminating with quark and anti-quark ends, correspond to mesons. Baryons are special field configurations composed of $N_c$ quarks with their color flux strings tied together (outer product of color indices) by the Levi-Civita antisymmetric tensor, $\epsilon_{i_1...i_{N_c}}$. In the physical ($N_c = 3$) case, baryons of flavor $(f_1, f_2, f_3)$ are represented by the color neutral and gauge invariant operator,

$$B_{f_1 f_2 f_3} = \overline{\Psi}_{i, f_1}(x_1)\overline{\Psi}_{i, f_2}(x_2)\overline{\Psi}_{i, f_3}(x_3)J^{i_1 i_2 i_3}(\Gamma_1, \Gamma_2, \Gamma_3),$$

(1)

where the quark color indices are contracted by the baryon Junction tensor

$$J^{i_1 i_2 i_3}(\Gamma_1, \Gamma_2, \Gamma_3) = \epsilon_{j_1 j_2 j_3}U^{i_{123}}(\Gamma_1)U^{i_{231}}(\Gamma_2)U^{i_{312}}(\Gamma_3),$$

(2)

that depends on the paths, $\Gamma_i$, connecting the quark at $x_i$ to an intermediate junction vertex point, $x$. All three paths are chromo-field flux lines oriented into the junction vertex as represented by black dots in Fig. 1. Anti-baryons can be constructed similarly with the help of an anti-Junction tensor, $\overline{\mathcal{J}}$, where all the flux lines are oriented away from the vertex. Note that because of the special, $\det U = 1$, constraint on the symmetry group, $SU(3)$, $J^{i_1 i_2 i_3}(\Gamma, \Gamma, \Gamma) = 1$. Thus color singlet states can be constructed from the color tensor links $U(\Gamma)$ not only by tracing and contracting with quark fields but also by contracting with baryon junctions. The paths from a physical junction vertex must be all nondegenerate. Paths are deformable according to Stoke’s theorem only if the background fields are pure gauge artifacts. In the physical, confining vacuum, or in a quark-gluon plasma, different paths correspond to configurations with different energy. In the ground state of a heavy quark baryon, the physical junction vertex ends up in the three quark plane, leading to a Y shaped chromo-field flux field configuration inside the baryon [4].

2 Archimedean polyhedra in QCD

The compelling theoretical arguments in favor of the existence of gauge junction and anti-junctions as inevitable components of the Standard Model led to the prediction [12] of $M^0_J = \text{Tr} \mathcal{J}\mathcal{J} = \epsilon U(\Gamma_1)U(\Gamma_2)U(\Gamma_3)\epsilon$, a new family of glueballs, with masses $O(N_c)$ larger than usual glueballs corresponding to a closed string. In addition, many new exotic states formed by a multitude of quarks and anti-quarks [3,6] were predicted to exist. So far none of these structures have been observed experimentally, probably because the decay widths of these structures is too large, due to their strong coupling to light meson and baryon anti-baryon states. These previously discussed QCD structures are
analogous to carbonic structures with low number of carbon atoms that do not possess any special geometric symmetry.

In high-energy baryon and nuclear collisions, the valence quarks carry a large fraction of the incident baryon’s momentum. Those quarks thus hadronize in the fragmentation regions which are typically within one unit of rapidity, $y = 0.5 \log\left[\frac{(E + p_z)}{(E - p_z)}\right]$, from the kinematic limits. However, baryon junctions invalidate this naive picture of baryon production, since gluons carry on the average only small fraction of the baryon’s momentum. Therefore, junction mechanism of baryon production (via exchange of the $M^0$ Regge trajectory) predicts a much higher probability of finding the conserved valence baryon number many units of rapidity away from the incident baryons [6]. In addition, junction dynamics also naturally predicts [7] a high probability that the valence baryon emerges with multiple strangeness quantum numbers, e.g. $\Xi^-(dss)$, $\Omega^- (sss)$, in the central rapidity region, since the final baryon is made by neutralizing the color of the gluon junction by pair production of quarks and antiquarks with arbitrary flavors. The baryon production data from SPS/CERN [7] and now RHIC/BNL [8] are consistent with these predictions and therefore lend experimental support to the important role that gluon junction dynamics plays in nuclear reactions. From a rehadronizing quark matter baryon junctions may pick up the valence quarks similarly as described by
the quark combinatorics of the ALCOR model that describes the production of multi-strange anti-baryon to baryon ratios at CERN SPS in simple terms [13]. The success of the ALCOR model implementation of quark combinatorics in predicting [14] the multistrange anti-baryon to baryon ratios at RHIC is thus consistent with a junction mechanism for the formation of baryons.

Motivated by Fullerenes, in this Letter we point out the existence of new geometric structures in QCD with high spatial symmetry. We determine the geometric structure and the characteristic “magic numbers” of these configurations, using analogies with carbon Fullere structures. We explore some of the interesting topological structures that can be created by QCD networks and closed cages that may be produced in high energy nuclear reactions joining multiple QCD junctions and anti-junctions. Although the QCD Lagrangian is CP even, we point out that the junction and anti-junction building blocks can be used construct CP odd configurations that may also serve as domain walls between inequivalent ($\theta$) QCD vacua.

In QCD, the orientation of flux lines going into (out of anti-) junctions restricts the set of allowed configurations. In particular, the number of junctions has to be equal with the number of anti-junctions on any closed path formed by the Wilson lines, which implies that QCD Fullerenes may have only even number of vertexes $V$, and a zero net baryon number. Recalling Euler’s formula, the number of faces ($F$), the number of edges ($E$) and the number of vertices ($V$) of a simple (genus 0) polyhedron is related by

$$V + F = 2 + E.$$  \hspace{1cm} (3)

Since each edge is sandwiched between a junction and anti-junction, each face must have an even number of edges. The number of faces,

$$F = N_4 + N_6 + N_8 + ...,$$  \hspace{1cm} (4)

is then a sum of the the number of squares ($N_4$), hexagons ($N_6$), etc. Each edge belongs to two faces :

$$E = (4N_4 + 6N_6 + 8N_8 + ...)/2,$$ \hspace{1cm} (5)

and each vertex belongs to three faces:

$$V = (4N_4 + 6N_6 + 8N_8 + ...)/3.$$ \hspace{1cm} (6)
The resulting Diophantic equations are solved by any number of hexagons and

\[ N_4 - \sum_{i=4}^{\infty} (i - 3)N_{2i} = 6. \]  

(7)

This implies that there is an infinite variety of Fullerene type of structures in QCD, similarly to the case of carbon Fullerenes.

We are particularly interested in the most symmetric geometric structures in QCD, based on the expectation that configurations with the highest geometric symmetry are the most stable ones, similarly to the case of the carbon Fullerenes. If we require that all the vertex positions are equivalent with each other, we have to find the so called Archimedean polyhedra with the constraint that all faces have even number of edges. Archimedean polyhedra can be characterized by the number of vertexes or, equivalently, by their vertex structure \((i, j, k)\) denoting that at each vertex one \(i\)-gon, one \(j\)-gon and one \(k\)-gon is joined. The simplest such geometric structure is the \(V = 8\) cube, with vertex structure \((4, 4, 4)\), denoting that three squares are joined at each vertex. The cube is followed by the \(V = 24\) truncated octahedron, with vertex-face structure \((4, 6, 6)\), denoting that at each vertex one square and two hexagons are joined. Allowing for octagon, and higher faces lead to only two more closed Archimedean polyhedra, \(V = 48\) \((4, 6, 8)\) and \(V = 120\) \((4, 6, 10)\), in which one square and one hexagon are joined to an 8 or 10 sided polygon at each vertex. These are the most symmetric QCD Fullerenes as illustrated in Fig. 2.

Infinite two dimensional tiling or fences can also be created, for example \((4, 8, 8)\), \((4, 6, 12)\) and the J-graphite \((6, 6, 6)\). In addition, as with carbon cages, there are of course many closed structures with less symmetry such as junction \(n\)-prisms \((4, 4, 2n)\) that can be constructed. Here we will not attempt to discuss the dynamics of elementary particle or heavy ion/nuclear collisions that may lead to the formation of QCD J-balls. In nuclear collisions, we simply assume that the observed copious production of baryons and anti-baryons (interpreted as junctions and anti-junctions) in central collisions is sufficient to allow such a configuration to form with finite probability amidst the “nuclear ashes” due to its relatively high binding energy. The carbon Fullerenes \(C_{60}\) and \(C_{70}\) were similarly found within the ashes of laser seared graphite. Another mechanism to create QCD Buckyballs may exist also, that has no analogy in Fullerene chemistry. In particular, high energy collisions of protons and anti-protons at the Fermilab Tevatron accelerator satisfy the conditions for zero net baryon number and high energy density, that are required to excite QCD Fullerenes out of the physical vacuum of strong interactions.

To estimate the relative binding energies of QCD Fullerenes, we consider the simplest model for the relative energy of J-balls consistent with QCD [6]. For a J-ball consisting of \(V/2\) junctions and \(V/2\) anti-junctions connected to
Fig. 2. The family of QCD Fullerenes, \((J_{\perp})_{V/2}\), with magic numbers \(V = 8, 24, 48\) and 120.

form a polyhedron with \(E\) edges with lengths \(l_i\) we take the following model Hamiltonian

\[
H(l_i, n_{v,i}; V, E) = \sum_{i=1}^{E} \left( a \frac{1}{l_i} + \kappa l_i \right) + \gamma \sum_{v=1}^{V} \sum_{i<j=1}^{3} n_{v,i} n_{v,j}
\]

(8)

where \(n_{v,i}, i = 1, 2, 3\) are the three unit vectors pointing away tangent to the edges at vertex \(v\). Implicit above is that the topology is defined by these unit vectors and that the flux tubes are straight lines between vertices. The first term is a “kinetic” or “vertex localization” energy, with coefficient \(a\) that is not yet precisely known. However, one can estimate that \(a \approx \pi \hbar\) from \(\omega = (2 \pi \hbar)/\lambda\) and assuming that \(\lambda = 2l\) that holds in the case of the lowest excitation for a string with two fixed ends. The second parameter of the effective Hamiltonian is the confining string tension, \(\kappa(T) \approx 1\) GeV/fm, a term that vanishes above the deconfinement temperature, \(T_c \approx 150\) MeV. The postulated “strain” term with strength \(\gamma\) is analogous to the Biot-Savart law in circuits and plays the role of bond angle strain in carbon nanostructures. In this model the relative binding energies are determined by the last term although its magnitude is not yet known from lattice QCD. We estimate below the possible range of \(\gamma\) and use these limiting values to give a semi-classical estimate of the mass range of the QCD Fullerenes.
For a vertex and face structure \( V \) and \((n_1, n_2, n_3)\) the total strain energy is

\[
\delta h_V = \frac{\Delta H_V}{V} = -\gamma \left[ \cos \left( \frac{2\pi}{n_1} \right) + \cos \left( \frac{2\pi}{n_2} \right) + \cos \left( \frac{2\pi}{n_3} \right) \right].
\]

(9)

For the \( V = 8 \) J-cube with face structure \((4, 4, 4)\) and \(\Delta h_8 = 0\). For the \( V = 24, 48, 120 \) J-balls, this strain energy per vertex is \(-1, -(1 + \sqrt{2})/2 = -1.207, -(3 + \sqrt{5})/4 = -1.309\) in units of \(\gamma\). The absolute minimum is reached for the junction graphite fence which is bound with \(-3/2\gamma\) per vertex. The \((4, 8, 8)\) and \((4, 6, 12)\) tiles are only bound by \(-\sqrt{2}\gamma\) and \(-0.5(1 + \sqrt{3})\gamma\) per vertex. In contrast, the \(n\) prisms \((4, 4, 2n)\) are bound by \(-\cos(\pi/n) > -1\). Note that the \((J, J)_1 = M^0_j\) is most unfavorable due to its maximum strain energy of \(6\gamma\). The \( V = 24 \) J-ball in Figs. 1, 2 with total strain energy \(-24\gamma\) may be particularly stable not only because of its relatively low strain energy but also due to its topological arrangement of junctions and anti-junctions that increases the potential barrier between adjacent \(J, J\) annihilation. The vertex structure of this \( V = 24 \) QCD Buckyball, \((4, 6, 6)\), is the closest to that of the carbon Buckyball \(C_{60}\) whose vertex structure is \((5, 6, 6)\).

2.1 Semiclassical mass estimates for QCD Fullerenes

Let us find the semi-classical values of the Hamiltonian \(H\) to give an estimate of the expected mass range of the \( V = 8, 24, 48 \) and 120 QCD Fullerenes. Let us first observe that the minimum of the \(H\) Hamiltonian can be determined from requiring that

\[
\frac{\partial H}{\partial l_i} = 0
\]

(10)

for all \(i = 1, ...V\), which implies that all the edges have the same length of

\[
l_i = l_j = l = \sqrt[\alpha]{\kappa} \approx 0.79 \text{ fm}.
\]

(11)

The mass of a QCD Fullerene can be semi-classically approximated by the value of \(H\) at this minimum,

\[
M_V = (\frac{3}{2}\sqrt{\alpha k} + \delta h_V) V
\]

(12)

Hence the mass of these QCD Fullerenes is always proportional to the number of vertexes \( V \) and the constant of proportionality is given by a sum of two terms. The first term is a kinetic term, that can be estimated as \(\frac{3}{2}\sqrt{\pi 0.197}\)
GeV ≈ 1.18 GeV, while the second strain term is a product of a known geometrical contribution and the unknown constant of proportionality $\gamma$. As $a > m_N = 0.940$ GeV, we find that without a strain term the mass of QCD Fullerenes were about 25% higher than that of a system consisting from $V/2$ nucleons and $V/2$ anti-nucleons, hence if $\gamma = 0$ these excitations are most likely unstable.

At what value of $\gamma = \gamma_c$ were at least some of the QCD Fullerenes stable? Including the possibility of tilings, the absolute minimum of the geometrical contribution to the strain term is $\sum_{i<j=1}^{3} n_i n_j = -1.5$, that can be achieved within a graphite like layer. Hence one obtains for the critical value of $\gamma$

$$\gamma_c = \sqrt{a_\kappa} - \frac{2}{3} m_N$$  \hspace{1cm} (13)

which leads to a numerical estimate of $\gamma_c \approx 0.16$ GeV. If in Nature $\gamma > \gamma_c$ then (at least some high mass) QCD Fullerenes are expected to be stable against decay to baryon anti-baryon pairs, while if $\gamma < \gamma_c$ all these objects are unstable for such decays and may exist only as short lived resonance states.

Let us now determine an absolute lower and an upper limit for the strain coefficient $\gamma$. If $\gamma$ were negative, the strings were attracted to each other and the $M_0$ state would be a stable bound state and it would be difficult to explain why this state has not been observed until now. Excluding this possibility, one obtains $0 \leq \gamma$. An upper limit on the possible values of $\gamma$ can be obtained from requiring that even for a graphite like tiling the mass of the Fullerene should be positive, which implies $\gamma < \sqrt{a_\kappa}$. Thus one obtains the following lower and upper limits for the mass of QCD Fullerenes:

$$\frac{3}{2} V \sqrt{a_\kappa} \leq M_V < \left[ \frac{3}{2} V - \cos \left( \frac{2\pi}{n_1} \right) - \cos \left( \frac{2\pi}{n_2} \right) - \cos \left( \frac{2\pi}{n_3} \right) \right] \sqrt{a_\kappa}. \hspace{1cm} (14)$$

Utilizing these limiting values, we obtain Table 1 that summarizes the mass range estimates for the most symmetric QCD Fullerenes utilizing the geometric strain coefficients determined by eq. (9).

Although Table 1 contains order of magnitude estimates only, we can already observe interesting patterns. In particular, the strain coefficient does not influence the mass estimate for the $V = 8$ QCD cube, and the estimated mass is much higher than that of 8 nucleons hence this and all the low lying QCD Fullerenes are expected to be unstable as they are even more strained than the cube. The first reasonable candidate would be a $V = B + \overline{B} = 24$ QCD truncated octahedron. The most stable candidates are expected to be the $V = 48$
Table 1

Estimated mass range for various QCD Fullerenes. $V$ stands for the number of vertexes, $(n_1, n_2, n_3)$ for the face structure at a vertex, $M_{\text{min}}$ and $M_{\text{max}}$ are the estimated lower and upper limits for the mass of the QCD Fullerene, together with the critical mass of stability $M_{\text{crit}}$. The diameter of the circumscribed sphere, $d$ was estimated from $l \approx 0.79$ fm and the geometrical structure.

QCD Great Rombicuboctahedron and the $V = 120$ QCD Great Rombicosidodecahedron. These structures are compact but less strained than similarly compact lower excitations. Their compact structure and their favourable strain term may stabilize all three of them in a large domain of the allowed parameter space.

3 CP odd J-ball states in QCD

The junction $n$-prisms can be regarded as a closed ribbon of $n$ $J\bar{J}$ pairs. Under simultaneous charge conjugation and parity transformation, these prisms are invariant and hence CP even as are all the junction Fullerenes shown in Fig. 2. However, other nontrivial topological configurations can be constructed which are not symmetric under CP. For example an odd number of $J\bar{J}$ pairs cannot be closed into a prism due to the oriented flux at the junctions, but after a twist to right or left can be connected into a Moebius strip. The two Moebius strips transform into each other and thus there exists a linear combination of the two that is odd under CP. Hence QCD $J$-ribbons can be characterized by a single “winding number” ($i$) that gives the number of twists before the ribbon is closed on itself. The topology of the excitations of QCD seems to be very interesting, because not only ribbons but also tubes can be formed. The ends can be closed with caps formed by squares, octagons and decagons, satisfying eq. (5), or can be open, ending on valence quarks. The QCD femto-tubes are analogous to the carbon nano-tubes, both may have interesting chiral properties. As carbon nano-tubes, the QCD femto-tubes can be characterized by two integers $(i, j)$, which gives the number of steps in the direction of the lattice vectors, that connect equivalent points on the surface of the tubes. Another interesting possibility is to close the $J$-tube on itself, creating a toroidal structure. The femto-tubes can be closed by connecting the two ends of a
long tube, and these ends can be rotated before the connection. This gives QCD femto-tori that can be characterized by 3 winding numbers, the \((i, j, k)\) femto-tori.

4 Summary

Fullerene type of pure glue topological configurations can be constructed in QCD. These “J-balls” are QCD femto-structures with the highest geometrical symmetry. All of the QCD Fullerenes have an equal number of junctions and anti-junctions, and may have specific geometrical and topological properties. The QCD Buckyballs are CP even, other QCD structures such as linear combinations J-Moebius ribbons can be constructed that are CP odd. Topological winding numbers can be introduced to characterize these states. The QCD femto-ribbons are characterized by a single integer \((i)\), the femto-tubes by a pair of integers \((i, j)\), while the QCD femto-tori by a triplet of integers, \((i, j, k)\).

We determined that the most symmetric (likely most stable) QCD Buckyball configurations have the magic numbers of baryons + anti-baryons \(B + \overline{B} = 8, 24, 48\) and \(120\). Although these configurations are likely unstable, they are expected to be more stable than clusters of baryons and anti-baryons with different junction numbers, and they may appear as peaks in the spectrum of \((B\overline{B})_n\) clusters with a given total baryon+antibaryon number. To create them, high initial energy densities and small net initial baryon number densities and large volumes are needed. Such conditions may exist in the mid-rapidity domain of central \(Au + Au\) collisions at RHIC or LHC as well as in diffractive collisions of protons and anti-protons at the Fermilab Tevatron accelerators.

We suggest to search for clusters of baryons and anti-baryons with multiparticle correlation patterns of the vertices of J-balls in rapidity slices. In addition, searches for CP violating domains at RHIC should look for unusual baryon anti-baryon correlations suggested by our J-Moebii structures. Baryon junction and anti-junction networks may also help to understand the structure of domain walls between different CP vacua in QCD.

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