Quantum Fragmentation

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Abstract. Phenomenological and theoretical aspects of fragmentation for elementary particles (resp. nuclei) are discussed. It is shown that some concepts of classical fragmentation remain relevant in a microscopic framework, exhibiting non-trivial properties of quantum relativistic field theory (resp. lattice percolation).

Introduction

At first sight, fragmentation appears to be a typical classical process [1], by contrast with a quantum (or microscopic) one. You take a “big” object of size $A$, you break it into pieces of different sizes and – if you are a scientist – you count the mean number $N_j$ of pieces of size $j$ after a certain number of events. In rather simple but general cases [2], one may introduce a time-dependent description in terms of successive binary splittings which can be formulated in terms of a “gain-loss” equation, namely:

$$\frac{dN_j}{dt} = \sum_{k=j+1}^{A} W_{jk} N_k - C_j N_j ,$$

(1)

where $W_{jk}$ is the binary fragmentation weight of an intermediate fragment of size $k$ into $j$ and $k-j$ and,

$$C_j \equiv \frac{1}{2} \sum_{\ell=1}^{j-1} W_{\ell j}$$

(2)

represents the total loss rate for the fragment species of size $j$. Indeed, there exist well-known papers [3] which discuss various mathematical solutions of equations of the type (1-2), with applications to e.g. depolymerisation, breaking of clusters etc...
For fragmenting quantum objects, such as elementary particles or nuclei, however, classical fragmentation concepts and equations are not necessarily relevant. In these cases, the fragmenting “big object” and the parameter $A$ correspond to a particle jet and its total energy before its fragmentation into pions and other particles. For nuclei, it is an excited nucleus of atomic mass $A$ fragmenting into smaller ones (including individual nucleons or $\alpha$ particles etc...). Note that in the former case, one has to replace the summation by an integral in equations (1-2), as also considered in Refs.[3].

There are some basic obstacles in front of us if we want to discuss quantum fragmentation in terms of classical concepts. For instance, for elementary particles and nuclei, the excited quantum state which characterizes the system before its fragmentation is governed by the same interactions (called virtual) than those (called real) responsible for its subsequent fragmentation. It is thus by no means obvious that the classical fragmentation structure could emerge from the quantum environment of the process. For instance, in field theory, the quantum fluctuations (e.g. loop Feynman diagrams) and more generally the renormalization procedure (necessary to give a realistic meaning to the perturbative calculations) do not necessarily lead to an equation like (1). Quite unexpectedly, equation (1) is however useful for Quantum Chromodynamics (QCD), the theory of the fundamental strong interactions between quarks and gluons, as we shall see further on.

Our aim in this contribution is to give a brief survey of how one can cast a bridge between quantum and classical concepts of fragmentation in this context. We will briefly analyze two specific cases where the phenomenological analysis can be supported by a theoretical model. In section 1, Quark jet fragmentation will be analyzed in the framework of Quantum Chromodynamics (QCD). In section 2, Nuclear multifragmentation will be discussed in the framework of 3-dimensional lattice percolation.

1. Quark Jet Fragmentation

The best occasion where we can observe and measure quark jet fragmentation is the $e^+e^-$ annihilation into hadrons (mainly pions and their decay products) at high energy when an intermediate $Z^0$ boson is formed, thereafter decaying into quark-antiquark ($q\bar{q}$) pairs. Such experiments have been performed at the LEP accelerator at CERN.

During such reactions, in a first step lasting less than $10^{-24}$ second, an intermediary $Z_0$ boson is formed and decays into a $q\bar{q}$ pair, often followed by the subsequent formation of a third (gluon) jet. These jets form many gluons and other $q\bar{q}$ pairs. This stage is well described by QCD calculations with a small effective coupling constant[4] and can thus be studied in a quantitative theoretical framework. Then, in a later stage of the fragmentation process, quarks and gluons recombine into hadrons in an unknown way, only described by modelization. The deep fundamental reason of this is that $\alpha_S$, the effective coupling constant of QCD, happens to be time-dependent as a consequence of quantum fluctuations. One has

$$\alpha_S = \frac{1}{b} \left[ \log \frac{Q}{\Lambda_{\text{QCD}}} \right]^{-1},$$  

(3)
where \( b, \Lambda_{\text{QCD}} \) are fundamental constants and \( Q \sim 1/\text{Time} \). One says that the coupling constant is “running” in this theory, being small at short times and becoming large later on. One also speaks of “asymptotic freedom” when time is short and “infrared slavery” at long times, since the elementary quanta, \( q, \bar{q}, g \), (called partons) are quasi free at short times and become tightly bound at long times and confined into hadrons. In this limit, the theory is in a non-perturbative regime and its complete solution is not yet known.

At short times, the predominance of quantum fluctuations and virtual interaction effects makes difficult a classical fragmentation picture of a quark jet. However such a description emerges from the calculations after using a set of non-trivial properties of QCD, the field theory of Gauge Fields describing strong interactions between partons. This theory possesses an internal symmetry group, local in space-time, which is the “color” group \( \text{SU}_3 \). We have no place for giving the full derivation of the gain-loss equation associated with QCD, but let us only describe it. The main issue lies in a system of equations which take the form of a continuous “gain-loss” expression similar to (1,2).

One writes:

\[
\frac{dD^B_A(z)}{d\xi} = \sum_C \int_0^1 \frac{dx}{x} P^C_A(x) \left\{ D^B_C(z/x) - x^2 \delta^C_A D^B_A(z) \right\},
\]

(4)

where \( D^B_A \) is the probability distribution of finding a “quantum” \( B \) or parton \( (g, q \text{ or } \bar{q}) \) in the fragmentation of the initial “quantum” \( A \) \( (g, q \text{ or } \bar{q}) \), with the fraction \( z \) of its total momentum. Note that Eq. (4) can be obtained, after some manipulation, from a continuum limit of (1) by choosing:

\[
\frac{1}{j} N_j = D(j/A); \quad W_{jk} = \frac{1}{k} P(j/k); \quad t = \xi.
\]

Following the analogy with the classical process obeying Equations (1),(2), one may interpret the first term in the integral as a “gain” term where the parton \( B \) is obtained via first fragmentation of \( A \) into an intermediate parton \( C \). Standard QCD calculations give a specific prediction for the weights \( P^C_A(x) \) and thus for the solution of equation (4). Note that the theory also leads to a precise re-definition of the “evolution” variable \( \xi \), namely

\[
\xi \equiv \int_Q^{Q_{\text{max}}} \frac{\alpha_s(Q) dQ}{2\pi} = \frac{1}{2\pi b} \ln \left( \frac{Q_{\text{max}}}{\Lambda_{\text{QCD}}} \right) \ln \left( \frac{Q}{\Lambda_{\text{QCD}}} \right).
\]

(5)

From various experimental analyses it has been shown that equation (4) gives a good description of the energy spectrum of jet fragmentation. However a model-dependent piece of information has to be added since one measures hadrons and not partons in the final state. In other physical configurations (structure functions instead of fragmentation) the same equation holds and can be tested with great success [4].

Note the interesting property of equation (4) that it can be exactly solved by the method of moments (or Mellin-transform). Introducing the moments \( [M_q]_A^B \equiv \int_0^1 dz \ z^q \ D^B_A(z) \), one gets in matrix form:

\[
[M_q] \equiv \exp \{ \xi [H_q] \},
\]

(6)
where the matrix elements $[H_q]_{BA}^B$ are the $q$–moments of the weights $P_{BA}^B$.

The emergence of a tree structure in jet fragmentation is not only based on the energy spectrum given by equation (4). Many other observables lead to the same structure, while the detailed analyses show that it always implies a non-trivial property of both the quantum and group invariance properties of the theory. As an illustration, it was recently shown [5] that the multiplicity fluctuations associated with jet fragmentation possess a dynamical structure, similar to the intermittency phenomena in hydrodynamics, which was predicted in particle physics some time ago [6]. This brings an interesting analogy between quantum fragmentation and intermittent fragmentation models, which appear in various classical or non classical systems such as spin-glasses, polymer diffusion, multi-particle production [7].

Note, however, that “differential” fragmentation observables beyond the energy spectrum and the functions $D_A^B(z)$ are more dependent on the unknown “hadronization” phase of partons and thus at present more model-dependent. The research is going on in this field.

2. Nucleus Multifragmentation

The physical understanding of nuclear multifragmentation is much less advanced than in QCD jet fragmentation. Experimentally, it is only recently that systematic data on the decay products of fragmented nuclei hence become available thanks to 4π–detectors at nuclear accelerators [8]. Even then, the difficulty remains of specifying without ambiguity the excited system which multifragments, and separating its fragments from the pre-equilibrium particles. Theoretically, the kinematical conditions of nuclear multifragmentation, e.g. the incident energy and the multicomponents of the final state, are far from a known regime of nuclear forces. One has thus to rely on models, which are useful for the experimental investigation and may lead to a deeper understanding of the multifragmentation phenomenon.

Among the proposed models, let us choose and discuss the one based on 3-dimensional percolation on a finite lattice[9,10]. Due to its particular simplicity, though unexplained on a purely nuclear-theoretical framework, it will allow us to develop on the links between a microscopic description and classical fragmentation concepts. This part of the talk comes from a recent study done in collaboration with Bertrand Giraud in Saclay[11].

In the 3–d percolation model, the excited nuclear system of mass $A$ (in nucleon mass units) is modeled by a finite lattice of volume $A$ with nucleon on sites and nearest-neighbour bonds. Multifragmentation results from a breaking of these bonds in proportion of the energy release in the system by the reaction. Event-by-event some bonds are randomly preserved, corresponding in average to a ratio $p$, ($0 < p < 1$) of all bonds, while a certain number of fragments are formed, giving rise to a statistical distribution of fragments as a function of their size $i$, ($1 < i < A$). This distribution is in good agreement with recent data, if one replaces the unknown parameter $p$ by an observable input for each event, e.g. the multiplicity of fragments. Another parameter is introduced corresponding to a site occupation probability, but we will stick to the
Our aim[11,12] is to answer the following question: is it possible to find a classical
time-dependent description of multifragmentation which would give, at least within
some approximation, the same prediction than 3–d percolation concerning the distribu-
tion of fragments? In some sense, we are looking for an eventual restoration of the
time variable in the percolation problem where such a reference scale is absent. More-
over, the question behind this is whether percolation could be described by a linear set
of equations similar to Eqns. (1-2). If such is the case, one could look for new scaling
laws, in much the same way as in the case of jet fragmentation where they correspond
to scaling properties of QCD.

Technically, our work[11] starts with the quest of a general, albeit approximate,
solution of the gain-loss equations (1-2). The idea is to exhibit properties which would
be independent of the particular choice of weights $W_{jk}$, (which we do not yet know for
percolation). Then, one looks for the same properties in some range of the percolation
model to test an eventual compatibility. Our conclusion is that indeed this compat-
ibility can be achieved during the short time evolution of the system. Let us sketch
how this can be proven.

As any such linear system, the solution of Eqns (1-2), is obtained via exponentiation
once the eigenvalues and eigenvectors of the matrix $[W_{jk} - C_j \delta_{jk}]$ are determined. Note
that the matrix is triangular, and thus the elements $C_j$ on the diagonale are the exact
eigenvalues. However the eigenvectors are unknown, except that they form also a
triangular matrix. In fact, we were able to prove that these vectors take the quite
general form of \textit{eigenmoments}, namely they are of the form $M_{q(j)}$, where $M_q$ is the
moment of rank $q$ of the mean distribution of fragments and $q(j) \geq 1, j = 1, ..., A$ are
particular, but not necessarily integer, values of the rank. One has:

$$q(1) \equiv 1 < q(2) < q(3) ... < q(A).$$

This property is obtained by inspection of large matrices for which the problem is
very similar to the QCD case (see Eq. (4-6)) where the moments $M_q$ give an exact
diagonalization in the space of particle momentum. What was verified also for matrices
of limited size, is that the diagonalization by moments remains true for a discretized
set of values of the rank, a set $q(j)$. Note however the model dependence of the set
$q(j)$ except for $q(1) \equiv 1$ which is dictated by mass conservation. A limitation of the
method was found in the case of the so-called “shattering transition”, see[3], which
needs a special treatment. With these limitations in mind, the eigenmoment property
is general enough to be tested, e.g. in the case of percolation.

For this purpose, we remark that, if they are identified as eigenmoments, the $M_q$’s
are linked by linear relations in Log-Log plots, and their explicit time dependence
disappears. We are thus led to display in the same way the moments obtained from
the percolation model, choosing for instance $M_2$ for reference, see the figure. Different
moments are displayed (with $q = 1, 1.5, 2, 3, 4, 5$) and show the interesting feature of a
quasi-linear dependence for the values $q = 3, 4, 5$, given the fact that for $q = 1$ (mass
conservation) and $q = 2$ (reference scale) the linear relation is trivial. It is clear from
this figure that a quasi-linear form is obtained between $p = 1$ and $p = p_c$, where $p_c$ is
the critical value above which, in the continuous limit, an \textit{infinite} percolation cluster
is formed. Indeed, the figure shows the dominant contribution of the cluster of largest mass to the averaged moments. This largest cluster is, for finite size problems, the representative of the infinite cluster when $p \geq p_c$.

Notice that the moments implied by the rate equations are the full moments, including the largest fragment, while in usual analyses of percolation models\cite{9,10}, scaling properties are investigated with moments modified by the subtraction of the largest cluster. Moreover, in such traditional analyses of percolation, the reference time scale is generally given by the moment $M_0$ or a similar variable related to the multiplicity of fragments. The comparison and compatibility of our approach with such analyses is an open problem of some interest.

**Conclusion and prospects**

The problem we want to discuss in conclusion of the study of the particle and nuclei fragmentation is whether classical concepts of fragmentation could serve as an unifying phenomenological picture at the microscopical level where quantum states and fields are involved. For this sake, let us discuss the striking common features and differences between the two examples we have treated.

In the case of elementary particles and field theory, it is known that only the short time evolution of a jet is accessible to perturbative calculations. More precisely, it is only the time derivative of the fragmentation functions (or momentum distributions of partons) which can be calculated exactly at first order in the quantum loop expansion. At any given fixed time, however, the knowledge of these functions depend on an expansion at all orders (for which the renormalization group properties can be invoked\cite{4}). At long times it involves the unknown transformation of partons into hadrons. In other terms, the change in the vacuum structure (from partons to hadrons) prevents one from a complete theoretical understanding of the fragmentation process.

In the case of nuclear multifragmentation, one does not possess a comparable theoretical framework. However, the indications from the 3–d percolation model shows that there could be a similar property at short times, namely the possibility of a multilinear evolution of fragmentation. This time range corresponds to the situation when the fragmenting force is mild enough to preserve the existence of at least one large cluster, that is when $p \leq p_c$. What seems to be remarkable is the complementarity of percolation with the previous case. Percolation is well determined in a given region of the parameter, namely near the critical percolation value $p_c$. In other terms, fragmentation is better determined at a given ”time”, while its ”time-dependence”, e.g. the relation between $p$-- and time-evolution is not trivial. This is just the opposite of the field theoretical case. It is tempting to confront the methods used in the two cases in such a way that the stated complementarity could hopefully be used to explore the shadow regions of both processes.

In conclusion, if fragmentation concepts could acquire some kind of universality, one can hope to find new methods to overcome the difficulties of the physical description of fragmentation in the quantum world. Some property of nuclear multifragmentation
could be useful for the hadronization problem of parton jets, as well as perturbative methods of Quantum Field Theory could help solving some nuclear fragmentation puzzles. More work in these directions is deserved.

Acknowledgments Thanks are due to the organizers of the Workshop on Fragmentation for the remarkable atmosphere they provided for discussions and exchanges between participants coming from very different fields.

References

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Percolation analyzed with the $M_2$ time scale

Relative strengths of moments $M_q$, $q = 0, 1, 1.5, 2, 3, 4, 5$, as functions of $M_2$ in a Log-Log plot. Data taken from 3-d bond percolation on a $6\times6\times6$ lattice. The corresponding values of the bond survival probability $p$ are shown on the horizontal axis. Its critical value is $p_c = .25$. Full lines: moments. Dashed lines: contributions of the largest cluster. Dashed-dotted line: the reference moment $M_2$. Notice that a linear behaviour is approximately obtained for $0 \leq p \leq p_c$ and $q = 3, 4, 5$ (Figure from [11]).