DISSIPATION IN OPEN TWO–LEVEL SYSTEMS:
PERTURBATION THEORY AND POLARON TRANSFORMATION

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We compare standard perturbation theory with the polaron transformation for non–linear transport of electrons through a two–level system. For weak electron–phonon coupling and large bias, there is good agreement between both approaches. This regime has recently been explored in experiments in double quantum dots.

1 Introduction

Semiconductor coupled quantum dots are well–defined artificial few–particle systems for the study of electron–electron correlations, quantum coherence, and quantum dissipation. These effects become visible in non–linear electron transport on an μeV energy scale when parameters like the interdot coupling and the coupling to external leads can be controlled by, e.g., external gate–voltages.

Double quantum dots in the regime of strong Coulomb blockade can be tuned into a regime that is governed by a (pseudo) spin–boson model (dissipative two–level system)

\[ H = \frac{\varepsilon}{2} \sigma_z + T_c \sigma_z^A + \sum_Q \omega_Q a_Q^\dagger a_Q, \quad A := \sum_Q g_Q \left( a_{-Q} + a_{Q}^\dagger \right), \quad (1) \]

where one additional ‘transport’ electron tunnels between a left (L) and a right (R) dot with energy difference \( \varepsilon \) and inter–dot coupling \( T_c \), where \( \sigma_z = |L\rangle\langle L| - |R\rangle\langle R| \) and \( \sigma_x = |L\rangle\langle R| + |R\rangle\langle L| \). Here, \( \omega_Q \) are the frequencies of phonons, and the \( g_Q \) denote interaction constants. Although not exactly solvable, the model is quite well understood for closed systems (isolated dots with one additional electron). The coupling to external leads offers the possibility to study its non–equilibrium properties, such as the inelastic stationary current through the dots.

2 Equations of Motion

We describe the dynamics of the double dot by a reduced statistical operator \( \rho(t) \), allowing for an additional ‘empty’ state and tunneling from a left reservoir at rate \( \Gamma_L \) into the left dot, and from the right dot to the right reservoir at rate \( \Gamma_R. \) Lowest order perturbation theory in these rates yields

\[ \frac{\partial}{\partial t} \rho_{LL}(t) = -i T_c [\rho_{LR}(t) - \rho_{RL}(t)] + \Gamma_L [1 - \rho_{LL}(t) - \rho_{RR}(t)] \]
\[ \frac{\partial}{\partial t} \rho_{RR}(t) = -i T_c [\rho_{RL}(t) - \rho_{LR}(t)] - \Gamma_R \rho_{RR}(t). \]
For the remaining equation for the off–diagonal element \( \rho_{LR} = \rho_{RL}^\ast \), one has to choose between perturbation theory in \( g_Q \) (weak coupling, PER), or in \( T_c \) in a polaron–transformed frame (strong coupling, POL). In general, no exact solution of the model is available: this is the case even for coupling to one bosonic mode only (\( g_Q \propto \delta_{Q,Q_0} \), Rabi Hamiltonian).

For the spin–boson problem with \( \Gamma_{R/L} = 0 \), it is well–known that POL is equivalent to a double–path integral ‘non–interacting blip approximation’ (NIBA) that works well for zero bias \( \epsilon = 0 \) but for \( \epsilon \neq 0 \) does not coincide with PER at small couplings and low temperatures. In the following, we compare both approaches for \( \Gamma_{R/L} \neq 0 \) and find nearly perfect agreement for very large \( \epsilon \gg T_c \), a regime that has been tested experimentally recently.

The standard Born and Markov approximation with respect to \( A \) yields

\[
\frac{d}{dt} \rho_{LR}^{\text{PER}}(t) = [i\epsilon - \gamma_p - \Gamma_R/2] \rho_{LR}(t) + [iT_c - \delta_-] \rho_{RR}(t) - [iT_c - \delta_+] \rho_{LL}(t).
\]

Here, the rates are

\[
\gamma_p := 2\pi T_c^2 \rho(\Delta) \coth (\beta \Delta/2), \quad \delta_{\pm} := -\frac{\epsilon T_c}{2} \rho(\Delta) \coth (\beta \Delta/2) \mp \frac{\pi T_c}{2} \rho(\Delta) \coth (\beta \Delta/2).
\]

\[
\rho(\omega) := \sum_Q |g_Q|^2 \delta(\omega - \omega_Q).
\]

(2)

where \( \Delta := \sqrt{\epsilon^2 + 4T_c^2} \) is the energy difference of the hybridized levels, and \( \beta = 1/k_B T \) the inverse phonon equilibrium bath temperature. Note that beside the off–diagonal decoherence rate \( \gamma_p \), there appear terms \( \propto \delta_{\pm} \) in the diagonals which below turn out to be important for the stationary current.

On the other hand, the polaron transformation leads to an integral equation

\[
\rho_{LR}^{\text{POL}}(t) = -\int_0^t dt' e^{i\epsilon(t-t')} \left[ \frac{\Gamma_R}{2} C(t-t') \rho_{LR}(t') + iT_c \{ C(t-t') \rho_{LL}(t') - C^*(t-t') \rho_{RR}(t') \} \right],
\]

where

\[
C(t) := \exp \left\{ -\int_0^\infty d\omega \rho(\omega) \omega^2 [ (1 - \cos \omega t) \coth (\beta \omega/2) + i \sin \omega t ] \right\}.
\]

(3)

3 Stationary Current

If an electron tunnels between two lateral dots, the interaction with 3d piezoelectric acoustic phonons leads to an orthogonality catastrophe (‘boson shake up effect’) which is determined by an effective phonon density of states

\[
\rho(\omega) = g\omega \left( 1 - \frac{\omega d}{c_s} \sin \left( \frac{\omega}{\omega_d} \right) \right) e^{-\omega/\omega_c} \theta(\omega),
\]

(4)

showing oscillations on a scale \( \omega_d := c_s/d \), where \( c_s \) is the speed of sound, \( d \) the distance between the centers of the two dots, \( g \) the dimensionless coupling constant,
and $\omega_c$ a high–energy cut–off. We argue that the assumption of sharply localized positions between which the additional electron tunnels is justified by the strong intra–dot electron–electron repulsion.

We compare results for the stationary electron current $I_{\text{stat}} = -e2T_c \text{Im} \hat{\rho}_{LR}(z = 0)$ as obtained from Laplace transforming the equations of motion within both PER and POL for small electron–phonon coupling. The result

$$I_{\text{stat}} = \frac{-e\Gamma_L \Gamma_R G_+}{\Gamma_L G_- + (\Gamma_L + \Gamma_R)G_+ - \Gamma_L \Gamma_R}.$$  

has to be used with either

$$G_+^{(\text{PER})} := 2T_c \text{Im} \frac{iT_c - \delta_+}{i\varepsilon - \gamma_p - \Gamma_R/2},$$

or

$$G_+^{(\text{POL})} := 2T_c \text{Im} \frac{-iT_c C_\varepsilon}{1 + (1/2)\Gamma_R C_\varepsilon}, \quad G_-^{(\text{POL})} := 2T_c \text{Im} \frac{-iT_c C^*_{\varepsilon}}{1 + (1/2)\Gamma_R C_{\bar{\varepsilon}}},$$

where $C_\varepsilon := \int_0^\infty dt e^{i\varepsilon t} C(t)$.

The function $C_\varepsilon$ is obtained for arbitrary large coupling constant $g$ by a double numerical integration. For our purpose here, it is sufficient to expand $C_\varepsilon$ as

$$C_\varepsilon = \frac{i}{\varepsilon} + \gamma(\varepsilon) + O(g^2), \quad \gamma(\varepsilon) := \frac{\pi \rho(|\varepsilon|)}{2} \left[ \frac{\varepsilon^2}{\varepsilon^2} \right] \left[ \coth(\beta |\varepsilon|/2) + \text{sgn}(\varepsilon) \right].$$

4 Discussion

In both PER and POL the current Eq. (5) is an infinite sum of contributions $G_+ (= I_{\text{stat}}/e$ in lowest order in $T_c$) and $G_-$ and therefore to infinite order in the inter–dot coupling $T_c$. Note that both expressions for $G_\pm$ coincide for vanishing electron–phonon coupling $g$.

We first point out that PER works in the correct eigenstate base of the hybridized system (level splitting $\Delta$), whereas the energy scale $\varepsilon$ in POL is that of the two isolated dots ($T_c = 0$). This difference reflects the general dilemma of two–level–boson Hamiltonians: either one is in the correct base of the hybridized two–level system and perturbative in $g$, or one starts from the ‘shifted oscillator’ polaron picture that becomes correct for $T_c = 0$. In fact, the polaron (NIBA) approach does not coincide with standard damping theory because it does not incorporate the square–root, non–perturbative in $T_c$ hybridization form of $\Delta = \sqrt{\varepsilon^2 + 4T_c^2}$.

However, for large $|\varepsilon| \gg T_c$, $\Delta \to |\varepsilon|$, and POL and PER should coincide. This is indeed the case for small $g$ where

$$-G_\pm \to 2T_c^2 \frac{\Gamma_R/2 + \gamma(\pm \varepsilon)}{\Gamma_R/4 + \varepsilon^2}.$$  

Thus, for large $|\varepsilon|$, the polaron approach works well even down to very low temperatures and small coupling constants. In particular, in the spontaneous emission regime of large positive $\varepsilon$ the agreement is very good. This regime is most interesting at low temperatures $T$ and has been tested in the non–linear transport experiment in great detail.
In Fig. 1, we compare both approaches and show the calculated stationary current, using realistic parameters. In the vicinity of $\varepsilon = 0$, the agreement between POL and PER gets worse as could be expected from the above argument. At higher temperatures, the agreement gets slightly better on the absorption side $\varepsilon < 0$. We conclude that our findings for the ‘open’ spin–boson model are in agreement with standard spin–boson physics. Furthermore, we see that for large bias $|\varepsilon| \gg T_c$ both perturbation theory and the polaron transformation approach practically coincide for small coupling $g$.

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