Impact of aspect ratio on a nanofluid-saturated porous enclosure

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Abstract. The aim of the present numerical simulation is to investigate the mixed convection flow and heat transfer characteristics in a two dimensional porous cavity filled with nanofluid for different aspect ratios. The top and bottom walls are assumed to have the uniform temperatures $\theta_0$ and $\theta_\infty$, respectively, with $\theta_0 > \theta_\infty$ while the vertical walls are kept to be adiabatic. The governing equations are solved by using finite volume method with the SIMPLE algorithm. The variations of isotherms, streamlines, mid-plane velocity profiles, and Nusselt numbers were discussed in detail over a wide range of pertinent parameters, viz., aspect ratio, Richardson number, Darcy number, and solid volume fraction. Although the addition of nanoparticles in the porous medium is to increase the overall heat transfer rate in most flow regimes, the overall heat transfer rate is appeared to decrease slightly or stay nearly the same with the increase of solid volume fraction for $Ar = 0.25$ and 0.5 porous cavities in the forced convection regime.

Keywords: aspect ratio / mixed convection / porous cavity / nanofluid

1 Introduction

The problem of convective heat transfer in porous media has been major topic for research studies due to its fundamental nature and the wide spectrum of engineering applications such as geothermal energy systems, oil recovery, subsoil water filtration, nuclear waste storage, and chemical separation processes. Numerous studies have been done on convective heat transfer inside the porous enclosures. Khanafer and Chamkha [1] studied laminar mixed convection flow in an enclosure filled with a fluid-saturated porous medium. Their results demonstrated the significant suppression of the convective currents in the presence of a porous medium. Jue [2] performed a numerical study on thermal convection flow in a fluid-saturated porous cavity with internal heat generation. He has summarized that a proper adjustment of the porous media is to control the heat transfer rate as well as the direction of heat transfer. Partially cooled and inclined porous rectangular enclosures were studied by Oztop [3]. It is concluded that inclination angle and aspect ratio are the dominant parameters on heat transfer and fluid flow in the porous enclosures. Misirlioglu [4] conducted a numerical study to investigate the mixed convection flow in a square cavity filled with porous medium. It is found that rotation of circular cylinder inside the cavity is more effective in the forced convection regime, and the heat transfer is almost independent from the Darcy number at high spin velocities. Vishnuvardhanarao and Das [5] numerically investigated the mixed convection flow in a parallel two-sided lid-driven differentially heated square porous enclosure. The average Nusselt number attains the value of 1 asymptotically when the Richardson number is gradually increased for Gr up to $10^3$. Chamkha and Al-Mudhaf [6] have performed numerical investigation of double-diffusive natural convection in inclined porous cavities with various aspect ratios and temperature dependent heat source or sink. It is found that the fluid flow and heat transfer inside the porous enclosure depend strongly on the buoyancy ratio, cavity inclination angle and the heat generation or absorption effects. Basak et al. [7] studied on mixed convection flows in a cavity filled with porous medium with linearly heated side walls. Their results have shown that the average Nusselt numbers of bottom, left and right walls of the cavity are found almost invariant with Gr for low Pr with all Da for linearly heated side walls or cooled right wall.

Since the fluids such as water, mineral oils, and ethylene glycol have a rather low thermal conductivity and do not meet the growing demand in the heat transfer exchange, it is required to develop the new types of fluids, which should be more effective in heat transfer performance. Nanofluids, base pure fluids with suspended metallic nanoparticles, are such a new kind of fluids to fulfill the rising demand in the heat transfer exchange. Khanafer and Vafai [8] presented a numerical study on heat transfer enhancement in a 2-D enclosure utilizing nanofluids. It is observed that the structure of the fluid flow is altered in the presence of...
nanoparticles. Convective heat transfer in 2-D rectangular cavity utilizing nanofluids was investigated by Jou and Tzeng [9]. They have shown that the overall heat transfer rate is increased with the increase in the buoyancy parameter and volume fraction of nanofluids. Hwang et al. [10] theoretically studied the thermal characteristics of natural convection in a rectangular cavity filled with a water-based nanofluid containing alumina. Ho et al. [11] observed that the heat transfer in a square cavity filled with nanofluids could be enhanced or diminished depending on the formulas used to estimate the dynamic viscosity of the nanofluid. Numerical simulations on heat transfer due to mixed convection with nanofluid have been investigated by Mansour et al. [12]. The investigation concluded that even though an increase in solid volume fraction is to decrease both of activity of the fluid motion and fluid temperature, the overall heat transfer is augmented on increasing the solid volume fraction. Muthamilselvan et al. [13] conducted a numerical analysis on the heat transfer enhancement of copper-water nanofluids in a lid-driven cavity. It is observed that both the aspect ratio and solid volume fraction affect the fluid flow and heat transfer rate in the enclosure. Ghasemi and Aminossadati [14] have reported a numerical study on convective heat transfer in triangular cavity filled with nanofluids. Their results have shown that a stronger flow circulation within the enclosure is observed when the motion of sliding wall is downward and hence, the higher heat transfer rate is obtained.

Even though several numerical studies of convection in heated enclosure containing nanofluids or porous medium are published, most of them concentrate separately on nanofluids and porous medium in cavities, and only a few of them consider the porous medium filled with nanofluids. Nield and Kuznetsov [15] studied the Cheng and Minkowycz's problem [16] for natural convective boundary layer flow of nanofluid in a porous medium. In their study, for the nanofluid the effects of Brownian motion and thermophoresis are incorporated and for the porous medium the Darcy model is used. Kuznetsov and Nield [17] developed a theory of double-diffusive nanofluid convection in porous media. They predicted that for the case when Soret and Dufour parameters are negligible, the non-oscillatory mode is expected when $\text{Ra}_n$ is positive, a situation which physically corresponds to the fact that for oscillations to occur two of the buoyancy forces have to be in opposite directions. Ahmad and Pop [18] reported on the steady of mixed convection boundary layer flow past a vertical flat plate embedded in a porous medium filled with nanofluids. Mittal et al. [19] performed a numerical study on mixed convection flow in porous medium filled nanofluid. Their numerical simulations have shown that a higher heat transfer rate is obtained by adding the nanoparticles to base fluid in the porous medium. Nguyen et al. [20] made a numerical analysis on laminar natural convective flow in non-Darcy porous enclosure filled with nanofluid. Their results have revealed that increasing the solid volume fraction decreases the overall heat transfer rate in Darcy flow regime at a high Rayleigh number and low Darcy number. Zhang et al. [21] presented a numerical study on MHD flow and radiation heat transfer of nanofluids in porous media with the effects of variable surface heat flux and chemical reaction. It is found that velocity and temperature fields are strongly affected in the presence of magnetic field and radiation effects.

Only few of published works focused on momentum and energy transfer in porous cavity filled with nanofluids. Complementary works are needed to understand the fluid flow and heat transfer characteristics of nanofluids in porous cavity. It is also found from the review of the above literature that mixed convection flow in porous cavity filled with nanofluid with various aspect ratios is not reported so far. To fulfill the above need, the present study aims to examine the laminar mixed convection flow of a nanofluid in porous cavity with different types of aspect ratios. Specifically, the aim of present work is to examine numerically the effects of Darcy number, Richardson number, aspect ratio, and solid volume fraction of nanoparticles.

2 Mathematical analysis

Figure 1(a) presents a schematic diagram of physical configuration of the present study. The system is considered to be steady, laminar, incompressible mixed convective flow and heat transfer in 2-D square porous cavity of height $H$ and length $L$ filled with nanofluid. The top and bottom walls are assumed to have the uniform temperatures $\theta_L$ and $\theta_U$, respectively, with $\theta_L > \theta_U$. The side walls are considered to be thermally insulated. The velocity components $u$ and $v$ are taken in $x$ and $y$ directions respectively. No slip condition is assumed between the fluid and two vertical walls. Further, the tangential fluid velocity is considered to equal the moving wall velocity. The top wall of the cavity is moving from left to right with constant speed $U_0$. The porous cavity is filled with nanofluid, which is made of a base fluid (water) and spherical nanoparticles (Cu). The fluid in the cavity is Newtonian and incompressible. The porous medium is taken as hydrodynamically and thermally isotropic and
Table 1. Thermophysical properties of water and nanoparticles.

|          | ρ (kg m⁻³) | Cₚ (J kg⁻¹ K⁻¹) | k (W m⁻¹ K⁻¹) | β × 10⁻⁵ (K⁻¹) |
|----------|------------|-----------------|---------------|----------------|
| Water    | 997.1      | 4179            | 0.613         | 21             |
| Copper Cu | 8933       | 385             | 401           | 1.67           |

Table 2. Applied formulae for the nanofluid properties.

- **Nano fluid properties**
- **Applied model**

| Property          | Formula                  | Formula                  |
|-------------------|--------------------------|--------------------------|
| Density           | \( \rho_{nf} = (1 - \chi) \rho_f + \chi \rho_s \) | \( \rho_{nf} = (1 - \chi) \rho_f + \chi \rho_s \) |
| Thermal diffusivity | \( \alpha_{nf} = \frac{\alpha_f}{\sqrt{\frac{\alpha_f}{\alpha_s}}} \) | \( \alpha_{nf} = \frac{\alpha_f}{\sqrt{\frac{\alpha_f}{\alpha_s}}} \) |
| Heat capacitance  | \( (\rho C_p)_nf = (1 - \chi)(\rho C_p)_f + \chi (\rho C_p)_s \) | \( (\rho C_p)_nf = (1 - \chi)(\rho C_p)_f + \chi (\rho C_p)_s \) |
| Dynamic viscosity | \( \mu_{nf} = \frac{(1 - \chi)^2}{\rho_s} \mu_f + \chi (\rho_s \mu_f) \) | \( \mu_{nf} = \frac{(1 - \chi)^2}{\rho_s} \mu_f + \chi (\rho_s \mu_f) \) |
| Thermal expansion coefficient | \( (\rho \beta)_nf = \chi \rho_f \beta_f + (1 - \chi) \rho_s \beta_s \) | \( (\rho \beta)_nf = \chi \rho_f \beta_f + (1 - \chi) \rho_s \beta_s \) |

The above dimensional form of mass, momentum and energy equations can be converted to non-dimensional form by using the following dimensionless parameters:

\[
\begin{align*}
X &= \frac{x}{L}, & Y &= \frac{y}{L}, & U &= \frac{u}{U_0}, & V &= \frac{v}{U_0}, \\
T &= \frac{\rho \beta_c}{\rho_f \beta_f} T, & Gr &= \frac{g \beta (\rho_f \rho_s)}{\rho_f \mu_s} L^4 \rho_f \beta_f, \\
P &= \frac{\rho_f U_0^2}{\rho_s \beta_f}, & Re &= \frac{\rho_f U_0}{\mu_f}, & Pr &= \frac{\mu_f}{\rho_f C_p}, & Da &= \frac{K}{L^2} \\
\end{align*}
\]

(5)

After non-dimensionalization, the above dimensional form of governing equations can now be written as follows:

\[
\begin{align*}
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} &= 0 \\
\frac{1}{\varepsilon^2} \left( U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right) &= -\rho_f \frac{\partial P}{\partial X} + \frac{\mu_{nf}}{\rho_f U} (\nabla^2 U) \\
&+ \frac{\rho_{nf} v_f}{\rho_f \mu_f} \frac{Re \rho_s v_f}{Re Da} (\nabla^2 U) \\
&- \frac{1.75 (U^2 + V^2)^{1/2}}{\sqrt{150}} U \\
\frac{1}{\varepsilon^2} \left( U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} \right) &= -\rho_f \frac{\partial P}{\partial Y} + \frac{\mu_{nf}}{\rho_f U} (\nabla^2 V) \\
&+ \frac{\rho_{nf} v_f}{\rho_f \mu_f} \frac{Re \rho_s v_f}{Re Da} (\nabla^2 V) \\
&- \frac{1.75 (U^2 + V^2)^{1/2}}{\sqrt{150}} V \\
&+ \frac{\rho_{nf} \beta_f}{\rho_f \beta_f} (Ri T) \\
\frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} &= \alpha_{nf} \frac{1}{Pr \cdot Re} (\nabla^2 T) \\
\end{align*}
\]

(6)-(9)

saturated with a fluid that is local thermal equilibrium (LTE) model with the solid matrix. It is assumed that the base fluid (water) and spherical nanoparticles (Cu) are in thermally equilibrium and no-slip velocity between them. The fluid physical properties are assumed to be constant except the density variation in the buoyancy term. The Boussinesq approximation is valid. The thermophysical properties of the base fluid and nanoparticles are shown in Table 1. The governing equations consisting of mass, momentum and energy equations for the mixed convection of the nanofluid in the 2-D porous cavity can be written in the dimensional form as follows:

\[
\nabla \cdot \mathbf{u} = 0 \tag{1}
\]

\[
\left( \frac{\mathbf{u}}{\varepsilon} \right) \cdot \nabla \mathbf{u} = \frac{\varepsilon}{\rho_{nf}} \left[ \frac{\partial p}{\partial \varepsilon} - \frac{\mu_{nf}}{\kappa} (\nabla^2 \mathbf{u}) - \frac{\mu_{nf}}{K} \mathbf{u} \right] \\
- \frac{F \rho_{nf} \mathbf{u} \mathbf{u}}{K^{1/2}} + (\rho \beta)_{nf} g(\theta - \theta_c) \right] \\
\mathbf{u} \cdot \nabla \theta = \frac{1}{(\rho C_p)_nf} \left( \nabla \cdot (\kappa \nabla \theta) \right) \tag{2}
\]

where \( \mathbf{u} \) is the Darcy velocity vector, \( p \) is the fluid pressure, \( \theta \) is the temperature, and \( \varepsilon \) is the porosity of the medium. The permeability \( K \) and Forchheimer's coefficient \( F \) can be written as

\[
K = \frac{\varepsilon^2 d_p^2}{a (1 - \varepsilon)^3}; \quad F = \frac{b}{\sqrt{\varepsilon} \varepsilon^{3/2}}
\]

where \( a = 150 \) and \( b = 1.75 \). The appropriate dimensional forms of the boundary conditions are given as follows:

\[
\begin{align*}
\begin{array}{lcl}
\text{u} &=& v = 0, \quad \theta = \theta_c, \quad y = 0 \\
u &=& U_0, \quad v = 0, \quad \theta = \theta_h, \quad y = H \\
u &=& v = 0, \quad \frac{\partial \theta}{\partial x} = 0, \quad x = 0 \\
u &=& v = 0, \quad \frac{\partial \theta}{\partial x} = 0, \quad x = L
\end{array}
\end{align*}
\]

(4)
The non-dimensional boundary conditions of the considered problem are given as follows:

\[ U = V = 0, \quad T = 0, \quad Y = 0 \]
\[ U = 1, \quad V = 0, \quad T = 1, \quad Y = Ar \]
\[ U = V = 0, \quad \frac{\partial T}{\partial X} = 0, \quad X = 0 \]
\[ U = V = 0, \quad \frac{\partial T}{\partial X} = 0, \quad X = Ar \]  

(10)

The properties of the nano fluids are given in Table 2. The local and average heat transfer rates of the cavity can be presented by means of the local and average Nusselt numbers. The local Nusselt number for the considered problem is calculated along the top heated wall and the average Nusselt number (\( \text{Nu}_{avg} \)) for overall heat transfer rate is obtained by integrating the local Nusselt number.

\[ \text{Nu} = -k_{eff} \frac{1}{k_f} \frac{\partial T}{\partial Y} \]  

(11)

\[ \text{Nu}_{avg} = \int_0^{Ar} \text{Nu} dX. \]  

(12)

Many researchers have cited the classical Maxwell [22] model in which only the particle volume concentration and the thermal conductivities of the particle are considered but the Brownian motion of the nanoparticles and the effect of solid-like nano-layers formed around nanoparticles are not taken into the account. To execute the above need the modified Maxwell [23] model considers a nano-layer with a solid-like structure formed by the liquid molecules close to solid surface. As far as the modified Maxwell model is concerned, \( k_{eff} \) is the effective thermal conductivity of the nano fluid for spherical nanoparticles, which is given as follows:

\[ \frac{k_{eff}}{k_f} = \frac{(k_{eq} + 2k_f) + 2(k_{eq} - k_f)(1 + \sigma)^3 \chi}{(k_{eq} + 2k_f) - (k_{eq} - k_f)(1 + \sigma)^3 \chi} \]  

(13)

where \( \sigma \) is the ratio of the thickness of nano-layer to the original radius of nanoparticles (\( h_{nl}/r_s \)) and \( k_{eq} \) is the equivalent thermal conductivity of nanoparticles and their layers:

\[ \frac{k_{eq}}{k_s} = \frac{2(1 - \eta) + (1 + \sigma)^3(1 + 2\eta)}{-(1 - \eta) + (1 + \sigma)^3(1 + 2\eta)} \]  

(14)

where \( \eta \) is the ratio of thermal conductivity of nano-layer upon the thermal conductivity of the nanoparticles (\( \eta = k_{nl}/k_s \)). In this study, it is assumed that \( h_{nl} = 2nm \), \( r_s = 3nm \) and \( k_{nl} = 100k_f \). Yu and Choi [23] obtained that for these conditions, the result of the modified Maxwell model are in good agreement with experimental results.

3 Numerical technique and validation

3.1 Numerical method

The governing equations (6)--(9) subject to the boundary conditions are discretized by the finite volume method (FVM) on a uniform staggered grid arrangement using...
SIMPLE algorithm of Patankar [24]. The third order accurate deferred QUICK scheme of Hayase et al. [25] and central difference scheme are applied for the convection and diffusion terms in the both momentum and energy equations. The uniform grid is selected in both $X$ and $Y$ directions. The solution domain consists of a number of control volumes at which discretization equations are applied. The governing equations are transferred into a system of algebraic equations through integration over each control volume. The solution of the discretized equations for each variable is obtained by TDMA line-by-line method. The grid independence tests are carried out using the grid points from $11 \times 21$ to $81 \times 161$ for $R_i = 100$, $Pr = 6.2$, $\varepsilon = 0.4$, $Da = 10^{-2}$, $Ar = 0.5$ and $\chi = 0.06$. The grid independence test which is shown in Figure 2(a) demonstrated that an $41 \times 81$ grid system is enough to obtain the desired accuracy of results.

3.2 Code validation

In order to check the accuracy of the present results, the present computational code has been validated by two published work in the literature. The direct validation of the present numerical model against the numerical results

![Isotherms (left) and streamlines (right) for different $R_i$ with $\varepsilon = 0.4$, $Da = 10^{-2}$, $Ar = 0.25$, and $\chi = 0.06$.](image)
of Roy and Basak [26] was performed. Comparisons of the local Nusselt number along the bottom wall for uniform heating are shown in Figure 2(b). In this case, the agreement is found to be excellent. The results for combined convection in lid driven enclosure filled with nano fluid were also compared with Mahmoodi et al. [27]. A comparison of the dimensionless vertical velocity component along the horizontal centerline of the cavity for \( \text{Ri} = 100 \) and \( x = 0.03 \) are executed in Figure 2(c) and show a good agreement. The results compared with the previous literature provide confidence to the accuracies of the present numerical solutions.

4 Results and discussion

In this paper, a numerical study has been presented to investigate the mixed convection flow and heat transfer characteristics in a two dimensional porous cavity filled with Cu-water nanofluid. The present computations were carried out to determine the effects of the aspect ratio \((0.25 \leq A_r \leq 2)\), the Richardson number \((0.01 \leq \text{Ri} \leq 100)\), the Darcy number \((10^{-4} \leq Da \leq 10^{-2})\), and the solid volume fraction \((0 \leq \chi \leq 0.06)\). Throughout the study, the porosity \( \varepsilon = 0.4 \) is fixed. The copper-water nanofluid is chosen as working fluid with the Prandtl number \( Pr = 6.2 \). The computational results are shown in terms of isotherms, streamlines, mid-plane velocity profiles, and local as well as average Nusselt numbers for various physical parameters.

The effects of Richardson number and aspect ratio on the fluid flow and thermal field for \( Da = 10^{-2} \) and \( \chi = 0.06 \) are depicted in Figures 3–6. For the aspect ratio \( A_r = 0.25 \) (Fig. 3), the flow field is described by a primary vortex near top moving lid and two weaker secondary eddies, which occurs one below the other in the remaining part of the cavity when \( \text{Ri} = 100 \). As \( \text{Ri} \) decreases \((\text{Ri} < 1)\), the primary vortex near top moving lid starts to grow bigger and two weaker secondary eddies are merged. Since this is slender cavity, the effect of the shear force can be observed only in one unit of the cavity height where the flow recirculation is prominent. In the remaining part of the cavity, the flow of nanofluid remains almost stagnant. The isotherms specify that a high temperature gradients are observed only at the location near top wall and conduction is the principal mode of heat transfer in the rest space of the cavity for all \( \text{Ri} \) varied. Figure 4 shows that the hot and cold walls of the porous enclosure come closer and convection gets stronger as the aspect ratio increases from 0.25 to 0.5. A tri-cellular flow structure is appeared with two clockwise rotating eddies at the top and bottom of the cavity and one anti-clockwise rotating vortex at the middle when \( \text{Ri} = 100 \). It is observed that the flow structure is
always of two symmetric eddies only at the top and bottom of the cavity for $Ri \leq 1$. A decrease in $Ri$ clusters the temperature gradients downwards markedly. The isotherms and streamlines for $Ar = 1$ and 2 are depicted in Figures 5 and 6. In the both aspect ratios $Ar = 1$ and 2, a symmetric behaviour is observed in the fluid flow and temperature field. It can be understood from these figures that for $Ri = 100$, two symmetrical counter rotating eddies are appeared at the top and bottom of the enclosure. The corresponding isotherms show high temperature gradients near top wall. When $Ri$ decreases, that is, when the buoyancy effect is overwhelmed by the mechanical effect of the sliding lid, the two symmetrical counter rotating eddies are merged, a main circulation fills the entire cavity, and two secondary cells are visible near bottom corners (see Figs. 5(c) and 6(c)). The isotherms depict that the temperature distributions are grouped near bottom surface of the cavity and it is very weak in the interior region of the cavity due to the vigorous effect of the top-moving lid.

Figure 7 shows the variations of horizontal and vertical velocity components of the porous cavity for different $Ri$ and $Ar$ when the other parameters are kept fixed. The
velocity profiles reveal that the effects of aspect ratio play a major role on the flow field. It is observed that for $Ar = 0.25$, $U$-velocity decreases with the decrease of $Ri$. On the other hand, $V$-velocity increases with a decrease in $Ri$. A similar behaviour of $U$ and $V$ velocity is observed for all aspect ratios when $Ri$ decreases. Since the fluid flow in bulk of the cavity remains stagnant for $Ar = 0.25$, $U$-velocity is almost zero in three-fourths of the porous cavity and attains its maximum in the rest space where the fluid flow is prominent. The mechanical effect of moving lid is not considerable enough when the cavity is slender. It can be verified clearly in Figure 7(a) and (b). As the aspect ratio increases from 0.25 to 0.5 (Fig. 7(c) and (d)), the magnitude of $U$ and $V$-velocity augments and the sign of $V$-velocity is reversed for all $Ri$ compared with the previous case $Ar = 0.25$. This enhancement of $U$ and $V$-velocity is due to the fact that the hot and cold walls of the porous enclosure come closer. Moreover, it is found that the mid-plane velocity increases significantly with the increase of the aspect ratio for all $Ri$ considered. It is shown in Figure 7(e)–(h).

Figure 6. Isotherms (left) and streamlines (right) for different $Ri$ with $\varepsilon = 0.4$, $Da = 10^{-2}$, $Ar = 2$, and $\chi = 0.06$.

Figure 8 illustrates the influence of the aspect ratio and Richardson number on the local Nusselt number for the fixed values of $Da = 10^{-2}$ and $\chi = 0.06$. For all $Ri$, the distribution of local Nusselt number begins with high value at the left end, decreases towards the right end, and attains the minimum near the right end of the $X$-axis when $Ar$ is fixed, as shown in Figure 8. It can be noticed that for the fixed value of $Ar$, an enhancement in the local Nusselt number is observed when $Ri$ varies from 100 to 0.01. Moreover, an increase in the aspect ratio increases the local heat transfer rate for all Richardson number varied. It is
Fig. 7. $U$ and $V$-velocity profiles at mid-plane of the cavity for different $Ar$ and $Ri$ with $Da = 10^{-2}$, $\varepsilon = 0.4$, and $\chi = 0.06$. 
interesting to note that for $Ar = 0.5$, the maximum of the local Nusselt number is obtained near the left end of the $X$-axis when $Ri = 1$ (see Fig. 8(b)). Figure 8 also reveals the fact that for $Ri = 1$ and 100, after the first half of the $X$-axis, the local Nusselt number converges almost equally and attains the same values when the aspect ratio is kept fixed. This is because the buoyancy force is substantial enough in comparison with shear force in the mixed and natural convection.

The effects of aspect ratio and solid volume fraction on the fluid flow and thermal field for $Da = 10^{-2}$ and $Ri = 1$ can be viewed in Figures 9–12. The streamlines in Figure 9 (a) confirm that for $x = 0$, a tri-cellular flow structure with a anti-clockwise eddy in the middle and two clockwise eddies near top and bottom of the cavity is appeared. As the solid volume fraction increases, the anticlockwise eddy in the middle of the cavity develops from its actual size and squeezes the two clockwise eddies near top and bottom of the cavity. The isotherms show that the temperature distributions are nearly parallel to the horizontal walls of the cavity, which indicates the conduction heat transfer mode throughout the cavity except the region at top of the enclosure where a mechanically induced force occurs. In addition, no significant change in temperature distribution is observed as $x$ increases. Figure 10 shows that when the aspect ratio is increased from 0.25 to 0.5 and $x = 0$, the flow structure is characterized by a primary cell in half size of the cavity and two secondary cells one bellow the other in the remaining part of the cavity. As $x$ increases, the recirculation of the primary vortex gets strengthened, two weaker cells are merged, and a anticlockwise cell appears in the bottom of the cavity. The corresponding isotherms demonstrate almost the same behaviour in order to compare with $Ar = 0.25$. However, convection gets reinforced when the aspect ratio is increased from 0.25 and 0.5. It can be seen from Figure 10. Figure 11 shows that when $x = 0$, the flow structure indicate a single recirculating vortex inside the cavity. Moreover, a secondary vortex emerges near left-bottom corner. Since the nanoparticles has the tendency to reinforce the movement of the fluid inside the porous cavity, the main recirculation cell gets strengthened and the secondary vortex is deteriorated as
the solid volume fraction increases. It is observed from Figure 11 that the temperature distributions are clustered towards the bottom of the cavity and show a steep temperature gradients along the left insulated wall. The thermal boundary layers elevate upwards as $x$ increases from 0 to 0.06, which indicates that addition of nanoparticles in porous enclosure is to reinforce the fluid flow and gives better heat transfer enhancement in the enclosure. A similar behaviour in streamlines and isotherms is observed for $Ar = 2$. It can be verified from Figure 12.

The effects of aspect ratio and solid volume fraction on velocity profiles for $Ri = 1$, $Da = 10^{-2}$ are depicted in Figure 13. The porous medium within the cavity has tendency to resist the flow motion of nanofluid. As a result, the flow velocity in the mid-plane of the cavity is almost zero on increasing the solid volume fraction for the slender porous cavity ($Ar = 0.25$). This can clearly be understood from the horizontal velocity profiles at the mid-section of the cavity for $Ar = 0.25$ (see Fig. 13(a)). The addition of nanoparticles increases $V$-velocity markedly whereas it has no significant effect on $U$-velocity when $Ar = 0.25$ and 0.5. For $Ar = 1$ and 2, no significant effect on velocity profiles is observed as the solid volume fraction increases. Furthermore, it is found that the flow velocity increases significantly with the increase of aspect ratio and solid volume fraction in the porous cavity.

The variations of the local Nusselt numbers along the hot moving lid are shown in Figure 14 for different aspect ratio and solid volume fraction. The effects of aspect ratio and solid volume fraction on the heat transfer rate are clearly shown in these plots. In general, the local Nusselt numbers along the hot wall start with high value at the left end and decrease monotonically to a small value near the right end for all the considered values of $\chi$ when $Ar$ is fixed. In addition, it can be understood that an increase in solid
Fig. 11. Isotherms (left) and streamlines (right) for different \( \chi \) with \( \varepsilon = 0.4, Da = 10^{-2}, Ri = 1, \) and \( Ar = 1 \).

(a) \( \chi = 0 \)

(b) \( \chi = 0.06 \)

Fig. 12. Isotherms (left) and streamlines (right) for different \( \chi \) with \( \varepsilon = 0.4, Da = 10^{-2}, Ri = 1, \) and \( Ar = 2 \).

(a) \( \chi = 0 \)

(b) \( \chi = 0.06 \)
Fig. 13. U and V-velocity profiles at mid-plane of the cavity for different $Ar$ and $\chi$ with $Da = 10^{-2}$, $\varepsilon = 0.4$, and $Ri = 1$. 
volume fraction increases the local Nusselt number for all aspect ratios. This reveals the fact that the heat transfer rate inside the porous cavity is directly affected on increasing the solid volume fraction. The maximum of local Nusselt number is obtained for \( Ar = 2 \), which indicates that the local heat transfer rate increases with the increase of aspect ratio in the porous enclosure. The overall heat transfer rate in terms of solid volume fraction for various aspect ratios and Richardson numbers is plotted in Figure 15. The overall heat transfer rate increases on increasing the aspect ratio. The curve of the average Nusselt number shows a linear variation with the solid volume fraction. An increase in average Nusselt number is obtained with the decrease of Richardson number for the fixed value of aspect ratio. Generally, the high permeability \( Da = 10^{-2} \) gives better heat transfer rate than the low permeability \( Da = 10^{-4} \).
But, in the slender porous cavity, i.e., in $Ar = 0.25$ and 0.5, high heat transfer rate is obtained for $Da = 10^{-4}$ compared with $Da = 10^{-2}$ when $Ri = 0.01$. This is shown in Figure 16. An increase in the overall heat transfer rate is observed by decreasing the Richardson number for the fixed values of aspect ratio and Darcy number.

5 Conclusion

In the present article, mixed convection flow of nanofluid in a square lid-driven cavity fully saturated with porous medium was numerically investigated for different aspect ratios. For $Ar = 0.25$ and 0.5, the fluid remains almost stagnant in the bulk of the cavity, which indicates the conduction dominated flow regime. Further, the convection gets strengthened with the increase of $Ar$. Consequently, the mid plane velocity increases with an increase in $Ar$. In the presence of nanoparticles, the magnitude of velocity and heat transfer rate increase remarkably with higher $Ar$ and $Da$. Many researchers have found that the
addition of nanoparticles in the porous medium is to increase the overall heat transfer rate in most flow regimes. To the contrary, the overall heat transfer rate is appeared to decrease slightly or stay nearly the same with increased solid volume fraction for $Ar = 0.25$ and 0.5 porous cavities in the forced convection regime. Since the resistance from the boundary friction is less at $Da = 10^{-2}$, the fluid circulation is prominent within the enclosure for $Da = 10^{-2}$ compared with $Da = 10^{-4}$. As a result, a better heat transfer rate is obtained for $Da = 10^{-2}$ than for $Da = 10^{-4}$. In other words, the porous medium of high permeability gives better heat transfer rate than the porous medium of low permeability. It is also found that the better heat transfer rate is obtained with the porous cavity of high aspect ratio in order to compare with the slender porous cavity. By adding the nanoparticles to the base fluid in the porous medium, the overall heat transfer rate increases significantly.

**Nomenclature**

- $g$: gravitational acceleration
- $C_p$: specific heat
- $Gr$: Grashof number
- $H$: enclosure height
- $L$: enclosure length
- $h_{nl}$: thickness of nano-layer
- $K$: permeability
- $Pr$: Prandtl number
- $N_{Navyg}$: average Nusselt number
- $Nu_l$: local Nusselt number
- $p$: fluid pressure
- $P$: dimensionless pressure
- $Pr$: Prandtl number
- $r_s$: radius of nanoparticles
- $Re$: Reynolds number
- $Re$: Richardson number
- $\theta$: temperature
- $T$: dimensionless temperature
- $u$: velocity vector
- $U, V$: dimensionless velocities in $x$- and $y$-direction respectively
- $U_0$: lid velocity
- $u, v$: velocities in $x$- and $y$-direction respectively
- $X, Y$: dimensionless Cartesian coordinates
- $x, y$: Cartesian coordinates
- $\Delta \theta$: temperature difference
- $Da$: Darcy number
- $d_p$: average particle size
- $F`: Forchheimer coefficient
- $\varepsilon$: porosity
- $\alpha$: thermal diffusivity
- $\beta$: thermal expansion coefficient
- $\mu$: dynamic viscosity
- $\nu$: kinematic viscosity
- $\rho$: density
- $\chi$: solid volume fraction
- $\eta$: ratio of thermal conductivity of nano-layers
- $\sigma$: ratio of the thickness of nano-layer
- $Ar$: aspect ratio

**Subscripts**

- $c$: cold wall
- $eff$: effective
- $eq$: equivalent
- $f$: fluid
- $h$: hot wall
- $nf$: nanofluid
- $nl$: nano-layer
- $s$: solid

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