Symmetry Breaking Operators for Line Bundles over Real Projective Spaces

The space of smooth sections of an equivariant line bundle over the real projective space $\mathbb{R}P^n$ forms a natural representation of the group $GL(n+1,\mathbb{R})$. We explicitly construct and classify all intertwining operators between such representations of $GL(n+1,\mathbb{R})$ and its subgroup $GL(n,\mathbb{R})$, intertwining for the subgroup. Intertwining operators of this form are called symmetry breaking operators, and they describe the occurrence of a representation of $GL(n,\mathbb{R})$ inside the restriction of a representation of $GL(n+1,\mathbb{R})$. In this way, our results contribute to the study of branching problems for the real reductive pair $(GL(n+1,\mathbb{R}), GL(n,\mathbb{R}))$.

The analogous classification is carried out for intertwining operators between algebraic sections of line bundles, where the Lie group action of $GL(n,\mathbb{R})$ is replaced by the action of its Lie algebra $\mathfrak{gl}(n,\mathbb{R})$, and it turns out that all intertwining operators arise as restrictions of operators between smooth sections.

Keywords: Symmetry breaking operators, real projective spaces, general linear group, intertwining operators, Harish-Chandra modules, principal series.

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