Mechanism Modeling and Simulink Simulation Analysis of Digital Hydraulic Cylinder

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Abstract. Based on the structure of the digital hydraulic cylinder and the principle of closed-loop control, the mechanism models of the stepper motor and the spool mechanism, the three-way spool valve, and the hydraulic cylinder are established respectively, and the overall mathematical model of the digital hydraulic cylinder is synthesized, and its Simulink simulation model is constructed. The stability and positioning accuracy of the system are analyzed. The results show that the error of this type of digital hydraulic cylinder is 0.0142mm for 1mm action and 0.0182mm for 2mm action. At the same time, it is found that the fluctuation of oil supply pressure will cause system jitter and reducing the frequency of stepper motor can improve the positioning accuracy of the cylinder.

Keywords: Digital hydraulic cylinder; Mathematical model; Simulink; Modeling and simulation.

1. Introduction
The traditional hydraulic cylinder is only an execution element, and it must be controlled by various hydraulic valves in the hydraulic circuit to achieve output. In digital hydraulic technology, a hydraulic valve is integrated in a hydraulic cylinder, and is simultaneously equipped with a stepper motor drive element to form a digital hydraulic cylinder. This kind of hydraulic cylinder can realize digital control of the piston rod movement, that is to say, the pulse frequency of the motor corresponds to the speed of the piston rod, the direction of rotation corresponds to the direction of expansion and contraction of the piston rod, and the number of pulses corresponds to the displacement.

In order to explore the performance indicators such as the positioning accuracy of digital hydraulic cylinders and optimize the performance of digital hydraulic cylinders, a digital hydraulic cylinder is taken as the research object. Based on the analysis of its structural principle, a mathematical model of digital hydraulic cylinder is established, The Simulink platform is used for simulation analysis, and the influencing factors and stability of digital hydraulic cylinder positioning accuracy are studied.

2. Structure and Principle of Digital Hydraulic Cylinder
The structure of the digital hydraulic cylinder is shown in Figure 1. It is mainly composed of stepper motor, slide valve, lead screw, feedback nut, piston rod, cylinder block and so on. The stepper motor is matched with one end of the spool of the spool valve by a sliding key, the other end of the spool is equipped with external threads to match the internal threads of the connecting sleeve, the connecting sleeve is fixedly connected to the lead screw by a pin, the piston rod is a hollow structure, and a wire is installed inside bar. The screw only rotates and does not move left and right, the feedback nut follows the piston and only moves left and right without rotation.
Assuming that the digital hydraulic cylinder piston is in a balanced state, when the stepper motor receives the pulse signal, it outputs a certain angular velocity or angular displacement. This movement is transmitted to the spool through the feather key. Under the action of the thread between the spool and the connection sleeve, the spool rotates and generates an axial displacement. If the spool moves to the left, the high-pressure oil circuit is connected to the rod-less cavity, and the oil return channel is connected to the rod-less cavity. Then, the pressure of the rod-less cavity rises, the piston and the feedback nut move to the right together, and at the same time, the lead screw is also started. The rotation occurs in proportion, and the direction of rotation is opposite to the running direction of the stepper motor. In this way, the lead screw will drive the connecting sleeve to rotate, the valve core will be moved to the right, the valve port will be closed, and the piston will be in a balanced state again. One stepping process ends. Similarly, if a reverse pulse signal is input and the stepping motor is reversed, the piston is retracted in the reverse direction.

Use the block diagram shown in Figure 2 to describe this process more clearly, input pulses to the stepper motor-the output motor output angle-the valve core rotates and generates axial displacement-the valve port is opened-the communication oil circuit-the piston begins to act when the force is unbalanced-the piston moved the screw in place-the spool moves in the opposite direction-the valve port is closed.

In this way, through the clever structure of the valve core, the lead screw and the feedback nut, a negative position feedback in a mechanical manner is realized, and a closed loop is formed inside, which shows the corresponding relationship between the number of pulses and piston displacement.

3. Digital Hydraulic Cylinder Mechanism Modeling

According to the structural principle of digital hydraulic cylinder, mathematical models of stepper motor rotation, screw screw feedback, three-way valve flow, and hydraulic cylinder movement are respectively established, and then the overall model is established, and the simulation results are analyzed by Simulink. The results obtained by the mathematical model can be compared with the software simulation results and experimental results, it lays the foundation for the software modeling and optimization analysis.

3.1. Rotating Model of Stepper Motor and Spool Mechanism

The input of the digital hydraulic cylinder is the pulse of the stepping motor, the output is the angular displacement, the input pulse amount is recorded as \( n \), and the output is the angle as \( \theta \). When \( n > 0 \), it
means that the motor is rotating forward, and when \( n<0 \), it means that the motor is rotating backward. The mathematical model of the stepper motor is
\[
T = T_m \sin \left( Z_r \left(n \theta_m / a - \theta \right) \right) = J \dot{\theta} + B \dot{\theta} + T_e
\]
where, \( \theta_m \) is the theoretical angular displacement, \( \theta \) is the motor output angle, \( a \) is the number of drive subdivisions, \( T_m \) is the maximum static torque, \( J \) is the motor rotor inertia, \( Z_r \) is the number of rotor teeth, and \( B \) is Rotor damping coefficient, \( T_e \) is the load torque, and \( T \) is the total torque.

Considering the three-way valve spool and the motor shaft as a rigid connection, the equation for the valve spool rotation is
\[
T_L = J_L \dot{\theta} + B_m \dot{\theta} + T_{x} + T_e
\]
where, \( J_L \) is the moment of inertia of the spool, \( B_m \) is the damping coefficient of the spool, \( T_{x} \) is the axial combined moment, and \( T_e \) is the total resistance moment.

The three-way valve spool is connected to the output shaft of the stepper motor. Therefore, the rotation equation of the spool and the rotation equation of the motor shaft are considered together, so that
\[
T = J \ddot{\theta} + B \dot{\theta} + T_e
\]
where, \( J \) is the total moment of inertia, \( J = J_L + J_r \), \( B \) is the total damping coefficient, \( B = B_e + B_m \), and \( T_e \) is the total resistance moment.

Simultaneous (1)-(3), we get
\[
J \ddot{\theta} + B \dot{\theta} + T_e - T_m \sin \left( Z_r \left(n \theta_m / a - \theta \right) \right) = 0
\]
Considering that \( \theta_m, \theta \) is small and \( \sin(Z_r(\theta_m-\theta)) \approx Z_r(\theta_m-\theta) \), then
\[
J \ddot{\theta} + B \dot{\theta} + T_e - T_m Z_r \left(n \theta_m / a - \theta \right) = 0
\]
After Laplace transformation of the above formula, the stepper motor transfer function can be obtained:
\[
\theta = \left( T_m Z_r / \left( J s^2 + B s + T_m Z_r \right) \right)
\]

3.2. Three-way Valve Model
This type of digital hydraulic cylinder uses a screw to complete the position feedback of the piston rod. The screw has high transmission efficiency and low friction between mechanical mechanisms. During the operation of the digital hydraulic cylinder, on the one hand, the stepper motor drives the spool to generate axial displacement through the rotating shaft, and on the other hand, the feedback mechanism drives the spool to move in reverse to complete the feedback, so the motion model of the spool is
\[
x_v = \theta t_2 / 2\pi - x_f t_1 / t_2
\]
where, \( x_v \) is the spool displacement, \( x_m \) is the theoretical displacement, \( x_f \) is the piston rod displacement, \( t_1 \) is the feedback nut pitch, and \( t_2 \) is the lead screw lead. For this type of digital hydraulic cylinder, \( t_1 = t_2 \).

There are many factors that affect valve port flow. To simplify the analysis, ignore some minor factors, and make the following assumptions.
(1) Oil supply pressure is constant and oil flow rate \( q_L = C_{o1} \omega x \sqrt{2(p_e - p_r) / \rho} \) is turn pressure is zero.
(2) Throttling losses at pipelines, valve ports, bends, etc. are ignored.
(3) The hydraulic oil is not compressible, and the density and temperature of the hydraulic oil are constant.

When \( x_v \geq 0 \), the valve flow equation is
$$q_L = C_d \omega x_v \sqrt{2(p_s - p_c)/\rho}$$  \hspace{1cm} (8)$$

When \(x_v < 0\), the valve flow equation is

$$q_L = C_d \omega |x_v| \sqrt{2 p_c/\rho}$$  \hspace{1cm} (9)$$

Where, \(q_L\) is the load flow of the three-way valve, \(p_s\) is the fuel supply pressure, \(p_c\) is the control cavity pressure, \(\omega\) is the orifice area gradient, and \(C_d\) is the three-way valve flow coefficient.

According to equations 8 and 9, this system is a valve-controlled asymmetric cylinder, which has the inherent non-linearity of the servo system. Considering that a three-way valve usually works near zero, its flow gain \(K_q\) and pressure gain \(K_c\) is

$$K_q = \frac{\partial q_L}{\partial x_v}, \quad K_c = \frac{\partial q_L}{\partial p_L}$$  \hspace{1cm} (10)$$

Calculate the gain using the zero point of time, that is

$$K_c = \frac{\pi r_c^2 \omega}{(64 \mu)}$$  \hspace{1cm} (11)$$

In the formula: \(r_c\) is the radial clearance between the valve core and the valve sleeve, and \(\mu\) is the dynamic viscosity of the oil.

$$K_q = \frac{C_d \omega p_c/\rho}{\sqrt{2}}$$  \hspace{1cm} (12)$$

The three-way valve flow model is a non-linear equation. After linearizing the equation, the flow equation is

$$\Delta q_L = K_q \Delta x_v - K_c \Delta p_L$$  \hspace{1cm} (13)$$

### 3.3. Hydraulic Cylinder Model

This system is a three-way valve differential control hydraulic cylinder. The valve port flow \(q_L\) will only flow into or out of the hydraulic cylinder's rod-less cavity, so only the rod-less cavity flow is modeled. At the same time, external leakage is usually small and relatively obvious, which can be ignored here.

The flow equation of the rod-less cavity of the hydraulic cylinder is

$$q_L = A_h \dot{x}_p + C_{ip} (p_c - p_s) + (V_0/\beta_e) \dot{p}_c$$  \hspace{1cm} (14)$$

In the formula, \(A_h\) is the piston area of the hydraulic control cavity, \(C_{ip}\) is the leakage coefficient in the hydraulic cylinder, \(V_c\) is the volume of the hydraulic control cavity, \(V_0\) is the initial volume of the hydraulic control cavity, and \(\beta_e\) is the equivalent volume elastic modulus of the oil.

The hydraulic cylinder piston and load dynamic balance equation is

$$A_h p_c - A_p p_s = m \ddot{x}_p + B_p \dot{x}_p + K x_p + F_h + F_L + F_f$$  \hspace{1cm} (15)$$

In the formula, \(A_p\) is the area of the rod cavity, \(m\) is the equivalent total mass of the load and the piston, \(B_p\) is the viscous damping coefficient, \(K\) is the spring stiffness of the load, \(F_h\) is the driving force of the feedback mechanism, \(F_L\) is the external load, and \(F_f\) is the friction force.

### 3.4. The Complete Model of Digital Hydraulic Cylinder

The formula (13) in the valve flow model, the formula (14) in the hydraulic cylinder flow model, and the formula (15) in the piston rod motion model are obtained by transformation.

$$q_L = K_q X_v(s) - K_c P_L(s)$$  \hspace{1cm} (16)$$

$$q_L = A_h sX_p(s) + C_{ip} p_L + V_0/\beta_e sP_L(s)$$  \hspace{1cm} (17)$$
The above three formulas can be used to obtain the displacement of the digital hydraulic cylinder piston as

\[ X_p = \frac{K_s / A_{x_s}(s) - K_{\omega_s}/A_{\beta s}}{s^2/\omega_n^2 + 2\zeta \omega_n s/\omega_n + 1} + F_L \]  

In the formula: \( K_{ce} = K_c + C_p \) is the total flow-pressure coefficient, \( \omega_n = \sqrt{\beta A_h V_0 m} \) is the natural frequency of the hydraulic system, and \( \zeta_h = K_{\omega_s}/2A_h \sqrt{\beta_s m/V_0} \) is the hydraulic damping ratio.

In addition, the expression of the output angle of the stepper motor according to formula (6) is

\[ \theta = \frac{\omega_n^2 \theta_m (s)}{s^2 + 2 \zeta \omega_n s + \omega_n^2} \]  

Where, \( \omega_n = \sqrt{T_m Z_r / J} \) is the natural frequency of the stepper motor, and \( \zeta = B \sqrt{2T_m Z_r J} \) is the damping ratio of the stepper motor.

4. Simulation Analysis

4.1. Determine Simulation Parameters and Build Models

By consulting the technical data and operation manual, the parameters in the simulation model of the digital hydraulic cylinder are determined.

Considering the actual situation, the valve core works near the zero position, but it does not stand still at the zero position, so the damping will increase rapidly after the valve core leaves the zero position. In this study, the hydraulic damping ratio \( \zeta_h \) is 0.2, which is more satisfactory for the actual situation of the system.

According to equations (7), (20), and (23), the simulation block diagram is shown in Figure 3, and the simulation model is established in Simulink.

4.2. System Stability Analysis

Analyzing the stability of the system is the basis for other analyses of the system. For a control system, when the external influence or internal parameters change, the original equilibrium position often deviates. When the interference is eliminated, the system cannot return to the original equilibrium position, then the system is determined to be unstable.

Here, the frequency characteristic analysis method is selected to analyze the stability of the digital hydraulic cylinder system, and the two indexes of the phase angle margin and the amplitude margin of the system are calculated to judge. Considering the influence of various non-linear factors and combining with the actual engineering, the stable conditions for this type of digital hydraulic cylinder system are set to the amplitude margin of 8-15dB, and the phase angle margin is greater than 45 degrees.

Through Simulink, you can directly draw the Bode diagram of the open-loop transfer function of the system, as shown in Figure 4. The results show that the system amplitude margin is 8.63dB and the phase angle margin is 80 degrees. According to the Nai’s criterion and stability requirements, the system stability is good.

**Figure 3.** Simulation block diagram.  
**Figure 4.** System Bird Diagram.
Step signals of 80 (displacement 1mm) and 160 (displacement 2mm) pulses are input respectively, and the system step response is shown in Figure 5. The results show that under two input conditions, the system reaches a steady-state operation time of about 0.2s, the overshoot is about 3% at 80 pulses, and the overshoot is about 4.5% at 160 pulses.

4.3. System Positioning Accuracy Analysis

The frequency of the digital hydraulic cylinder stepper motor is generally set to 400 Hz. Considering that each displacement cannot exceed 5mm of the valve core shoulder, it is generally 1mm or 2mm each time. When the simulation is run for 0.2 and 0.4s, the corresponding command displacement is 1mm and 2mm, the external load is set to zero, and the displacement curve is shown in Figure 6 and Figure 7. The results show that the error of 1mm action is 0.0142mm, and the error of 2mm action is 0.0182mm.

From the curve of valve opening in Figure 8, it can be seen that when the digital hydraulic cylinder works, because of the feedback mechanism, the system needs to adjust the size of the valve port to eliminate the error to reach a stable state. Constantly changing, the valve core is always in a dynamic adjustment state, and the size of the valve opening is constantly changing.

In addition, when the system is working, there is a pressure fluctuation in the oil source, which may also affect the operation of the oil cylinder. According to the oil supply condition of the oil source system, set the oil pressure \( p_s = 18 + 3 \sin(10 \pi t) \) in the model to simulate the fluctuation of the oil supply pressure. The simulation result is shown in Figure 9. It can be seen from the figure that the operating curve also fluctuates when the pressure sine wave fluctuates, which indicates that the pressure will cause the piston rod speed to jitter and affect the stability of the system operation.
When the stepper motor frequencies are set to 200 Hz, 400 Hz, and 600 Hz, the simulation operation is performed, and the positioning error table is shown in Table 1. It can be seen from the analysis in the table that the positioning error will increase with the increase of frequency. Among them, the frequency is 600 Hz, and the maximum error is 0.0514 mm when 160 pulses are input. Although the stepper motor frequency is usually set to 400 Hz, according to simulation results, the frequency of the stepper motor should be reduced as much as possible to meet other operating conditions to improve positioning accuracy.

Table 1. Table of displacement errors under different oil pressures.

| Frequency (Hz) | Error of 80 input pulses (mm) | Error of 160 input pulses (mm) |
|----------------|-------------------------------|-------------------------------|
|                | Retract | Reach out | Retract | Reach out |
| 200            | 0.0100  | 0.0107    | 0.0132  | 0.0147    |
| 400            | 0.0117  | 0.0142    | 0.0153  | 0.0182    |
| 600            | 0.0287  | 0.0317    | 0.0427  | 0.0514    |

5. Summary
This paper first introduces the structure and working principle of the digital hydraulic cylinder, and then based on the model of the stepper motor, hydraulic cylinder, three-way valve, and screw feedback mechanism, the overall mathematical model of the digital hydraulic cylinder is comprehensively obtained. The mathematical model is simulated in Simulink. The system stability is good through frequency domain analysis. Finally, the model is used to analyze the positioning accuracy of the digital hydraulic cylinder and its influencing factors.

The simulation results show that this type of digital hydraulic cylinder has an error of 0.0142mm for 1mm action and 0.0182mm for 2mm action. At the same time, it is found that fluctuations in fuel supply pressure will cause system jitter and reducing the frequency of the stepper motor can improve the positioning accuracy of the cylinder.

The modeling and simulation ideas and methods provided in this article can provide a technical approach for the analysis of complex electromechanical-hydraulic integration systems.

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