Determination of output link positioning error of tripod module using numerical method

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Abstract. The article presents a method for the determination of the tripod module output link positioning error, which is part of a robotic complex with relative manipulation mechanisms. The method of non-uniform coverings is used to solve this problem, which allows approximating the set of solutions of systems of equalities or inequalities describing the working tool position. The algorithm of approximation of the tripod module output link positioning error depending on the errors of the drives has been synthesized. As a result of modeling, an approximated error was obtained and graphically shown. The results can be used to select the geometric parameters of the tripod module and determine the accuracy of manufacturing robot parts, providing the technological errors of the working tool positioning specified by technological processes.

1. Introduction

Parallel robots have proven to be mechanisms with high performance, accuracy, and rigidity of the structure. They are being investigated by a large number of leading world scientists [1-5]. Parallel robots have a significant superiority compared with more distributed serial analogs in positioning accuracy, power, and permissible load, so they are increasingly used for machining parts of complex shape. One of the important problems in robotized complexes development based on the mechanisms of a parallel structure is to ensure the required machining accuracy. However, models and algorithms for compensating and determining errors that are used for conventional machines can not be used for parallel structure mechanisms. Therefore, new approaches need to be developed.

The accuracy of parallel robots is influenced by various factors, such as errors in manufacturing and assembly, kinematic errors in the actuators, elastic deformation, thermal deformation, control system errors [6].

In recent years, a large number of scientists have studied the accuracy of parallel robots [7, 8]. However, the works devoted to the study of accuracy does not take into account the output link installed on the mobile platform of the robot.

It is necessary at the design stage to analyze the relationship between the errors of the mechanical components of the robot and the accuracy of the output link to reduce production costs. Most approaches to analyzing the errors of parallel robots are based on solving the forward kinematics [9, 10]. Since in the general case it is impossible to obtain a forward kinematics solution for the mechanisms of a closed structure, the analytical conclusion cannot be calculated from the parameters. Even if the solution exists, it is difficult to work with it. The purpose of this article is to numerically assess the effect of drive errors
on the positioning error of a tool installed on a tripod module platform that is part of a robotic complex with relative manipulation mechanisms for finishing machining of complex shapes (Figure 1).

Figure 1. Kinematic scheme of a robotic complex: 1 - module tripod for tool installation; 2 - module for installation details

2. Algorithm for determining the error of positioning the output link

To solve the problem, the authors apply the method of non-uniform coverings [11-12], which allows approximating the set of solutions of systems of equalities or inequalities describing the position of the working tool.

To determine the accuracy of the output link (tool), it is necessary to develop a kinematic model of the mechanism. When developing the model, the following assumptions were made: all joints are perfect, i.e. their coordinate axes are perpendicular to each other and intersect at one point (hinge center); the position of the hinge centers is precisely known; drive rods have the same error.

Figure 2. Tripod scheme

The tripod module (Figure 2) includes three rods of variable length, which are connected by rotational hinges with the base and spherical hinges to the working platform. The base and working platform are equilateral triangles. A tool is attached to the working platform. When the rod lengths change, the working platform travels along the $Z_1$ axis by a distance $z_1$, and turns around the $X_1$ axis by the angle $\psi$ and around $Y_1$ the angle $\theta$. In addition, there are additional degrees of freedom - displacement along the axes $X_1$ at a distance $x_1$ and $Y_1$ at a distance $y_1$ and rotation relative to $Z_1$ through an angle $\alpha$, which are determined by the formulas [13]
\[ \alpha = \tan^{-1}\left(\frac{\sin \psi \sin \theta}{\cos \psi + \cos \theta}\right) \]

\[ x_1 = \frac{r}{2} \left(\cos \theta \cos \alpha + \sin \psi \sin \theta \sin \alpha - \cos \psi \alpha\right) \]

\[ y_1 = -r \cos \psi \sin \alpha \]

The input coordinates of the mechanism are the drive links \( l_1, l_2, l_3 \), and the output coordinates are the coordinates \( O' \) of the cutting edge of the tool: \( x_{O'}, y_{O'}, z_{O'} \). Coordinates \( O' \) in the moving coordinate system \( X_1'Y_1'Z_1' \):

\[ \mathbf{O}'_1 = \begin{bmatrix} r_t & 0 & h \end{bmatrix}^T \]

Since the inverse problem of kinematics has many solutions, to determine the accuracy of the output link, it is necessary to preliminarily determine the set of admissible values of linear and angular coordinates of the center \( O \) of the moving platform, and then determine the set of coordinates \( O' \) of the working tool for these values. Calculate the coordinates of \( O' \) in a fixed coordinate system \( X_1Y_1Z_1 \)

\[ \mathbf{O}'_1 = \mathbf{O}'_1 \mathbf{M}_{1',1} \]

where \( \mathbf{M}_{1',1} \) - is the transition matrix from the moving coordinate system \( X_1'Y_1'Z_1' \) to the fixed system \( X_1Y_1Z_1 \), which includes displacement matrices along the axes \( X_1, Y_1, Z_1 \) and turns around \( X_1, Y_1, Z_1 \) axes.

\[
\mathbf{M}_{1',1} = \begin{bmatrix}
\cos \theta \cos \alpha + \sin \psi \sin \theta \sin \alpha & \sin \theta \sin \alpha & \cos \theta \cos \psi & x_1 \\
\sin \psi \cos \theta \sin \alpha & \cos \psi & -\sin \psi & y_1 \\
\sin \psi \cos \theta \sin \alpha & -\sin \psi & \cos \psi & z_1 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

After the conversion, we get

\[
\mathbf{O}'_1 = \begin{bmatrix}
x_1 + M_{11r} + M_{13h} \\
y_1 + M_{21r} + M_{23h} \\
z_1 + M_{31r} + M_{33h}
\end{bmatrix} = \begin{bmatrix}
x_1 + (\cos \theta \cos \alpha + \sin \psi \sin \theta \sin \alpha) r_t + \sin \theta \cos \psi \alpha \\
y_1 + \sin \theta \sin \alpha - \sin \psi \alpha \\
z_1 + (\sin \psi \cos \theta \sin \alpha - \sin \theta \cos \alpha) r_t + \cos \theta \cos \psi \alpha
\end{bmatrix}
\]

Next, the authors introduce restrictions on the geometric parameters of the mechanism

\[
\begin{align*}
l_{i,nom} + EI & \leq l_i & \leq l_{i,nom} + ES, \\
l_{i,nom} & \leq l_{i,nom} & \leq l_{i,nom} + ES, \\
l_{i,nom} + EI & \leq l_i & \leq l_{i,nom} + ES,
\end{align*}
\]

where \( l_{i,nom} \) is the nominal length of the \( i \)-th rod, \( EI \) is the upper deviation, \( EL \) is the lower deviation, \( l_i \) is the length of the \( i \)-th rod, which is defined as

\[
l_i = \sqrt{(x_{Ei} - x_{Di})^2 + (y_{Ei} - y_{Di})^2 + (z_{Ei} - z_{Di})^2},
\]

where \( x_{Ei}, y_{Ei}, z_{Ei} \) are the coordinates of the centers of the hinges, \( p, E_i, x_{Di}, y_{Di}, z_{Di} \) are the coordinates of the centers of the hinges, \( p, D_i \) are the coordinates in the fixed coordinate system.

Get the system

\[
\begin{align*}
l_{1,nom} + EI - l_1 & \leq 0, \\
l_1 - l_{1,nom} - ES & \leq 0, \\
l_{2,nom} + EI - l_2 & \leq 0, \\
l_2 - l_{2,nom} - ES & \leq 0, \\
l_{3,nom} + EI - l_3 & \leq 0, \\
l_3 - l_{3,nom} - ES & \leq 0.
\end{align*}
\]

The authors define the coordinates of the hinges \( E_i \) in the moving coordinate system \( X_1'Y_1'Z_1' \): \( \mathbf{E}'_1 = [r \ 0 \ 0 \ 1]^T, \mathbf{E}'_2 = [-0.5r \ 0.5\sqrt{3}r \ 0 \ 1]^T, \mathbf{E}'_3 = [-0.5r \ -0.5\sqrt{3}r \ 0 \ 1]^T. \)

Denote in (1) \( M_{11} = \cos \theta \cos \alpha + \sin \psi \sin \theta \sin \alpha, M_{12} = -\cos \theta \sin \alpha + \sin \psi \sin \theta \cos \alpha, M_{13} = \sin \theta \cos \psi, M_{21} = \cos \psi \sin \alpha, M_{22} = \cos \psi \cos \alpha, M_{23} = -\sin \alpha, M_{31} = -\sin \psi \sin \alpha, M_{32} = \sin \alpha, M_{33} = \cos \theta \cos \psi \).

Express the coordinates of the hinges \( E_i \) in a fixed coordinate system \( X_1Y_1Z_1 \):

\[
\mathbf{E}_1 = \mathbf{M}_{1',1} \mathbf{E}'_1 = \begin{bmatrix}
x_1 + M_{11r} \\
y_1 + M_{21r} \\
z_1 + M_{31r} \\
1
\end{bmatrix} = \begin{bmatrix}
x_1 + M_{11r} \\
y_1 + M_{21r} \\
z_1 + M_{31r} \\
1
\end{bmatrix}
\]
\[ E_2 = M_{10} E'_2 = \begin{bmatrix} x_1 - 0.5r(M_{11} - \sqrt{3}M_{12}) \\ y_1 - 0.5r(M_{21} - \sqrt{3}M_{22}) \\ z_1 - 0.5r(M_{31} - \sqrt{3}M_{32}) \end{bmatrix} \]

\[ E_3 = M_{10} E'_3 = \begin{bmatrix} x_1 - 0.5r(M_{11} + \sqrt{3}M_{12}) \\ y_1 - 0.5r(M_{21} + \sqrt{3}M_{22}) \\ z_1 - 0.5r(M_{31} + \sqrt{3}M_{32}) \end{bmatrix} \]

where \( y_1 = -M_{12}r = -r \cos \psi \sin \alpha \).

The determination the coordinates of the hinges \( D_i \) in a fixed coordinate system \( X_1 Y_1 Z_1 \)

\[ D_1 = [R \ 0 \ 0 \ 1]^T \]
\[ D_2 = [-0.5R \ 0.5\sqrt{3}R \ 0 \ 1]^T \]
\[ D_3 = [-0.5R \ -0.5\sqrt{3}R \ 0 \ 1]^T \]

Substituting (5) and (6) in (4), it is obtained

\[ l_1 = \sqrt{(x_1 + M_{11}r - R)^2 + (z_1 + M_{31}r)^2}. \]
\[ l_2 = \left( (x_1 - 0.5r(M_{11} - \sqrt{3}M_{12}) + 0.5R)^2 + (y_1 - 0.5r(M_{21} - \sqrt{3}M_{22}) - \frac{\sqrt{3}}{2}R)^2 + \right) \]
\[ + \left( z_1 - 0.5r(M_{31} - \sqrt{3}M_{32}) \right)^2 \right)^{1/2}. \]
\[ l_3 = \left( (x_1 - 0.5r(M_{11} + \sqrt{3}M_{12}) + 0.5R)^2 + (y_1 - 0.5r(M_{21} + \sqrt{3}M_{22}) - \frac{\sqrt{3}}{2}R)^2 + \right) \]
\[ + \left( z_1 - 0.5r(M_{31} + \sqrt{3}M_{32}) \right)^2 \right)^{1/2}. \]

The coordinate space \( x', y', z' \) is characterized by array \( A \). Consider the construction of this array for some two-dimensional arbitrary area (Figure 3). The size of the array is determined by the approximation accuracy \( \delta \) and the interval of \( x \) and \( y \) coordinates. The array has \((\text{round} \left( \frac{x_{\text{max}}}{\delta} \right) - \text{round} \left( \frac{x_{\text{min}}}{\delta} \right) + 1)\) elements along the \( x \) axis and \((\text{round} \left( \frac{y_{\text{max}}}{\delta} \right) - \text{round} \left( \frac{y_{\text{min}}}{\delta} \right) + 1)\) elements along the \( y \) axis, initially all the elements of the array have the value 0. For the coordinates of each point \( A \) of the area, a row and a column of array \( B \) are calculated and 1 is assigned to this array element:

\[ B[\text{round} \left( \frac{x_A}{\delta} \right) - \text{round} \left( \frac{x_{\text{min}}}{\delta} \right) + 1, \text{round} \left( \frac{y_A}{\delta} \right) - \text{round} \left( \frac{y_{\text{min}}}{\delta} \right) + 1] = 1 \]

As a result, array elements with a value of 1 describe the area.
Taking into account formulas (2) - (3), (5) and (8) - (11), an algorithm for approximating the tripod module working tool error has been synthesized. For the approximation of the system of inequalities (5), an algorithm is used that works with the system of inequalities written in the general form:

\[
\begin{align*}
g_1(x) & \leq 0, \\
\vdots & \\
g_m(x) & \leq 0, \\
a_i & \leq x_i \leq b_i, i = 1, \ldots, n.
\end{align*}
\]

The initial box \(Q\), which contains all solutions of \(X\), is determined by the interval constraints \(a_i \leq x_i \leq b_i, i = 1, \ldots, n\). Consider an arbitrary box \(B\). Let \(m(B) = \max_{i=1,m} \min_{x \in B} g_i(x)\) and \(M(B) = \max_{i=1,m} \max_{x \in B} g_i(x)\). If \(M(B) \leq 0\), then \(B\) does not contain solutions of system (5). If \(M(B) \leq 0\), then each point of box \(B\) is a feasible solution. Therefore, it can be added to the cover as an inner box. In other cases, it is divided into two smaller boxes, if its diameter is greater than the specified approximation accuracy \(\delta\). Internal boxes describe the solutions of the system in the space \(\theta \psi z\). Then the solutions are transferred to the array \(\mathcal{P}\), which describes them in the coordinate space \(x, y, z\). The process of approximation of the system of inequalities (5) is repeated 2 times: taking into account the deviations of the geometric values of the lengths of the rods and for nominal sizes.

The algorithm works with two lists of six-dimensional boxes \(\mathcal{P}\) and \(\mathcal{P}_i\), while the list \(\mathcal{P}\) includes a set of boxes containing a set of \(O\) positions, the list \(\mathcal{P}_i\) includes a set of boxes containing a set of coordinates \(O\), satisfying condition (3). The algorithm works as follows:

1. At the beginning the internal approximation list \(\mathcal{P}_i\) is empty, the list \(\mathcal{P}\) consists of only one initial box \(Q\), that includes the whole range of angles \(\psi\) and \(\theta\) of the rotation of the platform \([-\pi/2, \pi/2]\) and the height of the platform \(z\).

2. Extract from the list \(\mathcal{P}_i\) a box \(B\).

3. Compute \(m(B)\), \(M(B)\) and the coordinates of \(O'(x_{\min}, y_{\min}, z_{\min})\) by the formula (2).

4. Compute the minimum and maximum values of the coordinates \(O'(x_{\min}, y_{\min}, z_{\min})\) and the maximum value of the coordinates \(O'(x_{\max}, y_{\max}, z_{\max})\).

5. If \(m(B) > 0\), \(B\) is excluded.

6. If \(M(B) \leq 0\), \(B\) is added to the \(\mathcal{P}_i\) list and the values \(x_{\min}, y_{\min}, z_{\min}, x_{\max}, y_{\max}, z_{\max}\) for the list \(\mathcal{P}_i\) are updated.

7. If \(d(Q_i) < \delta\), boxes \(Q_j, \ldots, Q_{m+j}\) are added to the \(\mathcal{P}_i\) list, the values \(x_{\min}, y_{\min}, z_{\min}, x_{\max}, y_{\max}, z_{\max}\) are updated \(\max\) for the \(\mathcal{P}_i\) list and proceeds to step 11.

![Figure 3](image-url)
8. In other cases, $B$ is divided into two equal boxes along the edge with the greatest length. These boxes are entered at the end of the list $\mathbb{P}$.

9. The algorithm is completed when the list $\mathbb{P}$ becomes empty; otherwise, steps 2–9 are repeated.

10. If $ES \neq 0$ or $EI \neq 0$ creates a binary array $A \left[ 1 \cdot (\text{round} \left( \frac{x_{\text{max}}}{\delta} \right) - \text{round} \left( \frac{x_{\text{min}}}{\delta} \right) + 1) \right]$, $i \cdot (\text{round} \left( \frac{y_{\text{max}}}{\delta} \right) - \text{round} \left( \frac{y_{\text{min}}}{\delta} \right) + 1)\ldots \right]$.  
11. Assign to all the array elements $A[i_x, i_y, i_z] = 0$, where $i_x \in 1, (\text{round} \left( \frac{x_{\text{max}}}{\delta} \right) - \text{round} \left( \frac{x_{\text{min}}}{\delta} \right) + 1), i_y \in 1, (\text{round} \left( \frac{y_{\text{max}}}{\delta} \right) - \text{round} \left( \frac{y_{\text{min}}}{\delta} \right) + 1), i_z \in 1, (\text{round} \left( \frac{z_{\text{max}}}{\delta} \right) - \text{round} \left( \frac{z_{\text{min}}}{\delta} \right) + 1)$.  
12. Remove the box $B_j$, $j \in 1, m_{1,1}$ from the $\mathbb{P}_i$ list and divide by a uniform grid on each of the axes. In this case, the dimension of the grid depends on the size of the box.

13. For all points $P_k \in B$ lying inside the box with coordinates $x_k, \psi_k, \theta_k$, we define the value $x_{aw}, y_{aw}, z_{aw}$.

14. If $ES \neq 0$ or $EI \neq 0$ we assign $A \left[ (\text{round} \left( \frac{x_{aw}}{\delta} \right) - \text{round} \left( \frac{x_{aw}}{\delta} \right) + 1), (\text{round} \left( \frac{y_{aw}}{\delta} \right) - \text{round} \left( \frac{y_{aw}}{\delta} \right) + 1)\ldots \right] = 1$

15. Otherwise, assign $A \left[ \text{round} \left( \frac{x_{aw}}{\delta} \right) - \text{round} \left( \frac{x_{aw}}{\delta} \right) + 1), (\text{round} \left( \frac{y_{aw}}{\delta} \right) - \text{round} \left( \frac{y_{aw}}{\delta} \right) + 1)\ldots \right] = 2$

16. Repeat steps 12-16 for all boxes $B_j$, $j \in 1, m_{1,1}$.

17. If $ES \neq 0$ or $EI \neq 0$, assign $ES = 0$ and $EI = 0$ and repeat steps 1-16.

18. Otherwise, the algorithm ends.

The simulation was performed for $l_1 = 520$ mm, $l_2 = 540$ mm, $l_3 = 500$ mm, $ES_i = -0.01$ mm, $EI_i = 0.01$ mm, $R = 430$ mm, $r = 100$ mm, $h = 150$ mm, $r_i = 6$ mm. The simulation results are presented in Figure 4. Purple parallelepipeds form the area in which p. $O'$ may be located, taking into account the rods errors, orange parallelepipeds with no errors. It can be seen from the figure that the error of the tripod module for the given parameters is $[-0.03 \text{mm}; + 0.03 \text{mm}]$ on each of the axes.

Figure 4. Simulation results: a - in the projection on the $XY$ plane, b - in the projection on the $XZ$ plane, c - in the projection on the $YZ$ plane.

3. Conclusion

To approximate the robot working tool positioning error, the developed algorithm is used, which includes transferring the constraints from the coordinate space of the moving tripod platform to the coordinate space of the tool cutting edge of the output link using an approximation set in the form of a three-dimensional array. The results can be used to select the geometric parameters of the module-tripod
and determine the accuracy of manufacturing parts of the robot at the design stage, ensuring the positioning errors of the working tool specified by technological processes.

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