A Supersymmetric Solution to CP Problems

Rabindra N. Mohapatra and Andrija Rašin

Department of Physics
University of Maryland
College Park, MD 20742

Abstract

We analyze the minimal supersymmetric left-right model with non-renormalizable interactions induced by higher scale physics and study its CP violating properties. We show that it: (i) solves the strong CP problem; (ii) predicts the neutron electric dipole moment well within experimental limits (thus solving the usual SUSY CP problem). In addition, it automatically conserves $R$-parity. The key points are that the parity symmetry forces the Yukawa couplings to be hermitean, while supersymmetry ensures that the scalar potential has a minimum with real higgs doublet vacuum expectation values. Gluino and B-L gaugino masses are automatically real. The observed CP violation in the kaon system comes, as in the Standard Model, from the Kobayashi-Maskawa-type phases. These solutions are valid for any value of the right-handed breaking scale $M_R$, as long as the effective theory below $M_R$ has only two Higgs doublets that couple fully to fermions. (i.e. the theory below $M_R$ is MSSM-like.) The potentially dangerous $SU(2)_L$ gaugino one-loop contributions to $\tilde{\Theta}$ below $M_R$ can be avoided if the left-right symmetry originates from a unified theory in which the $SU(2)_{L,R}$ gaugino masses are real. As an example, we show how the left-right symmetry can be embedded in an SO(10) theory.
1 Introduction

Quantum Chromodynamics (QCD) is now widely accepted as the theory of strong interactions. The periodic vacuum structure of QCD has however the unpleasant implication that strong interactions violate $CP$. This $CP$ violating interaction being flavor conserving only manifests itself as an electric dipole moment of the neutron and leads to a value far above the present experimental upper limit unless the associated $CP$ violating coupling (usually labelled as $\Theta$), which is left arbitrary by strong interaction dynamics, is somehow suppressed to the level of $10^{-9}$. This problem of fine tuning of the $\Theta$ parameter in gauge theories is known as the strong $CP$ problem \cite{1}. There are many solutions to the strong $CP$ problem\cite{1}: the most well-known of these is the Peccei-Quinn solution which requires the complete gauge theory of electroweak and strong interactions to respect a global chiral $U(1)$ symmetry. This symmetry must however be spontaneously broken in the process of giving mass to the W-boson and fermions leading to a pseudo-Goldstone boson in the particle spectrum known in the field as the axion. There are two potential problems with this otherwise beautiful proposal: (i) the axion has not been experimentally discovered as yet and the window is closing in on it; and (ii) if non-perturbative gravitational effects induced by black holes and wormholes are important in particle physics as is believed by some\cite{2}, then the axion solution would require fine tuning of the gravitationally induced couplings by some 50 orders of magnitude. This will make the axion theory quite contrived.

A second class of solution that does not lead to any near massless boson is to require the theory to be invariant under discrete symmetries\cite{3, 4}. In our opinion, the most physically motivated of such theories are the ones\cite{3} based on the left-right symmetric theories of weak interactions\cite{3}. These theories are based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ with quarks and leptons assigned in a left-right symmetric manner. Such models are also completely quark-lepton symmetric. To see how parity symmetry of the Lagrangian helps to solve the strong $CP$ problem, let us note that the physical QCD induced $CP$ violating phase can be written as

$$\Theta = \Theta + \text{Arg} \det(M_u M_d)$$

where $\Theta$ is the parameter in $F\tilde{F}$ part of the QCD Lagrangian, and $M_u, M_d$, are the up and down quark mass matrices respectively. Invariance under parity sets $\Theta = 0$ because
$F\bar{F}$ is odd under parity. Additionally, constraints of left-right symmetry imply that the Yukawa couplings of quarks responsible for the generation of quark masses are hermitean. If furthermore the vacuum expectation values (VEVs) of the Higgs fields responsible are shown to be real, then this would automatically lead to $\bar{\Theta} = 0$ at the tree level. If the one loop contributions also preserve the hermiticity of the quark mass matrices, then we have a solution to the strong $CP$ problem. In the nonsupersymmetric left-right models with nontrivial $CP$ violation, it is well known that in general VEVs of the Higgs field are not real. This, in the past led to suggestions that either new discrete symmetries be invoked together with left-right symmetry or new vectorlike fermions be added to the theory. Such theories also do not suffer from the Planck scale implied fine tunings. It always remained a challenge to solve the strong $CP$ problem only using left-right symmetry since often new additional symmetries invoked are not motivated from any other consideration.

A second $CP$ related problem is connected with the minimal supersymmetric standard model (MSSM), which is currently a subject of intense discussion next level of physics beyond the standard model and is the so called (usual) SUSY $CP$ problem. Namely, in the MSSM the complex phase in the gluino mass is arbitrary, and the one-loop gluino contribution to the neutron electric dipole moment is larger by two or three orders of magnitude than the experimental upper bound.

There have been many proposals in the literature to solve one or both of these problems. For instance, one recent suggestion is to consider a supersymmetric extension of the Peccei-Quinn symmetry which can solve the strong as well as the SUSY $CP$ problems. Another proposal in the context of grand unified models assumes $CP$ conserving gaugino masses at the GUT scale thereby solving only the SUSY $CP$ problem. Other proposals employ spontaneous breaking of $CP$ symmetry to achieve the same goal. None of the above approaches however address the important issue of $R$-parity conservation.

Our goal in this paper is to discuss a possible solution to both the strong $CP$ as well as the SUSY $CP$ problem in supersymmetry. The first point to note is that in supersymmetric theories, the $\bar{\Theta}$ receives an additional contribution from the phase of the gluino mass at the tree level:

$$\bar{\Theta} = \Theta + \text{Arg det}(M_u M_d) - 3\text{Arg} m_{\tilde{g}}$$

So any solution to strong $CP$ problem in supersymmetric theories must also require that the phase of the gluino mass must be naturally suppressed. Note that a solution to the
SUSY CP problem also requires the suppression of the same phase though to a lesser degree. Clearly therefore a solution to

In two recent letters[12, 13], it has been pointed out that if supersymmetry is combined with left-right symmetry, the strong CP problem is automatically solved without the need for any extra symmetry. Furthermore, in Ref. [12], it was pointed out that this model also provides a solution to the SUSY CP problem of MSSM, i.e. it does not lead to large electric dipole moment of the neutron. As a bonus, these models automatically conserve R-parity. In this paper we elaborate on the results of Ref.[12] and present some new ones which show the left-right scale independence of our result. We also discuss the question of possible embedding of left-right symmetry in grand unified theories.

This paper is organized as follows: in Sec 2, we discuss our supersymmetric solution to the strong CP problem; in Sec 3, we show how the solution remains regardless of whether the right-handed scale $M_R$ is in the TeV range or much higher; in Sec 4, we discuss our solution to the usual SUSY CP problem; in Sec 5, we show how the theory can be embedded into the $SO(10)$ model; in Sec 6, we give our conclusions. We discuss the question of potential minimization in Appendix A; show the reality of Higgs VEVs in Appendix B; list the evolution equations for Yukawa couplings for a general four doublet expansion of MSSM in Appendix C; and discuss the doublet-doublet splitting in Appendix D.

2 Supersymmetric Solution to the Strong CP Problem

Let us recall the arguments of Ref. [12] and see how the supersymmetric left-right model solves the strong CP problem at the scale $M_R$.

The gauge group of the theory is $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ with quarks and leptons transforming as doublets under $SU(2)_{L,R}$. In Table 1, we denote the quark, lepton and Higgs superfields in the theory along with their transformation properties under the gauge group. Note that we have chosen two bidoublet fields to obtain realistic quark masses and mixings (one bidoublet implies a Kobayashi-Maskawa matrix proportional to unity, because supersymmetry forbids $\hat{\Phi}$ in the superpotential).

The superpotential for this theory is given by (we have suppressed the generation
SU(2)_L × SU(2)_R × U(1)_{B-L} representation

| Fields | (2,1,+ 1/3) | (2,1,− 1/3) | (2,1,− 1) | (1,2,+ 1) | (2,2,0) | (3,1,+ 2) | (3,1,− 2) | (1,3,+ 2) | (1,3,− 2) |
|--------|--------------|--------------|------------|-----------|--------|----------|----------|----------|----------|
| Q      |              |              |            |           |        |          |          |          |          |
| Q^c    |              |              |            |           |        |          |          |          |          |
| L      |              |              |            |           |        |          |          |          |          |
| L^c    |              |              |            |           |        |          |          |          |          |
| Φ_{1,2}|              |              |            |           |        |          |          |          |          |
| ∆      |              |              |            |           |        |          |          |          |          |
| ∆^c    |              |              |            |           |        |          |          |          |          |
| ∆^c    |              |              |            |           |        |          |          |          |          |

Table 1: Field content of the SUSY LR model

\[
W = Y^{(i)}_q Q^T \tau_2 \Phi_i \tau_2 Q^c + Y^{(i)}_l L^T \tau_2 \Phi_i \tau_2 L^c + i(f L^T \tau_2 \Delta L + f_c L^c T \tau_2 \Delta^c L) + \mu_\Delta \text{Tr}(\Delta \bar{\Delta}) + \mu_{\Delta^c} \text{Tr}(\Delta^c \bar{\Delta}^c) + \mu_{ij} \text{Tr}(\tau_2 \Phi_i^T \tau_2 \Phi_j) + W_{NR}
\]

where \(W_{NR}\) denotes non-renormalizable terms arising from higher scale physics such as grand unified theories or Planck scale effects. At this stage all couplings \(Y^{(i)}_q, Y^{(i)}_l, \mu_\Delta, \mu_{\Delta^c}, f, f_c\) are complex with \(\mu_{ij}, f\) and \(f_c\) being symmetric matrices.

The part of the supersymmetric action that arises from this is given by

\[
S_W = \int d^4x \int d^2\theta W + \int d^4x \int d^2\bar{\theta} W^\dagger.
\]

The terms that break supersymmetry softly to make the theory realistic can be written as

\[
\mathcal{L}_{\text{soft}} = \int d^4\theta \sum_i m_i^2 \phi_i^4 \phi_i + \int d^2\theta \bar{\theta}^2 \sum_i A_i W_i + \int d^2\theta \bar{\theta}^2 \sum_i A_i^* W_i^\dagger + \int d^2\theta \bar{\theta}^2 \sum_p m_p \bar{W}_p W_p + \int d^2\theta \bar{\theta}^2 \sum_p m_p^* \bar{W}_p^* W_p^*.
\]

In Eq. 5, \(\bar{W}_p\) denotes the gauge-covariant chiral superfield that contains the \(F_{\mu\nu}\)-type terms with the subscript going over the gauge groups of the theory including SU(3)_c.
$W_i$ denotes the various terms in the superpotential, with all superfields replaced by their scalar components and with coupling matrices which are not identical to those in $W$. Eq. 5 gives the most general set of soft breaking terms for this model.

In Sec. 1 we saw that left-right symmetry implies that the first term in Eq. 1 is zero. Let us now see how supersymmetric left-right symmetry also requires the second term in this equation to vanish naturally. We choose the following definition of left-right transformations on the fields and the supersymmetric variable $\theta$

\[
\begin{align*}
Q & \leftrightarrow Q^c \\
L & \leftrightarrow L^c \\
\Phi_i & \leftrightarrow \Phi_i^\dagger \\
\Delta & \leftrightarrow \Delta^c \dagger \\
\bar{\Delta} & \leftrightarrow \bar{\Delta}^c \dagger \\
\theta & \leftrightarrow \bar{\theta} \\
\tilde{W}_{SU(2)} & \leftrightarrow \tilde{W}_{SU(2)}^* \\
\tilde{W}_{B-L,SU(3)} & \leftrightarrow \tilde{W}_{B-L,SU(3)}^*
\end{align*}
\]

Note that this corresponds to the usual definition $Q_L \leftrightarrow Q_R$, etc. With this definition of L-R symmetry, it is easy to check that

\[
\begin{align*}
Y^{(i)}_{q,l} & = Y^{(i)^\dagger}_{q,l} \\
\mu_{ij} & = \mu_{ij}^* \\
\mu_{\Delta} & = \mu_{\Delta c}^* \\
f & = f_c^* \\
m_{\lambda_{SU(2)} L} & = m_{\lambda_{SU(2)} R}^* \\
m_{\lambda_{B-L,SU(3)} C} & = m_{\lambda_{B-L,SU(3)} C}^* \\
A_i & = A_i^\dagger
\end{align*}
\]

We will make extensive use of equations (7) in this paper\footnote{Note that the dagger in the last equation for A-terms indicates that squark mass matrices $h$ are hermitian by L-R symmetry, although they of course do not have to be proportional to Yukawa mass matrices below some high scale.}. The first point to note
is that the gluino mass is automatically real in this model; as a result, the last term in the equation for $\Theta$ above is naturally zero. We now therefore have to investigate only the quark mass matrices in order to guarantee that $\Theta$ vanishes at the tree level. For this purpose, we note that the Yukawa matrices are hermitean and the mass terms involving Higgs bidoublets in the superpotential are real. If we can show that the vacuum expectation values of the bi-doublets are real, then the tree level value of $\Theta$ will be naturally zero.

As in [12], for $W_{NR}$ we will use a single operator $\lambda M [\text{Tr}(\Delta^c \tau_m \bar{\Delta}^c)]^2$, in order to be able to have vanishing sneutrino VEVs, as shown in Appendix A. The $M$ could be equal to $M_{Pl}$ or $M_U$. The other allowed non-renormalizable operators do not effect our result and could be easily included in our discussion.

In this case we have made a detailed analysis of the Higgs potential and find that, at the minimum of the potential, the $<\Phi_i>$ are always real. This result is not at all trivial because of large number of VEVs that enter and one might naively think that spontaneous $CP$ violation is possible. However, a recent analysis [14] has shown that a general supersymmetric model with two pairs of Higgs doublets (of which SUSY LR is a special case) cannot break $CP$ spontaneously. We give the application of this calculation to the SUSY LR case in Appendix B. It is now clear that the quark mass matrices are hermitean and therefore $\Theta = 0$ naturally at the tree level.

In Ref. [12] it was also shown that no strong $CP$ violating phase is generated at the one-loop level. Examples of one-loop diagrams are shown in Figures 1 and 2; the higgs diagram (Figure 1) and some of the gaugino diagrams (Figure 2a) generate only hermitean contributions, while the other gaugino diagrams (Figure 2b) is always real, if the gaugino masses are assumed to be real, as it happens when our model is embedded into a grand unified theory (see later). Thus in the total contribution at one-loop level the Yukawa matrices are still hermitean. We concluded that the first nonzero contribution to $\Theta$, if any, arises only at the two loop level, and is thus consistent with present limits.

---

2It is interesting that more general definitions of left-right transformations in the flavor sector are possible. For example, invariance under $Q \rightarrow U_1 Q^r$ and $Q^c \rightarrow U_2 Q^c$, where $U_1$ and $U_2$ are some $SU(3)$ matrices, gives non-hermitean Yukawa matrices, but which still have a real determinant.
3 Solution to the Strong $CP$ Problem Holds Below Scale $M_R$

We have seen how the supersymmetric left-right model solves the strong $CP$ problem at the scale of the $SU(2)_R$ breaking $M_R$. If this scale is of the order of the weak scale then we are done, because the mass matrices in the expression for $\Theta$ are defined at that scale, and no further phase can be generated. Let us investigate what happens if $M_R$ is some higher (intermediate) scale. Two questions must be answered:

- Does the determinant of the Yukawa matrices stay real below $M_R$?

- The one-loop contribution of the $SU(2)_L$ gaugino is no longer cancelled by the heavy $SU(2)_R$ gaugino. Can we avoid this contribution?

As we will show below the answer to both questions is yes.

Above the scale $M_R$ the hermiticity property of Yukawa couplings stays intact because L-R symmetry is not broken (see Appendix C). However, running of the Yukawa matrices below $M_R$ will necessarily spoil the hermiticity of Yukawa matrices because of breaking of parity (for example the right handed neutrino is excluded in running below $M_R$). Thus one might naively think that a nontrivial $\Theta$ will be generated, and that one must put constraints on $M_R$. However, we will now show that for the simplest case, when the field content below $M_R$ is that of the MSSM, namely two higgs doublets, the determinants of the Yukawa matrices stay real.

Let us denote by $y_i$ the Yukawa coupling of a Higgs doublet $H_i$ ($i=1,2$). In the MSSM the one-loop running of the Yukawa couplings is of the form:\cite{15}:

$$\frac{dy_i}{dt} = y_i T$$

where $T$ is a matrix in flavor space which is a sum of terms of the form $y_j^\dagger y_j$, $\text{Tr}(y_j^\dagger y_j)1$, $g_\alpha^21$ (see Appendix C). From (8) one can easily obtain the Jacobi identity for the determinant

$$\frac{d}{dt} \det y_i = \det y_i \text{Tr} T$$

\cite{15}

\footnote{Such scales can be desired in grand unification schemes with LR models as intermediate steps, because of the seesaw scenarios of neutrino masses.}
However, $\text{Tr} \mathbf{T}$ is always real, and since the determinant of $y_1$ is real at the scale $M_R$, it will be real at any scale below $M_R$. We conclude that although the Yukawa matrices will in general not be hermitean anymore at the lower scale, their determinants will nevertheless stay real.

The VEVs of the higgs doublets in MSSM can always be rotated so that both are real. Thus we conclude that $\bar{\Theta}_{\text{tree}} = 0$.

Let us consider the one-loop contributions to $\bar{\Theta}$ below $M_R$. Typical diagrams that contribute at scale $M_R$ are shown in Figs. 1 and 2. Since the running Yukawa matrices have a real determinant the diagrams that have at vertices only Yukawa matrices or bidoublet masses (which are real) will not contribute. However, since the right-handed gaugino will decouple below $M_R$, the phase in the diagram involving the mass of the left- gaugino will not cancel. The easiest way to circumvent this problem is to assume that the gaugino masses are real\[13\]. It is then easy to see that in the one-loop running the left gaugino mass stays real. Indeed, in Section 5 we show that the reality of the $SU(2)_{L,R}$ gaugino masses comes out naturally in an SO(10) model with a generalized left-right symmetry.

Let us next address the effect of the trilinear supersymmetry breaking term involving squarks and the Higgs boson (i.e. $h_u m_0 \tilde{Q} H_u \tilde{u}$ and the corresponding term with $u$ replaced by $d$) on $\bar{\Theta}$. Above the $M_R$ scale, the matrices $h_{u,d}$ are hermitean due to the constraint of left-right symmetry (like the $Y_{u,d}$). Therefore their contribution to $\bar{\Theta}$ involving the gluino at the one-loop level automatically vanishes above the scale $M_R$. (Here we used the fact that left-right symmetry requires that the gluino masses are real). As we extrapolate it down to the $M_Z$ scale using the renormalization group equations[15], we have to see if the det $h_{u,d}$ develop any imaginary part. A look at the one-loop renormalization group equation makes it clear that such an imaginary part (denoted by $\delta_A$) could develop; let us therefore estimate its effect on the gluino mass as well as the quark mass matrices. A rough order of magnitude of the CP violating phase in the gluino mass can be estimated as follows: since the $h_{u,d}$ are hermitean and proportional to the Yukawa couplings $Y_{u,d}$ at some scale above the $M_R$ scale, let us go to a basis where $Y_d$ and $h_d$ are diagonalized. Then we find that, at the scale of proportionality, if any one of the off-diagonal elements of $Y_u$ and (hence $h_u$) are set to zero, the theory becomes completely CP conserving and cannot generate a CP violating phase at any scale below
It is then clear that the one loop graph that generates a phase in the gluino mass can lead to the gluino phase \( \delta_\tilde{g} \) which is at most

\[
\delta_\tilde{g} \simeq \frac{V_{ub}V_{bc}V_{cd}V_{du}\alpha_s}{64\pi^3} \ln \frac{M_R}{M_Z}
\]

leading to \( \delta_\tilde{g} \leq 10^{-8} \) which is close to the upper limits on the \( \tilde{\Theta} \). Similar arguments can be given for the one loop contribution to the \( \tilde{Q}\tilde{Q}^c \) mass matrix to show that their contribution to \( \tilde{\Theta} \) is around \( 10^{-8} \).

It is worth pointing out at this stage that in the above discussion we have assumed that the theory below \( M_R \) is the MSSM (except of course the fact that the “obnoxious” R-parity violating terms are naturally absent). In Appendix D, we discuss one way of obtaining MSSM in the framework of our model.

At the end, let us consider what happens if we consider an effective four higgs doublet model below \( M_R \). The running of Yukawa couplings at one loop are listed in Appendix C. We note that the running of Yukawa matrices does not have a form of \( \langle 8 \rangle \). There are additional terms on the right hand side of the form \( y_j \text{Tr}(y_j^\dagger y_i) \) \( (i \neq j) \), thus invalidating the Jacobi identity for determinants. Indeed, such a term will in general produce phases of order \( V_{cb}^2/(16\pi^2) \approx 10^{-5} - 10^{-6} \). In this case, we also expect additional suppression coming from the fact at some very high scale, the theory becomes CP conserving if any off-diagonal element of \( Y_u \) is set to zero. Barring enough suppress from this, it may be necessary to impose some additional symmetry to suppress the Yukawa couplings of the second pair of Higgs doublets\[14\].

In conclusion, if the effective theory below \( M_R \) has the MSSM-like field content and if the left gaugino mass is real, no observable \( \tilde{\Theta} \) will be generated for all values of \( M_R \) from some intermediate scale (\( \approx 10^{12} \) GeV) all the way down to 1 TeV.

\[4\] Note however that in the second paper of Ref. \[14\], a general four higgs doublet model with arbitrary Yukawa couplings was considered, and the additional symmetry was needed to suppress too large CP violation in \( K\bar{K} \) mixing; strong CP violation was too large in that model. In our case Yukawa couplings have the constraint that they come from hermitian matrices at the \( M_R \) scale, and the additional global symmetry is enough to solve the strong CP problem.
4 Solution to the SUSY CP Problem

Let us now turn to the discussion of the SUSY CP problem. The main issue here is the potentially large contribution to the electric dipole moment of the neutron at the one-loop level. An analysis of the various aspects of the problem has been reviewed in Ref.\[16\]. In the standard parameterization of the MSSM interactions at the electroweak scale, the large contributions to \(d^n_e\) comes from two sources: the phases of the \((Am_\tilde{g})\) and \((\mu v_\nu d/v_d)\) terms. Another way to state this is to note that the first term originate from the same trilinear scalar susy breaking terms \(h_u,d\) discussed in the previous section, whereas the second term arises from the F-term contribution extrapolated down to the electroweak scale. We work in a basis where the diagonal block matrices in the squark \((\tilde{q} - \tilde{q}^c)\) mass matrices are diagonalized. We will then be interested in the 11 entry of the gluino one loop contribution to electric dipole moment operator for both the up and the down sector.

First point to note is that in our model, above the \(M_R\) scale, the hermiticity of \(h_u,d\) and \(Y_{u,d}\) together with the reality of the gluino mass implies that there is no one-loop contribution to \(d^n_e\). Garisto\[16\] has argued that if the above parameters are real at any high energy scale, their contribution to \(d^n_e\) remains small at the electroweak scale. For example, in our case as shown above, the phase of the gluino mass at the scale \(M_Z\) is of order \(10^{-8}\). As far as the the \(h_{u,d}\) and \(Y_{u,d}\) terms are concerned, we have not succeeded in showing that once extrapolated down to the \(M_Z\) scale, the 11 term of the gluino induced dipole moment matrix remains real. However, using the already stated argument above, the hermiticity of the Yukawa matrices above the scale \(M_R\) implies that that any departure from reality is at most of order \(\delta \equiv V_{ub}V_{bc}V_{cd}/(16\pi^2)ln(M_R/M_Z) \approx 10^{-8}\). We then expect that the maximum contribution to \(d^n_e\) from them to

\[
(d^n_e)^{max} \leq \frac{8e\alpha_s m_d}{27\pi M_\tilde{q}^2}\delta \leq 4 \times 10^{-31} \text{ ccm}
\]  

where we have assumed that \(m_d \approx 10\) MeV and \(M_\tilde{q} \approx 100\) GeV. This is safely within the present experimental upper limit. Thus our model provides simultaneously a solution to the SUSY CP problem without the need for any new symmetries.

Let us note that in this paper we do not address the usual SUSY flavor problems with \(K\bar{K}\) mixings, etc., that require a certain degree of fine tuning in the structure
of squark and quark matrices, since this is beyond the scope of our paper. Left-right symmetry implies that the Yukawa matrices be hermitean, but the flavor properties such as hierarchy and alignment must come from the underlying flavor theory, and we refer the reader to the existing solutions [17] which may be employed here as well.

5 \textit{SO}(10) Embedding and Reality of Weak Gaugino Masses

In this Section, we address the question of embedding the left-right model into an \textit{SO}(10) theory so that we not only have a grand unified version of our theory but also we guarantee the reality of the gaugino masses (i.e.\(m_{\lambda_{L,R}} = m_{\lambda_{L,R}}^*\)). The reality of the gaugino masses follow from the combination of two things: the requirement of left-right symmetry implies as shown earlier that \(m_{\lambda_{L}} = m_{\lambda_{R}}^*\); on the other hand \textit{SO}(10) unification implies that the two gauginos being part of the same 45 dimensional representation have equal mass. The main task for us in this section is to show that there exists a definition of left-right symmetry which preserves the hermiticity of the Yukawa couplings.

To prove the hermiticity of the Yukawa couplings, we will exhibit only the simplest model and not attempt to address the issues such as doublet triplet splitting etc. Let us consider the Higgs fields belonging to 10 (denoted by \(H\)), 45 (denoted \(A\)) and 126 (denoted by \(\Delta\))(plus \(\bar{\Delta}\)) representations. The superpotential involving all these fields can be written as:

\[
W_{\text{GUT}} = h^s_{ab}\psi_a^T B\Gamma_i\psi H_i + h^A_{ab}\psi_a^T B\Gamma_i\Gamma_j\Gamma_k\psi_b(H_i A_{jk})_{(Anti)}/M
+ h'_s\psi_a^T B\Gamma_i\psi H_j A_{ij}/M + f_{ab}\psi_a^T B\Gamma_i\Gamma_j\Gamma_k\Gamma_m\psi_b\Delta_{ijklm}
+\text{terms involving }\Delta \text{ in order } \frac{1}{M}
\]

It is well known that \(h^s\), \(h'\) and \(f\) are symmetric matrices whereas \(h^A\) is an antisymmetric combination since we have projected out the 120 dim. representation from the 10 and 45 product in the \(h^A\) term. Let us now define that under parity transformation

\[
\psi \rightarrow D\psi B^{-1}\psi^*, \quad H_i \rightarrow -D_H H_i^* D_H^{-1}, \quad A_{ij} \rightarrow -D_A A_{ij}^* D_A^{-1}, \quad \Delta \rightarrow -D_{\Delta}\Delta^* D_{\Delta}^{-1}
\]

Here \(B\) is the charge conjugation matrix for \textit{SO}(10); \(D\) is the operator that implements
the left-right transformation inside the $SO(10)$ multiplets $\psi$, $H$ etc. For instance operating on the $10$ dimensional representation $(H_i)$, it changes $H_7 \rightarrow -H_7$ and leaves all other components unchanged; Similarly in $45$, it changes the sign of all elements that carry the index $7$ etc. (Note that the choice of the $7$th component is basis dependent and we are working in a basis where $I_{3L} = \frac{1}{4} (\Sigma_{90} - \Sigma_{78})$ and $I_{3R} = \frac{1}{4} (\Sigma_{90} + \Sigma_{78})$, where $\Sigma_{ij}$ are the generators of $SO(10)$.)

Now, using the fact that $B^T = -B$ and $B^{-1} \tau_i B = -\Gamma_i^T$, it is then easy to show that $h_{ab}^s$, $h_{ab}'$ and $f_{ab}$ are real whereas $h_{ab}^A$ is imaginary. Together with the symmetricity properties this implies that all this matrices are hermitean.

Now if the $SO(10)$ symmetry is broken down to $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$ by the $45$ VEV as $\langle A \rangle = i\tau_2 \text{diag}(v, v, v, 0, 0)$, then the effective low energy theory has two bi-doublets and also general hermitean Yukawa coupling of quarks. This leads to the embedding of our solution to the strong CP problem in an $SO(10)$ model.

We do not discuss here the unification of gauge couplings in theories with $SU(2)_L \times SU(2)_R$ as intermediate symmetries, but note that examples of successful scenarios exist which could implement our mechanism [18].

## 6 Conclusion

We have shown that if the minimal supersymmetric extension of the standard model (MSSM) is embedded in the supersymmetric left-right model at higher energies, both the strong and weak CP problems of the MSSM are automatically cured. Adding this to the already known result that the R-parity conservation is restored as an exact symmetry in the SUSYLR model, thereby providing a naturally stable neutralino that can act as the cold dark matter of the universe, makes this embedding quite attractive. The left-right symmetry is then incorporated into an $SO(10)$ grand unified theory where the scale of right handed symmetry breaking may be quite high. We show how the conclusion about the vanishing of the strong CP parameter remains unchanged in this case.

---

5This is similar to the L-R definition [3]. This comes because, for example, strictly speaking $Q^c$ is not a doublet under $SU(2)_R$, but rather $Q^c \equiv \tau_2 Q^c$. So the L-R definition [3] in terms of the gauge multiplets would be: $Q \leftrightarrow \tau_2 Q^{c*}$. The operator $\tau_2$ plays a similar role as the operator $D$ above in the $SO(10)$ case.
Acknowledgments

A.R. thanks Markus Luty and Jogesh Pati for useful discussions. R.N.M. would like to thank Alex Pomarol for several important comments and discussions. This work was supported by the NSF grant No. PHY 9421385. The work of R.N.M. is also partially supported by the Distinguished Faculty Research award by the University of Maryland. R.N.M. would like to acknowledge the hospitality of the CERN Theory Division during the last part of the work.

APPENDIX A: Avoiding Sneutrino VEVs

In this Appendix we will show that if in the minimal SUSY LR model one includes non-renormalizable Planck scale induced terms, the ground state of the theory can be $Q^m$ conserving even for $<\tilde{\nu}^c>=0$. For this purpose, let us briefly recall the argument of Ref. [19]. The part of the potential containing $\tilde{l}^c$, $\Delta^c$ and $\bar{\Delta}^c$ fields only has the form (see Appendix B or [19])

$$V = V_0 + V_D,$$ (14)

where

$$V_0 = \text{Tr}[i\tilde{f}^\dagger L^c (L^c)^T \tau_2 + \mu^2_\Delta \bar{\Delta}^c |^2 + \mu_2^2 \text{Tr}(\Delta^c \bar{\Delta}^c) + \mu_2^2 \text{Tr}(\bar{\Delta}^c \Delta^c)] + \mu_3^2 \text{Tr}(\Delta^c \bar{\Delta}^c) + \mu_4 \bar{L}^c \tau_2 \Delta^c L^c,$$ (15)

and

$$V_D = \frac{g^2}{8} \sum_m |\tilde{L}^c \tau_m \bar{L}^c + \text{Tr}(2\Delta^c \tau_m \Delta^c + 2\bar{\Delta}^c \tau_m \bar{\Delta}^c)|^2 + \frac{g^2}{8} |\tilde{L}^c \bar{L}^c - 2 \text{Tr}(\Delta^c \bar{\Delta}^c - \bar{\Delta}^c \Delta^c)|^2.$$ (16)

Note that if $<\tilde{\nu}^c>=0$ then the vacuum state for which $\Delta^c = \frac{1}{\sqrt{2}} v\tau_1$ and $\bar{\Delta}^c = \frac{1}{\sqrt{2}} v'\tau_1$ is lower than the vacuum state $\Delta^c = v \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ and $\bar{\Delta}^c = v' \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$. However, the
former is electric charge violating. The only way to have the global minimum conserve electric charge is to have $<\bar{\nu}_c>\neq 0$. On the other hand, if we have non-renormalizable terms included in the theory, the situation changes: for instance, let us include non-renormalizable gauge invariant terms of the form (inclusion of other non-renormalizable terms simply enlarges the parameter space where our conclusion holds):

$$W_{NR} = \frac{\lambda}{M} [\text{Tr}(\Delta^c \tau_m \Delta^c)]^2.$$  \hfill (17)

This will change $V$ to the form:

$$V = V_0 + V_{NR} + V_D,$$ \hfill (18)

where $V_0$ and $V_D$ are given before and $V_1$ is given by

$$V_{NR} = \frac{\lambda \mu}{M} [\text{Tr}(\Delta^c \tau_m \Delta^c)]^2 + \frac{4\lambda \mu \Delta}{M} [\text{Tr}(\Delta^c \tau_m \Delta^c)][\text{Tr}(\Delta^c \tau_m \Delta^c)] + \Delta^c \leftrightarrow \bar{\Delta}^c + \text{etc.}$$ \hfill (19)

For the charge violating minimum above, this term vanishes but the charge conserving minimum receives a nonzero contribution. Note that the sign of $\lambda$ is arbitrary and therefore, by appropriately choosing $\text{sgn}(\lambda)$ we can make the electric charge conserving vacuum lower than the $Q^em$-violating one. In fact, one can argue that, since we expect $v^2 - v'^2 \approx \frac{f^2(M_{SUSY})^2}{16\pi^2}$ in typical Polonyi type models, the charge conserving minimum occurs for $f < 4\pi \left(\frac{4\lambda \mu \Delta}{M}ight)^\frac{1}{2} \frac{v}{M_{SUSY}}$. For $\lambda \approx 1$, $\mu \Delta \approx v \approx M_{SUSY} \approx 1\text{TeV}$ and $M = M_{Pl}$, we get $f \leq 10^{-3}$ if $v - M_{SUSY}$. We have assumed that the right handed scale is in the TeV range. The constraint on $f$ of course becomes weaker for larger values of $\mu \Delta$. We wish to note that a possible non-renormalizable term of the form $\lambda_1 \text{Tr}(\Delta^c \tau_m \Delta^c)\text{Tr}(\Phi_i \tau_m \Phi_j)\frac{1}{M_{Pl}}$ does induce a complex effective mass for the bidoublets but its magnitude is given by $\frac{v^2_{R}}{M_{Pl}}$ which for $v_R \leq 10^{10}/\lambda_1$ GeV gives a phase of order $10^{-9}$ and is negligible if $v_R$ is in the TeV range. Its presence therefore does not affect the solution to the strong $CP$ problem outlined in the paper for all values of $v_R$ from TeV up to some intermediate scale $\approx 10^{11} - 10^{12}$ GeV, depending on the value of $\lambda_1$.

Furthermore, it is also important to point out that since Planck scale effects are not expected to respect any global symmetries, the coupling parameters of the higher dimensional terms in Eq. (16) involving $\Delta$ and $\Delta^c$ will be different. This difference will help in the realization of the parity violating minimum as the global minimum of the theory.
APPENDIX B: Reality of Bidoublet VEVs

Here we show that the VEVs of the bidoublet Higgs fields in the supersymmetric left-right model are real. The scalar potential is given by

\[ V = V_F + V_{\text{soft}} + V_D + V_{NR}(\Delta^c, \bar{\Delta}^c), \]  

(20)

where

\[
V_F = \sum_p |Y_q^{(i)} p r \Phi_i \tau_2 Q^c_r|^2 + \sum_r |Y_l^{(i)} p r \Phi_i \tau_2 L^c_r|^2 + \sum_p |Y_l^{(i)} p r \Phi_i \tau_2 L^c_r|^2 + 2\mu_{ij}\Phi_j|^2 \\
+ \sum_p |Y_q^{(i)} p r \Phi_i \tau_2 Q^c_r|^2 + \sum_r |Y_l^{(i)} p r \Phi_i \tau_2 L^c_r|^2 + \sum_p |Y_l^{(i)} p r \Phi_i \tau_2 L^c_r|^2 + 2\mu_{ij}\Phi_j|^2 \\
+ |\mu_\Delta|^2 \text{Tr}(\Delta \bar{\Delta} + \Delta^c \bar{\Delta}^c),
\]

(21)

\[ V_{\text{soft}} = m_q^2 (\bar{Q}^T Q + \bar{Q}^c T Q^c) + m_{\Phi_i}^2 (\bar{\Phi}^T \Phi + \bar{\Phi}^c T \Phi^c) + m_{\Phi_i}^2 \Phi_i T \Phi_i \\
+ m^2_\Delta \text{Tr}(\Delta \bar{\Delta} + \Delta^c \bar{\Delta}^c) + m^2_\Delta \text{Tr}(\Delta \bar{\Delta} + \Delta^c \bar{\Delta}^c) \\
+ [A_{q,i} Y_q^{(i)} \bar{Q}^T \tau_2 \Phi_i \tau_2 Q^c + A_{l,i} Y_l^{(i)} \bar{L}^T \tau_2 \Phi_i \tau_2 L^c \\
+ A_{L,i} (f^T \bar{L}^T \tau_2 \Delta L + f^T \bar{L}^T \tau_2 \Delta^c L^c) + A_{\Delta} (\mu_\Delta \text{Tr}(\Delta \bar{\Delta}) + \mu_\Delta^* \text{Tr}(\Delta^c \bar{\Delta}^c)) + A_{\Phi} \mu_{ij} \text{Tr}(\tau_2 \Phi_i^T \tau_2 \Phi_j) + \text{h.c.}],
\]

(22)

\[ V_D = \frac{g^2}{8} \sum_m |\bar{L}^T \tau_m L + \text{Tr}(2\Delta^T \tau_m \Delta + 2\bar{\Delta}^T \tau_m \bar{\Delta} + \Phi^T \tau_m \Phi)|^2 \\
+ \frac{g^2}{8} \sum_m |\bar{L}^c T \tau_m L^c + \text{Tr}(2\Delta^c T \tau_m \Delta^c + 2\bar{\Delta}^c T \tau_m \bar{\Delta}^c + \Phi^T \tau_m \Phi)|^2 \\
+ \frac{g^2}{8} |\bar{L}^T \tau_m L^c - \bar{L}^T \bar{L} + 2\text{Tr}(\Delta^T \Delta - \Delta^c T \Delta^c - \bar{\Delta}^T \bar{\Delta} + \bar{\Delta}^c T \bar{\Delta}^c)|^2,
\]

(23)

and \( V_{NR} \) is defined in Appendix A.
We assume the following fields get the VEVs:

\[
<\Delta^c> = \begin{pmatrix}
0 & 0 \\
\Delta_0 e^{-i\beta} & 0 \\
0 & 0
\end{pmatrix},
<\bar{\Delta}^c> = \begin{pmatrix}
0 & \delta_0 \\
0 & 0
\end{pmatrix},
\]

and

\[
<\Phi_1> = \begin{pmatrix}
v_1 & 0 \\
0 & v_2 e^{i\delta_2}
\end{pmatrix},
<\Phi_2> = \begin{pmatrix}
v_3 e^{i\delta_3} & 0 \\
0 & v_4 e^{i\delta_4}
\end{pmatrix},
\]

where we have rotated away the nonphysical phases.

The VEV of the scalar potential is

\[
<V> = |2\mu_{11} v_1 + 2\mu_{12} v_3 e^{i\delta_3}|^2 + |2\mu_{11} v_2 e^{i\delta_2} + 2\mu_{12} v_4 e^{i\delta_4}|^2
+ |\mu_{\Delta}|^2 (\Delta_0^2 + \delta_0^2)
+ m_\Delta^2 \Delta_0^2 + m_\Delta^2 \delta_0^2
+ m_{\Phi_1}(v_1^2 + v_2^2) + m_{\Phi_2}(v_3^2 + v_4^2) + A_\Delta |\mu_\Delta| \Delta_0 \delta_0 \cos(\beta_\Delta + \text{Arg}(\mu_\Delta))
+ A_\Phi \mu_{11} 2 v_1 v_2 \cos \delta_2 + A_\Phi \mu_{12} 2 (v_1 v_4 \cos \delta_4 + v_2 v_3 \cos (\delta_2 + \delta_3))
+ A_\Phi \mu_{22} 2 v_3 v_4 \cos(\delta_3 + \delta_4) + <V_D> + V_{NR}(<\Delta^c>, <\bar{\Delta}^c>)
\]

where \(<V_D>\) is the VEV of the D-term

\[
<V_D> = \frac{g^2}{8} [v_1^2 + v_3^2 - v_2^2 - v_4^2]^2
+ \frac{g^2}{8} [2(\Delta_0^2 + \delta_0^2) + v_1^2 + v_3^2 - v_2^2 - v_4^2]^2
+ \frac{g^2}{8} [2(\Delta_0^2 + \delta_0^2)]^2.
\]

Note that the phases of the bidoublets \(\delta_i, i = 2, 3, 4\) come in the following terms

\[
v_1 v_i \cos \delta_i, \ i = 2, 3, 4
v_2 v_3 \cos(\delta_2 + \delta_3)
\]

\[
v_2 v_4 \cos(\delta_2 - \delta_4)
v_3 v_4 \cos(\delta_3 + \delta_4)
\]

Also, powers of the bidoublet VEVs which are higher than two come only in the D-term, and there in one only combination \(g(v) = v_1^2 + v_3^2 - v_2^2 - v_4^2\). This is exactly the situation
in general four Higgs doublet supersymmetric models with real mass parameters. In Ref. [14] it was shown, by using a simple geometrical interpretation for the minimum equations for the three phases, that the minimum in such a model is $CP$ conserving. Thus we conclude that in the SUSY LR model the VEVs of the doublets are real. This conclusion holds for general $A_{\phi i j}$, which can be different for different $i,j$.

The phase of the VEV of the triplet $\beta_\Delta$ is in general non-zero (e.g. induced by the phase of the coupling $\mu_\Delta$) but it does not couple to the VEVs of the doublets. Thus it is irrelevant since it does not enter the calculation of $\varTheta$ at the tree level or one loop.

APPENDIX C: One-loop Running of Yukawa Couplings

a) Four Higgs Doublet SUSY Model

Here we list the one-loop running of Yukawa couplings for a general four higgs doublet supersymmetric model. The Yukawa matrices of Higgses that couple to down quarks are denoted by $y_1$ and $y_3$, and similarly $y_2$ and $y_4$ for the up type quarks:

$$L_Y = Qy_1DH_1 + Qy_3DH_3 + Qy_2UH_2 + Qy_4UH_4 + Qy_1^eEH_1 + Qy_3^eEH_3 \quad (29)$$

\[
\begin{align*}
\frac{d}{dt}y_1 &= \frac{1}{16\pi^2}\{y_1[\text{Tr}(3y_1^ty_1 + y_1^ey_1^e) + 3y_1^ty_1 + y_2^ty_2 + y_3^ty_3 + y_4^ty_4] \\
&\quad + y_3[\text{Tr}(3y_3^ty_3 + y_3^ey_3^e) + 2y_3^ty_1] - y_1\left(\frac{7}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right)\} \\
\frac{d}{dt}y_2 &= \frac{1}{16\pi^2}\{y_2[\text{Tr}(3y_2^ty_2 + y_1^ty_1 + 3y_2^ty_2 + y_3^ty_3 + y_4^ty_4] \\
&\quad + y_4[\text{Tr}(3y_4^ty_2) + 2y_4^ty_2] - y_2\left(\frac{13}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right)\} \\
\frac{d}{dt}y_3 &= \frac{1}{16\pi^2}\{y_3[\text{Tr}(3y_3^ty_3 + y_3^ey_3^e) + y_1^ty_1 + y_2^ty_2 + 3y_3^ty_3 + y_4^ty_4] \\
&\quad + y_1[\text{Tr}(3y_1^ty_3 + y_1^ey_3^e) + 2y_1^ty_3] - y_3\left(\frac{7}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right)\} \\
\frac{d}{dt}y_4 &= \frac{1}{16\pi^2}\{y_4[\text{Tr}(3y_4^ty_4 + y_1^ty_1 + y_2^ty_2 + y_3^ty_3 + 3y_4^ty_4] \\
&\quad + y_2[\text{Tr}(3y_2^ty_4) + 2y_2^ty_4] - y_1\left(\frac{13}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2\right)\} \\
\frac{d}{dt}y_1^e &= \frac{1}{16\pi^2}\{y_1^e[\text{Tr}(3y_1^ty_1 + y_1^ey_1^e) + 3y_1^ty_1 + y_3^ty_3^e + y_3^ey_3^e] \\
&\quad + y_3^e[\text{Tr}(3y_3^ty_1 + y_3^ey_3^e) + 2y_3^ty_1] - y_1^e\left(\frac{9}{5}g_1^2 + 3g_2^2\right)\}
\end{align*}
\]
\[
\frac{d}{dt} y^e_3 = \frac{1}{16\pi^2} \left\{ y^e_3 [\text{Tr}(3y^\dagger_3 y_3 + y^e_3 y^e_3) + y^{\text{ef}}_1 y^e_1 + 3y^{\text{ef}}_3 y^e_3] \\
+ y^e_1 [\text{Tr}(3y^\dagger_1 y_3 + y^{\text{ef}}_1 y^e_3) + 2y^{\text{ef}}_1 y^e_3] - y^e_3 \left( \frac{9}{5} g_1^2 + 3g_2^2 \right) \right\} 
\]

The equations for a two higgs doublet model (i.e. the MSSM) are easily obtained by setting for example Yukawa matrices \( y_3 \), \( y_4 \) and \( y^e_3 \) to zero in the equations above. They indeed have the form of Eq. [8].

We see that the equations (30) part from the form of (8) because of higgs doublet wave function renormalization terms. (For example the term \( y_3 \text{Tr}(3y^\dagger_3 y_1) \) in the equation for \( y_1 \).) One can still write an equation in form of (8) with new terms in \( T \) which are not real in general. For example the phase will appear in \( y^{-1}_1 y_3 \text{Tr}(3y^\dagger_3 y_1) \), and it will depend on the structure of the Yukawa matrices how big the phase is.

b) SUSY LR Model

It is easy to generalize the above one-loop runnings for the case of Yukawa couplings in the SUSY LR model with two bidoublets:

\[
L_Y = QY_1 Q^c \Phi_1 + QY_2 Q^c \Phi_2 + LY^e_1 L^c \Phi_1 + LY^e_2 L^c \Phi_2
\]

We simply take \( y_1 = y_2 = Y_1 \) (and similarly for other Yukawas), add the right-handed neutrino and compute the contribution from gauge couplings. Alternatively, we use general formulas [15]. In any case we obtain:

\[
\frac{d}{dt} Y_1 = \frac{1}{16\pi^2} \left\{ Y_1 [\text{Tr}(3Y^\dagger_1 Y_1 + Y^{\text{ef}}_1 Y^e_1) + 4Y^\dagger_1 Y_1 + 2Y^\dagger_2 Y_2] \\
+ Y_2 [\text{Tr}(3Y^\dagger_2 Y_2 + Y^{\text{ef}}_2 Y^e_2) + 2Y^\dagger_1 Y_1 + 4Y^\dagger_2 Y_2] - Y_1 \left( \frac{4}{9} g_{B-L}^2 + 3g_L^2 + 3g_R^2 \left( \frac{16}{3} g_3^2 \right) \right) \right\} 
\]

\[
\frac{d}{dt} Y_2 = \frac{1}{16\pi^2} \left\{ Y_2 [\text{Tr}(3Y^\dagger_2 Y_2 + Y^{\text{ef}}_2 Y^e_2) + 2Y^\dagger_1 Y_1 + 4Y^\dagger_2 Y_2] \\
+ Y_1 [\text{Tr}(3Y^\dagger_1 Y_2 + Y^{\text{ef}}_1 Y^e_2) + 2Y^\dagger_2 Y_2] - Y_2 \left( \frac{4}{9} g_{B-L}^2 + 3g_L^2 + 3g_R^2 \left( \frac{16}{3} g_3^2 \right) \right) \right\} 
\]

\[
\frac{d}{dt} Y^{\text{ef}}_1 = \frac{1}{16\pi^2} \left\{ Y^{\text{ef}}_1 [\text{Tr}(3Y^{\text{ef}}_1 Y_1 + Y^{\text{ef}}_1 Y^{\text{ef}}_1) + 4Y^{\text{ef}}_1 Y_1 + 2Y^{\text{ef}}_2 Y_2] \\
+ Y^{\text{ef}}_2 [\text{Tr}(3Y^{\text{ef}}_2 Y_2 + Y^{\text{ef}}_2 Y^{\text{ef}}_2) + 2Y^{\text{ef}}_1 Y_1 + 4Y^{\text{ef}}_2 Y_2] - Y^{\text{ef}}_1 \left( g_{B-L}^2 + 3g_L^2 + 3g_R^2 \right) \right\} 
\]

\[
\frac{d}{dt} Y^{\text{ef}}_2 = \frac{1}{16\pi^2} \left\{ Y^{\text{ef}}_2 [\text{Tr}(3Y^{\text{ef}}_2 Y_2 + Y^{\text{ef}}_2 Y^{\text{ef}}_2) + 2Y^{\text{ef}}_1 Y_1 + 4Y^{\text{ef}}_2 Y_2] \\
+ Y^{\text{ef}}_1 [\text{Tr}(3Y^{\text{ef}}_1 Y_2 + Y^{\text{ef}}_1 Y^{\text{ef}}_2) + 2Y^{\text{ef}}_2 Y_2] - Y^{\text{ef}}_2 \left( g_{B-L}^2 + 3g_L^2 + 3g_R^2 \right) \right\} 
\]
It is easy to see that the hermiticity of Yukawa couplings is preserved throughout the running in the $SU(2) \times SU(2) \times U(1)_{B-L}$ phase (i.e. above $m_R$), as expected. This is in contrast to case a) where below $M_R$ running of matrices necessarily spoils hermiticity (both in the MSSM and the 4 Higgs doublet model), because then the LR symmetry is broken.

APPENDIX D: Doublet-Doublet Splitting

In this Appendix, we show how a left-right symmetric theory with two bidoublets above the scale $M_R$ reduces to the MSSM with only one pair of $(H_u,H_d)$. We will call this phenomenon doublet-doublet splitting. The simplest way to achieve this is by a fine tuning of the parameters of the superpotential involving the $\phi_1$ and $\phi_2$ fields - i.e. $\mu_{ij}$. To make this explicit, consider the part of the superpotential:

$$W_\phi = \sum_{ij} \frac{1}{2} \mu_{ij} Tr \phi_1^T \tau_2 \phi_2 \tau_2$$

(33)

where the symbols $a, b$ go over 1,2. This leads to the following superpotential in terms of the standard model doublets:

$$W_\phi = \mu_{11} H_u_1 H_d_1 + \mu_{22} H_u_2 H_d_2 + \mu_{12} (H_u_1 H_d_2 + H_u_2 H_d_1)$$

(34)

Now it is clear that, if the parameters $\mu_{ab}$ are so chosen that we have $\mu_{11} \mu_{22} - \mu_{12}^2 = 0$ and that each $\mu_{ij}$ are of order $v_R$, then below the scale $v_R$, the model has only two standard model doublets as in MSSM. The surviving doublets are then linear combinations of the of the original four doublets in the theory. If however one wanted “pure” doublets surviving below the $v_R$ scale (such as say, $H_u_1$ and $H_d_2$) then one can use the superpotential of the following type:

$$W'_\phi = \frac{\lambda_{12}}{M_{Pl}} \text{Tr} \phi_1^T \tau_2 \phi_2 \text{Tr} \Delta^c \tau_1 \Delta^c + \mu_{12} \text{Tr} \phi_1^T \tau_2 \phi_2 \tau_2$$

(35)

In this case fine tuning of the parameters $\lambda_{12} v_R^2 / M_{Pl} + \mu_{12} = 0$ leaves the pure low energy doublets $H_u_2$ and $H_d_1$.

---

6For example note that in the equation for $Y_1$, we have a sum of terms $Y_1 Y_2^\dagger Y_2 + Y_2 Y_2^\dagger Y_1$ which is hermitean if $Y_1$ and $Y_2$ are hermitean.
References

[1] R.D. Peccei and H. Quinn, Phys. Rev. Lett. 38, 1440 (1977); Phys. Rev. D16, 1791, (1977). J.E. Kim, Phys. Rev. Lett. 43, 103 (1979); M. Shifman, A. Vainshtein and V. Zakharov, Nucl. Phys. B166, 493 (1980); A.R. Zhitnitsky, Sov. J. Nucl. Phys. 31, 260 (1980); M. Dine, W. Fischler and M. Srednicki, Phys. Lett. 104B, 199 (1981). For recent reviews see J.E. Kim, Phys. Rep. 150, 1 (1987); H.Y. Cheng, ibid 158, 1 (1988); R.D. Peccei, in CP violation, edited by C. Jarlskog (World Scientific, Singapore, 1989)

[2] S. Giddings and A. Strominger, Nucl. Phys. B307, 854 (1988); R. Holman et al., Phys. Lett. 282B, 132 (1992); M. Kamionkowski and J. March-Russell ibid, 137; S. Barr and D. Seckel, Phys. Rev. D46, 539 (1992); R. Kallosh et al., Phys. Rev. D52, 912 (1995).

[3] M.A.B. Bég and H.S. Tsao, Phys. Rev. Lett. 41, 278 (1978); R.N. Mohapatra and G. Senjanović, Phys. Lett. 79B, 283 (1978); S. Barr and P. Langacker, Phys. Rev. Lett. 42, 1654 (1979); K.S. Babu and R.N. Mohapatra, Phys. Rev. D41, 1286 (1990); S. Barr, D. Chang and G. Senjanović, Phys. Rev. Lett. 67, 2765 (1991).

[4] H. Georgi, Hadron J. 1, 155 (1978). A. Nelson, Phys. Lett. 136B, 387 (1983); S. Barr, Phys. Rev. Lett 53, 329 (1984).

[5] J.C. Pati and A. Salam, Phys. Rev. D10, 275 (1974); R.N. Mohapatra and J.C. Pati, ibid D11, 566, 2558 (1975); G. Senjanović and R.N. Mohapatra, ibid D12, 1502 (1975).

[6] Z. Berezhiani, R.N. Mohapatra and G. Senjanović, Phys. Rev. D47, 5565 (1993).

[7] J. Ellis, S. Ferrara and D.V. Nanopoulos, Phys. Lett. 114B, 231 (1982); W. Buchmüller and D. Wyler, ibid 121B, 321 (1983); J. Polchinski and M.B. Wise, ibid 125B, 393 (1983).

[8] S. Dimopoulos and S. Thomas, SLAC-PUB-95-7010; hep-ph/9510220.

[9] S. Bertolini and F. Vissani, Phys. Lett. 324B, 164 (1994); S. Dimopoulos and L.J. Hall, Phys. Lett. 344B, 185 (1995); T. Imui et al., Nucl. Phys. B449, 49 (1995); R.
Barbieri, L.J. Hall and A. Strumia, Nucl. Phys. B449, 437 (1995); R. Barbieri, A. Romanino and A. Strumia, IFUP-TH-65-95, [hep-ph/9511303].

[10] K. S. Babu and S. Barr, Phys. Rev. Lett. 72, 2831 (1994).

[11] M. Dine, R. Leigh and A. Kagan, Phys. Rev. 48, 2214 (1993).

[12] R.N. Mohapatra and A. Rašin, Phys. Rev. Lett. 76, 3490 (1996), [hep-ph/9511391].

[13] R. Kuchimanchi, Phys. Rev. Lett. 76, 3486 (1996), [hep-ph/9511376].

[14] M. Masip and A. Rašin, Phys. Rev. D52, 3768 (1995); Nucl. Phys. B460, 449 (1996).

[15] See, for example S. P. Martin and M. T. Vaughn, Phys. Rev. D50, 2282 (1994), and references therein.

[16] R. Garisto, Nucl. Phys. B419, 279 (1994);

[17] H. Georgi, Phys. Lett. 169B, 231 (1986); L.J. Hall, V.A. Kostelecky and S. Raby, Nucl. Phys. B267, 415 (1986); M. Dine, A. Kagan and S. Samuel, Phys. Lett. 243B, 250 (1990); M. Dine, A. Kagan and R. Leigh, Phys. Rev. D48, 4269 (1993); Y. Nir and N. Seiberg, Phys. Lett. 309B, 337 (1993); J.S. Hagelin, S. Kelley and T. Tanaka, Nucl. Phys. B415, 293 (1994). A. Pomarol and D. Tommasini, CERN-TH-95-207, [hep-ph/9507462]. R. Barbieri, G. Dvali and L.J. Hall, LBL-38065, [hep-ph/9512388].

[18] K. Benakli and G. Senjanović, IC-95-140-REV, [hep-ph/9507219].

[19] R. Kuchimanchi and R.N. Mohapatra, Phys. Rev. D48, 4352 (1993); Phys. Rev. Lett. 75, 3989 (1995).
Figure 1: Higgs contribution to one-loop calculation of $\bar{\Theta}$. 
Figure 2: Examples of gaugino contributions to one-loop calculation of $\bar{\Theta}$. $V_{L,R}$ are left and right gauginos, respectively. The gaugino mass $m_{\lambda_L}$ is in general complex. There is an analogous graph to b) that involves right-handed gauginos.