Mixing of $\eta - \eta'$ in charge-exchange reactions and decays of mesons with heavy quarks

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Involving of elastic rescattering and annihilation of quark-antiquark pairs in a final state allows us to explain a dependence of ratio for cross sections of $\eta'$ and $\eta$ mesons versus transfer momentum in charge exchange reactions. We estimate the mixing angle of mesons with isoscalar states of $\bar{u}u + \bar{d}d$ and $\bar{s}s$ of hidden strangeness. The evaluation includes the consistent description of yield ratio for $\eta'$ and $\eta$ mesons in decays of $B^0$, $B^0_s$ and $J/\psi$ mesons.

I. INTRODUCTION

In the spectroscopy operating by quark quantum numbers, the neutral pseudoscalar mesons $\eta^{(\prime)}$ can be represented as the superpositions of isosinglet states

$$|\eta^{(\prime)}\rangle = \sin \phi |\bar{n}n\rangle + \cos \phi |\bar{s}s\rangle,$$
$$|\eta\rangle = \cos \phi |\bar{n}n\rangle - \sin \phi |\bar{s}s\rangle,$$

(1)

where $|\bar{s}s\rangle$ is composed of strange quark and antiquark, while $|\bar{n}n\rangle$ is the isosinglet of light $u$ and $d$ quarks:

$$|\bar{n}n\rangle = \frac{1}{\sqrt{2}} \{ |\bar{d}d\rangle + |\bar{u}u\rangle \}.$$

(2)

In such the representation one suggests the absence of any admixture of exotic glueball state with the same quantum numbers of parity and isospin with no valence quarks. This simplified representation should be also compared with a deeper consideration of mixing schemes that use quark currents and other strict notions, say, in [9, 10].

Numerous studies have been devoted to determination of mixing angle $\phi$ for the pseudoscalars, see, for instance, a review of quark model in [11] as well as original articles [12–18], also including a study within a holographic approach to quark physics in [19]. In the limit of flavor symmetry for three quarks, $SU_f(3)$, one can evidently get $\sin \phi \mapsto 2/\sqrt{6}$, so that $\eta^{(\prime)} \rightarrow \eta_0$, $\eta \rightarrow \eta_8$, i.e. one

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1 Models, involving the glueball admixing, lead to suppressed amplitudes of such the admixture $\eta^{(\prime)}$. This fact is natural because a matrix element of hamiltonian of non-perturbative mixing can be estimated as $\Lambda_{QCD} \sim 300 - 400$ MeV by the order of magnitude, while the glueball state takes the mass, which is $2.5 - 3$ GeV more heavy than $\eta^{(\prime)}$, at least. Therefore, the amplitude of mixing with the glueball is about $A_G \lesssim \Lambda_{QCD}/\Delta M \sim 0.1 \ll 1$. 
arrives to SU\(_f\)(3) singlet and SU\(_f\)(3) octet, respectively:

\[
\begin{align*}
|\eta_0\rangle &= \frac{1}{\sqrt{3}} \left\{ |\bar{u}u\rangle + |\bar{d}d\rangle + |\bar{s}s\rangle \right\}, \\
|\eta_8\rangle &= \frac{1}{\sqrt{6}} \left\{ |\bar{u}u\rangle + |\bar{d}d\rangle - 2 |\bar{s}s\rangle \right\}.
\end{align*}
\]

Such the rotation of neutral states of SU\(_f\)(3) singlet and SU\(_f\)(3) octet to the isosinglet states |\bar{nn}\rangle and |\bar{ss}\rangle with the angle of \(\cos^2 \phi_\star = \frac{1}{3}\) is usually called the ideal mixing: \(\phi_\star = \arctan \sqrt{2} \approx 54.7^\circ\). One usually introduces the mixing angle \(\theta\) by \(\theta = \phi - \phi_\star \approx \phi - 54.7^\circ\).

The difficulty of problem on the mixing is caused by the issue on how the mixing introduced in the spectroscopy is implemented in the description of dynamical processes and can been measured in a model-independent way. For instance, the measurement of cross section ratio for \(\eta^{(i)}\) mesons in the charge exchange reactions gives the result dependent on the transfer momentum \(t\) \[^{[20]}\]. It is problematic to make this fact straightforwardly compatible with simple spectroscopic notions, since the spectroscopy operates by some constant quantities, but dynamical functions.

In the present paper we show that the dynamical characteristics in the charge exchange reactions with the production of \(\eta^{(i)}\) are really consistent with the picture of meson mixing if we introduce the interaction in the final states for the isosinglets with taking into account the annihilation of quark-antiquark pairs. A relative contribution of such the annihilation acquires reasonable features in its magnitude and behavior with respect to the momentum transfer. This allows us to constrain a data region, wherein the mixing angle can be measured in the model-independent way (see Section \[II\]).

Further, in Sections \[III\] and \[IV\] we show that the measurements of \(\eta^{(i)}\) yields in radiative decays of \(J/\psi\) as well as in some channels of decays of neutral \(B\) mesons are also consistent with the mechanism taking into account the rescattering of quark-antiquark pair in the final state.

II. AMPLITUDE ANALYSIS OF \(t\)-DEPENDENCE FOR RATIO OF YIELDS OF \(\eta\) AND \(\eta'\) MESONS IN CHARGE EXCHANGE REACTIONS OF \(\pi^- p\) AND \(K^- p\)

The exchange by quantum numbers of flavors in the reactions \(\pi^- p \rightarrow n\eta(\eta')\) is shown\(^2\) in Fig. 1. It corresponds to Regge’s exchange with the trajectory possessing the quantum numbers of \(a\) mesons. The amplitude of \(M\) corresponds to the contribution when \(d\) quark is a spectator, while \(M_B\) denotes the term when the spectator is \(\bar{u}\) antiquark.

![Fig. 1](image)

FIG. 1: The exchange by quark flavors in the process of \(\pi^- p \rightarrow n\eta(\eta')\) with the production of |\bar{d}d\rangle and |\bar{u}u\rangle states.

The quark-antiquark state produced in the charge exchange reaction has the invariant mass in the range of 500 – 1000 MeV, hence, there is a strong non-perturbative interaction in the final

\[^{2}\text{See [21] about taking into account both terms in Fig.}\]
state. In the case of pseudoscalar color-singlet channel the interaction permits the annihilation of quark-antiquark state into the quark-antiquark state, say, by the pair of gluons or the one-gluon annihilation under the two-gluon exchange with the baryon (see Fig. 2), if the quark flavor is changed. Analogously, there should be the elastic scattering if the quark flavors are conserved. In this way, the two-gluon exchange with the baryon that corresponds to the pomeron exchange, leads to a $t$-dependence of the amplitude of interaction in the final state. Therefore, taking into account the interaction in the final state as pictured in Fig. 2 corresponds to the contribution of double reggeon exchange, namely, the reggeon-pomeron term.

$$\begin{align*}
M[\pi] &= M(|d\bar{d}|) + M_A(|d\bar{d}| + |u\bar{u}| + \sqrt{\lambda_s}|s\bar{s}|) + M_B(|u\bar{u}| + \sqrt{\lambda_s}|s\bar{s}|) + M_{AB}(|d\bar{d}| + |u\bar{u}| + \sqrt{\lambda_s}|s\bar{s}|) \\
&= M \{ |d\bar{d}| + A(|d\bar{d}| + |u\bar{u}| + \sqrt{\lambda_s}|s\bar{s}|) + B \{ |u\bar{u}| + A(|d\bar{d}| + |u\bar{u}| + \sqrt{\lambda_s}|s\bar{s}|) \} \, , \end{align*}$$

(3)

where $\lambda_s$ is the parameter of suppression for the strange quark production at the given invariant masses.

In this representation of amplitude we ignore the polarization effects related to spins of nucleons and an orientation of scattering plane. These effects can compose several percents of differential cross section, but they are not essential for our purpose to isolate main regularities connected to the mixing and $t$-dependence of measured quantities. Anyway, the offered amplitude is inevitably dictated by the processes with the quark quantum numbers, i.e. by the quark scheme of hadrons itself. In this respect, we label the quark amplitudes by the valence quark quantum states, $|qq\rangle$, for the visual clarity of notions.

For the sake of simplicity, we put $\lambda_s$ to be real, constant and equal to $\lambda_s = \frac{1}{2}$. These properties of parameter $\lambda_s$ can be justified in the following way:

First, the parameter describes the relative fraction of strange quark-antiquark pair creation in the fixed range of invariant masses corresponding to the mass interval of neutral mesons, so that we could suggest its value to be a constant.

Second, the strange quark fraction in the annihilation channel is independent on the transverse momentum, since such the dependence is factorized, and it can be suggested to be identical for different flavors of light quarks in the region of invariant masses greater than the current masses of light quarks, while the only difference for the strange quarks is given by the suppression of overall

FIG. 2: The interaction in the final state of $q\bar{q}$ pair with the annihilation.
probability for the creation of strange quark-antiquark pair in the mentioned interval of invariant masses.

Third, the magnitude of $\lambda_s$ can be evaluated empirically in other processes with the same mechanism with the annihilation channel in the final state interaction for the quark amplitudes. Such the extraction of $\lambda_s$ is presented in next Section, wherein $\eta(\prime)$ are produced in the radiative decays of $J/\psi$. We will see that the uncertainty of $\lambda_s$ is numerically significant, but, it is important, its variation certainly belongs to the range less than unit and the introduction of $\lambda_s$ is principal with no doubt. So, the chosen value of $\lambda_s$ is consistent with the other processes, while its variation due to the uncertainty will change the model parameters, of course, but this variation does not change our basic conclusions as we will see.

The ratio of $\eta$ and $\eta'$ yields does not depend on factor $M$, so that the overall behavior of this ratio is determined by complex amplitudes $A$ and $B$. In this way, in the case of $\pi^-p$ process the result does not depend on $B$, and the $t$-dependence of the ratio is completely given by the function of $A(t)$.

Indeed, the expansion in terms of the isosinglet state $|\bar{n}n\rangle$ according to (2) and the state with hidden strangeness $|\bar{s}s\rangle$ gives

$$\mathcal{M}[\pi] \sim (1 + B) \left\{ \frac{1}{\sqrt{2}} (1 + 2A) |n\bar{n}\rangle + A \sqrt{\lambda_s} |s\bar{s}\rangle \right\},$$

hence,

$$R_\pi = \left| \frac{\mathcal{M}[\pi \rightarrow \eta']}{\mathcal{M}[\pi \rightarrow \eta]} \right|^2 = \left| \frac{1}{\sqrt{2}} (1 + 2A) \tan \phi + A \sqrt{\lambda_s} \right|^2,$$

The exclusion of the final state interaction in the quark-antiquark state with the annihilation, i.e. $A \equiv 0$, leads to the quantity independent of $t$:

$$R_\pi \rightarrow \tan^2 \phi,$$

that contradicts to the experimental data. This fact directly shows that the amplitude of such interaction in the final state is not equal to zero. Moreover, at fixed mixing angle we can restore a form of function $A(t)$ by assuming a model for its complex phase.

Analogously, in accordance with diagrams in Fig. 3 the amplitude of charge exchange reaction

FIG. 3: The exchange by quark flavors in the process of $K^-p \rightarrow \Lambda\eta(\eta')$ with the production of $|s\bar{s}\rangle$ and $|u\bar{u}\rangle$ states.
\( K^{-}p \rightarrow \Lambda\eta(\eta') \) takes the form\(^3\)

\[
\mathcal{M}[K] = \mathcal{M}'(s\bar{s}) + \mathcal{M}'_{A}(|d\bar{d}| + |u\bar{u}| + \sqrt{\lambda_{s}}|s\bar{s}|) + \mathcal{M}'_{B}|u\bar{u}| + \mathcal{M}'_{AB}(|d\bar{d}| + |u\bar{u}| + \sqrt{\lambda_{s}}|s\bar{s}|)
\]

\( = \mathcal{M}' \{ |s\bar{s}| + \mathcal{A}(|d\bar{d}| + |u\bar{u}| + \sqrt{\lambda_{s}}|s\bar{s}|) + \mathcal{B} \{ |u\bar{u}| + \mathcal{A}(|d\bar{d}| + |u\bar{u}| + \sqrt{\lambda_{s}}|s\bar{s}|) \} \},
\]

wherein we suggest the invariance of high energy amplitudes with respect to the transformation of light quark flavors. Then,

\[
\mathcal{M}[K] \sim \frac{1}{\sqrt{2}} \{ \mathcal{B} + 2\mathcal{A} (1 + \mathcal{B}) \} |n\bar{n}\rangle + \left\{ 1 + \mathcal{A} \sqrt{\lambda_{s}} (1 + \mathcal{B}) \right\} |s\bar{s}\rangle,
\]

hence,

\[
R_{K} = \left| \frac{\mathcal{M}[K \rightarrow \eta]}{\mathcal{M}[K \rightarrow \eta']} \right|^{2} = \left| \frac{\sqrt{2}}{\sqrt{2}} \left\{ \mathcal{B} + 2\mathcal{A} (1 + \mathcal{B}) \right\} \tan \phi + \left\{ 1 + \mathcal{A} \sqrt{\lambda_{s}} (1 + \mathcal{B}) \right\} \right|^{2}.
\]

We assume that the absolute value of \( \mathcal{B} \) weakly depends on \( t \) at high energies, so that in calculations we put \( \mathcal{B} \) equal to a complex number with a phase which can depend on \( t \). Such the prescription allows us to study the dependence of amplitude on a single complex function \( \mathcal{A}(t) \). This assumption is quite rigid and it restricts a possibility to fit the experimental data, because the extraction of function \( \mathcal{A}(t) \) from \( R_{K} \) fixes a value of \( R_{K} \). However, the assumption is reasonable, since it allows us quite satisfactory to describe the behavior of \( R_{K} \).

Notice that in eqs. (6)–(8) for the \( K^{-}p \) scattering we have used the same notation \( \mathcal{B} \) for the fraction of amplitude with the spectator \( u \)-quark as it has been involved in the description of \( \pi^{-}p \) scattering, since for the sake of brevity we have missed the prime for this quantity in eqs. (6)–(8), because the ratio of cross sections under study in \( \pi^{-}p \) charge exchange reaction does not depend on \( \mathcal{B} \) at all\(^4\).

Moreover, we remind that in the Regge calculus the two processes with the different spectator quarks corresponds to the identical Regge trajectories, hence, we deal with the quark amplitude decomposition of the same amplitude. Therefore, it would be natural to expect that two quark amplitudes have got the same \( t \)-dependence, while their ratio in terms of factor \( \mathcal{B} \) does not depend on the transverse momentum at all, since we consider the same decomposition of unified Regge amplitude at different \( t \) in terms of two amplitudes with the definite exchange of quark quantum numbers. Thus, our assumption on the negligible \( t \)-dependence of fraction \( \mathcal{B} \) is justified and natural.

Let us note in the very beginning of analysis that we consider two versions of \( \mathcal{A}(t) \) behavior: \( \mathcal{A}(t) \) essentially grows or falls with the momentum transfer increase.

\[\text{A. Scenario I}\]

The significant increasing of \( \mathcal{A}(t) \) means that this contribution is practically negligible at low momentum transfers \( t \rightarrow t_{\text{min}} \), where \( t_{\text{min}} \rightarrow 0 \) at high energies of scattering, i.e. at energies much greater than masses of particles in the process, \( s \gg m^{2} \), so that in what follows we operate with a formal limit of \( t \rightarrow 0 \). In this way, the contribution of \( \mathcal{A}(t) \) becomes significant only at momentum

\(^3\) The term \( \mathcal{M}' \) corresponds to the contribution when the \( s \) quark is the spectator, while \( \mathcal{M}'_{A} \) corresponds to the term with the spectator \( \bar{u} \). The amplitude of annihilation channel is marked by subscript \( A \).

\(^4\) Therefore, we have not deal with an assumption about the SU(3)-flavor symmetry for the fraction \( \mathcal{B} \), it is just the economy of notations.
transfers comparable with the particle masses squared. Under such the assumptions the tangent of mixing angle is determined by the factor of $R_\pi = \tan^2 \phi$ at zero momentum transfer, and it is quite large, $\tan \phi \approx 0.75$. This version of behavior should lead to both a fall of $R_\pi$ factor and a reasonable value of $R_K$ raising versus $|t|$. We find that this scenario can be actualized if the following two conditions take place: at first, the imaginary part of factor $A(t)$ is negligibly small and, second, the absolute value and phase of $B$ at $t \to 0$ are adjusted in order to describe $R_K(t \to 0)$, while at $t \neq 0$ the phase of $B$ depends on the momentum transfer in the way to fit the $R_K$ data at constant fixed absolute value of $B$.

For instance, we find the model I

$$\tan \phi = 0.75, \quad B = \frac{8}{9} e^{-i\pi/2.3(1-f(a))}, \quad A = -a, \quad f(a) = \frac{5(5a)^{2.85}}{1 + 3(5a)^{2.85}},$$

where $a$ is a real positive parameter corresponding to the relative contribution of annihilation channel. The quality of data description in this model\(^5\) is shown in Fig. 4.

![Graph of $R_\pi$ and $R_K$ versus $-t$, GeV$^2$.](image)

**FIG. 4:** The description of charge exchange data [20] in Scenario I.

![Graph of $A(t)$ versus $-t$, (GeV/c)$^2$.](image)

**FIG. 5:** The dependence of $A(t)$ versus the momentum transfer in model I.

\(^5\) We do not show a statistical likelihood of the model because we have aimed to demonstrate the principal importance of annihilation channel in the final state. In addition, a justification of amplitude values extracted in the non-perturbative regime seems to be very problematic, so that the procedure of fitting the data could be improved to the ideal agreement without achieving a distinguished result.
The behavior of $A(t)$ (or simply the connection of parameter $a$ to the transfer momentum) in Scenario I is shown in Fig. 5. The characteristic values of amplitude $A(t)$ determine the magnitude of violation of OZI rule: the production of valence quark-antiquark pairs is suppressed with respect to the processes without such production. The accuracy of such approximation is about 10% in the amplitude, hence, about 1% in the probability, if the interference with the leading term is suppressed by relevant quantum numbers.

B. Scenario II

In this case, the factor of $A(t)$ significantly decreases with the growth of momentum transfer $|t|$. In this way, $\tan^2 \phi$ represents the bottom limit for $R_\pi$, i.e. it gets small values saturating $R_\pi$ at high momentum transfer. Currently, the degree of such the saturation can not be strictly established from the data because of valuable experimental uncertainties at high momentum transfers. Nevertheless, we can state that the small mixing angle in Scenario II would fit the experimental data with a low confidence level.

The most simple modeling corresponds to a constant phase of amplitude $A(t)$ equal or close to $\frac{\pi}{2}$, so that $R_\pi$ decreases with the momentum transfer growth. The opportunity of acceptable data description for $R_K$ at $B \equiv 0$ can be gotten in model II (see Fig. 6):

$$\tan \phi = 0.245, \quad B \equiv 0, \quad A = -a e^{i\pi/2}, \quad 0.33 < a < 0.97.$$  

(10)

Fig. 6: The description of charge exchange data in Scenario II.

Since the region of data is constricted by $|t| < 0.8$ GeV$^2$, model II does not reach the limit case of $A \to 0$ at large values of $|t|$, corresponding to the saturation of ratio $R_\pi \to \tan^2 \phi$. Moreover, in this scenario the value of $|A|$ corresponds to the magnitude of OZI rule breaking, and it takes an extremely unreal value of the order of unit at small transverse momenta (see Fig. 7).

It is worth to note that a variation of parameters in scenario II does not allows us to get the tangent of mixing angle greater than $\frac{1}{2}$.

C. Inference

The physical contents of Scenarios described above differ essentially: in version I, the elastic rescattering dominates over the annihilation interaction at $t \to 0$, so that the ratio of amplitudes,
i.e. $A(t)$ is suppressed and it grows versus the momentum transfer, while in version II the role of elastic channel increases with the momentum transfer $|t|$. To our opinion, Scenario I is the most realistic. Therefore, the ratio of cross sections for $\eta^{(f)}$ in the charge exchange reaction of $\pi p$ at $t \rightarrow 0$ gives the measured angle of mixing. The description of mixing in the $K$ reaction is model dependent.

Note that the main result of our study is the decomposition of charge exchange amplitude versus the quark quantum numbers with the involvement of annihilation fraction in the final state interaction, that cannot be neglected. We do empirically extract these decompositions in the case of kinematics of charge exchange reactions. The decomposition itself is universal. However, the same approach if used in other processes with the production of $\eta^{(f)}$-mesons with a different kinematics should result in a change of decomposition amplitudes, of course, as we will see in next Sections devoted to decays of heavy mesons, when one cannot talk on the $t$-channel of amplitude at all.

III. DECAYS OF $J/\psi \rightarrow \gamma \eta^{(f)}$

In radiative decays of $J/\psi$ to $\eta^{(f)}$, the mesons are produced via the annihilation, only (see Fig. 8). Two different kinds of diagrams are in action: the first is the radiative transition of $c\bar{c}$ pair to the pseudoscalar state with further annihilation via the channel with the quantum numbers of two gluons (the left diagram in Fig. 8), and the second is the mixing of vector $c\bar{c}$ state with the vector state composed of light quarks in the channel with quantum numbers of three gluons with the further radiative transition of vector state into the pseudoscalar meson (the right diagram in Fig. 8). The second mechanism involves the breaking down the isospin symmetry by the electromagnetic interaction, since the electric charges of $u$ and $d$ quarks are different. We suppose that this contribution to the isospin-symmetry breaking is irrelevant to our consideration based on the dominance of exact isospin symmetry, though more careful study was performed in [22], wherein both kinds of diagrams are discussed in the the respect of mixing problem$^6$. Therefore, we neglect the mixing of $J/\psi$ with the light vector states$^7$ and make calculations for the dominant

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$^6$ The paper [22] has been also focused on the radiative $\psi'$ decays.

$^7$ Otherwise, the isospin-symmetry breaking effects would be of the order of unit in the decays under consideration. In [22] the suppression of magnitude for the contribution of isospin-symmetry breaking in the radiative $j/\psi$ decays is estimated by the factor greater than 10.
diagram of first kind.

\[
\begin{align*}
\text{FIG. 8: Two kinds of diagrams for the radiative decays } & \ J/\psi \rightarrow \gamma \eta^{(l)} \text{ with the annihilation of two charmed quarks into the light quarks.}
\end{align*}
\]

In the mechanism taking into account the suppression of the strange quarks, that violates the flavor symmetry of SU\(_f\)(3), one can easily find that the ratio of \(\eta^{(l)}\) yields gets the form\(^8\)

\[
R_{\psi} = \frac{\Gamma[J/\psi \rightarrow \gamma \eta^{(l)}]}{\Gamma[J/\psi \rightarrow \gamma \eta]} = q \left| \frac{\sqrt{2} \tan \phi + \sqrt{\lambda_s}}{\sqrt{2} - \sqrt{\lambda_s} \tan \phi} \right|^2,
\]

(11)

wherein we define the factor

\[
q = \left( \frac{k_\gamma[J/\psi \rightarrow \eta^{(l)}]}{k_\gamma[J/\psi \rightarrow \eta]} \right)^3,
\]

which is caused by differences in phase spaces and matrix elements squared and it is proportional to the third degree of photon momentum \(k_\gamma\) in the rest frame of \(J/\psi\). Numerically, \(q \approx 0.81\), that indicates significant correction. Then, the ratio \(R_{\psi}\) essentially depends on the parameter of strange quark suppression, \(\lambda_s\), while the accuracy of experimental value \(R_{\psi}\) is much less than the accuracy of empirical estimate for \(\lambda_s\).

\[
\begin{align*}
\text{FIG. 9: The dependence of suppression factor } & \lambda_s \text{ for the strange sea versus the mixing angle } \phi \text{ according to (11) at the fixed experimental value of } R_{\psi}. \text{ The vertical shaded band restricts the region of tangent for the mixing angle as measured in the charge exchange reaction.}
\end{align*}
\]

\(^8\) See [23], wherein analogous radiative decays of light vector mesons are also considered.
Therefore, it is worth to draw the function connecting \( \tan \phi \) with \( \lambda_s \) according to (11). We show this function in Fig. 9 wherein we have pictured the region of variation for \( \lambda_s = 0.5 \pm 0.07 \) as one has usually adopted in the phenomenology involving the usage of quantity \( \lambda_s \).

Thus, we get

\[
\tan \phi = 0.733 \pm 0.045, \tag{12}
\]

which is in agreement with the value of mixing angle found in the charge exchange reactions above. However, the overlap with the region extracted from the charge exchange reactions results in the more strict estimate

\[
\tan \phi = 0.740 \pm 0.022. \tag{13}
\]

IV. DECAYS OF \( B_{(s)}^0 \rightarrow J/\psi \eta^{(r)} \)

In weak decays of neutral \( B \) mesons transformed to the charmonium \( J/\psi \) in association with \( \eta^{(r)} \), different pairs of light quarks are produced (see Fig. 10).

The interaction in the final state due to the annihilation results in the amplitudes in the form

\[
M_s \sim |s\bar{s}\rangle + A' \{|u\bar{u}\rangle + |d\bar{d}\rangle + \sqrt{\lambda_s}|s\bar{s}\rangle\}, \tag{14}
\]

\[
M_d \sim |d\bar{d}\rangle + A' \{|u\bar{u}\rangle + |d\bar{d}\rangle + \sqrt{\lambda_s}|s\bar{s}\rangle\}, \tag{15}
\]

for \( B_{(s)}^0 \) and \( B^0 \), respectively. Therefore, taking into account corrections caused by differences in values of relative momenta \( k[\eta] \), \( k[\eta'] \) in channels with \( \eta \) and \( \eta' \) in the rest frame systems for neutral \( B \) meson, we easily find\(^9\)

\[
R_s = \frac{\Gamma[B_{(s)}^0 \rightarrow J/\psi \eta']}{\Gamma[B_{(s)}^0 \rightarrow J/\psi \eta]} = q_s \frac{\sqrt{2A' \tan \phi + 1} + A'\sqrt{\lambda_s}}{\sqrt{2A' - (1 + A'\sqrt{\lambda_s}) \tan \phi}}^2 \tag{16}
\]

and

\[
R_d = \frac{\Gamma[B^0 \rightarrow J/\psi \eta']}{\Gamma[B^0 \rightarrow J/\psi \eta]} = q_d \frac{\frac{1}{\sqrt{2}}(1 + A') \tan \phi + A'\sqrt{\lambda_s}}{\frac{1}{\sqrt{2}}(1 + A') - A'\sqrt{\lambda_s} \tan \phi}^2, \tag{17}
\]

\(^9\) In the limit of \( A' \rightarrow 0 \) we arrive to formulae in [24].
where we define the factors

\[ q_s = \left( \frac{k[B_0^0 \to \eta]}{k[B_s^0 \to \eta]} \right)^3, \quad q_d = \left( \frac{k[B_0^0 \to \eta']}{k[B_0^0 \to \eta]} \right)^3, \]

which are caused by differences in phase spaces and matrix elements squared, which are proportional to relative momenta squared, since the decay takes place in p-wave. Numerically, these factors give the correction equal to 20%. In expressions (16)–(17) the contribution of annihilation amplitude \( A' \) is the parameter of model, because it cannot be extracted, for instance, from the charge exchange reactions, wherein there are the t-channel exchanges, which are absent in decays under consideration.

\[ \beta, \beta' \]

FIG. 11: The parametric curve for the phase of \( \beta \) versus the absolute value of relative amplitude for the annihilation of light quarks in the final state at the fixed value of \( R_s \) (see (16)–(17)).

The experimental data on \( R_s \) [25, 26] have the precision significantly better than that of \( R_d \): \( R_s \approx 0.90 \pm 0.09 \), while \( R_d \approx 1.11 \pm 0.49 \) (for the sake of simplicity of consideration, here we have taken the averaged values for statistical and systematic uncertainties of measurements). Under this fact we consider the model with fixed values of \( \tan \phi = 0.75 \) and \( \lambda_s = 0.5 \) in order to find a solution for complex number \( A' = a e^{i\beta} \) with real parameters of \( a \) and \( \beta \) in the equation with the fixed value of \( R_s \) equal to its central value. The solution is presented in Fig. 11 wherein one can see that the phase has the region of stability at \( \beta = \beta_* \). In order to enlarge a predictive power of model we have to aim a situation with a minimal possible spread of parameters in the model. Fortunately, this is reached in the case of extremal point in the region of correlations between the parameters: the phase can be fixed, while the magnitude gets a minimal variation. Taking the value of phase in the point of stability on Fig. 11 we show the prediction of model for the ratio of yields in Fig. 12.

Thus, the data on \( R_s \) are well described within the statistical uncertainties if we put

\[ A' = a e^{i\beta}, \quad a \in [0.4040; 0.4215], \]  

(18)

i.e at the constant phase and changing real amplitude\(^{10} \)\( a \). Therefore, the model predicts

\[ R_d = 0.943 \pm 0.015, \]  

(19)

\(^{10}\) At \( A' \equiv 0 \), i.e. in the case of switching off the mechanism of annihilation in the final state, \( R_s \) exceeds the experimental value by 70%.
FIG. 12: The ratio of $\eta^{(t)}$ yields in decays of neutral $B$ mesons into $J/\psi$ versus the real parameter $a$ (see (16)–(18)). The solid line gives $R_s$, the dotted line presents $R_d$, the shaded band shows the experimental value of $R_s$ with statistical uncertainties.

which is in a good agreement with the experimental result, of course. The accuracy of prediction is, at least, one order of magnitude less than the uncertainty of current data.

V. CONCLUSION

We have studied the mechanism generating the $t$-dependence of ratio for the cross sections of $\eta'$ and $\eta$ mesons in reactions of charge exchange for pions and kaons off protons with mixing of isoscalar components in $\eta^{(t)}$. We have shown that such the dependence appears under the introduction of interaction in the final state including the contribution of annihilation.

The annihilation channel is the only one that contributes to the yields of $\eta^{(t)}$ in radiative decays of $J/\psi$. This fact allows us to estimate the mixing angle in consistency with the data on the charge exchange reactions, if we take into account the uncertainty in the phenomenological value of factor for the suppression of strange quarks. Then,

$$\phi \approx 36.5 \pm 0.8^\circ,$$

that improves the accuracy in the value of mixing angle and is in agreement with results of other authors. The improvement of accuracy for the parameter of suppression of strange quarks, breaking down the symmetry of light quark flavors in the strong interactions, would allow us essentially to reduce the uncertainty in the mixing angle of isoscalar quark states for the case of neutral pseudoscalar mesons.

The account for the annihilation channel in the final state interaction allows us to describe the ratio of $\eta^{(t)}$ yields in decays of neutral $B$ mesons in the transition $B_0^{(s)} \rightarrow J/\psi$. In this description the value of mixing angle is consistent with the constraints obtained in the analysis for the charge exchange reactions. Then, accepting the data on $B_0^{(s)}$ we have predicted the ratio of yields in decays of $B^0$ with the uncertainty one order of magnitude less than the experimental uncertainty at present.
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