Josephson coupling through ferromagnetic heterojunctions with non-collinear magnetizations

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We study the Josephson effect in clean heterojunctions that consist of superconductors connected through two metallic ferromagnets with insulating interfaces. We solve the scattering problem based on the Bogoliubov–de Gennes equation for any relative orientation of in-plane magnetizations, arbitrary transparency of interfaces, and mismatch of Fermi wave vectors. Both spin singlet and triplet superconducting correlations are taken into account, and the Josephson current is calculated as a function of the ferromagnetic layers thicknesses and of the angle $\alpha$ between their magnetizations. We find that the critical Josephson current $I_c$ is a monotonic function of $\alpha$ when the junction is far enough from $0 - \pi$ transitions. This holds when ferromagnets are relatively weak. For stronger ferromagnets, variation of $\alpha$ induces switching between 0 and $\pi$ states and $I_c(\alpha)$ is non-monotonic function, displaying characteristic dips at the transitions. However, the non-monotonicity is the effect of a weaker influence of the exchange potential in the case of non-parallel magnetizations. No substantial impact of spin-triplet superconducting correlations on the Josephson current has been found in the clean limit. Experimental control of the critical current and $0 - \pi$ transitions by varying the angle between magnetizations is suggested.

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I. INTRODUCTION

The interplay between ferromagnetism and superconductivity in superconductor (S) – ferromagnet (F) hybrid structures attracts considerable interest for some time already. Variety of interesting theoretical predictions, such as the existence of $\pi$-state superconductivity in SF multilayers, has been confirmed experimentally. In SFS Josephson junctions, the Zeeman effect induces both a strong decay and oscillations of superconducting correlations inside the ferromagnet. The corresponding oscillations of the Josephson critical current with thickness of the ferromagnetic layer or the exchange energy have been calculated in both the clean and the dirty limit in the framework of quasiclassical theory of superconductivity. Experimental evidence for such oscillating behavior in weak ferromagnetic alloys came only recently. Similar experiments with strongly spin-polarized ferromagnets have been performed in spintronic setup. However, for strong ferromagnets and finite transparency of interfaces the quasiclassical treatment is no longer valid and Bogoliubov–de Gennes or Gor’kov equations have to be solved.

Heterostructures with superconductors coupled through inhomogeneous ferromagnets have been also extensively studied. The simplest example of such a structure contains a two-domain ferromagnet with collinear magnetizations, either parallel (P) or antiparallel (AP). In this case, besides spin singlet superconducting correlations, only triplet correlations with zero total spin projection exist. These correlations penetrate into the ferromagnet over a short length scale determined by the exchange energy. For non-collinear magnetizations, triplet correlations with nonzero spin projection are present as well; they are not suppressed by the exchange energy, and consequently they are long-ranged. It is predicted that triplet components should have a dramatic impact on the Josephson effect, displayed through a non-monotonic dependence of the critical current on angle between magnetizations. The triplet correlations were proposed as a possible explanation for recent observations of a Josephson current through half-metallic barriers. In diffusive Josephson junctions, the length scales associated with short- and long-range correlations are, respectively, $\xi_f = \sqrt{hD_f/\hbar_0}$ and $\xi_s = \sqrt{hD_f/k_BT}$, where $D_f$ is the diffusion constant in the ferromagnet, and thermal energy $k_BT$ is typically much smaller than the exchange energy $\hbar_0$. It is desirable to know if such interesting effects are also relevant in ballistic SF heterostructures, where the ferromagnet coherence length $\xi_f = \hbar v_f/(k_BT)$ is the only characteristic length, $v_f$ being the Fermi velocity.

In this paper we investigate the Josephson effect in a clean S1F1I2F2I3S heterojunction where the ferromagnetic interlayer consists of two mono-domain layers having a relative angle $\alpha$ between their in-plane magnetizations. Using the Bogoliubov–de Gennes formalism, we calculate the Josephson current and demonstrate that its critical value $I_c$ is a monotonic function of angle $\alpha$ when the junction is far enough from $0 - \pi$ transitions. However, this is possible when ferromagnets are weak. For stronger ferromagnets, $I_c(\alpha)$ is a non-monotonic function of $\alpha$ with characteristic dips related to the onset
of $0 - \pi$ transitions. This non-monotonicity is a simple consequence of a weaker influence of the exchange potential in the case of non-parallel magnetizations. No substantial impact of spin-triplet superconducting correlations on the Josephson current has been found in the clean limit. In fully transparent symmetric junctions the ferromagnetic influence is practically cancelled out for antiparallel magnetizations. This is not the case for finite transparency of interfaces when coherent geometrical oscillations of the critical Josephson current are superimposed on oscillations related to the transitions between 0 and $\pi$ states. In the limiting case of zero exchange energy our solutions reduce to the results for SI$_1$I$_2$N$_3$I$_4$S junctions with complex non-ferromagnetic normal-metal (N) interlayer, previously studied in the quasiclassical approach.

The paper is organized as follows. In Sec. II we present the model and solutions for the scattering problem used for calculation of the Josephson current. In Sec. III we discuss numerical results for the current-phase relation, and dependence of critical current on angle between magnetizations and thickness of ferromagnetic layers. We illustrate these results for various spin polarizations and transparencies of the interfaces. Concluding remarks are given in Sec. IV.

II. MODEL AND SOLUTION

We consider a clean SI$_1$I$_2$F$_1$I$_3$ heterojunction consisting of superconductors (S), two uniform mono-domain ferromagnetic layers (F$_1$ and F$_2$) with misorientation angle $\alpha$ between their magnetizations, and nonmagnetic interfacial potential barriers between metallic layers (I$_1$–I$_3$), see Fig. 1. Superconductors are described in the framework of standard BCS formalism, while for ferromagnets we use the Stoner model with an exchange energy shift $2h(r)$ between the spin subbands.

![FIG. 1: Schematics of an SI$_1$I$_2$F$_1$I$_3$S heterojunction. The magnetization vectors lie in the y-z plane and form the opposite angles $\pm\alpha/2$ with respect to the z-axis.](image)

Electron-like and hole-like quasiparticles with energy $E$ and spin projection $\sigma = \uparrow, \downarrow$ are described by wavefunctions $u_\sigma(r)$ and $v_\sigma(r)$, where $r$ is the spatial coordinate. Using the four-component wave function $\Psi(r) = [u_\uparrow(r), u_\downarrow(r), v_\uparrow(r), v_\downarrow(r)]^T$, we write the Bogoliubov–de Gennes equation as

$$\hat{H}\Psi(r) = E\Psi(r),$$

where

$$\hat{H} = \begin{pmatrix} \hat{H}_+(r) & \hat{A}(r) \\ \hat{A}^*(r) & -\hat{H}_-(r) \end{pmatrix},$$

while the $2 \times 2$ blocks are given by $\hat{H}_\pm(r) = H_0(r)\hat{1} - h(r)\sin\alpha(r)\hat{\sigma}_2 \mp h(r)\cos\alpha(r)\hat{\sigma}_3$ and $\hat{A}(r) = \Delta(r)\hat{1}$. Here, $\hat{\sigma}_3$ and $\hat{1}$ are the Pauli and unity matrix, respectively, and $H_0(r) = -\hbar^2\nabla^2/2m + W(r) + U(r) - \mu$. The chemical potential is denoted by $\mu$, $W(r) = \int_\alpha W(x-x_i)$ is potential of the barriers at interfaces, and $U(r)$ is the electrostatic potential. The $z$-axis is chosen to be perpendicular to the layers, whereas $x_1 = -d_1$, $x_2 = 0$, and $x_3 = d_2$ are coordinates of the interfaces. The electron effective mass $m$ is assumed to be the same throughout the layers. The difference $\mu - U(r)$ is equal to the Fermi energy of superconductors, $E_F^{(s)}$, or the mean Fermi energy of ferromagnets, $E_F^{(f)} = (E_F^s + E_F^f)/2$. Moduli of the Fermi wave vectors, $k_F^{(s)} = \sqrt{2mE_F^{(s)}/\hbar^2}$ and $k_F^{(f)} = \sqrt{2mE_F^{(f)}/\hbar^2}$ in S and F may be different in general. The in-plane, y-z, magnetizations of the neighboring F layers are not collinear in general, and the magnetic domain structure is described by the angle $\alpha(r)$ with respect to the z-axes: $\alpha(r) = -\alpha/2$ for $-d_1 < x < 0$ in F$_1$, and $\alpha(r) = \alpha/2$ for $0 < x < d_2$ in F$_2$. We assume equal magnitudes of the exchange interaction in ferromagnetic domains, $h(r) = h_0\Theta(x+d_1)\Theta(d_2-x)$, where $\Theta(x)$ stands for the Heaviside step function. We also assume that the two superconductors are identical, and neglecting self-consistency, we take the pair potential $\Delta(r)$ in the form

$$\Delta(r) = \Delta \left[ e^{-i\phi/2}\Theta(-x - d_1) + e^{i\phi/2}\Theta(x - d_2) \right],$$

where $\Delta$ is the bulk superconducting gap and $\phi$ is the macroscopic phase difference across the junction. The temperature dependence of $\Delta$ is given by $\Delta(T) = \Delta(0)\tanh(1.74\sqrt{T_c/T - 1})$.

Note that self-consistency may be safely neglected when the proximity effect is weak between S and F layers. This includes the situations with large tunnel barriers at interfaces, and/or narrow F constriction, and/or large Fermi velocities mismatch. For a planar junction geometry and good contacts between S and F layers, $\Delta$ will be suppressed at the vicinity of FS interfaces.

The parallel component of the wave vector, $k_F^\parallel$, is conserved due to translational invariance of the junction in directions perpendicular to the $x$-axis. Consequently, the wave function can be written in the form

$$\Psi(r) = \psi(x)e^{ik_F^\parallel r},$$
where \( \psi(x) = [u_r(x), u_s(x), v_r(x), v_s(x)]^T \) satisfies the boundary conditions

\[
\psi(x)_{x=0} = \psi(x)_{x=0} = \psi(x), \quad \frac{d\psi(x)}{dx}_{x=0} - \frac{d\psi(x)}{dx}_{x=0} = Z_ik_F(s) \psi(x),
\]

(2.5)

Here, \( Z_i = 2mW_i/h^2k_F(i = 1, 2, 3) \) are parameters that measure the strength of each insulating interface located at \( x_i = -d_1, 0, d_2 \).

The four independent solutions of the scattering problem for Eq. (2.1) correspond to the four types of quasiparticle injection processes: an electron-like or a hole-like quasiparticle injected from either the left or from the right superconducting electrode. When an electron-like quasiparticle is injected from the left superconductor, the solutions of Eq. (2.1) are

\[
u_r(x) = (e^{ik_x^+x} + b_r e^{-ik_x^+x})\bar{u} e^{-i\phi/2} + a_r e^{ik_x^-x} \bar{v} e^{-i\phi/2},
\]

(2.7)

\[
u_s(x) = (e^{ik_x^+x} + b_s e^{-ik_x^+x})\bar{u} e^{-i\phi/2} + a_s e^{ik_x^-x} \bar{v} e^{-i\phi/2},
\]

(2.8)

\[
u_l(x) = (e^{ik_x^+x} + b_l e^{-ik_x^+x})\bar{v} + a_l e^{ik_x^-x} \bar{u},
\]

(2.9)

\[
u_t(x) = (e^{ik_x^+x} + b_t e^{-ik_x^+x})\bar{v} + a_t e^{ik_x^-x} \bar{u},
\]

(2.10)

for the left superconductor \((x < 0)\), and

\[
u_r(x) = C_1 e^{iq_x^+x} \cos(\alpha/2) + C_2 e^{-iq_x^+x} \cos(\alpha/2) - iC_3 e^{iq_x^+x} \sin(\alpha/2) - iC_4 e^{-iq_x^+x} \sin(\alpha/2),
\]

(2.15)

\[
u_s(x) = -iC_1 e^{iq_x^+x} \sin(\alpha/2) - iC_2 e^{-iq_x^+x} \sin(\alpha/2) + C_3 e^{iq_x^+x} \cos(\alpha/2) + C_4 e^{-iq_x^+x} \cos(\alpha/2),
\]

(2.16)

\[
u_l(x) = iC_5 e^{iq_x^-x} \cos(\alpha/2) + iC_6 e^{-iq_x^-x} \cos(\alpha/2) + C_7 e^{iq_x^-x} \sin(\alpha/2) + C_8 e^{-iq_x^-x} \sin(\alpha/2),
\]

(2.17)

\[
u_t(x) = C_5 e^{iq_x^-x} \cos(\alpha/2) + C_6 e^{-iq_x^-x} \cos(\alpha/2) + iC_7 e^{iq_x^-x} \sin(\alpha/2) + iC_8 e^{-iq_x^-x} \sin(\alpha/2),
\]

(2.18)

for the right superconductor \((x > d_2)\), and

\( k^\pm = \sqrt{(2m/h^2)(E_F^2 \pm \Omega)^2 - k^2_\parallel} \)

(2.19)

in superconductors, and

\[ q_\sigma^\pm = \sqrt{(2m/h^2)(E_F^{(f)} + \rho_\sigma h_0 \pm \Omega) - k^2_\parallel} \]

(2.20)

in ferromagnetic layers. The sign + or - in the superscript corresponds to the sign of quasiparticle energy, whereas \( \rho_\sigma = +1(-1) \) is related to the spin projection \( \sigma = \uparrow(\downarrow) \).

Note that solutions for quasiparticles with opposite spin orientation in \( F \) layers are coupled due to misorientation of magnetizations. Solutions are decoupled for \( \alpha = 0 \) and \( \alpha = \pi \). In these cases spin triplet superconducting correlations with nonzero spin projections also vanish.

The case of transparent SFFS junctions with \( \alpha = \pi \) was treated previously by Blanter and Heikkin. For \( \alpha = 0 \), \( Z_2 = 0 \), and \( Z_1 = Z_3 \) solutions reduce to analytic expressions previously obtained for SIFS junctions with a net thickness of the ferromagnetic layer \( d = d_1 + d_2 \).

Complete solution of the scattering problem requires determination of 24 unknown coefficients: \( 4 \times 4 \) in superconducting electrodes \( (a_\sigma, b_\sigma) \) in the left \( S \), and \( c_\sigma, d_\sigma \) in the right \( S \) for two spin orientations) and \( 8 \times 8 \) in ferromagnetic electrodes \( (C_1, \ldots, C_8 \) in \( F_1 \) and \( C_1', \ldots, C_8' \) in \( F_2 \). When applied to the solutions given by Eqs. (2.1)–(2.18), the boundary conditions at three interfaces, Eq. (2.5), provide the necessary 24 equations. Analogously, one can find the solutions for other three channels of quasiparticle injection processes. However, the Andreev amplitudes in the first channel are sufficient to calculate the Josephson current.

### III. JOSEPHSON CURRENT

The stationary Josephson current can be expressed in terms of the Andreev reflection amplitudes, \( a_\sigma = a_\sigma(\phi) \), by using the temperature Green’s function formalism.

\[
I(\phi) = \frac{e\Delta}{2\hbar} \sum_{\sigma, k_{||}} k_{||} T \times \sum_{\omega_n} \frac{1}{2\Omega_n} (k_{\uparrow}^+ + k_{\downarrow}^-) \left( \frac{a_{\sigma\uparrow}(\phi)}{k_{\uparrow}} - \frac{a_{\sigma\downarrow}(-\phi)}{k_{\downarrow}} \right),
\]

(3.1)

where \( k_{\uparrow}^+, k_{\downarrow}^- \), and \( a_{\sigma\uparrow}(\phi) \) are obtained from \( k^+, k^- \), and \( a_{\sigma}(\phi) \) by the analytic continuation \( E \rightarrow i\omega_n \). The Matsubara frequencies are \( \omega_n = \pi k_B T(2n + 1) \), \( n = 0, \pm 1, \pm 2, \ldots \), and \( \Omega_n = \sqrt{\omega_n^2 + \Delta^2} \).

For simplicity, we illustrate our results for a single transverse channel, \( k_{||} = 0 \), symmetric junctions, \( d_1 = \ldots \)
$d_2 = d/2$, and equal Fermi wave vectors, $k_F^{(e)} = k_F^{(f)} = k_F$. Superconductors are characterized by fixed bulk value of the pair potential, $\Delta/E_F^{(e)} = 10^{-3}$. In order to adhere to realistic parameters all the examples are shown for $k_F d < 100$ since it still seems unlikely that the ballistic limit can be experimentally achieved for thicker ferromagnets.

For fully transparent interfaces, $Z_1 = Z_2 = Z_3 = 0$, and for a weak exchange field, $h_0/E_F = 0.01$, the current-phase relations are shown in Fig. 2 for three different values of misorientation angle $\alpha = 0$, $\pi/2$, $\pi$. The figure illustrates a junction with thin ferromagnetic interlayer, $k_F d = 30$, at low temperature $T/T_c = 0.1$. As expected, the $I(\phi)$ curve for $\alpha = \pi$ is the same as for the corresponding SNS junction ($h_0 = 0$) because the influence of opposite magnetizations in $F_1$ and $F_2$ practically cancels out. The $I(\phi)$ relations are anharmonic due to full transparency and low temperature. The junction is clearly in the 0-state for all values of misorientation angle; for such a small $h_0$ there are no oscillations of the critical current related to the $0 - \pi$ transitions for $k_F d < 100$, Fig. 3.

We have shown that the critical current increases mono-

![FIG. 2: (Color online) The current-phase relation $I(\phi)$ for $T/T_c = 0.1$, $h_0/E_F = 0.01$, $Z_1 = Z_2 = Z_3 = 0$, $d k_F = 30$, and three different values of the misorientation angle: $\alpha = 0$ (solid curve), $\alpha = \pi/2$ (dashed curve), and $\alpha = \pi$ (dash-dotted curve). The latter coincides with $I_c(d)$ for the corresponding SNS junction ($h_0 = 0$).](image)

![FIG. 3: (Color online) The critical current $I_c$ as a function of $d$ for $T/T_c = 0.1$, $h_0/E_F = 0.01$, $Z_1 = Z_2 = Z_3 = 0$, and three different values of the misorientation angle: $\alpha = 0$ (solid curve), $\alpha = \pi/2$ (dashed curve), and $\alpha = \pi$ (dash-dotted curve). The latter approximately coincides with $I_c(d)$ for the corresponding SNS junction (dotted curve). Inset: $I_c$ as a function of $\alpha$ for $d k_F = 30$.](image)

![FIG. 4: (Color online) The current-phase relation $I(\phi)$ for $T/T_c = 0.1$, $h_0/E_F = 0.1$, $Z_1 = Z_2 = Z_3 = 0$, $d k_F = 30$, and three different values of the misorientation angle: $\alpha = 0$ (solid curve), $\alpha = \pi/2$ (dashed curve), and $\alpha = \pi$ (dash-dotted curve). The latter approximately coincides with $I_c(d)$ for the corresponding SNS junction (dotted curve).](image)

![FIG. 5: (Color online) The critical current $I_c$ as a function of $d$ for $T/T_c = 0.1$, $h_0/E_F = 0.1$, $Z_1 = Z_2 = Z_3 = 0$, and three different values of the misorientation angle: $\alpha = 0$ (solid curve), $\alpha = \pi/2$ (dashed curve), and $\alpha = \pi$ (dash-dotted curve). Dotted curve represents $I_c(d)$ for the corresponding SNS junction ($h_0 = 0$).](image)
FIG. 6: (Color online) The critical current $I_c$ as a function of the misorientation angle $\alpha$ for $T/T_c = 0.1$, $h_0/E_F = 0.1$, and four sets of interface transparencies: $Z_1 = Z_2 = Z_3 = 0$ (solid curve), $Z_1 = Z_3 = 0$, $Z_2 = 1$ (dash-dotted curve), $Z_1 = Z_3 = 1$, $Z_2 = 0$ (dashed curve), and $Z_1 = Z_2 = Z_3 = 1$ (dotted curve). Panel (a): $dk_F = 30$. Panel (b): $dk_F = 60$. The dips in $I_c(\alpha)$ separate alternating 0 and $\pi$ states, beginning with a 0 state from the left.

tonically with increasing $\alpha$, from the SFS ($\alpha = 0$) to the SNS ($\alpha = \pi$) value. It is evident that spin triplet correlations, present for $0 < \alpha < \pi$, here do not induce a non-monotonic variation of $I_c(\alpha)$. For larger exchange energy, $h_0/E_F = 0.1$, the current-phase relations are shown in Fig. 4 for the same values of $\alpha$, $kp_d$, and $T/T_c$ as in Fig. 2. In Fig. 4, for $\alpha = \pi/2$ the $I(\phi)$ curve exhibits coexistence of 0 and $\pi$ states with a dominant contribution of the second harmonic. The critical Josephson current oscillates as a function of the ferromagnetic interlayer thickness due to the onset of $0 - \pi$ transitions, except for $\alpha$ close to $\pi$. The period of oscillations increases with $\alpha$. This is simply the effect of a weaker influence of the exchange potential in the case of non-parallel magnetizations. For example, the period of oscillations in $I_c$ is approximately twice bigger for $\alpha = \pi/2$ than for $\alpha = 0$. We have obtained the same result for parallel, but twice smaller, exchange energy ($\alpha = 0$ and $h_0/E_F = 0.05$). It can be shown (from the calculation of the junction free energy) that the dips in $I_c(\alpha)$ curves correspond to $0 - \pi$ transitions. Similar oscillations of the critical Josephson current as a function of $\alpha$ for fixed $kp_d$ (either 30 or 60) are shown in Fig. 4 as well as the influence of finite interfacial transparency. As already suggested for diffusive junctions, it can be clearly seen that the transition between 0 and $\pi$ states can be induced by varying the misorientation angle $\alpha$. This feature is of particular importance for experimental applications, since fine tuning of $0 - \pi$ transitions could be realized more easily by varying $\alpha$ than by varying thickness or temperature.

The influence of geometrical resonances is illustrated in Fig. 2 for $Z_1 = Z_3 = 0$, and (a) $Z_2 = 1$ and (b) $Z_2 = 10$. All other parameters are the same as in Fig. 4. Oscillations of the critical current due to geometrical resonances are superimposed on the oscillations related to the transitions between 0 and $\pi$ states. This effect is clearly visible in Fig. 2(b) for low transparency of the interface between magnetic domains ($Z_2 = 10$). We emphasize that now, due to the presence of a barrier between the F layers, the magnetic influence persists for $\alpha = \pi$, and $I_c(d)$ considerably differs from the results obtained for the corresponding SNINS junction. However, for identical domains, there are still no transitions to the $\pi$ state in the AP configuration. It can be seen that the amplitudes of geometrical oscillations of the supercurrent are significantly larger in SFIFS than in the corresponding SNINS junctions (dotted curves in Fig. 2). Rapid oscillation of $I_c(d)$ can be also seen in the case of finite transparency of interfaces between ferromagnetic layers and superconductors for any value of $\alpha$. This is due to the resonant amplification of the Josephson current by quasi-bound states entering the superconducting gap as the thickness $d$ of the interlayer is varied. The effect of finite transparency of all the interfaces is similar. In a more realistic case of planar (multichannel) junctions these geometrical oscillations of the critical current
are damped, while positions of maxima and minima are slightly shifted.\textsuperscript{25}

Transitions between 0 and $\pi$ states can be induced by changing the temperature of a junction with low transparency and strong ferromagnetic influence\textsuperscript{26-28}. This is illustrated in Fig. 8 for $Z_1 = Z_3 = 0$, and $Z_2 = 10$, $k_Fd = 43$, $h_0/E_F = 0.1$ and for three values of $\alpha = 0$, $\pi/16$, $\pi/2$. It can be seen that temperature dependence is very sensitive to the value of $\alpha$; for example, the temperature induced transition from 0 to $\pi$ state when $\alpha \approx 0$ is absent for $\alpha = \pi/2$.

![FIG. 8: (Color online) The critical current $I_c$ as a function of $T/T_\epsilon$ for $dk_F = 43$, $h_0/E_F = 0.1$, $Z_1 = Z_3 = 0$, $Z_2 = 10$ and three different values of the misorientation angle: $\alpha = 0$ (solid curve), $\alpha = \pi/16$ (dash-dotted curve), and $\alpha = \pi/2$ (dashed curve). For $\alpha \approx 0$ the characteristic nonmonotonic variation is related to the transition from 0 state (low $T$) to $\pi$ state (higher $T$) with dip at the transition. For $\alpha = \pi/2$ the junction is in 0 state at all temperatures.](image)

IV. CONCLUSION

We have studied the Josephson effect in clean superconductor-ferromagnet heterojunctions containing two mono-domain ferromagnetic layers with arbitrary transparency of the interfaces and any angle $\alpha$ between magnetizations, including two previously considered limiting cases $\alpha = 0$ and $\pi$.\textsuperscript{26,34} The Josephson current is calculated numerically via Bogoliubov-de Gennes formalism. We have found that transitions between 0 and $\pi$ states, resulting in characteristic dips in $I_c(\alpha)$ curves, can be induced by varying the relative orientation of magnetizations, like in diffusive junctions.\textsuperscript{25} However, this is simply the effect of a weaker influence of the exchange potential in the case of non-parallel magnetizations. No substantial impact of spin-triplet superconducting correlations on the Josephson current has been found in the clean limit. For weak ferromagnets, far from 0 -- $\pi$ transitions, the critical Josephson current monotonically depends on the angle between magnetizations. While in fully transparent junctions oscillatory dependence of the critical Josephson current on junction parameters is related only to the 0 -- $\pi$ transitions, for finite transparency of interfaces pronounced geometrical oscillations occur due to coherent contribution of quasiparticle transmission resonances (quasi-bound states) to the Andreev process.

In conclusion, due to recent progress in nanofabrication techniques\textsuperscript{25} the SI$_2$F$_1$F$_2$I$_2$S junction may be realized in the clean regime in a setup where the angle $\alpha$ could be tuned by a weak external magnetic field. We thus suggest experimental investigation of the predicted control of the critical current and 0 -- $\pi$ transitions by varying the angle between magnetizations in Josephson junctions with magnetic bilayers.

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