A sample of GRB radio afterglows inconsistent with the standard jet model

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ABSTRACT

We present a sample of 15 gamma-ray burst (GRB) afterglow light curves at radio frequencies, and compare them to the X-ray and/or optical properties of the afterglows and to the predictions of the standard jet/fireball model. Our sample has been chosen so that each afterglow exhibits a jet break at some frequency, usually X-ray. We examine the late-time decline of each burst in the radio and in the X-ray, and attempt to fit an analytical model based on the conventional GRB afterglow equations to each data set. We show that the GRBs in our sample are mostly incompatible with the light curve behavior predicted by conventional GRB afterglow theory. In particular, we observe a lack of visible jet breaks in the radio light curve, even when one is seen in the X-ray. No radio afterglow in this sample, at any time, shows the expected post-break decline of $\sim t^{-2}$, although a few remain consistent with the standard model if such a decline began soon after the observations. The observed decline in the radio is often described at least as well by a single power law as by the standard model, in some cases being consistent with the expected pre-break decline (assuming $\nu_{\text{radio}} > \nu_{m}$) until late times. Notably, signs of a jet break are visible in the millimeter-wave afterglow of GRB 161219B and GRB 111215A, perhaps suggesting that only lower radio-frequency afterglows behave anomalously. Nonetheless, the observed behavior conflicts with our current theoretical understanding of radio afterglows.

Keywords: gamma-ray burst: general — relativistic processes

1. INTRODUCTION

It is widely thought that the afterglow emission of a gamma-ray burst (GRB) originates as synchrotron radiation from electrons accelerated in the shock resulting from the interaction of the GRB jet with the circum-burst medium (CBM) (e.g. Paczynski & Rhoads 1993; Sari et al. 1998; Piran 2004). This standard fireball model predicts that the afterglow emission from X-ray to radio behaves as a series of (smoothly) connected power laws of the form $f_\nu \propto t^\alpha$, where the index $\alpha$ depends on the index of the electron energy distribution $p$ and on the frequency in relation to the (evolving) break frequencies in the afterglow spectrum (e.g. Granot & Sari 2002). These break frequencies are the self-absorption frequency $\nu_a$, the characteristic synchrotron frequency $\nu_m$ and the cooling frequency $\nu_c$. The relativistic nature of the jet and the resulting beaming of the emission from the jet is expected to cause an achromatic steepening of the light curve, known as a jet break, when the beaming angle becomes similar to the jet opening angle (Mészáros & Rees 1999; Rhoads 1999; Sari et al. 1999).

However, a problem with this interpretation was brought up in a series of papers showing that a population of radio-quiet GRBs exists (Hancock et al. 2013; Lloyd-Ronning & Fryer 2017; Lloyd-Ronning et al. 2019). It was also argued that these bursts (roughly a third of all GRBs) do not simply lack an observed radio afterglow due to insufficiently deep observations, but are indeed intrinsically radio-quiet. Lloyd-Ronning et al. (2019) further suggested that the two populations have different progenitor scenarios: the loud GRBs are typically longer and more luminous in $\gamma$-rays (especially at GeV energies where only loud GRBs are detected), and the loud bursts alone show an anti-correlation be-
tween prompt duration and redshift. Active galactic nuclei (AGN) exhibit a similar radio quiet vs. loud dichotomy (Xu et al. 1999). Chiaberge & Marconi (2011) have suggested that black holes with more mass and spin are responsible for the radio-loud AGN.

Another issue has been raised by Warren et al. (2017, 2018), who noted that a population of quasi-thermal electrons should exist in the jet in addition to the normally assumed power-law distribution, as the shock should only accelerate a fraction of the electrons (which may be as low as 0.01). The effect of this would be additional emission on top of the expected synchrotron spectrum – which should dominate in the radio while leaving higher frequencies unaffected – while νc should only accelerate a fraction of the electrons (which in the case of GRB 160625B, in particular, the radio break was not observed even at ~ 200 days. Furthermore, the observed radio light curves were better described by a single power law (∼ t−1) than by numerical afterglow models (van Eerten et al. 2012) in the decline phase. The only scenario permitted by the standard jet model (using equations from Rhoads 1999; Granot & Sari 2002) that produces a comparable behavior is a pre-break decline above νm. It was speculated that one possible solution might be a two-component jet where a wider, less energetic cocoon surrounds the core of the jet and would be responsible for the radio emission (suggested for GRB 030329 by Berger et al. 2003; Peng et al. 2005). Therefore radio emission from the 'main' component, commonly assumed to be the dominant source, may need to be suppressed somehow.

Based on the aforementioned studies, it has become clear that our understanding of the radio emission of GRB afterglows is incomplete. In this paper, we follow up on the work of K19, attempting to shed more light on this picture by examining the radio light curves of 15 GRBs with an observed light curve break in the optical or X-ray. We obtain this sample through a literature search, and compare the radio behavior to that of the higher frequencies, in particular concentrating on the radio decline rate, the presence or absence of an observed jet break, and the evolution of the peak frequency in the radio spectra. We show that the events in our sample, in general, and contrary to the expectations of the theory, do not show signs of a jet break at radio frequencies. We also attempt to fit each burst using the standard jet model; yet, in most cases a single power law with no break is a better description of the radio behavior. Two events do seem to exhibit a break at millimeter frequencies, however, and this is consistent with being simultaneous with their X-ray breaks. In Section 2 we present our sample and the methods we use to examine the properties of these GRBs through power-law fits and analytical modeling. Our results are presented in Section 3, and we discuss these findings in Section 4 and finally summarize our conclusions in Section 5. We adopt the notation \( F_\nu \propto t^\alpha \nu^2 \) for power-law light curves and spectra.

2. DATA AND ANALYSIS

2.1. The sample

The sample of GRBs we examine here includes the targets of K19, i.e. GRBs 160509A and 160625B, along with 13 other GRBs for which published radio and X-ray light curves exist in the literature, with the additional criterion that a break in the light curve consistent with a jet break (too steep to be the passage of \( \nu_c \)) must have been observed in at least one band – i.e. events such as GRB 130427A (Perley et al. 2014; De Pasquale et al. 2016) with no observed breaks were not included. This is not meant to be a complete or necessarily a representative sample of the full GRB population showing jet breaks; the objective of the study is to point out a widespread problem in radio light curve behavior with a sample of similarly behaving afterglows. The sample of GRBs and their main properties are summarized in Table 1. The sample includes a wide variety of GRBs including an ULGRB and a SGRB, three 'dark' GRBs with no detected optical afterglow, along with a broad range of redshifts (0.1475 ≤ z ≤ 4.954) and isotropic-equivalent energies (6.0 × 10^49 erg ≤ \( E_{iso} \) ≤ 3.0 × 10^54 erg).

2.2. Power-law fitting

We have examined the radio and X-ray light curves of our sample of GRBs as follows. With the exception of GRBs 990510 and 140903A, the Swift light curves were obtained from the Swift-XRT Lightcurve Repository and converted to flux densities at 5 keV using pimms2 and parameters from Swift (which in the case of GRB 130907A included a time-variable photon index). For the pre-Swift-era GRB 990510, X-ray fluxes were taken from Kuulkers et al. (2000) and converted to flux densi-

\(^1\) http://www.swift.ac.uk/xrt_curves/
\(^2\) http://cxc.harvard.edu/toolkit/pimms.jsp
ties using the reported parameters. The late-time \textit{Chandra} data associated with GRB 140903A were reported as flux densities at 1 keV; therefore we also convert the \textit{Swift} X-ray data to 1 keV flux densities using parameters in Troja et al. (2016). Radio data were obtained from the sources listed in Table 1. In some cases, to make the light curve fit a longer time period or make it more constraining, the light curves at some frequencies were augmented by interpolating between nearby frequencies, or by scaling to a nearby frequency assuming the density profile of the CBM (Mészáros & Rees 1999; Panaitescu & Mészáros 1999). The latter, which we refer to as the edge effect, is only due to the edge of the jet becoming visible after an initial pseudo-isotropic phase, with no lateral expansion.

In the radio we have used the frequencies with enough points for a fit after any observed rising phase and/or early features in the light curve dominated by a reverse shock (seen in e.g. GRBs 160509A, 160625B, and 161219B; Laskar et al. 2016, 2018b; Alexander et al. 2017; Troja et al. 2017; K19) – this means at least four points, so that we can see the shape of the light curve by eye. We make an exception in the case of GRB 140903A, where there are only three points available but they cleanly fit a single power law. We require an improvement at a $P_F > 0.95$ level, where $P_F$ is the probability of a smaller difference in variance if the fits are equally good, to accept the break as robust and at $P_F > 0.8$ to consider it ambiguous\(^3\). We use the fit to estimate $\alpha$ in the X-ray using standard closure relations; the break time $t_j$ if applicable; and whether the break is consistent with the expected post-break slope of $\alpha_2 = -p$ from lateral expansion at the speed of sound (Rhoads 1999) or with a steepening by $t^{-0.75}$ or $t^{-0.5}$ depending on the density profile of the CBM (Mészáros & Rees 1999; Panaitescu & Mészáros 1999). The latter, which we refer to as the edge effect, is only due to the edge of the jet becoming visible after an initial pseudo-isotropic phase, with no lateral expansion.

\(^{3}\)The notation $P_F$ is used instead of the conventional $p$ to avoid confusion with the index of the electron energy distribution $p$.

**Table 1.** GRB sample examined in this study.

| GRB  | $z$  | $E_{\text{iso}}$ (erg) | Type  | Reference(s) for radio/optical data |
|------|------|------------------------|-------|-----------------------------------|
| 990510 | 1.62 | $2.9 \times 10^{53}$   | Long  | van Eerten et al. (2012)\(^a\)   |
| 051022 | 0.8\(^b\) | $4.4 \times 10^{53}$   | Long  | Rol et al. (2007)                 |
| 070125 | 1.547 | $1.1 \times 10^{54}$   | Long  | Chandra et al. (2008)             |
| 090313 | 3.375 | $3.4 \times 10^{42}$   | Long  | Melandri et al. (2010)            |
| 100418A | 0.6235 | $1.0 \times 10^{32}$  | Long  | Moin et al. (2013); de Ugarte Postigo et al. (2018) |
| 110709B | ...  | ...                    | Long  | Zauderer et al. (2013)            |
| 111215A | 2.06\(^d\) | $1.4 \times 10^{53}$  | Long  | Zauderer et al. (2013); van der Horst et al. (2015) |
| 130907A | 1.238 | $3.0 \times 10^{34}$   | Long  | Veres et al. (2015)               |
| 140311A | 4.954 | $1.0 \times 10^{33}$   | Long  | Laskar et al. (2018a)             |
| 140903A | 0.351 | $6.0 \times 10^{49}$   | Short | Troja et al. (2016)               |
| 141121A | 1.469 | $8.0 \times 10^{52}$   | Ultra-long | Cucchiara et al. (2015) |
| 160509A | 1.17 | $8.6 \times 10^{53}$   | Long  | Laskar et al. (2016); K19         |
| 160625B | 1.406 | $3.0 \times 10^{54}$   | Long  | Alexander et al. (2017); (Troja et al. 2017); K19 |
| 161219B | 0.1475 | $1.8 \times 10^{30}$  | Long  | Laskar et al. (2018b)             |
| 171010A | 0.33 | $2.2 \times 10^{53}$   | Long  | Bright et al. (2019)              |

\(^{a}\)Compiled from Harrison et al. (1999); Israel et al. (1999); Bloom et al. (1999); Beuermann et al. (1999); Stanek et al. (1999), and Pietrzynski & Udalski (1999).

\(^{b}\)No optical afterglow was detected, but the GRB was localized to a likely host galaxy at $z \approx 0.8$.

\(^{c}\)No optical afterglow was detected.

\(^{d}\)No optical afterglow was detected – photometric host redshift given by van der Horst et al. (2015).
2.3. Analytical modeling

For each burst, we have also attempted to fit the standard afterglow model to the available data in order to check whether this model is able to reproduce the radio behavior. Even when no single (asymptotic) power-law segment predicted by the standard model matches the single or broken power-law fit, one can possibly obtain a good fit with a smooth transition from one slope to another, for example when break frequencies pass through the observed radio bands at certain times. We have developed a Python-based fitting code using the analytical representation of the evolution of a GRB synchrotron spectrum in Granot & Sari (2002) at times before the jet break. After the jet break we have used the analytical model of van der Horst (2007) (their Table 2.10; based on Rhoads 1999) for the evolution of each break frequency and the normalization of the flux.

As the break frequencies \( \nu_c, \nu_m, \) and \( \nu_a \) evolve differently with time, their order and thus the shape of the overall spectrum changes as well. Granot & Sari (2002) break the time evolution of the spectrum into five discrete regimes, determined by the order of the break frequencies. When one break frequency passes below another, the regime changes. Regime 1 corresponds to \( \nu_c > \nu_m > \nu_a \), regime 2 to \( \nu_c > \nu_a > \nu_m \), and regime 5 to \( \nu_m > \nu_c > \nu_a \). We assume a \( 5 \rightarrow 1 \rightarrow 2 \) evolution over time; however, as the model calculation begins at a certain time (0.001, 0.01 or 0.1 d depending on the first observed epoch), the spectrum is allowed to skip regime 5 (or even 1) if needed. The evolution of these regimes is determined by Equations (1) – (9) of Granot & Sari (2002), while their Table 2 lists the parameters of each break frequency and the flux at that frequency in each regime. Discontinuities in flux or break frequency across transitions from one spectral regime to another are eliminated as follows. We compute the break frequencies and normalization flux\(^4\) of the spectrum from model parameters only at the beginning of the model light curve. Their evolution over time is then described with broken power laws, with each break corresponding to times of transition between regimes (including the effects of a jet break). In addition, the spectrum itself is computed as a sum of the spectra in each regime, weighted using a function \( w_i \) for regime \( i \):

\[
\begin{align*}
    w_5 &= (1 + (t/t_{5 \rightarrow 1})^\eta)^{-1}, \\
    w_1 &= (1 + (t/t_{5 \rightarrow 1})^{-\eta})^{-1} + (1 + (t/t_{1 \rightarrow 2})^\eta)^{-1} - 1, \\
    w_2 &= (1 + (t/t_{1 \rightarrow 2})^{-\eta})^{-1}, \\
\end{align*}
\]

where \( t_{i \rightarrow j} \) is the transition time from \( i \) to \( j \), and \( \eta \) is a sharpness parameter that we fix at 2. As each break in the spectrum is soft, the resulting light curve has no sharp breaks.

We have used the Markov chain Monte Carlo (MCMC) package EMCEE (Foreman-Mackey et al. 2013) to find the best-fitting model parameters through \( \chi^2 \) minimization. The free parameters are \( p_i \), isotropic-equivalent kinetic energy \( E_{K,\text{iso}} \), the fraction of energy in electrons and magnetic fields \( e_e \) and \( e_B \), the jet opening angle \( \theta_j \), and the density of the CBM \( n_0 \) (ISM) or \( A_* \) (wind). For each of them, we have used a simple top-hat prior with upper and lower limits listed in Table 3. The jet break time \( t_j \) is determined by model parameters (van der Horst 2007):

\[
\begin{align*}
    t_j,\text{ISM} &= \frac{1 + z}{2} \left( \frac{\theta_j}{0.126} \right)^{8/5} n_0^{-1/5} \left( \frac{E_{K,\text{iso}}}{10^{52} \text{ erg}} \right)^{1/5}, \\
    t_j,\text{wind} &= \frac{1 + z}{2} \left( \frac{\theta_j}{0.166} \right)^4 A_*^{-1} \left( \frac{E_{K,\text{iso}}}{10^{52} \text{ erg}} \right). \\
\end{align*}
\]

We have also separately run each fit with \( t_j \) held fixed at the value we obtained using a broken power law fit, if applicable, as the fitting code is, in some cases, drawn toward unacceptably late break times by the radio data. The code was run with both constant-density and wind-type CBM for each target. Out of these, the best fit is then shown.

The optical, ultraviolet and infrared fluxes were corrected for Galactic reddening using the Schlaufy & Finkbeiner (2011) dust maps and the Cardelli et al. (1989) extinction law. A host galaxy extinction estimate was obtained from the source papers listed in Table 1, if available, and corrected for using the Pei (1992) extinction law for the Small Magellanic Cloud, which was assumed in these papers as well. Points attributed to early reverse shock features, supernovae or host galaxy contamination were discarded. The luminosity distance of each object was obtained using the redshifts in Table 1 – with the exception of GRB 110709B, where the real redshift is unknown, so a fairly typical value of \( z = 2.0 \) was assumed\(^5\). We used the cosmological parameters

\[
\begin{align*}
    \theta_j &= 2, \\
    p &= 0, \\
    \end{align*}
\]

\(^4\) This corresponds to the flux at the lowest-frequency break, i.e. \( f(\nu_b) \) in regimes 5 and 1 and \( f(\nu_m) \) in regime 2, in the Granot & Sari (2002) description.

\(^5\) The assumed value of the redshift will change some of the fit values (see Zauderer et al. 2013), but for our purposes this is not relevant as long as one redshift results in compatibility with the standard model.
in Bennett et al. (2014): $H_0 = 69.6$ km s$^{-1}$ Mpc$^{-1}$; $Ω_m = 0.286; ΩΛ = 0.714$. The best-fitting parameters and their uncertainties are listed in Table 4.

3. RESULTS

Below, we describe the behavior of each individual GRB in our sample and whether some scenario of the conventional afterglow theory (Rhoads 1999; Granot & Sari 2002) can account for all observations. In Figs. 1 through 15 we show the single or broken power-law fits and, if available, the best-fit analytical model, compared to the observed light curves of each individual object in our sample. We also show the radio spectral energy distributions (SEDs) of the objects where light curves at multiple (at least 3) radio frequencies are available, compared to best-fitting model spectra. In each light curve, the dashed vertical line corresponds to the jet break time in either the X-ray broken power law fit or our best-fit model. Any data points with open symbols were ignored in the analytical model fit, e.g. because of early reverse shock features or host galaxy contamination.

3.1. GRB 990510

For GRB 990510, only the optical shows evidence of a jet break (Fig. 1), but as the X-ray follow-up was much shorter, a break cannot be excluded in the X-ray either. Both the optical and X-ray pre-break light curves are consistent with $p \approx 2.4$ and $ν_c > ν_m$ in an ISM-type CBM, but in this case the optical post-break decline seems to require a break mechanism that involves a combination of the edge effect and partial lateral expansion, as suggested for GRB 100625B by K19 and seen in simulations by van Eerten & MacFadyen (2012). However, the post-optical-break radio decline can be fitted with $α_{\text{radio}} = −0.62 ± 0.10$, which is only compatible with the standard model at pre-break times in a wind CBM. Our best-fit analytical model is able to reproduce the radio and optical light curves with a smooth transition from $ν_a < 8.6$ GHz $< ν_m$ to $ν_m < ν_a < 8.6$ GHz, but in this case a $t^{−p}$ decline is predicted soon after the last radio observations. Although the best-fit model shows some deviation from the last radio point, we conclude that GRB 990510 is consistent with the standard model if a $t^{−p}$ decline began soon after the radio observations ended.

3.2. GRB 051022

A possible jet break was observed in the X-ray (with $P_F = 0.82$, the break is ambiguous); however, as this was a 'dark' burst, an optical afterglow was not detected (Fig. 2). The X-ray light curve is consistent with $p ≈ 2.5$ and $ν > ν_c$. The radio light curve has a large scatter, but Rol et al. (2007) attribute this to scintillation effects; a single power law with strong scintillation may thus fit the light curve. In this case $α_{\text{radio}}$ is consistent with the $−1/3$ expected for a post-break slope if $ν < ν_m$ and with full lateral expansion. Alternatively one can place a rise before $≈ 2$ d, but then $α_{\text{radio}} = −0.67 ± 0.28$. This is still consistent with $−1/3$ within $≈ 1.2σ$, however, if $ν_m$ stays above 4.9 GHz until $≈ 15$ d. Our model fit adequately reproduces the light curve; the best fit has a wind-type CBM and plausible physical parameters. Therefore GRB 051022 is consistent with the standard model, although a $t^{−p}$ decline is predicted soon after the last radio observations.

3.3. GRB 070125

The X-ray light curve is consistent with $p ≈ 2.0$ and $ν > ν_c$ – however, Chandra et al. (2008) place an optical break $≈ 3×$ later than the X-ray break. The radio light curve (Fig. 3) shows consistency with a single power law at all fitted frequencies after a peak between 10 and $≈ 30$ d, but at 8.5 GHz we see an ambiguous ($P_F = 0.852$) break at $88 ± 41$ d as well. The other frequency with an
| GRB  | Band  | Decline | $\alpha_1$ | $\alpha_2$ | $t_j$ | Notes |
|------|-------|---------|------------|------------|-------|-------|
| 990510 | 5 keV | SPL     | −1.43 ± 0.07 | ...        | ...   | Seems to continue past optical break |
|      | V     | BPL     | −0.94 ± 0.01 | −2.02 ± 0.03 | 1.3 ± 0.1 d | ... |
|      | i     | BPL     | −1.16 ± 0.04 | −1.81 ± 0.05 | 1.4 ± 0.2 d | ... |
| 8.6 GHz | SPL   | −0.62 ± 0.10 | ...        | ...        | ...   | ... |
| 051022 | 5 keV | BPL     | −1.42 ± 0.05 | −2.50 ± 0.27 | 2.7 ± 0.6 d | Ambiguous break |
| 4.9 GHz | BPL   | −0.46 ± 0.17 | ...        | ...        | ...   | Large scatter* |
| 070125 | 5 keV | BPL     | −0.71 ± 0.50 | −2.00 ± 0.99 | 1.1 ± 0.3 d | Large $\alpha_1$ error |
|      | 22.5 GHz | SPL  | −0.58 ± 0.05 | ...        | ...   | ... |
|      | 14.9 GHz | SPL  | −0.54 ± 0.23 | ...        | ...   | ... |
|      | 8.5 GHz | BPL?   | −0.33 ± 0.11 | −1.12 ± 0.32 | 88 ± 41 d | Ambiguous break |
|      | 4.9 GHz | SPL   | −0.29 ± 0.06 | ...        | ...   | ... |
| 090313 | 5 keV | BPL     | −1.04 ± 0.14 | −2.23 ± 0.17 | 1.0 ± 0.2 d | ... |
| 16 GHz | SPL   | −0.31 ± 0.01 | ...        | ...        | ...   | ... |
| 100418 | 5 keV | BPL     | −1.01 ± 0.08 | −1.85 ± 0.20 | 4.5 ± 1.2 d | ... |
|      | 90 GHz | SPL    | −0.60 ± 0.06 | ...        | ...   | ... |
|      | 8.5 GHz | SPL   | −1.05 ± 0.21 | ...        | ...   | ... |
|      | 5.0 GHz | SPL   | −0.41 ± 0.08 | ...        | ...   | ... |
| 110709B | 5 keV | BPL     | −0.91 ± 0.03 | −1.57 ± 0.04 | 0.65 ± 0.06 d | ... |
| 5.8 GHz | SPL   | −0.70 ± 0.10 | ...        | ...        | ...   | ... |
| 111215A | 5 keV | BPL     | −1.35 ± 0.03 | ...        | ...   | May break at the same time as 93 GHz |
| 93 GHz | SPL   | −0.20 ± 0.06 | −1.73 ± 0.38 | 15 ± 3 d | ... |
|      | 19.1 GHz | SPL  | −1.08 ± 0.04 | ...        | ...   | ... |
|      | 6.7 GHz | SPL   | −0.80 ± 0.09 | ...        | ...   | ... |
|      | 4.9 GHz | SPL   | −0.56 ± 0.04 | ...        | ...   | ... |
| 130907A | 5 keV | BPL     | −1.35 ± 0.02 | −2.20 ± 0.03 | 0.25 ± 0.02 d | ... |
| 15 GHz | SPL   | −0.71 ± 0.04 | ...        | ...        | ...   | ... |
| 11 GHz | SPL   | −0.88 ± 0.28 | ...        | ...        | ...   | ... |
| 140311A | 5 keV | BPL     | −1.14 ± 0.11 | −1.93 ± 0.48 | 1.3 ± 1.1 d | Ambiguous break |
| 24.5 GHz | SPL  | −0.88 ± 0.07 | ...        | ...        | ...   | ... |
|      | 19.2 GHz | SPL  | −0.80 ± 0.09 | ...        | ...   | ... |
|      | 13.5 GHz | SPL  | −0.63 ± 0.12 | ...        | ...   | ... |
| 8.6 GHz | complex | ... | ... | ... | ... | Late-time rebrightening |
| 140903A | 1 keV | BPL     | −0.95 ± 0.13 | −2.31 ± 0.18 | 0.7 ± 0.2 d | ... |
| 6.1 GHz | SPL   | −0.64 ± 0.03 | ...        | ...        | ...   | ... |
| 141121A | 5 keV | BPL     | −0.46 ± 0.11 | −2.21 ± 0.19 | 3.8 ± 0.5 d | ... |
|      | 15 GHz | SPL    | −0.61 ± 0.08 | ...        | ...   | ... |
|      | 7 GHz | complex | ...        | ...        | ...   | Hints of the shape of the 7 GHz light curve |
|      | 5 GHz | complex | ...        | ...        | ...   | Multiple peaks |
| 3 GHz | complex | ... | ... | ... | ... | Multiple peaks |
| 160509A | 5 keV | BPL     | −1.20 ± 0.06 | −1.96 ± 0.09 | 3.7 ± 0.8 d | ... |
| 9 GHz | SPL   | −0.92 ± 0.13 | ...        | ...        | ...   | ... |
| 6 GHz | SPL   | −0.91 ± 0.11 | ...        | ...        | ...   | ... |
| 160625B | 5 keV | BPL     | −1.24 ± 0.02 | −2.23 ± 0.15 | 22 ± 4 d | ... |
| 22 GHz | SPL   | −0.75 ± 0.12 | ...        | ...        | ...   | ... |
| 6.1 GHz | SPL   | −1.08 ± 0.11 | ...        | ...        | ...   | ... |
| 161219B | 5 keV | BPL     | −0.80 ± 0.01 | −1.64 ± 0.11 | 17 ± 3 d | ... |
| 104 GHz | BPL   | −0.61 ± 0.01 | ...        | ...        | ...   | SN-dominated points (> 2.7 d) ignored |
| 11 GHz | BPL   | −0.48 ± 0.05 | −1.47 | ... | Too few points for proper BPL fit |
| 5 GHz | BPL   | −0.63 ± 0.18 | ...        | ...        | ...   | ... |
| 171010A | 5 keV | BPL     | −1.29 ± 0.06 | −1.98 ± 0.27 | 3.8 ± 1.6 d | ... |
| 15.5 GHz | SPL  | −1.12 ± 0.05 | ...        | ...        | ...   | ... |

*Attributed to scintillation by Rol et al. (2007).

**Table 2.** Results of our single or broken power-law fits to the decline in the X-ray and radio light curves (in the case of GRBs 990510 and 161219B, also in the optical due to its relevance to the analysis), ignoring any rise in the light curve or features attributed to a reverse shock or late engine activity. In 'Decline', SPL corresponds to a single power law and BPL to a broken power law.
Table 3. Upper and lower limits of each free parameter in our model fits.

| Parameter | lower limit | upper limit |
|-----------|-------------|-------------|
| \( E_{\text{K, iso}} \) | \( 10^{49} \) erg | \( 10^{56} \) erg |
| \( p \) | 2.0 | 3.2 |
| \( n_0 \) | \( 10^{-5} \) cm\(^{-3} \) | \( 10^{4} \) cm\(^{-3} \) |
| \( A^* \) | \( 10^{-5} \) | \( 10^{4} \) |
| \( \epsilon_e \) | \( 10^{-5} \) | 1 |
| \( \epsilon_B \) | \( 10^{-5} \) | 1 |
| \( \theta_0 \) | 0.01 rad | 1.5 rad |

Figure 2. Our single and broken power law fits (upper panel) and best-fitting analytical model (lower panel) compared to the light curves of GRB 051022.

equally long follow-up, 4.9 GHz, does not share this feature, however, and the putative post-break slope is only \(-1.12 \pm 0.32\), inconsistent with post-break expectations but consistent with pre-break within the errors.

The spectral index is positive below 22.5 GHz until at least 87 d, indicating that \( \nu_m \) is located above or, at later times, possibly around, 22.5 GHz, but the slope of the light curve at 14.9 and 22.5 GHz before this time (\( \alpha \approx -0.55 \)) does not match the slope of \(-1/3\) predicted below \( \nu_m \) after the jet break, or indeed any pre- or post-

break expectation of the standard model when \( p \approx 2.0 \) (Rhoads 1999; Granot & Sari 2002).

While our fitting code can reproduce the X-ray and optical data, we are unable to obtain a good fit to the radio light curve after 20 days or so or to the spectra after 35 d. Instead of the observed behavior, a \( t^{-p} \) decline is predicted after this point. Chandra et al. (2008) acknowledge a lack of good radio fits to their model,

Figure 3. Our single and broken power law fits (upper panel) to the light curves of GRB 070125, and our best-fitting analytical model compared to the light curves (middle panel) and radio SEDs (lower panel).
but mostly bring up the early times, where they interpret the difference as scintillation effects. At late times, their model under-predicts the radio data points as well. Thus we do not see the light curve behavior predicted by standard theory.

### 3.4. GRB 090313

The X-ray behavior is consistent with $p \approx 2.2$ and $\nu > \nu_c$. The optical light curve exhibits a re-brightening feature, possibly explained by a decelerating fireball when $\nu_m$ lies below the optical bands (Melandri et al. 2010) – this ‘bump’ in the light curve was ignored in our model fits, which cannot accommodate it. The radio light curve (Fig. 4) peaks around $3 \times t_{j,X}$ and turns over onto a single power law with $\alpha_{66\text{GHz}} = 0.31 \pm 0.01$. This is close to $-1/3$ expected after a lateral expansion break when $\nu < \nu_m$, so GRB 090313 at first glance seems compatible with the standard model, if $t_{j,\text{radio}} \sim 3t_{j,X}$, which is consistent with simulations by van Eerten et al. (2011), and if $\nu_m$ stays above 16 GHz until $\sim 35$ d. However, our model fit cannot reproduce this behavior – the early ($< 0.1$ d) optical data constrain $\nu_m$ to be below the optical at that point, and it cannot stay above 16 GHz long enough. The predicted $t^{-B}$ decline is ruled out by the late-time radio data.

### 3.5. GRB 100418A

Based on the X-ray light curve (Fig. 5), we infer $p \approx 2.0$ and $\nu > \nu_c$, or alternatively $p \approx 2.3$ and $\nu_c > \nu > \nu_m$; in the latter case the CBM is restricted to ISM-like. The value of $\beta = -1.00 \pm 0.05$, reported by de Ugarte Postigo et al. (2018) to fit the spectrum from optical to X-ray at $\sim 2$ d, rules out the former unless $\nu_c$ is also below the optical bands. The radio light curve at 5 and 8.5 GHz peaks around 50 d ($10 \times t_{j,X}$) and at each

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**Table 4.** Best-fit analytical model parameters and their uncertainties for each GRB in our sample. Both types of CBM (ISM and wind) were attempted for each burst, and the better-fitting model is listed.

| GRB   | CBM | $E_{K,\text{iso}}$ (10$^{52}$ erg) | $p$ | $n_0$ (cm$^{-3}$) | $A_*$ | $\epsilon_c$ | $\epsilon_B$ | $\theta_j$ (rad) |
|-------|-----|----------------------------------|-----|------------------|------|--------------|--------------|-----------------|
| 090510 | ISM | $5.4^{+0.5}_{-3.5}$              | $2.26^{+0.01}_{-0.08}$ | 1270$^{+400}_{-800}$ | ...  | 0.48$^{+0.15}_{-0.04}$ | 2.6$^{+19.3}_{-6}$ | $10^{-5}$ 0.27 $\pm 0.01$ |
| 051022 | Wind| $39^{+25}_{-17}$                | 2.28 $\pm 0.04$ | ... | 0.22$^{+0.19}_{-0.09}$ | 0.23$^{+0.11}_{-0.06}$ | 3.4$^{+1.1}_{-0.6}$ | $10^{-5}$ 0.054 $\pm 0.023$ |
| 070125 | ISM | $0.097 \pm 0.005$               | $2.11 \pm 0.01$ | 102 $\pm 5$ | ... | 0.53 $\pm 0.02$ | 0.47 $\pm 0.02$ | 0.36 $\pm 0.01$ |
| 090313 | ISM | $18^{+1.6}_{-1.0}$              | 2.02$^{+0.06}_{-0.01}$ | 5.3$^{+4.1}_{-1.8}$ | ... | 0.64$^{+0.28}_{-0.40}$ | 4.9$^{+1.8}_{-2.9}$$10^{-3}$ | 0.08 $\pm 0.01$ |
| 110709B | ISM | $3.1^{+2.2}_{-0.9}$          | 2.16 $\pm 0.02$ | 1.5$^{+1.6}_{-1.4}$ | ... | 0.69$^{+0.22}_{-0.29}$ | 0.06$^{+0.24}_{-0.04}$ | 0.15$^{+0.04}_{-0.05}$ |
| 111215A | Wind | 6.7 $\pm 0.3$               | 2.26 $\pm 0.01$ | ... | 31.7 $\pm 0.5$ | 0.76 $\pm 0.03$ | 1.00 $\pm 0.01$$10^{-5}$ | 0.50 $\pm 0.01$ |
| 130907A | Wind | 30.4$^{+20.3}_{-10.0}$       | 2.15$^{+0.01}_{-0.02}$ | ... | 0.21 $\pm 0.01$ | 0.15 $\pm 0.01$ | 1.01$^{+0.01}_{-0.04}$$10^{-5}$ | 0.010 $\pm 0.001$ |
| 140311A | ISM | 273$^{+37}_{-36}$           | 2.04 $\pm 0.02$ | 7.4 $\pm 0.3$$10^{-5}$ | ... | 0.03$^{+0.04}_{-0.02}$ | 0.91$^{+0.17}_{-0.09}$ | 0.014 $\pm 0.001$ |
| 140903A | ISM | 1.8$^{+2.6}_{-1.0}$        | 2.15$^{+0.13}_{-0.04}$ | 0.06$^{+0.05}_{-0.03}$ | ... | 0.039$^{+0.045}_{-0.022}$ | 0.019$^{+0.170}_{-0.018}$ | 0.061$^{+0.042}_{-0.027}$ |
| 160509A | ISM | 14 $\pm 4$             | 2.04$^{+0.02}_{-0.01}$ | 4.1$^{+14.6}_{-2.9}$$10^{-4}$ | ... | 0.63$^{+0.18}_{-0.15}$ | 0.17$^{+0.19}_{-0.11}$ | 0.057$^{+0.016}_{-0.010}$ |
| 160625B | ISM | 60$^{+85}_{-37}$         | 2.35 $\pm 0.01$ | 3.6$^{+150.9}_{-3.6}$$10^{-3}$ | ... | 0.32$^{+0.37}_{-0.18}$ | 1.6$^{+13.6}_{-1.5}$$10^{-3}$ | 0.10$^{+0.03}_{-0.02}$ |
| 171010A | ISM | 0.79 $\pm 0.24$           | 2.26 $\pm 0.01$ | 1460$^{+400}_{-150}$ | ... | 0.54 $\pm 0.02$ | 3.6$^{+0.6}_{-0.5}$$10^{-5}$ | 0.51 $\pm 0.01$ |

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**Figure 4.** Our single and broken power law fits (upper panel) and best-fitting analytical model (lower panel) compared to the light curves of GRB 090313.
break and \( p \approx 1.8 \), meaning \( t_{j,90 \ GHz} \gtrsim 15t_{j,X} \). With such a low \( p \) we also need to use Eqs. (4)–(7) in Dai & Cheng (2001) for \( p < 2 \). We find that the 90 GHz and X-ray light curves are both consistent with \( p \approx 1.5 \) and an ISM-type CBM, with 5 keV > \( \nu_c \), and \( \nu_c > 90 \ GHz > \nu_m \), but the other radio frequencies are not. No other \( p < 2 \) scenario is consistent with both X-ray and millimeter frequencies.

Moin et al. (2013) suggest that the late peak epoch seems to require a model of late energy injection that re-vitalizes the forward shock. Thus our fitting code would be naturally incapable of reproducing the observed radio light curves or spectra, especially as the Granot & Sari (2002) model assumes \( p \geq 2 \), and we do not include a fit here. With such a re-energized shock one would still expect the evolution beyond the late-time peak to resemble the post-peak evolution of the ‘standard’ case; but a \( t^{-p} \) decline is not seen at any point in the radio. It is, however, possible that there is a second peak in the 90 GHz light curve around 25 d, in which case the slope before its onset could in principle be steeper; but this is not seen convincingly in the data. The peaks in

the radio light curves seem to roughly correspond to \( \nu_m \) passage based on the SED evolution; thus the late injection model remains plausible, but only if the transition to a \( t^{-p} \) decline takes place around the last observations.

3.6. GRB 110709B

This was a ‘dark’ burst that was not detected in the optical. The X-ray behavior (Fig. 6) is consistent with \( p \approx 1.9 \) and \( \nu > \nu_c \) using standard closure relations. Thus we again use Eqs. (4)–(7) of Dai & Cheng (2001) for \( 1 < p < 2 \). Here, we find an X-ray post-break slope consistent with \( p \approx 1.8 \) and \( \nu > \nu_c \) and either an edge effect in ISM (albeit only at 1.8σ), or with a break mechanism with partial lateral expansion (van Eerten & MacFadyen 2012). The radio light curve at 5.8 GHz declines consistently with a pre-break light curve when \( p \approx 1.8 \) and 5.8 GHz > \( \nu_m \) and in an ISM-type CBM, but if so, places a limit of \( t_{j,\text{radio}} \gtrsim 60t_{j,X} \). These results do not change when using standard closure relations, except that \( p \approx 1.9 \) instead. A wind scenario is incompatible with the radio assuming the single power law.
Both our fitting code and Zauderer et al. (2013) do, however, find a model that provides a reasonable fit to the radio if the peak of the light curve is soft, and the post-break light curve steepens into a $-p$ slope around 50 d. The latter is not seen in the available data, but it is plausible. The best fit found by Zauderer et al. (2013) includes a wind-type CBM; we find a roughly equally good fit with both types of CBM, but our best wind model requires a very high $\epsilon_e$ (almost 1), so we list the ISM model in 4. Thus GRB 110709B remains consistent with the standard model.

3.7. GRB 111215A

The early radio light curve of this ‘dark’ burst flattens toward higher frequencies (Fig. 7). A single pre-break power law consistent with $p \approx 2.4$ and $v > v_e$ fits the X-ray data until $t \sim 20$ d. Around this time, the 93 GHz light curve exhibits an unambiguous break if the upper limit at 41 d is considered. An X-ray break around the same time is also plausible. However, the 6.7 and 19.1 GHz light curves are both single power laws consistent with pre-break, 6.7 GHz > $v_m$, and $p \sim 2.2$ ($p \approx 2.2$ is also compatible with the X-ray, taking into account the intrinsic $p$ scatter proposed by Gompertz et al. 2018).

Based on the spectrum, $v_m$ is clearly located above 5 GHz until $\gtrsim 26$ d, perhaps as late as 45 d; while the light curve at 4.9 GHz already starts to decline before this. A decline is, however, allowed by the standard model if the X-ray light curve breaks around the same time as the 93 GHz one. The 93 GHz break is roughly simultaneous with the peak at lower radio frequencies and the end of X-ray observations, which also points toward an achromatic transition.

Zauderer et al. (2013) use the standard model with a break around the last X-ray observations; however, this does not provide a good fit to the late-time radio points. Our fitting code is able to reproduce the observations (both the light curves and the radio SEDs) reasonably well with a wind model and a slightly later jet break – although the early 93 GHz light curve shows some deviation. Similarly, van der Horst et al. (2015) are also able to fit the afterglow, placing a lower limit of $> 31$ d for the jet break. Thus we conclude that GRB 111215A is also consistent with the standard model.

3.8. GRB 130907A

The X-ray light curve exhibits a jet break at $\sim 0.2$ d (Fig. 8). The pre-break X-ray behavior is consistent with $p \approx 2.4$ and $v > v_e$. The post-break light curve slope is $\alpha_2 = -2.2 \pm 0.03$, which may not be steep enough for full lateral expansion, and may thus require a partial lateral expansion scenario or an intrinsic $p$ scatter.

The radio decline is a single power law. Veres et al. (2015) describe the radio evolution with a $-3(p - 1)/4$ decline, which would result in $p \approx 2.0$ – including $p$ scatter with $\sigma_p = 0.25$ (Gompertz et al. 2018) the burst is thus consistent with $p = 2.2$. However, in this case the radio break must again be delayed: $t_{j,\text{radio}} \gtrsim 170t_{j,X}$ – the longest relative delay in this sample. To account for this, Veres et al. (2015) attempted to explain the
burst without a jet break and to describe the behavior at all frequencies as being pre-break – in this scenario the break in the X-ray light curve would be a combination of a $\nu_c$ passage and a transition from wind-like CBM to ISM-like. They were unable to reconcile this scenario with the steepness of the X-ray decline, though, and also suggested a combination of a narrow jet responsible for the X-ray emission and a wide jet responsible for the lower frequencies.

Our fitting code is able to fit the X-ray and early optical data reasonably well, but fails to reproduce radio light curves after $\sim 10$ d. Similarly, the general shape of the radio SEDs before 10 d is reproduced, but not the last SED at 42 d. Instead of the observed radio behavior, the model predicts a $t^{-\alpha}$ decline in the radio at this point.

3.9. GRB 140311A

The presence of an X-ray break is ambiguous ($P_F = 0.85$), but the X-ray light curve (Fig. 9) is consistent with $p \approx 2.1$ and $\nu > \nu_c$; the large error on the post-break slope also makes it consistent with either lateral expansion or an edge effect. The radio light curves are mostly single power laws consistent with pre-break, $p \sim 2.1$ and 8.6 GHz $> \nu_m$; in this case we can set a limit of $t_{j,\text{radio}} \gtrsim 3t_{j,\text{x}}$. However, the light curve at 8.6 GHz peaks again at $\sim 40$ d, which is at odds with the expected behavior and with other frequencies.

The SED indicates $\nu_m$ is located at $\gtrsim 25$ GHz when the 24.5 GHz light curve peaks. However, the 8.6 and 13.5 GHz light curves also decline from this point on, while $\nu_m$ does not pass through 13.5 GHz until $\gtrsim 20$ d. A decline below $\nu_m$ is allowed by standard theory after the jet break, but in this case the slope of the light curve is steeper at all frequencies than the expected $-1/3$. Our best-fit model is also unable to reproduce the behavior of the radio SED. The light curves at higher frequencies can be fitted reasonably well, but once again the radio light curve rules out the $t^{-\alpha}$ decline that the model predicts.

3.10. GRB 140903A

This short GRB is exceptional in its class in that it has an observed X-ray break (Troja et al. 2016). The X-ray light curve (Fig. 10) is consistent with $p \approx 2.3$ and $\nu_c > \nu > \nu_m$ in an ISM-like CBM. However, the radio light curve at 6.1 GHz is inconsistent with this and would, instead, require a wind-type CBM (unlikely for short GRBs) and a fast-cooling spectrum ($\nu_m > \nu_c$) to be compatible with the standard model. We do note that the radio data are rather scarce for this burst. Our best-fit model does a good job with the optical and X-ray light curves, but cannot reproduce the radio light curve, including the late-time decline. Troja et al. (2016) show their broken power-law fits but not the light curves from their afterglow model, which requires a slightly off-axis jet.

3.11. GRB 141121A

This ultra-long GRB shows peculiarities both in X-ray and in radio (Fig. 11). The X-ray decline is a broken
Figure 9. Our single and broken power law fits to the light curves of GRB 140311A (upper panel), and our best-fitting analytical model compared to the light curves (middle panel) and radio SEDs (lower panel).

power law, and the decline after the last break at $\sim 3$ d matches expectations for a post-jet-break decline with $p \approx 2.2$ and full lateral expansion. The slow pre-break decline – almost a plateau – may be the product of late engine activity. The radio light curve at 15 GHz fits a single power law, but is inconsistent with $p \approx 2.2$, while the lower radio frequencies show a more complex light curve, interpreted by Cucchiara et al. (2015) using a model with a long-lasting reverse shock. However, even this model does not fit the early radio points very well. The possible influence of a reverse shock is also seen in the spectrum, which shows some evidence of multiple peaks until 33.3 d. For these reasons our fitting code is again naturally unable to reproduce the observations, and is therefore not used here.

3.12. GRB 160509A

This burst was well observed in the X-ray, but an optical break was not observed due to a high extinction (although K19 did see a late-time slope consistent with post-break expectations; see Fig.12). The radio light curves at 6 and 9 GHz (other frequencies were not followed up as long) are consistent with a pre-break slope assuming $6 \text{ GHz} > \nu_m$ in an ISM-type CBM and $p \approx 2.2$, which is also consistent with the X-ray. The break time in X-rays is $\sim 3.5$ d (K19), resulting in a limit of $t_{j,\text{radio}} > 20 \times t_{j,X}$. A reverse shock was detected in this GRB (Laskar et al. 2016). The spectrum initially shows a moving peak associated with the reverse shock, but after its passing ($\gtrsim 10$ d based on the
light curves) the spectrum becomes almost flat. This could in principle be due to a combination of reverse-shock contribution and a smooth $\nu_m$ break around the observed frequencies. Our best-fit model with an ISM-like CBM does reproduce a flat radio SED at late times, and fits the light curves at all frequencies reasonably well – although we note that K19 used the numerical fitting code boxfit (van Eerten et al. 2012), and their best fit was only consistent with the late-time points ($\gtrsim 30$ d), under-predicting the fluxes until then. Thus, provided that the $t^{-p}$ decline in the radio starts around the last observations, and the flat spectral shape around 10 d can be attributed to lingering contribution from the reverse shock, GRB 160509A is consistent with the standard model.

3.13. **GRB 160525B**

An optical and X-ray jet break was seen at $\sim 20$ d (K19). The radio light curve (Fig. 13) at 6.1 GHz is again consistent with a pre-break slope assuming $\nu > \nu_m$ in an ISM-type CBM, with a slope corresponding to $p \approx 2.4$, which is close to the value of $p \approx 2.3$ determined through the early X-ray and optical light curve. Thus a limit of $t_{j,\text{radio}} > 10 \times t_{j,\text{optical}}$ can be placed if this scenario holds. This burst also showed signs of a reverse shock (Alexander et al. 2017), at low frequencies ($\lesssim 5$ GHz) until $\sim 20$ d. After this, similarly to GRBs 130907A and 160509A, the radio SED becomes flat.

Our best-fit model is able to reproduce the observed behavior at high frequencies. However, the shapes of
the radio light curve and SED deviate somewhat from the model, and radio fluxes in general are consistently over-predicted by a factor of ~ 2 after ~ 20 d. Furthermore, the model once again predicts a $t^{-p}$ decline starting around the last observations.

3.14. **GRB 161219B**

The X-ray decline can be fitted with a broken power law, with a post-break slope of $-1.64 \pm 0.11$ (Fig. 14).

From the pre-break slope of $-0.80 \pm 0.01$ one obtains $p \approx 1.7$, $\nu > \nu_c$; in which case an ISM-type CBM with either an edge-effect or lateral expansion break is a good match. A value of $p$ close to this ($p \approx 1.8$) also matches the $r$-band pre-break decline of $-0.61 \pm 0.01$ (we cannot fit the optical post-jet-break afterglow as it is dominated by SN2016jca and a late-time host galaxy contribution). However, the Dai & Cheng (2001) closure relations, which should apply at such a low $p$, are not consistent with the early X-ray and optical slopes with any $p$ between 1 and 2. Furthermore, the radio light curve (after the proposed reverse shock no longer dominates; Laskar et al. 2018b) at the frequencies with the longest follow-up settles onto a power law inconsistent with this scenario. If $\nu_c > \nu_X$, on the other hand, the early X-ray slope is consistent with $p \approx 2.1$, but the optical light curve is not. The 104 GHz curve seems to break at or after $\sim 20$ d; there are not enough post-break epochs for a proper broken power law fit, but simply using the last two points, one obtains a post-break slope of $\alpha_{104\text{GHz}} = -1.47 \pm 0.31$, consistent with the X-ray slope. A steeper slope is also possible if the break occurs
later, though. This break is not seen at lower frequencies.

Our analytical model, being based on Granot & Sari (2002), is naturally unable to handle values of $p < 2$ that seem to be required by the optical and X-ray light curves. The model invoked by Laskar et al. (2018b) requires a refreshed reverse shock; however, even this model somewhat under-predicts the flux at late times, after their proposed jet break time. The spectrum does seem to require multiple peaks at least until 24.4 d, but the late-time spectra at $\geq 79.3$ d are consistent with $\nu_m$ being located at $\sim 5$ GHz. A separate, long-lasting spectral peak around 5 GHz could be related to the difference in behavior between $\sim 100$ GHz and lower frequencies. All in all, the radio emission is inconsistent with the standard model, and only (roughly) consistent with the proposed refreshed reverse shock model if the $t^{-p}$ decline started soon after the last radio observations.

3.15. GRB 171010A

An X-ray break is seen for this burst, but the radio behavior (Fig. 15) is described as unusual by Bright et al. (2019), who attempt to explain the light curve evolution using a steep CBM density profile, but nonetheless find that the evolution of break frequencies in the spectrum seems too slow for the standard model to explain. The X-ray light curve is consistent with $p \approx 2.4$ and the radio decline slope of $\alpha = -1.12 \pm 0.05$ is consistent with $p \approx 2.5$ and pre-break. However, the peak of the radio light curve is roughly simultaneous with the X-ray break, and thus GRB 171010A seems to exhibit another delayed radio break; we can place a limit of $t_{j, \text{radio}} \times 13 \times t_{j, \text{X}}$ in this case. This requires that $\nu_m < 15.5$ GHz at $t_{j, \text{X}}$, which is consistent with the first SED; but as Bright et al. (2019) point out, if the peak frequency is $\nu_m$, its evolution is slower than expected.

Our own best-fit model agrees with the findings of Bright et al. (2019): individual SEDs can be reproduced by it, but not all epochs simultaneously. The situation at 15.5 GHz is similar to the one with GRB 990510 radio data – instead of a single power law, our best fit is a smooth transition from $\nu_c < 15.5$ GHz $< \nu_m$ to $\nu_m < \nu_c < 15.5$ GHz. This prediction, however, deviates from the observed light curve even if the transition happens simultaneously with the late-time observations. At other frequencies the light curve is sparse and such comparisons are more difficult.

4. DISCUSSION

As the jet break is a geometric effect, in standard GRB jet theory we should see its effects at all frequencies – even when the radio emission source is a population of thermal electrons behind the shock wave as suggested by Warren et al. (2018). One expects a post-break decline of $\alpha_\text{radio} = -1/3$ or $\alpha_\text{radio} = -p$, depending on whether the frequency is below or above $\nu_m$, respectively (Rhoads 1999), assuming lateral expansion at the speed of sound. Without lateral expansion, one expects a steepening of the light curve decline by $t^{-0.75}$ in a constant-density CBM or by $t^{-0.5}$ in a wind CBM.
(Mészáros & Rees 1999; Panaitescu & Mészáros 1999). It is also possible to have a scenario where some lateral expansion is present, but the break is dominated by the edge effect (this was demonstrated in simulations by van Eerten & MacFadyen 2012). However, in the events of our sample, the situation may be more complicated.

In Table 5, we summarize how the radio light curve and SED evolve and whether they are consistent with the higher frequencies; we also describe how our best-fit analytical model fits the observations. In most of the GRBs in our sample, a single power law decline of the radio light curve, inconsistent with an expected post-break decline based on the properties of the X-ray or optical afterglow, persists until very late times. Even in the cases in our sample where the radio light curve does behave consistently with some scenario covered by the standard model or with our best-fit model, specifically GRBs 990510, 051022, 090313, 110709B, 111215A, 130907A, 160509A, 160625B, and 171010A, we never see a break onto a $F_\nu \propto t^{-p}$ decline even when it is seen in the X-ray light curve (except in the $\sim 100$ GHz band; see below) – and even when one considers the intrinsic scatter in $\nu$ suggested by Gompertz et al. (2018). Neither is a steepening associated with the relativistic edge effect observed in the radio even when it can apply to the X-ray (or optical) light curve. A simple transition to the non-relativistic phase can be excluded in many cases as well, as the expected slope is not seen in X-rays when the single power law behavior in the radio sets in, or the radio slope is not compatible with the expected Sedov-von Neumann-Taylor formulae. Furthermore, out of these eight cases, GRBs 990313, 130907A, and 171010A can be excluded based on our model fits, which fail to reproduce the radio behavior even when they work well at higher frequencies; even though the power-law slope is compatible with some scenario of the standard model, its timing is not consistent with the time evolution of the model spectrum.

GRBs 990510 and 051022 seem consistent with a model where the observed radio frequencies are located below $\nu_m$ until very late times, resulting in a smooth transition from $F_\nu \propto t^{-1/3}$ to $F_\nu \propto t^{-p}$ (Rhoads 1999) – especially if one considers the possibility of a jet break time that varies across spectral breaks (as seen in simulations by van Eerten et al. 2011). In the remaining cases mentioned above (GRBs 110709B, 111215A, 160509A, and 160625B), the single power-law fits, at least at some radio frequencies, are consistent with the pre-break slope expected if $\nu > \nu_m$ (Dai & Cheng 2001; Granot & Sari 2002), and with the peak being associated with the $\nu_m$ passage through the band, but this continues to epochs $> 10$ (or even $> 100$) times later than the jet break detected in the X-ray. Indeed, the radio emission mostly peaks after the X-ray break. As van Eerten et al. (2011) show, the break times may differ on different sides of the self-absorption frequency $\nu_a$ and/or the characteristic synchrotron frequency $\nu_m$, but in these simulations this effect was by a factor of a few, and in any case cannot explain the difference when the radio behavior is consistent with $\nu > \nu_m$. Nevertheless, our best-fit model in each case except GRB 160625B does reproduce the radio light curve with a smooth transition from $t^{-1/3}$ (post-break when $\nu < \nu_m$) to $t^{-p}$ ($\nu > \nu_m$); thus, five events (990510, 051022, 110709B, 111215A, and 160509A) in our sample remain consistent with the standard model.

However, it is noteworthy that even in these cases the predicted $t^{-p}$ phase is not reached by the observed light curve. This presents a possible problem for the scenario of a delayed jet break as well: these events remain consistent with a complete lack of a radio break. Another source of suspicion is the fact that three of the five cases consistent with the standard model lack any optical information, while GRB 160509A only has a scant few optical data points, resulting in a lack of constraints.

One explanation for the long-lasting single power-law decline of the radio emission might be the presence of a wider jet surrounding a more energetic narrow core, which dominates in the radio (similar to that suggested by Berger et al. 2003). The greater width of this jet would produce a later break time than that expected from the narrower main jet; however, this requires that the radio emission associated with the narrow jet must then be significantly weaker than what is observed. To investigate the plausibility of producing faint radio emission from the narrow jet within the confines of the standard model, we have re-run our model-fitting code for all bursts in our sample with good X-ray and optical coverage (GRBs 990510, 070125, 090313, 140311A, and 160625B), with all radio fluxes divided by ten. If the standard model can, in such a situation, approximately reproduce the lower radio fluxes with plausible parameters (even if the shape of the light curve is different), then the two-component scenario may be able to explain the light curve. If this is not the case, then the two-component model would likely over-predict the radio fluxes. For GRBs 070125 and 160625B, the resulting best fit does reproduce the approximate flux level, but requires $e_\nu \approx 1$ or $p > 3$; for GRB 990510 the radio flux is consistently overpredicted by a factor of a few; and for GRB 140311A the shapes of the higher-frequency light curves in turn become incompatible with the observations. However, the radio fluxes of GRB 090313 can be approximately reproduced with plausible parameters. This suggests that the two-component scenario...
may only be relevant for certain GRBs, or requires a mechanism to suppress the radio emission from the narrow jet. Furthermore, in the two-component model one would naively expect to observe some GRBs whose radio emission clearly exceeds what the standard model predicts based on higher-frequency emission. The lack of any such objects suggests either some sort of fine-tuning of the model, or that the model is inapplicable.

Even some of the ‘well-behaving’ radio light curves (the aforementioned GRBs 990510, 051022, and 110709B) may in fact be coincidental – in these cases we only have one radio frequency with enough points for fitting, and some other objects in the sample exhibit a frequency-dependent single power law decline\(^6\) that our model fits cannot reproduce. Thus, with light curves at more radio frequencies even these objects might be ruled out – we cannot say for sure. The same may apply to those bursts where the slope is consistent with pre-break expectations. A further complication is that, in some cases, the radio spectrum indicates that the decline

\(^6\) This dependency is not monotonic, however: there is no significant correlation between the slope and rest-frame frequency (with a Spearman correlation coefficient \(r_s = 0.16 \pm 0.13\)), and the scatter is large at all frequencies.

| GRB     | Radio behavior                                                                 | Model fit                                                                 |
|---------|--------------------------------------------------------------------------------|--------------------------------------------------------------------------|
| 990510  | SPL inconsistent with theory                                                   | Fair, but requires \(F_{\text{radio}} \propto t^{-p}\) after observations |
| 051022  | SPL consistent with \(\nu_m > 4.9\) GHz                                      | Good, but requires \(F_{\text{radio}} \propto t^{-p}\) after observations |
| 070125  | SPL inconsistent with theory; late peak at low frequencies                    | Cannot reproduce late radio data                                          |
| 090313  | SPL consistent with \(\nu_m > 16\) GHz, but too long                        | Cannot reproduce late radio data                                          |
| 100418A | SPL inconsistent with theory; late peak at low frequencies                    | ...                                                                      |
| 110709B | SPL consistent with pre-break until \(> 60\) \(t_{j,X}\)                    | Good, but requires \(F_{\text{radio}} \propto t^{-p}\) after observations |
| 111215A | Break at 93 GHz; \(\nu\)-dependent SPL otherwise                            | Good, but requires \(F_{\text{radio}} \propto t^{-p}\) after observations |
| 130907A | SPL consistent with pre-break until \(> 170\) \(t_{j,X}\); flat SED          | Cannot reproduce radio and optical simultaneously; requires \(F_{\text{radio}} \propto t^{-p}\) after observations |
| 140311A | Mostly SPL consistent with pre-break; SED inconsistent with light curve       | Cannot reproduce late radio data                                          |
| 140903B | SPL only consistent with \(\nu_m > \nu_e\) until late times                 | ...                                                                      |
| 141121A | Complex; multiple peaks                                                      | Good, but requires \(F_{\text{radio}} \propto t^{-p}\) after observations |
| 160509A | SPL consistent with pre-break until \(> 20\) \(t_{j,X}\); flat SED           | Deviation in light curve and SED at late times; requires \(F_{\text{radio}} \propto t^{-p}\) after observations |
| 160625B | SPL consistent with pre-break until \(> 10\) \(t_{j,X}\); flat SED           | ...                                                                      |
| 161219B | Break at \(\sim 100\) GHz; otherwise SPL inconsistent with theory           | Cannot reproduce SED evolution; requires \(F_{\text{radio}} \propto t^{-p}\) after observations |
| 171010A | SPL consistent with pre-break until \(> 13\) \(t_{j,X}\); \(\nu_m\) evolution too slow | ...                                                                      |

Table 5. Summary of our main findings. For each GRB, we briefly describe the radio behavior based on our single and broken power law fits, and whether it is consistent with the X-ray behavior in some scenario of the standard jet model. We also describe whether the best-fit analytical model is a reasonable match with the observations.

We point out that in at least one object, an achromatic late-time slope consistent with a post-jet-break radio decline with \(\alpha \sim -p\) has indeed been seen. This is the exceptional SGRB 170817A, which was also detected in gravitational waves as GW 170817 (e.g. Abbott et al. 2017). The early afterglow light curve was faint due to the GRB being seen off-axis, and thus the observed light was dominated by the accompanying kilonova AT2017gfo/SSS17a (e.g. Kilpatrick et al. 2017; Tanvir et al. 2017) in the first couple of weeks. However, at very late times (hundreds of days) an achromatic break was observed (Lamb et al. 2019; Hajela et al. 2019). This particular afterglow requires a structured jet model with \(p = 2.15^{+0.01}_{-0.02}\) and a relatively large observing angle of \(30.4^{+3.4}_{-3.0}\) deg, but it does conform to this model achromatically, unlike the events in this sample\(^7\). This object thus seems to be an exception rather than the rule.

Finally, GRBs 111215A and 161219B seem to exhibit a break in their millimeter light curves. In both cases

\(^7\) A two-component jet model also fits reasonably well (Lamb et al. 2019), but the authors tentatively favor a Gaussian structured jet model.
the available light curve is lacking in post-break points and the post-break slope is not well constrained, but a broken power law provides a better fit to the light curve than a single power law. The observed X-ray light curve of GRB 111215A does not show a break, but it is possible that it did occur soon after the Swift observations, or even around the time of the last observations. The 93 GHz break in GRB 111215A is consistent with this, and the millimeter break in GRB 161219B seems to occur shortly after the break time from our X-ray fit (Laskar et al. 2018b, fitting all observed light curves simultaneously, assume an achromatic break time). The millimeter light curve of GRB 161219B steepens to $\alpha_{2,104 \text{ GHz}} = -1.47 \pm 0.31$, which comes from the last two points. The lack of post-break points and a poorly constrained $t_{f,104 \text{ GHz}}$ mean it may have been steeper, but even the two-point measurement is consistent with the X-ray slope. For GRB 111215A the post-break slope is consistent with $\sim -2$. In neither of these cases is a similar break seen in the light curves at lower radio frequencies, although at least for GRB 111215A it may have occurred later. This may be a hint that in at least some cases, there is a boundary frequency where the behavior of the radio light curve changes significantly, and that future ALMA observations of bright and/or nearby GRBs can be used to probe the light curves close to this boundary. The same behavior is not seen in GRB 100418A, where the rest-frame millimeter light curve does not break, but if a boundary frequency exists, it is reasonable to expect it to vary.

5. CONCLUSIONS

We have examined a sample of 15 GRB afterglows with evidence of a jet break, but with otherwise varying properties ($E_{\text{iso}}, z, T_{90}$, and the presence or absence of an optical afterglow). We have used both single/broken power law fits to the light curves at individual frequencies and a fitting code based on the standard model as described by Granot & Sari (2002), Rhoads (1999) and van der Horst (2007).

In most cases, while conventional fireball/jet theory can provide a good fit to the X-ray (and optical) light curve, it does a much worse job with their radio light curves and/or SEDs. The decline of the radio light curve is mostly consistent with a single power law with no observed steepening corresponding to the jet break, which is observed in the X-rays in all but one case. Sometimes the single power law is consistent with theoretical predictions for a pre-break decline, however, even up to several tens of times later than the jet break time.

According to our model fitting code, the standard model is able to account for the observed behavior of five GRBs in the sample: GRBs 990510, 051022, 110709B, 111215A, and 160509A. However, even in these cases the radio observations show no clear sign of a jet break. Somewhat suspiciously, even when the standard model fits the data, it does so only if a steepening to $t^{-p}$ occurs soon after the last radio observations; furthermore, four of these bursts have little to no optical data to constrain the fits. However, at $\sim 100$ GHz, the signature of a jet break can be clearly seen in GRBs 111215A and 161219B (but not in GRBs 070125 or 100418A), providing a hint that the standard model may be inadequate only below some frequency limit around the millimeter band.

All in all, in individual cases, the model fit in the radio may seem adequate, especially considering the typical errors and often small numbers of data points per band in the literature, but looking at a larger sample of events, a pattern emerges. A single power law fit to the radio tends to describe the light curve at least as well as model fits based on the standard jet model, and, in general, no jet break is indicated by the radio data.

Although our sample is not complete, it was constructed essentially at random with a large variety of GRB properties, the only criterion being evidence of a jet-break in at least one band. Thus the large fraction of objects that do not behave in accordance with the standard jet model, along with no unambiguous cases of objects that do, is telling. Along with the radio-quiet GRB population (Hancock et al. 2013; Lloyd-Ronning & Fryer 2017; Lloyd-Ronning et al. 2019), this highlights our lack of understanding of this part of the GRB afterglow spectrum and, quite possibly, the relevant physics. This problem may be better investigated in the future through radio and millimeter follow-up programs which continue until long after an observed X-ray or optical break.

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