Threshold expansion of massive coloured particle cross sections
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Pair production of massive coloured particles in hadron collisions is accompanied by potentially large radiative corrections related to the suppression of soft gluon emission and enhanced Coulomb exchange near the production threshold. We recently developed a framework to sum both series of corrections for the partonic cross section using soft-collinear and non-relativistic effective theory. If it can be argued that the resummed cross section approximates the complete result over a significant kinematic range, an improvement of the hadronic cross section results, even when the production is not kinematically constrained to the threshold. This is discussed here for the case of top quark production.

1. Introduction
The rediscovery of the top quark at the Large Hadron Collider (LHC) in the near future will mark the beginning of an era of precision studies of the properties of the heaviest of all quarks. There is therefore currently much interest in predicting the production cross section and invariant mass distribution precisely \cite{1,2,3,4,5,6,7}, beyond the fixed-order NLO result \cite{8}, by extending and updating earlier calculations including soft gluon resummation \cite{9,10,11,12}, using recent results on NNLL resummation for massive particles \cite{13,14,15,16,17,18}.

In pair production of coloured particles an additional power-like threshold enhancement, formally stronger than the logarithmic enhancement related to soft gluons, arises due to the colour-Coulomb force. The question whether the standard resummation formalism must be modified in the presence of the strong Coulomb interaction, and whether both types of enhancements can be simultaneously resummed to all orders was not addressed in the above papers, which usually included the Coulomb correction at the one-loop level or assumed factorization of soft gluon and Coulomb effects. In \cite{16,19} we extended the momentum-space formalism for resummation \cite{20} based on soft-collinear effective theory (SCET) to the case of pair production, showing factorization of the partonic cross section of the form

\begin{equation}
\hat{\sigma}(\beta, \mu) = \sum_a \sum_{i,i'} H_{ai}^a(m_t, \mu) \times \int d\omega \sum_{R_\alpha} J_{R_\alpha}^a(E - \omega) W_{ii',R_\alpha}^a(\omega, \mu),
\end{equation}

with the top-quark velocity $\beta = (1 - 4m_t^2/\hat{s})^{1/2}$, $E = \sqrt{\hat{s}} - 2m_t \approx m_t \beta^2$ and $\sqrt{\hat{s}}$ being the partonic cm energy. Eq. (1) contains a multiplicative short-distance coefficient $H_{ai}^a$, in each colour (in higher orders also spin) configuration labelled by the irreducible representation $R_\alpha$, and a convolution of soft functions $W_{ii',R_\alpha}^a$ with functions $J_{R_\alpha}^a$, which contain Coulomb exchange to all orders. The factorization in this form implies that the soft and Coulomb corrections can both be summed.

The oral presentation covered a detailed discussion of the above factorization formula, our
results for squark-antisquark production at the next-to-leading logarithmic order, and for top-quark pair production. Since the former have already been documented in other proceedings articles [21,22], we focus here on our preliminary results for top quarks.

2. Top quark production

First we consider the inclusive partonic cross section for $t\bar{t}$-production, denoted by $\hat{\sigma}_t(\beta)$. The series of enhanced radiative corrections can be represented parametrically as

$$\hat{\sigma}_t(\beta) = \hat{\sigma}^{(0)}_t \sum_{k=0}^\infty \left( \frac{\alpha_s}{\beta} \right)^k \exp \left[ \ln \beta g_0(\alpha_s \ln \beta) \right]_{\text{(LL)}}$$

$$+ g_1(\alpha_s \ln \beta) + \alpha_s g_2(\alpha_s \ln \beta) + \ldots \right]_{\text{(NLL)}}$$

$$\times \left\{ 1(\text{LL, NLL}); \alpha_s, \beta(\text{NNLL}); \ldots \right\}, \quad (2)$$

where $\hat{\sigma}^{(0)}_t$ is the Born cross section. In terms of the fixed-order expansion the different orders of resummation refer to

LL : $\alpha_s \left\{ \frac{1}{\beta}, \ln^2 \beta \right\}; \alpha_s^2 \left\{ \frac{1}{\beta^2}, \ln^2 \beta, \ln^4 \beta \right\}; \ldots$

NLL : $\alpha_s \ln \beta; \alpha_s^2 \left\{ \frac{\ln^2 \beta}{\beta}, \ln^3 \beta \right\}; \ldots \quad (3)$

e tc. In obtaining the hadronic total cross section the partonic cross section is integrated over all $\beta$ up to the kinematic constraint $\beta_{\text{max}} = (1 - 4m_t^2/s)^{1/2}$, weighted by the parton luminosity. The threshold expansion is strictly valid for the hadronic cross section only for high masses $2m_t \rightarrow s$ such that $\beta_{\text{max}} \rightarrow 0$, but certainly not for tops at the Tevatron and the LHC with $\sqrt{s} = 7$ TeV and higher energy. Nevertheless, one sometimes finds that the threshold expansion provides a reasonable approximation even outside its domain of validity, so that it can be useful to include the threshold limit of higher-order terms in the perturbative expansion.

It is therefore interesting to investigate this issue at the next-to-leading order (NLO), where the exact result is known. Here, the $t\bar{t}$ invariant mass distribution peaks at about 380 GeV, corresponding to $\beta \approx 0.4$, but the average $\beta$ is even larger, see Table 1 for the gluon-gluon production channel. To check whether the threshold expansion can be a reasonable approximation we show in Figure 1 the $\beta$-integrand of the NLO correction to the hadronic cross section in the $gg$-channel

$$\frac{d\sigma_{t\bar{t}}}{d\beta} = \frac{8\beta m_t^2}{s(1 - \beta^2)^2} \mathcal{L}_{gg}(\beta) \hat{\sigma}^{(gg)}_t(\beta), \quad (4)$$

where $\mathcal{L}_{gg}$ is the gluon parton luminosity. The Figure displays the full NLO result $\approx$ and compares to it the approximation $\text{NLO}_{\text{sing}}$, which includes in addition the constant term $\beta^0$ in the $\beta$ expansion, see for instance Eq. (A.2) of Ref. [24]. (We use $m_t = 173.1$ GeV and set the renormalization and factorization scale equal to $\mu_r = \mu_f = m_t$.) We see that $\text{NLO}_{\text{approx}}$ provides a good approximation up to $\beta \approx 0.6$. In Table 1 we show the corresponding results for the integrated NLO cross sections with the approximations to the NLO correction as discussed above, now including all partonic production channels.

![Figure 1. $\beta$-integrand (in pb) for different approximations to the NLO correction to the hadronic cross section in the $gg$-channel for $t\bar{t}$ production in $pp$ collisions at $\sqrt{s} = 14$ TeV.](image)

We therefore make the assumption that the threshold expansion provides a good approximation for the integral over all $\beta$, i.e. the total
hadronic cross section. As shown, this works reasonably well for the gg-channel at NLO (but less well for the qT-channel). We expect that this approximation becomes better at NNLO, because the average is dominated by smaller β as the order increases due to the existence of more singular terms.

Let us now turn to our results for NLL resummation of the total hadronic cross section for t¯t-production. As mentioned above, we apply the SCET formalism [20], rather than working in Mellin-space [12]. Motivated by our previous findings, we integrate the resummed cross section over all values of β and do not switch off the threshold resummation outside its formal domain of validity. We include soft gluon and Coulomb gluon resummation and match the NLL resummed cross section to the full NLO result. Thus our first approximation, NLL+NLO, is given by

$$\sigma_{\text{NLL+NLO}}^{\text{approx}} = \sigma_{\text{NLO}}^{\text{NLL}} - \sigma_{\text{NLO}}^{\text{NLL}} \bigg|_{\text{NLO}} + \sigma_{\text{NLO}}^{\text{NLO}},$$  

(5)

which is NLO exact but further includes all NLL terms beyond NLO. Here and in the following, $$\sigma_{\text{NLL}}^{\text{NLO}}$$ is given by $$\sigma_{\text{NLL}}^{\text{NLO}}$$ expanded in $$\alpha_s$$ up to N(N)LO accuracy. Next we define NNLOapprox to be the sum of the the exact NLO result plus all singular terms in β at O($$\alpha_s^2$$), which were determined in Ref. [24]. Finally, our third and best approximation is given by

$$\sigma_{\text{NLL+NNLOapprox}}^{\text{approx}} = \sigma_{\text{NLO}}^{\text{NLL}} - \sigma_{\text{NLO}}^{\text{NLL}} \bigg|_{\text{NNLO}} + \sigma_{\text{NNLOapprox}}^{\text{NNLO}}.$$

(6)

Within the SCET formalism, there are several scales involved in the resummed cross section. The hard scale $$\mu_h$$, which is the scale of the hard matching coefficients, is taken to be equal to 2$$m_t$$. The soft scale $$\mu_s$$ is determined by minimizing the one-loop soft corrections. The Coulomb scale is chosen as $$\mu_C = \text{Max}(\alpha_s(\mu_C)C_F m_t, 2m_t \beta)$$. In order to estimate the scale uncertainty, we vary the scales $$\mu_f, \mu_s$$ and $$\mu_h$$ by a factor of 2. Our results for the t¯t cross section are summarized in Table 1 and Figure 2 using the MSTW2008nnlo parton distribution functions (PDFs) [25], and the ABKM09nnlo PDFs [26]. The two uncertainties shown explicitly in the Table stem from the variation of the scales (first error) and from the PDF uncertainty (second error). The two PDF sets lead to cross sections consistent within their stated uncertainties for the Tevatron, but for LHC energies the results for the MSTW08 set are larger by 2$$\sigma - 3\sigma$$, depending on the approximation one considers. This is due to a larger value of $$\alpha_s(M_Z)$$ and a larger value of the gluon PDF in the partonic threshold region $$s \approx 4m_t^2$$ for the MSTW08 PDF set. We can directly compare our results for the NLL + NNLOapprox approximation with Ref. [7]. The corresponding result is called $$\sigma_{\text{NNLO,β-exp.+potential}}$$ there and is in full agreement for $$\mu_f = m_t$$. We note that adding NLL resummation to NNLOapprox has only a permille

Table 1

| NLO results for different approximations | Tev. | LHC7 | LHC14 |
|-----------------------------------------|------|------|------|
| (β)gg, NLO                             | 0.41 | 0.49 | 0.53 |
| LO                                     | 5.25 | 101.9| 562.9|
| NLO                                    | 6.50 | 149.9| 842.2|
| NLOsing                                | 6.76 | 138.8| 751.2|
| NLOapprox                              | 7.45 | 159.0| 867.6|

Top pair production cross section in pb at the Tevatron (Tev.) and LHC with $$\sqrt{s} = 7$$ TeV and 14 TeV; MSTW2008nnlo PDFs [25].
effect. The scale dependence is reduced drastically once the singular terms at $O(\alpha_s^2)$ are included, from about 10% for the NLO result to (1 – 2)% for NNLOapprox+NLL. In Figure 2 we plot the dependence of the resummed cross section on the top mass and compare to the NLO result for $\sqrt{s} = 7$ TeV. The resummed cross section is enhanced by about 10%, and the combined PDF and scale uncertainty is reduced by roughly 50%.

3. Summary

We presented a progress report of our work on the combined soft and Coulomb resummation for top-quark pair production in hadron collisions. Including the singular terms near threshold at $O(\alpha_s^2)$ leads to an enhancement of the cross section and a significant reduction of the scale dependence. Summing NLL logarithms and the leading Coulomb corrections beyond this order is a minor effect. The complete NNLL resummation can be readily performed in the SCET plus NRQCD formalism and is in progress.

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REFERENCES

1. S. Moch and P. Uwer, Phys. Rev. D 78 (2008) 034003, arXiv:0804.1476 [hep-ph].
2. M. Cacciari, S. Frixione, M. L. Mangano, P. Nason and G. Ridolfi, JHEP 0809 (2008) 127, arXiv:0804.2800 [hep-ph].
3. N. Kidonakis and R. Vogt, Phys. Rev. D 78 (2008) 074003, arXiv:0805.3844 [hep-ph].
4. U. Langenfeld, S. Moch and P. Uwer, Phys. Rev. D 80 (2009) 054009, arXiv:0906.5273 [hep-ph].
5. K. Hagiwara, Y. Sumino and H. Yokoya, Phys. Lett. B 666 (2008) 71, arXiv:0804.1013 [hep-ph].
6. Y. Kiyo, J. H. Kühn, S. Moch, M. Steinhauser and P. Uwer, Eur. Phys. J. C 60 (2009) 375, arXiv:0812.0919 [hep-ph].
7. V. Ahrens, A. Ferroglia, M. Neubert, B. D. Pecjak and L. L. Yang, arXiv:1003.5827v1 [hep-ph].
8. P. Nason, S. Dawson and R. K. Ellis, Nucl. Phys. B 303 (1988) 607.
9. S. Catani, M. L. Mangano, P. Nason and L. Trentadue, Phys. Lett. B 378 (1996) 329 [hep-ph/9602208].
10. E. L. Berger and H. Contopanagos, Phys. Rev. D 54 (1996) 3085 [hep-ph/9603326].
11. N. Kidonakis, J. Smith and R. Vogt, Phys. Rev. D 56 (1997) 1553 [hep-ph/9608343].
12. R. Bonciani, S. Catani, M. L. Mangano and P. Nason, Nucl. Phys. B 529 (1998) 424 [Erratum-ibid. B 803 (2008) 234] [hep-ph/9801375].
13. N. Kidonakis, Phys. Rev. Lett. 102 (2009) 232003, arXiv:0903.2561 [hep-ph].
14. A. Mitov, G. F. Sterman and I. Sung, Phys. Rev. D 79, 094015 (2009), arXiv:0903.3241 [hep-ph].
15. T. Becher and M. Neubert, Phys. Rev. D 79, 125004 (2009) [Erratum-ibid. D 80, 109901 (2009)], arXiv:0904.1021 [hep-ph].
16. M. Beneke, P. Falgari and C. Schwinn, Nucl. Phys. B 828 (2010) 69, arXiv:0907.1443 [hep-ph].
17. M. Czakon, A. Mitov and G. F. Sterman, Phys. Rev. D 80, 074017 (2009), arXiv:0907.1790 [hep-ph].
18. A. Ferroglia, M. Neubert, B. D. Pecjak and L. L. Yang, Phys. Rev. Lett. 103, 201601 (2009), arXiv:0907.4791 [hep-ph].
19. M. Beneke, P. Falgari and C. Schwinn, PoS RADCOR2009 (2010) 011, arXiv:1001.4621 [hep-ph] and arXiv:1007.5414 [hep-ph].
20. T. Becher, M. Neubert and G. Xu, JHEP 0807 (2008) 030, arXiv:0710.0880 [hep-ph].
21. M. Beneke, P. Falgari and C. Schwinn, PoS PS-HEP2009 (2009) 319, arXiv:0909.3488 [hep-ph].
22. M. Beneke, P. Falgari and C. Schwinn, PoS RADCOR2009 (2010) 012, arXiv:1001.4627 [hep-ph].
23. M. Czakon and A. Mitov, Nucl. Phys. B 824 (2010) 111, arXiv:0811.4119 [hep-ph].
24. M. Beneke, M. Czakon, P. Falgari, A. Mitov and C. Schwinn, Phys. Lett. B 690, 483 (2010), arXiv:0911.5166 [hep-ph].
25. A. D. Martin, W. J. Stirling, R. S. Thorne and G. Watt, Eur. Phys. J. C 63, 189 (2009), arXiv:0901.0002 [hep-ph].
26. S. Alekhin, J. Blümlein, S. Klein and S. Moch, Phys. Rev. D 81 (2010) 014032, arXiv:0908.2766 [hep-ph].