

\textbf{CP- and T-Violation in the Decay $K_L \rightarrow \pi^+\pi^-e^+e^-$ and Related Processes*}

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\textbf{Abstract}

I review the theoretical basis of the prediction that the decay $K_L \rightarrow \pi^+\pi^-e^+e^-$ should show a large CP- and T-violation, a prediction now confirmed by the KTeV experiment. The genesis of the effect lies in a large violation of CP- and T-invariance in the decay $K_L \rightarrow \pi^+\pi^-\gamma$, which is encrypted in the polarization state of the photon. The decay $K_L \rightarrow \pi^+\pi^-e^+e^-$ serves as an analyser of the photon polarization. The asymmetry in the distribution of the angle $\phi$ between the $\pi^+\pi^-$ and $e^+e^-$ planes is a direct measure of the CP-odd, T-odd component of the photon’s Stokes vector. A complete study of the angular distribution can reveal further CP-violating features, which probe the non-radiative (charge-radius and short-distance) components of the $K_L \rightarrow \pi^+\pi^-e^+e^-$ amplitude.

Eight years ago, there appeared a report [1] by the E-731 experiment concerning the branching ratio and photon energy spectrum of the decays $K_{L,S} \rightarrow \pi^+\pi^-\gamma$. It was found that while the $K_S$ decay could be well-reproduced by inner bremsstrahlung from an underlying process $K_S \rightarrow \pi^+\pi^-$, the $K_L$ decay contained a mixture of a bremsstrahlung component (IB) and a direct emission component (DE), the relative strength being $DE/(DE + IB) = 0.68$ for photons above 20 MeV. The simplest matrix element consistent with these features is

\begin{equation}
\mathcal{M}(K_S \rightarrow \pi^+\pi^-\gamma) = e_f S \left( \frac{\epsilon \cdot p_+}{k \cdot p_+} - \frac{\epsilon \cdot p_-}{k \cdot p_-} \right),
\end{equation}

\begin{equation}
\mathcal{M}(K_L \rightarrow \pi^+\pi^-\gamma) = e_f L \left( \frac{\epsilon \cdot p_+}{k \cdot p_+} - \frac{\epsilon \cdot p_-}{k \cdot p_-} \right) + e_f \frac{f_{DE}}{M_K} \epsilon_{\mu\nu\rho\sigma} \epsilon^\mu k^\nu p_+^\rho p_-^\sigma.
\end{equation}

\*Talk given at the KAON 99 Conference, Chicago, June 21-26, 1999
where

\[ f_L \equiv \langle f_S | g_{Br} | g_{Br} \rangle = \eta_+ e^{i\delta_0(s=MK^2)}, \]
\[ f_{DE} \equiv \langle f_S | g_{M1} | g_{M1} \rangle = i(0.76)e^{i\delta_1(s)}. \] (2)

Here the direct emission has been represented by a CP-conserving magnetic dipole coupling \( g_{M1} \), whose magnitude \( |g_{M1}| = 0.76 \) is fixed by the empirical ratio \( DE/IB \). The phase factors appearing in \( g_{Br} \) and \( g_{M1} \) are dictated by the Low theorem for bremsstrahlung, and the Watson theorem for final state interactions. The factor \( i \) in \( g_{M1} \) is a consequence of CP T invariance. The matrix element for \( K_L \to \pi^+\pi^-\gamma \) contains simultaneously electric multipoles associated with bremsstrahlung (\( E_1, E_3, E_5 \ ... \)), which have \( CP = +1 \), and a magnetic \( M1 \) multipole with \( CP = -1 \). It follows that interference of the electric and magnetic emissions should give rise to CP-violation.

To understand the nature of this interference, we write the \( K_L \to \pi^+\pi^+\gamma \) amplitude more generally as

\[ \mathcal{M}(K_L \to \pi^+\pi^-\gamma) = \frac{1}{M_K^2} \{ E(\omega, \cos \theta) [\epsilon \cdot p_+ k \cdot p_+ - \epsilon \cdot p_- k \cdot p_-] + M(\omega, \cos \theta) \epsilon_{\mu \nu \rho \sigma} \epsilon^{\mu k^{\nu} p^{\rho} p_{-}^{\sigma}} \} \] (3)

where \( \omega \) is the photon energy in the \( K_L \) rest frame, and \( \theta \) is the angle between \( \pi^+ \) and \( \gamma \) in the \( \pi^+\pi^- \) rest frame. In the model represented by Eqs. (1) and (2), the electric and magnetic amplitudes are

\[ E = \left( \frac{2M_K}{\omega} \right)^2 \frac{g_{Br}}{1 - \beta^2 \cos^2 \theta} \]
\[ M = g_{M1} \] (4)

where \( \beta = (1 - 4m_\pi^2/s)^{1/2} \), \( \sqrt{s} \) being the \( \pi^+\pi^- \) invariant mass. The Dalitz plot density, summed over photon polarizations is

\[ \frac{d\Gamma}{d\omega d\cos \theta} = \frac{1}{512\pi^3} \left( \frac{\omega}{M_K} \right)^3 \beta^3 \left( 1 - \frac{2\omega}{M_K} \right) \sin^2 \theta \left[ |E|^2 + |M|^2 \right] \] (5)

Clearly, there is no interference between the electric and magnetic multipoles if the photon polarization is unobserved. Therefore, any CP-violation involving the interference of \( g_{Br} \) and \( g_{M1} \) is hidden in the polarization state of the photon.

The photon polarization can be defined in terms of the density matrix

\[ \rho = \left( \begin{array}{cc} |E|^2 & EM^* \\ EM^* M^* & |M|^2 \end{array} \right) = \frac{1}{2} \left( |E|^2 + |M|^2 \right) \left[ \mathbb{1} + \vec{S} \cdot \vec{\tau} \right] \] (6)
where $\vec{\tau} = (\tau_1, \tau_2, \tau_3)$ denotes the Pauli matrices, and $\vec{S}$ is the Stokes vector of the photon with components

$$
S_1 = 2 Re (E^* M) / (|E|^2 + |M|^2) \\
S_2 = 2 Im (E^* M) / (|E|^2 + |M|^2) \\
S_3 = (|E|^2 - |M|^2) / (|E|^2 + |M|^2)
$$

(7)

The component $S_3$ measures the relative strength of the electric and magnetic radiation at a given point in the Dalitz plot. The effects of $CP$-violation reside in the components $S_1$ and $S_2$, which are proportional to $Re (g_{B_s^0} g_{M_1})$ and $Im (g_{B_s^0} g_{M_1})$, respectively. Of these $S_1$ is $CP$-odd, $T$-odd, while $S_2$ is $CP$-odd, $T$-even. Physically, $S_2$ is the net circular polarization of the photon: such a polarization is known to be possible in decays like $K_L \rightarrow \pi^+ \pi^- \gamma$ or $K_{L,S} \rightarrow \gamma \gamma$ whenever there is $CP$-violation accompanied by unitarity phases \[\pi\]. To understand the significance of $S_1$, we examine the dependence of the $K_L \rightarrow \pi^+ \pi^- \gamma$ decay on the angle $\phi$ between the polarization vector $\vec{\epsilon}$ and the unit vector $\vec{n}_\pi$ normal to the $\pi^+ \pi^-$ plane (we choose coordinates such that $\vec{k} = (0, 0, k), \vec{n}_\pi = (1, 0, 0), \vec{p}_+ = (0, p \sin \theta, p \cos \theta)$ and $\vec{\epsilon} = (\cos \phi, \sin \phi, 0)$):

$$
\frac{d\Gamma}{d\omega \, d\cos \theta \, d\phi} \sim |E \sin \phi - M \cos \phi|^2 \sim 1 - [S_3 \cos 2\phi + S_1 \sin 2\phi]
$$

(8)

Thus the $CP$-odd, $T$-odd Stokes parameter $S_1$ appears as a coefficient of the term $\sin 2\phi$. The essential idea of Refs. \[\[\ref{4}, \ref{5}\] is to use in place of $\vec{\epsilon}$, the vector $\vec{n}_t$ normal to the plane of the Dalitz pair in the reaction $K_L \rightarrow \pi^+ \pi^- \gamma^* \rightarrow \pi^+ \pi^- e^+ e^-$. This motivates the study of the distribution $d\Gamma / d\phi$ in the decay $K_L \rightarrow \pi^+ \pi^- e^+ e^-$, where $\phi$ is the angle between the $\pi^+ \pi^-$ and $e^+ e^-$ planes.

To obtain a quantitative idea of the magnitude of $CP$-violation in $K_L \rightarrow \pi^+ \pi^- \gamma$, we show in Fig. \[\ref{fig:odd}\] the three components of the Stokes vector as a function of the photon energy \([6]\). These are calculated from the amplitudes \([\ref{4}]\) using weighted averages of $|E|^2, |M|^2, E^* M$ and $EM^*$ over $\cos \theta$. The values of $S_1$ and $S_2$ are remarkably large, considering that the only source of $CP$-violation is the $\epsilon$-impurity in the $K_L$ wave-function ($\epsilon = \eta_{+-}$). Clearly the $1/\omega^2$ factor in $E$ enhances it to a level that makes it comparable to the $CP$-conserving amplitude $M$. This is evident from the behaviour of the parameter $S_3$, which swings from a dominant electric behaviour at low $E_\gamma$ ($S_3 \approx 1$) to a dominant magnetic behaviour at large $E_\gamma$ ($S_3 \approx -1$), with a zero in the region $E_\gamma \approx 60 MeV$. To highlight the difference between the $T$-odd parameter $S_1$ and the $T$-even parameter $S_2$, we show in Fig. \[\ref{fig:even}\] the behaviour of the Stokes parameters in the “hermitian limit”: this is the limit in which the $T$-matrix or effective Hamiltonian
governing the decay $K_L \rightarrow \pi^+\pi^−\gamma$ is taken to be hermitian, all unitarity phases related to real intermediate states being dropped. This limit is realized by taking $\delta_0, \delta_1 \rightarrow 0$, and $\arg \epsilon \rightarrow \pi/2$. The last of these follows from the fact that $\epsilon$ may be written as

$$\epsilon = \frac{\Gamma_{12} - \Gamma_{21} + i (M_{12} - M_{21})}{\gamma_S - \gamma_L - 2i (m_L - m_S)}$$

(9)

where $H_{eff} = M - i\Gamma$ is the mass matrix of the $K^0\overline{K}^0$ system. The hermitian limit obtains when $\Gamma_{12} = \Gamma_{21} = \gamma_S = \gamma_L = 0$. As seen from Fig. 1b, $S_2$ vanishes in this limit, but $S_1$ survives, as befits a $CP$-odd, $T$-odd parameter. Fig. 1c shows what happens in the $CP$-invariant limit $\epsilon \rightarrow 0$. It is clear that we are dealing here with a dramatic situation in which a $CP$-impurity of a few parts in a thousand in the $K_L$ wave-function gives rise to a huge $CP$-odd, $T$-odd effect in the photon polarization.

We can now examine how these large $CP$-violating effects are transported to the decay $K_L \rightarrow \pi^+\pi^-e^+e^-$. The matrix element for $K_L \rightarrow \pi^+\pi^-e^+e^-$ can be written as

$$\mathcal{M}(K_L \rightarrow \pi^+\pi^-e^+e^-) = \mathcal{M}_{br} + \mathcal{M}_{mag} + \mathcal{M}_{CR} + \mathcal{M}_{SD}.$$  

(10)

Here $\mathcal{M}_{br}$ and $\mathcal{M}_{mag}$ are the conversion amplitudes associated with the bremsstrahlung and $M1$ parts of the $K_L \rightarrow \pi^+\pi^-\gamma$ amplitude. In addition, we have introduced an amplitude $\mathcal{M}_{CR}$ denoting $\pi^+\pi^-$ production in a $J = 0$ state (not possible in a real radiative decay), as well as an amplitude $\mathcal{M}_{SD}$ associated with the short-distance interaction $s \rightarrow d e^+e^-$. The last of these turns out to be numerically negligible because of the smallness of the CKM factor $V_{ts}V_{td}^*$. The $s$-wave amplitude $\mathcal{M}_{CR}$, if approximated by the $K^0$ charge radius diagram, makes a small ($\sim 1\%$) contribution to the decay rate. Thus the dominant features of the decay are due to the conversion amplitude $\mathcal{M}_{br} + \mathcal{M}_{mag}$.

Within such a model, one can calculate the differential decay rate in the form

$$d\Gamma = I(s_\pi, s_l, \cos \theta_l, \cos \theta_\pi, \phi)\, ds_\pi\, ds_l\, d\cos \theta_l\, d\cos \theta_\pi\, d\phi.$$ 

(11)

Here $s_\pi$ ($s_l$) is the invariant mass of the pion (lepton) pair, and $\theta_\pi$ ($\theta_l$) is the angle of the $\pi^+$ ($l^+$) in the $\pi^+\pi^-$ ($l^+l^-$) rest frame, relative to the dilepton (dipion) momentum vector in that frame. The all-important variable $\phi$ is defined in terms of unit vectors constructed from the pion momenta $\vec{p}_\pi$ and lepton momenta $\vec{k}_l$ in the $K_L$ rest frame:
\[
\sin \phi = \bar{n}_\pi \times \bar{n}_l \cdot \vec{z} \quad (CP = -, T = -),
\]
\[
\cos \phi = \bar{n}_l \cdot \bar{n}_\pi \quad (CP = +, T = +).
\]

In Ref. [4], an analytic expression was derived for the 3-dimensional distribution \( d\Gamma /ds_l ds_\pi d\phi \), which has been used in the Monte Carlo simulation of this decay. In Ref. [4], a formalism was presented for obtaining the fully differential decay function \( I(s_\pi, s_l, \cos \theta_l, \cos \theta_\pi, \phi) \).

The principal results of the theoretical model discussed in [4, 5] are as follows:

1. Branching ratio: This was calculated to be

\[
BR(K_L \to \pi^+ \pi^- e^+ e^-) = (1.3 \times 10^{-7})_{Br} + (1.8 \times 10^{-7})_{M1} + (0.04 \times 10^{-7})_{CR} \approx 3.1 \times 10^{-7},
\]

which agrees well with the result \((3.32 \pm 0.14 \pm 0.28) \times 10^{-7}\) measured in the KTeV experiment [7]. (A preliminary branching ratio \(2.9 \times 10^{-7}\) has been reported by NA48 [8]).

2. Asymmetry in \( \phi \) distribution: The model predicts a distribution of the form

\[
d\Gamma d\phi \sim 1 - (\Sigma_3 \cos 2\phi + \Sigma_1 \sin 2\phi)
\]

where the last term is \(CP\)- and \(T\)-violating, and produces an asymmetry

\[
\mathcal{A} = \frac{\left(\int_0^{\pi/2} - \int_\pi^{\pi/2} + \int_\pi^{3\pi/2} - \int_3^{2\pi/2}\right) \frac{d\Gamma}{d\phi} d\phi}{\left(\int_0^{\pi/2} + \int_\pi^{\pi/2} + \int_\pi^{3\pi/2} + \int_3^{2\pi/2}\right) \frac{d\Gamma}{d\phi} d\phi} = -\frac{2}{\pi} \Sigma_1.
\]

The predicted value [4, 5] is

\[
|\mathcal{A}| = 15\% \sin(\phi_{+-} + \delta_0(M_K^2) - \bar{\delta}_1) \approx 14\%
\]

to be compared with the KTeV result [7]

\[
|\mathcal{A}|_{KTeV} = (13.6 \pm 2.5 \pm 1.2)\%.
\]

The “Stokes parameters” \(\Sigma_3\) and \(\Sigma_1\) are calculated to be \(\Sigma_3 = -0.133, \Sigma_1 = 0.23\). The \(\phi\)-distribution measured by KTeV agrees with this expectation (after acceptance corrections made in accordance with the model). It should be noted that the sign of \(\Sigma_1\) (and of the asymmetry \(\mathcal{A}\)) depends on whether the numerical coefficient in \(g_{M1}\) is taken to be \(+0.76\) or \(-0.76\). The data happen to support the positive sign chosen in Eq. (4).

3. Variation of Stokes parameters with \(s_\pi\): As shown in Fig. 4d, the parameters \(\Sigma_1\) and \(\Sigma_3\) have a variation with \(s_\pi\) that is in close correspondence with the variation of \(S_1\) and \(S_3\) shown in Fig. 3. (Recall that the photon energy \(E_\gamma\) in \(K_L \to \pi^+ \pi^- \gamma\) can be expressed in terms of \(s_\pi\):
\( s_\pi = M_R^2 - 2M_K E_\gamma. \) In particular the zero of \( \Sigma_3 \) and the zero of \( S_3 \) occur at almost the same value of \( s_\pi \). This variation with \( s_\pi \) combined with the low detector acceptance at large \( s_\pi \), has the consequence of enhancing the measured asymmetry (23.3 ± 2.3\% in KTeV \([7]\), 20 ± 5\% in NA48 \([8]\)).

4. Generalized Angular Distribution: As shown in Ref. \([5]\), a more complete study of the angular distribution of the decay \( K_L \rightarrow \pi^+ \pi^- e^+ e^- \) can yield further \( CP \)-violating observables, some of which are sensitive to the non-radiative (charge-radius and short-distance) parts of the matrix element. In particular the two-dimensional distribution \( \frac{d\Gamma}{d\cos \theta_l d\phi} \) has the form

\[
\frac{d\Gamma}{d\cos \theta_l d\phi} = K_1 + K_2 \cos 2\theta_l + K_3 \sin^2 \theta_l \cos 2\phi + K_4 \sin 2\theta_l \cos \phi + K_5 \sin \theta_l \cos \phi + K_6 \cos \theta_l + K_7 \sin \theta_l \sin \phi + K_8 \sin 2\theta_l \sin \phi + K_9 \sin^2 \theta_l \sin 2\phi. \tag{18}
\]

Considering the behaviour of \( \cos \theta_l, \sin \theta_l, \cos \phi \), and \( \sin \phi \) under \( CP \) and \( T \), the various terms appearing in Eq. (18) have the following transformation:

| \( K_1, K_2, K_3, K_5 \) | \( CP \) | \( T \) |
|\( K_4, K_6 \) | \( - \) | \( + \) |
| \( K_8 \) | \( + \) | \( - \) |
| \( K_7, K_9 \) | \( - \) | \( - \) |

Note that \( K_{4,6,8} \) have \((CP)(T) = -\), a signal that they vanish in the hermitian limit. If only the bremsstrahlung and magnetic dipole terms are retained in the \( K_L \rightarrow \pi^+ \pi^- e^+ e^- \) amplitude, one finds \( K_4 = K_5 = K_6 = K_7 = K_8 = 0 \), the only non-zero coefficients being \( K_1 = 1 \) (norm), \( K_2 = 0.297 \), \( K_3 = 0.180 \), \( K_9 = -0.309 \). In this notation, the asymmetry in \( d\Gamma/d\phi \) is \( \mathcal{A} = 2 \frac{4K_9}{\pi 1 - \frac{3}{2}K_2} = -14\% \). The introduction of a charge-radius term induces a new \( CP \)-odd, \( T \)-even term \( K_4 \approx -1.3\% \), while a short-distance interaction containing an axial vector electron current can induce the \( CP \)-odd, \( T \)-odd term \( K_7 \). The standard model prediction for \( K_7 \), however, is extremely small.

We conclude with a list of questions that could be addressed by future research. In connection with \( K_{L,S} \rightarrow \pi^+ \pi^- \gamma \): (i) Is there a departure from bremsstrahlung in \( K_S \rightarrow \pi^+ \pi^- \gamma \) (evidence for direct \( E1 \))? (ii) Is there a \( \pi^+ / \pi^- \) asymmetry in \( K_L \rightarrow \pi^+ \pi^- \gamma \) (evidence for \( E2 \))? (iii) Is there a measurable difference between \( \eta_{+\gamma} \) and \( \eta_{+-} \) (existence of direct \( CP \)-violating \( E1 \) in \( K_L \rightarrow \pi^+ \pi^- \gamma \))? With respect to the decay \( K_L \rightarrow \pi^+ \pi^- e^+ e^- \): (i) Is there evidence of an \( s \)-wave amplitude? (ii) Is there evidence for \( K_4 \) or \( K_7 \) types of \( CP \)-violation? On the theoretical front: (i)
Can one calculate the $s$-wave amplitude, and the form factors in $K_L \to \pi^+\pi^-\gamma^*$? (ii) Can one understand the sign of $g_{M1}$? (iii) Can one explain why direct $E1$ in $K_S \to \pi^+\pi^-\gamma$ is so small compared to direct $M1$ in $K_L \to \pi^+\pi^-\gamma$ ($|g_{E1}/g_{M1}| < 5\%$)?

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Figure 1: (a) Stokes parameters of photon in $K_L \to \pi^+ \pi^- \gamma$; (b) Hermitian limit $\delta_0 = \delta_1 = 0$, $\arg \epsilon = \pi/2$; (c) $CP$-invariant limit $\epsilon \to 0$; (d) “Stokes parameters” for $K_L \to \pi^+ \pi^- e^+ e^-$. 