Reading the three–dimensional structure of a protein from its amino acid sequence

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Abstract

While all the information required for the folding of a protein is contained in its amino acid sequence\textsuperscript{[1]}, one has not yet learnt how to extract this information so as to predict the detailed, biological active, three–dimensional structure of a protein whose sequence is known\textsuperscript{[2–4]}. This situation is not particularly satisfactory, in keeping with the fact that while linear sequencing of the amino acids specifying a protein is relatively simple to carry out, the determination of the folded–native–conformation can only be done by an elaborate X–ray diffraction analysis performed on crystals of the protein or, if the protein is very small, by nuclear magnetic resonance techniques. Using insight obtained from lattice model simulations of the folding of small proteins (fewer than 100 residues), in particular of the fact that this phenomenon is essentially controlled by conserved contacts\textsuperscript{[5]} among strongly interacting amino acids, which also stabilize local elementary structures\textsuperscript{[6]} formed early in the folding process and leading to the (post–critical) folding core when they assemble together\textsuperscript{[7]}, we have worked out a successful strategy for reading the three–dimensional structure of a notional protein from its amino acid sequence.
An answer to the question of how to relate the protein primary structure to its full three–dimensional structure has long been awaited to come from a proper understanding of the mechanisms which are at the basis of the phenomenon of protein folding [8]. The fulfillment of such expectation is likely to revolution the therapeutic drug industry, let alone the chemistry of enzymatic processes. It may also be within reach, in any case, to the extent that simple lattice models of protein folding do capture some of the essential properties of real proteins.

Although Anfinsen and collaborators had conclusively shown [1] that the 1D–structure of a protein determines its 3D–structure, researchers had long been stymied in their efforts to predict the latter from the knowledge of the former, because of Levinthal’s paradox [9]: the number of all possible conformations of a polypeptide chain is too large to be sampled exhaustively. Nevertheless, proteins do fold into unique native states in seconds.

A major breakthrough in the study of protein folding was made by inventing a simple (although not oversimplified) model of protein folding, the so called inverse folding model [10], in which the quest of the relation existing between the 1D– and the 3D– structure of a notional protein is formulated in such a way that Levinthal’s paradox is circumvented. In essence, this model turns the problem of protein folding inside out. First, a native conformation is chosen and then, the notional protein designed by minimizing the energy of the system with respect to the amino acid sequence (for fixed composition). Using a 20–letter three dimensional lattice model for heteropolymers [10–18], and contact energies obtained from a statistical analysis of real proteins [19], one finds that good–folder–sequences are characterized by a large gap $\delta = E_n - E_c$ (compared to the standard deviation $\sigma$ of the contact energies) between the energy $E_n$ of the designed sequence in the native conformation and the lowest energy (threshold $E_c$) of the compact conformations structurally dissimilar to the native conformation. In other words, Monte Carlo (MC) simulations testify to the fact that designed sequences displaying a large normalized gap ($\xi = \delta / \sigma \gg 1$, quantity closely related to the z–score [20]) in the native conformation fold on short call [21]. The success of the inverse folding model is connected with the fact that good folders share a (small)
number of conserved contacts (which eventually form the folding nucleus of the protein), contacts which act among the most strongly interacting and best conserved amino acids (so called ”hot” and ”warm” sites in ref. [22]).

We now know [7,23] that this result is tantamount to saying that foldability is controlled by the presence of few, local elementary structures, stabilized by the conserved contacts and formed very early in the folding process, leading to the (post–critical) folding core (i.e., the minimum set of native contacts needed to ensure foldability [24,25]) when they assemble together. From this vantage point of view it is easy to see how to solve the ”inverse–inverse folding problem”, that is how to predict the 3D–structure of a protein, from its 1D–structure (for details we refer to Methods): 1) starting from a notional sequence and making use of the contact energies used in its design find the local elementary structures, 2) determine the possible folding cores by allowing the local elementary structures to interact among them, 3) relax the position of the remaining amino acids and determine the corresponding energy. The (single) compact structure which displays an energy smaller than $E_c$ is the native conformation. This in keeping with the fact that all sequences which in the compactation process display local elementary structures and thus a folding core will fold to a unique conformation, provided the associated total energy lies below $E_c$ [23]. We have applied this strategy to representative members of essentially all the classes of lattice designed sequences available in the literature: 27mers [26–28], 36mers [7,22,29–33] 48mers [24] and 80mers [10]. In all cases the predicted native structure coincides with the compact structure used to design the protein by carrying on it simulated annealing in sequence space.

An example of the results obtained by applying the strategy described above to a designed 36mer is shown in Fig.1. The results displayed in Figs. 1(b) and 1(c) testify to the fact that the designed sequence shown in Fig. 1(a) is a good candidate to code for a notional protein. In fact, the energy associated with the contacts stabilizing the three predicted local elementary structures (yellow dashed lines, Fig. 1(b) ) is $-2.66$ in the units we are using ($RT_{room} = 0.6$ kcal/mol), the interaction energy among these structures in the (post–critical) folding core (red dashed lines, Fig. 1(c) ) being equal to $-5.15$, the total energy of
this core thus being $-7.81$. Relaxing the amino acids not contained in the core, we find for the sequence shown in Fig. 1(a) that the system can lower its energy below $E_c$ (to a value $-16.5$, where $E_c$, calculated making use of the Random Energy Model [34] is $-14.1$) for the conformation shown in Fig. 1(d), which is thus predicted to be the native conformation of the sequence shown in Fig. 1(a). In fact, this conformation coincides with the one used in the literature to design the sequence under discussion. This sequence, known as $S_{36}$, folds into the native conformation shown in Fig. 1(d) with an (average) first passage time of $0.71 \cdot 10^6$ MC steps, following the hierarchy of steps described above (formation of elementary structures in $\approx 10^2$ MC steps, formation of the (post–critical) folding core in $\approx 0.7 \cdot 10^6$ MC steps and folding in $\approx 0.71 \cdot 10^6$ MC steps). These steps can be also observed in Fig. 1(e), where the time dependence of the native contacts for a particular run used to calculate the average folding time of the protein is shown as a function of the number of steps of the Monte Carlo simulation.

As documented by the results displayed in Fig. 2, even the 36mer sequence (cf. Fig. 2(a)) designed onto a conformation which displays as few local contacts as possible [35], surrenders its 3D structure (cf. Fig. 2(d)) to the local elementary structure–strategy discussed above. In this case, only one of the local elementary structures is closed, i.e. contains at least one internal interaction (yellow dotted lines, Fig. 2(b)), while the other two are open (elementary structures not containing any internal interaction). In Fig. 3 we display the results of the method applied to a 48mer. In this case, one of the two local elementary substructures looks like a piece of $\beta$–sheet, a result which testifies to the fact that within the framework of lattice model calculations, local elementary structures can be viewed as scars of (incipient) wild type secondary structures.

We have found that the method discussed above to read the 3D–structure of a notional protein from its 1D–amino acid sequence works not only for the designed sequences which fold fast, but also in the case where the design produces a non–folder. In fact, in such cases, the $1D \rightarrow 3D$ strategy (correctly) does not lead to a native structure.
Methods

1. Selection of the sequence

Sequences are selected within the framework of a 20 letter lattice model of proteins, making use of Monte Carlo simulations to minimize the energy of the chain in the native conformation with respect to the amino acid sequence and for fixed composition [10]. Operatively, the sequence is chosen among those displaying in the native conformation a sufficiently low energy $E_n$ such that the normalized energy gap or order parameter $\xi$ is much larger than 1. From these sequences, the three-dimensional native conformation is recovered through the 3-steps algorithm described below.

2. Search of elementary structures

We distinguish two classes of (local) elementary structures which control, within lattice model calculations, the phenomenon of protein folding: those which isolated do not display any interaction (no internal interaction), and those which do display at least one (internal) interaction (cf. Figs. 1(b), 2(b) and 3(b)). We shall refer to them generically as local elementary structures. When the need arises to distinguish between them, we shall call them ”open elementary structures” and ”closed elementary structures”, respectively [36].

Candidates to the role of open elementary structures are those displaying low values of the energy density $\epsilon_s = (j - i)^{-1} \sum_{i \leq l \leq j} \min_{k \not\in \{i,j\}} B_{\sigma(l)\sigma(k)}$, where $B_{\sigma(l)\sigma(k)}$ are the contact energies between the $l$th and the $k$th amino acid of the chain (Table 6 of ref. [19]). The calculations have been carried out with the condition $(j - i) \leq 10$. The results have been found to be stable with respect to an increase of the range of values of $(j - i)$. Candidates to closed elementary structures are formed by maximizing the value of $p(i, j) = (j - i)^{-\gamma} \exp(-\beta B_{\sigma(i)\sigma(j)})$, where $\gamma = 1.68$, $i + 2 \leq j \leq i + 8$ and $j - i$ is odd [38]. If local elementary structures build more than one internal contact (cf. e.g. Fig. 3(b)), the total value of $p$ associated with them is given by the product of the $p$-values associated with each of the contacts.

3. Search of the folding core.

The energy spectrum of the low-energy conformations which can be constructed mak-
ing use of the elementary structures is calculated through a complete enumeration of the conformations having an energy smaller than a given threshold. Starting from an elementary structure a second one is placed in all possible conformations with respect to the first one which have at least one contact between the two, and its energy recorded. To each of these conformations displaying an energy lower than some chosen energy, a new elementary structure is added, and the process repeated until the composite system contains all elementary structures. The calculations are repeated altering the order in which the different elementary structures are placed together and allowed to interact with each other. Making use of the resulting energy distribution of the conformations containing 2, 3, ..., all, of the local elementary structures, we select as potential (post-critical) folding cores of the notional protein, those conformations having an energy lying below a given threshold energy.

4. Relaxation of the monomers not belonging to the core

For each (potential) folding core, the variety of conformations of the remaining monomers are enumerated (a number which, e.g. for the 36mer shown in Figs 1 and 2, is of the order of $10^5$), and the total energy of the system calculated. The conformation which has energy lower than $E_c$ is the native conformation of the notional sequence. We know that this conformation is unique, in keeping with the fact that the (post-critical) folding core determines, in a unique fashion, the native conformation of a designed sequence [7,24,25].

5. Caveats and limitations

To calculate $E_c$ use is made of the Random Energy Model [34]. In this model the critical energy $E_c = N_c \sigma (2 \log \gamma)^{1/2}$ depends on the number $N_c$ of contacts of fully compact conformations (40 and 56 for chains with $N = 36$ and $N = 48$, respectively), on the number of such conformations per monomer ($\gamma = 1.8$ [39], $\gamma = 2.2$ [40]) and on the variance $\sigma$ of the contact energies (equal to 0.3 for the parameters suggested in Table 6 of ref. [19]). Using the average value $\gamma = 2$ one obtains $E_c = -14.1$ and $E_c = -19.8$ for $N = 36$ and $N = 48$ respectively, to be compared with the values of $-14.0$ and $-21.5$ calculated a posteriori making use of low temperature Monte Carlo simulations. This uncertainty on $E_c$ is of no consequence for the workings of the method in dealing with sequences with $\xi \gg 1$, but can
limit its predictive power for sequences whose native energy is close to $E_c$. 
FIGURES

FIG. 1. Elements in the prediction of the 3D-structure of a notional protein starting from its 1D-structure: (a) designed 36mer sequence (1D structure) given as the input to solve the 1D \( \rightarrow \) 3D puzzle, together with the 20x20 contact energies among the amino acids (Table 6 of ref. [19]), (b) local elementary structures obtained following protocol #1 of Methods. The amino acids participating in the contacts of these closed structures have been drawn in (a) in white colours, (c) only (post critical) folding core designed making use of the elementary structures and of the contact energies according to protocol #3 of Methods, to which is associated a compact conformation (shown in (d)) obtained by relaxing the amino acids not belonging to the core (according to protocol #4 of Methods), displaying an energy lower than \( E_c = -14.1 \), cf. protocol #5 of Methods). Accordingly, the conformation (d) is the predicted wild type (3D) native conformation of the notional 1D-sequence shown in (a) (where the hot, warm and cold sites of the protein in its native state, calculated according to ref. [22] are displayed in terms of red, yellow and green beads). In fact, this is the native conformation used in the literature to design the sequence (a). Furthermore, Monte Carlo simulations testify to the correctness of the prediction. In fact, evolving the sequence (a) starting from random elongated conformations, it always folds into the conformation (d) in \( \approx 0.7 \cdot 10^6 \) MC steps. In (e) the time evolution of all the native contacts of one particular run (in which the chain folds in 0.65 \( \cdot \) 10\(^6\) MC steps) is shown. In particular, that of the contacts associated with the local elementary structures (dashed yellow lines) and with the contacts between the elementary structures in the folding nucleus shown in (c) (red dotted lines). To be noted that the sequence shown in (a) and called \( S_{36} \) in the literature [7,22,24], was designed following the inverse folding-model according to protocol #1 of Methods using the structure shown in (e) as native conformation (for fixed amino acid composition). It has an energy \( E_n = -16.5 \), and thus a normalized gap \( \xi = 8.3 \).
FIG. 2. Same as in Fig. 1 but in this case the 36mer sequence has been designed on the native conformation (d) so as to minimize the local contacts of the native conformation and consequently of the folding core (protocol #1 of Methods). This situation which would look particular trying for a strategy based on local elementary structures, is solved by applying protocols #2-5 of Methods with the same ease than in the case of the 36mer shown in Fig. 1. In (b) are shown the resulting local elementary structures, two of which are open. All the amino acids participating in them are displayed in (a) in terms of white symbols. The same colour is used for the amino acid participating in the (internal) contacts of the closed structure. In (c) are shown the disposition of these structure to form the (unique) folding core with which is associated the (single) completely compact conformation (d) with energy lower then $E_c$. This conformation coincides with the one used in the literature to design the sequence shown in (a), the associated (native) energy being $E_n = -15.99$. Furthermore, Monte Carlo simulations testify to the correctness of the predicted native conformation (d). This can be seen in (e), where the time evolution of the native contacts for a particular run is displayed as a function of the MC steps of the folding simulations. The sequence in (a) folds into the structure shown in (d) in approximately $2 \cdot 10^6$ MC steps. The normalized gap associated with sequence (a) is $\xi = 6.3$. The hot, warm and cold sites of the protein in its native conformation calculated according to [22] are displayed in terms of red, yellow and green beads respectively.
FIG. 3. Same as in Figs. 1 and 2, but for a 48mer. With the designed sequence shown in (a) are associated the elementary structures shown in (b), the folding core (c) with which the native conformation shown in (d) is predicted (protocols #2-5 of Methods). This in fact is the conformation used in the literature [24] to design sequence (a) (protocol #1 of Methods). The correctness of the prediction is furthermore confirmed by Monte Carlo simulations. In fact, the sequence (a) folds into the native conformation (d) in approximately $3.3 \cdot 10^6$ MC steps (cf. Fig. (e)). The sequence (a) is associated with a normalized gap $\xi = 16.5$ in the native conformation (d). The amino acids shown in white in (a) are those participating in the contacts displayed by the closed structures shown in (b). The hot, warm and cold sites of the protein in the native conformation calculated according to [22] are displayed in terms of red, yellow and green beads respectively.
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(a) SQKWLERGATRIADGDLPVNGTYFSCKIMENVHLPA

(b) Diagram with nodes 3, 6, 11, 14, 4, 5, 12, 13, 27, 30, 28, 29

(c) Expanded diagram with additional connections and nodes

(d) 3D graph with nodes and connections

(e) Close-up of the 3D graph with node labels and connections:
- 27-30
- 5-28
- 11-14
- 12-5
- 3-6
- 14-27
- 6-11
- 13-28
- 6-27
- 4-29
(a) RASMKDKTVGIGHQLYLNFEGEGWCPAPDNTRVSLAI

(b)

(c)

(d)

(e)

20-31

6-31

5-20

4-25

0 t 2

5-28

5-22

4-23

1-6
(a) IMESQKWLCEMPAHWCVYTIQGLGNVNCNPNTREFD-SGRSKIQDAYLFH
(b) 
(c) 
(d) 
(e) 
34-37
33-38
33-42
10-15
6-11
3-6
2-7
35-40
33-40
9-16
7-10
6-33
5-34
4-35
3-40