Tolerance allocation using Monte Carlo simulation

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Abstract. Tolerance allocation is an important issue in product manufacturing, ensuring on
the one hand the interchangeability of parts in an assembly and on the other hand, it has a
major impact on the costs of manufacturing products and quality. This paper aim is to assign
in a 2D dimensional chain with, dimensional and geometric tolerances, dimensional and
angular (geometrical tolerances) values to dimensions in a variation interval in order to
obtain a design required variation in dimension and angle for the closing element of that
chain, using the Monte Carlo (MC) simulation. MC simulation randomly assigns values of
the dimensions of the dimensional chain (DC) - for their module and angle - to their
tolerance field, and then calculates the range for module and angle (R) caused at closing
dimension in a sufficiently large number of cases. The ratio of the desired tolerance (Tdo) to
the dimension chain closing element and the range (R) resulting from the simulation,
respectively Tdo/R are calculated and compared with a value that must be greater or at least
equal to an imposed value of an index dependent of the Cp - capability desired at the closing
dimension. At the desired value that meets design requirements, the tolerance allocation
process is completed.

1. Introduction

The task of placing +/- tolerances on each dimension of a CAD model or set of engineering can affect,
on the engineering design side, the fit and function of the final product, which can cause poor
performance and unsatisfied customers. On the manufacturing side, tolerance requirements determine
the selection of machines, tooling and fixtures; operator skill levels and setup costs; inspection
precision and gaging; and scrap and rework [1].

Any dimension of a part can be included in a dimensional chain (DC) and could be the closing
dimension (CD) of that dimensional chain. These tolerances of the (DC) closing elements (CDs) are
dependent on the tolerance of the component dimensions in the (DC). In order to obtain a lower
cost, it is necessary to obtain the tolerance admitted to the closing dimension CD having tolerance as
large as possible at dimensional chain component dimensions. Regarding tolerances, in order to
calculate them, the designers do a tolerance analysis, establishing how the closing dimension of
dimensional chain will vary given specific variations in parts and fixtures. Another issue is tolerance
allocation, which uses different allocation methods to decide values for allowed variations in the
parts and fixtures so that desired tolerances at the closing dimension level will not be exceeded.
Tolerance allocation is a design tool and its aim is to provide a rational basis for assigning
tolerances to dimensions. In the literature, there are many concerns about the analysis and allocation
of tolerances.
Among the existing methods in the literature are set out in [2, 3] the Proportional Scaling Method, which by proportional scaling allows bringing the tolerance zone of the closing dimension to the desired value. Researches on the analysis and allocation of tolerances have been made in [1], where the allocation of tolerances using weighting factors is presented. In this way can be controlled which tolerance fields are bigger and smaller. These scalars apply to the tolerance rather than to the dimensions in dimensional chain as in applied (MC) simulation. Research on the allocation of tolerances by optimizing the costs of obtaining them, are presented in [4-6]. Genetic algorithms to optimize tolerance allocation, uses the costs of obtaining the tolerances which is hard to do in practice. In [7] is presented an algorithm used for optimizing the tolerance based on concurrent objectives to minimize the manufacturing cost present worth of expected quality loss and quality loss. Also fuzzy based tolerance approaches are used, as in [8] combined with SDOF (small degrees of freedom).

2. 2D Dimensional Chain

In this paper is presented the dimensions tolerance allocation in a dimensional chain simulating the summation of the dimensions in dimensional chain and not of the dimensions’ tolerance. Dimension values in DC are randomly assigned in the tolerance fields allocated according to estimated distributions. Thus, the sum of the tolerances allocated by different methods like worst case, statistical case [1] is not used. Also, it is necessary to convert the geometric position tolerances into the angular deviations of the dimensions. This paper aim is to assign dimensional and angular (geometrical tolerances) values to dimensions in a 2D dimensional chain in a variation interval in order to obtain a design required variation in dimension and angle for the closing dimension of that chain using the MC simulation. The paper is structured as follows: in chapter 2 is illustrated the assembly dimensional chain, theoretical considerations related to calculating vector dimensional chain with closing dimension (CD) and the effect of geometric tolerance (angular and coaxial) on the tolerance analysis is discussed, in chapter 3 is presented the calculation of the dimensional chain closing dimension (CD) and its dimensional and angular tolerance field using MC simulation followed by the comparing of values obtained with the desired ones, and then the allocation of the tolerance fields using the MC simulation followed by the conclusions.

The allocation of tolerances for dimensional chains in 2D space can be done using the MC simulation. For example, the dimensional chain in figure 1 is analysed. This dimensional chain is dimensioned with both dimensional and geometric tolerances (deviations from perpendicularity, parallelism, concentricity, etc.). This dimensional chain contains both dimensions resulting from the actual manufacturing operations and dimensions that are obtained from the adjustment, in assembly operations. Also, all four dimensions C1-C4 are not fixed.

The dimensional chain C1,.., C4 has CD as closing dimension. The dimensional chain contains a part that is mounted in the tilted bore and whose nozzle has to respect the dimension prescribed for the closing dimension CD. The closing dimension is also dependent on the geometric tolerances shown on the drawing figure 1.

The vector equation for dimensional chain is:

\[ c_1' + c_2' + c_3' + c_4' = CD \]  \hspace{1cm} (1)

The dimensions C1-C4 – their module and angles with OX axis, lower deviation (Ei), upper deviation (Es) and operation are presented in table 1.
Figure 1. Assembly dimensional chain (CD – closing dimension of C1, C2, C3, C4 DC).

Table 1. The dimensions C1-C4 – their module and angles with OX axis, lower deviation (Ei), upper deviation (Es) and operation.

| Dimension in dimensional chain (DC) | Module [mm] | Angle Ox [degrees] | Ei [mm] | Es [mm] | Operation          |
|-------------------------------------|-------------|--------------------|--------|--------|--------------------|
| C1                                  | 30          | 120                | -0.05  | +0.05  | adjustment         |
| C2                                  | 42          | 110                | -0.06  | +0.06  | adjustment         |
| C3                                  | 27          | 290                | -0.04  | +0.04  | Cover manufacturing|
| C4                                  | 15          | 290                | -0.02  | +0.02  | Nozzle manufacturing|

The scalar equation for equation (1) is:

$$CD = C_1 \cdot \cos(\alpha_1) / C_2 \cdot \cos(\alpha_2) / C_3 \cdot \cos(\alpha_3) / C_4 \cdot \cos(\alpha_4)$$

Any dimension of dimensional chain (DC) is characterized by its size (module) and its orientation, respectively the angle to the Ox axis equation (3). So the module and angle of any dimension Ci will determine the module and the angle of the closing dimension of dimensional chain.

$$CD = f(C_i, \alpha_i)$$

The sensitivity (or transfer factor) of the CD closing dimension is defined as the ratio between the CD variation (either the module $\Delta CD_m$ or the angle $\Delta CD_\alpha$) and the dimension in dimensional chain variation (either its module $\Delta C_i$ or the angle $\Delta \alpha_i$). The sensitivities of CD against $C_i$ respectively $\alpha_i$ are as follows in equations 4(a)-(d):

$$S_{\Delta C_i} = \frac{\Delta CD_m}{\Delta C_i}$$

$$S_{\Delta \alpha_i} = \frac{\Delta CD_\alpha}{\Delta \alpha_i}$$

$$S_{\Delta \alpha_i} = \frac{\Delta CD_\alpha}{\Delta \alpha_i}$$

$$S_{\Delta \alpha_i} = \frac{\Delta CD_\alpha}{\Delta \alpha_i}$$
Where:
- $\Delta C_{D_n}$ is the modulus of closing dimension,
- $\Delta \alpha_n$ is the angle variation for closing dimension

For very small values of $\Delta C_n$ respectively $\Delta \alpha_n$, equations (4a)-(4d) constitutes the partial derivate of modulus and angle for closing dimension variable relative to modules and angles of dimensions comprised in dimensional chain.

Analysing the drawing in figure 1, the surface B, axis C and cylinder D have geometrical tolerances that could cause that the closing dimension C to not comply with the desired requirements.

Dimension C3, through its angle, comply with position geometric tolerance of B surface. The geometric tolerance of the C axis is influenced both by the position of the surface B and by the geometric tolerance allowed at the perpendicularity of the C axis on the surface B.

The angle of the C axis will be influenced by both the geometric tolerance of the surface B and the perpendicularity geometric tolerance of the C axis, the errors will be summed up.

The angle of the C4 dimension will be affected by the coaxial tolerance of the nozzle bore C2 against the cylinder D.

All these geometric position errors will influence both the size of the module and the position (angle) of the closing dimension CD. It is necessary to convert the geometric position tolerances into the angular deviations of the dimensions C3 and C4 respectively. Angle tolerance for surface B implies an angular error of dimension C3 as in figure 2. The perpendicularity tolerance of the axis C to the surface B will lead to an angular error of the dimension C3 as in figure 2. The two errors will be statistically summed to get the final error of the C3 dimension.

The angular error of dimension C3 due to geometric tolerance of surface B equation (5):

$$\delta_{u3} = \arctg \frac{\delta_{u3}}{l_3} [\text{rad}]$$  (5)

The angular error of dimension C3 due to geometric tolerance from the perpendicularity of the C axis to the surface B is:

$$\delta_{p3} = \arcsin \frac{\Delta \rho}{\epsilon_{m}} [\text{rad}]$$  (6)

Statistically summing up the two previous errors, will result the angular error of the C3 dimension:

$$\delta_{3} = \sqrt{\delta_{u3}^2 + \delta_{p3}^2} [\text{rad}]$$  (7)

The geometric coaxial error will generate an angular deviation of the C4 dimension that can be calculated according to figure 3:

$$\delta_{4} = \arctg \frac{\delta_{c4}}{l_4} [\text{rad}]$$  (8)
3. Tolerances allocation

In tolerance allocation, the assembly yield is specified as a design requirement. The component tolerances must then be set to assure that the resulting assembly yield meets the design requirement. The design requirements for analysed case are to obtain CD with dimensional tolerances field $T_{CDm}=0.08$ symmetrically versus CD mean, (respectively $\pm 0.04$) and angle tolerance for CD $T_{CDu}=0.5 \text{ [deg]}$, symmetrically versus CD angle mean $CD_{angle}=180 \text{ [deg]}$.

Monte Carlo simulation is realised in two cases - using initial tolerances and scaling factors and using initial tolerances and weighted scaling factors. Weight factors are established to related costs and difficulties in obtaining tolerances.

3.1. Closing dimension (CD) computing

The closing dimension CD module and angle with their tolerances resulted from dimensional chain $C_1$…$C_4$ is calculated using a Monte Carlo simulation. In order to apply the Monte Carlo simulation and the other calculations is used a software programme developed in Python language. The initial data needed: upper and lower deviations $E_i, E_s$ for $C_1$-$C_4$ dimensions, tolerances for angles, weighting cost coefficients for tolerances $1, \ldots, 5$, capability indexes for each of $C_1$-$C_4$ dimensions, type of variance for $C_1$-$C_4$ dimension (type 1, 2 or 3). Using the software, the analysed dimensional chain can be represented graphically in figure 4. The closing dimension has the values:

$CD_{m} = 30.0$, for module

$CD_{u}[\text{deg}] = 180.0$, for angle with OX axis in degrees

These values are compared to those required by design. Values of the dimensions of the DC (for their module and angle) to their tolerance field are randomly assigned in a large number of cases 10,000, and then calculate the range $R$ for dimension and angle at closing dimension figure 5.
3.2. Comparing the CD module and angle with required values

Initial tolerances analysis reveals that:
Module $z_i = \text{lower limit} = -2.05082669988$
Module $z_s = \text{upper limit} = 2.06306955778$
Angle $z_i = \text{lower limit} = -3.33266886377$
Angle $z_s = \text{upper limit} = 3.33978983102$
$R_{\text{mSim}} = 0.138228347019$ $R_{\text{uSim}} = 0.0084281884885$

The module is not verified, module range $R_m > T$ required
The angle is verified, angle range $R_u \leq T$ required

It can be seen that the closing dimension module exceeds the required tolerance field, resulting in a range greater than the allowed tolerance field. The angle of the closing dimension in return is in the specified limits. In this case, an allocation of tolerances fields for dimensional chain (DC) dimensions is required.

3.3. Proportional allocation tolerances using MC simulation for C1-C4 dimensions

In order to reallocate the tolerance field dimensional chains for dimensions C1-C4 it is necessary to calculate the C1-C4 dimensions sensitivities for modules and angles. Sensitivities are partial derivate of module and angle of closing dimensions relative to modules and angles of dimensions in dimensional chain. These sensitivities values are graphically represented for module and for the angle.
in figure 6. In proportional allocation tolerances case it will be maintained the proportions between tolerances in dimensional chain; they will be proportionally modified so that the tolerance field at closing dimension CD will be at limit, but included in the field of tolerance allowed for closing dimension.

The MC simulation randomly assigns values of the dimensions of the DC (for their module and angle) to their tolerance and then calculates the range Rm for module and the range Ru for angle at closing dimension CD in a sufficiently large number of cases n=10.000 cases.

The ratio of the desired tolerance (T do module or angle) to the closure element and the range (R) resulting from the simulation, respectively T do/R, whose value must be greater or at least equal to an imposed value (index dependent of the Cp capability desired at the closing dimension). If T/R<1 the tolerances will be decreased or if T/R>(1+DeltRat) will be increased (the term DeltRat controls the Cp or CPk index for closing dimension)

For the dimensional chain in figure 1, proportional allocation tolerances case is:

Modul Rat. Current T/R= 0.591585201569
Modul Rat. Current T/R= 3.51139423672
Modul Rat. Current T/R= 1.29666618152
Modul Rat. Current T/R= 1.07487152038
Modul Rat. Current T/R= 1.85278240774
Modul Rat. Current T/R= 1.24729552458
Modul Rat. Current T/R= 1.02629677674
Modul Rat. Current T/R= 2.10517665387
Modul Rat. Current T/R= 1.11460405864
Modul Rat. Current T/R= 2.13124191019
RatInitial= 0.513962635161 Modul
Rat.T/R= 1.24729552458
Tol. Ti Cz.Modul Prop= [ 0.04725001  0.05670001  0.03780001  0.0189 0.00164934  0.00020617 0.00032987  0.00022266]

Note that for module it was selected approx. 1.25 which corresponds to the proportional tolerance TypeM = Ti Cz, for the "i" dimensions of LD.

The first four values are the tolerance for the dimension modules in dimensional chain in [mm] and the last 4 for their angles in [rad]. In the same way are obtained the values for closing dimension angle:

Angle Rat. Current T/R= 1.08209279345
Angle Rat. Current T/R= 1.93615914453
Angle Rat. Current T/R= 1.31474703245
Angle Rat. Current T/R= 1.02908206941
Angle Rat. Current T/R= 1.97662571869
Angle Rat. Current T/R= 1.20635498499
Angle Rat. Current T/R= 1.03163101271
Angle Rat. Current T/R= 2.04173878017
Angle Rat. Current T/R = 1.28731705039
Angle Rat. Current T/R = 1.00736312318
RatInitial = 0.513962635161
Angle - Rat.T/R = 1.20635498499
Tol. Ti Cz.Angle Prop = [0.09031562, 0.10837875, 0.0722525, 0.03612625, 0.00315261, 0.00039408, 0.00063052, 0.0004256]

It can be seen that angle 1.21 corresponding to the proportional tolerance TypU = Ti Cz was selected for angle for "i" dimensions of dimensional chain.

From the tolerance corrected proportionally for T_ipM mode and T_ipU angle, the minimum tolerance will be selected, making them proportional tolerance T_ip:

T_ip = [0.04725001, 0.05670001, 0.03780001, 0.0189, 0.00164934, 0.00020617, 0.00032987, 0.00022266]

The first four values are the tolerances for the dimension modules in dimensional chain [mm] and the last 4 for their angles in [rad].

Verification by MC simulation of the tolerances established by the proportional method T_ip highlights the histograms shown in figure 7 it can be seen that the histogram of the modules and the angles are within the allowed tolerance fields.

![Figure 7](image)

**Figure 7.** Module histogram for proportional allocation and angle histogram for proportional allocation.

### 3.4. Weighted Proportional allocation tolerances using MC simulation for C1-C4 dimensions

On the basis of Winit weight coefficients (cost coefficients) are calculated the relative weighting coefficients Wk and then the proportional correction factors fpondM for the module, fpondU for the angle, according to which the proportional correction factors for tolerance fields of DC dimensions correction factors fpond - for the modul and fpondU for angle are calculated.

Applying the fpondM and fpondU factors to the desired tolerance field for the Tdom module and the Tdom angle respectively, primary tolerance fields for both module and angle are obtained. By proportional action, these primary tolerance fields will be modified proportionally. The sequence of calculation T / R ratio is:

Module Rat. Current T/R = 1.7326522497
Module Rat. Current T/R = 1.14463651206
Module Rat. Current T/R = 1.95096891889
Module Rat. Current T/R = 2.0986386695
Module Rat. Current T/R = 1.02727318793
Module Rat. Current T/R = 2.0451948458
Module Rat. Current T/R = 1.2861242638
Module Rat. Current T/R = 0.998377543074
Module Rat. Current T/R = 2.19864020555
Module Rat. Current T/R = 1.16202104043
RatInitial = 1.62170525591 Module
Rat.T/R = 1.20986386695
Tol. Ti Cz.Module POND-Prop = [0.02493108, 0.03324144, 0.0415518, 0.03324144,
0.00043513, 0.00058017, 0.00072522, 0.00058017]

Note that for module was selected 1.21 ratio which corresponds to the proportional tolerance fields
TippM = Ti Cz.Module pp, for the "i" dimensions of DC. The first four values are the tolerance fields for
the dimension modules in dimensional chain in [mm] and the last 4 for their angles in [rad]. In the
same way are obtained the values for closing dimension angle in weighted proportional allocation:

Angle Rat. Current T/R = 1.37773481248
Angle Rat. Current T/R = 1.18143010678
Angle Rat. Current T/R = 1.95026157943
Angle Rat. Current T/R = 1.20881191593
Angle Rat. Current T/R = 1.09379177402
Angle Rat. Current T/R = 1.8015037665
Angle Rat. Current T/R = 1.30975074445
Angle Rat. Current T/R = 1.07352132933
Angle Rat. Current T/R = 2.00482789054
Angle Rat. Current T/R = 1.13395069888
RatInitial = 1.62170525591
Angle - Rat.T/R = 1.20881191593
Tol. Ti Cz.Angle POND-Prop = [0.05946477, 0.07928636, 0.09910795, 0.07928636,
0.00103786, 0.00138381, 0.00172976, 0.00138381]

It should be noted that for angle was selected 1.21 ratio corresponding to the proportional tolerance
fields TippU = Ti Cz.Angle pp for the "i" dimensions of the DC.

From $T_{ippm}$ and $T_{ippu}$, the minimum tolerance is selected, which will become the tolerance for the
Weighted-Proportional case $T_{ipp}$.

$T_{ipp} = [0.02493108, 0.03324144, 0.0415518, 0.03324144,
0.00043513, 0.00058017, 0.00072522, 0.00058017]$

**Figure 8.** Allocated tolerances (1 initial tolerances,
2 proportional allocation and 3 weighted-
proportional allocation using MC simulation).

The first four values are the tolerance fields for the dimension modules in dimensional chain in
[mm] and the last 4 for their angles in [rad].

The graph with tolerance allocated by the proportional and weighted-proportional method,
compared to the initial ones, is shown in figure 8.

Verification by MC simulation of the tolerance fields established by the weighted-proportional
method $T_{ipp}$ highlights the histograms shown in figure 9.
4. Conclusions

It was presented a method that allocates tolerances dependent on the range of module and angle of closing dimension (the design requirement), much closer to reality. When allocating the tolerances, Cp capability indices for dimensions in dimensional chain and for the closing dimension are taken into account. It is observed that the tolerances obtained by the weighted proportional allocation tolerances using MC simulation for C1-C4 dimensions method have a similar pattern to the initial tolerances. The range of tolerances obtained by the weighted-proportional method is different from the initial tolerances, because the weighting of the tolerances was changed by using $W_{init}$ weight coefficients. It is noticed that the range $R_m$ and $R_u$ falls within the desired tolerance field for the closing dimension.

5. References

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