A Comparative Study of Homotopy Perturbation Method with Differential Transformation Method to Solve a Reaction Diffusion Equation

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Abstract
In this paper, the solution of Cauchy problems for the reaction–diffusion equations are obtained using the as two well-known methods. The first method is He’s Homotopy Perturbation Method (HPM) and other method is Differential Transformation Method (DTM). To explain the capability and reliability of the methods, some cases have been defined. The proper implementation of He’s homotopy perturbation method can awfully lessen the size of work if compared with the standing differential transformation method. Comparisons of the results obtained by the homotopy perturbation method and differential transformation method several examples are provided. The results obtained using HPM and DTM are same results obtain by variational iteration method and MATLAB solution and then the capability of each method are also discussed.

Keywords: Cauchy Reaction-diffusion Equation, Differential Transformation Method, Homotopy Perturbation Method, MATLAB, Variational Iteration Method

1. Introduction
Cauchy reaction-diffusion equations model many problems in mathematical physics, astrophysics, engineering and science. The study of nonlinear problems is of critical significance in all areas of Science and Engineering, as well as in other disciplines. Although it is very simple for us now to obtain the solutions of some problems by means of computers, it is very complicated to solve nonlinear problems either numerically or theoretically. Also it to find the exact solution for such type problems are is very difficult. There are several methods available to find exact, approximate, and numerical solution of reaction diffusion equations. Most of these methods are complicated symbolic computations. Over the last fifteen years, many methods used to solve a wide range of problems whose mathematical models involve algebraic, differential equation, integral equation, integro-differen-
tial equations, in the present work, we use two methods one is differential transformation method and other is He’s homotopy-perturbation method for the solution of Cauchy reaction diffusion equation. The differential transformation method is based on Taylor series expansion, introduced by Zhou in a study about electrical circuits. It gives exact values of the \( k \)th derivative of an analytical function at a point in terms of known and unknown initial and boundary conditions in a fast manner. Homotopy perturbation method was established in 1999 by He. The method is a powerful and efficient technique to obtain the solutions of linear and non-linear equations. The coupling of the perturbation and homotopy methods is known as homotopy perturbation method. Advantages of this method take the conventional perturbation method while eliminating its restrictions.

Ayaz worked the differential transformation method for partial differential equation and find closed
form series solutions for linear and non-linear initial value problems. The differential transforms method gave an analytical solution in polynomial form. It is different from the traditional high order Taylor series method. The Taylor series method is computationally used long time for higher orders. The differential transform method reduces the size of computational domain and applicable to many problems simply. Differential transform method and homotopy perturbation method are very efficient methods to obtain the approximate and analytic solutions of linear and non-linear homogeneous differential equations; delay differential equations as well as integral equations have already discussed.

Adomian defined a new approach to the heat equation which is an application of the decomposition method. Differential transformation in the differential equation for different applications were shown by Hassan, they were used the differential transformation technique to applied for solving Eigen value problems and partial differential equation. They were first used the one-dimensional differential transformation to obtain the Eigen-values and the normalized Eigen functions for the differential equation of the second and fourth-order, and the second used two-dimensional differential transformation to solve partial differential equation of the first and second-order with constant coefficients. The different applications to solve the reaction diffusion equations and differential equation by homotopy perturbation method have also discussed. Othman presented the solution of Cauchy reaction-diffusion problems by differential transformation method and variation iteration method.

2. Mathematical Formulations

In this paper, consider the time dependent one dimensional reaction-diffusion equation-

$$\frac{\partial u}{\partial t} = D \frac{\partial^2 u}{\partial x^2} + r \ u \quad (x, t) \in \Omega 
\subset \mathbb{R}^2$$  \hspace{1cm} (1)

where, $u$ be the function of $x$ and $t$ represent the concentration, $r$ be the function of $x$ and $t$ represent the reaction parameter and $D > 0$ represent the diffusion coefficient, subject to the initial or boundary conditions are

$$u(x, 0) = f(x), \quad x \in \mathbb{R}$$ \hspace{1cm} (2)

$$u(0, t) = g_0(t), \quad \left(\frac{\partial u}{\partial x}\right)_1 (x, t) = g_1(t), \quad t \in \mathbb{R}$$ \hspace{1cm} (3)

The combination of equations (1) and (2) are known as the characteristic Cauchy reaction-diffusion equation in the domain $\mathbb{R} \times \mathbb{R}$, while the combination of equations (1) and (3) are known as the non-characteristic Cauchy reaction-diffusion equation in the domain $\mathbb{R} \times \mathbb{R}$.

If $r = 1$ then the equation (1) becomes

$$\frac{\partial u}{\partial t} = D \frac{\partial^2 u}{\partial x^2} + u$$ \hspace{1cm} (4)

The equation (4) is known as Kolmogorov–Petrovsky–Piskunov equations.

Homotopy perturbation method can be used successfully finding the solution of linear and non-linear boundary value reaction-diffusion problems, and the system of differential equations. It may be concluded that this technique is very dominant and efficient in finding the analytical solutions for a large class of integral and differential equations. This technique provides more realistic series solutions as compared with the DTM and others techniques. However, DTM has some drawbacks, we obtain a series solution, or we can say a truncated series solution. This series solution does not exhibit the actual behaviors of the problem but gives a good guess to the correct solution in a very small region.

3. Methods

To obtain the solution of the reaction diffusion equations by following methods are given below.

3.1 Basic Idea of Differential Transformation Method

The basic idea of the differential transformation is introduced as follows:

3.1.1 Differential Transformation for One-dimension

The differential transformation of the $f^{th}$ derivative function $u(x)$ is define by:

$$U(j) = \frac{1}{j!} \bigg[ \frac{d^j u(x)}{dx^j} \bigg]_{x=x_0}$$ \hspace{1cm} (5)
Where \( U(x) \) be the transformed function of the original function \( u(x) \).

As discussed\(^\text{1,2}\) the inverse differential transformation of \( U(x) \) is define by:

\[
u(x) = \sum_{j=0}^{\infty} \frac{1}{j!} U(j)(x-x_0)^j \tag{6}
\]

In fact, from (4) and (5), we may obtain:

\[
u(x) = \sum_{j=0}^{\infty} \frac{1}{j!} \left( \frac{d^j u(x)}{dx^j} \right) \bigg|_{x=x_0} (x-x_0)^j \tag{7}
\]

The concept of differential transformation is derived from the Taylor series expansion at \( x = x_0 \) represent by equation (7).

### 3.1.2 Differential Transformation for Two-dimension

Similarly, consider a function \( u(x,y) \) of two variables \( x \) and \( y \) be analytic in the domain \( R \) and let \( (x, y) = (x_0, y_0) \) in this domain. The function \( u(x, y) \) represented a power series at center \( (x_0, y_0) \). Hence the differential transformation\(^\text{1,2}\) of function \( u(x,y) \) is given the following form:

\[
u(x,y) = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{1}{j! k!} \left( \frac{\partial^j \partial^k u(x,y)}{\partial x^j \partial y^k} \right) \bigg|_{(x_0,y_0)} (x-x_0)^j (y-y_0)^k \tag{8}
\]

where, \( U(k,h) \) is the transformed function of the original function \( u(x,y) \).

The original and transformed functions is represented by the lower case and upper case letters, and the transformation is known as T-function.

Table 1 represents the fundamental mathematical operations obtained by two-dimensional differential transform method.

| Original function | Transformed function |
|-------------------|----------------------|
| \( u(x,y) = x^m y^n \) | \( U(k,h) = \delta(k-m, h-n) = \begin{cases} 1 & k=m \text{ and } h=n \\ 0 & \text{otherwise} \end{cases} \) |
| \( u(x,y) = e^{ax + by} \) | \( U(k,h) = \frac{\alpha^k}{k!} \delta(k-m) \) |
| \( u(x,y) = \sin(ax + b) \) | \( U(k,h) = \frac{\alpha^k}{k!} \delta(k-m) \cos \left( \frac{h \pi}{2} + b \right) \) |

Table 1. Some operations of the two dimensional differential transformation
and the combination of equation (8) and (9) it may be accomplished that,

\begin{equation}
  u(x, y) = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{1}{j! k!} \left( x^{j} y^{k} \right) \tag{10}
\end{equation}

Substitute the value of \((x_0, y_0) = (0, 0)\) in equation (9), then (9) becomes:

\begin{equation}
  u(x, y) = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{1}{j! k!} U(j, k) x^{j} y^{k} \tag{11}
\end{equation}

Differential transformation method applies for solving equation (1)-(3).

According to the Differential transformation method, taking the differential transformation of equation (1) on both sides in the following form:

\begin{equation}
  U(j, k) = \frac{1}{(j + 1)!} \left[ x(j + 1)(j + 2) + \sum_{l=1}^{j} \frac{j!}{k!} U(k, l) x^{l} y^{j-l} + \frac{j!}{k!} U(k, l) \right] \tag{12}
\end{equation}

and taking the differential transformation of initial and boundary conditions equation (2) and (3) are given by:

\begin{align*}
  U(0, 0) &= F(0) \tag{13} \\
  U(0, k) &= G_{0}(k) \quad \text{and} \quad U(1, k) = G_{1}(k) \tag{14}
\end{align*}

By using (13) and (14) into (12), calculate \(U(k, h)\), and substitute all value \(U(k, h)\) into (11), then the solution \(u(x, y)\) will be consequently obtained.

3.2 Basic Ideas of He's Homotopy Perturbation Method

To illustrate the basic ideas of this method\cite{18-21}, consider the following non-linear differential equation:

\begin{equation}
  A(u) - g(r) = 0, \quad r \in \Omega \tag{15}
\end{equation}

with boundary conditions

\begin{equation}
  B \left( u, \frac{\partial u}{\partial n} \right) = 0, \quad r \in \Gamma, \tag{16}
\end{equation}

where, \(A\) is known as a general differential operator, \(B\) is a boundary operator, \(g(r)\) is known analytical function and \(\Gamma\) is known as boundary of the domain \(\Omega\).

The operator \(A\) can be separated into two portions \(L\) and \(N\), where \(L\) is the linear portion, while \(N\) is the nonlinear portion; and therefore equation (1) may be rewritten as:

\begin{equation}
  L(u) + N(u) - g(r) = 0 \tag{17}
\end{equation}

By the homotopy technique, we construct a homotopy as

\begin{equation}
  v(r, p; \Omega \times [0, 1] \rightarrow R \quad \text{which satisfies:}
  H(v, p) = (1 - p)L(v) + pL(u_{0}) + p[A(v) - g(r)] = 0, \quad r \in \Omega \tag{18 a}
\end{equation}

Or

\begin{equation}
  H(v, p) = L(v) - L(u_{0}) + pL(u_{0}) + p[N(v) - g(r)] = 0, \quad p \in [0, 1], \quad r \in \Omega \tag{18 b}
\end{equation}

where, \(p \in [0, 1]\) is an embedding parameter, while \(u_{0}\) represent the initial approximation of equation (1) which verified the boundary conditions, from equation (16) and (17), we will have:

\begin{equation}
  H(v, 0) = L(v) - L(u_{0}) = 0 \tag{19}
\end{equation}

\begin{equation}
  H(v, 1) = A(v) - g(r) = 0 \tag{20}
\end{equation}

The changing process of \(v(r, p)\) from \(u_{0}(r)\) to \(u_{p}(r)\) at \(p \rightarrow 0\) from zero to one. In this topology is called deformation, while \(L(v)\) and \(A(v) - g(r)\) are known as homotopy.

According to HPM, owing to the fact that the embedding parameter \(0 \leq p \leq 1\) consider as a “small parameter”, and suppose that the solution of equation (18a) and (18b) can be express as a power series in \(p:\)

\begin{equation}
  v = u_{0} + pv_{1} + p^{2}v_{2} + p^{3}v_{3} + \cdots \tag{21}
\end{equation}

Setting \(p \rightarrow 1\) results in the approximate solution of equation (17) as:

\begin{equation}
  u = \lim_{p \rightarrow 1} v = u_{0} + v_{2} + v_{3} + \cdots \tag{22}
\end{equation}

The combination of the homotopy method and perturbation method are known as homotopy perturbation method\cite{18-21} (HPM), which lessens the restrictions of the traditional perturbation methods. On the other hand, this technique can have complete benefits of the traditional perturbation techniques.

While the series (23) is convergent for maximum cases, however, the convergence rate depends on the non-linear operator \(A(v)\). The following opinions are proposed by He:

1. The second derivative of \(N(v)\) with respect to \(v\) must be small because the parameter \(p\) may be relatively
2. The norm of \(L^{-1}(\frac{\partial N}{\partial v})\) must be smaller than one so that the series converges.

Example 1: Consider the following the reaction diffusion\cite{22}
equation:
\[ \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - u \quad (x, t) \in \Omega \subset R \] (23)

With the initial and boundary conditions are
\[ u(x, 0) = e^{-x} + x = f(x), \quad x \in R, \] (24)
\[ u(0, t) = 1 = g_0(t) \] (25)
\[ \frac{\partial u}{\partial x}(1, t) = e^{-x} - 1 = g_1(t), \quad t \in R \] (26)

By the Differential Transformation Method (DTM).

Taking the differential transformation of equation (23), can be define by,
\[ U(j, k + 1) = \frac{1}{k + 1} (j + 1)(j + 2)U(j + 2, k) - U(j, k) \] (27)

The initial conditions (24) should be also transformed as following form
\[ \sum_{j=0}^{\infty} U(j, 0) x^j = \left( 1 - e^x \right) \sum_{j=0}^{\infty} \frac{1}{j!} x^j \] (28)

Equating the coefficient of \( x \) of equation (28) are given by
\[ U(j, 0) = \begin{cases} 
1, & \text{if } j = 0 \\
0, & \text{if } j = 1 \\
(-1)^j/j!, & \text{if } j = 2, 3, 4, \ldots
\end{cases} \] (29)

And the boundary condition (25) should be transformed the following form:
\[ U(0, k) = \delta(j, k) = \begin{cases} 
1, & \text{if } j = k = 0 \\
0, & \text{if otherwise}
\end{cases} \] (30)

and
\[ \sum_{j=0}^{\infty} U(j, k) x^j = \left( 1 - e^x \right) \sum_{j=0}^{\infty} \frac{1}{j!} x^j \] (31)

Equating the coefficient of \( t \) of equation (28) are given by
\[ U(j, k) = \begin{cases} 
0, & \text{if } j = 0, k = 0 \\
-1, & \text{if } j = 1, k = 1 \\
(-1)^k/k!, & \text{if } j = 1, k = 2, 3, 4, \ldots
\end{cases} \] (32)

For each value \( k, h \) putting equations (29), (30) and (32) into equation (28) and by recursive method, all other of \( U(j, k) \) are equal to zero, when substitute all values \( U(k, h) \) into (12) and get the series for \( u(x, t) \). Then reposition the series, and finding the following closed form solution:
\[ u(x, t) = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} U(j, k) x^j t^k \]
\[ u(x, t) = e^{-x} + x \]
\[ u(x, t) = x e^{-t} + e^{-x} \quad u(x, t) = x e^{-t} + e^{-x} \] (33)

By the HPM, construct the homotopy of equation (23) is given by:
\[ \frac{\partial u}{\partial t} - \frac{\partial u_0}{\partial t} = p \left( \frac{\partial^2 u}{\partial x^2} - u - \frac{\partial u_0}{\partial t} \right) \] (34)

and the initial approximation is:
\[ u_0(x, t) = u(x, 0) = e^{-x} + x \] (35)

Suppose that the solution of equation (34) in the following form
\[ u = u_0 + pu_1 + p^2u_2 + p^3u_3 + p^4u_4 + \cdots \] (36)

Substituting equation (36) into equation (34) and equating coefficient of \( p \), that it following form-
\[ p^0 : \frac{\partial u_0}{\partial t} - \frac{\partial u_0}{\partial t} = 0 \]
\[ p^1 : \frac{\partial u_1}{\partial t} = \frac{\partial^2 u_0}{\partial x^2} - u_0 - \frac{\partial u_0}{\partial t} \]
\[ p^2 : \frac{\partial u_2}{\partial t} = \frac{\partial^2 u_1}{\partial x^2} - u_1 \]

similarly, others.

By choosing \( u_0(x, t) = u(x, 0) = e^{-x} + x \), and solve the following above equations, and find the following approximation-
\[ u_1 = -x t \]
\[ u_2 = \frac{1}{2} x t^2 \]
\[ u_3 = -\frac{1}{6} x t^3 \]

similarly, others.

Further HPM obtain the series solutions of the following form:
\[ u = u_0 + u_1 + u_2 + u_3 + u_4 + \cdots \] (37)
\[ u = e^{-x} + x \left( 1 - \frac{1}{2} x^2 t + \frac{1}{6} x^3 t^2 - \cdots \right) = e^{-x} + xe^{-t} \] (38)
From equations (33) and (38), the solution of the given equation (23) to equation (26) by obtain the differential transformation method and homotopy-perturbation method are same results, it evidently appears that the solution remains closed form to the exact solution.

Figure 1(a) represents the solution for the (23) to equation (26) obtained by HPM or DTM of equation (33) or (38) from \( t = 0 \) to \( t = 1 \) and \( x = 0 \) to \( x = 1 \) while Figure 1(b) represents the by MATLAB solution of equation (23) to equation (26) equation from \( t = 0 \) to \( t = 1 \) and \( x = 0 \) to \( x = 1 \).

The accuracy of the HPM or DTM for the combined KPP equation is controllable, and absolute errors are very small with the present choice of \( t \) and \( x \). These results also obtain by MATLAB, and both the results are compare through the Figure 1(a) & (b)there are no visible differences in the two solutions. The exact solution HPM or DTM is the same as that obtained by VIM\(^{24,25}\) and ADM\(^{13,22}\).

**Example 2:** Consider the reaction-diffusion equation of the following:

\[
\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} + 2tu = 0, (x, t) \in \Omega \subset R,
\]

with initial and boundary conditions are

\[
u(x, 0) = e^x = f(x), \quad x \in R, \quad (40)
\]

\[
u(0, t) = e^{x - t^2} = g_0(t), \quad \frac{\partial u}{\partial x}(0, t) = e^{x - t^2} = g_1(t), \quad t \in R \quad (41)
\]

By the Differential Transform Method, Taking the transform of equation (39) can be written as the following form:

\[
U_{k+1} = \frac{1}{k+1} \left[ e_{k+1} + 2kU_k + \lambda \right] - e_{k+1} \sum_{p=1}^{k} \frac{\eta}{p!} e_{k-p}(x - t^2) (y - r_0, t^2) \quad (42)
\]

Taking the transform of initial condition (40) can be written as following form

\[
U(j, 0) = \begin{cases} \lambda, & \text{if } j = 0 \\ 0, & \text{if } j = 1, 2, 3, \ldots \end{cases} \quad (43)
\]

And taking the transform of boundary condition (41), can designed from the following Table 1.

Table 1.

| \( U(0, k) = U(1, k), \quad k = 0, 1, 2, 3, \ldots \) |
| \( U(0, 0) = 1 = U(0.1) \) |
| \( U(1.0) = 1 = U(1.1) \) |
| \( U(0.2) = \frac{3}{2} = U(1.2) \) |
| \( U(1.2) = \frac{7}{6} = U(0.3) \) |
| \( U(1.4) = \frac{25}{24} = U(0.4) \) |
| \( U(0.5) = \frac{27}{40} = U(1.5) \) |
| \( U(0.6) = \frac{331}{720} = U(0.6) \) |
| \( U(0.7) = \frac{331}{720} = U(0.7) \) |

| \( U(0.8) = \frac{1303}{5040} = U(1.2) \) |

Table 1 using for the values of equation (42), and by recursive technique, we get

\[
u(j, 1) = \frac{1}{j!}, \quad j = 2, 3, \ldots
\]

and

(a) HPM or DTM solution for the problem (1)(b)MATLAB solution of problem (1).
Substituting all values $U(k,h)$ into (10) and obtain series for $u(x,t)$. Then rearrange the solution, and get the find that following closed form solution:

$$u(x,t) = \sum_{k=0}^{\infty} p(k,h)x^{k}\lambda = e^{x+ct}$$  \hspace{1cm} (44)

By the homotopy perturbation method, construct the homotopy of equation (39):

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial t} = P\left(\frac{\partial^2 u}{\partial x^2} + 2\tau u - \frac{\partial u}{\partial t}\right)$$  \hspace{1cm} (45)

and the initial conditions are as follow:

$$u_0(x,0) = u(x,0) = e^{x}$$  \hspace{1cm} (46)

Suppose that the equation (45) solve by the following series form:

$$u = u_0 + u_1 + u_2 + u_3 + u_4 + \cdots$$  \hspace{1cm} (47)

Substituting equation (47) into equation (45) and separate coefficient of same power of $P$, we get:

$$p^0 : \quad \frac{\partial u_0}{\partial t} - \frac{\partial u_0}{\partial t} = 0$$

$$p^1 : \quad \frac{\partial u_1}{\partial t} = \frac{\partial^2 u_0}{\partial x^2} + 2\tau u_0 - \frac{\partial u_0}{\partial t}$$

$$p^2 : \quad \frac{\partial u_2}{\partial t} = \frac{\partial^2 u_1}{\partial x^2} + 2\tau u_1$$

similarly, others.

By choosing $u_0(x,t) = u(x,0) = e^{x}$, and solve the above equations, and find the following approximation:

$$u_1 = e^{x}(t + t^2)$$

$$u_2 = e^{x}\left(\frac{t^3}{2} + \frac{t^2}{2} \right)$$

$$u_3 = e^{x}\left(\frac{t^5}{6} + \frac{t^4}{2} + \frac{t^2}{2} + \frac{t^4}{6}\right)$$

Further HPM calculate the series solutions of the following form:

$$u = u_0 + u_1 + u_2 + u_3 + u_4 + \cdots$$  \hspace{1cm} (48)

$$u = e^{x+t^2}$$  \hspace{1cm} (49)

Figure 2(a) represents the HPM solution for the problem (1) of equation (44) or (49) from $t = 0$ to $t = 1$ and $x = 0$ to $x = 1$ while the Figure 2(b) represents the MATLAB solution of Problem (2) from $t = 0$ to $t = 1$ and $x = 0$ to $x = 1$.

From equations (44) and (49), the solution of the given equation (39) to equation (41) by obtain the differential transformation method and homotopy-perturbation method are same results, it evidently appears that the solution remains closed form to the exact solution.

Figure 2(a) Represents the solution for the (39) to equation (41) obtained by HPM or DTM of equation (44) or (49) from $t = 0$ to $t = 1$ and $x = 0$ to $x = 1$ while Figure 2(b) Represents the by MATLAB solution of (39) to equation (41) equation from $t = 0$ to $t = 1$ and $x = 0$ to $x = 1$.

The accuracy of the HPM or DTM for the combined KPP equation is controllable, and absolute errors are very small with the present choice of $t$ and $x$. These results also obtain by MATLAB, and both the results are compare through the Figure 1(a) & (b)and there are no visible differences in the two solutions. The exact solution HPM or DTM is the same as that obtained by VIM[14,25] and ADM[13,22].

4. Conclusion

This paper applied the differential transformation method and homotopy perturbation method to solve initial and boundary value problems of Cauchy reaction diffusion equation. It was shown that these methods are very efficient and powerful to get the approximate and exact solution. Too the results of our example show that both methods successfully can be alternative way for the solution of the reaction diffusion equations. In fact,
DTM and HPM are very efficient methods to find the approximate and analytic solutions of reaction diffusion equation, linear and nonlinear ordinary and partial differential equations, delay differential equations as well as integral equations. The results showed that the homotopy perturbation method is more powerful method to achieve a desired approximate solution by simple calculations is comparison to differential transformation method. The computations associated with the examples calculate the series obtained from the HPM or DTM were performed by MATLAB.

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