What can Machine Learning tell us about the background expansion of the Universe?

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Machine learning algorithms have revolutionized the way we interpret data in astronomy, particle physics, biology and even economics, since they can remove biases due to a priori chosen models. Here we apply a specific machine learning method, the genetic algorithms (GA), to cosmological data that describes the background expansion of the Universe, the Pantheon Type Ia supernovae and the Hubble expansion history $H(z)$ datasets. We obtain model independent and non-parametric reconstructions of the luminosity distance $d_L(z)$ and Hubble parameter $H(z)$ without assuming any dark energy model or a flat Universe. We then estimate the deceleration parameter $q(z)$, a measure of the acceleration of the Universe, and we make a $\sim 4.5\sigma$ model independent detection of the accelerated expansion, but we also place constraints on the transition redshift of the acceleration phase $(z_{tr} = 0.662 \pm 0.027)$. We also confirm a recently reported mild tension between the SnIa/quasar data and the cosmological constant $\Lambda$CDM model at high redshifts ($z \gtrsim 1.5$) and finally, we show that the GA can be used in complementary null tests of the $\Lambda$CDM via reconstructions of the Hubble parameter and the luminosity distance.

Cosmology has reached a stage of near percent level precision with a wide range of theoretical models that describe rigorous and accurate measurements. However, the explanation as to why the Universe is undergoing a period of accelerated expansion still remains an open question and the cause of this phenomenon is usually attributed to a dark energy (DE) component \cite{1}. The standard cosmological model contains the cosmological constant $\Lambda$ and a cold dark matter component (CDM) \cite{2} and is at the moment the best candidate to explain the accelerated expansion of the Universe as it is in excellent agreement with all of the current data sets \cite{3}.

However, there is a plethora of other models as well, many of which are included in the pipelines of upcoming surveys, such as Euclid \cite{4}. These models range from having a canonical scalar field \cite{5,7} to a scalar field with a generalized kinetic terms \cite{8,9} or a non-minimal coupling \cite{10,12}, being added to General Relativity (GR), coupled DE models \cite{13}, modifications of the Einstein-Hilbert action \cite{14}, the Chaplygin gas \cite{15} or extra dimensions \cite{16}. For further reviews see \cite{17,22}.

This huge landscape of DE models makes the interpretation of the cosmological observations difficult as the results, e.g. the value of the matter content of the Universe $\Omega_{m0}$, depend on the particular model chosen. For example, the Planck mission provides an accurate value for the matter density parameter today $\Omega_{m0} = 0.315 \pm 0.007$, see Ref. \cite{8}, however this value is specific to the $\Lambda$CDM model as it was obtained assuming the $\Lambda$CDM model to be the correct theory, hence is model dependent. To remove biases due to choosing an a priori defined model, it is important to use reconstruction techniques and model independent approaches, see for example \cite{28}. One such approach is the use of machine learning (ML) methods, which has already lead to many successes in cosmology \cite{24}. ML methods have been used to reduce the scatter in cluster mass estimates \cite{25}, to distinguish between standard and modified gravity theories from statistically similar weak lensing maps \cite{26}, and have been found to be useful for the next generation CMB experiments \cite{27}. N-body simulations \cite{28}, cosmological parameters inference \cite{29}, supernova classification \cite{30} and strong lensing probes \cite{31}.

In this Letter we will apply a particular ML method, the Genetic Algorithms (GA), which can be defined as a stochastic search approach. The GA have been used in many disciplines ranging from astrophysics, e.g. to determine the photometric redshift \cite{32}, to find the optimum parameters for cosmic ray injection and propagation \cite{33}, to fit dusty galaxies \cite{34}, to perform galaxy classification \cite{35}, in particle physics to constrain the MSSM \cite{36,37} or resonances in Lambda reactions \cite{38}, but also in finance \cite{39,40} and biology \cite{41}. More recently, they have also been applied to cosmology for data reconstruction \cite{23,42,47}. One of the most effective use of these methods is the reconstruction of null tests, i.e. pass/fail test made of variables of a theory that should always be constant for all values of the parameters, and can be used to test theories in a model independent way.

In light of the near future experiments that will gather a vast amount of data, such as Euclid and LSST, it is necessary to perform model independent tests to check for possible tensions that could be due to systematics or new physics. Specifically, null tests for $\Lambda$CDM have already been applied to the cosmological constant model \cite{25,45,49}, the growth-rate data \cite{50}, the cosmic curvature \cite{51,52} and to probe the scale-independence of the growth of structure in the linear regime \cite{53}. The null tests we will consider here are the $\Omega_m(z)$ statistic \cite{45,49} and a new null test derived from the luminosity distance, that we present here for the first time. We thus propose applying ML methods, in particular the GA, to perform a simultaneous fit to the Pantheon Type Ia supernovae (SnIa) data compilation \cite{54} and the $H(z)$
data compilation of Ref. [55] to obtain a model independent reconstruction of the luminosity distance \( d_L(z) \) and of the Hubble parameter \( H(t) \equiv \frac{\dot{a}}{a} \) where \( \dot{a}(t) \) is the scale factor in the Robertson-Walker metric, and the dot stands for a derivative with respect to the cosmic time \( t \).

In our analysis we use 1048 data points from the Pantheon set in the range \( z \in [0, 2.26] \), along with their covariances, and 36 points from the \( H(z) \) compilation in the range \( z \in [0, 2.34] \). Measurements of the Hubble expansion \( H(z) \) data are performed either by the differential age method or by the clustering of galaxies or quasars. The former is possible due to the redshift drift of distant objects over a decade or longer, since in GR the \( H(z) \) can also be expressed via the rate of change of the redshift \( H(z) = -\frac{1}{1+z} \frac{dz}{dt} \) [56]. The latter is related to the clustering of galaxies or quasars and it leads to measurements of \( H(z) \) by measuring the radial BAO peak [57].

The GA represents a method for non-parametric reconstruction of functions, based on the notions of grammatical evolution and the genetic operations of mutation and crossover[4]. To estimate the errors on the reconstructed quantities we use an analytical approach developed by [42, 44] where one calculates a path integral over the functional space that can be scanned by the GA. Regarding the error estimates of the ΛCDM model, we compared our approach against that of Ref. [55] for the SnIa data and we have found they are in excellent agreement.

We reconstruct the Hubble parameter by applying the GA to the \( H(z) \) data, while the value of the Hubble parameter \( H_0 \) was derived through minimizing the \( \chi^2 \) analytically as the \( \chi^2 \) is quadratic in \( H_0 \), see Ref. [55]. For the SnIa, due to the degeneracy between the absolute magnitude \( M \) and the Hubble parameter \( H_0 \), we used the value extracted from the \( H(z) \) data, given below. In both cases, no assumptions such as a flat Universe or a specific DE model were made, hence our results are almost completely model independent.

Note that sometimes the data are themselves model dependent, with an infamous example being the SnIa, as one must optimize parameters in the lightcurve function simultaneously with those of the assumed model. Furthermore, a covariance matrix is typically inferred based on an assumed background model, usually ΛCDM. However, since in our case the best-fit is close to ΛCDM and the errors are much larger than the effects of the model bias in the covariance, we can safely assume for now that these effects have a minimal effect to the minimization.

Finally, we performed several simulations with different random seed numbers to make sure we are not biasing our analysis due to the specific value of the random seed and we have also demanded that all functions, along with their derivatives, are continuous and have no singularities in the range covered by the data. The genetic evolution of several different initializations of the GA code with different seed random numbers for the SnIa data as a function of the generation number. In most cases the GA has converged very quickly in the evolutionary history and reaches a lower \( \chi^2 \) than ΛCDM does.

![FIG. 1. The genetic evolution of several different initializations of the GA code with different seed random numbers for the SnIa data as a function of the generation number. In most cases the GA has converged very quickly in the evolutionary history and reaches a lower \( \chi^2 \) than ΛCDM does.](image-url)
TABLE I. The $\chi^2$ for $\Lambda$CDM and GA using the Pantheon SNIa and $H(z)$ data.

| SNIa  | $H(z)$   |
|-------|----------|
| $\chi^2_{\Lambda\text{CDM}}$ | 1034.73 19.476 |
| $\chi^2_{\text{GA}}$       | 1034.30 17.683 |

parameter which is given by

$$q(z) = -\frac{\ddot{a}a}{a^2} = -1 + (1 + z)\frac{H'(z)}{H(z)},$$

where dots stand for derivatives with respect to the cosmic time $t$, while primes for derivatives with respect to the redshift $z$, where $a(t) = 1/t$. The advantage of this parameter over the DE equation of state $w(z)$ is that the former only requires the knowledge of $H(z)$ and not that of cosmological parameters such as $\Omega_{m0}$.

For the Universe to accelerate today, we require (due to historical reasons) that $q_0 < 0$, e.g. for the $\Lambda$CDM model we have $q_0,_{\Lambda\text{CDM}} = -1 + 3\Omega_{m0}/2 \approx -0.528 \pm 0.011$ for the Planck best-fit $\Omega_{m0} = 0.315$ and $q_0,_{\Lambda\text{CDM}} = -0.613 \pm 0.043$ for the $\Lambda$CDM best-fit to the $H(z)$ data of $\Omega_{m0} = 0.258 \pm 0.029$. Using the GA reconstruction of the Hubble parameter given by Eq. (3) we can calculate the deceleration parameter given by Eq. (4) and the result is given in Fig. 3. The present value of the deceleration parameter is found to be $q_0 \equiv q(z = 0) = -0.575 \pm 0.132$, a $\sim 4.5\sigma$ detection of the accelerated expansion of the Universe in a model-independent way.

We can also estimate the value of the transition redshift, i.e. the redshift where the deceleration parameter changes sign, see Refs [32][67] for a list of recent estimates. From the GA reconstruction we find that $z_{tr} = 0.062 \pm 0.027$, while for the $\Lambda$CDM the latter is equal to $z_{tr,\Lambda\text{CDM}} = -1 + 2^{1/3} (\Omega_{m0}^{2/3} - 1)^{1/3} = 0.632 \pm 0.018$ for Planck and $z_{tr,\Lambda\text{CDM}} = 0.791 \pm 0.091$ for the $H(z)$ $\Lambda$CDM best-fit. While the precision of these measurements seems worse than that of $\Lambda$CDM, in our case we have made very minimal assumptions and have not assumed any DE model.

We now focus on the reconstruction of the null tests for the $\Lambda$CDM model. The first null test we will consider is the Om($z$) statistic of Ref. [48], which only requires knowledge of the Hubble parameter $H(z)$ and allows us to discriminate $\Lambda$CDM from other DE models, see Refs [33].

![Fig. 2. Left: The $H(z)$ data compilation along with the $\Lambda$CDM best-fit (dashed line) and the GA best-fit (solid black line). Right: The difference between the GA best-fit distance modulus of the Pantheon SNIa data (black line) and that of the $\Lambda$CDM model (dashed line). The Pantheon SNIa data are shown as grey points in the background.](image)

![Fig. 3. The deceleration parameter given by Eq. (4) as reconstructed by using Eq. (3). The shaded region corresponds to the 1$\sigma$ errors, while the transition redshift $z_{tr}$ corresponds to the point where $q(z)$ crosses zero.](image)
It is defined as
\[ \Omega_m(z) = \frac{H(z)^2}{H_0^2} - 1 \quad (1+z)^3 - 1. \quad (5) \]

Here we also present a different, but at the same time complementary, null test of the ΛCDM by extracting the matter density \( \Omega_m \) from the luminosity distance instead of the Hubble parameter. To do this, we use the Lagrange inversion theorem which states that given an analytic function, we can estimate the Taylor series expansion of the inverse function, i.e. given the function \( y = f(x) \), where \( f \) is analytic at a point \( p \) and \( f(p) \neq 0 \) the theorem allows us to solve the equation for \( x \) and write it as a power series \( x = \tilde{y}(y) \), see [68].

We now apply the Lagrange inversion theorem to the luminosity distance \( d_L(z, \Omega_{m0}) \) and from now on we will restrict ourselves at late times, when DE dominates over the other components, such as radiation and neutrinos. Then, the analytical expression of the luminosity distance for the ΛCDM model, assuming a flat Universe but neglecting radiation and neutrinos, is given by
\[ d_L(z, \Omega_{m0}) = \frac{c}{H_0} (1+z) \int_0^z \frac{1}{H(x)} dx \]
\[ = \frac{c}{H_0} \frac{2(1+z)}{\sqrt{\Omega_{m0}}} \left( _2F_1 \left( \frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \frac{\Omega_{m0}-1}{\Omega_{m0}} \right) - \frac{\Omega_{m0}^2}{\Omega_{m0}^2 (1+z)^5} \right) \quad (7) \]

To derive the \( \Omega_{mL}(z) \) test we first do a series expansion on Eq. 7 around \( \Omega_{m0} = 1 \) and keep the first 10 terms in order to obtain a reliable unbiased estimation and avoid theoretical systematic errors. Then, we apply the Lagrange inversion theorem to invert the series and to write the matter density \( \Omega_{m0} \) as a function of the luminosity distance \( d_L \), i.e. \( \Omega_{mL} = \Omega_{mL}(z, d_L) \). For example, the first two terms of the expansion are
\[ \Omega_{mL}(a, d_L) = 1 - \frac{7a}{6 + \sqrt{a} (a^3 - 7)} + \cdots, \quad (8) \]

where the scale factor \( a \) is related to the redshift \( z \) as \( a = \frac{1}{1+z} \). This null test has the main advantage that it does not require taking derivatives of the data as we use the luminosity distance directly.

The reconstruction of both null tests of the ΛCDM model is shown in Fig. 4, in the left panel for the \( \Omega_H \) and the right panel for the \( \Omega_{mL} \) respectively. We find that both null tests are in agreement with ΛCDM at the 1σ level. While the errors of the distance modulus \( \mu(z) \) and the \( \Omega_{mL} \) test, shown in Figs. 2 and 4 respectively, seem somewhat larger compared to those in Refs. [23-24], the latter used the Union 2.1 set but did not include the systematic errors, thus underestimating the errors regions. So, even though the Pantheon set has roughly twice as many points than the Union 2.1, the inclusion of the systematic errors of the Pantheon in the analysis, brings the error estimates for \( \mu(z) \) and \( \Omega_{mL} \) to the same level as those in Refs. [23-24].

In summary, ML methods are revolutionizing the way we interpret data since they can help to remove biases due to choosing a priori a specific defined model. This is more important than ever as the endeavor to explain the accelerated expansion of the Universe has led to a plethora of DE models, which make the interpretation of the data difficult as the results are model dependent. This can lead to model bias, thus affecting the conclusions drawn about fundamental physics.

We have shown that the simultaneous GA fitting of SNIa and \( H(z) \) data can be used to reconstruct the ex-

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\(^2\) We use the notation \( \Omega_H \) with the subscript \( H \) to discriminate this null test from the one we will introduce later on and which is based on the luminosity distance \( d_L(z) \).
pansion history of the Universe and help determine the current deceleration parameter and transition redshift in a model independent fashion. We also find a $\sim 4.5\sigma$ detection of the accelerated expansion, contrary to recent claims by Ref. [69], and a mild tension with ΛCDM at high redshifts, in agreement with Ref. [60]. Finally, we showed that the GA can be used to reconstruct complementary null tests of the ΛCDM model via reconstructions of both the Hubble parameter and the luminosity distance.

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Numerical Analysis Files: The numerical codes used by the authors in the analysis of the paper will be released upon publication of the paper here.

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