Gluons as Goldstone Bosons when Flavor Symmetry is Broken Spontaneously

Nils A. Törnqvist

Department of Physics, POB 9, FIN–00014, University of Helsinki, Finland

(5 March 1997)

Abstract

The mechanism where flavor symmetry of the standard model is broken spontaneously is discussed within a QCD model with effective three-meson couplings. For sufficiently large coupling the model is unstable with respect to quantum loops from mesonic vacuum polarization. It is argued that color and gluons naturally can account for the Goldstone degrees of freedom expected when flavor symmetry is spontaneously broken.

Pacs numbers: 12.39.Ki, 11.30.Hv, 11.30.Qc, 12.15.Ff

As an introduction let me discuss a simple semiclassical analogue to the mechanism which I am to discuss, which is easy to understand intuitively, without any formulas: Consider classical balloons or balls, with fixed surface area, inflated by hot air. When cooled the inside pressure can fall below the outside pressure and the O(3) spherical balls must collapse spontaneously down to at least O(2) ellipsoids. Below the critical temperature the ellipsoids suddenly pick out preferred directions, the principal axes, and develop different moments of inertia. The O(3) symmetry is broken in the shapes, but the O(3) symmetry still remains in

\footnote{Deformed nuclei which are common among the lanthanides or actinides, e.g. $^{232}$Th, are actually examples of this O(3) symmetry breaking due to the balance of Coulomb and surface energies.}
the sense that all rotated states of the collapsed system have the same energy. The directions of collapse (1,2,3 in Fig.1) are of course arbitrary. This freedom of rotating the ground state of the collapsed systems correspond to the Goldstone degrees of freedom in a field theory with degenerate vacua. But here ”the vacuum” (the surrounding air) need not break the symmetry, only the shapes of the solutions break the spherical symmetry.

When considered as a local symmetry (”collapsed ellipsoids attached to each \(x\)-coordinate”) the directions of collapse can be \(x\)-dependent. Broken flavor symmetry, which I shall discuss below, corresponds to the broken \(O(3)\) symmetric shapes in this analogy and the different masses within flavor multiplets to the unequal moments of inertia along the principal axes in Fig.1. The moments of inertia, are like the masses of a flavor multiplet. They are independent of \(x\) as in a broken global symmetry. On the other hand the \(x\)-dependent freedom to rotate the ellipsoids, remains as an exact unbroken local symmetry. But both symmetries are defined within the same internal space.

In QCD color and flavor have a kind of complementary role: At short distances inside hadrons, the color and gluonic degrees of freedom are crucial, while the interactions are flavor independent. On the other hand at large distances the color degrees of freedom are absent, since hadrons are color singlets, while flavor symmetry and its breaking is evident in the mass spectrum. Conventionally one breaks flavor symmetry by adding, by hand, effective non-degenerate quark masses to the QCD Lagrangian, whereby the pseudoscalars obtain (small) masses and the degeneracy of all flavor multiplets is split. Most of the chiral quark masses are assumed to come from a short distance regime, where weak interactions, and the Higgs mechanism are relevant.

In two recent preprints I discussed a new mechanism where flavor symmetry was broken spontaneously by quantum effects, and where no quark mass splitting term is needed in the Lagrangian. Trilinear meson couplings can lead to unstable self-consistency equations for mesons dressed by the clouds of the same mesons. A natural question which arises in this context of spontaneous flavor symmetry breaking is: Where are the Goldstone bosons and the Goldstone degrees of freedom in such models. In this paper I suggest a natural
answer to this question within a scalar QCD model, when both color and flavor obey the same SU3 symmetry group.

Let a nonet of real meson fields be described by a $3 \times 3$ matrix $\Phi(x)$ such that an individual meson field is $\phi^\alpha = \text{Tr}[\Phi \Lambda^\alpha]$, or inversely $\Phi = \sum_\alpha \phi^\alpha \Lambda^\alpha$. Here $\Lambda^\alpha$ is a complete orthonormal set of nine real flavor matrices, normalized such that $\text{Tr}[\Lambda^\alpha \Lambda^\beta^\dagger] = \delta_{\alpha\beta}$. The simplest choice for these, which we denote $\Lambda^{ij}$, where $i, j$ run from 1 to $N_f$ and for which all matrix elements are 0 except the $(i,j)$’th matrix element which is 1 ($[\Lambda^{ij}]_{m,n} = \delta_{im} \delta_{jn}$).

The flavorless states $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, which are represented by $\Lambda^{ii}$, can of course mix through an orthogonal matrix $\Omega$, such that the diagonal matrices are replaced by $\sum_j \Omega_{ij} \Lambda^{jj}$. The mixing matrix $\Omega$ will be determined by our self-consistency equations. (With isospin exact $\Omega$ mixes $\Lambda^{11}$ and $\Lambda^{22}$ to $(\Lambda^{11} \pm \Lambda^{22})/\sqrt{2}$).

We can write for scalar QCD, with two degenerate scalar nonets $\Phi_C$, $C = +, -$ of opposite charge conjugation $C$ the flavor symmetric Lagrangean:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \sum_C \text{Tr}[D_\mu \Phi_C D_\mu \Phi_C^\dagger] - m_0^2 \sum_C \text{Tr}[\Phi_C \Phi_C^\dagger] + \mathcal{L}_{\text{int}},$$

where the first term is the usual pure gluonic term and $D_\mu = \partial_\mu - igT^a A^a_\mu$ is the usual covariant derivative and for $\mathcal{L}_{\text{int}}$ we chose

$$\mathcal{L}_{\text{int}} = g_F \text{Tr}[\Phi^\dagger_+ \Phi^*_- \Phi^\dagger_- \Phi_-] + g_D \text{Tr}[\Phi^\dagger_+ \Phi^*_- \Phi^\dagger_- \Phi_+].$$

We need the two nonets with opposite $C$ quantum numbers in order to have both F- and D- type couplings present, as in every more realistic model. (E.g. if one includes the ground state mesons, one has $C = +$ pseudoscalars (P) and $C = -$ vector mesons (V) mesons, with PPV and PVV couplings of both types.) Of course, we have to define the model as an effective theory with a flavor independent cutoff $\Lambda$ in order to render it finite.

The color gauge group is unbroken, i.e., none of the scalar fields develop a vacuum expectation value, since the mass terms assumed $m_0^2 \text{Tr}[\Phi^\dagger_+ \Phi^*_- \Phi^\dagger_- \Phi_-]$ have a quadratic minimum at the origin, and $< \phi^\alpha > = 0$. Thus the gluons remain massless as they should. Under a gauge transformation $U(x) = \exp(-i T^a \theta^a(x))$ the fields $\Phi$, the matrices $\Lambda^\alpha$, and the covariant derivative transform in a covariant way:
and the above Lagrangean is obviously gauge invariant. We drop in the following the index $C$ since our results are the same for both $C$’s, and its inclusion should be obvious from the context. The mass and three-meson coupling terms can also be written with the flavor indices $\alpha, \beta, \gamma$ explicit:

$$- \sum_\alpha m_\alpha^2 \Phi^\alpha \Phi^{\alpha\dagger} + \sum_{\alpha\beta\gamma} (g_F C_{\alpha\beta\gamma}^+ + g_D C_{\alpha\beta\gamma}^-) \Phi^{\alpha\dagger} \Phi^\beta \Phi^\gamma$$

where $C_{\pm}^{\alpha\beta\gamma}$ is a set of Clebsch-Gordan-like numbers relating different coupling constants

$$C_{\pm}^{\alpha\beta\gamma} = \text{Tr} [\Lambda^{\dagger \alpha} \Lambda^\beta \Lambda^\gamma]_\pm .$$

Chosing the $\Lambda$ matrices to be the $\Lambda^{ij}$, which we defined above before mixing ($\Omega = 1$), these numbers are simply 0 or $\pm 1$ or 2 according to:

$$C_{\pm}^{ijklmn} = \text{Tr} [\Lambda^{ijkl} \Lambda^{mn}]_\pm = \delta_{jk} \delta_{lm} \delta_{ni} \pm \delta_{jm} \delta_{nk} \delta_{li} .$$

Up til now flavor symmetry is assumed to be exact and the bare nonet members have the same mass $m_0$. But observe that the flavor fields $\Phi^\alpha = \text{Tr} [\Phi \Lambda^\alpha \Lambda^\dagger]$, and the constants $C_{\pm}^{\alpha\beta\gamma}$ of Eq.(7), are gauge invariant, and therefore we can break the flavor symmetry without destroying the gauge invariance, since each term in the sums in Eq. (6) is gauge invariant. Thus we can replace $m_0$, $g_F$ and $g_D$ by flavor dependent (but gauge independent) masses $m_\alpha$ and couplings $g_{\alpha\beta\gamma}$ in Eq.(6) and still have a gauge invariant theory. In fact, now the flavor breaking can be spontaneous since gauge invariance guarantees that any direction in the internal space chosen by the flavor matrices is equivalent, as is obvious from Eq. (4).

Now considering meson loops one must renormalize the mass and coupling terms. The loops shift the masses and induce mixings between the flavorless states by terms $\Delta m_{\alpha\beta}^2$ in the two-point functions or the inverse propagators:
\[ P_{\alpha\beta}^{-1}(s) = m_0^2 + \Delta m_{\alpha\beta}^2(s) - s \]  

(9)

where

\[ \Delta m_{\alpha\beta}^2(s) = \frac{g_F^2}{4\pi} \sum_{\gamma\delta} C_{-\alpha\gamma} \bar{C}_{-\beta\delta} F(s, m_{\gamma}^2, m_{\delta}^2, \Lambda) + \frac{g_D^2}{4\pi} \sum_{\gamma\delta} C_{+\alpha\gamma} \bar{C}_{+\beta\delta} F(s, m_{\gamma}^2, m_{\delta}^2, \Lambda) . \]  

(10)

The zeroes of \( \det[P_{\alpha\beta}^{-1}(s)] \) determine the meson masses, which must by self-consistency be the same as the masses \( m_\gamma, m_\delta \), which appear in the threshold positions. In Eq. (9) the constants \( C_{\pm\alpha\beta\gamma} \) contain the internal symmetry dependence, while the function \( F \) contains the kinematical dependence of the masses in the loop. We return to models for the latter later below. The constants \( C_{\pm\alpha\beta\gamma} \) satisfy the completeness relation

\[ \sum_{kl,mn} C_{\pm\alpha\beta\gamma} C_{\pm\alpha'\beta'\gamma'} + \sum_{ij} C_{\alpha\beta\gamma} C_{\alpha'\beta'\gamma'} = N_f \delta_{ij} \delta_{\alpha\alpha'} \delta_{\beta\beta'} + \delta_{ij} \delta_{\alpha'\alpha} \delta_{\beta\beta'} , \]  

(11)

as can easily be seen from their definition Eq.(7-8). In the case the solution is near the unmixed frame with \( \Omega = 1 \), one can write Eq.(10) to a good approximation as

\[ \Delta m_{ij,ij'}^2(s) = \delta_{ij'} \delta_{jj'} \frac{g_F^2 + g_D^2}{4\pi} \sum_k F(s, m_{ik}^2, m_{kj}^2, \Lambda) + \delta_{ij} \delta_{ij'} \frac{g_D^2 - g_F^2}{4\pi} F(s, m_{ii'}^2, m_{jj'}^2, \Lambda) . \]  

(12)

where we have replaced \( \alpha, \beta, \) etc. by the quark-line indices \( ij \) and \( i'j' \) etc. In terms of quark-line diagrams the first term represents a connected planar loop diagram, and the second a disconnected diagram. It is now easy to see that if the symmetry is unbroken and all bare masses = \( m_0 \), then all states with flavor get the same (normally negative) shift:

\[ \Delta m^2 = \frac{g_D^2 + g_F^2}{4\pi} N_f F(s, m_0^2 + \Delta m^2, m_0^2 + \Delta m^2, \Lambda) \]  

(13)

while for the flavorless states \( uu, dd, ss \) one has an extra nondiagonal piece (for \( N_f = 3 \)):

\[ \Delta m_{ii,ij}^2 = \left[ \frac{g_D^2 + g_F^2}{4\pi} \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} + \frac{g_D^2 - g_F^2}{4\pi} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \right] F(s, m_0^2 + \Delta m^2, m_0^2 + \Delta m^2, \Lambda) . \]  

(14)

The second term in (14) implies that the singlet is shifted differently from the octet if \( g_F \neq g_D \) (or if the \( C = + \) mesons are not degenerate with those with \( C = - \)). Thus when
summing over all intermediate states $\gamma, \delta$ one self-consistently gets the same mass shift function for all members of the octet when no symmetry breaking occurs. If $g_F = g_D$, the second nondiagonal term vanishes, and also the singlet is shifted equally, while if $g_F > g_D$ the singlet will be heavier than the octet. In general, there will always exist one solution, which is exactly flavor symmetric, and which for small couplings is also the stable solution. But, for sufficiently large couplings the symmetric solution becomes unstable and flavor symmetry is spontaneously broken! It is easiest to see how this symmetry breaking mechanism works in the special case when there is no initial singlet-octet splitting and $g_F = g_D = g$. Then only the first diagonal term proportional to $g_D^2 + g_F^2$ contributes in Eq.(12).

Let a small variation from the symmetric solution in the threshold masses be $\delta_{ij} = m_{ij}^2 - m_0^2$. This results in a shift $\delta_{ij}^\text{out}$ in the pole positions of Eq. (9). Self-consistency of course requires $\delta_{ij}^\text{out} = \delta_{ij}$ and stability $\delta_{ij}^\text{out} < \delta_{ij}$. The self-consistency condition requires that small deviations from the symmetric solution must satisfy the equal spacing rule: $\delta_{ij}^\text{out} - \delta_{ii}^\text{out} = (\delta_{jj}^\text{out} - \delta_{ii}^\text{out})/2$, while the instability condition $r = \delta_{ij}^\text{out}/\delta_{ij} > 1$ can be written

$$N_f \frac{g_F^2 + g_D^2}{4\pi} > \left[ \frac{\partial F}{\partial s} + \frac{\partial F}{\partial m_1^2} \right]^{-1}|_{s=m_1^2=m_2^2} .$$

Eq.(15) is my most important result. Here the left hand side is positive for any reasonable function $F$, which is essentially determined by the threshold behaviour. In particular, assuming $F$ to be given by its unitarity cut, $\text{Im}(F) \propto -k(s,m_1^2,m_2^2)/\sqrt{s} \ N(s)\theta(\Lambda - k)$, where $k$ is the 3-momentum in the loop, and $N(s) = 1$ for simplicity, the left hand side of Eq.(15) is given by the solid curve in Fig.2. If one has a P-wave behaviour at threshold, $\text{Im}(F) = k^3(s,m_1^2,m_2^2)/\sqrt{s} \ \theta(\Lambda - k)$ as for $\rho \to \pi\pi$, then the function $F$ is much more sensitive to the threshold position, and consequently the bound is stricter by more than an order of magnitude as seen by the dashed curve in Fig.2. The coupling constants in Eq.(15) are of course defined before symmetry breaking, and are not the physical renormalized phys-

---

2 The two partial derivatives are here finite for $\Lambda \to \infty$, $\partial F/\partial s|_{s=m_1=m_2} = 1/\pi - 2/\sqrt{27}$, and $\partial F/\partial m_1^2|_{s=m_1=m_2} = 1/\sqrt{27}$. For large $\Lambda$ the bound in Eq.(15) thus approaches 7.94.
ical couplings after the breakdown. Still, it is interesting to compare the bound with some typical physical coupling constants as done in Fig. 2. As can be seen \( g_{\rho\pi\pi} \) and \( g_{\sigma\pi\pi} \) are well above the bound indicating that the spontaneous symmetry breaking under discussion actually occurs in Nature. The value used in Fig. 2 for \( g_{\sigma\pi\pi} \) (assuming \( m_\sigma \approx \Gamma_\sigma \approx 0.5 \text{ GeV} \)) is more like a lower bound See ref. (3,4). Now of course one can argue that for such a large coupling the higher order loop diagrams invalidate the approximation \( N(s) = 1 \) (in the N/D terminology) made above. However, the fact that the bound is satisfied for all realistic cutoffs, \( \Lambda \) in an effective theory, suggests strongly that it holds also in the real world.

Once the inequality (15) is satisfied one has an unstable situation. Then, one must find out if there exists another solution which is stable. This cannot be done analytically, since the solution is highly nonperturbative and the function F is nonlinear even in the simplest possible model. Once one is off the symmetric situation, any mass splitting will feed into all other masses. In the first paper of Ref. (2) I did a numerical study of how the symmetry is broken, when \( N_f = 2, g_F = g_D = g \) is very large, and the the function F determined by its unitarity cut assumed to be \( \propto -k(s, m_1^2, m_2^2) \sqrt{s} \theta(\Lambda - k) \), where \( k \) is the three-momentum in the loop. It was shown that one finds for all \( \Lambda \) a stable flavor asymmetric solution which obeys approximately the equal spacing rule.

It is important to realize that once the equality is satisfied one has a discontinuous jump from the symmetric to the broken solution, - a small increase in \( g \) can give a large mass splitting within the multiplet. For \( N_f = 3 \) it turns out that generally not the whole SU3 group is broken, but that an SU2 subgroup remains unbroken, because of the nonlinearities entering once the breaking appears. Thus, once the mass splittings \( m_{s\bar{s}} - m_{s\bar{d}} \approx m_{s\bar{d}} - m_{u\bar{d}} \) are nonzero, then \( u\bar{d}, d\bar{u}, u\bar{u} \) and \( d\bar{d} \) remain degenerate. This is like in the mechanical analogue above with collapsing elastic spheres to ellipsoides: O(3) symmetry is broken, but normally there remains an exact O(2) symmetry around one of the principal axes.

The situation is more complicated when the singlet-octet splitting is broken already for the symmetric solution or \( g_F \neq g_D \). Then for any asymmetric solution also the mixing matrix \( \Omega \) deviates from ideal mixing, and furthermore, \( \Omega \) depends nonlinearly on all masses of the
actual solution. In which direction does the breaking then go? *A priori* the degenerate nonet mass could split into the 6 generally different masses of a nonet obeying $C$-symmetry, i.e. we have a 5-dimensional parameter space. This looks complicated, but in the common case that the effect from singlet-octet splitting and $(g_F^2 - g_D^2)/(g_F^2 + g_D^2)$ is small compared to the spontaneous breaking between $s\bar{s}$, $s\bar{u}$ and $d\bar{u}$ masses, one can see analytically the direction of the breaking. Considering the result above that an SU2 subgroup remains unbroken while strange states are split from nonstrange states one expects the solution to be near the ideally mixed frame, where isospin remains exact, while the $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ mix through the off-diagonal terms in Eq.(12). Instead of Eq.(14) one has a mass matrix of the form:

\[
\Delta m^{2}_{ii,jj} \propto \begin{pmatrix}
A & 0 & 0 \\
0 & A & 0 \\
0 & 0 & C
\end{pmatrix}
+ \frac{g_D^2 - g_F^2}{g_F^2 + g_D^2}
\begin{pmatrix}
a & a & b \\
0 & A & 0 \\
b & b & c
\end{pmatrix}
\rightarrow
\begin{pmatrix}
A & 0 & 0 \\
0 & A & 0 \\
0 & 0 & C
\end{pmatrix}
+ \frac{g_D^2 - g_F^2}{g_F^2 + g_D^2}
\begin{pmatrix}
0 & 0 & 0 \\
0 & 2a & \sqrt{2b} \\
0 & \sqrt{2b} & c
\end{pmatrix},
\]

(16)

where the second form is obtained after a rotation to pure isospin states. After further diagonalization one obtains mixing between $u\bar{u} + d\bar{d}$ and $s\bar{s}$.

In the second paper of Ref.(2) this mechanism was applied to an SU6 model involving pseudoscalar and vector nonets, where the nonet masses are degenerate before the spontaneous breaking. It was shown that the symmetry breaking was in the right direction and with a right order of magnitude in the mass splittings. Reasonable $\pi - K - \eta - \eta'$ and $\rho - \omega - K^* - \phi$ splittings was obtained when the scales were determined only by the $\pi$ and $\rho$ masses.

The main consequence of this spontaneous symmetry breaking is to generate splittings between $s\bar{s}$, $s\bar{d}$ and $u\bar{d}$ states. The same effect is obtained conventionally by inserting, by hand, into the theory a heavier effective $s$-quark mass than those of $u$, $d$.

Phenomenologically one can look at the instability I have discussed as a phenomenon where, for sufficiently large coupling, a flavor symmetric meson cloud around a meson is unstable. The meson clouds around hadrons rearrange themselves through "virtual decay"
such that stable, but flavor asymmetric clouds are formed.

In the suggested symmetry breaking mechanism flavor states are different self-consistent singlet solutions within the same internal (color) space. Flavor states do not carry color, nor do the Goldstone bosons carry flavor as was assumed in some early attempts \footnote{ } to break flavor symmetry spontaneously. The longitudinal and/or scalar, confined gluons can thus be identified with the Goldstone bosons. Color symmetry is then really the true unbroken symmetry behind flavor symmetry of strong interactions. When flavor symmetry is a good approximation it only means that the solutions happen to be near each others, because the bare states are (nearly) degenerate and the spontaneous symmetry breaking is absent or very weak. In contrast to conventional spontaneous symmetry breaking the new mechanism is clearly a quantum effect, where the symmetry breaking is in the solutions, not necessarily in the vacuum. I believe a better understanding of this spontaneous symmetry breaking could throw light on the confinement problem, which I have not addressed.

I have discussed only three flavors and three colors. Of course we know there are six flavors. In analogy with leptons three bare quarks should be massive because of weak interactions, but three may be (almost) massless. Extending my mechanism to 6 flavors all masses will be renormalized and shifted from naive expectations. Thus, with this new mechanism, the possibility remains open that with three (nearly) massless and three massive bare quarks one might understand the hadron mass spectrum.
REFERENCES

[1] J. Gasser and H. Leutwyler, Phys. Rep. 87 (1982) 77; H. Leutwyler, ”Light quark masses”, Bern preprint 1996, hep-th 9609463.

[2] N. A. Törnqvist, ”Breaking Flavour Symmetry Spontaneously” hep-ph/9610440 (revised), and Breaking SU3_f Down to Isospin” hep-ph/9612238 (revised), (To be submitted to Phys. Lett. B).

[3] N. A. Törnqvist and M. Roos, Phys. Rev. Lett. 76 (1996) 1575. N. A. Törnqvist, Z. Physik C68 647 (1995),

[4] T. Hatsuda and T. Kunihiro, Phys. Rep. 247, 223 (1994).

[5] H. Miyazawa, Proc. First Pacific Int. Summer School in Physics, ”Recent Developements in Particle Physics”, Ed. M. Moravcsik, Gordon & Breach 1966, p.22; S. Frautschi, ibid. p. 152.
FIG. 1. Classical spheres with fixed surface area collapsing to ellipsoids when the internal pressure is reduced (See text). The broken spherical shape and the different the moments of inertia of the collapsing ellipsoids correspond to the spontaneously broken global flavor symmetry, while the $x$-dependent $O(3)$-symmetry of rotating the ellipsoids correspond to the unbroken local symmetry.

FIG. 2. The instability limit on a dimensionless tri-meson coupling constant. The solid curve is for an S-wave transition of a scalar to two scalars, (like $\sigma \rightarrow \pi \pi$). The dashed curve is for a P-wave transition (like $\rho \rightarrow \pi \pi$). As shown the physical $g_{\rho \pi \pi}^2/4\pi = 2.5$ coupling constant, and a rough estimate for $g_{\sigma \pi \pi}^2/(4\pi m_\pi^2) \approx 15$ satisfy the instability condition for any reasonable value of the cut off (e.g. in $^3P_0$ decay models $\Lambda \approx 0.7$ GeV/c or $\Lambda/m_\pi \approx 5$).