A note on light velocity anisotropy

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ABSTRACT

It is proved that in experiments on or near the Earth, no anisotropy in the one-way velocity of light may be detected. The very accurate experiments which have been performed to detect such an effect are to be considered significant tests of both special relativity and the equivalence principle.

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1 Introduction

The anisotropy in the microwave background [1] has suggested the existence of a preferred frame $\Sigma$ which sees an isotropic background and of a corresponding anisotropy in the one-way velocity of light, when measured in our system $S$, which moves with respect to $\Sigma$ at the velocity of about 377 km/s. Possible consequences have been exploited from the theoretical point of view [2] [3]; many important and precise experiments have then been carried out with the purpose of detecting this anisotropy. No variation was observed at the level of $3 \times 10^{-8}$ [4], $2 \times 10^{-13}$ [5], $3 \times 10^{-9}$ [6], $2 \times 10^{-15}$ [7], $3.5 \times 10^{-7}$ [8], $5 \times 10^{-9}$ [9].

Our motion with respect to $\Sigma$ is a composition of the motions of the Earth in the solar system, of this system in our galaxy, of our galaxy inside a group of galaxies,... . The problem which arises is a very old one: may we perform, on or near the Earth, experiments to make evident our motion with respect to the preferred frame?

Historically, this question has been formulated in two steps, connected with the relativity principle and the equivalence principle, respectively.

The first step is due to Galilei, who excluded the possibility of performing, inside a ship cabin, experiments having the purpose of measuring the ship velocity with respect to the mainland. To compare the background radiation case with the Galilei proposal, if the Sun light entering inside the cabin through a port-hole is analysed, its black-body radiation spectrum would appear different from the one observed on the mainland.

The second step was introduced by Einstein through the equivalence principle [10]: At every space-time point in an arbitrary gravitational field it is possible to choose a "locally inertial coordinate system" such that, within a sufficiently small region of the point in question, the laws of nature take the same form as in unaccelerated Cartesian coordinate systems in absence of gravitation [11]. As a consequence, experiments inside a freely falling space cabin exhibit its relative motion only in the presence of inhomogeneities in the gravitational field.

In the following section we discuss the quoted experiments according to the two steps outlined above.

In section 2 we analyse the linear transformations due to Robertson and to Mansouri and Sexl, which generalize the Lorentz one and which have stimulated the experiments we are speaking of. We conclude that, due to the definitional role of light velocity, the linear transformations between inertial frames must have Lorentz form.
In section 3 we analyse the possibility of locally detecting anisotropies in the light velocity in the case of general relativity.

2 The preferred frame

The linear transformations between inertial frames have been analysed by very many authors (a list of them is given in ref. [12]) under hypotheses which include the requirement that they form a group, but do not include, a priori, the invariance of the light velocity. They conclude that these (Lorentz) transformations must be characterized by a velocity c, infinite in the Galilei limit, which, in principle, may take different absolute values in the different astronomical directions.

Robertson [2] and Mansouri and Sexl [3] have analysed the linear transformation between the preferred reference frame $\Sigma$ and another inertial frame $S$ which is moving with respect to it.

Robertson derives the following general linear transformation between the preferred system $\Sigma(x', y', z', t')$ and the frame $S(x, y, z, t)$, which moves along the $x$-direction which connects the respective origins $\Omega$ and $O$:

$$
x' = a_1 x + v a_0 t,
\quad y' = a_2 y,
\quad z' = a_2 z,
\quad t' = \frac{v a_1}{c^2} x + a_0 t,
$$

where $a_0$, $a_1$ and $a_2$ may depend on $v$. This transformation, which is expressed in terms of the parameter $v$ and which reduces to the identity when $v = 0$, is derived under the hypotheses that:

i) space is euclidean for both $\Sigma$ and $S$;

ii) in $\Sigma$ all clocks are synchronized and light moves with a speed $c$ which is independent of direction and position;

iii) the one-way speed of light in $S$ in planes perpendicular to the motion of $S$ is orientation-independent.

Notice that $O$ moves with respect to $\Sigma$ with velocity $v$, while $\Omega$ moves with respect to $S$ with velocity $\tilde{v} \equiv v a_0/a_1$; analogously, light, which is seen by $\Sigma$ to move according to the law $x' = c t'$, is seen by $S$ to move with velocity $\tilde{c} \equiv c a_0/a_1$, independently of the direction of $c$. If $c$ is the maximum speed in $\Sigma$, $\tilde{c}$ is the maximum speed in $S$; moreover $\tilde{v}/\tilde{c} = v/c$. In terms of these true velocities, equations (1) take the form

$$
t' = a_0 \left( t + \frac{\tilde{v}}{\tilde{c}^2} x \right),
\quad x' = a_1 (x + \tilde{v} t),
\quad y' = a_2 y,
\quad z' = a_2 z.
$$

(2)
The transformation is then the product of a Lorentz transformation and a scale transformation; the latter may be re-absorbed by a redefinition of the units.

If the length standard is established, in any frame, by giving an assigned value to the speed of light, then the light velocities in $\Sigma$ and $S$ are equal, $x$ is scaled by $a_0/a_1$ and the transformation between the $(x, t)$ variables takes a familiar form.

The fact that this transformation implies different light speeds in different directions in the $(x, y)$ plane is, a priori, admissible.

This case is typical of a tetragonal crystal; the light speeds may be different in the $x$ and in the $y$ directions, when measured with external standards; a suitable internal scaling of the $y$ variable would of course give the same value to the internal velocities, but the time required for the light to travel the crystal in the $y$-direction would be different from the one seen by an external observer. In this case the attribution of different light speeds for different directions is physically justified. But if we have no such a justification, the thing to do is to apply the Poincaré simplicity criterion and to attribute the same value to the light speed in different directions.

Mansouri and Sexl [3] analyse the linear transformation from a preferred frame $\Sigma(X, T)$ to another frame $S(x, t)$, which moves with respect to it at the velocity $v$, under the hypothesis that the synchronization is realized by clock transport. Their analysis is devoted both to one-dimensional transformations and to three-dimensional ones; we do not discuss here the last case, but the conclusions will apply as well.

To first order in $v$, the Mansouri and Sexl one-dimensional transformation takes the form:

$$
\begin{pmatrix}
x \\
t
\end{pmatrix} = \frac{1}{\sqrt{1 - (1 - \mu)^2c^2}} \begin{pmatrix}
1 & -v \\
(1-\mu)v/c^2 & 1
\end{pmatrix}
\begin{pmatrix}
X \\
T
\end{pmatrix},
$$

(3)

where $c$ is the isotropic light speed seen by $\Sigma$ and their quantity $2\alpha$ is substituted here by $-(1 - \mu)/c^2$ to make explicit the Lorentz and Galilei limits ($\mu = 0$ and $\mu = 1$, respectively). If $\mu \neq 0$, then a particle, which moves with velocity $u$ in $\Sigma$, appears to $S$ to move according the law:

$$
x = \frac{u - v}{1 - (1 - \mu)\frac{uv}{c^2}}t.
$$

(4)

The maximum speed of a frame with respect to $\Sigma$ is $c/\sqrt{1 - \mu}$, independently of the orientation; if something is seen by $\Sigma$ to move at this speed, it is
seen to move at the same invariant speed by all frames, independently of the orientation. On the other hand, light, which moves with respect to Σ according to \( X = \pm cT \), moves, in our frame \( S \), according to \( x = t(c\mp v)/(1\mp (1-\mu)v/c^2) \), the sign depending on the motion orientation. Light speed is no more the maximum one, and, what is more relevant, the one-way light velocities in \( S \) are different.

The undetectability of a possible dependence of the one-way velocity along a line on its orientation has been extensively discussed in literature \[13\] \[14\]. As a consequence, \( \mu = 0 \).

The last indisputable conclusion finds a confirmation in the following experimental fact: electrons in the large accelerator machines have now energies of \( \sim 100 \) Gev; at this energy, \( v^2/c^2 \sim 1 - 2 \cdot 10^{-11} \), but the electrons have not reached the light speed; we must have then \( \mu < 2 \cdot 10^{-11} \).

The situation does not change if we go beyond the lowest order and suppose that \( \mu \) is a function (even) of \( v \).

3 Local inertial systems

The above considerations refer to situations in which we are performing our experiments in frames which are seen by \( \Sigma \) to move inertially. Our conclusion is that, if the transformation between the preferred frame and our one is taken to be linear, then it must have Lorentz form.

On the other hand, the anisotropy of the primordial radiation strongly supports the existence of the preferred frame with respect to which we are moving. It must then be analysed how our state of non-inertial motion affects the experiments we are discussing.

The starting point is the fact that the background radiation intensity appears to be anisotropic to an observer \( O \), at the origin of a reference frame \( S \) in our region \( R \) of the universe, while it is isotropic from the point of view of an observer \( \Omega \), at the origin of a preferred reference frame \( \Sigma \).

In the last case, the absolute system \( \Sigma \) and the relative frame \( S \) of our region of the universe detect differences in the radiation background, but no differences in any local experiment. The region \( R \) behaves like the world inside an Einstein elevator; the Einstein equivalence principle states that, if \( \Omega \) and \( O \) perform, in their respective regions, identical experiments which are not influenced by the presence of local masses (Earth, Sun, ...), they obtain identical results. An immediate consequence is that the inertia principle is valid for all local inertial systems.
This concept is very clearly stated by Hans Reichenbach [13]:

According to Einstein, however, only these local systems are the actual inertial systems. In them the field, which generally consists of a gravitational and an inertial component, is transformed in such manner that the gravitational component disappears and only the inertial component remains.

Analogously, the local inertial systems associated to an Einstein elevator are connected by linear transformations characterized by an invariant velocity \( c \). So, our region \( R \) and another region \( R' \) in the universe have separate families of local inertial frames, characterized by identical light speeds, although these ones may appear different when measured by an asymptotic observer who sees how space curvature modifies in going from \( R \) to \( R' \). A well known example is given by the time delay measured in the Shapiro [15] and Reasenberg [16] experiments: an asymptotic observer detects a delay in the light trip, but any observer, who is in the region this ray is passing through, says that it is moving at the speed of \( c \).

Coming back to the experiments performed in presence of Earth and Sun, we do not exclude that local observers may see general relativistic effects induced by their masses [17]. The light behaviour is, however, locally influenced by the gravity only for a bending which is very small and difficult to detect [18]; this is not true for the motions of the satellites involved in some of the quoted experiments. All gravity effects due to the nearest relevant masses have been consistently taken into account in the previous experiments, which must be highly considered for their precision.

The conclusions of these experiments is that, apart from some very small local effects, the Lorentz transformation applies in our region \( R \), and that that our region belongs to a family of local inertial frames.

The force which induces the acceleration seen by some asymptotic observer is completely cancelled by the equivalence principle.

4 Conclusions

If there is no way to perform independent measures of lengths and light velocities; in other words, if the light velocity is used both for synchronizing clocks and for fixing the unit lengths, there is no way of locally detecting any dependence on the orientation of the length of a rod. The only thing to do is to use the Poincaré simplicity criterion and consider equal the lengths of the rods and the one-way speeds of light in the different directions.

In conclusion, isotropy in the one-way velocity of light is a matter of
definition.

However, the experiments quoted at the beginning, in particular those performed by J. Hall and coworkers at very sophisticated levels, cannot be considered simply significant improvements of classical special relativity tests.

As discussed in the introduction, this would surely be the case if the quoted experiments had been performed in a region where gravity effects are compensated. But the presence of an anisotropic background radiation, when interpreted as testimonial of an analogous anisotropic mass distribution, and the fact that these experiments find their explanation in the frame of the special relativity, strongly support the equivalence principle.

In conclusion we strongly suggest that the accurate conclusions of such experiments should be considered significant tests of both special relativity and the equivalence principle.

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References

[1] G.F. Smoot, M.V. Gorenstein and R.A. Muller, Phys. Rev. Lett. 39, 898 (1977)

[2] H.P. Robertson, Rev. Mod. Phys. 21, 378 (1949); H.P. Robertson and T.W. Noonan, Relativity and Cosmology, Saunders, Philadelphia, (1968) p. 69

[3] R.M. Mansouri and R.U. Sexl, J. Gen. Rel. Grav. 8, 497 (1977); see, in particular, pages 506-511; 8,515 (1977); 8,809 (1977)

[4] K.C. Turner and H.A. Hill, Phys. Rev. 134, B252 (1964)

[5] A. Brillet and J.L. Hall, Phys. Rev. Lett. 42, 549 (1979)

[6] E. Riis, L. Aaen Andersen, N. Bjerre, O. Poulsa, S.A. Lee, J. L. Hall, Phys. Rev. Lett. 60, 81, (1988)
[7] Dieter Hils and J.L. Hall, Phys. Rev. Lett. 64, 1697 (1990)

[8] T.P. Krisher et al., Phys. Rev. D, R731 (1990)

[9] Peter Wolf and Gérard Petit, Phys. Rev. A 56, 4405, (1997)

[10] C.M. Will, Theory and experiment in gravitational physics, Cambridge University Press, Cambridge (1993).

[11] Steven Weinberg, Gravitation and Cosmology, John Wiley, New York, N. Y. 1972

[12] B. Preziosi, N. Cim. 109 B, 1331, (1994)

[13] Hans Reichenbach, Philosophie der Raum-Zeit-Lehre Walter de Gruyter, Berlin and Leipzig, 1928, translated by M. Reichenbach and J. Freund as The Philosophy of Space and Time, Dover New York, N. Y. 1958

[14] Gruenbaum, A., Philosophical Problems of Space and Time, Reidel, Dordrecht Boston (1973)

[15] I.I. Shapiro et al., Phys. Rev. Lett. 26, 1132 (1971)

[16] R.D. Reasenberg et al. Astr. J. 234, 1219 (1979)

[17] S. Chandrasekhar, The Mathematical Theory of Black Holes, pages 328, 123 ff. and 98 ff. Clarendon Press, Oxford 1983

[18] B. Preziosi and G.M. Tino, J. Gen. Rel. Grav 30, 173 (1998)