The astrophysical reaction $^8\text{Li}(n,\gamma)^9\text{Li}$ from measurements by reverse kinematics

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We study the breakup of $^9\text{Li}$ projectiles in high energy (28.5 MeV/u) collisions with heavy nuclear targets ($^{208}\text{Pb}$). The wave functions are calculated using a single-particle model for $^9\text{Li}$, and a simple optical potential model for the scattering part. A good agreement with measured data is obtained with insignificant E2 contribution.

The use of the Coulomb dissociation method [1,2] has proven to be a useful tool for extracting radiative capture reaction cross section of relevance for nuclear astrophysics. In particular it appears that the Coulomb dissociation of $^9\text{Li}$ is very useful [3] for elucidating the role of the inhomogeneous nucleosynthesis in the Big Bang model - the formation of $^9\text{Li}$ via the $^8\text{Li}(n,\gamma)^9\text{Li}$ reaction. However, a few lingering questions still need to be addressed, including the importance of E2 excitations for the kinematics of the MSU experiment [3], performed at approximately 28.5 MeV/u. We attempt to resolve this issue by using a relatively simple but still realistic nuclear model that however yields a very good agreement with data and suggest that E2 excitations are negligible for the kinematical conditions of the MSU experiment.

For $^9\text{Li}$ assume that the $J_0 = 3/2^-$ ground state can be described as a $j_0 = p_{3/2}$ proton coupled to the $I_c = 2^+$ ground state of the $^8\text{Li}$ core. The spectroscopic factor for this configuration was taken as unity. The single particle states, $R_{E_xlj}(r)$, for the excitation energy $E_x = E_n + |E_0|$ ( = 4.05 MeV), where $E_n$ is the neutron-$^8\text{Li}$ relative energy, are found by solving the Schrödinger equation with a nuclear potential with spin-orbit of the form

$$V(r) = V_0 \left[1 - F_{s.o.} (1,s) \frac{r_0 d}{r dr}\right] f(r), \quad f(r) = \left[1 + \exp\left(\frac{r - R}{a}\right)\right]^{-1},$$

(0.1)

with the parameters $a = 0.52$ fm, $r_0 = 1.25$ fm, $R = 2.5$ fm, $F_{s.o.} = 0.351$ fm, and $V_0 = -45.3$ MeV. This reproduces the ground state bound energy $E_0 = -4.05$ MeV. The same set of parameters was used to calculate the continuum wavefunctions with energy $E_x$. The total wavefunction is constructed by coupling the single-particle states with the spin of $^8\text{Li}$. To compute the S-factors for the capture process $b + c \rightarrow a$ we have used the first-order perturbation theory and the reduced matrix elements for electric multipole transitions given by

$$\langle j_1 | \mathcal{M}(E\lambda) | j_0 \rangle = e_\lambda \langle -1 \rangle^{l_0+l_1+j_0-j} \left(\frac{2\lambda + 1}{4\pi (2j + 1)}\right)^{1/2} \left\langle j_0 \left| \frac{1}{2} \lambda^0 | \frac{1}{2} \right\rangle \right|$$

$$\times \int \left[1 + \left(-1\right)^{l_0+l_1+\lambda} \right] R_{E_xlj}(r) R_{E_nlj_0}(r) r^{\lambda+2} dr,$$

(0.2)

where $e_\lambda = Z_e(-A_x/A_n)^\lambda + Z_x e(A_b/A_n)^\lambda$ is the effective electric charge. We have considered $s, p, d$ and $f$ continuum states.

The response functions for the excitation of $^9\text{Li}$ are defined by ($b \equiv ^8\text{Li}$, $c \equiv n$)

$$\frac{dB(E\lambda; l_0j_0 \rightarrow E_xlj)}{dE_x} = \frac{m_{bc}}{\hbar^2 k} \left| \langle j_1 | \mathcal{M}(E\lambda) | j_0 \rangle \right|^2 / (2j_0 + 1)$$

(0.3)

They are presented in figure 1 for $E1$ and $E2$ excitations, as functions of the neutron energy, $E_n = \hbar^2 / 2m_{bc}$. They are dominated by s-wave components: the higher angular momentum waves become relevant for higher energies, as expected.

The cross sections for direct capture are given by

$$\sigma_{\text{dir}}(E_x) = 4\pi \int \frac{dB(E\lambda; l_0j_0 \rightarrow E_xlj)}{dE_x} dE_x$$

(0.4)

where $B(E\lambda; l_0j_0 \rightarrow E_xlj)$ is the $l_0$-multipole transition amplitude for the excitation $E_x$. The cross sections are presented in figure 2 for $E1$ and $E2$ excitations, as functions of the neutron energy, $E_n = \hbar^2 / 2m_{bc}$. They are dominated by s-wave components: the higher angular momentum waves become relevant for higher energies, as expected.
The figure 2 displays the direct capture cross section for the E1 transitions. The experimental data are from ref. [8]. Due to the factor \((E_x/hc)^{2\lambda+1}\) appearing in eq. (4) (and since \(B(E1)/\epsilon^2\) and \(B(E2)/\epsilon^4\) are of same order of magnitude, as seen from fig. 1), the E2 contribution to the radiative capture cross section is about 7 orders of magnitude smaller than the E1 contribution. For pedagogical purposes, we also show in figure 2 the hard sphere model of Lane and Lynn [4] for the direct capture cross section. The hard sphere model is the simplest model to calculate the reaction cross section for \((n, \gamma)\)-processes at low energies. This model has been used by other authors with reasonable success in some situations [4-6]. According to this model, the direct capture cross section for the \(n + A \rightarrow (A + 1) + \gamma\) reaction is given by

\[
\sigma^{(d.c.)}_\gamma = \sigma_{h.s.} \left[ 1 + \frac{R - a_s y (y + 2)}{R} \right]^2 \tag{0.5}
\]

where \(R\) is the target radius, which we take as \(R = 1.35 \ A^{1/3} \ \text{fm}\), \(y^2 = 2\mu_{nA} E_x R^2 / \hbar\), \(\mu_{nA}\) being the neutron+\(^8\)Li reduced mass, and \(A = 8\). The scattering length for the \(n + \ ^8\)Li system at low energies is \(a_s = 2.03 \ \text{fm}\) [7].

The hard-sphere, \(\sigma_{h.s.}\), cross section entering eq. (5) is given by

\[
\sigma_{h.s.} = \frac{0.062}{R\sqrt{E_n}} \left( \frac{Z}{A} \right)^2 \frac{2J_0 + 1}{6(2I_c + 1)} \cdot S \left[ \frac{y(y + 3)^2}{y + 1} \right]^2 \text{mb} \tag{0.6}
\]

with the neutron energy, \(E_n\), given in keV. In this equation, \(Z\) is the \(^8\)Li charge, \(I_c\) its spin, \(J_0\) the spin of \(^9\)Li, and \(S\) is the spectroscopic factor, which we take as unity. The hard sphere model yields a cross within only a factor of 2 difference from our results for E1 transitions, but with approximately the same energy dependence.

Since there is no data for the elastic scattering of \(^9\)Li on \(Pb\) targets at this bombarding energy, we construct an optical potential using an effective interaction of the M3Y type [8] modified so as to reproduce the energy dependence of total reaction cross sections, i.e. [9],

\[
t(E, s) = -\frac{i}{2t_0} \sigma_{NN}(E) \left[ 1 - i\alpha(E) \right] t(s) \tag{0.7}
\]

where \(t_0 = 421 \ \text{MeV}\) is the volume integral of the M3Y interaction \(t(s)\), \(v\) is the projectile velocity, \(\sigma_{NN}\) is the nucleon-nucleon cross section, and \(\alpha\) is the real-to-imaginary ratio of the forward nucleon-nucleon scattering amplitude. At 28.5 MeV/nuc, we use \(\sigma_{NN} = 20 \ \text{fm}^2\) and \(\alpha = 0.87\).

The optical potential is given by

\[
U(E, R) = \int d^3r_1 \ d^3r_2 \ \rho_p(r_1) \rho_p(r_2) \ t(E, s) \tag{0.8}
\]

where \(s = R + r_2 - r_1\), and \(\rho_p\) \((\rho_p)\) is the ground state density of the target (projectile).

Following ref. [10], the Coulomb amplitude is given by

\[
f_c = \sum \lambda \mu f^{(JM)}_{\lambda\mu} \tag{0.9}
\]

where

\[
f^{(JM)}_{\lambda\mu} = i^{1+\mu} Z_{Tc} e^{\mu \phi_T} \left( \frac{E_x}{\hbar c} \right)^\lambda \sqrt{2\lambda + 1} \ \exp \{-i\mu \phi\} \ \Omega_{\mu}(q) \times G_{E\lambda\mu}(\vec{r}) \langle JM | M_{E\lambda,-\mu} | J_0 M_0 \rangle \tag{0.10}
\]
\[ \Omega_\mu(q) = \int_0^{\infty} db \, b \, J_\mu(qb)K_\mu\left(\frac{E_x b}{\gamma \hbar}\right) \exp i\chi(b) . \]  

(0.11)

\( J_\mu(K_\mu) \) is the cylindrical (modified) Bessel function of order \( \mu \), and the functions \( G_\varepsilon \lambda \mu(c/v) \) are tabulated in ref. [1]. The angular momentum algebra connects \( \langle JM | M_{E\lambda,-\mu} | J_0 M_0 \rangle \) with the reduced matrix elements of eq. (2).

The eikonal phase, \( \chi(b) \), is given by

\[ \chi(b) = 2\eta \ln(kb) - \frac{1}{\hbar v} \int_{-\infty}^{\infty} dz \, U_{opt}(R) , \]  

(0.12)

where \( \eta = Z_P Z_T e^2 / \hbar v \), \( k \) is the projectile momentum, and \( R = \sqrt{b^2 + z^2} \). The optical potential, \( U_{opt} \), in the above equation is given by eq. (8).

The cross section for Coulomb excitation to the state with angular momentum \( J \) and excitation energy \( E_x \) is obtained by an average (and a sum) over the initial (final) angular momentum projections:

\[ \frac{d\sigma_\lambda}{d\Omega dE_x} = \frac{1}{2J_0 + 1} \sum_{M_0, M} \left| f^{(JM)}_{J\mu} \right|^2 . \]  

(0.13)

As explained in details in refs. [2][4][10], the above cross section can be factorized in terms of a product of virtual photon numbers and breakup cross sections by real photons, which by detailed balance are directly related to the radiative capture cross sections. Thus a measurement of \( d\sigma_\lambda / d\Omega dE_x \) can be used to obtain radiative capture cross sections of astrophysical interest. Experimentally, the nuclear contribution to the breakup cross section can be separated by repeating the measurement on light targets (see, e.g., ref. [3]).

At the bombarding energies of tens of MeV/nucleon, the \( E_2 \) virtual photon number is much larger than that of \( E_1 \). As a consequence, even when the \( E_2 \) contribution to the radiative capture cross section is small, they are amplified in Coulomb breakup experiments [2]. In order to infer the relevance of \( E_2 \) for the breakup of \( ^9\text{Li} \) (28.5 MeV) on lead targets, we plot in figure 3 the angle integrated cross section \( d\sigma_\lambda / dE_x \), as a function of the neutron energy relative to \( ^9\text{Li} \). We see that the \( E_2 \) contribution to the breakup cross section is 3 orders of magnitude smaller the \( E_1 \) contribution. We thus conclude that \( E_2 \) transitions are not relevant in the experiment of ref. [3]. Although we obtained this result by means of a single particle model for the \( ^8\text{Li}(n, \gamma)^9\text{Li} \) reaction, we do not expect that it would change appreciably with more sophisticated models.

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Figure Caption

Fig. 1 - Response function in units of $e^2 fm^2/MeV (e^2 fm^4/MeV)$ for $E1 (E2)$ transitions in the reaction $^9Li(\gamma,n)^8Li$, as a function of the neutron energy relative to the $^8Li$ core. The lower curves are the f (solid) and p+f (dashed) -waves contribution, while the upper curves display the contribution of s (solid) and s+d (dashed) -waves.

Fig. 2 - Radiative capture cross sections, in $\mu b$, for the reaction $^8Li(n,\gamma)^9Li$ in the direct capture model. The solid line is obtained with the s-waves, while the dashed line includes transitions to d-waves. The hard-sphere model of ref. [4] is also shown (dashed-dotted line). The experimental data are from ref. [3].

Fig. 3 - Coulomb breakup cross section $d\sigma/dE_n$ (in mb/MeV) for the reaction $^9Li (28.5 MeV/nucleon) + Pb \rightarrow ^8Li + n$, as a function of the neutron-$^9Li$ relative energy, in MeV. The E2 breakup contribution is multiplied by $10^3$ to be shown in the same graph.
\[ \frac{d\sigma_C}{dE} \left[ \text{mb MeV}^{-1} \right] \]