Ginzburg-Landau theory for the operation principle of superconducting delay-line inductance detectors

T Koyama\(^1\) and T Ishida\(^{1,2}\\)

\(^1\)Division of Quantum and Radiation Engineering, Osaka Prefecture University, Sakai, Osaka 599-8570, Japan
\(^2\)NanoSquare Research Institute, Osaka Prefecture University, Sakai, Osaka 599-8531, Japan
E-mail: tkoyama67@nifty.com

Abstract. Transmission of voltage pulses generated by local heating in a kinetic inductance detector made of a superconducting stripline is theoretically investigated on the basis of the TDGL equation. The effect of thermal diffusion is also incorporated into the electrodynamics of this system. We derive an equation that can describe the generation and transmission of voltage pulses in this detector. A pair of voltage pulses with opposite polarities are created when a spatiotemporal variation in the superconducting order parameter occurs in a small region of this superconducting stripline. We clarify the characteristic feature in the shape of the voltage pulse.

1. Introduction
Radiation detectors using superconducting circuits such as transition edge sensor (TES)[1, 2], superconducting nanowire single-photon detector (SNSPD)[3, 4] and microwave kinetic inductance detector (MKID)[5] have been widely utilized in various fields using radiations. Recently, a new type of neutron detector called Current-Biased Kinetic Inductance Detector (CB-KID), in which high spatial- and energy-resolved imaging is possible, has been intensively developed by Ishida’s group[6, 7, 8, 9, 10, 11, 12]. This neutron detector consists of two orthogonal meandering superconducting nanowires deposited on a basal plane composed of insulating and superconducting layer stacks. The S-I-S structure in this detector functions as an efficient microwave delay-line. The superconducting nanowires in this detector are further covered with a $^{10}$B-layer, which absorbs neutrons well. A neutron captured by a boron nucleus in the $^{10}$B-layer causes a nuclear reaction ($n+^{10}$B→$^{7}$Li+$^{4}$He) and then generates heat. As a result, the superconducting nanowires contacted with the $^{10}$B-layer are locally heated, that is, a hot spot is created in the nanowires. As discussed by Ishida’s group, a local heating of the current-biased superconducting nanowire causes variation of the kinetic inductance and creates voltage pulses traveling along the nanowire. Hence, one can read out the position and time of the neutron capture events from the arrival times of the pulses at the two nanowire’s ends.

In our previous paper[13] we formulated an electrodynamic theory for the operation principle of CB-KID. In this theory the Maxwell-London equation with a non-uniform London penetration depth is utilized to derive an equation that can describe the electromagnetic propagation modes in the detector having an S-I-S waveguide structure. It was shown that a transient hot spot appearing in the nanowire excites a pair of voltage pulses with opposite polarities and these pulses propagate with a Swihart velocity[14] toward the two nanowire’s ends.
In this paper we develop the previous theory for the operation principle of CB-KID on the basis of the time-dependent Ginzburg-Landau (TDGL) equation instead of the London equation. To incorporate the effect of heat flow from a hot spot the thermal diffusion equation in the superconducting state is also utilized. We derive an integral equation, which is similar to the one given in the previous paper, for the voltage propagation modes in this detector. The shape of the voltage pulses generated by a hot spot can be obtained by solving this equation. It is shown that the voltage pulse excited by a transient hot spot has a time width nearly equal to the lifetime of the hot spot.

2. Dynamics of the order parameter in a superconducting stripline

2.1. TDGL equation

Consider a long superconducting stripline with rectangular cross section of width $L$ and thickness $s$ ($L \gg s$), which is deposited on a basal plane made of a stack of insulating and superconducting layers and forms a structure which can be regarded as an S-I-S waveguide (Fig.1). In the following we use the coordinates, $x$, $y$ and $z$, as follows, $x \parallel$ the stripline, and $y, z \parallel$ the two sides of the rectangular cross section, and $-\infty < x < +\infty$, $-L/2 < y < L/2$ and $d < z < d + s$ with $d$ being the thickness of the insulating basal plane. Let us assume that the superconducting order parameter $\Psi(r, t)$ of the stripline obeys the TDGL equation, which is given in the absence of an external magnetic field as

$$\gamma \partial_t \Psi(r, t) = -\left[ -\frac{\hbar^2}{2m^*} \nabla^2 \Psi(r, t) + \alpha(T)\Psi(r, t) + \beta|\Psi(r, t)|^2\Psi(r, t) \right].$$

Here, the parameter $\gamma$ is the quasi-particle damping constant. Suppose that a high-energy particle, e.g. $^4$He ($\alpha$-particle), penetrates into the stripline and stops at a site inside it. In this event the temperature of the region around the site rises, i.e., $T \to T(r, t) = T + T_1(r, t)$, that is, one can expect that a hot spot is created inside the stripline by absorbing a high-energy particle. In this paper we study the case that an increase in temperature is not large in this incident, i.e., $T_1(r, t) \ll T_c$. To incorporate the effect of a hot spot into the dynamics of the order parameter we use the TDGL equation in which the parameter $\alpha(T)$ is extended as

$$\alpha(T) \to \alpha(T + T_1(r, t)) \simeq \alpha(T) + \alpha_1 T_1(r, t) + O(T_1(r, t)^2).$$

Furthermore, we express the order parameter as

$$\Psi(r, t)/\Psi_0 = 1 + \psi(r, t),$$
where $\Psi_0 = \sqrt{|\alpha(T)|/\beta}$ is the order parameter in the state without a hot spot and $\psi(r,t)$ represents the deviation from it. Substituting Eqs. (2) and (3) into Eq. (1), one finds the equation for $\psi(r,t)$ in the linear approximation as

$$\left[ D^{-1}\partial_t - \nabla^2 + \xi^{-2} \right] \psi(r,t) = -\eta T_1(r,t), \quad (4)$$

where the parameters, $D$, $\xi$ and $\eta$ are defined as

$$D^{-1} = 2m^*\gamma/h^2, \quad \xi = h/\sqrt{4m^*|\alpha(T)|}, \quad \eta = 2m^*\alpha_1/h^2. \quad (5)$$

Consider a case where the size of a hot spot is of the order of 1$\mu$m, which is much larger than the coherence length, $\xi \sim 0.1\mu$m. In this case, since one can expect $|\nabla \psi| \ll \xi^{-1}$, Eq.(4) is approximated as

$$\left[ D^{-1}\partial_t + \xi^{-2} \right] \psi(r,t) \simeq -\eta T_1(r,t). \quad (6)$$

This equation is easily solved as

$$\psi(r,t) = -D\eta \int_0^t d\tau e^{-D(t-\tau)/\xi^2} T_1(r,\tau), \quad (7)$$

when a hot spot appears at $t = 0$. We assume that the relaxation time of $T_1(r,t)$ is much longer than the quasi-electron relaxation time $\xi^2/D \equiv \tau_{qp}$. In this case Eq.(7) can be expanded as

$$\psi(r,t) = -D\eta \left\{ -\frac{\xi^2}{D} (e^{-t/\tau_{qp}} - 1) T_1(r,t) \right.$$ \n
$$\left. -\left(\frac{\xi^2}{D}\right)^2 [1 - (1 + \frac{Dt}{\xi^2})e^{-t/\tau_{qp}}] \partial_t T_1(r,t) + \cdots \right\}, \quad (8)$$

Then, in the lowest order approximation one finds the solution,

$$\psi(r,t) \simeq -\eta\xi^2[1 - e^{-t/\tau_{qp}}] T_1(r,t), \quad (9)$$

which indicates that the variation of the order parameter due to a hot spot is proportional to that of the local temperature. Using Eq.(9), one can derive the equation for the superconducting current flowing along the stripline in the presence of a hot spot as follows,

$$j_x(r,t) = -\frac{e^2\Psi_0^2}{m^*c} (1 + \psi(r,t))^2 [A_x(r,t) - \frac{hc}{e^*} \nabla x \varphi(r,t)]$$

$$\simeq -\frac{c}{4\pi\lambda_L^2} [1 - 2\eta\xi^2(1 - e^{-t/\tau_{qp}}) T_1(r,t)] A_x(r,t), \quad (10)$$

up to the first order in $T_1(r,t)$, where $A_x = A_x - (hc/e^*) \nabla x \varphi$ with $\varphi$ being the phase of the order parameter and $\lambda_L = \sqrt{m^*e^2/4\pi e^2\Psi_0^2}$ is understood to be the London penetration depth. Note that Eq.(10) corresponds to the generalized London equation given in our previous paper[13].

### 2.2. Thermal diffusion

Let us now construct a theory for the dynamics of the superconducting order parameter, taking account of the effect of thermal diffusion in the presence of a hot spot. For this we investigate the spatiotemporal variation of the temperature in the stripline having a hot spot, making use of the phenomenological argument given in [15, 16], which is based on the TDGL theory.
Suppose that a hot spot appears at a certain time in a finite region $V$ inside the stripline and it decays through the thermal diffusion along the stripline and also into the basal plane. Consider the free energy of the superconducting electrons in this region. In the presence of a hot spot it depends on time and can be expressed as $F = \int_V d\mathbf{r} (f_n(\mathbf{r},t) + f_s(\mathbf{r},t))$, where $f_n(\mathbf{r},t)$ and $f_s(\mathbf{r},t)$ represent the free energy densities, respectively, in the normal and superconducting states. Note that $\int_V d\mathbf{r} f_s(\mathbf{r},t) < 0$ in the superconducting state, while $\int_V d\mathbf{r} f_s(\mathbf{r},t) = 0$ in the normal state. The heat diffusion equation is derived from the relation, $dF/dt = \int_V d\mathbf{r} q(\mathbf{r},t)$ in the presence of a heat source, where $q(\mathbf{r},t)$ in the present case is understood to be the heat quantity supplied by a hot spot. Let us now calculate the time derivative $dF/dt$. For the normal component one can utilize the conventional relation in the phenomenological heat transport theory,

$$\frac{d}{dt} \int_V d\mathbf{r} f_n(\mathbf{r},t) = \int_V d\mathbf{r} \left\{ C_V \partial_t T(\mathbf{r},t) + \nabla \cdot j_n^Q(\mathbf{r},t) \right\},$$

(11)

where $C_V$ and $j_n^Q(\mathbf{r},t) = -\kappa_n \nabla T(\mathbf{r},t)$ are, respectively, the specific heat and the heat current in the normal state with $\kappa_n$ being the normal state thermal conductivity. For the superconducting component $f_s(\mathbf{r},t)$ we use the GL free energy and assume that the time dependence of the order parameter obeys the TDGL equation given in Eq.(1). Then, after some calculations one finds the relation,

$$\frac{d}{dt} \int_V f_s(\mathbf{r},t) = \int_V d\mathbf{r} \left\{ \partial_t \alpha(T(\mathbf{r},t)) |\Psi(\mathbf{r},t)|^2 - 2\gamma |\partial_t \Psi(\mathbf{r},t)|^2 + \nabla \cdot j_s^Q(\mathbf{r},t) \right\},$$

(12)

where $j_s^Q(\mathbf{r},t)$ is the heat current coming from the relaxation of the order parameter,

$$j_s^Q(\mathbf{r},t) = \frac{\hbar^2}{2m^*} (\nabla \Psi^*(\mathbf{r},t) \cdot \partial_t \Psi(\mathbf{r},t) + \partial_t \Psi^*(\mathbf{r},t) \cdot \nabla \Psi(\mathbf{r},t)).$$

(13)

From Eqs.(11) and (12) it follows the thermal diffusion equation in the superconducting state,

$$C_V \partial_t T(\mathbf{r},t) + \partial_t \alpha(T(\mathbf{r},t)) |\Psi(\mathbf{r},t)|^2 - 2\gamma |\partial_t \Psi(\mathbf{r},t)|^2 + \nabla \cdot [j_n^Q(\mathbf{r},t) + j_s^Q(\mathbf{r},t)] = q(\mathbf{r},t).$$

(14)

Furthermore, since $T_1(\mathbf{r},t) \ll T_c$, one can linearize Eq.(14) with respect to $T_1(\mathbf{r},t)$, noticing the fact that $|\partial_t \Psi|^2 \sim \mathcal{O}(T_1^2)$ and $j_s^Q \sim \mathcal{O}(T_1^4)$, as follows,

$$C_S \partial_t T_1(\mathbf{r},t) - \kappa_n \nabla^2 T_1(\mathbf{r},t) = q(\mathbf{r},t),$$

(15)

with $C_S = C_V + |\Psi_0|^2 \alpha_1$. Let us solve Eq.(15) in a stripline system such as CB-KID. Note that the stripline in CB-KID is very thin, $s \sim 40\text{nm}$, so that we neglect the temperature variation along the $z$-direction inside the stripline, i.e., $T_1(\mathbf{r},t) \simeq T_1(x,y,t)$. Furthermore, we introduce the approximation that the heat flow to the basal plane per unit time is proportional to the temperature difference between the strip line and the basal plane, i.e., $\partial_z j_s^Q \simeq \zeta(T(x,y,t) - T) = \zeta T_1(x,y,t)$, assuming that the temperature of the basal plane is kept $T$. Then, on the basis of the above argument we modify Eq.(15) for the present stripline system as follows,

$$C_S \partial_t T_1(\mathbf{r},t) - \kappa_n (\partial_z^2 + \partial_y^2) T_1(\mathbf{r},t) + \zeta T_1(\mathbf{r},t) = q_0 \delta(x) \delta(y - y_0) \delta(t),$$

(16)

in the case where a hot spot appears at $t = 0$ and its center is located at $\mathbf{r} = (0, y_0)$. Note that the width $L \sim 1\mu\text{m}$ of the stripline in CB-KID is not large. Accordingly, we solve Eq.(16) for the temperature averaged along the $y$-direction, i.e., $\bar{T}_1(x,t) = L^{-1} \int_{-L/2}^{L/2} dy T_1(x,y,t)$, under the boundary condition $\partial_y T_1|_{y=\pm L/2} = 0$. The equation for $\bar{T}_1(x,t)$ is easily obtained as
\[ C_S \partial_t \tilde{T}_1(x,t) - \kappa_n \partial^2_x \tilde{T}_1(x,t) + \zeta \tilde{T}_1(x,t) = q_0 \delta(x) \delta(t), \]  
and its solution is given as
\[ \tilde{T}_1(x,t) = \frac{q_0 x_h}{C_S} \sqrt{\frac{\tau_z}{t}} \exp \left[ - \frac{t}{\tau_z} - \frac{1}{2} \left( \frac{x}{x_h} \right)^2 \right] \Theta(t), \]
where \( \tau_z = C_S / \zeta, x_h = \sqrt{2 \kappa_n / \zeta} \) and \( \Theta(t) \) is the Heaviside step function. Thus, one understands that the time dependence of the order parameter is given as
\[ \psi(x,t) = -W [1 - e^{-t/\tau_{qp}}] \sqrt{\frac{\tau_z}{t}} \exp \left[ - \frac{t}{\tau_z} - \frac{1}{2} \left( \frac{x}{x_h} \right)^2 \right] \Theta(t), \]
with \( W = q_0 \eta c^2 x_h / C_S \). Eq. (19) indicates that the size of the hot spot is \( x_h \) and its relaxation time is equal to \( \tau_z \).

3. Voltage signals propagating along the stripline

In this section the electrodynamic response of a superconducting stripline to a transient hot spot is investigated on the basis of the TDGL theory developed in Sec. 2. In [13] we derived the equation for the superconducting phase difference in a thin superconducting stripline forming a S-I-S structure such as CB-KID, which is given as
\[ \frac{\lambda_L}{d} + \frac{\epsilon}{\hbar c} \partial^2_x \theta(x,t) - \partial^2_x \theta(x,t) = \frac{e^*}{\hbar c} A^F_x(x,d,t), \]
where \( \theta(x,t) \) is the superconducting phase difference between the stripline and the superconducting basal plane and \( \epsilon \) is the dielectric constant of the insulating layer with a thickness \( d \) (see Fig. 1). The vector potential \( A^F_x(x,d,t) \) in Eq. (20) can be eliminated by using the solution of the Maxwell equation in the region inside the stripline as shown in [13]. When the GL theory is used for the superconducting current flowing in the stripline, the Maxwell equation to determine \( A^F_x(x,d,t) \) is obtained as
\[ \partial_x B_y(x,z,t) = \lambda_{L}^{-2}(1 + \psi(x,z,t))^2 A^F_x(x,z,t). \]

In [13] we present a way to solve Eq. (21) under the suitable boundary condition at the interface between the superconducting stripline and the insulating basal plane. Then, following the procedure given in [13], one can derive the equation with a source term from Eq. (20) under the assumption \( \psi(x,z,t) \ll 1 \) as
\[ v^{-2} \partial^2_x \theta(x,t) - \partial^2_x \theta(x,t) = F(x,t), \]
where \( v \) is the velocity defined as \( v = d^{1/2}[d + \lambda_L (1 + \coth(s / \lambda_L)]^{-1/2} c^{-1/2} \) and the source term \( F(x,t) \), which stems from the appearance of a transient hot spot, is given as
\[ F(x,t) = (2 \lambda_L / d) \coth(s / \lambda_L) [\epsilon / c^2] \partial^2_x \theta(x,t) \cdot \psi(x,d,t) \]
\[ + 2 \lambda_L \coth(s / \lambda_L) [d + \lambda_L (1 + \coth(s / \lambda_L)]^{-1} \partial_x \theta(x,t) \cdot \partial_x \psi(x,d,t). \]

Note that \( F(x,t) = 0 \) for \( t < 0 \), when the hot spot appears at \( t = 0 \). Eq. (22) can be transformed into the integral form as
\[ \theta(x,t) = \theta_0(x,t) + \int_{-\infty}^{x} dx' \int_{-\infty}^{t} dt' g^{R}(x-x', t-t') F(x', t'), \]
where \( \theta_0(x, t) \) is a solution of the homogeneous equation, \([v^{2}\partial_{x}^{2} - \partial_{t}^{2}]\theta_0(x, t) = 0\), and \(G^R(x, t)\) is the retarded Green function satisfying the equation, \([v^{2}\partial_{x}^{2} - \partial_{x}^{2}]G^R(x, t) = \delta(x)\delta(t)\), i.e.,

\[
G^R(x, t) = \Theta(t) \int_{-\infty}^{\infty} \frac{dq}{2\pi} \frac{iv}{2q} [e^{iq(x-\nu t)} - e^{i(q+x+\nu t)}].
\]  

(25)

It is noted that Eq.(24) is an integral equation, since \(F(x, t)\) contains \(\theta(x, t)\). In the following we study the case in which a constant bias current \(I\) flows in the stripline and a hot spot appears at \((x, t) = (0, 0)\). In this case the phase difference \(\theta_0(x, t)\) is understood to come from the magnetic flux induced by the bias current. We express the magnetic flux \(B_0\) induced by the bias current as \(B_0 = LI\), introducing the inductance \(L\) phenomenologically. Then, using the Josephson relation, one can derive the phase difference, \(\theta_0(x, t) \equiv \theta_0(x)\), as

\[
\theta_0(x) = (e^*/\hbar c)(d + \lambda_L[1 + \coth(s/\lambda_L)])LIx + \text{const.}
\]  

(26)

Let us now solve Eq.(24). The equation can be solved systematically by means of the iteration method. We seek the lowest order solution, which is the first order with respect to \(\psi(x, t)\). The source term \(F(x, t)\) in Eq.(23) in this case is approximated as

\[
F(x, t) \simeq 2(e^*/\hbar c)\lambda_L \coth(s/\lambda_L) LI \partial_x \psi(x, t) \equiv (2e^*K/\hbar c) LI \partial_x \psi(x, t),
\]  

(27)

with \(K = (\lambda_L/c) \coth(s/\lambda_L)\), which is obtained by substituting \(\theta_0(x)\) into \(\theta(x, t)\) in Eq.(23). Furthermore, using the Josephson relation for the voltage difference, \(V(x, t) = -\int_{-\infty}^{d} dz E_x = -(\hbar/e^*)\partial_t \theta(x, t)\), one can obtain the lowest order solution for the voltage difference from Eqs.(24) and (27) as follows,

\[
V(x, t) = -2KLI \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dt' \partial_t G^R(x - x', t - t') \partial_x \psi(x', d, t').
\]  

(28)

The time derivative of the retarded Green function in Eq.(28) is derived from Eq.(25) as

\[
\partial_t G^R(x, t) = \frac{v^2}{2} [\delta(x - vt) + \delta(x + vt)] \Theta(t).
\]  

(29)

Then, Eq.(28) yields the solution,

\[
V(x, t) = -KLv^2 \int_{-\infty}^{\infty} dx' \left\{ \Theta(x - x') \partial_x \psi(x', t') |_{t' = (z^{(\pm)} - z^{(\mp)}) / v} \right. \\
+ \Theta(x' - x) \partial_x \psi(x', t') |_{t' = (z^{(\mp)} - x') / v} \} \cdot I,
\]  

(30)

with \(z^{(\pm)} = x \mp vt\). Note that Eq.(30) is expressed as \(V(x, t) = V^{(+)}(x, t) + V^{(-)}(x, t)\), where \(V^{(+)}(x, t)\) and \(V^{(-)}(x, t)\) are the voltage signals propagating in the \(+x\)- and \(-x\)-directions, respectively. It is easy to see that in the asymptotic region, i.e., \(x \to \pm \infty\) and \(|z^{(\pm)}| < \infty\), the voltage signals (30) depend only on the variables \(z^{(\pm)}\), i.e.,

\[
\lim_{x \to \pm \infty} V^{(+)}(x, t) = \hat{V}^{(+)}(z^{(\pm)}), \quad \lim_{x \to \pm \infty} V^{(-)}(x, t) = \hat{V}^{(-)}(z^{(\pm)}),
\]  

(31)

that is, the two voltage pulses propagate in the opposite directions with a constant speed \(v\). When Eq.(19) is utilized for \(\psi\), the voltage pulses, \(\hat{V}^{(\pm)}(z^{(\pm)})\), are given as

\[
\hat{V}^{(+)}(z^{(\pm)}) = -KLWv^2 \int_{0}^{\infty} dr (1 - e^{-r/\nu_{\theta}x_{h}})(\frac{r}{\nu_{\theta}})^{3/2} (\frac{r + z^{(+)}}{x_{h}})
\]
\[ \times \exp \left[ -\frac{r}{v \tau_z} - \frac{1}{2} \left( \frac{v \tau_z}{r} \right) \left( \frac{r + z^+}{x_h} \right)^2 \right] I = \tilde{L}^+(z^+)I, \]  

and

\[ \dot{V}^-(z^-) = KLWv^2 \int_0^\infty dr (1 - e^{-r/v \tau_{qp}}) \left( \frac{v \tau_z}{r} \right)^{3/2} \left( \frac{r - z^-}{x_h} \right)^2 \times \exp \left[ -\frac{r}{v \tau_z} - \frac{1}{2} \left( \frac{v \tau_z}{r} \right) \left( \frac{r - z^-}{x_h} \right)^2 \right] I = \tilde{L}^-(z^-)I, \]

where \( \tilde{L}^+(z^+) \) is understood to be the time derivative of the kinetic inductance of the stripline. It is also noted that the voltage pulses \( V^\pm \) have opposite polarities since \( V^+(z) = -V^-(z) \). Let us now calculate the voltage pulse \( V^-(z^-) \) given in Eq.(33). The integral in Eq.(33) can be evaluated as a function of the two dimensionless parameters, \( v \tau_{qp}/x_h \) and \( v \tau_z/x_h \). Suppose that the parameters, \( \tau_{qp} \), \( \tau_z \), \( v \) and \( x_h \), take values in the region of \( \tau_{qp} \sim 10^{-12}\text{sec} \), \( \tau_z \sim 10^{-11}\text{sec} \), \( v \sim 10^9\text{cm} \cdot \text{sec}^{-1} \) and \( x_h \sim 10^{-3}\text{cm} \), i.e., \( v \tau_{qp}/x_h \sim 1 \) and \( v \tau_z/x_h \sim 10 \), because the parameter values of the CB-KID made of Nb nanowires are expected to be in this region. Fig.2 shows the \( z^- \) dependence of the normalized voltage pulse, \( V_N(z^-) = \dot{V}^-(z^-)/KLWIv^2x_h \) for \( v \tau_{qp}/x_h = 5.0 \) and three different values of \( v \tau_z/x_h \). As seen in this figure, the voltage pulse steeply rises first and then gradually attenuates. Such a feature in the shape of the voltage pulse has been observed in several kinetic inductance detectors[12, 17]. It is also noticed that the width of the voltage pulse increases with \( \tau_z \), the relaxation time of the hot spot. From this result one may conclude that the voltage pulses generated by a transient hot spot in the kinetic inductance stripline have a time width of the order of the life time of the hot spot.

![Figure 2. Shape of the normalized voltage pulse propagating along the stripline.](image)

Let us next investigate the temperature dependence of the voltage pulse. We assume that the temperature dependence appears through the temperature variation of the London penetration depth only. In this case the velocity \( v \) and the coefficients, \( K \) and \( W \), which are included in the prefactor in Eq.(33), depend on temperature. In Fig.3 we plot the voltage pulses as a function of time for two different temperatures, \( T/T_c = 0 \) and 0.8. It is seen that the amplitude of the voltage pulse increases with temperature. This result indicates a possibility that the detector shows higher sensitivity at high temperatures. To obtain the voltage pulses on a more reliable footing in the high temperature region near \( T_c \) full numerical calculations will be required for the coupled equations (1) and (14), because the present theory is valid in the region \( |\psi(r,t)| \ll \Psi_0 \), which is well satisfied in the low temperature region, \( T \ll T_c \).
4. Summary

In summary we have constructed an electrodynamic theory for the operation principle of a kinetic inductance detector having a S-I-S waveguide structure on the basis of the TDGL theory. The heat flow effect from a hot spot is also incorporated into our theory, using the phenomenological thermal diffusion equation in the superconducting state. We have clarified the feature of the voltage pulses generated by local heating in this detector.

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