 Limits on primordial non-Gaussianities from BOSS galaxy-clustering data

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Abstract

We analyze the power spectrum and the bispectrum of BOSS galaxy-clustering data using the prediction from the Effective Field Theory of Large-Scale Structure at one-loop order for both the power spectrum and the bispectrum. With $\Lambda$CDM parameters fixed to Planck preferred values, we set limits on three templates of non-Gaussianities predicted by many inflationary models: the equilateral, the orthogonal, and the local shapes. After validating our analysis against simulations, we find $f_{NL}^{\text{equil}} = 245 \pm 293$, $f_{NL}^{\text{orth}} = -60 \pm 72$, $f_{NL}^{\text{loc}} = 7 \pm 31$, at 68% confidence level. These bispectrum-based constraints from Large-Scale Structure, not far from the ones of WMAP, suggest promising results from upcoming surveys.
# 1 Introduction

The SDSS-III Baryon Oscillation Spectroscopic Survey (BOSS) has provided us with a fantastic mapping of the clustering of galaxies in the nearby Universe [1]. The BOSS data, although modest in volume compared to forthcoming experiments such as DESI [2] or Euclid [3], are remarkable as they have revealed, and continue revealing, a wealth of cosmological information from the large-scale structure of the Universe.

In the last few years, a series of works have applied the Effective Field Theory of Large-Scale Structure (EFTofLSS) prediction at one-loop order to analyze the BOSS galaxy full shape (FS) of the Power Spectrum (PS) [4–6], and, more recently, of the correlation function [7, 8]. Ref. [4] has also analyzed the BOSS galaxy-clustering bispectrum monopole using the tree-level prediction (see also [9] for a recent generalization). By imposing a prior on Big Bang Nucleosynthesis, all ΛCDM cosmological parameters have been measured from these data. The precision reached on some of these parameters is remarkable. For example, the present amount of matter, \( \Omega_m \), and the Hubble constant (see also [10, 11] for subsequent refinements) have error bars that are not far from the ones obtained from the Cosmic Microwave Background (CMB) [12]. Clustering and smooth quintessence models have also been investigated, finding \( \lesssim 5\% \) limits on the dark energy equation of state \( w \) parameter using only late-time measurements [11, 13], which is again not far from the ones obtained with the CMB [12]. The measurements of the Hubble constant provide a new, CMB-independent, method for determining this parameter [4], and it is already comparable in precision with the method based on the cosmic ladder [14, 15] and CMB. Such a tool has allowed us to shed much light on the success (or lack thereof) of some models that were proposed to alleviate the tension in the Hubble measurements (see e.g. [16]) between the CMB and cosmic ladder [17, 18] (see also [19, 20]).

These results required an intense and years-long line of study to develop the EFTofLSS from the initial formulation to the level that allows it to be applied to data. We find it is
Figure 1: Summary plot of the 68% confidence level (CL) constraints on primordial non-Gaussianities obtained in this work. $P$ and $B$ represent the fact that the analysis uses either the power spectrum or the power spectrum and bispectrum. We find no evidence of primordial non-Gaussianity.

often the case that many of the works that were needed to reach this point are not properly acknowledged. For example, several authors cite Refs. [4, 5] for the ‘model’ to analyze the PS FS, but the EFT model that is used in [4, 5] is essentially the same as the one in [21], which is where it was first developed. We therefore find it fair to add the following footnote in every paper where the EFTofLSS is used to analyze observational data. Even though some of the mentioned papers are not strictly required to analyze the data, we, and we believe probably anybody else, would not have applied the EFTofLSS to data without all these intermediate results.\(^1\)

An observable that has been so-far unexplored in galaxy-clustering data analyses from the

\(^1\)The initial formulation of the EFTofLSS was performed in Eulerian space in [22, 23], and subsequently extended to Lagrangian space in [24]. The dark matter power spectrum has been computed at one-, two- and three-loop orders in [23, 25–34]. These calculations were accompanied by some theoretical developments of the EFTofLSS, such as a careful understanding of renormalization [23, 35, 36] (including rather-subtle aspects such as lattice-running [23] and a better understanding of the velocity field [25, 37]), of several ways for extracting the value of the counterterms from simulations [23, 38], and of the non-locality in time of the EFTofLSS [25, 27, 39]. These theoretical explorations also include an enlightening study in 1+1 dimensions [38]. An IR-resummation of the long displacement fields had to be performed in order to reproduce the Baryon Acoustic Oscillation (BAO) peak, giving rise to the so-called IR-Resummed EFTofLSS [40–44]. Accounts of baryonic effects were presented in [45, 46]. The dark-matter bispectrum has been computed at one-loop in [47, 48], the one-loop trispectrum in [49], and the displacement field in [50]. The lensing power spectrum has been computed at two loops in [51]. Biased tracers, such as halos and galaxies, have been studied in the context of the EFTofLSS in [39, 52–54, 21, 55, 56] (see also [57]), the halo and matter power spectra and bispectra (including all cross correlations) in [39, 53]. Redshift space distortions have been developed in [40, 58, 21]. Neutrinos have been included in the EFTofLSS in [59, 60], clustering dark energy in [61, 33, 62, 63], and primordial non-Gaussianities in [53, 64–66, 58, 67]. Faster evaluation schemes for the calculation of some of the loop integrals have been developed in [68]. Comparison with high-quality N-body simulations to show that the EFTofLSS can accurately recover the cosmological parameters have been performed in [4, 6, 69, 70]
EFTofLSS is the non-Gaussianity of the primordial fluctuations. Some inflationary models predict that the primordial fluctuations have a sizable non-Gaussian distribution. Though there are in principle many potential forms of non-Gaussianities, an extremely large class of models is covered by focusing on the three-point function, and by using three parameterizations for it. They are called ‘local’ (see [71] and references therein), ‘equilateral’ [72], and ‘orthogonal’ [73], and they correspond to the following forms of the three-point function of the primordial gravitational field, $\Phi_p$:

$$(\Phi_p(\vec{k}_1)\Phi_p(\vec{k}_2)\Phi_p(\vec{k}_3)) = (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) f^{(i)}_{\text{NL}} B^{(i)}_{\Phi_p}(k_1, k_2, k_3)$$

(1)

$$B^{\text{loc.}}_{\Phi_p}(k_1, k_2, k_3) = 2\Delta^{\Phi}_{k_3} \left( \frac{1}{k_1^3 k_2^3} + \frac{1}{k_1^3 k_3^3} + \frac{1}{k_2^3 k_3^3} \right) ,$$

$$B^{\text{equil.}}_{\Phi_p}(k_1, k_2, k_3) = 6\Delta^{\Phi}_{k_3} \left( -\frac{1}{k_1^3 k_2^3} - \frac{1}{k_1^3 k_3^3} - \frac{1}{k_2^3 k_3^3} - \frac{2}{k_1^2 k_2^2 k_3^2} + \left( \frac{1}{k_1 k_2 k_3} + \text{5 perms.} \right) \right) ,$$

$$B^{\text{orth.}}_{\Phi_p}(k_1, k_2, k_3) = 6\Delta^{\Phi}_{k_3} \left( (1 + p) \frac{\Delta(k_1, k_2, k_3)}{k_1^3 k_2^3 k_3^3} - p \frac{\Gamma(k_1, k_2, k_3) n}{k_1^3 k_2^3 k_3^3} \right) ,$$

(2)

with $p = 8.52$,

$$\Delta(k_1, k_2, k_3) = (-k_1 + k_2 + k_3)(k_1 - k_2 + k_3)(k_1 + k_2 - k_3) ,$$

(3)

and

$$\Gamma(k_1, k_2, k_3) = \frac{2}{3}(k_1 k_2 + k_2 k_3 + k_3 k_1) - \frac{1}{3}(k_1^2 + k_2^2 + k_3^2) ,$$

(4)

and where the primordial power spectrum is given by

$$P_{\Phi_p}(k) = \frac{\Delta^{\Phi}}{k_0^3} \left( \frac{k}{k_0} \right)^{n_s-1} ,$$

(5)

$n_s$ is the scalar tilt, and $k_0$ is the pivot scale.

For example, in the context of the Effective Field Theory of Inflation [74], the signal produced by the two leading interactions for the Goldstone boson of time translations, $\pi$, that parametrizes the inflaton fluctuations, $\dot{\pi}^3$ and $\pi(\partial_t \pi)^2$, is well described by a linear combination of $f^{\text{equil.}}_{\text{NL}}$ and $f^{\text{orth.}}_{\text{NL}}$ [73]. Many multifield inflationary models are well described by a combination that also includes $f^{\text{loc.}}_{\text{NL}}$ [71, 75–78]. Additionally, quasi-single field inflationary models [79, 80], where particles with mass of order of the inflationary Hubble rate are present, produce signals that in some kinematical limits are not well described by a combination of the three shapes above, but nevertheless have most of the signal to noise well reproduced by such a combination of shapes of three-point functions. In this paper, we develop the methods to analyze the three shapes above on the BOSS DR12 galaxy sample. Our results are summarized in Fig. 1.

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2Here and elsewhere, $\vec{k}$ is the wavevector, or Fourier partner of the position $\vec{x}$. We also write $k \equiv |\vec{k}|$ for the magnitude of vectors.
The paper is organized as follows. In the next section we summarize our methodology and provide our results. Most technical aspects are relegated to appendices. In App. A, we provide the most relevant formulas for the theory modeling, and, in App. B, we provide those for the likelihood and the data analysis. In App. C, we provide plots of the posteriors of the various main analyses performed in this work.

**Data sets:** We analyze the power spectrum and the bispectrum of the SDSS-III BOSS DR12 galaxy sample [1]. The power spectrum, window functions, and bispectrum, are measured from BOSS catalogs DR12 combined CMASS-LOWZ [81]. The covariances are estimated from the 2048 patchy mocks [82]. To test our analysis pipeline, we analyze the mean over the 84 Nseries ‘cut-sky’ mocks, which are full $N$-body simulations populated with a Halo Occupation Distribution model and selection function similar to the one of BOSS [1]. For the PS, we fit the monopole and quadrupole, up to $k_{\text{max}} = 0.23\, h\, \text{Mpc}^{-1}$, as determined in [4, 6, 11, 7]. We jointly fit the BOSS DR12 bispectrum monopole, up to $k_{\text{max}} = 0.23\, h\, \text{Mpc}^{-1}$, as we explain in the following. More details on our measurements can be found in App. B.

**Public codes:** The predictions for the full shape of the galaxy power spectrum in the EFTofLSS are obtained using **PyBird**: Python code for Biased tracers in Redshift space [11]. The linear power spectra were computed with the **CLASS** Boltzmann code v2.7 [83]. The posteriors were sampled using the **MontePython** v3.3 cosmological parameter inference code [84, 85]. The plots have been obtained using the **GetDist** package [86]. The power spectrum multipoles and the bispectrum monopole are measured using **Rustico** [87]. The PS window functions are measured as described in [88] using **fkpwin** [89] based on **nbodykit** [90].

### 2 Results and conclusions

**Main results:** We perform the analysis of the power spectrum monopole and quadrupole as well as of the bispectrum monopole of the BOSS DR12 galaxy-clustering data, by fixing $\Lambda$CDM parameters to Planck preferred values [12], and by scanning over $f_{\text{NL}}$. We use the EFTofLSS prediction at one-loop order both for the power spectrum and for the bispectrum. This allows us to reach relatively high wavenumber in both statistics, $k_{\text{max}} = 0.23\, h\, \text{Mpc}^{-1}$ for the power spectrum and the bispectrum, and therefore to extract much information from the data, as we will describe below. The development of a pipeline that allows us to analyze the one-loop bispectrum predicted by the EFTofLSS has required much theoretical work, and

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3publicly available at [https://data.sdss.org/sas/dr12/boss/lss/](https://data.sdss.org/sas/dr12/boss/lss/)

4Catalog available at [https://www.ub.edu/bispectrum/page11.html](https://www.ub.edu/bispectrum/page11.html)

5[https://github.com/pierrexyz/pybird](https://github.com/pierrexyz/pybird)

6[http://class-code.net](http://class-code.net)

7[https://github.com/brinckmann/montepython_public](https://github.com/brinckmann/montepython_public)

8[https://github.com/hectorgil/Rustico](https://github.com/hectorgil/Rustico)

9[https://github.com/pierrexyz/fkpwin](https://github.com/pierrexyz/fkpwin)

10[https://github.com/bccp/nbodykit](https://github.com/bccp/nbodykit)
the explanation of such techniques are presented in [91–93]. Here, instead, we will just give the essential details by focussing on the application of such techniques to the analysis of the BOSS data for primordial non-Gaussianities.

We first calibrate the model against the set of N-series simulations. These are simulations run with Gaussian initial conditions, such that the 84 N-series boxes represent a volume of approximately $\sim 50$ times the total BOSS volume. In order to estimate our theoretical error, we run our chains on data which is the average over all of the 84 N-series boxes, using the covariance corresponding to the total volume. We find

\begin{align}
35 < f_{\text{equil., sims}}^{\text{sims}} &< 169, \quad \text{or} \quad f_{\text{equil., sims}}^{\text{BOSS}} = 102 \pm 67, \quad \text{at 68\% CL}, \\
-59 < f_{\text{orth., sims}}^{\text{sims}} &< -17, \quad \text{or} \quad f_{\text{orth., sims}}^{\text{BOSS}} = -38 \pm 21, \quad \text{at 68\% CL}, \\
8 < f_{\text{loc., sims}}^{\text{sims}} &< 16, \quad \text{or} \quad f_{\text{loc., sims}}^{\text{BOSS}} = 12 \pm 4, \quad \text{at 68\% CL}.
\end{align}

Using the errors found above, and the errors that we will find on the BOSS data in Eqs. (10) - (12), we can estimate the theoretical systematic error in our analysis. First, let us in general write $f_{\text{NL}}^{s, \text{sims}} = \mu_s^{\text{sims}} \pm \sigma_s^{\text{sims}}$, and $f_{\text{NL}}^{s, \text{BOSS}} = \mu_s^{\text{BOSS}} \pm \sigma_s^{\text{BOSS}}$, where $\mu$ is the mean of the measurement, $\sigma$ is the 1$\sigma$ error, and $s$ stands for the shape $s \in \{\text{equil.}, \text{orth.}, \text{loc.}\}$. Next, we declare that the minimum theoretical error that we can detect is given by $\sigma_s^{\text{sims}}$, the 68\%-confidence intervals measured on the simulations. Then, it is meaningful to estimate the theoretical error of the EFTofLSS prediction, $\sigma_s^{\text{th., sys}}$, as the distance of the mean of the distribution to the zero value of $f_{\text{NL}}$ minus $\sigma_s^{\text{sims}}$, or $\sigma_s^{\text{sims}}$ if this number is negative. In equations, this is

$$\sigma_s^{\text{th., sys}} \lesssim |\mu_s^{\text{sims}}| - \sigma_s^{\text{sims}}, \quad (9)$$

or $\sigma_s^{\text{th., sys}} \lesssim \sigma_s^{\text{sims}}$ if this number is negative. In units of the standard deviation that we will find in the data, we therefore can estimate $\sigma_s^{\text{equil., th., sys}} \lesssim 0.12\sigma_{\text{equil.}}^{\text{sims}}$, $\sigma_s^{\text{orth., th., sys}} \lesssim 0.24\sigma_{\text{orth.}}^{\text{sims}}$, $\sigma_s^{\text{loc., th., sys}} \lesssim 0.26\sigma_{\text{loc.}}^{\text{sims}}$. This allows us to conclude that the errors associated to the modeling and the analysis methods are safely negligible. Contour plots associated to these analyses are presented in App. C.

Having found satisfactory results against simulations, we move to analyze the BOSS data. We find

\begin{align}
-48 < f_{\text{NL}}^{\text{equil., BOSS}} &< 538, \quad \text{or} \quad f_{\text{NL}}^{\text{equil., BOSS}} = 245 \pm 293, \quad \text{at 68\% CL}, \\
-132 < f_{\text{NL}}^{\text{orth., BOSS}} &< 12, \quad \text{or} \quad f_{\text{NL}}^{\text{orth., BOSS}} = -60 \pm 72, \quad \text{at 68\% CL}, \\
-24 < f_{\text{NL}}^{\text{loc., BOSS}} &< 38, \quad \text{or} \quad f_{\text{NL}}^{\text{loc., BOSS}} = 7 \pm 31, \quad \text{at 68\% CL}.
\end{align}

These constraints show no significant evidence of non-Gaussianity. We perform a few additional checks on the viability of these results. The $\chi^2$ of the best fit of the model to the data, considering only the parameters that are relevant as degrees of freedom, has a sufficiently large corresponding $p$-value $\sim 5\%$. Results for the joint analysis for $f_{\text{NL}}^{\text{equil.}} - f_{\text{NL}}^{\text{orth.}}$ give very similar results and the 68\% CL contour plot is given in Fig. 2. Contour plots for the posteriors associated to these analyses are presented in App. C.
It is interesting to investigate how it is possible to obtain such powerful limits. For the BOSS survey, we have an effective volume of about $V_{\text{eff}} \simeq 2.4 \text{(Gpc}/h)^3$. The wavenumber $k_{\text{shot}}$ at which the shot noise is comparable to the cosmic variance\(^\text{11}\) is about $k_{\text{shot}} \simeq 0.33h \text{ Mpc}^{-1}$ and therefore the number of available modes of the BOSS survey is $N_{\text{modes}} \sim V_{\text{eff}} f_{\text{NL}} k_{\text{shot}}^3/(2\pi)^3 \simeq V_{\text{eff}} k_{\text{shot}}^3/(6\pi^2) \sim 1.5 \cdot 10^6$. The number of modes we use, since our $k_{\max} \simeq 0.23h \text{ Mpc}^{-1}$, is thus $N_{\text{modes}} \sim 0.5 \cdot 10^6$. We expect a bound of order $f_{\text{NL}} \zeta \lesssim 1/\sqrt{N_{\text{modes}}}$: given that the amplitude of the primordial curvature fluctuation $\zeta$ is $\sim 3 \cdot 10^{-5}$, we find an expected limit $f_{\text{NL}} \lesssim 50$. Of course, this estimate does not take into account the information absorbed into the EFT parameters, the potential effect of shot noise which is not completely negligible at our wavenumbers, or of relative order one factors that go into the normalization of $f_{\text{NL}}$ for different shapes, but still, it gives a sense of the constraining power of this LSS survey.

It is worthwhile to put our results in a broader context. Within LSS, the local shape has been analyzed in [96], and more recently in [97, 98], and the local and the equilateral in [99]. All references analyze only the power spectrum of several data sets, and use the non-local bias [100–102] that is induced by primordial non-Gaussianities, with a characteristic and prominent $k^{-2}$ scale-dependence for the local shape. Ref. [99] obtains competitive constraints, but using information from the non-linear regime that depends on the astrophysics modeling. Refs. [97, 98] use an optimal estimator for non-Gaussianity and analyze the higher-redshift eBOSS quasar sample. Thus, it makes the most sense to compare our results with [96], which finds, on the DR5 LRG spectroscopic sample, $f_{\text{NL}}^{\text{loc}} = 70^{+73}_{-84}$ at 68% CL. When we

\[^{11}\] This estimate is done using the linear halo monopole power spectrum $P_{11,0}^h$ by computing $\bar{n} P_{11,0}(k_{\text{shot}}) = 1$ using the equations in App. A.1, $b_1 = 2.2$, $\bar{n} = 4 \cdot 10^{-4} \text{ (h Mpc}^{-1})^3$, and $f = 0.78$ at the redshift $z = 0.57$.\n
|   | BOSS | WMAP | Planck |
|---|------|------|--------|
| $f_{\text{NL}}^{\text{equil.}}$ | $245 \pm 293$ | $51 \pm 136$ | $-26 \pm 47$ |
| $f_{\text{NL}}^{\text{orth.}}$ | $-60 \pm 72$ | $-245 \pm 100$ | $-38 \pm 24$ |
| $f_{\text{NL}}^{\text{loc.}}$ | $7 \pm 31$ | $37.2 \pm 19.9$ | $-0.9 \pm 5.1$ |
analyze only the power spectrum without the perturbativity prior (as described in App. B) with the modeling provided by the EFTofLSS and on the DR12 galaxy sample, we find $f_{\text{NL}}^\text{loc} = 59 \pm 59$ at 68% CL. The two results look quite consistent, given that there are differences in the data version, modeling of the signal, and the choice of scales analyzed. The addition of the bispectrum together with the perturbativity prior gives a remarkable error reduction of $\simeq 1.9$.

Perhaps even more interesting is the comparison with the bounds from the CMB. Planck constraints [95] are still quite superior, but only by a factor that ranges from $\simeq 3.0$ for $f_{\text{NL}}^\text{orth}$, $\simeq 6.1$ for $f_{\text{NL}}^\text{loc}$ to $\simeq 6.2$ for $f_{\text{NL}}^\text{equil}$. In fact, for the orthogonal shape, our results are already stronger by a factor of $\simeq 1.4$ than the final results of WMAP [94], and they are just $\sim 36\%$ weaker for the local shape and $\sim 54\%$ for the equilateral shape. Given that BOSS happened prior to the large program of upcoming LSS experiments, these results are of great hope for the future power of LSS, perhaps through specifically designed surveys, in constraining primordial non-Gaussianities. As an illustration, the outlook for the nearest upcoming survey, DESI [2], is quite promising, where we can expect reductions of the error bars by factors of 2.5, 2.3, and 5.2, respectively for the equilateral, orthogonal, and local shapes [103].

We also comment on the improvement from the addition of the one-loop prediction on top of the tree-level one for the bispectrum. We find that, taking $k_{\text{max}} = 0.08 \text{ h/Mpc}$ as a reference for the tree-level analysis, our $f_{\text{NL}}$ constraints obtained with the one-loop theory up to $k_{\text{max}} = 0.23 \text{ h/Mpc}$ improve over the tree-level ones by a factor $\sim 1.5$ on the equilateral shape and $\sim 1.9$ on the orthogonal shape. This is summarized in Tab. 1.

Finally, we comment on the effect of the shot noise in the BOSS sample on our results. This is an interesting consideration since the shot noise is essentially determined by the number density of objects in the survey, and thus is (within reason) a part of the experimental design choice. As mentioned earlier, the optimal limit on $f_{\text{NL}}$ to expect from the BOSS survey at the $k_{\text{max}}$ up to which we analyze the data is better by a factor of few than the limit we have obtained. One reason for this is that at large $k$, shot noise starts to swamp out the usable signal (it is approximately 50% of the cosmic variance at $k_{\text{max}} \simeq 0.23 \text{ h/Mpc}^{-1}$). In order to better understand this, in Tab. 1 we show the size of the error bars on $f_{\text{NL}}^\text{equil}$ and $f_{\text{NL}}^\text{orth}$, using the tree-level and one-loop bispectrum, for the real BOSS data and for two synthetic BOSS experiments. The first synthetic BOSS data are generated by our one-loop best-fit theory model, including the realistic mean number density of galaxies $\bar{n} = 4 \cdot 10^{-4} (\text{h Mpc}^{-1})^3$. This data is analyzed with an analytic diagonal covariance matrix, using specifically the value $\bar{n} = 4 \cdot 10^{-4} (\text{h Mpc}^{-1})^3$. The second synthetic BOSS data are generated by our one-loop best-fit theory model, but with a value $\bar{n} = 1 (\text{h Mpc}^{-1})^3$ such that the shot noise is effectively zero. These data are analyzed also with an analytic diagonal covariance matrix, but now using the value $\bar{n} = 1 (\text{h Mpc}^{-1})^3$ to simulate a hypothetical experiment with negligible shot noise. Additionally, for this case, we use the same priors as the previous cases but with the priors involving $\bar{n}$ rescaled appropriately. The synthetic BOSS data with realistic value of $\bar{n}$ was

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12Assuming control over systematics in the DESI emission line galaxy sample.
Table 1: We give the improvements made when using the one-loop bispectrum \((k_{\text{max}} = 0.23 \, h \, \text{Mpc}^{-1})\) compared with the tree-level bispectrum \((k_{\text{max}} = 0.08 \, h \, \text{Mpc}^{-1})\) for the errors on \(f^{\text{equil}}_\text{NL}\) and \(f^{\text{orth}}_\text{NL}\) (\(\sigma^{\text{equil}}\) and \(\sigma^{\text{orth}}\), respectively) for various scenarios. In the first line, we analyze the real BOSS data described in this paper (the one-loop bispectrum columns are the results given in this paper). In the second line, we analyze synthetic data generated by our theory model with cosmological parameters determined by our best fit, including the realistic shot noise. Finally, in the third line, we analyze the same synthetic data, but with a large value of \(\bar{n}\), such that the shot noise is effectively zero. Columns marked \(\%\) give the percentage decrease in error bars due to using the one-loop bispectrum. All cases use the one-loop power spectrum. In general, we see that the EFT of LSS at one loop is expected to perform even better for experiments where the number density of objects is larger than that of BOSS.

done as a consistency check, since it should give approximately the same results as the real BOSS analysis, which indeed is true. As we can see, the decrease in error bars going from the tree-level to one-loop bispectrum for the real BOSS data is 33\% for \(f^{\text{equil}}_\text{NL}\), and 47\% for \(f^{\text{orth}}_\text{NL}\), while for the synthetic BOSS data with negligible shot noise, the decreases are 47\% and 57\% respectively. We thus conclude that in a more ideal BOSS experiment with very small shot noise, the decrease in error bars from using the one-loop bispectrum instead of the tree-level bispectrum would be about a factor of 1.4 better for \(f^{\text{equil}}_\text{NL}\) and a factor of 1.2 better for \(f^{\text{orth}}_\text{NL}\) than what we find in this work.

A note of warning: We end this section of the main results with a final note of warning. It should be emphasized that in performing this analysis, as well as the preceding ones using the EFT of LSS by our group [4, 6, 11, 17, 13, 7], we have assumed that the observational data are not affected by any unknown systematic error or undetected foregrounds. In other words, we have used the publicly available BOSS catalogs. Given the additional cosmological information that the theoretical modeling of the EFT of LSS allows us to exploit in BOSS data, it might be worthwhile to investigate if potential undetected systematic errors might affect our results. We leave an investigation of these issues to future work.

Note added: While this paper was being finalized, [104] appeared a few days before this work. Ref. [104] also analyzes the equilateral and orthogonal shape, but not the local shape. Regarding the overlapping shapes, the main difference is that [104] uses the tree-level predic-
tion for the bispectrum, while we use the one-loop prediction. Because of this, our bounds are much stronger (considering that we are analyzing the same experiment). In order to compare, we analyzed the equilateral shape using only the tree-level prediction, finding overall agreement. The most optimistic numbers quoted in [104], which assume dark-matter halo relations that fix the quadratic biases in terms of $b_1$, are $f_{\text{NL}}^{\text{equil}} = 260 \pm 300$ and $f_{\text{NL}}^{\text{orth}} = -23 \pm 120$. If we also fix the quadratic bias, we obtain the even stronger constraints $f_{\text{NL}}^{\text{equil}} = 81 \pm 209$, $f_{\text{NL}}^{\text{orth}} = -93 \pm 13$, and $f_{\text{NL}}^{\text{loc}} = 13 \pm 28$ (all numbers quoted here are at 68% CL). This again shows the constraining power of the one-loop bispectrum and the perturbativity prior. However, we do not think that fixing quadratic biases is a justified procedure from the EFTofLSS point of view (for a study of the effect of futuristic but perhaps realistic priors on bias parameters, see [103]). Fixing biases completely would make the EFTofLSS no longer manifestly correct.

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A EFTofLSS with non-Gaussianity

A.1 Biased tracers in redshift space

We first focus on biased tracers with Gaussian initial conditions, and introduce non-Gaussianities in the next subsection. The transformation of the halo overdensity $\delta_h$ (in position space) to redshift space is given by

$$\delta_{r,h}(\vec{x}) = \delta_h(\vec{x}) - \frac{\hat{z}^i \hat{z}^j}{aH} \partial_i \left((1 + \delta_h) v^j \right) + \frac{\hat{z}^i \hat{z}^j \hat{z}^k \hat{z}^l}{2(aH)^2} \partial_i \partial_j \left((1 + \delta_h) v^k v^l \right)$$

$$- \frac{\Pi_{n=1}^5 \hat{z}^i_n}{3!(aH)^3} \partial_i \partial_{i_2} \partial_{i_3} \left((1 + \delta_h) v^{i_4} v^{i_5} v^{i_6} \right) + \frac{\Pi_{n=1}^8 \hat{z}^i_n}{4!(aH)^4} \partial_i \partial_{i_2} \partial_{i_3} \partial_{i_4} \left(v^{i_5} v^{i_6} v^{i_7} v^{i_8} \right) + \ldots .$$

(13)

Next, we perturbatively expand the halo overdensity in redshift space

$$\delta_{r,h}(\vec{k}; \hat{z}) = \sum_n \delta_{r,h}^{(n)}(\vec{k}; \hat{z}) ,$$

(14)
Additionally, to them later) can be written as up to fourth order. The solutions (ignoring EFT counterterms for now, we will return to them later) are
\[ \hat{\delta}^{(1)}(\vec{k}; \hat{z}) = K_{1}^{r,h}(\vec{k}; \hat{z})\delta^{(1)}(\vec{k}) , \]
\[ \hat{\delta}^{(n)}(\vec{k}; \hat{z}) = \int_{\vec{k}_{1}, \ldots, \vec{k}_{n}} K_{n}^{r,h}(\vec{k}_{1}, \ldots, \vec{k}_{n}; \hat{z})\delta^{(1)}(\vec{k}_{1}) \cdots \delta^{(1)}(\vec{k}_{n}) , \]
(16)
for \( n \geq 2 \), which defines the \( n \)-th order symmetric halo kernels \( K_{n}^{r,h} \). The kernels \( K_{n}^{r,h} \) depend on the bias parameters \( \{b_{i}\} \) for \( i = 1, \ldots, 15 \) in the following way
\[ K_{1}^{r,h}[b_{1}] , \quad K_{2}^{r,h}[b_{1}, b_{2}, b_{5}] , \quad K_{3}^{r,h}[b_{1}, b_{2}, b_{3}, b_{5}, b_{6}, b_{8}, b_{10}] , \quad \text{and} \quad K_{4}^{r,h}[b_{1}, \ldots, b_{15}] . \]
(17)
For example,
\[ K_{1}^{r,h}(\vec{k}; \hat{z}) = b_{1} + f(\hat{k} \cdot \hat{z})^{2} , \]
(18)
is the Kaiser result, and explicit expressions for \( K_{2}^{r,h} \) and \( K_{3}^{r,h} \) can be found in [21], while the expression for \( K_{4}^{r,h} \) is available in [92] and is a conceptually straightforward extension to next order of the EFT bias expansion.

The observables that concern us here are the one-loop power spectrum and the one-loop bispectrum. The one-loop power spectrum is \( P_{11}^{r,h} + P_{1\text{-loop}}^{r,h} \), with \( P_{11}^{r,h} \) the tree-level power spectrum:
\[ P_{11}^{r,h}(k, \hat{k} \cdot \hat{z}) = (b_{1} + f(\hat{k} \cdot \hat{z})^{2})^{2}P_{11}(k) , \]
(19)
and \( P_{1\text{-loop}}^{r,h} \equiv P_{22}^{r,h} + P_{13}^{r,h} \), the one-loop contribution, with
\[ P_{22}^{r,h}(k, \hat{k} \cdot \hat{z}) = 2 \int_{\vec{q}} K_{2}^{r,h}(\vec{q}, \vec{k} - \vec{q}; \hat{z})^{2}P_{11}(q)P_{11}(|\vec{k} - \vec{q}|) , \]
(20)
\[ P_{13}^{r,h}(k, \hat{k} \cdot \hat{z}) = 6P_{11}(k)K_{1}^{r,h}(\vec{k}; \hat{z}) \int_{\vec{q}} K_{3}^{r,h}(\vec{q}, -\vec{q}, \vec{k}; \hat{z})P_{11}(q) . \]
The one-loop bispectrum is given by \( B_{211}^{r,h} + B_{1\text{-loop}}^{r,h} \), with the tree-level bispectrum:
\[ B_{211}^{r,h} = 2K_{1}^{r,h}(\vec{k}_{1}; \hat{z})K_{1}^{r,h}(\vec{k}_{2}; \hat{z})K_{2}^{r,h}(-\vec{k}_{1}, -\vec{k}_{2}; \hat{z})P_{11}(k_{1})P_{11}(k_{2}) + 2 \text{ perms.} , \]
(21)
(we have dropped the argument \( (k_{1}, k_{2}, \hat{k}_{1} \cdot \hat{z}, \hat{k}_{2} \cdot \hat{z}) \) on the bispectrum to remove clutter), and the one-loop contribution:
\[ B_{1\text{-loop}}^{r,h} = B_{222}^{r,h} + B_{321}^{r,h} + B_{411}^{r,h} , \]
(22)
\[ \int_{\vec{k}_{1}, \ldots, \vec{k}_{n}} \equiv \int_{\vec{k}_{1}} \cdots \int_{\vec{k}_{n}} \frac{d^{3}k_{1}}{(2\pi)^{3}} \cdots \frac{d^{3}k_{n}}{(2\pi)^{3}} , \quad \int_{\vec{k}_{1}, \ldots, \vec{k}_{n}}^{\vec{k}} \equiv \int_{\vec{k}_{1}, \ldots, \vec{k}_{n}} (2\pi)^{3}\delta_{D}(\vec{k} - \sum_{i=1}^{n} \vec{k}_{i}) . \]
(15)
Additionally, \( \hat{\delta}^{(1)} \) and \( P_{11} \) are the dark-matter linear overdensity field and power spectrum, respectively.
with
\[
B_{222}^{r,h} = 8 \int \frac{d^3q}{(2\pi)^3} P_1(q) P_1(|\vec{q}_2 - \vec{q}|) P_1(|\vec{q}_1 + \vec{q}|) \\
\times K_2^{r,h}(q, \vec{q}_1 + \vec{q}, \vec{q}_2 - \vec{q}; \hat{z}) K_2^{r,h}(\vec{q}_2 - \vec{q}, \vec{q}; \hat{z}) K_2^{r,h}(\vec{q}_1 + \vec{q}, \vec{k}_2 + \vec{q}; \hat{z})
\]
\[
B_{321}^{r,h,(I)} = 6 P_1(k_1) K_4^{r,h}(\vec{k}_1; \hat{z}) \int \frac{d^3q}{(2\pi)^3} P_1(q) P_1(|\vec{q}_2 - \vec{q}|) \\
\times K_3^{r,h}(q, \vec{q}_2 + \vec{q}, -\vec{k}_1; \hat{z}) K_2^{r,h}(\vec{q}_2 - \vec{q}, \vec{q}; \hat{z}) + 5 \text{ perms.},
\]
\[
B_{321}^{r,h,(II)} = 6 P_1(k_1) P_1(k_2) K_1^{r,h}(\vec{k}_1; \hat{z}) K_2^{r,h}(\vec{k}_2; \hat{z}) \int \frac{d^3q}{(2\pi)^3} P_1(q) K_3^{r,h}(\vec{q}_2, \vec{q}; \hat{z}) + 5 \text{ perms.},
\]
\[
B_{411}^{r,h} = 12 P_1(k_1) P_1(k_2) K_1^{r,h}(\vec{k}_1; \hat{z}) K_1^{r,h}(\vec{k}_2; \hat{z}) \int \frac{d^3q}{(2\pi)^3} P_1(q) K_4^{r,h}(\vec{q}_2, -\vec{q}, -\vec{k}_1, -\vec{k}_2; \hat{z}) + 2 \text{ perms.}.
\]

Next we turn to the counterterms that renormalize the one-loop power spectrum. Since we are working directly with biased tracers, we can introduce the counterterms directly into Eq. (13), as in [21] for example. The counterterms come from two sources. The first are response terms, which give a contribution:
\[
P_{13}^{r,h,ct}(\vec{k}, \hat{z}) = 2 K_1^{r,h}(\vec{k}; \hat{z}) K_1^{r,h,ct}(\vec{k}; \hat{z}) P_1(k),
\]
where
\[
K_1^{r,h,ct}(\vec{k}; \hat{z}) = \frac{k^2}{k_{NL}^2} \left( -c_{h,1} + f(\hat{k} \cdot \hat{z}) c_{\pi,1} - \frac{1}{2} f' (\hat{k} \cdot \hat{z})^2 c_{\pi,1} - \frac{1}{2} f'' (\hat{k} \cdot \hat{z})^2 c_{\pi,3} \right).
\]
We also have the stochastic terms, which give a contribution:
\[
P_{22}^{r,h,ct}(\vec{k}, \hat{z}) = \frac{1}{\bar{n}} \left( c_{St}^1 + \frac{k^2}{k_{NL}^2} c_{St}^2 + \frac{k^2}{k_{NL}^2} \frac{k^2}{k_{NL}^2} f(\hat{k} \cdot \hat{z})^2 \right).
\]
The one-loop bispectrum is renormalized by response terms in
\[
B_{411}^{r,h,ct} = 2 P_1(k_1) P_1(k_2) K_1^{r,h}(\vec{k}_1; \hat{z}) K_1^{r,h}(\vec{k}_2; \hat{z}) K_2^{r,h,ct}(-\vec{k}_1, -\vec{k}_2; \hat{z}) + 2 \text{ perms.},
\]
where $K_2^{r,h,ct}$ is given in [92], and
\[
B_{321}^{r,h,(II),ct} = 2 P_1(k_1) P_1(k_2) K_1^{r,h,ct}(\vec{k}_1; \hat{z}) K_1^{r,h}(\vec{k}_2; \hat{z}) K_2^{r,h}(\vec{q}_2, -\vec{q}, -\vec{k}_1, -\vec{k}_2; \hat{z}) + 5 \text{ perms.}.
\]
In a similar way, the stochastic contributions $B_{321}^{r,h,ct,(I)}$ and $B_{222}^{r,h,ct}$ are given in [92] and can be found by a straightforward application of EFTofLSS principles. Numerically, in our analysis, we use $k_{NL} = 0.7 \, h / \text{Mpc}$.

In redshift space, we analyze the power-spectrum monopole and quadrupole, and the bispectrum monopole. The power-spectrum multipoles are given by
\[
P_{\ell}^{r,h}(k) = \frac{2\ell + 1}{2} \int_{-1}^{1} d\mu P_{\ell}(\mu) P_{\ell}^{r,h}(k, \mu),
\]
where $\mathcal{P}_\ell$ are the Legendre polynomials, and the bispectrum monopole is given simply by the average over the redshift space angles [105–107] \(^{14}\)

$$B_{r,h}^{z}(k_1, k_2, k_3) = \frac{1}{4\pi} \int_{-1}^{1} d\mu_1 \int_{0}^{2\pi} d\phi B_{r,h}^{z}(k_1, k_2, k_3, \mu_1, \mu_2), \quad (30)$$

where $\mu_2 \equiv \mu_1 \hat{k}_1 \cdot \hat{k}_2 - \sqrt{1 - \mu_1^2} \sqrt{1 - (\hat{k}_1 \cdot \hat{k}_2)^2} \cos \phi$.

### A.2 Non-Gaussianity

We now pass to introduce the modifications needed if the primordial fluctuations are non-Gaussian. We describe non-Gaussianity in the primordial potential, $\Phi_p^{NG}$, in terms of a Gaussian auxiliary potential $\Phi_p^G$ as:

$$\Phi_p^{NG}(\vec{k}) = \Phi_p^G(\vec{k}) + f_{NL} \int_{\vec{k}_1, \vec{k}_2}^{\vec{k}} W(\vec{k}_1, \vec{k}_2) \Phi_p^G(\vec{k}_1) \Phi_p^G(\vec{k}_2), \quad (31)$$

which gives to leading order in $f_{NL}$ the primordial bispectrum:

$$B_{\Phi_p}(k_1, k_2, k_3) = 2f_{NL} \left(W(\vec{k}_1, \vec{k}_2)P_{\Phi_p}(k_1)P_{\Phi_p}(k_2) + 2 \text{ perms.} \right), \quad (32)$$

where $P_{\Phi_p}$ is the primordial power spectrum Eq. (5). A choice of $W$ that gives the desired bispectrum at leading order in $f_{NL}$ is

$$W(\vec{k}_1, \vec{k}_2) = \frac{1}{6} \frac{B_{\Phi_p}(k_1, k_2, |\vec{k}_1 + \vec{k}_2|)}{P_{\Phi_p}(k_1)P_{\Phi_p}(k_2)}, \quad (33)$$

Then, using the transfer function $T_\alpha$ defined by

$$\delta^{(1)}(\vec{k}, a) = T_\alpha(k, a)\Phi_p(\vec{k}), \quad (34)$$

we can convert Eq. (31) to

$$\delta^{(1)}_{NG}(\vec{k}, a) = \delta^{(1)}(\vec{k}, a) + f_{NL} \int_{\vec{k}_1, \vec{k}_2}^{\vec{k}} W(\vec{k}_1, \vec{k}_2) \frac{T_\alpha(|\vec{k}_1 + \vec{k}_2|, a)}{T_\alpha(k_1, a)T_\alpha(k_2, a)} \delta^{(1)}(\vec{k}_1, a)\delta^{(1)}(\vec{k}_2, a), \quad (35)$$

where $\delta^{(1)}$ is a Gaussian field.

There are two effects from non-Gaussianity on the perturbative expansion. The first comes from replacing all of the $\delta^{(1)}$ initial conditions in Eq. (16) with the non-Gaussian initial conditions Eq. (35). At the level to which we work, this simply produces a shift in the second-order kernel

$$\delta K^{h,r}_{2}(\vec{k}_1, \vec{k}_2; \hat{z}) = f_{NL} \int_{\vec{k}_1, \vec{k}_2}^{\vec{k}} \delta^{(1)}_{NG}(\vec{k}_1, a)T_\alpha(|\vec{k}_1 + \vec{k}_2|, a)\delta^{(1)}(\vec{k}_2, a) W(\vec{k}_1, \vec{k}_2), \quad (36)$$

\(^{14}\)We have corrected a factor of $1/(4\pi)$ in Eq. (14) of [107].
which therefore, at lowest order in $f_{NL}$ and $k/k_{NL}$ at which we work, modifies the bispectrum.

The second effect of $f_{NL}$ comes from allowing new terms (which appear non-local because of initial long-range correlations [100, 96, 101, 102, 53, 64, 58]) in the bias expansion. In order to define the new counterterms, and because of the smoothing procedure used to define the EFT, we look at the squeezed limit of the last term in Eq. (35). This defines a new long-wavelength field

$$\tilde{\phi}(k_L, a) \equiv \frac{W_{SL}(k_{NL} \cdot \vec{k}_L)}{T_a(k_L, a)} \delta^{(1)}(k_L, a), \quad (37)$$

which can now be used to build counterterms. The function $W_{SL}$ is defined by the squeezed limit of $W(k_s, \vec{k}_L)$ for $k_L \ll k_s$, with an angle average used for $W_{SL}^{\text{equil}}$. Explicitly, we have

$$W_{SL}(k_{NL}, \vec{k}_L) = w_{\beta} \left( \frac{k_L}{k_{NL}} \right)^{\beta}, \quad (38)$$

with

$$(\beta, w_{\beta})_{\text{loc}} = (0, 2/3), \quad (\beta, w_{\beta})_{\text{equil}} = (2, 4/3), \quad \text{and} \quad (\beta, w_{\beta})_{\text{orth}} = (2, 4(9 - 7p)/27). \quad (39)$$

Thus, the new terms that we can write in the renormalized halo density are

$$[\delta_{r,h}(\vec{k}; \hat{z})]^{f_{NL}} = b_1^{f_{NL}} f_{NL} \tilde{\phi}(k_L, a_{in}) + f_{NL} \int_{\vec{k}_1, \vec{k}_2} \left( b_1^{f_{NL}} \left( \frac{\vec{k}_1 \cdot \vec{k}_2}{k_1^2} + f \hat{z} \cdot (\vec{k}_1 + \vec{k}_2) \frac{\hat{z} \cdot \vec{k}_2}{k_2^2} \right) + b_2^{f_{NL}} \right) \tilde{\phi}(k_L, a_{in}) \delta^{(1)}(k_L). \quad (40)$$

where the first term in the second line is the flow term associated to the operator in $b_{f_{NL}}^{f_{NL}}$ [39] and $f$ is the linear growth rate. For the non-Gaussian bias parameters $b_1^{f_{NL}}$ and $b_2^{f_{NL}}$, we use the forms suggested in [108]

$$w_{\beta} b_1^{f_{NL}} = 2 \delta_c (b_1 - 1)$$

$$w_{\beta} b_2^{f_{NL}} = -2 \left( b_1 - 1 - \frac{13}{21} \delta_c (b_1 - 1) - 2 b_5 \delta_c \right). \quad (41)$$

where $\delta_c = 1.686$ is the critical collapse density.

Since, in our analysis, Eq. (41) relates the value of the non-local biases to the value of $f_{NL}$, it is important for them to be accurate for an accurate limit on $f_{NL}$. However, it is probably too optimistic to assume that they are exact. We test the role of possible inaccuracies in the following way. For $f_{NL}^{\text{equil}}$ and $f_{NL}^{\text{orth}}$, we analyze the data by setting $b_1^{f_{NL}}$ and $b_2^{f_{NL}}$ to zero, and find that the results are practically unchanged. For $f_{NL}^{\text{loc}}$, we expect a larger effect from these terms, as the scale dependence of the biases is more prominent. If we let the non-local biases scan with a Gaussian prior centered around the value obtained in Eq. (41) and with a width which is 60% of the same value to account for inaccuracies of the modeling, we find that the result on BOSS data is changed to $f_{NL}^{\text{loc}} = 20 \pm 49$, at 68% CL, while with the fixed

\[^{15}\]We neglect effects due to the angle $\hat{k}_S \cdot \hat{k}_L$ [64, 58].
bias relations in Eq. (41) we had obtained $f_{\text{NL}}^\text{loc} = 7 \pm 31$, at 68% CL.\textsuperscript{16} This shift is entirely dominated by the freedom in the linear non-local bias, but, as we see and considering the relatively large prior that we have allowed, it is not a large effect. We therefore conclude that our results are robust against mild uncertainties in the relation described by Eq. (41).

In Fig. 3, we show the posteriors of $f_{\text{NL}}^\text{loc}$ obtained varying $b_{\text{NL}}^1$ and $b_{\text{NL}}^2$ with Gaussian priors centered on the values given by Eq. (41) with relative widths of 60% and 200%. For large priors like 200%, we can see that the posterior becomes non-Gaussian, and takes on a characteristic hyperbola shape, due simply to the fact that we are actually bounding mostly the products $b_{\text{NL}}^1 f_{\text{NL}}^\text{loc}$ and $b_{\text{NL}}^2 f_{\text{NL}}^\text{loc}$. The resulting constraints on $f_{\text{NL}}^\text{loc}$ weaken slightly: the

\textsuperscript{16}As just described, letting the non-local biases vary by 60% when including the bispectrum leads to a $\sim 37\%$ increase in the error bar of $f_{\text{NL}}$. We can compare this to a power-spectrum-only analysis. In this case, $b_{\text{NL}}^i$ is completely degenerate with $f_{\text{NL}}$, appearing only in the combination $b_{\text{NL}}^i f_{\text{NL}}$. Because of this, we also put a flat prior $b_{\text{NL}}^i f_{\text{NL}} > 0.01$ because the limit $f_{\text{NL}} = 0$ corresponds to an infinite phase-space prior volume. We find, in the analysis with only the power spectrum and letting the non-local biases vary by 60%, that the error bar on $f_{\text{NL}}$ increases by $\sim 60\%$ as expected. This illustrates that the bispectrum (through Eq. (36)) breaks some degeneracies between $f_{\text{NL}}$ and the non-local biases.
60% number was quoted above, and for the prior of size 200% we obtain \( f_{NL}^{\text{loc}} = -2.3 \pm 83 \) at 68% CL. Notice that while the one-dimensional posterior for the 200% prior on \( f_{NL}^{\text{loc}} \) appears more narrow than the posterior with fixed bias relations, this is only because the former is non-Gaussian: indeed the 68% CL for the 200% prior is larger. We do not consider larger priors on Eq. (41) because of potential contamination from prior volume (or phase-space projection) effects \([109]\). All in all, it seems to us that, for bias models up to 60% or even 200% different from Eq. (41), our results are robust against these kinds of uncertainties in Eq. (41). For reference, quoting results for CMASS NGC, our current analysis gives \( b_{1}^{\text{NL}} f_{NL}^{\text{loc}} = 38 \pm 168 \) and \( b_{2}^{\text{NL}} f_{NL}^{\text{loc}} = 68 \pm 283 \) at 68% CL.

B Likelihood

Data: From the SDSS-III BOSS DR12 galaxy sample \([1]\), we make use of the power spectrum and the bispectrum that we measure as follows. To each galaxy we assign the standard FKP weights for optimality together with the correction weights described in \([81]\) for BOSS data and in \([82]\) for the patchy mocks. All celestial coordinates are converted to comoving distances assuming \( \Omega_{m}^{\text{fid}} = 0.310 \). The power spectrum and the bispectrum are measured with the estimator described in \([110–113]\) and \([114, 115, 48, 106]\) respectively, using the code Rustico \([87]\).\footnote{https://github.com/hectorgil/Rustico} For all the sky cuts we use a box consisting of \( 512^3 \) cells of side length \( L_{\text{box}} = 3500 \text{Mpc}/h \), with Piecewise Cubic Spline particle assignment scheme and grid interlacing \([116]\). The measurements are binned in \( \Delta n = 6 \) units of the fundamental frequency of the box \( k_f \), starting from the bin centered at \( n_{\text{min}} = 9 \), up to the one centered on \( n_{\text{max}} = 123 \), which correspond in frequencies to bins of size \( \Delta k \simeq 0.0108 h \text{Mpc}^{-1} \), with first and last bins centered on \( k_{\text{min}} = 0.016 h \text{Mpc}^{-1} \) and \( k_{\text{max}} \simeq 0.221 h \text{Mpc}^{-1} \), respectively. Importantly, we keep all bins whose centers form a closed triangle. Explicitly, we select the bins whose centers satisfy:

\[
(n_1, n_2, n_3), \quad n_1, n_2, n_3 = n_{\text{min}}, n_{\text{min}} + \Delta n, \ldots, n_{\text{max}}, \quad \text{if } n_1 \leq n_2 \leq n_3 \text{ and } n_3 \leq n_1 + n_2 . \tag{42}
\]

In this analysis, we use two redshift cuts \( 0.2 < z < 0.43 \) and \( 0.43 < z < 0.7 \), namely for LOWZ and CMASS, respectively, with two galactic cuts NGC and SGC each, for a total of four skies.

For the local shape, we find that for N-series, we can reliably take \( k_{\text{min}} = 0.005 h \text{Mpc}^{-1} \) for the power spectrum. However, for the data, as there can be effects at large scales like observational systematics beyond the ones simulated in the N-series (see for instance \([117]\)), we use \( k_{\text{min}} = 0.01 h \text{Mpc}^{-1} \) throughout. For the orthogonal shape and when fitting jointly with the equilateral shape, we re-bin the data and the covariance such that \( \Delta n = 12 \).
Likelihood: We use the following likelihood $L$ to describe the data:

$$-2 \log(L) = (D - T) \cdot C^{-1} \cdot (D - T). \quad (43)$$

Here $D$ is the data vector, constructed from the measurements of the power spectrum monopole and quadrupole, concatenated with the bispectrum monopole. $T$ is the corresponding EFTofLSS prediction, as described in App. A, containing also additional modeling aspects presented below in this appendix. Finally, $C^{-1}$ is the inverse covariance built from the 2048 patchy mocks, where we first concatenated the power spectrum multipoles with the bispectrum monopole of each realization. Given the finite number of mocks used to estimate the inverse covariance, we correct it with the Hartlap factor [118]. We have checked that, although the Hartlap factor is not a small correction, our estimation of the inverse covariance is unbiased: instead of the analysis with $40 + 1015$ bins (per skycut/patchy mock suite), we have explicitly checked that we obtain similar results by restricting the analysis to fewer bins $(100, 200, \ldots)$ constructed as the linear combinations that maximize the signal-to-noise of $f_{NL}$, following the method outlined in [119].

Posterior sampling: To sample the posteriors, we use the partially-marginalized likelihood as described in [4, 11], which already includes the bispectrum, as it was analyzed first in [4]. In this likelihood, the EFT parameters appearing only linearly in the predictions are marginalized analytically. We fix all cosmological parameters, but $f_{NL}$, to Planck preferred values [12]. We are thus left to sample using MCMC only $f_{NL}$, and, for each skycut, $b_1, b_2, b_5$. Therefore the sampling of the likelihood is extremely fast: a chain typically runs in 10 minutes on a single-core processor. All our results are presented with Gelman-Rubin convergence $R-1 < 1\%$, and are obtained with the Metropolis-Hasting algorithm as implemented in MontePython v3.3. We have checked that our results are robust upon change of the sampler to PolyChord [120, 121].

Prior: We impose an uninformative large flat prior on $f_{NL}$. For $b_1$, which is positive-definite, we choose a lognormal prior of mean 0.8 (since $e^{0.8} = 2.23$), and variance 0.8, such that $[0, 3.4]$ is the 68% bound for this prior on $b_1$. For the remaining EFT parameters, including $b_2$ and $b_5$, we use Gaussian priors centered on 0, with various widths $\sim O(b_1)$, to keep them within physical range.\(^{18}\) For the other parameters entering in the power spectrum, we use the same prior widths as described in [4], while for the additional parameters entering only in the bispectrum, we use prior widths as described in [91]. The only changes with respect to those references are for the stochastic parameters, where we use a prior of width of $1/2$ on $e_1$ and $e_5$, and a prior of width of 1 on $d_1$ (respectively $c_{1}^{St}$, $2c_{6}^{St}$, and $c_{1}^{(222)}$ in the notation of [92]). This choice is made to account for contributions degenerate with shot noise, such as corrections from fiber collisions [122]. We also assume a small correlation between the skycuts given the small redshift evolution of the EFT parameters and differences between the selection functions, as described in [91].

\(^{18}\)Note that $b_5$ is equal to $b_4$ in the notations of [4].
We further impose a prior on the size of the loop using considerations from the perturbative nature of our predictions, following [103]. Because it will capture most of the effect that we seek, we work in real space, i.e. with \( f = 0 \), and we use the superscript “\( h \)” to represent quantities in App. A with \( f = 0 \). The next order terms that we are not including in our prediction are two-loop terms. A good proxy for their sizes are

\[
|P_2^{\text{1-loop}}| \sim (P_{11}^{\text{1-loop}})^2 / P_{11}^{\text{1-loop}}, \quad |B_2^{\text{1-loop}}| \sim |B_{11}^{\text{1-loop}} P_{11}^{\text{1-loop}}| / P_{11}^{\text{1-loop}}.
\]

(44)

Around the highest wavenumber included in our analysis, these start to become sizeable with respect to the data error bars, such that

\[
|P_2^{\text{1-loop}}(k \sim k_{\text{max}})| \sim X \sigma_P^{k_{\text{max}}}, \quad |B_2^{\text{1-loop}}(k_1 \sim k_2 \sim k_3 \sim k_{\text{max}})| \sim X \sigma_B^{k_{\text{max}}},
\]

(45)

where \( \sigma_P^{k_{\text{max}}} \) and \( \sigma_B^{k_{\text{max}}} \) are the data error bars around \( k \sim k_{\text{max}} \) for the power spectrum and bispectrum, respectively, and \( X \) is related to our error tolerance (we choose \( X = 1/3 \) here). This is essentially the definition of \( k_{\text{max}} \). Thus, equating Eq. (44) and Eq. (45), a good proxy for the size of the one-loop terms around the highest wavenumber included in our analysis is

\[
P_{11}^{\text{1-loop}} \sim \sqrt{X \sigma_P^{k_{\text{max}}} P_{11}^{\text{1-loop}}(k_{\text{max}})}, \quad B_{11}^{\text{1-loop}} \sim X \sigma_B^{k_{\text{max}}} P_{11}^{\text{1-loop}}(k_{\text{max}}). \quad (46)
\]

Next, the expected scalings of the one-loop power spectrum and counterterm are well approximated by (see [103])

\[
S_1^{\text{1-loop}}(k) \sim b_1^2 P_{11}(k) \left( \frac{k}{k_{\text{NL}}} \right)^{3+n(k)},
\]

(47)

\[
S_{\text{ct}}^{\text{1-loop}}(k) \sim 2b_1 P_{11}(k) \left( \frac{k}{k_{\text{NL}}} \right)^2,
\]

(48)

where \( P_{11} \) is the linear matter power spectrum, and \( n(k) \equiv d \log P_{11}/d \log k \). Similarly, for the one-loop bispectrum, the expected scalings are

\[
S_1^{\text{1-loop}}(k_1, k_2, k_3) \sim B_{211}^{\text{1-loop}}(k_1, k_2, k_3) \sum_{i=1}^3 \left( \frac{k_i}{k_{\text{NL}}} \right)^{3+n(k_i)},
\]

(49)

\[
S_{\text{ct}}^{\text{1-loop}}(k_1, k_2, k_3) \sim 2b_1^2 \left( P_{11}(k_1) P_{11}(k_2) \left( \frac{k_3}{k_{\text{NL}}} \right)^2 + 2 \text{ cyc.} \right).
\]

(50)

By defining \( \sigma_P \propto \max(S_1^{\text{1-loop}}, S_{\text{ct}}^{\text{1-loop}}) \) and \( \sigma_B \propto \max(|S_1^{\text{1-loop}}|, S_{\text{ct}}^{\text{1-loop}}) \) normalized so that \( \sigma_P^{k_{\text{max}}} = 1 \) and \( \sigma_B^{k_{\text{max}}} = 1 \), the expected sizes for the one-loop contributions in our predictions are, for all wavenumbers,

\[
\sigma_P^{\text{1-loop}}(k) \sim S_P(k) P_{11}^{\text{1-loop}}(k), \quad \sigma_B^{\text{1-loop}}(k_1, k_2, k_3) \sim S_B(k_1, k_2, k_3) B_{11}^{\text{1-loop}}(k_1, k_2, k_3).
\]

(51)

Now, by choosing to penalize the likelihood when the size of the one-loop contribution exceeds its expectation in perturbation theory, potential unphysical predictions will not contribute to the posterior. We thus add the following ‘perturbativity’ prior to \(-2 \log \mathcal{L} \):

\[
\mathcal{P}_P = \frac{1}{2 N_{\text{bins}}} \sum_{i \in \text{bins}_P} \left( \frac{P_{11}^{\text{1-loop}}(k_i)}{\sigma_{P \text{ct}}^{\text{1-loop}}(k_i)} \right)^2,
\]

(52)
when analyzing the power spectrum at one-loop, and

\[
P_B = \frac{1}{2N^B_{\text{bins}}} \sum_{i \in \text{bins}_B} \left( \frac{B^h_{1\text{-loop}}(k^i_1, k^i_2, k^i_3)}{\sigma^B_{PF}(k^i_1, k^i_2, k^i_3)} \right)^2,
\]

when analyzing the bispectrum at one-loop. \(N^P_{\text{bins}}\) and \(N^B_{\text{bins}}\) are the total number of bins fit in the analysis of the power spectrum or of the bispectrum, respectively, such that a global departure of the size of the one-loop with respect to its expectation would penalize \(-2 \log \mathcal{L}\) by \(\mathcal{O}(1)\), independently of the number of bins.

For reference, we obtain the following constraints for our analysis with the one-loop bispectrum, but with no perturbativity prior: \(f_{\text{equil.}}^{\text{NL}} = -20 \pm 319\), \(f_{\text{orth.}}^{\text{NL}} = -67 \pm 86\), and \(f_{\text{loc.}}^{\text{NL}} = 78 \pm 50\) at 68\% CL.

**Modeling aspects:** To make contact with observations, we extend the EFTofLSS predictions to additional modeling aspects. For the power spectrum, we correct for the Alcock-Paczynski effect [123], window functions, and binning as done in PyBird [11], and we also include the integral constraints as described in [124].

For the bispectrum, the corresponding corrections are described in details in [91]. Here we give a quick summary of the additional modeling in the bispectrum. First, we implement the IR-resummation as in [91]. Second, the Alcock-Paczynski effect is corrected at tree level, while we estimate it to be negligible in the loop. Third, we implement the window function only at tree level and as approximately as in [4], and we estimate that the error, for the scales analyzed, is negligible with respect to BOSS error bars.

Finally, one can see from Eq. (42) that several bins we analyze contain fundamental triangles that are not closed (i.e. the bin centers do not form a triangle, but there are closed triangles within the bins). To properly account for them, and also for the fully closed lower \(k\)-bins, a binning scheme for the theory model is needed. The procedure we implement is outlined in detail in [91], and is found to be robust for the analysis of BOSS data. In brief, the tree-level part is binned exactly while the loop, being smaller, is evaluated on effective wavenumbers. In this work, we also bin exactly the pieces proportional to \(f_{\text{NL}}\), which turns out to be important especially for the local and the orthogonal shapes.

The goodness of all these approximations is confirmed by the fact that we find no evidence of significant theoretical systematic error in the N-series ‘cutsky’ simulations, that are mimicking the geometry of BOSS CMASS NGC. As we analyze them using predictions for the bispectrum with approximate window function and IR-resummation, with our binning prescription and with the correction for Alcock-Paczynski effect only at tree level in the bispectrum, we conclude that our cosmological results are robust against the modeling aspects discussed here.

See https://github.com/pierrexyz/fkpwin for the practical implementation of the geometrical effects.
Figure 4: Triangle plots obtained fitting the BOSS data in an analysis where we vary, together with the EFT parameters, only $f_{\text{equil}}^{\text{NL}}$ (red), only $f_{\text{orth}}^{\text{NL}}$ (blue), and jointly $f_{\text{equil}}^{\text{NL}}$ and $f_{\text{orth}}^{\text{NL}}$ (grey). The analysis on the BOSS data reveals no evidence of primordial non-Gaussianities of these kinds. $f_{\text{equil}}^{\text{NL}}$ and $f_{\text{orth}}^{\text{NL}}$ appear to have a correlation of $\sim -0.3$.

C Plots

In Figs. 4, 5, 6, and 7, we present the posteriors obtained in the main analyses of this work. The shown EFT parameters are the ones of CMASS NGC. As the contours for the other smaller skycuts look similar except that they have larger error bars, we do not shown them for clarity.

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Figure 5: Triangle plots obtained fitting the N-series data (with ‘$P$’ being ‘power spectrum’ and ‘$B$’ being ‘bispectrum’) in an analysis where we vary, together with the EFT parameters, $f_{\text{loc}}^\text{NL}$, or $f_{\text{orth}}^\text{NL}$. The N-series data represent the average over 84 boxes each of volume comparable to the BOSS one, and we show the results obtained with the total covariance of volume $V_{\text{tot}}$ or of volume corresponding to one box $V_{\text{BOSS}}$. This allows us to conclude that the analysis has no large theoretical systematic error.
Figure 6: Triangle plots obtained fitting the BOSS data (with ‘P’ being ‘power spectrum’ and ‘B’ being ‘bispectrum’) in an analysis where we vary, together with the EFT parameters, $f^\text{loc}_{\text{NL}}$. The analysis on the BOSS data reveals no evidence of primordial non-Gaussianity of this kind.
Figure 7: Triangle plots obtained fitting the N-series data (with ‘P’ being ‘power spectrum’ and ‘B’ being ‘bispectrum’) in an analysis where we vary, together with the EFT parameters, $f_{NL}^{loc}$. The N-series data represent the average over 84 boxes each of volume comparable to the BOSS one, and we show the results obtained with the total covariance of volume $V_{tot}$ or of volume corresponding to one box $V_{BOSS}$. This allows us to conclude that the analysis has no large theoretical systematic error.
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