Dynamics of nonspherical bubble in compressible liquid under the coupling effect of ultrasound and electrostatic field

Jin-Jie Deng a,b, Ri-Fu Yang b,*, Hai-Qin Lu c

a Department of Mechanical and Electrical Engineering, Yangjiang Polytechnic, Yangjiang 529500, China
b School of Physics, South China University of Technology, Guangzhou 510640, China
c College of Light Industry and Food Engineering, Guangxi University, Nanning 530004, China

ABSTRACT

A model for a nonspherical bubble in a compressible liquid under the coupling effect of ultrasound and electrostatic field was developed in this study. The following assumptions are made: (1) the bubble undergoes adiabatic oscillation; (2) the gravity of the liquid is negligible; (3) the bubble is insulating. If the speed of sound approaches infinity (c→∞), the equation set is reduced to the equation set for an incompressible liquid. We found that, under ultrasonic irradiation coupled with electric stress, a nonspherical bubble cannot oscillate steadily in the liquid. The bubble is bound to collapse during several cycles. The presence of electric stress reduces the surface tension at the bubble wall, which produces a larger maximum bubble-radius during the rarefaction cycle and a smaller minimum bubble-radius during the compression cycle. Consequently, during the collapse, both the gas pressure and the temperature in the bubble center increase substantially, if the bubble is exposed to both ultrasound and electrostatic field instead of ultrasound alone. In addition, the cavitation threshold of the bubble within an electrostatic field decreases significantly, compared to the bubble without an electrostatic field. In general, bubble cavitation occurs more easily and violently in the liquid after the introduction of an electrostatic field.

1. Introduction

Low-frequency ultrasonic irradiation is a powerful method that is often used to enhance important production/modification processes. It has been employed in both commercial production and research for many years[1–3]. Recently, some researchers reported that ultrasonic cavitation effect can be strengthened significantly by using an electrostatic field. According to a study by Geng et al. [4], the yield of flavonoids could be increased by 10% when ultrasound-synergized electrostatic field extraction was used instead of conventional ultrasound extraction. Furthermore, Jung et al. [5] used electric field-assisted ultrasonication to disintegrate waste-activated sludge. The group achieved an increase by about 47% of the disintegration rate compared to using ultrasonication alone. Results like these suggest that the synergistic effect of ultrasonic action and electrostatic force has a great potential to enhance the effect/yield of many physical or chemical processes, such as ultrasonic cleaning, waste-water treatment, extraction, oil molecule reactions, and sterilization. Nevertheless, few studies focused on revealing the mechanism of ultrasonic cavitation supported by an electrostatic field.

Many researchers reported significant progress in improving our understanding of the cavitation dynamics of the bubble and the effect of electric force on the bubble. Klapskii et al. [6] and Rath [7] developed two different models to describe the oscillation of a nonspherical bubble for both viscous liquid and nonviscous liquid. The models aimed to suit a native nonspherical bubble. A study by Zaghdoudi et al. [8] demonstrated that the bubble was elongated in the direction parallel to the uniform electric field and thus reached a large aspect ratio. Hongray et al. [9,10] provided a radial model for a spherical charged bubble and found that the charges distributed at the bubble interface were able to enhance the ultrasonic cavitation effect. In addition, various mathematical models were proposed to describe the oscillation of a spherical cavitation bubble in a compressible liquid [11–16].

However, none of these models can be used to successfully describe the behavior of a nonspherical bubble in a compressible liquid under the synergistic effect of ultrasound and electrostatic field. First, without consideration of the gravity of the liquid, the bubble should be spherical initially. It is stretched into an ellipsoid only after an electrostatic field.
was applied to the liquid. Hence, the model should be able to describe how electric stress deforms the bubble. Second, in our study, the bubble is assumed to be insulating, which means the electric stress cannot be explained by the charge distribution at the bubble wall. Third, during the collapse, the bubble interface velocity is very high, possibly even close to the speed of sound in the liquid, which means the liquid’s compressibility must be considered by the model. The required model cannot be obtained by simply modifying the model for a spherical bubble in a compressible liquid because the variation of the nonspherical term with time cannot be directly deduced from the latter.

This study aims to develop a model for a single nonspherical bubble in a compressible liquid under the coupling effect of ultrasound and electrostatic field. The interactions between adjacent bubbles are ignored. We also investigate the effect of an electrostatic field on the bubble in a compressible liquid because the variation of the nonspherical component for different electric field strengths. Furthermore, we applied the approximate equations to compare the variations of the bubble radius, the gas pressure and the temperature inside the bubble, and the cavitation threshold, with and without an electrostatic field. This practical model may be used to better describe the coupling effect of ultrasound and electrostatic field and provide better guidance for experiments and industrial applications.

2. Model for a nonspherical bubble

2.1. Equation for the bubble wall

To depict a nonspherical bubble, we introduce a nonspherical function \( S(\theta, t) \) with rotational symmetry, which is independent of the rotation angle \( \phi \). The bubble-wall equation, in spherical coordinates, is given by

\[
S(\theta, t) = a_0(t) + a_2(t) \rho_2(\cos \theta)
\] (1)

where \( \theta \) is the azimuth, and \( S(\theta, t) \) is the bubble radius, namely the distance from the bubble wall to the bubble center; \( \rho_2(\cos \theta) \) is the second-order Legendre function, which is

\[
\rho_2(\cos \theta) = \frac{1}{2} (3\cos^2 \theta - 1)
\] (2)

\( a_0(t) \) is the spherical radius component; \( a_2(t) \) is the coefficient of the nonspherical radius component, which describes the shape deformation of the bubble; \( \epsilon \) is a small parameter, which indicates that the nonspherical component is much smaller than the spherical component. Fig. 1 shows the section of a nonspherical bubble, which is elliptic.

2.2. Electric stress at the liquid–gas interface

When an electrostatic field is applied on the liquid region, it will generate an electric stress vector \( f_s \) (an electrostatic force vector per unit area) at the bubble interface, which consists of a gas-phase component \( f_t \) and a liquid-phase component \( f_l \), namely,

\[
f_s = f_t + f_l
\] (3)

The equations for \( f_t \) and \( f_l \) [17,18] are given by

\[
f_t = e_0\left(\varepsilon_d - 1\right)^2 \frac{3}{6} E^2 \n_t
\]

where \( E \) is the liquid-phase electric field strength and the gas-phase electric field strength; \( E_0 \) and \( E_l \) are the normal component and the tangential component of \( E \); \( \rho_l \) and \( \rho_g \) are the density of the liquid and the density of the bubble gas; \( n_t \) is the unit normal vector directed towards the gas phase, and \( n_t = -n_l \) (Fig. 1); \( e_l \) and \( e_g \) are the permittivity of the liquid and the permittivity of the bubble gas. For the gas medium, \( e_g \approx n_{in} \), where \( n_{in} \) is the vacuum permittivity. For a dielectric bubble, without electrical charges at the liquid–gas interface, the calculation result of Eq. (3) can be found in a study by Zaghdoudi et al. [8], which is as follows,

\[
f_s = -E_0\left(\varepsilon_d - 1\right)^2 \frac{3}{6} E^2 \n_t
\] (4)

Using the relations:

\( E_{in} = E_0 \cos \theta \), \( E_l = E_0 \sin \theta \)

Eq. (4) can be further transformed into

\[
f_s = -E_0\left(\varepsilon_d - 1\right)^2 \frac{3}{6} E^2 \rho_2(\cos \theta) \n_t
\]

where \( \varepsilon_d \) is the relative permittivity, \( \varepsilon_d = e_l/e_0 \); the expression of \( \rho_2(\cos \theta) \) see Eq. (2). Omitting the subscript of \( E \), we can rewrite the value of the electric stress vector as

\[
f_s = E_0\left(\varepsilon_d - 1\right)^2 \frac{3}{6} E^2 \rho_2(\cos \theta)
\] (6)

The positive direction of \( f_s \) points out of the bubble, while the negative direction of \( f_s \) points into the bubble.

2.3. Total pressure difference on the liquid region outside the bubble

From Section 2.2, we can conclude that the electric stress \( f_s \) acts on the bubble wall, stretching the bubble in the electric field direction and compressing the bubble in the direction vertical to the electric field. In
addition, in the electric field direction, the effect of electric stress should be opposite to the effect of the surface tension. The bubble is assumed to be spherical initially, which undergoes a deformation after the introduction of the electrostatic field to the liquid. Only the electric stress acts on the nonspherical radius component, while all other forces act on the spherical radius component. In other words, the electric stress on the nonspherical radius component, while all other forces act on the reduction of the electrostatic field to the liquid. Only the electric stress acts in the bubble; accordingly to simplify the calculation, we consider the bubble to be approximately spherical for the force analysis (see the right side of Eq. (7)). Rearranging Eq. (7) yields

\[ \Delta P = (p_0 + \frac{2\sigma}{R_0} - p_s - \varepsilon f_s) \left( \frac{R_0}{R} \right)^2 + p_s - p_0 - \frac{2\sigma}{R} - \frac{4\eta R}{R^2} + \varepsilon f_s - \varepsilon p_s \cos \theta \]  

where \( p_0 \) is the static pressure of the liquid, and \( p_s \) is the vapour pressure in the bubble; \( \sigma \) is the surface tension at the liquid–gas interface, \( \eta \) is the viscosity of the liquid, and \( \gamma \) is the adiabatic index; \( R_0 \) is the initial bubble radius, \( R \) is the radius of the bubble; for a nonspherical bubble, \( R = a(t); p_0 \) is the acoustic pressure amplitude, and \( a(t) \) is the angular frequency of the acoustic wave. For an elliptical bubble described by Eq. (1), the nonspherical term \( a(t)P_2(\cos \theta) \) is small, compared to the spherical term \( a(t) \). Accordingly, to simplify the calculation, we consider the bubble to be approximately spherical for the force analysis (see the right side of Eq. (7)). Rearranging Eq. (7) yields

\[ \Delta P = (p_0 + \frac{2\sigma}{R_0} - p_s - \varepsilon f_s) \left( \frac{R_0}{R} \right)^2 + p_s - p_0 - \frac{2\sigma}{R} - \frac{4\eta R}{R^2} + \varepsilon f_s - \varepsilon p_s \cos \theta \]

Introducing three notations:

\[ f_s = \left[ 1 - \left( \frac{R_0}{R} \right)^2 \right] \] \[ F_s = \frac{1 - \left( \frac{R_0}{R} \right)^2}{\varepsilon} f_s = f_s / P_2(\cos \theta) \]

\[ \Delta P_1 = (p_0 + \frac{2\sigma}{R_0} - p_s) \left( \frac{R_0}{R} \right)^2 + p_s - p_0 - \frac{2\sigma}{R} - \frac{4\eta R}{R^2} - \varepsilon f_s - \varepsilon p_s \cos \theta \]

then Eq. (8) can be written as

\[ \Delta P = \Delta P_1 + \varepsilon F_s \]

\[ = \Delta P_1 + \varepsilon f_s / P_2(\cos \theta) \]

2.4. Derivation of the mathematical model

Let \( u \) be the velocity of a liquid volume element and \( \Phi(r, \theta, t) \) be the corresponding potential velocity, so that

\[ u = \nabla \Phi(r, \theta, t) \]

Assuming that \( \Phi(r, \theta, t) \) can be written as

\[ \Phi(r, \theta, t) = \Phi_0(r, \theta, t) + \varepsilon \Phi_2(r, \theta, t) \]

where \( \Phi_0(r, \theta, t) \) and \( \Phi_2(r, \theta, t) \) are the spherical component and the nonspherical component of the velocity potential. In a compressible liquid, \( \Phi(r, \theta, t) \) should satisfy the following equation [19],

\[ \nabla^2 \Phi - \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} = 0 \]

where \( c \) is the speed of sound in the liquid. Substituting Eq. (14) into Eq. (15) yields

\[ \nabla^2 \Phi_0 - \frac{1}{c^2} \frac{\partial^2 \Phi_0}{\partial t^2} + \varepsilon \left( \nabla^2 \Phi_2 - \frac{1}{c^2} \frac{\partial^2 \Phi_2}{\partial t^2} \right) = 0 \]  

The above equality holds only if each order coefficient of \( \varepsilon \) equals zero, that is

\[ \nabla^2 \Phi_0 - \frac{1}{c^2} \frac{\partial^2 \Phi_0}{\partial t^2} = 0 \]

\[ \nabla^2 \Phi_2 - \frac{1}{c^2} \frac{\partial^2 \Phi_2}{\partial t^2} = 0 \]  

Assuming that the acoustic wave propagates within an infinitely large liquid region, then no reflected wave occurs in the liquid. Therefore, we may only take the outward traveling wave solutions of the Eqs. (17). Transforming Eq. (17a) into spherical coordinates, we obtain

\[ \frac{\partial^2 \Phi_0}{r^2 \partial \theta^2} - \frac{1}{r^2} \frac{\partial \Phi_0}{\partial \theta} + \frac{1}{r^2} \frac{\partial^2 \Phi_0}{\partial \theta^2} = 0 \]

where \( r \) is the distance from a point within the liquid region to the bubble center, \( r \approx s(\theta, t) \). The outward traveling wave solution of Eq. (18) is

\[ \Phi_0(r, t) = A_0 \sin(\omega t - kr) \]

where \( A_0 \) is a free constant; \( k \) is the wave number, \( k = \omega/c \). Furthermore, \( \Phi_2(r, \theta, t) \) is assumed to be of the following form,

\[ \Phi_2(r, \theta, t) = J(r, t)P_2(\cos \theta) \]

The reason is that \( \Phi_2(r, \theta, t) \) should be consistent with the nonspherical radius component \( a(t)P_2(\cos \theta) \) in form. Thus, the outward traveling wave solution of Eq. (17b) can be obtained as follows:

\[ \Phi_2(x, \theta, t) = C_2 \left\{ 3(\sin(\omega t - x) + \cos(\omega t - x)) \right\} \]

\[ - \sin(\omega t - x) \}

where \( C_2 \) is a free constant and \( x = kr \). The derivation of Eq. (21) is given in A. Similarly, by replacing \( kr \) with \( x \), we can rewrite Eq. (19) as

\[ \Phi_0(x, t) = A_0 \sin(\omega t - x) \]

When \( r \) is in the vicinity of the bubble radius, \( r \) is very small, then \( x \) is also very small. Hence, we have the following approximate equalities, \( \sin x \approx x, \cos x \approx 1 \)

Substituting them back into Eq. (22) and Eq. (21) respectively, we obtain

\[ \Phi_0(x, t) = A_0 \frac{1}{x} (\sin(\omega t - x) \cos x - \cos x \sin x) \]

\[ \approx A_0 \frac{1}{x} (\sin(\omega t - x) - x \cos x) \]

\[ = \frac{1}{x} A(t) - \frac{1}{\omega} \delta(t) \]

\[ \Phi_2(x, \theta, t) \approx \frac{P_2(\cos \theta)}{x^3} \left\{ 3(\sin(\omega t - x) \cos x + x \sin x) \right\} \]

\[ + 3(x \cos x - x^2 \cos x - x^3 \sin x) \}

\[ = \frac{P_2(\cos \theta)}{x^3} \left\{ (3 + 2x^2) \sin(x - x^2) \cos x + x^2 \sin x \cos x \right\} \]  

where
\[ A(t) = A_0 \sin \theta, \quad \dot{A}(t) = A_0 \cos \theta \]
\[ B(t) = C_0 \sin \theta, \quad \dot{B}(t) = C_0 \cos \theta \]

The gradient of the velocity potential at the bubble interface should be equal to the bubble interface velocity, namely,

\[
\frac{\partial \Phi}{\partial t} = \frac{dS(\theta, t)}{dt} = 0
\]

Substituting Eq. (14), Eq. (23) and Eq. (24) into Eq. (25) yields

\[
-\frac{A(t)}{kS^2(\theta, t)} + \epsilon \left[ 9B(t) - 2B(t) \right] P_2(\cos \theta) = \frac{dS(\theta, t)}{dt}
\]

Then substituting Eq. (1) and Eq. (27) into Eq. (26), using the Taylor series expansion of \( \epsilon \), and neglecting the Peano remainder \( o(\epsilon^2) \), we obtain

\[
\left[ \dot{a}_0(t) + \frac{1}{k a_0^2(t)} A(0) \right] + \epsilon \left[ \frac{2a_2(t)}{k a_0^2(t)} A(t) + \frac{9}{k^2 a_0^2(t)} B(t) \right] P_2(\cos \theta) = 0
\]

In the above equation, each order coefficient of \( \epsilon \) should be equal to zero, thus we obtain

\[
\dot{a}_0(t) + \frac{1}{k a_0^2(t)} A(0) = 0
\]

\[
\dot{a}_2(t) + \frac{2a_2(t)}{k a_0^2(t)} A(t) + \frac{9}{k^2 a_0^2(t)} B(t) = 0
\]

In addition, if the gravity of the liquid is negligible, then, at the bubble interface, \( \Phi(\theta, t) \) obeys the following fluid mechanics equation,

\[
\left[ \frac{\partial \Phi}{\partial t} + \frac{1}{2} \nabla \Phi \cdot \nabla \Phi \right]_{\theta = \theta(\theta)} = -\frac{p(r) - p(\infty)}{\rho} = -\Delta P = 0
\]

where the expression of \( \Delta P \) see Eq. (12). Eq. (31) is valid for both compressible liquid and incompressible liquid. The derivation of Eq. (31) is given in B. Following similar steps to Eq. (25), substituting Eq. (1), Eq. (14), Eq. (23) and Eq. (24) into Eq. (31), using the Taylor series expansion of \( \epsilon \), and only retaining the first two order terms for \( \epsilon \), we obtain

\[
\dot{a}_0(t) + \frac{1}{k a_0^2(t)} A(0) + \frac{A(t)}{k a_0^2(t)} = \frac{\Delta P}{\rho}
\]

\[
\dot{a}_2(t) + \frac{2a_2(t)}{k a_0^2(t)} A(t) + \frac{9}{k^2 a_0^2(t)} B(t) = 0
\]

Eliminating \( A(t), \dot{A}(t), B(t) \) and \( \dot{B}(t) \) from the equations consisted by Eq. (29), Eq. (30), Eq. (32) and Eq. (33), we obtain the model for a nonspherical bubble in a compressible liquid under the coupling effect of ultrasonic and electrostatic field,

\[
\frac{a_0(t) \dot{a}_0(t) + \frac{3}{2} a_2^2(t) + k a_0^2(t) \dot{a}_0(t)}{3} = \frac{\Delta P}{\rho}
\]

\[
\frac{N}{M} \dot{a}_0(t) \dot{a}_2(t) + Z \dot{a}_0(t) + Z \dot{a}_2(t) = \frac{f_c}{\rho}
\]

3. Numerical analyses and discussion

3.1. Numerical solution of the model for a nonspherical bubble

By solving the Eqs. (34) numerically, we obtain the temporal evolutions of the spherical radius component \( a_0(t) \) and the coefficient of the nonspherical radius component \( a_2(t) \) (Fig. 2). The parameters for Fig. 2 are as follows: the density of water \( \rho = 1000 \text{ kg} \cdot \text{m}^{-3} \), the adiabatic index \( \gamma = 1.4 \), the static pressure of water \( P_0 = 1.013 \times 10^5 \text{ Pa} \), the angular frequency of acoustic wave \( \omega = 2 \pi \times 10^3 \text{ rad} \cdot \text{s}^{-1} \), the period of acoustic wave \( T = 50 \mu \text{s} \), the viscosity of water \( \eta = 8.9 \times 10^{-4} \text{ kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1} \), the surface tension at the liquid–gas interface \( \sigma = 7.6 \times 10^{-2} \text{ N} \cdot \text{m}^{-1} \), the vapour pressure of water \( p_v = 2334.8 \text{ Pa} \), the acoustic pressure amplitude \( p_a = 1.35p_v \), the initial bubble radius \( R_0 = 4.5 \mu \text{m} \), the speed of sound \( c = 1480 \text{ m} \cdot \text{s}^{-1} \), the electric field strength \( E = 2 \text{ MV} \cdot \text{m}^{-1} \), the vacuum permittivity \( \varepsilon_0 = 8.854 \times 10^{-12} \text{ F} \cdot \text{m}^{-1} \), the relative permittivity \( \varepsilon_r = 78.3 \), \( a_0(0) = R_0 = 4.5 \mu \text{m} \) and \( \dot{a}_0(0) = a_2(0) \)
= a_2(0) = 0. In Fig. 2, a_2(t) increases to more than ten times its initial value during the rarefaction cycle. It then decreases sharply to its minimum within a short period of time, followed by a series of rebounds. Meanwhile, a_2(t) continues to increase rapidly with time over several periods.

Fig. 3 shows the variation of a_2(t) with time for different electric field strengths. At a higher electric field strength, a_2(t) increases faster, which causes the bubble to collapse more violently. Additionally, we consider a bubble (with the initial radius R_0 = 1 μm) to undergo ultrasonic cavitation at an electric field strength E = 0.2 MV m⁻¹, see Fig. 4. The spherical term a_0(t) vibrates slightly near the initial radius R_0. At the same time, a_2(t) still increases slowly over multiple periods. This causes the tiny bubble to expand to a very large size after multiple rarefaction cycles, and then collapse during a subsequent compression cycle. In conclusion, at an electric field strength E≥0.2 MV m⁻¹, a nonspherical bubble cannot oscillate steadily in the liquid. The bubble is bound to collapse during several cycles.

3.2. Effect of electrostatic field during bubble cavitation

In fact, it is impossible for the coefficient a_2(t) to increase without limits. Usually, a_2(t) undergoes a rapid increase during one or several expansion cycles before it decreases substantially with the spherical radius component a_0(t) in a subsequent compression cycle. After the collapse of the bubble, a_2(t) ceases to increase. The bubble shape becomes more spherical rather than nonspherical. Therefore, the Eqs. (34) cannot be used to directly compare the bubble cavitation with and without an electrostatic field. To this end, a feasible way is to take the mean value of the electric stress f_0 in half period, approximately transforming the nonspherical bubble into the spherical bubble. This method is based on the that the nonspherical radius component a_2(t)P_2(cosθ) is very small, compared to the spherical radius component a_0(t). The mean value for the integral of f_0 in half period is

\[ \bar{f}_0 = \frac{2}{R} \int_0^R f_0 \, d\theta \]

\[ \bar{f}_0 = \frac{2}{R} \int_0^R \epsilon_0 \left( \frac{\epsilon_0 - 1}{3} \right) E^2 P_2(\cos \theta) \, d\theta \]

\[ \bar{f}_0 = \epsilon_0 \left( \frac{\epsilon_0 - 1}{3} \right) E^2 \]

Noting that, in Section 3, all equations containing \( \bar{f}_0 \) can only be used to analyze the enhancement effect of an electrostatic field on the bubble cavitation qualitatively.

3.2.1. Effect of electrostatic field on the bubble radius

By inserting the term \( \bar{f}_0 \) into the right side of Eq. (34a), and replacing a_0(t) with R, we can rewrite Eq. (34a) as

\[ R \bar{f}_0 + \frac{3}{2} \frac{\gamma}{\rho} + k_0 R \bar{R} = \frac{\Delta P_M}{\rho} \]

where

\[ \Delta P_M = \left( p_0 + 2\frac{\sigma}{R_0} + p_r - \epsilon_0 \bar{f}_0 \right) \left( \frac{R_0}{R} \right)^3 \left( p_r - p_0 - \frac{2\sigma}{R} \right) - \frac{4\eta \bar{R}}{R} \epsilon_0 \bar{f}_0 \]

Eq. (37) is an approximate equation to describe the behavior of a cavitation bubble with an electrostatic field. In this equation, the condition that E = 0, namely \( \bar{f}_0 = 0 \), describes the behavior of a cavitation bubble without an electrostatic field. In the following analyses, we assume that \( \epsilon = 0.1 \). This assumption is only made to simplify the calculation. As long as \( \epsilon > 0 \), all the conclusions below will not change with the variation of \( \epsilon \) value. The variation of the bubble radius in the presence and absence of an electrostatic field are shown in Fig. 5. All parameters are the same as Fig. 2. The overall effect of the electric stress
is a significant reduction of the surface tension at the bubble interface. Hence, during the negative pressure phase, the bubble with electric stress expands more and has a larger maximum radius $R_{\text{max}}$ than without electric stress. When the positive pressure phase comes, a larger maximum bubble-radius $R_{\text{max}}$ leads to a larger collapse velocity and therefore a smaller minimum bubble-radius $R_{\text{min}}$. Hongray et al. have reported this phenomenon in one of their works [10]: The presence of the charges can effectively decrease the surface tension, which leads to a greater bubble expansion. However, the effect of electric stress is quite different with the effect of charges. For a conducting bubble, the charges reduce the surface tension by the electrostatic repulsion. For an insulating bubble, however, the electric stress reduces the surface tension by changing the bubble shape (stretching the bubble into an ellipsoid). Theoretically, at the end of the bubble collapse, the minimum radius $R_{\text{min}}$ can reach zero as the electric field strength increases infinitely. How stress expands more and has a larger maximum radius is a significant reduction of the surface tension at the bubble interface.

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For air, $h = R_0 / 8.54$, where $R_0$ is the initial bubble radius. Usually, $R_{\text{min}}$ can be $h$ only for a very large electric field strength.

3.2.2. Effect of electrostatic field on the gas pressure and the temperature inside the bubble

The gas pressure and the temperature inside the bubble are determined by the degree of compression of the bubble. Thus, the highest gas pressure and temperature in the bubble center are reached at the moment of total collapse, when the bubble is contracted to its smallest size. More precisely, with an electrostatic field, the gas pressure $P_\varepsilon$ and the temperature $T_\varepsilon$ in the bubble center [10,20] are given by

$$
P_\varepsilon = \left( p_0 + \frac{2\sigma}{R_0} - p_\varepsilon - \frac{\varepsilon f}{R_0^3} \right) \left( \frac{R_0^5}{R_0^5 - h^5} \right)^\gamma
$$

$$
T_\varepsilon = T_0 \left( \frac{R_0^5}{R_0^5 - h^5} \right)^{\gamma^{-1}}
$$

where $T_0$ is the ambient temperature, $T_0 = 298.15$ K, $R_0$ is the bubble radius with an electrostatic field, and $h$ is the van der Waals hard-core radius. For a bubble with the initial radius $R_0 = 4.5$ μm, $h = R_0 / 8.54 \approx 0.53$ μm. For $E = 0$, namely $f = 0$, the equations in the absence of an electrostatic field can be written as

$$
P_\varepsilon = \left( p_0 + \frac{2\sigma}{R_0} - p_\varepsilon \right) \left( \frac{R_0^5}{R_0^5 - h^5} \right)^\gamma
$$

$$
T_\varepsilon = T_0 \left( \frac{R_0^5}{R_0^5 - h^5} \right)^{\gamma^{-1}}
$$

where $R_0$ denotes the bubble radius in the absence of an electrostatic field. The temporal variations of $P_\varepsilon$ and $T_\varepsilon$, with and without an electrostatic field, are shown in Fig. 6 and Fig. 7 respectively. All parameters are the same as the parameters for Fig. 2. As depicted in these two figures, $P_\varepsilon$ shows a similar variation trend with $T_\varepsilon$. At the end of the collapse, the bubble with electric stress undergoes a more severe compression, which causes a higher internal gas pressure and temperature, compared to the bubble without electric stress. The similar effect induced by the charges at the interface has been reported by Hongray et al. [10].

3.2.3. Effect of electrostatic field on the cavitation threshold

Under electric stress, the critical radius $R_c$ for the bubble to enter the explosive expansion phase is given by

$$
R_c = \left[ \frac{3\gamma}{2\sigma} \left( p_0 + \frac{2\sigma}{R_0} - p_\varepsilon - \frac{\varepsilon f}{R_0^3} \right) R_0^5 \right]^{\frac{1}{\gamma}}
$$

where the viscous force of the liquid is neglected. Consequently, the corresponding cavitation threshold $P_c$ of the bubble is given by

$$
P_c = p_0 - \left( p_0 + \frac{2\sigma}{R_0} - p_\varepsilon - \frac{\varepsilon f}{R_0^3} \right) \left( \frac{R_0^5}{R_c} \right)^{\gamma} - p_\varepsilon + \frac{2\sigma}{R_c} = -\frac{\varepsilon f}{R_c}
$$

The derivations of $R_c$ and $P_c$ are given in C. The curves for $R_c$ vs. $R_0$ and $P_c$ vs. $R_0$ are shown in Fig. 8 and Fig. 9, respectively. All parameters are the same as Fig. 2. The condition that $E = 0$, namely $f = 0$, describes the dependence of $R_c$ and $P_c$ on $R_0$ without an electrostatic field. For both conditions, as the initial bubble radius $R_0$ increases from 0.5 μm to 10 μm, the critical radius $R_c$ increases from nearly 0.85 μm to nearly 29 μm, and the cavitation threshold $P_c$ decreases from nearly $2.3 \times 10^5$ Pa to nearly $1 \times 10^5$ Pa. This indicates that the ultrasonic cavitation occurs more easily for a bubble with a larger initial radius. In addition, after the electrostatic field activation, the cavitation threshold $P_c$ decreases significantly. This suggests that the acoustic pressure amplitude, which is needed to trigger a nuclei cavitation, is lower than without the electrostatic field. This can be interpreted as follows: The presence of electric stress reduces the surface tension at the bubble wall, which causes a decrease of the expansion resistance of the bubble during the rarefaction cycle. Similarly, the studies by Hongray et al. [9,10] revealed
that the presence of charges can also reduce the transient threshold pressures. In general, the bubble cavitation occurs more easily and violently in water after the introduction of an electrostatic field.

4. Conclusions

A dynamic model for a nonspherical bubble in a compressible liquid under the synergistic effect of ultrasound and electrostatic field was developed in this study. A nonspherical cavitation bubble with electric stress cannot exist steadily in the liquid. It is bound to collapse during several cycles. In the presence of electric stress, the bubble reaches a higher internal gas pressure and temperature at the moment of total collapse. In addition, the cavitation threshold decreases significantly when the bubble is exposed to an electrostatic field. Hence, ultrasonic irradiation coupled with an electrostatic field has a high potential to enhance the effect/yield of many physical or chemical processes. However, the model for a single nonspherical bubble may not be able to fully depict the true cavitation effect. When a number of adjacent bubbles undergo ultrasonic cavitation simultaneously, the interactions between the bubbles should be included in the theoretical model. Accordingly, the dynamics of a nonspherical bubble cluster, under ultrasonic action assisted by an electrostatic field, will be the subject of a future study.

Appendix A. Outward traveling wave solution of Eq. (17b)

Since \( \Phi_2(r, \theta, t) \) is independent of the rotation angle \( \phi \), we assume that

\[
\Phi_2(r, \theta, t) = D(t)Q(r)P_2(\cos \theta)
\]

Hence, transforming Eq. (17b) into spherical coordinates, we obtain

\[
r^2 \frac{d^2 Q(r)}{dr^2} + 2r \frac{dQ(r)}{dr} + (k^2r^2 - 6)Q(r) = 0
\]

(A.2a)

\[
\frac{d^2 D(t)}{dt^2} + \omega^2 D(t) = 0
\]

(A.2b)

The solution of Eq. (A.2b) is

\[
D(t) = C_1 \sin \omega t + C_2 \cos \omega t
\]

(A.3)

where \( C_1 \) and \( C_2 \) are free constants. Let be \( x = kr \) and \( y(x) = Q(r) \), Eq. (A.2a) can be transformed into

\[
x^2 \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} + (x^2 - 6)y = 0
\]

(A.4)

Eq. (A.4) is a spherical Bessel equation, of which one particular solution is

\[
y(x) = \frac{1}{x^3} [3(\sin x - \cos x) - x^2 \sin x]
\]

(A.5)

Substituting Eq. (A.3) and Eq. (A.5) into Eq. (A.1) yields

\[
\Phi_2(x, \theta, t) = \frac{1}{x^3} [3(\sin x - \cos x) - x^2 \sin x]
\times (C_1 \sin \omega t + C_2 \cos \omega t) \cdot P_2(\cos \theta)
\]

which is equivalent to the following traveling wave form,

\[
\Phi_2(x, \theta, t) = [C_3 J_1(x, t) + C_4 J_2(x, t)] \cdot P_2(\cos \theta)
\]

where \( C_3 \) and \( C_4 \) are free constants, and

Fig. 9. \( P_t \) vs. \( R_0 \) curves with and without an electrostatic field (\( R_0 \in [0.5 \mu m, 10 \mu m] \), \( p_0 = 1.013 \times 10^5 \) Pa).

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Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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\[
J_1(x, t) = \frac{1}{x^3} \{3 \sin(\omega t - x) + x \cos(\omega t - x) \} - x^2 \sin(\omega t - x) \\
J_2(x, t) = \frac{1}{x^3} \{3 \sin(\omega t + x) - x \cos(\omega t + x) \} - x^2 \cos(\omega t + x)
\]

Let be \( C_4 = 0 \), we obtain the outward traveling wave solution,
\[
\Phi_2(x, \theta, t) = C_3 \cdot \frac{1}{x^3} \{3 \sin(\omega t + x) + x \cos(\omega t + x) \} x^2 \sin(\omega t + x) \cdot P_2(\cos \theta)
\]

Appendix B. Derivation of Eq. (31)

By neglecting the gravity of the liquid, we obtain the following Euler equation,
\[
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} + \frac{1}{\rho} \nabla p = 0 \tag{B.1}
\]
where \( p \) and \( \nabla p \) are the pressure and the pressure gradient on a liquid volume element. When the fluid motion is irrotational, we obtain
\[
(\mathbf{u} \cdot \nabla) \mathbf{u} = \frac{1}{2} \nabla (\mathbf{u}^2)
\]
Hence, Eq. (B.1) can be rewritten as
\[
\frac{\partial \mathbf{u}}{\partial t} + \frac{1}{2} \nabla (\mathbf{u}^2) + \frac{1}{\rho} \nabla p = 0
\]
Substituting Eq. (13) into the above equation yields
\[
\frac{\partial}{\partial t} (\nabla \Phi) + \frac{1}{2} \nabla (\nabla \Phi)^2 + \frac{1}{\rho} \nabla p = 0
\]
namely,
\[
\nabla \left[ \frac{\partial \Phi}{\partial t} + \frac{1}{2} \nabla \Phi \right] + \frac{p}{\rho} = f(t)
\]
where \( f(t) \) is an arbitrary function of time. At the bubble wall, \( r = S(\theta, t) \), we have
\[
\left. \left[ \frac{\partial \Phi}{\partial t} + \frac{1}{2} \nabla \Phi \right] + \frac{p(r)}{\rho} \right|_{r=S(\theta, t)} = f(t) \tag{B.2}
\]
For \( r \to \infty, \Phi = 0 \), we have
\[
\frac{p(\infty)}{\rho} = f(t) \tag{B.3}
\]
Eq. (B.3) minus Eq. (B.2) yields
\[
\left. \left[ \frac{\partial \Phi}{\partial t} + \frac{1}{2} \nabla \Phi \right] \right|_{r=S(\theta, t)} - \left. \frac{p(r) - p(\infty)}{\rho} \right|_{r=S(\theta, t)} = - \frac{\Delta p}{\rho}
\]
Appendix C. Derivations of critical radius \( R_c \) and cavitation threshold \( P_c \)

Neglecting the viscous force of the liquid, we can obtain the liquid pressure \( p_l \) on a bubble with a radius \( R \) under electric stress as follows,
\[
p_l = \left( p_0 + \frac{2 \sigma}{R_0} - \rho \right) \left( \frac{R_0}{R} \right)^3 + \rho \frac{2 \sigma}{R} + \sigma
\]
Let be \( dp_l/dR = 0 \), we obtain the equation that the critical radius \( R_c \) satisfies,
\[
\left( \frac{3 \rho}{R_0} \left( p_0 + \frac{2 \sigma}{R_0} - \rho \right) \left( \frac{R_0}{R} \right)^3 + \frac{2 \sigma}{R} \right)^{3/4} = 0
\]
Hence, we obtain
\[ R_c = \left[ \frac{\Delta P}{2 \Delta \sigma} \left( p_0 + \frac{2 \sigma}{R_0} - p_r - \frac{\sigma}{R} \right) R_0^2 \right]^{\frac{1}{2}} \]  

Substituting Eq. (C.2) into Eq. (C.1), we obtain the critical pressure \( p_c \) as follows,

\[ p_c = \left( p_0 + \frac{2 \sigma}{R_0} - p_r - \frac{\sigma}{R} \right) \left( \frac{R_0}{R} \right)^{\frac{3}{2}} + p_r + \frac{2 \sigma}{R} - \frac{\sigma}{R} \]

Therefore, the corresponding cavitation threshold \( P_c \) is

\[ P_c = p_0 - p_c = p_0 - \left( p_0 + \frac{2 \sigma}{R_0} - p_r - \frac{\sigma}{R} \right) \left( \frac{R_0}{R} \right)^{\frac{3}{2}} - p_r + \frac{2 \sigma}{R} - \frac{\sigma}{R} \]

Appendix D. Supplementary data

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.ultsonch.2020.105371.