Testing the Formation Scenarios of Binary Neutron Star Systems with Measurements of the Neutron Star Moment of Inertia

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1. Introduction

The double pulsar system PSR J0737-3039A/B (Burgay et al. 2003; Lyne et al. 2004) and the double neutron star (NS) system containing PSR J1756-2251 (Ferdman et al. 2014) share the following properties, which suggest a specific, shared evolutionary history (see Burgay et al. 2003; Lyne et al. 2004; Piran & Shaviv 2005; Stairs et al. 2006; Ferdman et al. 2013, 2014; Iacolina et al. 2016). The second-born NSs in both systems have similar gravitational masses: $M_1 = 1.2489 \pm 0.0007 M_\odot$ for PSR J0737-3039B, and $1.230 \pm 0.007 M_\odot$ for the companion NS to PSR J1756-2251. The systems’ orbital eccentricities (0.088 and 0.18 respectively) are relatively low, as is the alignment angle between the first-born NS’s spin and the systems’ orbital angular momentum ($< 10^\circ$ and $< 35^\circ$, respectively). The systems’ transverse velocities are small ($\sim 10 \text{ km s}^{-1}$), and they are both located close to the galactic plane, making it statistically likely that the overall system velocities are small. The first-born NSs in both systems have very stable pulse profiles.

Taken together, these observations suggest that the second-born NS in each system was formed in a supernova which delivered a very small kick to the system—i.e., a symmetric explosion which occurred on timescales too short for instabilities to develop, resulted in very little core mass loss, and was sub-energetic (Scheck et al. 2004; Podsiadlowski et al. 2005; Kitaura et al. 2006; Wong et al. 2010). A strong candidate progenitor for such an explosion is a low-mass helium star progenitor whose massive hydrogen envelope has been stripped in binary interactions (Nomoto 1984; Dewi et al. 2002; Ivanova et al. 2003; Podsiadlowski et al. 2005), and which leads to an electron-capture supernova (ECSN; Podsiadlowski et al. 2005; Ferdman et al. 2013), the result of destabilization of an ONeMg core by electron-captures onto magnesium and neon nuclei (Miyaji et al. 1980; Nomoto 1987; Kitaura et al. 2006).

Models of the ECSN progenitor cores suggest the onset of the electron-capture instability occurs at a unique ONeMg core mass in the mass range of $1.366 \pm 0.001 M_\odot$ (Miyaji et al. 1980; Nomoto 1984, 1987; Podsiadlowski et al. 2005; Takahashi et al. 2013). Electron captures cause the core to contract, and O and Ne burning is ignited in the central regions and propagates outwards in a deflagration front (Schwab et al. 2015), processing material to nuclear statistical equilibrium, where further electron captures and photodissociation accelerates the collapse (Miyaji et al. 1980; Nomoto 1987; Takahashi et al. 2013). Whether the core collapses or the deflagration disrupts the core depends sensitively on the ignition density (Isern et al. 1991; Jones et al. 2016). If the core does collapse, the explosion proceeds via delayed explosion on short timescales (Mayle & Wilson 1988; Kitaura et al. 2006; Fischer et al. 2010), and 2D simulations suggest the explosion occurs before significant convection has had time to develop (Wanajo et al. 2011) and hence a symmetric explosion results. This, coupled with the steep density gradient at the core surface, leads to very little mass loss from the core; estimates of mass loss include of order $10^{-3} M_\odot$ (Podsiadlowski et al. 2005), $10^{-2} M_\odot$ (Kitaura et al. 2006), and $1.39 \times 10^{-2} M_\odot$ (1.14 $\times 10^{-2} M_\odot$) for the 1D (2D) models of Wanajo et al. (2009, 2011). Therefore the ONeMg progenitor core mass is a good estimate of the baryon mass $M_B$ of the resulting NS (Podsiadlowski et al. 2005). Indeed, PSR J0737-3039A and the companion to PSR J1756-2251 have
gravitational masses consistent with baryon masses $\sim 1.37 M_\odot$ when their gravitational binding energies are taken into account (Lattimer & Yahil 1989). Population synthesis calculations incorporating the various binary evolution channels that might lead to production of NSs via ECSNe show that J0737-3039B most likely formed in an ECSN, and the companion to PSR J1756-2251 is consistent with such a formation scenario (Andrews et al. 2015). Other systems with candidates for ECSNe formation also exist (Keith et al. 2009; Chen et al. 2011).

Establishing the existence of an ECSN pathway in stellar evolution is important in a nuclear physics context. The gravitational binding energy of a NS $BE = M_B - M_G$ is highly sensitive to the NS equation of state (EOS; Lattimer & Yahil 1989) and it was shown that the constraint on the binding energy of J0737-3039B from the assumption of an ECSN formation scenario gives constraints on EOS (Podsiadlowski et al. 2005) and the slope of the nuclear symmetry energy $L = 3n_0(dS/dn)_0$ (Newton & Li 2009), where the nuclear symmetry energy $S(n)$ is the difference between the energies of pure neutron matter (PNM) and symmetric nuclear matter (with a proton fraction of one half; SNM). $n$ is the baryon number density and $n_0 = 0.16$ fm$^{-3}$ nuclear saturation density. Tighter constraints on the nuclear symmetry energy will allow a number of important astrophysical and nuclear quantities to be more accurately determined such as the thickness and moment of inertia of the NS crust and the the neutron skin thickness of neutron-rich nuclei such as Pb208 and Ca48 (Tsang et al. 2012; Lattimer & Steiner 2014; Li et al. 2014).

Establishing the existence of an ECSN pathway is also very important in a number of astrophysical contexts. The population of short orbital and pulsar spin period Be/X-ray binaries is suggestive of an ECSN formation for the NS (Knigge et al. 2011), and the creation of such systems is signified by an increase in formation efficiency of Be/X binaries in starbursts of ages 20–60 Myr (Linden et al. 2009). The population of double NS systems is also consistent with the majority of second-formed NSs in such systems originating in ECSN (Schwab et al. 2010; Beniamini & Piran 2016). Upper limits on the contribution of ECSNe to all core-collapse supernova have been placed at 20%–30% (Poelarends et al. 2008; Wanajo et al. 2009) taking into account production channels involving single super-asymptotic giant branch (AGB) stars (Siess 2007; Poelarends et al. 2008; Pumo et al. 2009), and those that take into account binary interactions which may lead to systems such as J0737-3039A/B and J1756-2251 (Podsiadlowski et al. 2004, 2005; Ivanova et al. 2008; with the binary channels likely to dominate). Finding evidence that particular binary systems gave rise to an ECSN will substantially affect such population synthesis calculations, and estimates of the rates of binary NS mergers and resulting rate of detectable gravitational wave signals (see, for example, Belczynski et al. 2017; Chruslinska et al. 2017, 2018 for examples of how the ECSN channel plays into merger rate estimations, especially in the light of the first NS–NS merger discovery, GW170817).

Recently, the binary evolutionary pathway that leads to ECSNe has been subsumed into the broader category of ultra-stripped (US) supernovae (resulting from a helium star which has been stripped of most of its envelope by mass transfer onto a companion NS), which includes some iron core-collapse supernovae (US-FeCCSNe; Tauris et al. 2013). The small eject mass and resulting symmetric explosion and low kicks make such supernovae additional candidates for the progenitors of PSR J0737-3039A/B, PSR J1756-2251 and similar systems (Tauris et al. 2015). Binary evolution simulations suggest that US-FeCCSNe occur in He stars whose metal cores exceed 1.43 $M_\odot$ (Tauris et al. 2015). Unlike the progenitor core for US-ECSNe, there is no unique Chandrasekhar mass for US-FeCCSNe. Simulations suggest that the lowest mass progenitor metal cores of US-FeCCSNe, $\sim 1.45 M_\odot$, explode with core mass loss of $\sim 0.1 M_\odot$, producing an NS with baryon mass $\sim 1.35 M_\odot$ (Suwa et al. 2015). More exploration of the parameter space of progenitors of US-FeCCSNe and their resulting supernovae is necessary, but in this paper we will take $M_B = 1.35 M_\odot$ as the lowest baryon mass of NSs resulting from an US-FeCCSN.

Signatures of US-FeCCSNe and US-ECSNe as distinguished from ordinary FeCCSNe may show up in the pre-supernova neutrino signal, detectable either in specific events (Kato et al. 2015), or on average from relic neutrinos (Mathews et al. 2014). US-FeCCSNe/US-ECSNe are expected to be faint supernovae (explosion energies around $10^{52}$ erg Kitaura et al. 2006) and may be associated with certain supernova classifications: those that occur in binary systems from progenitors stripped of their hydrogen appear as sub-classes of Type-I supernovae such as type Ib/c (Podsiadlowski et al. 2005) or type Iax (Kitaura et al. 2006; Moriya & Eldridge 2016), whereas those from single super-AGB stars are expected to appear as Type II subclasses such as Type IIn-P (Kitaura et al. 2006; Smith 2013). Finally, US-ECSNe produce Ni and Fe abundances that can distinguish them from US-FeCCSNe (Wanajo et al. 2009; Smith 2013). In this paper we will demonstrate an additional method to distinguish systems which hosted an US-FeCCSN or US-ECSN from those that did not, and additionally could distinguish between the scenarios of US-FeCCSNe and US-ECSNe.

Strong constraints on the EOS can be obtained from a measurement of an NS’s moment of inertia (Ravenhall & Pethick 1994; Lattimer & Prakash 2001; Morrison et al. 2004; Bejger et al. 2005; Worley et al. 2008; Fattoyev & Piekarewicz 2010; Steiner et al. 2015; Raithel et al. 2016) or its tidal polarizability $\lambda$ (Hinderer et al. 2010; Postnikov et al. 2010; Panmarale et al. 2011; Fattoyev et al. 2013). It has been noted that a simultaneous measurement of the moment of inertia of PSR J0737-3039A and the binding energy of PSR J0737-3039B, would lead to a stronger constraint on the EOS (Morrison et al. 2004) compared to constraints from individual measurements. The measurement of the spin–orbit contribution to the precession of the periastron of the double pulsar system J0737-3039 could be accurate enough to infer the moment of inertia of pulsar A to within 10% in the next few years (Damour & Schafer 1988; Lattimer & Schutz 2005; Kramer & Wex 2009), although this depends on the accuracy of a number of post-Keplerian parameters of the system (Iorio 2009). The most recent estimates predict that such an accurate measurement will be possible in the next decade, taking into account next generation radio telescopes such as SKA (Kehl et al. 2016). It is estimated that Advanced LIGO-Virgo should obtain constraints on the tidal polarizability of an NS from the gravitational waveform of a binary NS–NS merger or NS–black hole merger (Read et al. 2009; Panmarale et al. 2011; Hotokezaka et al. 2016). Particularly, the dimensionless tidal polarizability $\lambda/M_\odot^2$ can be measured to a 1-\(\sigma\) accuracy of...
100 by an Advanced LIGO detection of a merger at a distance of 200 Mpc (Hotokezaka et al. 2016); this is equivalent to the dimensionless moment of inertia of $I/M_\odot^2$ of less than 10% accuracy independent of the stiffness of the EOS (Yagi & Yunes 2017). Recently, these hopes were bolstered by the first gravitational wave detection of an NS–NS merger and subsequent constraint on the tidal polarizability of Abbott et al. (2017).

Recently, universal (EOS-independent) relations between the moment of inertia and a number of other global NS parameters such as the mass quadrupole moment and tidal polarizability have been found: the so-called I-Love-Q relations (Yagi & Yunes 2013a, 2013b); see also Yagi & Yunes (2017) for a recent review on other universal relations. Particularly relevant for this work is the relation between moment of inertia and binding energy (Steiner et al. 2016) which, although not displaying quite the same universality as the I-Love-Q relations, nevertheless enables a moment of inertia measurement to place constraints on the binding energy of an NS. In this paper we illustrate how these relations enable a measurement of the moment of inertia of PSR J0737-3039A to potentially constrain the US-FeCCSN/US-EC SN scenario of PSR J0737-3039B and the companion to PSR J1756-2251, by examining in detail the relation between the moment of inertia of a 1.338 $M_\odot$ NS (corresponding to the mass of PSR J0737-3039A) and the binding energy of an NS with the masses of PSR J0737-3039B and the companion to PSR J1756-2251.

The organization of the rest of the paper is as follows. In Section (2) we describe the three models we employ to systematically explore the parameter space of the EOS at high and low densities. In Section (3) we describe the resulting correlations between the binding energy and moment of inertia, and between the binding energy and slope of the symmetry energy, and discuss the implications for testing the US-EC SN and US-FeCCSN scenarios. In Section (4) we present our conclusions.

2. Equations of State

In order to fully explore the relationship between the binding energy (BE) and moment of inertia $I$ of the NSs, we generate three families of EOSs that together explore the full range of EOS parameter space for low-mass NSs. As we shall see, the BE–$I$ relation, while strong, is not universal to the same level of accuracy as the I-Love-Q relations, so using three families of EOSs constructed in different ways allows for a rigorous test of the EOS dependence of the relation. We do not consider possible large first-order phase transitions resulting from the appearance of quark matter that may produce a third stable family of compact stars (Benić et al. 2015; Alford & Sedrakian 2017) since these are predicted to occur, if at all, only in NSs with significantly higher masses than the highest mass considered here, 1.338 $M_\odot$.

The proton fraction in the NS outer core is sufficiently small that the EOS is well approximated by the pressure of PNM. There has been significant recent progress (Gandolfi et al. 2012; Gezerlis et al. 2013; Hebeler et al. 2013) on computing the EOS of PNM from realistic nuclear forces, using the quantum Monte Carlo method and chiral effective theory interactions in many-body perturbation theory. The first two EOSs both use the parameterization of the results of the quantum Monte Carlo model from Gandolfi et al. (2012) given in Steiner & Gandolfi (2012), and we refer to this model as “GCR.” The limits on the parameters of that model, 12.5 MeV < $a$ < 13.5 MeV and 0.47 < $\alpha$ < 0.53 are as used in Steiner et al. (2015). These two parameters principally parameterize the two-nucleon part of the interaction. Also, as in Steiner et al. (2015), we reparameterize $b$ and $\beta$, parameters which control the three-nucleon interaction, in terms of the magnitude and slope of the symmetry energy $S$ and $L$. The slope $L$ is limited to be between 30 and 70 MeV because that is the range of $L$ predicted from the microscopic PNM calculations on which these EOS models are based (Gandolfi et al. 2012, 2014; Steiner et al. 2015).

The first two EOSs are the same below nuclear saturation density $n_0$ but differ at high-density. We attach the GCR results either to a set of three piecewise polytropes referred to as “GCR+Model A” in Steiner et al. (2013, 2015) or to a set of four line segments in the ($\rho$, $P$) plane, “GCR+Model C” (Steiner et al. 2013, 2015). This latter model is useful because it provides an alternative model which tends to favor stronger phase transitions in the core.

The third set of EOSs uses the Skyrme energy-density functional to construct the EOS near saturation density. In previous work (Fattoyev et al. 2012), we developed families of Skyrmes by taking a baseline parameterization and refitting the two purely isovector model parameters $x_0$ and $x_1$ to the latest results of PNM calculations (Gezerlis & Carlson 2010; Hebeler & Schweng 2010; Gandolfi et al. 2012, 2015; Lynn et al. 2016). The resulting re-fit Skyrme models follow a tight correlation $S = 0.1L + 26.4\text{ MeV}$ between the slope and magnitude of the symmetry energy, and we explore a wide range $20 < L < 120\text{ MeV}$. The fact that only the two purely isovector parameters are adjusted means that such adjustments leave the SNM properties unchanged (Chen et al. 2009). These Skyrme EOSs are used up to 1.5 $n_0$, and replaced at higher with two piecewise polytropes, with a total of two free parameters (after the transition to the first polytrope is fixed at 1.5 $n_0$ by the pressure of the Skyrme EOS there). In order to illustrate the importance of the density at which the first polytrope is attached, we also prepare versions of the Skyrme EOSs in which the first polytrope is attached at saturation density, $n_0$. This completes the third set of EOSs which we label “Skyrme+Poly.”

There are two main differences between the Skyrme+Poly EOSs and GCR+Models A,C. First, the Skyrme EOSs explore a wider range of possible values of the slope of the symmetry energy $L$ than GCR. The highest and lowest values of $L$ give poor fits to the current results of microscopic PNM calculations (Gandolfi et al. 2012, 2014; Hebeler et al. 2013) but are still potentially allowed by certain nuclear experimental observables (Fattoyev & Piekarewicz 2013). Part of the purpose of comparing two sets of EOS models spanning different ranges of $L$ is to demonstrate that the conclusions we reach are independent of the assumed range of $L$. Second, the Skyrme EOSs have a well defined symmetry energy curvature $K_{\text{sym}}$ which correlates linearly with $S$ and $L$ after the two isovector Skyrme parameters have been fit to PNM. A soft $L$ leads to a soft EOS up to at least 1.5 $n_0$ where the first polytrope is attached. In contrast, EOSs GCR+Models A,C adjoin the first of their polytropes or line segments at saturation density, thus decoupling $K_{\text{sym}}$ from $L$ and $S$. We shall also use the Skyrme+Poly EOSs with the first polytrope transition density set to $n_0$ to demonstrate the important effect on the binding energy of the decoupling of the super-saturation EOS from the sub-saturation EOS.
The parameter space for GCR+Model A and C EOSs is explored by performing a Markov chain Monte Carlo simulation as first outlined in Steiner et al. (2010) and implemented in Steiner (2014a, 2014b) using uniform priors in the model parameters and with the only astrophysical constraints on the NS maximum mass, \( M_{\text{max}} \geq 2.0 M_\odot \), and the constraint that matter is stable and causal. To obtain our final results we choose the smallest range in the EOS parameter space which encloses all of the EOS models, as done in Steiner et al. (2013).

The parameter space of the Skyrme+Poly EOSs is explored as follows. We select 7 baseline Skyrmes which lie in the ranges of binding energy, saturation density and incompressibility of symmetric nuclear matter as the GCR model, and which lie along the \( S - L \) correlation created by the re-fit to PNM calculations at values of \( L \) equal to 20, 40, 50, 60, 70, 80, and 120 MeV. Then, for each value of \( L \) (1) we adjust the two free polytropic parameters to obtain particular maximum masses starting at 2.0 \( M_\odot \) up to the mass at which causality is violated in the center of the star; (2) at this fixed value of \( L \) and \( M_{\text{max}} \), we adjust the two parameters to obtain the maximum and minimum moments of inertia \( I \) of a 1.4 \( M_\odot \) NS without the EOS violating causality at any point below the central density of the maximum mass model. For the Skyrme models, we are effectively using uniform priors in the quantities that parameterize the family of Skyrme+Poly equations of state: \( L, M_{\text{max}} \) and \( I \). The Skyrme+Poly EOSs with the transition density of the first polytrope set at \( n_0 \) are also prepared as outlined above.

In Figure 1 we show mass–radius relations for the GRC+Model C family of EOSs (red density distribution) and selected Skyrme+Poly EOSs with the transition density to the first polytrope set to 1.5 \( n_0 \) (solid black lines) and 1.0 \( n_0 \) (dashed black lines) respectively. The predicted radii span a wide range \( \sim 10 - 15 \) km. We are thus able to systematically explore a wide, representative range of NS models.

3. Results and Discussion

The binding energy is calculated as \( \text{BE} = M_B - M_G \) where \( M_G \) is the gravitational mass and \( M_B \) is the baryon mass, given by \( M_B = A m_B \) where \( A \) is the baryon number of the star, and \( m_B \) is the mass per baryon. For the ONeMg progenitor core of a US-ECSN, \( m_B \approx 931.5 \) MeV; for an Fe core, \( m_B = 930.4 \) MeV. The moment of inertia is calculated in the slow rotation approximation (Hartle 1967; Hartle & Thorne 1968), and the dimensionless tidal polarizability \( \lambda / M_\odot^2 \) is calculated from the dimensionless moment of inertia \( I / M_\odot^2 \) using the universal relations of Yagi & Yunes (2017).

In Figures 2 and 3, we plot the relations between the dimensionless binding energy \( \text{BE}/M_\odot \) and the dimensionless moment of inertia \( I / M_\odot^2 \). The results from the Skyrme+Poly models are given by the short black bands, with each line representing a different value of \( L = 20, 40, 50, 60, 70, 80, 120 \) MeV with the lower \( L \) giving the higher binding energy. Each band is the locus of points representing the equations of state with masses above 2.0 \( M_\odot \) and, for each maximum mass, equations of state with moments of inertia between the minimum and maximum allowed by causality. The Skyrme models with the transition density to the first polytrope lowered to saturation density, and with the smallest binding energy, are shown as the black points which appear at large \( I / M_\odot^2 \) and small \( \text{BE}/M_\odot \). The GCR+Model A EOSs are represented by the red density distribution, solid 68% confidence contour and dotted 95% confidence contours, and GCR+Model C is represented by the dashed 68% confidence contours and dashed–dotted 95% confidence contours. The moment of inertia is that of an NS of the mass of J0737-3039A, 1.338 \( M_\odot \), and we present results for binding energies of NSs with the mass of J0737-3039B, 1.25 \( M_\odot \), and the upper and lower limits to the mass of the companion to J1756-2251, 1.237 and 1.223 \( M_\odot \), in the left, center, and right columns respectively. A measurement of the moment of inertia of an NS of a particular mass constrains the EOS of all NSs, in turn constraining the binding energy of an NS of any other mass. In Figure 2, the colored bands represent the constraint on the binding energy from the assumption of a US-ECSN creation scenario for the pulsars under different assumptions of the mass lost from the core during the explosion \( \Delta M_{\text{ns}} \). The \( \Delta M_{\text{ns}} = 0 M_\odot \) band shows the binding energy range inferred from the progenitor mass range of 1.366–1.377 \( M_\odot \).
(Nomoto 1984; Podsiadlowski et al. 2005; Takahashi et al. 2013). Current modeling suggests $\Delta M_{\text{in}} \lesssim 1.5 \times 10^{-2} M_{\odot}$. In Figure 3 we show the same relations, now with the lower limit on the binding energy from the limited constraints from modeling of US-FeCCSN, shown as the gray bands.

We see that the general trend, as expected, is that higher binding energy (corresponding to more compact NSs) corresponds to smaller moment of inertia. The distribution of Skyrme+Poly models of different $L$ values are distributed relatively uniformly from high binding energy, small moment of inertia (small $L$) down to small binding energy, large moment of inertia (large $L$). However, when the transition to the first polytrope is lowered to saturation density, decoupling the sub-saturation EOS characterized by $L$ and the supersaturation density EOS, the smallest binding energies and largest moments of inertia cluster at the extreme lower right of the plot mostly independent of $L$.

For GCR+model A, the distribution of model parameters is also relatively uniform, and the 95% confidence contour overlaps with the higher $L$ Skyrme+Poly models ($L = 60$–$120 \text{ MeV}$). The GCR+Model C allows for stronger phase transitions which generate more models with large binding energy and small moment of inertia, reaching the part of the diagram populated by the low-$L$ Skyrme+Poly models, although the highest density of models is still found in the small binding energy, high moment of inertia part of the diagram.

Both GCR+models A and C have symmetry energy slopes between $L = 30 \text{ MeV}$ and $L = 70 \text{ MeV}$, yet most models overlap with the high-$L$ Skyrme+Poly EOSs $L > 70 \text{ MeV}$. This is illustrated by Figure 4, which shows the relationship between dimensionless binding energy $BE/M_G$ and the slope of the symmetry energy $L$, with the results from the different EOS models displayed in the same way as Figures 2 and 3. The binding energies of GCR+models A and C are systematically smaller than those of the Skyrme+Poly models at a given $L$. This is because GCR+models A and C allow for a weak or strong transition at saturation density, where the first polytrope or line segment is attached to the GCR EOS. For the Skyrme+Poly EOSs, a small value of $L$ at saturation density (soft EOS just below saturation density) will lead to a soft EOS up to $1.5 n_0$ and a larger binding energy. However, for GCR+models A and C, a small $L$ at saturation density is compensated in many models by a transition to a stiffer EOS between $n_0$ and $1.5 n_0$, and a correspondingly smaller binding energy. Since the requirement of $M_{\text{max}} > 2.0 M_{\odot}$ tends to select stiffer EOSs in this region, there are many more models in the model space with these characteristics, and this is reflected in the probability density plots in Figures 2 and 3. Therefore, the low-$L$ GCR+models A and C with a stiff first polytrope will match the binding energy of the higher-$L$ Skyrme+Poly EOSs. This is demonstrated by the black dots in Figure 4 obtained by lowering the transition density of the first polytrope in the Skyrme EOS to $1.0 n_0$, allowing that phase transition at a lower density allows the Skyrme+Poly EOSs to describe significantly smaller binding energies for a given value of $L$ than if described by the same nuclear matter EOS up to $1.5 n_0$. Allowing a smaller transition density to the first polytrope shifts the models to less compact configurations (lower binding energy and higher moment of inertia) for a fixed EOS up to that first transition density. The moment of inertia is sensitive most to the EOS just above saturation density, and so these
We can conclude that different behaviors of the EOS above saturation density can lead to very different binding energies and moments of inertia for the same value of \( L \), but that all EOSs give the same correlation between \( \text{BE}/M_G \) and \( I/M_G^2 \), demonstrating that they form a weaker version of the universal I-Love-Q relations.

Despite the fact that the correlation between \( I \) and \( \text{BE} \) is more robust than the correlation between \( L \) and \( \text{BE} \) explored in Newton & Li (2009), Figure 4 seems to bear out the prediction of Newton & Li (2009) that \( L < 70 \) MeV is consistent with the US-ECSN scenario assuming a mass loss of \( \approx 0.015 M_e \).

Some general conclusions can be made independent of a measurement of moment of inertia. Let us assume the NSs PSR J0737-3039B and the companion to PSR J1756-2251 were formed in a US-ECSN. From Figure 2, they (and any NS with a gravitational mass of \( \approx 1.25 M_e \)) have a minimum \( \text{BE}/M_G \) of \( \approx 0.07 \) or equivalently a minimum binding energy of \( \approx 0.085 M_e \). For PSR J0737-3039B (Figures 2(a), (b)) this corresponds to a maximum of \( \approx 0.03 \) \( M_e \) mass loss from the progenitor core. Taking the upper limit on the mass of the companion to PSR J1756-2251 (Figures 2(c), (d)) the maximum progenitor core mass loss is \( \approx 0.04 \) \( M_e \), and taking the lower limit (Figures 2(e), (f)) the maximum progenitor core mass loss is \( \approx 0.06 \) \( M_e \). These upper limits on the mass loss from these systems are 2–4 times larger than the current estimates of mass loss from supernova modeling of up to \( \approx 0.015 M_e \).

Let us now examine the consequences of specific measurement of the moment of inertia to within 10%. Corresponding ranges for the dimensionless tidal polarizability are given according to the I-Love-Q relations. For reference, the recent discovery of the merger of a double NS system resulted in a constraint on the dimensionless tidal polarizability of \( \lambda \lesssim 1600 \) (Abbott et al. 2017), corresponding to a dimensionless moment of inertia of \( I \lesssim 18 \).

(i) \( \text{BE}/M_G = 16 \pm 0.8 \) (a relatively stiff EOS on average; corresponding to a tidal polarizability of \( \lambda/M_G^3 \approx 850 \)–1220): in this case, the binding energy of the stars is constrained to be \( \approx 0.085–0.1 \) \( M_e \). Under the US-ECSN scenario, this constrains PSR J0737-3039B (Figures 2(a), (b)) to have been created by the collapse of a core from which \( \gtrsim 0.015 \) \( M_e \) has been lost; this is just inconsistent with current SN models, and would require updated

Figure 3. Same as Figure 2, but now the shaded band indicates the lower limit on the binding energy of an NS of the respective mass assuming it is born in a US-FeCCSN, as inferred from supernova modeling. A 10% measurement of the moment of inertia of J0737-3039A can rule out such a formation scenario, assuming current modeling of the supernova is robust and complete if the measured moment of inertia is sufficiently large, and can potentially distinguish this scenario from the US-ECSN scenario.

Figure 4. Binding energy of a 1.25 \( M_e \) NS vs. the slope of the nuclear symmetry energy \( L \). As in Figures 2 and 3, the short black lines are the results of the Skyrme EOSs with the first polytrope joined at 1.5 \( n_0 \), the black dots are the lowest binding energy results of the Skyrme EOSs with the first polytrope joined at 1.0 \( n_0 \), and the red density distribution and black solid (68% confidence) and dotted (95% confidence) contours are the results of GCR + Model A, and the dashed (68% confidence) and dashed–dotted (95% confidence) blue contours are the results of GCR+Model C. Constraints on the binding energy assuming an electron capture supernova origin for the respective pulsars are given by the colored bands as in Figure 2. The decoupling of the super-saturation density EOS from the sub-saturation density EOS at saturation density allows for models with significantly smaller binding energies to be explored for a given stiffness \( L \) of sub-saturation density EOS.

models will also give similar moments of inertia. Thus the correlation between \( \text{BE}/M_G \) and \( I/M_G^2 \) is preserved.

Furthermore, in GCR+Model A, models cluster at higher \( L \) (60–70 MeV) and smaller binding energy (there are more models with a stiffer symmetry energy that satisfy \( M_{\text{max}} > 2.0 M_e \)). For GCR+Model C, the possibility of a strong phase transition near saturation density more strongly decouples the high- and low-density EOSs, resulting in a binding energy that is largely independent of \( L \).
estimates or it would rule out the US-ECSN scenario. Taking the upper limit of the mass of the companion to PSR J1756-2251 (Figures 2(c), (d)), the mass loss is constrained to be $\gtrsim 0.025 M_{\odot}$. Taking the lower limit of the mass of the companion to PSR J1756-2251 (Figures 2(e), (f)), the mass loss is constrained to be $\gtrsim 0.045 M_{\odot}$ over three times the predicted mass loss from the progenitor core in the US-ECSN scenario. In all three cases, more core mass loss than is currently predicted for a US-ECSN explosion would be needed, or the US-ECSN scenario is inconsistent with this measurement of the moment of inertia. From Figure 3 we see that such a measurement would be marginally consistent with the production of J0737-3039B in a US-FeCCSN, but would rule out production of the NS companion to J1756-2251 in this way. An Advanced LIGO detection of a NS–NS merger at a distance of 200 Mpc should be able to measure the tidal polarizability to equivalent accuracy at the 2-$\sigma$ level (Hotokezaka et al. 2016).

(ii) $I/M_{\odot}^2 = 14 \pm 0.7$ (an EOS of intermediate stiffness on average; $\lambda/M_{\odot}^2 \approx 520–760$): in this case, the binding energy is constrained to be $\approx 0.09–0.11 M_{\odot}$. Under the US-ECSN scenario, this constrains PSR J0737-3039B (Figures 2(a), (b)) to have been created by the collapse of a core from which $\gtrsim 0.005 M_{\odot}$ has been lost, which is consistent with current supernova models. Taking the upper limit of the mass of the companion to PSR J1756-2251 (Figures 2(c), (d)), the mass loss is constrained to be $\gtrsim 0.025 M_{\odot}$. Taking the lower limit of the mass of the companion to PSR J1756-2251 (Figures 2(e), (f)), the mass loss is constrained to be $\gtrsim 0.04 M_{\odot}$. Therefore, more mass would need to be lost from the progenitor core of PSR J1756-2251 than is currently predicted during the US-ECSN explosion, or the US-ECSN scenario is inconsistent with this measurement of the moment of inertia. From Figure 3 we see that such a measurement would be marginally consistent with the production of J0737-3039B in a US-FeCCSN, but would rule out production of the NS companion to J1756-2251 in this way. An Advanced LIGO detection of a NS–NS merger at a distance of 200 Mpc should be able to measure the tidal polarizability to equivalent accuracy at the 2-$\sigma$ level (Hotokezaka et al. 2016).

(iii) $I/M_{\odot}^2 = 12 \pm 0.6$ (a relatively soft EOS on average; $\lambda/M_{\odot}^2 \approx 290–430$): in this case, the binding energy is constrained to be $\approx 0.105–0.14 M_{\odot}$. Under the US-ECSN scenario, this is consistent with between 0 and 0.02 $M_{\odot}$ being lost from the progenitor core of PSR J0737-3039B (Figures 2(a), (b)), and therefore is consistent with current supernova modeling. Taking the upper limit of the mass of the companion to PSR J1756-2251 (Figures 2(c), (d)), the measurement is consistent with between 0 and 0.03 $M_{\odot}$ being lost from the progenitor core, which is again consistent with current supernova modeling. Taking the lower limit of the mass of the companion to J1756-2251 (Figures 2(e), (f)), the measurement is consistent with between 0.015 and 0.05 $M_{\odot}$ being lost from the progenitor core, marginally inconsistent with current supernova modeling. Thus this measurement of the moment of inertia would be consistent with J0737-3039B being formed in an US-ECSN, and consistent with the companion to J1756-2251 being formed this way provided its mass does not fall very close to the current lower bound. From Figure 3 we see that such a measurement would be consistent with the production of J0737-3039B in a US-FeCCSN, and marginally consistent with the production of the NS companion to J1756-2251 in this way provided the mass of the pulsar lie in the higher half of the current uncertainty range. An Advanced LIGO detection of an NS–NS merger at a distance of 200 Mpc should be able to measure the tidal polarizability to equivalent accuracy at the 1-$\sigma$ level (Hotokezaka et al. 2016).

A small value for $I/M_{\odot}^2 \lesssim 12$ favors a strong phase transition at relatively low densities (model C) or very soft symmetry energy $L \lesssim 40$ MeV.

These results demonstrate the feasibility of ruling out formation scenarios based on a future moment of inertia measurement of an NS. They do rely, however, on the robust modeling of the progenitor core mass at the time of collapse, and the supernova itself, particularly the mass lost from the core during the explosion.

4. Conclusions

Evidence for or against the ultra-stripped electron-capture supernova (US-ECSN) or ultra-stripped iron-core collapse supernova (US-FeCCSN) formation scenarios of NSs in double NS systems has an important impact on population synthesis calculations and resulting estimates of rate of merger of double NS systems and resulting electromagnetic and gravitational wave signals. The US-ECSN formation scenario has implications for the number of Be-X-ray binaries and the interpretation of galactic starburst. Strong circumstantial evidence exists that PSR J0737-3039B and companion to J1756-2251 were formed in a US-ECSN or a US-FeCCSN. However, recent modeling of the progenitor OneMg cores of US-ECSN demonstrate the sensitivity of this scenario to electron-capture rates, and demonstrate the possibility that such cores will be entirely disrupted before they collapse to form NSs. In this paper we have demonstrated that a future measurement of the moment of inertia of an NS to within 10% can potentially provide strong evidence against such scenarios, or constrain certain details such as the mass lost from the collasping core during such supernova explosions.

A previous study Newton & Li (2009) using a wide variety of existing NS EOSs found a correlation between the slope of the nuclear symmetry energy $L$ and the binding energy of NSs, and used it to determine that $L \lesssim 70$ MeV if pulsar J0737-3039B was formed in a US-FeCCSN with up to $\sim 10^{-2} M_{\odot}$ mass from the progenitor core during collapse. The EOSs used had maximum masses of 1.44 $M_{\odot}$ and above, reflecting the highest accurately measured pulsar mass of the time. In this work we have used systematically constructed EOSs to explore a wider range of EOS parameter space, constrained only by causality and the requirement that $M_{\text{max}} > 2.0 M_{\odot}$. Two different approaches are taken to construct the EOSs. The first uses a simple parameterization of the EOS up to saturation density determined directly from the results of microscopic PNM calculations with $30 < L < 70$ MeV, supplemented above saturation density with either three polytropes (Model A) or, to accommodate the possibility of a strong phase transition, four line segments (Model C). The second uses Skyrme EOSs up
to 1.5 times saturation density, which are fit to the results of PNM calculations at a given value of $L$ which is allowed to vary over the wider range $20 < L < 120$ MeV, and supplemented with two polytropes above 1.5 $n_0$ which are adjusted to systematically give maximum masses above 2.0 $M_\odot$ and minimum and maximum moments of inertia. We also compare these models with those in which the first polytrope is attached at saturation density $n_0$ in order to systematically examine the effect of decoupling the sub-saturation EOS with the super-saturation EOS.

The most important point is that, although the correlation between $L$ and BE first considered in Newton & Li (2009) is no longer strong, now we are systematically exploring a wider range of EOS model space, the correlation between $BE/M_\odot$ and $I/M_\odot^2$ which, while not as strong as some of the other universal relations recently found, is still robust. In particular, differences in the EOS between the densities of 1.0 $n_0$ and 1.5 $n_0$ have a particularly large effect on the predicted value of the binding energy and moment of inertia individually, but not on the correlation between the two parameters. It is interesting to note, however, that the result from Newton & Li (2009), that symmetry energy slope values of $L > 70$ MeV favors mass loss greater than that predicted by current US-ECSN modeling, is still consistent with the results obtained here.

We find that if J0737-3039B was formed in a US-ECSN, no more than 0.03 $M_\odot$ could have been lost from the progenitor core, (more than twice the mass loss predicted by current supernova modeling) and if the companion to J1756-2251 was formed that way, no more than 0.06 $M_\odot$ could have been lost from the progenitor core during explosion. We demonstrate that a 10% measurement of the moment of inertia is sufficient to provide evidence for or against the US-ECSN or US-FeCCSN in these two systems, and any similar systems that might be discovered. If such a measurement favors the stiffer EOS models, it can be sufficient to rule out production of the NSs in the two formation scenarios, and if it favors the softer to intermediate EOS models, it is of sufficient accuracy to inform the details of the progenitor and supernova modeling of this particular formation channel, particularly the critical mass at which the ONeMg core becomes unstable in the US-ECSN scenario, and the mass lost from the core during the supernova. As more double NS systems are discovered in the SKA era, analyses like this one will strengthen our understanding of their possible formation pathways and corresponding consequences for astrophysics and nuclear physics. In particular, given the current predictions from supernova modeling, NSs with masses measured to be below around 1.22 $M_\odot$ can be ruled out as having been formed in US-ECSN. Currently there are eight more NSs in double NS systems with masses measured to be below 1.3 $M_\odot$ that would be amenable to this analysis (Özel & Freire 2016). For example, the NS J0453+1559c has a mass measured to be 1.174 ± 0.004 (Martinez et al. 2015), the lowest precisely measured NS mass, which is conclusively too low to have been produced in an US-ECSN given our current state of knowledge. The other NS masses currently have too large errors, but those should decrease with continued monitoring of the systems.

It is of crucial importance to determine the robustness of the prediction that less than 0.015 $M_\odot$ of mass is lost from the progenitor core, a prediction that results from a limited number of simulations. Particularly, is there a significant EOS dependence of this quantity? Additionally, more investigation of whether the core is disrupted by runaway nuclear burning, rather than collapsing, is necessary.

We have focused in this paper on the relation between binding energy and moment of inertia because of the prospect of a moment of inertia measurement of J0737-3039A within a decade. However, because of the universal I-Love-Q relations, a measurement of the tidal polarizability of an NS from the gravitational wave signal immediately prior to a binary NS merger to a similar degree of accuracy would also give similar constraints on the binding energy, and allow for similar discrimination of formation scenarios. Such accuracy could be obtained by Advanced LIGO for merger events within 200 Mpc. One can also use measurements of the NS radius and the relation between the binding energy and compactness to perform a similar analysis.

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