Lattice QCD Study of the Nucleon-Charmonium Interaction

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The $J/\psi$-nucleon interaction is studied by lattice QCD calculations. At the leading order of the derivative expansion, the interaction consists of four terms: the central, the spin-spin, and two types of tensor forces. We determine these spin-dependent forces quantitatively by using the time-dependent HAL QCD method. We find that the spin-spin force is the main cause of the hyperfine splitting between the $J = 1/2$ and the $J = 3/2$ states, while the two tensor forces have much smaller effects on the S-wave scattering processes.

**KEYWORDS:** lattice QCD, charmonium, spin-dependent forces

1. Introduction

One of the most important remaining problems in hadron physics is the identification of exotic hadrons, which have more complex structure than the conventional three-quark baryons or quark-antiquark mesons. Recently, the LHCb collaboration has found peak structures in the $J/\psi p$ invariant mass distribution from the weak decay $\Lambda_b \rightarrow J/\psi K^- p$ [1]. As the $K^- p$ contribution alone cannot describe the observed peaks [2], they are expected to be $uudc\bar{c}$ pentaquarks. The Breit-Wigner fit shows that there are two states, $P_c(4380)$ and $P_c(4450)$, with three acceptable spin-parity combinations $(3/2^-, 5/2^+), (3/2^+, 5/2^-), (5/2^+, 3/2^-)$.

It is important to study the $J/\psi p$ scattering to search for $P_c$ states. However, since experimental data in this channel is very limited, phenomenological approaches may end up with large ambiguity. To overcome this difficulty, we study the $J/\psi p$ scattering by using lattice QCD simulations without requiring further experimental data. Specifically we employ the approach introduced by the HAL QCD collaboration to calculate the $J/\psi N$ potential [3, 4]. Our previous report [5] shows that we need to use the time-dependent HAL QCD method to calculate the $J/\psi N$ potential properly.

The $J/\psi$-nucleon interaction is expected to be dominated by the exchanges of color-singlet gluon clusters, called the QCD van der Waals forces [6]. The possibility of $P_c(4450)$ as a deeply bound state of $\psi(2S)$ and $p$ has been discussed in this framework [7]. In order to check this scenario quantitatively, precise knowledge of the charmonium-nucleon interaction is necessary [8]. In particular, the $J/\psi N$ potential with spin-dependent forces is an essential key. The spin-dependent forces are omitted in many contexts, since they are suppressed by $O(1/m_c)$. Almost no information of the spin-dependent forces has been obtained so far. The spin-dependent potential is also important for a search for nucleus bound $J/\psi$ [9]. The aim of the present report is to perform a high-statistics lattice QCD calculation to obtain the spin-dependent $J/\psi N$ potential from the time-dependent HAL QCD method.
2. Method

In the time-dependent HAL QCD method, we start from the normalized four-point correlation function for the $J/\psi N$ system,

$$R(x - y, t - t_0) \equiv \left\langle 0 \left| N(x, t) \psi(y, t) \bar{f}(t_0) \right| 0 \right\rangle / e^{-(m_N + m_\psi)(t - t_0)},$$

(1)

where $N(x, t), \psi(y, t)$ and $\bar{f}(t_0)$ denote local interpolating operators for proton and $J/\psi$, and the corresponding wall-source operator. In actual lattice QCD calculations, the exponential factors involving the proton mass $m_N$ and the $J/\psi$ mass $m_\psi$ is replaced by the corresponding single-hadron correlation functions. The $J/\psi N$ non-local and energy-independent potential $U(r, r')$ satisfies the following time-dependent Schrödinger-like equation:

$$\left(\frac{\nabla^2}{2\mu} + \frac{\partial}{\partial t}\right) R(r, t) = \int d^3 r' U(r, r') R(r', t),$$

(2)

where $\mu = 1/(1/m_N + 1/m_\psi)$ is the reduced mass. On the left-hand side, small corrections of $O(k^4)$ ($k$ is related to the center of mass energy $W$ as $W = \sqrt{k^2 + m_N^2 + \sqrt{k^2 + m_\psi^2}}$) are neglected, as we have confirmed that the corrections are negligibly small. By taking the leading-order terms of the derivative expansion, the non-local potential is approximated by a local one, $U(r, r') = V(r)\delta^3(r - r')$. Here $V(r)$ has spin indices of $\rho$ and $J/\psi$. Our next task is to derive the spin structure of $V(r)$.

The nucleon spin operator is related to the Pauli matrices as $\sigma/2$, whereas the $J/\psi$ spin operator corresponds to the spin-1 equivalents of the Pauli matrices,

$$\Sigma_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & +i & 0 \end{pmatrix}, \quad \Sigma_2 = \begin{pmatrix} 0 & 0 & +i \\ 0 & 0 & 0 \\ +i & 0 & 0 \end{pmatrix}, \quad \Sigma_3 = \begin{pmatrix} 0 & -i & 0 \\ -i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

(3)

which satisfy the commutation relations $[\Sigma_i, \Sigma_j] = i\epsilon_{ijk} \Sigma_k$. The total spin operator $S$ is thus given as $S = \sigma/2 + \Sigma$, which has eigenvalues $1/2$ and $3/2$. We follow the arguments of Okubo and Marshak [10] to derive the general form of the $J/\psi N$ 2-body interaction at the leading order of the derivative expansion. We require that the potential satisfy the following six conditions: energy-momentum conservation, Galilei covariance, isospin invariance, total angular momentum conservation, Parity invariance, and time-reversal invariance. Then we find that the leading-order potential in the center of mass frame must have the form

$$V(r) = V_0(r) + V_4(r)\sigma \cdot \Sigma + V_{T1}(r)S_{12} + V_{T2}(r)T_{12}$$

(4)

where the relative coordinate is denoted by $r = r \hat{r}$, and $S_{12} = 3(\hat{r} \cdot \Sigma)(\hat{r} \cdot \Sigma) - \sigma \cdot \Sigma$ and $T_{12} = 3(\hat{r} \cdot \Sigma)^2 - \Sigma^2$ are the two tensor operators of the $J/\psi N$ interaction. The convergence of the derivative expansion must be checked numerically.

There are five $J/\psi N$ partial waves with total angular momentum $J = 1/2$ or $3/2$: $^2S_{1/2}$, $^4D_{1/2}$, $^4S_{3/2}$, $^2D_{3/2}$, and $^4D_{3/2}$, in the standard notation $2^{\pm}1J$. The four forces $V_0(r)$, $V_4(r)$, $V_{T1}(r)$ and $V_{T2}(r)$ are determined from the wave functions for four of the five states. In general the $J/\psi N$ system with $J^P = 1/2^-$ couples to the $\eta_c N$ system with the same quantum numbers; however we assume this transition is negligible due to the heavy charm quark mass.

3. Simulation Setup

We employ the $2 + 1$ flavor QCD gauge configurations on a $32^3 \times 64$ lattice generated by the PACS-CS collaboration [11]. They are generated with the renormalization group improved gauge
action at $\beta = 1.90$ and the non-perturbatively $O(\alpha)$ improved Wilson quark action at $\varepsilon_{SW} = 1.715$, which correspond to lattice spacing of $\alpha = 0.0907(13)$fm and the spatial volume of $L^3 = (2.9\text{fm})^3$. The hopping parameters are taken to be $\kappa_{ud} = 0.13700$ and $\kappa_s = 0.13640$. For the charm quark we employ the Tsukuba-type relativistic heavy quark (RHQ) action to remove the leading and next-to-leading order cutoff errors [12]. We use the same RHQ action parameters as in Ref. [13]. The periodic boundary condition is imposed in the spatial directions, while the Dirichlet boundary condition is imposed in the temporal direction at $t-t_0 = 32a$ to prevent inverse propagation. To improve statistics, we take average over 32 different source positions $t_0$. In this setup, the masses of pion, proton, and $J/\psi$ are found to be $m_\pi = 700(2)$MeV, $m_N = 1585(16)$MeV, and $m_\psi = 3139(12)$MeV, respectively.

4. Results

In Fig. 1, we show the $J/\psi N$ potentials $V_0(r)$, $V_4(r)$, $V_{T1}(r)$, and $V_{T2}(r)$ at $t-t_0 = 15a$. The left panel shows the potentials extracted from the combination of states $^{2S}_{1/2}, ^4S_{3/2}, ^2D_{3/2}, ^4D_{3/2}$, while the right panel is from $^{2S}_{1/2}, ^4D_{1/2}, ^4S_{1/2}, ^4D_{3/2})$. The difference of the two implies higher-order contributions of the derivative expansion. We see that $V_0(r) < 0$ and $V_4(r) > 0$ for all $r$, and there is no visible difference in $V_0(r)$ and $V_4(r)$ between the two panels. The strength of the tensor forces $V_{T1}(r)$ and $V_{T2}(r)$ are relatively smaller compared to $V_0(r)$ or $V_4(r)$. Also, we see significant differences in the strength of the tensor forces between the two panels in the small $r$ region. In our simulation setup, the D-wave signals are very small, and the tensor forces tend to contain larger ambiguity than the central and the spin-spin forces. It will be interesting to study the tensor forces with better precision by using a different source operator that couples to the D-wave states. Note that, in the present analysis we assume that the wave functions are the irreducible representations of rotational SO(3) symmetry. As a result, the potentials involve large ambiguities at points $(x, y, z) = (\pm n, \pm n, \pm n) (n \in \mathbb{Z})$, so that we have removed them in Fig. 1. The large errors at large distances will be due to the same reason. The temporal separation $t-t_0$ has to be large enough to achieve the elastic-state saturation. A necessary condition for this is that the potentials are not changed by small variation of $t$. We have confirmed that the latter condition is satisfied at $t-t_0 = 15a$, so that the elastic-state saturation is expected to be achieved.

By neglecting the $S-D$ mixing contributions from the tensor forces, the S-wave diagonal matrix elements of the potential in Eq. (4) are

$$\langle ^2S_{1/2} | V(r) | ^2S_{1/2} \rangle = V_0(r) - 2V_4(r), \quad (5)$$

$$\langle ^4S_{3/2} | V(r) | ^4S_{3/2} \rangle = V_0(r) + V_4(r). \quad (6)$$

Fig. 1. The $J/\psi N$ potentials $V_0(r)$, $V_4(r)$, $V_{T1}(r)$, and $V_{T2}(r)$ at $t-t_0 = 15a$. The left panel shows the potentials extracted from the states $^{2S}_{1/2}, ^4S_{3/2}, ^2D_{3/2}, ^4D_{3/2}$, while the right panel is from $^{2S}_{1/2}, ^4D_{1/2}, ^4S_{3/2}, ^4D_{3/2}$.
In Fig. 2(Left), we show these diagonal matrix elements together with the S-wave effective central potentials (for the definition of the effective central potentials, see Ref. [5]). We see that each diagonal matrix element coincides with great precision with the corresponding effective central potential. It implies that the hyperfine splitting of the $J/\psi N$ system between $J = 1/2$ and $J = 3/2$ is dominated by the spin-spin force, and the contribution from the tensor forces are much smaller.

To solve the Schrödinger equation for the scattering phase shift, we neglect the small tensor force contributions, so that the S-wave scattering is described by the effective central potentials with good precision. Shown in Fig. 2(Right) are the calculated phase shifts. For comparison we also include the corresponding result for the $\eta_c N$ scattering. We see that there is no two-body bound state below the charmonium-nucleon production thresholds. The scattering length $a$ and the effective range $r$ are given as $a(J/\psi N J = 1/2) = 0.656(71)$fm, $r(J/\psi N J = 1/2) = 1.105(16)$fm, $a(J/\psi N J = 3/2) = 0.380(48)$fm, $r(J/\psi N J = 3/2) = 1.476(39)$fm, $a(J/\psi N J = 1/2) = 0.246(26)$fm, and $r(J/\psi N J = 1/2) = 1.703(45)$fm.

5. Conclusions

We have studied the $J/\psi N$ scattering by the time-dependent HAL QCD method. We find that the $J/\psi N$ interaction at the leading order of the derivative expansion consists of the central, the spin-spin, and the two tensor forces. The spin-spin force strengthen the attraction for $J = 1/2$ and weaken the attraction for $J = 3/2$, causing the hyperfine splitting between the two states. The tensor forces are much weaker than the central and the spin-spin forces.

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