End-to-End Inverse Design for Inverse Scattering via Freeform Metastructures

Zin Lin\(^*\)\(^1\), Charles Roques-Carmes\(^2\), Raphaël Pestourie\(^1\), Marin Soljačić\(^{2,3}\), Arka Majumdar\(^{4,5}\), and Steven G. Johnson\(^1\)

\(^1\)Department of Mathematics, Massachusetts Institute of Technology, Cambridge MA 02138, USA
\(^2\)Research Lab of Electronics, Massachusetts Institute of Technology, Cambridge MA 02138, USA
\(^3\)Department of Physics, Massachusetts Institute of Technology, Cambridge MA 02138, USA
\(^4\)Department of Electrical and Computer Engineering, University of Washington, Seattle WA 98195, USA
\(^5\)Department of Physics, University of Washington, Seattle WA 98195, USA

June 17, 2020

Abstract

By co-designing a meta-optical frontend in conjunction with image processing backend, we demonstrate noise-robust subwavelength reconstruction of an image superior to an optics-only or computation-only approach. Our end-to-end inverse design couples the solution of the full Maxwell equations—exploiting all aspects of wave physics arising in subwavelength scatterers—with inverse-scattering algorithms in a single large-scale optimization involving \(\gtrsim 10^4\) degrees of freedom. The resulting structures scatter light in a way that is radically different from either a conventional lens or a random microstructure, and suppress the noise sensitivity of the inverse-scattering computation by several orders of magnitude.

1 Introduction

A conventional all-optical imaging system (Fig. Ia) maps each point in a “target” space onto a separate sensor pixel, directly producing a faithful image, but

\(^*\)ZinLin@mit.edu
generally requires bulky optics. In another extreme, a lens-free system (Fig. 1b) would directly detect a blurry image of the target and attempt to solve the subsequent “inverse scattering” problem (target reconstruction by, e.g., least square fitting), which may be very ill-conditioned and hence sensitive to noise [1–5]. In this paper, we introduce an end-to-end approach for inverse scattering (Fig. 1c), in which a compact meta-optical structure is generated by large-scale inverse design of the full Maxwell equations coupled with signal processing for target recovery. We show that noise-tolerant subwavelength (0.2λ) far-field reconstruction of a collection of point sources is possible even with an ultra-compact (2λ-thick) imaging device. Specifically, we design a meta-optical structure that generates well-conditioned (noise-robust) inverse-scattering problems, while exploiting a simple Tikhonov-regularization method (Sec. 3) to obtain subwavelength resolution without subwavelength focusing. Accomplishing this requires that the optical “inverse” design problem, involving large-scale optimization over ≈ 10^4 degrees of freedom, be coupled with the reconstruction algorithms (Sec. 2). That is, we perform “end-to-end” design in which the error \( L(\varepsilon, p) \) of the reconstructed targets is jointly minimized as a function of both the microstructure (\( \varepsilon \)) and the reconstruction parameters (\( p \)). Applying this approach to a two-dimensional (2D) example problem (Sec. 3), we obtain 0.22λ spatial resolution with a robust condition number (noise sensitivity) of only \( \approx 10 \), an improvement of \( 10^2 \)–\( 10^3 \) over the condition numbers for lens-free or random (diffusing [6]) scattering structures.

Recent work in end-to-end computational imaging achieved improved image quality using regularized least-square image reconstruction in conjunction with scalar diffraction theory (rather than the full Maxwell equations) to design a phase plate (i.e., treated as locally uniform and neglecting multiple scattering) [7]. Flat-optics meta-lenses [8–10], in contrast, have utilized more complete wave optics theory ranging from locally periodic [11,12] or overlapping [13] domain approximations to full Maxwell calculations [14,15] coupled with optimization-based inverse design [16–18], exploiting local resonances and multiple scattering to achieve diffraction-limited focusing [19,20]. However, these works specified the focal point and/or the desired wavefront a priori, even with more complex focal patterns chosen to facilitate subsequent computational processing [21–23], rather than performing a fully coupled end-to-end design. There is also a vast body of work on computational image reconstruction [24], but decoupled from the lens design (taking the optics as an immutable input rather than as design parameters). In contrast, we couple the full Maxwell equations with the post-processing reconstruction during the design process (Sec. 2), so that an optimal wavefront is determined for each source to maximize reconstruction accuracy. Specifically, we demonstrate imaging with subwavelength resolution, a feat not possible using previously reported end-to-end computational imaging. In order to perform this optimization, we employ standard adjoint techniques from photonic inverse design [16–18] combined with automatic-differentiation tools [25] to obtain the sensitivity to changes in structural parameters \( \varepsilon \) and reconstruction parameters \( p \).
Figure 1: Comparison of three imaging modalities. (a) In traditional all-optical imaging, a bulky optical system focuses each point of the target on a different sensor pixel: no signal processing is required, and a direct image is obtained at a cost of bulky optics being required. (b) In a lens-free system, the sensor directly records a blurry image while signal processing attempts to solve the resulting ill-posed (noise-sensitive) reconstruction problem. (c) In this work, we present an end-to-end inverse design approach, which optimizes a nanophotonic structure alongside the signal processing algorithm leading to a compact, noise-robust imaging system.

2 End-to-end Framework

Fig. 2 shows a schematic of our proposed framework within the specific context of optical imaging, although similar formulations are applicable to other wave-scattering problems such as spectroscopy, polarimetry or even optical computing. Here, the goal is to reconstruct a target $u$ in a region of interest $(L_x \times L_y \times L_z)$ from the captured image $v$ on a sensor. In between the sensor and the region of interest, we place a scattering structure, aka a photonic probe, $\varepsilon(r)$ to be designed, at a “working” distance $d_u = 5\lambda$ from the target (compact, but in the far field) and $d_v = \lambda$ from the sensor (where near-field effects may be relevant). The target region is voxelized into a $n_x \times n_y \times n_z$ grid and calibrated using $n$ point sources ($n = n_x n_y n_z$) [6], so that an arbitrary target residing within the region is described by an intensity vector $u = [u_1, ..., u_n]$ with a spatial resolution $\Delta r_u = L_r / n_r$. Here, we assume incoherent illumination of the target region (as is common for imaging) so that only intensities need to be considered [24]. For targets at “infinity,” such as a photographic scene, the region of interest is an angular field of view and one can consider plane-wave sources instead of point sources. The sensor has $m$ pixels with corresponding intensities (raw image) $v = [v_1, ..., v_m]$ given by the forward scattering model $v = G(\varepsilon)u + \eta$ where $G$ represents the solution of the Maxwell equations and $\eta$
Figure 2: A schematic of the end-to-end inverse design framework. The target region of interest is characterized by an intensity vector $u$ over a calibrated grid of $n$ voxels. The photonic probe has a dielectric profile $\varepsilon(r)$ (to be determined via inverse design). The sensor, with $m$ pixels, records the raw image $v$. $u$ and $v$ are related by the forward scattering model: $v = G(\varepsilon)u + \eta$, where $G$ is a $m \times n$ matrix whose columns are extracted from the solution of the full Maxwell equations, and $\eta$ is a noise vector (e.g. sensor noise). $v$ is then fed into a signal processing algorithm parametrized by a vector $p$; the overall performance is evaluated by a loss function $L$ (e.g. mean square deviation from the ground truth). The processing may involve any operations including matrix-vector multiplications, nonlinear kernels, integro-differential equations or artificial neural networks; in particular, we consider the inverse scattering problem of estimating $u$ through regularized least-square minimization. End-to-end inverse design seeks optimal $\varepsilon$ and $p$ that optimizes the entire work-flow including both the forward model and the inverse problem; the gradients are obtained by backpropagation and adjoint methods.

is an additive noise vector. For simplicity, we will consider zero-mean Gaussian white noise with non-zero variance ($\eta \sim \mathcal{N}(0, \sigma^2)$) [7], although our method can be easily adapted to other noise models (such as Poisson/shot noise) by
calibrating the camera beforehand. We consider a planar sensor, which is the most common configuration in imaging, but our framework can readily be extended to arbitrary sensor topologies. \( G \) is a \( m \times n \) matrix whose columns are essentially point spread functions (PSF) [6,26] computed from the underlying Maxwell equations given a structure \( \varepsilon(r) \).

The raw image \( v \) is fed into a signal-processing algorithm to approximately reconstruct \( u \), in our case by a regularized least-square fit. That is, we find \( \hat{u} \) such that

\[
\hat{u} = \arg \min_{\mu} \| G \mu - v \|^2 + R(\mu).
\]

Here, \( R \) is a regularization operator which serves to condition a typically ill-posed inverse problem; essentially, \( R \) incorporates any prior assumptions about \( u \) (such as smoothness or sparsity) which ensure that the inverse problem has a stable unique solution [1]. In particular, we choose a Tikhonov (\( L_2 \)) regularization \( R(\mu) = \alpha \| \mu \|^2 \) where \( \alpha > 0 \) is a regularization parameter to be determined, and \( \hat{u} \) has a closed-form solution

\[
\hat{u} = (G^T G + \alpha I)^{-1} G^T v
\]

The noise sensitivity of the reconstructed \( \hat{u} \) is characterized by the condition number \( \kappa(G) \) of the matrix \( G \), which is a dimensionless quantity \( \geq 1 \) that is roughly proportional to the ratio of the \( \| u - \hat{u} \| / \| u \| \) relative error to the input noise \( \| \eta \| / \| v \| \) [27]. \( (\kappa(G) \) can be computed as the ratio of the largest to smallest singular values of \( G \).) Many other variations are possible, such as \( L_1 \) “sparse” reconstruction [28] or artificial neural networks [29,30]. As we discuss in Sec. 4, our approach extends easily to such techniques, even if the reconstruction problem does not have a closed-form solution or it involves a vast number of free parameters to be determined. In general, a reconstruction algorithm is characterized by a vector \( p \) of \( P \) parameters; in this example, \( p = [\alpha] \) and \( P = 1 \).

The end-to-end inverse design seeks optimal choices of \( \varepsilon \) and \( p \), which are tightly coupled by the end-to-end work-flow, in order to minimize the difference between the reconstructed \( \hat{u} \) against the ground truth \( u \). Specifically, we define a loss function \( L(\varepsilon, p) \), here a mean-square error (MSE), such that

\[
L = \langle \| u - \hat{u} \|^2 \rangle_{u, \eta}
\]

where \( \langle \cdot \cdot \cdot \rangle_{u, \eta} \) denotes averaging (expected value) over many realizations of \( u \) and \( \eta \). The formulation can now be written as:

\[
\min_{\varepsilon, p} \quad L = \langle \| u - \hat{u} \|^2 \rangle_{u, \eta} \tag{1}
\]

\[
\hat{u} = (G^T G + \alpha I)^{-1} G^T v \tag{2}
\]

\[
v = G(\varepsilon) u + \eta \tag{3}
\]

Here, the PSF matrix \( G \) is extracted from the numerical solution of the Maxwell equations by any method.

In this paper, we consider the frequency-domain Maxwell equations with time-harmonic sources \( e^{-i\omega t} \) [31]:

\[
\nabla \times \nabla \times E - \omega^2 \varepsilon(r) E = i\omega J.
\]

solved by a finite-difference frequency-domain (FDFD) method [32]. For each voxel in the target region, \( J \) is chosen as a point-source situated at the center of the voxel and the corresponding PSF is obtained by simulating the integrated electric field intensities \( |E|^2 \) over the sensor plane. The optimization over \( \varepsilon, p \) require their gradients \( \frac{\partial L}{\partial \varepsilon}, \frac{\partial L}{\partial p} \), which can be found by back-propagation through
the signal processing stage and adjoint sensitivity analysis of the Maxwell equations. We numerically implement these gradients by coupling an open-source automatic-differentiation packages with our own Maxwell adjoint solvers.

3 Imaging at sub-wavelength resolutions

To demonstrate the capability of our framework, we consider an imaging problem at sub-wavelength resolutions. We consider a 2D problem $\varepsilon(x, z)$ (see Fig. 2) with $y$-polarized electric fields, so that the Maxwell equations are reduced to a scalar 2D Helmholtz equation. Specifically, we set $d_u = 5\lambda$ and $d_v = \lambda$ (see Fig. 2) where $\lambda$ is the operating wavelength. Also, we discretize a 1D 2$\lambda$-wide target region into $n = 10$ equi-spaced pixels with a resolution of $\Delta x_u = 0.222\lambda$; meanwhile, the probe and sensor have a width of 50$\lambda$ and the sensor contains $m = 50$ pixels with a pixel size $\Delta x_v = \lambda$. Although we have chosen these parameters for ease of demonstration, we note that this scenario is realizable using selective illumination, slit apertures, a high-speed scanning mode, and line sensors to produce 2D or even 3D images over a wide field of view. More importantly, this system illustrates the essential ingredients of many important applications as discussed in Sec. 4.

Although we have set $m > n$ (a nominally “over-determined” inverse problem), it is important to note that not any $\varepsilon(r)$ will lead to a well-conditioned (noise-robust) PSF matrix $G$. It is ill-advised to use a randomly-chosen $\varepsilon$ profile and directly invert $G$ because not every probe can resolve two point sources separated by a distance of 0.22$\lambda$ and project measurably-distinct noise-tolerant PSFs onto a coarse-resolution sensor ($\Delta x_v \gg \Delta x_u$) one wavelength away from the probe (small $d_v$ leaves little room for conventional magnification). For example, we checked that a uniform $\varepsilon$ leads to $G$ with a condition number $\kappa(G) \approx 10^4$; even a disordered $\varepsilon$ with rapidly-varying fine features yields $\kappa(G) \approx 1000$. Both of these values represent orders of magnitude amplification of input noise in the output reconstruction, indicating that radical re-design of $\varepsilon$ is required.

We show that our end-to-end framework can discover novel geometries $\varepsilon(r)$ with greatly reduced $\kappa(G)$, thereby rendering the inverse problem robust against noise. Here, the $\varepsilon$ degrees of freedom are a set of freeform variable heights at each pixel within a double-layer design region made up of a low-permittivity polymer material $\varepsilon_{\text{polymer}} \approx 2.3$ in air (see Fig. 3a). We have chosen these material settings because of rapidly maturing nano-scale 3D-printing technologies that would allow for exploration of such complex 3D geometries and are particularly suited for taking advantage of the full power of freeform topology optimization.

We employ stochastic gradient descent for optimizing $\varepsilon$ and $\alpha$ over $\geq 10^4$ iterations including random noise $\eta$; we found that $\alpha$ stays close to an initial choice of 0.5 while $\varepsilon$ evolves considerably during the course of optimization. In practice, we found that it works just as well to fix $\alpha$ with zero noise ($\eta = 0$) as to vary $\alpha$ under many realizations of $\eta$ (note that $\alpha$ is closely related to the
noise variance $\sigma^2$ [40]). Fig. 3a exhibits a double-layer optimized design in a 3D-printable polymer-matrix (for example, Nanoscribe IP-DIP [41,42]); each layer has thickness $\lambda/2$ and the minimum feature size is $\approx 0.04\lambda$, which may be challenging to fabricate at visible wavelengths but is feasible at longer wavelengths such as mid-wave and far-wave infra-red or even millimeter waves [43,44]. Fig. 3b demonstrates that optimization rapidly improves both MSE ($\approx 10^{-6}$) and $\kappa(G) \approx 10$. Fig. 3c shows that four randomly-chosen targets $u$ (with an averaged intensity $\langle u \rangle = 1$) can be faithfully reconstructed under various noise levels $\sigma$. Figs. 3d–m exhibit measurably distinct PSFs corresponding to a point source within each target voxel.

Our results suggest that a low-index photonic micro-structure with a highly complex geometry can faithfully reconstruct an image down to deeply subwavelength resolutions (albeit over a finite array of equi-spaced calibrated point sources), while maintaining a sufficiently high signal-to-noise ratio. From a fundamental-physics perspective, we note that even though the probe is close to the target, the former is clearly not in the near field of the latter (since $d_u > \lambda/2$), which means evanescent fields from the target have negligible amplitude at the probe. Instead, the sub-wavelength resolution is made possible by the ability of the computational probe to distinguish the subtle differences in spatial frequency components coming from adjacent point sources [45–47]. Therefore, our approach is unlike negative-index metamaterial superlenses [48,49] or super-oscillatory lenses [50], which seek perfect point-to-point physical image formation via amplification of evanescent waves or sub-diffraction-limit focal spots without the aid of computational reconstruction.

4 Summary and Outlook

The key conclusion of our paper is that optical metastructures designed in conjunction with signal processing result in non-obvious light scattering patterns that greatly ease the computational reconstruction. This results in devices far more compact compared to optics-only solutions while being robust to noise compared to computation-only designs. By solving the full (Maxwell) wave equations during the design process, our optimized structure can exploit all available wave physics (non-paraxial scattering, near-field interactions, resonances, dispersion, etc.). We illustrated this idea in the context of a specific subwavelength imaging system, but the same essential ideas can be readily applied to many other systems and computational processing techniques. In contrast to the many previous metasurface designs that have attempted to mimic and compete with traditional curved lenses [9], our scattered fields look nothing like a focal pattern and represent a functionality that is fundamentally distinct from that of conventional optics.

There are many other sensing/imaging problems that could benefit from this approach. Our designs in this paper closely resemble lab-on-a-chip microscopy. Related situations arise in ultra-compact opto-fluidic medical sensors, where the probe and sensor must be tightly integrated, the sample is situated only a few
Figure 3: (a) Topology-optimized double-layer photonic probe (ε ≈ 2.3). The probe is 50λ wide and λ thick, and is made up of freeform variable-height geometry. Note the scale bar. (b) Mean square error (MSE, blue line) and inverse condition number $\kappa^{-1}$ (red line) of the PSF matrix $G$, where $G$ is a $m \times n$ matrix with $m = 50$, $n = 10$. $\kappa^{-1}$ steadily increases to around 0.08 ($\kappa \approx 12$), showing that the reconstruction becomes robust against noise. (c) Randomly generated targets $u$ (solid lines) with average intensity $\langle u \rangle = 1$ and their reconstructed estimates $\hat{u}$ (open circles) at different noise levels $\sigma$. Note the target resolution $\Delta x_u \approx 0.22\lambda$. (d)–(m) Field intensities (solid lines, arbitrary units) at the sensor plane generated by calibrated point sources in the target region. The intensities are integrated over each sensor pixel ($\approx \lambda$ wide) to give the point spread functions (red dotted lines).
wavelengths away from the sensor, and scanning is naturally provided by sample flow \[51\]. Although our examples here were monochromatic, inverse design can easily be applied to broad-band problems, and we are especially excited about using it for computational spectroscopy \[52\], hyper-spectral imaging \[53\], and other broad-band sensing applications. Our framework can straightforwardly scale to 3D freeform structures, accommodate high-dimensional objects such as 4D spatio-spectral targets \[54\], plenoptic light-fields \[55\], high dynamic-range imaging \[56\], nonlinear pulse shaping \[57\], and quantum coherence engineering \[58,59\].

In this paper, our computational-reconstruction stage consisted of Tikhonov-regularized least-squares fitting, but end-to-end optical design can be coupled with many other computational techniques. In under-determined systems (many more targets than sensor pixels), a common approach is compressed sensing \[60\] for sparse targets, and techniques for end-to-end optimization with compressed sensing may include differentiable unrolled approximations \[61\] or epigraph formulations of basis pursuit denoising \[28\]. One could also employ deep learning (neural networks) for imaging and other cognitive tasks (e.g. passive ranging, object recognition); from the perspective of deep learning, the Maxwell solver is simply a specialized “network stage” that is differentiable (via adjoint methods) and hence composable with deep-learning software.

Apart from numerical and experimental endeavors, an important theoretical question is to identify the absolute limits to achievable dispersion (spatial or spectral) and condition numbers, given a desired resolution, a design volume \(V\), and a dielectric contrast \(\Delta \varepsilon\). Recent approaches for shape-independent bounds to light–matter interactions \[62–64\] may be capable of answering these questions.

Acknowledgements

Z.L, C.R.C, R.P, M.S and S.G.J were supported in part by the U. S. Army Research Office through the Institute for Soldier Nanotechnologies under award number W911NF-18-2-0048. Z.L and R.P were partially supported by the MIT-IBM Watson AI Laboratory under Challenge 2415. A.M. was partially supported by a Sloan Fellowship and by the National Science Foundation under award NSF-SNM-1825308.

References

[1] Albert Tarantola. *Inverse Problem Theory and Methods for Model Parameter Estimation*. SIAM, 2005.

[2] Patrick R Gill and David G Stork. Computationally efficient fourier-based image reconstruction in a lensless diffractive imager. In *Computational Optical Sensing and Imaging*, pages CM3E–4. Optical Society of America, 2015.
[3] M Salman Asif, Ali Ayremlou, Aswin Sankaranarayanan, Ashok Veeraraghavan, and Richard G Baraniuk. Flatcam: Thin, lensless cameras using coded aperture and computation. *IEEE Transactions on Computational Imaging*, 3(3):384–397, 2016.

[4] P Gill. Enabling a computer to do the job of a lens. *SPIE Newsroom, Sep*, 2013.

[5] Albert Wang, Patrick Gill, and Alyosha Molnar. Angle sensitive pixels in cmos for lensless 3d imaging. In *2009 IEEE Custom Integrated Circuits Conference*, pages 371–374. IEEE, 2009.

[6] Nick Antipa, Grace Kuo, Reinhard Heckel, Ben Mildenhall, Emrah Bostan, Ren Ng, and Laura Waller. DiffuserCam: Lensless single-exposure 3D imaging. *Optica*, 5(1):1–9, 2018.

[7] Vincent Sitzmann, Steven Diamond, Yifan Peng, Xiong Dun, Stephen Boyd, Wolfgang Heidrich, Felix Heide, and Gordon Wetzstein. End-to-end optimization of optics and image processing for achromatic extended depth of field and super-resolution imaging. *ACM Transactions on Graphics (TOG)*, 37(4):1–13, 2018.

[8] Nanfang Yu and Federico Capasso. Flat optics with designer metasurfaces. *Nature Materials*, 13(2):139, 2014.

[9] Mohammadreza Khorasaninejad, Wei Ting Chen, Robert C Devlin, Jae-won Oh, Alexander Y Zhu, and Federico Capasso. Metalenses at visible wavelengths: Diffraction-limited focusing and subwavelength resolution imaging. *Science*, 352(6290):1190–1194, 2016.

[10] Wei Ting Chen, Alexander Y Zhu, Vyshakh Sanjeev, Mohammadreza Khorasaninejad, Zhujun Shi, Eric Lee, and Federico Capasso. A broadband achromatic metalens for focusing and imaging in the visible. *Nature Nanotechnology*, 13(3):220, 2018.

[11] Raphaël Pestourie, Carlos Pérez-Arancibia, Zin Lin, Wonseok Shin, Federico Capasso, and Steven G Johnson. Inverse design of large-area metasurfaces. *Optics Express*, 26(26):33732–33747, 2018.

[12] Zin Lin, Victor Liu, Raphaël Pestourie, and Steven G Johnson. Topology optimization of freeform large-area metasurfaces. *Optics Express*, 27(11):15765–15775, 2019.

[13] Zin Lin and Steven G Johnson. Overlapping domains for topology optimization of large-area metasurfaces. *Optics Express*, 27(22):32445–32453, 2019.

[14] Zin Lin, Benedikt Groever, Federico Capasso, Alejandro W Rodriguez, and Marko Lončar. Topology-optimized multilayered metaoptics. *Physical Review Applied*, 9(4):044030, 2018.
[15] Haejun Chung and Owen D Miller. High-NA achromatic metalenses by inverse design. *Optics Express*, 28(5):6945–6965, 2020.

[16] Christopher M Lalau-Keraly, Samarth Bhargava, Owen D Miller, and Eli Yablonovitch. Adjoint shape optimization applied to electromagnetic design. *Optics Express*, 21(18):21693–21701, 2013.

[17] Jakob Søndergaard Jensen and Ole Sigmund. Topology optimization for nano-photonics. *Laser & Photonics Reviews*, 5(2):308–321, 2011.

[18] Sean Molesky, Zin Lin, Alexander Y Piggott, Weiliang Jin, Jelena Vucković, and Alejandro W Rodriguez. Inverse design in nanophotonics. *Nature Photonics*, 12(11):659, 2018.

[19] Elyas Bayati, Raphael Pestourie, Shane Colburn, Zin Lin, Steven G Johnson, and Arka Majumdar. Inverse designed metalenses with extended depth of focus. *ACS Photonics*, 7(4):873–878, 2020.

[20] Thaibao Phan, David Sell, Evan W Wang, Sage Doshay, Kofi Edee, Jianji Yang, and Jonathan A Fan. High-efficiency, large-area, topology-optimized metasurfaces. *Light: Science & Applications*, 8(1):1–9, 2019.

[21] Qi Guo, Zhujun Shi, Yao-Wei Huang, Emma Alexander, Cheng-Wei Qiu, Federico Capasso, and Todd Zickler. Compact single-shot metalens depth sensors inspired by eyes of jumping spiders. *Proceedings of the National Academy of Sciences*, 116(46):22959–22965, 2019.

[22] Shane Colburn and Arka Majumdar. Simultaneous achromatic and varifocal imaging with quartic metasurfaces in the visible. *ACS Photonics*, 2019.

[23] Shane Colburn and Arka Majumdar. Metasurface generation of paired accelerating and rotating optical beams for passive ranging and scene reconstruction. *ACS Photonics*, 2020.

[24] Gordon S Kino. *Acoustic Waves: Devices, Imaging, and Analog Signal Processing*. Prentice Hall, Englewood Cliffs, NJ, 1987.

[25] DD Dougal Maclaurin and M Johnson. Autograd: Efficiently computes derivatives of numpy code, 2015.

[26] Joseph W Goodman. *Introduction to Fourier Optics*. Roberts and Company Publishers, 2005.

[27] Lloyd N Trefethen and David Bau III. *Numerical Linear Algebra*, volume 50. Siam, 1997.

[28] Stephen Boyd, Stephen P Boyd, and Lieven Vandenberghe. *Convex Optimization*. Cambridge University Press, 2004.
[29] Christian Szegedy, Alexander Toshev, and Dumitru Erhan. Deep neural networks for object detection. In Advances in Neural Information Processing Systems, pages 2553–2561, 2013.

[30] Fausto Milletari, Nassir Navab, and Seyed-Ahmad Ahmadi. V-net: Fully convolutional neural networks for volumetric medical image segmentation. In 2016 Fourth International Conference on 3D Vision (3DV), pages 565–571. IEEE, 2016.

[31] John David Jackson. Classical Electrodynamics. American Association of Physics Teachers, 1999.

[32] Jian-Ming Jin. The Finite Element Method in Electromagnetics. John Wiley & Sons, 2015.

[33] Christopher M Bishop. Pattern Recognition and Machine Learning. Springer, 2006.

[34] Philipp J Keller, Annette D Schmidt, Joachim Wittbrodt, and Ernst HK Stelzer. Reconstruction of zebrafish early embryonic development by scanned light sheet microscopy. Science, 322(5904):1065–1069, 2008.

[35] Jian Wang. High Resolution 2D Imaging and 3D Scanning with Line Sensors. PhD thesis, Carnegie Mellon University Pittsburgh, PA, 2018.

[36] Aditi Udupa, Jinlong Zhu, and Lynford L Goddard. Voxelized topology optimization for fabrication-compatible inverse design of 3d photonic devices. Optics Express, 27(15):21988–21998, 2019.

[37] Tiemo Bückmann, Nicolas Stenger, Muamer Kadic, Johannes Kaschke, Andreas Frölich, Tobias Kenkerknecht, Christoph Eberl, Michael Thiel, and Martin Wegener. Tailored 3d mechanical metamaterials made by dip-in direct-laser-writing optical lithography. Advanced Materials, 24(20):2710–2714, 2012.

[38] Alan Zhan, Ricky Gibson, James Whitehead, Evan Smith, Joshua R Hendrickson, and Arka Majumdar. Controlling three-dimensional optical fields via inverse mie scattering. Science Advances, 5(10):eaax4769, 2019.

[39] Léon Bottou. Large-scale machine learning with stochastic gradient descent. In Proceedings of COMPSTAT’2010, pages 177–186. Springer, 2010.

[40] Finbarr O’Sullivan. A statistical perspective on ill-posed inverse problems. Statistical Science, pages 502–518, 1986.

[41] Timo Gissibl, Sebastian Wagner, Jachym Sykora, Michael Schmid, and Harald Giessen. Refractive index measurements of photo-resists for three-dimensional direct laser writing. Optical Materials Express, 7(7):2293–2298, 2017.
[42] Daniel B Fullager, Glenn D Boreman, and Tino Hofmann. Infrared dielectric response of Nanoscribe IP-Dip and IP-L monomers after polymerization from $250 \text{ cm}^{-1}$ to $6000 \text{ cm}^{-1}$. *Optical Materials Express*, 7(3):888–894, 2017.

[43] Calvin Yu, Shuting Fan, Yiwen Sun, and Emma Pickwell-MacPherson. The potential of terahertz imaging for cancer diagnosis: A review of investigations to date. *Quantitative Imaging in Medicine and Surgery*, 2(1):33, 2012.

[44] Philip Camayd-Muñoz, Conner Ballew, Gregory Roberts, and Andrei Faraon. Multifunctional volumetric meta-optics for color and polarization image sensors. *Optica*, 7(4):280–283, 2020.

[45] James L Harris. Resolving power and decision theory. *J. Opt. Soc. Am.*, 54(5):606–611, 1964.

[46] James L Harris. Diffraction and resolving power. *J. Opt. Soc. Am.*, 54(7):931–936, 1964.

[47] Sitan Chen and Ankur Moitra. Algorithmic foundations for the diffraction limit. *arXiv preprint arXiv:2004.07659*, 2020.

[48] John Brian Pendry. Negative refraction makes a perfect lens. *Physical Review Letters*, 85(18):3966, 2000.

[49] Xiang Zhang and Zhaowei Liu. Superlenses to overcome the diffraction limit. *Nature Materials*, 7(6):435–441, 2008.

[50] Fu Min Huang, Yifang Chen, F Javier Garcia de Abajo, and Nikolay I Zheludev. Optical super-resolution through super-oscillations. *Journal of Optics A: Pure and Applied Optics*, 9(9):S285, 2007.

[51] Xin Heng, David Erickson, L Ryan Baugh, Zahid Yaqoob, Paul W Sternberg, Demetri Psaltis, and Changhuei Yang. Optofluidic microscopy—a method for implementing a high resolution optical microscope on a chip. *Lab on a Chip*, 6(10):1274–1276, 2006.

[52] Zongyin Yang, Tom Albrow-Owen, Hanxiao Cui, Jack Alexander-Webber, Fuxing Gu, Xiaomu Wang, Tien-Chun Wu, Minghua Zhuge, Calum Williams, Pan Wang, et al. Single-nanowire spectrometers. *Science*, 365(6457):1017–1020, 2019.

[53] Chein-I Chang. *Hyperspectral Imaging: Techniques for Spectral Detection and Classification*. Springer Science & Business Media, 2003.

[54] Kristina Monakhova, Kyrollos Yanny, Neerja Aggarwal, and Laura Waller. Spectral DiffuserCam: Lensless snapshot hyperspectral imaging with a spectral filter array. *arXiv preprint arXiv:2006.08565*, 2020.
[55] Ren Ng, Marc Levoy, Mathieu Brédif, Gene Duval, Mark Horowitz, Pat Hanrahan, et al. Light field photography with a hand-held plenoptic camera. Computer Science Technical Report CSTR, 2(11):1–11, 2005.

[56] Erik Reinhard, Wolfgang Heidrich, Paul Debevec, Sumanta Pattanaik, Greg Ward, and Karol Myszkowski. High Dynamic Range Imaging: Acquisition, Display, and Image-based Lighting. Morgan Kaufmann, 2010.

[57] Govind P Agrawal. Applications of Nonlinear Fiber Optics. Elsevier, 2001.

[58] KW Murch, U Vool, D Zhou, SJ Weber, SM Girvin, and I Siddiqi. Cavity-assisted quantum bath engineering. Physical review letters, 109(18):183602, 2012.

[59] Robert Bennett. Inverse design of environment-induced coherence. arXiv preprint arXiv:2006.03816, 2020.

[60] David L Donoho. Compressed sensing. IEEE Transactions on Information Theory, 52(4):1289–1306, 2006.

[61] Karol Gregor and Yann LeCun. Learning fast approximations of sparse coding. In Proceedings of the 27th International Conference on International Conference on Machine Learning, pages 399–406, 2010.

[62] Mats Gustafsson, Kurt Schab, Lukas Jelinek, and Miloslav Capek. Upper bounds on absorption and scattering. New Journal of Physics, 2020.

[63] Sean Molesky, Pengning Chao, and Alejandro W Rodriguez. T-operator limits on electromagnetic scattering: Bounds on extinguished, absorbed, and scattered power from arbitrary sources. arXiv preprint arXiv:2001.11531, 2020.

[64] Zeyu Kuang, Lang Zhang, and Owen D Miller. Maximal single-frequency electromagnetic response. arXiv preprint arXiv:2002.00527, 2020.