Adjustable robust counterpart optimization model for internet shopping online problem

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Abstract. In this paper, a discussion on deriving the Adjustable Robust Counterpart for Internet Shopping Online (ARC-ISO) is discussed. The problem on Internet Shopping Online is considered as an application of Robust Maximum Flow Problem (RMFP) with circular demand. The decision variables which considered as the adjustable variable is the maximum flow from a destination node in a flow network which back to the source node. In this paper, when the source node is multiple, then it is possible to add a dummy node as a single source node. For the multiple destinations, the same design is done. The main challenge is when and how we assume the data can be uncertain and assumed to be lies within a box, an ellipsoidal or polyhedral uncertainty set. In this paper, the uncertain delivering time is assumed to be lied in a polyhedral uncertainty set.

1. Introduction
Nowadays, business activities are offered through the internet and known as online shopping. Referring to Blazewicz, et al [1] and also based to a survey on American internet user behavior from Horiggan et al in [2] show that from year to year, the online users population is growing rapidly. According Complete Digital 2020 Analysis as can be seen in [3], Indonesia’s data in 2020 shows that for the total population of 272.2 million, internet users in Indonesia consists of 338.2 million of mobile device users, 175.4 million internet users, and 160 million social media users. From this statistical data, it can be seen how big is the potential problem of internet shopping online in Indonesia.

As can be seen in Table 1, it states the art for the topics in optimization model for internet shopping online problem. The used methods for solving ISOP problem are genetic algorithm, mixed integer linear programming, multiobjective programming, heuristic algorithm also robust optimization.
Table 1. State of the art on optimization internet shopping online problem.

| Author          | Topic                                                                 | Optimization Method                  | Uncertain Data |
|-----------------|----------------------------------------------------------------------|--------------------------------------|----------------|
| Kim and Ahn [4] | Genetic Algorithm K-means clustering in an online shopping market      | Genetic Algorithm K-means            | No             |
| Blazewicz et al [1] | Internet shopping optimization problem with shipping costs | Minimizing costs                    | No             |
| Musial et al [5] | Price sensitive discounts                                              | Discount policy based on world observation | No             |
| Blazewicz et al [6] | Internet shopping with price sensitive discounts                     | Price sensitive discounts            | No             |
| Chung [7]       | Internet shopping optimization problem with delivery constraint.       | Multi-objective Optimization, Pareto Set | No             |
| Józefczyk and Lawrynowicz [8] | Heuristic Algorithm for Internet Shopping Online Problem with Sensitive Discount Price | Heuristic algorithm                   | No             |
| Verma et al [9] | Optimizing Online Shopping using Genetic Algorithm                    | Genetic Algorithm                    | No             |

In Chung [7], the determination of the is an upper bound of delivery time in objective function is not explained. In this paper, a discussion on how an upper bound of delivery time can be obtained via optimization model is proposed. This problem is considered as a problem which analogues to Adjustable Robust Counterpart for Maximum Flow Problem (see [10]). Since there must be uncertainty on delivery time, thus an adjustable robust counterpart formulation also discussed in this problem. The study of delivery time uncertainty in online internet shopping is discussed using an indefinite optimization approach. In this paper determination of maximum delivery time for the internet shopping online problem is considered as maximum flow problem with circular demand.

There are two problems that is discussed. First, how to construct the ARC-ISOP by considering ISOP as an uncertain maximum flow problem with circulated demand (MFPCD) as discussed by Kingsford [11]. Second, the adjustable robust counterpart optimization model is defined by consider a polyhedral uncertainty as discussed by Agustini et al [10] for uncertain delivering time, thus a new model for ISOP with ARC-MFPCD is obtained.

The paper is organised as follows. Discussion on Adjustable Robust Counterpart (ARC)Methodology is presented in Section 2. How the model of uncertain ISOP, robust counterpart and adjustable robust counterpart of ISOP are presented in Section 3. The paper is concluded by a recommendation for future research in Section 4.

2. Methods
2.1. Adjustable Robust Counterpart
Referring to [12], [13] and [14], in multistage optimization, the paradigm of RO, i.e., the "here and now" decision, can be relaxed. According to decision rules, some decision variables can be adjusted, which are an uncertain data function. Adjustable Robust Counterpart (ARC) is given
as in equation (1)-(3).

\[
\begin{align*}
\min_{x,y(\zeta)} & \quad c^T x \\
\text{s.t} & \quad A(\zeta)x + By(\zeta) \leq b, \quad \forall \zeta \in Z, \\
& \quad \forall \zeta \in Z,
\end{align*}
\]

where \(x \in \mathbb{R}^n\) is the first stage decision "here and now" made before \(\zeta \in \mathbb{R}^L\) is realized, \(y \in \mathbb{R}^k\) denotes a "wait and see" decision and \(B \in \mathbb{R}^{mxk}\) which shows a certain matrix coefficient. In practice, \(y(\zeta)\) is associated as an affine or a linear decision rules as

\[
y(\zeta) = y^0 + Q\zeta
\]

with \(y^0 \in \mathbb{R}^k\) and \(Q \in \mathbb{R}^{k \times L}\) is the decision rule coefficient, which is to be optimized. Thus, the reformulation of equation (1)-(3) can be seen in equations (5)-(7) as follows:

\[
\begin{align*}
\min_{(x,y^0,Q)} & \quad c^T x \\
\text{s.t} & \quad A(\zeta) + By^0 + BQ\zeta \leq b, \\
& \quad \forall \zeta \in Z.
\end{align*}
\]

2.2. Maximum Flow Problem with Circular Demand

Refers to Kingsford in [11], Maximum Flow Problem with Circular Demand (MFPCD) can be formulated from a problem of circulation with demand. It has assumptions that there are multiple sources and multiple sinks. Every sink node has a certain amount of flow as its supply. For every source node, a supply can be represented as a negative demand.

Maximum flow problem can be considered to present circulation with demands problem. This can be done by adding a new source \(s^*\) which connected to source nodes \(s\) such that there are edges \((s^*, s)\) from \(s^*\) to every node \(s \in S\). Do the same way to connect all sink nodes to a new sink \(t^*\), such that edge \((t, t^*)\) connects \(t^*\) to every node \(t \in T\). The edges capacity \(c_{(s^*, s)} = -d_s\) (since \(d_s < 0\)) and the edges capacity \(c_{(t, t^*)} = d_t\).

3. Result and Discussion

3.1. Internet Shopping Online Problem as an Optimization Model

Refers to Chung [7] who considers a delivery time constraint for ISOP. \(M = 1, \ldots, m\) is a set of shopping malls (shops) and \(N = 1, \ldots, n\) is a set of products (items) to buy in \(m\) shops. Let \(p_{ij}\) be a price of product \(i\) at shop \(j\) and \(f_j\) be a delivery cost at shop \(j\). Note that \(f_j\) is a fixed cost regardless of number of products to buy, \(d_{ij}\) is an expected delivery time of product \(i\) from shop \(j\) to customer after ordering. The objective of is to buy \(n\) products from \(m\) shops with least cost and minimum delivery time of complete products. In [7] \(x_{ij}\) is a binary decision variable whose value is one if product \(i\) is selected from shop \(j\) and zero otherwise. The binary decision variable whether there incurs a delivery cost at shop \(j\) or not is denoted by \(w_j\). The formulation of ISOP from Chung [7] has two objective functions (cost and delivery) as presented in the following mathematical optimization problem.

\[
\begin{align*}
\min & \quad \sum_i \sum_j p_{ij}x_{ij} + \sum_j f_jw_j, \\
\min & \quad \max_{i,j} d_{ij}x_{ij} \\
\text{s.t} & \quad \sum_j x_{ij} = 1, \forall i = 1, \ldots, n, \\
& \quad \sum_j x_{ij} \leq nw_j, j = 1, \ldots, m, ; \\
& \quad x_{ij}, w_{ij} \in \{0, 1\}.
\end{align*}
\]
The objective function (8) means that the purchasing cost including price of products and delivery cost is minimized. The objective function (9) aims is to minimize the delivery time of all products. Constraints (10) means that all products to buy must be selected from available shops and constraints (11) means that fixed delivery cost incurs whenever there is any product selection from shop. Constraints (12) means binary decision variables. Chung [7] reformulate (8) - (12) as (13) - (17) by put the objective function (9) into constraint (16), the $d_{\text{max}}$ is an upper bound of delivery time in objective function (9) . This is one of the general solution approaches to deal with the multi-objective optimization problem (see Deb in [15] and [16]).

\[
\min \sum_i \sum_j p_{ij}x_{ij} + \sum_j f_j w_j, \quad (13)
\]

\[
s.t \quad \sum_j x_{ij} = 1, \forall i = 1, \ldots, n, \quad (14)
\]

\[
\sum_j x_{ij} \leq nw_j, j = 1, \ldots, m, \quad (15)
\]

\[
d_{ij}x_{ij} \leq d_{\text{max}}, i = 1, \ldots, n \quad (16)
\]

\[
x_{ij}, w_{ij} \in \{0, 1\}. \quad (17)
\]

### 3.2. Delivering Time ISOP as Robust Maximum Flow Problem with Circular Demand (RMFP-CD)

Now, consider the ISOP (8) – (12), the transformation of ISOP into Maximum Flow Problem with Circular Demand is done by considering an addition of two dummy nodes as a source and a sink of ISOP. See Figure 1 and Figure 2. Recall the MFP problem as stated in [17] and extended by Kingsford in [11].

The problem of determining the optimal $d_{\text{max}}$, the upper bound of delivery time in objective function (9) can be stated as an Uncertain Maximum Flow Problem with Circular Demand (UMFP-CD). The formulation can be seen formulation (18)-(21).

\[
\max \ d_{\text{max}} \quad (18)
\]

\[
s.t \quad \sum_{i,j} a_{ij}z_{ij} = 0, \quad (19)
\]

\[
0 \leq z_{ij} \leq d_{ij}, \quad (20)
\]

\[
d_{ij} \in \mathcal{U}. \quad (21)
\]

In this problem, the optimal delivery time $z_{ij}^*$ must be determine where $d_{\text{max}}$ denotes the total maximum delivery time for the whole sales of all product $i$ which is delivered from shop $j$. Matrix $A$ is the incidence matrix for the ISOP network as illustrated in Figure 2. For each product $i$ and shop $j$, the delivery time is denoted by $d_{ij}$. Assume that this $d_{ij}$ is uncertain and belong to a polyhedral uncertainty $\mathcal{U}$. The choice for polyhedral uncertainty based on the resulted robust counterpart which yields in linear optimization with close-convex set feasible set.

To this end, firstly define the uncertain delivering time $d_{ij} \in \mathcal{U}$ as

\[
d_{ij} = \bar{d}_{ij} + P_{ij}\zeta, \forall \zeta \in \mathcal{Z} \quad (22)
\]

where $\bar{d}_{ij} \in \mathbb{R}^n$ is a nominal vector of delivering time, $P_{ij} \in \mathbb{R}^{nxL}$ is a confounding matrix, and $\zeta \in \mathbb{R}^L$ is a primitive uncertainty vector. Notes that $d_{ij}$ parameter is a vector of the right hand side of the constraint, then refers to [18], add an extra variable $\omega_{ij} = 1$ so that the $d_{ij}$ parameter becomes the coefficient of the $\omega_{ij}$ variable as in equation (20) follows:

\[
\omega_{ij} = 0, \forall i, j; \omega_{ij} = 1 \quad (23)
\]
Substitute equation (22) to equation (23) to get the model for the maximum flow problem with uncertainty in the following parameters:

\[
\begin{align*}
\max & \quad d_{\text{max}} \\
\text{s.t.} & \quad Az = 0, \\
& \quad z_{ij} - (\bar{d}_{ij} + P_{ij}\zeta)\omega_{ij} \leq 0, \forall i, j, \omega_{ij} = 1, \\
& \quad z_{ij} \geq 0, \forall i, j.
\end{align*}
\] (24-26)

The next step is to determine the adjustable decision variable from the maximum delivering time \( d_{\text{max}} \) which is can be considered as the adjustable variable to \( d_{\text{max}}(\zeta) \), i.e., the decision rules that depend on \( \zeta \) and can be defined as follows:

\[
d_{\text{max}}(\zeta) = \bar{d}_{\text{max}} + Q\zeta
\] (28)

where \( \bar{d}_{\text{max}} \in \mathcal{R} \) is the nominal vector of the amount of maximum deliverint time connected the dummy sink \( t \) to dummy source \( s \) in Figure 2, \( Q \in \mathcal{R}^{n \times L} \) is a confounding matrix, and \( \zeta \in \mathcal{R}^{L} \) is a primitive uncertainty factor. Substitute equation (28) to equation (24) to obtain an ARC model for the maximum flow problem as in equation (29)-(35) follows

\[
\begin{align*}
\max & \quad t \\
\text{s.t.} & \quad \bar{d}_{\text{max}} + Q\zeta - t \geq 0 \\
& \quad Az = 0; \\
& \quad d_{ij} - (\bar{d}_{ij} + P_{ij}\zeta)\omega_{ij} \leq 0, \forall i, j, \\
& \quad \omega_{ij} = 1, \\
& \quad z_{ij} \geq 0, \forall i, j, \\
& \quad \zeta \in \mathcal{Z}.
\end{align*}
\] (29-35)

3.3. ARC model for maximum delivering time ISOP with Polyhedral Uncertainty

Assume that the uncertain parameters and decision variables in the ARC model for the maximum delivering time problem are in the set of polyhedral uncertainty. Define the set of polyhedral uncertainty as follows:

\[
\mathcal{Z} = \{\zeta : p - D\zeta \geq 0\}
\] (36)
where $\zeta \in \mathbb{R}^L$ is a primitive uncertainty vector, $p \in \mathbb{R}^m$, and $D \in \mathbb{R}^{mxL}$. RC formulation for (18)-(21) constraints with the set of polyhedral uncertainty can be written as follows.

$$z_{ij} - (\bar{d}_{ij} + P_{ij}\zeta)^T \omega_{ij} \leq 0, \forall i,j$$

Using the duality theorem as stated in [18] to obtain the ARC in the case of polyhedral uncertainty, thus the following holds.

$$z_{ij} - \bar{d}_{ij}\omega_{ij} - \min_{y_k} \left\{d_k^T y_k : D_k^T y_k = P_{ij}^T \omega_{ij}, y_k \geq 0 \right\} \leq 0, \forall i,j$$

In the same way, the first constraint to equation (30) with the set of polyhedral uncertainty is equivalent to the equation (40).

$$\bar{d}_{\text{max}} + Q\zeta - t = \bar{d}_{\text{max}} + \max_{\zeta \in \mathbb{R}^L, D\zeta \geq 0} (Q\zeta) - t$$

which equivalent with

$$\bar{d}_{\text{max}} + d_x^T y_x - t \geq 0, D_x^T y_x = Q, y_x \geq 0.$$ 

Next, substituting equations (37) and (40) into the Optimization model (21), the ARC model with the set of polyhedral uncertainty for the maximum delivering time problem can be stated as in (43-51).

$$\max \quad t$$

$$s.t \quad \bar{d}_{\text{max}} + d_x^T y_x - t \geq 0$$

$$Az = 0;$$

$$z_{ij} - \bar{d}_{ij}\omega_{ij} - p_{ij}^T y_k \leq 0, \forall i,j,k$$

$$D_k^T y_k = P_{ij}^T \omega_{ij};$$

$$D_x^T y_x = Q;$$

$$\omega_{ij} = 1;$$

$$y_z, y_k \geq 0,$$

$$z_{ij} \geq 0, \forall i,j.$$ 

Notes that the problem (43)-(51) is a linear optimization problem with variables $y_z, y_k \geq 0, z_{ij} \geq 0, \forall i,j$. When the maximum delivering time $t$ can be determined, thus the complete Adjustable Robust Counterpart Model for Internet Shopping Online Problem (13)-(17) can be solved with $d_{\text{max}} = t$. The complete formulation for ARC for ISOP with polyhedral uncertainty set is the following (52)- (64).

$$\min \sum_i \sum_j p_{ij} x_{ij} + \sum_j f_j w_j,$$
s.t \[ \sum_{j} x_{ij} = 1, \forall i = 1, \ldots, n, \] \quad (53)
\[ \sum_{j} x_{ij} \leq nw_j, j = 1, \ldots, m, \] \quad (54)
\[ d_{ij} x_{ij} \leq t, i = 1, \ldots, n, \] \quad (55)
\[ \bar{d}_{max} + d_{T} y_k - t \geq 0 \] \quad (56)
\[ A z = 0; \] \quad (57)
\[ z_{ij} - d_{ij} \omega_{ij} - p_{T} y_k \leq 0, \forall i, j, k \] \quad (58)
\[ D_{T} y_k = P_{ij}^T \omega_{ij}; \] \quad (59)
\[ D_{x} y = Q \] \quad (60)
\[ \omega_{ij} = 1; \] \quad (61)
\[ y, y_k \geq 0, \] \quad (62)
\[ z_{ij} \geq 0, \forall i, j, t \geq 0 \] \quad (63)
\[ x_{ij}, w_{ij} \in \{0, 1\}. \] \quad (64)

As the problem (52) - (64) involved integer variable \( y_j \), thus the ARC-ISOP can be solved as two stage robust optimization problem. A decomposition can be employed. Refers to Chaerani \textit{et al} [19] one alternative procedure for two stage robust mixed integer linear programming is Benders Decomposition. See also Billionet \textit{et al} in [20] for a discussion on 2-Stage Robust MILP with continuous recourse variables.

4. Conclusion

This paper discusses the adjustable robust counterpart model for internet shopping online problem by considering uncertain delivering time. The uncertain delivering time is assumed to be lied in a polyhedral uncertain set. This has been done by determining the allowed maximum delivering time using an analogous the ARC model to maximum flow problem. In this paper, the suggested alternative procedure to solve the problem is by using two stage robust mixed integer linear programming with Benders Decomposition.

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