Analysis-suitable CAD Models based on Watertight Boolean Operations

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Isogeometric analysis has established a new paradigm in computational engineering that significantly improves the interoperability problem between design and analysis, but addressing the full scope of the issue requires adaptations of traditional CAD concepts as well. The recently introduced watertight Boolean operations resolve a fundamental inconsistency in the underlying representations of current CAD models. We investigate the suitability of the resulting geometries for analysis purposes; in particular, in the context of isogeometric boundary element methods. The results confirm that watertight Boolean operations yield CAD models that meet the requirements of numerical simulations.

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1 Introduction

Trimming procedures are ubiquitous in today’s engineering design since they provide the basis for fundamental geometric operations such as Boolean operations [1]. The resulting trimmed CAD models possess minor gaps and overlaps due to inevitable approximation errors and their current treatment in CAD systems. As a result, these models are not analysis-suitable in general. Trimming leads to several severe problems [2] and resolving them is a great challenge for the interaction of design and analysis.

An immense effort is necessary to make trimmed CAD models gap-free or “watertight” so that they become analysis-suitable. In the context of conventional simulations, this is an immediate task of the meshing process which leads to a complete reconstruction of the geometry. Alternatively, various isogeometric analysis concepts have been developed that aim to deal with the gaps during the simulation. In this contribution, we utilize another option, recently introduced in [3], that makes a step towards an analysis-driven design paradigm [4] and establishes a closer connection between design and analysis models. To be precise, geometric operations that lead to analysis-suitable CAD models are employed. The resulting boundary representations (B-reps) are directly analyzed using an isogeometric boundary element method.

2 Method

We combine watertight Boolean operations with isogeometric boundary methods to obtain an approach that allows a more holistic treatment of the engineering design process. The former resolves the trim problem already during the construction of the CAD object, while the latter elegantly circumvents the need to derive a volumetric representation out of a B-rep model.

2.1 Watertight Boolean operations

Watertight Boolean operations are a novel modeling strategy that applies standard user-defined tolerances of CAD systems to obtain watertight representations described by conventional tensor product splines [3]. The methodology uses a three-stage process to provide a well-defined mapping between a trimmed model and its gap-free version.

First, an analysis of the trimmed parametric spaces results in a proper partition of the visualized areas of the model’s surfaces into a set of rectangular regions \( \mathcal{R} \). At an intersection of two surfaces, \( C_{12} = S_1 \cap S_2 \), the corresponding \( \mathcal{R}^1_1 \) and \( \mathcal{R}^2_2 \) are subdivided such that an edge of \( \mathcal{R}^1_1 \) along \( C_{12} \) can be associated with only one \( \mathcal{R}^2_2 \). In the second step, a reparameterization of the trimmed domains is carried out. Thereby, the parameterization along intersections is unified for all surface involved, and each \( \mathcal{R} \) is described by a tensor product basis (without trimming). Finally, a model space update yields the control points of the reparameterized \( \mathcal{R} \). An essential idea of the overall procedure is to utilize the close relationship of a surface and its isocurves. For instance, the reparameterization is actually applied to a set of isocurves and their control points also determine the final surface control points.

The resulting models are watertight, consisting of un-trimmed surface patches connected with an explicit continuity, and accurate to the same model tolerance employed for a corresponding trimmed CAD model. The concept uses information...
computed during conventional Boolean operations, and thus it can be easily integrated into existing CAD systems utilizing B-rep data structures.

### 2.2 Isogeometric boundary element method

As a model problem, we consider the mixed Laplace problem in a bounded domain $\Omega \subset \mathbb{R}^3$ defined by its boundary $\Gamma := \partial \Omega$ divided into a Dirichlet part $\Gamma_D$ and a Neumann part $\Gamma_N$:

$$
\begin{align*}
-k \Delta u(x) & = 0 \quad \forall x \in \Omega \\
\text{Tr} u(x) & = u(y) = g_D(y) \quad \forall y \in \Gamma_D \\
\mathcal{T} u(x) & = q(y) = g_N(y) \quad \forall y \in \Gamma_N
\end{align*}
$$

with the conductivity $k$ and a given right-hand side, $g_D \in \Gamma_D$ and $g_N \in \Gamma_N$. The boundary trace $\text{Tr}$ maps $u(x)$ in the interior of $\Omega$ to $u(y)$ on $\Gamma$, and the co-normal derivative $\mathcal{T}$ transforms the heat potential $u(y)$ to boundary fluxes $q(y)$.

Using the fundamental solution to the Laplace operator for 3D, $U(x, y) = \frac{1}{4\pi|x-y|}$, we obtain the boundary integral equation

$$
c(x)u(x) = \int_{\Gamma} U(x, y) \ t(y) \, ds_y - \int_{\Gamma} \mathcal{T}(x, y) \ u(y) \, ds_y \quad \forall x, y \in \Gamma
$$

by utilizing the second Green’s identity and applying the boundary trace. The fundamental solution for the flux $\mathcal{T}$ results from the co-normal derivative with respect to $y$, i.e., $\mathcal{T}(x, y) = \mathcal{T}_y U(x, y)$.

Following the isogeometric paradigm, we discretize (2) using the basis functions provided by the CAD model that describes $\Gamma$. The system of equation is set-up by collocation at the Greville abscissae, and the integration is performed as detailed in [5].

### 3 Numerical results

Fig. 1 shows the example considered. The geometry is given by the Utah Teapot which has long served as a canonical test model in CAD. Although the teapot has an organic form, geometric accuracy can be measured when using the exact description of elements [6]. Note that in the original trimmed version (Fig. 1a) there is no connection between the lid and the body of the teapot. Fig. 1b illustrates the watertight Utah Teapot model constructed with a user-defined model tolerance of $10^{-3}$. The maximal geometric error obtained is $0.00076 < 10^{-3}$. The boundary conditions for the simulations are a constant flux at the bottom and homogeneous Dirichlet data at the top and the end of the lid. The resulting heat potential is illustrated in Fig. 1c. The only differences between the watertight CAD model and the simulation model are (i) a refined basis to capture the heat distribution and (ii) the additional interior ball which defines the teapot’s interior void and is not part of the original Utah Teapot description. Thus, it can be concluded that watertight Boolean operations generate analysis-suitable CAD models.

![Fig. 1: Numerical example: (a) original trimmed Utah Teapot CAD model, (b) analysis-suitable model obtained by watertight Boolean operations including the boundary conditions for the simulation, and (c) resulting isogeometric boundary element solution.](image)

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