Abnormal deflection of electrons crossing the boundary of two opposite magnetic fields

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Abstract: This paper reports an experiment about abnormal deflection of cathode-ray in odd-symmetric magnetic field. The measurement results show that during cathode-ray passes through odd-symmetric magnetic field, a deflection opposite to Lorentz force occurs at the boundary of two opposite magnetic fields. It can be explained by the inertial effect of the electron rotating on its axis in magnetic field, and Lorentz force is similar to the Magnus effect in fluid. In this paper, a mechanical model that replaces potential energy with rotational kinetic energy is used to calculate the force exerted on an electron and a proton under different conditions, and the Maxwell’s equations of electromagnetic field are derived.

Key words: Lorentz force; electron rotation; inertial effect; Magnus effect; Maxwell’s equations

1. Introduction.

Lorentz force is the basis of classical electromagnetism, but there is no ideal mechanical model for Lorentz force. Cathode-ray tube is easy to generate high-speed electron beam, and its speed can reach one tenth or more of the speed of light. It is an ideal device for measuring Lorentz force. Based on Thomson’s experiment of measuring electron charge-to-mass ratio with cathode-ray tube, this experiment uses modern digital photography technology to measure the characteristics of moving electron deflected by Lorentz force in magnetic field more accurately, and differential measurement results is used to improve reliability of the measurement.

The measurement results show that during cathode-ray passes through odd-symmetric magnetic field, a deflection opposite to Lorentz force occurs at the boundary of two opposite magnetic fields. This result, which is contradictory to electromagnetism, can be explained by the inertial effect of the electron rotating on its axis in magnetic field, and Lorentz force is similar to the Magnus effect in fluid. In this paper, a mechanical model that replaces potential energy with rotational kinetic energy is used to calculate the force exerted on an electron and a proton under different conditions, and the Maxwell’s equations of electromagnetic field are derived. The experimental measurement, related analysis and calculation are helpful to reveal the mechanical essence of Lorentz force.

2. Theoretical deflection of cathode-ray.

Firstly, the method of measuring the charge-to-mass ratio of electrons by magnetic focusing is briefly described; it is improved from Thomson’s charge-to-mass ratio experiment. As shown in Fig-1, an electron with electric quantity of \( q \) and mass of \( m \), its three-dimensional components of initial velocity are \( V_X, V_Y, V_Z \) respectively and \( V_Z = 0 \),

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when the electron enters a constant magnetic field parallel to the X-axis with magnetic induction intensity $B$, it does a uniform linear motion in the X-axis direction and a uniform circular motion in the YZ plane, trajectory of the electron is a helical line.

![Fig-1 Helical trajectory of an electron](image)

Assume that $r$ is radius of the uniform circular motion of the electron, the centripetal force is equal to the Lorentz force, so

$$\frac{mV_y^2}{r} = qV_yB$$

The time required for an electron to circle in YZ plane is $T$ and the angular velocity of uniform circular motion is $\omega_e$, so

$$T = \frac{2\pi r}{V_y} \quad \omega_e = \frac{2\pi}{T} = \frac{q}{m}B$$

$\omega_e$ is proportional to the charge-to-mass ratio $q/m$ of the electron and has nothing to do with the value of $V_y$. The direction of the uniform circular motion of the electron in the YZ plane changes while the direction of $V_y$ changes; the radius $r$ of the uniform circular motion of the electron in the YZ plane changes while the magnitude of $V_y$ changes. All electrons with different $V_y$ have the same angular velocity $\omega_e$ moving around their respective centres, all electrons starting from a same point will arrive at another same point after a period of $T$, and this phenomenon is called magnetic focusing. Projection of the trajectories of electrons on the YZ plane is shown in Fig-2.

![Fig-2 Projection of the trajectories of electrons](image)

In the experiment of measuring the charge-to-mass ratio of electron with cathode-ray tube, the velocity component $V_x$ of electron is generated by a constant accelerating voltage $U_x$, and the velocity component $V_y$ is generated by a periodically alternating voltage $U_y$, as shown in Fig-3. The alternating voltage $U_y$ causes the change of magnitude and direction of $V_y$, but while electrons reach screen of the cathode-ray tube, they revolve the same angle on the YZ plane and therefore the image on screen is always a straight line. Adjust the magnitude of the magnetic induction intensity $B$, the line image will rotate and be gradually shortened into a point. The above is the method of measuring charge-to-mass ratio of electron by magnetic focusing.
Assume that the cathode-ray tube in Fig-3 can be lengthened in the X-axis direction, the line image on screen will rotate with a tiny angle $d\theta$ when the distance of electrons movement increases by $dx$.

$$d\theta = \frac{\omega_e}{2} \cdot \frac{dx}{V_x} = \frac{qB}{2m} \cdot \frac{dx}{V_x} = \frac{q}{2mV_x} \cdot Bdx$$

Suppose that the direction of magnetic induction intensity $B$ is still parallel to the X-axis but its magnitude is a function $B_x(x)$ that varies with $x$, and then $d\psi$ becomes

$$d\theta = \frac{q}{2mV_x} \cdot B_x(x)dx$$

Fig-4 shows the main device used to measure deflection angle of the image. The diameter $D$ of the screen of cathode-ray tube is much smaller than the distance $L$ between electron gun and the screen. Near the screen of the cathode-ray tube, two identical and reverse-connected coils are used as excitation coils; they generate magnetic fields of equal magnitude in opposite directions. These two coils are called a coupled-tube, it can be moved left and right, and its axis overlaps with that of the cathode-ray tube.

Fig-4 The cathode-ray tube and the coupled-tube

Taking axis of the cathode-ray tube as the X-axis and direction of electron motion as the positive direction of the X-axis, the midpoint of the coupled-tube is the contact surface position of the two coils, taking the midpoint of the coupled-tube as the origin of X-axis. When the current of constant current source remains unchanged, magnetic induction intensity $B_x(x)$ of the coupled-tube is shown in Fig-5, $B_x(x)$ is an odd function.
Theoretical curves of magnetic induction intensity and deflection angle $\theta$

The line image of the cathode-ray is used as the reference line while the current of constant current source is zero. When position of the inner surface of the screen of the cathode-ray tube is $x$, deflection angle $\theta(x)$ of the line image is a function as follows

$$\theta(x) \approx \int_{-\infty}^{x} d\theta = \frac{q}{2mV_y} \int_{-\infty}^{x} B(\tau) d\tau$$

$B_x(x)$ is an odd function, therefore $\theta(x)$ is an even function as shown in Fig-5. $x = 0$ is its extreme point. Assume that the cathode-ray tube can be stretched in X-axis direction, while position of the inner surface of the screen moves from $x < 0$ to $x > 0$, it can be found that deflection angle of the line image increases gradually and reaches the maximum at $x = 0$, and then the line image begins to reverse and deflection angle decreases gradually, as shown in Fig-6.

Deflection angle variation

Keeping magnitude of the current of constant current source unchanged and changing the direction of the current, rotation characteristics of the line image should be exactly the same except direction of rotation. These results can be predicted by existing knowledge of electromagnetics.

The motion of electrons in a non-uniform constant magnetic field is complex, and the value of deflection angle $\theta(x)$ cannot be calculated directly by integration. The magnitude $V_y$ of velocity component of electron in the Y-axis direction is marked as the speed variable $v$. When deflection voltage in Y-axis direction is a periodic alternating signal, the image on cathode-ray tube screen is composed of afterglow generated by electrons with different speed $v$ impacting the fluorescent screen after being accelerated by voltage $U_x$ and deflected by magnetic field. Since the magnetic induction intensity of axial magnetic field in the coupled-tube changes with the distance from central axis, the image is not a straight line image but a curve image. In addition, in addition to the Lorentz force exerted on radially moving electrons in axial magnetic field, it is also necessary to consider the Lorentz force exerted on axially moving electrons in radial magnetic field.

Fig-6 describes that deflection angle $\theta(x)$ of line image reaches an extreme value at
3. Actual deflection of cathode-ray.

If we improve measuring accuracy, we will find that the measurement results are different from the predicted value discussed in previous. The extreme point of $\psi(x)$ will change to $x > 0$. While magnitude of the current of constant current source is unchanged but direction is changed, the distance from the extreme point of $\psi(x)$ to the position of $x = 0$ also changes, and the extreme point of $\psi(x)$ deviates more from $x = 0$ when the direction of magnetic field which electrons enter first is opposite to the direction of electrons motion, as shown in Fig.7, $b > a$.

![Fig.7 Actual curves of magnetic induction intensity and inclination angle of tangent](image)

Moving the cathode-ray tube so that position of the inner surface of the screen moves from $x < 0$ to $x > 0$, because the direction of Lorentz force related to $V_y$ on electrons changes at $x = 0$, the curve image should have begun to reverse at $x = 0$. However, the measurement results shown in Fig.7 indicate that the curve image continues to rotate for a distance in original direction, indicating that the electron is exerted a force opposite to Lorentz force within this distance, and this force is called as "reversed Lorentz force".

4. Method for measuring cathode-ray deflection.

Although the measurement result is difficult to explain with the existing electromagnetic knowledge, it is the result predicted in appendix A. In order to verify this prediction, we spent a lot of time in customizing several special cathode-ray tubes. The shape of the cathode-ray tube that can be used to measure is slender (about 470 mm in length, but the screen is only 20 mm in diameter, equivalent to a coin), and the exact value of thickness of the screen must be known (for example 1.20 mm), its photo is shown in Fig.8. The reason why diameter $D$ of the screen of the cathode-ray tube is much smaller than its length $L$ ($L > 16D$) is that the magnetic induction intensity $B(x)$ of coupled-tube must have enough decreasing distance to ensure that $B(x)$ is an odd function.
The deflection of curve image caused by the reversed Lorentz force is very weak, so that it is difficult to be measured. We used a high-pixel industrial camera to take photos of the screen of the cathode-ray tube, and the inclination angle of tangent of curve image can be calculated.

Fig-9 is a photo taken during the measurement, in order to facilitate calculation, the Y-axis is adjusted to a horizontal line, take it as an example to illustrate the method of calculating inclination angle of tangent. The curve image is an odd symmetrical curve. The inclination angle of tangent of the curve at the midpoint is the inclination angle $\psi$ of tangent corresponding to $v = 0$. Due to mechanical error and interference, the midpoint of the odd-symmetric curve in Fig-9 cannot be measured directly, but the midpoint of the odd-symmetric curve in Fig-9 is also the inflection point of the curve, the exact value of the inflection point can be calculated. For example, the midpoint of the curve can be roughly calculated first, and then the middle part curve (as shown in Fig-10) is taken for cubic fitting to obtain the curve equation.

The inclination angle of the tangent line of the cubic fitting curve of Fig-9 at each point is shown as Fig-11, the inclination angle of tangent corresponding to the minimum point is 27.949539 degrees. For each photo, the inclination angle $\psi$ of tangent that meets measurement accuracy can be calculated.
The main experimental equipment includes: (1) A slender cathode-ray tube with a length of about 470mm and a screen with diameter of 20mm. (2) A coupled-tube consists of two air core coils with inner diameter of 20 mm, length of 10.04 mm, line diameter of 0.20 mm and 448 turns. (3) An industrial camera with resolution of 16 megapixels. (4) A laser distance sensor with resolving power of 0.01mm and accuracy of 0.03mm. (5) High-precision sawtooth signal generator. (6) Current source and voltage source.

The main parameters includes: (1) Accelerating voltage is 2196 V. (2) Current of constant current source is 720 mA.

In order to improve measuring accuracy, following 32 measurement combinations were considered, and a total of 672 photos were taken.

(1) Four directions of electron motion: in the same direction as geomagnetic field (including geomagnetic declination and geomagnetic dip), in the opposite direction to geomagnetic field, in the same direction as earth rotation and in the opposite direction to earth rotation.

(2) Four combinations of connection and direction of coil A and coil B, as shown in Fig-12.

(3) Two ways to connect with constant current source.

There are a total of $4 \times 4 \times 2 = 32$ combinations. In order to facilitate measurement, keep position of the cathode-ray tube fixed and move the coupled-tube for measurement. Taking the axis of the cathode-ray tube as the X-axis and the direction of electron motion as the positive direction of the X-axis, the position of inner surface of the screen of the cathode-ray tube is $x = 0$ (the reference selected in experimental measurement is different from the reference selected in previous theoretical analysis, will cause the meaning of the sign of $x$ to change). Moving the coupled-tube makes midpoint of the coupled-tube move step by step from $x = -2.00mm$ to $x = +2.00mm$ with a step length of 0.20mm. One photo is taken every step, 21 photos are taken each combination, $21 \times 32 = 672$ photos are taken in total.

In order to improve reliability of the measurement, differential inclination angle $\psi$ of tangent is used for analysis. For each combination, inclination angle $\psi$ of tangent at $x = +2.00mm$ is used as the reference angle $\psi$ of tangent, absolute inclination angle $\psi$ of
tangent minus the reference angle $\psi$ of tangent is taken as relative inclination angle $\psi_r$ of tangent for every measurement point.

5. **Measurement result.**

The calculation results of these 672 photos show that the geomagnetic direction or the earth rotation direction or the connection method of the coupled-tube or AB direction of the coupled-tube have little influence on quantitative analysis, but the current direction of constant current source can influence qualitative analysis. The current direction of constant current source can be divided into two categories, one causes that direction of magnetic field which electrons enter first is the same as direction of electrons motion (as shown in Fig-7 left side), and the other causes that direction of magnetic field which electrons enter first is opposite to direction of electrons motion (as shown in Fig-7 right side). The calculation results of 672 photos are divided into two types according to the direction category of the current of constant current source, and mathematical average is used for each type of data, which can filter out most of noise and reduce error of final measurement results. After cubic fitting the mathematical average, the final measurement results can be obtained, as shown in Tab-1 and Fig-13.

**Tab-1 Measurement result table**

| Measure Point | Direction of magnetic field which electrons enter first is opposite to direction of electrons motion | Direction of magnetic field which electrons enter first is the same as direction of electrons motion |
|---------------|-------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------|
|               | Average of relative inclination angle of tangent | Fitting value of relative inclination angle of tangent | Average of relative inclination angle of tangent | Fitting value of relative inclination angle of tangent |
| -10           | 0.17763 | 0.17868 | 0.07829 | 0.07542 |
| -9            | 0.32319 | 0.31644 | 0.22246 | 0.22203 |
| -8            | 0.43593 | 0.43807 | 0.35306 | 0.352 |
| -7            | 0.54449 | 0.54368 | 0.46535 | 0.46547 |
| -6            | 0.63006 | 0.6334 | 0.55451 | 0.56256 |
| -5            | 0.69912 | 0.70733 | 0.63561 | 0.64343 |
| -4            | 0.75981 | 0.7656 | 0.70981 | 0.70819 |
| -3            | 0.8129 | 0.80831 | 0.75724 | 0.75698 |
| -2            | 0.83886 | 0.83559 | 0.79627 | 0.78995 |
| -1            | 0.85079 | 0.84755 | 0.81532 | 0.80721 |
| 0             | 0.85085 | 0.84431 | 0.81403 | 0.80892 |
| 1             | 0.83359 | 0.82598 | 0.79372 | 0.79519 |
| 2             | 0.78961 | 0.79267 | 0.76457 | 0.76617 |
| 3             | 0.74157 | 0.74451 | 0.71917 | 0.72199 |
| 4             | 0.67564 | 0.68161 | 0.66127 | 0.66279 |
| 5             | 0.6029 | 0.60409 | 0.58171 | 0.58869 |
| 6             | 0.50933 | 0.51206 | 0.50132 | 0.49984 |
| 7             | 0.40471 | 0.40563 | 0.39694 | 0.39637 |
| 8             | 0.2867 | 0.28493 | 0.27978 | 0.27841 |
As shown in Fig-13, the left curve is the relative inclination angle $\psi_r(x)$ of tangent when direction of the magnetic field which electrons enter first is opposite to direction of electrons motion, its cubic fitting equation is $0.0000193123667x^3 - 0.00754385862x^2 - 0.0108071624x + 0.844307228$, and its extreme point is $x = -0.714$, that is to say, when midpoint of the coupled-tube moves $0.714 \times 0.20 = 0.143mm$ from position of inner surface of the screen of the cathode-ray tube to direction of electron gun, relative inclination angle $\psi_r(x)$ of tangent of the curve image reaches its maximum value. The middle curve is the relative inclination angle $\psi_r(x)$ of tangent when direction of the magnetic field which electrons enter first is the same as direction of electrons motion, its cubic fitting equation is $0.0000224023665x^3 - 0.00771426295x^2 - 0.00603336354x + 0.80891566$, and its extreme point is $x = -0.390$, that is to say, when midpoint of the coupled-tube moves $0.390 \times 0.20 = 0.078mm$ from position of inner surface of the screen of the cathode-ray tube to direction of electron gun, inclination angle $\psi_r(x)$ of tangent of the curve image reaches its maximum value. The dashed line is a reference line symmetric to the right curve.

6. Conclusions.

As shown in Fig-14, the extreme point of the inclination angle $\psi(x)$ of tangent changes to $x > 0$ and $b > a$. 

| x  | 0.15401 | 0.15007 | 0.14682 | 0.14609 |
|----|--------|--------|--------|--------|
| y  | 0      | 0.00116| 0      | -0.00044|
According to the appendix A, the reason why extreme point of $\psi(x)$ changes to $x > 0$ is as follows, electrons rotate on their own axes in magnetic field, at the point of $x = 0$, the inertial effect of rotating electron (proportional to the moment of inertia of electron) causes that curve image continues to rotate for a distance in original direction. The deviation of extreme point of $\psi(x)$ from $x = 0$ cannot be caused by inertial effect of electron moving along tangential direction (proportional to the mass of electron), because Lorentz force calculated according to the existing electromagnetic theory has dropped to zero at the point of $x = 0$, and therefore the uniform linear motion of electrons will not cause inclination angle $\psi(x)$ of tangent to change. The shift of extreme point of $\psi(x)$ to $x > 0$ cannot be caused by the deflection of axially moving electrons in radial magnetic field, because near $x = 0$, the deflection direction of axially moving electrons in radial magnetic field is opposite to that of radially moving electrons in axial magnetic field of left coil, result of the deflection of axially moving electrons in the radial magnetic field is $x < 0$. When an electron moves along the helix in a non-uniform magnetic field, it will be subjected to a force pointing to the weak direction of the magnetic field, so, when the electron approaches $x = 0$ from the left side, it accelerates, and when it leaves $x = 0$, it decelerates, therefore, changes in the axial speed of electrons do not cause the extreme point of $\psi(x)$ to deviate from $x = 0$. Assuming that the extreme point of $\psi(x)$ shifted to $x > 0$ is caused by the mechanical error of cathode-ray tube or coils, then it should be $b = a$ instead of $b > a$.

According to the appendix A, the reason why $b > a$ is as follows, as shown in Fig-14 left, facing the screen of the cathode-ray tube, the electron rotates anticlockwise on its axis in the magnetic field generated by the left coil. According to the structure of an electron shown in Fig-A2 of appendix A, angular momentum of the high-speed rotating tiny ball points to centre of the electron. Fig-14 left taking a tiny ball as a rigid body, and analyzes its force when it revolves around centre O of the electron. The electron rotates anticlockwise, and the tiny ball moves from position A to position B, the increment of angular momentum $\Delta L$ of tiny
ball is shown in Fig-14 left. If pushing force \( F \) on the tiny ball is perpendicular to the paper and facing inward, then the direction of torque \( M \) generated by force \( F \) is opposite to the direction of \( \Delta L \), force \( F \) will block the rotation of the electron. Similarly, in Fig-14 right, the electron rotates clockwise on its axis in the magnetic field generated by the left coil. If pushing force \( F \) on the tiny ball is perpendicular to the paper and facing inward, then the direction of torque \( M \) generated by force \( F \) is the same as the direction of \( \Delta L \), force \( F \) will accelerate the rotation of the electron. An electron moves horizontally to the right at speed \( v_0 \) in the vacuum, the speed \( v_0 \) will gradually decrease, which equivalent to that the electron exerts a pushing force \( F \) towards is left. In Fig-14 left, this force \( F \) prevents the electron from rotating in the magnetic field generated by the left coil, which is equivalent to reducing the inertial effect of the electron rotating on its own axis. In Fig-14 right, this force \( F \) accelerates the rotation of the electron in the magnetic field generated by the left coil, which is equivalent to increasing the inertial effect of the electron rotating on its own axis. This is why \( b > a \). This result was not expected before the experimental measurement. It was a windfall.

672 photos taken during experiment measurement and related calculation results, as well as the code of Python program for calculation, can be downloaded at https://pan.baidu.com/s/1Eete__RbaZz3pYM3DzYQ_A, the download code is 24pf and the URL is case-sensitive. It is suggested that laboratories with good conditions should make more accurate measurements and error analysis.

Appendix A: Mechanical model of electromagnetic field

Abstract: In this paper, a mechanical model that replaces potential energy with rotational kinetic energy is proposed, it is based on rotating physical particle and can be used to explain why like charges repel but opposite charges attract. According to this mechanical model, electromagnetic field is only physical property generated by motions of the particle. Based on this mechanical model and Newtonian mechanics, the force exerted on an electron and a proton under different conditions is calculated, and therefore, electrostatic force and Lorentz force are calculated, mathematical expression of permittivity of vacuum and Maxwell's equations of electromagnetic field are derived, the mechanical essence of induced electric field and displacement current is described, it is explained that there is no causal relationship between changing electric field and changing magnetic field but concomitant relationship. Based on the mechanics model, it can be predicted that "when electrons cross the boundary of two opposite magnetic fields, deflection opposite to Lorentz force will occur due to inertial effect". The prediction is verified by experimental measurement of cathode-ray.

Key words: U-particle; electron; proton; electric field; magnetic field; electrostatic force; Lorentz force; Magnus effect; permittivity of vacuum; permeability of vacuum; induced electric field; displacement current; Maxwell’s equations

1 Mechanical model of U-particle

The purpose of this mechanical model is to replace potential energy with rotational kinetic energy, use Newtonian mechanics to analyze and calculate electromagnetic fields from the perspective of force and momentum, making electromagnetics more intuitive and easier to
understand. Correspondingly, Maxwell used analytical mechanics to analyze and calculate the electromagnetic field from the perspective of energy, requiring more abstract knowledge such as calculus of variations and potential energy.

U-1: Assuming that both electromagnetic field and gravitational field are physical properties generated by different motions of unknown physical particle, take a point A in three-dimensional space as reference point, and use linear time and linear space to calculate velocity. For any point in three-dimensional space, if momentum density of the particle is zero and translational kinetic energy density of the particle remains unchanged, then point A is called a stationary point, otherwise point A is called a moving point.

Explanation: The analysis of this mechanical model is limited to the case of a stationary frame of reference and low-speed movement of electrons/protons.

U-2: The physical particle assumed in U-1 is called universal particle, referred to as U-particle. Suppose that U-particle has following characteristics: (1) inertial mass of a single U-particle is a constant \( M_U \), \( M_U \) is much less than the inertial mass of an electron; geometric size of a single U-particle is a constant which is much smaller than that of an electron; (2) U-particle is uniformly distributed in three-dimensional space, and motion state of U-particle will change after collision between U-particles or collision between U-particle and electron/proton; (3) a U-particle with no translational motion is isotropic and has rotational kinetic energy inside; (4) the sum of translational kinetic energy and rotational kinetic energy of a U-particle is a constant \( E_U \).

Explanation: Absolute vacuum does not exist, the space is full of U-particles, and traditional vacuum just has no gas molecules. In addition to electron and proton, neutron can be seen as a combination of electron and proton, and motion state of U-particle will also change after collision between U-particle and neutron. Because electromagnetic field and gravitational field are physical properties generated by motion of U-particle, U-particle has only kinetic energy, and any other form of energy, including potential energy and heat energy, is different manifestation of kinetic energy of U-particle. The sum of translational kinetic energy and rotational kinetic energy of a specified U-particle is not necessarily a constant, the sum of translational kinetic energy and rotational kinetic energy of U-particle is a constant \( E_U \), which is the result of statistical average.

U-3: Because a U-particle with no translational motion is isotropic and has rotational kinetic energy inside, it can be assumed that there are two structural models of U-particle, Ue and Up. U-particle is an equivalent sphere O with following structure, the sphere is composed of many high-speed rotating tiny balls, and extension line of the tiny ball's rotation axis passes through the centre of sphere O. From the point of view of the centre of sphere O, all tiny balls rotate anticlockwise and angular momentum points to centre O, this kind of U-particle is called Ue. From the point of view of the centre of sphere O, all tiny balls rotate clockwise and angular momentum is backward to centre O, this kind of U-particle is called Up, as shown in Fig-A1.
Explanation: The structural model of U-particle is a little like dandelion flower ball.

U-4: An electron or a proton with no translational motion is isotropic, they have large amount of rotational kinetic energy stored inside. Suppose that an electron is an equivalent sphere O with following structure: radius of this sphere is a constant $R$, which is composed of many high-speed rotating tiny balls, and extension line of the tiny ball's rotation axis passes through the centre of sphere O. From the point of view of centre of the electron, all tiny balls rotate anticlockwise and angular momentum points to centre of the electron. A proton is a sphere with following structure: the sphere is composed of an inner layer and an outer layer. In process of U-particle crossing the outer layer, translational and rotational kinetic energy of the U-particle are not subject to additional effects. Radius of the inner layer is a constant $R$, and structure of the inner layer is similar to that of an electron, but rotation direction of tiny ball is opposite, that’s to say, from the point of view of centre of the proton, all tiny balls rotate clockwise and angular momentum is backward to centre of the proton, as shown in Fig-A2. The inner layer of a proton can also be called an anti-electron.

U-5: There are two U-particles, one of which has rotational kinetic energy of $E_U$ and the other has rotational kinetic energy of $U_t$, characteristics of collision between these two U-particles are as follows: (1) if the two U-particles are same kind of U-particle, then the rotational kinetic energy of both U-particles after collision is $(E_U - U_t)/2$, reduced rotational kinetic energy is transformed into translational kinetic energy; (2) if the two U-particles are different kind of U-particle, then the rotational kinetic energy of both U-particles after collision is $(E_U + U_t)/2$.

Explanation: According to the structural model of U-particle, in collision between the same kinds of U-particle, because rotation direction of tiny balls on collision surface is opposite, total rotational kinetic energy is reduced to $(E_U - U_t)$, each U-particle takes up half, the reduced rotational kinetic energy is transformed into translational kinetic energy. In collision
between different kinds of U-particle, because rotation direction of tiny balls on collision surface is the same, rotational kinetic energy of one U-particle increases and the other decreases, total rotational kinetic energy remains \((E_U + U_1)\) unchanged and each U-particle takes up half. Due to the complexity of structure and collision process of U-particle, the collision between two specified U-particles does not necessarily meet the characteristics described in U-5, the characteristic described in U-5 is the result of statistical average.

U-6: The characteristics of collision between U-particle and static electron/proton are as follows: (1) an electron swallows any one U-particle that collides with it, and then releases a Up at the collision point, the released Up has only rotational kinetic energy and translational kinetic energy is zero; (2) a proton swallows any one U-particle that collides with its inner surface, and then releases a Ue at the collision point, the released Ue has only rotational kinetic energy and translational kinetic energy is zero. Electron/proton is the converter that converts translational kinetic energy of U-particle into rotational kinetic energy of U-particle without the influence of other electron/proton.

**Fig-A3** Collision between U-particle and electron/proton

Explanation: As shown in Fig-A3, an Up is released after the electron swallows a U-particle, on contact surface between the Up and the electron, tiny balls rotate in the same direction, surface of the electron is covered by Up with rotational kinetic energy of \(E_U\). A Ue is released after the proton swallows a U-particle, on contact surface between the Ue and the proton, tiny balls rotate in the same direction, inner surface of the proton is covered by Ue with rotational kinetic energy of \(E_U\).

U-7: An isolated static electron swallows U-particle that collides with it and releases Up, these Up collide with other U-particles and converge to stable diffusion equilibrium state, rotational kinetic energy of the Up at a certain point is inversely proportional to the distance between the point and centre of the electron. An isolated static proton swallows U-particle that collides with its inner surface and releases Ue, these Ue collide with other U-particles and converge to stable diffusion equilibrium state, rotational kinetic energy of the Ue at a certain point is inversely proportional to the distance between the point and centre of the proton. Assume \(r\) is the distance between the Up and centre of the electron, then the decreasing function of rotational kinetic energy \(E_R(r)\) of Up is as follows

\[
E_R(r) = E_U \frac{R}{r} \quad (1)
\]

The gradient of rotational kinetic energy \(E_R(r)\) of Up is

\[
\nabla E_R(r) = -E_U \frac{R}{r^2} \hat{r} = - \frac{M_U C_U^2}{2} \frac{R}{r^2} \hat{r} \quad (2)
\]

\(C_U\) is the maximum speed of U-particle translational motion. The flux of rotational kinetic energy \(E_R(r)\) is proportional to \(\nabla E_R(r)\). The decreasing function of rotational kinetic energy of Ue around a proton is similar.
Fig-A4 Decreasing curves of rotational kinetic energy of U-particle

Explanation: Fig-A4 shows decreasing curves of rotational kinetic energy of Up around an electron and Ue around a proton, for mathematical proof of the curve, please refer to B-1 of appendix B "Mathematical calculation of random collision of U-particle", it is consistent with Fick’s law of diffusion. According to decreasing function of rotational kinetic energy of U-particle $E_R(r) = E_U \cdot R/r$, when $r = \infty$, $E_R = 0$, U-particle has only translational kinetic energy. $E_U = M_U \cdot C_0^2/2$, the following sections calculate $C_U$ is the speed of light. Since the radius R of an electron is very small, assuming that R is $10^{-16}$m, according to equation (1), when distance between U-particle and centre of the electron is 1 nm, rotational kinetic energy of the U-particle is far less than $E_U \cdot 10^{-7}$, and speed of the U-particle is close to $C_U$. In spherical coordinates system, the gradient of $E_R(r)$ is

$$\nabla E_R(r) = \frac{\partial E_R(r)}{\partial r} \cdot e_r = -E_U \cdot \frac{R}{r^2} \cdot e_r = -\frac{M_U \cdot C_0^2}{2} \cdot \frac{R}{r^2} \cdot e_r$$

$$\nabla^2 E_R(r) = \frac{1}{r^2} \cdot \frac{\partial}{\partial r} \left[ r^2 \cdot \left( -\frac{M_U \cdot C_0^2}{2} \cdot \frac{R}{r^2} \right) \right] = 0$$

U-8: The characteristics of collision between moving electron/proton and U-particle are as follows: (1) a moving electron swallows any U-particle that collides with it and then releases Up at the collision point, magnitude and direction of initial velocity of the Up are the same as that of the collision point on the electron surface, and the sum of translational and rotational kinetic energy of the Up is $E_U$; (2) a moving proton swallows any U-particle that collides with its inner surface, and then releases Ue at the collision point, magnitude and direction of initial velocity of the Ue are the same as that of the collision point on inner surface of the proton, and the sum of translational and rotational kinetic energy of the Ue is $E_U$. The momentum of macro motion of U-particle is spread by random collision. The relationship between velocity $v_0$ of an electron/proton moving at a low speed, macro velocity $v_U$ of U-particle and distance $r$ between the U-particle and centre of the electron/proton is

$$v_U(r) = v_0 \cdot \frac{R}{r}$$

Explanation: Macro velocity decreasing function of Up around an electron moving at a low speed is similar to decreasing function of rotational kinetic energy of Up released by a static electron. The mathematical proof of this decreasing function is referred to B-2 of appendix B "Mathematical calculation of random collision of U-particle".
2 Electrostatic force, gravitation, induced force, Lorentz force

There will be more mathematical calculations in the following sections, in order to reduce the trouble of marking vector specially, for the same variable, if it is not bold, it represents a scalar, and if it is bold, it represents a vector. For example, \( v \) represents magnitude of velocity, and \( \mathbf{v} \) represents velocity vector. For the spatial structure of three-dimensional rectangular coordinate system \((x, y, z)\), cylindrical coordinate system \((r, \phi, z)\), spherical coordinate system \((r, \theta, \phi)\), and divergence and curl operation in cylindrical coordinate system, please refer to appendix C "Three dimensional coordinate system and simplified operation of Hamilton operator". The constants and variables commonly used in this paper are shown in the table below.

| Constant | Description |
|----------|--------------|
| \( M_U \) | Inertial mass of a single U-particle |
| \( E_U \) | Total kinetic energy of a single U-particle |
| \( R \) | Radius of electron |
| \( C_U \) | Maximum translational speed of U-particle |
| \( \rho \) | Inertial mass density of U-particle in space |
| \( \rho_N \) | Quantity density of U-particle in space |
| \( Q_e \) | Absolute value of the electric quantity carried by a single electron |
| \( K_U \) | Coefficient less than 1, it is related to random diffusion of U-particle |
| \( K_d \) | The proportion of U-particle colliding with electron/proton directly |
| \( \varepsilon_0 \) | Permittivity of vacuum |
| \( \mu_0 \) | Permeability of vacuum |

| Variable | Description |
|----------|--------------|
| \( \rho_{ER} \) | Rotational kinetic energy density of U-particle |
| \( \rho_{PU} \) | Macro momentum density of U-particle |
| \( E_R \) | Rotational kinetic energy of U-particle |
| \( E_T \) | Translational kinetic energy of U-particle |
| \( v_i \) | The speed at which electrons move in a wire |
| \( v_0 \) | The velocity of movement of an electron outside a wire |
| \( v_U \) | The velocity of macro motion of U-particle, generally refers to Up |
| \( V_U \) | The microscopic translational velocity of U-particle |
| \( v_{Ue} \) | The velocity of macro motion of U-particle generated by the movement of a single electron |
| \( E_h \) | Hidden energy, which is is equal to rotational kinetic energy density of U-particle times the volume of a sphere with radius \( R \) |
| \( P_{em} \) | Electromagnetic momentum, which is equal to macro momentum density of U-particle times the volume of a sphere with radius \( R \), generally refers to Up |
| \( \lambda \) | Number of electrons in directional motion in per unit length wire |
| \( r \) | Distance between measuring point and electron/proton centre or a wire |
| \( \omega \) | Angular velocity of an electron |
| \( F_E \) | Electric field force |
| $F_B$ | Lorentz force |
|-------|---------------|
| $F_I$ | Induced force |
| $F_G$ | Gravitational force |
| $q$   | Electric quantity |
| $E$   | Electric field intensity |
| $B$   | Magnetic induction intensity |
| $\Phi_s$ | Electrostatic potential |
| $A$   | Magnetic vector potential |

U-9: As shown in Fig-A5, two static electrons A and B, electron A swallows U-particle that collides with it and releases $U_{pa}$, after collisions between this $U_{pa}$ and other U-particles, rotational kinetic energy of $U_{pa}$ at a certain point is inversely proportional to the distance between the point and centre of electron A. Electron B swallows U-particle that collides with it and releases $U_{pb}$, translational kinetic energy of $U_{pb}$ is zero and rotational kinetic energy is $E_U$. $U_{pb}$ becomes a "second-hand" U-particle Us after being collided by $U_{pa}$, when the Us collides with electron B, the resultant force on electron B is electrostatic force, which is generated by electron A. Electrostatic force between an electron and a proton or between two protons is similar. As shown in Fig-A6, two static proton A and B, due to the direct collision between $Ue_a$ and proton B, the resultant force on proton B is gravitation, which is generated by proton A. Gravitation on proton B generated by electron A is similar.

Explanation: As shown in Fig-A5, $U_{pb}$ becomes a "second-hand" U-particle Us after being collided by $U_{pa}$, the Us collides with electron B, and the effect of this indirect collision on electron B is electrostatic force. As shown in Fig-A6, $Ue_a$ collides directly with proton B, and the effect of this direct collision on proton B is gravitation.

![Fig-A5](image1) Electrostatic force is generated

![Fig-A6](image2) Gravitation is generated

U-10: The reason why two electrons repel each other is that rotational kinetic energy of $Ue$ between them is more and translational kinetic energy of second-hand Us colliding with electrons is more. The reason why two protons repel each other is that rotational kinetic energy of $Ue$ between them is more and translational kinetic energy of second-hand Us colliding with protons is more. The reason why an electron and a proton attracts each other is that rotational kinetic energy of U-particles between them is more and translational kinetic energy is less.
energy of second-hand Us colliding with electron or proton is less. Gravitation is the result of the direct collision between U-particle and electron/proton. Since the translational kinetic energy of U-particle released by an electron/proton increases with the distance between the U-particle and the electron/proton, the gravitation is always attractive force.

**Fig-A7** Electrostatic repulsion is generated

Explanation: As shown in Fig-A7, U-1, U-2, U-3, and U-4 are Up in equilibrium state around electron A when electron B does not exist, their rotational kinetic energy are \( U_1, U_2, U_3, U_4 \) respectively. U-5 and U-6 are Up released by electron B, and their rotational kinetic energy is \( E_U \). After U-3 collides with U-5, the second-hand Us gets the rotational kinetic energy as \( (E_U - U_3)/2 \) and its translational kinetic energy is \( (E_U + U_3)/2 \). After U-4 collides with U-6, the second-hand Us gets the rotational kinetic energy as \( (E_U - U_4)/2 \) and its translational kinetic energy is \( (E_U + U_4)/2 \), for the reason of \( U_3 > U_4 \), translational kinetic energy of the Us on the left-side of electron B is more than that of right-side and therefore generates greater pressure, so electron B moves to the right, it shows that two electrons repel each other. The force between two protons is similar.

**Fig-A8** Electrostatic attraction is generated

As shown in Fig-A8, U-1, U-2, U-3, and U-4 are Up in equilibrium state around electron A when proton B does not exist, their rotational kinetic energy are \( U_1, U_2, U_3, U_4 \) respectively. U-7 and U-8 are Ue released by proton B, and their rotational kinetic energy is \( E_U \). After U-3 collides with U-7, the second-hand Us gets the rotational kinetic energy as \( (E_U + U_3)/2 \) and its translational kinetic energy is \( (E_U - U_3)/2 \). After U-4 collides with U-8, the second-hand Us gets the rotational kinetic energy as \( (E_U + U_4)/2 \) and its translational kinetic energy is \( (E_U - U_4)/2 \), for the reason of \( U_3 > U_4 \), translational kinetic energy of the Us on the left-side of proton B is less than that of right-side and therefore generates
smaller pressure, so proton B moves to the left, it shows that an electron and a proton attracts each other.

Fig-A9 Gravitation is generated

As shown in Fig-A9, U-1, U-2, U-3, and U-4 are Ue in equilibrium state around proton A when proton B does not exist, their rotational kinetic energy are \( U_1, U_2, U_3, U_4 \) respectively, so their translational kinetic energy are \( (E_U - U_1), (E_U - U_2), (E_U - U_3), (E_U - U_4) \) respectively, gravitation is the result of their direct collision with proton B, for the reason of \( U_3 > U_4 \), translational kinetic energy of the Ue colliding with proton B directly is smaller than that of right-side and therefore generates smaller pressure, so proton B moves to the left, it shows that gravitation is always attractive force.

U-11: The distance between centres of two static electrons/protons is \( L \), electrostatic force between them is

\[
F_e = \frac{2\pi R^4 \rho C_0^2}{9} \frac{1}{L^2} = \frac{4\pi R^3 \rho_N}{9} \left| \left( \nabla E_r(L) \right)_r \right| \tag{4}
\]

\( \rho \) and \( \rho_N \) are the inertial mass density and quantity density of U-particle in space respectively. Electrostatic force is inversely proportional to square of the distance between two static electrons/protons, or proportional to the gradient of rotational kinetic energy of U-particle. The direction of electrostatic force is the line between the electrons/protons, like charges repel but opposite charges attract. Electrostatic force is differential force, and unidirectional component force is much greater than resultant force. When the distance between two electrons or two protons is less than \( 2R \), the electrostatic repulsion between them drops to zero.

Explanation: As shown in Fig-A5, relationship between rotational kinetic energy of Up_a around electron A and distance \( r \) between Up_a and electron A is equation (1). If the distance between a certain point on surface of electron B and centre of electron A is \( r \), at this point, electron B releases Up_b with rotational kinetic energy of \( E_U \), Up_b becomes a second-hand Us after being collided by Up_a and rotational kinetic energy of the Us is

\[
E_{RS}(r) = \frac{E_U - E_R(r)}{2} = \frac{E_U}{2} - \frac{RE_U}{2r}
\]

Translational kinetic energy of the Us is

\[
E_{RS}(r) = E_U - E_{RS}(r) = \frac{E_U}{2} + \frac{RE_U}{2r}
\]

Show as Fig-A10.
If the distance between a certain point on the inner surface of proton B and centre of electron A is \( r \), at this point, proton B releases \( Ue_b \) with rotational kinetic energy of \( E_U \). \( Ue_b \) becomes a second-hand Us after being collided by \( Up_a \) and rotational kinetic energy of the Us is

\[
E_{RS}(r) = \frac{E_U + E_R(r)}{2} = \frac{E_U}{2} + \frac{RE_U}{2r}
\]

Translational kinetic energy of the Us is

\[
E_{TS}(r) = E_U - E_{RS}(r) = \frac{E_U}{2} - \frac{RE_U}{2r}
\]

Show as Fig-A11.

Electrostatic repulsion between two electrons is calculated below. Electrostatic repulsion between two protons and electrostatic attraction between an electron and a proton can be calculated in the same way, magnitude of the force is the same.

As shown in Fig-A12, right plane of the cuboid is surface of electron B, its area is \( dS \) and
velocity component of second-hand U-particle Us in the X-axis direction is $V_x$, then the number of Us passing through left plane into surface of electron B in $dt$ interval is $N = V_x dt dS \rho_N$, these Us collide with electron B and are swallowed by electron B. The same number of U-particles are released on surface of electron B, and their rotational kinetic energy is $E_U$, momentum is zero, so the momentum increment of electron B in the X-axis direction is $N \cdot M_U V_x = V_x^2 dtdS \rho_N M_U$, it can be regarded as the result of the force $dF_e$ exerted on surface $dS$ of electron B in $dt$ interval, so $dF_e dt = V_x^2 dtdS \rho_N M_U$. The pressure on surface of electron B caused by Us collision is $p_s = dF_e / dS = \rho_N M_U V_x^2$. Since Us is isotropic in three-dimensional space, if the speed of the Us is $V_s$, then $V_x^2 = V_s^2 / 3$, translational kinetic energy of the Us is $E_{TS} = M_U V_s^2 / 2$, so

$$p_s = \frac{\rho_N M_U V_s^2}{3} = \frac{\rho V_s^2}{3} = \frac{2 \rho N E_{TS}}{3} \tag{5}$$

The Electrostatic repulsion on electron B generated by electron A is calculated as follows.

![Fig-A13 Repulsion between two electrons](image)

As shown in Fig-A13, the distance between centre of electron A and centre of electron B is $AB = L$. The area of shadowed surface of electron B is $dS = 2\pi R^2 \sin \theta \, d\theta$, using the value of $E_{TS}$ in Fig-A10, according to equation (5), pressure on point C of electron B is

$$p_s = \frac{2 \rho N E_{TS}}{3} = \frac{2 \rho N}{3} \left( \frac{E_U}{2} + \frac{RE_U}{2AC} \right) = \frac{\rho N E_U}{3} \left( 1 + \frac{R}{AC} \right) = \frac{\rho N M_U C_0^2}{6} \left( 1 + \frac{R}{AC} \right) = \frac{\rho C_0^2}{6} \left( 1 + \frac{R}{AC} \right)$$

When $p_s$ applies on $dS$, component force perpendicular to AB direction counteracts, and the horizontal force on electron B to the right is

$$F_e = \int_0^{\pi} p_s \cdot dS \cdot \cos \theta = \int_0^{\pi} \frac{\rho C_0^2}{6} \left( 1 + \frac{R}{AC} \right) \cdot 2\pi R^2 \sin \theta \, d\theta \cdot \cos \theta$$

$$= \frac{\pi R^2 \rho C_0^2}{3} \int_0^{\pi} \left( 1 + \frac{R}{\sqrt{(L - R \cos \theta)^2 + (R \sin \theta)^2}} \right) \sin \theta \cos \theta \, d\theta$$

$$= \frac{\pi R^2 \rho C_0^2}{3} \int_0^{\pi} \left( 1 + \frac{R}{\sqrt{L^2 + R^2 - 2LR\cos \theta}} \right) \sin \theta \cos \theta \, d\theta$$

Refer to D-1 of appendix D "Integral calculation related to electrostatic force", there is

$$F_e = \frac{2\pi R^4 \rho C_0^2}{9} \cdot \frac{1}{L^2}$$

According to equation (2)

$$\left( \nabla E_R(L) \right)_r = - \frac{M_U C_0^2}{2} \cdot \frac{R}{L^2}$$

$$F_e = \frac{2\pi R^4 \rho C_0^2}{9} \cdot \frac{1}{L^2} = \frac{4\pi R^3 \rho N}{9} \cdot \frac{M_U C_0^2 R}{2L^2} = \frac{4\pi R^3 \rho N}{9} \cdot \left| \left( \nabla E_R(L) \right)_r \right|$$

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When L is much greater than R, the component force to the right on the left hemisphere of electron B is

\[ hF_e = \int_0^{\pi/2} p_S \cdot dS \cdot \cos \theta = \frac{\pi R^2 \rho C_\theta^2}{3} \int_0^{\pi/2} \left( 1 + \frac{R}{\sqrt{L^2 + R^2 - 2LR\cos \theta}} \right) \sin \theta \cos \theta \, d\theta \]

\[ \approx \frac{\pi R^2 \rho C_\theta^2}{6} \]

For detailed calculation, please refer to D-2 of appendix D "Integral calculation related to electrostatic force". Suppose that radius of electron is \( R = 1.0 \times 10^{-16} \text{m} \), when the distance between two electrons is 1 m, the ratio of component force on half sphere of an electron to resultant force on the whole electron is

\[ \frac{hF_e}{F_e} \approx \frac{\pi R^2 \rho C_\theta^2}{6} \times \frac{9L^2}{2\pi R^4 \rho C_\theta^2} = \frac{3L^2}{4R^2} \gg 7.5 \times 10^{31} \]

It shows that electrostatic force is differential pressure, and unidirectional component force is much greater than resultant force. When calculating repulsion force between two electrons, assuming that rotational kinetic energy of the Ue around electron is exactly the same, and then its influence on differential force can be ignored.

When the distance L between two electrons is less than 2R, the two electrons overlap in space, as shown in Fig-A14.

**Fig-A14** Distance between two electrons is less than diameter of an electron

When \( p_S \) applies on dS, component force in upward and downward directions counteracts, and the horizontal force on electron B to the right is

\[ F_e = \int_0^{\pi} p_S \cdot dS \cdot \cos \theta = \frac{\pi R^2 \rho C_\theta^2}{3} \int_0^{\pi} \left( 1 + \frac{R}{\sqrt{L^2 + R^2 - 2LR\cos \theta}} \right) \sin \theta \cos \theta \, d\theta \]

Assume \( \alpha = \cos \alpha = L/2R \), then

\[ F_e = \frac{\pi R^2 \rho C_\theta^2}{18} (3x^2 + 4x - 6) \]

For detailed calculation, please refer to D-3 of appendix D "Integral calculation related to electrostatic force".

Solve the equation of \( x^2 + 4x - 6 = 0 \), so \( x \approx 0.8968 \), that is to say when \( L = 2R \times x \approx 1.8R \), \( F_e = 0 \). Electrostatic repulsion between two protons is equal to that between two electrons. Therefore, when the distance between two protons is less than 2 times of the electron radius, electrostatic repulsion between two protons drops to zero, which maybe helps to explain the strong interaction in nucleus.

When \( L > 2R \), assume \( x = L/2R \) also, then the repulsion between two electrons is
Assume

\[ F_e = \frac{2\pi R^4 \rho C_G^2}{9} \cdot \frac{1}{L^2} = \frac{\pi R^2 \rho C_G^2}{18} \cdot \frac{1}{x^2} \]

Then the curve of \( f(x) \) is shown in Fig-A15.

**Fig-A15** The change from electrostatic attraction to repulsion force

U-12: Gravitation on a proton generated by another proton is inversely proportional to square of the distance between them, or proportional to the gradient of rotational kinetic energy of U-particle. Direction of gravitation is the line between the two protons. Gravitation is an attractive force and differential force. If the proportion of U-particle colliding with electron/proton directly is \( K_d \), and distance between the two protons is \( L \), then gravitation on a proton generated by another proton is

\[ F_g = \frac{4\pi R^4 K_d \rho C_G^2}{9} \cdot \frac{1}{L^2} = \frac{8\pi R^3 K_d \rho N}{9} \cdot \left( \nabla E_p(L) \right) \]

Gravitation on a proton generated by an electron is similar.

Explanation: Similar to the calculation of electrostatic repulsion between electron A and B, gravitation between proton A and B can be calculated as follows. In Fig-A6, suppose that the distance between a certain point on the surface of proton B and center of proton A is \( r \), at this point, the magnitude translational velocity of \( U_e \) is \( V_D \) and translational kinetic energy is \( E_{TD} \), then

\[ E_{TD}(r) = E_U - E_r(r) = E_U - \frac{R E_U}{r} = \frac{M_U C_G^2}{2} \left( 1 - \frac{R}{r} \right) = \frac{M_U V_D^2}{2} \]

\[ V_D^2 = C_G^2 \left( 1 - \frac{R}{r} \right) \]

The curve of \( E_{TD}(r) \) is shown in Fig-A16.
Fig-A16 Translational kinetic energy enhancement curve of U-particle colliding directly

Assuming that the proportion of $U_e$ colliding with proton B directly is $K_d$, according to equation (5) for calculating electrostatic repulsion in U-11, the pressure generated by $U_e$ colliding with proton B directly is

$$p_D = \frac{K_d \rho V_D^2}{3} = \frac{K_d \rho C_H^2}{3} - \frac{K_d R \rho C_U^2}{3}$$

Gravitation on proton B generated by proton A is

$$\int_0^\pi p_D \cdot dS \cdot \cos \theta = \int_0^\pi \left( \frac{K_d \rho C_H^2}{3} - \frac{K_d R \rho C_U^2}{3} \right) \cdot 2\pi R^2 \sin \theta \cdot d\theta \cdot \cos \theta$$

$$= -\int_0^\pi \frac{K_d R \rho C_U^2}{3} \cdot \frac{3}{\sqrt{(L - R \cos \theta)^2 + (R \sin \theta)^2}} \cdot 2\pi R^2 \sin \theta \cdot \cos \theta \cdot d\theta$$

$$= -\frac{2\pi R^3 K_d \rho C_U^2}{3} \int_0^\pi \frac{\sin \theta \cdot \cos \theta \cdot d\theta}{\sqrt{L^2 + R^2 - 2LR \cos \theta}} = -\frac{2\pi R^3 K_d \rho C_U^2}{3} \cdot \frac{2R}{3L^2}$$

$$= -\frac{4\pi R^4 K_d \rho C_U^2}{9} \cdot \frac{1}{L^2}$$

For detailed calculation, please refer to D-1 of appendix D "Integral calculation related to electrostatic force". The negative sign indicates that gravitation is attractive force. According to equation (2)

$$\left(\mathbf{V}E_R(L)\right)_r = -\frac{M_d C_H^2}{2} \cdot \frac{R}{L^2}$$

The absolute value of gravitation is

$$F_g = \frac{4\pi R^4 K_d \rho C_U^2}{9} \cdot \frac{1}{L^2} = \frac{8\pi R^3 K_d \rho N}{9} \cdot \left| \frac{RM_d C_H^2}{2L^2} \right| = \frac{8\pi R^3 K_d \rho N}{9} \cdot \left| \left(\mathbf{V}E_R(L)\right)_r \right|$$

It can be seen from the above calculation process and results that gravitation on proton B generated by proton A is attractive force and differential force. Gravitation on proton B generated by electron A is similar. Electrostatic force $F_e$ is calculated according to equation (4), ratio of the electrostatic force on proton B generated by electron A to the gravitation on proton B generated by electron A is as follows

$$\frac{F_e}{F_g} = \frac{1}{2K_d}$$

The electrostatic force is much greater than the gravitational force, so in Fig-A6 only a very
small proportion of Uea collides with proton B directly and $K_d$ is a very small value.

U-13: A static electron/proton with zero translational velocity and angular velocity is pushed by U-particles with changing macro motion velocity, and this force is called the ideal induced force $F_i$, its value is proportional to the rate of change of macro velocity $v_u(t)$ of U-particles with respect to time. Direction of the force on a proton is the same as that of acceleration of Up and opposite to that of Ue. Direction of the force on an electron is the same as that of acceleration of Ue and opposite to that of Up. The force exerted on a proton is

$$F_i = \frac{2\pi R^3 \rho}{9} \cdot \frac{d\nu_u(t)}{dt} \quad (7)$$

**Fig-A17** The collision between macro moving Up and a static proton

As shown in Fig-A17, the static proton B is in an environment that macro velocity of Up is $\nu_u = V_2 - V_1$, direction of $V_2$ is positive direction of Z-axis. Before the collision between Upa1 and Ueb1, translational velocity of Upa1 is $V_1$, rotational kinetic energy of Upa1 is $E_{R1}$ and rotational kinetic energy of Ueb1 is $E_u$. After the collision, Ueb1 becomes a second-hand Us1 with rotational kinetic energy of $E_{RS1}$ and translational kinetic energy of $E_{TS1}$. Before the collision between Upa2 and Ueb2, translational velocity of Upa2 is $V_2$, rotational kinetic energy of Upa2 is $E_{R2}$ and rotational kinetic energy of Ueb2 is $E_u$. After the collision, Ueb2 becomes a second-hand Us2 with rotational kinetic energy of $E_{RS2}$ and translational kinetic energy of $E_{TS2}$.

$$E_{R1} = E_u - \frac{M_uV_1^2}{2}$$

$$E_{RS1} = \frac{E_u + E_{R1}}{2} = E_u - \frac{M_uV_1^2}{4}$$

$$E_{TS1} = E_u - E_{RS1} = \frac{M_uV_1^2}{4}$$

In the same way, it can be calculated

$$E_{TS2} = \frac{M_uV_2^2}{4} = \frac{M_u(V_1 + V_2)^2}{4}$$
When \( v_U > 0 \), \( E_{TS2} > E_{TS1} \), the pressure generated by second-hand U-particles on the bottom surface of the proton is greater than the pressure generated by second-hand U-particles on the top surface, therefore, proton B is pushed upward. When the upward velocity of proton B increases from zero to \( v_U \), proton B is in force equilibrium. No matter what kind of collision occurs between Upa and Ueb, when the upward velocity of proton B increases from zero to \( v_U \), the momentum change of proton B is the same. Therefore, when calculating the momentum change of proton B, the assumption of elastic collision between Upa and Ueb will not affect the calculation result. This calculation method is extended to the whole surface of proton B.

When the macro velocity of Upa is \( v_U(t) \), the macro momentum of Upa contained in a virtual sphere \( \Omega \) with radius \( R \) is

\[
P_v(t) = \iiint_{\Omega} v_u(t) * \rho d\Omega = \frac{4\pi R^3 \rho}{3} * v_u(t)
\]

When an object is stationary, the momentum entering its surface is equal to its momentum increment. According to B-3 of appendix B "Mathematical calculation of random collision of U-particle", the magnitude of momentum increment of proton B per unit time is

\[
\frac{dP(t)}{dt} = \frac{dP_v(t)}{dt} * \rho = \frac{4\pi R^3 \rho}{3} * \frac{dv_u(t)}{dt} * \rho = \frac{2\pi R^3 \rho}{9} * \frac{dv_u(t)}{dt}
\]

The upward pushing force on proton B is

\[
F_i(t) = \frac{dP(t)}{dt} = \frac{2\pi R^3 \rho}{9} * \frac{dv_u(t)}{dt}
\]

**Fig-A18** The collision between macro moving Up and a static electron

As shown in Fig-A18, the static electron B is in an environment that macro velocity of Up is \( v_U = V_2 - V_1 \), direction of \( V_2 \) is positive direction of Z-axis. Before the collision between Upa1 and Upb1, translational velocity of Upa1 is \( V_1 \), rotational kinetic energy of Upa1 is \( E_{R1} \) and rotational kinetic energy of Upb1 is \( E_{U} \). After the collision, Upb1 becomes a second-hand Us1 with rotational kinetic energy of \( E_{RS1} \) and translational kinetic energy of
Before the collision between Upa2 and Upb2, translational velocity of Upa2 is $v_2$, rotational kinetic energy of Upa2 is $E_{R2}$ and rotational kinetic energy of Upb2 is $E_U$. After the collision, Upb2 becomes a second-hand Us2 with rotational kinetic energy of $E_{RS2}$ and translational kinetic energy of $E_{TS2}$.

$$E_{R1} = E_U - \frac{M_U v_1^2}{2}$$
$$E_{RS1} = \frac{E_U - E_{R1}}{2} = \frac{M_U v_1^2}{4}$$
$$E_{TS1} = E_U - E_{RS1} = E_U - \frac{M_U v_1^2}{4}$$

In the same way, it can be calculated

$$E_{TS2} = E_U - \frac{M_U v_2^2}{4} = E_U - \frac{M_U (v_1 + v_U)^2}{4}$$

When $v_U > 0$, $E_{TS2} < E_{TS1}$, the pressure generated by second-hand U-particles on the bottom surface of the electron is less than the pressure generated by second-hand U-particles on the top surface, therefore, electron B is pushed downward. When the downward velocity of electron B increases from zero to $v_U$, electron B is in force equilibrium. No matter what kind of collision occurs between Upa and Upb, when the downward velocity of electron B increases from zero to $v_U$, the momentum change of electron B is the same. Therefore, when calculating the momentum change of electron B, the assumption of elastic collision between Upa and Upb will not affect the calculation result, but the result is negative. This calculation method is extended to the whole surface of electron B. The momentum of electron B in the $Z$-axis direction decreases, but the sum of the momentum in the $Z$-axis direction of Upa1 and Upa2 increases after collision, so it does not violate the momentum conservation.

Similar to momentum change of proton B, the magnitude of momentum increment of electron B per unit time is

$$\frac{dP(t)}{dt} = \frac{dP_v(t)}{dt} * \frac{1}{6} = \frac{2\pi R^3 \rho}{9} * \frac{dv_U(t)}{dt} * \frac{1}{6} = \frac{2\pi R^3 \rho}{9} * \frac{dv_U(t)}{dt}$$

The downward pushing force on electron B is

$$F_1(t) = \frac{dP(t)}{dt} = \frac{2\pi R^3 \rho}{9} * \frac{dv_U(t)}{dt}$$

Since the collision between U-particle and electron/proton is different from the collision between ordinary objects, the force exerted on the electron/proton is only $1/6$ of the collision force of the ordinary object.

For two parallel wires, when the current of one wire increases, the other wire will induce a current in the opposite direction, which seemingly violates the law of conservation of momentum. According to the above analysis, due to the existence of U-particles, the momentum is actually conserved. The phenomenon of electromagnetic induction seemingly violates the law of conservation of momentum, which makes it difficult for scientists in the nineteenth century to analyze electromagnetic field with Newtonian mechanics. Maxwell finally used analytical mechanics to analyze and calculate electromagnetic field from the perspective of energy. In the book of A Treatise on Electricity and Magnetism, he expressed his expectation for the mechanical model of electricity.
U-14: The motion state of U-particles around a electron/proton is the same, angular velocity of the electron/proton rotating on the axis of symmetry of itself is a constant $\omega$, if the electron/proton also has a translation velocity $v$, then the electron/proton is pushed by U-particle and the force is

$$F_\theta = -\frac{4\pi R^3 \rho}{9} * (v \times \omega) \quad (8)$$

This force is called ideal Lorentz force.

Explanation: As shown in Fig-A19, an electron rotates anticlockwise with a constant angular velocity $\omega$ in the positive direction of Z-axis in XYZ coordinate system, and moves in the direction of X-axis with velocity $v$. The electron is cut parallel to XY plane to form a circle with radius $r$, and $\varphi$ is the angle between 0 and $\pi$.

![Fig-A19 An electron rotating on its axis](image)

According to U-8, the velocity of Up released by the electron is

$$V_{1x} = v - \omega r \sin \varphi \quad V_{1y} = \omega r \cos \varphi$$

$$V_{2x} = v + \omega r \sin \varphi \quad V_{2y} = -\omega r \cos \varphi$$

$$V_2^2 - V_1^2 = (V_{2x}^2 + V_{2y}^2) - (V_{1x}^2 + V_{1y}^2) =$$

$$= [(v + \omega r \sin \varphi)^2 + (-\omega r \cos \varphi)^2] - [(v - \omega r \sin \varphi)^2 + (\omega r \cos \varphi)^2]$$

$$= 4\omega v r \sin \varphi = 4\omega v r \sin \theta \sin \varphi$$

The motion state of U-particle in surrounding environment is consistent, so the rotational kinetic energy of U-particles are the same, the electron with angular velocity $\omega$ enters the environment at translational velocity $v$.

Case 1: rotational kinetic energy of Up in surrounding environment is $E_{R_{\text{p}}}$

Suppose that the electron releases $U_{p_1}$ with velocity of $V_1$ and its rotational kinetic energy is $E_{R_1}$. $U_{p_1}$ becomes second-hand $U_{S_1}$ after being collided by Up in surrounding environment, rotational kinetic energy of the $U_{S_1}$ is $E_{R_{S1}}$, translational kinetic energy is $E_{T_{S1}}$ and the speed is $V_{S1}$, the pressure generated by $U_{S_1}$ is $p_{S1}$. The electron releases $U_{p_2}$ with velocity of $V_2$, $U_{p_2}$ becomes second-hand $U_{S_2}$ after being collided by Up in surrounding environment, the pressure generated by $U_{S_2}$ is $p_{S2}$. Using the method in U-5 to calculate rotational kinetic energy of second-hand U-particle after collision, the rotational kinetic energy of one of U-particles is required to be $E_U$, which is not satisfied here. A similar method can be used to approximately calculate the rotational kinetic energy of the second-hand U-particle as follows

$$E_{R_{S1}} = \frac{E_{R_{1}} - E_{R_{\text{p}}}}{2} = \frac{(E_U - M_U V_1^2/2) - E_{R_{\text{p}}}}{2} = \frac{E_U}{2} - \frac{E_{R_{\text{p}}}}{2} - \frac{M_U V_1^2}{4}$$
According to equation (5)

\[ E_{TS1} = E_U - E_{RS1} = \frac{E_U + E_{RP}}{2} + \frac{M_U V_1^2}{4} = \frac{M_U V_{S1}^2}{2} \]

\[ V_{S1}^2 = \frac{E_U + E_{RP}}{M_U} + \frac{V_1^2}{2} \]

It can be calculated similarly

\[ p_{S1} = \frac{\rho V_{S1}^2}{3} = \frac{\rho (E_U + E_{RP})}{3M_U} + \frac{\rho V_1^2}{6} \]

So

\[ p_{S2} - p_{S1} = \frac{\rho (V_2^2 - V_1^2)}{6} \]

Case 2: rotational kinetic energy of \( U_e \) in the surrounding environment is \( E_{Re} \)

Suppose that the electron releases \( U_{P1} \) with velocity of \( V_1 \) and its rotational kinetic energy is \( E_{R1} \). \( U_{P1} \) becomes second-hand \( U_{S1} \) after being collided by \( U_e \) in surrounding environment, rotational kinetic energy of the \( U_{S1} \) is \( E_{RS1} \), translational kinetic energy is \( E_{TS1} \) and the speed is \( V_{S1} \), the pressure generated by \( U_{S1} \) is \( p_{S1} \). The electron releases \( U_{P2} \) with velocity of \( V_2 \), \( U_{P2} \) becomes second-hand \( U_{S2} \) after being collided by \( U_e \) in surrounding environment, the pressure generated by \( U_{S2} \) is \( p_{S2} \). Using a method similar to U-5, the approximate calculation is as follows

\[ E_{RS1} = \frac{E_{R1} + E_{Re}}{2} = \frac{(E_U - M_U V_1^2 / 2) + E_{Re}}{2} = \frac{E_U}{2} + \frac{E_{Re}}{2} - \frac{M_U V_1^2}{4} \]

\[ E_{TS1} = E_U - E_{RS1} = \frac{E_U - E_{Re}}{2} + \frac{M_U V_{S1}^2}{4} = \frac{M_U V_{S1}^2}{2} \]

\[ V_{S1}^2 = \frac{E_U - E_{Re}}{M_U} + \frac{V_1^2}{2} \]

According to equation (5)

\[ p_{S1} = \frac{\rho V_{S1}^2}{3} = \frac{\rho (E_U - E_{Re})}{3M_U} + \frac{\rho V_1^2}{6} \]

It can be calculated similarly

\[ p_{S2} = \frac{\rho V_{S2}^2}{3} = \frac{\rho (E_U - E_{Re})}{3M_U} + \frac{\rho V_2^2}{6} \]

So

\[ p_{S2} - p_{S1} = \frac{\rho (V_2^2 - V_1^2)}{6} \]

The equation of \( p_{S2} - p_{S1} \) is the same regardless of whether the surrounding environment is \( U_p \) or \( U_e \), so

\[ p_{S2} - p_{S1} = \frac{\rho (V_2^2 - V_1^2)}{6} = \frac{\rho}{6} \cdot 4 \nu \omega R \sin \theta \sin \varphi = \frac{2R \rho \nu \omega \sin \theta \sin \varphi}{3} \]

In Fig-A19, area of shadowed surface is \( dS = R^2 \sin \theta \, d\theta \, d\varphi \), when \( p_s \) applies on \( dS \), component force in positive direction of Y-axis is \( dF_B = p_s \, dS \, \sin \theta \, \sin \varphi \). For the whole electron, component force in upward and downward directions counteracts. Suppose
that front hemisphere of the electron is a curved surface $\Sigma$, and then the resultant force on the electron in positive direction of Y-axis is

$$F_B = \iiint dF_B = \iiint p_S \ast dS \ast \sin \theta \sin \varphi = \iiint (p_{S2} - p_{S1}) \ast dS \ast \sin \theta \sin \varphi$$

$$= \iiint \frac{2R\rho \omega \sin \theta \sin \varphi}{3} \ast R^2 \sin \theta \vartheta \varphi \ast \sin \theta \sin \varphi$$

$$= \frac{2R^3 \rho \omega}{3} \int_0^\pi \sin^3 \theta \vartheta \varphi \int_0^\pi \sin^2 \varphi \varphi \varphi \varphi = \frac{2R^3 \rho \omega}{3} \ast \frac{4}{3} \ast \frac{\pi}{2}$$

$$= \frac{4\pi R^3 \rho}{9} \ast \nu \ast \omega$$

The resultant force on the electron in direction of X-axis is

$$\iiint p_S \ast dS \ast \sin \theta \cos \varphi = \frac{2R^3 \rho \omega}{3} \int_0^\pi \sin^3 \theta \vartheta \varphi \int_0^\pi \sin \varphi \cos \varphi \varphi \varphi \varphi = 0$$

The force $F_B$ is Lorentz force in classical electromagnetism, it is similar to the result of Magnus effect in fluid, and the banana ball shot by football players is the result of similar force. When $\nu$ is not perpendicular to $\omega$,

$$F_B = -\frac{4\pi R^3 \rho}{9} \ast (\nu \times \omega)$$

A negative sign indicates that the direction of $F_B$ is opposite to that of the vector product of $\nu \times \omega$. It can be seen from the derivation that equation (8) is the same for both electron and proton.

The Lorentz force calculated by the above formula is the result of approximate calculation, so the magnetic induction intensity $B$ cannot be accurately defined by Lorentz force. However, we can refer to the definition of magnetic induction intensity by Lorentz force in classical electromagnetism.

3 Superposition principle

From previous calculations of electrostatic force, gravitation, induced force, and Lorentz force, it can be seen that all these forces are the rate of change of momentum of U-particle with respect to time. Since momentum satisfies linear addition, these forces also satisfy linear addition.

From the calculation process of electrostatic force, it can be seen that when Up's $\nabla E_R$ or Ue's $\nabla E_R$ remains unchanged, the force direction on an electron is opposite to that on a proton. Direction of the force generated by Up's $\nabla E_R$ and Ue's $\nabla E_R$ on the same electric charge is opposite. For the convenience of calculations, it can be assumed that the signs of proton and electron are opposite, and the signs of Up's $E_R$ and Ue's $E_R$ are opposite. In this paper, take proton as positive and electron as negative, take Up's $E_R$ as positive and Ue's $E_R$ as negative. This negative sign indicates that direction of the force generated by Ue's $\nabla E_R$ and Up's $\nabla E_R$ on the same electric charge is opposite.

From the calculation process of induced force, it can be seen that when Up's $d\nu_U/dt$ or Ue's $d\nu_U/dt$ remains unchanged, the force direction on an electron is opposite to that on a proton. Direction of the force generated by Up's $d\nu_U/dt$ and Ue's $d\nu_U/dt$ on the same electric charge is opposite. For the convenience of calculations, this paper takes Up's $\nu_U$ as
positive and Ue's \( \nu_U \) as negative. This negative sign indicates that direction of the force generated by Ue's \( d\nu_U/dt \) and Up's \( d\nu_U/dt \) on the same electric charge is opposite.

Fig-A20 Two electrons close together

As shown in Fig-A20, Direction of macro motion of Up is upward. Assume that the macro speed \((V_c - V_a)\) of Up on the lower left side of the proton is greater than the macro speed \((V_b - V_a)\) of Up on the lower right side of the proton. After collision with Up1, Ue1 becomes a second-hand U-particle with translational kinetic energy of \( E_{TS1} \). After collision with Up2, Ue2 becomes a second-hand U-particle with translational kinetic energy of \( E_{TS2} \). Referring to U-13 analysis, it can be seen that \( E_{TS2} > E_{TS1} \), so the proton will rotate clockwise. If the proton in Fig-A20 is replaced by an electron, the electron rotates anticlockwise. Similarly, macro motion of Ue causes proton and electron to rotate in opposite directions. Although the Lorentz force on electrons and protons can be calculated by equation (8), the macro velocity of Up and the macro velocity of Ue cause the same electric charge to rotate in the opposite direction, resulting in the opposite direction of the Lorentz force. If rotation direction of an object is represented by right-hand rule, then, the rotational direction of proton is the same as the direction of curl \( \nabla \times \nu_U \) of Up's macro velocity.

Therefore, if we take proton as positive and electron as negative, take rotational kinetic energy \( E_R \) of Up as positive and rotational kinetic energy \( E_R \) of Ue as negative, take macro velocity \( \nu_U \) of Up as positive and macro velocity \( \nu_U \) of Ue as negative, then electrostatic force, gravitation, induced force, and Lorentz force satisfy linear calculations. Similarly, the gradient, divergence, and curl associated with them also satisfy linear operations.

In U-7, the decreasing function of rotational kinetic energy of U-particle is derived by applying mathematical theorem of random collision to an isolated electron and equilibrium system, but in the real physical environment, such as two or more electrons, it can neither satisfy the isolated electron nor keep the system in equilibrium state, therefore, the use of \( E_R(r) = E_U \times R/r \) is limited.

When an isolated electron is in stable diffusion equilibrium state, the flux \( \phi_e \) of rotational kinetic energy \( E_{R_e} \) of U-particle can be calculated in spherical coordinates according to equation (2) as follows, \( \Sigma \) is the closed surface encloses the electron.

\[
\phi_e \propto \iiint_{\Sigma} \nabla E_{R_e}(r) \cdot dS = \iiint_{\Sigma} -\frac{E_{R_e}r}{r^2} \cdot r^2 \sin \theta d\theta d\varphi = -4\pi r E_{U_e}
\]

It can be seen from the calculation results that the flux \( \phi_e \) of rotational kinetic energy is a constant. According to U-6, an electron is an energy converter which can convert translational kinetic energy into rotational kinetic energy without the influence of other electrons/protons, then, in the case of multiple electrons, the equation is still suitable for each individual electron.
In the case of N electrons, if the gradient of rotational kinetic energy of U-particle is $\nabla E_R(r)$, then the flux of rotational kinetic energy of U-particle can be calculated as follows. \( \Sigma \) is a closed surface that encloses these N electrons.

$$0 \propto \oint \nabla E_R(r) \cdot dS = \oint (\nabla E_{R1}(r) + \nabla E_{R2}(r) + \cdots + \nabla E_{RN}(r)) \cdot dS \propto N \cdot 0_e = \text{Constant}$$

U-15: Electrostatic force, gravitation, induced force, and Lorentz force are all the rate of change of momentum of U-particle with respect to time. Take proton as positive and electron as negative, take rotational kinetic energy $E_R$ of Up as positive and rotational kinetic energy $E_R$ of Ue as negative, take macro velocity $v_u$ of Up as positive and macro velocity $v_u$ of Ue as negative, then electrostatic force, induced force, and Lorentz force satisfy linear addition. Similarly, the gradient, divergence, and curl associated with them also satisfy linear addition. The principle of superposition in physics is to describe physical law by accurate mathematical addition, which has a small error with the actual physical results. The superposition principle can be used to describe low-speed motion, and it is assumed that the following equation is always correct for stationary electrons/protons.

$$\oint \nabla E_R(r) \cdot dS = \text{Constant} \quad (9)$$

\( \Sigma \) is a closed surface that encloses these electrons/protons.

Suppose the number of protons is positive, the number of electrons is negative, and the number of charges is the number of protons plus the number of electrons. The distance between two objects with charge number N and M is L, then, electrostatic force between them is

$$F_e = N \cdot M \cdot F_e = \frac{2\pi R^4 \rho C_0^2}{9} \cdot \frac{NM}{L^2}$$

It's hard to understand superposition principle of Coulomb force in classical electromagnetism, if there are N+1 electrons on a line, electron A is on the left and others N electrons on the right, no matter how many the N is, repulsive force from electron A to every electron is unchangeable, is electron A green giant? The superposition principle of electrostatic force is easy to understand by mechanical model based on U-particle, the force exerted on the N electron is generated by U-particles released by electron A. It's similar to gas pressure.

Equation (3) is based on the assumption that electrons move at low speeds, so the superposition principle is only suitable for low speed motion.

4 **Electrostatic field, gravitational field, constant magnetic field**

In classical electromagnetism, the unit of current Ampere is defined as follows: two parallel infinite long straight wires in vacuum, distance between them is 1 meter, current on the two wires is equal in magnitude and in the same direction, if attractive force exerted on per meter wire is equal to \( 2 \cdot 10^{-7} N \), then the current in each wire is 1 Ampere, equal to 1 Coulomb per second. The definition of Ampere is the bridge of quantitative calculation between Newtonian mechanics and electromagnetics. In classical electromagnetism, the electric quantity of an electron $Q_e \approx 1.602 \cdot 10^{-19} \text{C}$ and permittivity of vacuum $\varepsilon_0 = 8.854 \cdot 10^{-12} \text{C}^2/(\text{N} \cdot \text{m}^2)$ are measured by experiments. There is no concept of electric
quantity in mechanical model of U-particle. In order to verify correctness of mechanical model of U-particle by using existing achievements of classical electromagnetism, definition and unit of classical electromagnetism should be used uniformly, therefore, one electron in mechanical model of U-particle is equivalent to $1.602 \times 10^{-19}$ C in classical electromagnetism.

U-16: The gradient field of rotational kinetic energy of U-particle constitutes electrostatic field and gravitational field, they are essentially the same. Suppose that electric quantity of proton is positive and that of electron is negative, $q$ is electric quantity of protons plus that of electrons, absolute value of the electric quantity carried by one electron is $Q_e$, then electrostatic field intensity generated by $q$ at a distance of $r$ is

$$E_e(r) = \frac{2\pi R^4 \rho N}{9Q_e} * \nabla E_r(r) = \frac{2\pi R^4 \rho C_0^2}{9Q_e} * \frac{q}{r^2} * e_r$$  \hspace{1cm} (10)

$$\nabla \times E_s = 0$$  \hspace{1cm} (11)

Assuming that the gradient of rotational kinetic energy of Up is $\nabla E_{Rp}(r)$ and that of Ue is $\nabla E_{Re}(r)$, then the electrostatic field intensity is proportional to $\nabla E_{Rp}(r) + \nabla E_{Re}(r)$, and the gravitational field intensity is proportional to $\nabla E_{Rp}(r) - \nabla E_{Re}(r)$.

The negative value of the gradient of electrostatic potential $\varphi_e$ is electrostatic field intensity $E_s$, $E_s = -\nabla \varphi_e$. The electrostatic potential $\varphi_e(r)$ generated by electric quantity $q$ at a point with distance $r$ from it is

$$\varphi_e(r) = \frac{2\pi R^4 \rho C_0^2}{9Q_e} * \frac{q}{r} = -\frac{1}{3Q_e} * \left[ \rho_{ER}(r) * \frac{4\pi R^3}{3} \right] = -\frac{E_h(r)}{3Q_e}$$  \hspace{1cm} (12)

$$\rho_{ER}(r) = E_r(r) * \rho_N$$, where $\rho_{ER}(r)$ is rotational kinetic energy density of U-particle, and its symbol is the same as that of $E_r(r)$. Hidden energy $E_h(r)$ at a certain point is equal to rotational kinetic energy density $\rho_{ER}(r)$ of U-particle at the point times the volume of a sphere with radius $R$.

Explanation: The definition of electric field intensity in classical electromagnetism is the force exerted on unit positive charge in electric field. According to equation (4), the repulsive force of one proton to another is

$$F_e = \frac{2\pi R^4 \rho C_0^2}{9} * \frac{1}{r^2}$$

The number of protons equivalent to electric quantity $q$ is $N = q/Q_e$ and the number of protons of per unit positive charge is $M = 1/Q_e$, according to the superposition principle, the repulsion force exerted on unit positive charge which is generated by electric quantity $q$ is

$$F_E = N * M * F_e = \frac{q}{Q_e} * \frac{1}{Q_e} * F_e = \frac{2\pi R^4 \rho C_0^2}{9Q_e^2} * \frac{q}{r^2}$$

Electrostatic field intensity generated by electric quantity $q$ is

$$E_s = \frac{2\pi R^4 \rho C_0^2}{9Q_e^2} * \frac{q}{r^2}$$

U-particle around a proton is Ue, so take its rotational kinetic energy as a negative value. For a single proton, according to equation (1), $E_p(r) = -E_u * R/r$, for $N = q/Q_e$ protons, using the superposition principle

$$E_R(r) = -E_u * \frac{R}{r} * \frac{q}{Q_e}$$
The calculation method of electrostatic field intensity $\mathbf{E}_s$ in the above equation refers to the definition of electrostatic field intensity $\mathbf{E}_s$ in classical electromagnetism, but it can define the electrostatic field intensity $\mathbf{E}_s$ at a certain point. Therefore, this paper takes it as the accurate definition of electrostatic field intensity $\mathbf{E}_s$ under the mechanical model of U-particle. Because curl of gradient is always zero, the electrostatic field is irrotational field.

$$\nabla \times \mathbf{E}_s = 0$$

After the rotational kinetic energy of U-particle is given a positive or negative sign, for the electrostatic force on an electron/proton, the effect of Up's rotational kinetic energy gradient $\nabla \mathbf{E}_R (r)$ is the same as that of Ue's rotational kinetic energy gradient $\nabla \mathbf{E}_E (r)$, that is, the electrostatic field intensity is proportional to $\nabla \mathbf{E}_R (r) + \nabla \mathbf{E}_E (r)$. Since the gravitation is always attractive, for the gravitation on an electron/proton, the effect of Ue's rotational kinetic energy gradient $\nabla \mathbf{E}_E (r)$ is opposite to that of Up's rotational kinetic energy gradient $\nabla \mathbf{E}_R (r)$, that is, the gravitational field intensity is proportional to $\nabla \mathbf{E}_R (r) - \nabla \mathbf{E}_E (r)$.

Classical electromagnetism defines the electrostatic potential of a point as: the work done by electric field force when a unit positive charge travels from the point to infinity through any path. Referring to the definition of classical electromagnetism, under the mechanical model of U-particle, the electrostatic potential of electric quantity $q$ at a point with distance $r$ from it is defined as follows

$$\varphi_s (r) = \int_r^{\infty} E_s (x) dx = \int_r^{\infty} \frac{2\pi R^4 \rho_{C_0}^2}{9Q_e} \cdot \frac{q}{x^2} dx = \frac{2\pi R^4 \rho_{C_0}^2}{9Q_e} \cdot \frac{q}{r} = \frac{4\pi R^3 \rho}{9Q_e} \cdot \left( \frac{M_0 C_0^2}{2} \cdot \frac{R}{r} \cdot \frac{q}{Q_e} \right)$$

$$= \frac{4\pi R^3}{9Q_e} \cdot \rho \cdot \left( \frac{E_u}{R} \cdot \frac{R}{Q_e} \right) = \frac{4\pi R^3}{9Q_e} \cdot \rho \left[ -E_R (r) \right]$$

$$= -\frac{1}{3Q_e} \left[ \frac{\rho E_R (r)}{3} \cdot \frac{4\pi R^3}{3} \right] = -\frac{1}{3Q_e} \left[ \rho E_R (r) \cdot \frac{4\pi R^3}{3} \right]$$

The positive and negative sign of $\varphi_s (r)$ is the same as the sign of electric quantity $q$. For the mechanical model of U-particle, the rotational kinetic energy of U-particle that is infinitely far away from electrons/protons is zero, so the electrostatic potential is zero.

Hidden energy $E_h (r)$ at a certain point is equal to rotational kinetic energy density $\rho_{ER} (r)$ of U-particle at the point times the volume of a sphere with radius $R$, therefore, the above equation can also be written as $\varphi_s (r) = -E_h (r)/3Q_e$.

U-17: Infinite long straight wire in the direction of Z-axis, $\lambda$ electrons per unit length move in the wire at constant speed $v_i$, distance between point A and the wire is $r$, then the curl of the macro velocity $\nabla \cdot \mathbf{v}_u$ of Up at point A is

$$\nabla \times \mathbf{v}_u = \frac{2\pi R \lambda v_i}{r} \cdot \mathbf{e}_\phi$$

(13)

An electron/proton will rotate on its axis at point A. A proton rotates in the opposite direction to an electron at point A. The angular velocity of a proton is
The relationship between direction of electron angular velocity at point A and direction of current is right-handed helix. Hold the long straight wire with right hand, point four fingers in direction of angular velocity of the electron rotation, and direction of thumb is direction of the current. Velocity curl of \( \mathbf{U}_e \) and that of \( \mathbf{U}_p \) cause electrons or protons to rotate in the opposite direction.

\[
\omega = \frac{\nabla \times \mathbf{v}_U}{2} = \frac{R \lambda v_i}{r} \mathbf{e}_\varphi
\]  

Fig-A21 The velocity curl of \( \mathbf{U}_p \) and electron rotation

Explanation: As shown in Fig-A21, the infinite long straight wire overlaps with Z-axis, and direction of electron movement in the wire is the positive direction of Z-axis. Take a small length of wire \( dz \) on Z-axis, according to U-8, the movement of a single electron with velocity of \( v_i \) in \( dz \) causes the macro velocity of \( \mathbf{U}_p \) at point A to be \( \mathbf{v}_{Ue} = R v_i/l \). Since \( \mathbf{v}_{Ue} \) has only a component in \( \mathbf{e}_z \) direction, curl of \( \mathbf{v}_{Ue} \) is calculated with cylindrical coordinates. According to appendix C "Three dimensional coordinate system and simplified operation of Hamilton operator", curl of \( \mathbf{v}_{Ue} \) has only a component in direction of \( \mathbf{e}_\varphi \), and the magnitude is

\[
(\nabla \times \mathbf{v}_{Ue})_\varphi = -\frac{\partial}{\partial r} \left( \frac{R v_i}{l} \right) \frac{\partial l}{\partial r} = \frac{R v_i}{l^2} \frac{\partial (\sqrt{r^2 + z^2})}{\partial r} = \frac{R v_i \cos \theta}{l^2}
\]

Therefore, movement of a single electron with velocity of \( v_i \) in \( dz \) causes the velocity curl of \( \mathbf{U}_p \) at point A to be

\[
\nabla \times \mathbf{v}_{Ue} = \frac{R v_i \cos \theta}{l^2} \mathbf{e}_\varphi
\]

There are \( \lambda dz \) moving electrons in \( dz \) length, according to the superposition principle, their movement cause the velocity curl component in direction of \( \mathbf{e}_\varphi \) of \( \mathbf{U}_p \) at point A to be

\[
\lambda dz \cdot (\nabla \times \mathbf{v}_{Ue})_\varphi = \lambda d (r \tan \theta) \cdot \frac{R v_i \cos \theta}{l^2} \frac{R \lambda v_i \cos \theta}{r} d\theta
\]

All moving electrons in the infinite wire cause the velocity curl component in direction of \( \mathbf{e}_\varphi \) of \( \mathbf{U}_p \) at point A to be

\[
(\nabla \times \mathbf{v}_U)_\varphi = \int_{-\pi/2}^{\pi/2} \frac{R \lambda v_i \cos \theta}{r} d\theta = \frac{2 R \lambda v_i}{r}
\]

So

\[
\nabla \times \mathbf{v}_U = \frac{2 R \lambda v_i}{r} \mathbf{e}_\varphi
\]

In Fig-A21, the direction of velocity curl of \( \mathbf{U}_p \) at point A is perpendicular to the XZ plane inward. For the electron at point A, the velocity \( V_2 \) of \( \mathbf{U}_p \) on the left-side is bigger than \( V_1 \) on the right-side, referring to analysis of Fig-A20, upward force on the left side of the
electron is smaller than upward force on the right side, so the electron will rotate anticlockwise, and its angular velocity direction is opposite to the direction of velocity curl of Up at point A. Hold the long straight wire with right hand, point four fingers in direction of angular velocity of the electron rotation, and direction of thumb is direction of the current. Referring to analysis of Fig-A20, if the electron at point A is replaced by a proton, the proton will rotate clockwise. In the same way, velocity curl of Ue and that of Up cause electrons or protons to rotate in the opposite direction.

Mathematically, curl of the linear velocity of a rigid body is equal to twice the angular velocity, therefore, when angular velocity of the electron at point A is 1/2 of velocity curl of Up at point A, the electron is in equilibrium state. The angular velocity of an electron at point A is

$$\omega = \frac{(\mathbf{v} \times \mathbf{v}_U) \varphi}{2} = \frac{R \lambda v_i}{r}$$

U-18: The curl field of macro velocity of U-particle constitutes magnetic field, magnetic induction intensity \( B \) of the magnetic field generated by \( \mathbf{v} \times \mathbf{v}_U \) of macro velocity of Up is

$$B = -\frac{2\pi R^3 \rho}{9Q_e} \ast (\mathbf{v} \times \mathbf{v}_U) \quad \text{(15)}$$

The curl of magnetic vector potential \( A \) is magnetic induction intensity \( B \), \( \mathbf{B} = \mathbf{\nabla} \times \mathbf{A} \) and

$$A = -\frac{P_{em}}{6Q_e} = -\frac{1}{6Q_e} \ast \left( \frac{4\pi R^3 \rho}{3} \right) \quad \text{(16)}$$

Electromagnetic momentum \( P_{em}(r, t) \) at a certain point is equal to macro momentum density \( \rho_{PU} \) of Up at the point times the volume of a sphere with radius R.

Infinite long straight wire, \( \lambda \) electrons per unit length move in the wire at constant speed \( v_i \) to the positive direction of Z-axis, current in the wire is \( I \), distance between point A and the wire is \( r \), then the magnetic induction intensity at point A is

$$B = -\frac{4\pi R^4 \rho \lambda v_i}{9Q_e r} \ast \mathbf{e}_{\varphi} = -\frac{4\pi R^4 \rho}{9Q_e^2} \ast -\frac{I}{r} \ast \mathbf{e}_{\varphi} \quad \text{(17)}$$

The relationship between direction of magnetic induction intensity and direction of current is right-handed helix.

Explanation: In Fig-A21, angular velocity \( \omega \) of the electron rotation at point A is perpendicular to the XZ plane outward, if the electron moves upward at speed \( v \), according to equation (8), the electron is pushed by U-particle and the direction of \( F_B \) is to the left. When calculating the ideal Lorentz force in U-14, it is assumed that the states of U-particles around the electron are the same, but this assumption is impossible in the actual physical environment, because it is the different macro motion states of U-particles around the electron that leads to the angular velocity \( \omega \) of the electron. Although U-particle has a constant macro velocity in the Z-axis direction, the macro motion velocity of U-particle in direction of \( F_B \) is a constant, so there will be no additional momentum change in direction of \( F_B \), and \( F_B \) can still be approximately calculated by equation (8). Magnitude of \( F_B \) is

$$F_B = \frac{4\pi R^4 \rho}{9} \ast v \ast \omega$$

The definition of magnetic induction intensity \( B \) in classical electromagnetism comes from Lorentz force \( F_B = q \ast \mathbf{v} \times \mathbf{B} \). The electric quantity of an electron is \( q = Q_e \), therefore, the magnitude of magnetic induction intensity \( B \) is
According to equation (14):
\[
B = \frac{F_B}{qv} = \frac{4\pi R^3 \rho}{9} \ast \nu \ast \omega \ast \frac{1}{Q_e}v = \frac{4\pi R^3 \rho}{9Q_e} \ast \omega
\]

The wire with current \(I\), \(I = \lambda v_i \ast Q_e\). Magnitude of magnetic induction intensity of the long straight wire is
\[
B = \frac{2\pi R^3 \rho}{9Q_e} \ast (\nu \times \nu_u) = \frac{4\pi R^3 \rho}{9Q_e} \ast \frac{(\nu \times \nu_u)}{2} = \frac{4\pi R^3 \rho}{9Q_e} \ast \frac{R\lambda v_i}{r} = \frac{4\pi R^4 \rho \lambda v_i}{9Q_e r}
\]

In Fig-A21, direction of \(\nu \times \nu_u\) at point A is perpendicular to the XZ plane inward. Angular velocity of the electron rotation at point A is perpendicular to the XZ plane outward, if the electron at point A is replaced by a proton, angular velocity of the proton rotation is perpendicular to the XZ plane inward, according to equation (8), if the proton moves downward, force direction on the proton is to the left. According to the definition of Lorentz force \(F_B = q \ast \nu \times B\) in classical electromagnetism, if the proton moves downward and the force direction is to the left, then the direction of \(B\) is perpendicular to the XZ plane outward, therefore, the direction of \(B\) is opposite to that of \(\nu \times \nu_u\).
\[
B = \frac{2\pi R^3 \rho}{9Q_e} \ast (\nu \times \nu_u)
\]

The calculation method of magnetic induction intensity \(B\) in the above equation refers to the definition of magnetic induction intensity \(B\) in classical electromagnetism, but it can define the magnetic induction intensity \(B\) at a certain point. Therefore, this paper takes it as the accurate definition of magnetic induction intensity \(B\) under the mechanical model of U-particle, which does not need to be based on Lorentz force. This definition is still used when the current changes. The relationship between direction of magnetic induction intensity and direction of current is right-handed helix. Hold the long straight wire with right hand, point four fingers in direction of magnetic induction intensity, and direction of thumb is the direction of the current. The direction of magnetic induction intensity is the same as that of angular velocity of the electron rotation.

In electromagnetics, the curl of magnetic vector potential \(A\) is magnetic induction intensity \(B\), \(B = \nabla \times A\), so
\[
A = -\frac{2\pi R^3 \rho}{9Q_e} \ast \nu_u = -\left(\rho \nu_u \ast \frac{4\pi R^3}{3} \ast \frac{1}{6Q_e}\right) = -\left(\rho_{PU} \ast \frac{4\pi R^3}{3} \ast \frac{1}{6Q_e}\right) = -P_{em} \ast \frac{1}{6Q_e}
\]

Electromagnetic momentum \(P_{em}(r, t)\) at a certain point is equal to macro momentum density \(\rho_{PU}\) of Up at the point times the volume of a sphere with radius R, it is similar to the electromagnetic momentum assumed by Maxwell.

U-19: The permittivity and permeability of vacuum are
\[
\varepsilon_0 = \frac{9Q_e^2}{8\pi^2 R^4 \rho C_0^2} \quad (18)
\]
\[
\mu_0 = \frac{8\pi^2 R^4 \rho}{9Q_e^2} \quad (19)
\]
Explanation: In classical electromagnetism, when electric quantity of a charged object is \( q \) and distance from the charged object is \( r \), electrostatic field intensity is

\[
E_s = \frac{1}{4\pi \varepsilon_0} \frac{q}{r^2}
\]

According equation (10), when electric quantity of a charged object is \( q \) and distance from the charged object is \( r \), electrostatic field intensity is

\[
E_s = \frac{2\pi R^4 \rho C_U^2}{9Q_e^2} \frac{q}{r^2}
\]

Comparing the two equations, we can see that the permittivity of vacuum is

\[
\varepsilon_0 = \frac{9Q_e^2}{8\pi^2 R^4 \rho C_U^2}
\]

The mathematical expression of vacuum permittivity can be derived by the mechanical model base on U-particle, and in classical electromagnetism, the vacuum permittivity constant is only a value measured by experiment.

In classical electromagnetism, when current in long straight wire is \( I \) and distance from the wire is \( r \), magnetic induction intensity is

\[
B = \frac{\mu_0}{2\pi} \frac{I}{r}
\]

According equation (17), when current in long straight wire is \( I \) and distance from the wire is \( r \), magnetic induction intensity is

\[
B = \frac{4\pi R^4 \rho}{9Q_e^2} \frac{I}{r}
\]

Comparing the two equations, we can see that the permeability of vacuum is

\[
\mu_0 = \frac{8\pi^2 R^4 \rho}{9Q_e^2}
\]

U-20: The maximum speed \( C_U \) of U-particle translational motion equals the speed of light. Speed of electromagnetism and gravitation propagation equals the speed of light, and the speed of light \( C_U \) is a physical quantity with a constant value. Field constant is \( R^4 \rho = 3.68 \times 10^{-45} \, Kg.m \). There is no gravitation between two U-particles, and there is only inertial mass but no gravitational mass in U-particle.

Explanation: Based on the definition of current unit Ampere in classical electromagnetism, the permeability of vacuum can be determined as \( \mu_0 = 4\pi \times 10^{-7} \, N/A^2 \), according equation (18) and (19), the product of \( \varepsilon_0 \) and \( \mu_0 \) can be calculated by mechanical model of U-particle as follows

\[
\varepsilon_0 \mu_0 = \frac{9Q_e^2}{8\pi^2 R^4 \rho C_U^2} = \frac{8\pi^2 R^4 \rho}{9Q_e^2} \frac{1}{C_U^2} = \frac{1}{C_U^2}
\]

\[
C_U = \sqrt{\frac{1}{\varepsilon_0 \mu_0} = \sqrt{\frac{1}{8.854 \times 10^{-12} \times 4\pi \times 10^{-7}}} \approx 3.0 \times 10^8 \, m/s}
\]

That is to say, \( \varepsilon_0 \mu_0 \cdot C_U^2 = 1 \) can be confirmed by using the mathematical expressions of \( \varepsilon_0 \) and \( \mu_0 \) derived from mechanical model of U-particle. Using the value of \( \mu_0 \) determined by classical electromagnetism and the value of \( \varepsilon_0 \) measured by experiment, the maximum speed of U-particle translational motion can be directly calculated, it is equal to the speed of light.
According to U-7, rotational kinetic energy of U-particle is very small when distance between U-particle and centre of an electron/proton is 1 nm, that is to say, translational motion speed of U-particle is close to \( c_U \), so it can be approximately considered that translational motion speed of U-particle is \( c_U \). Therefore, the speed of propagation of any physical properties of U-particle, including electric field, magnetic field and gravitation, is \( c_U \). According to equation (19)

\[
\mu_0 = \frac{8\pi^2 R^4 \rho}{9Q_e^2}
\]

\[
R^4 \rho = \frac{9Q_e^2\mu_0}{8\pi^2} = \frac{9 \times (1.602 \times 10^{-19})^2 \times 4\pi \times 10^{-7}}{8\pi^2} \approx 3.68 \times 10^{-45}
\]

\( R^4 \rho \) can be called the field constant

\[
R^4 \rho \approx 3.68 \times 10^{-45} \text{ Kg.m}
\]

An electron is isotropic, but there is no ideal smooth boundary. On the sphere with centre of the electron and radius of R, collision between U-particle and the electron satisfies collision characteristics of mechanical model of U-particle. An electron cannot be regarded as a homogeneous sphere with radius R and the same internal density.

According to equation (4), when distance between two electrons is 1 meter, electrostatic repulsion between them is

\[
F_e = \frac{2\pi R^4 \rho C_0^2}{9} \times \frac{1}{L^2} = \frac{2\pi \times 3.68 \times 10^{-45} \times 9 \times 10^{16}}{9} \approx 2.3 \times 10^{-28} \text{ N}
\]

According to U-11, component force on half sphere of an electron is

\[
hF_e \gg F_e \times 7.5 \times 10^{31} \approx 2.3 \times 10^{-28} \times 7.5 \times 10^{31} \approx 1.73 \times 10^4 \text{ N}
\]

To make an analogy, the electron is like a shrimp at the bottom of the sea 8000 meters deep, although the pressure there is huge, but the shrimp only endure differential pressure, so the shrimp can move freely without being crushed to death. The pressure generated by U-particle around us is much bigger than this, is it exciting? Don't worry, U-particle only works on electrons/protons, it has no interest in your huge body 😊. The huge pressure caused by U-particle ensures the stability of structure of an electron/proton.

The physical properties of microscopic particles may seem extreme. For example, in a cathode-ray tube, an acceleration voltage about 2000 volts can accelerate electrons to \( 3 \times 10^7 \text{ m/s} \). The mass of an electron is \( M_e = 9.10956 \times 10^{-31} \text{ Kg} \), assuming the radius of electron is \( R = 1.0 \times 10^{-16} \text{ m} \), then the density of electron is

\[
\rho_e = \frac{M_e}{4\pi R^3/3} = \frac{3 \times 9.10956 \times 10^{-31}}{4\pi \times (1.0 \times 10^{-16})^3} \approx 2.2 \times 10^{17} \text{ Kg/m}^3
\]

The speed and density of electron are much higher than that of macroscopic object, so it is normal that the density of U-particle is much higher than that of electron. The density of U-particle in space can be estimated as follows by field constant

\[
\rho = \frac{3.68 \times 10^{-45}}{(1.0 \times 10^{-16})^4} \approx 3.68 \times 10^{19} \text{ Kg/m}^3
\]

The sum of translational kinetic energy of U-particle in the unit space is \( E = \rho C_0^2/2 \). If the speed of light \( c_U \) accumulates a change rate of \( 10^{-10} \) within 1 million years, that is, the change rate of the speed of light per second is

\[
\frac{dc_U}{dt} = \frac{10^{-10}}{10^6 \times 365 \times 24 \times 3600} \approx 3.17 \times 10^{-24}
\]
It is impossible for any cubic meter of space to continuously generate 3500 watts of power, which would result in a huge disaster. Therefore, the change rate of the speed of light per second is much smaller than $3.17 \times 10^{-24}$, and the speed of light $C_U$ can be approximated as a physical quantity with a constant value.

Although density of U-particle in space is huge, it does not affect movement of normal objects, including movement of our bodies. The reason is that U-particle only works on electron/proton, according to the analysis in U-13, for a stationary electron/proton, if acceleration of Up and Ue in surrounding environment are the same, the direction of the force caused by accelerating Up is opposite to that of the force caused by accelerating Ue, so resultant force of ideal induced forces exerted on the stationary electron/proton is zero. Similarly, if the macro velocities of Up and Ue in surrounding environment are zero, then the uniformly moving electron/proton is subjected to zero resultant force from U-particle. In addition, although density of U-particle in space is huge, it can be seen from the cause of gravitation that gravitation cannot be formed between two U-particles with inertial mass. Therefore, there is only inertial mass but no gravitational mass in U-particle.

### 5 Approximately stationary point

Suppose that at a certain moment $t$, an electron moves in the stationary environment of U-particle at speed $v(t)$, and there is no other electron/proton within infinite distance from this electron. According to U-8, a moving electron swallows U-particle that collides with it and releases Up at the collision point. Initial velocity of the Up is the same as that of collision point on the electron surface. Momentum of macro motion of the Up diffuses outward by random collision of U-particle. According to the law of conservation of momentum, momentum of the moving electron will gradually decrease to zero. Therefore, in equilibrium state, if an isolated electron takes itself as reference point, the electron is in stationary state defined by U-1.

Suppose that a big ball with the centre of O contains N electrons and N protons. These electrons/protons are uniformly distributed in the big ball and their relative position is fixed. There are no other electron/proton in infinite distance from the big ball, and the big ball is in equilibrium state. Taking point O as reference point and taking arbitrary point P in space, length of OP is a finite value, then velocity of macro motion of U-particle at point P is zero, and translational kinetic energy density of U-particle at point P remains unchanged, so point O is a stationary point.

If there is an electron far away from point P, distance between the electron and the big ball is L and speed of the electron is $v_0$, then point O is no longer a stationary point. However, when $v_0 \approx 0$, according to equation (3), the electron causes velocity of U-particle at point P is $v_U \approx 0$. If $N \gg 1$, according to the gradient superposition, translational kinetic energy gradient change ratio of U-particle at point P is about $1/N \approx 0$. That is to say, macro velocity of U-particle is approximately zero and translational kinetic energy density of U-particle is approximately unchanged. Therefore, the centre O of the big ball can still be regarded as an approximately stationary point.
When a small ball with electric quantity $q$ and mass $m$ moves to the position where the distance from the big ball is $L$ at the speed of $v_0$, and center O of the big ball is taken as an approximately stationary reference point, then according to the superposition of curl and gradient, the velocity curl or translational kinetic energy gradient change ratio of U-particle at point P increase with the increase of $q$, $m$ and $v_0$, and decrease with the increase of N and L.

Therefore, the center of an electrically neutral, massive planet far away from high speed charged object and satellites and planets and stars can be approximately a stationary point. If the planet does not rotate or rotates slowly, then any point on the planet can be approximately a stationary point. The earth meets these conditions, so in many cases, any point on the earth can be regarded as an approximately stationary point to study U-particle.

6 Changing electric field and changing magnetic field

An electron/proton is pushed by accelerating U-particles and the force is collectively called induced force. In the process of calculating ideal induced force, angular velocity of the stationary electron/proton is required to be zero, which is impossible in actual physical environment, because the curl of U-particles generated by moving electrons is not zero, this will cause angular velocity of the stationary electron/proton to be non-zero. In actual physical environment, when $v_U(r, t)$ varies with the parameter $t$, induced force on the stationary electron/proton is called the first type of induced force, and other induced forces are called the second type of induced force.

While an electron/proton rotating on its axis moves in U-particle environment, the pushing force on the electron/proton caused by the Magnus effect is collectively called Lorentz force. In the process of calculating ideal Lorentz force, the states of U-particle around the moving electron/proton are required to be the same, which is impossible in actual physical environment, because angular velocity of the electron/proton can be generated only when $v_U(r, t)$ varies with the parameter $r$. In actual physical environment, when $\mathbf{v} \times v_U(r, t)$ remains constant, Lorentz force on the moving electron/proton is called the first type of Lorentz force, and other Lorentz forces are called the second type of Lorentz force.

U-21: Infinite long straight wire, the current is zero before $t = 0$, and from $t = 0$, $\lambda$ electrons per unit length move in the wire in the $Z$-axis direction with constant acceleration $a$, speed of electrons in the wire is $v_l(t) = at$ at the moment of $t$. Suppose that the distance between point A and the wire is $r$, if $r < C_0 t$, then the curl of the macro velocity $v_U(r, t)$ of Up at point A is

\[
\nabla \times v_U(r, t) \approx \frac{2R \lambda a \sqrt{C_0^2 t^2 - r^2}}{C_0 r} \mathbf{e}_\varphi
\]

(21)

An electron/proton will rotate on its axis at point A. A proton rotates in the opposite direction to an electron at point A. The relationship between direction of electron angular velocity at point A and direction of current is right-handed helix. A proton rotates in the opposite direction to the electron at point A.
**Explanation:** As shown in Fig-A22, the infinite long straight wire overlaps with Z-axis. Because the distance \(l\) between each segment of wire \(dz\) and point A is different, according to U-20, after a time interval of \(l/C_U\), movement of electrons in the wire \(dz\) can affect motion of U-particle on point A, therefore, motion of Up on point A at the moment of \(t\) is caused by movement of electrons in the wire \(dz\) before the moment of \(t\). While \(t > l/C_U\), according to equation (3), the movement of a single electron in \(dz\) at the moment of \((t - l/C_U)\) causes that the macro velocity of Up at point A at the moment of \(t\) is

\[
v_{ue}(r, t) = \frac{R \cdot v_i(t - l/C_U)}{l} = \frac{R \left[ v_i(t) - a \cdot l/C_U \right]}{l} = \frac{Rv_i(t)}{l} - \frac{Ra}{C_U}
\]

Since \(v_{ue}(r, t)\) has only a component in \(e_z\) direction, using cylindrical coordinates to calculate curl of macro velocity \(v_{ue}(r, t)\) of Up caused by the movement of a single electron in \(dz\). \(\nabla \times v_{ue}(r, t)\) has only a component in direction of \(e_\phi\), and the magnitude is

\[
(\nabla \times v_{ue}(r, t))_\phi = -\frac{\partial}{\partial r}[v_{ue}(r, t)] = -\frac{\partial}{\partial r} \left[ \frac{Rv_i(t)}{l} - \frac{Ra}{C_U} \right] = \frac{Rv_i(t)}{l^2} \frac{\partial l}{\partial r}
\]

\[= \frac{Rv_i(t)}{l^2} \frac{\partial}{\partial r} \left( \sqrt{r^2 + z^2} \right) = \frac{Rv_i(t) \cos \theta}{l^2}
\]

There are \(\lambda dz\) moving electrons in the \(dz\) length, according to the superposition principle, their movement causes the velocity curl of Up at point A to be

\[
\lambda dz \ast (\nabla \times v_{ue}(r, t))_\phi = \lambda d(r \ast \tan \theta) \ast \frac{Rv_i(t) \cos \theta}{l^2} = \frac{R \lambda v_i(t) \cos \theta}{r} d\theta
\]

All moving electrons in the infinite wire cause the velocity curl of Up at point A to be

\[
(\nabla \times v_{ue}(r, t))_\phi = \int_{-\alpha}^{\alpha} \frac{R \lambda v_i(t) \cos \theta}{r} d\theta
\]

In the integration space, ensure \(l < C_U t\), that is

\[
\alpha = \sin^{-1} \left( \frac{\sqrt{C_U^2 t^2 - r^2}}{C_U t} \right)
\]

So

\[
(\nabla \times v_{ue}(r, t))_\phi = \frac{2R \lambda v_i(t)}{r} \sqrt{\frac{C_U^2 t^2 - r^2}{C_U t}} = \frac{2R \lambda a \sqrt{C_U^2 t^2 - r^2}}{C_U r}
\]

\[
\nabla \times v_{ue}(r, t) = \frac{2R \lambda a \sqrt{C_U^2 t^2 - r^2}}{C_U r} \ast e_\phi
\]

According to U-17, the relationship between direction of electron angular velocity at point A and direction of current is right-handed helix. A proton rotates in the opposite direction to the electron at point A.
U-22: A static electron/proton will be pushed by accelerating U-particles, the pushing force is the first type of induced force, which is proportional to the first derivative of the macro velocity of U-particle with respect to time. Induced electric field intensity $E_i$ is defined as follows. The first type of induced force is the result of induced electric field acting on stationary electron/proton.

$$E_i = \frac{2\pi R^3 \rho}{9Q_e} \frac{\partial v_U}{\partial t} = \frac{1}{6Q_e} \frac{\partial P_{em}}{\partial t}$$  \hspace{1cm} (22)

Relationship between induced electric field intensity and magnetic induction intensity at a stationary point satisfies

$$\nabla \times E_i = -\frac{\partial B}{\partial t}$$  \hspace{1cm} (23)

$$\nabla \cdot E_i = 0$$  \hspace{1cm} (24)

Induced electromotive force is equal to the line integral of induced electric field intensity.

**Fig-A23** Accelerated U-particle generates induced electric field

Explanation: As shown in Fig-A23, the infinite long straight wire overlaps with Z-axis, the current is zero before $t = 0$, and from $t = 0$, $\lambda$ electrons per unit length move in the wire in the Z-axis direction with constant acceleration $a$, speed of electrons in the wire is $v_i(t) = at$ at the moment of $t$. There is a static electron at point A. Distance between point A and the wire is $r$, macro velocity of Up at point A is $v_U(r, t)$, according to equation (21),

$$\nabla \times v_U(r, t) = \frac{2R\lambda a\sqrt{C_U^2 t^2 - r^2}}{C_U r} e_\phi$$

According to equation (15)

$$B(r, t) = -\frac{2\pi R^3 \rho}{9Q_e} \nabla \times v_U(r, t) = \frac{2\pi R^3 \rho}{9Q_e} \frac{2R\lambda a\sqrt{C_U^2 t^2 - r^2}}{C_U r} e_\phi$$

$$= -\frac{4\pi R^4 \rho \lambda a\sqrt{C_U^2 t^2 - r^2}}{9Q_e C_U r} e_\phi$$

$$\frac{\partial B(r, t)}{\partial t} = -\frac{4\pi R^4 \rho \lambda a}{9Q_e r} \frac{C_U t}{\sqrt{C_U^2 t^2 - r^2}} e_\phi$$

Because $v_{Ze}(r, t)$ has only a component in $e_z$ direction, in cylindrical coordinate system, $\nabla \times v_{Ze}(r, t)$ has only a component in direction of $e_\phi$, and the magnitude is

$$\left(\nabla \times v_U(r, t)\right)_\phi = -\frac{\partial v_U(r, t)}{\partial r}$$

According to equation (21)

$$\left(\nabla \times v_U(r, t)\right)_\phi = \frac{2R\lambda a\sqrt{C_U^2 t^2 - r^2}}{C_U r}$$

So
Solving the equation, it can be obtained

\[
v_u(r, t) = -\frac{2R\lambda a}{C_u} \left( \sqrt{C_u^2 t^2 - r^2} + C_u t \ln \frac{C_u t - \sqrt{C_u^2 t^2 - r^2}}{r} \right) + \text{constant}
\]

For detailed calculation, please refer to F-1 of appendix F "Calculation of partial derivatives related to induced electric field and displacement current". While \( r = C_u t \)

\[
v_u(r, t) = -\frac{2R\lambda a}{C_u} (0 + 0) + \text{constant} = 0
\]

So, \( \text{constant} = 0 \), and therefore

\[
v_u(r, t) = -\frac{2R\lambda a}{C_u} \left( \sqrt{C_u^2 t^2 - r^2} + C_u t \ln \frac{C_u t - \sqrt{C_u^2 t^2 - r^2}}{r} \right)
\] (25)

In the process of calculating ideal induced force, angular velocity \( \omega \) of the electron rotating on its axis is required to be zero, but \( \omega \) is not zero here. Since non-zero \( \omega \) will not cause additional momentum changes in the direction of \( \mathbf{F}_i \), equation (7) can still be used to approximately calculate the first type of induced force. If an electron moves at a uniform speed in the direction perpendicular to \( \mathbf{v}_u(t) \), the uniform motion will not cause additional momentum change in the direction of \( \mathbf{v}_u(t) \), and therefore, equation (7) can still be used to approximately calculate the second type of induced force exerted on an electron. According to equation (7)

\[
\mathbf{F}_i = -\frac{2\pi R^3 \rho}{9} \ast \frac{d\mathbf{v}_u(t)}{dt}
\]

The above formula requires that macro velocity \( v \) of \( \text{Up} \) is the same in the space occupied by the electron, that is, \( \mathbf{v}_u(r, t) \) will not change due to the change of variable \( r \). Because the space occupied by the electron is very small, the force \( \mathbf{F}_i(r, t) \) on the electron can be approximately calculated by \( \mathbf{v}_u(r, t) \) at the center of the electron.

\[
\mathbf{F}_i(r, t) = -\frac{2\pi R^3 \rho}{9} \ast \frac{\partial \mathbf{v}_u(r, t)}{\partial t}
\]

Induced electric field intensity \( E_i(r, t) = \mathbf{F}_i(r, t)/(-Q_e) \), so

\[
E_i(r, t) = \frac{2\pi R^3 \rho}{9Q_e} \ast \frac{\partial \mathbf{v}_u(r, t)}{\partial t} = \frac{1}{6Q_e} \ast \frac{4\pi R^3 \rho}{3} \ast \frac{\partial \mathbf{v}_u(r, t)}{\partial t} = \frac{1}{6Q_e} \ast \frac{\partial \mathbf{P}_{em}(r, t)}{\partial t}
\]

The calculation method of induced electric field intensity \( E_i \) in the above equation refers to the definition of induced electric field intensity \( E_i \) in classical electromagnetism, but it can define the induced electric field intensity \( E_i \) at a certain point. Therefore, this paper takes it as the accurate definition of induced electric field intensity \( E_i \) under the mechanical model of U-particle. The partial derivative of equation (25) with respect to time \( t \) is

\[
\frac{\partial \mathbf{v}_u(r, t)}{\partial t} = -2R\lambda a \ln \frac{C_u t - \sqrt{C_u^2 t^2 - r^2}}{r}
\]

For detailed calculation, please refer to F-1 of appendix F "Calculation of partial derivatives related to induced electric field and displacement current". So

\[
E_i(r, t) = \frac{2\pi R^3 \rho}{9Q_e} \ast \left( -2R\lambda a \ln \frac{C_u t - \sqrt{C_u^2 t^2 - r^2}}{r} \right) \ast \mathbf{e}_z
\]
Therefore

\[ E_i(r, t) = \frac{2\pi R^3 \rho}{9Q_e} \frac{\partial v_i(r, t)}{\partial t} = -\frac{4\pi R^4 \rho \lambda a}{9Q_e} \ln \frac{C_0 t - \sqrt{C_0^2 t^2 - r^2}}{r} \cdot e_z \] (26)

Since \( E_i(r, t) \) has only a component in \( e_z \) direction, in cylindrical coordinate system, \( \nabla \times E_i(r, t) \) has only a component in direction of \( e_\phi \), so

\[ \nabla \times E_i(r, t) = -\frac{\partial E_i(r, t)}{\partial r} \cdot e_\phi = \frac{4\pi R^4 \rho \lambda a}{9Q_e} \frac{C_0 t}{r\sqrt{C_0^2 t^2 - r^2}} \cdot e_\phi \]

For detailed calculation, please refer to F-2 of appendix F "Calculation of partial derivatives related to induced electric field and displacement current". Compare with

\[ \frac{\partial B(r, t)}{\partial t} = -\frac{4\pi R^4 \rho \lambda a}{9Q_e r} \frac{C_0 t}{\sqrt{C_0^2 t^2 - r^2}} \cdot e_\phi \]

So

\[ \nabla \times E_i = -\frac{\partial B}{\partial t} \]

\[ \nabla \cdot E_i(r, t) = \frac{\partial E_i(r, t)}{\partial z} = \frac{\partial}{\partial z} \left( -\frac{4\pi R^4 \rho \lambda a}{9Q_e} \ln \frac{C_0 t - \sqrt{C_0^2 t^2 - r^2}}{r} \right) = 0 \]

Fig-A24 shows a circle with infinite radius, and the circle in the middle represents an infinite long straight wire, horizontal line intersects with three circles at P1, P2 and P3 respectively. Compared with Fig-A23, current at point P2 is downward, according to analysis of Fig-A17, the proton at point P1 or P3 moves upward, according to analysis of Fig-A18, the electron at point P1 or P3 moves downward, that is, induced electric field blocks the growth of current in the middle circle. The negative sign of \( \nabla \times E_i = -\partial B/\partial t \) coincides with this "blocking" effect.

When current of infinite long straight wire increases, according to Lenz's law of classical electromagnetism, due to the increase of magnetic flux through the middle circle, direction of magnetic induction intensity generated by induced electric field is perpendicular to paper inward, therefore, direction of current generated by induced electric field is opposite to that of the infinite long straight wire, this is consistent with mechanical model of U-particle. Because \( \nabla \times E_i \neq 0 \), \( \oint E_i \cdot dl \neq 0 \), the result of \( \oint E_i \cdot dl \) is the induced electromotive force in classical electromagnetism.

Induced electric field is generated by the accelerated motion of U-particle, not by the change of velocity curl of U-particle. Both induced electric field and changing magnetic field are generated by moving electrons; there is a concomitant relationship between induced electric field and changing magnetic field not a causal relationship. According to classical electromagnetism, electric field and magnetic field are in same phase in electromagnetic wave, which is manifestation of non causal relationship but concomitant relationship between
changing electric field and changing magnetic field.

Mathematically, \( \nabla \times E_i = -\frac{\partial B}{\partial t} \) means that although the derivation order of electromagnetic momentum \( P_{em}(r, t) \) for time and space is different, the two second-order partial derivatives are equal, that is

\[
\frac{\partial}{\partial r} \left[ \frac{\partial P_{em}(r, t)}{\partial t} \right] = \frac{\partial}{\partial t} \left[ \frac{\partial P_{em}(r, t)}{\partial r} \right]
\]

According to equation (22)

\[
E_i(r, t) = \frac{1}{6Q_e} \frac{\partial P_{em}(r, t)}{\partial t}
\]

So

\[
\nabla \times E_i = -\frac{\partial E_i(r, t)}{\partial r} \cdot e_\phi = -\frac{\partial}{\partial r} \left[ \frac{1}{6Q_e} \frac{\partial P_{em}(r, t)}{\partial t} \right] \cdot e_\phi = -\frac{1}{6Q_e} \frac{\partial}{\partial r} \left[ \frac{\partial P_{em}(r, t)}{\partial t} \right] \cdot e_\phi
\]

According to equation (16)

\[
A = -\frac{P_{em}}{6Q_e}
\]

So

\[
B = \nabla \times A = \frac{1}{6Q_e} \frac{\partial P_{em}(r, t)}{\partial r} \cdot e_\phi
\]

\[
\frac{\partial B}{\partial t} = \frac{1}{6Q_e} \frac{\partial}{\partial t} \left[ \frac{\partial P_{em}(r, t)}{\partial r} \right] \cdot e_\phi
\]

U-23: Electrons move in a wire at a constant speed before \( t = 0 \), after \( t > 0 \), electrons move with constant acceleration. Relationship between electric field intensity \( E \), magnetic induction intensity \( B \) and conduction current density \( J_c \) satisfies

\[
\nabla \times B = \mu_0 \left( J_c + \varepsilon_0 \frac{\partial E}{\partial t} \right) = \mu_0 \left( J_c + \frac{Q_e}{4\pi RC_U^2} \frac{\partial^2 v_U}{\partial t^2} \right)
\]

(27)

Displacement current density \( J_d \) is defined as follows

\[
J_d(r, t) = \varepsilon_0 \frac{\partial E_i(r, t)}{\partial t} = \frac{Q_e}{4\pi RC_U^2} \frac{\partial^2 v_U(r, t)}{\partial t^2}
\]

(28)

It is proportional to the first derivative of induced electric field with respect to time, and also proportional to the second derivative of macro velocity of \( U \)-particle with respect to time.

Explanation: As shown in Fig-A25, the infinite long straight wire overlaps with Z-axis, \( \lambda \) electrons per unit length move upward in the wire at a constant speed of \( v_0 \) before \( t = 0 \), after \( t = 0 \), electrons move upward with constant acceleration \( \alpha \), speed of electrons moving in the wire is \( v(t) = v_0 + \alpha t \). Distance between point A and the wire is \( r \). Suppose that the current \( I(t) \) in the wire passes through surface \( \Sigma \) bounded by closed curve \( \Gamma \). The current \( I(t) \) is divided into constant current \( I_1(t) \) and variable current \( I_2(t) \), \( I_1(t) = -\lambda Q_e v_0 \).
\( I_2(t) = -\lambda Q_e a t, \quad I(t) = I_1(t) + I_2(t) = -\lambda Q_e v_i(t) \). The constant current and the variable current are calculated respectively.

For the constant current part, according to equation (17)

\[
B_1 = -\frac{4\pi R^4 \rho \lambda \nu_0}{9Q_e r} \cdot e_\varphi
\]

\[
\oint B_1(r,t) \cdot dl = \int \frac{4\pi R^4 \rho \lambda \nu_0}{9Q_e r} \cdot rd\varphi = -\frac{8\pi^2 R^4 \rho \lambda \nu_0}{9Q_e} \cdot \frac{8\pi^2 R^4 \rho l_1(t)}{9Q_e} = \mu_0 I_1(t)
\]

According to Stokes mathematical theorem

\[
\int [\nabla \times B_1(r,t)] \cdot dS = \oint B_1(r,t) \cdot dl
\]

Divide both sides of the above equation by \( \Sigma \) and take the limit that \( \Sigma \) tends to zero, when \( \Sigma \) tends to zero, \( r \) also tends to zero

\[
(V \times B_1(0,t))_z = \lim_{\Sigma \to 0} \frac{1}{\Sigma} \oint B_1(r,t) \cdot dl = \lim_{\Sigma \to 0} \frac{1}{\Sigma} \mu_0 I_1(t)
\]

While \( r > 0 \)

\[
V \times B_1(r,t) = \frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( -\frac{4\pi R^4 \rho \lambda \nu_0}{9Q_e r} \right) \right] e_z = 0
\]

For the variable current part, according to U-22

\[
B_2(r,t) = -\frac{4\pi R^4 \rho \lambda a \sqrt{C_0^2 t^2 - r^2}}{9Q_e C_0 r} e_\varphi
\]

\[
\oint B_2(r,t) \cdot dl = \int \frac{4\pi R^4 \rho \lambda a \sqrt{C_0^2 t^2 - r^2}}{9Q_e C_0 r} \cdot rd\varphi = -\frac{4\pi R^4 \rho \lambda a t}{9Q_e} \oint \sqrt{1 - \frac{r^2}{C_0^2 t^2}} d\varphi
\]

According to Stokes mathematical theorem

\[
\int [V \times B_2(r,t)] \cdot dS = \oint B_2(r,t) \cdot dl
\]

Divide both sides of the above equation by \( \Sigma \) and take the limit that \( \Sigma \) tends to zero, when \( \Sigma \) tends to zero, \( r \) also tends to zero

\[
(V \times B_2(0,t))_z = \lim_{\Sigma \to 0} \frac{1}{\Sigma} \oint B_2(r,t) \cdot dl = \lim_{\Sigma \to 0} \frac{1}{\Sigma} \left[ -\frac{4\pi R^4 \rho \lambda a t}{9Q_e} \oint d\varphi \right] = \lim_{\Sigma \to 0} \frac{1}{\Sigma} \left[ -\frac{8\pi^2 R^4 \rho \lambda a t}{9Q_e} \right] = \frac{8\pi^2 R^4 \rho \lambda a t}{9Q_e} = \lim_{\Sigma \to 0} \frac{1}{\Sigma} \mu_0 I_2(t)
\]

While \( r > 0 \)

\[
V \times B_2(r,t) = \frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( -\frac{4\pi R^4 \rho \lambda a \sqrt{C_0^2 t^2 - r^2}}{9Q_e C_0 r} \right) \right] e_z = \frac{4\pi R^4 \rho \lambda a}{9Q_e C_0 \sqrt{C_0^2 t^2 - r^2}} e_z
\]

According to equation (26)

\[
E_i(r,t) = \frac{2\pi R^3 \rho}{9Q_e} \frac{\partial v_i(r,t)}{\partial t} = -\frac{4\pi R^4 \rho \lambda a}{9Q_e} C_0 t - \frac{\sqrt{C_0^2 t^2 - r^2}}{r} \cdot e_z
\]

So

\[
\frac{\partial E_i(r,t)}{\partial t} = \frac{2\pi R^3 \rho}{9Q_e} \frac{\partial^2 v_i(r,t)}{\partial t^2} = \frac{4\pi R^4 \rho \lambda a}{9Q_e} \cdot \frac{C_0}{\sqrt{C_0^2 t^2 - r^2}} e_z
\]
For detailed calculation, please refer to F-3 of appendix F "Calculation of partial derivatives related to induced electric field and displacement current". Using the superposition principle, the electromagnetic effects of the constant current \( I_1(t) \) and the variable current \( I_2(t) \) are added. The constant current \( I_1(t) \) does not generate induced electric field, and the change rate of electrostatic field to time is zero, so

\[
\frac{\partial E(r,t)}{\partial t} = \frac{\partial E_z(r,t)}{\partial t} + \frac{\partial E_i(r,t)}{\partial t} = \frac{\partial E_z(r,t)}{\partial t} = \frac{4\pi R^4 \rho \lambda a}{9Q_e} \cdot \frac{C_U}{\sqrt{C_U^2 t^2 - r^2}} \cdot e_z
\]

While \( r = 0 \)

\[
\nabla \times B(0,t) = \nabla \times B_1(0,t) + \nabla \times B_2(0,t) = \lim_{t \to 0} \frac{1}{\Sigma} \cdot \mu_0 I_1(t) \cdot e_z + \lim_{t \to 0} \frac{1}{\Sigma} \cdot \mu_0 I_2(t) \cdot e_z
\]

\[
= \lim_{t \to 0} \frac{1}{\Sigma} \cdot \mu_0 I(t) \cdot e_z = \mu_0 J_c(t)
\]

While \( 0 < r < C_U t \)

\[
\nabla \times B(r,t) = \nabla \times B_1(r,t) + \nabla \times B_2(r,t) = \frac{4\pi R^4 \rho \lambda a}{9Q_e C_U \sqrt{C_U^2 t^2 - r^2}} \cdot e_z
\]

So

\[
\nabla \times B(r,t) = \frac{1}{C_U^2} \cdot \frac{\partial E(r,t)}{\partial t}
\]

Therefore, while \( 0 \leq r < C_U t \)

\[
\nabla \times B(r,t) = \mu_0 J_c(t) + \frac{1}{C_U^2} \cdot \frac{\partial E(r,t)}{\partial t} = \mu_0 \left[ J_c(t) + \varepsilon_0 \cdot \frac{\partial E(r,t)}{\partial t} \right] = \mu_0 \left[ J_c(t) + \varepsilon_0 \cdot \frac{\partial E_i(r,t)}{\partial t} \right]
\]

\[
= \mu_0 \left[ J_c(t) + \frac{9Q_e^2}{8\pi^2 R^4 \rho C_U^2} \cdot \frac{2\pi R^2 \rho}{9Q_e} \cdot \frac{\partial^2 \mathbf{v}_U(r,t)}{\partial t^2} \right]
\]

\[
= \mu_0 \left[ J_c(t) + \frac{Q_e}{4\pi R C_U^2} \cdot \frac{\partial^2 \mathbf{v}_U(r,t)}{\partial t^2} \right]
\]

Many teaching materials use charge-discharge model of parallel plate capacitor to illustrate displacement current, in fact, the key factor causing displacement current is the second derivative of the macro velocity of U-particle with respect to time. During the charging and discharging process of the parallel plate capacitor, moving speed of electron gradually decreases to zero after it reaching the parallel plate. The change of moving speed of electron leads to the change of macro speed of U-particle. Displacement current density \( J_d \) is

\[
J_d(r,t) = \varepsilon_0 \cdot \frac{\partial E_i(r,t)}{\partial t} = \frac{Q_e}{4\pi R C_U^2} \cdot \frac{\partial^2 \mathbf{v}_U(r,t)}{\partial t^2}
\]

The calculation method of displacement current density \( J_d \) in the above equation refers to the definition of displacement current density \( J_d \) in classical electromagnetism, therefore, this paper takes it as the accurate definition of displacement current density \( J_d \) under the mechanical model of U-particle. \( J_d \) is proportional to the first-order derivative of induced electric field intensity with respect to time, or \( J_d \) is proportional to the second-order derivative of macro velocity of U-particle with respect to time.

U-24: According to mechanical model of U-particle, Maxwell’s equations of electromagnetic field can be derived as follows
\[ \begin{align*}
\mathbf{V} \cdot \mathbf{E} &= \frac{\rho_q}{\varepsilon_0} \\
\mathbf{V} \cdot \mathbf{B} &= 0 \\
\mathbf{V} \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\mathbf{V} \times \mathbf{B} &= \mu_0 \left( J_c + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) 
\end{align*} \] (29)

\( \rho_q \) is volume density of electric charge.

Explanation: It is completely consistent with the results of classical electromagnetism. Most of its derivation process has been in previous chapters, which is summarized as follows.

According to equation (10)

\[ E_x = \frac{2\pi R^4 \rho C_0^2}{9Q_e^2} \cdot \frac{q}{r^2} \]

According to equation (18)

\[ \varepsilon_0 = \frac{9Q_e^2}{8\pi^2 R^4 \rho C_0^2} \]

Therefore, for the electrostatic field

\[ E_s = \frac{1}{4\pi \varepsilon_0} \cdot \frac{q}{r^2} \]

Suppose that electric quantity \( q \) is surrounded by a closed surface \( \Sigma \), volume inside the curved surface \( \Sigma \) is \( \Omega \), then

\[ \oint_S E_s \cdot dS = \frac{1}{4\pi \varepsilon_0} \cdot \frac{q}{r^2} \cdot r^2 \sin \theta \, d\theta \, dp = \frac{q}{\varepsilon_0} \]

According to Gauss mathematical theorem

\[ \iiint_\Omega (\mathbf{V} \cdot E_s) \cdot d\Omega = \iiint_S E_s \cdot dS \]

According to equation (24), the divergence of induced electric field intensity is \( \mathbf{V} \cdot \mathbf{E}_i = 0 \). Total electric field intensity is \( \mathbf{E} = \mathbf{E}_s + \mathbf{E}_i \), and \( \mathbf{V} \cdot \mathbf{E} = \mathbf{V} \cdot \mathbf{E}_s + \mathbf{V} \cdot \mathbf{E}_i = \mathbf{V} \cdot \mathbf{E}_s \), so

\[ \iiint_\Omega (\mathbf{V} \cdot \mathbf{E}) \cdot d\Omega = \int \int \int_\Omega (\mathbf{V} \cdot \mathbf{E}_s) \cdot d\Omega = \iiint_S E_s \cdot dS = \frac{q}{\varepsilon_0} \]

Divide both sides of the above equation by \( \Omega \) and take the limit that \( \Omega \) tends to zero

\[ \mathbf{V} \cdot \mathbf{E} = \lim_{\Omega \to 0} \frac{q}{\varepsilon_0} = \frac{\rho_q}{\varepsilon_0} \]

According to equation (15)

\[ \mathbf{B} = \frac{2\pi R^3 \rho}{9Q_e} \cdot (\mathbf{V} \times \mathbf{v}_0) \]

Since divergence of curl is always zero, hence

\[ \mathbf{V} \cdot \mathbf{B} = 0 \]

According to equation (11) \( \mathbf{V} \times \mathbf{E}_s = 0 \), according to equation (23) induced electric field intensity \( \mathbf{V} \times \mathbf{E}_i = -\partial \mathbf{B}/\partial t \), since \( \mathbf{V} \times \mathbf{E} = \mathbf{V} \times (\mathbf{E}_s + \mathbf{E}_i) = \mathbf{V} \times \mathbf{E}_i \), hence \( \mathbf{V} \times \mathbf{E} \) is equal to the negative value of the change rate of magnetic induction intensity, so

\[ \mathbf{V} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]

According to equation (27)
When the electron/proton moves in a changing magnetic field, it will be subjected to the second type of Lorentz force. The electric field with the same effect as the second type of Lorentz force is called equivalent motional electric field $E_m$. The second type of Lorentz force and the second type of induced force act on the moving electron/proton in changing magnetic field are approximately calculated by two different methods. Because they are the same force, the calculation results are exactly the same. Relationship between equivalent motional electric field intensity and magnetic induction intensity at the uniform moving point satisfies

$$\nabla \times E_m = -\frac{\partial B}{\partial t} \quad (30)$$
$$\nabla \cdot E_m = 0 \quad (31)$$

The equivalent motional electric field is different from the electrostatic field and induced electric field. It is electromagnetic effect with the uniform moving electron as the force object, while the electrostatic field and induced electric field are electromagnetic effects with the stationary electron as the force object. Motional electromotive force is equal to the line integral of equivalent motional electric field intensity.

Explanation: As shown in Fig-A26, the infinite long straight wire overlaps with Z-axis, $\lambda$ electrons per unit length move upward in the wire at constant speed $v_i$. Mathematically, the angular velocity of a rotating rigid body is independent of the velocity of the rigid body's uniform translational motion. Magnetic induction intensity is

$$B = -\frac{2\pi R^2 \rho}{9 \Omega_e} * (\nabla \times \mathbf{v}_B)$$

Therefore, at the same observation point, the magnetic induction intensity observed in the uniform motion state is the same as the magnetic induction intensity observed in the stationary state.

In Fig-A26 left, an electron at point A moves upward at a constant speed $v_0$, $r$ is constant, and the electron will be subjected to the first type of Lorentz force. According to equation (17), the magnetic induction intensity $B$ on the position of the electron is constant

$$B(r) = -\frac{4\pi R^4 \rho \lambda v_i}{9 \Omega_e} * e_{\phi}$$

$$\frac{\partial B(r)}{\partial t} = 0$$

According to equation (14), angular velocity of the electron rotation is
According to analysis in U-18, the first type of Lorentz force on the electron in the process of moving upward at a uniform speed can still be approximately calculated by equation (8), direction of Lorentz force is to the left and its magnitude is

$$F_B(r) = \frac{4\pi R^3 \rho}{9} * v_0 * \omega = \frac{4\pi R^3 \rho}{9} * v_0 * \frac{R \lambda v_i}{r} = \frac{4\pi R^4 \rho \lambda v_i v_0}{9r}$$

It is equivalent to that the moving electron on point A is affected by equivalent motional electric field intensity $E_m(r)$

$$E_m(r) = \frac{F_B(r)}{q_e} = \frac{4\pi R^4 \rho \lambda v_i v_0}{9Q_e r} * e_r$$

$\nabla \times E_m(r)$ is calculated with cylindrical coordinates

$$\nabla \times E_m(r) = \frac{1}{r} \begin{vmatrix} e_r & r * e_\phi & e_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ \frac{4\pi R^4 \rho \lambda v_i v_0}{9Q_e r} & 0 & 0 \end{vmatrix} = 0$$

So

$$\nabla \times E_m(r) = -\frac{\partial B(r)}{\partial t} = 0$$

$$\nabla \cdot E_m(r) = \frac{1}{r} \frac{\partial}{\partial r} \left( r * \frac{4\pi R^4 \rho \lambda v_i v_0}{9Q_e r} \right) = 0$$

An electron is subjected to the first type of Lorentz force in constant magnetic field. The curl and divergence of the equivalent motional electric field intensity equivalent to the first type of Lorentz force are zero.

In Fig-A26 right, an electron at point A moves to the right at a constant speed $v_0$, so the electron will be subjected to the second type of Lorentz force or the second type of induced force. According to equation (17)

$$B(r) = -\frac{4\pi R^4 \rho \lambda v_i}{9Q_e r} * e_\phi$$

As the electron moves to the right, distance $r$ between the electron and the wire varies with time $t$, so $B(r)$ on moving point A is a function of time $t$, suppose the function is $B(t)$.

$$\frac{\partial B(t)}{\partial t} = \frac{4\pi R^4 \rho \lambda v_i}{9Q_e r^2} * \frac{dr}{dt} * e_\phi = \frac{4\pi R^4 \rho \lambda v_i v_0}{9Q_e r^2} * e_\phi$$

(1) Calculate the second type of induced force $F_{\phi 2}$ on an electron

Because $\mathbf{v}_U(r)$ has only a component in $e_z$ direction, in cylindrical coordinate system, $\nabla \times \mathbf{v}_U(r)$ has only a component in direction of $e_\phi$, and the value is

$$\left( \nabla \times \mathbf{v}_U(r) \right)_\phi = -\frac{\partial v_U(r)}{\partial t}$$

According to equation (13)

$$\nabla \times \mathbf{v}_U = \frac{2R \lambda v_i}{r} * e_\phi$$

So
\[ \frac{\partial v_\rho(r)}{\partial r} = -\frac{2R\lambda v_{l}}{r} \]

For variables \( r, \phi \) and \( z \) of the cylindrical coordinates, left side of above equation is a partial derivative. Since above equation is not affected by the change of variable \( \phi \) and \( z \), when only the variable \( r \) is considered, left side of above equation is the derivative of \( dv_\rho(r)/dr \). As the electron moves to the right, distance \( r \) between the electron and the wire varies with time \( t \), so \( v_\rho(r) \) is a function of time \( t \), suppose the function is \( v_\rho(t) \).

Derivation according to composite functions is

\[ \frac{dv_\rho(t)}{dt} = \frac{dv_\rho(r)}{dr} * \frac{dr}{dt} = -\frac{2R\lambda v_{l}}{r} * v_0 \]

According to analysis in U-22, the second type of induced force on the electron in the process of moving to the right at a uniform speed can still be approximately calculated by equation (7)

\[ F_{l2} = -\frac{2\pi R^3 \rho}{9} * \frac{dv_\rho(t)}{dt} = -\frac{2\pi R^3 \rho}{9} * \left( -\frac{2R\lambda v_{l}}{r} * v_0 \right) * e_z = \frac{4\pi R^4 \rho \lambda v_{l} v_0}{9r} * e_z \]

(2) Calculate the second type of Lorentz force \( F_B \) on the electron

Suppose that the second type of Lorentz force \( F_B \) is approximately calculated using equation (8) for calculating the ideal Lorentz force, its magnitude is

\[ F_B = \frac{4\pi R^3 \rho}{9} * v_0 * \omega = \frac{4\pi R^3 \rho}{9} * v_0 * \frac{R\lambda v_{l}}{r} = \frac{4\pi R^4 \rho \lambda v_{l} v_0}{9r} \]

The direction of \( F_B \) is upward.

Therefore, \( F_B = F_{l2} \). The second type of Lorentz force and the second type of induced force act on the electron are approximately calculated by two different methods. Because they are the same force, the calculation results are exactly the same. The effect of \( F_B \) or \( F_{l2} \) on the electron is equivalent to the effect of equivalent motional electric field \( E_m \) on the electron

\[ E_m = \frac{F_{l2}}{-Q_e} = -\frac{4\pi R^4 \rho \lambda v_{l} v_0}{9Q_e r^2} * e_z \]

Since \( E_m \) has only a component in \( e_z \) direction, in cylindrical coordinate system, \( \nabla \times E_m \) has only a component in direction of \( e_\phi \)

\[ \nabla \times E_m = -\frac{\partial E_m}{\partial r} = -\frac{4\pi R^4 \rho \lambda v_{l} v_0}{9Q_e r^2} * e_\phi \]

So

\[ \nabla \times E_m = -\frac{\partial B(r)}{\partial t} \]

\[ \nabla \cdot E_m(r) = \frac{\partial}{\partial z} \left( -\frac{4\pi R^4 \rho \lambda v_{l} v_0}{9Q_e r} \right) = 0 \]

**Fig-A27** Direction of equivalent motional electric field and Lorentz force

As shown in Fig-A27, direction of magnetic induction intensity \( B \) of non-uniform magnetic
field is perpendicular to paper outward; conductor AB contacts the fixed conductor frame. When the conductor AB moves to the right by an external force $F_1$, since there are free electrons in the conductor AB, according to equation (8), the free electrons are exerted an upward Lorentz force and form a loop current. According to equation (8), upward moving electrons are exerted a Lorentz force $F_2$ to the left. $F_2$ is always in the opposite direction to $F_1$, that is to say equivalent motional electric field always blocks external force $F_1$. The negative sign of $\mathbf{V} \times \mathbf{E}_m = -\frac{\partial \mathbf{B}}{\partial t}$ coincides with this "blocking" effect. Because $\mathbf{V} \times \mathbf{E}_m \neq 0$, $\oint \mathbf{E}_m \cdot d\mathbf{l} \neq 0$, the result of $\oint \mathbf{E}_m \cdot d\mathbf{l}$ is the motional electromotive force in classical electromagnetism. When the conductor AB moves to the right, magnetic flux decreases because the area $S$ of conductor loop ABCD decreases, according to Lenz's law of classical electromagnetism, magnetic induction intensity generated by motional electromotive force should be perpendicular to paper outward, therefore, direction of current is ABCD, that is, direction of electrons movement in the moving conductor AB are upward, which is consistent with the results of mechanical model of U-particle.

According to the above calculation, the curl of the first type of Lorentz force is zero, and the curl of the first type of induced force is not zero. The second type of Lorentz force and the second type of induced force are essentially the same force, which are the same results obtained by two different calculation methods, and their curl is not zero.

In particular, the calculation of the equivalent motional electric field intensity shows that the electromagnetic induction law is also correct at the moving reference point by using the time and space length in the stationary reference system.

7 Summary

So far, in stationary frame of reference, Maxwell's equations of classical electromagnetism are derived by the mechanical model based on U-particle, physical laws used are conservation of momentum, conservation of kinetic energy and Newton's three laws of motion.

The electron/proton transforms translational kinetic energy of U-particle into rotational kinetic energy and acts as energy converter. This is manifestation of energy conservation, and also source of power in the world.

The electron/proton is exerted three kinds of forces: (1) electrostatic force and gravitation are generated by gradient of rotational kinetic energy of U-particle, electrostatic force constitutes electrostatic field and gravitation constitutes gravitational field; (2) the induced force on static electrons/protons generated by accelerating or decelerating motion of U-particle constitutes induced electric field; (3) Lorentz force is generated by Magnus effect of rotating electron/proton with translational velocity in U-particle environment. In addition, moving electron/proton causes macro velocity of U-particle, and curl of the macro velocity of U-particle constitutes magnetic field. Both electrostatic potential and gravitational potential are proportional to the rotational kinetic energy of U-particle, and magnetic vector potential is proportional to the macro momentum of U-particle. By replacing potential energy with rotational kinetic energy, the law of conservation of energy is simplified to the law of conservation of kinetic energy, and analytical mechanics can be more perfect without relying on potentials with action at a distance.
Both electromagnetic field and gravitational field are physical properties generated by motions of U-particle; they are fields in mathematical meaning. The statement that "electromagnetic field is a kind of matter" is not accurate. The reason is the same as that we can't say "dance is a kind of matter". Dance is only beautiful movement displayed by dancers. In order to avoid abusing definition of "matter", any object with zero inertial mass should not be defined as "matter". The statement that "changing electric field generates magnetic field" or "changing magnetic field generates electric field" is not accurate. The change of electric field and magnetic field are the result of electron/proton movement, and there is no causal relationship between them, but concomitant.

Ampere force is macro manifestation of Lorentz force. If two parallel straight wires A and B are applied with same direction current, movement of electrons in wire A causes velocity curl of U-particle at position of wire B, and electrons moving in wire B are pushed towards wire A by Lorentz force, as a result, two straight wires attract each other. As shown in Fig-A28, replace the two straight wires with two coils, number of turns of the two coils on cylinder is 1. When current in same direction is applied, the two coils attract each other, and when reverse current is applied, the two coils repel each other. Therefore, children can intuitively understand attraction or repulsion between magnets. Since clockwise rotation on front of paper turns into anticlockwise rotation on reverse side of the paper, children can intuitively understand why there is no "magnetic monopole".

![Fig-A28 Attraction and repulsion of current-carrying coils](image)

Whether it's electrostatic attraction, magnetic attraction, or gravitation, "attraction" is misleading, it mislead people to understand "attraction" as some kind of magic pulling force. It can be seen from this paper that the "attraction" comes from differential pushing-force of U-particle on electrons/protons.

The essence of magnetic field is curl of the macro motion of U-particle, and electrons/protons will rotate on its axis in magnetic field, a rotating electron/proton can store some mechanical energy like a flywheel, macro motion of the U-particle can store more mechanical energy, these energies are all magnetic field energies in classical electromagnetism.

![Fig-A29 Revolution and rotation of an electron](image)
As shown in Fig-A29, an electron makes a uniform circular motion under the action of Lorentz force in a constant magnetic field. If this circular motion is called revolution, the ratio of the angular velocity of the electron's rotation in the magnetic field to the angular velocity of revolution can be calculated. Suppose angular velocity of the electron revolution is \( \omega_{re} \), angular velocity of the electron rotation is \( \omega_{ro} \), inertial mass of an electron is \( M_e \), mass of U-particle contained in a virtual sphere with the same radius as electron is \( M_U \), magnetic induction intensity of the constant magnetic field is \( B \). Radius of the uniform circular motion of the electron is \( r \), period is \( T \), and linear speed is \( v \). Because centripetal force of an electron moving in a uniform circular motion is equal to Lorentz force,

\[
\frac{M_e \cdot v^2}{r} = Q_e v B
\]

\[
\frac{v}{r} = \frac{Q_e}{M_e} \cdot B
\]

\[
\omega_{re} = \frac{2\pi}{T} = 2\pi \cdot \frac{v}{2\pi r} = \frac{Q_e}{M_e} \cdot B
\]

According to equation (15)

\[
B = \frac{2\pi R^3 \rho}{9Q_e} \cdot 2\omega_{ro} = \frac{4\pi R^3 \rho}{9Q_e} \cdot \omega_{ro}
\]

\[
\omega_{re} = \frac{Q_e}{M_e} \cdot \frac{4\pi R^3 \rho}{9Q_e} \cdot \omega_{ro} = \frac{4\pi R^3 \rho}{3} \cdot \omega_{ro} = \frac{M_U \omega_{ro}}{3M_e}
\]

So

\[
\frac{\omega_{ro}}{\omega_{re}} = \frac{3M_e}{M_U}
\]

There is no electron or proton or conduction current in vacuum. Maxwell obtained the wave equation of electromagnetic field through the following mathematical calculation.

\[
\begin{align*}
\nabla \cdot E &= 0 \\
\nabla \cdot B &= 0 \\
\n\nabla \times E &= -\frac{\partial B}{\partial t} \\
\n\nabla \times (\nabla \times E) &= \nabla(\nabla \cdot E) - \nabla^2 E = -\nabla^2 E
\end{align*}
\]

Take curl on both sides of the third equation

\[
\nabla \times \left( \frac{\partial B}{\partial t} \right) = -\frac{\partial}{\partial t} (\nabla \times B) = -\frac{\partial}{\partial t} \left( \varepsilon_0 \mu_0 \frac{\partial E}{\partial t} \right) = -\varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}
\]

\[
\nabla^2 E = \varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}
\]

It can also be derived that

\[
\nabla^2 B = \varepsilon_0 \mu_0 \frac{\partial^2 B}{\partial t^2}
\]

The wave equation of mechanical wave is

\[
\frac{\partial^2 f}{\partial x^2} = \frac{1}{\nu^2} \frac{\partial^2 f}{\partial t^2}
\]

They are similar, and
\[
v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3.0 \times 10^8 \text{ m/s}
\]
That is to say, the calculated speed of electromagnetic propagation is equal to the measured speed of light, so Maxwell predicted that light is electromagnetic wave. However, mechanical wave has medium and stationary reference point of velocity, but what is the medium of electromagnetic wave? The speed of electromagnetic waves is equal to the speed of light, but what is the stationary reference point? At that time, physicists assumed that "Ether" was the medium and stationary reference point of electromagnetic wave propagation, but it has not been found so far. In this paper, U-particle can be used as the medium for Maxwell's electromagnetic wave propagation, and a new definition of stationary reference point is made in U-1, it avoids the absolute stationary point where the mathematical velocity of all U-particles is zero.

The stationary point defined by U-1 in this paper is an ideal point, which does not exist in reality. However, in many cases, the stationary point can be replaced by an approximately stationary point. It can be seen from U-13 that Up and Ue with the same acceleration cancel the induced force exerted on an electron or a proton, so the motion of an electrically neutral object will not affect the electron/proton. The geomagnetic field can be detected on the earth's surface, which indicates that the macro motion speed of U-particle is \( v_U \neq 0 \). However, because the geomagnetic field is very weak, it can be regarded as \( v_U \approx 0 \) for experiments or applications with low accuracy requirements. Although the Moon, the Sun and other celestial bodies will also affect translational kinetic energy density of U-particle on the earth's surface, the earth's influence is the most important. This is also the reason why we can feel the gravity of the earth, but can't feel the gravity of the Moon and the Sun. Therefore, we can regard the ground as the approximately stationary point of U-particle motion.

Both electromagnetic field and gravitational field are physical properties generated by motions of U-particle, and their changes take U-particle as the medium of transmission, so their propagation speed is exactly the same in the same U-particle environment. Propagation speed of the light emitted by a moving car is the same as that of the light emitted by a stopping car; propagation speed of the honking sound of a moving car is equal to that of a stopping car, and the two reasons are similar, because both lights propagate in the same U-particle environment, and both honking sounds propagate in the same air environment.

In stationary frame of reference, assuming that an object moves in X-axis direction at speed \( v \), the electron/proton in the object swallows any U-particle it collides with and then releases the U-particle with an initial velocity of \( V_x = v \). Suppose that the maximum speed of the U-particle in YZ plane is \( V_{yz} \), then \( v^2 + V_{yz}^2 = C_U^2 \), the maximum value of \( V_{yz} \) is reduced from \( C_U \) to \( \sqrt{C_U^2 - v^2} \), and the ratio is

\[
\gamma_1 = \frac{C_U}{\sqrt{C_U^2 - v^2}} = \frac{1}{\sqrt{1 - v^2/C_U^2}}
\]

Because all the forces including electromagnetism and gravitation are the result of the motion of U-particle, so the speed of physical processes inside moving objects will slow down, and moving clock will also slow down, and the slowing down ratio is \( \gamma_1 \). This is very interesting, the mechanical model of U-particle can help us intuitively and accurately understand the physical meaning of the Lorentz transformation.
Fig-A30 The model of Lorentz transformation

In two-dimensional space XOY shown in the left figure of Fig-A30, a car moving at a constant speed \( v \) to the right passes through point O at zero time, and a flash is emitted from point O at the same time. At time \( t \), the car arrives at point B, wavefront of the flash is a circle with point O as the center and radius \( C_0 t \). Take a point P on this circle, the projection of point P on X-axis is point Q, and the car arrives at point Q at time \( \tau \). Both \( t \) and \( \tau \) are measured by stationary clocks on the ground, \( OB = vt \), \( OP = C_0 t \), \( OQ = v \tau \).

If a tester uses a moving clock MC to measure in the car, the time for MC to move from point O to point Q is \( \tau' \). Now we need to find a lucky point \( G' \) on X-axis, the time for the moving clock MC to move from point O to point \( G' \) is \( t' \), and \( G'P = C_0 t' \). Both \( \tau' \) and \( t' \) are measured by the moving clock MC. This lucky point \( G' \) can easily make the tester standing at the position of clock MC have such an illusion: the ground is moving backwards, the time for point O to leave point \( G' \) is \( t' \), the time for the flash to travel from point \( G' \) to point P is also \( t' \), and the speed of light is \( C_0 \), that is, the speed of light in the moving reference frame is the same as that in the stationary reference frame. According to this wonderful illusion, using the time measured by the moving clock MC, we can use a simple method to correctly calculate the time for light to travel to point P.

Since the moving clock MC slows down, so \( \tau' = \tau \sqrt{1 - \frac{v^2}{C_0^2}} \). Suppose that \( x = v \tau \) and \( x' = v(\tau' - t') \), then \( x' + vt' = x\sqrt{1 - \frac{v^2}{C_0^2}} \), that is

\[
x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{C_0^2}}}
\]

This is an equation for the inverse Lorentz transformation. \( x \) is the length between OQ calculated using the time \( \tau \) measured by stationary clocks, abbreviated as stationary clock length, denoted as \( |OQ|_S = v \tau \). \( vt' = v \tau' + x' \) is the length between OQ calculated using the time \( \tau' \) measured by the moving clock, abbreviated as moving clock length, denoted as \( |OQ|_M = v \tau' \). Therefore \( |OQ|_M = |OQ|_S \sqrt{1 - \frac{v^2}{C_0^2}} \).

In the left figure of Fig-A30, draw a straight line AB perpendicular to X-axis through point B. AB intersects with the circle at point A. Connect OA and draw a perpendicular line QE perpendicular to OA through point Q, with the foot perpendicular to E, QE and OB intersect at point F. It can be proved that \( AF^2 - FQ^2 - QP^2 = 0 \) Triangle AEF is similar to QEO, so \( QE \ast EF = EO \ast AE \).

\[
AF^2 - FQ^2 - QP^2 = (AE^2 + EF^2) - FQ^2 - (PO^2 - OQ^2) \\
= (AO - EO)^2 + (EQ - FQ)^2 - FQ^2 - AO^2 + OQ^2 \\
= EO^2 - 2EO \ast AO + EQ^2 - 2EQ \ast FQ + OQ^2 \\
= 2(EO^2 - EO \ast AO + EQ^2 - EQ \ast FQ) = 2(EQ \ast EF - OE \ast AE) = 0
\]

Since \( OP = C_0 t \), \( OB = vt \), so \( AB = \sqrt{OA^2 - OB^2} = Ct \sqrt{1 - \frac{v^2}{C_0^2}} \). Triangle ABO is similar
to QEO, so $EQ = OQ * BA/OA = OQ \sqrt{1 - v^2/C^2_U}$, that is $EQ = |OQ|_m$. Previously calculated $|OQ|_m = vt' = vt' + x'$, so we can let $EF = vt'$ and $FQ = x'$. Triangle AEF is similar to ABO, so $AF = EF \times OA/0B = vt' \times C_u t/vt = C_u t'$. 

The right figure of Fig-A30 is the X'O'Y' coordinate system constructed on the basis of the left figure. Take X'-axis to coincide with X-axis, and point P remains unchanged, so the projection of point P on the X'-axis is still point Q. Take $G'Q = FQ = x'$ to get point G', then take $OG' = EF = vt'$ to get point O, make Y'-axis through point G' and regard point G' as point O'. Take $A'G' = AF = C_u t'$ on the Y'-axis, and make a circle with the point G' as the circle point and $C_u t'$ as the radius. Since it has been proved that $AF^2 - FQ^2 - QP^2 = 0$, so $A'G'^2 - G'Q^2 - QP^2 = 0$, that is $C_u t'^2 - x'^2 = QP^2$, it means that the point P is exactly located on the circle with point G' as the center and the radius $C_u t'$, that is, $G'P = C_u t'$. 

According to the left figure of Fig-A30, $QP^2 = OP^2 - OQ^2$, that is, $QP^2 = C_u t^2 - x^2$, so we get $C_u t^2 - x^2 = C_u t'^2 - x'^2$. Substituting the previously obtained equation for the inverse Lorentz transformation into this equation, we can get

$$C_u t^2 - \frac{(x' + vt')^2}{1 - v^2/C^2_U} = C_u t'^2 - x'^2$$

$$C_u t^2 - \frac{v^2 t'^2}{C^2_U} = (C_u t^2 - x^2)(1 - v^2/C^2_U) + (x' + vt')^2$$

$$= C_u t'^2 - x'^2 - v^2 t'^2 + \frac{v^2 x'^2}{C^2_U} + x'^2 + 2vt'x' + v^2 t'^2$$

$$= C_u t'^2 + \frac{v^2 x'^2}{C^2_U} + 2vt'x' = (C_u t' + vx'/C_u)^2$$

So

$$t = \frac{C_u t' + vx'/C_u}{\sqrt{C_u^2 - v^2/C^2_U}} = \frac{t' + vx'/C_u^2}{\sqrt{1 - v^2/C^2_U}}$$

Since the coordinates of point P on the Y-axis remain unchanged, so $y' = y$. Therefore, the inverse Lorentz transformation in two-dimensional space is

$$\begin{cases} x = \frac{x' + vt'}{\sqrt{1 - v^2/C^2_U}} \\ y = y' \\ t = \frac{t' + vx'/C_u^2}{\sqrt{1 - v^2/C^2_U}} \end{cases}$$

If an electron/proton suddenly disintegrates for some reason, gradient of rotational kinetic energy of U-particle $\nabla E_R(r)$ in the space around the electron/proton will suddenly change, which will lead to violent conversion between rotational kinetic energy and translational kinetic energy of U-particle. The phenomenon of an electron disintegration is disappearance of measurable inertial mass, and the phenomenon of violent conversion between rotational kinetic energy and translational kinetic energy of U-particle is output of huge energy. This may help to explain the relationship between mass and energy.

Suppose that mass of object A is $m$, and distance between point Q and object A is $L$, according to quation (2) and the superposition principle of U-15, rotational kinetic energy gradient of U-particle at point Q changes with the change of $m$ and $L$, so the U-particle environment around object A is changed. Therefore, velocity of light propagation around object A changes with the change of $m$ and $L$, which helps to explain the gravitational lens in astronomy. Following is an example to calculate the bending of starlight by the Sun.
According to G-2 and G-4 of appendix G "Calculation of light bending in spherically symmetric medium", relationship between radius vector and light is
\[
\frac{r \cdot \sin \varphi}{u} \equiv \text{constant} = c_0 \quad \frac{d\theta}{dr} = \frac{c_0 u}{r \sqrt{r^2 - c_0^2 u^2}}
\]
Among them, \( r \) is the length of radius vector, \( u \) is the speed of light, \( \theta \) is the angle between initial position vector and radius vector, \( \varphi \) is the angle between radius vector and light, \( \psi + \pi/2 \) is the angle between initial position vector and light.

As shown in Fig-A31, starlight is bent by the Sun and the starlight is perpendicular to OA at point A. The speed \( V \) of light at a certain point is exactly the micro motion speed of \( U \)-particle at that point. The Sun is electrically neutral. Suppose that the Sun contains \( N_S \) electrons in total, since the Sun is electrically neutral, according to equation (1) and the superposition principle, rotational kinetic energy of \( U_p \) and \( U_e \) are both
\[
E_R(r) = N_S \cdot E_U \cdot \frac{R}{r}
\]
In appendix B, if \( K_U = 1 \) is not taken, then
\[
E_R(r) = N_S \cdot K_U \cdot E_U \cdot \frac{R}{r}
\]
\[
E_T(r) = E_U - E_R(r) = E_U \left( 1 - \frac{N_S K_U R}{r} \right) = \frac{M_U C_U^2}{2} \left( 1 - \frac{N_S K_U R}{r} \right) = \frac{M_U V^2}{2}
\]
\[
V(r) = C_U \sqrt{1 - \frac{N_S K_U R}{r}}
\]
Suppose that radius of the Sun is \( R_s \), OA is taken as the initial position vector, then \( r_0 = R_s \), \( \varphi_0 = \pi/2 \), \( \psi_0 = 0 \), so
\[
V_0 = C_U \sqrt{1 - \frac{N_S K_U R}{R_s}}
\]
Take \( P_1 = N_S K_U R / R_s \). Assuming that the propagation speed \( u \) of light in \( U \)-particle is equal to the micro motion speed \( V \) of \( U \)-particle, according to G-2 of appendix G
\[
c_0 = \frac{r_0 \cdot \sin \varphi_0}{u_0} = \frac{R_s}{V_0} = \frac{R_s}{C_U \sqrt{1 - P_1}}
\]
\[
u(r) = C_U \sqrt{1 - \frac{P_1 R_s}{r}}
\]
\[
\sin \varphi = \frac{c_0 u}{r} = \frac{R_s}{r C_U \sqrt{1 - P_1}} \cdot C_U \sqrt{1 - \frac{P_1 R_s}{r}} = \frac{R_s}{r \sqrt{1 - P_1}} \sqrt{1 - \frac{P_1 R_s}{r}}
\]
Take $r = x \cdot R_s$, then

$$\sin \varphi = \frac{1}{x^{1/2} - P_1} \sqrt{1 - \frac{P_1}{x}} = \frac{1}{x^{1/2} - P_1} \sqrt{1 - \frac{P_1}{x^2}}$$

So

$$d\varphi = \frac{-2x + 3P_1}{2x\sqrt{x - P_1}\sqrt{x^3(1 - P_1) - x + P_1}} * dx$$

For the detail calculation process, please refer to G-5 of appendix G "Calculation of light bending in spherically symmetric medium". According to G-4 of appendix G

$$\frac{d\theta}{dr} = \frac{c_0 u}{r \sqrt{r^2 - c_0^2 u^2}}$$

Since $r = x \cdot R_s$, so

$$d\theta = \frac{x \cdot R_s dx}{x \sqrt{\frac{x^2 R_s^2}{c_0^2 u^2} - 1}} = \frac{dx}{x \sqrt{\frac{x^2 R_s^2}{1 - P_1} - \frac{P_1}{x}}} = \frac{\sqrt{x - P_1} dx}{x \sqrt{x^3(1 - P_1) - x + P_1}}$$

As shown in Fig-A31, $\psi + \pi/2 = \varphi + \theta$, these angles are functions of radius vector $r$, therefore

$$d\psi = d\varphi + d\theta = \frac{-2x + 3P_1}{2x\sqrt{x - P_1}\sqrt{x^3(1 - P_1) - x + P_1}} * dx + \frac{\sqrt{x - P_1}}{x \sqrt{x^3(1 - P_1) - x + P_1}} * dx$$

$$= \frac{P_1 dx}{2x\sqrt{x - P_1}\sqrt{x^3(1 - P_1) - x + P_1}}$$

When $x \geq 1 \gg P_1$

$$d\psi = \frac{P_1 dx}{2x\sqrt{x^3 - x}} = \frac{P_1 dx}{2x^2\sqrt{x^2 - 1}}$$

Take $x = 1/\sin y$, then

$$\frac{dx}{dy} = \frac{-\cos y}{\sin^2 y} = -x^2 \sqrt{1 - \sin^2 y} = -x^2 \sqrt{1 - \frac{1}{x^2}} = -x \sqrt{x^2 - 1}$$

$$\frac{dx}{x^2\sqrt{x^2 - 1}} = -x \sqrt{x^2 - 1} * dy * \frac{1}{x^2\sqrt{x^2 - 1}} = -\frac{dy}{x} = -\sin y dy$$

So

$$\psi \approx \int_{1}^{\infty} \frac{P_1 dx}{2x^2\sqrt{x^2 - 1}} = \frac{P_1}{2} \int_{1}^{\infty} \frac{dx}{x^2\sqrt{x^2 - 1}} = \frac{P_1}{2} \int_{\pi/2}^{0} -\sin y dy = \frac{P_1}{2}$$

According to astronomical observations, $2\psi = 1.75''$, that's to say

$$\psi = 0.875'' = \frac{0.875}{180} \cdot \pi = 4.24 \cdot 10^{-6} \text{ rad}$$

So, $P_1 = 2\psi = 2 \cdot 4.24 \cdot 10^{-6} = 8.48 \cdot 10^{-6}$. The following data are known: the mass of the Sun is $1.989 \cdot 10^{30} \text{ Kg}$, the radius of the Sun is $R_s = 6.955 \cdot 10^8 \text{ m}$, the mass of an electron is $1.6749 \cdot 10^{-27} \text{ Kg}$, the mass of a proton is $1.6726 \cdot 10^{-27} \text{ Kg}$, and the mass of an electron is $M_e = 9.10956 \cdot 10^{-31} \text{ Kg}$. The following two different methods are used to estimate the total number $N_e$ of electrons contained in the Sun.
The first method is to approximate an neutron as a combination of an electron and a proton, and the total number $N_s$ of electrons contained in the Sun is approximately

$$N_s = \frac{1.989 \times 10^{30}}{1.6749 \times 10^{-27}} = 1.188 \times 10^{57}$$

The second method assumes that both proton and neutron are composed of electrons and anti-electrons, so the total number $N_s$ of electrons contained in the Sun is approximately

$$N_s = \frac{1.989 \times 10^{30}}{2 \times 9.10956 \times 10^{-31}} = 1.092 \times 10^{60}$$

If the second method is used to estimate $N_s$, then

$$K_U R = \frac{P_1 R_s}{N_s} = \frac{8.48 \times 10^{-6} \times 6.95 \times 10^9}{1.092 \times 10^{60}} = 5.4 \times 10^{-57}$$

In appendix B, if $K_U = 1$ is not taken, then

$$\nu_U (r) = K_U \nu_i \frac{R}{r}$$

Therefore, equation (17) also has a coefficient $K_U$

$$B = -\frac{4\pi \rho R K_U \nu_i}{9 Q e r} \epsilon_0$$

The permeability of vacuum $\mu_0$ also has a coefficient $K_U$

$$\mu_0 = \frac{8\pi^2 R K_U \rho}{9 Q e^2}$$

The field constant also has a coefficient $K_U$, and $K_U R^4 \rho \approx 3.68 \times 10^{-45}$, therefore the inertia mass $M_U$ of U-particle contained in a sphere with radius of $R$ can be calculated

$$M_U = \frac{4\pi R^3 \rho}{3} = \frac{4\pi}{3} \frac{K_U R^4 \rho}{K_U R} = \frac{4\pi}{3} \frac{3.68 \times 10^{-45}}{5.4 \times 10^{-57}} = 2.85 \times 10^{12} Kg$$

The mass of $M_U$ is too large, so it is incorrect to explain the bending of starlight by the Sun using the mechanical model of U-particle mentioned above. The above calculations show that ignoring the existence of Ue and using linear addition to calculate the rotational kinetic energy of Up of an electrically neutral object alone will get wrong results. One solution to improve the mechanical model of U-particle is that electron/proton not only produces U-particle with rotational kinetic energy of $E_U$, but also produces tiny ball (abbreviated as TB) with rotational kinetic energy of $e_B$, rotation direction of TB is the same as that of the small ball on the surface of electron/proton. The result of the collision between TB and electron/proton is universal gravitation. Assuming that the mass $m$ of TB is much smaller than the mass $M_U$ of U-particle, the mass density of TB in space is $\rho_B$. Similar to the mechanical model of U-particle, the decreasing function of rotational kinetic energy $e_R$ of TB is

$$e_R (r) = K_B e_B \frac{R}{r}$$

Among them, $K_B$ is a coefficient related to random diffusion of TB, $e_B$ is a constant and $e_B = mc_0^2/2$. Assuming translational kinetic energy of TB is $e_T$ and micro motion speed of TB is $V_B$, then, the translational kinetic energy of TB around an isolated electron is

$$e_T (r) = e_B - e_R (r) = e_B \left(1 - \frac{K_B R}{r}\right) = \frac{mc_0^2}{2} \left(1 - \frac{K_B R}{r}\right) = \frac{mV_B^2}{2}$$

Both Up and Ue are isotropic, but the angular momentum direction of high-speed rotating ball
inside is opposite. Ignoring the existence of Ue and using linear addition to calculate the rotational kinetic energy of Up of an electrically neutral object, incorrect results were obtained. TB is a ball, and the rotating ball is not isotropic. Assuming that rotational kinetic energy of TB of electrically neutral objects can be calculated using linear addition, then rotational kinetic energy of TB near the Sun is

\[ e_T(r) = e_B - e_R(r) = e_B \left( 1 - \frac{2N_sK_BR}{r} \right) = \frac{mC_0^2}{2} \left( 1 - \frac{2N_sK_BR}{r} \right) = \frac{mV_B^2}{2} \]

\[ V_B(r) = C_U \sqrt{1 - \frac{2N_sK_BR}{r}} \]

Take \( P_2 = 2N_sK_BR/R_s \), then

\[ V_B(r) = C_U \sqrt{1 - \frac{P_2R_s}{r}} \]

Since the Sun is electrically neutral, \( \nabla E_B(r) + \nabla E_R(r) = 0 \), this does not impose strict constraints on the micro motion speed \( V \) of Up and Ue. TB collides with U-particle, so in equilibrium state, the micro motion speed \( V \) of Up and Ue is equal to the micro motion speed \( V_B \) of TB. Since \( N_s \) is large, the maximum micro motion speed \( V \) of U-particle nearby the Sun decreases from \( C_U \) to \( V_B \), and the ratio is

\[ \gamma_2 = \frac{C_U}{V_B} = \frac{1}{\sqrt{1 - \frac{P_2R_s}{r}}} \]

Therefore, the clock near the Sun will slow down by a ratio of \( \gamma_2 \). The TB around the Sun forms a spherical symmetric medium, and starlight is refracted when passing near the Sun. As shown in appendix G, the refraction is proportional to \( V_B \), so the proportion coefficient is \( \sqrt{1 - \frac{P_2R_s}{r}} \). If the factor of clock slowing is also taken into account, the proportion coefficient of refraction is \( \left( \sqrt{1 - \frac{P_2R_s}{r}} \right)^2 = 1 - \frac{P_2R_s}{r} \), which is equivalent to the speed of starlight propagation in TB being \( u(r) = C_U(1 - P_2R_s/r) \). According to G-2 of appendix G

\[ c_0 = \frac{r_0 \sin \varphi_0}{u_0} = \frac{R_s}{C_U(1 - P_2)} \]

\[ \sin \varphi = \frac{c_0u(r)}{r} = \frac{R_s}{rC_U(1 - P_2)} \cdot C_U \left( 1 - \frac{P_2R_s}{r} \right) = \frac{R_s}{r(1 - P_2)} \left( 1 - \frac{P_2R_s}{r} \right) \]

Take \( r = x \cdot R_s \), then

\[ \sin \varphi = \frac{1 - P_2/x}{(1 - P_2)x} = \frac{x - P_2}{(1 - P_2)x^2} \]

So

\[ \varphi = \sin^{-1} \left[ \frac{x - P_2}{(1 - P_2)x^2} \right] \]

\[ d\varphi = \frac{(1 - P_2)x^2 - 2(1 - P_2)(x - P_2)x}{(1 - P_2)^2 x^4} \cdot dx = \frac{(2P_2 - x)dx}{x\sqrt{(1 - P_2)^2 x^4 - (x - P_2)^2}} \]

According to G-4 of appendix G

\[ \frac{d\theta}{dr} = \frac{c_0u}{r\sqrt{r^2 - c_0^2 u^2}} \]
Since \( r = x \times R_s \), so

\[
\frac{R_s}{xR_s} \frac{x^2 R_s^2}{\sqrt{c_0^2 u^2} - 1} = x \frac{x^2 R_s^2}{\left(1 - \frac{R_s^2}{(1 - P_2)}x\right)^2 - 1} = \frac{(x - P_2)dx}{x\sqrt{(1 - P_2)^2 x^4 - (x - P_2)^2}}
\]

As shown in Fig-A31, \( \psi + \pi/2 = \phi + \theta \), these angles are functions of radius vector \( r \), therefore

\[
d\psi = d\phi + d\theta = \frac{(2P_2 - x)dx}{x\sqrt{(1 - P_2)^2 x^4 - (x - P_2)^2}} + \frac{(x - P_2)dx}{x\sqrt{(1 - P_2)^2 x^4 - (x - P_2)^2}}
\]

\[
d\psi = \frac{P_2 dx}{x\sqrt{x^4 - x^2}} = \frac{P_2 dx}{x^2\sqrt{x^2 - 1}}
\]

\[
\psi \approx \int_1^\infty \frac{P_2 dx}{x^2\sqrt{x^2 - 1}} = P_2 \int_1^{\infty} \frac{dx}{x^2\sqrt{x^2 - 1}} = P_2 \int_{\pi/2}^{0} -\sin y \, dy = P_2
\]

According to astronomical observations, \( 2\psi = 1.75'' \), that’s to say

\[
P_2 = \psi = 0.875'' = \frac{0.875}{3600} \times \frac{\pi}{180} \rad = 4.24 \times 10^{-6} \rad
\]

So

\[
K_B R = \frac{P_2 R_s}{2N_s} = \frac{4.24 \times 10^{-6} \times 6.955 \times 10^8}{2 \times 1.092 \times 10^{60}} = 1.35 \times 10^{-57}
\]

Assuming that the radius of electron is \( R = 10^{-16} m \), then \( K_B = 1.35 \times 10^{-41} \), which is a very small value.

Similar to U-12, the universal gravitation between two protons can be calculated as

\[
F_g = \frac{4\pi R^4 K_B \rho_B C_0^2}{9} \times \frac{1}{L^2}
\]

A single TB is not isotropic, and the random collisions of a large number of TBs are similar to the random collisions of gas molecules. Since the speed of TB's translational motion increases with the distance from electrons/protons, the gravitational force is always attractive.

The phenomenon that light of the star is bent by the Sun is caused by the increase of micro motion speed \( V \) of U-particle with the increase of distance \( r \) from the Sun. The universal gravitation and this phenomenon do not constitute a causal relationship but an accompanying relationship, just like changing magnetic field and induced electric field do not constitute a causal relationship but an accompanying relationship. In the process of deriving the Lorentz transformation above, we only consider the influence of macro motion speed \( v \) of U-particle, ignoring the influence of change of micro motion speed \( V \) of U-particle. Therefore, the Lorentz transformation itself is also an approximate calculation. Since the speed of all physical processes is affected by macro motion speed \( v \) and micro motion speed \( V \) of U-particle, the clock is a kind of instrument to display the speed of some physical processes, so the time measured by the clock is also affected by the motion speed of U-particle, the result is that the moving clock slows down, and the larger the gravity, the slower the clock.
In addition, the mechanical analysis and calculation in this paper are limited to the case of stationary frame of reference and low-speed movement of electrons/protons.

8 Prediction

As shown in Fig-A32, cathode-ray passes from left to right through an odd symmetrical magnetic field generated by two identical coils in reverse series connection, if electrons have velocity of up and down, a curve image appears on the right phosphor screen. Fix the position of coils and move cathode-ray tube from left to right, the curve image on the screen rotates as shown in Fig-A33. When the screen is at the point O in Fig-A32, inclination angle of tangent of the curve image is maximum, which can be predicted by classical electromagnetism.

![Fig-A32 Electrons pass through odd symmetrical magnetic field](image1)

![Fig-A33 Change of curve image on phosphor screen](image2)

According to mechanical model of U-particle in this paper, essence of Lorentz force is Magnus effect of electrons with both translational velocity and rotation on its axis in U-particle environment. In magnetic field, the electron will rotate on its axis, and the rotating electron has an inertial effect. Therefore, it can be predicted that if cathode-ray passes through the odd symmetric magnetic field from left to right, when the screen is at point A in Fig-A32, inclination angle $\psi(x)$ of tangent of the curve image will be maximum. This result, which is contradictory to classical electromagnetism, is due to the inertial effect of the electron rotating on its axis in magnetic field.

9 Experiment

The results of the actual measurement using cathode-ray tube are consistent with the prediction. Please refer to the paper “Abnormal deflection of electrons crossing the boundary
of two opposite magnetic fields” for details.

This paper is translated from Chinese into English with translation software. Changgen Zou, April 2023@ Nanjing, China.

Appendix B: Mathematical calculation of random collision of U-particle

First, simply calculate the random collision of ordinary particles.

As shown in Fig-B1, unit price of a gray ball is $x$ and unit price of a white ball is $y$. These balls have no other differences except for color and unit price. They diffuse randomly like gas molecules. Grey balls and white balls collide constantly, assuming that (1) their motion speed between two collisions is $v$ and distance is $\Delta x$, (2) the sum of quantity densities of gray and white balls is a constant $\rho_N$.

Take a virtual vertical interface at the coordinate $x_0$ on X-axis, which is a rectangle with area of $S$. Take cuboids with a thickness of $\Delta x$ on the left and right sides of the interface, and there are $\Delta x\rho_N$ small balls in each cuboid. The opportunities of small balls moving in six directions in space are equal, so within $\Delta t = \Delta x/v$ time interval, $\Delta x\rho_N/6$ small balls from the left cuboid pass through the interface and enter the right cuboid, while an equal number of small balls from the right cuboid pass through the interface and enter the left cuboid.

Assuming that the proportion of gray balls in the left cuboid is $p$, and the proportion of gray balls in the right cuboid is $q$, and $p > q$, $x > y$, the price flux per unit time passing through the interface from left to right is

$$\varphi = \frac{\left\{px + (1-p)y \right\} - \left\{qx + (1-q)y \right\} \ast \Delta x\rho_N/6}{\Delta t} = \frac{\Delta x}{3} \ast \frac{[px + (1-p)y]\rho_N - [qx + (1-q)y]\rho_N}{2\Delta x} \ast S$$

Where $[px + (1-p)y]\rho_N$ is the price density $\rho_L$ of the cuboid on the left side of the interface, and $[qx + (1-q)y]\rho_N$ is the price density $\rho_R$ of the cuboid on the right side of the interface, so

$$\varphi = \frac{\Delta x}{3} \ast \frac{\rho_L - \rho_R}{2\Delta x} \ast S = -\frac{\Delta x}{3} \ast \frac{\rho_R - \rho_L}{2\Delta x} \ast S$$

When $\Delta x$ is very small, $(\rho_R - \rho_L)/2\Delta x$ is the price density gradient $\partial \rho / \partial x$ at $x = x_0$. Take $D = \Delta x v/3$, then
\[ \phi = -D \cdot \frac{\partial \rho}{\partial x} \cdot S \]

Therefore, for diffusion caused by random motion, the price flux passing through the interface per unit time is equal to the gradient of the price density multiplied by the area and then multiplied by a proportional coefficient. The diffusion flux per unit time is proportional to the density gradient and area, which is manifestation of mathematical theorems in physical process, it is universal. The above equation is completely consistent with Fick's first diffusion law, which also indicates that Fick's first diffusion law is suitable for diffusion motion of all microscopic particles, including U-particles. When the interface is not perpendicular to the gradient, the above equation is \( \phi = -D \cdot \vec{v} \rho \cdot dS \).

**B-I**: An isolated static electron, decreasing function of rotational kinetic energy \( E_R(r) \) of Up is \( E_R(r) = E_U \cdot R/r \), \( r \) is the distance between the Up and centre of the electron. The flux of rotational kinetic energy \( E_R(r) \) per unit time is proportional to gradient of rotational kinetic energy density \( \nabla \rho_{ER}(r) \) times area.

Explanation: According to mechanical model of U-particle, when two Up collide with each other, angular velocity direction of tiny balls at the collision point is opposite, so the sum of rotational kinetic energy of the two Up after collision decreases. The Up released by an isolated electron is gradually far away from the centre of the electron after numerous collisions, and rotational kinetic energy of Up is gradually decreased to zero. This is equivalent to diffusion of rotational kinetic energy of Up from surface of the electron. In the stable diffusion equilibrium state, rotational kinetic energy diffused from a fixed closed surface is constant.

As shown in Fig-B2, it is assumed that there are two spheres A and B with radii of \( r_1 \) and \( r_2 \) outside the electron, because rotational kinetic energy flowing out from surface A is equal to rotational kinetic energy flowing into surface B in the diffusion equilibrium state, according to Fick's first diffusion law, the flux of rotational kinetic energy \( E_R(r) \) per unit time is proportional to gradient of rotational kinetic energy density \( \nabla \rho_{ER}(r) \) times area, so

\[
\frac{d\rho_{ER}(r)}{dr}\bigg|_{r_1} \cdot 4\pi r_1^2 = \frac{d\rho_{ER}(r)}{dr}\bigg|_{r_2} \cdot 4\pi r_2^2 = \text{Constant}
\]

Solving the equation and substituting two initial conditions, \( \rho_{ER}(r) = \rho_{E0} \) when \( r \) is radius \( R \) of electron and \( \rho_{ER}(r) = 0 \) when \( r \) is infinite, we can get the following results

\[ \rho_{ER}(r) = \rho_{E0} \cdot \frac{R}{r} \]
Suppose that the rotational kinetic energy diffused by a stationary electron per unit time is $N$ times $E_U$. According to Fick's first diffusion law, the diffusion flux is equal to density gradient multiplied by diffusion area multiplied by proportional coefficient $D$. Calculate using spherical coordinates

$$N \cdot E_U = -D \oint \nabla \rho_{ER}(r) \cdot dS = D \oint \frac{\rho_{ER}R}{r^2} * r^2 \sin \theta \, d\theta \, d\varphi = D \rho_{E0} R \int_0^\pi \sin \theta \, d\theta \int_0^{2\pi} d\varphi$$

$$= 4\pi RD \rho_{E0}$$

So

$$\rho_{E0} = \frac{NE_U}{4\pi RD}$$

Take the scale coefficient $K_U$ as

$$K_U = \frac{N}{4\pi RD \rho_N}$$

Then

$$\rho_{E0} = \frac{N}{4\pi RD \rho_N} * E_U \rho_N = K_U E_U \rho_N$$

So

$$\rho_{ER}(r) = K_U E_U \rho_N * \frac{R}{r} = \rho_N * E_R(r)$$

And therefore

$$E_R(r) = K_U E_U \cdot \frac{R}{r}$$

This can be understood as follows: A stationary electron swallows N Up with zero rotational kinetic energy per unit time and releases N Up with rotational kinetic energy of $E_U$. The rotational kinetic energy density $\rho_{ER}(r)$ of Up surrounding the electron decreases inversely with distance $r$ from center of the electron. When $r = R$, rotational kinetic energy density is $K_U E_U \rho_N$. $\rho_N$ is constant, so rotational kinetic energy $E_R(r)$ of Up decreases inversely with distance $r$ from center of the electron. When $r = R$, rotational kinetic energy is $K_U E_U$. $K_U$ must be less than or equal to 1, otherwise, it will cause rotational kinetic energy of U-particle to be higher than $E_U$. To simplify the mechanical model of U-particle, take $K_U = 1$, so

$$E_R(r) = E_U \cdot \frac{R}{r}$$

**B-2:** The relationship between velocity $v_l$ of an electron moving at a low speed, macro velocity $v_u$ of Up and distance $r$ between the Up and centre of the electron is $v_u(r) = v_l \cdot \frac{R}{r}$.

Explanation: An electron moves at a constant velocity of $v_l$, after the electron swallows U-particle that collides with it, it releases Up with initial velocity of $v_l$ at the collision point. After this Up collides with other U-particles, macro momentum $P_u$ of the Up changes. Since $v_l$ is much less than the speed $V$ of U-particle random collision, where $V$ is close to the speed of light, hence, $P_u$ diffuses isotropically in space depending on the random collision of U-particle, $P_u$ decreases gradually to zero with the increase of the distance $r$ between the Up and the electron. Take a curved surface $\Sigma$ which is fixed with relative position of the electron and encloses the electron, when the electron moves uniformly, the macro momentum diffused
from the closed surface $\Sigma$ is a fixed value. When $v_i \ll C_U$, it can be approximately assumed that an electron swallows N Up with zero rotational kinetic energy per unit time and the sum of momentum of these N Up is zero, and then releases N Up with rotational kinetic energy of $E_U$ and macro momentum of $M_U v_i$, so the diffusion of $P_U$ follows the same law as that of $E_U$ of Up released by a stationary electron, therefore, the macro momentum density $\rho_{PU}(r)$ of Up satisfies

$$\rho_{PU}(r) = \rho_{P0} \cdot \frac{R}{r}$$

$\rho_{P0}$ represents the value of $\rho_{PU}(r)$ when $r$ is electron radius R. Similar to B-1, the macro momentum diffused by a moving electron per unit time is N times $M_U v_i$, so

$$N \cdot M_U v_i = -D \int \mathbf{v}_{PU}(r) \cdot dS = D \int \frac{\rho_{P0} R}{r^2} \cdot r^2 \sin \theta \ d\theta \ d\varphi = D \rho_{P0} R \int_0^\pi \sin \theta \ d\theta \int_0^{2\pi} d\varphi$$

$$= 4\pi RD\rho_{P0}$$

So

$$\rho_{P0} = \frac{NM_U v_i}{4\pi RD} = \frac{N}{4\pi RD\rho_N} \cdot \frac{M_U v_i \rho_N}{R} = K_U v_i \rho$$

$$\rho_{PU}(r) = K_U v_i \rho \cdot \frac{R}{r} = \mathbf{v}_{PU}(r) \cdot \rho$$

So

$$\mathbf{v}_{PU}(r) = K_U v_i \cdot \frac{R}{r}$$

According to B-1, take $K_U = 1$, so

$$\mathbf{v}_{PU}(r) = v_i \cdot \frac{R}{r}$$

Suppose that when an electron moves at a low speed and the speed changes slowly, the above equation is still approximately correct. The result of a proton is the same.

**B-3**: The mass and radius of a white ball and a grey ball are both $m$ and $r$. As shown in Fig-B3, the sphere with centre of O and radius of R is surrounded by static white balls. $R \gg r$. Grey balls are uniformly distributed in space outside the sphere O. The probability of grey balls moving in all direction is equal, but the average velocity is vector $\mathbf{v}$, that is, the macro velocity of grey ball is $\mathbf{v}$. The gap between white balls is very small, and grey ball has no chance to directly collide with the sphere O. A grey ball collides with a white ball elastically, after each collision, the white ball enters the sphere O. The number of collisions between grey balls and white balls in unit time and unit area on surface of sphere O is a fixed value N. Under the above assumptions, white balls is randomly collided by grey balls and then enter the sphere O, the macro momentum entering the sphere O in unit time is $\mathbf{0}$. If there is no white ball in the above assumptions, and the sphere O that grey balls random collide with is a virtual sphere O, when grey balls collide with the virtual sphere, they enter directly the virtual sphere O, the macro momentum entering the virtual sphere O in unit time is $\mathbf{0}_V$, then $\mathbf{0} = \mathbf{0}_V / 6$. 

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Explanation: As shown in Fig-B3, macro velocity $v$ of grey balls is upward. Enlarge geometric size of white ball $B$ to radius of $(R + 2r)$, and surface of enlarged white ball $B$ is shown as the dotted line in Fig-B3. Since the probability of grey balls moving in all directions is equal, when $R \gg r$, component values of momentum after collision in different directions can be calculated by the white ball with the same mass but enlarged geometric size.

Suppose that the grey ball with centre of $A$ collides with the enlarged white ball $B$ with centre of $O$, and point $B$ overlaps with point $O$, as shown in Fig-B4 left, before collision, velocity of grey ball $A$ is $AE$, and white ball $B$ is stationary, and angle between $AE$ and $AB$ is $\theta$, then, after elastic collision, velocity of white ball $B$ is $BF = AE \cos \theta$, and the upward component of $BF$ is $BF \cos \theta = AE \cos \theta$. Since the probability of grey balls moving in all directions is equal, it is always possible to find a grey ball $C$ colliding with a white ball $D$ on the extension line of $AB$. Grey ball with centre of $C$ collides with the enlarged white ball whose centre is $D$, as shown in Fig-B4 right, before collision, velocity of grey ball $C$ is $CG$, and white ball $D$ is stationary, and angle between $CG$ and $CD$ is $\theta$, after elastic collision, velocity of white ball $D$ is $DH = CG \cos \theta$, and the downward component of $DH$ is $DH \cos \theta = CG \cos^2 \theta$. After collision between grey ball $A$ and white ball $B$, grey ball $C$ and white ball $D$, the sum of upward momentum of white ball $B$ and white ball $D$ is

$$AE \cos^2 \theta - CG \cos^2 \theta = (AE - CG) \cos^2 \theta = \nu \cos^2 \theta$$

Suppose that the lower hemisphere surface of sphere $O$ is $\Sigma$, the white ball enters the spherical $O$ after each collision between the grey ball and the white ball on surface of the spherical $O$. Calculated in spherical coordinates, the macro momentum $\theta$ entering the sphere $O$ in unit time is
If there is no white ball on surface of the virtual sphere O, after the grey ball randomly collides with the virtual sphere O, it enters directly the virtual sphere O, then, the macro momentum $\phi_v$ entering the virtual sphere O in unit time is

$$\phi_v = \iint \! mv \ast N dS = 4\pi R^2 Nmv$$

So

$$\frac{\phi}{\phi_v} = \frac{2\pi R^2 Nmv}{3 \ast 4\pi R^2 Nmv} = \frac{1}{6}$$

That is, the momentum entering the sphere O in unit time after random collision between grey balls and white balls is equal to $1/6$ of the momentum of grey balls entering the virtual sphere O randomly in unit time.

**Appendix C: Three dimensional coordinate system and simplified operation of Hamilton operator**

Explanation: the spatial structure of three-dimensional rectangular coordinate system $(x, y, z)$, cylindrical coordinate system $(r, \varphi, z)$, and spherical coordinate system $(r, \theta, \varphi)$ used in this paper is shown in Fig-B.

![Fig-C cylindrical coordinate and spherical coordinate](image)

Vector of cylindrical coordinate system

$$\mathbf{F} = F_r(r, \varphi, z) \ast \mathbf{e}_r + F_\varphi(r, \varphi, z) \ast \mathbf{e}_\varphi + F_z(r, \varphi, z) \ast \mathbf{e}_z$$

$$\nabla \cdot \mathbf{F} = \frac{1}{r} \frac{\partial}{\partial r} (r \ast F_r) + \frac{1}{r} \frac{\partial F_\varphi}{\partial \varphi} + \frac{\partial F_z}{\partial z}$$

$$\nabla \times \mathbf{F} = \frac{1}{r} \begin{vmatrix} \mathbf{e}_r & r \ast \mathbf{e}_\varphi & \mathbf{e}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ F_r & r \ast F_\varphi & F_z \end{vmatrix}$$

When $F_r(r, \varphi, z) = 0$ and $F_\varphi(r, \varphi, z) = 0$, the calculation of $\nabla \cdot \mathbf{F}$ in cylindrical coordinate system can be simplified as follows

$$\nabla \cdot \mathbf{F} = \frac{\partial F_z(r, \varphi, z)}{\partial z}$$
When \( F_r(r, \varphi, z) = 0 \) and \( F_\varphi(r, \varphi, z) = 0 \) and \( F_z(r, \varphi, z) \) is independent of \( \varphi \), the calculation of \( \nabla \times \mathbf{F} \) in cylindrical coordinate system can be simplified as follows

\[
\nabla \times \mathbf{F} = \frac{1}{r} \begin{vmatrix}
\mathbf{e}_r & r \mathbf{e}_\varphi & \mathbf{e}_z \\
\frac{\partial}{\partial r} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\
0 & 0 & r F_z
\end{vmatrix}
= \frac{1}{r} \frac{\partial F_z}{\partial \varphi} \mathbf{e}_r - \frac{1}{r} \frac{\partial F_r}{\partial \varphi} \mathbf{e}_\varphi - \frac{\partial F_z}{\partial r} \mathbf{e}_r = 0 - \frac{\partial F_z}{\partial r} \mathbf{e}_\varphi
\]

So

\[
\nabla \times \mathbf{F} = -\frac{\partial F_z(r, \varphi, z)}{\partial r} \mathbf{e}_\varphi
\]

When \( F_r(r, \varphi, z) = 0 \) and \( F_z(r, \varphi, z) = 0 \) and \( F_\varphi(r, \varphi, z) \) is independent of \( z \), the calculation of \( \nabla \times \mathbf{F} \) in cylindrical coordinate system can be simplified as follows

\[
\nabla \times \mathbf{F} = \frac{1}{r} \begin{vmatrix}
\mathbf{e}_r & r \mathbf{e}_\varphi & \mathbf{e}_z \\
\frac{\partial}{\partial r} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\
0 & r F_\varphi & 0
\end{vmatrix}
= \frac{1}{r} \frac{\partial}{\partial r} (r F_\varphi) \mathbf{e}_z - \frac{1}{r} \frac{\partial}{\partial \varphi} (r F_\varphi) \mathbf{e}_r
\]

\[
= \frac{1}{r} \frac{\partial}{\partial r} (r F_\varphi) \mathbf{e}_z
\]

So

\[
\nabla \times \mathbf{F} \equiv \frac{1}{r} \frac{\partial}{\partial r} \left[ r F_\varphi(r, \varphi, z) \right] \mathbf{e}_z
\]

Appendix D: Integral calculation related to electrostatic force

**D-1:**

\[
F_e = \frac{\pi R^2 \rho C_0^2}{3} \int_0^\pi \left( 1 + \frac{R}{\sqrt{L^2 + R^2 - 2LR\cos \theta}} \right) \sin \theta \cos \theta \, d\theta = \frac{\pi R^3 \rho C_0^2}{3} \int_0^\pi \frac{\sin \theta \cos \theta d\theta}{\sqrt{L^2 + R^2 - 2LR\cos \theta}} = \frac{\pi R^3 \rho C_0^2}{3} \int_{-1}^{1} \sqrt{L^2 + R^2 - 2LRu} \, du
\]

According to the integral table

\[
\int \frac{xdx}{\sqrt{ax + b}} = \frac{2(ax - 2b)\sqrt{ax + b}}{3a^2} + C
\]

So

\[
\int \frac{udu}{\sqrt{L^2 + R^2 - 2LRu}} = \frac{2(-2LRu - 2L^2 - 2R^2)\sqrt{L^2 + R^2 - 2LRu}}{3 \cdot 4L^2 R^2} + C
\]

\[
= \frac{(L^2 + R^2 + LR)(L - R) - (L^2 + R^2 - LR)(L + R)}{3L^2 R^2} + C
\]

\[
\int_{-1}^{1} \frac{udu}{\sqrt{L^2 + R^2 - 2LRu}} = \frac{(L^3 - R^3) - (L^3 + R^3)}{3L^2 R^2} = \frac{2R^3}{3L^2 R^2} = \frac{2R}{3L^2}
\]

\[
F_e = \frac{\pi R^3 \rho C_0^2}{3} \frac{2R}{3L^2} = \frac{2\pi R^4 \rho C_0^2}{9L^2} = \frac{1}{L^2}
\]
According to D-1
\[ h_e = \frac{\pi R^2 \rho C_D^2}{3} \int_0^{\pi/2} \left( 1 + \frac{R}{\sqrt{L^2 + R^2 - 2LR \cos \theta}} \right) \sin \theta \cos \theta \, d\theta \]
\[ = \frac{\pi R^2 \rho C_D^2}{3} \int_0^{\pi/2} \frac{2}{\sqrt{L^2 + R^2 - 2LR \cos \theta}} \sin \theta \cos \theta \, d\theta \]
\[ = \frac{\pi R^2 \rho C_D^2}{6} + \frac{\pi R^2 \rho C_D^2}{3} \int_0^{\pi/2} \frac{R \sin \theta \cos \theta \, d\theta}{\sqrt{L^2 + R^2 - 2LR \cos \theta}} \]
\[ = \frac{\pi R^2 \rho C_D^2}{6} + \frac{\pi R^2 \rho C_D^2}{3} \int_0^{\pi/2} \frac{\sin \theta \cos \theta \, d\theta}{\sqrt{L^2 + R^2 - 2LR \cos \theta}} \]

According to D-1
\[ \int \frac{udu}{\sqrt{L^2 + R^2 - 2LRu}} = -\frac{(L^2 + R^2 + LRu)\sqrt{L^2 + R^2 - 2LRu} + C}{3L^2 R^2} \]
\[ \int_0^1 \frac{udu}{\sqrt{L^2 + R^2 - 2LRu}} = -\frac{(L^2 + R^2 + LR)(L - R) - (L^2 + R^2)^{3/2}}{3L^2 R^2} \]
\[ = -\frac{(L^3 - R^3) - L^3(1 + R^2/L^2)^{3/2}}{3L^2 R^2} \]

When \( L \gg R \), according to Maclaurin's formula
\[ \left( 1 + \frac{R^2}{L^2} \right)^{3/2} \approx 1 + \frac{3R^2}{2L^2} \]
\[ \int_0^1 \frac{udu}{\sqrt{L^2 + R^2 - 2LRu}} \approx -\frac{(L^3 - R^3) - (L^3 + 3L^2R^2/2)}{3L^2 R^2} = \frac{R^3 + 3L^2R^2/2}{3L^2 R^2} \approx \frac{RL^2}{3L^2 R^2} = \frac{1}{2L} \]
\[ h_F_e \approx \frac{\pi R^2 \rho C_D^2}{6} + \frac{\pi R^2 \rho C_D^2}{3} \cdot \frac{1}{2L} = \frac{\pi R^2 \rho C_D^2}{6} \cdot \left( 1 + \frac{R}{L} \right) \approx \frac{\pi R^2 \rho C_D^2}{6} \]

D-3:
\[ F_e = \frac{\pi R^2 \rho C_D^2}{3} \int_0^{\pi} \left( 1 + \frac{R}{\sqrt{L^2 + R^2 - 2LR \cos \theta}} \right) \sin \theta \cos \theta \, d\theta \]
\[ = \frac{\pi R^2 \rho C_D^2}{3} \left( \int_0^{\pi} \cos \theta \cdot d\cos \theta + \int_0^{\pi} -\cos \theta \cdot d\cos \theta \right) \]
\[ = \frac{\pi R^2 \rho C_D^2}{3} \left( \int_{\cos \alpha}^{-1} -u \cdot du + \int_{\cos \alpha}^{\pi/2} \frac{-Ru \cdot du}{\sqrt{L^2 + R^2 - 2LRu}} \right) \]

Suppose \( x = \cos \alpha = L/2R \), then
\[ \int_{\cos \alpha}^{-1} -u \cdot du = \frac{x^2}{2} - \frac{1}{2} \]

According to D-1
\[ \int \frac{udu}{\sqrt{L^2 + R^2 - 2LRu}} = -\frac{(L^2 + R^2 + LRu)\sqrt{L^2 + R^2 - 2LRu} + C}{3L^2 R^2} \]
\[ \int_{\cos \alpha}^{-1} \frac{udu}{\sqrt{L^2 + R^2 - 2LRu}} = -\frac{L^3 + R^3 - (L^2 + R^2 + LR \cos \alpha)\sqrt{L^2 + R^2 - 2LR \cos \alpha}}{3L^2 R^2} \]
\[ = -\frac{L^3 + R^3 - (L^2 + R^2 + LR \cdot L/2R)\sqrt{L^2 + R^2 - 2LR \cdot L/2R}}{3L^2 R^2} \]
\[ = -\frac{L^3 + R^3 - (L^2 + R^2 + L^2/2)R}{3L^2 R^2} = -\frac{L^3 - 3L^2 R/2}{3L^2 R^2} = \frac{1}{2R} - \frac{L}{3R} = \frac{1}{2R} - \frac{2x}{3R} \]
Appendix E: Calculation of the point where the axial magnetic field drops to zero

In non-uniform magnetic field in an energized solenoid, radially moving electrons will be subject to Lorentz force in axial magnetic field, and axially moving electrons will also be subject to Lorentz force in radial magnetic field, therefore, it is difficult to calculate the point where the axial magnetic field drops to zero by accurately calculating electron motion trajectory in three-dimensional space. The trajectories of electrons with different radial velocities passing through the energized solenoid form a three-dimensional smooth and curved surface. Since convexity of the trajectory of a moving electron may change with the Lorentz force, the point where the axial magnetic field drops to zero can be obtained by mathematical calculation with the curved surface.

The trajectory of electron motion is a three-dimensional space curve, and its tangent is the direction of electron motion. The direction of electron motion cannot be represented only by tangent of projection of the space curve on XZ plane or YZ plane. The second-order derivative of the inflection point of a plane curve is zero, and correspondingly, on the tangent plane at the inflection point of a space curve, the space curve is projected, then the second-order derivative of the projection at the inflection point is zero.

\[
F_e = \frac{\pi R^2 \rho C_0^2}{3} \left( \frac{x^2}{2} - \frac{1}{2} + \frac{2x}{3} \right) = \frac{\pi R^2 \rho C_0^2}{18} (3x^2 + 4x - 6)
\]

\[
\int_{\cos \alpha}^{-1} \frac{-Ru \, du}{\sqrt{L^2 + R^2 - 2LRu}} = -\frac{1}{2} + \frac{2x}{3}
\]

As shown in Fig-E1, electrons with initial velocity in Y-axis direction pass from left to right through two identical and reverse-connected energized coils, and the radius \( r \) of each coil is approximately equal to its length \( L \). The axial magnetic field of left and right coils constitutes an odd symmetric axial magnetic field \( B_x \), and the radial magnetic field \( B_r \).
constitutes an even symmetric radial magnetic field $B_r$. According to electromagnetics, it can be calculated that for the same abscissa $x_i$, amplitude of $B_x$ increases with the increase of wheelbase $d$. The intersection point between the contact surface of two coils and X-axis is point C, the abscissa of point C is $x_c$. $x_c$ is the zero-crossing point of axial magnetic field $B_x$, and radial magnetic field $B_r$ remains constant at both sides of $x_c$. Electrons are subjected to Lorentz force $F_1$ in the axial magnetic field due to radial motion, and Lorentz force $F_2$ in the radial magnetic field due to axial motion. According to the left-hand rule of electromagnetics, it can be judged that near the left side of abscissa $x_c$, the direction of force $F_1$ is opposite to that of force $F_2$.

Electrons have periodic alternating velocity component in Y-axis direction move to the right at high speed to impact the phosphor screen, and the resulting afterglow forms a linear image. Since the magnetic induction intensity $B_x$ of axial magnetic field in two coils increases with wheelbase $d$, the image is not a straight line but an upturned curve.

Move the phosphor screen from left to right and take photos of the screen. The inclination of the curve image on the photos gradually increase and decrease after reaching the maximum value. The left figure of Fig-E2 shows three photo curves, each of which is an oddly symmetric curve $z(x_i, y)$. The right figure of Fig-E2 is slope curve $k(x_i, y) = \frac{\partial z}{\partial y}$ of the tangent corresponding to the photo curve in the left figure. Each slope curve is an even function, and the Y-axis coordinate of the minimum value point $D_i$ is zero.

These photo curves can form a three-dimensional smooth and curved surface $z(x, y)$ of electron motion trajectories, and slope curves of the tangents of these photo curves can form a three-dimensional smooth and curved surface $k(x, y) = \frac{\partial z}{\partial y}$, as shown in Fig-E3.
The coordinate of the minimum value point $D_i$ of each slope curve is $(x_i, 0, k_i)$. These minimum value points constitute the red plane smooth curve shown in Fig-E3, and the abscissa of the maximum value point of the red curve is $x_0$.

In order to facilitate calculation, the range of $(x, y)$ is limited to the first quadrant of XY plane. The excitation current of coils is controlled so that the inclination angle of all tangents of each photo curve does not exceed $\pi/2$. In this case, Lorentz force $F_1$ exerted on radially moving electrons in axial magnetic field has an upward or downward component. In addition, the geometry of coils are selected (for example, radius of the coil is approximately equal to its length) so that the slope curve of tangent of the photo curve is concave downward and slope of the slope curve increases with the increase of $|y|$, as shown in the right figure of Fig-E2. It will be proved that $x_c \geq x_0$.

As shown in Fig-E4, it is assumed that there is an electron trajectory passing through three points R, S and Q, and coordinates of these three points in XY plane are $(x_R, y_R)$, $(x_S, y_S)$ and $(x_0, y_Q)$ respectively. The abscissa of point Q is $x_0$, and its projection on XY plane is point Q2. The three points R, S and Q on curved surface $z(x, y)$ of Fig-E4 are also the three points R, S and Q on the left figure of Fig-E2. $\partial z/\partial y$ of these three points correspond to the three points R1, S1 and Q1 in the right figure of Fig-E2, and also is the three points R1, S1 and Q1 in Fig-E3. It can be seen from Fig-E3 that $x_R < x_S < x_0$, $y_R < y_S < y_Q$, $k(x, y_i) = \partial z/\partial y$ reaches the maximum value at $x = x_0$, and

$$\frac{\partial z(x_R, y_R)}{\partial y} < \frac{\partial z(x_S, y_S)}{\partial y} < \frac{\partial z(x_0, y_Q)}{\partial y}$$

Assuming that the zero-crossing point of axial magnetic field $B_x$ is $x_c = x_S$, the upward component of $F_1$ at point R near the left side of $x_S$ is greater than zero, therefore, the direction of electron motion will deflect upward and $\partial z/\partial y$ will continue to increase during movement of the electron from point R to point S. When the electron moves to point S, the upward component of $F_1$ is equal to zero, and $\partial z/\partial y$ reaches the maximum value. When the electron continues to move to point Q near the right side of $x_S$, the downward component of $F_1$ is greater than zero, therefore, the direction of electron motion will deflect downward and $\partial z/\partial y$ should decrease. So, if the zero-crossing point of axial magnetic field $B_x$ is $x_c = x_S$, the change of $\partial z/\partial y$ should be
\[
\frac{\partial z(x, y)}{\partial y} < \frac{\partial z(x, y)}{\partial y} > \frac{\partial z(x, y)}{\partial y}
\]

This contradicts the previous analysis, so the zero-crossing point of axial magnetic field \( B_x \) must be \( x_c \geq x_0 \). In addition, near both sides of \( x = x_0 \), the downward component of \( F_2 \) is greater than zero, which will cause the direction of electron motion to be deflected downward and \( \partial z / \partial y \) continues to decrease, therefore, the reason why \( \partial z / \partial y \) increases during the movement of electron from point S to point Q is \( F_1 \) rather than \( F_2 \).

It can be seen from Fig-E3 that the intersection line \( k(x, y) \) of curved surface \( k(x, y) \) and plane \( y = y_1 \) reaches the maximum value at \( x = x_0 \), that is, when \( x = x_0 \)

\[
\frac{\partial}{\partial x} \left( \frac{\partial z}{\partial y} \right) = 0
\]

\( z(x, y_1) = \int_0^{y_1} k(x, y) \, dy \) is the intersection line of curved surface \( z(x, y) \) and plane \( y = y_1 \). It can be seen from Fig-E2 and Fig-E3, \( z(x, y) \) reaches the maximum value at \( x = x_0 \), that is, \( \partial z / \partial x = 0 \) at \( x = x_0 \). So, on the curved surface \( z(x, y) \), when \( x = x_0 \)

\[
\begin{align*}
\frac{\partial}{\partial x} \left( \frac{\partial z}{\partial y} \right) &= 0 \\
\frac{\partial z}{\partial x} &= 0
\end{align*}
\]

(E)

The following will prove that an electron trajectory can always be found through point Q, making point Q as its inflection point. Assuming that the tangent line of this trajectory at point Q is line K, line K and straight line Q2Q constitute the orange tangent plane in Fig-E4. Assume that the intersection line of this tangent plane and XY plane is V-axis, and the angle between X-axis and V-axis is \( \alpha \). Take Q2 as the coordinate origin, create Q2Q as U-axis, then the projection of the electron trajectory on UV plane is function \( u = u(v) \). It can be seen from Fig-E4 that points on the tangent plane satisfy \( x = x_0 + v \cos \alpha \), \( y = y_0 + v \sin \alpha \) and \( z = u \), so

\[
\frac{du}{dv} = \frac{dz}{dv} = \frac{dx}{dv} \cdot \cos \alpha + \frac{dy}{dv} \cdot \sin \alpha = \frac{\partial z}{\partial x} \cdot \cos \alpha + \sin \alpha \cdot \frac{\partial z}{\partial y}
\]

\[
\frac{d^2z}{dv^2} = \cos^2 \alpha \frac{\partial^2z}{\partial x^2} + \sin^2 \alpha \cdot \frac{\partial}{\partial x} \left( \frac{\partial z}{\partial y} \right) \cdot \cos \alpha + \cos \alpha \cdot \frac{\partial}{\partial y} \left( \frac{\partial z}{\partial x} \right) \cdot \sin \alpha + \sin^2 \alpha \cdot \frac{\partial^2z}{\partial y^2}
\]

According to equation (E), the second-order mixed partial derivative at point Q is zero, so

\[
\frac{d^2z}{dv^2} = \cos^2 \alpha \frac{\partial^2z}{\partial x^2} + \sin^2 \alpha \cdot \frac{\partial^2z}{\partial y^2}
\]

According to equation (E), \( \partial z / \partial x = 0 \) at \( x = x_0 \), and \( z(x, y) \) reaches the maximum value at \( x = x_0 \), so \( \frac{\partial^2z}{\partial x^2} < 0 \) at point Q. It can be seen from the right figure of Fig-E2 that \( \frac{\partial^2z}{\partial y^2} > 0 \) at point Q corresponding to point Q1. Let \( d^2z / dv^2 = 0 \), then

\[
\alpha = \tan^{-1} \sqrt{\frac{\frac{\partial^2z}{\partial x^2}}{\frac{\partial^2z}{\partial y^2}}}
\]
Therefore, for point Q, if take above equation as the angle between the tangent plane and X-axis, then the tangent of the projection of the electron motion trajectory on the tangent plane at point Q is the direction of the electron motion, and point Q is the inflection point of the electron motion trajectory.

At any point Q on the intersection line of curved surface \( z(x,y) \) and plane \( x = x_0 \), an electron motion trajectory passing through point Q can always be found, making point Q as the inflection point of the electron motion trajectory, therefore, the intersection line of curved surface \( z(x,y) \) and plane \( x = x_0 \) is called as "inflection point line" of electron motion trajectories. The intersection point of inflection point line and X-axis is point P, and its abscissa is \( x = x_0 \).

The abscissa of the zero-crossing point of axial magnetic field \( B_x \) is \( x_c \geq x_0 \), and any point on the inflection point line can be the inflection point of a certain electron motion trajectory. Therefore, on X-axis, the point C where axial magnetic field drops to zero should overlap with point P, or point C is located on the right side of point P. This is result according to electromagnetic theory.

**Appendix F: Calculation of partial derivatives related to induced electric field and displacement current**

**F-1**: The known condition is

\[
\frac{\partial v_u(r,t)}{\partial r} = -\frac{2R\lambda a}{C_u} \frac{\sqrt{C_0^2 t^2 - r^2}}{r}
\]

For variables \( r, \varphi \) and \( z \) in cylindrical coordinates, the left side of above equation is partial derivative. Since \( r \) and \( t \) are independent variables, for variable \( t \), the left side of above equation is derivative \( dv_u(r) / dr \). According to the integral table

\[
\int \frac{\sqrt{a^2 - x^2}}{x} \, dx = \sqrt{a^2 - x^2} + a \ln \frac{a - \sqrt{a^2 - x^2}}{|x|} + \text{constant}
\]

So

\[
v_u(r,t) = -\frac{2R\lambda a}{C_u} \left( \sqrt{C_0^2 t^2 - r^2} + C_u t \ln \frac{C_u t - \sqrt{C_0^2 t^2 - r^2}}{r} \right) + \text{constant}
\]

The known condition is \( v_u(r,t) = 0 \) when \( r = C_u t \), so

\[
v_u(r,t) = -\frac{2R\lambda a}{C_u} (0 + 0) + \text{constant} = 0
\]

So, \( \text{constant} = 0 \), and then

\[
v_u(r,t) = \frac{2R\lambda a}{C_u} \left( \frac{C_0^2 t}{\sqrt{C_0^2 t^2 - r^2}} + C_u t \ln \frac{C_u t - \sqrt{C_0^2 t^2 - r^2}}{r} \right)
\]

\[
\frac{\partial v_u(r,t)}{\partial t} = \frac{2R\lambda a}{C_u} \left( \frac{C_0^2 t}{\sqrt{C_0^2 t^2 - r^2}} + C_u t \ln \frac{C_u t - \sqrt{C_0^2 t^2 - r^2}}{r} \right)
\]

The last item in brackets is simplified as
So

\[ \frac{\partial v_u(r,t)}{\partial t} = \frac{-2R\lambda a}{C_u} \cdot C_u \ln \frac{C_u t - \sqrt{C_u^2 t^2 - r^2}}{r} = \frac{-2R\lambda a}{C_u} \cdot C_u \frac{t - \sqrt{C_u^2 t^2 - r^2}}{r} \]

**F-2:**

Let

\[ \text{Partial}E = \ln \frac{C_u t - \sqrt{C_u^2 t^2 - r^2}}{r} \]

Then

\[ \frac{\partial (\text{Partial}E)}{\partial r} = \frac{r \ast r}{C_u t - \sqrt{C_u^2 t^2 - r^2}} \ast \frac{r \ast r}{C_u t - \sqrt{C_u^2 t^2 - r^2}} \]

\[ = \frac{1}{r \left( C_u t - \sqrt{C_u^2 t^2 - r^2} \right)} \cdot \frac{r^2 - C_u t \sqrt{C_u^2 t^2 - r^2} + C_u^2 t^2 - r^2}{r \sqrt{C_u^2 t^2 - r^2}} \]

\[ = \frac{1}{r \left( C_u t - \sqrt{C_u^2 t^2 - r^2} \right)} \cdot \frac{C_u t \left( C_u t - \sqrt{C_u^2 t^2 - r^2} \right)}{r \sqrt{C_u^2 t^2 - r^2}} = \frac{C_u t}{r \sqrt{C_u^2 t^2 - r^2}} \]

**F-3:**

\[ \frac{\partial (\text{Partial}E)}{\partial t} = \frac{C_u \sqrt{C_u^2 t^2 - r^2} - C_u t}{C_u t - \sqrt{C_u^2 t^2 - r^2}} = \frac{1 - \frac{C_u t}{C_u t - \sqrt{C_u^2 t^2 - r^2}}}{C_u t - \sqrt{C_u^2 t^2 - r^2}} = \frac{\sqrt{C_u^2 t^2 - r^2} - C_u t}{C_u t - \sqrt{C_u^2 t^2 - r^2}} \]

\[ = \frac{-C_u}{\sqrt{C_u^2 t^2 - r^2}} \]

So

\[ \frac{\partial^2 v_u(r,t)}{\partial t^2} = -2R\lambda a \cdot \frac{\partial (\text{Partial}E)}{\partial t} = \frac{2R\lambda a C_u}{\sqrt{C_u^2 t^2 - r^2}} \]

**Appendix G: Calculation of light bending in spherically symmetric medium**

According to Huygens principle, every point on the wavefront is a source of vibration. We can consider light as the propagation of vibration in a medium. As shown in Fig-G1, the
speeds of light in two mediums are \( u_1 \) and \( u_2 \) respectively. The wavefront AC of incident light is perpendicular to the incident light. After a period of \( \Delta t \), the vibration of point A reaches point D in lower medium, where \( AD = u_2 \cdot \Delta t \). At the same time, the vibration of point C reaches point B, where \( CB = u_1 \cdot \Delta t \). BD is the wavefront of refractive light, which is perpendicular to the refractive light. Assuming that the incident angle of light is \( \alpha \) and the refracting angle is \( \beta \), then \( CB = AB \cdot \sin \alpha \) and \( AD = AB \cdot \sin \beta \), so

\[
\frac{\sin \alpha}{u_1} = \frac{\sin \beta}{u_2} \quad \text{(G-1)}
\]

This is also the refraction law of waves.

![Fig-G2 Spherically symmetric medium](image_url)

In a spherically symmetric medium, the speed of light is only related to magnitude of radius. As shown in Fig-G2, take two layers of medium with equal thickness in the radial direction, \( HB = BC = \Delta r \). When \( \Delta r \) is very small, the refractive index of each layer of medium remains constant, so light travels along a straight line. In Fig-G2, the speeds of light in two layers of medium are \( u_1 \) and \( u_2 \) respectively, the direction of the light is GBDK, and the light is refracted twice. The first refractive point is B, and the normal line is radius vector OB. The angle between the radius vector and incident light is the incident angle \( \alpha \), and the refractive angle is \( \beta \). HG and CE are both perpendicular to the light. The second refractive point is D, and the normal line is radius vector OD. The angle between the radius vector and incident light is the incident angle \( \gamma \), and AF is perpendicular to the light. According to equation (G-1)

\[
\frac{\sin \alpha}{u_1} = \frac{\sin \beta}{u_2}
\]

\[
\frac{\Delta r}{\sin \alpha} = \frac{\Delta r}{\sin \beta} = \Delta r
\]

Multiply two equations to get \( HG/u_1 = CE/u_2 \). When \( \Delta r \) very small, arc AB and arc CD are approximately straight lines, and AB is parallel to CD, so the angle between AB and BD is equal to the angle between CD and BD, so \( AF/AB = CE/CD \). Since \( AB = OB \cdot \Delta \theta \) and \( CD = OD \cdot \Delta \theta \), so \( CE = AF \cdot OD/OB \), substitute it into \( HG/u_1 = CE/u_2 \) to get

\[
\frac{HG \cdot OB}{u_1} = \frac{AF \cdot OD}{u_2}
\]

\[
\frac{\Delta r \cdot \sin \alpha \cdot OB}{u_1} = \frac{\Delta r \cdot \sin \gamma \cdot OD}{u_2}
\]

\[
\frac{OB \cdot \sin \alpha}{u_1} = \frac{OD \cdot \sin \gamma}{u_2}
\]
Assuming \( \varphi \) is the angle between radius vector \( r \) and light, \( u \) is the speed of light in medium, when \( \Delta r \) infinity approaches zero

\[
\frac{r \cdot \sin \varphi}{u} \equiv \text{constant} = c_0 \quad (G-2)
\]

Some optical textbooks can also get the same results by using Fermat's principle and variational knowledge.

![Fig-G3 The geometric relationship of differentiation of light](image)

As shown in Fig-G3, the angle between tangent of light at point A and radius vector OA is \( \varphi \), and \( OA = r \). After the radius vector OA rotates a very small angle \( \Delta \theta \), it becomes the radius vector OC, point C is on the light and \( OC = r + \Delta r \). When \( \Delta \theta \) is very small, triangle ABC is approximately a right triangle, and the angle between OC and AC is approximately equal to \( \varphi \), so \( \tan \varphi \approx \frac{AB}{BC} \approx r \cdot \frac{\Delta \theta}{\Delta r} \), so

\[
\sin \varphi \approx \frac{r \Delta \theta}{\sqrt{(r \Delta \theta)^2 + (\Delta r)^2}} = \frac{r}{\sqrt{r^2 + (\frac{\Delta r}{\Delta \theta})^2}}
\]

\[
\sin \varphi = \frac{r}{\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}} \quad (G-3)
\]

Substitute equation (G-3) into equation (G-2) to get

\[
\frac{r^2}{u \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}} = c_0
\]

So

\[
\frac{dr}{d\theta} = \sqrt{\frac{r^4}{c_0^2 u^2} - r^2} = \frac{r}{c_0 u} \sqrt{r^2 - c_0^2 u^2}
\]

\[
\frac{d\theta}{dr} = \frac{c_0 u}{r \sqrt{r^2 - c_0^2 u^2}} \quad (G-4)
\]

\[
\sin \varphi = \frac{1}{\sqrt{1 - \frac{p}{x^2}}} \sqrt{\frac{1}{x^2} - \frac{p}{x^3}}
\]

Take derivative of both sides of the equation with respect to \( x \)
\[ \cos \varphi \cdot \frac{d \varphi}{dx} = \frac{-2 + 3 \left( \frac{x}{x^3} \right)}{2 \sqrt{1 - P} \left( \frac{1}{x^2} - \frac{P}{x^3} \right)} = \frac{-2x + 3P}{x^4} \cdot \frac{1}{2 \sqrt{1 - P} \left( x - \frac{P}{x} \right)} = \frac{(-2x + 3P)\sqrt{x^3}}{2x^4 \sqrt{1 - P} \sqrt{x - \frac{P}{x}}} \]

\[ \cos \varphi = \sqrt{1 - \sin^2 \varphi} = \sqrt{1 - \left( 1 - \frac{1}{1 - P} \left( \frac{1}{x^2} - \frac{P}{x^3} \right) \right)} = \sqrt{1 - \frac{x - P}{x^3 (1 - P)}} = \frac{\sqrt{x^3 (1 - P)} - x + P}{\sqrt{x^3 (1 - P)}} \]

\[ d \varphi = \frac{(-2x + 3P)\sqrt{x^3}}{2x^4 \sqrt{1 - P} \sqrt{x - \frac{P}{x}}} \cdot \cos \varphi \cdot dx = \frac{(-2x + 3P)\sqrt{x^3}}{2x^4 \sqrt{1 - P} \sqrt{x - \frac{P}{x}}} \cdot \frac{\sqrt{x^3 (1 - P)}}{\sqrt{x^3 (1 - P)} - x + P} \cdot dx \]

\[ = \frac{-2x + 3P}{2x \sqrt{x - \frac{P}{x}} \sqrt{x^3 (1 - P)} - x + P} \cdot dx \]

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