Cold Atoms For Testing Quantum Mechanics and Parity Violation In Gravitation

N.D. Hari Dass
Institute of Mathematical Sciences, Taramani, Madras-600113, INDIA.

I. INTRODUCTION

Techniques of Atom trapping and laser cooling have proved to be very important tools in probing many aspects of fundamental physics. In this talk I wish to present ideas on how they may be used to settle certain issues in the foundational aspects of quantum mechanics on the one hand(see II) and about some quantum gravitational interactions of matter that violate parity and time-reversal, on the other hand(see III).

II. TESTING QUANTUM MECHANICS OF "PROTECTIVE MEASUREMENTS"

A "protective" measurement is one that does not disturb the state of a quantum system, yet it gives full information about it [1]. This appears to go against the postulate of von Neumann. Further, if the state can yield any of the eigenvalues and Qureshi [2]), it is instructive to first consider a model of quantum mechanics as first of all, the outcome can be any of the eigenvalues and secondly, the state changes in an uncontrollable manner to the corresponding eigenstate. Repeated measurements subsequently will only keep yielding the same outcome as the first measurement and the average of the measurements bears no relation to the expectation value of A in the state before measurements. As stated in the jargon of quantum mechanics, a single state has no ontological meaning.

A. A Proposal by Aharonov, Anandan and Vaidman

In this context an extraordinary proposal was made by Aharonov, Anandan and Vaidman [3] who claimed that when is a non-degenerate eigenstate of some a priori unknown Hamiltonian, can be measured for a single quantum state without disturbing it, and that this can be done for arbitrary observables A. Since the original state is left undisturbed, one is free to perform a sequence of "protective" measurements with sufficiently many different observables and determine the single state! In other words, AAV claim to give an ontological meaning to the state of a single quantum system. Equivalently, they claim that the wavefunction is real and measurable even for single states and therefore, the ensemble interpretation is unwarranted for the circumstances mentioned before.

B. Conventional Measurements

To understand the results of AAV(for a detailed and critical analysis of protective measurements see Hari Dass and Qureshi [3]), it is instructive to first consider a model for conventional measurements which as mentioned earlier lead to a collapse of the wave function. Let us consider a quantum system described by a Hamiltonian and couple the system to an apparatus described by . Let be an operator, corresponding to the observable of the system we wish to measure, and let the interaction between the system and the apparatus (in what follows, we shall use the notion of an apparatus to indicate a quantum system to which full information about the system can be transferred) be of the form

where is an observable of the apparatus, and is the strength of the interaction normalized such that

Based on this credo, measurements on a single quantum state as opposed to measurements on an ensemble of states, has no significance in conventional interpretations of quantum mechanics as first of all, the outcome can be any of the eigenvalues and secondly, the state changes in an uncontrollable manner to the corresponding eigenstate. Repeated measurements subsequently will only keep yielding the same outcome as the first measurement and the average of the measurements bears no relation to the expectation value of A in the state before measurements. As stated in the jargon of quantum mechanics, a single state has no ontological meaning.
\[ \int dt g(t) = 1. \] The interaction is now taken to be nonzero only in a very short interval \([0, \tau]\). Let the system be in an initial state \(|\nu\rangle\) which is not necessarily an eigenstate of \(Q_S\), and let the apparatus be in a state \(|\phi(r_0)\rangle\), which is a wave packet of eigenstates of the operator \(R_A\) conjugate to \(Q_A\), centered at the eigenvalue \(r_0\). The interaction \(H_I\) is of such short duration, and assumed to be so strong that the effect of the free Hamiltonians of the apparatus and the system can be neglected. Then the combined wave function of the system and the apparatus at the end of the interaction can be written as

\[ |\psi(\tau)\rangle = e^{-\frac{i}{\hbar}Q_AQ_S}|\nu\rangle|\phi(r_0)\rangle. \quad (4) \]

If we expand \(|\nu\rangle\) in the eigenstates of \(Q_S\), \(|s_i\rangle\), we obtain

\[ |\psi(\tau)\rangle = \sum_i e^{-\frac{i}{\hbar}Q_A s_i} c_i |s_i\rangle |\phi(r_0)\rangle, \quad (5) \]

where \(s_i\) are the eigenvalues of \(Q_S\) and \(c_i\) are the expansion coefficients. The exponential term shifts the center of the wave packet by \(s_i\):

\[ |\psi(\tau)\rangle = \sum_i c_i |s_i\rangle |\phi(r_0 + s_i)\rangle. \quad (6) \]

This is an entangled state, where the position of the wave packet gets correlated with the eigenstates \(|s_i\rangle\). Detecting the center of the wave packet at \(r_0 + s_i\) will throw the system into the eigenstate \(|s_i\rangle\).

### C. Protective Measurements

Let us now consider the opposite limit where the interaction of the system with the apparatus is weak and adiabatic. Here the system is assumed to be in a non-degenerate eigenstate of its Hamiltonian, which is not known to the experimenter, and the interaction being weak and adiabatic, we cannot neglect the free Hamiltonians. Let the Hamiltonian of the combined system be

\[ H(t) = H_A + H_S + g(t)Q_AQ_S, \quad (7) \]

The coupling \(g(t)\) acts for a long time \(T\) and goes to zero smoothly before and after the interaction. It is also normalized as \(\int_0^T dt g(t) = 1\). Therefore, \(g(t) \approx 1/T\) is small and constant for the most part. If \(|t = 0\rangle\) is the state vector of the combined apparatus-system just before the measurement process begins, the state vector after \(T\) is given by

\[ |t = T\rangle = T e^{-\frac{i}{\hbar} \int_0^T H(\tau) d\tau} |t = 0\rangle, \quad (8) \]

where \(T\) is the time ordering operator. We divide the interval \([0, T]\) into \(N\) equal intervals \(\Delta T\), so that \(\Delta T = T/N\), and because the full Hamiltonian commutes with itself at different times during \([0, T]\), we can write equ(8) as

\[ |t = T\rangle = \left( \exp\left[ -\frac{i}{\hbar} \Delta T \left( H_A + H_S + \frac{1}{T} Q_AQ_S \right) \right] \right)^N |t = 0\rangle. \quad (9) \]

We illustrate the special case when \(Q_A\) commutes with the free Hamiltonian of the apparatus, i.e., \([Q_A, H_A] = 0\), so that we can have eigenstates \(|a_i\rangle\) such that \(Q_A |a_i\rangle = a_i |a_i\rangle\) and \(H_A |a_i\rangle = E_i |a_i\rangle\). Now \(|a_i\rangle\) are also the exact eigenstates of the instantaneous Hamiltonian \(H(t)\), in the apparatus subspace. So, the exact instantaneous eigenstates can be written in a factorized form \(|a_i\rangle |\mu\rangle\) where \(|\mu\rangle\) are system states which depend on the eigenvalue of \(Q_A\), i.e., they are the eigenstates of \(\frac{1}{\hbar} \hbar a_iQ_S + H_S\). Let us assume the initial state to be a direct product of a non-degenerate eigenstate of \(H_S\), \(|\nu\rangle\), and \(|\phi(r_0)\rangle\):

\[ |t = 0\rangle = |\nu\rangle |\phi(r_0)\rangle. \quad (10) \]

Introducing complete set of exact eigenstates in the above equation, the wave function at a time \(T\) can now be written as

\[ |t = T\rangle = \sum_{i, \mu} e^{\frac{i}{\hbar} E(a_i, \mu)N \Delta T} |a_i\rangle |\mu\rangle |\nu\rangle |\phi(r_0)\rangle, \quad (11) \]

where the exact instantaneous eigenvalues \(E(a_i, \mu)\) can be written as

\[ E(a_i, \mu) = E_i + \frac{1}{T} \langle Q_S | \mu \rangle a_i + \langle \mu | H_S | \mu \rangle. \quad (12) \]

It should be kept in mind that the expectation value \(\langle Q_S \rangle_T\) depends on the eigenvalue \(a_i\) of \(Q_A\). The sum over \(\mu\) in \((11)\) makes it appear as if the state is entangled. But the important point to notice is that the basis \(|\mu\rangle\) can be made to be arbitrarily close to the original basis, as the interaction is assumed to be weak, so that \(|\mu\rangle = |\mu\rangle + O(1/T) + \ldots\). In the large \(T\) limit, one can assume the states to be unperturbed, and retain only terms of \(O(1/T)\) in the energy (this is necessary as \(E(a_i, \mu)\) is multiplied by \(T\) in eqn(11)), which amounts to using first order perturbation theory. This yields eigenvalues of the form

\[ E(a_i, \mu) = E_i + \frac{1}{T} \langle Q_S | \mu \rangle a_i + \langle \mu | H_S | \mu \rangle + O(1/T^2). \]

\[ (13) \]

In addition to this, the sum over \(\mu\) disappears and only the term where \(\mu = \nu\) survives. Thus, we can write the apparatus part of the exponent again in the operator form

\[ |t = T\rangle \approx e^{-\frac{i}{\hbar} \int_0^T Q_A \langle Q_S \rangle_T - \langle H_S \rangle_T \langle \nu | \phi(r_0)\rangle}. \quad (14) \]

Now, it is easy to see that the second term in the exponent will shift the center of the wave packet \(|\phi(r_0)\rangle\) by an amount \(\langle \nu | Q_S | \nu \rangle\).
This shows that at the end of the interaction, the center of the wave packet $|\phi(r_0)\rangle$ shifts by $\langle \nu | Q_{S} | \nu \rangle$.

The idea behind this approximation is that in $|\mu |\rangle$ only one term is large and close to unity, and rest of the terms are very small, of the order $1/T$. Making $T$ very large, one can make the smaller terms arbitrarily close to zero. Thus, the state is effectively not entangled, and so the original wave function is not destroyed during the measurement. Looking at the position of the wave packet, one can determine the expectation value $\langle Q_{S} \rangle_{\nu}$. This, basically, is the essence of protective measurements.

D. Protective Measurements Do Not Work For Single Systems

As we saw in the last section, complete disentanglement of the apparatus and system wavefunctions only happens in the $T \to \infty$ limit, which though mathematically well defined, can never be realised physically. Of course the degree of entanglement can be made arbitrarily small in any physical realisation, but never zero. In the case of ensemble measurements, a tiny admixture in wavefunctions is of no consequence as it will only change the probability distribution by a very small amount.

But for a single system, even an extremely tiny entanglement with the apparatus can have a disastrous effect! This is because for single systems the outcome of the first measurement can be anything and not necessarily the outcome with the largest probability as calculated from the probability interpretation of quantum mechanics. In particular, the first outcome could throw the system into a state quite far from the initial state. Then one can not rely on successive protective measurements to yield any information on the original state. Stated differently, "protective" measurements on single states can never be done with 100% confidence. Hence the wavefunction can not be deemed to be real.

E. A Pragmatic Interpretation

Though the idea of a protective measurement does not work for single systems and hence fails to give any reality to the wavefunction, from a practical point of view, the proposal has some very interesting possibilities. When performed on an ensemble, the effect of the tiny entanglement becomes negligible and this means that the system ensemble which was pure to begin with, remains pure to a high degree even after the measurement! This is in contrast to the conventional measurement where a pure state density matrix $\rho_{\text{fin}} = |n\rangle\langle n| = \sum_{ij} c_i^* c_j |a_i\rangle\langle a_j|$ turns into the mixed state density matrix $\rho_{\text{fin}} = \sum_{i} |c_i|^2 |a_i\rangle\langle a_i|$ after measurement. This could have a lot of practical applications. Also, since the ideal protective measurement allows one to straightaway measure expectation values of arbitrary observables, in practice much smaller ensembles could yield accuracies comparable to what one obtains in conventional measurements. It is worth the while to experimentally test some of these consequences. In the next section we show, through a simple example, how cold atoms could be used for this purpose.

F. Cold Atoms for Testing Protective Measurements: An Example

Consider the set-up for a typical Stern-Gerlach experiment represented by the Hamiltonian

$$H = \frac{\vec{p}^2}{2M} - \mu B_0 \vec{\sigma} \cdot \vec{n} - \mu g(t) \vec{B}_i x \vec{\sigma} \cdot \vec{n}. \quad (16)$$

Here $M$ is the mass of, say, the silver atom. As before, $g(t)$ is taken to be $\frac{1}{2}$. It should be noted that $B_0 \vec{n}$ is an a priori unknown magnetic field. Consequently we shall not vary $B_0 \vec{n}$ during the experiment. The initial state is chosen to be

$$|t = 0\rangle = e^{i\theta 0y} |\uparrow\rangle; \quad \vec{\sigma} \cdot \vec{n} |\pm\rangle = \pm |\pm\rangle. \quad (17)$$

It should be emphasized that this initial state is a priori unknown. We first treat this case by ignoring the kinetic term for the silver atoms which is tantamount to assuming them to be heavy. Then, the Hamiltonian of eqn(16) is the Hamiltonian of the spin-1/2 particle in the effective magnetic field

$$\vec{B} = B_0 \vec{n} + \vec{B}_i \frac{x}{T} \vec{n}, \quad (18)$$

whose eigenstates are given by

$$H|\pm\rangle = \pm \mu B|\pm\rangle. \quad (19)$$

Consequently, the state at $t = T$ is given by

$$|t = T\rangle = |\cos \frac{\theta}{2} e^{i \theta BT} |+\rangle + \sin \frac{\theta}{2} e^{-i \theta BT} |-\rangle, \quad (20)$$

where $\theta$ is the angle between $\vec{B}$ and $\vec{n}$. As $T \to \infty$, $\theta \to 0$ and $|+\rangle \to |\uparrow\rangle$. Also

$$B \to B_0 + \vec{B}_i \frac{x}{T} \vec{n} \cdot \vec{n}. \quad (21)$$

Thus

$$|t = T\rangle \to e^{i \mu B_0 T} e^{i (\theta 0y + \mu B \vec{n} \cdot \vec{n})} |\uparrow\rangle. \quad (22)$$

Hence the momentum of the apparatus in the x-direction shifts by $\mu \vec{B}_i \vec{n} \cdot \vec{n} = \langle \mu \vec{B}_i \vec{n} \vec{n} \rangle \uparrow$, while the system remains in the same state to begin with. Now we incorporate the essential effect of the atom kinetic term which was ignored; this is to make the initial wave packet spread in a manner that is familiar from free particle quantum mechanics. So, we take the initial wavefunction to be
a plane wave in the y-direction but a wavepacket $\phi(\epsilon)$ centred around the origin and of width $\epsilon$ in the x,z-directions. Thus the wave function after the adiabatic interaction is

$$|t = T\rangle = e^{i\mu B_0 T} e^{i p_0 y} e^{i \mu \vec{B}_i \cdot \vec{n} x} |\dagger\rangle, |\phi(t)\rangle \quad (23)$$

where

$$\epsilon(T)^2 = \frac{1}{2}(\epsilon^2 + \frac{T^2}{M^2\epsilon^2}) \quad (24)$$

G. A Possible Experimental Set Up

Let us see if the previous example can be experimentally realised. We could consider a beam of cold atoms of width $\epsilon = 0.1 \text{ cm}$. In addition to the various conditions of weakness and adiabaticity of interactions, we have to ensure that the momentum due to the uncertainty $\epsilon$ is much less than the momentum picked up due to the measurement interaction. Otherwise, the spreading of the wavepacket will completely obliterate the effects of the measurement. Let us take $B_0 = 1 \text{ Gauss}$. Let the beam move in the y-direction and let the spatial extent of the inhomogeneous magnetic field in the x-direction be $L = 30 \text{ cms}$. Let the velocity of the cold atoms be $v$. Now we define a "critical velocity" $v_c$ by

$$\frac{\mu B_0 L}{v_c} = 1 \quad (25)$$

For $\mu = 1$ nuclear magneton, we get $v_c = 3 \times 10^5 \text{ cm/sec}$. The relation between the inhomogeneous field $B_i$ in the laboratory and $\vec{B}_i$ introduced in the last section is $B_i = \vec{B}_i \cdot \vec{n} / L$. This can be written slightly differently as

$$B_i = \frac{v}{v_c} 3 \vec{B}_i \times 10^{-7} \text{ Gauss cm}^{-1} \quad (26)$$

The condition of weakness of the inhomogeneous Stern-Gerlach field i.e $B_0 >> B_i x_{\text{max}}$ becomes, on taking $x_{\text{max}} = 1 \text{ cm}$

$$1G >> \frac{v}{v_c} 3 \vec{B}_i \times 10^{-7} G \quad (27)$$

If velocities as low as 1 cm/sec can be achieved for the atoms, $\vec{B}_i$ can taken to be $10^{11}$ giving a maximum inhomogeneous field strength of $\simeq 0.1G$. Now the momentum imparted to the beam in the x-direction by the measurement interaction is

$$P_{\text{meas}} = \frac{\mu L}{v_c} 3 \vec{B}_i 10^{-7} \text{ G cm}^{-1} \quad (28)$$

With the parameters we have chosen this is $3 \vec{B}_i 10^{-7} \text{ cm}^{-1}$ (we are now using the natural units $\hbar = c = 1$). With $B_i = 10^{11}$ we get $P_{\text{meas}} = 3 \times 10^4 \text{ cm}^{-1}$. As the momentum due to beam width uncertainty $\frac{\epsilon}{4}$ is $10 \text{ cm}^{-1}$, we see that the momentum imparted by interaction is much larger so the effect of the spreading of the wavepacket is negligible. If we take the mass of the atom to be 50 amu, the velocity imparted is $6 \times 10^{-2}$ cm/sec. If after the interaction, the atoms are allowed to stream freely for about 30 secs, the displacement is about 2 cm, much larger than the beam width.

Then we can use many of the known techniques to establish that most of the initial ensemble is intact and at the same time we would have measured the expectation value of an observable (in this case $\vec{S} \cdot \vec{n}$). The choice of parameters is only indicative and a more careful study is needed to optimise them.

III. TESTING PARITY VIOLATING QUANTUM GRAVITATIONAL INTERACTIONS

General Theory of Relativity (GR) is extraordinarily successful in accounting for almost all classical gravitational phenomena so far. This prompts one to ask whether this success implies the correctness of GR at the quantum level too. I’ll outline here how cold atom techniques can shed light on some aspects of this fundamental question.

The crux of GR is the equivalence principle (for the moment let us not bother to distinguish between the so called "strong" and "weak" versions of this), which asserts the local equivalence of the dynamical effects of gravitation and the kinematical effects of an accelerated frame. Stated otherwise, an observer freely falling in a gravitational field would, according to GR, feel that he is in an inertial frame and to that observer all laws of nature would look like laws in empty space.

To appreciate the rest of my talk, it is worthwhile elucidating what an inertial frame is. One way of characterising inertial frames is to say they are the frames in which a stationary source of light sends out spherical wavefronts i.e light propagates isotropically with constant velocity $c$.

The other characterisation of an inertial frame is to say they are the frames in which an isolated gyroscope, for example, maintains its total angular momentum $\vec{L}$.

We shall call the latter a "rotational" characteristic of inertial frames. The two characterisations are logically independent. One knows that momentum $\vec{P}$ and angular momentum $\vec{J}$ are independent generators of the Lorentz group (a group of relevance when the gravitational fields are weak).

So a natural question to ask is how does the gravitational field couple to the different generators of the Lorentz group. It is well known that the generator $\vec{J}$ can be split into an orbital part and an intrinsic part. Under classical circumstances i.e when the macroscopic averages of $\vec{J}_{\text{intrinsic}}$ is zero, only the orbital part is relevant and this is not independent of $\vec{P}$ in the sense that...
\[ L = \vec{r} \times \vec{p}. \]

In particular, knowing how point particle momenta couple to a gravitational field and modelling a gyroscope as a collection of point particles in rotation, one can work out the coupling of the "spin angular momentum" of the gyroscope to the gravitational field. It can be shown that this is indeed in accordance with GR.

Thus the precession of gyroscopes in a gravitational field is not a logically independent test of GR. On the other hand, with the advent of quantum mechanics and quantum field theory, we know that intrinsic angular momenta can not be consistently modelled by extended rotating objects.

According to GR, the intrinsic spins must respond the same way as classical orbital angular momenta to gravitational fields. Otherwise, in a freely falling frame there would be differential precessions and that would not look like flat space physics! But do they?

Unfortunately there is no experimental evidence for this mainly because for all objects with which classical GR tests are carried out, the intrinsic spins are thermodynamically averaged out viz star light, pulsars in binary systems etc.

It is the purpose of my talk here to argue that trapped atom and laser cooling techniques have reached a stage where we can begin to experimentally tackle these fundamental questions. It should be remarked that these new tests should not be confused with other tests like interference experiments etc. (It should be noted that the non-symmetric theories of gravitation, Poincare-gauge theories etc., should be replaced by the Leitner-Okubo parametrisation has generated a lot of confusion, we wish to remark on it. While this is fine as a phenomenological parametrisation for spin-1/2 particles, it lacks a certain universality in the sense that the rotation generators in the spin-1/2 representation only. Of course one could argue that for the general case, the rotation generators should be replaced by \( \vec{S}/\hbar \) but this is unnatural because of the explicit appearance of an inverse power of \( \hbar \). Our parametrisation avoids these problems. It can also be brought into the Leitner-Okubo form but with \( A_i \) no longer constants but depending on \( r \). Thus limits on \( A_i \) will be context dependent! In fact for massive particles, \( A_1 = \alpha \frac{m_r}{v} \) etc.

GR predicts that \( \alpha = 0, \beta = 0, \gamma = 2 \). Any deviation from these values implies a breakdown of the equivalence principle and as shown in [5] actually means even a breakdown of the very structure of GR. In particular, they imply violation of the equivalence principle as well as a violation of Local Lorentz Invariance (LLI), which is really the true symmetry content of GR. That these effects violate these hallowed principles of GR can be expressed in a visually striking manner by noting that even in the freely falling frame elementary particle spins would be precessing and hence not all effects of gravitation are equivalent, even locally, to the effects of being in an accelerated frame! They also imply that discrete symmetries like P, C, T are violated in gravitational interactions. In [5] I have presented a very detailed analysis of the significance of \( \alpha \neq 0, \beta \neq 0 \) and the severe restrictions such a circumstance would place on many theories of gravitation like GR and its variants, Einstein-Cartan theories, non-symmetric theories of gravitation, Poincare-gauge theories etc. (It should be noted that the non-symmetric theories I have in mind are those such that the antisymmetric part of the "metric" has its source only in quantum effects). Thus it is extremely important to settle the status of these parameters experimentally.

The high precision tests of GR shed no light on these parameters as intrinsic spins are thermodynamically averaged out to zero in such tests. One will have to perform gravitational experiments with polarisations etc. The experiments proposed by me are of that genre.

Of the three effects, all terms are of comparable magnitude for photons. For massive particles, the \( \alpha \)-term is \( c/v \) times bigger than the other terms. The main observable consequences of the \( \alpha \)-term are:

**Differential Acceleration:**

\[
\frac{a_+ - a_-}{a} = 2 \alpha \frac{\hbar}{mcR}
\]  

(31)

where \( a_\pm \) refer to the acceleration of particles with spin polarised parallel (anti-parallel) to the local gravitational field and \( R \) is the radius of the earth. In magnitude this works out to \( \sim 10^{-22}, 10^{-19}, 10^{-23} \) for neutrons, electrons, and atoms respectively.

**Spin Precession:**

The spin-vector \( \vec{S} \) will precess around the local gravitational field according to

\[
\frac{d\vec{S}}{dt} = \alpha \frac{GM}{cR^3} \vec{R} \times \vec{S}
\]  

(32)

This effect is independent of the mass of the particle and the precession rate is \( \sim 4.5 \text{ mHz} \).

**Energy Difference between polarised particles:**

Let us consider spin-1/2 particles. The energy difference between particles with spin along the direction of the gravitational field and those with spin anti-parallel to the local gravitational fields is given by:

\[
\Delta E = \frac{GM}{cR^3} \hbar \alpha
\]  

(33)

Numerically this is \( \sim 3 \alpha 10^{-23} \text{ eV} \).
Existing Limits
As mentioned earlier, the existence of these effects would imply differential acceleration of particles with spin along and opposite the local gravitational field. Unfortunately such experiments have not been conducted yet. However, by ascribing the current uncertainties in Eotvos type experiments to these effects one could arrive at the limit $\alpha < 10^{-9}$ which is not a very useful bound. It would be instructive to perform “fifth-force” type experiments with test objects where a substantial fraction of the intrinsic angular momentum of the body is due to the intrinsic spin-angular momentum of the composites (as opposed to the quantised orbital angular momentum).

Leitner and Okubo estimated $A_i$ by ascribing the uncertainties in the hyperfine splitting of hydrogen arising out of an uncertainty in the fine structure constant (at that time about 1 ppm) to possible non-electromagnetic phenomena. And ascribing them fully to the discrete symmetry violating potential they concluded $A_i < 10^{-11}$. While this sounds impressive, it implies $\alpha < 10^{-10}$ which is far from impressive! Currently, the fine structure constant is known to about .05 ppm but the discrepancy between theory and experiment is still at the level of 0.5 ppm.

In fact the most conservative limit on $\alpha$ from the Hfs in hydrogen can be obtained by ascribing the uncertainty in measurement to the extra effects considered here. This error is about 1 mHz. This translates to a limit of $\alpha < 200$.

In fact a good way of estimating these effects is by looking for differential bending of polarised electromagnetic waves at the solar limb. In fact such an effect was looked for by Harwit et al who, with the parameterisation of the potential given by

$$V_{eff} = \pm \alpha' V_0(r) \quad (34)$$

where $\pm$ refers to the two states of circular polarisation. The relation to our parameterisation is $\alpha' = \frac{\lambda}{2\pi}$ where $\lambda$ is the wavelength. Harwit et al found $\alpha' \approx 10^{-2}$ with 13 cm radio waves at the solar limb. Later Dennison obtained the more accurate value of $\alpha' \approx 10^{-6}$ which translates to $\alpha \approx 10^4$. As these experiments were done long ago, perhaps current state of the art may already give a chance to probe $\alpha \approx 1$.

A. Astrophysical Constraints

There have been several attempts to see if the parameters of discrete symmetry violations in gravitation could be constrained using astrophysical data. Almeida et al used the limits on differential propagation of neutrinos and photons to set the limits $|A_i| < 10^{-3}$ where $A_i$ are the parametrisations used by Leitner and Okubo. As remarked already, these are context dependent and their relation to $\alpha$, $\beta$ in this context is $A_1 = \alpha \frac{h c}{4\pi G}$, $A_2 = \beta \frac{h c}{4\pi G}$ where $b$ is the impact parameter relative to the galaxy and $E$ the energy of the particles. Their limit translates to $\alpha < 10^{30}$, not a very useful bound.

 Losecco et al tried to use the observed pulse-widths of pulsars to constrain the differential propagation times of polarised photons and obtained $A_1 < 2 \times 10^{-10}$, $A_2 < 6 \times 10^{-11}$. They thought that their limits on $A_1$ were poorer than what obtained, but again due to context dependence the two are not related. This translates to $\alpha < 10^{23}$.

Choudhury et al tried to estimate these parameters by looking at their contribution to helicity flip scattering of massive neutrinos and their consequent effect on the cooling of neutron stars. Assuming a mass of 1 KeV for $\tau$-neutrinos, they could constrain $\alpha$ to be less than 300.

Nodland and Ralston report that their is a systematic cosmological effect that rotates the polarisation of photons. It is very important to see whether such an effect can be explained on the basis of our model. There is also a recent work of Lue et al which has analysed some very specific models with P violation for their cosmological signatures. We plan to investigate these signatures from the point of view of our model. There is also considerable theoretical interest in P violations from the point of view of a variety of models, as well as for their implications for global space-time issues.

IV. ATOM TECHNIQUES

A. Mercury Cell

In the experiment of Venema et al two ground state Hg isotopes are trapped in the same cell. The electronic configuration being $^1S_0$, the two isotopes are fully “nuclear spin polarised”. The two isotopes under consideration are $^{199}Hg(I = 1/2)$ and $^{201}Hg(I = 3/2)$.

One of the most problematic backgrounds for experiments looking for small spin precession rates like the Ramsey type Electric Dipole Moment measurements or our proposed experiments to look for discrete symmetry violations in gravitation are the very tiny stray magnetic fields that mimic the effects we are trying to study. For the gravitational experiments even stray fields as small as $10^{-11}$ Gauss can simulate an effect equivalent to $\alpha \simeq 1$.

The advantage of trapping two isotopes in the same cell is that they will both experience nearly the same stray magnetic field. The effect of the stray magnetic field can then be eliminated through an appropriate combination of observables. In the actual set up ambient magnetic fields were reduced to the level of $< 20 \mu G$. In addition an uniform field of less than $10 mG$ was also applied in the $z$-direction. The direction of this field was flipped every hour. The Hamiltonian for the system can be cast as

$$H = -g_\mu_N \hat{I} \cdot \vec{B} + A \hat{I} \cdot \vec{r} \quad (35)$$

where $\hat{I}$ is the nuclear spin, $\vec{B}$ the applied uniform field, $\mu_N$, $A$ the nuclear gyromagnetic ratio and the quan-
tity parametrising the additional effects one is looking for, respectively. The effect of the stray magnetic field can be taken into account by changing the Hamiltonian to

$$H_{\text{stray}} = H - g_1 \mu_N \vec{B} \cdot \vec{B}_{\text{ran}}$$  \hspace{1cm} (36)$$

Then the observed frequency of the NMR-line is given by

$$\nu_{199} = -g_{199} \mu_N (B + B_{\text{ran}} < \cos \theta_{\text{ran}}>) + A_{199} \cos \phi$$

$$\nu_{201} = -g_{201} \mu_N (B + B_{\text{ran}} < \cos \theta_{\text{ran}}>) + A_{201} \cos \phi$$  \hspace{1cm} (37)$$

where $\phi, \theta_{\text{ran}}$ are the angles between the direction of the uniform magnetic field and the local gravitational field, random magnetic field respectively. In general $\theta_{\text{ran}}$ is time-dependent but we assume that it is constant over the time scale of $g_1 \mu_N B$. Now there are two observables (related to each other) that are insensitive to the stray magnetic field; these are i) $R = \nu_{199}/\nu_{201}$ and ii) $S = \frac{\nu_{199}}{g_{199}} - \frac{\nu_{201}}{g_{201}}$, which are given by:

$$R = \frac{g_{199}}{g_{201}} \left( 1 + \frac{1}{\mu_N B} (A_{201} - A_{199}) \cos \phi \right)$$

$$S = \frac{A_{201}}{g_{201}} - \frac{A_{199}}{g_{199}} \cos \phi$$  \hspace{1cm} (38)$$

For the gravitational context, $A = \alpha \frac{G M}{R^2 c^2} I$. Hence for this case

$$S = \alpha \frac{(GM)}{R^2 c^2} \cos \phi \left( \frac{3}{g_{201}} - \frac{1}{g_{199}} \right)$$  \hspace{1cm} (39)$$

Thus this method is not suitable when the gyromagnetic ratios of the isotopes are in proportion to their nuclear spins.

It is remarkable that the measurements are so accurate that proper account of earth’s rotation should be taken into account! The change in the observed frequency is $\nu \rightarrow \nu \pm \Omega_E \cos \theta$ where $\Omega_E$ is the earth’s angular velocity and $\theta$ the angle between the earth’s axis of rotation and the z-direction. Numerically $\Omega_E \approx 11.6 \mu Hz$. It is best to choose $\theta$ small as the dependence on it would be like $\theta^2$. This will help minimise errors due to mechanical misalignment. The systematic error due to this was found to be $\Delta = \delta R_+ + \delta R_- \approx 10^{-8}/\nu(\mu Hz$).

The quoted result for $\alpha$ is

$$\alpha = -0.12 \pm 0.14 \pm 0.15(Syst) \mu Hz$$  \hspace{1cm} (40)$$

In eV this translates to $\alpha \approx 2.2 \times 10^{-21} eV$. Consequently the implied limit on $\alpha$ is $\alpha \approx 70$.

Thus the accuracies of this experiment need to be pushed just a few orders of magnitude more before we can begin to probe $\alpha \approx 1$.

I list here some possibilities for future. Instead of Hg Vapour cell, one could imagine trapping two different isotopes in an atom. In discussions with Y. Takahashi at Kyoto University, the use of a Magneto-Optic Trap (MOT) has been identified. Here it does not seem too idealistic to expect to achieve the following parameters: Number of trapped atoms $\approx 10^8$, dipole force time $\approx 10^6 sec$, total measurement time $\approx 10^8 sec$, leading to statistical errors $\approx \mu Hz/\sqrt{N}$. Systematic errors are hard to estimate as yet. By also studying the full $\phi$-dependence, one may hope to get some control over the systematic errors. With all these features, it is hoped that we may begin to probe the $\alpha \approx 1$ region.

B. Atoms in Traps

Another class of experiments that can be very successful in putting useful limits on parameters like $\alpha$ are experiments where ions are trapped in a Penning trap. In the experimental set up of Weinland et al. [14] about 5000 atomic $^9 Be^+$ in its ground state characterised by $^2S_{1/2}$ are trapped in a Penning trap with magnetic field $B_0 \approx 0.8194$ Tesla. The hyperfine structure of the ground state is characterised by $F = 1$ where $\vec{F} = \vec{J}(electron) + \vec{I}(nuclear)$. The hyperfine transition that was driven was $(F = 1, m_F = 0) \rightarrow (F = 1, m_F = -1)$.

The ions were trapped and laser cooled to reduce doppler shifts. First, they are optically pumped to $m_F = 0$ using a combination of laser and rf coils. Then the required transition $(m_F = 0 \rightarrow m_F = -1)$ was driven via rf. Finally, the changes in the population of the two states were monitored by the changes in the scattered laser light. Changes in $n_0$ were looked for as the magnetic field $B_0$ was flipped. Again, the effect of the earth’s rotation has to be subtracted. The final result is

$$\Delta \nu_0 \approx -6.4 \pm 2.9 \pm 6.4 \mu Hz$$  \hspace{1cm} (41)$$

where the main source of error of $6.4 \mu Hz$ is due to the pressure shift variation when the magnetic field is flipped. Using quadrature, one can put the limit

$$\Delta \nu_0 < 13.4 \mu Hz$$  \hspace{1cm} (42)$$

which translates to the limit

$$\alpha < 300$$  \hspace{1cm} (43)$$

The authors of this experiment claim that the error due to pressure shift variation can be minimised by using cryogenic pumping. Also, the number of stored ions could be increased to $10^7$. The time to drive the $\nu_0$ transition could also be increased to $100 \text{secs}$. With all these improvements the limits on $\Delta \nu_0$ could be improved to $\approx 3nHz$ which will push the limits on $\alpha$ to $\approx 1$.

C. Experiments looking for Local Lorentz Invariance Violations

In the early sixties the so-called Hughes-Drever experiments looked for violations of Local Lorentz Invariance
which is at the heart of GR. These experiments looked for a particular manifestation of the violation of LLI viz. dependence of the inertia of a body on its velocity and orientation relative to some preferred frame. While the early limits on $\frac{\delta m}{m}$ were $\simeq 10^{-23}$, these have been improved substantially in recent times. Lamoreaux et al. [18] improved this to $\frac{\delta m}{m} \simeq 10^{-28}$ or equivalently to splittings of the order of 500 nHz. Most recently Berglund et al. [20] have obtained limits of 110 nHz for nucleons.

It is tempting to see if one could use these experiments to limit our $\alpha$ parameter also. It should be recalled that one of the consequences of the existence of $\alpha$ is indeed violation of the equivalence principle and of LLI. One could imagine the earth’s gravitational field providing the preferred direction.

In all these experiments nuclei are polarised and allowed to freely precess about the direction of a magnetic field which is fixed in the earth frame. As the earth rotates, the orientation of this magnetic field with respect to some preferred direction keeps changing with a 24 hr cycle (nearly) and these experiments look for changes in the Zeeman level splittings that are correlated with this cycle. Unfortunately, for testing our model these experimental set ups are not adequate as the angle between the magnetic field and the earth’s gravitational field does not change. Nevertheless the extreme sensitivities achieved in these experiments suggests readopting them to look for the type of effects that we have been discussing.

This underscores the fact that these experiments are looking at very specific models of LLI and are not looking at generic LLI violations. In some sense they are looking for kinematical effects incorporating LLI violations whereas our effects embody LLI violating dynamics. A wide class of LLI violating models have recently been analysed by Glashow et al [21].

I am thankful to Y. Takahashi, H. Funahashi and Akira Masaike of Kyoto University for many illuminating discussions on the feasibility of improving the limits on the parity violating parameters, and for Y. Takahashi for many discussions on realising protective measurements.

[1] Y. Aharonov, J. Anandan and L. Vaidman, Phys.Rev. A 47, 4616(1993)
[2] N.D. Hari Dass and Tabish Qureshi, Phys.Rev. A 59, 2590(1999) quant-ph/9805012
[3] N.D. Hari Dass, Phys.Rev.Lett 36(1976) 393.
[4] J. Leitner and S. Okubo, Phys. Rev. 136(1964) B1542.
[5] N.D. Hari Dass, Annals of Physics(NY) 107, No 1-2 (1977) 337. Jour.Gen.Rel.Grav. 8(1977) 89.
[6] N.F. Ramsey, in “Quantum Electrodynamics” ed, T. Kinoshita (World Scientific 1990)