Symbolic analysis of bursting dynamical regimes of Rulkov neural networks

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Abstract

Neurons modeled by the Rulkov map display a variety of dynamic regimes that include tonic spikes and chaotic bursting. Here we study an ensemble of bursting neurons coupled with the Watts-Strogatz small-world topology. We characterize the sequences of bursts using the symbolic method of time-series analysis known as ordinal analysis, which detects nonlinear temporal correlations. We show that the probabilities of the different symbols distinguish different dynamical regimes, which depend on the coupling strength and the network topology. These regimes have different spatio-temporal properties that can be visualized with raster plots.

Keywords: Neural networks; Neural encode information; Ordinal symbolic analysis.

1. Introduction

Neurons encode and transmit information in temporally correlated sequences of spikes [1, 2, 3]. Neurons can fire regularly (tonic spikes) or irregularly. In the first case, the spikes are periodic in time and the distribution of inter-spike intervals is very narrow; in the second case, the time intervals between spikes are irregular and their distribution is broad. Bursting is a dynamical regime in which a neuron fires groups or bursts of spikes and each burst is followed by a silent period before the next burst occurs.

Since spike correlations have a functional role in the neural code, improving or degrading information transmission [4, 5, 6, 7], relevant questions are how to detect temporal correlations in the spike sequences and how the couplings among the neurons affect them. While linear correlations can be detected and quantified by serial correlation coefficients [8, 9, 10, 11], nonlinear correlations need to be detected by using nonlinear techniques [12]. A popular one is symbolic ordinal analysis [13, 14], which applied to sequences of spikes detects nonlinear temporal correlations [15, 16, 17, 18, 19, 20, 21, 22, 23, 24].

When used to analyze burst sequences, this symbolic approach considers only the relative duration of the time intervals between bursts (inter-burst-intervals, IBIs), and transforms a sequence of IBIs into a sequence of symbols (known as ordinal patterns) using the ordinal rule that takes into account the temporal order of consecutive time intervals. For example, when analyzing the symbols defined by three consecutive intervals, the six possible order relations define six symbols whose frequencies of occurrence can reveal the presence of temporal order, in the form of over-expressed and/or less-expressed symbols. If the burst sequence is fully stochastic the frequency of occurrence of each symbol will be \(1/6\) (if the sequence is long enough). On the other hand, over (or less) expressed symbols (whose probabilities are significantly higher or lower than

Here we use ordinal analysis to investigate nonlinear correlations in a neuronal ensemble modeled by the popular Rulkov two-dimensional iterated map [25]. We use this map because it displays a variety of dynamical regimes and makes possible to simulate the behavior of relatively large neuronal ensembles [26]. We focus on the regime where individual neurons fire bursts of spikes and analyze temporal correlations among the IBIs. We find that ordinal analysis detects correlations in the IBI sequences that depend on the strength of the coupling, \(\epsilon\), between neurons and on the network topology that is varied from regular to small-world by changing the rewiring parameter, \(p\), defined Watts and Strogatz [27]. The analysis of the ordinal probabilities as a function of \(\epsilon\) and \(p\) reveals different dynamical regimes, which can be clearly visualized using network spatio-temporal plots (raster plots).

Complementing the symbolic analysis, we also use the Kuramoto order parameter [28] to investigate the synchronization features of the network in the parameter space of \(\epsilon \times p\), in which the role of the coupling and topology are considered. We find that the network displays a variety of complex spatio-temporal patterns, including phase-synchronized states [29, 30] and unsynchronized states where zig-zag structures are seen in the raster plots [31, 32, 33, 34].

The paper is organized as follows: Sec. 2 describes the neuron model and the network topology; Sec. 3 describes the methodology used to analyze the network dynamics; Sec. 4 presents the results, Sec. 5 presents the discussion, and Sec. 6 presents the conclusions.

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for each neuron.

ability exhibits the burst activity and panel (b) depicts the slow variable of the Rulkov map. The blue line ($\epsilon$) depicts the burst activity where the maxima of $y$ coincide with the start of the bursts.

### 3.1. Network construction

The adjacency matrix of the network, $A$, is constructed following the Watts-Strogatz procedure [27], where a network, initially regular, is gradually modified to a small-world one with random characteristics by replacing local connections with random ones. We start with a regular network and, as the rewiring parameter, $p$, increases, the local connections are replaced by random ones, which leads to the decrease of the average path length without change of the total number of connections. In this way, a small-world network is obtained, where the average path length is low and the clustering coefficient is high [27].

#### 3.2. Inter-burst intervals

As depicted in Fig. 1, the maximums of the slow variable $y$ allow to detect the times when the bursts of neuron $i$ start. Specifically, the $k$th maximum of $y_i$ that occurs at time $t_k$ indicates the start of the $k$th burst of neuron $i$.

The $k$th inter-burst interval (IBI) of the $i$th neuron is the time interval between two consecutive bursts:

$$IBI_{i,k} = t_{k+1} - t_k,$$

The mean value of the inter-burst interval, $\langle IBI \rangle$, is calculated by performing a temporal average over the IBI sequence of each neuron, $\langle IBI \rangle = \langle IBI_{i,k} \rangle$, followed by an average over all neurons: $\langle IBI \rangle = \langle IBI_{i,k} \rangle_{i}$. $\langle IBI \rangle$ is computed as the average of the IBI length across the sequence of the neuron and $\langle IBI \rangle_{i}$ is the average of the IBI length across the sequence of the neuron.

#### 3.3. Ordinal analysis

After calculating the IBI sequence of each neuron, $\{IBI_{i,k}\}$, we use ordinal analysis [13] to detect temporal structures in the IBI sequences. As discussed in the Introduction, ordinal analysis allows identifying patterns in complex datasets. This method takes into account the relative temporal ordering of the data values in a time-series and provides a way to compute a set of probabilities that characterize the time series.

For example, if we consider two consecutive IBI intervals, if the first one is longer than the next one we assign to that IBI the symbol “10”, otherwise we assign the symbol “01”. In this way, we have reduced the sequence of IBI time intervals to a sequence of two symbols. If we compare the relative ordering of three (or four) consecutive values of IBI, then six (or twenty-four) different symbols (known as ordinal patterns) can be defined. For example, three consecutive increasingly long IBIs give pattern 012, while three consecutive increasingly short IBIs give pattern 210. In this sense, an example of the pattern 012 is observed in Fig. 1 where a sequence of three increasing IBI values (197, 216, 280) is depicted.

The frequency of occurrence of the different patterns (evaluated from the sequences of IBIs of all the neurons, $\{IBI_{i,k}\}$) gives the set of ordinal probabilities, $P(i)$ with $\sum_{i} P(i) = 1$, that will be analyzed in the next section.

We consider patterns of length $K = 3$. As the number $N$ of patterns grows as $N = K!$, using a larger value of $K$ means that there is a large number of probabilities to calculate, which is computationally expensive (see [20] for a discussion of the data requirements). Ordinal patterns of length $K = 3$ are small-size patterns, but nevertheless, they yield relevant information about a system’s dynamics (see Ref. [16] for practical examples of how small ordinal patterns of length $K = 2$ or 3 are able to extract relevant information from data).
It has been recently shown that when a time series contains a significant number of equal values (in our case, equal IBIs), they can give rise to false conclusions regarding the presence of temporal structures. Because we chose parameters such that the neurons’ dynamics is chaotic, the IBIs do not have a large number of equal values (we have verified that the percentage of patterns that contain equal IBIs is less than 6%).

Shannon entropy computed from ordinal probabilities is known as Permutation Entropy. \( S = -N \sum_{i} P(i) \ln (P(i)). \) \( S \) is maximum when \( P(i) = 1/N \) \( \forall i \) and \( S = 0 \) when \( P(i) = 1, \) \( P(j) = 0 \) \( \forall j \neq i. \)

### 3.4. Phase synchronization quantifier

We use the Kuramoto order parameter to measure the degree of phase synchronization of the bursts of the neuronal ensemble. The Kuramoto order parameter is given by

\[
R(t) = \left| \frac{1}{N} \sum_{j=1}^{N} e^{i \theta_j(t)} \right|,
\]

where \( \theta_j(t) \) is the phase of neuron \( j \) at time \( t. \) The time-averaged parameter, \( \langle R \rangle, \) is \( \sim 1 \) if the neurons are phase-synchronized and \( \sim 0 \) if they are phase incoherent.

To associate a phase to the dynamics of each neuron, we use the slow variable, \( y. \) As depicted in Fig. 1, \( y \) is maximum when a burst starts, and therefore, the phase as a function of time can be defined as

\[
\theta(t) = 2\pi k + 2\pi \left( \frac{t - t_{kb}}{t_{k+1,ib} - t_{k,ib}} \right), \quad t_{k,ib} < t < t_{k+1,ib},
\]

where \( t_{k,ib} \) is the time where the \( k \)th burst of the \( i \)th neuron starts. The parameters considered in this study are such that all neurons show bursting behavior, which allows the use of \( y \) maximums to detect the beginning of the bursts, which in turn allows defining a phase without the need of reconstructing the dynamical evolution in the phase space.

### 3.5. Implementation

The model parameters are: \( \beta = 0.001 \) (equal for all neurons) and \( \alpha_i \in [4.1, 4.4] \) Gaussian distributed with mean value 4.25 and standard deviation 0.045. \( I_{noise} \) is uncorrelated for each neuron: \( I_{noise} \in [0.003, 0.065] \) is Gaussian distributed with mean value 0.035 and standard deviation 0.01. The coupling strength and the rewiring probability are considered control parameters, varied in the range \( \varepsilon \in [0.000, 0.099] \) and \( p \in [0.001, 1.000]. \)

The initial conditions are random in the interval \( x, y \in [0, 1]. \)

The Kuramoto parameter, \( \langle R \rangle, \) the mean IBI, \( \langle IBI \rangle, \) and the ordinal probabilities were calculated by averaging results from 10 simulations with different networks, different parameters and different initial conditions.

To investigate the role of the system size, we have simulated networks of 100, 500, and 1000 neurons with 400, 2000, and 4000 connections, respectively, which give the same average of four connections per neuron.

### 4. Results

The synchronization behavior is depicted in Fig. 3 where the Kuramoto parameter, \( \langle R \rangle, \) is displayed in color code as a function of the coupling strength, \( \varepsilon, \) and the rewiring connection probability, \( p \) for networks of 100, 500 and 1 000 neurons (panels (a)-(c) respectively).

A transition from non-synchronized to phase-synchronized state is observed. For \( p \) large enough \( (p > 0.02), \) the transition occurs as the coupling parameter increases. For \( \varepsilon \) large enough \( (\varepsilon > 0.025), \) the increase of \( p \) induces a similar synchronization transition. This behavior is observed for the three network sizes considered; however, the border is less defined.
when $N = 100$. When the coupling is too weak ($\varepsilon < 0.02$) or when the network is too regular ($p < 0.02$) the transition to phase synchronization does not occur (the Kuramoto parameter is $(R)_c < 0.6$).

Panels (d), (e), and (f) of Fig. 3 depict the mean value of inter-burst intervals, $\langle IBI \rangle$, as a function of $\varepsilon$ and $p$ for $N = 100$, 500, and 1000, respectively. In the three cases, we see that the coupling strength ($\varepsilon$) increases the mean inter-burst interval. On the other hand, we note that the rewiring probability does not have a large impact in $\langle IBI \rangle$.

The results obtained with ordinal analysis are presented in Fig. 4 that depicts the probabilities of observation of pattern 012 (panels (a), (b), and (c)) and pattern 210 (panels (d), (e), and (f)) as a function of the rewiring probability, $p$, and the coupling strength, $\varepsilon$ for networks of $N = 100$, $N = 500$, and $N = 1000$, respectively. In the three cases, we see that the coupling strength ($\varepsilon$) distinguishes the mean inter-burst interval. On the other hand, we note that the rewiring probability does not have a large impact in $P_{012}$.

$P_{210}$ (panels (d), (e), and (f)) uncovers more details, as in the low synchronization region that occurs for $p < 0.02$, it distinguishes different regions, for weak coupling ($\varepsilon < 0.03$) where pattern 210 is under-expressed ($P_{210} < 1/6$) and for stronger coupling ($\varepsilon > 0.60$) where 210 is over-expressed ($P_{210} > 0.19$). For higher $p$, a region where pattern 210 is less expressed is also observed. This scenario is robust for the three network sizes analyzed.

While the “trend” patterns 012 and 210 are often the most informative, the other patterns can yield interesting information as well. Figure 5 displays all the ordinal probabilities vs. the coupling strength, for three values of the rewiring parameter. Without coupling all the patterns have similar probabilities, consistent with the uniform distribution. Therefore, in spite of the fact that the dynamics of the uncoupled neurons is chaotic, no temporal structures are detected in the sequences of $IBI$s. Similar results are obtained when the neurons have identical parameters and are noise-free (not shown). In the presence of neuronal coupling temporal structures in the $IBI$ sequences emerge and we note that they depend on the network connectivity. For high enough coupling and low $p$ (when the network relatively regular) the “oscillation” patterns 021, 120, 102, and 201 are under expressed (their probabilities are $< 1/6$), while for high $p$ (when the network is close to random) these patterns are over expressed (their probabilities are $> 1/6$).

To investigate the reasons for the low or high values of the ordinal probabilities, we inspect the dynamics of the networks for different values of $\varepsilon$ and $p$ using raster plots, which are obtained from the fast variable of each neuron, $x_i$, considering the spike threshold of $x_i = 0$ and positive first derivative. Figure 6 depicts the raster plots for a network of $N = 1000$ neurons. Panel (a) for $\varepsilon = 0.005$ and $p = 0.010$ is representative of the non-synchronized state, where we see that the neural activity has low spatio-temporal coherence.

In contrast, panels (b) and (c) ($\varepsilon = 0.045$ and $p = 0.010$, and $\varepsilon = 0.090$ and $p = 0.010$, respectively) show a different situation, in which diagonal structures are observed, which resemble zig-zag fronts [31, 32, 33, 54]. In this situation, the network can depict groups of neurons with different mean frequencies [34]. The presence of horizontal structures in those plots would indicate that most of neurons start their bursts at similar times, that means, that they are phase synchronized. However, horizontal structures are almost absent in Figs. 6b and (c). Because the Kuramoto order parameter quantifies the level of phase synchronization in the network, the absence of horizontal structures explains why the Kuramoto parameter has a low value.
Figure 6: Raster plots representing the activity of a network of \( N = 1000 \) neurons. Panel (a) \((\epsilon = 0.005 \; \text{and} \; p = 0.010)\) depicts a non-synchronized state, panels (b) and (c) \((\epsilon = 0.045 \; \text{and} \; p = 0.010, \; \text{and} \; \epsilon = 0.090 \; \text{and} \; p = 0.010, \) respectively) show diagonal spatio-temporal structures, and panel (d) \((\epsilon = 0.090 \; \text{and} \; p = 0.500)\) represents a phase-synchronized state.

(for the parameters of Figs. 6(b) and (c), \( \langle R \rangle_i < 0.6 \)).

Figure 6(d) for \( \epsilon = 0.09 \) and \( p = 0.5 \) is representative of the phase-synchronized states, in which the horizontal structures are observed and \( \langle R \rangle \) is high. These structures show that the bursts begin at similar times.

An important question is the role of noise and heterogeneous neurons’ parameters. So far we have considered \( \alpha_i \) and \( I_{\text{noise}} \) Gaussian distributed. To determine how they affect the dynamics, we simulate 1 000 neurons and compare four cases:

(i) heterogeneous neurons with noise \((\alpha_i \text{ and } I_{\text{noise}} \text{ distributed as indicated in Sec. 3.5})\);

(ii) heterogeneous neurons without noise \((\alpha_i \text{ distributed as indicated above, } I_{\text{noise}} = 0)\);

(iii) identical neurons with noise \((\alpha_i = 4.25 \text{ and } I_{\text{noise}} \text{ distributed as indicated in Sec. 3.5})\);

(iv) identical neurons without noise \((\alpha_i = 4.25, I_{\text{noise}} = 0)\).

Figure 7 depicts the results. Here, panels (a), (b), (c), and (d) depict the mean value of Kuramoto parameter, \( \langle R \rangle_i \), as a function of \( \epsilon \) and \( p \) for the cases (i), (ii), (iii), and (iv), respectively. A very similar behavior is observed for all cases.

The second and third rows in Fig. 7 depict the mean value of inter-burst intervals, \( \langle IBI \rangle \), and the probability of pattern 012 respectively, and we again see a very similar behavior in the four cases cases. In contrast, the probability of pattern 210 (fourth row) uncovers differences between the dynamics of the deterministic (noise free) network and the dynamics of the stochastic network. For intermediate coupling \((0.01 < \epsilon < 0.04)\), without noise \( P(210) \) takes lower values \((<0.16)\) than with noise.

In the four cases considered we see that the probability of pattern 210 distinguishes, in the region \( p < 0.020 \) (almost regular networks), different dynamical behavior depending on the coupling strength. In contrast, \( \langle R \rangle \) only characterizes the dynamics as non-synchronized states.

To investigate the origin of the differences found in \( P(210) \) in the region \( p < 0.1 \) and \( 0.01 < \epsilon < 0.04 \), we analyze the spatio-temporal dynamics of the network. Figure 8 depicts the raster plot for \( \epsilon = 0.025, p = 0.01 \) when the neurons are heterogeneous and noisy (case i, panel a), and when they are identical and noise-free (case iv, panel b). We see a similar partially coherent spatio-dynamics; however, we can also notice subtle differences: panel (a) depicts less coherent spatial structures than panel (b), where more partially horizontal structures are observed. The similar dynamics is captured by the Kuramoto parameter that takes similar values, \( \langle R \rangle_i = 0.092 \) in (a) and \( \langle R \rangle_i = 0.098 \) in (b). The subtle differences might be the reason why \( P(210) \) is different in the two cases: \( P(210) = 0.16 \) in (a) and \( P(210) = 0.14 \) in (b).

To complement the analysis done at the macroscopic level of the global network, we now characterize the microscopic dynamics by analyzing the burst sequences of the individual neurons. Our goal is to determine if there is a statistical relation between the way a neuron is connected and the properties of its sequence of bursts. Specifically, we investigate how the number of links (i.e., the degree \( k \) of a neuron) affects the \( IBI \) sequence. In networks of coupled oscillators it was recently found, numerically and experimentally [42, 43], that there is a relation between the dynamical complexity and the degree: nodes with higher degree were found to have lower levels of complexity.

Considering a network of 1000 noisy heterogeneous neurons (case i), Fig. 9(a) shows that the mean \( IBI \) of a neuron, \( \langle IBI \rangle \), increases with the degree of the neuron, \( k_i \). In addition, the
Figure 7: Mean value of the Kuramoto parameter (top row); mean IBI (second row) and probabilities of patterns 012 (third row) and 210 (fourth row) for a network of 1 000 neurons, considering the four cases described in the text: noisy heterogeneous neurons (case i, left column); deterministic (noise-free) heterogeneous neurons (case ii, second column); noisy identical neurons (case iii, third column); deterministic identical neurons - (case iv, right column).

It is interesting to note that, for the parameters used in Fig. 9(a), the Kuramoto parameter has a large value, \( \langle R \rangle > 0.95 \), however, the mean IBI is not the same for all the neurons: it increases with the neuron’s degree. The raster plot, shown in Fig. 9(b), confirms that the neurons are synchronized in phase (horizontal structures are clearly visible), but the synchronization is not perfect. While in this plot the irregular fluctuations appear to be random, our analysis of the IBI distribution of each neuron (the fact that the mean value increases and the standard deviation decreases with the neuron’s degree) indicates that fluctuations are not only due to the added noise but have a dynamical origin, and in fact, similar results are found in identical and noise-free neurons (not shown).

5. Discussion

Through extensive simulations we have shown that ordinal analysis is a useful tool to analyze neuronal activity. We have shown that it can uncover subtle differences in the spatio-temporal dynamics, which are not seen by other indicators (such as the mean inter-burst-interval, or the Kuramoto parameter). The main advantages of this methodology are that it can be applied to raw, unprocessed data and is rather unaffected by the presence of noise, outliers or missing data [36]. In spite of having been extensively used to analyze biomedical signals (EEG, ECG, etc. [17, 18, 44]), few studies have applied ordinal analysis to neuronal spike trains. The data requirement is a main limitation, as (to the best of our knowledge) simultaneous intra-cellular recordings of the membrane potential of a large number of cells, during a period of time long enough to record 100s or 1000s of spikes, are not yet freely available. Therefore, efforts so far have focused in simulated neuronal spikes.

Another limitation of the ordinal methodology is that it does not consider amplitude information, i.e., the actual values of the data points are disregarded. Different generalizations have been proposed in order to also take into account amplitude information [45, 46, 47]. Because in neuroscience the standard way to characterize spike trains is through the analysis of the distribution of the inter-spike intervals, here we have complemented the results obtained with ordinal analysis, with the analysis of the IBI distribution, both, at the microscopic level of the individual neurons and at the macroscopic level of the global network.

In our network we have not detected any relation between the

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Standard deviation of IBI distribution of each neuron decreases with the degree (not shown) indicating that neurons that have several connections have a more regular bursting activity than neurons that have few connections.

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The degree of synchronization has been quantified with the degree of its neuron, $k_i$, for a network of $N = 1000$ heterogeneous neurons with noise (case (i)), $\varepsilon = 0.08$ and different values of $p$. Results are obtained from 10 simulations and the dispersion is represented by the filled area. Panel (b) depicts the raster plot for $\varepsilon = 0.08$ and $p = 0.8$. In this situation, the network is phase synchronized and $\langle R \rangle = 0.958$.

6. Conclusions

We have used symbolic ordinal analysis to investigate the dynamics of an ensemble of Rulkov neurons mutually coupled in a Watts-Strogatz network. We have considered parameters such that the individual neurons fire bursts of spikes, and we have characterized the sequences of the inter-burst-intervals (IBIs) by computing the probabilities of the ordinal patterns. We have found that these ordinal probabilities allow to identify different dynamical regimes, which depend on the coupling strength and the network topology. These regimes are not differentiated by the average $IBI$ or by the Kuramoto order parameter; however, different spatio-temporal structures were seen in the raster plots (such as non-synchronized states, phase-synchronized ones, and zig-zag structures). We have shown that our results are valid for different network sizes and are robust to the presence of noise and heterogeneous neurons’ parameters.

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