A quantum heat switch based on a driven qubit

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Heat flow management at the nanoscale is of great importance for emergent quantum technologies. For instance, a thermal sink that can be activated on-demand is a highly desirable tool that may accommodate the need to evacuate excess heat at chosen times, e.g. to maintain cryogenic temperatures or reset a quantum system to ground, and the possibility of controlled unitary evolution otherwise. Here we propose a design of such heat switch based on a single coherently driven qubit. We show that the heat flow provided by a hot source to the qubit can be switched on and off by varying external parameters, the frequency and the intensity of the driving. The complete suppression of the heat flow is a quantum effect occurring for specific driving parameters that we express and we analyze the role of the coherences in the free qubit energy eigenbasis. We finally study the feasibility of this quantum heat switch in a circuit QED setup involving a charge qubit coupled to thermal resistances. We demonstrate robustness to experimental imperfections such as additional decoherence, paving the road towards experimental verification of this effect.

In this letter, we show that a single coherently-driven charge qubit, can play the role of a heat switch, controlled by the parameters of the driving. When the qubit is connected to a hot and cold thermal reservoirs, the heat current from the hot reservoir is completely suppressed for a specific choice of driving parameters. Tuning either the intensity of the drive or its frequency around this working point allows to easily switch on or off this heat current. We show that the suppression of the heat flow is a quantum effect, occurring when the non-equilibrium steady-state reached by the qubit only differs from the thermal equilibrium state with the hot bath by special types of coherences that are out-of-phase with the driving field, and therefore store no energy. We first describe the setup under study, the associated dynamics and the expression for the heat flows. We then present the quantum switch effect and analyze the role of the coherences in the free qubit energy eigenbasis.

We consider a two-level quantum system (hereafter called qubit) of frequency $\omega_q$ weakly coupled to two thermal baths $R_h$ and $R_c$ of temperatures $T_h > T_c$. The dynamics of the qubit is governed by the Lindblad master equation

$$\dot{\rho} = -i[H_0, \rho] + {\mathcal L}_h[\rho] + {\mathcal L}_c[\rho],$$

where $H_0 = h\omega_0\sigma_z/2$ is the Hamiltonian of the qubit and

$${\mathcal L}_h,c[\rho] = \gamma_{h,c}(\bar{n}_{h,c} + 1)D_{\sigma_-}[\rho] + \gamma_{h,c}\bar{n}_{h,c}D_{\sigma_+}[\rho],$$

with $D_X[\rho] = X\rho X^\dagger - \frac{1}{2}\{XX^\dagger, \rho\}$ the dissipation super-operator. At steady state, a heat current flows from the hot bath to the qubit, given by

$$J_0^\infty = Tr\{\mathcal L}_h[\pi_0]H_0\},$$

where $\pi_0 = \frac{1}{2} + \frac{i}{2} \sigma_-$ is the steady state the qubit master equation, characterized by $z_0 = -(\gamma_h + \gamma_c)/\gamma_h(2\bar{n}_h + 1) + \gamma_c(2\bar{n}_c + 1)$. The solution reads:

$$J_0^\infty = h\omega_0(\bar{n}_h - \bar{n}_c)\frac{\gamma_h\gamma_c}{\gamma_h(2\bar{n}_h + 1) + \gamma_c(2\bar{n}_c + 1)},$$

which is positive (i.e. flowing from the hot bath to the cold bath) as expected and non-zero as long as $T_h > T_c$ such that $\bar{n}_h > \bar{n}_c$.

We now suppose that the qubit is quasi-ressonantly driven by a monochromatic field of frequency $\omega_d$, which can be modelled by adding a term $H_d(t) = h\gamma(\cos(\omega_d t)\sigma_- + e^{-i\omega_d t}\sigma_+)$ in its Hamiltonian, now time-dependent. We have denoted $\gamma$ the field-matter coupling strength. This term induces a rotation of the state of the qubit in the Bloch sphere along the rotating unit vector $\vec{i}(t) = (\cos(\omega_d t), \sin(\omega_d t), 0)$. In the limit where $g, |\delta| \ll \omega_0, \omega_d$, with $\delta = \omega_q - \omega_d$ the detuning, and provided the spectral densities of the reservoirs are flat around the frequency $\omega_0$, the dissipation induced by the bath is unchanged by the presence of the drive (see [23, 25]). Therefore, the evolution of the density operator of the qubit is ruled by the same master equation.
FIG. 1. Principle of the switch. The qubit is driven coherently quasi-resonantly at frequency $\omega_d = \omega_0 - \delta$, the driving-qubit coupling strength being denoted $g$, and coupled to two thermal reservoirs at temperatures $T_h > T_c$. When choosing setting the driving to fulfill $g = g^\ast(0)$ (see text) and $\delta = 0$, the heat flow $J_h$ provided by the hot reservoir is completely suppressed despite the qubit is not in thermal equilibrium with the hot bath.

as before (see Eq. 1) except that the Hamiltonian part of the dynamics is generated by $H_0 + H_d(t)$ instead of $H_d$. The competition between the driving and the dissipation results in a stationary orbit of the qubit’s state of the form $\pi(t) = U_{\text{rot}}^\dagger \tilde{\pi} U_{\text{rot}}$, where $U_{\text{rot}} = e^{i\omega_d t/2}$ is the unitary transformation to the frame rotating at the driving frequency and $\tilde{\pi} = (\mathbb{I} + \mathbf{r}_\infty \cdot \vec{\sigma})/2$ is the steady state reached by the qubit in such rotating frame. We have denoted $\mathbf{r}_\infty = (x_\infty, y_\infty, z_\infty)$ the steady-state Bloch vector in the rotating frame and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ the vector of Pauli matrices. The exact expression of the steady state can be found analytically, yielding:

$$x_\infty = -\frac{2\delta g(\gamma_h + \gamma_c)/\gamma_{\text{tot}}}{2g^2 + \gamma_{\text{tot}}^2 + 4\delta^2}, \quad (4a)$$

$$y_\infty = \frac{g(\gamma_h + \gamma_c)}{2g^2 + \gamma_{\text{tot}}^2 + 4\delta^2}, \quad (4b)$$

$$z_\infty = -\frac{(\gamma_h + \gamma_c)(2g^2 + \gamma_{\text{tot}}^2 + 4\delta^2)}{\gamma_{\text{tot}}(2g^2 + \gamma_{\text{tot}}^2 + 4\delta^2)}, \quad (4c)$$

with $\gamma_{\text{tot}} = \gamma_h(2n_h + 1) + \gamma_c(2n_c + 1)$.

Note that in contrast with $\rho_0$, the stationary orbit state carries non-zero average value of the coherences of constant modulus $|\langle e|\pi(t)|g\rangle| = \sqrt{x_\infty^2 + y_\infty^2}$ in the free-qubit energy eigenbasis $\{|e\}, \{|g\}\}$, where $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$. Those coherences are characterized by a contribution in phase with the driving $\dot{x}_\infty = \text{Tr}\{(\vec{u}(t) \cdot \vec{\sigma})\pi(t)\}$ and out of phase $\dot{y}_\infty = \text{Tr}\{[\vec{u}(t) \cdot \vec{\sigma}]\pi(t)\}$, with $\vec{u}(t) = (-\sin(\omega_d t), \cos(\omega_d t), 0)$ is a vector orthogonal to $\vec{u}(t)$.

**Stationary heat flow.**—In presence of the quasi-resonant drive, the heat flow, defined as the energy provided by the hot reservoir to the qubit, takes the value $J_h(t) = J_{cl}(t) + J_q(t) = \text{Tr}[H_0 L_h^\dagger \rho(t)] + \text{Tr}[H_d(t)L_h^\dagger \rho(t)]$. The contribution $J_{cl}(t) = -\gamma_h(2n_h + 1)(P_c(t) - \bar{n}_h/(2\bar{n}_h + 1))$, with $P_c(t) = \langle e|\rho(t)|e\rangle$ can be interpreted as the heat flow in the case of a classical two-level system, unable to carry coherences in the $\{|e\}, \{|g\}\}$ basis. The contribution $J_q(t) = -\gamma_h(2\bar{n}_h + 1)\hbar g(\dot{x}_\infty)/4$, with $\dot{x}_\infty = \text{Tr}\{(\vec{u}(t) \cdot \vec{\sigma})\rho(t)\}$, involves only the coherences in the $\{|e\}, \{|g\}\}$ basis and is therefore a genuinely quantum contribution. It encompasses the price for the reservoir to erase the coherences in phase with the driving, which contribute to the energy stored in the qubit’s state via the term $E_q(t) = \text{Tr}[H_d(t)\rho(t)] = \hbar g(\dot{x}_\infty)/4$. Conversely, the coherences out of phase $\dot{y}(t) = \text{Tr}\{[\vec{v}(t) \cdot \vec{\sigma}]\rho(t)\}$ do not contribute to the qubit’s energy and do not play any role in the heat flow. A more detailed analysis of this contribution and the thermodynamics of the driven qubit can be found in Ref. [25].

**Quantum heat switch.**—The stationary value of the heat flow can be controlled by engineering the steady-state of the qubit, which in turn can be adjusted by tuning the driving parameters, namely the coupling strength $g$ (determined by the driving intensity) and the detuning $\delta$. We first show that the classical part of the heat flow can be completely switched off. The key idea is that this contribution is zero if the population of the qubit in the $\{|e\}, \{|g\}\}$ basis matches the thermal equilibrium with the hot reservoir, i.e. $P_{e,c} = \bar{n}_h/(2\bar{n}_h + 1)$. For each fixed value of the detuning $\delta$, this can be realized for a particular value of the driven intensity $g^\ast(\delta)$ found by solving the equation $(x_\infty + 1)/2 = P_{e,c}$, yielding:

$$g^\ast(\delta) = \left[\frac{2\gamma_c}{\gamma_{\text{tot}}}(\gamma_{\text{tot}}^2 + 4\delta^2)(\bar{n}_h - \bar{n}_c)\right]^{1/2}.$$  \hspace{1cm} (5)

From the proportionality to the square root of the thermal occupation difference $n_h - n_c$, it is clear that the classical part of the heat current can be suppressed solely in the presence of a colder bath at temperature $T_c < T_h$.

Even when the population of the qubit matches its value at thermal equilibrium with the hot bath, the stationary state still differs from the thermal equilibrium state $\rho_{eq} = e^{-H_0/kT_h}/\text{Tr}\{e^{-H_0/kT_h}\}$ because of coherences in the $\{|e\}, \{|g\}\}$ basis. This results in a non-zero value of the stationary quantum contribution $J_q^\infty$ of the heat current. It is remarkable that the present setup allows to separate the classical and quantum contribution by canceling $J_{cl}^\infty$. Setting $g = g^\ast(\delta)$ and measuring the slight temperature variations of the hot reservoir provides a method to measure the quantum contribution to the heat flow (see also experimental proposal below).

The steady state quantum heat flow takes the value:

$$J_q^\infty = \hbar \delta \frac{\gamma_h \gamma_c}{\gamma_{\text{tot}}} (\bar{n}_h - \bar{n}_c).$$  \hspace{1cm} (6)

This contribution to the heat flow can therefore be switched off by simply driving the qubit at resonance, i.e. for $\delta = 0$. The total heat flow from the hot reservoir $J_h^\infty = J_{cl}^\infty + J_q^\infty$ can therefore be controlled externally by tuning the parameters of the drive. Surprisingly, when setting the drive parameters to $\rho_{eq}$, the heat flow provided by the hot reservoir is zero even though the qubit state differs from the thermal equilibrium state at temperature $T_h$, because of the non-zero
value of the out-of-phase coherences equal for these parameters to \( \tilde{\gamma}_c = (\bar{n}_h - \bar{n}_c)\gamma_c/\gamma_{\text{tot}} \) for this special value of the parameters, the power provided by the driving \( P^\infty = \text{Tr}\{\dot{H}_d(t)\pi(t)\} \) takes the value \( h\omega_0\gamma_c(\bar{n}_h - \bar{n}_c)/(2\bar{n}_h + 1) \). This value is positive, meaning that the switch must be sustained by a constant amount of power, which is eventually dissipated in the cold bath.

As an illustration of the quantum heat switch effect, we plot in Fig. 2 the value of \( J_h^\infty \) for \( g = g^*(0) \) when sweeping the detuning across resonance. The heat flow goes from 0 to the value \( \bar{n}_h = 5 \cdot 10^{-3}, \bar{n}_c = 10^{-2} \).

Hamiltonian of the qubit in the charge basis is given as

\[
H = E_C\delta n_g(t)\sigma_z - \frac{E_J}{2}\sigma_z.
\]

FIG. 2. Stationary heat flow \( J_h^\infty \) (solid blue) provided by the hot thermal reservoir and power injected by the drive (orange dashed) for \( g = g^*(0) \) (see Eq. 5) as a function of the detuning \( \delta \). Parameters: \( \gamma_h = \gamma_c = \gamma, \omega_0 = 100\gamma, \bar{n}_h = 5 \cdot 10^{-3}, \bar{n}_c = 10^{-2} \).

Implementation in a superconducting circuit: We now analyze the feasibility of the scheme in a typical superconducting quantum circuit setup. Superconducting qubits are versatile candidates to perform different quantum thermodynamic experiments as they can be controlled externally and measured with high precision. \[26-28\]

The superconducting qubit (two-level system) can be a transmon \[26\], a flux qubit \[29\] or a charge qubit \[30\]. In this letter, we discuss the implementation of the heat switch using a charge qubit, which is based on a superconducting island with very small capacitance. This island is capacitively coupled to the baths and driving electrode, and terminated by a superconducting loop made of two small Josephson junctions, as shown in Fig. 3.

Once quantized, the circuit behaves as an anharmonic oscillator such that the two lower levels can be addressed independently from other levels and treated as a qubit. For an appropriate choice of constant voltage \( V_{DC} \), the

FIG. 3. (a) Schematic experimental set up for the implementation of a quantum heat switch in a superconducting circuit. (b) First and second energy levels (for \( E_g > E_J \)) versus \( \delta n_g \) are plotted. We only drive the system very close to \( \delta n_g = 0 \).

where the charging energy \( E_C = e^2/2C_\Sigma \), the total capacitance of the island \( C_\Sigma = C_c + C_h + C_g + C_J \), and \( E_J = E_J(\Phi) \) is the Josephson energy that can be tuned with an external flux \( \Phi \). Note that with respect to the general analysis above, the free qubit quantization axis and the driving axis have been swapped to match usual conventions in the field.

Here \( \delta n_g(t) = C_gV_g/(2e) \) is driven near the avoided level crossing \( \delta n_g = 0 \), with a voltage \( V_g \cos(\omega_L t) \) and gate capacitance is \( C_g \). At \( \delta n_g = 0 \), the energy gap is \( E_J \). Further, two normal-metal resistors act as the heat baths whose temperatures can be controlled and measured. The qubit is capacitively coupled to the heat baths in order to achieve a weak coupling of the qubit with the environment. The transition rates are given by \( \gamma_{h,c} \approx C_{h,c}^2E_JR_{h,c}/(2hC_\Sigma^2R_Q) \), where \( R_Q = h/4e^2 \) is the superconducting resistance quantum. \[31-32\]. The temperatures of the baths can be taken in the range of 30 mK to 200 mK such that the populations in the higher excited states of the qubit can be ignored. For realistic parameters (see Supplement), the driving amplitude \( g \) can be of the order of 10 MHz.

Discussion and conclusion: The scheme presented is
still valid in presence of pure dephasing at rate $\gamma_\phi$. In this case, the value of the driving strength $g$ allowing to suppress the classical heat flow has an expression similar to Eq. (5), except replacing $\gamma_{tot}$ with $\gamma_{tot} + 2\gamma_\phi$ (see Supplement). The quantum heat flow is still canceled at resonance. In fact, the charge qubit setup may allow a good test of this property, since detuning the qubit away from the avoided level crossing makes the qubit much more sensitive to charge noise and therefore increases the dephasing rate \cite{33}.

In this letter, we have demonstrated that a driven quantum two-level system can be operated as a quantum heat switch: in this active device, tuning the external parameters of the drive activates or suppresses the heat current provided by a thermal reservoir. We have proposed an implementation of the scheme in a circuit QED setup and analyzed its feasibility in state-of-the-art setups.

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The new steady state of the qubit (in the rotating frame) is then:

\[ \dot{Y}(t) = AY(t) \]  

where

\[ A = \begin{pmatrix} -\gamma_h(n_h + 1) - \gamma_c(n_c + 1) & 0 & g & \gamma_h n_h + \gamma_c n_c \\ -\frac{3}{2}(2n_h + 1) - \frac{5}{2}(2n_c + 1) & \delta & \gamma_h n_h + \gamma_c n_c & 0 \\ \gamma_h(n_h + 1) + \gamma_c(n_c + 1) & 0 & -\frac{3}{2}(2n_h + 1) - \frac{5}{2}(2n_c + 1) & g \\ \end{pmatrix} \]  

Solving Eq. (8) for its stationary state allows to derive Eqs. (4a)-(4c).

Effect of additional decoherence channel

A dephasing channel causing a decay of the coherences in the \{(|e\rangle, |g\rangle)\} basis of the qubit at a rate \( \gamma_\phi \) can be modeled by an additional term \( \mathcal{L}_\phi[\rho] = (\gamma_\phi/2)\mathcal{D}_{\sigma_+}[\rho] \) in the master equation of the qubit Eq. (3). This results in a new evolution for \( Y(t) \), given by \( \dot{Y}(t) = A_1 Y(t) \), with

\[ A_1 = \begin{pmatrix} -\gamma_h(n_h + 1) - \gamma_c(n_c + 1) & 0 & g & \gamma_h n_h + \gamma_c n_c - \gamma_\phi \\ 0 & -\frac{3}{2}(2n_h + 1) - \frac{5}{2}(2n_c + 1) - \gamma_\phi & -\delta & \gamma_h n_h + \gamma_c n_c - \gamma_\phi \\ -\frac{3}{2}(2n_h + 1) - \frac{5}{2}(2n_c + 1) & \delta & -\frac{3}{2}(2n_h + 1) - \frac{5}{2}(2n_c + 1) & g \\ \gamma_h(n_h + 1) + \gamma_c(n_c + 1) & 0 & -g & -\gamma_h n_h - \gamma_c n_c \\ \end{pmatrix} \]  

The new steady state of the qubit (in the rotating frame) is then:

\[ \dot{\bar{x}}_\infty = -\frac{4\delta g(\gamma_h + \gamma_c)}{2g^2(\gamma_{tot} + 2\gamma_\phi) + \gamma_{tot}((\gamma_{tot} + 2\gamma_\phi)^2 + 4\delta^2)} \]  

\[ \dot{\bar{y}}_\infty = \frac{2g(\gamma_h + \gamma_c)(\gamma_{tot} + 2\gamma_\phi)}{2g^2(\gamma_{tot} + 2\gamma_\phi) + \gamma_{tot}((\gamma_{tot} + 2\gamma_\phi)^2 + 4\delta^2)} \]  

\[ \dot{\bar{z}}_\infty = -\frac{(\gamma_h + \gamma_c)((\gamma_{tot} + 2\gamma_\phi)^2 + 4\delta^2)}{2g^2(\gamma_{tot} + 2\gamma_\phi) + \gamma_{tot}((\gamma_{tot} + 2\gamma_\phi)^2 + 4\delta^2)} \]  

As before, we solve \((\ddot{z}_\infty + 1)/2 = P_c^h\) to find the value of \( g \) canceling the classical heat flow from the hot bath. We obtain:

\[ g^*(\delta) = \left[ \frac{\gamma_c}{\gamma_{tot} + 2\gamma_\phi} \left((\gamma_{tot} + 2\gamma_\phi)^2 + 4\delta^2\right) (\bar{n}_h - \bar{n}_c) \right]^{1/2}. \]  

The heat flow from the hot bath \( J_{h\infty}^\infty = \text{Tr}\{H\mathcal{L}_h[\rho]\} \) then reads for \( g = g^*(\delta) \):

\[ J_{h\infty}^\infty = J_{q\infty}^\infty = \hbar \delta \frac{\gamma_h \gamma_c}{\gamma_{tot} + 2\gamma_\phi} (\bar{n}_h - \bar{n}_c), \]  

such that as for \( \gamma_\phi = 0 \), the choice \( \delta = 0 \) allows to completely stop the heat current from the hot bath.
Feasible experimental parameters

The capacitances $C_h$ and $C_c$ must be chosen so that they do not exceed $C_J$, the Josephson junctions’ capacitance (because the charging energy should be high enough), while allowing reasonable coupling to resistors $R_h$ and $R_c$. Meanwhile, $C_g$ should satisfy $C_g \ll C_J$. The charging energy is $E_C = e^2/2C_\Sigma$. A choice of experimentally accessible parameters that may satisfy all these requirements is: $C_J = 2 \times 0.3$ fF (factor 2 because of two junctions), $C_h = C_c = 0.3$ fF, $C_g = 0.03$ fF. For the resistors, we can use a large range depending on the desired relaxation rate. Highly sensitive measurements need high resistances. For example, AuPd resistors, which enable NIS thermometry, allows to reach the resistance value around 1 kΩ, easily. With a set of parameters chosen so as to maximize $\gamma_{h,c}$, and supposing a typical Josephson energy $E_J/h = 6$ GHz, one obtains $\gamma_{h,c} \approx 28$ MHz, which can be easily lowered with the resistances values. Further, taking the temperatures $T_h = 120$ mK and $T_c = 80$ mK, the amplitude of the drive $g(0) \approx 11$ MHz, which can be varied by changing temperatures or $\gamma_{h,c}$. The driving frequency in resonance is $E_J/h$. 
