Moving Horizon Scheduling of an Air Separation Unit under Fast-Changing Energy Prices

Richard Pattison ∗ Cara R. Touretzky ∗ Ted Johansson ∗∗
Michael Baldea ∗ Iiro Harjunkoski ∗∗∗

* McKetta Department of Chemical Engineering, The University of Texas at Austin, USA
** Department of Chemical Engineering and Technology, KTH Royal Institute of Technology, Sweden
∗∗ ABB Corporate Research, Ladenburg, Germany

Abstract: Maximizing the benefits of time-of-use pricing for industrial electricity consumers requires varying production rates, such that energy use is shifted from peak price periods to off-peak times during the day. Assuming that excess capacity and product storage are available, production of energy intensive processes can be increased at off-peak times beyond nominal rates, and the stored product can be used at peak times when the production rate is lowered. Under these rapidly changing circumstances, scheduling calculations must take into consideration explicitly the dynamic model of the process, often rendering the scheduling problem intractable in practical amounts of time. To address this challenge, we introduce a class of scheduling-relevant low-order process models, which capture the closed-loop input-output behavior of a plant. We use these models to close the scheduling loop, whereby the scheduling problem is formulated over a moving horizon with feedback. We apply the theoretical concepts to an industrial-scale air separation unit model, demonstrating that variable production rate operation with product storage has the potential for significant operating cost savings while abiding by product quality and safety constraints.

Keywords: Scheduling, Flexible manufacturing, Low-order closed-loop dynamic models

1. INTRODUCTION

The deregulation of energy markets, along with inherent fluctuations in electricity demand, have led to the implementation of time-of-use electricity pricing, especially for large-capacity industrial consumers. Operating under such pricing structures calls for increased flexibility of production systems. Assuming that excess capacity and product storage facilities are available or can be installed at a reasonable cost, energy consumption should be lowered at “peak” times, when grid demand increases and energy tariffs are high, and increased at “off-peak” times, when energy prices drop. The amount of product generated in excess in the latter case is stored, and used to satisfy demand at peak times. The potential cost savings derived from applying this intuitive strategy can be significant; depending on the location, the discrepancy between off-peak and peak energy tariffs can be of an order of magnitude or more.

Optimal scheduling of production rate targets is critical for taking advantage of time-of-use pricing opportunities. However, operating under such fast changing market conditions poses unique challenges, since more frequent production target changes are required than usually considered in chemical production scheduling. Equivalentlty, the emerging overlap between the time scale of scheduling decisions and the time scale of the process response requires that scheduling calculations explicitly account for process dynamics.

Embedding a representation of the process dynamics in scheduling calculations is an onerous task (Baldea and Harjunkoski, 2014). The detailed dynamic models of process systems are almost invariably large scale, stiff, highly nonlinear and potentially discontinuous; as such, the (mixed-integer) program that corresponds to the integrated scheduling and dynamic optimization/control problem is likely very difficult (if not impossible) to solve in a practical amount of time. Furthermore, in order to account for updated processing conditions, and energy prices in particular, scheduling calculations must be repeated periodically.

In this paper, we address this significant challenge via a novel moving-horizon scheduling formulation incorporating low-order dynamic models. We rely on data-driven representations of the closed-loop evolution of scheduling-relevant process variables (states and outputs) (Pattison et al., 2015). We close the scheduling loop through feedback of the process variables, coupled with a state observer for updating the model states.

This framework is applied to an industrial-scale air separation unit (ASU) model. ASUs represent a major electricity consumer. In 2010, the cryogenic air product sector accounted for about 2.5% of the total industrial consumption of the United States (US EIA, 2010). ASU operations can therefore benefit significantly from time-of-use electricity pricing; moreover, modulating ASU production can have a positive impact on the grid. Indeed, several past studies have demonstrated that price fluctuations can be leveraged to reduce operating costs (Miller et al., 2008), and production scheduling under time-sensitive energy prices was investigated (Zhu et al., 2010; Pattison and Baldea, 2014; Ierapetritou et al., 2002; Karwan and Keblis,
2. BACKGROUND: SCHEDULING UNDER DYNAMIC CONSTRAINTS

Figure 1 presents the hierarchy of decision making for process operations. For simplicity, we focus on a class of chemical processes with a single product stream and storage capabilities. Decisions are separated by their time scale or time horizon. The planning layer establishes \( \tilde{y} \), the (long-term) targets of product grade and quantity/production rate. This forecast is determined by business units tasked with satisfying contractual agreements related to product quantity and quality. Scheduling determines how to meet the planning target by establishing the optimal sequence of setpoints for production \( (y^{sp}_p) \), inventory storage \( (u^{sp}_p) \), and inventory utilization \( (y^{sp}_inv) \) which minimize operating costs or maximize profit. The control layer manipulates specific variables \( (u_p \text{ and } u_{inv}) \) to track the setpoint trajectories.

Conventionally, scheduling methods rely on tabulated process transition information to account for process dynamics. For example, it is typically assumed that the process control system can adjust the plant output to a new setpoint within a fixed transition period. Moreover, the transition time is considered to be negligible compared to the amount of time that the process is operated at steady state. As a result, the closed-loop process dynamics (as defined by the control and process layers in Figure 1) are not explicitly modeled in the scheduling calculation. However, when setpoint changes are made on a time scale that is comparable to the dominant process time constant (as is necessary to capitalize on fast-changing electricity prices), it is likely that process will not settle to steady state in the time elapsed between consecutive setpoint changes. Thus, the closed-loop process dynamics and operating constraints must be considered in the scheduling problem formulation to ensure that the sequence of setpoint changes is feasible from a dynamic point of view.

The slot-based production scheduling problem for the class of systems considered can be formulated using a detailed dynamic process model as follows:

\[
\begin{align*}
\min_y & \quad y^{sp,n}_{p,\alpha,n} \cdot (\phi(p, y_p, y_{inv}, \tilde{y})) dt \\
\text{Subject To:} & \\
\text{Timing constraints} & \quad t^n_{end} = t^n_{start} + \tau^n \\
& \quad t^n_{start} = t^{n-1}_{end} \\
& \quad t^1_{start} = 0 \\
& \quad t^n_{end} = T_m \\
\text{Dynamic process model} & \quad f_p(x_p, x_{inv}, z_{inv}, u_{inv}, v_{inv}, t) = 0 \\
& \quad g_p(x_p, z_{inv}, u_{inv}, v_{inv}, t) = 0 \\
& \quad u_p = K_p(x_p, z_{inv}, y^{sp}_p, t) \\
\text{Storage system model} & \quad f_{inv}(z_{inv}, x_{inv}, z_{inv}, u_{inv}, v_{inv}, t) = 0 \\
& \quad g_{inv}(x_{inv}, z_{inv}, u_{inv}, v_{inv}, t) = 0 \\
& \quad u_{inv} = K_{inv}(x_{inv}, z_{inv}, y^{sp}_{inv}, t) \\
\text{Product split and mixing} & \quad v_{inv} = g_{split}(\alpha, y_p) \\
& \quad \tilde{y} = g_{mix}(y_p, y_{inv}, \alpha) \\
\text{Constraints} & \\
\text{Inventory:} & \quad h_{inv}(x_{inv}, z_{inv}, u_{inv}, t) \leq 0 \\
\text{Quality:} & \quad h_{product}(\tilde{y}, y_{inv}, \alpha) \leq 0 \quad \forall t \\
\text{Process:} & \quad h_{process}(x_p, z_{inv}, u_{inv}, y^{sp}_p, t) \leq 0 \\
\end{align*}
\]

The variables are defined in the nomenclature Table 1. In this formulation, the objective is to minimize the cost of electricity \( \phi \), which is a function of the process throughput, over the horizon for which price forecasts are available, \( T_m \). Equations (2) establish the timing of scheduling events. The decision variables (DVs) are the production setpoints in every time slot, which are converted into a continuous-time setpoint signal in (2). Dynamic models of the process (3), product storage tank (4), and product and inventory mixing and splitting equations (5) are included in the formulation. Constraints are imposed on inventory, product quality, and process operating states (6).

Explicitly embedding dynamic models and path constraints in the optimization formulation is challenging because detailed models are high dimensional and involve highly coupled and nonlinear equations. Finding a solution in a short time frame, so that the result would be useful for online implementation, is very difficult (Bansal et al., 2003; Biegler, 2007). Recent results suggest that approaches based on reduced-order models are required (Du et al., 2015; Pattison et al., 2015). In the following section, we discuss moving horizon production scheduling in the context of data-driven low-order dynamic models.

3. MOVING-HORIZON SCHEDULING WITH LOW-ORDER DYNAMIC MODELS

3.1 Scale-bridging models

Scheduling-oriented low-order models, which –owing to their mission of bridging the times scales between process dynamics and control, and scheduling decisions– we refer to as “scale-bridging models” (SBMs), must satisfy two key-requirements: i) relevance, in the sense that the dynamics captured must be
relevant to scheduling calculations, and, ii) accuracy, in the
sense that approximation must be an accurate representation of
the process dynamics.

In our recent work (Pattison et al., 2015), we have addressed
the relevance requirement by formulating the following propos-
iton:

**Proposition 1.** (based on Pattison et al. (2015)) Consider a
complex process with multiple operating constraints related to
the process performance, efficiency, and safety, and assume that
the process is closed-loop stable for all production rate/product
choice combinations. Then, the subset of constraints (and
SBMs) relevant to the scheduling calculation are the constraints
that closely approach or reach their corresponding bounds dur-
ing steady state operation and/or during transitions between
operation setpoints/targets.

Such scale-bridging models can be derived either from applying
appropriate transformations and model-reduction techniques to
first principles system representations (see, e.g., Baldea and
Daoutidis (2008); Jogwar et al. (2009); Touretzky and Baldea
(2014); Baldea and Daoutidis (2012)). Alternatively, system
identification techniques are available to derive scale-bridging
models from operating data. We note that the latter approach
is particularly advantageous for existing processes, where his-
torical data reflecting production target changes that cover the
range of potential production setpoints provide a rich source of
information for model identification (MacGregor and Cinar,
2012).

### 3.2 Scheduling with scale-bridging models

The optimization problem (1) can be reformulated to include
the low-order dynamics of scheduling-relevant variables and
constraints:

\[
\begin{align*}
\text{minimize} & \quad J = \int_0^{T_m} \phi(w, y_{\text{inv}}, \hat{y}) dt \\
\text{Subject to:} & \quad \text{Time slots (2)} \text{\hspace{1cm} Inventory model (4)} \text{\hspace{1cm} Production split/mixing ratio (5)} \text{\hspace{1cm} Process model:} \\
& \quad \text{Low-order dynamic process models (SBMs) for each identified variable } w \text{ (10):} \\
& \quad \text{Inventory: (6a)} \\
& \quad \text{Quality: } \hat{h}_{\text{product}}(w, \hat{y}, t) \leq 0 \quad \text{(8)} \\
& \quad \text{Process: } \hat{h}_{\text{process}}(w, y_{pp}, t) \leq 0 \quad \text{(9)}
\end{align*}
\]

This formulation alters the original problem considering the
full-order dynamic process model (1) by, i) considering only
the scheduling relevant subset of product quality and process
operating constraints ( \( \hat{h}_{\text{product}} \) and \( \hat{h}_{\text{process}} \)). Additionally, ii) the relevant variable trajectory predictions, \( w(t) \), are deter-
mined using low-order dynamic models rather than the detailed
dynamic process model (3).

In this work, we rely on Hammerstein-Weiner models of the
form (10) to capture the nonlinear dynamic behaviour of the
relevant variables. Here, \( \Psi \) and \( \Phi \) are nonlinear transformations
of the input (i.e., the production setpoints) and output. The
matrices \( A \), \( B \) and \( C \) describe a linear state-space model. The
single output variable \( w \) contains dynamic information of a
variable relevant to the scheduling calculation.

\[
\begin{align*}
\dot{u'} &= \Psi(y_{pp}^p) \quad \text{(10a)} \\
\dot{x} &= Ax + Bu' \quad \text{(10b)} \\
y' &= Cx \quad \text{(10c)} \\
w &= \Phi(y') \quad \text{(10d)}
\end{align*}
\]

We note that the optimal scheduling formulation is agnostic to
the form of the low-order dynamic models.

### 3.3 Moving horizon scheduling formulation

The solution of the scheduling problems formulated above,
whether using a full-order process model (3) or using the
low-order scale-bridging models (10), provides a sequence of
production targets that covers the entire horizon \( T_m \). While
sufficient in the ideal case when energy price forecasts are
available for an extended amount of time and the process model
is a perfect representation of the plant dynamics, this solution
is likely suboptimal in practical situations when energy prices
may fluctuate and the predicted process dynamics may not be
accurate owing to disturbances and plant-model mismatch.

The above observations are best captured by drawing an anal-
ylog with control systems. Namely, determining the schedule for
the entire horizon \( T_m \) amounts to an “open loop” solution which
lacks process feedback. Thus “closing the loop” for pro-
ductions scheduling requires incorporating process feedback.
Our approach is based on the receding horizon control frame-
work (and, more specifically, on economic model predictive
control (Amrit et al., 2013; Ellis et al., 2014)), in that we pro-
pose implementing the low-order formulation (7) on a periodic
basis, with the time horizon correspondingly shifted in time.

This effectively amounts to a periodic rescheduling, with the
energy price forecast being updated at each time point. More-
over, measurements of the process states are used to update
the states of the model at each time point. Our moving horizon
scheduling framework is described in the algorithm below:

1. Optimize the schedule using the low-order model over
   the horizon \( T_m \) for which price and demand forecasts are
   available
2. Implement the optimal schedule on the plant (here we
   use a detailed model to represent the plant) and track
   measurements of scheduling-relevant process variables
3. Use a state observer to determine the trajectories of the
   states of the scale-bridging models
4. When new price and demand forecasts are available, or a
   disturbance is detected, return to optimization problem (7) and:
   - Shift the time horizon by the amount of time elapsed
   - Update the initial conditions of the low-order model with
     the states of the observer
   - Update price and demand forecasts
   - Update endpoint constraints for inventory levels (to
     avoid depletion of product inventory)
5. Return to step 1

In conjunction with the Hammerstein-Wiener models (10), we
use a high-gain (Luenberger) observer to update the model
states \( x \):
\( u' = \Psi(y_{sp}^p) \) \hfill (11a)
\[
\dot{x} = Ax + Bu' + L(\hat{w} - w) \quad \text{(11b)}
\]
\( y' = Cx \) \hfill (11c)
\( w = \Phi(y') \) \hfill (11d)

where \( \hat{w} \) is the measurement of the scheduling relevant variable. The states \( \bar{x} \) are assumed to be observable.

### 3.4 Comparison to Economic MPC

While the objective of the two methods is the same, i.e., to minimize operating costs over a forecast horizon while meeting operational constraints, there are several important differences between our proposed moving horizon scheduling framework using SBMs and economic MPC (EMPC).

1. **Model Structure:** The SBMs used in this work are single-input multi-output, and capture the closed-loop dynamics of the scheduling-relevant variables in response to a product target sequence, i.e., the input is the production target (product flowrate or quality) and the outputs are the evolution of the scheduling-relevant variables assuming that a (supervisory) control system is effectively guiding the plant throughout the production target changes. In contrast, EMPC models are multi-input multi-output, where the inputs are the manipulated variables (or setpoints for distributed control loops) and the outputs are the controlled variables throughout the process.

   SBMs have a single input and only several outputs resulting in a relatively small, sparse model (Pattison et al., 2015). MPC systems often accounts for \( m \) manipulated variables and \( n \) control variables (where \( m \) and \( n \) can be on the order of tens to hundreds) resulting in a much larger, non-sparse process model (Qin and Badgwell, 2003).

2. **Execution Frequency:** An EMPC system must ensure that the process is stabilized throughout the horizon, and thus, it must be updated frequently (re-optimize the manipulated variable trajectories) to compensate for high frequency disturbances or plant-model mismatch. In contrast, the moving horizon scheduling framework assumes that a process control system is guiding (and stabilizing) the plant throughout transitions, and thus, the schedule can be re-optimized infrequently (e.g., when new price or demand forecasts are available). Conceivably, constraints could be imposed to ensure that the process remains within stability limits of the control system.

### 4. MOVING HORIZON ASU SCHEDULING

We consider an air separation unit (ASU) as shown in Figure 2. Inlet air is compressed to 6.8 bar then cooled and partially liquefied in a multistream heat exchanger (PHX). The partially liquefied air stream is fed to the bottom of a cryogenic distillation column where nitrogen, the lightest component, is concentrated in the distillate. The bottoms of the column are adiabatically expanded to provide cooling to condense the vapor at the top of the column. This occurs in an integrated reboiler/condenser where a minimum temperature driving force of 1.85°C must be maintained. The nitrogen gas product and a nitrogen waste stream from the reboiler are passed through the PHX and warmed against the condensing air stream. A separate liquefier is included in the process flowsheet, along with a liquid nitrogen storage tank and an evaporator to allow further modulation of the process throughput, by overproducing, liquefying and storing nitrogen product during periods of low electricity price and under-producing during periods of high electricity price while evaporating the stored product to meet demand. The detailed model of the process contains 6094 equations and 430 state variables; the reader is referred to the works of Cao et al. (2015); Pattison et al. (2015) for more details.

Our preliminary results (Pattison et al., 2015) indicated that solving the scheduling problem using the full-order dynamic process model is impractical; for example, obtaining a solution for a three day (72-hour) horizon required in excess of four days of CPU time.

In order to derive scale-bridging models for this process, we used Proposition 1 to identify the following scheduling-relevant variables (Pattison et al., 2015):

- production flowrate
- inlet air flowrate
- product purity (modeled in terms of impurity concentration)
- flooding fraction in the distillation column
- temperature difference across the reboiler/condenser
- pressure of the air exiting zone 1 of the PHX
- pressure of the air exiting zone 2 of the PHX
- liquid level in the reboiler

We derived Hammerstein-Wiener models capturing the dynamics of each variable. The models follow the same structure,
Fig. 4. Production rate setpoint. The nominal flowrate (demand) is 20mol/s and the production rate may vary by up to 20% having production rate target (setpoint) as the input, and the output being the predicted trajectory of the aforementioned variables. The model order and the functional form of the input and output nonlinearities were chosen to obtain the best fit.

4.1 Results

We assume that accurate 2-day forecasts of time-variable electricity prices are available daily at noon (Figure 3). Thus, the algorithm in section 3 was implemented with a rescheduling period of 1 day (when new forecasts are available) with a two-day prediction horizon. A terminal point constraint (Subramaniam et al., 2012) \((M_{inv} \geq 0.5 M_{inv0})\) was implemented to stabilize the inventory level throughout the horizon. The optimal production setpoint and inventory levels to meet a constant demand of 20mol/s with impurity levels below 2000ppm are shown in Figures 4 and 5. Markers indicate points where the schedule was recalculated. While there are eight scheduling-relevant constraints and models, we will focus our discussion on the impurity and temperature driving force across the reboiler/condenser because these are examples of variables that evolve, respectively, over very slow and fast time scales and are near their bounds.

As expected, the production flowrate setpoint increases when prices are low, thus the inventory level in the liquid nitrogen storage tank rises. Then, when prices are high, the production target setpoint is lowered to the minimum allowed value and stored product is used to satisfy the remaining demand. Note that the maximum production rate (24mol/s) is only reached on several brief occasions; this is due to the fact that several operating constraints are near their bound when production rate is increased significantly in a short time interval. The impurity level and reboiler/condenser temperature difference, shown in Figures 6 and 7, are near their bounds at these times. Overall, the variable capacity operation results in an electricity cost savings of $1620 (or 4.85%) over the 4-day horizon in comparison to a constant production profile set at the nominal rate.

In this case study, we assume that there are no disturbances, and plant-model mismatch is minimal. However, in reality, processes are always subject to disturbances, and this framework allows for a rapid rescheduling calculation online when a disturbance is detected. We note here that the average CPU time for each optimization calculation was around 30 min on a desktop computer, a significant reduction compared with the computational effort required to solve the equivalent problem based on the full-order model of the process.

5. CONCLUSIONS

In this work we have presented a framework for scheduling production of processes operating under fast-changing market conditions, notably, under rapidly varying electricity prices. Our approach is based on using low-order dynamic models derived from operating data to capture the dynamics of process variables relevant to scheduling calculations. Scheduling calculations are carried out in a receding horizon fashion; thus,
updated energy price forecasts can be exploited by repeating the scheduling calculation as soon as new forecasts become available. The online implementation on a detailed air separation unit model shows that significant cost savings can be achieved (4.85%) by optimally increasing the production rate during periods of low electricity price and storing excess liquefied product, then ramping down production when prices are high and gasifying stored product to meet demand. The computational performance of the proposed framework is excellent, and provides incentives for real-time implementation with frequent rescheduling.

| Table 1. Nomenclature |
|-----------------------|
| **Variable** | **Description** |
| $\alpha$ | Fraction of process output bypassing storage unit |
| $\alpha_{\text{sp}}$ | Setpoint for $\alpha$ |
| $t$ | Time |
| $I$ | Objective function |
| $\bar{y}$ | Production target characteristics |
| $\bar{y}_{\text{sp}}$ | Product stream characteristics |
| $y_p$, $y_{\text{inv}}$ | Process output and setpoint |
| $x_p$ | Process states |
| $u_p$ | Process manipulated variables |
| $v_p$ | Process inputs |
| $z_p$ | Process algebraic variables |
| $y_{\text{inv}}$, $y_{\text{sp}}$, $u_{\text{inv}}$, $v_{\text{inv}}$ | Inventory output and setpoint |
| $p$ | Process manipulated variables |
| $\tau$ | Inventory inputs |
| $w$ | Reduced set of scheduling-relevant variables |

| Parameter | Description |
|-----------|-------------|
| $n$ | Index for scheduling time slots |
| $N_c$ | Number of time slots in the scheduling horizon |
| $P$ | Price profile |
| $\tau$ | Slot duration |
| $T_m$ | Makespan/schedule horizon |

REFERENCES

Amrit, R., Rawlings, J.B., and Biegler, L.T. (2013). Optimizing process economics online using model predictive control. *Comp. Chem. Eng.*, 58, 334–343.

Baldea, M. and Daoutidis, P. (2008). Dynamics and control of process networks with high energy throughput. *Comp. Chem. Eng.*, 32, 1964–1983.

Baldea, M. and Daoutidis, P. (2012). *Dynamics and Control of Integrated Process Systems*. Cambridge University Press, Cambridge, UK.

Baldea, M. and Harjunkoski, I. (2014). Integrated production scheduling and process control: A systematic review. *Comp. Chem. Eng.*, 71, 377–390.

Bansal, V., Sakizlis, V., Ross, R., Perkins, J.D., and Pistikopoulos, E.N. (2003). New algorithms for mixed-integer dynamic optimization. *Comp. Chem. Eng.*, 27, 647–668.

Biegler, L.T. (2007). An overview of simultaneous strategies for dynamic optimization. *Chem. Eng. Process.: Process Intensification*, 46, 1043–1053.

Cao, Y., Swartz, C., and Baldea, M. (2011). Design for dynamic performance: Application to an air separation unit. *American Control Conference*, 2683–2688.

Cao, Y., Swartz, C., Baldea, M., and Blouin, S. (2015). Optimization-based assessment of design limitations to air separation plant agility in demand response scenarios. *Journal of Process Control*, 33, 37–48.

Du, J., Park, J., Harjunkoski, I., and Baldea, M. (2015). A time scale bridging approach for integrating production scheduling and process control. *Comp. Chem. Eng.*, 79, 59–69.

Ellis, M., Durand, H., and Christofides, P. (2014). A tutorial review of economic model predictive control methods. *J. Proc. Contr.*, 28, 1156–1178.

Ierapetritou, M., Wu, D., Vin, J., Sweeney, P., and Chigirinskii, M. (2002). Cost minimization in an energy-intensive plant using mathematical programming approaches. *Ind. Eng. Chem. Res.*, 41(21), 5262–5277.

Jogwar, S., Baldea, M., and Daoutidis, P. (2009). Dynamics and control of process networks with large energy recycle. *Ind. Eng. Chem. Res.*, 48, 6087–6097.

Karwan, M. and Keblis, M. (2007). Operations planning with real time pricing of a primary input. *Computers & operations research*, 34(3), 848–867.

MacGregor, J. and Cinar, A. (2012). Monitoring, fault diagnosis, fault-tolerant control, and optimization: Data driven methods. *Comp. Chem. Eng.*, 47, 111–120.

Miller, J., Luyben, W., and Blouin, S. (2008). Economic incentive for intermittent operation of air separation plants with variable power costs. *Ind. Eng. Chem. Res.*, 47(4), 1132–1139.

Pattison, R., Touretzky, C., Johansson, T., Harjunkoski, I., and Baldea, M. (2015). Production scheduling with low-order dynamic models: Framework and an air separation application. *Ind. Eng Chem. Res.*, Under Review.

Pattison, R.C. and Baldea, M. (2014). Optimal design of air separation plants with variable electricity pricing. *Foundations of Computer-Aided Process Design (FOCAPD)*, 393–398.

Qin, S.J. and Badgwell, T.A. (2003). A survey of industrial model predictive control technology. *Control engineering practice*, 11, 733–764.

Subramanian, K., Maravellas, C.T., and Rawlings, J.B. (2012). A state-space model for chemical production scheduling. *Comp. Chem. Eng.*, 47, 97–110.

Touretzky, C. and Baldea, M. (2014). Nonlinear Model Reduction and Model Predictive Control of Residential Buildings with Energy Recovery. *J. Proc. Contr.*, 24(6), 723–739.

US EIA (2010). Manufacturing energy consumption survey: Total consumption of electricity. Zhu, Y., Legg, S., and Laird, C. (2010). Optimal design of cryogenic air separation columns under uncertainty. *Comp. Chem. Eng.*, 34, 1377–1384.