A housing-demographic multi-layered nonlinear model to test regulation strategies

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Abstract

We propose a novel multi-layered nonlinear model that is able to capture and predict the housing-demographic dynamics of the real-state market by simulating the transitions of owners among price-based house layers. This model allows us to determine which parameters are most effective to smoothen the severity of a potential market crisis. The International Monetary Fund (IMF) has issued severe warnings about the current real-state bubble in the United States, the United Kingdom, Ireland, the Netherlands, Australia and Spain in the last years. Madrid (Spain), in particular, is an extreme case of this bubble. It is, therefore, an excellent test case to analyze housing dynamics in the context of the empirical data provided by the Spanish \textit{National Institute of Statistics} and other sources of data. The model is able to predict the mean house occupancy, and shows that i) the house market conditions in Madrid are unstable but not critical; and ii) the regulation of the construction rate is more effective than interest rate changes. Our results indicate that to accommodate the construction rate to the total population of first-time buyers is the most effective way to maintain the system near equilibrium conditions. In addition, we show that to raise interest rates will heavily affect the poorest housing bands of the population while the middle class layers remain nearly unaffected.

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1 Introduction

A crisis on the housing market could heavily reverse the current positive economical indicators [Kindleberger 2000, Garber 2000] and have a negative general impact in the world economy, as shown in the Asian crisis in 1997 [Quigley 2001]. Real-state prices have increased more than fifty percent in Australia, Ireland, United Kingdom and Spain from 1997 to 2004. These increases are hardly explainable in terms of economic fundamentals alone, even including record-low interest rates [Terrones et al. 2004]. Spain, in particular, holds high risk figures in terms of ratios of house prices to disposable income per worker (RPI) and house prices to rent (RPR). According to IMF calculations Spain has 3.6, 3.6, 2.5, 2.2, 1.8 times the RPI of Germany, Japan, United States, France and United Kingdom, respectively [Terrones et al. 2004]. Despite the overwhelming cost to acquire a house in Spain compared to all developed countries, house unit prices currently remain unaffected. In addition, in terms of the RPR, which is a good estimation of the yield of the investment, Spain has 3.42, 3.32, 1.9, 1.8, and 1.28 times the RPR of Germany, Japan, France, United States and United Kingdom, respectively [Terrones et al. 2004]. The RPI index has been proved to be a good reference of the distance to market equilibrium [Malpezzi 1999]. Therefore, Spain is a valid scenario to analyze the conditions to understand and control a housing market by means of nonlinear dynamical models.

In this paper we develop a mean field model of the house occupancy derived from a stochastic process as previously used in the study of epidemic dynamics [Bailey 1975]. This nonlinear model is employed as a tool to ascertain the possibility of sudden changes in the dynamics as a function of the control parameters. The model is based on dissecting the house population in several groups or layers. Each layer has a given number of houses, which can be either occupied or empty. Every family occupying a house in a given layer has a certain probability rate to migrate to any other layer. In addition, there exists a base pool of nonowners, which can jump to the housing layers at a certain probability rate. Thus the model captures the dynamics of the mean migrations between housing layers.

As a particular case, we analyze the house market in the city of Madrid in the framework of the model. The parameters of the model have been obtained from different sources such as the Spanish National Institute of Statistics, the Spanish Ministry, and some valuation companies [INE 1991, INE 2002, INE 2004, Sociedad de Tasación S.A. 2004].

Our main results may be summarized as follows:

- The model is able to predict the occupancy levels in 2001 given the parameters obtained from 1991 data. This is a good indication that the mean description is able to model the real dynamics.

- The model shows that the house market for the city of Madrid asymptotically evolves to an out-of-equilibrium condition. It is rather worrisome that the new housing units cannot be replenished by first-time buyers.

- Critical phenomena do not exist in the context of this model. Only smooth changes can be expected.
• A sudden increase in the interest rate will seriously affect the occupancy level of the lowest layer of the market, i.e. the poorest housing unit sector, while the middle layers remain nearly unaffected.

• According to the model, the most effective way to control the out-of-equilibrium situation of the Madrid house market is to down regulate the construction rate.

There is a large body of research in the housing market [Green and Malpezzi 2003, Malpezzi 1990], most of the analyses aiming at predicting house prices. We do not intend to estimate prices, our modeling efforts fit better with housing-demographic models [Mankiw and Weil 1989, Crone and Mills 1991]. The novelty of our approach lies on building a multi-layered nonlinear dynamical model that includes family migrations across layers of housing units. The whole population is separated in non-overlapping housing bands, which are estimated from census data. Our model neglects the random component of the stochastic process, because we use a mean field approach under the assumption of random mixing. The random mixing approximation basically states that any family can, in principle, uniformly access any house. This random mixing approximation allows to obtain a set of ordinary differential equations (ODEs) derived from a stochastic process. The formalism that we use here sets us apart from previous approaches.

2 Overview of the model

In this section we provide an overall description of the non-linear dynamical model for the house occupancy. The model equations will be explicitly derived in the next section. We divide the total housing population into different price bands or layers, and model the transitions of owners among these layers. Note that, in this paper, we only use the price as a tool to determine the different house bands. Let us assume that there are $N_{\text{layers}}$ layers. We define $N_i(t)$ as the total number of houses, or units, in layer $i$ at time $t$. Each of the layers has a certain number of occupied houses, $O_i(t)$. These are defined as houses occupied by owners. In addition to the house price bands, we consider a source of new buyers, which we call the base pool. It accounts for non-emancipated people, immigrants, and rented units.

A group of people living in the same house will be called a family. The family is the basic people unit in our model, and so we are modeling family jumps among house layers. Note that the number of families in a given layer $i$ equals the number of occupied houses in that layer, $O_i(t)$. The concept of family applies to non-emancipated people and immigrants as well. However, in these cases information concerning the total number of single individuals is more frequently available in census databases, so we must apply a correction factor when calculating the number of families ($\Sigma(t)$) in the base pool.

The migrations among house layers are modeled in terms of transition probabilities. We denote by $\mu_{ij}$ the probability rate for a family in house layer $j$ to move to house layer $i$. Equivalently, we denote the probability rate to move from the base pool to any of the house layers $i$ by $\eta_i$. Finally, there is a probability for any family to disappear, or die, leaving its
house empty. We call this probability the death rate, $\lambda_i$, which we assume to be constant for each layer $i$. Figure A shows an overall scheme of the model; figure B displays a list of the variables and parameters involved.

With all the above ingredients we pose a set of non-linear coupled equations for the mean occupancy of each level: $o_i(t) = O_i(t)/N_i(t)$. The model equations are derived from the mean field approximation to a stochastic process, as used in epidemic dynamics. Details are provided in the next section.

Figure 1: (A) Explanatory figure of the model and the main parameters required in our housing-demographic model. Each layer represents a price housing band. The dark circles represent occupied housing units. $\Sigma(t)$ represents the pool of units that can enter any of the vacant sites on any of the layers with some probability rate $\eta_i$. Each of the layers have a total number of housing units $N_i(t)$. The transition probability rate $\mu_{ij}$ denotes the probability of having transition from layer $j$ into layer $i$. Note that we omitted the rest of the transitions for the sake of the readability of the figure. (B) Glossary of variables and parameters of the nonlinear dynamical model.

### 3 Derivation of the model equations

The model proposed here is similar in derivation to the ones used in epidemic modeling [Bailey 1975, Huerta and Tsimring 2002] where the stochastic epidemic process is reduced to a set of ordinary differential equations (ODEs). These ODEs capture quite faithfully the behavior of the stochastic process behind epidemics. The main advantage of this approach is that the complexity of the process is simplified to a formalism that allows an easier understanding of the qualitative behavior. The parameters can be easily related to the end result of the stochastic process. In most cases the ODEs match well the stochastic process, although there are some others where the ODE description fails, for example, for finite-size effects. Overall, the ODE description is a very good framework to gain understanding that can complement very well stochastic modeling. Our main contribution is to bring these convenient tools to housing-demographic modeling.

To model the dynamics across housing layers we use two possible states for any house: occupied and empty. The total number of houses in each layer $i$, $N_i(t)$, evolves in time
according to a function estimated from the census data. Given the number of occupied units in layer $i$, $O_i(t)$, and the transition probability rate from layer $j$ to layer $i$, $\mu_{ij}$, the probability that a family jumps from layer $j$ to layer $i$ in the time interval $dt$ is $\mu_{ij}\{1 - O_i(t)/N_i(t)\} \, dt$. This probability already assumes that a family can only occupy an empty house in layer $i$. This is equivalent to the random mixing approximation widely used in epidemic modeling [Bailey 1975]. The net flow into layer $i$ is the difference between the number of incoming and outgoing families in the time interval $dt$:

$$F_i = \sum_{j=1}^{N_{\text{layers}}} \left[ O_j(t) \mu_{ij} \left\{ 1 - \frac{O_i(t)}{N_i(t)} \right\} - O_i(t) \mu_{ji} \left\{ 1 - \frac{O_j(t)}{N_j(t)} \right\} \right] \, dt$$

This flow equals the variation in the number of occupied states in layer $i$ during the time interval $dt$, so we can write the following set of ordinary differential equations (ODEs) for the layer occupancy:

$$\frac{dO_i(t)}{dt} = \sum_{j=1}^{N_{\text{layers}}} \left[ \mu_{ij} O_j(t) \left\{ 1 - \frac{O_i(t)}{N_i(t)} \right\} - \mu_{ji} O_i(t) \left\{ 1 - \frac{O_j(t)}{N_j(t)} \right\} \right]$$

It is critical to include the dynamical contribution of the base pool. The probability for a family to jump from the base pool to the layer $i$ in the time interval $dt$ is given by $\eta_i \{1 - (O_i(t)/N_i(t))\} \, dt$. Then, the net change in layer $i$ due to the flow from the base pool is simply $\Sigma(t)\eta_i \{1 - (O_i(t)/N_i(t))\} \, dt$. Finally, we will consider a family death rate for each layer, $\lambda_i$, which contributes to the variation in layer occupancy with the term $-\lambda_i O_i(t)dt$. The final model equations can be written as:

$$\frac{dO_i}{dt} = \eta_i \left( 1 - \frac{O_i}{N_i(t)} \right) \Sigma(t) - \lambda_i O_i + \sum_{j=1}^{N_{\text{layers}}} \left[ \mu_{ij} O_j \left\{ 1 - \frac{O_i}{N_i(t)} \right\} - \mu_{ji} O_i \left\{ 1 - \frac{O_j}{N_j(t)} \right\} \right]$$ (1)

Since we plan to analyze the asymptotic behavior of the equations, we define a new variable, $o_i(t) \equiv O_i(t)/N_i(t)$, which is the normalized occupancy level, bounded between 0 and 1. We can rewrite the set of equations [1] in terms of the normalized occupancy as:

$$\frac{do_i}{dt} = \eta_i (1-o_i) \frac{\Sigma(t)}{N_i(t)} - o_i \frac{d}{dt} \log N_i(t) - \lambda_i o_i + \sum_{j=1}^{N_{\text{layers}}} \left( \mu_{ij} o_j (1-o_i) \frac{N_j(t)}{N_i(t)} - \mu_{ji} o_i (1-o_j) \right)$$ (2)

There are three terms in these ODEs with explicit dependence on time. The first one is the drive from the base pool to saturation levels. If $\Sigma(t)$ grows faster than single layers do, then all layers will saturate. The third term with explicit dependence on $t$ is multiplied by $N_j(t)/N_i(t)$. This term implies that, if the size of layer $j$ grows much faster than layer $i$, in the asymptotic limit the layer $i$ will be totally full, with a huge demand in that layer that will quickly change the band location in the whole distribution of layers.
4 Test case: the city of Madrid

Madrid is a particularly extreme case of the real-state bubble in Spain [Terrones et al. 2004]. This fact, together with the availability of data to estimate model parameters, makes Madrid an interesting test case to be analyzed in the context of our mean field model. The two main sources of data used to feed the model parameters are the Spanish National Institute of Statistics (INE) and the Spanish Ministry, but we have also used data provided by valuation companies and real-state internet sites. First we will provide an overall description of the model parameter estimation in section 4.1. Then, in section 4.2, we will use these parameters in the model equations (2) to understand the implications of current market conditions.

4.1 Parameter estimation

First of all we must determine the price layers. Figure 2A represents the distribution of house prices in the city of Madrid in 1991 (data from INE 1991, Sociedad de Tasación S.A. 2004). This distribution provides the total number of houses per layer, \( N_i \), using 7 layers that account for the 99% of the total number of houses. The number of occupied houses in each layer, \( O_i \), is also provided by INE 1991, Sociedad de Tasación S.A. 2004. To choose the price size of each layer we find a compromise between two opposing criteria: i) the maximum number of layers in order to have a detailed distribution of the housing stock; and ii) the widest price size per band such that the transition probability rates between layers are not very small when normalized to the integration time scale. This second criterion intends to avoid finite size effect problems.

The time evolution of the number of houses per layer, \( N_i(t) \), is hardly available in public databases. Nevertheless, as shown in fig. 2B, the total population of houses is available at five different years since 1970 INE 2004. In that figure we can see that the total population of houses is very well fit by an exponential function of time. We will assume that the shape of the layers distribution is time invariant, and will apply the same exponential growth to all layers. This assumption is supported by two facts: (i) the evidence of self-regulation of each of the layers as shown in Linneman 1986 for the city of Philadelphia, i.e., undervalued houses compared to similar type of houses get more appreciated than the average; and (ii) the similarity of price distributions in the cities of Pitt County (North Carolina) Bin 2004 and Madrid (figure 2A).

The number of families in the base pool (\( \Sigma(t) \)), i.e., the subset of families that do not own a property, is estimated from the INE databases as the sum of: (i) the number of people that are not emancipated within the range of age where people usually emancipate (data from INE 2002, OBJOVI 2004); (ii) the number of families that rent an apartment (data provided by INE 2004); and (iii) the number of incoming families due to the immigration (data from INE 2004). These statistics do not provide family units, but they report single people data instead. So a headship factor of 2 has been applied when calculating \( \Sigma(t) \).

In fig. 2C we can see the time evolution of the three subsets that conform the base pool. The two main contributions to the pool have been decreasing in the last few years. On the
other hand the immigrant population has undergone a sharp increase. However, in contrast to the popular view, this subset of the population is not officially large, and it might just compensate for the negative tendency in the base pool size time evolution. In contrast to the total number of houses (fig. [2B]), there is no exponential growth of $\Sigma(t)$. In fact, we will assume it is a constant. This situation, if maintained, would lead the system to complete depletion as we will discuss below in the context of the model.

Other important parameters to be estimated are the transition probability rates from the base pool to the different layers, $\eta_i$, and the transition probability rates among layers, $\mu_{ij}$. The average transition probability rate from the base pool to any of the layers, $\bar{\eta}$, can be estimated from the total number of new occupancies in 1981, 1991 and 2001, as follows. The total number of occupied houses in 1981 was 699,557, in 1991 was 789,444 and in 2001 was 908,790 [INE 2004]. Therefore the occupancy rate from 1981 to 1991 was 8,989 new occupancies per year, and from 1981 to 1991 it was 11,935 new occupancies per year. The base pool population in 1981 is estimated in 568,300 families (we have data for the non-emancipated population since 1986 and we apply a headship per future household of 2), in 1991 it was 532,717 and in 2001 it was 543,323. Then an estimation of the probability rate of having one base pool family accessing any of the layers is 0.016 year$^{-1}$ from 1981 to 1991, and 0.022 year$^{-1}$ from 1991 to 2001. This rate has been increasing in the last few years, maybe due to the decrease in the interest rates.

To fit the probabilities $\mu_{ij}$, we assume that the occupancy levels in 1991 are stationary, and search for the values of $\mu_{ij}$ that provide these levels at equilibrium. The available data do not permit direct calculation of these probabilities, nevertheless it is possible to estimate their average by extrapolating the number of housing transactions in Spain (provided by analysis office by [BBVA 2003] and [OBJOVI 2004]) to the city of Madrid. If we take this total number of transactions and discount the probability of transition from the base pool and the estimation of housing transactions due to investments, we obtain $\bar{\mu} = 0.052$ year$^{-1}$. This basically states that the time it takes a family to change to a new house is 20 years on average for the city of Madrid. This is imposed as a constraint in the following calculations. Therefore, we will assume that the occupancy levels are slightly off from the equilibrium state and that the growth of the population is compensated with the growth of the total number of housing units. The occupancy level of each of the 7 layers at 1991 is given by $\hat{o} = (0.77, 0.56, 0.68, 0.62, 0.85, 0.62, 0.39)$. The parameter search is performed by means of a genetic algorithm with a fitness function that measures the Euclidean distance between $\hat{o}$ and the model equilibrium solution $o$. To reduce the search space we apply two constraints: (i) the average value of $\mu_{ij}$ is, as stated above, $\bar{\mu} = 0.052$ year$^{-1}$; and (ii) the derivative between close parameters is as small as possible. A total of 50 different simulations were run, the outcomes of three of them are shown in figure [3]. Although the specific values of the probabilities do not match for the three simulations, the shape and tendency maintain qualitative agreement. We use their average value when solving the model equations.

The last important estimation in our model is the family death rate for each layer. We have data of the absolute number of deaths per age and the age distributions per housing layer [INE 2004]. With these data, assuming a family is composed of two people very close
in age, we can calculate the instantaneous family death rate per layer (fig. 2D). We can see
that the lower layers have a higher death rate, because the average age of families in these
layers is higher. This can be explained by the slow integration of the old houses into the
lower price layers. On the other hand, newly constructed houses tend to fill higher layers.

Figure 2: (A) Distribution of house prices in the city of Madrid (Spain) during 1991, extracted from data
provided by \[\text{INE 1991}\] and \[\text{Sociedad de Tasación S.A. 2004}\]. (B) Fits to the logarithm of the total number
(circles), primary (squares), secondary (triangles), and empty (stars) houses. The data is provided for only
5 points in time \[\text{INE 2004}\]. The fits yield the following slopes: 0.012 $\pm$ 0.001 year$^{-1}$ (total), 0.009 $\pm$ 0.001
year$^{-1}$ (primary), 0.022 $\pm$ 0.001 year$^{-1}$ (secondary), and 0.05 $\pm$ 0.003 year$^{-1}$ (empty). In all the cases the
correlation factor of the linear regression is greater than 0.98. (C) Time evolution of the different subsets of
the base pool: non-emancipated families (stars), immigrant families (squares), and renting families (circles).
(D) Instantaneous death rate per layer, $\lambda_i$. Parameters estimated from 1991 data.

### 4.2 Results

For the problem under analysis, and taking into account the exponential growth of the total
number of houses for the city of Madrid, the model equations (2) can be rewritten as:

$$
\frac{d o_i}{dt} = \eta_i (1 - o_i) \sum_{k=1}^{N_{\text{layers}}} \frac{N_{\text{layers}}}{N_i} \exp k t - o_i (k + \lambda_i) + 
\sum_{j=1}^{N_{\text{layers}}} \left( \mu_{ij} o_j (1 - o_i) \frac{N_j}{N_i} - \mu_{ji} o_i (1 - o_j) \right)
$$

(3)
where \( k \) is the constant of the exponential growth fitted in figure 2B, and \( \hat{N}_i \) is the total number of houses in layer \( i \) in 1991 (figure 2A). Integrating the equations with the parameters of previous section, we can make predictions on the mean occupancy level. In particular, the decreasing trend of the occupancy levels from 1991 to 2001 is well predicted by the model using data from 1971 to 1991 (see figure 4A).

![Figure 3: Solutions of three different runs of the genetic algorithm used to estimate the transition probabilities. These plots show the values of the transition probabilities from any of the layers including the base pool (rows, y axis) to any of the 7 property layers (x axis) in ten years. To get the probabilities in one year the values must be divided by 10. The standard deviation with respect to the target value \( \hat{\phi} \) is in all the cases less than \( 10^{-4} \).](image)

A sufficient condition to keep the system asymptotically away from 0 (empty) or 1 (full) is the following

\[
N(t) = \left( N(0) + \int_0^t \left( \sum_i \eta_i \frac{1 - \bar{\delta}}{\bar{\delta}} e^{\bar{\lambda}_\tau \Sigma(\tau)} d\tau \right) e^{-\bar{\lambda} t} \right), \tag{4}
\]

where \( \bar{\delta} \) is the mean occupancy level, and \( \bar{\lambda} \) is the mean instantaneous household death rate. If the time evolution of the base population is constant, as it appears according to figure 2C, then \( N(t) \propto \exp(-\bar{\lambda} t) \). The optimal equilibrium occupancy levels in the housing market is out of the scope of this paper. Nevertheless it is certainly desirable to steer the system to a equilibrium condition whose asymptotic state is far from the 1-0 extremes. Equation (4)
intends to provide a simple framework for policy makers to regulate the system in a smooth way from a housing-demographic perspective, disregarding price value of the housing units.

Nevertheless, although equation (4) is useful due to its simplicity, the intrinsic dynamics of the system is more complicated. Each layer follows a different trend due to the integration of the ODEs. In order to provide a vacancy rate for each of the layers, i.e., the pace at which each layer becomes empty, we fit an exponential function of time to the predicted occupancy levels, such as $o_i(t) \propto \exp(\kappa_i t)$, for 20 years since 1991. Note that $o_i(t)$ is not an exponential decay function, but since we are mostly interested in short periods of time (10 to 50 years), the exponential fits are adequate. As it can be seen in Fig. 4B, layers 2 and 3 undergo the heaviest vacancy rate at current market conditions, while the other layers are in a more stable situation. This indicates that the middle class layers are subject to the heaviest speculative process, while the other layers may have a lower demand.

Figure 4: A. Occupation level for 4 different years. The occupation level has been obtained as the sum of paid property, property with mortgage, and property passed from parents to sons and daughters divided by the total number of housing units. The solid line represents the prediction of the model by using 1991 data. B. Decay rate of the occupation for each of the layers. This decay rate has been obtained by fitting the $o_i(t) \propto \exp(\kappa_i t)$ from 1991 to 2010.

5 Economical conditions and temperature control parameter

The economical conditions that may affect the transition probabilities among layers are incorporated into the model by means of a single parameter $T$, that plays a role similar to the temperature in statistical physics. For example, if the interest rates are raised, the probability of transition from one layer to the rest or from the base pool to any layer is decreased, which is modeled by a temperature decrement. Another example, if the RPI grows excessively, then the temperature is also lowered, with the consequent decrease of the transition probabilities. Unfortunately, we do not have data to quantify the dependence of transition probabilities on economical conditions. To be able to estimate them we would need data that indicates the number of mortgage requests and the amount loaned as a
function of time. This has to be provided by banks and we currently do not have such data. The general dependence of the transition probability rates are therefore unknown. Initially, we assume that the transition rates ($\mu_{ij}$ and $\eta_i$) have a linear dependence on the parameter $T$. Therefore, the parameters $\mu_{ij}$ and $\eta_i$ in equation 3 are replaced by $T\mu_{ij}$ and $T\eta_i$. This dependence basically implies that the economical conditions uniformly affect all the probability rates in the same way. The goal is to determine whether there is a value of the temperature parameter that leads to a sudden change on the occupancy levels. As long as the dependence of the probability rates on the temperature is an analytic function, the existence of critical behavior in the dynamical system should be preserved. In other words, if $\mu_{ij}(T)$ and $\eta_i(T)$ are discontinuous, then the criticality emerges from parameter dependence on $T$, not from the dynamical system under consideration. In this work, we can only concentrate on the dynamical system behavior.

First, to determine the effects of freezing the system by temperature reduction we fit the decay rate of the occupancy level to a exponential function as shown in the previous section (see Fig. 5A). The middle layers are more robust to temperature changes. The middle layers show more resilience to critical economical conditions while both extremes of the multilayer model are highly sensitive. This contrasts with the depletion resistance observed in Fig. 4B for layer one. Thus, the lowest layer is the most sensitive to changes in the economical conditions.

As an example of the sensitivity of the lowest layer we introduce in 2005 a sudden decrease in the temperature that can be seen as a strong increase in the interest rates. In Fig. 5B we can see that a sudden change in the temperature yields a sharp slope modification in layer 1. On the other hand, it is striking to see that layers 2 and 3 are not as heavily impaired.

In summary, using the 1991 market parameters and the nonlinear dynamics formalism proposed in this article, the housing market of Madrid is not subject to critical (non-
continuous) behavior on the temperature control parameter. It is clear that layers (2 and 3) are very resilient to temperature changes. Nevertheless, although criticality is absent, layer 1 undergoes the most rapid changes to variable economical conditions. The layers with most expensive houses can potentially undergo serious reversals under difficult economical conditions. By means of this multilayer model we can identify which sectors of the housing population would be more affected. An overall regulation of the market following equation (4) may not lead to uniform stabilization of the system. Multi-layer models can be used to detect the individual impact of global policies.

6 Testing regulation strategies

The temperature parameter is not a strong control parameter, since it does not lead the system into an asymptotic equilibrium. Therefore, the question of whether there is a parameter that can globally keep the system in a nearly asymptotic equilibrium remains unanswered. A possible candidate is the growth rate of the number of houses, $k$. As we have seen in section 4 and fig. 2B the house growth rate is exponential for the city of Madrid, which is in contrast with the nearly flat increase of the base pool population (fig. 2C). This situation is not sustainable, and the model can be useful to determine what to expect when regulating the house growth. We have integrated equations 3 with the parameter values obtained for 1991, suddenly changing the growth factor $k$ at 2005. Time to 10% depletion versus $k$ is shown in figure 6. As we can see, there is a power law dependence. Basically, the time required to a 10% depletion is proportional to $(1/k)$ (fits actually yield $k^{-1.01}$ to $k^{-1.04}$). This simple solution allows to easily estimate what the effects of house growth regulation will have in the market.

This housing regulation policy contradicts the intuition about a voiced general opinion that the land in Madrid should be deregulated to build more housing units and, therefore, decrease the overall prices. This suggestion might reduce the prices of the housing units, yet it could worsen the current situation in Madrid in the long term. From the housing-demographic point of view, and according to this nonlinear model, more deregulation of land can push the system even farther out of equilibrium.

It is interesting to note that regulation policies for the lowest and the higher layers are closer to each other than the middle ones. Policies to regulate the poorest layer will also contribute positively in regulating the upper ones. The middle layers follow a different dynamics on its own mostly due to the fact that they receive and send family units from both sides of the distribution.

7 Conclusion

As pointed out by Brian Arthur [Arthur 1999]: “...complexity economics, is not an adjunct to standard economic theory, but theory at a more general, out-of-equilibrium level.” In this paper, we develop a novel multi-layered nonlinear dynamical framework for modeling the
housing market dynamics. Using realistic data with the highest available precision, we show that the housing market for the city of Madrid is currently driving away from equilibrium. This model can be used as a testing tool to determine the global effects of policy changes by governments. It is a tool to determine the effect of control parameters for all potential types of dynamics even for out-of-equilibrium conditions. Traditional econometric tools approximate the dynamics near set points, which are estimated (sometimes believed) to be the equilibrium points. When the system gets out of equilibrium there is not much guidance about what to expect, except waiting till it gets near the equilibrium point again. General tools to estimate effects of policy changes in the long run can be useful. Here we give an example that is able to provide a good prediction of the global level of occupancy in Madrid in 2001 (see Fig. 4A).

Our original goal was to find out whether the current house-market conditions in Madrid could lead to a critical condition such that, while slowly moving a parameter value as the temperature, the levels of occupancy suddenly drop to low levels. Fortunately, we did not find the existence of such criticality in the context of this model that uses to the maximum possible extent realistic data. When the interest rate is increased, which reduces the temperature parameter in our model, the occupancy levels of the medium range housing are not seriously affected. The first layer suffers dramatic consequences, and the wealthier layers undergo serious readjustments.

Although this result appears to be good news (except for the poorest sector of the housing units), the fact is that the city of Madrid is in a serious out-of-equilibrium condition that asymptotically drives the levels of occupancy to 0. According to our model, interest rate corrections will not modify this condition. The pragmatical way to control this situation
is to individually regulate the amount of new construction for each of the housing layers. Smooth changes in the construction rate can have a positive effect slowing down the out-of-equilibrium condition.

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