Investigation of the influence of the dynamic system parameters on its frequency and modal spectrums

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Abstract. The influence of parameters of dynamic system on its frequency and modal spectra is investigated by the example of chain unbranched dynamic system with finite number of degrees of freedom. The general regularities between the parameters of the chain dynamic system and its frequency and modal spectra are determined. It is established that according to the own spectrum of the chain dynamic system the inflated estimation of the interval width of eigenfrequency change at variation of any parameter may be given. The criterion, allowing to determine the structural parameters of the system, which significantly affect the given natural frequencies, is obtained.

Introduction
The efficiency of power transmissions of wheeled and tracked vehicles is largely determined by their dynamic processes and, in particular, resonance effects. Therefore, it is not a wonder that a large number of fundamental research [1-3] and articles [4-22] are focused on the research of dynamic processes in the transmission and reduction of loads in them. One of the easiest ways to reduce the dynamic loads in the transmission is the change of its elastic-inertial parameters, which ensures the removal of dangerous resonant operating modes outside the operating range. There is a practice of solving problems of removing resonant operating modes of transmission outside its operating range. The influence of dynamic system parameters on its natural frequencies and oscillation modes is different. Therefore, the possibility of targeted change of frequency and modal spectra is determined by the choice of changing parameters and their number. It is necessary to determine the parameters that significantly affect the frequency spectrum. They are assigned as variable parameters. It is necessary to determine the parameters that significantly affect the frequency spectrum. They are assigned as variable parameters. Such an approach allows, on the one hand, to increase variation limits of the frequency range, and, on the other hand, to reduce problem dimensions (the number of variable parameters). Thus, the problem solving efficiency is increased.

The influence of dynamic system parameters on its frequency and modal spectra
Investigation of the influence of dynamic system parameters on its frequency and modal spectra was carried out as an example of a 7-mass chain unbranched system. Each of the system's parameters changed over a rather wide range. Calculation of frequency and modal spectra was carried out by the Givens method together with the inverse iteration method. The research results are shown on Fig. 1. The following diagrams describe a well-known feature: the natural frequencies of unbranched chain systems are different.
Fig. 1. Frequency spectrum, normalized amplitudes of free oscillations of \( x_{j,i} \) \( j \)-th mass at \( i \)-th natural frequency and sensitivity coefficients \( \varepsilon_{ji}^{\omega_i} \) natural frequency \( \omega_i \) to the change of parameter \( J_k \), \( (k=1,2) \) of the investigated dynamic system at variation of moments of inertia \( J_1 \) and \( J_2 \).
The unlimited increase or decrease of any taken by itself \( k \)–th parameter of system \( p_k \) (moment of inertia or compliance coefficient) occurs monotone decrease or increase of all natural frequencies of system respectively. For each natural frequency of the system, except the supreme frequency, there is a finite upper and lower limit of its change. The upper limit of change of the \( i \)–th natural frequency does not exceed the lower limit of change of the \( i+1 \)–th natural frequency.

A significant change of the random \( i \)–th natural frequency, except supreme one, is carried out at a limited interval of \([p_k^{\min}, p_k^{\max}]\) values of the \( k \)–th parameter. The width of this interval depends on the order number of the natural frequency, the difference between the upper and lower limits of this frequency change, the position of the variable parameter in the system, the specific numerical values of the system parameters and its structure.

As the natural frequency ordinal number increases (when frequencies are numbered in ascending order of frequency) and decreases the difference between the upper and lower limits of the frequency change, the width of the interval \([p_k^{\min}, p_k^{\max}]\), in which the frequency changes significantly decreases.

In cases where the lower limit of change of the natural frequency differs from zero and there is a finite upper limit, the ratio \( p_k^{\min}/p_k^{\max} \) does not exceed \( 10^2 \). If the lower limit of change of the first natural frequency is equal to zero, (it takes place in systems with fixed support), the interval \([p_k^{\min}, p_k^{\max}]\) is extended. And then \( p_k^{\min}/p_k^{\max} < 10^4 \).

The supreme natural frequency does not have a finite upper limit.

For any chain unbranched \( l \) mass system without restraint, the following ratios are correct

\[
0 = \omega_i^{(0)} < \lim_{p_1 \to \infty} \omega_i < \omega_i^{(0)} < \lim_{p_1 \to 0} \omega_i < \lim_{p_2 \to \infty} \omega_i < \omega_i^{(0)} < \lim_{p_2 \to 0} \omega_i < \omega_i^{(0)} < \lim_{p_3 \to \infty} \omega_i < \omega_i^{(0)} < \lim_{p_3 \to 0} \omega_i < \omega_i^{(0)} < \lim_{p_4 \to \infty} \omega_i < \omega_i^{(0)} < \lim_{p_4 \to 0} \omega_i < \omega_i^{(0)} < \lim_{p_5 \to \infty} \omega_i < \omega_i^{(0)} < \lim_{p_5 \to 0} \omega_i < \omega_i^{(0)} < \lim_{p_6 \to \infty} \omega_i < \omega_i^{(0)} < \lim_{p_6 \to 0} \omega_i < \omega_i^{(0)} < \lim_{p_7 \to \infty} \omega_i < \omega_i^{(0)} < \lim_{p_7 \to 0} \omega_i < \omega_i^{(0)} < \lim_{p_8 \to \infty} \omega_i < \omega_i^{(0)} < \lim_{p_8 \to 0} \omega_i < \omega_i^{(0)} < \lim_{p_9 \to \infty} \omega_i < \omega_i^{(0)} < \lim_{p_9 \to 0} \omega_i < \omega_i^{(0)} < \lim_{p_{10} \to \infty} \omega_i < \omega_i^{(0)} < \lim_{p_{10} \to 0} \omega_i < \omega_i^{(0)} ...
\]

where \( \omega_i^{(0)}, \omega_i^{(1)}, \ldots, \omega_i^{(n-1)} \) – natural frequencies of the system at initial values of its parameters.

Arbitrary \( i \)–th natural oscillation mode (when it's normalized \( \mathbf{x}_i^T \mathbf{x}_i = 1 \), where \( \mathbf{x}_i^T \) – transposed vector \( \mathbf{x}_i \)) change significantly at a limited parameter variation range \([p_k^{\min}, p_k^{\max}]\). The width of the \([p_k^{\min}, p_k^{\max}]\) range is slightly greater than the width of the \([p_k^{\min}, p_k^{\max}]\) range for the corresponding natural frequency. And the \([p_k^{\min}, p_k^{\max}]\) range is inside the \([p_k^{\min}, p_k^{\max}]\) range.

The dependence of the amplitudes of natural oscillations on the parameter is not always monotoneous. In case of unlimited increase or decrease of any parameter \( p_k \) the normalized amplitudes of natural oscillations tend to some limiting values.

The properties of the frequency spectrum considered allow express estimation of the influence of any parameter of the system on its natural frequencies.

The method is based on the following idea. When analyzing the influence of any \( k \) - th parameter \( p_k \) on the eigenstem spectrum of the dynamic system, four auxiliary dynamic systems are created. Each system has the same structure and dimension as the original system. All parameters of the auxiliary systems, except for \( k \) parameters calculated by the following equations:

\[
p_k^{(1)} = 10^{-4} p_k^{(0)}, \quad p_k^{(2)} = 10^4 p_k^{(0)}, \quad p_k^{(3)} = 0.5 p_k^{(0)}, \quad p_k^{(4)} = 2 p_k^{(0)},
\]
where \( p_k^{(0)} \) – value of the \( k \)-th parameter in the initial system,
\[ \]
\( p_k^{(l)} \) – value of the \( k \)-th parameter in the \( l \)-th auxiliary system,
are equal to the corresponding parameters of the initial system
\[ p_m^{(l)} = p_m^{(0)}, \quad m = 1, n_p (m \neq k), l = 1, 4, \] (3)
where \( n_p \) – system parameter number.

Using the natural spectra of the first two auxiliary systems, approximate estimates of the upper and lower limits of change in the natural frequencies of the system can be calculated by changing each \( k \)-th parameter \( p_k \) on the \((0, +\infty)\) range:
\[ \]
\[ \lim_{p_k \to 0} \omega_i \approx \omega_i^{(1)}, \]
\[ \lim_{p_k \to +\infty} \omega_i \approx \omega_i^{(2)}, \]
where \( \omega_i^{(1)}, \omega_i^{(2)} \) – \( i \)-th natural frequency 1-st and 2-nd auxiliary systems \((i = 1, n - 1 \text{ – for systems without fixed support and } i = 1, n \text{ – for systems with fixed support})\).

1.1. \textbf{By the difference}
\[ \Delta \omega_i^{(1-2)} = (\omega_i^{(1)} - \omega_i^{(2)}) \] (4)
and quotient
\[ \epsilon_{\omega_i}^{(1-2)} = \frac{\omega_i^{(1)}}{\omega_i^{(2)}} \] (5)
can be inferred by the possible effect of the \( k \)-th parameter on the \( i \)-th natural frequency. The larger these values, the more the natural frequency \( \omega_i \) changes when the parameter \( p_k \) varies over a rather wide range \([10^{-4} \cdot p_k^{(0)}, 10^4 \cdot p_k^{(0)}]\).

The parameter \( p_k^{(0)} \) belongs to the range \([p_k^{(0)}, p_k^{(0)}]\). At this range there is a significant change of frequency \( \omega_i \), can be set using the sensitivity coefficient \( \epsilon_{p_k}^{\omega_i} \) of natural frequency \( \omega_i \) to the change of parameter \( p_k \), calculated by the equation
\[ \epsilon_{p_k}^{\omega_i} = \frac{\omega_i^{(1)} - \omega_i^{(2)}}{\omega_i^{(3)} - \omega_i^{(4)}}. \]

The analysis of the change in the coefficient \( \epsilon_{p_k}^{\omega_i} \) from the parameter \( p_k \) (Figures 1 (d), 1 (f)) shows that when the parameter is inside the range \([p_k^{(0)}, p_k^{(0)}]\) (Fig. 2, (a)), where a change in the frequency \( \omega_i \) is significant, independent of \( \Delta \omega_i^{(1-2)} \) for all frequencies, except the supreme one, the sensitivity coefficient \( \epsilon_{p_k}^{\omega_i} \) does not exceed 4–5. When the point \( p_k^{(0)} \) on the numerical axis is outside the range \([p_k^{(0)}, p_k^{(0)}]\), but is close to its boundaries (Fig. 2, (b)), the inequality:
\[ \epsilon_{p_k}^{\omega_i} < 10 - 20. \]
If the distance of the point $p_k^{(O)}$ from the range $[p_k^{n_{k0}}, p_k^{u_{k0}}]$ boundaries (Fig. 2, (c)) is significant, the coefficient $\varepsilon_{pk}^{nu}$ increases to $6.8 \cdot 10^3$ and more. Therefore, to find the $k$ - th parameter $p_k^{(O)}$ inside the range $[p_k^{n_{k0}}, p_k^{u_{k0}}]$, where a significant change in the frequency $\omega_i$, occurs, or close to the boundaries of this range, it is necessary to the fulfillment of the condition:
\[
\varepsilon_{pk}^{nu} < \sigma = 5 - 20. \tag{6}
\]

Fig. 2. Possible positions of a point $p_k^{(O)}$ on a numerical axis relative to the range $[p_k^{n_{k0}}, p_k^{u_{k0}}]$

Estimation of sensitivity of natural frequencies and coupled modes of dynamic system oscillations to changes in its elastic-inertial parameters is possible by analyzing the values of the corresponding partial derivatives of the parameter [23-25]. However, if a similar approach is used, the point $p_k^{(O)}$ on the numerical axis is outside the range $[p_k^{n_{k0}}, p_k^{u_{k0}}]$ and in close to its left or right boundary (Fig. 2, (b)). The value of derivative $\frac{\partial \omega_i}{\partial p_k}$ at this point will be small. When the problem is solved due to the change of the frequency spectrum, the $k$ - th parameter can be wrongly removed from the list of variable parameters and thus the limits of the change of natural frequencies. As a result, the range of natural frequencies may be narrowed.

Conclusions
1. The general conformations between the parameters of the chain dynamic system and its frequency and modal spectrum are established.
2. It is determined that the eigen spectrum of a chain dynamic system can be used to give an inflated estimate of the range width of change in the natural frequencies when a randomly chosen parameter, $p_k$ (moment of inertia or compliance coefficient) is varied. Thus for any random $i$ - th natural frequency $\omega_i^{(O)}$ of the investigated system the following ratio is correct:
\[
\omega_i^{(O)} < \lim_{p_k \to 0} \omega_i < \omega_i^{(O)} < \lim_{p_k \to 0} \omega_i < \omega_{i+1} < \omega_{i+1}^{(O)}
\]
3. The criterion, allowing to find out the design parameters of the system, which significantly influence on the given natural frequencies, is obtained.

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