Asymptotic states of the bounce geometry

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Abstract

We consider the question of asymptotic observables in cosmology. We assume that string theory contains a landscape of vacua, and that metastable de Sitter regions can decay to zero cosmological constant by bubble nucleation. The asymptotic properties of the corresponding bounce solution should be incorporated in a nonperturbative quantum theory of cosmology. A recent proposal for such a framework defines an S-matrix between the past and future boundaries of the bounce. We analyze in detail the properties of asymptotic states in this proposal, finding that generic small perturbations of the initial state cause a global crunch. We conclude that late-time amplitudes should be computed directly. This would require a string theory analogue of the no-boundary proposal.
1 Introduction

The observed acceleration of the expansion of the universe has modified the cosmological constant problem. In Ref. [1] it was argued that string theory contains an exponentially large number of long-lived false vacua, all of which are realized dynamically as huge regions in an infinite universe, and some of which have an effective cosmological constant of order the observed value. Recent progress on stabilizing string compactifications [2–22] lends support to this scenario and provides increasingly concrete explorations of the stringy “landscape” [23].

Another, more conceptual problem is the question of observables in cosmology. A number of authors have pointed out that the definition of exact observables is problematic in cosmology (see, e.g., Refs. [24–30]) because of obstructions such as event horizons, thermal radiation, and entropy bounds. In particular, in Ref. [29] it was argued that an S-matrix cannot be measured by any real observer inside a large class of universes. However, it was also noted that event horizons do not automatically preclude the existence of other exact observables. In any case, if exact observables can be defined at all, they will be defined at asymptotically late time.

There is no reason why a rigorous quantum gravity theory with well-defined observables should exist for all imaginable cosmologies; indeed, it would be satisfying if its existence were to provide a criterion constraining the type of universe we find ourselves in. Assuming that the landscape picture is correct, it makes sense to search for a nonperturbative theoretical framework that rigorously defines its dynamics and observables.

A first step in this direction was taken in Ref. [32], which noted that the asymptotic structure of supersymmetric regions with vanishing cosmological constant may allow for constructions of observables that would be impossible in an eternal de Sitter universe. (Regions with negative vacuum energy do not share this property because they collapse when accessed cosmologically.) The corresponding semiclassical solution is the Coleman-De Luccia “bounce”. It consists of a bubble of a $\Lambda = 0$ vacuum inside de Sitter space. The bubble of true vacuum expands but the de Sitter region inflates fast enough that it is not consumed. Asymptotically at early and late times, the geometry has infinite regions of true and false vacuum. It was suggested that this system can be described quantum

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1 Disclaimers should include the fact that corrections to metastable de Sitter solutions are not yet controlled with the same level of rigor as in supersymmetric backgrounds. Because of our potential ignorance of vast sectors of string theory, current surveys of vacua may not be representative. For further criticism, see, e.g., Ref. [31].
mechanically in terms of an S-matrix relating asymptotic states in the early and late time \( \Lambda = 0 \) regions.

In the present paper we present results that suggest a partial modification of this strategy. In Sec. 3 we identify an entropy bound limiting the number of states in the semiclassical geometry expected to dominate the path integral. This is concrete evidence that the proposed S-matrix would have finite rank, as first suggested by Banks [28]. We point out that the past half of the Coleman-De Luccia evolution violates the second law of thermodynamics. We find that this makes it difficult to characterize the asymptotic states relevant to the proposed S-matrix.

In Sec. 4 we analyze how the entropy bound is reconciled at the semiclassical level with the infinite number of particles present in the universe at early and late times. Here we find that the picture is more complex than previously noted [28]. By construction, the time-symmetric Coleman-De Luccia solution involves an era in which the coarse grained entropy decreases. This leads to violent instabilities. Even a small perturbation in the initial state will generically produce a big crunch rather than a future asymptotic region. In this respect the Coleman-De Luccia geometry considered here is not better, but worse than eternal de Sitter space.

In Sec. 5 we characterize “allowed” states that correspond to a semiclassical evolution from a past to a future asymptotic region. Many such states do not appear to admit a simple description near the boundary at all. Those that do are best thought of as excitations of zero mass in a cosmological fluid. The task is to leave all fields outside a compact region undisturbed, in order to limit the crunch to a finite black hole. We achieve this by treating all matter as dust and creating an overdense region surrounded by an underdense region (a “halo”).

Working with such states is awkward, and the relation between their S-matrix elements and bulk observables accessible to physical observers is obscure. In light of the inhomogeneity of the allowed perturbations we found, the corresponding S-matrix is unlikely to describe the perturbations that are actually observed in the universe at late times. Unless the dust approximation is exact, even halo states involve infinitely fine-tuned small adjustments in the entire, infinite initial \( \Lambda = 0 \) region. Moreover, the choice of initial state has no natural interpretation and may be redundant. Note that all of these difficulties relate to the past asymptotic region.

Yet, in light of the difficulty in defining observables in cosmology, we would like to exploit the more advantageous asymptotic properties of the Coleman-De Luccia solution.
They are associated with the future asymptotic region. It is particularly promising that 
the solution contains an open (rather than closed or flat) FRW universe, since this cuts 
off the growth of perturbations after curvature domination. Arbitrarily large regions of 
arbitrarily flat space become available at late times. In particular, there will be well-
separated, noninteracting particles. Just as string theory computes the S-matrix in flat 
space, it may well compute amplitudes for out-states in the Coleman-De Luccia geometry.

Thus, our results suggest that the second-law-violating, contracting half of the Coleman-
De Luccia universe does more to obstruct than to facilitate a nonperturbative description 
of the landscape and its dynamics. They point instead to a framework in which amplitudes 
are computed for boundary conditions specified at late times only. An example of this in 
semiclassical gravity is the Hartle-Hawking wavefunction [33], defined by the saddlepoint 
approximation to a path integral over compact geometries. In the context of string theory, however, one would seek an exact nonperturbative definition of amplitudes in the 
asymptotic future.

2 Metric and causal structure

In this section we review the fully extended Coleman-De Luccia solution [34], or “bounce”. 
We present the metric and conformal diagram. More details can be found, e.g., in Refs. [29, 
32, 35]. We work in four dimensions for simplicity, but our analysis is independent of 
dimension aside from trivial factors.

The bounce is a solution to the Einstein equations in the presence of a scalar field 
potential with at least one false vacuum, as shown in Fig. 1. We assume that the cosmo-
logical constant vanishes exactly in the true vacuum. The simplest classical solutions are 
Minkowski space (where the scalar is everywhere in the true vacuum) and de Sitter space 
(with the scalar in the false vacuum). Both of these have maximal symmetry, generated 
respectively by the Poincaré group and the de Sitter group, $SO(1,4)$. The de Sitter cur-
vature radius $R$ is inversely related to the cosmological constant $\Lambda$ in the false vacuum, 
$R = \sqrt{3/\Lambda}$.

But there is also a field configuration in which the scalar crosses the barrier from true 
to false vacuum, forming a domain wall. The bounce is the most symmetric solution of 
the Einstein equation with this field configuration. Its isometry group is $SO(1,3)$. For 
simplicity, we will describe the solution in the thin wall limit where the width of the 
bubble is negligible compared to its minimum radius. (See Ref. [34] for the conditions on
the potential under which this limit is valid.)

Inside the domain wall, the spacetime is approximately empty, so the geometry is simply a region of Minkowski space glued across the thin domain wall to a portion of de Sitter space. This approximation actually requires infinite fine tuning, so it will important to include the effects of matter present in the $\Lambda = 0$ region. However, it is instructive to describe first the geometry of the idealized, vacuous bounce. It will then not be difficult to go beyond this excessive simplification.

The domain wall is a hyperboloid, whose worldvolume describes a three-dimensional de Sitter spacetime. Its minimum area is $4\pi r_0^2$, where the parameter $r_0$ is determined by the tension $\sigma$ of the domain wall and the (4-dimensional) de Sitter radius $R$,

$$r_0 = \frac{8\pi G\sigma R^2}{(4\pi G\sigma R)^2 + 1}.$$  \hspace{1cm} (2.1)

In the flat spacetime enclosed by the hyperboloid the position of the domain wall satisfies

$$r^2 - t^2 = r_0^2,$$  \hspace{1cm} (2.2)

where $r$ and $t$ are the standard Minkowski coordinates. The wall expands from a minimum radius $r_0$, at $t = 0$, to infinite size as $t \to \pm\infty$. To describe the motion of the domain wall from the de Sitter side, it is simplest to embed de Sitter space in 5-dimensional Minkowski space as the surface

$$X_{\mu}X^{\mu} = -R^2, \quad \mu = 0, 1, ..., 4.$$  \hspace{1cm} (2.3)

The domain wall is the intersection of the de Sitter space with the plane

$$X_4 = (R^2 - r_0^2)^{1/2}.$$  \hspace{1cm} (2.4)
The part of the space farther from the origin than the plane is discarded and replaced by a piece of flat space. An embedding picture of the solution is shown in Fig. 2.

Figure 2: The Coleman-De Luccia bounce geometry, as an embedding in Minkowski space. The flat piece has zero cosmological constant, while the curved piece has positive cosmological constant. A domain wall separates the two regions.

The solution is invariant under the symmetry group of the hyperboloid, $SO(1, 3)$. On the de Sitter side, the full $SO(1, 4)$ de Sitter symmetry is broken down to those boosts and rotations which leave $X_4$ unchanged. On the Minkowski side, the symmetry arises because the domain wall picks out an origin, breaking translation invariance while preserving the Lorentz group. The orbits of the symmetry group are surfaces of constant invariant distance from the origin. These surfaces can be spacelike separated from the origin, like the domain wall (region I of Fig. 3). They can also be timelike separated from the origin (region II of Fig. 3). The surviving $SO(1, 3)$ symmetry is not an artifact of the vacuous bounce. It is present in the full solution (even beyond the thin wall approximation).

If the thin wall approximation is well satisfied, the energy density in the center of the true vacuum bubble (point $b$ in Fig. 3) will almost vanish. But it will not be exactly zero, because the distance of $b$ to the domain wall is finite, so that $b$ lies on the exponential tail along which $\phi$ attempts to approach the true vacuum. By continuity, the same nonzero energy density will be present on the infinitesimally later point $c$, and hence on the entire symmetry orbit of $c$. The presence of a constant energy density on a hyperbolic spacelike slice means that region II (III) is not flat space, but an expanding (contracting) open FRW cosmology [34], with metric

$$ds^2 = -d\tau^2 + a^2(\tau)[d\rho^2 + \sinh^2 \rho d\Omega_{D-2}^2].$$  \hspace{1cm} (2.5)
Figure 3: Conformal diagram of the Coleman-De Luccia solution, including orbits of the symmetry group $SO(1,3)$. The thick orbit is the domain wall. The radius of spheres goes to zero at the left and right boundaries and to infinity on the remaining boundaries.

Thus, the idealization as flat spacetime is not even approximately correct in regions II and III, in the sense that the total energy in those regions is infinite.

As the field $\phi$ completes its descent into the true vacuum, it will need to dissipate this extra energy. For simplicity, we think of the conversion of potential energy to particles ("reheating") as occurring on a definite time slice of the FRW geometry, the reheating surface. By coupling $\phi$ to other fields, particles are produced at the reheating surface. The details can vary, but it is inevitable that somehow the initially uniform potential energy will evolve into an incoherent, highly entropic form as the universe expands. Because the symmetry orbits are noncompact hyperboloids, an infinite number of particles will be produced.

3 Entropy

The bounce solution has an infinite amount of matter, and hence, an infinite amount of coarse-grained entropy in the FRW regions. In this section, we apply the covariant entropy bound [36,37] to show that the total number of allowed microstates is nevertheless finite.

A light-sheet is a convergent null hypersurface orthogonal to an arbitrary two-dimensional
spatial surface with area $A$. The bound says that there can be at most $e^{A/4G}$ different quantum states on a light-sheet, i.e., the entropy will not exceed $A/4G$ (see Ref. [38] for a detailed review).

![Figure 4: A wedged conformal diagram [37] for the bounce geometry. Normal (>), <), trapped (∨), and anti-trapped (∧) regions are separated by apparent horizons (thick lines). Their shape is determined by matter sources (the domain wall, and the matter created by it in regions II and III). We treat the domain wall (dotted line) as a delta function source. $L_1$ and $L_2$ are light-sheets of the sphere $P$. The tightest entropy bound on the spacetime is obtained from $L$, a single light-sheet whose maximal area is the de Sitter horizon in the top right corner.]

In a spherically symmetric spacetime, each sphere can be classified depending on the contracting null directions, i.e., the directions in which light-sheets exist. The corresponding domains are indicated in Fig. 4 spheres are either trapped (∨), antitrapped (∧), or normal (> and <, with the open side pointing towards $r = 0$).

As is common for a cosmological solution, the bounce geometry contains light-cones of finite maximal area. By following the wedges back to the tips, one is led to an apparent horizon, an area maximal with respect to orthogonal light-rays. Such a surface, for example the sphere $P$ in Fig. 4 admits light-sheets in two opposing directions. These two light-sheets, $L_1$ and $L_2$, form a complete Cauchy surface, in the sense that every timelike curve
must pass through them. Hence the total number of states in the universe is bounded by \( e^{A_P/2G} \).

Note that \( P \) is a sphere on the worldvolume of the domain wall. Its area increases monotonically and without bound as \( P \) is moved down. This explains why we can have an infinite coarse-grained entropy at early times in the FRW region. We get the best possible bound by moving \( P \) all the way up until the light-sheet becomes \( L \). This is a single light-sheet starting at the late-time cosmological horizon of the de Sitter region (the top right corner). Hence the optimal bound on the total number of states is given by

\[
N \leq \exp \left( \frac{\pi R^2}{G} \right),
\]

where \( R \) is the de Sitter radius. (One might worry about what happened to the second light-sheet: is this estimate not off by a factor of 2? There is indeed a de Sitter horizon volume in the top right corner which is not captured by \( L \). However, the D-bound [39] implies that no matter entropy is present there.)

Thus there is a huge discrepancy: Unbounded coarse-grained entropy is allowed, and indeed present, at early and late times in the FRW regions II and III; yet, the total number of microstates allowed to pass though the light-sheet \( L \) in the center of the geometry is finite. It follows that of the infinite number of microstates corresponding to the initial macroscopic configuration, all but a final number will not reach the light-sheet \( L \). What enforces this pruning dynamically?

## 4 Instabilities

In this section we discuss why many initial states that are macroscopically identical to that of the background lead to a completely different future evolution. It is instructive to consider first the idealized case where no energy density is present in the true vacuum region of the background. This case turns out to be somewhat similar to de Sitter space [40, 41]: the background is stable against sufficiently small perturbations, but introducing too much matter will make it collapse to a big crunch. The extra mass produced near the time-symmetric slice is a good diagnostic for the permissibility of a perturbation.

Interestingly, this is no longer the case for the true bounce solution, which is filled with homogeneous matter energy density. We find that this background is far more unstable. The process whereby matter energy is absorbed into vacuum energy, permitting a bounce, is extremely delicate since it decreases coarse-grained entropy. Arbitrarily small
perturbations at early times typically suffice to derail this absorption completely, causing a big crunch. (On the other hand, one might imagine introducing enormous extra mass at early times; this could be consistent with a bounce as long as it all ends up in a coherent excitation of the field $\phi$.)

4.1 Instabilities of the vacuous bounce

In this subsection we assume that by fine tuning the potential, we have arranged that there are no particles in the background. In this case the matter entropy vanishes at all times.

In simple examples, it is possible to see how gravitational backreaction enforces the bound (3.6). For example, we could add a black hole at rest at the center of the flat region. We will see that there is a bound on how big the black hole can be.

It is simplest to analyze the situation on the time-symmetric slice $t = 0$; because we are adding a black hole at rest at the center it does not break the time reversal invariance of the background. The solution will be Schwarzschild out to the domain wall. Outside the domain wall, the geometry must be de Sitter or Schwarzschild-de Sitter because of the spherical symmetry. The radius of the domain wall, $r_0$, will be bigger [32] than the Schwarzschild radius of the mass, $2GM$. In order to match to de Sitter space along the time-symmetric slice, the domain wall radius must be less than the de Sitter radius $R$. In order to satisfy both of these constraints, we need $2GM < R$. A bound on the mass of the black hole is equivalent to a limit on the entropy,

$$S < \frac{\pi R^2}{G},$$

which agrees precisely with the prediction we derived from the covariant entropy bound.

Another example is to add $N$ photons to the initial state. Because the spatial slices are infinite, their wavelength can be made arbitrarily long. Hence $N$ can be large without causing large energy density. The entropy of the state is $N$, and if the entropy exceeds the entropy bound (3.6) then something must go wrong. What happens is that as the space contracts towards $t = 0$, the photons are blueshifted. At $t = 0$, the wavelength of the photons is limited by the size of the space, roughly the de Sitter radius $R$. $N$ photons with wavelength $R$ have energy $N/R$. If $N$ exceeds the entropy bound $S_{\text{max}}$ then the energy exceeds roughly $R/G$.

There must be a huge backreaction, because the biggest black hole that can fit in de
Sitter space has a mass of order
\[ M_{\text{BH}} \approx \frac{R}{G}. \] (4.8)

If we exceed the entropy bound, we are inserting more energy than can fit through the geometry, even if we allow black hole formation. The result must be a big crunch. Thus we find again that the conditions for avoiding a crunch and for avoiding a violation of the bound coincide.

Of course, not all states that lead to a crunch have large entropy. Consider adding two small particles of mass \( m \) which are comoving with respect to the open-FRW coordinate system picked out by the symmetries of the geometry. As we shall see, they can be more simply characterized from the point of view of Minkowski space, as two particles colliding at the event \( r = 0, t = 0 \), as shown in Fig. 5. The FRW coordinates cover a region of flat space, with metric
\[ ds^2 = -d\tau^2 + \tau^2(\rho^2 + \sinh^2 \rho \, d\Omega_{D-2}^2). \] (4.9)

We use Greek letters \( (\tau, \rho) \) for the FRW coordinates and Roman letters \( (r, t) \) for the usual flat space coordinates. The two coordinate systems are related by
\[ \tau^2 = t^2 - r^2. \] (4.10)
\[
\sinh^2 \rho = \frac{r^2}{t^2 - r^2}.
\] (4.11)

A particle at rest in FRW coordinates follows the trajectory

\[ r(t) = t \tanh \rho_0 , \] (4.12)

where \( \rho_0 \) is the value of its FRW radial coordinate. The trajectory passes through the origin at \( t = 0 \) no matter where in the FRW the particle began. So two particles at rest relative to the FRW coordinates will collide at the origin. If they start at opposite points on the 2-sphere, their relative velocity will be

\[ v_{\text{rel}} = 2 \tanh \rho_0 . \] (4.13)

The center of mass energy is

\[ E_{\text{cm}} = 2m \cosh \rho_0 , \] (4.14)

where \( m \) is the mass of the particles, so the energy is large if they start far apart. To avoid a global crunch, this energy must not exceed the mass of the maximal black hole, leading to the inequality

\[ m \cosh \rho_0 \lesssim \frac{R}{G} . \] (4.15)

This result is interesting because it indicates that two innocent perturbations far from each other tend to collide catastrophically.

### 4.2 Instabilities of the bounce

As discussed in Sec. 2, the FRW regions (II and III) of the bounce contain an infinite number of particles. The light-sheet \( L \) can contain only a finite number of particles, so as time evolves, all but a finite number of particles must be absorbed. For definiteness, we assume that in the background all particles are produced on a reheating surface in region II, so by time-reversal symmetry, they must be absorbed on a “recooling” surface in region III, prior to \( t = 0 \).

Recooling is an extremely delicate process that decreases the entropy drastically. It is comparable to a broken glass reassembling spontaneously, or all air molecules collecting in one small corner of a room. Only a miniscule subset of phase space trajectories correspond to these processes. Tiny perturbations do not take these special states into each other, but into states with a generic evolution. For example, a tiny mass added in the far past will gravitationally influence a vast number of other particles and disturb the recooling process.
in the entire future light-cone of the perturbation (see Fig. 6). If the perturbation is introduced sufficiently early, this will result in an enormous number of particles remaining unabsorbed. Their energy, not the energy of the initial perturbation, decides whether the universe will bounce or crunch.

In fact, recooling can be thrown off just by moving some particles around, leaving the energy invariant (at least at the perturbative level at which we might hope to define energy in this background); or by removing some particles from the background, effectively decreasing the initial energy and entropy.

But the entropy bound does not forbid all states; there should be a finite number that do not destroy the geometry. In the next section, we construct examples of such states and verify that their entropy approximately saturates the bound.

5 Allowed perturbations

In the saddlepoint approximation to the path integral, amplitudes between points near past and future infinity should correspond to non-crunching perturbations. In this section we provide approximate, semiclassical constructions of two classes of such states.
5.1 States defined at $t = 0$

A natural place to define amplitudes is the past and future FRW infinities. But we have seen in the previous section that it is tricky to identify in-states that do not collapse. At least at the semi-classical level, however, such states can be found by perturbing the geometry on the time-symmetric slice, $t = 0$. (In a full quantum theory, amplitudes would receive contributions from various configurations at $t = 0$.)

Unlike the FRW regions, the true vacuum portion of the time-symmetric slice is well approximated by empty flat space. Hence we can use the results of Sec. 4.1. There are small perturbations with a few extra particles, but most states will correspond to black holes. As discussed in Sec. 4.1, the largest black hole that fits has radius of order $R$, so the entropy of the possible perturbations will be of order $R^2/G$, in agreement with the bound (3.6).

When those states propagate forward, they undergo complex interactions with the particles produced at reheating. Some of the excitations will cross the event horizon of the FRW observer and enter region IV. In Ref. [32] it was conjectured that the evolution is nevertheless unitary. But if scattering off of black holes offers any guidance, the information will arrive at $I^+$ in extremely scrambled form.

Thus, it will be hard to look at an out-state and tell whether it originated from an allowed perturbation at $t = 0$, i.e., whether it corresponds semiclassically to a geometry with that contains also a past asymptotic region. But by time-reversal symmetry of the bounce background, this means it is also very difficult to set up an initial perturbation that will not crunch, but will rather bounce and produce a late-time geometry asymptotic to the future regions of the bounce.

In the next subsection, we shall see that it is nevertheless possible to characterize some of the suitable in-states approximately, and to verify once more that their entropy is in agreement with the bound (3.6).

5.2 Halo states

We are interested in an approximate construction, near $I^-$, of asymptotic states that can lead to a nonsingular bounce geometry. We will assume that the matter present in the FRW regions is dust. We will also assume that its density is low enough so that the FRW evolution is always curvature dominated.

The basic idea is to construct solutions which differ from the bounce in a finite region,
so that the delicate “recooling” process is only disturbed for a small number of particles. If we could neglect gravitational signals propagating out from the perturbation, this would be readily accomplished by adding, subtracting, or rearranging particles in a bounded region of space. But we must be careful to avoid any changes, no matter how small, in the gravitational field outside the region considered.

We achieve this by imposing two conditions. The first is that the perturbation be spherically symmetric, so that Birkhoff’s theorem applies. The second is that the total mass in the perturbed region remain unchanged compared to the background. A simple example is an overdense region mass surrounded by an vacuous shell, or “halo”.2

Our solution consists of three pieces joined along timelike hypersurfaces, as shown in Fig. 7. We take the outer piece to be a portion of the background, i.e., an open contracting dusty FRW universe, except for the region $\rho < \rho_0$, which we excise and throw away. [Here $\rho$ is the comoving radial coordinate; see Eq. (2.5).] By starting with this exterior piece and working our way in, the absence of any perturbation outside $\rho_0$ is guaranteed by construction. This will be important.

We take the middle piece to be vacuous. Then Birkhoff’s theorem implies that the middle piece is given by a portion of a Schwarzschild solution. The Schwarzschild radius characterizing this solution is that of the black hole which the excised piece (which we discarded) would have formed if complemented by a vacuum exterior. Equivalently, the correct solution is fixed by the requirement that the Schwarzschild geometry contain a geodesic identical to the path of the innermost dust shell of the outer piece. This geodesic, $\beta$, forms the outer limit of the middle piece.

We have thus obtained an open universe containing a black hole. The fully extended solution contains an Einstein-Rosen bridge to an asymptotically flat region. This would not describe a reasonable perturbation of the in-state. But this is easily fixed by bringing matter back at a smaller radius.

Consider a timelike radial geodesic, $\alpha$, in the middle piece, such that $\alpha$ begins at past timelike infinity and lies everywhere inside $\beta$, as shown in Fig. 7. The inner piece (inside of $\alpha$) can then be taken to be the central portion of an open dust-dominated FRW universe uniquely determined by the requirement that it contain the geodesic hypersurface $\alpha$ that

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2In reality, the dust approximation is never exact, and with realistic quantum fields it is impossible to suppress the propagation of signals out from the perturbed region entirely. Here we assume that the dust approximation is good and that the corrections (the weak signals leaking out) can be precisely cancelled by subtle perturbations of the global background, without affecting the basic geometry and coarse-grained entropy of our solutions.
Figure 7: Construction of an in-state which will *not* cause a global collapse. The full solution (c) is obtained by gluing a piece of the background solution (b) to a collapsing ball of dust surrounded by vacuum (a). The full solution keeps the region of (a) to the left of the line $\beta$ and the region of (b) to the right of $\beta$. $\alpha$ marks the boundary between the collapsing dust (shaded) and vacuum. In the center of the diagram the particles of the outside solution have been absorbed, rendering the choice of $\beta$ arbitrary (dashed line). In this region $\beta$ is not a geodesic, whereas $\alpha$ is a geodesic throughout the diagram. Heavy lines represent the domain wall (timelike) and the reheating/recooling surfaces (spacelike). The black hole horizon and singularity are shown in the top left corner.

forms its boundary. Note that the inner piece is *not* a portion of the original open FRW universe—it will have a different density at the time when curvature starts to dominate.\(^3\)

Because they involve underdense shells, we will call these states halo states. They differ from the background only within a finite comoving radius, and because they contain only dust, no signal will propagate out to the unperturbed region. Hence, it is easy to evolve the halo states from past infinity to the recooling surface. In the unperturbed region, the conversion of particles into vacuum energy will proceed exactly as in the vacuum.\(^4\)

\(^3\)One could also construct halo solutions with white-hole initial conditions by taking the inner piece to be a portion of a closed recollapsing dust dominated FRW universe.

\(^4\)At the recooling surface, spatial gradients will arise in the field which makes up the domain wall, allowing signals to propagate out. This is acceptable, since our goal was only to avoid disturbing the recooling. However, there will be corrections to our solution after recooling.
The halo region will not be absorbed, so that a collapsing ball of dust will enter the central region of the spacetime. Thus halo states form black holes, and black holes cannot fit if they are too large. It follows that we can estimate the number of (bouncing) halo states by the entropy of the black holes they form, which is of order $R^2/G$. Thus we find that the entropy bound is approximately saturated by the non-crunching states.

Of course, the idealization as dust is not realistic in quantum field theory. Even if the background outside $\rho_0$ is truly unperturbed near $I^-$, some weak signals will propagate out from the halo and disturb the absorption process on large scales. However, we do expect that there are allowed perturbations whose main feature is a halo of size $\rho_0$, plus extremely subtle additional perturbations outside the halo which cancel against such signals.

Not all allowed states will be of halo form. We have merely pointed out that the halo states appear to be the only allowed states that can be described simply near the boundary of spacetime, and that they can account for a significant portion of the allowed entropy. The halo states are highly inhomogeneous and thus of no relevance to a realistic universe. The other allowed states are extremely scrambled near infinity and hence hard to characterize.

As we discussed in the introduction, this limits the appeal of an S-matrix description (involving the past hat) as a framework for the landscape. But the attractive properties of the Coleman-De Luccia solution for defining observables [32]—the presence of noninteracting particles and low curvature at late times—remain to be exploited. They may yet allow for the construction of an exact quantum cosmology whose amplitudes correspond to asymptotic observables in the future hat.

Acknowledgements

We would like to thank T. Banks and L. Susskind for valuable discussions. This work was supported by the Berkeley Center for Theoretical Physics, by a CAREER grant of the National Science Foundation, and by DOE grant DE-AC03-76SF00098.

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