An integrated process targeting and continuous review system with sampling inspection

Firas M Tuffaha¹ and Mohammad M AlDurgam²

Abstract
It is common in the integrated targeting inventory literature to assume 100% inspection. Yet, sampling inspection is still a valid alternative in numerous situations. Inspection time has been assumed negligible in the literature of integrated inventory and sampling inspection. Neglecting inspection time is unrealistic, especially when rejected lots are sent for 100% inspection. This research work integrates process targeting, production lot-sizing and inspection. Given a scenario of a producer and distributor, the objective is to determine the optimal mean setting at the producer, the production lot size to be produced and shipped to the distributor and the reorder point at the distributor under a given sampling inspection plan. To the best of the authors' knowledge, sampling inspection and its associated costs are rarely addressed in integrated supply chain models, and have never been addressed in integrated models with controllable production rates.
Numerical illustrations using an efficient solution technique are presented to highlight the impact of various model parameters. The results indicated that inspection time has a significant impact on the total cost of the developed model, especially, when tightened inspection plans are used.

Keywords
Two-echelon supply chain, sampling inspection, process targeting, continuous review system

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Introduction
Supply chain management is the centralized management of suppliers, materials, production facilities, distribution and customers. This is all linked together through the forward flow of material and backward flow of information.¹ Production, quality and maintenance are the main interrelated areas in any supply chain system. Due to their interdependence and to achieve global optimality, many researchers have studied the integration of more than one of these areas.²,³ Joint economic lot-sizing is a primary aspect of coordination in supply chain, which was addressed for the first time by Goyal⁴ in the 1970. A research stream evolved over the following years to address different aspects of coordination and integration of decisions in supply chain. For a comprehensive review on integrated inventory models, the reader is referred to Glock.⁵

In this research work, we propose an integrated process targeting, joint economic lot-sizing and inspection model. The objective is to determine the process mean setting, reorder level and order quantity for a two-stage supply chain with sampling inspection. Products are manufactured through a variable production rate at the first stage then are sent in lots to a distributor’s warehouse at the second stage.
A stochastic $Q - R$ model is used by the distributor to control the inventory level. Sampling inspection is used by the distributor for monitoring the quality of incoming lots. Note this assumes a predefined/given sampling scheme.

The earlier relevant research work (e.g., Darwish$^{6,7}$) assumed 100% inspection. Yet, sampling inspection is still a practical necessity since it has lower cost and represents the only applicable option whenever destructive testing is needed. Therefore, in contrary to the aforementioned research work, we consider inspection and relax the assumption of negligible inspection time. Also, we assume that inspection time is linearly proportional to the size of the produced lot. The effect of the inspection time will be significant whenever lots are rejected and sent to 100% inspection. This will result in the extension of the lead time due to 100% inspection, and increased probability of running out of stock. We denote this time as the extended lead time.

The remainder of this paper is structured as follows: Section 2 provides a summary of the related literature. Section 3 illustrates the model assumptions and notations used throughout this paper. Section 4 provides a discussion on the effect of sampling inspection on the inventory model. Section 5 outlines the development of the proposed model. Section 6 presents the proposed efficient solution technique. Section 7 presents a numerical example and a thorough sensitivity analysis. Finally, Section 8 provides conclusions, remarks and recommendations for future research.

**Literature review**

The integrated joint economic lot-sizing and quality control literature spans a plethora of topics. This section presents two streams of research that are closely relevant to the work at hand, namely; I) research works that address the integrated targeting inventory models II) research works that address integrated inventory and inspection sampling models.

**Integrated process targeting and inventory control models**

Process targeting models aim at determining the optimal process parameters to minimize the cost of items that do not meet the required specifications.$^{8,9}$ Gong et al.$^{10}$ presented one of the earliest models that addressed the integration of process targeting and inventory control. They discussed the trade-offs in the process targeting problem and how it affects the process yield. Al-Fawzan and Hariga$^{11}$ studied the case of time-dependent process mean with lower specification limit on the produced items. Their developed model optimized process mean setting and both the lot sizes of the raw material and finished products. Later on, Hariga and Al-Fawzan$^{12}$ extended Al-Fawzan and Hariga’s$^{11}$ work to incorporate multiple markets, where the output of the production process can be sold to different customers in the same market, each with different quality requirements.

Darwish$^{13}$ extended the single-vendor single-buyer inventory-targeting problem to consider joint determination of process mean and lot-sizing. The author showed the impact of the process mean on the vendor’s yield rate, which affects the production lot size and the number of unequal-sized shipments from the vendor to the manufacturer. Later, Darwish and Abdulmalek$^{14}$ extended this problem to consider a time-varying process mean. Alkhedher and Darwish$^{15}$ addressed the integrated inventory-targeting problem with random demand. They assumed the produced items to have upper and lower specification limits and non-conforming items were scrapped. They concluded that demand variation should not be neglected in the integrated inventory-targeting problem. Darwish and Aldaihani$^{16}$ studied the integrated inventory-targeting problem for a single-vendor multi-newsvendor supply chain. They demonstrated the importance of integration through a illustrative examples. The vendor sets the process mean, produces, and then ships to newsvendors who, at the end of the selling period, either run short of supplies or return surplus items to the vendor at a known rate.

Recent research works include: the possibility of order processing time reduction by including an investment cost to reduce order processing time from normal to a minimum duration,$^{16}$ smart pricing and market segmentation,$^{17}$ and addressing the case of stochastic demand.$^{7}$

**Integrated inventory and inspection sampling models**

Lee and Rosenblatt$^{18}$ provided one of the first joint inspection and inventory planning models, the fraction of non-conforming items per lot was assumed fixed and known. By relaxing this assumption, Zhang and Gerchack$^{19}$ derived the optimal order quantity and fraction of lot to be inspected. Peters et al.$^{20}$ introduced an integrated inventory-inspection model, which determines the lot-sizing decision along with Bayesian quality control system, for a lot-by-lot attribute acceptance sampling plan.

Salameh and Jaber$^{21}$ extended the classical EOQ model to a situation where the purchased items have imperfect quality, subject to 100% screening, items of secondary quality are sold in batches at the end of the screening process. Goyal and Cárdenas-Barrón$^{22}$ provided a simplified way to solve Salameh and Jaber’s$^{21}$ model. AlDurham et al.$^{23}$ extended the classical production lot-sizing model to incorporate machining economics. Assuming 100% inspection, the authors demonstrated that machining parameters have an impact on the optimal lot size and the fraction of non-confirming units.

Cheung and Leung$^{24}$ developed a $Q - R$ model to manage the joint replenishment of a two-item inventory system. Ben-Daya et al.$^{25}$ developed an integrated stochastic inventory and inspection sampling model. They assumed that a
buyer buys items with imperfect quality and uses inspection sampling to get rid of defective items.

For the joint economic lot-sizing systems, Wu et al.\textsuperscript{26} considered the impact of inspection sampling, and order lead time reduction on a single-vendor single-buyer inventory system. Hsieh and Liu\textsuperscript{27} considered a single-supplier single-manufacturer supply chain. The supplier performs outbound quality inspection and the manufacturer performs inbound and out bound quality inspection (before sending items to his customers). They studied the quality investment and inspection strategies of the system in four non-cooperative games. Sharifi et al.\textsuperscript{28} studied the case of perishable products with imperfect quality and destructive testing. They determined the optimal lot size subject to an acceptance sampling plan.

Recently, Bouslah et al.\textsuperscript{29} addressed a joint lot-sizing and inspection system, with an unreliable manufacturing process. Duffuaa and El-Ga’aly\textsuperscript{30} presented a multi-objective process targeting model, the model aims at maximizing profit and product uniformity using the Taguchi loss function. They assumed a lower specification limit of the product and lot-by-lot acceptance sampling. Further, Duffuaa and El-Ga’aly\textsuperscript{31} showed that the inspection errors have a significant impact on the optimal problem solution. Recently, Duffuaa and El-Ga’aly\textsuperscript{32} extended Duffuaa and El-Ga’aly\textsuperscript{31} by considering multiple specification limits for two different quality grades of the same product. Chen et al.\textsuperscript{33} presented a modified Kapur and Wang’s model with unequal target value. Taguchi’s quadratic quality loss function correlated to process capability indices (Cpm and Cpmk) is assumed. The author showed that the model with given Cpm value has less specification tolerance, larger process mean, and lower expected cost than those of the modified model with specified Cpmk. In a recent study, AlDurgam\textsuperscript{3} developed a stochastic dynamic programming model to study the interactions between production lot-sizing, quality inspection and maintenance for a single stage system, the author demonstrated the interactions between the maintenance, production and quality inspection parameters. Furthermore, the author characterized the optimal production and inspection policies.

**Synthesis of both research streams**

Most of the articles cited in Section 2.1 assumed 100\% inspection. Also, most of the articles in Section 2.2 did not consider process targeting and neglected the effect of inspection time. In this article, we develop an integrated model that spans both research streams. Moreover, we introduce the concept of extended lead time as a consequence whenever the inspection time per unit is not neglected. The research works closest to the one at hand are those by Ben-Daya and Noman\textsuperscript{34} and Darwish et al.\textsuperscript{7}. Ben-Daya and Noman\textsuperscript{34} didn’t consider process targeting and assumed negligible inspection time. Darwish et al.\textsuperscript{7} did not consider sampling inspection. The proposed model can be viewed as an integration of the models by Ben-Daya and Noman\textsuperscript{34} and Darwish et al.\textsuperscript{7} and as an extension of the inspection model in Ben-Daya and Noman\textsuperscript{34} by addressing non-negligible inspection time and different inspection schemes. Our work contributes to the literature through addressing a two-echelon supply chain involving a producer and distributor with sampling inspection. To the best of the authors’ knowledge, sampling inspection and its associated costs are rarely addressed in integrated supply chain models, and have never been addressed in integrated models with controllable production rates. Moreover, inspection time is always overlooked and neglected in the literature. In this article, the effects of sampling inspection including the inspection time will be considered, especially, when the incoming lots are rejected and subjected to 100\% inspection.

**Model assumptions and notations**

In a two-echelon supply chain system, lots of fixed size ($Q$) are produced by a producer and dispatched to a distributor’s warehouse. The inventory system assumed here is a continuous review ($Q - R$) model with random demand. Let $X$ be the quality characteristic of interest for the product. It is assumed that $X$ has the nature of the larger the better. That is, the product is considered to be accepted if $X \geq L$; otherwise, the product is rejected. There are numerous examples in the literature of products with a lower sided quality characteristic, such as glass sheets produced by glass manufacturers\textsuperscript{35}; the gold plating industry, where the thicker the layer of gold the better; and the steel galvanization industry.\textsuperscript{36} The quality characteristic is assumed to follow a normal distribution with a pre-set parameter $\mu_x$ and constant variance $\sigma^2_x$. The probability of producing a nonconforming item will be

$$p = \Pr(X \leq L) = \Pr\left(Z \leq \frac{L - \mu}{\sigma_x}\right) = \int_{-\infty}^{\frac{L - \mu}{\sigma_x}} \varphi(z) \, dz.$$

This section will develop the mathematical model for a two-echelon supply chain system involving a producer and a distributor. Figure 1 shows the inventory model of the distributor over time under sampling inspection. In this work, dispatched lots from the producer to the distributor are subjected to sampling inspection. A sample of size $n$ is taken. If the number of defectives found is less than or equal to the threshold level, $c$, then the lot is accepted; otherwise, the lot is rejected. Rejected lots are sent to 100\% inspection. The duration of the 100\% inspection ($\Delta$) is not negligible, and is assumed to be linearly proportional to the lot size $Q$ (Figure1).

Sampling inspection trade-offs are extended to involve: the acceptance of defective items passing through the non-inspected portion of the accepted lots. The cost per unit of these items is $c_d$. And the time lost due to 100\% inspection of the rejected lots. During this time, the inventory level is
expected to drop to a lower level \((S' < S)\) due to the extension of the lead-time period with the additional 100% inspection duration (Figure 1). At the same time, the delivered lot size after 100% inspection \((Q')\) will be lower than the produced lot \((Q)\) due to 100% screening. The difference between \(Q\) and \(Q'\) will not be replaced during the same cycle, since a reset up and reproduction is required. Therefore, this shortage will be treated in a similar way to the shortage encountered during lead time, and it will be backordered in subsequent cycles at a cost of rejection per unit \(c_r\). Finally, the inspection cost per unit is considered. In this paper, the appropriate setting of the process parameters \(Q, R,\) and \(\mu_x\) is simultaneously determined, such that the overall system cost is minimized.

To develop the model, the following assumptions are used:

1. Shortages are backordered;
2. The demand pattern is modeled by a normal probability distribution;
3. Inspection is free of errors;
4. Defective items after 100% inspection are not discarded, but they will be backordered in subsequent cycles;
5. The sampling inspection duration is negligible, but the 100% inspection duration is not. It is considered to be deterministic in nature and assumed to be proportional to the lot size \(Q\);
6. The sampling inspection scheme is known and given.; and
7. In any cycle, the fraction of incoming defectives \((p)\) is constant and depends on the setting of the production process parameter \(\mu_x\).
The following notations will used throughout this paper:

- $A$: Setup cost of the producer per cycle.
- $c$: Number of defectives allowed per sample.
- $C$: Material cost per unit.
- $C_d$: Cost of accepting a defective item.
- $C_i$: Inspection cost per unit.
- $C_r$: Cost of rejecting a defective item during 100% inspection.
- $D$: Random variable that denotes demand rate.
- $h$: Inventory holding cost per unit of the finished item per unit of time.
- $l$: Lead-time duration.
- $L$: Lower specification limit.
- $n$: Sample size.
- $p$: Probability of producing a nonconforming item.
- $R$: Reorder point.
- $s$: Safety stock level before replenishment takes place.
- $q$: Number of defectives allowed per sample.
- $v$: Production rate of the producer.
- $x$: Random variable that denotes number of defectives per incoming lot.
- $y$: Random variable that denotes number of defectives per sample.
- $\Delta$: Deterministic variable that denotes 100% inspection duration.
- $\mu$: Process mean setting.
- $\pi$: Shortage cost per unit short per unit of time.
- $\sigma_s$: Standard deviation of the material used to produce an item.
- $\sigma_d$: Standard deviation of the demand per unit time.

Lemma. The joint probability distribution $Pr(x, y)$ can be stated as

$$Pr(x, y) = Pr(y).Pr(x - y),$$

where $y$ is a binomial random variable, $y = 0, \ldots, n$, $x - y$ is a binomial random variable, $x - y = 0, \ldots, Q - n$.

Proof: By replacing the right-hand side terms of $Pr(x, y)$ with their binomial and hypergeometric forms respectively, we have

$$Pr(x, y) = \binom{Q}{n} p^n q^{Q-n} \frac{X}{Q} \frac{Q-x}{n-y} \frac{n-y}{n}.$$

Simplifying the factorials, multiplying by $p^{-y} q^n q^{-y}$, and by further simplification:

$$Pr(x, y) = \binom{Q-n}{X-y} p^n q^{Q-n-x+y} \binom{n}{y} p^y q^{n-y},$$

or

$$Pr(x, y) = Pr(y).Pr(x - y).$$

In the following part, we follow a similar approach to that of Ben Daya et al.25 to evaluate our inspection-related costs of our model.

Case 1: If the lot is accepted. The cost of inspection is the cost of taking a sample of size $n$ and inspecting it; this is given by

$$C_i n \sum_{y=0}^{c} \sum_{x=y}^{Q-n+y} Pr(x, y) = C_i n Pr(y \leq c). \quad (1)$$

The cost of defective items passing through the non-inspected portion of the accepted lot is given by

$$C_d \sum_{y=0}^{c} \sum_{x=y}^{Q-n+y} (x - y)Pr(x, y)$$

Replacing $Pr(x, y)$ with $Pr(y).Pr(x - y)$, and letting $x - y = R$ gives

$$C_d \sum_{y=0}^{c} \sum_{R=0}^{Q-n} (R)Pr(y)Pr(R) = C_d (Q-n)p Pr(y \leq c). \quad (2)$$

Case 2: If the lot is rejected. The cost of inspection will cover not only the sample of size $n$ but also the remaining portion of the lot, $Q - n$, that has to go for 100% inspection. This is expressed as

$$C_i Q \sum_{y=c+1}^{n} \sum_{x=y}^{Q-n+y} Pr(x, y) = C_i Q - C_i Q Pr(y \leq c). \quad (3)$$
The rejected items resulting from 100% inspection will not be replaced during the same cycle, but instead, they will be backordered at a cost per unit \( C_r \). This cost could either be set to be equal to the penalty cost per unit short \( \pi \) or have a different value. The expected cost of rejected units after 100% inspection is given by

\[
C_r \sum_{y=0}^{n} \sum_{x=y}^{n+y} (x-y)Pr(x,y).
\]

Simplifying in a similar manner to (2) gives

\[
C_r(Q-n)pPr(y > c)
\]

or equivalently,

\[
C_r((Q-n)p - (Q-n)pPr(y \leq c))
\]

Combining the costs in (1) through (4) gives the following expression for the expected quality cost:

\[
\text{Quality costs} : C_sQ + C_r(Q-n)p + (Q-n)pPr(y \leq c)((C_d-C_r)p - C_r).
\]

### The effect of sampling inspection on the received lot size \( Q \)

The reduction in the lot size \( Q \) will be realized from the defectives discarded from the inspected sample if the lot is either accepted or rejected and from the defective items resulting from the 100% inspection of the uninspected portion of the rejected lot, that is,

\[
\sum_{y=0}^{n} \sum_{x=y}^{n+y} yPr(x,y) + \sum_{y=c+1}^{n} \sum_{x=y}^{n+y} (x-y)Pr(x,y)
\]

By further simplification,

\[
np + (Q-n)pPr(y > c)
\]

or equivalently,

\[
Qp - (Q-n)pPr(y \leq c)
\]

Therefore, the expected quantity delivered due to sampling is

\[
\bar{Q} = Q - [Qp - (Q-n)pPr(y \leq c)]
\]

### The effect on the safety stock level(s)

The safety stock level, \( s \), changes according to the acceptance or rejection of the lot. If the lot is accepted, then \( s = R - Dl \), while if the lot is rejected, the safety stock will drop further, and it will be given as \( R - D(l + \Delta) \). That is,

\[
s = \begin{cases} 
R - Dl, & \Pr(y \leq c) \\
R - D(l + \Delta), & \Pr(y > c)
\end{cases}
\]

Hence, the expected safety stock level is

\[
\bar{s} = R - Dl - D\Delta Pr(y > c)
\]

or equivalently,

\[
\bar{s} = R - D(l + \Delta) + D\Delta Pr(y \leq c).
\]

### The effect of sampling inspection on the inventory cycle time

The reduction in the lot size when the lot is accepted is due only to the defective items discarded from the sample. Thus, the delivered quantity is \( Q - np \). In contrast, the reduction in the delivered lot size due to rejection because of the defectives discarded from the inspected sample and defectives discarded from the remaining portion of the lot is \( Qp \). Hence, the quantity delivered is \( Q - Qp \). The cycle length, \( T \), is given as

\[
T = \begin{cases} 
\frac{Q - np}{D}, & \Pr(y \leq c) \\
\frac{Q - Qp}{D} + \Delta, & \Pr(y > c)
\end{cases}
\]

Hence,

\[
E(T) = \frac{\bar{Q} + D\Delta Pr(y > c)}{D}
\]

### Model development

In this section, we formulate the cost model by examining the costs associated with it for all the involved terms. The renewal theorem is used through adding inventory-related costs and dividing them by the expected cycle time in (8). The associated costs are given as follows.

1. **Setup and material costs per unit of time at the manufacturer:** A setup cost, \( A \), is incurred once per cycle. The expected cost of material used per lot is \( CQ_\mu \). Thus, the expected setup and material costs per unit of time are equal to \( (A + CQ_\mu) / E(T) \). Substituting for \( E(T) \) using (8) gives:

\[
\frac{D(A + CQ_\mu)}{\bar{Q} + D\Delta Pr(y > c)}
\]

2. **Holding cost per unit of time at the manufacturer and distributor:** The average on-hand inventory for the \( Q - R \) system at the distributor is \( Q/2 + s \). However, with sampling inspection, depending on whether the lot is accepted or rejected, there are two possible values for both \( Q \) and \( s \). In developing equations (7) and (8), we have considered the respective value of \( Q \) and \( s \) that will take place in both cases. Therefore, the average on-hand inventory can be shown to be \( E(Q)/2 + E(s) \) or \( \bar{Q} + \bar{s} \). For the production side, the inventory starts to build up at a rate of \( \nu \) once an order
is received and stops once \( Q \) units are produced. Thus, the average on-hand inventory per unit of time is \( \frac{Q}{E(T)} \) and the total holding cost per unit of time is

\[
h\left(\frac{Q}{2} + \bar{s} + \frac{Q^2}{2\nu E(T)}\right)
\]

Substituting (6), (7) and (8) gives

\[
h\left(\frac{Q - [Qp - (Q - n)pPr(y \leq c)]}{2} + R - D(l + \Delta) + D\Delta Pr(y \leq c)
\]

\[
+ \frac{DQ^2}{2\nu(Q - [Qp - (Q - n)pPr(y \leq c)] + \Delta DPr(y \leq c))}
\]

3. Shortage cost per unit of time at the distributor: The inventory system is subjected to shortages whenever the demand during lead time exceeds the reorder point, \( R \). The expected number of shortages per cycle for the \( Q - R \) system is given as

\[
S(z) = \sigma L(Z)
\]

where

\[
Z = \frac{R - \mu}{\sigma}
\]

and \( \mu = DI \) is the mean of the lead time demand, \( \sigma \) is the standard deviation of the lead time demand, and \( L(z) \) is the standardized normal loss function.

In our model, we must distinguish between shortages encountered when a lot is accepted versus those when a lot rejected. The distinction has to be made because of the additional 100% inspection period, \( \Delta \), in the case of lot rejection. The parameters when the lot is rejected are as follows:

\[
\mu_r = D(l + \Delta), \sigma_r = \sqrt{\frac{L + \Delta}{l} \sigma} \text{ and } Z_r = \frac{R - \mu_r}{\sigma_r}
\]

The suffixes \( r \) and \( a \) are added to denote rejection and acceptance, respectively; thus, shortages due to rejection are denoted by \( S_r(Z) \), whereas shortages under acceptance are denoted by \( S_a(Z) \). The difference between \( S_r(Z) \) and \( S_a(Z) \) is the use of different parameters. Hence, the shortage is expressed as

\[
S(Z) = \begin{cases} S_a(Z), & Pr(y \leq c) \\ S_r(Z), & Pr(y > c) \end{cases}
\]

Therefore, the expected shortage per unit of time is:

\[
\frac{\pi S(Z)}{E(T)} = \frac{\pi D(S_a(z)Pr(y \leq c) + S_r(z)Pr(y > c))}{Q + \Delta DPr(y > c)}
\]

The expected total cost per unit of time is the sum of the quality costs, setup costs, holding costs and shortage costs given by the equations (5), (9), (10) and (11), respectively, and by using (6), gives:

\[
TC(Q, R, \mu) = \frac{D(A + CQ\mu_a)}{Q(1 - (p - zD)(Pr(y > c))}
\]

\[
+ C,DQ + C,DQp + DQPr(y \leq c)((C_d - C_r)p - C_d)
\]

\[
+ h\left(\frac{Q - [Qp - (Q - n)pPr(y \leq c)]}{2} + R - D(l + \Delta) + D\Delta Pr(y \leq c)
\]

\[
+ \frac{DQ^2}{2\nu(Q - [Qp - (Q - n)pPr(y \leq c)] + \Delta DPr(y \leq c))}
\]

To simplify this expected total cost rate equation, we assume that the sample size, \( n \), is negligible relative to the lot size \( Q > n \). In addition, we assume that the inspection duration is linearly related to the lot size, that is, \( \Delta = \alpha Q \). Finally, as Figure 1 shows, \( l = Q/v \). This gives

\[
TC(Q, R, \mu) = \frac{D(A + CQ\mu_a)}{Q(1 - (p - zD)(Pr(y > c))}
\]

\[
+ C,DQ + C,DQp + DQPr(y \leq c)((C_d - C_r)p - C_d)
\]

\[
+ h\left(\frac{Q - [Qp - (Q - n)pPr(y \leq c)]}{2} + R - DQ
\]

\[
+ \frac{DQ^2}{2\nu(Q - [Qp - (Q - n)pPr(y \leq c)] + \Delta DPr(y \leq c))}
\]

**Model analysis and solution procedure**

This section presents an efficient solution technique for the model in Section 5. The decision variables are the order quantity \( Q \), the reorder level \( R \) and the process mean of the manufacturer (\( \mu \)). To determine the optimal values of these decision variables, we need to find an expression for each term at which the total cost vanishes. This is achieved by taking the derivative for each term and setting it to zero. By observing equation (12), it can be seen that \( \mu_r \) appears in various forms, either implicitly, as for the parameters \( p \), \( Pr(y \leq c) \), or even in an explicit manner. Therefore, it is not possible to have a first derivative for it. Working out the other two, we have

\[
\frac{\partial TC}{\partial Q} = 0.
\]
\[-AD + \pi D(S_e(R)Pr(y \leq c) + S_i(R)Pr(y > c)) \over BQ^2 + h \left( vB(1 - pPr(y > c) - 2BD(1 + zvPr(y > c)) + D \right) \over 2vB \right] \\
+ \pi D^2 \left( Pr(y \leq c)(1 - \Phi(Z)) + Pr(y > c)(1 + \infty v)(1 - \Phi(Z_r)) \right) = 0\]

where:
\[
B = 1 - (p - \alpha D)Pr(y > c)
\]
\[
Z = \frac{R - D(Q_i/\sigma)}{\sigma}, \quad Z_r = \frac{1}{\sqrt{1 + \infty}} \left( Z - \frac{QD}{\sigma} \right); \quad \text{applying the above for } Z \text{ and } Z_r, \text{ we have:}
\]
\[
Pr(y \leq c)\Phi(Z) + Pr(y > c)\Phi(Z_r) = \frac{\pi D - hQ^*B}{\pi D},
\]

The presence of Q in the Z terms (Z and Z_r) gave the last term in equation (13). The derivative of S_e(.) can be found by using its respective normal loss function, that is, S_e(z) = \sigma L(z) and then by using the chain rule. When taking the derivative with respect to R, there will be two cases for Z. It can be easily verified that the maximum value equation (14) can take is 1.0. Thus, to have a feasible solution for equation (14), the right-hand side, \[\frac{\pi D - hQ^*B}{\pi D}\]
has to be less than or equal to 1. Hence, \[hQB \leq \pi D, \text{ or } Q \leq \frac{\pi D}{hB}\] (15)

Note that B \geq 0. Equation (15) will set an upper limit for Q at any given pre-set value \(\mu_i\). The inspection time per unit \(\alpha\) has a clear effect on the feasible region of equation (14). If \(\alpha\) is too small (\(\alpha \rightarrow 0\)), then equation (14) reduces to \(\Phi(Z) = (\pi D - hQ^* B)/\pi D\) and a unique solution is found. In contrast, if \(\alpha\) is extremely large (\(\alpha \rightarrow \infty\)), then \(Z \rightarrow \infty\); therefore, equation (14) becomes \(Pr(y \leq c)\Phi(Z) = (\pi D - hQ^* B)/\pi D\). This leads to the distortion of the feasible space. Therefore, a unique feasible solution is no longer guaranteed.

Equations (13) and (14) are not explicit in the variables Q and R. Therefore, to find a root for both equations, a search method is used. For equation (14), we use a simple search method over the range of \(Z \in [-3, 3.9]\) at an increment of \(\delta\). The search starts at \(Z = -3\) and continues until the value obtained for the equation is close enough to the right-hand side within an acceptable margin of error. If such value does not exist, then set \(Z = 3.9\) and \(R = \frac{DQ}{v} + 3.9\) a. To determine the root for equation (13), the bisection method is used. This is a simple derivative-free method that requires bracketing the variable within two points with opposite function signs. To find the bracket limits, a simple search at a predetermined step size is used. This search will be conducted over the range \((0, \text{ minimum } \frac{\partial F}{\partial Q}, D)\) till a point K exists such that \(\frac{\partial F}{\partial Q} > 0\). If K does not exist, then \(Q^* = \text{ minimum } \frac{\partial F}{\partial Q}, D\). The procedure for determining the optimal solution calls for letting \(\mu_i\) be controlled by a line search method. Here, we use the interval-halving method. This method is simple, derivative free and exhibits rapid convergence (Bazaraa et al. 1993). We start by bracketing \(\mu_i\) such that \(\mu_i \in [L, L + 3\sigma_i]\). We believe the bracket limits are adequate as they ensure the proportion of defectives will vary between almost 0 and 50%. During any iteration of the interval-halving algorithm, the corresponding \(Q^*\) and \(R^*\) values are determined using an iterative procedure that cycles between equations (13) and (14) until the values of Q and R are close enough within two subsequent iterations. The starting point of \(Q = Q_o\) is assumed to be the economic production quantity, or \(Q_o = hB^{2}\). The iterative algorithm is as follows:

At any given \(\mu_i\) determine \(Q^*\) and \(R^*\) by the following procedure.

Step 1: Set \(Q_o\) to be equal to the EPQ = \(Q_o = hB^{2}\).
Step 2: Using the search method via equation (14), determine the value of \(R_i\) that corresponds to \(Q_i\), where \(i\) represents the iteration number.
Step 3: Using the bisection method via equation (13), determine a new value for \(Q_i\) that corresponds to \(R_i\) from step 2.
Step 4: Using the value of \(Q_i\) from step 3, compute a new value for \(R_i\) using the search method via equation (14).
Step 5: Repeat steps 3 and 4 until two successive values of \(R_i\) and \(Q_i\) are approximately equal.
Step 6: The last values for \(R_i\) and \(Q_i\) coming from step 5 are the optimal values of \(Q^*\), and \(R^*\) at any given \(\mu_i\).

**Numerical results**

Below is a numerical example solved using the method outlined in Section 6. The goal here is to study the effect of the main model parameters and highlight various managerial insights. Unless otherwise specified, the following data are assumed throughout this section: \(L = 1.00\), \(\sigma_x = 1 : 00\), \(\nu = 500\), \(A = 100\), \(D = 50 000\), \(C_i = 1.25\), \(h = 5\), \(C_d = 30\), \(C_s = 40\), \(\pi = 20\), \(\alpha = 0.25\) hr/\(\text{unit}\), \(n = 89\), \(c = 2\), \(\sigma_d = 2000\) and \(C = 1\). We assume that the system operates 10 hours per day and 260 days per year. The inspection time per unit has to be converted to a yearly basis,
considering the number of hours available per year. During our search for the proper Z value in equation (14) we used a fine value of $\delta = 0.0001$. The numerical illustrations are carried out in the following sequence: First, we study the effect of inspection time over the model decision variables, assuming various sampling plans. Then, we study the effect of selected cost parameters using a fixed sampling plan.

**Quality sampling plan and inspection time**

Sampling plans should be designed adequately to have sufficient segregation power when incoming lots’ quality deteriorates. Three different sampling plans are considered, each of which has the same sample size but different acceptance limits. The sample size used is fixed ($n = 89$), whereas the acceptance limits are $c = 2, 4$ or 6. The three sampling plans are denoted as tightened ($c = 2$), moderate ($c = 4$) and relaxed ($c = 6$). For each sampling plan, the inspection time per unit ($a$) is changed and their impact on the model’s performance is monitored. Figures 2–7 show

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**Figure 2.** Effect of inspection time ($a$) on the process mean setting ($\mu$) and lot probability of acceptance ($Pa$) under the tightened sampling plan.

**Figure 3.** Effect of inspection time ($a$) on the reorder level ($R$) and order quantity ($Q$) under the tightened sampling plan.

**Figure 4.** Effect of inspection time ($a$) on the process mean setting ($\mu_x$) and lot probability of acceptance ($Pa$) under the moderate sampling plan.

**Figure 5.** Effect of inspection time ($a$) on the reorder level ($R$) and order quantity ($Q$) under the moderate sampling plan.

**Figure 6.** Effect of inspection time ($a$) on the process mean setting ($\mu$) and lot probability of acceptance ($Pa$) under the relaxed sampling plan.

**Figure 7.** Effect of inspection time ($a$) on the reorder level ($R$) and order quantity ($Q$) under the relaxed sampling plan.
the results of varying the inspection time on the model decision variables under various types of sampling plans.

Figures 2 and 3 show the effect of varying the inspection time under tightened sampling plan. Figure (2) shows that, as \( x \) increases, the process mean setting \( (\mu_x) \) will increase to reduce the effects of the extended lead-time period related costs. This increase in \( \mu_x \) leads to an increase in the probability of acceptance \( (P_a) \). It is clear that the increase in inspection time made \( \mu_x \) increase over the range of \([3.41, 3.84]\). This increase in \( \mu_x \) values is attributed to the effect of the tightened sampling plan, The effects of the inspection time over \( Q \) and \( R \) is almost negligible as seen by Figure 3.

The same behavior has been reported under the moderate sampling plan. Again, as can be seen in Figure 4, the increase in the inspection time is accompanied by the increase of both \( \mu_x \) and \( P_a \). This increase is explained by the need to reduce the effects of the extended lead-time period related costs. In contrast to Figure 2, the variation of \( \mu_x \) was limited to fall over a smaller range of \([3.26, 3.46]\). This demonstrates how \( \mu_x \) can adapt to reduce the effects of inspection time under moderate sampling plans. The effects of the inspection time over \( Q \) and \( R \) is almost negligible as shown in Figure 5.

Figures 6 and 7 show the effect of inspection time for the relaxed sampling plan. In this case, the inspection duration \( (x > 0) \) has no effect on \( \mu_x, Q, \) or \( R \). The process mean setting \( \mu_x \) is almost set at 3.23, with minor upward deviations ensuring a 100% acceptance level \( (P_a = 1.00) \) at three significance levels.

Figure 8 shows the effect of inspection time over total cost under the three sampling plans. It is clear that, under the relaxed sampling plan, the inspection time has no effect on the total cost, and the total cost is the least among the three modes. In contrast, whenever the sampling plan is more tightened, as for the moderate and tightened plans, the total cost becomes higher.

The effect of the inspection time over \( Q \) and \( R \) under different sampling plans is negligible. The values of \( Q \) and \( R \) did not vary that much. It was almost fixed at 1140, 495 with some variation around these levels.

**Effects of the main cost parameters**

In this section, we use a moderate sampling plan \( (n = 89, c = 4) \) throughout the analysis. Tables 1–3 show the effects of the main cost parameters \( C_d, C_r, \) and \( C \) on the decision variables of the model. Table 1 presents the effect of \( C_d \). As seen by Table 1, the effect of \( C_d \) over \( Q \), and \( R \) is negligible.
Table 1. Effect of the variation of Cd on the main cost parameters over various inspection times.

| Cd   | \( \mu_x \) | R   | Q   | TC   | \(^{*}\)Pa |
|------|-------------|-----|-----|------|---------|
| 10   | 3.07        | 487.63 | 1153.80 | 174,430 | 0.971    |
| 0.008 | 3.12        | 504.69 | 1135.10 | 176,160 | 0.982    |
| 0.025 | 3.21        | 491.34 | 1105.10 | 179,100 | 0.993    |
| 0.050 | 3.27        | 482.01 | 1128.70 | 181,150 | 0.996    |
| 0.125 | 3.35        | 484.88 | 1143.40 | 183,960 | 0.998    |
| 0.250 | 3.41        | 485.93 | 1148.00 | 186,140 | 0.999    |
| 30   | 3.26        | 487.49 | 1153.50 | 189,750 | 0.967    |
| 0.008 | 3.28        | 482.01 | 1130.60 | 190,100 | 0.997    |
| 0.025 | 3.33        | 483.66 | 1139.10 | 190,780 | 0.998    |
| 0.050 | 3.36        | 484.82 | 1143.80 | 191,430 | 0.999    |
| 0.125 | 3.41        | 485.68 | 1147.40 | 192,570 | 1.000    |
| 0.250 | 3.46        | 486.01 | 1149.10 | 193,630 | 1.000    |
| 50   | 3.45        | 487.73 | 1154.00 | 198,970 | 1.000    |
| 0.008 | 3.45        | 485.90 | 1148.70 | 199,010 | 1.000    |
| 0.025 | 3.46        | 486.00 | 1149.10 | 199,120 | 1.000    |
| 0.050 | 3.48        | 485.87 | 1149.80 | 199,260 | 1.000    |
| 0.125 | 3.50        | 486.17 | 1149.90 | 199,590 | 1.000    |
| 0.25  | 3.53        | 486.24 | 1150.20 | 200,000 | 1.000    |

\(^{*}\)Pa: is the Probability of accepting incoming lots.

Table 2. Effect of the variation of Cr on the main cost parameters over various inspection times.

| Cr   | \( \mu_x \) | R   | Q   | TC   | \(^{*}\)Pa |
|------|-------------|-----|-----|------|---------|
| 10   | 3.25        | 487.35 | 1153.20 | 189,670 | 0.995    |
| 0.008 | 3.28        | 482.59 | 1131.00 | 190,040 | 0.996    |
| 0.025 | 3.32        | 481.27 | 1133.50 | 190,740 | 0.998    |
| 0.050 | 3.36        | 484.80 | 1143.70 | 191,410 | 0.999    |
| 0.125 | 3.41        | 485.88 | 1148.20 | 192,560 | 0.999    |
| 0.250 | 3.46        | 486.33 | 1149.90 | 193,630 | 1.000    |
| 40   | 3.26        | 487.49 | 1153.50 | 189,750 | 0.996    |
| 0.008 | 3.28        | 482.01 | 1130.60 | 190,100 | 0.997    |
| 0.025 | 3.33        | 483.66 | 1139.10 | 190,780 | 0.998    |
| 0.050 | 3.36        | 484.82 | 1143.80 | 191,430 | 0.999    |
| 0.125 | 3.41        | 485.68 | 1147.40 | 192,570 | 0.999    |
| 0.250 | 3.46        | 486.01 | 1149.10 | 193,630 | 1.000    |
| 60   | 3.26        | 487.49 | 1153.50 | 189,800 | 0.996    |
| 0.008 | 3.29        | 482.16 | 1131.30 | 190,140 | 0.997    |
| 0.025 | 3.33        | 483.59 | 1138.80 | 190,800 | 0.998    |
| 0.050 | 3.36        | 484.89 | 1144.00 | 191,440 | 0.999    |
| 0.125 | 3.42        | 485.97 | 1148.30 | 192,580 | 0.999    |
| 0.250 | 3.46        | 486.34 | 1149.90 | 193,640 | 1.000    |

\(^{*}\)Pa: is the Probability of accepting incoming lots.

Different values of \( C_d \) has been used with different inspection times and yet the values of \( Q \) and \( R \) were almost stable at 1140, and 480 with minor fluctuations. The effect of \( C_d \) over \( \mu_x \) and the total cost is clear. As \( C_d \) increases, and as \( x \) increases too, \( \mu_x \) will increase leading to an increase in the total cost.

As shown in Table 2, changing \( C_r \) over three different values (10, 40, 60) had no effect on the model performance.

The same results were evident at various inspection times. Recall that this experiment was conducted while other parameters were kept at their base levels. More specifically, \( C_d = 30 \) and \( C = 1 \). This shows that both \( C_d \) and \( C \) have more effect than \( C_r \). If \( C_d = 30 \) and \( C = 1 \), the probability of acceptance is always beyond 0.995, leaving negligible chance for \( C_r \) to be active. This explains the behavior seen in Table 2.

Table 3 shows the effect of the variation of the material cost per unit (\( C \)) on the model performance. It is clear that the behavior of varying \( C \) is almost the reverse compared with \( C_d \). When \( C = 0.5 \), as \( x \) increases, \( \mu_x \) will range between 3.52 and 3.59. At \( C = 1 \), and as \( x \) increases, \( \mu_x \) increases too, the behavior of varying \( C \) over process mean \( x \) ranges between 3.26 and 3.46. At \( C = 1.5 \), and as \( x \) increases, \( \mu_x \) will change from 3.13 to 3.39. In general, as \( x \) increases, \( \mu_x \) will increase too leading to higher material/total costs. Again the effect of \( C \) on \( Q \) and \( R \) is also negligible. The values of \( Q \) and \( R \) are also stable at 1140, 480 with some variations.

In this example, the effects of sampling plans, inspection time, \( C, C_d \), and \( C_r \) shows minor effect over \( Q \), and \( R \) values. However, their impact is clear over process mean setting \( \mu_x \) and the total cost of the model.

**Discussion and conclusion**

This paper presented a study on the effect of sampling inspection for a two-stage supply chain model that involves a producer and a distributor. The major contribution here is addressing a two-echelon supply chain involving a
producer and distributor with sampling inspection. The distributor’s warehouse was controlled by a $Q-R$ system. Relaxed assumptions were made for negligible inspection time. Two cases of inspection were addressed; (1) sampling inspection for dispatched lots, and (2) 100% inspection for rejected lots. For the latter case, the time any lot spends under 100% inspection is not negligible, and it was assumed to be proportional to the lot size $Q$. Furthermore, the lead time duration had to be extended to account for the 100% inspection time. The developed cost model is helpful to find the optimal values of the production process mean ($\mu_i$), and simultaneously, determines the proper setting of the order quantity ($Q$) and reorder level ($R$), under a given sampling plan with non-negligible 100% inspection time. In practice, sampling is still a viable alternative and in some scenarios, it is the only method that can be used to ensure the quality of incoming lots. Hence, sampling parameters can have large impact on the supply chain performance (especially the inspection time). For instance, whenever the inspection time is high, the supply chain will incur additional losses related to the extended lead-time period. To reduce the effects of these losses, the producer will need to improve the quality of his production (i.e., larger value of $\mu_i$ in our model). The same effect was observed for both $C_d$ and $C_r$. Yet, the material cost per unit, $C$, restricts the ability of the producer to improve the quality (i.e., larger value of $\mu_i$). The type of sampling plans was also found to have a significant effect on the performance of the supply chain. Even though they have the largest segregation power, adopting a tightened sampling plans is not always the best policy for the considered supply chain. In this case, the producer is forced to add extra material to avoid lots rejection. Therefore, higher total cost by the supply chain will be incurred. A similar analogy can be made for either moderate, or relaxed sampling policies. We believe that the best policy is not a specific one of the three presented modes. A dynamic sampling policy that uses a combination of them with the proper design of the switching rule will be more suitable. Proper determination of the switching rules between tightened, moderate and relaxed sampling plans will offer a strong, flexible sampling policy that considers producers’ commitment to quality and adopts for quality variations of incoming lots. At the same time, it balances the trade-offs among the relevant cost parameters. We believe that this could be an interesting topic for future research.

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ORCID iD

Mohammad M AlDurgam @ https://orcid.org/0000-0003-1742-7361

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