STARTING–POINT OF SUPERGRAVITY

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Abstract

In this report I wish to recall how the basic concepts and ingredients of supergravity were formulated by Dmitrij V. Volkov and the present author in 1973-74 under the first investigation of the super-Higgs effect.
1 Introduction

It is well-known that spontaneously broken gauge symmetries play a fundamental role for the construction of realistic models for the unified description of the different interactions (see, for example, [1]). Gauge symmetries imply the introduction of the massless gauge fields describing interactions, while spontaneously broken gauge symmetries mean that some gauge fields become massive, due to the Higgs effect [2], to describe some kind of forces. Gauge internal symmetries serve for the description of the strong and electroweak interactions and gauge space-time symmetries describe gravity.

The discovery of the extended super-Poincaré group [3, 4], nontrivially uniting the space-time and internal symmetries, opened a principal possibility for the unification of gravity with other interactions based on the internal symmetries. Since an irreducible representation of the super-Poincaré group, in contrast with the experience, contains bosons and fermions of the same masses, supersymmetry has to be broken. Therefore it was reasonable to consider a supersymmetric unification of gravity and other interactions on the basis of the spontaneously broken extended super-Poincaré gauge group.

The spontaneous breaking of the extended super-Poincaré global group leads to the massless Goldstone fermions with spin 1/2 (goldstinos) [3, 4]. Real fermionic gauge fields with spin 3/2 (gravitinos), which appear as superpartners of graviton with spin 2 under gauging of this group, are necessary ingredients of supergravity theory. The super–Higgs effect takes place in supergravity when the (extended) super-Poincaré gauge group is spontaneously broken.

If supergravity theory does not contain an interaction with other (matter) fields, then the super–Higgs effect results in that not only gravitinos become massive, by absorbing the goldstinos degrees of freedom, but also the space-time changes its own metric and topological properties: a nonzero cosmological constant appears. The super–Higgs effect has been introduced and investigated for the first time in 1973–74 in Refs. [5, 6], which in fact can be considered as a starting–point in the development of supergravity (see also [7]), since a notion of the super–Higgs effect is meaningless outside the framework of supergravity.

2 Spontaneously broken extended super-Poincaré gauge group

N–extended super–Poincaré group, whose spontaneous breaking leads to the appearance of goldstinos, has the following representation

$$G = K(\theta, x)H(l, u) = \begin{pmatrix} 1 & \theta & \frac{1}{2}\theta\bar{\theta} \\ 0 & 1 & \bar{\theta} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} L & 0 & 0 \\ 0 & U & 0 \\ 0 & 0 & (L^+)^{-1} \end{pmatrix},$$

(1)

where $\theta^a_k$ is a $2 \times n$ matrix with one spinor index $\alpha$ corresponding to the $SL(2, C)$ group and an index $k$ belongs to a representation of the internal symmetry group given in the form of
the $n \times n$ unitary matrix $U(u)$, \( \theta^{k\dot{\alpha}} = (\theta^\alpha_k)^+ \), $x^{\alpha\dot{\alpha}} = x^\mu \sigma_{\mu}^{\alpha\dot{\alpha}} \left( \sigma_{\mu}^{\alpha\dot{\alpha}} = (\bar{\sigma}_{\mu})^{\alpha\dot{\alpha}} = (1, -\bar{\sigma})^{\alpha\dot{\alpha}} \right)$ are relativistic Pauli matrices) is a Hermitian $2 \times 2$ matrix corresponding to the translation group and $L(l)$ is a $2 \times 2$ matrix of the $SL(2, C)$ group. Quantities $\theta^\alpha_k$ and $\theta^{k\dot{\alpha}}$, which will correspond to the goldstino fields, are Grassmann variables. Each of the multiplies $K$ and $H$ in (1) form subgroups and $H$ will be considered as a subgroup of vacuum stability.

The parameters $x$, $\theta$ and $\bar{\theta}$ form a homogeneous space (superspace) with respect to left shifts, i.e., they are transformed among themselves under a transformation $G_L$

\[ K(\theta', x')H(l', u') = G_L K(\theta, x)H(l, u). \]  

(2)

The expression $G^{-1}dG$ is invariant under left shifts (2) so that the Cartan forms, being coefficients of the group generators in the decomposition

\[ G^{-1}dG = i\omega^\mu \bar{\sigma}_\mu + \omega^a Q^k_a + \bar{Q}_{k\dot{\alpha}} \bar{\omega}^{k\dot{\alpha}} + \omega^{\mu\nu}(I_{\mu\nu} - I_{\mu\nu}^+) + \omega^a I_a, \]  

(3)

are invariant functions under transformations of $G_L$. In (3) the quantities $Q^k_a$ and $\bar{Q}_{k\dot{\alpha}}$ are generators of the spinor translations, $I_{\mu\nu}$ are generators of the $SL(2, C)$ group and $I_a$ ($a = 1, ..., N$) are generators of the internal symmetry group.

For a gauge group $G_L$, which parameters are functions of the point in superspace with coordinates $z = (x, \theta, \bar{\theta})$, the expression (3) ceases to be an invariant of $G_L$ and acquires an additional term under the transformations (2) $G' = G_L^G$

\[ G'^{-1}dG' = G^{-1}dG + G^{-1}G_L^{-1}dG_L G \]  

(4)

For the invariance restoration a gauge differential 1–form $A(d)$ on superspace has to be introduced

\[ A(d) = \begin{pmatrix} \omega(d) & \psi(d) & i\epsilon(d) \\ 0 & V(d) & \psi^+(d) \\ 0 & 0 & -\omega^+(d) \end{pmatrix} \]

with the following transformation law

\[ A'(d) = G_L A(d)G_L^{-1} + G_L dG_L^{-1}, \]  

(5)

where $\omega(d) = \omega^{\mu\nu}(d)I_{\mu\nu}$ is a Lorentz connection, $\psi(d) = \psi_k^\alpha(d)Q^k$ and $\psi^+(d) = \bar{Q}_{k\dot{\alpha}} \bar{\psi}^{k\dot{\alpha}}$ are Rarita–Schwinger gauge fields with spin 3/2 (gravitinos), $\epsilon(d) = \epsilon^\mu(d)\bar{\sigma}_\mu$ is a vierbein 1–form (graviton) and $V(d) = V^a(d)I_a$ are Yang–Mills gauge fields with spin 1.

Taking into account the transformation rules for the goldstinos (4) and gauge 1–form (5), it can readily be seen that the differential form

\[ A(d) = G^{-1}dG + G^{-1}A(d)G = H^{-1}(dH + \bar{A}(d)H) \]

is an invariant of the gauge group $G_L$. Differential 1-form

\[ \bar{A}(d) = K^{-1}(dK + A(d)K) = \bar{A}_K(d) + \bar{A}_H(d) \]
is not invariant but is transformed as follows
\[ \tilde{\AA}'_K = H_L \tilde{\AA}_K H_L^{-1}, \]  
\[ \tilde{\AA}'_H = H_L (dH_L^{-1} + \tilde{\AA}_H H_L^{-1}), \]  
where
\[ \tilde{\AA}_K = i \tilde{e}^\mu \tilde{\sigma}_\mu + \tilde{\psi}_k^a Q^k_a + \bar{Q}_k \tilde{\bar{\psi}}^k \]  
is a part of the gauge 1–form \( \tilde{\AA}(d) \) which belong to the class \( K \) and
\[ \tilde{\AA}_H = A_H = \omega(d) - \omega^+(d) + V(d) \]  
is a part of the form \( \tilde{\AA}(d) \) which belong to the subgroup \( H \).

Entering into \( \tilde{\AA}_K(d) \) 1–forms are
\[ \tilde{e}(d) = e(d) + Dx + \frac{i}{2} \left[ \theta (2 \tilde{\psi}(d) + D\bar{\theta}) - (2 \psi(d) + D\theta) \bar{\psi} \right], \]  
\[ \tilde{\psi}(d) = \psi(d) + D\theta, \quad \bar{\psi}(d) = \bar{\psi}(d) + D\bar{\theta}, \]
where covariant derivatives have the following forms
\[ Dx = dx + \omega(d)x + x\omega^+(d), \quad D\theta = d\theta + \omega(d)\theta - \theta V(d) \text{ and } D\bar{\theta} = d\bar{\theta} + \bar{\theta} \omega^+(d) + V(d)\bar{\theta}. \]

To construct kinetic terms for the gravitinos, graviton and Yang–Mills fields it will be necessary to use the following covariant 2–forms
\[ D(d,\delta)\tilde{\psi}(\delta) = d\tilde{\psi}(\delta) + \omega(d)\tilde{\psi}(\delta) - \tilde{\psi}(\delta)V(d), \]  
\[ R^{\mu\nu}(d,\delta) = d\omega^{\mu\nu}(\delta) - \delta \omega^{\mu\nu}(d) + 2 \left[ \omega^\mu_\lambda(d)\omega^\lambda^{\nu}(\delta) - \omega^\nu_\lambda(d)\omega^\lambda^{\mu}(\delta) \right], \]  
\[ F^a(d,\delta) = dV^a(\delta) - \delta V^a(d) + c_{bc}^a V^b(d)V^c(\delta), \]  
where \( c_{bc}^a \) are structure constants for the group of internal symmetry. It can easily be verified that 2–forms (10)–(12) are transformed under the gauge group \( G_L = K_L H_L \) homogeneously by means of the subgroup transformation \( H_L \) as well as the 1–form \( \tilde{\AA}_K(d) \) is transformed in accordance with (6).

### 3 Invariant action and the super–Higgs effect

The invariant action integral is derived from the expressions (8)–(12) as a sum with arbitrary constants of exterior products of forth order (or in the general case in the form of a homogeneous function of first order of such products) that are invariant under the Lorentz group and the internal symmetry group.

The simplest admissible invariant combinations, corresponding to the requirement that the degree of the field derivatives be minimal, are
\[ I_1 = i R^\hat{\beta}_\alpha(d_1, d_2) \wedge \tilde{e}^\beta_\gamma(d_3) \wedge \tilde{e}^\gamma_\delta(d_4) + h.c., \]  
\[ (13) \]
\[ I_2 = D(d_1) \wedge \tilde{\psi}^\alpha_k(d_2) \wedge \tilde{e}_{\alpha\delta}(d_3) \wedge \tilde{\psi}^{k\delta}(d_4) + h.c., \]
\[ I_3 = \tilde{\psi}^\alpha_k(d_1) \wedge \tilde{e}_{\alpha\delta}(d_2) \wedge \tilde{e}_{\beta\gamma}^\delta(d_3) \wedge \tilde{\psi}_{\gamma}^k(d_4) + h.c., \]
\[ I_4 = i\tilde{e}^\delta_{\alpha}(d_1) \wedge \tilde{e}^\gamma_{\beta}(d_2) \wedge \tilde{e}^\delta_{\delta}(d_3) \wedge \tilde{e}^\delta_{\gamma}(d_4), \]
\[ I_5 = \left[ F^a(d_1, d_2) \wedge \tilde{e}_{\alpha\delta}(d_3) \wedge \tilde{e}_{\gamma\delta}^\gamma(d_4) \right] \left[ F_a(d_5, d_6) \wedge \tilde{e}_{\beta\gamma}^a(d_7) \wedge \tilde{e}_{\delta}^\beta(d_8) \right] \times (I_4)^{-1}. \]

The quantities in (13)–(17) are given in the spinor representation a transition to which from the tensor representation is
\[ e^{\alpha\beta} = (\bar{\sigma}_\mu)^{\alpha\beta}\epsilon^\mu, \quad R^{\alpha}_{\beta} = (\bar{\sigma}^\mu\sigma^\nu)^{\alpha}_{\beta}R_{\mu\nu}, \quad R^{\dot{\alpha}}_{\beta} = (\bar{\sigma}^\mu\bar{\sigma}^\nu)^{\dot{\alpha}}_{\beta}R_{\mu\nu}, \]
where \( \sigma^{\mu}_{\dot{\alpha}\beta} = (1, -\bar{\sigma})^{\dot{\alpha}\beta}. \)

The invariant expression (13) contains as a term the Einstein–Cartan action for the gravitational field while the invariant (17) has a term for the Yang–Mills fields. The expression (16) contains the kinetic term for the goldstinos. The invariants (14) and (15) include respectively the kinetic and mass terms for Rarita–Schwinger real (Majorana) fields with spin 3/2 (gravitinos).

When all the field variables are parametrized as functions of \( x, \theta \) all the gauge differential forms \( A(d) \) can be reduced by a redefinition of the fields to the form
\[ A(d) = A_\mu(x)dx^\mu, \]
where \( A_\mu(x) \) are the redefined gauge fields.

In order to study the Higgs effect contained in the expressions (13)–(17) it is necessary to examine the structure of the Cartan forms in the definitions (8)–(12).

By a transition with the help of the left shift \( G_L = K^{-1}(x, \theta) \) to the gauge in which the evident dependence on the goldstinos \( \theta \) and space-time coordinates \( x \) disappears, the transformed gauge fields become
\[ \psi'(d) = \tilde{\psi}(d), \quad e'_{\mu}(d) = \tilde{e}_{\mu}(d). \]

In this gauge the following conditions \( \theta' = 0 \) and \( \alpha' = 0 \) are realized. This conditions are invariant under gauge transformations from the subgroup \( G_L = H(l, u) \) and therefore do not fix the gauge completely. Under the rest gauge transformations from the subgroup \( G_L = H(l, u) \) of vacuum stability the forms \( \tilde{\varphi}(d), \) which contain the forms \( \tilde{\psi}(d) \) and \( e_{\mu}(d), \) absorbing the goldstinos degrees of freedom and transforming homogeneously accordingly to (6), lost its gauging nature, while the forms \( \tilde{\varphi}(d), \) transforming in accordance with (7) inhomogeneously, remain truly gauge forms.

As a result of the redefinition (18), the invariants (13)–(17) correspond to the interaction of the gravitational field with cosmological term (16), massive gravitinos, and Yang–Mills fields. That corresponds to the super–Higgs effect, as a result of which the goldstinos disappear because of the redefinition (18) of the quantities associated with the gravitinos and metric and topological properties of space-time. Moreover, in contrast with
the Higgs effect for the group of internal symmetry, the kinetic term for the goldstinos transits not into the mass term of gravitinos but into the cosmological term, while the mass term of gravitinos appears from the term which in the absence of gauge fields is a total derivative.

Thus, a mechanism of the super–Higgs effect essentially differs than the Higgs effect for the Goldstone particles with spin zero which takes place in the case of a spontaneously broken group of internal symmetry.

4 Conclusion

Thus, we see that the main notions and ingredients of supergravity were formulated in our papers [5, 6].

1. It has been shown that the extended super-Poincaré gauge group leads to the supersymmetric generalization of gravity which has to include the real gauge fields with spin 3/2 as graviton superpartners. This fact was very unusual for those times, because till then the gauge fields possessed only integer spins. The graviton superpartners with spin 3/2 were called later on as gravitino fields.

2. We have written down our action as a sum of the five invariant terms $I_1, ..., I_5$ (13)--(17) with arbitrary constants before them. The invariants $I_1$ and $I_2$ contain respectively: the Einstein–Cartan action for gravity and a kinetic term for the gravitino fields which enter into the action for pure supergravity. The relation between the constants before the terms $I_1$ and $I_2$ has been established as a result of the powerful progress achieved by Ferrara, Freedman, van Nieuwenhuizen [8] and Deser, Zumino [9] under construction of the on–shell formulation for $N = 1$ supergravity.

The invariant terms $I_3$ and $I_4$, containing correspondingly a mass term of the gravitinos and a cosmological term, appear as a result of the super–Higgs effect.

3. The term $I_5$ for the Yang–Mills fields with spin 1 demonstrates a most important fact that the extended super-Poincaré gauge group gives a principle possibility for the nontrivial unification of gravity with the interactions based on the internal symmetries.

As the complete list of papers on the theme is very huge and essentially exceeds this report, I refer only to our works and to those very closely related to them. A very extensive list of references concerning the development of supergravity can be found in Ref. [10].

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