Cabibbo–Kobayashi–Maskawa Mixing in Superstring Derived Standard–like Models

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ABSTRACT

We examine the problem of three generation quark flavor mixing in realistic, superstring derived standard–like models, constructed in the free fermionic formulation. We study the sources of family mixing in these models and discuss the necessary conditions to obtain a realistic Cabibbo–Kobayashi–Maskawa (CKM) mixing matrix. In a specific model, we estimate the mixing angles and discuss the weak CP violating phase. We argue that the superstring standard–like models can produce a realistic CKM mixing matrix. We discuss the possible textures of quark mass matrices that may be obtained in these models.

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1. Introduction

The quest of theoretical physics for many years has been to understand the origin of fermion masses and mixing. The standard model which is consistent with all experiments to date, uses thirteen free parameters to parametrize the observed spectrum. Grand Unified Theories (GUTs) and Supersymmetric Grand Unified Theories (SUSY GUTs) reduce the number of free parameters and are able to explain inter–family relations between some of the masses. However, GUTs and SUSY GUTs can explain neither the mass hierarchy among the generations nor the origin and the amount of observed family mixing. Within the context of unified theories it is plausible that the number of generations and the structure of the fermion mass matrices have their origin in a more fundamental theory at the Planck scale. Indeed, the best known Planck scale theory, namely superstring theory [1], indicates that the number of chiral generations is related to the Euler characteristic of the compactified six dimensional space at the Planck scale. Therefore, it is important to examine whether realistic superstring models can lead to a qualitative understanding of the fermion mass matrices.

In Ref. [2,3,4] realistic superstring standard–like models were constructed in the four dimensional free fermionic formulation. The realistic models in the free fermionic formulation [2,3,4,5,6,7] have the attractive property of correlating the reduction to three generations with the factorization of the gauge group into observable and hidden sectors, and with the breaking of unwanted non–Abelian horizontal symmetries in the observable sector to $U(1)$ factors. A detailed discussion on the construction of the free fermionic standard–like models was given in Ref. [4]. The models of interest must possess the following properties:

1. The gauge group is $SU(3)_C \times SU(2)_L \times U(1)^n \times \text{hidden}$, with $N = 1$ space-time supersymmetry.

2. Three generations of chiral fermions and their superpartners, with the correct quantum numbers under $SU(3)_C \times SU(2)_L \times U(1)_Y$.

3. The spectrum should contain Higgs doublets that can produce realistic gauge
4. Anomaly cancellation, apart from a single “anomalous” $U(1)$ which is canceled by application of the Dine–Seiberg–Witten (DSW) mechanism [8].

The combined constraints (1–4) impose strong restrictions on the possible boundary condition basis vectors and GSO projection coefficients, and result in a class of realistic standard–like models with unique properties [4]. First, they produce three and only three generations of chiral fermions. Second, proton decay from dimension four and five operators is suppressed due to gauged $U(1)$ symmetries and a unique superstringy doublet–triplet splitting mechanism [4]. Finally, the standard–like models suggest an explanation for the fermion mass hierarchy [3,9,10]. At the tree level of the superpotential only the top quark gets mass. Mass terms for lighter quarks and leptons are obtained from higher order nonrenormalizable terms. The allowed nonrenormalizable terms in the fermion mass matrices are constrained by the horizontal symmetries that are derived in the standard–like models. The horizontal symmetries arise due to the compactification from ten to four dimensions. In the realistic free fermionic models the horizontal symmetries reflect the underlying $Z_2 \times Z_2$ orbifold compactification [10].

An important property of the superstring standard–like models is the absence of gauge and gravitational anomalies apart from a single “anomalous $U(1)$” symmetry. This anomalous $U(1)_A$ generates a Fayet–Iliopoulos term that breaks supersymmetry at the Planck scale [8]. Supersymmetry is restored and $U(1)_A$ is broken by giving VEVs to a set of standard model singlets in the massless string spectrum along the flat F and D directions [11]. The $SO(10)$ singlet fields in the nonrenormalizable terms obtain non–vanishing VEVs by the application of the DSW mechanism. Thus, the order $N$ nonrenormalizable terms, of the form $c f f h (\Phi/M)^{N-3}$, become effective trilinear terms, where $f, h, \Phi$ denote fermions, scalar doublets and scalar singlets, respectively. $M$ is a Planck scale mass to be defined later. The effective Yukawa couplings are given by $\lambda = c(\langle \Phi \rangle /M)^{N-3}$ where the calculable coefficients
are of order one [12]. In this manner quark mass terms, as well as quark mixing terms, can be obtained. Realistic quark masses and mixing can be obtained for a suitable choice of scalar VEVs.

In a previous letter [13], we studied the mixing between the two lightest generations. We showed that for a suitable choice of scalar singlet VEVs, a Cabibbo angle of the correct order of magnitude can be obtained in standard–like models. In this paper, we extend our analysis to the case of three generation mixing. We demonstrate that mixing among three generations and a weak CP violating phase of the correct order of magnitude can be obtained in the standard–like models. We illustrate our results in a specific model and discuss the general properties of our results that are expected to be valid for a large class of standard–like models.

The paper is organized as follows. In Section 2 we review the superstring standard–like models. We discuss the structure of the massless spectrum and emphasize the general properties of the standard–like models that are reflected in the generation mixing. In Section 3, we obtain the tree level superpotential and the nonrenormalizable terms. We discuss the form of the generation mixing nonrenormalizable terms in the standard–like models. We argue that the generation mixing terms reflect the general structure of the massless spectrum in these models. In section 4, we discuss the case of two generation Cabibbo mixing. In section 5, we extend our analysis to the case of three generation mixing. We discuss the possibility of obtaining mixing angles and weak CP violating phase of the correct order of magnitude. We present an F and D flat solution that yields a semi–realistic CKM matrix. In section 6, we discuss the relation between quark mass matrices in the standard–like models and ansatze for quark mass matrices. Our conclusions are summarized in section 7.

2. The superstring standard–like models

The superstring standard–like models are constructed in the four dimensional free fermionic formulation [14]. The models are generated by a basis of eight
boundary condition vectors for all world-sheet fermions. The first five vectors
in the basis consist of the NAHE set \{1, S, b_1, b_2, b_3\} [4,5]. The first five vectors
(including the vector 1) in the basis are

\[ S = (1, \cdots, 1, 0, \cdots, 0|0, \cdots, 0) \]  
\[ b_1 = (1, \cdots, 1, 0, \cdots, 0|1, \cdots, 1, 0, \cdots, 0) \]  
\[ b_2 = (1, \cdots, 1, 0, \cdots, 0|1, \cdots, 1, 0, \cdots, 0) \]  
\[ b_3 = (1, \cdots, 1, 0, \cdots, 0|1, \cdots, 1, 0, \cdots, 0) \]

with the choice of generalized GSO projections

\[ c \begin{pmatrix} b_i \\ b_j \end{pmatrix} = c \begin{pmatrix} b_i \\ S \end{pmatrix} = c \begin{pmatrix} 1 \\ 1 \end{pmatrix} = -1, \]  

and the others given by modular invariance.

The gauge group after the NAHE set is \( SO(10) \times E_8 \times SO(6)^3 \) with \( N = 1 \) space–time supersymmetry and 48 spinorial 16 of \( SO(10) \). The NAHE set is common to all the realistic models in the free fermionic formulation. The special properties of the NAHE set are emphasized in Ref. [4]. In short, the vectors \( b_1, b_2 \) and \( b_3 \) of the NAHE set perform several functions. First, they produce the chiral generations. Second, they split the observable and hidden sectors. Finally, they determine the chirality of the massless generations. Models based on the NAHE set correspond to models that are based on \( Z_2 \times Z_2 \) orbifold compactification with nontrivial background fields. This correspondence is best illustrated by adding the basis vector

\[ X = (0, \cdots, 0|1, \cdots, 1, 0, \cdots, 0) \]

to the NAHE set. The gauge group is extended to \( E_6 \times U(1)^2 \times E_8 \times SO(4)^3 \) with \( N = 1 \) supersymmetry and 24 chiral 27 of \( E_6 \). The same model is obtained in the
orbifold language by moding out an SO(12) lattice by a $Z_2 \times Z_2$ discrete symmetry with “standard embedding” [10]. The internal fermionic states $\{y, \omega | \bar{y}, \bar{\omega}\}$ correspond to the six left–moving and the six right–moving compactified dimensions in the orbifold language. In the construction of the standard–like models beyond the NAHE set, the assignment of boundary conditions to the set of internal fermions $\{y, \omega | \bar{y}, \bar{\omega}\}$ determines many of the properties of the low–energy spectrum such as the number of generations, the presence of Higgs doublets, Yukawa couplings, etc [4]. We would like to emphasize that many of the low energy properties are closely related to the $Z_2 \times Z_2$ orbifold compactification. In particular, each of the three chiral generations is obtained from a distinct twisted sector of the orbifold model. The horizontal symmetries of each generation correspond to the three orthogonal complex planes of the $Z_2 \times Z_2$ orbifold.

The standard–like models are constructed by adding three additional vectors to the NAHE set [2,3,4,6]. Three additional vectors are needed to reduce the number of generations to one generation from each sector $b_1$, $b_2$ and $b_3$. The three vectors that extend the NAHE set and the choice of generalized GSO projection coefficients for our model are given in Table 1 [2]. The observable and hidden gauge groups after application of the generalized GSO projections are $SU(3)_C \times U(1)_C \times SU(2)_L \times U(1)_L \times U(1)_6^*$ and $SU(5)_H \times SU(3)_H \times U(1)_2$, respectively. The weak hypercharge is given by $U(1)_Y = \frac{1}{3}U(1)_C + \frac{1}{2}U(1)_L$ and has the standard SO(10) embedding. The orthogonal combination is given by $U(1)_{Z'} = U(1)_C - U(1)_L$. The vectors $\alpha, \beta, \gamma$ break the $SO(6)_j$ horizontal symmetries to $U(1)_{r_j} \times U(1)_{r_{j+3}}$ ($j = 1, 2, 3$), which correspond to the right–moving world–sheet currents $\bar{y}_j^1 \bar{y}_j^{\ast 2}$ ($j = 1, 2, 3$) and $\bar{y}^3 \bar{y}^6, \bar{y}^1 \bar{\omega}^5, \bar{\omega}^2 \bar{\omega}^4$, respectively. For every right–moving $U(1)_r$ gauge symmetry there is a left–moving global $U(1)_l$ symmetry. The first three correspond to the charges of the supersymmetry generator $\chi^{12}, \chi^{34}$ and $\chi^{56}$. The last three, $U(1)_{\ell_{j+3}}$ ($j = 1, 2, 3$), correspond to the complexified left–moving fermions $y^3 y^6$, $y^1 \omega^5$ and $\omega^2 \omega^4$. Finally, the model contains six Ising model operators that are

\[
\begin{align*}
U(1)_C &= \frac{3}{2}U(1)_{B-L} \quad \text{and} \quad U(1)_L = 2U(1)_{T_{3R}}.
\end{align*}
\]
obtained by pairing a left–moving real fermion with a right–moving real fermion, \( \sigma^i_\pm = \{ \omega^1 \bar{\omega}^1, y^2 \bar{y}^2, \omega^3 \bar{\omega}^3, y^4 \bar{y}^4, y^5 \bar{y}^5, \omega^6 \bar{\omega}^6 \}_\pm \).

The basis vectors span a finite additive group \( \Xi = Z_2^7 \times Z_4 \). A general property of the free fermionic models, which are based on the NAHE set and that use a \( Z_4 \) twist to break the gauge symmetry from \( SO(2n) \) to \( SU(n) \times U(1) \), is the presence of the sectors \( b_j \) and \( b_j + 2\gamma + (I) \) \( j = (1, 2, 3) \) in the massless spectrum. The sectors \( b_j \) produce the chiral generations and the sectors \( b_j + 2\gamma + (I) \) produce representations of the hidden gauge groups that are \( SO(10) \) singlets with horizontal charges. The vector \( 2\gamma \), in effect, when added to the NAHE set, plays the role of the vector \( X \) in Eq. (3). It splits the \( \{ \bar{y}, \bar{\omega} \} \) right–moving fermionic states from \( \bar{\eta}_1, \bar{\eta}_2, \bar{\eta}_3 \), and breaks the horizontal symmetries from \( SO(6)_j \) to \( SO(4)_j \times U(1)_j \). However, rather than enhancing the observable gauge group from \( SO(10) \) to \( E_6 \), it breaks the hidden gauge group from \( E_8 \) to \( SO(16) \) and produces massless states in the vector representation of \( SO(16) \), from the sectors \( b_j + 2\gamma \). We will argue that this structure of the additive group, in these models, is the essential feature behind the generation mixing.

The full massless spectrum was presented in Ref. [2]. Here we list only the states that are relevant for the quark mass matrices. The following massless states are produced by the sectors \( b_{1,2,3}, S + b_1 + b_2 + \alpha + \beta, O \) and their superpartners in the observable sector:

(a) The \( b_{1,2,3} \) sectors produce three \( SO(10) \) chiral generations, \( G_\alpha = e^c_{L\alpha} + u^c_{L\alpha} + N^c_{L\alpha} + d^c_{L\alpha} + Q_\alpha + L_\alpha \) \( (\alpha = 1, \cdots, 3) \) where

\[
\begin{align*}
 e^c_L &\equiv [(1, \frac{3}{2}); (1, 1)]; & u^c_L &\equiv [(3, \frac{-1}{2}); (1, -1)]; & Q &\equiv [(3, \frac{1}{2}); (2, 0)] & (4a, b, c) \\
 N^c_L &\equiv [(1, \frac{3}{2}); (1, -1)]; & d^c_L &\equiv [(3, \frac{-1}{2}); (1, 1)]; & L &\equiv [(1, \frac{-3}{2}); (2, 0)] & (4d, e, f)
\end{align*}
\]

of \( SU(3)_C \times U(1)_C \times SU(2)_L \times U(1)_L \), with charges under the six horizontal \( U(1) \)s. We obtain from the sector \( b_1 \)

\[
(e^c_L + u^c_L)_{\frac{1}{2}, 0, 0, \frac{1}{2}, 0, 0} + (d^c_L + N^c_L)_{\frac{1}{2}, 0, 0, -\frac{1}{2}, 0, 0} + (L)_{\frac{1}{2}, 0, 0, \frac{1}{2}, 0, 0} + (Q)_{\frac{1}{2}, 0, 0, -\frac{1}{2}, 0, 0}; \quad (5a)
\]
the sector $b_2$,

$$(c_L^c + u_L^c)_{0, \frac{1}{2}, 0, 0, 0, 0} + (N_L^c + d_L^c)_{0, \frac{1}{2}, 0, 0, 0, 0} + (L)_{0, \frac{1}{2}, 0, 0, 0, 0} + (Q)_{0, \frac{1}{2}, 0, 0, 0, 0}. \quad (5b)$$

and the sector $b_3$,

$$(c_L^c + u_L^c)_{0, 0, 0, 0, 0, \frac{1}{2}} + (N_L^c + d_L^c)_{0, 0, 0, 0, 0, -\frac{1}{2}} + (L)_{0, 0, 0, 0, 0, \frac{1}{2}} + (Q)_{0, 0, 0, 0, 0, -\frac{1}{2}}. \quad (5c)$$

The vectors $b_1, b_2, b_3$ are the only vectors in the additive group $\Xi$ which give rise to spinorial 16 of $SO(10)$.

(b) The $S + b_1 + b_2 + \alpha + \beta$ sector gives

$$h_{45} \equiv [(1, 0); (2, 1)]_{-\frac{1}{2}, -\frac{1}{2}, 0, 0, 0, 0} \quad D_{45} \equiv [(3, -1); (1, 0)]_{-\frac{1}{2}, -\frac{1}{2}, 0, 0, 0, 0} \quad (6a, b)$$

$$\Phi_{45} \equiv [(1, 0); (1, 0)]_{-\frac{1}{2}, -\frac{1}{2}, -1, 0, 0, 0} \quad \Phi_1^\pm \equiv [(1, 0); (1, 0)]_{-\frac{1}{2}, -\frac{1}{2}, 0, \pm 1, 0, 0} \quad (6c, d)$$

$$\Phi_2^\pm \equiv [(1, 0); (1, 0)]_{-\frac{1}{2}, -\frac{1}{2}, 0, 0, \pm 1} \quad \Phi_3^\pm \equiv [(1, 0); (1, 0)]_{-\frac{1}{2}, -\frac{1}{2}, 0, 0, 0, \pm 1} \quad (6e, f)$$

(and their conjugates $h_{45}, \text{etc.}$). The states are obtained by acting on the vacuum with the fermionic oscillators $\bar{\psi}^{4, 5}, \bar{\psi}^{1, ..., 3}, \bar{\eta}^3, \bar{y}^3 \pm iy^6, \bar{y}^1 \pm i\bar{\omega}^5, \bar{\omega}^2 \pm i\bar{\omega}^4$, respectively (and their complex conjugates for $h_{45}$, etc.).

(c) The Neveu–Schwarz $O$ sector gives, in addition to the graviton, dilaton, antisymmetric tensor and spin 1 gauge bosons, scalar electroweak doublets and singlets:

$$h_1 \equiv [(1, 0); (2, -1)]_{1, 0, 0, 0, 0, 0} \quad \Phi_{23} \equiv [(1, 0); (1, 0)]_{0, 1, -1, 0, 0, 0} \quad (7a, b)$$

$$h_2 \equiv [(1, 0); (2, -1)]_{0, 1, 0, 0, 0, 0} \quad \Phi_{13} \equiv [(1, 0); (1, 0)]_{1, 0, -1, 0, 0, 0} \quad (7c, d)$$

$$h_3 \equiv [(1, 0); (2, -1)]_{0, 0, 1, 0, 0, 0} \quad \Phi_{12} \equiv [(1, 0); (1, 0)]_{1, -1, 0, 0, 0, 0} \quad (7e, f)$$

(and their conjugates $h_1, \text{etc.}$). Finally, the Neveu–Schwarz sector gives rise to three singlet states that are neutral under all the $U(1)$ symmetries. $\xi_{1, 2, 3} : \chi_{12}^{12} \bar{\omega}^3 \bar{\omega}^6 |0\rangle_0$, $\chi_{12}^{34} \bar{\omega}^5 \bar{\omega}^1 |0\rangle_0$, $\chi_{12}^{12} \bar{\omega}^5 \bar{\omega}^4 |0\rangle_0$.  


The sectors $b_i + 2\gamma + (I)$ ($i = 1, \ldots, 3$) give vector–like representations that are $SU(3)_C \times SU(2)_L \times U(1)_L \times U(1)_C$ singlets and transform as 5, 5 and 3, 3 under the hidden $SU(5)$ and $SU(3)$ gauge groups, respectively (see Table 2). As will be shown below, the states from the sectors $b_j + 2\gamma$ produce the mixing between the chiral generations. We would like to emphasize that the structure of the massless spectrum exhibited in Eqs. (5–7), and in Table 2, is common to a large number of free fermionic standard–like models. All the standard–like models contain three chiral generations from the sectors $b_j$, vector–like representations from the sectors $b_j + 2\gamma$, and Higgs doublets from the Neveu–Schwarz sector. The vector combination of $\alpha + \beta$ plus some combination of $\{b_1, b_2, b_3\}$, produces additional doublets and singlets, and exists in the models that were found to admit F and D flat solution [2,3,4], but not in the model Ref. [6]. We will show that mixing terms are obtained in all these models. We will argue that the source of the family mixing is a general characteristic of these models. It arises due to the basic set $\{1, S, b_1, b_2, b_3\}$ and the use of the $Z_4$ twist to break the symmetry from $SO(2n)$ to $SU(n) \times U(1)$.

In addition to the states above, the massless spectrum contains massless states from sectors with some combination of $\{b_1, b_2, b_3, \alpha, \beta\}$ and $\gamma + (I)$. These states are model dependent and carry either fractional electric charge or $U(1)_{Z'}$ charge. As argued in Ref. [9,15] the $U(1)_{Z'}$ symmetry has to be broken at an intermediate energy scale that is suppressed relative to the Planck scale. Therefore, the states from these sectors do not play a significant role in the quark mass matrices and we do not consider them in this paper.

The model contains six anomalous $U(1)$ gauge symmetries: $\text{Tr}U_1 = 24$, $\text{Tr}U_2 = 24$, $\text{Tr}U_3 = 24$, $\text{Tr}U_4 = -12$, $\text{Tr}U_5 = -12$, $\text{Tr}U_6 = -12$. Of the six anomalous $U(1)$s, five can be rotated by an orthogonal transformation and one combination remains anomalous. The six orthogonal combinations are given by [2],

\[
U'_1 = U_1 - U_2, \quad U'_2 = U_1 + U_2 - 2U_3, \quad (8a,b)
\]

\[
U'_3 = U_4 - U_5, \quad U'_4 = U_4 + U_5 - 2U_6, \quad (8c,d)
\]
\[ U'5 = U_1 + U_2 + U_3 + 2U_4 + 2U_5 + 2U_6, \]  
\[ U_A = 2U_1 + 2U_2 + 2U_3 - U_4 - U_5 - U_6, \]  
(8e)  
(8f)

with \( Tr(Q_A) = 180 \). The set of F and D constraints is given by the following equations:

\[ D_A = \sum_k Q_A^k |\chi_k|^2 = \frac{-g^2 e^{\phi_D}}{192\pi^2} Tr(Q_A) \]  
(9a)  
\[ D'^j = \sum_k Q'^j_k |\chi_k|^2 = 0 \quad j = 1 \ldots 5 \]  
(9b)  
\[ D^j = \sum_k Q^j_k |\chi_k|^2 = 0 \quad j = C, L, 7, 8 \]  
(9c)  
\[ W = \frac{\partial W}{\partial \eta_i} = 0 \]  
(9d)

where \( \chi_k \) are the fields that get VEVs and \( Q^j_k \) are their charges. \( W \) is the tree level superpotential.

### 3. The superpotential and mixing terms

We now turn to the superpotential of the model. Trilinear and nonrenormalizable contributions to the superpotential are obtained by calculating correlators between vertex operators [12]

\[ A_N \sim \langle V_1^f V_2^f V_3^b \cdots V_N^b \rangle \]  
(10)

where \( V_i^f \) (\( V_i^b \)) are the fermionic (scalar) components of the vertex operators. The non–vanishing terms are obtained by applying the rules of Ref. [12]. In order to obtain the correct ghost charge, some of the vertex operators are picture changed by taking

\[ V_{q+1}(z) = \lim_{w \to z} e^c(w) T_F(w) V_q(z) \]  
(11)
where \( T_F \) is the world–sheet super current given by

\[
T_F = \psi^\mu \partial_\mu X + i \sum_{I=1}^{6} \chi_I y^I \omega^I = T_F^0 + T_F^{-1} + T_F^{+1}
\]  

(12)

with

\[
T_F^{-1} = e^{-i\chi_{12}^2} \tau_{12} + e^{-i\chi_{34}^3} \tau_{34} + e^{-i\chi_{56}^5} \tau_{56} \quad T_F^{-1} = (T_F^{+1})^* 
\]

(13)

where \( \tau_{ij} = \frac{i}{\sqrt{2}}(y^i \omega^j + iy^j \omega^i) \) and \( e^{i\chi_{ij}} = \frac{1}{\sqrt{2}}(\chi^i + i\chi^j) \).

Several observations simplify the analysis of the potential non–vanishing terms. First, it is seen that only the \( T_F^{+1} \) piece of \( T_F \) contributes to \( A_N \) [12]. Second, in the standard–like models [2,3] the pairing of left–moving fermions is \( y^1 \omega^5, \omega^2 \omega^4 \) and \( y^3 y^6 \). One of the fermionic states in every term \( y^i \omega^i \) \((i = 1, \ldots, 6)\) is complexified and therefore can be written, for example for \( y^3 \) and \( y^6 \), as

\[
y^3 = \frac{1}{\sqrt{2}}(e^{iy^3 y^6} + e^{-iy^3 y^6}), \quad y^6 = \frac{1}{\sqrt{2}}(e^{iy^3 y^6} - e^{-iy^3 y^6}).
\]

(14)

Consequently, every picture changing operation changes the total \( U(1)_\ell = U(1)_{\ell_4} + U(1)_{\ell_5} + U(1)_{\ell_6} \) charge by \( \pm 1 \). An odd (even) order term requires an even (odd) number of picture changing operations to get the correct ghost number [12]. Thus, for \( A_N \) to be non–vanishing, the total \( U(1)_\ell \) charge, before picture changing, has to be an even (odd) number for even (odd) order terms. Similarly, in every pair \( y^i \omega^i \), one real fermion, either \( y^i \) or \( \omega^i \), remains real and is paired with the corresponding right–moving real fermion to form an Ising model operator. Every picture changing operation changes the number of left–moving real fermions by one. This property of the standard–like models [2,3] significantly reduces the number of potential non–vanishing terms.

At the cubic level the following terms are obtained in the observable sector [2],

\[
W_3 = \{ (u_L^c Q_1 \bar{h}_1 + N_{L_1}^c L_1 \bar{h}_1 + u_L^c Q_2 \bar{h}_2 + N_{L_2}^c L_2 \bar{h}_2 + u_L^c Q_3 \bar{h}_3 + N_{L_3}^c L_3 \bar{h}_3) \}
\]
\[ + h_1 \bar{h}_2 \Phi_{12} + h_1 \bar{h}_3 \Phi_{13} + h_2 \bar{h}_3 \Phi_{23} + \bar{h}_1 h_2 \Phi_{12} + \bar{h}_1 h_3 \Phi_{13} + \bar{h}_2 h_3 \Phi_{23} + \Phi_{23} \bar{\Phi}_{13} \Phi_{12} \\
+ \Phi_{23} \Phi_{13} \Phi_{12} + \Phi_{12}(\Phi_{1}^{+} \Phi_{1}^{-} + \Phi_{2}^{+} \Phi_{2}^{-} + \Phi_{3}^{+} \Phi_{3}^{-}) + \Phi_{12}(\Phi_{1}^{-} \Phi_{1}^{+} + \Phi_{2}^{-} \Phi_{2}^{+} + \Phi_{3}^{-} \Phi_{3}^{+}) \\
+ \frac{1}{2} \xi_{3}(\Phi_{45} \bar{\Phi}_{45} + h_{45} \bar{h}_{45} + D_{45} D_{45} + \Phi_{1}^{+} \Phi_{1}^{-} + \Phi_{1}^{-} \Phi_{1}^{+} + \Phi_{2}^{+} \Phi_{2}^{-} + \Phi_{2}^{-} \Phi_{2}^{+} + \Phi_{3}^{+} \Phi_{3}^{-} + \Phi_{3}^{-} \Phi_{3}^{+} + \Phi_{3}^{-} \Phi_{3}^{+}) + h_{3} \bar{h}_{45} \Phi_{45} + \bar{h}_{3} h_{45} \bar{\Phi}_{45} \} \\
\] (15)

with a common normalization constant \( \sqrt{2g} \).

It is seen that only Yukawa couplings of the \( + \frac{2}{3} \) charged quarks and neutral leptons appear in the tree level superpotential. This is a result of our choice of the basis vector \( \gamma \) given in Table 2 [16]. In the analysis of nonrenormalizable terms we impose the F–flatness restriction \( \langle \bar{\Phi}_{12}, \Phi_{12}, \xi_{3} \rangle \equiv 0 \) [9]. In addition, we take \( \langle \Phi_{23}, \bar{\Phi}_{45} \rangle = 0 \). At the cubic level there are two pairs of light Higgs doublets, which are combinations of \{\( h_{1}, \bar{h}_{2}, h_{45} \)\} and \{\( \bar{h}_{1}, \bar{h}_{2}, \bar{h}_{45} \)\} [9,10]. One additional pair receives heavy mass at the intermediate scale of \( U(1)_{Z'} \) breaking. The light Higgs representations, below this scale, may consist of \( h_{45} \) and a combination of \( \bar{h}_{1} \) or \( \bar{h}_{2} \) and \( \bar{h}_{45} \). As the mixing is dominantly in the down quark sector, we do not lose any generality by assuming the light Higgs representation to be \( \bar{h}_{1} \) and \( h_{45} \). Since the heavy Higgs doublets decouple at low energies, only the Yukawa couplings with \( \bar{h}_{i} \) remain in the superpotential given by Eq. (15). Therefore only the top quark gets a tree level mass. The other quarks get their masses from higher order nonrenormalizable terms which contain the light Higgs doublets.

At the quartic order there are no potential quark mass terms. At the quintic order the following mass terms are obtained,

\[
\begin{align*}
&d_{1} Q_{1} h_{45} \Phi_{1}^{+} \xi_{2} \\
&d_{2} Q_{2} h_{45} \Phi_{2}^{-} \xi_{1} \\
&u_{1} Q_{1} (\bar{h}_{45} \Phi_{45} \Phi_{13} + \bar{h}_{2} \Phi_{i}^{+} \Phi_{i}^{-}) \\
&u_{2} Q_{2} (\bar{h}_{45} \Phi_{45} \Phi_{23} + \bar{h}_{1} \Phi_{i}^{+} \Phi_{i}^{-}) \\
&(u_{1} Q_{1} h_{1} + u_{2} Q_{2} h_{2}) \frac{\partial W}{\partial \xi_{3}}.
\end{align*}
\] (16a,b) (16c) (16d) (16e)
At order $N = 6$ we obtain mixing terms for $-\frac{1}{3}$ charged quarks,

\begin{align}
\frac{d_3}{2} Q_2 h^{45} \Phi_{45} V_3 V_2, & \quad \frac{d_2}{2} Q_3 h^{45} \Phi_{45} V_2 V_3, \\
\frac{d_3}{2} Q_1 h^{45} \Phi_{45} V_3 V_1, & \quad \frac{d_1}{2} Q_3 h^{45} \Phi_{45} V_1 V_3,
\end{align}

(17a, b)

(17c, d)

At order $N = 7$ we obtain in the down quark sector,

\begin{align}
\frac{d_2}{2} Q_1 h^{45} \Phi_{45} (V_1 \bar{V}_2 + V_2 \bar{V}_1) \xi_i & \quad \frac{d_1}{2} Q_2 h^{45} \Phi_{45} (V_1 \bar{V}_2 + V_2 \bar{V}_1) \xi_i \\
\frac{d_1}{2} Q_3 h^{45} \Phi_{45} V_3 \bar{V}_1 \xi_2 & \quad \frac{d_3}{2} Q_1 h^{45} \Phi_{45} V_1 \bar{V}_3 \xi_2 \\
\frac{d_2}{2} Q_3 h^{45} \Phi_{45} V_3 \bar{V}_2 \xi_1 & \quad \frac{d_3}{2} Q_2 h^{45} \Phi_{45} V_2 \bar{V}_3 \xi_1,
\end{align}

(18a, b)

(18c, d)

(18e, f)

where $\xi_i = \{\xi_1, \xi_2\}$. In the up quark sector we obtain,

\begin{align}
\frac{u_1}{2} Q_2 h^{45} \Phi_{45} \{\bar{\Phi}_2^- (T_1 \bar{T}_2 + T_2 \bar{T}_1) + \Phi_1^+ (V_1 \bar{V}_2 + V_2 \bar{V}_1) \\
\frac{u_2}{2} Q_1 h^{45} \Phi_{45} \{\Phi_1^- (T_1 \bar{T}_2 + T_2 \bar{T}_1) + \Phi_2^+ (V_1 \bar{V}_2 + V_2 \bar{V}_1) \\
\frac{u_1}{2} Q_2 h^{45} \Phi_{45} \{\Phi_2^+ (T_1 \bar{T}_2 + T_2 \bar{T}_1) + \Phi_1^- (V_1 \bar{V}_2 + V_2 \bar{V}_1) \\
\frac{u_2}{2} Q_1 h^{45} \Phi_{45} \{\Phi_1^+ (T_1 \bar{T}_2 + T_2 \bar{T}_1) + \Phi_2^- (V_1 \bar{V}_2 + V_2 \bar{V}_1) \\
\frac{u_3}{2} Q_2 h^{45} \Phi_{45} \{\Phi_1^- T_1 \bar{T}_3 + \Phi_3^+ V_3 \bar{V}_1 \} & \quad \frac{u_1}{2} Q_3 h^{45} \Phi_{45} \{\Phi_3^- T_1 \bar{T}_3 + \Phi_1^+ V_3 \bar{V}_1 \} \\
\frac{u_3}{2} Q_1 h^{45} \Phi_{45} \{\Phi_3^+ T_1 \bar{T}_3 + \Phi_1^- V_3 \bar{V}_1 \} & \quad \frac{u_1}{2} Q_3 h^{45} \Phi_{45} \{\Phi_3^+ T_1 \bar{T}_3 + \Phi_1^- V_3 \bar{V}_1 \} \\
\frac{u_3}{2} Q_2 h^{45} \Phi_{45} \{\Phi_2^- T_2 \bar{T}_3 + \Phi_3^+ V_3 \bar{V}_2 \} & \quad \frac{u_2}{2} Q_3 h^{45} \Phi_{45} \{\Phi_3^- T_2 \bar{T}_3 + \Phi_2^+ V_3 \bar{V}_2 \} \\
\frac{u_3}{2} Q_2 h^{45} \Phi_{45} \{\Phi_2^- T_2 \bar{T}_3 + \Phi_3^+ V_3 \bar{V}_2 \} & \quad \frac{u_2}{2} Q_3 h^{45} \Phi_{45} \{\Phi_3^- T_2 \bar{T}_3 + \Phi_2^+ V_3 \bar{V}_2 \}
\end{align}

(19a)

(19b)

(19c)

(19d)

(19e)

(19f)

(19g)

(19h)

At order $N = 7$ we obtain generation mixing terms in the up and down quark sectors. The states that induce the mixing come from the sectors $b_j + 2\gamma$. In the up quark sector, mixing is obtained by 5, 5 and 3, 3 of the hidden $SU(5)$ and $SU(3)$ gauge groups, respectively. In the down quark sector, the mixing is only by the 3, 3 of the hidden $SU(3)$ gauge groups. At order $N = 8$ we obtain mixing in the down quark sector by the $SU(5)$ states from the sectors $b_j + 2\gamma$,

\begin{align}
\frac{d_3}{2} Q_1 h^{45} \Phi_{45} \{\Phi_1^- \bar{\Phi}_3^- + \Phi_3^+ \bar{\Phi}_1^- \} T_1 \bar{T}_3 & \quad \frac{d_1}{2} Q_3 h^{45} \Phi_{45} \{\Phi_1^+ \bar{\Phi}_3^- + \Phi_3^+ \bar{\Phi}_1^- \} T_3 \bar{T}_1
\end{align}

(20a)
The analysis of the nonrenormalizable terms up to order \( N = 8 \) shows that family mixing terms are obtained for all generations. Before the spontaneous symmetry breaking due to the scalar VEVs, there is no mixing because of the six generational gauge \( U(1)_r \) and the six global \( U(1)_\ell \) symmetries. In general the set of scalar VEVs break all of these symmetries and induce generation mixing by higher order nonrenormalizable terms.

We observe that the mixing arises due to the states from the sectors \( b_j + 2\gamma \). These sectors, and their relation to the sectors \( b_j \), is a general characteristic of the realistic free fermionic models that use a \( Z_4 \) twist. The mixing terms are of the form \( f_i f_j h \phi^n \), where \( f_i, f_j \) are fermion states from the sectors \( b_i, b_j \) with \( i \neq j \), \( h \) are the light Higgs representations and \( \phi^n \) is a string of standard model singlets. The fermion states from each sector \( b_j \) carry \( U(1)_{\ell_{j+3}} = \pm \frac{1}{2} \). The singlets from the NS sector and the sector \( b_1 + b_2 + \alpha + \beta \) all have \( U(1)_{\ell_{j+3}} = 0 \). Every picture changing operation changes the total \( U(1)_\ell = U(1)_{\ell_4} + U(1)_{\ell_5} + U(1)_{\ell_6} \) by \( \pm 1 \).

Thus, in order to construct nonrenormalizable terms which are invariant under \( U(1)_{\ell} \), we must tag to \( f_i f_j h \) additional fields with \( U(1)_{\ell_{j+3}} = \pm \frac{1}{2} \). We observe that the only available states are from the sectors \( b_j + 2\gamma \), which are in the fundamental representations of the hidden gauge group. Therefore, the family mixing due to these states is a general characteristic of these models.

We now comment on quark flavor mixing in other standard–like models. The terms in Eqs. (16–20) were obtained in the model of Ref. [2] (model 1) and similar terms are obtained in the model of Ref. [3] (model 2). The symmetries of these two models are the same and both contain the sector \( b_1 + b_2 + \alpha + \beta \) in the massless spectrum. In the observable sector, models 1 and 2 differ by the \( U(1)_{(\ell,r)_{j+3}} \) charges of the massless states from the sector \( b_j \). Consequently, in model 1 [2] bottom quark and tau lepton Yukawa couplings are obtained at the quintic order, while in model 2 [3] they are obtained at the quartic order [4]. The nonrenormalizable terms, Eqs. (16–20) may suggest that the condition of realistic

\[
d_3 Q_2 h_{45} \Phi_{45} \{ \Phi_2^+ \Phi_3^- + \Phi_3^+ \Phi_2^- \} T_2 T_3 \quad d_2 Q_3 h_{45} \Phi_{45} \{ \Phi_2^+ \Phi_3^- + \Phi_3^+ \Phi_2^- \} T_3 T_2 \quad (20b)
\]
quark flavor mixing requires the presence of the sector \( b_1 + b_2 + \alpha + \beta \) in the massless spectrum (or the presence of the scalar \( \Phi_{45} \)). However, by examining the quark flavor mixing terms in the model of Ref. [6] (model 3), we obtain at order \( N = 6 \) the non–vanishing terms (in the notation of Ref. [6])

\[
d_2 Q_1(h_2\Phi_{13} + h_3\bar{\Phi}_{12})V_2V_{11}, \quad d_1 Q_2(h_2\Phi_{13} + h_3\bar{\Phi}_{12})V_1V_{12}, \quad (21a, b) \\
d_3 Q_1(h_2\Phi_{13} + h_3\bar{\Phi}_{12})V_2V_{21}, \quad d_1 Q_3(h_2\Phi_{13} + h_3\bar{\Phi}_{12})V_1V_{22}, \quad (21c, d) \\
d_3 Q_2(h_2\Phi_{13} + h_3\bar{\Phi}_{12})\{V_{12}V_{21} + V_{13}V_{24}\}, \quad (21e) \\
d_2 Q_3(h_2\Phi_{13} + h_3\bar{\Phi}_{12})\{V_{11}V_{22} + V_{14}V_{23}\}. \quad (21f)
\]

The states \( V_i \) are the states from the sectors \( b_j + 2\gamma \) in the model of Ref. [6]. The terms in Eqs. (21) reflect the dependence of the mixing terms on the interplay between the sectors \( b_j \) and the sectors \( b_j + 2\gamma \), without the presence of a sector of the form \( \alpha + \beta \) in the massless spectrum.

4. Cabibbo mixing

In a previous letter [13], we showed that there are solutions to the F and D constraints which give non–negligible Cabibbo mixing between two generations. In principle generation mixing can arise from two different sources. The first one is due to condensates of the states which are in the vector representation of the hidden gauge group (see Table 2). \( T_i \) and \( V_i \) which transform as 5’s and 3’s under \( SU(5)_H \) and \( SU(3)_H \) form condensates when these gauge groups get strong, i.e. when

\[
\alpha_H(\Lambda_H) = \frac{\alpha_H(M)}{1 - (b/2\pi)\alpha_H(M)\ln(\Lambda_H/M)} \sim 1 \quad (22)
\]

where \( b = (n_f/2) - 3N \) and \( \alpha_H(M) \sim 0.06 \) [17]. The value of the scalar condensates \( \langle \bar{V}_iV_i \rangle \) or \( \langle \bar{T}_iT_i \rangle \) is \( \sim \Lambda_H^2 \), where \( \Lambda_H \) is given by Eq. (22). In our model, for the matter content of the hidden gauge groups, \( \Lambda_H \) turns out to be too small to give appreciable Cabibbo mixing. Even for the largest possible hidden gauge group \( SU(7)_H \), \( \Lambda_7 \) turns out to be an order of magnitude too small.
An alternative way to obtain mixing is by giving VEVs to vector representations of the hidden gauge groups by the F and D constraints given by Eq. (9). These VEVs will necessarily break the hidden gauge groups spontaneously. By choosing an appropriate solution one can easily get non-negligible mixing. In Ref. [13], we considered a solution to the F and D constraints with the following set of non-vanishing VEVs:

$$\{V_2, \bar{V}_3, \Phi_{45}, \Phi_{23}, \bar{\Phi}_{23}, \Phi_{13}, \Phi_{13}^+, \Phi_{2}^+, \Phi_{1}^+, \Phi_{13}^\pm, \xi_1, \xi_2\}, \quad (23)$$

with

$$|\langle V_2 \rangle|^2 = |\langle \bar{V}_3 \rangle|^2 = \frac{1}{5}|\langle \Phi_{45} \rangle|^2 = |\langle \bar{\Phi}_{45} \rangle|^2 = \frac{g^2}{16\pi^2} \frac{1}{2\alpha'} \quad (24a)$$

$$3|\langle \Phi_{23}^+ \rangle|^2 = 3|\langle \bar{\Phi}_{23}^+ \rangle|^2 = |\langle \Phi_{23}^- \rangle|^2 = |\langle \bar{\Phi}_{23}^- \rangle|^2 = |\langle \bar{\Phi}_{13} \rangle|^2 \quad (24b)$$

$$\frac{1}{4}|\langle \Phi_{13}^+ \rangle|^2 = \frac{1}{4}|\langle \bar{\Phi}_{13}^+ \rangle|^2 = |\langle \Phi_{13}^- \rangle|^2 \quad (24c)$$

$$|\langle \Phi_{23} \rangle|^2 = |\langle \bar{\Phi}_{23} \rangle|^2 = \frac{1}{3}|\langle \bar{\Phi}_{13} \rangle|^2 \quad (24d)$$

$$|\langle \Phi_{13} \rangle|^2 = |\langle \Phi_{13} \rangle|^2 - \frac{g^2}{8\pi^2} \frac{1}{2\alpha'} \quad (24e)$$

The VEVs of $\xi_1, \xi_2$ and $\bar{\Phi}_{13}$ are undetermined and remain free parameters to be fixed. For this solution, the up mass matrix $M_U$ is diagonal

$$M_u = \text{diag}(0, \langle \Phi_{13}^+ \bar{\Phi}_{13}^- \rangle/M^2, 1) v_1 \quad (25)$$

where $v_1 = \langle h_1 \rangle$ and the down mass matrix $M_D$ is given by

$$M_d \sim \begin{pmatrix} 0 & \frac{V_2 \bar{V}_3 \Phi_{45}}{M^2} & 0 \\ \frac{V_2 \bar{V}_3 \Phi_{45}}{M^2} & \frac{\bar{\Phi}_{13} \xi_1}{M^2} & 0 \\ 0 & 0 & \frac{\Phi_{13}^+ \xi_2}{M^2} \end{pmatrix} v_2 \quad (26)$$

where $v_2 = \langle h_{45} \rangle$ and we have used $\frac{1}{2}g\sqrt{2\alpha'} = \sqrt{8\pi}/M_{Pl}$, to define $M \equiv M_{Pl}/2\sqrt{8\pi} \approx 1.2 \times 10^{18}\text{GeV}$ [12]. We use the undetermined VEVs of $\bar{\Phi}_{13}$ and
\[ \xi_2 \] to fix \( m_b \) and \( m_s \) such that \( \langle \xi_1 \rangle \sim M \). We also take \( \tan \beta = v_1/v_2 \sim 1.5 \). Substituting the values of the VEVs above and diagonalizing \( M_D \) by a biunitary transformation we obtain the Cabibbo mixing matrix

\[
|V| \sim \begin{pmatrix} 0.98 & 0.2 & 0 \\ 0.2 & 0.98 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\] (27)

Since the running from the scale \( M \) down to the weak scale does not affect the Cabibbo angle by much [18], we conclude that realistic mixing of the correct order of magnitude can be obtained in this scenario.

A Cabibbo angle of the correct order of magnitude is obtained due to the non–vanishing VEVs of \( V_2 \) and \( \bar{V}_3 \) along the F and D flat directions, Eq. (24). From the nonrenormalizable terms, Eqs. (16–20), and the F and D flat solution, Eqs. (23,24), we conclude that mixing between the other generations can be obtained by giving a non–vanishing VEV to a state from the sector \( b_1 + 2\gamma \). Thus, in order to obtain realistic CKM mixing matrices, we must find F and D flat solutions with a non–vanishing VEV for at least one state from each sector \( b_j + 2\gamma \) (j=1,2,3).

5. KM mixing among three generations

In this section we consider the mixing between three generations obtained from the VEVs of hidden sector states. This can be accomplished by giving VEVs to one hidden sector state from each sector \( b_i + 2\gamma \). We impose several conditions on the VEVs that solve the F and D constraints. The VEVs should generate mass terms of the correct order of magnitude for the charm, strange, bottom and top quarks. This means that \( \Phi_1^+, \Phi_2^-, \Phi_i^+, \Phi_i^- \), \( \xi_1 \) and \( \xi_2 \) should get VEVs of the required magnitude. The light Higgs representations should include \( h_{45} \) and one \( \bar{h}_i \). This imposes vanishing VEVs on \( \Phi_{12}, \Phi_{12}, \Phi_{45}, \Phi_{23} \) and \( \xi_3 \). We allow a non–vanishing VEV only for one \( V_i \) or \( \bar{V}_i \) from every sector \( b_i + 2\gamma \). This guarantees that terms of the form \( h\bar{h}V_i\bar{V}_i\langle \phi \rangle^n \) will not render all the Higgs doublets superheavy and cause problems with electroweak–weak symmetry breaking. For illustrative purposes,
we require mixing terms in the down and up quark mass matrices. Therefore, in addition to the above fields also $\Phi_{45}, \Phi_{1,2,3}^+$, two $V_i$'s and one $\bar{V}_i$ should get VEVs. We require that $\lambda_b \sim \lambda_\tau$ at the unification scale, which imposes $\langle \Phi_1^+ \rangle \sim \langle \Phi_1^- \rangle$. We would like to stress that for different standard–like models, requiring realistic quark mass matrices imposes similar constraints. A solution that satisfies these requirements is given by the following set of non–vanishing VEVs:

$$\{\Phi_1^\pm, \Phi_1^\mp, \Phi_2^\pm, \Phi_3^\pm, \Phi_4^\pm, \phi_{13}, \phi_{23}, V_1, \bar{V}_2, \bar{V}_3, \xi_1, \xi_2\}$$

(28)

with

$$-\langle \Phi_3^- \rangle = \langle \Phi_1^- \rangle = \langle \bar{\Phi}_3^- \rangle = \frac{3}{\sqrt{10}} \frac{g}{4\pi} \frac{1}{\sqrt{2\alpha'}}$$

(29a)

$$|\langle \Phi_1^+ \rangle|^2 = |\langle \Phi_3^+ \rangle|^2 = |\langle \Phi_3^- \rangle|^2 = \frac{3}{\sqrt{5}} \left( \frac{\sqrt{2} - 1}{\sqrt{2}} \right) \frac{g}{4\pi} \frac{1}{\sqrt{2\alpha'}}$$

(29b)

$$-\langle \Phi_2^- \rangle = \langle \bar{\Phi}_2^- \rangle = \left( \frac{\sqrt{2} - 1}{\sqrt{2}} \right) \frac{g}{4\pi} \frac{1}{\sqrt{2\alpha'}}$$

(29c)

$$\frac{1}{6} |\langle \Phi_{45} \rangle|^2 = 2 |\langle \bar{\Phi}_{23} \rangle|^2 = 2 |\langle \bar{\Phi}_2^+ \rangle|^2 = \frac{g^2}{16\pi^2} \frac{1}{2\alpha'}$$

(29d)

$$\langle \phi_{13} \rangle = \left( \frac{\sqrt{2} - 1}{\sqrt{2}} \right) \left( \frac{3}{\sqrt{5}} \left( \frac{\sqrt{2} - 1}{\sqrt{2}} \right) \right) \frac{g}{4\pi} \frac{1}{\sqrt{2\alpha'}}$$

(29e)

$$|\langle \phi_{13} \rangle|^2 = |\langle \Phi_{13} \rangle|^2 + \frac{3}{2} \frac{g^2}{16\pi^2} \frac{1}{2\alpha'}$$

(29f)

$$|\langle V_1 \rangle|^2 = \frac{1}{3} |\langle \bar{V}_2 \rangle|^2 = \frac{1}{2} |\langle \bar{V}_3 \rangle|^2 = \frac{g^2}{16\pi^2} \frac{1}{2\alpha'}$$

(29g)

The VEVs of $\xi_1$ and $\xi_2$ are not constrained. With this set of VEVs, the up and down quark mass matrices, $M_u$ and $M_d$ are given by

$$M_u \sim \left( \begin{array}{ccc} 0 & \frac{V_1 \bar{V}_2 \Phi_{13} \Phi_4^+}{M^4} & \frac{V_1 \bar{V}_2 \Phi_{13} \Phi_4^+}{M^4} \\ \frac{V_3 \bar{V}_2 \Phi_{13} \Phi_4^+}{M^4} & \frac{\bar{\Phi}_2^- \Phi_1^+}{M^4} & V_1 \bar{V}_2 \Phi_{13} \Phi_4^+ \\ 0 & \frac{V_1 \bar{V}_2 \Phi_{13} \Phi_4^+}{M^4} & 1 \end{array} \right) v_1$$

(30)
and

\[
M_d \sim \begin{pmatrix}
0 & \frac{V_3 V_2 \Phi_{45}}{M^4} & 0 \\
\frac{V_1 V_2 \Phi_{12} \xi_i}{M^4} & \frac{\Phi_1 \xi_i}{M^2} & \frac{V_4 V_2 \Phi_{12} \xi_i}{M^4} \\
0 & \frac{V_1 V_2 \Phi_{12} \xi_i}{M^4} & \frac{\Phi_1 \xi_i}{M^2}
\end{pmatrix} v_2
\]  

(31)

with \(v_1, v_2\) and \(M\) as before. The up and down quark mass matrices are diagonalized by bi–unitary transformations

\[
U_L M_u U_R^\dagger = D_u \equiv \text{diag}(m_u, m_c, m_t), \\
D_L M_d D_R^\dagger = D_d \equiv \text{diag}(m_d, m_s, m_b),
\]

(32a, 32b)

with the CKM mixing matrix given by

\[
V = U_L D_L^\dagger.
\]

(33)

The VEVs of \(\xi_1\) and \(\xi_2\) are fixed to be \(\langle \xi_1 \rangle \sim M/12\) and \(\langle \xi_2 \rangle \sim M/4\) by the masses \(m_s\) and \(m_b\) respectively. Substituting the VEVs and diagonalizing \(M_u\) and \(M_d\) by a bi–unitary transformation, we obtain the mixing matrix

\[
|V| \sim \begin{pmatrix}
0.98 & 0.205 & 0.002 \\
0.205 & 0.98 & 0.012 \\
0.0004 & 0.012 & 0.99
\end{pmatrix}
\]

(34)

To study the effect of the renormalization from the unification scale to the electroweak scale we run the coupled renormalization group equations of the MSSM in matrix form [19]. The renormalization does not affect the mixing terms that correspond to the Cabibbo 2 \(\times\) 2 submatrix by much. The remaining elements, that mix the heavy generation with the lighter two generations are modified by up to thirty percent. Therefore, \(V\) is a CKM matrix with elements of the correct order of magnitude. The string model does not determine the flat direction (scalar VEVs) and therefore does not predict the matrix elements. Since our aim is only to demonstrate the possibility of obtaining a realistic CKM matrix and not to predict it, we do not pursue this point further.
In Eq. (34) only the magnitude of the CKM matrix elements appear since we took all the VEVs to be real. From Eq. (29) we see that the phases of the VEVs except for those of $\Phi_1^-, \bar{\Phi}_1^-, \Phi_2^-, \bar{\Phi}_2^-, \Phi_3^-, \bar{\Phi}_3^-, \Phi_{13}$ are not fixed by the F and D constraints. By giving phases to some of these VEVs we will be able to obtain a complex CKM matrix.

Consider the set of VEVs given in Eq. (29) where now we give phases to the VEVs of $V_1, \bar{V}_2$ and $V_3$ only:

$$\langle V_1 \rangle = e^{i\alpha} \frac{g}{4\pi} \frac{1}{\sqrt{2\alpha'}}$$  \hspace{1cm} (35a)

$$\langle \bar{V}_2 \rangle = e^{-i\beta} \sqrt{3} \frac{g}{4\pi} \sqrt{\frac{1}{2\alpha'}}$$  \hspace{1cm} (35b)

$$\langle V_3 \rangle = e^{i\gamma} \sqrt{2} \frac{g}{4\pi} \sqrt{\frac{1}{2\alpha'}}$$  \hspace{1cm} (35c)

where $\alpha$, $\beta$ and $\gamma$ are completely arbitrary. For this choice of VEVs the mass matrices $M_u$ and $M_d$ become complex. Only the two combinations $\alpha - \beta$ and $\gamma - \beta$ of these three angles appear in $M_u$ and $M_d$. As a result we can always set one of the angles to zero without loss of generality. For the choice $tan(\gamma - \beta) = 1$ and $tan(\alpha - \beta) = 1$ and using the freedom to change the phases of 5 quarks we obtain

$$V \sim \begin{pmatrix} 0.973 & 0.230 & 0.002e^{-i\frac{19}{3}\pi} \\ 0.230 & 0.973 & 0.01 \\ 0.0004e^{i\frac{1}{3}\pi} & 0.01 & 0.99 \end{pmatrix}$$  \hspace{1cm} (36)

This form of the CKM mixing matrix resembles the mixing matrix in the Chau–Keung parametrization [20]. The 11, 12, 21, 23 and 33 matrix elements are real (This can always be done by the phase transformations on the 5 quarks.) and the 22 and 32 elements have small phases that we have neglected. The parametrization–invariant CP violating quantity, $|J| = |Im(V_{ij}V_{ik}V_{*lj}V_{*lk})|$ for any $i \neq l, j \neq k$, is of the order of $10^{-6}$. Experimentally, in the Standard Model, $\delta$ is $20^\circ < \delta < 178^\circ$ [21]. There are different possible choices of the phases $\alpha$, $\beta$ and $\gamma$ which give different CP violating phases $\delta$. In general, as $\alpha$, $\beta$ and $\gamma$ are varied continuously,
one obtains a mixing matrix with varying phase $\delta$ and non-negligible phases also appearing in the 22 and 32 elements. We do not discuss further the dependence of $\delta$ on the flat directions as our aim is only to demonstrate the possibility of having a realistic phase in the string model.

6. Ansätze for mass matrices

The standard model uses ten free parameters to parametrize the quark masses and mixing. Several ansätze for the quark mass matrices have been proposed to reduce the number of free parameters. These ansätze assume the existence of discrete symmetries that force some of the entries in the quark mass matrices to vanish. The origin of these ansätze and of the symmetries that they assume to have is not explained. In this section, we discuss the relation between the quark mass matrices in the superstring standard-like models and a few of these ansätze.

Consider for example the Fritzsch ansatz [22] with

\[ M_u = \begin{pmatrix} 0 & a_u & 0 \\ a_u & 0 & b_u \\ 0 & b_u & c_u \end{pmatrix} \quad M_d = \begin{pmatrix} 0 & a_d & 0 \\ a_d^* & 0 & b_d \\ 0 & b_d^* & c_d \end{pmatrix} \] (37a,b)

with all elements of $M_u$ and $c_d$ real. From the mixing terms given by Eqs. (16–20) we see that a mass matrix of the form of Eq. (37b) cannot be obtained in our model. The reason is that in Eq. (37b) the elements 23 and 32 in $M_d$ are complex conjugates of each other, while in model 1, they (and therefore their phases) are equal. However, we can consider modifying Eq. (37b) by taking

\[ M_{\{u,d\}} = \begin{pmatrix} 0 & a & 0 \\ a & 0 & b \\ 0 & b & c \end{pmatrix} \] (38)

with $a$, $b$, $c$ complex. We see that this ansatz can be obtained from our model for a flat direction with $\langle \Phi_i^+ \Phi_i^- \rangle = 0$ and $\langle \Phi_2^- \rangle = 0$ and non-vanishing VEVs for $V_1$, 

20
\( \bar{V}_2 \) and \( V_3 \). Similarly, consider an ansatze of the form

\[
M_{\{u,d\}} = \begin{pmatrix}
0 & a \\
0 & b \\
a & b & c
\end{pmatrix}
\] (39a)

This form of mass matrices can be obtained in our model by a flat direction with the same conditions but VEVs for \( \bar{V}_1, V_2 \) and \( V_3 \). This form of mass matrices is not realistic as it gives vanishing up and down quark masses and zero Cabibbo angle. However, Eq. (39) illustrates the dependence of the vanishing mass matrix entries on the state from the sectors \( b_j + 2\gamma \) that obtain non–vanishing VEVs. Similarly, an ansatze of the form

\[
M_{\{u,d\}} = \begin{pmatrix}
0 & a & b \\
a & 0 & c \\
b & 0 & c
\end{pmatrix}
\] (39b)

can be obtained with non–vanishing VEVs for \( V_1, V_2 \) and \( \bar{V}_3 \).

The superstring standard–like models provide the possibility of obtaining up and down mass matrices of different textures. Non–vanishing VEVs for \( V_1, \bar{V}_2, V_3, \xi_1 \) and \( \xi_2 \) produce down quark mass that give a realistic CKM matrix. If we impose a flat F and D solution with vanishing VEV for \( \Phi^+_2 \) or \( \Phi^+_3 \), the up quark mass matrix will take the form

\[
M_u = \begin{pmatrix}
0 & a & 0 \\
0 & b & 0 \\
0 & c & 1
\end{pmatrix}
\] or \( M_u = \begin{pmatrix}
0 & 0 & 0 \\
0 & b & c \\
0 & c & 1
\end{pmatrix}
\] (40)

This form of up mass matrix results in vanishing up quark mass being the well known solution to the strong CP problem [23]. The down quark mass matrix is of the form of Eq. (38), and therefore \( m_u = 0 \) while \( m_d \neq 0 \). To ascertain if this solution to the strong CP problem is a possible solution within the context of the superstring standard–like models, one would have to check the possible sources for the 12, 21 and 11 entries in the up quark mass matrix. Those being
nonrenormalizable terms up to a sufficient order and possible contributions to the diagonal up quark mass term from VEVs that break $U(1)_{Z'}$ [9].

The vanishing mixing entries in Eqs. (30–31) arise because only one $V_i$ from each sector $b_i + 2\gamma$ obtained a non–vanishing VEV. If more than one state from any sector $b_i + 2\gamma$ obtain a non–vanishing VEV, for example $\{V_1, \bar{V}_2, V_3, \bar{V}_3\}$, then in general all the off–diagonal terms will be nonzero at some order of nonrenormalizable terms. However, one has to be careful not to generate Higgs mass terms that will render all the Higgs doublets superheavy. Thus, it is seen that the vanishing off–diagonal entries in the quark mass matrices arise due to effective discrete symmetries which are a result of the particular solution that we choose in our model. The mixing terms Eqs. (17–20) show that for different F and D flat directions different textures of mass matrices can be obtained. The resulting mass matrices are not necessarily symmetric. A sufficient condition for the down quark mass matrix to have off–diagonal terms is that two $V_i$ one $\bar{V}_i$, $\Phi_{45}$ and $\xi_i$ obtain non–vanishing VEVs. The up quark mass matrix may be diagonal for some choices of flat directions. The richness of the flat F and D flat solution space gives us the reason to hope that the superstring standard–like models can successfully reproduce the observed quark mixing and mass spectrum.

7. Conclusions

In this paper, we examined the three generation mixing among quark families in realistic, superstring derived standard–like models. Family mixing is induced by order $N > 3$ terms in the superpotential which respect all local and global symmetries of the string model. From the explicit mixing terms we observe that generation mixing arises from certain sectors of the spectrum, namely the sectors $b_j + 2\gamma$. The states in these sectors (which are in the vector representations of $SU(5)_H \times SU(3)_H$) obtain VEVs by the F and D flatness conditions which are essential to preserve SUSY close to the Planck scale. By examining the conservation of left–handed global $U(1)_\ell$ symmetries, we showed that the existence of the sectors $b_j + 2\gamma$ is a necessary condition for obtaining nonrenormalizable quark mixing terms.
Since the sectors $b_j + 2\gamma$ always exist in the free fermionic models with fermion generations from the twisted sectors $b_j$, and with a $Z_4$ twist (vector $\gamma$), generation mixing is a generic feature of these models. The question then is whether there exists a suitable F and D flat direction which gives a realistic CKM matrix.

There exist F and D flat directions that produce three generation mixing. We found one such flat direction given by Eq. (29) and calculated the mixing matrix from it. In addition, by giving phases to some of the scalar VEVs, one can obtain the weak CP violating phase, $\delta$, in the CKM matrix. In our model, we were able to obtain $\delta$ by giving phases only to the VEVs of $V_1$, $\bar{V}_2$ and $V_3$. We emphasize that the string model does not fix the flat direction and therefore does not predict the CKM matrix. Our aim is only to show that a realistic CKM matrix can be obtained in this class of models.

Can we improve our order of magnitude results? There are three ways that our results can be made more predictive. First, we can take into account the coefficients $c$ which enter the effective Yukawa couplings. In this paper we have taken them to be of order $O(1)$. They can, in principle, be calculated from the correlators of vertex operators for every nonrenormalizable term. Second, we can consider other phenomenological constraints on the model such as acceptable neutrino masses, baryon decay, very small FCNCs etc., in addition to the ones we took into account. These will further constrain the possible F and D flat directions and make the model more predictive. For example, the condition of realistic neutrino masses requires the existence of light $SO(10)$ singlet fermions which mix with the right–handed neutrinos, $N_i$ [15]. This condition puts additional constraints on the possible F and D flat directions which in turn constrain the possible CKM matrices in the model. The weak CP violating phase $\delta$ can always be obtained from the phases of $V_i$ and $\Phi_{45}$ which enter only the D flatness equations that are quadratic in VEVs. Therefore, their phases and those of $\xi_{1,2}$ (which do not appear in F and D flatness equations) are completely free. In this paper, we considered phases only for $V_i$ since this is the simplest possibility. A better fix on $\delta$ can be obtained by giving phases to all the VEVs above. The freedom in the flat directions gives us reason
to believe that realistic quark mixing and mass spectrum can be obtained from the superstring standard–like models.

The horizontal symmetries that arise in superstring models due to the compactification from ten to four dimensions constrain the allowed terms in the superpotential and consequently the terms in the fermion mass matrices. In this paper we examined the superstring derived standard–like models. These models are constructed in the free fermionic formulation and correspond to models that are based on $Z_2 \times Z_2$ orbifold compactification. We showed that the horizontal symmetries and the choice of flat directions in the application of the Dine–Seiberg–Witten mechanism constrain some of the entries in the quark mass matrices to vanish. Consequently different textures for the fermion mass matrices may be obtained from the standard–like models, that may naturally resolve some of the problems that exist in traditional GUTs. For example, the relation $\lambda_t = \lambda_b$ in $SO(10)$ models that forces a large value for $\tan \beta = v_1/v_2$, is broken in the superstring standard–like models and allows small values for $\tan \beta$. Similarly, the choice of flat directions may naturally lead to $m_u = 0$ with $m_d \neq 0$. Thus, the superstring standard–like models may provide simple and well motivated solutions to some of the fundamental problems in particle physics. We will expand upon the phenomenology derived from these models in future publications.

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