Neutrino Magnetic Moment, Large Extra Dimensions and High Energy Cosmic Neutrino Spectra

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Abstract

We point out that the presence of bulk neutrinos in models with large extra spatial dimensions can lead to observable flavour specific deformations in the spectra of extreme high energy cosmic neutrinos. These deformations are due to the spin precession of the high energy neutrinos in the background magnetic fields via electromagnetic interactions. Measurements with existing and proposed neutrino telescopes which are meant to detect high energy neutrinos can therefore provide a novel way to probe the size of extra hidden dimensions. We qualitatively illustrate the flavour suppression due to the Earth, Sun and intergalactic magnetic fields. An observable consequence of this precession could be an angular asymmetry for the extreme high energy neutrinos from the atmosphere and flavour specific deformations of the intergalactic neutrinos.
I. INTRODUCTION

The possibility that there may be extra hidden dimensions in Nature, first suggested by Kaluza and Klein, is an intriguing one. Recent developments in string theories have suggested that there may be fundamental reasons for their existence connected with the consistency and symmetry requirements of the string world sheet. This has led to an explosive revival of interest in the physics implications of extra dimensions and many investigations have been carried out in this area in the past twenty years.

A particularly interesting framework for these investigations is the so called brane-bulk setup where the standard model (SM) particles live in a 3+1 dimensional brane subspace of the full multi-dimensional universe whereas the gravity and other SM singlet fields reside in the entire 3+1+n dimensional space. This picture arises in the nonperturbative D-brane solutions of string theories and is thus believed to have solid theoretical foundation.

This picture allows a very novel approach to gravity according to which the Planck scale is not a fundamental scale in Nature but rather a function of the fundamental variable, the string scale $M_*$. The “radii of compactification” $R_i$ of the extra hidden dimensions and the familiar Planck scale $M_P \ell_p$ are related by

$$M_P^2 = M_*^{2+n}(2\pi)^n R_1 R_2 R_3 \cdots R_n .$$

The scenario $n = 1$, with all but one $R_i$ of order $M_*$ is ruled out for $M_* \sim \text{TeV}$ by the observed accuracy of Newton’s laws over the Earth-Sun distances. If only two extra dimensions are significantly larger than $M_*^{-1}$, i.e. $R_1 \simeq R_2 = R$ and $R_3 \simeq R_4 \simeq \cdots R_n = M_*^{-1}$, we get $R \simeq \text{millimeter}$ for $M_* \simeq \text{TeV}$. This scenario is allowed by the current data, since the inverse square law for gravity has not been tested for distances less than mm. Since the small radii $R_3, R_4$ have no effect on the physics, this is equivalent to a scenario with $n = 2$.

Alternatively, one may envision one large extra dimension with $R_1 \sim \text{mm}$, with the rest much smaller e.g. $r \lesssim 10^{-2}$ fm for $M_* \simeq 10$ TeV. The influence of such small dimensions cannot be felt at any of the current experiments, and hence this scenario of only one large extra dimension cannot be ruled out. This will be equivalent to a $n = 1$ scenario.

A basic difference between the two scenarios described above ($n=2$ and $n=1$) is that the Kaluza-Klein (KK) excitations of any particle in the bulk in the first scenario are closely spaced in two extra dimensions whereas in the second class of models it happens only in one dimension, making the phase space structure very different. The two scenarios then have different phenomenological consequences, and it is important to distinguish between them through their experimentally testable consequences. Perhaps more important than this is to test the possibility of large extra dimensions itself. The goal of this paper is to provide one such test in the domain of neutrinos.

The small mass of the neutrinos, for which there is now very strong evidence from solar and atmospheric neutrino data, is understood in these models in a way which is very different from the conventional 3+1 dimensional unified theories. The simplest possibility is to postulate the existence of SM singlet neutrinos in the bulk and couple them to the SM neutrinos in a gauge invariant way using the usual Higgs mechanism. This leads to naturally small masses for the SM neutrinos due to small overlap of the bulk neutrino wave function on the brane. The coupling of the SM neutrinos with the bulk neutrino provides a naturally massless sterile neutrino and a tower of KK neutrinos that also mix with the SM neutrinos.
This leads to a variety of interesting tests and constraints on the brane bulk models; for instance for tests using solar neutrinos, see [3] and tests using high energy muons, see [4]. There are many possible scenarios for neutrino masses in brane-bulk models. A common feature of all those models is the presence of a SM singlet neutrino mentioned above. Regardless of the details of the model, as long as the known neutrinos mix with the KK modes of the bulk neutrino, there is a magnetic moment connecting the brane neutrinos with the bulk $\nu_B$ modes [5]. We will show below that this can give rise to observable deformation of the high energy neutrino spectra. The flavour composition of the high energy neutrinos may also change; the final flavour content will depend on the neutrino mass hierarchy. This can provide valuable information about the extra dimensions, and possibly also about the hierarchy structure of the neutrino masses. These spectral deformations occur over a wide range of energies starting from one TeV to 1000 TeV and higher. The neutrino telescope facilities such as ANTARES and ICE CUBE, which can look for these extreme high energy muon and tau neutrinos can therefore provide valuable information on models with large extra dimensions.

Our paper is organised as follows. In the next section, we describe the basic framework of the model. We illustrate the enhancement of the neutrino magnetic moment with energies and give the expected strengths depending on the neutrino mass hierarchy. We then apply this result in section III for ultra-high energy neutrino spectrum and examine the changes in the individual neutrino fluxes. Although, these changes are dependent on the mass hierarchy they are independent of neutrino oscillations. We discuss the physical situations under which we can have flavour suppression and in particular, we focus on the spin precession effects due to the Earth, Sun and intergalactic magnetic fields. In section IV, we describe a useful angular asymmetry which can probe the effects of neutrino magnetic moments in extra-dimension scenarios. Finally, we conclude in section V by summarising the main results.

II. MINIMAL SCENARIO

We will work with the simplest scenario where there are only three extra neutrinos in the bulk and the usual three generations of fermions of the SM residing in the brane. The neutrino mass in these models arises from the brane bulk Yukawa interactions of the type

$$\mathcal{L} = \frac{h_{\alpha\beta}}{M_s^{n/2}} \int dy \, \delta(y) \, \bar{L}_\alpha H \nu_{B,\beta} (x, y) + \text{h.c.} .$$

(2)

If the bulk neutrino resides in $(3+1+n)$ dimensions, where $\delta(y)$ is a n-dimensional delta function and $dy$ is the n-dimensional measure in the hidden space. Taking the individual modes on the brane located at $y = 0$, after symmetry breaking one gets a Dirac mass matrix for the active neutrinos given by

$$m_{D,\alpha\beta} = \frac{h_{\alpha\beta} v M_s}{M_{Pl}} \sim 6 \times 10^{-5} \text{ eV} \, \frac{h_{\alpha\beta}}{\text{TeV}} .$$

(3)

These neutrinos also mix with the KK modes of the bulk neutrinos. Diagonalization of this mass matrix leads to three eigenvalues $m_i$ and a mixing matrix $U_{ai}$ that characterizes
the neutrino oscillations. The mixing with the bulk fermion modes is characterized for the case of one dimension by three parameters $\xi_i \simeq \sqrt{2}m_i R$. This minimal model induces a magnetic moment connecting $\nu_i$ from the brane to each of the bulk modes $\nu_B^{(k)}$, in addition to that given by the SM result \[4\]

$$\mu_i \simeq 1.6 \times 10^{-19} \mu_B \frac{\sqrt{2}m_i}{1 \text{ eV}},$$

where $\mu_B$ is the Bohr magneton. Note that the value of the magnetic moment is connected to the mass of the neutrino and through it to the observed neutrino oscillation parameters. We will exploit this connection in our search for extra dimensions using high energy neutrinos from the cosmic rays.

Before getting to the discussion of realistic extra dimensional scenarios for neutrinos, let us first outline the effect of the magnetic moment on a high energy neutrino travelling through a magnetic field. If the neutrino energy is denoted by $E$, then all KK modes of the bulk neutrino upto energy $E$ will be excited by the magnetic moment interaction. This enhances the effect of the spin flip due to magnetic moment over the case with no large extra dimension as in SM. In the generic case, the effective magnetic moment becomes \[5\]

$$\tilde{\mu} \approx 4h A_n \left( \frac{1}{2\pi} \frac{E}{M_*} \right)^{n/2} \times 10^{-8} \mu_B .$$

Here, $h$ is the strength of the brane-bulk Yukawa coupling, and $A_n$ is the volume of the positive hemisphere of a $n$ dimensional unit sphere. Note that $\tilde{\mu}$ is inversely proportional to the value of the fundamental scale $M_*$, whereas with a given value of the scale, it grows with the energy of neutrino. This last feature provides us with the possibility of having large effective magnetic moments for neutrinos at high energies, while still obeying the constraints provided by the experiments at lower energies.

The current strongest limits on the neutrino magnetic moment are $\mu_\nu \leq 1.6 \times 10^{-10} \mu_B$ at around 10 MeV from reactor experiments and SK solar neutrino data and $\mu_\nu \leq 10^{-12} \mu_B$ from supernova 1987a. Note that these limits now get translated to limits on the $\tilde{\mu}$, which depends on two parameters ($h, M_*$). So clearly, the limits can be satisfied by appropriate choice of $h$ and would therefore fail to provide any useful information on the string scale $M_*$. One may however hope to combine other terrestrial data to extract useful information.

A further remark is in order. If $E$ is higher than the string scale, one would excite the stringy modes rather than the KK modes, whose spacing is in general higher than that of the KK modes (expected to be of order $\sim M_*).$ As a result, for energies higher than the string scale, it is probably reasonable to use the $M_*$ as a cut-off on the phase space integral. In any case, as we will see below, for higher dimensional models, we expect $M_* \sim 500 - 1000$ TeV, which covers almost the entire expected range of neutrino energies (upto $10^{15}$ eV or so). We will assume in all our considerations that it is only the KK modes that are important. In fact, as the neutrino energy gets higher, our method probes higher values of the fundamental scale $M_*$. In this paper, we will focus on the effects due to $M_* \sim 10 - 10^9$ Tev and hence neutrino energies in the intermediate range, $10^{10}$eV $\leq E \leq 10^{15}$eV.
To get a feeling for the kind of $\tilde{\mu}$ one can expect in realistic examples consistent with known data, we will consider constraints coming from attempts to fit the observed solar and atmospheric neutrino oscillation data [9-11], although this does not account for the LSND anomaly. Since the expression for $\tilde{\mu}$ involves the Yukawa coupling $h$ and the string scale $M_\ast$ which are connected to the value of neutrino masses, acceptable neutrino mass patterns from oscillation data will also restrict the allowed values of $\tilde{\mu}$. First important point to recall is that in the minimal scenario described here the LSND observations cannot be accommodated [9,10]. However, it is possible to find a domain of parameter space in $h$ and $M_\ast$ that explains the neutrino data and does not violate the experimental constraints on $\tilde{\mu}$ mentioned above. Indeed, in the limits $h \to 0$ or $M_\ast \to \infty$, the oscillations between active neutrinos account for the solar and atmospheric anomalies successfully.

Since the bulk modes are sterile neutrinos, the recent SNO data on solar neutrinos and the Super-Kamiokande data on atmospheric neutrinos can be understood only if we choose the parameter region $\xi_i \ll 1$. The mixings are then dominated by the $U_{\alpha i}$ discussed in [9,10].

The individual neutrino masses can have one of the following patterns [12]:

(i) Normal hierarchy: $m_1 \ll m_2 \ll m_3$ ;
(ii) Inverted hierarchy $m_1 \sim m_2 \gg m_3$ ;
(iii) Degenerate: $m_1 \simeq m_2 \simeq m_3$ .

Here the neutrino mass eigenstates are defined such that the familiar atmospheric ($\Delta m^2_{\odot}$) and solar ($\Delta m^2_{A}$) mass-squared differences, i.e. $|\Delta m^2_{31}| \approx |\Delta m^2_{32}| \approx \Delta m^2_{A} \approx 3 \times 10^{-3}$ eV$^2$, and $|\Delta m^2_{21}| \approx \Delta m^2_{\odot}$.

Since the magnetic moment is directly given by the parameters $m_i$, the effective magnetic moments follow the same pattern as the mass hierarchies. Therefore, we can write the contribution due to the $i$th mass eigenstate as

$$\tilde{\mu}_i \approx 7 \left( \frac{1}{2 \pi} \frac{E}{M_\ast} \right)^{n/2} \left( \frac{\text{TeV}}{M_\ast} \right) \left( \frac{m_i}{\text{eV}} \right) \times 10^{-4} \mu_B .$$  \hspace{1cm} (6)

For instance, in case (i), we have $\tilde{\mu}_1 \ll \tilde{\mu}_2 \ll \tilde{\mu}_3$. In the case of normal hierarchy, if we choose the maximum value of the Yukawa coupling $h$ to be one, then atmospheric neutrino data implies $m_3 \approx 0.06$ eV, and hence $M_\ast \sim 10^3$ TeV. Using (6), we then get

$$\tilde{\mu}_3 \approx 5 \times 10^{-10} \mu_B (E/\text{TeV})^{1/2} \quad (n = 1) ,$$  \hspace{1cm} (7)

$$\tilde{\mu}_3 \approx 7 \times 10^{-12} \mu_B (E/\text{TeV}) \quad (n = 2) .$$  \hspace{1cm} (8)

Since the LMA solution to the solar neutrino data requires $m_2 \approx 3 \times 10^{-3}$ eV, we have

$$\tilde{\mu}_2 \approx 2.5 \times 10^{-11} \mu_B (E/\text{TeV})^{1/2} \quad (n = 1) ,$$  \hspace{1cm} (9)

$$\tilde{\mu}_2 \approx 3.5 \times 10^{-13} \mu_B (E/\text{TeV}) \quad (n = 2) .$$  \hspace{1cm} (10)

The value of $\tilde{\mu}_1$ is much smaller. Since the spin-flavour precession is proportional to $\tilde{\mu}_i$ the precession phase will be much smaller for $\nu_1$ and $\nu_2$ as compared to that for $\nu_3$. Similar observations can be made about the other mass hierarchies which we shall discuss below.
III. MAGNETIC MOMENT EFFECTS ON INTERGALACTIC HIGH ENERGY NEUTRINOS

Here, we consider the effects of the induced magnetic moment on high energy neutrinos in the presence of galactic magnetic fields through which they will travel, and as a result, may spin-precess to the bulk states. Depending on the amount of precession, this may lead to a depletion of various neutrino species. We assume that the precession phase gained by a mass eigenstate $\nu_i$ is

$$\phi_i(E) = \int \tilde{\mu}_i(E) B \, dl ,$$  \hspace{1cm} (11)

where $dl$ is the distance travelled through a magnetic field $B$. Physically, this is the phase gained in the mass basis, due to a transition from the active left-handed modes to the sterile right-handed bulk modes. The corresponding precession probability for the $i$th mass eigenstate is then

$$P_i(E) = \sin^2 \phi_i(E) .$$  \hspace{1cm} (12)

It is important to note that there are two simultaneous effects here: one due to the flavour mixing and another due to magnetic moment precession. The final flavor content at the detector therefore depends on the distance travelled and the energy. If an extremely long distance of travel is involved, two mass eigenstates with a mass squared difference, $\Delta m^2_{ij}$, can lose their coherence over a distance scale of $E/\Delta m^2_{ij}$ and travel as independent mass eigenstates. The coherence length scale is

$$L_c \sim 10^8 \text{cm} \left( \frac{E}{\text{TeV}} \right) \left( \frac{\text{eV}^2}{\Delta m^2_{ij}} \right) .$$  \hspace{1cm} (13)

Loss of coherence happens specifically when the distances involved are intergalactic distances. An example of this are neutrinos which are produced through sources like AGNs. The final flavour content will depend on whether coherence is lost or it remains. In this section, we will focus on intergalactic neutrinos, which are likely to lose coherence.

The neutrinos from cosmic sources are produced mainly from the decays of pions. This implies that the ratios of the production fluxes of the electron, muon and tau neutrinos are

$$F^{(0)}_e : F^{(0)}_\mu : F^{(0)}_\tau \approx 1 : 2 : 0 .$$  \hspace{1cm} (14)

As long as the mixing between $\nu_\mu$ and $\nu_\tau$ is maximal (which is strongly supported by the atmospheric neutrino data at SK), the three mass eigenstates have the production fluxes

$$F^{(0)}_1 : F^{(0)}_2 : F^{(0)}_3 \approx 1 : 1 : 1 .$$  \hspace{1cm} (15)

As already mentioned, due to loss of coherence, the three mass eigenstates travel independently and if there is no magnetic moment induced precession, the fluxes of different flavours at the detector are determined by flavour mixings and are given by

$$F_\alpha = \sum |U_{\alpha i}|^2 F_i .$$  \hspace{1cm} (16)
For bimaximal mixing case, the flavour composition of the neutrino flux at the detector will be

\[ F_e : F_\mu : F_\tau \approx 1 : 1 : 1 \ . \]  

(17)

The spin precession will modify this relation, since these neutrinos travel a distance of several \( \sim \) Mpc, even though the magnetic fields along the path are small \( (B \ll 10^{-8} \text{ Gauss}) \). We estimate the precession phase by assuming (for example) that the neutrinos encounter a coherent magnetic field of this magnitude for a typical distance of Mpc. This gives

\[ \phi_i \sim 10^{13} (\tilde{\mu}_i / \mu_B) \ . \]  

(18)

Since \( \tilde{\mu}_i \) depends on the neutrino mass, the phase \( \phi_i \) that a particular mass eigenstate precesses through can be large if the corresponding neutrino mass is “large”, and averaging for large phases would give \( \langle \sin^2 \phi_i \rangle \approx 0.5 \), and there will be a suppression of the flux \( F_i \) by a factor of two. If a particular mass is small, there will be no magnetic precession effect and no additional suppression. The resulting flavour composition of the detected neutrinos for different mass patterns of the neutrinos is given below.

(i) For the normal mass hierarchy, \( \tilde{\mu}_1 \ll \tilde{\mu}_2 \ll \tilde{\mu}_3 \). Hence, it is possible for \( \phi_3 \sim 1 \), whereas \( \phi_1 \ll \phi_2 \ll 1 \). In this case, we get

\[ F_1 : F_2 : F_3 = 1 : 1 : 0.5 \quad \text{or} \quad F_e : F_\mu : F_\tau = 1 : 0.75 : 0.75 \ . \]

If \( \phi_3 \gg 1, \phi_2 \sim 1 \) and \( \phi_1 \ll 1 \), we have for bimaximal mixing

\[ F_1 : F_2 : F_3 = 1 : 0.5 : 0.5 \quad \text{or} \quad F_e : F_\mu : F_\tau = 0.75 : 0.625 : 0.625 \ . \]

If on the other hand, the mixing of \( \nu_e \) with the other states is not large (this possibility is disfavoured, but not yet completely ruled out), we get

\[ F_1 : F_2 : F_3 = 1 : 0.5 : 0.5 \quad \text{or} \quad F_e : F_\mu : F_\tau = 1 : 0.5 : 0.5 \ . \]

If the precession of all three mass eigenstates is large, all the flavours are suppressed by a factor of two, i.e.

\[ F_1 : F_2 : F_3 = 0.5 : 0.5 : 0.5 \quad \text{or} \quad F_e : F_\mu : F_\tau = 0.5 : 0.5 : 0.5 \ . \]

(ii) For the inverted mass hierarchy, if \( \phi_1 \approx \phi_2 \sim 1 \) and \( \phi_3 \ll 1 \), then

\[ F_1 : F_2 : F_3 = 0.5 : 0.5 : 1 \quad \text{or} \quad F_e : F_\mu : F_\tau = 0.5 : 0.75 : 0.75 \ . \]

The second possibility is that the precession of all three mass eigenstates is large, in which case we get a uniform suppression of a factor of two as shown above.

(iii) If the masses of all three mass eigenstates are degenerate, then either all mass eigenstate undergo large precession giving an overall suppression by a factor of two, or none of the eigenstates undergo significant precession, so that the fluxes remain unchanged.
Note that in the case of inverted hierarchy and degenerate neutrino scenarios, the final fluxes do not depend on the extent of mixing of the electron flavour in $\nu_2$. These flavour specific deformations are to be contrasted with the conventional expectations in (17). Current and planned neutrino telescope experiments, which are going to measure the flux of high energy muon and tau type neutrinos (the latter by the so called double bang events) could be sensitive to these changes. Such measurements can be compared with theoretical predictions of the fluxes to draw conclusions about the extra dimensions.

One possible drawback regarding the above conclusion is that no reliable information is available about the intergalactic magnetic fields. For instance, if these magnetic fields are incoherent with a spatial spread shorter than the coherent length for spin precession, then the average spin precession phase gained by a neutrino mass eigenstate, $\langle \phi_i \rangle$, could be smaller. For instance, if the source distance is $D$ and the coherence length of the magnetic field is $\ell_c$, then the total length in the Eq. (11) can be replaced in the random walk approximation by $\sqrt{D\ell_c}$. This could lead to nontrivial effects for higher energy extragalactic neutrinos.

Furthermore, in the physical situation considered, we require neutrinos from cosmologically distant sources, wherein, the production mechanisms are source dependent and yet unclear [18]. This motivates us to analyse the effects due to the Earth’s magnetic field on neutrinos produced in the atmosphere, which we discuss in the next section.

IV. EFFECTS ON EXTREME HIGH ENERGY ATMOSPHERIC NEUTRINOS

In this section, we focus on neutrinos with energies below $E < 10^{15}$ eV produced in the atmosphere. The total neutrino spectrum in this energy range is expected to be dominated by the atmospheric neutrinos [19] that originate from the decays of pions that are generated from cosmic rays in Earth’s atmosphere. The spectrum of neutrinos then has the same shape as the cosmic ray spectrum, which has been well measured in this energy range and has a simple power law dependence. The magnitude of the flux may be fixed by extrapolating the flux of the atmospheric neutrino spectrum, which is measured for energies $\sim$ GeV, with the same power law. Using the energy range $E < 10^{15}$ eV has one more advantage – as mentioned before, this energy would be below the fundamental scale of the theory, so that the interactions with the stringy modes may be neglected. In the following, we address the spin precession of these high energy neutrinos due to the magnetic field of the Earth.

The neutrinos that are produced in the atmosphere of the Earth travel through the magnetic field of the Earth, which is $\sim 1$ gauss. If the distance travelled by these neutrinos before reaching the detector is $L$ km, using (18), we get

$$\phi_i \sim 30L (\text{km}) \left( \frac{\tilde{\mu}_i}{\mu_B} \right) \left( \frac{m_\nu}{eV} \right).$$

(19)

For $m_\nu \sim 0.06$ eV, $M_*= 10$ TeV, (6) implies that significant precession may be observed only for neutrinos travelling distances of $\sim 10^4$ km (through the earth). This will result in a zenith angle dependence which we discuss here. The “neutrino telescope” experiments like ICE CUBE [15] and ANTARES [16] are tuned to see neutrinos arriving through the Earth and can therefore test for this effect.
We show that magnetic precession of the kind we are discussing leads to a novel feature of a flavour specific nonzero up-down asymmetry for these neutrinos. To make this effect quantitative, let us define the angular asymmetry in the standard form for a neutrino, $\nu_\alpha$ as

$$A_\alpha(\theta_1, \theta_2) = \frac{R_\alpha(\theta_1) - R_\alpha(\theta_2)}{R_\alpha(\theta_1) + R_\alpha(\theta_2)},$$

(20)

where $\theta_i$ is the azimuthal angle. The asymmetry owes its origin to the different distances travelled by the neutrinos coming from the top and bottom of an Earth located detector as we explain below. In (20), $R_\alpha(\theta_i)$ is the rate of neutrinos $\nu_\alpha$ in a given direction cos $\theta_i$.

We will focus on the high-energy neutrinos (with $E \leq 10^{15}$ eV) a large fraction of which are induced by atmospheric air showers at a distance of a few tens of kilometers above the Earth. Qualitatively, if we examine (19), we see that $\phi_i \sim O(1)$ only for $L \geq 10^4$ Km. This implies that for neutrinos traversing the entire diameter of the Earth, spin precession can lead to an up-down asymmetry

$$A_{\alpha UD} = A_\alpha(\pi, 0) = \frac{R_\alpha(\theta_1 = \pi) - R_\alpha(\theta_2 = 0)}{R_\alpha(\theta_1 = \pi) + R_\alpha(\theta_2 = 0)},$$

(21)

The sign of this asymmetry is negative for $\nu_e$ and $\nu_\mu$ whereas for $\nu_\tau$, since $R_\tau(\theta_2 = 0) = 0$ we will have a unit positive value. In the following, we illustrate the expected flavour asymmetry for the three mass patterns. Note that since in this case the distance of travel is less than the coherence length $L_c$, the expected flavour fluxes are different from (17). Since there is not enough time/distance for flavour oscillation to have taken place, the original flavour ratio remains 1 : 2 : 0 rather than 1 : 1 : 1, which is for the intergalactic neutrinos. To derive the up-down asymmetry, note that the flavour and mass eigenstates are related through a unitary mixing matrix $U$ by

$$\nu_\alpha = U_{\alpha i} \nu_i.$$  

(22)

Now, the mass eigenstates precess to the right-handed bulk modes ($\nu_i^s$) in the background magnetic field and at the detector, we have for these states the relation

$$\nu_\alpha^d = U_{\alpha i} \nu_i^d ; \quad \nu_i^d = \nu_i \cos \phi_i + \nu_i^s \sin \phi_i.$$  

(23)

Therefore, the flavour fluxes at the detector, are easily seen to be

$$F_\alpha^d = P_{\alpha \beta} F_\beta$$  

with $P_{\alpha \beta} = |U_{\alpha 1} U_{\beta 1}^* \cos \phi_1 + U_{\alpha 2} U_{\beta 2}^* \cos \phi_2 + U_{\alpha 3} U_{\beta 3}^* \cos \phi_3|^2,$  

(24)

where $F_\beta$ denotes the initial flux at the production point which is taken to be a few tens of kilometers above the Earth. Using (24) and taking bimaximal mixing, we calculate the expected flavour fluxes at the detector for the three mass hierarchies. In doing these calculations, we have taken the averaging $\langle \cos \phi_i \rangle = 0$ and $\langle \cos^2 \phi_i \rangle = 0.5$.

The resulting values for $F_\alpha^d$ and $A_{\alpha UD}$ for the three different mass spectra are:

(i) For the normal mass hierarchy:

$$F_e^d : F_\mu^d : F_\tau^d = 1 : 0.75 : 0.75.$$
\[ A^{UD}_e : A^{UD}_\mu : A^{UD}_\tau = 0 : -0.45 : 1.0 \ . \]

(ii) For the inverted mass hierarchy:
\[ F^d_e : F^d_\mu : F^d_\tau = 0.50 : 0.75 : 0.75 \ . \]
\[ A^{UD}_e : A^{UD}_\mu : A^{UD}_\tau = -0.33 : -0.45 : 1.0 \ . \]

(iii) For the degenerate mass hierarchy:
\[ F^d_e : F^d_\mu : F^d_\tau = 0.5 : 1.0 : 0 \ . \]
\[ A^{UD}_e : A^{UD}_\mu : A^{UD}_\tau = -0.33 : -0.33 : 0 \ . \]

It is important to note that for energies close $10^{15}$ eV and higher, the neutrino weak interaction cross section matter in the Earth becomes important enough so that the spectrum gets suppressed for the upward going neutrinos [20]. But our effect can manifest also at lower energies.

V. COMMENTS AND CONCLUSION

There can also be other possible sources of spin precession for neutrinos traversing through space. For example, the precession effects may be large if we consider the neutrinos that travel close to the Sun on their way to the detector. These will be affected by the solar magnetic field, which is as much as $B_\odot \approx 10$ Gauss at the surface of the Sun. If we assume the spatial profile of the magnetic field to be given by an approximate power law ($B \sim r^{-\alpha}$), where $r$ is the distance from the Sun, and $\alpha \sim 1.25$, the precession undergone by a neutrino travelling from the Sun to the Earth would then be

\[ \phi_i \sim \bar{\mu}_i \int_{R_\odot}^{r_{\text{earth}}} \, dr \, B_\odot \left( \frac{r_\odot}{r} \right)^\alpha \approx \bar{\mu}_i B_\odot R_\odot \frac{1}{\alpha - 1} \]

\[ \sim 10^{11} \left( \frac{\bar{\mu}_i}{\mu_B} \right) \ . \]

The precession effects are likely to be large for one or more mass eigenstates. In principle, this could also lead to an angular asymmetry.

In summary, we have pointed out a novel flavour specific deformation of the extreme high energy cosmic neutrino spectra in the presence of large extra dimensions with low fundamental scale of particle interactions. We have specifically noted how extreme high energy atmospheric neutrinos detected on Earth would show an up-down asymmetry, unique in their origin due to the extra dimensional effect. The extreme high energy galactic neutrinos are also likely to show flavour specific deformations of their spectra. In contrast to the neutrinos produced in the Earth’s atmosphere, these galactic neutrinos are expected to show a uniform spherically symmetric flavour deformation. But, given the low rates of such high energy neutrinos, and the source uncertainties, we emphasise that the up-down asymmetry in [21] for atmospheric neutrinos with energies $E \leq 10^{15}$ eV will be statistically favoured. The upcoming neutrino telescope experiments such as ICE CUBE, NESTOR [21]...
and ANTARES. are particularly suited to detect these kinds of effects. An advantage of this method is that it can probe the fundamental scales much higher than the conventional collider methods discussed in the literature since the magnetic moments depend on both the fundamental scale $M_*$ and neutrino energy $E$.

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