Sub-Stream Fairness and Numerical Correctness in MIMO Interference Channels

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Abstract—Sub-stream fairness, fairness between all streams in the system, is a more restrictive condition than sub-stream fairness, fairness between all streams of each user. Thus sub-stream fairness alleviates utility loss as well as complexity and overhead compared to stream fairness. Moreover, depending on algorithmic parameters, conventional algorithms including distributed interference alignment (DIA) may not provide sub-stream fairness, and generate sub-streams with poor signal-to-interference plus noise ratios (SINRs), thus with poor bit error rates (BERs). To this end, we propose a distributed power control algorithm to render sub-stream fairness in the system, and establish initiatory connections between sub-stream SINRs, BERs, and rates. Algorithms have particular responses to parameters. In the paper, important algorithmic parameters are analyzed to exhibit numerical correctness in benchmarking. The distinction between separate filtering schemes that design each stream of a user separately and group filtering schemes that jointly design the streams of a user is also underscored in the paper. Finally, the power control law used in the proposed algorithm is proven to linearly converge to a unique fixed-point, and the algorithm is shown to achieve feasible SINR targets.

Index Terms—MIMO, interference channel, SINR, BER, rate, sub-stream fairness, algorithmic parameters.

I. INTRODUCTION

Fairness is important to ensure quality-of-service (QoS) in the system [12]. While power control to attain signal-to-interference plus noise ratio (SINR) fairness is well explored for downlink channels [13,14], the research for interference channels (ICs) is still at a primitive level due to NP-hardness of the problem in general [5]. Fairness in the system can be achieved by two complementary approaches, maximization of minimum SINR subject to power constraint or minimization of power subject to SINR constraint, and at three different levels, fairness between streams, users, or sub-streams. Explicitly, fairness between all streams, all users, or all streams of each user can be aimed. Both problems achieve optimal solutions when, depending on the intended level, streams’, users’, or sub-streams’ signal-to-interference plus noise ratios (SINRs) are attained with equality [6]. From more to less restrictive, stream, user, and sub-stream fairness come in order. Consequently, sub-stream fairness causes the least degradation in sum-rate, followed by user and stream fairness. Sum-rate is a prominent but only a commensurate metric with each stream’s SINR. In particular, a stream can contribute substantially to sum-rate, but can have poor SINR, thus have gross bit error rate (BER) [7]. For example, SINR ratios of sub-streams can range from 5 to 9 times, whereas rate ratios can be around 1.5 times only [1]. As well known, although SINR and rate metrics are coupled via the log function, the beamforming vectors that maximize the sum-rate do not necessarily maximize the sum-SINR. To this end, we propose an ad-hoc distributed power control algorithm (DPCA) to achieve SINR fairness at the sub-stream level by using the later approach, minimization of power subject to SINR constraint. Basically, transmit and receive beamforming vectors are initially obtained via a conventional beamforming scheme including SINR maximization (max-SINR) and distributed interference alignment (DIA) [8]. Then, our proposed power control algorithm is plugged and run in an ad-hoc manner. The outer loop of the proposed algorithm linearly searches for a feasible SINR target for each user, thus convergence is guaranteed. Since there is a maximum power constraint, the optimal power values may not be feasible if SINRs are not well balanced before power control applied. The goal of the proposed algorithm is to achieve sub-stream fairness while causing the least sum-rate degradation. Therefore, power saving is not the primary concern of our algorithm. The power control law used in the algorithm to achieve sub-stream fairness in ICs is a direct extension of standard power control law introduced in [9]. However, our paper establishes initiatory connections between sub-stream SINRs, bit error rate (BERs), and rates to some extent. In addition, our paper is the only paper except [10] to consider SINR targets as optimization variables as opposed to conventional approaches that propose schemes with preset SINR or rate targets.

Achieving sub-stream fairness in physical layer, e.g., scheduling is not considered, is addressed in this paper. Modifying conventional schemes including minimization of maximum mean square error (min-max-MSE) [11] and maximization of the proportional utility function [12] so to achieve sub-stream fairness can be rewarding compared with our simplistic proposed algorithm that suits practical applications. Min-max-MSE is computationally costly, and maximization of proportional utility is NP-hard even for multiple-input single-output (MISO) ICs, but if fact efficient algorithms can
be proposed. However, in this paper, our goal is to take initiative steps on emphasizing the importance of sub-stream fairness in the system and show its effects on system metrics, along with underscoring the importance of numerical details. Thus this paper provides the basis for designing more powerful schemes to achieve sub-stream fairness.

The prominent paper [13] proposed a new technique that was coined interference alignment (IA), and IA was shown to achieve the upper bound of non-interfering signaling dimensions in an interference channel (IC). Later, the limits of IA in multiple-input multiple-output (MIMO) ICs were mainly identified in [14] and in consecutive papers [15][16]. The numerical results of IA first appeared in [13], where the authors proposed a DIA algorithm. Subsequently modified DIA techniques that achieved improved sum-rate performances appeared in [17][18]. In [8], IC extension of the conventional max-SINR algorithm for point-to-point channels was also proposed, and max-SINR was shown to achieve higher sum-rate than DIA in the low to medium signal-to-noise ratio (SNR) regime. Although max-SINR lacks art in its design approach, its sum-rate results in homogenous MIMO ICs (MIMO IC is fully connected and channels are independent and identically distributed, i.i.d.) is surprisingly satisfactory. Excluding the weighted minimum mean square error (MMSE) technique [19] proposed for single stream MIMO ICs, designing a linear scheme that superposes max-SINR is still an open problem [7]. However, an important QoS metric, sub-stream fairness, i.e., fairness between streams of a user, was overlooked in all these works. It can be shown that as the number of streams per user increases, the SINR levels of some sub-streams can go very low implicating high decoding errors for those sub-streams. This phenomena can occur due to the inherent competition in the algorithm (e.g., separate stream design of max-SINR) and due to the preset parameters in the algorithm (e.g., stopping criterion of DIA). The results that showed the importance of sub-stream fairness and algorithmic parameters first appeared in [7].

In the literature, sub-stream fairness was only studied in limited number for the layers above physical layer. Since these papers are out of our scope, we do not cite them in the paper. To the best of our knowledge, our work is the first paper to consider sub-stream fairness in physical layer. Our goal is to achieve fairness at sub-stream level in the system with minimum sum-rate degradation. Algorithmic parameters shift the results considerably. For example, a predetermined iteration number or a negligible increment in the sum-rate can be the stopping criterion of an algorithm. While DIA can reasonably achieve sub-stream fairness for the later, the imbalance between sub-streams increases as the preset iteration number decreases. In fact, for homogenous ICs, in ergodic sense, the three different fairness approaches have similar outcomes but with complexity and overhead varieties. However, for a given channel, the three approaches have disparate outcomes, especially in the high SNR regime and with a low preset iteration number [7]. In other words, for a given homogenous IC, altruism degree matters. For example, stream fairness, thus user fairness, can be achieved at the cost of higher utility loss than sub-stream fairness. On the other hand, sub-stream fairness can be achieved while letting users preserve their ranks, i.e., user fairness is not achieved, with the reward of lesser complexity and overhead than stream fairness. Clearly, sub-stream fairness is less altruistic than stream fairness. For example, for a given channel, assume the system has SINR distribution with \( \text{SINR}_{\text{sys}} = \{1, 3\}, \{4, 6\}, \{7, 9\}, \) where \( \text{SINR}_k = \{\text{SINR}_{k,1}, \text{SINR}_{k,2}\} \) denotes sub-stream SINRs of user \( k \). Please note that the average SINR per stream and user is \( 5 \) and \( 10 \), respectively. The system can achieve \( \text{SINR}_{\text{sys}} = \{\{3, 3\}, \{3, 3\}, \{3, 3\}\} \), \( \text{SINR}_{\text{sys}} = \{\{3, 5\}, \{3, 5\}, \{3, 5\}\} \) and \( \text{SINR}_{\text{sys}} = \{\{2, 2\}, \{5, 5\}, \{7, 7\}\} \) for stream, user and sub-stream fairness, respectively.

Power control and beamforming are common approaches to achieve fairness. The reader is referred to [20] for beamforming design and to [21] for joint power control and beamforming design, and the references therein. In [20] and [5][21], authors propose decentralized and centralized algorithms for ICs, respectively. In this work, we focus on decentralized algorithms in consensus with ICs’ nature. In downlink channels, SINR duality can be achieved between downlink and uplink directions via various joint designs as summarized in [22]. At each iteration, SINRs are equalized via power control and incremented via beamforming, thus SINRs are maximized monotonically. Deploying this concept to ICs in a distributed manner is nontrivial, thus designing joint power control and beamforming with decentralized and linear features to achieve fairness in MIMO ICs is still an open problem. In this paper, we propose a practical and a distributed power control algorithm that can be attached to any conventional beamforming scheme, i.e., the proposed algorithm is ad-hoc, with a slightly increased algorithmic load. Simulation results show that the algorithm has narrow rate and SINR losses with achieved sub-stream fairness. It is shown in [7] that some sub-streams can have high decoding errors due to their poor SINR levels, but their contribution to sum-rate can still be substantial. Therefore, sum-rate results without parsed stream BERs cannot capture the whole picture. Our results also underline the ascending importance of sub-stream fairness as sub-stream number increases.

Finally, a power control algorithm that swiftly converges to a fixed-point is requisite in practice. By using recently introduced contractive interference functions [23], i.e., slightly modified versions of the well-known standard interference functions [9], we prove that the power law in the proposed algorithm has a linear convergence rate to a unique fixed-point making the algorithm preferable in practice.

The rest of the paper is organized as follows. In Section II we introduce the system model. In Section III we focus on sub-stream fairness and give the motivation. In this section, we present separate filtering schemes that design each stream of a user separately and group filtering schemes that jointly design streams of a user, and introduce our power control algorithm. In Section IV we introduce important algorithmic parameters that can significantly differentiate numerical results and show that a complete picture can only be depicted by varying these parameters in a benchmarking process. In Section V we present the numerical results. In Section VI we show that the proposed algorithm linearly converges to a unique fixed-point.
Notation: † and $^{-1}$ denote the complex conjugate and inverse matrix (if the matrix is full-rank) operations, respectively. Matrices are denoted by bold-face uppercase letters whereas vectors by bold-face lowercase letters. $\mathbf{1}$, $\mathbf{I}$, $\mathbf{0}$, and $\text{diag}[v_1, \ldots, v_{L}, v_L]$ denotes all ones vector, identity matrix, zero vector or matrix, and diagonal matrix with elements $v_i$ on its diagonal, respectively. $||| \cdot |||$ and $|| \cdot ||_1$ and $|| \cdot ||_\infty$ denote determinant, $l_1$-norm, and minimum operators, respectively, and for a given vector $\mathbf{v} > 0$, $|| \cdot ||_\infty$ denotes weighted maximum norm.

II. System Model

We consider a $K$-user IC, where there are $K$ transmitters and receivers with $M_k$ and $N_k$ antennas at node $k$, respectively. A transmitter has $d_k$ streams to be sent to its corresponding receiver. This system can be modeled as $y_k = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{x}_k + z_k$, $\forall k \in K \triangleq \{ 1, 2, \ldots, K \}$, where $y_k$ and $z_k$ are the $N_k \times 1$ received signal vector and the zero mean unit variance circularly symmetric additive white Gaussian noise (AWGN) at the $k^{th}$ receiver, respectively. $\mathbf{x}_k$ is the $M_k \times 1$ signal vector transmitted from the $l^{th}$ transmitter and $\mathbf{H}_{kl}$ is the $N_k \times M_l$ matrix of channel coefficients between the $l^{th}$ transmitter and the $k^{th}$ receiver. $E[||\mathbf{x}_k||^2] = p_l$ is the power of the $l^{th}$ transmitter.

The transmitted signal from the $l^{th}$ user is $\mathbf{x}_l = \mathbf{U}_l \sqrt{\mathbf{P}_l} \mathbf{d}_l$, where $\mathbf{U}_l = [\mathbf{u}_{l, 1}, \ldots, \mathbf{u}_{l, d_l}]$ is the $M_l \times d_l$ precoding (beamforming) filter, $\mathbf{d}_l$ is a $d_l \times 1$ vector denoting the $d_l$ independently encoded streams, and $\mathbf{P}_l = \text{diag}[p_{l, 1}, \ldots, p_{l, d_l}]$ is a $d_l \times d_l$ diagonal matrix consisting of sub-stream powers, $p_l = \sum_{j=1}^{d_l} p_{l,j}$. The $N_l \times d_l$ receiver matrix is denoted by $\mathbf{V}_l$.

III. Sub-stream Fairness

In this section, we give the motivation for achieving sub-stream fairness in MIMO ICs, and then take preliminary steps to introduce our algorithm. Particularly, we highlight the distinction between separate and group filtering schemes, and introduce a slightly modified SINR definition for separate filtering schemes. Based on this new definition, we present the standard interference function to be used in our proposed algorithm. Finally, we introduce our DPCA. We first begin with summarizing an algorithm that also dynamically sets the SINR targets.

In the literature, SINR targets are generally predetermined [5, 23], and to the best of our knowledge, setting SINR targets opportunistically is only studied in [10] by using the augmented Lagrangian penalty function (ALPF) method. The method imposes fairness constraint between streams

$$\text{SINR}_{k,l} - \text{min} \text{ SINR}^*_{k,l} \leq F, \ \forall k \in K, \ \forall l \in L \triangleq \{ 1, 2, \ldots, d_k \},$$

where \( \text{SINR}_k = [\text{SINR}_{k,1}, \ldots, \text{SINR}_{k,d_k}] \) is the vector of sub-stream SINRs of user $k$, \( \text{SINR}^*_k \) is the vector excluding the compared SINR, i.e., $\text{SINR}^*_k = \text{SINR}_k \setminus \{ \text{SINR}_{k,l} \}$, and $F$ is the fairness constraint. $F$ can be called SINR fairness offset, and it clearly limits the SINR difference between the streams. This method is a fair benchmark to our proposed algorithm since SINR targets are set as optimization variables as well. However, the ALPF algorithm is not ad-hoc, thus a good starting point for target SINR searching is critical for the algorithm. In [10], the authors use nonlinear search methods to find feasible starting points. An important advantage of ad-hoc algorithms is their ability of searching target SINRs linearly. Maximum sub-stream SINR per user achieved after beamforming is the upper bound to the maximum sub-stream SINR achieved after power control. Therefore, setting average SINR per user as the sub-stream SINR target is a good starting point for searching. ALPF algorithm can be extended to MIMO ICs and converted to an ad-hoc DPCA, but the extended algorithm is expected to be considerably slower than our DPCA since they use Lagrangian function whereas we use a simple standard interference function [25] for power control as will be explained in the end of this section.

Before proceeding further, the reader is recommended to con our earlier paper [7] that precedes the current paper. Unbeknownst to us, a study that uses standard interference function was published [26] concurrently with the submission of our earlier work [7]. The paper [26] follows a conventional motivation by proposing a power control algorithm to achieve predetermined rate targets per user with minimum power consumption, and serves as a rectification to an erroneous SINR definition in [27] as noted in [7].

A. Motivation

The importance of sub-stream fairness and algorithmic parameters is demonstrated in Fig. 1 and 2 via Monte Carlo (MC) simulations using 40 independent channel realizations, MC=40. Unless otherwise stated in the paper, we present numerical results for the IC with $K = 3$, $M_k = 4$, $N_k = 4$, and $d_k = 2$, $(4 \times 4, 2)^3$. The abbreviation Iter in the plots stands for the number of iterations, basically a downlink and an uplink iteration are counted as one. Iter=0 indicates that the number of iterations is not predetermined, and the algorithm stops when the increment in the sum-rate is negligible

$$|R_{\text{sum}}(n+1) - R_{\text{sum}}(n)| \leq \epsilon,$$ (1)

where $n$ stands for the iteration number, and $\epsilon$ is $10^{-6}$ in our simulations. For all simulations in the paper, we fix the number of random transmit beamforming initializations to one, presented numerical results are for channel use. SINR results and SNR values are in linear and dB scale, respectively.

As explained in [7] and as shown in Fig. 1 and 2, DIA achieves reasonable sub-stream fairness for Iter=0, but for fixed number of iterations, DIA cannot achieve balanced sub-streams. Numerical results of max-SINR and another conventional scheme minimization of the sum of mean-square errors (min-sum-MSE) are given in [7]. In [7], it is shown that max-SINR cannot achieve sub-stream fairness even for Iter=0, whereas min-sum-MSE still preserves fairness at a much better level even for a small number of iteration, Iter=50. As known, BER is influenced by the worst stream SINRs in the system. Thus, in general, min-sum-MSE can provide lower BER than max-SINR due to its stream fairness, and max-SINR can achieve lower BER than DIA since it additionally aims to maximize desired signal power. In [20], BER performances are compared for a small iteration number, Iter=16. Since
the imbalance between streams in max-SINR and DIA soar marginally more than min-sum-MSE as the iteration number decreases, BER gaps between these schemes are significant at Iter=16. These findings again indicate the influence of iteration number, as other algorithmic details do, on perceiving the complete picture.

In [7], the results of max-SINR algorithm with orthonormal beamforming vectors are presented. In [28,29], it is observed that without the orthogonalization step, the algorithm yields linearly dependent beamforming vectors. In other words, at least one stream of a user is shut off, thus a major sum-rate loss at high SNR is observed [29]. Note that MMSE receivers are assumed in [28,29], whereas we assume same type of filter structures are used at transmitters and receivers. For either of these assumptions, we could not observe dependent beamforming vectors as in [28,29], but the following. Algorithms require more iterations in the high SNR regime [30,31]. However, max-SINR without the orthogonalization step needs significantly high number of iterations, otherwise the algorithm saturates in the high SNR regime. Please note that required iteration number and rate of convergence (RoC) of an algorithm are coupled parameters. More on these parameters are discoursed in Sections IV and VI.

In Table I, the sum of SINR ratios of the $i^{th}$ to the $j^{th}$ stream, $\sum_{k=1}^{K} (\text{SINR}_{k,i}/\text{SINR}_{k,j})$, of DIA ($i = 1$ and $j = 2$), max-SINR ($i = 2$ and $j = 1$), and min-sum-MSE ($i = 2$ and $j = 1$) for Iter=50, $\emptyset$, and 50, respectively, are given

\begin{table}[h]
\centering
\caption{Sum of SINR ratios.}
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
DB & 0 & 10 & 20 & 30 & 40 & 50 & 60 \\
\hline
DIA & 1.18 & 1.23 & 2.07 & 3.19 & 7.45 & 21.88 & 70.94 \\
Max-SINR & 1.74 & 2.92 & 4.12 & 5.87 & 6.17 & 7.21 & 7.35 \\
Min-sum-MSE & 1.03 & 1.04 & 0.94 & 1.02 & 0.79 & 0.76 & 1.40 \\
\hline
\end{tabular}
\end{table}

B. Separate and Group Filtering Schemes

This section starts by defining the SINR of a stream. Stream SINR can be defined in two ways based on whether separate filtering (SF) or group filtering (GF) is applied [7]. SF methods such as max-SINR design each stream of a user separately, thus sub-streams are considered as interference on one another

\begin{equation}
\text{SINR}_{k,i}^{SF} = \frac{v_{k,l}^\dagger R_{k,i} v_{k,l}}{v_{k,l}^\dagger B_{k,i} v_{k,l}},
\end{equation}

for MC=40. As seen from the SINR and rate results of max-SINR in Table I of this paper and in Fig. 1(a) of [7], some sub-streams can have low SINRs, thus high BERs, but their contribution in sum-rate can be substantial. Please note that the first stream of DIA achieves higher SINR simply because beamforming vectors are assigned with eigenvectors having in order from smaller to larger eigenvalues of interference covariance matrix [8]. On the other hand, the second stream of max-SINR achieves higher SINR because Gram-Schmidt approach is used for QR decomposition.
where $B_{k,l} = Q_{k,l} + I_{N_k}$, $Q_{k,l} = \sum_{j=1}^{K} H_{kj} U_j P_j U_j^\dagger H_{kj}^\dagger - R_{k,l}$, and $R_{k,l} = p_{k,l} H_{kk} u_{k,l} u_{k,l}^\dagger H_{kk}^\dagger$ are interference plus noise, interference covariance, and covariance matrices of the $l$th stream of user $k$, respectively. Note that $B_{k,l}$ contains the intra-user interference. Max-SINR is apparently sub-optimal even for a point-to-point system with multiple streams since the streams are competing with each other. A better approach is to design beamforming vectors by allowing cooperation between beamforming vectors of a user [7].

$$\text{SINR}_{k,l}^\text{GF} = \frac{v_{k,l}^\dagger R_{k,l} v_{k,l}}{v_{k,l}^\dagger B_{k,l} v_{k,l}}, \quad (3)$$

where $B_k = Q_k + I_{N_k}$, $Q_k = \sum_{j=1, j \neq k}^{K} H_{kj} U_j P_j U_j^\dagger H_{kj}^\dagger$, and $R_{k,l} = H_{kk} U_k P_k U_k^\dagger H_{kk}^\dagger$ are interference plus noise, interference covariance, and covariance matrices of user $k$, respectively. For the max-SINR algorithm, it is shown in [7] that there is nearly no difference between reckoning and not reckoning intra-user interference when Shannon’s rate formula is used. In fact, for ICs, joint decoders such as maximum likelihood (ML) decoder are preferred since linear decoders such as hard decision decoder achieves severely poor BERs. Therefore, the motivation of max-SINR to consider intra-user interference is not clear. Consequently, for ICs, we use the following beamforming filter for the max-SINR algorithm

$$v_{k,l} = \frac{B_{k,l}^{-1} H_{kk} u_{k,l}}{||B_{k,l}^{-1} H_{kk} u_{k,l}||}, \quad (4)$$

In line with the above beamforming filter, the new SINR definition for SF schemes used in ICs can be given as

$$\text{SINR}_{k,l}^{\text{SF}'} = \frac{v_{k,l}^\dagger R_{k,l} v_{k,l}}{v_{k,l}^\dagger B_{k,l} v_{k,l}}, \quad (5)$$

The justification of (4) and (5) can also be shown in a different approach as follows. Consider the conventional max-SINR filter that reckons intra-user interference, thus the SINR definition in (2) is used. Now consider a GF scheme, thus SINR definition in (3) is used. As explained in [7], SF and GF achieve similar rate results when Shannon’s formula is used. On the other hand, the SINRs that SF and GF schemes achieve significantly deviate due to the difference in SINR formulas (4) and (5) while numerical results show that these schemes achieve similar BERs. These results indicate that evaluating beamforming filters and SINRs of SF schemes with ML decoders by (4) and (5) is more congenial. Note that when compared to $R_{k,l}$ (5), GF schemes can explore the extra degrees of freedom in $R_{k,l}$ [3].

C. Standard Interference Function

We briefly recall the well-known DPCA with maximum power per user $p_k$ constraint [11]

$$p_k^n = \min \left( \frac{\Gamma_k}{\text{SINR}_k^{n-1} p_k^{n-1}}, p_k \right), \quad (6)$$

where superscript $n$ is the iteration number, $p_k^n$ is the power, SINR$^{-1}$ is the SINR of user $k$, and $\Gamma_k$ is the SINR target. As seen in [32], each user updates its power following a simple decision function, i.e., min operator. Basically, a user increases its power if its SINR is below its SINR target and vice versa. Clearly the SINR target can be unmet due to the maximum power constraint.

Using the SINR constraint per stream $\text{SINR}_{k,l}^{\text{SF}'} \geq \Gamma_k$, the interference function for our problem is given as

$$I_{k,l}(p) = \Gamma_k \delta_{k,l}, \quad (7)$$

where $p = [p_{1,1}, \ldots, p_{1,d_1}, \ldots, p_{K,1}, \ldots, p_{K,d_K}]$ is the power vector of the system, $\Gamma_k = \text{SINR}_{k,l}^{\text{SF}'}$ is the SINR target set to average SINR of user $k$, and

$$\delta_{k,l} = \frac{p_{k,l}}{\text{SINR}_{k,l}^{\text{SF}'}}, \quad (8)$$

The interference function $I_{k,l}(p)$ can be shown to be standard, i.e., interference function satisfies monotonicity and scalability properties, following a similar approach in [26]. As mentioned in Section I, satisfying SINR duality, i.e., carrying a two-way joint design approach, is nontrivial in ICs. Presumably owing to this fact, the proposed algorithm in [26] is a one-way but joint power control and beamforming design, viz. power control is applied in only downlink direction after each beamforming design step. Finally, the new SINR definition introduced in this section will play an important role in Section VII for proving the RoC of the proposed algorithm.

D. Proposed Algorithm

Before presenting our algorithm, we first review some possible approaches to achieve sub-stream fairness. For joint power control and beamforming design to achieve sub-stream fairness, the problem can be formulated as

$$\begin{align*}
&\max_{u_k, v_k, p_k} \min_{l} \text{SINR}_{k,l}^{\text{SF}'} \\
&\text{subject to } \sum_{l=1}^{d_k} p_{k,l} \leq p_k, \forall k \in K \text{ and } \forall l \in L.
\end{align*}$$

Compared with stream fairness problems, the sub-stream problems are decoupled into $K$ sub-problems given SINR targets are feasible, thus the above problem is solved asynchronously among users. However, feasibility check is coupled among users and can be shown to be NP-hard [5]. Therefore, we focus on designing efficient algorithms for achieving locally optimal points. Please note that the cyclic coordinate ascent algorithm presented in [5] can be modified to achieve sub-stream fairness. This method is a fair benchmark to our proposed algorithm since it can also be applied in a distributed manner, but its complexity is higher than our simplistic algorithm. A centralized approach can be maximization of sum of all stream SINRs while SINR target per sub-stream and power constraint per user are met. Developing more advanced schemes for sub-stream fairness is our next research direction.

As well known, and mentioned before, locally optimal solutions are obtained when SINR constraints are active in
other words when SINR constraints are satisfied with equality. Moreover, without the min operator, the problem (10) is convex over transmit or receive filters and a closed-form solution exists [8], but the problem is not jointly convex over all beamforming matrices. Motivated by these results, the problem (2) can be divided into two sub-problems. Beamforming vectors can be first obtained via conventional schemes including max-SINR and DIA [8]. Then in the second sub-problem where beamforming vectors are fixed, and by applying active SINR constraints, power vectors can be obtained

$$\max_{p_k} \Gamma_k$$

subject to

$$\mathrm{SINR}_{k,l} = \Gamma_k \sum_{l=1}^{d_k} p_{k,l} \leq p_k, \forall k \in \mathcal{K} \text{ and } \forall l \in \mathcal{L}.$$ (10)

Given feasible SINR targets, the above problem (10) is still nontrivial [21]. After shedding light on current state of the problem, and addressing open points, we propose a practical scheme, named ad-hoc DPCA, to achieve sub-stream fairness. Basically, we unite the simple linear search for finding maximum possible SINR targets with the minimization of power subject to SINR constraint problem

$$\min_{p_k} \text{subject to } \mathrm{SINR}_{k,l} \geq \Gamma_k, \forall k \in \mathcal{K} \text{ and } \forall l \in \mathcal{L}.$$ (10)

that can be solved via conventional DPCA.

The proposed ad-hoc DPCA in Algorithm 1 opportunistically searches a feasible SINR target for each user. The algorithm can run asynchronously among users. In Algorithm 1 $p_k^n = [p_{k,1}^n, \ldots, p_{k,d_k}^n]$ is the power vector and $p_{k,l}^n$ is the power for the $l$th stream of user $k$ at iteration $n$, $p_k^n = \sum_{l=1}^{d_k} p_{k,l}^n$, $B_k^{-1} = Q_k^{-1} + I_{N_k}$ and $Q_k^{-1} = \sum_{j=1,j\neq k}^{K} H_{k,j} U_j^H P_j^{n-1} U_j H_{k,j}^T$ are interference plus noise and interference covariance matrices, respectively, $R_{k,l}^n = H_{k,l} u_k u_k^H H_{k,l}$ is akin to a covariance matrix, $P_{j,l}^{n-1} = \text{diag}[p_{j,1}^{n-1}, \ldots, p_{j,d_j}^{n-1}]$ is a diagonal matrix of sub-stream powers, $1 = \{1, \ldots, 1\}$ is all ones vector, and $\delta_k = [\delta_{k,1}, \ldots, \delta_{k,d_k}]$ is the vector of terms defined in (9), respectively.

The outer while loop in Algorithm 1 searches a feasible SINR target for each user. Since there is a maximum power constraint, the optimal power values may not be feasible if SINRs are not well balanced before power control applied. For example consider the system $\text{SINR}_{\text{sys}} = \{\{2,6\}, \{2,4\}, \{1,15\}\}$. Clearly, the sub-stream SINRs of the last user $k = 3$ are not well balanced. The optimal power values may not be feasible to achieve fairness between sub-streams, i.e., $\text{SINR}_{\text{sys,sub-str, opt}} = \{\{4,4\}, \{3,3\}, \{8,8\}\}$ may not be achieved, but $\text{SINR}_{\text{sys,sub-str}} = \{\{4,4\}, \{3,3\}, \{5,5\}\}$ may be achieved instead. The step 11 of the algorithm is the most critical part where powers of sub-streams are updated in order from the sub-stream with the lowest to the highest $\delta_{k,l}$. This way the sub-stream with the lowest $\delta_{k,l}$ can definitely reach the SINR target, while the sub-stream with the highest $\delta_{k,l}$ reaches to a maximum possible SINR value with the remaining power budget of the user. In the next iteration, the target SINR is the average of these achieved SINRs, thus the algorithm keeps iterating until the convergence of sub-stream SINRs. The convergence plot of the proposed algorithm is given in [7]. As shown in Section VI, our proposed algorithm has a fast convergence rate, and each iteration has small costs [7].

IV. ALGORITHMIC PARAMETERS

In this section, we briefly introduce some important algorithmic parameters that significantly affect the results and our perceptions in benchmarking. Different algorithms have different responses to algorithmic parameters. Therefore carefully scanning these parameters is important to benchmark entirely. For example, the sum-rate gap between max-SINR and DIA in low to medium SNR regime can be emphasized or both can be asserted as not achieving sub-stream fairness when screening of parameters shortfall. As shown in the previous section, iteration number is the critical factor for the later. For the former, the number of initializations of random transmit beamforming vectors is the critical parameter. Numerical results show that if more initializations are allowed, the sum-rate gap in the low to medium SNR regime between DIA and max-SINR is reduced. This can indicate that local sum-rate optimal points of DIA are more inhomogeneously distributed than those of max-SINR, thus more initializations can increase the chances of finding a better local optimum for DIA.

The number of iterations required for an algorithm to converge is coupled with the algorithm’s RoC as mentioned in Section III. RoC is shown to depend on SNR in [30,31], and it also depends on whether the algorithm provides fair sub-streams or not. If the algorithm cannot provide fair sub-streams, an utmost example can be some streams are completely shut off, as seen in DIA and max-SINR examples in previous sections, the algorithm can require less number of iterations in general. Conversely, the algorithm requires more iterations, i.e., the algorithm’s RoC is slower, when for example the system size increases, e.g., the numbers of

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**Algorithm 1 Ad-Hoc DPCA**

1. Evaluate SINR outcomes of a beamforming scheme, $\text{SINR}_{k,l}$
2. initialize $\text{SINR}_{k,l} = \text{SINR}_{k,l}^i, \forall k \in \mathcal{K}, \forall l \in \mathcal{L}$
3. check=0
4. while check<=1 do
5. $p_k^n = \frac{d_k}{\delta_k}$, $p_k^1 = 2p_k^n$, $\Gamma_k = \overline{\text{SINR}}_k, \forall k \in \mathcal{K}$
6. $n = 1/k$
7. while $\sum_{k,l} |\text{SINR}_{k,l}^n - \text{SINR}_{k,l}^{n-1}| < \epsilon$ do
8. $\delta_{k,l} = \frac{\sum_{k,l} \text{SINR}_{k,l}^n}{\sum_{k,l} \text{SINR}_{k,l}^{n-1}}$, $\forall k \in \mathcal{K}, \forall l \in \mathcal{L}$
9. $x = 2 \max(\delta_{k,l})$, $p_k^j = 0$, $\forall k \in \mathcal{K}$
10. for counter=1:6, do, $\forall k \in \mathcal{K}$
11. $[\rho, y] = \min(\delta_{k,l})$
12. $p_k^j = \min(\Gamma_k \delta_{k,y}, p_k - p_k^j)$
13. $p_k^j = p_k^j + p_k^y$, $\delta_{k,y} = x$
14. end for
15. $n = n + 1$
16. end while
17. Evaluate new SINRs $\text{SINR}_{k,l}^{'n}$ by using new power values $p_k^n, \forall k \in \mathcal{K}$
18. if $\sum_{k,l=1}^{K} |\text{SINR}_{k,l} - \text{SINR}_{k,l}^{'n}| \leq \epsilon$ then
19. check=1
20. end if
21. end while
users and fair sub-streams are increased. Sum-rate results
for different $\epsilon$ values of the stopping criterion (1) are given
in Table II for MC=20. For each scheme, the results in
the first and second rows are for $\epsilon = 10^{-6}$ and $10^{-2}$,
respectively. In the iteration number column of the table,
the iteration number of the last MC trial for SNR 40 dB
is given since in general high SNR regime requires more
iterations. In the (normalized) convergence speed column
of the table, the sum-rate difference normalized by the iteration
number difference, e.g., $\frac{72.2 - 59.6}{358 - 147}$, is given. From slower to
to faster convergence speed, min-sum-MSE, max-SINR without,
max-SINR with orthogonalization step, and DIA come in
order. Basically, schemes that provide sub-stream fairness tend
to be slower as in the cases of min-sum-MSE and max-SINR
without the orthogonalization step. The convergence speed
of DIA is still high, although for $\text{Iter}=0$ it achieves quite fair
sub-streams as mentioned before. Max-SINR is not a true
SINR maximizer as shown in the next section, thus max-SINR
and min-sum-MSE can have slow convergence speeds for
sum-rate maximization objective. Whereas DIA aims for the
minimization of interference, a more influential objective at
high SNR, thus it can have a fast convergence speed for
sum-rate maximization.

Max-SINR without the orthogonalization step can achieve
reasonable sub-stream fairness in low to medium SNR regime.
However, at high SNR along with low preset iteration number,
fairness cannot be achieved due to the inherent competition
between sub-streams. On the other hand, with the orthogo-
nalization step, max-SINR cannot achieve sub-stream fairness
as shown in [7] and in Section V of this paper. Some
flows achieve poor SINR levels although their contribu-
tions are insignificant. Therefore, without observing
streams with low SINRs thus with high BERs, the poor
sum-rate performance of max-SINR in the high SNR regime
seems to be improved by the orthogonalization step.

In Fig. 3 the sum-rate results of max-SINR algorithm for
different cases are presented for MC=20 and $\text{Iter}=1000$. In
the legend, we use $+$ and $-$ to indicate whether the feature
exists or not, respectively. For example, the $(\text{QR}+,\text{PC})$ legend
denotes the max-SINR scheme with orthonormal beamforming
vectors but without the power control (PC) algorithm. A simple
QR decomposition can be used to obtain orthogonal
beamforming vectors. As seen in the figure, orthogonalization
step seems to fix the high SNR region problem of conventional
max-SINR algorithm that has no power control. However, as
will be shown in Section V max-SINR without power control
generates sub-streams with low SINRs, thus with high BERs.
In the following sections, we use max-SINR algorithm with
orthogonal beamforming vectors since the RoC of max-SINR
without orthogonalization step is much slower.

![Fig. 3. Sum-rates of max-SINR for different cases.](image)

Finally, every detail in numerical results can be consid-
ered as algorithmic parameter, ranging from how sum-rate is
calculated, e.g., Shannon’s or sum of stream rates formula
is used, to MC number. As well-known, MC number, i.e.,
the total number of tested channels that are independently
generated, is another founding parameter to exhibit numeri-
cal correctness. A low set MC number for plotting ergodic
sum-rate and sum-SINR can give a false impression especially
for sub-streams as seen in Fig. 6 of this paper and in Fig. 4
of [7]. While running BER simulations that require high MC
tests, we evaluate the SINR values as well, and present the
results in Section V. Ultimately the ergodic rate and SINR
results are found to be more accurate than those simulations
with low MC number.

V. Numerical Results

For all algorithms except min-sum-MSE [20–34], we assume
the same type of filter structures are used at the transmitters
and receivers, e.g., max-SINR filter (4) is used for both
transmit and receive filters. In [20–34], authors show that
MMSE receive filter is optimal for the given transmit filter,
and obtain optimal transmit filter via Lagrangian solution.
On the other hand, our approach is useful, for example, for sensor
network transceivers. In addition, our approach isolates the
problem on the considered filter, e.g., max-SINR filter. For
better scrutinizing the rate and SINR results of the algorithms,
we save randomly initiated beamforming vectors and channels
into files, and feed the same files into benchmarked algorithms.
Since BER results require abundant number of random chan-
nels to be tested, which increases the file sizes thus run time,
we do not follow the same approach for BER results.

A. Targeting Sum-SINR

Max-SINR results that are presented in the literature are
generally obtained via targeting sum-rate maximization, e.g.,
algorithms stop based on condition (1), as opposed to a

| dB  | 0   | 10  | 20  | 30  | 40  | Iter. | Conv. |
|-----|-----|-----|-----|-----|-----|-------|-------|
| $\text{DIA}$ | 4.3 | 15.4 | 32.2 | 52.1 | 72.2 | 358   | 0.0602 |
|     | 3.7 | 13.6 | 27.1 | 38.2 | 59.6 | 147   |       |
| $\text{Max-SINR w/o}$ | 9.3 | 21.8 | 37   | 53.3 | 75.4 | 340   | 0.0624 |
| $\text{Max-SINR w/}$ | 9.3 | 21.2 | 34.6 | 45.9 | 63.3 | 146   |       |
| $\text{Min-sum-MSE}$ | 9.4 | 22  | 37.2 | 51.5 | 71.6 | 12880 | 0.0015 |
|     | 9.3 | 21.5 | 34.7 | 44.6 | 52.4 | 50    |       |
|     | 9.3 | 21.4 | 36.2 | 51.2 | 64.2 | 21153 | 0.0006 |
TABLE III
SUM-SINRs NORMALIZED BY GEVD RESULTS.

| dB  | 0  | 10 | 20 | 30 | 40 | 50 | 60 |
|-----|----|----|----|----|----|----|----|
| DIA | 0.74 | 1.43 | 1.68 | 1.84 | 1.82 | 1.88 | 1.91 |
| Max-SINR | 1.01 | 1.05 | 1.07 | 1.05 | 1.05 | 1.10 | 1.08 |
| GEVD | 0.99 | 1.00 | 1.01 | 1.00 | 1.00 | 0.98 | 1.01 |

The possible perception that the objective is sum-SINR maximization. Since there is no one-to-one correspondence between sum-SINR and sum-rate metrics, beamforming optimization based on sum-SINR maximization yields lower sum-rate, and vice versa. As mentioned before, improving the worst stream SINR in the system improves BER of the system. Therefore, the stopping criterion

$$|\text{SINR}_{\text{sum}}(n+1) - \text{SINR}_{\text{sum}}(n)| \leq \epsilon, \quad (11)$$

where $\text{SINR}_{\text{sum}} = \sum_{k=1}^{K} \sum_{l=1}^{d_k} \text{SINR}_{k,l}$ is the sum of SINRs, is also plausible. In Table III sum-SINR results are presented. The starred and unstarred algorithm denotes the algorithm with sum-SINR and sum-rate maximization objective, in other words with the stopping criterion (11) and (1), respectively. The results are normalized with the unstarred GEVD results. Although the starred max-SINR algorithm (max-SINR*) truly aims for SINR maximization, starred DIA (DIA*) achieves higher sum-SINR. Interestingly, we observe that the received signal power of DIA* is more than max-SINR* as opposed to expected. On the other hand, interference signal power of DIA* is slightly lesser than max-SINR*. This is another indicator that max-SINR scheme is far from being an optimal SINR maximizer.

B. BER Results

Bit error, sum-rate and SINR results of max-SINR with and without the proposed power control in Algorithm 1 for the system $(4 \times 4, 2)$ are presented in Fig. 4-6 and BER results for $(6 \times 6, 3)$ is presented in Fig. 5. To plot BER results in the paper, MC= $10^3, 10^4, 10^5$, and $10^6$ random ICs are tested for SNR values 0, 5, 10, and 15 dB, respectively. Channel coefficients are generated by i.i.d. zero-mean unit-variance complex Gaussian variables, QPSK modulation is used, and Iter=16 is chosen. As seen in Fig. 4(a), the proposed ad-hoc DPCA whose objective is sub-stream fairness can achieve stream fairness in ergodic sense with lesser algorithmic complexity and information exchange than the algorithms whose objectives are stream fairness. As seen in Fig. 5 sub-stream fairness is achieved at the cost of a reasonable sum-rate degradation.

BER results of max-SINR with and without power control are given in Fig. 6. Clearly, error rates in Fig. 6 are coherent with SINR results in Fig. 4. As seen in Fig. 6(b) for the given number of trials, two streams have no bit errors at SNR 15 dB, thus not plotted. To achieve leveled BERs in Fig. 6(a) similar to leveled SINRs in Fig. 4(a) many more MC simulations needed even for SNR 0 dB. Due to lengthy simulation times, we avoid such high MC numbers.

In Fig. 7 standard deviation of bit errors per stream is given. Basically, total number of bit errors per stream is obtained, and then standard deviation of these numbers is evaluated. Since more channels are tested as SNR increases, total number of errors per stream, thus standard deviation increases. However, Fig. 7 clearly shows the success of DPCA on keeping fairness in the system. In Table IV total and average number of bit errors are explicitly given at sub-stream and system levels. In the table, $BE_k = \{BE_{k,1}, BE_{k,2}\}$ denotes the total number of bit errors per stream of user $k$, e-$x$ denotes $10^{-x}$, which is a multiplicative factor, e.g., $\{8.035, 7.5\} (e-3, e-5) \approx \{8.035 \times 10^{-3}, 7.5 \times 10^{-5}\}$. From these results, we see that DPCA enforces bit errors to be distributed more homogeneously among the streams.

As the number of sub-streams increased, the imbalance between them increases, thus power control to achieve sub-stream fairness becomes more imperative as seen in Fig.
V. RATE OF CONVERGENCE

The proposed ad-hoc DPCA increases and decreases the algorithmic complexity and sum-rate with marginal gaps, respectively, while garnering sub-stream fairness thus improving BER. As shown before, our algorithm guarantees feasible SINR targets via linear search. Moreover, the power control law used in the algorithm converges to a unique fixed-point at a linear rate. For completeness, we review important definitions before presenting the proof, and begin with the definition of linear convergence. A sequence \( \{ x^n \} \in \mathbb{R}^N \) converges to \( x^* \)

\[
\lim_{n \to \infty} \frac{||x^n - x^*||}{||x^{n-1} - x^*||} = C,
\]

where \( ||\cdot|| \) is some norm defined in \( \mathbb{R}^N \). In a similar manner, for every initial vector \( x^0 \), the sequence \( \{ x^n \} \) generated by an iterative algorithm converges to \( x^* \) at a linear rate if

\[
||x^n - x^*|| \leq c^n||x^0 - x^*||
\]

is satisfied, and \( c \in [0,1) \). In other words, the distance \( ||x^n - x^*|| \) is always lesser than \( c^n||x^0 - x^*|| \), and decays exponentially.

| Level | Sub-stream | System |
|-------|------------|--------|
| 10 dB | Total: \( \{ 472.525, 472.522, 434.504 \} \) | 2929 |
|       | Av: \( \{ 2.36, 2.63, 2.17, 2.52 \} \) | 2.44e-3 |
|       | Total: \( \{ 1607.15, 1544.12, 1612.6 \} \) | 4796 |
|       | Av: \( \{ 8.04, 7.26, 8.06 \} \) | 4e-3 |
| 15 dB | Total: \( \{ 440.894, 447.911, 440.894 \} \) | 4026 |
|       | Av: \( \{ 2.24, 2.47, 2.24, 2.47 \} \) | 3.36e-4 |
|       | Total: \( \{ 2593.0, 2585.0, 2629.1 \} \) | 7808 |
|       | Av: \( \{ 1.3, 1.29, 1.31 \} \) | 6.51e-4 |
In [23], authors introduce contractive interference functions that do not need separate proof for the existence of fixed-points and that also give estimates on the convergence rates of algorithms. These features are not present in standard interference functions introduced in [9]. Contractive interference functions are obtained by slightly reformulating the last condition of standard interference functions, scalability,

\[ \forall \alpha > 1, \alpha I(p) > I(\alpha p), \]

while keeping the first two conditions, positivity and monotonicity, same. The last condition of contractive interference functions, contractivity, in [23] is given as

\[ \text{contractivity: There exists a constant } c \in [0, 1) \text{ and a vector } v > 0 \text{ such that } \forall \epsilon > 0, I(p) + \epsilon v \geq I(p + \epsilon v). \]

In [23], it is proven for contractive interference functions that for any initial power vector \( p^0 \), the sequence \( p^n = I(p^{n-1}) \) converges linearly to \( p^* \)

\[ ||p^n - p^*||_{\infty}^\gamma \leq c^n ||p^0 - p^*||_{\infty}^\gamma, \]

where \( ||.||_{\infty} \) denotes weighted maximum norm for a given vector \( v > 0 \), i.e., \( ||x||_{\infty} = \max_j |x_j|/v_j \). The reader is referred to [23] for further details. Next we show that interference function (7) is contractive.

The interference function (7) can be rewritten as

\[ I_{k,l}(p) = \sum_{d=1}^{d_j} \sum_{j=1}^{K} T_{k,j,d}p_{j,d} + N_k, \]

where \( N_k = \frac{1}{\alpha_k} \),

\[ T_{k,j,d} = \begin{cases} \frac{G_{k,j,d}}{\epsilon_k} & \text{if } j \neq k, \\ 0 & \text{if } j = k, \end{cases} \]

\[ G_{k,j,d} = |v^+_{k,j}H_{k,j}u_{j,d}|^2, \text{ and } \]

\[ G_{k,k} = |v^+_{k,k}H_{k,k}u_{k,k}|^2. \]

Henceforth, the contractivity condition is satisfied

\[ I_{k,l}(p + \epsilon v) = I_{k,l}(p) + \epsilon \sum_{d=1}^{d_j} \sum_{j=1}^{K} T_{k,j,d}v_j \\ \leq I_{k,l}(p) + \epsilon \sum_{d=1}^{d_j} ||T_d||_{\infty}^\gamma v_k, \]

where \( T_d \) is a \( K \times K \) matrix with entries in (12). Thus, (7) is a \( c \)-contractive interference function with \( c = \sum_{d=1}^{d_j} ||T_d||_{\infty}^\gamma \). The extension of the above proof to interference function with min operator in Algorithm [1] is straightforward. In Fig. 9, the distance to a fixed-point \( ||p^n - p^*||_{\infty} \), the vector \( v \) is chosen as all ones vector (\( v = 1 \)) thus omitted in notation, is plotted for a channel realization at SNR 30 dB.

**VII. CONCLUSION**

We developed an ad-hoc DPCA that achieves sub-stream fairness at the cost of a slightly increased algorithmic load. The instantaneous sum-rate degradation of the proposed algorithm is lesser since sub-stream fairness poses milder conditions than stream fairness. The proposed algorithm can achieve stream fairness in the ergodic sense as well. The algorithm guarantees feasible SINR targets via linear search. As opposed to common approach, the algorithm does not require preset SINR targets, and optimizes the targets. In the paper, the impacts of sub-stream fairness on BER results and algorithmic parameters on benchmarking processes are illustrated. Finally, via contractive interference functions, the power control law in the proposed algorithm is proven to linearly converge to a unique fixed-point.

In addition to the future research directions already pointed in the paper, theoretical modeling of numerical BER results presented in the paper is another important research direction. Finally, as the system size increases, e.g., increasing number of users and sub-streams, achieving theoretically promised results in practice becomes a challenging research problem, especially in the high SNR regime since by and large, algorithms require more iterations in this regime. Therefore, developing fast converging beamforming algorithms is consequential.

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