Klein Tunneling through Double Barrier in ABC-Trilayer Graphene

Abderrahim El Mouhafid,* Ahmed Jellal,* and Miloud Mekkaoui*

Klein tunneling and conductance for Dirac fermions in ABC-stacked trilayer graphene through symmetric and asymmetric double potential barrier are investigated. This was done by using the continuum model of two and six-bands. The numerical results show that the transport is sensitive to the height, the width, and the distance between the two barriers. It is found that the Klein paradox at normal incidence ($k_y = 0$) and resonant features at $k_y \neq 0$ in the transmission result from resonant electron states in the wells or hole states in the barriers. It is shown that such features strongly influence the ballistic conductance of the structures.

1. Introduction

Generally, graphene[1–4] is a 2D lattice of carbon atoms arranged in hexagonal geometry. Its stacking can be realized in different methods to engineer multi-layered graphene showing various physical properties. Typical examples of stacking includes order, Bernal (AB), and rhombohedral stacking (ABC).[5–10] In the first all carbon atoms of each layer are well-aligned, while the second and third have cycle periods composed, respectively, of two and three layers of non-aligned graphene. It was shown that the band structure, Klein tunneling, band gap, transport and optical properties of graphene depend on the way how its layers are stacked[5,9,11–22] and the applied external sources.[5,9,19,23–33]

Recently, trilayer graphene (TLG) has attracted more attention.[15,19,26,29,31,34–49] It has two distinct allotropes: the Bernal (ABA) and rhombohedral (ABC) stackings. ABA has atoms of the top layer lie exactly on top of the bottom layer. It possesses a dispersion relation as summation of the linear and quadratic dispersions corresponding to the single layer and bilayer, respectively. It has no opening gap under applied external electric field.[50] As for ABC, the atoms of one of the sublattices of topmost layer lie above the center of the hexagons of bottom layer.

ABC shows a dispersion relation approximately cubic with conduction and valence bands touching each other at a point close to the highly symmetric $K$ and $K'$ points.[5,35] Consequently, the way in which the layers are stacked modifies strongly the energy spectrum and transport properties of the resulted system. Moreover, for TLG it was shown that the emergence of a Klein tunneling strongly depends on the stacking order, being present only in ABC-TLG.[19,26,27] Experimentally, the transport properties of ABC- and ABA-TLG using dual locally gated field effect devices were investigated[37] with the observation of an opening gap. The giant conductance oscillations in ballistic TLG Fabry-Pérot interferometers was realized.[51] This resulted from the phase coherent transport through resonant bound states subjected to an electrostatic barrier.

Motivated by different achievements listed above, especially in ref. [19, 26, 27], we study the Klein tunneling of Dirac fermions in ABC-TLG scattered by a square double barrier. We stress that our contribution is a novel one, that nobody has investigated similar or the same problem to the best of our knowledge. More precisely, the transmission probabilities and conductance of electrons will be investigated by taking into account the full six bands energy spectrum. We analyze two interesting cases by making comparison between the incident energy $E$ and interlayer coupling parameter $\gamma_1$. Indeed, for $E < \gamma_1$, there is only one channel of transmission exhibiting resonances, even for incident particles of energy $E$ less than the barrier heights $V_j$, that is $E < V_j$, depending on the double barrier profile. For $E > \gamma_1$, we end up with three propagating modes resulted from nine possible ways of transmission. Subsequently, by using the transfer matrix method and current density we determine all transmission channels together with the conductance. Under various conditions, we numerically analyze and compare our results with literature.

The outline of the present paper is as follows. In Section 2, we establish a mathematical framework using the full band model to determine the eigenvalues and eigenvectors through double barrier. In Section 3, by matching the eigenspinors of different regions at interfaces and using the transfer matrix method together with the current density, we obtain nine channels of transmission and reflections as well as the associated conductance. We study two band tunneling for symmetric and asymmetric barrier with and without the interlayer potential difference. At high energy, we repeat the same task but by considering full band and emphasize what makes difference with respect to other case. In Section 4, we numerically study the basic features of the resulted conductance.
and show the effect of symmetric and asymmetric barrier. Finally, we briefly summarize our main findings.

2. Theoretical Formalism

2.1. Eigenvalues and Eigenvectors

Single layer graphene (SLG) has a hexagonal crystal structure of two sublattices A and B with the interatomic distance $a = 0.142$ nm$^{[52]}$ and the intra-layer coupling $\gamma_0 \approx 3$ eV$^{[33]}$. While TLG is a three stacked SLG (rhombohedral stacking) with an unit cell of six atoms.$^{[54]}$ Under the nearest-neighbor tight binding approximation, one can derive the Hamiltonian describing ABC-TLG near the K point$^{[35]}$

$$H = \hbar v_F \begin{pmatrix} \sigma \cdot k & \Gamma & 0 \\ \Gamma & \sigma \cdot k & \Gamma \\ 0 & \Gamma & \sigma \cdot k \end{pmatrix}$$

(1)

in the basis of the atomic orbital eigenfunctions

$$\Psi = (\psi_{A_1}, \psi_{A_2}, \psi_{A_3}, \psi_{B_1}, \psi_{B_2})^T$$

(2)

with $\sigma = (\sigma_z, \sigma_y)$ a vector of Pauli matrices, the Fermi velocity $v_F = 3\hbar \gamma_0 / (2\hbar)$ and the wave vector $k$. The interlayer coupling is

$$\Gamma = \frac{1}{\hbar v_F} \begin{pmatrix} 0 & 0 \\ 0 & \gamma_1 \\ 0 & 0 \end{pmatrix}$$

(3)

and $\gamma_1 = 0.4$ eV is the nearest neighbor coupling term between adjacent layers. Based on the profile of double barrier, see Figure 1a, we label all regions by $j = I (x \leq 0), j = II (0 < x \leq a), j = III (a < x \leq b), j = IV (b < x \leq c)$ and $j = V (x > c)$, with $a = b_1 + b_2 + b_3$. Thus, in the $j$th region the Hamiltonian Equation (1) reads as

$$H_j = \begin{pmatrix} V_j + \delta_j & \hbar v_F \pi & 0 & 0 & 0 \\ \hbar v_F \pi & V_j + \delta_j & 0 & 0 & 0 \\ 0 & 0 & V_j & 0 & 0 \\ 0 & 0 & 0 & V_j & 0 \\ 0 & 0 & 0 & 0 & V_j - \delta_j \end{pmatrix}$$

(4)

where $\pi = p_x + ip_y$, and $\pi^T = p_x - ip_y$ are the in-plane momenta, with $p_{x,y} = -i\hbar \partial_{x,y}$, $V_j$ is the electrostatic potential, and $\delta_j$ is the interlayer potential difference between top and bottom layers. In regions I, III, and V we have $V_j = \delta_j = 0$ but non-null in regions II and IV. Since the Hamiltonian Equation(4) commutes with the momentum $p_j$, then we can separate the eigenspinors as

$$\psi^j(x, y) = e^{i\phi_j} [\phi_{A_1}^j, \phi_{A_2}^j, \phi_{A_3}^j, \phi_{B_1}^j, \phi_{B_2}^j]^T$$

(5)

where $T$ stands for the transpose of the row vector. According to Figure 1a, we have basically two different sectors with zero (I, III, V) and nonzero (II, IV) potentials. Consequently, we derive a general solution in the second sector and require $V_j = \delta_j = 0$ to find that for the first one. To simplify our notation, we introduce the length scale $l = \frac{\hbar v_F}{eV} \approx 1.64$ nm, which represents the interlayer coupling length, and define $E_j = \frac{k}{\gamma_1}, V_j = \frac{V_j}{\gamma_1}, \delta_j = \frac{\delta_j}{\gamma_1}, k_j \rightarrow lk_j, r \rightarrow \frac{r}{l}$. We determine the solutions of the energy spectrum by solving the eigenvalue equation $H_j \psi^j = E^j \psi^j$. As for Equations (4)–(5) we obtain six coupled differential equations, given by

$$-i(\partial_\sigma + k_j)\phi_{A_1}^j = (e_j - \delta_j)\phi_{A_1}^j$$

(6a)

$$-i(\partial_\sigma - k_j)\phi_{A_1}^j = (e_j - \delta_j)\phi_{A_1}^j$$

(6b)
\[-i(\partial_x + k_j)\phi_j^i = \epsilon_j \phi_{\lambda_1}^i - \phi_{\lambda_2}^i \] (6c)

where \(a_n\) and \(b_n\) \((n = 1, 2, 3)\) are constants of normalization. The wave vector \(k_j\) along the \(x\)-direction for \(j\)th region is solution of the cubic equation

\[\left(k_j^3 + k_j^2\right)^3 - \left(k_j^2 + k_j^2\right)^2 + \left(k_j^2 + k_j^2\right) - h_j^1 = 0 \quad (12)\]

where we have defined the quantities

\[h_j^1 = (a'_j)^2 + (a'_j)^2 + (a'_j)^2 \quad (13a)\]

\[h_j^2 = (a'_j a'_j a'_j)^2 + (a'_j a'_j)^2 - a'_j(a'_j + a'_j) \quad (13b)\]

\[h_j^3 = (a'_j a'_j a'_j)^2 - a'_j a'_j a'_j(a'_j + a'_j) + a'_j a'_j \quad (13c)\]

with \(\epsilon_j = E_j - V_j\) and the wave vector \(k_j\) along the \(y\)-direction. We proceed further by decoupling the above set of equations. Indeed, we express Equations (6a) and (6f) as

\[\phi_{\lambda_1}^i = -i \frac{\partial_x}{\epsilon_j - \delta_j} (\partial_x + k_j) \phi_{\lambda_2}^i \] (7a)

\[\phi_{\lambda_2}^i = -i \frac{\partial_x}{\epsilon_j + \delta_j} (\partial_x - k_j) \phi_{\lambda_1}^i \] (7b)

which can be injected into Equations (6b) and (6e) to end up with two equations

\[\left(\partial_x^2 - k_j^2\right)\phi_{\lambda_2}^i + (\epsilon_j - \delta_j) \phi_{\lambda_1}^i = (\epsilon_j - \delta_j)^2 \phi_{\lambda_2}^i \] (8a)

\[\left(\partial_x^2 - k_j^2\right)\phi_{\lambda_1}^i - (\epsilon_j + \delta_j) \phi_{\lambda_2}^i = -(\epsilon_j + \delta_j)^2 \phi_{\lambda_1}^i \] (8b)

Now we substitute Equations (8a) and (8b), respectively, in Equations (6c) and (6d). After a straightforward algebra and by setting

\[a_j = 3x_j^2 + 2\delta_j \]

\[b_j = (\delta_j + \epsilon_j)(3x_j^2 + 3\delta_j^2 + (\delta_j^2 - 2)\epsilon_j - \delta_j \epsilon_j) \]

\[c_j = (\delta_j - \epsilon_j)(1 + (\delta_j^2 - \epsilon_j)(1 + \delta_j + \epsilon_j)\)

one finds a sixth-order differential equation for \(\phi_{\lambda_1}^i\)

\[
\begin{align*}
&\frac{d^6}{dx^6} + \left(a_j - 3k_j^2\right) \frac{d^4}{dx^4} - \left(2ak_jk_j^2 - 3k_j^2 - b_j\right) \frac{d^2}{dx^2} \\
&- k_j^2 + a_jk_j^2 - b_jk_j^2 + c_j \end{align*}
\]

\[\phi_{\lambda_1}^i = 0 \quad (10)\]

We show that the solution of Equation (10) can be expressed as a linear combination of plane waves, such as

\[\phi_{\lambda_1}^i = \sum_{n=1}^3 (a_n e^{ik_n x} + b_n e^{-ik_n x}) \quad (11)\]

with \(g_n^j = k_{j,n}^j + ik_{j,n}^j, f_n^j = k_{j,n}^j - ik_{j,n}^j, \mu_n^j = (a'_j)^2 - k_{j,n}^j - k_{j,n}^j, \eta_n^j = (a'_j)^2 - k_{j,n}^j - k_{j,n}^j, \rho_n^j = \alpha_j^2 \mu_n^j - \alpha_j^2, \lambda_n^j = \rho_n^j \eta_n^j - \alpha_j^2 \eta_n^j\). The energy spectrum of the Hamiltonian Equation (4) in the different regions is presented in Figure 1b. It consists of six energy bands symmetric at \(E = 0\) of which two touch each other at \(k = 0\). The dashed (solid) curves correspond to the energy of ABC-TLG inside (outside) the barriers. In the presence of the inter-layer bias \(\delta\), we note that the three bands are switched and a band gap is opened between them for the small potential in the case \(V_j = 0\) or for \(V_j = V_j^1\) when \(V_j = V_j^2\). Indeed for \(E < \gamma_1\), we have just one propagating mode that corresponds to the wave vector \(k_j\). As for \(E > \gamma_1\), there are three propagating modes correspond to the wave vectors \(k_j, k_j,\) and \(k_j\) as shown in Figure 1c.

It convenient to write the general solution in each region as

\[\psi(x,y) = Q^j M(x) C^j e^{ik_j y} \quad (15)\]
and the matrices are given by

\[
Q^l = \begin{pmatrix}
\delta_{j1} & \delta_{j1} & \delta_{j2} & \delta_{j3} & \delta_{j3} \\
\alpha_1 & \alpha_1 & \alpha_1 & \alpha_1 & \alpha_1 \\
\alpha_1 & \alpha_1 & \alpha_1 & \alpha_1 & \alpha_1 \\
\alpha_1 & \alpha_1 & \alpha_1 & \alpha_1 & \alpha_1 \\
\alpha_1 & \alpha_1 & \alpha_1 & \alpha_1 & \alpha_1
\end{pmatrix}
\]

\[
M^l = \begin{pmatrix}
e^{ik_1x} & 0 & 0 & 0 & 0 \\
0 & e^{-ik_1x} & 0 & 0 & 0 \\
0 & 0 & e^{ik_2x} & 0 & 0 \\
0 & 0 & 0 & e^{ik_3x} & 0 \\
0 & 0 & 0 & 0 & e^{-ik_3x}
\end{pmatrix}
\]

\[
C = \begin{pmatrix}
a^+_1 \\
b^+_1 \\
a^+_2 \\
b^+_3 \\
a^+_3 \\
b^+_3
\end{pmatrix}
\]

where the subscript \(j\) in Equation (18) refers to the corresponding wave vector and the superscript plus/minus indicates the right/left propagation or evanescent states. We are using the transfer matrix method because we are interested in the components of \(C\). For this purpose, let us specify the spinors in region I

\[
\phi^I_{A1} = \sum_{n=1}^3 \left( \delta_{n1} Q^I_{4,2n-1} e^{ik_1x} + r^n Q^I_{4,2n} e^{-ik_1x} \right)
\]

\[
\phi^I_{B1} = \sum_{n=1}^3 \left( \delta_{n2} Q^I_{4,2n-1} e^{ik_2x} + r^n Q^I_{4,2n} e^{-ik_2x} \right)
\]

\[
\phi^I_{A2} = \sum_{n=1}^3 \left( \delta_{n3} Q^I_{4,2n-1} e^{ik_3x} + r^n Q^I_{4,2n} e^{-ik_3x} \right)
\]

\[
\phi^I_{B2} = \sum_{n=1}^3 \left( \delta_{n1} Q^I_{4,2n-1} e^{ik_1x} + r^n Q^I_{4,2n} e^{-ik_1x} \right)
\]

\[
\phi^V_{A1} = t^I_1 Q^V_4 e^{ik_1x} + t^I_2 Q^V_5 e^{ik_2x} + t^I_3 Q^V_5 e^{ik_3x}
\]

\[
\phi^V_{B1} = t^I_1 Q^V_4 e^{ik_1x} + t^I_2 Q^V_5 e^{ik_2x} + t^I_3 Q^V_5 e^{ik_3x}
\]

\[
\phi^V_{A2} = t^I_1 Q^V_4 e^{ik_1x} + t^I_2 Q^V_5 e^{ik_2x} + t^I_1 Q^V_3 e^{ik_3x}
\]

\[
\phi^V_{B2} = t^I_1 Q^V_4 e^{ik_1x} + t^I_2 Q^V_5 e^{ik_2x} + t^I_3 Q^V_5 e^{ik_3x}
\]

where \(\delta_{i,j} (i = 1, 2, 3)\) is the symbol of Kronecker, \(s = 1, 2, 3\) indicates the propagating modes or evanescent waves, which are characterized by three different wave vectors \(k_1, k_2, k_3\). \(Q^I_{mn}\) are the elements of the matrix in Equation (16).

In regions I, III, and V the potential is null, then we immediately derive the relation

\[
Q^I M^{II}(x) = Q^{III} M^{III}(x) = Q^V M^V(x)
\]

analogue to Equation (15). These results will be used to deal with different issues related to our system. Indeed, we will compute all channels of transmissions and reflections together with the associated conductance.

2.2. Transmission Probabilities and Conductance

To determine the transmission and reflection we use the boundary conditions in addition to the current density. We start from the continuity of the spinors at different interfaces to obtain the coefficients in the incident and reflected regions

\[
C^I = \begin{pmatrix}
\delta_{11} & \delta_{12} & \delta_{13} \\
r^I_1 & r^I_2 & r^I_3 \\
d_{11} & d_{12} & d_{13}
\end{pmatrix}
\]

\[
C^V = \begin{pmatrix}
\delta_{21} & \delta_{22} & \delta_{23} \\
r^V_1 & r^V_2 & r^V_3 \\
d_{21} & d_{22} & d_{23}
\end{pmatrix}
\]
energ), the three propagating modes introduced the current density $\mathbf{J}$ associated to our system. This is

$$\mathbf{J} = v_F \psi \mathbf{a} \psi$$  \hspace{1cm} (28)

where $\mathbf{a}$ is a $6 \times 6$ matrix having three Pauli matrices $\sigma_i$ in diagonal and the remaining elements are nulls. Equation (28) allows to obtain the incident $J_{\text{inc}}$, transmitted $J_{\text{tr}}$, and reflected $J_{\text{ref}}$ current densities. Consequently, we get

$$T = \frac{|J_{\text{tr}}|}{|J_{\text{inc}}|} \quad \text{and} \quad R = \frac{|J_{\text{ref}}|}{|J_{\text{inc}}|}$$  \hspace{1cm} (29)

By using Equation (15), we explicitly determine $T$ and $R$

$$T_i^k = \frac{A_{ij}^k}{A_{ij}^*} |r_i|^2 \quad \text{and} \quad R_i^k = \frac{A_{ij}^*}{A_{ij}} |r_i|^2$$  \hspace{1cm} (30)

such that $A_{ij}^k$ and $A_{ij}^* \epsilon \mathbf{a}$ are elements of the diagonal matrix $\mathbf{A} = Q^* \mathbf{a} Q$ consisting of traceless $2 \times 2$ blocks where each one corresponds to a propagating mode $s$. These expressions can be explained as follows. Since we have six bands, the electrons can be scattered between them and then has to take into account the change in their velocities. With that, we find nine channels in transmission and reflection corresponding to three propagating modes $k_1$, $k_2$, and $k_3$ solutions of the cubic Equation (12).

Figure 1c shows the modes $k_1$, $k_2$, and $k_3$ with their transmission and reflection probabilities through double barrier structure. For $E < \gamma_1$ (low energies), there is only the propagating mode $k_1$, which gives rise to one transmission $T_1^1$ and one reflection $R_1^1$ through the two conduction bands touching at zero energy on the both sides of the double barrier. As for $E > \gamma_1$ (higher energy), the three propagating modes $k_1$, $k_2$, and $k_3$ are present, which leads to nine transmission $T_i^j$, $R_i^j$ ($i = 1, 2, 3$) and reflection $R_i^j$, $R_i^j$ channels, through the six conduction bands. There are three non scattered transmission channels denoted by $T_3^1$, $T_3^2$, $T_3^3$ for the propagation via $k_1$, $k_2$, $k_3$, respectively, in addition to six scattered transmission channels in which the particle enters via one channel and exits via another one. These will be specified as $T_1^1$, $T_2^1$, and $T_3^1$ for scattering from the $k_1$ band to the $k_2$, $k_3$ band to the $k_3$, and from the $k_2$, $k_3$ band to $k_3$, respectively. Also we will adopt the same definition for the reflection channels, that is, $R_1^1$, $R_2^1$, $R_3^1$. The nine channels are schematically depicted in Figure 1c. We note that the transmission $T_{1,2,3}^i$ and reflection $R_{1,2,3}^i$ probabilities obey the following equation

$$\sum_{i=1,2,3} \left( T_{1,2,3}^i + R_{1,2,3}^i \right) = 1$$  \hspace{1cm} (31)

For example, for the lower band in Figure 1c, $T_1^1 + R_1^1 + T_2^1 + R_2^1 = 1$ is well established.

Since we have found transmission probabilities, let us study the associated conductance. For this, we use the Landauer-Büttiker formula$^{[15]}$ and sum over all channels to end up with

$$G(E) = G_0 \frac{L_y}{2\pi} \int_{-\infty}^{\infty} dk_y \sum_{i=1,2,3} T_i^0(E, k_y)$$  \hspace{1cm} (32)

$L_y$ is the width in the $y$-direction of our system and $G_0 = 4e^2/h$ is the conductance unit, the factor four refers to the valley and spin degeneracy in graphene.

The obtained results so far will be numerically analyzed to discuss the basic features of our system and also make link with other published results. To achieve this goal, we consider two different cases in terms of the band tunneling at low and high energies.

3. Band Tunneling Analysis

We will distinguish two interesting cases resulted from our energy spectrum. Each case will be separately treated to underline their relevant properties. Comparisons with published works$^{[28,29,32,33]}$ will be established.

3.1. Tunneling at Low Energy

Figure 2 shows the contour plots of the transmission probability $T_i^1$ corresponds to the propagation from the channel $k_i$ in region I to the channel $k_i$ in region V at low energy as a function of the wave vector $k_i$ of the incident energy $E$ under various conditions. We address to the two different cases: i) the two barriers have the same height $V_2 = V_4$ (symmetric double barrier structure); and ii) different height such that $V_2 < V_4$ (asymmetric double barrier structure). In addition, we will consider the cases of without and with interlayer potential. More precisely, for (i) we choose $V_2 = V_4 = 0.4\gamma_1$ with $\delta_2 = \delta_4 = 0$ in Figure 2a and $\delta_2 = \delta_4 = 0.2\gamma_1$ in Figure 2b. Regarding (ii) we fix $V_2 = 0.4\gamma_1$ and $V_4 = 0.6\gamma_1$ such that $\delta_2 = \delta_4 = 0$ in Figure 2c and $\delta_2 = \delta_4 = 0.2\gamma_1$ in Figure 2d. It is interesting to note that the Van Duppen et al. results$^{[29]}$ can be derived from our analysis by considering the case (i) and requiring $b = c$ in our double barrier structure. As shown in Figure 2a,c for a normal incidence ($k_y = 0$) and
Figure 2. (Color online) Density plot of transmission probability $T_1$, for $\delta_2 = \delta_4 = 0$ (a,c) and for $\delta_2 = \delta_4 = 0.2 \gamma_1$ (b,d), versus the incident energy $E$ and the wave vector $k_y$ for $V_2 = V_4 = 0.4 \gamma_1$ (a,b) and for $V_2 = 0.4 \gamma_1$, $V_4 = 0.6 \gamma_1$ (c,d), with $b_1 = b_2 = \Delta = 10$ nm. The dashed white lines show the band inside the barrier whereas the black lines represent the band outside the barrier.

Figure 3. (Color online) (Top row) Density plot of the transmission probability $T_1$ for $V_2 = V_4 = 0.6 \gamma_1$, $E = \frac{4}{5} V_2$ and for the gap $\delta_2 = \delta_4 = 0$ versus a) $k_y$ and the width of the two barriers ($b_1 = b_2 = l$) and $\Delta = 10$ nm, b) $k_y$ and $\Delta$ for $b_1 = b_2 = 10$ nm, c) $k_y$ and $b_2$ with $b_1 = 5$ nm and $\Delta = 10$ nm. (Bottom row) The same as top row but for the gap $\delta_2 = \delta_4 = 0.1 \gamma_1$.

$\delta_2 = \delta_4 = 0$ the transmission is unit. Also it becomes independent of the incident energy or the barrier structure (symmetric or asymmetric), which is the same for the single barrier case. This is a manifestation of the Klein tunneling that is resulted from the conservation of pseudospin and occurs for rhombohedral stacks multilayers with an odd number of layers. As for $\delta_2 = \delta_4 = 0.2 \gamma_1$, we present Figure 2b with $V_2 = V_4 = 0.4 \gamma_1$ and Figure 2d with $V_2 = 0.4 \gamma_1$, $V_4 = 0.6 \gamma_1$. For single barrier, in AB-BLG and ABC-TLG, there is no resonance inside the induced gap in contrary to the double barrier as clearly seen in Figure 2b,d. Without $\delta_2$, $\delta_4$, and for $k_y \neq 0$ we still have a full transmission (very narrow resonances), even for $E < V_j$, which are symmetric in $k_y$. Such resonances get reduced and even disappear in Figure 2b,d due to the asymmetric structure of double barrier. As a result, the asymmetric structure of double barrier reduces these resonances resulted from the bound electrons in the well between the two barriers, which are similar to those obtained for AB-BLG.

Figure 3 presents the density plot of the transmission probability as a function of the wave vector $k_y$ and the width of two barriers for $V_2 = V_4 = 0.6 \gamma_1$, and $E = \frac{4}{5} V_2$, $\delta_2 = \delta_4 = 0$ and $\delta_2 = \delta_4 = 0.1 \gamma_1$, in top and bottom rows, respectively. For $k_y = 0$ we have a full transmission regardless of thickness $b_1$ and $b_2$, of the two barriers or distance $\Delta$ between them. In contrary, with increasing $l$, $\Delta$, and $b_2$, the transmission probability dramatically decreases, but some resonances still show up as depicted in Figure 3a–c. The transmission probability in Figure 3b is completely different compared to Figure 3a,c where a change of the position and number of resonances is noted. In Figure 3b for a gapless ABC-TLG, we observe the existence of a brighter region corresponding to higher transmission probability for a wide range of $k_y$ between $\pm 0.025$ nm$^{-1}$ for any width $\Delta$ of barrier well. Then $\Delta$ is the crucial parameter leading to determine the number of resonant peaks and their positions. It is also the case for the AB-bilayer graphene. As for the case of non-normal incidence, that is $k_x \neq 0$, with $\delta_2 = \delta_4 = 0$, we observe that the transmission
still equals unit. It comes as a result of the Fabry–Pérot oscillations and does not depend on the widths either $b_1$, $b_2$, or $\Delta$. A similar result is also established for the single barrier case in both systems made of AB-BLG$^{[28,33]}$ and ABC-TLG$^{[29]}$. Now for $\delta_2 = \delta_4 = 0.2 \gamma_1$ and the case of symmetric barrier the density plots in Figure 3 (bottom row) show that the Klein tunneling is suppressed at $k_y = 0$ for some values of $b_1$ or $b_2$, and $\Delta$. We note that most of resonances disappeared and split due to the band gap in the energy spectrum generated from the induced electric field. A full transmission frequently occurs for normal and non-normal incidence when $L$, $\Delta$, and $b_2$ increase as shown in Figure 3 for both gap and gapless ABC-TLG.

3.2. Tunneling at High Energy

At high energy, we consider all propagating modes by showing the transmission and reflection probabilities versus the incident energy $E$ and the wave vector $k_y$. The results are presented in Figure 4 for $V_2 = V_4 = 1.5 \gamma_1$ and Figure 5 with $V_2 = 1.3 \gamma_1$, $V_4 = 1.5 \gamma_1$. Here we have chosen $\delta_2 = \delta_4 = 0.2 \gamma_1$ and $b_1 = b_2 = \Delta = 10$ nm. The dashed black lines show the band inside the barrier whereas the white lines represent the band outside the barrier. These lines separate between different regions in the transmission and reflection probabilities. This can be explained by identifying the propagating modes inside and outside the barrier, see

Figure 4. (Color online) Density plot of transmission and reflection probabilities versus $E$ and $k_y$ with $V_2 = V_4 = 1.5 \gamma_1$, $\delta_2 = \delta_4 = 0.2 \gamma_1$, and $b_1 = b_2 = \Delta = 10$ nm. The dashed black lines show the band inside the barrier whereas the white lines represent the band outside the barrier.

Figure 5. (Color online) The same as Figure 4 but now for $V_2 = 1.3 \gamma_1$, $V_4 = 1.5 \gamma_1$. 

of the system unlike \( V_i \neq V_j, T_i^j \) are not symmetric and \( T_i^j \neq T_j^i \) (\( i \neq j \)). We observe that the barrier heights act by reducing the transmission probabilities. However, their effects become more intense inside the gap, which resulted from the fact that the available states outside the first barrier are in the same energy zone of the gap on the second barrier.

At normal incidence, and in contrast to the single barrier case\,[29] as shown in Figures 4 and 5 at \( k_t = 0 \) there is a non-null transmission \( T_i^j \) inside the gap in the energy spectrum. This resulted from the available states in the well between the barriers, similarly to the case AB-BLG\,[32] In addition, the Klein tunneling in \( T_i^j \) occurs when \( V_2 - \gamma_i < E < V_i - \gamma_i - \Delta_2, E > V_j + \delta_j \) for the symmetric and asymmetric barrier with \( \gamma_i' = 0.918\gamma_i \)[29] Outside these intervals the transmission \( T_i^j \) is nearly zero. For \( E < \gamma_i \), the transmission \( T_i^j = 0 \) for \( (i \neq j) \) while the reflection \( R_i^j \neq 0 \) and \( R_i^j = 0 \) for \( (i \neq j) \). It implies that at low energy the propagation from region 1 to region V is only valid by one channel \( k_j \), as in the case of the one barrier in AB-BLG\,[28,31] and ABC-TLG\,[29] We observe the suppression in transmission due to cloaking effect at non-normal incidence\,[29,56] which also exists for some states as a result of the available states in the well.

4. Conductance

The energy dependence of the conductance of ABC-TLG through symmetric and asymmetric double barrier structure is shown in Figure 7a,b respectively. The solid lines are the total conductance and the dashed ones are for the contributions of different propagation channels. The resonances that are visible in the transmission probabilities in Figures 4 and 5 due to the existence of the bound electron states in the well, seen previously, are responsible for the appearance of peaks in the conductance. These resonances are more important in Figure 7a for symmetric structure than in Figure 7b for asymmetric one. Several local maxima and minima are observed, which are strongly dependent on the structure of the barrier. It is clearly seen in Figure 7a,b that for \( E < \gamma'_i \) once the propagating states become available, the conductance shows a sharp increase to the unit in the regime where only one band \( T_1^i \) is available for contributing to the conductance. For \( E > \gamma'_i \) when more bands become possible, additional transmission channels contribute by increasing the conductance to almost perfect one with \( G_j^i = G_i^i, G_j^i = G_i^i \), and \( G_j^i = G_i^i \) in Figure 7a for symmetric structure. Contrary to the asymmetric structure shown in Figure 7b, we have \( G_j^i \neq G_i^i, G_j^i \neq G_i^i, \) and \( G_j^i \neq G_i^i \).

5. Conclusion

We have investigated the tunneling effect of electrons through symmetric and asymmetric double barrier potential in ABC-TLG system. Using the six bands model, we have derived the solutions of energy spectrum in all regions composing our system. By matching the eigenspinors at different interfaces, we have determined all possible channels of the transmission and reflection coefficients. Based on the double barrier structure, we have studied the transmissions by specifying two energy zones \( E < \gamma'_i \) (one propagating mode) and \( E > \gamma'_i \) (three propagating modes) in addition to various values of the barrier heights and widths.

Figure 6. (Color online) Density plot of transmission probabilities \( T_i^j \) (left panel) and \( T_i^j \) (right panel) versus \( E \) and \( k_y \) with \( b_1 = b_2 = \Delta = 10 \) nm for \( V_j = V_i = 1.5 \gamma_i \), and \( \delta_1 = \delta_2 = 0 \) (top row), \( V_2 = 1.3 \gamma_i, V_1 = 1.5 \gamma_i \), and \( \delta_1 = \delta_2 = 0 \) (middle row), \( V_2 = V_4 = 1.5 \gamma_i, \delta_2 = 0 \), and \( \delta_4 = 0.2\gamma_i \) (bottom row).
Additionally, we have compared our results with previous work for symmetric or asymmetric structure, a full transmission was found independently on the incident energy or barrier widths as the case for a single barrier. At $k_0 \neq 0$ it was seen that the transmission shows a sequence of resonances in the zone $E < V_j$, which is a consequence of the bounded electrons in the well between two barriers. To control the position and the number of these resonances, in both cases $E < \gamma_1$ or $E > \gamma_1$, it was stressed that one could use the well width between the two barriers rather than their thickness as shown in AB-BLG.

It was shown that the asymmetric structure of the double barrier reduces the transmission probabilities and removes the sharp resonant peaks as well. Furthermore, the symmetry of transmission is broken due to the existence of the interlayer potential difference in regions II and IV. However, we have observed that the resulting conductance for the double barrier turned into distinctive from that of the single barrier. This distinction manifests itself via the presence of many extra resonances that are related to the bound electron states in the well.

**Acknowledgements**

The generous support provided by the Saudi Center for Theoretical Physics (SCTP) is highly appreciated by all authors.

**Conflict of Interest**

The authors declare no conflict of interest.

**Data Availability Statement**

The data that support the findings of this study are available from the corresponding author upon reasonable request.

**Keywords**

double barriers, Klein tunneling, trilayer graphene

---

[1] K. S. Novoselov, A. K. Geim, S. V. Morozov, D. Jiang, Y. Zhang, S. V. Dubonos, I. V. Grigorieva, A. A. Firsov, Science 2004, 306, 666.
[2] K. S. Novoselov, A. K. Geim, S. V. Morozov, D. Jiang, M. I. Katsnelson, I. V. Grigorieva, S. V. Dubonos, A. A. Firsov, Nature 2005, 438, 197.
[3] Y. B. Zhang, Y. W. Tan, H. L. Stormer, P. Kim, Nature 2005, 438, 201.
[4] A. K. Geim, K. S. Novoselov, Nat. Mater. 2007, 6, 183.
[5] M. Aoki, H. Amawashi, Solid State Commun. 2007, 142, 123.
[6] M. S. Dresselhaus, G. Dresselhaus, Adv. Phys. 2002, 51, 186.
[7] M. Koshino, T. Ando, Solid State Commun. 2009, 149, 1123.
[8] S. H. Jhang, M. F. Craciun, S. Schmidmeier, S. Tokumitsu, S. Russo, M. Yamamoto, Y. Skourski, J. Wosnitza, S. Tarucha, J. Eroms, C. Strunk, Phys. Rev. B 2011, 84, 164408(R).
[9] M. Koshino, Phys. Rev. B 2010, 81, 125304.
[10] C. H. Lui, Z. Q. Li, Z. Y. Chen, P. V. Klimov, L. E. Brus, T. F. Heinz, Nano Lett. 2011, 11, 164.
[11] S. Latil, L. Henrard, Phys. Rev. Lett. 2006, 97, 036803.
[12] E. McCann, M. Koshino, Rep. Prog. Phys. 2013, 76, 056503.
[13] A. I. Cocemasov, D. L. Nika, A. A. Balandin, Phys. Rev. B 2013, 88, 035428.
[14] K. F. Mak, J. Shan, T. F. Heinz, Phys. Rev. Lett. 2010, 104, 176404.
[15] F. Guinea, A. H. C. Neto, N. M. R. Peres, Phys. Rev. B 2006, 73, 245426.
[16] A. A. Avetissyan, B. Partoens, F. M. Peeters, Phys. Rev. B 2009, 80, 195401.
[17] A. A. Avetissyan, B. Partoens, F. M. Peeters, Phys. Rev. B 2010, 81, 115432.
[18] C. L. Lu, C. P. Chang, Y. C. Huang, R. B. Chen, M. L. Lin, Phys. Rev. B 2006, 73, 144427.
[19] B. Van Duppen, F. M. Peeters, Europhys. Lett. 2013, 102, 27001.
[20] R. Chegel, Synth. Met. 2017, 223, 172.
[21] L. Hao, L. Sheng, Solid State Commun. 2009, 149, 1962.
[22] C. Yelgel, J. Phys.: Conf. Ser. 2016, 707, 012022.
[23] S. Bala Kumar, J. Guo, Appl. Phys. Lett. 2011, 98, 222101.
[24] M. I. Katsnelson, K. S. Novoselov, A. K. Geim, Nat. Phys. 2006, 2, 620.
[25] M. Ramezani Masiri, P. Vasilopoulos, F. M. Peeters, Phys. Rev. B 2010, 82, 115417.
[26] S. B. Kumar, J. Guo, Appl. Phys. Lett. 2012, 100, 163102.
[27] B. Van Duppena, F. M. Peeters, Appl. Phys. Lett. 2012, 101, 226101.
[28] B. Van Duppen, F. M. Peeters, Phys. Rev. B 2013, 87, 205427.
[29] B. Van Duppen, S. H. R. Sena, F. M. Peeters, Phys. Rev. B 2013, 87, 195439.
[30] J. M. Pereira, P. Vasilopoulos, F. M. Peeters, Phys. Rev. B 2009, 79, 155402.
[31] M. Mirzakhani, M. Zarenia, P. Vasilopoulos, F. M. Peeters, Phys. Rev. B 2017, 95, 155434.
[32] H. M. Abdullah, A. El Mouhabid, H. Bahlouli, A. Jellal, Mater. Res. Express 2017, 4, 025009.
[33] N. Benlakhouy, A. El Mouhabid, A. Jellal, Physica E 2021, 134, 114835.
[34] M. Koshino, E. McCann, Phys. Rev. B 2009, 80, 165409.
[35] F. Zhang, B. Sahu, H. K. Min, A. H. MacDonald, Phys. Rev. B 2010, 82, 035409.
[36] S. H. R. Sena, J. M. Pereira, F. M. Peeters, G. A. Farias, Phys. Rev. B 2011, 84, 205448.
[37] K. Zou, F. Zhang, C. Clapp, A. H. MacDonald, J. Zhu, Nano Lett. 2013, 13, 369.
[38] J. Jung, A. H. MacDonald, Phys. Rev. B 2013, 88, 075408.
[39] Salah Uddin, K. S. Chan, J. Appl. Phys. 2014, 116, 203704.
[40] Marcos G. Menezes, Rodrigo B. Capaz, Steven C. Louie, Phys. Rev. B 2014, 89, 035431.
[41] R. Ma, L. Sheng, M. Liu, D. N. Sheng, Phys. Rev. B 2012, 86, 115414.
[42] S. Yuan, R. Roldán, M. I. Katsnelson, Phys. Rev. B 2011, 84, 125455.
[43] M. G. Menezes, R. B. Capaz, S. G. Louie, Phys. Rev. B 2014, 89, 035431.
[44] L. J. Yin, W. X. Wang, Y. Zhang, Y. Y. Ou, H. T. Zhang, C. Y. Shen, L. He, Phys. Rev. B 2017, 95, 081402.
[45] Y. Barlas, R. Côté, M. Rondeau, Phys. Rev. Lett. 2012, 109, 126804.
[46] R. Côté, M. Rondeau, A. M. Gagnon, Y. Barlas, Phys. Rev. B 2012, 86, 125422.
[47] R. Xu, L. J. Yin, J. B. Qiao, K. K. Bai, J. C. Nie, L. He, Phys. Rev. B 2015, 91, 035410.
[48] Y.-P. Wang, X.-G. Li, J. N. Fry, H.-P. Cheng, Phys. Rev. B 2016, 94, 165428.
[49] M. F. Craciun, S. Russo, M. Yamamoto, J. B. Oostinga, A. F. Morpurgo, S. Tarucha, Nat. Nanotechnol. 2009, 4, 383.
[50] C. H. Lui, Z. Li, K. F. Mak, E. Cappelluti, T. F. Heinz, Nat. Phys. 2011, 7, 944.
[51] L. C. Campos, A. F. Young, K. Surakitbovorn, K. Watanabe, T. Taniguchi, P. Jarillo-Herrero, Nat. Commun. 2012, 3, 1239.
[52] B. Partoens, F. M. Peeters, Phys. Rev. B 2007, 75, 193402.
[53] F. Zhang, J. Jung, G. A. Fiete, Q. Niu, A. H. MacDonald, Phys. Rev. Lett. 2011, 106, 156801.
[54] F. Zhang, B. Sahu, H. Min, A. H. MacDonald, Phys. Rev. B 2010, 82, 035409.
[55] Y. M. Blanter, M. Büttiker, Phys. Rep. 2000, 336, 1.
[56] N. Gu, M. Rudner, L. Levitov, Phys. Rev. Lett. 2011, 107, 156603.