Semirelativistic quark-antiquark potential, mass spectra and weak decays into excited states of heavy-light mesons

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Abstract
Relativistic properties of $q\bar{q}$ potential, mass spectra of orbitally and radially excited $B$ and $D$ mesons as well as semileptonic decays of $B$ mesons to orbitally excited $D$ mesons are discussed in the framework of the relativistic quark model based on the quasipotential approach.

1. Relativistic properties of $q\bar{q}$ potential. In preceding papers [1,2] we have developed the relativistic quark model with the $(q\bar{q})$ potential consisting of the perturbative one-gluon exchange part and a nonperturbative one which is a mixture of the Lorentz scalar and vector confining potentials:

$$V(p, q; M) = \bar{u}_a(p)\bar{u}_b(-p)\left\{ \frac{4}{3}\alpha_s D_{\mu\nu}(k)\gamma^\mu\gamma^\nu_{ab} + V_{\text{conf}}^V(k)\Gamma_a\Gamma_b + V_{\text{conf}}^S(k) \right\} u_a(q)u_b(-q),$$

(1)
where \( k = p - q \), \( D_{\mu\nu} \) is the gluon propagator in the Coulomb gauge and \( \Gamma_\mu \) is the effective vector long-range vertex, containing both the Dirac and Pauli terms

\[
\Gamma_\mu = \gamma_\mu + \frac{i\kappa}{2m}\sigma_\mu k^\nu, \tag{2}
\]

\( u_{a,b}(p) \) are the Dirac bispinors. The parameter \((1 + \kappa)\) can be treated as the nonperturbative (long-range) chromomagnetic moment of the quark and \( \kappa \) as its anomalous part (flavour independent).

In the nonrelativistic limit the Fourier transform of eq. (1) gives the static potential

\[
V_0(r) = V_{\text{Coul}}(r) + V_{\text{conf}}^S(r) + V_{\text{conf}}^V(r), \tag{3}
\]

where

\[
V_{\text{Coul}}(r) = -\frac{4\alpha_s}{3r},
\]

\[
V_{\text{conf}}^S(r) = (1 - \varepsilon)(Ar + B) ; \quad V_{\text{conf}}^V(r) = \varepsilon(Ar + B), \tag{4}
\]

here \( \varepsilon \) is the mixing parameter.

Now assuming that both quarks are heavy enough we evaluate the \((v^2/c^2)\) relativistic corrections to the static potential (3), (4) paying special attention to retardation effects. The Fourier transform of the linear potential \( Ar \) in the momentum space looks like:

\[
A\int d^3r e^{-ik\cdot r} = -A\frac{8\pi}{|k|^4}, \quad k = p - q. \tag{5}
\]

The natural (though not unique) relativistic extension (dependent only on the four-momentum transfer) of expression (5) is to substitute \((-k^2) \rightarrow (k_0^2 - k^2)\). Now we should choose the procedure of fixing \( k_0 \). On the mass shell due to energy conservation we have \( k_0 = 0 \). So \( k_0 \) may be considered as the measure of deviation either from the mass shell or from the energy shell. We choose the second possibility and set \( k_0 \) equal to \( \epsilon_a(p) - \epsilon_a(q) \) or to \( \epsilon_b(q) - \epsilon_b(p) \). Then in the symmetrized form we get

\[
k_0^2 = -(\epsilon_a(p) - \epsilon_a(q))(\epsilon_b(p) - \epsilon_b(q)), \quad \epsilon_{a,b}(p) = \sqrt{p^2 + m_{a,b}^2}.
\]

This form is not unique and other possible expressions for \( k_0^2 \) are discussed in [3, 4]. In favour of choice (3) we mention the following arguments. It is
well-known that for the one-photon exchange contribution in QED only choice (6) in the Feynman (diagonal) gauge leads to the same correct result (the Breit-Fermi Hamiltonian) as the prescription $k_0 = 0$ in the Coulomb (or transverse Landau) gauge. The same is naturally true for the one-gluon exchange contribution in QCD. Moreover as shown in ref. [5] for any effective vector potential generated by a vector exchange and its couplings to conserved vector currents (vertices) there is the so-called instantaneous gauge which plays the role of the Coulomb gauge. In the instantaneous gauge the prescription $k_0 = 0$ reproduces the same result as the expansion in $k^2_0$ fixed by eq. (6) in the diagonal gauge used here. The other reason to utilize prescription (6) is the reproduction of the correct Dirac limit in this case [4].

Carring out $p^2/m^2$ expansion of the $(q\bar{q})$ potential (1) in configuration space we obtain for the spin-independet part the following expression [1, 6]:

$$V_{SI}(r) = V_0(r) + V_{VD}(r) + \frac{1}{8}\left(\frac{1}{m_a^2} + \frac{1}{m_b^2}\right)\Delta[V_{Coul}(r) + (1 + 2\kappa)V_{conf}(r)], \quad (6)$$

where $V_0(r)$ is given by eqs. (3), (4). The velocity-dependent part $V_{VD}(r)$ can be presented in the form

$$V_{VD}(r) = \frac{1}{m_am_b}\left\{p^2V_{bc}(r) + \frac{(p\cdot r)^2}{r^2}V_c(r)\right\}_W$$

$$+ \left(\frac{1}{m_a^2} + \frac{1}{m_b^2}\right)\left\{p^2V_{dc}(r) - \frac{(p\cdot r)^2}{r^2}V_c(r)\right\}_W \quad (7)$$

with

$$V_{bc}(r) = -\frac{2\alpha_s}{3r} + \left(\frac{1}{2} - \varepsilon\right)Ar + (1 - \varepsilon)B; \quad V_c(r) = -\frac{2\alpha_s}{3r} - \frac{1}{2}Ar;$$

$$V_{de}(r) = -\varepsilon Ar + \left(\frac{1}{4} - \varepsilon\right)B; \quad V_{e}(r) = 0. \quad (8)$$

Now we are able to test the fulfillment of the exact Barchielli, Brambilla, Prosperi (BBP) relations [7], which follow from the Lorentz invariance of the Wilson loop. In our notations these relations look like

$$V_{de} - \frac{1}{2}V_{bc} + \frac{1}{4}V_0 = 0; \quad V_c + \frac{1}{2}V_c + \frac{r}{4}\frac{dV_0}{dr} = 0 \quad (9)$$

One can easily find that the functions (8) identically satisfy relations (9) independently of values of the parameters $\varepsilon$ and $\kappa$. This is a highly nontrivial
result. For the perturbative one-gluon-exchange part of \( V_{VD} \) our expressions for \( V_b, \ldots, V_c \) are the same as in \[7\], but for the confining (long-range) part they are different. The terms with the Laplacian in \( (6) \) coincide only for \( \kappa = 0 \) and \( \varepsilon = 0 \), i.e. for purely vector confining interaction without the Pauli term in the vertex \( (2) \). Our expressions \( (6) \) for purely vector \( (\varepsilon = 0) \) and purely scalar \( (\varepsilon = 1) \) interactions and for \( \kappa = 0 \) coincide with those of ref. \[4\] except for the constant \( B \) term. Our \( B \) term for \( \varepsilon = 1 \) (scalar potential) is the same as in \[7\]. The \( B \) term from ref. \[4\] does not satisfy the BBP relations (it gives contribution \(-B/2\) only to \( V_{de} \)). Our result for the scalar \( (\varepsilon = 1) \) confining potential also differs from the one obtained in ref. \[8\], where the prescription \( k_0 = 0 \) was used and as a result the contribution of retardation was lost. The differences between our results and the results presented in ref. \[9\] originate from the use of specific models such as minimal area law, flux tube, dual superconductivity and stochastic vacuum.

The spin-dependent part of our potential \( (1, 6) \) (for \( \kappa = -1 \)) completely coincides with the one found in refs. \[10, 11\]. The Gromes relation is identically fulfilled. Our result supports the conjecture that the long-range confining forces are dominated by chromoelectric interaction and that the chromomagnetic interaction vanishes. It is also in accord with the flux tube and dual superconductivity picture \[9, 10\].

2. Mass spectra of heavy-light mesons. Many different approaches have been used for the calculation of orbital and radial excitations of heavy-light mesons \[11, 12, 13\]. However, almost in all of them the expansion in inverse powers not only of the heavy quark mass \( (m_Q) \) but also in inverse powers of the light quark mass \( (m_q) \) is carried out. The estimates of the light quark velocity in these mesons show that the light quark is highly relativistic \( (v/c \sim 0.7 \div 0.8) \). Thus the nonrelativistic approximation is not adequate for the light quark and one cannot guarantee the numerical accuracy of the expansion in inverse powers of the light quark mass. Here we present the results of the calculation of the masses of orbitally and radially excited \( B \) and \( D \) mesons without employing the expansion in \( 1/m_q \) (see \[14\] for details). Thus the light quark is treated fully relativistically. Concerning the heavy quark we apply the expansion in \( 1/m_Q \) up to the first order. Our numerical results are presented in Tables \[1\]-\[4\].

Let us compare the obtained results with model independent predictions of heavy quark effective theory (HQET). Heavy quark symmetry provides
Table 1: Mass spectrum of $D$ mesons with the account of $1/m_Q$ corrections in comparison with other quark model predictions and experimental data. All masses are given in GeV. We use the notation $(nL_J)$ for meson states, where $J$ is the total angular momentum of the meson.

| State | our [11] | [13] | experiment [15, 16] |
|-------|----------|------|---------------------|
| $1S_0$ | 1.875    | 1.88 | 1.8645(5)           |
| $1S_1$ | 2.009    | 2.04 | 2.0067(5)           |
| $1P_2$ | 2.459    | 2.50 | 2.460               |
| $1P_1$ | 2.414    | 2.47 | 2.415               |
| $1P_1$ | 2.501    | 2.46 | 2.585               |
| $1P_0$ | 2.438    | 2.40 | 2.565               |
| $2S_0$ | 2.579    | 2.58 |                    |
| $2S_1$ | 2.629    | 2.64 | 2.637(9)            |

relations between excited states of $B$ and $D$ mesons, such as

$$\bar{M}_{B_1} - \bar{M}_{D_1} = \bar{M}_{B_{s1}} - \bar{M}_{D_{s1}} = \bar{M}_B - \bar{M}_D = \bar{M}_{B_s} - \bar{M}_{D_s} = m_b - m_c, \quad (10)$$

where $\bar{M}_{B_1} = (3M_{B_1} + 5M_{B_2})/8$, $\bar{M}_B = (M_B + 3M_{B'})/4$ are appropriate spin averaged $P$- and $S$-wave states. We get from Tables 1-4 the following values of mass splittings

$$\bar{M}_{B_1} - \bar{M}_{D_1} = 3.29 \text{ GeV} \quad \bar{M}_{B_{s1}} - \bar{M}_{D_{s1}} = 3.30 \text{ GeV}$$
$$\bar{M}_B - \bar{M}_D = 3.34 \text{ GeV} \quad \bar{M}_{B_s} - \bar{M}_{D_s} = 3.33 \text{ GeV}, \quad (11)$$

in agreement with (10). There arise also the following relations between hyperfine splittings of levels

$$\Delta M_B \equiv M_{B_2} - M_{B_1} = \frac{m_c}{m_b} \Delta M_D \equiv \frac{m_c}{m_b} (M_{D_2} - M_{D_1}), \quad (12)$$

and the same for $B_s$ and $D_s$ mesons as well as for $P_1 - P_0$ states. Our model predictions for these splittings are displayed in Table 5.

In Tables 1-4 we compare our relativistic quark model results for heavy-light meson masses with the predictions of other quark models of Godfrey and Isgur [11], Isgur [13], Eichten, Hill and Quigg [12] and experimental data [15, 16]. All these quark models use the expansion in inverse powers both of the heavy $m_Q$ and light $m_q$ quark masses for the $Q\bar{q}$ interaction potential. In
Table 2: Mass spectrum of $D_s$ mesons with the account of $1/m_Q$ corrections in comparison with other quark model predictions and experimental data. All masses are given in GeV.

| State | our | [11] | [12] | experiment [13, 16] |
|-------|-----|------|------|--------------------|
| $1S_0$ | 1.981 | 1.98 | 1.968(6) |
| $1S_1$ | 2.111 | 2.13 | 2.1124(7) |
| $1P_2$ | 2.560 | 2.59 | 2.561 | 2.5735(17) |
| $1P_1$ | 2.515 | 2.56 | 2.526 | 2.535(4) |
| $1P_1$ | 2.569 | 2.55 |
| $1P_0$ | 2.508 | 2.48 |
| $2S_0$ | 2.670 | 2.67 |
| $2S_1$ | 2.716 | 2.73 |

ref. [14] some relativization of the potential has been put in by hand, such as relativistic smearing of coordinates and replacing the factors $1/m_q$ by $1/\epsilon_q(p)$. However, the resulting potential in this approach accounts only for some of the relativistic effects, while the others, which are of the same order of magnitude, are missing. The considerations of Refs. [13, 12] are closely related. The heavy quark expansion is extended to light $(u,d,s)$ quarks and the experimental data on $P$ wave masses of $K$ mesons are used to obtain predictions for $B$ and $D$ mesons.

In the paper [13] it is argued that the heavy quark spin $P$-wave multiplets with $j = 1/2$ ($0^+, 1^+$) and $j = 3/2$ ($1^+, 2^+$) in $B$ and $D$ mesons are inverted. The $2^+$ and $1^+$ states lie about 150 MeV below the $1^+$ and $0^+$ states. In the limit $m_Q \to \infty$, we find the same inversion of these multiplets in our model, but the gap between $j = 1/2$ and $j = 3/2$ states is smaller ($\sim 90$ MeV for $B$ and $D$ mesons and $\sim 70$ MeV for $B_s$ and $D_s$ mesons), and $1/m_Q$ corrections reduce this gap further. However, the hyperfine splittings among the states in these multiplets turn out to be larger than in [13]. As a result, the states from the multiplets for $D$, $D_s$ and $B_s$ mesons overlap in our model, however the heavy quark spin averaged centres are still inverted. We obtain the following ordering of $P$ states (with masses increasing from left to right): $B$ meson — $1P_1(\frac{3}{2}), 1P_2, 1P_0, 1P_1(\frac{1}{2})$; $D_s$ meson — $1P_0, 1P_1(\frac{3}{2}), 1P_2, 1P_1(\frac{1}{2})$; $D$ and $B_s$ mesons — $1P_1(\frac{3}{2}), 1P_0, 1P_2, 1P_1(\frac{1}{2})$. Thus only for $B$ meson we get the purely inverted pattern. Note that the model [14] predicts the
ordinary ordering of levels. The results of our model agree well with available experimental data.

**3. Semileptonic B decays to orbitally excited D mesons.** In ref. [14] we have applied the relativistic quark model to the consideration of semileptonic decays of B mesons to orbitally excited charmed mesons in the leading order of the heavy quark expansion. At the leading order of the heavy quark expansion the Isgur-Wise [18] functions $\tau_{3/2}$ and $\tau_{1/2}$ are as follows

$$\tau_{3/2}(w) = \frac{\sqrt{2}}{3} \frac{1}{(w+1)^{3/2}} \int \frac{d^3p}{(2\pi)^3} \bar{\psi}_{D(3/2)}(p + \frac{2\epsilon_q}{M_{D(3/2)}(w+1)}) \Delta \left[ -2\epsilon_q \frac{\partial}{\partial p} + \frac{p}{\epsilon_q + m_q} \right] \psi_B(p),$$

$$\tau_{1/2}(w) = \frac{1}{3\sqrt{2}} \frac{1}{(w+1)^{1/2}} \int \frac{d^3p}{(2\pi)^3} \bar{\psi}_{D(1/2)}(p + \frac{2\epsilon_q}{M_{D(1/2)}(w+1)}) \Delta \left[ -2\epsilon_q \frac{\partial}{\partial p} - \frac{2p}{\epsilon_q + m_q} \right] \psi_B(p),$$

where the arrow over $\partial/\partial p$ indicates that the derivative acts on the wave function of the $D^{**}$ meson. The last terms in the square brackets of these expressions result from the wave function transformation associated with the relativistic rotation of the light quark spin (Wigner rotation) in passing to the moving reference frame. These terms are numerically important and
Table 4: Mass spectrum of \(B_s\) mesons with the account of \(1/m_Q\) corrections in comparison with other quark model predictions and experimental data. All masses are given in GeV.

| State | our | \[11\] | \[12\] | experiment \[13, 16\] |
|-------|-----|------|------|---------------------|
| \(1S_0\) | 5.375 | 5.39 | 5.369(20) | |
| \(1S_1\) | 5.412 | 5.45 | 5.416(4) | ? |
| \(1P_2\) | 5.844 | 5.88 | 5.861 | 5.853(15) ? |
| \(1P_1\) | 5.831 | 5.86 | 5.849 | |
| \(1P_0\) | 5.859 | 5.86 | | |
| \(2S_0\) | 5.971 | 6.27 | | |
| \(2S_1\) | 5.984 | 6.34 | | |

Table 5: Hyperfine splittings of \(P\) levels. All values are given in MeV.

| States | \(\Delta M_D\) | \(\frac{m_c}{m_b}\Delta M_D\) | \(\Delta M_B\) | \(\Delta M_{D_s}\) | \(\frac{m_c}{m_b}\Delta M_{D_s}\) | \(\Delta M_{B_s}\) |
|--------|----------------|----------------|----------------|----------------|----------------|----------------|
| \(1P_2 - 1P_1\) | 45 | 14 | 14 | 45 | 14 | 13 |
| \(1P_1 - 1P_0\) | 63 | 20 | 19 | 61 | 19 | 18 |

lead to the suppression of the \(\tau_{1/2}\) form factor compared to \(\tau_{3/2}\). Note that if we had applied a simplified nonrelativistic quark model \[18\] these important contributions would be missing. Neglecting further the small difference between the wave functions \(\psi_{D(1/2)}\) and \(\psi_{D(3/2)}\), the following relation between \(\tau_{3/2}\) and \(\tau_{1/2}\) would have been obtained \[19\]

\[
\tau_{1/2}(w) = \frac{w + 1}{2} \tau_{3/2}(w). \tag{15}
\]

At the point \(w = 1\), where the initial \(B\) meson and final \(D^{**}\) are at rest, we find instead the relation

\[
\tau_{3/2}(1) - \tau_{1/2}(1) \approx \frac{1}{2} \int \frac{d^3p}{(2\pi)^3} \hat{\psi}_{D^{**}}(\mathbf{p}) \frac{p}{\epsilon_q + m_q} \psi_B(\mathbf{p}), \tag{16}
\]

obtained by assuming \(\psi_{D(3/2)} \approx \psi_{D(1/2)} \approx \psi_{D^{**}}\).

In Table 6 we present our numerical results for \(\tau_j(1)\) and its slope \(\rho_j^2\) in comparison with other model predictions \[19, 21, 22, 23, 11, 24\]. We see that most of the above approaches predict close values for the function.
Table 6: The comparison of our model results for the values of the functions $\tau_j$ at zero recoil of final $D^{**}$ meson and their slopes $\rho^2_{j}$ with other predictions.

|          | our | [19] | [21] | [22] | [23] | [20],[11] | [20],[24] |
|----------|-----|------|------|------|------|-----------|-----------|
| $\tau_{3/2}(1)$ | 0.49 | 0.41 | 0.56 | 0.66 | 0.54 | 0.52      |
| $\rho^2_{3/2}$  | 1.53 | 1.5  | 2.3  | 1.9  | 1.5  | 1.45      |
| $\tau_{1/2}(1)$ | 0.28 | 0.41 | 0.09 | 0.41 | 0.35 | 0.08      |
| $\rho^2_{1/2}$  | 1.04 | 1.0  | 1.1  | 1.4  | 2.5  | 1.0       |

$\tau_{3/2}(1)$ and its slope $\rho^2_{3/2}$, while the results for $\tau_{1/2}(1)$ significantly differ from each other. This difference is a consequence of a different treatment of the relativistic quark dynamics. Nonrelativistic approaches predict $\tau_{3/2}(1) \simeq \tau_{1/2}(1)$ (see [15]), while the relativistic treatment leads to $\tau_{3/2}(1) > \tau_{1/2}(1)$ (see [16]). Our results for the branching ratios of $B \to D_{1,2}(3/2)e\nu$ decays are consistent with available experimental data.

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References

[1] V.O. Galkin, A.Yu. Mishurov and R.N. Faustov, Yad. Fiz. 55 (1992) 2175 [Sov. J. Nucl. Phys. 55 (1992) 1207].
[2] R.N. Faustov and V.O. Galkin, Z. Phys. C 66 (1995) 119; R.N. Faustov, V.O. Galkin and A.Yu. Mishurov, Proc. Int. Seminars “Quarks ’88, 92, 94, 96”.
[3] D. Gromes, Nucl. Phys. B 131 (1977) 80.
[4] M.G. Olson and K.J. Miller, Phys. Rev. D 28 (1983) 674.
[5] W. Celmaster and F.S. Henyey, Phys. Rev. D 17 (1978) 3268.
[6] D. Ebert, R.N. Faustov and V.O. Galkin, hep-ph/9804335 (1998).
[7] A. Barchielli, N. Brambilla and G.M. Prosperi, Nuovo Cim. A 103 (1990) 59.
[8] T. Barnes and G.I. Ghandour, Phys. Lett. B 118 (1982) 411.
[9] N. Brambilla and A. Vairo, Phys. Rev. D 55 (1997) 3974.
[10] W. Buchmüller, Phys. Lett. B 112 (1982) 479.
[11] S. Godfrey and N. Isgur, Phys. Rev. D 32 (1985) 189; S. Godfrey and R. Kokoski, Phys. Rev. D 43 (1991) 1679.
[12] E.J. Eichten, C.T. Hill and C. Quigg, Phys. Rev. Lett. 71 (1993) 4116; Preprint FERMILAB-CONF-94/118-T (1994).
[13] N. Isgur, Phys. Rev. D 57 (1998) 4041.
[14] D. Ebert, R.N. Faustov and V.O. Galkin, Phys. Rev. D 57 (1998) 5663.
[15] R.M. Barnett et al., Particle Data Group, Phys. Rev. D 54 (1996) 1.
[16] P. Abreu et al. (DELPHI Collaboration), Phys. Lett. B 345 (1995) 598; Phys. Let. B 426 (1998) 231; D. Buskulic et al. (ALEPH Collaboration), Z. Phys. C 73 (1997) 601; R. Anastassov et al. (CLEO Collaboration), Phys. Rev. Lett. 80 (1998) 231;
[17] D. Ebert, R.N. Faustov and V.O. Galkin, Phys. Lett. B 434 (1998) 365.
[18] N. Isgur and M.B. Wise, Phys. Rev. D 43 (1991) 819.
[19] A.K. Leibovich et al., Phys. Rev. D 57 (1998) 308.
[20] V. Moréñas et al., Phys. Rev. D 56 (1997) 5668.
[21] A. Deandrea et al., hep-ph/9802308 (1998).
[22] A. Wambach, Nucl. Phys. B 434 (1995) 647.
[23] P. Colangelo, F. De Fazio and N. Paver, hep-ph/9804377 (1998).
[24] P. Cea et al., Phys. Lett. B 206 (1988); P. Colangelo, G. Nardulli and M. Pietroni, Phys. Rev. D 43 (1991) 3002.