Is life a thermal horizon?

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Abstract

This talk aims at questioning the vanishing of Unruh temperature for an inertial observer in Minkowski spacetime with finite lifetime, arguing that in the non-eternal case the existence of a causal horizon is not linked to the non-vanishing of the acceleration. This is illustrated by a previous result, the diamonds temperature, that adapts the algebraic approach of Unruh effect to the finite case. Written for the proceedings of DICE 2006, Piombino September 2006.

1 Introduction

For a uniformly accelerated observer with infinite lifetime, the vacuum state of a suitable quantum field theory in Minkowski spacetime $M$ appears as a thermal equilibrium state with temperature

$$T_U = \frac{\hbar a}{2\pi k_B c}.$$  (1)

This result, known as the Unruh effect, can be derived in at least three ways: a comparison of quantization schemes on various regions of $M$, an integration along the worldline of the observer of the interaction term between a detector and the vacuum, or a geometrical approach based on the modular properties of the region causally accessible to the observer. Those three approaches strongly rely on the eternity of the observer and in this talk we would like to discuss what is to be expected for a non-eternal observer. In particular, at the light of the analogy between Unruh and Hawking temperatures, it appears that for an inertial observer (i.e. with zero acceleration) with finite lifetime there is no clear reason to ask for the vanishing of the thermal effect. The reason is that for a non-eternal observer the presence of an horizon is not linked to the acceleration as in the eternal case. This point is discussed in section 2. In section 3 we give a presentation of the geometric approach to Unruh effect, based on the KMS formulation of statistical physics, the modular theory of Tomita-Takesaki and the algebraic formulation of quantum field theory. We then recall how to use these technics to treat the non-eternal case, yielding an "Unruh-effect for bounded trajectories" that has the striking property of being non-zero for zero acceleration.

This talk aims at underlining that the non-vanishing of the temperature for a finite lifetime inertial observer is not unexpected as it might seem at first sight, but is rather natural. In fact, and this will be our conclusion in section 4, if the thermal properties of the vacuum had to disappear for an inertial observer with finite lifetime, this would raise the following question (whose answer to our knowledge is not clear): what makes the horizon of a finite lifetime observer - its "life horizon" given by the intersection of the future cone of its birth with the past of its death - so different from the horizon of an eternal observer - a Rindler wedge - so that to kill the thermal property of the vacuum?
The importance of being eternal

Let us recall three main ways to Unruh effect:

- In Unruh’s original approach, \( T_U \) is obtained by observing that the vacuum for a quantization scheme on all \( M \) is not a pure state for an alternative (but as well defined) quantization prescription on the Rindler wedge \( W \).

- In DeWitt approach, one integrates all along the worldline of a uniformly accelerated and eternal observer the interaction term of a quantum system coupled to the vacuum. And it turns out that the system gets excited as if it was at rest in a thermal bath at temperature \( T_U \).

- Finally one can recover \( T_U \), independently of any detector prescription, by noting that the trajectory of a uniformly eternal accelerated observer coincides with the orbit of the point at coordinate \( t = 0 \) under the action of the vacuum modular group associated to the algebra of observables on the wedge region. By general result of modular theory, one knows that the vacuum is a thermal state at temperature \( \beta^{-1} \) with respect to the time evolution determined by the modular group, up to a rescaling

\[
\tau = -\beta s
\]

where \( \tau \) denotes the physical time and \( s \) the modular parameter. In case of the wedge \( W \), the comparison of the two parameterizations of the trajectory, one by \( s \) the other one by \( \tau \), precisely yield \( \beta = T_U^{-1} \).

Note that in those three approaches the eternity of the observer is an important requirement: either one needs to integrate the interaction term from \(-\infty \) to \(+\infty \) in order to recover a thermal distribution, or one uses some property of the wedge \( W \). The latest is physically relevant for it is the (whole and only) region of \( M \) with whom an eternal uniformly accelerated observer can interact. For a non eternal observer, \( W \) is no longer significant and considering a quantization scheme on \( W \) or the modular group of \( W \) has no more physical meaning.

Eternity of the observer is generally overcame by viewing \( T_U \) as a limit for asymptotic states. However such a limit is not always meaningful. Specifically \( T_U \) identifies to the Hawking temperature measured by an observer very close to the horizon of an eternal black hole (see [2] for instance) but not for a Kerr black hole. Quoting Wald [18]: “the difference in nature between the Unruh effect [...] and the Hawking effect of particle creation by black holes [...] is dramatically illustrated by considering the case of the Kerr metric [...] In essence this is because there is no analog of incoming thermal radiation from infinity with respect to the notion of time translations defined by the Killing field which generates the horizon, since this Killing field has spacelike orbits near infinity. However there is no corresponding difficulty with the derivation of particle creation in the case where gravitational collapse produces a Kerr black hole. In this framework the question of an Unruh effect for a non eternal observer becomes relevant independently of the asymptotic acceptation. A traditional answer is to consider that, at best, the excitation rate at finite time might give an indication on the duration of the interaction time between the detector and the thermal bath. However this answer raises more questions than it solves:

-first it makes no distinction between the fact that the vacuum seen from the accelerated point of view is a thermal state, and the fact that a detector may need an infinite amount of time to get in equilibrium with it.

-second it questions the origin of the effect. If, as suggested by the analogy with Hawking temperature, \( T_U \) emerges due to the presence of an horizon, then a finite lifetime observer should also see the vacuum as a thermal state. Indeed the double cone region (or diamond region)

\[
D = \text{birth}^+ \cap \text{death}^-
\]

*in cartesian coordinates \( W \) is the set of points such that \( x > |t| \) where \( x \) is the direction of the acceleration.

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where ± indicates future/past cones, acts for a non eternal observer as the wedge for an eternal one (see figure 1). Unless one is able to point out a difference in nature between $D$ and $W$ (is compactness such important in this framework?), one should be ready to accept that the vacuum is thermal also from a non-eternal accelerated point of view.

Assuming this point is not academic. In the eternal case the presence of an horizon coincides with the non vanishing of the acceleration: an inertial, eternal observer has access to the whole of Minkovki spacetime. There is no longer wedge to limit its horizon and $T_U$ vanishes. On the contrary in the finite lifetime case the two notions, horizon and acceleration, do not coincide. For such an observer $D$ acts like a causal horizon, whether he is accelerated or not. Hence there is no reason why an inertial observer with lifetime $T$ should not observe an Unruh temperature $T_U(T) \neq 0$. The only requirement to be compatible with the eternal case is

$$\lim_{T \to +\infty} T_U(T) = 0.$$  

### 3 The algebraic way and the non eternal case

Motivated by completely different reasons (namely, the issue of time in quantum gravity\textsuperscript{b} and a possible solution known as the thermal time hypothesis\textsuperscript{c}) we proposed in [11] an adaptation for Unruh effect in Minkovski spacetime for an observer with finite lifetime $T$. Our construction is based on the algebraic approach\textsuperscript{1,15} that we recall below.

Given a statistical system with algebra of observables $\mathcal{A}$ and Hamiltonian $H$, a state $\omega$ is said to be KMS with parameter $\beta$ if it satisfies

$$\omega(\alpha_t(a)b) = \omega(b\alpha_{t+i\beta}(a)) \quad a, b \in \mathcal{A}$$

(2)

where $\alpha_t(a) = e^{iHt}a e^{-iHt}$ is the time translation, extended to complex variable\textsuperscript{c}. It has been shown (see [7] for an complete presentation of the subject) that for a system with a finite number of degrees of freedom, being KMS with parameter $\beta$ is equivalent to being a Gibbs equilibrium state at temperature $\beta^{-1}$. Moreover contrary to Gibbs definition KMS properties are still meaningful at the thermodynamical limit. Therefore given a system with a well known time evolution $\alpha_t$ one defines an equilibrium state in the following way:

\textsuperscript{b}which in the present context can be stated as follows: assuming that covariance is preserved at the quantum level and that one is surrounded by a quantum superposition of states of the gravitational field, then one can a priori picks out any direction as the direction of time. How to combine this freedom at the quantum level with the locally unique intuition of physical flow of time at the classical level?

\textsuperscript{c}as well, one asks that the function $z \mapsto \omega(b\alpha_z(a))$ be analytic in the strip $0 < \text{Im} z < \beta$ for any $a, b \in \mathcal{A}$
An equilibrium state at temperature $\beta^{-1}$ is a state that satisfies the KMS condition—with parameter $\beta$—with respect to the time evolution $\alpha_t$.

In other terms, given a temperature and a time evolution, the KMS condition allows to characterize thermal equilibrium states,

\[
\begin{align*}
\text{time} & \longrightarrow \text{thermal state}. \\
\text{temperature} & \rightarrow \text{thermal state}. 
\end{align*}
\]

(3)

Now it happens that mathematics furnish a lot of KMS state. Specifically to any Von Neumann algebra $\mathcal{A}$ acting on an Hilbert space $\mathcal{H}$ is associated a canonical (up to unitary) 1-parameter group of automorphisms\[\sigma_s \in \text{Aut}(\mathcal{A})\]
built from a cyclic and separating vector $\Omega \in \mathcal{H}$. Explicitly the modular group $\sigma_s$ is

\[
\sigma_s(a) = \Delta^{is}a\Delta^{-is} \\
a \in \mathcal{A}, \ s \in \mathbb{R}
\]

where $\Delta$ is given by the polar decomposition of Tomita’s operator $S$ defined on $\mathcal{A}\Omega \subset \mathcal{H}$ by $Sa\Omega = a^*\Omega$. The remarkable point is that the state defined by $\Omega$ is KMS with respect to $\sigma_s$

\[
\langle \Omega, \sigma_s(a)b\Omega \rangle = \langle \Omega, b\sigma_{-s}(a)\Omega \rangle.
\]

(6)

Putting $\alpha_s = \sigma_{-\beta s}$ where $\beta$ is a fixed constant, one is back to (2) as soon as

\[
t = -\beta s.
\]

(7)

In other terms, as a reformulation of the KMS-definition, one has:\[7\]

A thermal state at temperature $\beta^{-1}$ is a state whose associated modular group $\sigma_s$ coincides with the time flow $\alpha_t$ up to rescaling (7).

This definition is less tractable than Gibbs’s one. But it has the advantage to give one solution to the issue of time, simply by inverting the arrow of (3)

\[
\begin{align*}
\text{thermal state} & \longrightarrow \text{time}. \\
\text{temperature} & \rightarrow \text{time}. 
\end{align*}
\]

(8)

Namely assuming that time flow is not known a priori but the system is in an equilibrium state $\omega$ at temperature $\beta^{-1}$, the thermal time hypothesis maintains that the flow of time is given by the modular flow associated to $\omega$, the physical time $t$ being related to the modular parameter $s$ by (7). For physic, the difficulty is of course to explicitly compute the modular flow. This has been done for the wedge region by Bisognano and Wichman\[1\] taking for $\mathcal{A}$ the algebra of local observables\[2\] on the wedge $W$ and for $\Omega$ the vacuum state, one has that the modular group coincides with the time flow of a uniformly accelerated observer $\mathcal{O}$. So the line of universe of $\mathcal{O}$ can be parameterized by its proper time $\tau$ or by the modular parameter $s$ and the ratio of the corresponding tangent vectors precisely yield Unruh temperature

\[
\frac{ds}{d\tau} = \text{constant} = -T_U.
\]

(9)

\[4\] obtained by smearing out on $W$ the fields viewed as operator valued distributions\[7,19\]
A natural question is whether the same analysis,

\[
\begin{cases}
\text{thermal state} \\ \text{time}
\end{cases} \rightarrow \text{temperature},
\]

is true for other regions of Minkowski spacetime. In [11] we have considered the modular group associated to the region causally connected to a uniformly accelerated observer \( O \) with lifetime \( T = 2\tau_0 \), namely a diamond-shape region \( D \)

\[
|\vec{x}| + |t| < L(a, \tau_0)
\]

where

\[
L(a, \tau_0) \doteq a^{-1} \text{argsinh} \tau_0
\]

is the size of the diamond, and coincides with (half of) the lifetime of the observer in case of zero acceleration

\[
2L(0, \tau_0) = 2\tau_0 = T.
\]

Assuming the field is conformally invariant, Hislop and Long [5] have shown that the line of universe of \( O \) is nothing but the orbit of one of its point under the action of the modular group. In other terms, as in the eternal case, the trajectory of \( O \) can be parameterized either by \( O \)'s proper time \( \tau \in [-\tau_0, \tau_0] \) or by the modular parameter \( s \in (-\infty, +\infty) \). But now the ratio of the two parameterizations is no longer constant since the proper time \( \tau \) is bounded while the modular parameter \( s \) is unbounded. Explicit computations detailed in [11] yield

\[
\frac{ds}{d\tau} = T(\tau_0, \tau) = T_U \frac{\sinh a\tau_0}{\cosh a\tau_0 - \cosh a\tau}. \tag{11}
\]

We have interpreted (11) as a temperature by noting that for given \( \tau_0 \) and acceleration \( a \), \( T(\tau_0, \tau) \) is almost a constant for most of \( O \)'s lifetime and takes the value observed in the middle of its life, \( T(\tau_0, \tau) \approx T(\tau_0, 0) \) or, written as a function of \( \tau_0 \) and \( a \),

\[
T(a, \tau_0) \doteq T_U \frac{\cosh a\tau_0 + 1}{\sinh a\tau_0}. \tag{12}
\]

The interesting point is that this temperature does not vanish for an inertial observer

\[
T(0, \tau_0) = \frac{2}{\pi T} \tag{13}
\]

as soon as the lifetime \( T \) is finite.

### 4 Conclusion

Several adaptations of Unruh effect have been proposed for an observer with a finite lifetime (see [11][13][14][10] for the most recent). By this one often means that the detector does interact with the vacuum only for a finite period of time. The result generally depends on the nature of the coupling with the vacuum as well as on the shape of its switching on/off (as explained by J. Louko in his talk). What seems to be commonly admit is that the vacuum is still thermal at temperature \( T_U \) but the detector has no time to reach the thermal equilibrium. Here we argued that the temperature of the vacuum as seen from a finite lifetime accelerated point of view:

- does not necessarily equal \( T_U \) and can be given by some corrections in \( T^{-1} \),

*the observer’s proper time \( \tau \) is measured from \(-\tau_0\) to \( \tau_0 \).*
has no reason to vanish for an inertial observer since, in the finite case, the presence of a diamond shape "life horizon" does not depend on the acceleration.

The thermal time hypothesis allows to derive a temperature for this "life horizon", whose interpretation is questionable: (11) is not a constant with respect to the proper time of the observer, but a function which is almost constant on most of its domain. Basically what the thermal time hypothesis allows is to conformally map the infinity of the lifetime to a (sharp) infinity of the temperature. What is best for physical interpretation is not clear to us at the moment. However, independently of this specific proposal, it remains that if life is not a thermal horizon for an inertial observer, then one should explain why $W$ leads to a thermalization of the vacuum whereas $D$ does not. The only obvious difference is that $D$ is compact while $W$ is not. So the vanishing of $T_U$ for inertial observer, eternal or not, would imply that compactness of the horizon has something to do with Unruh temperature. This could be interesting to confront this idea to general study on horizons. Otherwise it would be interesting to study whether the thermalization of the vacuum for inertial observer leads to some contradiction with known physics.

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