BV formulation of higher form gauge theories in a superspace

Sudhaker Upadhyay* and Bhabani Prasad Mandal†

Department of Physics,
Banaras Hindu University,
Varanasi-221005, INDIA.

We discuss the extended BRST and anti-BRST symmetry (including shift symmetry) in the Batalin-Vilkovisky (BV) formulation for two and three form gauge theories. Further we develop the superspace formulation for the BV actions for these theories. We show that the extended BRST invariant BV action for these theories can be written manifestly covariant manner in a superspace with one Grassmann coordinate. On the hand a superspace with two Grassmann coordinates are required for a manifestly covariant formulation of the extended BRST and extended anti-BRST invariant BV actions for higher form gauge theories.

Keywords: BV formulation; Higher form gauge theory; BRST transformation; superspace formulation.

I. INTRODUCTION

Two and higher form gauge theories play an important role in the different branches of physics [1, 2]. Low energy excitations in string theories contain states described by antisymmetric tensor fields [3, 4]. Various supergravity models are described in terms of antisymmetric tensor fields. The Abelian rank-2 tensor field plays crucial role in the study of classical string theories [5], in the theory of vortex motion in an irrotational,
incompressible fluid \([6, 7]\), in the dual formulation of the Abelian Higgs model \([8, 9]\), in studying supergravity multiplets \([10]\) and in anomaly cancellation of certain superstring theories \([11]\). The BRST symmetry is a fundamental tool for quantizing such theories and studying the renormalizability, unitarity and other aspects of different gauge theories \([12–15]\). On the other hand BV formulation is known to be one of the most powerful method of quantizing different gauge field theories, supergravity theories and topological field theories in Lagrangian formulation \([14–19]\). A superspace formalism for the BV action of 1-form gauge theories has been studied \([20, 21]\). It has been shown how an extended BRST and extended anti-BRST invariant formulation (including some shift symmetry) of the BV action for these theories \([20, 22]\), naturally leads to the proper identification of the antifields through equations of motion of auxiliary field variables. Recently, this formulation has been extended for higher derivative theories \([22]\). We intend to extend such formulation beyond 1-form gauge theories.

In this present article we discuss the BV formulation of extended BRST and extended anti-BRST invariant higher form gauge theories. The extended BRST and extended anti-BRST symmetry includes the shift symmetry of the fields. We further consider the superspace formulation of BV actions for 2-form and 3-form gauge theories. We show that the gauge-fixed Lagrangian density for such theories can be described in the superspace formulation in the extended BRST invariant manner by considering one Grassmann coordinate \(\theta\). On the other hand for manifestly extended BRST and extended anti-BRST invariant formulation of these theories, a superspace with two Grassmann coordinates is required.

The paper is organized as follows. We study the BV formalism in a superspace for 2-form gauge theory in section II. A superspace formulations for 3-form gauge theories are discussed in Sec III. Conclusions are drawn in the last section.
II. BV FORMULATION OF 2-FORM GAUGE THEORY IN SUPERSPACE

We intend to discuss the BV formulation of 2-form gauge theories in a suitably constructed superspace in this section. In particular, we consider some shift symmetry and the usual BRST symmetry to construct an extended BRST invariant BV action. Further, we develop an extended BRST invariant superspace formulation for such theory. The extended anti-BRST symmetry for this BV is also developed. Using all these formulation we finally construct the extended BRST and extended BRST and extended anti-BRST invariant BV action in a superspace.

A. Shift symmetry and an extended BRST invariant BV action

We start with the classical Lagrangian density for four dimensional Abelian rank-2 antisymmetric tensor field \( (B_{\mu \nu}) \) theory as

\[
\mathcal{L}_0 = \frac{1}{12} F_{\mu \nu \rho} F^{\mu \nu \rho},
\]

where the field-strength tensor \( (F_{\mu \nu \rho}) \) is defined as \( F_{\mu \nu \rho} \equiv \partial_\mu B_{\nu \rho} + \partial_\nu B_{\rho \mu} + \partial_\rho B_{\mu \nu} \). This Lagrangian density is invariant under the following gauge transformation

\[
\delta B_{\mu \nu} = \partial_\mu \zeta_\nu - \partial_\nu \zeta_\mu,
\]

where \( \zeta_\mu(x) \) an arbitrary vector field.

To quantize this theory using BRST technique, it is necessary to introduce two anticommuting vector fields \( \rho_\mu \) and \( \tilde{\rho}_\mu \), a commuting vector field \( \beta_\mu \), two anticommuting scalar fields \( \chi \) and \( \tilde{\chi} \), and the commuting scalar fields \( \sigma, \varphi \) and \( \tilde{\sigma} \). The BRST invariant effective Lagrangian density for this theory in a covariant gauge is then given by

\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_0 + \mathcal{L}_{gf},
\]
where the gauge-fixing and ghost part of the Lagrangian density is given as

\[ L_{gf} = -i\partial_\mu \tilde{\rho}_\nu (\partial^\mu \rho^\nu - \partial^\nu \rho^\mu) + \partial_\mu \tilde{\sigma} \partial^\mu \sigma + \beta_\nu (\partial_\mu B^{\mu\nu} + k_1 \beta^\nu - \partial^\nu \varphi) \]

\[ - i\tilde{\chi} \partial_\mu \rho^\mu - i\chi (\partial_\mu \tilde{\rho}^\mu - k_2 \tilde{\chi}), \] (2.4)

where the gauge-fixing and ghost part of the Lagrangian density is given as

\[ L_{gf} = -i\partial_\mu \tilde{\rho}_\nu (\partial^\mu \rho^\nu - \partial^\nu \rho^\mu) + \partial_\mu \tilde{\sigma} \partial^\mu \sigma + \beta_\nu (\partial_\mu B^{\mu\nu} + k_1 \beta^\nu - \partial^\nu \varphi) \]

\[ - i\tilde{\chi} \partial_\mu \rho^\mu - i\chi (\partial_\mu \tilde{\rho}^\mu - k_2 \tilde{\chi}), \] (2.4)

\[ k_1 \text{ and } k_2 \text{ are arbitrary gauge parameters. This effective theory is then invariant under following BRST transformation:} \]

\[ s_b B_{\mu\nu} = (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu), \quad s_b \rho_\mu = -i\partial_\mu \sigma, \quad s_b \sigma = 0, \quad s_b \tilde{\rho}_\mu = i\beta_\mu, \]

\[ s_b \beta_\mu = 0, \quad s_b \tilde{\sigma} = -\tilde{\chi}, \quad s_b \tilde{\chi} = 0, \quad s_b \varphi = \chi, \quad s_b \chi = 0. \] (2.5)

Now, we extend this symmetry using shift symmetry in BV formulation.

In the BV formalism, the gauge-fixing and ghost part of the Lagrangian density is generally expressed in terms of BRST variation of a gauge-fixed fermion. It is straightforward to write the \( L_{gf} \) given in Eq. (2.4) in terms of gauge-fixed fermion \( \Psi \) as

\[ L_{gf} = s_b \Psi, \] (2.6)

where the expression for \( \Psi \) is

\[ \Psi = -i[\tilde{\rho}_\nu (\partial_\mu B^{\mu\nu} - k_1 \beta^\nu) + \tilde{\sigma} \partial_\mu \rho^\mu + \varphi (\partial_\mu \tilde{\rho}^\mu - k_2 \tilde{\chi})]. \] (2.7)

To obtain the extended BRST invariant BV action for Abelian rank-2 tensor field theory we consider the following shifted Lagrangian BV action [24]

\[ \bar{L}_{gf} = L_{gf}(B_{\mu\nu} - \bar{B}_{\mu\nu}, \rho_\mu - \bar{\rho}_\mu, \tilde{\rho}_\mu, \sigma_\mu - \bar{\sigma}_\mu, \tilde{\sigma}_\mu - \bar{\tilde{\sigma}}_\mu, \beta_\nu - \bar{\beta}_\nu, \chi - \bar{\chi}, \tilde{\chi} - \bar{\tilde{\chi}}, \varphi - \bar{\varphi}) \]

\[ = -i(\partial_\mu \tilde{\rho}_\nu \partial^\mu \rho^\nu - \partial^\mu \tilde{\rho}^\mu - \partial^\nu \rho^\mu + \partial^\nu \tilde{\rho}^\mu) + (\partial_\mu \sigma - \partial_\mu \tilde{\rho}) (\partial^\nu \sigma - \partial^\nu \tilde{\rho}) \]

\[ + (\beta_\nu - \bar{\beta}_\nu)(\partial_\mu B^{\mu\nu} - \partial_\mu \bar{B}^{\mu\nu} + k_1 \beta^\nu - k_1 \beta^\nu - \partial^\nu \varphi + \partial^\nu \tilde{\varphi}) \]

\[ - i(\tilde{\chi} - \bar{\tilde{\chi}})(\partial_\mu \rho^\mu - \partial_\mu \tilde{\rho}^\mu) - i(\chi - \bar{\chi})(\partial_\mu \tilde{\rho}^\mu - \partial_\mu \tilde{\rho}^\mu - k_2 \tilde{\chi} + k_2 \bar{\tilde{\chi}}), \] (2.8)

which coincides with \( L_{gf} \) in Eq. (2.4) when all the bar fields vanish. Here we notice that the above Lagrangian density is invariant under the BRST transformation (2.5) for the fields \( \Phi - \bar{\Phi} \), where \( \Phi \) and \( \bar{\Phi} \) are generic notation for all fields and shifted fields respectively.

But in addition it is also invariant under the shift symmetry

\[ s_b \Phi(x) = \alpha(x), \quad s_b \bar{\Phi}(x) = \alpha(x). \] (2.9)
The BRST symmetry along with this shift symmetry form the extended BRST symmetry. The extended BRST transformation is then compactly written as

\[ s_b \Phi(x) = \alpha(x), \quad s_b \bar{\Phi}(x) = \alpha(x) - \beta(x). \] (2.10)

Here \( \beta(x) \) represents the original BRST transformation of collective fields \( \Phi \), whereas \( \alpha(x) \) corresponds to the shift transformation corresponding the collective fields \( \Phi \). This local shift symmetry needs to be gauge-fixed, which leads to an additional BRST symmetry [21]. The extended BRST symmetry transformation for the fields involved in the theory can be written explicitly as

\[ s_b B_{\mu \nu} = \psi_{\mu \nu}, \quad s_b \bar{B}_{\mu \nu} = \psi_{\mu \nu} - (\partial_{\mu} \rho_{\nu} - \partial_{\nu} \bar{\rho}_{\mu} - \partial_{\nu} \rho_{\mu} + \partial_{\nu} \bar{\rho}_{\mu}), \quad s_b \rho_{\mu} = \epsilon_{\mu}, \]
\[ s_b \bar{\rho}_{\mu} = \epsilon_{\nu} + i \partial_{\mu} \sigma - i \partial_{\nu} \bar{\sigma}, \quad s_b \bar{\rho}_{\mu} = \xi_{\mu}, \quad s_b \bar{\rho}_{\mu} = \xi_{\mu} - i \beta_{\mu} + i \bar{\beta}_{\mu}, \]
\[ s_b \sigma = \varepsilon, \quad s_b \bar{\sigma} = \varepsilon, \quad s_b \beta_{\mu} = \eta_{\mu}, \quad s_b \bar{\beta}_{\mu} = \eta_{\mu}, \quad s_b \sigma = \psi, \quad s_b \bar{\chi} = \eta, \]
\[ s_b \bar{\sigma} = \psi + \bar{\chi} - \bar{\chi}, \quad s_b \bar{\chi} = \eta, \quad s_b \bar{\varphi} = \phi - \chi + \bar{\chi}, \quad s_b \bar{\chi} = \Sigma, \]
\[ s_b \varphi = \phi, \quad s_b \bar{\chi} = \Sigma, \quad s_b \xi_i = 0, \quad \xi_i \equiv [\psi_{\mu \nu}, \epsilon_{\mu}, \varepsilon, \eta_{\mu}, \psi, \eta, \phi, \Sigma], \] (2.11)

where the fields \( \psi_{\mu \nu}, \epsilon_{\mu}, \xi_{\mu}, \varepsilon, \eta_{\mu}, \psi, \eta, \phi \) and \( \Sigma \) are introduced as ghost fields associated with the shift symmetry for the fields \( B_{\mu \nu}, \rho_{\mu}, \bar{\rho}_{\mu}, \sigma, \beta_{\mu}, \bar{\sigma}, \bar{\chi}, \phi \) and \( \chi \) respectively. Further, we add antighosts fields \( B_{\mu \nu}^*, \rho_{\mu}^*, \bar{\rho}_{\mu}^*, \sigma^*, \bar{\sigma}^*, \beta_{\mu}^*, \chi^*, \bar{\chi}^* \) and \( \varphi^* \) with opposite parity, corresponding to each of the fields with the following BRST transformations

\[ s_b B_{\mu \nu}^* = L_{\mu \nu}, \quad s_b \rho_{\mu}^* = M_{\mu}, \quad s_b \bar{\rho}_{\mu}^* = \bar{M}_{\mu}, \quad s_b \sigma^* = N, \]
\[ s_b \bar{\sigma}^* = \bar{N}, \quad s_b \beta_{\mu}^* = S_{\mu}, \quad s_b \bar{\beta}_{\mu}^* = \bar{S}_{\mu}, \quad s_b \chi^* = O, \quad s_b \bar{\chi}^* = \bar{O}, \]
\[ s_b \varphi^* = T, \quad s_b \chi_i = 0, \quad \chi_i \equiv [L_{\mu \nu}, M_{\mu}, \bar{M}_{\mu}, N, \bar{N}, S_{\mu}, O, \bar{O}, T], \] (2.12)

where the fields \( \chi_i \) are the Nakanishi-Lautrup type auxiliary fields.

Now, if we gauge fix the shift symmetry by putting all the bar fields to zero we will be able to recover our original theory. This can be achieved by choosing the following gauge-fixed Lagrangian density

\[ \bar{L}_{gf} = L_{\mu \nu} B_{\mu \nu} - B_{\mu \nu}^* (\psi_{\mu \nu} - \partial_{\mu} \rho_{\nu} + \partial_{\nu} \bar{\rho}_{\mu} + \partial_{\nu} \rho_{\mu} - \partial_{\nu} \bar{\rho}_{\mu}) + \bar{M}_{\mu} \bar{\rho}_{\mu} \]
\[ + \tilde{\rho}_\mu^* (\epsilon^\mu + i \partial^\mu \sigma - i \partial^\mu \bar{\sigma}) + M_\mu \tilde{\beta}^\mu + \rho_\mu^* (\xi^\mu - i \beta^\mu + i \tilde{\beta}^\mu) \]
\[ + N \tilde{\sigma} - \sigma^* \varepsilon + \bar{N} \bar{\sigma} - \bar{\sigma}^* (\psi - \tilde{\chi} + \bar{\chi}) + \bar{O} \bar{\chi} + \tilde{\chi}^* \Sigma + O \tilde{\chi} \]
\[ + \chi^* \eta + T \varphi - \varphi^* (\phi - \chi + \bar{\chi}) + S_\mu \tilde{\beta}^\mu - \beta_\mu^* \eta^\mu, \] (2.13)

which is invariant under the extended BRST symmetry transformations given in Eqs. (2.11) and (2.12).

Now, it is straightforward to check that using equations of motion of auxiliary fields \( \chi_i \) all the bar fields disappear from the above expression. The extended Lagrangian density \( \bar{L}_{gf} \) then can be cast in the following form:

\[ \bar{L}_{gf} = - B_{\mu \nu}^* (\psi_{\mu \nu} - \partial^\mu \rho^\nu + \partial^\nu \rho^\mu) + \tilde{\rho}_\mu^* (\epsilon^\mu + i \partial^\mu \sigma) + \rho_\mu^* (\xi^\mu - i \beta^\mu) \]
\[ - \sigma^* \varepsilon - \bar{\sigma}^* (\psi - \tilde{\chi}) + \bar{\chi}^* \Sigma + \chi^* \eta - \varphi^* (\phi - \chi) - \beta^* \eta^\mu. \] (2.14)

If the gauge-fixed fermion \( \Psi \) depends only on the original fields, then a general gauge-fixing Lagrangian density for Abelian rank-2 antisymmetric tensor field with original BRST symmetry will have the following form

\[ L_{gf} = s_b \Psi = s_b B_{\mu \nu} \frac{\delta \Psi}{\delta B_{\mu \nu}} + s_b \rho_\mu \frac{\delta \Psi}{\delta \rho_\mu} + s_b \tilde{\rho}_\mu \frac{\delta \Psi}{\delta \bar{\rho}_\mu} + s_b \sigma \frac{\delta \Psi}{\delta \sigma} \]
\[ + s_b \bar{\sigma} \frac{\delta \Psi}{\delta \bar{\sigma}} + s_b \beta_\mu \frac{\delta \Psi}{\delta \beta_\mu} + s_b \chi \frac{\delta \Psi}{\delta \chi} + s_b \bar{\chi} \frac{\delta \Psi}{\delta \bar{\chi}} + s_b \varphi \frac{\delta \Psi}{\delta \varphi}. \]
\[ = \psi_{\mu \nu} \frac{\delta \Psi}{\delta B_{\mu \nu}} + \epsilon_\mu \frac{\delta \Psi}{\delta \rho_\mu} + \xi_\mu \frac{\delta \Psi}{\delta \bar{\rho}_\mu} + \frac{\delta \psi}{\delta \sigma} + \xi_\mu \frac{\delta \psi}{\delta \beta_\mu} + \frac{\delta \psi}{\delta \chi} + \frac{\delta \psi}{\delta \bar{\chi}} + \frac{\delta \psi}{\delta \varphi}. \] (2.15)

Using the properties of the fields the above gauge-fixed Lagrangian density can further be expressed as

\[ L_{gf} = - \frac{\delta \psi}{\delta B_{\mu \nu}} \psi_{\mu \nu} + \frac{\delta \psi}{\delta \rho_\mu} \epsilon_\mu + \frac{\delta \psi}{\delta \bar{\rho}_\mu} \xi_\mu - \frac{\delta \psi}{\delta \sigma} \varepsilon \]
\[ - \frac{\delta \psi}{\delta \bar{\sigma}} \psi - \frac{\delta \psi}{\delta \beta_\mu} \eta_\mu + \frac{\delta \psi}{\delta \chi} \Sigma + \frac{\delta \psi}{\delta \bar{\chi}} \eta - \frac{\delta \psi}{\delta \varphi} \phi. \] (2.16)

Now, the total Lagrangian density \( \mathcal{L}_T = \mathcal{L}_0 + \mathcal{L}_{gf} + \bar{\mathcal{L}}_{gf} \) is then given as

\[ \mathcal{L}_T = \frac{1}{12} F_{\mu \nu \rho} F^{\mu \nu \rho} + B_{\mu \nu}^* (\partial^\mu \rho^\nu - \partial^\nu \rho^\mu) + i \bar{\rho}_\mu \partial^\mu \sigma - i \rho^*_\mu \bar{\beta}^\mu + \tilde{\sigma}^* \tilde{\chi} + \varphi^* \chi. \]
\[-(B^*_{\mu\nu} + \frac{\delta \Psi}{\delta B^{\mu\nu}}) \psi^{\mu\nu} + \left( \rho^*_{\mu} + \frac{\delta \Psi}{\delta \rho^\mu} \right) \xi^\mu + \left( \tilde{\rho}^*_{\mu} + \frac{\delta \Psi}{\delta \tilde{\rho}^\mu} \right) \epsilon^\mu - \left( \sigma^* + \frac{\delta \Psi}{\delta \sigma} \right) \varepsilon \]
\[-(\tilde{\sigma}^* + \frac{\delta \Psi}{\delta \tilde{\sigma}}) \psi + \left( \tilde{\chi}^* + \frac{\delta \Psi}{\delta \tilde{\chi}} \right) \Sigma + \left( \chi^* + \frac{\delta \Psi}{\delta \chi} \right) \eta - \left( \varphi^* + \frac{\delta \Psi}{\delta \varphi} \right) \phi \]
\[-(\beta^*_{\mu} + \frac{\delta \Psi}{\delta \beta^\mu}) \eta^\mu, \quad (2.17)\]

where we have used the Eqs. (2.1), (2.14) and (2.16). Integration over ghost fields associated with the shift symmetry leads to the following identification

\[
B^*_{\mu\nu} = -\frac{\delta \Psi}{\delta B^{\mu\nu}}, \quad \tilde{\rho}^*_{\mu} = -\frac{\delta \Psi}{\delta \tilde{\rho}^\mu}, \quad \rho^*_{\mu} = -\frac{\delta \Psi}{\delta \rho^\mu}, \quad \sigma^* = -\frac{\delta \Psi}{\delta \sigma}, \quad \tilde{\sigma}^* = -\frac{\delta \Psi}{\delta \tilde{\sigma}}, \quad \tilde{\chi}^* = -\frac{\delta \Psi}{\delta \tilde{\chi}}, \quad \chi^* = -\frac{\delta \Psi}{\delta \chi}, \quad \beta^*_{\mu} = -\frac{\delta \Psi}{\delta \beta^\mu}, \quad \varphi^* = -\frac{\delta \Psi}{\delta \varphi}. \quad (2.18)\]

Using above equation and the gauge-fixed fermion given in Eq. (2.7), we obtain the antifields associated with this theory as

\[
B^*_{\mu\nu} = -i \partial_\mu \tilde{\rho}_\nu, \quad \tilde{\rho}^*_{\mu} = -i \partial_\mu \tilde{\sigma}, \quad \rho^*_{\mu} = i (\partial^\nu B_{\nu\mu} - k_1 \beta_{\mu}), \quad \sigma^* = 0 \]
\[
\tilde{\sigma}^* = i \partial_\mu \rho^\mu, \quad \tilde{\chi}^* = 0, \quad \chi^* = -ik_2 \varphi, \quad \beta^*_{\mu} = ik_1 \tilde{\rho}_{\mu}, \quad \varphi^* = i (\partial_\mu \rho^\mu - k_2 \tilde{\chi}). \quad (2.19)\]

With these identification the total Lagrangian density reduces to the original theory for Abelian rank-2 tensor field given in Eq. (2.3). Now, we are able to write the gauge-fixing part of the total Lagrangian density in terms of the BRST variation of a generalized gauge-fixed fermion as follows

\[
\mathcal{L}_{gf} + \tilde{\mathcal{L}}_{gf} = s_b \left( B^*_{\mu\nu} \bar{B}^{\mu\nu} + \rho^*_{\mu} \bar{\rho}^\mu + \tilde{\rho}^*_{\mu} \tilde{\rho}^\mu + \sigma^* \sigma + \tilde{\sigma}^* \tilde{\sigma} + \beta^*_{\mu} \tilde{\beta}^\mu \right) + \chi^* \tilde{\chi} + \tilde{\chi}^* \chi + \varphi^* \varphi \equiv \mathcal{L}_0 + s_b \Phi^* \bar{\Phi}, \quad (2.20)\]

where the fields \( \Phi^* \) and \( \bar{\Phi} \) are the generic notation for antifields and corresponding shifted fields respectively. As expected the ghost number of \( \Phi^* \bar{\Phi} \) is \(-1\). We recover the BV action for Abelian 2-form gauge theory with the identification of antifields given in Eq. (2.19).

Next, we construct the superspace formulation of such BRST invariant extended theory.
B. Extended BRST invariant superspace formulation

In this section we develop a superspace formalism of the extended BRST invariant theory discussed in the previous section. For this purpose we consider a superspace with coordinates \((x^\mu, \theta)\) where \(\theta\) is fermionic coordinate. In such a superspace the “superconnection” 2-form can be written as 

\[
\omega^{(2)} = \frac{1}{2!} \Omega_{\mu\nu}(x, \theta)(dx^\mu \wedge dx^\nu) + \mathcal{M}_\mu(x, \theta)(dx^\mu \wedge d\theta) + \mathcal{N}(x, \theta)(d\theta \wedge d\theta),
\]

where \(d\) is an exterior derivative and is defined as \(d = dx^\mu \partial_\mu + d\theta \partial_\theta\). The requirement for super curvature (field strength \(F^{(3)} = d\omega^{(2)}\)) to vanish along the \(\theta\) direction restricts the component of the superfields to have following form

\[
\begin{align*}
\Omega_{\mu\nu}(x, \theta) &= \Omega_{\mu\nu}(x) + \theta s_b \Omega_{\mu\nu}, \\
\mathcal{M}_\mu(x, \theta) &= \mathcal{M}_\mu(x) + \theta s_b \mathcal{M}_\mu, \\
\mathcal{N}(x, \theta) &= \mathcal{N}(x) + \theta s_b \mathcal{N}.
\end{align*}
\]

Similarly, we define all the superfields corresponding to each fields involved in extended BV action as

\[
\begin{align*}
\mathcal{B}_{\mu\nu}(x, \theta) &= \mathcal{B}_{\mu\nu}(x) + \theta \psi_{\mu\nu}, \\
\bar{\mathcal{B}}_{\mu\nu}(x, \theta) &= \bar{\mathcal{B}}_{\mu\nu}(x) + \theta (\psi_{\mu\nu} - \partial_\mu \rho_\nu + \partial_\nu \rho_\mu - \partial_\nu \bar{\rho}_\mu), \\
\bar{\mathcal{M}}_\mu(x, \theta) &= \bar{\mathcal{M}}_\mu(x) + \theta (\epsilon_\mu - i \partial_\mu \sigma + i \partial_\mu \bar{\sigma}), \\
\bar{\mathcal{N}}(x, \theta) &= \bar{\mathcal{N}}(x) + \theta \bar{\epsilon}, \\
\mathcal{S}_\mu(x, \theta) &= \mathcal{S}_\mu(x) + \theta \eta_\mu, \\
\bar{\mathcal{S}}_\mu(x, \theta) &= \bar{\mathcal{S}}_\mu(x) + \theta \bar{\eta}_\mu, \\
\bar{\mathcal{N}}(x, \theta) &= \bar{\mathcal{N}}(x) + \theta \bar{\psi}, \\
\mathcal{O}(x, \theta) &= \mathcal{O}(x) + \theta \Sigma, \\
\bar{\mathcal{O}}(x, \theta) &= \bar{\mathcal{O}}(x) + \theta \bar{\Sigma}. \\
\end{align*}
\]
The super antifields also have the following form

\[
\bar{B}^\star_{\mu\nu} = B^\star_{\mu\nu} + \theta L_{\mu\nu}, \quad \bar{M}^\star_\mu = \rho^\star_\mu + \theta M_\mu, \quad \bar{\tilde{M}}^\star_\mu = \tilde{\rho}^\star_\mu + \theta \tilde{M}_\mu,
\]

\[
\bar{S}^\star_\mu = \beta^\star_\mu + \theta S_\mu, \quad \bar{N}^\star = \sigma^\star + \theta N, \quad \bar{\tilde{N}}^\star = \tilde{\sigma}^\star + \theta \tilde{N},
\]

\[
\bar{O}^\star = \chi^\star + \theta O, \quad \bar{T}^\star = \varphi^\star + \theta T, \quad \bar{\tilde{O}}^\star = \tilde{\chi}^\star + \theta \tilde{O}.
\] (2.24)

The Eqs. (2.23) and (2.24) enable us to write

\[
\frac{\delta}{\delta \theta} B^\star_{\mu\nu} B^{\mu\nu} = L_{\mu\nu} B^{\mu\nu} - B^\star_{\mu\nu} (\psi_{\mu\nu} - \partial^\mu \rho^\nu + \partial^\nu \rho^\mu - \partial^\nu \tilde{\rho}^\mu),
\]

\[
\frac{\delta}{\delta \theta} \bar{M}^\star_\mu M^\mu = M_\mu \tilde{\rho}^\mu + \rho^\mu (\epsilon^\mu + i \partial^\mu \sigma - i \partial^\mu \tilde{\sigma}),
\]

\[
\frac{\delta}{\delta \theta} \bar{\tilde{M}}^\star_\mu M^\mu = M_\mu \rho^\mu + \rho^\mu (\xi^\mu - i \beta^\mu - i \tilde{\beta}^\mu),
\]

\[
\frac{\delta}{\delta \theta} \bar{N}^\star N = N \tilde{\sigma} - \sigma^\star \epsilon, \quad \frac{\delta}{\delta \theta} \bar{\tilde{N}}^\star \tilde{N} = \tilde{N} \sigma - \tilde{\sigma}^\star (\psi - \tilde{\chi} + \tilde{\chi}),
\]

\[
\frac{\delta}{\delta \theta} \bar{O}^\star O = \bar{O} \bar{\chi} + \chi^\star \Sigma, \quad \frac{\delta}{\delta \theta} \bar{\tilde{O}}^\star \tilde{O} = O \tilde{\chi} + \chi^\star \eta,
\]

\[
\frac{\delta}{\delta \theta} \bar{T}^\star T = T \bar{\phi} - \varphi^\star (\phi - \chi + \tilde{\chi}), \quad \frac{\delta}{\delta \theta} \bar{S}^\star_\mu S^\mu = S_\mu \tilde{\beta}^\mu - \beta^\mu \eta^\mu.
\] (2.25)

Then the gauge-fixed Lagrangian density for shift symmetry given in Eq. (2.13) can be written in the superspace formulation as

\[
\bar{L}_{gf} = \frac{\delta}{\delta \theta} \left[ B^\star_{\mu\nu} B^{\mu\nu} + \bar{M}^\star_\mu M^\mu + \bar{\tilde{M}}^\star_\mu M^\mu + \bar{N}^\star N + \bar{\tilde{N}}^\star \tilde{N} + \bar{O}^\star O + \bar{\tilde{O}}^\star \tilde{O} + \bar{T}^\star T + \bar{S}^\star_\mu S^\mu \right].
\] (2.26)

The \( \bar{L}_{gf} \) remains invariant under the extended BRST transformation as it belongs to the \( \theta \) component of superfields. If the gauge-fixing fermion depends only on the original fields, then one can define the fermionic superfield \( \Gamma \) as

\[
\Gamma = \Psi + \theta s_b \Psi,
\]

\[
= \Psi + \theta \left[ - \frac{\delta \Psi}{\delta B^\star_{\mu\nu}} \psi_{\mu\nu} + \frac{\delta \Psi}{\delta \rho_\mu} \rho_\mu + \frac{\delta \Psi}{\delta \tilde{\rho}_\mu} \tilde{\rho}_\mu - \frac{\delta \Psi}{\delta \sigma} \epsilon \right]
\]

\[
- \frac{\delta \Psi}{\delta \tilde{\sigma}} \bar{\psi} - \frac{\delta \Psi}{\delta \beta_\mu} \eta_\mu + \frac{\delta \Psi}{\delta \chi} \Sigma + \frac{\delta \Psi}{\delta \chi} \eta - \frac{\delta \Psi}{\delta \varphi} \phi \right].
\] (2.27)

With these realization the original gauge-fixing Lagrangian density \( L_{gf} \) in the superspace formalism can be expressed as

\[
L_{gf} = \frac{\delta \Gamma}{\delta \theta}.
\] (2.28)
Further we notice that invariance of $L_{gf}$ under the extended BRST transformation is assured as it is $\theta$ component of fermionic superfield. Combining Eqs. (2.26) and (2.28) we can write the total Lagrangian density in this formalism as

$$L_T = L_0 + L_{gf} + \bar{L}_{gf},$$

$$= L_0 + \frac{\delta}{\delta \theta} \left[ B^\mu_{\nu\rho} B^{\rho\nu} + \tilde{M}^*_\mu M^\mu + \tilde{\tilde{M}}^*_\mu \tilde{M}^\mu + \tilde{N}^* \tilde{N} + \tilde{\tilde{N}}^* \tilde{\tilde{N}} + \tilde{O}^* \tilde{O} + \tilde{\tilde{O}}^* \right] \tilde{T}^* + S^* S^\mu + \Gamma. \tag{2.29}$$

If one performs equations of motion of auxiliary fields and ghost fields associated with the shift symmetry, this Lagrangian density reduces to the original BRST invariant Lagrangian density.

C. Extended anti-BRST invariant BV action

In the previous subsections we have analyzed the extended BRST symmetry of the Abelian rank-2 antisymmetric tensor field and have developed the corresponding superspace formulation. In this subsection we construct the extended anti-BRST invariant Lagrangian density for the same theory. For that we start with the anti-BRST symmetry transformation $(s_{ab})$, under which the Lagrangian density for the 2-form gauge theory given in Eq. (2.3), remains invariant, as

$$s_{ab} B^\mu_{\nu\rho} = (\partial_{\mu} \tilde{\rho}_{\nu} - \partial_{\nu} \tilde{\rho}_{\mu}), \quad s_{ab} \tilde{\rho}_{\mu} = -i \partial_{\mu} \tilde{\sigma}, \quad s_{ab} \tilde{\sigma} = 0, \quad s_{ab} \rho_{\mu} = -i \beta_{\mu},$$

$$s_{ab} \beta_{\mu} = 0, \quad s_{ab} \sigma = \chi, \quad s_{ab} \chi = 0, \quad s_{ab} \varphi = -\tilde{\chi}, \quad s_{ab} \tilde{\chi} = 0. \tag{2.30}$$

We note that the above anti-BRST transformation does not absolutely anticommute with the BRST transformation given in Eq. (2.5) i.e. $\{ s_b, s_{ab} \} \neq 0$. But one can achieve the absolutely anticommutativity of these by considering a Curci-Ferrari (CF) type restriction [26] in this theory. We will emphasize these with more details in the case of Abelian 3-form gauge theory in the next subsection.

The gauge-fixed anti-fermion $\bar{\Psi}$ (gauge-fixing fermion in case of anti-BRST transfor-
mation) for this theory is defined as
\[
\bar{\Psi} = i [\rho_\nu (\partial_\mu B^{\mu \nu} + k_1 \beta^\nu) - \sigma \partial_\mu \bar{\rho}^\mu + \varphi (\partial_\mu \rho^\mu + k_2 \chi)] ,
\]  
(2.31)
to write the gauge-fixing part of the Lagrangian density in terms of anti-BRST variation of \(\bar{\Psi}\) as
\[
L_{gf} = i s_{ab} [\rho_\nu (\partial_\mu B^{\mu \nu} + k_1 \beta^\nu) - \sigma \partial_\mu \bar{\rho}^\mu + \varphi (\partial_\mu \rho^\mu + k_2 \chi)] .
\]  
(2.32)
Following the same procedure as in the case of BRST transformation, we demand that
\[
s_{ab} (\Phi - \bar{\Phi}) \text{ reproduces the anti-BRST transformations of ordinary fields (} \Phi) \text{ for the Abelian rank-2 antisymmetric tensor field theory mentioned in Eq. (2.30), which leads to the following transformations}
\]
\[
s_{ab} \bar{B}_{\mu \nu} = B^*_{\mu \nu}, \quad s_{ab} B_{\mu \nu} = B^*_{\mu \nu} + (\partial_\mu \bar{\rho}_\nu - \partial_\nu \bar{\rho}_\mu - \partial_\nu \bar{\rho}_\mu + \partial_\nu \bar{\rho}_\mu),
\]
\[
s_{ab} \bar{\rho}_\mu = \bar{\rho}_\mu^*, \quad s_{ab} \bar{\rho}_\mu = \bar{\rho}_\mu^* - i \partial_\mu \tilde{\sigma} + i \partial_\mu \bar{\tilde{\sigma}} , \quad s_{ab} \bar{\bar{\rho}}_\mu = \rho_\mu^* ,
\]
\[
s_{ab} \bar{\beta}_\mu = \beta^*_\mu , \quad s_{ab} \bar{\sigma} = \sigma^* , \quad s_{ab} \bar{\sigma} = \bar{\sigma}^* , \quad s_{ab} \bar{\beta}_{\mu} = \beta^*_{\mu} ,
\]
\[
s_{ab} \bar{\chi} = \chi^* , \quad s_{ab} \bar{\psi} = \varphi^* , \quad s_{ab} \bar{\varphi} = \varphi^* = \bar{\chi} + \bar{\chi} , \quad s_{ab} \bar{\xi} = \xi^* ,
\]
\[
s_{ab} \bar{\bar{\chi}} = \bar{\chi}^* , \quad s_{ab} \bar{\xi} = 0 , \quad \xi^* \equiv [B^*_{\mu \nu} , \bar{\rho}_\mu^* , \bar{\sigma}^* , \bar{\beta}^*_{\mu} , \psi , \sigma^* , \chi^* , \varphi^* , \bar{\chi}^*].
\]  
(2.33)
The ghost fields associated with the shift symmetry have the following extended anti-BRST transformations,
\[
s_{ab} \bar{\psi} = L_{\mu \nu} , \quad s_{ab} \bar{\epsilon}_\mu = M_\mu , \quad s_{ab} \epsilon_\mu = \bar{M}_\mu , \quad s_{ab} \bar{\xi} = N ,
\]
\[
s_{ab} \bar{\psi} = \tilde{N} , \quad s_{ab} \bar{\eta}_\mu = S_\mu , \quad s_{ab} \bar{\Sigma} = O , \quad s_{ab} \bar{\eta} = \bar{O} , \quad s_{ab} \bar{\phi} = T ,
\]
\[
s_{ab} \bar{\bar{M}}_\mu = O , \quad s_{ab} \bar{\omega} = 0 , \quad \omega \equiv [L_{\mu \nu} , M_\mu , N , \bar{N} , S_\mu , O , \bar{O} , T] .
\]  
(2.34)
These transformations along with the transformations in Eq. (2.33) consist the extended anti-BRST transformations under which the total Lagrangian density including the shift fields remains invariant. These transformations will be helpful to establish the results in superspace formulation of BV action in the next section.
D. Extended BRST and anti-BRST invariant superspace formulation

To write a Lagrangian density that is manifestly invariant under the both extended BRST and extended anti-BRST transformations we need to define a superspace with two Grassmannian coordinates $\theta$ and $\bar{\theta}$. All the superfields in this superspace are the function of $(x_\mu, \theta, \bar{\theta})$. In this situation the “super connection” 2-form $(\omega^{(2)})$ and field strength $(F^{(3)})$ are defined as

$$\omega^{(2)} = \frac{1}{2!} B_{\mu \nu}(x, \theta, \bar{\theta})(dx^\mu \wedge dx^\nu) + M_\mu(x, \theta, \bar{\theta})(dx^\mu \wedge d\theta) + N(x, \theta, \bar{\theta})(d\theta \wedge d\theta)$$

$$+ \bar{M}_\mu(x, \bar{\theta}, \bar{\theta})(dx^\mu \wedge d\bar{\theta}) + \bar{N}(x, \bar{\theta}, \bar{\theta})(d\bar{\theta} \wedge d\bar{\theta}) + T(x, \bar{\theta}, \bar{\theta})(d\bar{\theta} \wedge d\bar{\theta})$$

$$F^{(3)} = d\omega^{(2)},$$

where the exterior derivative has the following structure $d = dx^\mu \partial_\mu + d\theta \partial_\theta + d\bar{\theta} \partial_{\bar{\theta}}$.

The requirement of vanishing the field strength corresponding to the extended theory along the directions $\theta$ and $\bar{\theta}$ produces the superfields to have the following forms:

$$B_{\mu \nu}(x, \theta, \bar{\theta}) = B_{\mu \nu}(x) + \theta \psi_{\mu \nu} + \bar{\theta}(B^*_{\mu \nu} + \partial_{\mu} \bar{\rho}_\nu - \partial_{\nu} \bar{\rho}_\mu - \partial_{\nu} \bar{\rho}_\mu + \partial_{\mu} \bar{\rho}_\mu)$$

$$+ \theta \bar{\theta}[L_{\mu \nu} + i(\partial_{\nu} \beta_{\mu} - \partial_{\nu} \beta_{\mu} - \partial_{\nu} \bar{\beta}_{\mu} + \partial_{\nu} \bar{\beta}_{\mu})],$$

$$\bar{B}_{\mu \nu}(x, \theta, \bar{\theta}) = \bar{B}_{\mu \nu}(x) + \theta \psi_{\mu \nu} - \partial_{\mu} \bar{\rho}_\nu + \partial_{\nu} \bar{\rho}_\mu - \partial_{\nu} \bar{\rho}_\mu + \bar{\theta} B^*_{\mu \nu} + \bar{\theta} L_{\mu \nu},$$

$$M_\mu(x, \theta, \bar{\theta}) = \rho_\mu(x) + \theta \epsilon_\mu + \bar{\theta}(\rho^*_\mu - i \beta_\mu + i \bar{\beta}_\mu) + \theta \bar{\theta} M_\mu,$$

$$\bar{M}_\mu(x, \bar{\theta}, \bar{\theta}) = \bar{\rho}_\mu(x) + \theta \epsilon_\mu - i \bar{\theta} \sigma + i \partial_\mu \bar{\sigma}) + \theta \rho^*_\mu + \theta \bar{\theta} M_\mu,$$

$$N(x, \theta, \bar{\theta}) = \sigma(x) + \theta \epsilon + \bar{\theta}(\sigma^* + \chi - \bar{\chi}) + \theta \bar{\theta} N, \quad \bar{N}(x, \theta, \bar{\theta}) = \bar{\sigma}(x) + \theta \epsilon + \sigma^* + \theta \bar{\theta} N,$$

$$\bar{M}_\mu(x, \theta, \bar{\theta}) = \bar{\rho}_\mu(x) + \theta \xi_\mu + \bar{\theta}(\bar{\rho}^*_\mu - i \partial_\mu \bar{\sigma} + i \partial_\mu \bar{\sigma}) + \theta \bar{\theta}(\bar{M}_\mu - i \partial_\mu \bar{\chi} + i \partial_\mu \bar{\chi}),$$

$$\bar{N}(x, \theta, \bar{\theta}) = \bar{\sigma}(x) + \theta \psi + \bar{\theta} \sigma^* + \theta \bar{\theta} N, \quad \bar{\bar{N}}(x, \theta, \bar{\theta}) = \bar{\bar{\sigma}}(x) + \theta (\psi - \bar{\chi} + \bar{\psi}) + \bar{\theta} \sigma^* + \theta \bar{\theta} \bar{N},$$

$$\bar{O}(x, \theta, \bar{\theta}) = \chi(x) + \theta \Sigma + \bar{\theta} \chi^* + \theta \bar{\theta} O, \quad \bar{O}(x, \theta, \bar{\theta}) = \bar{\chi}(x) + \theta \Sigma + \bar{\theta} \chi^* + \theta \bar{\theta} O,$$

$$\bar{O}(x, \theta, \bar{\theta}) = \bar{\chi}(x) + \theta \eta + \bar{\theta} \bar{\chi}^* + \theta \bar{\theta} O, \quad \bar{O}(x, \theta, \bar{\theta}) = \bar{\chi}(x) + \theta \eta + \bar{\theta} \bar{\chi}^* + \theta \bar{\theta} O,$$
Lagrangian density in BV formulation can be recovered. The antifields can be calculated. With these antifields the original gauge-fixed equations of motion for auxiliary fields and the ghost fields associated with shift transformations. Furthermore we define the super gauge-fixed fermion as

\[ L_{gf} = 1 \frac{\delta}{\delta \theta} \left[ \delta \phi + \bar{\phi}(\phi - \chi) + \theta \bar{\theta} T, \phi(x) + \theta \phi + \bar{\theta}(\phi^* - \bar{\chi} + \bar{\bar{\chi}}) + \theta \bar{\theta} T. \right. \]

\[ \bar{T}(x, \theta, \bar{\theta}) = \bar{\varphi}(x) + \theta (\phi - \chi) + \bar{\theta} \varphi^* + \bar{\theta} \bar{T}. \]  

From the structure of superfields, we calculate the following relations

\[ \frac{1}{2} \frac{\delta}{\delta \theta} \frac{\delta}{\delta \theta} \bar{B}_{\mu \nu} \bar{B}^{\mu \nu} = L_{\mu \nu} \bar{B}^{\mu \nu} - B_{\mu \nu}^* (\psi^{\mu \nu} - \partial^{\mu} \partial^{\nu} + \partial^{\mu} \rho^* - \partial^{\nu} \rho^*), \]

\[ \frac{\delta}{\delta \theta} \frac{\delta}{\delta \theta} \bar{M}_\mu \bar{M}^\mu = \bar{M}_\mu \bar{\rho}^\mu + \bar{\rho}^\mu (\epsilon^\mu + i \partial^\mu \sigma - i \partial^\mu \bar{\sigma}) + M_\mu \bar{\rho}^\mu + \rho^\mu (\xi^\mu - i \beta^\mu + i \bar{\beta}^\mu), \]

\[ \frac{1}{2} \frac{\delta}{\delta \theta} \bar{N} \bar{N} = \bar{N} \sigma - \sigma^e, \frac{1}{2} \frac{\delta}{\delta \theta} \bar{N} \bar{N} = \bar{N} \sigma - \bar{\sigma}^*(\psi - \bar{\chi} + \bar{\bar{\chi}}), \]

\[ \frac{1}{2} \frac{\delta}{\delta \theta} \bar{S}_\mu \bar{S}^\mu = S_\mu \bar{\beta}^\mu - \beta^\mu \eta^\mu. \]  

The Lagrangian density \( L_{gf} \) given in Eq. 2.13 can be written using the above relations as

\[ \bar{L}_{gf} = 1 \frac{\delta}{\delta \theta} \left[ \bar{B}_{\mu \nu} \bar{B}^{\mu \nu} + 2 \bar{M}_\mu \bar{M}^\mu + \bar{N} \bar{N} + \bar{N} \bar{N} + 2 \bar{O} \bar{O} + \bar{T} \bar{T} + \bar{S}_\mu \bar{S}^\mu \right]. \]  

This implies \( \bar{L}_{gf} \) is \( \theta \bar{\theta} \) component of the superfield in the square bracket of Eq. 2.39. Hence the \( \bar{L}_{gf} \) is invariant under extended BRST as well as extended anti-BRST transformations. Furthermore we define the super gauge-fixed fermion as

\[ \Gamma(x, \theta, \bar{\theta}) = \Psi + \theta b \Psi + \bar{\theta} s_b \Psi + \theta \bar{\theta} b s_b \Psi, \]  

\[ \text{to express the} \quad L_{gf} \quad \text{as} \quad \frac{\delta}{\delta \theta} \left[ \Gamma(x, \theta, \bar{\theta}) \right]. \]  

The \( \theta \bar{\theta} \) component of \( \Gamma(x, \theta, \bar{\theta}) \) vanishes due to equations of motion in the theories having both BRST and anti-BRST invariance.

The complete gauge-fixed Lagrangian density which is invariant under both extended BRST and extended anti-BRST transformations can therefore be written as

\[ L_{gf} + \bar{L}_{gf} = 1 \frac{\delta}{\delta \theta} \frac{\delta}{\delta \theta} \left[ \bar{B}_{\mu \nu} \bar{B}^{\mu \nu} + 2 \bar{M}_\mu \bar{M}^\mu + \bar{N} \bar{N} + \bar{N} \bar{N} + 2 \bar{O} \bar{O} + \bar{T} \bar{T} + \bar{S}_\mu \bar{S}^\mu \right] \]

\[ + \frac{\delta}{\delta \theta} \left[ \Gamma(x, \theta, \bar{\theta}) \right]. \]  

Using equations of motion for auxiliary fields and the ghost fields associated with shift symmetry the antifields can be calculated. With these antifields the original gauge-fixed Lagrangian density in BV formulation can be recovered.
III. BV FORMULATION OF 3-FORM GAUGE THEORY IN SUPERSPACE

The study of the higher dimensional 3-form gauge theories is important as it appears in the excitations of the quantized versions of strings, superstrings and related extended objects [1, 2]. In this section we will follow the same technique, as in the previous section, to develop the superspace formulation of extended BRST and anti-BRST invariant Abelian 3-form gauge theory in a covariant manner.

A. Extended BRST invariant BV action

Here we start with the classical Lagrangian density for the Abelian 3-form gauge theory as

\[ \mathcal{L}_0 = \frac{1}{24} H_{\mu\nu\eta\xi} H^{\mu\nu\eta\xi}, \]  

where the field strength (curvature) tensor in terms of totally antisymmetric tensor gauge field \( B_{\mu\nu\eta} \) is defined as

\[ H_{\mu\nu\eta\xi} = \partial_\mu B_{\nu\eta\xi} - \partial_\nu B_{\eta\xi\mu} + \partial_\eta B_{\xi\mu\nu} - \partial_\xi B_{\mu\nu\eta}. \]  

This Lagrangian density is invariant under the infinitesimal gauge transformation for the gauge field \( B_{\mu\nu\eta} \) can be written as

\[ \delta B_{\mu\nu\eta} = \partial_\mu \lambda_{\nu\eta} + \partial_\nu \lambda_{\eta\mu} + \partial_\eta \lambda_{\mu\nu}, \]  

where \( \lambda_{\mu\nu} \) is an arbitrary antisymmetric parameter. To write the absolutely anticommuting BRST and anti-BRST invariant BV action for 3-form theory, we consider the two equivalent candidates for the gauge-fixing part including ghost term of the the Lagrangian density as [27]

\[ \mathcal{L}_{gf}^B = (\partial_\mu B_{\nu\eta}) B_{\nu\eta} + \frac{1}{2} B_{\mu\nu} \tilde{B}^{\mu\nu} + (\partial_\mu \tilde{c}_{\nu\eta} + \partial_\nu \tilde{c}_{\eta\mu} + \partial_\eta \tilde{c}_{\mu\nu}) \partial^\mu c_{\nu\eta} \\
- (\partial_\mu \tilde{\beta}_\nu - \partial_\nu \tilde{\beta}_\mu) \partial^\mu \beta^\nu - BB_2 - \frac{1}{2} B_1^2 + (\partial_\mu \tilde{c}^{\mu\nu}) f_{\nu} - (\partial_\mu \tilde{c}^{\mu\nu}) \tilde{F}_{\nu}. \]
\[ L_{gf}^B = -(\partial_\mu B^{\nu\eta})\tilde{B}_{\nu\eta} + \frac{1}{2} B_{\mu\nu} \tilde{B}^{\mu\nu} + (\partial_\mu \beta^\nu) B_2 + (\partial_\mu \phi^\nu) B_1 - (\partial_\mu \tilde{\beta}^\mu) B, \]
\[ f_\mu + F_\mu = \partial_\mu c_1, \quad \tilde{f}_\mu + \tilde{F}_\mu = \partial_\mu \tilde{c}_1, \quad B_{\mu\nu} + \tilde{B}_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu. \]

Both the Lagrangian densities can be written in both BRST and anti-BRST exact terms as

\[ L_{gf}^B = s_b s_{ab} \left[ \frac{1}{2} c_2 c_2 - \frac{1}{2} \tilde{c}_1 c_1 - \frac{1}{2} c_{\mu\nu} c^{\mu\nu} - \tilde{\beta}_\mu \tilde{\beta}^\mu - \frac{1}{2} \phi_\mu \phi^\mu - \frac{1}{6} B_{\mu\nu\eta} B^{\mu\nu\eta} \right]. \]
\[ L_{gf}^{\tilde{B}} = -s_{ab} s_b \left[ \frac{1}{2} \tilde{c}_2 \tilde{c}_2 - \frac{1}{2} \tilde{c}_1 \tilde{c}_1 - \frac{1}{2} \tilde{c}_{\mu\nu} \tilde{c}^{\mu\nu} - \tilde{\beta}_\mu \tilde{\beta}^\mu - \frac{1}{2} \tilde{\phi}_\mu \tilde{\phi}^\mu - \frac{1}{6} B_{\mu\nu\eta} B^{\mu\nu\eta} \right]. \]

The absolute anticommuting BRST \((s_b)\) and anti-BRST \((s_{ab})\) transformations, which leave the Lagrangian densities given in Eqs. (3.3) and (3.5) invariant, are

\[ s_b B_{\mu\nu\eta} = \left( \partial_\mu c_{\nu\eta} + \partial_\nu c_{\eta\mu} + \partial_\eta c_{\mu\nu} \right), \quad s_{ab} c_{\mu\nu} = \partial_\mu \beta_\nu - \partial_\nu \beta_\mu, \quad s_b \tilde{c}_{\mu\nu} = B_{\mu\nu}. \]
\[ s_b \tilde{B}_{\mu\nu} = \left( \partial_\mu \tilde{c}_{\nu\eta} + \partial_\nu \tilde{c}_{\eta\mu} + \partial_\eta \tilde{c}_{\mu\nu} \right), \quad s_{ab} \tilde{c}_{\mu\nu} = \partial_\mu \tilde{\beta}_\nu - \partial_\nu \tilde{\beta}_\mu, \quad s_{ab} \tilde{c}_{\mu\nu} = \tilde{B}_{\mu\nu}. \]
\[ s_{ab} B_{\mu\nu} = \left( \partial_\mu \tilde{c}_{\nu\eta} + \partial_\nu \tilde{c}_{\eta\mu} + \partial_\eta \tilde{c}_{\mu\nu} \right), \quad s_{ab} c_{\mu\nu} = \partial_\mu \beta_\nu - \partial_\nu \beta_\mu, \quad s_{ab} \tilde{c}_{\mu\nu} = B_{\mu\nu}. \]

Since \( L_{gf}^B \) is BRST invariant it can be written in terms of BRST variation of a \( \Psi \) as

\[ L_{gf}^B = s_b \Psi = s_b \left[ -\frac{1}{2} \tilde{c}_2 B + \frac{1}{2} B_2 c_1 - \frac{1}{2} \tilde{c}_1 B_1 - \frac{1}{2} \left( \partial_\mu \tilde{\beta}_\nu - \partial_\nu \tilde{\beta}_\mu \right) c^{\mu\nu} \right]. \]
Such BV action also remains invariant under the extended BRST transformations of fields as follows

\[ \Psi = -\frac{1}{2} \tilde{\varepsilon} B + \frac{1}{2} \tilde{B} c_{\mu} - \frac{1}{2} \partial \beta - c_{\mu} - \tilde{c}_{\mu} \rightarrow c_{\mu} - \tilde{c}_{\mu}, \quad B_{\mu} \rightarrow B_{\mu} - \tilde{B}_{\mu}, \]

The requirement for an extended Lagrangian density in BV formulation is that the original Lagrangian density should be invariant under both the original BRST transformations and the shift transformations of the original fields. Therefore, we make the shift transformations as follows

\[ B_{\mu \nu \eta} \rightarrow B_{\mu \nu \eta} - \tilde{B}_{\mu \nu \eta}, \quad c_{\mu \nu} \rightarrow c_{\mu \nu} - \tilde{c}_{\mu \nu}, \quad \beta_{\mu \nu} \rightarrow \beta_{\mu \nu} - \tilde{\beta}_{\mu \nu}, \quad \beta_{\mu \nu} \rightarrow \beta_{\mu \nu} - \tilde{\beta}_{\mu \nu}, \quad F_{\mu} \rightarrow F_{\mu} - \tilde{F}_{\mu}, \]

where the bar fields are the shifted one corresponding to the original fields. The extended BRST invariant formulation of the BV action for this theory is achieved by choosing the following Lagrangian density:

\[ \mathcal{L}_{gf}^B = \mathcal{L}_{gf}(B_{\mu \nu \eta} - \tilde{B}_{\mu \nu \eta}, c_{\mu \nu} - \tilde{c}_{\mu \nu}, \beta_{\mu \nu} - \tilde{\beta}_{\mu \nu}, B_{\mu \nu} - \tilde{B}_{\mu \nu}, F_{\mu} - \tilde{F}_{\mu}, B - \tilde{B}, B_{1} - \tilde{B}_{1}, B_{2} - \tilde{B}_{2}, \]

Such BV action also remains invariant under the extended BRST transformations of fields as follows

\[ s_{\beta} \Phi(x) = \alpha(x), \quad s_{\beta} \Phi(x) = \alpha(x) - \beta(x), \]

where \( \Phi(x) \) represents the set of all fields and \( \Phi(x) \) represents the set of shifted ones. Here \( \beta(x) \) represents the original BRST transformation of collective fields \( \Phi \), whereas \( \alpha(x) \)
corresponds to some collective fields which generates the shift in fields. The extended BRST transformation for such theory is given explicitly in Appendix A (in Eqs. (A1) and (A2)).

We require all the shift fields to vanish in order to obtain the original theory. To achieve this goal, we choose the following gauge-fixing part of Lagrangian density for shift symmetry as

\[
\tilde{L}^B_{gf} = l_{\mu\nu}(\bar{B}^\mu\nu - B^*_{\mu\nu}(L_{\mu\nu} - \partial^\mu c^\nu - \partial^\nu c^\mu + \partial^\rho c^{\mu\nu} - \partial^\mu c^{\nu\rho} + \partial^\nu c^{\rho\mu} + \bar{m}_{\mu\nu}c^{\mu\nu})
+ c^*_{\mu\nu}(M^{\mu\nu} - \partial^\mu \beta^\nu + \partial^\nu \beta^\mu - \partial^\nu \beta^\mu) + m_{\mu\nu}\bar{c}^{\mu\nu} + c^*_{\mu\nu}(\bar{M}^{\mu\nu} - B^{\mu\nu} + \bar{B}^{\mu\nu})
+ n_{\mu\nu}\bar{B}^{\mu\nu} - B^*_{\mu\nu}N^{\mu\nu} + \bar{n}_{\mu\nu}\bar{B}^{\mu\nu} - \bar{E}^\mu_{\mu\nu}(\bar{N}^{\mu\nu} - \partial^\mu f^\nu + \partial^\nu f^\mu + \partial^\nu \bar{f}^\mu)
+ \beta^\mu(\bar{O}^\mu - \partial^\mu c_2 + \partial^\nu c_2) + \bar{c}_2(\bar{O}^\mu - \bar{F}^\mu + \bar{F}^\mu) + \bar{p}_\mu\bar{F}^\mu
+ \bar{f}_{\mu}(\bar{O}^\mu - \partial^\mu B_1 + \partial^\rho B_1) + \bar{c}_2 R + \bar{c}_2(\bar{R} - \bar{B}_2 + \bar{B}_2) + \bar{s}_1
+ \bar{c}_1(S + B - \bar{B}) + \bar{s}_1 + c_2^*(S - B_1 + \bar{B}_1) + \bar{t}_\mu\bar{g}^\mu - \phi^*_\mu(T^\mu - f^\mu + \bar{f}^\mu) + u\bar{B}
- B^*U + u\bar{B}_1 - B^*_1V + w\bar{B}_2 - B^*_2W,
\]

(3.16)

where the fields \(L_{\mu\nu}, M_{\mu\nu}, \bar{M}_{\mu\nu}, N_{\mu\nu}, \bar{N}_{\mu\nu}, \bar{O}_{\mu}, \bar{\bar{O}}_{\mu}, \bar{P}_{\mu}, \bar{\bar{P}}_{\mu}, \bar{Q}_{\mu}, \bar{\bar{Q}}_{\mu}, R, \bar{R}, S, \bar{S}, T_{\mu}, U, V, W\) are ghost fields associated with the shift symmetries for fields \(B_{\mu\nu}, c_{\mu\nu}, \bar{c}_{\mu\nu}, B_{\mu\nu}, \bar{B}_{\mu\nu}, \beta_{\mu}, \bar{\beta}_{\mu}, F_{\mu}, \bar{F}_{\mu}, f_{\mu}, \tilde{f}_{\mu}, c_2, \bar{c}_2, c_1, \bar{c}_1, \phi_{\mu}, B, B_1, B_2\) respectively and fields \(l_{\mu\nu}, m_{\mu\nu}, \bar{m}_{\mu\nu}, n_{\mu\nu}, \bar{n}_{\mu\nu}, o_{\mu}, \bar{o}_{\mu}, p_{\mu}, \tilde{p}_{\mu}, q_{\mu}, \bar{q}_{\mu}, r, \bar{r}, s, \bar{s}, t_{\mu}, u, v, w\) are Nakanishi-Lautrup type auxiliary fields corresponding to the antighost fields \(c^*_{\mu\nu}, \bar{c}^*_{\mu\nu}, B^*_{\mu\nu}, B^*_1, B^*_2\) respectively.

Such a Lagrangian density \(\tilde{L}^B_{gf}\) is invariant under the extended BRST symmetry transformations given in Eqs. (A1) and (A2) (see Appendix). All bar fields disappear when we use the equations of motion for auxiliary fields and the extended Lagrangian density takes the form

\[
\tilde{L}^B_{gf} = -B^*_{\mu\nu}(L_{\mu\nu} - \partial^\mu c^\nu - \partial^\nu c^\mu + \partial^\rho c^{\mu\nu} - \partial^\mu c^{\nu\rho} + \partial^\nu c^{\rho\mu} + \bar{m}_{\mu\nu}c^{\mu\nu})
+ c^*_{\mu\nu}(M_{\mu\nu} - \partial^\mu \beta^\nu + \partial^\nu \beta^\mu) + m_{\mu\nu}\bar{c}^{\mu\nu} + c^*_{\mu\nu}(\bar{M}_{\mu\nu} - B_{\mu\nu} + \bar{B}_{\mu\nu})
+ n_{\mu\nu}\bar{B}_{\mu\nu} - B^*_{\mu\nu}N_{\mu\nu} + \bar{n}_{\mu\nu}\bar{B}_{\mu\nu} - \bar{E}^\mu_{\mu\nu}(\bar{N}_{\mu\nu} - \partial^\mu f^\nu + \partial^\nu f^\mu + \partial^\nu \bar{f}^\mu)
+ \beta^\mu(\bar{O}_{\mu} - \partial^\mu c_2 + \partial^\nu c_2) + \bar{c}_2(\bar{O}_{\mu} - \bar{F}_{\mu} + \bar{F}_{\mu}) + \bar{p}_\mu\bar{F}_{\mu}
+ \bar{f}_{\mu}(\bar{O}_{\mu} - \partial^\mu B_1 + \partial^\rho B_1) + \bar{c}_2 R + \bar{c}_2(\bar{R} - \bar{B}_2 + \bar{B}_2) + \bar{s}_1
+ \bar{c}_1(S + B - \bar{B}) + \bar{s}_1 + c_2^*(S - B_1 + \bar{B}_1) + \bar{t}_\mu\bar{g}^\mu - \phi^*_\mu(T^\mu - f^\mu + \bar{f}^\mu) + u\bar{B}
- B^*U + u\bar{B}_1 - B^*_1V + w\bar{B}_2 - B^*_2W,
\]
\[ L_B^g = L_B^g = s_b \Psi[\Phi] = \Sigma(s_b \Phi) \frac{\delta \Psi}{\delta B}, \]

where \( \Phi \) is the generic notation for all fields in the theory. Keeping the fermionic/bosonic nature of fields in mind the above gauge-fixed Lagrangian density can be re-expressed as

\[
L_B^g = -\frac{\delta \Sigma}{\delta B_{\mu\nu\eta}} L_{\mu\nu\eta} + \frac{\delta \Sigma}{\delta c_{\mu\nu}} M_{\mu\nu} + \frac{\delta \Sigma}{\delta c_{\mu\nu}} N_{\mu\nu} - \frac{\delta \Sigma}{\delta \bar{B}_{\mu\nu}} \bar{N}_{\mu\nu}
- \frac{\delta \Sigma}{\delta \beta_{\mu}} \bar{O}_{\mu} + \frac{\delta \Sigma}{\delta F_{\mu}} P_{\mu} + \frac{\delta \Sigma}{\delta f_{\mu}} Q_{\mu} + \frac{\delta \Sigma}{\delta \bar{f}_{\mu}} \bar{Q}_{\mu}
+ \frac{\delta \Sigma}{\delta c_{2}} \bar{R} + \frac{\delta \Sigma}{\delta c_{1}} \bar{S} - \frac{\delta \Sigma}{\delta \phi_{\mu}} T_{\mu} - \frac{\delta \Sigma}{\delta B_{1}} U - \frac{\delta \Sigma}{\delta B_{2}} V - \frac{\delta \Sigma}{\delta \bar{B}_{B}} W. \tag{3.19}
\]

Combining all the Lagrangian densities of such theory given in Eqs. (3.5), (3.17) and (3.19), the total Lagrangian density for Abelian 3-form gauge theory in BV formulation can be written as

\[
L_T = L_0 + L_B^g + \tilde{L}_B^g,
= \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + B_{\mu\nu\eta} (\partial^{\mu} c^{\eta} + \partial^{\nu} c^{\mu} + \partial^{\eta} c^{\nu}) - \tilde{c}_{\mu} (\partial^{\mu} \beta^{\nu} - \partial^{\nu} \beta^{\mu})
- \tilde{c}_{\mu} B^{\mu} + \tilde{f}_{\mu} (\partial^{\mu} f^{\nu} - \partial^{\nu} f^{\mu}) + \beta^{\mu} \partial^{\nu} c_{2} + \tilde{f}_{\mu} \bar{F}^{\mu} + \bar{F}^{\mu} \partial^{\mu} B - f^{\mu} \partial^{\mu} B_{1}
- \tilde{c}_{2} B_{2} + \tilde{c}_{1} B - c_{1} B_{1} + \phi^{\mu} f^{\mu} - \left( B^{\mu\nu\eta} + \frac{\delta \Sigma}{\delta B_{\mu\nu\eta}} \right) L_{\mu\nu\eta} + \left( \tilde{c}_{\mu}^{\mu} + \frac{\delta \Sigma}{\delta c_{\mu}} \right) M_{\mu}
+ \left( c_{\mu}^{\nu} + \frac{\delta \Sigma}{\delta \bar{c}_{\mu}} \right) \bar{M}_{\mu} - \left( B^{\mu*} + \frac{\delta \Sigma}{\delta B_{\mu}} \right) N_{\mu}
- \left( \beta^{\mu*} + \frac{\delta \Sigma}{\delta \beta_{\mu}} \right) \bar{O}_{\mu} - \left( \tilde{f}^{\mu*} + \frac{\delta \Sigma}{\delta f_{\mu}} \right) \bar{P}_{\mu} + \left( f^{\mu*} + \frac{\delta \Sigma}{\delta f_{\mu}} \right) Q_{\mu} + \left( \tilde{c}^{\mu} + \frac{\delta \Sigma}{\delta \bar{c}_{\mu}} \right) \bar{R} + \left( c_{1} + \frac{\delta \Sigma}{\delta c_{1}} \right) \bar{S} - \left( \phi^{\mu*} + \frac{\delta \Sigma}{\delta \phi_{\mu}} \right) T_{\mu}
- \left( B^{*} + \frac{\delta \Sigma}{\delta B_{1}} \right) U - \left( B_{1}^{*} + \frac{\delta \Sigma}{\delta B_{1}} \right) V - \left( B_{2}^{*} + \frac{\delta \Sigma}{\delta B_{2}} \right) W. \tag{3.20}
\]
The antighost fields are identified as the antifields after eliminating ghost fields associated with the shift symmetry and are given as

\[ B^{\mu
u\eta} = \frac{1}{3} \left( \partial^\mu c^{\nu\eta} + \partial^\nu c^{\mu\eta} + \partial^\eta c^{\mu\nu} \right), \quad c^{\mu\nu} = -\frac{1}{2} B^{\mu
u}, \quad B^{\mu\nu} = -\frac{1}{2} \bar{c}^{\mu\nu}, \]

\[ \bar{c}^{\mu\nu} = \frac{1}{2} \left( \partial^\mu \bar{c}^\nu - \partial^\nu \bar{c}^\mu \right) - \partial_\eta B^{\mu\nu\eta}, \quad \bar{c}^{\mu\nu} = -\frac{1}{2} \partial^\mu \bar{c}_2, \quad \bar{c}^{\mu} = F^\mu + \partial_\nu c^{\mu\nu}, \]

\[ \bar{c}^{\mu} = \bar{\beta}^\mu, \quad f^{\mu} = \phi^\mu, \quad c_2^* = \frac{1}{2} B - \partial_\mu \beta^\mu, \quad c_1^* = -\frac{1}{2} B_2, \quad c_1^* = \frac{1}{2} B_1, \]

\[ \phi^{\mu} = \bar{\phi}^\mu, \quad B^* = \frac{1}{2} \bar{c}_2, \quad B_1^* = \frac{1}{2} \bar{c}_1, \quad B_2^* = -\frac{1}{2} \bar{c}_1, \quad [\bar{B}^{\mu\nu\eta^*}, F^{\mu*}, f^{\mu*}, \bar{c}^*] = 0. \quad (3.21) \]

We are now able to write the gauge-fixed part of total Lagrangian density as a BRST variation of a generalized gauge-fixing fermion

\[ L^B_{gf} + \bar{L}^B_{gf} = s_b \left( B^{\mu\nu\eta}_F \bar{B}^{\mu\nu\eta} + c^{\mu}_F \bar{c}^{\mu\nu} + \bar{c}^{\mu}_F c^{\mu\nu} + B^{\mu\nu}_F \bar{B}^{\mu\nu} + \beta^{\mu}_F \bar{\beta}^{\mu} + \bar{\beta}^{\mu}_F \beta^{\mu} + F^{\mu}_F \bar{F}^{\mu} + \bar{F}^{\mu}_F F^{\mu} \right. \]

\[ \left. + f^{\mu}_F \bar{f}^{\mu} + f^{\mu}_F \bar{f}^{\mu} + c^{\mu}_F \bar{c}^2 + c^2 F^2 + \bar{c}^2 F^2 + \bar{c}^2 F^2 + \bar{c}^2 F^2 + F^2 B^2 + B^2 \bar{B} + B^2 \bar{B} + B^2 \bar{B} \right), \]

\[ = s_b \left( \Phi^* \bar{\Phi} \right), \quad (3.22) \]

where the fields \( \Phi \) and \( \bar{\Phi} \) are the generic notation for all original fields and corresponding shifted fields respectively, the ghost number of the expression \( \Phi^* \bar{\Phi} = -1 \) as expected. Here we note the difference with the ordinary gauge-fixing fermion given as

\[ \Psi = -[B^{\mu\nu\eta} \bar{B}^{\mu\nu\eta} + \bar{c}^{\mu}_F c^{\mu\nu} + \bar{\beta}^{\mu}_F \beta^{\mu} + \phi^\mu \bar{\phi}^\mu + \bar{c}^2 + \bar{c}^2 F^2 + \bar{c}^2 F^2 + B^2 B^2]. \quad (3.23) \]

**B. Superspace formulation: Extended BRST invariant BV action**

In this subsection we develop a superspace formalism for extended BRST invariant BV action developed in the previous subsection. For this purpose we consider superspace with one fermionic parameter \( \theta \) and define the following superfields, \( \Upsilon \), in terms of generic fields \( \Phi \)

\[ \Upsilon(x, \theta) = \Phi(x) + \theta(s_b \Phi). \quad (3.24) \]

Explicit expressions for each superfield are listed in Appendix (see Eq. (A3)).
The gauge-fixing part of the Lagrangian density given in Eq. (3.5) is written, in this superspace formulation as

\[ \bar{L}_{gf}^{B} = \frac{\delta}{\delta \theta} \left[ \bar{B}_{\mu \nu \eta}^{*} \bar{B}^{\mu \nu \eta} + \tilde{\bar{c}}_{\mu \nu} \tilde{c}^{\mu \nu} + \bar{c}_{\mu \nu} \tilde{c}^{\mu \nu} + \bar{B}_{\mu \nu}^{*} \bar{B}^{\mu \nu} + \bar{B}_{\mu}^{*} \bar{B}^{\mu} + \bar{F}_{\mu}^{*} \bar{F}^{\mu} + \bar{f}_{\mu}^{*} \bar{f}^{\mu} \right. \\
\left. + \tilde{\bar{f}}_{\mu}^{*} \tilde{f}^{\mu} + \tilde{\bar{c}}_{1}^{*} \tilde{c}_{1} + \bar{c}_{1} \tilde{c}_{1} + \tilde{\bar{c}}_{2}^{*} \tilde{c}_{2} + \bar{c}_{2} \tilde{c}_{2} + \bar{B}^{*} \bar{B} + \bar{B}_{1}^{*} \bar{B}_{1} + \bar{B}_{2}^{*} \bar{B}_{2} \right]. \tag{3.25} \]

\( \bar{L}_{gf}^{B} \) remains invariant under the extended BRST transformation as it is \( \theta \) component of a superfield. The gauge-fixed Lagrangian density for the original symmetry can also be written in this formalism by defining \( \Gamma \) as

\[ \Gamma = \Psi + \theta s_{b} \Psi. \tag{3.26} \]

Assuming \( \Psi \) is a function of all original fields, we write

\[ \Gamma = \Psi + \theta \left[ \frac{\delta \Psi}{\delta B_{\mu \nu \eta}} L_{\mu \nu \eta} + \frac{\delta \Psi}{\delta c_{\mu \nu}} M_{\mu \nu} + \frac{\delta \Psi}{\delta \tilde{c}_{\mu \nu}} \tilde{M}_{\mu \nu} - \frac{\delta \Psi}{\delta \bar{B}_{\mu \nu}} N_{\mu \nu} - \frac{\delta \Psi}{\delta \bar{B}_{\mu \nu}} \tilde{N}_{\mu \nu} \right. \\
\left. - \frac{\delta \Psi}{\delta \beta_{\mu}} \tilde{O}_{\mu} + \frac{\delta \Psi}{\delta \bar{O}_{\mu}} \tilde{\bar{O}}_{\mu} + \frac{\delta \Psi}{\delta \tilde{F}_{\mu}} \tilde{\bar{P}}_{\mu} + \frac{\delta \Psi}{\delta \bar{P}_{\mu}} \tilde{P}_{\mu} + \frac{\delta \Psi}{\delta \tilde{f}_{\mu}} \tilde{\bar{Q}}_{\mu} + \frac{\delta \Psi}{\delta \bar{Q}_{\mu}} \tilde{Q}_{\mu} + \frac{\delta \Psi}{\delta \bar{c}_{2}} R \\
+ \frac{\delta \Psi}{\delta \bar{c}_{2}} \tilde{\bar{R}} + \frac{\delta \Psi}{\delta \bar{c}_{1}} S + \frac{\delta \Psi}{\delta \tilde{c}_{1}} \tilde{S} - \frac{\delta \Psi}{\delta \phi_{\mu}} \tilde{T}_{\mu} - \frac{\delta \Psi}{\delta \phi_{\mu}} \tilde{\bar{T}}_{\mu} \right]. \tag{3.27} \]

Thus, we write the original gauge-fixing Lagrangian density in the superspace formalism as

\[ \mathcal{L}^{B}_{gf} = \frac{\delta \Gamma}{\delta \theta}. \tag{3.28} \]

Once again, being the \( \theta \) component of a superfield, this is manifestly invariant under the extended BRST transformation. Now, we are able to write the total Lagrangian density in the superspace as

\[ \tilde{\mathcal{L}}^{B}_{gf} = \mathcal{L}_{0} + \mathcal{L}^{B}_{gf} + \mathcal{L}^{B}_{gf} \]
\[ = \mathcal{L}_{0} + \frac{\delta}{\delta \theta} \left[ \tilde{\Gamma}^{*} \tilde{\Gamma} \right] + \frac{\delta \Gamma}{\delta \theta}. \tag{3.29} \]

where \( \tilde{\Gamma}^{*} \) and \( \tilde{\Gamma} \) are the generic notation for the shift fields corresponding to the super antifields \( \Gamma^{*} \) and superfields \( \Gamma \) respectively. This Lagrangian density is manifestly invariant under the original BRST symmetry, after elimination of the auxiliary and ghost fields associated with the shift symmetry.
c. Extended anti-BRST invariant BV action

In the previous subsections we have analyzed the extended BRST symmetry for the Lagrangian density of the Abelian rank-3 antisymmetric tensor field and corresponding superspace formulation. In this subsection we study the extended anti-BRST symmetry for such theory.

The gauge-fixed anti-fermion $\bar{\Psi}$ for the theory is defined as

$$\bar{\Psi} = -\frac{1}{2}B_{2c2} + \frac{1}{2}B_{1c1} + \frac{1}{2}\tilde{c}_1B + \frac{1}{2}B_{\mu \nu}c^{\mu \nu} - \frac{1}{2}\tilde{c}_{\mu \nu}(\partial^\mu \beta^\nu - \partial^\nu \beta^\mu) + \tilde{F}_{\mu}\beta^\mu + \beta^\mu \partial^\mu c_2$$

$$+ \frac{1}{2}\phi^\mu f^\mu + \frac{1}{3}B_{\mu \nu \eta}(\partial^\mu c^{\nu \eta} + \partial^\nu c^{\mu \eta} + \partial^\eta c^{\mu \nu}).$$

So that the gauge-fixing part of the Lagrangian density can be written in terms of $\bar{\Psi}$ as

$$\mathcal{L}^B_{gf} = s_{ab}\bar{\Psi} = s_{ab}\left[-\frac{1}{2}B_{2c2} + \frac{1}{2}B_{1c1} + \frac{1}{2}\tilde{c}_1B + \frac{1}{2}B_{\mu \nu}c^{\mu \nu} - \frac{1}{2}\tilde{c}_{\mu \nu}(\partial^\mu \beta^\nu - \partial^\nu \beta^\mu) + \tilde{F}_{\mu}\beta^\mu$$

$$+ \beta^\mu \partial^\mu c_2 + \frac{1}{2}\phi^\mu f^\mu + \frac{1}{3}B_{\mu \nu \eta}(\partial^\mu c^{\nu \eta} + \partial^\nu c^{\mu \eta} + \partial^\eta c^{\mu \nu})\right].$$

Following the structure of Eq. (3.11) given in Appendix A, we demand that $s_{ab}(\Phi - \bar{\Phi})$ reproduces the anti-BRST transformations of ordinary fields (\Phi) for the Abelian rank-3 antisymmetric tensor field theory mentioned in Eq. (3.10), consequently we have following transformations

$$s_{ab}\tilde{B}_{\mu \nu \eta} = B^*_{\mu \nu \eta}, \quad s_{ab}B_{\mu \nu \eta} = B^*_{\mu \nu \eta} + \partial_\mu \tilde{c}_{\nu \eta} - \partial_\nu \tilde{c}_{\mu \eta} + \partial_\mu \tilde{c}_{\eta \mu} - \partial_\eta \tilde{c}_{\mu \nu} - \partial_\eta \tilde{c}_{\nu \mu} - \partial_\nu \tilde{c}_{\eta \mu},$$

$$s_{ab}\tilde{c}_{\mu \nu} = c^*_{\mu \nu}, \quad s_{ab}c_{\mu \nu} = c^*_{\mu \nu} + \partial_\mu \beta_{\nu} - \partial_\nu \beta_{\mu} - \partial_\nu \tilde{c}_{\mu} + \partial_\nu \tilde{c}_{\mu}, \quad s_{ab}\tilde{\beta}_{\mu} = \beta^*_{\mu},$$

$$s_{ab}c_{\mu \nu} = c^*_{\mu \nu} + \tilde{B}_{\mu \nu} - \tilde{B}_{\mu \nu}, \quad s_{ab}B_{\mu \nu} = B^*_{\mu \nu} + \partial_\mu \tilde{f}_{\nu} - \partial_\nu \tilde{f}_{\mu} - \partial_\nu \tilde{f}_{\mu} + \partial_\nu \tilde{f}_{\mu},$$

$$s_{ab}\tilde{B}_{\mu \nu} = B^*_{\mu \nu}, \quad s_{ab}\tilde{\beta}_{\mu} = \beta^*_{\mu}, \quad s_{ab}\beta_{\mu} = \beta^*_{\mu} + F_{\mu} - \tilde{F}_{\mu}, \quad s_{ab}\tilde{\beta}_{\mu} = \beta^*_{\mu},$$

$$s_{ab}\tilde{\beta}_{\mu} = \tilde{\beta}^*_{\mu} + \partial_\mu \tilde{c}_{2} - \partial_\mu \tilde{c}_{2}, \quad s_{ab}\tilde{F}_{\mu} = \tilde{F}^*_{\mu}, \quad s_{ab}\tilde{F}_{\mu} = \tilde{F}^*_{\mu} - \partial_\mu B_{2} + \partial_\mu \tilde{B}_{2},$$

$$s_{ab}\tilde{f}_{\mu} = \tilde{f}^*_{\mu}, \quad s_{ab}\tilde{f}_{\mu} = \tilde{f}^*_{\mu} - \partial_\mu B_{1} + \partial_\mu \tilde{B}_{1}, \quad s_{ab}\tilde{c}_{2} = \tilde{c}^*_{2}, \quad s_{ab}\tilde{c}_{2} = \tilde{c}^*_{2} + B - \tilde{B},$$

$$s_{ab}\tilde{c}_{1} = \tilde{c}^*_{1}, \quad s_{ab}\tilde{c}_{1} = \tilde{c}^*_{1} - B_{1} + \tilde{B}_{1}, \quad s_{ab}\tilde{c}_{2} = \tilde{c}^*_{2}, \quad s_{ab}\tilde{c}_{2} = \tilde{c}^*_{2} - B + \tilde{B},$$

$$s_{ab}\tilde{\phi}_{\mu} = \phi^*_{\mu}, \quad s_{ab}\tilde{\phi}_{\mu} = \phi^*_{\mu} + \tilde{f}_{\mu} - \tilde{f}_{\mu},$$

$$s_{ab}\tilde{\phi}_{\mu} = \tilde{\phi}^*_{\mu}, \quad s_{ab}\tilde{\phi}_{\mu} = \tilde{\phi}^*_{\mu} + \tilde{f}_{\mu} + \tilde{f}_{\mu},$$

$$s_{ab}\tilde{\phi}_{\mu} = \tilde{\phi}^*_{\mu}, \quad s_{ab}\tilde{\phi}_{\mu} = \tilde{\phi}^*_{\mu} - \tilde{f}_{\mu}.$$
\( s_{ab}B_1 = B_1^*, \ s_{ab}\bar{B}_2 = B_2^*, \ s_{ab}B_2 = B_2^*, \ s_{ab}\bar{B}_1 = B_1^*, \ s_{ab}\bar{B}_2 = \bar{B}_2^*, \ s_{ab}\bar{\Xi} = 0, \)

\[
\Xi \equiv [B_{\mu\nu\eta}^*, c_{\mu\nu}^*, B_{\mu\nu}^*, \beta_{\mu}^*, \bar{\beta}_{\mu}, F_{\mu}^*, \bar{F}_{\mu}^*, c_2^*, \phi_{\mu}^*, \bar{c}_2^*, \bar{\phi}_{\mu}, B_{\mu}^*, B_{\mu}^*, \bar{B}_{\mu}^*]. \tag{3.32}
\]

The ghost fields associated with the shift symmetry have the following extended anti-BRST transformations,

\[
s_{ab}L_{\mu\nu\eta} = l_{\mu\nu\eta}, \ s_{ab}M_{\mu\nu} = m_{\mu\nu}, \ s_{ab}\bar{M}_{\mu\nu} = \bar{m}_{\mu\nu}, \ s_{ab}N_{\mu\nu} = n_{\mu\nu},
\]

\[
s_{ab}\bar{N}_{\mu\nu} = \bar{n}_{\mu\nu}, \ s_{ab}Q_{\mu} = q_{\mu}, \ s_{ab}\bar{Q}_{\mu} = \bar{q}_{\mu}, \ s_{ab}T_{\mu} = t_{\mu}, \ s_{ab}U = u, \ s_{ab}V = v, \ s_{ab}W = w. \tag{3.33}
\]

\section{D. Extended BRST and anti-BRST invariant superspace formulation}

To write a Lagrangian density that is manifestly invariant under both extended BRST transformations and extended anti-BRST transformations in superspace formalism we introduce a superspace with two Grassmann parameters, \( \theta \) and \( \bar{\theta} \). The generic superfields in this superspace are defined as

\[
\Upsilon(x, \theta, \bar{\theta}) = \Phi(x) + \theta(s_b\Phi) + \bar{\theta}(s_{ab}\Phi) + \theta\bar{\theta}(s_b s_{ab}\Phi), \tag{3.34}
\]

where \( \Upsilon \) and \( \Phi \) are the generic notation for all the superfields and the fields respectively. The expressions of all the individual superfields are given in the Appendix A (see Eq. [A1]).

One can write the gauge-fixing Lagrangian density \( \bar{\mathcal{L}}_{gf}^B \) given in Eq. (3.5) in terms of these superfields as

\[
\bar{\mathcal{L}}_{gf}^B = \frac{1}{2} \frac{\delta}{\delta \theta} \frac{\delta}{\delta \bar{\theta}} \left[ \bar{B}_{\mu\nu\eta}B^{\mu\nu\eta} + 2\bar{c}_{\mu\nu}C^{\mu\nu} + \bar{B}_{\mu\nu}\bar{B}^{\mu\nu} + \bar{B}_{\mu}B^{\mu} + \bar{B}_{\mu}\bar{B}^{\mu} + \bar{B}_{\mu}\bar{B}^{\mu}
\right.
\]

\[
\left. + 2\bar{F}_{\mu}\bar{F}^{\mu} + 2\bar{\beta}_{\mu}\bar{\beta}^{\mu} + 2\bar{c}_2\bar{c}_2 + 2\bar{c}_1\bar{c}_1 + \bar{\phi}_{\mu}\bar{\phi}^{\mu} + \bar{B}\bar{B} + \bar{B}_1\bar{B}_1 + \bar{B}_2\bar{B}_2 \right]. \tag{3.35}
\]

This implies that the Lagrangian density \( \bar{\mathcal{L}}_{gf}^B \) is manifestly invariant under extended BRST and anti-BRST transformations. Furthermore, we define the gauge-fixing fermion
in superspace as
\[
\Gamma(x, \theta, \bar{\theta}) = \Psi + \theta s_b \Psi + \bar{\theta} s_{ab} \Psi + \theta \bar{\theta} s_{ab} \Psi.
\] (3.36)

The component of $\theta \bar{\theta}$ disappears from the above expression using equations of motion and therefore the Lagrangian density for the original fields can be written as
\[
\mathcal{L}_{gf}^\mu = \frac{\delta}{\delta \theta} \left[ \Gamma(x, \theta, \bar{\theta}) \right] = s_b \Psi.
\] (3.37)

This Lagrangian density is not only manifestly invariant under extended BRST transformations but also invariant under extended anti-BRST transformations. The complete gauge-fixed Lagrangian density can therefore be written as
\[
\mathcal{L}^\mu_{gf} + \tilde{\mathcal{L}}^\mu_{gf} = \frac{1}{2} \frac{\delta}{\delta \bar{\theta}} \frac{\delta}{\delta \theta} \left[ \tilde{B}_{\mu\nu\eta} \tilde{B}^{\mu\nu\eta} + 2 \tilde{\sigma}_{\mu} \tilde{\sigma}^{\mu} + 2 \tilde{B}_{\mu \nu} \tilde{B}^{\mu \nu} + \tilde{\beta}_{\mu} \tilde{\beta}^{\mu} + \tilde{\beta}^{\mu} \tilde{\beta}_{\mu} \\
+ 2 \bar{\tilde{F}}_{\mu} \tilde{F}^{\mu} + 2 \bar{\tilde{f}}_{\mu} \tilde{f}^{\mu} + 2 \bar{\tilde{c}}_2 \tilde{c}_2 + 2 \bar{\tilde{c}}_1 \tilde{c}_1 + \bar{\tilde{\phi}}_{\mu} \tilde{\phi}^{\mu} + \tilde{\phi} \bar{\tilde{B}} + \bar{B}_1 \tilde{B}_1 + \bar{B}_2 \tilde{B}_2 \\
+ \frac{\delta}{\delta \bar{\theta}} \left[ \Gamma(x, \theta, \bar{\theta}) \right]. \right)
\] (3.38)

Using equations of motion of auxiliary fields the tilde fields can made vanish and by integrating out the ghost fields for the shift symmetry we will get the explicit expressions for the antifields.

\section*{IV. CONCLUDING REMARKS}

Higher form gauge theories play a very important role in certain string theoretic and supergravity models. In this work we have considered the BV formulation of extended BRST and anti-BRST invariant (including some shift symmetry) 2-form and 3-form gauge theories. Antifields arise naturally in such formulation. We have further constructed a superspace formulation for these theories. We have shown that the BV action for 2-form as well as 3-form gauge theories can be written in a manifestly extended BRST invariant manner in a superspace with one fermionic coordinate. However, a superspace with two Grassmann coordinates are required for a manifestly covariant formulation of the extended BRST and extended anti-BRST invariant BV actions for higher form gauge theories. It will be interesting to extend this formulation for anomalous gauge theories.
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Appendix A: Mathematical details of Abelian 3-form gauge theory

1. Extended BRST transformation of fields

\[
\begin{align*}
    s_b B_{\mu \nu \eta} &= L_{\mu \nu \eta}, \quad s_b \bar{B}_{\mu \nu \eta} &= L_{\mu \nu \eta} - (\partial_\mu c_{\nu \eta} - \partial_\mu \bar{c}_{\nu \eta} + \partial_\nu c_{\eta \mu} - \partial_\nu \bar{c}_{\eta \mu} + \partial_\eta c_{\mu \nu} - \partial_\eta \bar{c}_{\mu \nu}), \\
    s_b c_{\mu \nu} &= M_{\mu \nu}, \quad s_b \bar{c}_{\mu \nu} = M_{\mu \nu} - (\partial_\mu \beta_\nu - \partial_\mu \bar{\beta}_\nu - \partial_\nu \beta_\mu + \partial_\nu \bar{\beta}_\mu), \quad s_b \tilde{c}_{\mu \nu} = \tilde{M}_{\mu \nu}, \\
    s_b \bar{c}_2 &= R, \quad s_b \bar{c}_1 &= S, \quad s_b \phi_\mu = T_\mu, \quad s_b \bar{\phi}_\mu = T_\mu - f_\mu + \tilde{f}_\mu, \quad s_b f_\mu = Q_\mu, \quad s_b \bar{f}_\mu = Q_\mu - B_2 - \bar{B}_2, \\
    s_b \tilde{c}_2 &= \tilde{R} - B_1 + \bar{B}_1, \quad s_b \tilde{c}_1 = \tilde{S} - B_1 + \bar{B}_1, \quad s_b \phi_\mu = T_\mu, \quad s_b \bar{\phi}_\mu = T_\mu - f_\mu + \tilde{f}_\mu, \quad s_b B = U, \quad s_b \bar{B} = V, \quad s_b \bar{B}_1 = V, \quad s_b B_1 = V, \quad s_b B_2 = W, \quad s_b \bar{B}_2 = W, \quad s_b \Omega = 0,
\end{align*}
\]

where \( \Omega \equiv [L_{\mu \nu \eta}, M_{\mu \nu}, \tilde{M}_{\mu \nu}, N_{\mu \nu}, \tilde{N}_{\mu \nu}, O_\mu, \tilde{O}_\mu, P_\mu, \tilde{P}_\mu, Q_\mu, \tilde{Q}_\mu, R, \tilde{R}, S, \tilde{S}, T_\mu, U, V, W]. \)
2. Extended BRST transformation of antifields

\[ s_b B_{\mu \nu}^* = l_{\mu \nu}, \quad s_b c_{\mu \nu}^* = m_{\mu \nu}, \quad s_b \bar{c}_{\mu \nu}^* = \bar{m}_{\mu \nu}, \quad s_b B_{\mu \nu}^* = n_{\mu \nu}, \]
\[ s_b \beta_{\mu}^* = o_{\mu}, \quad s_b \bar{\beta}_{\mu}^* = \bar{o}_{\mu}, \quad s_b \bar{F}_{\mu}^* = \bar{p}_{\mu}, \quad s_b \bar{F}_{\mu}^* = \bar{p}_{\mu}, \quad s_b f_{\mu}^* = q_{\mu}, \]
\[ s_b \bar{f}_{\mu}^* = \bar{q}_{\mu}, \quad s_b c_2^* = r, \quad s_b \bar{c}_2^* = \bar{r}, \quad s_b \bar{c}_1^* = s, \quad s_b \bar{c}_1^* = \bar{s}, \]
\[ s_b \phi_{\mu}^* = t_{\mu}, \quad s_b B_{1} = u, \quad s_b B_{2} = v, \quad s_b \bar{B}_{\mu \nu}^* = \bar{n}_{\mu \nu}, \quad s_b \Lambda = 0, \]
\[ \Lambda \equiv l_{\mu \nu}, m_{\mu \nu}, \bar{m}_{\mu \nu}, n_{\mu \nu}, o_{\mu}, p_{\mu}, \bar{p}_{\mu}, q_{\mu}, r, s, \bar{r}, s, t_{\mu}, u, v, w. \quad (A2) \]

3. Superfields for the extended BRST invariant theory

\[ B_{\mu \nu}(x, \theta) = B_{\mu \nu}(x) + \theta L_{\mu \nu}, \quad C_{\mu \nu}(x, \theta) = c_{\mu \nu}(x) + \theta M_{\mu \nu}, \]
\[ B_{\mu \nu}(x, \theta) = B_{\mu \nu}(x) + \theta (L_{\mu \nu} - (\partial_{\mu} c_{\nu \eta} - \partial_{\nu} c_{\mu \eta})), \quad \bar{C}_{\mu \nu}(x, \theta) = \bar{c}_{\mu \nu}(x) + \theta (M_{\mu \nu} - B_{\mu \nu}), \]
\[ C_{\mu \nu}(x, \theta) = c_{\mu \nu}(x) + \theta (M_{\mu \nu} - (\partial_{\mu} \beta_{\nu} - \partial_{\nu} \beta_{\mu})), \quad \bar{C}_{\mu \nu}(x, \theta) = \bar{c}_{\mu \nu}(x) + \theta (M_{\mu \nu} - B_{\mu \nu}). \]
\[ C_2(x, \theta) = \bar{c}_2(x) + \theta \bar{R}, \quad \bar{C}_2(x, \theta) = \bar{c}_2(x) + \theta (\bar{R} - B_2 + \bar{B}_2), \]
\[ C_1(x, \theta) = c_1(x) + \theta S, \quad \bar{C}_1(x, \theta) = \bar{c}_1(x) + \theta (S + B - \bar{B}), \]
\[ \Phi_\mu(x, \theta) = \phi_\mu(x) + \theta T_\mu, \quad \bar{\Phi}_\mu(x, \theta) = \bar{\phi}_\mu(x) + \theta (T_\mu - f_\mu + \bar{f}_\mu), \]
\[ B(x, \theta) = B(x) + \theta U, \quad \bar{B}(x, \theta) = B(x) + \theta U, \]
\[ B_1(x, \theta) = B_1(x) + \theta V, \quad \bar{B}_1(x, \theta) = \bar{B}_1(x) + \theta V, \]
\[ B_2(x, \theta) = B_2(x) + \theta W, \quad \bar{B}_2(x, \theta) = \bar{B}_2(x) + \theta W, \]
\[ B^*_\mu = B^*_\mu + \theta L^*_\mu, \quad \bar{C}^*_\mu = \bar{c}^*_\mu + \theta M^*_\mu, \quad \bar{C}^*_\mu = \bar{c}^*_\mu + \theta \bar{M}^*_\mu, \]
\[ \tilde{B}^*_\mu = \tilde{B}^*_\mu + \theta N^*_\mu, \quad \bar{\tilde{B}}^*_\mu = \bar{\tilde{B}}^*_\mu + \theta \tilde{N}^*_\mu, \quad \bar{B}^*_\mu = \beta^*_\mu + \theta O^*_\mu, \]
\[ \tilde{\bar{B}}^*_\mu = \tilde{\bar{B}}^*_\mu + \theta \tilde{O}^*_\mu, \quad \tilde{F}^*_\mu = \tilde{F}^*_\mu + \theta \bar{P}^*_\mu, \quad \bar{\tilde{F}}^*_\mu = \bar{\tilde{F}}^*_\mu + \theta \tilde{P}^*_\mu, \]
\[ \tilde{f}^*_\mu = f^*_\mu + \theta Q^*_\mu, \quad \bar{\tilde{f}}^*_\mu = \bar{\tilde{f}}^*_\mu + \theta \bar{Q}^*_\mu, \quad \bar{C}^*_2 = c^*_2 + \theta R, \]
\[ \tilde{c}^*_2 = \tilde{c}^*_2 + \theta \tilde{R}, \quad \bar{C}^*_1 = c^*_1 + \theta S, \quad \bar{\tilde{c}}^*_1 = \bar{c}^*_1 + \theta \tilde{S}, \]
\[ \bar{\tilde{\Phi}}^*_\mu = \bar{\tilde{\phi}}^*_\mu + \theta T^*_\mu, \quad \bar{\tilde{B}}^* = B^* + \theta U, \quad \bar{\tilde{B}}^*_1 = B^*_1 + \theta V, \quad \bar{\tilde{B}}^*_2 = B^*_2 + \theta W. \quad (A3) \]

4. Superfields for both extended BRST and anti-BRST invariant theory

\[ B_{\mu\eta}(x, \theta, \bar{\theta}) = B_{\mu\eta}(x) + \theta L_{\mu\eta} + \bar{\theta} (B^*_{\mu\eta} + \partial_\mu \bar{c}_\eta - \partial_\eta \bar{c}_\mu - \partial_\nu \bar{c}_\eta + \partial_\nu \bar{c}_\mu - \partial_\eta \bar{c}_{\mu\eta} + \partial_\eta \bar{c}_{\nu\mu}) \]
\[ - \partial_\eta \bar{c}_{\mu\nu} + \bar{\theta} (l_{\mu\eta} + \partial_\mu B_{\nu\eta} - \partial_\nu B_{\mu\eta} + \partial_\eta B_{\mu\nu} - \partial_\nu B_{\mu\eta} + \partial_\eta B_{\mu\nu} - \partial_\nu B_{\mu\eta}), \]
\[ C_{\mu\nu}(x, \theta, \bar{\theta}) = c_{\mu\nu}(x) + \theta M_{\mu\nu} + \bar{\theta} (c^*_{\mu\nu} + \tilde{B}_{\mu\nu} - \tilde{B}_{\mu\nu}) + \bar{\theta} (m_{\mu\nu} + \partial_\mu f_\nu - \partial_\nu f_\mu + \partial_\nu \bar{f}_\mu), \]
\[ \bar{B}_{\mu\eta}(x, \theta, \bar{\theta}) = \bar{B}_{\mu\eta}(x) + \theta (l_{\mu\eta} - (\partial_\mu c_{\eta\nu} - \partial_\mu \bar{c}_\eta + \partial_\nu c_{\eta\mu} - \partial_\nu \bar{c}_\eta + \partial_\eta c_{\mu\nu} - \partial_\eta \bar{c}_{\mu\nu})) \]
\[ + \bar{\theta} B^*_{\mu\eta} + \bar{\theta} l_{\mu\eta}, \]
\[ C_{\mu\nu}(x, \theta, \bar{\theta}) = c_{\mu\nu}(x) + \theta M_{\mu\nu} + \bar{\theta} (c^*_{\mu\nu} + \partial_\mu \bar{c}_\eta - \partial_\nu \bar{c}_\mu + \partial_\nu \bar{c}_\mu + \partial_\nu \bar{c}_\mu) \]
\[ + \bar{\theta} (m_{\mu\nu} + \partial_\mu \bar{F}_\nu - \partial_\nu \bar{F}_\mu - \partial_\nu \bar{F}_\mu + \partial_\nu \bar{F}_\mu), \]
\[ \tilde{C}_{\mu\nu}(x, \theta, \bar{\theta}) = \tilde{c}_{\mu\nu}(x) + \theta(M_{\mu\nu} - B_{\mu\nu} + \bar{B}_{\mu\nu}) + \bar{\theta} \tilde{c}_{\mu\nu} + \theta \bar{\theta} \bar{m}_{\mu\nu}, \]
\[ B_{\mu\nu}(x, \theta, \bar{\theta}) = B_{\mu\nu}(x) + \theta N_{\mu\nu} + \bar{\theta}(B^*_{\mu\nu} + \partial_{\mu} \tilde{f}_{\nu} - \partial_{\nu} \tilde{f}_{\mu} + \partial_{\nu} \tilde{f}_{\mu} + \partial_{\nu} \tilde{f}_{\mu}) + \theta \bar{\theta} n_{\mu\nu}, \]
\[ \tilde{B}_{\mu\nu}(x, \theta, \bar{\theta}) = \tilde{B}_{\mu\nu}(x) + \theta N_{\mu\nu} + \bar{\theta} B^*_{\mu\nu} + \theta \bar{\theta} n_{\mu\nu}, \]
\[ B_{\mu}(x, \theta, \bar{\theta}) = \beta_{\mu}(x) + \theta O_{\mu} + \bar{\theta}(\beta^*_{\mu} + F_{\mu} - F_{\mu}) + \theta \bar{\theta}(o_{\mu} + \partial_{\mu} B_{2} + \partial_{\mu} B_{2}), \]
\[ \tilde{B}_{\mu}(x, \theta, \bar{\theta}) = \tilde{B}_{\mu}(x) + \theta(\bar{N}_{\mu} - (\partial_{\mu} f_{\nu} - \partial_{\mu} \tilde{f}_{\nu} - \partial_{\nu} f_{\mu} + \partial_{\nu} \tilde{f}_{\mu})) + \bar{\theta} B^*_{\mu} + \theta \bar{\theta} \bar{n}_{\mu}, \]
\[ \tilde{B}_{\mu}(x, \theta, \bar{\theta}) = \tilde{B}_{\mu}(x) + \theta(\bar{O}_{\mu} - \bar{F}_{\mu} + \bar{F}_{\mu}) + \bar{\theta} \tilde{B}^*_{\mu} + \theta \bar{\theta} \bar{o}_{\mu}, \]
\[ \tilde{B}_{\mu}(x, \theta, \bar{\theta}) = \tilde{B}_{\mu}(x) + \theta(\bar{O}_{\mu} - \bar{F}_{\mu} + \bar{F}_{\mu}) + \bar{\theta} \tilde{B}^*_{\mu} + \theta \bar{\theta} \bar{o}_{\mu}, \]
\[ F_{\mu}(x, \theta, \bar{\theta}) = F_{\mu}(x) + \theta P_{\mu} + \bar{\theta} F^*_{\mu} + \theta \bar{\theta} p_{\mu}, \]
\[ \tilde{F}_{\mu}(x, \theta, \bar{\theta}) = \tilde{F}_{\mu}(x) + \theta(\bar{P}_{\mu} + \partial_{\mu} B - \partial_{\mu} \bar{B}) + \bar{\theta} F^*_{\mu} + \theta \bar{\theta} \bar{p}_{\mu}, \]
\[ \tilde{F}_{\mu}(x, \theta, \bar{\theta}) = \tilde{F}_{\mu}(x) + \theta \bar{P}_{\mu} + \bar{\theta} \tilde{F}^*_{\mu} - \partial_{\mu} B_{2} + \partial_{\mu} \bar{B}_{2} + \theta \bar{\theta} \bar{p}_{\mu}, \]
\[ \tilde{F}_{\mu}(x, \theta, \bar{\theta}) = \tilde{F}_{\mu}(x) + \theta \bar{P}_{\mu} + \bar{\theta} \tilde{F}^*_{\mu} + \theta \bar{\theta} \bar{p}_{\mu}, \]
\[ f_{\mu}(x, \theta, \bar{\theta}) = f_{\mu}(x) + \theta Q_{\mu} + \bar{\theta}(f^*_{\mu} - \partial_{\mu} B_{1} + \partial_{\mu} \bar{B}_{1}) + \theta \bar{\theta} q_{\mu}, \]
\[ \tilde{f}_{\mu}(x, \theta, \bar{\theta}) = \tilde{f}_{\mu}(x) + \theta Q_{\mu} + \bar{\theta} f^*_{\mu} + \theta \bar{\theta} q_{\mu}, \]
\[ \tilde{f}_{\mu}(x, \theta, \bar{\theta}) = \tilde{f}_{\mu}(x) + \theta Q_{\mu} + \bar{\theta} f^*_{\mu} + \theta \bar{\theta} q_{\mu}, \]
\[ \tilde{f}_{\mu}(x, \theta, \bar{\theta}) = \tilde{f}_{\mu}(x) + \theta Q_{\mu} + \bar{\theta} f^*_{\mu} + \theta \bar{\theta} q_{\mu}, \]
\[ c_{\mu}(x, \theta, \bar{\theta}) = c_{\mu}(x) + \theta C_{\mu} + \bar{\theta}(c^*_{\mu} - B_{1} + \partial_{\mu} \bar{B}_{1}) + \theta \bar{\theta} r_{\mu}, \]
\[ \tilde{c}_{\mu}(x, \theta, \bar{\theta}) = \tilde{c}_{\mu}(x) + \theta R + \bar{\theta}(c^*_{\mu} + B - \bar{B}) + \theta \bar{\theta} r_{\mu}, \]
\[ \tilde{c}_{\mu}(x, \theta, \bar{\theta}) = \tilde{c}_{\mu}(x) + \theta R + \bar{\theta}(c^*_{\mu} + B - \bar{B}) + \theta \bar{\theta} r_{\mu}, \]
\[ \tilde{c}_{\mu}(x, \theta, \bar{\theta}) = \tilde{c}_{\mu}(x) + \theta(\bar{R} - B_{2} + \bar{B}_{2}) + \bar{\theta} c^*_{\mu} + \theta \bar{\theta} \bar{r}_{\mu}, \]
\[ \tilde{c}_{\mu}(x, \theta, \bar{\theta}) = \tilde{c}_{\mu}(x) + \theta \bar{R} + \bar{\theta} c^*_{\mu} + \theta \bar{\theta} \bar{r}_{\mu}, \]
\[ C_{\mu}(x, \theta, \bar{\theta}) = c_{\mu}(x) + \theta S + \bar{\theta}(c^*_{\mu} - B_{1} + \partial_{\mu} \bar{B}_{1}) + \theta \bar{\theta} s_{\mu}, \]
\[ \tilde{C}_{\mu}(x, \theta, \bar{\theta}) = \tilde{c}_{\mu}(x) + \theta(S + B - \bar{B}) + \bar{\theta} c^*_{\mu} + \theta \bar{\theta} s_{\mu}, \]
\[ \tilde{C}_{\mu}(x, \theta, \bar{\theta}) = \tilde{c}_{\mu}(x) + \theta(S + B - \bar{B}) + \bar{\theta} c^*_{\mu} + \theta \bar{\theta} s_{\mu}, \]
\[ \tilde{C}_{\mu}(x, \theta, \bar{\theta}) = \tilde{c}_{\mu}(x) + \theta(S - B_{1} + \bar{B}_{1}) + \bar{\theta} c^*_{\mu} + \theta \bar{\theta} \bar{s}_{\mu}, \]
\[ \Phi_{\mu}(x, \theta, \bar{\theta}) = \phi_{\mu}(x) + \theta T_{\mu} + \bar{\theta}(\phi^*_{\mu} + \tilde{f}_{\mu} - \tilde{f}_{\mu}) + \theta \bar{\theta}(t_{\mu} + \partial_{\mu} B_{1} - \partial_{\mu} \bar{B}_{1}), \]
\[ \Phi(x, \theta, \bar{\theta}) = \bar{\phi}_\mu(x) + \theta(T_\mu - f_\mu + \bar{f}_\mu) + \bar{\theta} \phi^*_\mu + \theta \bar{\theta} t_\mu, \]

\[ B(x, \theta, \bar{\theta}) = B(x) + \theta U + \bar{\theta} B^* + \theta \bar{\theta} u, \]

\[ B(x, \theta, \bar{\theta}) = B(x) + \theta U + \bar{\theta} B^* + \theta \bar{\theta} u, \]

\[ B_1(x, \theta, \bar{\theta}) = B_1(x) + \theta V + \bar{\theta} B_1^* + \theta \bar{\theta} v, \]

\[ B_1(x, \theta, \bar{\theta}) = B_1(x) + \theta V + \bar{\theta} B_1^* + \theta \bar{\theta} v, \]

\[ B_2(x, \theta, \bar{\theta}) = B_2(x) + \theta W + \bar{\theta} B_2^* + \theta \bar{\theta} w, \]

\[ B_2(x, \theta, \bar{\theta}) = B_2(x) + \theta W + \bar{\theta} B_2^* + \theta \bar{\theta} w. \] (A4)

Form the above relations, we calculate

\[ \frac{1}{2} \frac{\delta}{\delta \theta} \frac{\delta}{\delta \bar{\theta}} \bar{B}_{\mu \nu} \bar{B}^{\mu \nu} = l_{\mu \nu} \bar{B}^{\mu \nu} - B^*_{\mu \nu} (L^{\mu \nu} - \partial^\mu \bar{c}^{\nu \eta} + \partial^\nu \bar{c}^{\mu \eta} - \partial^\nu \bar{c}^{\mu \eta} + \partial^\nu \bar{c}^{\mu \eta} - \partial^\nu \bar{c}^{\mu \eta}) \\
- \partial^\mu \bar{c}^{\nu \mu} + \partial^\nu \bar{c}^{\mu \nu} - \partial^\nu \bar{c}^{\mu \nu} + \partial^\nu \bar{c}^{\mu \nu} - \partial^\nu \bar{c}^{\mu \nu}) \\
+ \bar{c}^*_{\mu \nu} (\bar{M}^{\mu \nu} - B^{\mu \nu} + \bar{B}^{\mu \nu}), \]

\[ \frac{1}{2} \frac{\delta}{\delta \theta} \frac{\delta}{\delta \bar{\theta}} \bar{B}_{\mu \nu} \bar{B}^{\mu \nu} = n_{\mu \nu} B^{\mu \nu} - B^*_{\mu \nu} N^{\mu \nu}, \]

\[ \frac{1}{2} \frac{\delta}{\delta \theta} \frac{\delta}{\delta \bar{\theta}} \bar{B}_{\mu \nu} B^{\mu \nu} = o_{\mu \nu} \bar{B}^{\mu \nu} - B^*_{\mu \nu} (O^{\mu \nu} - \partial^\mu \bar{c}_2 + \partial^\nu \bar{c}_2), \]

\[ \frac{1}{2} \frac{\delta}{\delta \theta} \frac{\delta}{\delta \bar{\theta}} \bar{B}_{\mu} \bar{B}^{\mu} = \bar{o}_{\mu} \bar{B}^{\mu} - \bar{B}_2^{\mu} (\bar{O}^{\mu} - \bar{F}^{\mu} + \bar{F}^{\mu}), \]

\[ \frac{\delta}{\delta \theta} \frac{\delta}{\delta \bar{\theta}} \bar{f}_\mu \bar{f}^{\mu} = \bar{p}_\mu \bar{F}^{\mu} + p_\mu \bar{F}^{\mu} + \bar{F}^*_{\mu} (P^{\mu} + \partial^\mu B - \partial^\mu \bar{B}) + f_\mu \bar{P}^{\mu}, \]

\[ \frac{\delta}{\delta \theta} \frac{\delta}{\delta \bar{\theta}} \bar{c}_2 \bar{c}_2 = \bar{r} \bar{c}_2 + \bar{r} \bar{c}_2 + \bar{c}_2^* R + c_2^* (\bar{R} - B_2 + B_2), \]

\[ \frac{\delta}{\delta \theta} \frac{\delta}{\delta \bar{\theta}} \bar{c}_1 \bar{c}_1 = \bar{s} \bar{c}_1 + \bar{c}_1^* (S + B - \bar{B}) + \bar{s} \bar{c}_1 + \bar{c}_1^* (\bar{S} - B_1 + \bar{B}_1), \]

\[ \frac{1}{2} \frac{\delta}{\delta \theta} \frac{\delta}{\delta \bar{\theta}} \bar{\Phi}_\mu \bar{\phi}^{\mu} = t_\mu \bar{\phi}^{\mu} - \phi^{* \mu} (T^\mu - f^\mu + \bar{f}^\mu), \]

\[ \frac{1}{2} \frac{\delta}{\delta \theta} \frac{\delta}{\delta \bar{\theta}} \bar{B} \bar{B} = u \bar{B} - B^* U, \]

\[ \frac{1}{2} \frac{\delta}{\delta \theta} \frac{\delta}{\delta \bar{\theta}} \bar{B}_1 \bar{B}_1 = v \bar{B}_1 - B_1^* V, \]

\[ \frac{1}{2} \frac{\delta}{\delta \theta} \frac{\delta}{\delta \bar{\theta}} \bar{B}_2 \bar{B}_2 = w \bar{B}_2 - B_2^* W. \] (A5)
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