Multi-epoch X-ray burst modelling: MCMC with large grids of 1D simulations

Zac Johnston,∗ Alexander Heger, Duncan K. Galloway

1 School of Physics and Astronomy, Monash University, Victoria 3800, Australia
2 Monash Centre for Astrophysics, Monash University, Victoria 3800, Australia
3 Department of Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China
4 School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455, USA

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ABSTRACT
Type-I X-ray bursts are recurring thermonuclear explosions on the surface of accreting neutron stars. Matching observed bursts to computational models can help to constrain system properties, such as the neutron star mass and radius, crustal heating rates, and the accreted fuel composition, but systematic parameter studies to date have been limited. We apply Markov chain Monte Carlo methods to 1D burst models for the first time, and obtain system parameter estimations for the ‘Clocked Burster’, GS 1826−238, by fitting multiple observed epochs simultaneously. We explore multiple parameters which are often held constant, including the neutron star mass, crustal heating rate, and hydrogen composition. To improve the computational efficiency, we precompute a grid of 3840 Kepler models – the largest set of 1D burst simulations to date – and by interpolating over the model grid, we can rapidly sample burst predictions. We obtain estimates for a CNO metallicity of $Z_{\text{CNO}} = 0.010^{+0.005}_{-0.004}$, a hydrogen fraction of $X_0 = 0.74^{+0.02}_{-0.03}$, a distance of $d\sqrt{\xi_b} = 6.5^{+0.4}_{-0.6}$ kpc, and a system inclination of $i = 69^{+2}_{-3}$°.

Key words: X-rays: bursts – stars: neutron – stars: individual: GS 1826-238 – methods: numerical

1 INTRODUCTION

Type-I thermonuclear X-ray bursts are recurring flashes observed from accreting neutron stars (for reviews, see Lewin et al. 1993; Strohmayer & Bildsten 2006; Galloway & Keek 2017). In the host low-mass X-ray binary (LMXB) systems, a neutron star accretes material from a companion star with a mass of $M \lesssim 1 M_\odot$, which accumulates as a $\sim 10$ m envelope on the neutron star surface. Under the weight of accreting material, the base of the envelope is compressed by the extreme surface gravity of $g \sim 10^{14}$ cm s$^{-2}$ to the point of thermonuclear runaway. Within seconds, the layer is heated to $\sim 10^9$ K, generating a burst of X-rays before cooling to background levels over the following seconds to minutes. New fuel is accreted on top of the ‘ashes’, and the cycle repeats.

X-ray bursts have been the target of numerical calculations since the 1970s (e.g., Joss 1978; Taam & Pilkington 1979), and their diverse behaviour has been studied with a variety of computational models (e.g., Fujimoto et al. 1987; Woosley et al. 2004; Keek et al. 2012). By exploring model parameters and comparing the predictions with observations, the neutron star system properties can be inferred (e.g., Cumming 2003; Galloway et al. 2004; Keek & Heger 2017; Johnston et al. 2018).

One-dimensional (1D) burst codes are the best tools currently available for this purpose. With adaptive nuclear reaction networks and treatments for convective transport (e.g., Woosley et al. 2004), they can produce detailed simulations of burst energetics not possible in semi-analytic or one-zone models. Multi-dimensional burst simulations are also under active development, but computational costs limit the calculations to $\lesssim 1$ s of simulation time (e.g., Zingale et al. 2015; Cavecchi et al. 2016).

Nevertheless, targeted parameter explorations using 1D models have been relatively limited. Small sets of models are typically used, and many parameters, such as the gravity, fuel composition, and crustal heating, are often held constant. Due to the relatively unexplored parameter space, obtaining robust constraints on system properties is difficult. To encourage more directed modelling efforts, Galloway et al. (2017, hereafter, G17) presented a set of standardised burst observations. Their reference data set included three
epochs of bursts from GS 1826–238, famously dubbed the ‘clocked burster’ (e.g., Ubertini et al. 1999). The system’s reliability has made it a popular target for modelling (e.g., Galloway et al. 2004; Heger et al. 2007), and particularly for the study of the nuclear rp-process (e.g., Schatz et al. 1998; Fisker et al. 2008). The first study to make use of the G17 data set was Meisel (2018, hereafter, M18), who performed the first extended comparison of MESA burst models to GS 1826–238. M18 demonstrated the benefit of fitting multiple epochs by ruling out parameter combinations which otherwise agreed with individual epochs.

GS 1826–238 was discovered as a transient source with the Ginga X-ray telescope in 1988 (Makino 1988), and X-ray bursts were later discovered in 1997 (Ubertini et al. 1997, 1999). The system orbital period is not precisely known, but is thought to be roughly 2 h (Homer et al. 1998), implying a hydrogen-rich mass donor, consistent with the long-tailed bursts observed (in’t Zand et al. 2009). Despite the popularity of GS 1826–238 for modelling, ambiguity persists regarding the system properties. For example, Galloway et al. (2004, hereafter, G04) modelled bursts observed between 1997 and 2002 using a semi-analytic ignition model (Settle, first used in Cumming & Bildsten 2000). They reported that an accreted CNO mass fraction of \( Z_{\text{CNO}} = 0.001 \) best reproduced the trend of recurrence time, \( \Delta t \), versus accretion rate, \( \dot{M} \), but that the observed ratios were only consistent with a higher metallicity of \( Z_{\text{CNO}} = 0.02 \). Using Kepler, Heger et al. (2007, hereafter, H07) found good lightcurve agreement \(^1\) for \( Z_{\text{CNO}} = 0.02 \), and M18 found agreement for both \( Z_{\text{CNO}} = 0.01 \) and 0.02 using MESA. The accretion rates are typically inferred to be in the range \( \dot{M} = 0.05–0.08 \dot{M}_{\text{Edd}} \) (where \( \dot{M}_{\text{Edd}} = 1.75 \times 10^{-8} M_\odot \text{yr}^{-1} \); Heger et al. 2007; Galloway et al. 2008, 2017), but M18 reported improved model fits using twice as large accretion rates of \( \dot{M} = 0.1–0.17 \dot{M}_{\text{Edd}} \).

The inconclusive estimates are, we suggest, partly due to the limited parameter explorations to date, in addition to degeneracies between the model predictions. For example, the metallicity is often fixed at \( Z_{\text{CNO}} = 0.02 \), with an accreted hydrogen fraction of \( X_0 = 0.7 \). G04 and H07 used fixed crustal heating rates of \( Q_0 = 0.1 \) and 0.15 MeV nuc\(^{-1}\) respectively, whereas M18 considered \( Q_0 = 0.1, 0.5, \) and 1.0 MeV nuc\(^{-1}\). Both H07 and M18 assumed a fixed neutron star mass of \( M = 1.4 M_\odot \) and a radius of \( R = 11.2 \) km, whereas G04 assumed \( M = 1.4 M_\odot \) and \( R = 10 \) km. The earlier estimates for \( \dot{M} \) did not account for the possible effect of anisotropic emission (§ 2.6), which is dependent on the system inclination and disc morphology (Fujimoto 1988). Using the disc models of He & Keek (2016), M18 inferred an approximate inclination of 65–80°, suggesting that the X-ray emission is preferentially beamed away from the line of sight, allowing for larger \( \dot{M} \). To fully account for the complex dependencies between these model parameters and predictions, a more comprehensive analysis is required.

Markov chain Monte Carlo (MCMC) methods are algorithms capable of sampling complex probability distributions (for a comprehensive introduction, see MacKay 2003). The use of Bayesian statistics in astrophysics has seen a rapid expansion in recent years, but its application to X-ray burst modelling has been minimal. Most recently, Goodwin et al. (2019) applied MCMC methods to the semi-analytic burst code, SETTLE, to model bursts from the transient accretor, SAX J1808.4–3658. This system is also included in the G17 data set, as an example of helium bursts triggered during an accretion event. Pairing a semi-analytic model with MCMC is beneficial due to the computational speed required for drawing thousands of sequential samples. By contrast, 1D burst models can take several days to compute, and are, on their own, unsuitable for MCMC methods.

As we show here for the first time, this computational barrier can be overcome with the use of pre-compiled model grids. For burst properties that vary smoothly over the model parameters, interpolation can be used to sample bursts between existing models with little computational cost. We present the first application of MCMC methods to large grids of 1D burst models. By constructing a grid of 3840 KEPLER simulations, we are able to rapidly sample burst properties across twelve parameters. Using the data set from G17, we fit three epochs of burst data simultaneously, and obtain probability distributions for the system parameters of GS 1826–238.

In Section 2, we describe the KEPLER code and its recent updates, the epoch data used, the construction and interpolation of the model grid, and the setup of the MCMC model. In Section 3, we describe the model results, the posterior distributions, the predicted burst properties, and lightcurve comparisons. In Section 4, we discuss and compare the parameter estimates to previous works, discuss the limitations of the model, and describe potential improvements to the model. In Section 5, we provide concluding remarks and the future outlook.

2 METHODS

2.1 An update on Kepler

KEPLER (Weaver et al. 1978) is a one-dimensional (1D) hydrodynamics code capable of simulating a variety of regimes in stellar evolution and explosive nucleosynthesis (e.g., Woosley et al. 2002; Menon & Heger 2017). It has prominently been used for modelling X-ray bursts, reproducing observed behaviour, including burst energetics, recurrence times, and lightcurves (Woosley et al. 2004; Heger et al. 2007; Keek et al. 2012; Lampe et al. 2016). Because KEPLER has steadily been modified and improved over time, some descriptions in earlier works are now out of date. We here briefly summarise notable changes to the code and model setup.

To aid reproducibility and comparisons to other burst models, we used V2.2 of JINA REACLIB, the public database of nuclear reaction rates\(^2\) (Cyburt et al. 2010).

KEPLER burst models published prior to Johnston et al. (2018) used a setup file which erroneously multiplied the opacities by a factor of \( \approx 5 \). The original intent was to approximate the time-dilation effects of general relativity (GR; Appendix B) by artificially slowing down thermal transport.

\(^1\) we note that these models were discovered to use inadvertently large opacities; see § 2.1

\(^2\) https://jinaweb.org/reaclib/db/
Table 1. The observed burst data from three epochs of GS 1826–238, adapted from the reference set of G17. As an additional system constraint, we have included the Eddington flux, $F_{\text{Edd}}$, taken from the peak of a PRE burst observed in 2014 June (Chenevez et al. 2016).

To estimate the average recurrence time $\Delta t$, G17 collected multiple bursts from each epoch, from which we have obtained the burst rate $\nu$. They then extracted average lightcurves, from which the peak flux, $F_{\text{peak}}$, and fluence, $f_s$, could be determined. The persistent flux $F_p$ was averaged for each epoch, and the values listed here have incorporated the bolometric corrections estimated by G17.

| Epoch    | $\nu$ (day$^{-1}$) | $F_{\text{peak}}$ (10$^{-9}$ erg s$^{-1}$ cm$^{-2}$) | $f_s$ (10$^{-9}$ erg cm$^{-2}$) | $F_p$ (10$^{-9}$ erg s$^{-1}$ cm$^{-2}$) | $F_{\text{Edd}}$ (10$^{-9}$ erg s$^{-1}$ cm$^{-2}$) |
|----------|---------------------|-----------------------------------------------|---------------------------------|--------------------------------------|-------------------------------------|
| 1998 Jun | 4.67 ± 0.06         | 30.9 ± 1.0                                    | 1.102 ± 0.011                   | 2.108 ± 0.015                       | –                                   |
| 2000 Sep | 5.746 ± 0.014       | 29.1 ± 0.5                                    | 1.126 ± 0.016                   | 2.85 ± 0.03                        | –                                   |
| 2007 Mar | 6.799 ± 0.008       | 28.4 ± 0.4                                    | 1.18 ± 0.04                     | 3.27 ± 0.04                        | –                                   |
| 2014 Jun | –                   | –                                             | –                               | –                                   | 40 ± 3                              |

During the setup phase of the model envelope, before accretion and nuclear reactions are switched on, the thermal profile is initialised near equilibrium, in order to minimise simulation ‘burn-in’. In previous Kepler studies (and to our knowledge, all other burst studies in the literature), the only heat source included was the crustal heating, $Q_b \approx 0.15$ MeV nucleon$^{-1}$, as a boundary condition at the base of the model, at a column depth of $y \approx 1 \times 10^{12}$ g cm$^{-2}$. Additional heat generated by nuclear reactions, $Q_{\text{nuc}} \approx 5$ MeV nucleon$^{-1}$, around $y \approx 10^6$ g cm$^{-2}$, was assumed to largely escape the surface, and was neglected from the setup calculations. Minor heating of the deeper ocean, once the full nuclear simulation began, was expected to stabilise after the initial few bursts. For example, Woosley et al. (2004) discarded the first three bursts from analysis to address this ‘thermal inertia’, in addition to the related effect of ‘chemical inertia’ (§ 2.4).

During testing, however, we discovered that nuclear heating can indeed alter the thermal profile enough to influence burst ignition. Models which do not account for $Q_{\text{nuc}}$ during setup begin comparatively colder, producing a steadily-changing burst sequence as the envelope is heated by nuclear reactions towards a steady thermal state. This burn-in period can persist for dozens of bursts – much longer than previously assumed. To address this issue, we added during setup a heat source of $Q_{\text{nuc}} = 5$ MeV nucleon$^{-1}$ at a depth of $y \approx 8 \times 10^7$ g cm$^{-2}$ with a Gaussian spread of $y \approx 8 \times 10^6$ g cm$^{-2}$. This heat source is switched off once the full nuclear calculations begin (further detail is provided in Appendix A). The model burn-in was effectively eliminated, and the burst sequence was stabilised within the first few bursts, as was originally assumed. Further study is still required to explore the optimal configuration of this ‘pre-heating’, which is likely to depend on other model parameters, such as the composition, accretion rate, and crustal heating.

2.2 Observed data

We used observations of bursts from GS 1826–238 for three epochs: 1998 June, 2000 September, and 2007 March (Table 1). These observations were provided as part of a reference set for burst modelling by G17, using data from the MINBAR catalogue4. The epochs were selected by G17 for their burst consistency and the availability of high-precision X-ray lightcurves from the RXTE satellite.

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3 by Adam Jacobs, Michigan State University, pers. comm.

4 http://burst.sci.monash.edu/minbar
We also included the peak bolometric flux of $F_{\text{peak}} = (40 \pm 3) \times 10^{-9} \text{erg cm}^{-2} \text{s}^{-1}$, observed from a photospheric radius expansion (PRE) burst in 2014 (Chenevez et al. 2016). We have assumed that $F_{\text{peak}}$ corresponds to the local Eddington luminosity, $F_{\text{Edd}}$, for a mixed hydrogen/helium envelope (§ 2.7), and that $F_{\text{Edd}}$ is common to the 1998–2007 epochs.

### 2.3 Model grid

To model the bursts of GS 1826–238, we computed a regular grid of 3840 Kepler simulations over five model parameters: the accretion rate, $\dot{m}$, the accreted hydrogen mass fraction $X_0$, the accreted CNO-metallicity mass fraction, $Z_{\text{CNO}}$, the crustal heating rate, $Q_c$, and the surface gravitational acceleration, $g$. Note that for the Kepler model parameters, we give the local accretion rate per unit area, $\dot{m}$, because the global accretion rate, $M = 4\pi R^2 \dot{m}$, depends on the choice of $R$ (§ 2.6). The numerical parameters controlling zone resolution were held constant, following convergence tests to ensure consistent burst sequences. The grid points for each parameter are listed in Table 2.

Following a trial parameter exploration, we chose a grid that approximately covered the observed recurrence times of $3 \lesssim \Delta t \lesssim 6\,\text{h}$. The model grid represents over 100 000 CPU hours, and is the largest collection of 1D burst models to date, with the previous largest containing 464 Kepler models (Lampe et al. 2016).

Each model generated a sequence of 30–40 bursts (Fig. 1). Simulating a long sequence ensures a consistent burst train, and reduces the effect of model burn-in (§ 2.1). The entire grid contained a total of approximately 138 000 bursts. Modelling such large collections of bursts has been made possible by improved CPU speeds, which have reduced the computational cost from $\approx 24\,\text{h}$ per burst in 2003 to $\approx 1\,\text{h}$ per burst.

### 2.4 Extracting model bursts

Analysing bursts from Kepler models has been detailed previously (Woosley et al. 2004; Heger et al. 2007; Keek & Heger 2011; Lampe et al. 2016). We briefly summarise our analysis routine, for which we developed a new open source PYTHON package, PYBURST\(^5\).

The procedure identified bursts from local maxima in the model lightcurve, from which the the individual lightcurves were extracted and analysed (Fig. 1). The peak luminosity, $L_{\text{peak}}$, was taken from the lightcurve maximum. The burst energy, $E_b$, was the time-integrated luminosity over the lightcurve. The recurrence time, $\Delta t$, was the time since the previous burst as measured peak-to-peak, giving a burst rate of $\nu = 1/\Delta t$.

Our inclusion of a nuclear heat source during initialisation of the envelope had substantially reduced thermal burn-in (§ 2.1). Despite this improvement, the first burst still ignites in a chemically pristine envelope, which lacks the complex ashes later accumulated. Due to ‘chemical inertia’, the models typically also require a ‘chemical burn-in’ of several bursts to reach a quasi-stable bursting pattern.

To minimize these combined burn-in effects, we excluded the first 10 bursts of each model from analysis. In previous studies, only $\approx 3$ had typically been discarded (Woosley et al. 2004).

Modelled and observed X-ray bursts are occasionally followed by short waiting-time bursts ($\Delta t \lesssim 45\,\text{min}$), which are thought to be triggered by the ignition of unburned hydrogen (Keek & Heger 2017). These unusually weak bursts were not included in the data set of G17, and we excluded them from our analysis using the threshold of $\Delta t < 45\,\text{min}$.

The extracted properties of the remaining 20–30 bursts of each model were averaged, and the $1\sigma$ standard deviations were adopted as the model uncertainties. This process produced tabulated burst properties across the model grid parameters (Fig. 2), which could be sampled with minimal computational cost.

\(^5\) [https://github.com/zacjohnston/pyburst/](https://github.com/zacjohnston/pyburst/)
Table 2. The parameters of the model grid. Every combination was simulated, forming a regular grid in 5 dimensions. All models used a preheating value of $Q_{\text{pre}} = 5 \text{ MeV nucleon}^{-1}$ (§ 2.1). The accretion rate, $\dot{m}$, is the local rate per unit area, because the global accretion rate, $\dot{M}$, is dependent on the choice of $R$ (§ 2.6). The Eddington-limited accretion rate, $\dot{m}_{\text{Edd}} = 8.775 \times 10^6 \text{ g cm}^{-2} \text{s}^{-1}$ (assuming $M = 1.4 \text{ M}_\odot$, $R = 10 \text{ km}$, and $X = 0.7$), is simply used as a common reference point between models, and was not corrected for the $g$ or $X_0$ of each model. The values of $g$ correspond to Newtonian surface gravities for masses of $M = 1.4$, 1.7, 2.0, and $2.6 \text{ M}_\odot$, for a reference radius of $R = 10 \text{ km}$, but the actual mass and radius are free parameters (§ 2.6).

| Parameter | Name | Units | Grid Points | N |
|-----------|------|-------|-------------|---|
| $\dot{m}$ | Local accretion rate | ($\dot{m}_{\text{Edd}}$) | 0.06, 0.07, 0.08, 0.10, 0.12, 0.14, 0.16, 0.18 | 8 |
| $X_0$ | H mass fraction | – | 0.64, 0.67, 0.70, 0.73, 0.76 | 5 |
| $Z_{\text{CNO}}$ | CNO mass fraction | – | 0.0025, 0.005, 0.0075, 0.0125, 0.02, 0.03 | 6 |
| $Q_{\text{b}}$ | Crustal heating (MeV nucleon$^{-1}$) | – | 0.0, 0.2, 0.4, 0.6 | 4 |
| $g$ | Surface gravity ($10^{14} \text{ cm s}^{-2}$) | – | 1.858, 2.256, 2.654, 3.451 | 4 |

Total 3840

2.5 Grid interpolation

Computational speed is critical for the large-scale sampling used in MCMC methods. The smooth and monotonic behaviour of the burst properties (Fig. 2) allowed for the use of interpolation between model predictions. Using multivariate linear interpolation, we constructed a continuous function of the burst properties across the five grid parameters, $\dot{m}$, $Q_{\text{b}}$, $X_0$, $Z_{\text{CNO}}$, and $g$.

Any point within the grid bounds of Table 2 could then be quickly ($\ll 1$ s) sampled to predict the burst properties of interest: $\nu$, $L_{\text{peak}}$, and $E_b$. In contrast to the roughly 40–90 h required for each KEPLER simulation, this approach granted a considerable efficiency gain.

The interpolated burst properties were in the local neutron star frame of the KEPLER models. In order to compare with the observed data (Table 1) we converted these values to observable quantities. These calculations first accounted for the fact that KEPLER uses Newtonian gravity. A KEPLER model with a given Newtonian surface gravity, $g$, can be considered equivalent to a neutron star with an equal $g$ under GR, but a different ‘true’ mass and radius (e.g., Fujimoto 1988; He & Keek 2016). The anisotropy parameters are defined with

$$F_b = \frac{L_b}{4\pi d^2 \xi_b}, \quad F_p = \frac{L_p}{4\pi d^2 \xi_p},$$

where the subscripts ‘b’ and ‘p’ correspond to the burst and persistent emission, respectively.

Because $\xi_b$ and $\xi_p$ are degenerate with distance, we combined them into independent parameters: a modified distance, $d\sqrt{\xi_b}$, and the anisotropy ratio, $\xi_p/\xi_b$. We can later retrieve the absolute values for $\xi_b$, $\xi_p$, and $d$ by choosing an accretion disc model which relates the anisotropy to the system inclination, $i$ (§ 3.3).

2.7 Transforming to observable quantities

The free parameters can then be used to calculate observables from the local burst properties. We here signify observed quantities with the subscript ‘$\infty$’.

The burst rate as seen by a distant observer is time-dilated with

$$\nu_{\infty} = \frac{\nu}{1 + z}.$$  (3)

The observed peak flux is given by

$$F_{\text{peak,}\infty} = \frac{L_{\text{peak}}}{4\pi d^2 \xi_b(1 + z)^2}.$$  (4)

The observed fluence is given by

$$f_{b,\infty} = \frac{E_b}{4\pi d^2 \xi_b[1 + z]}.$$  (5)

where we use the redshift factor of $1 + z$ instead of $(1 + z)^2$, because fluence is time-integrated.

The two remaining observables, $F_{\text{Edd}}$ and $F_p$, are not predicted from the model grid, but are calculated analytically from the given parameters.

The observed Eddington flux is given by

$$F_{\text{Edd,}\infty} = \frac{L_{\text{Edd}}}{4\pi d^2 \xi_b(1 + z)^2}.$$  (6)

where $L_{\text{Edd}}$ is the local Eddington luminosity, given by

$$L_{\text{Edd}} = \frac{8\pi Gm_p c(1 + z)M}{\sigma_T (X_0 + 1)},$$  (7)

where $m_p$ is the mass of the proton and $\sigma_T$ is the Thompson scattering cross section. This equation assumes that the

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6 These corrections are not applicable to codes which already use GR surface gravity, for example, MESA.
radiation pressure is exerted on the electrons of an ionized plasma, and includes a scaling factor of $2/(X_0 + 1)$ to account for the charge per mass for a given composition of hydrogen and helium.

The observed persistent flux is given by
\[
F_{p, \infty} = \frac{L_p}{4\pi d^2 \xi_p (1 + z)^2},
\]
where $L_p = -4\pi R^2 \dot{m} \phi$ is the local accretion luminosity, and $\phi = -c^2 z/(1 + z) \approx -0.2 c^2$ is the gravitational potential at the neutron star surface.

### 2.8 Multi-epoch model

To model bursts from multiple epochs, we used our interpolated grid (§2.5) to predict the observed burst properties (§2.7) of three separate epochs from GS 1826–238 (§2.2). Our multi-epoch model contained both epoch-independent and epoch-dependent parameters.

The accreted composition and neutron star properties, $X_0$, $Z_{\text{CNO}}$, $g$, and $M$, are expected to remain unchanged between the observed epochs, and so global parameters are used. Global parameters were also used for the distance and anisotropy, $d\sqrt{\xi_b}$ and $\xi_p/\xi_b$, although the anisotropy factors $\xi_b$ and $\xi_p$ could feasibly evolve due to changes in the accretion disc. We leave the testing of epoch-dependent parameters of $\xi_b$ and $\xi_p$ for a future study. The six epoch-independent parameters are thus $X_0$, $Z_{\text{CNO}}$, $g$, $M$, $d\sqrt{\xi_b}$, and $\xi_p/\xi_b$.

Conversely, the accretion rate is expected to evolve between epochs, and so we used three parameters, $\dot{m}_1$, $\dot{m}_2$, and $\dot{m}_3$, where the subscripts 1–3 correspond to the 1998, 2000, and 2007 epochs, respectively. The crustal heating efficiency of star samples. We used this as the prior distribution underling distributions of the Gaia DR2 catalogue (Rybizki et al. 2018), we took a sample of 100 000 stars located inside the grid bounds of 0.01–15 kpc, and 0.1 $\leq \xi_p/\xi_b \leq 10$. The prior distribution for each parameter was set to $p(\theta) = 0$ outside these boundaries.

We used flat prior distributions for all parameters except $Z_{\text{CNO}}$, setting $p(\theta) = 1$ everywhere inside the boundaries. For $Z_{\text{CNO}}$, we estimated a prior distribution using a process similar to Goodwin et al. (2019). From a simulated catalogue of Milky Way stars, constructed to represent the underlying distributions of the Gaia DR2 catalogue (Rybizki et al. 2018), we took a sample of 100 000 stars located within 15 arcmin of GS 1826–238, and between a distance of 5–9 kpc. We then fit a beta distribution to $[\text{Fe/H}]$, obtaining the values of $\alpha = 10.1$ and $\beta = 3.5$ after translating to the interval $[-3.5, 1]$, which contained the vast majority of star samples. We used this as the prior distribution for $\log_{10}(Z_{\text{CNO}}/0.01)$, where we have assumed a solar CNO metallicity of 0.01 (Lodders et al. 2009). This distribution was applied inside the grid bounds of 0.0025 $\leq Z_{\text{CNO}} \leq 0.03$, which roughly corresponds to $-0.6 \leq \log_{10}(Z_{\text{CNO}}/0.01) \leq 0.5$.

For a given sample point in parameter space, $\theta$, the local burst properties were interpolated from the model grid (§2.5), from which the observables were predicted (§2.7).
The likelihood function, $p(D|\theta)$, from Equation (9), was then evaluated by comparing these predictions with the observed data, $D$, using

$$p(D|\theta) = \prod_x \frac{1}{\sqrt{2\pi(\sigma_x^2 + \sigma_d^2)}} \exp \left[ -\frac{(x - x_0)^2}{2(\sigma_x^2 + \sigma_d^2)} \right],$$

where $x$ is iterated over each observable of each epoch, $\sigma$ is the uncertainty, and the subscript '0' signifies the corresponding observed value from Table 1.

The MCMC model used an ensemble of 1000 walkers, which were initialised in a small ‘hyperball’ in parameter space. The algorithm was run for 20,000 steps, resulting in a total of $2 \times 10^7$ individual samples. The average computation time for each sample was $\approx 0.012$ s, for a total of $\approx 560$ CPU hours split over 8 cores. For comparison, each KEPLER simulation costs roughly 40–70 CPU hours.

We discarded the first 1000 steps as burn-in, after which the walkers had spread out across the domain of each parameter. To check convergence, we estimated the autocorrelation time ($\tau$) at multiple steps in the chain\(^8\), to ensure the total chain length was longer than 10 $\tau$.

### 3 RESULTS

We present here the distributions and estimates from the burst matching procedure, and discuss the implications for the system properties. The 2D marginalised posteriors for $\dot{m}_i$, $Q_{b,i}$, $X_0$, and $Z_{CNO}$, are shown in Fig. 3, with the 1D posteriors shown along the diagonal. The maximum likelihood estimates for the 1D posteriors are listed in Table 3. Unless otherwise stated, the uncertainties given for 1D parameter estimates are 68 per cent credible intervals, and the 2D contour levels are 38, 68, 87, and 95 per cent credible regions.

There is a strong correlation visible between the accretion rates of each epoch, and the crustal heating of each epoch. These correlations are expected, because for given ratios between the epoch burst properties, similar ratios are required between the epoch parameters. For example, the persistent flux, $F_p$, is calculated using Eq. (8), and is proportional to $\dot{m}$.

The CNO mass fraction, $Z_{CNO}$, is correlated with the hydrogen fraction $X_0$ – a common feature of such model-observation comparisons (e.g., Galloway & Cumming 2006; Goodwin et al. 2019). The correlation arises because multiple pairs of $X_0$ and $Z_{CNO}$ result in the same reduced hydrogen fraction at ignition.

The distributions of some parameters, for example $Q_{b,1}$, $X_0$, $M$, and $Z_{CNO}$, appear to be truncated by the prior boundaries. These limits could bias the results, potentially underestimating the full extent of the distributions. Some of these boundaries were chosen as expected natural limits, whereas others are simply limited by the model grid. For example, $X_0$ is truncated at the upper grid limit of $X_0 = 0.76$. Although models with larger $X_0$ could be added, they would substantially exceed the primordial mass fraction from Big Bang nucleosynthesis (Makki et al. 2019).

\[\text{Table 3. Maximum likelihood estimates for each 1D marginalised posterior. In addition to the twelve parameters explored directly by the MCMC routine (§ 2.8), we include the derived neutron star properties, } R \text{ and } z \text{ (§ 3.4), and the system properties predicted using a disc model of anisotropy, } i, \xi_0, \xi_p, \text{ and } d \text{ (§ 3.3). The subscripts of 1–3 correspond to the 1998, 2000, and 2007 epochs, respectively. The accretion rates are given as fractions of fixed reference points, } \dot{m}_{\text{Edd}} = 8.775 \times 10^4 \text{ g cm}^{-2} \text{ s}^{-1} \text{ and } M_{\text{Edd}} = 1.75 \times 10^{-8} M_\odot \text{ yr}^{-1}.\]

| Parameter | Units | Estimate |
|-----------|-------|----------|
| $\dot{m}_{1}$ | ($\dot{m}_{\text{Edd}}$) | 0.083$^{+0.013}_{-0.011}$ |
| $\dot{m}_{2}$ | ($\dot{m}_{\text{Edd}}$) | 0.114$^{+0.016}_{-0.017}$ |
| $\dot{m}_{3}$ | ($\dot{m}_{\text{Edd}}$) | 0.133$^{+0.018}_{-0.02}$ |
| $Q_{b,1}$ | (MeV nucleon$^{-1}$) | 0.36$^{+0.10}_{-0.2}$ |
| $Q_{b,2}$ | (MeV nucleon$^{-1}$) | 0.17$^{+0.10}_{-0.14}$ |
| $Q_{b,3}$ | (MeV nucleon$^{-1}$) | 0.15$^{+0.10}_{-0.11}$ |
| $X_0$ | (Mass fraction) | 0.74$^{+0.02}_{-0.03}$ |
| $Z_{CNO}$ | (Mass fraction) | 0.016$^{+0.005}_{-0.004}$ |
| $\Delta \sqrt{\xi_0}$ | (kpc) | 6.5$^{+0.4}_{-0.6}$ |
| $\xi_0/\xi_p$ | – | 1.57$^{+0.15}_{-0.19}$ |
| $i$ | (deg) | 69$^{+2}_{-2}$ |
| $\xi_p$ | – | 2.0$^{+0.2}_{-0.4}$ |
| $d$ | (kpc) | 5.8$^{+0.3}_{-0.4}$ |
| $g$ | $(10^{14} \text{ cm s}^{-2})$ | 2.8$^{+0.6}_{-0.4}$ |
| $M$ | ($M_\odot$) | > 1.7 |
| $R$ | (km) | 11.3$^{+1.3}_{-1.3}$ |
| $z$ | – | 0.39$^{+0.07}_{-0.07}$ |
| $M_1$ | ($M_{\text{Edd}}$) | 0.098$^{+0.012}_{-0.014}$ |
| $M_2$ | ($M_{\text{Edd}}$) | 0.132$^{+0.016}_{-0.017}$ |
| $M_3$ | ($M_{\text{Edd}}$) | 0.153$^{+0.017}_{-0.02}$ |

On the other hand, $Q_{b,1}$ appears truncated at the upper grid limit of 0.6 MeV nucleon$^{-1}$. This value is larger than the typical assumed heating of $\approx 0.15$ MeV nucleon$^{-1}$ (e.g., Heger et al. 2007), although the amount of crustal heating emerging into the envelope is poorly constrained, and a total of 1–2 MeV nucleon$^{-1}$ is potentially available (Haensel & Zdunik 2008). The CNO metallicity, $Z_{CNO}$, is also slightly limited by the lower grid boundary of $Z_{CNO} = 0.0025$. A future study could extend the model grid in these parameters, and examine the effect on the posteriors.

#### 3.1 Predicted observables

The distribution of burst properties predicted over the MCMC simulation corresponds to the posterior predictive distribution. This distribution represents the expected observations according to our model, given the posteriors of the model parameters. A random sample of 20,000 points were selected from the chain, and the predicted triplets of observables were extracted for each point using the multi-epoch model (§ 2.8). The distribution peaks and 68 per cent
credible intervals are plotted with the original observed data in Fig. 4. The predicted burst properties are consistent with the observed data, within uncertainties. Note that this comparison should not be confused with the 'best-fit', which MCMC methods are ill-suited to finding.

3.2 Crustal heating and accretion rate

By using independent crustal heating rates between epochs, we can examine the constraints on $Q_b$, as a function of $\dot{m}$. Theoretical models predict that the effective crustal heating is stronger at low accretion rates, and weaker at higher accretion rates due to neutrino losses (Cumming et al. 2006).

The posteriors of $Q_b$ and $\dot{m}$ for each epoch are plotted in Fig. 5. There is significant overlap between the distributions,
particulary between the 2000 and 2007 epochs, which have similar estimates for $Q_b$. The 1998 epoch, with the lowest inferred $\dot{m}$, covers similar $Q_b$ but is overall consistent with larger values, with a 68 per cent 2D credible region extending up to the grid boundary of 0.6 MeV nucleon$^{-1}$, compared to $\approx 0.3$ MeV nucleon$^{-1}$ for 2000 and 2007.

This comparison, though inconclusive, suggests an anticorrelation between $Q_b$ and $\dot{m}$, as expected, but further investigation is needed. Modelling burst epochs which span a larger range of accretion rates could help to constrain this relationship.

### 3.3 Distance and inclination

From $\xi_p/\xi_b$, we can derive constraints on the system inclination by adopting a disc model for anisotropy. Disc models have been presented by He & Keek (2016), which predicted the anisotropy according to the system inclination for multiple disc morphologies. We used their model of a thin, flat disc (Disc a) to predict the inclination, $i$, using $\xi_p/\xi_b$. The disc model also predicts $\xi_g$ and $\xi_u$, from which we could obtain the absolute distance, $d$. The posteriors for these quantities are plotted in Fig. 6, and the maximum likelihood estimates are listed in Table 3.

These estimates depend on the assumptions of the thin disc model, and only flat priors were used for $d\sqrt{\xi_p}$ and $\xi_p/\xi_b$. Exploring other priors, and other disc models, could yield different constraints.

### 3.4 Neutron star properties

We extract distributions for the neutron star properties using the MCMC parameters of $M$ and $g$. The neutron star radius is calculated by solving

$$g = \frac{GM}{R^2 \sqrt{1 - 2GM/(c^2R)}}$$  (11)

for $R$, given $M$ and $g$.

The gravitational redshift, $z$, is then calculated from $M$
and $R$ using Eq. (1). The posteriors for these quantities are plotted in Fig. 7, and the maximum likelihood estimates are listed in Table 3.

The highest probability density for $M$ is against the upper boundary of $M = 2.2 M_\odot$, indicating that the distribution is truncated. This upper limit was informed by the largest known neutron star mass (Linares et al. 2018), suggesting a possible bias in our model towards large masses. Additionally, the distribution for $g$ is constrained by both the upper and lower model grid boundaries.

We note that only flat prior distributions were used for $M$ and $g$, and thus did not include any expectations from theoretical EOS predictions, or from mass estimates of similar bursting systems (e.g., Özel et al. 2012). Exploring additional prior distributions, and expanding the model grid in $g$, is required before drawing stronger conclusions.

Despite the limitations, these results represent a step towards constraining the neutron star mass and radius using 1D burst models.

3.5 Global accretion rate

The model grid uses the local accretion rate per unit area, $\dot{m}$. The global accretion rate, $\dot{M} = 4\pi R^2 \dot{m}$, depends on the neutron star radius, which is determined by $g$ and the free parameter of $M$. Combining the posterior samples of $\dot{m}$ and $R$, we obtain estimates for $\dot{M}$, which are listed in Table 3. We give $\dot{M}$ as a fraction of the fixed Eddington rate, $\dot{M}_{\text{Edd}} = 1.75 \times 10^{-8} M_\odot \text{yr}^{-1}$, which is the equivalent of $\dot{m}_{\text{Edd}} = 8.775 \times 10^4 g \text{cm}^{-2} \text{s}^{-1}$ for $R = 10 \text{ km}$. We note again that this Eddington value is simply used as common reference points for convenience, and does not represent the ‘true’ Eddington limit.

3.6 Lightcurve sample

The burst data used by the MCMC model is an incomplete description of the full burst lightcurve. The two quantities extracted from the lightcurves were the fluence, $f_{\text{peak}}$, and the peak flux, $F_{\text{peak}}$. To test whether the full lightcurves of Kepler models remain consistent with the observations, we performed an additional set of simulations.

We took a random sample of 30 points from the MCMC chain, and for each point generated three new Kepler models using the sampled parameters $m_i$, $Q_{b,i}$, $X_0$, $Z_{\text{CNO}}$, and $g$. The result was a set of 90 Kepler simulations, representing a sample of 30 epoch triplets from the posterior distribution.

The modelled bursts were extracted using the same procedure as the original grid (§ 2.4). We calculated average burst lightcurves for each model, and converted them to observables using the corresponding samples of $M$ and $d\sqrt{R}$ (§ 2.7). These lightcurves are plotted with the observed lightcurves in Fig. 8.

There is good agreement between the modelled and observed lightcurves, particularly considering that the MCMC model was fitting the fluence and peak flux, and not the full lightcurves. This comparison suggests that these scalar quantities may be sufficient proxies for the overall lightcurve – at least for bursts with similar lightcurve morphologies.

Nevertheless, some lightcurve information is still lost with this method. For example, the morphology of the decay tail is not considered, which encodes further information about the $rp$-process, cooling of the envelope layers in’t Zand et al. (2009), and possible interactions between the burst flux and the disc (Worpel et al. 2015). Fitting additional lightcurve data, or even the entire lightcurve itself (§ 4), should remain a goal for future model comparisons.

3.7 Modelling single epochs

To test the benefit of fitting multiple epochs simultaneously, we performed three additional MCMC models, each fitting the data of a single epoch. The posterior distributions for the single-epoch chains are shown in Fig. 9 (coloured histograms), along with the original multi-epoch posteriors (black histograms).

Compared to fitting the epochs separately, the posterior distributions were generally more constrained when all three epochs were fit simultaneously. An exception appears to be $g$, although all four distributions for this parameter are heavily truncated at the boundaries, potentially interfering with the results. The parameter constraints also remain overall consistent between the multi-epoch and single-epoch chains.

This comparison supports the approach that fitting multiple epochs simultaneously can help to improve the degeneracies between the system parameters (as tested by M18).

4 DISCUSSION

We constrained system parameters for GS 1826–238 by comparing multi-epoch observations to the most extensive
set of 1D model predictions to date. All central values discussed here correspond to the maximum likelihood estimates of the 1D marginalised posteriors, with 68 per cent credible intervals (Table 3).

The global accretion rate estimates of $\dot{M} = 0.098^{+0.012}_{-0.014}$, $0.132^{+0.016}_{-0.02}$, and $0.153^{+0.017}_{-0.02}$ $M_\odot$yr$^{-1}$, are roughly double those initially suggested by G17 of 0.0513, 0.0692, and 0.0796 $M_\odot$yr$^{-1}$, respectively, although those estimates did not account for anisotropy or different values of gravity. Conversely, our central values are only $\approx 10$ per cent smaller than those reported by M18 of $\dot{n}_i = 0.11, 0.15$, and 0.17 $M_\odot$yr$^{-1}$, and are consistent within $2\sigma$. Planned comparisons of MESA and Kepler models could test whether their predictions are more consistent, given the improvements to Kepler described in § 2.1.

The crustal heating estimates for the 2000 and 2007 epochs of $0.17^{+0.14}_{-0.14}$ and $0.15^{+0.13}_{-0.11}$ MeV nucleon$^{-1}$ are centred near the canonical value of $Q_h = 0.15$ MeV nucleon$^{-1}$, though with broad credible intervals. On the other hand, the estimate for the 1998 epoch, with a lower accretion rate (§ 3.2), is roughly double, at $0.36^{+0.10}_{-0.10}$ MeV nucleon$^{-1}$, although 0.15 MeV nucleon$^{-1}$ still lies within $1\sigma$. In agreement with M18, crustal heating above $Q_h \approx 0.5$ MeV nucleon$^{-1}$ is disfavoured in all epochs, although the upper limit of our model grid is $Q_h = 0.6$ MeV nucleon$^{-1}$, compared to the 1.0 MeV nucleon$^{-1}$ considered by M18.

The CNO metallicity of $Z_{\text{CNO}} = 0.01^{+0.005}_{-0.004}$ is centred on the assumed solar value of 0.01. The chosen prior distribution was also centred near 0.01 (§ 2.9). The result broadly supports a solar metallicity, whereas previous studies typically used higher values of $Z_{\text{CNO}} = 0.02$ (e.g., M18; H07). Values below $\approx 0.005$ are disfavoured, such as the low-metallicity of $Z_{\text{CNO}} = 0.001$ suggested by G94. Our model grid, however, only extends down to $Z_{\text{CNO}} = 0.0025$, and could be expanded in future studies.

The accreted hydrogen fraction of $X_h = 0.74^{+0.02}_{-0.03}$ is larger than the commonly-assumed value of $X_h = 0.7$, which lies slightly outside $1\sigma$, but still within $2\sigma$ of our estimate. The $1\sigma$ credible interval extends up to $X_h \approx 0.76$, at odds with M18, who reported poor model fits for $X_h = 0.75$.

Studies which do not account for burst anisotropy in their distance estimates are implicitly reporting $d_V$. Our value of $d_V = 6.5^{+1.4}_{-1.0}$ kpc is consistent with the previous estimates of 6 kpc (M18), 6.1 kpc (G17), and (6.07 ± 0.18) kpc (H07). Our distance is larger than the estimate of (5.7 ± 0.2) kpc from Chenevez et al. (2016), which

Figure 8. A comparison of the average lightcurves from 30 triplets of Kepler simulations (blue curves, 90 models total) and the observed epoch lightcurves (black histograms). The parameters for these additional Kepler models were drawn from 30 random samples of the MCMC chain. There is overall good agreement between the modelled and observed lightcurves, despite the MCMC procedure fitting only the fluence and peak flux from the observed lightcurve. For clarity, only the average curves are plotted, and the $1\sigma$ variations cover a wider range, as reflected by the $F_{\text{peak}}$ values in Fig. 4.

Figure 9. The posteriors for three additional MCMC models, each fitting only one epoch (coloured histograms), along with the original multi-epoch model (black histogram). The shaded regions are 68 per cent credible intervals.
was obtained using the same 2010 $F_{\text{Edd}}$, but assumed a fixed mass of $M = 1.4 M_\odot$ and a radius of $R = 10$ km. Our distance is also consistent with earlier upper limits of 8 kpc (in’t Zand et al. 1999) and 7.5 ± 0.5 kpc (Kong et al. 2000), but is larger than the more recent upper limit of 4.0–5.5 kpc (Zamfir et al. 2012).

The anisotropy ratio of $\xi_p/\xi_0 = 1.57^{+0.15}_{-0.10}$ agrees with the original estimate of $\xi_p/\xi_0 = 1.55$ from H07, but not with the value of 3.5 from M18, although both of these studies explored fewer model parameters. Using a flat disc model from He & Keek (2016), we obtained from explored fewer model parameters. Using a flat disc model from He & Keek (2016), we obtained from $\xi_p/\xi_0$, a system inclination of $i = 69^{+25}_{-15}$°. This inclination is consistent with the upper limit of $i \lesssim 70$° from low-amplitude optical modulations (Homer et al. 1998), and the range of 40–70° suggested by Mescheryakov et al. (2004) from models of the disc size.

The gravitational redshift of $z = 0.39 \pm 0.07$ is larger than the commonly-approximated value of $z = 0.26$ from $M = 1.4 M_\odot$ and $R = 11.2$ km, and agrees with the value of 0.42 from M18, but is outside the inferred range of $z = 0.19 - 0.28$ for GS 1826–238 reported by Zamfir et al. (2012). The radius of $R = 11.3 \pm 1.3$ km is consistent with the rough upper limit of 9.0–13.2 km suggested by Zamfir et al. (2012).

This study is the first application of MCMC methods to 1D burst models featuring adaptive nuclear networks. As such, simplifying assumptions have been made and some care should be taken when interpreting the results.

It should be emphasized that our posterior statistics are fully dependent on the assumptions contained in the KEPLER models and our interpolation between their predictions. Although it is currently among the most advanced codes for simulating X-ray bursts, KEPLER is still the subject of ongoing refinements (§2.1), and is inherently limited to spherical symmetry. Additionally, using linear interpolation to sample between the models (§2.5) may introduce artificial ‘kinks’ at the grid points, potentially affecting the resulting distributions. Our comparisons of the posterior predictive distribution to the observed data (§3.1), and a sample of full lightcurves (§3.6), however, suggest that the interpolated models are not behaving unexpectedly.

The posterior distributions were truncated by some of the model grid boundaries, notably $X_0$, $Z_{\text{CNO}}$, $Q_b$, and $g$ (Fig. 3). Some of these boundaries are physically-motivated, for example $X_0 < 0.76$ and $Q_b > 0 \text{ MeV nucleon}^{-1}$, whereas others could realistically be extended, for example below $Z_{\text{CNO}} = 0.0025$ and above $Q_b = 0.6 \text{ MeV nucleon}^{-1}$. Large extensions of the five-dimensional model grid, however, are limited by computational costs.

The information we included in our prior distributions (§2.9) was relatively limited. All parameters except for $Z_{\text{CNO}}$ used flat priors. Aside from providing no additional constraints, flat priors could give undue weight to physically unrealistic regions of parameter space. For example, all combinations of $M$ and $g$ – and by extension, the corresponding $R$ and $z$ – were considered equally likely under the prior assumptions. This may have contributed to the possible bias towards large $M$ (§3.4).

The limitations discussed above can be investigated and improved upon in future work. Linear interpolation, while computationally fast, has limited accuracy. Other interpolation methods, such as cubic splines, could be explored, but care should be taken to avoid introducing artefacts. A parameter sensitivity study could also identify which grid parameters can afford fewer model points, reducing the total number of simulations required.

The observed values of $F_{\text{peak}}$ and $f_b$ were taken from observed lightcurves. The full burst lightcurves, however, encode additional information about the rates of heating and cooling, and the extent of rp-process burning in the tail. Fitting whole lightcurves could improve, or even significantly reshape, the posteriors. Implementing this approach, however, poses certain challenges. Whereas interpolating scalar quantities is straightforward, it is unclear how best to do so for lightcurves. If the lightcurves significantly change in morphology, interpolation could introduce nonphysical features. A possible alternative is to use machine learning to efficiently predict lightcurves between models, such as the methods recently applied to gravitational waveforms of neutron star mergers (Easter et al. 2019). Extra parametrizations of the lightcurve could also be used, by fitting curves to the burst tail (in’t Zand et al. 2009). Nevertheless, our test of a limited sample of full lightcurves (§3.6) suggests that $f_b$ and $F_{\text{peak}}$ may still serve as reasonable representations of the lightcurve.

A key benefit of MCMC methods is their ability to efficiently handle large numbers of parameters. Additional parameters not used in this work could also be explored. For example, using epoch-dependent anisotropy ratios, $\xi_p/\xi_0$, could test for possible changes in the accretion disc properties between epochs. When calculating the Eddington flux, $F_{\text{Edd}}$, we assumed that the hydrogen fraction was equal to the accreted fraction, $X_b$, but expansion of the outer layers during PRE may expose deeper hydrogen-poor layers, increasing $F_{\text{Edd}}$. This hypothesis could be tested by including the hydrogen composition for $F_{\text{Edd}}$ as an additional parameter.

5 CONCLUSION

We carried out Markov chain Monte Carlo (MCMC) simulations to model multi-epoch X-ray bursts from GS 1826–238. By precomputing a grid of 3840 KEPLER models, we interpolated the predicted burst properties and efficiently sampled the model parameter space. Applying the Bayesian framework of MCMC allowed us to systematically examine the relationships between the model parameters and the predicted burst properties. We obtained probability distributions for the properties of GS 1826–238, including the accretion rates, crustal heating rates, accreted composition, and surface gravity.

This work represents the most comprehensive use of 1D models on a burst source to date. We have explored model parameters which are often held constant in burst models, including the crustal heating, accreted hydrogen composition, surface gravity, the neutron star mass and radius, and the gravitational redshift. By using epoch-dependent parameters of $Q_b$, we have also tested the dependence of crustal heating on accretion rate (§3.2), suggesting a preference for stronger crustal heating at lower accretion rates.

Although we have focused on GS 1826–238, the methods presented here are applicable to other X-ray burst observations. Once the model grids are established, they can also be reused for similar systems. By incorporating new epoch data and expanding the grid parameters, we can analyse...
additional sources suggested by G17, such as the helium-burster, 4U 1820–30. Preliminary work is already underway to model PRE bursts from 4U 1820–30 with a new model grid, which we plan to present in a future publication.

This work demonstrates the largely uncharted potential of using 1D burst models for the parameter estimation of neutron star systems.

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SOFTWARE PACKAGES USED

pyburst, emcee, matplotlib, scipy, astropy, chainconsumer

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Multi-epoch X-ray burst modelling
APPENDIX A: MODEL PRE-HEATING

Excessive burn-in can occur during simulations if nuclear heating, $Q_{\text{nuc}}$, is neglected during the model setup phase (§ 2.1). Addressing the issue is not straightforward, however, because nuclear heating occurs throughout the envelope at difference rates, depending on the local conditions. By contrast, the flux from crustal heating, $Q_b$, is simply implemented as a lower boundary condition. Predicting $Q_{\text{nuc}}$ in advance is difficult prior to running the full simulation with a nuclear envelope.

We added a heat source during the setup of the envelope before the full burst simulation begins. For a chosen $Q_{\text{nuc}}$, the total heat flux is given by $F_{\text{nuc}} = Q_{\text{nuc}} \dot{m}$, which we distributed throughout the envelope as a Gaussian function, centred at a column depth of $y \approx 8 \times 10^7$ g cm$^{-2}$ with a spread of $\sigma \approx 8 \times 10^6$ g cm$^{-2}$. For comparison, $Q_b$ is implemented at the lower model boundary of $y \sim 10^{12}$ g cm$^{-2}$.

We tested this model setup with a heating of $Q_{\text{nuc}} = 5$ MeV nucleon$^{-1}$, approximately the energy yield for hydrogen burning. We used model parameters of $X_0 = 0.73$, $Z_{\text{CNO}} = 0.005$, $\dot{m} = 0.2$, and $Q_b = 0.05$ MeV nucleon$^{-1}$. The burn-in was largely eliminated from the resulting burst simulation (Fig. A1), in contrast to an identical model without preheating (effectively, $Q_{\text{nuc}} = 0$ MeV nucleon$^{-1}$). Previous studies typically discarded only the first $\approx 3$ bursts to account for model burn-in (e.g., Woosley et al. 2004). We demonstrate, however, that a 10–20 per cent discrepancy persists between the recurrence times even after 50 bursts. Nevertheless, further investigation is required into the sensitivity of models to preheating. Other bursting regimes, for example helium bursts, may require additional testing of the heating rates and depths.

APPENDIX B: GR CORRECTIONS

KEPLER uses Newtonian gravity to calculate the gravitational acceleration, given by

$$g = \frac{GM}{R^2}.$$  \hfill (B1)

In the GR regime of a neutron star, however, the surface gravity is given by

$$g = \frac{GM(1+z)}{R^2},$$  \hfill (B2)

where $z$ is the gravitational redshift given in Eq. (1).

Because the envelope is a thin shell ($\Delta R \ll R$), the surface gravity is approximately constant throughout the envelope. The Newtonian KEPLER model is equivalent to neutron stars under GR with different $M$ and $R$, but with the same $g$. There is a contour of $M$ and $R$ pairs which satisfy this constraint. For a chosen $M$ and $R$, the Newtonian KEPLER quantities can be corrected to the equivalent GR values. A more detailed description of these corrections can be found in appendix B of Keek & Heger (2011).

Keek & Heger (2011) defined the mass and radius ratios between the two regimes,

$$\varphi = \frac{M_{k}}{M_{g}}, \quad \xi = \frac{R_{k}}{R_{g}},$$  \hfill (B3)

where we here signify the Newtonian and GR quantities with the subscripts ‘k’, and ‘g’, respectively. Setting the requirement that $g$ must be equal under the two regimes, the above ratios are related by

$$\xi^2 = \varphi(1+z),$$  \hfill (B4)

where $z$ is evaluated for $M_k$ and $R_k$.

The ratio of the neutron star surface areas is given by $\xi^2$, and so the GR-corrected luminosity is given by

$$L_{k} = \xi^2 L_{g} = \varphi(1+z)L_{g}.$$  \hfill (B5)

For a given accretion rate per unit area, $\dot{m}$, the global accretion rate, $\dot{M} = 4\pi R^2\dot{m}$, is also scaled by the area ratio, $M_{k} = \xi^2 M_{g} = \varphi(1+z)M_{g}$. Both regimes are in the same local reference frame, and so $\Delta t$ and $\nu$ are not time-dilated.

These GR-corrected quantities were used to calculate the predicted observables in § 2.6 and 2.7.
APPENDIX C: MODEL DATA

The data used in this work are publicly available at Mendeley Data\(^9\). Included is a data table of the analysed model grid, listing the input parameters and summary burst properties of each model as Newtonian Kepler quantities (i.e., not corrected for GR). Additionally, the full MCMC chains are included as 3D arrays, containing 1000 walkers × 12 parameters × 20000 steps (including the initial 1000 burn-in steps which were discarded from our analysis). Further information on how to load and use this data is provided in the data repository.

The software tools used to extract the model burst properties, analyse the model grid, and manage the MCMC models, have been collected under a Python package called pyburst, which can be downloaded from https://github.com/zacjohnston/pyburst/.

This paper has been typeset from a \TeX/\LaTeX file prepared by the author.

\(^9\) https://data.mendeley.com/datasets/nmb24z6jrp/draft?a=9896f6b8-5d98-4bd1-b448-222eb0fa5b9b [to be replaced with permanent DOI URL in final manuscript]