Invesitgation of nonlinear deformation of solid and porous elements of three-dimensional structures

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Abstract. In this paper, a number of test problems for deforming three-dimensional bodies, made of solid and porous materials, are solved on the basis of the method for solving nonlinear problems of the mechanics of a deformable solid body based on defining relations connecting the increments of true stresses and strains within the framework of the finite-element discretization of the computational domain. A good agreement between the calculation results and the known solutions was noted, and some detected patterns of deformation of bodies made of porous material were noted.

1. Introduction

Recently, approaches to the analysis, design and optimization of complex technical systems based on the use of large-scale simulation computer models have been widely used in many spheres of human activity. Such approaches are well known in the aircraft and helicopter industry [1-7], the automotive industry, and the chemical and transport industry, construction and biology. In recent years, there has been an active use in technology of new materials, in particular, printed on 3D printers, fibrous composites, as well as new approaches are applied to the modeling of porous collectors and biomaterials in computational geo- and biomechanics [8-13]. One of these areas forms the approaches based on a comprehensive application in the construction of mechanical and mathematical models of experimental and numerical methods, as well as the use of scanning equipment to determine the microstructure of the material at all stages of its deformation.

The use of numerical methods of mechanics of a deformable solid allows solving a wide range of practical problems of deforming fine-grained porous media with regard to their microstructural changes. Despite the fact that numerical solutions of continuum equations are well known theoretically [14-18] and implemented practically in modern applied multiphysical systems, the specifics of the effect of changing the microstructure in the process of deformation on the macroscopic behavior of fine-grained porous materials, bone tissue, porous and fractured-porous geological rocks and composite materials entail significant difficulties in their practical application.

At this time, a considerable part of the computational methods is based on the discretization of the domain, moreover, mainly finite element and finite difference methods [19-23]. The solution of many practical problems of deforming fine-grained porous media, taking into account their microstructural changes, is possible with the help of numerical methods of mechanics of a deformable solid. A
deformable solid can be represented as a whole hierarchically formed system of deformation mechanisms according to the concept of mechanics of a deformable solid [24-26]. There are numerical solutions of continuum equations, practically implemented, for example, in a system such as ANSYS, but a great difficulty in practical implementation arises due to the peculiarities of the effect on macroscopic behavior of fine-grained porous materials, porous and fractured-porous geological rocks, and composite materials as a result of deformation of the microstructure.

2. Problem statement and numerical results

A variant of the method called the “modified Lagrange incremental theory” is proposed as a calculation algorithm. It is one of the options for implementing the incremental algorithm of the decision continuation process on the load parameter [27-30]. The process of deformation is represented as a sequence of equilibrium states that are realized at the corresponding loading levels.

The construction of the computational algorithm is based on the discretization of the computational domain in the framework of the finite element method. The finite element method [31-33] has been implemented, which allows, on the basis of the previously constructed equations of a consistent version of the theory of elasticity with small deformations and arbitrary displacements, to simulate the processes of deformation of structural elements of fine-grained porous materials, bone tissue, porous and fractured porous geological rocks and composite materials with taking into account their structural rebuilding under single and multiple loads. In the framework of the problem of elastoplastic deformation of elements of three-dimensional structures, models of isotropic and kinematic loading were used.

Based on the proposed algorithm, the solutions were solved: the problem of stretching a three-dimensional link, the problem of bending fixed at one end of a three-dimensional link and the Lamé problem of inflating a cylinder for solid and porous materials. For the porous material, the case was considered when the pores have a cubic shape and are uniformly distributed throughout the volume. The porosity was assumed to be equal \( m = 1/27 \), the results are given for the case when there are 10 pores across the thickness of the computational domain.

To illustrate the calculations carried out, Figures 1 and 2 show deformation diagrams for the case of tensile bar with different hardening parameters, which were chosen in the Odqvist form, for elastoplastic models with isotropic and kinematic hardening, respectively.

![Figure 1. Strain diagrams in a link with isotropic hardening](image1)

![Figure 2. Deformation diagrams in the link with kinematic hardening](image2)

Figure 3 shows the dependence of stresses on the axial displacements of the link end for elastic and elastoplastic models of a material with isotropic linear hardening for a solid and porous link.
3. Analysis of the results and conclusions

For the problem of stretching a solid link, one can note a good agreement between the results obtained by different methods. When decreasing $\epsilon ps$, estimating the magnitude of the quadratic deviation of the increment of the global vector of nodal displacements from the achieved value, the results of the solution obtained by the proposed method tend to the results obtained in ANSYS Program 14.0 and when $\epsilon ps$ reach 0.0001, they practically coincide with them. The results of solving this problem show that the technique implemented in the work adequately models both isotropic and kinematic hardening during elastoplastic deformation.

For the Lame problem, the radius of the surface separating the elastoplastic and elastic zones of the solution does not depend on whether the hardening law is used is isotropic or kinematic, since considered the same nature of the deformation. An increase in the tangential hardening modulus leads to a decrease in the radius delimiting the zones of elastic and elastoplastic deformation.

For the problem of bending a three-dimensional link, the level of plastic deformations achieved is higher than the level of elastic deformations. With isotropic hardening, the achieved plastic deformations practically do not decrease with further loading, and with kinematic hardening with further loading, they can decrease almost to zero. Increasing the tangential hardening modulus leads to an increase in the level of elastic deformations relative to the level of plastic deformations.

For the problem of stretching a porous link, it is possible to note a satisfactory coincidence of the Young's modulus obtained by the proposed numerical method for calculating the elastoplastic deformation of porous bodies and estimated theoretically. The difference in their definition is explained by a very rare QE mesh. The yield strength of a porous sample is naturally below the yield strength of a continuous sample. It can be noted that the onset of plastic deformation in the sample will most likely be determined by the minimum cross-sectional area of the sample. Therefore, its size can also be affected by the maximum pore size. The module of tangential hardening of a porous material is also reduced compared with the corresponding module for a continuous sample. But it can be noted that during unloading and further loading of the porous sample, this module remains unchanged. With an increase in the level of stress, the module of tangential hardening remains unchanged, but the unloading and subsequent loading follows a linear law, the modulus of which is much lower than the elastic modulus of the sample material.

Acknowledgments

The reported study was supported by Government of the Republic of Tatarstan research projects and RFBR research project No. 18-41-160018 and was funded by RFBR according to the research project №18-07-00964.
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