Numerical simulation of three-stage gas coupled pulse tube refrigerator

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Abstract: For its compact structure, small mass, no moving parts at low temperature, strong reliability and stability, Stirling pulse tube refrigerator is regarded as a major development direction of small refrigerator at low temperatures. In order to obtain lower no-load cooling temperature and higher cooling efficiency, multi-stage structure is often used in pulse tube refrigerator. In this paper, a model of three-stage gas-coupled pulse tube refrigerator with multi-bypass and double-inlet is designed by SAGE software. The effects of double-inlet and multi-bypass on the gas distribution of multi-stage pulse tube refrigerator are analyzed. The results show that the multi-bypass and double-inlet do not independently affect the minimum temperature of the refrigerator at no-load, and there is a coupling relationship between their opening. The numerical simulation results are of great value for the construction of a three-stage gas coupled pulse tube refrigerator prototype.

1. Introduction.

Stirling-type pulse tube cryocoolers (SPTC) are driven by a linear compressor with double opposed pistons, which uses pressure waves generated by the reciprocating motion of the pistons in the cylinder to directly drive the expansion and compression of the gas inside the pulse tube to produce the refrigeration effect, offering the advantages of long life, low vibration and high efficiency. SPTC use the same compressors as Stirling refrigerator, therefore, it can benefit from the researches of flexible bearings and hydrodynamic gas bearings in space Stirling refrigerator, which are considered a major development in low-temperature compact refrigerator.

There is some research literature on SPTC that have successfully obtained liquid helium cooling temperature. In 2008, Lockheed Martin\textsuperscript{[1]} developed a four-stage SPTC that uses the cold head with a
helium-4 to cool the first and second stages of a cryocooler with a helium-3, and the final four-stage cold end can provide 1-10 mW of cooling capacity at 4.5 K. In 2011, L.M. Qiu et al.\[2\] studied a three-stage SPTC driven by two compressors, achieving a no-load temperature of 4.97 K and a cooling capacity of 25 mW at 6 K. In 2018, Liubiao Chen et al.\[3\] studied a three-stage SPTC using liquid nitrogen pre-cooling, achieving a no-load temperature of 3.6 K and a cooling capacity of 6 W @4.2 K with a single compressor. In 2020, Hai-Zheng Dang et al.\[4\] studied a four-stage SPTC with a similar structure to the Lockheed Martin four-stage SPTC, which achieved a no-load temperature of 4.2 K and a cooling capacity of 25 mW at 5 K using helium-4 as the working gas.

However, the research of SPTC in the liquid helium temperature is still not perfect, and the use of more than two compressors increases the weight and size of the refrigerator, and also reduces the efficiency of the SPTC. Therefore, if a single compressor driven gas-coupled pulse tube refrigerator can reach the liquid helium temperature, it will further promote the miniaturization of the SPTC, which is a very meaningful research topic. In this paper, we propose to use SAGE software to numerically study a three-stage pulse tube refrigerator with a single linear compressor, in order to prepare for the next step of building an experimental prototype.

2. Model construction

Sage numerical calculation software is a refrigerator calculation software developed by David Gedeon. Sage has a visual interface and models each component individually according to its geometry and material. The components are connected by mass flow, pressure wave and energy flow, allowing for the design and optimization of the entire refrigerator.

A three-stage SPTC model was developed, as shown in Figure 1:

![Figure 1. Schematic diagram of the structure of SPTC with multi-bypass](image)

The parameters of each stage pulse tube refrigerator are as follows:

| Table 1. Pulse tube and inertia tube parameters |
|-----------------------------------------------|
| Inertia tube                                  |
| IT I     | 1m@2mm+2m@3mm+4m@5mm |
|----------|----------------------|
| IT II    | 1.84m@2mm            |
| IT III   | 0.88m@1mm+2.37m@2mm  |

|       | Length (mm) | Outer diameter (mm) | Inner diameter (mm) |
|-------|-------------|---------------------|---------------------|
| PT I  | 44mm        | 28mm                | 14mm                |
| PT II | 40mm        | 18mm                | 9mm                 |
| PT III| 53mm        | 12.5mm              | 5.5mm               |

3. Numerical calculation results

![Table 2. Optimization calculation results](image)

Figure 2. Effect of different multi-bypass openings on the coldest end temperature

Figure 2 gives the effect of unloaded temperature for different diameters of the multi-bypass2 corresponding to different diameters of the multi-bypass1. It can be seen that when the opening of the multi-bypass1 is changed, the optimal opening of the multi-bypass2 also changes.
Figure 3. Effect of multi-bypass2 opening on available energy loss in the three regenerators

AEfric: Available energy loss to viscous flow friction; AEQw: Available energy loss to radial directed solid-surface heat flow; AEQx: Available energy loss to axial heat flow

Figure 3 shows the effect of the multi-bypass2 opening on the energy loss inside the third stage heat exchanger. With the increase of the multi-bypass2 opening, the gas flow rate into the third stage heat exchanger decreases, as shown in Figure 4(a), so the available energy loss in the heat exchanger is significantly reduced. The irreversible heat transfer loss between gas and solid also decreases slightly with the increase of the multi-bypass opening.
Figure 4. Effect of multi-bypass2 opening on the gas flow of the pulse tube at each stage
Gas flow ratio: Ratio of gas flow through the cold head heat exchanger to the next stage of the accumulator to the gas flow through the previous stage of the accumulator

Figure 4 shows the effect of varying the opening of multi-bypass2 on the gas flow through the third stage cold head for different openings of multi-bypass1. It shows that the gas flow through the third stage cold head decreases with increasing the opening of the second stage multi-bypass without changing the opening of the first stage multi-bypass. However, when comparing different first-stage multi-bypass openings horizontally, it is a strange phenomenon that the gas flow through the third-stage cold head increases with the increase of the first-stage multiple bypass opening for the same second-stage multi-bypass opening.
Figure 5. Effect of multi-bypass opening on the phase difference between mass flow and pressure at each stage of the cold head

Δθ1: Phase difference between gas flow velocity and pressure wave in the first heat exchanger stage; Δθ2: Phase difference between gas flow velocity and pressure wave in the second heat exchanger stage; Δθ3: Phase difference between gas flow velocity and pressure wave in the third heat exchanger stage; T2: Third stage cold head temperature

Figure 5 shows that increasing the opening of the second-stage multiple bypass increases the phase difference between gas flow and pressure in the cold head of the third-stage refrigerator, slightly increases the phase difference between gas flow and pressure in the second-stage cold head. While a phase difference is about 35 degrees, the third-stage cold head can obtain the lowest unloaded temperature.

4. Summary and Prospects
The variation of the multi-bypass openings affects both the distribution of gas flow in the return heaters at each stage and the phase difference between the gas flow and the pressure wave. The distribution of gas flow further affects the magnitude of gas flow losses in the return heaters, while the phase difference between the pressure wave and the gas flow affects the work capacity of the gas flow. The combination of the two influences the no-load minimum temperature of the pulse tube refrigerator.

Based on the numerical calculation results, we will build an experimental prototype to investigate the effect of the first and second stage multiple bypass openings on the no-load minimum temperature of the pulsed tube chiller and explore the theoretical support, so that the numerical simulation results can be verified and corrected.

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