Longitudinal fluid response and pseudorapidity dependent flow in relativistic heavy-ion collisions

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Abstract

We study the pseudorapidity dependent hydrodynamic response in heavy-ion collisions. A differential hydrodynamic relation is obtained for elliptic flow. Using event-by-event simulations of 3+1D MUSIC, with initial conditions generated via a multi-phase transport (AMPT) model, the differential response relation is verified. Based on the response relation, we find that the two-point correlation of elliptic flow in pseudorapidity are separated into the fluid response and the two-point correlation of initial eccentricity.

Keywords: heavy-ion collisions, relativistic hydrodynamics, longitudinal fluid response

1. Introduction

The fluidity of the quark-gluon plasma (QGP) created in heavy-ion collisions has been discovered through the measurements of flow harmonics in multi-particle correlations. These flow harmonics can be understood as the fluid response to the decomposed azimuthal modes associated with the initial state geometrical deformations. For instance, it is noticed that the eccentricity of the initial density profile, $E_2$, is linear to the elliptic flow $V_2$ [1]. While this linear relation has been well studied by relativistic hydrodynamics [2, 3, 4], a pseudorapidity dependent hydrodynamic response relation between $V_2$ and $E_2$ is absent in the community, until some recent studies [5, 6]. In the current proceeding, we generalize the linear response relation for the second flow harmonics, to a pseudorapidity dependent hydrodynamic response. With event-by-event simulations of 3+1D MUSIC with respect to initial condition from AMPT, the pseudorapidity dependent response relation is confirmed. Given the pseudorapidity dependent hydrodynamic response, we are able to explore the relation between the two-point correlation of elliptic flow in pseudorapidity and that of initial eccentricity.

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2. Framework

To study the pseudorapidity dependent hydrodynamic response, we generalize the linear response relation as

\[ V_2(\xi) = \int_{-\infty}^{\infty} d\xi' G(\xi - \xi') E_2(\xi'), \]

where \( \xi \) is the pseudorapidity and \( \xi' \) is the space-time rapidity. Although the response function \( G(\xi - \xi') \) implies a boost invariant background, the broken boost invariant symmetry in realistic heavy-ion collisions can be accounted for by perturbations. In Eq. (1), the pseudorapidity dependent flow is defined as \( V_2(\xi) = v_2(\xi)e^{i\Phi_2(\xi)} \) and the initial eccentricity is \( E_2(\xi) = -\int d^2x_\perp \rho(x_\perp, \xi)(x + iy)^2 \int d^2x_\perp \rho(x_\perp, \xi)(x + iy)^2 \).

To identify the response function, it is advantageous to work in a momentum space through a Fourier transformation, namely, for the elliptic flow \( \tilde{V}_2(k) = \int_{-\infty}^{\infty} d\xi V_2(\xi)e^{-ik\xi} \) and initial eccentricity \( \tilde{E}_2(k) = \int_{-\infty}^{\infty} d\xi E_2(\xi)e^{-ik\xi} \). In terms of the wave-number \( k \), Eq. (1) becomes

\[ \tilde{V}_2(k) = \tilde{G}(k)\tilde{E}_2(k). \]

In the small wave number limit with \( |k| \ll k^* \), corresponding to the hydrodynamic regime, the response function can be expanded in series of \( k \). Up to the second order, the expansion is

\[ \tilde{G}(k) = G_0 + ikd_1 - k^2d_2 + O(k^3), \]

which in the \( \xi \)-space amounts to

\[ V_2(\xi) = G_0E_2(\xi) + d_1\frac{dE_2(\xi)}{d\xi} + d_2\frac{d^2E_2(\xi)}{d\xi^2} + O\left(\frac{d^3}{d\xi^3}\right). \]

To obtain these expansion coefficients \( G_0 \)'s, we define new sets of flow observable variables and initial eccentricity variables weighted with powers of \( \xi \) (or \( \xi' \)): \( V_2^{(n)} = \int d\xi\xi^nV_2(\xi)/n! \) and \( E_2^{(n)} = \int d\xi\xi^nE_2(\xi)/n! \). Using integration by parts repeatedly, with these new variables the generalized linear response relation Eq. (1) can be rewritten as

\[ V_2^{(n)} = \sum_{i=0}^{n} (-1)^i G_i E_2^{(i-n)}. \]

Note that the leading order relation \( V_2^{(0)} = G_0E_2^{(0)} \) is the familiar linear response relation of elliptic flow, with the response coefficients being calculated in event-by-event hydrodynamic simulations as [4]: \( G_0 = \langle V_2^{(0)}E_2^{(0)}\rangle \langle E_2^{(0)}E_2^{(0)}\rangle \), where and in the following the angular brackets indicate event average. Following a similar procedure, a set of linear relations can be realized between \( E_2^{(n)} \) and \( V_2^{(n)} = \sum_{i=0}^{n} (-1)^i G_i E_2^{(i-n)} \), with \( G_i \) calculated recursively.

The pseudorapidity dependent response relation Eq. (1) is non-local, which implies that the generation of \( V_2 \) at one pseudorapidity receives contributions from other space-time rapidities. This effect can be shown in the analysis of two-point correlations. We define

\[ \langle (\Delta \xi)^2 \rangle \equiv \frac{\int d\xi' d\xi'' (V_2(\xi)V_2(\xi'))(\xi' - \xi)^2}{\int d\xi' d\xi'' (V_2(\xi)V_2(\xi'))}, \]

to characterize the length of the two-point correlation measured via elliptic flow at different pseudorapidities. With the response relation derived in Eq. (5), it can be proved that

\[ \langle (\Delta \xi)^2 \rangle = \langle (\Delta \xi)^2 \rangle + 4G_2/G_0. \]

The length of the initial state eccentricity two-point correlation \( \langle (\Delta \xi)^2 \rangle \) is defined according to Eq. (6) through \( E_2(\xi) \).
3. Results and discussion

To verify the pseudorapidity dependent response relation, we perform event-by-event hydrodynamic simulations, for the Pb-Pb collision system with $\sqrt{S_{NN}} = 2.76$ TeV at the LHC, within centrality class 30-40%. The 3+1D MUSIC [7, 8] is used with respect to random 3D initial conditions generated by the AMPT model [9]. The initial density profile is obtained in a similar method as in [10]. The pseudorapidity dependent elliptic flow $V_2(\zeta)$ is calculated from thermal pions.

Given the results from hydrodynamic simulations, the linear relation between $E_2^{(0)}$ and $V_2^{(n)} - \sum_{i=0}^{n-1} (-1)^i G_i E_2^{(n-i)}$ can be examined, which can be used to determine the constant response coefficients. Fig. 1 shows the results from the hydrodynamic simulations of approximately 5000 events, with a constant $\eta/s = 0.08$. Only the case of $n = 1$ and $n = 2$ are presented, but they are representative to illustrate the linear relations, and the fact that odd order $G_n$’s vanish. The absolute values of even order $G_n$’s increase exponentially, which implies the convergence of the hydrodynamic gradient expansion in Eq. (3).

Fig. 2 shows the numerical results of about 1000 events from hydrodynamic simulations with different values of $\eta/s$ from 0.001 to 0.2. The obtained length of the two-point correlation of $V_2$ defined in Eq. (6) is plotted as a function of $\eta/s$, which agrees with the expectation from Eq. (7) within statistical errors. This indicates that the increase of the two-point correlation length in elliptic flow comparing to that in the initial eccentricity is purely an effect of fluid dynamics. One can also see that the two-point correlation at the final stage is reduced as the increase of $\eta/s$. This can be understood as a direct consequence of sound propagation, reflected in the ratio $G_2/G_0$ [11]. Since sound propagation is damped as a result of fluid dissipation, the two-point correlation at the final stage is reduced.

In summary, we have derived the differential formulation of pseudorapidity dependent hydrodynamic response which is different from some previous studies (cf. [12, 13]). The formulation is expected to be applicable to other flow harmonics as well, when nonlinear response is not dominant in flow generation [4, 14]. Through the event-by-event simulations of 3+1D MUSIC, with initial conditions generated via the AMPT model, the differential response relation is verified. We also find that the two-point correlation of elliptic flow comparing to that in the initial eccentricity is purely an effect of fluid dynamics.

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Fig. 2. Two-point correlation length of $V_2$ and the expectations from Eq. (7), as a function of $\eta/s$, where the initial two-point correlation length is $\langle (\Delta \xi^2) \rangle = 12.81 \pm 0.08$. Figure taken from Ref. [5].

References

[1] J.-Y. Ollitrault, Anisotropy as a signature of transverse collective flow, Phys. Rev. D46 (1992) 229–245. doi:10.1103/PhysRevD.46.229.

[2] Z. Qiu, U. W. Heinz, Event-by-event shape and flow fluctuations of relativistic heavy-ion collision fireballs, Phys. Rev. C84 (2011) 024911. arXiv:1104.0650, doi:10.1103/PhysRevC.84.024911.

[3] H. Niemi, G. S. Denicol, H. Holopainen, P. Huovinen, Event-by-event distributions of azimuthal asymmetries in ultrarelativistic heavy-ion collisions, Phys. Rev. C77 (5) (2013) 054901. arXiv:1212.1008, doi:10.1103/PhysRevC.77.054901.

[4] J. Noronha-Hostler, L. Yan, F. G. Gardim, J.-Y. Ollitrault, Linear and cubic response to the initial eccentricity in heavy-ion collisions, Phys. Rev. C93 (1) (2016) 014909. arXiv:1511.03896, doi:10.1103/PhysRevC.93.014909.

[5] H. Li, L. Yan, Pseudorapidity dependent hydrodynamic response in heavy-ion collisions arXiv:1907.10854, doi:10.1016/j.physletb.2020.135248.

[6] R. Franco, M. Luzum, Rapidity-dependent eccentricity scaling in relativistic heavy-ion collisions arXiv:1910.14598.

[7] B. Schenke, S. Jeon, C. Gale, (3+1)D hydrodynamic simulation of relativistic heavy-ion collisions, Phys. Rev. C82 (2010) 014903. arXiv:1004.1408, doi:10.1103/PhysRevC.82.014903.

[8] B. Schenke, S. Jeon, C. Gale, Elliptic and triangular flow in event-by-event (3+1)D viscous hydrodynamics, Phys. Rev. Lett. 106 (2011) 042301. arXiv:1009.3244, doi:10.1103/PhysRevLett.106.042301.

[9] Z.-W. Lin, C. M. Ko, B.-A. Li, B. Zhang, S. Pal, A Multi-phase transport model for relativistic heavy ion collisions, Phys. Rev. C72 (2005) 064901. arXiv:nucl-th/0411110, doi:10.1103/PhysRevC.72.064901.

[10] H. Li, L.-G. Pang, Q. Wang, X.-L. Xia, Global $\Lambda$ polarization in heavy-ion collisions from a transport model, Phys. Rev. C96 (5) (2017) 054908. arXiv:1704.01507, doi:10.1103/PhysRevC.96.054908.

[11] J. I. Kapusta, B. Muller, M. Stephanov, Relativistic Theory of Hydrodynamic Fluctuations with Applications to Heavy Ion Collisions, Phys. Rev. C85 (2012) 054906. arXiv:1112.6405, doi:10.1103/PhysRevC.85.054906.

[12] P. BoÄck, W. Broniowski, A. Olszewski, Hydrodynamic modeling of pseudorapidity flow correlations in relativistic heavy-ion collisions and the torque effect, Phys. Rev. C91 (2015) 054912. arXiv:1503.07425, doi:10.1103/PhysRevC.91.054912.

[13] A. Sakai, K. Murase, T. Hirano, Rapidity decorrelation from hydrodynamic fluctuations, Nucl. Phys. A982 (2019) 339–342. arXiv:1807.06254, doi:10.1016/j.nuclphysa.2018.08.012.

[14] D. Teaney, L. Yan, Non lineairities in the harmonic spectrum of heavy ion collisions with ideal and viscous hydrodynamics, Phys. Rev. C86 (2012) 044908. arXiv:1206.1905, doi:10.1103/PhysRevC.86.044908.