Shell-model investigation of odd-mass nuclei in the $^{132}$Sn region

H. Naïdja†,⋆, F. Nowacki†
† Université de Strasbourg, CNRS, IPHC UMR 7178, F-67000 Strasbourg, France
⋆ Université Constantine 1, LPMS, 1 route Ain El Bey, 25000 Constantine, Algeria
E-mail: nhouda@gmail.com

Abstract. New shell-model calculations are performed in the mass region around $^{132}$Sn robust core, where the low-lying states energies and the magnetic dipole moments of even-odd ($^{135,137}$Te, $^{137,139}$Xe, $^{141}$Ba), and odd-even ($^{135}$Sb, $^{135,137}$I, $^{139,139}$Cs, $^{139,141}$La) nuclei are obtained with the N3LOP effective interaction. The position of $5/2^+$ state in $^{135}$Sb is discussed with the variation of the proton gap between $0g_{7/2}$ and $1d_{5/2}$ orbits.

1. Introduction

The study of odd nuclei with few neutrons and protons above $^{132}$Sn doubly magic nucleus constitute a challenging topic for experimentalists and theorists, and provide them an important source of information. Experimentally [1, 2, 3, 5, 4, 6], this class of nuclei can yield information about β-decay rates, half-lives $T_{1/2}$ and single particle excitation energies. For nuclear theorists [7, 8, 9] they remain a severe test of their Hamiltonians, and the observed single particle energies can serve in the calculations of the two-body matrix elements.

In this context, before starting the study of odd-mass nuclei, the analysis of even ones has been necessary. The first investigations were carried out for $^{134,136,138,140}$Sn isotopes, where the energy levels, the isomeric transitions and the masses were widely exposed in [10, 11, 12]. The non-sub-shell closure at $N = 90$ in $^{140}$Sn was discussed and proved in [12], where increasing the neutron gap $1f_{7/2} - 2p_{3/2}$ was incompatible with the observed energy levels and the transition rates in tin isotopes.

Another important step was the survey of the spectroscopic properties of open neutron and proton systems with $52 \leq Z \leq 60$ and $82 \leq N \leq 88$ [13, 14, 15, 16]. One of the most interesting content in this study was to demonstrate the first signs of collectivity in $N = 86$ and 88 isotones, with the signature of soft γ bands supported by the quadrupole properties and the deformation parameters. It is worth noticing that the calculations performed by the way of new effective interaction N3LOP exhibit an overall excellent agreement with the available experimental data.

The main task in this part of work is to extend our shell-model study of $^{132}$Sn mass region, by the calculation of the energy levels and the magnetic moments of odd-mass nuclei with $52 \leq Z \leq 58$ and $82 \leq N \leq 85$, using the same effective interaction N3LOP applied for even nuclei. It was important for us to return to the question about the position of $5/2^+$ state in $^{135}$Sb, where its sharply lowered position was not fully understood.

All these applications constitute the best test for different matrix elements components: neutron-neutron, proton-proton, and neutron-proton, of our N3LOP effective interaction.
This report is organized as follows: after a brief insight into the model space and the employed effective interaction N3LOP (section 2), the results of the energy levels and the magnetic moments are presented in detail in sections 3 and 4 respectively. The conclusions are collected in section 5.

2. Model space and interaction
The calculations are performed in the model space spanned by the 1f7/2, 0h9/2, 1f5/2, 2p3/2, 2p1/2, 0i13/2 neutron orbitals and the 0g7/2, 1d5/2, 1d3/2, 2s1/2, 0h11/2 proton orbitals referred to \( r_{4}h - r_{5i} \), taken above the robust core \(^{132}\text{Sn}\). The corresponding neutrons and protons single particle energies are borrowed from \(^{133}\text{Sn}\) and \(^{134}\text{Sb}\) experimental data \([17]\). However the energies of 0i13/2 neutron and 2s1/2 proton orbital energies are empirical values taken from \([18]\) and \([19]\) respectively.

The diagonalizations are achieved using ANTOINE and NATHAN shell-model codes \([20, 21]\), where the full configuration space is included for all the nuclei of interest, except for \(^{143}\text{Ce}\) where we adopt a truncation scheme by limiting the occupation numbers of 0h11/2 proton orbital.

The realistic \( V_{NN} \) interaction is derived from the Chiral effective field theory potentials N3LO \([22]\) employing the low momentum approach \( V_{\text{low-}} \), by integrating \( V_{NN} \) down to a cutoff momentum \( \Lambda = 2.2\text{fm}^{-1} \). The renormalized interaction is adapted to the model space by many body perturbation theory techniques, including all the \( Q \)-box folded-diagrams up to the second order \([23]\). After, some monopole and multipole modifications to the realistic interaction were necessary: first of all, we reduced slightly (by about 120 keV) the 1f7/2 neutron-neutron pairing matrix elements, to obtain the isomeric transitions in the \(^{134,136,138}\text{Sn}\) isotopes, where the initial realistic interaction failed to reproduce the isomeric transition in the mid-shell nucleus \(^{136}\text{Sn}\). This quenching is due to seniority mixing effects discussed extensively in our previous works \([10, 11, 12]\). After, we did some adjustments to the proton-proton and proton-neutron monopole matrix elements, in order to reproduce the energy of the single particle states of \( N = 82 \) and 83 isotones.

The same effective interaction dubbed hereafter N3LOP already applied in the survey of the spectroscopic properties and collectivity of even-even chains of \( N = 86 \) and 88 isotones with \( 52 \leq Z \leq 60 \) \([13, 14, 15, 16]\), is also employed in this part, to study odd-mass nuclei above \(^{132}\text{Sn}\) core.

3. Energy levels
3.1. Odd-even nuclei
The energy levels of \(^{135}\text{Sb}, \, ^{135,137}\text{I}, \, ^{135,139}\text{Cs}\) and \(^{139,141}\text{La}\) displayed in figures 1-4 and calculated using N3LOP interaction are fairly well reproduced compared to the data \([17]\), and some tentative spin-parity assignment are confirmed by our calculations (i.e \(^{141}\text{La}\) and \(^{139}\text{Cs}\)). However a discrepancy of order 200 keV appeared in some high excited states like 11/2\(^+\) and 15/2\(^+\) states in \(^{137}\text{Cs}\). The 7/2\(^+\) ground state in \(^{135}\text{Sb}\) and \(^{135,137}\text{I}\) is interpreted as the occupation of \( \nu(1f7/2)^{n} \times \pi(0g7/2)^{m} \) (\( n \) and \( m \) are the neutron and the proton valence numbers), while in \(^{137}\text{Cs}\) and \(^{139,141}\text{La}\) it is characterized by \( \nu(1f7/2)^{n} \times \pi(1d5/2)^{5}(1d3/2)^{(m-5)} \).

One of the striking remarks in this investigation, is the good position of 5/2\(^+\) state in all nuclei in particular in \(^{135}\text{Sb}\), which was unexpected behavior discussed before in many theoretical and experimental works. According to the experimentalists in \([5]\), the fall down in energy of the first excited 5/2\(^+\) state in \(^{135}\text{Sb}\) was viewed as a downship of 1d5/2 proton orbit relative to 0g7/2, which was justified by log ft values, and was interpreted to the formation of the neutron-skin. In this context many shell-model calculations were performed to explain this experimental surprising information. In \([7]\) using an empirical interaction the 5/2\(^+\) energy state was overestimated by about 300 keV. To overcome this difficulty A. Brown et al. proposed in \([24]\) a downship of

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As we have proposed and detailed in [6], this behavior could be explained by the fact that the CD-Bonn effective interaction calculations [8] overestimated this state by about 100 keV, without any modification to their CD-Bonn effective interaction.

After this briefly review of some shell-model calculations, it was important for us to check this unusual behavior of 5/2\(^+\) state in \(^{135}\)Sb. As it is shown in figure [1], its position calculated using N3LOP interaction is consistent with the data, characterized by the large weight (43\%) of \(\nu(1f_7/2)^2 \times \pi 1d_5/2\) configuration with 18\% of \(\nu(1f_7/2)^2 \times \pi 0g_{7/2}\). We turn attention that this coherence is not originated from the monopole corrections made in our interaction, where the same result is obtained using the initial realistic interaction \(V_{low-k}\).

Since the position of 5/2\(^+\) state is sensitive to the variation of the proton gap between 0\(g_{7/2}\) and 1\(d_5/2\), we report in Fig. [5] the splitting between the 0\(g_{7/2}\) and 1\(d_5/2\) effective single particle energies (espe). As it is displayed, the gap decreases by increasing the neutron number, until the inversion at N=90, where we expect that 5/2\(^+\) state becomes the ground state in \(^{141}\)Sb. As we have proposed and detailed in [6], this behavior could be explained by the fact that the neutron-proton monopole part \(V^0_{1f_7/20g_{7/2}}\) is more attractive than \(V^0_{1f_7/21d_5/2}\), which leads to the reduction of the 0\(g_{7/2} - 1d_5/2\) proton gap by increasing the neutron numbers.
Figure 5. The variation of the $0g_{7/2}$ and $1d_{5/2}$ protons effective single particle energies.

Figure 6. The energy levels of $^{135,137}$Te

Figure 7. The energy levels of $^{137,139}$Xe

Figure 8. The energy levels of $^{139,141}$Ba

Figure 9. The energy levels of $^{141,143}$Ce
3.2. Even-odd nuclei
Similar to the previous section, the energy levels of even-odd nuclei: $^{135,137}$Te, $^{137,139}$Xe, $^{139,141}$Ba and $^{141,143}$Ce are gathered in figures 6-9. A close agreement within 100keV with the measured data is visible. Moreover, some spin with negative parity attribution are confirmed by our calculations (i.e $^{135,137}$Te). The $7/2^-$ ground state in $^{135,137}$Te and $^{137}$Xe is characterized by the occupation of $\nu(1f_{7/2})^n \times \pi(0g_{7/2})^m$, and it is marked by the main configuration $\nu(1f_{7/2}) \times \pi(1d_{5/2})^{(m-2)}(1d_{3/2})^2$ in $^{139}$Ba and $^{141}$Ce. In $^{139}$Xe and $^{141}$Ba where stronger octupole correlations seem to be present [25], the $3/2^-$ ground state is distinguished by $\nu(1f_{7/2})^3 \times \pi(0g_{7/2})^{(m-2)}(1d_{5/2})^2$.

4. Magnetic dipole moment
The Electromagnetic properties constitute a severe test of our shell-model wave functions. thereby we have assembled in figure 10 the ground states magnetic dipole moments $\mu$ of some odd nuclei, where they are calculated using two sets of spin and orbital $g$ factors, and compared to the scarce available data [26]. Making use of the same effective $g-$ factors used for even-even nuclei in our earlier works [14, 15] ($g_s^p, g_l^p$) = (3.250, 1.069) and ($g_s^n, g_l^n$) = (−1.506, 0.019) for protons and neutrons respectively, an agreement with the experimental data [26] is far more satisfactory than using the free $g-$ factors $(g_s^p, g_l^p) = (5.5857, 1.0)$ for protons and $(g_s^n, g_l^n) = (−3.8263, 0.0)$ for neutrons. As it was argued in [14, 15], the justification of this discrepancy is the missing of some spin-orbit partner in our model space ($r4h−r5i$). However, the disagreement between the calculated value of $\mu(7\frac{1}{2}^-)$ in $^{141}$Ce and the experimental one remains unexplained.

5. Conclusions
Within the shell-model framework, we have deepened our survey of the $^{132}$Sn mass region to odd-mass nuclei. The low-lying state energies of odd-even and even-odd nuclei are established, employing the same effective N3LOP interaction used before in the investigation of even-even nuclei [13, 14, 15]. The experimental spin with negative and positive parity states are well reproduced, and predictions are given in some cases. Also, a good description of the magnetic moments is obtained using effective $g-$factors. In addition, we have probed as well the position of $5/2^+$ state in $^{135}$Sb, which is influenced by the splitting between $0g_{7/2}−1d_{5/2}$ proton orbits.

Finally, at this step of calculations, we conclude that the inspection of the spectroscopic
properties of even-even, odd-even and even-odd nuclei is established with success employing the same N3LOP effective interaction. Moreover, the results are a good support for experimentalists by affirming their new measurements, and a starting point for theorists to develop other interactions. Therefore, these results and interaction provide a good starting point to extend our calculations to higher mass nuclei, where stronger deformations (quadrupole an octupole) are already observed experimentally in rare-earth nuclei.

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