The Evolution of Lyman α Absorbers in the Redshift Range 0.5 < z < 1.9

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Abstract. We investigate the evolution and the statistical properties of the Ly α absorbers of the intergalactic medium (IGM) in the largely unexplored redshift range z = 0.5–1.9. We use high-resolution (R ≳ 30 000) UV (STIS) and optical (VLT/UVES and Keck/HIRES) spectra of nine bright quasars with zem < 1.94. The Ly α lines detected in the lines of sight (LOS) towards these quasars are evaluated with a software package which determines simultaneously the quasar continuum and the line profiles. The main results for the combined Ly α line sample are summarized as follows:

1. The evolution of the number density of the absorbers can be described by the power law \( \frac{dn}{dz} \propto (1+z)^{\gamma} \). The number density of the low column density lines (\( N_{HI} = (10^{12.90} - 10^{14.00}) \text{ cm}^{-2} \)) decreases with decreasing \( z \) with \( \gamma = 0.74 \pm 0.31 \) in the interval \( z = 0.7 - 1.9 \). A comparison with results at higher redshifts shows that it is decelerated in the explored redshift range and turns into a flat evolution for \( z \to 0 \). The stronger absorbers (\( N_{HI} > 10^{13.64} \text{ cm}^{-2} \)) thin out faster (\( \gamma = 1.50 \pm 0.45 \)). The break in their evolution predicted for \( z = 1.5 - 1.7 \) cannot be seen down to \( z = 0.7 \). On the other hand, a comparison with values from the literature for the local number density gives a hint that this break occurs at lower redshift.

2. The distribution of the column densities of the absorbers is complete down to \( N_{HI} = 10^{12.90} \text{ cm}^{-2} \). It can be approximated by a single power law with the exponent \( \beta = 1.60 \pm 0.03 \) over almost three orders of magnitude. \( \beta \) is redshift independent.

3. The Ly α lines with lower column densities as well as the higher column density lines show marginal clustering with a 2σ significance over short distances (\( \Delta v < 200 \text{ km s}^{-1} \) and \( \Delta v < 100 \text{ km s}^{-1} \), respectively). We do not see any difference in the clustering with either column density or redshift.

4. The distribution of the Doppler parameters has a mean value of \( \overline{b} = (34 \pm 22) \text{ km s}^{-1} \). This value is typical for the analyzed region. It does not change significantly with \( z \).

Key words. Cosmology: observations – intergalactic medium – quasars: Ly α forest

1. Introduction

It is traditional to decompose spectra of the Ly α forest into individual absorbers that can be fit with Voigt profiles, each with a redshift \( z_i \), a column density of neutral hydrogen \( N_{HI,i} \) and a Doppler parameter \( b \). Examining a sufficiently large number of LOS to background quasi-stellar objects (QSOs), one can derive the distributions of these parameters as well as dependences between them and in this way deduce characteristics of the Ly α forest. Observers have often focused on the evolution of the number density of Ly α lines, usually approximated by the power law

\[
\frac{dn}{dz} = \left( \frac{dn}{dz}_0 \right) (1+z)^{\gamma}
\]

[Sargent et al., 1980]. At high redshift (\( z > 1.5 \)), the evolution of the strong lines (\( \log N_{HI} > 14.00 \)) is steep (\( \gamma = 2 - 3 \)), transiting into a flat evolution (\( \gamma = 0.1 - 0.3 \)) at lower redshift. This abrupt slow-down in the evolution, first detected by observers [Bahcall et al., 1991; Impey et al., 1996, Weymann et al., 1998] and later also seen in simulations [Theuns et al., 1998; Davé et al., 1999], is suspected to happen at \( z = 1.5 - 1.7 \).

However, since the apparent break in the evolution rate seems to occur just at the transition from high-resolution optical spectra to low-resolution spectra taken with the HST, the lo-
2. Observations

2.1. Selection of the quasars

Table 1 gives an overview of the quasars whose spectra we selected to analyze the Lyman $\alpha$ forest. We list their emission redshifts $z_{em}$ and their apparent $B$ magnitudes $m_B$. We selected QSOs with emission redshifts suitable to show the evolution of the Lyman $\alpha$ forest at $1 < z < 2$. In Table 2, we give the spectral resolution and the approximate $S/N$ of the spectral regions that we examined. The resolution of the spectra of all nine quasars is high enough to fully resolve all Lyman $\alpha$ lines. The $S/N$ varies from one spectrum to the next, and within each spectrum, so the minimum observable column density depends strongly on the spectrum and the wavelength.

### Table 1. Parameters of the quasars

| QSO             | $z_{em}$ | $m_B$ [mag] |
|-----------------|----------|-------------|
| PG 1634+706     | 1.34     | 14.9        |
| PKS 0232-04     | 1.44     | 16.6        |
| PG 1630+377     | 1.48     | 16.5        |
| PG 0117+213     | 1.50     | 16.1        |
| HE 0515-4414    | 1.73     | 15.0        |
| HE 0141-3932    | 1.80     | 16.2        |
| HE 2225-2258    | 1.89     | 16.3        |
| HS 0747+4259    | 1.90     | 15.8        |
| HE 0429-4901    | 1.94     | 16.2        |

with HST/STIS with the E230M echelle mode ($R \sim 30,000$), and in the optical with UVES ($R \sim 50,000$) for HE 0515-4414 and with Keck/HIRES ($R \sim 38,000$) for HS 0747+4259.

The sample was extended with the UVES spectra of three bright quasars from the Hamburger ESO Survey taken in the service mode with the VLT. These three were HE 0141-3932, HE 2225-2258 [Wisotzki et al., 2000] and HE 0429-4901 (not yet published). The last was taken with a slit width of $d = 0.8''$, the other two with $d = 1.0''$. The Lyman $\alpha$ lines detected in these spectra strengthen the statistics for $1.5 < z < 2.0$.

Finally, the QSO sample was supplemented in the UV ($0.5 < z < 1.5$) with data from the STIS archive. For this purpose, the spectra of four quasars — PKS 0232-04 [Shimmins et al., 1966], PG 1630+377 [Noguchi et al., 1980] and PG 0117+213 (Schmidt & Green, 1983) from an observing program of B. Jannuzi (HST proposal 8673/cycle 9) and PG 1634+706 (Schmidt & Green, 1983) from two programs of S. Burles (7292/7) and B. Jannuzi (8312/8) — were downloaded from the STIS data base. These data were also taken with the E230M echelle mode resulting in the same resolution applicable to the other STIS spectra.

Table 2 lists the observing dates, exposure times and quality informations for all the QSO spectra.

The data reduction was performed using the pipelines of UVES and STIS on their respective data. The Keck data of HS 0747+4259 was reduced by J. O'Meara using an internal pipeline and the data reduction software package of T. Barlow. For the HE 0515-4414 STIS spectra another rework had to be made: the radiative flux was in places smaller than zero because an inaccurate correction of the noise background of the CCD chips was used in the pipeline. The problem was solved by measuring the signal needed to make the minimum flux in saturated lines zero. A slightly reduced background value (varying from order to order, and extrapolated to orders that lacked saturated lines) was then subtracted from the raw data flux, resulting in positive flux values for the entire spectrum, apart from random photon noise.

The vacuum and the barycentric corrections to the wavelength scales were performed with the data analysis package MIDAS. Multiple exposures of each QSO were added, weighting each flux value $F_{\lambda,i}$ by the inverse variance (noise) $\sigma_{F_{\lambda,i}}^{-2}$.

2.2. Spectra and data reduction

Because the Lyman $\alpha$ line is shifted into the optical for $z \gtrsim 1.5$, a complete analysis of the Lyman $\alpha$ forest at median and high redshifts requires both optical and UV spectra. For two of the nine quasars, HE 0515-4414 and HS 0747+4259 (discovery of both objects published in Reimers et al., 1998; for the Lyman $\alpha$ forest in the LOS to HE 0515-4414 alone see Janknecht et al., 2002), data in both spectral regions have been obtained: in the UV
We also excluded redshift regions within $\Delta v \sim 2500 - 5000 \, \text{km s}^{-1}$ of the emission redshifts where the ionization of hydrogen is enhanced by the proximity effect. We calculated the maximum redshifts, given in Table 5 from the absolute magnitudes of the QSOs.

For HS 0747+4259, there is a gap of approximately 145 Å between the UV and the optical spectra, while for HE 0515-4414 there is an overlap of roughly 50 Å. In this overlap, we used only lines which can be identified in both spectra (nine altogether) and we chose the line parameters from UVES because they are more accurate. Six lines are only found in one spectrum (three in the STIS and three in the UVES spectrum, respectively), all with column densities $\lesssim 10^{13.20} \, \text{cm}^{-2}$. We suppose that these are predominantly artefacts.

Absorbers with constant comoving number density and constant proper sizes will have a changing density per unit redshift. We define the absorption distance $\Delta X$ so that non-evolving absorbers have a constant density per unit $X$ (Tytler, 1982). Assuming the $\Lambda$CDM model,

$$\Delta X = \frac{1 + z}{\sqrt{\Omega_M (1 + z) + \Omega_\Lambda (1+z)^3}} \Delta z$$

(Misawa et al., 2002), where $\Omega_M$ and $\Omega_\Lambda$ are the density parameters for matter and for dark energy, respectively.

Given a set of QSO spectra, in general there are multiple LOS which sample a specific redshift range. We take this into account by converting the $z$ into $X$ values and summing the $\Delta X$ values to obtain the total absorption distance.

Table 4 shows the wavelength and redshift regions (columns two and three) that we investigated for all quasars after removal of the proximity zones and low $S/N$ orders. The fourth and fifth columns give the computed redshift path $\Delta z$ and the absorption distance $\Delta X$. The total absorption distance for all nine examined quasar LOS is $\Delta X \sim 12.897$. In the sixth column we give the number of detected Ly $\alpha$ absorption lines, 1325 altogether.

In Fig. 1 we show the redshift intervals of the LOS, while in Fig. 2 we show the distribution of all 1325 Ly $\alpha$ lines as a function of redshift and column density, giving qualitative information on the contents of the sample. It can be clearly seen that the redshift interval analysed in this paper is well sampled.

### 3. Voigt profile fits

The analysis of absorption lines detected in quasar spectra is usually performed in three steps. First the lines have to be identified as certain atomic transitions that have been redshifted to the observed wavelengths. Then they are modelled. Finally they are accepted or rejected for the line sample.

For the line identification we used the empirical fact that the H$\alpha$ Doppler parameter $b$ is rarely smaller than 15 km s$^{-1}$ and never $\lesssim 10$ km s$^{-1}$, while for metal lines almost always...
Using the redshifts of the strongest H\textsc{i} systems, we could search for metal lines. We found many additional metal lines at the redshifts of the common Crv 1548/1550 doublets.

The absorption line list derived in this way still contains a few unidentified lines, noise dips interpreted as absorption lines by mistake, unidentified blends, and wrong identifications. However, at these low redshifts, and especially in the high-resolution UVES spectra with their high $S/N$ we guess that such errors are less than 5% of the total list. Besides, the unidentified or wrongly identified lines are typically those with very low column densities and/or low Doppler parameters, which are either rejected by applying the significance level (see below) or do not belong in our Ly\textsc{\alpha} line sample. On the other hand, the blends are a problem that is easily underestimated.

We fit the Ly\textsc{\alpha} lines with two different programs: the lines found in the HST/STIS and VLT/UVES spectra of HE 0515-4414 were fit with the MIDAS software package FITLYMAN (Fontana & Ballester, 1995), which needs a normalized spectrum as input. In order to obtain this, the effective background continuum of the quasar was defined with MIDAS by specifying points in line-free regions and fitting these by a polynomial. An improvement to FITLYMAN is the program CANDALF developed by R. Baade. In CANDALF, the continuum is determined simultaneously with the line fitting procedure. With this parallel approach, the continuum can also be defined reliably in spectral regions where it is hidden by a high line density. CANDALF was used for the analysis of all quasar spectra except for HE 0515-4414 whose spectra were already fit before the development of CANDALF.

Extensive investigations of complex absorption line ensembles, several with partly blended atomic transitions and lines of multiplets fit simultaneously, were carried out with both pro-
grams to check their consistency. The comparison of the ionic parameters determined with FITLYMAN and with CANDALF showed that they were consistent within 1σ in almost all cases. CANDALF converges more easily compared with FITLYMAN in regions with high line densities, finds the global fit minimum with larger probability, and its fit errors are a little bit smaller; however the choice of the fitting program does not have any significant influence on the results of the analysis of the whole Ly α sample (Section 5).

FITLYMAN and CANDALF both assume Voigt profiles convolved with the instrumental profile. The programs adjust three independent parameters per line to minimize the χ²: z, N_HI and b which comprises the thermal and the turbulent broadening of the lines. The general fit strategy, especially for blends, was to start with a single line and to add further components as long as the χ² decreased significantly. Different atomic transitions of the same ion were fit with identical values for the curve of growth where log ⟨R⟩ decreased significantly. Di ff erential distribution function of the hydrogen column density can be seen at 0.5 < z < 1.9.

4. Individual LOS

Before we analyze the Ly α line sample from all nine quasar LOS in Section 5, we briefly mention some interesting characteristics in the individual LOS.

4.1. HE 0515-4414

In the LOS to the quasar HE 0515-4414 a damped Ly α (DLA) system can be seen at z ~ 1.15. Because of this absorber and the brightness of the quasar (m_B = 15.0 mag), it was already a topic of several studies with different goals by the Hamburg group (de la Varga et al., 2000; Reimers et al., 2001, 2003; Janknecht et al., 2002; Quast et al., 2004; 2006; Levshakov et al., 2003).

4.2. HS 0747+4259

The metal line spectrum of HS 0747+4259 has been studied in a separate paper (Reimers et al., 2006) which concentrated on the Ovi absorbing systems in this LOS.

4.3. HE 0141-3932

HE 0141-3932 is an unusual quasar in several respects: its spectrum contains — atypical for quasars — an extremely weak Ly α emission line. In addition the emission lines of different ions give rather different redshifts, depending on the degree of ionization (e.g., Tytler & Fan, 1992). Furthermore, the quasar has several associated absorption systems presumably built by gas ejected from the QSO. The chemical composition of this gas is rather atypical. These special characteristics of HE 0141-3932 are discussed in detail in Reimers et al. (2005) and in Levshakov et al. (2005).

5. Analysis and Discussion

5.1. Column density distribution

The differential distribution function of the hydrogen column densities f(N_HI) is usually defined as the number n of Ly α absorption lines per column density interval ΔN_HI, and per absorption distance ΔX (ΔX from eq. 6; Tytler, 1987). Since our data contains more than one LOS, we need the sum of the individual ΔX_i over all LOS:

\[ f(N_{HI}) = \frac{n}{\Delta N_{HI} \Sigma_{i=1}^{N} \Delta X_i}. \]

In Fig. 4 the distribution function for the lines of all nine LOS is plotted against the column density. Note that the error bars on log f increase with increasing column density because of the decreasing number of lines. Fig. 4 shows that the data points at the highest column densities have a low statistical significance. The fits plotted in this diagram were derived by weighting the data points with 1/σ_e^2 log f. The absorption lines were binned in intervals Δ log N_HI = 0.1. As expected, the linearity of log f breaks off at low column densities. This presumably reflects a selection effect: the weak lines are hidden by the noise of the spectra.

In general, the column density distribution function can be well approximated by the power law

\[ f(N_{HI}) = A \cdot N_{HI}^{-\beta}, \]
Table 4. Fit parameters of the column density distribution for different boundary conditions. Given are (from left to right): the log $N_{\text{H}}$, range of the data points considered in the fit; the interval width; the fit parameters log $A$ and $\beta$ with their 1σ errors.

| $\log N_{\text{H}}$ | $\Delta \log N_{\text{H}}$ | log $A$ | $\beta$ |
|---------------------|-----------------|-------|-------|
| 12.90–15.70         | 0.1             | 9.4 ± 0.3 | 1.60 ± 0.03 |
| 12.90–17.20         | 0.1             | 9.3 ± 0.3 | 1.59 ± 0.02 |
| 13.00–15.50         | 0.5             | 10.0 ± 0.4 | 1.64 ± 0.03 |
| 13.00–16.50         | 0.5             | 9.9 ± 0.3 | 1.63 ± 0.02 |

Fig. 3. Distribution of the Ly $\alpha$ column densities in intervals $\Delta \log N_{\text{H}} = 0.1$, statistical error of log $f$ and fits to the distribution for different fit regions. For the calculation of the fit parameters, the data points were weighted with their error.

where $\beta$ is the (negative) slope and log $A$ is the intercept of the logarithmic representation of $f(N_{\text{H}})$.

We set a completeness limit $\log N_{\text{H}} = 12.90$ for the analyzed line ensemble, while we chose $\log N_{\text{H}} = 15.70$ for the upper limit (see Fig. 3). Using these boundaries and fitting the model to the column density distribution, we derived log $A = 9.4 ± 0.3$ and $\beta = 1.60 ± 0.03$. This result is consistent with other studies of the column density distribution at lower log $N_{\text{H}}$, at comparable $z$: Dobrzycki et al. (2002) found $\beta = 1.6 - 1.7$ performing a curve of growth analysis, Hu et al. (1995) determined $\beta = 1.46$ for log $N_{\text{H}} = 12.30 - 14.50$, and Kim et al. (2001) derived an exponent $\beta = 1.70 - 1.74$ with varying column density regions.

Occasionally it has been claimed that at least two power laws are necessary to accurately describe the column density distribution over the observed range of column density (Carswell et al. 1987; Giallongo et al. 1993; Meiksin & Madau 1993; Pettitae et al. 1993; Penton et al. 2004). If we extend our fit region to higher column densities ($\log N_{\text{H}} = 12.90 - 17.20$), the distribution does not flatten ($\beta = 1.59 ± 0.02$). However, our sample is too small at large column densities to see minor changes in this $N_{\text{H}}$ region.

Table 4 summarizes the results for the fit parameters log $A$ and $\beta$ for different column density regions and interval widths.

Fig. 4. Distribution of the Ly $\alpha$ column densities in intervals $\Delta \log N_{\text{H}} = 0.5$, statistical error of log $f$ and fit to the distribution. The data points were weighted with the error for the fit.

Of course the choice of the size of the intervals $\Delta \log N_{\text{H}}$ in which the lines are collected is somewhat arbitrary. Fig. 4 is based on a larger step size $\Delta \log N_{\text{H}} = 0.5$. For $\log N_{\text{H}} = 13.00 - 16.50$, the distribution can be well approximated by $\beta = 1.63 ± 0.02$ (again weighted with $\sigma_{\log f}^2$). This is within 1σ consistent with the slope for the interval width 0.1 in the comparable range $\log N_{\text{H}} = 12.90 - 17.20$ (only two lines lie in the interval $\log N_{\text{H}} = 16.50 - 17.20$), which makes the point that the choice of the interval width does not influence the result.

Frequently a dependence of the index $\beta$ on $z$ has been postulated by observers (Kim et al. 1997, 2001, 2002; Davé & Tripp 2001; Heap et al. 2002; Misawa 2002). Davé & Tripp (2001) found $\beta = 2.04 ± 0.23$ and Heap et al. (2002) derived $\beta = 2.02 ± 0.21$ for $z \sim 0$ each, while Kim et al. (2001) calculated $\beta = 1.72 ± 0.16$ for $z = 1.6$ and $\beta = 1.38 ± 0.08$ for $z = 2.1$. Telfer et al. (2002) got $\beta = 1.41 ± 0.05$ for $z = 2.3$ and Hu et al. (1995) and Kim et al. (1997) found $\beta = 1.46$ and $\beta = 1.4$, respectively, both for $z = 3$ (all values calculated for the low column density region log $N_{\text{H}} ≤ 14$ where the number of lines make a reliable judgement possible). Simulations of Theuns et al. (1998), e.g., suggested a clear relationship $\beta(z)$, too. These results are contradicted by the comparatively low gradients of Penton et al. (2000, 2004) for the local universe ($\beta = 1.72 ± 0.06$ and $\beta = 1.65 ± 0.07$, respectively). However, the difference might be caused by the analysis method since Penton et al. (2000, 2004) assumed a constant Doppler parameter for the line fits.

In order to search for a possible dependence $\beta(z)$, we examined the column density distribution function separately for three $z$ intervals of the same width (0.5–1.0, 1.0–1.5, 1.5–2.0). For this purpose, the LOS to the quasars had to be divided according to the chosen $z$ intervals, and the absorption distance $\Delta X$ had to be computed for the individual intervals following (2). We selected $\Delta \log N_{\text{H}} = 0.1$ for the interval width. The distributions for all three redshift regions and the fits to them are presented in Fig. 5. In each case the fits take into account the
Table 5. Fit parameters of the column density distribution for different redshift ranges. Indicated are (from left to right): the \( z \) interval; the log \( N_{\text{HI}} \) range of the data points considered in the fit; the fit parameters log \( A \) and \( \beta \) with their 1\( \sigma \) errors.

| \( z \)      | log \( N_{\text{HI}} \) | log \( A \) | \( \beta \) |
|-------------|----------------|------------|-----------|
| 0.5 – 1.0   | 12.90 – 15.70 | 9.1 ± 0.7  | 1.57 ± 0.05 |
| 1.0 – 1.5   | 12.90 – 15.70 | 9.1 ± 0.5  | 1.58 ± 0.04 |
| 1.5 – 2.0   | 12.90 – 15.70 | 8.7 ± 0.6  | 1.55 ± 0.04 |

Table 6. Parameters of different \( b \) distributions. We give the number of Ly \( \alpha \) lines detected, the average value, the median of \( b \), and on the right a parameter that represents the mode.

| \( z \)      | \( n \) | \( b \) [km s\(^{-1}\)] | \( b_{\text{median}} \) [km s\(^{-1}\)] | \( b_{\text{err}} \) [km s\(^{-1}\)] |
|-------------|-------|----------------|----------------|----------------|
| 0.5 – 2.0   | 1325  | 34 ± 22        | 28             | 22.7           |
| 0.5 – 1.0   | 270   | 35 ± 23        | 29             | 18.9           |
| 1.0 – 1.5   | 595   | 33 ± 22        | 28             | 23.5           |
| 1.5 – 2.0   | 460   | 34 ± 22        | 28             | 23.5           |

5.2. Distribution of Doppler parameters

To investigate the distribution of the Doppler parameters, we counted the number of lines per Doppler parameter interval in interval widths \( \Delta b = 5 \) km s\(^{-1}\). Fig. [6] shows how \( b \) is distributed across the complete line sample (without the 24 broadest lines with \( b > 100 \) km s\(^{-1}\)). The distribution has a typical form: nearly Gaussian with a maximum at \( b = (20–25) \) km s\(^{-1}\), an average value \( \bar{b} = (34 ± 22) \) km s\(^{-1}\) lying above the maximum, and a long tail to higher Doppler widths. A large fraction of the lines with \( b > 100 \) km s\(^{-1}\) might represent unresolved blends of several components, whereas the steep decline at small Doppler parameters is caused by the \( SL \) limit (lines with small \( b \) values have small equivalent widths and are more likely to be missed) and by our Ly \( \alpha \) selection threshold \( b > 10 \) km s\(^{-1}\).

The average and median of \( b \) fit the results from other studies in comparable redshift regions very well: [Kim et al. 2001] found \( b_{\text{median}} = 28 \) km s\(^{-1}\) for \( \langle z \rangle = 1.6 \), while [Kim et al. 2002] derived \( \bar{b} = 32.6 \) km s\(^{-1}\) for \( \langle z \rangle = 2.2 \).

Frequently a dependence \( b(z) \) is claimed in the sense of a Doppler width increasing with decreasing redshift. In the high redshift range [Lu et al. 1996] found \( \bar{b} = (23 ± 8) \) km s\(^{-1}\) for \( \langle z \rangle = 3.7 \). [Kim et al. 1997] derived \( b_{\text{median}} = 26 \) km s\(^{-1}\) for \( \langle z \rangle = 3.35 \) and \( b_{\text{median}} = 30 \) km s\(^{-1}\) for \( \langle z \rangle = 2.31 \), respectively, and [Hu et al. 1995] found \( \bar{b} = (28 ± 10) \) km s\(^{-1}\) for \( \langle z \rangle = 2.9 \).

The average value of this study for \( \langle z \rangle = 1.31 \) appears to support this trend. Going to \( z = 0 \), the evolution of \( b \) possibly continues: [Penton et al. 2000] derived \( \bar{b} = (38 ± 16) \) km s\(^{-1}\) for the local universe.

Therefore we investigated whether an evolution of \( b \) with \( z \) can be determined within the \( z \) range examined here. In Fig. [7] the \( b \) distribution is shown for three different \( z \) ranges of the same width, while Table [6] gives parameters of the distributions. The highest average value \( \bar{b} = (35 ± 23) \) km s\(^{-1}\) for the interval \( z = 0.5 – 1.0 \) lies only marginally above those in the higher...
redshift regions. We see no evolution in the \( b \) mean and median over redshifts 0.5 to 2. In general, the question of whether the Doppler width changes with \( z \) is not resolved: \cite{Kim2002} also did not see any evolution of \( b \) in the interval \( \langle z \rangle = 3.3 \rightarrow 2.1 \) in their data, nor did \cite{Davé2001} \( (b = 25 \text{ km s}^{-1} \) for \( \langle z \rangle = 0.2 \)), whose results do not fit the evolution pattern outlined above.

In addition to the mean and median, we also list a parameter that represents the mode, or most common \( b \) value, because this tends to be less sensitive to the \( S/N \) and the details of line and continuum fitting. We list values for the parameter \( b_{\sigma} = 1.0574 \cdot b_{\text{peak}} \) from \cite{Hui1999}. \cite{Tytler2004} and \cite{Jena2005} found that the \cite{Hui1999} fitting formula with a single parameter \( b_{\sigma} = (23.6 \pm 1.5) \text{ km s}^{-1} \) gives excellent fits to the \( b \) distribution at \( 1.5 < z < 2.4 \). Here we obtain essentially the same value for our two higher redshift bins, 1.0 – 1.5 and 1.5 – 2.0, and a smaller value at lower redshifts 0.5 – 1.0. The \( b \) value distribution is significantly wider than the fitting function for the two lower redshift bins, and hence these two \( b_{\sigma} \) values are not well determined. Both these two lower redshift bins show many more lines with \( b < 15 \text{ km s}^{-1} \) than we see at \( z > 1.5 \). We know, for example from \cite{Kirkman2005}, that as the \( S/N \) of a spectrum drops, as it does here dramatically at low \( z \), we will measure an excess of both low and high \( b \) values. Hence, all the changes that we see may arise from the changing \( S/N \). We do not claim to detect any change in the intrinsic \( b \) distribution with \( z \).

### 5.3. Clustering of Ly \( \alpha \) absorbers

The clustering properties of the baryonic matter can be investigated by computing the correlation between the Ly \( \alpha \) absorbers along the LOS. The degree of clustering of the H\( \text{I} \) absorbers can be expressed by the two-point velocity correlation function

\[
\xi (\Delta v) = \frac{n_{\text{obs}}(\Delta v)}{n_{\text{sim}}(\Delta v)} - 1
\]  

\cite{Sargent1980} where \( n_{\text{obs}} \), the number of observed Ly \( \alpha \) line pairs in a given velocity separation bin \( \Delta v \), is compared with the number of expected line pairs \( n_{\text{sim}} \) derived in the same velocity difference bin in a spectrum with randomly placed lines. For two absorbers at the redshifts \( z_1 \) and \( z_2 \), the velocity interval at mean redshift in the rest frame is given by

\[
\Delta v = \frac{c(z_2 - z_1)}{1 + \frac{z_2 + z_1}{2}}.
\]  

We determined \( n_{\text{sim}} \) from Monte Carlo simulations by distributing our line sample — in accordance with the derived number density evolution (see Subsection 5.4) — randomly over the spectrum, counting the line pairs for various velocity
Fig. 8. Two-point velocity correlation function for the weaker absorbers, up to 1000 km s$^{-1}$, in 100 km s$^{-1}$ bins. The correlation function is shown by the solid line. For orientation $\xi = 0$ is marked with a dotted line. Dashed and dot-dashed lines represent the 1$\sigma$ and 2$\sigma$ Poisson errors, related to $\xi = 0$, respectively.

Fig. 9. Two-point velocity correlation function for the weaker absorbers, up to 10 000 km s$^{-1}$, in 1 000 km s$^{-1}$ bins. Line symbols are the same as in Fig. 8.

splittings, repeating this procedure 1000 times, and computing the average.

When the clustering properties in multiple LOS are investigated, each quasar spectrum must first be analyzed separately. Subsequently, the individual numbers of pairs $n_{\text{obs}}$ and $n_{\text{sim}}$ of the $j$ analyzed LOS can be added in order to build a total $\xi$ based on improved statistics:

$$\xi(\Delta v) = \frac{\sum_{j=1}^{J} n_{\text{obs},j}(\Delta v)}{\sum_{j=1}^{J} n_{\text{sim},j}(\Delta v)} - 1. \quad (9)$$

For varying intervals $\Delta z$ and $\Delta \log N_{\text{H}I}$, we counted $n_{\text{obs}}$ and simulated $n_{\text{sim}}$ for every LOS. To investigate the clustering on small scales, we chose $\Delta v_{\text{max}} = 1000$ km s$^{-1}$ using a binning of 100 km s$^{-1}$, and we defined $\Delta v_{\text{max}} = 10 000$ km s$^{-1}$ (in 1 000 km s$^{-1}$ bins) for a study of the long-scale clustering properties. We calculated $\xi$ according to (9) with $j = 10$ (because the LOS to HS 0747+4259 was formally treated as two LOS: one for $z < 1.44$ and one for $z > 1.56$; compare with Table 3). We estimated the error on the clustering signal with error propagation, assuming a Poisson error for $n_{\text{sim}}$ and setting $\sigma_{n_{\text{obs}}} = 0$. We think that this is an acceptable approximation because $\sigma_{n_{\text{obs}}}$ follows directly from $\sigma_{\lambda}$ or $\sigma_{z}$, respectively, and the line positions are very well known compared with the values of the investigated line separations $\Delta z$. Furthermore, $n_{\text{obs}}$ and $n_{\text{sim}}$ have similar values in most cases.

Since Cristiani et al. (1995) found that clustering is stronger for higher column density absorbers, we also looked for this trend. We took the conventional definition $\log n_{\text{H}I} > 13.64$ for the strong lines, and we chose $\log n_{\text{H}I} = 12.90 - 14.00$ for the low column density absorbers in accordance with the completeness limit of our column density distribution function (see Subsection 5.1).
In Figs. 15 and 16 we show $\xi(\Delta v)$ for the low column density absorbers on scales up to 1000 km s$^{-1}$ and 10 000 km s$^{-1}$, respectively. We detect a marginal signal above the $2\sigma$ level only for $\Delta v = 150$ km s$^{-1}$, 1 500 km s$^{-1}$ and 6 500 km s$^{-1}$. Apart from that, $\xi(\Delta v)$ is always below the $1\sigma$ level. The excess at $\Delta v = 150$ km s$^{-1}$ ($\xi = 0.32, 1\sigma = 0.13$) suggests a clustering of the weak lines over short distances. The signal at $\Delta v = 50$ km s$^{-1}$ ($\xi = 0.02, 1\sigma = 0.10$) might be too weak because some weaker lines remain unidentified due to blends: resolving these blends into superpositions of several lines would result in far more line pairs at low $\Delta v$.

As expected, we also found slight clustering signals for the high column density absorbers, however only over short distances: $\xi = 1.04 > 2\sigma$ for $\Delta v = 50$ km s$^{-1}$ and $\xi = 0.36 > 1\sigma$ for $\Delta v = 150$ km s$^{-1}$ (Fig. 11). At higher distances (up to 1 000 km s$^{-1}$), the strong absorbers are obviously uncorrelated. The large-scale investigation of the stronger lines provides marginal signals exceeding the $1\sigma$ level at $\Delta v = 500$ km s$^{-1}$ and $\Delta v = 7 500$ km s$^{-1}$ (Fig. 10).

The clustering of the high and low column density absorbers on short scales contradicts our own results presented in Janknecht et al. (2002), where no clustering signal at all was found in the LOS to HE 0515-4414 alone, independent of the investigated column density ranges and velocity splittings. Individual LOS might not offer a sufficient statistical basis to detect a weak clustering signal. This is supported by Fig. 12 where we plot the correlation function $\xi$ for some individual LOS of our sample. Although the four LOS with the highest numbers of strong lines are considered ($> 40$), occasionally an at most marginal excess above the $1\sigma$ level can be seen for $\Delta v < 200$ km s$^{-1}$, but no value at all above the $2\sigma$ level.

Most studies found significant clustering on short scales, even to some extent for the low column density lines (Kim et al. 2001; Cristiani et al. 1995; Ulmer 1996; Hu et al. 1995).

Previous observations suggested a clustering signal increasing with decreasing redshift (Kim et al. 2001; Ulmer 1996). This is made plausible by the fact that the structures in the universe develop by gravitational pull over the course of time. In addition, at low redshift $\geq 1/3$ of all Ly $\alpha$ absorbers are associated with galaxies (Lanzetta et al. 1995) which are known to cluster on these scales.

We also searched for a dependence of the correlation function on $z$. For this purpose, we defined three $z$ intervals of the same width ($0.5 - 1.0, 1.0 - 1.5$ and $1.5 - 2.0$). As described above we computed $\xi$ and estimated the $1\sigma$ and $2\sigma$ errors separately for each $z$ interval and for distances up to $\Delta v_{\text{max}} = 1 000$ km s$^{-1}$ in 100 km s$^{-1}$ bins. The results are illustrated in Figs. 13 and 14. For the weaker lines, the $\xi \approx 0$ value for $\Delta v < 100$ km s$^{-1}$ is again probably caused by blending effects, while for $\Delta v > 200$ km s$^{-1}$ no clustering signal can be recognized. For $\Delta v = 150$ km s$^{-1}$ the strongest clustering is found in the highest redshift range: $\xi = 0.44 > 2\sigma$. At lower $z$, $\xi$ is roughly at the $1\sigma$ level ($\xi = 0.18, 1\sigma = 0.19$ for $z = 1.0 - 1.5; \xi = 0.33, 1\sigma = 0.30$ for $z = 0.5 - 1.0$), i.e., for $z < 1.5$, $\xi$ doesn’t change relative to $\sigma$.

The high column density lines show a similar picture for $\Delta v = 50$ km s$^{-1}$: starting at a $\sim 2\sigma$ level ($\xi = 1.40, 1\sigma = 0.79$), the clustering degree decreases strongly with decreasing redshift. However, the opposite trend can be seen at $\Delta v = 150$ km s$^{-1}$ where the signal increases from $\xi \approx 0$ to $\xi \approx 1.5\sigma$ when going from the highest to the middle $z$ interval. Besides, neither the high nor the low column density absorbers show a clustering signal lying clearly above the $2\sigma$ level.

In summary, no uniform evolutionary behaviour of $\xi$ can be established. The rising correlation of the absorbers with
decreasing redshift suggested by Kim et al. (2001) and Ulmer (1996) cannot be confirmed.

5.4. Evolution of the number density

As for the clustering properties, a different behaviour is also expected for the number density evolution of the stronger and weaker Lyα lines. We analyze this evolution using the two column density ranges from Subsection 5.3 separately.

In practice, the value for the differential number density \( \frac{dn}{dz} \) can be deduced by choosing an interval width \( \Delta z \) and counting the number of absorption lines \( n \) in the individual intervals. When the line samples of several LOS which in general cover different \( z \) regions are combined, it must be taken into account, of course, that a varying number of LOS \( n_L \) falls into a certain redshift interval. (Here, \( n_L \) can be estimated from Fig. 1). Thus, we defined a density \( \frac{dn}{dz} \) per LOS. Since the fluctuations between the LOS are large, only \( z \) intervals with \( n_L > 1 \) were considered. This is the case for \( 0.7 \leq z \leq 1.9 \).

We computed \( \frac{dn}{dz} \) for the two sub-groups as a function of \( z \). On the basis of the Levenberg Marquard algorithm, we fit the power law (1) to the data points of \( \frac{dn}{dz}(z) \) and determined \( (\frac{dn}{dz})_0 \) and \( \gamma \) in this way.

Figs. 15 and 16 show the data points for the low and high column density absorbers, respectively. In addition to the data points, the best fits to them with their 2\( \sigma \) confidence limits are presented.

For the weak absorbers, the number density decreases with decreasing redshift with \( \gamma = 0.74 \pm 0.31 \). The outliers in the interval \( z = 0.7 - 0.8 \) might be due to the small size of our sample in this region (see Figs. 11 and 12). The stronger lines evolve faster with an exponent \( \gamma = 1.50 \pm 0.45 \) which is only just consistent (within 1\( \sigma \)) with the exponent of the weak lines.

Fig. 17 gives an extensive comparison of the results of this work for the weaker absorbers (here defined by
Fig. 15. Number density evolution of the weak Ly $\alpha$ absorbers ($12.90 < \log N_{\text{HI}} < 14.00$). Given are the data points (binned with $\Delta z = 0.05$), the best fit and the $2\sigma$ confidence limits.

Fig. 16. Same as in Fig. 15 for the strong Ly $\alpha$ absorbers ($\log N_{\text{HI}} > 13.64$).

$\log N_{\text{HI}} = 13.10 - 14.00$ to facilitate comparison with other results) with different contributions to the number density evolution from the literature measured in varying $z$ regions. Fig. 18 does the same for the stronger absorbers. In the double-logarithmic presentations the symbols correspond to the number densities measured in different studies. The Ly $\alpha$ lines of this work are binned in intervals of $\Delta z = 0.2$ in order to have interval sizes comparable with the other investigations. The data points of this study are shown with filled circles; the error bars result from a $1\sigma$ poisson error of $n$.

Regarding the evolution of the weaker absorbers (Fig. 17), it becomes clear that the LOS analyzed here fall into a $z$ region which has not yet been examined so far, due to a lack of high-resolution UV data. The best fit to the data points of this work gives $\gamma = 0.78 \pm 0.27$.

The decrease of the line density with decreasing redshift is obviously decelerated in our investigated $z$ interval, as is revealed from a comparison with higher redshifts: for $z > 1.5$, Kim et al. (2002) derived $\gamma = 1.18 \pm 0.14$ from a best fit to the data points of different studies (presented as dashed line in Fig. 17). Excluding two LOS, they found $\gamma = 1.42 \pm 0.16$ which is no longer in agreement with the result found here.

Interestingly, an extrapolation of our fit line for $z \rightarrow 0$ is consistent within $1\sigma$ with the study of Penton et al. (2004) for the local universe (see dotted line in Fig. 17), while an extrapolation of the Kim et al. (2002) fit line misses the Penton et al. (2004) data point by $\approx 2\sigma$. (Williger et al. 2003) examine only
a single LOS). We interpret this as further evidence for a changing evolution of the lower column density lines in the interval $z = 1 - 2$.

Furthermore, the comparison with the investigations at higher and at lower $z$ suggests a continuous and moderate transition to the present, rather than a sharp break.

As already derived from Figs. 13 and 16, the high column density absorbers show a steeper gradient in the evolution diagram with $\gamma = 1.66 \pm 0.57$ (Fig. 13). A remarkable data point occupies the interval $z = 0.9 - 1.1$. Without this outlier the evolution index would be much higher and its error much lower ($\gamma = 2.11 \pm 0.21$). A detailed analysis of this redshift range which is covered by six of the nine LOS (see Fig. 11) shows that the high line density is not caused by a fluctuation in an individual LOS, but that the $\frac{\Delta \ln N}{\Delta z}$ lies clearly above the level of the adjacent intervals in nearly all six LOS. Obviously we detect a coincidental line accumulation within this interval.

Just as for the weaker lines, our $\gamma$ is lower compared with the results of investigations at higher $z$: Kim et al. (2002) found $\gamma = 2.47 \pm 0.18$ by fitting a straight line to the data points of varying studies for $z > 1.5$ which is inconsistent within 1$\sigma$ with the value computed here. In addition, the number density continuously decreases in the analyzed range $z = 0.7 - 1.9$ (apart from the outlier) without showing the frequently postulated break in the evolution at $z = 1.5 - 1.7$. Weymann et al. (1998) Dobrzycki et al. (2002) Impev et al. (1998). Possibly this slow-down in the evolution, which was also successfully simulated by theoreticians (Theuns et al. 1998, Davé et al. 1999), was determined in a later phase ($z < 0.7$) beyond the redshift region examined here. An indication for this might be that at lower $z$ the thinning out of the absorbers obviously continues to decelerate: in contrast to the weaker absorbers, an extrapolation of our fit line for $z \rightarrow 0$ would lead to a distinctly too low absorber density at $z = 0$, compared with Penton et al. (2004) and Weymann et al. (1998) (see Fig. 13).

If we fit the data points of Weymann et al. (1998) with a straight line (shown in Fig. 13), $\gamma = 0.13 \pm 0.06$, it does not intersect our fit line before $z = 0.6$. Also, leaving out our outlier leads to a point of intersection at $z = 0.8$. Thus, in the case that the break in the number density evolution took place, the redshift value at which it occurred has to be corrected downwards.

However, it must be noted that Weymann et al. (1998) used the equivalent width $W_{\lambda}$ instead of the column density as the observable. The conversion formula

$$W_{\lambda,\text{rest}} \, [\text{Å}] = 8.85 \times 10^{-21} \left(\lambda_{\text{rest}} \, [\text{Å}]\right)^2 f N \, [\text{cm}^{-2}]$$

(10)

(f oscillator strength) only applies to unsaturated absorption lines and depends on the Doppler parameter. Occasionally, $b = 25 \, \text{km} \, \text{s}^{-1}$ is assumed in the literature. Penton et al., 2000, 2004. Then the lower limit for the equivalent width $W_{\lambda,\text{rest}} = 240 \, \text{mÅ}$ used by Weymann et al. (1998) corresponds to a minimum column density log $N_{\text{HI},\text{min}} = 14.00$. Applying this higher cut-off to the evolution diagram of the stronger absorbers would shift our data points in Fig. 13 significantly downwards. As a result of this, the intersection of our fit line with the one to the Weymann et al. (1998) data points (and therefore the evolution break estimated from it) had to be corrected to a higher redshift. For this reason, the uncertainty in the $W_{\lambda}$-$N_{\text{HI}}$ conversion makes the exact localization of the slow-down difficult to determine.

Nevertheless, newer studies of Kim et al. (2001), Kim et al. (2002) (whose fit line hits that of the Weymann et al. (1998) data points at $z = 1.1$; see Fig. 13) or Penton et al. (2004) also favoured the break in the evolution taking place at no higher redshift than $z = 1.0 - 1.2$. Supported by these results, we conclude that a transition in the evolution of the higher column density lines indeed took place, but much later than assumed before. In simulations the sudden slow-down in the evolution results directly from the decline of the UV background at $z = 2$ (Davé et al. 1999). However, the QSO dominated UV background of Haardt & Madau (1996) is usually assumed in these calculations, and thus the contribution of the galaxies is possibly underestimated. With the latter as main source of the UV radiation field for $z < 2$, the radiation field would decrease more slowly and the break would naturally be corrected to a lower $z$.

All results for the evolution of the number density of Ly $\alpha$ absorbers in this subsection have to be considered in view of the high scatter of the data points. This scatter could not be significantly reduced compared with our study of the single LOS to HE 0515-4414 (Janknecht et al. 2002), though we enhanced the examined total redshift path length $\Delta z$ by roughly a factor 6. For all nine observed QSOs we obtained high-resolution spectra ($R \geq 30,000$) where the Ly $\alpha$ forest is completely resolved (because $b_{\text{LSR}} > 10 \, \text{km} \, \text{s}^{-1}$). Therefore, we suppose that the variations from LOS to LOS are caused physically (rather than methodically or statistically), reflecting a strongly inhomogeneous appearance of the Ly $\alpha$ forest. Indeed, Kim et al. (2002) and Tytler et al. (2004) (for $z < 2.5$ and for $z = 1.9$, respectively) and Impev et al. (1999) for the local universe ($z < 0.2$) found hints for a strong cosmic variance too.

The reason for the large variations of $\frac{\Delta \ln N}{\Delta z}$ between individual LOS falling into the same redshift ranges is probably the advanced stage of structure formation in the universe at $z = 2$. The structures are gravitationally evolved: over- and underdensities have been formed. In individual LOS this can be recognized in the form of strongly varying number densities, in particular of the high column density absorbers. The structure formation obviously starts to dominate over the Hubble expansion, which causes the general decline of the number density.

5.5. Effective optical depth at $z \approx 2$

We examined the effective optical depth $\tau_{\text{eff}}$ and the average normalized flux $\langle F_{\text{norm}} \rangle$, which are related to each other by the equation

$$\tau_{\text{eff}} = -\ln \left(\frac{F}{F_{\text{cont}}}\right) \equiv -\ln \langle F_{\text{norm}} \rangle,$$

(11)

where $F_{\text{cont}}$ is the continuum flux. We calculated $\tau_{\text{eff}}$ and $\langle F_{\text{norm}} \rangle$ for the maximum redshift range of our LOS, $z = 1.81 - 1.91$. The LOS to three of our quasars fall into the chosen interval: those to HE 2225-2258, HE 0429-4901 and
We have analyzed the combined sample of all Ly $\alpha$ absorption lines which were detected in the spectra of nine quasars over a total redshift path length $\Delta z = 5.176$ within the range $z = 0.5 - 1.9$. We classified 1325 absorption features as Ly $\alpha$ lines. Examining the distributions of their fit parameters $\log N_{HI}$, $b$ and $z$, we find the following:

1. The column density distribution might be complete down to $\log N_{HI} = 12.90$. It can be approximated over roughly three orders of magnitude ($\log N_{HI} = 12.90 - 15.70$) by a simple power law. The slope $\beta = 1.60 \pm 0.03$ is consistent with other studies in comparable redshift intervals. We do not see any change in $\beta$ with $z$.

2. The distribution of the Doppler widths has the expected form of nearly Gaussian with an additional tail to high $b$ and an average value $\bar{b} = (34 \pm 22) \text{ km s}^{-1}$. We cannot detect any evolution of the Doppler parameter distribution with $z$ in the examined redshift phase.

3. The weaker ($\log N_{HI} = 12.90 - 14.00$) as well as the stronger Ly $\alpha$ lines ($\log N_{HI} > 13.64$) show marginal clustering with a $2\sigma$ significance on short velocity intervals ($\Delta v < 200 \text{ km s}^{-1}$ and $\Delta v < 100 \text{ km s}^{-1}$, respectively). However, the clustering signal is too weak to be seen in individual LOS. We do not see any change with column density or redshift.

4. Using the customary power law $\frac{dN}{dz} \propto (1+z)^{\gamma}$, the evolution of the number density of the weaker Ly $\alpha$ lines declines with $\gamma = 0.74 \pm 0.31$ with decreasing $z$ within the range $z = 0.7 - 1.9$. From comparisons with investigations of other authors at higher and lower redshifts we conclude that the decrease of the weaker lines is decelerated in the phase $z = 1 - 2$, turning into a nearly flat evolution for $z \rightarrow 0$, without showing any hint for a sharp break in the evolution.

5. The number density of the stronger absorbers also decreases with decreasing $z$ ($\gamma = 1.50 \pm 0.45$ for $z = 0.7 - 1.9$), faster than for the weaker lines, though the trends are consistent with each other within $1\sigma$.

6. The decline of the line density of the stronger absorbers is also decelerated compared with higher redshifts ($z = 1.5 - 4.0$). The break in the evolution predicted for $z = 1.5 - 1.7$ cannot be detected, however. The deduced $z$ dependence of the number density as well as comparisons with other studies for the local universe rather suggest a later slow-down ($z < 0.7$), followed by an approximately flat evolution. The precise $z$ value of the break in the evolution probably has to be corrected somewhat upwards due to the uncertain conversion between equivalent width (used by Weymann et al. [1998] as observable) and column density.

7. Though having analyzed nine high-resolution ($R \geq 30,000$) quasar spectra altogether, the scatter of the data points in the diagrams of the number density evolution is large. As Tytler et al. (2004) showed, the strong variations between the LOS are consistent with the effects of normal structure formation.

8. An analysis of the distribution of three quasars with LOS in the upper redshift range ($z = 1.81 - 1.91$) gives $\langle F_{\text{norm}} \rangle = 0.895 \pm 0.020$ for the average normalized flux and $\tau_{\text{eff}} = 0.111 \pm 0.020$ for the effective optical depth, in good agreement with the values from Kirkman et al. (2004).

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