On particle–like jets

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Abstract

Under which conditions does a jet appear as a particle–like signal from the hidden realm of quarks and gluons? Motivated by this question jet clustering conditions are formulated, in order to characterize jet clustering algorithms, which can be used for a determination of particle–like jets. Jets are understood as particle–like, if they behave like free particles. The simplest solution to the jet clustering conditions leads to a new jet algorithm: a Lorentz invariant generalization of the JADE algorithm. It is found that this generalization amplifies hadronization effects in certain phase space regions in such a way, that hadronization models might become testable in jet physics at the electron–proton collider HERA. Moreover, a method is suggested, which can be used at HERA, in order to determine a region in the phase space, where hadronization effects from the proton remnant are small and where parton jets are particle–like.

1 Introduction

Quarks and gluons do not exist: they cannot be investigated experimentally as free particles. While the mass of a free particle is an observable, the mass of a quark is not well defineable. Only gauge invariant collections of quarks and gluons are physically meaningful. Because of this confinement property it is difficult to obtain observable predictions from QCD. Various strategies were developed to obtain approximate predictions. A perturbation theory at short distances can be formulated, since QCD is asymptotically free. It provides deep insights in the dynamical structures of QCD. But the hadronization of quarks and gluons cannot be explained within the perturbative approach. For a description of hadronization phenomenological models have to be used.
In high–energy collision experiments hadron jets, i.e. spatially isolated clusters of hadrons, are observed. An interesting question is: under which conditions does the direction of hadron jets agree with the direction of partons from perturbative QCD? An attempt to answer this question, has to take into account that partons are colored and hadrons are colorless.

Monte Carlo event generators were developed, which connect perturbative QCD and hadronization models, like LEPTO\textsuperscript{1}. They can be used to study the influence of hadronization. Parton jets and hadron jets are determined event by event by applying a jet algorithm to the outgoing partons of perturbative QCD and the hadrons from the phenomenological hadronization model, respectively. The differences between parton jets and hadron jets are caused by hadronization and are called hadronization effects. Hadronization effects depend strongly on the jet algorithm, see e.g. \cite{2}, \cite{3}, \cite{4}, \cite{5}, \cite{6}, \cite{7}.

A jet algorithm is a tool, to analyze the hadronic final state and to provide a basis for testing the underlying theory. There are many jet algorithms. One reason to create a jet algorithm is, to make the calculation of special aspects of the theory possible, see e.g. the Geneva algorithm \cite{5} and the kT–algorithm \cite{6}. Jet algorithms motivated by QCD–calculations allow a detailed view into certain regions of the phase space, but they might not be suited equally in different regions. For example, in the deep inelastic scattering (DIS) region at the electron–proton collider HERA it is expected, that the kT–algorithm describes jets the better, the more they are separated from the proton remnant, i.e. the more the proton remnant “factorizes” from other jets. In section 2 a new jet algorithm is derived, not from explicit QCD–calculations, but from general physical considerations. This jet algorithm is a Lorentz invariant generalization of the JADE algorithm. It can be applied to all areas of jet physics, as long as the jet masses are small. Using the DJANGO Monte Carlo\textsuperscript{2} hadronization effects are compared in section 3 for the JADE algorithm and the new jet algorithm. A variable is introduced, which assigns a Lorentz invariant distance between the jets and the incoming proton. It is shown that jet masses influence strongly the hadronization effects, if the jets are close (with respect to this variable) to the incoming proton. The new jet algorithm seems to be a sensitive tool to test hadronization models in jet physics.

### 2 Formulation of jet clustering conditions

We are interested in a description of jets, which behave like free particles and which we call particle–like jets. In this section we formulate four conditions for jet clustering algorithms. They lead to a new jet algorithm, which can be used

\footnote{LEPTO links leading order QCD matrix elements (ME) of the order $\alpha_s$, parton shower (PS) and the LUND string fragmentation model.}

\footnote{DJANGO is an interface between HERACLES, a Monte Carlo for deep inelastic lepton–proton scattering including electroweak radiation corrections \cite{8}, and LEPTO.}
to determine particle–like jets.

Clustering algorithms have been developed in [10], [11], [12]. We concentrate on clustering algorithms of the following kind [13]. Consider a collision event consisting of \( N \) hadrons with momenta \( p_1, \ldots, p_N \) and assign a distance \( d_{ij}, 1 \leq i < j \leq N \). By a renumbering of the hadrons one always manages that \( d_{1N} \) is the smallest of all distances. If \( d_{1N} \) is smaller than some reference distance

\[
d^2_{1N} \leq y_{\text{cut}} M^2,
\]

recombine hadron \( N \) with hadron \( 1 \) and assign a 4–tuple to the recombined hadrons. We call the 4–tuple pseudo–momentum and characterize it by the symbol \( p_1 \oplus p_N \). (In general it is not justified to consider \( p_1 \oplus p_N \) as a momentum.) \( M \) is called reference mass and \( y_{\text{cut}} \) is called jet resolution parameter. This clustering procedure is repeated until none of the distances \( d_{ij} \) fulfils the inequality (1). The remaining hadron clusters define the jet content of the event. These jets are called hadron jets.

In perturbative QCD jet algorithms can be applied to partons in order to obtain results, which are free of infrared and collinear singularities (see e.g. [15]). The resulting jets are called parton jets.

Commonly used clustering algorithms (see e.g. [3], [4], [5], [6], [7] and references therein) differ by the reference mass \( M \), the distance \( d_{ij} \), and the recombination prescription \( p_i \oplus p_j \).

Hadrons, produced in a high–energy particle collision, propagate almost independently. Consequently, the forces between hadron jets are small. Parton jets, on the other hand, are influenced by hadronization effects because of the color forces between the partons. If the color forces between parton jets are small, parton jets behave like free particles and are characterizable by kinematical quantities.

We assume that a momentum is sufficient to describe a particle–like jet and that internal degrees of freedom, like spin, can be neglected. An event consisting of \( n \) jets, where each jet is particle–like, is called particle–like \( n \)–jet. Energy–momentum conservation suggests, to identify the total momentum of a particle–like \( n \)–jet with the total momentum of its hadrons or partons, respectively. What can be said about the momentum of an individual jet within a particle–like \( n \)–jet event? Consider an event consisting of 2 hadrons with the momenta \( p_1 \) and \( p_2 \). If the 2 hadrons are recombined, the only way to guarantee momentum conservation is, to assign the momentum \( p_1 + p_2 \) to the 1–jet. By induction one is led to the

A) Recombination condition: The pseudo–momentum has to be identified with

\[3\] The jet clustering algorithm ARCLUS assigns a distance \( d_{ijk} \) to all triples of hadrons [14].
In the current calculations of perturbative QCD parton jets are massless (see e.g. [15], [18] and references therein). Recombination prescriptions were proposed in the literature which guarantee that the masses of the hadron jets are also zero, e.g. $p_i \oplus p_j = (|\vec{p}_i + \vec{p}_j|, \vec{p}_i + \vec{p}_j)$. These prescriptions are arbitrary, because there are uncountable many ways to map the hadron momenta into massless hadron jets. This arbitrariness it reduced, but not completely, if one restricts to recombination prescriptions which lead to small hadronization effects. One is still left with the problem that the recombination prescription is not universal, since it depends on a test variable, which is used as a measure for the strength of the hadronization effects. Moreover, if the pseudo–momentum is not simply the sum of the two recombined momenta, $p_i \oplus p_j \neq p_i + p_j$, the physical meaning of the pseudo–momentum is not clear and in particular one cannot expect, that the direction of a hadron jet can be identified with the direction of a parton jet.

Another way to treat the masses of jets, is to compare data with a more complex evaluation of QCD matrix elements. This, of course, requires the calculation of higher than leading-order corrections.

The reference distance on the right hand side of (1) is defined as the product of the reference mass squared $M^2$ and the jet resolution parameter $y_{\text{cut}}$. If the jet resolution parameter $y_{\text{cut}}$ is chosen sufficiently large, the right hand side of (1) can be made larger than every distance squared $d_{ij}^2$ and, consequently, all hadrons momenta $p_1, \ldots, p_N$ are finally recombined into a 1–jet with the momentum $p_1 + \ldots + p_N$. In the hadronic center of mass frame a 1–jet is at rest and its energy is the mass of the hadronic final state. Let $y_{\text{cut,triv}}$ be the smallest $y_{\text{cut}}$ value, at which the hadrons of an event are recombined into a 1–jet. Because a 1–jet is a trivial jet, it does not make sense to consider values of the jet resolution parameter higher than $y_{\text{cut,triv}}$, i.e. the jet resolution parameter has to be restricted to the interval $0 \leq y_{\text{cut}} \leq y_{\text{cut,triv}}$. By a suitable rescaling of the reference mass $M$, one always achieves that $y_{\text{cut,triv}} = 1$.

B) Boundary condition: The reference mass $M$ has to be chosen such that, for $y_{\text{cut}} = 1$, all hadrons are recombined into a 1–jet and that, for $y_{\text{cut}} < 1$, only higher order $n$–jets, $n > 1$, are generated.

A simple solution to this condition is obtained by an identification of the square of the reference mass with the sum of the squares of all distances between the hadrons

$$M^2 = \sum_{i<j} d_{ij}^2.$$  \hspace{1cm} (3)
This choice is not unique. For example one could weight each distance \( d_{ij} \) in (3) with a factor \( w_{ij} \geq 1 \), but \( w_{ij} = 1 \) is distinguished, since it leads to the smallest reference mass \( M \).

The reference mass \( M \) changes, in general, from clustering step to clustering step. On the one hand this might cause problems in theoretical calculations, but on the other hand there is a lot of freedom to tune the reference mass, e.g. by means of the weights \( w_{ij} \).

Finally we formulate two conditions on the distance \( d_{ij} \).

Almost all commonly used jet algorithms are not Lorentz invariant. Consequently the jet predictions of these algorithms depend on the chosen reference frame. This causes experimental problems, if the laboratory frame does not agree with the reference frame, suited for a non–Lorentz invariant jet algorithm. Due to the strong energy imbalance between the electrons and protons at HERA, the laboratory frame does not belong to one of the theoretically preferred reference frames, like the hadronic center of mass frame or the Breit frame. Besides the experimental errors in the measurement of jets, a considerable source of additional uncertainty is introduced at HERA through an event by event measurement of certain kinematical observables. These kinematical observables are needed in order to determine the transformation between the laboratory frame and the reference frame of a non–Lorentz invariant jet algorithm. The additional uncertainty might reduce the applicability of a non–Lorentz invariant jet algorithm.

Lorentz invariant jet algorithms are very convenient, at least from an experimental point of view, since they can be used in the laboratory frame. The laboratory frame is the only natural frame to study detector acceptance effects. To find the proper acceptance cuts is one of the most important and most difficult experimental tasks. But also from a theoretical point of view Lorentz invariant jet algorithms are preferred, if one is interested in particle–like jets and if one assumes, that particle–like jets belong to the observables of QCD. Since QCD is Lorentz invariant\(^4\), the determination of particle–like jets cannot depend on the inertial reference frame where the jet algorithm is applied. This leads us to the

C) Lorentz invariance condition: The distance \( d_{ij} \) has to be independent of the inertial reference frame.

In QCD the radiation of collinear partons is more probable than the radiation of non–collinear partons, if one lets aside infrared partons. Collinear partons are recombined with the highest priority, if one assigns a vanishing distance between them. This motivates the

\(^4\) A short discussion of the problem, whether the Lorentz invariance might be broken in the vacuum of gauge theories, can be found in [12].
D) **Collinearity condition**: The distance between collinear jets should vanish

\[ d_{ij} = 0, \quad \text{if} \quad p_i^\mu = \lambda p_i^\mu. \]  

Collinearity is understood as collinearity of 4-momenta. The collinearity factor \( \lambda \) has to be positive, because of the positivity of the energy.

Let us analyze the consequences of the clustering conditions C) and D).

Because of the Lorentz invariance condition C) the distance \( d_{ij} \) is a function of the invariants \( p_i^2, p_j^2, p_ip_j \). We are therefore able to expand

\[ \frac{d_{ij}^2}{M^2} = a_0 + \sum_{k=1}^{3} a_k r_k + \frac{1}{2!} \sum_{k,l=1}^{3} a_{kl} r_k r_l + \ldots \]  

into powers of the invariants

\[ r_1 = \sqrt{\frac{p_i^2}{M^2}}, \quad r_2 = \sqrt{\frac{p_j^2}{M^2}}, \quad r_3 = \sqrt{\frac{p_ip_j}{M^2}}. \]  

For a moment let us assume that the jet masses \( m_j, m_i \) are small compared to the reference mass \( M \). If in addition the hadrons are almost infrared \((p_i \sim 0)\), or almost collinear \((p_j^\mu \sim \lambda p_i^\mu \) with a collinearity factor \( \lambda \) of the order of 1), one also has \( p_ip_j \ll M^2 \). Then it is allowed to stop the expansion (5) at the quadratic level. It turns out that the clustering condition D) determines the distance \( d_{ij} \) up to an overall normalization factor \( c = a_{33} \)

\[ d_{ij}^2 = c \left( (p_i + p_j)^2 - (m_i + m_j)^2 \right), \]  

i.e. we are able to derive a distance from the clustering conditions C) and D), which is unique (up to normalization) for small jet masses.

The normalization factor \( c \) has to be positive, otherwise the distance \( d_{ij} \) is not a real number (to see this, use \( p_ip_j \geq m_im_j \)). If the reference mass is identified with \( M \), the normalization factor \( c \) does not influence the inequality (1) of the clustering algorithm. In this case one can set \( c = 1 \).

In the limit of massless jets the distance (7) becomes proportional to the distance of the JADE algorithm [16], [15], [13]

\[ d_{ij}^2 \rightarrow c2E_iE_j(1 - \cos \theta_{ij}), \quad \text{as} \quad m_i, m_j \rightarrow 0. \]  

The distance \( d_{ij} \) would not become the JADE distance in the limit of small jet masses, if the invariants \( r_i \) in (6) were defined without the square root.

During a clustering procedure the mass of a particle–like jet increases. Practically one arrives at jet masses which are large compared to the masses of the individual hadrons within a jet and not negligible against the reference mass.
Beyond the first clustering step the JADE algorithm violates Lorentz invariance more and more, because its distance $2E_iE_j(1−\cosθ_{ij})$ is Lorentz invariant only for massless jets.

The distance of the E-clustering algorithm \[16\], \[15\], $d^2_{ij} = (p_i + p_j)^2$, is Lorentz invariant, but it does not fulfil the collinearity condition D), if the jets are massive.

It should be noticed that (7) does not define a distance in the classical sense. If it were a classical distance the triangle inequality $d_{ij} + d_{jk} \geq d_{ik}$ would hold, but the triangle inequality is violated because of the collinearity condition D). To see this, consider e.g. three hadrons with momenta $p_1, p_2, p_3$ and assume that two of them are collinear: $p_3 = \lambda p_2$; then the sum of two of the distances, $d_{13} + d_{23} = \sqrt{\lambda}d_{12}$, is smaller than the third distance, $d_{12}$, for all $\lambda < 1$.

Combining (3) and (7) the square of the reference mass becomes (with $c=1$)

$$M^2 = W^2 - (m_1 + m_2 + \ldots)^2$$

where $W^2 = (p_1 + \ldots + p_N)^2$ is the square of the mass of the hadronic final state. We see that the reference mass $M$ can be chosen smaller than the mass of the hadronic final state $W$ without contradicting the boundary condition B).

If the jet masses $m_i, m_j$ are large, higher order corrections in (5) have to be taken into account and the distance $d_{ij}$ is no longer unique because the condition C) and D) are not violated if the right hand side of (7) is multiplied by any positive function $f(m_i, m_j, p_i p_j)$.

The distance (7) with $c = 1$, the reference mass (8) and the recombination prescription (2) define a new jet clustering algorithm, which is the simplest algorithm allowed in the setting. This jet algorithm can be applied to all areas of jet physics, as long as the jet masses are small compared to the reference mass. It is suited for the investigation of hadronization effects around remnants and the determination of particle-like parton jets, as is demonstrated in the next section in the case of deep inelastic electron-proton scattering. Theoretical calculations based on this jet algorithm are well defined, since collinear and infrared singularities are identifiable. For certain theoretical problems, e.g. the exponentiation problem, it might be indicated, to modify the distance (7) by higher order mass corrections and/or to use a different reference mass $M$.

### 3 Hadronization effects

In this section we present a study of hadronization effects for the JADE algorithm, called jet algorithm 1, and its simplest generalization to massive jets, as presented in the last section, called jet algorithm 2. The jet algorithm 1 is not Lorentz invariant and needs the specification of an inertial system. We consider it in the hadronic center of mass frame. Using the DJANGO Monte Carlo we generate 5000 deep inelastic scattering events of 26.7 GeV...
electrons and 820 GeV protons. DJANGO is able to take into account corrections from the bremsstrahlung of the electron. Let \( p_{\text{prot}}, p_e, p_{e'}, p_\gamma \) be the momenta of the incoming proton, the incoming electron, the scattered electron and the bremsstrahlungs photon, respectively, and let us define \( Q^2 = -q^2 \) and \( Q^2_{\text{had}} = -q^2_{\text{had}} \), where \( q = p_e - p_{e'} \) and \( q_{\text{had}} = q - p_\gamma \). The events are generated with \( Q^2 > 80 \text{ GeV}^2 \). This choice restricts \( x_{\text{Bj}} = Q^2_{\text{had}}/(2p_{\text{prot}}q_{\text{had}}) \) to \( x_{\text{Bj}} > 10^{-3} \).

LEPTO offers different options to study various aspects of the electron–proton scattering: a matrix element (ME) option to generate 1+1 or 2+1 parton jets\(^5\) according to leading order QCD matrix elements of order \( \alpha_s \), a parton shower (PS) option, and a fragmentation option for the generation of hadrons out of partons based on the LUND string fragmentation model \[19\]. Since these options can be combined freely, partons at two different levels can be extracted from LEPTO: a ME–parton level and a PS-parton level. In the following we identify the parton level with the ME–parton level. In LEPTO the generation of the ME–partons is based on the JADE algorithm. We generated the ME–partons with the jet resolution parameter \( y_{\text{cut}} = \text{PARL}(8) = 0.008 \). The parton jets and hadron jets are determined with the higher value \( y_{\text{cut}} = 0.02 \), in order to guarantee, that the available phase space for the 2+1 jets of the jet algorithm 2 at \( y_{\text{cut}} = 0.02 \) is (for the most part) included in the phase space for the 2+1 jets of the jet algorithm 1 at \( y_{\text{cut}} = 0.008 \). (According to the PROJET Monte Carlo \[20\] contributions from 3+1 jets are small at \( y_{\text{cut}} = 0.02 \).) We order the parton jets and the hadron jets according to their distance to the proton \( d_{\text{jet,prot}} \). The jet, which is closest to the proton direction, is considered as the remnant jet, the next jet is called jet 1.

First we concentrate on the hadron level and compare the transverse momentum \( p_T \) distributions of the jet 1 for the jet algorithms 1 and 2 (grey distributions in figure 1a and 2a; the hatched part shows hadronization effects, see later). The \( p_T \) distributions differ strongly. Both jet algorithms show up a maximum around \( p_T = 10 \text{ GeV} \), but the jet algorithm 2 finds much more low \( p_T \) jets, i.e. the jet algorithm 2 is much more sensitive in the low \( p_T \) region, than the jet algorithm 1. The different predictions of the low \( p_T \)–behavior is caused by the mass of the jets, although the mean mass of the hadron jets is only of the order of 10 % of the reference mass \( M \).

The low \( p_T \)–behavior of the jet 1 is dominated by hadronization effects. This can be read off from the figures 1b and 2b, which show the scatter plots of the transverse momentum \( p_T \) of the jet 1 (hadron level (HL) versus parton level (PL)) for the jet algorithm 1 and the jet algorithm 2, respectively. From the figures 1b and 2b follows moreover, that the major part of the hadronization effects is characterizeable by a transverse momentum \( p_T(\text{HL}) < 5 \text{ GeV} \).

The transverse momentum of the jets is often used to control hadronization effects. We suggest another method. Consider the square of the distance

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\(^5\)The proton remnant is counted as “+1” jet.
between the proton and jet1 scaled by the square of the reference mass

\[ z_p^* = \frac{d^2_{\text{jet1,prot}}}{M^2}. \]

The figures 1c and 2c show the influence of the cut

\[ z_p^* > 0.3 \] (9)
on the scatter plots 1b and 2b. As can be seen a lot of the offdiagonal events are removed for both jet algorithms, i.e. the cut (9) specifies a region in the phase space, where parton jets can be considered as particle–like, at least with respect to the transverse momentum of the jet 1. A \( p_T \)–cut acts “horizontally” in the figures 1b and 2b, a \( z_p^* \)–cut acts almost “diagonally”. Contrary to a \( p_T \)–cut, which depends on the reference frame, a \( z_p^* \)–cut is Lorentz invariant and can always be applied in the laboratory frame. The hatched distributions in the figures 1a and 2a show the transverse momentum of the jet 1 for those events, which do not fulfill the \( z_p^* \)–cut (9). By a measurement of the \( p_T \)–distribution of the jet 1, the increase below \( p_T = 6 \) GeV, predicted by the jet algorithm 2, might be observable at HERA. Since the increase is caused by hadronization effects, the jet algorithm 2 could be useful to test hadronization models experimentally.

The scaled distance between the the jet1 and the remnant

\[ z_p^* = \frac{d^2_{\text{jet1,remnant}}}{M^2} \]
is less suited practically to identify hadronization effects, since most of the remnant escapes undetected in the beampipe. While the momentum of the proton is known, the momentum of the remnant can only be reconstructed from the missing momentum in the detector.

Finally we show, that a \( z_p^* \)–cut is also useful to control hadronization effects for 2+1 jets. 2+1 jets are important, since they can be used to measure the strong coupling constant \( \alpha_s \). Let us consider the variable

\[ \eta = \frac{Q_{\text{had}}^2}{Q_{\text{had}}^2 + (p_{\text{jet1}} + p_{\text{jet2}})^2}. \]

The figures 3 and 4 show the \( \eta \)–scatter plots (hadron level versus parton level) for the 2+1 hadron jets of the jet algorithms 1 and 2, respectively. In the figures 3a and 4a no hadronization cuts are applied. The correlations are very bad, indicating strong hadronization effects for both jet algorithms. After applying the \( z_p^* \)–cut (9), the correlations are considerably improved (figures 3b and 4b). Since strong \( z_p^* \)–cuts lower the statistics, one has to look for a compromise between the strength of the hadronization effects and the statistics of the data. The “best” \( z_p^* \)–cut is the cut, at which the uncertainty for the reconstruction of the parton level from the hadron level is comparable to the uncertainty for the reconstruction of the hadron level from the detector level.

Another important variable to characterize 2+1 jets is

\[ z_p = \frac{p_{\text{prot}} p_{\text{jet1}}}{p_{\text{prot}} Q_{\text{had}}}. \]
Since in the limit of small masses

\[ z_p^* \rightarrow \frac{z_p}{1 - x_{Bj}} \]

one can consider \( z_p^* \) as a generalization of \( z_p \) to massive jets. The mean mass of hadron jets is of the order of 10% of the reference mass \( M \) and therefore \( z_p \) differs not very much from \( z_p^* \) practically. This means that a measurement of \( z_p \) cannot be performed below the \( z_p^* \)-cut.

In summary the hadronization effects of the JADE algorithm and its simplest Lorentz invariant generalization to massive jets differ strongly around the direction of the incoming proton. By adjusting the \( z_p^* \)-cut properly the differences disappear almost. The hadronization effects from the proton remnant are controllable without any special treatment of the proton remnant during the clustering procedure. The Lorentz invariant generalization of the JADE algorithm, derived in section 2, is sensitive to hadronization effects and might be useful in jet physics to test hadronization models experimentally.

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Figure 1: Jet algorithm 1: a) $p_T$ distribution (in [GeV]) of jet 1 (hadron level) in the hadronic center of mass frame. The hatched part shows hadronization effects with $z_p^* < 0.3$. b) Scatter plot of the transverse momentum $p_T$ (in [GeV]) of jet 1 in the hadronic center of mass frame: without a $z_p^*$-cut. c) As b) but with the cut $z_p^* > 0.3$. 
Figure 2: Jet algorithm 2: a) $p_T$ distribution (in [GeV]) of jet 1 (hadron level) in the laboratory frame. The hatched part shows hadronization effects with $z_\rho^* < 0.3$. b) Scatter plot of the transverse momentum $p_T$ (in [GeV]) of jet 1 in the laboratory frame: without a $z_\rho^*$-cut. c) As b) but with the cut $z_\rho^* > 0.3$. 
Figure 3: Jet algorithm 1: scatter plot of $\eta$ for 2+1 hadron jets: a) without a $z_p^*$-cut, b) with the cut $z_p^* > 0.3$. 
Figure 4: Jet algorithm 2: scatter plot of $\eta$ for 2+1 hadron jets: a) without a $z_p^*$-cut, b) with the cut $z_p^* > 0.3$.