Application of Recurrent Neural Networks in the numerical analysis of reinforced concrete structures considering polymorphic uncertainty

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The realistic modelling of structures is fundamental for their numerical simulation and is mainly characterized by the physical approach and the consideration of the available data by an adequate uncertainty description. The key idea in the contribution at hand is the consideration of polymorphic uncertainty in the numerical structural analysis and in the mechanical modelling for reinforced concrete structures, which are characterized by a combination of heterogeneous concrete and different types of reinforcement (e.g. steel bars or woven carbon fibres). Typically, the reinforcement is determined by another length scale compared to the overall structural size. The formulation and development of a computational homogenization approach, considering the different homogeneous and heterogeneous characteristics of a macroscopic structure, are essential for a realistic numerical computation. In recent years, focal point of research was on structural analysis considering uncertain material or geometry parameters. Probabilistic approaches are dominating the uncertainty consideration currently, although they are associated with specific disadvantages and limitations. In this contribution, a generalized uncertainty model is utilized in order to take variability, impression as well as inaccuracy, vagueness and incompleteness into account. This allows a separated evaluation of the influence for each uncertainty source on the results. Therefore, polymorphic uncertainty models are applied and developed by combining aleatoric and epistemic uncertainty descriptions, resulting (e.g. in the formulation of the uncertainty model “fuzzy probability based randomness”. The information of the different length scales is considered to be uncertain, e.g. the geometry or the material properties of a representative volume element (RVE) at the meso scale. Subsequently, the uncertainty of a macro structure is derived from uncertain results on the meso structure. In the contribution, a parameterized RVE for concrete structures, including cement phase as well as aggregates, is presented. Various material parameters are considered as uncertain, which results in uncertain effective quantities. Assuming, that an uncertain quantity is substitutable by a combination of a representative measure and multiple uncertainty characterizing measures, an approximation of both type of quantities by a recurrent neural network is carried out. The surrogate models are utilized as constitutive description in a numerical structural analysis on macro level, providing uncertain structural responses.

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1 Introduction

In this contribution, a brief overview of multi-scale analysis is presented, incorporating uncertain material parameters on every length scale. The multidisciplinary solution does embed computer science methods in form of machine learning as well as numerical structural analysis and uncertainty quantification. The content is given accordingly. Therefore a compact introduction of utilized polymorphic uncertainty models is provided, followed by the presentation of the computational workflow of multi-scale analysis in case of present uncertainties. Fundamentals of the underlying neural network are described and conclusively utilized in structural example.

2 Polymorphic uncertainty models

It is common to distinguish between two general characteristics of uncertainty, namely aleatoric and epistemic uncertainties. Where aleatoric uncertainty models, such as randomness, incorporate the variability of data or measurements, epistemic uncertainty considers e.g. incompleteness due to lack of knowledge or a small amount of available data.

Interval variables can be utilized in cases of e.g. few to none data samples or if no realistic assessment of certain data points is possible or reasonable. In contrast to determinisitic values, the characteristic function of an interval variable (see [1], [2], [3]) is defined as

$$\chi_I : \mathbb{R} \rightarrow \{0, 1\}, x \mapsto \begin{cases} 1, & x \in I \\ 0, & x \notin I \end{cases}.$$ (1)

Subsequently, an interval variable can be defined by its bounds

$$I = [x_l, x_r] = \mathbb{I}, \ x_l, x_r \in \mathbb{R} \land x_l < x_r.$$ (2)

If it seems reasonable to assess weightings (e.g. due to expert knowledge) to certain parameter ranges, an interval variable can be extended to a fuzzy-variable, where the corresponding membership function or characteristic function is stated as

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\( \mu : \mathbb{R} \mapsto [0, 1] \) according to [1], [2], [4]. A resulting fuzzy set (or fuzzy variable, introduced by [5]) is then represented by discrete \( \alpha \)-levels, whereas, in case of a one-dimensional fuzzy variable, each \( \alpha \)-level shall be expressed as an interval variable (see Eq. (2))

\[
A^k_{\alpha} \subseteq \mathbb{R}, \quad A^k_{\alpha} = [x_l, x_r]_{\alpha} = (A^k)_{\alpha}.
\]

(3)

By accepting the concept of an interval-based representation of \( \alpha \)-levels, it becomes obvious that an interval analysis as uncertainty quantification method could be extend to a fuzzy analysis, simply by repetitive utilization and previous possibility assessment. A combination with probabilistic approaches towards polymorphic uncertainty methods is extensively discussed e.g. in [2], [4], [6].

According to e.g. [6], fuzzy-probability based random variables (fp-r) can be assumed as a polymorphic uncertainty model, combining aleatoric (probability distribution) and epistemic (fuzzy set) features in one fp-r variable. Hereby, fuzzy valued distribution parameters are defined, since few data samples lead to incompleteness and do not justify a fully probabilistic approach. A two parametric uncertain cumulative distribution function (cdf) can be, subsequently, denoted as

\[
\hat{F}_X = \{\{ F_{\theta_1 \times \theta_2} \mid \theta_1 \in \hat{\theta}_{1,\alpha}, \theta_2 \in \hat{\theta}_{2,\alpha} \}\}_{\alpha \in [0, 1]}.
\]

(4)

3 Homogenization approach incorporating uncertainties

One approach in multi-scale analysis of concrete structures is the separation into two length scales (macro and meso), whereas the macro scale defines the overall homogeneous macroscopic structure and the meso scale the heterogeneity by means of a representative volume element (RVE) containing aggregates, pores, cement phase and optionally different types of reinforcements. On both scales, a separate boundary value problem (BVP) is formulated, where as the scale transition from meso to macro scale is referred to as homogenization, since the stress state on the minor length scale is condensed in an effective stress vector. The effective properties \( \square \) at macro scale are determined by the volume average of the corresponding quantity at meso scale.

The effective coefficients of the stiffness tensor (if Hooke’s law is assumed on both length scales, see [7]) are determined by numerical material testing (see [8]). By introducing the Hill-Mandel condition [9] (in case of small strains) \( \sigma \vdash \varepsilon = \tilde{\sigma} : \varepsilon \), boundary conditions, such as e.g. linear displacement boundary conditions, could be derived \( u(x) = \varepsilon_0 x \quad \forall x \in \partial B \). Uniform traction boundary conditions as well as periodic boundary conditions are applicable as well, whereas it is notable that the interchange of boundary conditions yields a change in effective stiffness values.

As it is depicted in the following scheme (see Fig. 1), the obtained effective stress strain dependencies for various loading conditions are approximated by neural networks. The uncertain stress response is decomposed with a representative measure and an uncertainty quantifying measurement. Hereby, the separate treatment of the representative measure (mean value, solution of deterministic FEM) and the uncertainty are enabled for the further numerical simulation.

![Fig. 1: Proposed solution scheme incorporating uncertainty and RNNs](image)

Despite the reinforcement, the accurate modelling of aggregates is crucial in order to determine the representative material parameters of the composite structure. In civil engineering, Fuller’s-Curve or grading curve defines the distribution of different size aggregates within one concrete mixture.

Based on [10], the aggregates are simplified by ellipsoids, where the principle radii are determined with respect to the medium radius \( r_2 \) and two realisations \( u_1, u_3 \) of uniformly distributed random variables. The flatness of the ellipsoids is defined by the parameter \( m \). The grading curve contains the volume-percentage of aggregates passing through a sieve of predefined size. Due to the sequential decrease of the sieve diameters, the grading curve represents the mass or volume percentage of certain mineral size classes \( k \) (defined by minimal \( d_{\text{min},k} \) and maximal diameter \( d_{\text{max},k} \) in the entire aggregate volume. On basis of a logarithmic size distribution, the principle diameter \( d_{2,k} \) for each mineral size class is utilized as sampling variable within the bounds of the related mineral size class. A sampling scheme is carried out for each mineral size class in a decreasing order, where each aggregate is evaluated in order to avoid intersections with existing aggregates, pores or reinforcement elements. In Fig. 2, a resulting numerical model is depicted, which is utilized for the computation of the effective stiffness. As an example, an uncertain effective stress strain dependency is depicted in Fig. 3.
4 Recurrent Neural Networks

Based on the research in [11], [12], recurrent neural networks with long short-term memory (LSTM) cells are considered. Deep learning capabilities and temporal preservation of input data relations enable the utilization especially for plasticity or more generally speaking – nonlinear material behaviour. In Fig. 4, an LSTM-cell with its time-dependent flow of data is depicted. The cell consists of four major gates for input and cell state information. Namely, the input $i_t = \sigma (w_i \cdot x_t + r_i \cdot h_{t-1} + b_i)$, forget gate $f_t = \sigma (w_f \cdot x_t + r_f \cdot h_{t-1} + b_f)$ and output gate $o_t = \sigma (w_o \cdot x_t + r_o \cdot h_{t-1} + b_o)$ transfer the nodal information $h_t = o_t \odot \tanh (c_t)$ through time, whereas the cell gate $\tilde{c}_t = \tanh (w_c \cdot x_t + r_c \cdot h_{t-1} + b_c)$ forwards the information in order to update the cell state $c_t = f_t \odot c_{t-1} + i_t \odot \tilde{c}_t$.

If the proposed utilization of RNNs within homogenization is expanded to considered uncertainty. The analysis procedure remains basically unchanged, but is extended by multiple evaluations of the uncertain RVE under various effective boundary conditions. The resulting representative measures and uncertainty quantifying formulations with respect to a strain tensor are hereby approximated by two separate recurrent neural networks (RNN), which substitute the deterministic constitutive law as well as the underlying uncertainty on the mesoscale. At the current state of research and development, this approach is investigated for quasi-static loading conditions. The utilization of an RNN with LSTM cells provides the approximation of consecutive (or time dependent) input data, due to the capability of LSTM cells to adjust cell parameters with respect to temporal sequences. A scheme for the sequencing is depicted in Fig. 5. As highlighted, effective stresses of the previous time steps $t_{1-3}$ (if sequence contains four time steps) are considered as input quantity of the network in order to predict the current stress state at time $t_4$. To provide convergence in the prediction of stresses, noise is added to the preprocessed input data.

Training is carried out with shuffled time sequences of a variable predetermined length. For both networks (representative and uncertainty quantifying effective properties), the preprocessing of training data as well as network topology are equal. Essential steps of the described procedure for multi-scale analysis incorporating uncertainties are summarized in Fig. 1.
5 Example

The general usability of a constitutive law, approximated by an RNN, is investigated with a simple macro structure (see Fig. 6). The cubic structure is discretized by 3x3x3 linear 8-node elements and linear boundary conditions are applied at each surface (normal and shear loading) simultaneously. The purpose of this initial example is the observation of the phenomenological correspondence of the stress response rather than the agreement with stress strain dependencies governed by analytical material law solutions. Furthermore, the convergence behaviour has been investigated in order to evaluate the overall applicability in the context of FEM. As the depicted stress response in Fig. 7 highlights, the RNN provides as qualitative representation of the mechanical behaviour incorporating von-Mises plasticity. Due to nonlinear activation functions within the LSTM-cells, the purely bilinear characteristic of the plasticity law cannot be maintained. Nevertheless, the discrepancy is comparatively small.

Fig. 6: Deformed example structure

Fig. 7: Predicted stress over load step for normal $\sigma_{11}$, $\sigma_{22}$ and shear $\sigma_{12}$ stresses

6 Conclusion and outlook

In this contribution, the framework for multi-scale numerical analysis with RNNs has been presented, whereas the RNN substitutes the constitutive law on the macro scale and is trained with effective stress strain dependencies from a meso scale RVE. The proposed approach allows a separate consideration of uncertainty for each length scale. As shown, uncertain effective quantities on meso scale are consolidated by uncertainty quantifying measures and approximated by RNNs, too. This leads to uncertain macro scale quantities even if the macro BVP is assumed to be deterministic. The 3D example validates the applicability of RNNs within an FEM-framework. Regarding reinforced concrete structures, uncertain numerical simulations of the reinforced RVE meso structure are necessary to obtain a training data set containing a sufficient amount of samples.

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