NOVEL A-B TYPE OSCILLATIONS IN A 2-D ELECTRON GAS IN INHOMOGENEOUS MAGNETIC FIELDS

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Abstract

We present results from a quantum and semiclassical theoretical study of the $\rho_{xy}$ and $\rho_{xx}$ resistivities of a high mobility 2-D electron gas in the presence of a dilute random distribution of tubes with magnetic flux $\Phi$ and radius $R$, for arbitrary values of $k_f R$ and $F = e\Phi/h$. We report on novel Aharonov-Bohm type oscillations in $\rho_{xy}$ and $\rho_{xx}$, related to degenerate quantum flux tube resonances, that satisfy the selection rule $(k_f R)^2 = 4F(n + \frac{1}{2})$, with $n$ an integer. We discuss possible experimental conditions where these oscillations may be observed.

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Transport in a two-dimensional electron gas (2DEG) in the presence of weak inhomogeneous magnetic fields has recently been the subject of considerable interest, both experimental [1–3] and theoretical [4]. This situation has been achieved experimentally by gating the 2DEG system with a type-II superconducting layer. Abrikosov vortices are then produced by applying an external magnetic field perpendicular to the plane of the layers. In the ballistic transport regime and for low fields, when the density of vortices is small, clear modifications to the Hall resistance in the quantum regime have been measured [3]. Previous theoretical studies of this problem were restricted to the asymptotic quantum regime $k_f R \gg 1$ [4] and classical limit $k_f R \ll 1$ [5]. However, the experiments have covered the interesting intermediate $k_f R$ crossover regime, with $F = 1/2$. Here, $k_f$ is the Fermi wave vector and $F = \Phi/\Phi_0$ with, $\Phi_0 = h/2$ the flux quantum.

In this paper we present a full solution to this problem for arbitrary $k_f R$ and $F$. Our results identify a series of novel quantum oscillations in the galvanomagnetic properties of the 2DEG, that appear to be within the reach of an experimental confirmation. These oscillations can be seen at intermediate ranges of $k_f R$ and $F(> 1/2)$ values and are related to the Aharonov-Bohm (AB) effect. The intermediate ranges of $k_f R$ are experimentally already achievable (e.g. [1,3]). At the end of the paper we discuss two experimental set-ups that have been suggested to produce larger values of $F$. Here we are interested in the experimental situation considered in [3] where the 2-D electrons move ballistically between the flux tubes and the dominant transport mechanism can be assumed to come from electrons scattering off individual flux tubes. Under these conditions, as a first approximation, we can apply the results of linear response theory in the Born approximation [6]. These results are formally the same as those obtained with the Boltzmann equation [4]. The weak nonlocal localization limit has already been considered experimentally and theoretically [2].

There are three important physical contributions to the electronic transport properties of this system: (i) For finite $R$ the Lorentz force that leads to an asymmetry in the scattering process. (ii) A diffractive force, relevant in the $0 < k_f R < 1$ regime and first considered by Iordanskii [7], that also yields a transversal contribution to the transport, and (iii) the
standard AB contribution \[8\]. The Iordanskii term in $\rho_{xy}$, which is not taken into account in the differential cross section, is due to the scattering of electrons by finite radius flux tubes and has essentially the same origin as the AB effect \[4\], for both are topological in nature and due to the long range properties of the vector potential. This means, as we see below, that the contribution from (ii) to the Hall resistance only depends on the value of $F$ and not on the specific magnetic flux profile chosen in the analysis.

The modification to the Hall resistivity, $\rho_{xy}$, due to the inhomogenous field can be represented by a Hall coefficient, $\alpha$, which is defined by the expression \[3\]

$$\rho_{xy} = \alpha(k_f R, F) \frac{B}{n_e e},$$

where $n_e$ is the electron density and $B$ the magnetic field. In the Born approximation of the Kubo formula the transport coefficients are expressed in terms of the scattering cross section $f(\phi)$, with $\phi$ the electronic scattering angle \[3,4\]. Explicit limiting values of $\alpha$ have been calculated in the extreme quantum $\alpha(k_f R \ll 1) = \frac{1}{2\pi F} \sin (2F\pi)$ and semi-classical limits $\alpha(k_f R \gg 1) = 1$ \[4,5\]. We use these two results as constraints to be satisfied in our calculations. Previous studies were restricted to these two limits because of mathematical difficulties in the evaluation of the scattering amplitude $f(\phi)$ in the whole $k_f R$ and $F$ ranges. These difficulties were identified by Khaetskii \[4\] and are essentially related to the singularity in the AB and Iordanskii scattering in the forward direction. In the AB case $f_{AB}(\phi \sim 0) \sim 1/\sin(\phi/2)$, which would lead to an infinite $\alpha$ \[3,4\].

Below we present the results of an explicit evaluation of $f(\phi)$ in the whole range of $k_f R$ and $F$ values. More importantly, we use these results to calculate $\alpha$ and the magnetoresistance $\rho_{xx}$ in the extended parameter range. Since the calculational problems arise in the forward scattering region we consider the regularized scattering amplitude

$$f_\epsilon(\phi) = \frac{e^{-i\pi/4}}{\sqrt{2\pi k_f}} \sum_{m=-\infty}^{\infty} e^{im\phi} e^{-|m|\epsilon} [e^{2i\delta_m} - 1],$$

with $\epsilon$ the regularization parameter, which is taken to zero at the end of the calculations. Here $\delta_m$ is the phase shift associated with the $m^{th}$ partial wave and can be written as
\[ \delta_m = \delta^{AB}_m - \tilde{\delta}_m. \] The \( \delta^{AB}_m = \frac{\pi}{2} (|m| - |m + F|) \) accounts for the AB phase shift, and \( \tilde{\delta}_m = \tan^{-1}(b_m/a_m) \) for the remaining contribution to the scattering. The coefficients \( a_m \) and \( b_m \) are obtained from the asymptotic wave function solution to the Schrödinger equation \[ \psi(r \to \infty, \theta) \approx \sum_{m=-\infty}^{\infty} [a_m J_\nu(k_f r) + b_m N_\nu(k_f r)] e^{im\theta}, \] which has the required form for incoming plane waves and outgoing circular waves. Here \( J_\nu(k_f r) \) and \( N_\nu(k_f r) \) are the Bessel and Neumann functions of order \( \nu = |m + F| \) (see Ref. [10] for more details).

Including the contributions (i-iii) we can then write the Hall coefficient as,

\[ \alpha = \frac{k_f^2}{2\pi F} \lim_{\epsilon \to 0} \int_{0}^{2\pi} \sin \phi |f_\epsilon(\phi)|^2 d\phi + \frac{1}{\pi F} \sin (F\pi). \] (3)

The first term is the regularized Boltzmann contribution while we wrote the second topological term following Iordanskii [7]. An important property of this expression is that it fully reduces to the extreme quantum and semiclassical results mentioned above. In this expression, as long as \( \epsilon \) is finite, there is no singularity in \( f_\epsilon(\phi) \) and we can perform the integral. After evaluating this integral using Eq.(2), and from the general fact that \( [\delta_{m+1} - \delta_m] \to 0 \) in the limit \( |m| \to \infty \), we get the \textbf{finite} result

\[ \alpha = -\frac{1}{2\pi F} \sum_{m=-\infty}^{\infty} \sin [2(\delta_{m+1} - \delta_m)]. \] (4)

This equation is one of the main results of this paper, for it provides an algorithm to calculate \( \alpha \) for arbitrary values for the parameters \( k_f R \) and \( F \). The number of terms needed in the sum will depend on the parameter range considered.

The corresponding linear response theory result for the magnetoresistance \( \Delta \rho_{xx} \) is

\[ \frac{\Delta \rho_{xx}(F)}{\rho_{xx}(0)} = \frac{\tau_i}{\tau} - 1 = N_F \ell_i \lim_{\epsilon \to 0} \int_{0}^{2\pi} (1 - \cos \phi) |f_\epsilon(\phi)|^2 d\phi \]

\[ = \frac{2}{k_f N_F \ell_i} \sum_{m=-\infty}^{\infty} \sin^2 (\delta_{m+1} - \delta_m) \] (6)

where \( \tau \) is the time between electronic flux tube collisions, within the Kubo-Born approximation, and \( N_F \) the concentration of magnetic flux tubes. When the magnetic field is zero
the transport scattering time is \( \tau_i = \ell_i / v_f \), with \( v_f \) the Fermi velocity and \( \ell_i \) the impurity scattering elastic mean free path, which is assumed to be much larger than \( 2R \).

We now proceed to present our results from the direct evaluation of Eqs.(4) and (6). Later in this paper we present a physical explanation of these results using a semi-classical analysis. In plotting these results we have used the typical experimental parameter values given in the captions. In Fig.1(a) we show \( \alpha \) as a function of \( k_f R \) for different values of \( F \). The curve \( F = 1/2 \) corresponds to an extended range of Fig. 3 in Ref. 3. We note that for values of \( F \leq 1 \), \( \alpha \) is a monotonic function of \( k_f R \). Notice, however, that for \( F = 3/4 \), \( \alpha \) can become negative for small values of \( k_f R \), which comes from our careful treatment of the extreme quantum region. For \( F \geq 2 \) we see clear oscillations in the \( \alpha \) vs \( k_f R \) curves 14. For \( F = 10 \), for example, we can clearly identify sharp oscillations of \( \alpha \) vs \( k_f R \), although their absolute value is smaller. The number of oscillations as a function of \( k_f R \) is equal to the integer part of \( F \), \( \lfloor F \rfloor \). For \( F = 10 \), there are ten oscillations (five of them not shown occur for \( k_f R > 15 \)). Moreover, for small \( k_f R \) there are narrow oscillations superimposed on the first few oscillations. In Fig.1(b) we show the Hall resistivity as a function of \( F \) and \( k_f R = 10 \). For small values of \( F \) we see the classical linear behavior of \( \rho_{xy} \) up to a maximum value, after which it decreases as \( F \) increases. We note that the quantum curve, obtained using Eq.(4), decreases non monotonically as \( F \) increases and even becomes negative for values of \( F \sim 50 \). Finally, in Figs.2(a) and 2(b) we show the corresponding results for \( \Delta \rho_{xx}(F)/\rho_{xx}(0) \) as a function of both \( k_f R \) and \( F \). In Fig. 2(a) for \( F \leq 1 \) we see that \( \Delta \rho_{xx}(F)/\rho_{xx}(0) \) is a monotonic decreasing function of \( k_f R \), while for larger values of \( F \) the magnetoresistance becomes an oscillatory function of \( k_f R \). In Fig.2(b) we note the sharp resonances that occur exactly at the same values of the minima in \( \rho_{xy} \).

We now provide a physical interpretation of these results in the semi-classical limit. To understand the semi-classical analysis, we start by discussing the classical problem 15. We note that the energy, \( E = \frac{1}{2} m^* v_f^2 \), and the total (particle+field) angular momentum, \( J = m^* v_f b - e \Phi \), are constants of the motion. Here \( b \) is the impact parameter and \( m^* \) the electron’s effective mass. The impact parameter is defined as \( b = y(t \to -\infty) \), where \( y \)
is along the direction of the current. The **classical** Hall coefficient is characterized by the important parameter \( \beta \equiv \omega_c T = e\Phi/(2\pi m^* v_f R) = F/k_f R \). Different scattering events have different total angular momenta and different \( \beta \) parameters. When \( \beta \ll 1 \), the electron trajectories are only slightly affected by the magnetic flux tube. As \( \beta \) increases the Lorentz force becomes important until a critical \( \beta_c \), above which trapped orbits can exist. The particular quantitative value of \( \beta_c \) depends upon the particular flux tube profile. For our flux-tube model \( \beta_c = 1/2 \), and the trapped circular orbits have radius \( r_o = R/2\beta \). Both the quantum and classical scattering problems can be separated into angular and radial components. The radial component of the classical equation is, as usual, a one-dimensional problem with effective potential, \( V_{\text{eff}}(r,b) = \left[J + \beta \int_0^r r'B(r') dr'\right]^2 / 2r^2 \). Here the magnetic field of the flux tube is \( \vec{B}(r) = B(r)\vec{z} \), and we have rescaled energies by \( m^* v_f^2 \), the angular momentum by \( 2\pi mv_f R = \hbar k_f R \), and distances by \( R \). In these units \( J = b - \beta \) and the flux tube radius is equal to 1. In the inset of Fig.1(b) we show curves for \( V_{\text{eff}}(r,b) \) for different values of \( J \). We see that electrons with different \( J \)'s, or impact parameters, experience different effective potentials. As the total angular momentum decreases, \( V_{\text{eff}}(r,b) \) develops a potential barrier with height \( [J + \beta]^2 / 2 \), which decreases rapidly. We can show that for \( \beta \geq \beta_c = 1/2 \) there is a range of \( J \)'s for which there can be trapped circular orbits inside the flux tube from \( J_1 = 1/4\beta \) decreasing to \( J_2 = 1 - \beta \). As \( J \) decreases from \( J_1 \) to \( J_2 \), the center of the electronic circular orbit shifts from \( r_1 = 0 \) to \( r_2 = 1 - r_0 = 1 - 1/2\beta \). This range of possible total angular momenta is such that the circular orbit stays completely within the flux tube. Classically, these circular orbits can not be reached by a scattering process. However, quantum mechanically the scattering electron can tunnel through this potential barrier and form a quasi-bound state inside the flux tube for a finite time, and then escape again. In the classical calculation of the Hall resistivity and magnetoresistivity shown by dashed lines in the figures, we computed the classical differential cross-section which was used in Eqs.(1), (3) and (6), in place of \( |f(\phi)|^2 \).

In the semi-classical analysis we associate a classical circular orbit to each quasi-bound state. Using the standard Bohr-Sommerfeld quantization condition it is easy to show that the
quasi-bound states are degenerate and occur at quantized values of the energy, \( E_n/\hbar \omega_c = (k_f R)^2/4F = n + 1/2 \), with \( n \) an integer. This result is the analog of the Landau level condition in a homogeneous magnetic field. The factor \( n + 1/2 \) gives the total number of flux quanta enclosed by the circular orbit. Since the quanta of flux in the tube is equal to \( F \), the quantum number \( n \) ranges from 0 to \([F] - 1\). Moreover, the quantized total angular momentum, \( J_m = m\hbar \), leads to a degeneracy of the \( n \) levels. This degeneracy is equal to the total number of quantized circular orbits which we can put inside the flux tube. From the range of classically allowed circular orbits mentioned above, we deduce that the allowed \( m \) values start at \( m_1 = [(k_f R)^2/4F] = n \) and decrease down to \( m_2 = [k_f R - F] + \delta \), with \( \delta = 1 \) if \([k_f R - F] > 0\) and \( \delta = 0 \) if \([k_f R - F] \leq 0\). Therefore, we conclude that the degeneracy is equal to \( m_1 - m_2 + 1 = n + 1 + [F - k_f R] - \delta \). In the figures, the arrows indicate the position of the principal quantum number \( n \) calculated using the selection rule \((k_f R)^2 = 4F(n + 1/2)\).

We observe that they are remarkably well aligned with some maxima and minima of \( \rho_{xx} \) and \( \rho_{xy} \). Furthermore, we have numerically determined that each resonance in \( \rho_{xx} \) and \( \rho_{xy} \) occurs at a preferential angular momentum \( J_m = m\hbar \) for some \( m \). We arrived at this conclusion by evaluating the time delay \( t_D^m(k_f R, F) = 2\hbar(\partial \delta_m/\partial E) = (2R/v_f)[\partial \delta_m/\partial (k_f R)] \) and we found that as a function of \( m \), \( t_D^m \) becomes sharply peaked for one particular value of \( m \) each time the pair \((k_f R, F)\) corresponds to a resonance in the transport coefficients. In the figures we have indicated the values of \( m \) for the resolved resonances. The minima and maxima in \( \rho_{xy} \) correspond to quasi-bound states due to the tunneling of the electron into the flux tube.

Semi-classically, as \( m \) decreases the resonances correspond to rotationally asymmetric orbits leading to larger \( \rho_{xy} \), as observed in Fig.1(b). Note that the number of resonances observed for a particular quasi-bound state level should be equal to the degeneracies of this level. However, near \( m_1 \), the amplitude of the resonances is suppressed due to the exponentially small tunneling probability through the potential barrier \( = [J + \beta]^2/2 \). For example note that \( \rho_{xy} = 0 \) at \( F = 50 \) in Fig.1(b). On the other hand, for each quasi-bound state level, the observed resonances with the smallest \( m \) (\( m=3, 0, -7, -51 \) in Fig.1(b)) correspond precisely to the value \( m_2 \) derived semi-classically.
We now consider the possible experimental conditions necessary to observe the galvanomagnetic oscillations described in this paper. The variation of $k_f R$ in the ranges of interest has already been achieved \[1,3\]. New techniques need to be developed to produce larger values of $F$ inside the flux tubes. We discuss a couple of possibilities that have already been suggested to us. The general idea is to have the usual Hall bar shown schematically in the inset of Fig.2(a), with the inhomogenous magnetic field produced by a dilute distribution of magnetic flux tubes of strength $F$. One possible way to get larger values of $F$ experimentally is by depositing randomly located submicron size *superconducting dots or pillars* on top of the 2DEG, in a manner similar to the way the dot and antidot systems have been fabricated \[11,12\]. Alternatively, one may drill randomly located submicron holes in the superconducting layer by using electron beam lithography \[13\]. In both cases, by following a magnetic field cooling technique the magnetic flux may be pinned inside the dots thus trapping a large bundle of flux quanta. As in the antidot systems we do not expect that the oscillations found here will be significantly affected by temperature or Coulomb effects, provided the temperatures are sufficiently low and the charging energy effects are not significant for the superconducting pillars fabricated.

In conclusion, we have presented a detailed analysis of the transport properties of a 2-D electron gas system in the presence of a dilute gas of randomly located magnetic flux tubes for arbitrary values of $k_f R$ and $F$. The main result from our analysis is the presence of novel AB-like oscillations in the galvanomagnetic properties of the system. These oscillations are explained in terms of the degenerate resonant levels, satisfying the selection rule $(k_f R)^2 = 4F(n + \frac{1}{2})$, due to the effective trapping potentials produced by the flux tubes. A more extensive presentation of the results described here will appear elsewhere \[16\].

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[15] M. Carreau, J. V. José, and C. Rojas (preprint)

[16] M. Carreau, and J. V. José (in preparation)
FIGURES

FIG. 1. (a) Hall coefficient $\alpha$ as a function of $k_f R$. Here $R = 100$ nm, the density of flux tubes is $N_F = 10^5$ mm$^{-2}$ and the electron concentration is $n_e = 3.98 \times 10^{10}$ cm$^{-2}$, with $\Phi = \pi R^2$. Here $n$ denotes the flux tube resonances and $m$ the angular momentum degeneracies. (b) same as in (a) for the Hall resistivity $\rho_{xy}$ as a function of $F$. The dashed line corresponds to the classical results. The inset shows the effective potential $V_{\text{eff}}$ for different values of the total angular momentum (from top to bottom $J=1.25$, 0.5, 0.2, 0, -0.25, -0.75)

FIG. 2. $\Delta \rho_{xx}(F)/\rho_{xx}(0)$ vs $k_f R$ (a) and vs $F$ (b) for the same parameter values as in Fig.1, with mean free path $\ell_i = 2 \mu$m. The inset shows the system considered in this paper formed by a Hall bar with a dilute random distribution of perpendicular magnetic flux tubes of strength $F\Phi_0$. See text for further details.