Electromagnetic modeling and design of a novel class of complementary split-ring resonators

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Abstract
This research study reports the assessment of complementary split ring resonators based on Gielis transformation as basic elements for the design of high-performance microwave components in printed technology. From the electromagnetic simulation of said structures, suitable equivalent circuit models are extracted and analyzed. Physical prototypes are fabricated and tested for design validation. The obtained results confirm that the adoption of supershaped geometries enables the synthesis of very compact scalable microwave filters.

KEYWORDS
complementary split-ring resonator, equivalent circuit, Gielis formula, metamaterials, microwave filter

1 | INTRODUCTION

Split ring resonators (SRRs) have been originally proposed as metamaterial structures featuring double negative medium properties, whereas their complementary variant complementary split ring resonators (CSRRs) are typically utilized as negative-permittivity components in planar left-handed structures for synthesis of artificial transmission lines and complex microwave passive components such as multiband impedance inverters. In particular, the use of SRRs and CSRRs as basic resonant units in planar microwave filters has gained a progressively growing interest thanks to their electrically small size when compared to conventional resonators, this enabling the realization of semilumped filtering structures with high performance and controllable characteristics.

Canonical circular or square geometries are commonly adopted for the design of the considered class of resonators. More recently, Sierpinsky fractal geometries have been proposed for the realization of miniaturized CSRRs still with a square-like form factor. It is to be noticed that squares and circles may be regarded as special cases of the Gielis equation, so the question arises whether higher-order supershaped geometries could provide benefits or additional degrees of freedom in the synthesis of highly compact planar microwave filters. The goal of this study is therefore to analyze the performance of bandpass CSRR filters whose geometry is based on Gielis formula.

The article is organized as follows. A detailed description of the geometry of the considered CSRR unit cells, as well as of the equivalent circuit models used to investigate the relevant electromagnetic properties is given in section 2. Measurement results are presented and discussed in section 3. Finally, the concluding remarks and outline of future work are summarized in section 4.

2 | SUPERSHAPED CSRR

2.1 | Geometrical foundation of the design

Gielis formula is a generalization of Lamé equation that can be used to describe a wide variety of complex shapes found...
in Nature. Its general expression is reported in Ref. 13. In this study, we are making use of the following particular case:

\[
\rho = r_m(\varphi) = \frac{1}{2} \lim_{n \to +\infty} \left( \left| \cos \frac{mn}{4} \right| + \left| \sin \frac{mn}{4} \right| \right) \frac{n}{C_{12/C12/C12/C12}}
\]

\[
= \frac{1}{2} \min \left\{ \left| \sec \frac{mn}{4} \right|, \csc \frac{mn}{4} \right\}.
\]

(1)

with \( \rho \) and \( \varphi \) denoting the usual polar coordinates defined with respect to a background Cartesian reference frame \( Oxy \), and where \( m \) is a positive integer parameter which defines the number of pseudovertices featured by the general closed curve \( C_m \) described by (1).

It is known from the basic theory of differential geometry that the length and area of \( C_m \) are given, respectively, by:

\[
\ell_m = \int_0^{2\pi} \sqrt{r_m^2(\varphi) + \left( \frac{dr_m(\varphi)}{d\varphi} \right)^2} \, d\varphi,
\]

(2)

\[
A_m = \frac{1}{2} \int_0^{2\pi} r_m^2(\varphi) d\varphi.
\]

(3)

By making use of (2) and (3), it can be found out, after simple algebra, that the considered class of supershaped curves is characterized by unitary area \( A_m = 1 \) for \( m > 0 \), whereas the length can be computed as:

\[
\ell_m = \sqrt{8 + \frac{m^2}{2} + 4 \left[ F\left(\frac{\pi}{4}, 1-\frac{m^2}{16}\right) - E\left(\frac{\pi}{4}, 1-\frac{m^2}{16}\right) \right]},
\]

(4)

with \( F(\cdot, \cdot) \) and \( E(\cdot, \cdot) \) being the elliptic integral of the first and second kind, respectively. It is apparent from (4) that \( \ell_m \) increases asymptotically with linear law, namely \( \ell_m \propto (\sqrt{2}-1)m \) as \( m \to +\infty \) (see Figure 1).

2.2 Physical implementation

The microwave filters investigated in this study consist of a microstrip transmission line loaded with a supershaped CSRR of general Gielis order \( m \) etched in the relevant ground plane, as shown in Figure 2A.

The considered structure is realized on dielectric laminate having relative permittivity \( \varepsilon_r = 3.78 \), and loss tangent \( \tan\delta = 0.025 \), the thickness of the metal layers being
\[ t = 35 \, \mu m. \] The two concentric ring slots forming the CSRR unit are separated by distance \( s \), and are characterized by width \( d \) and gap \( g \). Conversely, the microstrip line printed on the circuit substrate features a width \( w = 1.835 \, mm \), which is selected in such a way as to achieve a nearly 50 \( \Omega \) characteristic impedance.

The design and full-wave analysis of the complete assembly has been carried out by making use of the commercially available electromagnetic solver CST Microwave Studio.\(^{14}\) In this way, the characteristics of the structure in terms of stop-band frequency \( f_s \) and peak insertion loss \( IL_{\text{max}} \) have been evaluated as a function of the geometrical parameters \( s \), \( d \), \( g \), and \( m \).

From the visual inspection of Figure 3, it is apparent that \( f_s \) is decreasing as \( s \), \( d \), \( g \) become smaller, and the Gielis order \( m \) increases. Conversely, under the same conditions, \( IL_{\text{max}} \) tends to decrease. It is worth noting here that the general supershaped CSRR as described by (1) degenerates in the conventional circular and square one for \( m \rightarrow 0 \) and \( m = 4 \), respectively.
Following the design stage, several physical demonstrators of the considered filtering structures (see Figure 2A), with \( s = d = g = 0.75 \text{ mm} \) and \( m \) ranging from 5 to 10, have been fabricated. As it can be noticed in Figure 2B, SMA adapters are soldered at both ports of the device, and adopted as connector interface to external coaxial cables for \( S \)-parameter measurements.

### 2.3 Equivalent circuit models

Thanks to the small electrical size of CSRRs at the resonance (\( \sim \lambda/10 \)), the considered structures can be conveniently described by means of suitable lumped-element equivalent circuits. Several models have been proposed in the scientific literature. In this respect, Baena et al. proposed the circuit topology shown in Figure 4A,\(^6\) where the CSRR unit is modeled as a parallel resonant tank with inductance \( L_c \) and capacitance \( C_c \) and, therefore, characterized by resonant frequency:

\[
 f_0 = \frac{1}{2\pi \sqrt{L_c C_c}}. \quad (5)
\]

In said schematic, the capacitance \( C \) models the coupling between the CSRR and the microstrip line which, in turn, is characterized by inductance \( L \). It is worth mentioning that the inductance \( L_c \) is mainly dependent on the length \( \ell_m \) of the CSRR embedded in the design, whereas the capacitance \( C \) is largely affected by the relevant area which, for the proposed class of structures, is an invariant.

At frequency \( f_0 \), the shunt admittance is zero and, therefore, the impedance measured at the any port of the network is set by the host line inductance \( L \) which, in turn, can be readily estimated by using well-known computer-aided design (CAD) formulas available in the scientific literature. Conversely, the transmission-zero frequency of the filter, at which the condition \( S_{21} = 0 \) holds true, is readily found to be:

\[
 f_z = \frac{1}{2\pi \sqrt{L_c (C + C_c)}}. \quad (6)
\]

Finally, upon denoting the series and shunt impedances of the equivalent T-network of the structure as \( Z_1 \) and \( Z_2 \), respectively, one can verify, after simple mathematical manipulations, that:

\[
 Z_1(f_{\pi/2}) + Z_2(f_{\pi/2}) = 0, \quad (7)
\]

with \( f_{\pi/2} \) being the frequency where the phase of the transmission coefficient \( S_{21} \) is equal to zero. By combining (5) and (6) with (7), the parameters of the circuit model in Figure 4A can be extracted from the numerically simulated \( S \)-parameter data.

Li et al. proposed, for the considered structure, the alternative equivalent circuit topology shown in Figure 4B,\(^7\) that consists of a series \( LC \) resonator (\( L_1, C_1 \)) with a capacitance \( C_2 \) connected in parallel, as well as two sections of 50 \( \Omega \) transmission lines having length \( l \) at both sides. This model is characterized by a higher degree of accuracy over a broader frequency band of operation, though it relies on a negative-value parameter as outlined hereinafter.

Three independent equations are needed for the synthesis of the circuit in Figure 4B. The first one is given by the resonant condition of the whole shunt branch of the equivalent T-network that reflects in the zero of the input reflection
Finally, the length deality can be easily addressed by integrating resistive elements in the corresponding physical structure. However, this nonideal behavior is responsible for an unwanted impedance deviation.

As for the equivalent circuits, both proposed models can be fitted to the phase of the scattering parameters. In this way, one can verify that:

\[
C_2 = \frac{Y_0 f_2^2 - f_3^2}{\pi f_s f_1^2 - f_2^2}, \quad (8)
\]

\[
C_1 = \left[\left(\frac{f_1}{f_2}\right)^2 - 1\right]C_1, \quad (9)
\]

\[
L_1 = \frac{1}{(2\pi f_2)^2}C_1, \quad (10)
\]

where \(Y_0\) is the characteristic admittance of the input lines. It is apparent from (10) that the \(C_2\) is negative as \(f_1 < f_3 < f_2\). Finally, the length \(l\) of the microstrip line sections embedded in the series branches of the equivalent circuit is determined by fitting the phase of the scattering parameters.

As it appears in Figure 4, the considered equivalent circuit networks do not account for loss mechanisms occurring in the corresponding physical structure. However, this nonideality can be easily addressed by integrating resistive elements in the resonant LC tanks included in both models.

### 3 NUMERICAL RESULTS AND MEASUREMENTS

Following the procedure described in section 2, equivalent circuit models of the considered supershaped CSRR filters have been extracted starting from the numerical simulation data. The circuit parameters are given in Table 1 as a function of the Gielis order \(m\) of the considered class of CSRRs decreases, with a nearly linear progression, as \(m\) becomes larger. A good agreement between numerical and experimental results can be noticed in Figure 5 and Table 2. However, from the visual inspection of Figure 5, it is apparent that a deviation occurs in terms of simulated and measured insertion-loss bandwidth, this being attributable to a mismatch between the actual loss tangent featured by the dielectric laminate used for the manufacturing of the filter prototypes (see Figure 2B) and the nominal value adopted in the numerically based electromagnetic model of the various structures.

As an example, Figure 6 shows the frequency-domain behavior of the magnitude of the coupling coefficient \(|S_{21}|\) and input reflection coefficient \(|S_{11}|\) of the supershaped CSRR of order \(m = 9\), as simulated with CST Microwave Studio, measured on physical prototypes, and evaluated using the equivalent circuits shown in Figure 4. The agreement between numerical and experimental results is pretty good, minor discrepancies in return loss being attributed to tolerances in the fabrication process and parasitic effects associated with the microstrip-line-connector transition which is responsible for an unwanted impedance deviation.

As for the equivalent circuits, both proposed models can predict the stop-band behavior of supershaped CSRRs with a reasonable accuracy, though Li’s equivalent circuit shows a better agreement with simulated data across a broader range of frequencies, this being beneficial for a more effective design process of microwave systems by means of CAD software applications. This is however achieved at the expense of a higher complexity level associated with the integration of microstrip line sections and, possibly, a negative capacitor (see Table 1) in the network. Conversely, Bae-na’s circuit provides a useful physical insight in terms of the equivalent intrinsic inductance and capacitance of CSRRs.

### 4 CONCLUSIONS

A new class of supershaped CSRR filters has been presented, and the relevant circuit behavior studied extensively. It has been demonstrated that the transmission-zero frequency of the considered structures can be easily tuned by properly selecting their Gielis order, while keeping other characteristics nearly unchanged. Novel designs based on the periodic or...
aperiodic clustering of supershaped CSRRs are being investigated to synthesize high-performance filter structures.

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