Could Grover’s quantum algorithm help in searching an actual database?

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Abstract
I investigate whether it would technologically and economically make sense to build database search engines based on Grover’s quantum search algorithm. The answer is not fully conclusive but in my judgement rather negative.

1 Introduction
In the quantum computing community different views about the usefulness of Grover’s algorithm for actual database search have been expressed informally but so far nothing has been published on the subject. Probably most authors (e.g. [1], [2]) who refer to Grover’s quantum search algorithm as “database search” think of the oracle as a “virtual database” and don’t mean to imply actual database search.

The main argument against building a database search engine based on Grover’s algorithm is of course that the whole addressing system would have to work in superposition, thus it would have to be built out of (presumably expensive) quantum hardware which would have roughly the size of the database. At this point one wonders whether a (partially) parallel classical computer with many processors would not be a better solution. Here I analyze and compare in more detail the cost and performance of quantum and classical search engines.

2 Grover’s algorithm
Grover’s algorithm [1], [3] a priori solves a somewhat artificial problem: We are given some quantum hardware around which there is a black box so we don’t know how it works. The hardware outputs a 0 or a 1 for any of a large number of possible inputs. The task is to find an input for which the output is 1. If the number $n_1$ of such inputs is known, Grover’s algorithm finds one of them
querying the black box (also called “oracle”) some $\pi/4\sqrt{N/n_1}$ times. Classically the number of queries would on average be more like $N/n_1$.

2.1 what is it good for?

Practically there are computational problems where we don’t know better that to try through a large number of possibilities to find one that suits us. On the other hand often (possibly heuristic) classical algorithms are used which give a much better than square root improvement over a simple “exhaustive” search.

I have been told about the following examples of problems where no (much) better algorithm than a simple unstructured search is known and where thus Grover’s algorithm could help:

- breaking a DES (data encryption standard) key
- certain (particularly hard) instances of the SAT (satisfiability of boolean formulae) problem
- hard instances of coloring a map with just 4 colors, a task which is guaranteed to be possible by the 4-color theorem

3 Comparing classical and quantum search engines

3.1 preliminary considerations

An existing database (e.g. everything available via the internet) of course can’t be searched with Grover’s algorithm as the addressing system (the internet) is not quantum. Thus here I only consider purpose built databases including an addressing system and search engine.

Also we only consider unordered databases as clearly an ordered database can be searched in logarithmic (in the number of datasets) time.

Actually I only consider databases which can’t be ordered because ordering a databases once and for all is not such a big task and can actually be done while the datasets are acquired at a cost of $\log N$ per dataset where $N$ is the size of the database.

A database can’t be ordered when our search criteria are too complex. Still, often indexing according to different search criteria is possible which allows searching in logarithmic time for a not too big variety of search criteria. Here I consider the case where our search criteria are so complex and varied that this is not possible.
3.2 definition of the problem

We have a database consisting of $N$ datasets, each of size $d$ (say $d$ bits). The search criteria are binary functions to be computed from the datasets which tell us whether a dataset meets our criteria or not. The size (e.g. number of gates) of a processor needed to compute the search criterion is $p$ and the computation time is $t$. I don’t consider the possibility of space time tradeoffs between these 2 quantities.

3.3 how to compare quantum and classical in a fair way

I compare cost and performance of the best classical search engine I can think of to the best quantum engine (based on Grover’s algorithm) I can think of. The database is of course the same in both cases. We could then e.g. compare the performance (speed) of the quantum and classical solutions for a given and equal cost of the machines. A problem is that we don’t know the (probably high) cost of future quantum hardware.

Alternatively we could compare for a given performance (= search speed). Again a problem is that we don’t know how the clock rate of quantum hardware will compare to that of classical hardware. Actually from the various quantum hardware proposals one get’s the impression that it might be rather slow due to the slowness of 2-qubit gates.

Due to these difficulties arising from our ignorance concerning the cost and speed of future quantum hardware, I will simply consider a quantum and a classical engine of same size (same number of gates) and compare the number of clock cycles they need to find a desired dataset. (Given my pessimism concerning cost and speed of quantum hardware, this gives a big advantage to the quantum solution.) In this setting it is clear that a quantum solution will be faster, the question is only by how much.

3.4 the architecture of my search engines

The architecture of the best classical and quantum search engines I can think of is essentially the same. The addressing system is a binary tree, at each node of which there is a switch. At the top (the root) there is a central processor and at the bottom (the leaves) we have the $N$ datasets. As I consider (partially) parallel search engines, I also place a processor at each node on some intermediate level. These are identical processors which compute the search criterion. Each of these processors sits atop $n$ datasets.
My considerations are not really affected if instead of a binary tree we have a tree with some other small number of fan-outs at each node. On the other hand things may change if the fan-out becomes large, as I will discuss later.

3.5 performance of the classical search engine

The size (number of gates) of the classical search engine is (S for “space”):

$$S_c = \frac{N}{n} p + N$$.

Again $N$ is the size of the database, $n$ the number of datasets per processor and $p$ the size of a processor. Thus the first term is the hardware in the processors and the second term is the addressing system which clearly is about of the size of the database. Note that here I assume that the switches at the nodes of the addressing system have the same cost as the gates in the processors. This seems reasonable as long as either could perform the other’s task, as then we simply use the cheaper of the two. This seems to be true as long as the switches in the addressing tree have only few states (=small fan-out).

The search engine works by having all processors in parallel search through their $n$ datasets, each time computing the search criterion in $t$ time steps. I neglect the (logarithmic) time it takes to propagate the answer from a successful processor to the central processor. I also neglect the time it takes a processor
to retrieve one of its datasets, assuming that this is less than the time it takes
to compute the search criterion. Then the search time is simply:

\[ T_c = nt \quad . \]  

(2)

### 3.6 performance of the quantum search engine

As stated above, I take the quantum search engine to be of the same size as the
classical one, thus I choose the same parameter \( n \), so:

\[ S_q = S_c = \frac{N}{n} p + N \quad . \]  

(3)

Where I neglect the (probably small) factors by which a reversible processor is
larger and uses more time steps.

Each (now quantum) search criterion processor now runs Grover’s algorithm
on its \( n \) datasets. The processor retrieves the contents of the datasets in quan-
tum parallelism and also computes the search criterion in quantum parallelism.
After roughly \( \sqrt{n} \) such steps it finds a dataset meeting our search criteria if
there is one. The propagation of this result to the central processor can now
be done classically. Thus the addressing system has to be quantum only below
the level of the processors, but that doesn’t help much as that’s of course where
most hardware is. The search time is now about:

\[ T_q = \sqrt{n} \quad t \quad . \]  

(4)

### 3.7 comparison

We still have the free parameter \( n \) which is equivalent to the size and thus to
the cost we want to invest. If we want to use as little hardware as possible, then
we don’t use any parallelism and have only a central processor, thus \( n = N \). In
this case we get the familiar quantum speed-up of \( \sqrt{N} \), even though we don’t
get a square root of the number of classical steps \( tN \) but only of the number \( N \)
of datasets.

I don’t think it is reasonable to use only as little hardware as possible as
with a relatively small additional investment we can get big speed-ups. As we
have already invested in an addressing system the size of the database, I think
it makes sense to invest a similar amount into parallel processors for computing
the search criterion. Equal investment means:

\[ \frac{N}{n} p = N \quad \Rightarrow \quad n = p \quad . \]  

(5)

As \( n \) can’t be larger than \( N \) we really have \( n = \text{min}(p, N) \). Now the quantum
speed-up becomes:
\[ T_c/T_q = \min(\sqrt{p}, \sqrt{N}) \] (6)

Thus it depends on the complexity of the search criterion. For even larger investment in hardware (smaller \( n \)) the quantum speed-up becomes still smaller.

3.8 (quasi-) analog addressing systems

So far I have only considered tree-like addressing systems. In (classical) reality addressing systems often look different. Hard disks are addressed in a quasi-analog manner where the read-write head can be in a large number of positions. A second analog variable in this system is time which allows us to retrieve the correct dataset by simply reading at the right time. If such addressing systems (quantum or classical) with many-state elements are much cheaper than tree like ones, the comparison shifts in favor of the quantum solution.

3.8.1 a quasi-analog quantum addressing system

Imagine e.g. a 2- or even 3-dimensional optical storage medium (the database) which can be addressed by a light beam (intersecting light beams in the 3-dimensional case). A quantum addressing system might now produce a photon in a superposition of many positions, thus querying the database in quantum parallelism. This superposition might be produced by mirrors being in a superposition of different classical positions. After going through the optical medium the photon could e.g. go back through the same set of mirrors.

The optical medium would shift the phase, change the polarization or modify another property of the photon depending on the stored data. Note that as in any reasonable setup the (of course classical) database would act as an exterior field on the quantum information, thus avoiding unwanted entanglement with it.

If inexpensive ways could be found to put a photon into a position-superposition according to the content of a quantum-address register, Grover’s algorithm might really prove to be useful in building search engines.

4 Short summary

I assume tree-like addressing systems and assume that quantum hardware will be as cheap and fast as classical hardware. Then by arguing that we should invest as much hardware into parallel processors as there is anyways in the addressing system, I get that the speed-up of a quantum search engine over a classical search engine would be \( \min(\sqrt{p}, \sqrt{N}) \), where \( N \) is the size of the database and \( p \) is the size of a processor needed to compute the search criterion on the datasets.
5 Conclusion

Even under favorable assumptions about quantum hardware the speed-up of a quantum search engine is limited not only by the size of the database but also by the complexity of the search criterion. Furthermore it is not clear (at least not to me) that there are any databases of practical interest which can’t be indexed intelligently so that at least the vast majority of queries can be done very quickly. Therefore I would rather not expect Grover-based database search ever to be of much use.

Still it is possible that there is a technological “window of opportunity” for such solutions. By this I mean a time when certain advanced technologies are available while others are still not. In the long run it is not clear whether passive hardware (storage) will continue to be much cheaper than active hardware (processors, addressing systems). It seems that at this point the database search problem would anyways go away.

I would like to thank Daniel Gottesman who has contributed to my analysis through discussions. He is less pessimistic than me about the usefulness of Grover’s algorithm for database search.

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