Title page:

Extreme geomagnetic activities: A statistical study

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Abstract

Statistical distributions are investigated for magnetic storms, sudden commencements (SCs), and substorms to identify the possible amplitude of the one in 100-year and 1000-year events from a limited data set of less than 100 years. It is found that majorities of events essentially follow the log-normal distribution, as expected from the random output from a complex system. However, it is uncertain that rare events follow the same log-normal distributions, and rather follow the power-law distributions. Based on the statistical distributions, the probable amplitudes of the 100-year (1000-year) events can be estimated for magnetic storms, SCs, and substorms as approximately 750 nT (1100 nT), 230 nT (450 nT), and 5000 nT (6200 nT), respectively. The possible origin to cause the statistical distributions are also discussed, consulting the other space weather phenomena such as solar flares, coronal mass ejections, and solar energetic particles.

Keywords

Magnetic storms, sudden commencements, substorms, solar flares, coronal mass
Main Text

1. Introduction

It is important to understand the characteristics and possible amplitudes of extreme events of substorms, sudden commencements (SCs), and magnetic storms to mitigate the space weather hazard, especially from geomagnetically induced currents (Kataoka and Ngwira, 2016). However, it is still hard to predict the amplitude of unprecedented extreme events by physics-based simulations, and the statistical analysis is necessary to estimate the quantitative amplitude of possible extreme events.

One of the extreme geomagnetic activity events was observed associated with an episodic solar flare on 1 September 1859 (Carrington, 1958), which has been considered as a measure of extreme events. A low-latitude magnetometer measured ~1600 nT spike during the magnetic storm on 2 September 1859 (Tsurutani et al., 2003). Siscoe et al. (2006) estimated the 1-hour averaged value as $-\text{Dst} = 850$ nT, which is well below the
theoretical upper limit of $-\text{Dst} = 2500 \text{ nT}$ (Vasyliunas, 2011). From the statistical comparison among several space weather phenomena of magnetic storms, solar flares, and coronal mass ejections, Riley (2012) estimated a probability of 12% to have another Carrington event in coming 10 years. Kataoka (2013) applied the same analysis to the 90-year list of magnetic storms and estimated the probability of another Carrington storm in 10 years as 4–6%.

It is possible that we will have extreme magnetic storms even larger than the Carrington storm in future. For example, from the detailed analysis of an auroral painting from Kyoto, Japan, Kataoka and Iwahashi (2017) estimated that the amplitude of a historical magnetic storm occurred in September 1770 can be comparable to or even larger than the Carrington storm. As the latest example, the amplitude of a magnetic storm in May 1921 was estimated to be comparable to the Carrington event (Hapgood, 2019; Love et al., 2019), suggesting that the Carrington-class events may be more frequent than previously expected.
Another useful concept to define the extreme events is so-called “one in 100-year event”. Tsubouchi and Omura (2007) applied an extreme value theory to estimate the amplitude of the 100-year event as \(-\text{Dst} = 550-750 \text{ nT}\). More recently, Love et al. (2015) estimated the 100-year event as \(-\text{Dst} = 880 \text{ nT}\), i.e. the Carrington event corresponds to the 100-year event. The best efforts have been repeatedly conducted to estimate 100-year event and even 1000-year event by carefully extrapolating the tail distributions, although such results are highly uncertain especially from the limited data set of the Dst index for only a half century (Love, 2020).

In this study, acknowledging an advantage of Japan’s long-term monitoring effort of geomagnetic activities at Kakioka Magnetic Observatory (KAK), their compiled event lists of magnetic storms and SCs are used for the statistics to estimate the possible amplitudes of the 100-year and 1000-year events. Using the KAK lists, the possible power-law distributions of the amplitude of magnetic storms as well as SCs were discussed by Minamoto et al. (2015).
The similar statistical analysis can also be applied for substorms as well. It has been discussed for a long time that the amplitude of substorms basically follows log-normal distributions (e.g., Liou et al., 2018). From the statistical analysis, Nakamura et al. (2015) estimated the possible maximum amplitude of substorms as \( \text{AE} = \sim 4100 \text{ nT} \).

The purpose of this paper is to estimate the possible amplitude of extreme events of magnetic storms, SCs, and substorms from those statistical distributions. The event lists used in this study are explained in detail in Section 2. The method of analysis how to fit the log-normal and power-law distributions to the data set is described in Section 3.

Obtained results are provided in Section 4. The possible origins to cause the log-normal and power-law distributions are discussed in Section 5, by consulting the statistical distributions of the other space weather phenomena such as solar flares, coronal mass ejections (CMEs), and solar proton events (SEPs). Finally, concluding remarks are described in Section 6.

2. Data set
The event lists of magnetic storms and SCs are available at the KAK website (https://www.kakioka-jma.go.jp/obsdata/metadata/ja/products/list/event/kak). The lists are manually accumulated everyday by the professional operators at KAK. The occurrence properties of the identified magnetic storms and SCs are displayed in Figures 1 and 2. Although the lists essentially include the local variation, the long-term 96-year data is very unique to investigate the 100-year and 1000-year extreme events. Further, in the view-point of space weather countermeasure against the extreme events, the local enhancement itself is also of great interest to mitigate the possible maximum hazard.

The standard data for measuring the amplitude of substorms are the AE index. However, the AE index becomes unreliable when large substorms occurred at lower magnetic latitude than that of the AE stations located at high latitude of 65-70 deg. Recently, the SME index was developed from globally distributed magnetometers ranging from 40 deg to 80 deg magnetic latitude (Gjerloev, 2012) to provide a better replacement to evaluate the amplitude of such a large substorm events. The substorm
list was also provided from SuperMAG website (http://supermag.jhuapl.edu/). The data set used in this study is 34-year data from January 1986 when the number of SME stations was large enough (>30 stations) to better identify extreme events. A total of \(~6\times10^4\) substorm events were identified in the list for the 34-year time interval. The substorm amplitude of each event was calculated as the 15-min mean value of the SME index starting 10 min after the substorm onset (Newell and Gjerloev, 2011). The basic occurrence property and the amplitude distribution of the SME index was documented by Newell and Gjerloev (2011).

In this paper, some more space weather event lists are consulted to discuss the possible origins of the statistical distributions. The event list of solar flares is available at NOAA’s website (https://www.ngdc.noaa.gov/stp/solar/solarflares.html). The event list of CMEs is available at NASA’s website (https://cdaw.gsfc.nasa.gov/CME_list/). The list of SEP events is available at NOAA and NASA (https://umbra.nascom.nasa.gov/SEP/).
3. Data set

In general, interactions among many elements or their non-linearity bring out a new characteristic from the complex network. In the complex system, rare events tend to follow a power-law distribution. The power-law distribution of the event amplitude $x$ is defined as

$$f(x) = A x^{-\alpha},$$

where $\alpha$ denotes the spectral index and $A$ is a constant. A useful way to investigate the rare events is a cumulative distribution function (CDF) which is defined as

$$N(x) = N_T \int_{x}^{\infty} f(x')dx' = \frac{N_T A}{\alpha - 1} x^{-\alpha+1}.$$  

We use the maximum likelihood estimation (Riley 2012; Kataoka 2013) to obtain the slope as

$$\alpha - 1 = N_T \left\{ \sum_{i=1}^{N_c} \ln\left( \frac{x_i}{x_{\text{min}}} \right) \right\}^{-1},$$

where the $x_{\text{min}}$ is the minimum value to be used for the fitting.

Rare events always come arise from the majority, and the majority usually follows a log-normal distribution in a complex system. The log-normal distribution is defined as
where $\mu$ is the geometric mean value and $\sigma$ is the standard deviation. The CDF of the log-normal distribution can be written as

$$N(x) = N_T \int_x^\infty f(x')dx' = \frac{N_T}{2} \left[1 - \text{erf} \left(\frac{\ln x - \mu}{\sqrt{2} \sigma}\right)\right],$$

where $N_T$ is the total number of events and the error function is

$$\text{erf}(x) = \frac{2}{\sqrt\pi} \int_x^\infty \exp(-y^2)dy.$$}

A standard method of minimum variance fitting (scipy.optimize.curve_fit) is used in this study to find the best-fit CDF.

In order to estimate the possible amplitudes of the 100-year or 1000-year events from a limited data set of less than 100 years, both the log-normal CDF (equation 2) and power-law CDF (equations 5) are fitted to the data set. The time-stationarity of the distributions is then assumed to extend the limited time interval of the data set to 100 years and 1000 years. The log-normal distribution gives relatively conservative estimates, while the power-law distribution generally gives upper-limit estimates (Riley and Love, 2017).
4. Results

4.1. Magnetic storms

Figure 3 shows that the majority of magnetic storms roughly follows the log-normal distribution, and the rare population of \(-dH > 200\) nT follows the power-law. Note that the log-normal fit for above 200 nT level can be meaningful to give an estimate of the extreme amplitude because those large storms were caused by only CMEs, which is essentially different from another population of relatively weak magnetic storms associated with coronal holes. In other words, the excess from the log-normal distribution at relatively weak level can be interpreted to be the magnetic storms associated with corotating interaction regions (Richardson et al. 2005; Kataoka and Miyoshi, 2006).

The log-normal fit for \(>200\) nT storms gives the 100-year and 1000-year events as \(-dH = 750\) nT and 1100 nT, respectively. The power-law fit gives the 100-year and 1000-year events as \(dH = 1100\) nT and 2200 nT, respectively. The largest amplitude in the list
is $dH = 661 \text{ nT}$ that occurred on 24 March 1940. Note also that the record largest event of $dH > 700 \text{ nT}$ on 4 July 1941 at KAK was not used in this study because of its ambiguity. The largest events of $dH > 400 \text{ nT}$ are listed in Table 1. The 13 March 1989 storm is the largest Dst event since 1957 with the peak of final Dst index of $-589 \text{ nT}$.

Nagatsuma (2015) estimated the intensity of the southward $B_z = -50 \text{ nT}$ for the March 1989 storm, which is about the largest intensity of interplanetary magnetic field at 1 AU.

4.2. Sudden commencements

Figure 4 shows that SC events follow a log-normal distribution within the amplitude range from 5 nT to 70 nT, while the largest SC events deviate from the log-normal distribution and follow a power-law distribution. The log-normal fit gives the 100-year and 1000-year events as $dH = 160 \text{ nT}$ and $240 \text{ nT}$, respectively. The power-law fit gives the 100-year and 1000-year events as $dH = 230 \text{ nT}$ and $450 \text{ nT}$, respectively. The largest amplitude is $dH = 220 \text{ nT}$ that occurred on 13 November 1960. Note also that the record largest event of $dH = 310 \text{ nT}$ on 24 March 1940 was not in the KAK list.
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because it happened at the main phase of the magnetic storm (Araki 2014). The largest events of dH >100 nT are listed in Table 2.

4.3. Substorms

Figure 5 shows that the substorm events essentially follows the log-normal distribution. The log-normal fit gives the 100-year and 1000-year events as 5000 nT and 6000 nT, respectively. The power-law fit gives the 100-year and 1000-year events as 6200 nT and 8500 nT, respectively. The record largest amplitude is SME = 3929 nT that occurred on 30 October 2003 during the so-called Halloween event. The largest events of SME >3000 nT are listed in Table 3.

4.4 Solar flares, CMEs, and SEP

Consulting some other space weather phenomena, the same statistical analysis is applied to solar flares, CMEs, and SEP events. The results shown in Figures 6, 7, 8 basically follow the analysis of Gopalswamy (2018), and the only difference is the types
of distributions fitted to the CDF. Gopalswamy (2018) used the Weibull distribution (exponential curve) and power-law distribution, while this study uses the log-normal and power-law distributions to give somewhat larger estimates as follows.

The log-normal fit shown in Figure 6 gives the 100-year and 1000-year event sizes are X70 and X200, respectively. These values are larger than the estimates of Gopalswamy (2018) in which the 100-year and 1000-year event sizes are ~X40 and ~X100, respectively. The log-normal fit shown in Figure 7 gives the 100-year and 1000-year speed as 4500 km/s and 6000 km/s, while Gopalswamy (2018) gives the 100-year and 1000-year event as 3800 km/s and 4700 km/s, respectively. The log-normal fit shown in Figure 8 gives the 100-year and 1000-year events as ~2.5×10^5 pfu and ~1.5×10^6 pfu, while the extrapolated curve of Gopalswamy shows the 100-year and 1000-year events as ~2×10^5 pfu and ~1×10^6 pfu, respectively.

5. Discussions

In summary, conservative amplitudes can be estimated from the log-normal distribution,
while the possible excess (likely upper limit to the behavior of the tail) can be estimated from the power-law distribution. The possible amplitudes of 100-year and 1000-year events based on the log-normal and power-law distributions are summarized in Table 4.

The possible origins of the statistical distributions are briefly discussed as follows. The power-law fitting is likely meaningful for rare events, and there are possible reasons to cause the excess from log-normal distribution at the tail. For SCs, the main cause of the excess of rare events from log-normal distribution are spikes (Araki, 1997; 2014), which can be interpreted as the amplification of the preliminary impulse phase due to the velocity-jump effect over the density-jump effect, based on the parameter survey of a global magnetohydrodynamic (MHD) simulation (Kubota et al. 2015). Although it has been well known that the amplitudes of SC are proportional to the change of dynamic pressure, density-jump effect dominates the change of dynamic pressure for weak SCs, while velocity-jump effect dominates for large SCs. The rapidly changing large amplitude spike appears when the shock downstream speed becomes high (Kubota et al., 2015). In addition, we must admit that there are a few more missing extreme events
from the statistics. For example, an extreme SC event on 24 March 1940 (Araki, 2014) is not included in the KAK list, because it occurred during the main phase of magnetic storms. This particular example reminds us of the complex interaction among multiple CMEs and ambient solar wind to enhance the geo-effectivity (e.g., Kataoka et al., 2015; Shiota and Kataoka, 2016) which may also additionally contribute to the further excess from the standard log-normal distribution.

For magnetic storms, unusual spikes may also cause the excess, although the physics and time-scale are totally different from SCs of course. There is an example that a huge spike of >1600 nT appeared in the Carrington storm, in which additional ionospheric current or field-aligned current may locally contribute (Akasofu and Kamide, 2005). Even if the spikes are not originated from the ring current, it does matter to prepare against the possible hazards. Those spikes may also contribute to make an excess from the log-normal extrapolation at the tail.

It was clearly demonstrated that substorms of the Earth’s magnetosphere essentially
follow the log-normal distribution (Figure 5). On the other end of the solar-terrestrial system, solar flares and CMEs resemble the plasma explosions against substorms. Here we discuss whether there are similarities among their statistical distributions.

It is interesting to note here that the majority of solar flares ranging from C-class to X10-class follows a power-law distribution rather than a log-normal distribution, although C1 class and the tail beyond X10 flares show deviations (Figure 6). The fewer samples of C1 class flares may be the missing counts when the background level is comparable to C1 flares during highly active conditions. Note also that we have only a few samples for >X10 flares in the last 40 years. The power-law distribution may be interpreted, considering a difference between the Earth’s magnetosphere and sunspots’ magnetosphere, i.e. the active region. Active regions have a large variation of the spatial scales ranging multi orders of magnitude, and the fractal reconnection patterns naturally arise in the scale-free MHD system, in contrast to the only-one magnetospheric system of the Earth. In other words, if significant limitations in the scale-free system exist, log-normal distribution may clearly appear.
Figures 7 and 8 show that CMEs and SEP events essentially follow the log-normal distributions. For SEP events, an additional factor of a complex interplanetary propagation of energetic particles may broaden the distribution from the standard log-normal distribution. These different distributions from solar flares can be originated from the fact that only selective active regions can launch CMEs against the strong solar gravitation, and only selective CMEs can launch SEPs.

In a simplified view, geomagnetic activities are the product of the interaction between the solar wind and magnetosphere. The solar wind parameters follow log-normal distributions (Burlaga and Lazarus, 2000; Burlaga, 2001), while the Earth’s magnetic moment does not essentially change in short time. This is one of the reasons that majority of the geomagnetic activities follows the log-normal distribution rather than the power-law distribution. In other words, if the magnetic moment changes rapidly and follows the log-normal distribution like sunspots, the occurrence of substorms may follow a power-law distribution for a certain range. That can be tested by a global MHD
Recently, it became possible to continuously run the global MHD simulation of the magnetosphere for more than several months, using the observed solar wind data as input to reproduce a number of substorms. For example, the occurrence properties of the simulated substorms was statistically compared against the observed one for a whole month in January 2015 (Haiducek et al., 2017; 2020). Future works should also include the similar direction with different simulation code to examine the difference of the statistical distributions.

6. Conclusions

It was found that the amplitudes of magnetic storms, SCs, and substorms essentially follow the log-normal distributions, with the rare events showing a possible excess from the log-normal distributions, which follow the power-law distributions. This is interpreted as a natural consequence as a random output from a complex system. Based on both the log-normal and power-law distributions, the amplitudes of the 100-year
(1000-year) events can be estimated for magnetic storms, SCs, substorms as approximately 750 nT (1100 nT), 230 nT (450 nT), and 5000 nT (6200 nT), respectively.

**Declarations**

**Ethics approval and consent to participate**

Not applicable.

**Consent for publication**

Not applicable.

**List of abbreviations**

CDF: Cumulative distribution function

CME: Coronal mass ejection

KAK: Kakioka Magnetic Observatory

MHD: Magnetohydrodynamics

SC: Sudden commencement

SEP: Solar energetic particle
Availability of data and materials

The event lists of magnetic storms and SCs are available at the website of Kakioka Magnetic Observatory (https://www.kakioka-jma.go.jp/obsdata/metadata/en/products). Substorm list and SME index data is available at SuperMAG website (http://supermag.jhuapl.edu/).

The event list of solar flares is provided from NOAA/NGDC (https://www.ngdc.noaa.gov/stp/solar/solarflares.html). The event list of CMEs is provided from NASA/GSFC (https://cdaw.gsfc.nasa.gov/CME_list/). The event list of SEP is provided from NOAA/SWPC (ftp://ftp.swpc.noaa.gov/pub/indices/SPE.txt).

Competing interests

No competing interests

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Authors' contributions
RK conducted the research and wrote the manuscript.

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**Figure legends**

Figure 1: Magnetic storm events observed at Kakioka Magnetic Observatory for 96-year time interval from May 1924 to May 2020. The histogram is also shown at right-hand side.

Figure 2: Sudden commencement events observed at Kakioka Magnetic Observatory for 96-year time interval from May 1924 to May 2020. The histogram is also shown at right-hand side.

Figure 3: Cumulative distribution function of 1921 magnetic storms observed at KAK for time interval May 1924 to May 2020. Log-normal and power-law fits are shown by
thick and thin solid curves, respectively.

475 Figure 4: Cumulative distribution function of 1854 SCs observed at KAK for time
interval May 1924 to May 2020. Log-normal and power-law fits are shown by thick and
thin solid curves, respectively.

479 Figure 5: Cumulative distribution function of ~6×10^4 substorms identified by
SuperMAG for time interval 1986 to 2019. Log-normal and power-law fits are shown
by thick and thin solid curves, respectively.

483 Figure 6: Cumulative distribution function of flares for time interval 1975 to 2016. Log-
normal and power-law fits are shown by thick and thin solid curves, respectively.

486 Figure 7: Cumulative distribution function of CMEs for time interval 1996 to 2020.
Log-normal and power-law fits are shown by thick and thin solid curves, respectively.
Figure 8: Cumulative distribution function of SEP events at GOES for time interval 1976 to 2020. Log-normal and power-law fits are shown by thick and thin solid curves, respectively.