Simulations of Baryon Oscillations

Eric Huff¹, A.E. Schulz¹, Martin White¹,²
David J. Schlegel², Michael S. Warren³

¹Departments of Physics and Astronomy,
University of California, Berkeley, CA 94720
²Lawrence Berkeley National Laboratory,
1 Cyclotron Road, Berkeley, CA
³Theoretical Astrophysics Division, Los Alamos National Laboratory

Abstract

The coupling of photons and baryons by Thomson scattering in the early universe imprints features in both the Cosmic Microwave Background (CMB) and matter power spectra. The former have been used to constrain a host of cosmological parameters, the latter have the potential to strongly constrain the expansion history of the universe and dark energy. Key to this program is the means to localize the primordial features in observations of galaxy spectra which necessarily involve galaxy bias, non-linear evolution and redshift space distortions. We present calculations, based on mock catalogs produced from high-resolution N-body simulations, which show the range of behaviors we might expect of galaxies in the real universe. We investigate physically motivated fitting forms which include the effects of non-linearity, galaxy bias and redshift space distortions and discuss methods for analysis of upcoming data. In agreement with earlier work, we find that a survey of several Gpc³ would constrain the sound horizon at $z \sim 1$ to about 1%.

1 Introduction

Recently four groups (1), using data from the Sloan Digital Sky Survey⁶, published evidence for features in the matter power spectrum on scales of

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¹ ehuff@astro.berkeley.edu
² aschulz@astro.berkeley.edu
³ mwhite@berkeley.edu
⁴ djschlegel@lbl.gov
⁵ msw@lanl.gov
⁶ http://www.sdss.org/
100 Mpc. These features, long predicted, hold the promise of another route to understanding the expansion history of the universe and the influence of dark energy (2).

Oscillations in the baryon-photon fluid at \( z \sim 10^3 \) lead to a series of almost harmonic peaks in the matter power spectrum, or a bump in the correlation function, arising from a preferred scale in the universe: the sound horizon. (A description of the physics leading to the features can be found in (3) or Appendix A of (4); a comparison of the Fourier and configuration space pictures is presented in (5).) It was pointed out in Refs. (6; 7) that this scale could be used as a standard ruler to constrain the distance-redshift relation, the expansion of the universe and dark energy. Numerous authors (8) have now observed that a high-\( z \) galaxy survey\(^7\) covering upwards of several hundred square degrees could place interesting constraints on dark energy. Key to realizing this is the ability to accurately predict the physical scale at which the oscillations appear in the power spectrum plus the means to localize those primordial features in observations of galaxy spectra which necessarily involve galaxy bias, non-linear evolution and redshift space distortions. The former problem seems well in hand (11; 12). Preliminary investigations of the latter problem were presented in Refs. (10; 13; 14; 15; 16). We continue these investigations in this paper using a large set of high resolution N-body simulations.

The outline of the paper is as follows: §2 describes our N-body simulations and the construction of the mock galaxy catalogs using halo model methods. It also presents some basic properties of the galaxy clustering. §3 introduces the models for the 2-point function that we consider in this paper. §4 introduces a new configuration space band-power statistic which we believe is useful for BAO work while §5 introduces our fitting methodology. Our primary results are described in §6. Some preliminary investigations of the reconstruction method of (17) are described in §7 and our conclusions are presented in §8.

2 Simulations

We need an “event generator” which can be used to develop methods for going from observations of galaxies to cosmology. Ideally this tool would encode many of the complications we expect in real observations but be based on a known cosmology. To this end we use a series of simulations of a \( \Lambda \)CDM cosmology (\( \Omega_M = 0.3 = 1 - \Omega_A, \ \Omega_B = 0.046, \ h = 0.7, \ n = 1 \) and \( \sigma_8 = 0.9 \)). The linear theory mass spectrum was computed by evolution of the coupled Einstein, fluid and Boltzmann equations using the code described in (18).

\(^7\) It is even possible that such oscillations could be seen in the Ly-\( \alpha \) forest (9) or in very large cluster surveys (10).
comparison of this code to CMBfast (19) is given in (11).) For this model the sound horizon \( s = 143 \text{ Mpc} \) or \( 100.2 \, h^{-1}\text{Mpc} \).

For tests in which we need large numbers of runs (i.e. computing covariance matrices) we use mock catalogs based on Gaussian density fields. We first generate a Gaussian density field with the desired power spectrum (in our case the linear theory spectrum) on a \( 512^3 \) grid in a box of side \( 1.1 \, h^{-1}\text{Gpc} \). A ‘galaxy’ is then placed at random in the cell with a probability proportional to \( \exp[3\nu - 0.3\nu^2] \) for \( \nu \geq 0 \) and \( \exp[3\nu] \) for \( \nu < 0 \), where \( \nu = \delta/\sigma \) is the peak height. Similar techniques are used to construct mock galaxy redshift surveys in (20). The non-linear mapping of the Gaussian density field mocks up the action of gravity, inducing extra power on small scales and correlating different scales in Fourier space. The resulting ‘galaxy’ field has a power spectrum with roughly constant bias on large scales and excess power on small scales, though the form does not match in detail the more realistic catalogs produced with the halos found in N-body simulations.

A more realistic catalog can be produced using N-body simulations which have enough spatial and force resolution to resolve the halos hosting the galaxies of interest for BAO surveys. The basis for these calculations is a sequence of high force resolution N-body simulations in a periodic, cubical box of side \( 1.1 \, h^{-1}\text{Gpc} \). These simulations were carried out with the HOT (21) and TreePM (22) codes. In all we ran 3 simulations, with different randomly generated Gaussian initial conditions, which evolved \( 1024^3 \) particles of mass \( 10^{11} \, h^{-1}\text{M}\odot \) from \( z = 34 \) to \( z = 1 \). The Plummer softening was \( 35 \, h^{-1}\text{kpc} \) (comoving).

The simulations were chosen to be the largest we could practically run several realizations of, while retaining sufficient force resolution to resolve the halos likely to host the galaxies of interest. This allowed us to simulate a volume comparable to that of proposed future surveys (23) at \( z = 1 \). While observational campaigns could also study baryon oscillations at higher redshift (e.g. \( z = 3 \)) going to earlier times becomes increasingly difficult for simulations. The volume for a given survey area increases and the characteristic mass scale of halos decreases to earlier times, making the required dynamic range infeasible for direct simulation at present. Thus we will focus our attention here on \( z = 1 \).

For each output we generate a catalog of halos using the Friends-of-Friends

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\(^{8}\) We caution the reader that the definition of the sound horizon, like that of the epoch of last scattering, is one of convention. Unfortunately several conventions exist in the literature. Along with fitting formulae of limited accuracy this makes it difficult to compare quoted numbers at the percent level. We define \( s \) as the integral of the sound speed up to the redshift where \( \tau = 1 \), excluding the contribution from \( z < z_{\text{reion}} \).
Fig. 1. (Upper) The cumulative mass function of halos from the 3 simulations at $z = 1$. We define $M$ as $1.03 \times$ the sum of the masses of the particles in the FoF group. The last point plotted in each case is the mass of the largest halo in the box. The solid line shows the fit of Ref. (25) while the dotted line shows the results with $a = 0.8$ and $p = 0.25$ as discussed in the text. The horizontal dashed lines show the range of number densities of most interest for this work. (Lower) The ratio of the N-body results to the fit with $a = 0.8$ and $p = 0.25$.

algorithm (24) with a linking length of $b = 0.175$ in units of the mean inter-particle spacing. This procedure partitions the particles into equivalence classes, by linking together all particle pairs separated by less than a distance $b$. The groups correspond roughly to all particles above a density of about $3 / (2 \pi b^3) \approx 90$ times the background density and we keep all groups with more than 16 particles. Increasing the “friends-of-friends” mass of the groups by a few percent gives a good match to the analytic mass function of Ref. (25). However we find that even with this increase the $z = 1$ results are better fit if we change the parameters in the analytic mass function ($p$ which controls
\[ \bar{n} = 10^{-3.0} h^3 \text{Mpc}^{-3} \]

\[ \log_{10} M_{\text{min}} \quad \log_{10} M_1 \quad \langle b \rangle \quad \log_{10}\langle M \rangle \]

\begin{array}{cccc}
12.8 & 13.0 & 2.4 & 13.6 \\
12.7 & 13.5 & 1.9 & 13.3 \\
12.6 & 14.0 & 1.8 & 13.1 \\
12.6 & 14.5 & 1.7 & 13.1 \\
\end{array}

\[ \bar{n} = 10^{-3.5} h^3 \text{Mpc}^{-3} \]

\[ \log_{10} M_{\text{min}} \quad \log_{10} M_1 \quad \langle b \rangle \quad \log_{10}\langle M \rangle \]

\begin{array}{cccc}
13.3 & 13.0 & 3.1 & 13.9 \\
13.1 & 13.5 & 2.5 & 13.7 \\
13.0 & 14.0 & 2.3 & 13.5 \\
13.0 & 14.5 & 2.1 & 13.4 \\
\end{array}

Table 1
The HOD parameters of Eq. 1 for some of our catalogs at \( z = 1 \). Masses are in units of \( h^{-1} M_\odot \). Within each set the catalogs have a fixed space density, \( \bar{n} \). The large-scale bias, \( \langle b \rangle \), and the galaxy weighted mean halo mass, \( \langle M \rangle \), are also listed.

We make mock catalogs using two different procedures. First we apply a simple threshold mass to the group catalogs, taking our tracers to be the central galaxies of halos above the mass threshold. To allow the inclusion of satellites we choose a mean occupancy of halos: \( N(M) \equiv \langle N_{\text{gal}}(M_{\text{halo}}) \rangle \). Each halo either hosts a central galaxy or does not. For each halo we define a galaxy to live at the minimum of the halo potential with probability \( p = \min[1, N(M)] \).

The central galaxy inherits the velocity of the halo, which we take to be the average velocity of the halo particles weighted by the square of the potential. This weighting emphasizes particles near the halo center and allows the central galaxy to move with respect to the center-of-mass (com) of the halo, but the difference between the com velocity and the central galaxy velocity is typically only a few tens of \( \text{km s}^{-1} \). Following Ref. (26), if \( N(M) > 1 \) the mean number of satellites, \( N_{\text{sat}} = N(M) - 1 \), is computed for the halo and a Poisson random number, \( n_{\text{sat}} \), drawn. Then \( n_{\text{sat}} \) dark matter particles, chosen at random, are anointed as galaxies. Our fiducial model thus has the satellite galaxies tracing the dark matter in the halo. The galaxy velocity is taken to be the particle velocity, thus the satellites have no velocity bias. However since the central galaxy is nearly at rest with respect to the halo, the population as a whole has a different velocity field than the dark matter. The characteristic mass, \( M_* \), for our models at \( z = 1 \) is \( 2 \times 10^{11} h^{-1} M_\odot \). As all of our tracer galaxies live in halos more massive than \( M_* \) they have biases greater than 1.

We follow (14) and choose a simple two-parameter form for \( N(M) \):

\[ N(M) = \Theta(M - M_{\text{min}}) \frac{(M - M_{\text{min}}) + M_1}{M_1} \]  

where \( \Theta(x) \) is the Heaviside step function. If we take \( M_1 \to \infty \) our catalog
\[ n = 10^{-3.0} h^3 \text{Mpc}^{-3} \]

| $\log_{10} M_{\text{min}}$ | $\log_{10} M_1$ | $\langle b \rangle$ | $\log_{10} \langle M \rangle$ |
|--------------------------|--------|---------|-----------------|
| 12.8                     | 13.0   | 2.0     | 13.3            |
| 12.7                     | 13.5   | 1.9     | 13.2            |
| 12.6                     | 14.0   | 1.8     | 13.2            |
| 12.6                     | 14.5   | 1.8     | 13.1            |

| $\log_{10} M_{\text{min}}$ | $\log_{10} M_1$ | $\langle b \rangle$ | $\log_{10} \langle M \rangle$ |
|--------------------------|--------|---------|-----------------|
| 13.2                     | 13.0   | 2.6     | 13.7            |
| 13.1                     | 13.5   | 2.4     | 13.6            |
| 13.0                     | 14.0   | 2.3     | 13.5            |
| 13.0                     | 14.5   | 2.2     | 13.4            |

Table 2

The HOD parameters used for $z = 1$ catalogs with $n_{\text{sat}} \propto M^{1/2}$. Units and labels are as in Table 1.

reduces to the catalog of halos more massive than $M_{\text{min}}$. By holding $n$ fixed we can specify a 1-parameter sequence of models with varying $M_{\text{min}}$, large-scale bias, $\langle b \rangle$, or galaxy weighted mean halo mass $\langle M \rangle$ (see Table 1). In comparing the models with different HODs it is important to remember that we hold $n$ fixed within each sequence, so variations in mean halo mass, satellite fraction, bias etc are highly correlated. To test the dependence on the slope of the satellite contribution we also ran some models where $n_{\text{sat}} \propto M^{1/2}$. The parameters of these models are listed in Table 2. Both theoretical (27) and observational (28) results suggest that at higher redshift $M_{\text{min}} \approx M_1$. These models have the larger biases, mean halo masses and satellite fractions.

While the galaxy models above are not prescriptive, or likely even close to “right”, they are physically well motivated, easy to adjust and lead to galaxy catalogs with non-linear, scale-dependent, stochastic biasing and redshift space distortions – many of the complications we will face in observations of the universe.

The statistics of counts in cubical cells allow us to infer the stochasticity of the bias and the degree to which the 1-point distribution of the galaxies is Poisson. We find that the galaxy-mass cross-correlation coefficient (29), $r$, rises from $0.6 - 0.7$ (depending on sample) on scales of $1 h^{-1}\text{Mpc}$ to $> 95\%$ on scales larger than $10 h^{-1}\text{Mpc}$. The variance of the counts divided by their mean, which would be unity for a Poisson distribution, exceeds one on scales $1 - 30 h^{-1}\text{Mpc}$ with the largest value ($\approx 20$) on the largest scales. This excess is easily understood as extra power coming from large-scale clustering of the galaxies. On Mpc scales the value is very close to Poisson for the less biased galaxies and close to 2 for the more biased samples.
3 BAO models

The cosmological signal that interests us is primarily contained in the two-point statistics of the galaxy density field, and we shall concentrate on these statistics henceforth. We begin by considering measurements in real space, such as would be relevant to photometric or 2D surveys, and then include redshift space distortions which are relevant for spectroscopic surveys.

3.1 Real space

There are now several models in the literature relating the (non-linear) galaxy power spectrum to the (linear) dark matter power spectrum. We have critically compared these to the power spectra and correlation functions measured in our mock surveys. To our knowledge the performance of these models in matching the shape of the power spectra and correlation function has not been compared to mock catalogs produced with a wide range of HOD schemes. In particular we find that the correlation function is a very discriminating statistic, because it is sensitive to translinear scales. As we discuss in §6 some of the models do not match the clustering of our mock galaxy samples, particularly for highly biased or rare samples.

We now describe the five models we’ve investigated in this paper. The simplest is the linear bias model, which is often motivated by arguments like those presented in (30). The linear bias model asserts that

\[ \Delta_{\text{gal}}^2(k) = b^2 \Delta_{\text{lin}}^2(\alpha k) \] (2)

where \( \Delta^2(k) \) denotes the dimensionless power spectrum, or variance per \( \ln k \):

\[ \Delta^2(k) \equiv \frac{k^3 P(k)}{2\pi^2} \] (3)

The parameter \( b \) in Eq. 2 is the large scale galaxy bias and \( \Delta_{\text{lin}}^2(k) \) is the linear dark matter power spectrum. In this study we have introduced the parameter \( \alpha \), which scales the wave number in Eqs. 2, 5, 6 and 7, to parameterize small changes in the cosmology that result in a stretch in the baryon signature. We introduce this parameter in order to study potential degeneracies between the other model parameters, which depend on the HOD, and the inferred cosmology. When \( \alpha \neq 1 \) the inferred length scale differs from the true length scale, leading to an incorrect estimate of the sound horizon and hence the constraints on dark energy. We will use biases and errors on \( \alpha \) as an indicator of how well the sound horizon can be measured. To translate the error in \( \alpha \) into an error on dark energy parameters we need to make further assumptions.
As a rough rule of thumb: if we assume a constant equation of state, \( w \), for the dark energy the fractional error in \( w \) is five times that in \( \alpha \).

It has been observed several times in the literature (14; 15; 31) that the large-scale bias may not be constant at the 1% level. Halo bias (32) will have a small scale dependence even for weakly non-linear scales. The distribution of galaxies within dark matter halos and halo exclusion effects also lead to small changes in large-scale power. The shifting of galaxy positions on \( \sim 10 \, h^{-1}\text{Mpc} \) scales leads to a smearing of the amplitude of the oscillations in the power spectrum. Several attempts have been made to model these effects.

In (33) a method was introduced to empirically fit the scale dependence, using the form

\[
\Delta^2(k) = \Delta^2_{\text{ref}}(k) \left[ 1 + A k \exp \left\{ - \left( \frac{k}{k_s} \right)^{1.4} \right\} \sin \left( \frac{2\pi k}{k_A} \right) \right]
\]  

Here \( A \) and \( k_A \) are fit parameters in the decaying sinusoid used to characterize the baryon oscillations, \( k_s \equiv 0.1 \, h \, \text{Mpc}^{-1} \) is a constant and \( \Delta^2_{\text{ref}}(k) \) is a reference spectrum including the effects of Silk damping but excluding the oscillations. The reference spectrum is from the fitting formula in (35). Since we are fitting non-linear power spectra we additionally allow a linear ramp in power when doing our fits. This approximates the broad-band power removal suggested by (33) without correlating the errors. The fits do prefer a positive slope to this extra factor. In Eq. (4) a shift in \( k_A \) corresponds to a shift in the sound horizon, so \( k_A \) replaces the \( \alpha \) in our previous expression. While the true spectrum cannot be accurately fit by Eq. (4), if we concentrate around the second peak \( k_A \approx 0.058 \, h \, \text{Mpc}^{-1} \) provides a good fit to the peak position for our input model. An alternative definition, used by (33), is \( k_A = 2\pi/s \approx 0.063 \, h \, \text{Mpc}^{-1} \) a difference of about 10%. If we use the fitting function, Eq. (26), of (34) for \( s \) instead we find \( k_A \approx 0.060 \, h \, \text{Mpc}^{-1} \); midway between the former two values. The latter approximation comes closest to our best fit \( k_A \) (see below) so we shall use that – but we note again the uncertainty in quoted values of \( s \) in the literature. In our fitting we shall assume that \( \Delta_{\text{ref}} \) is known, and use the correct cosmological parameters for our runs.

A recent analysis of the clustering of Luminous Red Galaxies (LRGs) in the Sloan Digital Sky Survey (SDSS) instead used the fitting function (36; 37)

\[
\Delta^2(k) = b^2 \Delta^2_{\text{lin}}(k) \frac{1 + Qk^2}{1 + ak} \quad \text{for } k < 0.5 \, h \, \text{Mpc}^{-1}
\]  

In this description, the parameter \( Q \sim 10 \) governs the scale dependence of the bias and \( a = 1.7 \, h^{-1}\text{Mpc} \) is a constant (\( a \) becomes \( 1.4 \, h^{-1}\text{Mpc} \) in redshift space). To test for shifts in the sound horizon we again replace \( k \) with \( \alpha k \). Note that for small \( k \) this model looks like the quadratic, multiplicative bias
model used in (14).

In (38), motivated by analytic arguments using the halo model, the scale dependence was modeled by adding a term to the expression in Eq. 2 proportional to $k^3$. In (15) a similar treatment was used for slightly different reasons, involving $k^3$ times a quadratic in $k$. For $k \ll 1$ the leading order term will dominate and the two expressions are similar. We consider

$$\Delta_{\text{gal}}^2(k) = b^2 \Delta_{\text{lin}}^2(\alpha k) e^{-\frac{(\alpha k/k_2)^2}{2}} + \left(\frac{\alpha k}{k_1}\right)^3 \Delta_{\text{ref}}^2(\alpha k).$$  \hspace{1cm} (6)$$

The parameters $b$ and $k_1$ are the galaxy weighted large-scale halo bias and the amplitude of the 1-halo term, and the parameter $k_2$ governs a suppression due to halo profiles and exclusion. We introduce $k_2$ because very little clustering power on small scales is due to galaxies in separate halos. It is most necessary for those models with low $M_1$, i.e. many satellite galaxies, the strongest 1-halo terms, the largest mean halo mass, but the shape of the transition between the 2- and 1-halo terms is not well constrained. We will consider a different transition below. Since the baryon oscillation signal is present in $\Delta_{\text{lin}}^2(k)$, it is clear that the one halo term, which is a description of the impact of non-linear physics, decreases the contrast of the oscillations. Because the parameters $b$, $k_1$, and $k_2$ all depend on the HOD, the contrast of the oscillations will depend somewhat on the type of galaxies being selected in the BAO survey, and on the mean redshift of the survey.

Another way of viewing the effects of non-linearities and galaxy bias is found in (5). That analysis seeks to describe the gradual erasure of the acoustic peak signature by considering the distribution of Lagrangian displacements of galaxies. The authors of (5) showed that to simultaneously model the smearing due to galaxy displacement, and also the correct level of small scale power, it is useful to add back the broad-band power that is filtered out by the exponential suppression in Eq. 6. The fitting form from (35) is again used for this purpose leading to

$$\Delta_{\text{gal}}^2(k) = b^2 \Delta_{\text{lin}}^2(\alpha k) e^{-\frac{(\alpha k/k_2)^2}{2}} + \left(\frac{\alpha k}{k_1}\right)^3 + \left(1 - e^{-\frac{(\alpha k/k_2)^2}{2}}\right) b^2 \Delta_{\text{ref}}^2(\alpha k).$$  \hspace{1cm} (7)$$

The role of $k_2$ in this form differs from its role in Eq. 6 in that here it controls the erosion of the baryon wiggles due to galactic motions on $\sim 10\, h^{-1}\text{Mpc}$ scales, while preserving the overall shape of the two halo portion of the power spectrum in the trans-linear regime.

### 3.2 Redshift space

Little work has been done on extending these models to redshift space. In redshift space the power is enhanced, by a $z$-dependent factor, on large scales
due to supercluster infall (39) and suppressed on small scales due to virial motions within halos. To include the dependence on the angle with respect to the line of sight, \( \mu \equiv \cos \theta = \hat{k} \cdot \hat{r} \), we decompose \( \Delta^2(\vec{k}) \) into Legendre polynomials, \( P_\ell(\mu) \), as

\[
\Delta^2(\vec{k}, \mu) \equiv \sum_{\ell=0}^{\infty} \Delta^2_\ell(k) P_\ell(\mu)
\]

so

\[
\Delta^2_\ell(k) = \frac{2\ell + 1}{2} \int_{-1}^{+1} d\mu \; \Delta^2(\vec{k}, \mu) P_\ell(\mu)
\]

Symmetry along the line-of-sight implies that the coefficients for odd \( \ell \) modes vanish.

On very large scales (\( k \to 0 \)) supercluster infall modifies the power spectrum as (39; 40)

\[
\Delta^2_{\text{red}}(k, \mu) = \Delta^2_{\text{real}}(k) \left( 1 + \beta \mu^2 \right)^2,
\]

where \( \mu \equiv \hat{k} \cdot \hat{r} \) and \( \beta \equiv f(\Omega)/b \simeq \Omega^{0.6}/b \) assuming a scale-independent bias. Here \( f \) is the dimensionless linear growth rate of the growing mode in linear perturbation theory which can be approximated as \( \Omega^{0.6} \) (41). The coefficients of the first two multipole moments are thus

\[
\Delta^2_0(k) = \left( 1 + \frac{2}{3} \beta + \frac{1}{5} \beta^2 \right) \Delta^2_{\text{real}}(k)
\]

and

\[
\Delta^2_2(k) = \left( \frac{4}{3} \beta + \frac{4}{7} \beta^2 \right) \Delta^2_{\text{real}}(k).
\]

To capture the smaller-scale angular dependence one often adds a high-\( k \) cut-off; popular choices include Lorentzian, Gaussian or exponential, e.g.

\[
\Delta^2_{\text{red}}(k, \mu) = \Delta^2_{\text{real}}(k) \left( 1 + \beta \mu^2 \right)^2 e^{-k\sigma|\mu|}
\]

We will refer to these modifications collectively as “streaming models” (40). In general \( \beta \) and \( \sigma \) are regarded simply as parameters to be fit to the data. The analytic expressions for the resulting multipole moments are straightforward but lengthy and will not be reproduced here.

A more empirical model for the angular dependence is explored in (42), who found the quadrupole-to-monopole ratio in N-body simulations can be fit by

\[
\frac{\Delta^2_2}{\Delta^2_0} = \frac{\frac{4}{3} \beta + \frac{4}{7} \beta^2}{1 + \frac{2}{3} \beta + \frac{1}{5} \beta^2} \left[ 1 - \left( \frac{k}{k_{\text{rs}}} \right)^{1.22} \right],
\]

9 This statement assumes a \( k \)-space picture. In configuration space, on large scales, overdensities are the sites of convergent flows, so redshift space distortions ‘sharpen’ structure. The correlation function thus falls off more quickly along the line-of-sight than transverse to it.
where \( k_{rs} \) is a free parameter analogous to the \( \sigma \) parameter in streaming models. Though the authors of (42) do not discuss the shape of the small-scale downturn in \( \Delta^2_\ell(k) \), an additional parameter needs to be introduced if we are to predict the run of \( \Delta^2_\ell(k) \) with \( k \). Further discussion of the interplay between the large-scale enhancement and small-scale suppression can be found in (43; 44).

In §6.3, we will investigate the accuracy with which each of these forms reproduces the large- and intermediate-scale \( (k \leq 0.2 \, h \, \text{Mpc}^{-1}) \) angular dependence of the power spectrum in redshift space. Our approach also allows us to examine the effects of the HOD on the redshift-space distortions, with the hope of using this additional information to reduce degeneracies between cosmology and galaxy physics. We do not consider in this paper how modeling of the anisotropic clustering allows us to constrain the line-of-sight and transverse distance scales separately.

### 4 A configuration space band-power estimator

For each of the forms in Eqs. 2-7 there is a corresponding model for the correlation function \( \xi(r) \); the probability, in excess of random, of finding a pair of tracers at separation \( r \). The correlation function is related to the dimensionless power spectrum as

\[
\xi(r) = \int \frac{dk}{k} \Delta^2(k) j_0(kr) \simeq \int \frac{dk}{k} \Delta^2(k) \left[ 1 - \frac{(kr)^2}{6} + \cdots \right]
\]

where \( j_0 \) is the zeroth spherical Bessel function and the series expansion is valid for \( kr \ll 1 \). Most of the scale dependence in the galaxy bias can be traced to the fact that galaxies and dark matter transition from one to two halo dominance at disparate scales (38). In Fourier space this is spread over a range of \( k \), but in configuration space the 1-halo term is confined to scales much smaller than the scales of interest to us. Beyond \( 2 - 3 \, h^{-1}\text{Mpc} \) more than 99.9% of the contribution to \( \xi(r) \) comes from the 2-halo term for all of our models. For this reason we expect less scale dependence in the bias in configuration space than in Fourier space (see also (16)), as shown in Fig. 2.

Formally the correlation function, \( \xi(r) \), and the power spectrum, \( P(k) \), are a Fourier transform pair. However the simulation volume is a periodic cube and our signal has support only for \( k \)-modes which are integer multiples of the fundamental mode in each dimension. Because of this restriction the correlation function computed in the box differs from the true continuum correlation function on scales approaching the box size. The modulation in power is non-trivial even on 100 Mpc scales where we would like to work. Fortunately the
Fig. 2. An estimate of the scale-dependence of the bias in configuration space. The different symbols are $\Delta \xi$ (see text) for different HOD models (from Table 1) each divided by a constant bias to match near $100 \, h^{-1}$Mpc. The degree to which the shapes match indicates how well each can be considered a constant times the dark matter correlation function. The lower panel shows the residuals from one of the models, taken as a template.

We choose to proceed by computing a quantity containing the same information as $\xi(r)$ but which is less sensitive to the low-$k$ modes. Specifically we compute

$$\Delta \xi(r) \equiv \bar{\xi}(< r) - \xi(r) = \frac{3}{r^3} \int_0^r x^2 \, dx \, \xi(x) - \xi(r)$$

for which

$$\Delta \xi(r) = \int \frac{dk}{k} \, \Delta^2(k) \, j_2(kr) \simeq \int \frac{dk}{k} \, \Delta^2(k) \left[ \frac{(kr)^2}{15} - \frac{(kr)^4}{210} + \cdots \right]$$

where $j_2$ is the spherical Bessel function of order 2. We have effectively formed a band-pass filter in $k$-space; the slowly varying, long-wavelength contribution is significantly down-weighted while retaining the sensitivity to $100$ Mpc scales. Only 3% of the weight in the window function comes from scales below $kr = 1$, so if we include the fact that $\Delta^2(k)$ falls steeply as $k \to 0$ we see that modes
near the fundamental contribute little to $\Delta \xi(r)$. We believe that this configuration space band-power measurement is of considerable value in analyzing the acoustic oscillation signal. Computing $\Delta \xi(r)$ is straightforward once $\xi(r)$ has been computed, either in simulations or from galaxy survey volumes. In surveys of the sky, the correlation function is computed by assuming a mean density given by the ratio of total object number to the survey volume. This necessarily drives the observed correlation function to zero at survey sized scales; a difficulty referred to as the integral constraint. The quantity $\Delta \xi(r)$ is inherently less sensitive to the integral problem, and is thus observationally useful apart from comparison to theory via simulations. A comparison of the relative robustness of $\xi(r)$ and $\Delta \xi(r)$ is presented in §6.

As an aside we note that further low-$k$ suppression can be obtained from a linear combination of $\xi(r)$, $\bar{\xi}(< r)$ and $\bar{\bar{\xi}}(< r)$ with

$$\bar{\bar{\xi}}(< r) \equiv \frac{5}{r^3} \int_0^r x^4 dx \xi(x) = \frac{5}{2} \left[ \bar{\xi}(< r) - \frac{3}{r^3} \int_0^r x^4 dx \xi(x) \right]$$

for which the window function is $j_4(x)$, going as $x^4$ for $x \ll 1$. The generalization to higher orders is straightforward but will not be needed here.

We do not expect any of the models of $\Delta^2(k)$ to be accurate on small scales. Since $\bar{\xi}(< r)$ is a cumulant, using the quantity $\Delta \xi(r)$ in place of $\xi(r)$ has traded uncertainty at large scales for uncertainty at small scales. However, there is a distinct advantage because we know the functional form of the change in $\Delta \xi(r)$ at large separations due to any modulation of small scale power. Assuming that the models of $\xi(r)$ are accurate at large $r$ we can write the change in $\Delta \xi(r)$ from inaccurately modeled (or measured) small scale power as an additive term

$$\Delta \xi(r) = \Delta \xi_{\text{model}}(r) + \frac{A}{r^3} \quad \text{with} \quad A \equiv 3 \int_0^r r^2 dr \left[ \xi(r') - \xi_{\text{model}}(r') \right].$$

For example, if the extra power were pure shotnoise, $(k/k_1)^3$, it would give $A/r^3 = (3\pi/2)(k_1 r)^{-3}$. No matter what the small scale physics is doing to the true value of $\xi(r)$, the integral quickly asymptotes to a constant value for all separations greater than $O(10 h^{-1}\text{Mpc})$, for which the models are a good fit. It is useful to know the functional form of the modification; it allows us to marginalize over the parameter $A$ when connecting the observed galaxy correlation function to the HOD through $b$ and $k_2$, and to the cosmology, through $\alpha$.

Finally we point out that the generalization of $\Delta \xi(r)$ to redshift space is straightforward and gives the same kernel for the quadrupole as the monopole allowing easy interpretation of their ratio. Writing the redshift space coordi-
nate as $s$ with the cosine of the angle to the line of sight as $\mu$ we can define

$$\xi(s, \mu) \equiv \sum_{\ell=0}^{\infty} \xi_{\ell}(s) P_{\ell}(\mu)$$  \hspace{1cm} (20)

where $P_{\ell}(\mu)$ is a Legendre polynomials of order $\ell$ and

$$\xi_{\ell}(s) = i^\ell \int \frac{dk}{k} \Delta_{\ell}(k) j_{\ell}(ks)$$  \hspace{1cm} (21)

Note that the sign of the quadrupole term in the correlation function is opposite that in the power spectrum. Again we can reduce the dependence of $\xi(s, \mu)$ on low-$k$ modes by defining

$$\Delta \xi(s, \mu) \equiv \bar{\xi}(< s) - \xi(s, \mu)$$  \hspace{1cm} (22)

which leads to a replacement of the $j_0$ in the monopole expression with $j_2$ as in Eq. (17). Both $\Delta \xi_0$ and $\Delta \xi_2$ then probe the same $k$-modes. The hexadecapole can be isolated by using $\xi$, $\bar{\xi}$ and $\tilde{\xi}$ as in Eq. (18).

5 Methodology

5.1 Fourier space

To compute the galaxy power spectrum for the periodic, constant time, slices we first assign our tracers to the nearest grid point of a regular, periodic, $256^3$ Cartesian mesh and Fourier transform the resultant density field\(^\text{10}\). The resulting $P(k)$ is corrected for the assignment to the grid using the appropriate window function and the Poisson shot noise is subtracted. The power is averaged in bins spaced linearly in $k$. The average $P(k)$ is plotted at the position of the average $k$ in each bin. We obtain approximate error bars both by counting the number of modes in each bin and by dividing the data into 8 disjoint octants and using the octant-to-octant variance from the 3 different simulations – for a total of 24 sub-samples. The latter method underestimates the errors on large scales, while the former neglects the mode coupling which occurs on small scales, the extent of which depends on the galaxy sample under consideration. When computing the power spectrum of each octant we set the remaining 7 octants to zero and correct for the windowing factor of 8 in power and the modified shot noise when doing the FFT. For HODs with $M_1 \gg M_{\text{min}}$ we find that mode counting and sub-sample variance agree very

\(^{10}\text{We get the same results by using Cloud-in-Cell interpolation onto a 512}^3\text{ grid.}\)
well with
\[
\frac{\delta P}{P} = \sqrt{\frac{2}{N_{\text{mode}}}} \left(1 + \frac{1}{nP(k)}\right).
\]  
(23)

The higher order shot-noise terms (59) are subdominant for the range of scales of interest to us. As \( M_1 \) approaches \( M_{\text{min}} \) there is excess variance on small scales compared to the mode counting predictions. This is not unexpected. For models with \( M_1 \simeq M_{\text{min}} \) the galaxy power spectrum goes non-linear at relatively low \( k \), with the extra power being transferred from larger scales by gravitational collapse. The number of modes driving the variance is thus smaller than one would estimate by mode counting at the measured \( k \).

From the 8 octants for each of 3 simulations we are able to compute the covariance matrix of \( P(k) \). We find that for our chosen bin size the bins are almost independent at the scales of interest for the oscillations. Once the data become significantly non-linear mode coupling induces a correlation. For \( k < 0.2 \, h \, \text{Mpc}^{-1} \) the correlation coefficient is below 5%, reaching 10% by \( k \simeq 0.25 \, h \, \text{Mpc}^{-1} \). The degree of correlation between adjacent bins in Fourier space is, however, a function of the HOD. The more highly biased catalogs have more significant correlations between adjacent bins on larger scales. For catalogs with a bias of \( b = 3.5 \) for example, adjacent bins have correlations exceeding 0.2 for \( k > 0.7 \, h \, \text{Mpc}^{-1} \). However, since the correlations are so small in the \( k \)-range of interest we use Eq. (23) in the fits.

We compute our power spectra in redshift space assuming the distant observer approximation for all outputs and use the periodicity of the simulation to remap positions. For the isotropically averaged spectra the power ratios do not exactly recover the results of Ref. (39) on large scales. Whether we should expect to reach the those limits on the scales relevant to baryon oscillations remains in doubt – see Ref. (45) and references therein for further discussion. On small scales we are able to compute the \( \Delta^2 \ell \) by direct summation on the Cartesian \( k \) grid, however on large scales we do not have enough modes. For this reason we perform a least squares fit for the \( \Delta^2 \ell \) up to \( \ell = 6 \) for each of the \( k \) bins. The line-of-sight angular dependence of the power spectrum on large scales is simple, as expected: the resulting Legendre coefficients above \( \ell = 4 \) are small for \( k \leq 0.3 \, h \, \text{Mpc}^{-1} \).

5.2 Configuration space

We can compute \( \xi(r) \) either by directly counting pairs as a function of separation in the periodic volume, using FFT techniques in the periodic volume, or counting pairs and using the Landy & Szalay (46) estimator
\[
\xi(r) = \frac{\langle DD \rangle - 2\langle DR \rangle + \langle RR \rangle}{\langle RR \rangle} \quad \text{pairs in } (r;dr) \quad (24)
\]
in sub-volumes with vacuum boundary conditions – here $D$ refers to the data, $R$ refers to a random catalog with the same selection and the angled brackets indicate the number of pairs in a given shell ($r; dr$). A comparison of the different techniques is shown in Fig. 3. For the FFT method we use a Fourier grid with $512^3$ points. Both CIC and NGP charge assignment (47) give the same answer for $\xi(r)$ well above the grid scale. For the Landy & Szalay method we used a random catalog with $10 \times$ as many points as the data – increasing the number did not alter the results – but in computing $\langle RR \rangle$ we only searched for pairs around the first $N_{\text{data}}$ random points. Since the $\langle DR \rangle$ point is limited to $N_{\text{data}}$ points there is no loss in accuracy from limiting the $\langle RR \rangle$ term similarly but the cost scales as $N_{\text{data}} \times N_{\text{random}}$ rather than $N_{\text{random}}^2$. In what follows we shall use the direct pair counting estimate of the correlation function, in bins of width $3 \, h^{-1}\text{Mpc}$. We note in passing that estimating $\xi$ from future surveys will be non-trivial as we wish to work at very large scales with fine radial resolution.

Formally $\bar{\xi}(< r)$ can be estimated in a similar manner to $\xi(r)$ except using counts within spheres of radius $r$ rather than shells at $r$ of radius $dr$. Unfortunately this method is prone to strong boundary effects (in the case of the sub-volumes with vacuum boundary conditions) because the simulation volume available to each shell in the sphere is a strong function of radius. Computing $\xi$ in shells and then integrating up to find $\bar{\xi}$ is much more stable, and we do this for all of our estimators. We tried several interpolation schemes and the results were insensitive to our choice. We use linear interpolation between the measured $\xi$ points in the results below.
Fig. 4. The fractional error in $\xi(r)$ (thin, upper) and $\Delta \xi(r)$ (thick, lower) computed in a smaller volume compared to the quantities in the full simulation. In each case we divided each simulation into $2^3$ (solid), $3^3$ (dashed) or $4^3$ (dotted) sub-cubes and computed $\xi(r)$ or $\Delta \xi(r)$ using the estimator of (46) (see text). The curves display the deviation from the average computed with periodic boundary conditions in the full simulation.

We expect $\Delta \xi(r)$ to be less susceptible to finite volume effects than $\xi(r)$. To verify this we divided each simulation into $2^3$, $3^3$ or $4^3$ sub-cubes of side $L/2$, $L/3$ or $L/4$ and computed $\xi(r)$ and $\Delta \xi(r)$ using the estimator of (46). We average the estimates over sub-cubes and simulations and compare them in Fig. 4 to the average result computed using direct counting in the full box with periodic boundary conditions for each simulation. At larger $r$ the results become noisy, however it is clear that while both $\xi(r)$ and $\Delta \xi(r)$ are insensitive to the sub-division at small $r$, $\Delta \xi(r)$ suffers far less than $\xi(r)$ from small volume effects at larger $r$. For reference the analytically predicted error on $\Delta \xi$, averaged over all 3 simulations for this model, is 4-6% over the range $100-150 \, h^{-1}\text{Mpc}$, close to the difference seen in Fig. 4 at higher $r$.

The different estimators of $\xi(r)$ differ at small radius, and for this reason our results for $\Delta \xi(r)$ can differ by an $r^{-3}$ term at large $r$. As discussed earlier we also expect our models of $\Delta^2(k)$ to misestimate $\Delta \xi(r)$ by a term $A/r^3$. For this reason we shall drop the 1-halo contribution in $\Delta^2(k)$ in favor of an
additive $r^{-3}$ term in $\Delta \xi$ when doing the fits. This term also soaks up any error in computing $\Delta \xi$ arising from uncertainties in our estimate of $\xi(r)$ on small scales. We shall marginalize over constant $A$ when fitting our theories to the observed $\Delta \xi(r)$ points.

The correlation function errors are highly correlated between different scales. Although we can estimate $\text{Cov} [\Delta \xi(r_1), \Delta \xi(r_2)]$ from sub-divisions of the full volume we found that the covariances are so strong, and the numerically estimated covariance matrix so noisy, that the results are unstable. For this reason we compute the covariance analytically, making several approximations. First we assume that the small-scale fluctuations contributing to $A$ are independent of the large-scale fluctuations contributing to $\Delta \xi(r)$ at large $r$. For the large scale modes we assume Gaussian errors on $P(k)$ with different $k$-modes being independent. Then it is straightforward to show that

$$\text{Cov} [\Delta \xi(r_1), \Delta \xi(r_2)] = \frac{(2\pi)^2}{V} \int \frac{dk}{k^4} \Delta^4(k) j_2(k r_1) j_2(k r_2) + \frac{\sigma_A^2}{r_1^3 r_2^3} + \text{s.n.}$$

where $V$ is the volume of the survey, in our case the simulation volume, $\sigma_A^2$ is the variance of $A$ and s.n. indicates shot-noise terms. Note that the first term scales as $V^{-1}$, indicating that all of the errors on $\xi$ tend to zero as $V \to \infty$ but remain highly correlated. This is to be contrasted with the errors on the power spectrum, which remain $O(1)$ for each $k$-mode but are uncorrelated between increasingly finely spaced $k$-modes as $V \to \infty$. The shot-noise term proportional to $1/\bar{n}$ is

$$\frac{4}{V \bar{n}} \int \frac{dk}{k} \Delta^2(k) j_2(k r_1) j_2(k r_2)$$

For $r_1 > 0$ and $r_2 > 0$ the $1/\bar{n}^2$ shot-noise terms are

$$\frac{1}{2\pi V \bar{n}^2} \left[ \frac{3}{r_1^3} \Delta \xi(r_2) + \frac{3}{r_2^3} \Delta \xi(r_1) + \frac{3}{r_>^3} \Delta \xi(r_) + \frac{1 + \xi(r_1)}{r_1^2} \delta(r_1 - r_2) \right]$$

where $r_<$ the lesser of $r_1$ and $r_2$ and $r_>$ is the greater. The $\delta$-function in the last term is rendered finite when we estimate $\xi$ in bins of finite width. For small bins we can replace $\delta(r_1 - r_2)$ with the inverse of the bin width. Since $\xi \ll 1$ at large scales this gives $r_1^{-2} \Delta r^{-1}$ along the diagonal. Finally the $1/\bar{n}^3$ term is simply

$$\left( \frac{1}{4\pi} \right)^2 \frac{1}{V \bar{n}^3} \frac{3}{r_1^3} \frac{3}{r_2^3}$$

While we show in §6 that making this Gaussian assumption does not bias the results for $\alpha$, in future it would be better to use a large set of mock catalogs to estimate the covariance. From our limited numbers of realizations, and using the non-linearly processed Gaussian fields, we found that the shape and amplitude of the analytic expression were within $O(25\%)$ of the numerical results.
Fig. 5. Contours of $\xi(r_p, \pi)$ (left) and $\Delta \xi(r_p, \pi)$ (right) for one of our $b \simeq 2$ catalogs. Contours are equally spaced in log, and dotted lines indicate negative values. Here $r_p$ measures separations across the line-of-sight and $\pi$ along it.

for scales near $100\, h^{-1}$Mpc. The agreement at smaller scales was considerably worse.

In configuration space there are no peculiar issues with estimating the $\mu$ dependence of $\xi(s, \mu)$ on the scales of interest. We bin $\xi(s, \mu)$ in 15 bins in $|\mu|$ and then sum to get $\xi_\ell(s)$. As was found for $P(k)$, the modes beyond the quadrupole are small at large $s$. Performing a least squares fit to $\ell = 0, 2$ and 4 yields the same result to better than a percent for $\ell = 0, 2$ and a few percent for the very small $\ell = 4$ mode. We find in general that $\Delta \xi$ is much closer to spherical than $\xi$ due to the large contribution from $\bar{\xi}(s)$. For example from $75\, h^{-1}$Mpc to $125\, h^{-1}$Mpc, $\Delta \xi_2/\Delta \xi_0$ falls from approximately 0.05 to $\simeq 0.02$ for our fiducial $b \simeq 2$ catalog. The contours of both $\xi$ and $\Delta \xi$ are shown in Figure 5.

6 Results

6.1 Dark matter

We begin by presenting\textsuperscript{11} the power spectrum of the dark matter, in real space, at the present epoch ($z = 0$) from one of the simulations: Fig. 6. Also on the plot we show the results of two ansätze for non-linear spectra, that of Peacock & Dodds (51) based on an idea by Hamilton et al. (52), and another based on halo model ideas (53). The former is seen to be a bad approximation.

\textsuperscript{11}We thank Wayne Hu for suggesting we make this point explicitly.
Fig. 6. The real space power spectrum of the dark matter at $z = 0$ for one of our simulations along with two ansätze commonly used in the literature (see text). The lower panel shows the ratio of the fits and N-body points to the smooth spectrum of (35).

as it implicitly assumes that there exists a 1−1 mapping between linear and non-linear power. While not an issue for smoothly varying spectra, this causes problems when the spectrum contains features such as the baryon oscillations. In reality mode coupling erases features, whereas the mapping procedure enhances them. We could reduce some of the discrepancy by using a broad band measure of the slope in the fitting function, but the underlying problem still remains. The halo-model based methods perform better in this regard, as expected (54), since they model the non-linear power with an integral over the linear theory power spectrum. None of the fitting formulae approach percent level accuracy in the non-linear regime.

6.2 Galaxies

Now we turn to the mock galaxy catalogs. We show the results at $z = 1$ for one of our HOD prescriptions, with $\bar{n} = 10^{-3} h^3 \text{Mpc}^3$ and $b \approx 2$, in Fig. 7 along with the predictions of linear theory multiplied by $b^2$. The power is biased
Fig. 7. The real space power spectrum of one of our mock galaxy catalogs with $b \simeq 2$ at $z = 1$. The solid line shows the predictions of linear theory, multiplied by $b^2$. on large scales and shows a clear excess on small scales. We shall now try to fit this behavior using the models described in §3. Our results for the sound horizon parameter $\alpha$ are reported in Table 3.

We have tried several methods for performing these fits. We fit to both $P(k)$ and $\Delta \xi(r)$. For the power spectrum we use errors from Eq. (23), since they agree with the errors estimated from the dispersion among the octants. For the correlation function we use the analytic expression of §5. The multidimensional fitting was done using the Levenberg-Marquardt algorithm (60). We experimented with several implementations and found good convergence with both analytic and numerically computed derivatives. From these fits we also obtain an estimate of the parameter covariance matrix from the curvature of the likelihood around the best fit. To test the Gaussianity of the likelihood surface we also ran Markov-Chain Monte-Carlo fits (see e.g. (61) for an introduction) for the power spectra for each model. We provide comparisons of each of these methods for the different models below.

We begin with the linear bias model (Eq. 2) fit to $P(k)$ over the range $0.02 < k < k_{\text{max}} \simeq 0.15 \, h\, \text{Mpc}^{-1}$. We impose the lower $k$ cutoff so that there are enough independent modes in the bin that the Rayleigh distribution is close
to symmetric. In performing this and subsequent fits we convolve the known $k$-space sampling with the model at each bin to sample the theory in the same manner as the data, although this has only a small effect on our results. We found that the strength of the small-scale excess is a strong function of the HOD parameters\textsuperscript{12}. The sense is as expected – larger small-scale excess for models with $M_1$ closer to $M_{\text{min}}$ or steeper power-law slopes for the satellite term in $N(M)$. When $M_1 \approx M_{\text{min}}$ multiple galaxies reside in a single halo, enhancing small-scale power. Because the 1- and 2-halo terms shift by different amounts in going from dark matter to galaxies, the bias is both larger and more scale-dependent (38) with the galaxy 1-halo term dominating at larger characteristic scales than it does in the dark matter. When $M_1 \gg M_{\text{min}}$ the HOD has a long ‘plateau’, with many halos containing only a single galaxy, and the galaxy bias is smaller and less scale-dependent (recall we hold $\bar{n}$ fixed in our models). For the former class of HODs, the 1-halo term can become significant at a level of interest to BAO surveys at $k$ comparable to or even below the standard choice for $k_{\text{max}}$. This results in biases in the sound horizon of up to 10\%, many times the formal fitting error. Reducing $k_{\text{max}}$ further to compensate for these cases is problematic because – as has been pointed out before (8) – the information content of a BAO experiment is very sensitive to this cutoff. Thus safely fitting the power spectrum in this way requires throwing away a significant amount of useful information. To get around this one must accurately model the smaller scale or 1-halo effects.

For the catalogs with $\bar{n} = 10^{-3} h^3 \text{Mpc}^{-3}$ and the models of Eqs. (5-7) we find we can safely fit up to $k_{\text{max}} \approx 0.3 h \text{Mpc}^{-1}$ before we notice a bias coming from incorrectly modeled small-scale physics. We show the best fit $\alpha$, and the fit error, as a function of $k_{\text{max}}$ for several models in Fig. 8. Note that the points are not independent because the information content is cumulative in $k_{\text{max}}$. Beyond the range plotted the bias in $\alpha$ becomes significant. The fitting form of Eq. (4) does not fare as well. To be conservative in what follows we choose $k_{\text{max}} = 0.15 h \text{Mpc}^{-1}$ for the fits to Eq. (4) and $k_{\text{max}} = 0.25 h \text{Mpc}^{-1}$ for the other models. Beyond this $k_{\text{max}}$ our assumption of Gaussian uncorrelated errors becomes increasingly suspect.

For the lower-density catalogs with $\bar{n} = 10^{-3.5} h^3 \text{Mpc}^{-3}$, we find similar results when fitting the same models. Adopting a small-scale cutoff of $k_{\text{max}} = 0.25 h \text{Mpc}^{-1}$ does not tend to bias the fitting results. Significant differences between the fit results at differing densities only begin to appear when dealing with highly biased populations.

Fitting the form of Eq. (4) to the data in the range $0.02 \leq k \leq 0.15 h \text{Mpc}^{-1}$ and assuming the ‘correct’ $\Delta_{\text{ref}}$ we find the best fitting $k_A$. Dividing the ‘true’

\textsuperscript{12}In principle the form of the HOD for the sample being selected can be fit from the smaller scale clustering data that we are not analyzing here.
Fig. 8. The sound horizon vs. $k_{\text{max}}$ for two different HODs: (left) $\log_{10} M_{\text{min}} = 12.8$ and $\log_{10} M_1 = 13.0$ and (right) $\log_{10} M_{\text{min}} = 12.7$ and $\log_{10} M_1 = 13.5$. Open squares are Eq. (5), solid triangles Eq. (6) and open circles Eq. (7). Points have been offset horizontally for clarity.

$k_A \simeq 0.06 \, h\text{Mpc}^{-1}$ by the best fit we find $\alpha \simeq 1.00$ for essentially all of our HOD models, with a mild trend for higher $\alpha$ (about $1\sigma \approx 1\%$) for the less biased samples with the smaller satellite fractions. If we extend $k_{\text{max}}$ beyond $0.15 \, h\text{Mpc}^{-1}$ we find $\alpha$ increases and becomes inconsistent with unity. Using all 3 of our runs, for a total survey volume of $3.8 \times 10^9 \, h^{-3}\text{Gpc}^3$, we are able to constrain $\alpha$ to $\mathcal{O}(1\%)$ for our catalogs – consistent with the expectations of simple error propagation.

The model of Eq. (5) provides a reasonable fit to the data over the relevant $k$ range: $0.02 \leq k \leq 0.25 \, h\text{Mpc}^{-1}$. The Markov chains converge very rapidly for this model, and the best fits for $b$ and $\alpha$ were insensitive to the precise value of $a$ chosen. This suggests that the first few elements of a Taylor series expansion in the multiplicative bias would work as well. The marginalized distributions for $b$, $Q$ and $\alpha$ are reasonably well fit by Gaussians, with $\alpha = 0.996 \pm 0.005$ for our $b \simeq 2$ catalog for example. The best fit provided by the Levenberg-Marquardt algorithm for this model is $\alpha = 0.998 \pm 0.004$. Compared to the halo model forms (Eqs. 6, 7) the best fitting models with Eq. (5) have less intermediate scale power. To compensate, the value for $b$ tends to be a few percent higher ($b \approx 2.17$ for this model). This also leads to a slightly higher $\chi^2$ for the fit. Comparing the galaxy $P(k)$ to that of the dark matter we find $b(k)$ rises from 2.05 near the fundamental mode to 2.2 at $k = 0.2 \, h\text{Mpc}^{-1}$ and 2.3 at $k = 0.4 \, h\text{Mpc}^{-1}$. The best fit bias is thus $\mathcal{O}(10\%)$ higher than the $k \to 0$ value.

Figure 9 shows the residuals around the fit to $P(k)$ when using Eq. (6) and the catalog with $b \approx 2$. As is evident from the figure, Eq. (6) is a good representation of the real-space power spectrum. Comparisons between the fit results for different HODs show that $k_1$ (the 1-halo parameter) and $b$ (the large-scale bias) vary as described in (38) with little scatter. The Markov
chains show the marginalized $\alpha$ distribution is consistent with unity, within $1\sigma$, for all of the catalogs. We also show in Fig. 9 the best fit model using the $\Delta\xi(r)$ data for $80\hmpc < r < 140\hmpc$. The agreement between the $P(k)$ and $\Delta\xi(r)$ best fits is within $1\sigma$ for the bias, $k_2$ and the sound horizon. It is difficult to meaningfully compare the values of $k_1$, but the fit prefers a negligible 1-halo term for models with large $M_1$ as expected. For the model of Eq. (6) the predicted shape $\Delta\xi$ falls below the data for smaller $r$. For this reason the values of the parameters returned are sensitive to the range of $r$ chosen in the fit. In general the fits to $\Delta\xi$ have slightly worse $\chi^2$ values than for $P(k)$, but the shape of the $\chi^2$ surface for $\alpha$ is similar.

We show marginalized error contours (68 and 95% enclosed probability) for the 4 parameters of our $P(k)$ fit to the same catalog in Fig. 10. These regions are derived from our Markov chains; the error ellipses derived from the curvature of the likelihood at maximum are similar except for $k_1$ where the distribution is not well fit by a Gaussian. Except for the large-scale bias, $b$, the contours show little degeneracy between the cosmology, as parameterized by $\alpha$, and the properties of the galaxy sample. The $b-\alpha$ degeneracy is more pronounced for galaxy populations with larger 1-halo terms. As the small-scale power excess is increased, and the acoustic features are more washed out, reducing the sound horizon (increasing $\alpha$) becomes a way of shifting power from large scales to small, and is thus difficult to distinguish from a change in the amplitude of the 1-halo term. The MCMC derived marginalized likelihoods are close to Gaussian for $b, k_2$ and $\alpha$, but there is a significant tail to low $k_1$. 

Fig. 9. (Left) The real space power spectrum, $\Delta^2 (k)$, for one of our HOD models along with the halo model based fit of Eq. (6). The points are derived from the mock galaxy catalog, the dotted and dashed lines show the biased linear theory and shot-noise terms for the best fit with the solid line being their sum. The lower panel shows the ratio of the theory and points to the smooth spectrum of (35) along with the fit residuals. (Right) The correlation function, $\Delta\xi$, along with the best fit (solid) and the best fit to the power spectrum (dotted). Note the errors on $\Delta\xi$ have been computed analytically and so have been placed on the best fit curve rather than the data (stars). The errors are highly covariant.
Fig. 10. The marginalized errors in each of the fit parameters of Eq. (6) for one of our catalogs with $b \simeq 2$. Contours represent 68% and 95% enclosed probability.

| $\log_{10} M_{\text{min}}$ | $\log_{10} M_1$ | Eq. (5)       | Eq. (6)       | Eq. (7)       |
|-----------------------------|-----------------|---------------|---------------|---------------|
| 12.8                        | 13.0            | 1.003 ± 0.005 | 0.998 ± 0.007 | 0.999 ± 0.008 |
| 12.7                        | 13.5            | 0.996 ± 0.005 | 0.998 ± 0.006 | 1.000 ± 0.007 |
| 12.6                        | 14.5            | 0.991 ± 0.005 | 0.998 ± 0.006 | 0.999 ± 0.006 |

Table 3
Results of fitting our catalogs to the various functional forms for three typical choices of HOD parameters at $\bar{n} = 10^{-3.0} h^3 \text{Mpc}^{-3}$. We quote the parameters of a Gaussian fit to the marginalized $\alpha$ distribution from our Markov chains.

The form of Eq. (7) fits $\Delta \xi$ better to lower $r$ than Eq. (6). As for Eq. (6) there is a slight trend for HOD models with higher $M_1$ to have lower $\alpha$, but again the best fits are within 1% of unity for all of our catalogs. The marginalized parameter distributions from the MCMC run on the $P(k)$ data are close to Gaussian except for $k_1$ and $k_2$, with the latter poorly constrained for this model. For example, the marginalized distribution for $\alpha$ for the $b \simeq 2$ data plotted above is well fit by $\alpha = 1.000 \pm 0.007$.

Finally we compare in Fig. 11 the marginalized errors in $b$ and $\alpha$ from the Markov chain fits to the catalog in Fig. 7. The value of $b$ preferred by Eq. (5) is slightly high, as discussed above.
Fig. 11. The marginalized errors in $b$ and $\alpha$ computed from the Markov chains fit to our $b \simeq 2$ mock catalog for several of the models discussed above: Eq. (4) dot-dashed, Eq. (5) dotted, Eq. (6) dashed and Eq. (7) solid. Contours represent 68% and 95% enclosed probability.

6.3 Redshift space clustering

The results so far have been in real space, as appropriate for surveys measuring projected clustering statistics. We now turn to measures in redshift space. We begin by discussing the angle-averaged power spectrum and correlation function. The constraints on $\alpha$ should thus be interpreted as an average of the transverse and line-of-sight distances, or approximately a shift in $(D^2H^{-1})^{1/3}$.

Since the form of Eq. (4) did not perform well for the real space tests we begin by considering Eq. (5). In terms of the angle-averaged clustering pattern in redshift space we have found that for $k < 0.25\, h\text{Mpc}^{-1}$ the exponential, Lorentzian or Gaussian streaming models fit the small-scale downturn to sufficient accuracy for our purposes, though they each prefer different values of $\sigma$. The goodness of fit is best for the Gaussian suppression. If we multiply Eq. (5) by a Gaussian suppression and a $1 + 2\beta/3 + \beta^2/5$ prefactor (here $\beta \equiv \Omega^{0.6}/b$) we obtain good fits to most of the mock catalogs. Taking our $b \simeq 2$ catalog for example, the marginalized real space constraint is $\alpha = 0.996 \pm 0.005$ while

$^{13}$ We neglect the contribution of $\Delta_2^2$ to the covariance of $\Delta_0^2$. For our models, on the scales of relevance, this is a good approximation.
the redshift space constraint is $\alpha = 0.997 \pm 0.008$. For the bias we obtain $b = 2.17 \pm 0.01$ vs. $b = 2.14 \pm 0.02$ while the small-scale suppression is only poorly constrained. The difference in $b$ reflects the inapplicability of the large-scale enhancement factor. With this $k_{\text{max}}$ there is a very slight bias in $\alpha$ for the highest $b$ HOD model. The results are essentially unchanged if we use a Lorentzian suppression model, but if we use exponential suppression there is a 1% downward bias on $\alpha$ even for the $b \approx 2$ catalog and the goodness of fit is noticeably worse.

For the forms Eqs. (6, 7) we get good fits for either a Gaussian or exponential suppression. In each case the marginalized $\alpha$ distribution is consistent with unity with an uncertainty of just under 1%. The form of Eq. (7) does slightly better, as in the real space case.

Now let us consider the angle dependence of the clustering. Figure 5 shows the anisotropy in configuration space, i.e. for $\xi$ and $\Delta \xi$ for one of our models. Figure 12 shows the quadrupole-to-monopole ratio in Fourier space for a number of our HOD models with $\bar{n} = 10^{-3} h^3 \text{Mpc}^{-3}$. As can be seen in the figure, we find a strong HOD dependence to the redshift space clustering of galaxies. In the limiting case of halos-only (i.e. models with no satellite contribution) there is little small-scale suppression even on scales as small as $1.5 h \text{Mpc}^{-1}$.

Further insight into this behavior comes from considering the deviation between the true halo velocity in the simulation and that which would be predicted using linear theory from the (smoothed) density field in the simulation at $z = 1$ (see also (43; 62; 63; 64) for similar calculations). This is shown in Fig. 13 for two smoothing scales. When the density field is estimated from the dark matter particles in the simulation the rms deviation in each component of $v$ is between 50 and 80 km/s with smaller deviations for higher mass halos and smaller smoothing scales. If the velocity is estimated from the halo catalog (weighting all halos equally) then the rms rises to between 90 and 260 km/s. The increase in the rms comes from the neglect of the mass contributed by the lower mass halos, the fact that more massive halos tend to live in denser environments and the assumption of equal weights per halo (43; 63). The fact that the true velocities differ only slightly (the equivalent of a few Mpc) from those predicted by linear theory suggests we should see little small-scale suppression in the redshift space halo power spectrum on the scale of interest.

The other extreme case – the strongest small-scale deviation from the Kaiser approximation – occurs where the mean galaxy-weighted halo mass and the satellite fraction are at their highest. For these HOD models the galaxies preferentially sample the highest peculiar velocities. The other HODs occupy a continuum between these cases that is well explained by the fraction of satellite galaxies and the mean galaxy-weighted halo mass. Overplotted in Fig. 12 are the fits to the functional form derived from a streaming model.
Fig. 12. (Left) The quadrupole-to-monopole ratio vs. scale for a number of our models at $z = 1$ along with streaming model fits (see text). The streaming model tends to overestimate $Q/M$ at low-$k$ for the more biased models (lower panels). (Right) The same models with the fit of Ref. (42). In our catalogs the high $k$ suppression closely tracks the mean galaxy weighted halo mass (see text).

Fig. 13. The true halo velocity in the simulations compared to the linear theory prediction from a smoothed density field with Gaussian width $5 \, h^{-1}\text{Mpc}$ (left) or $10 \, h^{-1}\text{Mpc}$ (right). The thick lines show the histogram of velocity differences in each component when the density field is estimated from the halo positions, the thin lines when the density field is estimated directly from the dark matter particles in the simulation.

with exponential small-scale suppression. We also attempted to fit streaming models with Gaussian and Lorentzian cutoffs, but these were generally poorer fits to the data than the exponential cutoff. The streaming model fits shown are acceptable fits to the simulation quadrupole-to-monopole ratios only when the latter varies slowly with $k$; our more highly biased models (lower panels)
are not well-represented. In Fig. 12 we also compare our results with the fitting formula of Eq. (14). This is quite a good fit to the simulations over the range of interesting scales.

Additional constraints on power spectrum model parameters can be gleaned from the redshift-space distortions. One option is to use the linear theory parameter $\beta$ as a simultaneous constraint on the cosmology and the galaxy bias (recall the large-scale bias, $b$, was the parameter most degenerate with $\alpha$, see Fig. 10). We examined this possibility, and found that even on the largest scales in the simulations, the best fits to the multipole moments did not approach the linear theory value of $\beta$ with enough precision for this to be useful.

One property of the angular dependence of the redshift-space distortions that is well constrained is the scale of the quadrupole zero-crossing – represented by $k_{nl}$ in the empirical fit of (42), and by $\sigma$ in the streaming model. This quantity is in fact very tightly correlated with the choice of HOD parameters. In this sense, at least one independent constraint on the relevant galaxy physics derived from redshift-space distortions is fairly insensitive to the choice between these models.

We do not consider using the angular dependence of the redshift space distortions to fit separately for the line-of-sight and transverse distance scales. Before further considering redshift space distortions we want to include the (light-cone) evolution of clustering.

7 Reconstruction

The common lore is that surveys targeting galaxy populations with a large 1-halo term (small values of $k_1$), or surveys at low $z$ will have fewer ‘useful’ modes than high redshift surveys or surveys of objects which singly occupy their halos. Recent work (17) has suggested that it may be possible to partially mitigate these effects and reconstruct the baryon oscillation signal despite the corrosion due to non-linear collapse, even for surveys at low redshifts.

The original work of (17) did not make their measurements on ‘galaxy’ catalogs made by populating halos with a range of occupation distributions. We have cross-checked their method using some of our catalogs and found very consistent results. We show the results from one of our simulations and models in Fig. 14 in both configuration and Fourier space. We present the results here without redshift space distortions to better show the degree to which reconstruction gains signal. In agreement with (17) we find that the real space correlation function at low redshift is considerably sharpened by the recon-
Fig. 14. The power spectrum (left) and correlation function (right) for one of our boxes at 3 different redshifts, \( z = 0.3, 0.5 \) and 1.0, when using the reconstruction method of Ref. (17). For the correlation function we show both \( \xi(r) \) (lower curves) and \( \Delta \xi(r) \) (upper curves). For the power spectrum we have plotted the signal compared to the no-oscillation form of Ref. (35). The squares are the original signal, the triangles show a reconstruction using a smoothed field with \( R_s = 20 \, h^{-1}\text{Mpc} \) and the circles with \( R_s = 10 \, h^{-1}\text{Mpc} \).

Since the reconstruction procedure is inherently non-linear, we also tested whether it induces correlations between otherwise uncorrelated modes. To do this we used the 100 non-linearly processed Gaussian density fields described in §12. For each we computed the power spectrum before and after reconstruction and hence the covariance matrix. On the scales of relevance for the acoustic oscillations the procedure does not seem to introduce significant correlations. Further investigations of reconstruction will be deferred to a future publication.

8 Conclusions

The coupling of baryons and photons by Thomson scattering in the early universe leads to a rich structure in the power spectra of the CMB photons and the matter. The study of the former has revolutionized cosmology and allowed precise measurement of a host of important cosmological parameters. The study of the latter is still in its infancy, but holds the potential to constrain the nature of the dark energy believed to be causing the accelerated expansion of the universe.

Future large redshift surveys offer the opportunity to measure a characteristic scale in the universe: the sound horizon at the time of photon-baryon decon-
pling. This standard ruler, which we can calibrate from observations of the CMB, may allow us to tightly constrain the evolution of the scale factor and determine the nature of dark energy. To ensure the success of these efforts we need to improve our understanding of the theoretical underpinnings of the method and generate simulated universes which can be used to refine and test our observational strategies.

In this paper we have made a first attempt to go all the way from (mock) observations to constraints on the sound horizon for a number of galaxy catalogs which display non-linear, scale dependent and stochastic bias. We have performed fits in configuration and Fourier space for a number of models which have been proposed in the literature. We investigate the shape of the likelihood function, parameter degeneracies and the range of validity of the fits. We find that the forms of Eqs. (5-7) fare quite well and lead to unbiased estimates of the sound horizon in both real and redshift space. In agreement with earlier work (8), we find that a survey of several Gpc$^3$ would constrain the sound horizon at $z \sim 1$ to about 1%.

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References

[1] D.J. Eisenstein, et al., ApJ, 633, 560 (2005) [astro-ph/0501171]; G. Huitsi, A&A, 446, 43 [astro-ph/0505441]; G. Huitsi, A&A, 449, 891 [astro-ph/0512201]; N. Padmanabhan et al., preprint [astro-ph/0605302]; C. Blake et al., preprint [astro-ph/0605303].
[2] D.J. Eisenstein, New Astronomy Reviews, 49, 360 (2005).
[3] D.J. Eisenstein, W. Hu, J. Silk, A. Szalay, ApJ 494, L1 (1998).
[4] A. Meiksin, M. White, J.A. Peacock, MNRAS 304, 851 (1999).
[5] D.J. Eisenstein, H. Seo, M. White, ApJ, in press [astro-ph/0604361]
[6] A. Cooray, W. Hu, D. Huterer, M. Joffre, ApJ 557, L7 (2001).
[7] D.J. Eisenstein, in Wide-field multi-object spectroscopy, ASP Conference Series, ed. A. Dey., (2003).
[8] W. Hu, Z. Haiman, Phys. Rev. D68, 063004 (2003); C. Blake, K. Glazebrook, ApJ 594, 665 (2003); H.-J. Seo, D.J. Eisenstein, ApJ 598, 720 (2003); K. Glazebrook, C. Blake, Astrophys. J., 631, 1 (2005).
[9] M. White, in proceedings of the UC Davis meeting on Cosmic Inflation [astro-ph/0305474]
[10] R. Angulo, et al., MNRAS, 362, L25 (2005).
[11] U. Seljak, N. Sugiyama, M. White, M. Zaldarriaga, Phys. Rev. D68, 83507 (2003).
[12] D. Eisenstein, M. White, Phys. Rev. D70, 103523 (2004); M. White, New Astronomy Reviews, in press [astro-ph/0606643].
[13] V. Springel, et al., Nature 435, 629 (2005).
[14] M. White, Astroparticle Physics, 24, 334 (2005).
[15] H.-J. Seo, D.J. Eisenstein, Astrophys. J., 633, 575 (2005).
[16] J. Guzik, G. Bernstein, preprint [astro-ph/0605594]
[17] D.J. Eisenstein, H. Seo, E. Sirko, D. Spergel, preprint [astro-ph/0604362]
[18] M. White, D. Scott, ApJ 459, 415 (1995); W. Hu, D. Scott, N. Sugiyama, M. White, Phys. Rev. D52, 5498 (1995); W. Hu, M. White, Phys. Rev. D56, 596 (1997); W. Hu, U. Seljak, M. White, M. Zaldarriaga, Phys. Rev. D57, 3290 (1998).
[19] U. Seljak, M. Zaldarriaga, ApJ, 469, 437 (1996).
[20] S. Cole, S. Hatton, D.H. Weinberg, C.S. Frenk, MNRAS, 300, 945 (1998).
[21] M.S. Warren, J.K. Salmon, in Supercomputing ’93, 12, Los Alamos IEEE Comp. Soc., (1993).
[22] M. White, ApJS 143, 241 (2002).
[23] K. Glazebrook et al., White paper for the DETF [astro-ph/0507457]
[24] M. Davis, G. Efstathiou, C.S. Frenk, S.D.M. White, ApJ 292, 371 (1985).
[25] R.K. Sheth, G. Tormen, MNRAS 349, 1464 (2004).
[26] A.V. Kravtsov, et al., ApJ 609, 35 (2004).
[27] C. Conroy, R.H. Wechsler, A.V. Kravtsov, ApJ, in press [astro-ph/0512234]
[28] R. Yan, D. Madgwick, M. White, ApJ 598, 848 (2003); R. Yan, M. White, A. Coil, ApJ 607, 739 (2004).
[29] A. Dekel, O. Lahav, MNRAS 520, 24 (1999).
[30] R.J. Scherrer, D.H. Weinberg, ApJ 504, 607 (1998).
[31] A. Cooray, MNRAS 348, 250 (2004).
[32] G. Efstathiou et al., MNRAS 235, 715 (1988); S. Cole, N. Kaiser, MNRAS 237, 1127 (1989); H.J. Mo, S.D.M. White, MNRAS 282, 347 (1996).
[33] C. Blake, K. Glazebrook, ApJ 594, 665 (2003);
[34] D.J. Eisenstein, W. Hu, ApJ 496, 605 (1998).
[35] D.J. Eisenstein, W. Hu, ApJ 511, 5 (1999).
[36] N. Padmanabhan et al., preprint [astro-ph/0605302];
[37] S. Cole et al., MNRAS 362,505 (2005).
[38] A. Schulz, M. White, Astroparticle Physics, 25, 172 (2006). [astro-ph/0510100]
[39] N. Kaiser, MNRAS 227, 1 (1987).
[40] A.J.S. Hamilton, “Linear redshift distortions: A review”, in “The Evolving Universe”, ed. D. Hamilton, pp. 185-275 (Kluwer Academic, 1998) [astro-ph/9708102]
[41] P.J.E. Peebles, “The large-scale structure of the universe, Princeton University Press (Princeton, 1980); Eq. 14.8.
[42] S.J. Hatton, S. Cole, MNRAS 310, 113 (1999).
[43] R.K. Sheth, A. Diaferio, MNRAS, 322, 901 (2001).
[44] M. White, MNRAS 321, 1 (2001); U. Seljak, MNRAS 325, 1359 (2001).
[45] R. Scoccimarro, Phys. Rev. D70, 083007 (2004).
[46] S.D. Landy, A.S. Szalay, ApJ 412, 64 (1993).
[47] R.W. Hockney, J.W. Eastwood, Computer Simulation using Particles McGraw-Hill, New York (1980).
[48] G.M. Bernstein, ApJ 424, 569 (1994).
[49] D.J. Eisenstein, M. Zaldarriaga, ApJ 546, 2 (2001)
[50] J.D. Cohn, New Astronomy 11, 226 (2006).
[51] J.A. Peacock, S.A. Dodds, MNRAS, 280, L19 (1996).
[52] A.J.S. Hamilton, P. Kumar, E. Lu, A. Matthews, ApJ, 374, L1 (1991).
[53] R.E. Smith, et al., MNRAS, 341, 1311 (2003).
[54] U. Seljak, MNRAS, 318, 203 (2000).
[55] H.-J. Seo, D.J. Eisenstein, ApJ 598, 720 (2003).
[56] I. Zehavi, et al., ApJ 621, 22 (2005).
[57] N. Kaiser, MNRAS 219, 785 (1986).
[58] J.S. Bullock, R.H. Wechsler, R.S. Somerville, MNRAS 329, 246 (2002).
[59] A. Meiksin, M. White, MNRAS 308, 1179 (1999).
[60] W.H. Press, B.P. Flannery, S.A. Teukolsky, W.T. Vetterling, “Numerical Recipes”, Cambridge University Press (Cambridge, 1992)
[61] W.R. Gilks, S.R. Richardson, D.J. Spiegelhalter, “Markov chain Monte Carlo in practice”, Chapman & Hall (Florida, 1996)
[62] R.A.C. Croft, G. Efstathiou, MNRAS, 268, L23 (1994).
[63] J. Colberg, et al., MNRAS, 313, 229 (2000).
[64] N.D. Padilla, C.M. Baugh, MNRAS, 329, 431 (2002).