Neutrino Magnetic Moment Behaviour in a Renormalizable Model

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Abstract

We have shown in [1] that a neutrino magnetic moment form factor $\mu_\nu(q^2)$ could be considerably amplified at low momentum transfer, $q^2 \leq m_N^2$, at the cost of introducing an extra light neutral fermion $N$ with mass $m_N$ and nonzero magnetic moment. It was assumed that the magnetic moment of $N$ would originate in a renormalizable way at a heavy scale $M$. While the enhancement of the neutrino magnetic moment was unambiguous, we stressed that in this effective Lagrangian approach an uncertainty persisted about the behaviour of $\mu_\nu(q^2)$ in the interval $m_N^2 \ll q^2 \ll M^2$. This is not unexpected in presence of a nonrenormalizable effective theory (a particle with bare magnetic moment). We show in a simple renormalizable model for the magnetic moment of particle $N$ how a 2 loop calculation solves the ambiguity. In the domain $m_N^2 \ll q^2 \ll M^2$ we confirm the result obtained in [1] using a sharp cut-off. It is amusing that the correct results are given, as expected, through dimensional regularization in the full theory, but NOT in the effective lagrangian approach.

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1 Introduction

Paper [1] considered the possibility of a rapid variation of the neutrino magnetic form factor at small momentum transfer. Such a rapid variation would allow comparatively large magnetic (transition) moments at negligible momentum transfer (absorption/emission of almost real photon by neutrino and neutrino propagation in external magnetic field) despite severe bounds from $\nu e$-scattering experiments and the lack of pair production at $e^+e^-$ colliders.

A neutrino magnetic form factor appears at the 1-loop level and varies with momentum transfer with a typical scale set by the virtual particles masses. In scattering experiments momentum transfer varies from several MeV (for $\bar{\nu}_e$ from reactors) to 100 MeV ($\nu_\tau$ scattering), and a low-energy enhancement can only be achieved if very light particles run in the loop and couple to the photon. The only existing charged particle with a mass in the right range is the electron, and it indeed couples with $\nu_e$. However, the real momentum scale is set by the $W$-boson running in the graph, and its huge mass determines the value of $q^2$ at which the neutrino magnetic form factor starts to diminish. The way out of this was suggested in ref [1] and consists in using a virtual neutral light spinor particle with nonzero magnetic moment and a light scalar instead of charged particles in the loop. Such particles could have escaped detection, and the origin of the magnetic moment of the new fermion (a non-renormalizable interaction) could be found at a much higher scale, involving particles of mass $M \gg M_W$.

The result of the calculation of the neutrino magnetic form factor was given in [1] in the following form:

$$\mu_\nu = \frac{f^2 \mu_N}{16\pi^2} I(m_N, m_\varphi, m_\nu, q^2) \;,$$

where $\varphi$ is the light scalar particle while $f$ is Yukawa coupling constant. The interaction $f\bar{\nu}N\varphi$ induces $\mu_\nu$ at one loop (in this paper we consider both $\nu$ and $N$ as Dirac fermions) (this coupling breaks the elecroweak SU(2) symmetry, but this can indeed occur after the usual standard model symmetry breaking). In general $I \sim 1$, but in the special case where $\nu$ and $N$ are nearly degenerate while the scalar mass is negligible, $(m_\nu \approx m_N = m, m_\varphi \ll m_\nu - m_N)$ a logarithmic enhancement takes place at small $q^2$ [1]:

$$I = 4 \ln \frac{m}{2(m_N - m_\nu)} - \frac{7}{2} \; , \; q^2 \lesssim m^2$$

$$I = -1/2 \; , \; q^2 \gg m^2 \; .$$
When deriving the expressions (2) and (3) we dealt both with ultraviolet-convergent and -divergent integrals over the virtual particles 4-momentum $k_\mu$. The ultraviolet divergent integral has the following form:

$$ J_\mu = \int \frac{d^4k}{(k^2 + a^2)^3} \hat{k} \sigma_{\mu\nu} \hat{k} q_\nu, \quad \sigma_{\mu\nu} = \frac{1}{2}(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu). $$

(4)

We calculated this integral in 4 space-time dimensions with a sharp symmetrical cut-off $\Lambda$ and got zero when averaging over the directions of 4-momentum $k_\mu$:

$$ \gamma_\alpha \frac{1}{2}(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu) \gamma_\alpha = 2g_{\mu\nu} - 2g_{\nu\mu} = 0 $$

(5)

We also remarked that using instead dimensional regularization adds a non-trivial constant due to a $d - 4$ factor from the $\gamma$-matrices algebra ($\gamma_\alpha \frac{1}{2}(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu) \gamma_\alpha = (d - 4)\frac{1}{2}(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)$) multiplies the pole in $1/(d - 4)$ from the integration over $|k|$. So, in this way of performing calculations the function $I$ shifts by a constant. This is of course neither unexpected nor discouraging, since we are dealing with the effective interaction of a particle $N$ with bare magnetic moment, which is nonrenormalizable. In [1] it was naturally assumed that the $N$ magnetic moment was due in turn to a loop with virtual heavy charged particles of mass $M$, $M \gtrsim 100$ GeV. At scale $M$ we have a perfectly renormalizable theory while for $q \ll M$ we had to deal with a nonrenormalizable effective theory. In such an effective theory the logarithmic term in (2) which originates from small momentum of virtual particles is calculated unambiguously while the precise value of the constant may depend on how the theory looks at the scale $M$.

Quite intriguing is the fact that the dimensional regularization of the effective one loop calculation discussed above produces exactly an additional factor $\delta I = +1/2$, which would result in a precocious decrease of the magnetic moment, long before the $M$ scale is reached: $I \sim m^2/q^2$ for $q^2 > m^2$!

We thus have three possibilities:

a) The constant term in $I$ depends on the form of the theory at scale $M$ at which magnetic moment of particle $N$ is determined;

b) The constant term is universal and equals 0 (as dimensional regularization suggests), leading to precocious power suppression of the magnetic form factor

c) The constant term is universal and equals $-1/2$ as suggested by our naive sharp symmetric cut-off, that is the superficially ultraviolet divergent contribution from the loop vanishes identically.

To determine which of these possibilities is realized, we study in the present note the simplest renormalizable model for inducing the magnetic moment of
we assume a simple Yukawa coupling of $N$ with charged spinor and scalar fields of equal masses $M$.

In the next Section we will calculate the magnetic moment of $N$ in this model. This will also bring us to discuss the charge form factor of $N$. The 2-loops diagrams which produce the ordinary neutrino magnetic formfactor will be calculated in the third Section. Finally a comparison with the result obtained in Sect. 2 will allow us to present expressions for the neutrino magnetic formfactor in a form, given by eq. (1). We will see that option (c) realizes; which confirms the exact expression given in [1].

At the end of this introduction let us make the following technical remark: as we are interested here only in the neutrino magnetic form factor behavior in the domain $m^2 \ll q^2 \ll M^2$ we will neglect the masses of $\nu$ and of $N$ in what follows.

2 Electromagnetic form factors of the particle $N$.

The coupling of $N \bar{N}$ pair with a photon is generated by two one-loop diagrams (see Fig. 1). The corresponding amplitude is (in order to simplify our formulas we take the masses $M$ of the heavy virtual spinor and scalar particles to be equal):

\[
M_{\mu} = \int \frac{d^4 p(i)^6 \sqrt{4\pi\alpha}}{(2\pi)^4(p^2 - M^2)} \bar{N} \frac{1}{\hat{p} + \hat{k}_2 - M} \gamma_\mu \frac{1}{\hat{p} + \hat{k}_1 - M} N +
\]

\[
+ \int \frac{d^4 p(i)^6 \sqrt{4\pi\alpha}}{(2\pi)^4[(p - k_1)^2 - M^2][(p - k_2)^2 - M^2]} \times
\]

\[
\times \bar{N} \frac{1}{\hat{p} - M} N (2p - k_1 - k_2)_\mu ,
\]

where for simplicity we also put the Yukawa coupling equal to unity. In what follows we will neglect mass of the external particle $N$. Performing integration we obtain the charge and magnetic formfactors of $N$. Here we meet with the first surprise – due to the loop correction $N$ seems to get finite non-zero charge. Such a behavior is obviously forbidden by charge conservation. However, multiplying $M_{\mu}$ by $(k_1 - k_2)_\mu$ to check electromagnetic current conservation we encounter differences of linear divergent integrals. So in spite of finiteness of the result for the amplitude $M_{\mu}$, to preserve gauge invariance we need a proper regularization scheme. Using dimensional regularization we automatically obtain zero charge for $N$. After simple transformations using Feynman parametrization for the
propagators we get:

\[
M_\mu = -i \int \frac{2ydydx^d}{(2\pi)^4[p^2 + 2x(1-x)y^2(k_1k_2) + M^2]^3} \times \\
\times \bar{N}\left[\frac{4-d}{d} \gamma_\mu p^2 - M^2 \gamma_\mu - 2x(1-x)y^2 \times \\
\times (k_1k_2)\gamma_\mu + 4x(1-x)y^2(k_1k_2)\gamma_\mu + (k_1 + k_2)_\mu M\right]N .
\]

The sum of the first three terms in brackets is zero the fourth term generates a charge form factor while the last term is the magnetic form factor we want to study (for massless fermions \((k_1 + k_2)_\mu \bar{N}N = \bar{N}\sigma_{\mu\nu}q_\nu N\)). For small momentum transfer \(q^2 \equiv (k_1 - k_2)^2 \ll M^2\) we get:

\[
M_\mu = \frac{i\sqrt{4\pi\alpha}}{192\pi^2} \frac{q^2}{M^2} \bar{N}\gamma_\mu N - \frac{i\sqrt{4\pi\alpha}}{32\pi^2 M} \bar{N}N(k_1 + k_2)_\mu ,
\]

here the first term describes the charge radius of particle \(N\), while the second describes its magnetic moment.

To end this Section let us note that for large momentum transfer \(q^2 \gg M^2\) the magnetic form factor falls down as expected like \(\sim M^2/q^2 \ln^2(q^2/M^2)\), while the charge form factor tends to a (non-zero) constant. (It is interesting to consider the charge form factor in the light of dispersion relations. As the imaginary part of the form factor falls down, an unsubtracted dispersion relation can be written. It produces a form factor which falls down for \(q^2 \gg M^2\), but contains nonzero charge for \(N\). Subtracting this "charge" we get constant behavior for \(q^2 \gg M^2\).)

### 3 Neutrino magnetic form factor for \(q^2 \gg m^2\).

The two Feynman diagrams shown in Fig. 2 are in turn responsible for the ordinary neutrino magnetic form factor. Since we neglect both \(\nu\) and \(N\) masses, helicity flip (necessary for a magnetic form factor) can only occur in the inner fermion line (with mass \(M\)). As the first part of calculation we perform the integral over momentum \(p\). Taking into account only the terms proportional to \(M\) (they are the only ones contributing to the magnetic moment) we obtain:

\[
A_\mu = i \int \frac{2ydydx}{32\pi^2} \frac{M[k_2\gamma_\mu + \gamma_\mu k_1 - (k_1 + k_2)_\mu]\sqrt{4\pi\alpha}}{[M^2 - y(1-y)(k-xk_2 - (1-x)k_1)^2 + 2yx(1-x)k_1k_2]} ,
\]

where \(0 < y, x < 1\). When expression (9) is inserted into the external loop we get as expected from renormalizability an extra suppression of the integrand.
over \( k_\mu \) in the domain \( k^2 > M^2 \). Taking into account the propagators of fields \( \phi \) and \( N \) we observe that the integral over \( k_\mu \) is now u.v. convergent by power counting. In this way, our microscopic model of the \( N \) particle magnetic moment regularizes the expression for the light neutrino magnetic form factor. This leads as announced to the convergence of the analog of integral (4) in the 2-loop approach; so even making the full integration in space-time dimensions \( d \neq 4 \) we get zero for the integral proportional to \( \hat{k}\sigma_{\mu\nu}\hat{k} \). In this way the result obtained in [1] for the ordinary neutrino magnetic moment is justified; our option c) realizes.

Armed with the qualitative arguments given above let us proceed with the calculation of the second loop. Making use of Feynman parametrization for propagators of particles \( N \) and \( \phi \) (we use parameters \( u \) and \( v \), \( 0 < u, v < 1 \)) after simple transformations we get:

\[
T_\mu = f^2 \int \frac{2ydydx\sqrt{4\pi\alpha}}{32\pi^2} \frac{d^4k2vdu}{(2\pi)^4[k^2 - 2k_1k_2uv(1-u)]^3} \times
\]

\[
M\bar{\nu}_2[-(k_1 + k_2)_\mu 2k_1k_2u(1-u)v^2 + \hat{k}((k_1 + k_2)_\mu - \hat{k}_2\gamma_\mu - \gamma_\mu\hat{k}_1)\hat{k}]\nu_1
\]

\[
\frac{M^2 - y(1-y)(k - xk_2 - (1-x)k_1 + uvk_2 + (1-u)vk_1)^2 + 2xy(1-x)k_1k_2]}{[M^2 - y(1-y)(k - xk_2 - (1-x)k_1 + uvk_2 + (1-u)vk_1)^2 + 2xy(1-x)k_1k_2]}
\]

The second term in brackets in the nominator produces corrections to the neutrino magnetic moment of order \( \frac{1}{M} \times \frac{q^2}{M^2} \) and is negligible for \( q^2 \ll M^2 \). To calculate the contribution of the first term, let us begin with comparatively small momentum transfer, \( q^2 \ll M^2 \). For such momentum transfer the second bracket in the denominator equals \( M^2 \) (let us mention that integral over \( k_\mu \) is u.v. convergent for the part of \( T_\mu \) now considered) and we readily get:

\[
T_\mu = \frac{i\sqrt{4\pi\alpha}f^2}{32\pi^2M} \bar{\nu}_2\nu_1(k_1 + k_2)_\mu \frac{1}{32\pi^2}, \text{ or } I = -\frac{1}{2} \text{ for } q^2 \ll M^2 . \tag{11}
\]

For large momentum transfer, \( q^2 \gg M^2 \) the second bracket in denominator in (10) is proportional to \( q^2 \) which means that the neutrino magnetic moment falls down, \( I \sim M^2/q^2 \) according to general expectations for a renormalizable field theory.

4 Conclusions.

In this paper we demonstrated that an effective theory with a neutral spinor particle \( N \) with nonzero magnetic moment allows to calculate the induced neutrino magnetic form factor in the domain \( q^2 \ll M^2 \), where \( M \) is scale at which \( N \)
magnetic moment is generated if loop calculations are made with naive ultraviolet cut-off. This is fully consistent with a dimensional regularization treatment of the full renormalisable underlying theory, but not of the effective model.

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References

[1] J.-M.Frere, R.B.Nevzorov, M.I.Vysotsky, ULB-TH/96/14, ITEP 38-96, [hep-ph/9608266].
Figure 1: These diagrams generate the coupling of a $N\bar{N}$ pair with a photon.
Figure 2: These two-loop diagrams contribute to the neutrino magnetic form factor.