Uniform Asymptotic Stability for Convection-Reaction-Diffusion Equations in the Inviscid Limit Towards Riemann Shocks

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Abstract

In this talk, I will present a result obtained in a recent paper about the study of the stability in time of a family \((U_\epsilon)_{0<\epsilon<\epsilon_0}\) of traveling waves solutions to

\[
\partial_t u + \partial_x f(u) = g(u) + \epsilon \partial_x^2 u
\]

that approximate a given Riemann shock, and we aim at showing some uniform asymptotic orbital stability result of these waves under some conditions that guarantee the asymptotic orbital stability of the corresponding Riemann shock, as proved in a previous work of V. Duchêne and L. M. Rodrigues.

Even at the linear level, to ensure uniformity in \(\epsilon\), the decomposition of the Green function associated with the (fast-variable) linearization about \(U_\epsilon\) of the above equation into a decreasing part and a phase modulation is carried out in a highly non-standard way.

Furthermore, we introduce a multi-scale norm depending in \(\epsilon\) that is the usual \(W^{1,\infty}\) norm when restricted to functions supported away from the shock location. To avoid the use of arguments based on parabolic regularization that would preclude a result uniform in \(\epsilon\), we close nonlinear estimates on this norm through some suitable maximum principle.