Mathematical model of the interaction of plasma with the inner surface of a hollow cathode in a vacuum plasma torch

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Abstract. The analysis of the equation of energy balance on the inner surface of a hollow cathode enabled us to establish a decline in electric current in the cathode down to zero value, taking place due to the interaction of the electronic component of plasma with its magnetic field. A formula of the depth of plasma influx inside the cathode channel was developed. The spatial position of the active zone and the current density on the surface of the current transfer were determined using an experimental dependency.

1. Introduction

The voltage of discharges in vacuum plasma torches with hollow cathodes, which in nominal modes work with diffuse cathode spots, is significantly higher than the voltage of free-burning arcs. The higher voltage of the hollow cathode discharge in high-current modes ($I > 500$ A) is explained by the following factors. The near-cathode voltage decline is determined by the processes occurring within the cathode channel and the potentials of ionization of the plasma gas, and happens to approximate the voltage on all of the discharge length between the cathode and the anode. In this case, the electrons of thermal emission gain maximum energy, which they expend partially on the ionization of gas during their radial oscillation within the cathode channel and partially on the heating of secondary electrons and the formation of the reverse electron current on the cathode. Meanwhile, the ionization of the gas in the cathode channel is carried out via direct electric impact in the basic state [1, 2].

Positive space charge created by ions strengthens the field near the cathode. The negative charge of the electrons partially compensates this effect. The plasma in the cathode channel acts as a source of ion current on the cathode, which takes part in the generation of positive space charge.

Balance relations, together with other equations describing the near-cathode processes, comprise a united system of equations.

2. Methods and models

The interaction of metastable plasma with the surface of the cathode channel ensures self-sustainment of the cathode temperature $T_1 = f(z)$, which is determined by the energetic balance of the cathode [3, 4]

$$q_i + q_i^r + q_e = q_{em} + q_R + \lambda$$

where $q_i$, $q_i^r$, $q_{em}$ - are the densities of the streams of energies brought by ions and reverse current electrons and carried away by the emission electrons; $q_R$, $q_e$, $q_{em}$ - are the reradiation streams within the channel, radiations from the outer surface of the cathode, heat conductivity along axis Z and heat conductivity expended on the evaporation of the cathode material.
Putting the definitive parameters of the interconnected system into (1), we have the following equation of energetic balance [2, 5]

\[ j_1 \left( 2T_e + U_e - \varphi + \varphi_i \right) + j_2' \left( 2T_e + \varphi \right) = q_r + \varphi j_{em}, \]  

(2)

where \( U_e, \varphi \) - are the near-cathode voltage decline and work function, \( \varphi_i \) - the ionization energy, \( T_e \) - the temperature of the electrons, \( j_1, j_2', j_{em} \) - the densities of ion current, reverse electron current and thermal emission current.

The equation (2) enables us to estimate the degree of nonlinearity of the discharge parameters along the cathode. As we go from the point of the introduction of the plasma gas to the end of the cathode, the intensity of the magnetic field of plasma and accordingly of the magnetic pressure near the inner surface of the cathode grows. Sources [6, 7] prove that the sum of gas-dynamic pressure, velocity head of the gas and the magnetic pressure in metastable mode is a constant quantity. With the increase of magnetic pressure towards the end of the cathode, a decrease in gas-dynamic pressure and gas density will take place. With the decrease in plasma density, the temperature of the electrons, \( \varphi \) - the ionization energy, \( T_e \) - velocity head of the gas and the magnetic pressure induction. Furthermore, the condition of force balance will be calculated based on [8, 9] the permanence of the sum of gas and magnetic pressures \( \alpha_0 eT_e n + B_o^2 \)

\[ \int \frac{F}{2\mu_0} = p_0 \]  

(2)

where \( \mu_0 \) is the probability of absorption of the electron by the surface of the cathode.

The value of \( p_0 \) equals the gas pressure at the point of the plasma gas stream entering the discharge \( (p_0 = n_0 \alpha_0 eT_e) \), \( B_o \) - magnetic field induction. The magnetic field induction near the surface of the cathode shall be \( B_o = \mu_0 \int z dz = \mu_0 F \). Then the balance equation of the pressures should be [6, 7]

\[ F + LF^2 = C, \]

(3)

where \( L = \frac{\mu_0 K}{2 \alpha_0 eT_e \varphi}, C = \frac{p_0 K}{\alpha_0 eT_e \varphi} - q_r \).

The solution of the equation (3) was found to be \( F = (C / L)^{0.5} \left( \sqrt{CL} \cdot z \right) \), which leads to the following equation for the current density \( j_z = \frac{dF}{dz} = \frac{C}{\text{ch}^2 \left( \sqrt{CL} \cdot z \right)} \). Therefore, the density of the current on the length of the cathode has a maximum which equals \( j_{max} \approx C \) and \( j_z = f(z) \) declines quickly enough towards end of the cathode (figure 1).

\[ \text{Figure 1.} \] Distribution of current density from the open end of the cathode \( (z=0) \) to the temperature maximum in the active zone: \( z_1 \) - current 1200 A; \( z_2 \) - current 800 A

The determined relations show that with a change in current or plasma gas consumption the distance \( \ell \) between the active zone and the end of the cathode changes as well.

The presented qualitative estimation of the distribution \( j(z) \) for the cathode does not take into account the area between the cathode holder and the maximum current density in the active zone. The
solution of the balance equation (2) for this area provides the characteristic \( j(z) \) for the fore front of the current density curve.

3. Results and discussion

Experimental definition of the position of the active zone (coordinates \( z_1 \) and \( z_2 \) in figure 1) was carried out during the operation of plasma torches in normal high-current modes. A probe made up of a fine tungsten needle was pressed to the cathode at a controlled distance from the cathode holder, measuring the voltage decline between the point of contact with the discretely moving probe and the cathode holder, the potential of which was assumed to equal zero. The results of the measurements are presented in figure 2. The dependencies \( U = f(z) \) can be divided in three sections. In section I, the current only flows through metal. The relation between specific electric resistance and temperature defines the nonlinearity of this characteristic. Figure 3 shows these relations for porous tungsten, of which the cathodes are made. In section III, where the current only flows through plasma, the cathode's potential does not change and the voltage decline does not depend on the position of the probe. Section II is where the current is transferred from plasma to the cathode.

![Figure 2. Distribution of the voltage decline along the length of the cathode with \( d_1 = 10^{-2} \text{ m} \), argon consumption \( G = 1.3 \cdot 10^{-5} \text{ kg/c} \) under currents 800 A (1) and 1200 A (2), \( \delta = 2 \cdot 10^{-3} \text{ m} \) (800 A), 5 \( \cdot 10^{-3} \text{ m} \) (1200 A)](image)

![Figure 3. Dependence of the electrical resistance of tungsten on the temperature: 1 – solid W; 2–4 – porous W (P = 0.2 (2), 0.25 (3), 0.35 (4))] (image)

The voltage decline between the probe and the cathode holder, with the probe positioned arbitrarily, is defined by the expression

\[
U = \int_0^z \frac{1}{S} I(z) \rho(z) \, dz,
\]

(4)

Where \( I(z) \) – the current flowing through the metal of the cathode; \( \rho(z) \) – the local specific electric resistance; \( S \) – the area of the cathode wall section. From (4) we get

\[
\frac{\partial U}{\partial z} = \frac{1}{S} I(z) \rho(z).
\]

(5)

Differentiating the expression (5) with coordinate \( z \), we have

\[
\frac{\partial I(z)}{\partial z} = \frac{\partial}{\partial z} \left( \frac{S}{\rho(z)} \frac{\partial U}{\partial z} \right).
\]

(6)
The density of the current flowing from the plasma onto the inner surface of the cathode for section II (figure 2) is linked to the derivative of the current flowing through metal $\partial I/\partial z$ by the following expression

$$j_c = \frac{1}{n d_1} \frac{\partial I}{\partial z},$$

where $d_1$ – the diameter of the cathode channel. Using (6) and (7) and carrying out the differentiation according to the dependency $\rho(T_c) = \rho_0(1 + \alpha T_c)$, we have

$$j_c = \frac{d_2^2 - d_1^2}{4d_1 \rho(T_c)^2} \left[ \rho(T_c) \frac{\partial^2 U}{\partial z^2} - \rho_0 \alpha \frac{\partial T_c}{\partial z} \frac{\partial U}{\partial z} \right],$$

where $T_c = (T_1 + T_2)/2$ – the local average temperature of the inner wall of the cathode in the electric exchange zone, $d_1, d_2$ – the inner and outer diameter of the cathode wall, $\alpha$ - the thermal coefficient of specific resistance of the cathode material for the temperature range of section II (figure 2).

With the established local cathode temperature $T_c(z)$ and the distribution of potential along its length $U(z)$ the position of the active zone and the density of the current in it were determined using the expression (8). Figure 4 shows the distributions of temperature and current density along the length of the cathode on its inner surface $T_2(z)$ and the dependency $U(z)$ (figure 2).

![Figure 4. Distribution of temperature $T_2$ and current density $j_c$ on the inner surface of cathode: current 1 $\sim$ 1200 A, 2 $\sim$ 800 A; $d_1 = 10^{-2}$ m, $G = 1.3 \times 10^5$ kg/c](image)

The determined decline of current from plasma on the cathode to zero value further down the stream of plasma and plasma gas (coordinates $z_1$ or $z_2$ on figure 1) requires a further analysis of the development of the physical processes. Let us show that at the core of this phenomenon (the absence of current transfer between the plasma and the cathode) lies the interaction between the electronic component of the plasma and its natural magnetic field [8].

The angle between the vector of the strength of the electric field and the vector of the current in plasma $\beta_c$ within the magnetic field is determined by the expression

$$\tan \beta_c = \omega_e \tau_e,$$

where $\omega_e$ is the cyclotron frequency of the electrons, $\tau_e$ - time between the collisions

In cathodes of larger diameter, if the plasma introduced into the cathode channel possesses potential close to that of the anode, the vector of the strength of the electric field near the cathode surface is directed parallel to the radius, and the vector of the induction of the magnetic field is perpendicular to the radius. Assuming that the heating of the cathode is mainly carried out by means of ion bombardment and the position of the active zone is defined by the vector of the ion current, similarly to (9) we can write down

$$\tan \beta_i = \omega_i \tau_i.$$

A dynamic balance between the stream of plasma gas and the inner plasma is observed within the cathode channel, since plasma is permeable to quick gas molecules under lower pressures. The molecules of neutral gas partially enter the plasma, become ionized by the oscillating electrons, and, as a result, the density and pressure of plasma increase. On the other hand, the charged components of plasma in this area end up within a stream of cold gas, where they are recombined. Thus, a dynamic
balance is ensured, for which an equality of the pressures of gas and plasma can be written down: \( p_e = p_p \). The pressure of the plasma depends mainly on the pressure of the electrons \( p_e = n_e kT_e \), and the stream (consumption) of the plasma gas defines the concentration of the atoms of neutral gas \( n_g = G/m_a \nu_s S \), where \( \nu_s \) – the gas velocity. As the plasma is magnetized by its natural magnetic field, the displacement of ions is determined by the cyclotron frequency of ions \( \omega_i \), the average ion-ion and ion-atom impact time \( \tau_i = \lambda_i / \nu_s = \left[ 2 \sqrt{2} \cdot \nu_s \left( n_i \sigma_{ia} + n_n \sigma_{ia} \right) \right]^{-1} \) and the Debye screening radius of electric potential \( r_0 = \sqrt{kT_e / 2ne^2} \). Then the length of plasma influx into the cathode channel is defined as

\[
\ell = r_0 \tan \beta_i = r_0 \omega_i \nu_s = \frac{kT_e \sqrt{\epsilon_0 \mu_0}}{\sqrt{2 \omega_i m_i}} \left[ \frac{G \sigma_{ia}}{m_n \nu_s S} + \frac{p_p \sigma_{ia}}{kT_e} \right]^{-1},
\]

(11)

where \( m_i \) and \( \omega_i \) are the mass and average thermal speed of ions; \( G \) – the plasma gas consumption; \( \nu_s \) – the speed of the gas stream; \( \sigma_{ia}, \sigma_{ia} \) – the cross-sections of the impact of ions with neutral atoms and ions, \( m_n \) – the mass of a neutral atom; \( p_p \) – the plasma pressure.

Considering that the plasma pressure equals the electron pressure \( p_p = p_e = \mu_e I_e / \left( 4 \pi^2 R^2 \right) \), let us find the dependency of the depth of plasma influx on the consumption of plasma gas and working current

\[
\ell = \frac{kT_e \sqrt{\epsilon_0 \mu_0}}{2 \omega_i m_i} \left[ \frac{G \sigma_{ia}}{m_n \nu_s S} + \frac{\mu_e I_e^2 \sigma_{ia}}{4 \pi^2 R^2 kT_e} \right]^{-1}.
\]

(12)

As follows from the equation (12), if the role of the second part of the denominator is small, a rise in plasma gas consumption \( G \) in low current mode leads to \( \ell \) decreasing and the active zone shifting towards the open end of the cathode (as observed in the experiment). With an increase in current, the quantitative influence of the first summand of the denominator decreases, and in high-current modes the changes in gas consumption \( (\Delta G = \pm 0.1 G_{nom}) \) have little effect on the position of the active zone inside the cathode channel. A rise in the discharge current leads to the active zone shifting towards the open end of the cathode [10].

4. Conclusion

The modes with diffuse cathode spots are observed in hollow cathode discharges when the cathode surface is co-axial with the discharge column and a metastable area of plasma is created in the cathode channel. The stability of these modes is supported by the presence of radially oscillating electrons so that the ionization of plasma gas is occurring on all of the cathode channel section within the active zone.

References

[1] Cherednichenko V S Pavlenko L K and Zagorskii A V 1999 Pysical phenomena in a hollow cathode and powder interaction with the vacuum arc Thermal plasma torches and technologies (Cambridge International Science Publishing) vol 1 pp 161-172
[2] Cherednichenko V S and Yudin B I 2008 Plasma furnace for processing powder materials Proc. of the 16th Int. Conf. of Electricity Application in Modern World (Krakow, Poland) pp 131-132
[3] Cherednichenko M V, Serikov V A, Butakov E B and Urbakh A E 2017 Erosion of plasma generator electrodes for various power supplies Russian Metallurgy (Metally) 6 pp 527-531
[4] An’shakov A S, Cherednichenko M V, Serikov V A and Domarov P V 2019 Operating modes of vacuum plasmastrons with hollow cathodes IOP Conf. Series: Materials Science and Engineering vol 560 012100
[5] Cherednichenko A V, Cherednichenko V S, Bikeev R A and Serikov V A 2019 Electrothermal processing of tantalum capacitor powders IOP Conf. Series: Materials Science and
Engineering vol 560 012100

[6] Cherednichenko V S and Yudin B I 2008 Thermophysical processes in vacuum plasma electric furnaces for heating powdered materials Russian Metallurgy (Metally) 7 583-587

[7] Becker K H, Kersten H, Hopwood J and Lopez J L 2010 Microplasmas: scientific challenges & technical opportunities The European Physical Journal D 60 437-439

[8] Anders A and Oks E 2007 Charge-state-resolved ion energy distribution functions of cathodic vacuum arcs: a study involving the plasma potential and biased plasmas J. Appl. Phys. 101 043304

[9] Raiser Y P 1991 Gas Discharge Physics (Berlin: Springer)

[10] Bikeev R A and Cherednichenko V S 2015 Simulation of electromagnetic processes in three-phase electric arc furnaces Russian Metallurgy (Metally) 8 454-470