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A CLASS OF ANISOTROPIC (FINSLER-) SPACE-TIME GEOMETRIES

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Abstract

A particular Finsler-metric proposed in [1,2] and describing a geometry with a preferred null direction is characterized here as belonging to a subclass contained in a larger class of Finsler-metrics with one or more preferred directions (null, space- or timelike). The metrics are classified according to their group of isometries. These turn out to be isomorphic to subgroups of the Poincaré (Lorentz-) group complemented by the generator of a dilatation. The arising Finsler geometries may be used for the construction of relativistic theories testing the isotropy of space. It is shown that the Finsler space with the only preferred null direction is the anisotropic space closest to isotropic Minkowski-space of the full class discussed.

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1. Introduction

In Refs [1, 2] proceeding from the group

\[
\begin{align*}
x^{0'} &= \left(\frac{1-v/c}{1+v/c}\right)^{r/2} \frac{x^0-v/c}{\sqrt{1-v^2/c^2}} \\
x^{1'} &= \left(\frac{1-v/c}{1+v/c}\right)^{r/2} \frac{x^1-v/c}{\sqrt{1-v^2/c^2}}
\end{align*}
\] (1)

the function

\[
f = \left(\frac{x^0 - x^1}{x^0 + x^1}\right)^r \left[(x^0)^2 - (x^1)^2\right]
\] (2)

was found as a function invariant under the transformation (1). After that the corresponding 2-dimensional Finsler-metric

\[
ds = \left(\frac{dx^0 - dx^1}{dx^0 + dx^1}\right)^{r/2} \left[(dx^0)^2 - (dx^1)^2\right]^{1/2}
\]

was written and then generalized to 4-dimentional space in a coordinate independent manner with the help of a constant vector \(\nu\) satisfing \(\nu_a\nu_b\eta^{ab} = 0\):

\[
ds = \left[\frac{(\nu_a dx^a)^2}{\eta_{cd} dx^c dx^d}\right]^{r/2} \left(\eta_{ij} dx^i dx^j\right)^{1/2},
\] (3)

where \(\eta_{ab}\) is the Minkowski metric (Latin indices run form 0 to 3). Actually \(\nu\) does not transform as a 4-vector under the transformations belonging to an 8-parameter isometry group of the space (3); it is an absolute preferred null vector in Finsler-space. \(r\) is called “parameter of anisotropy”; obviously \(0 \leq r < 1\) must hold. In fact, from the present tests of special relativity, \(r \approx 10^{-10}\).

The purpose of this note is (i) to uniquely characterize the Finsler metric (3) among other constructive possibilities, and (ii) to discuss further such metrics describing geometries with inbuilt preferred directions (not necessarily lightlike) according to the isometry groups occurring. From such metrics theories may be developped for experiments testing the isotropy of space or preferred system theories. After introducing the general class of Finsler-metrics investigated here, in Section 2, we study the Lie-algebra of their isometry groups. In Section 3, the metric (3) is reobtained while in Section 4 Finsler-metrics with isometry groups smaller than the 8-parameter group
are discussed. In Section 5 we derive the group of finite transformations for one of the cases discussed.

2. A special class of Finsler-metrics and its isometry group

We now use coordinates in which \( \eta_{ab} := \text{diag}(+1, -1, -1, -1) \) and introduce a second symmetric, constant tensor of rank 2 \( a_{cd} \) (i.e. \( \frac{\partial a_{cd}}{\partial x^i} = 0 \)). The variable

\[
v := \frac{a_{cd} dx^c dx^d}{\eta_{ij} dx^i dx^j}
\]

is of degree of homogeneity zero. Any metric of the form

\[
ds = \psi(v) \sqrt{\eta_{ab} dx^a dx^b}
\]

with arbitrary function \( \psi(v) \) such that \( \lim_{v \to \infty} ds \) exists, then is of degree one and can be used as a Finsler metric. For the further calculations we employ the more convenient form

\[
ds^2 = \phi(v) \eta_{ab} dx^a dx^b.
\]

\( \phi = 1 \) leads back to Minkowski space while \( \phi = v \) leads to the pseudo-Riemannian metric

\[
ds^2 = a_{cd} dx^c dx^d
\]

i.e. to a metric the rank and signature of which are not fixed from the outset. Except for these two limiting cases, (6) represents a proper Finsler-metric. It is tempting, however, to try and use a dual interpretation of (6) as a pseudo-Riemannian, conformally flat metric with conformal factor

\[
\phi \left( \frac{a_{cd} \dot{x}^c \dot{x}^d}{\eta_{ab} \dot{x}^a \dot{x}^b} \right),
\]

where \( \dot{x}^c = \frac{dx^c}{d\lambda} \), \( \lambda \) any arbitrary parameter of the curve \( x^j = x^j(\lambda) \). In this dual interpretation \( x^\lambda \) and \( \dot{x}^\lambda \) would be considered as independent variables such that \( \frac{\partial \phi}{\partial x^j} = 0 \). From the point of view of physics, the immediate problem is then which of the two metrics \( ds^2 \) and \( d\ell^2 := \eta_{ab} dx^a dx^b \) measures times and lengths. In accordance with the Finslerian approach we take here, \( ds \) is to stand for the space-time interval measured by clocks and rigid rulers. Consequently, we will have to cope with the occurrence of a second metric (i.e. the
Minkowski metric $\eta_{ab}$ devoid of physical significance. In a way, a new type of bi-metric theories would result with one of the metrics being Finslerian, the other Lorentzian. However, in this paper, only formal groundwork for physical theories to be built on is laid.

Starting now from metric components $\gamma_{ij}(x^k, dx^k)$ and deriving the Lie-derivative

$$- \mathcal{L}_X \gamma_{ij} := \gamma_{ij}(x^k, dx^k) - \gamma_{ij}(x^k, dx^k),$$

where $x^k = x^k + \xi^k(x^\ell)$ and $X = \xi^k \frac{\partial}{\partial x^k}$ is the generator of the infinitesimal coordinate transformation, we arrive at

$$- \mathcal{L}_X \gamma_{ij} = \gamma_{ij,k} \xi^k + \gamma_{i\ell} \xi^\ell + \gamma_{\ell j} \xi^\ell + \frac{\partial \gamma_{ij}}{\partial \dot{x}^\ell} \xi_{,k} \dot{x}^k$$

(9)

a formula to also be found in Yano [3]. In place of $\dot{x}^\ell$ we may also write $dx^\ell$. An isometry of the Finsler-metric with components $\gamma_{ij}$ is defined by

$$\mathcal{L}_X \gamma_{ij} = 0 .$$

(10)

If now the Ansatz (6), i.e. $\gamma_{ij} = \phi(v) \eta_{ij}$ is introduced into equation (10), we obtain

$$0 = \eta_{ac} \xi_c^a + \eta_{bc} \xi_c^b + 2v \frac{\phi'}{\phi} \eta_{ab} \left[ \frac{a_{ij}}{a_{\ell m} \dot{x}^\ell \dot{x}^m} - \frac{\eta_{ij}}{\eta_{\ell m} \dot{x}^\ell \dot{x}^m} \right] \dot{x}^j \xi_{,k} \dot{x}^k$$

(11)

where $\phi' := \frac{d\phi}{dv}$. Eq. (11) may be decomposed into

$$\xi_{,\alpha}^0 - \xi_{,0}^\alpha = 0 ,$$

$$\xi_{,\alpha}^\beta + \xi_{,\beta}^\alpha = 0 ,$$

$$\xi_{,0}^\alpha = \xi_{,1}^1 = \xi_{,2}^2 = \xi_{,3}^3 ,$$

(12.1-3)

and

$$\xi_{,3}^3 + \frac{v \phi'}{\phi} \left[ \frac{a_{ij}}{a_{\ell m} \dot{x}^\ell \dot{x}^m} - \frac{\eta_{ij}}{\eta_{\ell m} \dot{x}^\ell \dot{x}^m} \right] \dot{x}^j \xi_{,k} \dot{x}^k = 0$$

(12.4)

($\alpha, \beta = 1, 2, 3, \ i, j, ... = 0, ..., 3$).

The integration of (12.1-3) is straightforward and leads to

$$\xi^0 = \sum_{\beta=1}^3 f_{\beta 0} x^\beta + \alpha(x^0),$$

$$\xi^\gamma = \dot{\alpha}(x^0) x^\gamma + \sum_{\delta=1}^3 \Omega^\gamma_\delta x^\delta + f_{\gamma 0} x^0 + f_{\gamma \gamma}$$

(13)
with \( \gamma = 1, 2, 3 \) and the skew matrix

\[
\Omega^\beta_\gamma = \begin{pmatrix}
0 & \Omega^1_2 & \Omega^1_3 \\
-\Omega^1_2 & 0 & \Omega^2_3 \\
-\Omega^1_3 & -\Omega^2_3 & 0
\end{pmatrix}.
\]

Thus, 9 constants (group parameters) \( f_{\beta 0}, f_{\gamma \gamma}, \Omega^\beta_\gamma \) and one arbitrary function \( \alpha(x^0) \) emerge. Obviously, the parameters \( f_{\beta 0} \) correspond to Lorentz boosts, \( \Omega^\beta_\gamma \) to space rotations while \( f_{11}, f_{22} \) and \( f_{33} \) parametrize space translations. It is to be noted that, in principle, all parameters could be taken to also be functions of the independent variables \( \dot{x}^\ell \). The generator connected with \( \alpha(x^0) \) is

\[
T_0 := \alpha(x^0) \frac{\partial}{\partial x^0} + \dot{\alpha}(x^0) \left[ x^1 \frac{\partial}{\partial x^1} + x^2 \frac{\partial}{\partial x^2} + x^3 \frac{\partial}{\partial x^3} \right]. \tag{14}
\]

If \( T_0 \) and the other nine generators are to form a Lie algebra then

\[
\alpha = \alpha_0 x^0 + \alpha_1 \tag{15}
\]

with \( \alpha_0, \alpha_1 \) constants (or functions of \( \dot{x}^\ell \)) results. Consequently, at most we obtain the 10-parameter Poincaré group plus an additional dilatation generator

\[
\hat{T}_0 := x^0 \frac{\partial}{\partial x^0} + x^1 \frac{\partial}{\partial x^1} + x^2 \frac{\partial}{\partial x^2} + x^3 \frac{\partial}{\partial x^3}. \tag{16}
\]

Up to now eq. (12.4) has not yet been solved. In the following, however, we shall use eq. (15) for solving eq. (12.4).

3. Most general matrix \( a_{ij} \) permitting the 8-paramter isometry group

The Lie algebra generators belonging to the transformation generalized to full 3-dimensional space, derive from [1,2]:

\[
\begin{align*}
\xi^0 &= - \sum_{\beta=1}^{3} \epsilon_\beta x^\beta - r \epsilon_3 x^0, \\
\xi^1 &= \epsilon_1 (-x^0 + x^3) + \epsilon_4 x^2 - r \epsilon_3 x^1, \\
\xi^2 &= \epsilon_2 (-x^0 + x^3) - \epsilon_4 x^1 - r \epsilon_3 x^2, \\
\xi^3 &= - \epsilon_3 x^0 - \epsilon_4 x^1 - \epsilon_2 x^2 - r \epsilon_3 x^3. \tag{17}
\end{align*}
\]
Inspection shows that the algebra is formed by two independent Lorentz boosts, one space rotation and a third Lorentz boost combined with the dilatation generator $\hat{T}_0$. Insertion of eq. (17) into eq. (12.4) leads, after a straightforward but lengthy integration procedure, to the following result

$$\phi = u^{1/3} F(s, t)$$

where $u := \left[\frac{\nu_a \nu_b \eta_{ab}}{\eta_{cd} \dot{x}^c \dot{x}^d}\right]^{2r/(1-r)}$, $s := \left(\frac{\nu_a \dot{x}^a}{\eta_{ij} \dot{x}^i \dot{x}^j}\right)^{2r/(1-r)}$, $t := \left(\frac{\nu_{\ell}}{\nu_b} \eta_{ij} \dot{x}^i \dot{x}^j\right)^{2r/(1-r)}$.

and $\nu_a \nu_b \eta^{ab} = 0$ while $F(s, t)$ is an arbitrary function. In order that $F$ is of degree of homogeneity zero we must have

$$F(s, \lambda^{2/3} t) = \lambda^{(2/3) - 2r} F(s, t),$$

whence follows by a well known procedure that

$$\tilde{F}(s) = s^{r - \frac{2}{3}} \text{ leads back to the metric of eq. (3).}$$

It does not contain the $x^a$-variables and thus also permits the spacetime translation group as isometry. Moreover, if we require this larger group to be the group of isometries from the start, then in place of (18) the following form of $\phi$ results

$$\phi = \left[\frac{(\nu_a \dot{x}^a)}{\eta_{ij} \dot{x}^i \dot{x}^j}\right]^{2r/(3(1-r))} \tilde{F}\left[\left(\frac{\nu_a \dot{x}^a}{\nu_{\ell}} \eta_{ij} \dot{x}^i \dot{x}^j\right)^{2r/(1-r)}\right].$$

The demand of degree of homogeneity zero for $\phi$ then uniquely leads back to Bogoslovsky’s metric (3). It is characterized uniquely by the requirement that one obtain a genuine Finsler-metric with the isometry group $G_8$ described above. As can be seen from the corresponding Lie-algebra, despite of the occurrence of the dilatation generator $\hat{T}_0$ a subalgebra of the Poincaré algebra prevails. The matrix $a_{ij} = \nu_i \nu_j$ with $\nu_a \nu_b \eta^{ab} = 0$.

4. Finsler-metrics with a different type of space-time anisotropy

We now can approach the solution of eq. (12.4) in two ways: either we can give the matrix $a_{ij}$ and then determine the isometry group or, we can give
the isometry group and determine $a_{ij}$. In practice, a combination of both methods is suitable. For example, we may start from the known subgroups of the Lorentz group [4].

By inserting eqs. (13) and (15) into eq. (12.4) we obtain a set of ten algebraic equations

$$\alpha_0 X_{ij} + \frac{v \phi'}{\phi} Y_{ij} = 0$$

(22)

where

$$X_{ii} = a_{ii} \quad \text{(no summation on } i)$$
$$X_{ij} = 2a_{ij}, \ i \neq j$$

and

$$Y_{00} := f_{10}a_{10} + f_{20}a_{20} + f_{30}a_{30}$$
$$Y_{01} := f_{10}(a_{00} + a_{11}) + a_{21}f_{20} + a_{31}f_{30} - a_{20}\Omega_{12} - a_{30}\Omega_{13}$$
$$Y_{02} := f_{10}a_{12} + f_{20}(a_{00} + a_{22}) + a_{32}f_{30} + a_{10}\Omega_{12} - a_{30}\Omega_{23}$$
$$Y_{03} := f_{10}a_{13} + f_{20}a_{23} + f_{30}(a_{00} + a_{33}) + a_{10}\Omega_{13} + a_{20}\Omega_{23}$$
$$Y_{11} = a_{01}f_{10} - a_{21}\Omega_{12} - a_{31}\Omega_{13}$$
$$Y_{12} := (a_{11} - a_{22})\Omega_{12} - a_{32}\Omega_{13} - a_{31}\Omega_{23} + a_{02}f_{10} + a_{01}f_{20}$$
$$Y_{13} := a_{21}\Omega_{23} + (a_{11} - a_{33})\Omega_{13} - a_{23}\Omega_{12} + a_{03}f_{10} + a_{01}f_{30}$$
$$Y_{23} := a_{12}\Omega_{13} + a_{13}\Omega_{12} + (a_{22} - a_{33})\Omega_{23} + a_{03}f_{20} + a_{02}f_{30}$$
$$Y_{22} := a_{12}\Omega_{12} - a_{32}\Omega_{13} + a_{02}f_{20}$$
$$Y_{33} := a_{13}\Omega_{13} + a_{23}\Omega_{23} + a_{03}f_{30}.$$  

(23)

Now, if eq. (22) is to hold for arbitrary $\phi(v), \alpha_0$ must be a function of $v$. Within the dual interpretation described in section 2 above, this is a distinct possibility. Within our work here, we restrict $\alpha_0$ (and all other parameters) to be constant. Consequently, either $\alpha_0 = 0$, or $\frac{v \phi'}{\phi} = \text{const}$ must hold. In the first case, the dilatation generator drops out while in the second case

$$\phi = \phi_0 v^r$$

(24)

results with $r$ being the integration constant.
As our first example we consider the matrix $a_{ij}$ of signature zero

$$a_{ij} = \begin{pmatrix} a_{00} & 0 & a_{11} \\ a_{11} & a_{00} & 0 \\ 0 & -a_{00} & a_{11} \end{pmatrix}$$

with $a_{00} \neq a_{11}, a_{00}, a_{11} > 0$. We also set $\alpha_0 = 0$. Then $Y_{ij} = 0$ and eq. (23) lead to the 6-parameter isometry group generated by the space-time translations, a space-rotation $R_3$ with parameter $\Omega^{1}_{2}$, and a Lorentz boost $B_3$ (parameter $f_{30}$) with $[R_3, B_3] = 0$. Geometrically this situation corresponds to two preferred 2-flats.

In the second example, we assume two preferred directions; i.e. we set

$$a_{ij} = 2\nu_{(i}\mu_{j)}$$

We work in the coordinate system in which

$$\nu_i \overset{\ast}{=} \nu_0 \delta^0_i + \delta^3_i \nu_3, \quad \mu_j \overset{\ast}{=} \mu_0 \delta^0_j + \delta^1_j \mu_1$$

$$a_{ij} \overset{\ast}{=} 2 \left[ \nu_0 \mu_0 \delta_i^0 \delta_j^0 + \nu_0 \mu_1 \delta_i^0 \delta_j^1 + \nu_0 \mu_3 \delta_i^0 \delta_j^3 + \nu_1 \mu_3 \delta_i^1 \delta_j^3 \right],$$

where the symmetrization bracket comes with the usual factor $1/2$. Thus, in (22) and (23) we set $\alpha_0 = 0, \ a_{00} \cdot a_{01} \cdot a_{03} \cdot a_{13} \neq 0$ all other $a_{ij} = 0$. Two possible solutions emerge.

i) Both absolutely preferred directions $\nu_i$ and $\mu_j$ are on the null cone of the Finsler-metric (6). The isometry group is a 2-parameter abelian subgroup of the homogeneous Lorentz group completed with the space-time translations. The generators of the abelian subalgebra of the Lorentz group result from the combination of three Lorentz-boosts and three space rotations given by:

$$f_{10} = e\Omega^1_3, \quad f_{20} = \epsilon\Omega^2_3, \quad f_{30} = -\delta\Omega^1_3,$$

$$\Omega^2_3 = -\epsilon \cdot \delta \Omega^1_2,$$

with $e^2 = \epsilon^2 = 1, \ \nu_0 = \epsilon \nu_3, \ \mu_0 = \delta \mu_1$.

$\Omega^1_2$ and $\Omega^1_3$ are the independent parameters.

ii) None of the two preferred absolute directions lies on the light cone. The isometry group is a one-parameter subgroup of the homogeneous Lorentz
group plus the space-time translation group. The single generator of the first mentioned subgroup is a combination of a Lorentz boost and two rotations

\[ f_{20} = -\frac{\mu_1}{\mu_0} \Omega^1_2, \quad f_{10} = f_{30} = 0 \]
\[ \Omega^2_3 = -\frac{\nu_0}{\nu_3} \frac{\mu_1}{\mu_0} \Omega^1_2, \quad \Omega^1_3 = 0. \]  

(28)

As a final example, we consider a preferred spacelike direction \( \nu_a \). Using coordinates in which \( \nu^*_a = \delta^1_a \), i.e. \( \eta^{ab} \nu_a \nu_b = -1 \), we obtain from \( Y_{ij} = 0 \) with \( a_{11} = 1 \) and all other \( a_{ij} = 0 \):

\[ f_{10} = 0, \quad \Omega^1_2 = \Omega^1_3 = 0. \]

The 3-parameter subgroup with parameters \( f_{20}, f_{30} \) and \( \Omega^2_3 \) remains. In contrast, the preferred timelike direction \( \nu_a \) would retain the group of space-rotations as an isometry group but lose all boosts.

5. Finite transformations

It is no problem to find the groups of finite transformations belonging to (27) or (28). In the first case, after taking \( \epsilon = \delta = \mu_1 = \nu_3 = 1 \), the generators of the abelian subgroup of the Lorentz group are

\[ X_1 = (x^1 - x^3) \frac{\partial}{\partial x^0} + (x^0 + x^3) \frac{\partial}{\partial x^1} - (x^0 + x^1) \frac{\partial}{\partial x^3} \]  

(29)

and

\[ X_2 = x^2 \left( \frac{\partial}{\partial x^0} - \frac{\partial}{\partial x^1} - \frac{\partial}{\partial x^3} \right) + (x^0 + x^1 + x^3) \frac{\partial}{\partial x^2}. \]  

(30)

By summing up the Taylor series

\[ x^i + u \ X_\alpha x^i + \frac{u^2}{2!} (X_\alpha)^2 x^i + \ldots \quad (\alpha = 1, 2) \]

we obtain the finite transformations corresponding to eqs. (29) and (30), respectively

\[ x'^\alpha = x^\alpha + \sinh u [\delta^\alpha_0 (x^1 - x^3) + \delta^\alpha_1 (x^0 + x^3) - \delta^\alpha_3 (x^0 + x^1)] \]
\[ + (\cosh u - 1) [\delta^\alpha_0 (x^1 + x^3 + 2x^0) - \delta^\alpha_1 (x^0 + x^3) - \delta^\alpha_3 (x^0 + x^1)] \]  

(31)
and
\[ x^\alpha' = x^\alpha + \sin \varphi [x^2(\delta_0^\alpha - \delta_1^\alpha - \delta_3^\alpha) + (x^0 + x^1 + x^3)\delta_2^\alpha] + (1 - \cos \varphi) [x^0 + x^1 + x^3](\delta_0^\alpha - \delta_1^\alpha - \delta_3^\alpha) - x^2\delta_3^\alpha]. \tag{32} \]

\( u \) and \( \varphi \) are the group parameters. We set \( \sinh u = \gamma v/c \), \( \cosh u = \gamma \) with \( \gamma := (1 - \nu^2/c^2)^{-1/2} \) and introduce new coordinates
\[ y^0 = x^0 + x^1, \quad y^1 = x^1, \quad y^2 = x^2, \quad y^3 = x^0 + x^3. \]

Eq. (31) then can be written as
\[ y^0' = \gamma(1 + \frac{v}{c})y^0 \]
\[ y^1' = y^1 \left(1 - \frac{1 - \nu/c}{1 + \nu/c}\right)y^3 \]
\[ y^2' = y^2 \]
\[ y^3' = \gamma \left(1 - \frac{v}{c}\right)y^3. \tag{33} \]

Similarly, by introducing new coordinates
\[ y^0 = x^0 + x^1, \quad y^1 = x^0 + x^1 + x^3, \quad y^2 = -x^2, \quad y^3 = x^0 + x^3 \]
we obtain from eq. (32)
\[ y^0' = y^0 \]
\[ y^2' = \cos \varphi y^1 + \sin \varphi y^2 \]
\[ y^2' = -\sin \varphi y^1 + \cos \varphi y^2 \]
\[ y^3' = y^3. \tag{34} \]

The space-time translations have to be added to eqs. (33) and (34). An invariant function thus is
\[ f \left( \frac{y_0 y^3}{(y^0)^2 - (y^1)^2 - (y^2)^2 - (y^3)^2} \right) [(y^0)^2 - (y^1)^2 - (y^2)^2 - (y^3)^2] \]
such that we arrive at Finsler metric
\[ ds = \left[ \frac{dy_0 dy_3}{\eta_{ab} dy^a dy^b} \right]^{s/2} \sqrt{\eta_{ij} dy^i dy^j} \]
with the anisotropy parameter \( s \).

6. Conclusions

By introducing space-time geometries with preferred directions, in general, Lorentz boosts or space rotations are lost as space-time isometries. Consequently, angular momentum conservation and/or conservation of the center of mass is no longer fully guaranteed. Thus, such geometries may be used to construct relativistic theories with anisotropic space or with preferred reference systems (preferred directions in velocity space). To be close enough to present empirical data, Finsler-geometries with the largest possible isometry groups (as subgroups of the Poincaré group) are to be preferred. As no 5-parameter subgroup of the homogeneous Lorentz-transformations exists [4], the largest nontrivial such isometry group is an 8-parameter group. The Finslerian special relativity theory of Refs [1, 2] is built on such an 8-parameter group. It is unique, because only one non-isomorphic 4-parameter subgroup of the homogeneous Lorentz group does exist. There are, however, other possibilities for Finsler geometries with 7-parameter isometry groups as exemplified by the formalism of this paper and which could also be used for the construction of relativistic test theories for anisotropy or a preferred system of reference.

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