Magnetic excitation in relativistic heavy ion collisions

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In this note we study the conversion of nucleons into deltas induced by a strong magnetic field in ultraperipheral relativistic heavy ion collisions. The interaction Hamiltonian couples the magnetic field to the spin operator, which, acting on the spin part of the wave function, converts a spin 1/2 into a spin 3/2 state. We estimate this transition probability and calculate the cross section for delta production. This process can in principle be measured, since the delta moves close to the beam and decays almost exclusively into pions. Forward pions may be detected by forward calorimeters.

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I. INTRODUCTION

In relativistic heavy ion collisions we observe extreme phenomena. One of them is the production of the strongest magnetic field of the universe [1,2]. This field is so intense because the charge density is large, because the speed of the source is very close to the speed of light and also because we probe it at extremely small distances (a few fermi) of the source. After the first estimates of \( \vec{B} \), there has been a search for observable effects of this strong field [3]. The first and most famous is the Chiral Magnetic Effect (CME) [4]. Unfortunately, after ten years of experimental searches there are no conclusive results. From the theoretical point of view the study of the CME is quite complex. It is desirable to look also for conceptually simpler effects of the magnetic field, such as the one discussed in this note.

We are going to study ultraperipheral relativistic heavy ion collisions (UPC’s), in which the two nuclei do not overlap [6]. Since there is no superposition of hadronic matter, the strong interaction is strongly suppressed and the collision becomes essentially a very clean electromagnetic process almost without hadronic background. The few produced particles are mostly in the central rapidity region. In UPC’s we do not expect to see produced particles at very large rapidities. So far, the only particles measured in this region are neutrons originated from electromagnetic excitation of the nuclei [7], a process in which the energy exchanged between the nuclei is enough to fragment them but too small for particle production.

We will argue that forward pions are very likely to be produced by magnetic excitation (ME) of the nucleons in the nuclei. The strong magnetic field produced by one nucleus induces magnetic transitions, such as \( N \to \Delta \) (where \( N \) is a proton or a neutron), in the nucleons of the other nucleus. The produced \( \Delta \) keeps moving together with the nucleus (or very close to it) and then decays almost exclusively through the reaction \( \Delta \to N + \pi \). From the kinematics of this two-body decay we know that the pion has momentum around \( \approx 200 \text{ MeV} \) in the \( \Delta \) rest frame. Thus, in the cms frame it has a very large longitudinal momentum and very large rapidity. Since there is no other competing mechanism for forward pion production, the observation of these pions would be a signature of the magnetic excitation of the nucleons and also an indirect measurement of the magnetic field. In what follows we will show that ME has a very large cross section.

II. FORMALISM

Magnetic excitation in the context of heavy ion collisions was first considered in [5], where it was argued that in the presence of a strong magnetic field the transition \( \eta_c \to J/\psi \) might happen. Unfortunately, the transition rate was extremely small. This was due to the presence of the charm quark mass in the denominator of the transition amplitude. Here we revisit the idea, using the same formalism and applying it to the nucleon-delta transitions in ultraperipheral collisions. Now, instead of the heavy quark mass we have a light quark mass in the denominator and, as will be seen, the transition rate becomes large.

Under the influence of a strong magnetic field, a nucleon is converted into a \( \Delta \). For the sake of definiteness let us consider the transition \( |n \uparrow \rangle \to |\Delta^0 \uparrow \rangle \). The amplitude for this process is given by:

\[
a_{fi} = -i \int_{-\infty}^{\infty} e^{iE_{f'}t'} \langle \Delta^0 \uparrow | H_{\text{int}}(t') | n \uparrow \rangle dt'
\]  

(1)
FIG. 1: Coordinate system with magnetic field along the z direction. The projectile nucleus moving with velocity $\vec{v}$ at impact parameter $b$. The blue circle represents a nucleon at rest.

where $\hbar = 1$ and where

$$E_{fi} = \frac{m_{\Delta}^2 - m_n^2}{2m_n}$$

(2)

In the above equation $m_\Delta$ and $m_p$ are the $\Delta$ and nucleon masses respectively. The interaction Hamiltonian is given by:

$$H_{int} = -\vec{\mu}.\vec{B}$$

(3)

The magnetic dipole moment of the nucleon is given by the sum of the magnetic dipole moments of the corresponding constituent quarks:

$$\vec{\mu} = \sum_{i=u,d} \vec{\mu}_i = \sum_{i=u,d} \frac{q_i}{m_i} \vec{S}_i$$

(4)

where $q_i$ and $m_i$ are the charge and mass of the quark of type $i$ and $\vec{S}_i$ is the spin operator acting on the spin state of this quark. In Fig. 1 we show the system of coordinates and the moving projectile. The projectile moves along the $x$ direction and the magnetic field is in the $z$ direction. Because of the symmetry of the problem there is no magnetic field in the collision plane. Since we are studying an UPC, we will, for simplicity, assume that the projectile-generated field is the same produced by a point charge. The field is given by [3]:

$$B_z = \frac{1}{4\pi} \frac{qv\gamma(b - y)}{((\gamma(x - vt))^2 + (y - b)^2 + z^2)^{3/2}}$$

(5)

In the above expression $\gamma$ is the Lorentz factor, $b$ is the impact parameter along the $y$ direction, $v \simeq 1$ is the projectile velocity and the projectile electric charge is $q = Ze$.

The interaction Hamiltonian acts on spin states. The relevant ones are:

$$|p \uparrow\rangle = \frac{1}{3\sqrt{2}}[udu(\uparrow\uparrow + \downarrow\downarrow - 2 \uparrow\downarrow) + duu(\uparrow\uparrow + \downarrow\downarrow - 2 \uparrow\downarrow) + uud(\uparrow\uparrow + \downarrow\downarrow - 2 \uparrow\downarrow)]$$

(6)

$$|p \downarrow\rangle = \frac{1}{3\sqrt{2}}[udu(\downarrow\downarrow + \downarrow\downarrow - 2 \uparrow\downarrow) + duu(\downarrow\downarrow + \downarrow\downarrow - 2 \uparrow\downarrow) + uud(\downarrow\downarrow + \downarrow\downarrow - 2 \uparrow\downarrow)]$$

(7)
\[|n \uparrow\rangle = \frac{1}{3\sqrt{2}}[dud(\uparrow\uparrow + \uparrow\downarrow - 2 \uparrow\downarrow) + udd(\downarrow\downarrow + \uparrow\uparrow - 2 \uparrow\downarrow) + ddu(\uparrow\downarrow + \downarrow\uparrow - 2 \downarrow\uparrow)] \quad (8)\]

\[|n \downarrow\rangle = \frac{1}{3\sqrt{2}}[dud(\uparrow\downarrow + \downarrow\uparrow - 2 \downarrow\uparrow) + udd(\downarrow\downarrow + \uparrow\uparrow - 2 \uparrow\downarrow) + ddu(\uparrow\downarrow + \downarrow\uparrow - 2 \downarrow\uparrow)] \quad (9)\]

\[|\Delta^+ \uparrow\rangle = \frac{1}{3}(uud + udu + duu)(\uparrow\uparrow + \uparrow\downarrow + \downarrow\uparrow) \quad (10)\]

\[|\Delta^+ \downarrow\rangle = \frac{1}{3}(uud + udu + duu)(\downarrow\uparrow + \uparrow\downarrow + \downarrow\uparrow) \quad (11)\]

\[|\Delta^0 \uparrow\rangle = \frac{1}{3}(ddu + dud + udd)(\uparrow\uparrow + \uparrow\downarrow + \uparrow\downarrow) \quad (12)\]

\[|\Delta^0 \downarrow\rangle = \frac{1}{3}(ddu + dud + udd)(\downarrow\uparrow + \downarrow\downarrow + \uparrow\downarrow) \quad (13)\]

With these ingredients we can compute all the matrix elements: \(|\Delta^+ \uparrow|H_{\text{int}}|p \uparrow\rangle\), \(|\Delta^+ \downarrow|H_{\text{int}}|p \downarrow\rangle\), \(|\Delta^0 \uparrow|H_{\text{int}}|n \uparrow\rangle\) and \(|\Delta^0 \downarrow|H_{\text{int}}|n \downarrow\rangle\). The required matrix elements can be obtained by substituting Eq. (4) into Eq. (3) and then calculating the sandwiches of Eq. (3) with the spin states given above. For example, in the case of \(|\Delta^0 \uparrow|H_{\text{int}}|n \uparrow\rangle\) we have:

\[\langle \Delta^0 \uparrow | H_{\text{int}} | n \uparrow \rangle = -\frac{B_z}{m} \sum_{q=u,d} \langle \Delta^0 \uparrow | e_q S_q | n \uparrow \rangle \quad (14)\]

where we have assumed that all the light quarks have the same constituent quark mass \(m\). As the charges of the quarks up and down are known to be \(e_u = 2e/3\) and \(e_d = -e/3\), applying the \(S_z\) operator on the neutron wavefunction yields:

\[e_q S_q | n \uparrow \rangle = \frac{1}{6\sqrt{2}} \left[ \frac{2e}{3} |d^* u^* d^\uparrow\rangle + \frac{2e}{3} |d^* u^* d^\downarrow\rangle + \frac{4e}{3} |d^* u^* d^\uparrow\rangle + \frac{2e}{3} |u^* d^* d^\uparrow\rangle + \frac{2e}{3} |u^* d^* d^\downarrow\rangle \right. \]

\[\left. + \frac{8e}{3} |u^* d^* d^\uparrow\rangle + \frac{2e}{3} |d^* d^* u^\uparrow\rangle + \frac{2e}{3} |d^* d^* u^\downarrow\rangle + \frac{8e}{3} |d^* d^* u^\uparrow\rangle \right] \quad (15)\]

Multiplying the above expression by the \(\Delta\) wavefunction we obtain:

\[\langle \Delta^0 \uparrow | H_{\text{int}} | n \uparrow \rangle = -\frac{\sqrt{2}Be}{3m} \quad (16)\]

Evaluating all the possible nucleon-delta transition matrix elements we find:

\[\langle \Delta^0 \uparrow | H_{\text{int}} | n \uparrow \rangle = \langle \Delta^+ \downarrow | H_{\text{int}} | p \downarrow \rangle = -\frac{\sqrt{2}Be}{3m} \]

\[\langle \Delta^0 \downarrow | H_{\text{int}} | n \downarrow \rangle = \langle \Delta^+ \uparrow | H_{\text{int}} | p \uparrow \rangle = \frac{\sqrt{2}Be}{3m} \]

The cross section for a single \(N \rightarrow \Delta\) transition is given by:

\[\sigma = \int |a_{fi}|^2 d^2 b = 2\pi \int |a_{fi}|^2 b db \quad (17)\]
where we have used cylindrical symmetry \( d^2b = b^2 db \). Inserting the above matrix element into (11) and using it in the above expression we find:

\[
\sigma = \frac{Z^2 e^4}{9 \pi m^2} \left( \frac{E_{fi}}{v\gamma} \right)^2 \int_R^\infty \left[ K_1 \left( \frac{E_{fi} b}{v\gamma} \right) \right]^2 b db
\]

(18)

where \( K_1 \) is the modified Bessel function and \( \gamma = \sqrt{s/(2m_{\pi})} \). When the target is a nucleus all nucleons have the same squared transition amplitude. Hence the nuclear target provides a nucleon flux enhancing the cross section which is, in a first approximation, given by:

\[
\sigma_A \approx A \sigma
\]

(19)

Before presenting the numerical evaluation of (18) and (19) it is useful to make some analytical estimates. The typical strength of the magnetic fields can be roughly estimated from (5) (or taken from [4]) and is given by:

\[
eB \approx \frac{\gamma \alpha_{em} Z}{R_A^2}
\]

(20)

where \( R_A \) is the nuclear radius. The matrix elements can be approximated as:

\[
\langle \Delta | H_{int} | n \rangle = \langle \Delta | \vec{B} \cdot \vec{B} | n \rangle = \langle \Delta | \frac{eBS_z}{m} | n \rangle \approx \frac{\gamma \alpha_{em} Z}{m R_A^2}
\]

(21)

Inserting the above expression into (1), assuming that \( E_{fi} \) is small and that \( H_{int}(t') \) is different from zero only during the collision, i.e., in the time interval \( \Delta t \approx R_A \), the transition amplitude simplifies to:

\[
a_{fi} \approx \langle \Delta | \frac{eBS_z}{m} | n \rangle R_A = \frac{\gamma \alpha_{em} Z}{m R_A}
\]

(22)

Finally the single \( N \to \Delta \) transition cross section reads:

\[
\sigma \approx |a_{fi}|^2 R_A^2 \approx \left( \frac{\gamma \alpha_{em} Z}{m R_A} \right)^2 R_A^2 \approx \frac{\gamma^2 \alpha_{em} Z^2}{m^2}
\]

(23)

III. RESULTS AND DISCUSSION

Let us consider a lead (\( Z = 82 \)) - proton collision. We will use \( m = 0.36 \) GeV, \( R = 7 \) fm and \( v = 1 \). Integrating (18) and plotting it as a function of \( \sqrt{s} \) we obtain the result shown in Fig. 2. The cross section obtained in our calculations illustrates the enormous effect that the magnetic field can produce in ultraperipheral collisions. The “pocket formula” (23) gives the correct order of the magnitude.

Whenever the nucleon excitation to a delta resonance occurs, this latter will decay with 99 \% probability into a nucleon and a pion. Hence, the above number is also the cross section for pion production through \( \Delta \) decay. Since the magnetic field does not transfer any momentum to the nucleon, when it is converted to a \( \Delta \) the resonance keeps moving together with the nuclear target. When the \( \Delta \) decays, the outgoing pion has a momentum of \( p_\pi \approx 200 \) MeV in the target rest frame. Hence the pion will escape from the nucleus but will be moving in the same forward rapidity region. Our result indicates that forward pion production has a large cross section and could be observed by the ALICE collaboration in a similar way it was done for neutrons in Ref. [7].

Our calculation can be improved. In particular we should use the correct spatial charge distributions instead of the pointlike charge approximation. Moreover we should include recoil effects due to the electric field. Finally we should compute the pion rapidity and transverse momentum distributions. This would be crucial to study the possibility of detecting them. Work along this direction is already in progress.

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FIG. 2: Magnetic excitation cross section.

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