Average SER analysis of two-hop WP DF relay system under $\kappa - \mu$ shadowed fading

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Abstract
This paper investigates the performance of a two-hop decode-and-forward relaying system when the source node is energy-constrained and it harnesses energy using the radio frequency signal radiated by the relay node. Data transmission at the energy-constrained source node is enabled by the energy harvested at the node. The analysis is presented for two modulation schemes: coherent $M$-ary phase-shift keying and orthogonal non-coherent $M$-ary frequency-shift keying. Analytical and asymptotic expressions for the average symbol error rate are derived for the considered modulation schemes under $\kappa - \mu$ shadowed fading. The results are plotted to investigate the system's performance with variation in modulation order, fading parameters, and relay location.

1 | INTRODUCTION

Wireless energy harvesting (EH) has grabbed the interest of a vast group of researchers as it can facilitate seamless energy transfer to energy-constrained nodes and extend communication lifetime [1]. Radio frequency (RF) signal communicated over a wireless medium is regarded as a source of energy that can be efficiently harvested at a node using proper hardware. The energy harvested at a node decreases drastically with an increase in distance from the RF signal source. Hence, wireless powered (WP) (or RF-based EH) systems find application in networks supporting communication among nodes located in the neighbourhood of each other. In addition, relay-based communication can be useful in further extending the coverage range for the nodes which are difficult to reach otherwise. The relaying protocols commonly considered for forwarding the data are amplify-and-forward (AF) and decode-and-forward (DF) [2]. In AF protocol, the signal received at the relay is amplified before it is broadcasted by the node whereas the received data is first decoded and then re-encoded before it is communicated to the subsequent node.

1.1 | Background and motivation

The literature on WP communication systems mainly considers two approaches for performance analysis—information-theoretic and communication-theoretic [3, 4]. In the information-theoretic approach expressions for the outage probability, capacity, and/or throughput are analysed. The works in [3–9] are among the few which provide communication-theoretic analysis to deduce average bit error rate/average symbol error rate (SER). The communication-theoretic approach provides more realistic results as it involves information on modulation and demodulation techniques.

The very first work on performance analysis of a WP relay system is presented in [10]. Information-theoretic analysis of a three-node AF relay system with RF-based EH at the relay node is considered. Subsequently, work on EH at relay node(s) has been extended to systems like parallel relaying [4], multi-hop relaying [11], two-way relaying [12, 13], MIMO (multiple-input multiple-output) [14], physical layer security [15, 16] etc. Extensive research has been done on the WP relay system with EH at relay node(s). However, limited work is available on an RF-based system with EH at the source node(s) [3, 9, 17–24]. The work in [17] considers a machine-to-machine communication system for data transmission; the energy-constrained machine-type communication devices rely on the energy harvested using the RF signals transmitted by the machine-type gateway devices. Furthermore, the works in [18, 19] provide analysis for uplink transmission in cellular systems considering a hybrid relay node. The hybrid relay node provides wireless power to the energy-constrained source node and assists in forwarding source data.
to the destination. The other works available in the literature consider a general scenario of a three-node relay system [3, 9, 20–24]. The basic idea is to enable end-to-end communication when low power devices are energy-constrained. One specific application of such three-node relay systems can be in medical implants, where a low power source device (such as pacemaker, glucose sensor etc.) is implanted in a patient’s body. The implanted device sends vital information to a doctor (destination) with the help of a nearby located hub. The hub acts as a relay between the implanted device and the device at the doctor’s end. Also, it provides wireless energy to the body implant. Furthermore, these three-node WP relay systems and their variants having multiple nodes can find applications in systems including Internet of Things (IoT) entities, wireless body area network (WBAN), device-to-device (D2D) communications, machine-to-machine communications, and uplink cellular transmission.

In the literature, a variety of fading models are considered to characterize the wireless medium. Rayleigh fading model is a usual consideration for scattering environment and Rician fading is more practical in case of communication with Line-of-Sight (LoS) component. The generalized Nakagami-\(m\) channel model reduces to Rayleigh for fading parameter \(m = 1\) and approximates to Rician fading for \(m > 1\). Rayleigh, Rician, and Nakagami-\(m\) distributions model homogeneous fading environments [25]. In practice, non-homogeneous generalized fading models, including, \(\eta - \mu\) fading, \(\nu - \mu\) fading, and \(\kappa - \mu\) shadowed fading provide a better fit to the experimental data [25–27]. Using experimental setups in [26] and [27], it is demonstrated that \(\kappa - \mu\) and \(\kappa - \mu\) shadowed fading models provide a more accurate and reliable fit to WBAN and D2D communication environments, respectively. The experimental results also suggest that the generalized fading models when compared to the basic homogeneous fading models are more suitable for short-range communications. Furthermore, using the state-of-art technologies the applicability of WP systems is limited to short-range communications. Hence, it is more appropriate to analyse WP systems’ behaviour under the consideration of the generalized fading models. The listed generalized fading models reduce to the basic ones on substituting the corresponding fading parameter as shown in [28].

In the work available in the literature on performance of WP relay systems with EH at source node(s), the analysis is presented for AF and DF relay systems in [19, 20] and [3, 9, 17, 18, 21–24], respectively. The information-theoretic approach is considered in [17–24] to analyse throughput, capacity, and/or outage probability, while average SER for \(M\)-ary modulation schemes is deduced in [3, 9]. The effects of fading are ignored in [24]. Rayleigh fading environment is assumed in [9, 17, 19–23] and mixed Rician-Rayleigh fading in [18]. Recently, the average SER expressions for Nakagami-\(m\) fading is deduced in [3].

Generalized \(\eta - \mu\), \(\nu - \mu\), and \(\kappa - \mu\) shadowed fading models are considered in [29–33] to examine the performance of WP relay systems with EH at the relay node(s). To the best of our knowledge, no work on WP relay systems with EH at source node under consideration of the generalized fading models is reported in the literature. This paper considers a three-node WP relay system with a source node wirelessly powered by the relay node and links are affected by \(\kappa - \mu\) shadowed fading. The end-to-end average SEFs are deduced considering two modulation schemes, namely, \(M\)-ary phase-shift keying (\(M\)-PSK) modulation with coherent detection and orthogonal \(M\)-ary frequency-shift keying (\(M\)-FSK) modulation with non-coherent detection.

In the literature, probability density function (PDF) and moment generating function (MGF)-based approaches are mainly considered for averaging the instantaneous SER expressions. MGF is used to get moments of the corresponding random variable. It can also be applied to simplify the complicated analysis. The MGF-based approach is used for \(M\)-PSK modulated data, whereas the PDF-based approach is considered for the orthogonal \(M\)-FSK modulation scheme by this study.

1.2 Contributions

The work presented in this paper is an extension of our previous work [3], where the system is considered to be under Nakagami-\(m\) fading. In the present paper, to provide a more accurate and reliable analysis, we consider that the system is under \(\kappa - \mu\) shadowed fading. The main contributions of this paper include (i) consideration of a three-node WP relay system with EH at the source node under generalized \(\kappa - \mu\) fading, (ii) derivation for the corresponding average SEFs when data is \(M\)-PSK and orthogonal \(M\)-FSK modulated, (iii) high signal-to-noise ratio (SNR) approximation of the average SEFs to get simplified expressions, and (iv) examining the performance through numerical results with variation in different system parameters. In addition, another major contribution of this paper is PDF and MGF expressions for the product of two independent and non-identically distributed \(\kappa - \mu\) shadowed random variables.

The derived average SER expressions are novel and can be useful in investigating the system’s behaviour for a wide range of channel conditions by varying the fading parameters. Furthermore, this work enriches the theoretical aspects by filling the gaps in this domain of research.

1.3 Organization

This paper is organized into five sections. Section 2 provides a description of the system considered. The communication-theoretic approach is used to deduce performance of the system for \(M\)-ary modulation schemes in Section 3. The average SER expressions obtained in Section 3 are plotted in Section 4 for varying parameters affecting performance of the system. Concluding remarks based on the numerical results plotted in Section 4 are stated in Section 5.

2 SYSTEM MODEL

We consider a WP two-hop relay system containing an energy-constrained source node \(S\), a relay node \(R\), and a destination
node \( D \). All three nodes operate in half-duplex mode and feature a single antenna at each node. In odd time slots, node \( S \) transmits the data to node \( R \), which broadcasts processed data in the following even slot. We consider node \( R \) employs fixed DF protocol to process received data, that is, information detected at the node is forwarded over the channel. In even slots, the data transmitted by the relay \( R \) is received at nodes \( S \) and \( D \), which utilize the signals for EH and data decoding, respectively. The harvested energy is consumed by the source \( S \) for transmitting the data in subsequent (odd) slot. This cycle continues until all the data at node \( S \) are communicated to node \( D \). We consider that energy harvested at source \( S \) is processed using the harvest-use approach, that is the energy harvested in a slot is completely consumed and it cannot be stored beyond the current slot duration [1]. In addition, we consider that the circuitry for transmitting and receiving information, and processing energy at a node consumes negligible power [34]. The average SER of the system is analysed for \( M \)-PSK and orthogonal \( M \)-FSK modulated symbols. At the receiving ends, coherent detection and non-coherent detection are performed for \( M \)-PSK and orthogonal \( M \)-FSK modulation, respectively. Each symbol has the support of \([0, T_s]\) and is transmitted with equal a priori probability. We define the constellation \( S = \{S_1, ..., S_M\} \), where \( S_1, ..., S_M \) are the constellation points for the \( M \) symbols [3]. The inter-node links are independent and \( \kappa - \mu \) shadowed faded.

The baseband equivalent of received RF signals at node \( R \) in the odd slot and at nodes \( S \) and \( D \) in even slot are given by

\[
\gamma_{SR} = \sqrt{P_R T_s} \begin{pmatrix} d_{SR} \end{pmatrix}^{-\alpha_{SR}} b_{SR} x + n_R
\]

(1)

and

\[
\gamma_{RD} = \sqrt{P_R T_s} \begin{pmatrix} d_{RD} \end{pmatrix}^{-\alpha_{RD}} b_{RD} \hat{x} + n_R, \quad j = S, D,
\]

(2)

respectively. \( x \) is symbol transmitted in the odd slot with power \( P_R \) and \( \hat{x} \) is the detected symbol at node \( R \) which is then forwarded in even slot with power \( P_R \), where \( x \in S \) and \( \hat{x} \in S \). Symbols \( x \) and \( \hat{x} \) have unit energy. \( b_\varphi \) is the complex channel gain of \( \varphi \in \{SR, RS, RD\} \) link, \( (d_\varphi)^{-\alpha_\varphi} \) is path loss of \( \varphi \) link occurred at distance \( d_\varphi \) meters with path loss exponent \( \alpha_\varphi \) (typically, \( \alpha \in (2, 6) \) for wireless channels). The noise components \( n_R \) and \( n_j \) are uncorrelated and zero-mean complex Gaussian with power spectral density (PSD) \( N_0 \) watt per Hertz (W/Hz).

Here, energy harvested at node \( S \) is directly related to the transmission power \( P_R \) at node \( R \) in even time slot of the previous transmission. Therefore at the start of communication, the node \( R \) can convey an arbitrary RF signal to harvest energy at node \( S \). In case the noise component is negligible, the energy harvested at node \( S \) can be quantified using (2) and it can be approximated as [5]

\[
E_S \approx \eta P_R T_s |b_{SR}|^2 (d_{SR})^{-\alpha_{SR}}
\]

(3)

where \( \eta \) is efficiency of the energy conversion unit. Now \( E_S = P_R T_s \), thus using (3) the transmission power at node \( S \) is given by

\[
P_S \approx \eta P_R |b_{SR}|^2 (d_{SR})^{-\alpha_{SR}}.
\]

(4)

### 2.1 PDF and MGF of instantaneous SNRs

In this subsection, we derive the PDF and MGF of the instantaneous received SNRs \( \gamma_{SR} \) and \( \gamma_{RD} \) for source-relay and relay-destination links, respectively. The received SNRs are the ratios of the signal energy and noise energy at the relay and destination nodes. Using (1), (2), and (4), the instantaneous received SNRs are given as

\[
\gamma_{SR} = b_{SR} |\gamma_{SR}|^2 |b_{SR}|^2 \quad \text{and} \quad \gamma_{RD} = b_{RD} |\gamma_{RD}|^2
\]

(5a)

respectively, where \( b_{SR} = \eta P_R T_s / (N_0 d_{SR}^{\alpha_{SR}} + \sigma_{SR}) \) and \( b_{RD} = \eta P_R T_s / (N_0 d_{RD}^{\alpha_{RD}} + \sigma_{RD}) \). The corresponding average SNRs are \( \bar{\gamma}_{SR} = E[\gamma_{SR}] = b_{SR} \lambda_{SR} \alpha_{SR} \) and \( \bar{\gamma}_{RD} = E[\gamma_{RD}] = b_{RD} \lambda_{RD} \alpha_{RD} \).

\( E[|b_\varphi|^2] \) is mean power of the channel gain \( b_\varphi \). In general, the uplink and downlink channels in time-division duplexing (TDD) are assumed to be reciprocal. However, owing to non-reciprocity problems, the channel conditions can vary for different slots [35]. We consider the channel conditions of \( RS \) and \( SR \) links in consecutive time slots can vary independently and, hence, the channel gains \( b_\varphi \) and the path loss exponents \( \alpha_{RD} \) and \( \alpha_{SR} \) are regarded as different. The PDF of power variable for \( \kappa - \mu \) shadowed faded link gains are given by [36, Equation 4]

\[
f_{|b_\phi|^2}(|b_\phi|^2) = \frac{\mu_\phi^{\mu_\phi} \mu_\phi^{-\mu_\phi} (1 + \kappa_\phi)^\mu_\phi}{\Gamma(\mu_\phi) \lambda_\phi(\mu_\phi \kappa_\phi + \mu_\phi)^\mu_\phi} \times \left( \frac{|b_\phi|^2}{\lambda_\phi} \right)^{\mu_\phi-1} \exp \left( -\frac{\mu_\phi(1 + \kappa_\phi)|b_\phi|^2}{\lambda_\phi} \right) \times \frac{1}{\Gamma(\mu_\phi)} \left( \frac{\mu_\phi^{\mu_\phi} \mu_\phi^{-\mu_\phi} (1 + \kappa_\phi)^\mu_\phi}{(\mu_\phi \kappa_\phi + \mu_\phi)^\mu_\phi}, \right)
\]

(6)

where \( \kappa_\phi, \mu_\phi, \mu_\phi \) are fading parameters of link \( \phi \), \( \lambda_\phi = E[|b_\phi|^2] \) is the mean power of \( |b_\phi|^2 \), and \( \Gamma(\cdot) \) represents confluent hypergeometric function [38]. Parameter \( \kappa_\phi \) is ratio of the total power of the LOS components and the total power of non-LOS components, parameter \( \mu_\phi \) corresponds to the number of clusters, and parameter \( m_\phi \) captures the shadowing effect of the fading environment. Statistical characterization of \( \kappa - \mu \) shadowed fading is elaborated in [36].

Deriving the average SER using the PDF in (6) requires dealing with the integration of integrands including confluent hypergeometric function. This makes it difficult to obtain...
closed-form expressions of average SERs. To simplify the derivation, we use the series form representation of the PDF given in [37]. In [37], the series form representation of confluent hypergeometric function (38) is used to get the PDF as

\[ f_{|b_{SR}|^2}(|b_{SR}|^2) = \sum_{i=0}^{\infty} p_{\varphi,\mu_{\varphi}}(\varphi, \mu_{\varphi}) i^{\lambda_{\varphi}} \times |b_{SR}|^{2(i+\mu_{\varphi}-1)} \exp(-\varphi |b_{SR}|^2), \]  

(7)

where \( \varphi \) and \( \mu_{\varphi} \) are given by

\[ \varphi = \frac{\mu_{\varphi}(1 + \mu_{\varphi})}{\lambda_{\varphi}} \]  

and

\[ p_{\varphi,\mu_{\varphi}} = \frac{(m_{\varphi})_i}{\Gamma(m_{\varphi})_i} \left( \frac{\mu_{\varphi} \mu_{\varphi} + m_{\varphi}}{\mu_{\varphi} \mu_{\varphi} + m_{\varphi}} \right)^i \left( \frac{m_{\varphi}}{\mu_{\varphi} \mu_{\varphi} + m_{\varphi}} \right)^m \]

respectively, with \( n = 1 \) for \( \varphi = R \), \( n = 2 \) for \( \varphi = SR \), and \( n = 3 \) for \( \varphi = RD \). \( \chi_0 = \Gamma(\alpha + 1/\Gamma(\alpha)) \) represents Pochhammer symbol. The infinite series PDF given in (7) can be approximated to a series containing finite number of terms, provided the approximation error is below a desired value. For instance, the approximated PDF with \( N_n \) terms is given by

\[ f_{|b_{SR}|^2}^{N_n}(|b_{SR}|^2) = \sum_{i=0}^{N_n} p_{\varphi,\mu_{\varphi}}(\varphi, \mu_{\varphi}) i^{\lambda_{\varphi}} \times |b_{SR}|^{2(i+\mu_{\varphi}-1)} \exp(-\varphi |b_{SR}|^2) \]  

(8)

and the corresponding approximation error can be expressed as

\[ \epsilon_{N_n} < f_{|b_{SR}|^2}(|b_{SR}|^2) - f_{|b_{SR}|^2}^{N_n}(|b_{SR}|^2) \]

\[ = \sum_{i=N_n+1}^{\infty} p_{\varphi,\mu_{\varphi}}(\varphi, \mu_{\varphi}) i^{\lambda_{\varphi}} \times |b_{SR}|^{2(i+\mu_{\varphi}-1)} \exp(-\varphi |b_{SR}|^2). \]  

(9)

It is shown in Appendix A.2 that the series given in (8) converges with an increase in \( N_n \). Equation (A.17) can be utilized to numerically evaluate the desired number of summation terms.

Next, the SNR \( \gamma_{SR} \) in (5a) is a scaled version of the product of two independent random variables \( |b_{RS}|^2 \) and \( |b_{SR}|^2 \). Its CDF (cumulative distribution function) can be obtained as

\[ F_{\gamma_{SR}}(\gamma_{SR}) = \Pr[|b_{SR}|^2|b_{RS}|^2 \leq \gamma_{SR}] \]

\[ = \int_0^\infty \Pr[|b_{SR}|^2 \leq \left( \frac{\gamma_{SR}}{b_{RS}} \right)] f_{|b_{SR}|^2}(t) dt \]

\[ = \int_0^\infty \int_0^{\gamma_{SR}/b_{RS}} f_{|b_{SR}|^2}(\gamma_{SR}/b_{RS} | t) f_{|b_{SR}|^2}(t) dt. \]  

(10)

In the above relation, we have substituted \( |b_{RS}|^2 = t \) without actually deviating from the final result. Hence, the PDF of \( \gamma_{SR} \) can be obtained by taking the first-order derivative of (10) with respect to \( \gamma_{SR} \) as

\[ f_{\gamma_{SR}}(\gamma_{SR}) = \int_0^\infty \frac{1}{b_{RS}} f_{|b_{SR}|^2}(\gamma_{SR}/b_{RS}) f_{|b_{SR}|^2}(t) dt. \]  

(11)

Now, on substituting (8) in (11) and interchanging the order of summations and integration, we have

\[ f_{\gamma_{SR}}(\gamma_{SR}) = \frac{1}{b_{SR}} \sum_{i=0}^{N_1} \sum_{j=0}^{N_2} p_{\gamma_{SR}}(\gamma_{SR}) i^{\lambda_{\gamma_{SR}}} \times (\gamma_{SR})^{\lambda_{\gamma_{SR}}} \times \int_0^\infty t^{\lambda_{\gamma_{SR}}-1} \exp\left( - \frac{\gamma_{SR} \gamma_{SR}}{b_{SR}} - \frac{\gamma_{SR}}{b_{SR}} \right) dt, \]  

(12)

where \( \gamma = l_s - l_i + \mu_{SR} - \mu_{RS} \). Using (39), the PDF in (12) can be expressed as

\[ f_{\gamma_{SR}}(\gamma_{SR}) = \frac{2a_{SR}}{\gamma_{SR}} \sum_{i=0}^{N_1} \sum_{j=0}^{N_2} \left( \frac{a_{SR} \gamma_{SR}}{\gamma_{RD}} \right)^{\lambda_{\gamma_{SR}}-1} \times (p_{\gamma_{SR}}) K_{\lambda_{\gamma_{SR}}-1} \left( \frac{a_{SR} \gamma_{SR}}{\gamma_{RD}} \right), \]  

(13)

where \( a_{SR} = \mu_{RS} \mu_{SR}(1 + \kappa_{SR}) \) and \( K_{\gamma}(\gamma) \) is the \( \gamma \)-order modified Bessel’s function of the second kind. Here, \( a_{SR} \) is obtained using the expression for \( \varphi \) given in (7). The SNR \( \gamma_{RD} \) is a scaled version of \( |b_{RS}|^2 \). Hence, using the property of the random variables, the PDF of \( \gamma_{RD} \) is obtained using (5b) and (8) as

\[ f_{\gamma_{RD}}(\gamma_{RD}) = \frac{a_{RD}}{\gamma_{RD}} \sum_{i=0}^{N_1} \left( \frac{a_{RD} \gamma_{RD}}{\gamma_{RD}} \right)^{\lambda_{\gamma_{RD}}-1} \times (p_{\gamma_{RD}}) \exp\left( - \frac{a_{RD} \gamma_{RD}}{\gamma_{RD}} \right), \]  

(14)

where \( a_{RD} = \mu_{RD}(1 + \kappa_{RD}) \).

Next, the MGF of the instantaneous SNR \( \gamma_{SR} \) is given by [5, Equation (17)]

\[ M_{\gamma_{SR}}(\gamma_{SR}) = \int_0^\infty \exp(-\gamma_{SR} \gamma_{SR}) f_{\gamma_{SR}}(\gamma_{SR}) d\gamma_{SR}. \]  

(15)

Hence, the MGF of \( \gamma_{SR} \) can be obtained by substituting (13) in (15). On interchanging the order of summations and integration
the MGF is given by

\[
M_{y_{SR}}(\zeta) = \frac{2d_{SR}}{y_{SR}} \sum_{l_1=0}^{N_1} \sum_{l_2=0}^{N_2} p_{RS,l_1} p_{SR,l_2} \left( \frac{d_{SR}}{y_{SR}} \right)^{(\zeta-1)} 
\times \int_0^\infty y_{SR}^{\zeta-1} \exp \left( -y_{SR} \right) y_{SR} \left( 2\sqrt{2y_{SR}} \right) d\gamma_{SR},
\tag{16}
\]

where \( \zeta = (l_1 + \mu_{RS} + l_2 + \mu_{SR})/2 \) and \( \alpha' = (l_2 + \mu_{SR} - l_1 - \mu_{RS}) \). Substituting \( y_{SR} = r^2 \) and using (A.3), the MGF in (16) is simplified as

\[
M_{y_{SR}}(\zeta) = \frac{2d_{SR}}{y_{SR}} \sum_{l_1=0}^{N_1} \sum_{l_2=0}^{N_2} p_{RS,l_1} p_{SR,l_2} \Gamma(\mu_{RS} + l_1) \Gamma(\mu_{SR} + l_2) 
\times \left( \frac{d_{SR}}{y_{SR}} \right)^{l_1+\mu_{RS}+l_2} \exp \left( -\frac{d_{SR}}{y_{SR}} \right) 
\times W_{1-\mu_{RS}+l_1-\mu_{SR}+l_2}^{\mu_{SR}+l_2-\mu_{RS}-l_1} \left( \frac{d_{SR}}{y_{SR}} \right),
\tag{17}
\]

where \( W_{\psi}(\cdot) \) is Whittaker’s function. Similarly, on putting (14) in (15) and using (A.4), the MGF of \( y_{RD} \) is written as

\[
M_{y_{RD}}(\zeta) = \sum_{l_0=0}^{N_3} p_{RD,l_0} \Gamma(\mu_{RD} + l_0) \left( 1 + \frac{a_{RD}}{a_{RD}} \right)^{-l_0-\mu_{RD}}.
\tag{18}
\]

\section{Performance Analysis}

The end-to-end average SER of the system can be given by [3, Equation (19)]

\[
P_o = P_{o,SR} + P_{o,RD} - P_{o,SR}P_{o,RD} - \sum_{\nu=1}^{M-1} p_{\nu} p_{\nu}^o,
\tag{19}
\]

where \( P_{o,\nu} \) is the average SER and \( P_{P,\nu}^o \) for \( \nu = \{1, \ldots, M-1\} \) are paired error probability when a symbol is mapped to one of the remaining \( M-1 \) symbols [38] for \( \nu \) link.

\subsection{Analytical end-to-end average SER}

\subsection{M-PSK modulation}

Using [3, Equations (20) and (21)], the error terms \( P_{o,\nu} \) and \( P_{P,\nu}^o \) in (19) can be represented in integral form as

\[
P_{o,\nu} = \frac{1}{\pi} \int_0^{\phi_1} M_{\nu} \left( \frac{g_{\nu}}{\sin^2 \theta} \right) d\theta
\quad \text{and}
\tag{20}
\]

\[
P_{P,\nu}^o = \frac{1}{2\pi} \int_0^{\phi_1} M_{\nu} \left( \frac{g_{\nu}}{\sin^2 \theta} \right) d\theta,
\tag{21}
\]

respectively, where \( \phi_0 = \pi(M-1)/M \), \( g_0 = \sin^2(\pi - \phi_0) \), \( \phi_1 = (\pi - 2\pi\nu/M + \pi/M) \), \( g_1 = \sin^2(\pi - \phi_1) \), \( \phi_2 = (\pi - 2\pi\nu/M - \pi/M) \), \( g_2 = \sin^2(\pi - \phi_2) \), and \( M_{\nu} \) is the MGF of \( y_{\nu} \). On putting (20) and (21) in (19) and using MGFs (17) and (18), the end-to-end average SER for M-PSK modulation scheme under \( \kappa - \mu \) shadowed fading can be evaluated. Numerical evaluation of integrals in the expression is done using MATLAB.

\subsection{Orthogonal M-FSK modulation}

The average SER in (19) reduces to [3, Equation (27)] for orthogonal M-FSK modulated data, that is

\[
P_o = P_{o,SR} + P_{o,RD} - \frac{M}{M-1} P_{o,SR}P_{o,RD}.
\tag{22}
\]

Equation (22) requires the computation of \( P_{o,\nu} \), which can be done by averaging the conditional SER [3, Equation (28)]. The error term \( P_{o,\nu} \) can be analysed by taking expectation of the conditional SER (A.5) with respect to the PDF in (17). The simplified form of expression for \( P_{o,\nu} \) can be obtained using (A.3) and following the steps used in deriving (17), we get

\[
P_{o,\nu} = \sum_{l_1=0}^{N_1} \sum_{l_2=0}^{N_2} \sum_{\psi=1}^{M-1} (-1)^{\nu+1} \left( \frac{M-1}{\psi + 1} \right) \left( p_{RS,l_1} p_{SR,l_2} \right)
\times \Gamma(\mu_{RS} + l_1) \Gamma(\mu_{SR} + l_2) \exp \left( \frac{(\nu + 1)a_{SR}}{\psi_{SR}} \right)
\times \left( \frac{(\nu + 1)a_{SR}}{\psi_{SR}} \right)^{-l_1+1-\mu_{SR}+l_2} 
\times W_{1-\mu_{RS}+l_1-\mu_{SR}+l_2}^{\mu_{SR}+l_2-\mu_{RS}-l_1} \left( \frac{(\nu + 1)a_{SR}}{\psi_{SR}} \right).
\tag{23}
\]

On averaging (A.5) using the PDF of \( y_{RD} \) (18) and (A.4), the error term \( P_{o,RD} \) in (22) is given by

\[
P_{o,RD} = \sum_{l_0=0}^{N_3} \sum_{\psi=1}^{M-1} (-1)^{\nu+1} \left( \frac{M-1}{\psi + 1} \right) \Gamma(\mu_{RD} + l_0)
\times p_{RD,l_0} \left( 1 + \frac{\psi_{RD}}{\psi + 1} \right)^{-l_0-\mu_{RD}}.
\tag{24}
\]

On substituting (23) and (24) in (22), the end-to-end average SER for orthogonal M-FSK modulation scheme under \( \kappa - \mu \) shadowed fading can be obtained.
Note that the deduced exact average SER expressions in this subsection involve either computation of Whittaker’s function or its integration, which is complex and time-consuming. In order to simplify expressions for the average SER, high SNR approximations are obtained.

### 3.2 Asymptotic end-to-end average SER

In order to simplify the expressions of average SER at high SNRs asymptotic approximations are obtained. The average SER in (19) can be approximated as

$$P_{e}^{\infty} = P_{eSR}^{\infty} + P_{eRD}^{\infty}$$  \hspace{1cm} (25)

where $P_{e}^{\infty}$, $P_{eSR}^{\infty}$, and $P_{eRD}^{\infty}$ are high SNR approximations of $P_{e}$, $P_{eSR}$, and $P_{eRD}$, respectively. The approximations for the two modulation schemes are deduced in this subsection.

#### 3.2.1 M-PSK modulation

To obtain $P_{e}^{\infty}$ in (25), an approximate expression of the PDF $f_{YSR}(Y_{SR})$ is used to get $P_{eSR}^{\infty}$. The PDF in (13) has a series containing $\mathcal{K}(\cdot)$, which can be approximated using (43) for the two cases: (a) $r = 0$ and (b) $r > 0$, where $r = (l_{2} + \mu_{SR} - l_{1} - \mu_{RD})$. Hence, the approximate expressions of the PDFs for the cases (a) and (b) are

$$f_{YSR}(Y_{SR}) \approx \sum_{S_{1}} \sum_{S'_{2}} (p_{RCA} p_{SR,l} p_{RD})^{\mu_{p}}$$

$$\times Y_{SR}^{\mu_{p} - 1} \ln \left( \frac{\gamma_{SR}}{4\sigma_{SR}^{2} Y_{SR}} \right)$$  \hspace{1cm} (26)

and

$$f_{YSR}(Y_{SR}) \approx \sum_{S'_{1}} \sum_{S'_{2}} (p_{RCA} p_{SR,l} p_{RD})^{\mu_{p}}$$

$$\times Y_{SR}^{\mu_{p} - 1} \ln \left( \frac{\gamma_{SR}}{4\sigma_{SR}^{2} Y_{SR}} \right)$$  \hspace{1cm} (27)

respectively, where $S_1$ and $S_2$ are sets of values for $l_1$ and $l_2$ when $r = 0$, sets $S'_1$ and $S'_2$ correspond to the values of $l_1$ and $l_2$ for $r > 0$, $\mu_{p} = (\mu_{RS} + l_{1} + \mu_{SR} + l_{2})/2$, and $\mu_{p} = (\mu_{RD} + l_{1} + \mu_{RS} + l_{2} - |\mu_{SR} + l_{1} - \mu_{RS} - l_{1}|)/2$. The PDFs in (26) and (27) can be unified as

$$f_{YSR}(Y_{SR}) \approx \sum_{S_{1} \cup S'_{1}} \sum_{S_{2} \cup S'_{2}} (p_{RCA} p_{SR,l} p_{RD})^{\mu_{p}}$$

$$\times \left( \frac{\sigma_{SR}}{\gamma_{SR}} \right)^{\mu_{p}} Y_{SR}^{\mu_{p} - 1} \ln \left( \frac{\gamma_{SR}}{4\sigma_{SR}^{2} Y_{SR}} \right)$$  \hspace{1cm} (28)

where $\epsilon$ is Euler’s number, $\mu_{p} = \mu_{p} + \rho_{1} = 0$, and $\rho_{2} = 1$ for case (a), and $\mu_{p} = \mu_{p} + \rho_{2} = 1$, and $\rho_{2} = 0$ for case (b). The representation $\{S_{n}, S'_{n}\}$ suggests for the selection of cases (a) and (b) depending on the values of $l_1$ and $l_2$.

Putting (28) in (15) and applying (A.7), the approximate expression for MGF is given by

$$M_{YSR}(\epsilon) \approx \sum_{S_{1} \cup S'_{1}} \sum_{S_{2} \cup S'_{2}} (p_{RCA} p_{SR,l} p_{RD})^{\mu_{p}}$$

$$\times \left( \frac{\sigma_{SR}}{\gamma_{SR}} \right)^{\mu_{p}} \left( 1 + \rho_{2} \ln \left( \frac{\gamma_{SR}}{4\sigma_{SR}^{2} Y_{SR}} \right) - \rho_{2} \psi(\mu_{p}) \right),$$  \hspace{1cm} (29)

where $\psi(\cdot)$ is digamma function. Substituting (29) in (20), the approximate average SER $P_{eSR}^{\infty}$ is

$$P_{eSR}^{\infty} \approx \frac{1}{\pi} \sum_{S_{1} \cup S'_{1}} \sum_{S_{2} \cup S'_{2}} (p_{RCA} p_{SR,l} p_{RD})^{\mu_{p}}$$

$$\times \Gamma(\mu_{p}) \left( 1 + \rho_{2} \ln \left( \frac{\gamma_{SR}}{4\sigma_{SR}^{2} Y_{SR}} \right) - \rho_{2} \psi(\mu_{p}) \right)$$

$$\times \left( \frac{\sigma_{SR}}{\gamma_{SR}} \right)^{\mu_{p}} Y_{SR}^{\mu_{p} - 1}$$  \hspace{1cm} (30)

where $I = \int_{0}^{\theta} (\sin^{2}(\theta))^{\mu_{p}} d\theta$ and $J = \int_{0}^{\theta} (\sin^{2}(\theta))^{\mu_{p}} d\theta$. Integrals $I$ and $J$ can be numerically evaluated in MATLAB or approximately solved using [5, Equation (22)], (A.8), and (A.9) in terms of the beta function and digamma function.

For $\frac{\sigma_{SR}}{\gamma_{SR}} \gg 1$, (18) can be approximated as

$$M_{YSR}(\epsilon) \approx \sum_{l_{1}=0}^{N_{S}} p_{POD,3} \Gamma(\mu_{RD} + l_{3}) \left( \frac{a_{RD}}{\gamma_{SR}^{\mu_{RD} + l_{3}}} \right).$$  \hspace{1cm} (31)

Substituting (31) in (20), we get

$$P_{eSR}^{\infty} \approx \frac{1}{\pi} \sum_{l_{1}=0}^{N_{S}} p_{POD,3} \Gamma(\mu_{RD} + l_{3}) \left( \frac{a_{RD}}{\gamma_{SR}^{\mu_{RD} + l_{3}}} \right) \left( \kappa \right),$$  \hspace{1cm} (32)

where $\kappa = \int_{0}^{\theta} (\sin^{2}(\theta))^{\mu_{RD} + l_{3}} d\theta$ can be simplified by approximating it in terms of beta function using [5, Equation (22)] and (A.8).

The high SNR approximation of the average SER can be found on replacing (30) and (32) in (25). Thus

$$P_{e}^{\infty} \approx \frac{1}{\pi} \sum_{l_{1}=0}^{N_{S}} A(l_{1}, l_{2}) \left( B(l_{1}, l_{2}) + \rho_{2} \ln(Y_{SR}) \right) I - \rho_{2} J$$

$$+ \sum_{l_{1}=0}^{N_{S}} C(l_{3}) \kappa$$  \hspace{1cm} (33)
where
\[
A(l_1, l_2) = \frac{p_{RSA} p_{SR2} (\Gamma(\mu_r)) \Gamma(\mu_r)}{\pi} \left( \frac{a_{SR}}{g_{SR}} \right)^{\mu_r},
\]
\[
B(l_1, l_2) = 1 + \rho_2 \ln \left( \frac{\delta_0}{4 \delta a_{SR}} \right) - \rho_2 \psi(\mu_r), \quad \text{and}
\]
\[
C(l_3) = \frac{p_{RD} \Gamma(\mu_{RD} + l_3)}{\pi} \left( \frac{a_{RD}}{g_{RD}} \right)^{\mu_{RD} + \delta_1}.
\]

We observe that the integrals \( I, J, \) and \( K \) in the asymptotic average SER (33) can be simplified in terms of beta and digamma functions, which are inbuilt functions in mathematical software and can be represented efficiently in series form. There is no need to compute time-consuming numerical integration as in (20) and (21) for the exact analysis. This implies the asymptotic analysis is considerably simplified in terms of computational efficiency.

### 3.2.2 Orthogonal M-FSK modulation

For orthogonal \( M \)-FSK modulated data, \( P^\infty_{e,SR} \) is derived by taking expectation of the conditional SER (A.4) using the PDF in (28) and (A.6) as
\[
P^\infty_{e,SR} \approx \sum_{l_1=0}^{N_1} \sum_{l_2=0}^{N_2} \left[ \frac{p_{RSA} p_{SR2} (\Gamma(\mu_r)) \Gamma(\mu_r)}{\pi} \left( \frac{a_{SR}}{g_{SR}} \right)^{\mu_r} \right]
\]
\[
\times \sum_{l=1}^{M-1} \frac{(-1)^{l+1} (M-1)}{v+1} \left( \frac{v+1}{v} \right)^{\mu_r}
\]
\[
\times \left( 1 + \rho_2 \ln \left( \frac{\varepsilon_{SR}}{4 (v+1) a_{SR}} \right) - \rho_2 \psi(\mu_r) \right).
\]

For \( \varepsilon_{RD}/((v+1)\mu_{RD}) \gg 1, \) (24) is reduced to
\[
P_{e,RD} = \sum_{l_1=0}^{N_1} \sum_{l_2=0}^{N_2} \frac{(v+1)}{v} \left( \frac{v+1}{v} \right)^{\mu_r}
\]
\[
\times \left( \frac{v+1}{v} \right)^{\mu_{RD} + \delta_1}.
\]

On putting (34) and (35) in (25), the high SNR approximation of the end-to-end average SER is given by
\[
P^\infty_{e,SR} \approx \sum_{l_1=0}^{N_1} \sum_{l_2=0}^{N_2} \left[ A(l_1, l_2; M) + \rho_2 B(l_1, l_2; M) \ln(\varepsilon_{SR}) \right]
\]
\[
+ \sum_{l_3=1}^{\infty} \frac{C(l_3; \varepsilon_{RD})}{(\varepsilon_{RD})^{\mu_{RD} + \delta_1}},
\]
where
\[
A(l_1, l_2; M)
\]
\[
= B(l_1, l_2; M) \left( 1 + \rho_2 \ln \left( \frac{v(v+1)^{-1}}{4 \delta a_{SR}} \right) - \rho_2 \psi(\mu_r) \right),
\]
\[
B(l_1, l_2; M) = (p_{RSA} p_{SR2}) \Gamma(\mu_r) \left( \frac{a_{SR}}{g_{SR}} \right)^{\mu_r}
\]
\[
\times \sum_{l=1}^{M-1} \frac{(-1)^{l+1} (M-1)}{v+1} \left( \frac{v+1}{v} \right)^{\mu_r},
\]
and
\[
C(l_3; M) = (p_{RD}) \Gamma(\mu_{RD} + l_3) \sum_{l=1}^{M-1} \frac{(-1)^{l+1} (M-1)}{v+1} \left( \frac{v+1}{v} \right)^{\mu_r}
\]
\[
\times \left( \frac{v+1}{v} \right)^{\mu_{RD} + \delta_1}.
\]

We further observe that the asymptotic average SER (36) does not contain Whittaker’s function as in (23). Hence, the average SER for orthogonal \( M \)-FSK modulated data can be computed efficiently using (36).

### 3.3 Diversity order (DO)

The DO for any communication system is described as the order of improvement in the performance with an increase in SNR. Generally, it is quantified as the negative slope of the average SER curve at high SNR on the log–log axis. The DO of the considered system can be analysed on substituting the approximate expressions of the average SER is given by (33) and (36) in \( \text{DO} = -\lim_{\gamma \to \infty} \ln P^\infty_{e,SR}/\ln \gamma \) [5]. Since the expressions for asymptotic average SERs is a sum of two error terms \( P^\infty_{e,SR} \) and \( P^\infty_{e,RD} \), the DO is dominated by the one with the smallest order of \( \gamma \) in the denominator. Hence, we have
\[
\text{DO} = \min \{ \text{DO}_1 + \text{DO}_2 \},
\]
where \( \text{DO}_1 = -\lim_{\gamma \to \infty} \ln P^\infty_{e,SR}/\ln \gamma \) and \( \text{DO}_2 = -\lim_{\gamma \to \infty} \ln P^\infty_{e,RD}/\ln \gamma \). Furthermore, the error factors \( P^\infty_{e,SR} \) and \( P^\infty_{e,RD} \) are in series form. Only the first term (corresponding to \( l_1 = 0 \)) of these two has the smallest power of \( \gamma \) in the denominator. Hence, only these first two terms would add to the resultant DO. Considering \( \ln(\gamma) \) is sufficiently larger than the other factors in \( P^\infty_{e,SR} \) and after doing some algebraic manipulations, we get
\[
\text{DO} = \min \left\{ (\mu_r - \rho \ln(\gamma)) \right\},
\]
where \( \mu_r = (\mu_{SR} + \mu_{RS})/2, \rho = 1 \) for case (a) and \( \rho = 0 \) for case (b). At significantly high SNRs, the system possesses DO = \( \min(\mu_r, \mu_{RD}) \).
4 | NUMERICAL RESULTS

We present plots for the average SER of the system described in Section 2. The suitable ranges of various parameters considered for the analysis. It is evident that $\kappa - \mu$ shadowed fading provides flexibility to analyse the system’s performance for a wide range of channel conditions attributed by the fading parameters $\kappa_\varphi$, $\mu_\varphi$, and $m_\varphi$. In this section, we choose specific values of these fading parameters to visualize their impact on the performance. However [27, Table I], and other experimental results could be of more help while considering a specific scenario. Furthermore, the transmission power of low-power devices can be in the range of micro Watts (\(\mu\)W) and high as a few Watts. The noise PSD is usually considered in the range of $-70$ to $-120$ dBm and transmission bandwidth falls in the range of MHz. The path loss exponents for wireless communication can be considered in the range of 2–6.

The results are obtained considering power transmitted at node $R$ is $P_R \in [-10, 30]$ dBm ($P_R \in [100, \mu W, 1 W]$), noise PSD $N_0 = -70$ dBm ($N_0 = 10^{-10}$ W/Hz), and symbol bandwidth $B$ (in MHz) (commonly used in wireless systems). Assuming $B = 1$ MHz (symbol duration $T_s = 1/B$ is in range of microseconds), the noise power is $10^{-3}$ W. Furthermore, we consider path loss exponent of each link is $\alpha_\varphi = 3$ and distance $d_\varphi = 1$ m, unless otherwise stated. The variance of channel gains is taken as unity, that is, $\lambda_\varphi = 1$.

The average SER expression for $M$-PSK modulation scheme is deduced as (19), where the terms are defined in (20) and (21) with MGFs in (17) and (18). In the case of orthogonal $M$-FSK modulation scheme, the average SER is obtained on putting (23) and (24) in (22). The factors $P_{\varphi,SR}$ and $P_{\varphi,RD}$ in the average SER expression are represented in a series form which converges faster with an increase in the number of terms. It is observed through numerical evaluation in MATLAB that for an increase in $\kappa_\varphi \mu_\varphi / m_\varphi$, more number of terms are required in (8) to get close to the actual PDF. Typically, for the fading parameters considered in this paper, one would get the desired PDF using (8) when the number of terms is around 100. Furthermore, the average SER expressions are obtained on evaluating the integrals involving the PDF (8). It is observed that the required number of summation terms reduces after each integration. Let $P_1 = (N_1 N_2)$ and $N_3$ terms are required for the computation of $P_{\varphi,SR}$ and $P_{\varphi,RD}$, respectively. The required number of terms in the series varies with change in the value of fading parameters $\kappa_\varphi$, $\mu_\varphi$, and $m_\varphi$. Also, for $M$-PSK and orthogonal $M$-FSK modulations different values of $P_1$ and $N_3$ are required to attain the same level of accuracy. Without going into details of the computation for the required number of terms depending on various parameters, analytical results are obtained which ensure the accuracy up to the fifth decimal point with respect to the simulation results. For $M$-PSK and orthogonal $M$-FSK, (17) and (23), respectively, are twofold summations. The number of terms $P_1$ is equally divided for the two summations, that is, the upper limit of each summation is $N_1 = N_2 = P_1 / 2 = 10$. Using (26) and (27), asymptotic average SERs are also evaluated and plotted to check accuracy of the approximation.

In Figures 1 and 2, plots of the average SER versus relay transmission power $P_R$ are presented for the two modulation schemes. The plots are shown for different modulation order and the fading parameters. In Figure 1, the results are plotted for parameters $\kappa_\varphi = 1$, $\mu_\varphi = 1$, and $m_\varphi = 1$ and the parameters $\kappa_\varphi = 1$, $\mu_\varphi = 2$, and $m_\varphi = 3$ are considered in Figure 2, $\varphi \in \{SR, RS, RD\}$. The analytical results of the two modulation schemes closely match with the simulation results for all modulation orders. Hence, our analysis of the system is validated. Moreover, it is found that the asymptotic results at high $P_R$ (directly related to SNR) provide a good approximation. Furthermore, it can be observed from the plots $M$-PSK performs better than orthogonal $M$-FSK for $M = 2$ and $M = 4$. However, for $M = 8$, $M$-FSK has a slight edge in performance when compared to $M$-PSK. Because for the same average energy, degradation in performance with an increase in modulation order is high in case of $M$-PSK than orthogonal $M$-FSK. Thus...
as expected M-PSK outperforms M-PSK for higher modulation order. This observation is more distinguishable for the larger value of the fading parameters as seen in Figure 2.

Note that the terms $P_{e,SR}^\infty$ and $P_{e,RD}^\infty$ in the asymptotic average SER expressions should be between zero and unity (∈ [0, 1]). However, due to the approximation involved in deriving these expressions, they may violate the axiom by taking values that are negative or more than unity at low SNRs and/or with variation in different parameters. Specifically, at low SNRs, the term $P_{e,SR}^\infty$ can be negative, whereas a positive value greater than unity can be observed for $P_{e,RD}^\infty$. The term $P_{e,SR}^\infty$ results in negative values when the factors $-\psi_2(\mu_e) + J$ and $-\psi_2(\mu_e)$ in (30) and (34), respectively dominate over the other factors. Moreover, at low SNRs, the factors ln($\delta_{SR}/(4\alpha_{e,SR})$) and ln($\delta_{SR}/(4\psi_{e,SR})$) in (30) and (34), respectively can be negative, which further adds in overall negative value of $P_{e,SR}^\infty$. On the other side, the term $P_{e,RD}^\infty$ is always positive but it can be greater than unity depending on the values of SNR and other parameters due to the factors $\delta_{RD}/(\delta_{SR})^{\mu_{RD}+i}$ and $(\psi_{SR}^{\infty}/(\psi_{SR}^{\infty}))^{\mu_{RD}+i}$ in (32) and (35), respectively. Thus, the asymptotic average SER expression obtained on summing the terms $P_{e,SR}^\infty$ and $P_{e,RD}^\infty$ for the considered modulation schemes at low SNRs may be less than zero or greater than unity, depending on which one of the terms dominates over the other. The same can be seen in Figures 1 and 2. In Figure 1, the resultant summation is found to be greater than unity at low SNRs, which implies that $P_{e,SR}^\infty$ dominates over $P_{e,RD}^\infty$. As the average SER becomes greater than unity, it cannot be shown in the figure having y-axis range between zero and unity. The irregular shape of the curve at medium SNRs is due to the transition from low to high SNR region. Similarly, in Figure 2, as $P_{e,SR}^\infty$ dominates over $P_{e,RD}^\infty$ at low SNRs, the resultant average SER is negative. The negative values cannot be shown in the figure as it corresponds to positive values only.

In Figure 3, we plot average SER versus source-to-relay distance of 4-PSK for variation in $\mu_e$, $\varphi \in \{SR, RS, RD\}$. We consider that nodes are located on a plane satisfying the distance relation $d_{SR} + d_{RD} = \beta d_{SD}$, for $\beta \geq 1$. We assume $\beta = 1.1$. Let $\mu_RS = \mu_RS, \mu_RS = \mu_RS$, and $m_RS = m_RS$ for case (a). In Figure 3(a), we consider fading parameters $\mu_{RD} = 2, \mu_{e,SR} = 2, \mu_{e,SR} = 2$ and plot average SER for varying $\mu_e \in \{1, 2, 3\}$ when other fading parameters are constant, that is, $\mu_e = 2, \mu_e = 2, m_e = 2$. The relay location for optimal performance moves closer to the source with gain in $\mu_{RD}$. Thus we conclude that the optimal relay location is closer to the node with poor quality in order to facilitate reliable communication between the nodes. Alternatively, we can also say that (i) for $d_{SR} < 1$ (relay is located closer to source than destination) increment in $\mu_{RD}$ has a better impact on performance than with gain in $\mu_e$, and (ii) for $d_{SR} > 1$ (relay is located closer to destination than source) the effect of $\mu_e$ on performance is more than that due to $\mu_{RD}$. These observations hold for all modulation orders of the considered modulation schemes for the cases (a) and (b).

In Figure 3, we compared the performance with variation in $\mu_e$ and $\mu_{RD}$. Next, the effects of change in other fading parameters for the SR link are compared in Figure 4. The assumptions regarding node placement and fading parameters are the same as viewed in Figure 3. In Figure 4(a), plots showing variation in average SER with $\mu_e$ and $m_e$ are shown for fixed $\kappa_{e,SR}, \mu_{RD}$, and $m_{RD}$. We observe that raise in $\mu_e$ has more advantages in terms of performance rather than those due to increment in $m_e$. Similarly, in Figure 4(b), the performance is compared for varying $\mu_e$ and $\kappa_e$ while considering constant values of $m_e, \mu_{RD}$, and $m_{RD}$. The performance for increment in $\mu_e$, dominates over that due to an increase in $\kappa_e$. Moreover, on comparing Figures 4(a) and (b), it can be observed that performance gain is more for increment in $m_e$ than $\kappa_e$. These results also hold for variation in fading parameters of RD link. The analysis is valid for different modulation orders of the considered schemes.
While it is 2 for curves 4 and 5. These results validate the concept of a decode-then-forward (DF) relay system. We observe that DF for curves 1, 2, and 3 is 1, while it is 2 for curves 4 and 5. This shows that the DF system is more effective in improving the system's performance.

We have deduced the average SER of a DF relaying system with source node WP through RF signal broadcasted at the relay node. We consider the scenario when the source node provides wireless power to the energy-constrained relay node. The optimal relay location in [39] lies between the source and the destination nodes, while it is close to the source node in [5]. Compared to these two cases, the optimal relay location with EH at the source node lies between the source and destination nodes with an inclination towards the node having poor link quality.

In Figure 5, the average SER is plotted against the relay transmission power $P_R$ for different values of the fading parameters. The negative slope of the plots is observed with variation in $\mu_{SR} = \mu_{RS} = \mu$ and $\mu_{RD}$ for $\kappa = 1$ and $m = 1$. The plots indicate the DO for curves 1, 2, and 3 is 1, while it is 2 for curves 4 and 5. These results validate the correctness of the derived analytical expression for DO in [37].

![FIGURE 5](image-url)  
**FIGURE 5**  
Average SER $P$ versus relay transmission power $P_R$ of 4-PSK for $\kappa = 1$, $m = 1$, $\varphi \in \{SR, RS, RD\}$ and varying $\mu$ and $\mu_{RD}$.

5 CONCLUSION

We have deduced the average SER of a DF relaying system with source node WP through RF signal broadcasted at the relay node. M-PSK and orthogonal M-FSK modulation schemes are considered. The system is assumed to be affected under $\kappa - \mu$ shadowed fading. The obtained average SER expressions are validated with simulation results and hence can be utilized to investigate the system’s performance with variation in modulation order/scheme, link quality, and relay location. Through numerical results, we observed that (i) M-PSK performs better than orthogonal M-FSK for $M \leq 4$ and the later outperforms for $M \geq 8$ and (ii) the optimal relay location is in between the source and destination nodes with an inclination towards the node with poor link quality which increases on the decrement in its quality. Among fading parameters $\kappa$, $\mu$, and $m$ of each link, the increment in $\mu$ has the best impact on the performance improvement, while increment in $\kappa$ has the least impact. Simplified expressions of the average SER for the considered modulation schemes are obtained at high SNRs using asymptotic approximations which are later used to obtain the DO of the system.

REFERENCES

1. Sudevalayam, S., Kulkarni, P.: Energy harvesting sensor nodes: Survey and implications. IEEE Commun. Surv. Tutorials 13(3), 443–461 (2011)
2. Liu, K., et al.: Cooperative Communications and Networking. Cambridge University Press, Cambridge (2009)
3. Kumar, P., Dhaka, K.: Performance of wireless powered DF relay system under Nakagami-$\alpha$ fading: Relay assists energy-constrained source. IEEE Syst. J. 14(2), 2497–2507 (2020)
4. Liu, P., et al.: Energy harvesting noncoherent cooperative communications. IEEE Trans. Wireless Commun. 14(12), 6722–6737 (2015)
5. Kumar, P., Dhaka, K.: Performance analysis of wireless powered DF relay system under Nakagami-$\alpha$ fading. IET Trans. Veh. Technol. 67(8), 7073–7085 (2018)
6. Babaei, M., et al.: BER performance of wireless-powered dual-hop AF relaying. In Proceedings of IEEE Microwave Theory and Techniques in Wireless Communications, Riga, Latvia, 1-2 October, vol. 1, pp. 43–46 (2019)
7. Jin, X., et al.: Performance analysis of a wireless energy-harvesting cooperative system with precoding spatial modulation. IET Commun. 13(15), 2369–2374 (2019)
8. Burch, L., et al.: Error probability analysis of NOMA-based relay networks with SWIPT. IEEE Commun. Lett. 23(7), 1223–1226 (2019)
9. Kumar, P., Dhaka, K.: Performance analysis of wireless powered decode-and-forward relay system. In Proceedings of IEEE National Conference Communications (NCC), Bangalore, India, 20-23 February, pp. 1–6 (2019)
10. Nasir, A.A., et al.: Relaying protocols for wireless energy harvesting and information processing. IEEE Trans. Wireless Commun. 12(7), 3622–3636 (2013)
11. Chen, E., et al.: Multihop cooperative relaying with energy harvesting from cochannel interferences. IEEE Commun. Lett. 21(5), 1199–1202 (2017)
12. Mamaghani, M.T., et al.: Secure two-way transmission via wireless-powered untrusted relay and external jammer. IEEE Trans. Veh. Technol. 67(9), 8451–8465 (2018)
13. Mamaghani, M.T., Abbas, R.: Security and reliability performance analysis for two-way wireless energy harvesting based untrusted relaying with cooperative jamming. IET Commun. 13(4), 449–459 (2019)
14. Li, B., et al.: Transceiver design for AF MIMO relay systems with a power splitting based energy harvesting relay node. IEEE Trans. Veh. Technol. 69(5), 2376–2388 (2020)
15. Sharma, S., et al.: Secrecy outage of a multi-relay cooperative communication network with accumulation of harvesting energy at relays. IET Commun. 13(18), 2986–2995 (2019)
16. Mamaghani, M.T., Hong, Y.: On the performance of low-altitude UAV-enabled secure AF relaying with cooperative jamming and SWIPT. IEEE Access 7, 153060–153073 (2011)
17. Yang, Z., et al.: Energy efficient resource allocation in machine-to-machine communications with multiple access and energy harvesting for IoT. IEEE Internet Things J. 5(1), 229–245 (2018)
18. Luo, S., et al.: Throughput of wireless-powered relaying systems with buffer-aided hybrid relay. IEEE Trans. Wireless Commun. 15(7), 4790–4801 (2016)
19. Chen, H., et al.: Wireless-powered cooperative communications via a hybrid relaying. In Proceedings of IEEE Information Theory Workshop (ITW), Hobart, Australia, 2–5 November, pp. 666–670 (2014)
20. Hou, J., et al.: Bidirectional wireless information and power transfer with an energy accumulating relay. IEEE Access 6, 57257–57266 (2018)
21. Velkov, Z.H., et al.: Rate maximization of decode-and-forward relaying systems with RF energy harvesting. IEEE Commun. Lett. 19(12), 2290–2293 (2015)
22. Mishra, D., De, S.: Optimal time allocation for RF powered DF relay-assisted cooperative communication. Electron. Lett. 52(14), 1274–1276 (2016)
23. Huang, X., Ansari, N.: Optimal cooperative power allocation for energy-harvesting-enabled relay networks. IEEE Trans. Veh. Technol. 65(4), 2424–2434 (2016)
24. Zlatanov, N., et al.: Capacity of the two-hop relay channel with wireless energy transfer from relay to source and energy transmission cost. IEEE Trans. Wireless Commun. 16(1), 647–662 (2017)
25. Yacoub, M.D.: The \( \kappa - \mu \) distribution and the \( \eta - \mu \) distribution. IEEE Antennas Propag. Mag. 49(1), 68–81 (2007)
26. Cotton, S.L., et al.: The \( \kappa - \mu \) distribution applied to the analysis of fading in body to body communication channels for fire and rescue personnel. IEEE Antennas Wirel. Propag. Lett. 7, 66–69 (2008)
27. Cotton, S.L.: Human body shadowing in cellular device-to-device communications: Channel modeling using the shadowed \( \kappa - \mu \) fading model. IEEE J. Sel. Areas Commun. 33(1), 111–119 (2015)
28. Chun, Y.J., et al.: A comprehensive analysis of 5G heterogeneous cellular systems operating over \( \kappa - \mu \) shadowed fading channels. IEEE Trans. Wireless Commun. 16(11), 6955–7010 (2017)
29. Hussain, A., et al.: Non-linear energy harvesting dual-hop DF relaying system over \( \eta - \mu \) fading channels. Int. J. Adv. Comput. Sci. Appl. 9(6), 423–426 (2018)
30. Hussain, A., et al.: Energy harvesting in opportunistic relaying network with multiple antennas. Int. J. Comput. Sci. Netw. Secur. 18(5), 125–129 (2018)
31. Badarneh, O.S., et al.: Wireless energy harvesting in cooperative decode-and-forward relaying networks over mixed generalized \( \eta - \mu \) and \( \kappa - \mu \) fading channels. Trans. Emerging Telecommun. Technol. 29(2), 1–18 (2017)
32. Hussain, A., et al.: Energy harvesting relaying network in a delay-tolerant transmission mode over \( \kappa - \mu \) shadowed fading channels. Int. J. Comput. Sci. Netw. Secur. 18(3), 119–125 (2018)
33. Rabe, K., et al.: Full-duplex energy-harvesting enabled relay networks in generalized fading channels. IEEE Wireless Commun. Lett. 8(2), 384–387 (2019)
34. Nasir, A.A., et al.: Throughput and ergodic capacity of wireless energy harvesting based DF relaying network. In Proceedings of IEEE International Conference on Communications, Sydney, Australia, 10–14 June, pp. 4066–4071 (2014)
35. Racesi, O., et al.: Estimation and mitigation of channel non-reciprocity in massive MIMO. IEEE Trans. Signal Process. 66(10), 2711–2723 (2018)
36. Paris, J.F.: Statistical characterization of \( \kappa - \mu \) shadowed fading. IEEE Trans. Veh. Technol. 63(5), 518–526 (2014)
37. Parthasarathy, S., Ganti, R.K.: Coverage analysis in downlink Poisson cellular network with \( \kappa - \mu \) shadowed fading. IEEE Wireless Commun. Lett. 6(1), 10–13 (2017)
38. Selvaraj, M.D., Mallick, R.K.: Error analysis of the decode and forward protocol with selection combining. IEEE Trans. Wireless Commun. 8(6), 3086–3094 (2009)
39. Lin, W., et al.: SER performance analysis and optimal relay location of cooperative communications with distributed Alamouti code. In Proceedings of Annual Conference on Information Sciences and Systems, Baltimore, MD, USA, 18–20 March, pp. 646–651 (2009)
40. Gradshteyn, I.S., Ryzhik, I.M.: Table of Integrals, Series and Products. 6th ed. Academic Press, New York (2000)
41. Prudnikov, A.P., et al.: Integrals and Series: Elementary Functions. Vol. 3, 4th ed. Gordon and Breach Science, London, Great Britain (1998)
42. Prudnikov, A.P., et al.: Integrals and Series: Special Functions. Vol. 2, 3rd ed. Gordon and Breach Science, New York (1992)
43. Lathi, B.P.: Modern digital and analog communication systems. 3rd ed. Oxford University Press, New York (1998)
44. Abramowitz, M., Stegun, I.A.: Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables. Government Printing Office, Washington (1970)

**APPENDICES A**

### A.1 Mathematical equations used in the analysis

The mathematical equations used for derivation in Sections 2 and 3 are listed below:

- **Confluent hypergeometric function in series form** can be represented using \([40, \text{Equation (9.210.1)}]\) as
  \[
  _{1}F_{1}[a, b; z] = \sum_{l=0}^{\infty} \frac{(a)_{l}}{(b)_{l}} \frac{z^{l}}{l!}.
  \]

- **Integral form representation for Bessel’s function of the second kind** using \([41, \text{Equation (2.3.16.1)}]\) is given by
  \[
  \int_{0}^{\infty} x^{\lambda-1} \exp(-px-q/x)dx = 2\left(\frac{2}{p}\right)^{\frac{\lambda}{2}} K_{\lambda}(2\sqrt{pq}).
  \]

- **Integral representation of Whittaker’s function** can be written as \([42, \text{Equation (2.16.8.4)}]\):
  \[
  \int_{0}^{\infty} x^{\eta-1} \exp(-sx^{2})K_{\eta}(sx)dx = s^{-\frac{1}{2}} \Gamma\left(\frac{m+n}{2}\right) \times \Gamma\left(\frac{m-n}{2}\right) \exp\left(\frac{s^{2}}{8}\right) W_{\eta/2,\eta/2}\left(\frac{s^{2}}{4}\right).
  \]

- **Integral form representation of gamma function** is \([40, \text{Equation (3.381.4)}]\):
  \[
  \int_{0}^{\infty} x^{\lambda-1} \exp(-sx)dx = s^{-\lambda} \Gamma(m).
  \]

- In orthogonal \(M\)-FSK modulation with non-coherent detection, the conditional SER expression for link \(\varphi\) is expressed as \([43, \text{Equation (13.59c)}]\):
  \[
  P_{c,\varphi}(y_{\varphi}) = \sum_{l=1}^{M-1} \left(\frac{y_{\varphi}}{l+1}\right) \left(\frac{M-1}{l}\right) \exp\left(-\frac{y_{\varphi}}{l+1}\right).
  \]

An approximation of \(K_{\lambda}(t)\) for \(t \to 0\) can be obtained using \([44, \text{Equations (9.6.6), (9.6.8) and (9.6.9)}]\) as
\[
K_{\lambda}(t) = \begin{cases} 
-\ln(t), & r = 0 \\
\frac{1}{2} \Gamma(|r|) \left(\frac{1}{2}ight)^{|r|}, & |r| > 0.
\end{cases}
\]
A finite integral involving product of algebraic, logarithmic, and exponential functions is simplified using \[40, \text{Equation (4.352.1)}\] as

\[
\int_0^\infty \chi^{m-1} \ln(\chi) \exp(-\chi) \, d\chi = s^{-m} \Gamma(m) \left( \psi(m) - \ln(s) \right).
\]  
(A.7)

Using \[40, \text{Equation (3.621.1)}\], a finite integration of sine function raised to a power can be expressed in terms of beta function as

\[
\int_0^{\pi/2} \sin^{m-1}(\theta) \, d\theta = 2^{m-2} B(m/2, m/2).
\]  
(A.8)

Using \[40, \text{Equation (4.387.2)}\], a finite integral of expression involving trigonometric functions can be written in terms of digamma function as

\[
\int_0^{\pi/2} \ln(\sin(\theta)) \sin^{m-1}(\theta) \, d\theta = \frac{\sqrt{\pi} \Gamma(m/2)}{4 \Gamma((m+1)/2)} \left( \psi(l/2) - \psi((m+1)/2) \right).
\]  
(A.9)

Ratio of two gamma functions is represented as \[44, \text{Equation (6.1.47)}\]

\[
\frac{\Gamma(a+n)}{\Gamma(b+n)} = n^{-i} \left( 1 + \frac{(a-b)(a+b-1)}{2n} + o(n^{-2}) \right).
\]  
(A.10)

Using \[44, \text{Equations (6.3.1), (6.3.5), and (6.3.16)}\], the first-order differentiation of gamma function is

\[
\frac{d}{dt} \left( \Gamma(t) \right) = \Gamma(t) \left( -\zeta + \sum_{i=1}^{\infty} \frac{t}{i(i+t)} - \frac{1}{t} \right),
\]  
(A.11)

where \(\zeta = 0.577216\) is Euler-Mascheroni constant \[44, \text{Equation (6.1.3)}\].

### A.2 Convergence of the PDF in (8)

The approximative expression for the error in (9) can be rewritten as

\[
\varepsilon_N < \sum_{i=1}^{\infty} \frac{\ln|b_i|}{\Gamma(m)_i^{(m+n+1, +\mu \lambda)}} \times |b_i^{(m+n+1, +\mu \lambda)}| \exp(-\zeta_i |b_i^{(m+n+1, +\mu \lambda)}) |b_i^{(m+n+1, +\mu \lambda)}|^{m-n-1}.
\]  
(A.12)

Using the relations for \(c_{\omega} \) and \(p_{\omega, i} \), given in (7), the approximation of error in (A.12) is expanded (after dropping the subscript and substituting \( |b_i^{(m+n+1, +\mu \lambda)}| = t \)) as

\[
\varepsilon_N < \sum_{i=1}^{\infty} \frac{\ln|b_i^{(m+n+1, +\mu \lambda)}|}{\Gamma(m)_i^{(m+n+1, +\mu \lambda)}} \exp(-\zeta_i |b_i^{(m+n+1, +\mu \lambda)}) |b_i^{(m+n+1, +\mu \lambda)}|^{m-n-1} \times \left( \frac{\mu (1+\zeta/t)}{\lambda} \right)^{i+\mu} \exp(-\frac{\mu (1+\zeta/t)}{\lambda} t)
\]  
(A.13)

where

\[
D = \sum_{i=1}^{\infty} \frac{(m)_i}{\Gamma(m)_i^{(m+n+1, +\mu \lambda)}} \left( \frac{\mu (1+\zeta/t)}{\lambda} \right)^{i+\mu} \exp(-\frac{\mu (1+\zeta/t)}{\lambda} t).
\]

Now rewriting (A.13) in summation form on substituting \( \zeta = \mu^2 (1+\zeta)/(\mu \lambda) \) and taking the first term common, we get

\[
\varepsilon_N < D \times \left( \frac{\ln|b_i^{(m+n+1, +\mu \lambda)}|}{\Gamma(m)_i^{(m+n+1, +\mu \lambda)}} \exp(-\zeta \ln b_i^{(m+n+1, +\mu \lambda)}) \right)
\]  
(A.14)

In (A.14), it is observed that convergence of \(\varepsilon_N\) is mainly limited by the terms \(T_1(N)\) and \(T_2(N)\). It is shown below that these two terms converge with an increase in \(N\).

First, we proceed with the term \(T_1(N)\), which is expanded using relation \(a \approx \Gamma(n+1)/\Gamma(n)\) and (A.10) as

\[
T_1(N) = \frac{a N^m}{\Gamma(\mu + N + 1)} \left( 1 + \frac{b}{2 N} + o(|N|^{-2}) \right)^{m-n+1},
\]  
(A.15)

where \(a = \Gamma(\mu)/\Gamma(\mu + 1)^2\) and \(b = N + 1\). Ignoring the term \(o(|N|^{-2})\) and applying first-order differentiation on both sides of (A.15) with respect to \(N\), then using (A.11) followed by rearranging the terms, we get

\[
\frac{d}{dN} T_1(N) \approx \frac{a N^m}{\Gamma(\mu + N + 1)} \times T_3(N),
\]  
(A.16)

where

\[
T_3(N) \approx \left( \frac{3}{2} + N \right) \ln(z) + \left( \frac{3m}{2N} + (m+1)z \right) - \left( \frac{3}{2} + N \right) z \times \left( -\zeta + \sum_{i=1}^{\infty} \frac{(\mu + N + 1) - 1}{i(i+\mu + N + 1)} \right).
\]  
(A.17)
It is obvious that $T_1(N)$ is said to be decreasing with an increase in $N$ when its derivative $dT_1(N)/dN$ is negative. It can be seen from (A.17) that a value of $N$ can be obtained such that $T_3(N) < 0$. However, simplifying this for $N$ in terms of other parameters is tedious. So, we skip the derivation for $N$ on stating that $T_1(N)$ decreases with an increase in $N$.

Next, we analyse variation in $T_2(N)$ with respect to $N$ on considering its two consecutive terms, given by

$$r_{q+1} = \frac{(m)_{q+1}(N + 1)! (\mu)_{N+1}}{(q + 1)! (\mu)_{q+1}(m)_{N+1}} \xi^{q+1}$$ (A.18)

and

$$r_q = \frac{(m)_q(N + 1)! (\mu)_{N+1}}{(q)! (\mu)_q(m)_q}_{N+1} \xi^q$$ (A.19)

for $q = (N + 1), (N + 2), (N + 3), ...$. The series $T_2(N)$ is said to be converging or decreasing with increase in $N$; in general, if the ratio $r_q = r_{q+1}/r_q$ decrease with increase in $N$. Dividing (A.18) by (A.19) and utilizing the relation $(a)_n = \Gamma(a + n)/\Gamma(a)$, we get the ratio

$$r_{q} = \frac{\Gamma(m + q + 1)\Gamma(\mu + q)\Gamma(q + 1)}{\Gamma(\mu + q + 1)\Gamma(m + q)\Gamma(q + 2)} \xi.$$

Now applying (A.10), we have

$$r_q = \frac{\xi}{(q + 1)^{\mu-q}} \left[ 1 + \frac{(m - \mu)(m + \mu + 1)}{2q} + o(|q|^{-2}) \right]$$

$$\times q^{\mu-q} \left[ 1 + \frac{(\mu - m)(m + \mu - 1)}{2q} + o(|q|^{-2}) \right].$$ (A.21)

which after rearranging the terms and ignoring the higher order terms can be represented as

$$r_q \approx \frac{\xi}{(q + 1)^{\mu-q}} \left[ 1 + \frac{(m - \mu)}{q} \right] \xi.$$ (A.22)

This suggests that the ratio $r_q$ decreases with $q$. Thus, the term $T_2(N)$ decreases with increase in $N$.

Since the terms $T_1(N)$ and $T_2(N)$ in (A.14) decrease with increase in $N$, we can state that the finite series form of the PDF given in (8) converges with an approximation error for a sufficiently higher value of $N$ (or $N_n$).