Study of the process $e^+e^− \rightarrow K^+K^−$ in the center-of-mass energy range 1010–1060 MeV with the CMD-3 detector

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The process $e^+e^− \rightarrow K^+K^−$ has been studied using $1.7 \times 10^6$ events from a data sample corresponding to an integrated luminosity of 5.7 pb$^{-1}$ collected with the CMD-3 detector in the center-of-mass energy range 1010–1060 MeV. The cross section measured with an about 2% systematic uncertainty and it is used to calculate the contribution to the anomalous magnetic moment of the muon $\mu$ [K$^+K^−$] = (19.33 ± 0.40) × 10$^{-10}$, and to obtain the $\phi$(1020) meson parameters. We consider the relationship between the $e^+e^− \rightarrow K^+K^−$ and $e^+e^− \rightarrow K_S^0K_L^0$ cross sections and compare it to the theoretical expectations.

I. INTRODUCTION

Investigation of $e^+e^−$ annihilation into hadrons at low energies provides unique information about interactions of light quarks. A precise measurement of the $e^+e^− \rightarrow K^+K^−$ cross section in the center-of-mass energy range E$_{c.m.}$=1010–1060 MeV allows to obtain the $\phi$(1020) meson parameters and to estimate a contribution of other light vector mesons, $\rho$(770), $\omega$(782), to the process studied. The $e^+e^− \rightarrow K^+K^−$ cross section, particularly in the $\phi$ meson energy region, is also required for a precise calculation of the hadronic contribution to the muon anomalous magnetic moment, $\alpha(M_Z)$ [4].

The most precise cross section measurements performed by the CMD-2 [2] and BaBar [3] experiments have tension at the level of more than 5% (about 2.6 standard deviations) in the $\phi$ meson energy region.

Another motivation for the study arises from the comparison of the charged $e^+e^− \rightarrow K^+K^−$ and neutral $e^+e^− \rightarrow K_S^0K_L^0$ final states. A significant deviation of the ratio of the coupling constants $\frac{\sigma_{e^+e^− \rightarrow K^+K^−}}{\sigma_{e^+e^− \rightarrow K_S^0K_L^0}}$ from a theoretical prediction based on previous experiments (see the discussion in Ref. [2]) requires a new precision measurement of the cross sections.

II. CMD-3 DETECTOR AND DATA SET

The Cryogenic Magnetic Detector (CMD-3) is a general purpose detector installed in one of the two interaction regions of the VEPP-2000 collider [5] and is described elsewhere [6]. A detector tracking system consists of a cylindrical drift chamber (DC) and a double-layer cylindrical multiwire proportional chamber (Z-chamber), both installed inside a thin (0.2 X$_0$) superconducting solenoid with a 1.3 T field. The DC comprises of 1218 hexagonal cells and allows to measure charged particle momentum with a 1.5–4.5% accuracy in the 100–1000 MeV/c momentum range, it also provides a measurement of the polar ($\theta$) and azimuthal ($\phi$) angles with a 20 mrad and 3.5–8.0 mrad accuracy, respectively. Amplitude information from the DC wires is used to measure the ionization losses $dE/dx$ of charged particles with a $\sigma_{dE/dx} < dE/dx >\approx 11−14\%$ accuracy for minimum ionization particles (m.i.p.). The Z-chamber with cathode strip readout is used to calibrate a DC longitudinal scale.

An electromagnetic calorimeter comprised of a liquid xenon volume of a 5.4 radiation length (X$_0$) thickness followed by CsI crystals (8.1 X$_0$) outside of the solenoid in the barrel part and BGO crystals (14.4 X$_0$) in the endcap parts [7, 8]. A flux return yoke of the detector is surrounded by scintillation counters to veto cosmic events.
The beam energy $E_{\text{beam}}$ is monitored by using the back-scattering-laser-light system [9, 10], which determines $E_{\text{c.m.}}$ at each energy point with an about 0.06 MeV systematic accuracy.

Candidate events are recorded using signals from two independent trigger systems. One, a charged trigger, uses information only from DC cells indicating presence of at least one charged track, while another, a neutral trigger, requires an energy deposition in the calorimeter above $E_{\text{beam}}/2$ or presence of more than two clusters above 25 MeV threshold.

To study a detector response for the investigated processes and to obtain a detection efficiency, we have developed a Monte Carlo (MC) simulation of the detector based on the GEANT4 [11] package. Simulated events are subject to all reconstruction and selection procedures. MC includes photon jet radiation by initial electron or positron (ISR) calculated according to Ref. [12].

The measurement of the $e^+e^- \rightarrow K^+K^-$ cross section presented here is based on a data sample collected at 24 energy points with a $5.7 \text{ pb}^{-1}$ integrated luminosity (IL) in the energy range $E_{\text{c.m.}} = 1010–1060 \text{ MeV}$ in 2012 and 2013.

III. EVENT SELECTION

Selection of $e^+e^- \rightarrow K^+K^-$ candidates is based on the detection of two collinear tracks satisfying the following criteria:

- The tracks originate from the beam interaction region within 20 cm along the beam axis (Z-coordinate) and within 1 cm in the transverse direction.
- The polar and azimuthal collinearity are required to have $\Delta\theta = |\theta_{K^+} + \theta_{K^-} - \pi|$, $\Delta\phi = |\phi_{K^+} - \phi_{K^-} - \pi| < 0.45$ radians. The distributions of these parameters for data and MC at $E_{\text{beam}} = 530 \text{ MeV}$ are shown in Figs. [12] where the MC sample is normalized to data, and arrows demonstrate the applied restriction. Two additional bumps in the $\Delta\theta$ distribution are caused by a significant contribution of $K^+K^-\gamma$ events, where $\gamma$ is emitted from the initial state (radiative return to the $\phi$ resonance).
- The tracks are required to have an average polar angle in the range $1 < \theta_{\text{ave}} = (\theta_{K^+} + \pi - \theta_{K^-})/2 < \pi - 1$ radians. The polar angle distribution is shown in Fig. [3] (a) where arrows demonstrate the applied restriction. Tracks out of the selected range do not pass all DC layers and are detected less efficiently (see the discussion in Sec. [VI]).
- Momenta of both tracks are required to be close to each other: $|p_1 - p_2|/|p_1 + p_2| < 0.3$.
- The average momentum of two tracks is required to be in a range depending on $E_{\text{beam}}$ to minimize the background-to-signal ratio. An example of this restriction for $E_{\text{beam}}=530 \text{ MeV}$ is shown in Fig. [4] by arrows:

| Energy (MeV) | RMS | Mean |
|-------------|-----|------|
| 1010        | 0.1796 | 0.0009208 |
| 1060        | 0.1796 | 0.0009208 |

IV. DETECTION EFFICIENCY

The detection efficiency, $\epsilon_{\text{MC}}$, is determined from MC by dividing the number of MC simulated events, after

![FIG. 1: The polar collinearity $\theta_{K^+} + \theta_{K^-} - \pi$ for data (points) and MC (shaded histogram) at $E_{\text{beam}} = 530 \text{ MeV}$.](image-url)
The azimuthal collinearity $|\phi_{K^+} - \phi_{K^-}| - \pi$ for data (points) and MC (shaded histogram) at $E_{\text{beam}} = 530$ MeV.

The average polar angle $\theta_{\text{aver}} = (\theta_{K^+} + \pi - \theta_{K^-})/2$ distribution for data (points) and MC (shaded) at $E_{\text{beam}} = 509.5$ MeV. The MC histogram is normalized to six central bins of the data distribution.

The ionization losses vs momentum for positive tracks for data at $E_{\text{beam}} = 530$ MeV. The line shows selection of signal kaons.

The approximation of the distribution of average $Z$-coordinates of selected tracks at $E_{\text{beam}} = 505$ MeV. The long-dotted line corresponds to the signal, the solid line to the background. The shaded histogram shows the background distribution obtained using events at $E_{c.m.} = 984$ MeV.
and to take into account this effect we apply the following correction:

\[
1 + \delta^{\text{en.spr.}}(E_{\text{c.m.}}) = \frac{1}{\sqrt{2\pi \sigma_{E_{\text{c.m.}}}}}. \tag{3}
\]

\[
\int \frac{dE'_{\text{c.m.}}}{\sigma_{\text{born}}(E'_{\text{c.m.}})} \frac{\sigma_{\text{born}}(E'_{\text{c.m.}})(1 + \delta^{\text{rad.}}(E'_{\text{c.m.}}))e^{-\frac{(E'_{\text{c.m.}} - E_{\text{c.m.}})^2}{2\sigma_{E_{\text{c.m.}}}}}}{\sigma_{\text{born}}(E'_{\text{c.m.}})(1 + \delta^{\text{rad.}}(E'_{\text{c.m.}}))},
\]

which depends on the cross section \(\sigma_{\text{born}}\), radiative correction \((1 + \delta^{\text{rad.}})\), and is calculated by iterations in the same way as \(\epsilon_{\text{MC}}\) and \((1 + \delta^{\text{rad.}})\). The calculated \((1 + \delta^{\text{en.spr.}})\) value for each energy point is shown in Fig. 7 by circles (right scale), and has the maximum value of 1.026±0.006 at the peak of the \(\phi\) resonance.

The trigger efficiency, \(\epsilon_{\text{trig}}\), is studied using responses of two independent triggers, charged and neutral, for selected signal events, and is found to be close to unity \(\epsilon_{\text{trig}} = 0.998 \pm 0.001\) for applied selections.

The resulting cross section is listed in Table II at each energy point and shown in Fig. 8. The statistical error includes fluctuations of signal and Bhabha events, used for the luminosity calculation, and fluctuations of the uncertainty on the c.m. energy measurement, \(\delta E_{\text{c.m.}}\), calculated as \(|\frac{\delta \sigma_{\text{born}}}{\sigma_{\text{born}}}| \times \delta E_{\text{c.m.}}\).

VI. SYSTEMATIC UNCERTAINTIES

The uncertainty on the \(e^+e^- \rightarrow K^+K^-\) cross section is dominated by the accuracy of the detection efficiency \(\epsilon_{\text{det}}\) calculation.

The systematic uncertainty of the data-MC ratios in Eq. (1) is estimated by applying different selection requirements on the “test” events and does not exceed 1%, however, for five energy points with \(E_{\text{c.m.}} > 1030\) MeV we increase the uncertainty to 2%.

The data-MC difference in the polar angle distributions of kaons is shown in Fig. 4(b) by circles. The observed difference is due to incorrect simulation of detector resolution, angular dependence of the track reconstruction and trigger efficiency, and uncertainty on the calibration of the DC longitudinal scale. We tune our simulation to match the detector angular and momentum resolutions (see Fig. 1), study angular dependence of the track reconstruction efficiency using a single-track “test” sample, and study response of two independent triggers as a function of the track polar angle. The data-MC ratio of the polar angle distributions after applied corrections is shown in Fig. 4(b) by squares.

To estimate influence of the remaining angular uncertainty on the measured cross section we divide all data into three independent samples with \(\theta \in [0.95 : 1.35], [1.35 : \pi-1.35]\) and \([\pi-1.35 : \pi-0.95]\) radians. By separately calculating all parameters in Eq. (2) for three

FIG. 6: The EXP-MC ratio of the single-track efficiencies for positive \(\overrightarrow{\text{en}}\) (squares) and negative kaons \(\rightarrow\overrightarrow{\text{mc}}\) (circles) for data collected in 2012 and 2013 runs.

FIG. 7: Radiative corrections \(1 + \delta^{\text{rad.}}\) (squares, left scale) and corrections \(1 + \delta^{\text{en.spr.}}\) for the spread of collision energy (points, right scale).
regions and comparing the obtained cross sections we estimate the corresponding uncertainty as the average difference of the samples to be 1%.

To check the quality of the DC chamber scale calibration we extrapolate the reconstructed kaon tracks from DC to ZC and compare it with the position of the ZC response: a possible systematic uncertainty is less than 0.3%.

The total systematic uncertainty in the reconstruction efficiency is estimated as 1.6%, but increased to 2.5% for five energy points for $E_{c.m.}>1030$ MeV.

To estimate uncertainty on the background subtraction procedure we use the data accumulated at the energy point $E_{c.m.} = 984$ MeV below the reaction threshold. Applying our selection criteria we obtain the number of background events, $N_{984}$, and then estimate the number of background events for each energy point using the in-
TABLE I: The c.m. energy $E_{c.m.}$, number of selected signal events $N$, uncorrected and corrected detection efficiencies $\epsilon_{MC}$ and $\epsilon_{det}$, radiative correction factor $1 + \delta^{\text{rad}}$, correction for the spread of collision energy $1 + \delta^{\text{spr}}$, integrated luminosity $IL$, and Born cross section $\sigma$ of the process $e^+e^- \rightarrow K^+K^-$ where only statistical errors are presented.

| $E_{c.m.}$, MeV | $N$ events | $\epsilon_{MC}$ | $\epsilon_{det}$ | $1 + \delta^{\text{rad}}$ | $1 + \delta^{\text{spr}}$ | $IL$, nb$^{-1}$ | $\sigma$, nb |
|----------------|-------------|-----------------|-----------------|-----------------|-----------------|----------------|------------|
| 1010.47 ± 0.01 | 21351 ± 145 | 0.439 0.441 | 0.735 | 0.993 | 946.05 ± 1.44 | 69.87 ± 0.50 |
| 1012.96 ± 0.01 | 26882 ± 164 | 0.485 0.493 | 0.728 | 0.988 | 485.36 ± 1.04 | 152.45 ± 1.01 |
| 1015.07 ± 0.02 | 6031 ± 78  | 0.520 0.510 | 0.718 | 0.987 | 479.31 ± 0.93 | 341.10 ± 5.11 |
| 1016.11 ± 0.01 | 41260 ± 201 | 0.510 0.513 | 0.712 | 0.978 | 192.11 ± 0.66 | 575.08 ± 3.84 |
| 1017.15 ± 0.02 | 176768 ± 421 | 0.515 0.517 | 0.706 | 0.983 | 478.99 ± 1.04 | 993.19 ± 5.02 |
| 1017.16 ± 0.02 | 22243 ± 149 | 0.517 0.524 | 0.706 | 0.985 | 60.15 ± 0.30 | 984.71 ± 8.89 |
| 1018.05 ± 0.03 | 27973 ± 529 | 0.521 0.519 | 0.706 | 0.993 | 478.34 ± 1.04 | 1584.27 ± 11.00 |
| 1019.12 ± 0.02 | 270045 ± 520 | 0.525 0.524 | 0.721 | 1.026 | 328.62 ± 0.86 | 2228.59 ± 8.13 |
| 1019.21 ± 0.03 | 44051 ± 209 | 0.525 0.531 | 0.724 | 1.022 | 52.75 ± 0.34 | 2230.81 ± 18.14 |
| 1019.40 ± 0.04 | 30539 ± 174 | 0.526 0.533 | 0.730 | 1.024 | 36.05 ± 0.29 | 2233.66 ± 22.07 |
| 1019.90 ± 0.02 | 391083 ± 626 | 0.527 0.527 | 0.752 | 1.017 | 472.34 ± 1.04 | 2127.07 ± 6.46 |
| 1021.22 ± 0.03 | 134598 ± 365 | 0.532 0.533 | 0.829 | 0.994 | 228.34 ± 0.72 | 1325.01 ± 9.01 |
| 1021.31 ± 0.01 | 27717 ± 165 | 0.531 0.540 | 0.835 | 0.993 | 46.85 ± 0.33 | 1308.31 ± 12.50 |
| 1022.08 ± 0.03 | 89487 ± 299 | 0.532 0.530 | 0.885 | 0.991 | 201.62 ± 0.68 | 933.95 ± 6.81 |
| 1022.74 ± 0.03 | 41756 ± 204 | 0.534 0.536 | 0.928 | 0.988 | 116.71 ± 0.52 | 710.23 ± 5.86 |
| 1023.26 ± 0.04 | 19718 ± 140 | 0.536 0.545 | 0.961 | 0.991 | 62.91 ± 0.38 | 595.03 ± 6.56 |
| 1025.32 ± 0.04 | 7023 ± 84  | 0.537 0.538 | 1.077 | 0.995 | 36.32 ± 0.29 | 334.77 ± 5.55 |
| 1027.96 ± 0.02 | 24236 ± 156 | 0.540 0.536 | 1.200 | 0.997 | 195.83 ± 0.67 | 191.64 ± 1.74 |
| 1029.09 ± 0.02 | 5786 ± 76  | 0.542 0.550 | 1.244 | 0.997 | 52.94 ± 0.35 | 159.94 ± 2.95 |
| 1033.91 ± 0.02 | 11752 ± 108 | 0.546 0.535 | 1.392 | 0.998 | 175.55 ± 0.64 | 89.65 ± 1.24 |
| 1040.03 ± 0.05 | 9143 ± 95  | 0.551 0.553 | 1.509 | 0.999 | 195.91 ± 0.68 | 55.87 ± 0.94 |
| 1049.86 ± 0.02 | 14818 ± 122 | 0.553 0.536 | 1.604 | 0.999 | 499.59 ± 1.09 | 34.47 ± 0.47 |
| 1050.86 ± 0.04 | 4441 ± 67  | 0.554 0.559 | 1.609 | 0.999 | 146.31 ± 0.59 | 33.89 ± 0.84 |
| 1059.95 ± 0.02 | 4594 ± 68  | 0.553 0.543 | 1.640 | 0.999 | 198.86 ± 0.69 | 25.93 ± 0.64 |

The difference in the calculated number of background events and events obtained by the approximation of the Z-coordinate distribution (see Sec. III) gives less than 0.3% uncertainty of the cross section: this value is used as an estimate of the corresponding systematic uncertainty.

A significant part of selected signal events include ISR photons, which should be taken into account in $\epsilon_{det}$ and $1 + \delta^{\text{rad}}$. The photon spectrum is calculated by a convolution of the radiator function $IL$ and Born cross section $\sigma^{\text{born}}(E_{c.m.})$ which is known with uncertainties discussed above. By varying $N_{\text{exp}}, IL$, $E_{c.m.}$, and $\epsilon_{det}$ in Eq. (2) according to their uncertainties and repeating the calculation of $\sigma^{\text{born}}(E_{c.m.})$ and $1 + \delta^{\text{rad}}$, we estimate the uncertainty on the cross section as 0.1% (0.8% for energy points with $E_{c.m.} > 1030$ MeV). These values are quadratically summed with the 0.1% theoretical uncertainty of the radiator function.

The systematic uncertainties contributing to the measured cross section are listed in Table II and the quadratic sum gives 2.0% (2.8% for $E_{c.m.} > 1030$ MeV) as the total systematic uncertainty.

TABLE II: Summary of systematic uncertainties on the $e^+e^- \rightarrow K^+K^-$ cross section measurement

| Source | Uncertainty, % |
|--------|----------------|
| Signal selection | 0.3 |
| Detection efficiency | 1.6(2.5) |
| Radiative correction | 0.15(0.80) |
| Energy spread correction | 0.3 |
| Trigger efficiency | 0.1 |
| Luminosity | 1.0 |
| Total | 2.0(2.8) |

VII. APPROXIMATION OF THE $e^+e^- \rightarrow K^+K^-$ CROSS SECTION

The measured cross section defined by Eq. (2) includes a vacuum polarization factor, Coulomb interaction between $K^+K^-$, and final-state radiation of real photons $\gamma_{FSR}$. We approximate the energy dependence of the cross section according to the vector meson dominance (VMD) model as a squared sum of the $\rho$, $\omega$, $\phi$-like am-
plitudes:  

\[
\sigma(s) = \sigma_{e^+e^- \to K^+K^-}(s) = \frac{8\pi\alpha}{3s^{3/2}} P^2 \left( \frac{Z(s)}{m^2_\phi} \right) D(s) \left( \frac{g_\phi g_{\phi KK}}{D_\phi(s)} \right) + r_{\rho,\omega} \times \left[ \frac{g_{\rho\omega} g_{\rho KK}}{D_\rho(s)} + \frac{g_{\omega\omega} g_{\omega KK}}{D_\omega(s)} + A_{\phi',\phi',\omega'} \right] \tag{5}
\]

where \( s = E_{c.m.}^2 \), \( p_K \) is a kaon momentum,

\[
Z(s) = \frac{\pi\alpha/\beta}{1 - \exp(-\pi\alpha/\beta)} \left( 1 + \frac{\alpha^2}{4\beta^2} \right) \tag{6}
\]

is the Sommerfeld-Gamov-Sakharov factor that can be obtained by solving the Schrödinger equation in a Coulomb potential for a P-wave final state with velocity \( \beta = \sqrt{1 - 4m^2_K/s} \). \( D_V(s) \) is the inverse propagator of the vector state \( V \):  

\[
D_V(s) = m_V^2 - s - i\sqrt{s} \Gamma_V(s). \tag{7}
\]

Here \( m_V \) and \( \Gamma_V \) are mass and width of the major intermediate resonances: \( V = \rho(770), \omega(782), \phi(1020) \). For the energy dependence of the \( \phi \) meson width we use

\[
\Gamma_\phi(s) = \Gamma_\phi \left( B_{K^+K^-} - \frac{m^2_\phi}{s} F_{K^+K^-}(s) \right) + B_{rK^+K^-} \left( \sqrt{s} F_{rK^+K^-}(m^2_\phi) + B_{rK^+K^-} \left( \frac{F_{sK^+K^-}(m^2_\phi)}{s} F_{sK^+K^-}(m^2_\phi) \right) \right)
\]

where \( F_{K^+K^-} = (s/4 - m^2_K)^{3/2} \), \( F_{rK^+K^-} = (\sqrt{1 - m^2_\phi/s})^3 \), and for the \( F_{rK^+K^-}(s) \) calculation the model assuming the \( \phi \to \rho \pi \to \pi^+\pi^-\pi^0 \) decay is used [20]. The magnitudes of \( \Gamma_\phi(s) \) and \( \Gamma_\omega(s) \) are calculated in the same way using the corresponding branching fractions [21]. The coupling constants of the intermediate vector meson \( V \) with initial and final states can be presented as:

\[
g_{s\phi K^+K^-} = g_{s\phi K^+K^-} = g_{s\phi K^+K^-} = \frac{\sqrt{2}}{\sqrt{2}}.
\]

Taking into account a possible breaking of these assumptions, both \( g_{s\phi K^+K^-} \) and \( g_{s\phi K^+K^-} \) are multiplied by the common complex constant \( r_{\rho,\omega} \).

The amplitude \( A_{\phi',\phi',\omega'} \) denotes a contribution of the higher vector mesons \( \omega(1420), \rho(1450), \omega(1650), \phi(1680) \) and \( \rho(1700) \) to the \( \phi(1020) \) mass region. Using BaBar [3] and SND [17] data above \( \sqrt{s} = 1.06 \) GeV for the process \( e^+e^- \to K^+K^- \) we extract a contribution of these states.

We perform a fit to the \( e^+e^- \to K^+K^- \) cross section with floating \( m_\phi, \Gamma_\phi, \Gamma_\phi \), \( B_{\phi\to K^+K^-} \) (or alternatively \( B_{\phi\to e^+e^-} \times B_{\phi\to K^+K^-} \)) and \( r_{\rho,\omega} \) parameters: the fit yields \( \chi^2/ndf = 25/20 \) (\( P(\chi^2) = 20\% \)). The fit result is shown in Fig. 4. Figures 4 show the relative difference between the obtained data and the fit curve. Only statistical errors are shown and the width of the band corresponds to the systematic uncertainty of the cross section. In Fig. 4(a) we compare our result with the previous Novosibirsk measurements [2, 17, 18] while Fig. 4(b) shows a comparison with the recent BaBar experiment [3]. The obtained parameters of the \( \phi \) meson in comparison with the values of other measurements are presented in Table III. The first uncertainties are statistical and the second are systematic, resulting from the 60 keV accuracy in the \( E_{c.m.} \) measurements and errors listed in Table III. From the fit we obtain \( \Re(r_{\rho,\omega}) = 0.95 \pm 0.03 \) while an imaginary part is compatible with zero. The contributions of the \( \rho \) and \( \omega \) intermediate states (\( \sigma(s) - \sigma(s) |_{r_{\rho,\omega}=0} \)) and higher excitations (\( \sigma(s) - \sigma(s) |_{A_{\phi',\phi',\omega'}=0} \)) are demonstrated in Fig. 4 as an inset.

To study model dependence of the results, we perform alternative fits with the \( A_{\phi',\phi',\omega'}=0 \) amplitude in Eq. (5), or with an additional floating phase of the \( \phi \) meson amplitude, or with the form of the inverse propagator \( D_V(s) = m_V^2 - s - i\sqrt{s} \Gamma_V(s) \) instead of Eq. (7). The variations of the \( \phi \) meson parameters in these fits are used as an estimate of the model-dependent uncertainty presented as third errors in Table III.

As shown in Fig. 4 the obtained results have comparable accuracy but are not consistent, in general, with the previous data.

The difference with the CMD-2 [2] measurement can be explained by the overestimation of the value of the trigger efficiency for slow kaons in the previous experiment. The positive trigger decision of the CMD-2 required the presence of one charged track in DC in coincidence with the corresponding hits in the Z-chamber, and with at least one cluster in the CsI calorimeter with the energy deposition greater than 20 MeV. But slow kaons stop in the first wall of the Z-chamber and only decay or their nuclear interaction products can make hits in the Z-chamber or leave energy in the calorimeter. The trigger efficiency about 90% was obtained actually by simulation, using recorded information from detector cells.

In contrast to the CMD-2 experiment, the new CMD-3 detector has two independent trigger systems, the Z-
chamber is excluded from the decision, and a charged (total) trigger efficiency is close to 100%. The CMD-3 detector has the same Z-chamber and much more detailed information, and by including to our selection requirements of hits in the Z-chamber and presence of energy deposition greater than 20 MeV in the barrel calorimeter, we obtain a significantly larger trigger efficiency correction than the value obtained in the CMD-2 analysis [2]. A reanalysis of CMD-2 data is expected.

Our value of $\Gamma_{\phi \to e\gamma} B_{\phi \to K^+ K^-}$ is larger than the BaBar result by 1.8 standard deviations while the obtained value of $B_{\phi \to e\gamma} B_{\phi \to K^+ K^-}$ is larger than the PDG one, predominantly based on the CMD-2 measurement, by 2.7 standard deviations. The difference between the calculation of $\sigma$ and $\sigma_{\text{prod}}$ of electron-positron annihilation into $K^+ K^-$ and $K^0 L^0$ final states. The difference between them comes from the kinematic effect of the $K^+$ mass and $K^0$ mass difference and the Coulomb interaction between $K^+$ and $K^-$ mesons [0]. At the $\phi$ peak the Coulomb factor, $Z(m_\phi^2)$, contributes 4.2% to the total cross section. We correct the $e^+ e^- \to K^+ K^-$ cross section for the above two effects and calculate the difference with the $e^+ e^- \to K^0 L^0$ cross section:

$$D_{c/n} = \sigma_{e^+ e^- \to K^+ K^-} \times \frac{\delta^2 s_{K^0}(s)}{s_{K^0}(s)} \times \frac{1}{Z(s)} - \delta^2 s_{K^0 L^0} \times \sigma_{e^+ e^- \to K^0 L^0},$$

where the factor $\delta^2 s_{K^0 L^0}$ is introduced to account for a possible remaining systematic uncertainty in two measurements: most of the common uncertainties cancel in the difference. The experimental value of $D_{c/n}$ is shown in Fig. [11] by points with error bars, where the cross section of the production of neutral kaons is taken from our recent measurement [22]. The shaded area in the figure corresponds to the systematic uncertainties.

The deviation of $D_{c/n}$ from zero mostly comes from the different structure of the amplitudes of non-resonant isovector states, dominated by the $\rho$ meson, for the processes with charged and neutral kaons. Indeed, instead of relations in Eq. [5] for the charged final state the coupling constants of the $\omega(782)$ and $\rho(770)$ with the $K^0 L^0$ final state are:

$$g_{\omega K^0 L^0} = -g_{\rho K^0 L^0} = -g_{\phi K^0 L^0} \sqrt{2},$$

where the $\rho$-meson term has a different sign. So, the magnitude of $D_{c/n}$ in Eq. [10] is proportional to $g_{\rho K^0 L^0} K_{L}^0$, that allows to see experimentally the $\rho$ meson contribution to K-meson production.

We fit $D_{c/n}$ using Eq. [10] with two floating parameters, $r_{\rho/\omega}$ and $\delta K^0 L^0$, discussed above. The mass, width of the $\phi$ meson and $\Gamma_{\phi \to e\gamma} B_{\phi \to K^+ K^-}$ are fixed at the values obtained in Sec. [VII] also $\Gamma_{\phi \to e\gamma} B_{\phi \to K^+ K^-}$ is fixed at 0.428 keV according to Ref. [22]. The fit result is shown by a solid line in Fig. [10] a) and, in more detail, in insets to Fig. [10] (b, c) and yields:

$$r_{\rho/\omega} = 0.954 \pm 0.027,$$

$$\delta K^0 L^0 = 0.9964 \pm 0.0014,$$

$$\chi^2/ndf = 22.2/22.$$

We obtain good description of data by the fit. A small deviation of $r_{\rho/\omega}$ from unity demonstrates the precision ($\approx 5\%$) of relations [10] and confirms that the contribution from the $\rho(770)$ meson to $D_{c/n}$ dominates in the

VIII. CONTRIBUTION TO $a_\mu$.

Using the result for the $e^+ e^- \to K^+ K^-$ cross section we compute the contribution of this channel to the anomalous magnetic moment of the muon $a_\mu$ via a dispersion relation in the energy region $2 \cdot m_K < E_{c.m.} < 1.06$ GeV. According to Ref. [1], for the leading-order approximation we obtain:

$$a_\mu^{K^+ K^-} = \left( \frac{4 m_\mu}{3 \pi} \right)^2 \int_{4 m_K^2}^{(1.06 \text{ GeV})^2} \frac{d s}{s^2} K(s) \times$$

$$\times \frac{\sigma(e^+ e^- \to K^+ K^-) \cdot |1 - \Pi(s)|^2}{\sigma_0(e^+ e^- \to \mu^+ \mu^-)} = 19.33 \pm 0.040_{\text{stat}} \pm 0.40_{\text{syst}} \pm 0.04_{\text{VP}} \times 10^{-10},$$

where $K(s)$ is the kernel function, the factor $|1 - \Pi(s)|^2$ excludes the effect of leptonic and hadronic vacuum polarization (VP), and the Born cross section $\sigma_0(e^+ e^- \to \mu^+ \mu^-) = 4 \pi \alpha^2 / 3 s$. The first uncertainty is statistical, the second one corresponds to the systematic uncertainty of $\sigma(e^+ e^- \to K^+ K^-)$ and the third is the uncertainty of the VP factor (0.2% [24]). We integrate Eq. (9) using the model for the cross section obtained in the previous section. Then, in order to avoid a model uncertainty, the difference between the experimental cross section values and used model is integrated using the trapezoidal method.

The value should be compared with the recent result of the BaBar collaboration $a_\mu^{K^+ K^-} = 18.64 \pm 0.10_{\text{stat}} \pm 0.13_{\text{syst}} \pm 0.03_{\text{VP}} \times 10^{-10}$ [20] calculated in the same energy range. The difference between the calculation of $a_\mu$ based on our data and on the most precise previous measurement by BaBar is 1.6σ.
TABLE III: The resulting parameters obtained from the cross section fit in comparison with previous experiments.

| Parameter | CMD-3 | Other measurements |
|-----------|-------|---------------------|
| $m_\phi$, MeV | $1019.469 \pm 0.020 \pm 0.060 \pm 0.010$ | $1019.461 \pm 0.019$ (PDG2016) |
| $\Gamma_\phi$, MeV | $4.249 \pm 0.015$ | $4.266 \pm 0.031$ (PDG2016) |
| $\Gamma_{\phi \to e\mu} B_{\phi \to K^+K^-}$, keV | $0.670 \pm 0.022$ | $0.634 \pm 0.008$ (BaBar) |
| $B_{\phi \to e\tau} B_{\phi \to K^+K^-}$, $10^{-5}$ | $15.789 \pm 0.033 \pm 0.527 \pm 0.120$ | $14.24 \pm 0.30$ (PDG2016) |

energy range under study. The deviation of $\delta K_\phi^2 K_\phi^2$ from unity (0.36%) shows the level of possible remaining systematic uncertainty of the cross section measurements.

Additionally, from the comparison of the charged and neutral cross sections we can obtain the ratio of the coupling constants:

$$R = \frac{g_{\phi K^+K^-}}{g_{\phi K_\phi^2 K_\phi^2} \sqrt{Z(m_\phi^2)}} = \frac{B(\phi \to K^+K^-)}{B(\phi \to K_\phi^2 K_\phi^2)} \cdot \frac{1}{Z(m_\phi^2)} \frac{\beta_{K_\phi^2}^3}{\beta_{K_\phi^2}^3} = 0.990 \pm 0.017,$$

where the common parts of systematic uncertainties originating from those on the luminosity, radiative and energy spread corrections, are also reduced. As expected from isospin symmetry of u- and d-quarks, the value of $R$ is consistent with unity.

Additionally to the Coulomb interaction taken into account by the factor $Z(s)$, the final-state radiation of real photons, according to Ref. [28], decreases the total $e^+e^- \to K^+K^-(\gamma)$ cross section by about 0.4% at the $\phi$ meson mass. This effect partially explains the deviation of $R$ from unity.

**X. CONCLUSION**

Using CMD-3 data in the $E_{c.m.}=1010–1060$ MeV energy range we select $1.7 \times 10^6$ events of the process $e^+e^- \to K^+K^-$, and measure the cross section with an about 2% systematic error. Using the fit in the VMD model the following values of the $\phi$ meson parameters have been obtained:

$$m_\phi = 1019.469 \pm 0.061 \text{ MeV}/c^2$$
$$\Gamma_\phi = 4.249 \pm 0.015 \text{ MeV}$$
$$\Gamma_{\phi \to e\mu} B_{\phi \to K^+K^-} = 0.670 \pm 0.022 \text{ keV}$$

We calculate the contribution of the obtained cross section to the anomalous magnetic moment of the muon $\alpha_{\mu}^{K^+K^-} = (19.33 \pm 0.40) \times 10^{-10}$ in the energy range from threshold to $\sqrt{s} = 1.06$ GeV.

The observed deviation of the $\rho(770)$ and $\omega(782)$ amplitudes, $r_\rho/\omega = 0.95 \pm 0.03$, from a naive theoretical prediction allows to estimate the precision of the used VMD-based phenomenological model as better than 5%. The obtained ratio $\frac{g_{\phi K^+K^-}}{g_{\phi K_\phi^2 K_\phi^2} \sqrt{Z(m_\phi^2)}} = 0.990 \pm 0.017$ is consistent with isospin symmetry.

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FIG. 10: The difference of the charged and neutral cross sections defined as $D_{c/n} = \sigma_{e^+e^-\rightarrow K^+K^-} \times \frac{\rho^{\pm}_{0}(s)}{\rho^{\pm}_{\pm}(s)} \times \frac{1}{\sigma_e} - \delta K^0_L \times \sigma_{e^+e^-\rightarrow K^0_L K^0_L}$. The shaded area corresponds to systematic uncertainties in data, solid line - to the fit described in the text.

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