Leptonic CP problem in left-right symmetric model

Ravi Kuchimanchi

We find using the minimal left-right symmetric model that the presence of leptonic CP violation can radiatively generate a strong CP phase at the one-loop level itself, which can be beyond the current bounds established by the neutron electric dipole moment experiments. If there are no axions, this leads to the testable prediction that leptonic CP violation must be negligibly small (Dirac phase $\delta_{CP} = 0$ or $\pi$), in a wide and interesting region of parameter space.

Introduction –

One of the most attractive extensions of the standard model is the left-right symmetric model [1] based on $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P$ that restores parity as a good symmetry of the Lagrangian. The model requires the introduction of 3 right handed neutrinos, which are parity partners of the left handed neutrinos, and thereby provides a strong reason for neutrino masses and mixings. Interestingly, the QCD vacuum angle $\theta_{QCD}$ is absent in the left-right and mixings. Interestingly, the QCD vacuum angle which are parity partners of the left handed neutrinos, $(\bar{\theta})$ radiatively till the third loop and is negligibly small (parity odd, and the strong CP phase $\bar{\theta}$ (that contributes to the neutron’s electric dipole moment) is calculable in terms of the other parameters of the Lagrangian.

In the standard model, if we set the tree-level strong CP parameter $\tilde{\theta}$ to zero by hand, it is not produced radiatively till the third loop and is negligibly small ($\tilde{\theta} \sim 10^{-16}$) [2]. In the left-right symmetric model, the parity-odd $\bar{\theta}$ would be zero had $P$ remained unbroken. However a single CP violating quartic term in the Higgs potential generates a large $\bar{\theta}$ at the tree level once parity is spontaneously broken. If we select the coupling $\alpha_{2}$ of this quartic term to be real (CP conserving) at the tree level, so that $\bar{\theta}$ is zero, we can ask in which loop-order a non-real phase will get generated. Just like in the standard model, it was shown that even in the left-right model, $\bar{\theta}$ is not generated up to the third or fourth loop [3]. However this calculation (see last two paragraphs of Ref [4]) in the left-right model had only looked at CP violating radiative corrections from the CKM phase of the quark sector, and not at potential corrections from CP violation in the leptonic sector.

In this letter we show that CP violation in the leptonic sector can radiatively generate a complex phase in $\alpha_{2}$ and thereby the strong CP phase $\bar{\theta}$, at the one-loop level itself. Moreover if the Dirac-type Yukawa terms in the leptonic sector are similar to their quark sector counterparts, for a wide region in parameter space that includes all of type 2, and some interesting regions of type 1 seesaw mechanism, the strong CP phase generated from the leptonic phases exceeds the bound $\tilde{\theta} \leq 10^{-10}$ set by neutron EDM experiments. Thus we predict that leptonic CP violation must be absent or unobservably small (the leptonic CP phases being measured must be consistent with 0 or $\pi$) in the left-right symmetric model with the above provisions.

Since we take $\alpha_{2}$ to be real at a relaxation or cut-off scale above the $P$ breaking scale, it is worth noting that there are axionless solutions to the strong CP problem that restore CP symmetry, so that $\alpha_{2}$ is naturally real at the relaxation scale, while the CKM phase is generated after spontaneous CP breaking [5]. These solutions are discussed towards the end of the letter, and they will now also need to address the problem identified in this work, of leptonic CP phases radiatively generating the strong CP phase in one loop.

That a complex $\alpha_{2}$ is generated from phases in the leptonic sector, should have shown up in the one-loop Renormalization Group Equations of the left-right symmetric model which were evaluated in [6]. However the RG equations obtained in that work do not contain the contribution to the imaginary part of $\alpha_{2}$ (denoted by $\lambda_{11}$ in [6] and $\alpha_{2I}$ in this work) from the phases in leptonic Yukawa matrices. More recent work such as [7] also does not find CP violating one loop contributions to $\bar{\theta}$, if $\alpha_{2}$ is real or CP conserving at tree level. Our result is a significant departure from all previous works which concluded or assumed that $\alpha_{2I}$ (and therefore $\bar{\theta}$), once set to zero at the tree-level, does not pick up CP violating radiative corrections at the one loop level in the LR model.

Moreover, the excessive one-loop corrections we find are not suppressed even for large values of right handed scale, such as $v_{R} \sim 10^{14} GeV$, and imply that the smallness of $\bar{\theta}$ may not be natural, even in a technical sense. The consequence is that in important regions of parameter space of the non-supersymmetric LR model the strong CP problem must be solved by introducing an axion, or else the leptonic CP violating phases must vanish, which can be due to an axionless solution.

Connection between Strong and Leptonic CP violation –

We consider the minimal Left-Right symmetric model [8] based on $G_{LR} = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P$, with scalar triplets $\Delta_{R}$ (1, 1, 3, 2) and $\Delta_{L}$ (1, 3, 1, 2), and bi-doublet $\phi$ (1, 2, 2, 0). Under parity (P), the space-time coordinates $(x, t) \rightarrow (-x, t)$, $\phi \rightarrow \phi^{\dagger}$ and subscripts $L \leftrightarrow R$ for all other fields (see for example [3]). The scalar fields have the form

$$\phi = \left( \begin{array}{c} \phi_{1}^{o} \\ \phi_{2}^{\dagger} \\ \phi_{2}^{o} \\ \phi_{1}^{\dagger} \end{array} \right), \quad \Delta_{L,R} = \left( \begin{array}{cc} \delta_{L,R}^{+} / \sqrt{2} & \delta_{L,R}^{++} / \sqrt{2} \\ \delta_{L,R}^{-} & -\delta_{L,R}^{++} / \sqrt{2} \end{array} \right).$$

As is well known, all parameters of the Higgs potential
term $\alpha$ where $\tilde{\text{mass}}$ ratio. As noted in [8], $\alpha$ part of therefore gets generated from the non-Hermitian mass
diagrams of Figure I so that
\[
\alpha_{21} \sim \frac{i}{16\pi^2} \text{Tr} \left( f^1 f \left[ h^\ell, \tilde{h}^\ell \right] \right) \ln (v_R/M_{Pl})^2
\] (5)
where we note that $i[h^\ell, \tilde{h}^\ell]$ and $f^1 f$ are Hermitian matrices, and the cut-off has been taken to be at the Planck scale $M_{Pl} > v_R$. Note that there is also a pair of diagrams with the arrows in the internal lines of Figure I reversed (with their appropriate couplings) that also gives the same contribution. Note that there is no suppression in eq. [5] by a factor such as $(v_{wk}/v_R)^2$ or $(v_R/M_{Pl})^2$, and hence this contribution can be dangerously large, even if $v_R$ scale is well above the TeV scale.
The above equation, along with eq. [3], generates the strong $CP$ phase in the quark sector from leptonic Yukawas and $CP$ violation therein, and provides the following severe constraint on the leptonic sector that is being probed by neutrino experiments:
\[
\left| \text{Tr} \left( f^1 f \left[ h^\ell, \tilde{h}^\ell \right] \right) \right| \lesssim 3 \times 10^{-11}
\] (6)
where we have substituted $\theta \lesssim 10^{-10}, m_\ell/m_b \sim 40, \alpha_3 \lesssim 1$ and took the logarithm to have a generic value $\sim 10$.
Taking the Hermitian conjugate of (6) it can be seen that if $f^1 f, h$, and $\tilde{h}$ have all real matrix elements (conserve $CP$), then the left hand side vanishes. On the other hand, as we will now see, if they have complex phases these can be constrained by the above equation.
We will now consider the well motivated case where the leptonic Dirac Yukawa couplings are similar to their quark counterparts. This for example would be the case if there is an ultra-violet completion with a semi-unified theory such as the Pati-Salam model with $SU(4)_c \times SU(2)_L \times SU(2)_R$, or grand unified theory such as $SO(10)$. This implies that the matrix $[h^\ell, \tilde{h}^\ell]$ has some off-diagonal matrix elements that are of the order $\sim V_{\text{mix}}(m_\ell/m_b) \gtrsim 3 \times 10^{-4}$, in a basis in which either $h^\ell$ or $\tilde{h}^\ell$, like its quark counterpart is diagonal. Even if there is no unification, that the quark and leptonic Dirac type Yukawa matrices could be similar is hinted by the fact that the charged lepton masses are similar to the down sector quark masses.
With the above, equation [6] implies that some off diagonal matrix elements
\[
\left| (f^1 f)_{ij} \right| \lesssim 10^{-7}
\] (7)
if there are $O(1)$ $CP$ phases present in $f^1 f$ or in $[h^\ell, \tilde{h}^\ell]$.
The Yukawa matrix $f$ leads to Majorana mass terms for neutrinos once $\tilde{\delta}_R^\ell$ picks up a large $VEV \sim v_R/\sqrt{2}$ and $\delta_L^\ell$ picks up an induced $VEV \sim \gamma v_{wk}/v_R$ where $\gamma$ which is symbolically $\beta/\rho$ is obtained from real quartic couplings $\beta_i$, and $\rho_i$ of the Higgs potential [8] (which has terms such as $\rho_i^2 \text{Tr}(\Delta_{R}^\ell\Delta_{R})^2 + R \to L$, and is real at tree level. Since no symmetries can protect $\rho_i$, we have
in general, $\rho_i \gtrsim 0.01$ and therefore, $|\gamma| \lesssim 100$. The light neutrino mass matrix is given by the well known seesaw mechanism [10] and has the form

$$M_\nu = \frac{\nu_{wk}^2}{v_R} \left[ \gamma f - h_D \left( \frac{1}{f} \right) h_D^T \right]$$

(8)

where $h_D = (k_1 h^f + k_2 e^{-i\alpha} h^f)/\nu_{wk}$ is the Dirac type Yukawa matrix for the neutrinos.

If the first term in the square brackets of eq. (8) dominates over the second term, we have a Type 2 seesaw mechanism. Taking the third generation Yukawas to be larger than the rest of the Yukawas, for the first term to dominate, we must have $f_{33} < h_{D33}/\sqrt{|\gamma|}$. Since we have assumed that leptonic Dirac type and quark Yukawas are similar, we take $h_{D33} \simeq h_i \gtrsim 0.3$. Substituting $|\gamma| \lesssim 100$ we have for Type 2 seesaw mechanism, $f_{33} \gtrsim 0.03$. Since $M_\nu \approx f \gamma v_{wk}^2/v_R$ for type 2 seesaw, and we know by light neutrino experiments that the leptonic mixing angles are large we have for the off diagonal matrix elements, $f_{3j} \sim f_{33}$ to $f_{33}/10$. Thus we obtain $|f_{j1} f_{j3}| \gtrsim 10^{-4}$.

Comparing the above with eq. (7) we can see that the leptonic Dirac phase $\delta_{CP}$ cannot be of the order 1, and must be less than $10^{-3}$ from its $CP$ conserving value of 0 or $\pi$. Note that in this case of type 2 seesaw, the Majorana CP violating phases are unconstrained since they do not occur in $M_\nu M_\nu^T$ or in $[h^f, h^f]$. Currently experiments are being planned or under way to measure $\delta_{CP}$ with a sensitivity 5° to 10° (or $\sim 0.1$) [11] similar to the sensitivity achieved for the $CKM$ phase. The absence of a measurable $\delta_{CP}$ (modulo $\pi$) for the above well motivated case, is a key prediction of this work.

We now consider the case of type 1 seesaw where the second term in eq. (8) dominates. Substituting $h_{D33} \sim 0.3$ we find that for type 1 seesaw, $f_{33} v_R \sim 10^{14}$ GeV so that $h_{D33}^2 v_{wk}^2/(f_{33} v_R) \sim \sqrt{\Delta m_{32}^2} \sim 0.05$ eV, where we have used the mass squared difference of light neutrinos [11] $\Delta m_{32}^2 \sim 0.0023$ eV$^2$ and $v_{wk} \sim 246$ GeV.

If $v_R \sim 10^{18}$ GeV, we must have $f_{33} \sim 10^{-4}$ and we can see that the off-diagonal terms of $f^T f$ will satisfy eq. (7), and there is no constraint on the leptonic $CP$ phases. On the other hand if $v_R \sim 10^{15}$ to $14$ GeV, then $f_{33} \sim 0.1$ to 1. For type 1 seesaw since $f$ is more hierarchical, we take $f_{23} \sim f_{33}/1000$ ($f_{23}$ would be about $f_{33}/100$ but we allow for an additional factor of 10 since the phases in $f_{23}$ could be order $\delta_{CP}/10$). Comparing now with eq. (7) we once again find that leptonic $CP$ phases cannot be order 1 and must be constrained to be $\lesssim 10^{-2}$ to 4, modulo $\pi$. This is another key prediction. Note also that for type 1 seesaw, since both Majorana and Dirac phases can be present in $f^T f$, all the $CP$ phases are constrained.

So far we have looked at cases where quark and leptonic Dirac Yukawas are similar. We now relax this assumption to consider an example where $v_R$ can be at the $TeV$ scale. If all neutrino Dirac Yukawas including those of the third generation such as $h_{D3j} \sim 10^{-5}$ to $10^{-6}$, so that they are similar to the electron’s Yukawa coupling, then $v_R$ can be $O(\text{TeV})$ and $f_{33} \sim 1$, without the second term in eq. (8) giving a contribution greater than the observed 0.0023 eV$^2$ for light neutrino mass squared differences. Moreover, $[h^f, h^f]$ can have off-diagonal elements of the order $10^{-6} \times 10^{-2} = 10^{-8}$, since both $h_D$ and charged fermion masses arise from a combination of $h^f$ and $h^f$. For type 2 seesaw, $f_{3j} \sim f_{33}/10 \sim 1/10$ and so we find that the left hand side of eq. (6) is $\sim 10^{-9} \delta_{CP}$ and therefore $\delta_{CP}$ cannot be order 1 and is $\sim 1/30$. On the other hand for type 1 seesaw, $\delta_{CP} \sim 1$ is allowed and would result in a neutron EDM that could be potentially observed in future.

**Strong CP solution** – We have not assumed anything beyond the minimal left-right symmetric model to obtain the connection between leptonic and strong CP violation, and therefore the result may be of fundamental importance. It is clear that if the leptonic $CP$ phases are $O(1)$, then the strong $CP$ phase may have to be fine tuned to cancel excessive one-loop radiative corrections, making it technically unnatural. On the other hand if leptonic phases and $\alpha_{2I}$ are all zero at the tree level, there could be an underlying symmetry reason. This motivates us to look at solutions of the strong CP problem.

If we extend the model by adding an axion, then $\tilde{\theta}$ dynamically relaxes to zero [12], independent of the leptonic phases. However since $P$ sets the QCD vacuum angle to zero, historically it has been hoped that there would be an axionless solution in the LR model. An early attempt was made in [13] by adding an additional bi-doublet, and invoking a discrete symmetry. However the discrete symmetry also sets the $CKM$ phase to zero, and therefore the problem remained unsolved. Later it was noted that $\alpha_2$ is automatically absent in SUSYLR models and the strong CP problem can be thus solved [14]. However it was shown for the SUSYLR solution, that without any further constraints, the radiatively generated $\tilde{\theta} \sim 10^{-8}$ to $10^{-10}$ [15] which is uncomfortably close to the experimental bound, while a solution in the LR model (without supersymmetry), continued to be elusive [16] at the turn of the century.

Recently, progress was made by adding one heavy vectorlike quark family to the minimal LR model, and breaking both $P$ and $CP$ spontaneously so that $\alpha_{2I}$ naturally vanishes at the $CP$ restoration scale [5]. $CP$ is spontaneously broken by the VEV of a $P$ even, $CP$ odd real scalar singlet whose Yukawa couplings mix the usual and vectorlike quarks and generate the $CKM$ phase. Since $P$ is not broken by the singlet VEV, the resultant tree-level quark mass matrices are Hermitian, thus solving the strong $CP$ problem without requiring supersymmetry.

The interesting thing is that in the minimal version of the above solution, since a vectorlike lepton family
is not introduced, no CP violation is generated in the lepton sector! Thus it predicts that not only $\alpha_2$, but also the Dirac ($\delta_{CP}$) and Majorana leptonic phases vanish (modulo $\pi$), as noted in [17] and detailed in [18]. It is remarkable that the solution addresses perfectly the issues raised in this work.

However if a vectorlike lepton family is introduced then leptonic CP phases can be generated. This work shows that they would in turn generate too high a strong CP phase, in a wide and interesting region of parameter space, and vectorlike lepton family is disfavored.

Neutron EDM in axionless LR solution – If $v_R << M$, we just have the minimal LR model below $M$ (mass of the vector-like quark family or equivalently the scale of CP breaking). Radiative corrections from heavy quarks introduce a slight non-Hermiticity in the light quark mass matrices and generate a finite and calculable $\theta$. This was estimated in [5, 17], and it was found that up to logarithmic factors contribution from terms at the one-loop level are of the form,

$$\bar{\theta} \sim \frac{1}{16\pi^2} \left( \text{Product of Yukawas} \right) \left( \frac{v_R}{M} \right)^2$$

where the term in the round brackets is essentially a product of a string of up and down quark Yukawa matrices with the standard model Higgs doublet, and includes one Yukawa inverse. The two loop contribution has terms of a similar form with a longer Yukawa string in the product and an additional factor of $4\pi^{-2}$.

If $M \sim M_{Pl} \sim 10^{18} \text{GeV}$ and $v_R \sim 10^{14}$ to $10^{15} \text{GeV}$, then the above implies $\bar{\theta} \sim 10^{-10}$ to $10^{-12}$ if the product of Yukawas in the round brackets is $\sim h_i V_{ts} \sim 1/100$. Depending on the smallness of some unknown Yukawas involving the heavy quarks, the Yukawa product and hence $\bar{\theta}$ can be lesser.

An important feature in the above radiative corrections due to the heavy quarks is the suppression factor $(v_R/M)^2$. This is because as $M \to \infty$ the vector like quarks decouple. Additionally, if there are Planck scale corrections due to non-renormalizable terms, they will induce a $\theta_{Pl} \sim \lambda v_R^2/(MM_{Pl})$, where $\lambda$ is a dimensionless parameter. Thus the radiatively generated $\theta$ due to vector like quarks, and Planck scale corrections are both suppressed by $(v_R/M_{Pl})^2$ for $M \sim M_{Pl}$. Even if $v_R$ is small enough that both are undetectable, we still have the testable prediction that leptonic phases vanish (modulo $\pi$) in the axionless model.

Conclusions – We have shown that presence of leptonic CP violation can radiatively generate an excessive strong CP phase at the one loop level in the left-right symmetric model, that is beyond the limits already established by the neutron EDM experiments for the following interesting regions of parameter space:

- for any value of the right-handed symmetry breaking scale $v_R$ with Type 2 seesaw.
- for $v_R \lesssim 10^{15}\text{GeV}$ with Type 1 seesaw.

- if the Dirac Yukawa couplings of the neutrinos, $h_{D_{3\nu}} \sim 10^{-5}$ to $10^{-6}$, so that $v_R$ is at TeV scale, then for Type 2 seesaw.

While deriving the above, we assumed that the third generation Yukawas are greater than those of the other generations, but similar computations can be made for inverted hierarchy.

In the above significant regions of parameter space, the strong CP problem must be solved using an axion, or else the leptonic CP phases (particularly the Dirac phase $\delta_{CP}$) must vanish (modulo $\pi$). Remarkably, there is already an axionless strong CP solving LR model that in its minimal version, predicts the absence of leptonic CP phases (modulo $\pi$), in which a measurable strong CP phase $\sim 10^{-10}$ to $10^{-12}$ can be radiatively generated for a range of Yukawa couplings, if the parity breaking scale $v_R \sim 10^{14}$ to $10^{15} \text{ GeV}$ and CP breaking scale $M \sim 10^{18} \text{GeV}$.

In general, we can think of a theory as being afflicted with a leptonic CP problem if leptonic phases generate an excessive strong CP phase radiatively or through RGE running from higher scales. It makes the smallness of $\bar{\theta}$ technically unnatural and requires that either strong CP problem be solved using an axion, or predicts the vanishing of leptonic CP violation in problematic regions of parameter space. There can be axionless solutions that naturally set both $\bar{\theta}$ as well as leptonic phases to zero (modulo $\pi$) at the tree level.

I thank Rabi Mohapatra for helpful comments on the manuscript.

ravi@aidindia.org

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