Low-frequency oscillations in hydroturbines caused by cavitation together with the phase transitions effects

P A Kuibin
Kutateladze Institute of Thermophysics SB RAS, 1 Lavrentyev ave., Novosibirsk, 630090, Russia
E-mail: kuibin@itp.nsc.ru

Abstract. Among the unsteady phenomena taking place in hydroturbines one can separate oscillations caused by a cavitation bubble behind the runner. Usually a concept of cavitation compliance is used to model the cavity pulsations (Chen et al.). There was demonstrated the destabilizing effect of the diffuser and swirl. Kuibin et al. had found that the effect of swirl on stability is much smaller than that described by Chen et al. In the present work, the same analysis is repeated, but taking into account influence of the evaporation and condensation on dynamics of the cavity appearing in the draft tube. During analysis a simplified spherical cavity form was considered.

1. Introduction
The hydroturbines during their operation undergo unsteady phenomena of electrical, mechanical and hydrodynamical nature. The description of hydrodynamical phenomena is complicated by the possibility of existence of two-phase areas due to cavitation or involvement of air in the water flow. The reviews on approaches to solve the unsteady problems in hydro power stations can be found for example, in PhD thesis by Nicolet [1] or in monograph by Dörfler et al. [2]. One of interesting and simple enough method for analysis of instability generated due to the presence of the cavitation / air area behind the turbine runner was developed by Chen et al. [3]. The authors used one-dimensional model together with the concept of cavitation compliance and revealed the destabilizing effect of the draft tube diffuser and swirl. The same authors [4] considered water as compressible medium and found generation of high-frequency unstable modes in the penstock. Chen et al. [3, 4] had shown that one can control the flow instability as well as its eigen frequencies by changing the parameters of the diffuser or the swirl intensity, or the volume of the cavity. Kuibin et al., [5], Kuibin and Zakharov [6] tried to develop one-dimensional approach with help of more correct usage of the Bernoulli’s equation and application of different vortex models for evaluation of the swirl effect. They demonstrated much weaker influence of swirl on the stability conditions. Moreover, this influence decreases with gas/vapor cavity growth.

When considering the cavitation phenomenon, a reasonable question arises, how the effects of phase transition, evaporation and condensation influence on the instability parameters. Just this question is the main goal of the present research. For analysis we take the main equations derived by Chen et al. [3] and introduce new terms responsible for the phase transition effect. For simplicity a spherical cavity form was considered.
2. Analytical model

Consider a system modelling a hydro power station water passage consisting of a penstock of length $L_i$ with cross-sectional area $A_i$, a turbine runner (TR) and a draft tube (DT) with the area of the inlet cross-section $A_c$ and the exit cross-sections $A_e$ (see figure 1). As a model of cavitational area we put volume $V_c$ behind the turbine. Two main equations were derived by Chen et al. [3] for description of the flow rate and pressure. When there exist the cavitational bubble, the flow rate in the penstock $Q_1$ differs from the flow rate in the draft tube $Q_2$.

\[
Q_2 - Q_1 = \frac{dV_c}{dt} = -\rho C \frac{L_c}{A_c} \frac{d^2 Q_2}{dt^2} + \rho C \frac{D - \zeta_2}{A_e^2} Q_2 \frac{dQ_2}{dt} + 2\rho C \alpha \cot \beta_2 \left( \frac{Q_1}{S} \cot \beta_2 - U_2 \right) \frac{dQ_1}{dt}
\]  

(1)

Here $\rho$ is the liquid density; $L_c = \int (A_e/A(s)) ds$ is the DT effective length; $D = (A_e/A_c)^2 - 1$ is the diffusor factor, $\zeta_2$ is the DT loss factor ($\zeta_2$ is assumed to be constant). All terms of the right-hand part of equation (1) contain multiplier $C = -\partial V_c/\partial p_c$, the cavitation compliance. The last term in equation (1) reflects the effect of flow swirl behind the runner. The quantity in brackets is the characteristic circumferential flow velocity at the outlet from TR

\[c\theta_2 = (Q_1/S) \cot \beta_2 - U_2\]

$U_2$ is the peripheral velocity at the runner exit, $\beta_2$ is the angle of inclination of the blade at the exit from the TR, $S$ is the runner exit area. $\alpha$ is the pressure coefficient responsible for the swirl effect, it can be represented through difference between the ambient pressure $p_a$ in the zone with cavitational bubble and pressure inside the bubble $p_c$

\[\alpha = \frac{p_a - p_c}{\rho c^2 \theta_2^2}\]

(2)

The second main equation from [3] links the pressure at the system inlet, $p_i$, with the ambient pressure, $p_a$, and with the exit pressure, $p_e$

\[p_i - \rho \frac{L_i}{A_i} \frac{dQ_i}{dt} - \rho \frac{\zeta_T}{2A_e^2} Q_1^2 = p_a + \rho \frac{L_e}{A_e} \frac{dQ_e}{dt} + \rho \frac{\zeta_2 - D}{2A_e^2} Q_2^2\]

(3)

At a constant rotation speed of runner and fixed angle of opening of the guide apparatus, the turbine can be considered as a resistance with a constant loss coefficient $\zeta_T$, which depends on the opening of the guide device.

Assume that cavitational bubble has a spherical form of radius $R_c$. The equation for the rate of bubble radius changing due to phase transition reads [7, 8]

\[
\frac{dR_c}{dt} = -\frac{dV_c}{\rho \gamma} - \frac{R_c}{3\gamma p_c} \frac{dp_c}{dt}
\]

(4)

\[\frac{dR_c}{dt} = -\frac{dV_c}{\rho \gamma} - \frac{R_c}{3\gamma p_c} \frac{dp_c}{dt}\]

Figure 1. Scheme of the hydraulic part of the hydroturbine.
Here \( q_l \) is the density of heat flux from the bubble into liquid, \( \rho_c \) is the vapor density, \( \kappa \) is the latent heat of vapor generation, \( \gamma \) is the vapor adiabatic exponent. Equation (1) was derived in [3] through the time derivative of the cavity pressure \( Q_2 - Q_1 = dV_c/dt = -C \frac{d\rho_c}{dt} \). Thus, to take into account the phase transitions effects we find additional rate of the cavity pressure change in time and add new term to the right-hand side of equation (1)

\[
C \frac{3\gamma \rho_c}{\rho_c \kappa} \left[ q_l + \frac{d\rho_c}{dt} \right]
\]

In view of the expression for the bubble volume, \( V_c = \frac{4\pi R_c^3}{3} \), we obtain the modified continuity equation

\[
\left( Q_2 - Q_1 \right) \left( 1 - C \frac{\gamma \rho_c}{V_c} \right) = -\rho C \frac{L_v}{A_c} \frac{d^2Q_2}{dr^2} + \rho C \frac{D - \xi_2}{A_c^2} Q_2 \frac{dQ_2}{dt} + 2\rho C \alpha \cot \beta_2 S \left( \frac{Q_1}{S} \cot \beta_2 - U_2 \right) \frac{dQ_2}{dt} + C \frac{3\gamma \rho_c}{\rho_c \kappa} q_l
\]

It is necessary to determine also the density of heat flux from the bubble into liquid \( q_l \). It will follow from the heat transfer equation on the liquid temperature \( T_l \)

\[
\frac{\partial T_l}{\partial t} + \frac{R_c^2}{r^2} \frac{dR_c}{dt} \frac{\partial T_l}{\partial r} = \frac{a_l}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T_l}{\partial r} \right)
\]

with boundary conditions

\[
T_l \big|_{r=R_c} = T_c \big|_{r=R_c} = T_s(\rho_c), \quad q_l \big|_{r=R_c} = \lambda_l \left( \frac{\partial T_l}{\partial r} \right)_{r=R_c} = q_{lc}
\]

The coefficient \( a_l \) is the thermal diffusivity of the liquid. It is evident that temperature in cavity equals the saturation temperature.

Equations (3), (5), (6) allow to fulfill the stability analysis for \( Q_1(t) \) and \( Q_2(t) \). For linearization we assume \( Q_1 = \bar{Q}_1 + \bar{Q}_1 e^{i\omega t} \), \( Q_2 = \bar{Q}_2 + \bar{Q}_2 e^{i\omega t} \) as well as \( \rho_c = \bar{\rho}_c + \bar{\rho}_c e^{i\omega t} \), \( \gamma = \bar{\gamma} + \bar{\gamma} e^{i\omega t} \), \( V_c = \bar{V}_c + \bar{V}_c e^{i\omega t} \), \( R_c = \bar{R}_c + \bar{R}_c e^{i\omega t} \), \( T_l = \bar{T}_l + \bar{T}_l e^{i\omega t} \). The amplitude coefficients are much smaller than the corresponding quantities with overbar. As for the heat flux, we will have only pulsating part \( q_l = \bar{q}_l e^{i\omega t} \).

First, consider linearized equation (6)

\[
joT_l = \frac{d_l}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T_l}{\partial r} \right)
\]

This equation can be rewritten as a second-order differential equation

\[
\frac{\partial^2 T_l}{\partial r^2} + \frac{2}{r} \frac{\partial T_l}{\partial r} + k^2 T_l = 0
\]

with \( k^2 = -j\omega/a_l \). The solution of this equation has been found and represented through elementary functions

\[
T_l = A \frac{\sin(kr)}{r} + B \frac{\cos(kr)}{r}
\]

The constants A and B should be found from the boundary condition. We obtain
\[
A = \bar{R}_e \bar{T}_s \sin \left( k\bar{R}_e \right) + \left( \frac{\bar{q}_l}{\lambda_l} \bar{R}_e^2 + \bar{R}_e \bar{T}_s \right) \frac{\cos \left( k\bar{R}_e \right)}{k\bar{R}_e}, \quad B = \bar{R}_e \bar{T}_s \cos \left( k\bar{R}_e \right) - \left( \frac{\bar{q}_l}{\lambda_l} \bar{R}_e^2 + \bar{R}_e \bar{T}_s \right) \frac{\sin \left( k\bar{R}_e \right)}{k\bar{R}_e} \]
\]

Finally, for the pulsating heat flux we have formula
\[
\bar{q}_l = A_k \frac{\cos \left( k\bar{R}_e \right)}{\bar{R}_e} - A \frac{\sin \left( k\bar{R}_e \right)}{\bar{R}_e^2} - B_k \frac{\sin \left( k\bar{R}_e \right)}{\bar{R}_e} - B \frac{\cos \left( k\bar{R}_e \right)}{\bar{R}_e^2} \]

Equations (3) and (5) after linearization yield a set of homogeneous linear equations in terms of \( \tilde{Q}_1 \) and \( \tilde{Q}_2 \). Note, that relations between \( \bar{q}_l \) and \( \tilde{Q}_1 \), \( \tilde{Q}_2 \) can be found from the steady solution of equation (6)
\[
\frac{d\bar{q}_l}{d\bar{V}_e} \bar{V}_e = -\frac{\lambda_l}{\bar{R}_e} \frac{dT_e}{d\bar{V}_e} \frac{dp_e}{d\bar{V}_e} \bar{V}_e = \frac{1}{C} \frac{d\bar{T}_e}{d\bar{V}_e} \frac{d\bar{V}_e}{d\bar{V}_e} = \frac{1}{C} \frac{d\bar{T}_e}{d\bar{V}_e} \frac{\bar{Q}_2 - \bar{Q}_1}{j\omega} \]

The dependence \( p_c(T_e) \) is known from the Handbook of Physical Properties of Liquids and Gases [9] which usually is approximated by some analytical function, for example
\[
p_c(T_e) = 10^{A_1} - B_2((C_1 + T_e) \]

The temperature here is measured in Celsius degrees and the pressure in Pa.

As a result, we write down the characteristic equation by setting the determinant of the coefficient matrix of the linear equation to zero:
\[
-\rho C \frac{L_x}{A_x} \frac{L_y}{A_y} (j\omega)^4 + \rho C \frac{L_x}{A_x} \frac{\zeta_T}{A_T} \bar{Q} - \rho C \frac{D - \zeta_2}{\lambda_2} \frac{L_x}{A_x} \frac{L_y}{A_y} + 2 \rho C \alpha \frac{\cos \beta_2}{S} \left( \frac{\bar{Q}}{S} \frac{\cos \beta_2 - U_2}{U_2} \frac{L_x}{A_x} \right) \frac{(j\omega)^3}{j\omega} \]

\[
+ \left[ \frac{C}{V_e} \frac{\bar{L}_x}{A_x} + \frac{L_x}{A_x} \bar{L}_x \right] + \rho C \frac{D - \zeta_2}{\lambda_2} \frac{L_x}{A_x} \frac{L_y}{A_y} - 2 \rho C \alpha \frac{\cos \beta_2}{S} \left( \frac{\bar{Q}}{S} \frac{\cos \beta_2 - U_2}{U_2} \frac{L_x}{A_x} \right) \frac{(j\omega)^2}{j\omega} \]

\[
+ \left[ \frac{C}{V_e} \frac{\bar{L}_x}{A_x} + \frac{L_x}{A_x} \bar{L}_x \right] + \frac{3 \gamma p_c}{R_e^2} \frac{\lambda_l}{\rho c \kappa} \frac{d\bar{T}_e}{d\bar{V}_e} \left( \frac{L_x}{A_x} + \frac{L_y}{A_y} \bar{L}_y \right) \frac{(j\omega)}{j\omega} \]

\[
+ \frac{3 \gamma p_c}{R_e^2} \frac{\lambda_l}{\rho c \kappa} \frac{d\bar{T}_e}{d\bar{V}_e} \left( \frac{L_x}{A_x} + \frac{L_y}{A_y} \bar{L}_y \right) \frac{(j\omega)}{j\omega} = 0 \]

The complex frequency consists of two parts: \( \omega = \omega_R + i\omega_I \). The real part, \( \omega_R \), is the frequency, and the imaginary part, \( \omega_I \), is the damping rate (decrement) of the perturbation. When \( \omega_I \) is positive, one has stable disturbances and controversy, at \( \omega_I < 0 \) the amplitude of disturbances will grow.

Unlike the paper by Chen et al. [3], where similar equation was of the third order and had analytical solution, here we obtained the equation of forth order. Nonetheless these equations have some common features. At \( \gamma = 0 \) equation (15) converts to equation (13) from [3]. This circumstance allows to use the analytical solution of equation (13) from [3] and slowly changing value of \( \gamma \) from zero to its natural value to find solution of equation (15).

Unfortunately, the model constructed obeys some lacks. First of all, the attention should be payed on the bubble radius (or volume) in denominators in number of coefficients. So, we have limitation on consideration of large enough size of bubbles. Nonetheless we try to analyze the effect of phase transitions on the disturbances development.

3. Results of calculations

Some examples demonstrating influence of the phase transition effects on the stability characteristics are presented in figure 2. For the base we took stability analysis made by Kuibin and Zakharov [6] for
two sets of parameters and for the smooth model of the vortex simulating flow behind the turbine runner (Model II in [6]), namely at vortex core size equal \( \varepsilon = 0.5 \, R \left( \pi R^2 = A_c \right) \), cavity radius \( r_c = 0.5 \, \varepsilon \) and \( r_c = 0.9 \, \varepsilon \). As seen, at small flow rate the frequencies of unstable disturbances are close to ones found in [6]. The range of flow rates, when the oscillations are possible become narrower in both cases for small cavity ([0.591 \ldots 0.744] instead of [0.552 \ldots 0.712]) and bigger one ([0.572 \ldots 0.700] instead of [0.520 \ldots 0.778]). For the first case this interval is shifted to area of higher flow rates. When the phase transition takes place the perturbances become more stable at small flow rates. At \( \tilde{Q} > 0.4 \) and moderate cavity size the behavior of the increment dependence on the flow rate qualitatively changes. At \( \tilde{Q} = 0.446 \) the disturbances become neutral and at \( \tilde{Q} > 0.446 \) – stable. As for the branch of high flow rates, \( \tilde{Q} > 0.744 \), we have both situations, perturbances with lesser \( \omega_I \) at \( \tilde{Q} < 0.875 \) and more stable at \( \tilde{Q} > 0.875 \). For large cavity size the disturbances in presence of the phase transitions are found to be stable practically at any flow rate excluding range of [0.219 \ldots 0.322] with very small increments.

**Figure 2.** The frequencies \((a, c)\) and increments/decrements \((b, d)\) of the disturbances vs flow rate. \((a, b)\): \( \varepsilon = 0.5 \, R \), \( r_c = 0.5 \, \varepsilon \); \((c, d)\): \( \varepsilon = 0.5 \, R \), \( r_c = 0.9 \, \varepsilon \). The black lines correspond to calculations from [6], blue lines – present calculations.

**4. Conclusion**

In the paper a model is constructed for description of the phase transition influence on the low-frequency oscillations arising in flow in hydro turbine in presence of the cavitational bubble size behind the turbine runner. It was found that effect is strong enough on the perturbances increments or decrements. In the same time influence on the frequencies is relatively weak. The model needs in future development.

**References**

[1] Nicolet C 2007 Hydroacoustic modelling and numerical simulation of unsteady operation of hydroelectric systems, Thesis EPFL n° 3751

[2] Dörfler P, Sick M and Coutu A 2013 Flow-induced pulsation and vibration in hydroelectric machinery Springer-Verlag London
[3] Chen C, Nicolet C, Yonezawa K, Farhat M, Avellan F and Tsujimoto Y 2008 *J. Fluids Eng.* **130** 041106

[4] Chen C, Nicolet C, Yonezawa K, Farhat M, Avellan F and Tsujimoto Y 2009 *J. Fluid Machinery and Systems* **2** 260

[5] Kuibin P, Pylev I, Zakharov A 2012 *IOP Conf.Series:Earth and Environmental Science* **15** 022001

[6] Kuibin P, Zakharov A 2019 *IOP Conf.Series:Earth and Environmental Science* **288** 012100

[7] Nakoryakov V, Pokusaev B, Shreiber I 1992 *Wave dynamics of gas- and vapor-liquid media* (New York, Begell House Publ.)

[8] Franc J-P, Michel J-M 2004 *Fundamentals of Cavitation* (Dordrecht-Boston-London, Kluwer)

[9] Vargaftik N 1975 *Handbook of Physical Properties of Liquids and Gases. Pure Substances and Mixtures* (Berlin-Heidelberg-New York, Springer-Verlag)