In this paper we consider the influence of dust charge fluctuations on damping of the dust-ion-acoustic waves. Fluid approximation of longitudinal electrostatic waves in unmagnetized plasmas is considered. We show that for a weak acoustic wave the attenuation depends on a phenomenological charging coefficient.

**Introduction**

Plasmas with dust grains are of interest both for the cosmic space as well as for the laboratory plasmas. Examples include cometary environments, planetary rings, the interstellar medium, the Earth’s and other planets magnetospheres [1]. Dust has been found to be a determinant component of rarefied plasmas used in the microelectronic processing industry [2], and it may also be present in the limiter regions of fusion plasmas due to sputtering of the carbon by energetic particles. It is interesting to note the recent flurry of activity in the dusty plasma research. It has been driven largely by discoveries of the role of dust in quite different settings: the ring of Saturn [1] and the plasma processing device [2]. Dusty plasmas contain, beside positive ions and electrons, large particles usually negatively charged. They are conglomerations of the ions, electrons and neutral particles. These large particles, to be called grains, have atomic numbers $Z_d$ in the range of $10^4 - 10^6$ and their mass $m_d$ can be equal to $10^6$ of the proton mass or even much more. In the considered dusty plasmas, the size of grains is small compared with average distance between the grains. The ratio of charge to mass for a given component of plasma determines its dynamics. We note that for dusty plasmas, ratio of electrical charges of grains to their masses is usually much smaller than in the case of multispecies plasmas with negative ions and hence, here comes the first of the crucial differences between multispecies plasmas with negative ions and dusty plasmas. Because dynamics of the dusty plasma components, electrons, positive ions and dust grains is quite different in the time and length scales considered here, then the equations for these components of the dusty plasma may be different. If it is assumed that all grains have equal masses and charges steady in time, therefore the dust-ion-acoustic and dust-acoustic dispersion relations are obtained on the basis of fluid [3, 4] or kinetic [5] models. In this case we have assumed, for simplicity, that all grains have equal masses and charges, but charges are not constant in time - they may fluctuate in time. The dust charges are not really independent of the variations in the plasma potentials. Here, even in the fluid theory, appear the crucial differences between the ordinary multispecies plasmas and the dusty plasmas. All modes will influence the charging mechanism, and feedback will lead to several new interesting and unexpected phenomena. The charging of the grains depends on local plasma characteristics. If the waves disturb these characteristic, then charging of the grains is affected and the grain charge is modified, with a resulting feedback on the wave mode.
The simplest cases to deal with are the parallel electrostatic modes in an unmagnetized plasma. Then the electric field $E$ is one-dimensional and may be represented by the electric potential $\phi$: $E = -\partial \phi / \partial x$. We consider a problem when the temperature of electrons $T_e$ is much greater than the temperature of ions $T_i$: $T_e \gg T_i$. In such simplified situations, fluctuations in time of the number density of electrons $\delta n_e$ can occur due to the grains of the dust loosing or picking up some electrons. Dust charge fluctuations in time give rise to purely damped acoustic modes when the streams of particles are absent. Here we solve the continuity equations in approximation for the source term vanishing at equilibrium.

1 Fluctuation of dust grains in dusty plasmas

As a result of fluctuating dust charges in dusty plasmas, many new problems can appear which are in partly treatment by Verheest. We consider a specific problem when the temperature of electrons is much greater than the temperature of ions and we also assume that the mass of grains with fluctuating charges may be approximated by constant values. In this case the continuity equations for specimens of dusty plasmas can be written in the form:

$$\frac{\partial n_d}{\partial t} + \frac{\partial (n_d u_d)}{\partial x} = 0,$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i u_i)}{\partial x} = 0,$$

$$\frac{\partial n_e}{\partial t} + \frac{\partial (n_e u_e)}{\partial x} = S_e.$$  \hspace{1cm} (2.1)

Due to the possible fluctuations of the dust charges we can express the conservation of charge in the dusty plasma by:

$$\frac{\partial}{\partial t} \left( -n_e + n_d q_d + n_i e \right) + \frac{\partial}{\partial x} \left( -n_e e u_e + n_d q_d u_d + n_i e u_i \right) = 0,$$  \hspace{1cm} (2.2)

where $q_d$ is the charge of grain of dust. This can be rewritten with the help of the continuity equation (2.1 - 2.3) as:

$$n_d \left( \frac{\partial}{\partial t} + u_d \frac{\partial}{\partial x} \right) q_d = e S_e.$$  \hspace{1cm} (2.3)

On the other hand, the charge of grain of dust fluctuation is given by:

$$\frac{dq_d}{dt} = \left( \frac{\partial}{\partial t} + u_d \frac{\partial}{\partial x} \right) q_d = I_i (n_i, q_d) + I_e (n_e, q_d),$$  \hspace{1cm} (2.4)

where $I_i (n_i, q_d)$ and $I_e (n_e, q_d)$ are the ionic and electronic charging current, respectively. When we combine (2.3) and (2.4), we get

$$e S_e = n_d I_e (n_e, q_d) + n_d I_i (n_i, q_d).$$  \hspace{1cm} (2.5)
In equilibrium dusty plasma, the total charging current vanishes:

\[ I_{i0} + I_{e0} = 0, \quad (2.6) \]

where \( I_{i0} \) and \( I_{e0} \) denotes the equilibrium charging current for ions and electrons, respectively. Therefore we can expand (2.5) as a function of \( n_e, q_d \) and \( n_d \) using (2.6) and hence in linear approximation for \( S_e \) vanishing at equilibrium, it is given by:

\[ S_e = -\nu_e \delta n_e - \mu_e \delta q_d, \quad (2.7) \]

where \( \nu_e, \mu_e \) denotes charging fluctuation coefficients while \( \delta n_e \) and \( \delta q_d \) denotes fluctuation electron number density and fluctuation charges of grains from their equilibrium values respectively.

## 2 Dumping dust-acoustic wave

Now we add to the continuity equations (2.1), the equations of motion written explicitly for a three-component dusty plasma.

\[
\begin{align*}
\left( \frac{\partial}{\partial t} + u_d \frac{\partial}{\partial x} \right) u_d &= -\frac{q_d}{m_d} \frac{\partial \phi}{\partial x}, \\
\left( \frac{\partial}{\partial t} + u_i \frac{\partial}{\partial x} \right) u_i + \frac{c_{si}^2}{n_i} \frac{\partial n_i}{\partial x} &= \frac{e}{m_i} \frac{\partial \phi}{\partial x}, \\
\left( \frac{\partial}{\partial t} + u_e \frac{\partial}{\partial x} \right) u_e + \frac{c_{se}^2}{n_e} \frac{\partial n_e}{\partial x} &= \frac{e}{m_e} \frac{\partial \phi}{\partial x}.
\end{align*}
\]

Here \( \phi, c_{sc}, c_{si} \) \( c_{si}^2 = \frac{k_B T_i}{m_i}, c_{se}^2 = \frac{k_B T_e}{m_e} \) if the electron obeys an ideal gas law, \( m_i, m_e, k_B, T_e, T_i \) are electric potential, thermal velocity of electrons and ion, mass of ion and electron, Boltzmann constant, temperature of electrons and ions, respectively. The fluid equations are supplemented by Poisson’s equation:

\[ \varepsilon_0 \frac{\partial^2 \phi}{\partial x^2} = en_e - cn_i - q_d n_d, \quad (3.2) \]

where \( \varepsilon_0 \) denotes free space permittivity.

Assuming sufficiently small disturbances of plasma equilibrium, we linearize the relevant equations around the equilibrium and Fourier transform. Taking into account the conservation of total charge and the fact that the phase velocities of wave are smaller than the thermal velocity of electrons and larger than the thermal velocities of ions and dust: \( c_{sd}, c_{si} \ll \frac{\omega}{k} \ll c_{sc} \), where \( \omega \) and \( k \) denotes the wave frequency and wave number, we obtain - after some calculations - dispersion relation for the dust-ion-acoustic waves including the charge fluctuation of dust:

\[ \omega = \omega_0 - i \frac{k^2 \lambda_{De}^2 \omega_p^2}{2 \omega_0^2 (1 + k^2 \lambda_{De}^2)^2} \nu_e, \quad (3.3) \]
where

\[ \omega_0 = \sqrt{\frac{k^2 \lambda_{De}^2 \omega_{pi}^2}{1 + k^2 \lambda_{De}^2} + k^2 c_s^2} \]

(3.4)

\[ \lambda_{De} = \sqrt{\frac{\varepsilon_0 k_B T_e}{N_{0e} e^2}} \]

is the electron Debye length, \( \omega_{pi} = \sqrt{\frac{N_{0i} e^2}{\varepsilon_0 m_i}} \) is the ion plasma frequency. The equilibrium number density is related by quasi-neutrality of dusty plasma relation: \( N_{0i} = N_{0e} + Z_d N_{0d} \), where \( q_d = e Z_d \) and \( Z_d \) - is the atomic number grain of dust.

For \( k \to 0 \) we have the following dispersion relation for the dust-ion-acoustic waves (DIAW):

\[ \omega = \omega_0 - i \nu_e \frac{\lambda_{De}^2}{2 (\lambda_{De}^2 + \lambda_{Di}^2)} \approx k c_s - \frac{1}{2} i \nu_e. \]

(3.5)

This equation describes the dumped dust-ion-acoustic waves including the charge fluctuation. In our approximation: \( T_e >> T_i \), the dumping of dust-ion-acoustic waves is independent of the parameter \( \mu_e \).

3 Conclusions

The paper deals with a small dust charge fluctuations. In the case considered here, when the temperature of electrons is much greater than the temperature of the ions: \( T_e >> T_i \) and \( T_e \) is not great enough for further ionization of the ions, we show that attenuation of the acoustic wave depends only on one phenomenological coefficient \( \nu_e \). The value of this coefficient depends mainly on the temperature of electrons.

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