Conductance of the single-electron transistor: A comparison of experimental data with Monte Carlo calculations

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We report on experimental results for the conductance of metallic single-electron transistors as a function of temperature, gate voltage and dimensionless conductance. In contrast to previous experiments our transistor layout allows for a direct measurement of the parallel conductance and no ad hoc assumptions on the symmetry of the transistors are necessary. Thus we can make a comparison between our data and theoretical predictions without any adjustable parameter. Even for rather weakly conducting transistors significant deviations from the perturbative results are noted. On the other hand, path integral Monte Carlo calculations show remarkable agreement with experiments for the whole range of temperatures and conductances.

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I. INTRODUCTION

The usual single-electron transistor (SET) layout consists of two ultrasmall tunnel junctions with conductances $G_{s,d}$ and capacitances $C_{s,d}$, respectively. The junctions in series are biased by a voltage $U_{sd}$ and the island between the tunneling barriers is coupled to a gate voltage $U_g$ via a capacitance $C_g$.

This setup has been used as the basic device to study the Coulomb blockade. Its transport properties are governed by two dimensionless parameters. Firstly, the dimensionless inverse temperature $\beta E_c$ relates $\beta = (k_B T)^{-1}$ to the charging energy $E_c = e^2/(2C_s)$ with $C_s = C_s + C_d + C_g$ being the total island capacitance. This parameter determines how far the electrostatic blockade of the source-drain current is lifted by thermal excitations. Secondly, the dimensionless parallel conductance $g = (G_s + G_d)/G_K$, $G_K = e^2/h$ being the conductance quantum, measures how much the quantization of the island charge is smeared by quantum fluctuations. The charge on the island can be controlled by means of the gate voltage $U_g$. The linear-response conductance $G$ between source and drain is a periodic function of $U_g$ taking on values between $G_{\text{min}}$ and $G_{\text{max}}$. Since the control of a device by Coulomb blockade exploits a large difference between $G_{\text{min}}$ and $G_{\text{max}}$ a detailed understanding of the washout of Coulomb blockade effects by thermal and quantum fluctuations is crucial for the optimal design of fast and reliable Coulomb blockade devices.

Theoretical work has mainly focused on the limits of small $g$ or $\beta E_c$, respectively. In the weakly conducting regime, $g \ll 1$, perturbation theory should be sufficient to describe the experimental data. For $\beta E_c \ll 1$ it is more suitable to formulate the problem in terms of a path integral which may be evaluated semiclassically. Only recently the use of path integral Monte Carlo (PIMC) techniques was proposed to calculate the conductance of the SET over the whole range of experimentally accessible parameters. Especially the regime of large parallel conductance $g$ is of interest for technological applications.

This recent work has pointed to a shortcoming of previous experiments. The first step of a comparison between theory and experiment is the determination of the parameters $g$ and $E_c$. For the SET layout described above one can obtain only the series conductance $G_s$ of the two junctions which does not suffice to calculate the dimensionless parallel conductance $g$ without further assumption. Joyez et al. assumed that their SETs were symmetric, i.e. built up of two identical tunnel barriers which turned out to be a good approximation for the first three samples measured in the experiment. However, comparing the data for the high conductance sample of Ref. to their PIMC simulation results, Göppert et al. found strong indications that the tunnel barriers could be asymmetric.

Since an unambiguous determination of the relevant parameters is a prerequisite of a thorough comparison between theoretical and experimental results we have developed a transistor design with four tunnel junctions connected to the island (see Fig. 1). This arrangement allows the determination of the individual resistances of each junction and therefore the direct measurement of the tunneling strength parameter $g$. It is operated as SET by connecting two of the junctions in parallel to form source and drain, respectively.

In Sec. 1 we present experimental details about the sample fabrication, the determination of the parameters and the conductance measurements. In Sec. 2 we give the path integral formulation which is used for the imaginary time quantum Monte Carlo simulation. We summarize how the conductance can be calculated from the simulation data by use of the singular value decomposition (SVD) analytical continuation scheme. In Sec. 4 we compare experimental and theoretical results. At low
II. EXPERIMENT

Seven samples with varying tunneling strengths were investigated. The samples were fabricated from aluminum by standard e-beam lithography in combination with two-angle shadow evaporation. The evaporation is performed by electrical heating a tungsten wire which holds a drop of aluminum. Tunnel barriers of different strengths could be produced by a variation of the oxygen pressure applied to the evaporation chamber between the two evaporation steps.

We used two different layouts: firstly, the usual SET design with two junctions, forming a small island in between, and a straight gate finger from the side pointing towards the island, secondly, the design which is shown on the SEM picture in Fig. 1. In this design one can determine the individual tunnel resistances by measuring the current in response to an applied voltage bias across different combinations of the four tunnel junctions. The measurements presented in this article are performed on four standard SETs (samples I–IV), one four-contact sample (sample VII) and two samples which also have the four-contact layout, but turned out to have only three working tunnel junctions (samples V and VI). Nevertheless, this is sufficient to determine all individual tunnel resistances. The sample parameters are given in Table I. The measurements are performed in a top-loading dilution refrigerator in the temperature range from 25 mK to 18 K. The samples are mounted within a well shielded metallic cavity. All electrical wiring into the cavity is made of highly resistive leads (32 Ω/m, diameter 0.23 mm) which are fed through stainless steel capillaries with an inner diameter of 0.34 mm and a length of 1 m. The capillaries are wound up in a compact coil which is held in thermal equilibrium with the sample. These feedthroughs form resistive co-axial cables. They provide a damping exceeding 200 dB in the frequency range from 20 GHz to 6 THz, as calculated by classical electrodynamics taking the skin effect into account. The validity of the used formulae was checked experimentally in the frequency range up to 20 GHz using a spectrum analyzer. Additional rf-filtering is performed at room temperature at the entrance to the cryostat.

To measure the conductance of the transistors they are biased with a voltage and the resulting current is measured with an operational amplifier at the top of the cryostat. The resolution of the current measurement is better than 100 fA. To gain resolution and to circumvent the 1/f-increase of noise at low frequencies an AC component of ≈ 10 Hz is added to the biasing DC voltage simultaneously and the resulting AC component of the current is measured with a lock-in amplifier. The DC measurement is used to ensure that the measurement is performed at vanishing bias and stays in the linear-response regime.

To determine the conductance $G_Σ$ of two tunnel junctions in series, we measure the $I_{sd}U_{sd}$ characteristic up to a bias voltage of maximal ±20 mV. We define the asymptotic slope at large bias voltages as $G_Σ$. The four-junction design allows six different configurations of two tunnel junctions connected in series. From the corresponding set $G_Σ,i$, $i = 1, ..., 6$, the individual tunnel junction conductances can be derived with simple algebra. Their values are given in Table I. Thus this layout enables us to actually measure the coupling strength parameter $g$ directly. The SET investigated in the following experiments was formed by using all four tunnel junctions, where source and drain were made by connecting two tunnel junctions in parallel, respectively (see Fig. 1).
TABLE I: Parameters of the samples I–VII. $G_{cl}$ denotes the high temperature conductance of the SET. The gate capacitance $C_g$ is extracted from the period of the Coulomb oscillations within an accuracy of about 1%. Samples V–VII consist of at least three tunnel junctions. The individual conductances $G_{s,s'}$ and $G_{d,d'}$ for these samples were evaluated by simple algebra from values of $G_S$, which had been measured at different pairs of tunnel junctions as described in the text. For the 2-junction samples I–IV, the given value of $g$ equals $(G_s + G_d)/(2G_K)$, an expression valid for symmetric SETs only. For the asymmetric sample I, the value derived from a comparison with the second-order perturbation theory is given in brackets (see Fig. 2).

### III. THEORY

#### A. Path integral formulation

The linear DC conductance $G$ represents a transport coefficient which can be expressed by correlation functions of the system using a Kubo formula. Defining the current through the SET as the average of the current through the source and drain junctions $I = (I_s + I_d)/2$ the conductance $G$ may be connected to the spectrum of the current autocorrelation function $F(t) = \langle I(t)I(0) \rangle$, i.e.

$$G = \frac{\beta}{2} \hat{F}(\omega = 0) \quad (1)$$

with $\hat{F}(\omega)$ denoting the Fourier transform of $F(t)$. The calculation of the current correlator may be done in the phase representation i.e. in terms of the phase variable $\varphi$ which is conjugate to the charge $q$ on the island. For imaginary time $\tau$ one gets

$$F(\tau) = 4\pi G_{cl} \alpha(\tau) C(\tau) \quad (2)$$

with the classical high temperature conductance $G_{cl}$ and

$$\alpha(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \hat{\alpha}(\omega) e^{-\tau \omega}, \quad \hat{\alpha}(\omega) = \frac{\hbar}{2\pi} \frac{\omega}{1 - e^{-\hbar \omega}} \quad (3a)$$

$$\hat{\psi}(\omega) = \frac{\hbar}{2\pi \beta} \int_{-\infty}^{\infty} d\tau \alpha(\tau) \sin \left[ \frac{\varphi(\tau) - \varphi(0)}{2} \right] \quad (4)$$

The cosine correlation function $C(\tau) = \langle \cos [\varphi(\tau) - \varphi(0)] \rangle$ has the formally exact path integral representation

$$C(\tau) = \frac{1}{Z} \sum_{k=-\infty}^{\infty} \int_{\varphi(0)=0}^{\varphi(\beta \hbar) = 2\pi k} D\varphi e^{-\frac{1}{2} \beta \hbar S[\varphi] + \pi i k n_g} \times$$

with the partition function

$$Z = \sum_k \int D\varphi e^{-\frac{1}{2} \beta \hbar S[\varphi]} e^{2\pi i k n_g} \quad (5)$$

and the dimensionless gate voltage $n_g = (U_g C_g)/e$. The Euclidian action $S[\varphi] = S_C[\varphi] + S_T[\varphi]$ splits into the Coulomb action

$$S_C[\varphi] = \int_{0}^{\beta \hbar} d\tau \frac{\hbar^2 \dot{\varphi}^2(\tau)}{4E_c} \quad (6)$$

which describes the charging of the island and the tunneling action

$$S_T[\varphi] = 2g \int_{0}^{\beta \hbar} d\tau \int_{0}^{\beta \hbar} d\tau' \alpha(\tau - \tau') \sin^2 \left[ \frac{\varphi(\tau) - \varphi(\tau')}{2} \right] \quad (7)$$

that expresses the influence of tunneling processes on the dynamics of the phase variable $\varphi$.

Equation (3) may serve as the starting point for analytical calculations or numerical work. In the latter case the interval $[0, \beta \hbar]$ is divided into $N$ Trotter slices and the multidimensional integral is calculated using Monte Carlo methods. Since the action $S[\varphi]$ is real, the Metropolis algorithm can be applied for an important sampling of the configurations $\{\varphi_i = \varphi(\tau_i) | i = 0, ..., N\}$ and the winding number $k$.

We did Monte Carlo simulations for fixed tunneling strength $g = 4.75$ over a range of inverse temperatures $\beta E_c \in [0.5, 21.0]$ which will be compared to our experimental findings in Section IV. For each temperature the system was equilibrated during several million Monte Carlo steps. Measurements were then carried out for another 5–32 million sweeps depending on the value of $\beta E_c$.

Especially for low temperatures it is necessary to increase the number of measurements to get reliable statistics of the data. We ensured that the error of the correlation function is always less than 3%. Over the whole range
of temperatures we chose $N = 200$ Trotter slices fulfilling the convergence criterion $N \geq 5 \beta E_c$ used in earlier work. The conductance was calculated for 200 values of the dimensionless gate voltage spanning the range $n_g \in [0, 0.5]$.

Additionally we examined single electron transistors with different parallel conductances $g \in [2.0, 15.0]$ for fixed temperature $1/(\beta E_c) = 0.05$. Here we chose $N = 250$ Trotter slices and did up to 23 million measurement sweeps. We found that the convergence was slower for small values of the dimensionless conductance.

### B. Analytic continuation

Having calculated the cosine correlation function $C(\tau)$ using the PIMC method we still have to solve Eqs. (1) and (2) for the linear conductance $G$. With the positive and symmetric spectral function a $A(\omega) = C(\omega)(1 - e^{-\beta \omega})/\omega$ one can write the Fourier transformation of $C(\tau)$ in imaginary time as

$$C = KA$$

(8)

where the integral operator $K$ is given by

$$(KA)(\tau) = \frac{1}{2\pi} \int_0^\infty d\omega \frac{\omega \cosh \left(\frac{\beta h}{2} - \tau\right)}{\sinh \left(\frac{\beta h}{2}\omega\right)} A(\omega)$$

(9)

and Eqs. (1) and (2) can be combined to give

$$G = \frac{\beta h G_{cl}}{2\pi} \int_0^\infty d\omega \frac{\omega^2}{\cosh(\beta h \omega) - 1} A(\omega)$$

(10)

Before we can calculate $G$ using Eq. (10) it is necessary to invert the integral transform of Eq. (8) to obtain $A(\omega)$. This problem is similar to the analytical continuation of a function from the imaginary to the real axis. It belongs to the class of ill-posed inverse problems. A straightforward inversion of Eq. (8) fails, because the integral operator $K$ is almost singular and the statistical error in the data $C(\tau)$ is strongly amplified, making the result $A(\omega)$ meaningless. To deal with this problem, we use a recently developed method based upon the singular value decomposition of the integral operator.

From the SVD we get the singular system of $K$ which fulfills

$$K u_j(\omega) = \sigma_j v_j(\omega)$$

$$K^\dagger v_j(\tau) = \sigma_j u_j(\tau)$$

$$\sigma_0 \geq \sigma_1 \geq \sigma_2 \geq \ldots \geq 0$$

The functions $u_j(\tau)$ and $v_j(\omega)$ are called the right and left singular vectors, respectively. The real and positive $\sigma_j$ are the singular values of $K$. The formal solution to the inverse problem (8) is given as

$$A(\omega) = \sum_{j=0}^{\infty} \frac{c_j}{\sigma_j} v_j(\omega)$$

(12)

with the coefficients

$$c_j = \int_0^{\beta \hbar} d\tau C(\tau) u_j(\tau).$$

(13)

Since the numerical calculation of the correlation function $C(\tau)$ can only be done for a discrete set of points and additionally introduces a statistical error, the expansion coefficients $c_j$ have a limited accuracy. Taking into account that for an ill-posed problem such as (8) the singular values $\sigma_j$ vanish exponentially with increasing $j$ it becomes obvious that only the first few terms in the expansion (12) contain meaningful information whereas the higher terms will just corrupt the result by amplifying the noise of the data. This idea is used in the truncated SVD approach which truncates the summation in (12) by neglecting all terms for which $\sigma_j/\sigma_0$ is smaller than the statistical error of the correlation function $C(\tau)$.

Apart from Eq. (8) we still have supplementary information, namely that the spectral function $A(\omega)$ is positive and symmetric. To ensure positiveness one may use a "triangular window" i.e. in the truncated singular value decomposition one multiplies the singular values with a weight factor which falls off linearly from one to zero. Previous studies have shown that this implementation of positiveness reduces the resolution of the method. Recent work has shown how to use supplementary information of positiveness to enhance the resolution of the singular value decomposition method. The idea is to determine additional expansion coefficients which cannot be inferred from the inverse problem. They can be fixed by the constraints that the result shall be positive and the difference to the truncated SVD solution shall be minimal. Further details about the implementation of the method can be found in Ref. 20.

The approach used in this work is limited only by the statistical error in the correlation function. The error of our PIMC calculation and the subsequent analytical continuation was estimated as follows. First we determined the statistical error of the Monte Carlo data. Then we produced an ensemble of data sets with different realizations of the error by adding Gaussian distributed random noise of the given size. The error bars shown in Figs. 4, 5 and 7 were then calculated from the maximum and minimum result of the analytical continuation.

### IV. RESULTS AND DISCUSSION

In this section we present experimental data and theoretical results for the minimum and maximum linear response conductance, $G_{\text{min}}$ and $G_{\text{max}}$, respectively, as a function of the dimensionless parallel conductance $g$ and the temperature. Before we turn to the comparison between experimental and theoretical data we would like to summarize our findings about the determination of the experimental parameters.

The four-junction layout allows us to determine the dimensionless parallel conductance unambiguously offering
FIG. 2: $G_{\text{max}}$ and $G_{\text{min}}$ normalized to the high temperature conductance $G_{\text{cl}}$ for sample I (●) and II (○) as a function of the normalized temperature. $G_{\text{max}}$ and $G_{\text{min}}$ are the maximum and minimum linear response conductance observed as a function of the gate voltage $U_g$, respectively. Lines correspond to the predictions of a second-order perturbative expansion in $g$ (solid line: $g = 1.4$, dashed line: $g = 1.1$, dashed-dotted line: $g = 0.8$). The data are best described by $g = 1.1$ (●) and $g = 1.4$ (○).

the possibility of a direct comparison between experiment and theory. On the other hand measurements carried out on the two-junction samples I and II (see Fig. 2) clearly demonstrate a problem also encountered in Ref. 10. The assumption that the conductance is distributed equally among both tunnel junctions would lead to $g_{\text{I}} = 0.80$ and $g_{\text{II}} = 1.39$ for the samples I and II in contradiction to their almost identical temperature dependence of both $G_{\text{min}}$ and $G_{\text{max}}$. Since the maximum conductance $G_{\text{max}}$ in the weak tunneling regime should fall off proportional to $g \ln(\beta E_c/\pi)$ for low temperature. Fig. 2 shows that the correct parallel conductances of those samples are almost equal. Our four-junction samples allow us to check this assumption directly (see below). We found that the tunnel junctions are not identical although they are produced simultaneously during shadow evaporation. This is not surprising as we tried to produce contacts with oxide barriers as thin as possible and very small junction areas. At the borderline between functioning and broken contacts the unavoidable variations in the fabrication procedure become visible as large fluctuations in the conductances of different junctions.

In previous experiments the determination of the charging energy $E_c$ was proposed. As already mentioned in Ref. 2 the determination of the charging energy from the offset of the $I_{\text{sd}}U_{\text{sd}}$ curve is not very accurate. For sample V and VII we have analyzed the subgap resonances observed in the SET in the superconducting state to obtain the renormalized charging energy $E_c^\text{S}$ as described in Ref. 1. Joyez et al. also give a perturbative result which connects $E_c^\text{S}$ to the bare charging energy $E_c$. Unfortunately it is only valid up to $O(g^2)$. In our experiments we found that this is not sufficient for a description of high conductance single-electron transistors. In our opinion the best method for the determination of the charging energy is a comparison of the high-temperature experimental data to semiclassical calculations. Apart from the full semiclassical expressions we tested the high-temperature expansion

$$\frac{G}{G_{\text{cl}} - G} = \frac{3k_B T}{E_c} + \frac{27g(3)}{2\pi^4} - \frac{2}{5}$$

using only data with $k_B T \gg gE_c/2\pi^4$ (see Fig. 3). At a dimensionless conductance of $g = 4.75$ the difference in the charging energy between the full semiclassical result and Eq. (14) was less than 1%. In contrast to earlier work sufficient experimental data at high temperatures are available for a determination of $E_c$. Moreover, Eq. (14) provides a consistency check for the dimensionless parallel conductance $g$ which we have determined independently.

Having determined the experimental parameters we can turn to the comparison with the theoretical results. First of all we want to compare the data measured on sample I, III and V–VII with perturbation theory in second order in the parallel conductance $g$. Fig. 3 shows the maximum and minimum conductance of the single-electron transistor as a function of temperature. For sample I and V with their moderate $g = 1.1$ and $g = 1.40$, respectively, good agreement can be stated for $G_{\text{max}}$. At higher $g$ (sample III, VI and VII) deviations of increasing size are visible. Such deviations are not surprising as the perturbation expansion is not justified in this parameter range. For the minimum conductance $G_{\text{min}}$ we get deviations from the second-order result for all values of $g$ investigated. At the lowest temperatures the determination of the minimum conductance $G_{\text{min}}$ is limited by the resolution of the lock-in signal which is of the order $10^{-3}G_0$. However the deviations are also prominent at $1/(\beta E_c) = 0.05$ where the lock-in signal has a sufficient
FIG. 4: Maximum (a) and minimum (b) conductance normalized to the high temperature conductance $G_{cl}$ for samples I, V, VI, VII, and III with (from top to bottom) $g = 1.10, 1.40, 1.85, 4.75$, and $5.38$ as a function of the normalized temperature. Together with the experimental data (●) the predictions of the perturbation theory in second order (—) are shown for all samples. For sample VII the results of the Monte Carlo calculations (■) are given additionally. For the sake of clarity the curves are plotted in (a) with a vertical offset increasing by 0.2 from curve to curve, in (b) the different datasets are multiplied by $10^2, 10^4$, etc.

Accuracy. The discrepancy between perturbation theory and our data increases with $g$, indicating that higher-order corrections have to be included. With third-order perturbation theory deviations for $G_{\text{min}}$ are reduced but still visible.

Besides renormalization group methods the quantum Monte Carlo approach is the only method which can cover the whole range of parameters that is accessible in the experiment. For sample VII ($g = 4.75$), which is beyond the perturbative regime, we perform a detailed comparison.

In Fig. 5 we show the gate voltage dependent conductance for inverse temperatures ranging from $1/(\beta E_c) = 0.048$ to $1/(\beta E_c) = 2.0$. We find that the experimental Coulomb oscillations are very well described by the Monte Carlo calculations. Minor discrepancies at some temperatures can be attributed to the fact that the temperatures used in the simulations do not exactly match those of the experimental data. For the lowest temperatures of the experiment it was not possible to get converged Monte Carlo results for the whole range of gate voltages in reasonable time.

In Fig. 6 the data are analyzed in terms of the minimum and maximum conductance. Once again the accordance is remarkable. For high $\beta E_c$ the maximum conductance could not be determined by our Monte Carlo calculations. Here a limitation of the Monte Carlo procedure becomes obvious. For low temperatures more terms of the winding number summations in Eqs. (3) and (4) are relevant leading to phase cancellations due to the factors $\exp(2\pi i k n_g)$ which are especially strong at $n_g = 0.5$, i.e. for the maximum conductance. Thus the convergence of the Monte Carlo procedure gets slower with decreasing temperature and the data cannot be determined as accurately. Since the analytic continuation is sensible to the statistical error of the data, reliable results for $G_{\text{max}}$ could not be obtained with reasonable effort and the data points for $G_{\text{max}}$ at the lowest temperatures have been omitted. This is not the case for the minimum conductance $G_{\text{min}}$ as can be seen in the inset of Fig. 6. Here no phase cancellations occur and the experimental and theoretical data match nicely even on a logarithmic plot. We can also observe that in contrast to the perturbation theory in second order (cf. Fig. 4), the Monte Carlo approach gives an accurate description of $G_{\text{min}}$ at low temperatures.

Finally we have examined the maximum and minimum conductance of single-electron transistors for varying tunneling strength $g$ at a fixed temperature $1/(\beta E_c) = 0.05$. The results are shown in Fig. 7. Besides our experimental data the results of Joyez et al. [1] are shown. According to Ref. [1] the data of Joyez et al. at $g = 7.5$ has been displayed at $g = 10$. The Monte Carlo data include also the earlier results of Goppert et al. [3]. Once again we observe a reasonable accordance between theory and experiment. For $g > 8$ the comparison is hampered by uncertainties of the experimental parameters $g$ and $E_c$. The parallel conductance of earlier experiments was system-
FIG. 5: Coulomb oscillations for sample VII ($g = 4.75$, thin lines) compared with Quantum Monte Carlo calculations (thick lines). The conductance $G$ normalized to the high temperature conductance $G_{cl}$ is shown as a function of the dimensionless gate voltage $n_g = (G/cU_c)/e$ for the temperatures as given in the right margin. The calculations were done at temperatures (from bottom to top) 0.048, 0.1, 0.2, 0.22, 0.25, 0.33, 0.4, 0.5, 0.67, 1.0 and 2.0 $E_c/k_B$.

FIG. 6: Maximum and minimum linear response conductance $G_{\text{min}}$ and $G_{\text{max}}$ for sample VII ($g = 4.75$) normalized to the high temperature conductance $G_{cl}$. The experimental data (●) are compared to the results of our PIMC simulation (□). Data points for $G_{\text{max}}$ at the lowest temperatures have been omitted (see text). The inset shows the same data on a logarithmic scale for better comparison of $G_{\text{min}}$. Errorbars are only shown if they exceed the symbol size.

FIG. 7: Maximum and minimum conductance for $1/(\beta E_c) = 0.05$ as a function of the tunneling strength $g$ for all examined samples (●) in comparison with PIMC calculations (□). Included are also the experimental data of Joyez et al. (■) and the results of perturbation theory in second order (○) and third order (♦). Errorbars are only shown if they exceed the symbol size.

Automatically underestimated by the assumption of symmetry of the SET while for the determination of the charging energy $E_c$ sufficient high temperature data were missing.

Also shown are the predictions of the perturbation expansion in second and third order in $g$. The range of validity of the second-order perturbative approach is limited to $g \leq 2.5$ where the maximum conductance $G_{\text{max}}$ drops with increasing $g$. The plateau and the following increase of the maximum conductance can not be described by perturbation theory. Also for $G_{\text{min}}$ deviations occur at $g > 2.5$ while the Monte Carlo approach gives excellent results up to $g = 10$.

V. CONCLUSION

We have presented experimental results for the conductance of single-electron transistors as a function of temperature and dimensionless gate voltage. The employed four-junction layout for the SET allows for an unambiguous determination of the physical parameters $g$ and $E_c$. Thus we were able to clarify and eliminate problems encountered in earlier experiments. In particular, comparison with theory can be made without any adjustable parameter. We have compared the experimental findings with perturbation theory in second order in $g$ and with results of PIMC simulations.

Comparison with perturbation theory was made for the maximum and minimum conductance of five SETs with different tunneling strength. At $g < 1.85$ we found good agreement for the maximum linear conductance with second-order perturbation theory for the whole range of temperatures. Surprisingly even for such low-conductance SETs deviations from perturbation theory for $G_{\text{min}}$ are pronounced at low temperatures. In con-
trast, a detailed comparison with PIMC data for $g = 4.75$
revealed good agreement between experiment and theory outside the perturbative regime. Further comparison
showed that at $g = 4.75$ not only $G_{\text{max}}$ and $G_{\text{min}}$ but also
the form of the Coulomb peaks could be described very
well by our simulations for temperatures $1/(\beta E_c) \geq 0.05$.

Finally we have presented a comparison of experimental and theoretical results for the maximum and minimum conductance as a function of the tunneling strength $g$ for a fixed temperature. The experimental data for this comparison are combined from experiments on different SETs including also the results of earlier experiments by Joyez et al.. For $g < 8$ we find that the PIMC data is in
good accordance with experiment whereas perturbation theory in second order shows significant deviations for
$g > 2.5$. Also the available data of third-order perturbation theory still show rather large deviations for $G_{\text{max}}$.

The good agreement found in our study is an affirmation that the path integral formulation in combination with the Monte Carlo method allows for an accurate description of the minimum conductance $G_{\text{min}}$ over

the whole range of experimentally accessible parameters. Limitations of the Monte Carlo method for the calculation of the maximum conductance $G_{\text{max}}$ exist at low temperatures and small conductances due to slow convergence. Nonetheless $G_{\text{max}}$ as well as the entire shape of the Coulomb peaks can be described well in a range of parameters which lies outside the perturbative regime.

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