Omnidirectional total transmission at the interface associated with an anisotropic dielectric-magnetic metamaterial

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Abstract

Based on the Ewald-Oseen extinction theorem, the omnidirectional total transmission of waves incident from vacuum into an anisotropic dielectric-magnetic metamaterial is investigated. It is shown that the omnidirectional total transmission need not limit at the interface associated with the conventional nonmagnetic anisotropic medium. The recent advent of a new class of anisotropic dielectric-magnetic matermaterial make the omnidirectional total transmission become available.

It is found that the inherent physics underlying the omnidirectional total transmission are collective contributions of the electric and magnetic responses.

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I. INTRODUCTION

The phenomena of reflection and refraction of light at the interface of two transparent media are widely used for steering light in many optical devices. There has been much discussion on realizations of omnidirectional total reflection \([1, 2, 3, 4, 5, 6]\). While the omnidirectional total transmission, where wave is completely transmitted for arbitrary incident directions, attracted little attention. Recently, the omnidirectional total refraction at the planar interface associated with conventional nonmagnetic uniaxial crystal have been studied \([7, 8]\).

The advent of a new class of anisotropic dielectric-magnetic metamaterial with negative permittivity and permeability has attained considerable attention \([9, 10, 11, 12, 13]\). Lindell \textit{et al.} \([14]\) have extended that anomalous negative refraction can occur at the interface associated with an anisotropic dielectric-magnetic metamaterial, which does not necessarily require that all tensor elements of permittivity \(\varepsilon\) and permeability \(\mu\) have negative values. The study of such anisotropic metamaterial have recently received much interest and attention \([15, 16, 17, 18, 19, 20]\).

The question thus naturally arises: whether there exists any type of interface associated the anisotropic dielectric-magnetic material support the omnidirectional total transmission. In the present work, we present an investigation on the omnidirectional total transmission from vacuum into anisotropic dielectric-magnetic metamaterial. We find that the omnidirectional total transmission need not limit at the interface associated with conventional nonmagnetic uniaxial crystal. We want to explore how the omnidirectional total transmission shows up at microscopic leave.

II. THE EWALD-OSEEN EXTINCTION THEOREM

In molecular optics theory, a bulk material can be regarded as a collection of molecules (or atoms) embedded in the vacuum. Under the action of an incident field, the molecules oscillate as electric and magnetic dipoles and emit radiations. The radiation field and the incident field interact to form the new transmitted field in the material and the reflection field outside the material \([21]\). Let us carry this idea one step further: the metamaterial is structured into subunits. In the case of electromagnetic radiation this usually means that the subunits must be much smaller than the wavelength of radiation. Then the unit cells of
FIG. 1: Schematic diagram for how the reflected and transmitted fields of E-polarized waves are generated by the incident field and radiated fields of dipoles. In the vacuum, the reflected field $E_r = E_{r,e} + E_{r,m}$, while the transmitted field $E_t = E_{i} + E_{t,e} + E_{t,m}$ in the anisotropic metamaterial.

Metamaterials can be modelled as the molecules (or atoms) in ordinary materials. For anisotropic metamaterials one or both of the permittivity and permeability are second-rank tensors. To simplify the proceeding analysis, we assume the permittivity and permeability tensors are simultaneously diagonalizable:

$$\begin{align*}
\varepsilon &= \begin{pmatrix}
\varepsilon_x & 0 & 0 \\
0 & \varepsilon_y & 0 \\
0 & 0 & \varepsilon_z
\end{pmatrix}, \\
\mu &= \begin{pmatrix}
\mu_x & 0 & 0 \\
0 & \mu_y & 0 \\
0 & 0 & \mu_z
\end{pmatrix}.
\end{align*}$$

(1)

where $\varepsilon_i$ and $\mu_i$ are the permittivity and permeability constants, respectively. We choose the $z$ axis to be normal to the interface, the $x$ and $y$ axes locate at the plane of the interface.

Let us consider a monochromatic electromagnetic field $E_i = E_{i0} \exp[i(k_i \cdot r - \omega t)]$ incident from vacuum into the anisotropic metamaterial. The reflected and transmitted fields can be express as $E_r = E_{r0} \exp[i(k_r \cdot r - \omega t)]$ and $E_t = E_{t0} \exp[i(k_t \cdot r - \omega t)]$, respectively. The incident angle is given by $\theta_i = \tan^{-1}[k_{ix}/k_{iz}]$, and the refractive angle of the transmitted wave vectors is decided by $\theta_t = \tan^{-1}[k_{tx}/k_{tz}]$. For compactness, let us first explore the E-polarized incident waves.
Following the Ewald-Oseen extinction theorem, the total radiated field $E_{\text{rad}}$ is the sum of the contribution from all electric dipoles $E_{\text{rad}}^e$ and that from all magnetic dipoles $E_{\text{rad}}^m$. While the incident field is assumed to permeate to the medium without being affected by the interface and the properties of that medium \[21\]. Hence the reflected field in vacuum $(-\infty < z < 0)$ can be expressed in the terms of the collective operations of the electric and magnetic responses:

$$E_r = E_{\text{rad}}^e + E_{\text{rad}}^m. \quad (2)$$

While the transmitted field is the superstition of the incident field and all the radiated field induced by the dipoles:

$$E_t = E_{\text{rad}}^e + E_{\text{rad}}^m + E_i. \quad (3)$$

In the other words, the re-emission of the electric and magnetic dipoles to the half of the space $(0 \leq z < +\infty)$ extinguishes the incident field and produces the transmitted field.

The electric fields radiated by electric dipoles and magnetic dipoles are respectively decided by \[21\]

$$E_{\text{rad}}^e = \nabla (\nabla \cdot \Pi_e) - \varepsilon_0 \mu_0 \frac{\partial^2 \Pi_e}{\partial t^2}, \quad (4)$$

$$E_{\text{rad}}^m = -\mu_0 \nabla \times \frac{\partial \Pi_m}{\partial t}. \quad (5)$$

Here $\Pi_e$ and $\Pi_m$ are the Hertz vectors,

$$\Pi_e(r) = \int \frac{P(r')}{\varepsilon_0} G(r - r')dr', \quad (6)$$

$$\Pi_m(r) = \int M(r') G(r - r')dr'. \quad (7)$$

$P$ is the dipole moment density of electric dipoles and $M$ is that of magnetic dipoles, which are related to the transmitted fields by $P = \varepsilon_0 \chi_e \cdot E_t$ and $M = \chi_m \cdot H_t$, where the electric susceptibility $\chi_e = \varepsilon / \varepsilon_0 - 1$ and the magnetic susceptibility $\chi_m = \mu / \mu_0 - 1$. The Green function is

$$G(r - r') = \frac{\exp (ik_i |r - r'|)}{4\pi |r - r'|}, \quad (8)$$

where $k_i = k_{ix} \hat{x} + k_{iz} \hat{z}$ is the incident wave vector. Inserting Eq. (8) into Eqs. (6) and (7), then using the delta function definition and contour integration method, the Hertz vectors can be evaluated as\[22, 23\]

$$\Pi_e = \begin{cases} -\chi_e \cdot E_0 \exp (ik_r \cdot r), & -\infty < z < 0 \\
\chi_e \cdot E_0 \exp (ik_i \cdot r) + \frac{\chi_e \cdot E_0 \exp (ik_t \cdot r)}{2k_{iz}(k_{iz} - k_{tz})} & 0 \leq z < \infty \end{cases} \quad (9)$$
\[ \Pi_m = \begin{cases} 
-\chi_m \cdot \mathbf{H}_t \exp (ik_r \cdot \mathbf{r})/2k_{iz}(k_{iz} + k_{iz}), & -\infty < z < 0 \\
\chi_m \cdot \mathbf{H}_0 \exp (ik_i \cdot \mathbf{r})/2k_{iz}(k_{iz} - k_{iz}) + \chi_m \cdot \mathbf{H}_0 \exp (ik_t \cdot \mathbf{r})/k_i^2 - k_i^2, & 0 \leq z < \infty 
\end{cases} \] (10)

where \( \mathbf{k}_r = k_{iz}\hat{x} + k_{iz}\hat{z} \) and \( \mathbf{k}_t = k_{iz}\hat{x} + k_{iz}\hat{z} \) are the reflected and transmitted wave vectors, respectively. Here we have used the Faraday’s law \( \mathbf{H}_t = (\mathbf{k}_t \times \mathbf{E}_t)/(\omega \mu) \) which can also be established by the molecular theory.

The contributions from the electric and magnetic dipoles form the reflected field \( \mathbf{E}_r \). Applying Eqs. (9) and (10) for \(-\infty < z < 0\) to Eqs. (4) and (5), we obtain

\[ \mathbf{E}^e_{rad} = \frac{\mathbf{k}_r \times \{ \mathbf{k}_r \times (\chi_e \cdot \mathbf{E}_t) \}}{2k_{iz}(k_{iz} + k_{iz})} \] (11)

\[ \mathbf{E}^m_{rad} = \frac{\mathbf{k}_r \times \{ \chi_m \cdot \left[ \mu_0 \mu^{-1} \cdot \left( \mathbf{k}_t \times \mathbf{E}_t \right) \right] \}}{2k_{iz}(k_{iz} + k_{iz})} \] (12)

where \( \mathbf{k}_r = k_{iz}\hat{x} - k_{iz}\hat{z} \) is the refractive wave vector. For E-polarized incident waves, combining Eqs. (9) and (10) with Eq. (3), we can obtain the following dispersion relation

\[ \frac{k_{iz}^2}{\varepsilon_y\mu_z} + \frac{k_{iz}^2}{\varepsilon_y\mu_x} = \omega^2. \] (13)

It should be mentioned that the same dispersion relation can be obtained from the Maxwell equations.

Next, let us review the transmission in isotropic nonmagnetic media. In classical electrodynamics, a simple generalization shows that the zero reflection occurs when H-polarized waves incident from vacuum into an isotropic nonmagnetic medium (\( \mu_I = 1 \)). The total transmission takes place at an incident angle satisfying \( \theta_i + \theta_r = \pi/2 \). Such an angle, determined by \( \theta_B = \tan^{-1}[\sqrt{\varepsilon_I}] \), is called the Brewster angle [21].

The Ewald-Oseen extinction theorem allow us to investigate how the Brewster angle generates. From the point of view of molecular optics, we know that the radiated fields would never occur along the axes of electric and magnetic dipoles. The inherent physics for Brewster angle in isotropic nonmagnetic media is that the axes of electric dipoles align with the direction of the reflected wave. While in the anisotropic dielectric-magnetic material, the physics origins are significantly different. The Brewster angle occurs when the magnitudes of \( \mathbf{E}^e_{rad} \) and \( \mathbf{E}^m_{rad} \) are equal. Setting \( \mathbf{E}_r = 0 \) in Eq. (2), the Brewster angle can be expressed as

\[ \theta_B = \sin^{-1}\left( \frac{\mu_0(\varepsilon_y\mu_0 - \varepsilon_0\mu_x)}{\varepsilon_0(\mu_0^2 - \mu_x\mu_z)} \right). \] (14)
FIG. 2: Radiated electric field amplitudes for a E-polarized wave incident from the vacuum into anisotropic dielectric-magnetic material: (a) $\varepsilon_y = 1, \mu_x = 2, \mu_z = 2$; (b) $\varepsilon_y = -1, \mu_x = -2, \mu_z = -2$. Note that the polarized incident wave exhibit a Brewster angle when $E_{r0} = 0$.

For the purpose of illustration, let us choose the positive anisotropic parameters in Fig. 2(a), while select the corresponding negative anisotropic parameters in Fig. 2(b). It can be seen from the Fig. 2(a) that $E_{e\text{rad}} \equiv 0$ when we choose $\varepsilon_y = \varepsilon_0$. It means only the induced magnetic dipoles contribute to the reflected field. In this special case, the condition for Brewster angle requires that the axes of magnetic dipoles align with the direction of the reflected wave. However, compared with Fig. 2(a), the values of $E_{e\text{rad}}$ and $E_{m\text{rad}}$ in Fig. 2(b) are much larger and never reach zero. Note that the directions of $E_{e\text{rad}}$ and $E_{m\text{rad}}$ are always reversed. When $E_{e\text{rad}} + E_{m\text{rad}} = 0$ is satisfied, E-polarized incident wave exhibits a Brewster angle.

III. OMNIDIRECTIONAL TOTAL TRANSMISSION

From the point of view of molecular optics, the microscopic interpretation of the omnidirectional total transmission lies in the cancellation of two extremely large fields radiated by different types of induced dipoles:

$$E_{e\text{rad}} + E_{m\text{rad}} = 0$$  \hspace{1cm} (15)
FIG. 3: Radiated electric field amplitudes for a E-polarized wave incident from the vacuum into anisotropic dielectric-magnetic material: (a) $\varepsilon_y = 1.25$, $\mu_x = 1.25$, $\mu_z = 0.8$; (b) $\varepsilon_y = -1.25$, $\mu_x = -1.25$, $\mu_z = -0.8$. Note that $E_{r0}$ is always zero, since the radiated fields of the oscillating electric and magnetic dipoles cancel each other for any incident angle.

An alternative view is that the re-emission of the electric and magnetic dipoles induced by incident field cancel out in vacuum. To some extent, any incident angle can be considered to be a Brewster angle. Substituting Eqs. (11) and (12) into Eq. (2), we can obtain

$$\varepsilon_0(\mu_0^2 - \mu_x\mu_z) \sin^2 \theta_I = \mu_z(\varepsilon_y\mu_0 - \varepsilon_0\mu_x).$$

(16)

The omnidirectional total transmission means that Eq. (16) should be satisfied for arbitrary incident angle $\theta_I$. So the conditions for omnidirectional total transmission for E-polarized waves can be obtained as

$$\frac{\varepsilon_0}{\mu_0} = \frac{\varepsilon_y}{\mu_x}, \quad \frac{\varepsilon_0^2}{\mu_0^2} = \mu_x\mu_z.$$

(17)

In this case, $E_{r0}^e \equiv 0$ because $E_{r}\text{rad}^e$ and $E_{r}\text{rad}^m$ have the same magnitude but exhibit the opposite signs.

Next we want to investigate the omnidirectional total transmission of H-polarized incident wave. The appearance of the omnidirectional total transmission for the H-polarized waves is due to the reverse roles of electric and magnetic response. Evidently, interchanging $\varepsilon$ and $\mu$ in Eq. (16), we can can get

$$\mu_0(\varepsilon_0^2 - \varepsilon_x\varepsilon_z) \sin^2 \theta_I = \varepsilon_z(\varepsilon_0\mu_y - \varepsilon_x\mu_0).$$

(18)
Analogously, the conditions for omnidirectional total transmission can be obtained as

\[
\frac{\varepsilon_0}{\mu_0} = \frac{\varepsilon_x}{\mu_y}, \quad \varepsilon_0^2 = \varepsilon_x\varepsilon_z.
\]

(19)

To obtain a better physical picture of the omnidirectional total transmission, we plot the radiated fields induced by the electric and magnetic dipoles in Fig. 3. The radiated fields \(E_{\text{rad}}^e\) and \(E_{\text{rad}}^m\) always have the same magnitude but exhibit the opposite directions. Hence, the resulting reflected fields cancel each other for any incident angle.

Comparing Fig. 3(a) and Fig. 3(b) show that, although the phenomenon of omnidirectional total transmission is formally identical \((E_{r0} \equiv 0)\), the microscopic origin are significantly different. It should be emphasized that the radiated fields \(E_{\text{rad}}^e\) and \(E_{\text{rad}}^m\) in Fig. 3(b) are much larger the counterparts in Fig. 3(a). Hence the molecular optics theory is useful for uncovering the inherent secret in anisotropic metamaterials.

IV. CONCLUSION

In conclusion, we have investigated the omnidirectional total transmission of waves incident from vacuum into anisotropic dielectric-magnetic metamaterials. The omnidirectional total transmission need not limit at the interface conventional nonmagnetic anisotropic crystal. If certain conditions are satisfied, the anisotropic dielectric-magnetic metamaterials provide more available option to realize the omnidirectional total transmission. We have shown that the inherent physics underlying the omnidirectional total transmission are collective operations of the electric and magnetic responses. We expect many potential applications based on the total omnidirectional direction discussed above. They can, for example, be used to construct reflection absent lens, radar-absorbing material and light bending device.

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