Magnetoplasmon spectrum of two-dimensional helical metals

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Abstract. We investigate theoretically the magnetoplasmon spectrum of two-dimensional helical metals. The effects of the Zeeman energy and Coulomb interaction on the collective excitation modes are examined through the charge-density excitation (CDE) and spin-density excitation (SDE) intensities. At low excitation frequencies, the CDE is always accompanied by an SDE and is almost unaffected by the change in the Zeeman energy or Coulomb interaction intensity. However, at some higher excitation frequencies, the CDE and SDE can be decoupled and a strong SDE with a weak CDE can be realized by tuning either the Zeeman energy or the Coulomb interaction intensity.

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1. Introduction

Topological insulators (TIs) have recently attracted much attention because of the special band structures and the quantum Hall states they reveal [1–12]. A TI is characterized by an insulating gap in the bulk and gapless edge or surface states, which are protected by time-reversal symmetry. The two-dimensional (2D) TI displaying the quantum spin Hall effect was realized in HgTe quantum wells a few years ago [5, 6]. Recently, a class of 3D TIs were predicted in stoichiometric materials (such as Bi$_2$Se$_3$ and Bi$_2$Te$_3$) and experimentally observed through scanning tunneling microscopy and angle-resolved photoemission spectroscopy [7–9]. The topologically protected surface states of 3D TIs can be described by a 2D massless Dirac Hamiltonian, as the graphene system. There are two major differences between graphene and the 3D TI. Firstly, graphene has four Dirac cones due to spin and valley degeneracy, while the 3D TI has an odd number of Dirac cones. Secondly, for Dirac electrons in graphene, the Pauli matrices in the Dirac Hamiltonian act on the sublattice pseudospin. For the 3D TI, they are related to a real spin and give rise to a spin-momentum locking effect [9–12]. Thus the 2D electron system on the topological surface is called a helical metal. When a magnetic field is applied to the 3D TI, the Landau quantization of the topological surface states happens. The linear dispersion of the massless Dirac electrons can be confirmed by the square-root dependence of Landau levels on magnetic field [13, 14].

The collective excitation of the helical metal has been investigated by Raghu et al [11]. They found that a spin density wave always accompanies the charge density wave, which would be detected by means of the spin-grating experimental setup. For the 3D TI subject to a magnetic field, the magnetoplasmom spectrum is also expected to reveal some spin-related information, as in graphene or 2D electron gas systems modulated by spin–orbit interactions [15–18]. Therefore, in this work, we examine the features of the magnetoplasmon spectrum of the helical metal and its corresponding charge-density excitation (CDE) and spin-density excitation (SDE) intensities. In the framework of the random phase approximation (RPA), we find that at low excitation frequencies, the CDE and SDE are coupled together and are almost unaffected by the Zeeman energy or the Coulomb interaction intensity. However, at some higher excitation frequencies, there are collective excitation modes for which the CDE and SDE intensities show remarkable changes with the Zeeman energy and the Coulomb interaction intensity. Note that the Zeeman energy is proportional to the effective $g$-factor of electrons, which can be tuned by means of gate voltages [19, 20]. In addition, the Coulomb interaction intensity is sensitive to the dielectric environments and can be modulated through the effective background dielectric constant [21–23]. Therefore, they provide possible ways to manipulate the SDE and CDE in 2D helical electron systems.

2. Model

We consider a 3D TI system with an infinite flat surface subjected to a perpendicular magnetic field $\mathbf{B} = B \hat{z}$. The low-energy surface states can be described by the 2D Dirac Hamiltonian [10]

$$H = \hbar v_F \mathbf{\sigma} \cdot [(\mathbf{p} + eA) \times \hat{z}] + E_z \sigma_z,$$

where $v_F$ is the Fermi velocity, $\mathbf{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is the vector of three Pauli matrices, $\mathbf{p} = (p_x, p_y)$ is the electron momentum, $A = (0, Bx, 0)$ is the vector potential in the Landau gauge.
and \( E_Z = g^* \mu_B B/2 \) is the Zeeman energy with the Bohr magneton \( \mu_B \) and electron effective Lande factor \( g^* \). The corresponding eigen energy and wavefunction of \( H \) are

\[
E_0 = -E_Z, \quad E_n^\pm = \pm \sqrt{2n^2 \hbar e B n + E_Z^2}, \quad n = 1, 2, \ldots,
\]

\[
\psi_0(x, y) = \frac{1}{\sqrt{L_y A_n}} \exp(i k_y y) \left( \phi_0((x - x_0)/l_c) \right),
\]

\[
\psi_n^+(x, y) = \frac{1}{\sqrt{L_y A_n}} \exp(i k_y y) \left( \lambda_n \phi_n-1((x - x_0)/l_c) \right),
\]

\[
\psi_n^-(x, y) = \frac{1}{\sqrt{L_y A_n}} \exp(i k_y y) \left( \phi_n-1((x - x_0)/l_c) \right), \quad n = 1, 2, \ldots,
\]

where \( \pm \) denotes the conduction band and valence band, \( x_0 = -k_y l_c^2 \), \( l_c = [\hbar/(eB)]^{1/2} \) is the magnetic length, \( \lambda_n = \sqrt{2n \hbar v_F/(E_n^+ - E_Z)} \), \( A_n = 1 + |\lambda_n|^2 \) and \( \phi_n(x) = N_n \exp(-x^2/2)H_n(x) \) is the Hermite function.

The charge and spin density–density correlation functions in the RPA are

\[
\chi_{\rho} \left( \mathbf{q}, \omega \right) = D^0 \left( \mathbf{q}, \omega \right) \left[ 1 - V(q) D^0 (\mathbf{q}, \omega) \right]^{-1},
\]

\[
\chi_{s} \left( \mathbf{q}, \omega \right) = S^0 \left( \mathbf{q}, \omega \right) \left[ 1 - V(q) D^0 (\mathbf{q}, \omega) \right]^{-1},
\]

where the Coulomb interaction \( V_q = 2\pi \epsilon^2/(\epsilon q) \), \( \epsilon \) is the background dielectric constant, \( q = |q| \) with \( q = (q_x, q_y) \). \( D^0 (\mathbf{q}, \omega) = \sum_{s, \sigma} D^0_{s, \sigma} \) and \( S^0 (\mathbf{q}, \omega) = \sum_{s} [D^0_{s, \sigma} - D^0_{s, -\sigma} + 2V(q) (D^0_{s, -\sigma} D^0_{s, -\sigma} - D^0_{s, \sigma} D^0_{s, \sigma})]. \) The single-particle part of the density–density correlation function \( D^0_{s, \sigma} (\mathbf{q}, \omega) \) can be written as

\[
D^0_{s, \sigma} (\mathbf{q}, \omega) = \frac{1}{2\pi l_c^2} \sum_{n, n'} f(E_n^+ - E_n') \frac{f(E_n^+)}{l_c^2 q^2} \frac{1}{2}
\]

\[
\mathcal{M}_{s, \sigma} (\mathbf{q}, \omega) = e^{-g} x^m \left[ (\lambda_n^s)^{\delta_{s, \sigma}} (\lambda_n^{s'})^{\delta_{s', \sigma}} \sqrt{L_{n, s}^m(x)} \right]^{-2},
\]

in which \( \lambda_n = \sqrt{2n \hbar v_F/(E_n^+ - E_Z)} \), \( n = \max(n, n') \), \( n_s = \min(n, n') \), \( m = n_S - n_s \) and \( L_{n, s}^m(x) \) is the associated Laguerre polynomial. The relative intensities of the CDE and SDE at \( \omega = \omega_q \) are

\[
I_{\text{CDE}} = | \lim_{\omega \to \omega_q} (\omega - \omega_q) \chi_{\rho} (\mathbf{q}, \omega) |,
\]

\[
I_{\text{SDE}} = | \lim_{\omega \to \omega_q} (\omega - \omega_q) \chi_{s} (\mathbf{q}, \omega) |.
\]

For convenience, \( l_c \) and \( E_c = \hbar v_F / l_c \) are taken as the units of length and energy. The Coulomb interaction intensity is characterized by \( r_c = 2\pi \epsilon^2/(\epsilon \hbar v_F) \). For a typical value \( v_F = 5 \times 10^5 \text{ m s}^{-1} \) in \( \text{Bi}_2\text{Se}_3 \) material and a magnetic field \( B = 1.0 \text{ T} \), we have \( E_c \simeq 9 \text{ meV} \). The Fermi energy level is assumed to be in the conduction band and \( n_F \) is the index of the highest filled Landau levels.
Figure 1. Magnetoplasmon spectrum (solid black curves) of the 2D helical metal. The contour plot indicates the imaginary part of the RPA dielectric function, in which the nonzero region denotes the particle–hole continuum. The units for frequency $\omega_q$ and wave vector $q$ used here and hereafter are $\omega_c = v_F/l_c$ and $q_c = 1/l_c$. The Fermi level index is (a) $n_F = 1$ and (b) $n_F = 3$. $B = 1.0$ T, $E_Z = 0$.

3. Numerical results

In our numerical computation, the infinite sums for the single-particle density–density correlation function in equation (6) have been truncated at $n_{\text{max}} = 25$. We have checked that when a larger $n_{\text{max}} = 50$ is chosen, the changes in the correlation function are less than 0.2%. In figure 1, we show the magnetoplasmon spectrum (solid black curves) of the helical metal and its particle–hole continuum. The contour plot indicates the imaginary part of the RPA dielectric function. The region with nonzero values denotes the particle–hole continuum. We can see that the collective excitation modes become closer to each other with the rise of excitation frequencies or the increase of $n_F$. When $n_F = 1$ (figure 1(a)), there are only four collective excitation modes in this low-frequency region. All of them are excitations between energy levels in the conduction band. It is known that in a 2D electron gas, because of the equal distance of adjacent Landau levels, each collective excitation mode is contributed by several allowed Landau level transitions. However, in the helical metal, due to the nonuniform distribution of Landau levels, the collective excitation modes in the low-frequency region are caused by a single Landau level transition. For a system with a higher $n_F$, more collective excitation modes appear in the low-frequency region. The contribution of more than one Landau level transition results in an enhancement of the electron–hole excitation at particular frequencies (see the yellow and red regions in figure 1(b)).

The Zeeman energy in figure 1 is set at zero. From expression (2) we can see that the introduction of a small $E_Z$ (for $B = 1.0$ T and $g^* \sim 23$ in Bi$_2$Se$_3$, we have $E_Z \sim 0.07E_c$) leads to a slight shift of the energy levels. Therefore, the collective excitation modes in the low-frequency excitation region shown in figure 1(a) are almost unaffected by a small $E_Z$. At some higher excitation frequencies, there are excitation modes caused by transitions from the valence band to the conduction band and may contain contributions of more than one Landau level transition. Such excitation modes will show different behaviors when the Zeeman energy changes. In figure 2, the collective excitation modes at higher frequencies are plotted for several different Zeeman energies. The two special excitation frequencies considered here are $\omega_1 \sim E_Z^+ - E_1^-$ and $\omega_2 \sim E_Z^+ - E_2^-$.
Figure 2. Magnetoplasmon spectrum of the 2D helical metal (solid black curves) at higher frequencies ($\omega_1 \sim E_2^+ - E_1^+$ for the left panels and $\omega_2 \sim E_4^+ - E_1^+$ for the right panels) for different Zeeman energies (a, b) $E_Z = 0$, (c, d) $E_Z = -0.06$ and (e, f) $E_Z = 0.12$. $n_F = 1$, $r_c = 1.0$, $B = 1.0$ T.

It is seen that the electron–hole continuum changes greatly with the Zeeman energy. For $E_Z = 0$, the electron–hole continuum is shown as a single horizontal line at both $\omega_1$ and $\omega_2$. When $E_Z = -0.06$, the single line at $\omega_1$ splits into two lines, while it does not split at $\omega_2$, as shown in figures 2(c) and (d). When $E_Z = 0.12$, the electron–hole continuum is unchanged at $\omega_1$, whereas it splits at $\omega_2$.

From figure 2 we know that the frequency feature of the collective excitation modes has only a slight variation with $E_Z$. The effect of $E_Z$ is much more clearly manifested through the CDE intensity $I_{\text{CDE}}$ and SDE intensity $I_{\text{SDE}}$ of the collective excitation modes, which are plotted in figure 3. Here excitations at frequencies $\omega_0 \sim E_2^+ - E_1^+$, $\omega_1$ and $\omega_2$ are chosen for comparison. The collective excitation mode at $\omega_0$ is shown in figure 1(a) (the lowest black curve), and its CDE and SDE intensities at several values of $E_Z$ are shown in figures 3(a) and (d). In this case the SDE and CDE intensities show similar behavior and vary slightly with $E_Z$. Such a feature indicates that a CDE is always accompanied by an SDE, which has been illustrated in the nonmagnetic spin-plasmon mode of the helical metal [11]. For the collective excitations at $\omega_1$ and $\omega_2$, the CDE and SDE intensities show remarkable variations with $E_Z$. At $\omega_1$, $I_{\text{CDE}}$ is very small when $E_Z = 0$ and reaches its maximum when $E_Z = -0.06$ (see figure 3(b)). Unlike $I_{\text{CDE}}$, for all considered values of $E_Z$ except $E_Z = -0.06$, the $I_{\text{SDE}} - q$ curve shows similar features (see figure 3(e)). For $E_Z = -0.06$, $I_{\text{SDE}}$ exhibits a more rapid variation with the wave vector $q$ and has a zero point at $q_0 \sim 3$. The disappearance of the SDE at $q = q_0$ indicates the decoupling of the CDE and SDE, while the turning of $I_{\text{SDE}}$ here denotes a change in the spin polarization direction in the SDE. The variation of $I_{\text{SDE}}$ with $E_Z$ is more clearly seen in the collective excitation mode at
Figure 3. The corresponding CDE intensity (panels (a)–(c)) and SDE intensity (panels (d)–(f)) of the collective excitation modes and the ratio between them (panels (g)–(i)) at the frequencies $\omega_0$ (left panels), $\omega_1$ (middle panels) and $\omega_2$ (right panels) for different Zeeman energies. $r_c = 1.0$, $B = 1.0$ T.

$\omega_2$, as shown in figure 3(f). It has more abrupt turnings for $E_Z = 0$ and $E_Z = -0.06$ at wave vector $q_0$ (about 3.5 and 4, respectively). For other values of $E_Z$, $I_{\text{SDE}}$ has no zero point. We also note that for $E_Z = 0$ and $E_Z = -0.06$, when $q$ passes $q_0$, $I_{\text{SDE}}$ increases quickly with $q$ and reaches a relatively large value at large wave vectors (about 4.5). The ratio between $I_{\text{SDE}}$ and $I_{\text{CDE}}$ is further plotted in figures 3(g)–(i), which would more straightforwardly illustrate the feature of the collective excitation modes. $I_{\text{SDE}}/I_{\text{CDE}}$ at $\omega_0$ varies slightly around 1 for all $E_Z$ values, as shown in figure 3(g). For this kind of collective excitation modes, the CDE and SDE are always locked together and are almost immune to the change in the Zeeman energy. The variation of $I_{\text{SDE}}/I_{\text{CDE}}$ at $\omega_1$ is drastic for different Zeeman energies (see figure 3(h)). When $E_Z = 0$, it increases sharply with $q$ to values more than 2, which indicates a strong SDE with a weak CDE. In the case of $E_Z = -0.06$, $I_{\text{SDE}}/I_{\text{CDE}}$ decreases with $q$ to zero for $q \leq q_0$. For the other three values of $E_Z$ considered, $I_{\text{SDE}}/I_{\text{CDE}}$ always varies around 1. For $I_{\text{SDE}}/I_{\text{CDE}}$ at $\omega_2$, we can see from figure 3(i) that it decreases to zero first and increases to around 1 again for $E_Z = 0$ and $E_Z = -0.06$, which indicates a coupling–decoupling–coupling transition of the SDE and CDE.

So far, we have considered only the effect of the Zeeman energy. Actually, the Coulomb interaction intensity $r_c$ also affects the collective excitation modes. In figure 4, we plot the SDE and CDE intensities of the collective excitation modes and the ratio between them at frequencies $\omega_0$, $\omega_1$ and $\omega_2$ for different $r_c$ values. $E_Z$ is fixed at zero. At $\omega_0$, the CDE and SDE intensities decrease synchronously when $r_c$ increases from 0.5 to 3 (see figures 4(a) and (d)), while $I_{\text{SDE}}/I_{\text{CDE}}$ remains around 1 for all considered $r_c$ values (figure 4(g)). This is another indication of the so-called spin-magnetoplasmon. For the collective excitation at $\omega_1$,
even though both $I_{CDE}$ and $I_{SDE}$ decrease with $r_c$, they exhibit different decaying tendencies. Accordingly, $I_{SDE}/I_{CDE}$ changes greatly with $r_c$, as shown in figure 4(h). For $r_c = 0.5$, $I_{SDE}/I_{CDE}$ remains around 1. For a large value $r_c = 3$, $I_{SDE}/I_{CDE}$ increases quickly with $q$ from a small value $\sim 0.3$ to a large value $\sim 8$. Thus at large (small) wave vectors, the magnetoplasmon is dominated by the SDE (CDE). At $\omega_2$, a pure CDE ($I_{SDE}/I_{CDE} = 0$ at a wave vector $q_0$) can be achieved for all values of $r_c$ and $q_0$ decreases with $r_c$ (see figure 4(i)). Under a strong Coulomb interaction, the magnetoplasmon behaves mainly as SDE at large $q$.

Finally, we give a brief remark on the similarities and differences in the magnetoplasmon excitation between a conventional 2D electron gas and a helical metal. In the case of a 2D electron gas without spin–orbit interaction [24–26], the SDEs only occur when the two spin states are unequally occupied. They are similar to the low-frequency collective excitation modes between the conduction band energy levels in a helical metal. When spin–orbit interactions are taken into account, there will be SDEs even when the two spin states (spin–orbit modified) are equally occupied. However, the intensities of the SDE and CDE always have a distinctive difference, which indicates a weak coupling between the SDEs and CDEs. Only in a helical metal, as shown above, can there be strong coupling or decoupling between the CDEs and SDEs.

4. Conclusions

In summary, we have examined the features of the magnetoplasmon spectrum of the 2D helical metal and its corresponding CDE and SDE intensities. The effects of the Zeeman energy and the Coulomb interaction intensity on the collective excitation modes are investigated in detail. We find that at low excitation frequencies, the CDE and SDE are locked together and are almost
immune to the change in the Zeeman energy or the Coulomb interaction intensity. At some higher excitation frequencies the collective excitation modes show remarkable changes with the Zeeman energy and the Coulomb interaction intensity. For some values of $E_Z$, the SDE intensity of this kind of collective modes exhibits a rapid variation with the wave vector and turns into zero at a special wave vector $q_0$. This disappearance of the SDE indicates a decoupling of the CDE and SDE. The spin polarization direction for the SDE is changed when the wave vector passes $q_0$. A strong SDE with a weak CDE can also be realized by the tuning of the Zeeman energy or the Coulomb interaction intensity. Because of the tunability of the effective $g$-factor and the background dielectric constant in 2D systems, the variations of collective excitations in TIs with the Zeeman energy and Coulomb interaction intensity have promising applications in spintronics.

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