A PARTIAL REGULARIZATION METHOD FOR NETWORK COMPRESSION

A PREPRINT

E Zhenqian*
School of Data Science
Fudan University
Shanghai 200000, China

Gao Weiguo†
School of Data Science
Department of Mathematical Science
Fudan University

September 4, 2020

ABSTRACT

Deep Neural Networks have achieved remarkable success relying on the developing availability of GPUs and large-scale datasets with increasing network depth and width. However, due to the expensive computation and intensive memory, researchers have concentrated on designing compression methods in order to make them practical for constrained platforms. In this paper, we propose an approach of partial regularization rather than the original form of penalizing all parameters, which is said to be full regularization, to conduct model compression at a higher speed. It is reasonable and feasible according to the existence of the permutation invariant property of neural networks. Experimental results show that as we expected, the computational complexity is reduced by observing less running time in almost all situations. It should be owing to the fact that partial regularization method involves a lower number of elements for calculation. Surprisingly, it helps to improve some important metrics such as regression fitting results and classification accuracy in both training and test phases on multiple datasets, telling us that the pruned models have better performance and generalization ability. What’s more, we analyze the results and draw a conclusion that an optimal network structure must exist and depend on the input data.

Keywords Network Compression · (Sparse) Group Regularization · Partial Regularization

1 Introduction

Deep neural networks have attracted lots of attention for their great success in many tasks regardless of the disadvantage that they require enormous memory and time. It is because that they contain millions or even billions of parameters, especially in the fully-connected layers. Practical evidence has proved that parameters in such over-parametric networks are not equally important and most of them are redundant [1, 2]. Thus, in principle, compact architectures could do as good a job as deep ones. Moreover, the extremely high complexity of neural networks makes it more prone to overfitting, which is the case that the generalization gap between training and test error is too large. On the other hand, training shallow networks from scratch may struggle to handle non-linearities as effectively as deeper ones since it is unlikely to approximate high-dimensional functions and can be easily trapped in bad optima [3]. Hence, model compression for deep architectures, or more precisely determining the best number of parameters, is indeed a challenging but necessary task that should be studied. Recently, it has already made much progress by the deep learning community. Those works do serve to reduce the storage and computational costs.

Roughly speaking, the wider and deeper the network, the higher its capacity is. So obviously, there are two ways to increase the capacity of neural networks, one is adding hidden layers and the other is adding hidden units. However, in order to solve the problem efficiently and not to be trapped in such overfitting dilemma, scientists always need to do a trade-off between the depth and width when designing the network structure manually. A recent deduction from

*email: 18110980004@fudan.edu.cn
†email: wggao@fudan.edu.cn
researchers is that increasing the depth is more effective because it not only increases the number of units but also the embedding depths of the functions. Therefore, the over-parametric phenomenon is largely due to the width, at least in our experiments. If so, we don’t need to penalty all parameters in each layer since any whole layer is not required to be removed.

When considering the model compression task, some researchers tried to impose group sparsity regularization on deep neural networks by setting the variables in a group to all zero which amounts to canceling the influence of a particular neuron and thus removing it entirely. As a consequence, those classical methods don’t rely on the success of learning an over-size neural network to later reduce its parameters. So instead, it helps to automatically determine the number of neurons and select the remaining ones with the corresponding parameters in each layer simultaneously as the machine is learning.

In this paper, we show how a simple modification, being called as partial regularization, can be used efficiently for network compression without sacrificing the performance. Thanks to the permutation invariant property in neural networks, we can easily implement our idea with only small changes of codes for full regularization methods. We compare the results of both methods on multiple datasets and find that our work does have some advantages. It brings improvements on some important metrics that we are interested in on both training and test stages. At the same time, it reduces the computational costs since less entries are involved in the calculation of regularization terms. It satisfies the fact that algorithm containing less elements seems to have less computation complexity from a mathematical point of view. In short, applying partial regularization on the parameters based on the underlying permutation invariant property of deep neural networks, at least fully connected layers, is more powerful than full regularization.

2 Regularization Methods

Given a training dataset consisting of N instances \((x_i, y_i)_{1 \leq i \leq N}\), the training objective for learning the parameters of the network can be expressed as:

\[
\min_{\theta} \frac{1}{N} \sum_{i=1}^{N} L(y_i, f(x_i, \theta)) + \lambda R(\theta)
\]

where \(L(\cdot, \cdot)\) is a proper loss function, \(R(\cdot)\) is the regularizer and \(\lambda\) is the regularization factor that balances the two terms.

The most popular choice of parameter norm penalty is the \(l_2\)-norm in which the sum of squares of parameters values is added to the loss function: \(R_{l_2}(\theta) = ||\theta||_2^2\). However, weight decay can only enforce sparsity by artificially forcing all weights that are lower than a certain threshold in absolute terms to zero so that its sparsity effect might be negligible. In order to obtain a sparse model where large portion of weight is zeroed out, \(R(\cdot)\) needs to be a sparsity-inducing regularizer \(R_{l_1}(\theta) = ||\theta||_1\). However, the remaining connections are not as good as with \(l_2\)-norm regularization, resulting in slight performance reduction after retraining.

Recall that our goal is to perform network compression by starting from an over-parametric network and canceling the influence of some neurons. None of the standard regularizers could achieve the goal. Therefore, [6] made use of group sparsity regularization:

\[
R_{GL}(\theta) = \sum_{l=1}^{L} \sqrt{|p_l|} \sum_{n=1}^{N_l} ||\theta^n_l||_2
\]

where \(|p_l|\) amounts for the varying group size of the parameters for each neuron in layer \(l\). Group sparsity has the effect of complete elimination of some neurons so that it is an efficient regularization to learn compact structures during training. We could apply it to automatically determine the parameters in neural networks, such as the number of neurons and layer depths in fully-connected layers.

However, the formulation in Eq. [2] might still be sub-optimal since we lose guarantees of sparsity at the level of single connections among those remaining after removing some of the groups. To solve this problem, [7] and [8] proposed a more general penalty in order to select groups and predictors within a group. Researchers have found that optimal solutions can be achieved by applying a single regularization factor between group lasso penalty and \(l_1\)-norm regularization. In [9], they first exploit the following composite ’sparse group Lasso’ (SGL) penalty for their use of network compression:

\[
R_{SGL}(\theta) = (1 - \alpha)R_{GL}(\theta) + \alpha R_{l_1}(\theta),
\]

where \(\alpha \in [0, 1]\) sets the relative influence of both terms and \(\alpha = 0\) brings us back to the regularization strategy of Eq. [2].
Learning the right connections is an iterative process. Pruning followed by a retraining is one iteration, and the minimum number of connections could be found after many such iterations. After one such iteration, neurons with zero input connections or zero output connections may be safely pruned and then other connections from that neuron should be further removed. Therefore, the retraining phase finally arrives at the result where dead neurons will have both zero input connections and zero output connections. Removing all these neurons yields a more compact architecture than an original, over-parametric one without loss of performance since they have no contribution to the output.

3 Partial Regularization Method

![Figure 1: Original Neural Network (Left), Compact Neural Network (Right).](image)

We now present our partial regularization method, starting with a simple introduction to an interesting property in neural networks. In the presence of multiple hidden neurons, the permutation invariant property appears, i.e., the model of observed data is invariant with respect to exchange of arbitrary two hidden neurons. In addition, this property is a common feature in many modern neural network architectures. It plays an important role in understanding the computation performance of a broad class of neural networks with two or more hidden units.

As a matter of fact, we impose penalty on all the parameters only if we want to remove the whole layer. In practice, we usually encounter the application of full regularization since the initial network is artificially set to be really too big. The goal of network compression is to make it as small as possible. However, designed network for each problem is not so large that removing one or more whole hidden layers may have an extremely bad effect to the final results. Because of this, we propose an idea of only penalizing part of the parameters of neural networks. Fig. 1 gives a visual representation of our partial regularization method, containing an original network with one input layer, two hidden layers and one output layer which takes $x_0$ as input and $y_0$ as output. Without loss of generality, we can artificially set a series of coefficients in order to remove the two particular neurons of hidden layer 1 that are highlighted in red color. Once the selected neurons are removed, the corresponding outgoing weights represented by blue dashed line should not be crucial to the outputs. Therefore, we don’t need to penalize all the outgoing connections of hidden layer 1 if we have done the neuron-cut procedure. This is why partial regularization would be faster because it requires less computation for the regularization terms. Of course, we can also impose the penalty on the input layer unless the original network has only one single input neuron.

To this end, we introduce a new hyper-parameter $\beta_n$ to modify group lasso regularization method (2):

$$R(\theta) = \sum_{l=1}^{L} \sqrt{p_l} \sum_{n=1}^{N} \beta_n ||\theta^n_l||_2$$

(4)

Following the derivation in the previous section, we exploit sparse group lasso regularization and rewrite its regularizer as follows:

$$R(\theta) = \sum_{l=1}^{L} \left( (1 - \alpha) \sqrt{p_l} \sum_{n=1}^{N} \beta_n ||\theta^n_l||_2 + \alpha \sum_{n=1}^{N} \beta_n ||\theta^n_l||_2 \right)$$

(5)
Clearly, to realize our idea, we could set the new hyper-parameter $\beta_n$ to be a diagonal matrix with some entries being zeros mathematically. If so, the regularization term contains a smaller number of elements than that of full regularization. Furthermore, we can set the number of zeros in $\beta_n$ artificially to determine the number of entries left in the regularization terms. Positions of zeros make no difference to the final results by taking advantage of the permutation invariant property in neural networks, as long as the ratio is identical. We will discuss later about the effects of different initialization of $\beta_n$.

However, when we conduct experiments with partial regularization on the group lasso variants adopting layer weights in [10], training performances are better but test performances become worse, implying that the pruned model doesn’t have a better generalization ability. We deduce that it may have a conflict between these two tricks so that we do not combine together.

4 Experiments

In this section, we evaluate our proposal on different datasets with a strong focus on the performance of reducing computation complexity and improving network metrics. We begin with a simple toy example and then move on to famous and large real-world datasets which need to use deeper and larger networks. In particular, we demonstrate the effectiveness of applying partial regularization with comparison of those with full regularization. Note that hardware and program optimizations can further boost the performance but are not covered in this work.

4.1 Implementation Trick

In the mathematical formula, we introduce a new hyper-parameter $\beta_n$ operating on the regularization terms. Based on our proposed method, we choose it to be a diagonal matrix $\beta$ with some entries being 1 and the remaining being 0. However, if we multiply the network parameters with such a diagonal matrix, the corresponding rows of matrix product will be all zeros and cannot be processed successfully in Tensorflow [11]. Therefore, we decide to adopt a trick of slicing the parameter matrix only on the first dimension to the required shape. By this way, specific number of parameters are involved to be updated in each iteration, same as the former matrix multiplication idea. For group lasso method and its sparse variant, we assume that each group only has one neuron so that $p_l$ is set to be the width of parameter $\theta_l$. Due to the fluctuation of running time, we choose the average among three tries under the same settings.

The following tables contain metrics showing training and test performances, neurons after pruning, sparsity and total running time. Sparsity reports the ratio of the number of parameters that are smaller than a fixed threshold over the total.

| Dataset      | Regularization Factor | Neurons       | Batch Size |
|--------------|-----------------------|---------------|------------|
| Boston       | 0.001                 | 13/40/30/1    | -          |
| Toy Example  | 0.001                 | 1/50/50/1     | -          |
| SDD          | 0.0001                | 48/40/40/30/11| 500        |
| MNIST        | 0.0001                | 784/400/300/100/10 | 400      |
| FASHION-MNIST| 0.0001                | 784/400/300/100/10 | 400      |

4.2 Regression Tasks

We first employ a neural network to approximate a quadratic function as the toy example. Table 2 shows that partial regularization gives rise to lower training and test loss within less running time. It indicates that networks after pruning by partial regularization method achieve better fitting results than those of the competitors more quickly. Red curves on the right-hand side are definitely closer to the original function colored in blue, as shown in Fig 2.

Similarly, we see from Table 3 that modified methods perform better on the training data of Boston House Pricing dataset. In addition, test performances are also improved. It tells us that partial regularization provides better predictions of the housing prices by a pruned model with a better generalization ability. More importantly, it takes less time to those comparable results and serves to reduce more neurons in some layers of the networks.

Here we present an observation which is not presented but does exist in the experiments. When conducting further training with the pruned network and comparing the total running steps, we find that it takes fewer steps to comparable
performance. Simultaneously, if we train a neural network of the same small size with random initialization, it will always cause worse results. As a direct consequence, it’s better to do online model compression and continue training the slim network with current parameters of the remaining neurons.

Table 2: Results on Toy Example

| Reg | Training loss | Test Loss | Neurons   | Sparsity       | Time(s) |
|-----|---------------|-----------|-----------|----------------|---------|
| GL  | 1.51E-03      | 0.36E-02  | [1, 30, 2, 1] | [0.96, 0.9988, 0.96] | 4.86    |
| iGL | 8.24E-05      | 0.12E-02  | [1, 29, 10, 1] | [0.8, 0.8, 0.8]    | 4.66    |
| SGL | 1.46E-03      | 0.33E-02  | [1, 49, 2, 1] | [0.96, 0.9988, 0.96] | 5.67    |
| iSGL| 1.03E-04      | 0.14E-02  | [1, 50, 10, 1] | [0.8, 0.8, 0.8]    | 5.13    |

Table 3: Results on Boston House Pricing Dataset

| Reg  | Training loss | Test Loss | Neurons   | Sparsity       | Time(s) |
|------|---------------|-----------|-----------|----------------|---------|
| GL   | 1.03E-02      | 1.05E-02  | [12, 3, 1, 1] | [0.9827, 0.9992, 0.9667] | 38.10   |
| iGL  | 0.99E-02      | 0.89E-02  | [10, 25, 12, 1] | [0.2327, 0.7267, 0.6] | 36.77   |
| SGL  | 1.02E-02      | 1.03E-02  | [13, 40, 1, 1] | [0.9808, 0.9992, 0.9667] | 38.27   |
| iSGL | 1.13E-02      | 0.89E-02  | [13, 35, 10, 1] | [0.2385, 0.7217, 0.6667] | 38.13   |

Figure 2: Blue: Plot of $y = -x^2$ in [-1, 1], Red: Fitting Results

4.3 Classification Tasks

We choose a variety of datasets to show that partial regularization is a general technique for improving the classification performance and generalization ability of neural networks. Since the number of neurons for each layer has great difference, we compute several results for different ratios of zeros in $\beta$ for classification tasks and report the values under a particular $\beta$ for some metrics to show the effectiveness of our modified methods.
As we can see, no matter which dataset we choose, partial regularization always makes the training better, even with a great improvement under an appropriate choice of the new hyper-parameter. In the same way, it also has a good impact on test performances. Actually, it is not difficult to find such $\beta$ because almost all we choose in our experiments could provide us better results for most circumstances. Another thing we want to emphasize is that partial regularization really helps to keep the most important neurons and connections. Furthermore, it still achieves a more compact network with little or even neglected loss on metrics with comparison to the original network.

When we take a look at the last column of each table, partial regularization brings a minor acceleration on the running time. However, the speed-ups are not such great as we imagine and deduce by mathematical representations. Even though partial regularization contains less parameters which need to be trained, it may owing to that the running time may largely come from data processing steps, background program occupation in GPUs when running codes or other unknown reasons. At the same time, the computation is more complicated since partial regularization reserve more neurons.

We now analyze the sensitivity of partial regularization method with respect to $\beta$. Our observation is that there is no monotone increasing or decreasing trend in test classification accuracy with different choices of $\beta$ in Fig[3]. Therefore, we could conclude that for any network on a specific dataset, there must exist an optimal diagonal matrix $\beta$ such that partial regularization could assist to get a compact network along with highly acceptable accuracy and generalization ability. At the same time, artificial settings of $\beta$ with arbitrary positions of zeros do not largely affect the performances so that it verifies the permutation variant property of neural networks conversely.

One main drawback of partial regularization occurs when considering the performance of network compression on large datasets. According to our experiments, we find that neural networks compressed by partial regularization contain much more neurons than those of full regularization. Maybe the original methods, especially group lasso method, remove so many neurons including some relatively important ones. Much useful information are ignored and cannot be transferred to the following layers so that it hurts the performances.

### Table 4: Results on SDD ($\text{Ratio of Zeros} = \frac{1}{8}$)

| Reg  | Training Acc | Test Acc  | Neurons     | Sparsity  | Time(s) |
|------|--------------|-----------|-------------|-----------|---------|
| GL   | 0.9889       | 0.9874    | [47, 25, 11, 10] | [0.59, 0.84, 0.92, 0.71] | 507     |
| iGL  | 0.9943       | 0.9928    | [48, 20, 8, 15]  | [0.55, 0.85, 0.90, 0.65] | 489     |
| SGL  | 0.9883       | 0.9875    | [47, 22, 9, 9]   | [0.67, 0.89, 0.93, 0.74] | 528     |
| iSGL | 0.9960       | 0.9929    | [47, 24, 9, 12]  | [0.57, 0.79, 0.85, 0.67] | 508     |

### Table 5: Results on MNIST ($\text{Ratio of Zeros} = \frac{1}{8}$)

| Reg  | Training Acc | Test Acc  | Neurons     | Sparsity  | Time(s) |
|------|--------------|-----------|-------------|-----------|---------|
| GL   | 0.9598       | 0.9574    | [522, 19, 28, 33] | [0.99, \approx 1.0, 0.99, 0.68] | 73      |
| iGL  | 0.9722       | 0.9640    | [605, 345, 79, 39] | [0.87, 0.88, 0.88, 0.69] | 68      |
| SGL  | 0.9569       | 0.9541    | [756, 341, 27, 33] | [0.99, \approx 1.0, 0.99, 0.69] | 74      |
| iSGL | 0.9696       | 0.9592    | [759, 384, 81, 46] | [0.87, 0.88, 0.88, 0.70] | 74      |

### Table 6: Results on FASHION-MNIST ($\text{Ratio of Zeros} = \frac{1}{4}$)

| Reg  | Training Acc | Test Acc  | Neurons     | Sparsity  | Time(s) |
|------|--------------|-----------|-------------|-----------|---------|
| GL   | 0.8528       | 0.8352    | [589, 7, 9, 16] | [\approx 1.0, \approx 1.0, \approx 1.0, 0.84] | 328     |
| iGL  | 0.9171       | 0.8577    | [748, 260, 84, 27] | [0.76, 0.76, 0.76, 0.74] | 307     |
| SGL  | 0.8650       | 0.8467    | [771, 304, 10, 17] | [\approx 1.0, \approx 1.0, \approx 1.0, 0.84] | 340     |
| iSGL | 0.9131       | 0.8536    | [778, 365, 91, 27] | [0.76, 0.76, 0.76, 0.74] | 321     |

5 Conclusions

In this work, we have introduced partial regularization method to modify group lasso regularization and its sparse variant for network compression task. With the help of the permutation invariant property in neural networks, we could penalize less parameters to obtain a better network structure with almost same or better capacity. Since regularization
methods are frequently used recently and the original manually-designed networks contain a lot of parameters, we believe that partial regularization could bring an acceleration and improvement when compared to full regularization. To this end, we have imposed a new hyper-parameter $\beta_n$ on the weights and biases of fully-connected layers in deep neural networks. Our experiments have demonstrated the advantages of partial regularization for tasks including those of regression and classification types. We conclude that if we choose $\beta$ properly, it has been found useful to improve the training performances and test performances with less or at least equal running time consistently. This means that partial regularization could not only achieve better results, but also increase the generalization ability of the pruned networks. By the way, it yields a more compact architecture than the initial over-parametric network, thus saving both memory and computation costs. Moreover, for any specific dataset, there must exist an optimal network along with small errors and good generalization ability.

The limitation is that we have not shown the benefits of partial regularization on other different architectures including convolutional and recurrent neural networks. We wish to explore in future contributions for the influence in the aforementioned circumstances. In addition, to deal with the drawback of more reserved parameters, we could think of applying another modification on our partial regularization methods in a proper way to obtain an optimal network structure with highly acceptable performance, according to the elegant idea in [12].

Acknowledgement

The authors thank Qianxiao Li, Yingzhou Li and Li Zhou for helpful discussions and insightful comments.

References

[1] Yu Cheng, Felix X. Yu, Rogerio S. Feris, Sanjiv Kumar, Alok Choudhary, and Shi-Fu Chang. An exploration of parameter redundancy in deep networks with circulant projections. In Proceedings of the IEEE International Conference on Computer Vision (ICCV), December 2015.
[2] Misha Denil, Babak Shakibi, Laurent Dinh, Marc’Aurelio Ranzato, and Nando De Freitas. Predicting parameters in deep learning. In *Advances in neural information processing systems*, pages 2148–2156, 2013.

[3] Guido F Montufar, Razvan Pascanu, Kyunghyun Cho, and Yoshua Bengio. On the number of linear regions of deep neural networks. In *Advances in neural information processing systems*, pages 2924–2932, 2014.

[4] Anders Krogh and John A Hertz. A simple weight decay can improve generalization. In *Advances in neural information processing systems*, pages 950–957, 1992.

[5] Robert Tibshirani. Regression shrinkage and selection via the lasso. *Journal of the Royal Statistical Society: Series B (Methodological)*, 58(1):267–288, 1996.

[6] Ming Yuan and Yi Lin. Model selection and estimation in regression with grouped variables. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 68(1):49–67, 2006.

[7] Jerome Friedman, Trevor Hastie, and Robert Tibshirani. A note on the group lasso and a sparse group lasso. *arXiv preprint arXiv:1001.0736*, 2010.

[8] Noah Simon, Jerome Friedman, Trevor Hastie, and Robert Tibshirani. A sparse-group lasso. *Journal of Computational and Graphical Statistics*, 22(2):231–245, 2013.

[9] Simone Scardapane, Danilo Comminiello, Amir Hussain, and Aurelio Uncini. Group sparse regularization for deep neural networks. *Neurocomputing*, 241:81–89, 2017.

[10] Jose M Alvarez and Mathieu Salzmann. Learning the number of neurons in deep networks. In *Advances in Neural Information Processing Systems*, pages 2270–2278, 2016.

[11] Martín Abadi, Ashish Agarwal, Paul Barham, Eugene Brevdo, Zhifeng Chen, Craig Citro, G.s Corrado, Andy Davis, Jeffrey Dean, Matthieu Devin, Sanjay Ghemawat, Ian Goodfellow, Andrew Harp, Geoffrey Irving, Michael Isard, Yangqing Jia, Rafal Jozefowicz, Lukasz Kaiser, Manjunath Kudlur, and Xiaoxiang Zheng. Tensorflow: Large-scale machine learning on heterogeneous distributed systems. 03 2016.

[12] Jaehong Yoon and Sung Ju Hwang. Combined group and exclusive sparsity for deep neural networks. In *Proceedings of the 34th International Conference on Machine Learning-Volume 70*, pages 3958–3966. JMLR. org, 2017.