Adding a Helper Can Totally Remove the Secrecy Constraints in Interference Channel

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Abstract

In many communication channels, secrecy constraints usually incur a penalty in capacity, as well as generalized degrees-of-freedom (GDoF). In this work we show an interesting observation that, adding a helper can totally remove the penalty in sum GDoF, for a two-user symmetric Gaussian interference channel. For the interference channel where each transmitter sends a message to an intended receiver without secrecy constraints, the sum GDoF is a well-known “W” curve, characterized by Etkin-Tse-Wang in 2008. If the secrecy constraints are imposed on this interference channel, where the message of each transmitter must be secure from the unintended receiver (eavesdropper), then a GDoF penalty is incurred and the secure sum GDoF is reduced to a modified “W” curve, derived by Chen recently. In this work we show that, by adding a helper into this interference channel with secrecy constraints, the secure sum GDoF turns out to be a “W” curve, which is the same as the sum GDoF of the setting without secrecy constraints. The proposed scheme is based on the cooperative jamming and a careful signal design such that the jamming signal of the helper is aligned at a specific direction and power level with the information signals of the transmitters, which allows to totally remove the penalty in GDoF due to the secrecy constraints. Furthermore, the estimation approaches of noise removal and signal separation due to the rational independence are used in the secure rate analysis.

I. INTRODUCTION

Since Shannon’s seminal work in 1949 [1], information-theoretic secrecy has been studied for many years in many communication channels, such as wiretap channels [2]–[4], broadcast channels [5]–[8], multiple access channels and interference channels [5], [9]–[31]. In the pioneer work by Wyner in 1975 [2], the notion of secure capacity was introduced via a wiretap channel with secrecy constraint. For this wiretap channel, Wyner showed that there is a penalty in capacity due to the secrecy constraint. Later on, it has been shown that this insight also holds true for the other channels (see, e.g., [5], [14], [15], [19], [21], [24]–[26], [30]).

This work focuses on the secure sum generalized degrees-of-freedom (GDoF, a capacity approximation) of a two-user symmetric Gaussian interference channel, and shows an interesting observation that the penalty in GDoF due to the secrecy constraints can be totally removed by adding a helper. For the symmetric Gaussian interference channel without secrecy constraints, the sum GDoF is characterized as a well-known “W” curve (cf. [32]). If the secrecy constraints are imposed on this interference channel, where the message of each transmitter must be secure from the unintended receiver (eavesdropper), the work in [31] showed that the secure sum GDoF is then reduced to a modified “W” curve (see Fig. 1). It reveals that there is a penalty in GDoF in a large regime due to the secrecy constraints. Interestingly, this work shows that adding a helper can totally remove the penalty in GDoF due to the secrecy constraints (see Fig. 1). In other words, by adding a helper, the secure sum GDoF of the interference channel derived in this work is exactly the same as the sum GDoF of the interference channel without secrecy constraints.

The proposed scheme is based on cooperative jamming and signal alignment. Specifically, the helper sends a cooperative jamming signal at a specific direction and power level to confuse the eavesdroppers, while keeping legitimate receivers’ abilities to decode their desired messages. The role of helper(s) in secure communications has been studied extensively in the literature (see [5], [9], [21], [24], [33] and references therein). The helper can be the node transmitting its own confidential message (cf. [5], [9].
α
2
GDoF of interference channel

Secure GDoF of interference channel without any helper

Secure GDoF of interference channel with a helper

[Etkin,Tse,Wang08]

Chen18

GDoF of interference channel

Fig. 1. Secure sum GDoF vs. α for the two-user symmetric Gaussian interference channel with a helper, where α corresponds to the interference-to-signal ratio (all link strengths in decibel scale). Note that the secure sum GDoF of the interference channel with a helper, derived in this work (cf. Theorem 1), is the same as the sum GDoF of the interference channel without secrecy constraints (cf. [22]). Without any helper, there is a penalty in GDoF due to the secrecy constraints (cf. [31]).

[21], [24]), as well as the node without transmitting any message (cf. [21], [33]). The helper considered in our setting refers to the latter case, which is able to totally remove the penalty in GDoF due to the secrecy constraints. For the proposed scheme, the estimation approaches of noise removal and signal separation due to rational independence (cf. [34]) are used in the secure rate analysis.

The remainder of this work is organized as follows. Section II introduces the system model. Section III presents the main result of this work. Section IV describes the proposed scheme via an example. The achievability proof is given in Sections V-VIII and the appendices, while the converse proof is given in Section IX. Section X concludes this work. Throughout this work, I(●), H(●) and h(●) denote the mutual information, entropy and differential entropy, respectively. Z, Z+ and R denote the sets of integers, positive integers and real numbers, respectively. o(●) comes from the standard Landau notation, where f(x) = o(g(x)) implies that \( \lim_{x \to \infty} f(x)/g(x) = 0 \). Logarithms are in base 2.

II. System model

In this setting, we consider a two-user Gaussian interference channel with a helper and confidential messages (see Fig. 2). The output of the channel at receiver k at time t is

\[
y_k(t) = \sum_{\ell=1}^3 2^{m_{k\ell}} h_{k\ell} x_\ell(t) + z_k(t), \quad k = 1, 2
\]  

where \( x_\ell(t) \) is the normalized channel input at transmitter \( \ell \) under the power constraint \( \mathbb{E}|x_\ell(t)|^2 \leq 1 \); \( z_k(t) \sim \mathcal{N}(0, 1) \) is the additive white Gaussian noise at receiver k; \( m_{k\ell} \) is a nonnegative integer; and \( h_{k\ell} \in (1, 2) \) is a normalized channel coefficient, for \( k = 1, 2 \) and \( \ell = 1, 2, 3 \). In our setting, transmitter 3 is the helper. By following the convention in [31], we let \( P = \max_k \{2^{2m_{kk}}\} \), and define

\[
\alpha_{k\ell} = \frac{\log 2^{m_{k\ell}}}{\frac{1}{2} \log P} \quad k = 1, 2, \quad \ell = 1, 2, 3.
\]  

Then, we can rewrite the channel model in (1) as

\[
y_k(t) = \sum_{\ell=1}^3 \sqrt{P^{\alpha_{k\ell}}} h_{k\ell} x_\ell(t) + z_k(t), \quad k = 1, 2
\]  

where \( \alpha_{k\ell} \geq 0 \) denotes the channel strength of the link between transmitter \( \ell \) and receiver k. In the rest of this work, we will consider the channel model in (3). It is assumed that all the channel parameters \( \{\alpha_{k\ell}, h_{k\ell}\}_{k,\ell} \) are available at each node. For the symmetric case, it is assumed that

\[
\alpha_{11} = \alpha_{22} = 1, \quad \alpha_{21} = \alpha_{12} = \alpha_{13} = \alpha_{23} = \alpha, \quad \alpha \geq 0.
\]
In this interference channel, a confidential message $w_k$ is sent from transmitter $k$ to receiver $k$, where $w_k$ is uniformly chosen from a set $\mathcal{W}_k \Delta \{1, 2, \ldots, 2^{nR_k}\}$, for $k = 1, 2$. At transmitter $k$, a stochastic function $f_k : \mathcal{W}_k \times \mathcal{W}_k' \to \mathbb{R}^n$, $k = 1, 2$ is used to map the message $w_k \in \mathcal{W}_k$ to a transmitted codeword $x_n^k = f_k(w_k, w_k') \in \mathbb{R}^n$, where $w_k' \in \mathcal{W}_k'$ denotes the randomness in this mapping, and $w_k'$ is available at transmitter $k$ only. At transmitter 3 (the helper), the following function $f_3 : \mathcal{W}_3' \to \mathbb{R}^n$ is used to generate $x_3^n = f_3(w_3')$, where the random variable $w_3' \in \mathcal{W}_3'$ is available at transmitter 3 only. We assume that the random variables $\{w_1, w_1', w_2, w_2', w_3'\}$ are mutually independent. A secure rate pair $(R_1, R_2)$ is said to be achievable if for any $\epsilon > 0$ there exists a sequence of $n$-length codes such that each receiver can decode its own message reliably and the messages are kept secret such that

$$I(w_1; y_2^n) \leq n\epsilon, \quad I(w_2; y_1^n) \leq n\epsilon.$$  

(4)

The secure capacity region, denoted by $C$, is the closure of the set of all achievable $(R_1, R_2)$ secure rate pairs. We define the secure sum capacity as:

$$C_{\text{sum}} \overset{\Delta}{=} \sup \left\{ R_1 + R_2 \mid (R_1, R_2) \in C \right\}.$$  

(5)

We also define the secure sum GDoF as

$$d_{\text{sum}} \overset{\Delta}{=} \lim_{P \to \infty} \frac{C_{\text{sum}}}{\frac{1}{2} \log P}.$$  

(6)

Note that the GDoF is a form of capacity approximation. It is more general than degrees-of-freedom (DoF), as the latter considers only a specific case with $\alpha_{k\ell} = 1, \forall k, \ell$.

**III. MAIN RESULT**

This section provides the main result of this work, for the secure communication over a two-user symmetric Gaussian interference channel with a helper.

**Theorem 1.** Considering the two-user symmetric Gaussian interference channel with a helper defined in Section II for almost all the channel coefficients $\{h_{kk}\} \in (1, 2)^{2\times 3}$, the optimal secure sum GDoF is characterized as

$$d_{\text{sum}} = \begin{cases} 
2(1 - \alpha) & \text{for } 0 \leq \alpha \leq \frac{1}{2} \\
2\alpha & \text{for } \frac{1}{2} \leq \alpha \leq \frac{2}{3} \\
2(1 - \alpha/2) & \text{for } \frac{2}{3} \leq \alpha \leq 1 \\
\alpha & \text{for } 1 \leq \alpha \leq 2 \\
2 & \text{for } \alpha \geq 2.
\end{cases}$$

(7a) (7b) (7c) (7d) (7e)
In this setting defined in Section II, Theorem 1 reveals that adding a helper can totally remove the secrecy constraints, in the sense that the secure sum GDoF of the interference channel with a helper and with secrecy constraints is the same as the sum GDoF of the interference channel without any helper and without secrecy constraints (a “W” curve, see [32] and Fig. 1). The optimal secure sum GDoF is achieved by a cooperative jamming scheme. The achievability is described in Section V. The converse proof is provided in Section IX. Before providing the achievability, we describe the proposed scheme via an example in the following section.

IV. SCHEME EXAMPLE

In this section, we will describe the outline of the proposed scheme via an example. We will focus on the specific case with \( \alpha = 3/4 \), for the two-user symmetric Gaussian interference channel with secrecy constraints and with a helper defined in Section II. Note that, for the two-user symmetric Gaussian interference channel without any secrecy constraints, the sum GDoF is 5/4 when \( \alpha = 3/4 \) (cf. [32]). If the secrecy constraints are imposed on this interference channel, where the message of each transmitter must be secure from the unintended receiver, the secure sum GDoF is reduced to 1 when \( \alpha = 3/4 \) (cf. [31]). Therefore, there is a penalty in GDoF due to the secrecy constraints. Interestingly, we will show that adding a helper can totally remove the penalty in GDoF due to the secrecy constraints, that is, by adding a helper the secure sum GDoF is increased to 5/4, which is exactly the same as the sum GDoF of the interference channel without secrecy constraints. For the proposed scheme, pulse amplitude modulation (PAM) and signal alignment will be used in the signal design, and the estimation approaches of noise removal and signal separation will be used in the rate analysis. The scheme is motivated by the cooperative jamming scheme proposed in [31] for a different setting, i.e., a two-user interference channel with confidential messages but without any helper.

For this case with \( \alpha = 3/4 \), the transmitted signals are designed as (removing the time index):

\[
\begin{align*}
x_1 &= h_{23}h_{12}v_{1,c} + \sqrt{P - 3/4} \cdot h_{23}h_{12}v_{1,p} \\ x_2 &= h_{13}h_{21}v_{2,c} + \sqrt{P - 3/4} \cdot h_{13}h_{21}v_{2,p} \\ x_3 &= h_{12}h_{21}u_3,
\end{align*}
\]

where \( v_{k,c} \) and \( v_{k,p} \) are two signals that carry the message of transmitter \( k \), for \( k = 1, 2 \), and \( u_3 \) is the jamming signal from transmitter 3. The random variables \( \{v_{1,c}, v_{1,p}, v_{2,c}, v_{2,p}, u_3\} \) are mutually independent. Specifically, for \( \Omega(\xi, Q) \triangleq \{\xi \cdot a : a \in \mathbb{Z} \cap [-Q, Q]\} \) denoting the PAM constellation set, the above random variables are independently and uniformly drawn from their PAM constellation sets, given as

\[
\begin{align*}
v_{k,c} &\in \Omega(\xi = 2\gamma \cdot \frac{1}{Q}, \quad Q = P^{3/8-\epsilon}) \\ v_{k,p} &\in \Omega(\xi = \gamma \cdot \frac{1}{Q}, \quad Q = P^{1/4-\epsilon}) \\ u_3 &\in \Omega(\xi = 2\gamma \cdot \frac{1}{Q}, \quad Q = P^{3/8-\epsilon})
\end{align*}
\]
respectively, for \( k = 1, 2 \), where \( \epsilon > 0 \) can be made arbitrarily small, and \( \gamma \) is a finite constant such that \( \gamma \in \left(0, \frac{1}{8\sqrt{2}} \right) \). Based on our signal design, \( v_{k,c} \) carries \( 3/8 \) GDoF and \( v_{k,p} \) carries \( 1/4 \) GDoF, that is, \( \mathbb{H}(v_{k,c}) = \frac{3}{8}\log P + o(\log P) \) and \( \mathbb{H}(v_{k,p}) = \frac{1}{4}\log P + o(\log P) \), when \( \epsilon \to 0 \), for \( k = 1, 2 \). The above signal design satisfies the average power constraints, \( \mathbb{E}[|x_1|^2] \leq 1 \), \( \mathbb{E}[|x_2|^2] \leq 1 \) and \( \mathbb{E}[|x_3|^2] \leq 1 \), which will be proved in the next section in detail. Then, the received signals at the receivers 1 and 2 are given as (without time index)

\[
y_1 = \sqrt{P}h_{11}h_{12}v_{1,c} + \sqrt{P^{1/4}}h_{11}h_{23}h_{12}v_{1,p} + \sqrt{P^{3/4}}h_{12}h_{13}(v_{2,c} + u_3) + h_{12}h_{13}v_{2,p} + z_1
\]

\[
y_2 = \sqrt{P}h_{22}h_{13}v_{2,c} + \sqrt{P^{1/4}}h_{22}h_{13}h_{21}v_{2,p} + \sqrt{P^{3/4}}h_{21}h_{23}(v_{1,c} + u_3) + h_{21}h_{23}v_{1,p} + z_2.
\]

At receiver 1, the information signal \( v_{2,c} \) of transmitter 2 is aligned with the jamming signal \( u_3 \) of the helper. At receiver 2, the information signal \( v_{1,c} \) of transmitter 1 is aligned with the jamming signal \( u_3 \) of the helper. As we will see later on, the penalty in GDoF due to the secrecy constraint will be minimized with this signal alignment design. Fig. 3 describes the rate (GDoF) and power of some signals at receivers 1 and 2 when \( \alpha = 3/4 \).

As we will discuss in detail in the next section, the secure rate pair

\[
R_1 = \mathbb{I}(v_1; y_1) - \mathbb{I}(v_1; y_2|v_2)
\]

\[
R_2 = \mathbb{I}(v_2; y_2) - \mathbb{I}(v_2; y_1|v_1)
\]

is achievable by the proposed scheme with a careful codebook design and message mapping. Let us begin with the secure rate \( R_1 \) expressed in (16). On one hand, we expect the term \( \mathbb{I}(v_1; y_1) \) to be sufficiently large; on the other hand, we expect the term \( \mathbb{I}(v_1; y_2|v_2) \) to be sufficiently small. Let \( \hat{v}_{1,c} \) and \( \hat{v}_{1,p} \) be the estimates for \( v_{1,c} \) and \( v_{1,p} \) respectively from \( y_1 \), and let \( \Pr[\{v_{1,c} \neq \hat{v}_{1,c}\} \cup \{v_{1,p} \neq \hat{v}_{1,p}\}] \) denote the corresponding estimation error probability. Then the term \( \mathbb{I}(v_1; y_1) \) can be lower bounded by

\[
\mathbb{I}(v_1; y_1) \geq \mathbb{I}(v_1; \hat{v}_{1,c}, \hat{v}_{1,p})
\]

\[
= \mathbb{H}(v_1) - \mathbb{H}(v_1|\hat{v}_{1,c}, \hat{v}_{1,p})
\]

\[
\geq \mathbb{H}(v_1) - (1 + \Pr[\{v_{1,c} \neq \hat{v}_{1,c}\} \cup \{v_{1,p} \neq \hat{v}_{1,p}\}] \cdot \mathbb{H}(v_1))
\]

\[
= (1 - \Pr[\{v_{1,c} \neq \hat{v}_{1,c}\} \cup \{v_{1,p} \neq \hat{v}_{1,p}\}]) \cdot \mathbb{H}(v_1) - 1
\]

where (18) results from the Markov chain \( v_1 \to y_1 \to \{\hat{v}_{1,c}, \hat{v}_{1,p}\} \); (19) results from Fano’s inequality. Note that, based on our signal design, \( \{v_{1,p}, v_{1,c}\} \) can be reconstructed from \( v_1 \), and vice versa, because \( v_1 \) takes the form of \( v_1 = v_{1,c} + \sqrt{P^{-3/4}} \cdot v_{1,p} \) and the minimum of \( v_{1,c}/2 \) is bigger than the maximum of \( \sqrt{P^{-3/4}} \cdot v_{1,p} \). In this case \( v_{1,c} \) carries \( 3/8 \) GDoF and \( v_{1,p} \) carries \( 1/4 \) GDoF, which implies that \( v_1 \) carries \( 5/8 \) GDoF. The term \( \mathbb{H}(v_1) \) in (20) then becomes

\[
\mathbb{H}(v_1) = \frac{5/8}{2} \log P + o(\log P).
\]

In Sections V and VIII we will prove that \( v_{1,c} \) and \( v_{1,p} \) designed in (11)-(13) can be estimated from \( y_1 \) by using the estimation approaches of noise removal and signal separation, and the corresponding estimation error probability is

\[
\Pr[\{v_{1,c} \neq \hat{v}_{1,c}\} \cup \{v_{1,p} \neq \hat{v}_{1,p}\}] \to 0 \quad \text{as} \quad P \to \infty
\]

for almost all the channel coefficients \( \{h_{k,l}\} \in (1, 2]^{2 \times 3} \). The proof outline of (22) will be provided later on in this section. By incorporating (21) and (22) into (20), it gives

\[
\mathbb{I}(v_1; y_1) \geq \frac{5/8}{2} \log P + o(\log P)
\]
for almost all the channel coefficients \( \{ h_{k\ell} \} \in (1, 2)^{2 \times 3} \).

On the other hand, \( \mathbb{I}(v_1; y_2|v_2) \) can be bounded as
\[
\mathbb{I}(v_1; y_2|v_2) \leq \log(2\sqrt{65}).
\]
(24)

The detailed proof is provided in the next section. This term \( \mathbb{I}(v_1; y_2|v_2) \) can be considered as a penalty term in the secure rate \( R_1 \). The result in (24) reveals that this penalty is sufficiently small, based on our careful signal design. In the proposed scheme, the jamming signal of the helper is aligned at a specific direction and power level with the information signals of the transmitters, which minimizes the rate penalty due to the secrecy constraints. Finally, with the results in (23) and (24), the secure rate \( R_1 \) is lower bounded by
\[
R_1 = \mathbb{I}(v_1; y_1) - \mathbb{I}(v_1; y_2|v_2) \geq \frac{5/8}{2} \log P + o(\log P)
\]
and similarly, the secure rate \( R_2 \) is lower bounded by \( R_2 \geq \frac{5/8}{2} \log P + o(\log P) \), for almost all the channel coefficients \( \{ h_{k\ell} \} \in (1, 2)^{2 \times 3} \). Then the secure GDoF pair \( (d_1 = 5/8, d_2 = 5/8) \) can be achieved by the proposed cooperative jamming scheme for almost all the channel coefficients in this case with \( \alpha = 3/4 \).

In the following we provide the proof outline of (22). After some manipulations, \( y_1 \) in (14) can be rewritten as
\[
y_1 = P^{3/8+c} \cdot 2\gamma \cdot (\sqrt{P^{1/4}g_0q_0 + g_1q_1}) + \tilde{z}_1
\]
where \( x_s \triangleq \sqrt{P^{1/4}}g_0q_0 + g_1q_1, \tilde{z}_1 \triangleq \sqrt{P^{1/4}}h_{11}h_{23}h_{12}v_{1,p} + h_{12}h_{21}h_{13}v_{2,p} + z_1, g_0 \triangleq h_{11}h_{23}h_{12}, g_1 \triangleq h_{12}h_{21}h_{13} \)
\[
g_0 \triangleq Q_{\max} \cdot v_{1,c}, q_1 \triangleq Q_{\max} \cdot (v_{2,c} + v_3), \text{ and } Q_{\max} \triangleq P^{\frac{3/8}{2}}.\]
One important step is to estimate \( x_s \) from \( y_1 \) by treating other signals as noise. This step is called as noise removal. After correctly decoding \( x_s \), we can recover \( g_0 \) and \( g_1 \) from \( x_s \), because \( g_0 \) and \( g_1 \) are rationally independent. This step is called as signal separation (cf. [34]). As we will show in Section VIII the average power of the virtual noise \( \tilde{z}_1 \) is bounded by
\[
\mathbb{E}[\tilde{z}_1|^2 \leq \kappa P^{1/4}
\]
for a finite constant \( \kappa \). To estimate \( x_s \) from \( y_1 \), we show in Section VIII that the minimum distance of \( x_s \), denoted by \( d_{\min} \), is bounded by
\[
d_{\min} \geq \delta P^{-1/16}
\]
(25)
for all the channel coefficients \( \{ h_{k\ell} \} \in (1, 2)^{2 \times 3} \setminus \mathcal{H}_{\text{out}} \), where \( \mathcal{H}_{\text{out}} \subseteq (1, 2)^{2 \times 3} \) is an outage set and the Lebesgue measure of this outage set, denoted by \( \mathcal{L}(\mathcal{H}_{\text{out}}) \), satisfies
\[
\mathcal{L}(\mathcal{H}_{\text{out}}) \leq 1792\delta \cdot P^{-\frac{7}{2}}
\]
(26)
for some constants \( \delta \in (0, 1] \) and \( \epsilon > 0 \). The proofs of (25) and (26) build on the conclusion of Lemma 1 (see below). The results of (25) and (26) reveal that, the minimum distance of \( x_s \) is sufficiently large for almost all the channel coefficients in the regime of large \( P \), that is \( \mathcal{L}(\mathcal{H}_{\text{out}}) \to 0 \) as \( P \to \infty \). At this point we can decode the sum \( x_s = \sqrt{P^{1/4}}g_0q_0 + g_1q_1 \) from \( y_1 \) by treating other signals as noise (noise removal), with vanishing error probability. After that we can recover \( g_0 = Q_{\max} \cdot v_{1,c} \), as well as \( v_{1,c} \), from \( x_s \) because \( g_0 \) and \( g_1 \) are rationally independent (rational independence). By removing the decoded \( x_s \) from \( y_1 \), \( v_{1,p} \) can be estimated with vanishing error probability, resulting in
\[
\Pr[\{ v_{1,c} \neq \hat{v}_{1,c} \} \cup \{ v_{1,p} \neq \hat{v}_{1,p} \}] \to 0 \quad \text{as} \quad P \to \infty
\]
for almost all the channel coefficients in the regime of large \( P \). The lemma used in the proofs of (25) and (26) is given as follows.
Lemma 1. Let \( \beta \in (0, 1) \), \( \tau \in \mathbb{Z}^+ \) and \( \tau > 1 \), and \( A_0, A_1, Q_0, Q_1 \in \mathbb{Z}^+ \). Define the event
\[
B(q_0, q_1) \triangleq \{(g_0, g_1) \in [1, \tau]^2 : |A_0 g_0 q_0 + A_1 g_1 q_1| < \beta\}
\] (27)
and set
\[
B \triangleq \bigcup_{q_0, q_1 \in \mathbb{Z} : (q_0, q_1) \neq 0, |q_0| \leq Q_k} B(q_0, q_1).
\] (28)

Then the Lebesgue measure of \( B \), denoted by \( \mathcal{L}(B) \), is bounded by
\[
\mathcal{L}(B) \leq 8(\tau - 1)\beta \min \left\{ \frac{Q_1 Q_0}{A_1}, \frac{Q_0 Q_1}{A_0}, \frac{Q_0 \tau}{A_1}, \frac{Q_1 \tau}{A_0} \right\}.
\]

Proof. See Section VI. \( \square \)

V. ACHIEVABILITY

In this section, for the two-user symmetric Gaussian interference channel defined in Section II, we provide a cooperative jamming scheme, focusing on the regime of \( \alpha > 1/2 \). Note that when \( 0 \leq \alpha \leq 1/2 \), the optimal secure sum GDoF \( d_{\text{sum}} = 2(1 - \alpha) \) is achievable by a scheme without any helper and without cooperative jamming (cf. [31]). In the proposed scheme, pulse amplitude modulation and signal alignment will be used. In what follows, we describe the details of codebook generation and signal mapping, PAM constellation and signal alignment, and secure rate analysis.

1) Codebook generation and signal mapping: At transmitter \( k, k = 1, 2 \), it generates a codebook given as
\[
B_k \triangleq \left\{ v^n_k(w_k, w'_k) : w_k \in \{1, 2, \ldots, 2^n R_k\}, w'_k \in \{1, 2, \ldots, 2^n R'_k\} \right\}
\] (29)
where \( v^n_k \) are the codewords. All the elements of the codewords are independent and identically generated according to a distribution that will be defined later on. In (29), \( w'_k \) is the confusion message that is used to guarantee the security of the confidential message \( w_k \). \( R_k \) and \( R'_k \) are the rates of \( w_k \) and \( w'_k \), respectively, which will be defined later on (see (40) and (41)). To transmit the message \( w_k \), transmitter \( k \) selects a sub-codebook \( B_k(w_k) \) that is defined as
\[
B_k(w_k) \triangleq \left\{ v^n_k(w_k, w'_k) : w'_k \in \{1, 2, \ldots, 2^n R'_k\} \right\}, \quad k = 1, 2
\]
and then randomly selects a codeword \( v^n_k \) from \( B_k(w_k) \) according to a uniform distribution. In this scheme, the chosen codeword \( v^n_k \) will be mapped to the channel input such that
\[
x_k(t) = h_{\ell 3} h_{k \ell} v_k(t)
\] (30)
for \( \ell, k = 1, 2, \ell \neq k \), and \( t = 1, 2, \ldots, n \), where \( v_k(t) \) is the \( t \)-th element of the codeword \( v^n_k \).

2) PAM constellation and signal alignment: Specifically, each element of the codeword at transmitter \( k, k = 1, 2 \), is designed to take the following form
\[
v_k = \sqrt{P^{-\beta_{v_k,c}} \cdot v_{k,c}} + \sqrt{P^{-\beta_{v_k,p}} \cdot v_{k,p}}
\] (31)
which implies that the channel input in (30) can be rewritten as
\[
x_k = \sqrt{P^{-\beta_{v_k,c}} \cdot h_{\ell 3} h_{k \ell} v_{k,c}} + \sqrt{P^{-\beta_{v_k,p}} \cdot h_{\ell 3} h_{k \ell} v_{k,p}}
\] (32)
(removing the time index for simplicity) for \( \ell, k = 1, 2, \ell \neq k \). At the helper (transmitter 3), it sends a cooperative jamming signal designed as
\[
x_3 = \sqrt{P^{-\beta_{u_3}} \cdot h_{12} h_{21} u_3}.
\] (33)
In (32) and (33), $v_{k,c}$, $v_{k,p}$ and $u_3$ are independent random variables which are uniformly drawn from their PAM constellation sets

$$v_{k,c} \in \Omega(\xi = \gamma_{v_{k,c}} \cdot \frac{1}{Q}, Q = P^{\lambda_{v_{k,c}}})$$  \hspace{1cm} (34)

$$v_{k,p} \in \Omega(\xi = \gamma_{v_{k,p}} \cdot \frac{1}{Q}, Q = P^{\lambda_{v_{k,p}}})$$  \hspace{1cm} (35)

$$u_3 \in \Omega(\xi = \gamma_{u_3} \cdot \frac{1}{Q}, Q = P^{\lambda_{u_3}})$$  \hspace{1cm} (36)

respectively, for $k = 1, 2$, and $\gamma_{v_{k,c}}, \gamma_{v_{k,p}}$ and $\gamma_{u_3}$ are some finite constants designed as

$$\gamma_{v_{1,c}} = \gamma_{v_{2,c}} = \gamma_{u_3} = 2\gamma_{v_{1,p}} = 2\gamma_{v_{2,p}} = 2\gamma \in \left(0, \frac{1}{4\sqrt{2}}\right].$$  \hspace{1cm} (37)

Table I provides the designed parameters $\{\beta_{v_{k,c}}, \beta_{v_{k,p}}, \beta_{u_3}, \lambda_{v_{k,c}}, \lambda_{v_{k,p}}, \lambda_{u_3}\}_{k=1,2}$ for different cases of $\alpha$ in the proposed scheme. Based on our signal design (see (32)-(37)), it can be checked that the power constraint $E|x_k|^2 \leq 1$ is satisfied for $k = 1, 2, 3$. For example, since $v_{1,c}$ and $v_{1,p}$ are uniformly drawn from $\Omega(\xi = 2\gamma \cdot \frac{1}{Q}, Q = P^{\lambda_{v_{1,c}}})$ and $\Omega(\xi = \gamma \cdot \frac{1}{Q}, Q = P^{\lambda_{v_{1,p}}})$ respectively, we have

$$E|v_{1,c}|^2 = \frac{8\gamma^2 \cdot Q^2}{3Q + 1} \sum_{i=1}^{Q} i^2 \leq \frac{8\gamma^2}{3}$$  \hspace{1cm} for $Q = P^{\lambda_{v_{1,c}}}$

$$E|v_{1,p}|^2 = \frac{2\gamma^2 \cdot Q^2}{3Q + 1} \sum_{i=1}^{Q} i^2 \leq \frac{2\gamma^2}{3}$$  \hspace{1cm} for $Q = P^{\lambda_{v_{1,p}}}$

which implies that

$$E|x_1|^2 \leq 16 \times \frac{8\gamma^2}{3} + 16 \times \frac{2\gamma^2}{3} = \frac{160\gamma^2}{3} \leq 1$$

where $h_{\ell k} \in (1, 2], \forall \ell, k$ and $\gamma \in \left(0, \frac{1}{8\sqrt{2}}\right]$. Similarly, one can check that $E|x_2|^2 \leq 1$ and $E|x_3|^2 \leq 1$. Based on the above signal design, the received signals at the receivers 1 and 2 take the following forms (without the time index)

$$y_1 = \sqrt{P^{1-\beta_{v_{1,c}}}} h_{11} h_{23} h_{12} v_{1,c} + \sqrt{P^{1-\beta_{v_{1,p}}}} h_{11} h_{23} h_{12} v_{1,p}$$
$$+ h_{12} h_{21} h_{13}(\sqrt{P^{\alpha-\beta_{v_{2,c}}}} v_{2,c} + \sqrt{P^{\alpha-\beta_{u_3}}} u_3)$$
$$+ \sqrt{P^{\alpha-\beta_{v_{2,p}}}} h_{12} h_{21} h_{13} v_{2,p} + z_1$$  \hspace{1cm} (38)

$$y_2 = \sqrt{P^{1-\beta_{v_{2,c}}}} h_{22} h_{13} h_{21} v_{2,c} + \sqrt{P^{1-\beta_{v_{2,p}}}} h_{22} h_{13} h_{21} v_{2,p}$$
$$+ h_{21} h_{12} h_{23}(\sqrt{P^{\alpha-\beta_{v_{1,c}}}} v_{1,c} + \sqrt{P^{\alpha-\beta_{u_3}}} u_3)$$
$$+ \sqrt{P^{\alpha-\beta_{v_{1,p}}}} h_{21} h_{12} h_{23} v_{1,p} + z_2.$$  \hspace{1cm} (39)

Based on our signal design, at receiver 1 the signal $v_{2,c}$ is aligned with the jamming signal $u_3$, while at receiver 2 the signal $v_{1,c}$ is aligned with the jamming signal $u_3$. 

Then the received signals at the receivers 1 and 2 are given by

\[ y_1 = \sqrt{P} h_{11} h_{23} v_{1,c} + \sqrt{P^{1-\alpha}} h_{12} h_{13} v_{2,c} + u_3 + h_{12} h_{13} v_{2,p} + z_1 \]

\[ y_2 = \sqrt{P} h_{22} h_{13} v_{2,c} + \sqrt{P^{1-\alpha}} h_{21} h_{12} v_{1,c} + u_3 + h_{21} h_{12} v_{1,p} + z_2. \]

| \( \frac{1}{2} < \alpha \leq \frac{2}{3} \) | \( \frac{2}{3} < \alpha \leq 1 \) | \( 1 < \alpha \leq 2 \) | \( 2 \leq \alpha \) |
|------------------------|------------------------|------------------------|------------------------|
| \( \beta_{v_1,c}, \beta_{v_2,c} \) | 0 | 0 | 0 |
| \( \beta_{u_3} \) | 0 | 0 | 0 |
| \( \lambda_{v_1,c}, \lambda_{v_2,c} \) | 0 | 0 | 0 |
| \( \lambda_{v_1,p}, \lambda_{v_2,p} \) | 0 | 0 | 0 |

3) Secure rate analysis: We define the rates \( R_k \) and \( R'_k \) as

\[ R_k = \mathbb{I}(v_k; y_k) - \mathbb{I}(v_k; y_k | v_k) - \epsilon \]

\[ R'_k = \mathbb{I}(v_k; y_k | v_k) - \epsilon \]

for some \( \epsilon > 0, \ell, k = 1, 2, \ell \neq k \). Given the above codebook design and signal mapping, the result of [24, Theorem 2] (or [5, Theorem 2]) implies that the rate pair \( (R_1, R_2) \) defined in (40) and (41) is achievable and messages \( w_1 \) and \( w_2 \) are secure, i.e., \( \mathbb{I}(w_1; y_2^\ell) \leq n\epsilon \) and \( \mathbb{I}(w_2; y_1^\ell) \leq n\epsilon \).

In what follows we will focus on the regime of \( \alpha > 1/2 \) and analyze the secure rate performance of the cooperative jamming scheme. We will consider four cases: \( \frac{1}{2} < \alpha \leq \frac{2}{3}, \frac{2}{3} < \alpha \leq 1, 1 < \alpha \leq 2 \) and \( 2 < \alpha \). We will first consider the cases of \( \frac{1}{2} < \alpha \leq \frac{2}{3} \) and \( 2 < \alpha \), in which a successive decoding method will be used in the rate analysis. Later on we will consider the rest two cases, in which the estimation approaches of noise removal and signal separation due to rational independence will be used in the rate analysis.

A. Rate analysis when \( 1/2 < \alpha \leq 2/3 \)

For the case with \( 1/2 < \alpha \leq 2/3 \), the parameters are designed as

\[ \beta_{v_1,c} = \beta_{v_2,c} = 0 \]

\[ \lambda_{v_1,c} = \lambda_{v_2,c} = 2\alpha - 1 - \epsilon \]

\[ \beta_{v_1,p} = \beta_{v_2,p} = \alpha \]

\[ \lambda_{v_1,p} = \lambda_{v_2,p} = 1 - \alpha - \epsilon \]

\[ \beta_{u_3} = 0 \]

\[ \lambda_{u_3} = 2\alpha - 1 - \epsilon \]

where \( \epsilon > 0 \) can be set arbitrarily small. In this case, the transmitted signal at transmitter \( k, k = 1, 2, 3 \), is

\[ x_1 = h_{23} h_{12} v_{1,c} + \sqrt{P^{-\alpha}} h_{23} h_{12} v_{1,p} \]

\[ x_2 = h_{13} h_{21} v_{2,c} + \sqrt{P^{-\alpha}} h_{13} h_{21} v_{2,p} \]

\[ x_3 = h_{12} h_{21} u_3. \]

Then the received signals at the receivers 1 and 2 are given by

\[ y_1 = \sqrt{P} h_{11} h_{23} h_{12} v_{1,c} + \sqrt{P^{1-\alpha}} h_{11} h_{23} h_{12} v_{1,p} \]

\[ + \sqrt{P^{-\alpha}} h_{12} h_{13} (v_{2,c} + u_3) + h_{12} h_{13} v_{2,p} + z_1 \]

\[ y_2 = \sqrt{P} h_{22} h_{13} h_{21} v_{2,c} + \sqrt{P^{1-\alpha}} h_{22} h_{13} h_{21} v_{2,p} \]

\[ + \sqrt{P^{-\alpha}} h_{21} h_{12} h_{23} (v_{1,c} + u_3) + h_{21} h_{12} h_{23} v_{1,p} + z_2. \]
Given that \( v \) be lower bounded by a decoding method, given the design in (34)-(36) and (42)-(47). From the steps in (18)-(20), \( \text{Pr} \) (see (31), (42) and (43)). From \( \alpha \), Fig. 4 depicts the rate (GDoF) and power of some signals at receiver 1 when \( \alpha = 2/3 \).

Fig. 4 depicts the rate (GDoF) and power of some signals at receiver 1 when \( \alpha = 2/3 \).

For the proposed cooperative jamming scheme, the secure rate pair \( (R_1, R_2) \) defined in (40) and (41) can be achieved. For \( \epsilon \to 0 \), this secure rate pair can be written as

\[
R_1 = \mathbb{I}(v_1; y_1) - \mathbb{I}(v_1; y_2 | v_2) \tag{50}
\]

\[
R_2 = \mathbb{I}(v_2; y_2) - \mathbb{I}(v_2; y_1 | v_1). \tag{51}
\]

First, we focus on the lower bound of \( R_1 \) expressed in (50) and seek to get a tight lower bound on \( \mathbb{I}(v_1; y_1) \). For this case, \( v_1 \) is designed as

\[
v_1 = v_{1,c} + \sqrt{P^{-\alpha}} \cdot v_{1,p}
\]

(see (31), (42) and (43)). From \( y_1 \) expressed in (48), we can estimate \( \{v_{1,c}, v_{1,p}\} \) by using a successive decoding method, given the design in (34)-(36) and (42)-(47). From the steps in (18)-(20), \( \mathbb{I}(v_1; y_1) \) can be lower bounded by

\[
\mathbb{I}(v_1; y_1) \geq (1 - \text{Pr}\{v_{1,c} \neq \hat{v}_{1,c} \cup \{v_{1,p} \neq \hat{v}_{1,p}\}\}) \cdot \mathbb{H}(v_1) - 1. \tag{52}
\]

Given that \( v_{1,c} \in \Omega(\xi = \gamma_{v_{1,c}} \cdot \frac{1}{Q}, \ Q = P^{2\alpha - 1} - 1) \) and \( v_{1,p} \in \Omega(\xi = \gamma_{v_{1,p}} \cdot \frac{1}{Q}, \ Q = P^{1 - \frac{1}{2} - \alpha}) \), the rates of \( v_{1,c} \) and \( v_{1,p} \) are computed as follows:

\[
\mathbb{H}(v_{1,c}) = \log(2 \cdot P^{2\alpha - 1} - 1)
\]

\[
\mathbb{H}(v_{1,p}) = \log(2 \cdot P^{1 - \frac{1}{2} - \alpha} + 1) \tag{54}
\]

In this case, \( \{v_{1,p}, v_{1,c}\} \) can be reconstructed from \( v_1 \), and vice versa. This fact, together with (53) and (54), gives

\[
\mathbb{H}(v_1) = \mathbb{H}(v_{1,c}) + \mathbb{H}(v_{1,p})
\]

\[
= \frac{\alpha - 2\epsilon}{2} \log P + o(\log P). \tag{55}
\]

To further derive the lower bound on \( \mathbb{I}(v_1; y_1) \) from (52), we provide an upper bound on the error probability \( \text{Pr}\{v_{1,c} \neq \hat{v}_{1,c} \cup \{v_{1,p} \neq \hat{v}_{1,p}\}\} \), described in the following lemma.

**Lemma 2.** When \( 1/2 < \alpha \leq 2/3 \), given the signal design in (34)-(36) and (42)-(47), the error probability of estimating \( \{v_{k,c}, v_{k,p}\} \) from \( y_k \), \( k = 1, 2 \), is

\[
\text{Pr}\{v_{k,c} \neq \hat{v}_{k,c} \cup \{v_{k,p} \neq \hat{v}_{k,p}\}\} \to 0 \quad \text{as} \quad P \to \infty. \tag{56}
\]

**Proof.** See Section VII. \( \square \)

By combining (52), (55) and Lemma 2, \( \mathbb{I}(v_1; y_1) \) can be lower bounded as

\[
\mathbb{I}(v_1; y_1) \geq \frac{\alpha - 2\epsilon}{2} \log P + o(\log P). \tag{57}
\]
Let us now focus on the term $\mathbb{I}(v_1; y_2 | v_2)$ in (50), which can be upper bounded by
\[
\mathbb{I}(v_1; y_2 | v_2) \\ \leq \mathbb{I}(v_1; y_2, v_{1,c} + u_3 | v_2) \\ = \mathbb{H}(v_1) - \mathbb{H}(v_1 | v_{1,c} + u_3) \\ \leq \log(4 \cdot P^{2\alpha-1} + 1) - \log(2^{2\alpha-1} + 1) \\ \leq 1 + \frac{1}{2} \log(2\pi e \times 65) - \frac{1}{2} \log(2\pi e) \\ = \log(2\sqrt{65})
\]
where (59) follows from the fact that $v_1, v_2, u_3$ are mutually independent; (60) stems from that $\{v_k, p, v_{k,c}\}$ can be reconstructed from $v_k$ for $k = 1, 2$; (61) holds true because $\mathbb{H}(u_3) = \log(2^{2\alpha-1} + 1)$ and $\mathbb{H}(v_{1,c} + u_3) \leq \log(4 \cdot P^{2\alpha-1} + 1)$; (62) follows from the derivation that $\log(h_{21} h_{12} h_{23} v_{1,p} + z_2) \leq \frac{1}{2} \log(2\pi e (\log(1^2 + h_{12}^2 + h_{23}^2 + 2 v_{1,p}^2 + 2 z_2^2))) \leq \frac{1}{2} \log(2\pi e \times 65)$. Finally, by incorporating (57) and (63) into (50), we can bound the secure rate $R_1$ as
\[
R_1 = \mathbb{I}(v_1; y_1) - \mathbb{I}(v_1; y_2 | v_2) \geq \frac{\alpha - 2\epsilon}{2} \log P + o(\log P).
\]
Let $\epsilon \to 0$, then the secure GDoF $d_1 = \alpha$ is achievable. Due to the symmetry, $d_2 = \alpha$ is also achievable by the proposed cooperative jamming scheme when $1/2 < \alpha \leq 2/3$.

B. Rate analysis when $\alpha \geq 2$

For the case with $\alpha \geq 2$, the parameters are designed as
\[
\beta_{v_1,c} = \beta_{v_2,c} = 0, \quad \lambda_{v_1,c} = \lambda_{v_2,c} = 1 - \epsilon \\
\beta_{v_1,p} = \beta_{v_2,p} = \infty, \quad \lambda_{v_1,p} = \lambda_{v_2,p} = 0 \\
\beta_{u_3} = 0, \quad \lambda_{u_3} = 1 - \epsilon.
\]
In this case, the transmitted signals are designed as
\[
x_1 = h_{23} h_{12} v_{1,c}, \quad x_2 = h_{13} h_{21} v_{2,c}, \quad x_3 = h_{12} h_{21} u_3.
\]
Then the received signals at the receivers 1 and 2 become
\[
y_1 = \sqrt{P} h_{11} h_{23} h_{12} v_{1,c} + \sqrt{P} \alpha h_{12} h_{23} v_{2,c} + u_3 + z_1 \\
y_2 = \sqrt{P} h_{22} h_{13} h_{21} v_{2,c} + \sqrt{P} \alpha h_{21} h_{13} h_{12} h_{23} (v_{1,c} + u_3) + z_2.
\]
Figure 5 depicts the rate and power of some signals at receiver 1 when $\alpha = 2$. 
From (40) and (41), the secure rates 
\[ R_1 = I(v_1; y_1) - I(v_1; y_2|v_2) \]
and 
\[ R_2 = I(v_2; y_2) - I(v_2; y_1|v_1) \]
can be achieved by the proposed scheme. For this case, \( v_k \) is designed as
\[ v_k = v_{k,c}, \quad k = 1, 2. \]

From \( y_k \) expressed in (68) and (69), we can estimate \( v_{k,c} \) by using a successive decoding method, for 
\( k = 1, 2. \) Lemma 3 provides a result on the error probability for this estimation.

**Lemma 3.** When \( \alpha \geq 2 \), given the signal design in (34)-(36) and (64)-(67), the error probability of estimating \( v_{k,c} \) from \( y_k \) is
\[ Pr[v_{k,c} \neq \hat{v}_{k,c}] \rightarrow 0 \quad \text{as} \quad P \rightarrow \infty. \] (70)

**Proof.** See Appendix A

Considering the lower bound on the secure rate \( R_1 \), \( I(v_1; y_1) \) can be lower bounded by
\[ I(v_1; y_1) \geq \left(1 - Pr[v_{1,c} \neq \hat{v}_{1,c}] \right) \cdot H(v_1) - 1 \] (71)
by following the steps in (18)-(20). In this case with \( v_1 = v_{1,c} \) and \( v_{1,c} \in \Omega(\xi = \gamma_{v_{1,c}} \cdot \frac{1}{Q}, \quad Q = P^{\frac{1}{2\epsilon}}) \),
the rate of \( v_1 \) is given by
\[ H(v_1) = H(v_{1,c}) = \log(2 \cdot P^{\frac{1}{2\epsilon}} + 1). \] (72)

By combining (72) and Lemma 3, the lower bound of \( I(v_1; y_1) \) is given as
\[ I(v_1; y_1) \geq \frac{1 - \epsilon}{2} \log P + o(\log P). \] (73)

By following the steps (58)-(63), \( I(v_1; y_2|v_2) \) can be bounded by
\[ I(v_1; y_2|v_2) \leq I(v_1; y_2, v_{1,c} + u_3|v_2) = I(v_1; v_{1,c} + u_3|v_2) + I(v_1; y_2, v_{1,c} + u_3|v_2) = H(v_1, z_2|v_2, v_{1,c} + u_3) = H(v_{1,c} + u_3) - H(u_3) \leq \log(4 \cdot P^{\frac{1}{2\epsilon}} + 1) - \log(2 \cdot P^{\frac{1}{2\epsilon}} + 1) \leq 1. \] (74)

Finally, with the results in (73) and (74), we can bound the secure rate \( R_1 \) as
\[ R_1 = I(v_1; y_1) - I(v_1; y_2|v_2) \geq \frac{1 - \epsilon}{2} \log P + o(\log P). \]

Let \( \epsilon \rightarrow 0 \), then the secure GDoF \( d_1 = 1 \) is achievable. Due to the symmetry, \( d_2 = 1 \) is also achievable by the proposed cooperative jamming scheme when \( \alpha \geq 2 \).
C. Rate analysis when $2/3 \leq \alpha \leq 1$

The rate analysis for this case is different from that for the previous two cases. In the previous two cases, a successive decoding method is used in the rate analysis. In this case, we will use the estimation approaches of noise removal and signal separation due to rational independence that will be discussed later on.

For this case with $2/3 \leq \alpha \leq 1$, the parameters are designed as
\begin{align}
\beta_{v_{1,c}} &= \beta_{v_{2,c}} = 0, \quad \lambda_{v_{1,c}} = \lambda_{v_{2,c}} = \alpha/2 - \epsilon, \\
\beta_{v_{1,p}} &= \beta_{v_{2,p}} = \alpha, \quad \lambda_{v_{1,p}} = \lambda_{v_{2,p}} = 1 - \alpha - \epsilon, \\
\beta_{u_3} &= 0, \quad \lambda_{u_3} = \alpha/2 - \epsilon,
\end{align}

where $\epsilon > 0$ can be set arbitrarily small. In this case, the transmitted signals are designed as
\begin{align}
x_1 &= h_{23}h_{12}v_{1,c} + \sqrt{P - \alpha} \cdot h_{23}v_{1,p} \\
x_2 &= h_{13}h_{21}v_{2,c} + \sqrt{P - \alpha} \cdot h_{13}v_{2,p} \\
x_3 &= h_{12}h_{21}u_3.
\end{align}

Then the received signals at the receivers 1 and 2 take the same forms as in (48) and (49). Fig. 3 describes the rate and power of some signals at receivers 1 and 2 when $\alpha = 3/4$.

In this proposed scheme, the secure rates $R_1 = \mathbb{I}(v_1; y_1) - \mathbb{I}(v_1; y_2|v_2)$ and $R_2 = \mathbb{I}(v_2; y_2) - \mathbb{I}(v_2; y_1|v_1)$ are achievable (see (40) and (41)). Let us bound the secure rate $R_1$ first. By following the steps in (18)-(20), $\mathbb{I}(v_1; y_1)$ can be lower bounded by
\begin{equation}
\mathbb{I}(v_1; y_1) \geq (1 - \text{Pr}\{v_{1,c} \neq \hat{v}_{1,c} \cup \{v_{1,p} \neq \hat{v}_{1,p}\}\}) \cdot \mathbb{H}(v_1) - 1. \quad (81)
\end{equation}

For this case, the rates of $v_{1,c}$, $v_{1,p}$ and $v_1 = v_{1,c} + \sqrt{P - \alpha} \cdot v_{1,p}$ are computed as
\begin{align}
\mathbb{H}(v_{1,c}) &= \log(2 \cdot P^{\alpha/2 - \epsilon} + 1) \quad (82) \\
\mathbb{H}(v_{1,p}) &= \log(2 \cdot P^{1-\alpha/2} + 1) \quad (83) \\
\mathbb{H}(v_1) &= \frac{1 - \alpha/2 - 2\epsilon}{2} \log P + o(\log P). \quad (84)
\end{align}

To further derive the lower bound on $\mathbb{I}(v_1; y_1)$ from (81), we provide an upper bound on the error probability $\text{Pr}\{v_{1,c} \neq \hat{v}_{1,c} \cup \{v_{1,p} \neq \hat{v}_{1,p}\}\}$, described in the following lemma.

**Lemma 4.** When $2/3 \leq \alpha \leq 1$, given the signal design in (34)-(36) and (75)-(80), then for almost all the channel coefficients $\{h_{k\ell}\} \in (1,2]^ {2 \times 3}$, the error probability of estimating $\{v_{k,c}, v_{k,p}\}$ from $y_k$, $k = 1, 2$, is
\begin{equation}
\text{Pr}\{v_{k,c} \neq \hat{v}_{k,c} \cup \{v_{k,p} \neq \hat{v}_{k,p}\}\} \rightarrow 0 \quad \text{as} \quad P \rightarrow \infty. \quad (85)
\end{equation}

**Proof.** In this proof we use the approaches of noise removal and signal separation. The full details are described in Section [VIII].

By combining (81), (84) and Lemma 4, $\mathbb{I}(v_1; y_1)$ can be lower bounded by
\begin{equation}
\mathbb{I}(v_1; y_1) \geq \frac{1 - \alpha/2 - 2\epsilon}{2} \log P + o(\log P) \quad (86)
\end{equation}
for almost all channel coefficients $\{h_{k\ell}\} \in (1,2]^ {2 \times 3}$. By following the steps in (58)-(60), $\mathbb{I}(v_1; y_2|v_2)$ can be bounded as
\begin{equation}
\mathbb{I}(v_1; y_2|v_2) \leq \log(2\sqrt{55}). \quad (87)
\end{equation}

Finally, with the results in (86) and (87), the secure rate $R_1$ is lower bounded by
\begin{equation}
R_1 = \mathbb{I}(v_1; y_1) - \mathbb{I}(v_1; y_2|v_2) \geq \frac{1 - \alpha/2 - 2\epsilon}{2} \log P + o(\log P)
\end{equation}
which implies the following secure GDoF $d_1 = 1 - \alpha/2$, as well as $d_2 = 1 - \alpha/2$ due to the symmetry, for almost all the channel coefficients $\{h_{k\ell}\} \in (1,2]^ {2 \times 3}$, in this case with $2/3 \leq \alpha \leq 1$. 
The rate analysis for this case also uses the approaches of noise removal and signal separation. For this case, the parameters are designed as

\[ \beta_{v_1,c} = \beta_{v_2,c} = 0, \quad \lambda_{v_1,c} = \lambda_{v_2,c} = \alpha/2 - \epsilon \]  
(88)  
\[ \beta_{v_1,p} = \beta_{v_2,p} = \infty, \quad \lambda_{v_1,p} = \lambda_{v_2,p} = 0 \]  
(89)  
\[ \beta_{u_3} = 0, \quad \lambda_{u_3} = \alpha/2 - \epsilon. \]  
(90)

In this case, the transmitted signals are designed as

\[ x_1 = h_{23}h_{12}v_{1,c}, \quad x_2 = h_{13}h_{21}v_{2,c}, \quad x_3 = h_{12}h_{21}u_3. \]  
(91)

Then the received signals at the receivers take the forms as in (68) and (69).

In the following we will bound the secure rates expressed in (40) and (41). For this case, \( v_k \) is designed as \( v_k = v_{k,c}, k = 1, 2 \). We can estimate \( v_{k,c} \) from \( y_k \) by using the approaches of noise removal and signal separation, \( k = 1, 2 \). Lemma 5 presents a result on the error probability for this estimation.

**Lemma 5.** When \( 1 \leq \alpha \leq 2 \), given the signal design in (34)-(36) and (88)-(91), then for almost all channel coefficients \( \{h_{k\ell}\} \in (1, 2]^{2 \times 3} \), the error probability of estimating \( v_{k,c} \) from \( y_k, k = 1, 2 \), is

\[ \Pr[v_{k,c} \neq \hat{v}_{k,c}] \rightarrow 0 \quad \text{as} \quad P \rightarrow \infty. \]  
(92)

**Proof.** In this proof we use the approaches of noise removal and signal separation. The full details are described in Appendix B. \( \square \)

In the following we will show that by using Lemma 5 the secure GDoF pair \( (d_1 = \alpha/2, d_2 = \alpha/2) \) can be achieved for almost all the channel coefficients \( \{h_{k\ell}\} \in (1, 2]^{2 \times 3} \) by the proposed cooperative jamming scheme. In this case with \( v_{1,c} \in \Omega(\xi = 2\gamma \cdot 1/Q, Q = P^{\alpha/2-\epsilon}) \) and \( v_1 = v_{1,c} \), the rate of \( v_1 \) is computed as

\[ \mathbb{H}(v_1) = \mathbb{H}(v_{1,c}) = \log(2 \cdot P^{\alpha/2-\epsilon} + 1). \]  
(93)

and then, \( \mathbb{I}(v_1; y_1) \) can be bounded by

\[ \mathbb{I}(v_1; y_1) \geq (1 - \Pr[v_{1,c} \neq \hat{v}_{1,c}]) \cdot \mathbb{H}(v_1) - 1 \]  
(94)
\[ = \alpha/2 - \epsilon \log P + o(\log P) \]  
(95)

for almost all the channel coefficients \( \{h_{k\ell}\} \in (1, 2]^{2 \times 3} \), where (94) follows from the steps in (18)-(20); and (95) results from (93) and Lemma 5. By following the steps related to (74), \( \mathbb{I}(v_1; y_2|v_2) \) can be bounded as

\[ \mathbb{I}(v_1; y_2|v_2) \leq 1. \]  
(96)
The final step is to incorporate (95) and (96) into (40). It then gives the lower bound on $R_1$

$$R_1 = \mathbb{I}(v_1; y_1) - \mathbb{I}(v_1; y_2|v_2)$$

$$\geq \frac{\alpha/2 - \epsilon}{2} \log P + o(\log P)$$

(97)

which implies the following secure GDoF $d_1 = \alpha/2$, as well as $d_2 = \alpha/2$ due to the symmetry, for almost all the channel coefficients $\{h_{k\ell}\} \in (1, 2)^{2 \times 3}$, in this case with $1 \leq \alpha \leq 2$.

VI. PROOF OF LEMMA I

In this section, we will prove Lemma I. Let $\beta \in (0, 1]$, $\tau \in \mathbb{Z}^+$ and $\tau > 1$, $A_0, A_1, Q_0$ and $Q_1 \in \mathbb{Z}^+$. Define the event

$$B(q_0, q_1) \triangleq \{(g_0, g_1) \in (1, \tau]^2 : |A_0|q_0 + A_1|g_1| < \beta\}$$

and set

$$B \triangleq \bigcup_{q_0, q_1 \in \mathbb{Z} : (q_0, q_1) \neq 0, |q_k| \leq Q_k, \forall k} B(q_0, q_1).$$

For $(q_0, q_1) \in \{(q_0, q_1') : (q_0, q_1') \neq 0, q_0' q_1' \in \mathbb{Z}, |q_0| \leq Q_0, |q_1| \leq Q_1\}$, we will consider the following three cases: $(q_0 \neq 0, q_1 \neq 0)$, $(q_0 \neq 0, q_1 = 0)$, and $(q_0 = 0, q_1 \neq 0)$.

Let us consider the case with $(q_0 \neq 0, q_1 \neq 0)$ first. In this case, assuming that $A_0|q_0| \geq \tau A_1|q_1| + 1$, then it gives

$$|A_0|q_0 + A_1|g_1| \geq A_0|q_0| A_1|g_1|$$

$$A_0|q_0| - \tau A_1|q_1|$$

$$\geq 1$$

$$\geq \beta$$

(98)

which contradicts the event $|A_0|q_0 + A_1|g_1| < \beta$ defined in $B(q_0, q_1)$. Therefore, without loss of generality we will consider

$$A_0|q_0| \leq \tau A_1|q_1|$$

(99)

for the case with $(q_0 \neq 0, q_1 \neq 0)$.

For the first case with $(q_0 \neq 0, q_1 \neq 0)$, as shown in Fig. 7, the set $B(q_0, q_1)$ has one strip with slope $-A_0/(A_1 q_1)$ and width $2\beta/(A_1|q_1|)$. The area of this set is upper bounded by

$$\mathcal{L}(B(q_0, q_1)) \leq (\tau - 1) \cdot \frac{2\beta}{A_1|q_1|} = \frac{2(\tau - 1)\beta}{A_1|q_1|}.$$  

(100)
For the second case with \((q_0 \neq 0, q_1 = 0)\), it holds true that \(|A_0q_0 + A_1q_1| = |A_0q_0| \geq 1 \geq \beta\) for any \((q_0, q_1) \in (1, \tau)^2\), which implies that

\[
\mathcal{L}(B(q_0, 0)) = 0 \tag{101}
\]

based on the definition of \(B(q_0, q_1)\). Similarly, for the last case with \((q_0 = 0, q_1 \neq 0)\), it holds true that

\[
\mathcal{L}(B(0, q_1)) = 0. \tag{102}
\]

By combining \((99)-(102)\), \(\mathcal{L}(B)\) can be upper bounded by

\[
\mathcal{L}(B) = \mathcal{L}\left( \bigcup_{q_0, q_1 \in \mathbb{Z}: \left(\begin{array}{c} q_0, q_1 \in \mathbb{Z}; \\
(q_0, q_1) \neq (0), \\
|q_k| \leq Q_k \forall k
\end{array}\right)} B(q_0, q_1) \right)
\]

\[
\leq \sum_{q_0, q_1 \in \mathbb{Z}: \left(\begin{array}{c} q_0, q_1 \in \mathbb{Z}; \\
(q_0, q_1) \neq (0), \\
|q_k| \leq Q_k \forall k
\end{array}\right)} \mathcal{L}(B(q_0, q_1))
\]

\[
= \sum_{q_0 \in \mathbb{Z} \setminus \{0\}: |q_0| \leq Q_0} \mathcal{L}(B(0, q_1)) + \sum_{q_0 \in \mathbb{Z} \setminus \{0\}: |q_0| \leq Q_0} \mathcal{L}(B(q_0, 0))
\]

\[
+ \sum_{q_1 \in \mathbb{Z} \setminus \{0\}: |q_1| \leq Q_1} \sum_{q_0 \in \mathbb{Z} \setminus \{0\}: |q_0| \leq Q_0} \mathcal{L}(B(q_0, q_1))
\]

\[
= \sum_{q_1 \in \mathbb{Z} \setminus \{0\}: |q_1| \leq Q_1} \sum_{q_0 \in \mathbb{Z} \setminus \{0\}: |q_0| \leq Q_0} \mathcal{L}(B(q_0, q_1)) \tag{103}
\]

where \((103)\) results from \((101)\) and \((102)\). Note that

\[
|\{q_0 \in \mathbb{Z} \setminus \{0\}: |q_0| \leq Q_0, A_0|q_0| \leq \tau A_1|q_1|\}|
\]

\[
\leq 2 \min\left\{ Q_0, \frac{\tau A_1|q_1|}{A_0} \right\}. \tag{104}
\]

With this, we can bound the term in \((103)\) as

\[
\sum_{q_1 \in \mathbb{Z} \setminus \{0\}: |q_1| \leq Q_1} \sum_{q_0 \in \mathbb{Z} \setminus \{0\}: |q_0| \leq Q_0} \mathcal{L}(B(q_0, q_1))
\]

\[
\leq 2Q_1 \cdot 2 \min\left\{ Q_0, \frac{\tau A_1|q_1|}{A_0} \right\} \cdot \frac{2(\tau - 1)\beta}{A_1|q_1|} \tag{105}
\]

\[
= 8(\tau - 1)\beta \min\left\{ \frac{Q_1 Q_0}{A_1|q_1|}, \frac{Q_1 \tau}{A_0} \right\}
\]

\[
\leq 8(\tau - 1)\beta \min\left\{ \frac{Q_1 Q_0}{A_1}, \frac{Q_1 \tau}{A_0} \right\} \tag{106}
\]

where \((105)\) follows from \((100)\) and \((104)\). Therefore, \((103)\) can be further upper bounded by

\[
\mathcal{L}(B) \leq 8(\tau - 1)\beta \min\left\{ \frac{Q_1 Q_0}{A_1}, \frac{Q_1 \tau}{A_0} \right\}. \tag{107}
\]

Due to symmetry, by interchanging the roles of \(A_0\) and \(A_1\), and interchanging the roles of \(Q_0\) and \(Q_1\), \(\mathcal{L}(B)\) can also be upper bounded by

\[
\mathcal{L}(B) \leq 8(\tau - 1)\beta \min\left\{ \frac{Q_0 Q_1}{A_1}, \frac{Q_0 \tau}{A_0} \right\}. \tag{108}
\]
By combining the results in (107) and (108), we finally bound $\mathcal{L}(B)$ as

$$\mathcal{L}(B) \leq 8(\tau - 1)\beta \min\left\{ \frac{Q_1Q_0}{A_1}, \frac{Q_0Q_1}{A_0}, \frac{Q_0\tau}{A_1}, \frac{Q_1\tau}{A_0} \right\}. \quad (109)$$

VII. PROOF OF LEMMA 2

This section provides the proof of Lemma 2. In this proof we will use [31, Lemma 1] described below.

**Lemma 6.** [31, Lemma 1] Consider the channel model $y = \sqrt{P\alpha_1}h_1x + \sqrt{P\alpha_2}g + z$, where $x \in \Omega(\xi, Q)$ is the random variable, $z \sim N(0, \sigma^2)$, and $g \in S_g$ is a discrete random variable such that

$$|g| \leq g_{\max}, \quad \forall g \in S_g$$

for a given set $S_g \subset \mathcal{R}$. In the above model $g_{\max}$, $\sigma$ and $h$ are positive and finite constants independent of $P$, and $\alpha_1$ and $\alpha_2$ are two positive parameters such that $\alpha_1 - \alpha_2 > 0$. By setting $Q$ and $\xi$ such that

$$Q = \frac{P\gamma}{2g_{\max}}, \quad \xi = \gamma \cdot \frac{1}{Q}, \quad \forall \alpha \in (0, \alpha_1 - \alpha_2) \quad (110)$$

where $\gamma > 0$ is a finite constant independent of $P$, then the probability of error for decoding a symbol $x$ from $y$ is

$$\Pr(e) \to 0 \quad as \quad P \to \infty. \quad (111)$$

We will provide the proof of Lemma 2 by focusing on the case with $k = 1$, as the case with $k = 2$ can be proved in a similar way. In this proof, we estimate $v_{1,c}$, $v_{2,c} + u_3$ and $v_{1,p}$ from $y_1$ expressed in (48) by using a successive decoding method. At first, $v_{1,c} \in \Omega(\xi = \gamma v_{1,c} \cdot \frac{1}{Q}, \ Q = P^{\frac{2\alpha_1-1}{2}})$ will be estimated by treating the other signals as noise. Let us rewrite $y_1$ in (48) as

$$y_1 = \sqrt{P}h_{11}h_{23}h_{12}v_{1,c} + \sqrt{P\alpha_2}g + z_1 \quad (112)$$

where

$$g \triangleq h_{12}h_{21}h_{13}(v_{2,c} + u_3) + \sqrt{P^{1-2\alpha}h_{11}h_{23}h_{12}v_{1,p}} + \sqrt{P^{-\alpha}h_{12}h_{21}h_{13}v_{2,p}}. \quad (113)$$

For this case with $1/2 < \alpha < 2/3$, it is true that $|g| \leq 4\sqrt{2}$ for any realizations of $g$. At this point, by using the result of Lemma 6 it implies that the probability of error for estimating $v_{1,c}$ from $y_1$ is

$$\Pr[v_{1,c} \neq \hat{v}_{1,c}] \to 0, \quad as \quad P \to \infty. \quad (114)$$

The decoded $v_{1,c}$ can be removed from $y_1$, which can allow us to estimate $v_{2,c} + u_3$ from the following observation

$$y_1 - \sqrt{P}h_{11}h_{23}h_{12}v_{1,c} = \sqrt{P^\alpha}h_{12}h_{21}h_{13}(v_{2,c} + u_3) + \sqrt{P^{1-\alpha}}g' + z_1 \quad (115)$$

where $g' \triangleq h_{11}h_{23}h_{12}v_{1,p} + \sqrt{P^{\alpha-1}h_{11}h_{23}h_{12}v_{2,p}}$ and $v_{2,c} + u_3 \in 2 \cdot \Omega(\xi = \gamma v_{2,c} \cdot \frac{1}{Q}, \ Q = P^{\frac{2\alpha_1-1}{2}})$ for $2 \cdot \Omega(\xi, Q) \triangleq \{ \xi \cdot a : \ a \in \mathcal{Z} \cap [-2Q, 2Q] \}$. It is true that $|g'| \leq 2\sqrt{2}$ for any realizations of $g'$. Define $\hat{s}_{vu}$ as the estimate of $s_{vu} \triangleq v_{2,c} + u_3$. From the result of Lemma 6, it reveals that

$$\Pr[s_{vu} \neq \hat{s}_{vu}|v_{1,c} = \hat{v}_{1,c}] \to 0, \quad as \quad P \to \infty. \quad (116)$$

which, together with (112), gives

$$\Pr[s_{vu} \neq \hat{s}_{vu}] \leq \Pr[s_{vu} \neq \hat{s}_{vu}|v_{1,c} = \hat{v}_{1,c}] + \Pr[v_{1,c} \neq \hat{v}_{1,c}] \to 0 \quad as \quad P \to \infty. \quad (117)$$

Similarly, by removing the decoded $v_{2,c} + u_3$ from $y_1$, $v_{1,p} \in \Omega(\xi = \gamma v_{1,p} \cdot \frac{1}{Q}, \ Q = P^{\frac{1-\alpha_2}{2}})$ can be estimated with vanishing error probability, i.e.,

$$\Pr[v_{1,p} \neq \hat{v}_{1,p}] \to 0 \quad as \quad P \to \infty. \quad (118)$$
By combining (112) and (116), it gives that the probability of error for estimating \( \{v_{1,c}, v_{1,p}\} \) from \( y_1 \) is

\[
Pr[\{v_{1,c} \neq \hat{v}_{1,c}\} \cup \{v_{1,p} \neq \hat{v}_{1,p}\}] \to 0 \quad \text{as} \quad P \to \infty
\]

which completes the proof for the case of \( k = 1 \). Due to the symmetry, the case of \( k = 2 \) can be proved in a similar way.

VIII. PROOF OF LEMMA 4

The proof of Lemma 4 is provided in this section. In this proof, \( v_{1,c}, v_{2,c} + u_3 \) and \( v_{1,p} \) will be estimated from \( y_1 \) (see (48)) by using the approaches of noise removal and signal separation. At first, the following two symbols

\[
v_{1,c} \in \Omega(\xi = \gamma v_{1,c}, Q = P^{\alpha/2+\epsilon})
\]

\[
v_{2,c} + u_3 \in 2 \cdot \Omega(\xi = \gamma v_{2,c}, Q = P^{\alpha/2-\epsilon})
\]

will be estimated simultaneously from \( y_1 \) by treating the other signals as noise. Let us rewrite \( y_1 \) in (48) as

\[
y_1 = \sqrt{P}h_{11}h_{23}h_{12}v_{1,c} + \sqrt{P}h_{12}h_{13}(v_{2,c} + u_3) + \tilde{z}_1
\]

\[
= P^{\alpha/2+\epsilon} \cdot 2\gamma \cdot (\sqrt{P}g_0 + g_1) + \tilde{z}_1
\]

where \( \tilde{z}_1 = \sqrt{P}h_{11}h_{23}h_{12}v_{1,p} + h_{12}h_{13}v_{2,p} + z_1 \) and

\[
g_0 = h_{11}h_{23}h_{12}, \quad g_1 = h_{12}h_{13}v_{1,c}, \quad Q_0 = \frac{Q_{\max}}{2\gamma}, \quad Q_{\max} = \frac{P^{\alpha/2-\epsilon}}{2}
\]

for a given constant \( \gamma \in \left(0, \frac{1}{8\sqrt{2}}\right) \) (see (37)). In this setting, \( g_0, g_1 \in \mathbb{Z}, |g_0| \leq Q_{\max}, |g_1| \leq 2Q_{\max}, \) and \( \sqrt{P}g_0 \in \mathbb{Z}^+ \), based on our definitions \( P \triangleq \max_k \{2^{m_{kk}}\} \) and \( \sqrt{P}g_{kk} \triangleq 2^{m_{kk}}, k, \ell = 1, 2, \).

Let \( \hat{g}_0 \) and \( \hat{g}_1 \) be the estimates of \( g_0 \) and \( g_1 \), respectively, from the observation \( y_1 \) expressed in (117). Specifically, we use an estimator that minimizes

\[
|y_1 - \sqrt{P}g_0 + g_1|\]

We now consider the minimum distance

\[
d_{\min}(g_0, g_1) \triangleq \min_{g_0, g_1 \in \mathbb{Z}^+} \left|P^{\alpha/2+\epsilon}g_0 + g_1\right|
\]

between the signals generated by \( (g_0, g_1) \) and \( (\hat{g}_0, \hat{g}_1) \). Later on, Lemma 7 (see below) shows that the minimum distance \( d_{\min} \) is sufficiently large for almost all the channel coefficients \( \{h_{kk}\} \in \{1, 2\}^{2 \times 3} \) when \( P \) is large, with the signal design in (34)-(36) and (75)-(80). Let us now provide Lemma 7, the proof of which is based on the result of Lemma 1 (see Section IV).

**Lemma 7.** Consider the signal design in (34)-(36) and (75)-(80) in the case of \( \alpha \in [2/3, 1] \). Let \( \delta \in (0, 1] \) and \( \epsilon > 0 \). Then the minimum distance \( d_{\min} \) defined in (118) is bounded by

\[
d_{\min} \geq \delta P^{- \frac{3\alpha /2 -1}{2}}
\]
for all the channel coefficients \( \{ h_{k\ell} \} \in (1, 2)^{2 \times 3} \setminus H_{\text{out}} \), where \( H_{\text{out}} \subseteq (1, 2)^{2 \times 3} \) is an outage set and the Lebesgue measure of the outage set, denoted by \( \mathcal{L}(H_{\text{out}}) \), satisfies

\[
\mathcal{L}(H_{\text{out}}) \leq 1792 \delta \cdot P^{-\frac{1}{2}}.
\] (120)

**Proof.** Consider the case of \( \alpha \in [2/3, 1] \). Let us set

\[
\beta \triangleq \delta P^{-\frac{3\alpha/2-1}{2}}, \quad A_0 \triangleq \sqrt{P^{1-\alpha}}, \quad A_1 \triangleq 1
\]

\[
Q_0 \triangleq 2Q_{\text{max}}, \quad Q_1 \triangleq 4Q_{\text{max}}, \quad Q_{\text{max}} \triangleq P^{\frac{\alpha/2-\epsilon}{2}}
\]

for some \( \epsilon > 0 \) and \( \delta \in (0, 1] \). Recall that \( g_0 \triangleq h_{11} h_{23} h_{12}, g_1 \triangleq h_{12} h_{21} h_{13} \). Let \( \tau \triangleq 8 \). Define the event

\[B(q_0, q_1) \triangleq \{(g_0, g_1) \in (1, \tau]^2 : |A_0 g_0 q_0 + A_1 g_1 q_1| < \beta\}\]

and set

\[B \triangleq \bigcup_{q_0, q_1 \in \mathbb{Z} : (q_0, q_1) \neq 0, |q_k| \leq Q_k \forall k} B(q_0, q_1).
\]

We now bound the Lebesgue measure of \( B \) by using the result of Lemma 1 (see Section IV), given as

\[
\mathcal{L}(B) \leq 8(\tau - 1) \beta \min\left\{ \frac{8Q_{\text{max}}^2}{1}, \frac{8Q_{\text{max}}^2}{1}, \frac{2Q_{\text{max}}}{\sqrt{P^{1-\alpha}}}, \frac{4Q_{\text{max}}^2}{1}, \frac{4Q_{\text{max}}}{\sqrt{P^{1-\alpha}}}, \frac{4Q_{\text{max}}}{\sqrt{P^{1-\alpha}}} \right\}
\]

\[
\leq 8(\tau - 1) \beta \cdot Q_{\text{max}} \cdot \min\{8Q_{\text{max}}, \frac{8Q_{\text{max}}}{\sqrt{P^{1-\alpha}}}, \frac{2Q_{\text{max}}}{\sqrt{P^{1-\alpha}}}, 4\tau, \frac{4\tau}{\sqrt{P^{1-\alpha}}}, \frac{4\tau}{\sqrt{P^{1-\alpha}}} \}
\]

\[
\leq 8(\tau - 1) \beta \cdot Q_{\text{max}} \cdot P^{\frac{\alpha/2-1}{2}} \cdot 4\tau
\]

\[= 32\tau(\tau - 1) \beta \cdot P^{\frac{3\alpha/2-1-\epsilon}{2}}
\]

\[= 1792 \delta \cdot P^{-\frac{1}{2}}.
\] (121)

From our definition, the set \( B \) is the collection of \( (g_0, g_1) \), where \( (g_0, g_1) \in (1, \tau]^2 \). For any \( (g_0, g_1) \in B \), there exists at least one pair \( (q_0, q_1) \in \{q_0, q_1 : q_0, q_1 \in \mathbb{Z}, (q_0, q_1) \neq 0, |q_0| \leq Q_0, |q_1| \leq Q_1\} \) such that \( |A_0 g_0 q_0 + A_1 g_1 q_1| < \delta P^{-\frac{3\alpha/2-1}{2}} \). Thus, we can consider the set \( B \) as an outage set. For any pair \( (g_0, g_1) \) that is outside the outage set \( B \), we have the following conclusion:

\[d_{\text{min}}(g_0, g_1) \geq \delta P^{-\frac{3\alpha/2-1}{2}}, \quad \text{for} \quad (g_0, g_1) \notin B.
\]

Let us now define \( H_{\text{out}} \) as a set of the \( (h_{11}, h_{21}, h_{12}, h_{22}, h_{13}, h_{23}) \in (1, 2)^{2 \times 3} \) such that the corresponding pairs \( (g_0, g_1) \) are in the outage set \( B \), i.e.,

\[H_{\text{out}} \triangleq \{(h_{11}, h_{21}, h_{12}, h_{22}, h_{13}, h_{23}) \in (1, 2)^{2 \times 3} : (g_0, g_1) \in B\}.
\]
At this point, with the connection between $\mathcal{H}_{\text{out}}$ and $B$, we can bound the Lebesgue measure of $\mathcal{H}_{\text{out}}$ as
\[
\mathcal{L}(\mathcal{H}_{\text{out}}) = \int_{h_{11}=1}^{2} \int_{h_{21}=1}^{2} \int_{h_{12}=1}^{2} \int_{h_{22}=1}^{2} \int_{h_{13}=1}^{2} \int_{h_{23}=1}^{2} \mathbbm{1}_{\mathcal{H}_{\text{out}}}(h_{11},h_{21},h_{12},h_{22},h_{13},h_{23}) dh_{23} dh_{13} dh_{12} dh_{22} dh_{12} dh_{11} \\
= \int_{h_{11}=1}^{2} \int_{h_{21}=1}^{2} \int_{h_{12}=1}^{2} \int_{h_{22}=1}^{2} \int_{h_{13}=1}^{2} \int_{h_{23}=1}^{2} \mathbbm{1}_{B}(h_{11}h_{23}h_{12},h_{12}h_{21}h_{13}) dh_{23} dh_{13} dh_{12} dh_{22} dh_{12} dh_{11} \\
\leq \int_{h_{11}=1}^{2} \int_{h_{21}=1}^{2} \int_{h_{12}=1}^{2} \int_{h_{22}=1}^{2} \int_{h_{13}=1}^{2} \int_{h_{23}=1}^{2} \mathbbm{1}_{B}(g_{0},g_{1}) \cdot \frac{1}{h_{11}h_{21}h_{21}h_{13}} dg_{0} dg_{1} dh_{22} dh_{12} dh_{12} dh_{11} \\
= \mathcal{L}(B) dh_{22} dh_{12} dh_{21} dh_{11} \\
\leq 1792\delta \cdot P^{-\frac{3}{2}} dh_{22} dh_{12} dh_{21} dh_{11} \\
= 1792\delta \cdot P^{-\frac{3}{2}} 
\] (122)

where (123) follows from the result in (121); in the above derivations we use the following definitions

\[
\mathbbm{1}_{\mathcal{H}_{\text{out}}}(h_{11},h_{21},h_{12},h_{22},h_{13},h_{23}) = \begin{cases} 
1 & \text{if } (h_{11},h_{21},h_{12},h_{22},h_{13},h_{23}) \in \mathcal{H}_{\text{out}} \\
0 & \text{if } (h_{11},h_{21},h_{12},h_{22},h_{13},h_{23}) \notin \mathcal{H}_{\text{out}}
\end{cases}
\]

and

\[
\mathbbm{1}_{B}(g_{0},g_{1}) = \begin{cases} 
1 & \text{if } (g_{0},g_{1}) \in B \\
0 & \text{if } (g_{0},g_{1}) \notin B
\end{cases}
\]

Then, we complete the proof of Lemma 7.

The result of Lemma 7 reveals that, the minimum distance $d_{\text{min}}$ is sufficiently large for all the channel coefficients $\{h_{kl}\} \in (1,2)^{2 \times 3}$ except for a bounded set $\mathcal{H}_{\text{out}} \subseteq (1,2)^{2 \times 3}$ whose Lebesgue measure satisfies

\[
\mathcal{L}(\mathcal{H}_{\text{out}}) \to 0, \quad \text{as } P \to \infty.
\]

In the following, we will consider the channel coefficients $\{h_{kl}\} \in (1,2)^{2 \times 3}$ that are not in the outage set $\mathcal{H}_{\text{out}}$. With this channel condition, the minimum distance $d_{\text{min}}$ is bounded by

\[
d_{\text{min}} \geq \delta P^{-\frac{3\alpha/2-1}{2}}.
\]

Let us now go back to the expression of $y_{1}$ in (117), which can also be described as

\[
y_{1} = P^{-\frac{\alpha}{2+\epsilon}} \cdot \frac{2\gamma}{\bar{g}} \left( \sqrt{P^{1-\alpha}} g_{0}q_{0} + g_{1}q_{1} \right) \\
+ \sqrt{P^{1-\alpha}} \left( h_{11}h_{23}h_{12}v_{1,p} + \frac{1}{\sqrt{P^{1-\alpha}}} h_{12}h_{21}h_{13}v_{2,p} \right) + z_{1} \\
= P^{-\frac{\alpha}{2+\epsilon}} \cdot 2\gamma \cdot \bar{x}_{s} + \sqrt{P^{1-\alpha}} \bar{g} + z_{1} \\
\]

(127)

where

\[
x_{s} \triangleq \sqrt{P^{1-\alpha}} g_{0}q_{0} + g_{1}q_{1}
\]
and \( \tilde{g} \triangleq h_{11}h_{23}h_{12}v_{1,p} + \frac{1}{\sqrt{P^{1-\alpha}}}h_{12}h_{21}h_{13}v_{2,p} \), with
\[
|\tilde{g}| \leq \tilde{g}_{\text{max}} \triangleq \sqrt{2} \quad \forall \tilde{g}
\]

In the first step we will decode the sum \( x_s = \sqrt{P^{1-\alpha}}g_0q_0 + g_1q_1 \) from \( y_1 \) by treating other signals as noise. After correctly decoding \( x_s \), we can recover \( q_0 \) and \( q_1 \) from \( x_s \) because \( g_0 \) and \( g_1 \) are rationally independent. This step is called as noise removal. After correctly decoding \( x_s \) from \( y_1 \) by choosing the point close to \( x_s \). Then, the error probability for decoding \( x_s \) from \( y_1 \) in (127) is
\[
\Pr[x_s \neq \hat{x}_s] \leq \Pr[|z_1 + P^{1-\alpha} \tilde{g}| > P^{\alpha/2+\epsilon} \cdot 2\gamma \cdot \frac{d_{\text{min}}}{2}] \\
\leq \Pr[z_1 > P^{\alpha/2+\epsilon} \cdot 2\gamma \cdot \frac{d_{\text{min}}}{2} - P^{1-\alpha} \tilde{g}_{\text{max}}] \\
+ \Pr[z_1 > P^{\alpha/2+\epsilon} \cdot 2\gamma \cdot \frac{d_{\text{min}}}{2} - P^{1-\alpha} \tilde{g}_{\text{max}}] \\
= 2 \cdot Q\left(P^{\alpha/2+\epsilon} \cdot 2\gamma \cdot \frac{d_{\text{min}}}{2} - P^{1-\alpha} \tilde{g}_{\text{max}}\right) \\
\leq 2 \cdot Q\left(P^{\alpha/2+\epsilon} (\gamma \delta P^2 - \sqrt{2})\right)
\]

where the \( Q \)-function is defined as \( Q(a) \triangleq \frac{1}{\sqrt{2\pi}} \int_a^\infty \exp(-s^2/2) \, ds \); (128) follows from that \( |\tilde{g}| \leq \tilde{g}_{\text{max}} \triangleq \sqrt{2}, \forall \tilde{g} \); (129) is from that \( d_{\text{min}} \geq \delta P^{-3\alpha/2-1} \). By using the identity that \( Q(a) \leq \frac{1}{2} \exp(-a^2/2), \forall a \geq 0 \), and together with (129), we can have the following bound
\[
\Pr[x_s \neq \hat{x}_s] \leq \exp\left(-\frac{P^{1-\alpha} (\gamma \delta P^2 - \sqrt{2})^2}{2}\right)
\]

when \( \gamma \delta P^2 - \sqrt{2} \geq 0 \). Therefore, when \( P \to \infty \), it implies that \( \gamma \delta P^2 - \sqrt{2} \geq 0 \), which then gives the following conclusion on the error probability for decoding \( x_s \) from \( y_1 \)
\[
\Pr[x_s \neq \hat{x}_s] \to 0 \quad \text{as} \quad P \to \infty.
\]

After decoding \( x_s = \sqrt{P^{1-\alpha}}g_0q_0 + g_1q_1 \) correctly from \( y_1 \), we can recover the symbols \( g_0 \) and \( g_1 \) because \( g_0 \) and \( g_1 \) are rationally independent.

In the second step, the decoded \( x_s \) can be removed from \( y_1 \) and then \( v_{1,p} \) can be decoded from
\[
y_1 - P^{\alpha/2+\epsilon} \cdot 2\gamma \cdot x_s = \sqrt{P^{1-\alpha}}h_{11}h_{23}h_{12}v_{1,p} + h_{12}h_{21}h_{13}v_{2,p} + z_1.
\]

Given that \( v_{1,p}, v_{2,p} \in \Omega (\xi = \gamma \cdot \frac{1}{Q}, Q = P^{1-\alpha/2+\epsilon}) \) and \( h_{12}h_{21}h_{13}v_{2,p} \leq \frac{1}{\sqrt{2}} \), the result of Lemma 6 reveals that the error probability for decoding \( v_{1,p} \) is
\[
\Pr[v_{1,p} \neq \hat{v}_{1,p}] \to 0 \quad \text{as} \quad P \to \infty.
\]

At this point, it holds true that the error probability of estimating \( \{v_{1,c} , v_{1,p}\} \) from \( y_1 \) is
\[
\Pr[\{v_{1,c} \neq \hat{v}_{1,c} \} \cup \{v_{1,p} \neq \hat{v}_{1,p}\}] \to 0 \quad \text{as} \quad P \to \infty
\]

for all the channel coefficients \( \{h_{k\ell}\} \in (1,2)^{2x3} \setminus \mathcal{H}_{\text{out}} \), with Lebesgue measure \( \mathcal{L}(\mathcal{H}_{\text{out}}) \) satisfying
\[
\mathcal{L}(\mathcal{H}_{\text{out}}) \to 0, \quad \text{as} \quad P \to \infty.
\]

Due to the symmetry, the case of \( k = 2 \) can be proved in a similar way.
IX. CONVERSE

In this section, we provide the converse proof of the secure sum GDoF in Theorem 1 focusing on the two-user symmetric Gaussian interference channel with a helper defined in Section II. The converse proof is based on the following two steps:

1) The secure capacity region of the interference channel with a helper and with secrecy constraints, denoted by $C$, is outer bounded by the capacity region of the interference channel with a helper but without secrecy constraints (denoted by $C_{h,ns}$), i.e.,

$$C \subseteq C_{h,ns}$$

due to the fact that secrecy constraints will not enlarge the capacity region.

2) The capacity region of the interference channel with a helper is outer bounded by the capacity region of the interference channel without a helper (denoted by $C_{nh,ns}$), i.e.,

$$C_{h,ns} \subseteq C_{nh,ns}$$

because the helper’s signal that is independent of the other transmitters’ signals, will not enlarge the capacity region for this setting without secrecy constraints.

Therefore, it implies that

$$C \subseteq C_{nh,ns}.$$  

In other words, the capacity region $C_{nh,ns}$ (and the GDoF region respectively) of the interference channel without a helper and without secrecy constraints (see [32]), will sever as the outer bound of the secure capacity region $C$ (and the secure GDoF region respectively) of the interference channel with a helper and with secrecy constraints.

X. CONCLUSION

For the two-user symmetric Gaussian interference channel, this work revealed an interesting observation that adding a helper can totally remove the secrecy constraints, in terms of GDoF performance. In the proposed scheme, the cooperative jamming and a careful signal design are used such that the jamming signal of the helper is aligned at a specific direction and power level with the information signals of the transmitters. It turns out that, the penalty in GDoF due to the secrecy constraints can be totally removed. In the rate analysis, the estimation approaches of noise removal and signal separation due to the rational independence are used. In the future work, we will extend the result to the other communication channels with a helper.

APPENDIX A

PROOF OF LEMMA 3

In this section we will provide the proof of Lemma 3. We first focus on the proof for the first user ($k = 1$). In the following, $v_{2,c} + u_3$ and $v_{1,c}$ can be estimated from $y_1$ by using a successive decoding method with vanishing error probability, where $y_1$ is expressed in (68). At first, $v_{2,c} + u_3 \in 2 \cdot \Omega(\xi = \gamma_{v_{2,c}} \cdot 1, Q = P\frac{1}{2})$ can be estimated from $y_1$ by treating the other signals as noise, where $y_1$ in (68) can be described as

$$y_1 = \sqrt{P}h_{12}h_{21}h_{13}(v_{2,c} + u_3) + \sqrt{P}h_{11}h_{23}h_{12}v_{1,c} + z_1.$$  

In this case with $\alpha \geq 2$, we have $|h_{11}h_{23}h_{12}v_{1,c}| \leq \sqrt{2}$ for any realizations of $v_{1,c}$. Let $\hat{s}_{vu}$ be the estimate of $s_{vu} \triangleq v_{2,c} + u_3$. From Lemma 4, it is true that

$$\Pr[s_{vu} \neq \hat{s}_{vu}] \rightarrow 0, \quad \text{as} \quad P \rightarrow \infty.$$
Once \( v_{2,c} + u_3 \) is decoded, it can be removed from \( y_1 \). After that, \( v_{1,c} \in \Omega(\xi = \gamma_{v_{1,c}} \cdot \frac{1}{Q}, \, Q = \frac{P^{\frac{1}{2}-\epsilon}}{2} ) \) can be estimated from the following observation

\[
y_1 - \sqrt{P\alpha} h_{12} h_{13} (v_{2,c} + u_3) = \sqrt{P} h_{11} h_{12} v_{1,c} + z_1
\]

with vanishing error probability, i.e.,

\[
Pr[ v_{1,c} \neq \hat{v}_{1,c} ] \to 0 \quad \text{as} \quad P \to \infty
\]

(134) which completes the proof for the case of \( k = 1 \). Similarly, the case of \( k = 2 \) can be proved due to the symmetry.

**APPENDIX B**

**PROOF OF LEMMA [5]**

In this section we will provide the proof of Lemma [5] which is similar to that of Lemma [4]. In this case with \( \alpha \in [1, 2] \), the two symbols

\[
v_{1,c} \in \Omega(\xi = 2\gamma \cdot \frac{1}{Q}, \, Q = \frac{P^{\frac{1}{2}}}{2} )
\]

\[
v_{2,c} + u_3 \in 2 \cdot \Omega(\xi = 2\gamma \cdot \frac{1}{Q}, \, Q = \frac{P^{\frac{1}{2}}}{2} )
\]

will be estimated from \( y_1 \) by using the approaches of noise removal and signal separation. The expression of \( y_1 \) is given in (68), which can be rewritten as

\[
y_1 = \sqrt{P} h_{12} h_{13} (v_{2,c} + u_3) + z_1
\]

\[
= \sqrt{P^{1-\alpha/2+\epsilon}} \cdot 2\gamma \cdot (\bar{q}_0 \bar{q}_1 + \sqrt{P^{\alpha-1}} \bar{g}_1 \bar{q}_1) + z_1
\]

\[
= \sqrt{P^{1-\alpha/2+\epsilon}} \cdot 2\gamma \cdot \bar{x}_s + z_1
\]

(135) where \( \gamma \in (0, \frac{1}{8\sqrt{2}}) \), \( \bar{x}_s \triangleq (\bar{q}_0 \bar{q}_1 + \sqrt{P^{\alpha-1}} \bar{g}_1 \bar{q}_1) \), \( \bar{q}_0 \triangleq h_{11} h_{23} h_{12} \), \( \bar{g}_1 \triangleq h_{12} h_{13} \) and

\[
\bar{q}_0 \triangleq \frac{Q_{\max}}{2\gamma} \cdot v_{1,c}, \quad \bar{q}_1 \triangleq \frac{Q_{\max}}{2\gamma} \cdot (v_{2,c} + u_3), \quad Q_{\max} \triangleq \frac{P^{\frac{1}{2}}}{2}.
\]

In this setting, \( \bar{q}_0, \bar{q}_1 \in \mathbb{Z}, \, |\bar{q}_0| \leq Q_{\max}, \, |\bar{q}_1| \leq 2Q_{\max}, \, \sqrt{P^{\alpha-1}} \in \mathbb{Z}^+ \) based on our definitions. Let us define the minimum distance for \( \bar{x}_s \) as

\[
\bar{d}_{\min}(\bar{q}_0, \bar{q}_1) \triangleq \min_{\bar{q}_0, \bar{q}_1' \in \mathbb{Z} | [-Q_{\max}, Q_{\max}]} | \bar{q}_0 (\bar{q}_0 - \bar{q}_0') + \sqrt{P^{\alpha-1}} \bar{g}_1 (\bar{q}_1 - \bar{q}_1') |.
\]

(136)

A lower bound on the minimum distance \( \bar{d}_{\min} \) is given in the following lemma.

**Lemma 8.** Consider the signal design in (34)-(36) and (88)-(91) for the case with \( \alpha \in [1, 2] \). Let \( \delta \in (0, 1] \) and \( \epsilon > 0 \). The minimum distance \( \bar{d}_{\min} \) defined in (136) is bounded by

\[
d_{\min} \geq \delta P^{-\frac{1-\alpha}{2}}
\]

(137) for all the channel coefficients \( \{h_{kt}\} \in (1, 2)^{2 \times 3} \setminus \mathcal{H}_{\text{out}}, \) where \( \mathcal{H}_{\text{out}} \subseteq (1, 2)^{2 \times 3} \) is an outage set and the the Lebesgue measure of the this outage set, denoted by \( \mathcal{L}(\mathcal{H}_{\text{out}}) \), satisfies

\[
\mathcal{L}(\mathcal{H}_{\text{out}}) \leq 1792\delta \cdot P^{-\frac{1}{2}}.
\]

(138)

**Proof.** The proof of this lemma is similar to that of the Lemma [7] Considering the case of \( \alpha \in [1, 2] \), let us set

\[
\beta \triangleq \delta P^{-\frac{1-\alpha}{2}}, \quad \bar{A}_0 \triangleq 1, \quad \bar{A}_1 \triangleq \sqrt{P^{\alpha-1}}
\]
Similarly, the result in (143) can also be extended to the case of satisfying $g$ for some $\epsilon > 0$ and $\delta \in (0, 1]$. Recall that $\bar{g}_0 \triangleq h_{11}h_{23}h_{12}$ and $\bar{g}_1 \triangleq h_{12}h_{21}h_{13}$. Let $\tau \triangleq 8$. We define the event

$$B(\bar{q}_0, \bar{q}_1) \triangleq \{ (\bar{q}_0, \bar{q}_1) \in (1, \tau]^2 : |\bar{A}_0\bar{q}_0\bar{q}_0 + \bar{A}_1\bar{q}_1\bar{q}_1| < \beta \}$$

and set

$$\bar{B} = \bigcup_{\bar{q}_0, \bar{q}_1 \in \mathbb{Z} : (\bar{q}_0, \bar{q}_1) \neq 0, m_{x, k} \leq \bar{q}_k} B(\bar{q}_0, \bar{q}_1).$$

By using Lemma 1, we bound the Lebesgue measure of $\bar{B}$ as

$$L(\bar{B}) \leq 8(\tau - 1)\beta \min\left\{ \frac{8q_{\max}^2}{\sqrt{P_{\alpha - 1}}}, \frac{2q_{\max}^2}{\sqrt{P_{\alpha - 1}}} \right\} \leq 8(\tau - 1)\beta \cdot Q_{\max} \cdot \min\left\{ \frac{8q_{\max}^2}{\sqrt{P_{\alpha - 1}}}, \frac{2q_{\max}^2}{\sqrt{P_{\alpha - 1}}} \right\} \leq 8(\tau - 1)\beta \cdot Q_{\max} \cdot \frac{2\tau}{4\tau} = 32\tau(\tau - 1)\beta \cdot Q_{\max} \cdot \frac{2\tau}{4\tau} = 32\tau(\tau - 1)\beta \cdot Q_{\max} \cdot \frac{1}{2} \cdot \frac{2\tau}{4\tau} = 1792\delta \cdot P^{-\frac{1}{2}}.$$  \hspace{1cm} (141)

We can consider set $\bar{B}$ as an outage set. For any pair $(\bar{q}_0, \bar{q}_1)$ that is outside the outage set $\bar{B}$, we can conclude that $d_{\min}(\bar{q}_0, \bar{q}_1) \geq \delta P^{-\frac{1}{2}}$. Let us now define $\mathcal{H}_{\text{out}}$ as a set of the $(h_{11}, h_{21}, h_{12}, h_{22}, h_{13}, h_{23}) \in (1, 2)^{2 \times 3}$ such that the corresponding pairs $(\bar{q}_0, \bar{q}_1)$ are in the outage set $\bar{B}$, that is,

$$\mathcal{H}_{\text{out}} \triangleq \{ (h_{11}, h_{21}, h_{12}, h_{22}, h_{13}, h_{23}) \in (1, 2)^{2 \times 3} : (\bar{q}_0, \bar{q}_1) \in \bar{B} \}.$$  \hspace{1cm} (142)

With the connection between $\mathcal{H}_{\text{out}}$ and $\bar{B}$, one can follow the steps in (122)-(124) to bound the Lebesgue measure of $\mathcal{H}_{\text{out}}$ as:

$$L(\mathcal{H}_{\text{out}}) \leq 1792\delta \cdot P^{-\frac{1}{2}}.$$  \hspace{1cm} (143)

Let us now go back to the expression of $y_1$ in (135), i.e.,

$$y_1 = \sqrt{P^{1-\alpha/2+\epsilon}} \cdot 2\gamma \cdot \bar{x}_s + z_1.$$  \hspace{1cm} (144)

Note that the minimum distance for $\bar{x}_s$ is $d_{\min}$ defined in (136). Lemma 8 shows that this minimum distance for $\bar{x}_s$ is bounded by $d_{\min} \geq \delta P^{-\frac{1}{2} - \frac{1}{2}}$ when the channels are not in the outage set, i.e., $\{h_{kk}\} \notin \mathcal{H}_{\text{out}}$. This implies that we can estimate $\bar{x}_s$ from $y_1$ with vanishing error probability as $P \to \infty$. Since $\bar{q}_0$ and $\bar{q}_1$ are rationally independent, we can recover the symbols $\bar{q}_0$ and $\bar{q}_1$ from $\bar{x}_s = (\bar{q}_0\bar{q}_0 + \sqrt{P_{\alpha - 1}}\bar{q}_1\bar{q}_1)$ after decoding $\bar{x}_s$. Then, the error probability for decoding $v_{1,c}$ can be concluded as

$$\Pr[\hat{v}_{1,c} \neq \hat{v}_{1,c}] \to 0 \quad \text{as} \quad P \to \infty$$  \hspace{1cm} (145)

which is true for all the channel coefficients $\{h_{kk}\} \in (1, 2)^{2 \times 3} \setminus \mathcal{H}_{\text{out}}$, with Lebesgue measure $L(\mathcal{H}_{\text{out}})$ satisfying

$$L(\mathcal{H}_{\text{out}}) \to 0, \quad \text{as} \quad P \to \infty.$$  \hspace{1cm} (146)

Similarly, the result in (145) can also be extended to the case of $k = 2$ due to the symmetry.
[34] A. S. Motahari, S. O. Gharan, M. A. Maddah-Ali, and A. K. Khandani, “Real interference alignment: Exploiting the potential of single antenna systems,” *IEEE Trans. Inf. Theory*, vol. 60, no. 8, pp. 4799 – 4810, Aug. 2014.