Abstract: Metamaterials offer exciting opportunities that enable precise control of amplitude, polarization and phase of the light beam at a subwavelength scale. A gradient metasurface consists of a class of anisotropic subwavelength metamaterial resonators that offer abrupt amplitude and phase changes, thus enabling new applications in optical device design such as ultrathin flat lenses. We propose a highly efficient gradient metasurface lens based on a metal-dielectric-metal structure that operates in the terahertz regime. The proposed structure consists of slotted metallic resonator arrays on two sides of a thin dielectric spacer. By varying the geometrical parameters, the metasurface lens efficiently manipulates the spatial distribution of the terahertz field and focuses the beam to a spot size on the order of a wavelength. The proposed flat metasurface lens design is polarization insensitive and works efficiently even at wide angles of incidence.

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1. Introduction

Subwavelength metamaterials with smart and rational designs are one of the fastest growing fields over the past decade due to their unconventional electromagnetic, acoustic, thermal, or mechanical properties that are not readily available in naturally occurring materials [1,2]. The field of metamaterial research has provided an unprecedented ability to manipulate electromagnetic waves ranging from negative index of refraction [3], invisibility cloaking [4,5], to super-focusing [6]. Recent progress has shown that flat metasurfaces with reduced dimensionality can engineer abrupt amplitude, phase, and polarization changes which has facilitated new physics with enhanced functionalities for device applications distinctly different from those observed in their three-dimensional counterparts. Intriguing integrated devices, such as deflectors [7], vortex plates [8], aberration-free or dual-polarity lenses [9,10], and zoom plates [11] were demonstrated based on this concept.

In this article, we propose a high performance flat metasurface lens for the terahertz waves. A regular lens is an indispensable tool for various fundamental and practical applications. The lenses are usually manufactured using materials such as glass or dielectrics. However, a lens based on metamaterial structures can be flat and optically thin without aberration and could focus light beyond the diffraction limit [12–17]. Recently, metamaterial-based gradient index (GRIN)-type lenses including Eaton lens [14], Maxwell fish-eye lens [18], and Luneburg lens [19] were reported, which provide a large refractive index and sufficient bandwidth. However, the complex retrieval algorithm of this type of lenses is a major hindrance to realistic applications [20]. According to the Fermat’s principle, the wavefront of light beam can be modified by controlling its phase. Hence, smart metasurfaces are capable of providing the required phase change and enable the development of novel
ultrathin flat lenses with unique functionalities and enhanced performance [21,22]. Here, we present a systematic numerical study of a flat metasurface lens design consisting of a metal-dielectric-metal structure functioning in the terahertz regime. The proposed periodic slot structure allows high transmission in a broad terahertz band. By varying the geometric parameters of each slot unit, a large phase change is obtained at different positions that enables the gradient metasurface to focus terahertz radiation down to a spot size of approximately one wavelength. From here on, we would address this lens as a *terahertz metasurface lens* (TML). The TML is also found to be relatively insensitive to wide angles of incidence, as well as the incident linear polarization states of the terahertz radiation.

2. Design and numerical analysis

![Diagram of the unit cell and lens design](image)

Figure 1(a) illustrates the schematic of a unit cell of the metasurface structure at normal incidence excitation and Fig. 1(b) shows the whole lens design. A pair of 200-nm thick patterned metallic cladding layers is separated by a dielectric spacer of thickness $T$. Each metallic layer was perforated with an array of square-loop slots with inner patch width $W$, periodicity $P$ and length $L$. The square-loop slot pattern is a symmetric structure that exhibits excellent transmission, polarization insensitivity and works efficiently even at wide angles of incidence. The spacer is a 50-μm-thick benzocyclobutene (BCB) layer with a permittivity of $\varepsilon = 2.67$ and a loss tangent of $\delta = 0.012$. The spacer thickness has been optimized such that the effect of near-field coupling between the resonators on two sides could be ignored. The metal layer is made from aluminum with the conductivity of $\sigma = 3.72 \times 10^7$ S/m. The spectral response of the unit cell was calculated using CST Microwave Studio TM simulations. The incident wave is a plane wave with the electric field polarization along the $x$ axis, as shown in Fig. 1(a).

In the TML design, the patch width $W$ is varied to tailor the required phase change while the periodicity of the unit cell is kept constant. For simplicity, only three cases with $W = 0, 40$, and 80 μm are discussed here to compare their transmission and phase response. As shown in Figs. 2(a)–2(f), when the operation frequency is chosen to be 0.84 THz, the incident electric field induces polarized surface currents and dipolar responses in the metallic patch as well as the outer square loop, forming localized surface plasmon (LSP) and dipole localized surface plasmon (DLSP) modes [23,24]. The LSP (DLSP) leads to a transmission field enhancement determined by the parameter $W$ [25,26]. As the two metasurfaces with the same pattern are stacked together on two sides of the 50-μm-thick spacer, a broadband transmission window is observed in Fig. 2(g). Additionally, in Fig. 2(h), we notice that the corresponding phase change through a unit cell has a monotonic trend with parameter $W$. Thus by varying $W$ one
could simply control the transmission and phase of the terahertz wave as it propagates through the metasurface lens. The number of stacked metasurfaces allows another degree of design freedom to control the transmission amplitude and the bandwidth around a specific frequency.

Here, the effects of the metasurface stack are elaborated in more detail. Depending on the spacer thickness, the interaction between the metasurface and the incident light could be very different, even though their far-field transmission properties are similar. Due to the relatively thick spacer layer, the enhanced broadband transmission of the unit cell is attributed to the electric LSP and DLSP as explained earlier. In the recent work [27,28], the high transmission of similar unit cells was attributed to the electric and magnetic dipoles, behaving as a Huygens’ surface metamaterial. The magnetic dipole was formed by the fishnet structure where the front and back metasurfaces were tightly coupled via their near fields. The unit cell in our design has 50-μm-thick spacer layer and there is no longitudinal near-field coupling between different metasurfaces and thus there is no formation of any magnetic dipole between the layers. The absence of near-field coupling due to thicker spacer layer would also allow for much higher tolerance to the alignment errors while fabricating the real device.

In order to focus the incident terahertz wave, the phase change through the metasurface is varied along the lens radius by changing the parameter $W$. The transmission and phase of unit cells with different structural size $W$ are shown in Fig. 3(a). It is observed that the broad passband feature in the transmission spectra stays high ($\sim 75\%$) for a large range of width $W$. 

![Fig. 2. (a)-(c) Simulated electric field distribution at 0.84 THz for the three different designs with $W = 0, 40,$ and 80 μm, respectively. (d)-(f) Corresponding surface current at 0.84 THz. (g) Transmission and (h) phase change in TML for the three different designs.](image)
The red circles represent the transmission through TML from $W = 0$ to 80 $\mu$m, where the lowest transmission is found for the structure with $W = 52$ $\mu$m. The blue triangles indicate the gradual phase change with increasing width $W$.

Fig. 3. (a) Transmission (red circles) and phase (blue triangles) of the unit cells with different patch widths, $W$. (b) Patch width and phase change of the metasurface structure as a function of the radius number. The red circles and the blue triangles represent the phase change $\Delta \varphi$ through the lens and the patch width $W$ along the radial direction, respectively.

According to the Fermat’s theorem, the transmitted terahertz wave through the lens at different radii should meet the phase relationship as described below,

$$\Delta \varphi = \left( \sqrt{f^2 + r^2} - f \right) \frac{2\pi}{\lambda}, \quad (1)$$

where $\Delta \varphi$ represents the phase change and $f$ is the focal length. The above equation has three variables and if one of them is known, the relationship between the other two could be easily obtained. From Eq. (1) we found that $f$ is directly related to the working wavelength while their relation is not simply being reciprocal since $\Delta \varphi$ is also determined by the wavelength in a more complicated way. For a better focusing effect, we need more than two metasurfaces to engineer the required phase change. Thus a terahertz lens design with three metasurfaces and two spacer layers (50 $\mu$m thick) was chosen. With this design, we set the lens radius to $r = 2.1$ mm and obtained a focal length of 4.7 mm according to Eq. (1).

The optimized phase change $\Delta \varphi$ and patch width $W$ with respect to the radial position are depicted in Fig. 3(b). It is observed that the phase change along the radial direction shows a hyperbolic behavior as the patch width decreases gradually from the center of the lens to its edge. At the edge of the TML, the patch is completely removed from the pattern, leaving a bare square hole supporting the LSP resonance, as shown in Figs. 2(a) and 2(d). The metasurface lens structure contains 42 unit cells with different $W$ and is 4.2 mm long in both dimensions.
Fig. 4. (a) Two dimensional plots of the calculated electric field distributions of the transmitted wave at normal incidence. (b) Corresponding electric field intensity at the focus for normal incidence. (c) and (d) Electric field distributions at oblique incidences. The incidence angles are 15° and −25°, respectively. For clarity, the scale bar is shown on the right side.

The electromagnetic properties of the proposed TML were numerically demonstrated by using CST Microwave Studio. The incident terahertz beam is a linearly polarized plane wave (along the x axis). We used open boundary conditions in the numerical calculation. Figure 4(a) shows the two-dimensional plot of the calculated electric field distribution of the transmitted wave at normal incidence. As expected, the wave front is focused to a point that is 4.84 mm behind the lens. The simulated focal length is slightly longer than the theoretical prediction, which is attributed to the difference between the sophisticated numerical model and the simple analytical Eq. (1).

The focused spot size is rather small and is of the same order as the incident terahertz wavelength. The corresponding electric field intensity at the focus is shown in Fig. 4(b), where the electromagnetic energy is concentrated at the focal point. Compared to the incident wave, the focused terahertz wave is about 5.2 times enhanced in the intensity. We observe that the terahertz wave propagates through the metasurfaces with high transmission efficiency. Due to symmetric design of the TML unit cell, the response of metasurface lens is polarization independent. We also calculated the electric field distributions under different polarizations. When the polarization direction is along the y axis or any other direction in the x-y plane, the same convergence and focusing could be achieved. Therefore, the focusing effect of the TML is insensitive to the polarization direction of the incident light.

Moreover, we also examined the incidence angle behavior of the designed lens by changing the incidence angle to 15° in the y-z plane and to −25° in the x-z plane, as shown in Figs. 4(c) and 4(d), respectively. The off axis aberration of TML is not very clearly visible for oblique terahertz wave incidence angle of less than 15°. However, it becomes more obvious for larger oblique incidence angle [29]. We calculated the electric field distributions at other incidence angles and found that the maximum oblique incidence angle for which focusing could still be achieved is around 30°.

The spot size of the focused beam is extremely important in lens design since it is directly related to the imaging resolution of the lens. As shown in Fig. 5(a), the electric field magnitude in the z-axis increases gradually and reaches a maximum value at z = 4.84 mm. In Fig. 5(b), the normalized transverse electric field distributions at the respective focal lengths of three different frequencies are shown. As expected, the strongest focus is achieved at 0.84 THz since the metasurface unit cell is optimized for this specific frequency, where the spot diameter is d = 450 μm corresponding to 1.26λ₀. The diffraction-limited spot size of the...
The terahertz wave is given by $d_{\text{in}} = \lambda_0 f / D = 1.15 \lambda_0$, where $f$ is the focal length, $D$ is the diameter of the lens and $\lambda_0$ is the designed wavelength. Even at other frequencies we observe the convergence and focusing of the terahertz wave, which implies that TML is indeed a broadband device.

![Fig. 5](image.png)

**Fig. 5.** (a) Electric field magnitude at the $z$-axis. (b) Normalized electric field distribution of different frequencies in the focal plane.

![Fig. 6](image.png)

**Fig. 6.** (a) Two-dimensional plot of the electric field magnitude of the triple-layer structure. (b) Electric field distribution of the divergence structure.

Since the phase change caused by a single layer metasurface is very limited, we carried out the two-layer design. When the number of layers is further increased, the focusing effect is much more enhanced. The electromagnetic field distribution of the triple-layer structure is calculated, as shown in Fig. 6(a). Compared to the two-layer structure, the focus spot is smaller and the focal length is shorter. It is shown that the performance of the triple-layer structure is better than that of the two-layer and single-layer design in terms of the focused spot size. However, the only drawback is that the transmission drops down to approximately 40% while it was 75% in the two-layer design. Thus, there is a tradeoff between tight focusing and transmission of the terahertz wave propagation through the TML.

As the convergence and focusing depend on the phase change achieved at the unit cell level, it is straightforward to design other interesting on-demand optical components by engineering the building blocks. For example, using the same unit cell design based on Eq. (1), we could set the patch width to increase gradually from middle to edge along the radial direction. The wave front would be hyperbolic and the electric field would become divergent to form a virtual focus on the left hand side of the lens, as seen in Fig. 6(b).
3. Theoretical modeling and analysis

As shown in Fig. 7(a), the blue line indicates the simulated electric field amplitude along the \( y \) axis in the \( y-z \) plane at \( z = 1 \) mm, which shows that the electric field amplitude is weak at the center and relatively strong on both sides. Comparing to the transmission trend of the unit cells along the radial direction shown in Fig. 3(a), we found that the electrical field distribution propagating through the TML could not be attributed only to the transmission difference of each unit cell. The diffraction effect of the unit cell needs to be considered when understanding the performance of the terahertz lens. According to the Huygens' principle, an element can be seen as a secondary source and the unit cell is equivalent to a point source. It gives rise to a diffraction effect at the rear surface of the structure. Due to particular distribution of phase and amplitude of the structure, the effect could be equivalent to the combined effect of a zone plate and an echelle grating.

In order to verify our interpretation, we departed from the Fresnel diffraction formula to look through the field distribution [30]. In order to simplify the calculation, a one-dimensional approximation is applied to describe the field distribution,

\[
\tilde{E}(y) = \frac{\exp(ikz)}{i\lambda z_1} \exp\left(\frac{ik}{2\lambda z_1} y^2\right) \int \tilde{E}(y_1) \exp[-i2\pi(y_1 - \frac{y}{\lambda z_1})] \exp\left(\frac{ik}{2\lambda z_1} y_1^2\right) dy_1,
\]

where \( \tilde{E}(y) \) and \( \tilde{E}(y_1) \) are the complex amplitudes of electric field in the observation plane \((z = 1 \) mm\) and rear surface of structures respectively, \( z_1 \) is the distance between the two planes, \( y_1 \) and \( y \) are the space coordinates corresponding to the two planes. Because the structure consists of the unit cell pixels, here we treat the analytical formula as a summation instead of an integral,

\[
\tilde{E}(y) = \frac{\exp(ikz)}{i\lambda z_1} \exp\left(\frac{ik}{2\lambda z_1} y^2\right) \sum_{D} E(y_1) \exp[i(nkT + \Delta \phi)] \exp[-i2\pi(y_1 - \frac{y}{\lambda z_1})] \exp(\frac{ik}{2\lambda z_1} y_1^2) \Delta y_1,
\]

where \( n \) and \( T \) are the relative refractive index and the thickness of the lens, respectively. \( D \) is the diameter of the lens structure. \( E(y_1) \) is the transmission amplitude of each unit at the rear surface, and \( \exp[i(nkT + \Delta \phi)] \) corresponds to the phase difference via each unit cells for each layer.

The calculated result is shown as the red line in Fig. 7(a) which agrees well with the blue line obtained by simulation. To verify the theoretical model, we calculated the field intensity of the focus plane at 0.84 THz, as shown in Fig. 7(b), where the simulated result matches very well with the calculation. It is shown that the theoretical model generally explains the field amplitude distribution at the rear surface. We also found that when frequency is far away from the design specific frequency, for example at 0.95 THz, the characteristic of a relatively weak behavior at the center and relatively strong on both sides is more obvious.
Fig. 7. (a) and (b) The blue lines indicate the electric field amplitude at 0.84 THz of axial length at $z = 1$ mm and focus plane, respectively. The red lines show the results calculated from Eq. (3).

4. Summary

An engineered TML proposed for the terahertz wave focusing is systematically investigated. The two-layer design shows high transmission and strong focusing of the terahertz radiation at a desired frequency. Spatial distributions of the electromagnetic field at normal and oblique angles of incidence clearly show that the metasurface design works well over a broad range of frequencies. The symmetric structure is polarization insensitive and works efficiently up to $30^\circ$ wide angle of incidence. Quantitative analysis of the magnitude of electric field shows that the metasurface lens focuses the terahertz radiation to a spot diameter of $1.26\lambda_0$. We also achieved tighter focusing but a lower transmission by using the three-layer metasurface design. The proposed design concept has the flexibility to be tailored for on-demand lensing and imaging applications.

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