Threshold Effects and Lorentz Symmetry

Orfeu Bertolami

Instituto Superior Técnico, Depto. Física,
Av. Rovisco Pais, 1049-001 Lisboa, Portugal

Abstract. Evidence on the violation of Lorentz symmetry arises from the observation of cosmic rays with energies beyond the GZK cutoff, \( E_{\text{GZK}} \approx 4 \times 10^{19} \text{eV} \), from the apparent transparency of the Universe to the propagation of high energy gamma radiation and from the stability of pions in air showers. These three paradoxes can be explained through deformations of the relativistic dispersion relation. Theoretical ideas aimed to understand how Lorentz symmetry may be broken and phenomenologically interesting deformations of the relativistic dispersion relation may arise are briefly discussed.

1 Introduction

Invariance under Lorentz transformations is one of the most fundamental symmetries of physics and is a key feature of all known physical theories. However, recently, evidence has emerged that this symmetry may not be respected in at least three different phenomena:

i) Observation of ultra-high energy cosmic rays with energies \([12,13,14]\) beyond the Greisen-Zatsepin-Kuzmin (GZK) cutoff, \( E_{\text{GZK}} \approx 4 \times 10^{19} \text{eV} \). These events, besides challenging our knowledge of mechanisms that allow accelerating cosmic particles to such high energies, may imply in a violation of Lorentz invariance as only through this violation that new threshold effects may arise and the resonant scattering reactions with photons of the Cosmic Microwave Background (CMB), e.g. \( p + \gamma_{\gamma_{\gamma}} \rightarrow \Delta_{1232} \), are suppressed \([6,7,8,9]\). An astrophysical solution to this paradox is possible and imply identifying viable sources at distances within, \( D_{\text{Source}} \lesssim 50 - 100 \text{ Mpc} \) \([10,11]\), so that the travelling time of the emitted particles is shorter than the attenuation time due to particle photoproduction on the CMB. Given the energy of the observed ultra-high energy cosmic rays (UHECRs) and the Hilla’s criteria \([12]\) on the energy, size and intensity of the magnetic field to accelerate protons, \( E_{18} \leq \frac{4}{3} \beta B(\mu G)L(kpc) \) - where \( E_{18} \) is the maximum energy measured in units of \( 10^{18} \text{eV} \), \( \beta \) the velocity of the shock wave relative to \( c \) - it implies that, within a volume of radius \( 50 - 100 \text{ Mpc} \) about the Earth, only neutron stars, active galactic nuclei, gamma-ray bursts and cluster of galaxies are feasible acceleration sites \([12,13]\). This type of solution has been recently suggested in Ref. \([14]\), where it was also argued that the near isotropy of the arrival directions of the observed UHECRs can be attributed to extragalactical magnetic fields near the Milky Way that are strong enough to deflect and isotropize the incoming directions of UHECRs from sources within...
$D_{\text{Source}}$. This is a debatable issue and it is worth bearing in mind that scenarios for the origin of UHECRs tend to generate anisotropies (see e.g. Ref. [15]). A further objection against this proposal is the mismatch in the energy fluxes of observed UHECRs and of the potential sources as well as the lack of spatial correlations between observed UHECRs and candidate sources (see e.g. [16] and references therein).

ii) Observations of gamma radiation with energies beyond $20 \, \text{TeV}$ from distant sources such as Markarian 421 and Markarian 501 blazars [17,18,19]. These observations suggest a violation of Lorentz invariance as otherwise, due to pair creation, there should exist a strong attenuation of fluxes beyond 100 $\, \text{Mpc}$ of $\gamma$-rays with energies higher than 10 $\, \text{TeV}$ by the diffuse extragalactic background of infrared photons [20,21,22,23].

iii) Studies of longitudinal evolution of air showers produced by ultra high-energy hadronic particles seem to suggest that pions are more stable than expected [24].

Violations of the Lorentz symmetry may lead to other threshold effects associated to asymmetric momenta in pair creation, photon stability, alternative Čerenkov effects, etc [25,26].

On the theoretical front, work in the context of string/M-theory has shown that Lorentz symmetry can be spontaneously broken due to non-trivial solutions in string field theory [27], from interactions that may arise in braneworld scenarios where our 3-brane is dynamical [28], in loop quantum gravity [29,30], in noncommutative field theories $^1$ [31], and in quantum gravity inspired spacetime foam scenarios [33]. The resulting novel interactions may have striking implications at low-energy [34,35,37,38,39]. Putative violations of the Lorentz invariance may also lead to the breaking of CPT symmetry [40]. An extension of the Standard Model (SM) that incorporates violations of Lorentz and CPT symmetries was developed in Ref. [41].

2 Possible Solutions for the Observational Paradoxes and Experimental Bounds

Potential violations of fundamental symmetries naturally raise the question of how to experimentally verify them. In the case of CPT symmetry, its violation can be experimentally tested by various methods, such as for instance, via neutral-meson experiments [42], Penning-trap measurements and hydrogen-antihydrogen spectroscopy [43]. The breaking of CPT symmetry also allows for a mechanism to generate the baryon asymmetry of the Universe [44]. In what concerns Lorentz symmetry, astrophysics plays, as we have already seen, an essential role. Moreover, it will soon be possible to make correlated astrophysical

$^1$ We mention however, that in a model where a scalar field is coupled to gravity, Lorentz invariance may still hold, at least at first non-trivial order in perturbation theory of the noncommutative parameter [32].
observations involving high-energy radiation and, for instance, neutrinos, which will make viable direct astrophysical tests of Lorentz invariance [9,20,45].

The tightest experimental limit on the extent of which Lorentz invariance is an exact symmetry arises from measurements of the time dependence of the quadrupole splitting of nuclear Zeeman levels along Earth’s orbit. Experiments of this nature can yield an impressive upper limit on deviations from the Lorentz invariance, $\delta < 3 \times 10^{-22}$ [46], and even more stringent bounds according to Ref. [47].

On very broad terms, proposals to explain the three abovementioned paradoxes rely on deformations of the relativistic dispersion relation, that can be written, for a particle species $a$, as:

$$E_a^2 = p_a^2 c_a^2 + m_a^2 c_a^4 + F(E_a, p_a, m_a, c_a),$$

where $c_a$ is the maximal attainable velocity for particle $a$ and $F$ is a function of $c_a$ and of the relevant kinematical variables.

For instance, Coleman and Glashow [7] proposed to explain the observation of cosmic rays beyond the GZK limit assuming that each particle has its own maximal attainable velocity and a vanishing function $F$. This is achieved studying the relevant interaction between a CMB photon and a proton primary yielding the $\Delta(1224)$ hadronic resonance. A tiny difference between the maximal attainable velocities, $c_p - c_\Delta \equiv \epsilon_{p\Delta} \simeq 1.7 \times 10^{-25}c$, can explain the events beyond the GZK cutoff. This bound is three orders of magnitude more stringent than the experimental one. A bound from the search of neutrino oscillations can also be found, even though less stringent, $|\epsilon| \lesssim \text{few} \times 10^{-22}c$ [48]. Interestingly these limits can be turned into bounds on parameters of the Lorentz-violating extension of the SM [9]. As discussed in Ref. [9], a characteristic feature of the Lorentz violating extension of the SM of Ref. [41] is that it gives origin to a time delay, $\Delta t$, in the arrival of signals brought by different particles that is energy independent, in opposition to what is expected from other models (see [37] and references therein), and has a dependence on the chirality of the particles involved as well:

$$\Delta t \simeq \frac{D}{c} \left[ (c_{00} \pm d_{00})_i - (c_{00} \pm d_{00})_j \right],$$

where $c_{00}$ and $d_{00}$ are the time-like components of the CPT-even flavour-dependent parameters that have to be added the fermion sector of the SM so to exhibit Lorentz-violating interactions [41]

$$\mathcal{L}^{\text{CPT-even}}_{\text{Fermion}} = \frac{i}{2} \bar{\psi} \gamma^\mu \frac{\partial}{\partial x} \psi + \frac{i}{2} d_{\mu \nu} \bar{\psi} \gamma_5 \gamma^\mu \frac{\partial}{\partial x} \psi \ .$$

The $\pm$ signs in Eq. (2) arise from the fact that parameter $d_{\mu \nu}$ depends on the chirality of the particles in question, and $D$ is the proper distance of the source.

The function $F$ arising from this SM Lorentz violating extension is given by [9]:

$$F = -2c_{00}E^2 \pm 2d_{00}Ep \ ,$$

where $c_{00}$ and $d_{00}$ are the time-like components of the CPT-even flavour-dependent parameters that have to be added the fermion sector of the SM so to exhibit Lorentz-violating interactions [41].
with \( c_a = c \) for all particles.

It has been argued that some quantum gravity and stringy inspired models (see [30, 37, 38, 39]) lead to modifications of the dispersion relation of the following form:

\[
F = -k_a \frac{p_3}{M_P},
\]

where \( k_a \) is a constant and \( M_P \) is Planck’s mass. This deformation can explain the three discussed paradoxes [30, 37, 38, 39, 25].

At very high energies, deformation (5) can be approximately written as

\[
F \simeq -\frac{E^3}{E_{QG}},
\]

where \( E_{QG} \) is a quantum gravity scale. For photons this leads to the following dispersion relation

\[
pc = E \sqrt{1 + \frac{E}{E_{QG}}},
\]

from which bounds on the quantum gravity scale [20, 37, 38, 45] can be astrophysically determined, the most stringent being [49]

\[
E_{QG} > 4 \times 10^{18} \text{ GeV}.
\]

Another aspect of the problem of violating a fundamental symmetry like Lorentz invariance concerns gravity. A putative violation of Lorentz symmetry renews the interest in gravity theories that have intrinsically built in this feature. From the point of the post-Newtonian parametrization the theory that most closely resembles General Relativity is Rosen’s bimetric theory [50]. Indeed, this theory shares with General Relativity the same values for all post-Newtonian parameters [51]

\[
\beta = \gamma = 1; \quad \alpha_1 = \alpha_3 = \zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = \xi = 0,
\]

except for parameter \( \alpha_2 \) that signals the presence preferred-frame effects (Lorentz invariance violation) in the \( g_{00} \) and \( g_{0i} \) components of the metric. Naturally, this parameter vanishes in General Relativity, but in Rosen’s bimetric theory it is given by

\[
\alpha_2 = \frac{f_0}{f_1} - 1,
\]

where \( f_0 \) and \( f_1 \) are the asymptotic values of the components of the metric in the Universe rest frame, i.e. \( g_{\mu\nu}^{(0)} = diag(-f_0, f_1, f_1, f_1) \), which must be close to the Minkowski metric. A non-vanishing \( \alpha_2 \) implies that angular momentum is not

\footnote{This theory has some difficulties as in its simplest form it does not admit black hole solutions and it is unclear to which extent it is compatible with cosmology.}
conserved. Bounds on this parameter are obtained from the resulting anomalous torques on the Sun, whose absence reveals that $\alpha_2 < 4 \times 10^{-7}$ \cite{52}. It is worth remarking that the other parameters leading to preferred-frame effects are bound by the pulsar PSR J2317+1439 data, that lead to $\alpha_1 < 2 \times 10^{-4}$ \cite{51}, and from the average on the pulse period of millisecond pulsars, which gives $\alpha_3 < 2.2 \times 10^{-20}$ \cite{53}. It is clear that Rosen’s theory deserves a closer examination.

3 Conclusions and Outlook

Lorentz and CPT symmetries may be spontaneously broken in string theory and in some quantum gravity inspired models. Modifications to the relativistic dispersion relation arising from these models allow for explaining the three paradoxes associated to threshold effects in ultra high-energy cosmic rays, pair creation in the propagation of TeV photons and its interaction with the diffuse gamma radiation background, and the longitudinal evolution of high energy hadronic particles in extensive air showers. Confirmation that these phenomena signal the breaking of Lorentz symmetry is an exciting prospect as it would constitute in an unequivocal indication of physics beyond the SM. Near future observations that will be carried by extensive detectors such as by the Auger Observatory \cite{54} may unfold interesting questions and challenges to theory.

References

1. N. Hayashida et al., (AGASA Collab.), Phys. Rev. Lett. 73, 3491 (1994); M. Takeda et al., (AGASA Collab.), Phys. Rev. Lett. 81, 1163 (1998).
2. D.J. Bird et al., (Fly’s Eye Collab.), Phys. Rev. Lett. 71, 3401 (1993); Astrophys. J. 424, 491 (1994); 441, 144 (1995).
3. M.A. Lawrence, R.J.O. Reid and A.A. Watson (Haverah Park Collab.), Journ. Phys. G17, 733 (1991).
4. N.N. Efimov et al., (Yakutsk Collab.), ICRR Symposium on Astrophysical Aspects of the Most Energetic Cosmic Rays, eds. N. Nagano and F. Takahara (World Scientific, 1991).
5. K. Greisen, Phys. Rev. Lett. 16, 748 (1966); G.T. Zatsepin, V.A. Kuzmin, JETP Lett. 41, 78 (1966).
6. H. Sato, T. Tati, Prog. Theor. Phys. 47, 1788 (1972).
7. S. Coleman, S.L. Glashow, Phys. Lett. B405, 249 (1997); Phys. Rev. D59, 116008 (1999).
8. L. Gonzales - Mestres, hep-ph/9905430.
9. O. Bertolami, C.S. Carvalho, Phys. Rev. D61, 103002 (2000).
10. F.W. Stecker, Phys. Rev. Lett. 11, 1016 (1968).
11. C.T. Hill, D. Schramm, T. Walker, Phys. Rev. D36, 1007 (1987).
12. A.M. Hillas, Ann. Rev. Astron. Astrophys. 22, 425 (1984).
13. J.W. Cronin, Rev. Mod. Phys. 71, S165 (1999).
14. G.R. Farrar, T Piran, Phys. Rev. Lett. 84, 3527 (2000).
15. A.V. Olinto, astro-ph/0003013.
16. O. Bertolami, Gen. Rel. Grav. 34, 707 (2002).
17. F. Krennrich et al., Astrophys. J. 560, L45 (2001).
18. F.A. Aharonian et al., *Astron. and Astrophys.* **349**, 11A (1999).
19. A.N. Nikishov, Sov. Phys. JETP **14**, 393 (1962); J. Gould, G. Schreder, Phys. Rev. **155**, 1404 (1967); F.W. Stecker, O.C. De Jager, M.H. Salmon, Astrophys. J. **390**, L49 (1992).
20. G. Amelino-Camelia, J. Ellis, N.E. Mavromatos, D.V. Nanopoulos, S. Sarkar, Nature **393**, 763 (1998).
21. P.S. Coppi, F.A. Aharonian, Astropart. Phys. **11**, 35 (1999).
22. T. Kifune, Astrophys. J. Lett. **518**, L21 (1999).
23. W. Kluźniak, astro-ph/9905308; R.J. Protheroe, H. Meyer, Phys. Lett. B**493**, 1 (2000); G. Amelino-Camelia, T. Piran, Phys. Rev. D**64**, 036005 (2001).
24. E.E. Antonov et al., Pisma ZhETF **73**, 506 (2001).
25. T.J. Konopka, S.A. Major, New J. Phys. **4**, 57 (2002).
26. T. Jacobsen, S. Liberati, D. Mattingly, hep-ph/0209264.
27. V.A. Kostelecký, S. Samuel, Phys. Rev. D**39**, 683 (1989); Phys. Rev. Lett. **63**, 224 (1989).
28. G. Dvali, M. Shifman, hep-th/9904021.
29. R. Gambini, J. Pullin, Phys. Rev. D**59**, 124021 (1999).
30. J. Alfaro, H.A. Morales-Tecotl, L.F. Urrutia, Phys. Rev. Lett. **84**, 2183 (2000).
31. S.M. Carroll, J.A. Harvey, V.A. Kostelecký, C.D. Lane, T. Okamoto, Phys. Rev. Lett. **87**, 141601 (2001).
32. O. Bertolami, L. Guisado, Phys. Rev. D**67**, 025001 (2003).
33. L.J. Garay, Phys. Rev. Lett. **80**, 2508 (1998).
34. O. Bertolami, Class. Quantum Grav. **14**, 2748 (1997).
35. O. Bertolami, D.F. Mota, Phys. Lett. B**455**, 96 (1999).
36. H. Sato, astro-ph/0005218.
37. G. Amelino-Camelia, T. Piran, Phys. Lett. B**497**, 265 (2001).
38. N. Mavromatos, gr-qc/0009045.
39. R. Aloisio, P. Blasi, P.L. Ghia, A.F. Grillo, astro-ph/0001258.
40. V.A. Kostelecký, R. Potting, Phys. Rev. D**51**, 3923 (1995); Phys. Lett. B**381**, 389 (1996).
41. D. Colladay, V.A. Kostelecký, Phys. Rev. D**55**, 6760 (1997); Phys. Rev. D**58**, 116002 (1998).
42. D. Colladay, V.A. Kostelecký, Phys. Lett. B**344**, 259 (1995); Phys. Rev. D**52**, 6224 (1995); V.A. Kostelecký, R. Van Kooten, Phys. Rev. D**54**, 5585 (1996).
43. R. Bluhn, hep-ph/0006033.
44. O. Bertolami, D. Colladay, V.A. Kostelecký, R. Potting, Phys. Lett. B**395**, 178 (1997).
45. S.D. Biller et al., Phys. Rev. Lett. **83**, 2108 (1999).
46. S.K. Lamoreaux, J.P. Jacobs, B.R. Heckel, F.J. Raab, E.N. Fortson, Phys. Rev. Lett. **57**, 3125 (1986).
47. V.A. Kostelecký, C.D. Lane, Phys. Rev. D**60**, 116010 (1999).
48. E.B. Brucker et al., Phys. Rev. D**34**, 2183 (1986); S.L. Glashow, A. Halprin, P.I. Krastev, C.N. Leung, J. Panteleone, Phys. Rev. D**56**, 2433 (1997).
49. G. Amelino-Camelia, gr-qc/0212002.
50. N. Rosen, Gen. Rel. Grav. **4**, 435 (1973); Ann. Phys. (New York) **84**, 455 (1974); Gen. Rel. Grav. **9**, 339 (1978).
51. C.M. Will, “Theory and Experiment in Gravitational Physics” (Cambridge University Press, 1993); “The Confrontation between General Relativity and Experiment: A 1998 Update. gr-qc/9811036.
52. K. Nordtvedt, Astrophys. J. 320, 871 (1987).
53. J.F. Bell, T. Damour, Class. Quantum Grav. 13, 3121 (1996).
54. L. Anchordoqui, T. Paul, S. Reucroft, J. Swain. astro-ph/0206072