Fermionic decays of sfermions in the MSSM: a full one-loop calculation

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We present a full one-loop calculation of the electroweak corrections to the partial decay widths of the fermionic modes of sfermions $\Gamma(\tilde{f} \rightarrow f' \chi)$. The main technical points of the renormalization are presented, and the main features of the results are discussed.

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1. Introduction

Supersymmetry (SUSY) is one of the firmest candidates for physics beyond the Standard Model (SM) of particle interactions. If new particles are discovered at the LHC, the forthcoming step will be to establish their nature. This step could be performed at a high energy $e^+e^-$ linear collider. One of the basic predictions of SUSY is the equality of the couplings in the different sectors: the SM gauge couplings must be equal to the superpartners gauge couplings and to the Yukawa coupling between a particle and its superpartner.

We present a full one-loop computation of the electroweak (EW) corrections to the sfermion partial decay width

$$\Gamma(\tilde{f} \rightarrow f' \chi),$$

\(\tilde{f}\) being a sfermion, \(f'\) a SM fermion and \(\chi\) a chargino \((\chi_{1,2}^\pm)\) or a neutralino \((\chi_{0,1,2,3}^0)\). The process \(\tilde{f} \rightarrow f' \chi\) probes the fermion-sfermion-gaugino/higgsino coupling, and can be used to test the SUSY relations. As we will see, the radiative corrections induce finite shifts in these relations which are non-decoupling.

The QCD corrections to the process \(\tilde{f} \rightarrow f' \chi\), in the framework of the Minimal Supersymmetric Standard Model (MSSM), were computed in \cite{1}. However, the electroweak effects are much more cumbersome as their computation requires the renormalization of the whole MSSM Lagrangian. A first estimate was given in \cite{2}, where the contribution from the Yukawa couplings of bottom quarks decaying into chargino-neutralinos was analyzed within the so-called higgsino approximation. The electroweak effects have been further elaborated in \cite{3}.

2. Renormalization and radiative corrections

The computation to one-loop level of the partial decay width \(\Gamma(\tilde{f} \rightarrow f' \chi)\) requires the renormalization of the full MSSM Lagrangian, taking into account the relations among the different sectors and the mixing parameters. We choose to work in an on-shell renormalization scheme, in which the renormalized parameters are the measured quantities. The SM sector is renormalized according to the standard on-shell SM $\alpha$-scheme \cite{4}, and the MSSM Higgs sector (in particular the renor-
ormalization of tan $\beta$) is treated as in [3].

As far as the sfermion sector is concerned, each squark doublet contains 5 independent parameters. We choose to use the following set of input parameters:

$$(m_{\tilde{b}_1}, m_{\tilde{b}_2}, \theta_b, m_{\tilde{t}_1}, \theta_t), \quad m_{\tilde{f}} > m_{\tilde{f}}.$$ (2)

The remaining parameters are computed as a function of those in (3). In particular, the trilinear soft-SUSY-breaking couplings read:

$A_{\langle b,t \rangle} = \mu (\tan \beta, \cot \beta) + \frac{m_{\tilde{f}}^2 - m_{\tilde{f}}^2}{2m_{\tilde{f}}} \sin 2\theta_f,$ (3)

with $\tan \beta = v_2/v_1$, the ratio of the vacuum expectation values of the two Higgs boson doublets. The mass parameters in (2) are defined to be on-shell. For the mixing angle counterterm we adopt the following convention:

$$\delta \theta_f = \frac{1}{2} \frac{\Sigma^{ij}_L(m_{\tilde{f}}^2) + \Sigma^{ij}_R(m_{\tilde{f}}^2)}{m_{\tilde{f}}^2 - m_{\tilde{f}}^2}.$$ (4)

being $\Sigma^{ij}(k^2)$ the mixing self-energy between sfermions $i$ and $j$. The convention (3) is known to introduce gauge parameter dependence in the definition of the mixing angle, but the result is the same as the gauge independent computation of the self-energies in (1) within the 't Hooft-Feynman gauge [3]. Since the heaviest topsquark mass is not an input parameter, it receives finite radiative corrections:

$$\Delta m_{\tilde{t}_1} = \delta m_{\tilde{t}_1} + \Sigma^{\tilde{t}_1}(m_{\tilde{t}_1}^2),$$ (5)

where $\delta m_{\tilde{t}_1}$ is a combination of the counterterms of the parameters in (2), and the counterterms of the gauge and Higgs sectors.

The chargino/neutralino sector contains six particles, but only three independent parameters ($M$ and $M'$ – the soft-SUSY-breaking $SU(2)_L$ and $U(1)_Y$ gaugino masses– and the higgsino mass parameter $\mu$). The situation in this sector is quite different from the sfermion case, since in this case no independent counterterms for the mixing matrix elements can be introduced. We stick to the following procedure: First, we introduce a set of renormalized parameters ($M, M', \mu$) in the expression of the chargino and neutralino matrices ($\mathcal{M}$ and $\mathcal{M}'$), and diagonalize them by means of unitary matrices $M_D = U^* \mathcal{M} V^*$, $M_D' = N^* \mathcal{M}' N^*$. Now $U$, $V$ and $N$ must be regarded as renormalized mixing matrices. The counterterm mass matrices are then

$$\delta M_D = U^* \delta \mathcal{M} V^*, \quad \delta M_{D'} = N^* \delta \mathcal{M}' N^*,$$ (6)

which are non-diagonal. At this point, we introduce renormalization conditions for certain elements of $\delta M_D$ and $\delta M_{D'}$. In particular, using the unitarity properties of $U$ and $V$ we can write the following set of equations

$$M_1 \delta M_1 + M_2 \delta M_2 = M \delta M + \mu \delta \mu + \delta M_W^\dagger,$$

$$M_1 M_2 (M_1 \delta M_2 + M_2 \delta M_1) = \left[ (M_\mu - M_W^\dagger s_{2\beta}) \right] \times \left[ M \delta M + \mu \delta M \right]$$ (6)

$$- M_W^\dagger \delta s_{2\beta} - s_{2\beta} \delta M_W^\dagger.$$ (6)

$M_1$ and $M_2$ are the chargino on-shell masses, and $\delta M_i$ is a shortcut for the following combination of chargino self-energies:

$$\frac{\delta M_i}{M_i} = - \frac{1}{2} \left( \Sigma^{\tilde{t}_i}(M_i^2) + \Sigma^{\tilde{t}_i}(M_i^2) \right) - \Sigma^{\tilde{b}_i}(M_i^2).$$ (7)

Conditions (4) and (5) are equivalent to one-loop on-shell renormalization of the chargino masses. As a last renormalization condition, we require on-shell renormalization for one of the neutralinos, which we choose the lightest one, and find the expression for the $M'$ counterterm:

$$\delta M' = \frac{1}{N_{1\alpha}} (\delta M_1^\alpha - \sum_{\alpha, \beta \neq 1} N_{1\beta}^* \delta M_{\alpha \beta} N_{1\beta}^*),$$ (8)

where $\delta M_1^\alpha$ is as in (4), but for the lightest neutralino $(\chi_{10}^0)$. Solving eqs. (4) for $\delta M$ and $\delta \mu$, and using eq. (8) for $\delta M'$, the renormalization conditions for the chargino/neutralino sector are complete. The other neutralino masses receive radiative corrections. In this framework the renormalized one-loop chargino/neutralino 2-point functions are non-diagonal. Therefore one must take into account this mixing either by explicitly the reducible $\chi_i - \chi_0$ mixing diagrams, or by means of external mixing wave-function terms, e.g. the left- and right-chiral form factors for the

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2 Throughout this work we make use of third generation notation. The notation is as in [3].
creation of a neutralino $\chi_0^0$ must be multiplied by,

$$Z_{R}^{0\beta\alpha} = \frac{1}{M_R^2 - M_Z^2} \times \left[ M_\beta M_\alpha \Sigma_{SR}^0 (M_0^2) + M_\alpha M_\beta \Sigma_{SL}^0 (M_0^2) + M_\beta M_\alpha \Sigma_{SR}^0 (M_0^2) + M_\alpha M_\beta \Sigma_{SL}^0 (M_0^2) \right], \quad (9)$$

Here $\Sigma_{\pm\alpha}^0(k^2)$ are the renormalized mixing two-point functions $\chi_0^\pm \to \chi_0^0$. Equivalent expressions can be worked out for the chargino sector. See Refs. [7] for different (but one-loop equivalent) approaches to the renormalization of the chargino/neutralino sector.

The complete one-loop computation consists of:
\begin{itemize}
  \item renormalization constants for the parameters and wave functions in the bare Lagrangian,
  \item one-loop one-particle irreducible three-point functions,
  \item mixing terms among the external charginos and neutralinos,
  \item soft- and hard- photon bremsstrahlung.
\end{itemize}

All kind of MSSM particles are taken into account in the loops: SM fermions, sfermions, electroweak gauge bosons, Higgs bosons, Goldstone bosons, Fadeev-Popov ghosts, charginos, neutralinos. The computation is performed in the 't Hooft-Feynman gauge, using dimensional reduction for the regularization of divergent integrals. The loop computation itself is done using the computer algebra packages FeynArts 3.0 and FormCalc 2.2 [8, 9]. The numerical evaluation of one-loop integrals makes use of LoopTools 1.2 [10, 11].

3. Results

The results show the very interesting property that none of the particles of the MSSM decouples from the corrections to the observables [12]. This can be well understood in terms of renormalization group (RG) running of the parameters and SUSY breaking. Take, e.g., the effects of squarks in the electron-electron-photino coupling. Above the squark mass scale ($Q > m_{\tilde{s}}$) the electron electromagnetic coupling ($\alpha(Q)$) is equal (by SUSY) to the electron-electron-photino coupling ($\tilde{\alpha}(Q)$), and both couplings run according to the same RG equations. At $Q = m_{\tilde{s}}$ the squarks decouple from the RG running of the couplings. At $Q < m_{\tilde{s}}$, $\alpha(Q)$ runs due to the contributions from pure quark loops, but $\tilde{\alpha}(Q)$ does not run anymore, and it is frozen at the squark scale, that is: $\tilde{\alpha}(Q < m_{\tilde{s}}) = \alpha(m_{\tilde{s}})$. Therefore, when comparing these two couplings at a scale $Q < m_{\tilde{s}}$, they differ by the logarithmic running of $\alpha(Q)$ from the squark scale to $Q$: $\tilde{\alpha}(Q) / \alpha(Q) - 1 = \beta \log(m_{\tilde{s}} / Q)$.

The above discussion has two important consequences:
\begin{itemize}
  \item 1) the non-decoupling can be used to extract information of the high-energy part of the SUSY spectrum: one can envisage a SUSY model in which a significant splitting among the different SUSY masses exists, e.g. $m_{\tilde{q}} \gg m_{\tilde{t}}$, where the sleptons lie below the production threshold in an $e^+e^-$ linear collider, but the squarks are above it. By means of high precision measurements of the lepton-slepton-chargino/neutralino couplings one might be able to extract information of the squark sector of the model, to be checked with the available data from the LHC.
  \item 2) By the same token, it means that the value of the radiative corrections depends on all parameters of the model, and we can not make precise quantitative statements unless the full SUSY spectrum is known. This drawback can be partially overcome by the introduction of effective coupling matrices, which can be defined as follows. The subset of fermion-sfermion one-loop contributions to the self-energies of gauge-boson, Higgs-bosons, Goldstone-bosons, charginos and neutralinos form a gauge invariant finite subset of the corrections. Therefore these contributions can be absorbed into a finite shift of the chargino/neutralino mixing matrices $U$, $V$ and $N$ appearing in the couplings: $U_{\gamma\gamma} = U + \Delta U_{\gamma\gamma}$, $V_{\gamma\gamma} = V + \Delta V_{\gamma\gamma}$, $N_{\gamma\gamma} = N + \Delta N_{\gamma\gamma}$. In this way we can decouple the computation of the universal (or super-oblique) corrections. These corrections contain the non-decoupling log-
arithmetic from sfermion masses. We have been able to derive an analytic expression containing these logarithms in a simple case. We have computed the electron-selectron contributions to $\Delta U^{(f)}$ and $\Delta V^{(f)}$ matrices, assuming zero mixing angle in the selectron sector ($\theta_s = 0$), we have identified the leading terms in the approximation $m_{\tilde{e}_L}, m_{\tilde{e}} \gg (M_W, M_t) \gg m_e$, and analytically cancelled the divergences and the renormalization scale dependent terms; finally, we have kept only the terms logarithmic in the slepton masses. The result for $\Delta U^{(f)}$ reads as follows:

$$\Delta U^{(f)}_{11} = \frac{2}{4\pi s_W} \log \left( \frac{M^2}{M_X^2} \right) \left[ \frac{\alpha_s}{6} - U_{12} \frac{\sqrt{2} M_W (M_{\tilde{e}_L} + \mu s_b)}{3(M^2 - \mu^2)} \left( M^2 - M^2 + \frac{3}{2} M^2 M_{\tilde{W}}^2 + \frac{3}{2} M^2 M_{\tilde{W}}^2 + M_{\tilde{W}}^4 + M_{\tilde{W}}^4 c_{4\beta} \right) + (\mu^2 - M^2) M_{\tilde{W}}^2 + 4 M \mu M_{\tilde{W}}^4 s_{2\beta} \right]$$

$$\Delta U^{(f)}_{12} = \frac{2}{4\pi s_W} \log \left( \frac{M^2}{M_X^2} \right) U_{11} \times$$

$$\frac{M_{\tilde{W}}^2 (M_{\tilde{e}_L} + \mu s_b)}{3 \sqrt{2} (M^2 - \mu^2) (M^2 - M^2)} \left( M^2 - \mu^2 \right)^2 + \left( \begin{array}{c} (M^2 - \mu^2)^2 + 4 M^2 M_{\tilde{W}}^2 + 4 M \mu M_{\tilde{W}}^2 + 2 M^4 + 2 M^4 c_{4\beta} + 8 M \mu M_{\tilde{W}}^4 s_{2\beta} \end{array} \right)$$

$M_{\tilde{e}_L}^2$ being the soft-SUSY-breaking mass of the $(\tilde{e}_L, \tilde{\nu})$ doublet, whereas $M_X$ is a SM mass. In the on-shell scheme for the SM electroweak theory we define parameters at very different scales, basically $M_X = M_W$ and $M_X = m_e$. These wide-ranging scales enter the structure of the counterterms and so must appear in eq.(10) too. As a result the leading log in the various terms of this equation will vary accordingly. For simplicity in the notation we have factorized $\log M_{\tilde{e}_L}^2/M_X^2$ as an overall factor. In some cases this factor can be very big, $\log M_{\tilde{e}_L}^2/M_X^2$; it comes from the electron-selectron contribution to the chargino-neutralino self-energies.

Although above we have singled out the non-decoupling properties of sfermions, we would like to stress that the whole spectrum shows non-decoupling properties. By numerical analysis we have been able to show the existence of logarithms of the gaugino mass parameters ($M/M_X$ and $M'/M_X$), and the Higgs mass ($M_h/M_X$). However, due to the complicated mixing structure of the model, we were not able to derive simple analytic expressions containing these non-decoupling logarithms. Note that in any observable which includes the fermion-sfermion-chargino/neutralino Yukawa couplings at leading order we will have this kind of corrections, therefore the full MSSM spectrum must be taken into account when computing radiative corrections, since otherwise one could be missing large logarithmic contributions of the heavy masses.

As for the non-universal part of the contributions, we summarize here the main features:

a) The corrections grow as the logarithm squared of the decaying particle mass, due to the presence of electroweak Sudakov double-logs.

b) The corrections show multiple threshold singularities when varying any of the parameters of the model, however they are well behaved in the regions between thresholds. Therefore, in order to give a quantitative value of the corrections the correlation between the different particle masses must be known.

c) The third sfermion family contains a large contribution that can be traced back to the presence of corrections similar to the threshold-like corrections to the bottom-quark (and τ-lepton) Yukawa coupling $\Delta m_{(b, \tau)}$ [12]. This contribution contains (aside from $\Delta m_{(b, \tau)}$) terms from the sfermion mixing self-energies $\tilde{f}_L - \tilde{f}_R$, and

|     | $\chi_1^0$ | $\chi_2^0$ | $\chi_3^0$ | $\chi_4^0$ | $\chi_1^+$ |
|-----|------------|------------|------------|------------|------------|
| Tree | 0.272      | 0.092      | 0.047      | 0.014      | 0.575      |
| QCD  | 0.308      | 0.104      | 0.031      | 0.018      | 0.538      |
| Total| 0.291      | 0.092      | 0.031      | 0.018      | 0.568      |

Table 1

Branching ratios of bottom-squarks into charginos and neutralinos for the parameter set [13]. Shown are: the tree-level value, the QCD-corrected value, and the value including EW corrections. Branching ratios below $10^{-3}$ are not shown.
The three-point vertex functions contain the soft SUSY-breaking trilinear Higgs-sfermion-sfermion couplings $A_f$. If $A_f$ is large, it would induce large corrections (even larger than 100%!)
However, as long as $A_f$ is in the perturbative coupling regime, the corrections stay below 20%.

Points c) and d) play a complementary role in the large $\tan \beta$ regime, the corrections stay below 20%.

A small mass splitting (or small mixing angle) means small $\Delta m_{(b,\tau)}$. However a large $A_f$ value would both spoil perturbativity and generate charge and/or colour breaking vacua. Therefore, at large $\tan \beta$ the bottom-squarks and $\tau$-sleptons must have significant splitting and mixing angle, providing large corrections of the type c) above.

The $\Delta m_{(b,\tau)}$ corrections play also an important role in the phenomenology of the MSSM Higgs bosons at large $\tan \beta$, showing non-decoupling properties for $M_{SUSY} \gg M_{SM}, M_{H^\pm}$, as stressed e.g. in [13]. The $f_a-f^\prime-\chi$, coupling, on the other hand, exhibits a new kind of non-decoupling properties, which are independent of the value of $\tan \beta$.

As an example of the numerical value of the corrections, we present in Table 1 the tree-level and corrected branching ratios for bottom-squarks, for the following set of input parameters:

\begin{align}
  m_t &= 175 \text{ GeV} , m_b = 5 \text{ GeV} , \tan \beta = 4 , \\
  m_{\tilde{b}_2} &= m_{\tilde{d}_2} = m_{\tilde{u}_2} = 300 \text{ GeV} , \\
  m_{\tilde{t}_1} &= m_{\tilde{d}_1} = m_{\tilde{e}_1} = m_{\tilde{b}_2} + 5 \text{ GeV} , \\
  m_{\tilde{e}_2} &= 290 \text{ GeV} , m_{\tilde{\tau}_2} = 300 \text{ GeV} , \\
  \theta_b &= \theta_d = \theta_u = \theta_e = 0 , \theta_t = -\pi/5 , \\
  \mu &= 150 \text{ GeV} , M = 250 \text{ GeV} , \\
  M_{H^\pm} &= 120 \text{ GeV} , m_{\tilde{g}} = 500 \text{ GeV} .
\end{align}

The results of Table 1 show that a high precision measurement of the partial decay branching ratios of sfermions, to be performed at a high energy $e^+e^-$ linear collider, is sensitive to the EW corrections. From Table 1 it is also clear that the EW corrections can induce a change on the branching ratios of the leading decay channels comparable to the QCD corrections. Therefore both contributions must be taken into account on equal footing in the analysis of the phenomenology of sfermions.

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