Magnetic field induced flow pattern reversal in a ferrofluidic Taylor-Couette system

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We investigate the dynamics of ferrofluidic wavy vortex flows in the counter-rotating Taylor-Couette system, with a focus on wavy flows with a mixture of the dominant azimuthal modes. Without external magnetic field flows are stable and pro-grade with respect to the rotation of the inner cylinder. More complex behaviors can arise when an axial or a transverse magnetic field is applied. Depending on the direction and strength of the field, multi-stable wavy states and bifurcations can occur. We uncover the phenomenon of flow pattern reversal as the strength of the magnetic field is increased through a critical value. In between the regimes of pro-grade and retrograde flow rotations, standing waves with zero angular velocities can emerge. A striking finding is that, under a transverse magnetic field, a second reversal in the flow pattern direction can occur, where the flow pattern evolves into pro-grade rotation again from a retrograde state. Flow reversal is relevant to intriguing phenomena in nature such as geomagnetic reversal. Our results suggest that, in ferrofluids, flow pattern reversal can be induced by varying a magnetic field in a controlled manner, which can be realized in laboratory experiments with potential applications in the development of modern fluid devices.

Reversal of a fluid flow upon parameter changes or perturbation is closely related to intriguing phenomena such as geomagnetic reversal, a drastic change in a planet’s magnetic field where the positions of magnetic north and south are interchanged1. Typically, for a planet there is dynamo action in which convection of molten iron in the core produces electric currents, generating geomagnetic field. The reversal of the molten iron flow direction can cause the geomagnetic field to switch the poles. Computational fluid models incorporating the interaction between electromagnetism and fluid dynamics, e.g., in the Earth’s interior, were developed2–6 to account for the complete flip flop of the geomagnetic field which can occur within a few 10000 years of each other. There was also a recent experimental study of liquid metal in which global field reversals occurred at irregular time intervals7. To study the dynamical mechanism and controlled generation of flow reversal is of interest.

In this paper, we report magnetic-field induced flow pattern reversals in the classic Taylor-Couette system (TCS)8, which can exhibit a large number of flow structures of distinct topologies and has been an experimental and computational paradigm for investigating many fundamental phenomena in fluid dynamics for decades9–13. In our study, we consider ferrofluid14 in between the cylinders, the dynamics of which constitute an area of interest with a variety of applications ranging from embedded fluidic devices in computer hard drives to laboratory experiments designed to probe into the fundamentals of geophysical flows15,16. Generally, a ferrofluid consists of a conventional fluid with embedded nano-sized, magnetized particles. In the absence of any external magnetic field, the magnetic moments of the nanoparticles are randomly oriented, leading to zero net magnetization for the entire fluid. However, an external magnetic field can have a drastic effect on the fluid and its dynamics. Usually, the axial component of the magnetic field does not tend to change the physical properties of the fluid but it can shift the various bifurcation points of the flow structures and patterns. For example, when the magnetic field is entirely axial, the onset of the basic rotational state in TCS tends to shift toward a larger value of the bifurcation parameter17,18. The transverse component of the external magnetic field, however, additional to a shift alters the physical properties of the fluid dramatically17,19,20, leading to characteristic or even fundamental changes in the underlying hydrodynamics. Especially, we demonstrate that, as the magnitude of the magnetic field

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is systematically changed, the system can exhibit repeated reversals in the ferrofluidic wavy vortex flow pattern in between the two rotating cylinders.

The TCS with conventional fluid or with ferrofluid but without external magnetic field typically possesses a large number of solutions with distinct dynamical properties, some of which can coexist in a wide range of parameters. For example, it was shown earlier that different wavy states can occur \( \text{for different same parameters (e.g., the Reynolds number Re and the radius ratio between the two cylinders). It was also discovered that, as Re is increased, the wavy flows can become supercritical, and the wave speed and the angular velocity tend to decrease monotonically and approach an asymptotic value. It was also discovered that for TCS with a wide gap between the inner and outer cylinders (e.g., radius ratio below \( < 0.8 \)), the waviness is usually dominated by low azimuthal modes of low speed and angular frequency. For TCS with a ferrofluid, as an external magnetic field is applied, one might intuitively expect the basic rotational state to be stabilized with an increasing angular velocity. However, our study demonstrates that the flow dynamics can become much more complicated than this intuitive picture would suggest. We note that, recent studies on the transition to turbulence in ferrofluidic flows suggest the possibility to control turbulence through an applied magnetic field.

Our work was motivated by the recent discovery in TCS with conventional fluid that the azimuthal waviness can change the direction of the rotation. In particular, as the radius ratio is increased, the wavy flows can become co-rotating, and a reversal in the direction of rotation can occur, which is typically associated with a change in the azimuthal symmetry. For example, accompanying the flow pattern reversal, transitions from three-fold to two-fold or even to one-fold azimuthal waviness were observed. Curiosity thus demanded that we ask what might happen when the conventional fluid is replaced by a ferrofluid and a magnetic field is present. We find that, as the strength of the magnetic field is increased, flow pattern reversal can occur. However, in contrast to the TCS with conventional fluid, the reversals are in fact smooth transitions without any change in the azimuthal symmetry of the underlying flows. In particular, with variation in the strength of either axial or transverse magnetic field, the fluid motion can change from pro-grade to retrograde wavy flow patterns separated by stable interim, standing-wave solutions with zero angular velocities. These standing waves are part of an other class of flow states, mixed-ribbon. For example, the azimuthal contributions to the wavy flows of azimuthal wave numbers \( m = 2 \) and \( m = 3 \) are maintained with variations only in their strength. A more striking phenomenon is that, as the strength of a transverse magnetic field is increased, the flow pattern can change its direction twice - from pro-grade to retrograde and back to pro-grade, a phenomenon that has not been observed in any study of the TCS. Our results suggest that flow pattern reversals in the ferrofluidic TCS can be controlled through an external magnetic field, which is not only fundamentally interesting but also relevant for practical development of novel fluid devices.

Results

Nomenclature. In this work we focus on toroidally closed wavy solutions, meaning that the axisymmetric Fourier mode, i.e. the azimuthal wavenumber, \( (m = 0) \) is always the strongest/largest. The corresponding flows possess different (pronounced) azimuthal modes for non-axisymmetric contributions \( (m \neq 0) \). For most wavy flows studied, there are two different azimuthal modes, \( m_z, m_s \), typically with unequal contributions. To specify the flow patterns, we use the following notations: \( \text{WVF}_{m_z, m_s} \) for a wavy vortex flow solution with dominant \( (m_z) \) azimuthal wavenumber \( m_z \) and subordinated \( (m_s) \) azimuthal wavenumber \( m_s \) (apart from the strongest mode \( m = 0 \)). Of particular interest are wavy solutions \( \text{WVF}_{3,2}, \text{WVF}_{3,2}, \text{WVF}_{2,3}, \text{WVF}_{3,2}, \text{WVF}_{2,3} \), and \( \text{WVF}_{2,3} \), with the last having azimuthal contribution than \( \text{WVF}_{2,3} \). Both flow patterns exhibit a pronounced \( m = 2 \) contribution, which is visible in the azimuthal vorticity \( \eta \) (in the \( \theta, \phi \) plane) at mid-height, as shown in Fig. 1(b) on an unrolled cylindrical surface in the annulus at mid-gap. The more complex pattern of \( \text{WVF}_{3,2} \) illustrates a stronger influence of the \( m = 3 \) contribution than \( \text{WVF}_{2,3} \). Both flow patterns exhibit a pronounced \( m = 2 \) contribution, which is visible in the azimuthal vorticity \( \eta \) (in the \( \theta, \phi \) plane) at mid-height, as shown in Fig. 1(c). The centerline of the vortices for \( \text{WVF}_{3,2} \) is located closer to the inner cylinder than the one for \( \text{WVF}_{2,3} \), which can also be seen in the vector plots \( [u(r, \phi), w(r, \phi)] \) of the radial and axial velocity components in a constant \( \theta \) plane, as shown in Fig. 1(d). In order to gain insights, we highlight the two points that mark the centerline of vortices within the illustrated plane. However, it is important to note that the position as well as the entire flow profile are \( \theta \)-dependent. Nonetheless, the azimuthal averaged position (about the whole cylinder) of the centerline of vortices for \( \text{WVF}_{3,2} \) is closer to the inner cylinder than for \( \text{WVF}_{2,3} \) [See movie files movie1. avi and movie2.avi in Supplementary Materials (SMs), where the pro-grade rotation can be identified in 1D isosurfaces and contours of the respectively structure].

With respect to the comparative values of \( m \) between the flow states \( \text{WVF}_{2,3} \) and \( \text{WVF}_{3,2} \), we show in Fig. 2 variations with time \( t \) of the dominant flow field mode amplitudes \( u_{1,1}, u_{2,2}, u_{3,3} \), which are present in \( \text{WVF}_{2,3} \)
Since we focus on wavy flows with toroidal closed symmetry, the azimuthal component $m = 0$ is always dominant (cf., axis scaling in Fig. 2) and the wavy modulation actually originates from the higher azimuthal modes (e.g., $m = 2$ and $m = 3$). Depending on the amplitudes of these modes, especially the amplitude ratio, the combined solution can exhibit predominantly a 2-fold (WVF$_{2,3}$) or a 3-fold (WVF$_{3,2}$) symmetry. To characterize the corresponding states we used the time-averaged value $m$. For example, in Fig. 2(a), $π_{2,1}$ is significantly larger than $π_{3,1}$ so that the flow is designated as WVF$_{2,3}$. In general, in the TCS various combinations of different azimuthal wave numbers are possible, which can result in complex, mixed states such as mixed-cross-spiral

**Figure 1. Flow structures in absence of any magnetic field.** Flow states of WVF$_{2,3}$ (left panels) and WVF$_{3,2}$ (right panels) for $s_z = 0$ and $s_y = 0$. (a) Isosurfaces of $η = ± 240$. Red (yellow) color indicates positive (negative) vorticity. For clear visualization, two periods are plotted in the axial direction. The arrows below the snapshots illustrate the rotational direction (appearing laterally on the cylinders), which is pro-grade for both wavy flows. Note, here and in the following the rotation direction always refers to the pattern rotation. Contours of the flow structures: (b) radial velocity $u(θ, z)$ on an unrolled cylindrical surface in the annulus at mid-gap, (c) azimuthal vorticity $η$ in the $(r, θ)$ plane at mid-height, and (d) vector plots $(u(r, z), w(r, z))$ of the radial and axial velocity components in a constant $θ = π$ plane, including the azimuthal vorticity $η$. The two points mark the centerline of vortices within the illustrated plane. Note that this position is $θ$-dependent but the azimuthal averaged position for WVF$_{2,3}$ is closer to the inner cylinder than for WVF$_{3,2}$. All contours are color coded from red (dark gray - minimum) to yellow (light gray - maximum). See also movie files movie1.avi and movie2.avi in SMs.
patterns. Differing from such mixed states, the predominant mode in our system is always the azimuthal symmetric \( m = 0 \) mode, so the underlying flow structures remain toroidally closed. It is possible that, when some parameters are systematically varied, a WVF3,2 solution can change to WVF3,2, and vice versa.

Pro-grade and retrograde flows in presence of a magnetic field. In TCS the rotational direction of the flow pattern is defined in terms of the relative speed of rotation of the inner and outer cylinders (i.e., the Reynolds numbers \( Re_1 \) and \( Re_2 \), respectively). Without any magnetic field, the most common case is that the flow pattern follows the rotational direction of the inner cylinder. Only for quite strong counter-rotations (e.g., larger \( Re_2 \) value as compared to the value of \( Re_1 \)) does the flow pattern follow the rotational direction of the outer cylinder. We fix both \( Re \) values with the ratio \( Re_2 / Re_1 \approx -0.4143 \). Without any magnetic field, both wavy flows are “normal” in the sense that they rotate along the rotational direction of the inner cylinder, i.e., pro-grade motion.

Figure 3 illustrates the variations in the angular velocity \( \Omega \) of the wavy states in the presence of axial \( \dot{z} \neq 0 \) but \( \dot{x} = 0 \), (a) and transverse \( \dot{x} \neq 0 \) but \( \dot{z} = 0 \), (b) magnetic field. Vertical dotted and dashed lines indicate the points of non-rotating flow pattern, i.e., standing waves, which are the reversal points of the motion for WVF2,3 and WVF3,2, respectively. The horizontal black line indicates zero angular velocity. For axial field strength \( \dot{z} \geq 0.812 \), there is wavy flow WVF2 without any \( m = 3 \) contribution, where the short thin gray line marks the bifurcation point. With axial field only WVF3,2 changes its propagating direction for \( \dot{z} \approx 0.427 \). With transverse field both wavy flows change their propagation directions: WVF3,2 at \( \dot{x} \approx 0.209 \) and WVF2,3 twice, first for \( \dot{x} \approx 0.309 \) to retrograde and second for \( \dot{x} \approx 0.772 \) back to prograde behavior. For WVF2,3 the minimal angular velocity \( \Omega_{\text{min}} = -0.325 \) is achieved for \( \dot{x} \approx 0.58 \).
the same results obtained previously\textsuperscript{27}, taking into consideration the fact that a magnetic field tends to stabilize the basic rotational state\textsuperscript{27,28}. In the large $s_z$ regime, the angular velocity of WVF$_{2,3}$ decreases monotonously which, for $s_z \approx 0.427$, becomes zero, effectively turning WVF$_{2,3}$ into a standing wave. Note, that these are special kind of standing waves, mixed-ribbons\textsuperscript{28}. For $s_z \gtrsim 0.427$, the flow pattern becomes retrograde, rotating in the opposite direction to that of the inner cylinder [See movie file movie4.avi in SM for WVF$_{2,3}$ at $s_z = 1.0$. For a transverse magnetic field, WVF$_{2,3}$ exhibits qualitatively similar changes with the transition from pro-grade to retrograde motions occurring at $s_z \gtrsim 0.209$, as shown in Fig. 3(b) [See movie file movie7.avi in SM for WVF$_{2,3}$ at $s_z = 1.0$]. The difference between the cases of transverse and axial magnetic field is that, for the former, the magnitude of the angular velocity $\Omega$ for larger $s_z$ values is much higher than that for the latter.

For the flow WVF$_{2,3}$, the behavior of the angular velocity $\Omega$ is dramatically different for different field directions. For a transverse field, as $s_z$ is increased, $\Omega$ decreases continuously, vanishes for $s_z \approx 0.309$, and becomes negative as $s_z$ is increased further [See movie file movie5.avi in SM for WVF$_{2,3}$ at $s_z = 0.7$], similar to the behavior of the flow WVF$_{3,2}$. As $s_z$ is further increased, $\Omega$ reaches minimum at $s_z \approx 0.58$ and begins to increase from the minimum afterwards. For $s_z \approx 0.772$, $\Omega$ is zero again and becomes positive as $s_z$ is increased [See movie file movie6.avi in SM for WVF$_{3,2}$ at $s_z = 1.0$]. Thus the flow pattern of WVF$_{3,2}$ reverses twice: from pro-grade to retrograde and back to pro-grade. This is quite different from the situation of an axial magnetic field, where no flow pattern reversal takes place for WVF$_{2,3}$ and the pattern remains to be pro-grade. Another difference is that, in the presence of a transverse magnetic field, there is always contribution to the flow pattern from the $m = 3$ mode. (The relative contributions from the $m = 2$ and $m = 3$ modes will be detailed below.) Increasing or decreasing the $Re$ values, we expect the curves for $\Omega$ to move upwards or downwards, respectively. For certain value of $Re,\Omega(\text{WVF}_{2,3})$ can no longer reach zero. In such a case, no flow pattern reversal would occur. For instance, in the parameter regime of weakly counter-rotating cylinders, we find that the curve for WVF$_{2,3}$ Fig. 2(d) will move away from $\Omega = 0$ towards large, positive values of $\Omega$, rendering prograde the underlying flow.

**Bifurcations with magnetic field.** Figure 4 shows the variation in the radial velocities of the wavy flows in the presence of an applied transverse ($s_x$) or axial ($s_z$) magnetic field. In order to characterize the flow structures, we examine radial flow field amplitudes $|u_m|$ at mid-gap and display the contributions from the dominant, axisymmetric ($m = 0$) mode, as well as those from the dominant non-axisymmetric ($m = 0$) modes embedded in the underlying flow structure. For the parameter setting in Fig. 4, the dominant modes are $(0,1),(2,1)$ and $(3,1)$ for either WVF$_{2,3}$ and WVF$_{3,2}$ flows. We see that a purely axial field does not change the structure of the flow pattern in the real space nor the mode structure in the Fourier plane $(m, n)$\textsuperscript{27,28}. In contrast, when the magnetic field has a finite transverse component $s_x \neq 0$, the structures are changed due to excitation of higher-order modes $m = \pm 2$, as described in detail in ref. 17.

For increasing axial field strength $s_x$, all mode amplitudes for WVF$_{2,3}$ and WVF$_{3,2}$ decrease monotonically, as shown in Fig. 4(a). However, as the transverse component of the magnetic field is strengthened, all the mode amplitudes increase monotonically, as shown in Fig. 4(b). In fact, similar behaviors occur for the unstable TVFs. Examining the corresponding frequencies of the complex mode amplitudes [Fig. 4(c,d)], we see that $\omega_{1,2}(\text{WVF}_{2,3})$ ’s approaching zero at $s_z \approx 0.427$ or $s_z \approx 0.209$ exhibits a similar behavior with respect to axial or transverse magnetic field: it is the critical point at which the rotational direction of the wavy flow pattern WVF$_{2,3}$ reverses. For sufficiently large value of $s_z$, $\omega_1(\text{WVF}_{2,3})$ crosses zero and becomes negative, enforcing the entire flow pattern in the retrograde direction and resulting in a large negative value of the angular velocity $\Omega$ [cf., Fig. 3(d)]. We also find that the angular velocity $\Omega$ of WVF$_{2,3}$ is determined by the two frequencies $\omega_{2,1}(\text{WVF}_{2,3})$ and $\omega_{3,1}(\text{WVF}_{2,3})$. The first reversal in the flow pattern direction is associated with the vanishing of $\omega_{2,1}(\text{WVF}_{2,3})$ at $s_z \approx 0.309$, and the second reversal occurs when $\omega_{3,1}(\text{WVF}_{2,3})$ approaches zero for the second time at $s_z \approx 0.772$. The first time that $\omega_{2,1}(\text{WVF}_{2,3})$ becomes zero at $s_z \approx 0.378$ only leads to an enhancement of the retrograde behavior with no effect on the flow pattern direction. The frequency $\omega_{3,1}(\text{WVF}_{2,3})$ reaches its minimum at $s_z \approx 0.503$, which is different from the value of $s_z$ for which the angular velocity is minimized [cf., Fig. 3]. The reason is that both frequencies $\omega_{2,1}(\text{WVF}_{2,3})$ and $\omega_{3,1}(\text{WVF}_{2,3})$ are negative but the magnitude of $\omega_{3,1}(\text{WVF}_{2,3})$ is larger. A general observation is that increasing $s_z$ reduces the flow complexity but an increase in $s_z$ plays the opposite role, i.e., making the flow more complex.

**Flow patterns in an axial magnetic field.** Figure 5 shows the isosurfaces of the azimuthal vorticity $\eta$ over two axial wavelengths for wavy flows at two values of the axial magnetic field strength: $s_z = 0.7$ and $s_z = 1.0$ [See movie files movie3.avi and movie4.avi in SM.]. The spatiotemporal structures are qualitatively the same as for the case without any magnetic field (Fig. 1). As $s_z$ is increased, modulations in all flow structures become weaker. The $m = 3$ contribution to the flow is either reduced (Fig. 4) or vanishes completely, as is visible from the pattern of WVF, for $s_z = 1.0$. In general, an axial magnetic field weakens the waviness of the underlying flows.

Figure 6 shows the contour plots for the same wavy flows and values of $s_z$ as in Fig. 5. As shown in Fig. 6(a), the behavior of the radial velocity $u(\theta, r)$ on an unrolled cylindrical surface at mid-gap demonstrates a mixture of the azimuthal modes $m = 2$ and $m = 3$ for WVF$_{2,3}$ and WVF$_{3,2}$. The corresponding plot for WVF$_{3,2}$ contains the $m = 2$ mode as the only non-axisymmetric component. The symmetries are apparent in the contour plots of the azimuthal vorticity in the $(r, \theta)$ plane, as shown in Fig. 6(b). The clear two-fold symmetry can be seen for WVF$_{3,2}$. The predominance of the $m=2$ mode can also be seen from the behavior of WVF$_{2,3}$ [Fig. 6(a)], but such a dominance is not apparent in WVF$_{3,2}$ [Fig. 6(b)] because it contains contributions from both $m = 2$ and $m = 3$ modes [cf., Fig. 4]. For $s_z=1.0$, WVF$_{3,2}$ is dominated by the $m = 3$ mode. From the vector plots of the radial and axial velocity
Flow patterns in a transverse magnetic field. Figure 7 shows the isosurfaces of the azimuthal vorticity $\eta$ over two axial wavelengths for wavy flows for transverse magnetic field strength $s_x$. [See movie files movie5.avi, movie6.avi and movie7.avi in SM.] Compared with the structures under an axial magnetic field, [Fig. 5], we observe more complex wavy flows. The isosurfaces of $\eta$ are much more intertwined in a transverse magnetic field compared to the ones in an axial magnetic field [Fig. 5]. The increased complexity can be better seen in the contour plots in Fig. 8 for WVF2,3 and WVF3,2. The plots of the radial velocity $u$ on an unrolled cylindrical surface at mid-gap [Fig. 8(a)] illustrate the more complex flow structures in the transverse magnetic field due to the azimuthal modes $m = 2$ and $m = 3$ in the WVF2,3 and WVF3,2 flow. For $s_x = 0.7$, both WVF2,3 and WVF3,2 contain a strong contribution from the $m = 2$ mode, which is visible in the azimuthal vorticity plot in the $(r, \theta)$ plane [Fig. 8(b)]. In fact, WVF3,2 for $s_x = 1.0$ does not exhibit any predominant symmetry due to similar amplitudes of the $m = 2$ and $m = 3$ modes [Fig. 4(b)]. In general, the increase in the complexity for larger field strength $s_x$ can be attributed to the enhanced mode amplitudes (cf., Fig. 4) resulting from the intrinsic stimulation of higher ($m = \pm 2$) modes under a transverse magnetic field17.

Behavior of the angular momentum and torque. To better characterize the flow pattern reversal phenomenon, we examine the behaviors of the angular momentum and torque for a variety of flow structures. Figure 9(a–e) show the mean (axially and azimuthally averaged) angular momentum $L(r) = r \langle v(r) \rangle \frac{\rho}{\mu} \Omega$ scaled with the inner Reynolds number, versus the radius $r$ for three different combinations of the magnetic field strength $s_x$ and $s_z$, respectively. The black short dashed curve shows the angular momentum for the unstable TVF and wavy flows show the typical behav-
ior that the angular momentum is transported outwards from the inner cylinder. All curves have similar shape with increased slope (gradient) of $L_r$ near the boundaries and reduced gradient in the interior. For any combination of the field strength $s_z$ and $s_x$, the unstable TVF has the steepest gradient of $L_r$ at the outer boundary, due to the stronger torque on the outer cylinder than the wavy flows and CCF. For sufficiently large values of the magnetic field strength, e.g., $s_z \geq 0.35$ or $s_x \geq 0.4$, the unstable TVF has the steepest slope at the inner boundary layer, as shown in Fig. 9(a–e).

In the absence of any magnetic field [Fig. 9(a)], the gradient of $L_r$ near the inner cylinder is the largest for WVF$_{2,3}$, the smallest for WVF$_{3,2}$, and intermediate for the unstable TVF. In this case, WVF$_{2,3}$ (WVF$_{3,2}$) has the largest (smallest) torque on the inner cylinder. For WVF$_{3,2}$, there is a plateau in the gradient near the central region, but the unstable TVF exhibits the plateau at larger values of $L_r$. In the presence of an axial or a transverse magnetic field, the gradients of $L_r$ for both types of wavy flows are quite similar and differ only slightly in the fluid interior. In general the plateaus for WVF$_{2,3}$ are at slightly higher values of $L_r$ compared with those for WVF$_{3,2}$, with larger differences in an axial than in a transverse magnetic field. Near the boundaries the difference diminishes. In general, a magnetic field decreases the gradient of $L_r$ near the inner cylinder, consequently reducing the torque on the boundary layers.

The behaviors of dimensionless torque $G = \nu J^\omega$ with field strength $s_z$ and $s_x$, are shown in Fig. 9(g). In calculating the torque we used the fact that for a flow between infinite cylinders the transverse current of the azimuthal motion, $J^\omega = r^3 \left[ (\nu \omega)_{A,1} - \nu \left( \partial_r \omega \right)_{A,1} \right]$ (with $\langle \ldots \rangle_A \equiv \int_{A,1} \langle \ldots \rangle_r dr$), is a conserved quantity$^{29}$. Thus the dimensionless torque is the same at the inner and the outer cylinders. As the axial magnetic field strength $s_z$ is increased, the torque $G$ decreases monotonically for WVF$_{3,2}$ but it increases monotonically for WVF$_{2,3}$ and unstable TVF. When a transverse magnetic field is applied, $G$ decreases with $s_z$ for all flow types examined. We thus see that, while either an axial or a transverse magnetic field can stabilize the basic flow states, their effects on the dynamical behaviors of the flows can be quite different.

Conclusions

The phenomenon of flow pattern reversal is interesting as it is relevant to intriguing natural phenomena such as geomagnetic reversal. We study computationally the reversal of ferrofluidic wavy flows in the classic counter-rotating Taylor-Couette system. In absence of any magnetic field all wavy flows are pro-grade in the sense that they rotate in
the same direction as the inner cylinder. However, an axial or a transverse magnetic field can slow down the flow, leading to standing waves and subsequently to reversal into a retrograde flow state. For an axial magnetic field, there can be at most one reversal. However, for a transverse magnetic field, a second reversal can occur at which the flow pattern becomes pro-grade again. All these can occur when a single parameter, the magnetic field strength, is increased. We elucidate the structural properties of the flow by examining the variations in the significant mode amplitudes and frequencies with the magnetic field. A general finding is that a transverse magnetic field can be more effective in generating flow pattern reversal, due to the stimulation of higher modes17 (cf., Figs 4 and 7).

Figure 10 summarizes our findings in terms of the sequences of wavy flow patterns with either increasing axial magnetic field strength \( s_z \) or increasing transverse magnetic field strength \( s_x \), where large arrows indicate the rotating directions of the corresponding flow pattern, which appear lateral on the cylinder, and the sequences are the same for small \( s_x \) and \( s_z \) values. We see that WVF3,2 has a single reversal in its direction of motion with \( s_x \) or \( s_z \).

WVF2,3 does not exhibit any flow pattern reversal under an axial magnetic field but it changes the rotating direction twice with continuous increase in the strength \( s_x \) or \( s_z \). WVF2,3 does not exhibit any flow pattern reversal under an axial magnetic field but it changes the rotating direction twice with continuous increase in the strength \( s_x \) or \( s_z \) of a transverse magnetic field.

It may be interesting to investigate the effects of combined axial and transversal magnetic field. It has been known that interactions among the modes can increase the flow complexity17. It may also be useful to study more realistic system sizes because previous experimental20 and computational19 studies revealed that an applied magnetic field can change the number of vortices, or the wavenumber, in the bulk. It would be insightful to study whether this can occur for wavy flows, especially in terms of the effects on the azimuthal component. Moreover the effects of magnetic fields on other wavy flows with different topology as helical wavy spiral states30 may be interesting, particularly for realistic axial boundary conditions.

We hope that our computational results will stimulate experimental works on ferrofluidic wavy flows. Since the setting of our computation and the choices of the simulation parameters are experimentally motivated, it may be feasible to realize flow pattern reversal in experiments. For example, our ferrofluid APG933 and the typical magnetic field of 100 [kA/m] (about 0.968 in terms of \( s_x \) or \( s_z \)) are realizable in laboratories. Exploiting other ferrofluids such as those based on Cobol can reduce the required magnetic field strength20. Control of flow pattern reversal through variations of the external magnetic field appears promising.

**Methods**

**Ferrohydrodynamical equation of motion.** Consider a TCS consisting of two concentric, independently rotating cylinders with an incompressible, isothermal, homogeneous, mono-dispersed ferrofluid of kinematic viscosity \( \nu \) and density \( \rho \) within the annular gap. The inner and outer cylinders of radii \( R_1 \) and \( R_2 \) rotate at the angular speeds \( \omega_1 \) and \( \omega_2 \), respectively. The boundary conditions at the cylinder surfaces are of the non-slip type but axially periodic boundary conditions of period (length) \( \Gamma \) are used. The system can be described in the cylindrical coordinate system \((r, \theta, z)\) with the velocity field \( \mathbf{u} = (u, v, w) \) and the corresponding vorticity...
\[ \nabla \times \mathbf{u} = (\xi, \eta, \zeta). \] We set the radius ratio of the cylinders and the parameter \( \Gamma \) to typical values used in experiments, e.g., \( R_1/R_2 = 0.5 \) and \( \Gamma = 1.6 \), where the latter corresponds to an axial wavenumber \( k = 3.927 \). A homogeneous magnetic field \( H = H_\perp, e_\perp, H_\parallel, e_\parallel \) is applied, where \( H_\perp \) and \( H_\parallel \) are the field strengths in the transverse \((x = r \cos \theta)\) and axial directions \((z)\), respectively. To keep the setting as simple as possible for uncovering new phenomena, we assume that the magnetic field is either transverse or axial, i.e., either \( H_\parallel = 0 \) or \( H_\perp = 0 \), as the presence of both transverse and axial components in the field can give rise to unnecessary complications.

The gap width \( d = R_2 - R_1 \) is chosen as the length scale and the diffusion time \( \nu/d^2 \) serves as the time scale. The pressure is normalized by \( \rho \nu^2/d^2 \), and the magnetic field \( H \) and magnetization \( M \) can be normalized by the quantity \( \rho \mu_0 \nu^2/d \), where \( \mu_0 \) is the permeability of free space. These considerations lead to the following non-dimensionalized hydrodynamical equations:

\[
(\partial_t + \mathbf{u} \cdot \nabla) \mathbf{u} - \nabla p = (\mathbf{M} \cdot \nabla) \mathbf{H} + \frac{1}{2} \nabla \times (\mathbf{M} \times \mathbf{H}), \nabla \cdot \mathbf{u} = 0.
\] (1)

The boundary conditions on the cylindrical surfaces are \( u(r_1, \theta, z) = (0,Re_1,0) \) and \( u(r_2, \theta, z) = (0,Re_2,0) \), where the inner and outer Reynolds numbers are \( Re_1 = \omega r_1 d/\nu \) and \( Re_2 = \omega r_2 d/\nu \), respectively, \( r_1 = R_1/(R_2 - R_1) \) and \( r_2 = R_2/(R_2 - R_1) \) are the non-dimensionalized inner and outer cylinder radii, respectively. To be concrete, we consider counter-rotating cylinders and fix the Reynolds numbers at \( Re_1 = 350 \) and \( Re_2 = -145 \). The rotation ratio \( \beta = Re_2/Re_1 \) of the cylinders is about \(-0.4143 \).

We need to solve Eq. (1) together with an equation that describes the magnetization of the ferrofluid. Using the equilibrium magnetization of an unperturbed state where homogeneously magnetized ferrofluid is at rest and the mean magnetic moment is orientated in the direction of the magnetic field, we have \( M_0^\parallel = \chi \mathbf{H} \). The magnetic susceptibility \( \chi \) of the ferrofluid can be approximated with the Langevin's formula, where we set the initial value of \( \chi \) to be 0.9 and use a linear magnetization law. The ferrofluid studied corresponds to APG93333. We consider the near equilibrium approximations of Niklas with small \( \|M - M_0^\parallel\| \) and small magnetic relaxation time \( \tau : |\nabla \times \mathbf{u}| \ll 1 \). Using these approximations, one can obtain the following magnetization equation:
where

\[ \tau \chi = \frac{1}{\mu} \left( \frac{\rho_0 H^2}{6 \mu \Phi} \right) \]

(3)

is the Niklas coefficient, \( \mu \) is the dynamic viscosity, \( \Phi \) is the volume fraction of the magnetic material, \( S \) is the symmetric component of the velocity gradient tensor, and \( \lambda_2 \) is the material-dependent transport coefficient, which we choose to be \( \lambda_2 = 4/3s^2 \). Using Eq. (2), we can eliminate the magnetization from Eq. (1) to obtain the following ferrohydrodynamical equations of motion:

\[
\begin{align*}
(\partial_t + u \cdot \nabla) u - \nabla^2 u + \nabla p_M &= -\frac{s_N^2}{2} \left[ \nabla \cdot \left( F + \frac{4}{5} \nabla H \right) \right] + H \times \nabla \times \left( F + \frac{4}{5} \nabla H \right),
\end{align*}
\]

(4)

where \( F = (\nabla \times u)/2 \times H, p_M \) is the dynamic pressure incorporating all magnetic terms that can be expressed as gradients, and \( s_N \) is the Niklas parameter [Eq. (6)]. To the leading order, the internal magnetic field in the ferrofluid can be approximated as the externally imposed field, which is reasonable for obtaining dynamical solutions of the magnetically driven fluid motion. Equation (4) can then be simplified as

\[
\begin{align*}
(\partial_t + u \cdot \nabla) u - \nabla^2 u + \nabla p_M &= -\frac{s_N^2}{2} \left[ \nabla^2 u - \frac{4}{5} (\nabla \cdot (SH)) \right] \\
&\quad - H \times \frac{1}{2} \nabla \times (\nabla \times u) - H \times (\nabla^2 u) \\
&\quad + \frac{4}{5} \nabla \times (SH)
\end{align*}
\]

(5)

This way, the effect of the magnetic field and the magnetic properties of the ferrofluid on the velocity field can be characterized by a single parameter, the magnetic field or the Niklas parameter, with

\[ s_N^2 = s_N^2 \left[ s_N^2 \right], \]

where

\[ s_N^2 = \frac{\tau \chi \mu}{\Phi} \]

and

\[ s_N^2 = \frac{1}{\mu} \left( \frac{\rho_0 H^2}{6 \mu \Phi} \right) \]

(3)

is the Niklas coefficient, \( \mu \) is the dynamic viscosity, \( \Phi \) is the volume fraction of the magnetic material, \( S \) is the symmetric component of the velocity gradient tensor, and \( \lambda_2 \) is the material-dependent transport coefficient, which we choose to be \( \lambda_2 = 4/3s^2 \). Using Eq. (2), we can eliminate the magnetization from Eq. (1) to obtain the following ferrohydrodynamical equations of motion:

\[
\begin{align*}
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\end{align*}
\]

(4)

where \( F = (\nabla \times u)/2 \times H, p_M \) is the dynamic pressure incorporating all magnetic terms that can be expressed as gradients, and \( s_N \) is the Niklas parameter [Eq. (6)]. To the leading order, the internal magnetic field in the ferrofluid can be approximated as the externally imposed field, which is reasonable for obtaining dynamical solutions of the magnetically driven fluid motion. Equation (4) can then be simplified as

\[
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&\quad - H \times \frac{1}{2} \nabla \times (\nabla \times u) - H \times (\nabla^2 u) \\
&\quad + \frac{4}{5} \nabla \times (SH)
\end{align*}
\]

(5)

This way, the effect of the magnetic field and the magnetic properties of the ferrofluid on the velocity field can be characterized by a single parameter, the magnetic field or the Niklas parameter, with

\[ s_N^2 = s_N^2 \left[ s_N^2 \right], \]

where

\[ s_N^2 = \frac{\tau \chi \mu}{\Phi} \]

(3)
Figure 9. Angular momentum and torque. (a–e) Angular momentum $L(r) = r \left( v(r) \right) / Re_i$ scaled with the inner Reynolds number versus the radius $r$ for the solutions: WVF$_{2,3,}$ (WVF$_2$ for $s_z = 1.0$), WVF$_{3,2}$, unstable TVF, and CCF for $s_x$ and $s_z$ values as indicated. (f,g) Variation with $s_z$ and $s_x$ of the dimensionless torque $G = v^2 \omega$ (see text for details) for WVF$_{2,3,}$ (WVF$_2$) and WVF$_{3,2}$. 
For the parameter regimes considered, the choice 

\[ s_H \chi_{N}^2 \]

with Fourier spectral 

\[ \phi_{mn} \]

provides adequate 

\[ \text{obtained from the Fourier decomposition in the axial} \]

\[ \text{and (explicit) time splitting.} \]

The variables can be expressed as 

\[ f(r, \theta, z, t) = \sum_{m=-m_{\text{max}}}^{m_{\text{max}}} \phi_{m,n}(r, t) e^{i m \theta}, \]

where \( f \) denotes one of \( \{u, v, w, p\} \). For the parameter regimes considered, the choice \( m_{\text{max}} = 10 \) provides adequate accuracy. We use uniform grids with spacing \( br = bz = 0.05 \) and time-steps \( \Delta t < 1/3800 \). For diagnostic purposes, we also evaluate the complex mode amplitudes \( \phi_{m,n}(r, t) \) obtained from the Fourier decomposition in the axial direction 

\[ \phi_{m,n}(r, t) = \sum_{n} \phi_{m,n}(r, t) e^{i n k z}, \]

where \( k = 2\pi/\Gamma = 3.927 \).

**Numerical scheme for ferrohydrodynamical equation.** The ferrohydrodynamical equations of motion Eq. (4) can be solved\(^{17,19,37}\) by combining a second-order finite-difference scheme in \( \theta \) and (explicit) time splitting. The variables can be expressed as 

\[ s_x = \frac{2(2 + \chi) H_{c} c_{N}}{(2 + \chi)^2 - \chi^2 \eta^2}, \quad \text{and} \quad s_z = H_{c} c_{N}. \]  

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Author Contributions
S.A., Y.D. and Y.C.L. devised the research project. S.A. performed numerical simulations and analyzed the results. S.A., Y.D. and Y.C.L. wrote the paper.

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