CONSTRAINING BOUNCING COSMOLOGY CAUSED BY CASIMIR EFFECT

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We constrain the Friedman Robertson Walker (FRW) model with “radiation-like” contribution to the Friedmann equation against the astronomical data. We analyze the observational limitations on a $(1+z)^4$ term from supernovae type Ia (SNIa) data, Fanaroff-Riley type IIb (FRIIb) radio galaxy (RG) data, baryon oscillation peak and cosmic microwave background radiation (CMBR) observations. We argue that it is not possible to determine the energy densities of individual components scaling like radiation from a kinematic astronomical test. The bounds for density parameter for total radiation-like term can be obtained. We find different interpretations of the presence of scaling like radiation term: the FRW universe filled with a massless scalar field in a quantum regime (the Casimir effect), the FRW model in a semi-classical approximation of loop quantum gravity, the FRW model in the Randall Sundrum scenario with dark radiation or cosmological model with global rotation. In this paper we mainly concentrate on the Casimir effect arising from quantum effects of the scalar field. This contribution can describe decaying part of cosmological constant. We discuss the back reaction of gravity on Casimir-type force which is a manifestation of the vacuum fluctuations of the quantum scalar field at finite temperature. It is shown that while the Casimir energy gives rise to the accelerating Universe, the cosmological constant term is still required. We argue that a small negative contribution of a radiation-like term can reconcile the tension between the observed primordial $^4$He and $D$ abundance. Moreover the presence of such contribution can also remove the disagreement between the Hubble parameter $H_0$ values obtained from both SNIa and Wilkinson Microwave Anisotropy Probe (WMAP) satellite data.

1. Introduction

From the recent measurements of distant supernovae type Ia \cite{1, 2} we deduce that the universe is in an accelerating phase of expansion. The effective and simple explanation of this current state of the Universe requires that one third of total energy in the universe is non-relativistic dust (dark matter) and two third is a constituent of negative pressure (dark energy). While cosmological constant remains the simplest explanation of observations of distant supernovae, there appeared serious problem in this context like: why is the vacuum energy so much smaller than we except from effective quantum field calculations? \cite{3}. To better understand how vacuum energy contributes to the cosmological constant, we consider the Casimir effect which is a purely quantum field theory phenomenon. We discuss the back reaction of gravity on Casimir-type force which is a manifestation of the vacuum fluctuations of the quantum scalar field at finite temperature. It is shown that while the Casimir energy gives rise to accelerating Universe, the cosmological constant is still required. The FRW model with a Casimir type force contains a term which scales like negative radiation $((-)(1+z)^4)$. 

There are different interpretations of the presence of such a term: cosmological model with global rotation, Friedmann-Robertson-Walker (FRW) model in the Randall Sundrum scenario with dark radiation, FRW universe filled with a massless scalar field in a quantum regime (Casimir effect), or FRW model in a semi-classical approximation of loop quantum gravity. To constrain the “radiation-like” contribution to the Friedmann equation, we use a variety of astronomical observations, such as SNIa data \cite{1, 2}, FRIIb RG data \cite{4}, baryon oscillation peak and CMBR observations. Although the obtained bounds on total density parameters strongly limit the presence of this term in the Friedmann equation, this does not mean that it is not present, as it is impossible to determine the energy density of sepa-
rate components. We note that the CMBR and big-bang nucleosynthesis (BBN) offer stringent conditions on this term, which can be regarded as an established upper limit on any individual components of negative energy density, and therefore on the Casimir effect, global rotation, discreteness of space following loop quantum gravity or brane dark radiation. The plan of the paper is as follows. In the next section, we summarize different interpretations to the possibility radiation like term in Friedmann equation. We provide observational constraints on the radiation like term in Section III. Finally we conclude our work in Section IV.

2. Different interpretations of the presence of negative radiation like term in the \( H^2(z) \) relation.

2.1. Casimir effect

Ishak [20] distinguished old and new cosmological problems. While the old problem is related to the order of magnitude of cosmological constant the new problem is related to this magnitude being of the same order as the matter density during the present epoch. He pointed out the relevance of experiments focusing on the Casimir effect in gravitational and cosmological context [20]. He argued that while the geometrical cosmological constant has no quantum properties, the vacuum energy has both gravitational and quantum properties. In this context the Casimir effect [5] seems to be relevant because it can tell us how vacuum energy contributes to the cosmological constant [6] [7].

The Casimir effect can be derived from the general principles of quantum theory of electromagnetic field [8] for two uncharged, perfectly conducting plates in vacuum. They should attract each other with the force \( F \propto d^{-4} \), where \( d \) is distance between the plates. This prediction was verified experimentally and yielded, for electric bodies, reasonable agreement to the theory [8]. The Casimir effect is a simple observational consequence of the existence of quantum fluctuations [9]. The Casimir force between conducting plates leads to a repulsive force, like the positive cosmological constant. It is worthy to mention that Casimir type of contribution arising from the tachyon condensation is possible [10].

Moreover, different laboratory experiments were designed to measure the Casimir effect with increased precision and then strengthen the constraints on correction to Newtonian gravitational law [11]. Therefore, the measurement of the thermal Casimir force is promising to obtain stronger constraints on non-Newtonian gravity. Although the Casimir force is very weak (for typical values of \( d = 0.5 \mu m \) and area of 1 cm², the Casimir force is \( \simeq 0.2 \) dyn) it becomes measurable with a high degree of precision [12].

For a survey of recently obtained results in the Casimir energy studies see [8]. The Casimir effect can be regarded as a manifestation of the quantum fluctuations on the geometry and the topology of the system boundaries although the Casimir force can be also present with the system with no boundaries and a compact topology. Therefore, if our Universe has non-trivial topology then every quantum fields will generate a Casimir-type force which many authors study on the FRW background space-time. If we consider a massless scalar field, conformally coupled to gravity, on the background of the static Einstein universe then its Casimir energy has been shown to have the form \( \alpha/a^4 \) with the value of \( \alpha = 1/(480\pi^2) \) [13]. While for all such fields \( \alpha \) is positive because such models obey the strong energy condition, the computation of Casimir energy in the cosmological context leads to the conclusion that the Casimir energy of the scalar fields could drive the inflation in the flat universe with toroidal topology [14]. It has been recently demonstrated that there exists a family of quantum scalar fields which give rise to a repulsive Casimir force in a closed universe [15]. They “produce” Casimir energy scaling like radiation (\( \alpha < 0 \)), and violating the strong energy condition. However, we must remember that all calculations of the Casimir effect are performed under the assumption of a quasi adiabatic approximation and their generalization to the case of the non-static FRW models is extremely difficult.

The finite temperature quantum effects of massless scalar fields on the background of spacetimes of cosmological models have been considered by many authors for many years in the context of the Kaluza-Klein theories and dynamical reduction of extra dimension process. The main result of these investigations was a universal quantum correction in low and high temperatures. Assuming the so called adiabatic approximation it can be shown that while at low temperature the Casimir energy is always negative and proportional to \( \alpha/a^{4+d} \) where \( d \) is the number of extra dimensions, and is positive and proportional to \( \alpha/a^{4+d} \) at high temperatures [16]. Therefore if \( d = 0 \) we can recover results for the standard cosmology which indicate that quantum effects in cosmological models at finite temperatures have a universal asymptotic in both high and low temperatures. The Casimir energy at low temperatures scales like radiation but is still negative. Finally the Casimir effect is significant when the topology of the Universe is not trivial [17]. Also Casimir type energy can be produced from some extra dimensions [18] [19]. Recently the relevance of the Casimir effect in the context of dark energy problem has been pointed out [20] [21] [22] [23] [24].

When the field occupies some bounded region of the configuration space then its spectrum is discrete and general vacuum energy \( E_0 \) depends on eigenfrequencies \( \omega_n \) which can be determined from the geometry of boundary \( \partial M \): \( E_0(\partial M) = \frac{1}{2} \sum \omega_n \). While the total zero-point energy of the vacuum is infinite in the presence of boundaries it is modified and \( E_0 = E_0(\partial M) - E_0(0) \) In the case considered by Casimir, when electro-
magnetic field is confined between two parallel conducting plates the contribution of unbounded Minkowski space $-E_0(0)$ should be subtracted. The generally to obtain the physical value of the vacuum energy we must remember that in quantum field theory (i.e., in the case of infinite number of degrees of freedom) the observable quantity is not zero point energy itself, but only its excess, caused by boundaries [8].

Recently, the idea that the Casimir effect is responsible for UV ultraviolet cut-off that renders the total vacuum energy finite was rule out [25]. Therefore the Casimir effect cannot be treated as a natural cut off leading to the observed cosmological constant value.

The cosmological significance of the Casimir effect was pointed out in many contexts since the pioneering paper of Zeldovich and Starobinski [13]. Also, the Casimir effect has been used as an effective mechanism of compactification of extra dimensions in the Kaluza-Klein cosmological models — so called dynamical reduction mechanism [16, 26].

It was demonstrated that nontrivial topology of a physical space as well as an internal space can lead to compactification of an extra dimension [27, 28], (for contemporary context see also [18]. In these investigations the quantum field theory at finite temperature is used in calculation of quantum effects and back-reaction arising from massless scalar fields. It is assumed that the metric of background space time (FRW space time) is static and conditions for thermodynamical equilibrium of matter fields are satisfied due to their interactions with the thermal bath. As a result we obtain universal approximation of quantum distribution function for massless scalar bosons at high and low temperatures. The assumption of the quasi-static approximation (characteristic time of quantum process is much smaller than the characteristic time of cosmic evolution) enables us to determine the thermodynamical characteristic. As a result we obtain at low temperatures: $p(M^3) = (-)\rho$, $\rho = (-) \frac{|\alpha|}{a^4}$, $p'(M^D) = \frac{4}{3}p$, where $p$ and $p'$ are pressure on physical $(M^3)$ and internal $(M^D)$ spaces. $D$ is the dimension of space with additional dimensions, $a$ is the scale factor.

Because in this case the energy momentum tensor is traceless (massless scalar field) we define the energy momentum tensor as $T_{\mu\nu} = \text{diag}(\rho, -p, -p, -p, -p', -p', -p')$. In the standard space of topology $\mathcal{R} \times M^3$, where $M^3$ is homogeneous and isotropic space with Robertson-Walker metric we know that quantum effects of scalar field are equivalent to the effect of fluid with pressure $p = \frac{k}{a^2}$ and $\rho = (-) \frac{|\alpha|}{a^4}$. It is a universal approximation effect of quantum fields originating from massless scalar fields at low temperatures (Casimir effect). Analogical result was recently obtained by Herdeiro and Sampaio [13]. The back-reaction problem is considered by taking the semi-classical equation with $\Lambda$

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}^{\text{matter}} + <T_{\mu\nu}^\phi>.$$  \hfill (1)

Considering R-W symmetry and matter content in the form of perfect fluid with energy density $\rho$ and pressure $p$ we obtain

$$\ddot{a}^2 + k = \frac{2a^2}{3},$$  \hfill (2)

$$\ddot{a} = -\frac{1}{6}(\rho + 3p)a,$$  \hfill (3)

where dot means differentiation with respect the cosmological time and $\rho$ is the effective energy density

$$\rho_{\text{eff}} = \Lambda + \frac{\alpha}{a^4},$$  \hfill (4)

where $\alpha$ is the constant of the Casimir force scaling like radiation, positive for conformally coupled scalar field in the cosmological context as it was verified long time ago by Zeldovich and Starobinski [14]. Casimir energy is negative in a flat universe with toroidal topology. In this case ($\alpha < 0$) it is produced accelerating phase of expansion of the universe because the strong energy condition $\rho_{\text{eff}} + 3p_{\text{eff}} > 0$ is violated.

$$\rho_{\text{eff}} = \Lambda - \frac{|\alpha|}{a^4} + \frac{\rho_m, 0}{a^4}, \quad p_{\text{eff}} = -\Lambda - \frac{|\alpha|}{3a^4}. \hfill (5)$$

For our aims it is important that the effects of Casimir energy with a negative value of $\alpha$ which scales like radiation can contribute into the $H^2(z)$ relation—crucial for any kinematic test. It is also interesting that the same type of contribution to the effective energy density can be produced by loop quantum theory effects in semi-classical quantum cosmology [29, 30, 31]. These effects give rise to evolitional scenario in which the initial singularity is replaced by a bounce.

2.2. Other interpretations of the Casimir-type term

There are many different interpretations of the term in the Friedmann equation which diminishes with the cosmic scale factor like $a^{-4}$. The first interpretation comes from the generalized Friedmann equation on the brane in the Randall and Sundrum scenario [32, 33]. In the brane world scenario our universe is some sub-manifold which is embedded in a higher-dimensional space-time called bulk spaces. While the physical matter fields are confined to this sub-manifold called brane, the gravity can reside in the higher dimensions. This brane paradigm was first proposed by Arkani-Hamed et al. [34] as a means to reconcile the hierarchy problem between the weak scale and the new Planck scale $M_{\text{pl}}$. Randal and Sundrum [32] solved the analogous problem between the weak scale and the size of extra dimension by introducing non-compact extra dimensions. In their model our universe is represented by a three brane embedded in a 5 dimensional anti de Sitter space. The cosmological evolution of such brane universes was extensively investigated by several authors (see for example [35]). This way, the Einstein equations restricted
to the brane reduce to some generalization of the FRW equation. Two additional terms contribute into the $H^2$ relation \[36\]. The $\rho^2$ term arises from the imposition of a junction condition for the scale factor on the surface on the brane. This term decays rapidly as $a^{-6}$ for dust or as $a^{-8}$ for the radiation dominated early universe. This term should be significant only in the very early universe \[36 \, 37\].

The second term is of considerable interest for us because it scales like radiation with a negative constant $\alpha$. Hence it is called dark radiation. This term arises from the non-vanishing electric part of the five dimensional Weyl tensor. Mathematically both negative and positive values of $\alpha$ are possible.

Dark radiation strongly effects both BBN and CMBR. It was demonstrated by Ichiki et al. \[38\] that BBN limits the possible contribution from dark radiation just before $e^+e^-$ annihilation epoch. They gave limits on the possible contribution of dark radiation as $-1.23 < \rho_{dr}/\rho_\gamma \leq 0.11$ from BBN and $-0.41 < \rho_{dr}/\rho_\gamma \leq 0.105$ at the 95% confidence level from CMBR measurements. Let us note that small negative contribution of dark radiation can also reconcile the tension between the observed $^4\text{He}$ and $D$ abundances \[39\].

Another interpretation of the presence of the negative radiation like term is rotation in the Newtonian cosmology. When we consider Newtonian cosmology following Senovilla et al. \[40\] then we can define, homogeneous Newtonian cosmology as $\rho$ and $p$ having no spatial dependence i.e $\rho = \rho(t)$ and $p = p(t)$ while we assume that the velocity vector fields depends linearly on the spatial coordinates. In such a case we obtain equation which represents shear-free Newtonian cosmologies with expansion and rotation which is well known as the Heckmann-Schücking model \[41\].

\[
\dot{a}^2 = \frac{\rho(t_0)}{3a} - \frac{2\omega^2}{3a^2} + C \tag{6}
\]

where $C$ is an arbitrary constant. We interpret it in terms of curvature constant although in the Newtonian spacetime the curvature is zero. For our aims it is important that the effect of rotation produce negative term scaling like $(1 + z)^4$ in the Newtonian analogue of the Friedmann equation.

In the Newtonian cosmology in contrast to general relativity effect of rotation are not necessary related to non-vanishing shear. The homogeneous universe with non-vanishing shear basing on general relativity may expand and rotate relative to local gyroscopes. The problem of rotation between the rotation of the universe and origin of the rotation of galaxies was investigated in \[42\] and \[43 \, 44 \, 45\]. Also, the role of rotation of objects in the Universe, their significance and astronomical measurements was recently addressed by \[46 \, 47\].

Figure 1: The 68.3% and 95.4% confidence levels (obtained from combined analysis of SN+RG+SDSS+CMBR) on the $\left(\Omega_{m,0}, \Omega_{\Lambda,0}\right)$ plane.

Figure 2: The location of the first peak $l_1$ as a function of $-\Omega_{dr,0}$. Note that $l_1 \approx 220$ for $h = 0.73$ favour $\Omega_{dr,0} \simeq 0$, while for $h = 0.65$ and $h = 0.67.6$ lead to $\Omega_{dr,0} \neq 0$.

3. Observational constraints on the FRW model parameter with the radiation-like term

Cosmological model are usually tested against observations. One of the most popular test is based on the luminosity distance $d_L$ of the supernovae Ia as a function of redshift \[48\]. However for the distant SNIa, one can not directly observe their luminosity distances $d_L$ but their apparent magnitudes $m$ and redshifts $z$. The absolute magnitude $M$ of the supernovae is related to its absolute luminosity $L$. Since we could obtain the following relation between distance modulus $\mu$, the hu-
minority distance \(d_L\), the observed magnitude \(m\) and the absolute magnitude \(M\):

\[
\mu \equiv m - M = 5 \log_{10} d_L + 25 = 5 \log_{10} D_L + M
\]

(7)

where \(D_L = H_0 d_L\) and \(M = -5 \log_{10} H_0 + 25\). We could compute the luminosity distance of a supernova as the function of redshift:

\[
d_L(z) = (1+z) \frac{c}{H_0} \frac{1}{\sqrt{\Omega_{k,0}}} \int_0^z \frac{dz'}{H(z')}
\]

(8)

where

\[
\left(\frac{H}{H_0}\right)^2 = \Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{r,0}(1+z)^4
\]

\[+ \Omega_{\Lambda,0}(1+z)^4 + \Omega_{\Lambda,0}.
\]

\(\Omega_{k,0} = -\frac{k}{H_0^2}\) and \(F(x) \equiv (\sinh(x), x, \sin(x))\) for \(k < 0, k = 0, k > 0\), respectively. We assumed \(\Omega_{r,0} + \Omega_{\Lambda,0} = 0.248 h^{-2} \times 10^{-5} + 1.77 h^{-2} \times 10^{-5} \approx 0.0001\) [80].

Daly and Djorgovski [49] suggested to include in the analysis not only supernovae but also radio galaxies (see also [50] [51] [52]). In such a case, it is useful to use the coordinate distance defined as

\[
y(z) = \frac{H_0 d_L(z)}{c(1+z)}.
\]

(10)

Daly and Djorgovski [49] have compiled a sample comprising the data on \(y(z)\) for 157 SNIa in the Riess et al. Gold dataset [1] and 20 FRIIb radio galaxies. In our data sets we also include 115 SNIa compiled by Astier et al. [2].

It is clear from eq. (9) and (10) that the coordinate distance does not depend on the value of \(H_0\). Unfortunately we do not know coordinate distance \(y(z)\) for supernovae. This distance must be computed from the luminosity distance (or the distance modulus \(\mu\)) and for such a computation a knowledge of the value of \(H_0\) is required. For both supernovae sample we choose the values of \(H_0\) which were used in the original papers. We used the distance modulus presented in Ref. [1] [2] for the calculation of the coordinate distance. For each sample we choose the values of \(H_0\) appropriate to the data sets. For Riess et al.’s Gold sample we fit the value of \(h = 0.646\) as the best fitted value and this value is used for calculation of coordinate distance for SNIa belonging to this sample. In turn the value \(h = 0.70\) was assumed in the calculations of the coordinate distance for SNIa belonging to Astier et al.’s sample, because the distance moduli \(\mu\) presented in Ref. [2] Tab. 8 was calculated with such an arbitrary value of \(h = 0.70\). The error of the coordinate distance can be computed as

\[
\sigma^2(y_i) = \left(\frac{10^{\mu_i}}{c(1+z)10^5}\right)^2 \times \left(\sigma^2(H_0) + \left(\frac{H_0 \ln 10}{5}\right)^2 \sigma^2(\mu_i)\right)
\]

(11)

where \(\sigma_i(\mu_i)\) denotes the statistical error of distance modulus determination (note that for Astier et al.’s sample the intrinsic dispersion was also included) and \(\sigma(H_0) = 0.8\) km/s Mpc denotes error in \(H_0\) measurements.

We included to our constraints obtain from extragalactic analysis: the baryon oscillation peaks (BOP) detected in the Sloan Digital Sky Survey (SDSS) [53]. They found that value of \(A\)

\[
A = \sqrt{\frac{\Omega_{m,0}}{\Omega_{k,0}}} \left(\frac{1}{z_{1}}\right)^2 \int_{0}^{z_{1}} \frac{dz}{E(z)}
\]

(12)

where \((E(z) \equiv H(z)/H_0\) and \(z_{1} = 0.35\) is equal \(A = 0.469 \pm 0.017\). The quoted uncertainty corresponds to one standard deviation, where a Gaussian probability distribution has been assumed.

Another constraint which we also include in our analysis is the so called the (CMBR) “shift parameter”

\[
R = \sqrt{\frac{\Omega_{m,0}}{\Omega_{k,0}}} y(z_{sdss}) = \sqrt{\frac{\Omega_{m,0}}{\Omega_{k,0}}} \int_{0}^{z_{sdss}} \frac{dz}{E(z)}
\]

(13)

where \(R_0 = 1.716 \pm 0.062\) [54].

In our combined analysis, we can obtain a best fit model by minimizing the pseudo-\(\chi^2\) merit function [55]

\[
\chi^2 = \chi^2_{SN+RG} + \chi^2_{SDSS} + \chi^2_{CMBR}
\]

\[= \sum_i \left(\frac{y_i^{\text{obs}} - y_i^{\text{th}}}{\sigma_i(y_i)}\right)^2 + \left(\frac{A_{\text{mod}} - 0.469}{0.017}\right)^2 + \left(\frac{R_{\text{mod}} - 1.716}{0.062}\right)^2,
\]

(14)

where \(A_{\text{mod}}\) and \(R_{\text{mod}}\) denote the values of \(A\) and \(R\) obtained for a particular set of the model parameter. For Astier et al. SNIa [2] sample additional error in z measurements were taken into account. Here \(\sigma_i(y_i)\) denotes the statistical error (including error in z measurements) of the coordinate distance determination.

Constraints for the cosmological parameters, can be obtain by minimizing the following likelihood function \(L \propto \exp(-\chi^2/2)\). One should note that when we are interested in constraining a particular model parameter, the likelihood function marginalized over the remaining parameters of the model should be considered [55]. This method was used in the case of negative \((1+z)^4\) type contribution only in the paper [52]. In the present paper we avoid from constrain that a priori the \((1+z)^4\) term must be negative.

Our results are presented in Table 1. Table 1 refers to the minimum \(\chi^2\) method, whereas Table 3 shows the results from the marginalized likelihood analysis.

From our combined analysis (SN+RG+SDSS+CMBR) we obtain as the best fit a flat (or nearly flat universe) with \(\Omega_{m,0} \simeq 0.3\), and \(\Omega_{\Lambda,0} \simeq 0.7\). For the dark radiation term, we obtain the stringent bound \(\Omega_{\text{Tot,0}} = \ldots\)
$\Omega_{r,0} + \Omega_{dr,0} \in (-0.00017, 0.00857)$ at the 95% confidence level. It lead for limit on dark radiation $\Omega_{dr,0} \in (-0.00027, 0.0847)$ ($\Omega_{Totr,0} \in (-0.00017, 0.0857)$. This results mean that the positive value of dark radiation term is preferred ($\Omega_{dr,0} > 0$), however small negative contribution of dark radiation is also available. One should note that when we used only SN and RG data only, we obtain value $\Omega_{m,0}$ close to zero what seems to be unrealistic. It is the reason that we repeat our analysis with prior $\Omega_{m,0} = 0.3$ [60]. In such a case, we again obtain flat (or nearly flat universe) as a best fit. Contribution of dark energy is small and positive, but also in this case small negative contribution of dark radiation is available. From combined analysis we obtain limit on dark radiation $\Omega_{dr,0} \in (-0.00037, 0.00727)$ ($\Omega_{Totr,0} \in (-0.00027, 0.00737)$). Our results shows that in the present epoch contribution of the dark radiation, if it exist, is small and gives only small corrections to the $\Lambda$CDM model in the low redshift.

The AIC is defined in the following way [60]
\[
AIC = -2 \ln L + 2d
\]
(15)
where $L$ is the maximum likelihood and $d$ is a number of the free model parameters. The best model with a parameter set providing the preferred fit to the data is that minimizes the AIC. It is interesting that the AIC also arises from an approximate minimization of the Kulbak-Leibner information entropy [62].

The BIC introduced by Schwarz [61] is defined as
\[
BIC = -2 \ln L + d \ln N
\]
(16)
where $N$ is the number of data points used in the fit. Comparing these criteria, one should note that the AIC tends to favour models with large number of parameters unlike the BIC, because the BIC penalizes new parameters more strongly. It is the reason that the BIC provides a more useful approximation to the full statistical analysis in the case of no priors on the set of model parameters [59]. It makes this criterion especially suitable in the context of cosmological applications.

Please note that while the AIC is useful in obtaining upper limit to the number of parameters which should be incorporated to the model, the BIC is more conclusive. Of course only the relative value between the BIC of different models has statistical significance. The difference of 2 is treated as a positive evidence (and 6 as a strong evidence) against the model with the larger value of the BIC [63]. If we do not find any positive evidence from information criteria the models are treated as a identical and eventually additional parameters are treated as not significant. The using of the BIC seems to be especially suitable whenever the complexity of reference does not increase with the size of data set. Liddle [58] noted that in cosmology, a new parameter is usually a quantity set to zero in a simpler base model and if the likelihood function is a continuous function of its parameters it will increase as the parameter varies in either the positive or negative direction. The problem of classification of the cosmological models on the light of information criteria on the base of the astronomical data was discussed in our previous papers [65, 66, 67, 68, 69].

Our results are presented in Table 5. It is clear that in the light of informative criterion model with dark energy do not increase fit significantly. It confirm our conclusion that dark energy term, if it exist, is small in the present epoch. Using the prior $\Omega_{m,0} = 0.3$ [50] does not change our conclusion. It shows that using such a prior in the light of information criteria is realistic.

Please also note that if $\Omega_{Totr,0} = \Omega_{r,0} + \Omega_{dr,0} < 0$, then we obtain a bouncing scenario [70, 71, 72] instead of a big bang. For $\Omega_{m,0} = 0.3$, $\Omega_{dr,0} = -0.00027$ and $h = 0.65$ bounces ($H^2 = 0$) appear for $z \simeq 1800$. In this case, the BBN epoch never occurs and all BBN predictions would be lost.

This results shows that more stronger constraints for model parameters is required. To obtain such a constraints on the model parameters, it is useful to use the CMBR observations. The hotter and colder spots in the CMBR can be interpreted as acoustic oscillations in the primeval plasma during the last scattering. In such a case the locations of the peaks in the CMBR power spectrum are very sensitive to variations in the model parameters. Therefore, the location of these peaks can also be used for constraining the parameters of cosmological models. The acoustic scale $\ell_A$ which gives the locations of the peaks is defined as
\[
\ell_A = \pi \frac{\int_0^{z_{dec}} dz' \frac{dz'}{H(z')}}{\int_\infty^{z_{dec}} c_s H(z')} = \ell_{A,0}
\]
(17)
where, for the flat model, equation [49] reduces to
\[
H(z) = H_0 \left[ \Omega_{m,0}(1 + z)^3 + \Omega_{r,0}(1 + z)^4 + \Omega_{dr,0}(1 + z)^4 + \Omega_{\Lambda,0} \right]^{1/2},
\]
(18)
where $c_s$ is the speed of sound in plasma. Knowing the acoustic scale we can determine the location of the $m$-th peak $\ell_m = \ell_A(m-\phi_m)$ where $\phi_m$ is the phase shift caused by the plasma driving effect. The CMBR temperature angular power spectrum provides the locations of the first two peaks $\ell_1 = 220.1^{+0.8}_{-0.8}$, $\ell_2 = 546^{+10}_{-10}$ [73]. Using three years of WMAP data, Spergel et al. obtained that the Hubble constant $H_0 = 73$ km/s Mpc, the baryonic matter density $\Omega_{b,0} = 0.0222 h^{-2}$, and the matter density $\Omega_{m,0} = 0.128 h^{-2}$ [74], which are in good agreement with the observation of position of the first peak (see Fig. 2 but lead (assuming the $\Lambda$CDM model) to a value $\Omega_{m,0} = 0.24$. It mean that there is disagreement between $H_0$ values obtained from SNIa and WMAP. We compute the location of the first peak as a function of $\Omega_{dr,0}$ assuming $H_0 = 65$ km/s Mpc ($\Omega_{m,0} = 0.3$). Separately we repeat our computation using the latest Riess et al. result which obtain $\Omega_{m,0} = 0.28$ ($H_0=67.6$ km/s Mpc) [75].

From Fig. 2 it is easy to see that we can obtain agreement with the observation of the location of the first peak for non-zero values of the parameter $\Omega_{dr,0}$ (Fig. 2) both for $\Omega_{m,0} = 0.28$ and $\Omega_{m,0} = 0.30$. We obtain $-1.05 \times 10^{-5} < \Omega_{dr,0} < -0.5 \times 10^{-5}$ at the 95% confidence level for the case $\Omega_{m,0} = 0.3$ while $-0.75 \times 10^{-5} < \Omega_{dr,0} < -0.25 \times 10^{-5}$ at the 95% confidence level for the case $\Omega_{m,0} = 0.28$ Please note that our limits are stronger than that obtained by Ichiki et al. [38], which provides bounds of $-7.22 \times 10^{-5} < \Omega_{dr,0} \leq 0.65 \times 10^{-5}$ (in the case of the BBN) and $-2.41 \times 10^{-5} < \Omega_{dr,0} \leq 0.62 \times 10^{-5}$ (in the case of the CMBR). In all cases the obtained values of $\Omega_{dr,0}$ are in agreement with the result obtained from the combined analysis because the 2$\sigma$ confidence interval for this parameter obtained from this analysis contains the area allowed from the CMBR. Most important conclusion is, that while the combined analysis allowed the possibility that $\Omega_{dr,0}$ is equal to zero, the CMBR location of the first peak seems to exclude this case both for $h = 0.65$ and $h = 0.67$.

4. Conclusion

In the paper we analysed the observational constraints on the $(1+z)^4$-type contribution in the Friedmann equation. Because it the present paper we mainly concentrate on Casimir effect arising from quantum effects of the scalar field the constraints for negative $(1+z)^4$-type contribution are in our special interest. The analysis of SNIa data as well as both SNIa and FRIIb radio galaxies (with and without priors going from baryon oscillation peaks and CMBR “shift parameter”) shows that the values of $\chi^2$ statistics are lower for model with radiation like term, than for the $\Lambda$CDM model. However, information criteria show that using such a term does not increase the quality of the fit significantly. BIC even favour $\Lambda$CDM model over our model of Bouncing Cosmology (with dark radiation). However this preference is weak. This results show that $(1+z)^4$ term is not significant in the present epoch of the Universe.

We show that there are several interpretations of the $(1+z)^4$-type contribution and we discussed different proposals for the presence of such a term. Unfortunately, it is not possible, with present kinematic astronomical test, to determine the energy densities of individual components scales like radiation. However we show that some stringent bounds on the value of this total contribution can be given. Combined analysis of SNIa data and FRIIb radio galaxies using baryon oscillation peaks and CMBR “shift parameter” give rise to a concordance universe model which is almost flat with $\Omega_{m,0} \simeq 0.3$. From the above-mentioned combined analysis, we obtain an constraint for the term which scales like radiation $\Omega_{Totr,0} \in (-0.00017, 0.00857)$ which leads to bounds on the dark radiation term $\Omega_{dr,0} \in (-0.00027, 0.0847$) at the 95% confidence level. This is a stronger limit than obtained previously by us from SNIa data only [74]. Our model with a small contribution of dark radiation type can also resolve the disagreement between $H_0$ values obtained from SNIa and WMAP. For $\Omega_{m,0} = 0.3$ ($H_0 = 65$ km/s Mpc) we find new stringent limits on the negative component scaling like radiation from the location of the peak in the CMBR power spectrum, $-1.05 \times 10^{-5} < \Omega_{dr,0} < -0.5 \times 10^{-5}$ at the 95% confidence level. This bound is stronger than that obtained from BBN and CMBR by Ichiki et al. [38].

In this paper we have especially studied the advance of initial singularity using back reaction gravity quantum effect at low temperatures (Casimir effect). Casimir force arising from the quantum effect of massless scalar field give rise to a $-1(1+z)^4$ correction whose effect depends upon the geometry and nontrivial topology of the space. Typically this type of correction is thought to be important at the late time of evolution of the universe. We have shown that Casimir effect could remove initial singularity which would be replaced by bounce. However, from the observational limits we obtain that bounce does not appear which means that a big bang scenario is strongly favoured instead of bounce. Our limit $-\Omega_{dr,0} < 1.05 \times 10^{-5}$ obtain from position of the first peak in the power spectrum of CMBR leads to $\Omega_{Totr,0} = \Omega_{dr,0} + \Omega_{r,0} > 0$. This implies that $H^2(z)$ is always greater than zero ($H^2(z) > 0$) and bounce does not appear which means that a big bang scenario is strongly favoured instead of bounce.

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Table 1: Results of the statistical analysis of the model with radiation like term obtained from $\chi^2$ best fit. The upper section of the table represents the constraint $\Omega_{k,0} = 0$ (flat model).

| sample                     | $\Omega_{k,0}$ | $\Omega_{m,0}$ | $\Omega_{Totr,0}$ | $\Omega_{\Lambda,0}$ | $\chi^2$ |
|----------------------------|----------------|----------------|-------------------|----------------------|----------|
| SN                        | -              | 0.01           | 0.133             | 0.807                | 295.9    |
| SN+RG                     | -              | 0.11           | 0.123             | 0.767                | 319.5    |
| SN+RG+SDSS                | -              | 0.28           | 0.022             | 0.698                | 320.1    |
| SN+RG+SDSS+CMBR           | -              | 0.30           | 0.00054           | 0.699                | 322.3    |
| SN                        | 0.01           | 0.00           | 0.186             | 0.804                | 295.9    |
| SN+RG                     | 0.08           | 0.00           | 0.169             | 0.751                | 319.5    |
| SN+RG+SDSS                | -0.11          | 0.28           | 0.049             | 0.781                | 319.5    |
| SN+RG+SDSS+CMBR           | 0.03           | 0.29           | 0.00270           | 0.657                | 320.9    |

Table 2: Results of the statistical analysis of the model with radiation-like term. The values of the model parameters are obtained from marginalized likelihood analysis. We present maximum likelihood value with 68.3% confidence ranges. The upper section of the table represents the constraint $\Omega_{k,0} = 0$ (flat model).

| sample                     | $\Omega_{k,0}$ | $\Omega_{m,0}$ | $\Omega_{Totr,0}$ | $\Omega_{\Lambda,0}$ | $\Omega_{\text{Totr}}$ | $\Omega_{\text{Totr}}$ |
|----------------------------|----------------|----------------|-------------------|----------------------|-------------------------|-------------------------|
| SN                        | -0.28±0.25     | 0.00±0.09      | 0.091±0.004       | 0.86±0.12            | 0.80±0.12               | 0.86±0.12               |
| SN+RG                     | -0.19±0.24     | 0.00±0.055     | 0.081±0.058       | 0.81±0.12            | 0.82±0.13               | 0.82±0.12               |
| SN+RG+SDSS                | -0.11±0.15     | 0.00±0.02      | 0.051±0.042       | 0.78±0.12            | 0.81±0.13               | 0.81±0.13               |
| SN+RG+SDSS+CMBR           | 0.06±0.04      | 0.28±0.02      | 0.000334±0.000267 | 0.64±0.04            | 0.65±0.03               | 0.65±0.03               |

Table 3: Results of the statistical analysis of the model with the radiation like term obtained from $\chi^2$ best fit with the assumption $\Omega_{m,0} = 0.3$. The upper section of the table represents the constraint $\Omega_{k,0} = 0$ (flat model).

| sample                     | $\Omega_{k,0}$ | $\Omega_{m,0}$ | $\Omega_{Totr,0}$ | $\Omega_{\Lambda,0}$ | $\chi^2$ |
|----------------------------|----------------|----------------|-------------------|----------------------|----------|
| SN                        | -0.19          | 0.30           | 0.056             | 0.834                | 295.9    |
| SN+RG                     | -0.13          | 0.30           | 0.042             | 0.788                | 319.5    |
| SN+RG+SDSS                | -0.09          | 0.30           | 0.034             | 0.756                | 320.7    |
| SN+RG+SDSS+CMBR           | 0.03           | 0.30           | 0.00182           | 0.668                | 321.4    |

Table 4: Results of the statistical analysis of the model with the radiation like term. The values of the model parameters are obtained from the marginalized likelihood analysis with the assumption $\Omega_{m,0} = 0.3$. We present maximum likelihood value with 68.3% confidence ranges. The upper section of the table represents the constraint $\Omega_{k,0} = 0$ (flat model).

| sample                     | $\Omega_{k,0}$ | $\Omega_{m,0}$ | $\Omega_{Totr,0}$ | $\Omega_{\Lambda,0}$ | $\chi^2$ |
|----------------------------|----------------|----------------|-------------------|----------------------|----------|
| SN                        | -0.19±0.16     | 0.30           | 0.057±0.043       | 0.87±0.12            | 0.87±0.12 |
| SN+RG                     | -0.13±0.16     | 0.30           | 0.042±0.040       | 0.82±0.12            | 0.82±0.12 |
| SN+RG+SDSS                | -0.09±0.15     | 0.30           | 0.034±0.038       | 0.61±0.11            | 0.61±0.11 |
| SN+RG+SDSS+CMBR           | 0.04±0.03      | 0.30           | 0.00217±0.00173   | 0.65±0.03            | 0.65±0.03 |
Table 5: The values of the AIC and BIC for the ΛCDM model and Bouncing Cosmology model (with dark radiation) without and with prior $\Omega_{m,0} = 0.3$. The upper section of the table represents the constraint $\Omega_{k,0} = 0$ (flat model).

| sample               | ΛCDM   | BC     | BC($\Omega_{m,0} = 0.3$) |
|----------------------|--------|--------|--------------------------|
|                      | AIC    | BIC    | AIC          | BIC    | AIC    | BIC    |
| SN                   | 299.5  | 303.1  | 299.9        | 307.1  | 299.3  | 302.9  |
| SN+RG                | 322.4  | 326.1  | 323.5        | 330.9  | 322.2  | 325.9  |
| SN+RG+SDSS           | 324.4  | 328.1  | 324.1        | 331.5  | 323.0  | 326.7  |
| SN+RG+SDSS+CMBR      | 324.5  | 328.2  | 326.3        | 333.7  | 324.3  | 328.0  |
| SN                   | 300.0  | 307.2  | 301.9        | 312.7  | 299.9  | 307.1  |
| SN+RG                | 323.5  | 330.9  | 325.5        | 336.5  | 323.5  | 330.9  |
| SN+RG+SDSS           | 325.1  | 332.5  | 325.5        | 336.5  | 324.7  | 332.1  |
| SN+RG+SDSS+CMBR      | 326.5  | 333.9  | 326.9        | 337.9  | 325.4  | 332.8  |