Reliability Analysis of Loess Landslide Stability Under Earthquake

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Abstract. Traditionally, the stability coefficient of the fixed value analysis method is used as the evaluation index of the stability of the loess landslide under the earthquake. This method does not consider the randomness and variability of the calculated parameters, which cannot reflect the real stability of the landslide. Based on this reality, this paper establishes the reliability analysis model of loess landslide stability under earthquake, and the model is used to evaluate the seismic stability of a loess landslide.

1. Introduction

The stability state of loess landslide is controlled by many factors or variables, such as the structure of landslide rock and soil, strength and deformation characteristics, geometry of sliding surface, seismic force, groundwater and so on [1,2]. And these variables have uncertainty, that is, the so-called random variables. The reliability analysis of loess landslide under earthquake action is to summarize the deformation characteristics and genetic mechanism of landslide, select scientific and reasonable probability distribution function, regard various uncertain factors as basic random variables, build mathematical model of landslide damage, select scientific and reasonable reliability evaluation means, and make stability and reliability analysis of landslide on the basis of field engineering geological investigation and analysis evaluation.

At present, the methods of reliability analysis mainly include point estimation method, Monte Carlo method and first or second moment method. Among them, the first-order and second-order moment method is easy to calculate, which can directly derive the analytical formula of reliability index, but has limitations on the distribution form of variables. Monte Carlo simulation method is not limited by the analysis conditions, and has low requirements for the distribution form of limit state functions and variables. It can simulate the main states and characteristics of the landslide system. Under sufficient simulation times, a relatively accurate failure probability and reliability index can be obtained [3-5]. Therefore, Monte Carlo method is used to calculate the reliability of loess landslide stability under earthquake.

2. Reliability analysis model of loess landslide stability under earthquake

2.1. Establishment of function

In order to establish the reliability analysis model of loess landslide stability under earthquake, the first step is to select the state function of landslide stability calculation. In this paper, based on the unbalanced force transfer method, the state function of loess landslide stability calculation is
established. When establishing the analysis model, based on the theory of unbalanced force transfer method and considering the effect of horizontal seismic force, the corresponding calculation model and formula are established. Assuming that the landslide material meets the Mohr Coulomb strength criterion, the landslide body is divided vertically, assuming that the resultant force of the inter slope forces is parallel to the bottom of the previous slope body, and combining with the balance condition of the forces, the downward calculation is carried out one by one.

\[
R_i - W_i \cos \alpha_i - P_{i+1} \sin(\alpha_i - \alpha_{i+1}) + W_i A \sin \alpha_i = 0
\]

\[
S_i + P_i - P_{i+1} \cos(\alpha_i - \alpha_{i+1}) - \left(\frac{R}{S}\right)W_i \sin \alpha_i - \left(\frac{R}{S}\right)W_i A \cos \alpha_i = 0
\]

According to the Mohr Coulomb criterion:

\[
S_i = C_i L_i + R_i \tan \phi_i
\]

From formula (1), formula (2) and formula (3), it can be solved that:

\[
P_i = P_{i-1} \psi_j + \left(\frac{R}{S}\right)S_i - R_i
\]

In the formula:

\[
R_i = W_i (\cos \alpha_i - A \sin \alpha_i) \tan \phi_i + C_i L_i
\]

\[
S_i = W_i (\sin \alpha_i + A \cos \alpha_i)
\]

\[
\psi_j = \cos(\alpha_i - \alpha_{i+1}) - \sin(\alpha_i - \alpha_{i+1}) \tan \phi_{i+1}
\]

In solving the specific problem, first assume that \(F_j = R / S\) is a certain value, and then from the first to the next, until the last thrust \(P_n = 0\). \(F_j\) is the stability coefficient of landslide.

According to formula (5) and formula (6), it can be concluded that:

\[
R = \sum_{i=1}^{n} (R_i \prod_{j=i}^{i-1} \psi_j) + R_n
\]

\[
S = \sum_{i=1}^{n} (S_i \prod_{j=i}^{i-1} \psi_j) + S_n
\]

\(R\) is the anti sliding force of the landslide; \(S\) is the sliding force of the landslide; \(R_i\) is the anti sliding force of the block \(i\); \(S_i\) is the sliding force of the i-th block, \(W_i\) is the weight of the i-th block, kN / m; \(C_i\) is the cohesion of the i-th sliding surface (belt), kPa; \(L_i\) is the length of the i-th sliding surface (belt), m; \(\phi_i\) is the internal friction angle of the i-th sliding surface (belt) (°); \(\alpha_i\) is the inclination angle of the i-th sliding surface (°); \(A\) is the horizontal acceleration of the earthquake, g; \(\psi_j\) is the transfer coefficient from the residual sliding force of the i-th block to the i+1 block.

The reliability state equation of loess landslides can be obtained by substituting the expressions of \(R_n\) and \(S_n\) into equation (8) and equation (9):

\[
Z = R - S = \sum_{i=1}^{n} (R_i \prod_{j=i}^{i-1} \psi_j) + R_n - \sum_{i=1}^{n} (S_i \prod_{j=i}^{i-1} \psi_j) - S_n
\]

\[
= \sum_{i=1}^{n} W_i (\cos \alpha_i - A \sin \alpha_i) \tan \phi_i + C_i L_i \prod_{j=i}^{i-1} \psi_j + W_i (\cos \alpha_i - A \sin \alpha_i) \tan \phi_i + C_i L_i
\]

\[
- \sum_{i=1}^{n} W_i (\sin \alpha_i + A \cos \alpha_i) \prod_{j=i}^{i-1} \psi_j - W_i (\sin \alpha_i + A \cos \alpha_i)
\]

The reliable index \(\beta\) of state function \(Z = R - S\) is:
\[ \beta = \frac{\mu_z}{\sigma_z} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \]  

When \( R \) and \( S \) are normal distribution, the failure probability is as follows: 
\[ P_f = 1 - \Phi(\beta) \]  

2.2. Computing method 

The theoretical basis of reliability analysis of landslide is probability theory and mathematical statistics based on uncertainty theory. The uncertainty factors affecting the stability of landslide include the uncertainty of physical and mechanical parameters, the uncertainty of calculation model and the uncertainty of environmental factors. According to the analysis of previous research results, the most important factors affecting the reliability of loess landslides are cohesion \( c \), internal friction angle \( \phi \) and seismic acceleration \( A \). The weight of soil is properly weighted according to different soil layers as a constant. Therefore, this paper mainly studies the influence of shear strength parameters \( c \), \( \phi \) and seismic horizontal acceleration \( A \) as random variables on the reliability of loess landslide stability. A large number of previous research results show that the mechanical parameters of rock and soil, such as cohesion \( c \), internal friction angle \( \phi \), are approximately in accordance with normal or lognormal distribution. Therefore, in the process of reliability analysis of loess landslides under the action of seismic forces, the probability distribution types of cohesion \( c \) and internal friction angle \( \phi \) of the soil parameters in the sliding zone are fitted as normal distribution. 

For earthquakes, the frequency, time, space and intensity are all random. The study area is in the high intensity earthquake area, and the frequency of earthquakes is relatively high. According to the relevant research, it is shown that, the horizontal acceleration \( A \) of earthquake is a random variable, and conforms to the distribution of maximum type II. Its cumulative distribution function and probability density function are respectively:

\[ F(A) = \exp \left[ -\left( \frac{A}{\eta} \right)^2 \right] \]  

\[ f(A) = -\frac{2A}{\eta^2} \exp \left[ -\left( \frac{A}{\eta} \right)^2 \right] \]  

In equation (14), \( \eta \) is the scale parameter. 
\[ \eta = 0.38\tau_s \]  

\( \tau_s \) is the peak value of ground horizontal acceleration \( (m/s^2) \). 

The relationship between the maximum ground horizontal acceleration and the seismic intensity \( I \) is \( I_g 100\tau_s = 1.1\eta_2 - 0.01 \). In the process of reliability analysis of loess landslides under the earthquake action in this paper, the seismic horizontal acceleration \( A \) accords with the distribution of maximum II type. After the equivalent normalization, it is transformed into the random variable of normal distribution, and then \( \mu_s \cdot \mu_s \) and \( \sigma_s \cdot \sigma_s \) are calculated according to the probability distribution. 

Based on Monte Carlo method, for general landslides, the number of simulation \( N= 5000-10000 \) can meet the accuracy requirements of stability evaluation. In this paper, the calculation times of reliability analysis of loess landslide stability under earthquake action are 8000 times, and 8000 groups of values of cohesion \( c \), internal friction angle \( \phi \) and seismic horizontal acceleration \( A \) in normal distribution are randomly generated. Then, these values are substituted into function functions in turn, and the reliability index \( \beta \) of loess landslide stability under earthquake action can be calculated by using MATLAB software, Thus, the failure probability \( P_f \) of landslide can be further solved.
3. Reliability analysis of the stability of a loess landslide under earthquake

A typical section (Figure 1) is selected for reliability analysis of a loess landslide, and the formula (10) is used as the function for calculation.

Figure 1. Stability calculation section of loess landslide.

The probability distribution type of cohesion $c$ and internal friction angle $\phi$ in the sliding zone of loess landslide is fitted as normal distribution. Combined with the indoor test results, the calculation parameters are shown in Table 1. The seismic horizontal acceleration $A$ conforms to the distribution of maximum type II, which is transformed into the random variable of normal distribution after the equivalent normalization. According to the national standard No.1 revision sheet of seismic ground motion parameter zoning map of China (GB110386-2001) implemented in June 2008, the seismic intensity of landslide area is VⅢ degree, and the design basic seismic acceleration is 0.20g.

Table 1. Statistical results of soil parameters in a landslide slide zone.

| Parameter          | Mean value (kPa) | Standard deviation | Coefficient of variation | $c$、$\phi$ correlation coefficient |
|--------------------|------------------|--------------------|--------------------------|-----------------------------------|
| Cohesive force $C$ | 20.1             | 3.81               | 0.152                    | -0.3                              |
| Internal friction angle $\phi$ | 12.3          | 2.13               | 0.117                    | -0.3                              |

Using Monte Carlo method, the number of calculation times is 8000, and 8000 groups of normal distribution cohesion $c$, internal friction angle $\phi$ and horizontal acceleration $A$ (A value is taken as 0 when the earthquake action is not considered) are randomly generated, and then substituted into the function in turn. The calculation is carried out by using MATLAB software, and the instability probability of a loess landslide under seismic and natural conditions is obtained. The calculation results are shown in Table 2.

Table 2. Calculation results of probability and reliability index of a loess landslide.

| Working condition | Mean value of stability | Standard deviation of stability | Reliability index | Instability probability | Stability evaluation |
|-------------------|-------------------------|--------------------------------|-------------------|-------------------------|---------------------|
| Seismic condition | 1.19                    | 0.38                           | 0.451             | 32.7%                   | Less stable, medium risk |
| Natural condition | 1.45                    | 0.48                           | 1.39              | 9.1%                    | Basically stable, low risk |

It can be seen from Table 2 that the probability of instability of a loess landslide in natural state is 9.1%, which is basically stable. The failure probability of a loess landslide under the action of earthquake force is 32.7%, which is under stable. The calculation results are consistent with the actual situation.
4. Conclusion
Considering the randomness of geotechnical parameters and seismic acceleration, a reliability method of loess landslide stability under seismic action is proposed, and the model is applied to the seismic stability evaluation of a loess landslide. The calculation results are consistent with the actual situation, which proves the correctness of the reliability analysis model of loess landslide stability under the seismic action.

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