Neutrino Masses in SUSY Models

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Abstract

In this talk we assume the conventional see-saw mechanism, and construct a hierarchical pattern of three active neutrinos with bi-maximal mixing. In order to enforce the hierarchy we use the single right handed neutrino dominance mechanism which is a very nice way to ensure a neutrino mass hierarchy in the presence of large mixing angles. We show how this mechanism can be organised in the framework of a $U(1)$ family symmetry, then discuss a realistic string-inspired model which includes such a symmetry.
1 Introduction

The problem of understanding the quark and lepton masses and mixing angles represents one of the major unsolved questions of the standard model. Recently additional information on the fermion mass spectrum has come from the measurement of the atmospheric neutrino masses and mixing angles by Super-Kamiokande \cite{1}. The most recent data disfavors mixing involving a sterile neutrino, and finds a good fit for $\nu_\mu \to \nu_\tau$ mixing with $\sin^2 2\theta_{23} > 0.88$ and a mass square splitting $\Delta m^2_{23}$ in the $1.5 - 5 \times 10^{-3} \text{ eV}^2$ range at 90\% CL \cite{2}. Super-Kamiokande has also provided additional support for solar neutrino mixing. The most recent Super-Kamiokande data does not show a significant day-night asymmetry and shows an energy independent neutrino spectrum, thus it also disfavors the sterile neutrino mixing hypothesis, the just-so vacuum oscillation hypothesis, and the small mixing angle (SMA) MSW \cite{3} solution \cite{4}. The preferred solution at the present time seems to be the large mixing angle (LMA) MSW solution, although a similar solution with a low mass splitting (the LOW) solution is also possible. A typical point in the LMA MSW region is $\sin^2 2\theta_{12} \approx 0.75$, and $\Delta m^2_{12} \approx 2.5 \times 10^{-5} \text{ eV}^2$ \cite{5}.

If one accepts the recent data as evidence for neutrino masses and mixing angles, then the obvious question is how these can be accommodated in the standard model, or one of its supersymmetric extensions. The simplest possibility to account for the smallness of the neutrino masses is the see-saw mechanism \cite{6} in which one introduces right-handed neutrinos which acquire very large Majorana masses at a super-heavy mass scale. When one integrates out the right-handed neutrinos the “normal sized” Dirac Yukawa couplings, which connect the left-handed to the right-handed neutrinos, are transformed into very small couplings which generate very light effective left-handed physical Majorana neutrino masses. Given the see-saw mechanism, it is natural to expect that the spectrum of the neutrino masses will be hierarchical, since
Having assumed the see-saw mechanism and a hierarchical neutrino mass spectrum, the next question is how such large (almost maximal) lepton mixing angles such as $\theta_{23}$ could emerge? There are several possibilities that have been suggested in the literature. One possibility is that it happens as a result of the off-diagonal 23 entries in the left-handed Majorana matrix being large, and the determinant of the 23 sub-matrix being accidentally small, leading to a neutrino mass hierarchy with large neutrino mixing angles \[8\]. Another possibility is that the neutrino mixing angles start out small at some high energy scale, then get magnified by renormalization group (RG) running down to low energies \[9\]. A third possibility is that the off-diagonal elements of the left-handed neutrino Majorana matrix are large, but the 23 sub-determinant of the matrix is small for a physical reason, as would be the case if a single right-handed neutrino were providing the dominant contribution to the 23 sub-matrix \[10, 11, 12, 13\]. We shall refer to these three approaches as the accidental, the magnification and the single right-handed neutrino dominance (SRHND) mechanisms, respectively.

A promising approach to understanding the fermion mass spectrum is within the framework of supersymmetric (SUSY) unified theories. Within the framework of such theories the quark and lepton masses and mixing angles become related to each other, and it begins to be possible to understand the spectrum. The simplest grand unified theory (GUT) is $SU(5)$ but this theory in its minimal version does not contain any right-handed neutrinos. Nevertheless three right-handed neutrinos may be added, however this is not guaranteed due to the unknown structure of the heavy Majorana matrix, and for example an inverted neutrino mass hierarchy could result although this relies on some non-hierarchical couplings in the Dirac Yukawa matrix \[8\].
and in this theory it is possible to have a large 23 element on the Dirac neutrino Yukawa matrix without introducing a large 23 element into any of the charged fermion Yukawa matrices. The problem of maintaining a 23 neutrino mass hierarchy in these models may be solved for example by assuming SRHND. Another possibility within the framework of SU(5) is to maintain all the off-diagonal elements to be small, but require the 22 and 32 elements of the Dirac neutrino Yukawa matrix to be equal and the second right-handed neutrino to be dominant, in which case SRHND again leads to a large 23 neutrino mixing angle with hierarchical neutrino masses. However the drawback of SU(5) is that it does not predict any right-handed neutrinos, which must be added as an afterthought.

From the point of view of neutrino masses, the most natural GUTs are those like SO(10) that naturally predict right-handed neutrinos. However within the framework of SO(10) the quark masses and mixing angles are related to the lepton masses and mixing angles, and the existence of large neutrino mixing angles is not expected in the minimal versions of the theory in which the Higgs doublets are in one (or two) 10’s (ten dimensional representations of SO(10)) and each matter family is in a 16. Nevertheless various possibilities have been proposed in SO(10) in order to account for the large neutrino mixing angles. Within the framework of minimal SO(10) with third family Yukawa unification, it has been suggested that if two operators with different Clebsch coefficients contribute with similar strength then, with suitable choice of phases, in the case of the lepton Yukawa matrices one may have large numerical 23 elements, which add up to give a large lepton mixing angle, while for the quarks the 23 elements can be small due to approximate cancellation of the two contributing operators. This is an example of the accidental mechanism mentioned above, where in addition one requires the quark mixing angles to be small by accident, although it remains to be seen if the LMA MSW solution could be understood in this framework.

\(^3\)We use Left-Right (LR) convention for Yukawa matrices in this paper.
Moving away from minimal $SO(10)$, one may invoke a non-minimal Higgs sector in which one Higgs doublet arises from a $10$ and one from a $16$, and in this framework it is possible to understand atmospheric neutrino mixing [17]. Alternatively, one may invoke a non-minimal matter sector in which parts of a quark and lepton family arise from a $16$ and other parts from a $10$, and in these models one may account for atmospheric and solar neutrinos via an inverted mass hierarchy mechanism [18].

We have recently discussed [19] neutrino masses and mixing angles in a particular string-inspired minimal model based on the Pati-Salam $SU(4) \times SU(2)_L \times SU(2)_R$ (422) group. As in $SO(10)$ the presence of the gauged $SU(2)_R$ predicts the existence of three right-handed neutrinos. However, unlike $SO(10)$, there is no Higgs doublet-triplet splitting problem since in the minimal model both Higgs doublets are contained in a $(1,2,2)$ representation. Moreover, since the left-handed quarks and leptons are in the $(4,2,1)$ and the right-handed quarks and leptons in the $(4,1,2)$ representations, the model also leads to third family Yukawa unification as in minimal $SO(10)$. Although the Pati-Salam gauge group is not unified at the field theory level, it readily emerges from string constructions either in the perturbative fermionic constructions [21], or in the more recent type I string constructions [22], unlike $SO(10)$ which typically requires large Higgs representations which do not arise from the simplest string constructions. The question of fermion masses and mixing angles in the string-inspired Pati-Salam model has already been discussed for the case of charged fermions [23, 24], and later for the case of neutrinos [25]. For the neutrino study [23] it was assumed that the heavy Majorana neutrino mass matrix was proportional to the unit matrix, and only small neutrino mixing angles were considered. Later on a $U(1)_X$ family symmetry was added to the model, in order to understand the horizontal hierarchies, although in this case the neutrino spectrum was not analysed at all [26].
We discussed [19] neutrino masses and mixing angles in the string-inspired Pati-Salam model supplemented by a $U(1)_X$ flavour symmetry. The model involves third family Yukawa unification and predicts the top mass and the ratio of the vacuum expectation values $\tan \beta$, as we also recently discussed in Ref. [27]. It was already known that the model can provide a successful description of the CKM matrix and predicts the down and strange quark masses, although our updated analysis differed from that presented previously [26] partly due to the recent refinements in third family Yukawa unification [27], but mainly as a result of the recent Super-Kamiokande data which has important implications for the flavour structure of the model. In fact our main focus was on the neutrino masses and mixing angles which were not previously discussed at all in this framework. We assumed a minimal version of the model, and avoided the use of the accidental cancellation mechanism, which in any case has difficulties in accounting for bi-maximal neutrino mixing. We also showed that the mixing angle magnification mechanism can only provide limited increases in the mixing angles, due to the fact that the unified third family Yukawa coupling is only approximately equal to 0.7 [27] and is therefore too small to have a dramatic effect. Instead, we relied on the SRHND mechanism, and we showed how this mechanism may be implemented in the 422 model by appropriate use of operators with “Clebsch zeros” resulting in a natural explanation for atmospheric neutrinos via a hierarchical mass spectrum. We specifically focused on the LMA MSW solution since this is preferred by the recent fits.

2 Single Right-handed Neutrino Dominance (SRHND)

Consider two RH neutrinos for simplicity. Then let us write the (Dirac) Yukawa couplings in the LR basis as

$$Y_\nu = \begin{pmatrix} 0 & a & d \\ 0 & b & e \\ 0 & c & f \end{pmatrix}$$

(1)
and the heavy Majorana mass matrix as

\[ M_{RR} = \begin{pmatrix} 
0 & 0 & 0 \\
0 & X & 0 \\
0 & 0 & Y 
\end{pmatrix} \]  \hspace{1cm} (2)

Then using the see-saw formula for the light effective Majorana mass matrix

\[ m_{LL} = Y_\nu M_{RR}^{-1} Y^T_\nu v_2^2 \] where \( v_2 \) is the Higgs doublet vacuum expectation value,

\[ m_{LL} = \begin{pmatrix} 
\frac{d^2}{Y} + \frac{a^2}{X} & \frac{de}{Y} + \frac{ab}{X} & \frac{df}{Y} + \frac{ac}{X} \\
\frac{e^2}{Y} + \frac{b^2}{X} & \frac{ef}{Y} + \frac{bc}{X} \\
\frac{f^2}{Y} + \frac{c^2}{X} & \frac{e}{Y} & \frac{f}{Y} \end{pmatrix} v_2^2 \]  \hspace{1cm} (3)

Now suppose that the third right-handed neutrino contributions \( \sim \frac{1}{Y} \) dominate the 23 block of \( m_{LL} \), then we get an automatic hierarchy even for large 23 mixing:

\[ \text{det}[m_{LL}]_{23} = m_{\nu_2} m_{\nu_3} \sim 0 \Rightarrow m_{\nu_2} / m_{\nu_3} \ll 1 \]  \hspace{1cm} (4)

This physical mechanism responsible for the neutrino mass hierarchy is called SRHND.

In the limit that only a single right handed neutrino contributes the determinant clearly exactly vanishes and we have \( m_{\nu_2} = 0 \) exactly. In the actual situation the third right-handed neutrino dominates, and sub-dominant contributions from the second right-handed neutrino give a small finite mass \( m_{\nu_2} \neq 0 \) (suitable for the MSW solution to the solar neutrino problem) whilst maintaining the 23 mass hierarchy. To be precise we have,

\[ m_{\nu_1} = 0, \] \hspace{1cm} (5)

\[ m_{\nu_2} \sim \frac{(b - c)^2}{X} v_2^2 \] \hspace{1cm} (6)

\[ m_{\nu_3} \sim \frac{(d^2 + e^2 + f^2)}{Y} v_2^2 \] \hspace{1cm} (7)

The mixing angles may easily be estimated to be [12]

\[ \tan \theta_{23} \sim \frac{e}{f} \sim 1, \] \hspace{1cm} (8)

\[ \tan \theta_{13} \sim \frac{d}{\sqrt{e^2 + f^2}} \ll 1 \] \hspace{1cm} (9)
\[
\tan \theta_{12} \sim \frac{a}{b-c} \sim 1
\]  

(10)

Thus by a suitable choice of Yukawa couplings it is possible to have a large 12 angle suitable for the LMA MSW solution and a large 23 angle suitable for atmospheric oscillations, while maintaining a small 13 angle consistent with the CHOOZ constraint. Note that the bi-maximal mixing angle scenario does not threaten the 23 mass hierarchy which is maintained by SRHND. In other words, providing SRHND is in operation, the hierarchy is guaranteed in the presence of large mixing angles. This is a very nice feature of SRHND.

3 \textit{U}(1) Family Symmetry

Introducing a \textit{U}(1) family symmetry \cite{28}, \cite{29}, \cite{30}, \cite{31} provides a convenient way to organise the hierarchies within the various Yukawa matrices. For definiteness we shall focus on a particular class of model based on a single pseudo-anomalous \textit{U}(1) gauged family symmetry \cite{30}. We assume that the \textit{U}(1) is broken by the equal VEVs of two singlets \(\theta, \bar{\theta}\) which have vector-like charges \(\pm 1\) \cite{30}. The \textit{U}(1) breaking scale is set by \(\langle \theta \rangle = \langle \bar{\theta} \rangle\) where the VEVs arise from a Green-Schwartz mechanism \cite{32} with computable Fayet-Illiopoulos D-term which determines these VEVs to be one or two orders of magnitude below \(M_U\). Additional exotic vector matter with mass \(M_V\) allows the Wolfenstein parameter \cite{33} to be generated by the ratio \cite{30}

\[
\frac{\langle \theta \rangle}{M_V} = \frac{\langle \bar{\theta} \rangle}{M_V} = \lambda \approx 0.22
\]  

(11)

The idea is that at tree-level the \textit{U}(1) family symmetry only permits third family Yukawa couplings (e.g. the top quark Yukawa coupling). Smaller Yukawa couplings are generated effectively from higher dimension non-renormalisable operators corresponding to insertions of \(\theta\) and \(\bar{\theta}\) fields and hence to powers of the expansion parameter in Eq.\(\text{[11]}\) which we have identified with the Wolfenstein parameter. The number of
powers of the expansion parameter is controlled by the $U(1)$ charge of the particular operator. The fields relevant to neutrino masses $L_i$, $N_p^c$, $H_u$, $\Sigma$ are assigned $U(1)$ charges $l_i$, $n_p$, $h_u = 0$, $\sigma$. From Eqs.[4], the neutrino Yukawa couplings and Majorana mass terms may then be expanded in powers of the Wolfenstein parameter,

$$M_{RR} \sim \begin{pmatrix}
\lambda^{2|n_1+\sigma|} & \lambda^{n_1+n_2+\sigma} & \lambda^{n_1+n_3+\sigma} \\
\text{.} & \lambda^{2|n_2+\sigma|} & \lambda^{n_2+n_3+\sigma} \\
\text{.} & \text{.} & \lambda^{2|n_3+\sigma|}
\end{pmatrix}$$

(12)

The conditions which ensure that the third dominant neutrino is isolated require that the elements $\lambda^{n_1+n_3+\sigma}$, $\lambda^{n_2+n_3+\sigma}$ be sufficiently small.

The neutrino Yukawa matrix is explicitly

$$Y_\nu \sim \begin{pmatrix}
\lambda^{|l_1+n_1|} & \lambda^{|l_1+n_2|} & \lambda^{|l_1+n_3|} \\
\lambda^{|l_2+n_1|} & \lambda^{|l_2+n_2|} & \lambda^{|l_2+n_3|} \\
\lambda^{|l_3+n_1|} & \lambda^{|l_3+n_2|} & \lambda^{|l_3+n_3|}
\end{pmatrix}$$

(13)

The requirement of large 23 mixing and small 13 mixing becomes

$$|n_3 + l_2| = |n_3 + l_3|, \quad |n_3 + l_1| - |n_3 + l_3| = 1 \text{ or } 2$$

(14)

4 Examples

Let us consider an example of charges which lead to SRHND with bimaximal mixing,

$$l_i = (-2, 0, 0), n_i = (-2, 1, -1), \sigma = 1$$

(15)

then this leads to

$$m_{LL} \sim \begin{pmatrix}
\lambda^5 + 2\lambda^5 & \lambda^3 + \lambda^5 + \lambda^3 & \lambda^3 + \lambda^5 + \lambda^3 \\
\text{.} & \lambda + 2\lambda^3 & \lambda + \lambda^3 + \lambda^3 \\
\text{.} & \text{.} & \lambda + 2\lambda^3
\end{pmatrix}$$

where the first entry in each element corresponds to the $1/Y$ contributions coming from the right-handed neutrino $N_{3R}$, and clearly dominates the 23 block by a factor of $\lambda^2$. In this case it does not dominate the other elements outside the 23 block. The 13 element from $N_{3R}$ is suppressed relative to the 23 block elements by a factor of $\lambda^2$, leading to a CHOOZ angle of this order. The subdominant entries in the 12,13,22,23 elements are of the same order, leading to a large 12 angle suitable for LMA MSW.
5 Neutrino Masses and Mixing Angles in a Realistic String-Inspired Model \[19\]

Consider the Pati-Salam gauge group:

\[ SU(4) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1) \]  

(16)

with fermions in the superfield multiplets

\[ F^i_{L,R} = \begin{pmatrix} u & u & u & \nu \\ d & d & d & e^- \end{pmatrix}^{i}_{L,R} \]  

(17)

The model has the following features:

- predicts three right-handed neutrinos
- has third Family Yukawa Unification
- gives vertical hierarchies from Clebsch coefficients
- horizontal hierarchies from \( U(1) \) family symmetry
- this gauge group has emerged from explicit type I string constructions.

The Higgs \( h \) contains the two MSSM Higgs doublets

\[ h = \begin{pmatrix} h_1^0 & h_2^+ \\ h_1^- & h_2^0 \end{pmatrix} \]  

(18)

The Higgs \( H, \bar{H} \) break the Pati-Salam group and \( \theta, \bar{\theta} \) break \( U(1) \) family symmetry.

\[ H, \bar{H} = \begin{pmatrix} u_H & u_H & u_H & \nu_H \\ d_H & d_H & d_H & e_H \end{pmatrix}, \ldots \]  

(19)

\[ < H > = < \bar{H} > = < \nu_H > \sim M \sim 10^{16} \text{GeV} \]  

(20)

\[ < \theta > = < \bar{\theta} > \sim M \sim 10^{16} \text{GeV} \]  

(21)

\[ SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R \otimes U(1) \]
\[ \rightarrow SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \]

The fermion mass operators (responsible for Yukawa matrices) are:

\[
(F^i_L F^j_R) h \left( \frac{HH}{M^2} \right)^n \left( \frac{\theta}{M} \right)^p
\]  

(22)

The Majorana mass operators (responsible for \( M_{RR} \)) are:

\[
(\bar{F}^i_R \bar{F}^j_R) \left( \frac{HH}{M^2} \right) \left( \frac{\bar{H}H}{M^2} \right)^m \left( \frac{\theta}{M} \right)^q
\]  

(23)

The \( U(1) \) family symmetry charges we assume are:

\[
F^1_L = 1, \quad F^2_L = 0, \quad F^3_L = 0
\]

\[
\bar{F}^1_R = 4, \quad \bar{F}^2_R = 2, \quad \bar{F}^3_R = 0
\]

Approximate Structure of the Matrices:

\[
\lambda_u(M_X) \sim \begin{pmatrix} \delta^3 \epsilon^5 & \delta^2 \epsilon^3 & \delta^2 \epsilon \\ \delta^3 \epsilon^4 & \delta^2 \epsilon^2 & \delta^3 \\ \delta^3 \epsilon^4 & \delta^2 \epsilon^2 & 1 \end{pmatrix}
\]

\[
\lambda_d(M_X) \sim \begin{pmatrix} \delta^5 \epsilon^5 & \delta^4 \epsilon^3 & \delta^2 \epsilon \\ \delta^5 \epsilon^4 & \delta^4 \epsilon^2 & \delta^3 \\ \delta^4 \epsilon^4 & \delta^2 \epsilon^2 & 1 \end{pmatrix}
\]

\[
\lambda_e(M_X) \sim \begin{pmatrix} \delta^5 \epsilon^5 & \delta^4 \epsilon^3 & \delta^2 \epsilon \\ \delta^5 \epsilon^4 & \delta^4 \epsilon^2 & \delta^3 \\ \delta^4 \epsilon^4 & \delta^2 \epsilon^2 & 1 \end{pmatrix}
\]

\[
\lambda_\nu(M_X) \sim \begin{pmatrix} \delta^3 \epsilon^5 & \delta^3 \epsilon^3 & \delta^2 \epsilon \\ \delta^3 \epsilon^4 & \delta^2 \epsilon^2 & \delta \\ \delta^3 \epsilon^4 & \delta^2 \epsilon^2 & 1 \end{pmatrix}
\]

\[
M_{RR}(M_X) \sim \begin{pmatrix} \delta \epsilon^8 & \delta \epsilon^6 & \delta \epsilon^4 \\ \delta \epsilon^6 & \delta \epsilon^4 & \delta \epsilon^2 \\ \delta \epsilon^4 & \delta \epsilon^2 & 1 \end{pmatrix}
\]

where \( \delta = \frac{<H>}{M^2} \), \( \epsilon = \frac{<\theta>}{M} = \frac{<\bar{\theta}>}{M} \).

If we assume similar expansion parameters \( \delta \sim \epsilon \sim \lambda \sim 0.22 \) then the above matrices lead to SRHND with the third right-handed neutrino giving the dominant contribution the 23 block of \( m_{LL} \).
The neutrino masses at low energies are:

\[m_{\nu_1} = 4.84 \times 10^{-8} \text{ eV},\]
\[m_{\nu_2} = 5.79 \times 10^{-3} \text{ eV},\]
\[m_{\nu_3} = 5.39 \times 10^{-2} \text{ eV}.\]

It is interesting to consider the renormalisation group evolution of neutrino mixing angles at different scales:

\[Q = M_X \sim 3 \times 10^{16} \text{ GeV}\]
\[\sin^2(2\theta_{12}) = 0.828, \sin^2(2\theta_{23}) = 0.890, \sin^2(2\theta_{13}) = 0.025\]

\[Q = M_{\nu_3} \sim 3 \times 10^{14} \text{ GeV}\]
\[\sin^2(2\theta_{12}) = 0.832, \sin^2(2\theta_{23}) = 0.908, \sin^2(2\theta_{13}) = 0.027\]

\[Q = M_Z\]
\[\sin^2(2\theta_{12}) = 0.853, \sin^2(2\theta_{23}) = 0.943, \sin^2(2\theta_{13}) = 0.027\]

6 Conclusions

We have assumed the conventional see-saw mechanism, and constructed a hierarchical pattern of three active neutrinos with bi-maximal mixing. In order to enforce the hierarchy we used the SRHND mechanism. This is a very nice way to ensure a neutrino mass hierarchy in the presence of large mixing angles.

We showed how this mechanism can be organised in the framework of a \(U(1)\) family symmetry, then went on to discuss a realistic string-inspired model which includes such a symmetry. The main features of our approach may be summarised as follows:
• SRHND is a natural and general mechanism for yielding hierarchical neutrino masses in the presence of large 23 mixing.

• SRHND underpins success of string-inspired SUSY Pati-Salam.

• “Clebsch zero” operators play key role

• SRHND models are stable under radiative corrections, but atmospheric mixing can be increased by several per cent.

• SRHND underpins several other models in the literature [14, 15].

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