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A deformation model and draping behavior analysis of plain weave fabric with low-twist yarn on continuous surface

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Abstract

Fiber-reinforced composites have been widely applied in aerospace, transportation and other industrial applications. An effective method shaping the complex forms is draping plain fabrics on the mandrel surface. However, it is a challenge to realize and predict accurately the deformation of fabric. To establish an accurate deformation mode, plain weave fabrics with low-twist yarn were classified into a stable structure that are not easy to deform and an unstable structures with large deformability based on microstructure characteristics. A variable microstructure unit-cell model has been established to analyze the variation of fabric geometry and performance during deformation. To analyze the draping behavior of fabric with large deformation, a four-node unit was used to mesh the fabric, and then each node on the fabric was mapped to the mandrel surface. The deformation of fabric after draping was simulated by the continuous change of the unit mesh. Analysis results showed that the thickness of preform decreases with the increase of the major-axis of the yarn cross-section, and the formability of fabric increases with the increase of fabric pitch. The accuracy and effectiveness of the geometric mapping method are verified by the fabric draping experiment.

1. Introduction

Fiber-reinforced composites have found wide applications in aerospace, transportation, sports, and marine industries because of their high specific modulus and strength, high fatigue resistance [1]. To address a wide range of applicability, it is most common to fabricate these lay-ups in a plain state and then apply them on a surface having the aspired geometry. This forming process is called draping [2]. The draping of woven fabrics is an important concern in the manufacturing of composite material products [3]. Messiry et al [4] developed a mathematical model to express the drape coefficient and tested the drape of fabric with different weaves structures. The results indicated that the arrangement of thread interlacements affects the forming of the folds and fabric drapability. Breen et al [5] proposed an interacting-particle model based on the microstructure of woven fabric to recover quantitative mechanical information and reproduce the unique largescale draping characteristics of a range of fabrics. Kown et al [6] used drape evaluation system to study the effect of fabric properties on drape parameters. The results showed that tensile, bending, and shear properties of fabrics are closely related to each drape parameter. Tan et al [7] prepared two types of fabric. The tensile, bending, drape and bursting properties of two fabrics were characterized. Results showed that the basic structure of the fabric has impacted on the mechanical properties of the fabric significantly.

Mathematics models are vital to analyze the structure and performance variation tendency of plain fabric during deformation [8]. Chen et al [9] proposed a stress resultant shell approach to simulate the draping of textile composite reinforcements with continuous fibers. Han et al [10] implemented a draping simulation technique that can reflect the mechanical behavior of uncured woven fabric prepregs. The drapability of fabric is important in evaluating the deformation behavior of preform [11]. The deformation problems of fabrics have been studied by many researchers. Bai et al [12] proposed a fibrous shell approach based on the quasi-inextensibility of
the fibers to correctly model the deformation during forming. Buyukbayraktar [13] studied the effect of change of angle between warp and weft yarns during the shear deformation on the drape, bending and forming properties of the product. Dileep et al [14] compared two analytical approaches to describe the deformation and the resulting strains. The results showed that the wrinkling prediction calculations depend on the pre-deformation. Nasri et al [15] investigated the behavior of woven composite fabrics under large deformation. A model which can predict the initial phase during tensile loading was established.

Most of the numerical simulations of dry textile reinforcements forming are based on a macroscopic approach and continuous models whose behavior is assumed to be elastic. However, experience shows that under loading stresses, inelastic deformations are observed [16, 17]. This research aims to find plain fabric structure with deformation ability and study the variation trend of fabric geometry and performance. The permanent deformation mechanism of low-twist plain fabric was analyzed based on the quasi-plastic...
characteristics. The geometry and performance variation trend of preform were analyzed by establishing the mapping relation between plain fabric and mandrel.

2. The quasi-plastic deformation mechanism of plain fabric with low-twist yarn

The inelastic deformation behavior of fabric before the elastic stage under loading stresses was defined as quasi-plastic deformation in this research. There are two types of quasi-plastic deformation during the forming process, small axial deformation under axial tension, as shown in figure 1(a), and large angular deformation under diagonal stretch as shown in figure 1(b). The mechanism of the small axial deformation is fiber slip (yarn cross-section deformation) caused by unstable fiber arrangement under external load. The large angular deformation is caused by the change of interleaving angle between warp and weft.

3. Structure classification and deformation behavior of plain weave fabric

The plain weave fabric could be classified in two categories according to their microstructure and deformation behavior during forming process.

3.1. Small deformability structure (Type-I)

The structure which the center lines (ellipse-I and ellipse-II) of all adjacent and parallel yarns in the fabric are tangent is defined as type-I structure, as shown in figure 2(a). The major and minor axes of the yarn cross-section are \(2a\) and \(2b\), respectively. Plain weave fabric has biaxial deformation characteristics. Due to the inextensibility of the fiber, there is ultimate state under compression and tension during the forming process. The shape of the yarn cross-section changes after loading, but the area remains unchanged, it can be expressed as \(\pi a_1 b_1 = \pi a_2 b_2 = \pi ab\). The compression and tensile deformation of fabric could be classified as biaxial and uniaxial as shown in figures 2(b), (c), \(R\) and \(SR\) represent the radii of cylinder and sphere, respectively. This kind of structure is suited for being draped on a concave surface due to its ability of relatively large compressive deformation compared to other

![Figure 3](image-url)
structures. However, the quasi-plastic deformation of the fabric is limited. Thus, the plain weave fabric with a type-I structure is suitable only for draping the small curvature surface.

3.2. Large deformability structure (Type-II)
The structure which the central lines (ellipse-I and ellipse-II) of all adjacent and parallel yarns in the fabric are not tangent is defined as large deformability structure (type-II), as shown in figure 3(a). The deformation coefficient of the yarn cross-section is defined as \( k = \frac{a_k}{b_k} \), where \( a_k \) and \( b_k \) are half the length of the major and minor axis during deformation. When the fabric is in the initial state before deformation, the corresponding value is represented by \( k_0, a_0 \) and \( b_0 \), and the common tangent \( l_t > 0 \) as shown in figure 3(a). The fabric’s axial tensile and compressive deformation leads to the change in coefficient \( k \), as shown in figure 3(b). The fabric shows intrinsic structural deformation and yarn rotation when subjected to diagonal stretching as shown in figure 3(c).

4. Variable microstructure unit-cell (VMU) model of plain weave fabric

4.1. Basic assumptions
The following assumptions have been made to establish the model.

(1) The yarns in the fabric are low-twist yarn.
(2) The plain weave fabric consists of inextensible fibers.
(3) The elliptical shape is considered for yarn cross-section.

4.2. The VMU model of type-I structure
The VMU of plain weave fabric is shown in figure 4(a). \( L \) and \( H \) represent VMU length and height, respectively. The mathematical model of yarn in VMU is shown in figure 4(b). Ellipses I and II are congruent figures. The length of a single yarn in VMU is \( l \), and it consists of \( e', l_1, \) and \( f' \). The fiber volume fraction may then be calculated with equation (1).
Figure 5 shows the geometric relationship when the fabric deforms to the same degree along the \( x \) and \( y \) axes. Take the center of left ellipse as the origin of the coordinate system, the common tangent point of ellipses I and II is \( (L/2, -b/L) \), the VMU length can be expressed as equation (2).

\[
\begin{aligned}
L &= \sqrt{3} (a + b) \\
H &= 4b
\end{aligned}
\]  

(2)

The initial yarn length \( l_0 \) before deformation could be expressed as equation (3):

\[
l_0 = 2 \times \int_{-\pi/2}^{\pi/2} \sqrt{[(a_0 + b_0) \sin \alpha]^2 + (2b_0 \cos \alpha)^2} \, d\alpha
\]  

(3)

where \( a_0 \) and \( b_0 \) represent the semi-major axis and semi-minor axis of the yarn cross-section before deformation, respectively. \( \theta_0 \) is initial angle between horizontal and a connecting line of yarn center point, as given by

\[
\theta_0 = \arctan \left( \frac{-2b_0}{\sqrt{3} (a_0 + b_0)} \right)
\]

The yarn length \( l_k \) after deformation can be expressed as equation (4):

\[
l_k = 2 \times \int_{-\pi/2}^{\pi/2} \sqrt{[(a_k + b_k) \sin \alpha]^2 + (2b_k \cos \alpha)^2} \, d\alpha
\]  

(4)

where \( a_k \) and \( b_k \) are the semi-major axis and semi-minor axis of the yarn cross-section after deformation, respectively.

According to the above derivation, when \( l_k < l_0 \), the fabric can be compressed and deformed. The inequality can be expressed as \( f(k) \leq 0 \), where \( k = a_k / b_k \). Therefore, the range of \( k \) can be obtained as \([k_{\text{min}}, k_0]\). Consider a type-I structure with initial parameters \( S = 0.5 \, \text{mm}^2, k_0 = a_0 / b_0 = 2, l_0 = 1.8121 \, \text{mm} \). Figure 6 shows the variation in yarn length \( l_k \) with \( k \).
change of yarn length $l_k$ with deformation coefficient $k$. It should be noted that the structure could deform into type-II when $l_k < l_0$.

Figure 7 shows the geometric relationship when the fabric deforms to the different degree along the $x$ and $y$ axes. $k_1$ ($k_1 > k_0$) and $k_2$ ($k_2 < k_0$) are cross-section deformation coefficients along the $x$ and $y$ axis, respectively. According to the geometric relationship, the length ($L_1$), width ($W_1$) and height ($H_1$) of VMU can be expressed by yarn cross-section parameters $(a_i, b_i)$ as follows:

$$\begin{align*}
L_1 &= 2 \sqrt{(b_1 + b_2)^2 - b_i^2 \left(\frac{a_i + b_i}{b_1 + b_2}\right)}, \\
W_1 &= 2 \sqrt{(b_1 + b_2)^2 - b_i^2 \left(\frac{a_2 + b_1}{b_1 + b_2}\right)}, \\
H_1 &= 2(b_1 + b_2) \\
\end{align*}$$

The yarn length $(l_1', l_2')$ in VMU can be calculated as:

$$\begin{align*}
l_1' &= 2 \int_{-\pi/2}^{\pi/2} \sqrt{((a_1 + b_2)\sin\varphi)^2 + ((b_1 + b_2)\cos\varphi)^2} \, d\varphi, \\
l_2' &= 2 \int_{-\pi/2}^{\pi/2} \sqrt{((a_2 + b_1)\sin\varphi)^2 + ((b_1 + b_2)\cos\varphi)^2} \, d\varphi
\end{align*}$$

Equation (7) can be obtained

$$l_1' + l_2' = f(K, S)$$

where $K = a_1/a_2 = b_2/b_1$. 

![Figure 8. Variation of yarn axis ratio with single yarn length.](image1)

![Figure 9. VMU of type-II structure.](image2)
Figure 8 shows the relationship between $K$ and yarn length in VMU during deformation. When $K = 1$, $l_1' - l_2' = 0$, the plain weave fabric is the type-I structure. When $K < 1$, $l_1' - l_2' = 0$, plain weave fabric can be transformed into type-II structure. The deformability decreases with the increases of $K$.

4.3. The VMU model of type-II fabric structure

Figure 9 shows the VMU model of the type-II fabric. The geometric center point is $(L/2, -b)$. The tangency point of the yarn central line and ellipse-I is $(x_0, y_0)$. $\theta$ is the angle between the horizontal and the line connecting the tangency point and center.

The initial yarn length before deformation can be expressed as equation (8)

$$l = 2l_m + l_i$$

where $l_m = \int_0^{\theta} \sqrt{(a + b \sin \theta)^2 + (2b \cos \theta)^2} \, d\theta$

Equation (9) can be obtained by substituting the above equation into equation (8).

$$l = 2 \left( \int_{-\pi/2}^{\theta} \sqrt{(a + b \sin \theta)^2 + (2b \cos \theta)^2} \, d\alpha + \sqrt{(x_0 - L/2)^2 + (y_0 + b)^2} \right) + l_i$$

The deformation state of the fabric can be indicated by $k$, when the fabric is compressed, $k < a_0/b_0$, and when it is stretched, $k > a_0/b_0$. The deformation ultimate state of type-II structure is the type-I structure. Limit value of unit length $L_{lim} = \sqrt{3} (a_{lim} + b_{lim})$. The total length of yarn remains unchanged during deformation.

Consider a type-II structure with yarn cross-section area $S = 0.5 \text{ mm}^2$, initial deformation coefficient $k_0 = a_0/b_0 = 2$, yarn length $L_2 = 2 \text{ mm}$. The initial length $l_0$ of VMU can be calculated as $l_0 = 2.7824 \text{ mm}$. Figure 10 shows the changes of parameters in the process of deformation. The fabric thickness decreases nonlinearly with the increase of the major-axis of the yarn cross-section, as shown in figure 10(a). The void of fabric increases with the decreases of the major-axis as shown in figure 10(b). With the major-axis of the yarn elliptical section increases nonlinearly, the element length increases nonlinearly as shown in figure 10(c). The results show that there is inelastic deformation during the deformation of type-II fabric structure. The quasi-plastic deformation of this type of fabric in draping process cannot be avoided.
Compared with type-I, the fabric of type-II has greater angular deformability. The analysis of its angular deformation behavior has guiding significance for the drape on convex surfaces. The angular deformation of the plain fabric causes the rotation of yarn. Figure 11 shows the angular deformation of type-II VMU. The angular deformation could be divided into two stages, first compressive deformation, and then yarn rotation. The fabric reaches the quasi-plastic ultimate state after the yarn rotates to the critical angle $\alpha_{\text{min}}$.

Figure 12 shows the projection of VMU before and after deformation in figure 11. The mathematical relationship of structural parameters in figure 12 is given in equation (10). The critical angle $\alpha_{\text{min}}$ can be obtained from equation (10).
Figure 13 shows the relationship between the critical angle $\alpha_{\text{min}}$ and the length of the pitch, the results show that the larger the initial pitch of the fabric, the smaller the critical angle of the yarn. In other words, the angular deformability increases with the increase of pitch length.

Consider a type-II structure with yarn cross-section area $S_0 = 0.5 \text{ mm}^2$, initial deformation coefficient $k_0 = a_0/b_0 = 2$, yarn length $L_0 = 2 \text{ mm}$. The initial length $l_0$ of VMU can be calculated as $l_0 = 2.0899 \text{ mm}$. The range of yarn deformation coefficient can be calculation as $k \in [0.096, 0.127]$. The range of yarn rotation angle is $\alpha \in [58^\circ 29', 90^\circ]$. The critical angle is 58$^\circ 29'$. The critical angle decreases with the decreases of $k$ as shown in figure 14(a). Figures 14(b) and (c) show that the length of pitch and thickness of fabric increase with decreases of $k$.

5. Analysis of plain weave fabric draping on continuous surface

Type-II plain weave fabrics are suited for being draped on continuous surfaces due to their ability of large deformation. The draping process is highly influenced by the quasi-plastic deformation behavior. Evaluating the deformation behavior of plain fabric to drape more accurately can improve the process accuracy and product quality. The work presented in this article focuses solely on the method for analyzing the type-II fabric drape on rotating curved surfaces. The deformation of fabric during the draping process is non-uniform and continuous due to the change of surface curvature. The study of geometric mapping relation between plain fabric and mandrel surface is based on the VMU structure and the fabric deformation mechanism deduced above.

Figure 14. The variation of structure parameters with deformation coefficient: (a) The critical angle-deformation coefficient curve; (b) The pitch length-deformation coefficient curve; (c) The fabric thickness-deformation coefficient curve.
5.1. Mathematical model of continuous surface

Let the generatrix be parametrically represented by \( f_1(x, z) \). The surface formed by the rotation of the generatrix around the \( z \)-axis is defined as \( f_2(\pm \sqrt{x^2 + y^2}, z) \). When the curved generatrix is \( z = -px^2 + H \), the surface can be expressed as \( x^2 + y^2 = (H-z)/p \). As shown in figure 15.

*line1* is a tangent line of a point on the generatrix in the \( x-o-z \) plane as shown in figure 16. *line2* passes through point \( A(x_1, z_1) \) and is perpendicular to *line1*. \( A(x_1, z_1) \) is the intersection of two straight lines. The plane formed by point \( A \) and \( y \)-axis can be defined as \( (z_1/x_1)x + z = 0 \). The intersection line between the plane and rotating curved surface can be expressed as \( f(x, y, z) \).

Figure 15. Mathematical model of continuous surface.

Figure 16. Geometric relations in the continuous surface.

Figure 17. Division of symmetric zones on the curved surface.
5.2. Establishment of geometric mapping relation
The surface shown in figure 17 can be divided into two symmetrical surfaces, thus, only half of the surface needs to be calculated. The fabric mapping to curves $f(y, z)$ and $f(x, y, z)$ deforms uniformly in the axial direction. However, mapping of other zones is accompanied by angular deformation of the fabric cells.

Draping fabric on different positions will produce different degrees of angular deformation. Figure 17 shows that zones I, III and II, IV are symmetrical about the $y$-axis. Therefore, only I and III need to be analyzed. The elements in the fabric space are set as four node elements. Fabric space consists of continuously changing elements.

Set point $A$ as the origin, the node in zone-I is $W(i, j)$. $i$ represents the step along $f(x, y, z)$, $j$ represents the step along $f(y, z)$ as shown in figure 17. As mentioned above, mapping of fabric elements on $W(0, j)$ and $W(i, 0)$ produce uniform intrinsic deformation. Divide the curves $f(x, y, z)$ and $f(y, z)$ equally by the element length after deformation $L_0$. Take a series of nodes $W(0, m)$ and $W(n, 0)$ as the starting nodes of draping, where $m$ and $n$ represent the number of units to drape on $f(x, y, z)$ and $f(y, z)$, respectively. $x_{i,j}, y_{i,j}, z_{i,j}$ represents the coordinates of the nodes $W(i, j)$ on the surface. Search for the fourth node of the element using three known nodes such as

### Table 1. Mechanical properties of fabric.

| Type      | Weave         | Grammage g m$^{-2}$ | Thickness (mm) | Fiber count (10 mm) |
|-----------|---------------|---------------------|----------------|--------------------|
| 12 K 400 g| Plain weave   | 400                 | 0.46           | 2.5                |

![Figure 18. Mandrel with continuous surface.](image)

![Figure 19. Fabric draping experiment and mapping analytical model.](image)
A(0,0), W_f(0,1), and W_z(1,0), then determine the initial yarn angle \( \alpha_{1,1} \), and calculate the corresponding element length \( L_{1,1} = g(\alpha_{1,1}) \). Loop over the above steps for all nodes in zones I and III to obtain all angles \( \alpha_{i,j} \) and element length \( L_{i,j} \) satisfy the following deformation compatibility equations

\[
\begin{align*}
\bar{P}_{i,j} \bar{P}_{i+1,j} &= (x_{i,j} - x_{i+1,j}, y_{i,j} - y_{i+1,j}, z_{i,j} - z_{i+1,j}) \\
\bar{P}_{i,j} \bar{P}_{i,j+1} &= (x_{i,j+1} - x_{i+1,j}, y_{i,j+1} - y_{i+1,j}, z_{i,j+1} - z_{i+1,j}) \\
|P_{i,j} P_{i,j+1}| &= \sqrt{(x_{i,j+1} - x_{i+1,j})^2 + (y_{i,j+1} - y_{i+1,j})^2 + (z_{i,j+1} - z_{i+1,j})^2} \\
\alpha_{i,j} &= \arccos(\nabla P_{i,j+1} \times P_{i,j+1}/(|P_{i,j} P_{i,j+1}| \times |P_{i,j+1} P_{i,j+1}|)) \\
(x_{i+1,j+1} - x_{i,j+1})^2 + (y_{i+1,j+1} - y_{i+1,j})^2 + (z_{i+1,j+1} - z_{i+1,j})^2 &= L_{i,j}^2 \\
(x_{i+1,j+1} - x_{i,j+1} + y_{i+1,j+1} - y_{i,j+1} + z_{i+1,j+1} - z_{i,j+1}) &\in f'_{ij}(x, y, z)
\end{align*}
\] (11)

5.3. Experimental verification

In case to verify the acceptability of the geometric mapping method, the draping experiment was carried out with type-2 structure fabric. Consider a curved surface obtained by rotating the generatrix \( z = 200–0.05y^2 \) about the \( z \)-axis. The mandrel surface is made by 3D printing technology as shown in figure 18. A plain weave carbon fiber fabric (12K, Yufeei, carbon fibre, China) was used for the draping. The properties of fabric are given in table 1. The cross-sectional area of yarn \( S = 0.5 \text{ mm}^2 \), and the initial deformation coefficient of yarn \( k_0 = 2 \).

Figure 19 shows the numerical and experimental results of fabric draping. There are no angle deformations of the yarn unit cell in the axial deformation zone, which is consistent with the element deformation in the calculation results. The variation of pitch length during draping is consistent with the theoretical analysis. The fiber density at different positions of the surface is not uniform, and the local coverage may be reduced. This is caused by the inelastic deformation of plain fabric in the draping process, which shows the quasi-plastic characteristics of the fabric. The expansion of curved surface as shown in the figure, this approach opens new possibilities for draping complex geometries by cutting the designed fabric into corresponding shapes.

6. Conclusions

Based on the assumption of intextensible fibers and yarn elliptical section, a variable microstructure unit-cell model capable of predicting the macroscopic quasi-plastic behavior of plain weave fabric has been established. A comparison of deformation behaviors between different fabric structure types was presented. A geometric mapping method is proposed to analyze the deformation behavior of plain weave fabric during the draping. It has been shown that the deformability of type-I structure is small, so it is not suited for being draped on a large curvature surface. Type-II fabric structure has large deformability and is suitable for draping on a continuous surface. The void of the fabric increases with the decreases of the major-axis of the yarn cross-section, the thickness of preform decreases with the increase of the major-axis, and the angular deformability of plain weave fabric increases with the increase of pitch. These results have implications for the design of the fabric structure used for the draping process.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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