T-violating Triple-Product Correlations in Charmless $\Lambda_b$ Decays

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Abstract

Using factorization, we compute, within the standard model, the T-violating triple-product correlations in the charmless decays $\Lambda_b \to F_1 F_2$, where $F_1$ is a light spin-$\frac{1}{2}$ baryon and $F_2$ is a pseudoscalar ($P$) or vector ($V$) meson. We find a large triple-product asymmetry of 18% for the decay $\Lambda_b \to pK^-$. However, for other classes of $\Lambda_b \to F_1 P$ decays, the asymmetry is found to be at most at the percent level. For $\Lambda_b \to F_1 V$ decays, we find that all triple-product asymmetries are small (at most $O(1\%)$) for a transversely-polarized $V$, and are even smaller for longitudinal polarization. Our estimates of the nonfactorizable contributions to these decays show them to be negligible, and we describe ways of testing this.

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Over the past two decades, there has been a great deal of theoretical work examining CP violation in the $B$ system. Most of this work has focused on the decays of $B$ mesons. The main reason is that the indirect CP-violating asymmetries in $B$-meson decays can be used to extract the interior angles of the unitarity triangle ($\alpha$, $\beta$ and $\gamma$) with no hadronic uncertainty [1]. The knowledge of these angles will allow us to test the standard model (SM) explanation of CP violation. In order to make such measurements, the $B$-factories BaBar and Belle have been built. These machines produce copious numbers of $B^0$–$\bar{B}^0$ pairs, and have now provided the first definitive evidence for CP violation outside the kaon system: $\sin 2\beta = 0.78 \pm 0.08$ [2].

On the other hand, in the coming years machines will be built which are capable of producing large numbers of $\Lambda_b$ baryons. These include hadron machines, such as the Tevatron, LHC, etc., as well as possibly a high-luminosity $e^+e^-$ machine running at the $Z$ pole. People have therefore started to examine the SM predictions for a variety of $\Lambda_b$ decays. This is a worthwhile effort, since it is conceivable that certain types of new physics will be more easily detectable in $\Lambda_b$ decays than in $B$ decays. For example, $\Lambda_b$'s can be used to probe observables which depend on the spin of the $b$-quark, whereas such observables will be unmeasurable in $B$-meson decays.

One class of observables which may involve, among other things, the $b$-quark spin is triple-product correlations. These take the form $\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3)$, where each $v_i$ is a spin or momentum. These triple products are odd under time reversal (T) and hence, by the CPT theorem, also constitute potential signals of CP violation. By measuring a nonzero value of the asymmetry

$$A_T \equiv \frac{\Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) > 0) - \Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) < 0)}{\Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) > 0) + \Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) < 0)},$$

(1)

where $\Gamma$ is the decay rate for the process in question, one can establish the presence of a nonzero triple-product correlation. Note that there is a well-known technical complication: strong phases can produce a nonzero value of $A_T$, even if there is no CP violation (i.e. if the weak phases are zero). Thus, strictly speaking, the asymmetry $A_T$ is not in fact a T-violating effect. Nevertheless, one can still obtain a true T-violating signal by measuring a nonzero value of

$$\tilde{A}_T \equiv \frac{1}{2} (A_T - \bar{A}_T),$$

(2)

where $\bar{A}_T$ is the T-odd asymmetry measured in the CP-conjugate decay process.

Recently, T-violating triple-product correlations were calculated for the inclusive quark-level decay $b \to s\bar{u}u$ [3]. In that calculation, all final-state masses were neglected. Ignoring triple products which involve three spins, only two non-negligible triple-product asymmetries were found. They are: (i) $\vec{p}_u \cdot (\vec{s}_u \times \vec{s}_u)$ or $\vec{p}_\bar{u} \cdot (\vec{s}_\bar{u} \times \vec{s}_\bar{u})$, and (ii) $\vec{s}_b \cdot (\vec{p}_u \times \vec{p}_u)$. While the former triple product can be probed in $B \to V_1V_2$ decays, where $V_1$ and $V_2$ are vector mesons, the latter can only be measured in $\Lambda_b$ decays, since the spin of the $b$ quark is involved.

In this paper we study, within the SM, the triple products in charmless two-body $\Lambda_b$ decays which are generated by the quark-level transitions $b \to s\bar{u}u$ or $b \to d\bar{u}u$. These decays are of the type $\Lambda_b \to F_1F_2$, where $F_1$ is a light spin-$\frac{1}{2}$ baryon, such
as $p$, $\Lambda$, etc., and $F_2$ is a pseudoscalar ($P$) or vector ($V$) meson. Such $T$-violating triple-product correlations, along with other $P$-violating asymmetries, have been studied for hyperon decays [4], but relatively little work has been done to study CP violation in $\Lambda_b$ decays.

The decays $\Lambda_b \to F_1 P$ are similar to hyperon decays. As we will see, there is a triple-product correlation in such decays of the form $\vec{s}_{\Lambda_b} \cdot (\vec{s}_{F_1} \times \vec{p})$, where $\vec{s}_{\Lambda_b}$ and $\vec{s}_{F_1}$ are the polarizations of the $\Lambda_b$ and $F_1$, respectively, and $\vec{p}$ is the momentum of one of the final-state particles in the rest frame of the $\Lambda_b$. On the other hand, $\Lambda_b$ decays can also include a vector meson in the final state, which is not kinematically accessible for hyperon decays. The decay $\Lambda_b \to F_1 V$ can give rise to a variety of triple-product correlations involving the spin of the $\Lambda_b$ and/or $V$.

Many of these triple products involve the spin of the $\Lambda_b$. Perhaps the easiest way to obtain this quantity is to produce the $\Lambda_b$ baryons in the decay of an on-shell $Z$ boson. This is because, in the decay $Z \to b\bar{b}$, the $b$-quarks have a large average longitudinal polarization of about $-94\%$. According to heavy-quark effective theory, this polarization is retained when a $b$-quark hadronizes into a $\Lambda_b$, and recent measurements of the average longitudinal polarization of $b$-flavored baryons produced in $Z^0$ decays (measured through their decay to $\Lambda_c \ell \nu_\ell X$) is consistent with this conclusion [3]. Thus, the so-called GigaZ option ($2 \times 10^9 Z$ bosons per year [3, 4]) of a high-luminosity $e^+e^-$ collider running at the $Z$ peak would be a particularly good environment for measuring triple-product correlations in $\Lambda_b$ decays. However, even if the spin of the $\Lambda_b$ cannot be measured at a given machine, some of the triple-product correlations in $\Lambda_b \to F_1 V$ do not involve the polarization of the initial state. Thus, triple products can be measured at a variety of facilities in which a large number of $\Lambda_b$ baryons is produced.

We begin our analysis by studying the nonleptonic decay $\Lambda_b \to F_1 P$. The general form for this amplitude can be written as

$$\mathcal{M}_P = A(\Lambda_b \to F_1 P) = i\bar{u}_{F_1}(a + b\gamma_5)u_{\Lambda_b}.$$

In order to make contact with the conventional notation for hyperon decay, we note that, in the rest frame of the parent baryon, the decay amplitude reduces to

$$A(\Lambda_b \to F_1 P) = i\chi_{F_1}(S + P\hat{\sigma} \cdot \hat{p})\chi_{\Lambda_b},$$

where $\hat{p}$ is the unit vector along the direction of the daughter baryon momentum, and $S = \sqrt{2m_{\Lambda_b}(E_{F_1} + m_{F_1})}a$ and $P = -\sqrt{2m_{\Lambda_b}(E_{F_1} - m_{F_1})}b$, where $E_{F_1}$ and $m_{F_1}$ are, respectively, the energy and mass of the final-state baryon $F_1$. The decay rate and the various asymmetries are given by

$$\Gamma = \frac{\hat{p}}{8\pi m_{\Lambda_b}}(|S|^2 + |P|^2), \quad \alpha = \frac{2\Re(S^*P)}{|S|^2 + |P|^2}, \quad \beta = \frac{2\Im(S^*P)}{|S|^2 + |P|^2}, \quad \gamma = \frac{|S|^2 - |P|^2}{|S|^2 + |P|^2}.\quad (5)$$

(Note: above, the quantities $\alpha$, $\beta$ and $\gamma$ should not be confused with the CP phases of the unitarity triangle, which have the same symbols.)

The calculation of $|\mathcal{M}_P|^2$ in Eq. (3) yields

$$|\mathcal{M}_P|^2 = (|a|^2 - |b|^2)(m_{F_1}m_{\Lambda_b} + p_{F_1} \cdot s_{\Lambda_b}p_{\Lambda_b} \cdot s_{F_1} - p_{F_1} \cdot p_{\Lambda_b} s_{F_1} \cdot s_{\Lambda_b}).$$

2
\[(|a|^2 + |b|^2)(p_{F_1} \cdot p_{\Lambda_b} - m_{F_1} m_{\Lambda_b} s_{F_1} \cdot s_{\Lambda_b}) + 2 \text{Re}(ab^*) (m_{\Lambda_b} p_{F_1} \cdot s_{\Lambda_b} - m_{F_1} p_{\Lambda_b} \cdot s_{F_1}) + 2 \text{Im}(ab^*) e_{\mu\nu\rho\sigma} p_{\rho}^F s_{\nu}^F p_{\sigma}^\Lambda \cdot s_{\Lambda_b} \cdot s_{\Lambda_b} \cdot s_{\Lambda_b}. \]  

It is the last term above which gives a triple-product correlation. (It corresponds to \(\beta\) in Eq. (5).) In the rest frame of the \(\Lambda_b\), it takes the form \(\vec{p}_{F_1} \cdot (\vec{s}_{F_1} \times \vec{s}_{\Lambda_b})\).

In order to estimate the size of this triple product, we will use factorization to calculate \(\text{Im}(ab^*)\) at the hadron level. The starting point is the SM effective hamiltonian for charmless hadronic \(B\) decays [8]:

\[
H_{eff}^q = \frac{G_F}{\sqrt{2}} [V_{ub} V_{us}^* (c_1 O_1^q + c_2 O_2^q) - \sum_{i=3}^{10} V_{tb} V_{ts}^* c_i^q O_i^q] + h.c.,
\]  

where

\[
O_1^q = \bar{q} \gamma_\mu L u \bar{u} \gamma^\mu L b, \quad O_2^q = \bar{q} \gamma_\mu L u \bar{u} \gamma^\mu L b,
\]

\[
O_{3(5)}^q = \bar{q} \gamma_\mu L b \sum_q \bar{q}^\prime \gamma^\mu L(R) q', \quad O_{4(6)}^q = \bar{q} \gamma_\mu L b \beta \sum_q \bar{q}^\prime \gamma^\mu L(R) q',
\]

\[
O_{7(9)}^q = \frac{3}{2} \bar{q} \gamma_\mu L b \sum_q e_q^\prime \bar{q}^\prime \gamma^\mu R(L) q', \quad O_{8(10)}^q = \frac{3}{2} \bar{q} \gamma_\mu L b \sum_q e_q^\prime \bar{q}^\prime \gamma^\mu R(L) q' .
\]  

In the above, \(q\) can be either a \(d\) or an \(s\) quark, depending on whether the decay is a \(\Delta S = 0\) or a \(\Delta S = -1\) process, \(q' = d, u\) or \(s\), with \(e_q\) the corresponding electric charge, and \(R(L) = 1 \pm \gamma_5\). The values of the Wilson coefficients \(c_i\) evaluated at the scale \(\mu = m_b = 5\) GeV, for \(m_t = 176\) GeV and \(\alpha_s(m_Z) = 0.117\), are [9]:

\[
c_1 = -0.324 \quad c_2 = 1.151 , \quad c_3 = 0.017 , \quad c_4 = -0.037 , \quad c_5 = 0.010 , \quad c_6 = -0.045 , \quad c_7 = -1.24 \times 10^{-5} , \quad c_8 = 3.77 \times 10^{-4} , \quad c_9 = -0.010 , \quad c_{10} = 2.06 \times 10^{-3} .
\]  

In our analysis we will also consider the gluonic dipole operator in which the gluon splits into two quarks, giving the effective operator

\[
H_{11} = i \frac{G_F}{\sqrt{2}} \frac{\alpha_s(\mu)}{2 \pi k^2} m_b(\mu) c_{11} V_{ub} V_{ts}^* \sum(p_s) \sigma_{\mu\nu} R T^a b(p_b) \bar{q}(p_2) \gamma^\mu T^a q(p_1) k^\nu ,
\]  

where \(k = p_b - p_s\) and \(c_{11} = 0.2\) [10]. It is often useful to write this in the Fierz-transformed form

\[
H_{11} = - \frac{G_F}{\sqrt{2}} \frac{\alpha_s(\mu)}{16 \pi} m_b^2(\mu) c_{11} \frac{N_c^2 - 1}{N_c^2} V_{ub} V_{ts}^* \left[ \delta_{\alpha\beta} \delta_{\alpha'\beta'} - \frac{2 N_c}{N_c^2 - 1} T^a_{\alpha\beta} T^a_{\alpha'\beta'} \right] \sum_i T_i ,
\]  

where

\[
T_1 = 2 \bar{s}_\alpha \gamma_\mu L q_\beta \bar{q}_\alpha' \gamma^\mu L b_\beta' - 4 \bar{s}_\alpha R q_\beta \bar{q}_\alpha' L b_\beta' ,
\]

\[
T_2 = 2 \left( \frac{m_s}{m_b} - \frac{m_s}{m_b} \right) \bar{s}_\alpha \gamma_\mu L q_\beta \bar{q}_\alpha' R b_\beta' - 4 \frac{m_s}{m_b} \bar{s}_\alpha L q_\beta \bar{q}_\alpha' R b_\beta' ,
\]

\[
T_3 = \left( \frac{m_b + m_s}{m_b} \right) \bar{s}_\alpha \gamma_\mu L q_\beta \bar{q}_\alpha' R b_\beta' + \bar{s}_\alpha R q_\beta \bar{q}_\alpha' \gamma^\mu R b_\beta' ,
\]

\[
T_4 = \left( \frac{m_b + m_s}{m_b} \right) \left[ \bar{i} \bar{s}_\alpha \sigma_{\mu\nu} R q_\beta \bar{q}_\alpha' \gamma_\mu R b_\beta' - i \bar{s}_\alpha \gamma_\mu L q_\beta \bar{q}_\alpha' \sigma_{\mu\nu} R b_\beta' \right] ,
\]
in which we have defined $\sigma_{\mu \nu} = \frac{i}{2} [\gamma_{\mu} \gamma_{\nu} - \gamma_{\nu} \gamma_{\mu}]$.

We now apply the effective hamiltonian to specific exclusive $\Lambda_b$ decays. We will focus on those processes for which factorization is expected to be a good approximation, namely colour-allowed decays. We begin with $\Lambda_b \to pK^-$, which is a $b \to s\bar{u}u$ transition. Factorization allows us to write

$$A(\Lambda_b \to pK^-) = \sum_{O, O'} \langle K^- | O | 0 \rangle \langle p | O' | \Lambda_b \rangle .$$

(13)

It is straightforward to show that the operators in $H_{eff}^s$ and $H_{11}$ lead to two classes of terms in the decay amplitude: (a) $\langle K^- | \bar{s}\gamma^\mu(1 + \gamma_5)u | 0 \rangle \langle p | \bar{u}\gamma_\mu(1 + \gamma_5)b | \Lambda_b \rangle$, and (b) $\langle K^- | \bar{s}(1 + \gamma_5)u | 0 \rangle \langle p | \bar{u}(1 + \gamma_5)b | \Lambda_b \rangle$. For the first of these, we define the pseudoscalar decay constant $f_K$ as

$$i f_K q^\mu = \langle K | \bar{s}\gamma^\mu(1 - \gamma_5)u | 0 \rangle ,$$

(14)

where $q^\mu \equiv p_{\Lambda_b}^\mu - p_p^\mu = p_K^\mu$ is the four-momentum transfer. For the second, one can show that

$$\langle K^- | \bar{s}(1 + \gamma_5)u | 0 \rangle = \mp \frac{f_K m_K^2}{m_s + m_u} , \quad \langle p | \bar{u}(1 + \gamma_5)b | \Lambda_b \rangle = \frac{q^\mu}{m_b} \langle p | \bar{u}\gamma_\mu(1 + \gamma_5)b | \Lambda_b \rangle .$$

(15)

(In the second matrix element, we have neglected $m_u$ compared to $m_b$.) Thus, factorization leads to the following form for the $\Lambda_b \to pK^-$ amplitude:

$$A(\Lambda_b \to pK^-) = i f_K q^\mu \langle p | \bar{u}\gamma_\mu(1 - \gamma_5)b | \Lambda_b \rangle X_K + i f_K q^\mu \langle p | \bar{u}\gamma_\mu(1 + \gamma_5)b | \Lambda_b \rangle Y_K .$$

(16)

Like any CP-violating observable, a nonzero triple product can arise only if there are two interfering amplitudes. This will occur only if both $X_K$ and $Y_K$ are nonzero. Since all the operators $O_1$–$O_{10}$ involve a left-handed $b$-quark, it is clear that $X_K \neq 0$ in the SM. Furthermore, though it is less obvious, one can also have $Y_K \neq 0$. Consider, for example, the operator $O_6$ of Eq. (13). After performing Fierz transformations, this can be written as

$$O_6 \sim \bar{s}(1 + \gamma_5)u \bar{u}(1 - \gamma_5)b .$$

(17)

However, according to Eq. (13), $\langle p | \bar{u}(1 - \gamma_5)b | \Lambda_b \rangle$ can be related to $\langle p | \bar{u}\gamma_\mu(1 + \gamma_5)b | \Lambda_b \rangle$. Thus, $Y_K$ receives contributions from operators such as $O_6$. We find

$$X_K = \frac{G_F}{\sqrt{2}} \left[ V_{ub} V_{us}^* a_2 - \sum_{q=u,c,t} V_{qb} V_{qs}^* (a_4^q + a_6^q) - V_{tb} V_{ts}^* a_d \left( 1 + \frac{2 E_K}{m_b} \right) \right] ,$$

$$Y_K = -\frac{G_F}{\sqrt{2}} \left[ \sum_{q=u,c,t} V_{qb} V_{qs}^* (a_6^q + a_8^q) + \frac{5}{4} V_{tb} V_{ts}^* a_d \right] \chi_K ,$$

(18)

with

$$\chi_K = \frac{2 m_K^2}{(m_s + m_u) m_b} , \quad a_d = \frac{\alpha_s(\mu)}{16 \pi} \left( \frac{m_b^2(\mu)}{k^2} \right) c_{11} \frac{N_c^2 - 1}{N_c^2} .$$

(19)
In the above, we have defined $a^q_i = c^q_i + c^{q+1}_{i-1}$ for $i$ odd and $a^q_i = c^q_i + c^{q+1}_{N_q}$ for $i$ even. We estimate the average gluon momentum in the dipole operator to be

\[ \langle m_b^2 / k^2 \rangle = \int \phi_K(x) m_b^2 / k^2 dx, \]

where the gluon momentum in the heavy-quark limit is $k^2 = m_b^2 (1 - x)$ and $\phi_K$ is the kaon light-cone distribution. Choosing the asymptotic form $\phi_K = 6x(1 - x)$, we find $\langle m_b^2 / k^2 \rangle = 3$, which leads to $a_d = 0.0021$.

Now, the vector and axial-vector matrix elements between the $\Lambda_b$ and $p$ can be written in the general form

\[
\begin{align*}
\langle p | \bar{u} \gamma^\mu b | \Lambda_b \rangle &= \bar{u}_p \left[ f_1 \gamma^\mu + i \frac{f_2}{m_{\Lambda_b}} \sigma^{\mu\nu} q_\nu + \frac{f_3}{m_{\Lambda_b}} q^\mu \right] u_{\Lambda_b} \\
\langle p | \bar{u} \gamma^\mu \gamma_5 b | \Lambda_b \rangle &= \bar{u}_p \left[ g_1 \gamma^\mu + i \frac{g_2}{m_{\Lambda_b}} \sigma^{\mu\nu} q_\nu + \frac{g_3}{m_{\Lambda_b}} q^\mu \right] \gamma_5 u_{\Lambda_b},
\end{align*}
\]

where the $f_i$ and $g_i$ are Lorentz-invariant form factors. Heavy-quark symmetry imposes constraints on these form factors. A systematic expansion of these form factors, including $1/m_b$ corrections, has been calculated [1]: in the $m_b \to \infty$ limit, one obtains the relations

\[
f_1 = g_1, \quad f_2 = g_2 = f_3 = g_3.
\]

Using the above expressions, we find that the parameters $a$ and $b$ of Eq. (3) can be written as

\[
\begin{align*}
a_K &= f_K (X_K + Y_K) \left[ (m_{\Lambda_b} - m_p) f_1 + f_2 \frac{m_K^2}{m_{\Lambda_b}} \right] , \\
b_K &= f_K (X_K - Y_K) \left[ (m_{\Lambda_b} + m_p) g_1 - g_2 \frac{m_K^2}{m_{\Lambda_b}} \right].
\end{align*}
\]

According to Eq. (3) the triple product in $\Lambda_b \to pK^-$ is proportional to $\text{Im}(a_K b_K^*)$, which is in turn proportional to $\text{Im}(X_K Y_K^*)$. Since $X_K$ and $Y_K$ are both nonzero, and have different weak phases [Eq. (13)], we expect a nonzero triple-product asymmetry in $\Lambda_b \to pK^-$ of the form $\bar{p}_p \cdot (\bar{s}_p \times \bar{s}_{\Lambda_b})$. At first sight, this appears to contradict the results of Ref. [3], since no triple products involving two spins were found in the quark-level decay $b \to s\bar{u}u$. However, note that $Y_K$ is proportional to $\chi_K$, which is formally suppressed by $1/m_b$. Thus, in the limit $m_b \to \infty$, one has $Y_K = 0$, so that the triple-product correlation will vanish. This agrees with the conclusions of Ref. [3], which neglects the masses of the final-state quarks (i.e. the limit $m_b \to \infty$ is implicitly assumed).

However, the key point is that, for finite $m_b$, $\chi_K$ is not small because of the presence of the chiral enhancement term $m_K^2/(m_s + m_u)$. In fact, for $m_s = 100$ MeV and $m_u = 5$ GeV, $\chi_K \sim 1$, and hence is clearly non-negligible. The triple-product asymmetry of $\bar{p}_p \cdot (\bar{s}_p \times \bar{s}_{\Lambda_b})$ may therefore be sizeable. Note that this triple product requires the measurement of both the $\Lambda_b$ and the $p$ polarizations. If the measurement of the proton polarization is not possible, one can instead consider a final state with an excited nucleon, such as $\Lambda_b \to N(1440)K^-$. In this case the polarization of the $N(1440)$ can be determined from its decay products. (Alternatively, one can consider the decay $\Xi_b \to \Sigma^+ K^-$, where $\Xi_b$ has quark content $bus.$)
Note also that in Eq. (18) we have included the up- and charm-quark penguin pieces, proportional to $V_{ub}V_{ub}^*$ and $V_{cb}V_{cb}^*$ respectively. These are generated by rescattering of the tree-level operators in the effective Hamiltonian in Eq. (7). As we will see, the contributions from these rescattering terms are very important. The coefficients associated with these terms are given by

$$c_{3,5}^i = -c_{4,6}^i / N_c = P_i^s / N_c , \quad c_{7,9}^i = P_i^e , \quad c_{8,10}^i = 0 , \quad i = u, c ,$$ (23)

where $N_c$ is the number of colours. The leading contributions to $P_i^s, e$ are given by

$$P_i^s = (\alpha_s / 8\pi) c_2 (10 / 9 + G(m_i, \mu, q^2)) \quad \text{and} \quad P_i^e = (\alpha_s / 9\pi) (N_c c_1 + c_2) (10 / 9 + G(m_i, \mu, q^2)) ,$$

in which the function $G(m, \mu, q^2)$ takes the form

$$\int_0^1 x(1-x) \ln \frac{m^2 - x(1-x)q^2}{\mu^2} \, dx ,$$ (24)

where $q$ is the momentum carried by the virtual gluon in the penguin diagram. Of course, we are really interested in the matrix elements of the various operators for the decay $\Lambda_b \rightarrow pK^-$, and so the coefficients in Eq. (23) should be understood to be

$$\frac{c_{i}^{u,c}}{\bar{c}_i} = \frac{\langle pK^- | c_i^{u,c}(q^2)O_i | \Lambda_b \rangle}{\langle pK^- | O_i | \Lambda_b \rangle} .$$ (25)

We will henceforth drop the distinction between $c_{i}^{u,c}$ and $\bar{c}_i^{u,c}$, with the understanding that it is the $c_{i}^{u,c}$ which appear in the amplitude.

The analysis of other colour-allowed $\Lambda_b$ decays follows straightforwardly from that for $\Lambda_b \rightarrow pK^-$. For example, consider $\Lambda_b \rightarrow p\pi^-$, which is generated by the quark-level decay $b \rightarrow d\bar{u}u$. The amplitude for $\Lambda_b \rightarrow p\pi^-$ is given by Eq. (3), with

$$a_\pi = f_\pi (X_\pi + Y_\pi) \left[ (m_{\Lambda_b} - m_p) f_1 + f_3 \frac{m^2_\pi}{m_{\Lambda_b}} \right] ,$$

$$b_\pi = f_\pi (X_\pi - Y_\pi) \left[ (m_{\Lambda_b} + m_p) g_1 - g_3 \frac{m^2_\pi}{m_{\Lambda_b}} \right] ,$$ (26)

where

$$X_\pi = \frac{G_F}{\sqrt{2}} \left[ V_{ub}V_{ud}^* a_2 - \sum_{q=u,c,t} V_{qb}V_{qd}^* (a_1^q + a_{10}^q) - V_{tb}V_{td}^* a_d (1 + \frac{2E_\pi}{m_b}) \right] ,$$

$$Y_\pi = -\frac{G_F}{\sqrt{2}} \left[ \sum_{q=u,c,t} V_{qb}V_{qd}^* (a_6^q + a_8^q) + \frac{5}{4} V_{tb}V_{td}^* a_d \right] \chi_\pi ,$$ (27)

with

$$\chi_\pi = \frac{2m^2_\pi}{(m_d + m_u)m_b} .$$ (28)

Finally, we consider the decay $\Lambda_b \rightarrow \Lambda\eta(\eta')$ (12), which is dominated by a colour-allowed $b \rightarrow s$ penguin transition (there is also a small colour-suppressed tree contribution). For the decay $\Lambda_b \rightarrow \Lambda\eta$ we get

$$a_\eta = f_\pi (X_\eta + Y_\eta) \left[ (m_{\Lambda_b} - m_\eta) f_1 + f_3 \frac{m^2_\eta}{m_{\Lambda_b}} \right] ,$$

$$b_\eta = f_\pi (X_\eta - Y_\eta) \left[ (m_{\Lambda_b} + m_\eta) g_1 - g_3 \frac{m^2_\eta}{m_{\Lambda_b}} \right] ,$$ (29)
\[ X_\eta = \frac{G_F}{\sqrt{2}} \left[ V_{ub} V_{us}^* a_1 r_1 - \sum_{q=u,c,t} V_{qb} V_{qs}^* (r_1 A_q + r_2 B_q) - V_{tb} V_{ts}^* r_2 a_d \left( 1 + \frac{2E_\eta}{m_b} \right) \right], \]

\[ Y_\eta = -\frac{G_F}{\sqrt{2}} \left[ \sum_{q=u,c,t} V_{qb} V_{qs}^* \left( \frac{1}{2} a_0^2 - \frac{1}{2} a_0 \right) + \frac{5}{4} V_{tb} V_{ts}^* a_d \right] r_2 \chi_\eta, \] (30)

with

\[ A_q = 2a_3^q - 2a_5^q - \frac{1}{2} a_0^q + \frac{1}{2} a_1^q, \]

\[ B_q = a_3^q + a_4^q - a_5^q + \frac{1}{2} a_0^q - \frac{1}{2} a_1^q - \frac{1}{2} a_0^q, \]

\[ \chi_\eta = \frac{m_0^2}{m_s m_b}. \] (31)

In the above, we have defined \( r_1 = f_\eta^u / f_\pi \) and \( r_2 = f_\eta^s / f_\pi \), with

\[ i f_\eta^u p_\eta^u = \langle \eta | \bar{u} \gamma^\mu (1 - \gamma_5) u | 0 \rangle = \langle \eta | \bar{d} \gamma^\mu (1 - \gamma_5) d | 0 \rangle, \]

\[ i f_\eta^s p_\eta^s = \langle \eta | \bar{s} \gamma^\mu (1 - \gamma_5) s | 0 \rangle. \] (32)

The amplitude for \( \Lambda_b \to \Lambda \eta' \) has the same form as Eq. (30) with the replacement \( \eta \to \eta' \). Note that the polarization of the final-state \( \Lambda \) can be measured via its decay \( \Lambda \to p \pi^- \).

The above analysis has been performed within the framework of factorization. Before turning to estimates of the size of the triple-product asymmetries, it is useful at this point to address the issue of nonfactorizable corrections. Nonfactorizable effects are known to be important for hyperon and charmed-baryon nonleptonic decays, but are expected to be negligible for non-leptonic \( \Lambda_b \) decays. An unambiguous signal for the presence of nonfactorizable effects would be the observation of the decay \( \Lambda_b \to \Delta^+ K^- (\pi^-), \Lambda_b \to \Sigma \eta (\eta'), \) or \( \Lambda_b \to \Sigma \phi \). This is because, for the factorizable contribution, the light diquark in the \( \Lambda_b \) baryon remains inert during the weak decay. Thus, since the light diquark is an isosinglet, and since strong interactions conserve isospin to a very good approximation, the above \( \Lambda_b \) decays are forbidden within factorization [13].

One way to estimate the size of nonfactorizable corrections is by using the pole model. In this model, one assumes that the nonfactorizable decay amplitude receives contributions primarily from one-particle intermediate states, and that these contributions then show up as simple poles in the decay amplitude. An example of intermediate single-particle states is the ground-state positive-parity baryons. Consider the decay \( \Lambda_b \to p K^- \). One nonfactorizable contribution is described by the diagram in which there is a \( \Lambda_b \to \Sigma^0 \) weak transition through a \( W \) exchange, followed by the strong decay \( \Sigma^0 \to p K^- \). The pole contribution to the parity-violating amplitude, \( a \), in Eq. (3) is known to be small for charmed-baryon decays [14], and we assume this to be the case here as well. For the parity-conserving amplitude, \( b \), in Eq. (3), we can then write

\[ b_{\text{nonfac}} \sim V_{ub} V_{us}^* \frac{\langle \Sigma^0 | H_w | \Lambda_b \rangle}{m_{\Lambda_b} - m_{\Sigma^0}} g_{\Sigma^0 p K^-}. \] (33)
where $g_{\Sigma^0 pK^-}$ is the strong-coupling vertex which will depend on the energy of the emitted kaon. We can use heavy-quark and flavour $SU(3)$ symmetry to set $\langle \Sigma^0 | H_w | \Lambda_b \rangle \sim \langle \Sigma^+ | H_w | \Lambda_c \rangle$. Writing the weak matrix element $\langle \Sigma^+ | H_w | \Lambda_c \rangle = g_{\Sigma^0 pK^-} \sqrt{2} m^3$, we obtain

$$b_{\text{nonfac}} \sim \frac{m^2}{f_K (m_{\Lambda_b} - m_{\Sigma^0})(m_{\Lambda_b} + m_p)} g_{\Sigma^0 pK^-},$$

(34)

where we have chosen the tree-level term for $A_{fac}$. Since the emitted kaon is hard and since the quarks inside it are energetic, the strong coupling $g_{\Sigma^0 pK^-} \sim \alpha_s (\mu \sim E_K \sim m_b)$. In other words, the offshell $\Sigma^0$ has to emit a hard gluon to create a $\bar{u}u$ pair to form the $pK^-$ final state. The matrix element $m$ can either be estimated using a model \[14\], or obtained from a fit to the charmed baryon decay $\Lambda_c \to \Sigma^0 \pi^+ \[15\]$. In both cases one obtains $m \sim 0.1 - 0.2$ GeV, so that, from Eq. (34), the nonfactorizable corrections are found to be tiny. Arguments for small nonfactorizable effects in $\Lambda_b$ decays can also be made based on the total width calculations \[11\].

To summarize: for colour-allowed $\Lambda_b \to F \pi$ decays, we find that the triple-product correlation $\text{Im}(ab^*) \vec{p}_{F_1} \cdot (\vec{s}_{F_1} \times \vec{s}_{\Lambda_b})$ can be nonzero. The next step is to calculate the size of the asymmetry $A_T$ in Eq. (4) for the various decays.

We begin with $\Lambda_b \to pK^-$. For this decay, we use the expressions for $a_K$ and $b_K$ found in Eq. (22). We note that the $f_3 (g_3)$ term is suppressed relative to the $f_1 (g_1)$ term by a factor $m_K^2 / m_{\Lambda_b}^2 \sim 0.01$, and so can be neglected (and similarly for the $g_3$ piece). Furthermore, we take $f_1 = g_1$ [Eq. (21)], in which case all dependence on this form factor cancels in $A_T$ [Eq. (4)]. The quantities $a_K$ and $b_K$ depend on the parameters of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, whose values are taken to be

$$\rho = 0.17, \quad \eta = 0.39.$$

(35)

For the chiral enhancement term $\chi_K$ [Eq. (14)], we take $\chi_K = 1$.

In order to estimate the value of rescattering terms, one has to choose a value of $q^2$. We consider two possibilities:

Model 1 : $q^2 = \frac{m_b^2}{4}$, \quad Model 2 : $q^2 = \frac{m_b^2}{2}$.

(36)

Taking $m_c = 1.4$ GeV, $m_u = 6$ MeV and $m_b = 5$ GeV, and writing $c_4^u = |c_4^u|e^{i\delta^c}$, we find

Model 1 : $|c_4^u| = 0.02$, \quad $|c_4^c| = 0.02$, \quad $\delta^c = 51^\circ$,

Model 2 : $|c_4^u| = 0.021$, \quad $|c_4^c| = 0.015$, \quad $\delta^c = 0$.

(37)

(In accordance with CPT, we set the phase of $c_4^u$ to zero \[17\].)

Before presenting the numerical analysis, it is useful to anticipate the results. Referring to Eqs. (5), (22) and (18), we expect the triple-product asymmetry to be of order

$$\frac{2\text{Im}(a_K b_K^*)}{|a_K|^2 + |b_K|^2 + 2\text{Re}(a_K b_K^*)} \approx \frac{\text{Im}(X_K Y_K^*)}{|X_K|^2} \approx \frac{a_2 a_0 |\lambda|^2}{a_2^2 \lambda^4 + a_4^2} = 24\%.$$

(38)
Of course, this is a back-of-the-envelope estimate, but it does indicate that we can expect a reasonably large asymmetry, even when the rescattering effects are included.

The fundamental reason for this is the following: the triple product is due mainly to the interference of the $V_{ub}V_{us}^*$ piece of $X_K$ (we refer to this as $T$, the “tree”) and the $V_{tb}V_{ts}^*$ piece of $Y_K$ ($P$, the “penguin”). Like any CP-violating quantity, the asymmetry will therefore be maximized when the two interfering amplitudes are of comparable size. A quick calculation of these two quantities in Model 2 above shows that $|T/P| = 0.35$. The two amplitudes are therefore similar in size, leading to the sizeable asymmetry estimate above.

We have performed the phase-space integration for $\Lambda_b \to pK^-$ using the computer program RAMBO. For Model 1, we find that $A_T = -20.8\%$ and $\bar{A}_T = +15.0\%$, leading to a T-violating asymmetry of $A_T^{\pi K} = -17.9\%$. In Model 2, since the strong phase vanishes, one necessarily has $A_T = -A_T$, and we find $A_T^{\pi K} = -19.1\%$. (We note in passing that the rescattering effects are quite important. Without them, the asymmetry would be $A_T^{\pi K} = -26.1\%$. Thus, their inclusion leads to a correction in the asymmetry of about 25%) These numbers are all consistent with the estimate in Eq. (38). We therefore conclude that the SM predicts a sizeable triple-product asymmetry in the decay $\Lambda_b \to pK^-$. (Note that, since the estimate in Eq. (38) uses only the values of the Wilson coefficients and the CKM matrix elements, we expect a large asymmetry even if nonfactorizable contributions are present.)

There is one digressionary remark which is worth making here. From the measurement of $\epsilon_K$, the CP-violating parameter in the kaon system, we know that the product $B_K\eta$ is positive, where $B_K$ is the kaon bag parameter and $\eta$ is the CP-violating CKM parameter. It is usually assumed that $B_K > 0$, so that $\eta$ is also positive, and the unitarity triangle points up. However, there is no experimental evidence yet that $B_K > 0$. The T-violating triple-product asymmetry in $\Lambda_b \to pK^-$ is proportional to $\eta \cos(\delta)$, where $\delta$ is a strong phase. If one assumes that $|\delta| < 90^\circ$, which is strongly favoured theoretically, then the triple product asymmetry measures the sign of $\eta$. This provides a cross check to the information obtained from the kaon system.

Turning now to the decay $\Lambda_b \to p\pi^-$, we have applied this same analysis as above. Taking $m_d = m_u = 6$ MeV, we have $\chi_{\pi} = 0.65$. In this case, the tree amplitude $T$ is larger than the penguin amplitude $P$, with $|P/T| = 0.08$. Because these two interfering amplitudes are less comparable in size than was the case for $\Lambda_b \to pK^-$, we expect a correspondingly smaller asymmetry. This is indeed what is found. In Model 1, we have $A_T = 6.3\%$ and $\bar{A}_T = -4.5\%$, so that $A_T^{\pi \pi} = 5.4\%$. Model 2 gives a similar asymmetry: $A_T^{\pi \pi} = 5.6\%$.

For the decays $\Lambda_b \to \Lambda\eta$ and $\Lambda_b \to \Lambda\eta'$, we have to define the quark content and mixing of the physical $\eta$ and $\eta'$ mesons. We use the Isgur mixing [18]:

$$\langle \eta \rangle = \frac{1}{\sqrt{2}}[N - S] \quad , \quad \langle \eta' \rangle = \frac{1}{\sqrt{2}}[N + S] , \quad (39)$$

where $N = \langle u\bar{u} \rangle + \langle d\bar{d} \rangle \sqrt{2}$ and $S = \langle s\bar{s} \rangle$. $SU(3)$ symmetry then gives

$$f_\eta^u = f_\pi/2 \quad , \quad f_\eta^s = -f_\pi/\sqrt{2} \quad , \quad f_\eta'^u = f_\pi/2 \quad , \quad f_\eta'^s = f_\pi/\sqrt{2} , \quad (40)$$
where $f_\pi = 131$ MeV. We also take $\chi_0 = 0.6$ and $\chi_{0'} = 1.8$. For both decays the interfering amplitudes are very different in size: $|T/P| = 0.03$ and 0.01 for the $\Lambda\eta$ and $\Lambda\eta'$ final states, respectively. We can therefore expect to obtain tiny triple-product asymmetries, and this should hold even if nonfactorizable effects are present. For $\Lambda_b \to \Lambda\eta$ we have $A_{\Lambda\eta}^{\Lambda_b} = 0.6\%$ (Model 1) or 0.9\% (Model 2), while for $\Lambda_b \to \Lambda\eta'$, $A_{\Lambda\eta'}^{\Lambda_b} = -0.6\%$ (Model 1) or $-0.5\%$ (Model 2). It is unlikely that such tiny asymmetries can be measured. However, this also suggests that these processes might be good areas to search for new physics [19].

We now turn to the decays $\Lambda_b \to F_1 V$. The general decay amplitude can be written as [20]

$$M_V = \text{Amp}(\Lambda_{F_1} \to BV) = \bar{u}_{F_1} \epsilon_\mu^* \left((p_{\Lambda_b}^\mu + p_{F_1}^\mu)(a + b\gamma_5) + \gamma^\mu(x + y\gamma_5)\right) u_{\Lambda_b} ,$$

where $\epsilon_\mu^*$ is the polarization of the vector meson. In the rest frame of the $\Lambda_b$, we can write $p_V = (E_V, 0, 0, |\vec{p}|)$ and $p_{F_1} = (E_{F_1}, 0, 0, -|\vec{p}|)$. Thus, it is clear that $\epsilon_\mu^* \cdot (p_{\Lambda_b} + p_{F_1})$ will be nonzero only for a longitudinally-polarized $V$. This will be important in what follows.

The calculation of $|M_V|^2$ gives the following triple-product terms:

$$|M_V|^2_{t.p.} = 2 \text{Im}(ab^*) |\epsilon_V \cdot (p_{\Lambda_b} + p_{F_1})|^2 \epsilon_{\mu\rho\sigma} p_{F_1}^\rho s_{F_1}^\mu s_{F_1}^\sigma$$

$$+ 2 \text{Im}(xy^*) \epsilon_{\alpha\beta\mu\nu} \left[\epsilon_V \cdot s_{F_1} p_{\Lambda_b}^\alpha p_{\Lambda_b}^\beta s_{\Lambda_b}^\mu \epsilon_V^* - \epsilon_V \cdot p_{F_1} s_{F_1}^\alpha p_{\Lambda_b}^\beta s_{\Lambda_b}^\mu \epsilon_V^* - \epsilon_V \cdot p_{\Lambda_b} s_{\Lambda_b}^\alpha p_{F_1}^\beta s_{\Lambda_b}^\mu \epsilon_V^*\right]$$

$$+ 2 \epsilon_V \cdot (p_{\Lambda_b} + p_{F_1}) \epsilon_{\alpha\beta\mu\nu} \left[\text{Im}(ax^* + by^*) p_{F_1}^\alpha p_{\Lambda_b}^\beta s_{\Lambda_b}^\mu \epsilon_V^* + \text{Im}(bx^* + ay^*) p_{F_1}^\alpha p_{\Lambda_b}^\beta s_{\Lambda_b}^\mu \epsilon_V^* - \text{Im}(ax^* - by^*) p_{F_1}^\alpha p_{\Lambda_b}^\beta s_{\Lambda_b}^\mu \epsilon_V^* - \text{Im}(ay^* - bx^*) s_{F_1}^\alpha p_{\Lambda_b}^\beta s_{\Lambda_b}^\mu \epsilon_V^*\right] .$$

Note that if we sum over the polarization of the vector meson, we essentially reproduce the results found for $\Lambda_b \to F_1 P$. That is, there is only one triple product, which takes the form $\epsilon_{\mu\rho\sigma} p_{F_1}^\rho s_{F_1}^\mu s_{F_1}^\sigma$.

As usual, we use factorization to calculate the coefficients $a$, $b$, $x$ and $y$. Consider first the decay $\Lambda_b \to pK^*$. We define the decay constant $g_{K^*}$ as

$$m_{K^*} g_{K^*} \epsilon^*_\mu = \langle K^* | \bar{s} \gamma_\mu u | 0 \rangle .$$

In general, factorization allows us to write

$$A(\Lambda_b \to pK^*) = m_{K^*} g_{K^*} \left\{ \epsilon^*_\mu \langle p | \bar{u} \gamma^\mu(1 - \gamma_5)b | \Lambda_b \rangle X_{K^*} + \epsilon^* \langle p | \bar{u} \gamma^\mu(1 + \gamma_5)b | \Lambda_b \rangle Y_{K^*} + \epsilon \cdot (p_{\Lambda_b} + p_p) q_{\mu} \langle p | \bar{u} \gamma^\mu(1 - \gamma_5)b | \Lambda_b \rangle A_{K^*} + \epsilon \cdot (p_{\Lambda_b} + p_p) q_{\mu} \langle p | \bar{u} \gamma^\mu(1 + \gamma_5)b | \Lambda_b \rangle B_{K^*} \right\} .$$

The coefficients $X_{K^*}$, $Y_{K^*}$, $A_{K^*}$ and $B_{K^*}$ can be calculated using the effective hamiltonian. As noted earlier, $A_{K^*}$ and $B_{K^*}$ are nonzero only for a longitudinally-polarized $K^*$. 

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Consider first the operators $O_1-O_{10}$. Since all of these lead to $K^{*-}$ matrix elements of the form in Eq. (42), none of them can contribute to $A_{K^*}$ and $B_{K^*}$. Furthermore, one can show that none of these give $Y_{K^*} \neq 0$ either. For example, consider again the operator $O_6$, which led to $Y_K \neq 0$. Because $\langle K^{*-}\mid \bar{s}(1+\gamma_5)u\mid 0 \rangle = 0$, $O_6$ will not contribute to $Y_{K^*}$. Thus, within factorization, if we restrict ourselves only to the operators $O_1-O_{10}$, the only nonzero coefficient is $X_{K^*}$, which means that all triple products vanish, since there is only a single decay amplitude.

In order to generate triple products in $\Lambda_b \rightarrow pK^{*-}$, it is necessary to consider the dipole operator $O_{11}$, whose effective coefficient is rather small (Eq. (44): $a_d = 0.0021$). However, there is an important observation one can make. The contributions of $O_{11}$ to $Y_{K^*}$, $A_{K^*}$ and $B_{K^*}$ all involve the tensor matrix element for $K^{*-}$, which we define as

$$-ig_{K^*}^{-T}\left[\varepsilon^*_{pK^*} - \varepsilon^*_u p_{K^*}^u\right] = \langle K^*|\bar{s}\gamma_{\mu}u|0\rangle. \tag{45}$$

Now, in the rest frame of the $\Lambda_b$, we can write $p_{K^*} = (E_{K^*}, 0, 0, |\vec{p}_{K^*}|)$. In the heavy-quark limit, in which $E_{K^*} \gg m_{K^*}$, the longitudinal polarization vector can be written approximately as

$$\varepsilon_{\mu}^{\lambda=0} \approx \frac{1}{m_{K^*}} \left(p_{K^*}^{\mu} + \frac{m_{K^*}^2}{2E_{K^*}} n^{\mu}\right), \tag{46}$$

with $n^{\mu} = (-1, 0, 0, 1)$. From Eq. (45), we see that the piece of $\varepsilon_{\mu}^{\lambda=0}$ which is proportional to $p_{K^*}^{\mu}$ will not contribute to the matrix element. Thus, in the heavy-quark limit, we have $A_{K^*} \approx B_{K^*} \approx 0$. Furthermore, we expect that the value of $Y_{K^*}$ for a longitudinally-polarized $K^{*-}$ meson will be suppressed relative to that for a transversely-polarized $K^{*-}$ by about $m_{K^*}/2E_{K^*} = 16\%$. As we will see, $Y_{K^*}$ is already small for a transversely-polarized $K^{*-}$, so that $Y_{K^*} \approx 0$ for longitudinal polarization. Therefore, any triple products in $\Lambda_b \rightarrow pK^{*-}$ should be largest for a transversely-polarized $K^{*-}$, although we expect even these to be small.

Considering separately the longitudinal ($\lambda = 0$) and transverse ($\lambda = \perp$) polarizations of the final-state vector meson, we find

$$X_{K^*}^{\lambda=\perp} = \frac{G_F}{\sqrt{2}} \left[ V_{ub}V_{us}^*a_2 - \sum_{q=u,c,t} V_{qb}V_{qs}^* (a_q^2 + a_{10}^q) - V_{tb}V_{ts}^* \frac{a_d}{2} \right],$$

$$X_{K^*}^{\lambda=0} = \frac{G_F}{\sqrt{2}} \left[ V_{ub}V_{us}^*a_2 - \sum_{q=u,c,t} V_{qb}V_{qs}^* (a_q^2 + a_{10}^q) - V_{tb}V_{ts}^* a_d \left(1 + \frac{2E_{K^*}}{m_b}\right) \right],$$

$$Y_{K^*}^{\lambda=\perp} = \frac{G_F}{\sqrt{2}} V_{tb}V_{ts}^* a_d,$$

$$Y_{K^*}^{\lambda=0} \approx 0, \tag{47}$$

where

$$z \equiv \frac{E_{K^*} g_{K^*}^-}{m_{K^*} g_{K^*}^+} \tag{48}.$$
For $g_{K^*} = 226$ MeV and $g_{K^*}^T = 160$ MeV \cite{21}, $z = 2.23$. To a good approximation, the quantities $a$, $b$, $x$ and $y$ of Eq. (41) can then be expressed as

\[
\begin{align*}
a_{K^*}^\lambda &= m_{K^*} g_{K^*} \frac{f_2}{m_{\Lambda_b}} [X_{K^*}^\lambda + zY_{K^*}^\lambda] , \\
b_{K^*}^\lambda &= -m_{K^*} g_{K^*} \frac{g_2}{m_{\Lambda_b}} [X_{K^*}^\lambda - zY_{K^*}^\lambda] , \\
x_{K^*}^\lambda &= m_{K^*} g_{K^*} [f_1 - \frac{m_p + m_{\Lambda_b}}{m_{\Lambda_b}} f_2] [X_{K^*}^\lambda + zY_{K^*}^\lambda] , \\
y_{K^*}^\lambda &= -m_{K^*} g_{K^*} [g_1 + \frac{m_{\Lambda_b} - m_p}{m_{\Lambda_b}} g_2] [X_{K^*}^\lambda - zY_{K^*}^\lambda].
\end{align*}
\]

(49)

There are several points to be deduced from the above results. First, since $Y_{K^*}^{\lambda=0} \approx 0$, the coefficients $a$, $b$, $x$ and $y$ all have the same phase for a longitudinally-polarized $K^*$. Thus all triple products involving a longitudinal $K^*$ are expected to vanish. Furthermore, since $v \cdot p_{\Lambda_b} = 0$ for a transversely-polarized $K^*$, most of the triple products in Eq. (42) are expected to vanish in the SM. The only potential nonzero triple-product correlations are

\[
2 \text{Im}(xy^\ast) \epsilon_{\alpha\beta\mu\nu} \left[ \varepsilon_{K^* \cdot \eta} s_p p_{p'}^\ast p_{\Lambda_b}^\ast s_{\Lambda_b}^\ast \varepsilon_{K^*}^\ast + \varepsilon_{K^* \cdot \eta} s_{\Lambda_b} p_{p'}^\ast p_{\Lambda_b} \varepsilon_{K^*}^\ast \varepsilon_{K^*} \right].
\]

(50)

Since these both require the measurement of all three spins, this result is consistent with the results of Ref. \cite{3}.

Second, and more importantly, both of these asymmetries only arise due to the interference between the (small) dipole term $Y_{K^*}^{\lambda=\mp}$ and the $V_{ub}V_{us}^\ast$ piece of $X_{K^*}^{\lambda=\pm}$. Thus, by analogy with Eq. (38), we estimate the size of the asymmetries to be roughly

\[
\frac{2 \text{Im}(xy^\ast)}{|x|^2 + |y|^2 + 2 \text{Re}(xy^\ast)} \approx \frac{z \text{Im}(X_{K^*}^{\lambda=\pm} Y_{K^*}^{\lambda=\ast})}{|X_{K^*}^{\lambda=\pm}|^2} \approx \frac{za_2 a_d d}{a_2^2 \lambda^4 + a_4^2} \approx 2\%,
\]

(51)

which would be very difficult to measure. (Essentially, the asymmetry is reduced compared to that in $\Lambda_b \to pK^-$ by the factor $z|a_d/a_6| = 0.11$.) Furthermore, the decay $\Lambda_b \to pK^{*\mp}$ is dominated by the longitudinally-polarized $K^{*\mp}$; the rate for the production of a transversely-polarized $K^{*\mp}$ is suppressed by the factor $(m_{K^*}/E_{K^*})^2 = 0.1$. Thus, even if the asymmetry were larger, it would still be difficult to detect, given the small rate.

We therefore conclude that any measurement of a sizeable triple-product asymmetry in the decay $\Lambda_b \to pK^{*\mp}$ is an unequivocal signal of new physics \cite{19}. (As noted in the case of $\Lambda_b \to pK^-$ decay, if the measurement of the proton polarization is difficult, one can consider a final state with an excited nucleon such as $\Lambda_b \to N(1440)K^{*\mp}$. In this case, the polarization of the $N(1440)$ can be determined from its decay products. Alternatively, one can consider $\Xi_b \to \Sigma^+ K^{*\mp}$, for which the above conclusions should also hold.)

The decay $\Lambda_b \to pp^\mp$ is similar to $\Lambda_b \to pK^{*\mp}$, and its amplitude can be obtained from Eqs. (11), (19) and (17) with the replacements $V_{is} \to V_{id}$ and $K^* \to \rho$. However, here too the asymmetry is expected to be smaller than that in $\Lambda_b \to p\pi^\mp$ by the
factor \( |a_d/a_b| = 0.11 \), yielding an asymmetry of less than 1%. Finally, the pure penguin process \( \Lambda_b \to \Lambda \phi \), is dominated by a single weak amplitude, so that all its triple-product asymmetries vanish. (As mentioned earlier, the observation of the decay \( \Lambda_b \to \Sigma \phi \) would indicate the existence of nonfactorizable contributions and the possible presence of a significant \( V_{ub}V_{us}^* \) piece in the amplitude.)

To summarize, we have examined the predictions of the standard model for T-violating triple-product asymmetries in \( \Lambda_b \to F_1 F_2 \) decays, where \( F_1 \) is a light spin-\( \frac{1}{2} \) baryon, and \( F_2 \) is a pseudoscalar (P) or vector (V) meson. In \( \Lambda_b \to F_1 P \) decays, there is only a triple product possible. In the rest frame of the \( \Lambda_b \), it takes the form \( \vec{p}_{F_1} \cdot (\vec{s}_{F_1} \times \vec{s}_{\Lambda_b}) \), where \( \vec{p}_{F_1} \) is the 3-momentum of the \( F_1 \), and \( \vec{s}_{F_1} \) and \( \vec{s}_{\Lambda_b} \) are the spins of the \( F_1 \) and \( \Lambda_b \), respectively. On the other hand, in \( \Lambda_b \to F_1 V \) decays, since all three particles have a non-zero spin, there are several possible triple products.

Using factorization, we find the following results. First, for \( \Lambda_b \to F_1 P \) decays, the SM predicts a large asymmetry (\( \sim 18\% \)) only for \( \Lambda_b \to pK^- \). This is due to the presence of the chiral enhancement term \( m_K^2/(m_s + m_u) \) in the amplitude, which compensates the \( 1/m_b \) suppression. The asymmetry in \( \Lambda_b \to p\pi^- \) is smaller (\( \sim 5\% \)), and for the decays \( \Lambda_b \to \Lambda \eta, \Lambda \eta' \), it is less than 1\%. Second, for \( \Lambda_b \to F_1 V \) decays with a transversely-polarized \( V \), the asymmetries are quite small: for \( \Lambda_b \to pK^{*-} \) and \( \Lambda_b \to p\rho^- \) they are \( O(1\%) \) and \( < 1\% \), respectively. The asymmetries involving a longitudinally-polarized \( V \) are expected to be roughly 15\% smaller than those for a transversely-polarized \( V \), so that they are effectively unmeasurable. There are no asymmetries in \( \Lambda_b \to \Lambda \phi \) since this decay is dominated by a single weak amplitude. The fact that, within the SM, the triple-product asymmetries in many decays are tiny suggests that this is a good area to search for new physics.

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