Multifluid magnetohydrodynamic turbulence in weakly ionised astrophysical plasmas

Turlough P. Downes\textsuperscript{1,2,3}

\textsuperscript{1}School of Mathematical Sciences, Dublin City University, Glasnevin, Dublin 9, Ireland.
\textsuperscript{2}School of Cosmic Physics, Dublin Institute for Advanced Studies, 31 Fitzwilliam Place, Dublin 2, Ireland.
\textsuperscript{3}National Centre for Plasma Science and Technology, Dublin City University, Glasnevin, Dublin 9, Ireland.

Abstract. Initial results from simulations of 4-fluid MHD turbulence in molecular clouds are presented. The species included in the simulations are ions, electrons, negatively charged dust grains and neutrals. The results indicate that, on length scales of a few tenths of a parsec, multifluid effects have a significant impact on the properties of the turbulence. In particular, the power spectra of the velocity and magnetic fields are significantly softened, while the PDF of the densities of the charged and neutral fluids are appreciably different. Indeed, the magnetic field strength displays much less spatial structure on all length scales up to 1 pc than in the ideal MHD case. The assumptions of ideal MHD therefore appear to be inadequate for simulating turbulence in molecular clouds at these length scales.

1. Introduction

It is generally believed that molecular clouds are turbulent (see the reviews of Mac Low & Klessen 2004; Elmegreen & Scalo 2004). Observationally (e.g. Larson 1981) this turbulence appears to be supersonic, with RMS Mach numbers of anything up to 10, and either super-Alfvénic or trans-Alfvénic. The amplitude of this turbulence makes it likely to be an important ingredient in our understanding both of the dynamics of molecular clouds and of the process of star formation (Elmegreen 1993) and hence developing an understanding of this phenomenon is of considerable interest.

Many authors have addressed the issue of MHD turbulence in the context of molecular clouds using both the ideal MHD approximation (e.g. Mac Low et al. 1998; Mac Low 1999; Ostriker et al. 2001; Glover & Mac Low 2007; Lemaster & Stone 2009) and, more recently, various flavours of non-ideal MHD (Oishi & Mac Low 2006; Li et al. 2008; Kudoh & Basu 2008; Downes & O’Sullivan 2009, 2011). Ambipolar diffusion has been found to cause greater temporal variability in the turbulence statistics. Clearly of lesser significance, the Hall effect is in principle capable of inducing topological changes in the magnetic field which are quite distinct to any influence caused by ambipolar diffusion.

The non-ideal MHD effects caused by multifluid physics occur on relatively small length scales (fractions of a parsec). However, since three dimensional MHD turbu-
Downes & O'Sullivan (2009, 2011) performed simulations of decaying, non-ideal MHD and multifluid MHD, molecular cloud turbulence incorporating parallel resistivity, the Hall effect and ambipolar diffusion. They found that the Hall effect has surprisingly little impact on the behaviour of the turbulence: it does not affect the energy decay at all and has very limited impact on the power spectra of any of the dynamical variables, with the exception of the magnetic field at very short lengthscales.

In this paper we use the HYDRA code (O'Sullivan & Downes 2006, 2007) to investigate driven, isothermal, multifluid MHD turbulence in a system consisting of 1 neutral and 3 charged fluids. The aim of this work is to determine the influence of multifluid effects on the behaviour of driven turbulence in molecular clouds. An added benefit of our properly multifluid approach is that we can self-consistently deduce the behaviour of the charged species and, in principle, make links with observations such as those of Li & Houde (2008).

2. The model

Molecular clouds are, to a great extent, weakly ionised systems. This allows us to make several simplifying assumptions, as follows: the bulk flow velocity is the neutral velocity; the majority of collisions experienced by each charged species occur with neutrals; the charged species’ inertia is unimportant and, finally, the charged species’ pressure gradient is unimportant. Making these assumptions allows us to derive a relationship a generalised Ohm’s law (e.g. Falle 2003; Ciolek & Roberge 2002), and this allows us to avoid having to calculate the electric field explicitly from the charge distribution.

The resulting weakly ionised, multifluid MHD equations are a nonlinear system of PDEs (see Ciolek & Roberge 2002; Falle 2003; O’Sullivan & Downes 2007) and, in general, must be tackled using simulations. The method used by HYDRA to integrate these equations has been elucidated previously in the literature (e.g. O’Sullivan & Downes 2006, 2007) and we refer the reader to these papers for a more in-depth discussion. The method is explicit and scales extremely well on massively parallel systems, displaying strong scaling from 8192 up to 294,912 cores with 73% efficiency on the JUGENE BlueGene/P system.

3. Numerical set-up

We run our turbulence simulations in a cube of side 1 pc with periodic boundary conditions on all faces. The density and magnetic field are initially uniform with values of $10^6$ cm$^{-3}$ and 1 mG respectively. The (isothermal) sound-speed is chosen to be $3.3 \times 10^4$ cm s$^{-1}$ or, equivalently, the temperature is 30 K. The four fluids in the system are initially at rest and the properties of the charged species are chosen to approximate those of dust grains, ions and electrons with the dust grains making up 1% of the mass density of the neutrals. We require charge neutrality and that the resistivities in the generalised Ohm’s Law be as described in Wardle & Ng (1999). The grid used is uni-
form and made up of $N_g^3$ grid points with $N_g$ points in each of the $x$, $y$ and $z$ directions. We emphasise here that this work is directed at understanding the nature of multifluid MHD turbulence. While a physical system with the properties just described may well be subject to dynamically important self-gravity, it seems reasonable that the optimal way of approaching understanding the behaviour of molecular clouds is to attempt to understand the turbulence itself before including the effect of self-gravity which will, undoubtedly, significantly impact the behaviour of the system.

To drive turbulence we add velocity increments to the neutral velocity at each time-step. The incremental velocity field, $\delta u$, is defined in a manner similar to Lemaster & Stone (2009). Each component of $\delta u$ is generated from a set of waves with wave numbers, $3 \leq k \leq 4$ where $k = |k|$. The amplitudes of these waves are drawn from a Gaussian random distribution with mean 1.0 and deviation 0.33 while the phases of the waves are drawn from a uniform distribution between 0 and $2\pi$. A new realisation of the incremental velocity field is generated after $10^{-3} t_c$ has elapsed since the previous realisation was generated.

The rate of injection of energy is the same for all simulations in this work and is given by $\dot{E} = 300 \rho_0 L^2 c_s^3$ where $\rho_0$ is the initial (uniform) density, $L$ is the length of side of the computational domain and $c_s$ is the sound speed. In our simulations this energy injection rate yields steady state turbulence with an RMS Mach number of around 5 and an Alfvénic Mach number (with respect to the unperturbed magnetic field) of around 1.2.

The nomenclature used here to refer to the simulations is defined as follows: each simulation is referred to as either mf-xxx or mhd-xxx with “mf” standing for multifluid and “mhd” standing for ideal MHD. The “xxx” gives the value of $N_g$.

4. Resolution study

Figure 1 contains the Mach number as a function of time and the compensated power spectra of the neutral velocity for three simulations (mf-128, mf-256 and mf-512) in our resolution study. While there are differences between mf-128 and mf-256 it is clear that the differences between mf-256 and mf-512 are minor, indicating that both simulations resolve the multifluid physics occurring in this system reasonably well.
5. Results

In this section we analyse some aspects of the turbulence generated in our simulated systems. We compare the properties of the mf-512 simulation with that of mhd-512 in order to gain a sense of the differences introduced by multifluid effects at the 1 pc level. The left panel of Figure 1 indicates that for \( t \geq 0.12 t_c \) the turbulence has reached a quasi-steady steady state. Any time-averaged quantities discussed in this section are averaged over times after \( t = 0.12 t_c \).

5.1. Energy dissipation

Figure 2 contains plots of the behaviour of the mean, mass-weighted RMS Mach number (see Lemaster & Stone 2009; Downes & O’Sullivan 2011) and the perturbed magnetic energy as functions of time. It is clear that the Mach number reached in each case is very similar and in fact the difference in time-averaged RMS Mach number after \( t = 0.12 t_c \) is less than 1%. Therefore, at these length scales and with this numerical resolution, we do not detect any difference in the energy dissipation rate due to multifluid effects. It is worth noting, however, that the perturbed magnetic energy in mf-512 is lower than that in mhd-512 (Figure 2, right panel) which indicates that the energy in the turbulence is less weighted toward the magnetic field in a multifluid MHD system than in an ideal MHD one.

The work of Downes & O’Sullivan (2009, 2011) implies that the energy dissipation rate is higher in multifluid MHD systems. Their simulations were carried out in a cube of length 0.2 pc and the decay rate of the turbulence is undoubtedly higher in their multifluid simulation than in the ideal case. While this point is worthy of further study, it may be the case that the difference in the length scales, which determines the physical resolution of the simulations, may be partly responsible for the apparent discrepancy in the results presented here and those of Downes & O’Sullivan (2009, 2011).

5.2. Power spectra

We now turn to the power spectra of the various quantities of interest in these simulations. The nature of these spectra will give us some idea of the influence of the multifluid effects on the turbulence. We time-average all the power spectra discussed here over the time-interval \([0.12 t_c : 0.22 t_c]\).
Multifluid MHD turbulence in molecular clouds

Figure 3. Time-averaged, compensated power spectra of the velocity field (left panel) and the magnetic field strength (right panel) for mhd-512 and mf-512. The velocity spectra are compensated with $k^{1.5}$ while the magnetic field spectra are compensated with $k^2$.

Figure 3 contains plots of the velocity and magnetic field strength power spectra for mhd-512 and mf-512. It is clear that the velocity field in mhd-512 has less structure at length scales below about 0.05 pc than that of mf-512. The difference between the two systems is more dramatic in the power spectra of the magnetic field strength where simulation mf-512 displays considerably less structure at virtually all length scales below the driving scale. This is, perhaps, what we would expect given the fact that the multifluid effects manifest themselves most directly in the induction equation.

The left panel of figure 4 contains plots of the power spectra of the densities for all of the fluids in mf-512. At these length scales the electrons and ions have almost identical power spectra while the neutral fluid has considerably more power at all length scales below $\sim 0.1$ pc. These differences are due to the fact that the electrons and ions are more closely tied to the dynamics of the magnetic field than are the neutrals. The magnetic field, as a result of ambipolar diffusion, has less power at shorter length scales in particular (see Figure 3), and hence the species closely tied to it also have less fine-scale structure. Interestingly the density PDFs of the fluids in mf-512 (right panel of figure 4) indicate that, though all fluids are distributed in an approximately lognormal way, the distributions of the charged species are considerably more peaked and narrower. This is another manifestation of the effect just noted: since the magnetic field is somewhat decoupled from the bulk (neutral) flow it is, in particular, not compressed or rarefied by the turbulent flow as much as the neutral flow. This means that the species tied to the magnetic field will not be compressed or rarefied as much as the neutrals and hence the PDF of their densities will have a lower standard deviation than that of the neutrals.

6. Conclusions

In this work we present 4-fluid MHD simulations of turbulence in molecular clouds, neglecting self-gravity. The aim is to investigate the impact of multifluid MHD effects on the turbulence itself while the inclusion of self-gravity will be the subject of future work. The results indicate that, on the length scales at which the simulations were run ($\sim 1$ pc) multifluid effects have an appreciable effect on both the velocity and density power spectra, and there is a significant difference between the PDFs of the neutral and charged species.
Figure 4. Time-averaged, compensated power spectra of the densities of the various fluids in mf-512 (left panel) and the probability distribution functions of the logarithm of their densities (right panel).

This all implies that, even on fairly large length scales, multifluid effects are dynamically important in molecular cloud turbulence.

Acknowledgments. This material is based upon works supported by the Science Foundation Ireland under Grant No. 07/RFP/PHYF586. We thank the DEISA Consortium (www.deisa.eu), funded through the EU FP7 project RI-222919, for support within the DEISA Extreme Computing Initiative. The authors also wish to acknowledge the SFI/HEA Irish Centre for High-End Computing (ICHEC) for the provision of computational facilities and support.

References

Ciolek, G. E., & Roberge, W. G. 2002, ApJ, 567, 947. arXiv:astro-ph/0112569
Downes, T. P., & O’Sullivan, S. 2009, ApJ, 701, 1258. 0909.4226
— 2011, ApJ, 730, 12. 1101.3429
Elmegreen, B. G. 1993, ApJ, 419, L29+
Elmegreen, B. G., & Scalo, J. 2004, ARA&A, 42, 211. arXiv:astro-ph/0404451
Falle, S. A. E. G. 2003, MNRAS, 344, 1210. arXiv:astro-ph/0308396
Glover, S. C. O., & Mac Low, M.-M. 2007, ApJ, 659, 1317. arXiv:astro-ph/0605121
Kudoh, T., & Basu, S. 2008, ApJ, 679, L97. 0804.4303
Larson, R. B. 1981, MNRAS, 194, 809
Lemaster, M. N., & Stone, J. M. 2009, ApJ, 691, 1092. 0809.4005
Li, H.-b., & Houde, M. 2008, ApJ, 677, 1151. 0801.2757
Li, P. S., McKee, C. F., Klein, R. I., & Fisher, R. T. 2008, ApJ, 684, 380. 0805.0597
Mac Low, M.-M. 1999, ApJ, 524, 169. arXiv:astro-ph/9809177
Mac Low, M.-M., & Klessen, R. S. 2004. Reviews of Modern Physics, 76, 125. arXiv:astro-ph/0310193
Oishi, J. S., & Mac Low, M.-M. 2006, ApJ, 638, 281. arXiv:astro-ph/0510366
Ostriker, E. C., Stone, J. M., & Gammie, C. F. 2001, ApJ, 546, 980. arXiv:astro-ph/0008454
O’Sullivan, S., & Downes, T. P. 2006, MNRAS, 366, 1329. arXiv:astro-ph/0511478
— 2007, MNRAS, 376, 1648. arXiv:astro-ph/0612580
Wardle, M., & Ng, C. 1999, MNRAS, 303, 239. arXiv:astro-ph/9810468