A Comment on the Light-Cone Vacuum in 1+1 Dimensional Super-Yang-Mills Theory

F. Antonuccio\textsuperscript{(a)}, S. Pinsky\textsuperscript{(a)} and S. Tsujimaru\textsuperscript{(b)}

\textsuperscript{(a)} Department of Physics,  
The Ohio State University,  
Columbus, OH 43210, USA

and

\textsuperscript{(b)} Max-Planck-Institut für Kernphysik,  
69029 Heidelberg, Germany

Abstract

The Discrete Light-Cone Quantization (DLCQ) of a supersymmetric gauge theory in 1+1 dimensions is discussed, with particular attention given to the inclusion of the gauge zero mode. Interestingly, the notorious ‘zero-mode’ problem is now tractable because of special supersymmetric cancellations. In particular, we show that anomalous zero-mode contributions to the currents are absent, in contrast to what is observed in the non-supersymmetric case. An analysis of the vacuum structure is provided by deriving the effective quantum mechanical Hamiltonian of the gauge zero mode. It is shown that the inclusion of the zero modes of the adjoint scalars and fermions is crucial for probing the phase properties of the vacua. We find that the ground state energy is zero and thus consistent with unbroken supersymmetry, and conclude that the light-cone Fock vacuum is unchanged with or without the presence of matter fields.
1 Introduction

A possibly surprising outcome of recent developments in string/M theory are the proposed connections between non-perturbative objects in string theory, and supersymmetric gauge theories in low dimensions \[1, 2\]. It is therefore of interest to study directly the non-perturbative properties of super-Yang-Mills theories in various dimensions.

Recently, a class of 1+1 dimensional super Yang-Mills theories has been studied using a supersymmetric form of Discrete Light-Cone Quantization (SDLCQ) \[3, 4, 5, 6\]. This formulation has the advantage of preserving supersymmetry after discretizing momenta, and admits a very natural and straightforward algorithm for extracting numerical bound state masses and wave functions \[7, 8\]. Although a technical necessity, the omission of zero-momentum modes in these numerical computations raises many doubts about the consistency of such a quantization scheme. Little is in fact known about the precise effects of dropping the zero-momentum mode at finite compactification radius, but it is generally believed that such effects disappear in the decompactified limit \[6\].

There are instances, however, when we would like to know the measurable effects of a finite spatial compactification \[2\]. In this work, we will deal with measurable effects that reflect the spatial compactification induced by DLCQ. This is accomplished by explicitly incorporating the gauge zero mode in the DLCQ formulation of a supersymmetric gauge theory. It turns out that this is tantamount to including a quantum mechanical degree of freedom corresponding to ‘quantized electric flux’ around the compact direction. The implications of this on the vacuum structure of the theory is discussed.

The supersymmetric gauge theory we consider may be obtained by dimensionally reducing \( \mathcal{N} = 1 \) super-Yang-Mills from 2+1 to 1+1 dimensions \[3\]. For simplicity, we choose SU(2) to be the gauge group. The DLCQ formulation of this theory consists of an adjoint scalar field (represented as a 2 × 2 Hermitian matrix field), a corresponding adjoint fermion field, and several zero-mode (or quantum mechanical) degrees of freedom to be discussed later. To maintain supersymmetry one must impose periodic boundary conditions, so all three color degrees of freedom of the fermion and boson fields will have zero modes. In addition, periodic boundary conditions prevent us from adopting the light-cone gauge, \( A^+ = 0 \), so we choose instead the gauge \( \partial^- A^+ = 0 \), which allows \( A^+ \) to have a zero mode.

For the gauge group SU(2), the field degrees of freedom may be labeled as +, − and 3, corresponding to the three generators of the group. In general, the ± components of the matter fields depend on the gauge zero mode and exhibit a spectral flow under large gauge transformations. In addition, there is a transformation that combines a large gauge transformation and a Weyl transformation, and is known to be a symmetry of the theory. There are two important consequences of this. The zero modes of the ± components of the fermion and boson fields transform into non-zero momentum modes, and give rise to a degenerate vacuum in the theory. For this reason it is inconsistent to omit these modes, and so we will formally include them in our formulation. We will

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\[\text{i.e. arising from the three generators of SU(2).}\]
not, however, discuss the dynamical implications of including such degrees of freedom, although some discussion on this and related issues appeared recently [6].

In this work, we concentrate on the effect of including the quantum mechanical degree of freedom represented by the gauge zero mode. This zero mode corresponds to a quantized color electric flux that circulates around the compact direction $x^-$. The problems associated with this zero mode have already been studied in two dimensional gauge theories involving adjoint scalars, and theories with adjoint fermions [9, 10, 11, 12, 13]. The consequences of including these modes are quite drastic. These theories are known to possess anomalies in the 3-component of the current, and a simple consequence is that the charges in these theories are time dependent. This makes it difficult – if not impossible – to define a consistent theory. In contrast, owing to special supersymmetric cancellations between boson and fermion currents, no such anomalies arise in the supersymmetric theory studied here, and so a DLCQ formulation becomes sensible and tractable.

In general, one finds a contribution after normal ordering the Hamiltonian that is a function only of the gauge zero-mode. This term acts as a vacuum potential and leads to a non-zero vacuum energy. When the gauge theory without matter fields is solved, however, the only degree of freedom is the quantum mechanical gauge zero mode, in which the vacuum potential plays no role. The ground state energy is thus zero. However, this simple picture of the vacuum may be drastically altered if we consider the addition of matter. For the supersymmetric case studied here, we show that there is no vacuum potential, and that the ground state has zero energy even in the presence of matter fields.

This paper is organized as follows. In Section 2, we briefly describe the DLCQ procedure of the 1+1 dimensional supersymmetric Yang-Mills theory in the modified light-cone gauge. In Section 3, the point splitting regularization designed to preserve symmetry under large gauge transformations is applied to the current operator. In Section 4, we discuss the vacuum structure of the theory by deriving the quantum mechanics of the gauge zero mode. We conclude in Section 5 with a brief discussion.

# 2 Gauge Fixing in DLCQ

We consider the supersymmetric Yang-Mills theory in 1+1 dimensions [14] which is described by the action

$$S = \int d^2x \text{tr} \left( -\frac{1}{2} F_{\mu\nu} F^{\mu\nu} + D_\mu \phi D^\mu \phi + i \bar{\Psi} \gamma^\mu D_\mu \Psi - ig \phi \bar{\Psi} \gamma_5 \Psi \right),$$

where $D_\mu = \partial_\mu + ig[A_\mu, \cdot]$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$. All fields are in the adjoint representation of the gauge group SU(2). A convenient representation of the gamma matrices is $\gamma^0 = \sigma^2$, $\gamma^1 = i\sigma^1$ and $\gamma^5 = \sigma^3$ where $\sigma^a$ are the Pauli matrices. In this representation the Majorana spinor is real. We choose Cartan basis, $(\tau^+, \tau^-, \tau^3)$, defined
by $[\tau^+, \tau^-] = \tau^3$ and $[\tau^3, \tau^\pm] = \pm \tau^\pm$, with the normalization $\text{tr}(\tau^+\tau^-) = \text{tr}(\tau^3\tau^3) = 1/2$. In terms of this basis the matrix valued fields may be decomposed as follows:

$$A^\mu = A^\mu_+ \tau^+ + A^\mu_- \tau^- + A^\mu_3 \tau^3.$$  

(2)

Henceforth the lower index refers to the gauge group component.

We now introduce the light-cone coordinates $x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^1)$. The longitudinal coordinate $x^-$ is compactified on a finite interval $x^- \in [-L, L]$ [13, 4] and we impose periodic boundary conditions on all fields to ensure unbroken supersymmetry.

The light-cone gauge $A^+ = 0$ can not be used in a finite compactification radius, but the modified condition $\partial_- A^+ = 0$ [9] is consistent with the light-like compactification. We can make a global rotation in color space so that the zero mode is diagonalized $V(x^+) = v(x^+) \tau^3$ [9]. The gauge zero mode corresponds to a (quantized) color electric flux loop around the compactified space.

The modified light-cone gauge is not a complete gauge fixing. In fact, the large gauge transformation $U(x) = \exp(-\frac{ixnx^-}{L}\tau^3)$ generates shifts in the zero mode

$$v(x^+) \rightarrow v(x^+) + \frac{n\pi}{gL}, \quad n \in \mathbb{Z},$$

(3)

while preserving the gauge condition $\partial_- A^+ = 0$. To completely fix the gauge one therefore fixes $v$ to be in the finite interval $0 < v < \pi/gL$. It is convenient to introduce the dimensionless variable $z = gLv/\pi$ as well. Other intervals give gauge “copies” [16] of this domain which is called the fundamental domain. In addition, we can explicitly see that the Wilson loop $\cos(2Lgv)$, is invariant under (3).

With this gauge choice the quantization is straightforward. The details of this light-cone formulation may be found in the literature [10, 11, 3, 5, 8]. Here we provide only the results which are useful for later purposes. The quantization proceeds in two steps. First, we must resolve the constraints to eliminate the redundant degrees of freedom. There are two constraints in the theory,

$$-D_- A^- = gJ^+, \quad (4)$$

$$\sqrt{2}iD_- \chi = g[\phi, \psi], \quad (5)$$

where $\Psi \equiv (\psi, \chi)^T$ and the current operator is

$$J^+(x) = \frac{1}{i}[\phi(x), D_- \phi(x)] - \frac{1}{\sqrt{2}}\{\psi(x), \psi(x)\}. \quad (6)$$

The first equation is the Gauss-law constraint, and in components takes the form

$$-\partial_-^2 A^-_3 = gJ^+_{3}, \quad (7)$$

$$-(\partial_- + igv)^2 A^-_+ = gJ^+_{+}, \quad (8)$$

$$-(\partial_- - igv)^2 A^-_- = gJ^+_{-}. \quad (9)$$

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We may therefore eliminate any dependence of $A^-$ and $\chi$ in favor of the physical degrees of freedom ($\phi, \psi, v$). However the kernel of the operator $D_-$ has to be treated separately. For example, the zero mode of (7) which is associated with the kernel of $\partial_-$ provides us with the condition $\int dx^- J^+ = 0$ which must be imposed on the Fock space to select the physical states in the quantum theory.

The next step is to derive the commutation relations for the physical degrees of freedom. As in the ordinary quantum mechanics, the zero mode $v$ has a conjugate momentum $p = 2L\partial_x v$ and the commutation relation is $[v, p] = i\hbar$. The off-diagonal components of the scalar field are complex valued operators with $\phi^{+} = (\phi^{-})^\dagger$. The canonical momentum fields conjugate to $\phi^{-}$ and $\phi^{+}$ are $\pi^{+} = (\partial_- + igv)\phi^{+}$ and $\pi^{+} = (\partial_- - igv)\phi^{+}$, respectively. They satisfy the canonical commutation relations

$$[\phi^{+}(x), \pi^{-}(y)]_{x^{+}=y^{+}} = [\phi^{-}(x), \pi^{+}(y)]_{x^{+}=y^{+}} = \frac{i}{2}\delta(x^{-} - y^{-}).$$

(10)

On the other hand, the quantization of the diagonal component $\phi_3$ needs care. As mentioned in [10], the zero mode of $\phi_3$, the mode independent of $x^-$, is not an independent degree of freedom but obeys a certain constrained equation [15, 10, 12]. Except the zero mode, the commutation relation is canonical

$$[\phi_3(x), \partial_\phi \phi_3(y)]_{x^{+}=y^{+}} = \frac{i}{2}\left[\delta(x^{-} - y^{-}) - \frac{1}{2L}\right].$$

(11)

Finally, the canonical anti-commutation relations which we should impose on fermion fields are [11]

$$\{\psi^{+}(x), \psi^{-}(y)\}_{x^{+}=y^{+}} = \frac{1}{\sqrt{2}}\delta(x^{-} - y^{-}).$$

(12)

We will also omit the zero mode of $\psi_3$, but simply note here that it has a special role in ensuring supersymmetry. This will be discussed in future work [17]. The corresponding relations are then

$$\{\psi_3(x), \psi_3(y)\}_{x^{+}=y^{+}} = \frac{1}{\sqrt{2}}\left[\delta(x^{-} - y^{-}) - \frac{1}{2L}\right].$$

(13)

The commutation relations for the Fourier modes, and the form of the light-cone Hamiltonian will be given in the following Sections.

3 Current Operators

The resolution of the Gauss-law constraint is a necessary step for obtaining the light-cone Hamiltonian. The expression for the current operator is, however, ill-defined unless an appropriate definition is specified, since the operator products are defined at the same point. We shall use the point-splitting regularization which respects the symmetry of the theory under the large gauge transformation.
The mode-expanded fields at the light-cone time $x^+ = 0$ are

$$\phi_-(x) = (\phi_+(x))^\dagger = \frac{1}{\sqrt{4\pi}} \left( \sum_{n=0}^\infty b_n u_n e^{-ik_nx^-} + \sum_{n=1}^\infty d_n^\dagger v_n e^{ik_nx^-} \right),$$

$$\phi_3(x) = \frac{1}{\sqrt{4\pi}} \sum_{n=1}^\infty \frac{1}{\sqrt{n}} (a_n e^{-ik_nx^-} + a_n^\dagger e^{ik_nx^-}),$$

$$\psi_+(x) = (\psi_-(x))^\dagger = \frac{1}{2\sqrt{2L}} \left( \sum_{n=0}^\infty B_n e^{-ik_nx^-} + \sum_{n=1}^\infty D_n^\dagger e^{ik_nx^-} \right),$$

$$\psi_3(x) = \frac{1}{2\sqrt{2L}} \sum_{n=1}^\infty \left( A_n e^{-ik_nx^-} + A_n^\dagger e^{ik_nx^-} \right),$$

where $k_n = n\pi/L$, $u_n = 1/\sqrt{|n+z|}$ and $v_n = 1/\sqrt{|n-z|}$ [16]. The (anti)commutation relations for Fourier modes are found in [10, 11] and take the form

$$[b_n, b_m^\dagger] = \text{sgn}(n+z)\delta_{n,m}, \quad [d_n, d_m^\dagger] = \text{sgn}(n-z)\delta_{n,m},$$

$$\{B_n, B_m^\dagger\} = \{D_n, D_m^\dagger\} = \delta_{n,m}, \quad [a_n, a_m^\dagger] = \{A_n, A_m^\dagger\} = \delta_{n,m}.$$  \hspace{1cm} (15)

The large gauge transformations are denoted by $T_n$ which act as $T_n z T_n^{-1} = z + n$. The charged fields receive the phase rotations

$$T_n \phi_\pm T_n^{-1} = \phi_\pm e^{\pm i\pi n x^-},$$

$$T_n \psi_\pm T_n^{-1} = \psi_\pm e^{\mp i\pi n x^-},$$

while the color neutral fields, $\phi_3$ and $\psi_3$, are unchanged under $T_n$. One can observe that the large gauge transformation preserves periodic boundary conditions and are tantamount to a spectral flow. This gauge symmetry is an example of the Gribov ambiguity [10] and can be fixed once we restrict ourselves to one of the “copies”. The fundamental domain $0 < z < 1$ is one of such gauge choices, and this completes the gauge fixing.

The theory has another symmetry called the Weyl conjugation symmetry, denoted by $R$,

$$RzR^{-1} = -z, \quad R\phi_\pm R^{-1} = \phi_\mp, \quad \text{and} \quad R\psi_\pm R^{-1} = \psi_\mp.$$ \hspace{1cm} (18)

Although the Weyl symmetry is no longer the symmetry of the theory after the “gauge fixing” there still exists a symmetry of the gauge-fixed theory which is a particular combination of the large gauge transformation and the Weyl conjugation $S = T_1 R$. In fact $S$ maps the fundamental domain onto itself. This operator can be chosen to satisfy $S^2 = 1$ and is used in classifying the vacua [18, 13].

Let us now discuss the definition of singular operator products in the current (6). We define the current operator by point splitting,

$$J^+ \equiv \lim_{\epsilon \to 0} \left( J^+_{\phi}(x; \epsilon) + J^+_{\psi}(x; \epsilon) \right),$$ \hspace{1cm} (19)

\text{5}$u_n$ and $v_n$ are well-defined in the fundamental domain. Similarly, $(\partial_- \pm igv)^2$ in the Gauss-law constraint have no zero modes in this domain.
where the divided pieces are given by
\[
J^+\phi(x; \epsilon) = \frac{1}{i} \left[ e^{-i\frac{\pi}{L} \tau^3} \phi(x^- - \epsilon) e^{i\frac{\pi}{L} \tau^3}, D_- \phi(x^-) \right] \tag{20}
\]
\[
J^+\psi(x; \epsilon) = -\frac{1}{\sqrt{2}} \left\{ e^{-i\frac{\pi}{L} \tau^3} \psi(x^- - \epsilon) e^{i\frac{\pi}{L} \tau^3}, \psi(x^-) \right\}. \tag{21}
\]

An advantage of this regularization is that the current transforms covariantly under the large gauge transformation. In fact, it is easy to show that \( J^{+3} \) is invariant under the large gauge transformation while the others transform covariantly as \( J^{+\pm} \rightarrow J^{+\pm} e^{\mp i\frac{\pi}{L} \tau^3 x^-} \).

It is straightforward to evaluate (20) and (21) and they have previously been discussed separately [10, 11]. With a slight modification they are found to be
\[
\lim_{\epsilon \rightarrow 0} J^+\phi(x; \epsilon) = \tilde{J}^+\phi(x) - \frac{1}{2L} (z + \frac{1}{2}) \tau^3, \tag{22}
\]
\[
\lim_{\epsilon \rightarrow 0} J^+\psi(x; \epsilon) = \tilde{J}^+\psi(x) - \frac{1}{2L} (\bar{z} + \frac{1}{2}) \tau^3, \tag{23}
\]

where \( \tilde{J}^+\phi \) and \( \tilde{J}^+\psi \) are the naive normal ordered currents. To be more precise, we have omitted the zero modes of the color 3 sectors in which the notorious constrained zero mode [15] appears. On the other hand, the zero modes of the color charged sectors are explicitly incorporated and their effects are found in (22) and (23) as constant terms which are independent of \( z \). As can be seen, \( J^+\phi \) and \( J^+\psi \) acquire extra \( z \) dependent terms, so called gauge corrections. Integrating these charges over \( x^- \), one finds that the charges are time dependent. Of course this is an unacceptable situation, and implies the need to impose special conditions to single out ‘physical states’ to form a sensible theory. The important simplification of the supersymmetric model is that these time dependent terms cancel, and the full current (19) becomes
\[
J^+(x) = \tilde{J}^+\phi + \tilde{J}^+\psi - \frac{1}{2L} \tau^3. \tag{24}
\]

The regularized current is thus equivalent to the naive normal ordered current up to an irrelevant constant. Similarly, one can show that \( P^+ \) picks up gauge correction when the adjoint scalar or adjoint fermion are considered separately but in the supersymmetric theory it is nothing more than the expected normal ordered contribution of the matter fields.

In one sense these results are a consequence of the well known fact that the normal ordering constants in a supersymmetric theory cancel between fermion and boson contributions. The important point here is that these normal ordered constants are not actually constants, but rather quantum mechanical degrees of freedom. It is therefore not obvious that they should cancel. Of course, this property profoundly effects the dynamics of the theory.
4 Vacuum Energy

The wave function of the vacuum state for the supersymmetric Yang-Mills theory in 1+1 dimensions has already been discussed in the equal-time formulation [19]. An effective potential is computed in a weak coupling region as a function of the gauge zero mode by using the adiabatic approximation. Here we analyze the vacuum structure of the same theory in the context of the DLCQ formulation.

The presence of zero modes renders the light-cone vacuum quite nontrivial, but the advantage of the light-cone quantization becomes evident: the ground state is the Fock vacuum for a fixed gauge zero mode and therefore our ground state may be written in the tensor product form

\[ |\Omega\rangle \equiv \Phi[z] \otimes |0\rangle, \tag{25} \]

where we have taken the Schrödinger representation for the quantum mechanical degree of freedom \( z \) which is defined in the fundamental domain. In contrast, to find the ground state of the fermion and boson for a fixed value of the gauge zero mode turns out to be a highly nontrivial task in the equal-time formulation [19].

Our next task is to derive an effective Hamiltonian acting on \( \Phi[z] \). The light-cone Hamiltonian \( H \equiv P^- \) is obtained from energy momentum tensors, or through the canonical procedure. In terms of the dimensionless operator \( \hat{H} \equiv \frac{4\pi^2}{g^2 L} H \), it is schematically given by

\[
\hat{H} = -\frac{1}{\sin^2(\pi z)} \frac{\partial}{\partial z} \sin^2(\pi z) \frac{\partial}{\partial z}, \tag{26}
\]

\[
+ \frac{4\pi^2}{g^2 L} \int_{-L}^{L} dx \, \text{tr} \left( -g^2 J^+ \frac{1}{D^-} J^+ + \frac{ig^2}{\sqrt{2}} \oint \phi^1 \phi \right), \tag{27}
\]

where the first term is the kinetic energy of the gauge zero mode, and in the second term the zero modes of \( D_- \) are understood to be removed. Note that the kinetic term of the gauge zero mode is not the standard form \(-d^2/dz^2\) but acquires a nontrivial Jacobian which is nothing but the Haar measure of SU(2). The Jacobian originates from the unitary transformation of the variable from \( A^+ \) to \( v \), and can be derived by explicit evaluation of a functional determinant [20, 18]. In the present context it is found in [12].

Projecting the light-cone Hamiltonian onto the Fock vacuum sector we obtain the quantum mechanical Hamiltonian

\[
\hat{H}_0 = -\frac{1}{\sin^2(\pi z)} \frac{\partial}{\partial z} \sin^2(\pi z) \frac{\partial}{\partial z} + V_{JJ} + V_{\phi\psi}, \tag{28}
\]

where the reduced potentials are defined by

\[
V_{JJ} \equiv -\frac{4\pi^2}{L} \int_{-L}^{L} dx \, \langle \text{tr} J^+ \frac{1}{D^-} J^+ \rangle, \tag{29}
\]

\[
V_{\phi\psi} \equiv \frac{4i\pi^2}{\sqrt{2}L} \int_{-L}^{L} dx \, \langle \phi^1 \phi \rangle \frac{1}{D^-} \left[ \phi^1 \phi \right], \tag{30}
\]

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respectively. As stated in the previous section, the gauge invariantly regularized current turns out to be precisely the normal ordered current in the absence of the zero modes. It is now straightforward to evaluate $V_{JJ}$ and $V_{\phi\psi}$ in terms of modes. One finds that they cancel among themselves as expected from the supersymmetry:

$$V_{JJ} = -V_{\phi\psi} = \frac{1}{4} \sum_{n,m=1} \left[ \frac{1}{n(m+z)} + \frac{1}{m(n-z)} + \frac{1}{(n-z)(m+z)} \right].$$  

(31)

Thus we arrive at

$$\hat{H}_0 = -\frac{1}{\sin^2(\pi z)} \partial \sin^2(\pi z) \partial.$$

(32)

In order to solve the eigenvalue problem, it is useful to define the new wavefunction in analogy with the radial wavefunctions. The appearance of the Jacobian seems vital since it determines the behavior of the $\tilde{\Phi}[z]$ at the edges of the fundamental domain: it leads to the boundary condition

$$\tilde{\Phi}[z = 0] = \tilde{\Phi}[z = 1] = 0.$$

(34)

The eigenvalue problem $\hat{H}_0 \Phi[z] = E \Phi[z]$ now turns out to be

$$\left( -\frac{d^2}{dz^2} - \pi^2 \right) \tilde{\Phi}[z] = E \tilde{\Phi}[z],$$

(35)

which can be solved easily. One can find that the ground state energy is precisely zero $E_0 = 0$ and the corresponding vacuum wave function is

$$\tilde{\Phi}[z] = \sqrt{2} \sin(\pi z).$$

(36)

We have thus found within the present assumption that the ground state has a vanishing vacuum energy, suggesting that the supersymmetry is not broken spontaneously.

Note that an emergence of the “effective potential” $-\pi^2$ in (35) is essential to this conclusion. In non-supersymmetric theories this constant energy is simply disregarded since it merely shifts all energy eigenvalues by the same amount. Then the eigenvalue problem (35) becomes formally the same as the original one $\hat{H}_0 \Phi[z] = E \Phi[z]$ but with the standard kinetic term $-d^2/dz^2$ supplemented by the boundary condition $\Phi[z = 0] = \Phi[z = 1] = 0$ (not for $\tilde{\Phi}[z]$). In supersymmetric theories, however, such constant energy cannot be discarded by hand since it is a part of dynamics closely related to the vacuum structure.
5 Discussion

We have performed a detailed analysis of the quantum mechanical degrees of freedom represented by the gauge zero mode of a supersymmetric SU(2) gauge theory in $1 + 1$ dimensions. This theory may be obtained by dimensionally reducing $\mathcal{N} = 1$ super-Yang-Mills from $2 + 1$ dimensions, and consists of one adjoint fermion and one adjoint scalar field periodically identified in the $x^-$ light-cone coordinate.

The remaining zero mode degrees of freedom that were not treated here are known to give rise to vacuum effects that are crucial in understanding the effects of topological field configurations due to the non-trivial center of SU(2), and the existence of two-fold vacua $|\Omega_\pm\rangle$, along the lines investigated in [13]. Evidently, an understanding of all the zero-mode degrees of freedom awaits future work.

We have, however, carefully taken account of the gauge zero mode. In this context the gauge zero mode is a quantum mechanical degree of freedom. In general, when one normal orders the operators of the theory one finds contributions that depend only on this quantum mechanical degree of freedom. These term are anomalies and profoundly effect the structure of the theory. In theories with only fermions or only bosons, these anomalies yield time dependent charges and a non-zero vacuum energy. In the supersymmetric theory presented here, these anomalies are seen to cancel and the operators are all well behaved. In particular, the charges are time independent and the ground state with matter is the same state that one finds when the theory is quantized without matter. In as much as the ground state energy is zero we conclude that the gauge zero mode does not break supersymmetry.

Finally, we remark that the properties of Matrix String Theory [2] – which is defined as $1+1 \mathcal{N} = 8$ super-Yang-Mills theory on a circle – depend crucially on the measurable effects produced by the space-like compactification. These effects are intimately tied with the dynamics of non-perturbative objects in Type IIA string theory known as D0 branes. It would be interesting to consider the DLCQ formulation of the same Yang-Mills theory, and to establish – if possible – any connection with the Matrix String proposal. The simplicity of the light-cone Fock vacuum, owing to special supersymmetry cancellations, might present a tractable approach to non-perturbative string theory.

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