Abstract

In this report we derive the couplings of the Randall-Sundrum radion to the standard model higgs boson. We then use these couplings to determine the $J=0$ partial wave amplitude for the process $hh \rightarrow h\phi$. We find that at very high energies (i.e. $s \gg m_h^2, m_\phi^2$) the s wave partial wave unitarity is violated if $m_h > m_c \approx \sqrt{16\pi \langle \phi \rangle v}$ where $\langle \phi \rangle$ is the radion vev. Interestingly this bound is independent of the radion mass to the leading order. We also consider the high energy behaviour of the transition amplitudes for some other processes in the RS scenario and compare them with their SM behaviour.
Recently several radical proposals based on extra dimensions have been put forward to explain the large hierarchy between the Planck scale and the weak scale. Among them the Randall-Sundrum RS model [1] is most interesting because it proposes a five dimensional world with a non-factorizable metric

\[ ds^2 = e^{-2k\pi r_c} g^{\mu\nu} dx_\mu dx_\nu - r_c^2 d\theta^2. \]  

Here \( r_c \) measures the size of the extra dimension which is an \( S^1/\mathbb{Z}_2 \) orbifold. \( x^\mu \) are the coordinates of the four dimensional space-time. \(-\pi \leq \theta \leq \pi\) is the coordinate of the extra dimension with \( \theta \) and \(-\theta\) identified. Two branes extending in the \( x^\mu \) or space time direction are placed at the orbifold fixed points \( \theta = 0 \) and \( \theta = \pi \). \( k \) is a mass parameter of the order of the fundamental five dimensional Planck mass \( M \). Randall and Sundrum showed that any field with a mass parameter \( m_0 \) in the fundamental five dimensional theory gets an effective four dimensional mass given by \( m = m_0 e^{-k\pi r_c} \). Thus for \( kr_c \approx 12 \) the weak scale is generated from the Planck scale by the exponential warp factor of the model.

In the original proposal of Randall and Sundrum the compactification radius was determined by the vacuum expectation value (vev) of a scalar field \( T(x) \). However the modulus field \( T(x) \) was massless and therefore its vev was not stabilized by some dynamics. Goldberger and Wise [2] showed that by introducing a scalar field in the bulk with interactions localized on the two branes it is possible to generate a potential for \( T(x) \). They also showed that this potential can be adjusted to yield a minimum at \( kr_c \approx 12 \) without any extreme fine tuning of parameters.

In the Randall-Sundrum model the SM fields are assumed to be localized on the visible brane at \( \theta = \pi \). However the SM action is modified due to the exponential warp factor. Small fluctuations of the modulus field \( T(x) \) about its vev \( r_c \) then gives rise to non-trivial couplings of the modulus field with the SM fields. It was shown in Ref.[3] that small fluctuations in the radion field \( \hat{\phi} \) couples with the SM fields on the visible brane through
the Lagrangian

\[ L_I = \frac{T^\mu_{\mu}}{\langle \phi \rangle} \hat{\phi}. \]  

(2)

Here \( T^\mu_{\mu} \) is the trace of the energy momentum tensor of the SM fields localized on the visible brane. \( \hat{\phi} \) is a small fluctuation of the radion field from its vev and is given by

\[ \phi = f e^{-k\pi T(x)} = \langle \phi \rangle + \hat{\phi}. \quad \langle \phi \rangle = f e^{-k\pi r_c} \]

is the vev of \( \phi \) and \( f \) is a mass parameter of the order of \( M \).

In this report shall derive the couplings of the radion field with the SM higgs field in the linearized approximation. We shall then use these couplings to calculate the transition amplitude for the process \( hh \rightarrow h\phi \) at very high energies i.e. when \( s \gg m_h^2, m_\phi^2 \). By requiring the \( J = 0 \) partial wave amplitude for this process satisfies the unitarity constraint we then derive an upper bound on the higgs mass. Interestingly this unitarity bound on the higgs mass is independent of the radion mass to leading order. The subleading terms has a dependence on \( \frac{m_\phi^2}{s} \) but it is only logarithmic and it vanishes as \( s \rightarrow \infty \). The reasons behind considering the process \( hh \rightarrow h\phi \) are the following:

i) This process does not occur in the SM.

ii) The transition amplitude for this process is free from bad high energy behaviour and leads to a unitarity bound on \( m_h \) that is independent of \( m_\phi \).

iii) It does not receive any contribution from the tower of Kaluza-Klein modes of the graviton and the stabilizing bulk scalar. The transition amplitude for this process is therefore simple to compute and is free from the uncertainties associated with processes that receive contribution from the tower of Kaluza-Klein modes of the graviton.

The couplings of the radion field to the SM higgs field localized on the brane at \( \theta = \pi \) is completely determined by general covariance. The action for the SM higgs field in the Randall-Sundrum model is given by

\[ S = \int d^4x \sqrt{-g_v} \left[ \frac{1}{2} g^{\mu\nu} \partial_\mu h \partial_\nu h - V(h) \right]. \]  

(3)
Here \( g^{\mu\nu} \) is the induced metric on the visible brane. In the absence of graviton fluctuations about the background metric it is given by
\[
g^{\mu\nu} = e^{2k\pi T(x)} \eta^{\mu\nu} = \left( \frac{\phi}{f} \right)^{-2} \eta^{\mu\nu}
\]
where \( \eta^{\mu\nu} \) is the Minkowski metric.
\[
\sqrt{-g_v} = \sqrt{-\det(g_v)} = e^{-4k\pi T(x)} = \left( \frac{\phi}{f} \right)^4.
\]
The tree level higgs potential \( V(h) \) is given by
\[
V(h) = \frac{\lambda}{4} (h^4 + 4h^3v + 4h^2v^2). \tag{4}
\]
The mass of the higgs scalar can be determined from the above potential and is given by \( m_h^2 = 2\lambda v^2 \). The couplings of the RS radion to the higgs field is therefore given by
\[
S = \int d^4x \left[ \frac{1}{2} \eta^{\mu\nu} \frac{\phi}{\langle \phi \rangle}^2 \partial_\mu h \partial_\nu h - \frac{\lambda}{4} (\frac{\phi}{\langle \phi \rangle})^4 (h^4 + 4h^3v + 4h^2v^2) \right]. \tag{5}
\]
Rescaling \( h \) and \( v \) according to \( h \to \frac{\phi}{\langle \phi \rangle} h \) and \( v \to \frac{\phi}{\langle \phi \rangle} v \) the canonically normalized action becomes
\[
S = \int d^4x \left[ \frac{1}{2} \eta^{\mu\nu} \frac{\phi}{\langle \phi \rangle}^2 \partial_\mu h \partial_\nu h - \frac{\lambda}{4} (\frac{\phi}{\langle \phi \rangle})^4 (h^4 + 4h^3v + 4h^2v^2) \right]. \tag{6}
\]
Expanding \( \phi \) about its vev and keeping terms only up to linear order in \( \hat{\phi} \) we get
\[
S = \int d^4x \left[ \frac{1}{2} \eta^{\mu\nu} \partial_\mu h \partial_\nu h - \frac{\lambda}{4} (h^4 + 4h^3v + 4h^2v^2) \right]
\]
\[
+ \int d^4x \left[ \eta^{\mu\nu} \partial_\mu h \partial_\nu h - \frac{\lambda}{4} (h^4 + 4h^3v + 4h^2v^2) \right] \frac{\hat{\phi}}{\langle \phi \rangle} + .. \tag{7}
\]
Since the trace of the energy momentum tensor of the higgs field is given by
\[
T^\mu_\mu = \eta^{\mu\nu} \partial_\mu h \partial_\nu h - \lambda (h^4 + 4h^3v + 4h^2v^2).
\tag{8}
\]
we find that small fluctuations in the radion field from its vev couples to the higgs field on the visible brane through the trace of its energy-momentum. Using the classical equation of motion \( \partial^2 h = -\lambda(h^3 + 3h^2v^2 + 2hv^2) \) for the higgs field in the above expression for \( T^\mu_\mu \) the higgs-radion interaction can be written as
\[
L_I = -[m_h^2 h^2 + \lambda vh^3] \frac{\hat{\phi}}{\langle \phi \rangle} \tag{9}
\]
The radion coupling to the higgs field therefore has a three point vertex and a four point vertex. We shall find later that the four point vertex actually leads to unitarity violation at high enough energies. We shall now use these couplings to determine the transition amplitude for the process \( hh \to h\phi \) at very high energies i.e. when \( s \gg m_h^2, m_{\phi}^2 \).

Let \( M_1, M_2, M_3 \) denote the transition amplitudes due to s, t, u channel higgs exchanges and \( M_4 \) denote the transition amplitude for due to the four point \( hhh\phi \) vertex. We then find that at very high energies to leading order in \( \frac{m_h^2}{s} \) and \( \frac{m_{\phi}^2}{s} \):

\[
M_1 = \frac{6\lambda v}{\langle \phi \rangle} \frac{m_h^2}{s - m_h^2} \approx \frac{6\lambda v m_h^2}{\langle \phi \rangle} \frac{1}{s}.
\]

\[
M_2 = \frac{6\lambda v}{\langle \phi \rangle} \frac{m_h^2}{t - m_h^2} \approx -\frac{12\lambda v m_h^2}{\langle \phi \rangle} \frac{1}{s} \frac{1}{s} \frac{1}{\alpha - \beta x}.
\]

\[
M_2 = \frac{6\lambda v}{\langle \phi \rangle} \frac{m_h^2}{u - m_h^2} \approx -\frac{12\lambda v m_h^2}{\langle \phi \rangle} \frac{1}{s} \frac{1}{s} \frac{1}{\alpha + \beta x}.
\]

and

\[
M_4 = \frac{\lambda v}{\langle \phi \rangle}.
\]

where \( \alpha = 1 - \frac{m_h^2 + m_{\phi}^2}{s} \) and \( \beta = 1 - \frac{3m_h^2 + m_{\phi}^2}{s} \). \( x = \cos \theta \) where \( \theta \) is the angle between the outgoing h and one of the incoming h. We shall consider energies far above the higgs pole to avoid the strong rise in the transition amplitude near the pole. It is clear that as \( s \to \infty \) the terms that constitute a potential threat to unitarity are the ones that diverge with s or at least remains constant. We find from the above expressions that as \( s \to \infty \), the first three transition amplitudes tend to zero. However \( M_4 \) remains a constant. Although the total transition amplitude has acceptable high energy behaviour, it can lead to unitarity violation for sufficiently large values of \( \lambda \) or small \( \langle \phi \rangle \). In other words either \( m_h \) must be sufficiently small or \( \langle \phi \rangle \) must be large enough to avoid unitarity violation. In order to determine the unitarity bound on \( m_h \) implied by the process \( hh \to h\phi \) we shall follow Lee,
Quigg and Thacker [4] and confine our attention to the J=0 partial wave amplitude. We find that

\[
a_0 \approx \frac{\lambda v}{16\pi\langle\phi\rangle}\left[1 + 6\frac{m_h^2}{s} - 12\frac{m_h^2}{s\beta} \ln \frac{\alpha + \beta}{\alpha - \beta}\right]
\approx \frac{\lambda v}{16\pi\langle\phi\rangle}\left[1 + 6\frac{m_h^2}{s} + 12\frac{m_h^2}{s\beta} \ln \frac{m_h^2}{s}\right]
\approx \frac{\lambda v}{16\pi\langle\phi\rangle},
\]

(11)
as \frac{m_h^2}{s} \to 0. The unitarity constraint \(|a_0| < \frac{1}{2}\) then implies that \(m_h^2 < 16\pi\langle\phi\rangle v\).

In other words the higgs mass must be less than \(4\sqrt{\pi\langle\phi\rangle} v\) so that the J=0 partial wave amplitude does not violate unitarity. For \(\langle\phi\rangle \approx 1\ Tev\) the unitarity bound on \(m_h\) is about 3.5 Tev. We would like to note firstly that in SM the unitarity bound on \(m_h\) that follows from the somewhat analogous process \(hh \to hh\) is given by \(m_h < \sqrt{8\pi\langle\phi\rangle} v \approx 0.7\ Tev\) which is somewhat smaller than the value obtained in this report. Secondly although the processes \(W^+_L W^-_L \to W^+_L W^-_L, W^+_L W^-_L \to Z_L Z_L, W^+_L W^-_L \to hh\) and \(Z_L Z_L \to hh\) have acceptable high energy behaviour in the SM model they exhibit bad high energy behaviour in the context of the Randall-Sundrum scenario. The bad high energy behaviour arises from the diagram that involves the exchange of radion. These processes however also receive contributions from the s channel exchange of Kaluza-Klein gravitons which could give rise to transition amplitudes with much worse high energy behaviour. On the other hand the process \(hh \to hh\) exhibits acceptable high energy behaviour both in the SM and the RS scenario if we neglect the contribution of KK gravitons. The s wave transition amplitude for the latter process however depends both on \(m_h\) and \(m_\phi\) in the RS scenario. The unitarity constraint \(|a_0| < \frac{1}{2}\) therefore gives a bound on \(m_h\) that depends on \(m_\phi\). This explains the reason why we chose the process \(hh \to h\phi\) which gives a unitarity bound on \(m_h\) that is independent of \(m_\phi\).

The four point vertex \(hhh\phi\) in the higgs-radion interaction Lagrangian is crucial for the unitarity bound discussed in this paper. The presence of this term in \(T^\mu_\mu\) is easily
understood. The electro-weak symmetry breaking also breaks the conformal symmetry. The conformal symmetry is broken not only by the higgs mass term in the potential but also the trilinear higgs interaction term. The unitarity violation discussed in this paper signals that the higgs-radion coupling becomes strong and non-perturbative as $\lambda$ approaches its unitarity limit value. Therefore higher order radiative corrections and non-perturbative effects become important and they could significantly change the unitarity bound presented here. The estimation of such non-perturbative corrections although important will not be considered in this paper.

References

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4. B. W. Lee, C. Quigg and H. B. Thacker, Phys. Rev. D 16 282 (1977).