Abstract. Fragmentation functions (FFs) describe the formation of final state hadrons from final state quarks or gluons. Precise knowledge of these functions is a key ingredient in extracting spin-dependent parton distribution functions from spin asymmetries observed in polarized Semi-Inclusive Deep Inelastic Scattering (SIDIS) and in inclusive hadron production in polarized proton-proton collisions. However, fragmentation functions can not be determined from first principles Quantum Chromodynamics (QCD) and have to be extracted from experimental data. The Belle experiment at KEK in Japan has acquired large data samples from hadron production in $e^+e^-$ annihilation. Precision measurements of identified hadron yields from Belle can be used to improve the precision of the presently available hadron fragmentation data. In this talk Belle results on spin-dependent fragmentation functions will be summarized and the status of the spin-averaged hadron yield analysis will be presented.

1. Precision Measurement of Multiplicities for Identified Hadrons

1.1. Motivation and Outline

Unpolarized hadron fragmentation functions are used in the QCD analysis of polarized Semi-Inclusive Deep Inelastic Scattering (SIDIS) and polarized proton-proton scattering (pp) experiments. The goal of the QCD analysis of SIDIS and pp data is the extraction of quark and gluon helicity distributions. Two recent studies have extracted hadron fragmentation functions from the available world data from hadron production in $e^+e^-$ annihilation [1], [2], SIDIS and pp [2]. In both studies the authors point out the importance of a future precision data sample from $e^+e^-$ annihilation at low center of mass compared to the existing data sets from the Large Electron-Positron Collider (LEP) at CERN. This data set will in particular improve the knowledge of the gluon fragmentation functions. A precision measurement to obtain said data sets is presently carried out for charge-resolved pion and kaon multiplicities in $e^+e^-$ annihilation at a center of mass energy of 10.52 GeV at the Belle experiment at KEK, Japan. The status of the analysis is presented in this paper.

The multiplicities are measured in dependence of $z = 2E_{had}/\sqrt{s} = E_{had}/E_{parton}$, the hadron energy relative to half of the center-of-mass energy. The measured hadrons are produced in reactions $e^+e^- \rightarrow q\bar{q}$, where $q = \{u, d, s, c\}$. The measured distributions are corrected for systematic effects, among which particle misidentification and acceptance effects are found to be most significant.

1.2. Real-Data-Based Corrections to the Particle Identification in Belle

For identified charged hadrons, measured particle yields need to be corrected for particle misidentification. For charged hadron yields in Belle this corrections range between 5 and 10%
PID detectors to select pions, Figures a) and b) show distributions without and with PID likelihood cuts from the Belle.

The ratio of the hatched areas yields \( p_{(K^- \rightarrow \pi^-)} = 0.111 \pm 0.004 \) for negatively charged kaons with laboratory frame momentum \( 1.4 < p_{\text{lab}} < 1.6 \text{ GeV/c} \) and laboratory frame azimuthal angle \( 77.9 < \theta_{\text{lab}} < 89.0 \) degrees.

for pions and 10 to 35% for kaons depending on hadron momentum. The correction is performed through an unfolding technique based on inverse 5x5 particle identification (PID) probability matrices. The PID probability matrices were obtained from analyzing decays of resonances in which the identity of the decay products was obtained through purely kinematic means. Forming samples of tracks with known PID makes it possible to determine PID probabilities, \( p_{(i \rightarrow j)} \) by imposing cuts based on the PID detectors of the Belle detector on the tracks in the samples. Complete PID probabilities where obtained from data for particles \( i = \{e, \mu, \pi, K, p\} \). This data-driven PID correction method avoids the dependence on the modeling of the Belle PID detectors in GEANT [3]. Decays of \( D^*, \Lambda \) and \( J/\psi \) particles have been analyzed. Figure 1 shows fits of experimental data distributions from the reconstructed decay \( D^* \rightarrow \pi + D^0 \rightarrow \pi + (K\pi) \) which allow to extract the PID probability \( p_{(K^- \rightarrow \pi^-)} \).

With the method of kinematically reconstructing unstable particles, a good understanding of the Belle PID performance over a wide kinematic range is obtained. For regions not accessible to this method, an extrapolation algorithm is used to dispose of PID information for all of the kinematic area of interest. Figure 2 shows examples of PID probabilities for selected kinematic areas extracted from the studied decay \( D^* \rightarrow \pi + D^0 \rightarrow \pi + (K\pi) \).

The PID probability matrices are inverted and then applied to the measured yield vector \( \overrightarrow{N}_{\text{meas}} = (N_e, N_\mu, N_\pi, N_K, N_p) \) to obtain the PID-corrected yield vector \( \overrightarrow{N}_{\text{PID-corr}} \). All uncertainties of the extracted PID probabilities are propagated through the inversion process by using a Monte Carlo technique and are assigned to the PID-corrected yields as systematic uncertainties.

Figure 3 a) shows a selection of the presently available world data for unidentified charged hadron yields at different center-of-mass energies ranging from the TASSO data at \( \sqrt{s} = 12 \text{ GeV} \) to the LEP data up to \( \sqrt{s} = 202 \text{ GeV} \). Above \( z = 0.6 \) the existing data generally have wide \( z \)-bins and large uncertainties. Figure 3 b) shows relative systematic and statistical uncertainties for charged pion and kaon yields from Belle after performing the PID corrections; the systematic uncertainties include uncertainties from the PID corrections.
Figure 2. Particle ID probabilities against $p_{\text{lab}}$ for $0.4 < \cos \theta_{\text{lab}} < 0.5$. The PID probabilities have been extracted from the decay $D^* \rightarrow \pi + D^0 \rightarrow \pi + (K\pi)$, Panel a) lists the PID probabilities extracted and panel b) shows the obtained values for the particle identification probabilities $p_{(\pi \rightarrow j^-)}$, where $i = \{e, \mu, \pi, K, p\}$.

Figure 3. a) Compilation of unidentified charged particle multiplicities from $e^+e^-$ annihilation processes measured by different experiments at different center-of-mass energies. For the purpose of plotting, the distributions have been scaled by $c(\sqrt{s}) = 10^i$, where $i$ ranges from $i = 0$ ($\sqrt{s} = 12$ GeV) to $i = 13$ ($\sqrt{s} = 202$ GeV). The plot has been adapted from reference [4]. b) Expected systematic and statistical uncertainties after PID-correction for the Belle analysis. Projected uncertainties are shown for charge-separated pion and kaon multiplicities.
1.3. Outlook
The measurement of identified hadron multiplicities in Belle versus normalized energy $z$ will cover the range 0.2 < $z$ < 1.0. Studies of systematic uncertainties in the PID, the momentum smearing and from acceptance effects are in progress. The leading uncertainty is expected to arise from systematic uncertainties connected with the PID correction. The overall systematic uncertainties are expected to remain below 3% (5%) for $\pi$ ($K$) spectra for $z < 0.6$, and to increase with $z$ to 10% (17%) for $\pi$ ($K$) spectra through $z \sim 0.9$, respectively.

2. Measurement of the Collins fragmentation function for charged hadrons at Belle
Spin-dependent, chiral-odd fragmentation functions can be used to extract transverse spin quark distributions (so called transversity distributions) in the nucleon from polarized SIDIS and $pp$ scattering experiments. One candidate for such a chiral-odd function is the Collins fragmentation function. Introduced by Collins [8], the Collins FF describes the fragmentation of a quark with transverse spin into an unpolarized hadron. The spin orientation of the quark translates into an experimentally accessible azimuthal dependence of the hadron transverse momentum with respect to the jet axis. In Belle, reactions $e^+e^- \rightarrow q\bar{q} \rightarrow (h)(h)X$ have been studied by identifying one charged hadron in each hemisphere of a two-jet event. The electron and positron beams at Belle are unpolarized and the initial quark spin direction is unknown. Thus the azimuthal modulations average out to zero in the sum over events and Collins measurements of azimuthal distributions of hadrons in one jet hemisphere alone would yield a zero result. However, a correlation measurement between back-to-back hadrons in opposite jet hemisphere is sensitive to the product of the Collins effect in the two hemispheres.

Azimuthal Collins correlations have been measured in Belle for $\pi^+\pi^-$ pairs. The yield of two pions from opposite jet hemispheres $N(\phi_1 + \phi_2)$ has been analyzed versus the sum of the pion angles, $\phi_1 + \phi_2$. The angles $\phi_i$, $i = 1, 2$, are taken between the transverse momentum of the hadron with respect to the jet axis and the event plane defined by the beam axis and the jet axis. Schematically the Collins effect is accessible through the amplitude of the modulation $a_{12}$ in the di-hadron yield: $N(\phi_1 + \phi_2) \sim a_{12}\cos(\phi_1 + \phi_2)$, $a_{12} \sim H_1^+(z_1)H_1^+(z_2)$.

However, the presence of QCD radiative effects in the $\cos(\phi_1 + \phi + 2)$ moment and the presence of acceptance effects makes it impossible to access the Collins asymmetry in $e^+e^-$ directly through $a_{12}$. Instead measurements of ratios $A_{12} = a_{12}^{\pi^+\pi^-} / a_{12}^{(\pi^+,\pi^-),(\pi^-,\pi^-),(\pi^+,\pi^-)}$ were found to cancel acceptance and first order QCD radiative effects while retaining sensitivity to the Collins effect.

Results for $A_{12}$ for two-pion-correlations from a dataset of 547 $fb^{-1}$ taken at center-of-mass energies of 10.52 and 10.58 GeV have been published [5]. [6]. As can be seen in Figure 4 a), large significant modulations have been found, indicating significant spin effects in $e^+e^-$ fragmentation. These results have already been used in the first extraction of transversity distributions [7].

3. Measurement of the interference fragmentation function for charged hadrons at Belle
Similar to the Collins fragmentation function, the interference fragmentation function (IFF) is a chiral-odd function which can be used to access transversity distributions in the nucleon in SIDIS and proton-proton measurements. At Belle, the product of two IFF can be measured by identifying one hadron pair in either hemisphere in a two-jet event $e^+e^- \rightarrow q\bar{q} \rightarrow (hh)(hh)X$. First proposed by Collins, Heppelman and Ladinsky [10], the IFF describes the fragmentation of a polarized quark into two hadrons correlated via partial wave interference. The corresponding measurement observable is an azimuthal dependence in the hadron pair production cross-section.

The IFF does not depend on transverse momenta and therefore its factorization and evolution can be described in a collinear approach. This makes the IFF an attractive alternative compared to the Collins fragmentation function for the extraction of parton transversity distributions.
to the average hadron pair yield and fitted with azimuthal modulations. Different from the axis approximated by the thrust axis are extracted. The raw azimuthal yields are normalized interaction plane defined by the incoming lepton momenta and the quark-antiquark momentum Similar to the Collins analysis, an IFF measurement requires a correlation measurement between two fragmenting partons in the two hemisphere, where

\[ a_{12}(z) = \frac{E_{\text{pair}}}{E_{\text{parton}}} \]

represents the fractional energy of the hadron pair and \( m \) its invariant mass. Preliminary results for charged pion pairs obtained from a dataset of 672 fb\(^{-1}\) containing 711 \( 10^6 \) \( \pi^+\pi^- \) pairs collected at the \( \Upsilon(4S) \) resonance have already been shown at the Dubna Spin workshop 2009 [9]. It can be seen in Figure 4 b) that azimuthal modulations rise with the fractional energy of the hadron pair significantly up to 10% for highest \( z \). Publication of the final results as well as the analysis of other hadron combinations are in progress.

**References**

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