ELECTRON ACCELERATIONS AT HIGH MACH NUMBER SHOCKS: TWO-DIMENSIONAL PARTICLE-IN-CELL SIMULATIONS IN VARIOUS PARAMETER REGIMES

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ABSTRACT

Electron accelerations at high Mach number collisionless shocks are investigated by means of two-dimensional electromagnetic particle-in-cell simulations with various Alfvén Mach numbers, ion-to-electron mass ratios, and the upstream electron $\beta_e$ (the ratio of the thermal pressure to the magnetic pressure). We find electrons are effectively accelerated at a super-high Mach number shock ($M_A \sim 30$) with a mass ratio of $M/m = 100$ and $\beta_e = 0.5$. The electron shock surfing acceleration is an effective mechanism for accelerating the particles toward the relativistic regime even in two dimensions with a large mass ratio. Buneman instability excited at the leading edge of the foot in the super-high Mach number shock results in a coherent electrostatic potential structure. While multi-dimensionality allows the electrons to escape from the trapping region, they can interact with the strong electrostatic field several times. Simulation runs in various parameter regimes indicate that the electron shock surfing acceleration is an effective mechanism for producing relativistic particles in extremely high Mach number shocks in supernova remnants, provided that the upstream electron temperature is reasonably low.

Key words: acceleration of particles – cosmic rays – plasmas – shock waves

1. INTRODUCTION

The origin of cosmic rays has been a longstanding issue in astrophysics. Rapid progress in understanding their accelerators has been made by observations of radio and X-ray synchrotron emissions from the shell of supernova remnants (SNRs; Koyama et al. 1995; Winkler & Long 1997; Bamba et al. 2003; Long et al. 2003). Those observations have indicated that electrons are strongly accelerated up to TeV energies out of the thermal population in the vicinity of non-relativistic collisionless shocks.

Diffusive shock acceleration (DSA) is a widely accepted theory responsible for particle acceleration at shocks, which nicely predicts a power-law energy spectrum (e.g., Blandford & Eichler 1987). This theory has achieved many successes in explaining energized ions, for example, at Earth’s bow shock (Scholer et al. 1980). However, it cannot be straightforwardly applied to electron accelerations at SNRs: In the DSA process, particles are scattered by MHD waves back and forth across the shock. The process is thus unlikely to occur for thermal electrons whose gyroradius is much smaller than the wavelength of the MHD waves or the shock width (ion inertia length). They are strongly tied to the magnetic field line and convect downstream adiabatically. This small-scale property of the electrons makes it challenging to explain the electron accelerations by DSA at SNR shocks. A pre-acceleration mechanism for the electrons that enables DSA to work subsequently has been proposed for the last decade in order to overcome this difficulty, which is now known as the injection problem.

In supercritical quasi-perpendicular shocks ($M_A > 2.7$, where $M_A$ is the Alfvén Mach number), it has been pointed out that plasma microscopic instabilities play important roles in energizing electrons (Papadopoulos 1988; Cargill & Papadopoulos 1988; Scholer et al. 2003; Scholer & Matsukiyo 2004). In the particular case of high Mach number shocks, electrostatic instabilities arise due to velocity difference between the reflected ions and the incident (upstream) ions and electrons (Papadopoulos 1988; Cargill & Papadopoulos 1988). Buneman instability is initially excited by the reflected ion beam interacting with the upstream electrons. The instability strongly heats the electrons perpendicular to the magnetic field, which then gives a favorable condition for destabilizing the ion-acoustic instability in the foot region. Particle-in-cell (PIC) simulations, which follow ion and electron motions along with electromagnetic field development in a self-consistent manner, have confirmed these ideas. Furthermore, rapid acceleration of the electrons by the convective electric field accompanied with particle trapping by the Buneman instability was found to contribute to non-thermal electron production (Shimada & Hoshino 2000; McCombs et al. 2001; Hoshino & Shimada 2002; Shimada et al. 2010). This electron shock surfing acceleration has been considered to be a very efficient mechanism at extremely high Mach number shocks such as SNR shocks that inject thermal electrons to DSA.

It is only recently that evolutions of non-relativistic shocks have been investigated by means of two-dimensional PIC simulations (Umeda et al. 2009; Amano & Hoshino 2009a; Lembège et al. 2009; Kato & Takabe 2010; Riquelme & Spitkovsky 2011). The efficiency of the electron shock surfing acceleration in multi-dimensions has been discussed in different physical parameter regimes, such as the ion-to-electron mass ratio, Alfvén Mach number, upstream electron $\beta_e$ (the ratio of the thermal pressure to the magnetic pressure), and magnetic field configurations; the two-dimensional PIC simulation still requires large computational resources and only allows limited simulation runs. Consequently, the results lead to different conclusions. The simulation study with a relatively low-mass ratio ($M/m = 25$) concluded that the shock surfing acceleration is an effective mechanism even in two dimensions in a high ($M_A = 14$) Alfvén Mach number shock (Amano & Hoshino 2009a). Kato & Takabe (2010) examined an extremely high Alfvén Mach number ($M_A = 130$) shock with a high upstream electron $\beta_e$ condition. In their simulation result, the downstream electron energy spectrum was almost Maxwellian. They attributed the lack of the non-thermal population to the in-plane magnetic field configuration which was different from...
the out-of-plane configuration adopted by Amano & Hoshino (2009a). Riquelme & Spitkovsky (2011) conducted simulation runs with various mass ratios and concluded that the Buneman instability is suppressed in a large mass ratio case \((M/m = 400)\) under the condition of \(M_A = 7\), and the electron shock surfing acceleration will not be realized at SNR shocks. Those contrary conclusions regarding the electron shock surfing acceleration at SNRs are mainly due to the limited choices of the mass ratio, Alfvén Mach number, and the upstream electron \(\beta_e\), which should be carefully chosen based on the theoretical estimations (Papadopoulos 1988; Matsukio 2010).

In this paper, we present two-dimensional electromagnetic PIC simulations of collisionless shocks with various mass ratios, Alfvén Mach numbers, and the upstream electron \(\beta_e\) conditions in order to clarify the efficiency of the electron shock surfing acceleration at SNR shocks. The parameters are carefully chosen in accordance with the linear and nonlinear theories of the Buneman instability in the foot region, which are reviewed in Section 2. In Section 3, the numerical setup and our two-dimensional PIC code are described. In Section 4, we first overview a simulation result in which the maximum energy gain of the electrons is observed. Electron acceleration processes are analyzed in detail. Then, dependence of the acceleration mechanisms on the physical parameters is discussed. The results are summarized and discussed in the last section.

2. UNSTABLE CONDITION AND THE SATURATION LEVEL OF THE BUNEMAN INSTABILITY IN THE FOOT REGION

The electron shock surfing acceleration is basically supported by the electrostatic field excited by the Buneman instability at the leading edge of the foot region. The free energy of the instability is the relative drift motion between the incoming electrons and the reflected ions. The region becomes unstable when the relative speed exceeds the thermal speed of the electron, \(\Delta V/v_{te} > 1\). In applying the unstable condition to the foot region, the velocities of the incoming ion and electrons and the reflected ions are expressed as

\[
V_i = V_0, \quad V_r = -V_0, \quad V_e = \frac{1 - \alpha}{1 + \alpha} V_0,
\]

by assuming the zero net current condition

\[
n_0 V_i + n_r V_r - (n_0 + n_r) V_e = 0,
\]

where \(n_0\) is the upstream number density, \(n_r\) is the number density of the reflected ion, \(V_0\) is the upstream speed, \(V_i\) is the incoming ion speed, \(V_r\) is the incoming electron speed, and \(V_e\) is the reflected ion speed. \(\alpha = n_r/n_0\) is the density ratio of the reflected and incoming ions, which is typically \(\sim 0.2\) for high Mach number shocks (Papadopoulos 1988; Leroy et al. 1981, 1982; Leroy 1983). Using \(\Delta V = V_e - V_r\), the unstable condition reads

\[
M_A \geq \frac{1 + \alpha}{2} \sqrt{\beta_e (\frac{M}{m})^\frac{2}{3}},
\]

where \(M_A\) is the Alfvén Mach number, \(M\) and \(m\) are the ion and electron rest mass, and \(\beta_e\) is the ratio of the electron pressure to the magnetic pressure (plasma beta).

Given the Buneman instability being excited, the electrons are accelerated by the convective electric field while they are trapped in the electrostatic potential well. The maximum energy gain is determined by force balance between the trapping force by the saturated Buneman instability \((q E_{BI})\) and the Lorentz force \((qvB/c)\) for escaping. To be accelerated toward relativistic speeds \((v \sim c)\), the electric force must be greater than the Lorentz force (Hoshino & Shimada 2002):

\[
\frac{E_{BI}}{B_0} \geq 1.
\]

The saturation level of the Buneman instability \((E_{BI})\) can be estimated by introducing an energy conversion rate \(C\) of the electron drift energy density as

\[
\frac{E_{BI}^2}{8\pi} = \frac{1}{2} mn_0 \Delta V^2 C.
\]

The conversion rate \(C\) has been analyzed by the nonlinear theory of the Buneman instability (Ishihara et al. 1980) and found to depend weakly on the mass ratio \(C \sim (m/M)^{1/3}\). In multi-dimensions, the saturation level is reduced to \(\sim 25\%\) of the above estimate due to the resonant wave–particle interactions with waves oblique to the beam direction (Amano & Hoshino 2009b). Using \(\Delta V = V_e - V_r\) and Equation (7), and taking into account the multi-dimensionality, Equation (6) leads to

\[
M_A \geq (1 + \alpha) \left(\frac{M}{m}\right)^\frac{2}{3}.
\]

Equation (5) indicates that the electron shock surfing acceleration occurs at a sufficiently high Alfvén Mach number shock in a low beta plasma. Furthermore, for non-thermal relativistic electrons to be observed, Equation (8) shows that the mass ratio restricts the Alfvén Mach number more severely with a power of 2/3 rather than 1/2 in the unstable condition, indicating a need for numerical simulations of super-high Alfvén Mach number shocks with larger mass ratios, for instance, \(M_A > 26\) for \(M/m = 100\). In other words, simulations with artificial small mass ratios would easily give an unrealistic electron acceleration in relatively low Alfvén Mach number shocks.

3. NUMERICAL SETUP

Two-dimensional electromagnetic PIC simulation code is used for examining shock evolutions. The code is parallelized via domain decomposition using the Message Passing Interface library and OpenMP and runs efficiently on massively parallel supercomputer systems.

Particles are continuously injected from one boundary \((x = 0)\) with a speed \(V_0\) toward the other end \((x = L_x)\) where the particles are reflected (injection method), resulting in the shock formation that propagates toward \(x = 0\). Number density in the upstream is 40 particles per cell for each species (ion and electron). The injected plasma carries the \(z\) component of the magnetic field \((B_0)\) and the convective electric field \(E_z = E_0 = V_0 B_0\). Thus, we deal with purely perpendicular shocks in the downstream rest frame. The periodic boundary condition is applied in the \(y\)-direction. Simulation box sizes in the \(x\)- and \(y\)-directions are \(L_x = 12 V_0/\Omega_{pi} = 12 M_A \lambda_i\) and \(L_y = 5 \lambda_i\), respectively, where \(M_A = V_0/\Omega_{ci}\), \(\Omega_{ci} = q B_0/M e\) is the ion gyro frequency, \(\lambda_i = c/\omega_{pi} = c\sqrt{m/4\pi n_0 q^2}\) is the ion inertia length, \(V_A = B_0/\sqrt{4\pi M n_0}\) is the Alfvén speed, and
$n_0$ is the number density in the upstream region (40 particles per cell). The notations $c$ and $q$, respectively, represent the speed of light and elementary charge. The grid size $\Delta h$ and the time step size $\Delta t$ are set as $\Delta h = \lambda_D$ and $\omega_{pe}\Delta t = 0.025$, where $\lambda_D = \sqrt{T_e/4\pi n_0 q^2}$ is the Debye length in the upstream region, $T_e$ is the electron temperature, and $\omega_{pe} = \sqrt{4\pi n_0 q^2/m}$ is the electron plasma frequency. In the following, we use the units of $\Omega_{ce}^{-1}$, $c/\omega_{pi}$, $n_0$, $B_0$, $c$, and $mc^2$ for time, space, number density, electromagnetic fields, velocity, and energy, respectively, unless otherwise stated.

We carry out several runs with various ion-to-electron mass ratios ($M/m$), the Alfvén Mach numbers ($M_A = V_0/V_A$), and the electron plasma $\beta_e = 8\pi n_0 T_e/B_0^2$, where the ratio of the electron plasma to gyro frequencies ($\omega_{pe}/\Omega_{ce}$) and the temperature ratio of the ion to the electron ($T_i/T_e$) are fixed to $\omega_{pe}/\Omega_{ce} = 10.0$ and $T_i/T_e = 1$, respectively. Two simulation runs use the mass ratios of 25 and 100 for fixed values of $M_A = 10$ (respectively, $M_A \sim 14$ and 16 in the shock rest frame) and $\beta_e = 0.5$ in order to examine the mass ratio dependence of the electron acceleration (Runs A and B). Parameters in Run A are the same as those used by Amano & Hoshino (2009a). Another simulation run uses $M/m = 100$ with $M_A = 20$ ($M_A \sim 30$) for comparison with the lower Alfvén number cases (Run C). We also examine a hot electron case with $\beta_e = 4.5$ for comparison with the high Alfvén Mach number and a lower electron temperature case (Run D). We examine in total four simulation runs (Runs A–D) with various $M_A$, $M/m$, and $\beta_e$ to assess the importance of these parameters to electron acceleration mechanisms. The upstream parameters of the four simulation runs are summarized in Table 1. The largest computational resources are used in Run C, in which $5 \times 10^9$ particles are followed in a simulation domain with $24001 \times 1024$ grid points.

| Table 1 | Upstream Conditions of Simulation Runs and Resultant Electron Accelerations |
|---------|----------------------------------|
| $M_A$  | $M/m$  | $\beta_e$ | $\omega_{pe}/\Omega_{ce}$ | Equation (5) | Equation (8) | Ele. Accel. |
| Run A  | 14.4  | 25  | 0.5  | 10 | 2.1 | 10.3 | BI |
| Run B  | 16.2  | 100 | 0.5  | 10 | 4.2 | 25.8 | Weak |
| Run C  | 30.0  | 100 | 0.5  | 10 | 4.2 | 25.8 | BI + adiabatic |
| Run D  | 30.0  | 100 | 4.5  | 10 | 12.7 | 25.8 | Weak |

Figure 1(b) shows the phase space plot for ions. The ions in the foot consist of the incoming cold population and the relatively hot ion moving toward the upstream region, which is specularly reflected by the shock potential at $X \sim 74 c/\omega_{pi}$. At the leading edge of the reflected ion, electrons are preferentially accelerated immediately after entering the foot as seen in the energy–space diagram in Figure 1(c). Just behind the region around $X \sim 63 c/\omega_{pi}$, the maximum energy of the electrons increased to $\gamma \sim 2$. They are then further heated in $X > 66.5 c/\omega_{pi}$ in the foot. In the vicinity of the overshoot, the electron energy again increased up to $\gamma \sim 9$.

Zoomed-in spatial profiles surrounded by the white dashed line in Figure 1(a) are shown in Figures 1(d) and (e). At the edge of the foot, we can identify fine scale structures of the electron number density in Figure 1(d), whose spatial size is much smaller than the ion inertia scale, i.e., the electron scale that is embedded in the MHD scale shock structure. This fine structure is accompanied by electrostatic fields. Figure 1(e) shows the electrostatic field strength $|E_{es}| = |−\nabla \phi|$ given by solving the Poisson equation of $\nabla^2 \phi = −V \cdot E$. The amplitude reaches twice as large as the background magnetic field ($B_0$), which is much larger than the upstream convective electric field $|E_{es}/B_0| = V_0/c = 0.2$. Coherent wave trains lie in the $x$-$y$ plane rather than exhibiting two-dimensional isolated structures (Amano & Hoshino 2009a). This strong electrostatic field at the edge of the foot is caused by a linear instability between the incoming electron and the reflected ion, which one usually refers to as Buneman instability. The Buneman instability heats the electrons perpendicular to the magnetic field and sets a favorable condition for the ion-acoustic instability, which further energizes the electrons in $X > 66.5 c/\omega_{pi}$ in the foot.

In order to identify the instability in the two-dimensional plane as the Buneman instability, we focus on the closed-up region of interest in Figures 1(d) and (e). Figure 2(a) shows the zoomed-in profile of the electrostatic potential energy $q\phi$. In this region, the cold upstream and the reflected gyrating ions constitute the distribution function shown in Figure 2(b). The number of reflected ions with respect to the upstream population is $n_r/n_0 = 0.22$ as expected in previous high Mach number shock simulations (Leroy et al. 1982; Leroy 1983).

The relative drift speed between the two populations is $\Delta V \sim 0.4 c$. This velocity difference is much larger than the upstream electron thermal speed of $0.07 c$, thus satisfying the unstable condition of the Buneman instability (Equation (5)). Figure 2(c) shows the Fourier amplitude of the electrostatic potential of Figure 2(a) in the $k_x$–$k_y$ space in which a Hann window is applied in the $x$-direction. One can identify a strong peak appearing around $(k_x, k_y) = (0.6, 0.3) \omega_{pe}/\Delta V$. This peak corresponds to the coherent wavy structure that obliquely lies in the $x$–$y$ plane, and $|k| = \sqrt{k_x^2 + k_y^2} = 0.7 \omega_{pe}/\Delta V$ agrees with the fastest growing mode at $k_x, k_y, \Delta V/\Omega_{pe} \sim 1$ predicted by the linear analysis of the Buneman instability (Amano & Hoshino 2009b).

4. SIMULATION RESULTS

4.1. Electron Acceleration in Run C

4.1.1. Overview of 2D Shock Structure

We first present an overview of a two-dimensional perpendicular shock structure in Run C in which the maximum energy gain of the electron was observed. In the following results, the data at a time of $\Omega_{ce} T = 6$ are used, after which the energy spectrum of the electron in the downstream region is not significantly changed.

Figure 1(a) shows a two-dimensional spatial profile of the $z$ component of the magnetic field. The structure is characterized by the foot region, which extends from $X = 63 c/\omega_{pi}$ to the ramp and the overshoot around $X = 74 c/\omega_{pi}$, and the downstream region in $X > 74 c/\omega_{pi}$. The width of the foot (the shock transition region) is well scaled by the gyroradius of the reflected ion ($L \sim 0.7 V_0/\Omega_{ce}$) (Leroy 1983; Shimada et al. 2010).

In Run C, a high energy tail of the electron energy distribution function which extends to $\gamma \sim 9$ is observed. Figure 3 shows the energy spectra of the electron in the foot and downstream regions. Particles are sampled in regions $64 c/\omega_{pi} \leq X \leq 71 c/\omega_{pi}$ for the foot spectrum and $74 c/\omega_{pi} \leq X \leq 88 c/\omega_{pi}$ for the downstream spectrum. In the foot region, non-thermal...
electrons are already produced. The energy distribution consists of the cold, non-accelerated, and accelerated populations. The highest energy in this region is $\gamma \sim 3.5$. In the downstream region, a clear two-component distribution is found. One is the thermal electrons whose temperature is $T_e/mc^2 = 0.13$, and the other consists of accelerated particles whose energy reaches $\gamma \sim 9$. Acceleration mechanisms responsible for these non-thermal electrons are found in two ways as described in the following.

First, the electrons are accelerated at the edge of the foot by the shock surfing acceleration as manifested in Figures 1 and 2. When a particle enters the unstable region of the Buneman instability, it is accelerated by the convective electric field while being captured by the electrostatic potential. The present two-dimensional simulation shows in Figure 4(a) that a sampled electron can escape from the potential well after gaining a fraction of energy and drift downstream. Since the length of the potential well is limited to the order of the ion inertia length, the particle can escape from the well even though the electric field strength is sufficiently large. This is the multi-dimensionality effect. Nevertheless, it can enter the unstable region from the downstream side again since the gyroradius of the accelerated particle becomes larger than before. Then, the second surfing acceleration is realized. In the trajectory in Figure 4(b), the energy gain is more effective than in the previous encounter, and the energy reaches $\gamma \sim 2$. The third encounter with the unstable region is possible in the same manner, and the energy increases to $\gamma \sim 3$ (Figure 4(c)). Accelerated particles typically experience shock surfing acceleration three times, as shown in this example. The present acceleration mechanism can be applied to lucky particles that will constitute the non-thermal population, and most of the particles that will constitute the thermal population drift toward the downstream without being captured, as shown with cyan lines in Figure 4. Nevertheless,
Figure 2. (a) Zoomed-in spatial profile of the electrostatic potential energy. The color code is normalized by the upstream kinetic energy. (b) Velocity distribution function of the ions sampled in the region in panel (a). (c) Fourier power spectrum of the electrostatic potential in two dimensions. Wave number in each axis ($k_x$ and $k_y$) is normalized by $\frac{\Delta V_0}{\omega_{pe}}$, where $\Delta V_0 = 0.4c$. (A color version of this figure is available in the online journal.)

Figure 3. Energy spectra of the electron in the foot (gray) and downstream (black) regions obtained in Run C. The dashed line is a fitted Maxwell distribution to the downstream spectrum.

shock surfing acceleration is so effective that it can energize the electrons from $\gamma \sim 1$ to the relativistic regime of $\gamma \sim 3$ within a timescale of $\approx 20\ \Omega_{ge}^{-1}$, or $\approx 0.2\ \Omega_{ge}^{-1}$ in this particular case with $M/m = 100$.

Time histories of the energy and the magnetic moment (the first adiabatic invariant) normalized by $U_0^2/2B_0 = V_0^2/2(1-V_0^2/c^2)B_0$ are shown in Figure 5. The energy history of the accelerated particle is threefold, corresponding to Figures 4(a)–(c). The maximum energy gain is found at $\Omega_{ge}T \sim 517$ when $\gamma - 1$ is rapidly increased from 0.05 to 1.0 within a timescale of $5\ \Omega_{ge}^{-1}$ (Figure 5(a)). Since the shock surfing acceleration is due to the direct acceleration by the DC electric field, the magnetic moment also rapidly increases as the energy is increased (Figure 5(b)). After the third surfing acceleration at $\Omega_{ge}T \sim 530$, the energy and the magnetic moment are kept constant until $\Omega_{ge}T = 550$. On the other hand, the non-accelerated particle remains at the initial energy level while the normalized magnetic moment fluctuates around one; neither obvious non-adiabatic nor adiabatic accelerations work on this particle as it travels in the foot region.

The maximum energy achieved by the surfing acceleration at the edge of the foot is up to $\gamma \sim 3$, which is not as sufficiently large as the maximum energy of $\gamma \sim 9$ in the downstream (see also Figure 9). An additional acceleration process is also found as the particle approaches the shock surface. Figure 6(a) shows a trajectory of the accelerated electron in Figure 4 at $\Omega_{ge}T = 580$ after the first acceleration process. The particle undergoes the grad-B drift in the $-y$-direction while being deflected at the overshoot. Figure 6(b) shows that the acceleration starts when the particle encountered with the shock surface at $\Omega_{ge}T \sim 560$. After this time, the energy variation exhibits a sawtooth pattern rather than a sinusoidal one in $\Omega_{ge}T = 540$–560; the energy increases rapidly and decays gradually. During the acceleration, the magnetic moment stays at a certain level on average, but it is highly oscillatory (Figure 6(c)). Figure 6(d) shows the running average of the magnetic field at the particle’s position. The
Figure 4. Trajectories of accelerated (red) and non-accelerated (cyan) electrons superposed on the two-dimensional profile of the electrostatic field strength. The open square locates the position of the particle at each time step following the trajectory back for $15 \Omega_\gamma^{-1}$. A number located beside the open square denotes $\gamma - 1$. Snapshots are taken at times of (a) $\Omega_\gamma T = 513.5$, (b) $\Omega_\gamma T = 522.5$, and (c) $\Omega_\gamma T = 532.5$, respectively.

An animation and a color version of this figure are available in the online journal.

particle experiences larger magnetic fields as it is energized in time. These overall signatures in Figures 6(b)–(d) indicate that the acceleration at the shock surface is basically an adiabatic process.

Figures 7(a) and (b) show the magnetic and electric fields’ profiles with respect to the overshoot position averaged in the $y$-direction (solid lines). The region in the vicinity of the overshoot is enlarged. The electric field is calculated in the shock rest frame based on the electron frozen-in condition (the generalized Ohm’s law including the Hall term) as

$$E_y'(x) = E_y(x) - V_{sh} B_z(x),$$

where $V_{sh}$ is the electron convective velocity in the $x$-direction at the overshoot position. The amplitude of the magnetic field at the overshoot becomes 20 times as large as the upstream value within a scale of the ion inertia length, giving rise to the fast grad-$B$ drift speed of the relativistic electron. Since the direction of the convective electric field is opposite to the drift motion, the electron can gain energy as it travels in the $y$-direction. This is similar to the typical (ion) shock drift acceleration, but for the relativistic electron (Armstrong et al. 1985; Krauss-Varban et al. 1989). Furthermore, the electron is accelerated by the strong negative $E_y$ in the ramp ($X = -0.1 c/\omega_{pe}$) unlike the typical shock drift acceleration. This electric field is induced by time variations of the shock structure, that is, the
Figure 5. Time histories of (a) the energy ($\gamma - 1$) and (b) the magnetic moment. Solid and dashed lines depict the accelerated and non-accelerated particles, respectively. Stars in panel (a) indicate the times in Figure 4.

Figure 6. (a) Trajectory of the accelerated particle in the same format as in Figure 4. The snapshot is taken at $\Omega_{pe} T = 580$. Time histories of (b) $\gamma - 1$, (c) the magnetic moment, and (d) $B_z$ at the particle’s position. Running average for $15 \Omega_{pe}^{-1}$ is applied to the magnetic field profile (d). (An animation and a color version of this figure are available in the online journal.)

Figure 7. Comparisons of (a) $B_z$, (b) $E_y$, and (c) $-c \nabla \times E|_z$ around the overshoot between Run A (dashed line) and Run C (solid line). The spatial profile across the shock is averaged in the $y$-direction with respect to the overshoot position at each $y$-position.

4.2. Mass Ratio and Alfvén Mach Number Dependencies

We have shown the shock surfing acceleration and additional adiabatic accelerations at the shock surface in Run C in which the maximum energy gain of the electron is observed. Here, we examine these mechanisms in the other simulation runs with various ion-to-electron mass ratios ($M/m$) and Alfvén Mach numbers ($M_A$).

Figure 8 shows a comparison of the electrostatic field strength in the leading edge of the foot. In Run A with $M/m = 25$ and $M_A \sim 14$, the strong electrostatic field is excited, indicating the Buneman instability (Figure 8(a)). The spatial profile is not like a coherent wave form in Run C (Figure 8(c)), but consists of isolated peaks distributed in the $x$–$y$ plane (Amano & Hoshino 2009a). The amplitude of the electrostatic field reaches $|E_{est}/B_0| \sim 2.4$, which satisfies the trapping condition of the electron (Equation (6)). For a reduced mass ratio, the condition of the acceleration toward the relativistic regime (Equations (6))

shock reformation (Lembége & Savoini 1992; Scholer et al. 2003; Lembége et al. 2009). Figure 7(c) shows the $z$ component of $-\nabla \times E = \partial B/\partial t$. It takes large positive and negative values in the ramp in the shock rest frame, indicating that a new shock front is forming ahead of the existing overshoot position. Since the inductive electric field strength is larger than the convective electric field ($|E_{ind}| \sim 0.3 B_0$), the betatron acceleration is also effective for the relativistic electron in the present simulation run.
and (7)) is relaxed to a relatively low Alfvén number case. For instance, $M/m = 25$ requires $M_A > 10.5$. Run A is indeed the case since $M_A \sim 14$.

In Runs A and C, the Buneman instability is highly activated and the electrons are accelerated in the foot since the trapping condition is satisfied in both simulation runs. However, differences arise in the number and the maximum energy of the non-thermal population in the downstream. We have shown in the previous subsection that the second acceleration process is found at the shock surface in Run C, which further energizes the pre-accelerated electron from $\gamma \sim 3$ up to $\gamma \sim 7$ adiabatically. The key for this acceleration is the magnetic and electric fields’ profiles around the overshoot. Figure 7(a) compares the magnetic field structures in Runs A and C. The peak amplitude of the magnetic field at the overshoot is $B_z/B_0 \sim +8.0$ in Run A, which is much smaller than the peak of $B_z/B_0 \sim +20.0$ in Run C. The inductive electric field in the ramp found in Run C is not evidently found in Run A (Figures 7(b) and (c)); thus, the betatron acceleration by the reformation process is not effective. The pre-accelerated relativistic electron in Run A cannot gain much energy through the adiabatic accelerations at the shock surface.

If the mass ratio is increased with the Alfvén Mach number being fixed, one expects the Buneman instability to be weakly saturated, or even completely stabilized from Equations (5) and (7). In Run B, $M/m = 100$ and $M_A \sim 16$; therefore, the trapping condition $M_A > 25.8$ is not satisfied while the unstable condition $M_A > 4.2$ is realized (Table 1). Figure 8(b) reflects these conditions that the amplitude of the electrostatic field is $|E_{est}|/B_0 \sim 0.5$ even though the Buneman instability seems to become unstable. Thus, the electrons can easily escape from the potential well after moderate energization, and the efficient acceleration to the relativistic regime is not expected in this run.

**4.3. Effect of Upstream Electron Temperature**

Upstream electron temperature also affects the activity of the electrostatic field in the foot; the Buneman instability is expected to be weakly saturated or even completely stabilized in a high electron $\beta_e$ condition. To assess this effect, we simply increase $\beta_e$ from 0.5 in Run C to 4.5 with $M/m$ and $M_A$ being fixed in Run D. In this case, both the trapping condition $M_A > 25.8$ and the unstable condition $M_A > 12.7$ are satisfied (Table 1). The resultant electric field profile in the leading edge of the foot is

![Figure 8](image_url)
shown in Figure 8(d). The amplitude of the electrostatic field is so weak ($|E_{\text{el}}|/B_0 \sim 0.5$) that any coherent waves are not recognized out of the thermal noise of the hot electron. As in the case of Run B, the non-thermal electrons are hardly found in the downstream.

4.4. Energy Spectra in the Downstream Region

We summarize our simulation runs by showing the energy spectrum of the electron in the downstream region. Particles are sampled in a rectangular region of $0.7 V_0/\Omega_{gi} \times L_s$ behind the overshoot. Figure 9(a) compares the results from Runs A–C. The energy spectrum in Run C exhibits a high energy tail that deviates from the thermal population and reaches $\gamma \sim 9$, as is shown in Figure 3. Similar non-thermal electron production is observed in Run A, in which the Buneman instability is highly activated at the edge of the foot as well. These simulation runs satisfy the trapping condition of Equation (6). When the trapping condition does not hold by increasing the mass ratio, the non-thermal population is significantly reduced. This is clearly seen in Run B when compared with Run A. It is also notable that fitted Gaussian profiles give the electron temperatures of $T_e/mc^2 = 0.062$ for Run A and $T_e/mc^2 = 0.033$ for Run B. Although the Mach number is the same in both runs, the downstream condition is greatly altered by the activity of the Buneman instability in the foot, which heats the electrons perpendicular to the magnetic field.

Figure 9(b) shows the effect of the upstream electron temperature on the non-thermal electron production by comparing the results from Runs C and D. Even though both the unstable and trapping conditions are satisfied in Runs C and D, the non-thermal population in the downstream in Run D is also significantly reduced. Thus, the upstream electron temperature ($\beta_e$) must be another important parameter that determines the efficiency of the electron acceleration. The fitted Gaussian profile for Run D gives the electron temperature of $T_e/mc^2 = 0.11$, which is slightly smaller than in Run C.

In Table 1, the results are summarized in relation to the upstream conditions.

5. SUMMARY AND DISCUSSION

We have examined electron acceleration mechanisms at high Mach number shocks by means of two-dimensional PIC simulations with various ion-to-electron mass ratios, Alfvén Mach numbers, and the upstream electron $\beta_e$. We found electrons are effectively accelerated at a super-high Mach number shock ($M_A \sim 30$) with a mass ratio of $M/m = 100$ and $\beta_e = 0.5$. The electron shock surfing acceleration is found to be an effective mechanism for accelerating the particles toward the relativistic regime even in two dimensions with a large mass ratio. The Buneman instability in the super-high Mach number shock resulted in a coherent electrostatic potential structure which enabled efficient trapping of the electrons. While multidimensionality allows the electrons to escape from the trapping region, they can interact with the strong electrostatic field several times. This multiple interaction enables effective shock surfing accelerations, as has been found in the small mass ratio case (Amano & Hoshino 2009a).

The conditions of the electron shock surfing acceleration toward the relativistic regime have been derived from one-dimensional arguments (Papadopoulos 1988; Cargill & Papadopoulos 1988; Ishihara et al. 1980). Those simple estimations still hold in the present two-dimensional simulations, as summarized in Figure 10. While our four simulation runs all satisfy the unstable condition of the Buneman instability (dashed lines), the shock surfing acceleration becomes effective in two simulation runs (denoted by “Run A” and “Run C” in Figure 10) whose upstream conditions are also well above the trapping condition of the electron (solid line). A similar aspect holds in recent two-dimensional PIC simulations with parameters different from those in our simulation runs. For example, the electron shock surfing acceleration does not play an important role in producing non-thermal electrons in shocks with $M_A = 7$, $M/m = 100$, 400, and $\beta_e = 0.5$ in which the trapping condition is not met (Riquelme & Spitkovsky 2011).

An exception was found for high electron $\beta_e$ cases. In our high $\beta_e$ run (denoted by “Run D” in Figure 10), the Buneman instability is destabilized in the foot region. However, its peak amplitude is not so large that electrons can escape from the trapping region before reaching the relativistic regime. A similar conclusion was also drawn by Kato & Takabe (2010), in which $M_A \sim 130$, $M/m = 30$, and $\beta_e = 26$ are used (see also Figure 10). Although their linear analysis revealed that the foot region in their simulation result was unstable due to the Buneman instability, the resultant energy spectrum in the downstream region was almost Maxwellian, like what we see in our high $\beta_e$ run. These results indicate that we cannot simply understand the high electron $\beta_e$ simulations from Figure 10.
which is based on the linear and nonlinear theories of cold plasma, and detailed analysis of the saturation mechanism of the Buneman instability with finite electron temperature effects is necessary. Extrapolating from the recent and present two-dimensional PIC simulations, Figure 10 indicates that the electron shock surfing acceleration is an effective mechanism for producing relativistic particles in extremely high Mach number shocks in SNRs, provided that the upstream electron temperature is reasonably low.

In addition to the shock surfing acceleration in the foot, the adiabatic accelerations are found to be effective for the pre-accelerated, relativistic electrons in the super-high Mach number shock. The process is a combination of the shock drift acceleration and the betatron acceleration by the time variation of the ramp due to the shock reformation process. Since the mechanism itself is an adiabatic one, the maximum attainable energy by this process is limited by the amplitude of the magnetic field at the overshoot:

$$\frac{\gamma_{\text{ft}}^2}{B_{\text{ft}}} = \frac{\gamma_{\text{ov}}^2}{B_{\text{ov}}},$$

$$\gamma_{\text{ov}} = \gamma_{\text{ft}} \sqrt{\frac{B_{\text{ov}}}{B_{\text{ft}}}},$$

where the subscripts “ft” and “ov” denote the foot and the overshoot, respectively. $\gamma_{\text{ft}}$ is the energy after the shock surfing acceleration in the foot. The magnetic field strength at the leading edge of the foot does not strongly depend on the Alfvén Mach number and our simulation runs exhibit $B_{\text{ft}} \sim 2B_{\text{0}}$, which agrees with the multi-fluid model of Leroy (1983). In Leroy’s model, the field strength at the overshoot was also estimated to leading order as

$$B_{\text{ov}} \sim \alpha B_{\text{0}} M_A^2,$$

where we assume $\alpha$ is a constant. This scaling can be applied to our simulation runs up to $M_A = 30$; for instance, $B_{\text{ov}} = 8.0B_{\text{0}}$ for $M_A = 14.4$ in Run A and $B_{\text{ov}} = 20.0B_{\text{0}}$ for $M_A = 30.0$ in Run C give $\alpha \sim 0.4$. Equations (11) and (12) are combined to give the maximum energy attained by the adiabatic acceleration around the overshoot as

$$\gamma_{\text{ov}} = \sqrt{0.2 M_A^2 \gamma_{\text{ft}}},$$

which suggests that the electrons accelerated in the foot can be further energized by an order of magnitude in extremely high Mach number shocks in SNRs. Note, however, that the high energy particles reside in the vicinity of the shock surface and their energy decreases as they are transmitted in the downstream far away from the overshoot owing to the conservation of the magnetic moment. Non-adiabatic processes, such as a pitch angle scattering of the particles by electromagnetic waves at the shock surface, are necessary in order to observe the relativistic electrons in the entire downstream region.

In our two-dimensional purely perpendicular shock simulations with the out-of-plane magnetic field configuration, only drift waves along the shock surface are allowed to exist. Growth of the drift waves (e.g., the lower hybrid drift instability) is observed in the super-high Mach number shock (not shown); however, relationships between the waves and the electron heating or acceleration were not clearly found in the present study. A similar point of view must be introduced if one examines the in-plane magnetic field configuration in which electromagnetic waves are allowed to propagate along the shock surface. In supercritical shocks, two-dimensional hybrid simulations have revealed ion-scale rippled structures at the shock surface associated with strong electromagnetic perturbations, and the ripple is observable only in the in-plane magnetic field configuration (Lowe & Burgess 2003; Burgess 2006). In this case, the angle between the shock normal and the magnetic field direction changes locally, thereby, the reflection rate of the ions is enhanced in some locations feeding more energy to the Buneman instability. Such a connection between the rippled shock structure and the electron acceleration has been reported (Umeda et al. 2009). The discussions are, however, limited in relatively low Alfvén Mach number and small mass ratio cases, and understanding how the ion-scale rippled structure and the electron-scale Buneman instability coexist in high Mach number shocks remains the next step.

Note that a two-dimensional simulation can deal with either the two-dimensional electron surfing acceleration in the out-of-plane magnetic field configuration or the rippled structure in the in-plane configuration. In the latter case, the surfing acceleration occurs one dimensionally and the electrons cannot escape from the trapping region. The acceleration efficiency will be different from the present out-of-plane case. Therefore, a unified understanding of electron accelerations in high Mach number shocks will be made possible only by a three-dimensional PIC simulation, which will be reported in the near future by virtue of a peta-scale supercomputer system.

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