Effective dark energy through spin-gravity coupling

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Abstract

We investigate cosmological scenarios with spin-gravity coupling. In particular, due to the spin of the baryonic and dark matter particles and its coupling to gravity, they probe an effective spin-dependent metric, which can be calculated semi-classically in the Mathisson-Papapetrou-Tulczyjew-Dixon formalism. Hence, the usual field equations give rise to modified Friedmann equations, in which the extra terms can be identified as an effective dark-energy sector. Additionally, we obtain an effective interaction between the matter and dark-energy sectors. In the case where the spin-gravity coupling switches off, we recover standard ΛCDM cosmology. We perform a dynamical system analysis and we find a matter-dominated point that can describe the matter era, and a stable late-time solution corresponding to acceleration and dark-energy domination. For small values of the spin coupling parameter, deviations from ΛCDM concordance scenario are small, however for larger values they can be brought to the desired amount, leading to different dark-energy equation-of-state parameter behavior, as well as to different transition redshift from acceleration to deceleration. Finally, we confront the model predictions with Hubble function data.

1. Introduction

Modified gravity is one of the two ways one may follow in order to explain universe acceleration [1, 2] (the other one being the introduction of the dark-energy sector [3, 4]), while it has been also proved to be efficient in alleviating the two famous tensions of ΛCDM cosmology, namely the \( H_0 \) and the \( \sigma_8 \) ones [5]. However, a crucial additional advantage of modified gravity is that it may have improved renormalizability and thus is closer to a quantum description [6].

One possibility is the addition of quantum corrections to the action [7–9]. Other approaches to quantum gravity consider that the standard energy-momentum dispersion relation is deformed near the Planck scale, since this may arise from string field theory [10], loop quantum gravity [11], and non-commutative geometry [12]. Furthermore, in the context of “doubly general relativity” [13–16] the modification of the dispersion relation leads to an effective spacetime metric which depends on the energy and momentum of the probe particle. Since spacetime is represented by a one-parameter family of metrics, parametrized by the energy of the probe particle, this semiclassical approach is called Gravity’s Rainbow [13]. Applications of Gravity’s Rainbow to dark energy and inflation can be found in [17–23].

However, interestingly enough, an effective spacetime metric can alternatively arise in the context of a semiclassical description of the spinning particle in an arbitrary gravitational field [24–27], without the need of a modified dispersion relation. For instance, in [27] (see also Refs. [28, 29]) the authors have constructed a Lagrangian formulation for the Mathisson-Papapetrou-Tulczyjew-Dixon (MPTD) equations [30–33], in the context of a semiclassical vector model for the spin space. Hence, in the minimal-coupling prescription of gravity, a spinning particle effectively probes a different geometry, lying within the general class of Riemann-Cartan geometry, which is determined by an effective metric \( g^{(eff)}_{\mu\nu} \) that depends on its spin.

In particular, in vector models of spin the basic variables are the non-Grassmann vector \( \omega^\mu \) and its conjugated momentum \( \pi_\mu \). The spin-tensor is constructed using these variables as \( S^\mu_\nu = 2 (\omega^\mu \pi_\nu - \omega^\nu \pi^\mu) \). Then, one starts from the free theory in flat space, for which there is a Lagrangian formulation without auxiliary variables [27–29], and the minimal coupling to gravity is achieved by covariantization of this Lagrangian [27–29]. After constructing the Hamiltonian formulation, one can eliminate the momenta from the Hamilton equations by using the mass-shell condition. Thus, a closed system for the equations of motion (the Lagrangian form of MPTD equations) is obtained, with the emergence of the effective metric \( g^{(eff)}_{\mu\nu} \) [27].

The effective metric, produced along the world-line of the particle through interaction of the spin with gravity,
is given by [27]
\[ g^{(\text{eff})}_{\mu\nu} = g_{\mu\nu} + \frac{1}{8\pi m} \left( S^\alpha_{\nu} \theta_{\sigma\mu} + S^\sigma_{\nu} \theta_{\sigma\mu} \right) + \left( \frac{1}{8\pi m^2} \right)^2 S^\alpha_{\sigma\mu} S^\sigma_{\alpha\nu}, \]
(1)
where \( m \) is the mass of the particle. In the above expression, the spin-tensor of the particle is defined as \( S^\mu_{\nu} = 2 (\omega^\mu \pi^\nu - \omega^\nu \pi^\mu) = (S^{00} = D^1, S^{01} = 2\epsilon^{ijk} S_k) \), where \( D^1 \) is the dipole electric moment and \( S^3 \) is the three-dimensional spin-vector [34, 35], and it satisfies the relation \( S_{\mu\nu} S^{\mu\nu} = 8\pi = \text{const.} \), with \( \sigma \) the absolute spin which is a constant of motion. Finally, the tensor
\[ \theta_{\mu\nu} \equiv R_{\alpha\beta\mu\nu} S^{\alpha\beta}, \]
(2)
where \( R_{\alpha\beta\mu\nu} \) is the Riemann tensor related to the physical metric \( g_{\mu\nu} \), quantifies the coupling between spin and gravity.

In the present manuscript we desire to investigate the implications of this effective spin-dependent metric in the context of cosmology, by deriving the corresponding modified Friedmann equations. In particular, we identify the extra spin-gravity coupling terms as an effective dark-energy sector. We mention here that since the spin-gravity coupling above is based on the coupling of Riemann tensor to spin, one expects that is would be larger in the early Universe, or around black holes. Nevertheless, since the coupling is present, even at late-times it can play a role if it generates a collective effect from all spinning dark-matter particles of the Universe. In some sense the situation is similar to modified gravity, where the modification at late-times (where curvature is small) is extremely small, and thus it is impossible to be observed in Solar System experiments or in scales below galaxy clusters, however, in the whole Universe collectively, it can lead to deviations from \( \Lambda \)CDM paradigm that can improve cosmological behavior. Finally, note that concerning dark matter there are many theories which suggest that it could correspond to massive higher spin particles, typically found in string theory, and this could lead to enhanced spin-gravity coupling effects [8].

The plan of the paper is the following: In Section 2 we present the construction at hand, extracting the modified Friedmann equations and the effective dark energy sector. In Section 3 we investigate the resulting cosmological behavior, performing a dynamical system analysis and elaborating the model numerically. Finally, in Section 4 we summarize the obtained results. Throughout the manuscript, we adopt natural units \( c = \hbar = 1 \) and we use the metric signature \((-,-,+,+)\).

2. Modified Friedmann equations and effective dark energy

In this section we apply the above formulation in a cosmological framework, namely we consider the back-ground metric \( g_{\mu\nu} \) to be a flat Friedmann-Robertson-Walker (FRW) one, with form
\[ ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j, \]
(3)
with \( a(t) \) the scale factor. Concerning the matter sector, we consider baryonic and dark matter particles corresponding to the standard perfect-fluid energy-momentum tensor
\[ T_{\mu\nu} = [(\rho_m + \rho_p) U_\mu U_\nu + g_{\mu\nu} p_m], \]
(4)
with \( \rho_m \) and \( p_m \) the energy density and pressure respectively, while the four-velocity of the fluid is \( U_\mu = (-1, 0, 0, 0) \) such that \( U_\mu U^\mu = -1 \). Finally, concerning the field equations we consider the ones of standard general relativity, namely
\[ G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \kappa^2 T_{\mu\nu}, \]
(5)
where \( G_{\mu\nu} \) is the Einstein tensor, \( \kappa^2 = 8\pi G \) is the gravitational constant, and \( \Lambda \) is the cosmological constant.

As we mentioned in the Introduction, due to the spin-gravity coupling the matter particles feel the effective metric \( g^{(\text{eff})}_{\mu\nu} \) of [1]. In order to calculate it one starts with the calculation of the averaged effective metric \( \langle g^{(\text{eff})}_{\mu\nu} \rangle \) [29]. The volume element contains a large number of particles with a randomly oriented spin distribution. Several different authors have studied the effects at cosmological scales of matter distributions that locally contain a large number of randomly oriented spin particles, and they have found that the microscopic gravitational field equations can assume a pseudo-Einsteinian form that includes spin corrections terms [21, 29]. Definitely, one needs an averaging procedure for these fluctuating terms in the microscopic domain. This is similar to what is done when obtaining the macroscopic Maxwell equations.

The above averaging procedure typically leads to zero spin average and zero spin gradient, however to non-zero average for the spin-squared terms arising in the field equations of the theories with spin-gravity couplings [24]. Therefore, the averaged effective metric is obtained by substituting \( \langle \right \rangle \) into \( \langle \left \rangle \) and then averaging over all possible directions of the three-dimensional spin-vector \( \vec{S}(t) \) and the dipole electric moment \( \vec{D}(t) \), namely \( \langle \vec{S} \cdot \vec{S} \rangle = \vec{S}^2 \), \( \langle \vec{D} \cdot \vec{D} \rangle = \vec{D}^2 \) and \( \langle \vec{S} \rangle = \langle \vec{D} \rangle = 0 \). In the flat FRW metric, under the assumption that the absolute spin \( S_{\mu\nu} S^{\mu\nu} = 8\pi = \text{const.} \), we have the relations \( \vec{S}(t)^2/m^2 = 3\alpha/4a(t)^2 \) and \( \vec{D}(t)^2/m^2 = 6\beta/a(t)^2 \), where \( \alpha \) and \( \beta \) are constants with dimensions of mass\(^{-2}\), and thus we find \( 8\pi = \langle S_{\mu\nu} S^{\mu\nu} \rangle = 6m^2(\alpha - 2\beta) \) [29]. Hence, we finally result to the averaged effective metric \( g_{\mu\nu} \equiv \langle g^{(\text{eff})}_{\mu\nu} \rangle \) given by
\[ ds^2 = (-1 + F_1 ) dt^2 + a(t)^2 (1 + F_2 ) \delta_{ij} dx^i dx^j, \]
(6)
where
\[ F_1 = 3\beta \left( \dot{H} + H^2 \right) \left[ 1 + \frac{(5\alpha - 19\beta)(\dot{H} + H^2)}{20} \right], \]
(7)
\[ F_2 = \beta \dot{H} \left( \frac{19\beta \dot{H} + 2(19\beta - 5\alpha) H^2}{20} - 1 \right) + \frac{(38\beta^2 - 30\alpha \beta + 19\alpha^2) H^4}{40} + (\alpha - \beta) H^2, \] (8)

with \( H \equiv \dot{a}/a \) the Hubble function and with dots denoting derivative terms. Hence, in the case where the dipole electric moment is absent we have \( \beta = 0 \), and thus \( F_1 = 0 \).

By substituting the averaged effective metric \( g^{\text{eff}}_{\mu\nu} \) into the field equations (5), we finally obtain the modified Friedmann equations

\[ \dot{F}_2 = \dot{F}_2 + \frac{6(1 + F_1) H}{2(1 - F_1)(1 + F_2)} \frac{1}{1 + F_2} + 2H^2 \frac{H^2}{(1 - F_1)} = \frac{(\kappa^2 \rho_m + \Lambda)}{3}, \] (9)

\[ \dot{F}_2 = \frac{4(1 + F_2) H}{1 + F_2} + \frac{H^2}{(1 - F_1)} + \frac{2}{(1 - F_1)(1 + F_2)} \frac{\dot{F}_2 + 4(1 + F_2) \dot{H}}{4(1 + F_2)^2} F_2. \] (10)

Note that in the case where the spin-gravity coupling switch off (in the spinless limit, or for a Minkowski spacetime, where \( \theta_{\mu\nu} = 0 \)), namely when \( \alpha \) and \( \beta \) become zero, we obtain that \( F_1 = F_2 = 0 \) and thus we recover standard cosmology. Hence, although spinless particles follow geodesics related to the metric \( g_{\mu\nu} \), in theories with spin-gravity coupling, such as the Einstein-Cartan-(Sciama-Kibble) theory, spinning particles follow non-geodesic paths when moving on gravitational fields [30, 31, 38, 39]. The study of spinning particles in general relativity is an old subject [30-32, 39] and currently the theory is under detailed investigation by many groups.

The Friedmann equations (9,10) can be rewritten in the standard form

\[ 3H^2 = \kappa^2 (\rho_{de} + \rho_m), \] (11)

\[ -2\dot{H} = \kappa^2 (\rho_m + p_m + \rho_{de} + p_{de}), \] (12)

where we have defined an effective dark energy sector with energy density and pressure

\[ \kappa^2 \rho_{de} = \frac{3F_1 H^2}{(F_1 - 1)} + \Lambda + \frac{3}{4(1 + F_2)^2 (F_1 - 1)} \frac{\dot{F}_2}{F_2}, \] (13)

\[ \kappa^2 p_{de} = -\frac{F_1}{(F_1 - 1)} \frac{2H + 3H^2}{F_1} - \frac{\dot{F}_2}{4(F_1 - 1)(1 + F_2)^2} - \Lambda + \frac{1}{2(1 - F_1)} \left( \frac{1}{1 + F_2} \frac{\dot{F}_2 + 6(1 - F_1) H^2}{F_2} + 2H \dot{F}_1 \right). \] (14)

Furthermore, we can introduce the effective dark-energy equation-of-state parameter as

\[ w_{de} = \frac{p_{de}}{\rho_{de}}. \] (15)

In the averaged effective metric \( [g^{\text{eff}}_{\mu\nu}] \) the matter perfect fluid \([\rho, p]\) acquires a modified four-velocity vector \( U_\mu = (-\sqrt{1 - F_1}, 0, 0, 0) \), such that \( U_\mu U^\mu = -1 \). Thus, the energy-momentum tensor for the spinning matter particles is

\[ T^{\mu\nu} = (\rho_m + p_m) U^{\mu} U^{\nu} + G^{\mu\nu} p_m, \] (16)

where \( G^{\mu\nu} \) is the averaged effective metric \( [g^{\text{eff}}_{\mu\nu}] \). From the conservation law \( \nabla_\mu T^{\mu\nu} = 0 \), where the covariant derivative \( \nabla_\mu \) satisfies the metric compatibility condition \( \nabla_\mu G^{\mu\nu} = 0 \), we obtain the continuity equation

\[ U^\mu \nabla_\mu \rho_m = - (\rho_m + p_m) \nabla_\mu U^{\mu}, \] (17)

while the perfect fluid equation of motion is

\[ U^\mu \nabla_\mu U^{\nu} = \frac{1}{\rho_m + p_m} (U^\mu \nabla_\mu p_m U^{\nu} + \nabla^{\nu} p_m). \] (18)

Hence, in terms of the physical metric \( g_{\mu\nu} \) we obtain the perfect fluid equation as

\[ U^0 \dot{U}^0 = \frac{1}{2} \frac{\dot{F}_1}{1 - F_1} (U^0)^2, \] (19)

with \( U^0 = 1/\sqrt{1 - F_1} \). Therefore, in the case \( F_1 \neq 0 \) and \( F_1 \neq \text{const} \), we do not obtain \( U^0 \dot{U}^0 = 0 \) which is the standard geodesic motion, as we mentioned above.

Hence, the matter conservation equation in the effective metric is written as

\[ \dot{\rho}_m + 3H (\rho_m + p_m) = Q, \] (20)

where

\[ Q = -3 (\rho_m + p_m) \frac{\dot{F}_2}{2(1 + F_2)}, \] (21)

and this can be also cross-checked by differentiating \( [f] \) and inserting into \( [10] \). Consequently, \( \rho_{de} \) and \( p_{de} \) obey the evolution equation

\[ \dot{\rho}_{de} + 3H (\rho_{de} + p_{de}) = -Q. \] (22)

Interestingly enough, the spin-gravity coupling gives rise to an effective interaction between the matter and dark energy sectors, and thus the present scenario exhibits the advantages of interacting cosmology [40,53]. As expected, in the case where the spin-gravity coupling switches off we have \( Q = 0 \) and thus we recover standard, non-interacting, cosmology.

We mention here that the physical metric is \( g_{\mu\nu} \) (for instance photons follow null geodesics in terms of \( g_{\mu\nu} \) and thus the CMB, SNIa etc physics is done with \( g_{\mu\nu} \)). The basic idea of the class of theories with spin-gravity couplings is that the coupling makes dark-matter particles feel the effective metric \( g^{\text{eff}}_{\mu\nu} \), and their interaction with \( g^{\text{eff}}_{\mu\nu} \), namely the spin-gravity interaction, from the point of view of the physical metric \( g_{\mu\nu} \) appears as extra terms in the standard Friedmann equations (i.e. the Friedmann equations in terms of the physical metric), which in our
model provides the effective dark-energy sector. Hence, one cannot perform a coordinate transformation to the averaged effective metric \( \tilde{\Lambda} \) and bring it to the standard FRW form, since this transformation freedom has already be spent in order to bring \( g^{\mu\nu} \) to the the standard FRW form.

Lastly, concerning cosmological investigations it proves convenient to introduce the total equation-of-state parameter as

\[
w_{\text{tot}} = \frac{\rho_{\text{de}} + \rho_m}{\rho_{\text{de}} + \rho_m},
\]

which is related to the deceleration parameter \( q \) through

\[
q \equiv -1 - \frac{\dot{H}}{H^2} = \frac{1}{2} (1 + 3w_{\text{tot}}),
\]
as well as the density parameters

\[
\Omega_m = \frac{\kappa^2 \rho_m}{3H^2}, \quad \Omega_{\text{de}} = \frac{\kappa^2 \rho_{\text{de}}}{3H^2}.
\]

### 3. Cosmological behavior

In this section we investigate the cosmological behavior of the scenario at hand, by using the Friedmann equations \([11]\) and \([12]\), alongside the conservation equation \([20]\). As a starting model we focus on the case where the dipole electric moment is absent, namely \( \beta = 0 \), thus from \([7]\)–\([8]\) we obtain

\[
F_1 = 0,
\]
\[
F_2 = \frac{19}{40} \alpha^2 H^4 + \alpha H^2.
\]

Substituting these into \([13]\) and \([14]\), we find

\[
\kappa^2 \rho_{\text{de}} = \Lambda - 6\alpha H^2 \tilde{H} - 3\alpha^2 H^2 \tilde{H}^2,
\]
\[
\kappa^2 \rho_{\text{de}} = -\Lambda + 6\alpha H^2 \tilde{H} + 2\alpha \tilde{H}^2 + \frac{27\alpha^2 H^2 \tilde{H}^2}{10} + 2\alpha H \tilde{H},
\]

and then

\[
w_{\text{de}} = -1 + \frac{20\alpha H \tilde{H} + 20\alpha \tilde{H}^2 - 3\alpha^2 H^2 \tilde{H}^2}{10 (\Lambda - 3\alpha^2 H^2 \tilde{H}^2 - 6\alpha H^2 \tilde{H})}.
\]

Hence, we can verify that

\[
\dot{\rho}_{\text{de}} + 3H(\rho_{\text{de}} + \rho_m) = -Q,
\]
\[
\rho_m + 3H(\rho_m + \rho_m) = Q,
\]

with

\[
Q = -3\alpha (\rho_m + \rho_m) H \tilde{H}.
\]

Therefore, in an expanding universe, for \( \alpha > 0 \) we have \( Q > 0 \) and thus energy is transferred from dark energy to dark matter sector \([54]\).

In order to study the cosmological dynamics of the scenario we introduce the following set of dimensionless variables

\[
X = \kappa H, \quad Y = -\frac{\dot{H}}{H^2}, \quad Z = \Omega_m.
\]

Therefore, the set of cosmological equations can be rewritten in an autonomous form as

\[
\frac{dX}{dN} = -XY, \quad \frac{dY}{dN} = -3Y(w_m + 1 - Y) - \tilde{\Lambda}(w_m + 1) + \frac{2\alpha X^4}{20} + \frac{3}{20} \tilde{\alpha}(10w_m + 9)X^2Y^2 + \frac{3(w_m + 1) - 2Y}{2\alpha X^2},
\]

with \( w_m \equiv p_m/\rho_m \) the matter equation-of-state parameter. Moreover, we find that

\[
Z = (\tilde{\alpha} X^2 Y - 1)^2 - \frac{\tilde{\Lambda}}{3X^2}.
\]

Furthermore, in terms of these phase-space variables we obtain

\[
w_{\text{de}} = w_m + \frac{X^2 [3(w_m + 1) - 2Y]}{3\tilde{\alpha} X^4 Y (\tilde{\alpha} X^2 Y - 2) - \tilde{\Lambda}},
\]

and

\[
w_{\text{tot}} = -1 + \frac{2Y}{3},
\]

and therefore accelerated expansion occurs for \( 0 \leq Y < 1 \).

We proceed by performing a dynamical system analysis, in order to extract the global behavior of the scenario, i.e. to investigate the late-time asymptotic behavior independently of the Universe initial conditions \([55]\). In order to extract the critical points of the autonomous system \([35]\) we impose the conditions \( dX/dN = dY/dN = 0 \), and in order to study their stability we calculate the eigenvalues of the involved perturbation matrix. We find the following fixed points:

- **Point (a):** \( \{X_c = 0, Y_c = \frac{3(w_m + 1)}{2}, Z_c = 1\} \). This point represents the dark-matter dominated era for which \( \Omega_m = 1 \) and \( \Omega_{\text{de}} = 0 \), while \( w_{\text{tot}} = w_{\text{de}} = w_m \).

- **Point (b):** \( \{X_c = \sqrt{\frac{2}{3}} Y_c, Y_c = 0, Z_c = 0\} \). This fixed point corresponds to a dark-energy dominated accelerating solution for which \( \Omega_{\text{de}} = 1 \) and \( \Omega_m = 0 \), with \( w_{\text{tot}} = w_{\text{de}} = -1 \).

Considering small perturbations around the critical points such that \( X(N) = X_c + \delta X(N) \) and \( Y(N) = Y_c + \delta Y(N) \), with \( \delta X(N) \ll X_c \) and \( \delta Y(N) \ll Y_c \), we define the functions \( F(X, Y) = dX/dN \) and \( G(X, Y) = dY/dN \) and then use them to construct the perturbation matrix \( M = \{\{\delta F/\delta X, \delta F/\delta Y\}, \{\delta G/\delta X, \delta G/\delta Y\}\}\}(X_c, Y_c) \). Therefore we obtain:
• Point (a): The perturbation matrix is divergent and thus this critical point corresponds to unstable solution that describes the intermediate-time matter era of the Universe.

• Point (b): The eigenvalues are given by

\[
\mu_1 = -\frac{3}{\alpha \Lambda}, \quad \mu_2 = -3(w_m + 1).
\] (40)

The above eigenvalues satisfy the stability conditions \(\mu_1 < 0\) and \(\mu_2 < 0\) for \(\Lambda > 0\) and \(\alpha > 0\), and therefore this point is a stable node, i.e. a stable late-time solution.

Let us now perform a numerical elaboration of the cosmological equations. We use the stability conditions obtained above, and the current observational values of the cosmological parameters [64]. Additionally, it proves convenient to introduce the redshift parameter \(z = 1/a - 1\) (we set the present scale factor to \(a = a_0 = 1\) as the independent variable, and for the present density parameters at redshift \(z = 0\) we impose \(\Omega^{(0)}_{m} \simeq 0.69\) and \(\Omega^{(0)}_{de} \simeq 0.31\) [67].

![Figure 1: The matter and dark-energy density parameters \(\Omega_{m}(z)\) and \(\Omega_{de}(z)\) as functions of the redshift \(z\), for \(\tilde{\alpha} = 0.01\) (dotted curve), \(\tilde{\alpha} = 0.1\) (dashed curve) and \(\tilde{\alpha} = 1\) (solid curve). We have imposed \(\Omega^{(0)}_{de} = \Omega_{de}(z = 0) \simeq 0.69\) and \(\Omega^{(0)}_{m} = \Omega_{m}(z = 0) \simeq 0.31\) in agreement with observations [64].](image1)

In Fig. 1 we depict the behavior of the matter and dark-energy density parameters as functions of the redshift, and as we can see we obtain the expected thermal history, with the successive matter and dark-energy epochs. Additionally, in Fig. 2 we present the behavior of the effective dark-energy equation-of-state parameter \(w_{de}(z)\) for different values of the parameter \(\tilde{\alpha} \equiv \alpha/\kappa^2\). From this figure one can clearly see the effect of the spin-gravity coupling. In particular, while at present times, as well as in the future \((z \to -1)\), \(w_{de}\) stabilizes at the cosmological constant value \(-1\), at earlier times it deviates from this value, and thus the evolution of the present scenario deviates from that of ΛCDM concordance model.

![Figure 2: The dark-energy equation-of-state parameter \(w_{de}\) as function of the redshift \(z\), for \(\tilde{\alpha} = 0.01\) (dotted curve), \(\tilde{\alpha} = 0.1\) (dashed curve) and \(\tilde{\alpha} = 1\) (solid curve). We have imposed \(\Omega^{(0)}_{de} = \Omega_{de}(z = 0) \simeq 0.69\) and \(\Omega^{(0)}_{m} = \Omega_{m}(z = 0) \simeq 0.31\) in agreement with observations [64].](image2)

In Fig. 3 we show how the spin-gravity coupling can affect the behavior of the deceleration parameter. In particular, the transition from the decelerated to the accelerated regime is sensitive to the value of the parameter \(\tilde{\alpha}\). For large values of \(\tilde{\alpha}\) (e.g. \(\tilde{\alpha} \gtrsim 10\)), this transition happens at a redshift \(z \gtrsim 0.78\), long before \(z \approx 0.65\) which is suggested by observations [64], while for smaller \(\tilde{\alpha}\) values we obtain transition redshift values that deviate only slightly from the ΛCDM value. Finally, in Fig. 4 we present the predictions of our model for the Hubble function. As we observe, for \(\tilde{\alpha} \gtrsim 10\) there is a significant deviation with respect to the ΛCDM predictions, however for smaller \(\tilde{\alpha}\) values we can obtained the desired deviation from ΛCDM scenario.

![Figure 3: The deceleration parameter \(q\) as a function of the redshift \(z\), for \(\tilde{\alpha} = 10\) (dotted curve), \(\tilde{\alpha} = 1\) (dot-dashed curve), \(\tilde{\alpha} = 0.1\) (dashed curve) and \(\tilde{\alpha} = 0.01\) (solid curve), as well as for the ΛCDM scenario. We have imposed \(\Omega^{(0)}_{de} = \Omega_{de}(z = 0) \simeq 0.69\) and \(\Omega^{(0)}_{m} = \Omega_{m}(z = 0) \simeq 0.31\) in agreement with observations [64].](image3)
the matter clustering behavior and thus alleviate the \(\sigma_8\) tension. These interesting and necessary studies lie beyond the scope of the present work and are left for future projects.

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