Neutrino Magnetic Moments and Atmospheric Neutrinos\textsuperscript{1}

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Abstract. I review the history on neutrino magnetic moments and apply the neutrino magnetic moment idea to constrain its bound from Super-Kamiokande neutrino oscillation data.

INTRODUCTION

For a long time, it was assumed that neutrinos have vanishing quantities: $m_\nu = 0$, $Q_{\text{em}} = 0$, and $\mu_\nu = 0$. Among these the neutrino mass problem has attracted the most attention, and finally we might have an evidence for nonzero neutrino mass \textsuperscript{1}. The other important neutrino property to be exploited is the electromagnetic property, in particular the magnetic moment.

The reason for vanishing neutrino mass was very naive in 50’s and 60’s: the hypothesis of the $\gamma_5$-invariance. Under the $\gamma_5$-invariance, $\nu = \pm \gamma_5 \nu$, $\nu$ appears only in one chirality. In the standard model (SM), this is encoded as no right-handed neutrino.

In gauge theory models, the story changes because one can calculate the properties of the neutrino at high precision. In SM, one cannot write a mass term for $\nu$ in $d \leq 4$ terms. To write a mass term for $\nu$, one has to introduce $d \geq 5$ terms, or introduce singlet neutrino(s). The two-component neutrino we consider in the left-handed doublet can be Weyl or Majorana type.

If it is a Weyl neutrino, it satisfies $\nu_i = a_i \gamma_5 \nu_i$ where $a_i = 1$ or $-1$. Then the magnetic moment term is given by $\bar{\nu}_i \sigma_{\mu\nu} q^\nu \nu_j - \bar{\nu}_j \sigma_{\mu\nu} q^\nu \nu_i \to -a_i a_j [\bar{\nu}_i \sigma_{\mu\nu} q^\nu \nu_j - \bar{\nu}_j \sigma_{\mu\nu} q^\nu \nu_i]$. Therefore, to have a nonvanishing magnetic moment (or mass), we must require $a_i a_j = -1$, i.e. the existence of right-handed singlet neutrino(s).

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For Majorana neutrinos, $\psi_c = C\psi^*$, it is possible to write a mass term without introducing right-handed neutrino(s).

Thus, it is possible to introduce neutrino masses and magnetic moments in the SM, by a slight extension of the model. The question is how large they are.

For the detection of neutrino masses and oscillations, there have been numerous studies from solar-, atmospheric-, reactor-, and accelerator-neutrino experiments. The effect of neutrinos in cosmology was also used to get bounds on neutrino masses. On the other hand, for the neutrino magnetic moment astrophysical constraints gave useful bounds.

Usually, the bound of the neutrino magnetic moment is given in units ($f$) of Bohr magneton ($\mu_B$),

$$\mu_{\nu_i} = f_i \mu_B. \quad (1)$$

**HISTORY AND THE KNOWN BOUNDS**

The first significant bound on magnetic moment of $\nu_e$ was given from astrophysics, $|f_e| < 10^{-10}$, by Bernstein et al. [2]. A better bound on $f_e$ was obtained from SN1987A, $|f_e| < 10^{-13}$ [3].

For the muon neutrino, the useful bound was obtained from the neutral current data [4], $|f_\mu| < 0.8 \times 10^{-8}$. The first bound on the transition magnetic moment was given also from the neutral current experiment [5].

For the tau neutrino, $|f_\tau| < 1.3 \times 10^{-7}$ has been obtained recently [6].

For the theoretical side, it has been known from early days that it is possible to generate large magnetic moments for neutrinos [7]. Note that the see-saw mass for neutrinos appear as in Fig. 1. Here, there does not exist any charged particle and hence there is no contribution to magnetic moment at this level. Thus to have a large neutrino magnetic moments, one needs a Feynman diagram of Fig. 2 type, where we introduced a heavy lepton $L$ coupling to $W$ via

$$\left( L^0 \right)_L, \left( L^- \right)_L, \left( \nu_l \right)_L, \left( L^0 \right)_L, \left( L^- \right)_L. \quad (2)$$

**FIGURE 1.** A see-saw mechanism for neutrino masses.
Then one can easily estimate the magnetic moment of neutrinos as \[7\]

\[
f \text{ or } f' = \frac{G_F m_L m_e}{2\sqrt{2}\pi^2} abI \left( \frac{1}{2} + \frac{1}{2} \delta_{\nu\nu'} \right)
\] (3)

where \(abI\), which is a function of mixing and Feynman integral, is of order 1. One can also draw Feynman diagrams with charged scalars in the loop with appropriate Yukawa interactions introduced. This kind of diagrams generally introduce transition magnetic moment of order \(f' \sim m_L m_e / M^2\) where \(M\) is the mass of the intermediate scalar or gauge boson. Note that without extra charged leptons \(m_L\) should be neutrino mass, rendering an extremely small \(\mu\).

**NC, \(\mu'_\nu\), AND SINGLE \(\pi^0\) PRODUCTION BY \(\nu^\text{ATM}_\mu\)**

The Super-K collaboration has reported the ratio

\[
R_{\pi^0/e} = \frac{(\pi^0/e)_{\text{data}}}{(\pi^0/e)_{\text{MC}}} = 0.93 \pm 0.07 \pm 0.19
\] (4)

which is consistent with 1 at present. However, one may narrow down the experimental errors and can observe whether it is different from 1 or not. One assumes that \(\nu_e\) is not oscillated in the atmospheric neutrino data sample, and hence the MC electrons are estimated with the standard CC cross section. The denominator is calculated assuming that the NC is the same for the cases with and without neutrino oscillation. So \(R_{\pi^0/e}\) is expected to be 1 if there is no oscillation of SM neutrinos to sterile neutrinos. If there exist oscillation of \(\nu_\mu\) to sterile neutrinos, then one expects that \(R_{\pi^0/e} < 1\).

However, in our recent work [8] we pointed out that one should be careful to draw a firm conclusion on this matter because if a sizable transition magnetic moment of \(\nu_\mu\) exists then one expects a different conclusion.

\(f'\) is the transition moment.

**FIGURE 2.** A Feynman diagram for neutrino magnetic moment.
For the study of NC, the single $\pi^0$ production is known to be very useful. Most dominant contribution to the single $\pi^0$ production at the atmospheric neutrino energy is through $\Delta$ production,
\[
\nu + N \rightarrow \nu + \Delta, \quad \Delta \rightarrow N + \pi^0.
\] (5)
In this calculation, we used the form factors given in Ref. [9]. For $E_\nu < 10$ GeV or the kinetic energy of recoil nucleon < 1 GeV, the process $\nu + N \rightarrow \nu' + N$ is difficult to observe at Super-K. So the $\pi^0$ production is the cleanest way to detect NC interactions through Cherenkov ring (from $\pi^0$ decay) at Super-K.

For transition magnetic moment parametrized by $f'$, $i f' \mu_B \bar{u}(l')\sigma_{\mu\nu}q^\nu u(l)_{\nu'}$ where $q = l - l'$, the single $\pi^0$ production cross section through $\Delta$ production is given in Ref. [8]. In Fig. 3, we show the result (the ratio of the neutrino magnetic moment contribution and the NC contribution) as a function of neutrino energy.

Note that the magnetic moment part is more important at low $q^2$ region due to the photon propagator. In principle, one can distinguish neutrino magnetic moment interactions from the NC interactions. From Fig. 3, if we require $r_{f'/NC} \leq 0.13$, then we obtain a bound $f' \leq 2.2 \times 10^{-9}$.

In conclusion, the transition magnetic moment $f'$ can be large. For $\nu'$ heavy, it is not restricted by SN1987A bound. But atmospheric neutrinos of 1–10 GeV can produce $\nu'$, and can mimic NC data [5]. Before interpreting NC effects from atmospheric neutrino data, one has to separate out the $\mu_{\nu'}$ contribution.

**MODELS WITH LARGE $\mu_{\nu}$**

Before closing, we point out $\mu_{\nu}$ and solar neutrino problem. One possibility to reduce solar $\nu_e$ flux is to precess $\nu_{eL}$ to $\nu_{eR}$ with a large $\mu_{\nu}$ in a strong magnetic field [10]. But this idea seems to be ruled out by the nucleosynthesis argument [11], $\mu_{\nu} < 10^{-11}\mu_B$, and the SN1987A argument [3], $\mu_{\nu} < 10^{-13}\mu_B$. The SN1987A bound is coming from the energy loss mechanism: if $\nu_{eR}$ is created, it takes out energy out of the core. But if it is trapped, then the bound does not apply.\(^3\)

\(^3\) The transition magnetic moments to $\nu'$ are not restricted by these bounds for a heavy enough $\nu'$, but then the transition magnetic moment cannot account for the solar neutrino deficit.

![FIGURE 3. The ratio of $\mu'_{\nu_e}$ and NC contributions as a function of $E\nu_{\nu_e}$. See Ref. [7] for details.](image)
The solar neutrino problem requires $\mu_{\nu_e} \sim 10^{-11}\mu_B$ which is considered to be large. To use the idea of trapping, the oscillation is

$$\nu_{eL} \to \nu_{\mu R}^\prime.$$ \hfill (6)

Namely, we use the Konopinski-Mahmoud scheme where $\nu_{\mu R}^\prime$ is the weak interaction partner of $\mu_R^\prime$, implying the oscillated neutrino participates in weak interactions; hence trapped in the supernova core. This picture is very restrictive as suggested by models given in Ref. [12]. These models try to get a large neutrino magnetic moment while keeping the neutrino mass small. The $SU(2)_V$ symmetry in which $(\nu_e, \nu_{\mu R}^\prime)$ forms a doublet under $SU(2)_V$ is introduced for this purpose. In this case, $\mu$-number minus electron-number is conserved. The reason for vanishing mass is that $\nu^T C \nu^e$ is symmetric under $SU(2)_V$, i.e. the mass term is a triplet under $SU(2)_V$ and hence is forbidden. On the other hand the magnetic moment term, $\nu^T C \sigma_{\alpha\beta} \nu^e F^{\alpha\beta}$, is a singlet and is allowed, as given in Fig. 4. But, why $SU(2)_V$? It is like the dilemma asked in any fermion mass ansatz problem.

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\begin{figure}
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\includegraphics[width=0.5\textwidth]{figure4.png}
\caption{Neutrino magnetic moment with $SU(2)_V$.}
\end{figure}