Prediction of the velocity of air flow by dimensional analysis for drying application

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Abstract. Drying is a process that is widely used in the food processing industry. There are several factors that influence during the drying process, one of which is the speed of air flow in the dryer unit. The air flow velocity inside drying unit can occur by forced airflow and natural airflow. Natural air flow is the flow of air created due to the difference in pressure caused by differences in temperature and air density, so there is no thrust by the blower. This study aims to estimate the natural airflow velocity in modeling system unit. This modeling system unit has a source of heat energy that can be used to adjust the room temperature level of the heat generator, and is equipped with a chimney that has an adjustable height and diameter. This research was conducted experimentally, through empirical mathematical model development using dimensional analysis. The variables measured include: heat generator room temperature (TS), system outside air temperature (TL), chimney diameter (d), chimney height (H), exhaust air flow velocity (V). The results of this study obtained mathematical model of airflow velocity. The mathematical model showed good agreement due to measurement results with average of errors was about 9%.

1. Introduction
Drying is a process to reduce the moisture content of materials, with one of the goals of preserving agricultural products. Agricultural products at harvest time have a high-water content, this condition will cause these agricultural products to decline in quality, damage, and even decompose. Referring to this fact, drying activities are indispensable to support the development of agricultural industrial activities, from small scale industries (home industries) to large scale industries [1-3].

In order for the drying process to obtain optimal results, both technically and economically, the drying process must be able to measure the rate / velocity of the air flow in the dryer unit. The measurement of the air flow rate in the dryer unit often experiences problems, due to the limitations of the measuring instrument. To solve this problem, a modeling formula is needed to calculate the air flow velocity in the drying unit system. By knowing the speed of air flow in the system, the drying time can be predicted before the drying process takes place [4,5].

The mathematical model developed is a mathematical model empirically using dimensionless number analysis (π). Analysis with dimensionless numbers is very useful for research based on engineering, for the following reasons. (1) A dimensionless number is not sensitive to size because it has no units; (2) The size of the research can be reduced in the form of modeling, so that research costs can be saved; (3) The number of variables used can be reduced as much as the basic variables, so that it will reduce the complexity of the research [6-8].
The mathematical modeling that will be produced later can not only be used to estimate the drying rate, but can also be used as a reference for designing the dryer. The air flow rate predicted in this study is the air flow rate with the natural air flow velocity (without the thrust energy from the blower). The variables studied in this study were the temperature, diameter, and height of the chimney. The purpose of this study is to develop a mathematical model to estimate the velocity of air flow in the dryer unit.

2. Materials and methods

2.1. Materials and instruments
The materials used for this research are a box model of a heat generator equipped with an air intake, an electric heat generator, a chimney, and an exhaust outlet. The heat generator box can be seen in Figure 1. The measuring instruments used include: a temperature measuring device (thermocouple), and a measuring device for air flow velocity. heat generating box system.

![Heat generating box system](image)

**Figure 1.** Heat generating box system

2.2. Methods

2.2.1. Measurement of air flow velocity. The velocity data of the chimney outlet air flow was measured by performing 3 variations of the heat generator room temperature, 3 variations of chimney height, and 3 variations of chimney diameter. When measuring the outlet air velocity, the ambient air temperature is also taken. After the measurement data is obtained, a model development is carried out.

2.2.2. Development of mathematical model. Value of air flow velocity (V) in the system of heating generator unit is depend on the following parameters: chimney diameter (d), chimney height (H), air temperature in heating room (Ts), environment temperature (TL), earth gravitation (g). Therefore, mathematical equation for calculating the V was written in the following form:

\[ V = f(d, H, Ts, TL, g) \]
The equation (1) have 6 basic variables, with 3 basic dimension of length (L), time (θ), and temperature (T). It can be solved by dimensional analysis with variables involved in the analysis are presented in Table 1.

**Table 1. Dimension variables**

| No. | Name of Variable                | Symbol | Unit  | Dimension |
|-----|--------------------------------|--------|-------|-----------|
| 1.  | Exhaust air flow velocity      | V      | m/s   | Lθ⁻¹      |
| 2.  | Chimney diameter               | d      | m     | L         |
| 3.  | Chimney height                 | H      | m     | L         |
| 4.  | Heating room temperature       | Ts     | C     | T         |
| 5.  | Environment temperature        | TL     | C     | T         |
| 6.  | Earth gravitation              | g      | m/s²  | Lθ⁻²      |

Solution of dimensional analysis was made using “Buckingham π Theorem” [6 - 8]. Relation form of dimensionless number (π) function was written in the following form:

\[ \sum \text{Number of } \pi = \sum \text{Number of basic variable} - \sum \text{Number of basic dimensions} \]

Hence : \( \pi = 6 - 3 = 3 \)

**Dimensional equation :**

\[
V^c_1 d^c_2 H^c_3 T_s^c_4 T_L^c_5 g^c_6 = 0
\]

\[
(Lθ^-1)(L)(L)(T)(T)(Lθ^-2)
\]

L : \( c_1 + c_2 + c_3 + c_6 = 0 \) ....... (1)

T : \( c_4 + c_5 = 0 \) ........................ (2)

\( θ : -c_1 -2c_6 = 0 \) ........................ (3)

There are 6 main variables to be searched for, but only 3 equations are available, so to solve the equation, 3 variables must be assigned a value (any)

**For determining \( \pi_1 \)**

for example, the values given are \( c_1, c_3, \) and \( c_5 \), so that the remaining variables are \( c_2, c_4, \) and \( c_6 \)

The matrix of the remaining variables must have a non-zero determinant

\[
\begin{matrix}
  c_2 & c_4 & c_6 \\
  1 & 0 & 1 \\
  0 & 1 & 0 \\
  0 & 0 & -2 \\
\end{matrix}
\]

L : \( \text{det} = 1 \)

T : \( \text{det} = -2 \)

(\( \text{det} \) is not equal to zero) → meet the independent conditions

\( c_1, c_3, \) and \( c_5 \) are assigned any values, for example \( c_1 = 1, c_3 = 0, \) & \( c_5 = 0, \) then get into the dimensional equation

\( (1) : 1 + c_2 + 0 + c_6 = 0 \)

\( (2) : c_4 + 0 = 0 \)

\( (3) : -1 -2c_6 = 0 \)

will be obtained → \( c_4 = 0; c_6 = -1/2; c_2 = -1/2 \)

\( \pi_1 = V^c_1 d^c_2 H^c_3 T_s^c_4 T_L^c_5 g^c_6 \)
so that: \[ \pi_1 = V \ d \ H \ Ts \ TL \ g \]
\[ \pi_1 = \frac{V}{(d \cdot g)^{1/2}} \]

**For determining \( \pi_2 \)**

\( c_1, c_3, \) and \( c_5 \) are assigned any values,

for example \( c_1 = 0, \ c_3 = 1, \) & \( c_5 = 1, \) then get into the dimensional equation

(1): \( 0 + c_2 + 1 + c_6 = 0 \)

(2): \( c_4 + 1 = 0 \)

(3): \( 0 - 2c6 = 0 \)

will be obtained \( \Rightarrow c_4 = -1; \ c_6 = 0; \ c_2 = -1 \)

\[ \pi_2 = V \ d \ H \ Ts \ TL \ g \]
\[ \begin{bmatrix} 0 & -1 & 1 & -1 & 1 & 0 \end{bmatrix} \]

so that: \( \pi_2 = \frac{H \cdot TL}{d \cdot Ts} \)

**For determining \( \pi_3 \)**

\( c_1, c_3, \) and \( c_5 \) are assigned any values,

for example \( c_1 = 0, \ c_3 = 0, \) & \( c_5 = 1, \) then get into the dimensional equation

(1): \( 0 + c_2 + 0 + c_6 = 0 \)

(2): \( c_4 + 1 = 0 \)

(3): \( 0 - 2c6 = 0 \)

will be obtained \( \Rightarrow c_4 = -1; \ c_6 = 0; \ c_2 = 0 \)

\[ \pi_3 = V \ d \ H \ Ts \ TL \ g \]
\[ \begin{bmatrix} 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix} \]

so that: \( \pi_3 = \frac{TL}{Ts} \)

Hence the basic function can be written in the following form:
\[ \pi_1 = F(\pi_2, \pi_3) \]

which can be written in the following form:
\[ \frac{V}{(d \cdot g)^{1/2}} = F\left[\frac{H \cdot TL}{d \cdot Ts}, \frac{TL}{Ts}\right] \]

3. **Results and discussions**

Data from measurement of the laboratory experiment, by using 3 variations of heat generator temperature, 3 variations of chimney height, and 3 variations of chimney diameter can be seen in Table 2.
Table 2. Measurement data of research results

| No | (d) (m) | (H) (m) | (Ts) (C) | (TL) (C) | (g) (m/s^2) | (V) (m/s) |
|----|---------|---------|----------|----------|-------------|----------|
| 1  | 0.06    | 0.50    | 100      | 30       | 10          | 3.8      |
| 2  | 0.10    | 0.50    | 99       | 29       | 10          | 3.0      |
| 3  | 0.12    | 0.50    | 98       | 28       | 10          | 2.9      |
| 4  | 0.06    | 0.75    | 102      | 31       | 10          | 4.8      |
| 5  | 0.10    | 0.75    | 101      | 30       | 10          | 3.9      |
| 6  | 0.12    | 0.75    | 98       | 29       | 10          | 3.7      |
| 7  | 0.06    | 1.00    | 100      | 30       | 10          | 6.5      |
| 8  | 0.10    | 1.00    | 99       | 28       | 10          | 5.8      |
| 9  | 0.12    | 1.00    | 100      | 30       | 10          | 4.1      |
| 10 | 0.06    | 0.50    | 150      | 32       | 10          | 9.5      |
| 11 | 0.10    | 0.50    | 152      | 31       | 10          | 7.5      |
| 12 | 0.12    | 0.50    | 151      | 30       | 10          | 6.9      |
| 13 | 0.06    | 0.75    | 150      | 29       | 10          | 12.1     |
| 14 | 0.10    | 0.75    | 149      | 28       | 10          | 10.5     |
| 15 | 0.12    | 0.75    | 151      | 30       | 10          | 9.1      |
| 16 | 0.06    | 1.00    | 150      | 30       | 10          | 13.2     |
| 17 | 0.10    | 1.00    | 149      | 28       | 10          | 12.3     |
| 18 | 0.12    | 1.00    | 148      | 29       | 10          | 11.1     |
| 19 | 0.10    | 0.50    | 200      | 30       | 10          | 16.5     |
| 20 | 0.12    | 0.50    | 198      | 31       | 10          | 14.4     |
| 21 | 0.10    | 0.75    | 199      | 30       | 10          | 18.0     |
| 22 | 0.12    | 0.75    | 198      | 28       | 10          | 16.8     |
| 23 | 0.10    | 1.00    | 200      | 30       | 10          | 20.5     |
| 24 | 0.12    | 1.00    | 201      | 31       | 10          | 19.6     |

Six basic variable which used:
Ts = heat generator room temperature; TL = system outside air temperature; d = chimney diameter; H = chimney height; V = exhaust air flow velocity; g = gravitation of earth

Using data of Table 2, can be calculated for every value of dimensionless number (π). Next step, using the value of dimensionless number (π) be solved by matrix solution of linear programming [7 - 9]. Matrix solution of linear programming was used to obtain value of b2, b3, and than to get b1, as follow:

\[
\begin{bmatrix}
\sum (\pi_3 - \bar{\pi}_3)^2 \\
\sum (\pi_3 - \bar{\pi}_3)(\pi_2 - \bar{\pi}_2)
\end{bmatrix}
\begin{bmatrix}
\pi_1 \\
\pi_2
\end{bmatrix}
= \begin{bmatrix}
\sum (\pi_2 - \bar{\pi}_2)(\pi_1 - \bar{\pi}_1) \\
\sum (\pi_3 - \bar{\pi}_3)(\pi_1 - \bar{\pi}_1)
\end{bmatrix}
\]

\[\bar{\pi} = \text{average value of } \pi\]
\[b_1 = \pi_1 - (b_2)\bar{\pi}_2 - (b_3)\bar{\pi}_3\]
\[\pi_1 = b_1 + (b_2)\pi_2 + (b_3)\pi_3\]

Arranged a logaritmic equation that correspoding to equation (2), and solved by matrix solution, as follow:

\[\log \pi_1 = f(\log \pi_2, \log \pi_3)\]
\[\log \pi_1 = b_1 + (b_2)\log \pi_2 + (b_3)\log \pi_3\]
\[ \pi_1 = 10^{b_1} \pi_2^{b_2} \pi_3^{b_3} \]

Be obtained a mathematical model of air flow velocity (V)

\[ \pi_1 = 10^{-1.29} \pi_2^{0.817} \pi_3^{-3.05} \]

\[ \frac{V}{(d.g)^{0.5}} = 10^{-1.29} \left( \frac{H.TL}{d.Ts} \right)^{0.817} \left( \frac{TL}{Ts} \right)^{-3.05} \]

\[ V = 10^{-1.29} \left( \frac{H.TL}{d.Ts} \right)^{0.817} \left( \frac{TL}{Ts} \right)^{-3.05} (d.g)^{0.5} \] ................................. (3)

For knowing accuracy of the mathematical model above, the equation of math model be evaluated by comparing between observation data and prediction value of airflow velocity. The Result can be seen as follow in Graph (Figure 2), and Table 3

**Figure 2.** The relationship between Vprediction and Vobservation (average error 9.03 %)

Based on the mathematical model obtained (equation 3), it can be seen that the magnitude of the value of the exhaust airflow velocity (V) is largely determined by the ratio of ambient temperature (TL) and heat generator room temperature (Ts). This can be seen from the greatest power of the ratio equation TL and Ts, which is equal to 3.05. So, the higher the difference between heating room temperature (Ts) and ambient air temperature (TL), will cause an increase in the velocity of airflow out of the chimney. This logic is very rational, because the higher the air temperature will cause the relative humidity (RH) to decrease, the smaller the relative humidity results in the smaller the air density. As a result, the lower air density will cause the air to become lighter and lift upward and cause airflow out of the chimney [10 - 14].
Table 3. The measurement results and estimated velocity of air flow out of the chimney

| No. | Vpred (m/s) | Vobs (m/s) | error (%) |
|-----|-------------|------------|-----------|
| 1   | 3.3         | 3.8        | 13.16     |
| 2   | 2.96        | 3.0        | 1.33      |
| 3   | 2.96        | 2.9        | 2.07      |
| 4   | 4.47        | 4.8        | 6.88      |
| 5   | 4.00        | 3.9        | 2.56      |
| 6   | 3.81        | 3.7        | 2.97      |
| 7   | 5.82        | 6.5        | 10.46     |
| 8   | 5.65        | 5.8        | 2.59      |
| 9   | 4.67        | 4.1        | 13.90     |
| 10  | 7.07        | 9.5        | 25.58     |
| 11  | 6.65        | 7.5        | 11.33     |
| 12  | 6.66        | 6.9        | 3.48      |
| 13  | 12.27       | 12.1       | 1.40      |
| 14  | 11.12       | 10.5       | 5.90      |
| 15  | 9.27        | 9.1        | 1.87      |
| 16  | 14.39       | 13.2       | 9.02      |
| 17  | 14.07       | 12.3       | 14.39     |
| 18  | 12.09       | 11.1       | 8.92      |
| 19  | 13.21       | 16.5       | 19.94     |
| 20  | 11.33       | 14.4       | 21.32     |
| 21  | 18.19       | 18.0       | 1.06      |
| 22  | 19.8        | 16.8       | 17.86     |
| 23  | 23.27       | 20.5       | 13.51     |
| 24  | 20.64       | 19.6       | 5.31      |

To determine the sensitivity level of the variables involved in this study, a sensitivity analysis was carried out on the mathematical model that had been built. The results of the sensitivity analysis of the mathematical model can be seen in Table 4. Based on the results of the sensitivity analysis, it was found that the variable room temperature of the heat generator (Ts) was the most sensitive variable, with sensitivity in the form of an increase in air flow velocity (dV) of 23.72% for every 10% increase in the room temperature of the heat generator (Ts). This fact informs us that if you want to change the speed of the air flow out of the outlet chimney, the most influential thing is to change the temperature of the air in the heat generator room. The next sensitivity is the ambient temperature (TL), the lower the ambient temperature will cause an increase in the chimney exit air flow (dV), with an increase of 19.17% for every 10% decrease in ambient air temperature [7, 15].
Table 4. Sensitivity analysis of airflow velocity (V) to changes in variable increase of 10%

| (m) (d) | (m) (H) | (C) (Ts) | (C) (TL) | (m/s²) π² | π3 | Vpred (m/s) | ΔV (%) |
|--------|--------|---------|---------|----------|----|-------------|--------|
| 0.1 0.5 | 99 29  | 10 1.465 | 0.293  | 2.963   |
| 0.11 0.5 | 99 29  | 10 1.331 | 0.293  | 2.875  -2.98 |
| 0.1 0.55 | 108.9 29 | 10 1.611 | 0.293  | 3.203  8.10 |
| 0.1 0.5 | 99 31.9 | 10 1.611 | 0.322  | 2.395 -19.17 |
| 0.1 0.5 | 99 11  | 10 1.465 | 0.293  | 3.108  4.88 |

4. Conclusion
The developed mathematical model has an average error of 9.03%. This mathematical model can be used as a reference for calculating the airflow velocity in a dryer application with a natural air flow system. In the 6 variables studied in this experiment, the change in temperature in the heating room (Ts) is the variable most sensitive to changes in air flow velocity (V)

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