Obtaining the size distribution of fault gouges with polydisperse bearings

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We generalize the recent study of random space-filling bearings to a more realistic situation, where the spacing offset varies randomly during the space-filling procedure, and show that it reproduces well the size-distributions observed in recent studies of real fault gouges. In particular, we show that the fractal dimensions of random polydisperse bearings sweep predominantly the low range of values in the spectrum of fractal dimensions observed along real faults, which strengthen the evidence that polydisperse bearings may explain the occurrence of seismic gaps in nature. In addition, the influence of different distributions for the offset is studied and we find that the uniform distribution is the best choice for reproducing the size-distribution of fault gouges.

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I. INTRODUCTION

In the early nineties the question of the possibility to tile an arbitrarily large strip of space-filling roller bearings without friction nor slipping was addressed [1], motivated by the study of real systems such as seismic gaps. Seismic gaps are regions along a fault zone where earthquakes do not take place and therefore they could be explained by sheared plates on a space-filling bearing [2]. Fault zones are typically self-similar and the mechanical origin of the power-law in the particle size distribution was associated to the particle’s fracture probability which has been proposed to be controlled by the relative size of its nearest neighbors [3, 4]. More recently, a geophysical model [5] explained the different values of the fractal dimension, ranging from \( d_f = 2.6 \) to \( d_f \approx 3 \), by taking into account the fault gouge strain. While such model explains the dynamical origin of different power-laws, there is still the question if the space-filling bearing scenario is able to reproduce such empirical results in a simple and systematic way.

By reproducing with space-filling bearings the same particle size distributions observed in fault zones one can strengthen the hypothesis that the existence of seismic gaps in fault zones may be related to the emergence of particular geometrical arrangements of their composing rocks, due to local fragmentation during tectonic motion. Figure 1 illustrates two random space-filling bearings in two and three dimensions.

Pioneering studies with space-filling bearings were done using deterministic procedures in two [1] and three [6] dimensions and also using random algorithms [7]. However, up to now specific initial configurations and fixed parameter values were addressed. In this paper we present a general algorithm to construct realistic space-filling bearings that allows to reproduce the range of fractal dimensions observed in fault gouges [3, 4, 5, 8]. The model is parameterized by a unique parameter that controls the strength of fragmentation and takes into account ensemble averages. Despite the wider freedom in the parameters and initial configurations, the systems presents robust results in what concerns the fractal dimension. In particular, we will show that by varying the range of admissible values of the control parameter one finds fractal dimensions observed in fault gouges.

We start in Section II by describing in some detail the procedure to generate random space-filling bearings, introducing a parameter that accounts for fragmentation at the local scale. In Sec. III and IV we describe the results for two and three dimensions respectively, with special emphasis on the size distribution and the fractal dimension. Discussions and conclusions are given in Sec. V.

FIG. 1: (Color Online) Illustration of a random space-filling bearing in (a) three and in (b) two dimensions. Discs and spheres of the same color do not touch each other. The random space-filling bearing starts with a large disc or sphere, maximizing polydispersity (see text).
II. THE RANDOM SPACE-FILLING OF PARTICLES

In this Section we will start by revisiting previous procedures [7] for constructing random space-filling packings and bearings and then introduce the necessary ingredients to obtain a fully random space-filling bearing.

Random bearings in two and three dimensions are constructed in the following way. First, one starts by randomly distributing a small number $N_0$ of discs or spheres within a given range of sizes, without touching each other. Second, one fills the empty spaces in the system by introducing iteratively the biggest possible disc or sphere in the neighborhood of some empty region. Third, one resizes some disc or sphere in order for the packing to be bi-chromatic (bearing condition), i.e. only two colors are needed to color all discs in such a way that no discs of the same color touch each other. This guarantees the bearing condition: particles are able to roll on each other without friction or slipping. Figures (a) and (b) give illustrative examples of such random bearings in two dimensions.

The filling procedure is done by choosing randomly a void within the inter-disc free space and then fitting the biggest disc in it, i.e. fit the disc that touches the three nearest discs in the neighborhood, as illustrated in Fig. 2a. For the three dimensional case one considers spheres touching the four nearest neighbors.

The coloring procedure is done by attributing a proper color to the introduced disc. In the case that the three neighboring discs have the same color one attributes the other color to the new disc. Otherwise, one chooses only one of the neighbors to be in contact with the new disc, and the new disc shrinks to a size with radius $r = \alpha r_0$ ($0 \leq \alpha \leq 1$), where $r_0$ is the radius before shrinking and gets a different color as the disk it touches. Figure 2b illustrates this coloring procedure.

Parameter $\alpha$ is our control parameter. For constant $\alpha = 1$ one obtains the particular case where bearing cannot be guaranteed. In this case, frustrated contacts emerge when particles are forced to rotate [7], which would eventually lead to the fragmentation of the discs into smaller ones. Recently a new method to implement realistic grain fracture in three-dimensional simulations of granular shear was proposed [9], based on breakable bonds between particles within a medium. We keep the model simple, by using instead the reduction factor $\alpha$ that mimics the effect of fragmentation: by shrinking a particle originally with frustrated contacts we mimic its fragmentation into smaller particles that will fill the empty space left after the fragmentation.

The algorithm described above was previously [7, 10] used in two and three dimensions by fixing a given initial configuration with $N_0$ discs having sizes within a given fixed range and also by using a fixed reduction factor $\alpha$ during the filling and coloring stages. The density of the packing was studied as a function of the number $N$ of existing spheres as well as the cumulative particle size distribution. It was found that the cumulative distribution obeys a power-law, namely

$$N(r) \equiv \int_{r}^{\infty} n(q)dq \sim r^{-d_f},$$

where $d_f$ is the fractal dimension of the bearing [12].

Next, we introduce the additional points to strengthen previous findings and improve the algorithm described above.

First, there is the statistical significance of the results, and their sensitiveness to initial configurations, i.e. to the initial range $[r^* - \delta/2, r^* + \delta/2]$ of sizes. This initial configuration may influence the polydispersity of the system and consequently the attained distribution after filling the entire system. As we will see, large number $N_0$ of initial discs typically influence how the density increases during the space-filling procedure.

To take this point into account we enable the construction procedure to start with an initial configuration having a single arbitrarily large disc (or sphere) and perform ensemble averages on a significant number of initial configurations. Concerning the single initial large disc, one should notice that it does not suffice one single disc to introduce a second one, because in two-dimensions each inserted disc needs to have at least three neighbors (four neighbours for three-dimensions). However, as illustrated in Fig. 2a, such starting disc can be introduced into the system by previously distributing a few very small ‘seed-discs’ in the system and then following the algorithm described above. The number of such seeds is small, and therefore they do not affect significantly the cumulative size distribution. Their role is that the average distance between them is eventually of the order of the system size, enabling the introduction of a first disc with the size of the order of the system size. Of course, depending on the number and distribution of the seeds, the first initial discs may have also a

![FIG. 2](Color Online) Sketch of the construction of a random space-filling bearing. (a) A new disc (blue) is randomly inserted in the system and is shifted and enlarged to the maximal accessible size (dashed circle) without overlapping neighboring discs. Then, (b) it is reduced by a factor $0 < \alpha \leq 1$, keeping a single contact point with one of the neighbors and assuming the opposite color. (c) To start the space-filling with a large disc (brown) one needs to place previously a few small ‘seeds’ (green) in the system (grey) and then proceed as in (a) and (b) [see text].
alistic assumption. Indeed, we show that the typical range of that a certain range of admissible values for by analyzing samples of gouges in real situations, one expects centered around different values, namely around $\rho^*$.

FIG. 3: Density and size distribution in two-dimensional random space-filling packings. (a) Density $\rho$ of the packing as a function of the total number of discs $N$ for different values of $\alpha = 0.2, 0.6$ and 1.0, together with (b) the distribution of the radius $r$ of the discs. In both cases, one starts from a fixed initial configuration of $N_0 = 40$ discs with radius in the range $[0.08R, 0.14R]$ with $R = 1$ being the radius of the system (see Fig. 1b). The minimal radius of the initial set of discs is indicated as $r_m \sim 0.08$ (see text). Fitting of the power-law range in (b), the distribution $N(r) = br^{-d_f}$ yields the fractal dimension $d_f$ as a function of $\alpha$ plotted in the inset. In (c) one plots the density as a function of $N$ for the initial sets of $N_0 = 40$ discs in $[0.05R, 0.10R]$, $[0.15R, 0.20R]$ and $[0.20R, 0.25R]$, i.e. ranges with the same interval width but centered around different values, namely around $r^* = 0.075$, 0.175 and 0.225 respectively and fixing $\alpha = 0.6$. In the inset of (c) the density $\rho(N)$ is plotted for initial ranges $[r^* - \frac{\delta}{2}, r^* + \frac{\delta}{2}]$ having the same center $r^* = 0.075$ but different widths $\delta = 0.2, 0.4, 0.6$ and 0.8. In (d) we plot distribution $N(r)$ of the spheres as a function of $r$ for the same conditions as in (c). In all cases $N = 10^5$ discs.

size within the initial range of sizes. In this way, one generalizes the previous procedure [2] and maximizes the admissible polydispersity.

Second, we also introduced a criterion to increase computational efficiency of our algorithm. The neighborhood were the neighboring discs (or spheres) are searched for must be chosen conveniently. We propose to choose a size that decreases with the increase of the density $\rho$, since the denser the packing the smaller are the empty spaces to put new discs. Therefore, the radius $r_n$ of the neighborhood of a given random point introduced in the system at iteration $n$, is updated as $r_n = \frac{1}{\rho_n} (r_{sys} - r_{max})$, where $r_{sys}$ and $r_{max}$ are the radii of the system and of the biggest disc or sphere in it, respectively.

Third, we also consider the control parameter $\alpha$ to vary randomly within a tunable range of values. In particular, we argue that though a tentative value of constant $\alpha$ could be obtained by analyzing samples of gouges in real situations, one expects that a certain range of admissible values for $\alpha$ is the most realistic assumption. Indeed, we show that the typical range of fractal dimensions observed in real fault gouges is in this way reproduced.

III. THE TWO-DIMENSIONAL CASE

We start this section by addressing the two-dimensional random space-filling bearing and systematically reviewing the behavior of the packing for different, but fixed, values of $\alpha$ and study the effect of fixed initial size ranges (no maximal admissible polydispersity). Figure 3 shows for this case the density $\rho(N)$ and cumulative size distribution $N(r)$ of two-dimensional random space-filling packings.

Figure 3a shows the density $\rho$ as a function of the number $N$ of discs for $\alpha = 1.0$ (packing) and also for $\alpha = 0.2$ and 0.6 (bearings) separately, starting from $N_0 = 40$ discs with radius in the range $[r^* - \frac{\delta}{2}, r^* + \frac{\delta}{2}]$ having $r^* = 0.11$ and $\delta = 0.06$. As expected, the convergence $\rho \to 1$ as $N$ increases is faster for larger values of $\alpha$. We consider one fixed initial configuration with $N_0$ initial discs, and therefore the different
curves coincide for $N < N_0$.

In Fig. 3b we plot the distribution of the radius $r$ of the discs, where $r_m = r^* - \frac{\Delta r}{2}$ is the minimal radius of the initial set of discs, and the deviation from the power law for $r > r_m$ is due to the initial configuration. Below this value $r_m$, the size distribution obeys a power-law $N \sim r^{-d_f}$, where $d_f$ is the fractal dimension of the packing, plotted in the inset as a function of $\alpha$ (symbols). As one sees from the inset, the fractal dimension typically takes values in the range $1.2 < d_f < 1.4$, differently from the values found in two-dimensional cuts of fault gouges ($\sim 1.6 \pm 0.1$). There is a maximum of $d_f$ for $\alpha = 0.5$ that can be explained from the definition of $\alpha$ in the algorithm described above. For an $\alpha < 0.5$, to each new disc introduced there is a remaining free space characterized by $\alpha' > 0.5$ such that $\alpha + \alpha' = 1$, and similarly for $\alpha > 0.5$.

Both Figs. 3a and 3b consider the same initial configuration. To study the influence of the initial configurations, we plot in Fig. 3c: the density $\rho(N)$, fixing $\alpha = 0.6$, similar to previous works[10], and using different size ranges for the initial sets of $N_0 = 40$ discs, namely in $[0.05R, 0.10R]$, $[0.15R, 0.20R]$ and $[0.20R, 0.25R]$, i.e. ranges with the same width $\Delta = 0.06$ but centered around different values, namely around $r^* = 0.075, 0.175$ and 0.225 respectively.

Since different initial configurations are now used, the density is no longer the same below $N_0$ as in Fig. 3a. Further, one observes that the density converges to one for increasing the value of $r^*$. In the inset the density $\rho(N)$ is plotted by fixing $r^* = 0.075$ and starting with initial configuration having different widths, namely $\delta = 0.2, 0.4, 0.6$ and 0.8. In these cases the density gives always similar dependencies on $N$. Therefore, the average size $r^*$ of the initial configuration is the important parameter to tune the density of the packing. Its width can be varied without changing significantly the results.

In Fig. 3d we plot the accumulative size distribution $N(r)$ of the discs for the same conditions as in Fig. 3c. The value of the exponent remains almost constant, $d_f \sim 1.35$. In other words, the fractal dimension is not very sensitive to the initial configuration, and the parameter on which the fractal dimension depends more strongly must be indeed $\alpha$.

Since $\alpha$ is also the parameter controlling the fragmentation of discs with frustrated contacts (see above), we will now study it more deeply. When $\alpha$ is able to vary randomly, the fragmentation of the largest disc in the free holes can be regarded as a random process by its own. We next consider $\alpha$ to be each time randomly selected from a fixed interval $[\alpha^* - 0.5 \Delta \alpha/2, \alpha^* + 0.5 \Delta \alpha/2]$. We will show that when enabling $\alpha$ to take different values for each particle shrinking, one obtains fractal dimensions similar to the ones observed in fault gouges[3, 4, 5].

To this end, we put everything together, namely $\alpha$ varying in the middle range of admissible values, a large initial disc and an ensemble average over a significant number of initial configurations. The results for density and size distribution are shown in Fig. 4 where one considers three initial seeds in the range $r_m = 2r_m \sim R/1000$, with $R$ the size of the system and $\alpha$ varies randomly in the range $[0.5 - \Delta \alpha/2, 0.5 + \Delta \alpha/2]$.

FIG. 4: Averaging (a) the density $\rho$ and (b) the distribution size $N(r)$ over 100 initial configurations starting from large discs ($r \sim R/2$) and with $\alpha$ varying in a range $[0.5 - \Delta \alpha/2, 0.5 + \Delta \alpha/2]$ with $\Delta \alpha = 0.2, 0.4, 0.6$ and 0.8. The inset of (b) shows that the fractal dimension in $N(r) \sim r^{-d_f}$ is almost independent of $\Delta \alpha$ yielding $d_f \sim 1.54$ which is, within the numerical errors of real fault gouges (see text).
of discs for the inset where the fractal dimension of the first large disc. For all the four cases the dependence towards cause they hinder the occurrence of large discs.

Averages over a sample of 100 initial configurations. Since initialization, filling and coloring procedure are now all random, we call these systems fully random space-filling bearings.

Figure 4b shows the density as a function of the number \( N \) of discs for \( \Delta \alpha = 0.2, 0.4, 0.6 \) and 0.8. One sees an abrupt transition above \( N = 3 \) (initial seeds), due to the introduction of the first large disc. For all the four cases the dependence of \( \rho \) on the range of \( \alpha \)-values is similar, with the convergence towards \( \rho = 1 \) being slightly slower for narrower ranges, because they hinder the occurrence of large discs.

Figure 4b shows the size distribution \( N(r) \) for each of the four ranges. All the distributions almost coincide, as shown in the inset where the fractal dimension \( d_f \) taken from \( N(r) \sim r^{-d_f} \) is almost constant \( (d_f \sim 1.54) \). This value is larger than the one obtained when \( \alpha \) is kept constant (see Fig. 3). Notice that the value of the fractal dimension for \( \Delta \alpha = 0 \), though corresponding to the case of constant \( \alpha = 0.5 \), is different from the one plotted in the inset of Fig. 3b, since the constructing procedure of the bearing is slightly different (see Sec. II).

Since the above value is obtained from a significantly larger sample of initial configurations and all the parameters \( \alpha \) and position of the discs are randomly selected, we will consider this value \( d_f \sim 1.54 \) as the characteristic exponent of the size distribution for fully random two-dimensional space filling bearings. The average characteristic value \( d_f \) obtained lies in the range of values measured of the fractal dimension measured in real fault gouges \((D = 1.6 \pm 0.1)\) \([3,4]\), as indicated with a dashed line and shadow region in the inset of Fig. 3b.

Till now, the value of \( \alpha \) was considered to vary uniformly within a certain range of values. If the probability distribution for choosing \( \alpha \) values is Gaussian similar results are obtained, were the standard deviation \( \sigma \) of the distribution plays a similar role as the width \( \Delta \alpha \) used in Fig. 4.

However, if we take values within a range, say \( \alpha \in [0.1, 0.9] \), and chose them according to a power-law distribution \( P(\alpha) \sim \alpha^{-\beta} \) the fractal dimension changes significantly, as shown in Fig. 5. Even for small values of the exponent \( \beta \), e.g., \( \beta = 0.5 \), the fractal dimension decreases when compared to the value obtained for the uniform distribution \((\beta = 0)\) and remains approximately constant at \( d_f \sim 1.43 \).

Notice that \( \beta = 0 \) in Fig. 5 corresponds to \( \Delta \alpha = 0.8 \) in Fig. 5b. If the power-law distribution selects values in a range with a different width, a similar decrease of the fractal dimension is observed when comparing with the uniform distribution case. Therefore, one can conclude that a reasonable choice for constructing space-filling bearings with fractal dimension similar to the one observed in fault gouges is by taking a random value of \( \alpha \) uniformly distributed in a certain range around 0.5.

### IV. THE THREE-DIMENSIONAL CASE

As described above in Sec. II a three-dimensional version of fully random space-filling bearings is obtained in a similar way as for discs with the single difference that the introduction of new spheres takes into account four nearest neighbors. In this Section we address the case of three-dimensional space-filling bearings as a more realistic approach to fault gouges, and study how well the two-dimensional model approximates three-dimensional systems of spheres.

Recently \([3,4]\), it was found that grain fracture simulations produce a comminuted granular material similar to the one observed in real fault gouges. From those simulations, it followed that comminution rate and survival of large grains

![Figure 5: The fractal dimension as a function of the exponent \( \beta \) when the value of \( \alpha \) is chosen according to a power-law \( P(\alpha) \sim \alpha^{-\beta} \) in a range \( \alpha \in [0.1, 0.9] \). Although within the error bars, the fractal dimension is somewhat lower compared to Fig. 4b where the distribution of chosen \( \alpha \) values is uniform (see text).](image1)

![Figure 6: The fractal dimension as a function of \( \alpha \) for three-dimensional random space-filling bearings when \( \alpha \) is kept constant. A similar procedure as the one illustrated in Fig. 3a is used, with new spheres being introduced touching the four nearest neighbors (see text). The dashed line indicates one typical value \( d_f \sim 2.58 \) found in some real fault gouges.](image2)
is sensitive to applied normal stress, with a fractal dimension of the resultant grain size distributions in the range \(d_f \in [2.3 \pm 0.3, 2.9 \pm 0.5]\), that agrees with the observations of three-dimensional samples of real gouges where typically \(d_f \sim 2.58\).

In three dimensional space-filling bearings with a constant value of \(\alpha\) the fractal dimension lies above the observed values in fault gouges. In Fig. 6 we plot typical values of \(d_f\) as a function of \(\alpha\). The fractal dimension of such bearings is typically larger than \(d_f \gtrsim 2.58\) (dashed line) with values within the range \(d_f \in [2.60 \pm 0.10, 2.74 \pm 0.15]\) and, similarly to the two-dimensional case, the maximum of \(d_f\) is reached for \(\alpha \sim 0.5\).

As summarized above in the introduction, it was recently found [5] that fractal dimensions \(\sim 2.6\) are observed for low-strain gouges. In regions subject to larger shear strain the fractal dimension is significantly larger, \(\lesssim 3\). Therefore, the particle size or mass dimensions were proposed as a way to distinguish between regions with different strain strengths [5]. From Fig. 6 one sees that a similar range of values for the fractal dimension is also found for space-filling bearings.

Furthermore, the explanation relating the fractal dimension of fault zones and their strain strength assumes that fragmentation is controlled by nearest neighboring particle contact and that a particle is most likely to split into smaller particles with a particle of similar size, yielding a larger fractal dimension \(\sim 3\). In the case of our construction procedure for space-filling bearings this would correspond to the case of \(\alpha \sim 0.5\). Indeed, from Fig. 6 one observes that the maximum of the fractal dimension is reached for such \(\alpha\) values yielding \(d_f = 2.74 \pm 0.15\).

When varying \(\alpha\) randomly in a range around 0.5 and study
the dependence of the space-filling bearing on the width $\Delta \alpha$ of the range $[0.5 - \Delta \alpha / 2, 0.5 + \Delta \alpha / 2]$. In Fig. 7a one sees that the density increases faster for larger $\Delta \alpha$, similarly to what was shown in Fig. 4b. As for the fractal dimension, Fig. 7b shows that it decreases slightly when compared with the case of constant $\alpha$ ($\Delta \alpha = 0$). Therefore, increasing the width $\Delta \alpha$ of the range of admissible values for $\alpha$ one is able to reduce the fractal dimension of the bearing.

Similarly to the situation of measures taken in fault gouges, the two-dimensional cross section of such three-dimensional space-filling bearings should have a fractal dimension within the range of the observed empirical values in real fault gouges ($d_f = 1.6 \pm 0.1$ [3, 4]). By averaging several different two-dimensional cross sections of the 3D bearings we plot in Fig. 8a the size distribution of a typical two-dimensional cross section for the different values of $\Delta \alpha$. In Fig. 8b we observe that only for very wide ranges of $\alpha$ values it is possible to obtain a fractal dimension similar to the one observed on fault zones.

V. DISCUSSION AND CONCLUSIONS

In this work we studied the size-distribution of random space-filling bearings with large polydispersity, showing that it reproduces well the size-distribution found in fault gouges. Focusing on the dependence of the bearings fractal dimension on the spacing offset, we have shown that the fractal dimensions of such bearings sweep the low range of values observed in real faults. Since recently it has been reported that the fractal dimension varies in space along fault gouges [5], our findings enables us to conjecture that the occurrence of seismic gaps, where earthquakes are absent and therefore behave similarly to roller bearings, may occur in regions where the fractal dimension lies in the low range of admissible values, namely $d_f \in [2.5, 2.75]$.

To compute an accurate value for the exponent characterizing random bearings, we introduced a general algorithm that allows $\alpha$ to vary randomly in a wide range of admissible values, typically $0 < \alpha < 1$ and start the space-filling procedure from one unique large disc (or sphere), maximizing the range of admissible sizes in the bearing.

With such model we were able to show that bearings have a fractal dimension with values within the range of values in real fault. Since it is known [5] that along a specific fault gouge the fractal dimension varies typically between $\sim 2$ and $\sim 3$, our results support the hypothesis that seismic gaps, occurring only in certain particular locations of the fault, could be explained by this simple geometrical model.

Further, we also interpret the control parameter $\alpha$ for the bearing property as a measure of the fragmentation strength, and introduce simple criteria to improve the computational efficiency of previous space-filling packing algorithms.

To improve further our findings we should also take the effect of gravity into account. Moreover, concerning the space-filling bearings by themselves, other questions raise, namely their contact network correlations, which should help to understand the range of observed fractal dimensions. These and other points will be addressed elsewhere.

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