The development of teaching material: Rigorous mathematical thinking in a geometry classroom

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Abstract. This study aimed to develop a teaching material, a textbook, using a Rigorous Mathematical Thinking (RMT) approach. It was constructed by considering some activities involving problem-solving, problem posing, open-ended problem, connection problem, conjecture, creativity task, mini lab, and drawing skill. The subject in this study was the undergraduate mathematics students who enrolled in the geometry subject in the University of Wijaya Kusuma Surabaya. A textbook was developed because of the necessity. It was developed using the Mediated Learning Experience based on the Rigorous Mathematical Thinking. This study employed Plomp's Research Design model consisting of three phases: the preliminary research, prototyping phase, and assessment phase. However, this paper focused only on the validation of the textbook involving five experts. Students have been mediated to construct their understanding by using mental operation. One important task about creative thinking including in the textbook dealing with the trapezium definition. Students were asked to create a new definition of trapezium which is different from the preceding definition they learned at school. The results showed that the development of student book using Rigorous Mathematical Thinking approach was valid.

1. Introduction
Geometry contains symbols which are challenging to be understood by students without guidance, directions, and mediation from teachers or geometry experts. The symbols can be used to help students to solve geometry problems. However, students with low capabilities in geometry still experience some difficulties in solving them. Therefore, it is necessary to provide a geometry expert to mediate the learning process of geometry concepts. Mediator stimulates students to optimally utilize their mathematical psychological tools for a good understanding of the concepts so that they will not encounter any difficulties in solving the problems. The textbook is one of the teaching materials used to assist the learning process to achieve the objectives. It is categorized as a handbook written by the relevant experts satisfying the rules of textbooks, being officially published and disseminated [8].

A geometry textbook is designed to utilize the mathematical psychological tool as one of the aspects emphasized on teaching processes involving Rigorous Mathematical Thinking (RMT) interventions [1]. Vygotsky stated that the development of the higher mental process of students depends on the mediation in the interaction with the environment. Vygotsky emphasized that the symbolic mediator is tailored to the students’ specific socio-cultural context. Based on the personal experience, there are some issues
found in a geometry classroom, including: (1) the delay of students in following the lecturers’ explanations, (2) students who cannot draw the cross line ask the other students who also cannot draw the line intercept resulting in the student was not successful in drawing (3) students do not try to search other resources instead they only rely on the lecturers, and (4) students’ ability to look at the spatial perspective using a textbook is lacking, resulting in the classroom learning being organized, especially in designing the learning activities playing the mediator role between students [3] and the lecturer's role is changed from being an information provider to the source of the mediated learning experience [4].

The Rigorous Mathematical Thinking (RMT) paradigm enables to produce a textbook which culturally describes the abstract representations to build an understanding, transforming, generalizing, generating the conceptualization and understanding of coherent and logical ideas as well as planning how to use ideas to facilitate problem solving, to reduce the new knowledge in writing and systematic in the form of textbooks. Some rules of writing textbooks are using scientific references, writing clearly, correctly, nor abstractly or out of the field of knowledge; using language effectively; presenting an attractive style of the layout; well organized in term of both the content and the writing standards including the design of the cover, colour, and graphics [5]. Thus, the textbook contents must satisfy the scientific truth and in line the description of the geometry course. Also, the language used in the textbook must align with the rules of excellent language and legibility. The textbook must be arranged in a systematic and coherent hierarchy of science. The layout and the design are tailored to the students as the users.

The textbook discussed in this paper used the Plomp development model consisting of the preliminary investigation phase, the design phase, the realization/construction phase, and the phase of test, evaluation, and revision, and implementation.

2. Method

The process of developing the geometry textbook used the Mediated Learning Experience based on the Rigorous Mathematical Thinking. It is developed by referring to the Plomp’s Research Design model involving three phases: the preliminary research, prototyping phase, and assessment phase. Preliminary research phase was conducted by doing the prerequisite analysis, the context analysis, literature study, and create the development of concepts. In the prerequisite analysis phase, we collected some information as the requirement from books or the internet. Then, we selected and grouped the materials according to the descriptions of the geometry courses aligned with the curriculum. In the phase of the context analysis, the researcher performed student and material analysis. The literature study was conducted to obtain a learning theory suitable for developing the geometry textbook, in this study the theory of Rigorous Mathematical Thinking (RMT) was adopted. The illustration of the textbook development process can be seen in Figure 1.
The next activity in this study was gathering some information related to what, why, who, where, which, and how. According to Figure 1, the final result is a draft of the geometry textbook for students. Thus, the next step was to determine the geometry materials to be compiled. Based on the theory referred, the RMT-based geometry textbook should include the cognitive development phase, the content as a development process, and the conceptual cognitive construction practice. The implementation activities undertaken were the preparation of the early prototype of geometry material draft. Next, the draft contents were then compiled into a geometry book draft.

Following the previous step, we developed an instrument to validate the textbooks including the content feasibility components such as the dimensions of knowledge and skill dimensions. The instruments also had the feasibility of presentation consisting of the presentation techniques, supporting the materials and learning presentation. The feasibility of the content in the knowledge dimension included the scope of the materials consisting of the coverage, breadth, and depth. The accuracy of the material dealt with the accuracy of the facts/notation/symbols, concepts/definitions, principles (theorems, axioms, propositions, properties, rules, laws), as well as procedures/algorithms, samples, and questions. Based on the law and legislation, the contents of the textbook is a subject of the intellectual property that should be free from racism, pornography as well as gender, area, and occupation bias. The skill dimensions criteria included the coverage and activity accuracy. The feasibility component of the content in the presentation technique involved the systematic consistency criteria presented in the chapter as well as the presentation of logic and demands. To support the material, the criteria of the suitability and accuracy of illustrations was needed, including the organization and the concept maps at the beginning of the chapter, summary and exercise problems at the end of each chapter, and the precision of numbering and naming of the tables, figures, and appendix. The presentation of learning consisted of the criteria of students’ active engagement and focused on the students. The final presentation criteria comprise of the preliminary criteria, table of contents, glossary, bibliography, and index.
3. Result and discussion

Table 1 presents the result of the validation process conducted by five selected validators. Overall, the total average validity is 2.545. It means that the textbook is valid and it can be used as a geometry teaching material.

| Table 1. Validation Result. |
|-----------------------------|
| Aspect                     | Criteria          | Validator | Mean Criteria | Mean Aspect |
| Knowledge Dimension         |                  |          | 1 | 2 | 3 | 4 | 5 | |
| Feasibility of Content      |                  |          | 1 | 2 | 3 | 4 | 5 | |
| Skills Dimension            |                  |          | 1 | 2 | 3 | 4 | 5 | |
| Presentation technique      |                  |          | 1 | 2 | 3 | 4 | 5 | |
| Presentation Support        |                  |          | 1 | 2 | 3 | 4 | 5 | |
| Learning Presentation       |                  |          | 1 | 2 | 3 | 4 | 5 | |
| Completeness of Presentation|                  |          | 1 | 2 | 3 | 4 | 5 | |

The total average of validity 2.545
The average score provided by the validators was 2 for the reference/source, text, table, figure, and appendix. The follow-up effort involved a thorough examination of the font, the table, the source, the number of images, the appendix as well as index and glossary. The average score of the feasibility of content in knowledge dimension with material coverage criterion was 2.5. To improve the scores, the researcher added some illustrations, new information, relevant concepts and challenging activities. The variations in the presentation obtained the score of 2.5, some illustrations, examples, and non-examples were added to improve the variations. Also, fixing the order of summaries, clarifying the phrase, and re-examining the items that have been and have not been summarized were some of the efforts to improve the summary sections.

In addition, based on the expert discussion conducted in Aceh on 26-28 June 2018, some revision was required, namely: 1) the layout should be revised based on the standard of textbook writing proposed by the ministry of higher education (DIKTI), (2) there should be an accurate naming, scale and image cropping, (3) the order of image number should be revised and (4) it is necessary to add glossaries.

4. Conclusion
Based on the results of the field analysis of the Geometry textbook, it is concluded that the textbook is valid to be used as literature on geometry course, especially for students of Wijaya Kusuma Surabaya University. Based on the validation conducted by five validators and discussion results, the average score of validation is 2.545 indicating that the textbook is valid and can be used as a geometry textbook with some revision. The validity of the textbook is assessed regarding knowledge and skills. The instruments are assessed for the materials, presentation, legibility and the overall illustrations of the contents of the Geometry book model.

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