Effects of Anisotropy on the Sign-Changeable Interacting Tsallis Holographic Dark Energy

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A spatially homogeneous and anisotropic Bianchi type I universe is considered while it is filled by pressureless dark matter (DM) and Tsallis holographic dark energy (DE) interacting with each other throughout a sign-changeable mutual interaction. Various infra-red (IR) cutoffs are studied, and it has been obtained that while the current universe can classically be stable for some cases, all models display classical instability by themselves at the future \((z \to -1)\). Moreover, we find out that some models can cross the phantom line. In order to have a more comprehensive study, the statefinder diagnostic and the \(\omega_D - \omega_D^\prime\) plane are also investigated showing that the model parameters significantly affect the evolution trajectories in the \(r - s\) and \(\omega_D - \omega_D^\prime\) planes.

\section{I. INTRODUCTION}

Cosmological observations such as type Ia supernova \[1,2\], WMAP \[3,4\] and Large Scale Structure (LSS) \[5–7\] indicate that our Universe is now experiencing a phase of accelerated expansion \[8\]. This accelerated phase of the Universe expansion is describable by a mysterious source of energy (DE) whose energy density \(\rho_{\text{DE}}\) and pressure \(P_{\text{DE}}\) filling about 0.73 percent of the Universe, while \(\rho_{\text{DE}} + 3P_{\text{DE}} < 0\) meaning that its equation of state (EoS) parameter, \(\omega_{\text{DE}} = P_{\text{DE}}/\rho_{\text{DE}}\), must satisfy the condition \(\omega_{\text{DE}} < -1/3\) \[23\]. An interesting attempt to find a physical origin for DE is called the holographic dark energy (HDE) making a relation between the system entropy and the UV cutoff (or equally the energy density of quantum fields in vacuum) \[26\].

Since gravity is a long-range interaction, some physicists tried to study the cosmological evolution by using the generalized entropy formalism in which the Bekenstein entropy (as the backbone of the HDE hypothesis \[26\]) is not always met \[27,63\]. Recently, using generalized entropy formalisms, some new HDE models have been introduced \[64,66\]. Tsallis holographic dark energy (THDE) is one of these new attempts, based on Tsallis entropy \[67\], which can provide suitable description for the current universe in various cosmological setups \[68–72\]. It is also worth mentioning that although THDE is not stable at the classical level \[64\], a more comprehensive study on its stability may consider the global features of the metric perturbations \[72,74\].

On the other hand, observations admit a mutual interaction between DE and DM \[73,87\], and the \(CMB + BAO + SN + H_0\) data suggests that the sign of mutual interaction has probably been changed during the cosmic evolution in the \(0.45 \leq z \leq 0.9\) interval \[75\]. This result motivates physicists to consider sign-changeable interactions, including the deceleration parameter \[76,87\]. Although the FLRW metric is a powerful tool in modeling the universe, and is in accordance with the cosmological principle \[8\], it does not compatible with the early universe anisotropy, and also the anisotropy of the cosmos in scales smaller than 100-Mpc and also \[8\]. Thus, a comprehensive study on the cosmic evolution should consider the anisotropy effects. The Bianchi models are some interesting attempts to model the anisotropy of the early universe \[88–103\]. Motivated by the above arguments, we are going to study the cosmic evolution of a Bianchi type I (BI) model filled by DE and DM interacting with each other throughout a sign-changeable interaction. In our models, THDE with various IR cutoffs is used to model DE.

The paper is organized as follows. In the next section, we present the general remarks on the BI universe filled by mutually interacting DE and DM. The results of considering the Hubble horizon as the IR cutoff is investigated in sections III. The cases using the event and particle horizons as the IR cutoffs are studied in sections IV) and V, respectively. Two other famous IR cutoffs, including the GO \[104,105\] and Ricci \[106,108\] cutoffs, will be addressed in sections VI and VII, respectively. The last section is devoted to a brief summary.

\section{II. GENERAL FRAMEWORK}

The BI metric, which includes the anisotropy of early cosmos, is written as \[88,103\]
\begin{equation}
ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)dy^2 - C^2(t)dz^2,
\end{equation}
where \(A(t), B(t)\) and \(C(t)\) are functions of cosmic time. Hence, the FLRW metric is recovered whenever \(A = B = C\) \[88,103\]. For this metric, i) \(V^3 \equiv ABC\) denotes the spatial volume. ii) \(a = (ABC)^{1/3}\) is defined as the average scale factor. iii) \(H = \frac{1}{3}(H_1 + H_2 + H_3)\) is the generalized mean Hubble parameter, where \(H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{B}}{B}\) and \(H_3 = \frac{\dot{C}}{C}\).
the shear scalar, respectively. Moreover, $\sigma$ the interaction sign is also changed. Indeed, for whenever the universe phase expansion is changed, its evolution phase are changed, which controls the current universe dynamics, it is eval-

In this manner, the corresponding Friedmann equations (the BI equations) take the form \[109, 111\]

\begin{align*}
3H^2 - \sigma^2 &= \frac{1}{m_p^2} (\rho_m + \rho_D), \quad (2) \\
3H^2 + 2 \dot{H} + \sigma^2 &= -\frac{1}{m_p^2} (\rho_D), \quad (3)
\end{align*}

in which $m_p^2 = 1/(8\pi G)$ and $\sigma$ are the Planck mass and the shear scalar, respectively. Moreover, $\sigma^2 = 1/2\sigma_{ij}\sigma^{ij}$ while $\sigma_{ij} = u_{ij,j} + \frac{1}{2} (u_{ij,k}u^{kj} + u_{jk}u^{ki}) + \frac{1}{4} \theta (g_{ij} + u_{ij})$ is called the shear tensor, which describes the rate of distortion of the matter flow, and $\theta = 3H = w_{ij}^j$ denotes the scalar expansion. Defining the critical density $\rho_c = 3m_p^2H^2$, and introducing the dimensionless density parameters

$$
\Omega_m = \frac{\rho_m}{\rho_c}, \quad \Omega_D = \frac{\rho_D}{\rho_c}, \quad \Omega_\sigma = \frac{\sigma^2}{3H^2},
$$

the first BI equation can be rewritten as

$$
\Omega_m + \Omega_\Lambda = 1 - \Omega_\sigma.
$$

It is worthwhile mentioning that the shear scalar is describable, using the average Hubble parameter, as $\sigma^2 = \sigma_0^2 H^2$ in which $\sigma_0$ is a constant \[88, 103\]. In the presence of a mutual interaction ($Q$) between the cosmos sectors, the continuity equation is decomposed as \[76, 77\]

\begin{align*}
\dot{\rho}_m + 3H\rho_m &= Q, \quad (7) \\
\dot{\rho}_D + 3H(1+\omega_D)\rho_D &= -Q. \quad (8)
\end{align*}

Following \[76, 77\], we assume $Q = 3b^2qH\rho_D(1+u)$ where $b^2$ is coupling constant, $u = \frac{\rho_m}{\rho_D}$ and $q$ is the deceleration parameter defined as

$$
q = -\frac{\ddot{a}}{aH^2} = -1 - \frac{\dot{H}}{H^2}. \quad (9)
$$

Thus, whenever the universe phase expansion is changed, the interaction sign is also changed. Indeed, for $Q < 0$ ($Q > 0$), there is an energy flow from DM (DE) to DE (DM). At the classical level, for a stable model, the sound speed square $v_s^2$ is positive \[112\]. For DE candidate, which controls the current universe dynamics, it is evaluated as

$$
v_s^2 = \frac{d^2 P_D}{d\rho_D^2} = \frac{\dot{P}_D}{\dot{\rho}_D} = \frac{\rho_D}{\rho_D} \dot{\omega}_D + \omega_D. \quad (10)
$$

A new set of completely geometrical parameters \{r, s\}, dubbed the statefinder, has been introduced by Sahni et al \[113\]

$$
s = \frac{r-1}{3(q-1/2)}, \quad r = \frac{\ddot{a}}{aH^3}, \quad (11)
$$

which obviously show the statefinder pair depend only on the scale factor and it’s time derivatives up to the third order. Combining the above relation with each other, we can also write

$$
r = 2q^2 + q - \frac{\dot{q}}{H}. \quad (12)
$$

Another way to analyse the cosmic evolution has been introduced in Ref. \[113\], based on the set of parameters \{\omega_D, \omega_D\} (prime denotes derivative respect to $x = \ln a$). This approach works in the $\omega_D - \omega_D'$ plane, in which $\omega_D > 0$ and $\omega_D < 0$ present the thawing region, while $\omega_D < 0$ and $\omega' < 0$ present the freezing region \[114\].

![FIG. 1: $\Omega_D$ versus redshift parameter $z$ for sign-changeable interacting THDE with Hubble radius as the IR cutoff.](image1)

![FIG. 2: $\omega_D(z)$ for sign-changeable interacting THDE with Hubble radius as the IR cutoff.](image2)
III. SIGN-CHANGEABLE INTERACTING THDE WITH HUBBLE RADIUS AS IR CUTOFF IN BI MODEL

THDE is defined as

$$\rho_D = BL^{2\delta - 4},$$ \hspace{1cm} (13)

leading to

$$\rho_D = BH^{-2\delta + 4},$$ \hspace{1cm} (14)

where $B$ is an unknown parameter, if we consider the Hubble radius $H^{-1}$ as the IR cutoff $L$. The time derivative of relation (14) leads to

$$\dot{\rho}_D = \rho_D(-2\delta + 4)\frac{\dot{H}}{H},$$ \hspace{1cm} (15)

combined with the time derivative of the first Friedmann equation and Eq. (17) to reach

$$\frac{\dot{H}}{H^2} = \frac{-3\Omega_D(b^2(1 + u) + u)}{2 - 2\Omega_H + \Omega_D(3b^2(1 + u) - (-2\delta + 4))},$$ \hspace{1cm} (16)

where

$$u = \frac{\rho_m}{\rho_D} = \frac{\Omega_m}{\Omega_D} = -1 + \frac{1}{\Omega_D(1 - \Omega_H)}.$$

By inserting equation (16) in relation (15), the deceleration parameter $q$ is found out

$$q = -1 + \frac{3\Omega_D(b^2(1 + u) + u)}{2 - 2\Omega_H + \Omega_D(3b^2(1 + u) - (-2\delta + 4))}.$$ \hspace{1cm} (18)

We can also find the EoS parameter, by combining Eqs. (15), (8) and (18) as

$$\omega_D = -1 - b^2q(1 + u) + (1 + q)(-2\delta + 4/3).$$ \hspace{1cm} (19)

With the help of Eqs. (6), (15) and the $\dot{\Omega}_D = \Omega'_DH$ relation, one reaches

$$\Omega'_D = -2\Omega_D(-\delta + 1)(1 + q),$$ \hspace{1cm} (20)

in which prime denotes derivative with respect to $\ln(a)$. Using the time derivative of Eqs. (15) and (19), we can find $v_s^2$ and the $(r, s)$ pair as the statefinder parameters. Since these expressions are too long, we do not present them here, study the evolution of these quantities via figures. The model behavior has been depicted in Figs. (1-9) for the initial conditions $\Omega_D(z = 0) = 0.73$, $H(z = 0) = 67$, $b^2 = .01$ and $\Omega_H = .001$. These figures indicate that although the model is unstable at the classical level ($v_s^2 < 0$), it can cover the current accelerated universe for $\omega_D \geq -1$.

IV. SIGN-CHANGEABLE INTERACTING THDE WITH EVENT HORIZON AS IR CUTOFF IN BI MODEL

If the future event horizon, defined as

$$R_h = a(t)\int_t^\infty dt/a(t),$$ \hspace{1cm} (21)
FIG. 6: The evolution of the statefinder parameter $s$ against $z$ for sign-changeable interacting THDE with Hubble radius as the IR cutoff.

FIG. 7: The evolution of $r$ versus $s$ for sign-changeable interacting THDE with Hubble radius as the IR cutoff.

leading to

$$\dot{R}_h = HR_h - 1,$$  \hspace{1cm} (22)

is used as the IR cutoff ($L = R_h$), then

$$\rho_D = BR_h^{2\delta - 4}. \hspace{1cm} (23)$$

By combining the time derivative of the above equation with Eq. (22), we find

$$\dot{\rho}_D = \rho_D(2\delta - 4)H(1 - F), \hspace{1cm} (24)$$

where $F = (\frac{3\Omega_D H^{2\delta - 2}}{H})^\frac{1}{\delta - 1}$. Inserting Eq. (24) in Eq. (8), we easily reach at

$$\omega_D = -1 - b^2 q(1 + u) - \left(\frac{2\delta - 4}{3}\right)(1 - F). \hspace{1cm} (25)$$

FIG. 8: $r$ against $q$ for sign-changeable interacting THDE with Hubble radius as the IR cutoff.

FIG. 9: The $\omega_D - \omega_D'$ diagram for sign-changeable interacting THDE with Hubble radius as the IR cutoff. Here, we have taken $\Omega_D(z = 0) = 0.73$, $H(z = 0) = 67$, $b^2 = .01$ and $\Omega_{\sigma} = .001$

Additionally, combining the time derivative of Eq. (2) with Eqs. (24) and (8), one obtains

$$\frac{\dot{H}}{H^2} = \frac{\Omega_D(-3b^2(1 + u) - 3u + (2\delta - 4)(1 - F))}{2 - 2\Omega_{\sigma} + 3b^2\Omega_D(1 + u)}, \hspace{1cm} (26)$$

and

$$q = -1 - \Omega_D(-3b^2(1 + u) - 3u + (2\delta - 4)(1 - F)) \frac{2 - 2\Omega_{\sigma} + 3b^2\Omega_D(1 + u)}{2} \hspace{1cm} (27)$$
Using the $\Omega_D$ expression and Eq. (24), we find

$$\Omega_D' = \Omega_D ((2\delta - 4)(1 - F) + 2(1 + q)).$$  \hspace{1cm} (28)

Although $v_s^2$, $r$ and $s$ can be found out by by taking the time derivative of Eqs. (25) and (27), since they are too long relations, we do not present them here, and only plot them. In Figs. (10-18), the model behavior is depicted for the initial conditions $\Omega_D(z = 0) = 0.73$, $H(z = 0) = 67$, $\delta = 0.1$, $B = 2.4$ and $\Omega_\sigma = 0.001$ indicating that, unlike the previous case, the phantom behaviors is unavoidable.

V. SIGN-CHANGEABLE INTERACTING THDE WITH PARTICLE HORIZON AS IR CUTOFF IN BI MODEL

The particle horizon, introduced as \cite{106}

$$\dot{R}_p = HR_p + 1,$$  \hspace{1cm} (29)
FIG. 15: The evolution of $s$ against $z$ for sign-changeable interacting THDE with event horizon as IR cutoff.

FIG. 16: The evolution of $r$ versus $s$ for sign-changeable interacting THDE with event horizon as IR cutoff.

is considered as the IR cutoff in this section. In this manner, Eq. (13) leads to

$$\rho_D = BR_p^{\frac{2\delta - 4}{4}},$$

(30)

and

$$\dot{\rho}_D = \rho_D (2\delta - 4) H (1 + F),$$

(31)

the time derivative of $\rho_D$. Combining Eq. (31) with the

FIG. 17: The evolution of $r$ versus $q$ for sign-changeable interacting THDE with event horizon as IR cutoff.

FIG. 18: The $\omega_D - \omega'_D$ diagram for sign-changeable interacting THDE with event horizon as IR cutoff.
conservation law (3), one obtains
\[ \omega_D = -1 - b^2 q (1 + u) - \left( \frac{2 \delta - 4}{3} \right) (1 + F). \] (32)

In addition, by using Eqs. (2), (31) and (9), the deceleration parameter \( q \) is found out as
\[ q = -1 - \frac{\Omega_D(-3b^2(1+u) - 3u + (2\delta - 4)(1 + F))}{2 - 2\Omega_\sigma + 3b^2\Omega_D(1 + u)}. \] (33)

Moreover, one can insert \( \Omega_D = \frac{\rho_D}{3m_pH^2} \) in Eq. (31) to reach at
\[ \Omega'_D = \Omega_D((2\delta - 4)(1 + F) + 2(1 + q)). \] (34)

The same as the previous sections, since the expressions of \( v^2_s \), \( r \) and \( s \) are too long, we only plot them and do not write them here. In Figs. (19-27), the system parameters have been plotted by employing the initial conditions \( \Omega_D(z = 0) = 0.73 \), \( H(z = 0) = 67 \), \( \delta = 2.4 \), \( B = 2.4 \) and \( \Omega_\sigma = 0.001 \).
VI. SIGN-CHANGEABLE INTERACTING THDE WITH GO HORIZON AS IR CUTOFF IN BI MODEL

Bearing Eq. (13) in mind, and employing the GO cutoff \cite{104, 105}, the energy density of THDE is given by

\[
\rho_D = (\alpha H^2 + \beta \dot{H})^{-\delta - 2},
\]

rewritten as

\[
\frac{\dot{H}}{H^2} = \frac{1}{\beta} \left( \frac{(3m_p^2 \Omega_D)^{\frac{1}{3-\delta}}}{H^{\frac{3-\delta}{3-\alpha}}} - \alpha \right). 
\]

Now, by using relation \cite{9}, we have

\[
q = -1 - \frac{1}{\beta} \left( \frac{(3m_p^2 \Omega_D)^{\frac{1}{3-\delta}}}{H^{\frac{3-\delta}{3-\alpha}}} - \alpha \right). \quad (37)
\]

In addition, inserting the time derivative of Eq. (2) in
Eq. (7), one obtains
\[
\dot{\rho}_D = \frac{\dot{H}}{H^2} (2 - 2\Omega_{\sigma} + 3b^2\Omega_D(1 + u)) + 3b^2\Omega_D(1 + u) + 3(1 - \Omega_{\sigma} - \Omega_D),
\] (38)
combined with
\[
\dot{\Omega}_D = \frac{\rho_D}{3M_p^2 H^2} - 2\Omega_D \frac{\dot{H}}{H},
\] (39)
to reach at
\[
\dot{\Omega}_D = (3b^2\Omega_D(1 + u) + 3(1 - \Omega_{\sigma} - \Omega_D) - (1 + q)(2 - 2\Omega_{\sigma} - 2\Omega_D + 3b^2\Omega_D(1 + u))).
\] (40)

The EoS parameter \(\omega_D\) of THDE is found out as
\[
\omega_D = -1 - \frac{1}{3\Omega_D} (3(1 - \Omega_{\sigma} - \Omega_D) - (1 + q)(2 - 2\Omega_D)),
\] (42)
by combining Eq. (38) with Eq. (8). Figs. 28-36 show the behavior of the model parameters during the cosmic evolution by presuming \(\Omega_D(z = 0) = 0.73, H(z = 0) = 67, \alpha = 0.8, \beta = 0.5, b^2 = 0.01\) as the initial conditions.

VII. SIGN-CHANGEABLE INTERACTING THDE WITH RICCI HORIZON AS IR CUTOFF IN BI MODEL

Let us use the Ricci cutoff \[108\] for describing cosmological parameters. It leads to
\[
\rho_D = \lambda(2H^2 + \dot{H})^{-\delta+2},
\] (43)
where \(\lambda\) is the unknown HDE constant as usual \[106, 108\], for THDE. By rewriting Eq. (43) as
\[
\frac{\dot{H}}{H^2} = \left(\frac{(3\lambda^{-1}m_p^2\Omega_D)^{\frac{1}{2\delta}}}{H^{\frac{\delta-2}{\delta}}} - 2\right),
\] (44)
and using relation (9), we get
\[
q = -1 - \left( \frac{(3\lambda^{-1}m_0^2\Omega_D)}{H^2 z^2} - 2 \right). \tag{45}
\]
Moreover, combining Eqs. (38) with (39), one finds
\[
\Omega_D' = (3b^2\Omega_D(1+u) + 3(1-\Omega_\sigma-\Omega_D) - (1+q)(2-2\Omega_\sigma-2\Omega_D + 3b^2\Omega_D(1+u))). \tag{46}
\]

Following the recipe of the previous section, the EoS parameter is calculated as
\[
\omega_D = -1 - \frac{1}{3\Omega_D} (3(1-\Omega_\sigma-\Omega_D)-(1+q)(2-2\Omega_D)). \tag{47}
\]

In Fig. (37-38), the model parameters have been plotted for the initial conditions \(\Omega_D(z=0)=0.73\), \(H(z=0)=67\), \(\delta=1\), \(\lambda=1.5\) and \(\Omega_\sigma=.001\) and \(\Omega_\sigma=.001\).

**VIII. CLOSING REMARKS**

A Bianchi type I universe was considered filled by DM and DE interacting with each other throughout a sign-changeable interaction. In our study, THDE plays the
FIG. 37: The evolution of $\Omega_D$ versus redshift parameter $z$ for sign-changeable interacting THDE with Ricci horizon.

FIG. 38: The evolution of $\omega_D$ versus redshift parameter $z$ for sign-changeable interacting THDE with Ricci horizon as IR cutoff in BI model. Here, we have taken $\Omega_D(z = 0) = 0.73$, $H(z = 0) = 67$, $\delta = 1$, $\lambda = 1.5$ and $\Omega_{\sigma} = .001$.

FIG. 39: The evolution of the deceleration parameter $q$ versus redshift parameter $z$ for sign-changeable interacting THDE with Ricci horizon as IR cutoff.

FIG. 40: The evolution of the squared of sound speed $v_s^2$ versus redshift parameter $z$ for sign-changeable interacting THDE with Ricci horizon as IR cutoff.

FIG. 41: The evolution of the statefinder parameter $r$ versus the redshift parameter $z$ for sign-changeable interacting THDE with Ricci horizon as IR cutoff.

FIG. 42: The evolution of the statefinder parameter $s$ versus the redshift parameter $z$ for sign-changeable interacting THDE with Ricci horizon as IR cutoff.
role of DE and various IR cutoffs, including the Hubble, event and particle horizons as well as the GO and Ricci cutoffs, have been used to study the evolution of the universe. We tried to present a comprehensive study by addressing diverse parameters such as $q, H, r, \omega_D$, and etc. Although suitable dynamics can be obtained for the models, the classical stability analysis ($v^2_s$) shows that the models are always unstable at the $z \to -1$ limit. It is also worthwhile mentioning that the models may show stability by themselves at the classical level for the current universe (where $z \to 0$), depending on the unknown parameters of model such as $\delta$.

FIG. 43: The evolution of the statefinder parameter $r$ versus $s$ for sign-changeable interacting THDE with Ricci horizon as IR cutoff.

FIG. 44: The evolution of the statefinder parameter $r$ versus the deceleration parameter $q$ for sign-changeable interacting THDE with Ricci horizon as IR cutoff.

FIG. 45: The $\omega - \omega_D$ diagram for sign-changeable interacting THDE with Ricci horizon as IR cutoff.

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