Deep subwavelength beam propagation in extremely loss-anisotropic metamaterials

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Received 9 January 2013, accepted for publication 25 March 2013
Published 17 April 2013
Online at stacks.iop.org/JOpt/15/055105

Abstract
Metal–dielectric multilayer metamaterials with extreme loss-anisotropy, in which the longitudinal component of the permittivity tensor has an ultra-large imaginary part, are proposed and designed. Diffraction-free deep subwavelength beam propagation and manipulation, due to the nearly flat iso-frequency contour, is demonstrated in such loss-anisotropic metamaterials. It is also shown that deep subwavelength beam propagation can be realized in practical multilayer structures with large multilayer periods, when the nonlocal effect is considered.

Keywords: multilayer metamaterials, extreme loss-anisotropy, deep subwavelength propagation, nonlocal effect

(Some figures may appear in colour only in the online journal)

1. Introduction
Light beams with subwavelength confinement and propagation beyond the diffraction limit are highly desirable for many optical integration applications. To enable the subwavelength optical confinement, optical materials supporting ultra-high wavevectors are necessary. Since the refractive indices of natural materials are quite limited at optical frequency, metamaterials with artificially engineered subwavelength meta-atoms are designed to exhibit ultra-high refractive indices so as to achieve large wavevectors [1, 2]. To obtain diffraction-free deep subwavelength beam propagation, a nearly flat iso-frequency contour (IFC) curve over a broad range in $k$-space is required, so that all spatial components will propagate with the same phase velocity along the longitudinal direction [3]. Extremely anisotropic metamaterials with an infinite real part of permittivity have been theoretically proposed to achieve flat IFCs and consequently subwavelength beam propagation without diffraction [4, 5].

Recently, metal–dielectric multilayer metamaterials with an indefinite permittivity tensor have been utilized to demonstrate intriguing applications of negative refraction [6], subwavelength imaging [7], enhanced photonic density of states [8], and broadband light absorbers [9], together with ultra-high refractive indices for subwavelength optical waveguides [10] and indefinite cavities [11]. In this work, we propose the concept of extreme loss-anisotropy in metal–dielectric multilayer metamaterials, where the longitudinal component of the permittivity tensor has an ultra-large imaginary part. Diffraction-free deep subwavelength beam propagation and manipulation is demonstrated in such loss-anisotropic metamaterial, due to the nearly flat IFC.

2. Theory and discussion
The inset of figure 1 shows the metal–dielectric multilayer structure, where titanium oxide ($\text{Ti}_3\text{O}_5$) with a dielectric constant of 5.83 and silver (Ag) are chosen [12]. When the thickness of each layer is infinitely small, the multilayer structure can be regarded as an anisotropic effective medium with a permittivity tensor of

\begin{equation}
\begin{aligned}
\epsilon_x &= f_d \epsilon_d + f_m \epsilon_m, \\
\epsilon_y &= (f_d/\epsilon_d + f_m/\epsilon_m)^{-1},
\end{aligned}
\end{equation}

(1)
where \( \varepsilon_d \) and \( \varepsilon_m \) are the permittivities of titanium oxide and silver, and \( f_d \) and \( f_m \) \((f_d + f_m = 1)\) are the filling ratios of titanium oxide and silver, respectively. The silver filling ratio \( f_m \) is 0.45, and its permittivity is taken from the experimental results [13]. The dependence of the permittivity tensor on wavelength \( \lambda_0 \) is shown in Figure 1(a), which is calculated using equation (1) according to the effective medium theory. It is found that the longitudinal permittivity component \( \varepsilon_x \) shows a strong resonance at \( \lambda_0 = 406.1 \text{ nm} \) (at position II), where \( \text{Im}(\varepsilon_y) \) has a peak of more than 230, while \( \text{Re}(\varepsilon_y) \) flips its sign quickly across the resonance from negative maximum (at position I) to positive maximum (at position III). Previously, ultra-large \( \text{Re}(\varepsilon_y) \) (at position III) has been utilized to realize subwavelength beam propagation without diffraction [4, 5]. However, the behavior of ultra-high \( \text{Im}(\varepsilon_y) \) has not been considered before. It is intuitively thought that a large \( \text{Im}(\varepsilon_y) \) will increase the beam propagation loss. In contrast, it will be demonstrated that the extreme loss-anisotropy with an ultra-large imaginary part can enable low-loss diffraction-free subwavelength beam propagation. For TM-polarized light with non-vanishing \( E_x, E_y \) and \( H_z \) field components, the corresponding IFC is determined by

\[
 k_x^2/\varepsilon_y + k_y^2/\varepsilon_x = k_0^2, \tag{2}
\]

where \( k_x \) is the transverse \( k \)-vector and \( k_y \) is the propagation \( k \)-vector. Figure 1(b) shows that an ultra-flat IFC over a large \( k_x \) range is supported at the resonance wavelength \( \lambda_0 = 406.1 \text{ nm} \) (position II), with \( \varepsilon_x = 1.06 + 0.098i \) and \( \varepsilon_y = 10.48 + 231.70i \). The propagation \( k \)-vector \( \text{Re}(k_y) \) remains a constant for all different \( k_y \), while the imaginary part of the propagation \( k \)-vector \( \text{Im}(k_y) \) is quite small. To understand the behavior of the flat IFC, equation (2) can be rewritten as

\[
 k_y = \sqrt{\varepsilon_x \left( k_0^2 - k_y^2/\varepsilon_y \right)} \approx \sqrt{\varepsilon_x k_0} - \sqrt{\varepsilon_x \frac{k_y^2}{2k_0^2} \varepsilon_y}, \tag{3}
\]

where the approximation \(|\varepsilon_y| \gg 1\) has been used to derive the above formula. Since \( \varepsilon_y \) is dominated by its real part, and \( \varepsilon_x \) is dominated by its imaginary part at the resonance wavelength, \( \text{Re}(k_y) \approx \sqrt{\varepsilon_x k_0} \) and \( \text{Im}(k_y) \approx \sqrt{\varepsilon_x k_0^2}/(2k_0 \text{Im}(\varepsilon_y)) \), which shows that \( \text{Re}(k_y) \) is independent of \( k_y \) and \( \text{Im}(k_y) \) is weakly proportional to \( k_y^2 \). Moreover, the propagation loss \( \text{Im}(k_y) \) is inversely proportional to \( \text{Im}(\varepsilon_y) \) and the large material loss \( \text{Im}(\varepsilon_y) \) actually enables the low-loss beam propagation. The ultra-large \( \text{Im}(\varepsilon_y) \) in extremely loss-anisotropic metamaterial not only gives rise to an ultra-flat IFC over a broad \( k_x \) range, but also results in ultra-small beam propagation loss.

To further illustrate the importance of large \( \text{Im}(\varepsilon_y) \), the comparisons of IFC curves at three different wavelengths of I, II and III are shown in Figure 1(c), which corresponds to negatively maximized \( \text{Re}(\varepsilon_y) \), maximized \( \text{Im}(\varepsilon_y) \) and maximized \( \text{Re}(\varepsilon_y) \), respectively. This indicates that the maximized \( \text{Im}(\varepsilon_y) \) case results in the flattest IFC with almost zero curvature, while the other two IFCs show positive and negative curvatures, respectively. These behaviors can be clearly understood using equation (3), in which a purely real \( \varepsilon_y \) will contribute to \( \text{Re}(k_y) \) and influence the curvature of the IFC (a positive \( \varepsilon_y \) leads to a negative curvature and vice versa). It is clear that the maximized \( \text{Im}(\varepsilon_y) \) case with zero curvature will achieve diffraction-free subwavelength beam propagation.

Next, diffraction-free deep subwavelength beam propagation will be demonstrated from a numerical simulation based on the finite element method (FEM). Here, it is worthwhile to define the minimal waist size of the light beam which can propagate inside the multilayer structure without diffraction. It is known from equation (2) that \( k_x^2/\varepsilon_y \ll k_0^2 \) is required to obtain a nearly flat IFC, thus the minimal beam waist size \( w_{\text{min}} \) turns out to be \( 2\pi/\max(k_x) = \lambda_0 \sqrt{|\varepsilon_y|} \).

Figure 2 shows the propagation of ultra-narrow Gaussian beams (with a waist size of 40 nm ~ 0.1\( \lambda_0 > w_{\text{min}} \)) inside loss-anisotropic metamaterials with different geometries at...
the resonance wavelength \( \lambda_0 = 406.1 \text{ nm} \). The effective medium results and the realistic multilayer results are shown in figures 2(a)–(c) and (d)–(f), respectively. Figures 2(a) and (d) show that ultra-narrow Gaussian beams can propagate over a long distance without any wavefront distortion. Two subwavelength beams with 150 nm center-to-center distance remain well defined as the beams propagate across the multilayer from the bottom to the top. It is emphasized that the subwavelength beam confinement is entirely due to the unique loss-anisotropic property, and the beam path is solely determined by the launching location. This is distinct from the situation in a subwavelength waveguide, where the mode is confined by the waveguide boundary.

Besides the straight beam propagation, the flow of light can be flexibly modeled by controlling the local metamaterial properties. For instance, the beam path can be manipulated by gradually varying the direction of multilayers, since the direction of beam propagation is always vertical to the multilayer interface. The designed geometries for achievement of 90° and 180° bending of subwavelength beams are shown in figures 2(b)–(c) and (e)–(f), for both the effective media and the multilayer structure. In the effective medium calculation, the anisotropic permittivity tensor depends on the tilted angle of the multilayer interface. In the local coordinate \((u, v)\), the components of the permittivity tensor can still be determined by the mixing formula in equation (1). The permittivity tensor expression in the global coordinate \((x, y)\) is related to that in the local coordinate as

\[
\mathbf{\epsilon} = \begin{pmatrix} \epsilon_u \cos^2 \theta + \epsilon_v \sin^2 \theta & (\epsilon_u - \epsilon_v) \sin \theta \cos \theta \\ (\epsilon_u - \epsilon_v) \sin \theta \cos \theta & \epsilon_u \sin^2 \theta + \epsilon_v \cos^2 \theta \end{pmatrix},
\]

where \(\theta\) is the local tilted angle of the multilayer with respect to the \(+x\) axis, and \(\epsilon_u\) and \(\epsilon_v\) are the local permittivity tensor components along and normal to the multilayer, respectively. The simulation results indicate that the flow of light can indeed be manipulated while maintaining the deep subwavelength beam confinement and diffraction-free propagation. For the results of 180° bending shown in figures 2(c) and (f), it is noted that the wavefront becomes tilted at the output section (but the energy flow is still vertical to the interface as a result of the flat IFC). This is due to the fact that the light traveling at the inner side undergoes a shorter optical path than the light traveling at the outer side. The phase difference arising from the light path difference leads to the beam wavefront tilting.

Figure 2 shows that the multilayer structure simulation results agree very well with the EMT results, indicating that the multilayer structure with period \(a = 20 \text{ nm} \) \( (f_m = 0.45) \) can represent the loss-anisotropic effective medium well. However, the fabrication of such thin layers is very challenging in reality (but possible [14]). It is interesting to study the properties of a metal–dielectric multilayer with a large period \(a\), where the nonlocal effect has to be taken into account [15]. The dispersion relation describing the realistic multilayer structure for TM-polarized light is

\[
\cos \left[ k_y (a_m + a_d) \right] = \cos (k_m a_m) \cos (k_d a_d) - \gamma_{TM} \sin (k_m a_m) \sin (k_d a_d),
\]

which is derived by treating the layered structure as a one-dimensional photonic crystal. Here, \(\gamma_{TM} = (\epsilon_g k_m^2 / \epsilon_m k_m + \epsilon_m k_m / \epsilon_d k_m) / 2\), \(k_m = \sqrt{\epsilon_m k_0^2 - k_g^2}\), and \(k_d = \sqrt{\epsilon_d k_0^2 - k_g^2}\). The IFCs corresponding to a realistic multilayer structure with \(a = 40 \text{ nm}\) are shown in figure 3(a). It is found that the IFC curve at \(\lambda_0 = 406.1 \text{ nm}\) is no longer flat due to the nonlocal effect. That is to say, the effective permittivity tensor becomes strongly wavevector dependent, so that the permittivity components \(\epsilon_x\) and \(\epsilon_y\) will be functions of not only the frequency but also the \(k\)-vector. The frequency corresponding to the flattest IFC will then be shifted. For the multilayer structure with \(a = 40 \text{ nm}\), it turns out that the flattest IFC occurs at \(\lambda_0 = 418.3 \text{ nm}\) (which can be mathematically determined by finding the working wavelength with zero IFC curvature), as shown in figure 3(a). The propagation of subwavelength Gaussian beams at the two wavelengths in the multilayer structure with \(a = 40 \text{ nm}\) is shown in figure 3(b). As can be expected from the IFC curves in figure 3(a), the Gaussian beams at \(\lambda_0 = 406.1 \text{ nm}\) suffer strong diffraction, resulting in distorted beam profiles. In comparison, the beam profiles remain well defined for the Gaussian beams at \(\lambda_0 = 418.3 \text{ nm}\).
Figure 3. (a) The IFCs for a realistic multilayer structure with $a = 40$ nm at two different wavelengths. (b) The Gaussian beam propagation at the two wavelengths. Diffraction-free beam propagation is achieved after taking into account the nonlocal effect induced wavelength shift.

3. Conclusion

In conclusion, extremely loss-anisotropic metamaterial is designed using metal–dielectric multilayer structures. The IFC corresponding to such metamaterial turns out to be ultra-flat over a broad $k$-vector range. This unique property is then utilized to obtain diffraction-free deep subwavelength beam propagation. Furthermore, it is shown that the propagation of light beams can be manipulated flexibly by tuning the direction of the multilayer structure. Moreover, the nonlocal effect occurring in multilayer structures with large multilayer periods is investigated. It is found that diffraction-free beam propagation is still possible after taking into account the nonlocal effect. This study is very attractive for many applications such as optical imaging, optical integration, and on-chip optical communication.

Acknowledgments

This work was partially supported by the Department of Mechanical and Aerospace Engineering and the Intelligent Systems Center at Missouri S&T, the University of Missouri Research Board, the Ralph E Powe Junior Faculty Enhancement Award, and the National Natural Science Foundation of China (61178062 and 60990322).

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References

[1] Shin J, Shen J T and Fan S 2009 Three-dimensional metamaterials with an ultrahigh effective refractive index over a broad bandwidth Phys. Rev. Lett. 102 093903
[2] Choi M, Lee S H, Kim Y, Kang S B, Shin J, Kwak M H, Kang K Y, Lee Y H, Park N and Min B 2011 A terahertz metamaterial with unnaturally high refractive index Nature 470 369–73
[3] Han S, Xiong Y, Genov D, Liu Z, Bartal G and Zhang X 2008 Ray optics at a deep-subwavelength scale: a transformation optics approach Nano Lett. 8 4243–7
[4] Catrysse P B and Fan S 2012 Deep sub-wavelength beam propagation, beam manipulation and imaging with extreme anisotropic meta-materials Proc. Quantum Electronics and Laser Science Conf. (San Jose, California, 6 May 2012) QTu1G (doi:10.1364/QELS.2012.QTu1G.7)
[5] Catrysse P B and Fan S 2011 Transverse electromagnetic modes in aperture waveguides containing a metamaterial with extreme anisotropy Phys. Rev. Lett. 106 223902
[6] Yao J, Liu Z, Liu Y, Wang Y, Sun C, Bartal G, Stacy A M and Zhang X 2008 Optical negative refraction in bulk metamaterials of nanowires Science 321 930
[7] Liu Z, Lee H, Xiong Y, Sun C and Zhang X 2007 Far-field optical hyperlens magnifying sub-diffraction-limited objects Science 315 1686
[8] Krishnamoorthy H N S, Jacob Z, Narimanov E, Kretzschmar I and Menon V M 2012 Topological transitions in metamaterials Science 336 205–9
[9] Cui Y, Fung K H, Xu J, Ma H, Jin Y, He S and Fang N X 2012 Ultrabroadband light absorption by a sawtooth anisotropic metamaterial slab Nano Lett. 12 1443–7
[10] He Y, He S, Gao J and Yang X 2012 Nanoscale metamaterial optical waveguides with ultrahigh refractive indices J. Opt. Soc. Am. B 29 2559–66
[11] Yang X, Yao J, Rho J, Yin X and Zhang X 2012 Experimental realization of three-dimensional indefinite cavities at the nanoscale with anomalous scaling laws Nature Photon. 6 450–4
[12] Rho J, Ye Z, Xiong Y, Yin X, Liu Z, Choi H, Bartal G and Zhang X 2010 Spherical hyperlens for two-dimensional sub-diffractional imaging at visible frequencies Nature Commun. 1 143
[13] Johnson P B and Christy R W 1972 Optical constants of the noble metals Phys. Rev. B 6 4370–9
[14] Chen W, Thoreson M D, Ishii S, Kildishev A V and Shalaev V M 2010 Ultra-thin ultra-smooth and low-loss silver films on a germanium wetting layer Opt. Express 18 5124–34
[15] Elser J, Podolskiy V A, Salakhutdinov I and Avrutsky I 2007 Nonlocal effects in effective-medium response of nanolayered metamaterials Appl. Phys. Lett. 90 191109