Profile of the student’s mathematical reasoning ability in solving geometry problem

E S Wahyuni¹, Susanto¹, A F Hadi²
¹Department of Mathematics Education, Faculty of Teacher Training and Education, University of Jember, Indonesia.
²Department of Mathematics, FMIPA, University of Jember, Indonesia

Email : sriwahyunie91@gmail.com, susantouj@gmail.com, afhadi@unej.ac.id

Abstract. This research aimed to describing profile of the students’ mathematical reasoning ability in solving geometry problem. This research used descriptive qualitative. The samples of the research consisted of 33 Vocational High School students of XI TKJ B class who were chosen based on the results of reasoning ability test. The students’ profile data in solving geometry problem were collected through the interview of the students’ works. The results obtained were analyzed based on the indicators of reasoning ability including a) formulating Presumption; b) performing mathematics manipulation; c) checking the validity of an argument; d) drawing a conclusion. The results of the research revealed that (1) the students with high reasoning ability tended to master the indicators of problem solving with a good category, (2) the students with average reasoning ability tended to master the indicators of problem solving with a fairly good category, (3) the students with low reasoning ability tended to master the indicators of problem solving with poor category.

1. Introduction
In general, mathematics is a scientific discipline with a high failure rate in national examination, even in the international study, many students do not understand the use of mathematics. The importance of having mathematical reasoning ability in the students is basically in line with the vision of mathematics, especially to fulfill needs in the future. Mathematical learning is directed to provide opportunities for the development of reasoning ability, awareness of the usefulness of mathematics, fostering a sense of confidence, objective and open attitude in facing the ever-changing future[1].

Based on the interview results with some students conducted by the researchers at SMK Al-Maliki Sukodono, it was found that the students’ reasoning abilities were still low. First, the teacher only taught the mathematical concept and operation without connecting with the real situation. Second, the process of mathematics learning did not seem reasonable with a simple structure. The implications are that the teachers must change their roles, no longer as the scientific and indoctrinering authority, but to become a facilitator who guides the students toward the formation of knowledge by themselves.

The process of mathematical problem solving is different from the process of solving mathematical question. This difference is contained in the term of problem and question. If a mathematical task can be solved immediately, it means the task belongs to the routine task and not a problem. A mathematical task belongs to a problem if it cannot be solved directly but through several other relevant activities. The difficulty in problem solving of mathematics in the form of a story is included in the geometry material. The difficulty of geometry material can influence the difficulty of other parts in geometry material, because many units in geometry are interconnected. During this time,
many students are still confused to understand the intent contained in the story problem and the difficulty to determine the mathematical model of the story.

The research by [2] showed that the concept of geometry could be understood well if the students had good reasoning abilities. Likewise, the students’ reasoning abilities can be improved by exercises which need reasoning activities like a problem solving of geometry subject. In addition, [9] revealed that through the experience of learning geometry, increased problem solving skill, reasoning and ease in learning various mathematical topics, as well as various other sciences. Geometrical learning also improved the students’ wants to learn mathematics, improved problem solving skill, reasoning and ease in learning various mathematical topics and other sciences.

This research analyzed more deeply the profile of the students’ mathematical reasoning abilities in solving geometry problem. The indicators of mathematical reasoning ability in this research was a part of “4C”, that included in the thinking activities. Reasoning is the line of thought adopted to produce assertions and reach conclusions in task solving. It is not necessarily based on formal logic, thus not restricted to proof, and may even be incorrect as long as there are some kinds of sensible (to the reasoner) reasons backing it [3]. Reasoning as the relationship between the process of analysis, synthesis, experiment planning, drawing conclusion, generalizing, evaluation, proving and generalizing unusual problems. Based on those two statements, it can be concluded that reasoning is a thinking process that connects the process of analyzing, synthesis, experiment planning, generalizing, evaluating and proving to a conclusion [4].

Reasoning in mathematics is a basic mathematical skill that is needed for several objectives in understanding the concepts of mathematics, using mathematical ideas and flexibility procedures, as well as reconstructing [5]. Whereas, according to mathematical reasoning is a process of drawing a conclusion to several ideas and its relationship to solve mathematical problems. Thus, mathematical reasoning is a basic of mathematics in understanding the concepts, ideas and procedures to draw a conclusion [2]. The students’ mathematical reasoning has four components, covering mathematical communication, basic mathematical skill and logical thinking [5]. These components can be used to measure whether the students’ reasoning abilities are good or not. According to Bao reasoning ability includes analyzing evidence and data, thinking critically and logically, exploring problems, formulating and testing Presumption, controlling and manipulating variables, and evaluating experimental results [6]. The indicators of reasoning ability are presented in Table 1.

| Reasoning Indicator                  | Description                                                                 |
|--------------------------------------|-----------------------------------------------------------------------------|
| Formulating Presumption              | Using language, notation, and mathematical structures to present mathematical ideas.  
                                         | Determining pictures/sketches                                                |
| Performing Mathematics manipulation  | Understanding the concepts and interpreting data                           |
                                         | Using flexible mathematical ideas and procedures.                           |
                                         | Building logical arguments.                                                  |
| Checking the validity of an argument | Looking for the relationships of various representations of concepts and procedures |
                                         | Understanding the relationships between mathematical topics                |
                                         | Making generalizations                                                       |
| Drawing a conclusion                 | Using analogies, implications, and syllogism                                |
                                         | Using reasoning in solving problems                                          |

Based on the interviews with several vocational students, it was found that students’ mathematical reasoning abilities were still low. This was indicated by students having difficulty in determining geometric concepts, even if it was expressed in the form of variables.
Reasoning is an important process and is used by students in solving students' mathematical problems. The abstract nature of mathematics and its contribution to analytical thinking have the potential to enable training of well-equipped problem solvers [7]. According to Raynal & Rieunier, 1997, p 295 in [8], problem solving represents higher intellectual activity and is considered by the majority of experts as the most complex level of cognitive activity that mobilizes at the same time, all individual intellectual abilities: memory, perception, reasoning, conceptualization, language and they both involve emotions, motivation, confidence, and the ability to control situations. This is in line with the opinion of [2] which states that solving problems in geometry is a psychological activity (especially intellectuals) to find solutions to geometric problems faced by using integratively all the provisions of mathematical knowledge (geometry) that has been owned.

| Reasoning Indicator | Problem Solving Stage       | Description                                                                 |
|---------------------|----------------------------|----------------------------------------------------------------------------|
| Formulating         | Informing the problem      | • Able to re-communicate a mathematical problem with their own words verbally, in writing, pictures, or diagrams. |
| Presumption         |                            | • Able to mention things that are known and ask about the problem.           |
|                     |                            | • Able to find clues to problems                                            |
|                     |                            | • Able to compile clues based on information obtained on the problem.       |
|                     | Finding a clue             | • Creating models or graphs or tables or images. Then it is observed to assist in developing the chosen problem solving strategy. |
|                     | Formulating a strategy     | • Able to guess the answers and problem solving procedures                  |
|                     | Applying the strategy      | • Able to dig up information to help solve problems                         |
|                     | re-checking               | • Able to use the guessed answers and select the problem solving procedures |
|                     |                            | • Able to use information obtained to assist in solving the problems       |
|                     |                            | • Able to do mathematical manipulation                                      |
|                     |                            | • Able to re-check the results of alternative problem solving               |
|                     |                            | • Able to draw valid conclusions based on alternative problem solving obtained |

2. Method
The method used was a type of qualitative descriptive research. This research was conducted at Al-Maliki Sukodono Vocational High School in the odd semester of 2017/2018. The research subjects were 33 students of XI TKJ B class with purposive sampling technique that was with certain considerations and objectives [10]. The first thing the researchers did was to compile and validate the research instruments in the form of RPP, LKS, and tests. The research instrument was validated by three validators who were three lecturers of Mathematics Education from University of Jember. Validation was conducted to check the validity of the instruments used in this research. If the instrument was valid then the instrument could be used. And if it was not valid then the instrument was revised according to the validators’ suggestions. Next, the researchers determined the research area, coordinated with the mathematics teacher to determine the participants and the research schedule. The reasoning ability test consisted of 4 items with a time allocation of 90 minutes and the all answers were in accordance with the guidelines for mathematical reasoning scoring. To determine the high, medium and low groups, the researchers used the average value and standard deviation of the test scores of students' mathematical reasoning abilities. The data analysis technique was qualitative descriptive analysis. Data on students’ works were processed and analyzed qualitatively. The technique used to test and ensure data validity was triangulation, which is a data collection technique.
that is combining various techniques of collecting data and existing data sources [10]. The data about students’ works results was processed and analyzed qualitatively. The technique used to test and make sure that the validity of the data was triangulation, which is a data collection technique which combines some other data collection techniques and the available data source [10].

![Diagram of Research Procedures](image)

**Figure 1.** Research Procedures
3. Results and Discussion
The mathematical reasoning indicators in solving mathematics problem used in this research were: (1) formulating Presumption, (2) performing mathematics manipulation, (3) checking the validity of an argument, (4) drawing conclusion or generalizing. From those reasoning indicators, the students’ mathematical reasoning ability level was described in the following Table 3.

Table 3. The Category of Students’ Reasoning Ability in solving Problem

| Reasoning Indicator                          | Category  | Subject No |
|---------------------------------------------|-----------|------------|
| Formulating Presumption                     | High      | S01        |
| Performing Mathematics manipulation         |           |            |
| Checking the validity of an argument        |           |            |
| Drawing conclusion or generalizing          |           |            |
| Formulating Presumption                     | Moderate  | S02        |
| Performing mathematics manipulation         |           |            |
| Checking the validity of an argument        |           |            |
| Involve only one indicator that is Formulating Presumption | Low      | S03        |

Figure 2. The Answer of High Category of Student's Reasoning Ability

Based on Figure 2, the result of the research showed the profile of the students with high category in mathematical reasoning in solving geometry problem. The students had shown reasoning ability by exposing four indicators of reasoning.

Furthermore, the students with low category of reasoning ability had not been able to expose four categories of reasoning. The students’ answers were shown in the following Figure 3.
The following were the results of the interview with the teacher and student whose answer in Figure 2.

| Code | Description |
|------|-------------|
| Teacher | Do you know what the meaning of question number 1 is? |
| Student | Yes Mam. |
| Teacher | Now, please tell me about question number 1 in your own words. |
| Student | There are 2 holes, red and yellow flags, the player with red flag \(200\sqrt{2} \text{m} \) and the distance between red flag and yellow flag is 100 m. |
| Teacher | Is that all? |
| Student | Yes Mam |
| Teacher | Alright... Then what is the question ask? |
| Student | It is how far the distance between the golf player and the red flag. |
| Teacher | What are you need to answer the question number 1? |
| Student | Phytagoras formula, Mam |
| Teacher | Why do you need this formula? |
| Student | To answer the question no 1, Mam. |
| Teacher | If it is so, could you please explain the steps to solve the question no 1? |
| Student | The solution is by using phytagoras formula |

Therefore, it can be concluded that the student did not have good reasoning ability. He could not understand the question so that he failed in describing the known problem into a proper sketch form as well as finding and explaining the solution used based on the invention toward the calculation process. The information was one that should be described or structured to identify the formed relation in order to reveal the problem solution. Furthermore, in rechecking stage, the student could not write the conclusion. This was because the student did not recheck his final result as well as its effect toward the accuracy of the final answer.
The student who had high category of mathematical reasoning fulfilled the following reasoning indicators: (1) formulating Presumption, (2) performing mathematics manipulation, (3) checking the validity of the argument, (4) drawing conclusion or generalizing. Based on the research result, it was found that the existence of various students reasoning in solving geometrical mathematics problem was the student with high category of reasoning tend to fulfill the indicators well, the student with moderate category of reasoning tend to fulfill the indicators as quite good, and the student with low category of reasoning tend to fulfill the reasoning indicators not well because in evaluating the problem solving the student was less able to arrange and apply the strategy of problem solving.

Student: The solution is by using phytagoras formula

Teacher: So, it means that the answer is correct?
Student: Yes, Mam.
Teacher: Are you sure with the final calculation?
Student: I’m sure.

Therefore, it can be concluded that the student did not have good reasoning ability. He was not able to understand the question properly which made him fail to describe the question he knew in the form of sketche correctly, look for and explain the solution used based on the process of calculation correctly. This information should have been described or structured to identify the relationship which was formed so he was capable to prove his problem solving. Furthermore, at the re-examination stage, the student could not write conclusion. It happened as the student did not re-check his final answer obtained and his effect on the accuracy of the final answer.

The students who had high mathematical reasoning abilities would fulfill the reasoning indicators as follows: (1) formulating Presumption, (2) performing mathematics manipulation, (3) checking the validity of an argument, (4) drawing conclusion or generalizing. Several findings were found in this research in which they covered the reasoning diversity of each student in solving mathematical geometry problems, they belonged to high-reasoning and tended to fulfill the indicators of reasoning well, the students with moderate reasoning ability tended to fulfill reasoning indicators quite well, while the students on low reasoning ability met the reasoning indicators poorly as the students who were under low category when they evaluated problem solving was less able to develop and implement strategies for problem solving steps.

The diagram of the level of students’ reasoning ability in solving geometry problems on low, moderate and high categories is presented on Figure 4.

![Figure 4. The Achievement and Test Result of Students’ Reasoning Ability](image-url)
Based on Figure 4 shows that reasoning abilities of students 15% in the high category, 67% in the ability of the medium category, and 18% in the low category ability. The Results of the Discussion on Mathematical Reasoning Ability in Solving Problems on each component of reasoning are as follows:

**High-Reasoning Ability**
At the stage of formulating presumption, the subject was capable to write down the targets of the elements that he knew and write the unit correctly. In Performing Mathematics manipulation, the subject could describe the question he knew in the form of sketches, it is shown from the student’s skill to apply existing problem into sketches. In checking the validity of an argument, the subject was able to connect the elements he knew to the formula of cosine rule so that the solution needed to be proven based on the question and the subject could apply the known element to the formula of cosine rule so that the desired answer from the problem would be obtained. At the stage of drawing conclusions, the subject re-checked the final answer obtained and the effect on the accuracy of the final answer and wrote the conclusion appropriately.

**Moderate-Reasoning Ability**
At the stage of formulating presumption, the subject was capable to write down the targets of the elements that he knew and write the unit correctly. In Performing Mathematics manipulation, the subject could describe the question he knew in the form of sketches, it is shown from the student’s skill to apply existing problem into sketches. In checking the validity of an argument, the subject was able to connect the elements he knew to the formula of Pythagoras so that the solution is proven from the problem and the subject is unable to prove the correct solution, this can be seen from the inability of the subject to connect the known elements with the formula Pythagoras so that no solution can be obtained from the question. At the stage of drawing conclusions, the subject do not write conclusions correctly and only write the unit. This is because the subject does not re-check the final results obtained and its effect on the accuracy of the final answer.

**Low-Reasoning Ability**
The subject was still not good in solving the problem, so that he required the habituate of problem solving. In formulating Presumption, the subject was able to write down the targets from known elements and write down the unit even though it was not quite right. In Performing Mathematics manipulation, the subject could outline the known question into sketch even though it was less precise having no sign on each vertex. In checking the validity of an argument, the subject could not search and explain the solution he used, he was only capable to guess and imagine the process of calculation. Furthermore, in drawing conclusions, subjects could not write conclusion. This was due to the subject who disd not re-check the final answer and its effect on the accuracy of the final answer.

4. **Conclusion and Suggestion**
Based on the results this research and discussion, it can be concluded that the students’ mathematical reasoning ability in solving problems on high categories is able to fulfill the indicators of reasoning well. The students who are on moderate category are able to fulfill the indicators of reasoning quite well. On the contrary, the students who belong to low-reasoning ability are less able to fulfill the indicators of reasoning.

The researchers suggest the other researchers to explore further based on this research on different level and material but with the same or different point of view concerning the students’ reasoning profiles. The results of this research can also be used to conduct developmental research based on the findings obtained in this research.

**Acknowledgment**
We gratefully acknowledge the support from Faculty of Teacher Training and Education - the University of Jember, especially Postgraduate Program of Mathematics Education- University of Jember of year 2018.
References

[1] Sumarmo U 2012 Character education and the development of mathematical thinking and
disposition in mathematics learning Mathematics Education pp 1–26.

[2] Sukayasa 2009 Reasoning and Problem Solving in Geometry Learning Pros. Semin. Nas.
Penelitian, Univ. Negeri Yogyakarta pp 545–552.

[3] Lithner J 2008 A research framework for creative and imitative reasoning. Educ. Stud. Math 67
pp 255–276.

[4] Mariana N, Siahaan P and Utari S 2018 Scientific reasoning profile of junior secondary school
students on the concept of static fluid J. of Physic pp 2–6.

[5] Sumarsih, Budiyono and Indriati D 2018 Profile of mathematical reasoning ability of 8 th grade
students seen from communicational ability, basic skills, connection, and logical thinking J.
Phys. 1008 pp 0–10.

[6] Zhou S, Han J, Koenig K, Raplinger A, Pi Y, Li D, Xiao H, Fu Z, and Bao L 2016 Assessment of
scientific reasoning: The effects of task context, data, and design on student reasoning in
control of variables Think. Ski. Creat. 19 pp 175–187.

[7] Saglam Y and Dost S 2014 Preservice Science and Mathematics Teachers’ Beliefs about
Mathematical Problem Solving Procedia - Soc. Behav 116 pp 303–306.

[8] Caprioara D 2015 Problem Solving - Purpose and Means of Learning Mathematics in School
Procedia - Soc. Behav. Sci. 191 pp 1859–1864.

[9] Chairani Z 2013 Implications of Van Hiele’s Theory in Geometry Learning Lantern of the
Educational Scientific Journal 8 pp 20–29.

[10] Sugiyono 2017 Metode Penelitian Kombinasi (Mixed Methods) Bandung: Alfabeta