Modelling and Stability of Cylindrical Grid Shells

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Abstract. The constructive forms of the lattice surfaces are analyzed and the rational system of the cylindrical shell is chosen. The grid with cells having longitudinal, transverse and diagonal rods was studied. The use of nodes with a rigid connection of elements was taken into account. The features of the stability loss of cylindrical mesh shells within the given geometric parameters are determined. The regularities of systems behaviour under the action of operational loads are founded. It is revealed that with the increase of span and other sizes the coefficient of stability margin of the structure decreases. The forms of the possible onset of the critical state of the shells with increasing imposed load were obtained.

1. Introduction
The shape of cylindrical mesh shells can have a circular or parabolic shape in the cross section. The greatest bearing capacity has an elongated spatial grid. The system with the faces located on a circle is considered less expensive. Taking into account the importance of taking into account the economic factor, preference is given mainly to shells with the same faces. As evidenced by previous studies, [1] the circular mesh is vulnerable to the actions of applied loads. There is a risk of loss of stability of the surface. The paper, [2] discusses peculiar properties of the behaviour of two-layer systems. But they are less susceptible to deformation processes. More sensitive are single cylindrical meshes [3–9] with the same type of rods. Therefore, the proposed article aims to investigate the stability of single-layer cylindrical grids with different ratios of geometric parameters.

2. Grid modelling
For the formation of the shell, the experience of research in the field of creating a rational rod spatial lattice was taken into account. Computer models with different geometrical parameters are constructed. The size ratios are found in the modelling process and in accordance with the design recommendations. For each design scheme is given the material and stiffness characteristics of the rods. Boundary conditions on the contour are defined and accepted. The calculation was carried out and the imposed loads are applied.

2.1. Features of shaping
By analysing the operation of all possible grids the most efficient system with longitudinal and transverse elements was chosen. The required stiffness of the shell is achieved by setting the downward diagonal rod in each cell on all the mesh area. We used the method of rotation of the surfaces and having the data about the elements, form and fill type of the mesh is constructed corresponding finite element
models. The structure is formed on the basis of the fixed angle of the circumscribed circle \((\alpha = 120^\circ)\) taking into account the same number of panels along the length and faces in the direction of the arc of the circle \((n = m = 12)\). The main parameters (width \(B\), length \(L\), radius of curvature \(r\), cell size \(l \times h\), height \(f\)) are shown in Figure 1.

\[
\alpha/2
\]

\[
m
\]

\[
 Fragment
t of lattice
 surfac e

\[
Belt
(Longitudinal edge)
\]

\[
Rack
(Transverse edge)
\]

\[
Oblique strut
(Descending diagonal)
\]

**Figure 1.** Plan and cross-section of the mesh shell with geometric parameters

### 2.2. Relationships of sizes

The geometry of the grid was created in the framework of the relations of the calculated parameters, which are recommended for design. The angle \(\alpha\) of the circumscribed circle, the radius of curvature \(r\) and the width \(B\) of the shell is set. During the formation process the sizes of cell \(l \times h\), height \(f\) and length \(L\) of the structure are determined. The main parameters are shown in Table 1.

| \(B, m\) | \(L, m\) | \(r, m\) | \(l \times h, m\) | \(f, m\) |
|---------|---------|---------|----------------|-------|
| 18      | 25.92   | 10.4    | 2.16 \times 1.81 | 5.2   |
| 24      | 34.32   | 13.9    | 2.86 \times 2.42  | 6.95  |
| 30      | 43.32   | 17.3    | 3.61 \times 3.02  | 8.65  |

**Table 1.** Geometric parameters of shell models

### 2.3. The parameters of rods

Characteristics of rigidity of the rods were assigned on the basis of available basic types of section of rolling profiles. Steel round thin-walled tubes are used as mesh surface elements \((D/t = 12,5\ldots40)\). Was accepted limitation on diameter \(D \leq 0,15\) m.

Depending on the direction of the rods in space (along, across and diagonally) three type sizes of elements are selected.
2.4. Boundary conditions

Based on the requirement of geometric immutability, a hinged method of fastening the structure along the contour (a) with the imposition of a ban in two directions on the angular support units (b) and a complete prohibition of movements in one corner point (c) is presented (show in Figure 2).

![Options of fastening mesh shell in places the corner support nodes](image)

Figure 2. Options of fastening mesh shell in places the corner support nodes

In the mathematical form of fastening schemes of the surface can be written as

\[ Z = 0; \quad X = Z = 0; \quad X = Y = Z = 0. \quad (1) \]

The presented expressions (1) allow avoiding the occurrence of horizontal pressure from the action of loads.

2.5. The applied loads

Loads from its own weight are determined by the application computer program automatically after setting the stiffness characteristics of the rods. External distributed \( q \) and concentrated \( F \) forces are reduced to a system of unfavourable combinations and applied to the shell structure. Their calculated values using the load reliability coefficient \( \gamma_f \) are increased by 5% \( (\gamma_f = 1.05) \).

3. Materials and methods of research

The previously developed constructive forms of lattice surfaces are analyzed and the rational system of cylindrical shell is chosen. The grids with cells having longitudinal, transverse and diagonal rods are taken as a basis. The use of nodes with a rigid connection of elements is taken into account. The ratio of geometric parameters is chosen in such a way as to eliminate overvoltage, load as much as possible and most evenly include all the rods of the structure. The acceptance of the required number of cells is based on ensuring the stability of the grids within the framework of the requirements for the bearing capacity. For the rods of the reticulated surfaces steel of S235 is appointed. Studies for a given angle of the circumscribed circle \( \alpha = 120^\circ \) and the radius of curvature \( r = 10.4…20.8 \) m are carried out. These values allowed to reduce the vulnerability of the inclined parts of the shell and to determine the possible limit of applicability of overall dimensions \( B = 18…36 \) m and \( L = 25.92…51.84 \) m. The angle between the diagonal and longitudinal elements in the plane of the faces is obtained within 45°. The calculated finite element models are constructed using the surfaces of rotation. Finite element models are constructed using surfaces of rotation. Perturbations are introduced to determine the sensitivity to imperfections.

4. Results and discussion

The desire to evenly load the mesh surface justified the location of the descending diagonal elements. The verifications of the bearing capacity of the shells were quite acceptable and corresponded to the conditions of the limiting states.

\[ \sigma_i \leq R_y \gamma_c; \quad \delta_i \leq \delta_u; \quad \delta_{\text{max}} \leq \delta_u. \quad (2) \]

where \( \sigma_i \) – tension in rods; \( R_y \) – design resistance of steel of rods; \( \gamma_c \) – the coefficient of operating conditions of the rods; \( \delta_i, \delta_{\text{max}}, \delta_u \) – actual, maximum, and limit movement of nodes, respectively.

The change in geometry did not lead to the loss of stability of the structure.
The sensitivity of the mesh surface was directly dependent on the radius of curvature $r$ and the height $f$ of the shell. This state is reflected by the number of $n$ half-waves in the direction of the arc and the coefficient $\lambda$ of supply of stability of the grid

$$n = 3; \lambda \in [0, \Lambda] \tag{3}$$

The choice of the calculated value of the critical load $q_{cr}$ is carried out under the condition

$$q_{cr, \text{min}} \leq q_{cr} \leq q_{cr, \text{max}} \tag{4}$$

Therefore, the load is taken in the calculations, which is equivalent to the minimum critical $q_{cr, \text{min}}$. Otherwise, the stability of schemes of shells will not be ensured.

As studies have shown, the value of the critical load was $18 \ldots 26\%$ more than the value of the imposed load. The revealed forms of the possible occurrence of the critical state of the shells indicated the presence or absence of internal resources and the size of the energy barrier of the structures.

In the static calculation, taking into account the full application of the load on the system, within the requirements of the norms, the maximum vertical displacements of nodes are obtained. However, when calculating the stability of the grid surfaces, a totally different shape change was determined. In figure 3 shows the schemes of deformation of the shells considered with increasing imposed load.

![Figure 3. Changing of the shape of three models of mesh shell surfaces](image-url)
Obtained from the results of studies of regularities in the behaviour of the considered systems is reflected in the identical nature of the change in the geometry. Despite the same number of half-waves, the specified range of dimensional parameters influenced the value of the coefficient of supply of stability. In the longitudinal direction, the change in shape occurred in one half-wave. The systems did not have to overcome a significant energy barrier. However, they were in different degrees of vulnerability, as evidenced by the obtained coefficients of supply of stability 10.31; 3.76; 1.73. This factor was decisive for the installation of the border increase in span and other overall dimensions.

The General condition of the mesh surfaces also showed a weakening with an increase in the maximum force in the element and of the greatest vertical movement of the node.

5. Conclusions
According to the researches results, the features of the stability loss of cylindrical mesh shells within the given geometric parameters are determined. The regularities of systems behaviour under the action of operational loads are determined. It is revealed that with the increase of span and other sizes the coefficient of stability margin of the structure decreases.

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