The effect of rotation on the heat transfer between two nanoparticles

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Abstract. Quantizing the electromagnetic vacuum and medium fields of two nanoparticles, we investigate the heat transfer between them. One of the particles has been considered to rotate by angular velocity \( \omega_0 \). The effect of rotation on the absorbed heat power by the rotating nanoparticle is discussed. The results for angular velocities much smaller than the relaxation frequency \( \Gamma \) of the dielectrics are in agreement with the static nanoparticles, however increasing the angular velocity \( \omega_0 \) in comparison to the relaxation frequency of the dielectrics \( \omega_0 \geq \Gamma \) generates two sidebands in the spectrum of the absorbed heat power. The well-known near-field and far-field effects are studied and it is shown that the sidebands peaks in far-field are considerable in comparison to the main peak frequency of the spectrum.

1 Electromagnetic field quantization

The Lagrangian describing the whole system contain a term represent the electromagnetic vacuum field plus terms modelling the dielectrics and their interaction with the electromagnetic vacuum field. Following the method introduced in [13,23], we study the heat transfer to the rotating NP and its physical consequences.

We consider the following Lagrangian for the mentioned system,

\[
\mathcal{L} = \frac{1}{2} \epsilon_0 (\partial_i A_i)^2 - \frac{1}{2\mu_0} (\nabla \times A)^2 + \frac{1}{2} \int_0^\infty d\nu \left( [\partial_i X^1 + \omega_0 \partial_\nu X^1]^2 - \nu^2 (X^1)^2 \right) - \epsilon_0 \int_0^\infty d\nu \int_0^\infty \epsilon_{ij} (\nu, t) X^1_j \partial_i A_i + \epsilon_0 \int_0^\infty d\nu \left( [\partial_i X^2]^2 - \nu^2 (X^2)^2 \right) - \epsilon_0 \int_0^\infty d\nu f_{ij} (\nu, 0) X^2_j \partial_i A_i, \tag{1}
\]

where \( X^1 \) and \( X^2 \) are the dielectric fields describing the first (rotating) and second NPs respectively and \( \omega_0 \) is the angular velocity of the rotating NP. NPs are considered to be in local thermodynamical equilibrium at temperatures \( T_1 \) and \( T_2 \) respectively. \( f_{ij} (\nu, t) \) is the coupling tensor between the electromagnetic vacuum field and the medium fields \( X^1 \) and \( X^2 \). As the NPs are considered to be totally

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Fig. 1. A rotating nanoparticle located on axis z a distance d from a same static NP at the origin.

similar, they should have identical coupling tensor while to take care of the rotation of NP.1, its coupling tensor \( f_{ij}(\nu, t) \) is considered to be time dependent and for NP.2 it is time-independent as \( f_{ij}(\nu, t = 0) \) where

\[
f_{ij}(\nu, t) = \begin{pmatrix} f_{xx}(\nu) \cos(\omega_0 t) & f_{xy}(\nu) \sin(\omega_0 t) & 0 \\ f_{yx}(\nu) \sin(\omega_0 t) & f_{yy}(\nu) \cos(\omega_0 t) & 0 \\ 0 & 0 & f_{zz}(\nu) \end{pmatrix}.
\]

To derive the Lagrangian equation (1), the coordinate derivative and field transformation between rotating and fixed frames are used. As the electromagnetic fields are non-rotating fields, there is no need to modify them.

The response function \( \chi_{kk}(\omega) \), corresponding to set \( \omega_0 = 0 \), can be obtained in terms of the diagonal components of the coupling tensor \( f_{ij}(\nu, t) \) as [23],

\[
\chi_{kk}^0(\omega) = \epsilon_0 \int_0^\infty d\nu \frac{f_{kk}^2(\nu)}{\nu^2 - \omega^2}.
\]

The response functions of the rotating NP in the laboratory frame can be written in terms of \( \chi_{kk}^0(\omega) \),

\[
\chi_{zz}(\omega, m) = \chi_{zz}^0(\omega - m\omega_0),
\]

\[
\chi_{xx}(\omega, m) = \chi_{yy}(\omega, m),
\]

\[
\chi_{xy}(\omega, m) = \frac{1}{2} [\chi_{xx}^0(\omega + m\omega_0) + \chi_{xx}^0(\omega - m\omega_0)],
\]

\[
\chi_{yx}(\omega, m) = -\chi_{xy}(\omega, m),
\]

\[
= \frac{1}{2} [\chi_{xx}^0(\omega + m\omega_0) - \chi_{xx}^0(\omega - m\omega_0)],
\]

where \( \omega_\pm = \omega \pm \omega_0 \).

Defining \( P_i^k(r, t) = \epsilon_0 \int_0^\infty d\nu f_{ij}(\nu, t) X_i^k(r, t, \nu) \), as the electric polarization components of NPs and using equation (1), we obtain the equations of motion for the electromagnetic and matter fields as

\[
P^1(r, \omega) = P^{N,1}(r, \omega) + \epsilon_0 \chi^1(\omega, -i\partial_\nu)v, 
\]

\[
P^2(r, \omega) = P^{N,2}(r, \omega) + \epsilon_0 \chi^0(\omega, \nu) E, 
\]

\[
\{\nabla \times \nabla \times - \omega^2 c^2 - \omega^2 c^2 \chi^1(\omega, -i\partial_\nu) - \omega^2 c^2 \chi^0(\omega)\} \cdot E
\]

\[
= \mu_0 \omega^2 (P^{N,1} + P^{N,2}), 
\]

where \( P^{N,1} \) and \( P^{N,2} \) are the fluctuating or noise electric polarizations correspond to the fluctuating or noise matter fields \( X^{N,1} \) and \( X^{N,2} \) for the rotating and static NPs respectively. One can expand them in terms of ladder operators as

\[
X_i^{N,1}(\rho, \varphi, z, \nu, t) = \sum_m [e^{im\varphi} e^{i(\nu-m\omega)t} a_{i,m}^\dagger(\rho, z, \nu) + e^{-im\varphi} e^{-i(\nu-m\omega)t} a_{i,m}(\rho, z, \nu)], 
\]

\[
X_i^{N,2}(\rho, \nu, t) = e^{i\nu t} b_i^\dagger(\rho, \nu) + e^{-i\nu t} b_i(\rho, \nu). 
\]

In case of holding the NPs in thermal equilibrium at temperature \( T_1 \) and \( T_2 \), we have

\[
\langle a_{i,m}(\rho, z, \nu) a_{j,m'}(\rho', z', \nu') \rangle_T = \frac{\hbar}{4\pi\nu} n_{T_1}(\nu) \delta_{mm'} \delta_{ij},
\]

\[
\delta(\nu - \nu') \delta(\rho - \rho') \delta(z - z'),
\]

\[
\langle b_i^\dagger(\rho, \nu), b_j(\rho', \nu') \rangle_T = \frac{\hbar}{2\nu} n_{T_2}(\nu) \delta_{ij} \delta(\nu - \nu') \delta(\rho - \rho'),
\]

where \( n_T(\omega) = [\exp(\hbar\omega/kT) - 1]^{-1} \).

Using (4) and the dyadic Green’s tensor \( G_{ij} \), we find

\[
E_i(r, \omega) = E_i^0(r, \omega) + \mu_0 \omega^2 \int_{V_1} d\nu' G_{ij}(r, \nu', \omega) P_j^{N,1}(r', \nu) 
\]

\[
+ \mu_0 \omega^2 \int_{V_2} d\nu' G_{ij}(r, \nu', \omega) P_j^{N,2}(r', \omega),
\]

where the first term on the right-hand side of (7) corresponds to the fluctuations of the electric field in electromagnetic vacuum, while the second and third terms are the induced electric field due to the fluctuations of the electric polarization of the NPs. Mitsui and Aoki reported an observation of spontaneous quantum fluctuations in photon absorption by atoms where it is a kind of heat transfer [24].

2 Heat transfer

The rate of work done by the electromagnetic field on a differential volume \( dv \) of a dielectric is given by \( \mathbf{j} \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B}) dv \) where \( \mathbf{j} = \partial_t \mathbf{p} - \nabla \times (\mathbf{v} \times \mathbf{p}) \) is the current density in matter. In non-relativistic regime, we ignore the terms containing velocity \( \mathbf{v} \); therefore, the radiated power of the rotating NP can be written as

\[
\langle P \rangle = \int_V \int_{-\infty}^{\infty} dw \frac{dw}{2\pi} e^{-i(\omega + \omega')t} (i\omega) \times (\mathbf{p}^{N,2}(r, \omega) + \mathbf{p}^{N,2}(r', \omega')).
\]

The radiated power \( \langle P \rangle \) contains all emitted and absorbed energy of the rotating NP due to the interaction with electric field \( \mathbf{E} \). One of the terms contains the fluctuating electrical polarization of the static NP at the origin \( \langle \mathbf{p}^{N,2}(r, \omega) \cdot \mathbf{p}^{N,2}(r', \omega') \rangle \), which we will focus on in the following, is responsible for the heat transfer power \( \langle P \rangle_{HT} \)
from the static NP at the origin to the rotating NP on the axis $z$ a distance $d$ from the origin. Using equations (7), and (8), we derive

$$
\langle P \rangle_{HT} = \frac{2\hbar}{\pi} \int_0^\infty \frac{d\omega}{\omega^5} \Im [\alpha^\omega_{ij}(\omega)] \Im [\alpha^0_{kj}(\omega)] n_{T2}(\omega) \\
\times \int d\mathbf{r}' G_{ik}(\mathbf{r}, \mathbf{r}', \omega) G_{kj}^*(\mathbf{r}, \mathbf{r}', \omega),
$$

(9)

where $\Im [\alpha^\omega_{ij}(\omega)] = V \Im [\chi_{ij}(\omega, m = 0)]$ and $V$ represents the volume of NPs. One can find a proper dyadic Green’s tensor $G_{ij}$ for equation (7) as

$$
G_{ij}(\mathbf{r}, \mathbf{r}', \omega) = \frac{\epsilon^{ikR}}{R^4 k^2} \left[ (k^2 R^2 + i k R - 1) \delta_{ij} \\
- (k^2 R^2 + 3 i k R - 3) \frac{R_i R_j}{R^2} \right],
$$

(10)

where $\mathbf{R} = \mathbf{r} - \mathbf{r}'$ and $k = \omega/\epsilon$. In this work, the interparticle distance $d$ between NPs are chosen to be much bigger than their radius, therefore, it is a good approximation to consider them as point-like particles. On the other hand, using this approximation, the components of dyadic Green’s tensor $G_{ij}$ can be simplified to

$$
G_{xx}(0, d\hat{z}, \omega) = G_{yy}(0, d\hat{z}, \omega) = \frac{\epsilon^{ikd}}{d^4 k^2} (k^2 d^2 + i k d - 1),
$$

$$
G_{zz}(0, d\hat{z}, \omega) = \frac{2\epsilon^{ikd}}{d^4 k^2} (1 - i k d),
$$

(11)

and all other components are vanished.

### 3 Discussion and results

To find some numerical results, the NPs are considered to be made of silicon carbide (SiC) with similar radius ($a = 2$ nm), where the dielectric function is given by the oscillator model [25],

$$
\varepsilon(\omega) = \varepsilon_\infty \left( 1 + \frac{\omega_L^2 - \omega^2}{\omega_T^2 - \omega^2 - i \Gamma \omega} \right),
$$

(12)

with $\varepsilon_\infty = 6.7$, $\omega_L = 1.823 \times 10^{14}$ rad/s, $\omega_T = 1.492 \times 10^{14}$ rad/s, and $\Gamma = 8.954 \times 10^{11}$ rad/s.

It has been seen that in a cavity optomechanics, a mirror oscillating with frequency $\omega_0$, the optical sidebands are created around the incoming light frequency $\omega$,

$$
\omega' = \omega \pm \omega_0,
$$

(13)

although, in this work we are far from the cavity optomechanics, but both cases contain a kind of oscillation frequency $\omega_0$ on the matter fields. Thus, one can expect identical results on the spectrum of the absorbed heat transfer, where interestingly, it is supported by Figure 2. It shows that, increasing the angular velocity $\omega_0$, can cause a couple of sidebands on the spectrum of the absorbed heat transfer by rotating NP from the static NP, where the sidebands appeared around the remarkable peak frequency $\omega$ of the absorbed heat transfer spectrum of non-rotating NPs. The sidebands frequencies are given by equation (13). While the spectrum, for angular velocities smaller than the relaxation frequency $\Gamma$ of dielectrics, is the same as the spectrum for non-rotating NPs. Having a discontinuous behavior for $\omega_0 < \Gamma$ and $\omega_0 > \Gamma$ is not surprising. In fact, the relaxation frequency $\Gamma$ is well related to the response time of the dielectric to an external electric field. So any change that happens faster than this response time may cause new behaviors.

The effect of static NP temperature $T_2$ on the spectrum of the absorbed heat transfer of rotating Np has been depicted in Figure 3. As a result of that, the temperature of the static NP will affect the absorbed heat transfer of rotating and non-rotating NP in the similar way.

To focus more on the effect of rotation in the near and far field of the static NP, the total absorbed heat power of rotating NP as a function of the interparticle distance $d$ has been depicted in Figure 4, where it provides a dependence on $d^{-6}$ in the near field and on $d^{-2}$ in the far field as.
reported previously for non-rotating NPs [17,26] but interestingly, for \( \omega_0 = 5 \times 10^{13} \), in the mean while between the near field to the far field, interparticle distances \( 10^{-6} \) to \( 10^{-4} \) m, the total absorbed heat power of rotating NP depends on \( d^{-4} \). In case of non-rotating NPs, as one can see in Figure 4, there is no such effect and also no one has reported such functionality of total absorbed heat power of a NP.

Figure 4a shows a transition on the total absorbed heat power of rotating NP normalized by the total absorbed heat power of a non-rotating NP. Also, there is a remarkable maximum around one micron on that. As it appeared where the interparticle distance equals to the wavelength of the peak frequency of the absorbed radiation spectrum, it might be the effect of a resonance. As a consequence of transition between near field and far field of the static NP, One can expect, the effect of rotation on the absorbed heat power of a rotating NP can be quite different in near field and far field of the static NP. Therefore, we renew the plot of Figure 2, where it was depicted in the near field, in far field of the static NP. In Figure 5 surprisingly, the contribution of sidebands peaks has raised in compare to the main peak of the spectrum. This future may find a lot of applications, e.g., in designing the new optomechanical systems.

4 Conclusions

In conclusion, the effect of rotation on the heat transfer between two NPs has been analysed. Two sidebands appeared around the peak frequency of the heat transfer spectrum of non-rotating NPs. While the peaks of the sidebands are so small in compare to the main peak in the near field, they are considerable in the far field even up to the same order of magnitude in compare to the main peak frequency of the spectrum. It has been shown that the total absorbed heat power of a rotating NP, with angular velocities \( \omega \), larger than the relaxation frequency \( \Gamma \) of the dielectric, experience a new regime between the near field and the far field where it provides the dependence on \( d^{-4} \) and the range of this interesting regime decreases by the angular velocity of the NP and finally vanishes for the angular velocities smaller than the relaxation frequency \( \Gamma \) of the dielectric.

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Author contribution statement

Both authors contributed equally to the paper.

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