Effect of Shear Reinforcement on Punching Shear Capacity of Reinforced Concrete Flat Plates

Osama Mohamed, Rania Khattab
Abu Dhabi University, College of Engineering, Abu Dhabi, UAE
Osama.mohamed@adu.ac.ae

Abstract. Punching or two-way shear is a critical failure mode that must be accounted for in reinforced concrete (RC) slabs that are not supported by beams, known as flat plates. The often gravity-driven failure occurs around supporting columns may also be influenced by lateral forces. This paper presents the findings of a numerical study that evaluated the effect on punching shear strength of shear reinforcement bar diameter, and stirrups distribution length (L), measured from the face of the supporting column. The study was conducted on seven 3D nonlinear finite element models representing RC flat plates subjected to concentric vertical loads. Each of the seven models simulates the punching shear behaviour of interior slab-column connections with different shear reinforcement bar diameters and/or reinforcement distance from the face of the supporting column. Numerical investigation showed that increasing the stirrup diameter causes a significant increase in the ultimate two-way shear capacity. On the other hand, increasing the stirrups distribution length provides a moderate increase in the ultimate punching shear capacity. Furthermore, punching shear assessment using 3D finite element analyses were in good agreement with the mathematical models of the ACI-318-2019 building code.

1. Introduction
Punching or two-way shear is a critical failure mode in reinforced concrete slabs that are not supported by beams, known as flat plates. Two-way shear failure may occur at the slab-column junction and without any warning which may lead to localized or even progressive collapse of the structure [1]. The punching shear failure of this connection takes place when a plug of concrete is pushed out of the slabs immediately under the loaded area. Transfer of forces between the slab and the column usually governs the design of the structural system [2]. Experimental studies by Alaa and Walter [3] on continuous flat plates concluded that flexural steel parallel to the free edge of an exterior column will reach yielding while flexural reinforcement perpendicular to the free edge does not yield, with the exception of bars above the exterior column. The authors also noted that the ACI318 provisions for two-way shear are more conservative for flat plates without shear reinforcement than for flat plates with shear reinforcement. Shu et al [4] successfully developed finite element models using continuum nonlinear elements to predict the two-way shear strength of RC flat slabs without shear reinforcement. The proposed models were able to predict experimental data with smaller scatter and deviation compared to Eurocode 2. Polak, et al [5] conclude that bent bars inclined of 135° that are used to resist tension at the bottom of the beam near mid-span may improve the ultimate punching shear resistance of the slab when they are well anchored. The flexural tensile reinforcement in flat slabs above the column contributes to the two-way shear capacity of the slab. Studies demonstrated that the contribution of tension steel to punching shear strength is under-estimated by ACI 318-14 and the ECP 203-2007 [6]. Sulaiman et al. [7] indicated that the punching capacity increases with the increase in the number of branches of closed
stirrups of the same diameter. A gain of 10% in punching capacity be achieved for a number of branches equal to 4 compared with the control specimen [7]. Tests by Jang and Kang [8] concluded that the ability of shear reinforcement to enhance punching shear capacity of flat plates is highly dependent on the amount of flexural reinforcement above column. The authors concluded that when the flexural reinforcement ratio is below a certain threshold, flexural damage decreases the punching shear capacity. The authors observed that two-way shear capacity may be increased by increasing flexural reinforcement rather than only shear reinforcement. Shear reinforcement was reported to affect the bond of fiber-reinforced polymer composite (FRP) reinforcing bars. Studies by El-Ghandour et al. [9] on flat slabs reinforced with carbon FRP (CFRP) bars concluded that bond of tensile flexural reinforcement to surrounding concrete is enhanced is slippage is decreased with the presence of shear reinforcement. The authors noted that shear reinforcement prevents splitting of concrete around flexural reinforcement and thereby increases the strength of the slab above the supporting column. Experimental studies by Nguyen-Minh and Rovnak [10] on flat plates reinforced with GFRP bars concluded that increasing glass FRP (GFRP) longitudinal reinforcement ratio increases the two-way shear strength significantly. The authors developed a prediction formula for two-way shear capacity of flat slabs reinforced with GFRP longitudinal bars that takes into account the effects of reinforcement ratio, span/depth ratio, and depth of compression zone. A review of experimental data by Mohamed and Khattab [11] showed that CAN/CSA S806-12 accurately predicts the two-way shear capacity of flat plates reinforced with various types of FRP bars. ACI 440.1R-15 appears to underestimate the punching shear capacity of flat plates.

2. Punching shear design in building codes and standards

2.1. American Concrete Institute ACI-318-19 Model [12]

The two-way shear strength, $v_c$, of flat slab structures is given by Eqn. (1) and is based on critical perimeter located at a distance $0.5d$ from the perimeter of the loaded area.

$$v_c = \min \left\{ \begin{array}{l}
0.17 \left( 1 + \frac{z}{\beta} \right) \lambda_s \lambda' \sqrt{f'_c} \frac{b_0 d}{b_0} \\
0.083 \left( 2 + \frac{c_d}{b_0} \right) \lambda_s \lambda' \sqrt{f'_c} \frac{b_0 d}{b_0} \\
0.33 \lambda_s \lambda' \sqrt{f'_c} \frac{b_0 d}{b_0}
\end{array} \right\}$$

(1)

where: $\beta$ = supporting column aspect ratio (long side/short side); $\lambda$ = a modifier for lightweight concrete; $f'_c$ = compressive strength of concrete cylinders, not exceed 68 MPa; $c_d = 40$ for interior columns, 30 for exterior columns, and 20 for corner columns; and $\lambda_s$ = size effect factor, accounting for member depth. Ultimate shear force in flat plates without shear reinforcement should satisfy Eqn. (2):

$$v_u \leq \phi v_c$$

(2)

where: $\phi$ is the reduction factor for shear equal to 0.75. The ultimate shear in flat plates with shear reinforcement shall be calculated using Eqn. (3).

$$v_c = 0.17 \lambda_s \lambda' \sqrt{f'_c} \frac{b_0 d}{b_0} + \frac{A_s f_yt}{s} d \leq v_{c,\text{max}}$$

(3)

$$v_{c,\text{max}} = \phi 0.5 \sqrt{f'_c} \frac{b_0 d}{b_0}$$

(4)

where: $b_0$ = perimeter of the critical section for two-way shear; $d$ = effective depth from extreme compression face to centroid of longitudinal tension reinforcement; $f_yt$ = yield strength of shear reinforcement; and $s$ = spacing of transverse reinforcement, measured centre-to-centre.

Figure 1 shows the critical section for punching shear without shear reinforcement, as well as the critical section outside of the shear reinforcement.
Figure 1. Critical perimeter with and without shear reinforcement for interior column according to ACI 318-19 [9]

2.2. British Standard BS EN 1992-1-1 [13]
The British Standard [13] considers the critical perimeter for two-way shear to be located at 1.5d from the perimeter of the loaded area. The two-way shear capacity of the slab without transverse reinforcement, $V_{rd,c}$, is calculated according to Eqn (5), subject to the minimum value described by Eqn. (6).

$$V_{rd,c} = C_{rd,c}k(100\rho_1f_{ck})^{1/3} \geq V_{min}$$  \hspace{1cm} (5)

$$V_{min} = 0.035k^{3/2}f_{ck}^{1/2}$$  \hspace{1cm} (6)

where: $f_{ck}$ is the cylindrical compressive strength of concrete (MPa), $C_{rd,c} = 0.18/\gamma_c$. $\gamma_c$ is a reduction factor equal 1.5; $k$ is a dimensionless parameter that takes into account the size effect and is defined by the expression $k = 1 + \sqrt{\frac{200}{d}} \leq 2d$; $\rho_j$ is the longitudinal reinforcement ratio defined by the expression $\rho_j = \sqrt{\rho_{1y}\rho_{1z}} \leq 0.02$; $\rho_{1y}$ and $\rho_{1z}$ are the ratios of the flexural reinforcement in both directions. The two-way shear strength, $V_{rd,cs}$, of flat slab including shear reinforcement, is given by Eqn (7).

$$V_{rd,cs} = 0.75V_{rd,c} + 1.5(\frac{d}{s})A_{sw}f_{ywd,ef}(\frac{1}{u_1d})\sin\alpha$$  \hspace{1cm} (7)

where: $s$ is the spacing of shear reinforcement (mm), $A_{sw}$ is the area of shear reinforcement around the column for one perimeter (mm$^2$); $u_1$ is the length of the basic control perimeter; $\alpha$ is the angle between the shear reinforcement and the plane of the slab; $f_{ywd, ef}$ is the effective design strength of...
punching shear reinforcement (MPa), calculated according to \( f_{ywd,ef} = 250 + 0.25d \leq f_{ywd} \); \( f_{ywd} \) is the design yield strength for punching shear reinforcement.

3. Details of specimens and finite element simulations

In this study, finite element models were created for ten RC flat slab specimens of dimensions 3000 mm \( \times \) 3000 mm \( \times \) 150 mm (thickness). Each slab is centrally loaded through using 300 mm \( \times \) 300 mm column that 1000 mm high. Each RC slab is reinforced longitudinally with a uniform mesh of #10 bars @ 150 mm at the top and bottom surfaces, as shown in figure 2. Punching shear reinforcement stirrups are variable.

Concrete compressive strength was 30 MPa while yield strength of reinforcing bars was 570 MPa. The density of concrete and steel was 2400 and 7850 Kg/m\(^3\), respectively. Slab \( S \) was a control slab without shear reinforcement. Slabs \( S1, S2, \) and \( S3 \) were reinforced for two-way shear using closed stirrups having diameters of 8, 10, and 12 mm respectively. Stirrup spacing for all slabs was maintained at \( d/2 = 65 \) mm (where \( d \) is the effective depth = 130 mm). The first stirrup was placed at a distance \( d/2 = 65 \) mm from the column face, then placement of stirrups continued to a distance equals to \( 2d = 260 \) mm from the face of the column. Slabs \( S4, S5, \) and \( S6 \) were reinforced in a similar manner to the other three slabs (\( S1, S2, \) and \( S3 \)), but placement of stirrups continued to a distance equals to \( 4d \) (560 mm) from the face of the column. Details of specimens and descriptions of shear reinforcement are shown in Table 1.

![Figure 2. Details of modelled flat slab and Concrete column](image-url)
Table 1. Details of specimens and description of shear reinforcement

| Specimens Number | Shear Reinforcement Details |
|------------------|----------------------------|
|                  | Stirrup diameter (mm) | Spacing (S) (mm) | Length (L) (mm) |
| S (Control)      | ___                 | ___             | ___            |
| S1               | 8                   | 65              | 260            |
| S2               | 10                  | 65              | 260            |
| S3               | 12                  | 65              | 260            |
| S4               | 8                   | 65              | 520            |
| S5               | 10                  | 65              | 520            |
| S6               | 12                  | 65              | 520            |

The numerical models were created using the commercial finite element package ABAQUS [14] which offers 3D static, quasi-static, or dynamic nonlinear simulation capabilities.

3.1. Elements type
The concrete element is the solid hexahedral linear brick element (C3D8R) with eight-nodes which uses reduced integration. The element provides reasonable stress and strain values without being computationally expensive. C3D8R is not stiff in bending and doesn’t create or affected by mesh sensitivity during iterations. Traditional two-node truss (T3D2) element was used to model reinforcing bars. A perfect bond between concrete and reinforcing bars was assumed and accounted through an embedded constraint technique. When the reinforcing bars are embedded, the transitional degrees of freedom will automatically be eliminated since they have merged with concrete nodes.

3.2. Concrete damage plasticity
To account for material nonlinearity, concrete damage plasticity (CDP) model was adopted. CDP is a continuum, plasticity-based, damage model which assumes two failure mechanisms consisting of the compressive crushing and tensile cracking (see figure 3 and figure 4). Continuum mechanics concepts permit the simulation of the complicated nonlinear behavior of concrete, a quasi-brittle material, to evaluate the structural behavior by putting this material altogether to construct the structural members. In this context, CDP is considered one of the best models to represent the complex behavior of concrete by implementing isotropic damaged elasticity along with isotropic compressive and tensile plasticity. For plasticity parameters, the dilation angle $\Psi$ is considered 36 degrees, the shape factor $K_c$ is 0.667, the stress ratio $\sigma_{00}/\sigma_{c0}$ is 1.16, the eccentricity $e$ is 0.1, and the viscosity parameter $\mu$ is 0.01. With regard to the damage state in the CDP input, the damage value for compression and tension is in the range of 0.0 to 0.99. Input parameters for steel material definition including density and elastic-plastic behavior. Young’s modulus for steel is $E_s = 200$ GPa and Poisson’s ratio is $\nu = 0.3$. 
3.3. Boundary condition and loading
To model pin supports, nodal points were restrained in the translational (U) directions U1, U2, and U3 while the rotational directions are left free. In the simulation procedure, a constant gravity load was applied to the specimen in the form of pressure load over the column. Figure 5a shows the finite element model of the specimens (S1) concrete dimensions, location of supported steel plates, and boundary conditions. Figure 5b shows the finite element model of the specimens (S1) reinforcement of column, slab, and punching shear reinforcement stirrups.

3.4. MESHING
The last step in creating the numerical model is to mesh the structural system. Increasing the number of elements in a finite element model will increase accuracy but will also increase the time necessary to solve the equations. Element size 25x25x25 mm produced reasonable results as demonstrated in subsequent sections.
4. Results and Discussions

4.1. Load-Deflection Curve

The load and deflection curves for the seven flat plates (S and S1 to S6) are shown in figure 6. The curves have the same pattern until the failure load is reached. All specimens reinforced with punching shear reinforcement increased the load-carrying capacity compared to the control slab (S) that was not reinforced for shear resistance. It can be seen in figure 6 as well that increase in the diameter of the punching shear reinforcement increases the slab stiffness. The deflection of specimen S1, S2, and S3 is equal to 86.04, 88.24, and 83.64 mm respectively, while for S4, S5, and S6 are equal to 78.73, 83.43, and 90.98 mm respectively. Increase in stirrup diameter of punching shear stirrups reinforcement from #8 for S1 and S4 to #12 for S3 and S6 decrease deflection of the slab at the same amount of load.

Figure 6. Load - Deflection Curve
4.2. Ultimate Load (Failure Load)
Table 2 shows the ultimate load for all specimens and the percentage increase in ultimate load capacity compared to control slab (S). The ultimate punching load increased significantly by 49.26%, and 61.78% for specimens S3, S6, respectively compared to the control specimen. In addition, increase in stirrup diameter from #8 to #10 increased the ultimate load from 369.53 kN to 409.03 kN. The punching shear capacity increased compared to the control slab (S) with an increase in stirrup diameter from #8 (S1) to #10 (S2), and finally to #12 (S3). Figure 7 shows the ultimate load for different punching shear reinforcement distribution length, L, measured from the face of the column. In the case of #8 shear reinforcement diameter, the increase in ultimate load was 1.74% when the distribution of punching shear stirrups reinforcement was increased from 2d to 4d. In the case of #10 shear reinforcement, the increase in ultimate load was 6.61%. When #16 shear reinforcement was used, the increase in ultimate strength was 8.39% when the distribution of punching shear stirrups reinforcement was increased from 2d to 4d.

| Specimens Number | $V_{FE}$ (kN) | % Increase in punching shear capacity |
|------------------|---------------|-------------------------------------|
| S (Control)      | 319.87        | -----                               |
| S1               | 369.53        | 15.53                               |
| S2               | 409.03        | 27.88                               |
| S3               | 477.44        | 49.26                               |
| S4               | 375.97        | 17.54                               |
| S5               | 436.05        | 36.32                               |
| S6               | 517.48        | 61.78                               |

**Figure 7.** Ultimate load for different punching shear stirrups reinforcement distribution length (L).

4.3. Comparisons of Simulation Results with Predicted Quantities based on Selected Building Codes
The predicted ultimate capacity ($V_{code}$) was calculated according to ACI 318-2019 [12] and BS EN 1992-1-1 [13] punching shear equations. The ratios between ultimate loads from the FE model ($V_{FE}$) to the predicted values ($V_{code}$) are reported in Table 3. Design codes showed conservative predictions with average $V_{FE}/V_{code}$ of 1.07±0.04 and 1.03±0.08 respectively. BS 8110-97 produced slightly unconservative predictions for the slabs S1, S2, and S3 with an average $V_{FE}/V_{code}$ of 0.99, 0.96, and 0.97.
respectively. The table shows that the finite element models prepared using ABAQUS [14] produced the ultimate capacity in good agreement with the ACI 318-2019 [12].

Table 3. Comparisons between different codes

| Specimens Number | d (mm) | $V_{FE}$ (kN) | $V_{ACI}$ (kN) | $V_{EN}$ (kN) | $V_{ACI}/V_{EN}$ |
|------------------|--------|---------------|----------------|----------------|-----------------|
| S (Control)      | 130    | 319.87        | 303            | 270            | 1.06            | 1.18            |
| S1               | 130    | 369.53        | 360            | 372            | 1.03            | 0.99            |
| S2               | 130    | 409.03        | 387            | 427            | 1.06            | 0.96            |
| S3               | 130    | 477.44        | 459            | 491            | 1.04            | 0.97            |
| S4               | 130    | 375.97        | 360            | 372            | 1.04            | 1.01            |
| S5               | 130    | 436.05        | 387            | 427            | 1.13            | 1.02            |
| S6               | 130    | 517.48        | 459            | 491            | 1.13            | 1.05            |
| Average          |        | 400.03        | 393.67         | 411.77         | 1.07            | 1.03            |
| SD               |        |               |                |                | 0.04            | 0.08            |

5. Conclusions
This paper presents the findings of a numerical study to evaluate the two-way shear capacity of flat plates with various shear reinforcement diameters and length of distribution measured from the face of the supporting column. Seven flat slabs were reinforced with closed-stirrup two-way shear reinforcement while one control slab didn’t have shear reinforcement. The two-way shear capacity and deformations of the seven flat slab specimens were compared to a control slab. Based on the numerical results, the following conclusions are noted:

1. Increase in diameter of the two-way shear reinforcing stirrups increases the stiffness of the reinforced concrete slab.
2. Increase in diameter of the punching shear reinforcing stirrups increases the ultimate shear capacity of the reinforced concrete flat slab.
3. Ultimate punching shear capacities obtained using the finite element models are in agreement with the mathematical models in ACI-318-19.
4. Extending shear reinforcing stirrup for longer distances (2d to 4d) from face of supporting column only increases the ultimate punching shear capacity slightly.

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