The Implied Volatility Analysis: The South African Experience

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Abstract

In this paper, we analyse the South African implied volatility in various setting. We assess the information content in SAVI implied volatility using daily markets data. Our empirical application is focused on the FTSE/JSE Top 40 index and we emphasize our models performance in distinct sub-periods. Our results are compared with VIX/VXN and S&P 500/NASDAQ 100 data in some points which are taken as our benchmark. We find a significant negative relationship between returns and volatility, in line with the results found in other markets. Finally, the link between SAVI, VIX and VXN are undertaken to examine the equity market transmission with respect to uncertainty.

1 Introduction

In this article, we analyze the SAVI and assess its information content regarding realized volatility, the returns of the underlying equity index, RiskMetric and Garch-type models using different day time horizon. The implied volatility is the option market’s forecast of future return volatility over the remaining live of the option. The last decades have witnessed an increase in the amount of literature in the field of volatility forecasting and estimation. The principal motivation consolidating this branch of research is that volatility is the substratum in every financial application. Being that; in financial econometrics, a great amount of research has focused on the ARCH-type models that use past disturbances to model the variance of a time series (see e.g. Engle, 1982 and Bollerslev, Chou, and Kroner, 2001) for Arch type models, stochastic volatility models, (see e.g Taylor, 1986). More recently, realized volatility models is used intensively in finance (see e.g Andersen, Bollerslev, Diebold and Ebens, 2001). Finally there is some class of volatility based on kernel approach of which the use is very limited in finance see Florens-Zmirou (1993), Kenmoe and Sanfelici (2013), for an introduction and some applications. All these models used the historical asset price series to predict the future volatility of underlying asset An alternative way of predicting volatility relies on exploring the information contained in the options asset. When options price are available, one can compute the stock’s market volatility equating option price and the pricing model. As noted by Frijns et al., (2010), this implied

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volatility is a forward looking and represents the market’s estimate of the future volatility of the underlying asset for the remaining lifetime of the option. Whaley (1993) proposes a leading work in which the implied volatilities of various near-the-money options on the S&P 100 index is used to construct an implied volatility index (VIX). In 2000, the Nasdaq Volatility index (VXN) that is derived from the implied volatility of Nasdaq-100 index (NDX) options was introduced by CBOE. Although the principal advantage of implied volatility is to evaluate market future volatility, it has also been used to explain the behavior of stock market returns. In the effort of explaining the importance of modeling and predicting asset volatility, many authors examined the relationship between implied volatility and volatility forecast based on historical returns in order to deliver the unbiased and efficient forecasts of future volatility (see, e.g. Fleming et al., 1995, Giot, 2005: Simon 2003 among others). Implied volatility can be sued for as an underlying asset in derivatives products, the first example in this sense is due to Deutsche BÃ¶rse in 1998, with the futures contract on the German implied volatility as the underlying asset (VDAX), while CBOE, list futures on VIX on March 26, 2004 and introduced options on VIX on February 24, 2006. Alternatively, implied volatility index can serve as input variable in computing Value-At-Risk (VaR) see Giot (2005). Finally, international integration where the proxies are implied volatility indices spillovers has been questioned by several authors in recent finance literature (see Siriopoulos and Fasas, 2012, AijÃ¶, 2008 Konstantini, Skiadopoulos and Tzagkaraki, 2008 Nikkinene and Sahlstrom, 2004). In this paper, a thorough analysis of the South African implied Volatility (henceforth: SAVI) is undertaken. To check our findings, we also compute some of our results on the VIX and VXN. Therefore, these two implied volatility indices will be considered as our benchmark. The VIX is computed and made available by the Chicago Board of Options Exchange (CBOE) and have either found large consensus and either used intensively both by academics and practitioners. The VIX index use a weighted average of implied volatilities computed from a total of eight call and put near-the-money, nearby and second nearby American options contract on the underlying S&P 100 index by construction. This method guarantee that the index give the implied volatility of hypothetical at-the-money option with constant maturity of 22 trading days to expiry. The first VIX /VXN index launched in 1993 was calculated with options based on S&P 100 and used the Black-Scholes/Merton model. In September 2003 the CBOE launched the new VIX/VXN index. It used the data of S&P 500 and based on the concept of fair value of future variance developed by Demetrfi et al.,(1999a) and is computed directly from market observables, which are independent of any pricing model, such as the prices of out-of-money call and put and interest rate, this is known as model free implied volatility. These constraints are out of reach for emerging option markets that are less liquid than the U.S counterpart. The SAVI was launched in 2007 by Johannesburg Stock Exchange (JSE) as an index design to measure the market’s expectation of the 3-month market volatility. The construction is based on an more restrictive technique to VIX method so as to respect the liquidity constraints inherent in emerging markets. Two years later, in 2009, the SAVI index was adjusted by the volatility skew to reflect the new way of measuring the expected volatility. In an efficient market where options price reflect all available information, the level of implied volatility is an indicator of the best assessment of the expected volatility of the underlying stock market over the remaining life of the options Giot (2005). The majorities of empirical studies have focused on the U.S and other developed economies where the investigations of implied volatility have been conducted intensively. The theoretical and empirical experiments produced in emerging markets are very limited and concentrated especially on the Greece (see, Skiadopoulos, 2004, Siriopoulos and Fasas, 2012) and for South-Korea Ting (2007). This limitation can be explained by the fact that many emerging countries do not have well established derivative
markets. Contrary to this direction, the current article contribute to the enrichment of the existing literature by focusing our research on the evaluation of the information content in the SAVI as the Implied volatility sentiment in the major African markets. For this purpose, we analyze the properties of the SAVI, the Johannesburg Stock Exchange which is the largest stock market in Africa. Although there exist a South African Implied Volatility, there are few studies exploring its properties and its application is rarely available in the literature. instance Pillay and Shannon(2006) use the old methodology of VIX, which is based on the Black and Scholes, while kotz et al (2009) introduced the SAVI using model-free approach. In Both article the empirical analysis is very limited. Wandmacher and Bradfield (1998) examined empirically the Black and Scholes assumption of a constant volatility in the South African market. Finally, Samouilhan and Shannon (2008) used regression and some loss functions to compare the performance between historical volatilities (computed with GARCH-type) and implied volatility. One question which deserves our attention is to find the relationship between SAVI, VIX, and VXN which is known as implied volatility spillover. There are voluminous literature on the linkages and interactions between international stock prices and volatility. But little had been doing concerning the transmission of implied volatilities across the developed markets and developing markets. Gemmill and Kamiyama (1997) examined there is a transmission across the Japanese, British, and American markets over the period 1992-95. Skia dopoulos (2004) investigated the linkage between Greek implied volatility, VIX, VXN. Implied volatility propagation is of great importance to options portfolio managers since it affects options price and hedging ratio, and it can serve as a tool to indicate changes in expected volatility. Given the nature of South African Stock markets in Africa, it is worth investigating the implied volatility spillover between the leading African stock market (which is an emergent market) and a developed market. This study is the first step in that direction, at least in our knowledge and it examines the properties of a measure of implied volatility in the South African stock market. Following previous study, we assess the efficiency, information content and absence of bias of competing volatility forecast with respect to ex-post observed realized volatility computed form daily returns. Furthermore, we consider the volatility forecast based on the SAVI implied volatility index, RiskMetrics and GJR-GARCH models and assess their relevance in efficiency, information content and absence of bias. For each volatility forecast, we consider various forecasting horizons. One important result of this article is that we found an asymmetric negative relationship between SAVI and the underlying stock index returns, this result is in line with the previous findings.

We structured the article as follow: in section 2 the statistical properties of SAVI and FTSE/JSE TOP 40 are presented. Section 3 scrutinizes the relationship between implied volatility and stock market returns. While Section 4 examines the transmission of implied volatility between the South African and the U.S markets. In Section 5 we presents the econometric framework and the encompassing regressions. Finally, summary and conclusion are postponed in section 6 and 7.

2 DATA AND DESCRIPTIVE STATISTICS

In this section we describe the construction of the SAVI, the data used in this article and presents some statistical properties.
2.1 Construction of SAVI

The South African Futures Exchange (SAFEX), launched SAVI, in 2007, as an index designed to measure the market’s expectation of the 3-month volatility. The SAVI is based on the FTSE/JSE Top 40, which is a capitalization index comprising the 40 most liquid stocks trading in South African Johannesburg Stock Market (JSE) and it is determined using at-the-money option price. The SAVI is calculated on a daily basis, calculated the 3-month at-the-money volatility and uses bid-ask price. Until April 2009 the SAVI was constructed using the methodology proposed by Whaley (2000). In May 2009, the SAVI, has been updated to reflect the new way of measuring the expected 3-month volatility. The new SAVI is also based on the FTSE/JSE Top 40 index, but is not only determined using the at-the-money volatilities but using the volatility skew. This renders the new index more efficient since it incorporates a market crash protection volatility premium. The new SAVI is calculated as the weighted average prices of calls and puts over a wide range of strike prices, that expires in 3-month’s time. In short,

\[
\text{newSAVI} = \sqrt{\sum_{i=1}^{n=F} W_i P_i(K_i) + \sum_{i=n}^{\infty} W_i C_i(K_i)}
\]

where \( F \) is the current forward value of FTSE/JSE Top 40 index level, determined using the risk-free interest rate and the dividend yield. \( F \) marks the price boundary between the liquid puts options \( P_i(K_i) \) and the calls \( C_i(K_i) \), with strikes \( K \). The price of the call and put option are determined using the traded market volatility skew that inspires in 3 month’s time. The 3 month volatility skew, \( \sigma_K(0,T) \), is computed using the time weighted interpolation function defined by

\[
\sigma_K(0,T) = \sqrt{\left\{ T_2 \sigma_K^2(0,T_2) \left[ \frac{N_1}{N_2-N_1} \right] + T_1 \sigma_K^2(0,T_1) \left[ \frac{N_1}{N_2-N_1} \right] \right\} \frac{N_0}{N_3}}
\]

where, \( N_0 \) is the numbers of days in the year (365 is the South African convention), and \( N_3 \) is the number of days from the value date to the 3 months date, \( N_1 \) and \( N_2 \) being the date of the near skew, and nearest skew, from the 3 months skew expiry date, respectively. The weights used in equation (1) are published by Derman et al (1999). In this article, the period spanning from 4 May 2009 to 6 December 2012 has been set as SAVI 2, while SAVI1 starts from the creation of the index to 23 April 2009.

2.2 Some properties of the implied volatility

We first present some properties of the SAVI and the FTSE/JSE Top 40 index returns. Daily data of South African stock market and SAVI spanning from April 2007 to December 2012 are used. In figure 1, we present time-series plot of the SAVI and FTSE/JSE TOP 40. All data have been downloaded from Bloomberg. Negative correlation between the underlying index and the SAVI can be guessed. Table 1 shows the summary statistic, (viz mean, skewness, kurtosis, and the results from first and second order autocorrelation) and the volatilities indices. The distributional properties of return and volatilities appear non-normal. Given that the sampling distribution for skewness and kurtosis are normal with mean 0 and standard deviation of \( \sqrt{6/T} \) and \( \sqrt{24/T} \) respectively with T the sample size, none of this distribution is well approximated by normal distribution. The SAVI reaches his highest value on October 2008 and the minimum
was attained on December 2012. While, the FTSE/JSE Top 40 reaches its maximum value on December 2008 and its minimum value on October 2008. The mean of SAVI is 25.77% and the positive skewness is indicating a longer tail and the kurtosis which exceeds the normal distribution. Furthermore, given that the mean of daily returns is almost zero, we can guess that there is no trend in SAVI prices. The autocorrelation for the level of implied volatility index is positive, indicating that the autocorrelation function decays exponentially to zero and suggest a long-memory process. On the other hand, evidence of negative autocorrelation in the daily returns process suggest a mean reversing process. The significant of autocorrelation is tested using Ljung-Box Q-Statistic. In the same Table, we summarize the statistics of the returns of the FTSE/JSE Top 40 and the change \( \Delta SAVI = SAVI_t - SAVI_{t-1} \) of the implied volatility indices, as well as their cross-correlations. The sample mean for the implied volatility indices is zero indicating the absence of a trend. The departure from normal distribution also enhance that extreme movements in the volatility changes are more probable than using the normal distribution. The cross-correlations confirm the existence of leverage effect, despite the fact that it is rather weak. The correlation between FTSE/JSE Top 40 returns and the changes in SAVI is about -0.61 while those of the benchmark market are -0.75 and -0.73 for S&P 500 and NASDAQ respectively. As matter of comparison, Skiadopoulos, 2004 find a negative correlation between FTSE/20 and Greek implied volatility with values -0.16 and -0.17. correlation for both level and the first differences are computed and reported in Table 3. We can see that SAVI is more correlated with VXN rather than VIX (0.90 as opposed to 0.89). On the other hand, the correlation of the changes of VIX (VXN) and SAVI is almost null -0.05 (-0.023). The correlation between the changes of VIX and VXN is 0.95. The results indicate that the implied volatility indices are correlated. This means that South African market can reflect more information presents in U.S markets. We can notice that the first order autocorrelation for the \( \Delta SAVI \) is low, positive, statistically significant. In order to investigate whether the implied volatility index time-series are stationary, the Augmented Dickey-Fuller (ADF) unit root test at eight lags (we use Schwarz criterion to determine the number of lags) and without a time trend are reported in Table 2. The null hypothesis of a unit root is rejected at 1% significance level for the implied volatilities. After differencing, all the series are stationary. For accompanying the ADF test we computed the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test, both for level and first different series. This test intended to to test the null hypothesis of stationarity (no unit root) against the alternative non-stationarity (unit root). We reported the results in the same table with those of ADF test. The KPSS test results, at levels are consistent with those of the ADF the T-statistic are significant both at 1% and 5% confidence level, thus rejecting the null hypothesis of stationarity for all implied volatility indices. Different results are obtained for the first difference of the index series. For SAVI, VIX and VXN, the first differenced series are proved to be stationary. This allow us to use a number of tests in our series without imposing strict conditions.

### 2.3 Day of the week effect on the SAVI

We test the presence of the day of the week effect on the South African implied volatility, this will allow us to check if SAVI contains any seasonal or predictable design. We investigate the day of the week effect by estimating the following equation, i.e.,

\[
\Delta SAVI_t = \sum_{j=1}^{5} \alpha_j D_{jt} + \varepsilon_t
\]  \hspace{1cm} (2.2)
Where \( j \) indicates the day of the week (\( j=1 \) is Monday, and so on) and \( D_{jt} \) is a dummy variable equal to one on day \( j \) and zero otherwise. \( \varepsilon_t \) is a random term and the \( \alpha_j \) are coefficients to be estimated using ordinary least square (OLS). We report regression results in Table 4. The results from the entire period revealed that, the volatility significantly increases from Friday to Monday and insignificant decrease can be noticed from Tuesday to Wednesday and from Thursday to Friday. Only Monday dummy variable is significant in whole period, with the volatility index increasing by about 8.1\%. Tuesday has the lowest variation -0.031. In the first sub-period, which ranges from 04/05/2007 to 24/04/2009, the results confirm that Tuesday remains the lowest volatile day, Monday always exhibit positive volatility. In the last sub-period, spanning from 28/04/2009 to 06/04/200), Monday dummy variable is significant and exhibits an increase of volatility of about 10\%. We notice a trend of increase in general over the week-end, a finding similar to that Fleming et al.(1995). However, the presence of this seasonality is weak considering the adjusted \( R^2 \) of the regression (-0.0006) and regression for the two sub-periods (-0.005) and (0.0002). Significant days effects are found only for Monday, hence it is difficult to ascertain evidence of seasonality in the SAVI.

### 3 Relationship between implied volatility stock index and underlying stock market.

In this section, we examine the intertemporal relationship between FTSE/JSE Top 40 and the SAVI. Black (1976) and Christie (1982) documented a strong negative association between stock market returns and expected volatility observed empirically. In addition, an asymmetry relationship between return and volatility has been evidenced in Black (1976) and Schwert (1989,1990), that is the increase in expected volatility is related to a given decrease in the stock market price is larger than the respective decrease in expected volatility related to an equal increase of the stock market price. Black (1976) motivated the asymmetry by leverage effect; this hypothesis states that when the stock price of a company decrease, the debt-to-equity ratio increases, leading to higher volatility of equity returns. One other explanation known as volatility feedback effect is due to Campbell and Hentshel (1992) and French et al.,(1987). This hypothesis relies on the fact that, an anticipated increase in volatility leads to an increase in the asset’s risk premium and thus a higher expected return, which cause current price to decline. Within the implied volatility index literature, the evidence of a strongly asymmetric negative contemporaneous relationship between the implied volatility index and underlying returns is well documented. In their article for VIX construction and its properties, Fleming et al.,(1995) examine for the first time the intertemporal relationship between VIX and S&P 100 returns. In the same vein, whaley (2000), Giot (2005), among other found the presence of a statistically significant negative contemporaneous relationship between VIX and S&P 500 returns. Studies dealing with other volatility indices are consistent with the results associated to the VIX we can enumerate among other the works of Simon (2003) and Giot (2005) for the Nasdaq 100 Volatility Index(VXN), Skiadopoulos (2004) and Siriopoulos and Fassas (2012) for the Greek GVIX, Frijns et al., (2010) for the Australian AVX, Gonzalez and Novales (2009) for the Spanish VIBEX-NEW. In this article, we find a contemporaneous significant negative relationship between variation of implied volatility (SAVI) and stock market returns. The results are reported in Table 5. We test this relationship by following the multivariate regression adopted in Fleming et al., (1995). We perform a regression of the logarithmic SAVI changes on leads, lags and contemporaneous FTSE/JSE Top 40. To assess whether there is an asymmetric
contemporaneous relationship between the SAVI and FTSE/JSE Top 40 returns, the absolute return of lag zero has been added to the regression. Finally, we also include a lagged value of the change in the SAVI to check the first-order auto-correlation. The same exercise has been computed for VIX and S&P 500 and VXN and Nasdaq 100. We perform the following regression:

\[ \Delta hIMP_t = \alpha + \beta_{-2}r_{t-2} + \beta_{-1}r_{t-1} + \beta_0r_t + \beta_0^{AV}|r_t| + \beta_{+1}r_{t+1} + \beta_{+2}r_{t+2} + \gamma \Delta hIMP_{t-1} + \varepsilon_t \]  

(3.1)

where \( \Delta hIMP_t = \ln(SAVI_t) + \ln(SAVI_{t-1}) \)

For the empirical analysis, contrarily to Fleming et al (1995), we use the return of the SAVI rather than its level. This selection is based on three reasons: first, both academics and practitioners are interested in the changes or innovations of expected volatility. Second, if stock prices follow a random walk, assessing the relationship stock and volatility indices in levels may prove to be spurious. Finally, implied volatility indices’ levels also appear to be near random walk Fleming et al (1995). From the empirical results of Eq. (3) over the whole sample period, we notice that the coefficient of the contemporaneous, signed change is small and negative, -0.808 with a t-statistic of -27.60 and the value of the p-value indicates that it is significant at 10%. The lag coefficients are negative and significant while the lead coefficient are positive and insignificant but much smaller in magnitude than the contemporaneous coefficient. So, while a negative relationship exists between changes in expected volatility and past stock returns, the opposite hold for future stock returns. Finally, the \( \beta_0^{AV} \) is positive and smaller than \( \beta_0 \); in magnitude his value is 0.168 and the corresponding t-statistic is 4.00. The estimate of \( \beta_0^{AV} \) displayed in Table 5 shows a significant asymmetry in relationship between volatility variations and contemporaneous stock market returns. If the stock markets return is positive, the coefficient driving the change in volatility is \( \beta_0^{+} = \beta_0 + \beta_0^{AV} \), namely -0.6400. A stock market increase is expected to follow a decrease in the volatility index. On the other hand, if the stock return is negative, the coefficient is \( \beta_0^{-} = \beta_0 - \beta_0^{AV} \), viz -0.9760. A stock market decline is expected to follow an increase in volatility index. The difference in the magnitudes of the coefficients, however, indicates the asymmetry. In order to test the consistency of our findings, the Eq. (2) is applied in the two sub-samples. In the second sub-period, the results are consistent with the full sample results discussed above. The significant and negative contemporaneous coefficient is found and the \( \beta_0^{AV} \) coefficient is lesser than the full sample one. Results obtained from the first sub-sample are slightly different, although the contemporaneous coefficient is still negative and significant and coefficients are bigger than the previous ones. Lead-lag coefficient is all negative except for \( \beta_{+1} \). By way of comparison, we regress Eq. (2) on the data of S&P 500 and Nasdaq 100, the regression results for VIX (VXN) and S&P 500 (Nasdaq 100) are reported in the same table to ease the comparison. The results are in line with the previous studies which advocates that \( \beta_0 \) have to be negative. Indeed, the results show that for the analysed sample period there are significant negative relationship between contemporaneous indexes returns and the corresponding volatility (the coefficients of \( \beta_0 \) is significant with t-statistic -40.04 (-38.43) ). The coefficient \( \beta_0^{AV} \) is positive and significant t-statistic 5.08 (4.83). Irrespectively of the sample and indices we find a negative \( \gamma \) which reveals a negative first–order autocorrelation in the volatility change. The intercepts are all negative and significant. The values of the adjusted \( R^2 \) are comprised between 0.392 (SAVI) and 0.579 (VIX).
4 Linkages between implied volatility

In this section, we examine the transmission effects of implied volatility across the CBOE and the JSE markets. Towards this end, the relationship between VIX, VXN, and SAVI is studied. Figure 2 shows the evolution of VIX, VXN, and SAVI over the period 2007-2012. Implied volatility indices seem to behave similarly across markets. To analyse the linkage between the two markets, Granger causality test and vector autoregressive analysis (henceforth: VAR) are applied. The econometric analysis of implied volatility crossovers could be done either in levels or in changes. In our work, we applied the method on the first difference thank to the stationarity of implied volatility change. In addition, this properties of the time series are not affected by the change of the time origin. This methods provide large information for the analysis of the relationship between implied volatility indices and can be useful for implementing suitable trading strategy. Preliminary analysis of the cross–dynamics of the implied volatility indices and the contemporaneous and one lag cross Since the article of Sims (1980), multivariate data analysis in the context of vector autoregressive process become popular in econometrics. VAR models describe the endogeneous variables relying of their own history, apart from deterministic regressors. In this article, the following VAR(p) system is used:

\[ \Delta X_t = \alpha + \sum_{i=1}^{n} \beta_i \Delta X_{t-i} + \varepsilon_t \] (4.1)

To make the formula clearer, we write it explicitly for the different implied volatilities.

\[ \Delta VIX = \alpha^{SAVI} + \sum_{i=1}^{n} b_i^{SAVI} \Delta SAVI_{t-i} + \sum_{i=1}^{n} \varepsilon_i^{VIX} \Delta VIX_{t-i} + \sum_{i=1}^{n} d_i^{VXN} \Delta VXN_{t-i} + \varepsilon_t \] (4.2)

\[ \Delta VIX = \alpha^{VIX} + \sum_{i=1}^{n} b_i^{VIX} \Delta VIX_{t-i} + \sum_{i=1}^{n} c_i^{VXN} \Delta VXN_{t-i} + \sum_{i=1}^{n} d_i^{SAVI} \Delta SAVI_{t-i} + \varepsilon_t \] (4.3)

\[ \Delta VXN = \alpha^{VXN} + \sum_{i=1}^{n} b_i^{VXN} \Delta VXN_{t-i} + \sum_{i=1}^{n} d_i^{VIX} \Delta VIX_{t-i} + \sum_{i=1}^{n} d_i^{SAVI} \Delta SAVI_{t-i} + \varepsilon_t \] (4.4)

Where, \( \Delta SAVI, \Delta VIX, \Delta VXN \) are the daily first differences of implied volatility indices, and are also the endogenous variables, \( \alpha \) is a 3X1 vector of intercept, \( b_i, c_i, d_i \) are matrices of coefficients to be estimated and \( \varepsilon_i \) is a random vector of innovations serially uncorrelated and \( n \) defines the lag order to the system. We estimate the coefficient by means of OLS techniques. Before analysing the Granger causality and impulse response, the optimal lag length of the VAR(p) system has to be determined, for this end, we computed Akaike’s (AIC) and Schwartz’s (SIC) information, final prediction error,(FPE) and Lukepohl’s modified likelihood ratio (LR) test. While Akaike’s (AIC) information and final prediction error, (FPE) suggest the lag length eight to be adapted for the VAR(p) model, Schwartz’s (SIC) information and Lukepohl’s modified likelihood ratio (LR) test suggest lag lengths of two respec tively. Successively, we have used the diagnostic test of Breitung et al.,(2004) to identify the rightful number of lags. As the Breitung et al.,(2004) test shows that VAR(2) specification are too restrictive, we finally opt to use VAR(8), so the lag eight is applied in our examination. Time difference can jeopardize our results, as the
U.S underlying markets open for trading as the continuous trading in JSE is about to stop, that is 16:00 local time—which is also the ending time for the calculation of the option closing prices. Given that closing prices are used to construct the volatility indices, “contemporaneous” refers to the same calendar date t even though the US and JSE indices are measured differs. The results for the Ganger causality test are between implied volatility changes of SAVI, VIX, VXN and reported in Table 7. The statistics displayed are for lag order eight. A close look of Granger test for the lags eight advocates that the Granger VIX causes the implied volatilities of VXN and SAVI at 1% significance level, indicating that the expectations of the future volatilities, as measured by the implied volatility are transmitted form the VIX to VXN and SAVI. The results also show that implied volatility of the VXN and SAVI are linked, however the transmission power is slightly lower. In the Table 6, the summary of the estimations of VAR system are displayed. The F-statistics, show that the VAR(8) model is highly significant, as the p-values are less than 0.003. Moreover the adjusted R-squared stand between 0.017 and 0.13. We notice that, the results backed out by VIX and VXN are in general more robust and have the higher coefficients. Moreover, they present the highest adjusted R-squared which respectively 0.17 and 0.079. The results of contemporaneous residual correlations between the markets shows that VIX and VXN are highly correlated (0.95). We find a low, though significant correlation between the SAVI and VIX(VXN) residuals -0.019 (-0.013). We can observe that there is a contemporaneous spillover effect between the changes between SAVI and both US indices. The coefficients of the lagged values are including the intercept. We found that these do not have any additional forecasting power. Therefore an investor can take advantage of the information contained in the value of the changes of SAVI in order to develop a appropriate strategy.

5 The relation between implied and stock market volatility

It is well accepted between academicians and practitioners that the implied Black-Scholes volatility computed from the market options and the model-Free counterpart are good estimates of the forward expectation of the volatility on the underlying asset price. Therefore, in this section we present different volatility measures that will be included in a general regression equation. Namely, implied volatility, the realized volatility, RiskMetrics approach and GARCH-type volatility.

5.1 A. Implied volatility

We first assess the information content of the implied volatility indexes at a relatively short time-horizon (5, 10 and 22 days). In our analysis, although mimicking relevant literature in that field, we analyze the full sample which span from 4/5/2007 to 6/12/2012. We proxy the measure of implied volatility by taking the level of the SAVI, implied volatility index, for the FTSE/JSE Top 40. By definition the forward-looking time horizon is equal to 66 trading days and the implied volatility indices are expressed in annualized terms. We tackle the unavailability of no implied volatility term structures using the square root of time rule as in Giot (2005) and Frijns et al. (2010) i.e to switch from a time horizon of 66 days to the required h interval. Hence the h-day forward-looking forecast on day t is given by the following relation: 

$$IMP_{h,t} = \sqrt{\frac{h}{360}}VOL_t$$

where $VOL_t$ is substituted by SAVI. Thus $IMP_{5,t}$ is the expected volatility over the $[t+1; t+5]$ period. The implied (VOL) volatility forecasts are therefore simple re-scale version of the VOL series.
5.2 Realized volatility

Given daily returns \( r_t = \ln(\frac{P_t}{P_{t-1}}) \) for the FTSE/JSE Top 40 index, the forward-looking realized volatility over a time horizon of \( h \) days is computed by taking the square root of the sum of the (future) squared returns over this \( h \)-day period. The forward-looking realized volatility \( RV_{h,t} \) at a hypothetic time \( t \), for a generic time period \([t+1,t+h]\) can be computed as:

\[
RV_{5,t} = \sqrt{\sum_{j=1}^{h} r_{t+j}^2}
\]  

(5.1)

Note that this volatility measure is computed ex-post, i.e. at time \( t+h \) when all returns have been observed. In this study, three values of \( h \) are used, viz 5, 10 and 22. For the encompassing regressions estimated below, we define realized volatility computed from nonoverlapping data. Christensen and Prabhala (1998) point out that: the use of realized volatility computed from overlapping data in regression analysis yields potentially big estimation problems as the regression’s residuals will be strongly auto-correlated. Therefore, the measure of realized volatility computed using Eq. (7) and using all \( RV_{t,t} \) for \( t = 1, \ldots, T \) yields strongly correlated volatility measures. Hence we also define realized volatility measures computed from non-overlapping squared returns data. While Eq. (7) is still valid, we no longer compute it for all \( t = 1, \ldots, T \) but for a subset of those times such that the newly defined \( RV_{h,t} \) use unique data.

5.3 RiskMetrics and GARCH-type volatility

The last class of forecaster we consider is based on the RiskMetrics approach. This approach can be extended to derive the well-known GARCH (1,1). According to RiskMetrics specification, the volatility is defined as:

\[
rm_t = (1 - \lambda)r_{t-1}^2 + \lambda rm_{t-1}
\]  

(5.2)

Where \( \lambda \) captures the persistence in volatility and it is set to 0.94. Note that \( rm_t \) is an unconditional measure of daily variance and no volatility term structure is available in this model. The process in Eq. (8) is an interactive process and need to be initialized at some point. The forecast we obtain from the above equation are one day forecast. To obtain the multiple day forecast we rescale the daily forecast as : \( RM_{h,t} = \sqrt{hrm_t} \) where \( RM_{h,t} \) is the \( h \)-day volatility forecast according to RiskMetrics approach (Giot, 2005) used the same modification. Finally, we construct forecast based on ARCH type model, this type of volatility forecasting is intensively used in finance. Under this Arch types model, the return process is generated by \( r_t = \mu_t + \varepsilon_t \) where \( \mu_t \) is the conditional mean process encompassing autoregressive and moving average terms and \( \varepsilon_t = z_t \sqrt{\sigma_t} \) such that \( z_t \) is IID(0,1) and \( \sigma_t \) is the time varying conditional volatility process to be estimated. Symmetric GARCH The standard GARCH(1,1) process can be modelled as

\[
r_t = \varepsilon_t | \Theta \sim N(0,1)
\]

\[
\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2
\]

Where \( \sigma_t \) and \( \varepsilon_{t-1}^2 \) are the conditional and unconditional variances of \( \varepsilon_t^2 \), and the long-run variance \( \omega \) is given by \( \varepsilon^2(1 - \beta - \alpha) \). This GARCH model is symmetric in that negative and positive shocks have the same effect on volatility \( \sigma_t \) (time varying conditional) volatility process to be estimated.

Asymmetric GJR-GARCH
In order to capture the asymmetry, GJR-GARCH pioneered by Glosten et al. (1993) incorporates the asymmetry (or leverage effect) into the GARCH framework by use of an indicator function. A representation satisfying the GJR-GARCH type can be written down as follows:

$$\sigma_i^2 = \omega + (\alpha \varepsilon_i^2 + \varphi I_{\varepsilon_{t-1} < 0} \varepsilon_{t-1}^2) + \beta \sigma_{t-1}^2$$  \hspace{1cm} (5.3)

Where the dummy variable $I_{\varepsilon_{t-1} < 0}$ is such that $I_{\varepsilon_{t-1} < 0} = 1$ if $\varepsilon_{t-1} < 0$ and $I_{\varepsilon_{t-1} < 0} = 0$ otherwise. For example, in GJR-GARCH model, positive and negative shocks have an impact of $\alpha$ and $\alpha + \varphi$, respectively.

We can analyze the performance of the different forecaster by regressing any competing volatility of the realized volatility using the following formula:

$$RV_{k,t} = \alpha + \beta X_{k,t} + \eta_{k,t}$$  \hspace{1cm} (5.4)

Where $X_{k,t}$ is any of the volatility forecasters defined above. For $X_{k,t}$ to be an unbiased forecaster of $RV_{k,t}$ we need $\alpha = 0$ and $\beta = 1$. Furthermore, we evaluate the adjusted $R^2$ of Eq. (5.4) to assess the predictive power of every competing volatility forecasters. The estimates of Eq. (10) are reported in Table 8. When we consider Implied volatility as proxied by SAVI, we notice that it contains information regarding future realized volatility. $\alpha$ is negative and significantly different from zero in all cases indicating a significant bias. The $\beta$ coefficient, is significantly different from one in many cases. Looking at the results for the different forecast horizons reveals an interesting pattern. The adjusted $R^2$ for the 5 days-ahead forecast is reasonable high, with a value of approximately 0.50. Evidence from panel A show that, better forecast using SAVI can be made at the 10 and 22 days horizons. In panel B of the Table 8 we reported the results of the estimates based on RiskMetrics approach. According to our findings, the RiskMetrics approach does not produce unbiased forecast of the future volatility in general, since either $\alpha$ or $\beta$ are significantly different from zero and one. Predictive power for the RiskMetrics seem to be higher than the SAVI one. Finally, Panel C of Table 8 reports the estimates for GJR-GARCH model. The GJR-GARCH model performs reasonably well since it produces unbiased estimates. The models have the highest predictive power with adjusted $R^2$ ranging from 0.71 to 0.96. The SAVI seems to produce worse forecasts than RiskMetrics and GJR-GARCH model which it is the best predictor, but when confronting with respect to the parameters estimates SAVI perform the best. The next step of our analysis is to compare the efficiency of the volatilities estimates to that of historic realized volatility. For this end we estimate the following encompassing regressions where we test the performance of one forecasting method against the other.

$$RV_{k,t} = \alpha + \beta X_{h,t} + \gamma X_{k,t} + \eta_{k,t}$$  \hspace{1cm} (5.5)

Where $X_{h,t}$ and $X_{k,t}$ are the estimates based on two different forecasting approaches. The significance of $\beta$ and $\gamma$ will indicate whether one forecasting approach dominates the other. Alternatively, if both $\beta$ and $\gamma$ are significant then the both forecasting approaches complement each other and the best forecast can be made by using both forecasting approaches contemporaneously. The estimates on Eq. (5.4) are reported in Table 9 we report the coefficients and the statistic test in the brackets and the adjusted $R^2$. In panel A, we consider the encompassing regression confronting the RiskMetrics approach to GJR-GARCH. The coefficients are significant for all horizons. This implies that both approaches are complementary. The high adjusted $R^2$ confirmed our findings. This is furthermore higher than the adjusted $R^2$ for individual regressions reported in the Table 8. Panel B reports the encompassing regressions between
RiskMetrics approach and the SAVI. We find that in that case \( \gamma \) is negative and significant. This results contrast the previous findings available in the literature see Giot (2005), Frijns et al., (2010). In panel C, we compare the GARCH forecast to the forecast based on implied volatility. We notice an opposite result with confront to RiskMetrics forecast. \( \gamma \) is positive for all horizons. However, the GJR-GARCH contains additional information beyond the SAVI, but this discrepancy decreases at the longer period. In all cases we also find a highly significant SAVI series, indicating the importance of the SAVI series.

### 5.4 Discussion

It’s interesting to assess whether or not the empirical results from econometric analysis are in line with the findings observed in other markets. We find a statistically significant negative relationship between the change in the level of implied volatility index and the returns of underlying equity returns. We further find evidence for the asymmetry in the relationship between index returns and \( \Delta h IMP_t \). The coefficient \( \beta_{AV}^0 \) is positive and significant confirming the existence of asymmetric relationship. The results of the sub-samples are similar to those for the full sample. The contemporaneous coefficient remains negative and highly significant for all sub-periods. Results from Granger (1969) causality test suggest that the implied volatility term structure of the VIX and VXN cause the implied volatility term structure of the SAVI, indicating that the expectation of the future volatilities, as measured by implied volatility term structures are transmitted from VIX and VXN to SAVI while the reverse does not hold. However since the U.S market is not in the same time zone, some caution is needed when interpreting the results. We conduct a VAR test to obtain more information about the predictive ability of the variables and the time structures of analysis. The F-statistic show that the VAR(8) is highly significant (very low p-value). However, we notice that the returns of VIX are in general statistically more robust. The results from Table 5 show the interaction of the three indices, proving that there is, of course, no effect either on VIX or VXN from SAVI. Finally, we notice that, the implied volatility term structure of the VIX has an significant impact on the implied volatility term structure of the VXN and SAVI, this is consistent for earlier findings of linkage of the implied volatility Aijo (2008). Finally, results in term of forecasting show that, SAVI contain important information. On its own accord it has, the highest power relative to the competing forecaster (GJR-GARCH and RiskMetric). When using encompassing regression the alternative forecaster generally dominate the SAVI, but his inclusion increases the R-adjusted. We find that RiskMetric approach move in the oppositely with respect to the SAVI, whereas the GJR-GARCH move in the same way and is more informative when encompassing regression is considered.

### 6 concluding remarks

In this article, we examine the information content in the South Africa implied volatility as measure of volatility of different settings. The motivation of this study being that South African market is the most important market in Africa and one lending financial market of developing countries. We find a negative correlation between FTSE/JSE Top 40 returns and SAVI, this is confirm by the negative and asymmetric contemporaneous relationship between the returns of underlying asset and the volatility which are in line with the results found in other markets. As note by other authors the SAVI can be used as a gauge of investor’s fear. The analysing of the relationship between SAVI and the implied proxies of the U.S markets by means of
vector autoregression (VAR) analysis and granger causality tests show a spillover effect between the U.S markets and the South African one. However, lead-lag effects are very weak. In general, the results suggest that the SAVI contains important information about South African main principal market index (FTSE/JSE Top 40) and it can provide valuable information to market participants. A large study encompassing the other emergent markets should be helpful in shedding a light on the behaviour of the implied volatility in these market. This topic is currently under investigation. An analysis of the SAVI using high frequency data can also be performed.

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