A note on the stability of nonlinear differential-algebraic systems

Pierluigi Di Franco1 Giordano Scarciotti1 Alessandro Astolfi1,2

1Department of Electrical and Electronic Engineering, Imperial College
London, London SW7 2AZ, UK (e-mail: pierluigi.di-franco13@ic.ac.uk,
g.scarciotti@ic.ac.uk, a.astolfi@ic.ac.uk)
2Dipartimento di Ingegneria Civile e Ingegneria Informatica,
Università di Roma “Tor Vergata”, Via del Politecnico 1, 00133
Roma, Italy

Abstract: The problem of the stability analysis for nonlinear differential-algebraic systems is addressed using tools from classical control theory. Exploiting Lyapunov Direct Method we provide linear matrix inequalities to establish stability properties of this class of systems. In addition, interpreting the differential-algebraic system as the feedback interconnection of a dynamical system and an algebraic system, a sufficient stability condition has been derived using the small-gain theorem. The proposed techniques are illustrated by means of simple examples.

Keywords: Differential-algebraic systems, descriptor systems, nonlinear systems, Lyapunov functions, small-gain theorem.

1. INTRODUCTION

Analysis and control of differential-algebraic (DA) systems (also known as descriptor systems, singular systems or semi-state systems) have been the subject of increasing interest from the research community in the last decades. Differential-algebraic systems arise, for example, in multi-body mechanical systems where they may represent environmental constraints or constraints related to kinematic joints, see Blajer (1992). For example, large mechanical systems involving thousands of bodies can be modeled as smaller purely differential subsystems interconnected via algebraic constraints, see Pogorelov (1998). A similar modeling approach applies to the interconnection of large-scale electrical networks, where differential-algebraic equations arise from the application of the Kirchoff’s laws, see Riaza (2008). Other examples of differential-algebraic systems arise in social economic systems and chemical processes, see Dai (1989) and Kunkel and Mehrmann (2006). Despite significant advances in numerical analysis and simulation of DA systems, see Brenan et al. (1995), the problem of stability analysis and control for general DA systems remains open. Attempts to study DA systems have been undertaken by developing an equivalent ordinary differential equations (ODE) representation. ODE representations require multiple differentiation of the algebraic equations and further mathematical manipulations which poorly suit with the large-scale dimension of many engineering problems. Another approach consists in studying stability properties of DA systems in their original formulation by extending tools from classical and modern control theory. Successful attempts have been made for the case of linear time-invariant DA systems: for example in Müller (2006) an inertia theorem is presented, while the observer design problem is addressed in Müller and Hou (1991) and in Darouach and Boutayeb (1995). Contributions in extending optimal control theory to nonlinear DA systems are in Glad and Sjöberg (2006), where the Hamilton-Jacobi equation is directly formulated for nonlinear DA systems, and in Sjöberg et al. (2007a), where a sampled-data nonlinear model predictive control scheme with guaranteed stability is presented. Stability analysis with Lyapunov methods is studied in Wang and Zhang (2012) for nonlinear DA systems with delays. In Wu and Mizukami (1995) the Lyapunov stability theory is extended to DA systems and a class of state feedback controllers that guarantee asymptotic stability of uncertain DA systems is proposed. In Conti et al. (2004) the stability analysis with guaranteed domain of attraction and control of DA nonlinear systems is studied by means of Lyapunov functions based on the linear matrix inequality framework. For purely differential systems significant advances have been done in studying the nonlinear equivalent of the $H_\infty$ control problem. One first contribution to this problem was given by van der Schaft (1992), which showed that the $L_2$-induced norm can be calculated from the solution of a Hamilton-Jacobi-Isaacs equation or inequality. Other major contributions to nonlinear output feedback $H_\infty$ control come from Isidori and Astolfi (1992) and Isidori and Kang (1995). However, few results have been developed on $H_\infty$ schemes for DA systems, see for instance Wang et al. (2002), in which some necessary and sufficient conditions for the existence of a controller solving the $H_\infty$ control problem for nonlinear DA systems are provided. More recently, $H_\infty$ control and robust adaptive control for a class of nonlinear DA systems with external disturbances and parametric uncertainties have been studied in Sun and Wang (2013).

The objective of this paper is to present some stability conditions for nonlinear differential-algebraic systems ex-

* This work was supported in part by Imperial College London under the Junior Research Fellowship Scheme.
tending classical tools from nonlinear control theory. In Section 2 two theorems based on Lyapunov Direct Method are introduced to study the local stability of DA systems. At the end of the section a simple example illustrates the results. In Section 3 we reformulate the differential-algebraic system as the feedback interconnection of a purely differential system and a purely algebraic system. With this approach we are able to exploit the small-gain theorem for the stability analysis of DA systems. From the main result we derive simple conditions to achieve global stability and we establish connections with the linear case. At the end of the section two examples are presented: one academic example and a nonlinear system describing an electrical circuit. Finally, Section 4 contains some concluding remark.

**Notation.** We use standard notation. The superscripts \( T \) and \(-T\) represent the transposition operator and the transposition of the inverse operator, respectively. \( I \) represents the identity matrix. The symbol \( I_r(0) \) denotes a ball of radius \( r > 0 \) and center \( x = 0 \). The symbols \( \mathbb{R}^n_0 \) and \( \mathbb{R}_{>0} \) indicate, respectively, the set of strictly positive real numbers and the set of non-negative real numbers. Given a function \( f: \mathbb{R}^n \rightarrow \mathbb{R} \) and a manifold \( \mathcal{M} \), the symbol \( f|_{\mathcal{M}} \) indicates the restriction of \( f \) to \( \mathcal{M} \). Given a matrix \( A \) the symbols \( \sigma(A) \) and \( \hat{\sigma}(A) \) represent the smallest and largest singular value, respectively, of the matrix \( A \). In this notation \( \sigma(A) \) indicates the spectrum of the matrix \( A \). The symbol \( \|\Sigma\|_\infty \) represents the \( H_\infty \) norm of the linear system \( \Sigma \). The function \( \sin(c) \) is the cardinal sine function, i.e., \( \sin(x) = \frac{\sin(x)}{x} \).

2. LYAPUNOV DIRECT METHOD

In this section we provide some stability conditions for nonlinear DA systems using Lyapunov Direct Method. We conclude the section with an example.

2.1 Stability analysis

Consider a continuous-time, autonomous, differential-algebraic system in semi-explicit form, described by the equations

\[
\begin{align*}
\dot{x}_1 &= f(x_1, x_2) \\
0 &= g(x_1, x_2),
\end{align*}
\]

where \( x_1(t) \in \mathbb{R}^n \) and \( x_2(t) \in \mathbb{R}^m \) denote the states of the system at time \( t \) and \( f \) and \( g \) are smooth mappings. Before undertaking the stability analysis, some clarifications are required on the nature of such a problem. Given a DA system, the differential index \( \nu \) is the minimum number of differentiation steps required to transform the DA system into an ODE system. As outlined in Tarraf and Asada (2002) any solution of the differential-algebraic system with index \( \nu \) must lay on the solution manifold

\[
\mathcal{M} = \left\{(x_1, x_2) : \frac{\partial^k g(x_1, x_2)}{\partial x_2^k} = 0, \ k = 1, ..., \nu - 1 \right\}
\]

and satisfy the algebraic equation in (1) for all time. Note that the solution manifold is not attractive (invariant) in general. Hence, any perturbation of the state may cause the solution to diverge from the manifold. In conclusion, the stability property is addressed for perturbations of the solutions corresponding to consistent initial conditions, i.e. which remain on the manifold. Assuming that the origin is an equilibrium point, system (1) can be rewritten in the form

\[
\begin{align*}
\dot{x}_1 &= A_{11}(x_1, x_2)x_1 + A_{12}(x_1, x_2)x_2, \\
0 &= A_{21}(x_1, x_2)x_1 + A_{22}(x_1, x_2)x_2,
\end{align*}
\]

where

\[
\begin{align*}
A_{11} : \mathbb{R}^{n_1+n_2} &\rightarrow \mathbb{R}^{n_1 \times n_2}, & A_{12} : \mathbb{R}^{n_1+n_2} &\rightarrow \mathbb{R}^{n_1 \times n_2}, \\
A_{21} : \mathbb{R}^{n_1+n_2} &\rightarrow \mathbb{R}^{n_2 \times n_1}, & A_{22} : \mathbb{R}^{n_1+n_2} &\rightarrow \mathbb{R}^{n_2 \times n_2}.
\end{align*}
\]

We also assume that \( A_{11}, A_{12}, A_{21} \) and \( A_{22} \) are smooth functions. For the sake of clarity the explicit dependence on the nature of such a problem. Given a DA algebraic system as the feedback interconnection of a linear case. At the end of the section two examples are presented: one academic example and a nonlinear system describing an electrical circuit. Finally, Section 4 contains some concluding remark.

**Theorem 1.** Consider system (3). Assume that the matrix \( A_{22} \) is square and has full rank for all \((x_1, x_2) \in I_R(0, 0) \subseteq \mathbb{R}^{n_1+n_2} \). Consider the Lyapunov function

\[
V = x_1^T P x_1,
\]

with \( P \in \mathbb{R}^{n_1 \times n_1} \), a symmetric and positive definite matrix. If \( V|_{\mathcal{M}} > 0 \) and there exists \( \alpha > 0 \) such that

\[
\begin{align*}
A_{11}^T P + PA_{11} + (\alpha A_{12}^T - PA_{22}) A_{22} - A_{22}^T PA_{22} - A_{22}^T P < 0,
\end{align*}
\]

for all \((x_1, x_2) \in I_R(0, 0) \subseteq \mathbb{R}^{n_1+n_2} \), then the origin is a locally asymptotically stable equilibrium point.

Consider now the case in which one wishes to avoid computing the inverse of the matrix \( A_{22} \).

**Theorem 2.** Consider system (3) and the Lyapunov function

\[
V = x_1^T P x_1,
\]

with \( P \in \mathbb{R}^{n_1 \times n_1} \), a symmetric and positive definite matrix. If \( V|_{\mathcal{M}} > 0 \) and there exist constants \( \alpha > 0 \) and \( \gamma > 0 \) such that

\[
\begin{align*}
A_{11}^T P + PA_{11} + 2\alpha A_{22} - \frac{1}{\gamma} PA_{12} A_{12}^T P < 0, \\
-2\alpha A_{22} A_{22} + 2\gamma I < 0,
\end{align*}
\]

for all \((x_1, x_2) \in I_R(0, 0) \subseteq \mathbb{R}^{n_1+n_2} \), then the origin is a locally asymptotically stable equilibrium point.

2.2 Example 1

Consider the DA system in Scarciotti (2018) described by the equations

\[
\begin{align*}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
-4 & 2 \\
-1 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
+ \begin{bmatrix}
1 + \tilde{\mu} \\
0
\end{bmatrix}
\begin{bmatrix}
x_3 \\
x_4
\end{bmatrix},
\end{align*}
\]

where \( \tilde{\mu} \in \mathbb{R} \) is a constant parameter. Note that the matrix \( A_{22} = I \) is invertible. The linear matrix inequality in (5) becomes

\[
\begin{align*}
0 &-3 \quad 0 \\
1 - \mu &-\epsilon -2 \quad + 2\alpha \quad -\epsilon
\end{align*}
\]

which depends on the parameters \( \tilde{\mu} \) and \( \epsilon \). Select, for instance,
Consider again system (3) with

$$\dot{x} = A_{11}(x, u)x_1 + A_{12}(x, u)w, \quad z = A_{21}(x, u)x_1,$$

with input $w(t) \in \mathbb{R}^{n_1}$ and output $z(t) \in \mathbb{R}^{n_2}$, and

$$\dot{y} = x_2, \quad \dot{z} = A_{22}(x_1, x_2)x_2,$$

with input $v = z$ and output $y = w$, see also Fig. 2. We now provide a preliminary result.

**Lemma 1.** Consider system (14). Suppose

$$\max_{(x_1, x_2) \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2}} \sigma(A_{22}^{-1}(x_1, x_2)) \leq k,$$

for some $k > 0$. Then system $\Sigma_2$ has finite $\mathcal{L}_2$-gain.

Note that while the differential equation in (3) is not affected by adding the algebraic equation multiplied by a function $\Gamma : \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \rightarrow \mathbb{R}^{n_1 \times n_2}$ of the state, the system in (13) is affected by the addition of such a term. Performing this operation we obtain

$$\dot{x}_1 = A_{11}(x_1, w)x_1 + A_{12}(x_1, w)w + \Gamma(x_1, w)(A_{21}(x_1, w)x_1 + A_{22}(x_1, w)w),$$

or, after some rearrangements,

$$\dot{x}_1 = (A_{11}(x_1, w) + \Gamma(x_1, w) A_{21}(x_1, w))x_1 + (A_{12}(x_1, w) + \Gamma(x_1, w) A_{22}(x_1, w))w.$$  

We now provide a small-gain theorem for the stability analysis of DA systems.

**Theorem 3.** Consider system (3) and assume that condition (15) holds. Suppose that the systems $\Sigma_1$ is detectable in a closed and bounded set $\Omega$ and there exists $\Gamma$ such that the $\mathcal{L}_2$-gain $\gamma_1$ of the modified system described by equation (17) is

$$\gamma_1 < \frac{1}{\max_{(x_1, x_2) \in \Omega} \sigma(A_{22}^{-1}(x_1, x_2))}.$$  

Then the origin is a locally asymptotically stable equilibrium point.

**Remark 2.** The choice of $\Gamma$ in equation (17) plays an important role in the calculation of the $\mathcal{L}_2$-gain of system $\Sigma_1$. As we show in Section 3.4, if a linear system $\Sigma_1$ is not asymptotically stable, then a proper choice of the matrix $\Gamma$ transforms $\Sigma_1$ in an asymptotically stable system.

From the previous theorem the next result follows.

**Corollary 1.** Consider system (3). Suppose the following conditions hold.

$(C_1)$ There exists $\Gamma$ such that
\[
\frac{\partial}{\partial t} [(A_{11}(x_1, w) + \Gamma(x_1, w)A_{21}(x_1, w))x_1 + (A_{12}(x_1, w) + \Gamma(x_1, w)A_{22}(x_1, w))w] = 0.
\] (19)

(C2) \(x_1 = 0\) is a locally asymptotically stable equilibrium of the system
\[
\dot{x}_1 = A_{11}(x_1, 0) + \Gamma(x_1, 0)A_{21}(x_1, 0)x_1.
\] (20)

(C3) \(A_{22}(0, x_2)\) is invertible.

Then the origin is an asymptotically stable equilibrium of system (3).

Remark 3. Consider the following linear differential-algebraic system
\[
\dot{x}_1 = A_{11}x_1 + A_{12}x_2, \quad 0 = A_{21}x_1 + A_{22}x_2,
\] (21)
where \(A_{11} \in \mathbb{R}^{n_1 \times n_1}, A_{12} \in \mathbb{R}^{n_1 \times n_2}, A_{21} \in \mathbb{R}^{n_2 \times n_1}\) and \(A_{22} \in \mathbb{R}^{n_2 \times n_2}\) are constant matrices. Assume that the matrix \(A_{22}\) is invertible, then condition (C1) of Corollary 1 holds for \(\Gamma = -A_{12}A_{22}^{-1}\). In addition, condition (C2) is equivalent to
\[
\text{Re}(\lambda_i) < 0 \quad \forall \lambda_i \in \sigma(A_{11} - A_{12}A_{22}^{-1}A_{21}).
\] (22)

Note that (22) is a necessary and sufficient condition for asymptotic stability of the linear DA system (21) (provided \(A_{22}\) is invertible).

3.3 Example 2

Consider system (8). Such a system can be described as the interconnection of the two systems
\[
\Sigma_1: \begin{cases} 
\dot{x}_1 = 0 \quad & \begin{bmatrix} 0 & 2 \\ -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 + \mu & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}, \\
\dot{x}_2 = 0 \quad & \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},
\end{cases}
\] (23)

and
\[
\Sigma_2: \begin{cases} 
\dot{y}_1 = 0 \quad & \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}, \\
\dot{y}_2 = 0 \quad & \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}.
\end{cases}
\] (24)

Since the matrix
\[
A_{11} = \begin{bmatrix} 0 & 2 \\ -4 & -2 \end{bmatrix}
\] (25)
has all eigenvalues with negative real part, system \(\Sigma_1\) is asymptotically stable. Note that the function \(\epsilon = \sin(x_2)\) is bounded thus an upper bound on the \(\mathcal{L}_2\)-gain of system \(\Sigma_1\) can be calculated as
\[
\gamma_1 = \max_{\epsilon \in [-1, 1]} \|\Sigma_1\|_\infty.
\] (26)

Fig. 3 shows the value of \(\gamma_1\) as a function of \(\tilde{\mu}\), from which it is clear that
\[
\gamma_1 < 1, \quad \forall \tilde{\mu} \in (-1.44, -0.56).
\] (27)

Since \(A_{22} = I\) it follows that \(\gamma_2 = 1\). Hence, by Theorem 3 the origin is a (globally) asymptotically stable equilibrium point of system (8) for all \(\tilde{\mu} \in (-1.44, -0.56)\). Note that the set of \(\tilde{\mu}\) for which the equation is stable is different from that in Example 1.

Fig. 3. Plot of \(\gamma_1\) for different values of \(\tilde{\mu}\).

3.4 Example 3

The result presented in Section 3 is validated on the model studied in Sjöberg et al. (2007b) and Scarciotti (2018), which describes the electrical circuit shown in Fig. 4. The model is described by the differential-algebraic equations
\[
\dot{u}_C = \frac{i_1}{2} \left( 1 + 10^{-1}u_C \right) + \frac{i_2}{2} \left( 1 + 10^{-2}u_C \right), \\
\dot{i} = -u_C - u_L, \\
C \dot{u}_C = u - u_R - u_C - u_L, \\
\Phi = \Phi - \arctan(i),
\] (28)

where \(u\) is an ideal voltage source, \(u_R\) is the voltage of the nonlinear resistor, \(i_1\), \(i_2\) and \(u_C\), represent, respectively, the currents and the voltage of the capacitor, and \(\Phi\) and \(u_L\) represent, respectively, the saturated flux and the voltage of the inductor. In the following analysis we assume that the ideal voltage source \(u\) is set to zero. System (28) can be written according to the notation used in Section 3, where \((\dot{x}_1, \dot{x}_2) = (u_C, \Phi)\) are the dynamic variables, \((x_3, x_4, x_5, x_6, x_7) = (i, i_1, i_2, u_L, u_R)\) are the algebraic variables and
\[
A_{11} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ -1 & 0 \end{bmatrix}
\]

Fig. 4. The electrical circuit in Scarciotti (2018).
Since the matrix $A_{11}$ is zero, i.e., system $\Sigma_1$ is not asymptotically stable, the alternative description (17) is required to apply Theorem 3. Consider, for instance, the choice

$$A_{12} = \begin{bmatrix} 0 & 1 & \frac{1}{2} & 1 & 1 \\ \frac{1}{2} & 1 & 0 & 0 & 1 \\ \frac{1}{2} + 0.1\bar{x}_1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

and

$$A_{22} = \begin{bmatrix} \arctan(\bar{x}_3) & 0 & 0 & 0 & 0 \\ -10\bar{x}_3^2 - 5 & 0 & 0 & 1 & 0 \\ 1 & -1 & -1 & 0 & 0 \\ 0 & 1 & 1 + 0.1\bar{x}_1 & 1 + 0.01\bar{x}_1 & 0 \\ 0 & 0 & 0 & -1 & -1 \end{bmatrix}.$$ 

First note that the matrix $A_1$ is constant and has all eigenvalues with negative real part, hence the modified system $\Sigma_1$ is asymptotically stable. The $L_2$-gain of system $\Sigma_1$ is the smallest $\gamma_1$ which satisfies the linear matrix inequality, see Doyle et al. (1989),

$$A^TP + PA + \frac{PB^TBP}{\gamma_1^2} + CCT \leq 0,$$

where

$$\hat{B} = \max_{|x| < 0.1} B(x, w),$$

for some symmetric and positive definite matrix $P$. For instance, equation (34) is satisfied for

$$P = \begin{bmatrix} 53 & 5 \\ 5 & 1.5 \end{bmatrix}$$

and $\gamma_1 = 0.1225$. Moreover,

$$\max_{|x| < 0.1} \hat{\sigma}(A_{22}^{-1}(\bar{x}_1, \bar{x}_3)) = 7.3677,$$

hence from Lemma 1 it follows that system $\Sigma_2$ has finite gain. Since

$$\gamma_1 = 0.1225 < \frac{1}{1.3677} = 0.1357,$$

all conditions of Theorem 3 are satisfied and the origin is a locally asymptotically stable equilibrium point of system (28).

4. CONCLUSION

In this paper we have extended the Lyapunov Direct Method to nonlinear DA systems by proposing two theorems. In addition, interpreting the DA system as the feedback interconnection of a dynamical system and an algebraic system, sufficient stability conditions have also been derived using the small-gain theorem. We have also shown that the proposed results yield necessary and sufficient stability conditions when applied to linear DA systems. Finally, we have provided three examples to validate the theoretical results.

REFERENCES

W. Blajer. Index of differential-algebraic equations governing the dynamics of constrained mechanical systems. *Applied Mathematical Modelling*, 16(10):70–77, 1992.

K. Brenan, S. Campbell, and L. Petzold. *Numerical Solution of Initial-Value Problems in Differential-Algebraic Equations*. Society for Industrial and Applied Mathematics, 1995.

D. F. Coutinho, A. S. Bazanella, A. Trofino, and A. S. e Silva. Stability analysis and control of a class of differential-algebraic nonlinear systems. *International Journal of Robust and Nonlinear Control*, 14(15):1301–1326, 2004.

L. Dai. *Singular Control Systems*. ser. Lecture Notes in Control and Information Sciences. Springer Berlin Heidelberg, 1989.

M. Darouach and M. Boutayeb. Design of observers for descriptor systems. *IEEE Transactions on Automatic Control*, 40(7):1323–1327, 1995.

J. C. Doyle, K. Glover, P. P. Khargonekar, and B. A. Francis. State-space solutions to standard $H_2$ and $H_\infty$ control problems. *IEEE Transactions on Automatic Control*, 34(8):831–847, Aug 1989.

T. Glad and J. Sjöberg. Hamilton-Jacobi equations for nonlinear descriptor systems. In *2006 American Control Conference*, pages 1027–1031, June 2006.

A. Isidori and A. Astolfi. Disturbance attenuation and $H_\infty$-control via measurement feedback in nonlinear systems. *IEEE Transactions on Automatic Control*, 37(9):1283–1293, Sep 1992.

A. Isidori and Wei Kang. $H_\infty$ control via measurement feedback for general nonlinear systems. *IEEE Transactions on Automatic Control*, 40(3):466–472, Mar 1995.

H. K. Khalil. *Nonlinear systems*. Prentice Hall, Upper Saddle River, NJ, 1996.

P. Kunkel and V. L. Mehrmann. *Differential-Algebraic Equations: Analysis and Numerical Solution*. ser. EMS textbooks in mathematics, European Mathematical Society, 2006.

P. C. Müller. Modified Lyapunov equations for LTI descriptor systems. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, 28:448–452, 2006.

P. C. Müller and M. Hou. On the observer design for descriptor systems. In *Proceedings of the 30th IEEE Conference on Decision and Control*, 1991, pages 1960–1961 vol.2, 1991.

D. Pogorelov. Differential-algebraic equations in multi-body system modeling. *Numerical Algorithms*, 19(1):183–194, 1998.
R. Riaza. *Differential-Algebraic Systems: Analytical Aspects and Circuit Applications*. World Scientific Publishing Co Pte ltd, River Edge, NJ, USA, 2008.

G. Scarciotti. Steady-state matching and model reduction for systems of differential-algebraic equations. *To appear in IEEE Transactions on Automatic Control*, 2018.

J. Sjöberg, R. Findeisen, and F. Allgöwer. Model predictive control of continuous time nonlinear differential-algebraic systems. *7th IFAC Symposium on Nonlinear Control Systems*, 40(12):48–53, 2007a.

J. Sjöberg, K. Fujimoto, and T. Glad. Model reduction of nonlinear differential-algebraic systems. *IFAC Proceedings Volumes*, 40(12):176–181, 2007b. 7th IFAC Symposium on Nonlinear Control Systems.

L. Sun and Y. Wang. An undecomposed approach to control design for a class of nonlinear descriptor systems. *International Journal of Robust and Nonlinear Control*, 23(6):695–708, 2013.

D.C. Tarraf and H.H. Asada. On the nature and stability of differential-algebraic systems. In *Proceedings of the 2002 American Control Conference (IEEE Cat. No.CH37301)*, volume 5, pages 3546–3551, Anchorage, AK, USA, 2002.

A. J. van der Schaft. $L_2$-gain analysis of nonlinear systems and nonlinear state-feedback $H_\infty$ control. *IEEE Transactions on Automatic Control*, 37(6):770–784, 1992.

A. J. van der Schaft. *$L_2$-Gain and Passivity Techniques in Nonlinear Control*. Springer-Verlag New York, Inc., Secaucus, NJ, USA, 1st edition, 1996.

H. S. Wang, C. F. Yung, and F. R. Chang. $H_\infty$ control for nonlinear descriptor systems. *IEEE Transactions on Automatic Control*, 47(11):1919–1925, Nov 2002.

P. Wang and J. Zhang. Stability of solutions for nonlinear singular systems with delay. *Applied Mathematics Letters*, 25(10):1291–1295, 2012.

H. Wu and K. Mizukami. Lyapunov stability theory and robust control of uncertain descriptor systems. *International Journal of Systems Science*, 26(10):1981–1991, 1995.