Theoretical Limits on Agegraphic Quintessence from Weak Gravity Conjecture

Xiang-Lai Liu, Jingfei Zhang and Xin Zhang
Department of Physics, College of Sciences, Northeastern University, Shenyang 110004, China

ABSTRACT

In this paper, we investigate the possible theoretical constraint on the parameter $n$ of the agegraphic quintessence model by considering the requirement of the weak gravity conjecture that the variation of the quintessence scalar field $\phi$ should be less than the Planck mass $M_p$. We obtain the theoretical upper bound $n \lesssim 2.5$ that is inconsistent with the current observational constraint result $2.637 < n < 2.983$ (95.4% CL). The possible implications of the tension between observational and theoretical constraint results are discussed.
1 Introduction

In 1998, two independent supernovae (SN) observation groups found that our universe is undergoing an accelerated expansion at the present stage, through the observations of distant type Ia supernovae [1]. This implies that there exists a mysterious component, dark energy, which has large enough negative pressure, responsible for the cosmic acceleration. Many other astronomical observations, such as surveys of the large scale structure (LSS) [2] and measurements of the cosmic microwave background (CMB) anisotropy [3], also firmly indicate that dark energy is the dominant component in the present-day universe. It is commonly believed that exploring the nature of dark energy is one of the focuses in the realm of both cosmology and theoretical physics today.

The most obvious candidate for dark energy is the famous Einstein’s cosmological constant $\lambda$ which has the equation of state $w = -1$. However, as is well known, the cosmological constant is plagued with the “fine-tuning” and “cosmic coincidence” problems [4]. Another promising candidate for dark energy is the dynamical scalar field, a slowly varying, spatially homogeneous component. An example of scalar-field dark energy is the so-called quintessence [5], a scalar field $\phi$ slowly evolving down its potential $V(\phi)$. Provided that the evolution of the field is slow enough, the kinetic energy density is less than the potential energy density, giving rise to the negative pressure responsible to the cosmic acceleration. So far, in order to alleviate the cosmological-constant problems and explain the accelerated expansion, a wide variety of scalar-field dark energy models have been proposed. Besides quintessence, these also include phantom, $k$-essence, tachyon, ghost condensate and quintom amongst many. However, we should note that the mainstream viewpoint regards the scalar-field dark energy models as a low-energy effective description of the underlying theory of dark energy.

It is generally believed by theorists that we cannot entirely understand the nature of dark energy before a complete theory of quantum gravity is established. However, although we are lacking a quantum gravity theory today, we still can make some efforts to explore the nature of dark energy according to some principles of quantum gravity. The holographic dark energy model [6] is just an appropriate example, which is constructed in light of the holographic principle of quantum gravity theory. That is to say, the holographic dark energy model possesses some significant features of an underlying theory of dark energy. More recently, a new model consistent with the holographic principle, the agegraphic dark energy model, has been proposed in [7], which takes into account the uncertainty relation of quantum mechanics together with the gravitational effect in general relativity.

While, by far, a complete theory of dark energy has not been established presently, we can, however, speculate on the underlying theory of dark energy by taking some principles
of quantum gravity into account. The agegraphic dark energy model is no doubt a tentative
in this way. Now, we are interested in that if we assume the holographic/agegraphic vacuum
energy scenario as the underlying theory of dark energy, how the low-energy effective scalar-
field model can be used to describe it. In this direction, some work has been done, see, e.g.,
[8-11]. The agegraphic versions of scalar-field models, such as quintessence and tachyon, have
been constructed [10,11]. In this paper, we focus on the canonical scalar-field description of
the agegraphic dark energy, namely, the “agegraphic quintessence” [10].

In recent years, cosmological-constant/dark-energy problem has been studied by string the-
orists within the string framework. It is generally considered that string theory is the most
promising consistent theory of quantum gravity. Based on the KKLT mechanism [12], a vast
number of metastable de Sitter vacua have been constructed through the flux compactification
on a Calabi-Yau manifold. These string vacua can be described by the low-energy effective
theories. Furthermore, it is realized that the vast series of semiclassically consistent field theo-
ries are actually inconsistent. These inconsistent effective field theories are believed to locate in
the so-called “swampland” [13]. The self-consistent landscape is surrounded by the swampland.
Vafa has proposed some criterion to the consistent effective field theories [13]. Moreover, it was
conjectured by Arkani-Hamed et al. [14] that the gravity is the weakest force, which helps rule
out those effective field theories in the swampland. Arkani-Hamed et al. pointed out [14] that
when considering the quantum gravity, the gravity and other gauge forces should not be treated
separately. For example, in four dimensions a new intrinsic UV cutoff for the U(1) gauge theory
with single scalar field, \( \Lambda = g M_p \), is suggested, where \( g \) is the gauge coupling [14]. In [15],
the weak gravity conjecture together with the requirement that the IR cutoff should be smaller
than the UV cutoff leads to an upper bound for the cosmological constant. In addition, for the
inflationary cosmology, the application of the weak gravity conjecture shows that the chaotic
inflation model is in the swampland [16]. This conjecture even implies that the eternal inflation
may not be achieved [17]. Furthermore, Huang conjectured [18] that the variation of the infla-
ton should be smaller than the Planck scale \( M_p \), and this can make stringent constraint on the
spectral index.

More recently, the weak gravity conjecture has been applied to the dark-energy problem.
It is suggested that the variation of the quintessence field value \( \phi \) should be less than \( M_p \).
This criterion may give important theoretical constraints on the equation-of-state parameter of
quintessence models, and some of these constraints are even stringent than those of the present
experiments [19]. The criterion \( |\Delta \phi(z)|/M_p \leq 1 \) has also been used to put theoretical constraints
on other canonical scalar-field dark energy models; see, e.g., [20-22]. In this paper we shall in-
vestigate the possible theoretical limits on the parameter \( n \) of the agegraphic quintessence from
the weak gravity conjecture.
In the next section, we will briefly review the new agegraphic dark energy model proposed in [23]. In Sec. 3, we will give the possible theoretical limits on the parameter $n$ of the agegraphic quintessence model from the weak gravity conjecture. Conclusion will be given in Sec. 4.

## 2 Agegraphic Dark Energy Model

Holographic dark energy models arise from the holographic principle [24] of quantum gravity. The holographic principle determines the range of validity for a local effective quantum field theory to be an accurate description of the world involving dark energy, by imposing a relationship between the ultraviolet (UV) and infrared (IR) cutoffs [25]. As a consequence, the vacuum energy becomes dynamical, and its density $\rho_{\text{de}}$ is inversely proportional to the square of the IR cutoff length scale $L$ that is believed to be some horizon size of the universe, namely, $\rho_{\text{de}} \propto L^{-2}$.

The original holographic dark energy model [6] chooses the future event horizon size as its IR cutoff scale, so the energy density of holographic dark energy reads

$$\rho_{\text{de}} = 3c^2 M_p^2 R_{\text{eh}}^{-2},$$

where $c$ is a constant, and $R_{\text{eh}}$ is the size of the future event horizon of the universe. This model is successful in explaining the cosmic acceleration and in fitting the observational data. There are also other two versions of holographic dark energy, namely, the agegraphic dark energy model [7, 10, 11, 23, 26] and the holographic Ricci dark energy model [9, 27]. In this paper, we focus on the agegraphic dark energy model.

The agegraphic dark energy model discussed in this paper is actually the new version of the agegraphic dark energy model [23] (sometimes called the new agegraphic dark energy model in the literature) which suggests to choose the conformal age of the universe

$$\eta = \int_0^\eta \frac{dt'}{a} = \int_0^\eta \frac{da'}{Ha'^2}$$

as the IR cutoff, so the energy density of agegraphic dark energy is

$$\rho_{\text{de}} = 3n^2 M_p^2 \eta^{-2},$$

where $n$ is a constant which plays the same role as $c$ in the original holographic dark energy model.

The corresponding fractional energy density is given by

$$\Omega_{\text{de}} = \frac{n^2}{H^2 \eta^2}.$$  \hspace{1cm} (2.3)

Taking derivative for Eq. (2.3) with respect to $x = \ln a$, and considering Eq. (2.1), we obtain

$$\Omega_{\text{de}}' = 2\Omega_{\text{de}} \left( \epsilon - \frac{\sqrt{\Omega_{\text{de}}}}{na} \right).$$  \hspace{1cm} (2.4)
where $\epsilon \equiv -\dot{H}/H^2$. The Friedmann equation reads

$$3M_p^2H^2 = \rho_m + \rho_{de},$$  \hspace{1cm} (2.5)

or equivalently,

$$E(z) \equiv \frac{H(z)}{H_0} = \left( \frac{\Omega_{m0}(1 + z)^3}{1 - \Omega_{de}} \right)^{1/2}.$$  \hspace{1cm} (2.6)

From Eqs. (2.5), (2.2), (2.3) and $\dot{\rho}_m + 3H\rho_m = 0$, we have

$$\epsilon = \frac{3}{2}(1 - \Omega_{de}) + \frac{\Omega_{de}^{3/2}}{na}. \hspace{1cm} (2.7)$$

Hence, we get the equation of motion for $\Omega_{de}$, i.e.,

$$\Omega_{de}' = \Omega_{de}(1 - \Omega_{de}) \left( 3 - \frac{2}{n} \frac{\sqrt{\Omega_{de}}}{a} \right), \hspace{1cm} (2.8)$$

and this equation can be rewritten as

$$\frac{d\Omega_{de}}{dz} = -\Omega_{de}(1 - \Omega_{de}) \left( 3(1 + z)^{-1} - \frac{2}{n} \frac{\sqrt{\Omega_{de}}}{a} \right). \hspace{1cm} (2.9)$$

From Eqs. (2.2), (2.3) and $\dot{\rho}_{de} + 3H(1 + w_{de})\rho_{de} = 0$, we obtain the equation of state (EoS) of the agegraphic dark energy

$$w_{de} = -1 + \frac{2}{3n} \frac{\sqrt{\Omega_{de}}}{a}. \hspace{1cm} (2.10)$$

Now, we pause for a while to make some additional comments on the old version of the agegraphic dark energy model [7]. In the old model, the IR cutoff of the theory is taken as the age of the universe, $t = \int_0^a \frac{da}{Ha}$. However, for this choice, there are some internal inconsistencies in the model; see [23] for detailed discussions. In the matter-dominated epoch with $\Omega_{de} \ll 1$, one has $a \propto t^{2/3}$, thus $t^2 \propto a^3$. So, in this epoch, $\rho_{de} \propto t^{-2} \propto a^{-3}$. Since $\rho_m \propto a^{-3}$, one has $\Omega_{de} \approx \text{const.}$, which is in conflict with $\Omega_{de} \propto a^3$ obtained from the differential equation governing the evolution of dark energy [23]. What’s more, from $\rho_{de} \propto t^{-2}$, the agegraphic dark energy tracks the dominated components (either pressureless matter or radiation). Therefore, the agegraphic dark energy never dominates. This is of course unacceptable. Accordingly, the new version of the agegraphic dark energy model was proposed [23] by replacing the age $t$ with the conformal age $\eta$, for eliminating the inconsistencies in the old version. This is the reason why we only consider the new agegraphic dark energy model in this paper.

## 3 Agegraphic Quintessence and Its Possible Theoretical Limits from Weak Gravity Conjecture

For a single-scalar-field quintessence model, the potential energy density $V(\phi)$ is a function of the scalar field $\phi$. If the field is spatially homogeneous, namely, the spacial curvature of field
can be neglected, the field equation can be expressed as
\[ \ddot{\phi} + 3H \dot{\phi} + \frac{dV}{d\phi} = 0, \] (3.1)
where the dot denotes the derivative with respect to the cosmic time. The energy density and the pressure are
\[ \rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi), \]
\[ p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi), \] (3.2)
so the EoS parameter is
\[ w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2} \dot{\phi}^2 - V(\phi)}{\frac{1}{2} \dot{\phi}^2 + V(\phi)}, \] (3.3)
which generally varies with time. The range for the EoS parameter of the quintessence is \( w_\phi \in [-1, 1] \). If the scalar field varies slowly in time, namely, \( \dot{\phi}^2 \ll V \), the field energy approximates the effect of Einstein’s cosmological constant with \( p_\phi \approx -\rho_\phi \).

Using Eq. (3.3), we find a relationship between the potential of quintessence and its kinetic energy
\[ V(\phi) = \frac{\dot{\phi}^2}{2} \frac{1 - w_\phi}{1 + w_\phi}. \] (3.4)
The energy density takes the form
\[ \rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) = \frac{\dot{\phi}^2}{1 + w_\phi}. \] (3.5)
We assume, without loss of generality, \( dV/d\phi > 0 \), so that \( \dot{\phi} < 0 \). Thus, Eq. (3.5) reads
\[ \dot{\phi} = -\sqrt{(1 + w_\phi)\rho_\phi}. \] (3.6)
In the agegraphic quintessence model [10], the quintessence scalar field is viewed as an effective description of the agegraphic dark energy, so the scalar-field energy density \( \rho_\phi \) and EoS \( w_\phi \) are identified with those of the agegraphic dark energy, \( \rho_{de} \) and \( w_{de} \), respectively.

Integrating Eq. (3.6), we obtain
\[ \frac{\Delta \phi(z)}{M_p} = \int_{\phi(0)}^{\phi(z)} d\phi/M_p \]
\[ = \int_0^z \sqrt{3[1 + w_{de}(z')]} \Omega_{de}(z') \frac{dz'}{1 + z'}, \] (3.7)
where \( \Omega_{de} \) and \( w_{de} \) are given by Eqs. (2.9) and (2.10) for the agegraphic quintessence model. If we fix the field amplitude at the present epoch \((z = 0)\) to be zero, \( \phi(0) = 0 \), then Eq. (3.7) can be rewritten as
\[ \frac{\phi(z)}{M_p} = \int_0^z \sqrt{3[1 + w_{de}(z')]} \Omega_{de}(z') \frac{dz'}{1 + z'}. \] (3.8)
As suggested in [28], quintessence models can be divided into two classes, “thawing” models and “freezing” models. Thawing models depict those scalar fields that evolve from $w = -1$ but grow less negative with time as $dw/d\ln a > 0$; freezing models, whereas, describe those fields that evolve from $w > -1$ and $dw/d\ln a < 0$ to $w \to -1$ and $dw/d\ln a \to 0$. The agegraphic dark energy mimics a cosmological constant at the late time, so it belongs to the freezing quintessence models [10]. A particular feature of this model is that it is actually a single-parameter model: the differential equation of $\Omega_{de}$, namely, Eq. (2.9), is governed by a single parameter $n$, provided that the initial condition is taken to be $\Omega_{de}(z_{ini}) = n^2(1 + z_{ini})^{-2}/4$ at any $z_{ini}$ which is deep enough into the matter-dominated epoch. Following [23], here we take $z_{ini} = 2000$.

Figure 1: The scalar field evolution for the single-field agegraphic quintessence model. The theoretical requirement $|\Delta \phi(z)|/M_p \leq 1$ places a constraint on this model, $n \leq 2.5$, which is inconsistent with the current observational constraint $2.637 < n < 2.983$.

It should be mentioned that the agegraphic dark energy model has been constrained strictly by using the latest observational data including the Constitution sample of SN, the shift parameter of the CMB given by the five-year Wilkinson Microwave Anisotropy Probe (WMAP) observations, and the baryon acoustic oscillation (BAO) measurement from the Sloan Digital Sky Survey (SDSS) [29]. The analysis of these observational data gives the fitting results [29]: for 68.3% confidence level, $n = 2.807^{+0.087}_{-0.086}$, for 95.4% confidence level, $n = 2.807^{+0.176}_{-0.170}$.

Consider the theoretical constraint on the single-field agegraphic quintessence model in which the variation of the canonical scalar field $|\Delta \phi(z)|$ is required not to exceed the Planck scale $M_p$. Figure 1 shows the constraint result: $n \leq 2.5$, which is a surprising result because this limit is so stringent and somewhat inconsistent with the result obtained from the current
observational data. According to the current observations, at 95.4% confidence level, we have $2.637 < n < 2.983$ \cite{29} that refuses to accommodate the theoretical limit $n \lesssim 2.5$.

One may naturally ask whether the multi-field agegraphic quintessence model could loosen the theoretical limit and eliminate the above tension between theoretical and observational limits. In the following, we shall give a clear answer to this question.

Let us consider a quintessence scalar-field model containing $N$ scalar fields $\phi_i$ with independent potential $V_i(\phi_i)$ for $i = 1, \ldots, N$. Thus, for each scalar field $\phi_i$, we have

$$\ddot{\phi}_i + 3H\dot{\phi}_i + \frac{dV_i(\phi_i)}{d\phi_i} = 0,$$

where the dot denotes the derivative with respect to the cosmic time. The total energy density and the pressure of the fields are

$$\rho_{\phi} = \frac{1}{2} \sum_{i=1}^{N} \dot{\phi}_i^2 + \sum_{i=1}^{N} V_i(\phi_i),$$

$$p_{\phi} = \frac{1}{2} \sum_{i=1}^{N} \dot{\phi}_i^2 - \sum_{i=1}^{N} V_i(\phi_i).$$

For simplicity, we assume that $\phi_1 = \phi_2 = \ldots = \phi_i = \ldots = \phi_N \equiv \varphi$ and $V_1(\phi_1) = V_2(\phi_2) = \ldots = V_i(\phi_i) = \ldots = V_N(\phi_N) \equiv V(\varphi)$. Then, the total energy density and the pressure of the scalar fields can be rewritten as

$$\rho_{\phi} = N \left( \frac{1}{2} \dot{\varphi}^2 + V(\varphi) \right),$$

$$p_{\phi} = N \left( \frac{1}{2} \dot{\varphi}^2 - V(\varphi) \right),$$

and the EoS parameter can be expressed as

$$w_{\phi} = \frac{p_{\phi}}{\rho_{\phi}} = \frac{\frac{1}{2} \dot{\varphi}^2 - V(\varphi)}{\frac{1}{2} \dot{\varphi}^2 + V(\varphi)}.$$

Using Eqs. (3.11) and (3.12), we obtain

$$\dot{\varphi} = -\sqrt{\frac{1}{N} (1 + w_{\phi}) \rho_{\phi}}.$$

Note that in this expression we have assumed $dV/d\varphi > 0$, in accordance with the previous discussion. Next, we identify the scalar-field energy density $\rho_{\phi}$ and EoS $w_{\phi}$ with those of the agegraphic dark energy, $\rho_{de}$ and $w_{de}$, respectively. Integrating Eq. (3.13), we obtain

$$\frac{|\Delta \varphi(z)|}{M_p} = \int_{\varphi(0)}^{\varphi(z)} \frac{d\varphi}{M_p} = \int_{0}^{z} \sqrt{\frac{3(1 + w_{de}(z'))}{N} \Omega_{de}(z')} \frac{dz'}{1 + z'}.$$
It is easy to see that in this case the amplitude of $|\Delta \varphi(z)|$ is suppressed by a factor $N^{-1/2}$, so it seems that the tension between the theoretical and observational limits in the single-field case could be avoided in such a multi-field model.

Fixing the field amplitude at the present epoch to be zero, $\varphi(0) = 0$, from Eq. (3.14) we get

$$\frac{\varphi(z)}{M_p} = \int_0^\infty \sqrt{\frac{3[1 + w_{de}(z')]\Omega_{de}(z')}{N}} \frac{dz'}{1 + z'}.$$

(3.15)

Furthermore, using Eqs. (3.11) and (3.12), we obtain

$$V(\varphi) = \frac{1}{2N}(1 - w_\phi)\rho_\phi = \frac{\rho_{c0}}{2N}(1 - w_\phi)\Omega_\phi E^2,$$

(3.16)

or equivalently,

$$\frac{V(\varphi)}{\rho_{c0}} = \frac{1}{2N}(1 - w_{de})\Omega_{de} E^2,$$

(3.17)

where $\rho_{c0} = 3M_p^2 H_0^2$ is today’s critical density of the universe.

By far, we have constructed a multi-field quintessence model mimicking the agegraphic dark energy. Figure 2 is an example of multi-field agegraphic quintessence model corresponding to $n = 2.8$. In the left panel, we plot the evolution of the scalar field $\varphi(z)$; the corresponding potential $V(\varphi)$ can be found in the right panel. From this figure, it is clear to see that bigger $N$ indeed gives rise to smaller $|\Delta \varphi|$. So, if we use $|\Delta \varphi(z)|/M_p \leq 1$ to constrain the multi-field agegraphic quintessence model, the tension between the theoretical and observational limits in the single-field case would be removed. However, does the criterion $|\Delta \varphi(z)|/M_p \leq 1$ still hold in a multi-field model? Unfortunately, the answer is NO. It has been demonstrated by Huang [30] that for an effective canonical scalar field theory with $N$ species the weak gravity conjecture requires that the maximal variation of the scalar field satisfies the bound $|\Delta \varphi|/M_p \leq 1/\sqrt{N}$, i.e., the upper bound of the field variation in the multi-field case is also suppressed by a factor $N^{-1/2}$.

Therefore, obviously, the weak gravity conjecture for the multi-field agegraphic quintessence model must give the same theoretical limit result as the single-field model. For clarity, we illustrate two concrete examples, $N = 2$ and 3, in Fig. 3. In this figure, we explicitly show that the same theoretical limit $n \leq 2.5$ is obtained for the multi-field agegraphic quintessence model from weak gravity conjecture.

\[\text{In Ref. [30] the author proposes the weak gravity conjecture for a multiple scalar field theory and finds that the variation of the canonical scalar field is bounded by the gravity scale } \Lambda_G = M_p/\sqrt{N}, \text{ when an unimportant coefficient 2 is ignored. In this paper we also ignore this unimportant coefficient in order to keep the whole work consistent.}\]
Figure 2: Multi-field agegraphic quintessence model corresponding to $n = 2.8$. In the left panel, we plot the evolution of the scalar field $\varphi(z)$; in the right panel, we show the corresponding potential $V(\varphi)$. Note that in the right panel the potential $V(\varphi)$ is in unit of $\rho_{c0}$ and the field $\varphi$ is in unit of $M_p$. It is clear to see from this figure that bigger $N$ indeed gives rise to smaller $|\Delta \varphi|$.

Figure 3: The scalar field evolution $\varphi(z)$ for the multi-field agegraphic quintessence model. Here, we show two concrete examples, $N = 2$ and 3. The weak gravity conjecture for multi-field theory indicates that the same theoretical limit $n \lesssim 2.5$, independent of $N$, will be given.
Therefore, the agegraphic quintessence model is facing an awkward situation that the theoretical limit derived from weak gravity conjecture, \( n \lesssim 2.5 \), is not in accordance with the observational constraint result, \( 2.637 < n < 2.983 \) (95.4% CL). In fact, if the weak gravity conjecture is taken seriously, some low-energy effective field theories have been demonstrated to be in the swampland. For example, not only the chaotic inflation model is in the swampland [16], but also the assisted chaotic inflation might not be in the landscape [30]. The \( N \)-flation, a possible realization of the assisted chaotic inflation in string theory, is shown to be just semiclassically self-consistent, not really self-consistent, if the weak gravity conjecture is correct [30]. In the present paper, we find that the weak gravity conjecture leads to a tension between the theoretical and observational limits for the agegraphic dark energy model. This inconsistency can be explained as that the agegraphic dark energy might not be described by a consistent low-energy effective scalar field theory. Of course, one may argue that this assertion looks too strong, since after all the theoretical and observational limits both locate in around \( n \sim 2 - 3 \), which can also be viewed as that they are, to some extent, not in conflict. So, for the sake of rigorous, we do not exclude other possibilities such as that the current observational data cannot provide precise constraint on the agegraphic dark energy model, and the future accurate data perhaps would change the constraint result to be consistent with the theoretical limit.

4 Conclusion

To summarize, in this paper we have investigated the theoretical limits on the parameter \( n \) of the agegraphic quintessence model by considering that the variation of the quintessence scalar field \( \phi \) should be less than the Planck mass \( M_p \). The agegraphic dark energy can mimic the behavior of a quintessence scalar-field dark energy, so the quintessence model can be used to effectively describe the agegraphic dark energy. In this paper, we have tested the single-field and multi-field agegraphic quintessence models by using the weak gravity conjecture. We believe that the low-energy effective field theory is not applicable in the trans-Planckian field space.

We have shown that for both single-field and multi-field agegraphic quintessence models the weak gravity conjecture leads to the same theoretical limit, \( n \lesssim 2.5 \), which is inconsistent with the current observational constraint \( 2.637 < n < 2.983 \) (95.4% CL). The requirement that the variation of the field should be less than the Planck scale from weak gravity conjecture may arise from the consistent theory of quantum gravity, so in this sense the theoretical result obtained in this paper can, to some extent, be viewed as the prediction of quantum gravity. The tension between theoretical and observational limits implies that the agegraphic dark energy could not be described by a consistent low-energy effective scalar field theory. Of course, other possible reasons for the tension still exist, for example, perhaps the current observational data
cannot provide precise constraint on the agegraphic dark energy model, and the future accurate data might change the constraint result to be consistent with the theoretical limit from weak gravity conjecture.

Acknowledgements

We would like to thank the referee for providing us with many helpful suggestions. This work was supported by the National Natural Science Foundation of China under Grant Nos. 10705041 and 10975032.

References

[1] A. G. Riess et al. [Supernova Search Team Collaboration], Astron. J. 116, 1009 (1998) [arXiv:astro-ph/9805201]; S. Perlmutter et al. [Supernova Cosmology Project Collaboration], Astrophys. J. 517, 565 (1999) [arXiv:astro-ph/9812133].

[2] M. Tegmark et al. [SDSS Collaboration], Phys. Rev. D 69, 103501 (2004) [arXiv:astro-ph/0310723]; K. Abazajian et al. [SDSS Collaboration], Astron. J. 128, 502 (2004) [arXiv:astro-ph/0403325]; K. Abazajian et al. [SDSS Collaboration], Astron. J. 129, 1755 (2005) [arXiv:astro-ph/0410239].

[3] D. N. Spergel et al. [WMAP Collaboration], Astrophys. J. Suppl. 148, 175 (2003) [arXiv:astro-ph/0302209]; D. N. Spergel et al. [WMAP Collaboration], Astrophys. J. Suppl. 170, 377 (2007) [arXiv:astro-ph/0603449].

[4] S. Weinberg, arXiv:astro-ph/0005265; T. Padmanabhan, Curr. Sci. 88, 1057 (2005) [arXiv:astro-ph/0411044]; E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D 15, 1753 (2006) [arXiv:hep-th/0603057]; E. V. Linder, Am. J. Phys. 76, 197 (2008) [arXiv:0705.4102 [astro-ph]]; V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D 9, 373 (2000) [arXiv:astro-ph/9904398]; P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. 75, 559 (2003) [arXiv:astro-ph/0207347]; P. J. Steinhardt, in Critical Problems in Physics, edited by V. L. Fitch and D. R. Marlow (Princeton University Press, Princeton, NJ, 1997).

[5] P. J. E. Peebles and B. Ratra, Astrophys. J. 325, L17 (1988); B. Ratra and P. J. E. Peebles, Phys. Rev. D 37, 3406 (1988); C. Wetterich, Nucl. Phys. B 302, 668 (1988); M. S. Turner and M. J. White, Phys. Rev. D 56, 4439 (1997) [astro-ph/9701138]; R. R. Caldwell, R. Dave and P. J. Steinhardt, Phys. Rev. Lett. 80, 1582 (1998) [astro-ph/9708069]; I. Zlatev, L. M. Wang and P. J. Steinhardt, Phys. Rev. Lett. 82, 896 (1999) [astro-ph/9807002].
[6] M. Li, Phys. Lett. B 603, 1 (2004) [hep-th/0403127]; Q. G. Huang and M. Li, JCAP 0503, 001 (2005) [arXiv:hep-th/0410095]; JCAP 0408, 013 (2004) [arXiv:astro-ph/0404229]; Q. G. Huang and Y. G. Gong, JCAP 0408, 006 (2004) [arXiv:astro-ph/0403590]; X. Zhang, Int. J. Mod. Phys. D 14, 1597 (2005) [arXiv:astro-ph/0504586]; X. Zhang and F. Q. Wu, Phys. Rev. D 72, 043524 (2005) [arXiv:astro-ph/0506310]; Phys. Rev. D 76, 023502 (2007) [arXiv:astro-ph/0701405]; B. Wang, Y. Gong and E. Abdalla, Phys. Lett. B 624, 141 (2005) [arXiv:hep-th/0506069]; J. Zhang, X. Zhang and H. Liu, Eur. Phys. J. C 52, 693 (2007) [arXiv:0708.3121 [hep-th]]; X. Zhang, Phys. Lett. B 683, 81 (2010) [arXiv:0909.4940 [gr-qc]]; M. Li, X. D. Li, S. Wang, Y. Wang and X. Zhang, JCAP 0912, 014 (2009) [arXiv:0910.3855 [astro-ph.CO]].

[7] R. G. Cai, Phys. Lett. B 657, 228 (2007) [arXiv:0707.4049 [hep-th]].

[8] X. Zhang, Phys. Lett. B 648, 1 (2007) [arXiv:astro-ph/0604484]; Phys. Rev. D 74, 103505 (2006) [arXiv:astro-ph/0609699]; J. Zhang, X. Zhang and H. Liu, Phys. Lett. B 651, 84 (2007) [arXiv:0706.1185 [astro-ph]]; J. Zhang and Y. X. Gui, arXiv:0910.1200 [astro-ph.CO]; J. P. Wu, D. Z. Ma and Y. Ling, Phys. Lett. B 663, 152 (2008) [arXiv:0805.0546 [hep-th]]; W. Zhao, Phys. Lett. B 655, 97 (2007) [arXiv:0706.2211 [astro-ph]].

[9] X. Zhang, Phys. Rev. D 79, 103509 (2009) [arXiv:0901.2262 [astro-ph.CO]].

[10] J. Zhang, X. Zhang and H. Liu, Eur. Phys. J. C 54, 303 (2008) [arXiv:0801.2809 [astro-ph]].

[11] J. Cui, L. Zhang, J. Zhang and X. Zhang, Chin. Phys. B 19, 019802 (2010) [arXiv:0902.0716 [astro-ph.CO]].

[12] S. Kachru, R. Kallosh, A. D. Linde and S. P. Trivedi, Phys. Rev. D 68, 046005 (2003) [arXiv:hep-th/0301240].

[13] C. Vafa, arXiv:hep-th/0509212.

[14] N. Arkani-Hamed, L. Motl, A. Nicolis and C. Vafa, JHEP 0706, 060 (2007) [hep-th/0601001].

[15] Q. G. Huang, M. Li and W. Song, JHEP 0610, 059 (2006) [hep-th/0603127].

[16] Q. G. Huang, JHEP 0705, 096 (2007) [hep-th/0703071].

[17] Q. G. Huang, M. Li and Y. Wang, JCAP 0709, 013 (2007) [arXiv:0707.3471 [hep-th]].

[18] Q. G. Huang, Phys. Rev. D 76, 061303 (2007) [arXiv:0706.2215 [hep-th]].

[19] Q. G. Huang, Phys. Rev. D 77, 103518 (2008) [arXiv:0708.2760 [astro-ph]].

[20] Y. Z. Ma and X. Zhang, Phys. Lett. B 661, 239 (2008) [arXiv:0709.1517 [astro-ph]].
[21] X. Wu and Z. H. Zhu, Chin. Phys. Lett. 25, 1517 (2008) [arXiv:0710.1406 [astro-ph]].

[22] X. Chen, J. Liu and Y. Gong, Chin. Phys. Lett. 25, 3086 (2008) [arXiv:0806.2415 [gr-qc]].

[23] H. Wei and R. G. Cai, Phys. Lett. B 660, 113 (2008) [arXiv:0708.0884 [astro-ph]].

[24] G. ’t Hooft, arXiv:gr-qc/9310026; L. Susskind, J. Math. Phys. 36 6377 (1995).

[25] A.G. Cohen, D.B. Kaplan and A.E. Nelson, Phys. Rev. Lett. 82, 4971 (1999).

[26] H. Wei and R. G. Cai, Phys. Lett. B 663, 1 (2008) [arXiv:0708.1894]; I. P. Neupane, Phys. Rev. D 76, 123006 (2007) [arXiv:0709.3096 [hep-th]]; Y. W. Kim, H. W. Lee, Y. S. Myung and M. I. Park, Mod. Phys. Lett. A 23, 3049 (2008) [arXiv:0803.0574 [gr-qc]]; L. Zhang, J. Cui, J. Zhang and X. Zhang, Int. J. Mod. Phys. D 19, 21 (2010) [arXiv:0911.2838 [astro-ph.CO]]; X. L. Liu and X. Zhang, Commun. Theor. Phys. 52, 761 (2009) [arXiv:0909.4911 [astro-ph.CO]].

[27] C. Gao, F. Q. Wu, X. Chen and Y. G. Shen, Phys. Rev. D 79, 043511 (2009) [arXiv:0712.1394 [astro-ph]]; C. J. Feng and X. Zhang, Phys. Lett. B 680, 399 (2009) [arXiv:0904.0045 [gr-qc]]; C. J. Feng, Phys. Lett. B 670, 231 (2008) [arXiv:0809.2502 [hep-th]]; L. Xu, W. Li and J. Lu, Mod. Phys. Lett. A 24, 1355 (2009) [arXiv:0810.4730 [astro-ph]].

[28] R. R. Caldwell and E. V. Linder, Phys. Rev. Lett. 95, 141301 (2005) [arXiv:astro-ph/0505494].

[29] M. Li, X. D. Li, S. Wang and X. Zhang, JCAP 0906, 036 (2009) [arXiv:0904.0928 [astro-ph.CO]].

[30] Q. G. Huang, Phys. Rev. D 77, 105029 (2008) [arXiv:0712.2859 [hep-th]].