On the exotic eikosiheptaplet

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We use the chiral quark soliton model to estimate masses and widths of the two eikosiheptaplets (27-plets of SU(3) flavor) of spin 3/2 and 1/2 that emerge in the rigid rotator quantization. We use as input: hyperon decays, Θ⁺ mass and width. While 2Σ³/2 has small widths (although much larger than the values allowed by the partial wave analysis), 2Σ¹/2 has large decay widths to antidecuplet. However exactly for this decay channels the widths are suppressed in the large \( N_c \) limit.

I. INTRODUCTION

One of the most puzzling results of the chiral quark soliton model (\( \chi \) QSM) for exotic baryons consists in a very small hadronic decay width, governed by the decay constant \( G_\text{\( \pi \)} \). While the small mass of exotic states is rather generic for all chiral models \([1,2,3]\) the smallness of the decay width appears as a subtle cancelation of three different terms \([3]\) that contribute to \( G_\text{\( \pi \)} \). We are therefore trapped between two extremes. On one hand \( \Delta \) decay width which is suppressed in large \( N_c \) limit is numerically rather large, above 100 MeV, on the other hand \( \Theta^+ \) decay width which scales like \( N_c^0 \), is numerically tiny, below 1 MeV. If narrow pentaquarks exist, large \( N_c \) argument is not enough to claim consistency of the model, some degree of cancelation in the decay coupling is needed. In this paper we investigate this problem for the next exotic SU(3) representation, namely 27-plet, called eikosiheptaplet.

Following the prescription of Adkins, Nappi and Witten \([4]\) (criticized recently in Ref.\([5]\)) the decay width in solitonic models is calculated in terms of a matrix element \( \mathcal{M} \) of the collective axial current operator corresponding to the emission of a pseudoscalar meson \( \varphi \):

\[
\hat{\mathcal{O}}^{(8)} = 3 \times \text{const.} \times p^i_\varphi \times \sum_{i=1}^{3} \left( a_1 D^{(8)}_{\varphi i} + a_2 d_{bac} D^{(8)}_{\varphi b} \hat{S}_c + \frac{a_3}{\sqrt{3}} D^{(8)}_{\varphi 8} \hat{S}_i \right)
\]

\(= 3 \times p^i_\varphi \times \sum_{i=1}^{3} \left( G_0 D^{(8)}_{\varphi i} - G_1 d_{bac} D^{(8)}_{\varphi b} \hat{S}_c - \frac{G_2}{\sqrt{3}} D^{(8)}_{\varphi 8} \hat{S}_i \right)\)

where in the last line of eq.\([1]\) we have displayed the operator in the form often used in the literature. Here \( D^{(8)}_{\varphi i} \) are SU(3) Wigner matrices, \( \hat{S}_i \) collective spin operator, \( p^i_\varphi \) meson momentum (for more details on the collective quantization and baryon wave functions see e.g. Ref.\([3]\)). Constants \( a_{1,2,3} \) are constructed from the so called moments of inertia that are calculable in \( \chi \) QSM \([2,3]\). A multiplicative constant has to be fixed from the generalized Goldberger-Treiman relation \([3,4]\).

The predictive power of the model independent approach for exotic baryons is, however, hampered by the fact that only one linear combination constructed from two free parameters \( a_1, a_2 \), namely

\[a_1 - \frac{1}{2} a_2,\]

elects the hyperon decay widths, whereas for the decay widths of exotic states both \( a_1 \) and \( a_2 \) are needed separately. The same problem occurs for baryonic masses \([2,3]\) where no information on the exotica can be retrieved from the regular baryon spectra alone (and similarly for magnetic moments \([11]\)).

One is therefore forced to introduce some additional assumptions to fix the remaining coefficient. In the original work of Ref.\([2]\) masses were fixed by a requirement that nucleon resonance \( N^*(1710) \) was a member of antidecuplet. Decay widths were estimated with the help of hyperon semileptonic decays and \( g_{eNN} \) used as an input with some other simplifying assumptions. A complete phenomenological analysis in this spirit can be found in Ref.\([12]\).

Another possibility to constrain the undetermined parameter is to go beyond the SU(3) symmetry limit and include higher order symmetry breaking terms \([3]\). Why going off the symmetry limit may be at all of help? The answer is very simple: the baryonic wave functions belong no longer to pure SU(3) multiplets, but contain \( m_s \) dependent admixtures of higher representations. For example a nucleon contains an admixture of antidecuplet cryptoexotic nucleonic state. As a result, the matrix el-
em of any operator (e.g. the decay operator (1)) con-
tains – apart from the leading term – exotic transi-
tions from antidecuplet to octet as a nonleading correction. By fitting the decay rates with \( m_s \) corrections one is therefore able to constrain the otherwise undetermined parameter.

The first estimate of the \( \Theta^+ \) mass in the Skyrme model has been done in this way already long ago [2]. More recently magnetic moments [11] and \( \Theta^+ \) decay width [9] have been evaluated by applying the above mentioned procedure. There all higher representations are treated as stable hadronic states, rather than as wide resonances. In particular admixture of eikosiheptaplet (27-plet) is of the order of 20-30\%.

It is therefore of importance to check whether the eikosiheptaplet may be indeed considered as a (semi) stable exotic representation. Not only can it mix with ordinary baryons, but it contains a number of exotic states that may be of interest by themselves, the isostriplet of \( \Theta \) states being the most prominent example. Since transitions to exotic representations enter through representation mixing which itself is of the order of \( m_s \), (semi) stability of eikosiheptaplet has to be valid in the leading order of perturbartive expansion in the strange quark mass. Therefore in our analysis of the decay widths we work in the chiral limit.

In chiral models all baryon representations have positive parity and spin corresponding to the isospin of states with \( Y = 1 \). For eikosiheptaplet that means that we have two distinct representations, one of spin 3/2 and the second one of spin 1/2 (i.e. \( 2\tau_{3/2} \) and \( 2\tau_{1/2} \) respectively), the latter being heavier. In this work we shall concentrate on the lightest states, namely on the isospin triplet \( \Delta_{27} \) of isospin 3/2 and on \( N_{27} \) states of isospin 1/2. These states are light (for \( 2\tau_{3/2} \) and have been looked for in various experiments. Apart from still unconfirmed reports by STAR [15] recent partial wave (PW) analysis of meson-nucleon scattering data put stringent limits on possible existence of \( \Theta_{27} \) and \( \Delta_{27} \) states [14]. These states may be incorporated into the PW analysis provided that their widths are of the order of tens keV. As we shall see \( \chi \) QSM predicts that their widths are of order of magnitude larger. Although still small on a hadronic scale, they are much too large to be accommodated by PW analysis.

Throughout this paper we shall assume that \( \Theta^+ \) exists with mass 1535 MeV and width smaller than 1 MeV. This input allows us to constrain all models parameters except \( \Sigma_{\pi N} \). If additionally we assume that \( \Sigma_{3/2} \) has mass \( \sim 1860 \) MeV, also pion-nucleon sigma term is fixed \( \Sigma_{\pi N} = 73 \) MeV.

There have been already a few calculations in the literature of the eikosiheptaplet masses and widths in chiral soliton models [16]–[23]. In this paper we use the mass formula of Ref. [16]. Generically the mass of the lowest \( I = 1 \) multiplet of \( \Theta_{27} \) states in \( 2\tau_{3/2} \) is almost degener-
ate with \( \Theta^+ \) of \( \Xi \). On the contrary, \( 2\tau_{1/2} \) is substantially heavier.

As far as widths are concerned our calculations differ in three aspects from those of Refs. [16]–[19]. Firstly, in Ref. [16] one considers only the leading \( G_0 \)-term, whereas in Refs. [17]–[19] the constant \( G_2 \) has been neglected. Indeed, \( G_2 \) (or more precisely \( a_3 \)) is small as it is directly related to the singlet axial current. Even though it is really small, it can be safely neglected only if there is no cancelation between \( G_0 \) and \( G_1 \), so that the pertinent linear combination of \( G_0 \) and \( G_1 \) is much larger than \( G_2 \) itself. In the decays of antidecuplet, \( 2\tau_{3/2} \rightarrow 8+ \) meson and \( 2\tau_{1/2} \rightarrow 10+ \) meson strong cancelations are indeed present and \( G_2 \) cannot be neglected. In this paper we use \( a_3 \) extracted from the chiral limit fits to the semileptonic hyperon decays that is definitely not consistent with zero. Secondly, we use the Goldberger-Treiman relation to fix the constant entering eq. (1), so that \( G_{0,1,2} \) depend on the decay in question, whereas in Refs. [16]–[19] they were considered as universal. Thirdly, instead of calculating the decay widths and masses for a fixed choice of model parameters we explore the residual freedom within the model and calculate the range of values, rather than only one number. Finally some calculations [13] took partially into account the effects of the symmetry breaking which is neglected in our paper.

We show that \( 2\tau_{3/2} \) is in a sense ”well behaving” having small widths to octet with most other channels kinematically suppressed. On the contrary, \( 2\tau_{1/2} \) has large decay widths to antidecuplet, with small decay widths to other channels. However, precisely in the case of \( 2\tau \rightarrow \Xi \) transition the phase space is formally suppressed in the large \( N_c \) limit. The situation reminds the decay of \( \Delta \) and \( \Theta^+ \), the first one being numerically suppressed, but formally damped in the large \( N_c \) limit with the second one being numerically small but \( \mathcal{O}(1) \) as far as \( N_c \) counting is concerned.

The paper is organized as follows. In Section 2 we give an overview of the nonrelativistic formalism to calculate the decay widths using the generalized Goldberger-Treiman relation. We fix two out of three axial constants and define model parameters. Finally we calculate the masses of antidecuplet and eikosiheptaplet. In Section 3 we express antidecuplet and decuplet amplitudes entering the decay widths through couplings \( G_{10} \) and \( G_{10} \) and the SU(3) isoscalar factors. By fixing \( \Theta^+ \) decay width to be below 1 MeV we constrain the axial coupling parameter space and give results for the decay widths of other members of antidecuplet. In Section 4 we repeat the calculations from the preceding section for eikosiheptaplets of spin 3/2 and 1/2. We perform phenomenological analysis of the pertinent decay couplings – the analogs of \( G_{10} \) and \( G_{10} \) – and calculate the decay widths. We summarize our findings in Sect. 5. Some useful group-theoretical formulae are collected in the Appendix.
II. GENERAL FORMALISM

Throughout this paper we shall use the nonrelativistic formula for the decay width \( \Gamma_B \rightarrow B' + \phi \)

\[
\Gamma_B \rightarrow B' + \phi = \frac{1}{8\pi} \frac{p_\phi}{M_{B'} M} \frac{M'}{M^2} \frac{1}{8\pi} \frac{p_\phi^3}{M M'} A^2. \tag{2}
\]

The “bar” over the amplitude squared denotes averaging over initial and summing over final spin and over isospin. Anticipating linear momentum dependence of the decay amplitude \( \mathcal{M} \)

\[
\mathcal{M}_B \rightarrow B' + \phi = \langle \mathcal{R}_{S', B'}^q, B' \rangle \hat{\mathcal{O}}_{(8)} \langle \mathcal{R}, S, B \rangle \tag{3}
\]

we have introduced reduced amplitude \( \mathcal{A} \) where the momentum of the outgoing meson

\[
p_\phi = \frac{\sqrt{(M^2 - (M' + m_\phi)^2)(M^2 - (M' - m_\phi)^2)}}{2M} \tag{4}
\]

has been factored out. Here \( \mathcal{R} \) stands for the SU(3) representation and \( S \) for spin. In Refs.\[3\] following the approach of Ref.\[25\] \( M/M' \) in eq.(2) was replaced by \( (M + M')^2/4 \) and furthermore the additional factor \( M/M' \) was inserted to sum up certain kinematical effects. We will not make such alterations in the following. Instead, we will apply the generalized Goldberger-Treiman relation that allows to relate the axial constants \( a_{1,2,3} \) to the constants \( G_{0,1,2} \) by means of the following relation \[6\]:

\[
G_0 = - \frac{M + M'}{3f_{\phi}} a_1, \quad G_{1,2} = \frac{M + M'}{3f_{\phi}} a_{2,3} \tag{5}
\]

where \( M \) and \( M' \) stand for the baryonic masses involved in the decay \( B \rightarrow B' + \phi \) and \( f_{\phi} \) denotes pseudoscalar meson decay constant in the normalization where \( f_\pi = 93 \) MeV, \( f_K = 115 \) MeV and \( f_\eta = 1.2 f_\pi \) \[20\] (we neglect \( \eta - \eta' \) mixing). The use of eq.(5) makes constants \( G_{0,1,2} \) decay dependent in contrast to previous analysis where they were considered to be universal, with possible modification of the formula for the width \[2\].

In contrast to the early exploratory works we now know for sure that if \( \Theta^+ \) exists it is light and its width is small. Therefore we use these two pieces of information to constrain the mass and the decay width of \( \Theta^+ \) for which we take \( M_\Theta = 1535 \) MeV and \( \Gamma_{\Theta \rightarrow N + K} \sim 1 \) MeV. With these parameters fixed we calculate the decay widths of decuplet, antidecuplet and ekiosheptaplet and discuss uncertainties of our results coming from the \( m_s \) corrections. In this respect we differ from Ref.\[3\] where \( m_s \) corrections were used – as explained in the Introduction – to constrain input parameters.

In order to fix the input parameters \( a_{1,2,3} \) we use a fit from Refs.\[3, 27\] where one uses two linear combinations of known hyperon decays, that in \( \chi QSM \) are free of the linear \( m_s \) corrections

\[
a_1 - \frac{1}{2} a_2 = -2.675, \quad a_3 = 0.678. \tag{6}
\]

With these parameters one obtains:

\[
g_A^{(3)} = 1.27, \quad g_A^{(8)} = 0.43, \quad g_A^{(0)} = 0.68. \tag{7}
\]

These values overshoot present experimental results, especially for \( g_A^{(0)} \) (that ranges between 0.15 – 0.35 \[28\]). It should be, however, remembered that \( g_A^{(0)} \) is sensitive to the corrections of higher order in \( m_s \) that pull it down with respect to the chiral limit estimate (see Fig.2 in \[27\]).

Let us stress that parametrization \[6\] is theoretically very appealing, because one does not need to refit leading order parameters \( a_{1,2,3} \) when \( m_s \) corrections are included. Nevertheless the overall quality of the fit is of course better when full formula with \( m_s \) corrections is used \[6\]. To check sensitivity of our results to the fitting procedure, we have also used different set of parameters (that will be called fit 2 in the following) which better fits \( g_A^{(0)} \) in the leading order:

\[
a_1 - \frac{1}{2} a_2 = -5.4, \quad a_3 = 0.3 \tag{8}
\]

which gives:

\[
g_A^{(3)} = 1.27, \quad g_A^{(8)} = 0.36, \quad g_A^{(0)} = 0.3. \tag{9}
\]

Contenting ourselves with input parameters \[6\] we can check our formalism against the hadronic data. Firstly, let us compute the pion-nucleon coupling constant \( g_{\pi NN} \) that for both fits reads

\[
g_{\pi NN} = \frac{7}{10} (G_0 + \frac{1}{2} G_1 + \frac{1}{14} G_2) = 12.8 \quad \tag{10}
\]

vs. experimental value of 13.1 – 13.3 \[12\]. Here the numerical result has been obtained by putting \( M = M' = M_N \) in eq.(5). Secondly, anticipating results of the next section, we can also quote our prediction for the decay width of \( \Delta \) obtained by means of eq.(2)

\[
\Gamma_\Delta = 104 \quad (106) \text{ MeV} \quad \tag{11}
\]

in fair agreement with experiment (the number in parenthesis refers to the parameters of eq.(8)). Note that one may improve this result by including a phenomenological factor \( M_\Delta/M_N \) \[3, 25\] that would scale eq.(11) up to 134 MeV. Also \( m_s \) corrections increase the \( \Delta \) width (in this case by 25% – 30% \[13\]).

For the decays of exotic states we have to know \( a_1 \) and \( a_2 \) separately. We therefore parameterize

\[
a_1 = \rho, \quad a_2 = 5.352 + 2 \rho, \quad a_3 = 0.68. \tag{12}
\]

It follows from the phenomenological analysis of Ref.\[9\] that the realistic range for \( \rho \) lies within \(-3 \) to \(-1.9 \). In what follows we shall fix \( \rho \) to fit the “experimental” width for \( \Theta^+ \). As will be shown in eq.(25), if we require \( \Gamma_{\Theta^+} < 1 \) MeV then \( \rho_1 = -1.98 < \rho < \rho_2 = -1.814 \).

For comparison we will also use fit 2

\[
a_1 = \rho, \quad a_2 = 5.4 + 2 \rho, \quad a_3 = 0.3 \tag{13}
\]
varying ρ within the limits ρ1 = −1.933 < ρ < ρ2 = −1.767. All numerical results in the following will be presented for fit 1 [12], modifications due the second choice of input parameters [13] will be discussed in Sect.V.

Finally, in order to use formula (2) we have to specify masses of exotic states. To this end we parameterize all exotic masses in terms of one parameter: ΣπN, i.e. the pion nucleon sigma term that we will vary within the range of 40 – 70 MeV.

In chiral quark soliton model baryon masses can be read off from the collective hamiltonian

\[ \hat{H} = M_d + \frac{1}{2I_1} S(S + 1) \]

\[ + \frac{1}{2I_2} (C_2(SU(3)) - S(S + 1) - \frac{N^2}{12}) + \hat{H}' \]

where the symmetry breaking hamiltonian takes the following form:

\[ \hat{H}' = \alpha D_{8s}^{(8)} + \beta Y + \frac{\gamma}{\sqrt{3}} D_{8i}^{(8)} \hat{S}_i. \]

Matrix elements of \( \hat{H}' \) can be found e.g. in Refs. [12, 18]. For \( M_\Theta = 1535 \) MeV the model parameters take the following values (in MeV) as functions of \( \Sigma_{\pi N} \) [12, 13]:

\[ \frac{1}{I_2} = 152.4, \quad \frac{1}{I_2} = 608.7 - 2.9 \Sigma_{\pi N}. \]

and

\[ \alpha = 336.4 - 12.9 \Sigma_{\pi N}, \]

\[ \beta = -336.4 + 4.3 \Sigma_{\pi N}, \]

\[ \gamma = -475.94 + 8.6 \Sigma_{\pi N}. \]

Numerical results for antidecuplet obtained with the help of Eqs. (1)–(17) are summarized in Table I.

| \( \Sigma_{\pi N} \) | 42 MeV | 55 MeV | 73 MeV |
|----------------|--------|--------|--------|
| \( \Theta \) | 1535   | 1535   | 1535   |
| \( N \)   | 1709   | 1681   | 1642   |
| \( \Sigma \) | 1883   | 1827   | 1750   |
| \( \Xi_{3/2} \) | 2057   | 1974   | 1857   |

Our choice for the values of \( \Sigma_{\pi N} \) in Table I is not accidental. For \( \Sigma_{\pi N} = 42 \) MeV the mass of the cryptoexotic nucleon resonance corresponds to the original choice of [32], who associated it with the known resonance \( N^* (1710) \). Almost for sure this choice is now ruled out, and this implies that the new, narrow (as we will see below) nucleon resonance needs to be yet discovered. There are several candidates for such states found both in partial wave analysis [20], \( \eta \) photoproduction on nucleon (see Ref. [30] and references therein), and at STAR [31]. Next, the value of 55 MeV corresponds to \( \Sigma_{\pi N} \) calculated within the model [32], and moreover it is the value for which one of the symmetry breaking parameters [15] \( \gamma \approx 0 \). Let us note that \( \gamma = 0 \) in the nonrelativistic limit. Finally for \( \Sigma_{\pi N} = 73 \) MeV the mass of \( \Xi_{3/2} \) corresponds to the estimate of NA49 [33]. This is also the value preferred by the recent analysis of \( \pi N \) scattering [34].

For eikosiheptaplet the masses (in MeV) are listed in the Table II.

| \( \Sigma_{\pi N} \) | 42 MeV | 55 MeV | 73 MeV |
|----------------|--------|--------|--------|
| \( \Theta \) | 1568   | 1999   | 1593   |
| \( \Delta \) | 1721   | 2213   | 1642   |
| \( N \) | 1715   | 2158   | 1721   |
| \( \Gamma \) | 1875   | 2439   | 1978   |
| \( \Sigma \) | 1866   | 2358   | 1837   |
| \( \Lambda \) | 1862   | 2318   | 1856   |
| \( \Xi_{3/2} \) | 2018   | 2558   | 2186   |
| \( \Xi \) | 2011   | 2521   | 1986   |
| \( \Omega \) | 2160   | 2677   | 2115   |

Table II deserves a few comments. The first two columns corresponding to \( \Sigma_{\pi N} = 42 \text{ MeV} \) are in agreement with the numerical values from Ref. [16] where \( N^* (1710) \) was taken as input. The last two columns corresponding to the antidecuplet masses: \( M_{\Theta^+} = 1535 \) and \( M_{\Xi_{3/2}} = 1860 \) are in agreement with Refs. [17, 18, 19]. Finally, let us observe that – as can be also seen from Fig.1 – the spin 1/2 eikosiheptaplet is squeezed for smaller values of the hypercharge making the heaviest isospin submultiplets almost degenerate. On the other hand \( \Theta_{27} \) in \( 27_{3/2} \) is only a few tens of GeV above the \( \Theta^+ \) of antidecuplet.

![FIG. 1: Spectrum of eikosiheptaplet (in GeV) of spin 3/2 (left) and spin 1/2 (right) for \( \Sigma_{\pi N} = 73 \text{ MeV} \). Note large splittings of equal hypercharge multiplets.](image-url)

TABLE II: Masses of eikosiheptaplets for different values of \( \Sigma_{\pi N} \)

| \( \Sigma_{\pi N} \) | 42 MeV | 55 MeV | 73 MeV |
|----------------|--------|--------|--------|
| \( \Theta \) | 1568   | 1999   | 1593   |
| \( \Delta \) | 1721   | 2213   | 1642   |
| \( N \) | 1715   | 2158   | 1721   |
| \( \Gamma \) | 1875   | 2439   | 1978   |
| \( \Sigma \) | 1866   | 2358   | 1837   |
| \( \Lambda \) | 1862   | 2318   | 1856   |
| \( \Xi_{3/2} \) | 2018   | 2558   | 2186   |
| \( \Xi \) | 2011   | 2521   | 1986   |
| \( \Omega \) | 2160   | 2677   | 2115   |

Note: If you need any further assistance, please let me know!
III. DECAY CONSTANTS FOR DECUPLET AND ANTIDECUPLET

The matrix elements for decuplet and antidecuplet with \( S_3 = S'_3 = 1/2 \) read:

\[
A(B_{103/2} \rightarrow B'_8 + \varphi) = 3 \begin{pmatrix} 8 & 8 \\ \varphi & B' \end{pmatrix} \frac{2}{\sqrt{15}} \times G_{10},
\]

(18)

\[
A(B_{\overline{10}1/2} \rightarrow B'_8 + \varphi) = -3 \begin{pmatrix} 8 & 8 \\ \varphi & B' \end{pmatrix} \frac{1}{\sqrt{15}} \times G_{\overline{10}}.
\]

(19)

where

\[
G_{10} = G_0 + \frac{1}{2}G_1, \quad G_{\overline{10}} = G_0 - G_1 - \frac{1}{2}G_2.
\]

(20)

In order to have an estimate of the width [22] the authors of Ref.3 calculated \( G_{\overline{10}} \) in the nonrelativistic limit of \( \chi \)QM [33] and got \( G_{\overline{10}} \equiv 0 \). It has been shown [30] that this cancellation between terms that scale differently with \( N_c \) (\( G_0 \sim N_c^{3/2}, \quad G_1,2 \sim N_c^{1/2} \)) is actually consistent with large \( N_c \) counting, since in fact

\[
G_{\overline{10}} = G_0 - \frac{N_c + 1}{4}G_1 - \frac{1}{2}G_2
\]

(21)

where the explicit \( N_c \) dependence comes from the SU(3) Clebsch-Gordan coefficients calculated for large \( N_c \) (note that for arbitrary \( N_c \) baryons are built from \( N_c \) quarks rather than from 3). In the nonrelativistic limit (NRL) \[30\]:

\[
G_0 = -(N_c + 2)G, \quad G_1 = -4G, \quad G_2 = -2G, \quad G \sim N_c^{1/2}.
\]

(22)

Similar cancelations occur also for the decays of the eikosiheptaplet [37]. From now on we will keep \( N_c = 3 \).

Following steps described in the Appendix we obtain the averaged matrix elements

\[
\overline{A}^2(B_{103/2} \rightarrow B'_8 + \varphi) = \frac{6}{5} \begin{pmatrix} 8 & 8 \\ \varphi & B' \end{pmatrix} \frac{10}{B} \times G_{10}^2,
\]

(23)

\[
\overline{A}^2(B_{\overline{10}1/2} \rightarrow B'_8 + \varphi) = \frac{3}{5} \begin{pmatrix} 8 & 8 \\ \varphi & B' \end{pmatrix} \frac{10}{B} \times G_{\overline{10}}^2.
\]

(24)

where the squares of the isoscalar factors (the quantities in the square brackets in Eqs. (23,24)) are listed in Table III.

In Fig.1 we plot scaled coupling constants \( G_{10} \) and \( G_{\overline{10}} \) (i.e. without Goldberger-Treiman factors \( (M + M')/3f_\pi \) [53] as functions of parameter \( \rho \), where \( \rho \) is given by eq. (12). As already explained in the Introduction \( G_{10} \) is constant, as the \( \rho \) dependence cancels out, while \( G_{\overline{10}} \) steeply decreases reaching zero for \( \rho_0 = -1.897 \). This is a reflection of the nonrelativistic cancelation [22] observed for the first time in Ref.3. It is obvious that by an appropriate choice of \( \rho \) in the vicinity of \( \rho_0 \) we can make \( G_{\overline{10}} \) arbitrarily small. By plugging in parameters (6,12) into (5) and (20) we get that

\[
\Gamma_{\Theta} < 1 \text{ MeV} \Rightarrow \rho_1 = -1.98 < \rho < \rho_2 = -1.814.
\]

(25)

In Table IV we list the decay widths for the remaining members of antidecuplet for \( \rho = -1.98 \) (or equivalently \(-1.814\)) for various choices of the masses from Table I parameterized by the pion-nucleon sigma term \( \Sigma_{\pi N} \):

We see from Table IV that the widths of cryptoexotic nucleon and \( \Sigma \) resonances exceed 1 MeV, the width of \( \Xi_{1/2} \) is even larger, however within the limits set by NA49. It is important to observe that the estimate from Ref.13 is almost 4 times bigger; it is difficult to comment why because the authors of Ref.13 give no details of their width calculation. One has to remember that the entries in Table IV constitute in fact the upper limits, since the widths scale as \( (\rho - \rho_0)^2 \) (with \( \rho_0 = -1.897 \)),

TABLE III: Isoscalar factors squared for the decays of decuplet and antidecuplet

| Decay                | \( \Omega \rightarrow K + \Xi \) | \( \Theta \rightarrow K + N \) |
|----------------------|----------------------------------|-------------------------------|
| \( \Xi^* \rightarrow \pi + \Xi \) | \( N \rightarrow \pi + N \) | 1/4                           |
| \( \Xi^* \rightarrow \eta + \Xi \) | \( N \rightarrow \eta + N \) | 1/4                           |
| \( \Xi^* \rightarrow K + \Lambda \) | \( N \rightarrow K + \Lambda \) | 1/4                           |
| \( \Xi^* \rightarrow K + \Sigma \) | \( N \rightarrow K + \Sigma \) | 1/4                           |

TABLE IV: Decay widths in MeV for the decays of antidecuplet

| Decay                | \( B_{\overline{10}} \rightarrow \varphi + B'_8 \) | \( \Gamma_{B_{\overline{10}} \rightarrow \varphi + B'_8} \) [MeV] |
|----------------------|----------------------------------|-------------------------------|
| \( \Theta \rightarrow K + N \) | 0.95 0.95 0.95 | 3.25                         |
| \( N \rightarrow \pi + N \) | 4.18 3.77 3.25 | 0.99 0.80 0.56               |
| \( N \rightarrow \eta + N \) | 0.24 0.14 0.04 | 0.02 0.01 0.01               |
| \( N \rightarrow K + \Lambda \) | 1.95 1.53 1.04 | 4.40 3.57 2.59               |
| \( N \rightarrow K + \Sigma \) | 2.24 1.77 1.22 | 0.54 0.25 0.01               |
| \( \Xi_{1/2} \rightarrow \pi + \Xi \) | 8.41 6.01 3.44 | 1.01 0.60 0.10               |
| \( \Xi_{1/2} \rightarrow K + \Sigma \) | 4.52 2.89 1.20 | 4.18 3.77 3.25               |
and can be arbitrarily decreased with an appropriate choice of $\rho$. In the situation when the leading contributions are small, $m_s$ corrections become important, that issue has been studied in Ref. [13].

**IV. DECAY CONSTANTS FOR EIKOSIHEPTAPLET**

In this Section we shall consider decays of eikosiheptaplet (27) that can have either spin 1/2 or 3/2, the latter being lighter. Matrix elements for the decays of eikosiheptaplet of $S = 3/2$ (and with $S_3 = 1/2$) read:

$$A(B_{27_{3/2}} \rightarrow B'_0 + \varphi) = 3 \left( \begin{array}{c} 8 \cr 8 \end{array} \bigg| \begin{array}{c} 27 \cr B' \end{array} \right) \frac{2\sqrt{2}}{9} \times G_{27},$$

$$A(B_{27_{3/2}} \rightarrow B'_{10} + \varphi) = -3 \left( \begin{array}{c} 8 \cr 10 \end{array} \bigg| \begin{array}{c} 27 \cr B' \end{array} \right) \frac{\sqrt{10}}{36} \times F_{27},$$

$$A(B_{27_{3/2}} \rightarrow B'_{10} + \varphi) = 3 \left( \begin{array}{c} 8 \cr 10 \end{array} \bigg| \begin{array}{c} 27 \cr B' \end{array} \right) \frac{\sqrt{30}}{9} \times E_{27},$$

where

$$G_{27} = G_0 - \frac{1}{2} G_1,$$

$$F_{27} = G_0 - \frac{1}{2} G_1 - \frac{3}{2} G_2,$$

$$E_{27} = G_0 + G_1.$$

(26)

For $S = 1/2$ and $S_3 = 1/2$ we have:

$$A(B_{27_{1/2}} \rightarrow B'_0 + \varphi) = -3 \left( \begin{array}{c} 8 \cr 8 \end{array} \bigg| \begin{array}{c} 27 \cr B' \end{array} \right) \frac{\sqrt{10}}{45} \times H_{27},$$

$$A(B_{27_{1/2}} \rightarrow B'_{10} + \varphi) = -3 \left( \begin{array}{c} 8 \cr 10 \end{array} \bigg| \begin{array}{c} 27 \cr B' \end{array} \right) \frac{\sqrt{5}}{9} \times G'_{27},$$

$$A(B_{27_{1/2}} \rightarrow B'_{10} + \varphi) = 3 \left( \begin{array}{c} 8 \cr 10 \end{array} \bigg| \begin{array}{c} 27 \cr B' \end{array} \right) \frac{7\sqrt{2}}{36} \times H'_{27},$$

where

$$H_{27} = G_0 - 2G_1 + \frac{3}{2} G_2,$$

$$G'_{27} = G_0 - 2G_1,$$

$$H'_{27} = G_0 + \frac{11}{14} G_1 + \frac{3}{14} G_2.$$

(28)

In Fig. 2 we plot scaled coupling constants (i.e. without Goldberger-Treiman factors $(M + M')/3f_\pi$ [32]) for decays of $27_{3/2}$ and $27_{1/2}$ together with $G_{10}$ and $G'_{10}$ (solid lines) as functions of parameter $\rho$, where $\rho$ is given by eq. (12). Together with aforementioned suppression of $G'_{10}$ we see strong suppression of $F_{27}$ (corresponding to $27_{3/2} \rightarrow 10_{3/2} + \varphi$) and $H_{27}$ (corresponding to $27_{1/2} \rightarrow 8_{1/2} + \varphi$) for the same range of $\rho$. Interestingly, both $F_{27}$ and $H_{27}$ vanish [37] in the nonrelativistic limit exactly as $G'_{10}$. In our parametrization they cross zero for the parameter $\rho$ in the range (25). Somewhat smaller suppression is seen for spin changing transitions $G_{27}$ (corresponding to $27_{3/2} \rightarrow 8_{1/2} + \varphi$) and $G'_{27}$ (corresponding to $27_{1/2} \rightarrow 10_{3/2} + \varphi$). Interestingly, in the nonrelativistic limit there is a partial cancelation in these couplings, namely the leading $N_c$ coefficients cancel out [37]. Finally, the remaining couplings $E_{27}$ (corresponding to $27_{3/2} \rightarrow 10_{1/2} + \varphi$) and $H'_{27}$ (corresponding to $27_{1/2} \rightarrow 10_{1/2} + \varphi$) are not suppressed (they are neither suppressed in the nonrelativistic limit). However, decays to antidecuplet have much smaller phase space, and they are totally switched off for $27_{3/2}$. It is remarkable that our simple phenomenological parametrization [61,12] respects – for the $\rho$ values of interest [28] – the large $N_c$ suppression in the nonrelativistic limit.

Averaging over spin and isospin, as described in the
TABLE V: Isoscalar factors squared for the decays of eikosiheptaplet

| Decay          | Isoscalar Factors Squared |
|----------------|---------------------------|
| $27 \rightarrow 8 + 8$ | $C^2$                     |
| $27 \rightarrow 8 + 10$ | $C^2$                     |
| $27 \rightarrow 8 + 12$ | $C^2$                     |
| $\Theta \rightarrow K+N$ | $1$                        |
| $\Theta \rightarrow K+\Delta$ | $1$                        |
| $N \rightarrow \eta+N$ | $9/20$                    |
| $N \rightarrow \pi+N$ | $1/20$                    |
| $N \rightarrow K+\Sigma$ | $1/20$                    |
| $N \rightarrow K+\Lambda$ | $9/20$                    |
| $\Delta \rightarrow \eta+\Delta$ | $9/16$                    |
| $\Delta \rightarrow \pi+\Delta$ | $5/16$                    |
| $\Delta \rightarrow K+\Sigma$ | $1/8$                     |
| $\Delta \rightarrow K+\Lambda$ | $1/4$                     |

Appendix gives:

$$\langle B_{27a/2} \rightarrow B'_{8} + \varphi \rangle = \frac{4}{9} \left[ \begin{array}{c} 8 \\ 8 \\ B' \end{array} \right] \left[ \begin{array}{c} 8 \\ 8 \\ B \end{array} \right] \times G_{27}^2,$$

$$\langle B_{273/2} \rightarrow B'_{10} + \varphi \rangle = \frac{25}{72} \left[ \begin{array}{c} 8 \\ 10 \\ B' \end{array} \right] \left[ \begin{array}{c} 8 \\ 10 \\ B \end{array} \right] \times F_{27}^2,$$

$$\langle B_{275/2} \rightarrow B'_{10} + \varphi \rangle = \frac{5}{3} \left[ \begin{array}{c} 8 \\ 10 \\ B' \end{array} \right] \left[ \begin{array}{c} 8 \\ 10 \\ B \end{array} \right] \times G_{27}^2,$$

(30)

where the quantities in the square brackets denote SU(3) isoscalar factors. For $27_{1/2}$ we get

$$\langle B_{271/2} \rightarrow B'_{8} + \varphi \rangle = \frac{2}{45} \left[ \begin{array}{c} 8 \\ 8 \\ B' \end{array} \right] \left[ \begin{array}{c} 8 \\ 8 \\ B \end{array} \right] \times H_{27}^2,$$

$$\langle B_{271/2} \rightarrow B'_{10} + \varphi \rangle = \frac{2}{9} \left[ \begin{array}{c} 8 \\ 10 \\ B' \end{array} \right] \left[ \begin{array}{c} 8 \\ 10 \\ B \end{array} \right] \times G_{27}^2,$$

$$\langle B_{271/2} \rightarrow B'_{10} + \varphi \rangle = \frac{49}{72} \left[ \begin{array}{c} 8 \\ 10 \\ B' \end{array} \right] \left[ \begin{array}{c} 8 \\ 10 \\ B \end{array} \right] \times H_{27}^2.$$

(31)

The squares of the relevant SU(3) isoscalar factors are listed in Table V.

Now we are ready to calculate the decay widths for $27_{3/2}$. In fact only decays to the octet baryons have non-vanishing widths, we list them in the Table VI ($^n$ denotes the decay width below 1 MeV, whereas ”$^m$” means that the decay is kinematically forbidden).

Decays of $27_{3/2}$ to decuplet are kinematically forbidden except of the decays $N_{27} \rightarrow \pi + \Delta$ and $\Delta_{27} \rightarrow \pi + \Delta$ which have widths smaller than 1 MeV. All decays to antidecuplet are kinematically forbidden. We can therefore conclude that eikosiheptaplet of spin 3/2 has widths small enough to justify the rigid rotor quantization. Not only are the widths numerically smaller than the one of $\Delta$, but also in the large $N_c$ limit with the partial nonrelativistic cancelation taking place, $\Gamma_{27_{3/2}} \rightarrow 8 + 8 \rightarrow 0$.

Our results for $\Theta_{27}$ presented in Table VI are smaller than the estimate of Ref. [18]. Although the widths of the order of tens of MeV can be considered small, one has to remember that partial wave analysis requires $\Delta_{27}$ and $\Theta_{27}$ widths to be of the order of 100 keV [14].

We have concentrated here on the lightest states of eikosiheptaplet that have been looked for in PW analysis [14]. Obviously, we can easily calculate widths for the plethora of the remaining states of eikosiheptaplet. We have checked that for other states widths are smaller than the one of $\Delta_{27}$ quoted above. Assuming $\Sigma_{27} = 73$ MeV we get the following upper bounds for the partial widths of the next isospin multiplets

$$\Gamma_{\Lambda \rightarrow \eta + \Lambda} \sim 42 \text{ MeV},$$
$$\Gamma_{\Sigma \rightarrow \pi + \Lambda} \sim 75 \text{ MeV},$$
$$\Gamma_{\Xi_{1/2} \rightarrow K + \Lambda} \sim 68 \text{ MeV},$$
$$\Gamma_{\Xi_{3/2} \rightarrow \pi + \Xi} \sim 74 \text{ MeV}.$$  

(32)

For $27_{1/2}$ we expect larger widths because the available phase-space is much larger. Interestingly, this is not the case for the decays to octet. The reason is that $H_{27}$ responsible for these decays is strongly suppressed in the relevant range of $\rho$. Indeed, $H_{27}$ crosses zero at $\rho = -1.937$ i.e. within the range [25]. Moreover, the overall group theoretical factor in eq. (31) is suppressed by factor of 13 with respect to the decays of antidecuplet [24]. These two suppressions overcome the increase of
TABLE VIII: Decay widths in MeV for the decays of $27_{1/2}$ to decuplet.

| $27_{1/2} \rightarrow 8 + 10$ | $\Gamma$ [MeV] | $\Gamma$ [MeV] |
|-----------------------------|--------------|--------------|
| $\rho$                      | $\rho_1$     | $\rho_2$     |
| $\Sigma_{11N}$             | 42 55 73     | 42 55 73     |
| $\Theta_{27} \rightarrow K + \Delta$ | 21 17 12 | 86 69 48 |
| $N_{27} \rightarrow \pi + \Delta$ | 24 19 14 | 97 78 56 |
| $N_{27} \rightarrow K + \Sigma^*$ | 18 11 4   | 75 45 16    |
| $\Delta_{27} \rightarrow \eta + \Delta$ | 30 24 18 | 126 103 74 |
| $\Delta_{27} \rightarrow \pi + \Delta$ | 43 37 31 | 178 155 127 |
| $\Delta_{27} \rightarrow K + \Sigma^*$ | 4 3 2    | 16 12 8     |

TABLE IX: Decay widths in MeV for the decays of $27_{1/2}$ to antidecuplet.

| $27_{1/2} \rightarrow 8 + 10$ | $\Gamma$ [MeV] | $\Gamma$ [MeV] |
|-----------------------------|--------------|--------------|
| $\rho$                      | $\rho_1$     | $\rho_2$     |
| $\Sigma_{11N}$             | 42 55 73     | 42 55 73     |
| $\Theta_{27} \rightarrow \pi + \Theta_{10}$ | 658 523 365 | 697 554 387 |
| $\Theta_{27} \rightarrow K + \Theta_{10}$ | – – –     | – – –        |
| $N_{27} \rightarrow \eta + \Theta_{10}$ | – – –    | – – –        |
| $N_{27} \rightarrow \pi + \Theta_{10}$ | 500 364 215 | 530 385 228 |
| $N_{27} \rightarrow K + \Sigma_{10}$ | – – –     | – – –        |
| $N_{27} \rightarrow K + \Theta_{10}$ | 72 20 –    | 76 21 –      |
| $\Delta_{27} \rightarrow \pi + \Theta_{10}$ | 579 510 424 | 614 541 449 |
| $\Delta_{27} \rightarrow K + \Sigma_{10}$ | – – –     | – – –        |

the phase-space volume and the decay widths are comparable to those of $\Theta_{10}/_{1/2}$. Similar effect takes place for the decays to decuplet (although the decay constant $G_{27}$ does not cross zero in the relevant range of $\rho$) and the decays are comparable to those of $27_{3/2} \rightarrow 8 + 8$.

Unfortunately there is no suppression for the decays of $27_{1/2}$ to antidecuplet. Indeed, the relevant coupling $H_{27}$ is as large as $G_{10}$ (responsible for $\Delta$ decay) – see Fig. 2 - and the phase space is also not suppressed: for $\Theta_{27} \rightarrow \pi + \Theta_{10}$ the pion momentum is of the order of 300 \(\div\) 400 MeV depending on $\Sigma_{11N}$. Hence the resulting widths are large.

Therefore one would be tempted to conclude that $27_{1/2}$ cannot be considered as a semi stable multiplet and its description in terms of the rigid rotor fails, at least in the situations where the transitions $27_{1/2} \rightarrow \Theta_{10}$ are of importance. This statement is however not supported by the $N_c$ counting \cite{37}. We shall come back to this issue in the next Section.

V. SUMMARY

In the present paper we have studied masses and decay widths of exotic baryon eikosiheptaplets (i.e. $27_{\text{plets}}$) of spin $3/2$ and $1/2$ that follow from the chiral quark-soliton model in the rigid rotator quantization approach.

We have also reexamined widely studied by now antidecuplet that we use as an input that constrains model parameters. Rigid rotator quantization predicts a tower of stable exotic representations of different spins and positive parity, antidecuplet, eikosiheptaplet, \cite{37} being most prominent examples. Question arises, where does the rigid rotator approach break? Leaving aside fundamental problems based on claims in the literature that the rigid rotator approach to exotica is not compatible with large $N_c$ expansion for QCD \cite{38}, we have taken more modest phenomenological approach. If the widths of the baryonic states calculated within the model exceed certain critical value, that can be taken to be above the $\Delta$ resonance width (one has to remember that $\Delta$ can be considered as a well behaved stable state in the large $N_c$ limit), then the model becomes inconsistent. There are two sources that contribute to the increase of the width with the increase of the dimensionality of the SU(3) flavor representation. One is obvious: for higher representations the pertinent states are heavier and the phase space is larger. The second source is the coupling. For antidecuplet there is only one coupling corresponding to the transition $\Theta_{10} \rightarrow 8$, that is excessively small due to the cancelation found in Ref.\cite{6} and discussed in some detail in Sect. 3. For higher representations there are more couplings corresponding to different transitions and some of them are not suppressed. For eikosiheptaplet couplings to antidecuplet are not suppressed. Obviously if the phase space is large and the coupling is not suppressed then the widths are large. In other cases one has to perform explicit calculations to see what is the interplay between the rising phase space and small coupling.

We have addressed this question by applying the so called model-independent approach \cite{4} in which the general group theoretical structure is taken from the model, while the parameters are fitted to appropriate data.

We have used as an input nonexotic masses and the mass of $\Theta^+$, semi-leptonic decay constants and the assumption that $\Gamma_{\Theta^+} < 1$ MeV. The residual freedom was parameterized by the value of the pion-nucleon sigma term.

We have confined our analysis to eikosiheptaplet (i.e. $27_{\text{-plet}}$) that is the only exotic representation (apart from antidecuplet) appearing in the direct product of two octets. For this reason eikosiheptaplet might be produced in meson-nucleus scattering and could subsequently decay to meson-nucleon or meson-hyperon final states.

Our findings can be shortly summarized as follows. Based on group theory alone, eikosiheptaplet can decay into octet, decuplet and exotic antidecuplet. However, for $27_{3/2}$ regular octet is kinematically the only allowed channel (with two exceptions discussed in Sect. 4). Furthermore, transition $27 \rightarrow 8$ is governed by a small decay coupling, $G_{27}$. Therefore eikosiheptaplet of spin $3/2$ has widths of the order of a few tens MeV with one exception, namely $\Delta_{27}$ for which $\Gamma \approx 70 \div 170$ MeV. For spin $1/2$ the situation is different. Decays to octet and decuplet have small transition couplings and the resulting widths are small (see Tables \cite{VII} and \cite{VIII}). For the decays to
antidecuplet the coupling is large. Therefore whenever the
decay is possible the widths are of the order of 500
MeV. This might be interpreted as the signal that the
model breaks down and that the assumption that 27_{1/2}
is stable cannot be justified phenomenologically.
The situation is, however, more complicated. Since the
mass difference
\[ \Delta_{27_{1/2} → \pi} = \frac{1}{T} \sim O(1/N_c) \]  
(33)
as calculated from eq. \[ \text{(13)} \] is suppressed in the large \( N_c \)
limit, so is the meson momentum \( \langle 1 \rangle \). Therefore the
widths that depend on the third power of momentum
may be suppressed in the large \( N_c \) limit despite the fact
that they are numerically large. That this is indeed the
case was shown in Ref. \[ \text{(37)} \] where \( e.g. \)
\[ \Gamma_{0}^{27_{1/2} → 7} \sim O(1/N_c^2). \]  
(34)
On the contrary, for the transitions of 27_{1/2} to octet
which are numerically suppressed (remember that the
pertinent coupling \( H_{27}^{(2)} \) vanishes in the NR limit
\[ \text{(25)} \]) the \( N_c \) scaling is different, \( e.g. \) \[ \text{(37)} \]:
\[ \Gamma_{0}^{27_{1/2} → K + N} \sim O(1). \]  
(35)
Obviously numerical results presented in Sect. IV
depend on the choice of input parameters. We have studied
this sensitivity by employing another set of parameters
\[ \text{(13)} \] that corresponds to more realistic \( g_A^{(0)} \). The
decay widths of antidecuplet do not change, since we re-
quire that \( \Gamma_{0}^{27_{1/2} → \pi} < 1 \) (which is equivalent to slightly dif-
f erent range of the parameter \( \rho : \rho_1 = -1.933 < \rho < \rho_2 = -1.767 \)) and this condition fixes all remaining decay
widths. For eikosiheptaplet some differences appear. For
the transitions of 27_{3/2} to octet the decay widths for fit
2 \[ \text{(13)} \] are smaller by a few MeV. More drastic changes
appear for 27_{1/2}. The reason is that the small change in
the coupling is magnified by a large phase space factor.
Indeed, for the decays to octet, presented in Table VII,
the decay widths for fit 2 \[ \text{(13)} \] are larger by a factor 10 \div 6
(first number refers to \( \rho = \rho_1 \) whereas the second one to
\( \rho = \rho_2 \) for fit 2). Although this enhancement seems large,
the absolute values are still small on a typical hadronic
scale. Less drastic enhancement occurs for the decays to
decuplet presented in Table VIII, the decay widths for fit 2
are larger by a factor 2 \div 1.4. Finally, large decay
widths to antidecuplet remain almost the same as for the
fit 1 \[ \text{(12)} \]. We see therefore that despite some numerical
 uncertainties due to the choice of input parameters
the general pattern persists and our conclusions still hold.
Summarizing: there is no simple way to judge the quali-
ity of the rigid rotator approach to the eikosiheptaplet.
On the basis of phenomenology alone one would conclude
that 27_{1/2} is unstable because of the large numerical val-
ues of the decay widths to antidecuplet. On the other
hand precisely these decays are damped in the large \( N_c \)
limit, similarly to the decays of \( \Delta \) resonance. Other de-
cays, like the decays to octet scale as \( O(N_c^0) \) but their
numerical values are group theoretical factors. For eiko-
siheptaplet of spin 3/2 all kinematically allowed decays
have widths small enough to justify the rigid rotor quan-
tization. Not only are the widths numerically smaller
than the one of \( \Delta \), but also in the large \( N_c \) limit partial
nonrelativistic cancelation takes place and the pertinent
couplings are suppressed.

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APPENDIX A: SUMMING OVER SPINS AND
ISOSPINS
We shall use the identity
\[ \frac{1}{2S + 1} \sum_{S_3} \left( \begin{array}{c|c} 1 & S' \\ m' & S_3' \end{array} \right) \left( \begin{array}{c|c} 1 & S' \\ m & S_3 \end{array} \right) = \frac{1}{3} \delta_{m m'} \]  
(A1)
to average over the initial spin (and in the same time
to sum over the final spin). For spin 3/2 the amplitude
for 10_{3/2} \rightarrow 8_{1/2} + \varphi and 27_{3/2} \rightarrow 8_{1/2}, \overline{T}0_{1/2} + \varphi
is proportional to
\[ \left( \begin{array}{c|c} 1 & 3/2 \\ 0 & -1/2 \end{array} \right) \frac{3/2}{-1/2} = \sqrt{\frac{2}{3}} \]  
(A2)
hence
\[ \frac{1}{2S + 1} \sum_{S_3} |A(3/2 \rightarrow 1/2)|^2 = \frac{1}{2} |A(3/2 \rightarrow 1/2)|^2. \]  
(A3)
For 27_{3/2} \rightarrow 10_{3/2} + \varphi the amplitude is proportional to
\[ \left( \begin{array}{c|c} 1 & 3/2 \\ 0 & -1/2 \end{array} \right) \frac{3/2}{-1/2} = \sqrt{\frac{1}{15}} \]  
(A4)
and
\[ \frac{1}{2S + 1} \sum_{S_3} |A(3/2 \rightarrow 3/2)|^2 = 5 |A(3/2 \rightarrow 3/2)|^2. \]  
(A5)
Finally, for \( \overline{T}0_{1/2} \rightarrow 8_{1/2} + \varphi \) and 27_{1/2} \rightarrow 8_{1/2}, \overline{T}0_{1/2} + \varphi
the amplitude is proportional to
\[ \left( \begin{array}{c|c} 1 & 1/2 \\ 0 & -1/2 \end{array} \right) \frac{1/2}{-1/2} = \sqrt{\frac{1}{3}} \]  
(A6)
and for $27_{1/2} \rightarrow 10_{3/2} + \phi$ to

$$\begin{pmatrix} 1 \\ 3/2 \\ 0 \\ -1/2 \\ 1/2 \\ -1/2 \end{pmatrix} = -\sqrt{3/2}.$$  \hspace{1cm} (A7)

Hence

$$\frac{1}{2S+1} \sum_{\mathcal{S}_3} |A(1/2 \rightarrow 1/2, 3/2)|^2 = |A(1/2 \rightarrow 1/2, 3/2)|^2.$$  \hspace{1cm} (A8)

Similarly we shall average over initial isospin and sum over the final isospin using the formula

$$\frac{1}{2I+1} \sum_{I'_{\phi' \mathcal{S}'_3} I_{\mathcal{S}_3}} \begin{pmatrix} I_{\phi'} \\ I_{\mathcal{S}'_3} \end{pmatrix} = 1.$$  \hspace{1cm} (A9)

[1] L.C. Biedenharn and Y. Dothan, *Monopolar Harmonics in SU(3)*, as eigenstates of the Skyrme-Witten model for baryons, E. Gotsman and G. Tauber (eds.), *From SU(3) to gravity*, p. 15-34; L.C. Biedenharn, Y. Dothan and A. Stern, Phys. Lett. B 146, 289 (1984).

[2] M. Praszalowicz, talk at *Workshop on Skyrmions and Anomalies*, M. Ježabek and M. Praszalowicz eds., World Scientific 1987, page 112 and Phys. Lett. B 575 (2003) 234 [hep-ph/0308114].

[3] D. Diakonov, V. Petrov and M. V. Polyakov, Z. Phys. A 359 (1997) 305 [arXiv:hep-ph/9703373].

[4] G.S. Adkins, C.R. Nappi and E. Witten, Nucl. Phys. B 228 (1983) 552.

[5] H. Weigel, [arXiv:hep-ph/0703072].

[6] A. Blotz, D. Diakonov, K. Goeke, N.W. Park, V. Petrov and P.V. Pobylitsa, Nucl. Phys. A 555, 765 (1993).

[7] H.C. Kim, M. Praszalowicz and K. Goeke, Phys. Rev. D 61, 114006 (2000) [arXiv:hep-ph/9910282].

[8] M. Wakahatsu and T. Watabe, Phys. Lett. B 312, 184 (1993).

[9] G.S. Yang, H.C. Kim and K. Goeke, Phys. Rev. D 75, 094004 (2007) [arXiv:hep-ph/0701168].

[10] G.S. Adkins and C.R. Nappi, Nucl. Phys. B 249, 507 (1985).

[11] G.S. Yang, H.C. Kim, M. Praszalowicz and K. Goeke, Phys. Rev. D 70, 114002 (2004) [arXiv:hep-ph/0410042].

[12] J.R. Ellis, M. Karliner and M. Praszalowicz, JHEP 0405, 002 (2004) [arXiv:hep-ph/0404127].

[13] M. Praszalowicz, Acta Phys. Polon. B 35, 1625 (2004) [arXiv:hep-ph/0402038].

[14] Y.I. Azimov, R.A. Arndt, L.L. Strakovsky, R.L. Workman and K. Goeke, Eur. Phys. J. A 26 (2005) 79 [arXiv:hep-ph/0504022].

[15] J. Ma, APS meeting, 5 Jan. 2004. S. Kabana, RHIC and AGS user’s meeting, BNL, 10-15 July 2004.

[16] D. Borisuyk, M. Faber and A. Kobushkin, [arXiv:hep-ph/0307370].

[17] B. Wu and B. Q. Ma, Phys. Rev. D 69, 077501 (2004) [arXiv:hep-ph/0312041].

[18] D. Borisuyk, M. Faber and A. Kobushkin, Ukr. J. Phys. 49, 944 (2004) [arXiv:hep-ph/0312213].

[19] B. Wu and B. Q. Ma, Phys. Lett. B 586, 62 (2004) [arXiv:hep-ph/0312326].

[20] H. Weigel, Eur. Phys. J. A 21, 133 (2004) [arXiv:hep-ph/0404173].

[21] G. Duplancic, H. Pasagic and J. T rampetic, JHEP 0407, 027 (2004) [arXiv:hep-ph/0405029].

[22] Q. Zhou and B. Q. Ma, Eur. Phys. J. A 28, 345 (2006) [arXiv:hep-ph/0606282].
T.D. Cohen, Phys. Rev. D 70, 014011 (2004)