Classification of Different Branes at Angles

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Abstract

In this paper, we consider two D-branes rotated with respect to each other, and argue that in this way one can find brane configurations preserving $\frac{1}{16}$ of SUSY. Also we classify different brane configurations preserving $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{16}$, $\frac{1}{8}$, $\frac{1}{16}$ of SUSY.
1. Introduction

Branes and their different configurations have played crucial role in developing dualities of string theories and M-theory. These branes are BPS states which preserve half of the available supersymmetry [1]. The possible bound state of D-branes with F(undamental)-strings or with themselves or NS_{5}-branes [2-4] have been of special interest in testing duality conjectures in string theory or low energy effective field theories.

The stability of these bound states is supported by no-force condition induced by the supersymmetry preserved by that special configuration of branes. Due to the no-force conditions these branes form marginally bound states. These marginal bounds have been widely studied through solutions of low energy effective SUGRA theories in 10 and 11 dimensions and also by SUSY algebra arguments [5-13].

The non-marginal bound states of branes can also be constructed. In string theory these are bound states of D_{p}-brane with F-string [3] or D_{5}-branes with NS_{5}-branes [2,3] and in M-theory, they are as bounds states of M_{2} and M_{5}-branes [14]. All the non-marginal cases are believed to be BPS excited states of D_{p}-branes or NS_{5}-branes or M_{5}-branes respectively [15]. These states as discussed in [3,14] preserve $\frac{1}{2}$ of the SUSY charges present, like individual D_{p}-branes or NS_{5}-branes or M_{5}-branes.

In Ref.[5] p-branes making two or three angles have been studied and shown that in a special configuration with two angles, i.e. when two angles are equal, $\frac{1}{8}$ of SUSY survives. A more general argument on M_{5}-branes at angles is given by Townsend [16], where configurations of branes preserving $\frac{3}{16}$ and also branes preserving $\frac{1}{8}$ of SUSY have been studied through SUSY algebra arguments.

In this paper I find a new family of D-branes at angles which preserve $\frac{1}{16}$ of SUSY. These branes are rotated with respect to each other at 4-angles. By means of different dualities we can go to M-theory level at this level we argue that two M_{5}-branes rotated at 4 angles can preserve $\frac{1}{16}$ of SUSY. Finally we classify all the brane configurations with various fractions of SUSY. This article is organized as following:

In section 2, we introduce two $D_{4}$-branes making 4 angles, in string theory . These branes are rotated with respect to each other by rotations of $U(4)$ of SO(8) rotation group and by means of open strings stretched between such branes, calculate their interactions. Vanishing conditions of this amplitude which is a sign of remaining some SUSY, is studied and some special cases which reproduces the previous works is discussed.

In section 3, I consider other different D-branes ($(p,p')$-branes) at angles and again study non-interacting configurations.
In section 4, the results of sections 2, 3 are checked with SUSY algebra arguments. In this way we can generalize our string theoretic results to NS\(_5\)-branes too, and also by means of dualities regenerate results of Ref. [16] and find the \(\frac{1}{16}\) configuration in M-theory.

At last in section 5, We classify these results and gather them in a table which is organized in fraction of preserved SUSY: \(\frac{1}{2}, \frac{1}{4}, \frac{3}{16}, \frac{1}{8}, \frac{1}{16}\).

### 2. D-branes at angles in string theory

Consider two \(D_p\)-branes, world volume of one of them spans 01...\(p\) space and the second is rotated with respect to it. For \((p<5)\) in the most general case they can be described by \(p\) angles \((\theta_1, \theta_2, ..., \theta_p)\) \((0 \leq \theta_i \leq \pi)\), which show the Abelian rotations in \(U(p)\) which is a subspace of \(SO(2p)\) rotation group, i.e. \(\theta_i\) \((1 \leq i \leq p)\) shows rotation in \((i, p+i)\) plane.

These branes can be introduced by the following rotated boundary conditions on open strings ending on each brane [17]:

\[
\begin{align*}
\sigma = 0 \quad & \begin{cases} 
X^\mu = 0 & \mu = p + 1, ..., 9 \\
\partial_\sigma X^\mu = 0 & \mu = 0, ..., p 
\end{cases} \\
\sigma = \pi \quad & \begin{cases} 
X^\mu = Y^\mu & \mu = 2p + 1, ..., 9 \\
\partial_\sigma X^\mu = 0 & \mu = 0 \\
X^i \sin \theta_i + X^{(p+i)} \cos \theta_i = 0 \\
\partial_\sigma X^i \cos \theta_i - \partial_\sigma X^{(p+i)} \sin \theta_i = 0 & i = 1, ..., p.
\end{cases}
\end{align*}
\]

So for two \(D_2\)-branes 2 angles, for \(D_3\)-branes 3 angles and for \(D_4\)-branes 4 angles are the most general cases. For \((p \geq 5)\) the most general case is described by \((9 - p)\) angles and again all the arguments of \(p\)-branes holds for \((9 - p)\), more precisely the most general case is governed by \(\min(p, 9 - p)\) angles.

It is easy to see that the case which is described by only two (or three) angles (for arbitrary \(p\) greater or smaller than 4) is obtained from 4 angles case by putting two (or one) of them to zero, so we will focus on the 4 angles case.

Along the calculations given in [17] one can find the mode expansion of the \(X^\mu\) and also the world sheet fermions \(\psi^\mu_\pm\) for NS and R sectors. To calculate the corresponding interaction between two branes we should find amplitude of one closed string exchange, this amplitude
can be expressed by one loop vacuum amplitude of open strings stretched between branes:

\[ A = 2 \int \frac{dt}{2t} \text{Tr}e^{-tH}, \]  

(3)

where H is the open string Hamiltonian. The Tr should be performed on momentum modes and oscillator modes, which are allowed with proper GSO projection. The amplitude then is calculated to be

\[ A(\theta_i) = 2 \int \frac{dt}{2t} (8\pi^2 \alpha')^{-1/2} e^{-\frac{v^2}{2\pi\alpha'}} (\text{NS} - \text{R}), \]  

(4)

where NS, R are given by

\[
\text{R} = \prod_{j=1}^{4} \frac{\Theta_2(i\theta_j t \mid it)}{\Theta_1(i\theta_j t \mid it)} - \prod_{j=1}^{4} \frac{\Theta_1(i\theta_j t \mid it)}{\Theta_1(i\theta_j t \mid it)},
\]

(5)

\[
\text{NS} = \prod_{j=1}^{4} \frac{\Theta_3(i\theta_j t \mid it)}{\Theta_1(i\theta_j t \mid it)} - \prod_{j=1}^{4} \frac{\Theta_4(i\theta_j t \mid it)}{\Theta_1(i\theta_j t \mid it)}
\]

(6)

As it is proved in Appendix, the following identity between \( \Theta \)-functions:

\[
\prod_{j=1}^{4} \frac{\Theta_2(i\theta_j t \mid it)}{\Theta_1(i\theta_j t \mid it)} - \prod_{j=1}^{4} \frac{\Theta_1(i\theta_j t \mid it)}{\Theta_1(i\theta_j t \mid it)} = \prod_{j=1}^{4} \frac{\Theta_2(i\theta_j t \mid it)}{\Theta_1(i\theta_j t \mid it)} - \prod_{j=1}^{4} \frac{\Theta_3(i\theta_j t \mid it)}{\Theta_1(i\theta_j t \mid it)}
\]

(7)

holds provided:

\[ \theta_1 + \theta_2 + \theta_3 + \theta_4 = 0 \quad 0r \quad \theta_1 \pm \theta_2 = \theta_3 \pm \theta_4 \quad 0r \quad \theta_1 + \theta_3 = \theta_2 + \theta_4. \]  

(8)

Hence the amplitude vanishes to all orders of \( t \) when (8) is satisfied, i.e. in the case that branes make 4 angles related by (8), a certain fraction of SUSY must remain. We will argue in section 4 that, this configuration in general (for an arbitrary choice of angles \( \theta_i \) upto the condition (8 ) ) preserves \( \frac{1}{16} \) of SUSY, which has been sought for [16].

The small \( t \) limit of the integrand of (4), which shows the contribution of the massless closed strings exchanged between branes, reads to be:

\[
A(\theta_i) = V_0[\sum_{i=1}^{4} \cos 2\theta_i - 4 \prod_{i=1}^{4} \cos \theta_i + 4 \prod_{i=1}^{4} \sin \theta_i],
\]

(9)

where \( V_0 \) is proportional to \( T_\mu^2 = (4\pi^2 \alpha')^{3-p} \). The first term in the bracket is graviton and dilaton contribution and rest is RR contributions. Now let us consider some important special cases which reproduces the interaction configuration with less number of angles associated with the families with more SUSY.

\[ ^1 \text{These conditions are upto an integer factor of } 2\pi. \]
special cases

a) \( \theta_1 = \theta_2 = \theta_3 = \theta_4 = 0 \), \( A_0(0) = 0 \), which recovers results of [1].

b) \( \theta_2 = \theta_3 = \theta_4 = 0 \), \( A_1(\theta_1) = 2V_0(1 - \cos \theta_1)^2 \), in which the RR contribution is proportional to \(-\cos \theta_1\), and graviton and dilaton proportional to \((1 + \cos^2 \theta_1)\). This is what discussed in detail in [17].

c) \( \theta_3 = \theta_4 = 0 \), \( A_2(\theta_1, \theta_2) = 2V_0(\cos \theta_2 - \cos \theta_1)^2 \).

d) \( \theta_4 = 0 \), \( A_3(\theta_1, \theta_2, \theta_3) = 2V_0[\cos(\theta_1 + \theta_2) - \cos \theta_3][\cos(\theta_1 - \theta_2) - \cos \theta_3] \).

e) \( \theta_1 = \theta_2 = \theta_3 = \theta_4 = \pi/2 \), \( A_4(\pi/2) = 0 \).

f) \( \theta_2 = \theta_3 = \theta_4 = \pi/2 \), \( A_5(\theta_1) = 2V_0(1 - \sin \theta_1)^2 \).

g) \( \theta_3 = \theta_4 = \pi/2 \), \( A_6(\theta_1, \theta_2) = 2V_0(\sin \theta_2 - \sin \theta_1)^2 \).

h) \( \theta_4 = \pi/2 \), \( A_7(\theta_1, \theta_2, \theta_3) = 2V_0[\cos(\theta_1 + \theta_2) - \sin \theta_3][\cos(\theta_1 - \theta_2) - \sin \theta_3] \).

As we see \( A_1, A_5 \) vanish only for \( \theta_1 = 0 \) or \( \pi/2 \) respectively, so we have not a stable brane configuration related by only one independent rotation of \( U(4) \). \( A_2, A_6 \) vanish for \( \theta_1 = \theta_2 \) and \( \theta_1 = -\theta_2 \) (for \( A_2 \)) and \( \theta_1 = \pi - \theta_2 \) (for \( A_6 \)). These cases make a one parameter family of stable branes. \( A_3, A_7 \) vanish for \( \theta_1 \pm \theta_2 = \pm \theta_3 \) and \( \theta_1 \pm \theta_2 = \pi/2 \pm \theta_3 \) respectively. They describe a two parameter family of stable branes. All the above families are special cases of three parameter families given by (8).

3. \((p,p')\) branes at angles

Untill now we have only considered two \( D_p \)-branes case. In this section, we show that \((p,p')\) branes making angles can be studied as special case of the general 4 angles case.

Let us denote \( p - p' = \Delta \) \((p > p')\); parallel \((p,p')\) branes can be introduced by the following boundary conditions:

\[
\begin{align*}
\sigma &= 0 \quad \left\{ \begin{array}{ll}
X^\mu &= 0 \quad \mu = p' + 1, \ldots, 9 \\
\partial_\sigma X^\mu &= 0 \quad \mu = 0, \ldots, p' 
\end{array} \right. \\
\sigma &= \pi \quad \left\{ \begin{array}{ll}
X^\mu &= Y^\mu \quad \mu = p + 1, \ldots, 9 \\
\partial_\sigma X^\mu &= 0 \quad \mu = 0, \ldots, p
\end{array} \right.
\end{align*}
\]

which for \( \Delta = 2 \) is exactly the boundary conditions of (1),(2) for \( \theta_1 = \pi/2, \theta_2 = \theta_3 = \theta_4 = 0, \ldots, p \).
generally $\Delta = 2n$, is equivalent to

$$
\begin{cases}
\theta_i = \pi/2 & i \leq n \\
\theta_i = 0 & \text{otherwise}.
\end{cases}
$$

So all the results of $(p, p')$ branes at angles can be obtained from the 4 angle calculations. More precisely their amplitude are different only in the tension coefficient which is $T_p T_{p'}$ for $(p, p')$ and $T_p^2$ for $(p, p)$ case. Hence the most general case of rotated $\Delta = 2$ amplitude is proportional to $A_7$, for $\Delta = 4, 6$ proportional to $A_6, A_5$ respectively, in the spcial cases discussed above. For $\Delta = 8$ case the amplitude vanishes which is described by $A_4$ case.

4. SUSY arguments

In this section first we briefly review the SUSY constraints:

As we know any p-brane which its world volume spans 012..$p$ preserves $\frac{1}{2}$ of 32 super charges of type II or M-theory. For D-branes of type II they are associated with constraints

$$
\Gamma_{01..p} \epsilon_L = \epsilon_R, 
$$

where $\epsilon_L, \epsilon_R$ are parameters of left and right moving spinors of type II theories.

Any rotated $D_p$-brane must satisfy the condition

$$
(R \Gamma_{01..p} R^{-1}) \epsilon_L = \epsilon_R. 
$$

So each $\epsilon_L$ satisfied both (13), (14) gives the fraction of SUSY which is preserved. In our case, which $U(p)$ rotations of $SO(2p)$ is considered

$$
R \Gamma_{01..p} R^{-1} = R^2 \Gamma_{01..p}. 
$$

Then (14) can be replaced by

$$
R^2 \epsilon = \epsilon. 
$$

For 4 angle rotations described in section 2 ($((U(1))^4$ members of $U(4)$) the rotation $R^2$ is written as

$$
R^2 = exp(\sum_{i=1}^{4} \theta_i \Gamma_{i,i+4}) = \prod_{i=1}^{4} (\cos \theta_i + \Gamma_{i,i+4} \sin \theta_i).
$$

$^2$ metric is $(- + ....++)$ and $\Gamma_0 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ and $\Gamma_i = \begin{pmatrix} 0 & \gamma_i \\ \gamma_i & 0 \end{pmatrix}$, where $\gamma_i$ are real symmetric matrices.
Carrying out the calculations, we find that \( (R^2 - 1) \) has zero eigen-values if and only if
\[
\Gamma_{i,i} \Gamma_{j,j} + 4 = \epsilon, \quad i \neq j,
\]
which yields the conditions of (8) on \( \theta_i \) angles, required for the vanishing of the interaction. The number of conserved super charges then can be obtained from (18) equations, in the most general from these equations can be explicitly written as
\[
\begin{align*}
A\epsilon &= \epsilon, \quad A = \Gamma_{1526} \\
B\epsilon &= \epsilon, \quad B = \Gamma_{1537} \\
C\epsilon &= \epsilon, \quad C = \Gamma_{1548}.
\end{align*}
\]
Since the matrices \( A, B, C \) commute with each other, are traceless and their square is equal to 1; and also as \( AB, AC \) are traceless, the above equations will be satisfied simultaneously only for one \( \epsilon \) out of 16.

Hence in general the 4 angle contribution provided that (8) holds, preserves \( \frac{1}{16} \) of SUSY. There are some special cases such as \( \theta_1 = \theta_2, \theta_3 = \theta_4 \) which allows more SUSY, i.e. \( \frac{1}{8} \).

Although we have done our calculations for D-branes in string theory, the results on angles can be extended to the other kinds of branes (\( NS_5 \) or \( M_5^- \) branes) through SUSY arguments, i.e. two \( NS_5 \) or \( M_5^- \)-branes at angles satisfying (8) preserves \( \frac{1}{16} \) of SUSY.

Besides SUSY algebraic arguments, which are true for NS or M five branes, dualities can also relate configurations containing \( NS_5 \) or \( M_5^- \)-branes to those of D-branes and hence our D-branes results are generalized to these cases. Corresponding dualities are T-duality of IIA, IIB theories. Under this duality a stable system of D-branes goes to another stable configuration which has the same number of super charges. If we T-dualize a brane in an arbitrary direction \( D_p \)-brane goes to bound state of F-string with a \( D_{p+1} \)-brane [3], which is a non-marginal bound state and preserves the same SUSY fraction as \( D_p \)-brane does, i.e. \( \frac{1}{2} \).

So we can find configurations of two branes in which each brane is a bound state of F-string and D-brane. The same argument holds for Hodge duals( electric, magnetic duals) of the above configurations where every bound state of F-string and \( D_p \)-brane goes to \( NS_5 \) and \( D_{6-p} \)-brane bound state.

In type IIB theory there are also \( (m,n) \) string or five brane bound states [2] which preserves \( \frac{1}{2} \) of SUSY (Such branes are non-marginal bound states).

The M-theory branes \( (M_2,M_5^-) \)-branes) can be understood as U-duals of IIA branes. This duality like, T-duality of IIA and IIB preserves number of conserved super charges, so one
expects to find non-marginal bound states of $M_2$, $M_5$-branes (duals of $D_4$-brane, F-string bound state) with $\frac{1}{2}$ of SUSY [15]. Other $M_5$-brane configurations are duals of $D_4$-brane configurations we have considered here.

5. Summery

In this section, we summerize the results obtained partly by others (mainly in [5,16]) and partly in this article ordered in the fraction of SUSY. Unless it is mentioned explicitly, the results are true for D-branes, $NS_5$-branes and $M_5$-branes.

\[i) \frac{1}{2} \text{ Supersymmetry}\]

Parallel branes or non-marginal bound states of ($D_p$-branes, F-strings) and ($NS_5$-branes, D-branes) and ($M_5$-branes and $M_2$-branes). Also KK-monopoles of type II or M-theory on $S^1$ [18].

\[ii) \frac{1}{4} \text{ Supersymmetry}\]

In angles arguments:

- $\theta_1 = \theta_2 = \theta_3 = 0$, $\theta_4 = \pi/2$.  
- $\Delta = 2$, $\theta_1 = \theta_2 = 0$, $\theta_3 = \pi/2$.  
- $\Delta = 4$, $\theta_1 = \theta_2 = 0$.  
- $\Delta = 6$, $\theta_1 = \pi/2$.  
- $\Delta = 8$.

- $NS_5$, $D_4$: $\theta_1 = \theta_2 = \theta_3 = 0$, $\theta_4 = \pi/2$ in type IIA [19].
- $NS_5$, $D_3$: $\theta_1 = \theta_2 = 0$, $\theta_3 = \pi/2$ in type IIB [20].
- $NS_5$, $D_2$: $\theta_1 = 0$, $\theta_2 = \pi/2$ in type IIA and
- $M_5$, $M_2$: $\theta_1 = 0$, $\theta_2 = \pi/2$ in M-theory level [21].
- $NS_5$, $D_1$: $\theta_1 = \pi/2$ in type IIB [22].

\[iii) \frac{3}{16} \text{ Supersymmetry}\]

- $\theta_1 = \theta_2 = \theta_3 = \theta_4 \neq \pi/2$. 

8
iv) $\frac{1}{8}$ Supersymmetry

\[
\begin{align*}
\theta_1 \pm \theta_2 &= \pm \theta_3, \theta_4 = 0. & \theta_1 = \theta_2 \neq 0, \theta_3 = \theta_4 \neq 0. \\
\Delta &= 2, \theta_1 = \theta_2 \neq 0, \theta_3 = \pi/2. & \Delta = 2, \theta_1 = \pi - \theta_2, \theta_3 = \pi/2. \\
\Delta &= 4, \theta_1 = -\theta_2. & \Delta = 4, \theta_1 = \pi - \theta_2.
\end{align*}
\]

v) $\frac{1}{16}$ Supersymmetry

\[
\begin{align*}
\theta_1 + \theta_2 + \theta_3 + \theta_4 &= 0. & \theta_1 + \theta_2 = \theta_3 + \theta_4. \\
\Delta &= 2, \theta_1 + \theta_2 + \theta_3 + \pi/2 = 0. & \Delta = 2, \theta_1 + \pi/2 = \theta_2 + \theta_3.
\end{align*}
\]

Appendix:

New $\Theta$-function identity

Here we present a proof for the following identity between Jacobi $\Theta$-functions:

\[
\prod_{j=1}^{4} \Theta_3(\nu_j | \tau) - \prod_{j=1}^{4} \Theta_4(\nu_j | \tau) = \prod_{j=1}^{4} \Theta_2(\nu_j | \tau) - \prod_{j=1}^{4} \Theta_1(\nu_j | \tau)
\]

provided that $\sum_i \nu_i = 0$ or sum of any two of them is equal two others.

To prove we denote that: 1) Each $\Theta$-function is doubly periodic with respect to $\nu_i$.

2) It has certain zeros and poles.

As it is mentioned in [23,24] in order to prove the identity it is enough to check both sides first, to be doubly periodic (with periods to 1, $\tau$) with respect to $\nu_1$, second, to have same zeros and poles, then the ratio of both sides is a constant and the value of this constant is determined by using special values of $\nu_i$ (e.g. $\nu_i = 0$). So we check these conditions:

1) periodicity:

Let

\[
\Theta_a(\nu_1 | \tau) \Theta_a(\nu_2 | \tau) \Theta_a(\nu_3 | \tau) \Theta_a(\nu_1 + \nu_2 + \nu_3 | \tau) \equiv F_a
\]

Under $\nu \to \nu + 1$, $F_a$ is not changed for $a = 1, 2, 3, 4$.

Under $\nu \to \nu + \tau$, $F_a$ goes to $A^4(\nu) F_a$, where $A(\nu) = e^{-i\pi(2\nu+\tau)}$.

Hence the identity is doubly periodic with periods $(1, \tau)$.

2) Zeros and Poles:

The poles can be checked in small $i\tau$ limit ($q = e^{-i\pi \tau} \approx 1$). This limit could be studied by modular transformation on large $i\tau$ limit ($q \approx 0$). The $\nu$ dependence of this limit is
proportional to the four angle amplitude (9) which is true when the corresponding relation between \( \nu_i \) holds.

The zeros can be checked as following. Let us consider the identity as:

\[
F_2 + F_4 - F_1 = F_3.
\]

We check that for every root of the right hand side, left hand side vanishes. If we also check that for any root of \( F_2 \) the identity holds, then we can be sure about a one to one correspondence between zeros. As we know \( \Theta_3 \) vanishes at \( \nu = m + \frac{1}{2} + (n + \frac{1}{2})\tau \ m, n \in Z \) [23]. Replacing this in left hand side and using half period behaviour of \( \Theta \)-functions we find that left hand side vanishes provided that the following identity holds:

\[
\Theta_2(0 \mid \tau)\Theta_2(\nu_1 \mid \tau)\Theta_2(\nu_2 \mid \tau)\Theta_2(\nu_1 + \nu_2 \mid \tau) = \\
\Theta_3(0 \mid \tau)\Theta_3(\nu_1 \mid \tau)\Theta_3(\nu_2 \mid \tau)\Theta_3(\nu_1 + \nu_2 \mid \tau) - \Theta_4(0 \mid \tau)\Theta_4(\nu_1 \mid \tau)\Theta_4(\nu_2 \mid \tau)\Theta_4(\nu_1 + \nu_2 \mid \tau).
\]

which is given in [24] ( The above identity can also be proved by the same method we presented here: 1) checking the periodicity with periods \((1, \tau)\). 2) Proving the one to one correspondence between zeros and poles.). So the ratio of the both sides is constant, if we put all \( \nu_i \) to be zero we find that this constant is one. QED.

Acknowledgement:
Author would like to thank H. Arfaei for fruitful discussions.

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