More on the matter of 6D SCFTs

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A R T I C L E   I N F O

Article history:
Received 14 May 2015
Accepted 18 May 2015
Available online 21 May 2015
Editor: L. Alvarez-Gaumé

A B S T R A C T

M5-branes probing an ADE singularity lead to 6D SCFTs with (1, 0) supersymmetry. On the tensor branch, the M5-branes specify domain walls of a 7D super Yang–Mills theory with gauge group \( G \) of ADE-type, thus providing conformal matter for a broad class of generalized quiver theories. Additionally, these theories have \( G \times G \) flavor symmetry, and a corresponding Higgs branch. In this Note we use the F-theory realization of these theories to calculate the scaling dimension of the operator parameterizing sevenbrane recombination, i.e. motion of the stack of M5-branes off of the orbifold singularity. In all cases with an interacting fixed point, we find that this operator has scaling dimension at least six, and defines a marginal irrelevant deformation.

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1. Introduction

An open question in the study of 6D superconformal field theories (SCFTs) is to determine the spectrum of operators and their scaling dimensions. In recent work, a number of new 6D SCFTs have been constructed by compactifying F-theory on non-compact singular elliptically fibered Calabi–Yau threefolds. This has already led to a classification of (1, 0) theories without a Higgs branch [1], and has also been extended to specific theories with a Higgs branch [2] (see also [3]). For earlier work on the construction of (1, 0) theories see e.g. [4–12]. The F-theory realization in particular provides a powerful way to characterize many aspects of these theories.

In this Note we extract the scaling dimension of certain operators in the \( T(\mathcal{G}, \mathcal{N}) \) theories of Ref. [2]. These theories are given by a stack of \( N \) M5-branes probing an ADE singularity \( C^2 / \Gamma_C \) with \( \Gamma_C \) a finite ADE subgroup of SU(2), separating the M5-branes along the line transverse to this singularity, we see that each M5-brane specifies a domain wall in the 7D super Yang–Mills theory defined by the ADE singularity [2]. Here, the flavor symmetry of the system is \( \mathcal{G}_L \times \mathcal{G}_R \), with both factors isomorphic to \( \mathcal{G} \), the corresponding ADE-type Lie group.

Our aim will be to extract the dimension of the operator parameterizing motion of the M5-branes off of the orbifold singularity. In the F-theory realization of these SCFTs, this corresponds to a brane recombination operation, and we shall refer to it as such. We use the holomorphic geometry of F-theory to extract data about this branch of the theory.

2. Collisions and scaling dimensions

We consider the collision of two singularities in an F-theory compactification, each supporting an ADE-type gauge group \( \mathcal{G} \). At the intersection, we have a superconformal matter sector. We would like to know the scaling dimension of the operator associated with Higgsing the flavor symmetry by brane recombination. So, to begin, we consider the local geometries for such collisions:

\[
\begin{align*}
(E_8, E_8) : y^2 &= x^3 + (uv)^5 \\
(E_7, E_7) : y^2 &= x^3 + (uv)^3 x \\
(E_6, E_6) : y^2 &= x^3 + (uv)^4 \\
(D_p, D_p) : y^2 &= (uv)x^2 + (uv)^p - 1 \\
(A_k, A_k) : y^2 &= x^2 + (uv)^{k+1},
\end{align*}
\]

where \( u \) and \( v \) are local coordinates of the base. In all cases but the A-type, the collision leads to a singular geometry requiring further blowups in the base. Performing such a blowup, we get at least one additional exceptional curve, which can be wrapped by a D3-brane. When this curve collapses to zero size, we get a tensionless string.

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http://dx.doi.org/10.1016/j.physletb.2015.05.046
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Now, this string is a BPS object, so its tension is given by the exact formula:
\[ \text{tension} = \int \frac{J}{\Sigma} \] (6)
with \( J \) the Kähler form of the Calabi–Yau. On the other hand, the Calabi–Yau geometry tells us that the scaling of the holomorphic three-form \( \Omega \) is related to the Kähler form via \( J \wedge J \wedge J \sim \Omega \wedge \Omega \). So, we learn that the holomorphic three-form scales with mass as:
\[ [\Omega] \sim \text{tension}^{3/2} \sim \text{mass}^3. \] (7)

Our plan will be to use this to extract operator scaling dimensions for our system.

The argument we will give is well-known in the context of lower-dimensional systems, and has been used to extract the scaling dimension of the Coulomb branch parameter for \( \mathcal{N} = 2 \) SCFTs in four dimensions, as in Ref. [13]. In four dimensions, the homogeneity argument continues to work for \( \mathcal{N} = 1 \) systems in four dimensions which have a Coulomb branch [14], but the absolute scaling must be determined via a-maximization [15]. Here, the novelty is that the Calabi–Yau geometry informs us of the Higgs branch, and that we are extending this analysis to higher-dimensional field theories.

As just mentioned, the next step in our analysis will involve relating the scaling dimension of the holomorphic three-form back to the scaling dimension of operators in the SCFT. To this end, let us recall that the brane recombination operation is controlled by activating vevs for operators. In the M-theory description, this corresponds to moving the M5-branes off the orbifold singularity. This breaks the \( G_1 \times G_R \) global symmetry down to the diagonal subgroup \( G_{\text{diag}} \). In all the cases above, this amounts to the substitution:
\[ u v \mapsto u v + r. \] (8)

One should view the parameter \( r \) as the vev of a singlet of \( G_{\text{diag}} \) which is built from an operator \( O_{\text{rec}} \) of the original CFT. It is natural to expect that just as in the weakly coupled setting, \( O_{\text{rec}} \) transforms in the \( \text{adj}_G \otimes \text{adj}_G \) representation of \( G_1 \times G_R \). However, determining this representation is not crucial for our present considerations; All that matters is that the decomposition of \( O_{\text{rec}} \) into irreducible representations of \( G_{\text{diag}} \) contains a singlet.

Further support for this picture comes from the BPS equations of motion for the flavor branes, which are controlled by the Hitchin system coupled to defects [16]:
\[ F + \left[ \Phi, \Phi^\dagger \right] = \mu_{\mathbb{R}} \delta_p \quad \text{and} \quad \bar{J}_A \Phi = \mu_C \delta_p, \] (9)
where \( p \) denotes the collision of \( u = 0 \) and \( v = 0 \) in the base. Vevs of operators in the CFT translate to moment maps in the Hitchin system, which in turn translate to complex structure deformations. This in turn leads to deformations such as the brane recombination operation of line (8).

Now, in the configuration of F-theory collisions, it follows from the symmetry of the system that the coordinates \( u \) and \( v \) have the same scaling dimension. Further, we see that \( r \), and thus \( O_{\text{rec}} \) has twice the scaling dimension of \( u \).

But this can be determined directly from the geometry! To see how to extract this, observe that the holomorphic three-form is given by:
\[ \Omega = \frac{dx}{y} \wedge du \wedge dv. \] (10)
So, once we fix the relative scaling dimensions for the coordinates of the threefold, the absolute scaling of \( \Omega \) will allow us to extract the scaling dimension of the brane recombination operators.

Let us now proceed to the various collisions. Consider first the geometries:
\[ y^2 = x^3 + (uv)^k \] (11)
for \( k = 3, 4, 5 \), which respectively cover \( D_4, E_6 \) and \( E_8 \). Homogeneity yields the scaling relations:
\[ y \sim x^3, \quad x \sim L^2, \quad r \sim (uv)^\frac{k}{6}, \] (12)
for some scaling parameter \( L \). Relating this back to the scaling of the holomorphic three-form, we find:
\[ (E_8, E_6) : \dim r = 18 \] (13)
\[ (E_6, E_8) : \dim r = 9 \] (14)
\[ (D_4, D_4) : \dim r = 6. \] (15)

By a similar token, for the cases:
\[ y^2 = x^3 + (uv)^l x \] (16)
we learn, for \( l = 3 \) (i.e. \( E_7 \)) and \( l = 2 \) (i.e. \( D_4 \)):
\[ (E_7, E_7) : \dim r = 12 \] (17)
\[ (D_4, D_4) : \dim r = 6, \] (18)
and in the case of colliding D-type singularities, we get:
\[ (D_6, D_6) : \dim r = 6. \] (19)

Finally, in the case of colliding A-type singularities, our method is not really valid. The reason is that the fiber at the collision is still in Kodaira–Tate form, so there is nothing to blow up. This means we have no physical string coming from a D3-brane wrapped on a collapsing \( \mathbb{P}^1 \), and consequently, no way to fix the absolute scaling of the holomorphic three-form. Indeed, there is not even an interacting fixed point in this case.

Next, we proceed to all of the \( T(G, N) \) theories. This is obtained by starting with the collision of two \( G \)-type singularities and performing a further quotient by \( (u, v) \mapsto (\zeta u, \zeta^{-1} v) \) for \( \zeta = \exp(2\pi i/N) \). This leads to an additional singularity at the point \( u = v = 0 \) in the base of the F-theory geometry. Now, the important point for us is that the coordinates \( u \) and \( v \) are no longer valid in the quotient geometry, but \( u^N \) and \( v^N \) are. This means that a full brane recombination amounts to the substitution:
\[ uv \mapsto (uv)^N + r(N), \] (20)
where \( r(N) \) is the vev of our new brane recombination operator. By homogeneity, a similar scaling analysis now reveals:
\[ \dim r(N) = N \cdot \dim r(N_{1}). \] (21)

Another way to see this same scaling behavior is to consider a resolution of the \( C^2/Z_N \) singularity. When we do this, we partially move onto the tensor branch of the theory, reaching our generalized quiver theory with \( (G, G) \) conformal matter between each symmetry factor. The singularity resolves to \( N - 1 \) compact \( \mathbb{P}^1 \)'s, each of which supports a seven-brane with gauge group \( G \). So for each such curve (and the non-compact ones as well), introduce
homogeneous coordinates \([u_i, v_i]\) for \(i = 1, \ldots, N + 1\). The patch \(u_i = 1\) indicates the north pole, and \(v_i = 1\) indicates the south pole. Here, \(i = 1\) (resp. \(N + 1\)) denotes the curve for \(G_L\) (resp. \(G_R\)). For example, in the case of \(E_8\) singularities, each local collision will look like:

\[
y^2 = x^3 + (u_1v_{i+1})^5
\]

(22)

for \(i = 1, \ldots, N\). The local recombination operation for each pair is:

\[
u_1v_{i+1} \mapsto u_1v_{i+1} + r_{i,i+1}.
\]

(23)

Performing one such recombination corresponds to moving that particular M5-brane off of the ADE singularity. In the M-theory picture, the number of domain walls in the 7D SYM theory defined by the ADE singularity goes down by one, and in the F-theory picture the number of compact \(\mathbb{P}^1\)'s goes down by one. The recombination vev of equation (20) is now given by the product \(r_{(N)} \sim r_{1,2} \cdots r_{N,N+1}\). Based on this, it is tempting to view the aggregate recombination operator as the composite \(\mathcal{O}_{(N)} = \mathcal{O}_{1,2} \cdots \mathcal{O}_{N,N+1}\), in the obvious notation. Note that for \(N > 1\), the operator \(\mathcal{O}_{1,N+1}\) is not gauge invariant, though its Casimirs will be.

Summarizing, the scaling dimension of the brane recombination operator is given by:

\[
\begin{array}{cccc}
\text{(E}_8, \text{E}_8) & (\text{E}_7, \text{E}_7) & (\text{E}_6, \text{E}_6) & (\text{D}_p, \text{D}_p) & (\text{A}_k, \text{A}_k) \\
\text{dim} \gamma_{(N)} & 18N & 12N & 9N & 6N & 3N \\
\end{array}
\]

(24)

Here, \(N \geq 1\) for all entries but the A-type case, where \(N \geq 2\). Indeed, as we already mentioned, we need at least one collapsing \(\mathbb{P}^1\) to apply our scaling argument.

Interestingly, in all cases where our analysis applies, we expect to have an interacting conformal fixed point in which the scaling dimension of this operator is at least six. Moreover, we see that when it is exactly six (as can occur in the A- and D-series), a perturbation of the SCFT by a would-be marginal operator breaks some flavor symmetries. From a generalization of the argument in Ref. [17], we learn that such deformations are marginal irrelevant.

Acknowledgements

J.J.H. thanks M. Del Zotto, A. Tomasiello, D.R. Morrison and C. Vafa for helpful discussions and collaboration on related work. J.J.H. also thanks M. Del Zotto, T. Dumitrescu, A. Tomasiello, and C. Vafa for comments on an earlier draft. J.J.H. thanks the organizers of the workshop "Frontiers in String Phenomenology" for kind hospitality at Schloss Ringberg during the completion of this work. The work of J.J.H. was supported in part by NSF grant PHY-1067976.

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Corrigendum

Corrigendum to “More on the matter of 6D SCFTs” [Phys. Lett. B 747 (2015) 73–75]

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ARTICLE INFO

Article history:
Available online 17 August 2020
Editor: Editor: L. Alvarez-Gaumé

The original version of this paper actually performs a computation in a 5D Kaluza–Klein (KK) regulated theory. This result must then be lifted back to six dimensions to extract the corresponding scaling dimensions in 6D SCFTs. This means that when a dimension is stated, it is for the 5D KK theory and must be multiplied by 4/3 to obtain the value in the 6D SCFT.

The author would like to apologize for any inconvenience caused.