Two-body $Z'$ decays in the minimal 331 model

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The two-body decays of the extra neutral boson $Z_2$ predicted by the minimal 331 model are analyzed. At the three-level it can decay into standard model particles as well as exotic quarks and the new gauge bosons predicted by the model. The decays into a lepton pair are strongly suppressed, with $\text{Br}(Z_2 \rightarrow l^+l^-) \approx 10^{-2}$ and $\text{Br}(Z_2 \rightarrow \nu\bar{\nu}) \approx 10^{-3}$. In the bosonic sector, $Z_2$ would decay mainly into a pair of bilepton gauge bosons, with a branching ratio below the 0.1 level. The $Z_2$ boson has thus a leptonphobic and bileptophobic nature and it would decay dominantly into quark pairs. The anomaly-induced decays $Z_2 \rightarrow Z_{1\gamma}$ and $Z_2 \rightarrow Z_1Z_1$, which occurs at the one-loop level are studied. It is found that $\text{Br}(Z_2 \rightarrow Z_{1\gamma}) \approx 10^{-9}$ and $\text{Br}(Z_2 \rightarrow Z_1Z_1) \approx 10^{-9}$ at most. As for the $Z_2 \rightarrow W^+W^-$ and $Z_2 \rightarrow Z_1H$ decays, with $H$ a relatively light Higgs boson, they are induced via $Z' - Z$ mixing. It is obtained that $\text{Br}(Z_2 \rightarrow W^+W^-) \approx 10^{-2}$ and $\text{Br}(Z_2 \rightarrow Z_1H) \approx 10^{-3}$. We also examine the flavor changing neutral current decays $Z_2 \rightarrow tc$ and $Z_2 \rightarrow tu$, which may have branching fractions as large as $10^{-9}$ and $10^{-5}$, respectively, and thus may be of phenomenological interest.

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I. INTRODUCTION

Extra neutral gauge bosons can arise when the standard model (SM) group is extended with additional gauge symmetries or embedded into a larger gauge group. The phenomenology of this particle has been the subject of significant interest in the literature [1]. Current experimental data have been used to get lower bounds on its mass and the $Z' - Z$ mixing angle $\theta$. Though it is not possible to obtain model-independent bounds, current limits from collider [2] and precision [3] experiments imply that $m_{Z_2} \gtrsim 500$ GeV and $\sin \theta \lesssim 10^{-3}$. However, this situation may be dramatically different in some models with nonuniversal flavor gauge interactions, such as topcolor assisted technicolor [4], noncommuting extended technicolor [5], or the ununified standard model [6]. In this context, it was found that the extra $Z_2$ boson must be heavier than 1 TeV [7]. Such a gauge boson may be detected at the Tevatron Run-2. Furthermore, it is expected that the next generation of colliders will be able to produce a $Z_2$ boson with a mass up to 5 TeV [8]. It has also been argued [9] that hints of deviations in atomic parity violation [10] and the NuTeV experiment [11] can be better explained if the SM is enhanced with an additional $Z_2$ boson.

In this work we are interested in studying some phenomenological aspects of the extra $Z_2$ boson predicted by the minimal 331 model [12, 13], which is based on the $SU_C(3) \times SU_L(3) \times U_X(1)$ gauge group and has some interesting features [14], such as the possibility of yielding signals of new physics at the TeV scale. In particular, the effects of this model on $R_b$ [15] and $t\bar{t}$ production at hadron colliders [16] were analyzed recently. Our main goal is to investigate the two-body decay channels of the $Z_2$ boson into fermions and gauge bosons. Particular emphasis will be given to the rare anomaly-induced decay modes $Z_2 \rightarrow Z_{1\gamma}$ and $Z_2 \rightarrow Z_1Z_2$, where $Z_1$ stands for the SM $Z$ gauge boson.

In the 331 model the lepton spectrum is the same as in the SM, but it is arranged in antitriplets of the gauge symmetry $SU_L(3)$. The quark sector is also arranged in the fundamental representation of this group, which requires the introduction of three new quarks. An interesting feature of the model is that anomalies cancel out when all of the generations are summed over, which means that the family number must be a multiple of the color number, which suggest a possible approach to solve the generation replication problem. In this context, the rare $Z_2 \rightarrow Z_{1\gamma}$ and $Z_2 \rightarrow Z_1Z_1$ decays are a unique prediction of the model since they are well defined only if the theory is anomaly free. Although the decay of a $Z$ boson into two photons or gluons is the subject of interest in the context of noncommutative

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quantum field theories \cite{17}, it is worth mentioning that this class of decays are forbidden in standard field theories by angular momentum conservation and Bose statistics (Landau-Yang theorem \cite{18}).

Along with the \(Z_2\) boson, four additional charged gauge bosons are predicted by the minimal 331 model: two singly charged bosons \(Y^\pm\) and two doubly charged ones \(Y^{\pm\pm}\). These gauge bosons carry two units of lepton number and so have been classified as bileptons \cite{19}. The new gauge bosons together with the exotic quarks are endowed with mass at the first stage of spontaneous symmetry breaking (SSB), when \(SU_L(3) \times U_X(1)\) is broken into \(SU_L(2) \times U_Y(1)\) \cite{20,21}. Since \(SU_L(2)\) is completely embedded into \(SU_L(3)\), the couplings between the SM and the extra gauge bosons are determined by the coupling constant \(g\) associated with the \(SU_L(2)\) group and the weak angle \(\theta_W\) \cite{21,22}. Very interestingly, the mass of the new gauge bosons are bounded from above due to a theoretical constraint which yields \(\sin^2 \theta_W = s_{\theta_W}^2 \leq 1/4\) \cite{18,22}. The fact that the value of \(s_{\theta_W}^2\) is very close to \(1/4\) at the \(m_{Z_2}\) scale leads to an upper bound on the scale associated with the first stage of SSB, which translates directly into a bound on the \(Z_2\) mass \cite{18,22}. It was found that the condition \(s_{\theta_W}^2 (m_{Z_2}) \leq 1/4\) can be translated into the upper bound \(m_{Z_2} \leq 3.1\) TeV \cite{23}. Taking into account this bound and the SSB hierarchy determined by the minimal Higgs sector of the model, it was found that the bilepton masses cannot be heavier than \(m_{Z_2}/2\) \cite{23}, thereby allowing the \(Z_2 \to Y^\pm Y^{\mp}\), and \(Z_2 \to Y^{\pm\pm} Y^{\pm\mp}\) decay modes. The latter is very interesting due to its distinctive signature \(Y^{\pm\pm} \to 2l^\pm\).

The upper limit imposed on \(s_{\theta_W}^2\) also has some interesting dynamical consequences. It turns out that the couplings of the \(Z_2\) boson to a lepton pair are given by the \(X\) and \(T^8\) generators of \(U_X(1)\) and \(SU_L(3)\), respectively. \(X\) is multiplied by the relatively large factor \(1/\sqrt{1 - 4s_{\theta_W}^2}\), and \(T^8\) by its inverse. The key point is that the coupling of \(Z_2\) to a lepton pair is determined exclusively by \(T^8\) because the leptons are in an \(X = 0\) representation. This is to be contrasted with the couplings to quark pairs, which are dominated by a term proportional to \(1/(1 - 4s_{\theta_W}^2)^2\) as they have a quantum number \(X \neq 0\). Consequently, the extra \(Z\) boson of this model is leptophobic \cite{18}. It is important to stress that a similar situation arises in the case of the couplings to bileptons as they do not carry quantum number \(X\). As will be seen below, these couplings turn out to be proportional to \(\sqrt{1 - 4s_{\theta_W}^2}\), and thus the \(Z_2\) boson is also bileptophobic. It is clear then that the leptonic and bileptonic decay modes of the \(Z_2\) boson are expected to be rather suppressed as compared to the quark pair decays. The fact that the \(X\) quantum number of the third-generation quarks differs from that of the first two generations leads to flavor changing neutral current (FCNC) effects mediated by the \(Z_2\) boson. Since the \(Z_2 ftq^c (q' = u, c)\) couplings are generated at the tree level, it is worth discussing these rare FCNC transitions of the \(Z_2\) boson.

Another decay mode which is of phenomenological interest is \(Z_2 \to W^- W^+\). Though the \(Z_2\) boson cannot couple directly to SM bosonic particles, the \(Z_2 \to W^- W^+\) decay can be induced at the tree level through the \(Z' - Z\) mixing. A similar situation arises in the case of the \(Z_2 \to Z_1 H\) decay, with \(H\) a relatively light Higgs boson. In this case, a SM-like coupling \(HZ_1 Z_1\) is expected and thus the \(Z_2 \to Z_1 H\) mode can be induced via the \(Z' - Z\) mixing. The transition probabilities for the \(Z_2 \to W^- W^+\) and \(Z_2 \to Z_1 H\) decays are proportional to \(s_{\theta_W}^2\) and thus one would expect that they are negligibly small. However, the longitudinal components of the \(W\) and \(Z_1\) bosons give rise to the factors \(m_{Z_2}/m_W^4\) and \((m_{Z_2}/m_{Z_1})^2\), which might compensate the suppression effect coming from the insertion of the mixing angle. As will be seen below, the \(Z_2 \to W^- W^+\) decay can be significantly enhanced if \(m_{Z_2} \gg m_{Z_1}\). Though the widths for these decays do not depend on specific details of the model as they are determined entirely by the SM couplings and the \(\theta\) mixing angle, their branching ratios are model-dependent indeed.

The rest of the paper is organized as follows. A brief review of the minimal 331 model is presented in Sec. \textit{II} including all the Feynman rules necessary for our calculation. Sec. \textit{III} is devoted to the calculation of the two-body decays \(Z_2 \to Z_1 \gamma\), \(Z_1 Z_1\), \(W^+ W^-\), \(Y^+ Y^-\), \(Y^{\pm\pm} Y^{\pm\mp}\), and \(ff\), included the rare one \(Z_2 \to t q'\). Special emphasis is given to the anomaly-induced decays \(Z_2 \to Z_1 \gamma\) and \(Z_2 \to Z_1 Z_1\) modes. In Sec. \textit{IV} we discuss the branching fractions associated with each decay mode. Finally, our conclusions are presented in Sec. \textit{V}.

\section{The Minimal 331 Model}

The \(SU_C(3) \times SU_L(3) \times U_X(1)\) model has been discussed to some extent in the literature \cite{12,13,14,21,22,23}. We will only focus on those features which are relevant for the present discussion. In this model the gauge interactions are non flavor-universal since fermion generations are represented differently under the \(SU_L(3)\) group. The leptons are accommodated as antitriplets of \(SU_L(3)\):

\[ L_{1,2,3} = \begin{pmatrix} e \\ \nu_e \\ \nu_e \end{pmatrix}, \begin{pmatrix} \mu \\ \nu_\mu \\ \mu^c \end{pmatrix}, \begin{pmatrix} \tau \\ \nu_\tau \\ \tau^c \end{pmatrix} : (1,3^*,0). \]  

(1)

Notice that the leptons do not carry \(X\) quantum numbers. In order to cancel the \(SU_L(3)\) anomaly, the same number of fermion triplets and antitriplets is necessary. This requires to arrange two quark generations as triplets and the
other one as an antitriplet. It is customary to choose the third generation as the one transforming as a triplet in order to distinguish the new dynamic effects in the physics of the quark top from that of the lighter generations. Accordingly, the three generations are specified as follows:

\[
Q_{1,2} = \begin{pmatrix} u \\ d \\ D \end{pmatrix}, \quad \begin{pmatrix} c \\ s \\ S \end{pmatrix}: (3,3,-1/3), \quad Q_3 = \begin{pmatrix} b \\ t \\ T \end{pmatrix}: (3^*,2/3),
\]

\[
d^e, \quad s^e, \quad b^e: (3^*,1,1/3), \quad D^e, \quad S^e: (3^*,1,4/3),
\]

\[
u^e, \quad c^e, \quad t^e: (3^*,1,-2/3), \quad T^e: (3^*,1,-5/3).
\]

where \(D, S,\) and \(T\) are exotic quarks with electric charge \(-4/3, -4/3,\) and \(5/3,\) respectively.

The Higgs sector is comprised by three triplets and one sextet of \(SU_L(3):\)

\[
\phi_Y = \begin{pmatrix} \Phi_Y \\ \phi^0 \end{pmatrix}: (1,3,1), \quad \phi_1 = \begin{pmatrix} \Phi_1 \\ \delta \end{pmatrix}: (1,3,0), \quad \phi_2 = \begin{pmatrix} \Phi_2 \\ \rho^- \end{pmatrix}: (1,3,-1),
\]

\[
H = \begin{pmatrix} T \\ \Phi^3_3/\sqrt{2} \\ \eta^- \end{pmatrix}: (1,6,0).
\]

To break \(SU_L(3) \times U_X(1)\) into \(SU_L(2) \times U_Y(1),\) only the \(\phi_Y\) scalar triplet of \(SU_L(3)\) is required. The hypercharge is identified as a linear combination of the broken generators \(T^8\) and \(X: Y = \sqrt{3}(\lambda^8 + \sqrt{2}X\lambda^3),\) with \(\lambda^8\) a Gell-Mann matrix and \(\lambda^3 = \sqrt{2/3}\text{diag}(1,1,1).\) The next stage of SSB occurs at the Fermi scale and is achieved by the two triplets \(\phi_1\) and \(\phi_2.\) The sextet \(H\) is necessary to provide realistic masses for the leptons [21]. In these expressions \(\Phi_Y, \Phi_1, \Phi_2 = i\sigma^2\Phi_3^*,\) and \(\Phi_3\) are all doublets of \(SU_L(2)\) with hypercharge 3, 1, 1, and 1, respectively. On the other hand, \(T\) is a \(SU_L(2)\) triplet with \(Y = +2,\) whereas \(\delta^-, \rho^-,\) and \(\eta^-\) are all singlets of \(SU_L(2)\) with hypercharge \(-2,-4,\) and \(+4,\) respectively [21, 22]. The extra \(Z_2\) boson, the bilepton and the exotic quarks get masses at the first stage of SSB through the vacuum expectation value \(\langle \phi_Y \rangle_0 = (0,0,u/\sqrt{2}).\) The bileptons form an \(SU_L(2)\) doublet with hypercharge +3. The spectrum of physical gauge particles is the following. The charged gauge bosons are given by

\[
Y^{++}_\mu = \frac{1}{\sqrt{2}}(A^4_\mu - iA^5_\mu),
\]

\[
Y^+_\mu = \frac{1}{\sqrt{2}}(A^5_\mu - iA^4_\mu),
\]

\[
W^+_\mu = \frac{1}{\sqrt{2}}(A^3_\mu - iA^2_\mu).
\]

with \(m_{Y^+,+}^2 = g^2/4(u^2 + v_2^2 + 3v_3^2), m_{Y^+}^2 = g^2/4(u^2 + v_1^2 + v_2^2)\), and \(m_W^2 = g^2/4(v_1^2 + v_2^2 + v_3^2).\) The hierarchy of the SSB yields a splitting between the bilepton masses given by \(m_{Y^+,+}^2 - m^2_{Y^+,+} \leq 3m_W^2.\)

In the neutral sector, the gauge fields \((A^3, A^8, X)\) define three mass eigenstates \((A, Z_1, Z_2)\) via the following rotation

\[
\begin{pmatrix} A_\mu \\ Z_\mu' \\ Z_\mu'' \end{pmatrix} = \begin{pmatrix} s_{\thetaW} \sqrt{3}s_{\thetaW} \sqrt{1 - 4s^2_{\thetaW}} \\ c_{\thetaW} - \sqrt{3}s_{\thetaW}t_{\thetaW} - t_{\thetaW}\sqrt{1 - 4s^2_{\thetaW}} \\ 0 - \sqrt{\frac{1 - 4s^2_{\thetaW}}{s_{\thetaW}}} \end{pmatrix} \begin{pmatrix} A^3_\mu \\ A^8_\mu \\ X_\mu \end{pmatrix},
\]

with

\[
\begin{pmatrix} Z_{1\mu} \\ Z_{2\mu} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} Z_\mu \\ Z'_\mu \end{pmatrix},
\]

where the mixing angle is

\[
\sin^2\theta = \frac{m_{Z_1}^2 - m_{Z_2}^2}{m_{Z_1}^2 - m_{Z_1}'^2}.
\]
with \( m_Z^2 = m_W^2/c_W^2 \) and \( Z_1 \) standing for the SM \( Z \) boson.

On the other hand, by matching the gauge coupling constants at the first stage of SSB, it is found that

\[
g_Z^2 g^2 = \frac{6s_W^2(m_Z)}{1 - 4s_W^2(m_Z)}
\]

which means that \( s_W^2(m_Z) \) has to be smaller than 1/4. It was found that this condition implies that the extra \( Z \) boson cannot be heavier than 3.1 TeV \[28\]. From this result and the symmetry-breaking hierarchy \( u \gg v_1, v_2, v_3 \), it is inferred that the bileptons have masses smaller than \( m_{Z_2}/2 \) \[28\]. In this way, all the new gauge boson masses are bounded from above. As a consequence, the \( Z_2 \rightarrow Y^+Y^-, Y^+Y^+Y^- \) decays would always be kinematically allowed.

Some comments concerning the Yang-Mills sector are relevant for the subsequent discussion. Besides the sector associated with the electroweak group, the Yang-Mills Lagrangian associated with \( SU_L(3) \times U_X(1) \) gives rise to two new terms, which can be written in a manifest \( SU_L(2) \times U_Y(1) \)-invariant form \[22\]:

\[
\mathcal{L}_{SMNP} = \frac{1}{2} (D_\mu Y - D_\nu Y)^\dagger (D^\mu Y^\nu - D^\nu Y^\mu) - Y^\dagger \mu (igW_{\mu\nu} + ig'B_{\mu\nu}) Y^\nu
\]

\[
- \frac{i\sqrt{3}g\sqrt{1 - 4s_W^2}}{2c_W} Z_\mu^i [Y^\dagger \nu (D^\mu Y^\nu - D^\nu Y^\mu) - (D^\mu Y^\nu - D^\nu Y^\mu)^\dagger Y^\nu],
\]

\[
\mathcal{L}_{NP} = \frac{1}{4} Z_\mu^i Z^\mu_{\nu\lambda} - \sqrt{3}g\sqrt{1 - 4s_W^2} Z_\mu^i Y^\dagger \nu Y^\mu - \frac{3g^2 (1 - 4s_W^2)}{4c_W^2} Z_\mu^i Y^\dagger \nu (Z^\mu_{\nu\lambda} Y^\lambda - Z^\nu_{\nu\lambda} Y^\lambda)
\]

\[
+ \frac{g^2}{2} (Y^\dagger \mu Y^\dagger \nu) (Y^\mu_\alpha Y^\nu_\beta - \frac{1}{2} Y^\nu_\beta Y^\mu_\alpha) + \frac{3g^2}{4} (Y^\dagger \mu Y^\dagger \nu) (Y^\nu_\alpha Y^\mu_\alpha - Y^\nu_\beta Y^\mu_\beta) + \frac{g^2}{2} (Y^\dagger \mu Y^\dagger \nu) (Y^\mu_\alpha Y^\nu_\beta - \frac{1}{2} Y^\nu_\beta Y^\mu_\alpha),
\]

where \( W_{\mu\nu} = \sigma^I W_{\mu\nu}^I, B_{\mu\nu} = YB_{\mu\nu}/2, \) and \( D_\mu = \partial_\mu - igW_\mu - ig'B_{\mu} \) is the covariant derivative associated with the electroweak group, with \( Y^\dagger = (Y^-_1, Y^-_2) \) the \( SU_L(2) \) bilepton doublet. The bileptophic nature of the \( Z_2 \) boson becomes clear from these Lagrangians, in which the \( Z'Y \) and \( Z'Z'Y \) couplings appear multiplied by the factors \( \sqrt{1 - 4s_W^2} \) and \( 1 - 4s_W^2 \), respectively. This is a direct consequence from the fact that the bileptons belong to an \( X = 0 \) representation.

### A. Feynman rules

We will present now the Feynman rules necessary for our calculation. The current sector is determined by the covariant derivative associated with \( SU_L(3) \times U_X(1) \), which is given by

\[
D_\mu = \partial_\mu - ig\frac{\lambda^a}{2} A_\mu^a - ig\frac{\lambda^0}{2} X_\mu,
\]

in the fundamental representation. \( \lambda^a (a = 1, ..., 8) \) are the Gell-Mann matrices and \( \lambda^0 = \sqrt{2/3} \text{diag}(1,1,1) \). In terms of physical fields, \( D_\mu \) can be written as

\[
D_\mu = \partial_\mu - ig\frac{\lambda^a}{2} (\lambda_{12} W_{\mu}^a + \lambda_{45} Y_{\mu}^{a+} + \lambda_{67} Y_{\mu}^{a+} + \text{H.c.}) - i\epsilon Q A_\mu
\]

\[
- \frac{ig}{2c_W} (c_W^2 \lambda^3 - s_W^2 Y) Z_\mu + \frac{ig}{2c_W} \sqrt{1 - 4s_W^2} (\lambda^8 - \frac{3\sqrt{2} s_W^2 \lambda^9 X}{1 - 4s_W^2}) Z_\mu^i,
\]

where \( \lambda_{ab} = \frac{1}{2} (\lambda^a + i\lambda^b) \). From the presence of the factor \( \sqrt{1 - 4s_W^2} \) in the term involving the extra \( Z \) boson, the leptophobic nature of \( Z' \) becomes clear.

In terms of the \( \{A, Z, Z'\} \) fields we can write the neutral currents as follows

\[
\mathcal{L}^{NC} = ie \sum_f Q_f (f \gamma_{\mu} f) A_\mu + \frac{ig}{2c_W} \sum_f \left( f \gamma_{\mu} (g_{fZ} f_{AZ} \gamma_5 f) Z_\mu + f \gamma_{\mu} (g_{fZ} f_{AZ} \gamma_5 f) Z'_{\mu} \right).
\]

The coefficients \( g_{fZ} \) are listed in Table \ref{tab:leptophic_coup}. It turns out that the couplings of the SM \( Z \) boson to exotic quarks are vector-like. These results can be easily translated into the mass eigenstates \( \{Z_1, Z_2\} \) by means of the relations

\[
g_{VZ_1} = \cos \theta_{g_{VZ}} - \sin \theta_{g_{VZ}},
\]

\[
g_{VZ_2} = \sin \theta_{g_{VZ}} + \cos \theta_{g_{VZ}},
\]
TABLE I: Structure of the neutral currents in the minimal $3_{3}1$ model. The couplings are given in terms of the \{A, Z, Z'\} fields.

| Fermion | $Q_f$ | $g_{AZ}$ | $g_{AZ'}$ | $g_{AZ''}$ | $g_{AZ'''}$ |
|---------|-------|----------|-----------|-------------|-------------|
| $t^-$   | $-1$  | $-\frac{1-4\gamma_2}{2}$ | $-\frac{1}{2}$ | $\sqrt{2}\frac{1-4\gamma_2}{2\sqrt{3}}$ | $-\frac{\sqrt{3}}{2\sqrt{3}}$ |
| $\nu_1$ | $0$   | $\frac{1}{2}$ | $\frac{1}{2}$ | $\sqrt{1-4\gamma_2}$ | $\frac{\sqrt{1-4\gamma_2}}{2\sqrt{3}}$ |
| $u, c$  | $\frac{3}{2}$ | $\frac{3-8\gamma_2}{6}$ | $\frac{1}{2}$ | $\frac{1-6\gamma_2}{2\sqrt{3}}$ | $\frac{1}{2\sqrt{3}}$ |
| $d, s$  | $-\frac{1}{2}$ | $\frac{3-4\gamma_2}{6}$ | $-\frac{1}{2}$ | $\frac{1-9\gamma_2}{2\sqrt{3}}$ | $\frac{1}{2\sqrt{3}}$ |
| $D, S$  | $-\frac{3}{2}$ | $\frac{8\gamma_2}{3}$ | $0$ | $\frac{1-9\gamma_2}{2\sqrt{3}}$ | $\frac{1}{2\sqrt{3}}$ |
| $b$     | $-\frac{1}{2}$ | $\frac{3-4\gamma_2}{6}$ | $-\frac{1}{2}$ | $\frac{1-2\gamma_2}{2\sqrt{3}}$ | $\frac{1}{2\sqrt{3}}$ |
| $t$     | $\frac{3}{2}$ | $\frac{3-8\gamma_2}{6}$ | $\frac{1}{2}$ | $\frac{1+4\gamma_2}{2\sqrt{3}}$ | $\frac{1}{2\sqrt{3}}$ |
| $T$     | $\frac{5}{2}$ | $-\frac{10\gamma_2}{3}$ | $0$ | $\frac{1+11\gamma_2}{2\sqrt{3}}$ | $\frac{1}{2\sqrt{3}}$ |

$$g_{AZ_1} = \cos \theta g_{AZ} - \sin \theta g_{AZ'\prime},$$  
$$g_{AZ_2} = \sin \theta g_{AZ} + \cos \theta g_{AZ'\prime}.$$  

However, with the exception of the $Z_2WW$ and $Z_2Z_1H$ couplings, we will ignore $Z' - Z$ mixing effects as they cannot affect significantly the branching fractions of the main decay channels.

As far as FCNCs are concerned, they are essentially mediated by the $Z_2$ boson as the couplings of the SM quarks to $Z_1$ are proportional to $\sin \theta$ and thus are quite suppressed. These effects arise solely from the left-handed sector and are a result from the different $X$ quantum number assignments existing among the fermion families. We will focus on $Z_2$ transitions involving the top quark. Thus, the FCNC Lagrangian for the up sector can be written as

$$\mathcal{L}_{FCNC} = \frac{g}{2c_W} \left( -\sin \theta Z^\mu_1 + \cos \theta Z^\mu_2 \right) \delta_L V^a_{L} \bar{V}_{L} \gamma_{\mu} P_L U_b,$$

where

$$\delta_L = \frac{2}{\sqrt{3}} \frac{\gamma_2}{\sqrt{1-4\gamma_2}},$$

$V_{ab}$ is the unitary matrix relating gauge states to mass eigenstates, and $U_a = u, c, t$. In the right-handed sector there is no FCNC as these fermions transform identically.

The $Z_2W^+W^-, Z_2Y^+Y^-$, and $Z_2Y'^+Y'^-$ couplings arise from the Lagrangian

$$\mathcal{L}_{Z_2VV'} = igcw g_{Z_2V} \left( Z_2^2 \left( V_{\alpha\beta}^V V^\alpha - V_{\alpha\beta} V^\alpha \right) + Z_2 \varepsilon_{\alpha\beta} V^\alpha V^\beta \right),$$

where $V = W^+, Y^+, Y'^+$, and

$$g_{Z_2V} = \begin{cases} -\sin \theta, & V = W^+ \\ \sqrt{2(1-4\gamma_2)}, & V = Y^+, Y'^+ \end{cases}$$

The $Z_2V_{\mu}(k_1)V_{\nu}^+(k_2)$ vertex can be written as

$$\Gamma_{\lambda\mu\nu}^{Z_2V} (k_1, k_2) = -igcw g_{Z_2V} \left( 2k_{1\mu}g_{\lambda\nu} - 2k_{\mu\nu}g_{\lambda\nu} + (k_2 - k_1)\lambda g_{\mu\nu} \right),$$

where all the 4-momenta are incoming.
III. TWO-BODY DECAYS OF Z₂

We now turn to the widths for the decay modes \(Z_2 \rightarrow Z_1 \gamma, Z_1 Z_1\), \(YY, WW, Z_1 H\), and \(\bar{f} f\). The \(Z_2 \rightarrow Z_1 \gamma, Z_1 Z_1\) decays will be discussed more carefully to clarify some subtleties associated with the triangle anomaly.

A. Anomaly-induced \(Z_2 \rightarrow Z_1 \gamma, Z_1 Z_1\) decays

As will be seen below, the amplitudes associated with the \(Z_2 \rightarrow Z_1 \gamma, Z_1 Z_1\) decays are model-independent as they are dictated by gauge invariance and Bose symmetry, respectively, whereas their magnitude depends on the fermion content of the theory and the anomaly cancellation mechanism. In the 331 model, the couplings of \(Z_2\) to fermions are determined by the coupling constant \(g\) and the weak angle only. Also, we recall that the distinctive feature of the 331 model is that the triangle anomaly does not cancel out within each generation, but when all the generations are considered. The \(Z_2 \rightarrow Z_1 \gamma, Z_1 Z_1\) decays have been already studied in the context of a superstring-inspired \(E_6\) model [27].

1. The decay \(Z_2 \rightarrow Z_1 \gamma\)

The \(Z_2 \rightarrow Z_1 \gamma\) decay receives contributions from all the charged fermions via the two triangle diagrams shown in Fig. 1 where we also show the notation for the 4-momenta of the participating particles. The invariant amplitude can be written as

\[
\mathcal{M}(Z_2 \rightarrow Z_1 \gamma) = \Gamma_{\lambda \mu \nu}^2 Z_1 \gamma R_{\lambda \mu \nu}, \tag{28}
\]

where \(R_{\lambda \mu \nu}\) is a term involving the polarization vectors associated with the final states:

\[
R_{\lambda \mu \nu} = \epsilon^{\lambda}(k_3, \lambda_3)\epsilon^{\mu}(k_1, \lambda_1)\epsilon^{\nu}(k_2, \lambda_2). \tag{29}
\]

The \(\Gamma_{\lambda \mu \nu}^2 Z_1 \gamma\) tensor is well-defined only if the theory is anomaly free. In this case, its form is determined by electromagnetic gauge invariance and must satisfy the following Ward identity

\[
k_1^f \Gamma_{\lambda \mu \nu}^2 Z_1 \gamma = 0. \tag{30}
\]

It turns out that the anomaly cancels out after the following identity is used

\[
\sum_f Q_f N_C^f (g_{VZ}^f g_{AZ}^f + g_{VZ}^f g_{AZ}^f) = 0, \tag{31}
\]

where \(Q_f\) is the electric charge in units of the positron charge and \(N_C^f\) is the color index. We stress that the sum runs over all the charged fermions as the anomaly is not cancelled within each generation. This identity can be straightforwardly verified using the values given in Table I [27] Not all the terms appearing in the calculation are independent. In order to obtain a gauge invariant result, it is necessary to use Shouten’s identity, which leads to

\[
k_1 \epsilon_{\mu \nu \alpha} k_1^\alpha k_2^\beta + k_1 \epsilon_{\mu \nu \alpha} k_1^\alpha k_2^\beta + k_1 \cdot k_2 \epsilon_{\mu \nu \alpha} k_1^\alpha = 0. \tag{32}
\]

We thus obtain

\[
\Gamma_{\lambda \mu \nu}^2 Z_1 \gamma = \frac{e^2}{4 \pi \alpha^2 W} \left(\frac{m_2^2}{m_2^2 - m_{Z_1}^2}\right)^2 \sum_f Q_f N_C^f (g_{VZ}^f g_{AZ}^f + g_{VZ}^f g_{AZ}^f) AP_{\lambda \mu \nu}, \tag{33}
\]
where the loop amplitude \( A \) is given by
\[
A = B_0(1) - B_0(2) - 2 \left(1 - \frac{m^2_{Z_2}}{m^2_{Z_1}}\right) m^2_{\gamma} C_0(0, 1, 2).
\] (34)

where \( B_0(1) = B_0(m^2_{Z_1}, m^2_{Z_2}, m^2_{\gamma}) \), \( B_0(2) = B_0(m^2_{Z_2}, m^2_{\gamma}, m^2_{Z_1}) \), and \( C_0(0, 1, 2) = C_0(0, m^2_{Z_1}, m^2_{Z_2}, m^2_{\gamma}, m^2_{\gamma}) \) are Passarino-Veltman scalar functions written in the notation of Ref. [20]. In addition
\[
P_{\lambda\mu\nu} = \frac{1}{m^2_{Z_2}} \left( k_{2\mu} \epsilon_{\lambda\alpha\beta} k_1^{\alpha} k_1^{\beta} - k_{1\nu} \epsilon_{\lambda\mu\beta} k_1^{\mu} k_2^{\beta} + k_1 \cdot k_2 \epsilon_{\lambda\mu\alpha} k_1^{\alpha} - (k_1 \cdot k_2 - k_2^{\mu} \epsilon_{\lambda\mu\alpha} k_1^{\alpha}\right),
\] (35)

which is manifestly gauge invariant. It is important to notice that \( A \) vanishes when \( Z_2 \) and \( Z_1 \) are identical, which is consistent with the fact that a self-conjugate vector boson cannot have static electromagnetic properties.

Once the amplitude is squared, the following decay width is obtained
\[
\Gamma(Z_2 \rightarrow Z_1\gamma) = \frac{\alpha^3 m_{Z_1}}{192\pi^2 s_W^4} \left( m_{Z_2} \right)^2 \left( m_{Z_1}^2 - m_{Z_2}^2 - m_{\gamma}^2 \right) \left| \sum_f N_C^f \left( g_{VZ}^f g_{AZ}^f + g_{VZ}^f g_{AZ}^f \right) A \right|^2.
\] (36)

We would like to point out that this result is proportional to \( m_{Z_1}^2 \) rather than to \( m_{Z_2}^2 \), as one would expect. Notice also that there is a suppression factor \( m_{Z_1}/m_{Z_2} \). These peculiarities arise from the Lorentz form of the amplitude. In fact, the squared amplitude is proportional to
\[
|P_{\lambda\mu\nu} R^{\lambda\mu\nu}|^2 = 2m^2_{Z_1} \left( 1 + \frac{m^2_{Z_2}}{m^2_{Z_1}}\right) \left( 1 - \frac{m^2_{Z_2}}{m^2_{Z_1}}\right)^2.
\] (37)

Since \( P_{\lambda\mu\nu} \) is determined by electromagnetic gauge invariance, this result is model-independent and consistent with Landau-Yang theorem, which requires that \( |A|^2 = 0 \) for \( m_{Z_1} = 0 \). This means that this decay mode is expected to be rather suppressed if \( Z_2 \) is much heavier than \( Z_1 \).

2. The decay \( Z_2 \rightarrow Z_1 Z_1 \)

Although a vector boson cannot decay into a pair of massless vector bosons (Landau-Yang theorem), it can decay into a pair of massive vector bosons. The diagrams which contribute to the \( Z_2 \rightarrow Z_1 Z_1 \) decay are shown in Fig. 1. The corresponding amplitude can be written as
\[
\mathcal{M}(Z_2 \rightarrow Z_1 Z_1) = \Gamma^2_{Z_2 Z_1} (k_1, k_2) T^{\lambda\mu\nu},
\]
where
\[
T^{\lambda\mu\nu} = \epsilon^{\lambda}(k_3, \lambda_3) \epsilon^{\mu\nu}(k_1, \lambda_1) \epsilon^{\sigma\nu}(k_2, \lambda_2).
\] (39)

The form of \( \Gamma^2_{Z_2 Z_1} \) is dictated by Bose symmetry, which means that it must be symmetric under the interchange \( k_{1\mu} \leftrightarrow k_{2\nu} \),
\[
\Gamma^2_{Z_2 Z_1} (k_1, k_2) = +\Gamma^2_{Z_2 Z_1} (k_2, k_1).
\] (40)

In order to cancel the anomaly, it is necessary to use the identity
\[
\sum_f N_C^f \left\{ \left( g_{VZ}^f \right)^2 + \left( g_{AZ}^f \right)^2 \right\} g_{AZ}^{f\prime} + 2g_{VZ}^f g_{VZ}^{f\prime} g_{AZ}^f = 0,
\] (41)

where the sum runs over all the fermions. The amplitude can be written as
\[
\Gamma^2_{\lambda\mu\nu} (k_1, k_2) = -\frac{2e\alpha}{\pi s_W} \frac{m^4_{Z_2}}{(m^2_{Z_2} - 4m^2_{Z_1})^2} \left( (A_1 + A_2 + A_3) P_{1\lambda\mu\nu} + A_1 P_{2\lambda\mu\nu} \right),
\] (42)
where

\[ A_1 = \sum_f N_C \left\{ \left( g_{VZ}^f \right)^2 + \left( g_{AZ}^f \right)^2 \right\} g_{AZ}^f + 2g_{VZ}^f g_{V'Z}^f g_{AZ}^f \left\{ \frac{1}{2} \left( 1 + \frac{2m_{Z_1}^2}{m_{Z_2}^2} \right) (B_0(1) - B_0(2)) \right. \]

\[ - \left( m_{Z_1}^2 \left( 1 - \frac{m_{Z_1}^2}{m_{Z_2}^2} \right) + m_f^2 \left( 1 - \frac{4m_{Z_1}^2}{m_{Z_2}^2} \right) \right) C_0(1, 1, 2) \right\}, \]

\[ A_2 = 4 \sum_f N_C \left( g_{AZ}^f \right)^2 \left\{ -\frac{2m_f^2}{m_{Z_2}^2} (B_0(1) - B_0(2)) + \left( 1 - \frac{2m_{Z_1}^2}{m_{Z_2}^2} \right) m_f^2 C_0(1, 1, 2) \right\}, \]

\[ A_3 = 4 \sum_f N_C g_{VZ}^f g_{V'Z}^f g_{AZ}^f \left( 1 - \frac{4m_{Z_1}^2}{m_{Z_2}^2} \right) m_f^2 C_0(1, 1, 2), \]

with \( C_0(1, 1, 2) = C_0(m_{Z_1}^2, m_{Z_1}^2, m_{Z_2}^2, m_f^2, m_f^2, m_f^2) \). The Lorentz tensors \( P_{1\lambda\mu\nu} \) and \( P_{2\lambda\mu\nu} \) read

\[ P_{1\lambda\mu\nu} = \frac{1}{2} \left( 1 - \frac{4m_{Z_1}^2}{m_{Z_2}^2} \right) \epsilon_{\lambda\mu\nu\sigma}(k_1 - k_2)^\sigma, \]

\[ P_{2\lambda\mu\nu} = \frac{1}{m_{Z_2}^2} \left( k_1^\lambda \epsilon_{\mu\nu\sigma\beta} k_2^\alpha k_2^\beta - k_2^\lambda \epsilon_{\mu\nu\sigma\beta} k_1^\alpha k_1^\beta \right), \]

which are clearly symmetric under the interchange \( k_1^\mu \leftrightarrow k_2^\nu \). When the amplitude is squared, the contraction of \( T^{\lambda\mu\nu} \) with \( P_{\lambda\mu\nu} \) yields a term proportional to \( m_{Z_2}^2 (m_{Z_2}^2 / m_{Z_1}^2) \), which means that this decay is essentially determined by the longitudinal component of the \( Z_1 \) boson, which is to be contrasted with the case of the \( Z_2 \rightarrow Z_1 \gamma \) mode, which yields a squared amplitude proportional to \( m_{Z_1}^2 \). Since \( P_{1\lambda\mu\nu} \) and \( P_{2\lambda\mu\nu} \) are determined only by Bose symmetry, this behavior is model-independent indeed.

The decay width can be written as

\[ \Gamma(Z_2 \rightarrow Z_1 Z_1) = \frac{\alpha^3 m_{Z_2}}{24 \pi^2 s_W^2} \left( m_{Z_2} \right)^2 \sqrt{1 - \frac{4m_{Z_1}^2}{m_{Z_2}^2}} \left| A_1 + A_2 - \frac{4m_{Z_1}^2}{m_{Z_2}^2 - 4m_{Z_1}^2} A_3 \right|^2, \]

where we have introduced a factor of \( 1/2 \) to account for two identical particles in the final state. From this expression it can be seen that the decay width vanishes when \( m_{Z_1} = 0 \) and \( g_{AZ}^f = 0 \), which is consistent with Landau-Yang theorem.

**B. The decay \( Z_2 \rightarrow V^V \)**

We turn now to the analysis of the \( Z_2 \rightarrow V^V \) decays, with \( V = W^-, Y^-, \) or \( Y^{-+} \). The respective amplitude can be written as

\[ \mathcal{M}(Z_2 \rightarrow V^V) = \frac{Z_{\lambda\mu\nu}^{Z_2 V^V}(k_1, k_2) e^\lambda(k_3, \lambda_3) e^{+\mu}(k_1, \lambda_1) e^{+\nu}(k_2, \lambda_2)}{\lambda_{\lambda\mu\nu}} \]

where the \( \Gamma_{\lambda_{\lambda\mu\nu}}^{Z_2 V^V}(k_1, k_2) \) is given in Eq. (47). The longitudinal component of the \( V \) boson gives rise to a term proportional to \( m_{Z_2}^2 (m_{Z_2}^2 / m_V^2) \), which plays a very important role in the case of the \( Z_2 \rightarrow W^+ W^- \) channel. The decay widths for the \( Z_2 \rightarrow V^V \) modes read

\[ \Gamma(Z_2 \rightarrow W^+ W^-) = m_{Z_2} \left( \frac{\alpha}{3} \right) \left( \frac{G_F}{s_W^2} \right)^2 \sin^2 \theta f(x_W), \]

\[ \Gamma(Z_2 \rightarrow Y Y^+) = m_{Z_2} \frac{\alpha(1 - 4s_W^2)}{s_W^2} f(xy), \]

where \( x_V = 4m_V^2 / m_{Z_2}^2 \), and

\[ f(x) = \frac{\sqrt{1 - x}}{x^2} \left( 1 + 4x - \frac{17}{4} x^2 - \frac{3}{4} x^3 \right). \]
The $Z_2 W^+ W^-$ vertex has been studied in several contexts. The production of the $Z_2$ boson followed by the $Z_2 \rightarrow W^+ W^-$ decay has been studied within theories in which the extra $Z$ boson does not couple to the fermions \[27\] and also in the context of the $E_8 \times E'_8$ superstring theory \[28\]. The implications of a relatively heavy extra $Z$ boson on this decay mode within the context of string $E_8$ inspired theories were investigated in Ref. \[29\]. The magnitude of the $Z_2 \rightarrow W^+ W^-$ decay width depends crucially on the size of $\theta$, which is expected to be very small. From general grounds, one can assume that $\theta \approx \delta m^2 / m_{Z_2}^2 \ll 1$, with $\delta m^2$ a model-dependent quantity. This means that $\theta$ may be small because either $m_{Z_2} \gg m_{Z_1}$, or $\delta m^2$ is very small. Thus, a relatively light $Z_2$ boson is still compatible with a very small mixing angle provided that $\delta m^2$ is small. Most analysis from precision experiments support this scenario. In fact it has been found that $\sin \theta \lesssim 10^{-3}$ for $m_{Z_2} \gtrsim 500$ GeV (for an exception to this behavior see Ref. \[31\]).

Another interesting possibility arises when $Z_2$ is very heavy, which yields a very small mixing angle even if $\delta m^2$ is of the order of $m_{Z_1}^2$. In this case, $\theta \approx m_{Z_2}^2 / m_{Z_2}^2$, which is very small because $m_{Z_2} \gg m_{Z_1}$. In this scenario, the suppression effect coming from $\sin \theta$ would be compensated by the factor $1 / x_W^2$, and thus the $Z_2 \rightarrow W^- W^+$ mode might be as important as the fermionic channels \[29\]. In general, if $\delta m^2$ is either very small or of order $O(m_{Z_1}^2)$, the $Z_2 \rightarrow W^+ W^+$ mode is favored for $m_{Z_2} \gg m_{Z_1}$. As for the decay $Z_2 \rightarrow YY$, it is expected to have a relatively small branching ratio, mainly due to the birephobic character of $Z_2$ and also to phase space.

C. The decay $Z_2 \rightarrow Z_1 H$

The minimal Higgs sector of the 331 model has been already studied in the literature \[31\]. The three triplets and the sextet lead to 22 physical states: 5 neutral CP-even, 3 neutral CP-odd, 4 singly charged, and 3 doubly charged. It is expected that at least one of the neutral CP-even states resembles the behavior of the SM Higgs boson. In principle, the $Z_2$ boson may decay into a pair of singly or doubly charged Higgs bosons, but in order to simplify our analysis we will assume that these channels are not kinematically allowed. One interesting channel is $Z_2 \rightarrow Z_1 H$, with $H$ a relatively light Higgs boson. Since a SM-like $H Z_1 Z_1$ coupling is expected, the $Z_2 \rightarrow Z_1 H$ decay can be induced via $Z' - Z$ mixing. In this case the $Z_2 Z_1 H$ coupling will be proportional to $\sin \theta$. Naively, one could think that this suppression effect may be compensated by the $(m_{Z_2} / m_{Z_1})^2$ factor arising from the longitudinal component of the $Z_1$ boson. However, this is not the case because the factor $m_{Z_1}^2$ gets cancelled with the factor $m_{Z_2}^2$ coming from the SM-like coupling. Accordingly, the decay width is given by

$$\Gamma(Z_2 \rightarrow Z_1 H) = m_{Z_2} \frac{\alpha s_{2\theta}}{4 s_{1W}^2} \left(1 - x^2 - y^2 \right)^2 + 12 x^2 - 4 x^2 y^2 \right) \sqrt{(1 - (x + y)^2)(1 - (x - y)^2)},$$

(53)

with $x = m_{Z_1} / m_{Z_2}$ and $y = m_H / m_{Z_2}$. It can be seen that, in contrast with the $Z_2 \rightarrow W^+ W^-$ channel, this mode does not depend on the $m_{Z_1} / m_{Z_2}$ ratio and thus it is insensitive to the effects of a heavy $Z_2$ boson.

D. The decays $Z_2 \rightarrow f \bar{f}$

From the Feynman rules given above, it is straightforward to obtain the width for the flavor conserving $Z_2 \rightarrow \bar{f} f$ decay, which is given by

$$\Gamma(Z_2 \rightarrow \bar{f} f) = m_{Z_2} \frac{\alpha N_C}{3 s_{2W}^2} \sqrt{1 - 4 m_f^2 / m_{Z_2}^2} \left|g_{f'Z_2}' \right|^2 \left(1 + 2 m_f^2 / m_{Z_2}^2 \right) + \left|g_{f'Z_2}' \right|^2 \left(1 - 4 m_f^2 / m_{Z_2}^2 \right).$$

(54)

By simplicity we will take the diagonal elements of the mixing matrix of the quark sector equal to one. As already mentioned, the leptonic decays of the $Z_2$ boson are expected to be strongly suppressed. On the contrary, from Table \[4\] it is evident that the decay widths to quark pairs have associated an enhancement factor $1 / \sqrt{1 - 4 s_{1W}^2}$, although the decays $Z_2 \rightarrow q \bar{q}$, with $q = d, s$, and $t$ would be dominated by the associated vector coupling since the axial one is proportional to the factor $\sqrt{1 - 4 s_{1W}^2}$ and thus it is negligible.

The decay width for the FCNC transition $Z_2 \rightarrow q' q'$ can be written as

$$\Gamma(Z_2 \rightarrow q' q' + q' q') = 2 m_{Z_2} N_C a_{2L}^2 / 3 s_{2W}^2 \left|V_{3q'}^* V_{3q} \right|^2 \left(1 - x_t^2 \right)^2 \left(1 - \frac{1}{2} x_t^2 (1 + x_t^2) \right),$$

(55)

with $q' = u, c$ and $x_t = m_t / m_{Z_2}$. The $q'$ mass has been neglected.
IV. DISCUSSION

One interesting feature of the 331 model is that the masses of the new gauge bosons not only can be bounded from below using the available experimental data, but also from above due to the constraint $s_W^2 < 1/4$ imposed by the model. This condition translates into the upper bound $m_{Z_2} < 3.1$ TeV. Though this bound can be relaxed by introducing more complex Higgs sectors \(31\), in the following discussion we will assume that $m_{Z_2} \lesssim 3$ TeV. Even more, using the symmetry-breaking hierarchy $u \gg v_1, v_2, v_3$, this bound can be translated into an upper bound on the bileptons masses, which is given by $m_{\nu} < m_{Z_2}/2$. This constraint will be assumed for the bileptons masses, which in addition will be taken as degenerate. This means that the decays of $Z_2$ into bilepton pairs would always be kinematically allowed.

Currently the most stringent lower bound on the doubly charged bilepton arises from muonium-antimuonium conversion, $e^+e^- \rightarrow e^-\mu^+$, which yields the limit $m_{\nu_{++}} \geq 850$ GeV \(32\). It has been argued however that this bound can be evaded in a more general context since it relies on very restrictive assumptions \(33\). For instance, the scalar contributions were not considered in the analysis of Ref. \(32\) though they may give rise to strong cancellations, thereby relaxing the bound on the bilepton masses. Another strong limit, $m_{\nu_{++}} > 750$ GeV, arises from fermion pair production and lepton-flavor violating decays \(34\). In addition, the bound $m_{\nu_{++}} > 440$ GeV was derived from limits on the muon decay width \(35\). We would like to stress that all the above bounds are model dependent, and so the existence of lighter bilepton gauge bosons is still allowed. As far as the exotic quark mass is concerned, the lower bound $m_Q > 240$ GeV was derived from the search of SUSY at the Tevatron. This bound would reach the level of 320 GeV at the Run-2 \(36\).

We now consider two scenarios which are very illustrative of the possible behavior of the $Z_2$ boson in the 331 model. Throughout our analysis we will assume that the bileptons are degenerate and have a mass $m_{\nu} = 500$ GeV. We will also assume that the exotic quarks are degenerate and consider two specific values for their mass $m_{\nu}$. In the first scenario we will consider that $m_Q = 500$ GeV, whereas in the second scenario, we will take $m_Q = m_{Z_2}$. For the mass of the $Z_2$ boson, the range $1$ TeV < $m_{Z_2}$ < 3 TeV will be considered. As far as the mixing angle $\theta$ is concerned, it has been constrained within the context of the 331 model from precision experiments at the Z-pole and neutral current experiments. It was found that $-0.0006 < \theta < 0.0042$ for $m_{Z_2} > 490$ GeV \(23\). In both scenarios we will use $\sin \theta \approx \theta = 10^{-3}$ and $m_H = 120$.

To begin with, we will discuss the flavor conserving decays of the $Z_2$ boson, whereas the study of the FCNC transitions will be deferred as it is necessary to make some assumptions for the mixing-matrix elements $|V_{3q}, V_{3l}|^2$. The branching ratios for the decay modes $Z_2 \rightarrow Q\bar{Q}, q\bar{q}, l^+l^-, \nu\bar{\nu}, YY, WW, Z_1H, Z_1Z_1$, and $Z_1\gamma$, are displayed in Fig. 5 as a function of $m_{Z_2}$ in the scenario in which $m_Q = 500$ GeV. In this Fig., $Z_2 \rightarrow q\bar{q}$ and $Z_2 \rightarrow Q\bar{Q}$ stand for the decay to all the exotic quarks and all the SM quarks, respectively. It can be observed that these modes are dominant, being $Br(Z_2 \rightarrow Q\bar{Q})$ larger than $Br(Z_2 \rightarrow q\bar{q})$. On the contrary, the $Z_2 \rightarrow l^+l^-$ mode is marginal, with a branching ratio of the order of $10^{-3}$. The invisible mode is still more suppressed, with $Br(Z_2 \rightarrow \nu\bar{\nu}) \approx 10^{-4}$. This illustrates the leptophobic nature of the $Z_2$ boson. As can be noticed, there is a difference of one order of magnitude between the branching ratios for the two last modes, which can be explained from the fact that the vector coupling of the $Z_2$ boson to charged leptons has associated an additional factor of 3 (see Table I). As for the $Z_2 \rightarrow W^+W^-$ mode, its branching ratio may reach the level of $10^{-3}$ for large values of $m_{Z_2}$. Since this mode is very sensitive to the mass of the extra Z boson, it is expected to have a larger branching fraction if $Z_2$ is heavier. In contrast, $Br(Z_2 \rightarrow H_2)$ is of the order of $10^{-5}$ at most and it is almost independent of the value of the $Z_2$ mass. As far as the anomaly-induced decays are concerned, $Br(Z_2 \rightarrow Z_1Z_1)$ is of the order of $10^{-6}$ and is less sensitive to the mass of the extra Z boson than $Br(Z_2 \rightarrow Z_1\gamma)$. This fact can be explained from the fact that while the $Z_2 \rightarrow Z_1Z_1$ decay width decreases with $m_{Z_2}$, the loop amplitudes decrease in the same proportion with $m_{Z_2}$. A somewhat different behavior is found for $Br(Z_2 \rightarrow Z_1\gamma)$, which is more sensitive to $m_{Z_2}$ and lies in the range $10^{-9} - 10^{-11}$ for the $m_{Z_2}$ values shown in Fig. 5. Finally, in the scenario in which $m_Q = m_{Z_2}$, the respective branching ratios are shown in Fig. 6. In this case, the decay to exotic quarks is not kinematically allowed and thus the main decay channel is $Z_2 \rightarrow q\bar{q}$. In general terms, the branching ratios for all the remaining decay channels are slightly larger than those shown in Fig. 5.

It is interesting to compare our results for the anomaly-induced decays with those found in Ref. \(27\), which were obtained within the context of a superstring-inspired $E_6$ model. In that case, it was found that $Br(Z_2 \rightarrow Z_1Z_1) \sim 10^{-5}$ and $Br(Z_2 \rightarrow Z_1\gamma) \sim 10^{-6}$ for a relatively light $Z$ boson with a mass of the order of 0.5 TeV, though these branching ratios were obtained by considering the $Z_2 \rightarrow e^+e^-$ mode as the dominant one. We believe that our results are realistic since they were obtained within the context of a model that generates nontrivial couplings in the neutral current sector, which in addition depends on known parameters. Though in this model the $Z_2$ boson has a leptophobic character, it should be emphasized that these contributions are not important in general as these decays are only sensitive to heavy fermions. As can be seen in Table I both the vector and axial couplings of $Z_2$ to SM and exotic quarks are as important as those existing in the SM for the $Z_1$ boson. We do not expect that these branching ratios can be substantially enhanced within the context of other renormalizable theories.
We now turn to the FCNC decays of the $Z_2$ boson. In order to get an estimate for the corresponding branching fractions, it is necessary to assume some value for the unknown coefficient $|V_{3q}^* V_{3t}|^2$. In principle, a bound on the $Z_2 t u$ coupling can be indirectly obtained from the $\kappa_{tu\gamma}$ form factor associated with the one-loop vertex $tu\gamma$, for which the bound $|\kappa_{tu\gamma}| < 0.27$ was set recently [37]. In addition to the SM contribution, which turns out to be very suppressed [38], in the 331 model the $t \to u\gamma$ decay receives contributions from the $Z_2tu$ coupling and the charged flavor changing currents mediated by the bilepton gauge bosons and the charged scalars. Both contributions were analyzed in the context of the $b \to s\gamma$ decay [39]. Unfortunately the resulting bound on $|V_{3u}^* V_{3t}|$ is very poor due mainly to the large discrepancy between the scales $m_t$ and $m_{Z_2}$. In fact, considering only the $Z_2tu$ contribution to $\kappa_{tu\gamma}$, the above bound leads to

$$|V_{3u}^* V_{3t}| < 15.5 \frac{m_{Z_2}^2}{m_t^2}, \quad (56)$$

which happens to be very poor even for a relatively light $Z_2$ boson. In order to estimate the branching fractions for the FCNC decays, we will take a different approach, which consists in assuming that the $V_{3u}^* V_{3t}$ and $V_{3c}^* V_{3t}$ coefficients are of the same order of magnitude as those associated with the SM one-loop induced couplings $Ztu$ and $Ztc$, which are proportional to the products of the Kobayashi-Maskawa matrix elements $V_{tb} V_{ch}$ and $V_{tb}^* V_{ab}$. We then assume the following scenario

$$|V_{3u}^* V_{3t}|^2 \sim |V_{tb} V_{ch}|^2 \approx 2.3 \times 10^{-5}, \quad (57)$$
$$|V_{3c}^* V_{3t}|^2 \sim |V_{tb}^* V_{ch}|^2 \approx 1.9 \times 10^{-3}, \quad (58)$$

where the values of the Kobayashi-Maskawa matrix elements were taken from Ref. [40]. To show that this is a reasonable assumption, let us to consider the FCNC transitions in the down sector, for which there are already more acceptable bounds [28]. According to our assumption, the respective coefficients are

$$|V_{3d}^* V_{3s}|^2 \sim |V_{td} V_{ts}|^2 \approx 3.8 \times 10^{-7}, \quad (59)$$
$$|V_{3d}^* V_{3b}|^2 \sim |V_{td} V_{tb}|^2 \approx 1.9 \times 10^{-4}, \quad (60)$$
$$|V_{3c}^* V_{3b}|^2 \sim |V_{tc}^* V_{tb}|^2 \approx 1.9 \times 10^{-3}, \quad (61)$$

which is to be contrasted with the limits derived from $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ mixing [23]:

$$|V_{3d}^* V_{3s}| < 2.5 \times 10^{-5}, \quad (62)$$
$$|V_{3d}^* V_{3b}|^2 < 4 \times 10^{-4}. \quad (63)$$

Bearing in mind the above assumption, we can estimate the branching ratios for $Z_2 \to tu$ and $Z_2 \to tc$. In the limit of massless $t$, which is a good assumption for the range of values that we are considering for $m_{Z_2}$, the branching ratio for the FCNC transitions can be written in terms of that of the $Z_2 \to \tilde{t}t$ mode as follows:

$$Br(Z_2 \to \tilde{t}q + \tilde{q}t) = \frac{2 |g_L^Z V_{3q} V_{3t}|^2}{|g_L^Z|^2 + |g_A^Z|^2} Br(Z_2 \to \tilde{t}t) < 3 |V_{3q}^* V_{3t}|^2 Br(Z_2 \to \tilde{t}t). \quad (64)$$

In the scenario with $m_Q = 500$ GeV, $Br(Z_2 \to \tilde{t}t) \sim 0.1$, so $Br(Z_2 \to \tilde{t}u + \tilde{u}t) \sim 7 \times 10^{-6}$ and $Br(Z_2 \to \tilde{t}c + \tilde{c}t) \sim 6 \times 10^{-4}$. The situation is not very different when $m_Q = m_{Z_2}$. In such a scenario $0.1 < Br(Z_2 \to \tilde{t}t) < 0.5$, which yields $7 \times 10^{-6} < Br(Z_2 \to \tilde{t}u + \tilde{u}t) < 3.5 \times 10^{-5}$ and $6 \times 10^{-4} < Br(Z_2 \to \tilde{t}c + \tilde{c}t) < 3 \times 10^{-3}$.

V. SUMMARY

The detection of an extra $Z$ boson would constitute a clear evidence that the SM gauge group needs to be extended. The phenomenology of this particle is rather model dependent and it is necessary to investigate its physical properties within the context of some specific models. In this work we have presented a study on the main decay channels of the extra $Z$ boson predicted by the minimal 331 model, including the anomaly-induced decays $Z_2 \to Z_1 \gamma$ and $Z_2 \to Z_1 Z_1$. In the 331 model the couplings of the extra neutral gauge boson to the fermions and the extra charged gauge bosons are defined in terms of known parameters. In particular, the leptophobic and bileptophobic nature of the $Z_2$ boson is a direct result arising from the quantum number assignment rather than an ad hoc imposition. These features, together with the fact that the new gauge boson masses are bounded from above and the unique mechanism of anomaly cancellation, provide a very interesting scenario to study the physics of an extra $Z$ boson.
FIG. 2: Branching ratios for the two-body decays of the $Z_2$ boson as a function of $m_{Z_2}$ in the scenario in which $m_Q = 500$ GeV, $m_Y = 500$ GeV and $\sin \theta = 10^{-3}$.

FIG. 3: Branching ratios for the two-body decays of the $Z_2$ boson as a function of $m_{Z_2}$ in the scenario in which $m_Q = m_{Z_2}$, $m_Y = 500$ GeV and $\sin \theta = 10^{-3}$.

The two-body decay modes $Z_2 \to \overline{Q}Q$, $\overline{q}q$, $l^+l^-$, $\nu_l\nu_l$, $YY^\dagger$, $W^+W^-$, $Z_1H$, $Z_1Z_1$, and $Z_1\gamma$ were studied. Also, we outlined some scenarios which may arise from the existence of tree-level FCNC transitions. In particular we concentrate on the $Z_1 \to tc$ and $Z_2 \to tu$ decay modes, which may be important as the third generation quarks have a different quantum number assignment. For the purpose of the numerical analysis, the bileptons were assumed to be degenerate and their masses were taken in the range $2m_Y < m_{Z_2}$. As far as the exotic quark masses, two scenarios were considered: one in which the decay $Z_2 \to \overline{Q}Q$ is kinematically allowed and another in which these particles have masses of the order of $m_{Z_2}$. It was found that the $Z_2$ boson would decay dominantly into SM and exotic quarks, with a combined branching ratio near to 100%. The remaining decay modes can be considered as rare since are considerably suppressed. In particular, the leptophobic and bileptophobic nature of the $Z_2$ boson was discussed. It was found that $Br(Z_2 \to YY^\dagger) \sim 10^{-2} - 10^{-1}$ and $Br(Z_2 \to l^+l^-) \sim 10^{-2}$ for $m_{Z_2}$ in the range 1 TeV - 3 TeV. In the case of the invisible decay $Z_2 \to \nu_l\nu_l$, its branching fraction is one order of magnitude below than that to charged leptons. As far as the anomaly-induced decays $Z_2 \to Z_1Z_1$ and $Z_2 \to Z_1\gamma$ are concerned, they are marginal, with a branching ratio of the order of $10^{-6}$ and $10^{-9}$, respectively. The tree level FCNC effects mediated by the $Z_2$ boson were studied in the up sector of the model. The branching fractions for the $Z_2 \to tc$ and $Z_2 \to tu$ decays were estimated by assuming a reasonable scenario for the elements of the FCNC mixing matrix. It was found that the corresponding branching ratios can reach the level of $10^{-3}$ and $10^{-5}$, respectively. Finally, the $Z' - Z$ mixing induced decay $Z_2 \to W^+W^-$ has a branching ratio which is sensitive to the $Z_2$ mass and may be up to $10^{-2}$ for $m_{Z_2} = 3$ TeV. This decay might be enhanced in models which allow a very heavy extra $Z$ boson. In contrast, the $Z_2 \to Z_1H$ decay has a branching ratio of the order of $10^{-5}$ and is much less sensitive to a heavier extra $Z$ boson. This decay is
expected to be strongly suppressed.

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