Magnetic susceptibility of the quark condensate via holography

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(Dated: January 13, 2010)

We discuss the holographic derivation of the magnetic susceptibility of the quark condensate. It is found that the susceptibility emerges upon the account of the Chern-Simons term in the holographic action. We demonstrate that Vainshtein’s relation is not exact in the hard wall dual model but is fulfilled with high accuracy. Some comments concerning the spectral density of the Dirac operator are presented.

PACS numbers: 11.25.Tq, 11.40.Ha, 11.15.Ex, 12.38.Aw

I. INTRODUCTION

The analysis of the QCD properties by holographic methods is one of the most promising approaches to the description of the strong coupling region. The unique holographic model for QCD has not been found yet hence there is no hope to get the generic quantitative predictions at present. However there are some QCD results which seem to be independent on the details of the dual geometry hence one could consider these universal objects or relations to test the holographic picture. On the other hand it is instructive to analyze if some relation is universal indeed testing it in the different holographic geometries.

The simplest relation to be tested is the Gell-Mann-Renes-Oaks model one which was shown to be true in all holographic models of QCD like hard wall models \cite{1}, soft wall model \cite{2} or Sakai-Sugimoto model \cite{3}. The main focus in our paper is the magnetic susceptibility of the quark condensate describing the response of the QCD vacuum on the external magnetic field. It was introduced in \cite{4} in the context of the sum rules and investigation of its numerical value was performed in \cite{5, 6, 16, 17}. More recently using QPE arguments Vainshtein \cite{7} obtained the expression for the susceptibility in terms of the known QCD quantities. However the status of this relation is questionable since both sides of the correspondence have different anomalous dimensions and it is not clear if the higher states could influence the answer. (see \cite{18, 19})

In this paper we shall analyze the magnetic susceptibility in the holographic setting and shall focus mostly at the simplest hard wall model \cite{1, 8} (introduced in \cite{21, 22}). We shall consider the calculation of the three-point function similar the two-point calculations in \cite{8, 9} and form-factor calculations in \cite{10, 11}. We shall consider the special kinematics of the three-point function related to the susceptibility. It turns out that the only nontrivial contribution to the correlator comes from the Chern-Simons term in the dual action and substituting the solutions to the classical equations of motion we get the result for the susceptibility which is close to the Vainshtein’s relation.

The paper is organized as follows. First in Section II we calculate the three-point function and the magnetic susceptibility of quark condensate in the AdS/QCD hard wall model. In Section III we survey some other approaches to the calculation of this value, namely via Chiral Perturbation theory and via the relation with the Dirac operator spectrum density. The conclusion is given in Section IV. To make the paper self consistent, we state in the Appendix some results of \cite{1, 9}, which we will use.

II. HARD WALL ADS/QCD MODEL.

A. Chern-Simons action and 3-point function

Our aim is to calculate the correlation function of two vector and one axial currents. To make the holographic calculation we take the simple "hard wall" AdS/QCD model \cite{1, 8}. We will work in notation of \cite{9} and use some results, calculated in \cite{1, 9} (see also Appendix). The holographic action involves kinetic and Chern-Simons term:

\[ S = S_M(A_L, A_R) + S_{CS}(A_L) - S_{CS}(A_R) \]

where

\[ S_{CS}(A) = \frac{N_c}{24\pi^2} \int Tr \left( AF^2 - \frac{1}{2} A^3 F + \frac{1}{10} A^5 \right) \]

It is clear that only terms containing 3 gauge fields \((A, V, V)\) are relevant for the calculation of correlator:

\[ Tr(A_L F_L F_L - A_R F_R F_R) \rightarrow \frac{1}{AVV} 2Tr(V F_V F_A + V F_A F_V + AF_V F_V) \]

The classical solutions for the fields have the form:

\[ V_\mu^2(z, Q) = \hat{V}_\mu(z, Q) \cdot V_{\mu\nu}(Q^2, z) \]

\[ A_\mu^2(z, Q) = \hat{A}_\mu(z, Q) \cdot A_{\mu\nu}(Q^2, z) \]

\[ A_{\mu\nu}(Q^2, z) = P(Q)_{\mu\nu} a_{\perp}(Q^2, z) + P(Q)_{\mu\nu} a_{\parallel}(Q^2, z) \]

\[ V_{\mu\nu}(Q^2, z) = P(Q)_{\mu\nu} \phi(Q^2, z) \]
where $\tilde{V}_s^a(Q)$ and $\tilde{A}_s^a(Q)$ provide the sources for the operators. We assume the currents to have arbitrary charges with respect to $SU(2)$ group, so the gauge group structure of the result is:

$$\langle AV\tilde{V}\rangle = \frac{\delta^3}{\delta A_3^{\mu} V^{\delta}} S_{CS} = \frac{N_C}{12\pi^2} \int (T A T V V) \cdot (V \tilde{F} F A + \tilde{V} F A_3 + AF_3 \tilde{F})$$

We will work with the bulk-to-boundary propagator similarly to calculation in [10]. The equation is [9]:

$$\langle A_{\mu}(k_1) V_{\nu}(k_2) \tilde{V}_{\rho}(k_3) \rangle = -\frac{N_C}{\pi^2} \langle T A T V V \rangle \delta^4(k_1 + k_2 + k_3)\epsilon_{\mu\nu\rho\sigma}^{\perp} \int dz (ik_{2\sigma}) a v \tilde{v} - (ik_{3\sigma}) a v \tilde{v}$$

B. Solution for $A_\perp$

Let us consider the equation of motion for $A_\perp$ at small $Q^2$ similarly to calculation in [10]. The equation is [9]:

$$\left[ \partial_z \left( \frac{1}{z} \partial_z A^\mu_z \right) + \frac{A^\mu_z}{z} - \frac{R^2 g_5^2 \Lambda^2 v^2}{z^3} A^\mu_z \right]_{\perp} = 0$$

We will work with the bulk-to-boundary propagator $a_{\perp}(z)$, which is defined in [11] and denote $R^2 g_5^2 \Lambda^2 = k^2 = 3$ ($\Lambda^2$). Using variable $y = \frac{z}{\Lambda^2} \alpha^3 z^3 = \alpha^3 z^3$, we get the equation

$$\partial^2_y a + \frac{1}{3y} \partial_y a - a = \frac{Q^2}{9\alpha^2} y^{-4/3} a$$

$$+ \frac{2k^2 m^2}{9\alpha^4} y^{-2/3} a + \frac{m^2}{9\alpha^2} y^{-4/3} a$$

which is an inhomogeneous modified Bessel equation. We introduce here the dimension parameter $\alpha$ which equals $(\frac{k^2}{\Lambda^2})^{1/3} = 395 MeV$ (see [13], [14]). One can argue that

Substituting Fourier components of fields [11] and introducing the tensors

$$\epsilon_{\mu\nu\rho\sigma}^{\perp} = \epsilon^{\mu\rho\sigma} P_{\mu \sigma}^{\perp} (k_1) P_{\nu \rho}^{\perp} (k_2) P_{\nu \rho}^{\perp} (k_3)$$

we get (denote $\tilde{v} = v(k_3)$, $\dot{v} = \partial_z v$):

$$\langle A_{\perp \mu}(k_1) V_{\nu}(k_2) \tilde{V}_{\rho}(k_3) \rangle = -\frac{N_C}{\pi^2} \langle T A T V V \rangle \delta^4(k_1 + k_2 + k_3)\epsilon_{\mu\nu\rho\sigma}^{\perp} \int dz (ik_{2\sigma}) a v \tilde{v} - (ik_{3\sigma}) a v \tilde{v}$$

We can add a surface term $(-ik_3 + ik_{2\sigma})$$\partial_z (a_{\perp} v \tilde{v})$ in the action, in order to make the 3-point function vanish, if one of vector momenta tends to zero. This will lead us to the expression:

$$\langle A_{\perp \mu}(k_1) V_{\nu}(k_2) \tilde{V}_{\rho}(k_3) \rangle = -\frac{N_C}{\pi^2} \langle T A T V V \rangle \delta^4(k_1 + k_2 + k_3)\epsilon_{\mu\nu\rho\sigma}^{\perp} \int dz (ik_{2\sigma}) a v \tilde{v} - (ik_{3\sigma}) a v \tilde{v}$$

Similarly

$$\langle A_{\parallel \mu}(k_1) V_{\nu}(k_2) \tilde{V}_{\rho}(k_3) \rangle = -\frac{N_C}{\pi^2} \langle T A T V V \rangle \delta^4(k_1 + k_2 + k_3)\epsilon_{\mu\nu\rho\sigma}^{\parallel} \int dz (ik_{2\sigma}) a v \tilde{v} - (ik_{3\sigma}) a v \tilde{v}$$

the last term is negligible and solution to homogeneous part is

$$a^{(0)}(y) = Fy^{1/3}[AI_{1/3}(y) + BK_{1/3}(y)],$$

where constants are fixed by the conditions on the IR boundary $y_m = \alpha^3 z^3 = 1.82$ (see [13], [14]):

$$\partial_z a(z)|_{z=z_m} = 3\alpha g^2 y^3 \partial_y a(y)|_{y_m} = 0$$

$$A = K_{2/3}(y_m); \quad B = I_{-2/3}(y_m)$$

and UV boundary:

$$a(z)|_{z=\infty} = Fy^{1/3} By^{-1/3} \Gamma(1/3) 2^{2/3} = 1$$

$$F = \frac{2^{2/3}}{B \Gamma(1/3)}.$$

Given this solution (which corresponds to Q=0), we can compute $f$, using the recipe, described in [11] (see
\[ f_\pi^2 = -\frac{R}{g_\sigma^2} \frac{\partial_\alpha a(z)}{z} \big|_{z=0, Q=0} = \frac{R}{g_\sigma^2} 1.815 \alpha^2 \sim (85 \text{MeV})^2 \]  
(5)

C. Solution for \( A_{||} \)

To obtain the longitudinal part of the 3-point function we need to find bulk-to-boundary propagator in the pseudoscalar sector. It is the solution to equations \( [9] \):

\[ \partial_z \left( \frac{1}{z} \partial_z \varphi^a \right) + \frac{R^2 g_\sigma^2 v^2}{z^3} (\pi - \varphi^a) = 0 \]  
(6)

\[ Q^2 \partial_z \varphi^a + \frac{R^2 g_\sigma^2 v^2}{z^2} \partial_z \pi^a = 0, \]  
(7)

where \( \varphi \) is related to the longitudinal part of \( A_\mu \) as \( A_{||\mu} = \partial_\mu \varphi \). We introduce the function \( \psi(z) = \varphi(z) - \pi(z) \), and eliminate \( \pi(z) \) from the system to get an equation on \( \psi \) with the dimensionless variable \( t = \alpha z \):

\[ t \partial_t \left( \frac{1}{t} \partial_t \psi \right) - \frac{k^2}{t^2} \psi - t \partial_t \left( \frac{1}{t} \frac{q}{t^2} + \partial_t \psi \right) = 0, \]

where \( q = Q^2 / \alpha^2 \). Now we can substitute \( v(t) = \frac{z}{\alpha} t^3 + \frac{m}{\alpha} t \) and write down terms up to the first order in \( m / \alpha \) and \( q \), assuming \( Q^2 \) to be small enough.

\[ \partial_y^2 \psi + \frac{1}{3} \partial_y^2 \psi - \psi = \frac{2 k^2 m \sigma}{9 \alpha^4} y^{-2/3} \psi \]
\[ + \frac{q}{9 y^{-4/3}} \left[ 1 - \frac{4 m}{3 y^2} \right] \psi + O \left( q^2, \frac{m \sigma}{\alpha^4} \right), \]  
(8)

where \( y = t^3 \).

The homogeneous solution, subject to the boundary conditions \( \pi(\epsilon) = 0, \varphi(\epsilon) = 1, \partial_\epsilon \pi(z_m) = \partial_\epsilon \varphi(z_m) = 0 \) is the same as for \( a_{\perp \mu} \) as expected at \( Q^2 = 0 \). The Green function of this equation is:

\[ G(u, v) = \frac{u^{1/3} v^{1/3}}{AD - BC} [AI_{1/3}(u) + BK_{1/3}(u)] \]
\[ \times [CI_{1/3}(v) + DK_{1/3}(v)] \]

with \( C \) and \( D \) defined by the condition:

\[ G(y, y')|_{y=\epsilon} = 0 \]  
\[ C = -K_{1/3}(\epsilon); \quad D = I_{1/3}(\epsilon). \]

It satisfies the equation

\[ \left[ \partial_y^2 + \frac{1}{3 y} \partial_y - 1 \right] G(y, y') = \delta(y - y') \frac{1}{y^{1/3}}. \]

We can compute the correction due to the quark mass in \([5]\). It is obtained by the integral:

\[ \psi^{(m)}(y) = \frac{2 k^2 m \sigma}{9 \alpha^4} \int_{y_m}^{y} y^{1/3} G(y, y') y'^{-2/3} \psi^{(0)}(y') \]
\[ = \frac{2 k^2 m \sigma}{9 \alpha^4} F y'^{1/3} [AI_{1/3}(y') + BK_{1/3}(y')] \int_{y_m}^{y} y^{1/3} [CI_{1/3}(y') + DK_{1/3}(y')] \]
\[ + \frac{2 k^2 m \sigma}{9 \alpha^4} F y'^{1/3} [CI_{1/3}(y') + DK_{1/3}(y')] \int_{y_m}^{y} y^{1/3} [AI_{1/3}(y') + BK_{1/3}(y')] \]

D. Magnetic susceptibility of the quark condensate

In this Subsection we calculate the magnetic susceptibility \( \chi \) of the quark condensate defined as

\[ \langle \bar{q} \sigma_{\mu \nu} q \rangle_F = \chi \langle \bar{q} q \rangle F_{\nu \mu} \]  
(10)

In order to find magnetic susceptibility , we study the 3-point function

\[ \langle A_{||\mu}(-Q) V_{\nu} (Q - k_3) \tilde{V}_\rho (k_3) \rangle \]

in the limit \( k_3 \to 0 \), according to \([7]\) where the following expression for the susceptibility has been obtained

\[ \chi = -\frac{N_c}{8 \pi^2} \frac{1}{f_\pi^2}. \]  
(11)
Consider the classical solutions for the vector fields, calculated in [1, 4].

\[ v(Q, z) = Qz \left( K_1(Qz) + \frac{K_0(Qz^2 m)}{I_0(Qz^2 m)} I_1(Qz) \right) \sim \frac{1}{Q^2} \]

and substitute them into the correlator (3):

\[ \chi \text{ get} \]

which can be matched the OPE of [7]:

\[ \langle A_{\mu}(Q) V_\nu(Q - k_3) \bar{V}_\rho(k_3) \rangle = \frac{N_C}{\pi^2} \langle T_A T_V T_\psi \rangle \epsilon^{\mu \rho \sigma \nu} (ik_3) \int dz a_{\parallel}(z) v(Q, z) = \frac{N_C}{\pi^2} \langle T_A T_V T_\psi \rangle \epsilon^{\mu \rho \sigma \nu} (ik_3) \]

\[ \times \int dz \left[ \varphi^{(0)}(z) + \varphi^{(m)}(z) \right] [Q^2 z K_0(Qz)] \]

and this expression can be matched the OPE of [7]:

\[ \langle A_{\mu}(Q) V_\nu(Q - k_3) \bar{V}_\rho(0) \rangle = \langle T_A T_V T_\psi \rangle \epsilon^{\mu \rho \sigma \nu} (ik_3) \left[ \frac{N_C}{2 \pi^2} - 1.075 \frac{N_c m \langle \bar{q} q \rangle}{Q^2 f_\pi^2} + O(1/Q^4) \right] \]

This comparison allows us to determine the magnetic susceptibility of quark condensate \( \chi \)

\[ \chi = -2.15 \frac{N_c}{8 \pi^2} \frac{1}{f_\pi^2} \quad (12) \]

in close agreement with the result of Vainshtein [11]. This agreement is parametrical, but not numerical, because due to the small value of \( f_\pi \) in our model, we get \( \chi_{\text{mod}} = 11.5 \text{ Gev}^{-2} \) which is too large. Anyway, tuning the parameters of the model (mainly \( \langle \bar{q} q \rangle \)) can help fix \( f_\pi \) to its real value and get the reasonable numerical agreement with Vainshtein’s \( \chi_{\text{Vains}} = 8.9 \text{ Gev}^{-2} \). We’ve checked that such variation of parameters do not affect coefficient in (12) significantly. Namely, its value changes less than 5%, then the parameter \( y_m \) which is proportional to \( \langle \bar{q} q \rangle / m_\rho^3 \) is varied in the wide range from 1 to 8.

### III. OTHER APPROACHES

#### A. 3-point function in ChPT

We can compute the same 3-point function in the Chiral Perturbation theory [13] and compare the result with AdS/QCD. Note that the chiral Lagrangian is derived in the Sakai-Sugimoto [3] model hence the comment below can be considered as the justification of the Vainshtein [6] relation. To obtain the \( \langle AVV \rangle \) correlator, we consider the Wess-Zumino-Witten term in ChPT action and turning on axial and vector external currents,

\[ Z_{\chi PT} = \int d^4x L_2 + Z_{WZW} \]

\[ L_2 = \frac{F^2}{4} \langle D_\mu U D^\mu U^\dagger \rangle, \]

with \( D_\mu = \partial_\mu U - i r_\mu U + i U l_\mu \)

\[ Z[U, l, r]_{WZW} = \]

\[ = - \frac{i N_c}{240 \pi^2} \int_{M^5} d^5x e^{y_{km}} \langle \Sigma_1^L \Sigma_2^L \Sigma_3^L \Sigma_4^L \Sigma_m^L \rangle \]

\[ - \frac{i N_c}{48 \pi^2} \int d^4x \epsilon^{\mu \nu \alpha \beta} \left( W(U, l, r)^{\mu \nu \alpha \beta} - W(1, l, r)^{\mu \nu \alpha \beta} \right) \]
The standard definition of the spectral density reads

$$\rho(\lambda) = \langle V^{-1} \sum_n \delta(\lambda - \lambda_n) \rangle_A$$

where $V$ is Euclidean volume and the averaging over the gluon ensemble is assumed. The value of the spectral density at the origin is fixed by the Casher-Banks relation $[15]$ while the linear term was determined comparing the different calculations of the correlator of the scalar currents $[15]$. In the perturbation theory the spectral density behaves as $\rho(\lambda) \propto \lambda^3$ that is starting from the third order the universality is lost because of the mixing with the perturbative modes. We would like to note that the magnetic susceptibility is sensitive to the last "non-perturbative" quadratic $\lambda^2$ term in the spectral density.

To explain this point let us consider the "two-point loop diagram" with tensor and vector vertexes in terms of the eigenfunctions and eigenvalues of the Dirac operator. The simple inspection shows that the susceptibility is expressed in terms of two different contributions. The first "diagonal" contribution reads as

$$m \int d\lambda \frac{\rho(\lambda)}{(\lambda^2 + m^2)^2}$$

while the second "nondiagonal" contributions involves the following integrals

$$\int d^4x \bar{u}(x) \gamma_a \gamma^\lambda \delta(x')$$

and double integrals over the eigenvalues $\int d\lambda \int d\lambda'$. The "diagonal" contribution is IR divergent and this divergence is expected to be canceled by the "nondiagonal" terms amounting to a kind of sum rules. On the other hand it is clear that quadratic term in the spectral density yields the finite contribution. It is not clear if the "nondiagonal" terms yields the IR finite contribution as well. This point does not allow us to write down the coefficient in front of the $\lambda^2$ in the spectral density immediately. One can not also exclude that more careful treatment of the IR divergences should involve the derivation a kind of the effective action with the tensor insertion. We hope to discuss these issues elsewhere.

### IV. CONCLUSION

In this paper we have derived the expression for the magnetic susceptibility of the quark condensate in the holographic QCD model. We have demonstrated that this object captures nontrivial anomalous properties of the dual model encoded in the Chern-Simons term. It vanishes if the CS term is not taken into account. The second important lesson concerns the validity of Vainshtein’s relation which is not exact but is fulfilled with the high accuracy.

The numerical value of the susceptibility do not coincide with recent estimations from the instanton liquid model $[16, 17]$, sum rules fit $[18]$ and phenomenology of D-meson decays $[19]$. But it is calculated at significantly less energy scale: for our calculation $Q \ll 1150$ Mev, while others are calculated at $Q \sim 1$ Gev, so we do not find the contradiction. This also allows us to compare the result with Vainshtein’s whose normalization point is about 0.5 Gev.

The only parameter the coefficient in Vainshtein’s relation depends on is the IR cut-off scale however the de-
pendence is very smooth. It would be interesting to discuss the soft wall model and derive the dependence of the magnetic susceptibility on temperature and chemical potential.

**ACKNOWLEDGMENTS**

We are grateful to I. Denisenko and P. Kopnin for the useful discussions. The work of A.G. was supported in part by grants, INTAS-1000008-7865, PICS-07-0292165 and of A.K. by Russian President’s Grant for Support of Scientific Schools NSh-3036.2008.2, by RFBR grant 09-02-00308 and Dynasty Foundation.

**Appendix**

In Appendix, we state some results of [1, 2] concerning the "hard wall" AdS/QCD model. The 5D coupling constant $g_5$ is fixed by the 2-point function of vector currents in [1]

$$\frac{g_5^2}{R^2} = \frac{12\pi^2}{N_c}.$$  \hspace{1cm} \text{(A.1)}

The position of the IR boundary $z_m$ is related to the $\rho$-meson mass [1].

$$z_m = \frac{1}{323 MeV}.$$  \hspace{1cm} \text{(A.2)}

The parameter $\sigma$ is coupled with the value of quark condensate (we take the value $\langle \bar{q}q \rangle = (230\text{MeV})^3$) and equals [2]:

$$\sigma = \frac{N_f \langle \bar{q}q \rangle}{3R^2\Lambda^2} = (460\text{MeV})^3.$$  \hspace{1cm} \text{(A.3)}

We shall also fix the constant $\Lambda$ correcting calculation made in [9]. First, compute the leading order solutions to the equation of motion for the pseudoscalar fields. These are solutions to the equations of motion [6, 7] with fixed boundary value of $\phi$ at $z = \epsilon$. Differentiating [7] and substituting $\partial_z \phi$ from [5] we get:

$$\partial_z^2 v^2 \frac{v^2}{x^3} - \left(\partial_z^3 \frac{v^2}{x^3}\right) \frac{v^2}{x^2} \partial_z \frac{v^2}{x^3} \partial_z \pi - Q^2 \frac{v^2}{x^2} \partial_z \pi - \frac{g_5^2 R^2 \Lambda^2 v^2}{x^4} \partial_z \pi = 0.$$  \hspace{1cm} (A.5)

We need to solve it near the boundary, so substitute asymptotic value $v(z) = mz |_{z=0}$ and denoting $x = Qz$ it takes the form:

$$\partial_z^2 \frac{1}{x} \partial_z \pi + \frac{1}{x} \partial_z^3 \frac{1}{x} \partial_z \pi - \frac{1}{x} \partial_z \frac{1}{x} \partial_z \pi + \frac{g_5^2 R^2 \Lambda^2 m^2}{Q^2} \frac{1}{x} \partial_z \pi = 0.$$  \hspace{1cm} (A.6)

At large $Q^2$ we neglect the last term and obtain the modified Bessel equation with $\lambda = 0$. Hence the solution for $\pi(z)$ reads as:

$$\pi(z) = A' Qz J_1(Qz) + B' Qz K_1(Qz) - C'.$$

and using [5] we immediately obtain the solution for $\phi$:

$$\phi(z) \rightarrow \frac{\sqrt{2} \Lambda^2 m^2}{Q^2} \, \phi(0) \, e^{-\frac{Q^2}{2}}.$$  \hspace{1cm} (A.7)

The boundary condition on $\phi$ at $z = \epsilon$ fixes the constant $B'$:

$$\phi(0, q) = \phi_0(q) = -\frac{g_5^2 R^2 \Lambda^2 m^2}{Q^2} B' + B'$$

$$B' = \frac{1}{1 - \frac{g_5^2 R^2 \Lambda^2 m^2}{Q^2}} \phi(0, q)$$

therefore we finally get:

$$\phi(z) \big|_{z=\epsilon} = \phi_0(q)$$

$$\frac{\partial_z \phi(z)}{z} \big|_{z=\epsilon} = -\frac{g_5^2 R^2 \Lambda^2 m^2}{Q^2} B' \phi_0(q) \frac{Q^2}{2} \ln(Q^2 \epsilon^2)$$

$$\pi(z) \big|_{z=\epsilon} = 0$$

We can compute the 2-point function of pseudoscalar currents, using the relation:

$$\partial_\mu (\bar{q} \gamma_5 \gamma_\mu q) = 2 m_q (\bar{q} \gamma_5 q).$$

which yields us the source for pseudoscalar current

$$2 m_q \phi \leftrightarrow (\bar{q} \gamma_5 q).$$

In order to obtain the 2-point function, we vary the action twice with respect to $2 m_q \phi(0)$ and find:

$$\delta S_\pi = \int d^4 x \frac{R}{g_5^2} \left[ \delta \phi_q \frac{\partial_z \delta \phi_q}{z} \right]_{z=\epsilon} - \Lambda^2 R^3 \left[ \delta \phi_q \frac{\partial_z \phi_q}{z} \right]_{z=\epsilon}$$

$$= \int \epsilon (q_1 + q_2) \left( \frac{RQ^2}{g_5^2} \left[ \delta \phi(q_1, z) \frac{\partial_z \phi(q_2, z)}{z} \right]_{z=\epsilon} - \Lambda^2 R^3 m^2 \left[ \delta \phi(q_1, z) \frac{\partial_z \phi(q_2, z)}{z} \right]_{z=\epsilon} \right),$$

hence, the pseudoscalar correlator is:

$$\langle J_\pi^a (q) J_\pi^b (q) \rangle =$$

$$= 2 \delta^{ab} \frac{1}{4 m^2} \frac{RQ^2}{g_5^2} \left[ -\frac{g_5^2 R^2 \Lambda^2 m^2}{Q^2} B' \frac{Q^2}{2} \ln(Q^2 \epsilon^2) \right] \rightarrow$$

$$\rightarrow m = 0 \frac{\delta^{ab}}{4} \frac{R^3 \Lambda^2 m^2}{Q^2} \ln(Q^2 \epsilon^2)$$

Comparing with the QCD value [12]:

$$\langle J_\pi^a (q) J_\pi^b (q) \rangle = \delta^{ab} \frac{N_c}{16 \pi^2} Q^2 \ln(Q^2 \epsilon^2)$$

we find

$$\Lambda^2 R^3 = \frac{N_c}{16 \pi^2} = \frac{R}{3g_5^2}$$

$$k^2 = R^2 \Lambda^2 m^2 = 3$$  \hspace{1cm} (A.4)
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