Power corrections to the event shape in $e^+e^-$ annihilation.

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We study power corrections to the event shapes in $e^+e^-$ annihilation in the two jets kinematic region, $e \ll 1$. We argue that for $e \sim \Lambda_{QCD}/Q$ all power corrections of the form $1/(Qe)^n$ have to be taken into account for an arbitrary $n$. This is achieved by introducing a new universal distribution, the shape function, which describes the energy flow in the final state. The event shape differential distributions are given by a convolution of the shape function with perturbatively resummed cross-sections. Choosing a physically motivated ansatz for the shape function, we observe a good agreement of our predictions with available data on the differential thrust, $C$-parameter and heavy-jet mass distributions and their first few moments.

1 Introduction

The experimental data for the event shape distributions in $e^+e^-$ annihilation deviate from perturbative QCD predictions by corrections suppressed by a power of the hard scale $1/Q^p$ with the exponent $p$ depending on the observable. These power corrections are associated with hadronization effects in the final states. Their contribution to the differential event shape distributions and the mean values has a different form. The leading hadronization corrections to the mean values can be parameterized by a single nonperturbative scale, whereas for the differential distribution in the end-point region, $e \sim \Lambda_{QCD}/Q$, one has to take into account an infinite set of the power corrections of the form $1/(eQ)^n$ for arbitrary $n$. Denoting by $e$ a generic event shape variable ($e = 1 - T, \rho, C$), one can write the expression for the differential distribution with perturbative and hadronization corrections included in the following general form

$$\frac{1}{\sigma_{tot}} \frac{d\sigma}{de} = \frac{d\sigma_{pert}}{de} + \frac{1}{Q^p} F_{hadr}(Q, e).$$

Here, $F_{hadr}(Q, e)$ receives both perturbative and nonperturbative contributions, which need to be disentangled in order to understand the physical origin of power corrections. We shall study the event shape distributions in the two-jet region $0 \leq e \leq e_{max}$, with the upper limit $e_{max}$ separating the regions with two and three jets in the final states. For the event shapes under consideration pQCD predictions are known to two-loop accuracy:

$$\frac{d\sigma_{pert}}{de} = \frac{\alpha_s(Q)}{2\pi} A_e(e) \theta(e_{max} - e) + \left(\frac{\alpha_s(Q)}{2\pi}\right)^2 B_e(e) + O(\alpha_s^3(Q))$$

(1)

with $A_e(e)$ and $B_e(e)$ being coefficient functions. Due to enhancement of soft and collinear gluon emissions, these functions become divergent in the end point region, $e \to 0$,

$$A_e(e) \to 0 \sim 0 \frac{4C_F}{e} \left(\ln \left(\frac{e}{e_0}\right) - 3/4\right)$$

with $t_0 = \rho_0 = 1$ and $C_0 = 6$. Moreover, large Sudakov logs $\alpha_s^N \ln^{2N-n}(e)/e$, $n \geq 0$ are present to all orders of perturbation theory. They spoil the convergence of perturbative expressions and need to be resummed. This has been done to the NLL accuracy\(^\text{[3]}\). It turned out that the resummed pQCD predictions do not fit the data in the end-point region even at the $Z$-mass scale (Fig. \textbf{1}[a]), thus calling for a better understanding of the underlying nonperturbative physics.

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2 Event shape distributions

To understand the structure of the power corrections to the event shape distributions, we first have to identify the relevant scales. In the end-point region, the final states in the $e^+e^-$ annihilation consist of two narrow quark jets surrounded by soft gluon radiation. Going through QCD analysis of such states, one finds the dynamics is driven by two different scales, the so-called soft scale, $Q_e$, and collinear one, $Q\sqrt{e}$. The soft scale $Q_e$ sets up a typical momenta of the soft gluons, while the collinear scale $Q\sqrt{e}$ determines the transverse size of the outgoing fast jets. Nonperturbative corrections to the distributions appear suppressed by powers of both scales. We notice, however, that in the end point region, $e \sim \Lambda_{QCD} Q_e$, the following hierarchy of the scales holds, $Q_e \ll Q\sqrt{e} \ll Q$. This suggests to neglect power corrections on the larger scale $Q\sqrt{e}$ and resum all corrections on the smallest scale $Q_e$. The resulting differential distribution can be written as

$$\frac{1}{\sigma_{tot}} \frac{d\sigma}{de} = \sigma_0(\alpha_s(Q), \ln(e), 1/Qe) + \mathcal{O}(1/Q^2 e).$$  \hspace{0.5cm} (2)

This approximation amounts to neglecting the internal structure of two jets and treating them as two fast classical color charges moving in the direction of jet momenta and emitting soft gluons in the final states. The resulting expression for the differential distribution, $\sigma_0(\alpha_s(Q), \ln(e), 1/Qe)$, can be described using Wilson loop approach \[^{5,6}\]. By the construction, it should resum both large perturbative Sudakov logarithms and power corrections on the smallest scale $Q_e$

$$\sigma_0 = \frac{d\sigma_{PT}}{de} + \sum_{k=1}^{\infty} \frac{\lambda_k}{(Qe)^k} \Sigma_k(\alpha_s(Q), \ln(e)).$$  \hspace{0.5cm} (3)

Here, $\lambda_k$ are some nonperturbative scales depending on the choice of the event shape variable, and $\Sigma_k(\alpha_s(Q), \ln(e))$ are perturbatively calculable coefficient functions. The equation (3) can be thought of as a generalization of the OPE expansion to the event shape distributions. For $e \gg \lambda_{QCD} / Q$, the leading nonperturbative corrections come only from the first term in the sum (3). Their net effect on the distribution is the shift of the pQCD spectrum towards larger values of the event shape \[^{3,4,7}\].

The IR renormalons models give the same prediction with $\lambda_1$ being universal nonperturbative parameter related to the integral of the coupling constant over small momentum region.

For $e \sim \frac{\Lambda_{QCD}}{Q}$ all terms in (3) become equally important and need to be resummed. This is achieved by introducing a new nonperturbative distribution, the so-called shape function. Taking the thrust as an example we have \[^{1}\]

$$\frac{1}{\sigma_{tot}} \frac{d\sigma}{dt} = \int_0^{Qt} df_t(e) \frac{d\sigma_{pert}}{dt}(t - \epsilon/Q).$$  \hspace{0.5cm} (4)

Here, $f_t(e) = \int d\epsilon_L d\epsilon_R f(\epsilon_L, \epsilon_R) \delta(\epsilon - \epsilon_L - \epsilon_R)$ with the shape function $f(\epsilon_L, \epsilon_R)$ being a universal nonperturbative distribution describing the energy flow into the left and right hemispheres in $e^+e^-$ annihilation final states. The universality of $f(\epsilon_L, \epsilon_R)$ implies that it does not depend on the hard scale $Q$.

The shape function receives contributions of two different types

$$f(\epsilon_L, \epsilon_R) = f_{incl}(\epsilon_L)f_{incl}(\epsilon_R) + \delta f_{non-incl}(\epsilon_L, \epsilon_R)$$  \hspace{0.5cm} (5)

\[^{1}\text{Similar formulae hold for the other event shape variables.}\]
The first one is the inclusive contribution from gluons produced and decaying into the same hemisphere. This kind of event is partially taken into account by IR renormalons models. The second type of contribution corresponds to non-inclusive events, that is when gluon is produced in one hemisphere but decays into particles flowing into another hemisphere. The non-inclusive contribution describes the cross talk between two hemispheres and we expect that this effect is important for non-inclusive variables like the heavy-jet mass.

The shape function is a well defined nonperturbative QCD distribution, which in present can not be calculated from the first principles. Therefore, confronting our predictions with the data we shall rely on particular ansatz for this function. Choosing the ansatz, we require that the shape function should be positively definite, vanish at high energies and have a power like behaviour at the origin due to the phase space suppression. This leads to

\[
f(\epsilon_L, \epsilon_R) = \frac{N(a, b)}{\Lambda^2} \left( \frac{\epsilon_L \epsilon_R}{\Lambda^2} \right)^{a-1} \exp \left( -\frac{\epsilon_L^2 + \epsilon_R^2 + 2b\epsilon_L \epsilon_R}{\Lambda^2} \right).
\]

Here, \(N(a, b)\) is the normalization constant, \(a\), \(b\) and \(\Lambda\) are nonperturbative parameters that have to be fixed by comparing our predictions for the event shape distributions with the data. The parameter \(a\) determines how fast the shape function vanishes at the origin, \(\Lambda\) sets up a typical scale of the soft radiation and \(b\) controls the size of non-inclusive corrections, so that \(\delta f_{\text{non-incl}}(\epsilon_L, \epsilon_R) = 0\) at \(b = 0\). We fix the parameters by confronting the predictions for the heavy-jet-mass and \(C\)–parameter distributions with the data at \(Q = M_Z\) (see Fig. 1, where we present the predictions for heavy-jet mass distribution.)

\[
a = 2 \quad b = -0.4 \quad \Lambda = 0.55 \text{ GeV}.
\]

To test universality of the shape function, one can use these values of the parameters to compare the same distributions with the data over a wide energy interval, \(35 \text{ GeV} \leq Q \leq 189 \text{ GeV}\). As was shown in, a good agreement is observed over the whole range of the event shapes including the end-point region \(e \sim \Lambda_{\text{QCD}}/Q\).

![Image](image_url)

Figure 1: (a) Heavy-jet mass distributions at \(Q = M_Z\) with and without power corrections included. (b) Comparison of the QCD predictions for heavy jet mass at different energies from top to bottom \(Q(\text{GeV}) = 35, 44, 91, 133, 161, 172, 183\).
3 Moments

Using the obtained expressions for the differential distribution for the heavy-jet mass and the $C$-parameter, we can calculate their lowest moments. This calculation can be performed provided that the moments are dominated by the contribution of two-jet final states. One can verify that this is the case for the first and the second moments. The first two moments of various event shape distributions have been measured by the LEP experiments. The obtained values deviate from the predictions of IR renormalon models, especially for the second moments.

Going through the calculation of the second moment of the $C$-parameter and heavy-jet mass distribution we find

$$
\langle \rho^2 \rangle = \langle \rho^2 \rangle_{PT} + \frac{\lambda_1}{Q^2} \langle \rho \rangle_{PT} + \frac{\lambda_2 + \delta \lambda_2}{4Q^2}
$$

$$
\langle c^2 \rangle = \langle c^2 \rangle_{PT} + \frac{3\pi \lambda_1}{2Q} \left[ 2(c)_{pert} - 4.30 \frac{\alpha_s(Q)}{2\pi} \right] + \frac{9\pi^2}{4Q^2} \left[ 1 - 11.46 \frac{\alpha_s(Q)}{2\pi} \right]
$$

(7)

where $\lambda_2$ is equal to the second moment of the shape function, $\lambda_2 = \langle \epsilon^2 \rangle = 1.7$ GeV$^2$ and $\delta \lambda_2$ measures non-inclusive contribution to the heavy-jet mass,

$$
\delta \lambda_2 = \langle (\epsilon_L - \epsilon_R)^2 \rangle \left[ 1 + 4 \int_0^{\rho_{max}} d\rho \rho \left( \frac{d\sigma_{PT}}{d\rho} \right)^2 \right]
$$

Here, $\langle (\epsilon_L - \epsilon_R)^2 \rangle = 0.14$ GeV$^2$ and $\int_0^{\rho_{max}} d\rho \rho \left( \frac{d\sigma_{PT}}{d\rho} \right)^2$ varies from 2.19 for $Q = 10$ GeV to 1.85 for $Q = 100$ GeV. We notice that non-inclusive correction $\delta \lambda_2$ does not affect the second moment of the $C$-parameter. The reason for this is that the heavy-jet mass is less inclusive observable with respect to a single jet than the $C$-parameter and, therefore, it is more sensitive to the cross-talk effects between two hemispheres. Non-inclusive effects provide important contribution to the second moment of the heavy-jet mass $\sim 14\%$. These corrections have not been taken into account in the IR-renormalons models, since there the two hemispheres were supposed to be uncorrelated. Finally, using (7) we observe a good agreement of the QCD predictions with the available data for the second moments of the heavy-jet and $C$-parameter distributions (see Fig. (2)).
4 Conclusion

The shape function approach allows to reveal the physical meaning of the nonperturbative power corrections to the event shape distributions in the $e^+e^-$—final states. The emerging structure of power corrections to the event shapes looks as follows. For $e \gg \Lambda_{QCD}/Q$ the main nonperturbative effect is the shift of the pQCD spectrum towards larger values of the shape variable, $e \to e - \lambda_e/Q$, with the scale $\lambda_e$ given by the first moment of the shape function. In the end point region, $e \sim \Lambda_{QCD}/Q$, all power corrections $\sim 1/(Qe)^n$ become equally important. They can be resummed in a new distribution – the shape function which is a universal nonperturbative distribution describing the energy flow into two hemispheres in the final state. It takes into account both inclusive and non-inclusive corrections with the latter being especially important for the observable like heavy-jet mass, which are not completely inclusive with respect to a single jet. Choosing ansatz for the shape function, we observed a good agreement with the data for the differential thrust, $C$—parameter and heavy-jet mass distributions as well as their mean values and the second moments.

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