Two–component galaxy models: phase–space constraints

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The properties of the analytical phase–space distribution function (DF) of two–component spherical self–consistent galaxy models, where one density distribution follows the Hernquist profile, and the other a $\gamma = 0$ model, with different total masses and core radii (H0 models), presented in Ciotti (1998, C98), are here summarized. A variable amount of radial Osipkov–Merritt (OM) orbital anisotropy is allowed in both components. The necessary and sufficient conditions that the model parameters must satisfy in order to correspond to a model where each one of the two distinct components has a positive DF (the so–called model consistency) are analytically derived, together with some results on the more general problem of the consistency of two–component $\gamma_1 + \gamma_2$ models. The possibility to add in a consistent way a black hole (BH) at the center of radially anisotropic $\gamma$ models is also discussed. In the particular case of H0 models, it is proved that a globally isotropic Hernquist component is consistent for any mass and core radius of the superimposed $\gamma = 0$ halo; on the contrary, only a maximum value of the core radius is allowed to the $\gamma = 0$ component when a Hernquist halo is added. The combined effect of halo concentration and orbital anisotropy is successively investigated.

1. Introduction

In the study of stellar dynamical models the fact that the Jeans equations have a physically acceptable solution is not a sufficient criterion for the validity of the model: the essential requirement to be met is the positivity of the DF of each distinct component. A model satisfying this minimal requirement is called a consistent model. In order to recover the DF of spherical models with anisotropy, the OM technique has been developed (Osipkov 1979; Merritt 1985), and numerically applied (see, e.g., Ciotti & Pellegrini 1992, CP92; Carollo, de Zeeuw, & van der Marel 1995; Ciotti & Lanzoni 1997, CL97). In the OM framework, a simple approach in order to check the consistency of spherically symmetric, multi–component models (avoiding the recovering of the DF itself), is described in CP92. It is now accepted that a fraction of the mass in galaxies is made of a dark component, whose density distribution – albeit not well constrained by observations – differs from that of the visible one (see, e.g., Bertin et al.1994; Carollo, et al.1995; Buote & Canizares 1997; Gerhard, et al.1998). Moreover, there is an increasing evidence of the presence of massive BHs at the center of most (if not all) elliptical galaxies (see, e.g., Harms et al.1994; van der Marel et al.1997; Richstone 1998). Unfortunately, only few examples of two–component systems in which both the spatial density and the DF are analytically known are at our disposal, namely the Binney–Evans model (Binney 1991; Evans 1993), and the two–component Hernquist model (HH model, Ciotti 1996, C96). It is therefore of interest the result proved in C98 that also the DF of H0 models with OM anisotropy is completely expressible in analytical way. This family of models is made by the superposition of a density distribution following the Hernquist profile (Hernquist 1990), and another density distribution following the $\gamma = 0$ profile [see eq. (3.5)], with different total masses and core radii. OM orbital anisotropy is allowed in both components. Strictly related to the last point above, is the trend shown by the numerical investigations of CP92, i.e., the difficulty of consistently superimposing
a centrally peaked distribution to a centrally flat one. More specifically, CP92 showed numerically that King (King 1972) or quasi–isothermal density profiles can not be coupled to a de Vaucouleurs (de Vaucouleurs 1948) model, because their DFs run into negative values near the model center. On the contrary, the DF of the de Vaucouleurs component is qualitatively unaffected by the presence of centrally flat halos. From this point of view, the C96 work on HH models is complementary to the investigation of CP92: in the HH models the two density components are both centrally peaked, and their DF is positive for all the possible choices of halo and galaxy masses and concentrations (in the isotropic case). The implications of these findings have been not sufficiently explored. For example, one could speculate that in presence of a centrally peaked dark matter halo, King–like elliptical galaxies should be relatively rare, or, viceversa, that a galaxy with a central power–law density profile cannot have a dark halo too flat in the center. In fact observational results on the central surface brightness profiles of elliptical galaxies (see, e.g., Jaffe et al.1994; Möller, Stiavelli, & Zeilinger 1995; Lauer et al.1995), and bulges of spirals (Carollo & Stiavelli 1998), as well as high–resolution numerical simulations of dark matter halos formation (Dubinsky & Carlberg 1991; Navarro, Frenk, & White 1997) seem to point in this direction. In C98, I explore further the trend emerged in CP92 and in C96, considering the analytical DFs of the H0 models and determining the structural and dynamical limitations imposed to them by dynamical consistency.

2. The consistency of multi–component systems

For a multi–component spherical system, where the orbital anisotropy of each component is modeled according to the OM parameterization, the DF of the density component $\rho_k$ is given by:

$$f_k(Q_k) = \frac{1}{\sqrt{8\pi^2}} \frac{d}{dQ_k} \int_0^{Q_k} \frac{d\rho_k}{d\Psi_T} \sqrt{\Psi_T - \Psi_k}, \quad \rho_k(r) = \left(1 + \frac{r^2}{r_{ak}^2}\right) \rho_k(r), \quad (2.1)$$

where $\Psi_T(r) = \sum_k \Psi_k(r)$ is the total relative potential, $Q_k = E - L^2/2r_{ak}^2$, and $0 \leq Q_k \leq \Psi_T(0)$. $E$ and $L$ are respectively the relative energy and the angular momentum modulus per unit mass, $r_a$ is the anisotropy radius, and $f_k(Q_k) = 0$ for $Q_k \leq 0$. If each $f_k$ is non negative over all the accessible phase–space, the system is consistent. In C92 it was proved that

**Theorem**: A necessary condition (NC) for the non negativity of $f_k$ given in eq. (2.1) is:

$$\frac{d\rho_k(r)}{dr} \leq 0, \quad 0 \leq r \leq \infty. \quad (2.2)$$

If the NC is satisfied, a strong (SSC) and a weak sufficient condition (WSC) for the non negativity of $f_k$ are respectively:

$$\frac{d}{dr} \left[ \frac{d\rho_k(r) r^2 \sqrt{\Psi_T(r)}}{M_T(r)} \right] \geq 0, \quad \frac{d}{dr} \left[ \frac{d\rho_k(r) r^2}{M_T(r)} \right] \geq 0, \quad 0 \leq r \leq \infty. \quad (2.3)$$

Some considerations follow looking at the previous conditions. The first is that the violation of the NC is connected only with the radial behavior of $\rho_k$ and the value of $r_{ak}$, and so this condition applies independently of any other interacting component added to the model. Even when the NC is satisfied, $f_k$ can be negative, due to the radial behavior of the integrand in eq. (2.1), which depends on the total potential, on the particular $\rho_k$, and on $r_{ak}$: so, a range of permitted values of $r_{ak}$ satisfying the NC must be discarded. Naturally, the true critical anisotropy radius is always larger than or equal to that given
by the NC, and smaller than or equal to that given by the SSC (WSC). To summarize: a model failing the NC is certainly inconsistent, and a model satisfying the SSC (WSC) is certainly consistent; the consistency of a model satisfying the NC and failing the SSC (WSC) can be proved only by direct inspection of the DF.

3. Results and conclusions

Both density distributions defining the H0 models belong to the family of the $\gamma$ models (Dehnen 1993):

$$\rho(r) = \frac{3 - \gamma}{4\pi} \frac{M r_c}{r^\gamma (r_c + r)^{4-\gamma}}, \quad 0 \leq \gamma < 3,$$

where $M$ is the total mass and $r_c$ a characteristic scale–length. The main results obtained in C98 can be summarized as follows:

(1) The NC, WSC, and SSC that the model parameters must satisfy, in order to correspond to an H0 system for which the two physically distinct components have a positive DF, are analytically derived using the method introduced in CP92. Some conditions are obtained for the wider class of two–component $\gamma_1 + \gamma_2$ models (of which the H0 models are a special case). In particular, it is shown that the DF of the $\gamma_1$ component in isotropic $\gamma_1 + \gamma_2$ models is nowhere negative, independently of the mass and concentration of the $\gamma_2$ component, whenever $1 \leq \gamma_1 < 3$ and $0 \leq \gamma_2 \leq \gamma_1$. As an interesting application of this result, it follows that a BH of any mass can be consistently added at the center of any isotropic member of the $\gamma$ models family, when $1 \leq \gamma < 3$. Two important consequences follow. The first is that the consistency of isotropic HH (or H+BH) models proved in C96 using an “ad hoc” technique is not exceptional, but a common property of a large class of two–component $\gamma$ models; for example, also isotropic two–component Jaffe [Jaffe 1983, $\gamma = 2$ in eq. (3.4)] or Jaffe+BH models can be safely assembled. The second is that in two–component isotropic models, the component with the steeper central density distribution is usually the most robust against inconsistency.

(2) It is shown that an analytical estimate of a minimum value of $r_a/r_c$ for one–component $\gamma$ models with a massive (dominant) BH at their center can be explicitly found. As expected, this minimum value decreases for increasing $\gamma$.

(3) It is shown that the analytical expression for the DF of H0 models with general OM anisotropy can be found in terms of elliptic functions; the special cases in which each one of the two density components are embedded in a dominant halo are also discussed.

(4) The region of the parameter space in which H0 models are consistent is explored using the derived DFs: it is shown that, at variance with the H component, the $\gamma = 0$ component becomes inconsistent when the halo is sufficiently concentrated, even in the isotropic case. This is an explicit example of the negative result found by CP92 described in the Introduction.

(5) The combined effect of halo concentration and orbital anisotropy is finally investigated. The trend of the minimum value for the anisotropy radius as a function of the halo concentration is qualitatively similar in both the components, and to that found for HH models in C96: a more diffuse halo allows a larger amount of anisotropy. A qualitatively new behavior is found and explained investigating the DF of the $\gamma = 0$ component in the halo–dominated case for high halo concentrations. It is analytically shown the existence of a small region in the parameter space where a sufficient amount of anisotropy can compensate the inconsistency produced by the halo concentration on the structurally analogous – but isotropic – case.
As a final remark, it can be useful to point out some general trends that emerge when comparing different one and two-component models with OM anisotropy, as those investigated numerically in CP92 and CL97, and analytically in C96 and C98. The first common trend is that OM anisotropy produces a negative DF outside the galaxy center, while the halo concentration affects mainly the DF at high (relative) energies. The second is that the possibility to sustain a strong degree of anisotropy is weakened by the presence of a very concentrated halo. The third is that in two-component models, in case of very different density profiles in the central regions, the component with the flatter density is the most “delicate” and can easily be inconsistent: particular attention should be paid in constructing such models.

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