Cost Restrained Hybrid Attacks in Power Grids
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Abstract—The frequent occurrences of cascading failures in power grids have been receiving continuous attention in recent years. An urgent task for us is to understand the cascading failure vulnerability of power grids against various kinds of attacks. We consider a cost restrained hybrid attack problem in power grids, in which both nodes and links are targeted with a limited total attack cost. We propose an attack centrality metric for a component (node or link) based on the consequence and cost of the removal of the component. Depending on the width of cascading failures considered, the attack centrality can be a local or global attack centrality. With the attack centrality, we further provide a greedy hybrid attack, and an optimal hybrid attack with the Particle Swarm Optimization (PSO) framework. Simulation results on IEEE bus test data show that the optimal hybrid attack is more efficient than the greedy hybrid attack. Furthermore, we find counterintuitively that the local centrality based algorithms are better than the global centrality based ones when the cost constraint is considered in the attack problem. Our work can help in the robustness optimization of power systems by revealing their worst-case attack vulnerability and most vulnerable components.

Index Terms—Cascading failure, network attack, power grid, constrained optimization.

I. INTRODUCTION

In power grids, a small-scale broken down of components can result in large-scale cascading failures or blackouts that cause catastrophic consequences. In the past year, large blackouts have hit many places in the world including the US, Australia, Argentina, and Venezuela. The scale and frequency of blackouts are continuously growing due to the increasing interconnection in power infrastructures. The causes of the cascading failures can be the faults at power stations, damage to electric transmission lines, and deliberate network attacks. It is of great significance to have a profound understanding of the cascading failure vulnerability of power grids.

The topic of cascading failures has received great attention from the area of network science. A branch of research work was focused on the modeling and analysis of cascading failures [1]. Ren et al. [2] proposed a cascading failure model for communication networks, in which each overloaded element has a probability of failure. Jiang et al. [3] provided a heuristic algorithm to discover key nodes in cascading failures. Chen et al. [4] further performed a critical node analysis on interdependent power and communication networks to identify the vital nodes in cascading failures. Lee et al. [5] considered the data injection attacks and provided the associated load redistribution model in cascading failures. Pu et al. [6] proposed a link centrality measure based on both topological and electrical properties and further studied the cascading failures induced by the link centrality based attacks.

On the other hand, various works have been devoted to the defense of cascading failures. Jiang et al. [7] proposed a local load redistribution mechanism with the given node capacity distribution, which is applied to the mitigation of cascading failures. Zhou et al. [8] investigated the impending breakdown prediction of the network by monitoring the critical indicators, and further provided a node addition strategy to prevent the collapse of the network under cascading failures. In order to restore the network during cascading failures, Huang et al. [9] gave the result-oriented and resource-oriented restoration approaches respectively. Xu et al. [10] discussed the optimization of the allocation of the limited recovery resources to repair the failed nodes in cascading failures. Tu et al. [11] studied how the topological metrics affect the robustness of power grids and used the simulated annealing method to find the optimal network topology against cascading failures. Wang et al. [12] discussed three optimization algorithms aiming at enhancing network robustness against cascading failures included by link attacks.

As far as we know, the existing work considered attacking either nodes or links that the hybrid attack including both types of components has been rarely reported, which, however, happens in real situation. More importantly, an attack usually has a cost in any situation, and the costs for attacking a node and a link are usually different. Since removing a node is equivalent to removing all the links associated with the node, it is reasonable to assume that the cost for attacking this node is equal to the cost for attacking all the links incident from the node. Therefore, from an adversary’s perspective, it needs to optimize the selection of nodes and links for attacks when the total attack cost is given, in order to achieve a large attack performance.

In this paper, we study the robustness of power grids against the cost restrained hybrid attacks and their resulting cascading failures. The contributions of our work are summarized as follows.

- We formalize a cost restrained hybrid attack problem. In particular, we seek the best combination of nodes and links in power grids with a given total attack cost, removing which will lead to the largest scale of cascading failure. We prove that this optimization problem is NP-hard.
- Based on the damage and cost of attacking a component, we propose an attack centrality metric. With this metric, we further provide greedy hybrid attack algorithms and optimal hybrid attack algorithms, which embody the particle swarm optimization (PSO) framework.

The rest of this paper is organized as follows. We provide the cascading failure model in Section II. In Section III, we present the cost restrained hybrid attack problem. In Section
IV, we introduce our attack centrality measurement and its related greedy and optimal attacks. Simulation results are presented in Section V. Finally, we conclude our work in Section VI.

II. MODEL OF CASCADEING FAILURES

To obtain the topological structure of a power grid, we can take the base stations and transmission lines simply as nodes and links respectively, and then the corresponding network topology can be represented by an undirected graph \( G(V, E) \), where \( V \) and \( E \) denote the sets of nodes and links, respectively. The grid network has then \( N = |V| \) nodes and \( M = |E| \) links accordingly.

Following Ref. [1], we consider two types of nodes: generation nodes and consumer nodes. Then, the corresponding admittance matrix of a power grid can be written as

\[
\begin{bmatrix}
  \ldots & Y_{ij} & \ldots \\
  \ldots & 0 & \ldots \\
  \ldots & -Y_{ji} & \ldots \\
  \ldots & \ldots & \ldots \\
\end{bmatrix}
\begin{bmatrix}
  v_i \\
  v_j \\
  I_j \\
  \ldots \\
\end{bmatrix}
= 
\begin{bmatrix}
  v_i \\
  v_j \\
  I_j \\
  \ldots \\
\end{bmatrix},
\]

which is obtained according to the Kirchhoff’s law equations. In Eq. (1), the subscripts \( i \) and \( j \) represent generation node and consumer node, respectively; \( v \) and \( I \) represent the voltage and external injected current, respectively; the element \( Y_{ij} \) is the admittance of the link connecting nodes \( i \) and \( j \), and \( Y_{ij} = 0 \) if nodes \( i \) and \( j \) are not connected. Moreover, we have \( y_i = 1 \) for a generation node \( i \) and \( Y_{jj} = -\sum_{s \neq j} Y_{js} \) for a consumer node \( j \), which are ensured by the Kirchhoff’s law. Usually, the grid topology, link admittances, voltages of generation nodes, and current consumptions of consumer nodes are given as the prior information. Then, the voltages of consumer nodes can be obtained using Eq. (1).

Following Ref. [1], we define that the load on link \((s, d)\) is equal to its current flow, \( I_{sd} = (v_s - v_d) \times Y_{sd} \); the load on node \( s \) is \( v_s \times I_{ss} \), where \( I_{ss} \) is the total current flowing out of node \( s \). The capacity (maximum allowed load) of a node is defined as \((1 + \alpha)\) times its initial load, and the capacity of a link is set to be \((1 + \beta)\) times its initial load, where \( \alpha \) and \( \beta \) are safety margins of nodes and links, respectively. If the load of a component exceeds the given capacity, the component is considered to be failed and will be removed from the grid.

In power grid, an initial small-scale failure of components can further cause the overload of other components and eventually the cascading failures. The whole process of a cascading failure in the power grid, can be modeled as follows:

1) Initially, some components are removed from the grid, which usually represents the initial attack.

2) The load of each remaining component is recalculated, and if the current load of a component exceeds the given capacity, it will be removed immediately from the grid.

3) It is possible that the grid are split into multiple isolated subgrids after the removal of components. For a subgrid containing no generator nodes, all components in this subgrid are considered as unserved and are removed accordingly.

4) Repeat steps 2) and 3) until all existing components operate under capacity constraints.

A simple way for quantifying the consequence or damage of an attack on the grid is to calculate the fraction of the failed components. Usually, we exclude the number of components manually removed at the beginning to fairly evaluate the performance of an attack. Based on this idea, the damage of an attack is given by

\[
\Phi = \frac{N_{\text{unserved}} - N_{\text{attacked}}}{D - N_{\text{attacked}}},
\]

where \( N_{\text{unserved}} \) is the total number of unserved components after cascading failures; \( N_{\text{attacked}} \) is the number of initially removed components; \( D \) is the number of all components at the beginning, which equals to \( N + M \).

III. PROBLEM FORMULATION

In our cost constraint hybrid attack model, we remove a selected set of components containing both nodes and links from the grid under a cost constraint, which acknowledges the fact that usually removing a node or link has a cost. Generally, the larger the admittance of a circuit, the larger the price of the circuit is [13]. Thus, we assume the cost of removing a link is \( \gamma \) times its admittance. Since the removal of a node is equivalent to the removal of all its incident links, we further assume that the attack cost of a node is the sum of the cost of all the links incident on the node. For component \( i \), we denote its attack cost by \( c_i \). The maximum total attack cost allowed in the attack is given to be proportional to the sum of the attack cost of all components in the grid,

\[
C_0 = \theta \sum_{i=1}^{D} c_i,
\]

where \( \theta \in [0, 1] \) is the control parameter of total attack cost \( C_0 \). The larger \( \theta \) generally means more components can be removed in the attack.

We define the solution of an attack as \( X = [x_1, x_2, \ldots, x_D] \), where \( x_i = 1 \) if component \( i \) is attacked, otherwise \( x_i = 0 \). The cost of the attack solution \( X \) can be written as

\[
C(X) = \sum_{i=1}^{D} c_i \times x_i.
\]

**Definition 1 (Optimal Component Set Problem):** Given an attack cost \( C_0 \), find the optimal component set, the removal of which will result in the largest damage \( \Phi \).

The optimal component set problem (OCSP) can be formalized as

\[
\begin{align*}
\text{maximize} & \quad \Phi(X) \\
\text{subject to} & \quad C(X) \leq C_0.
\end{align*}
\]

Note that the number of attacked components \( K \) is not given as precondition, which eventually depends on the optimal attack solution.

**Theorem 1:** The OCSP is \( NP \)-hard.

**Proof:** For a meaningful attack, the number of failed components in the cascading failure should be significantly
larger than the number of components manually removed at the beginning, i.e., \( N_{\text{unserved}} \gg N_{\text{attacked}} \). Thus, the damage \( \Phi \) is approximately linearly dependent on \( N_{\text{unserved}} \) based on Eq. (2). In other words, the OCSP is essentially equivalent to seeking an optimal constrained hybrid attack that maximizes \( N_{\text{unserved}} \). On the other hand, the problem of maximizing \( N_{\text{unserved}} \) is similar with the 0/1 knapsack problem [14]: given \( N \) items numbered 1 to \( N \), each with a weight \( w_i \) and a value \( v_i \), seek a collection \( S \) of items such that \( \sum_{i \in S} w_i \leq W \) (the total weight constraint) and the total value \( \sum_{i \in S} v_i \) is as large as possible. In the problem of maximizing \( N_{\text{unserved}} \), if we set \( D = N \), \( c_i = w_i \) and \( N_{\text{unserved}}(i) = v_i \) (\( N_{\text{unserved}} \)) represents the number of unserved components caused by the removal of component \( i \), then this problem is equivalent to the 0/1 knapsack problem. Note that we can artificially fix the double-counting of failed components in the calculation of \( N_{\text{unserved}}(i) \). Collectively, the 0/1 knapsack problem can be reduced to the OCSP in polynomial time. Since the 0/1 knapsack problem is a well known \( \text{NP-hard} \) problem, the OCSP is also \( \text{NP-hard} \).

\[ \text{IV. ATTACK CENTRALITY AND ITS APPLICATIONS IN ALGORITHM DESIGN} \]

In this section, we first propose an attack centrality metric, which considers both the damage and cost of removing a component. Then, we employ the attack centrality to design greedy and optimal algorithms. We also discuss the computational cost of the proposed algorithms.

\[ \text{A. Attack centrality} \]

Different from previous work, where the attack consequence was the only concern, we consider the attack consequence and attack cost simultaneously in our work. Specifically, we define the attack centrality of a component as the ratio of the total attack cost of the failed components caused by the removal of the component to the attack cost of the component itself. For component \( i \), its attack centrality is

\[ \psi(i) = \frac{\sum_{j \in \Omega_i} c_j}{c_i}, \]

where \( \Omega_i \) is the set of components failed in the cascading failures triggered by the removal of component \( i \). This centrality metric essentially indicates the cost-effectiveness of attacking a single component. Note that in the simulation, the computational cost increases with the increase of the width of the cascading failure, and the total cascading width may be different for different components. Thus, we consider two special cases of cascading width, i.e., width one and the total width of a cascading failure, corresponding to the local and global attack centralities, respectively. In other words, to measure the local centrality of a component, we only consider the failed components of width one in the cascading failure caused by the removal of the component, while considering all failed components in the whole cascading process to calculate its global centrality.

\[ \text{B. Greedy hybrid attack} \]

In the greedy attack, we first rank all components in the decreasing order of their attack centrality. Then, we select the components from the top of the ranking under the given constraint of the total attack cost. Finally, we remove the selected components simultaneously from the grid, and evaluate the subsequent cascading failure based on Eq. (2).

The proposed local and global attack centralities are employed in the greedy attack and thus yield two greedy attack algorithms, which are called local centrality based greedy hybrid attack (LC-GHA) and global centrality based greedy hybrid attack (GC-GHA), respectively. We also consider the random hybrid attack (RHA) for the comparative purpose, in which we randomly remove a set of components under the total attack cost limit.

The time complexity of the greedy hybrid attack is \( O(D) \), when the width of a cascading failure is ignored due to its randomness. In practice, LC-GHA always has a much lower computational cost than GC-GHA, since only width one of a cascading failure is considered in the calculation of attack centrality for LC-GHA. Also, RHA has the lowest computational complexity, since it does not need to calculate attack centrality.

\[ \text{C. Optimal hybrid attack} \]

Since the OCSP is \( \text{NP-hard} \), we use the particle warm optimization (PSO) framework to seek its optimal solution. The general procedure of the PSO in our attack scenario is as follows.

1) Initialize \( n \) particles. A particle \( i \) is represented by a \( D \)-dimensional vector \( X_i \), in which each element is independently set to a random value in \([0, 1]\). Let \( \tilde{X}_i \) be a replica of \( X_i \). We then set the largest \( K \) elements in \( \tilde{X} \) under the total attack cost constraint to 1 and the rest to 0.

2) Calculate \( \Phi(\tilde{X}_i) \) by using Eq. (2) for each particle \( i \), which is also called the fitness in PSO. Then, update the local and global optimal solutions for the particle swarm.

3) Update the velocity and position of the particles. Repeat step 2) until the maximum number of iterations \( t_{\text{max}} \) is reached.

The velocity and position of particle \( i \) update with the following equations:

\[ \begin{cases} v_{i}^{t+1} = w_i^{t} v_{i}^{t} + c_1 r_1^{t} (p_{\text{best}}^{t} - x_{i}^{t}) + c_2 r_2^{t} (g_{\text{best}}^{t} - x_{i}^{t}), \\ x_{i}^{t+1} = x_{i}^{t} + v_{i}^{t+1}, \\ w_{i}^{t+1} = w_{\text{max}} - \frac{t (w_{\text{max}} - w_{\text{min}})}{t_{\text{max}}}, \end{cases} \]

In the above equations, \( v_{i}^{t} \) is the velocity of node \( i \) at time step \( t \); \( x_{i}^{t} \) represents the position of node \( i \) at time step \( t \); \( w_{i}^{t} \) is the inertia coefficient at time step \( t \); \( w_{\text{max}} \) and \( w_{\text{min}} \) are respectively the maximum and minimum values of \( w \); \( c_1 \) and \( c_2 \) are respectively the cognitive coefficient and the social learning coefficient; both \( r_1^{t} \) and \( r_2^{t} \) are random values in \([0, 1]\), which are independently generated at time step \( t \).
The pseudocode of the centrality based optimal hybrid attack algorithm, named as OHA, can find out the optimal attack solution $X_{opt}$ under a given total attack cost constraint for a sufficiently large number of iterations. Nevertheless, in real situations $t_{max}$ is always limited, and the quality of the solution is not guaranteed. Therefore, we further improve the OHA by applying our attack centrality to the optimization of the initial solution. Specifically, in the initialization phase, we randomly select one particle and set the corresponding $X$ with the values of the attack centrality of the components. Moreover, inspired by the genetic algorithm [15], we utilize the mutation mechanism to reset the particles to avoid their falling into the local optima. Specifically, after each iteration, the mutation values of the attack centrality of the components, i.e., $x_i = \psi(k)$. Assume $X_i$ is a replica of $X_i$, and then set the largest $K$ elements in $\tilde{X}_i$ to 1 and the rest to 0 with the constraint $C(\tilde{X}_i) \leq C_0$.

For particle $\mathbf{X}$, calculate its fitness $\Phi(\mathbf{X})$.

For each particle $\mathbf{X}$, do

1. Update its fitness $\Phi(\mathbf{X})$.
2. Update the global optimal value $g_{best}$ of the particle swarm.
3. For $t = 1 : t_{max}$ do
4. Generate a random number $b \in [0, 1]$.
5. For $j = 1 : n$ do
6. If $j == b$ then
7. $X_j \leftarrow g_{best}$.
8. Select an element of $X_j$ and conduct the mutation from 0 to 1 or 1 to 0;
9. End if
10. Update the velocity and position of particle $X_j$.
11. Copy $X_j$ to $\tilde{X}_j$ and set the largest $K$ elements in $\tilde{X}_j$ to 1 and the rest to 0 under the cost limit;
12. Calculate the fitness $\Phi(\tilde{X}_j)$;
13. Update the optimal solution $p_{j_{opt}}$ of particle $j$ and the global optimal solution $g_{best}$ of the particle swarm;
14. End for
15. Return the optimal $K$-component set corresponding to $g_{best}$.

V. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed algorithms on IEEE 118 bus and IEEE 162 bus data by using MATLAB. The topologies of the networks and the generation nodes are all given in the data. For the simplification purpose, we set that the voltage of generation nodes is 1 p.u. (per unit), the current of the consumer nodes is 1 p.u., and the admittances of the links follow a normal distribution with an average value of 11 p.u. and a standard deviation of 2 p.u., which basically follow ref. [11]. The cost parameter $\gamma$ is set to be 0.3. The safety margins are usually small and thus set to be $\alpha = \beta = 0.2$ in the experiments. The parameters for the particle swarm are empirically set as $n = 50$, $c_1 = c_2 = 2$, $w_{max} = 0.9$, $w_{min} = 0.4$ and $t_{max} = 200$.

For each histogram, the light and dark colors correspond to the fractions of links and nodes among all the attacked components, respectively. We see from Fig. 1 that instead of single type of components, both links and nodes are attacked for all the three algorithms to achieve a good attack performance. This indicates that when attack cost is also a major concern, it is necessary to weigh all types of components for a cost-effective attack.

Furthermore, the damage $\Phi$ of LC-GHA is larger than the GC-GHA and RHA for the same attack cost. The evidence that LC-GHA is better than GC-GHA is quite counterintuitive, since the global centrality calculated with more information and computational cost is expected to be more effective than local centrality in a general sense. In our cost constrained hybrid attack model, the attack centrality of a component is a trade-off between the attack performance and attack cost. This yields that a node with a larger attack performance may not have a larger attack centrality than a link with a smaller attack performance, since node generally has a larger attack cost than link. Therefore, the links with large attack performance especially in the GC-GHA are attacked with high priority, which is why the fraction of attacked links is always larger.
than that of attacked nodes in the greedy attacks. The larger tendency to links in the GC-GHA compared to that in the latter. RHA always has the lowest efficiency due to its nature of randomness.

Next, we evaluate the performance of optimal hybrid attacks, i.e., LC-OHA and GC-OHA, with a comparison with OHA. Fig. 2 shows the relation between the damage $\Phi$ and the number of iterations for the three optimal hybrid attacks. We can see clearly that with the increase of the iterations, the damage generally increases and then converges for all the optimal hybrid attacks. Interestingly, LC-OHA converges faster with a higher damage than GC-OHA, which together with its lower computational cost demonstrates its advantage over GC-OHA. The OHA always has the lowest efficiency owing to no optimization of initialization.

Finally, we compare the performance of all the greedy and optimal hybrid attacks. The results of damage vs. cost constraint are given in Fig. 3, which are the average of 100 independent realizations. We see that the optimal hybrid attacks noticeably outperform the greedy hybrid attacks for achieving a larger attack performance under an arbitrary cost constraint. The decreasing order of efficiency is LC-OHA > GC-OHA > OHA > LC-GHA > GC-GHA > RHA, which further confirms that the local attack centrality is better than the global one in the cost-constrained hybrid attack problem. Note that generally the greedy attacks have a much smaller computational cost than the optimal attacks. In addition, we observe that with the relaxation of cost limit, the damage $\Phi$ first increases and then decreases after the peak for all hybrid attacks. When the allowed total attack cost is small, the damage increases with the cost, i.e., the scale of failed components in the cascading failure increases with the growth of the attack cost. While when the cost constraint is large enough, the number of failed components (except those removed at the beginning) tends to be stable as the attack cost grows, which accounts for the decrease of $\Phi$ (see Eq. (2)).

VI. CONCLUSION

In summary, we have first discussed the cost constrained hybrid attacks and their induced cascading failures in power grids. The formulated optimization problem, which is proved to be NP hard, aims at finding the optimal set of components to be attacked under a given attack cost constraint. We proposed an attack centrality measure by considering the cost-effectiveness of attacking a component, and further provided the attack centrality based greedy algorithms and optimal algorithms, which embody the framework of PSO. The performance of these algorithms have been validated through the experiments on IEEE bus data. A promising future direction would be to develop a multiple-object optimization framework for the cost-restrained attack problem. Our work helps in assessing the vulnerability of power grid infrastructures.

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