Lyα-Emitting Galaxies as a Probe of Reionization: Large-Scale Bubble Morphology and Small-Scale Absorbers

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ABSTRACT
The visibility of Lyα-emitting galaxies during the Epoch of Reionization (EoR) is controlled by both diffuse H I patches in large-scale bubble morphology and small-scale absorbers. To investigate their impact on Lyα photons, we apply a powerful and novel combination of analytic and numerical calculations to three scenarios: (i) the ‘bubble’ model, where only diffuse H I outside ionized bubbles is present; (ii) the ‘web’ model, where H I exists only in overdense self-shielded gas; and (iii) the more realistic ‘web-bubble’ model, which contains both. Our analysis confirms that there exists a degeneracy between the ionization structure of the intergalactic medium (IGM) and the global H I fraction inferred from Lyα surveys, as the three models suppress Lyα flux equally with (very) different neutral fractions. We argue that a joint analysis of the Lyα luminosity function and the rest-frame equivalent width distribution/Lyα fraction of Lyα- and UV-selected galaxies can break this degeneracy and provide constraints on the reionization history and its topology. We further show that constraints can improve if we consider the full shape of the \(M_{UV}\)-dependent redshift evolution of the Lyα fraction of Lyman break galaxies. Contrary to conventional wisdom, we find that (i) a drop of Lyα fraction larger for UV-faint than for UV-bright galaxies can be reproduced with web and web-bubble models and therefore does not provide exclusive evidence of patchy reionization, and (ii) the IGM-transmission PDF is unimodal for bubble models and bimodal in web models. Our analysis further highlights the importance of galaxy-absorber cross-correlation and its systematic measurement from the post-reionized era to the EoR. Comparing our grid of models to observations, the neutral fraction at \(z \sim 7\) is likely to be of order of tens of per cent when interpreted with bubble or web-bubble models. Alternatively, we obtain a conservative lower limit on the neutral fraction of \(\sim 1\%\) in the web models, if we allow for a drop in the photoionization rate by a factor of \(\sim 100\) from the post-reionized universe.

Key words: intergalactic medium – galaxies: high-redshift – cosmology: theory – line: formation – radiative transfer – dark ages, reionization, first stars

1 INTRODUCTION
The Epoch of Reionization (EoR) and Cosmic Dawn are the least explored frontiers in observational cosmology and extragalactic astrophysics (Loeb & Furlanetto 2013). Galaxy surveys are one of the most important pillars of modern cosmology, allowing us to study high-redshift galaxy formation and the reionization process of the intergalactic medium (IGM). Surveys of high-redshift galaxies using Lyman-break drop-out technique (Lyman Break Galaxies, LBGs) (e.g. McLure et al. 2011; Ellis et al. 2013; Bouwens et al. 2015) and narrow-band filter targeting Lyα emission (Lyman Alpha Emitters, LAEs) (e.g. Malhotra & Rhoads 2004; Hu et al. 2010; Ouchi et al. 2010) have provided a deep sample of objects, indicating that reionization requires many faint galaxies below the sensitivity limit of the surveys (Robertson et al. 2013). Furthermore, observations of QSO spectra (e.g. Fan et al. 2006; Becker et al. 2015) and the Cosmic Microwave Background (CMB) (Hinshaw, Larson & Komatsu 2013; Planck Collaboration et al. 2015; Zahn et al. 2012) offer hints that reionization is mostly completed at \(z \gtrsim 6\).

However, beyond such indications, our present observational constraints on the EoR are still scarce, regarding both the reionization history and its topology/morphology. While 21 cm experiments with radio interferometers such as LOFAR1, MWA2, GMRT3, PA-
The challenge in using Lyα emitting galaxies as a probe of reionization lies in correctly interpreting observations (Ono et al. 2012; Curtis-Lake et al. 2012; Stark, Ellis & Chiu 2010; Treu et al. 2013; Caruana et al. 2014; Iliev et al. 2014; Cassata et al. 2015). The reduced visibility of Lyα emission from galaxies at $z > 6$ has already been used to infer the global $H$ i fraction of the IGM (e.g. Dijkstra, Mesinger & Wyithe 2011; Jensen et al. 2014). However, a robust interpretation is still uncertain because of the complex radiative transfer of both ionizing and Lyα photons. The Lyα transfer involves a wide range of scales including (i) the interstellar medium (ISM), where dust and gas distribution and kinematics determine the escape fraction of Lyα photons as well as their spectral line profile (e.g. Verhamme, Schaerer & Maselli 2006; Gronke, Bull & Dijkstra 2015; Hutter et al. 2014); (ii) the circum-galactic medium (CGM), i.e. the direct environment of galaxies out to a few hundred kpc (e.g. Dijkstra, Lidz & Wyithe 2007; Zheng et al. 2010, 2011; Laursen, Sommer-Larsen & Razoumov 2011); and (iii) the IGM, which during reionization contains diffuse neutral gas surrounding large ionized bubbles which themselves contain dense, self-shielding gas clouds. In order to obtain robust constraints on the global $H$ i fraction, it is essential to understand the cosmological Lyα RT on all these scales.

The precise ionization structure of the IGM, i.e. the topology of reionization, is not only characterized by the size, abundance and distribution of large-scale ionized bubbles, but also by the small-scale dense $H$ i absorbers self-shielded against the external ionizing sources. Interpretations of Lyα-emitting galaxies contain (often implicit) assumptions about the ionization structure of the IGM, mostly because of the difficulty to cover the entire dynamic range that is required to properly describe both the small-scale dense $H$ i absorbers and the large-scale diffuse neutral IGM in reionization simulations. Two extreme assumptions, described in the first two bullets below, have been commonly adopted in the literature. Here we introduce the following terminology:

- **Bubble model**: in this model small-scale $H$ i absorbers are neglected. Under this assumption, the global $H$ i fraction measures the $H$ i content of the diffuse neutral IGM outside ionized bubbles. We refer to this as the ‘bubble model’.
- **Web model**: here only the small-scale $H$ i absorbers are considered. As this overdense gas largely traces the large-scale cosmic web, we refer to it as the ‘web model’.
- **Web-bubble model**: reality is a combination of the two extreme configurations above. We refer to cases that contain both neutral phases (diffuse and clumped) of gas as the hybrid ‘web-bubble model’. One can visualize this as the more common bubble model, but with ‘impurities’ in the ionized bubbles in the form of small-scale neutral islands.

Most previous works interpreting the observed reduction in Lyα flux from $z > 6$ galaxies have favoured a very high value of $H$ i fraction, as high as $\sim 50\%$ at $z \sim 7$ (e.g. McQuinn et al. 2007; Dijkstra, Mesinger & Wyithe 2011; Jensen et al. 2013). These studies used large-scale reionization simulations which did not have the spatial resolution to resolve the self-shielded small-scale absorbers.

The lack of self-shielding gas inside ionized bubbles in large-scale reionization simulations is clearly problematic: Lyα forest observations indicate that in the post-reionized universe, i.e. $z < 5$, $H$ i gas is locked up in damped Lyα systems (DLA) and Lyman-limit systems (LLS) (e.g. Wolfe, Gawiser, & Prochaska 2005). Self-shielded absorbers (LLSs and DLAs) are also expected to reside inside ionized bubbles during reionization (and possibly with larger number densities, see e.g. Bolton & Haehnelt 2013). The first investigations of hybrid web-bubble models have recently been reported (Mesinger et al. 2015; Choudhury et al. 2015). Interestingly, these papers still favour large values for the $H$ i fraction, as high as $\sim 40\%$ at redshift $z = 7$.

In this paper we investigate the impact of large-scale patchy reionization and small-scale $H$ i absorbers on the observed Lyα flux of galaxies, and its implication on the $H$ i fraction measurements from Lyα surveys. We explore a unique combination of cosmological hydrodynamical, radiative transfer simulations and analytic models. Our analytic framework is powerful as it facilitates the interpretation of the results of our simulations, and provides us with a tool to quickly explore a large range of parameters describing the reionization and Lyα transfer processes in future work.

The paper is organized as follows. Section 2 briefly reviews the cosmic history of the $H$ i content in the universe, ranging from the epoch of reionization to the post-reionized universe. In Section 3 we present our analytic framework of cosmological Ly$\alpha$ radiative transfer. In Section 4 we describe the methodology employed to generate the reionization models (bubble, web, and web-bubble models), as well as the intrinsic and apparent mock catalogue of Lyα-emitting galaxies. Section 5 shows our results. The conclusions and discussion about implications on Lyα-emitting galaxy surveys are then presented in Section 6.

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2 COSMOLOGICAL $H$ i CONTENT

In this section, we review the redshift evolution of the $H$ i content both during and after reionization. This can be quantified either by the mass-weighted $\langle f_{\text{HI}} \rangle M$ or the volume-weighted $\langle f_{\text{HI}} \rangle V$ neutral fraction. A compilation of current estimates in the literature is shown in Fig. 1.
2.1 Observational Constraints on H I in the Post-Reionization Epoch

The left panel of Fig. 1 clearly indicates that the post-reionized universe still contains neutral islands of gas in the form of self-shielding LLSs and DLAs. The abundance of the H I gas is generally quantified in terms of the H I column density distribution function (CDDF), \( f(N_{\text{HI}}, z) \), which is defined as \( \frac{\partial^2 N_{\text{HI}}}{\partial N_{\text{HI}} \partial z} H(z) \) (Wolfe, Gawiser, & Prochaska 2005), where \( N_{\text{HI}} \) is the number of Lya\( \alpha \) absorbers per unit H I column density and unit redshift, \( H(z) = H_0(\Omega_m(1+z)^3 + \Omega_{\Lambda} )^{1/2} \), and \( H_0 \) is the Hubble parameter today. The mass-weighted H I fraction embedded in small-scale absorbers is estimated from observations of \( f(N_{\text{HI}}, z) \) (see Appendix A).

The left panel of Fig. 1 further shows the mass-weighted H I fraction embedded in each type of neutral gas reservoir: H I gas is the high-column density Lya\( \alpha \) absorbers, mainly DLAs. The \( f_{\text{HI}} \) dominates embedded in DLAs stays approximately constant over \( 2 < z < 5 \), while the H I fraction embedded in super-LLS and LLS, which is the second dominant H I gas reservoir, increases with redshift. The diffuse IGM, represented by the Lya\( \alpha \) forest absorbers, is highly ionized and remains a minor reservoir of neutral gas.

2.2 Observational Constraints on H I During Reionization

In the right panel of Fig. 1, we have compiled various inferred values of the volume-weighted H I fraction available in the literature from CMB (Planck Collaboration et al. 2014, 2015), Gunn-Peterson optical depth (Fan et al. 2006), dark pixels (McQuinn, Mesinger & Fan 2011), Gamma Ray Burst afterglow (McQuinn et al. 2008), quasars (QSOs) near zone (Bolton et al. 2011), Schroeder, Mesinger & Haiman 2013), Lya\( \alpha \) luminosity function, equivalent width distribution, Lya \( \alpha \) fraction, and correlation function (Ouchi et al. 2010, Dijkstra, Mesinger & Wyithe 2011, Jensen et al. 2013, Mesinger et al. 2015, Choudhury et al. 2015). We also show our suggested constraint using the Lya\( \alpha \) luminosity function.
The goal of this section is to highlight the need for a hybrid web-bubble model to interpret high-z galaxy observations. We present theoretical estimates of $\langle f_{HI} \rangle_V$ using analytic models for the three different classes of ionization structure in the IGM. These calculations illustrate the redshift evolution of $\langle f_{HI} \rangle_V$ in the web (§2.3.1), bubble (§2.3.2), and hybrid web-bubble (§2.3.3) model.

### 2.3.1 $HI$ Fraction in the Web Model

In the web model, $\langle f_{HI} \rangle_V$ is expected to increase with increasing redshift due to decreasing photoionization rate, and/or increasing mean gas density (by Hubble expansion). $\langle f_{HI} \rangle_V$ can be estimated as (e.g. Miralda-Escudé, Haehnelt & Rees 2000; Bolton & Haehnelt 2007)

$$
\langle f_{HI} \rangle_V = \int_0^{\Delta_z} x_{HI}(\Delta_z) P(\Delta_z) d\Delta_z + \int_{\Delta_z}^\infty P(\Delta_z) d\Delta_z, \quad (1)
$$

where $\Delta_z$ is the baryon overdensity, $P(\Delta_z)$ is the volume-weighted overdensity probability distribution function and $\Delta_z = \Gamma z^2/3$ is the density threshold above which the gas self-shields against the UV background (Schaye 2001; Furlanetto & Oh 2005).

$\Gamma(z) = \Gamma(z = 4.75)$

$x_{HI}(\Delta_z) = x_{HI}(\Delta_t^0) n_{HI}^{com}(1 + z)^3 f_e \Delta_z/\Gamma$ is the neutral fraction obtained assuming local photoionization equilibrium with a uniform photoionization rate $\Gamma (s^{-1})$, $n_{HI}^{com}$ is the average comoving hydrogen number density, $\alpha$ is the case A recombination rate at temperature $T$, and $f_e$ is the electron fraction per hydrogen atom. The first and second term on the right hand side of equation (1) are the volume-weighted $HI$ fraction embedded in residual $HI$ in the diffuse IGM and the self-shielded gas, respectively.

We consider two models for the redshift evolution of the photoionization rate $\Gamma$: the CONST model assumes a constant $\Gamma = \Gamma(z = 4.75)$, while the DEC model assumes a photoionization rate decreasing with increasing redshift, i.e. $\Gamma(z) = \Gamma(z = 4.75)/(1 + z)/5.75^{1.5}$. Calverley et al. [2011]

The blue and cyan lines in Fig. 2 show an example of the redshift evolution of $\langle f_{HI} \rangle_V$ with the two different contributions from residual $HI$ in the diffuse IGM (residual $HI$; cyan) and neutral self-shielded gas (self-shielded $HI$; blue). The global $HI$ fraction is clearly dominated at all redshifts by the self-shielded gas. While, as expected, the $HI$ fraction increases with redshift due to the larger mean gas density, $\langle f_{HI} \rangle_V$ increases more markedly in the DEC model due to the lower photoionization rate.
\( \dot{n}_{\text{ion}}(z) = \dot{n}_{\text{ion}}(z = 4.75)((1 + z)/5.75)^{-1.5} \). This choice of redshift evolution is made to bracket the possible range of parameters satisfying the [Fan et al. 2006] constraints.

In the left panel of Fig. 2, the redshift evolution of \( \langle f_{\text{in}} \rangle \nu \) in the bubble model (red lines) shows a rapid change at \( z \sim 6 - 8 \), when \( \langle f_{\text{in}} \rangle \nu \) plummets to zero once reionization ends. A smooth transition to the post-reionized IGM, where small-scale absorbers must exist, is clearly absent from these models as no H I gas is present inside ionized bubbles. The behaviour in the CONST and DEC cases is very similar, with an earlier reionization in the former case, where a larger photoionization rate is present.

### 2.3.3 H I Fraction in the Web-Bubble Model

In the web-bubble model we assume that (i) gas inside ionized bubbles behaves as in the web model, and (ii) gas outside ionized bubbles is fully neutral. These assumptions lead to

\[
\langle f_{\text{in}} \rangle \nu = 1 - \left[ 1 - \int_{0}^{\Delta_{av}} x_{\text{in}}(\Delta_{b}) P(\Delta_{b}) d\Delta_{b} - \int_{\Delta_{av}}^{\infty} P(\Delta_{b}) d\Delta_{b} \right] Q_{\nu},
\]

where the terms in square brackets are the H I fraction inside the ionized bubbles.

The redshift evolution of \( \langle f_{\text{in}} \rangle \nu \) in the web-bubble model is shown in the right panel of Fig. 2 for a case in which the residual H I inside ionized bubbles is calculated including only the diffuse (gray lines) or the self-shielded (black lines) gas. The assumed values of the photoionization rate and ionizing photon emissivity are the same as used in the previous sections. A comparison between the left and right panels of the figure shows that the web-bubble model produces a smooth transition from the bubble model (patchy reionization) during the EoR to the web model (dominated by small-scale absorbers) in post-reionization.

Hence, to coherently explain and interpret present observations, a unified framework that includes both large-scale bubbles and small-scale absorbers is essential because (i) the presence of small-scale absorbers at lower-\( z \) is evident from observations (Fig. 1), and (ii) a smooth transition from a patchy reionization to a post-reionized IGM with small-scale absorbers is only possible within a hybrid web-bubble model (Fig. 2).

### 3 COSMOLOGICAL Ly\( \alpha \) RADIATIVE TRANSFER

In this section we present the formalism adopted to follow the cosmological Ly\( \alpha \) transfer through the reionization models discussed above.

The general equation describing line transfer in the Lagrangian fluid frame is [Mihalas & Mihalas 1984; Castor 2004; Meiksin 2009; Dijkstra 2014]

\[
\frac{1}{c} \frac{dI_{\nu}}{dt} + \mathbf{n} \cdot \nabla I_{\nu} - \frac{H + \mathbf{n} \cdot \nabla \mathbf{v}}{c} I_{\nu} + \frac{3}{c} \frac{H}{c} I_{\nu} = -\sigma_{\text{tot}} n \varphi_{\nu} I_{\nu} + \sigma_{\text{ion}} n \mathcal{R}(\nu, \nu') I_{\nu'} + \varepsilon_{\nu},
\]

where \( I_{\nu} \) is the specific intensity, \( J_{\nu} \) is the angle-averaged intensity, \( \varepsilon_{\nu} \) is the Ly\( \alpha \) emissivity, \( \mathbf{v} \) is the peculiar velocity, \( \mathbf{n} \) is the unit direction vector of rays, \( \sigma_{\text{tot}} = 0.011 \text{ cm}^{2} \text{ Hz} \) is the Ly\( \alpha \) cross section, and \( \varphi_{\nu} \) is the line profile of the Ly\( \alpha \) resonance line (units Hz\(^{-1}\)). The \( \mathbf{n} \cdot \nabla \mathbf{v} \) term is the Doppler shift effect and \( \mathcal{R}(\nu, \nu') \) the redistribution function describing the resonant scattering of Ly\( \alpha \) photons.

There are generally no analytic solutions to equation (5). However, by performing a separation of scales, the problem can be simplified: multiple scattering effects are predominant on ISM scales because the surface brightness of Ly\( \alpha \) photons that are scattered back into the line-of-sight at IGM scales is typically negligibly small. As scatterings on such small ISM scales can be effectively treated as a modification of the intrinsic line profile, and the scattering term can be overall neglected [Laursen, Sommer-Larsen, & Razoumov 2011].

Equation (5) can then be readily integrated along a line-of-sight to give the so-called ‘\( e^{-r_{\alpha}} \) approximation’ ([e.g. Haardt & Madau 1996; Meiksin 2009; McQuinn et al. 2007]. In this approximation, the Ly\( \alpha \) flux \( F_{\alpha} \) observed from a Ly\( \alpha \)-emitting galaxy at redshift \( z_{s} \) is given by

\[
F_{\alpha} = \frac{L_{\alpha}}{4\pi D_{L}^{2}(z_{s})} \int S_{\nu}(\nu_{e}) e^{-\tau_{\alpha}(\nu_{e})} d\nu_{e} = \frac{L_{\alpha} T_{\text{IGM}}}{4\pi D_{L}^{2}(z_{s})},
\]

where \( \nu_{e} \) is the frequency of the Ly\( \alpha \) photon when it is emitted, \( D_{L}(z_{s}) \) is the luminosity distance, \( L_{\alpha} \) is the intrinsic bolometric Ly\( \alpha \) luminosity (in units of erg s\(^{-1}\)), \( S_{\nu}(\nu_{e}) \) (in units of Hz\(^{-1}\)) is the effective intrinsic line profile (including the effect of the ISM/CGM) normalized such that \( \int S_{\nu}(\nu_{e}) d\nu = 1 \), \( T_{\text{IGM}} = \int S_{\nu}(\nu_{e}) e^{-\tau_{\alpha}(\nu_{e})} d\nu_{e} \) denotes the IGM transmission factor [Dijkstra, Mesinger, & Wyithe 2011], and the Ly\( \alpha \) optical depth \( \tau_{\alpha}(\nu_{e}) \) is

\[
\tau_{\alpha}(\nu_{e}) = \sigma_{\alpha} \int_{0}^{\nu} d\nu' n_{\text{H}_{2}}(\nu')(\varphi_{\nu} \left[ T, \nu_{e} \left( 1 - \frac{v_{\text{disp}}(\nu_{e})}{c} \right) \right] \).
\]

where \( T \) is the gas temperature and \( v_{\text{disp}} = H(z_{s}) \nu + v_{\text{flow}} \) is the sum of the Hubble flow and the peculiar velocity. It is customary to express \( \nu_{e} \) in terms of a velocity shift, i.e. \( \Delta \nu/c = 1 - \nu_{e}/\nu_{e} \). In the following we will use this convention.

We would like to note here that by using the \( e^{-r_{\alpha}} \) approximation we ignore photons that scatter back into the line-of-sight, which would give rise to a low surface brightness ‘fuzz’ [Laursen, Sommer-Larsen, & Razoumov 2011] compared the \( e^{-r_{\alpha}} \) approximation to a full Monte-Carlo Ly\( \alpha \) radiative transfer approach finding that the \( e^{-r_{\alpha}} \) approximation provides a good description of the transfer through the IGM as long as this is assumed to start at a distance larger than 1.5 times the virial radius of the dark matter halo hosting a Ly\( \alpha \) galaxy. We have verified that this condition is met throughout our work.

We also introduce the mean IGM transmission factor and effective optical depth to characterize the typical impact of the intergalactic environment around Ly\( \alpha \)-emitting galaxies. The mean Ly\( \alpha \) flux of many Ly\( \alpha \)-emitting galaxies is \( \langle F_{\alpha} \rangle \approx \langle L_{\alpha} \rangle / \langle T_{\text{IGM}} \rangle / (4\pi D_{L}^{2}) \), where

\[
\langle T_{\text{IGM}} \rangle \approx \int \langle S_{\nu}(\nu_{e}) \rangle e^{-\tau_{\alpha}^{\text{eff}}(\nu_{e})} d\nu_{e}
\]

is the mean IGM transmission factor and \( \tau_{\alpha}^{\text{eff}} = -\ln(e^{-\tau_{\alpha}}) \) is the effective optical depth (e.g. [Haardt & Madau 1996]). Here we have assumed that the intrinsic line profiles of Ly\( \alpha \) galaxies and the optical depth of the IGM are uncorrelated, i.e. that \( \langle S_{\nu}(\nu_{e}) e^{-\tau_{\alpha}^{\text{eff}}(\nu_{e})} \rangle \approx \langle S_{\nu}(\nu_{e}) \rangle e^{-\tau_{\alpha}^{\text{eff}}(\nu_{e})} \).

The optical depth contribution from different intervening IGM absorbers (the diffuse neutral IGM outside ionized bubbles and the small-scale absorbers) is additive, i.e. \( \tau_{\alpha} = \tau_{\text{bubble}} + \tau_{\text{web}} \). The same applies to the effective optical depth, i.e. \( \tau_{\alpha}^{\text{eff}} = \tau_{\text{bubble}}^{\text{eff}} + \tau_{\text{web}}^{\text{eff}} \). In the bubble [web] model of reionization we ignore \( \tau_{\text{bubble}} \) [\( \tau_{\text{web}} \)], while in the web-bubble model we include both. These two terms are discussed in more detail in the following sections.
3.1 \( Ly\alpha \) Opacity from Large-Scale \( H\alpha \) Patches

In the bubble model the \( Ly\alpha \) optical depth is due to diffuse expanding neutral IGM outside ionized bubbles (\( H\alpha \) patches). The \( Ly\alpha \) optical depth of a homogeneous \( H\alpha \) patch extending between comoving distance from a \( Ly\alpha \)-emitting galaxy \( R_1 \) and \( R_2 \), can be written as [Mesinger & Furlanetto 2008; Dijkstra 2014]

\[
\tau_{\text{patch}}(v_\alpha, R_1, R_2) = \tau_{GP} \int_{x(v_\alpha, R_2)}^{x(v_\alpha, R_1)} \phi(x)dx,
\]

where \( \phi = \Delta v_D \varphi_\nu \) is the dimensionless line profile, \( \tau_{GP} = c\sigma_\alpha n_\text{HI} m_p \approx 4.44 \times 10^6 [(1 + z_s)/7.6]^{3/2} \) is the Gunn-Peterson optical depth, and \( x(v_\alpha, R) = \{v_\alpha(1 - H(z_s)R/[1(1 + z_s)c]) - v_\alpha\}/\Delta v_D \). \( \Delta v_D = \frac{2k_B T_s}{m_p} \) is the Doppler width, with \( k_B \) Boltzmann constant and \( m_p \) proton mass.

In general, the \( Ly\alpha \) optical depth along a line-of-sight in the bubble model is given by:

\[
\tau_{\text{bubble}}(v_\alpha) = \sum_{i=1}^{n_b} \tau_{\text{patch}}(v_\alpha, R_{1,i}, R_{2,i}),
\]

where \( R_{i,1} \bullet [R_{2,i}] \) is the near [far] side of the edge of the \( i \)-th \( H\alpha \) patch. The effective optical depth through an ensemble of \( H\alpha \) patches is

\[
e^{-\tau_{\text{bubble}}(v_\alpha)} = \int e^{-\tau_{\text{bubble}}(v_\alpha)} P[\tau_{\text{bubble}}(v_\alpha)]d\tau_{\text{bubble}}(v_\alpha),
\]

where \( P[\tau_{\text{bubble}}(v_\alpha)] \) denotes the probability distribution for \( \tau_{\text{bubble}}(v_\alpha) \), which must be obtained from cosmological realizations of the bubble model.

There is a simpler limiting analytic case if we assume an ensemble of single large \( H\alpha \) patches along all lines-of-sight. In the limit of a large \( H\alpha \) patch \( (R_2 \to \infty) \), the optical depth along a line-of-sight is \( \tau_{\text{patch}}(v_\alpha, R_1) \approx \tau_{GP} \frac{\Delta v_D}{4\pi v_\alpha} \frac{\alpha}{\Omega} \left[1 - \left(\frac{4\pi \alpha R_1}{H(z_s)}(1 + z_s)c\right)^2 \right]^{-1} \), where \( \Delta v_D = 6.25 \times 10^5 \text{ s}^{-1} \) is the damping coefficient [e.g. Miralda-Escude 1998; McQuinn et al. 2008]. Then, for an ensemble of large \( H\alpha \) patches we can evaluate the effective optical depth as

\[
e^{-\tau_{\text{bubble}}(v_\alpha)} \approx \int e^{-\tau_{\text{patch}}(v_\alpha, R_1)} P(R_1)dR_1,
\]

where \( P(R_1)dR_1 \) is the probability to find the near side of a \( H\alpha \) patch at a distance \( R_1 \) from a \( Ly\alpha \)-emitting galaxy (for a related definition of bubble size distribution, see [Mesinger & Furlanetto 2007]). We model \( P(R_1) \) as a Schechter function, \( P(R_1) \propto R_1^{-1} \exp(-R_1/R_1) \), normalized as \( \int P(R_1)dR_1 = 1 \). \( \alpha_1 \) and \( R_1 \) are free parameters. We compare this analytic estimate of the effective optical depth to numerical calculations in §5.2.

3.2 \( Ly\alpha \) Opacity from Small-Scale Absorbers

In the web model the \( H\alpha \) gas is distributed in a collection of self-shielded absorbers. Each absorber is characterized by its \( H\alpha \) column density, \( N_{\text{HI}} \), and its proper velocity, \( v_\alpha \), relative to a given \( Ly\alpha \)-emitting galaxy. The \( Ly\alpha \) optical depth through a single absorber is

\[
\tau_{\text{abs}}(v_\alpha) = \sigma_\alpha N_{\text{HI}} \varphi_\nu \left[ T_s(v_\alpha) \left(1 - \frac{v_\alpha}{c}\right)\right],
\]

where \( T_s \) denotes the gas temperature of an absorber.

We introduce a novel analytic model of the \( Ly\alpha \) opacity from small-scale absorbers as follows. The effective optical depth of an ensemble of \( H\alpha \) absorbers surrounding a \( Ly\alpha \)-emitting galaxy is (see Appendix B for a derivation)

\[
\tau_{\text{web}}(v_\alpha) = \int dN_{\text{HI}} \frac{\partial^2 N_{\text{HI}}}{\partial v_\alpha \partial z} \left| \frac{dz}{dv_\alpha} \right| \times
\]

\[
\int \frac{dv_\alpha}{H(z_s)} [1 + \xi(v_\alpha, N_{\text{HI}})] \left[1 - e^{-\tau_{\text{abs}}(v_\alpha, v_\nu, N_{\text{HI}})} \right],
\]

where \( \xi(v_\alpha, N_{\text{HI}}) \) is the galaxy-absorber correlation function in velocity space. We refer to a Gaussian streaming model (GSM) for \( \xi(v_\alpha, N_{\text{HI}}) \) when

\[
1 + \xi(v_\alpha, N_{\text{HI}}) = \frac{aH\nu r_{H1}}{\sqrt{2\pi}\sigma_{H1}^2} (1 + \xi(v_{12})) \exp \left[ -\frac{(v_\alpha - aH\nu r_{H1})^2}{2\sigma_{H1}^2} \right],
\]

where \( r_{H1} \) is the comoving separation between a galaxy and an absorber, \( \xi(v_{12}) \) is the real-space galaxy-absorber correlation function. \( (v_{12}(r_{12})) \) is the mean radial pairwise velocity [pairwise velocity dispersion] between galaxy-absorber pairs, and \( a = (1 + z_s)^{-1} \) is the scale factor.

3.2.1 The Region of Influence

As the optical depth depends on \( v_\alpha \), it is useful to calculate the ‘critical’ velocity, \( v_{\text{crit}} \), at which the optical depth of an absorber to a \( Ly\alpha \) photon emitted at frequency \( v_\nu \) becomes unity for a given \( H\alpha \) column density, i.e. \( \tau_{\text{abs}}(v_\alpha = v_{\text{crit}}) = 1 \). In fact, to first order, the \( Ly\alpha \) visibility is only affected by small-scale absorbers moving away from a central \( Ly\alpha \)-emitting galaxy with \( v_\alpha < v_{\text{crit}} \). We refer to the region that contains these absorbers as the ‘region of influence’. For high-column density absorbers such as LLS/DLA, the above condition is met in the wing of the absorption line profile \( \varphi_\nu \approx \Lambda [4\pi^2 (v_\alpha - v_\nu/c) - v_\nu/c]^2/\nu_\alpha^2 \). From the Lorentz wing it follows that for an absorber with \( H\alpha \) column density \( N_{\text{HI}} \),

\[
\frac{v_{\text{crit}}}{c} = 1 - \frac{v_\nu}{v_\alpha} \left(1 - \frac{\sigma_\alpha N_{\text{HI}}}{4\pi^2 c^2} \right).
\]

If we set \( v_\alpha = v_\nu \), then \( v_{\text{crit}} = c \sqrt{\frac{\sigma_\alpha N_{\text{HI}}}{4\pi^2 c^2}} = 507.3(N_{\text{HI}}/10^{20} \text{ cm}^{-2})^{1/2} \text{ km s}^{-1} \). For a pure Hubble flow, the critical velocity corresponds to the comoving distance

\[
D_{\text{crit}} = \frac{v_{\text{crit}}}{H_0} \left[\Omega_m(1 + z)^3 + \Omega_\Lambda\right]^{1/2}.
\]

As a reference, \( D_{\text{crit}} = 3.5(N_{\text{HI}}/10^{20} \text{ cm}^{-2})^{1/2}h^{-1}\text{Mpc} \) at \( z = 7 \).

Armed with the analytic framework of \( Ly\alpha \) transfer described above to aid the understanding of our results, in the next section we perform cosmological hydrodynamical, radiative transfer simulations and derive a mock survey of \( Ly\alpha \)-emitting galaxies with various reionization models.

4 SIMULATIONS

In this section we describe the simulations used to model the observability of high redshift \( Ly\alpha \)-emitting galaxies, and the mock galaxy catalogue obtained from them.
We emphasize that the purpose of these simulations is not to produce the best possible reionization model, but to explore the impact of large-scale patchy reionization features (i.e., ionized bubbles) and small-scale absorbers on the observability of Lya-emitting galaxies and on the inference of $\langle f_{\text{HI}} \rangle$ using Lya surveys.

4.2 Bubble Models

We use the cosmological radiative transfer code CRASH (Ciardi et al. 2001) and (Maselli, Ferrara & Ciardi 2003; Maselli, Ciardi & Kanekar 2009) to generate our bubble models. For the full RT computation, we post-process the density and temperature fields on the 256$^3$ cells of the hydrodynamical simulation. While our box size is not sufficient to include the largest ionized bubbles present during the later stages of reionization, this does not affect the goal of the paper.

The model for the ionizing sources is based on the one described in Ciardi et al. (2001), the volume averaged ionizing emissivity, $n_{\text{ion}}$ (photons s$^{-1}$ cm$^{-3}$), at $z > 6$ is parameterized as $n_{\text{ion}}(z) = 10^{50.89} \alpha(z)^{2.24} \left( \frac{\Gamma_{-12}(z=6)}{0.19} \right)$, where $\alpha(z) = ae^{b(z-9)} \left[ a - b e^{n(z-9)} \right]^{-1}$, with $a = 14/15$ and $b = 2/3$ (see Bolton & Haehnelt 2007). The values of $\Gamma_{-12}(z = 6)$ are shown in Table 1. We assume that the ionizing emissivity is produced by galaxies with a power-law spectrum of slope $\alpha_{\text{gal}} = 3$, and we distribute it among all haloes proportionally to their mass.

We ran the radiative transfer simulation using 10 gas density and temperature snapshots from $z = 15$ to $z = 5$ equally spaced in redshift, including both hydrogen and helium with a number fraction 0.92 and 0.08, respectively. For each source, we emit $10^6$ photon packets distributed according to the power-law spectrum with 29 frequency bins sampled from 13.6 eV to 200 eV.

Finally, we produce a catalogue of bubble models for different values of $\Gamma_{-12}(z = 6)$. Slices through these models are shown in the top panels of Figs. 3 and 4. In Fig. 3 the maps of HI number density clearly show that the global HI fraction increases as $\Gamma_{-12}(z = 6)$ decreases (from left to right), as expected. More specifically, a volume-weighted HI fraction of $\langle f_{\text{HI}} \rangle$ is $0.365, 0.676$ and $0.990$ obtained at $z = 7$ for $\Gamma_{-12}(z = 6) = 0.380, 0.190$ and 0.019, respectively. Furthermore, the figures show the characteristic feature of patchy reionization, i.e., large-scale bubbles.

Since there is a one-to-one correspondence between $\Gamma_{-12}(z = 6)$ and $\langle f_{\text{HI}} \rangle (z = 7)$, we will use them interchangeably to specify the model.

4.2.2 Web Models

We use the prescription of Rahmati et al. (2013) to account for self-shielding gas in the web models, which consists of a fitting function matched to their full RT transfer simulation. This prescription assumes photoionization equilibrium in each cell of the simulation with a modified background (see below), i.e.

$$\alpha_{\text{A}}(T) n_{\text{HII}} n_e = \Gamma_{\text{Rah}} n_{\text{HII}}$$

where $n_e$ is the electron number density, and $\Gamma_{\text{Rah}}$ is the modified photoionization rate. The neutral fraction in each cell is then given by $z_{\text{HI}} = (\gamma + 2 - \sqrt{(\gamma + 2)^2 - 4})/2$, where $\gamma = \Gamma_{\text{Rah}}/(\alpha_{\text{A}} n_{\text{HII}} f_e)$. The factor $f_e = n_e/n_{\text{HII}}$ is 1 for a pure hydrogen medium, while $f_e > 1$ if helium is included. We assume

| bubble model | $\Gamma_{-12}(z=6)$ | $\langle f_{\text{HI}} \rangle (z=7)$ |
|--------------|---------------------|-----------------------------------|
| B1           | 0.380               | 0.365                             |
| B2           | 0.190               | 0.676                             |
| B3           | 0.019               | 0.990                             |

\[ \text{Table 1. List of reionization models. The columns indicate, from left to right, the model and its name, the photoionization rate in terms of } \Gamma_{-12} = \Gamma / 10^{-12} \text{ s}^{-1} \text{ as assumed at } z = 6 \text{[7]} \text{ in the bubble [web] model, and the resulting volume-weighted H I fraction, } \langle f_{\text{HI}} \rangle, \text{ at } z = 7. \]
Figure 3. Neutral hydrogen number density, $n_{\text{HI}}$, at $z = 7$ in slices of our simulations for the bubble (B1, B2 and B3; top panels) and web (W1, W2 and W3; bottom panels) models detailed in Table 1. Each snapshot is a $x$-$y$ slice at $12.5h^{-1}\text{Mpc}$ with $97.7h^{-1}\text{ckpc}$ thickness. Panels in the same column give a similar suppression of the Ly$\alpha$ visibility in the observed Ly$\alpha$ luminosity functions shown in Fig. 6.

Figure 4. Same as in Fig. 3 but for the local H I fraction $x_{\text{HI}}$. 
that the IGM temperature is $T \sim 10^{4}$K due to photoionization heating. The modified local photoionization rate is given by

$$\Gamma_{\text{goh}} = 0.98 \left[ 1 + \left( \frac{n_{\text{H}}}{n_{SS}} \right)^{1.64} \right]^{-2.28} + 0.02 \left[ 1 + \left( \frac{n_{\text{H}}}{n_{SS}} \right)^{-0.84} \right]^{-2.28},$$

where $n_{SS}$ is the density at which the gas starts to be self-shielded

$$n_{SS} = 6.73 \times 10^{-3} \Gamma_{-12}^{-2/3} \left( \frac{T}{10^{4} \text{K}} \right)^{0.17} \text{cm}^{-3}.$$

To compute $n_{\text{goh}}$ using the above prescription we use the density field of the finest AMR level 102$^4$ from the hydrodynamical simulation.

The values adopted for the uniform photoionization rate $\Gamma_{-12}$ are found in Table I. Slices through our web models are reported in the lower panels of Figs. 3 and 4. Similarly to the bubble models, the maps show a higher neutral fraction for decreasing photoionization rate. However, the ionization structure of the IGM is significantly different, as the neutral gas is concentrated in high density peaks where small-scale absorbers, whose distribution follows the structure of the cosmic web, reside.

4.2.3 Web-Bubble Models

We generate the web-bubble models at $z = 7$ as follows. First, we take a full RT simulation used to generate the bubble models (B1 and B2). Then, we recalculate the local $\text{H} \text{I}$ fraction inside the ionized bubbles according to the web model with a photoionization rate $\Gamma_{-12}(z = 7) = 0.01$ and 0.005 (W2 and W3) on the finest AMR grid. In practice, $x_{\text{HI}}$ is calculated locally as the maximum between the values obtained from the bubble and the web model. Our web-bubble models are catalogued in Table I.

Slices through the web-bubble models are shown in Fig. 5 in terms of $x_{\text{HI}}$ map. As expected, the evolution of $x_{\text{HI}}$ with photoionization rate is the same as the one in the web and bubble models. Quantitatively, though, the neutral fraction here is slightly higher than the one in the corresponding bubble models due to the contribution of small-scale absorbers (see Table I). In addition, the ionization structure of the IGM looks like a combination of the one from the bubble and web models, as the small-scale absorbers appear as impurities inside large-scale ionized bubbles.

4.3 Mock Galaxy Catalogue

The observed Ly$\alpha$ luminosity of a galaxy is related to its intrinsic Ly$\alpha$ luminosity via the IGM transmission factor $T_{\text{IGM}}$ as $L_{\alpha}^{\text{IGM}} = T_{\text{IGM}} L_{\alpha}$ (this is discussed more in detail in § 3 and see also Appendix D for more technical aspects). We stress that 'intrinsic' here refers to the Ly$\alpha$ luminosity that a galaxy would have if the IGM were transparent. As our main results are insensitive to the precise model for the intrinsic luminosity function (LF), we only briefly describe the methodology applied to generate the intrinsic mock galaxy catalogue.

We use the abundance matching technique (e.g. Peacock & Smith 2000; see also Appendix C) to populate dark matter haloes with Ly$\alpha$ emitting galaxies. We find the relation between halo mass and intrinsic Ly$\alpha$ luminosity by equating the observed cumulative Ly$\alpha$ luminosity function $n(> L_{\alpha})$ (in units of cMpc$^{-3}$) at $z = 5.7$ (Ouchi et al. 2008) to the simulated halo mass function $dn(> M_{h})/dM_{h}$ at $z = 7$,

$$n(> L_{\alpha}) = f_{\text{duty}} \int_{M_{h}(L_{\alpha})}^{\infty} \frac{dn(> M_{h})}{dM_{h}} dM_{h},$$

where $f_{\text{duty}}$ is the duty cycle and $M_{h}(L_{\alpha})$ is the halo mass corresponding to a Ly$\alpha$ luminosity $L_{\alpha}$. We thus assume that the intrinsic Ly$\alpha$ luminosity function at $z = 7$ is equal to the observed one at $z = 5.7$, and that the difference between $z = 5.7$ and $z = 7$ is entirely due to the IGM. We therefore constrain the IGM opacity using the variation of the Ly$\alpha$ LF relative to that in the post-

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16 As a comparison, we have also calculated $n_{\text{goh}}$ using a threshold method, in which all the cells with gas density above $n_{SS}$ are assumed to be fully neutral, otherwise the neutral fraction is computed assuming photoionization equilibrium with $\Gamma$ rather than $\Gamma_{\text{goh}}$, i.e. $\alpha(T)n_{\text{H}}n_{e} = \Gamma n_{\text{H}}$. We note that while mapping between assumed photoionization rate and the abundance of small-scale absorbers changes depending on the prescription, as long as $(f_{\text{HI}})_{\nu}$ embedded inside small-scale absorbers is similar, the result is insensitive to the self-shielding prescription. Hence, the quantity that more directly impact the observation of Ly$\alpha$-emitting galaxies is the number density of small-scale absorbers rather than the photoionization rate.

17 Note that web models are not equivalent to outside-in reionization scenarios. They simply show the region of the universe that is reionized early in an inside-out scenario, with residual self-shielded H I.

18 While in principle the photoionization rate inside bubbles is not independent of bubble size, we take this as a convenient free ad hoc parameter to adjust the abundance of self-shielded absorbers inside bubbles.
reionization Universe. The abundance matching technique gives a semi-empirical relation between the halo mass and the intrinsic Lyα luminosity for each $f_{\text{HI}}$ (examples are shown in Fig. C1). In our fiducial case we use $f_{\text{HI}} = 1$. We then populate each halo with a single Lyα-emitting galaxy of intrinsic Lyα luminosity given by the $M_h - L_{\alpha}$ relation.

Because observations are available only down to $\log_{10}[L_{\alpha}/(\text{erg s}^{-1})] \approx 42.5$, to extend the calculations to lower luminosities we extrapolate assuming a faint-end slope of $1.5$ (Ouchi et al. 2008, but see Gronke et al. 2015, Dressler et al. 2015) for both theoretical and observational support for significantly steeper slopes of $\approx 2.2$). We note that, because of the small box size (which is needed to include small-scale absorbers and the faint galaxies responsible for reionization), the simulated LFs only extend to $\log_{10}[L_{\alpha}/(\text{erg s}^{-1})] \approx 42.8$.

We model the Lyα transfer in the ISM/CGM through the Lyα spectral line profile (e.g. Dijkstra, Mesinger & Wyithe 2011, Jensen et al. 2014, Choudhury et al. 2015), by assuming a Gaussian profile with circular velocity $v_{\text{circ}} = 20.4h^{1/3}(M_h/10^8M_\odot)^{1/3}[(1+z)/7.0]^{1/2}$ km s$^{-1}$ (Santos 2004, Zheng et al. 2010), shifted redward by $\Delta v = 600$ km s$^{-1}$ to mimic the effect of scattering through a galactic wind (Dijkstra & Wyithe 2010, Dijkstra, Mesinger & Wyithe 2011). This is rather arbitrary, but Steidel et al. (2010) and Willott et al. (2015) justify a number between 200-800 km s$^{-1}$. While the quantitative results are affected by this choice, the qualitative conclusions in this paper remain valid. We point out that our model assumes a universal line profile and shift, while a distribution is more likely. Since $T_{\text{TGM}}$ is highest for redshifted Lyα lines, this can bias samples of Lyα-selected galaxies to larger $\Delta v$.

5 RESULTS

5.1 Lyα Luminosity Function

We first show the impact of large-scale H I patches and small-scale absorbers on the Lyα luminosity function in Fig. 6, which contains the differential intrinsic Lyα luminosity function of galaxies (black solid line) together with the predicted apparent luminosity function for our bubble (red lines), web (blue lines), and web-bubble (green lines) models with different values of $\langle f_{\text{HI}} \rangle_V$. Fig. 6 shows that

- The predicted luminosity function decreases with $\langle f_{\text{HI}} \rangle_V$ as naturally expected, because more neutral hydrogen in the universe increases the overall opacity to Lyα photons.
- The relative abundance of large-scale bubbles and small-scale absorbers decreases with decreasing $\langle f_{\text{HI}} \rangle_V$. This decrease is in part due to the increase in the consumption of neutral hydrogen in the bubbles and web-bubble models. However, the effect is also due to the large-scale bubbles absorbing more Lyα photons, as can be seen in Fig. 7, which shows the fraction of Lyα photons absorbed in the large-scale bubbles and small-scale absorbers as a function of redshift. The fraction of absorbed photons increases with decreasing $\langle f_{\text{HI}} \rangle_V$, which indicates that the bubbles and web-bubble models are more effective at absorbing Lyα photons than the web model.
absorbers is a key factor to estimate the observed Lyα luminosity function. The bubble, web, and web-bubble models predict almost identical luminosity functions for vastly different \( \langle f_{\text{HI}} \rangle \). For example, a bubble model with \( \langle f_{\text{HI}} \rangle \sim 0.676 \) (B2) gives rise to a luminosity function that is practically indistinguishable from that of a web model with \( \langle f_{\text{HI}} \rangle \sim 0.032 \) (W3) or of a web-bubble model with \( \langle f_{\text{HI}} \rangle = 0.373 \) (B1+W2). This was first pointed out by Bolton & Haehnelt (2013).

- The presence of small-scale absorbers inside ionized bubbles provides an opacity additional to that from the neutral patches between large-scale bubbles. This is clear comparing e.g. the LFs from B1 (dashed red line) to those from B1+W2 (dotted green) or B1+W3 (dashed green).

- Web models with \( \langle f_{\text{HI}} \rangle \sim 10^{-2} \) correspond to bubble models with \( \langle f_{\text{HI}} \rangle \sim 10^{-1} \), Table 1 indicates that this requires \( \Gamma \lesssim 10^{-14} \text{s}^{-1} \). For example, the red dashed (B1) and blue dashed (W2) lines in Fig. 8 show that \( \Gamma_{-12}(z = 7) \approx 0.01 \) is needed for a web model to produce a LF similar to that of a bubble model with \( \langle f_{\text{HI}} \rangle \sim 0.4 \). This is in agreement with Mesinger et al. (2015).

- Comparing the simulations to the observations of Ouchi et al. (2008, 2010) and Konno et al. (2014), we conclude that at \( z = 6.6 \), \( 40\% \lesssim \langle f_{\text{HI}} \rangle \lesssim 70\% \) for the bubble model, \( \langle f_{\text{HI}} \rangle \sim 1\% \) for the web model, and \( \langle f_{\text{HI}} \rangle \lesssim 40\% \) for a web-bubble model. At \( z = 7 \) we have instead \( 70\% \lesssim \langle f_{\text{HI}} \rangle \lesssim 99\% \), \( \langle f_{\text{HI}} \rangle \gtrsim 3\% \), and \( 40\% \lesssim \langle f_{\text{HI}} \rangle \lesssim 70\% \), respectively. The inferred HI fraction thus highly depends on the reionization model adopted.

While the aim of this paper is to present a proof of concept and we defer to future work more rigorous and precise constraints on the \( \text{H} \) i fraction, these results are in excellent agreement with existing work (Bolton & Haehnelt 2013; Mesinger et al. 2015; Choudhury et al. 2015) and underline the importance of understanding the precise ionization structure of the IGM during the EoR in terms of both large-scale bubble features and small-scale absorbers. In the following, we use the simulations described in § 3 and the analytic formalism outlined in § 3.1 to gain more insight into the Lyα RT and the inference of \( \langle f_{\text{HI}} \rangle \) from observed Lyα-emitting galaxies.

5.2 The Red Damping Wing in Bubble Models

We now analyse the Lyα red damping wing opacity to quantify the impact of large-scale \( \text{H} \) i patches on the visibility of Lyα-emitting galaxies. The red lines in the top panel of Fig. 7 show the mean transmission \( \exp(-\tau_{\text{damp}}) \) as a function of \( \Delta v \) for three different values of \( \langle f_{\text{HI}} \rangle \) (B1, B2, and B3 from left to right). We evaluate the effective optical depth directly as an average of \( \exp(-\tau_{\text{damp}}) \) using line-of-sight skewers from galaxies extracted from the simulations. The shaded region indicates the 1σ dispersion \( \sigma^2_{\tau_{\text{damp}}} = \langle \exp(-\tau_{\text{damp}}) \rangle - \langle \exp(-\tau_{\text{damp}}) \rangle^2 \rangle \). We have used 1185 lines-of-sight, i.e. equivalent to the number of galaxies in the simulation box.

The damping wing becomes more opaque with increasing neutral fraction and decreasing \( \Delta v \). The opacity varies significantly between different lines-of-sight as indicated by the large dispersion of \( \sigma_{\tau_{\text{damp}}} \approx 0.2 \).

5.2.1 Comparison to the Analytic Model

To see how well the red damping wing can be captured by the analytic approximation, in Fig. 8 we compare the results from our B2 model to those obtained using equation (12) with \( \alpha_1 = 0.5 \) and \( R_s = 1.7, 3.0, 5.0, 10.0\text{h}^{-1}\text{cMpc} \). Note that the case with \( R_s = 1.7 \) (thickest black dashed line) represents the Schechter function fit to the simulated \( P(R_s) \) distribution. Fig. 9 shows that the Schechter function fit is indeed a good approximation to the
simulation, in which the distance to the near-side of the closest H i patch peaks at ~ 5h^{-1}cMpc from a galaxy.

The comparison in Fig. 8 clearly indicates that the analytic model is too crude to capture the red damping wing behaviour found in the simulations, and systematically overestimates the optical depth, although the bubble size distribution is modelled reasonably well. The discrepancy highlights that the opacity is coming indeed from the neutral gas distributed among multiple H i patches, rather than in a single large H i patch, as assumed in the analytic model of equation (12). This in fact leads to an overestimate of the neutral gas and thus of the opacity.

In addition, the single large H i patch approximation is also responsible for a different shape of the damping wing, because the optical depth scales as Δν^{-1} (e.g. Miralda-Escude 1998). On the other hand, the presence of multiple ionized bubbles in the simulations makes the medium more transparent, and hence the damping wing profile steeper. This implies that, unless the analytic approximation is improved to take into account the complex ionized bubble distribution, (semi-)numerical simulations of patchy reionization are required to properly model the Lyα opacity in the diffuse neutral IGM.\(^{20}\)

20 One obvious improvement of the analytic model would be to introduce an outer radius \(R_2\) for the first diffuse neutral patch, and construct a PDF for \(R_2\) which can then be included into equation (12) to give

\[ e^{-\tau_{\text{eff}}^\text{bub}(v_c)} \approx \int P(R_2|R_1) \int e^{-\tau_{\text{patch}}(v_c,R_1,R_2)} P(R_2|R_1), \quad (22) \]

where \(P(R_2|R_1)\) denotes the conditional probability of \(R_2\) given \(R_1\). We have started to include such improvement in our model. However, due to the difficulty in finding an analytic fitting function for \(P(R_2|R_1)\), we have deferred this to a future work.

5.3 The Red Damping Wing in Web Models

The ensemble of small-scale absorbers can also form a damping wing feature in the effective optical depth towards Lyα galaxies as shown in the bottom panel of Fig. 7 where the blue lines and the shaded areas refer to the mean transmission \(\exp(-\tau_{\text{eff}})\) and to the 1σ dispersion \(\sigma^2_{\text{web}} \approx \langle (e^{-\tau_{\text{web}}}-e^{-\tau_{\text{web}}^\text{eff}})^2 \rangle\) as a function of \(\Delta \nu\).

Similarly to the bubble model, Fig. 9 indicates that the damping wing in web models becomes more opaque with increasing neutral fraction and decreasing \(\Delta \nu\). Neutral fractions \(\langle f_\alpha \rangle\) which is \(\sim 10^{-2}\) (W2 and W3), i.e. much higher than the one in the post-reionized universe (which is \(\langle f_\alpha \rangle \sim 10^{-3}\)), are required to produce a ~ 60–80% reduction of Lyα visibility at \(\Delta \nu = 600 \text{ km s}^{-1}\). On the other hand, the effective optical depth in W1 (which has a neutral fraction closer to \(\sim 10^{-4}\)) is \(e^{-\tau_{\text{eff}}} > 0.9\) at \(\Delta \nu = 600 \text{ km s}^{-1}\), i.e. it hardly affects the Lyα visibility. The scatter around the effective optical depth is again large, with \(\sigma_{\text{web}} \sim 0.2\).

Finally, a comparison between the effective optical depth in the web and bubble models (e.g. B2 vs. W3 in Fig. 7) shows that small-scale absorbers can produce a profile and scatter of the red damping wing similar to those of the bubble models. This explains the similarity in the Lyα LFs observed through the large-scale bubbles and small-scale absorbers.

5.3.1 Comparison to the Analytic Model

Fig. 10 compares the simulation and the analytic effective optical depth described by equation (14) in § 5.2. The black dashed lines refer to the analytic model without the effect of clustering and velocity field, i.e. \(\xi_\nu = 0\) in equation (14), while the black solid line uses the Gaussian streaming model for \(\xi_\nu\), i.e. equation (15). The

21 This justifies the calibration of the intrinsic model discussed in § 4.2. However, the left bottom panel of Fig. 8 shows that \(\exp(-\tau_{\text{eff}}^\text{web}) \sim 0.6\) at \(\Delta \nu \sim 100 \text{ km s}^{-1}\) even with a neutral fraction as small as the one in the post-reionized universe. For galaxies that have \(\Delta \nu < 200 \text{ km s}^{-1}\) the impact of small-scale absorbers at \(z < 6\) should therefore be taken into account.
analytic model employs a factor of 2-10 boost to the extrapolated CDDF fit of Becker & Bolton (2013) at $z \approx 7$ (hereafter BB13 CDDF) to mimic the rapidly increasing abundance of small-scale absorbers. Our fiducial value is $2 \times$ CDDF (see §5.3.2 discussion on the reason of this choice).

Fig. 10 shows clearly that we cannot reproduce the results from our simulation by only changing the CDDF amplitude, while the agreement is much better if we simultaneously change the CDDF amplitude and include the galaxy-absorber correlation function in velocity-space (see §5.3.3 for the reason of the discrepancy at $\Delta v < 200 \text{ km/s}$). In other words, both the abundance of small-scale absorbers and their velocity-space clustering around galaxies play a key role in determining the Ly$\alpha$ visibility. In the following sections, we discuss in more detail the impact of (i) changing the CDDF (§5.3.2) and (ii) the galaxy-absorber clustering (§5.3.3).

### 5.3.2 CDDF and $N_{\text{HI}}$-Dependence of the Optical Depth

We first justify the artificial boosting factor of the power-law CDDF adopted in the analytic model. Fig. 11 compares the CDDF obtained in our web model simulation to the BB13 CDDF with a factor of 1, 2 and 5 boost. The adopted boosts broadly mimic the increase in simulated CDDF amplitude due to lower photoionization rate/higher neutral fraction ($\Gamma = 10^{-14} \text{ s}^{-1}$ and $5 \times 10^{-15} \text{ s}^{-1}$ for W2 and W3), although the slope is not properly reproduced. The fiducial choice of 2 (corresponding to $\Gamma \sim 10^{-14} \text{ s}^{-1}$) approximately represents the CDDF amplitude in the range $10^{19} \text{ cm}^{-2} < N_{\text{HI}} < 10^{20.5} \text{ cm}^{-2}$, which gives the highest contribution to the red damping opacity.

This predominance can be clearly seen in Fig. 12 which shows the ratio between the analytic effective optical depth from absorbers with column density below $N_{\text{HI}}^{\max}$ and below $N_{\text{HI}}^{\max} = 10^{21.3} \text{ cm}^{-2}$, in a case with $\xi_v = 0$. More than 80 per cent of the optical depth redward of line centre ($\Delta v = 300 \text{ km/s}$) comes numbers converge for larger projected lengths. The effect is minor in the other bins.

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22 We have computed the CDDF by taking the projected column density over 10 cells. The highest $N_{\text{HI}}$ bins ($\log_{10} N_{\text{HI}}/\text{cm}^{-2} \sim 21.3$ and 20.6) are about $\sim 0.3$ dex larger than those calculated with a single cell, but the

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23 The inclusion of the velocity-space correlation function, for example inflowing low column density absorbers, would enhance the contribution of lower column density absorbers to the optical depth.
from absorbers with $N_{\text{HI}} > 10^{19} \text{cm}^{-2}$, because of their prominent damping wings. On the other hand, at $\Delta v = -300 \text{ km/s}$ (i.e. blue-ward of the line resonance) lower column density absorbers with $N_{\text{HI}} < 10^{19} \text{cm}^{-2}$ can contribute $\sim 50$ per cent to $v_{\text{web}}$ via resonant absorption.

This strong dependence of the optical depth on the column density of absorbers is insensitive to the assumption about the shape of the CDDF, as shown by a comparison between the solid and dashed lines in Fig. 12 which refer to models using a CDDF from Becker & Bolton (2013) and O’Meara et al. (2013), respectively. In both cases $N_{\text{HI}} > 10^{19} \text{cm}^{-2}$ absorbers dominate the red damping wing, with a difference of only $\sim 10$ per cent.

Hence, the red damping wing opacity mainly depends on the abundance of strong H I absorbers, e.g. super-LLSs and DLAs, around Ly$\alpha$-emitting galaxies. Their rapid increase (stronger than what expected from a simple extrapolation to $z > 6$ of lower-$z$ CDDF) provides a large red damping wing opacity.

5.3.3 Galaxy-Absorber Correlation Function in Velocity-Space

The galaxy-absorber correlation function in velocity space, $\xi_v$, is another key factor in the formation of the red damping wing. In the Gaussian streaming model of equation (15), $\xi_v$ depends both on (i) the real-space correlation function, $\xi(r_{12})$, and (ii) the galaxy-absorber pairwise mean velocity field $\langle v_{12}(r_{12}) \rangle$, and pairwise velocity dispersion $\sigma_{12}(r_{12})$.

The simulated real-space galaxy-absorber correlation function at $z = 7$ is shown in Fig. 13 together with the LBG-DLA correlation function observed by Cooke et al. (2006b) at $z \sim 3$. The simulated $\langle r_{12} \rangle$ is obtained by correlating the position of the galaxies and of the cells with $x_{\text{HI}} > 0.9$ (which represent for us self-shielded absorbers) using the $\xi(r_{12}) = DD/RR - 1$ estimator (Davis & Peebles 1983). Clustering of self-shielding gas in the vicinity of Ly$\alpha$-emitting galaxies is clearly important, and the simulated real-space correlation function appears (maybe surprisingly) similar to its lower-redshift observed counterpart. We thus adopt the Cooke et al. (2006b) correlation function for our Gaussian streaming model in Fig. 14.

The mean pairwise velocity between Ly$\alpha$-emitting galaxies and absorbers defined above is shown in the top panel of Fig. 14 both in terms of the proper peculiar velocity $\langle v_{12}(r_{12}) \rangle$ (blue lines) and of the total proper velocity $H(z_s)r_{12}/(1 + z_s) + \langle v_{12}(r_{12}) \rangle$ (cyan lines). The solid black line is the best-fit curve to the mean pairwise velocity, $\langle v_{12}(r_{12}) \rangle = -v_{\text{web}}/\left[1 + (r_{12}/r_v)^{\gamma_v}\right]$ where $v_{\text{web}} = 133 \text{ km/s}, r_v = 6.3h^{-1}\text{Mpc}$ and $\gamma_v = 6.2$. This is adopted to evaluate the Gaussian streaming model in Fig. 10. For simplicity, rather than using a fit to the curve, we assume a constant pairwise velocity dispersion equal to its mean, i.e. $\sigma_{12} = 100 \text{ km/s}$.

As shown in Fig. 10 the impact of the galaxy-absorber correlation function in velocity-space provides an additional boost of effective optical depth relative to the model with $\xi_v = 0$. In fact, the enhanced clustering of absorbers around galaxies (Fig. 10) renders the IGM more opaque. Furthermore, the cosmological inflow of absorbers onto galaxies (Fig. 14) causes a departure from the Hubble flow in the immediate surroundings of galaxies and enhances the velocity-space clustering (the slower the total outflow velocity in the proper unit is, the more opaque to Ly$\alpha$ photons the gas becomes, as it is less redshifted out of resonance). This can increase $\tau_{\text{web}}$, preferentially at the lower $\Delta v$. Thus, the effective optical depth including a velocity-space galaxy-absorber clustering...
is larger and steeper than the one including only a change in the CDDF amplitude (with $\xi_\nu = 0$).

Overall, Fig. [10] shows that the simulation and the analytic model agree at $\Delta v > 400$ km/s, while our analytic approximation overestimates the opacity at $\Delta v < 300$ km/s, probably because we assume that the same galaxy-absorber correlation function applies to the full column density range of absorbers. This may lead to low column density absorbers with a $\xi(f_{12})$ which is too large. To address this issue, it is necessary to investigate in more detail the column density dependent clustering, the pairwise velocity field with outflow, and/or the effect of photoionization from the central galaxy.

### 5.4 Ly$\alpha$ Red Damping Wing in Web-Bubble Models

The top panels of Fig. [15] show the effective optical depth in the hybrid web-bubble models directly calculated from the simulations, together with the 1$\sigma$ dispersion of optical depth among different lines-of-sight. Not surprisingly, the red damping wing becomes more opaque towards higher neutral fractions, and the scatter from sight-line to sight-line is large. The red and blue lines show the contributions to the total simulated optical depth from the bubble and web models used to construct the web-bubble models, and the black lines show the sum of these two contributions, i.e. $\tau_{\text{web}}^{\text{eff}} + \tau_{\text{bub}}^{\text{eff}}$. (Bottom panels) Ratio of effective optical depth between web and bubble models used for the corresponding web-bubble models. This shows the impact of large-scale bubbles and small-scale absorbers on the total optical depth as a function of $\Delta v$.

The below two points underline the importance of correctly modelling small-scale absorbers within the large-scale bubble morphology. This section concludes our discussion on the average impact of large-scale neutral patches (§5.2) and small-scale absorbers (§5.3) on the Ly$\alpha$ red damping wing opacity in a unified web-bubble framework.

24 A slight discrepancy arises because the simple sum counts twice the neutral gas outside ionized bubbles (in the form of H I patches in bubble models and small-scale absorbers in web models), while in the simulations small-scale absorbers are present only when ionized bubbles by construction. The simple sum is thus expected to result in a slightly higher optical depth.
5.5 Probability Distribution Functions for $\langle T_{\text{IGM}} \rangle$

Fig. 16 shows the distribution of the IGM transmission factor, $\langle T_{\text{IGM}} \rangle$, along the line-of-sight to a Ly$\alpha$-emitting galaxy as a function of the host halo mass for models B2, W3, and B1+W2. These models have been chosen because they have a similar LF (see Fig. 9 and effective optical depth (see Figs. 7 and 15), and therefore a similar average Ly$\alpha$ visibility. The black lines are the average IGM transmission factor $\langle T_{\text{IGM}}(M_h) \rangle$ for each halo mass bin.

In the bubble model plotted in the top panel of Fig. 16 $\langle T_{\text{IGM}}(M_h) \rangle$ increases with $M_h$, as massive [small] haloes typically reside in large [small] ionized bubbles (in the highest mass bins the trend is reversed because of the poor statistics). At the same time, there exists a population of lower mass haloes clustered around the more massive ones, which is therefore also embedded within large ionized bubbles. This explains the large scatter exhibited by $T_{\text{IGM}}$ for low halo masses. Furthermore, in bubble models sightlines to most (if not all) galaxies pass through $\text{H}_\text{I}$ patches, meaning that the intrinsic luminosity of most galaxies is reduced, and explaining the unimodality of the $T_{\text{IGM}}$ distribution (something that was pointed out previously by Jensen et al. 2014 and Mesinger et al. 2015).

As in web models self-shielding absorbers cluster around the more massive haloes ($\S$ 5.3.3), $\langle T_{\text{IGM}}(M_h) \rangle$ decreases with increasing $M_h$. The still present large scatter in the distribution now appears to be bimodal, with a peak at $T_{\text{IGM}} \sim 1$ and a second one at $T_{\text{IGM}} \sim 0$. These peaks correspond to cases in which a line-of-sight intersects an absorber or not. Differently from what happens in the bubble model where the intrinsic luminosity of all galaxies is reduced, here a suppression is [is not] present depending on whether a small-scale absorber is [is not] aligned with a galaxy, hence the bimodality. Our results are consistent with those by Mesinger et al. (2015), who also find that a bimodal distribution is a characteristic of the attenuation by small-scale absorbers.

In hybrid web-bubble models the IGM transmission factor is a product of large-scale bubbles and small-scale absorbers. Because of the different mass-dependence of $T_{\text{IGM}}$ in the two models, the total IGM transmission factor here depends only weakly on $M_h$, and no clear unimodality or bimodality in the distribution is visible. For example, the sight-lines present in the web model with $T_{\text{IGM}} \sim 1$ are now more opaque due to the absorption from the $\text{H}_\text{I}$ patches between large-scale bubbles.

It is therefore clear that the conditional $T_{\text{IGM}}$-PDF at a given halo mass, $P(T_{\text{IGM}}|M_h)$, or in short the $T_{\text{IGM}} \sim M_h$ relation, differs for web, bubble and web-bubble models. In the next section, we search for observational signatures of this variation in the intergalactic environment.

5.6 Simultaneously Constraining the $\text{H}_\text{I}$ Fraction and the Topology of Reionization

We now examine the prospect of observationally constrain the global $\text{H}_\text{I}$ fraction and the topology of reionization simultaneously by combining various statistics of Ly$\alpha$ emitting galaxies.

5.6.1 The Equivalent Width Distribution

Fig. 17 shows the cumulative probability distribution of the rest-frame equivalent width (REW), following the method of Dijkstra.
The observed Lyα offers a test to differentiate reionization models. The T in the Ly line profile 
functions. A degeneracy is present between web and bubble models, the H I fraction. Similarly to what observed for the Lyα luminosity given a halo mass. A comparison between equations (23) and (24) shows a difference on $P(T_{IGM}|M_h)$ this is because the Lyα LF is constructed from Lyα selected LAEs, while the REW-PDF is constructed from continuum selected galaxies. In fact, Dijkstra & Wyithe 2012 and Gronke et al. 2015 have shown that selection by Lyα line flux enhances the contribution of UV-faint galaxies (at fixed Lyα flux), which are absent from continuum selected samples. As such UV-faint galaxies should preferentially reside in low mass haloes, this difference in selection function would introduce a different dependence in the $T_{IGM} = M_h$ relation that may lead to a drop in the observed Lyα LF different from the one in the REW distribution.

Hence, a combined analysis of Lyα LF and REW distribution may allow to constrain the H I fraction and the topology of reionization. We can already do this analysis. The upper limit at REW = 75 slightly favours the bubble or web-bubble models with $\langle f_{HI>0} \rangle \sim 68\%$ or $\sim 37\%$. If we include this constraint, the neutral fraction is favored to be of order of tens of per cent. This constraint is very weak because of a large uncertainty due to the interloper contamination. Moreover, the same observations favour bimodal quenching of the Lyα visibility, which is associated with web-models. This argument simply illustrates that a combined analysis of Lyα LF and REW-PDF can shed light on the history and topology of reionization.

### Lyα Fraction of Lyman-Break Galaxies

The power of such joint analysis can be strengthened once the $M_{UV}$-dependent Lyα fraction of LBGs measurement is included as well. The Lyα fraction of LBGs (hereafter $X_{Lyα})$ is defined as the fraction of LBGs with a UV magnitude $M_{UV}$ and Lyα REW greater than a given value. We generalize the method of Dijkstra, Mesinger & Wyithe 2011 (see also Dijkstra & Wyithe 2012) to calculate the Lyα fraction of LBGs as

$$X_{Lyα}(>REW|M_{UV}) = \int_0^1 P_{Lyα}(>REW/|T_{IGM}|M_{UV})P(T_{IGM}|M_{UV})dT_{IGM}.$$  \hspace{1cm} (26)

where $P_{Lyα}(>REW|T_{IGM}|M_{UV}) \equiv e^{-REW_{Lyα}/REW_{Lyα}(M_{UV})}$ is the intrinsic Lyα fraction, $REW_{Lyα}(M_{UV})$ is a characteristic REW (see Appendix B for more details), and the conditional $T_{IGM}$ probability distribution function at a given $M_{UV}$ is

$$P(T_{IGM}|M_{UV}) = \int P(T_{IGM}|M_h)P(M_h|M_{UV})dM_h.$$  \hspace{1cm} (27)

### 5.6.2 Lyα Fraction of Lyman-Break Galaxies

The intrinsic REW distribution is $P_{REW}(REW) = \exp(-REW/\alpha_{REW})$, with $\alpha_{REW} = 50$ Dijkstra, Mesinger & Wyithe 2011 and $REW_{Lyα} = REW/T_{IGM}$. The probability distribution function of the IGM transmission factor, $P(T_{IGM}) \propto \int P(T_{IGM}|M_h)dn(M_h)/dM_hdM_h$, is constructed from the simulations.

In all models, the observed REW distribution is decreased in comparison to the intrinsic one by an amount which increases with the H I fraction. Similarly to what observed for the Lyα luminosity function, a degeneracy is present between web and bubble models, with, for example, B1 and W2 providing similar REW distributions.

However, the degeneracy can be partially broken if the REW distribution is combined with the Lyα LF. In fact, while models B2, W3 and B1+W2 are degenerate in Lyα LF (see Fig. B) they produce distinguishable observed REW distributions. Although this is not always the case (for example, B1 and W2 show similar curves both in the Lyα LF and the REW distribution), such a combined analysis offers a test to differentiate reionization models.

The argument above can be better understood by noting that the observed Lyα LF and REW distribution depend differently on the $T_{IGM} - M_h$ relation. To see this, we first express the Lyα LF in terms of $P(T_{IGM}|M_h)$ as

$$dn(L_{α}^{obs}|dL_{α}) = \int P(L_{α}^{obs}|M_h)dn(M_h)/dM_hdM_h.$$  \hspace{1cm} (24)

where

$$P(L_{α}^{obs}|M_h) = \int_0^1 P_{int}(L_{α}^{obs}/T_{IGM}|M_h)P(T_{IGM}|M_h)dT_{IGM}.$$  \hspace{1cm} (25)

$P_{int}(L_{α}|M_h)$ is the intrinsic conditional probability distribution of the Lyα luminosity given a halo mass. A comparison between equations (23) and (24) shows a difference on $P(T_{IGM}|M_h)$. This is because the Lyα LF is constructed from Lyα selected LAEs, while the REW-PDF is constructed from continuum selected galaxies. In fact, Dijkstra & Wyithe 2012 and Gronke et al. 2015 have shown that selection by Lyα line flux enhances the contribution of UV-faint galaxies (at fixed Lyα flux), which are absent from continuum selected samples. As such UV-faint galaxies should preferentially reside in low mass haloes, this difference in selection function would introduce a different dependence in the $T_{IGM} - M_h$ relation that may lead to a drop in the observed Lyα LF different from the one in the REW distribution.

Hence, a combined analysis of Lyα LF and REW distribution may allow to constrain the H I fraction and the topology of reionization. We can already do this analysis. The upper limit at REW = 75 slightly favours the bubble or web-bubble models with $\langle f_{HI>0} \rangle \sim 68\%$ or $\sim 37\%$. If we include this constraint, the neutral fraction is favored to be of order of tens of per cent. This constraint is very weak because of a large uncertainty due to the interloper contamination. Moreover, the same observations favour bimodal quenching of the Lyα visibility, which is associated with web-models. This argument simply illustrates that a combined analysis of Lyα LF and REW-PDF can shed light on the history and topology of reionization.

Explicitly, we use $P_{int}(L_{α}|M_h) = \delta_M |L_{α} - L_{α}(M_h)|$ as we assume a one-to-one mapping between $L_{α}$ and $M_h$ based on the abundance matching technique.

Note that equation 24 implicitly assumes that the intrinsic REW distribution is independent of halo mass. We can, of course, generalize this modelling to include the halo mass dependence, but because this in general differs from the one of the Lyα luminosity, the dependence of the two statistics on the $T_{IGM} - M_h$ relation is expected to differ as well.

### Figure 17

Cumulative probability distribution of the rest-frame equivalent width at $z = 7$. The black line is the intrinsic REW distribution and the coloured lines refer to the observed REW distributions predicted from simulations: bubble model B1 (red dashed), B2 (red solid) and B3 (red dotted); web model W1 (blue dotted), W2 (blue dashed) and W3 (blue solid); web-bubble model B1+W2 (green solid), B1+W3 (green dashed), B2+W2 (green dotted) and B2+W3 (dotted-dashed). The black circles are the observations of Pentericci et al. (2014) without interloper correction (if the interloper correlation is taken into account the data points can be higher by $\sim 20\%$).
We construct our intrinsic model assuming that UV-bright LBGs populate more massive haloes, and consider a case with a correlation betweenREW and $M_{UV}$ ($M_{UV}$-dependent model) and one with no correlation (uncorrelated model). The $M_{UV}$-dependent model is our fiducial case because observations suggest that such correlation exists (Stark, Ellis & Chiu 2010; Jiang et al. 2013) but see Nilsson et al. (2009). More details are provided in Appendix E.

- the bubble model shows an upturn of Ly$\alpha$ fraction at UV-bright LBGs (typically defined as those with $M_{UV} < -20.25$), while the web model shows a monotonic decrease of Ly$\alpha$ fraction for increasing UV-bright LBGs. This qualitative change in the shape of the $M_{UV}$-dependent Ly$\alpha$ fraction is robust against different intrinsic models of REW.
- In the $M_{UV}$-dependent model (solid lines), the drop in the observed Ly$\alpha$ fraction compared to the intrinsic one is larger for UV-faint LBGs ($M_{UV} > -20.25$) than for UV-bright LBGs in all models. The common expectation that the drop of Ly$\alpha$ fraction of UV-faint LBGs is larger than the one of UV-bright LBGs occurs only in bubble models (Ono et al. 2012). This is true only if the intrinsic REW and $M_{UV}$ are uncorrelated (dashed lines).

The upturn of the $M_{UV}$-dependent Ly$\alpha$ fraction can be understood as an imprint of the $T_{IGM} - M_\ast$ relation (see §5). In fact, because UV-bright LBGs in bubble models are more likely to be surrounded by large ionized bubbles, the probability that their Ly$\alpha$ emission is visible (i.e. that they are associated to larger $T_{IGM}$) is higher than for UV-faint LBGs. On the other hand, in web models the small-scale absorbers cluster more strongly around UV-bright LBGs, lowering their Ly$\alpha$ visibility. The upturn of Ly$\alpha$ fraction, therefore, does not happen in web models. As a consequence, the qualitative change in the shape of the $M_{UV}$-dependent Ly$\alpha$ fraction can be used as an indicator of the (possible) presence of large-scale bubbles.

On the other hand, a drop of the Ly$\alpha$ fraction for UV-faint LBGs larger than for UV-bright LBGs cannot be used as a decisive evidence of patchy reionization. In fact, while in the uncorrelated case (dashed lines) we indeed see a larger drop for UV-faint LBGs only for the bubble model, in the $M_{UV}$-dependent case (solid lines) such drop is visible for all models. The simplest explanation for this is that, because order approximation, the neutral IGM suppresses the Ly$\alpha$ emission by re-scaling the characteristic REW as $\langle T_{IGM} \rangle \langle \text{REW}_c(M_{UV}) \rangle$ (see also Appendix E). UV-faint galaxies (with an intrinsically larger REW$_c$) experience a larger reduction in number above a given REW than the UV-bright galaxies (with intrinsically small REW$_c$) do.

In summary, the analysis of the $M_{UV}$-dependent Ly$\alpha$ fraction of LBGs provides a powerful diagnostic tool to characterize the impact of large-scale bubbles and small-scale absorbers when properly interpreted. Hence, when combined with the Ly$\alpha$ LF, it offers an opportunity to constrain the H$\alpha$ fraction and the topology of reionization simultaneously. While the aim of the present paper is to highlight the potential of this diagnostics, we plan to use it more extensively in a future study.

6 DISCUSSION & CONCLUSIONS

The visibility of Ly$\alpha$-emitting galaxies during the Epoch of Reionization is controlled by both diffuse H$\alpha$ patches in the IGM, and small-scale self-shielding absorbers around galaxies. It is therefore important to correctly include small-scale absorbers inside large-scale ionized bubbles. In this work we have explored the impact of both large-scale bubbles and small-scale absorbers on the visibility of the population of Ly$\alpha$-emitting galaxies at $z > 6$, using a powerful combination of an analytic approach and hydrodynamical simulations, which covers the full range of models explored in recent investigations (Jensen et al. 2013; Bolton & Haehnelt 2013; Mesinger et al. 2015; Choudhury et al. 2015). We have considered the IGM Ly$\alpha$ RT in three different classes of IGM ionization structure, namely (i) the bubble model, where only large-scale ionized bubbles due to patchy reionization are present, (ii) the web model, where only small-scale absorbers are considered, and (iii) the web-bubble model, which includes both small-scale absorbers and large-scale bubbles.

Our main conclusions are:

- The observed Ly$\alpha$ LF evolution from $z = 5.7$ to $z \sim 7$ requires a neutral fraction $\langle f_m \rangle_V \sim 60 - 80\%$ in bubble models,
A sole analysis of the Lyα luminosity function or of the distribution of rest frame equivalent width cannot put a stringent constraint on the reionization history. The Lyα LF function and the REW-PDF can be equally suppressed in bubble, web, and web-bubble models, yet with very different global H i fractions. Hence, there is a fundamental degeneracy between the ionization structure of the IGM and the global H i fraction inferred from Lyα surveys (see § 5.1).

We showed in § 5.6 that a joint analysis of the Lyα LF and the REW-PDF of LBGs can improve the constraints on the neutral fraction by breaking the degeneracy with the topology of reionization.

The Lyα fraction of LBGs can be a powerful diagnostic to study the relative importance of large-scale H i patches and small-scale absorbers in the IGM. We caution that a drop in Lyα fraction that is larger for UV-faint LBGs than for UV-bright LBGs (as in Ono et al. 2012) can be reproduced with web and web-bubble models, and does not provide exclusive evidence for patchy reionization. Instead, we argue that the shape of the MUV-dependent Lyα fraction may provide more insight into the topology of reionization (see e.g. Fig. 15).

For example, an upturn of Lyα fraction for UV-bright LBGs can be caused by large-scale ionized bubbles, but also by an increase in the UV background around UV-bright galaxies, which reduces the abundance of small-scale absorbers. Interestingly, this upturn may already have been observed at 4.5 < z < 6 (Stark, Ellis & Chiu 2010), and may reflect large fluctuations in the UV background. These fluctuations have been proposed to explain observations of the cumulative effective optical depth distribution at z > 5 in the spectra of high-redshift QSOs (Becker et al. 2015, Chardin et al. 2015).

Our analytic formalism shows that the Lyα damping wing opacity from small-scale absorbers is highly influenced by the clustering and the pairwise velocity field of galaxy-absorber pairs (see § 5.3). Absorbers with NHI > 10^{19} cm^{-2}, i.e. super-LLS/DLAs, provide the largest contribution to the red damping wing at ∆ν > 300 km s^{-1}, while lower column density absorbers are important at smaller ∆ν. Understanding the galaxy-absorber correlation functions and their velocity fields can improve the robustness with which the reionization history can be constrained using Lyα emitting galaxies. Direct observational constraints on H i CDDF and galaxy-absorbers correlation function (and as a function of NHI) can therefore be very useful. A possible approach is to extend to the range 3 < z < 7 the survey strategy that searches for Lyα-emitting galaxies in the foregrounds of high-redshift QSOs, similar to the observation of Cooke et al. (2006b). Keck Baryonic Structure Survey (Rudie et al. 2012), Turner et al. (2014), and VLT LBG Redshift Survey (Crighton et al. 2011). This observational strategy is already within reach at z ∼ 5 (Diaz et al. 2014).

We showed that the total effective optical depth in web-bubble models can be written as the sum of those in web and bubble models, i.e. τeff ≈ τ_{bub} + τ_{web} (see § 5.4). This is an important result as fast semi-numeric simulations can be used to generate τ_{bub}. These simulations can then be complemented with (improved) analytic or possibly empirical prescriptions for τ_{web} (as in § 5.3) to efficiently generate more realistic web-bubble models.

Web, bubble and web-bubble models produce different IGM-PDFs (§ 5.3). Bubble models show a unimodal IGM-PDF, while small-scale self-shielding absorbers in the web-model have a bimodal IGM-PDF. The modality of the hybrid web-bubble model depends on which component dominates the IGM opacity. Pentericci et al. (2014) have provided observational evidence for bimodal quenching of Lyα flux (see Treu et al. 2012, 2013 for details on the procedure). Our results imply that bimodal quenching indicates an influence of small-scale absorbers on the Lyα visibility (also see Mesinger et al. 2015), which is opposite to the common interpretation.

In conclusion, in this paper we have shown that a joint analysis of different statistics of Lyα emitting galaxies (e.g. Lyα LF, REW distribution, Lyα fraction of LBGs, correlation function), can break degeneracies associated with individual probes. It should therefore be possible to constrain simultaneously the global H i fraction and the reionization topology, when armed with a suit of models of reionization in which both large-scale bubble morphology and small-scale absorbers are included.

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absorbers, such as DLA, LLS, and diffuse IGM. The mass-weighted neutral fraction in the post-reionized universe can be estimated from DLA/LLS surveys and Ly\alpha absorbers, such as DLA, LLS, and diffuse IGM.

APPENDIX A: THE MASS-WEIGHTED NEUTRAL FRACTION IN THE POST-REIONIZED UNIVERSE

The mass-weighted HI fraction in the post-reionized universe can be estimated from DLA/LLS surveys and Ly\alpha forest observations, which measure the HI column density distribution function. As follows, this quantity can then be converted into the HI fraction embedded as Ly\alpha absorbers, such as DLA, LLS, and diffuse IGM.
The proper number density of HI gas in the universe, \(n_{\text{HI}}^\text{prop}(z)\), is expressed as (cf. Meiksin 2009)

\[
n_{\text{HI}}^\text{prop}(z) = \int N_{\text{HI}} \left| \frac{\partial^2 N_{\text{HI}}}{\partial N_{\text{HI}} \partial z} \right| \frac{dz}{dV} \, dN_{\text{HI}}, \tag{A1}
\]

\[
= \frac{(1+z)^3 H_0}{c} \int N_{\text{HI}} f(N_{\text{HI}}, z) dN_{\text{HI}},
\]

where \(l_p\) is the proper distance, \(dV/dz = c/H(z)(1+z)\). Therefore, the fraction of neutral hydrogen over the total hydrogen atoms in the entire universe, \(\langle f_{\text{HI}} \rangle_M\), is given by \(\langle f_{\text{HI}} \rangle_M = n_{\text{HI}}^\text{prop}(z)/n_{\text{HI}}^\text{prop}(z)\) \(^22\)

\[
\langle f_{\text{HI}} \rangle_M = \frac{8\pi Gm_H}{3H_0(1-Y)H_0} \int_{N_{\text{HI}}^\text{min}}^{N_{\text{HI}}^\text{max}} N_{\text{HI}} f(N_{\text{HI}}, z) dN_{\text{HI}}, \tag{A2}
\]

where \(m_H\) is the mass of a hydrogen atom and \(n_{\text{HI}}^\text{prop}(z) = \frac{3H_0^2(1-Y)H_0}{4\pi c(1+z)^3} = 2.057 \times 10^{-7} (1+z)^3 \left(\frac{\Omega_m^2}{0.25}\right) \text{cm}^{-3}\) for a helium abundance \(Y = 0.25\). The upper and lower limits of the integration specify whether the H I content is embedded in the Ly\(\alpha\) forest absorbers (\(\log_{10}[N_{\text{HI}}/\text{cm}^{-2}] < 17\)), Lyman-limit systems (\(17 < \log_{10}[N_{\text{HI}}/\text{cm}^{-2}] < 20.3\)), or damped Ly\(\alpha\) systems (\(20.3 < \log_{10}[N_{\text{HI}}/\text{cm}^{-2}]\)). We integrate equation (A2) using the fitting functions to the observed CDDFs, \(f(N_{\text{HI}}, z)\). We use the CDDF fitting functions from Kim et al. (2002) for the Ly\(\alpha\) forest absorbers, Péroux et al. (2005) for the LLS range, and Prochaska, Herbert-Fort, & Wolfe (2005) for the DLAs range. The observed \(f(N_{\text{HI}}, z)\) and the various fits are shown in Fig. A1.

**APPENDIX B: EFFECTIVE OPTICAL DEPTH OF DYNAMICAL SMALL-SCALE ABSORBERS**

The opacity from small-scale absorbers is determined by the phase-space distribution function of galaxy-absorber pairs, \(f(r_{12}, v_{12}, N_{\text{HI}})\), where \(r_{12}\) is the comoving separation and \(v_{12}\) is the peculiar pairwise radial velocity of pairs.

The line transfer is sensitive to the clustering in total velocity space, \(v_c = aHr_{12} + v_{12}\). The probability to find an absorber within \(v_c\) and \(v_c + dv_c\) and column density \(N_{\text{HI}}\) and \(N_{\text{HI}} + dN_{\text{HI}}\) is \(p(v_c, N_{\text{HI}})dv_N\), then, the effective optical depth is given by \(\tau_{\text{eff}} = \int p(v_c, N_{\text{HI}}) \left[ 1 - e^{-\tau_{\text{abs}}(v_c, N_{\text{HI}})} \right] dv_c dN_{\text{HI}}\). \(\tag{B1}\)

\[p(v_c, N_{\text{HI}})\] is related to the phase-space distribution function of galaxy-absorber pairs through the transformation of variables \(r_{12}, v_{12}\) and \(v_c\),

\[
p(v_c, N_{\text{HI}}) = \int \delta_D \left[ v_c - (aHr_{12} + v_{12}) \right] f(r_{12}, v_{12}, N_{\text{HI}}) dv_{12} dr_{12} = \int p(v_c - aHr_{12}, N_{\text{HI}}) p(v_c, N_{\text{HI}}) dv_{12} dr_{12}, \tag{B2}
\]

where \(\delta_D\) is the Dirac delta function. For the second equality, we have used \(f(r_{12}, v_{12}, N_{\text{HI}}) = p(v_{12} | r_{12}, N_{\text{HI}}) p(v_{12}, r_{12}, N_{\text{HI}})\),

\[28\] The fraction of total number of neutral hydrogen, \(N_{\text{HI}}^{\text{tot}}\), over the total hydrogen atom counts, \(N_{\text{HI}}\), is given by the mass-weighted neutral fraction \(f_{\text{HI}} = N_{\text{HI}}^{\text{tot}}/N_{\text{HI}} = \int x_{\text{HI}} n_{\text{HI}} dV / \int n_{\text{HI}} dV = \int x_{\text{HI}} dV / \int dV = (f_{\text{HI}})\). The volume-weighted and mass-weighted neutral fraction are identical only for a homogeneous IGM: \(\langle f_{\text{HI}} \rangle_M = \int x_{\text{HI}} \rho dV / \int \rho dV = \int x_{\text{HI}} dV / \int dV = (f_{\text{HI}})\).

\(\sim 2.4\) and \(z \sim 3.7\) use the compilation of data presented in O’Meara et al. (2013).

where \(p_0(r_{12}, N_{\text{HI}})dv_{12}\) is the conditional probability to find an absorber with peculiar pairwise velocity between \(v_{12}\) and \(v_{12} + dv_{12}\) at given pair separation \(r_{12}\) and column density \(N_{\text{HI}}\) and \(p_0(r_{12}, N_{\text{HI}})dv_{12} dv_{12}\) is the probability to find an absorber in the range \(r_{12} + dr_{12}\) and \(N_{\text{HI}} + dN_{\text{HI}}\). The real-space correlation function \(\xi(r_{12}, N_{\text{HI}})\) of absorbers around galaxies gives

\[
\rho_i(r_{12}, N_{\text{HI}}) = \frac{\partial^2 N}{\partial N_{\text{HI}} \partial z} \frac{dz}{dr} \left[ 1 + \xi(r_{12}, N_{\text{HI}}) \right], \tag{B3}
\]

where \(|dr/dz| = c/H(z)_s\). Substituting into equation (B2),

\[
p_i(v_c, N_{\text{HI}}) = \frac{\partial^2 N}{\partial N_{\text{HI}} \partial z} \frac{dz}{dr} \left[ \frac{1}{aH} + \xi(v_c, N_{\text{HI}}) \right], \tag{B4}
\]

where we have defined the absorber-galaxy correlation function in velocity space as

\[
1 + \xi(v_c, N_{\text{HI}}) = \frac{aH}{\int dr_{12} [1 + \xi(r_{12}, N_{\text{HI}})] p_0(v_c - aHr_{12} | r_{12}, N_{\text{HI}}). \tag{B5}
\]

Thus, the effective optical depth is

\[
\tau_{\text{eff}} = \int dN_{\text{HI}} \frac{\partial^2 N}{\partial N_{\text{HI}} \partial z} \frac{dz}{dr} \left[ 1 + \xi(v_c, N_{\text{HI}}) \right] \left[ 1 - e^{-\tau_{\text{abs}}(v_c, N_{\text{HI}})} \right]. \tag{B6}
\]

All the quantities are evaluated at redshift \(z = z_s\). By rearranging we obtain equation (13).

In the absence of clustering, \(\xi_v = 0\), the effective optical depth (B6) reduces to the well-known expression for the Poisson-distributed absorbers \(\tau_{\text{web}} = \int dz \int dN_{\text{HI}} \frac{dz}{dr} \frac{\partial^2 N}{\partial N_{\text{HI}} \partial z} (1 - e^{-\tau_{\text{abs}}}) \) (e.g. Haardt & Madau 1996).
We show two examples of the velocity-space correlation function $\xi$. For a pure Hubble flow $v_c = aH_{12}$, $p_\nu(v_{12}|r_{12}, N_\alpha) = \delta_D(v_{12})$. Thus, $\xi_\nu(v_c) = \zeta(r_{12} = 2000)$. Furthermore, a Gaussian streaming model is a simple generalization where the conditional pairwise peculiar velocity PDF is modelled as

$$p_\nu(v_{12}|r_{12}, N_\alpha) = \frac{1}{\sqrt{2\pi} \sigma_{12}(r_{12})} \exp\left[-\frac{(v_{12} - \langle v_{12}\rangle)\rangle^2}{2\sigma_{12}^2(r_{12})}\right],$$

where $\langle v_{12}\rangle$ and $\sigma_{12}$ are the radial pairwise peculiar velocity and velocity dispersion, respectively.

APPENDIX C: ABUNDANCE MATCHING

The abundance matching technique gives a semi-empirical relation between the halo mass and the Ly$\alpha$ luminosity for each $f_{duty}$ as shown in Fig. C1. The red lines are the result of matching the simulated halo mass function at $z = 7$ with the observed $z = 5.7$ Ly$\alpha$ luminosity function [Ouchi et al., 2008] assuming a duty cycle $f_{duty} = 0.1$ and 1.

Fig. C1 shows that, given a halo mass, a higher duty cycle requires a brighter Ly$\alpha$ luminosity to match the observed $z = 5.7$ Ly$\alpha$ luminosity function, and that a simple functional form, e.g. $L_\alpha \propto M_h$, $M_h^2$, cannot match the semi-empirical relation.

In our model, the intrinsic Ly$\alpha$ luminosity of each galaxy (halo) is assigned according to the $L_\alpha - M_h$ relation with $f_{duty} = 1$ in Fig. C1.

APPENDIX D: Ly$\alpha$ RT THROUGH THE IGM: COMPUTING THE LINE-OF-SIGHT SKEWERS AND OPTICAL DEPTH

We compute the Ly$\alpha$ optical depth in the red damping wing as follows. The density, temperature, velocity and local H I fraction fields along skewers originating at the location of halos and parallel to the z-axis are extracted from the hydrodynamical and radiative transfer simulations. To obtain a converged numerical integration of the optical depth, the sampling size of the skewers, $\delta l$, must be sufficiently fine. To be on the safe side, the Doppler core of the Voigt line profile should be resolved. In the velocity space this is $\delta v/c = \Delta v_D/\nu_\alpha = 4.286 \times 10^{-7}(T/K)^{1/2}$. Therefore, the velocity space resolution must be $\delta v \approx 0.13(T/1K)^{1/2}$ km/s, which corresponds to a real space resolution of $\delta l = \delta v/H(z_a) \approx 0.17(T/K)^{1/2}$ pkpc at $z_a = 7$ with our cosmological parameters. If this criterion is not met, scattering by Doppler core could be missed. Although the Doppler core scattering is important in low density regions to produce Ly$\alpha$ forest absorption blueward of the rest-frame Ly$\alpha$ line, here we are interested only in the red damping wing and the Lorentz wing scattering. Therefore, a converged evaluation of the optical depth in the red damping wing can still be obtained without strictly meeting this resolution criterion. Nonetheless, the sampling of the line-of-sight skewers must be sufficiently fine, and a sub-sampling within a cell of the cosmological hydrodynamical simulations is required to obtain a convergence in equation (D1).

To this aim, we have assumed that the density, ionization, temperature and peculiar velocity fields are constant within each cell, while the Hubble flow is allowed to vary. This is required to recover the analytic solution and to obtain a numerically converged optical depth in the limit of homogeneous expanding IGM.

The discretized form of the optical depth is then integrated at each frequency point $\nu_\alpha$ using the line-of-sight skewers according to

$$\tau_\alpha(\nu_\alpha) = \sum_{i=1}^N \sigma_{12} n_{\nu_\alpha}(l_i) \varphi_\nu \left[T_{\alpha}(l_i) \left(\frac{1 - v_{\nu_\alpha}(l_i)}{c}\right)\right] \delta l. \quad (D1)$$

The maximum proper length of the line-of-sight skewers influences the far redward optical depth, as a lower length would result in more transmission. We choose the maximum proper length of the skewer to be 12 pkpc. If a skewer exits the simulation box, a random cell in a random face of the box is chosen, and the line-of-sight is followed until the maximum proper length is reached. We have verified that for a homogeneous expanding IGM, the result at $\Delta v \sim 1000$ km/s has a discrepancy of $\sim 8\%$ relative to the analytic solution of the optical depth. Because the IGM will become more ionized as Ly$\alpha$ photons travel through the medium and because we retain the same redshift output to extract the line-of-sight skewers, we choose the maximum length of our skewer samples to be 12 pkpc.

The lower bound of the optical depth integration is chosen to be 300 $h^{-1}$ckpc. As a reference, the virial radius of a halo with mass $M_h$ is $R_{vir} \approx 78.5 (M_h/10^{11} h^{-1} M_{\odot})^{1/3} h^{-1}$ckpc, i.e. we exclude from the calculation the gas contained within a halo, as well as all the structures on scales smaller than the Jeans length because they are not well resolved in our simulations.

APPENDIX E: INTRINSIC Ly$\alpha$ FRACTION

We write the intrinsic Ly$\alpha$ fraction as $X_{\nu_\alpha}^{\mathrm{intr}}( > \mathrm{REW}_{\nu_\alpha}|M_{UV}) = e^{-\mathrm{REW}_{\nu_\alpha}|M_{UV}}$ where $\mathrm{REW}_{\nu_\alpha}|M_{UV}$ is the characteristic REW.

The $M_{UV}$-dependent model and uncorrelated model differ in their functional form of $\mathrm{REW}_{\nu_\alpha}|M_{UV}$, as the latter assumes a constant $\mathrm{REW}_{\nu_\alpha}|M_{UV} = 50$, while the former uses the $\mathrm{REW}_{\nu_\alpha}|M_{UV}$ obtained from the best-fit to the Ly$\alpha$ fraction of LBGs observed at $3 < z < 6$ (Stark, Ellis & Chu, 2010), i.e. $X_{\nu_\alpha}^{\mathrm{intr}}( > \mathrm{REW}|M_{UV}, z = 7) = X_{\nu_\alpha}^{\mathrm{REW}+\mathrm{FW}}( > \mathrm{REW}|M_{UV})$.

Furthermore, for $P(M_h|M_{UV})$ we assume a one-to-one mapping between UV magnitude and halo mass, i.e. $P(M_h|M_{UV}) = \delta_D(M_h - M_h(M_{UV}))$. The $M_h$ - $M_{UV}$ relation is given by $M_h(M_{UV}) = M_h^* \times 10^{-(M_{UV} - M_{UV}^*)/2.5}$ where $M_h^* = 10^{10} M_{\odot}$.
and $M_{UV}^* = -19$. We note that this relation tends to assign masses which are typically lower than those derived from observations. For example, $M_h(M_{UV} = -20) = 2.5 \times 10^{10} M_\odot$, which is much lower than the mass of LBGs hosts inferred from clustering analysis, i.e. $M_h \sim 3 \times 10^{11} - 10^{12} M_\odot$ (e.g. Kashikawa et al. 2006). Since we expect the dependence of $T_{IGM}$ on halo mass to extends in the range $11 < \log_{10} M_h / M_\odot < 12$, we assume the sampling of the $T_{IGM} - M_h$ relation at low mass haloes to mimic the realistic host halo mass of observed LBGs.