Resummation for direct photon
and $W + \text{jet}$ production

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Abstract
I discuss the resummation of soft gluon contributions to direct photon and
$W + \text{jet}$ production and I present some results for the next-to-next-to-leading
order expansions of the resummed cross sections for these processes near threshold.

1. Introduction

Direct photon production and $W + \text{jet}$ production are processes of great interest for a number of
reasons. Direct photon production is important for determinations of gluon distributions while $W + \text{jet}$
production is relevant for estimating backgrounds
for the resummation of soft-gluon contributions

corrections are potentially large. A formalism
emission to all orders in perturbative QCD. These
correctors of the final state in these processes one can resum
production [4, 5, 1, 6, 7, 8] in hadronic processes

First, we discuss resummation for direct photon
production (NLO); this is expected on theoretical grounds [3].

scale dependence relative to next-to-leading order
(NNLO), thus providing new analytical higher order

been expanded up to next-to-next-to-leading order
see Refs. [1, 2]). The resummed cross sections have

2. Direct photon production

First, we discuss resummation for direct photon production in hadronic processes

The factorized cross section may be written as a convolution of parton distributions $\phi$ with the
partonic hard scattering $\hat{\sigma}$

Near partonic threshold $\hat{\sigma}$ includes large logarithmic
terms that can be resummed to all orders in
perturbation theory. If we take moments of the
above equation, with $N$ the moment variable, we
can write the partonic cross section as

where in the second line of Eq. (3) we have introduced a refactorization in terms of new center-of-mass parton distributions $\psi$ and a jet function $J$ that absorb the collinear singularities from the
incoming partons and outgoing jet $\gamma \rightarrow f_1, f_2$, respectively; a soft gluon function $S$ that describes
noncollinear soft gluon emission $\rightarrow f_1, f_2$, and a short-distance hard scattering function $H$.

We may then solve for $\hat{\sigma}_{f_1 f_2 \rightarrow \gamma}(N)$ in Eq. (3). After resuming the $N$-dependence of all the
functions in the above equation, we may write the resummed cross section as

The exponent $E^{(f_i)}$ resums the ratio $\psi/\phi$ while
$E^{(f_i)}$ resums the $N$-dependence of the outgoing jet
$E^{(f_i)}$. The dependence on the scale is given by

Finally, Re$\Gamma_S$ is the real part of the soft
anomalous dimension $\Gamma_S$ which is determined from renormalization group analysis of the soft function $S$ and has been calculated explicitly at one-loop [11, 11]. Because of the simpler color structure, $\Gamma_S$ is here a simple function, in contrast to heavy
quark and jet production where it is a matrix in color space \[ \alpha \bar{q}g \] are [7, 8].

Since the direct photon production cross section has been calculated only up to NLO, expansions of the resummed result to higher orders provide new analytical predictions. Here we present the NNLO expansion for the partonic subprocess

\[ q(p_1) + g(p_2) \rightarrow \gamma(p_\gamma) + q(p_f). \]

We define the kinematical variables \( s = (p_1 + p_2)^2 \), \( t = (p_1 - p) \), \( u = (p_2 - p) \), and \( v \equiv 1 + t/s \). The threshold region is given in terms of the variable \( w \equiv -u/(s + t) \) by \( w = 1 \). Then at nth order in \( \alpha_s \) we encounter plus distributions of the form \( \ln^m(1-w)/(1-w) \) with \( m \leq 2n - 1 \).

The NLO soft gluon corrections in the \( \overline{\text{MS}} \) scheme for this partonic channel are given by

\[
vw(1-v)\frac{d\sigma_{qq \rightarrow q\gamma}^{NLO}}{dv dw} = \sigma_{qq \rightarrow q\gamma}^{B} \frac{\alpha_s(\mu_R^2)}{\pi} \times \left\{ \left( C_F + 2C_A \right) \left[ \ln(1-w) \right]_+ \right. \\
+ \left. \left[ C_F \left( -\frac{3}{4} + \ln v \right) - C_A \ln \left( \frac{1-v}{v} \right) \right] \right. \\
+ \left. \left[ C_F - C_A \right] \ln \left( \frac{\mu_F^2}{\mu} \right) \right] \left[ \frac{1}{1-w} \right]_+ \\
+ O(\delta(1-w)) \right\}, \tag{6}
\]

with \( \mu_F \) and \( \mu_R \) the factorization and renormalization scales. Here the Born term is [13]

\[
\sigma_{qq \rightarrow q\gamma}^{B} = \frac{1}{N_c^2} \pi \alpha_s(\mu_R^2) q_v^2 T_{qq} v, \tag{7}
\]

where \( T_{qq} = 1 + (1-v)^2 \). Agreement is found with the exact NLO cross section in Ref. [13]. All \( \delta(1-w) \) terms can be obtained by matching to the exact NLO result.

The NNLO \( \overline{\text{MS}} \) soft gluon corrections for \( qg \rightarrow q\gamma \) are [13]

\[
vw(1-v)\frac{d\sigma_{qq \rightarrow q\gamma}^{NNLO}}{dv dw} = \sigma_{qq \rightarrow q\gamma}^{B} \frac{\alpha_s^2(\mu_R^2)}{\pi^2} \times \left\{ \left( \frac{1}{2} C_F + 2C_A \right)^2 \left[ \ln^3(1-w) \right]_+ \right. \\
+ \left. \left[ \frac{3}{2} C_F^2 \left( -\frac{3}{4} + \ln v - \ln \left( \frac{\mu_F^2}{s} \right) \right) \right] \right. \\
+ \left. \left[ 3 C_A \left( \ln \left( \frac{v}{1-v} \right) - \ln \left( \frac{\mu_F^2}{s} \right) \right) \right] \right. \\
+ \left. \left[ \frac{3}{2} C_F C_A \left( -\ln(1-v) + 3 \ln v - 3 \ln \left( \frac{\mu_F^2}{s} \right) \right) \right] \right. \\
+ \left. O(\delta(1-w)) \right\}, \tag{8}
\]

with \( \beta_0 = (11C_A - 2n_f)/3 \). We have also derived all NNLL terms in the NNLO expansion by matching with the exact NLO cross section [13]. Analogous results have been obtained for the partonic channel \( q\bar{q} \rightarrow g\gamma \).

In Fig. 1 we show some numerical results for the direct photon production cross section as a function of the photon \( p_T \) and compare with the experimental results from the E706 Collaboration at Fermilab [4]. We see that the sum of the exact NLO cross section and NNLO approximate corrections shows a much reduced dependence on the factorization scale relative to the exact NLO cross section alone. However, the NNLO cross section is still below the E706 data.

3. \( W + \) jet production

In this section, we discuss resummation for \( W + \) jet production [15, 16] in hadronic processes

\[ h_1 + h_2 \rightarrow W + X. \tag{9} \]

The construction of the resummed cross section is completely analogous to what we presented in the previous section for direct photon production, Eq. (4). The soft anomalous dimensions are the same.
as for direct photon production. In this section we expand the NLL resummed cross section for $W + \text{jet}$ production up to NNLO and perform a comparison with the NLO fixed-order results of Refs. [16, 17]. We give explicit results for the partonic subprocess

$$q(p_1) + \bar{q}(p_2) \rightarrow W(\ell) + g(p_3).$$

(10)

The threshold region is given in terms of the variable $s_2 = s + t + u - Q^2$ by $s_2 = 0$, and we find plus distributions of the form $[\ln^2(s_2/Q^2)/s_2]_+$ at every order in perturbative QCD.

The NLO $\overline{\text{MS}}$ soft gluon corrections for this partonic process at NLL accuracy are [1, 15]

$$E_Q \frac{d^2\sigma_{qq\rightarrow gW}^{NLO}}{d^2Q} = \sigma_{qq\rightarrow gW} B \frac{\alpha_s(\mu_R^2)}{\pi} \times \left\{ \left(4C_F - C_A\right) \left[ \ln(s_2/Q^2) \right] \right\} + \left[ \left(2C_F - C_A\right) \ln\left(\frac{s_2Q^2}{tu}\right) - \frac{3}{4} \beta_0 \right] + \left(2C_F - C_A\right) \ln\left(\frac{\mu^2}{Q^2}\right) \left[ \frac{1}{s_2} \right] + \mathcal{O}(\delta(s_2)).$$

(11)

Here the Born term is

$$\sigma_{qq\rightarrow gW} = \frac{\alpha_s(\mu_R^2)C_F}{sN_c} (|L_{f_2f_1}|^2 + |R_{f_2f_1}|^2) \times \left( \frac{u}{t} + \frac{t}{u} + 2Q^2/s \right),$$

(12)

with $L$ and $R$ the left- and right-handed couplings of the $W$ boson to the quark line of flavor $f$ [17]. The NLO expansion is in agreement with the exact one-loop results in Ref. [17]. All $\delta(s_2)$ terms can be obtained by matching to the exact NLO result.

The NNLO $\overline{\text{MS}}$ soft gluon corrections for the partonic process $qq \rightarrow gW$ are [13]

$$E_Q \frac{d^2\sigma_{qq\rightarrow gW}^{NNLO}}{d^2Q} = \sigma_{qq\rightarrow gW} B \left( \frac{\alpha_s(\mu_R^2)}{\pi} \right)^2 \times \left\{ \frac{1}{2} \left(4C_F - C_A\right)^2 \left[ \ln^2(s_2/Q^2) \right] \right\} + \left( \frac{3}{2} \left(4C_F - C_A\right) \left[ 2C_F \ln\left(\frac{\mu^2}{Q^2}\right) \right. \right. + \left. \left. (2C_F - C_A) \ln\left(\frac{tu}{s_2Q^2}\right) \right) \right\} + \frac{3}{2} \left(5C_F - 3C_A\right) \left[ \ln^2(s_2/Q^2) \right] + \mathcal{O}(\delta(s_2)).$$

(13)

We can also derive all NNLL terms in the NNLO expansion by matching with the exact NLO cross section [15]. Analogous results have been obtained for the partonic channel $gq \rightarrow gW$ [16, 17]. In future work we plan to study the numerical significance of resummation for $W+$ jet production.

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