Optimal Pricing and Load Sharing for Energy Saving in Communications Cooperation

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Abstract

In this paper, we propose a pricing mechanism for the uplink communication cooperation to save the energy of mobile terminals (MTs) in wireless cellular network. Under the uncertainties of the other MTs’ channel and battery conditions, a source MT in low battery level or bad channel condition is allowed to select and pay another MT in proximity to help forward its data package to the base station (BS). We formulate the source MT’s pricing and load sharing problem as an optimization problem to minimize its expected energy cost. When the source MT cannot split its data package for a certain multimedia application, we motivate the selected relay MT to forward the whole data package by obtaining the optimal pricing through a dichotomous search algorithm. When the source MT can split the data package, we jointly optimize the pricing and load sharing with the relay MT and propose an alternating optimization algorithm that achieves near-optimal solution. Extensive numerical results are provided to show that our proposed pricing mechanism can significantly decrease the source MT’s expected energy cost, and load sharing is more cost-efficient when the size of the data package is large and the average number of helping MTs is small.

I. INTRODUCTION

With the recent development in the smart phones and the multimedia applications, the cellular network is experiencing an exponential increase in the wireless data traffic and MTs consume a lot more energy than before. Considering the limited battery capacity, MTs need to be charged more frequently and this has become the biggest customer complaint for smart phones [1]. In this regard, MT-side energy saving can help resolve the energy shortage and the shutdown of the MTs and also improve the connectivity and reliability of the wireless network. As such, reducing the energy consumption for the MT is of critical importance. Furthermore, it has been shown in [2] and [3] that, among all the energy-consuming applications and modules of the MTs, the communications modules cover a large proportion of the total energy consumption. Hence, this gives us the motivation to investigate the energy saving for the MTs in data communications.

Communications cooperation is an effective way to save energy for the MTs and has been thoroughly investigated in the literature of wireless sensor networks and cellular networks [4]–[6]. In particular, [4] discusses the optimal modulation to minimize the total energy consumption for the cooperative MIMO scheme to reduce the energy consumption of the sensor nodes. [5] considers the optimal timer-based relay selection scheme for the minimization of the sum energy consumption and maximization of the network lifetime. [6] has proposed a space-time coding scheme for the MTs to cooperatively transmit to the base station under given outage and capacity requirements such
that total transmit energy is minimized. Although communications cooperation has long been proposed for energy saving, it is hard to be realized in reality due to the lack of incentives that motivate the MTs to cooperate. From this perspective, prior works ([7], [8]) have also proposed the incentive mechanisms in cooperative communications. Specifically, [7] studies the optimal relay selection and the resource allocation problem formulated as a Stackelberg game, where the utilities of the source and relay MTs are maximized. [8] studies the dynamic bargaining-based cooperative spectrum sharing between a primary user (PU) and a secondary user (SU), where the PU shares spectrum to the SU and SU helps relay the signal of the PU in return.

Different from the previous works, in this paper, we exploit the heterogeneities of the battery levels and channel conditions of the MTs and propose the energy-saving communication cooperation between MTs through pricing mechanism. For the design of this pricing mechanism, there are two main challenges to be addressed. First, incentive design is required to ensure that both the source and relay MTs can benefit from the cooperation. Second, it is also challenging to obtain the optimal pricing policy due to the lack of the exact information about the relay MTs’ battery level and channel condition. In this paper, in order to motivate the relay MT, we propose a pricing mechanism that the source MT gives a certain type of compensation to the relay MT for the help it receives, which can be in the form of currency or credits in a multimedia application.

The main contributions of this paper are summarized as follows:

- **Incentive mechanism for communications cooperation to save energy**: In Section III under the uncertainties of the helping MTs’ battery levels and channel conditions, we propose a pricing mechanism for the communications cooperation between the source and relay MTs that can lead to a win-win situation. Load sharing is also investigated to save the energy.

- **Pricing only and joint pricing and load sharing for non-splittable and splittable data traffic**: First, for the case of non-splittable data traffic in Section IV-A we formulate the problem as a convex optimization problem and minimize the expected cost of the source MT by finding the optimal price via the dichotomous search method. Next, in the case of splittable data traffic with both pricing and load sharing in Section IV-B we propose an alternating optimization algorithm to obtain the near-optimal solution.

- **Performance Evaluation**: Numerical examples are given in Section V and it is shown that, compared to the benchmark scheme of direct data transmission, our proposed cooperative data transmission scheme can effectively decrease the expected energy cost of the source MT. Moreover, load sharing can further decrease the energy cost especially when the size of the data package is large and the average number of helping MTs is low.

II. SYSTEM MODEL

As shown in Fig. 1 we consider the uplink data transmission in a cellular network, where there are K MTs within a single cell, denoted by the set \( K = \{1, 2, \cdots, K\} \). We assume that the locations of the MTs follow a two-dimensional Homogeneous Poisson Point Process (HPPP) with spatial density \( \lambda \). We also assume that the MTs within the cell initiate their data traffic independently with probability \( \rho \). Then, according to the Marking Theorem [9], the source MTs that have data to transmit also form an HPPP with density \( \rho \lambda \) and the remaining idle MTs

\(^1\)Our results can be extended to the case of multiple cells by applying our results in each cell independently.
from another HPPP with density \((1 - \rho)\lambda\). We denote these sets of source MTs and idle MTs as \(\mathcal{K}_S\) and \(\mathcal{K}_I\), respectively, such that \(\mathcal{K}_S \cup \mathcal{K}_I = \mathcal{K}\) and \(\mathcal{K}_S \cap \mathcal{K}_I = \emptyset\).

For a certain source MT \(i \in \mathcal{K}_S\), it can associate with another idle MT (if any) within the distance of \(d\) as its relay MT for helping relay the data package, where \(d\) is the range of the short range communications (SRC) such as Zigbee [10], Smart Bluetooth [11], etc. We denote this set of idle MTs within the distance of \(d\) from the source MT \(i\) as its helping MTs by the set \(\mathcal{H}_i \subset \mathcal{K}_I\), \(i \in \mathcal{K}_S\), where \(|\mathcal{H}_i| = N_i\) is the number of MTs within the set. It follows that \(N_i\) is a Poisson random variable with mean \(\mu_{N_i} = (1 - \rho)\lambda \pi d^2\), \(i \in \mathcal{K}_S\) and its probability mass function (PMF) is given by

\[
\Pr(N_i = n) = \frac{\mu_{N_i}^n}{n!} e^{-\mu_{N_i}}, \ n = 0, 1, \cdots, \ i \in \mathcal{K}_S.
\] (1)

It can be observed from (1) that the PMF of the number of MTs \(N_i\) is proportional to the range of the SRC \(d\), the probability of an MT remaining idle \(1 - \rho\) and the spatial density \(\lambda\) of the MT.

A. MTs’ Data Transmission Model

The energy consumption of the MTs depend on the transmission model and the channel condition. We consider a narrow-band fading channel for all MTs. We denote \(n_k\) as the noise for the received signal of MT \(k \in \mathcal{K}\) at the BS. It is assumed that \(n_k\) is independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian (CSCG) random variable denoted by \(n_k \sim \mathcal{CN}(0, \sigma_k^2)\) with \(\sigma_k^2\) denoting the power of the noise. We also denote \(h_k\) as the complex baseband channel coefficient from MT \(k\) to the BS. We assume that \(h_k\) follows a simplified channel model incorporating the large-scale power attenuation with loss exponent \(\alpha > 2\) and the small-scale Rayleigh fading. More specifically, we denote \(r_k\) as the distance between MT \(k \in \mathcal{K}\) and the BS, and \(r_0\) as the reference distance, respectively. Then, the channel coefficient \(h_k\) is expressed as

\[
h_k = \bar{h}_k \left(\frac{r_k}{r_0}\right)^{-\alpha}, \ k \in \mathcal{K},
\] (2)

where \(\bar{h}_k \sim \mathcal{CN}(0, 1), \ k \in \mathcal{K}\) is an i.i.d. CSCG random variable with zero mean and unit variance modeling the small-scale Rayleigh fading, and \(G\) is a constant modeling the path-loss between the MT and the BS at the
reference distance $r_0$. Therefore, the channel gain between the MT $k$ and the BS is

$$g_k = |h_k|^2 = \eta_k G \left( \frac{r_k}{r_0} \right)^{-\alpha}, \quad k \in \mathcal{K},$$

(3)

where $\eta_k \sim \exp(1)$ is an exponential random variable with unit rate modeling the power envelope of the Rayleigh fading. Furthermore, we assume that the distance $d$ for the SRC is small. Then, the source MT $i$ and its helping MT $j \in \mathcal{H}_i$ have roughly the same distance to the BS ($r_i = r_j, \quad j \in \mathcal{H}_i, \quad i \in \mathcal{K}_S$). Hence, for the source MT $i \in \mathcal{K}_S$, the channel gain between its helping MT $j \in \mathcal{H}_i$ and the BS is

$$g_j = \eta_j G \left( \frac{r_i}{r_0} \right)^{-\alpha},$$

(4)

where the Rayleigh fading coefficient $\eta_j$ is still i.i.d. distributed across the helping MTs.

B. MTs’ Energy Consumption

The energy consumption of the MT depends on the uplink channel condition and the size of the data package. We denote $D_i$ as the size of the data package for the source MT $i, \quad i \in \mathcal{K}_S$ to be transmitted in one time slot. Without loss of generality, we normalize the data transmission rate to the bandwidth available at the MT, which is assumed to be unity. To accomplish the uplink transmission of (normalized) data rate $D_i$, source MT $i \in \mathcal{K}_S$ can choose between the two transmission modes as follows:

- **Direct Transmission Mode (DT Mode):** In this mode, the source MT $i \in \mathcal{K}_S$ transmits the data package with rate $D_i = \log_2(1 + \frac{\bar{E}_{i}^{(S)}}{N_0})$ to the BS directly, where $\bar{E}_{i}^{(S)}$ is the average energy per symbol. Hence, the required energy for transmitting the data package $D_i$ is

$$\bar{E}_{i}^{(S)} = \frac{N_0}{g_i}(2^{D_i} - 1), \quad i \in \mathcal{K}_S.$$  

(5)

- **Cooperative Transmission Mode (CT Mode):** In this mode, source MT $i \in \mathcal{K}_S$ splits its package $D_i$ into two parts with $D_i = D_{i}^{(S)} + D_{i}^{(R)}$: $D_{i}^{(S)}$ for the source to transmit directly to the BS and $D_{i}^{(R)}$ for its relay MT to transmit. For the part of data $D_{i}^{(S)}$, similar to (5), the required energy for source MT $i$ is

$$\bar{E}_{i}^{(S)} = \frac{N_0}{g_i}(2^{D_{i}^{(S)}} - 1), \quad i \in \mathcal{K}_S.$$  

(6)

Then, for the other part of data $D_{i}^{(R)}$, as shown by the blue dash line in Fig. 1, the source MT first transmits it to its selected relay MT and then the relay MT decodes and forwards the signal to the BS. Due to the small SRC radius $d$, the transmit power for the source MT to communicate with its relay MTs is also small. In practice, SRC technologies, such as Bluetooth, Zigbee, etc., offer high communication data rate with low transmit power. We thus assume the energy consumption of this short range data transmission equals zero and the required transmission time is zero. Hence, if MT $j$ is selected as the relay MT, the energy consumption for MT $j$, is

$$\bar{E}_{j}^{(R)} = \frac{N_0}{g_j}(2^{D_{j}^{(R)}} - 1), \quad j \in \mathcal{H}_i, \quad i \in \mathcal{K}_S.$$  

(7)

\(^{2}\text{For example, the typical range of Zigbee is 10 – 20 meters [10] and the range of the most common Class 2 Bluetooth device is 10 meters [11].}\)

\(^{3}\text{The analysis can be easily extended to the case that the energy consumption and the transmission time are constants without major changes in the results.}\)
C. Cooperative Data Transmission Protocol

In order to save energy for the source MTs, we propose the 4-step protocol for cooperative data transmission as follows.

1. When an MT has data to transmit, it chooses between the CT mode and DT mode according to the criterion to be specified later in the paper.
2. When the DT mode is selected, the source MT transmits the signal to the BS directly. When the CT mode is selected, it initiates the relay association by broadcasting the proposed payment information and the size of the data to be relayed to all its helping MTs.
3. The helping MTs (if any) calculate their own utilities and accept the offer if the utility for the cooperation can be more than a threshold. If the condition is satisfied, an acceptance notification will be sent to the source MT.
4. If multiple helping MTs accept the relay request, the source MT randomly choose one MT from them as the relay MT and transmits the data with the CT mode. If there is no MT accepting the request, the source MT uses DT mode to send the data package to the BS directly.

In the following, we elaborate on the protocol design and specify the criterion for the mode selection of the source MT.

III. Problem Formulation for Communications Cooperation

For different battery levels, an MT has different valuations towards the remaining energy in the battery. The cost for energy consumption is in general larger when the battery level is lower. We define the cost per unit energy \( \zeta_k \) for each MT \( k \in K \) as a function of its battery level \( B_k \), i.e.,

\[
\zeta_k = f(B_k), \quad k \in K,
\]

where \( B_k \in [0,B_{\text{max}}] \) is the battery level of MT \( k \) with its range from zero to the full storage \( B_{\text{max}} \), and \( f : [0,B_{\text{max}}] \rightarrow [0,\zeta_{\text{max}}] \) is a monotonically decreasing function of \( B_k \) whose range is from zero to the maximum energy cost \( \zeta_{\text{max}} \). In the following, we adopt a specific linear cost function for analysis tractability\(^4\)

\[
\zeta_k = \zeta_{\text{max}} - \frac{\zeta_{\text{max}}}{B_{\text{max}}} B_k, \quad k \in K.
\]

We assume that the battery state \( B_j \) of the helping MT \( j \in H_i \) is known by the source MT \( i \in K_S \) as uniform distribution, i.e. \( B_j \sim U[0,B_{\text{max}}] \). Thus, the cost \( \zeta_j, \, j \in H_i \) is also known to the source MT \( i \in K_S \) as uniform distribution, i.e. \( \zeta_j \sim U[0,\zeta_{\text{max}}] \).

Furthermore, we assume that the sizes of the data package \( D_i^{(S)} \) and \( D_i^{(R)} \) for transmission are relatively small that \( \zeta_k, \, k \in K \) remains constant during the data transmission. If a helping MT \( j \in H_i \) is selected by the source MT \( i \in K_S \) as the relay MT, it will receive a price \( \pi_i \) for forwarding MT \( i \)’s data package \( D_i^{(R)} \). Hence, the net utility of the helping MT \( j \) participating in the cooperation is \( \pi_i - \zeta_j E_j^{(R)} \), where \( E_j^{(R)} \) is the energy consumption\(^5\)

\(^4\)Multiple relay association incurs a large amount of overheads. In order to keep the overhead low, in this paper we assume single relay association.

\(^5\)Extension to the other non-negative monotonically decreasing functions is unlikely to change the main insights of this paper.

\(^6\)We assume that the battery level \( B_j \) of helping MT \( j \) is uniformly distributed for analysis tractability. The analysis for other distributions will be technically more involved but offers essentially the same engineering insights.
for transmitting the data package $D_i^{(R)}$ defined in (7). Because the association process will incur overheads due to signaling and processing, we assume that the helping MTs have a reservation utility of $\epsilon \geq 0$ for accepting the request. That is, helping MT $j$ will accept the relay request from source MT $i$ as long as its utility is larger than $\epsilon$, i.e., $\pi_i - \zeta_j E_j^{(R)} \geq \epsilon$. Therefore, the utility of the helping MT $j \in \mathcal{H}_i$ for source MT $i \in \mathcal{K}_S$ is the difference between the price and the energy cost if the difference is larger than $\epsilon$ and zero otherwise, which is defined as

$$U_j = \begin{cases} 
\pi_i - \zeta_j E_j^{(R)}, & \text{if } \pi_i - \zeta_j E_j^{(R)} \geq \epsilon, \\
0, & \text{otherwise.} 
\end{cases}$$

(10)

For the source MT $i$, if there is at least one helping MT accepting the price $\pi_i$, the cost of the source MT $i$ is the sum of the price $\pi_i$ and energy cost for direct transmission $\zeta_i E_i^{(S)}$. Otherwise, it needs to transmit the whole package $D_i$ directly to the BS at the cost of $\zeta_i \bar{E}_i^{(S)}$. Thus, the energy cost of source MT $i \in \mathcal{K}_S$ is

$$C_i = \begin{cases} 
\pi_i + \zeta_i E_i^{(S)}, & \text{if } \exists j \in \mathcal{H}_i \text{ s.t. } \pi_i - \zeta_j E_j^{(R)} \geq \epsilon, \\
\zeta_i \bar{E}_i^{(S)}, & \text{otherwise.} 
\end{cases}$$

(11)

To motivate the participation of the source MT $i \in \mathcal{K}_S$ and relay MT $j \in \mathcal{H}_i$ in the cooperation, the price $\pi_i$ should satisfy

$$\epsilon \overset{(a)}{\leq} \pi_i \overset{(b)}{\leq} \zeta_i \bar{E}_i^{(S)} - \zeta_i E_i^{(S)} ,$$

(12)

where inequality (a) is the minimum requirement for attracting helping MTs in the cooperation and inequality (b) ensures cost reduction for the source MT. Note that $\zeta_i \bar{E}_i^{(S)} \geq \epsilon$ must hold for the feasibility of the CT mode, that is, the value of the saved energy at the source MT must be larger than the reservation utility of the source MT. In the sequel, we assume this condition always holds to avoid the trivial non-cooperative case.

We assume that all the channel gains $g_j$ and battery states $B_j$ of the relay MTs $j \in \mathcal{H}_i$ are independent. We also assume that the source MT $i \in \mathcal{K}_S$ only knows the distributions of the number of helping MTs $N_i$ given in (1), channel gains $g_j$ in (4), and battery states $B_j$, and has to decide price $\pi_i$ and shared data package $D_i^{(R)}$ under such uncertainties. We denote $\Pr(\pi_i - \zeta_j E_j^{(R)} \leq \epsilon)$ as the probability that helping MT $j \in \mathcal{H}_i$ rejects the request given by the source MT $i \in \mathcal{K}_S$. Given the set of helping MTs $\mathcal{H}_i$, the conditional expected cost of the source MT $i$ for transmitting data package $D_i$ is

$$\mathbb{E}[C_i|\mathcal{H}_i] = \left( 1 - \prod_{j \in \mathcal{H}_i} \Pr(\pi_i - \zeta_j E_j^{(R)} \leq \epsilon) \right)$$

$$\times \left( \pi_i + \zeta_i E_i^{(S)} \right) + \left( \prod_{j \in \mathcal{H}_i} \Pr(\pi_i - \zeta_j E_j^{(R)} \leq \epsilon) \right) \zeta_i \bar{E}_i^{(S)}$$

$$= \left( 1 - \prod_{j \in \mathcal{H}_i} \Pr(\pi_i - \zeta_j E_j^{(R)} \leq \epsilon) \right)$$

$$\times \left( \pi_i + \zeta_i E_i^{(S)} - \zeta_i \bar{E}_i^{(S)} + \zeta_i \bar{E}_i^{(S)}, i \in \mathcal{K}_S, \right)$$

(13)

where the expectation is taken over the two possible outcomes of successful and un-successful association in (11). By further considering all possibilities of $\mathcal{H}_i$ for source MT $i$ in (1), the expected cost of the source MT $i$ for
transmitting the data package $D_i$ can be obtained with the law of iterated expectation, i.e.,

$$
\mathbb{E}[C_i] = \mathbb{E}[\mathbb{E}[C_i | H_i]] \\
= \sum_{n=0}^{\infty} \Pr(N_i = n) \mathbb{E}[C_i | H_i], \quad i \in K_S.
$$

(14)

Then, we formulate the optimization problem that minimizes the expected cost of the source MT $i \in K_S$ over the price $\pi_i$ and relay data $D_i^{(R)}$ as follows:

$$(P1): \min_{\pi_i \geq 0, D_i^{(R)} \geq 0} \mathbb{E}[C_i]$$

s.t. $\epsilon \leq \pi_i \leq \zeta_i E_i^{(S)} - \zeta_i E_i^{(S)}$,

$$D_i^{(S)} + D_i^{(R)} = D_i.$$ 

(15) (16)

Next, we discuss the criterion for the mode selection of source MT $i$, which has been introduced in Section II-C. For the source MT to choose the CT mode, its cost reduction from the direct transmission must be larger than a threshold denoted by $\gamma_i$, which incorporates overheads such as signaling, signal processing, etc. Hence, the condition for the source MT to choose the CT mode for the data transmission is

$$\zeta_i E_i^{(S)} - \mathbb{E}[C_i^*] \geq \gamma_i,$$

(17)

where $\mathbb{E}[C_i^*]$ is the minimum expected cost obtained in (P1). In the next section, we discuss the optimal solution for problem (P1) to obtain the $\mathbb{E}[C_i^*]$ under two cases: non-splittable and splittable data package.

### IV. Proposed Solutions for Problem (P1)

In this section, we derive the solution to problem (P1). With the energy consumption $E_j^{(R)}$ for the helping MT $j \in H_i$ defined in (7), the probability of successful association between source MT $i \in K_S$ and its potential relay MT $j \in H_i$ in (13) is

$$\Pr(\pi_i - \zeta_j E_j^{(R)} \geq \epsilon) = \Pr \left( \frac{\zeta_j}{\eta_j} \leq \frac{G_{P_1}^{-\alpha}(\pi_i - \epsilon)}{N_0 r_0^{-\alpha}(2D_i^{(R)} - 1)} \right)$$

$$= \Pr(\theta_j \leq w_i),$$

(18)

where $\theta_j = \frac{\zeta_j}{\eta_j}$ is a random variable denoting the aggregate private information about the battery state and channel condition of the helping MT $j$, and we denote

$$w_i = \frac{G_{P_1}^{-\alpha}(\pi_i - \epsilon)}{N_0 r_0^{-\alpha}(2D_i^{(R)} - 1)}.$$ 

(19)

For convenience, we normalize the energy price of MT $j$ in (9) and assume it to be uniformly distributed as $\zeta_j \sim U[0, 1]$. Also, with the assumption that $\eta_j$ is exponentially distributed as $\eta_j \sim \exp(1)$, the probability of successful association between the source MT $i \in K_S$ and helping MT $j \in H_i$, which is complimentary to $\Pr(\pi_i - \zeta_j E_j^{(R)} \leq \epsilon)$ in (13), is

$$\Pr(\pi_i - \zeta_j E_j^{(R)} \geq \epsilon) = \Pr(\theta_j \leq w_i) = \Pr(\eta_j \geq \frac{\zeta_j}{w_i})$$

$$= \int_0^{\frac{\zeta_j}{w_i}} e^{-\eta_j} d\eta_j = w_i \left( 1 - e^{-\frac{\zeta_j}{w_i}} \right).$$

(20)
Hence, with the results in (13) and (20), we simplify the objective of problem (P1) in (14) as
\[ E[C_i] = \sum_{n=0}^{\infty} \Pr(N_i = n) \left\{ \left[ 1 - \left( 1 - w_i \left( 1 - e^{-\frac{1}{w_i}} \right) \right) ^n \right] \times \left( \pi_i + \zeta_i E_i^{(S)} - \zeta_i E_i^{(S)} \right) + \zeta_i E_i^{(S)} \right\}. \]  
\[ (21) \]

With the objective function of problem (P1) reduces to (21), we discuss the convexity of problem (P1) by the following proposition.

**Proposition 4.1:** Problem (P1) is marginally convex with respect to $\pi_i$ and $D_i^{(R)}$.

**Proof:** Please refer to Appendix A for the details.

In the following, we provide the solutions to problem (P1) under two cases, depending on whether the source MT is able to split the traffic load or not.

**A. Optimal pricing for non-splittable data**

First, we discuss the case where the data package is not splittable due to the lack of necessary functionalities at the source MT. In this case, all the data are transmitted by the relay MT (i.e., $D_i^{(S)} = 0$ and $D_i^{(R)} = D_i$). With $w_i$ given in (19) reduced to $w_i = \frac{Gr_i - \alpha_i}{N_0^r - \alpha_i (2^{\alpha_i} - 1)}$, problem (P1) becomes the following problem without load sharing:
\[
(P1-w/oLS) : 
\begin{align*}
\min_{\pi_i \geq 0} & \sum_{n=0}^{\infty} \Pr(N_i = n) \left\{ \left[ 1 - \left( 1 - w_i \left( 1 - e^{-\frac{1}{w_i}} \right) \right) ^n \right] \times \left( \pi_i - \zeta_i E_i^{(S)} \right) + \zeta_i E_i^{(S)} \right\} \\
\text{s.t.} & \quad \epsilon \leq \pi_i \leq \zeta_i E_i^{(S)},
\end{align*}
\]  
\[ (22) \]

Because the data transmitted by the relay MT is fixed as $D_i^{(R)} = D_i$, according to Proposition 4.1, the problem is convex with respect to $\pi_i$. Therefore, for the uni-variable convex optimization problem, the optimal solution can be obtained by setting the first-order derivative to zero. However, the objective function of problem (P1-w/oLS) is complex for which the derivative is hard to obtain. Hence, we propose Algorithm 1 based on the derivative-free dichotomous search [12] to obtain the optimal solution for problem (P1-w/oLS).

**B. Joint pricing and load sharing for splittable data**

Next, we discuss the general case where the data package is splittable at the source MT in problem (P1). According to Proposition 4.1, the objective function of problem (P1) is convex with respect to $\pi_i$ with $D_i^{(R)}$ fixed and to $D_i^{(R)}$ with $\pi_i$ fixed. Hence, based on the dichotomous search algorithm in Algorithm 1, we propose Algorithm 2 that minimizes the expected cost of the source MT $i$ with alternating optimization.

For Algorithm 2, it starts with the optimal solution obtained in Algorithm 1. The algorithm then proceeds by iteratively optimizing and updating $\pi_i$ and $D_i^{(R)}$ with the other fixed until the stopping condition is satisfied. It should be noted that the algorithm always converges to a certain value within the range of $\delta$ from at least the locally optimal solution. This is because each iteration of the algorithm reduces the objective value and the objective function in problem (P1) is bounded.
Algorithm 1: One-dimensional dichotomous search algorithm for solving problem (P1-w/oLS). $0 < \tau \ll 1$

1. initialize $\pi_i^{(l)} := \epsilon, \pi_i^{(h)} := \zeta_i E_i^{(S)}, \tau := |\pi_i^{(l)} - \pi_i^{(h)}|$;

2. while $L > \delta$ do

3. $\tilde{\pi}_i^{(l)} := \frac{1}{2}(\pi_i^{(l)} + \pi_i^{(h)}) - \tau L, \tilde{\pi}_i^{(h)} := \frac{1}{2}(\pi_i^{(l)} + \pi_i^{(h)}) + \tau L$;

4. $E[C_i^{(l)}] := E[C_i^{(l)}(\tilde{\pi}_i^{(l)})], E[C_i^{(h)}] := E[C_i^{(l)}(\tilde{\pi}_i^{(h)})]$;

5. if $E[C_i^{(l)}] < E[C_i^{(h)}]$ then

6. $\pi_i^{(h)} := \tilde{\pi}_i^{(h)}$;

else if $E[C_i^{(l)}] > E[C_i^{(h)}]$ then

7. $\pi_i^{(l)} := \tilde{\pi}_i^{(l)}$;

else

8. $\pi_i^{(l)} := \tilde{\pi}_i^{(h)}$ and $\pi_i^{(h)} := \tilde{\pi}_i^{(l)}$;

9. $L := |\pi_i^{(l)} - \pi_i^{(h)}|$;

10. $\pi_i^* := (\pi_i^{(h)} + \pi_i^{(l)})/2$;

11. $E[C_i^*] := E[C_i(\pi_i^*)]$;

Algorithm 2: Alternating optimization algorithm for solving problem (P1).

1. initialize $n := 0, D_i^{(R)} := D_i, E[C_i^{(0)}] := \zeta_i E_i^{(S)}$;

2. while $|E[C_i^{(n)}] - E[C_i^{(n-1)}]| > \delta$ do

3. Optimize the objective of problem (P1) with respect to $\pi_i$ by dichotomous search with $D_i^{(R)}$ fixed;

4. $\pi_i := \pi_i^*(D_i^{(R)})$;

5. Optimize the objective of problem (P1) with respect to $D_i^{(R)}$ by dichotomous search with $\pi_i$ fixed;

6. $D_i^{(R)} = D_i^{(R)*}(\pi_i)$;

7. $n := n + 1$;

V. Numerical Results

In this section, we give numerical examples to evaluate the performance of the proposed incentive mechanism. The simulation setup is given in Table I. We consider the simulation of a single source MT with the average number of helping MTs as $\mu_{N_s} = 1$ and $\mu_{N_r} = 4$, respectively. By ignoring the cases with low probability, we consider a finite support of $\{0, 1, \cdots, 5\}$ for $\mu_{N_s} = 1$ and $\{0, 1, \cdots, 10\}$ for $\mu_{N_r} = 4$, respectively.

| TABLE I: Simulation Setup | $\sigma^2 = -90$ dBm |
|---------------------------|------------------|
| Power of the noise        | $r_i = 400$ m    |
| Distance from the source MT to BS | $\alpha = 3$    |
| Path-loss exponent        | $G = -70$ dB     |
| Path-loss at $r_0$        | $r_0 = 10$ m     |
| Reference distance        | $\epsilon = 1$  |
| Relay MT reservation utility | $\zeta_i = 0.9$ |
| Unit cost of the source MT energy | $\eta_i = 0.5$ |
| Realization of the channel gain | $\gamma_i = 1$  |
First, we consider the minimization of the expected cost for non-splittable traffic in Section IV-A by Algorithm 1. The simulation result is shown in Fig. 2. The expected energy cost of the source MT first decreases and then increases with the pricing $\pi_i$. Intuitively, a higher price increases the probability to attract a relay MT nearby but will eventually increase the cost when the relay probability is already high. The two ends of each curve in Fig. 2 achieve the same cost as they correspond to the two extreme prices ($\pi_i = \epsilon$, $\pi_i = \zeta_i \bar{E}(S)$) for the constraint in (22). The optimal prices $\pi_i^*$ for the green, red, and blue curves are 2.2, 3.4, and 3.8 and the reductions from the direct transmission $\zeta_i \bar{E}(S) - \mathbb{E}[C_i^*]$ are 0.4, 3.5 and 1.3, respectively. Hence, according to the condition for choosing the CT mode in (17), for the three cases, the source MT will choose DT mode, CT mode and CT mode, respectively. Thus, for source MT $i \in K_S$, as the package size $D_i$ increases or the average number of relay MTs $\mu_i$ decreases, a higher price $\pi_i^*$ is needed to compensate for the larger energy consumption. Comparing the two cases with $D_i = 3$ bits/sec/Hz, it can be observed that the optimal pricing is higher and the minimum cost is larger in the case of lower average number of helping MTs. We also notice that for all of these three cases, we have $\mathbb{E}[C_i^*] > \pi_i^*$. This is due to the lack of the exact information about the relay MT’s channel condition and battery level, which leads to the possibilities that the source MT may not be able to successfully associate with a relay MT.

Next, we consider the joint optimization of $\pi_i$ and $D_i^{(R)}$ as in Section IV-B with Algorithm 2. The simulation result is shown in Fig. 3 with the solid line. For comparison, the result without load sharing by Algorithm 1 is shown by the three dash lines. Exhaustive searches on $\pi_i$ with quantization of 0.5 and $D_i^{(R)}$ with quantization of 0.1 in the respective feasible regions are conducted for the three cases and the minimum expected costs are 647.2, 168.3 and 27.5, respectively. The expected cost at iteration $\{0\}$ denotes the cost by the direct transmission. The procedure in Algorithm 2 is executed 5 times and the uni-variable dichotomous search sub-routine is executed 10 times, with sub-routines $\{1, 3, 5, 7, 9\}$ for minimization with respect to $\pi_i$ in Line 3 and sub-routines $\{2, 4, 6, 8, 10\}$ for that with respect to $D_i^{(R)}$ in Line 5 in Algorithm 2. The simulation results show that the convergence is fast and the converged energy costs for the three cases are 647.2, 168.3, and 27.5, respectively, which are the same as the results in exhaustive search. It can also be observed that, in addition to the optimal pricing, load sharing can indeed further reduce the expected energy cost. Compared to the case without load sharing, the cost reductions with load sharing are 2.4, 1.6 and 1.5 times of those without load sharing, respectively. Therefore, load sharing is
more cost-effective when the size of the data package is large and the average number of helping MTs is low.

VI. CONCLUSIONS

This paper studies the pricing and load sharing for the energy saving of MTs in wireless cooperative communications. We minimize the expected energy cost of the source MTs considering two scenarios where the data package is splittable or not for the source MT. For the case of non-splittable data package, the pricing from the source MT to the relay MT is optimized for the minimization of the expected cost with dichotomous search. For the case of splittable data package, we propose an alternating optimization algorithm to optimize both the pricing and load sharing.

APPENDIX A

PROOF OF PROPOSITION 4.1

We first prove that the objective function is a convex function with respect to \( \pi_i \) with \( D_i^{(R)} \) fixed. In the first place, we prove that \( \psi_j(\pi_i) = 1 - w_i \left( 1 - e^{-\pi_i} \right) \) is a convex function with respect to \( \pi_i \). By taking the second-order derivative of \( \psi_j(\pi_i) \), we can have

\[
\psi_j''(\pi_i) = \frac{\epsilon N_0 r_0^{-\alpha} (2D_i^{(R)} - 1)e^{-\frac{\pi_i}{\epsilon}}}{G r_i^{-\alpha}(\pi_i - \epsilon)^3}, \quad j \in \mathcal{R}_i, \ i \in \mathcal{K}_S.
\]  

(23)

Since \( \psi_j''(\pi_i) > 0 \), \( \psi_j(\pi_i) \) is a convex function.

We then denote \( \psi_I(\pi_i) = 1 - \psi_j(\pi_i)^\mathcal{N}_i \) and \( \psi_{II}(\pi_i) = \pi_i + \zeta_i E_i^{(S)} - \zeta_i E_i^{(S)} \), where \( \zeta_i E_i^{(S)} \) is the energy cost for direct transmission. It can be verified that \( \psi_I(\pi_i)^\mathcal{N}_i \) is also convex because \( \psi_j(\pi_i) \) is monotonically decreasing and convex. Hence, \( \psi_I(\pi_i) \) is concave and monotonically increasing. Then, the second-order derivative of \( \psi_I(\pi_i)\psi_{II}(\pi_i) \) can be expressed as

\[
\psi_I''(\pi_i)\psi_{II}(\pi_i) + 2\psi_I'(\pi_i)\psi_{II}'(\pi_i) + \psi_I(\pi_i)\psi_{II}''(\pi_i).
\]  

(24)

First, it can be verified that \( \psi_{II}'(\pi_i) = 0 \) since \( \psi_{II}(\pi_i) \) is a linear function. Then, because \( \psi_I(\pi_i) \) and \( \psi_{II}(\pi_i) \) are both monotonically increasing functions, it can be obtained that \( \psi_I'(\pi_i)\psi_{II}'(\pi_i) > 0 \). Also, due to the fact that
$\psi''_I(\pi_i) < 0$ and $\psi''_II(\pi_i)$ is negative, $\psi''_I(\pi_i)\psi''_II(\pi_i)$ is also positive. Therefore, it can be verified that (24) is positive and thus $\psi_I(\pi_i)\psi''_II(\pi_i)$ is a convex function. Finally, because the objective function of problem (P1-w/oLS) is a non-negative linear combination of convex functions $\psi_I(\pi_i)\psi''_II(\pi_i) + \zeta_i \bar{E}_i^{(S)}$, it is also a convex function. Hence, we have proved that the objective function is convex with respect to $\pi_i$ with $D_i^{(R)}$ fixed.

Finally, by a similar approach, we can also prove that the objective is convex with respect to $D_i^{(R)}$ with fixed $\pi_i$. By combining the two cases, Proposition 4.1 thus follows.

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