I. INTRODUCTION

The production of light nuclei, such as deuteron, triton, $^3\text{He}$, $^3\Lambda\text{H}$, and their antiparticles, at the CERN Large Hadron Collider (LHC) as measured by the ALICE Collaboration [1–6], has attracted considerable attention recently (for a review see, e.g., [7]). The thermal model had a spectacular success in predicting their yields in central heavy ion collisions with the same thermal parameters as used for all other hadrons [8–10]. Alternate explanations based on the coalescence picture [14–16], contain in general, more parameters than the thermal model and a final picture is still not clear.

In this paper, we focus on the quality of the hadron resonance gas (HRG) model description of the yields of light nuclei in small systems and discuss in detail their dependence on charged particle multiplicity. The model predictions are compared to yields that have been measured recently by the ALICE Collaboration in pp and pA collisions in events with different charge particle multiplicities, $dN_{ch}/dy$.

To this end, we employ the baryon canonical ensemble (BCE) approach for the reduction (or enhancement) of yields in the HRG model [17, 18]. The description of the production of light ions in the thermal model can be severely affected by the exact conservation of the baryon number especially at lower beam energies and small systems. This important fact was first noted by Hagedorn [19] in the context of anti-$^3\text{He}$ production in low energy $p\bar{p}$ collisions. There, the implementation of the exact baryon number conservation in the partition function reduces the yield of anti-$^3\text{He}$ by seven orders of magnitude bringing it close to the experimental value.

To take into account the baryon number conservation in the HRG model, we developed an extension of the THERMUS code [20] to include the baryon canonical ensemble. The BCE formulation is especially relevant for describing multi-baryon states produced in events with low values of the accompanying charged particle multiplicity [21].

To account for exact baryon conservation in the presence of multi-baryon states, we will follow the procedure outlined in [18] for strange particles production. We focus on the exact baryon number conservation, where the conservation of all other charges are included in the grand canonical ensemble with all chemical potentials put equal to zero. However, the case of $^3\Lambda\text{H}$ yielding the strangeness canonical effects is also included. The chemical freeze-out temperature is fixed at $T = 156.5$ MeV, a value supported by fits to hadronic yields produced in central Pb-Pb collisions at the LHC [9, 22], that also coincides with the chiral crossover temperature obtained by Lattice Quantum Chromodynamics (LQCD) calculations [23, 24]. Thus, the only parameters left open in the BCE formulation of the HRG model are the volume of the system, $V_A$, and the baryon canonical volume $V_C$ where exact baryon number conservation $B = 0$ is fulfilled. We restrict our considerations to pp and pA collisions as the canonical corrections become negligible for large systems as seen in Pb-Pb collisions where yields of all baryons and light nuclei are well described by the HRG model formulated in the grand canonical ensemble [9].

II. BARYON CANONICAL ENSEMBLE

The data on light nuclei are obtained in a kinematical region where net quantum numbers like net baryon number and net strangeness are zero as witnessed by the particle-antiparticle symmetry observed in the central rapidity region at the LHC. To take this into account we focus on a system which has zero net baryon number inside a correlation volume $V_C$. The HRG partition function is constructed by inserting a Kronecker $\delta$ function,
thus enforcing zero baryon number:
\[ Z^C_{B=0} = \text{Tr} \left[ e^{-H/T} \delta_{(B,0)} \right]. \] (1)

After using the Fourier representation of the delta function, the canonical partition function can be written as
\[ Z^C_{B=0} = e^{B_0} \int_0^{2\pi} d\phi \exp \left\{ \sum_{b=1}^{3} \left\{ B_b e^{i b \phi} + B_{-b} e^{-i b \phi} \right\} \right\}, \] (2)

where \( B_b = \sum_k z_{k,b} \) is the sum of all single-particle partition functions, \( z_{k,b} = V_C n^b_k(T) \), of hadrons with baryon number \( b \) while \( Z_{-b} \) is the corresponding sum for all antiparticles with baryon number \( -b \). Here, \( n^b_k(T) \) is the particle density and \( V_C \) is the baryon canonical volume. The \( B_0 \) is the partition function containing all hadrons with zero baryon number. The above expression can be also written as [25]:

\[ Z^C_{B=0} = e^{B_0} \int_0^{2\pi} d\phi \exp \left\{ \sum_{b=1}^{3} \left\{ \sqrt{B_b B_{-b}} e^{i b \phi} \right. \right. \]
\[ \left. \left. \times \left[ \frac{B_b}{B_{-b}} e^{i b \phi} + \frac{B_{-b}}{B_b} e^{-i b \phi} \right] \right\} \right\}, \] (3)

and since at the LHC energies the chemical potentials are all equal to zero, one has \( B_t = B_{-t} \).

Using the well known series expansion for Bessel functions:
\[ \exp \left\{ \frac{x}{2} \left( t + \frac{1}{t} \right) \right\} = \sum_{m=-\infty}^{\infty} I_m(x) t^m \] (4)

and performing the integral, one can rewrite the baryon canonical partition function in Eq. (1) as a series of Bessel functions [25–28],
\[ Z^C_{B=0} = e^{B_0} \sum_{n,p=-\infty}^{\infty} a_p^n a^-_{1-2n-3p} I_n(x_2) \times I_p(x_3) I_{-2n-3p}(x_1). \] (5)

where
\[ a_i = \sqrt{B_i/B_{-i}}, \] (6)
\[ x_1 = 2 \sqrt{B_i/B_{-i}}. \] (7)

In this way we take into account all baryonic states with baryon number \( \pm1, \pm2 \) and \( \pm3 \). Thus, we include deuterons, tritons, \(^3\)He and their antiparticles but not particles with higher baryon number like \(^4\)He and its antiparticle. Note that in the case considered here, where all chemical potentials are zero, one gets \( a_i = 1 \) for all \( i \).

The resulting yields of particle carrying baryon number \( b \) in the baryon canonical ensemble are then given by the following expression:
\[ \langle N^b_k \rangle_A = \frac{z^A_{k,b}}{Z^C_{B=0}} \sum_{n,p=-\infty}^{\infty} a_p^n a^-_{1-2n-3p-b} I_n(x_2) \times I_p(x_3) I_{-2n-3p-b}(x_1). \] (8)

Furthermore, we parameterize \( z^A_{k,b} = V_A n^b_k(T) \), where \( V_A \) is the volume in the acceptance window. We also include the resonance contributions to \( z_{k,b} \).

It is important to distinguish two volumes in the analysis of yields. One (\( V_A \)) is the fireball volume determined by the experimentally measured charged particle yields within a unit rapidity, the other one is the correlation volume (\( V_C \)) of exact baryon conservation; these two are in general, not the same [18, 19, 28]. For example, in the recent baryon canonical model analysis of net-proton number fluctuations measured in central Pb-Pb collisions by ALICE Collaboration, it was found, that the baryon number conservation is long-range in rapidity and corresponds to full rapidity coverage, i.e. conservation is global [29, 30].

From Eq. (8) it is clear that the yields are determined by the chemical freeze-out temperature \( T \) and two volume parameters: \( V_A \) which appears as an overall factor determining the normalization of the yield and \( V_C \) of the baryon number conservation which appears in the arguments of the Bessel functions.

In the following, we apply the above HRG model formulated in the BCE to describe the yields of protons.

**FIG. 1.** Radius (a) and volumes (b) obtained from the thermal model as a function of charged particle multiplicities. The solid circles and triangles represent the correlation volume using the BCE formalism. The open circles represent the acceptance radius (Phys.Rev.C 103, 014904 (2021)). The solid (dotted) line represents the linear fit to the acceptance (baryon canonical correlation) volume using the strangeness canonical ensemble [18]. The solid (dotted) line represents the acceptance radius and volumes (b) obtained from the thermal model as a function of charged particle multiplicities. The solid circles and triangles represent the correlation volume using the BCE formalism. The open circles represent the acceptance radius (Phys.Rev.C 103, 014904 (2021)).
and (multi-)baryonic light nuclei and their behavior with charged particle multiplicity as observed by the ALICE Collaboration in different colliding systems and collision energies at the LHC.

III. RESULTS AND DISCUSSIONS

The system under consideration is same as was used to describe strange particle yields and their dependence on the charged particle multiplicity in pp and pA collisions at the LHC energies $13, 31, 32$. Thus, we fixed the freeze-out temperature at $T = 156.5$ MeV, and the acceptance volume $V_A$ at mid-rapidity to the values as were obtained in $13$. In Fig. 1, we show the radius parameter and the corresponding fireball volume $V_A$, and their dependence on $dN_{ch}/d\eta$ from Ref. $13$. Also shown in this figure is the strangeness canonical volume parameter that was extracted within the HRG model formulated in the canonical ensemble to successfully describe the strange and multi-strange hadron yields and their observed systematics $13$.

In the thermal model analysis of the production yields of baryons and multi-baryon states in pp and pA collisions one needs to account for the exact baryon number conservation introduced in the BCE in Eq. $8$. With the fireball volume in the acceptance window $V_A$ introduced in Fig. 1 and the freeze-out temperature from Refs. $22$, we are left with the baryon canonical volume parameter $V_C$ to quantify data. We calculated $V_C$ by fitting pions, protons and deuterons yields as measured by the ALICE Collaboration for different multiplicity classes in pp and pA collisions $2, 3, 33, 34$. The results are summarized in Fig. 1 where the solid circles and solid triangles represent the results for pp collisions at $\sqrt{s} = 13$ TeV and pPb collisions at $\sqrt{s} = 5.02$ TeV, respectively.

It is observed, that for low multiplicity, the canonical volume is only slightly larger than the acceptance volume at mid-rapidity, indicating that in pp and pPb collisions, the baryon number conservation is not necessarily extended to the full rapidity range. This is in contrast to the results of baryon canonical model analysis of net-proton fluctuation data in central Pb-Pb collisions at the LHC $18, 30$. One of the possible reasons could be we consider here a fully integrated $p_t$ yields data, while the net-proton fluctuations in central Pb-Pb collisions were measured over a limited $p_t$ window. Furthermore, the full phase space rapidity distributions of protons and baryons in pp, pPb, and Pb-Pb collisions are very different and at present not known for events with different $dN_{ch}/dy$. With future measurements of proton fluctuations in pp and pPb collisions, one could verify whether the statistical model in canonical ensemble can provide a consistent description of particle yields and fluctuations observable.

The volume deduced from the BCE tends to be also larger than the strangeness canonical volume deduced from yields of strange particles $13$. We note, however, that canonical volume parameters shown in Fig. 1 were extracted under the assumption that strangeness and baryon number are conserved independently. When the exact baryon and strangeness conservation are included simultaneously in the canonical ensemble then yields of particles are not calculated anymore following Eq. $8$. They are rather obtained from the partition function described by the double integrals over the $U_B(1) \times U_S(1)$ group with the weight function $\exp (S[\phi_B, \phi_S, T, V_C])$ which has only one canonical volume parameter $V_C$. Such an effective common volume parameter accounts simultaneously for the exact conservation of the baryon number and strangeness in a system and quantifies yields of hadrons carrying baryon and strangeness quantum numbers $20$.

We find, an approximate linearity of canonical correlation volume as a function of the multiplicity as seen in Fig. 1. Thus, we fit the baryon canonical volume as a function of charged particle multiplicity with a linear function,

$$ V_C \simeq 27.3 + 2.9 \times \frac{dN_{ch}}{dy} \quad (9) $$
The above parametrization is also shown in Fig. 1 as dotted lines. We note, however, that such a linear fit is valid only in the \(dN_{ch}/d\eta\) range given by the above considered data in pp and pA collisions. From the fits to acceptance volume \(V_A\) made for each multiplicity bin it was also shown that it can be well parameterised as linear function of \(dN_{ch}/d\eta\), as \([18]\):

\[
V_A \simeq 1.55 + 3.0 \times \frac{dN_{ch}}{d\eta}.
\] (10)

This linear dependence of \(V_A\) is only valid for \(dN_{ch}/d\eta\) values greater than two.

Having established the \(dN_{ch}/d\eta\) dependence of volume parameters \(V_A\) and \(V_C\), and fixing the chemical freeze-out temperature at \(T = 156.5\) MeV, the yields of (multi-) baryon states are obtained in the BCE from Eq. 5. In the actual calculations the conservation of all other charges is included in the grand canonical ensemble with vanishing chemical potentials.

The model results as a function of charged particle multiplicity are shown in Figs. 2 and 3 and compared with data. The various symbols represent the experimentally measured yields in pp and pA collisions from the ALICE experiment \([1–6, 33, 34]\). The quality of the BCE model description of the data is quantified in Figs. 2b and 2c by showing the ratios of experimental data to model results.

The pions, protons and deuterons production is well described by the BCE model for all multiplicities considered. This is very transparent in Fig. 2 for particle yields and in Fig. 3 for their ratios. In pA collisions the above yields data are consistent with the model results within experimental uncertainties. This is also the case for pp data with \(dN_{ch}/d\eta > 10\). For lower multiplicities in pp collisions the agreement is at the level of two standard deviations.

The data for proton, deuteron and \(^3\)He capture also basic properties of the BCE model, namely the so called canonical suppression effect \([26, 28]\), i.e. the suppression of production yields of baryonic states in low-multiplicity relative to high-multiplicity events. In addition, this suppression increases with the baryon number of the state. This property is very transparent in Fig. 3 where the yields of protons, deuterons, and \(^3\)He are normalized to pion yields, effectively suppressing the dependence on the volume parameter \(V_A\).

Although, the qualitative trend of \(^3\)He production as a function of multiplicity is nicely predicted by the BCE model, nevertheless the model results overpredict the observed yields. This is transparent in Fig. 2 for \(^3\)He yields as well as in their ratios to pions, protons and deuterons as shown in Fig. 3. It is particularly interesting to note, that for \(dN_{ch}/d\eta < 50\) the above deviations of the BCE model results from data are independent of the charge particle multiplicity. Indeed, in Fig. 2c we see that the ratio of experimental yields over the BCE model predictions for \(^3\)He and even for \(^3\)Λ, is within uncertainties constant. This ratio is fitted to be, \(\lambda = 0.45 \pm 0.03\).

When rescaling the model results for \(^3\)He and \(^3\)Λ with this factor \(\lambda\) (see dashed lines in Figs. 2 and 3) the data are nicely reproduced for all values of \(dN_{ch}/d\eta\).

We note, however, that since \(^3\)Λ carries a strange quantum number \(|S| = 1\), thus in small multiplicity events its yields are also subjects of additional suppression due to exact strangeness conservation (SC). The resulting strangeness suppression of \(^3\)Λ yields is also quantified in Figs. 2 and 3, and is seen to be small in the
parameter range considered. At the measured value of \( A_3^3 \text{He} \), the strangeness suppression is at the percentage level whereas at lower \( dN_{ch}/dy \) it increases up to \( \approx 15\% \). Such suppression is hardly visible on log plots.

The above-observed differences between the BCE model predictions and \( ^3\text{He}, \, A_3^3 \text{H} \) yields data by a constant multiplicative factor can be interpreted as being due to deviations from chemical equilibrium. In events with small \( dN_{ch}/dy \) the yields of multi-baryons states such as \( ^4\text{He} \) and \( A_3^3 \text{H} \) appear in thermal but not in chemical equilibrium. This can be quantified by the off-chemical equilibrium fugacity factor \( \lambda \) which in the present case is nearly independent of charged particle multiplicity. Considering, however, that in central Pb-Pb collisions the yields of \( ^3\text{He} \) and \( A_3^3 \text{H} \) are found to be consistent with the HRG model results in chemical equilibrium, one expects that the above fugacity parameter must depend on \( dN_{ch}/dy \) and will converge to unity for sufficiently large multiplicities.

\[ \text{IV. CONCLUSION} \]

We have analyzed the yields of protons, deuterons, \( ^3\text{He} \), and \( A_3^3 \text{H} \) in the thermal fireball constrained by an exact baryon number conservation. We have applied the hadron resonance gas (HRG) model formulated in the canonical ensemble concerning baryon number conservation, including contributions of all baryons and multibaryon states. The model predictions have been compared with recent yields data at mid-rapidity obtained by the ALICE Collaboration in pp and pPb collisions at \( \sqrt{s} = 13 \text{ TeV} \) and \( \sqrt{s_{NN}} = 5.02 \text{ TeV} \), respectively. We have focused on charged particle multiplicity (\( dN_{ch}/d\eta \)) dependence of protons and light-nuclei production. The fireball thermal parameters, the freeze-out temperature, and the volume at mid-rapidity, as well as, its \( dN_{ch}/dy \) dependence, were taken the same as obtained in the HRG model analysis of strange particle production in the corresponding system [15]. Thus the only parameter left in this analysis was the correlation volume \( V_C \) of an exact baryon number conservation. The extracted value of \( V_C \) from the fit to pion, proton and deuteron yields is nearly linearly-dependent on \( dN_{ch}/d\eta \), and for \( 2 < dN_{ch}/d\eta < 50 \).

We have shown that the observed yields of protons and deuterons and their \( dN_{ch}/d\eta \) dependence are well quantified by model predictions. Also the qualitative trend of data, i.e. the relative suppression of baryons yields with decreasing \( dN_{ch}/d\eta \) and its increase with the baryon content of the state is well reproduced by the thermal model with exact baryon-number conservation. However, on the quantitative level, the yields of \( ^3\text{He} \), and \( A_3^3 \text{H} \) are overpredicted by the constant multiplicative factor which has been interpreted as the off-chemical equilibrium fugacity factor. Thus, in small systems with \( dN_{ch}/d\eta < 50 \) the yields of hadronic states carrying baryon number \( |B| = 1 \) and 2 appear to be consistent with multiplicities expected in thermal and chemical equilibrium, where as light nuclei like \( ^3\text{He} \), and \( A_3^3 \text{H} \) are lacking chemical equilibrium population. This is, however, not the case for large systems like in central Pb-Pb collisions where the yields of nuclei and anti-nuclei including (anti-)hypernuclei have been shown to be consistent with chemical equilibrium thermal model predictions.

With the forthcoming data in these, including production of \( ^4\text{He} \) and \( A_3^3 \text{H} \) at \( dN_{ch}/dy < 40 \), the above conjecture of possible off-chemical equilibrium effects in the light-nuclei production in small systems can be verified further or their thermal production in small multiplicity events can be excluded.

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