ON THE RIEMANN ZETA-FUNCTION
AND THE DIVISOR PROBLEM IV

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Abstract. Let \( \Delta(x) \) denote the error term in the Dirichlet divisor problem, and \( E(T) \) the error term in the asymptotic formula for the mean square of \(|\zeta(\frac{1}{2} + it)|\). If \( E^*(t) = E(t) - 2\pi \Delta^*(t/2\pi) \) with \( \Delta^*(x) = -\Delta(x) + 2\Delta(2x) - \frac{1}{2}\Delta(4x) \), then it is proved that

\[
\int_0^T |E^*(t)|^3 \, dt \ll \varepsilon T^{3/2+\varepsilon},
\]

which is (up to ‘\( \varepsilon \)’) best possible, and \( \zeta(\frac{1}{2} + it) \ll t^{\rho/2+\varepsilon} \) if \( E^*(t) \ll t^{\rho+\varepsilon} \).

1. Introduction and statement of results

This paper is the continuation of the author’s works [5], [6], where the analogy between the Riemann zeta-function \( \zeta(s) \) and the divisor problem was investigated. As usual, let the error term in the classical Dirichlet divisor problem be

\[
\Delta(x) = \sum_{n \leq x} d(n) - x(\log x + 2\gamma - 1),
\]

and the error term in the mean square formula for \(|\zeta(\frac{1}{2} + it)|\) be defined by

\[
E(T) = \int_0^T |\zeta(\frac{1}{2} + it)|^2 \, dt - T \left( \log \left( \frac{T}{2\pi} \right) + 2\gamma - 1 \right).
\]

Here, as usual, \( d(n) \) is the number of divisors of \( n \), \( \zeta(s) \) is the Riemann zeta-function, and \( \gamma = -\Gamma'(1) = 0.577215\ldots \) is Euler’s constant. The analogy between

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ζ(s) and the divisor problem is more exact if, instead with ∆(x), we work with the modified function ∆∗(x) (see M. Jutila [8], [9] and T. Meurman [11], [12]), where

\begin{equation}
\Delta^*(x) := -\Delta(x) + 2\Delta(2x) - \frac{1}{2}\Delta(4x) = \frac{1}{2} \sum_{n \leq 4x} (-1)^n d(n) - x(\log x + 2\gamma - 1).
\end{equation}

M. Jutila (op. cit.) investigated both the local and global behaviour of the difference

\begin{equation}
E^*(t) := E(t) - 2\pi \Delta^*(\frac{t}{2\pi}).
\end{equation}

This function may be thought of as a discrepancy between E∗(t) and Δ∗(x). In particular Jutila in [9] proved that

\begin{equation}
\int_0^T (E^*(t))^2 \, dt \ll T^{4/3} \log^3 T,
\end{equation}

which was sharpened in [6] by the author to the full asymptotic formula

\begin{equation}
\int_0^T (E^*(t))^2 \, dt = T^{4/3} P_3(\log T) + O_\varepsilon(T^{7/6+\varepsilon}),
\end{equation}

where P_3(y) is a polynomial of degree three in y with positive leading coefficient, and all the coefficients may be evaluated explicitly. Here and later ε denotes positive constants which are arbitrarily small, but are not necessarily the same ones at each occurrence, while a \ll_\varepsilon b (same as a = O_\varepsilon(b)) means that the \ll-constant depends on \varepsilon. In Part II of [5] it was proved that

\begin{equation}
\int_0^T |E^*(t)|^5 \, dt \ll_\varepsilon T^{2+\varepsilon},
\end{equation}

while in Part III we investigated the function R(T) defined by the relation

\begin{equation}
\int_0^T E^*(t) \, dt = \frac{3\pi}{4} T + R(T),
\end{equation}

and proved, among other things, the asymptotic formula

\begin{equation}
\int_0^T R^2(t) \, dt = T^2 Q_3(\log T) + O_\varepsilon(T^{11/6+\varepsilon}),
\end{equation}

where Q_3(y) is a cubic polynomial in y with positive leading coefficient, whose all coefficients may be evaluated explicitly.
The asymptotic formula (1.9) bears resemblance to (1.6), and it is proved by a similar technique. The exponents in the error terms are, in both cases, less than the exponent of $T$ in the main term by $1/6$. This comes from the use of [6, Lemma 3], and in both cases the exponent of the error term is the limit of the method. Our first new result is an upper bound for the third moment of $|E^*(t)|$, which does not follow from any of the previous results. This is

**THEOREM 1.** We have

\[(1.10) \quad \int_0^T |E^*(t)|^3 \, dt \ll \varepsilon T^{3/2+\varepsilon}.\]

In view of (1.6) it follows that, up to ‘$\varepsilon$’, (1.10) is best possible.

**Corollary 1.** We have

\[
\int_0^T |\zeta(\frac{1}{2} + it)|^8 \, dt \ll \varepsilon T^{3/2+\varepsilon}.
\]

The last result is, up to ‘$\varepsilon$’, the sharpest one known (see [3, Chapter 8]). It follows from Theorem 1.4 of [5, Part II], which says that the bound

\[(1.11) \quad \int_0^T |E^*(t)|^k \, dt \ll \varepsilon T^{c(k)+\varepsilon}\]

implies that

\[(1.12) \quad \int_0^T |\zeta(\frac{1}{2} + it)|^{2k+2} \, dt \ll \varepsilon T^{c(k)+\varepsilon},\]

where $k \geq 1$ is a fixed real number.

**Corollary 2.** We have

\[(1.13) \quad \int_0^T (E^*(t))^4 \, dt \ll \varepsilon T^{7/4+\varepsilon}, \quad \int_0^T |\zeta(\frac{1}{2} + it)|^{10} \, dt \ll \varepsilon T^{7/4+\varepsilon}.\]

The first bound in (1.13) follows by the Cauchy-Schwarz inequality for integrals from (1.7) and (1.10). The second bound follows from (1.11)–(1.12) with $k = 4$ and represents, up to ‘$\varepsilon$’, the sharpest one known (see [3, Chapter 8]). The first exponent in (1.13) improves on $16/9 + \varepsilon$, proved in [5, Part I].
Corollary 3. If, for $k > 0$ a fixed constant and $1 \ll G = G(T) \ll T$,

$$J_k(T, G) := \frac{1}{\sqrt{\pi G}} \int_{-\infty}^{\infty} |\zeta(\frac{1}{2} + iT + iu)|^{2k} e^{-\left(u/G\right)^2} du,$$

then

$$\int_T^{2T} J_1^4(t, G) \, dt \ll_\varepsilon T^{1+\varepsilon}$$

holds for $T^{3/16} \leq G = G(T) \ll T$.

Namely it was proved in [6] that, for $T^\varepsilon \ll G = G(T) \leq T$ and fixed $m \geq 1$ we have

$$\int_T^{2T} J_1^m(t, G) \, dt \ll G^{-1-m} \int_{-G \log T}^{G \log T} \left( \int_T^{2T} |E^*(t + x)|^m \, dt \right) \, dx + T \log^{2m} T.$$

Thus (1.14) follows from (1.13) and (1.15) with $m = 4$, and improves on the range $T^{7/36} \leq G = G(T) \ll T$ stated in Theorem 1 of [6], since $3/16 < 7/36$.

Both (1.6) and (1.10) imply that, in the mean sense, $E^*(t) \ll_\varepsilon t^{1/6+\varepsilon}$. The true order of this function is, however, quite elusive. If we define

$$\rho := \inf \left\{ r > 0 : E^*(T) = O(T^r) \right\},$$

then we have unconditionally

$$1/6 \leq \rho \leq 131/416 = 0.314903\ldots,$$

and there is a big discrepancy between the lower and upper bound in (1.17). The lower bound in (1.17) comes from the asymptotic formula (1.6), which in fact gives $E^*(T) = \Omega(T^{1/6}(\log T)^{3/2})$. The upper bound comes from the best known bound for $\Delta(x)$ of M.N. Huxley [2] and $E(T)$ of N. Watt (unpublished). It remains yet to see whether a method can be found that would provide sharper bounds for $\rho$ than for the corresponding exponents of $E(T)$ and $\Delta(x)$. This is important, as one can obtain bounds for $\zeta(\frac{1}{2} + it)$ from bounds of $E^*(t)$. More precisely, if as usual one defines the Lindelöf function for $\zeta(s)$ (the famous Lindelöf conjecture is that $\mu(\frac{1}{2}) = 0$) by the relation

$$\mu(\sigma) = \liminf_{t \to \infty} \frac{\log |\zeta(\sigma + it)|}{\log t}$$

for any $\sigma \in \mathbb{R}$, then we have
THEOREM 2. If $\rho$ is defined by (1.16) and $\mu(\sigma)$ by (1.18), then we have

$$\mu(\frac{1}{2}) \leq \frac{1}{2}\rho.$$  

(1.19)

It may be remarked that, if $\rho \leq 1/4$ holds, then $\theta = \omega$, where

$$\theta = \inf \left\{ c > 0 : E(T) = O(T^c) \right\}, \quad \omega = \inf \left\{ d > 0 : \Delta(T) = O(T^d) \right\}.$$  

\[ \text{Namely as } \theta \geq 1/4 \text{ and } \omega \geq 1/4 \text{ are known to hold (this follows e.g., from mean square results, see [4]) } \theta = \omega \text{ follows from (1.4) and } \omega = \sigma, \text{ proved recently by Lau–Tsang [10], where} \]

$$\sigma = \inf \left\{ s > 0 : \Delta^*(T) = O(T^s) \right\}.$$  

\[ \text{The reader is also referred to M. Jutila [8] for a discussion on some related implications. The limit of (1.19) is } \mu(\frac{1}{2}) \leq 1/12 \text{ in view of (1.17).} \]

The plan of the paper is as follows. In Section 2 the necessary lemmas are given, while the proofs of Theorem 1 and Theorem 2 will be given in Section 3.

2. THE NECESSARY LEMMAS

In this section we shall state the lemmas which are necessary for the proof of our theorems.

LEMMA 1 (O. Robert–P. Sargos [13]). Let $k \geq 2$ be a fixed integer and $\delta > 0$ be given. Then the number of integers $n_1, n_2, n_3, n_4$ such that $N < n_1, n_2, n_3, n_4 \leq 2N$ and

$$|n_1^{1/k} + n_2^{1/k} - n_3^{1/k} - n_4^{1/k}| < \delta N^{1/k}$$

is, for any given $\varepsilon > 0$,

$$\ll_{\varepsilon} N^\varepsilon (N^4 \delta + N^2).$$  

This Lemma (with $k = 2$) is crucial in treating the fourth power of the sums in (2.5) and (2.12).

LEMMA 2. Let $T^\varepsilon \ll G \ll T/\log T$. Then we have

$$E^*(T) \leq \frac{2}{\sqrt{\pi G}} \int_0^\infty E^*(T + u) e^{-u^2/G^2} du + O_\varepsilon(GT^\varepsilon),$$  

(2.2)
and

\[ E^*(T) \geq \frac{2}{\sqrt{\pi G}} \int_0^\infty E^*(T-u) e^{-u^2/G^2} \, du + O(\epsilon (GT^\epsilon)). \]

**Lemma 2.** Follows on combining Lemma 2.2 and Lemma 2.3 of [4, Part I].

The next lemma is F.V. Atkinson’s classical, precise asymptotic formula for \( E(T) \) (see [1], [3] or [4]).

**Lemma 3.** Let \( 0 < A < A' \) be any two fixed constants such that \( AT < N < A'T \), and let \( N' = N'(T) = T/(2\pi) + N/2 - (N^2/4 + NT/(2\pi))^{1/2} \). Then

\[ (2.3) \quad E(T) = \Sigma_1(T) + \Sigma_2(T) + O(\log^2 T), \]

where

\[ (2.4) \quad \Sigma_1(T) = 2^{1/2}(T/(2\pi))^{1/4} \sum_{n \leq N} (-1)^n d(n)n^{-3/4} e(T, n) \cos(f(T, n)), \]

\[ (2.5) \quad \Sigma_2(T) = -2 \sum_{n \leq N'} d(n)n^{-1/2}(\log T/(2\pi n))^{-1} \cos(T \log T/(2\pi n) - T + \pi/4), \]

with

\[ (2.6) \quad f(T, n) = 2T \text{arsinh} \left( \sqrt{\pi n/(2T)} \right) + \sqrt{2\pi nT + \pi^2 n^2} - \pi/4 \]

\[ = -\frac{1}{4}\pi + 2\sqrt{2\pi nT} + \frac{1}{6}\sqrt{2\pi^3 n^{3/2}T^{-1/2}} + a_5 n^{5/2} T^{-3/2} + a_7 n^{7/2} T^{-5/2} + \ldots, \]

\[ (2.7) \quad e(T, n) = (1 + \pi n/(2T))^{-1/4} \left\{ (2T/(\pi n))^{1/2} \text{arsinh} \left( \sqrt{\pi n/(2T)} \right) \right\}^{-1} \]

\[ = 1 + O(n/T) \quad (1 \leq n < T), \]

and \( \text{arsinh} \ x = \log(x + \sqrt{1 + x^2}). \)

**Lemma 4 (M. Jutila [8, Part II]).** For \( A \in \mathbb{R} \) a constant we have

\[ (2.9) \quad \cos \left( \sqrt{8\pi nT} + \frac{1}{6}\sqrt{2\pi^3 n^{3/2}T^{-1/2}} + A \right) = \int_{-\infty}^\infty \alpha(u) \cos(\sqrt{8\pi n}(\sqrt{T} + u) + A) \, du, \]
where $\alpha(u) \ll T^{1/6}$ for $u \neq 0$,

\begin{equation}
\alpha(u) \ll T^{1/6} \exp(-bT^{1/4}|u|^{3/2})
\end{equation}

for $u < 0$, and

\begin{equation}
\alpha(u) = T^{1/8}u^{-1/4}\left(d \exp(ibT^{1/4}u^{3/2}) + \bar{d} \exp(-ibT^{1/4}u^{3/2})\right) + O(T^{-1/8}u^{-7/4})
\end{equation}

for $u \geq T^{-1/6}$ and some constants $b (> 0)$ and $d$.

We need also an explicit formula for $\Delta^*(x)$ (see [3, Chapter 15]). This is

**LEMMA 5.** For $1 \leq N \ll x$ we have

\begin{equation}
\Delta^*(x) = \frac{1}{\pi \sqrt{2}} x^{1/4} \sum_{n \leq N} (-1)^n d(n)n^{-3/4} \cos(4\pi \sqrt{n}x - \frac{1}{4}\pi) + O(x^{3/4 + \varepsilon}N^{-1/2}).
\end{equation}

3. **Proofs of the theorems**

The proof of (1.10) of Theorem 1 is based on the method of [5]. We seek an upper bound for $R = R(V,T)$, the number of points

\begin{equation}
\{t_r\} \in [T,2T] \ (r = 1, \ldots, R), \quad V \leq |E^*(t_r)| < 2V \quad (|t_r - t_s| \geq V \text{ if } r \neq s).
\end{equation}

We consider separately the points where $E^*(t_r)$ is positive or negative. Suppose the first case holds (the other one is treated analogously), using in either case the notation $R$ for the number of points in question. Then from Lemma 2 we have

\begin{equation}
V \leq E^*(t_r) \leq \frac{2}{\sqrt{\pi G}} \int_0^\infty E^*(t_r + G + u) e^{-u^2/G^2} \, du + O_\varepsilon(GT^\varepsilon),
\end{equation}

and the integral may be truncated at $u = G \log T$ with a very small error. We may suppose that $V$ satisfies

\begin{equation}
T^{1/6} \leq V \leq T^{1/4}.
\end{equation}

Indeed, if

\begin{align*}
I_1(T) &:= \int_{T,|E^*| \leq T^{1/6}} |E^*(t)|^3 \, dt, \\
I_2(T) &:= \int_{T,|E^*| \geq T^{1/4}} |E^*(t)|^3 \, dt,
\end{align*}

and
then from (1.6) it follows that

\[(3.4) \quad I_1(T) \leq T^{1/6} \int_T^{2T} |E^*(t)|^2 \, dt \ll T^{3/2} \log^3 T,\]

while from (1.7) we obtain that

\[(3.5) \quad I_2(T) \leq T^{-1/2} \int_T^{2T} |E^*(t)|^5 \, dt \ll \varepsilon T^{3/2 + \varepsilon}.\]

Thus supposing that (3.3) holds we estimate

\[I(V, T) := \int_{T,V \leq |E^*(t)| \leq 2V} |E^*(t)|^3 \, dt\]

by splitting the interval \([T, 2T]\) into \(R (= R(V, T))\) disjoint subintervals \(J_r\) of length \(\leq V\), where in the \(r\)-th of these intervals we define \(t_r (r = 1, \ldots, R)\) by

\[|E^*(t_r)| = \sup_{t \in J_r} |E^*(t)|.\]

The proof of Theorem 1 will be a consequence of the bound

\[(3.6) \quad R \ll \varepsilon T^{3/2 + \varepsilon} V^{-4},\]

provided that (3.1) holds (considering separately points with even and odd indices so that \(|t_r - t_s| \geq V (r \neq s)\) is satisfied). Namely we have

\[(3.7) \quad I(V, T) \ll V \sum_{j=1}^{R} |E^*(t_r)|^3 \ll \varepsilon V T^{3/2 + \varepsilon} V^{-4} V^3 = T^{3/2 + \varepsilon},\]

and from (3.4), (3.5) and (3.7) we obtain

\[(3.8) \quad \int_T^{2T} |E^*(t)|^3 \, dt \ll \varepsilon T^{3/2 + \varepsilon}.\]

The bound (1.10) follows from (3.8) if one replaces \(T\) by \(T 2^{-j}\) and sums the corresponding results for \(j = 1, 2, \ldots\).

We continue the proof of Theorem 1 by noting that, like in [5, Part I], the integral on the right-hand side of (3.2) is simplified by Atkinson’s formula (Lemma 3) and the truncated formula for \(\Delta^*(x)\) (Lemma 5). We take \(G = c V T^{-\varepsilon}\) (with sufficiently small \(c > 0\)) to make the \(O\)-term in (3.2) \(\leq \frac{1}{2} V\), and then we obtain

\[(3.9) \quad V \ll \sum_{j=4}^{6} V^{-1} T^{\varepsilon} \int_0^{G \log T} \sum_j (t_r + G + u) e^{-u^2/G^2} \, du \quad (r = 1, \ldots, R),\]
where we choose $X = T^{1/3 - \varepsilon}$, $N = TG^{-2}\log T$ and, similarly to [5], for $t \asymp T$ we set (in the notation of Lemma 3)

$$
\sum_4(t) := t^{1/4} \sum_{X < n \leq N} (-1)^n d(n)n^{-3/4}e(t + u, n)\cos(f(t + u, n)),
$$

$$
\sum_5(t) := t^{1/4} \sum_{X < n \leq N} (-1)^n d(n)n^{-3/4} \cos(\sqrt{8\pi n(t + u)} - \pi/4),
$$

$$
\sum_6(t) := t^{-1/4} \sum_{n \leq X} (-1)^n d(n)n^{3/4} \cos(\sqrt{8\pi n(t + u)} - \pi/4).
$$

The sums in (3.10)–(3.11) over $n$ are split into $O(\log T)$ subsums over the ranges $K < n \leq K' \leq 2K$. We denote these sums by $\Sigma_j(t, K)$ and let $\varphi(t)$ denote a smooth, nonnegative function supported in $[T/2, 5T/2]$, such that $\varphi(t) = 1$ when $T \leq t \leq 2T$. There must exist a set of $M = M(K)$ points $\{\tau_m\} \in \{t_r\}$ such that $M(K) \gg R/\log T$ for some $j, K$, so that it suffices to majorize $M(K)$, which we shall (with a slight abuse of notation) henceforth denote again by $R$. The contribution of $\sum_6(t, K)$ is estimated by raising the relevant portion of (3.9) to the fourth power and summing over $r$, noting that $|t_r - t_s| \geq V(r \neq s)$, so that the sum of integrals over the intervals $[t_r + G, t_r + G + G\log T]$ is majorized by the integral over $[T/2, 5T/2]$. We proceed as in [5, Part I and Part II] integrating by parts, and using $\varphi(\ell)(t) \ll \ell T^{-\ell}$ ($\ell \geq 0$). It transpires, when we develop $\sum_6^4(t, K)$ and set

$$
\Delta := \sqrt{n_1} + \sqrt{n_2} - \sqrt{n_3} - \sqrt{n_4},
$$

that the contribution of $\Delta \geq T^{3/2}$ is negligible (i.e., it is smaller than $T^{-A}$ for any given $A > 0$). The contribution of $\Delta < T^{3/2}$ is treated by Lemma 1 and trivial estimation of the ensuing integral. We obtain

$$
RV^4 \ll V^{-1}T^\varepsilon \sup_{|u| \leq G\log T} \int_{T/2}^{2T} \varphi(t) \sum_6^4(t, K) \, dt
\ll \varepsilon T^{1+\varepsilon}V^{-1} \sup_{|u| \leq G\log T, |\Delta| \leq T^{3/2}} T^{-1}K^4(K^{1/2}|\Delta| + K^2)
\ll \varepsilon T^{4\varepsilon - 4}\left(T^{-1/2}X^{13/2} + X^5\right) \ll \varepsilon T^{5/3 + \varepsilon}V^{-1},
$$

since $K \ll X = T^{1/3 - \varepsilon}$. This gives, since (3.3) holds,

$$
R \ll \varepsilon T^{5/3 + \varepsilon}V^{-5} \ll \varepsilon T^{3/2 + \varepsilon}V^{-4},
$$

which is the desired bound (3.6).
The contributions of $\sum_4(t, K)$ and of $\sum_5(t, K)$ are estimated analogously, with the remark that in the case of $\sum_4(t, K)$ one has to use Lemma 4 to deal with the complications arising from the presence of $\cos(f(t + u, n))$, coming from (2.5). This procedure was explained in detail in [5, Part I and Part II]. The non-negligible contribution of $\sum_5(t, K)$ will, again by raising the relevant expression to the fourth power, be for $\Delta \leq T^{\varepsilon-1/2}$ again. The application of Lemma 1 gives in this case

$$RV^4 \ll \varepsilon V^{-1} T^{1+\varepsilon} T K^{-3} (K^4 T^{-1/2} + K^2)$$

$$\ll \varepsilon T^{2+\varepsilon} V^{-1} (K^{1/2} T^{1/2} + K^{-1})$$

$$\ll \varepsilon T^{3/2+\varepsilon} V^{-1} K^{1/2} + T^{5/3+\varepsilon} V^{-1},$$

because $K \gg X = T^{1/3-\varepsilon}$ holds. For $K \leq V^2$ the bound (3.12) reduces to (3.6), and we are done. If $V^2 < K \leq T^{1+\varepsilon} V^{-2}$ (note that $V^2 < T^{1+\varepsilon} V^{-2}$ holds by (3.3)), then the relevant expression is squared, and not raised to the fourth power. We obtain

$$RV^2 \ll \varepsilon V^{-1} \max_{|u| \leq G \log T} \int_{T/2}^{5T/2} \varphi(t) \sum_5^2(t, K) \, dt$$

$$= T^{1/2} V^{-1} \max_{|u| \leq G \log T} \int_{T/2}^{5T/2} \varphi(t) \times$$

$$\sum_{K < m, n \leq 2K} (-1)^{m+n} d(m) d(n) (mn)^{-3/4} e^{i \sqrt{8 \pi (t+u) (\sqrt{m} - \sqrt{n})}} \, dt$$

$$\ll T^{3/2} V^{-1} \sum_{m > K} d^2(m) m^{-3/2} + T^{1+\varepsilon} K^{-3/2} V^{-1} \sum_{K < m \neq n \leq 2K} |\sqrt{m} - \sqrt{n}|^{-1}.$$
since, from (1.3) and $d(n) \ll \varepsilon n^\varepsilon$, it is seen that

$$\Delta^*(T + H) - \Delta^*(T) = O(H \log T) + \frac{1}{2} \sum_{4T < n \leq 4(T + H)} (-1)^n d(n) \ll \varepsilon HT^\varepsilon$$

holds for $1 \ll H \ll T$. Therefore (3.13) implies that

$$|\zeta(\frac{1}{2} + iT)|^2 \ll \varepsilon T^{\theta + \varepsilon},$$

and this gives $\mu(\frac{1}{2}) \leq \frac{1}{2} \rho$, as asserted.

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