Coherence plays a crucial role in the behavior of a system, and is central to our understanding of laser, matter wave and condensed-matter physics. The achievement of quantum degeneracy in ultracold one-dimensional (1D) atomic clouds either in magnetic \cite{1}, or optical traps \cite{2}, or maintained close to microfabricated surfaces known as ‘atom chips’ \cite{3} has led to a unique experimentally controllable system for fundamental studies of coherence in low dimensional systems. Potential applications of such systems include ‘on-chip’ atom interferometers \cite{4,5}, and continuously-operating atom lasers \cite{6}. 1D systems differ from their 3D counterparts, in that low-dimensional systems are generally prone to large phase fluctuations, which tend to destroy long-range phase coherence \cite{5,7,8,9,10,11,12}. Nonetheless, harmonically-confined 1D gases can maintain significant coherence across the system size at sufficiently low temperatures, whereas, at higher temperatures, coherence is limited to smaller regimes. In this case, the system contains a quasi-condensate \cite{5}.

The suppression of coherence with reduced dimensionality has been observed experimentally in very elongated 3D Bose-Einstein Condensates (BECs) \cite{13,14,15,16,17}. In these systems, the equilibrium coherence properties were found to be largely insensitive to density fluctuations, even at temperatures close to the critical temperature, where most of the atoms are in the thermal cloud. This can only be true, provided phase fluctuations set in at temperatures much lower than density fluctuations, which is typically considered as the only attainable experimental limit in such systems.

In this paper, we demonstrate that the opposite regime, in which density fluctuations set in at a lower temperature than phase fluctuations, is actually within current experimental reach. We determine optimum conditions for reaching this novel regime for two atomic species, highlighting in particular the case of $^{23}$Na, which appears to be a better candidate for such experiments. The interplay between density and phase fluctuations is examined by comparing appropriately defined characteristic temperatures for their respective onset. Such an approach has already been used in other systems to examine the relative interplay of phase and density fluctuations of the order parameter of the system. A striking example is the case of high-$T_c$ superconductors, for which the interplay between amplitude and phase fluctuations is believed to be related to the ‘pseudogap’ phase \cite{18}. The present study probes the entire ‘crossover’ between the previously observed regime of dominant phase fluctuations, and the currently unexplored regime with dominant density fluctuations in weakly-interacting 1D ultracold atomic Bose gases. This is performed by varying the number of atoms in the system (Fig. 1), in close analogy to the well-known phase-amplitude-fluctuations interplay studied in high-$T_c$ superconductors as a function of doping \cite{19}.

FIG. 1: Ratio $T_0/T_n$ highlighting interplay between density and phase fluctuations, as a function of atom number. Line $T_0 = T_n$ indicates crossover between regimes of dominant phase ($T_0 < T_n$) and density fluctuations. Circles: $^{23}$Na results for $\omega_z = 2\pi \times 10$ Hz, $\omega_L = 2\pi \times 2500$ Hz. $^{87}$Rb results also shown for same trap configuration (dashed line) and for $\omega_z = 2\pi \times 50$ Hz (squares) Inset: Corresponding characteristic temperatures $T_0$ (open symbols) and $T_n$ (filled).
In atomic gases, such characteristic temperatures can be determined as follows: The temperature for phase fluctuations, $T_\phi$, is obtained by measuring the normalized off-diagonal one-body density matrix, also known in the language of quantum optics as the first order correlation function. Roughly speaking, $T_\phi$ corresponds to the temperature at which the latter quantity decays to zero at the edge of the quasi-condensate [9], thus marking an approximate boundary between the regimes of ‘true’ condensation and quasi-condensation [10]. On the other hand, the temperature for density fluctuations can be determined by measuring the quasi-condensate fraction. In particular, we define $T_n$ as the temperature at which there is a 10% quasi-condensate depletion, since this can be readily measured experimentally. For a given experimental configuration, both $T_\phi$ and $T_n$, and therefore their ratio, are fixed, and experiments so far have only been performed in the limiting regime $T_\phi/T_n \ll 1$ (see later). However, the flexibility of atomic gas experiments enables complete tunability of the ratio $T_\phi/T_n$, thus allowing one to probe the full interplay between density and phase fluctuations. This is explicitly shown in Fig. 1, for the particular example of variable atom number at fixed trap configuration. As evident from this figure, the novel regime of dominant density fluctuations, which can be approximately defined by $T_\phi > T_n$, is attainable for a broad range of experimental parameters, both for $^{23}$Na and $^{87}$Rb. The corresponding characteristic temperatures for both species are shown in the inset.

The equilibrium properties of the system are expected to depend on the interplay between $T_\phi$ and $T_n$. In particular, the role of density fluctuations on the coherence of the gas can be determined by looking at the difference in the measured coherence length of a depleted quasi-condensate, from calculations based on the corresponding undepleted system with the same total atom number. Experiments to date have inferred a negligible effect of density fluctuations on the equilibrium coherence properties [13, 14, 15, 16, 17], based on good agreement between measured data and a theory which neglects density fluctuations [13, 21]. Surprisingly, this was argued to be the case even at temperatures close to the effective ‘transition’ temperature, where most of the quasi-condensate is known to be depleted by thermal excitations. In this paper, we demonstrate that such a conclusion is only correct in the regime $T_\phi \ll T_n$, which is in fact the only regime explored experimentally to date. Even within this regime, more recent work [17] indicates a currently unaccounted for reduction of 20% in the coherence length, which we attribute to density fluctuations. We also demonstrate that this reduction can be largely enhanced by appropriate choice of parameters.

**Theory:** Early theoretical work addressing trapped 1D systems treated phase fluctuations exactly, but neglected density fluctuations [4], thus applicable only in the limit of an undepleted quasi-condensate. Quasi-condensate depletion was subsequently included in a self-consistent treatment of both phase and density fluctuations, leading us to the construction of the 1D phase diagram in the weakly-interacting limit [10]. The resulting theory was shown to be free of both infrared and ultraviolet divergences and therefore valid in any dimension. Importantly, and in stark contrast to other existing theories [4, 11, 12], such an equation of state enables a direct \textit{ab initio} determination of quasi-condensate density profiles. In the harmonic trap, densities are obtained in the local density approximation, the effective temperature-dependent quasi-condensate size $R_{TF}(T)$ is obtained in the Thomas-Fermi limit, and the coherence of the system is investigated via the first-order normalized correlation function.

All quasi-condensate experiments performed to date [13, 14, 15, 16, 17] have been analyzed on the basis of the theory of [4, 21] which entirely neglects both quantum and thermal fluctuations. Although the effect of quantum fluctuations is small for experimentally relevant conditions, thermal fluctuations clearly become important with increasing temperature, as evident from the experimentally observable quasi-condensate depletion. It is therefore not a priori clear why such density fluctuations have not been observed to couple to the phase fluctuations of the system. In order to ascertain the importance of density fluctuations on the equilibrium properties of the system in an \textit{ab initio} manner, we henceforth compare...
the theory of \[10\], upon neglecting the quantum fluctuations, to the conventional 1D theory which additionally ignores thermal quasi-condensate depletion \[2\] (a, b). The general expression for the first-order correlation function thus becomes \(g^{(1)}(0, z) = \exp \left(-\langle[\hat{\chi}(z) - \chi(0)]^2\rangle / 2\right)\), where \(\hat{\chi}(z)\) is the phase operator, satisfying \[10\]

\[
\langle[\hat{\chi}(z) - \chi(0)]^2\rangle = \frac{4\pi T^4}{R_{TF}(T)} \sum_{j=0}^{\infty} 2N(h\omega_j) 	imes \left[A_j^2 \left(P_j(z/R_{TF}(T)) - P_j(0)\right)^2 - B_j^2 \left(P_j(z/R_{TF}(T)) - P_j(0)\right)^2\right].
\]

Here \(P_j(z)\) are Legendre polynomials of order \(j\), and \(A_j = \sqrt{\left(j + 1\right)/2} \mu'/\hbar\omega_j, B_j = \left(\sqrt{\left(j + 1\right)/2} \hbar\omega_j/\mu'\right)/2\). The frequencies are given by \(\omega_j = \sqrt{j(j + 1)/2} \omega_z \mu'\). The 'renormalized' chemical potential including quasi-condensate depletion, \(\mu^*\), is the effective 1D scattering length and \(l_{c}\) the harmonic oscillator length corresponding to longitudinal confinement \(\omega_z\). \(N(h\omega_j)\) is the usual Bose distribution function. Density fluctuations are explicitly maintained in the above expression, via the temperature-dependent quasi-condensate size \(R_{TF}(T)\) appearing in the prefactor. The general expression quoted above can be readily reduced to the conventional theory \[11\] which ignores quasi-condensate depletion by replacing \(R_{TF}(T)\) by the corresponding zero temperature quasi-condensate size \(R_{TF}(0)\) at the same total atom number, and ignoring the \(B_j\) contributions. Since \(R_{TF}(T) \approx \sqrt{\mu'}\), the former step is equivalent to replacing the renormalized chemical potential at temperature \(T\) by the corresponding zero-temperature one for the same total atom number. Then, the 'classical' approximation \(N(h\omega_j) \approx k_B T/h\omega_j\) leads to the definition of the characteristic temperature \(T_{\phi} = (\hbar\omega_z)^2 N/k_B \mu^*\) \[12\]. All presented results maintain the full Bose distribution function \(N(h\omega_j) = [\exp(\beta h\omega_j) - 1]^{-1}\), and are in the weakly-interacting regime, with \(\gamma = 1/(n\xi^2) < 10^{-3}\) \[13\].

**Results:** The effect of density fluctuations on the equilibrium properties of the system is best assessed by investigating the latter at the characteristic temperature, \(T_{\phi}\), where phase fluctuations set in. This study, shown in Fig. 2, reveals marked differences in both density profiles (top figures) and correlation functions (bottom) between the regimes of dominant phase and density fluctuations. In particular, inclusion of density fluctuations leads to significant quasi-condensate depletion, reducing both the central quasi-condensate density and the quasi-condensate size \(R_{TF}(T) < R_{TF}(0)\). Moreover, within the region of validity of the finite temperature correlation functions, i.e., \(z < R_{TF}(T)\), density fluctuations are shown to lead to a potentially significant reduction in the coherence.

To quantify this reduction further, Fig. 3 shows the dependence of the fractional deviation of the coherence length induced by the presence of density fluctuations (a) on the 'crossover' parameter \(T_{\phi}/T_n\) and (b) on appropriately scaled temperature. The coherence length is defined as the distance at which the correlation function decays to half its value, i.e., \(g^{(1)}(0, L_{coh}(T)) = 0.5\). The neglect of density fluctuations leads to a large overestimate of the coherence length. Importantly, this occurs already at the crossover region \(T_{\phi} \approx T_n\) even at temperatures as low as \(T_{\phi}\) at which density fluctuations are usually considered negligible \[2\]. Using the approximate scaling \(T_n \approx 0.2 T_{\phi}\) valid for the examined parameters, we conclude from Fig. 3(a) that density fluctuations should be observable at very low temperatures, even within experimental uncertainties, provided \(T_{\phi}/T_n > 0.1\). The approximately linear increase in temperature of \(\delta L_{coh}(T)\) plotted in Fig. 3(b) is universal when temperatures are scaled in terms of \(T_n\), or equivalently, \(T_{\phi}\). However, corresponding features differ noticeably in terms of \(T/T_{\phi}\), suggesting that \(T_{\phi}\) is not the most appropriate scaling parameter for such studies.

The dependence of the coherence length, scaled to the zero-temperature system size, on appropriately scaled temperature is shown in Fig. 4. Filled (open) symbols correspond to the cases \(T_{\phi} > T_n\) (\(T_{\phi} < T_n\)), with (black) or without (grey) density fluctuations included. The coherence length decreases approximately exponentially with temperature, consistent with experimental findings \[10\]. Importantly, however, in both cases, density fluctuations lead to a considerable shift of the curves to lower values. This is also true when temperatures are scaled in terms of \(T_n\) (Fig. 4(b)), as usually done experimentally \[13, 14, 15, 16, 17\], in which case results without density fluctuations fall roughly on a universal curve (inset). However, inclusion of density fluctuations is found to lead to a noticeable downward shift of the correlation function, already in the regime \(T_{\phi} < T_n/2\) (black circles). The magnitude of this shift clearly increases with
The proposed regime is achieved when a number of competing conditions are satisfied: Firstly, 1D thermal excitations require \( kT < \hbar \omega_{\perp} \), implying a relatively large transverse confinement of the order of a few kHz, which is easily achievable in experiments with atom chips [3]. The condition for 1D quasi-condensation, \( \mu < \hbar \omega_{\perp} \), translates into a condition for the maximum number of atoms at given trap confinement, namely \( N^2 (\omega_z^2 / \omega_{\perp}^2) < C_1 \). Finally, the requirement for dominant density fluctuations, \( T_{\phi} > T_n \), places a competing condition \( N^2 (\omega_z^2 / \omega_{\perp}^2) < C_2 \).

For \(^{23}\text{Na}\), we find \( C_1 \approx \text{few} \times 10^9 \text{Hz} \) and \( C_2 \approx \text{few} \times 10^{12} \text{Hz} \). At the chosen confinement \( \omega_{\perp} = 2\pi \times 10 \text{Hz} \) and \( \omega_z = 2\pi \times 2.5 \text{kHz} \), the 1D quasi-condensate condition is the most restrictive for the maximum number of atoms required, whereas the opposite is true for the same aspect ratio \( \omega_{\perp} / \omega_z \) but larger \( \omega_{\perp} = 2\pi \times 10 \text{kHz} \).

Throughout this work, we have focused on \(^{23}\text{Na}\), on the grounds that such experiments may be easier because \(^{23}\text{Na}\) enters the regime of dominant density fluctuations with a relatively large atom number \( N > 1000 \) for a broad parameter range. However, this comes at the expense of a stringent requirement on low temperatures, typically around 20nK. The same regime is also attainable with \(^{87}\text{Rb}\), for which \( \eta \approx 10 \). Although \(^{87}\text{Rb}\) requires lower aspect ratios and higher temperatures than \(^{23}\text{Na}\), it nonetheless requires a reduced number of atoms, with \( N < 700 \) even for optimized aspect ratios. The optimum candidate for such an experiment thus depends on the flexibility of the experimental set-up, with respect to cooling efficiency and atom detection techniques.

The regime of dominant density fluctuations typically requires the phase fluctuation temperature, \( T_{\phi} \), to be comparable to the effective ‘transition’ temperature \( T_c \). In this regime, the suppressed phase fluctuations are dominated by their coupling to density fluctuations. Such a regime may prove useful for potential applications, due to the presence of large coherence even at relatively high temperatures \( T \sim 0.5 T_c \). However, the effect of density fluctuations on the coherence length can already be observed at the much weaker approximate condition \( T_{\phi} > 0.1 T_n \), which increases the value \( C_2 \) by a factor of 10.

In conclusion, we proposed a new regime of quasi-one-dimensional weakly-interacting atomic gases, in which density fluctuations set in at a lower temperature than phase fluctuations. This regime was shown to be experimentally accessible for small atom numbers, low temperatures and moderate aspect ratios. It can be experimentally identified by the noticeable reduction in the equilibrium coherence length of the quasi-condensate, whose extent depends critically on the ratio of the phase degeneracy temperature \( T_{\phi} \) to the effective transition temperature \( T_c \). Given the pronounced nature of density fluctuations in homogeneous systems, an alternative observation of dominant density fluctuations can be performed in the new box-like trap [23]. The unprecedented experimental

![FIG. 4: Temperature dependence of the scaled coherence length \( L_{coh}(T) / R_{TF}(0) \) on (a) \( T / T_c \) and (b) \( T / T_{\phi} \) for parameters of Fig. 2: \( T_{\phi} / T_c \approx 0.1 \) (open symbols) and 0.4 (filled), without (grey) or with (black) density fluctuations. Inset: Deviation in regime \( 0.4 < T / T_{\phi} < 1.4 \).](image-url)
control offered by ultracold atomic gases may offer further insight into the interplay between phase and density fluctuations in analogous condensed-matter systems.

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