ρ-meson properties in medium

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Abstract

Properties of $\rho$-meson in symmetric nuclear matter are investigated in a light-front constituent quark model (LFCQM), using the in-medium inputs calculated by the quark-meson coupling (QMC) model. The LFCQM used in this study was already applied for the studies of the electromagnetic properties of $\rho$-meson in vacuum, namely, the charge $G_0$, magnetic $G_1$, and quadrupole $G_2$ form factors, electromagnetic charge radius, and electromagnetic decay constant. Using the two different density dependence of the regulator mass in medium, we predict that the charge radius, and quadrupole moment are enhanced as increasing the nuclear matter density, while the magnetic moment is slightly quenched. Furthermore, we predict the value $Q^2_{\text{zero}}$, which crosses zero of the charge form factor, $G_0(Q^2_{\text{zero}}) = 0$ ($Q^2 = -q^2 > 0$ with $q$ being the four-momentum transfer), decreases as increasing the nuclear matter density by the two different density dependence of the regulator mass. On the other hand, for the electromagnetic decay constant of the $\rho$-meson, the two different density dependence of the regulator mass predict the opposite density dependence. Namely, as increasing the nuclear matter density, the naive treatment with the density independent regulator mass as in the vacuum, predicts the increase of the decay constant, while the other that assumes the same density dependence of the regulator mass as that of the in-medium constituent quark mass, predicts the decrease of the decay constant. Thus, although the other physical quantities are predicted to have similar density dependence by the two different density dependence of the regulator mass applied, the density dependence of the $\rho$-meson electromagnetic decay constant are predicted to have opposite density dependence, and the facts suggest that the in-medium $\rho$-meson decay constant needs to be investigated further in the future.

Key words: $\rho$-meson in medium, Electromagnetic form factors, Symmetric nuclear matter, Light-front constituent quark model, Quark-meson coupling model
1 Introduction

One of the fundamental objectives in hadronic physics is to understand the structure of hadrons in terms of the quark and gluon degrees of freedom, the basis of quantum chromodynamics (QCD). The Standard Model (SM) of elementary particles contains QCD as the strong interaction theory, and practicing QCD to understand the hadron structure is an important part of understanding SM. However, it is not straightforward to apply QCD directly to study the properties of hadrons such as mesons, baryons, and tetra-quarks, the bound state systems of quarks and gluons, in particular in the low energy nonperturbative region (see Refs. [1,2]). Despite many successes of QCD which is believed as the correct quantum field theory of strong interaction [3,4], the hadronic properties in the low-energy region cannot be directly extracted naively. To overcome the difficulties, effective treatments of QCD, such as constituent quark models (CQM) and light-front treatment of hadrons have been developed, and achieved impressive success [5-27]. In particular, we emphasize the cases of spin-1 vector particles [28-46], that are of relevant for the present study of $\rho$-meson.

For the $\rho$-meson, experimental data are very scarce at present. In Refs. [47,48] some properties of the $\rho$-meson in medium were discussed based on the experiment data. Recently, in Refs. [49,50] the data from BaBar Collaboration [51] for the $e^+e^- \rightarrow \rho^+\rho^-$ reaction in vacuum were analyzed to study the electromagnetic properties of $\rho$-meson based on perturbative QCD.

Light-front quantum field theory (LFQFT) on which the present study bases, is able to incorporate the following two important aspects simultaneously, namely, QCD and the picture of constituent quark model. It is a natural approach to calculate physical observables based on the quark degrees of freedom [3,4]. In the light-front approach one can have relativistic wave functions of hadronic bound states described in terms of quarks and gluons, and the approach is suitable for understanding the hadron substructure focusing on nonperturbative aspect [28,52,53,54,55,56].

Based on the advantages mentioned above for the light-front approach, we use a light-front constituent quark model (LFCQM) which was already applied for the studies of $\rho$-meson electromagnetic properties in vacuum [33,57,58]. We extend the preliminary study made in symmetric nuclear matter for the $\rho$-meson properties in medium [59], and elaborate in the present work. In this work we use a different regularization treatment for the calculation in nuclear medium. Namely, we assume the same density dependence of the regularization mass as that of the in-medium constituent quark mass. By the use of this density dependent regularization mass, we calculate the $\rho$-meson electromagnetic charge $G_0$, magnetic $G_1$, and quadrupole $G_2$ form factors,
electromagnetic square charge radius \( r_{\rho}^2 \), and \( \rho^0 \to e^+e^- \) decay constant \( f_\rho \) in symmetric nuclear matter. The results obtained using the density independent regularization mass were presented in Ref. \[59\]. Then, the results of the two different regularization treatments applied in medium are compared and discussed with those of Ref. \[59\]. However, we emphasize that, except for the density dependence of the \( \rho \)-meson decay constant, the results of two different regularization treatments predict the similar density dependence of the \( \rho \)-meson in-medium electromagnetic properties. Since the density dependence of the \( \rho \)-meson decay constant shows the opposite density dependence, this needs to be studied further by using other model, and/or developing a proper in-medium regularization method in the future.

We remind that, we use the plus-component of the electromagnetic current \( J_\rho^+ \) in this study, considering the situation that the \( \rho \)-meson is immersed in symmetric nuclear matter \[60,61,62\]. (For comprehensive reviews on hadronic and quark properties in nuclear medium, e.g., see Refs. \[60,61\].) The model for the \( \rho \)-meson in vacuum we use \[35\] in this study, is constrained by the “angular condition” for the light-front electromagnetic current matrix elements \[28,29,35,57,58,63\]. Any light-front-based models of spin-1 particles should satisfy the angular condition, which was originally discussed in Ref. \[64\].

2 Quark-meson coupling model

To describe symmetric nuclear matter (many nucleon system), we rely on the quark-meson coupling (QMC) model, which bases on the quark degrees of freedom, and calculate necessary in-medium inputs for the quarks and hadrons to implement in the light-front constituent quark model, similarly to the studies made for pion and kaon \[62,65,66,67\] and nucleon \[24\]. Detail of the light-front constituent quark model we use in this study is described in Refs. \[8,9,23\] (and for nucleon in Ref. \[24\]).

The QMC model was invented by Guichon \[68\] using the MIT bag model, and Frederico et al. \[69\] using a relativistic confining harmonic potential. The model was successfully applied for studying the properties of finite nuclei \[70\], and the properties of hadrons in medium \[71,72\]. (See Ref. \[71\] for other approaches similar to the QMC model used in the present study.) In the QMC model the meson and baryon internal structure is modified in medium due to the surrounding medium, by the self-consistent exchange of the scalar-isoscalar (\( \sigma \)), vector-isoscalar (\( \omega \)), and vector-isovector (\( \rho \)) meson fields directly coupled to the relativistic, confined light-quarks in the nucleon (hadron), rather than to the point-like nucleon (hadron). We briefly explain next the main feature of the QMC model, which is used to calculate the in-medium inputs necessary to study the \( \rho \)-meson electromagnetic properties in symmetric nuclear matter.
We consider a system of infinite, uniform, spin and isospin saturated symmetric nuclear matter in the Hartree mean field approximation in the rest frame of matter. Thus, irrelevant $\rho$-meson filed is suppressed in the following, since the total isospin of the matter is zero, and in the Hartree approximation $\rho$-meson mean filed becomes zero. In addition quantities with an asterisk, $^*$, will stand for those in medium hereafter. The effective Lagrangian density of the QMC model at the hadron level may be given by [71],

$$
\mathcal{L} = \bar{\psi}[i\gamma \cdot \partial - m_N^*(\hat{\sigma}) - g_\omega \hat{\omega}^\mu \gamma_\mu] \psi + \mathcal{L}_{\text{meson}},
$$

with $\mathcal{L}_{\text{meson}}$ being the free meson Lagrangian,

$$
\mathcal{L}_{\text{meson}} = \frac{1}{2}(\partial_\mu \hat{\sigma} \partial^\mu \hat{\sigma} - m_\sigma^2 \hat{\sigma}^2) - \frac{1}{2} \partial_\mu \hat{\omega}_\nu (\partial^\mu \hat{\omega}^\nu - \partial^\nu \hat{\omega}^\mu) + \frac{1}{2} m_\omega^2 \hat{\omega}^\mu \hat{\omega}_\mu.
$$

In the above $\psi$, $\hat{\sigma}$ and $\hat{\omega}$ are respectively the nucleon, Lorentz-scalar-isoscalar $\sigma$, and Lorentz-vector-isoscalar $\omega$ field operators, with $g_\omega$ being the nucleon-$\omega$ coupling constant, while the nucleon-$\sigma$ effective coupling which depends on the $\hat{\sigma}$ (or nuclear density) is defined by,

$$
m_N^*(\hat{\sigma}) = m_N - g_\sigma(\hat{\sigma}) \hat{\sigma},
$$

where the effective nucleon mass is denoted by $m_N^*$. Then, the nucleon density $\rho_N$, the nucleon Fermi momentum $k_F$, the nucleon scalar density $\rho_s$, and the effective nucleon mass $m_N^*$ are related by,

$$
\rho_N = \frac{4}{(2\pi)^3} \int d^3k \theta(k_F - |k|) = \frac{2k_F^3}{3\pi^2},
$$

$$
\rho_s = \frac{4}{(2\pi)^3} \int d^3k \theta(k_F - |k|) \frac{m_N^*(\sigma)}{\sqrt{m_N^2(\sigma) + k^2}},
$$

where $m_N^*(\sigma)$ is the value of the effective nucleon mass at a given density, calculated by the QMC model [70,71].

The Dirac equations for the light quarks and light antiquarks in the bag of hadron $h$ in nuclear matter at the position $x = (t, \mathbf{r})$ with $|\mathbf{r}| \leq$ bag radius, are given by [71],

$$
\begin{align*}
\left[i\gamma \cdot \partial_x - (m_q - V_q^\rho) \mp \gamma^0 \left(V_q^\omega + \frac{1}{2} V_q^\rho \right)\right] \begin{pmatrix} \psi_u(x) \\ \psi_d(x) \end{pmatrix} &= 0, \\
\left[i\gamma \cdot \partial_x - (m_q - V_q^\rho) \mp \gamma^0 \left(V_q^\omega - \frac{1}{2} V_q^\rho \right)\right] \begin{pmatrix} \psi_u(x) \\ \psi_d(x) \end{pmatrix} &= 0,
\end{align*}
$$

(4)
where, $V_\sigma^q = g^q_\sigma \sigma$, $V_\omega^q = g^q_\omega \omega$, and $V_\rho^q = g^q_\rho \rho$ are respectively the constant mean field potentials with the corresponding quark-meson coupling constants, $g_\sigma$, $g_\omega$ and $g_\rho$, and the Coulomb interactions are neglected, because the nuclear matter is described in the strong interaction. The vector meson mean fields appearing in $V_\omega^q$ and $V_\rho^q$, correspond respectively to the expectation values evaluated in symmetric nuclear matter are, $\omega^\mu = (\omega, 0)$ and $\rho^\mu_i = (\delta_i^3 b, 0)$. In addition SU(2) symmetry for the light-quark masses, $m_q = m_\bar{q} = m_u = m_d$, is assumed.

The normalized, static solution for the ground state light quark ($q$) and light antiquark ($\bar{q}$) in the hadron $h$ may be written as $\psi_{q,\bar{q}}(x) = N_{q,\bar{q}} e^{-ix_{q,\bar{q}} \cdot t/R^*_h} \psi_{q,\bar{q}}(r)$, where $N_{q,\bar{q}}$ and $\psi_{q,\bar{q}}(r)$ are the normalization factor and corresponding spin and spatial part of the wave function. The bag radius in medium of the hadron $h$, $R^*_h$, is determined by the stability condition for the mass of the hadron against the variation of the bag radius [71], to be shown in Eq. (6). The eigenenergies in units of $1/R^*_h$ are given by,

$$
\begin{pmatrix}
\epsilon_u \\
\epsilon_{\bar{u}} \\
\epsilon_d \\
\epsilon_{\bar{d}}
\end{pmatrix}
= \Omega_q^* \pm R^*_h \left( V_\omega^q + \frac{1}{2} V_\rho^q \right),
$$

$$
\begin{pmatrix}
\epsilon_u \\
\epsilon_{\bar{u}} \\
\epsilon_d \\
\epsilon_{\bar{d}}
\end{pmatrix}
= \Omega_q^* \pm R^*_h \left( V_\omega^q - \frac{1}{2} V_\rho^q \right),
$$

(5)

where, $\Omega_q^* = \Omega_{\bar{q}}^* = [x_q^2 + (R^*_h m_q^*)^2]^{1/2}$, with $m_q^* = m_q - g^q_\sigma \sigma$, and $x_q$ being the lowest mode bag eigenvalue. Because we consider symmetric nuclear matter with the Hartree approximation, $V_\rho^q$ is zero also at the quark level, thus we will ignore hereafter.

The mass of the low-lying hadron $h$ in symmetric nuclear matter, is calculated with the bag radius stability condition,

$$m^*_h = \sum_{j=q,\bar{q}}^{n_q} \frac{n_j \Omega_j^* - z_h}{R^*_h} + \frac{4}{3} \pi R^*_h^3 B, \quad \frac{\partial m^*_h}{\partial R^*_h} \bigg|_{R^*_h=R^*_h} = 0,$$

(6)

with $n_q$ ($n_{\bar{q}}$) being the light-quark (light-antiquark) number.

Now we study the $\rho$-meson electromagnetic properties in symmetric nuclear matter using the inputs calculated by the QMC model. To do so, we must rely on the $\rho$-meson model in vacuum which is successful and simple enough to handle in extracting the main in-medium properties. As already mentioned, we use the light-front constituent quark model for the $\rho$-meson developed in Ref. [35].
The model uses the light-quark constituent quark mass value in vacuum $m_q = m_{ar{q}} = 430$ MeV. Using this value we calculate the corresponding symmetric nuclear matter properties with the QMC model. By fitting the nuclear matter saturation properties, namely the binding energy of 15.7 MeV at the saturation density $\rho_0 = 0.15$ fm$^{-3}$, we obtain the corresponding quark-meson coupling constants. The coupling constants, and some quantities calculated in the QMC model at $\rho_0$ are listed in Table 1. For a comparison, the same quantities obtained in the standard QMC model with $m_q = 5$ MeV, are also listed in Table 1 as already mentioned.

| $m_q$ (MeV) | $g_{\sigma}/4\pi$ | $g_{\omega}/4\pi$ | $m_N^*$ | $m_q^*$ | $K$ | $z_N$ | $z_{\rho}$ | $B^{1/4}$ |
|------------|------------------|------------------|--------|--------|-----|-------|---------|----------|
| 5          | 5.39             | 5.30             | 754.6  | -135.6 | 279.3| 3.295 | 1.907   | 170.0    |
| 430        | 8.73             | 11.94            | 565.3  | 245.7  | 361.4| 5.497 | 2.939   | 69.8     |

One of the noticeable differences among the quantities calculated with the standard value $m_q = 5$ MeV and those with the value $m_q = 430$ MeV, is the nuclear incompressibility, $K$. It yields a larger value of $K = 361.4$ MeV with $m_q = 430$ MeV, while $K = 279.3$ MeV with $m_q = 5$ MeV. The corresponding energy density per nucleon for $m_q = 430$ MeV, $(E_{\text{Total}}/A) - m_N$, is shown in Fig. 1 upper panel.

Concerning the solid line shown in Fig. 1 upper panel, the curvature of the energy density versus $\rho/\rho_0$ is larger than that for $m_q = 5$ MeV. Thus $(E_{\text{Total}}/A) - m_N$ varies faster than that for $m_q = 5$ MeV as the nuclear matter density increases. (See Ref. [71] for the curve of $(E_{\text{Total}}/A) - m_N$ with $m_q = 5$ MeV.) Next, we show also in Fig. 1 lower-left panel the effective light-quark mass $m_q^* = m_q - g_\sigma^\sigma \sigma$, scalar potential $V_\sigma^\sigma = g_\sigma^\sigma \sigma$, and vector potential $V_\omega^\omega = g_\omega^\omega \omega$ felt by the light quarks, and in lower-right panel the effective $\rho$-meson mass $m_\rho^*$. (See Eq. (6) for $m_\rho^*$ with $h \rightarrow \rho$.) Note that, the $\rho$-meson effective mass $m_\rho^*$ shown in Fig. 1 lower-right panel, corresponds to that of naive SU(6) quark models, and readers must not be confused with that of the $\rho$-meson mean field (operator) appearing in the QMC model. In addition we have neglected...
the width of the $\rho$-meson, following the usual practices of naive SU(6) quark models.

3 $\rho$-meson electromagnetic form factors

A general expression of the electromagnetic current for a $\rho$-meson (spin-1 particle) is given by [73]:

$$J^{\mu}_{\alpha \beta} = -\left[ F_1(Q^2)g_{\alpha \beta} - F_3(Q^2)\frac{q_\alpha q_\beta}{2m^{2}_\rho}\right] P^{\mu} - F_M(Q^2)\left(q_\alpha g^{\mu}_{\beta} - q_\beta g^{\mu}_{\alpha}\right), \quad (7)$$

where $m_\rho$ is the mass of the $\rho$-meson, $q^{\mu}$ the four-momentum transfer with $Q^2 = -q^2 > 0$, and $P^{\mu} \equiv (p_i + p_f)^{\mu}$, the sum of the initial ($p_i$) and final ($p_f$) momenta. The electromagnetic current, $J^{\mu}_{ji} \equiv \epsilon^i_\alpha J^{+\alpha}_{i\beta} \epsilon^j_\beta$, in an impulse approximation is given by,
\[ J_{ji}^\mu = i \int \frac{d^4 k}{(2\pi)^4} \frac{Tr[\epsilon_i^\alpha \Gamma_\alpha(k, k - p_f)(\not k - \not p_f + m_q)\gamma^\mu(\not k + \not p_i + m_q)]}{(k^2 - m_q^2 + i\epsilon)((k - p_f)^2 - m_q^2 + i\epsilon)} \]

where \( m_q \) is the quark mass, and without loss of generality, the four-momenta of the initial and final states of the \( \rho \)-meson may respectively be chosen by \( p_i^\mu = (p^0, -q/2, 0, 0) \) and \( p_f^\mu = (p^0, q/2, 0, 0) \) with the four-momentum transfer defined by \( q^\mu = (0, q, 0, 0) \) to satisfy the Drell-Yan condition \([29,35]\). In Eq. (8), \( \epsilon_i^\alpha \) and \( \epsilon_i^\beta \) are the final- and initial-state \( \rho \)-meson polarization vectors, respectively given by,

\[ \epsilon_x^\mu = (\sqrt{\eta}, \sqrt{1 + \eta}, 0, 0), \quad \epsilon_y^\mu = (0, 0, 1, 0), \quad \epsilon_z^\mu = (0, 0, 0, 1), \]

and

\[ \epsilon_x^\mu = (-\sqrt{\eta}, \sqrt{1 + \eta}, 0, 0), \quad \epsilon_y^\mu = (0, 0, 1, 0), \quad \epsilon_z^\mu = (0, 0, 0, 1), \]

with \( \eta = Q^2/m_{\rho}^2 \). The electromagnetic current, Eq. (8), is divergent, and in order to make \( J_{ji}^\mu \) finite, a regulator function \( \Lambda(k, p) \) is used \([35]\):

\[ \Lambda(k, p) = \frac{1}{((k - p)^2 - M_R^2 + i\epsilon)^2}. \]

Here, the regulator mass value \( m_R \) to yield \( M_R^2 \) (see Eq. (15) for \( m_R \) and the \( M_R^2 \) mass operator definition), is chosen to reproduce the experimental value of the \( \rho \)-meson decay constant \( f_\rho \) (see Eq. (19) for the definition) extracted from the \( \rho^0 \to e^+e^- \) decay width \([74]\). The \( \rho\bar{q}q \) vertex with spinor structure is modeled by \([35]\),

\[ \Gamma^\mu(k, k') = \gamma^\mu - \frac{m_\rho}{2} \frac{k^\mu + k'^\mu}{p \cdot k + m_\rho m_q - i\epsilon}, \]

where the \( \rho \)-meson is on-mass-shell, and its four momentum is \( p^\mu = k^\mu - k'^\mu \) with the quark momenta \( k^\mu \) and \( k'^\mu \) \([29,35]\).

Working with the light-front coordinate, \( a^\mu = (a^+ = a^0 + a^3, a^- = a^0 - a^3, \vec{a}_{\perp} = (a^1, a^2)) \), the light-front \( \rho \)-meson wave function is obtained, after substituting with the on-mass-shell condition \( k^- = (k_{\perp}^2 + m_q^2)/k^+ \) in the quark propagator (see Ref. \([35]\) for details),

\[ \Phi_i(x, \vec{k}_{\perp}) = \frac{N^2}{(1-x)^2(m_\rho^2 - M_0^2)(m_\rho^2 - M_R^2)^2} \tilde{\epsilon}_i^{\gamma - \not k/M_0 + m_q} \]

where, \( x = k^+/p^+ \). The polarization state is given by \( \tilde{\epsilon}_i \). The wave function corresponds to an S-wave state \([52]\). The square of the free mass operator \( M_0^2 \), and the regulator mass operator \( M_R^2 \), are given by:
\[ \begin{align*}
M_0^2 &= \frac{k_{\perp}^2 + m_q^2}{x} + \frac{(\vec{p} - \vec{k})_{\perp}^2 + m_q^2}{1 - x} - \vec{p}_{\perp}^2, \quad (14) \\
M_R^2 &= \frac{k_{\perp}^2 + m_q^2}{x} + \frac{(\vec{p} - \vec{k})_{\perp}^2 + m_R^2}{1 - x} - \vec{p}_{\perp}^2. \quad (15)
\end{align*} \]

3.1 Angular condition and electromagnetic form factors

For the spin-1 particles in the light-front approach, matrix elements of the plus-component of electromagnetic current, \( J^+ \), is constrained by the angular condition equation with the light-front spin basis \([28,35,64]\):

\[ \Delta(q^2 = -Q^2) = (1 + 2\eta)I_{11}^+ + I_{1-1}^+ + -\sqrt{8\eta}I_{10}^+ - I_{00}^+ = 0. \quad (16) \]

The relations among the light-front basis \( I_{m'm}^+ \) and those of the instant form spin basis \( J_{ji}^+ \) (\( j,i = x,y,z \)), can be made by the Melosh rotation matrix. (See Refs. \([29,35]\) for details.)

With the angular condition Eq. (16), it is possible to arrange the electromagnetic form factors to form with different linear combinations \([28,35]\) by eliminating some matrix elements \( I_{m'm}^+ \). However, some linear combinations break the covariance as well as the rotational symmetry. This is due to the zero mode contributions, or pair term contributions \([29,75,77]\). In Ref. \([29]\) a careful analysis was made for the origins of the zero-mode contributions for the matrix elements of the electromagnetic current of spin-1 particles, in particular for the \( \rho \)-meson.

It was demonstrated that the zero mode contributions are canceled out in the combinations of the electromagnetic current matrix elements of Grach et al. \([64]\), by numerically in Ref. \([35]\), and by analytically in Ref. \([29]\). The reason is that the electromagnetic matrix element of the current, \( I_{00}^+ \), was eliminated by the angular condition \([28,35,36,64]\). For some prescriptions in the literature, the zero-mode or non-valence contributions needed to be added in order to recover the full covariance \([18,29,35,75]\).

With the prescription of Ref. \([64]\), the electromagnetic form factors of \( \rho \)-meson are given by both in the light-front spin basis \( I_{m'm}^+ \) and the instant form spin basis \( J_{ji}^+ \).
\[ G_0 = \frac{1}{3} \left[ (3 - 2\eta) I_{11}^+ + 2 \sqrt{2\eta I_{11}^+} + I_{11}^+ \right] = \frac{1}{3} \left[ J_{xx}^+ + 2J_{yy}^+ - \eta J_{yy}^+ + \eta J_{zz}^+ \right], \]
\[ G_1 = 2 \left[ I_{10}^+ - \frac{1}{\sqrt{2\eta}} I_{10}^+ \right] = \left[ J_{yy}^+ - J_{xx}^+ - \frac{J_{zz}^+}{\sqrt{\eta}} \right], \]
\[ G_2 = \frac{2\sqrt{2}}{3} \left[ -\eta I_{11}^+ + \sqrt{2\eta} I_{10}^+ - I_{11}^+ \right] = \frac{\sqrt{2}}{3} \left[ J_{xx}^+ - (1 + \eta) J_{yy}^+ + \eta J_{zz}^+ \right]. \]

The electromagnetic form factors \( G_0, \ G_1 \) and \( G_2 \) above, are related by the covariant form factors \( F_1, \ F_M \) and \( F_3 \) of Eq. (7) [73]:

\[ G_0 = F_1(Q^2) + 2 \frac{\eta}{3} G_2(Q^2), \quad \text{(see below)}, \]
\[ G_1 = F_M(Q^2), \]
\[ G_2 = F_1(Q^2) - F_M(Q^2) + (1 + \eta) F_3(Q^2). \] (18)

The \( \rho \)-meson decay constant \( f_\rho \) is defined by [18,53],
\[ <0|\bar{q}(0)\gamma^\mu q(0)|\phi_\rho(\lambda)> = \epsilon^\mu_\lambda m_\rho f_\rho, \]
where \( \epsilon^\mu_\lambda \) is the polarization vector of the corresponding \( \rho \)-meson state \( \phi_\rho(\lambda) \).

Note that \( f_\rho \) defined above has the mass dimension one, the same as the usual definition of the pion decay constant (but without a factor \( \sqrt{2} \)). Here, we use the plus-component of the electromagnetic current with \( \lambda = z \) in the rest frame of the \( \rho \)-meson, and \( \epsilon^+_z = 1 \) with \( \epsilon^\mu = (\epsilon^+_z, \epsilon^-_z, \vec{\epsilon}_\perp) = (1, -1, \vec{0}) \) [18]. The result is independent of the choice of \( \lambda \).

We calculate also the \( \rho \)-meson magnetic moment \( \mu_\rho \), quadrupole moment \( Q_{2\rho} \) (note the definition below), and electromagnetic square charge radius \( <r^2_\rho> \).

They are obtained by the following expressions [76]:

\[ 1 = G_0(0), \quad \text{(charge normalization)}, \]
\[ \mu_\rho = G_1(0) = F_M(0), \]
\[ Q_{2\rho} = \lim_{Q^2 \to 0} 3\sqrt{2} \frac{G_2(Q^2)}{Q^2} \]
\[ <r^2_\rho> = \lim_{Q^2 \to 0} -6 \frac{[G_0(Q^2) - 1]}{Q^2} = -6 \frac{dG_0(Q^2)}{dQ^2} \Big|_{Q^2=0}. \] (23)

4 Results

In the following we present the results for the in-medium \( \rho \)-meson properties calculated in symmetric nuclear matter, namely in-medium electromagnetic
charge \((G^s_0)\), magnetic \((G^1_1)\), and quadrupole \((G^2_2)\) form factors, electromagnetic square charge radius \(<r^s_{\rho}^2>\), and electromagnetic \(\rho\)-meson decay constant \(f_{\rho}^*\). These are calculated by the light-front constituent quark model, using the in-medium inputs obtained by the QMC model as already explained.

Before presenting the results, we briefly remind below how the \(\rho\)-meson properties are calculated in symmetric nuclear matter. As explained in section \[\text{2}\] the light-quark and light-antiquark self-energies in symmetric nuclear matter are modified by the Lorentz-scalar-isoscalar \(\sigma\) and Lorentz-vector-isoscalar \(\omega\) mean fields. More specifically, in the Hartree approximation, the light-quark mass term acquires the attractive Lorentz potential \(V^q_{\sigma}\), while the time component of the light-quark (light-antiquark) four-momentum acquires the repulsive (attractive) mean field potential \(V^q_{\omega}\). Namely, the four-momentum \(p^\mu\) of the light-quark (light-antiquark) is modified by,

\[
p^\mu \rightarrow p^\mu + V^\mu = p^\mu + \delta^\mu_0 V^0_{\omega}(p^\mu - \delta^\mu_0 V^0_{\omega}),
\]

and both the light-quark and light-antiquark masses are modified by \(m_q \rightarrow m^*_q = m_q - V^q_{\sigma}\) and \(m_{\bar{q}} \rightarrow m^*_{\bar{q}} = m_{\bar{q}} - V^q_{\omega}\). These mean field potentials are constrained by the nuclear matter saturation properties (see Fig. 1 upper panel). Then, using the in-medium modified light-quark (light-antiquark) properties, as well as the effective \(\rho\)-meson mass obtained in the QMC model (see Fig. 1 lower-right panel), we calculate the in-medium \(\rho\)-meson electromagnetic properties. Similar approach has already been applied for the studies of pion \([62]\), kaon \([67]\) and nucleon \([24]\) properties in symmetric nuclear matter.

In the loop integral appearing in the calculations of electromagnetic form factors or the decay constant, we shift the momentum, \(k'^\mu = k^\mu + \delta^\mu_0 V^0 \rightarrow k^\mu\), and the vector potentials cancel out for the light-quark and light-antiquark systems such as pion and \(\rho\)-meson.

Furthermore, since the effective \(\rho\)-meson mass decreases as increasing the nuclear matter density in the QMC model (see Fig. 1 right panel), the sum of the effective quark masses \((m^*_q + m^*_{\bar{q}})\) forming the \(\rho\)-meson bound state, must be larger than the in-medium \(\rho\)-meson mass \((m^*_\rho)\), namely the binding energy \((B^*)\) to be positive, the same condition as in vacuum, to be discussed in detail later. We summarize in Table \[\text{2}\] some quantities calculated for the \(\rho\)-meson in symmetric nuclear matter.

To understand better the bound state nature in the present light-front constituent quark model, we discuss the binding energy, which should be positive in order to yield the bound state for the quark-antiquark composite system. The binding energy of the \(\rho\)-meson in medium \(B^*\) is defined by,

\[
B^* = m^*_q + m^*_{\bar{q}} - m^*_\rho.
\]

The binding energy calculated in symmetric nuclear matter is shown in Fig. 2 versus the nuclear matter density \(\rho_N/\rho_0\) (upper panel), versus the effective light-quark mass \(m^*_{\bar{q}}\) (lower-left panel), and versus the \(\rho\)-meson effective mass \(m^*_\rho\) (lower-right panel). The dependence of the binding energy on the nuclear matter density, effective quark mass and effective \(\rho\)-meson mass, are all nearly linear and smooth. As the nuclear matter
Table 2
Quantities associated with the in-medium $\rho$-meson properties. Effective light-quark mass ($m_q^*$) and in-medium $\rho$-meson mass ($m_\rho^*$) are quoted in [GeV], while the $\rho$-meson electromagnetic square charge radius $<r_{\rho}^2>$ is in [fm$^2$], the electromagnetic decay constant $f_\rho^*$ in [MeV], the magnetic moment $\mu_\rho^*$ in units of $[e/2m_\rho]$, quadrupole moment $Q_{2\rho}^*$ in [fm$^2$], and the momentum $Q_{\text{zero}}^2$ for $G_0(Q_{\text{zero}}^2) = 0$ in [GeV$^2$]. The experimental value for $\Gamma_{ee} \equiv \Gamma(\rho^0 \rightarrow e^+ e^-) = 7.04 \pm 0.06$ keV in vacuum ($\rho_N = 0$) is taken from Ref. [74]. The numbers given in brackets are the results obtained with the use of the density independent regulator mass, $m_R = 3.0$ GeV.

| $\rho_N/\rho_0$ | $m_q^*$ | $m_\rho^*$ | $<r_{\rho}^2>$ | $f_\rho^*$ | $\mu_\rho^*$ | $Q_{2\rho}^*$ | $Q_{\text{zero}}^2$ |
|-----------------|---------|------------|----------------|-----------|-------------|--------------|--------------|
| 0               | 0.430   | 0.770      | 0.267          | 153.627   | 2.20        | -0.0590      | 2.96         |
|                 | (0.270) | (153.669)  | (2.20)         | (-0.0595) | (2.96)      |
| 0.01            | 0.427   | 0.767      | 0.255          | 150.451   | 2.20        | -0.0594      | 2.94         |
|                 | (0.296) | (163.206)  | (2.20)         | (-0.0639) | (2.74)      |
| 0.10            | 0.410   | 0.738      | 0.287          | 150.381   | 2.20        | -0.0636      | 2.74         |
|                 | (0.352) | (166.148)  | (2.19)         | (-0.0721) | (2.43)      |
| 0.25            | 0.381   | 0.692      | 0.364          | 121.523   | 2.18        | -0.0716      | 2.41         |
|                 | (0.433) | (174.499)  | (2.19)         | (-0.0817) | (2.14)      |
| 0.50            | 0.333   | 0.618      | 0.560          | 108.720   | 2.18        | -0.0876      | 1.93         |
|                 | (0.505) | (179.325)  | (2.18)         | (-0.0884) | (1.97)      |
| 0.80            | 0.278   | 0.538      | 1.364          | 78.323    | 2.12        | -0.1067      | 1.44         |
|                 | (1.214) | (178.739)  | (2.14)         | (-0.1028) | (1.53)      |
| 0.85            | 0.268   | 0.527      | 1.483          | 68.788    | 2.12        | -0.1128      | 1.43         |
|                 | (1.353) | (189.556)  | (2.12)         | (-0.1134) | (1.48)      |
| 0.90            | 0.260   | 0.514      | 2.013          | 58.696    | 2.10        | -0.1128      | 1.34         |
|                 | (1.857) | (195.042)  | (2.10)         | (-0.1152) | (1.40)      |

Exp. [74] for $\Gamma_{ee} (\rho_N = 0) = 152 \pm 8$

density increases, the binding energy $B^*$ decreases, and the density beyond about $\rho/\rho_0 = 0.90$ it becomes negative, and does not yield the $\rho$-meson bound state in the present model. (See also Table 2.)

We comment on the limitation of the light-front constituent quark model applied in this study [35,57,58]. Since to yield the bound state in the con-
Fig. 2. Binding energy $B^*$ [GeV] of the $\rho$-meson versus nuclear matter density $[\rho_N/\rho_0]$ (upper panel), versus effective light-quark mass $m_q^*$ [GeV] (lower-left panel), and versus effective $\rho$-meson mass [GeV] (lower-right panel).

The constituent quark model, the binding energy $B$ of the quark-antiquark meson bound state must satisfy $B > m_q + m_{\bar{q}}$ in vacuum as well as in medium. Thus, the constituent quark mass values in vacuum for the $\rho$-meson case $[35,57,58]$ of $m_q = 430$ MeV and pion case of $m_q = 220$ MeV $[8,62]$ are determined by the best fit for each case to reproduce the experimental data with the regulator mass values $m_R$. Thus, the present models, although established very well in vacuum, cannot study for example $\rho \to \pi \gamma$ transition on the same footing even in vacuum. In this study, we focus on the possible property changes of $\rho$-meson itself in medium, and cannot study the reactions involving the mesons with two different constituent light quark mass values in the corresponding different mesons consistently, as the same reason in vacuum case.

Next, we discuss the $\rho$-meson electromagnetic decay constant in medium, $f_{\rho}^*$. The $\rho$-meson decay constant gives direct information on the structure of the $\rho$-meson bound state wave function at the origin. It is associated with the non-perturbative regime of QCD. The $\rho$-meson decay constant is calculated via Eq. (19) (see also Ref. [18]). However, the experimental data for $\rho$-meson in vacuum are very scarce $[7,4]$ compared with those of the other light mesons such as pion $[8]$. The parameters of the model are fitted to the empirically
extracted $\rho$-meson decay constant from the decay width in vacuum \cite{74,77} (see also Table 2). Namely, the light-quark (= light-antiquark) mass of $m_q = 0.430$ GeV, and two cases of the regulator masses, density independent case $m_R = 3.0$ GeV, and density dependent case, $m_R^* = (m_q^*/m_q)m_R$. Using these values, the $\rho$-meson decay constant in vacuum obtained is $f_\rho = 153.657$ MeV, close to the empirical value of $152 \pm 8$ MeV \cite{74}. Although we have no clue for the in-medium regulator mass $m_R^*$, or its density dependence, as a first trial we assume the density dependence $m_R^* = (m_q^*/m_q)m_R$, the same as that of the in-medium light quark constituent quark mass, which may be regarded as natural. We remind that, to calculate the in-medium decay constant $f_\rho^*$, we use the in-medium modified polarization vector $\epsilon_{\lambda=z}^{\mu}$ (but $\lambda = z$ is unmodified in medium), $\rho$-meson effective mass $m_\rho^*$, and effective quark mass $m_q^*$ in evaluating the both sides of Eq. (19).

We show in Fig. 3 (left panel) the density dependence of the $\rho$-meson decay constant in medium $f_\rho^*$, calculated using the density independent regulator mass $m_R = 3.0$ GeV, the same as applied in Ref. \cite{59}. While in the right panel we show the result obtained with using the density dependent regulator mass, $m_R^* = (m_q^*/m_q)m_R$. The solid lines are those interpolated to be able to see easier the density dependence. The density dependence of $f_\rho^*$ with the density independent $m_R$ is not smooth compared to that of the binding energy $B^*$ in Fig. 2 (upper panel). As we will show later, this feature is also noticeable compared with the density dependence of the $\rho$-meson electromagnetic form factors and the square charge radius, which have smooth density dependence. On the other hand, the density dependence of $f_\rho^*$ calculated with $m_R^* = (m_q^*/m_q)m_R$ shows smoother density dependence. Furthermore, the trends of the density dependence of the electromagnetic form factors etc., are similar for the results calculated with the two different regulator mass treatments, $m_R$ fixed, and $m_R^* = (m_q^*/m_q)m_R$. In this respect, we may be somehow safe in our predictions for those physical quantities to be discussed later.

Using the two different density dependent $f_\rho^*$, we calculate the $\rho^0$-meson decay width to $e^+e^-$, with the formula,

$$\Gamma^*(\rho^0 \rightarrow e^+e^-) = \frac{4\pi \alpha_e^2}{3 m_\rho^* f_\rho^*} f_\rho^*,$$

(24)

where $\alpha_e$ is the electromagnetic fine structure constant. Note that, the decrease (increase) of $f_\rho^*$ as increasing the nuclear matter density is the similar (opposite) behavior to that of the pion decay constant $f_\pi^*$ \cite{62}, which decreases in both the space component and the time component \cite{78} as increasing the nuclear matter density.

In Fig. 4 we show the density dependence of the decay width ratio to the vacuum, $\Gamma^*(\rho^0 \rightarrow e^+e^-)/\Gamma(\rho^0 \rightarrow e^+e^-)$, with the density independent (left panel) and density dependent (right panel) regulator mass, where $\Gamma(\rho^0 \rightarrow$
Fig. 3. Density dependence of the $\rho$-meson decay constant $f_{\rho}^*$ calculated with the

Fig. 4. Density dependence of the decay width ratios $\Gamma^*(\rho^0 \to e^+ e^-)/\Gamma(\rho^0 \to e^+ e^-)$, calculated with the fixed $m_{R}$ (left panel), and with $m_{R}^* = (m_{q}/m_{q}) m_{R}$ (right panel), where in vacuum $\Gamma(\rho^0 \to e^+ e^-) = 7.04 \pm 0.06$ keV \cite{74}.

$e^+ e^- = 7.04 \pm 0.06$ keV \cite{74} in vacuum.

The results for the in-medium decay width, $\rho^0 \to e^+ e^-$, show opposite density dependence, increase (left panel) and decrease (right panel) as increasing the nuclear matter density, reflecting respectively the density dependence of $f_{\rho}^*$ (left panel) and (right panel) directly shown in Fig. 3. This density dependence of the $\rho^0 \to e^+ e^-$ in nuclear medium (in nuclei) may be indirectly observed in experiment, and can give important information to draw a more solid conclusion on the density dependence of $f_{\rho}^*$.

We note that, the behavior of increasing $f_{\rho}^*$ with the increase of the square charge radius $<r_{\rho}^*^2>$ which will be shown later in Fig. 6 may be consistent with the correlation observed in vacuum in Ref. \cite{41}. Thus, we are in a difficult position to draw any conclusions on the density dependence of $f_{\rho}^*$ at this moment. We need to wait some useful information from experiment.

Now we discuss the main results of this article, $\rho$-meson electromagnetic form factors in symmetric nuclear matter. As we mentioned already the results
In Fig. 5 we show the $\rho$-meson charge $|G_0|$ (upper-left panel), magnetic $G_1$ (upper-right panel), and quadrupole $G_2$ (lower-left panel) form factors for several densities, and the $Q^2_{\text{zero}}$ for $G_0(Q^2_{\text{zero}}) = 0$ versus $m^*_\rho$ (lower-right panel) in symmetric matter.

given below are calculated with the density dependent regulator mass, $m^*_R = (m^*_q/m_q)m_R$.

In Fig. 5 we show the $\rho$-meson charge $|G_0(Q^2)|$ (upper-left panel), magnetic $G_1(Q^2)$ (upper-right panel), quadrupole $G_2(Q^2)$ (lower-left panel) form factors for several nuclear matter densities, and the zero, $Q^2_{\text{zero}} = -q^2_{\text{zero}}$, to give $G_0(Q^2_{\text{zero}}) = 0$ for the charge form factor, versus the effective $\rho$-meson mass $m^*_\rho$ (lower-right panel). For the zero, $Q^2_{\text{zero}}$, we also show that the result obtained with the density independent $m_R$.

The $\rho$-meson electromagnetic form factors in symmetric nuclear matter $G^*_0$, $G^*_1$ and $G^*_2$ are strongly modified as increasing the nuclear matter density. The modification of $|G_0^*|$ shows two distinct features: (i) faster decrease near $Q^2 = 0$ relative to that in vacuum, which implies the increase of the charge radius in symmetric nuclear matter, and (ii) the position of $Q^2_{\text{zero}}$ decreases as increasing the nuclear matter density almost linearly for the two cases of the regulator mass, the density dependent and independent. The present model has $Q^2_{\text{zero}} \simeq 3$ GeV$^2$ in vacuum [35], where in the literature the values in vacuum are spread in the region, $3$ GeV$^2 < Q^2_{\text{zero}} < 5$ GeV$^2$ [28,36,37,42,46,49,75].
The fact that $G_0$ has the zero, is similar to the spin-one deuteron case, a composite system of spin-1/2 particles of proton and neutron \cite{73,79,80}.

The $Q^2_{\text{zero}}$ versus $m_\rho^2$ shown in the lower-right panel in Fig. 5 is very interesting and may be noticeable. The use of the two different regulator mass $m_R$ and $m_R^* = (m_\rho^*/m_q)m_R$, both show the nearly linear dependence.

For the $\rho$-meson magnetic moment in medium $\mu_\rho^* = G_1^*(0)$, the medium modification is the very small quenching (see Table 2). In vacuum, $\mu_\rho = 2.20$ in the present model \cite{81} (and $\mu_\rho = 2.16$ in Ref. \cite{81}), while $\mu_\rho^* = 2.10$ at $\rho_N/\rho_0 = 0.90$. This is in contrast with the nucleon case, which has been demonstrated to be enhanced as increasing the nuclear matter density \cite{24,82}. This is probably due to the difference in the Lorentz structure between the spin-1 and spin-1/2 particles (see Eqs. (7) and (21)).

The $\rho$-meson quadrupole moment $Q_{2\rho}$ calculated via Eq. (22), is also sensitive to the effects of the nuclear medium (see Table 2). The quadrupole form factor $G_2^*$ shown in Fig. 5 (lower-left panel), changes behavior from quenching to enhancement for the densities larger than $\rho/\rho_0 = 0.75$. The enhancement of the quadrupole moment $Q_{2\rho}^*$ as increasing the nuclear matter density (see Table 2), means that the $\rho$-meson electromagnetic charge distribution deviates more from spherical symmetry in symmetric nuclear matter.

Next, we discuss the $\rho$-meson electromagnetic square charge radius in symmetric nuclear matter $< r_{\rho}^* >$ calculated with Eq. (23). The results are shown in Fig. 6 versus nuclear matter density (upper panel), versus the effective quark mass $m_q^*$ (lower-left panel), and versus the effective $\rho$-meson mass $m_\rho^*$ (lower-right panel). These are shown for the two cases, with the density independent (filled circles) and density dependent (filled squares) regulator mass. The solid line in each panel is a line obtained by fitting to the points calculated, for helping to see easier.

Three features shown in Fig. 6 for $< r_{\rho}^* >$ can be understood as follows. As the nuclear matter density increases, the charge form factor $G_0^*$ decreases faster near around $Q^2 = 0$ as shown in Fig. 5 upper-left panel, and the derivative with respective to $Q^2$ becomes negatively larger at $Q^2 = 0$ (see Eq. (23)) to yield larger $< r_{\rho}^* >$. As increasing the nuclear matter density, both $m_q^*$ and $m_\rho^*$ decrease as shown in Fig. 1 lower-left and lower-right panels, respectively. Furthermore, as the nuclear matter density increases, $m_q^*$ and $m_\rho^*$ as well as the binding energy $B^*$ decrease as shown in Fig. 2. The decrease in the binding energy yields a looser bound state, thus resulting in the increase of $< r_{\rho}^* >$. 
Fig. 6. \( \rho \)-meson electromagnetic square charge radius as well as the fitted curves, versus nuclear matter density \([\rho_N/\rho_0]\) (upper panel), versus effective quark mass \(m_q^*\) (lower-left panel), and versus effective \(\rho\)-meson mass \(m_\rho^*\). The filled circles and filled squares are respectively the results obtained with the density independent and density dependent regulator mass.

5 Summary and conclusion

We have studied the \( \rho \)-meson electromagnetic properties in symmetric nuclear matter with a light-front constituent quark model using the in-medium inputs calculated by the quark-meson coupling model. Similar approach was already applied in the studies of the pion, kaon, and nucleon properties in symmetric nuclear matter.

In this study we have applied a density dependent regulator mass, while in our previous study we used the density independent regulator mass. The results obtained in this extended study predict the similar density dependence of the \( \rho \)-meson electromagnetic properties, except for the density dependence of the \( \rho \)-meson electromagnetic decay constant. The present result shows the decrease of the \( \rho \)-meson electromagnetic decay constant, opposite behavior to the initial study obtained with the density independent regulator mass. Thus we need to wait relevant experimental data from which we can draw a more
definite conclusion on the density dependence of the $\rho$-meson decay constant.

Except for the $\rho$-meson decay constant, we first predict the $\rho$-meson electric, magnetic, and quadrupole form factors to vary faster in symmetric nuclear matter as increasing the nuclear matter density than those in vacuum, versus the (negative of) four-momentum transfer squared.

Second, we predict that, as increasing the nuclear matter density, the $\rho$-meson charge radius and modulus of the quadrupole moment increase (enhanced), while the magnetic moment is slightly quenched. The quenching of the magnetic moment shows the opposite behavior compared with that of the spin-1/2 Dirac particle (known to be enhanced), probably due to the difference in the Lorentz structure. The enhancement of the quadrupole moment in symmetric nuclear matter means that the $\rho$-meson charge distribution is more deviate from a spherical symmetric distribution in symmetric nuclear matter. Furthermore, we predict the value of “zero”, the value of the (negative of) four-momentum transfer squared to cross zero of the $\rho$-meson charge form factor (positive value), becomes smaller as increasing the nuclear matter density. This feature is also the same as that obtained with the use of the density independent regulator mass.

Although the present situation does not allow us to have many experimental data in vacuum as well as in nuclear medium, we hope further advances in experiments will provide us with more relevant data on the $\rho$-meson properties in vacuum and in a nuclear medium (a nucleus). In particular information on the density dependence of the $\rho$-meson electromagnetic decay constant, such as the corresponding decay width in-medium even though indirect manner, may be very useful.

For a future prospect, we need to study different scheme of the regularization, and/or the density dependence of the regulator mass, and also plan to extend the similar approach to study the in-medium properties of $K^-$, $D^-$, $K^{*-}$, $D^{*-}$ and $B$-mesons.

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