Counting $4\pi$’s in Strongly Coupled Supersymmetry

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Abstract

We extend the “naïve dimensional analysis” arguments used in QCD for estimating the strengths of operators in chiral Lagrangians to strongly coupled supersymmetric theories. In particular, we show how to count factors of $4\pi$—an inexact science, but nevertheless a useful art when such theories are used to model real particle physics.
1 Introduction

Recently there has been a surge of interest in constructing models of new physics in terms of strongly coupled supersymmetry (SUSY). These strong interactions typically produce dynamical SUSY breaking, composite quarks and leptons, or both \cite{1,2}. The low energy descriptions of such models inevitably involve an effective field theory, an expansion in local operators with unknown coefficients. Discussions of the phenomenology of such models require estimates of the sizes of the operator coefficients which control parameters of direct experimental interest, such as squark and gaugino masses, CKM mixing angles, etc. Usually we are thwarted in this endeavor by our ignorance of underlying strong dynamics; nevertheless estimates can be made on the basis of dimensional analysis and $4\pi$ counting. In refs. \cite{2,3}, we exploited a dimensional analysis scheme generalized from QCD; in this note we make our analysis explicit—the analysis itself is model independent.

The sizable mass gap between the pions and all other hadronic states in QCD leads to a profitable analysis of low energy hadronic physics in terms of an effective field theory, the chiral Lagrangian. Like all effective field theories, the chiral Lagrangian is constructed as an expansion in local operators constrained by low energy symmetries; each operator is multiplied by a characteristic mass scale to an appropriate power, times a dimensionless coefficient. In order to estimate the effect of operators neglected in a calculation, it is useful to have a method for estimating the sizes of these dimensionless coefficients. Such a method was introduced by Weinberg \cite{4} and discussed in detail in ref. \cite{5}. The method is predicated on the assumption that an effective field theory of strongly interacting fields has operator coefficients such that radiative corrections are no larger than $O(1)$ times tree level coefficients. Assuming that the radiative corrections are in fact of the same size as tree level coefficients leads to “naïve dimensional analysis” (NDA) estimates for the size of interactions.

We begin by explaining the power counting arguments for conventional field theories in a manner which differs somewhat from that in the literature, using the language of the Wilsonian renormalization group. We also discuss operator matching and the inclusion of light, weakly coupled fields. We then turn to the supersymmetric generalization. In our conclusions we discuss some of the assumptions behind NDA estimates.

1.1 Naïve dimensional analysis

We begin by assuming that we have some strongly interacting theory that we would like to match onto an effective theory at a scale $\Lambda$. The effective action which describes the interactions of any massless scalar fields $\Phi$ and fermion fields $\Psi$ in a derivative expansion is assumed to be characterized by a single dimensionful scale $\mu^2$. In a non-

\footnote{This assumption could conceivably be wrong, \textit{e.g.} for strongly coupled theories which are near an infrared fixed point, such as “walking technicolor” \cite{6}. Also, we are neglecting the possibility of...}
supersymmetric theory, the $\Phi$ fields will be massless only if they are Goldstone bosons, while the fermions can be protected from acquiring mass by chiral symmetry. The effective theory is described in terms of a Wilsonian effective action at the scale $\Lambda$:

$$S_\Lambda = \frac{1}{g^2} \int d^4x \Lambda^4 \hat{\mathcal{L}}_{\Lambda} \left( \frac{\Phi'}{\Lambda}, \frac{\Psi'}{\Lambda^{3/2}}, \frac{\partial}{\Lambda} \right),$$

(1.1)

where $g$ is a dimensionless parameter which we will determine. Terms in $\hat{\mathcal{L}}_{\Lambda}$ are assumed to have $O(1)$ coefficients, and the scale $\Lambda$ is the matching scale between the UV and IR descriptions of the theory, or equivalently, the mass scale of degrees of freedom that have been integrated out. (The primes on the fields $\Phi'$, $\Psi'$ are a reminder that kinetic terms may not have a canonical normalization in this basis.) Upon integrating out modes in the momentum shell $[e^{-1}\Lambda, \Lambda]$, the operator coefficients in $\mathcal{L}$ will flow to new values. Contributions to the operator coefficients of the effective action at $L$ loops will be of characteristic size

$$\frac{1}{g^2} \left( \frac{g^2}{16\pi^2} \right)^L$$

(1.2)

(see ref. [7] for example) where we have included a factor of $1/16\pi^2$ for each loop. We will have a “natural” theory if these renormalizations are no larger than tree level, which requires $g \lesssim 4\pi$; the NDA assumption is that this inequality is saturated,

$$g \sim 4\pi.$$  

(1.3)

If $g$ were to be smaller, we would assign it to weak rather than strong coupling. We will assume that eq. (1.3) holds throughout this paper, and examine this assumption in our conclusions.

We may rescale the fields to recast the effective action (1.1) into a form with conventionally normalized kinetic terms:

$$\Phi' = g\Phi, \quad \Psi' = g\Psi$$

(1.4)

so that the effective action (1.1) becomes

$$S_\Lambda = \int d^4x \frac{\Lambda^4}{g^2} \hat{\mathcal{L}}_{\Lambda} \left( \frac{g\Phi}{\Lambda}, \frac{g\Psi}{\Lambda^{3/2}}, \frac{\partial}{\Lambda} \right).$$

(1.5)

The above results may be compared with discussions of power counting in the chiral Lagrangian by using the correspondence

$$\Lambda \rightarrow \Lambda_\chi, \quad \frac{\Lambda}{g} \rightarrow f_\pi$$

(1.6)

small dimensionless numbers such as $1/N_c$.

\[2\] We are concerned only with factors of coupling constants and $4\pi$’s; consequently this normalization may differ from a truly conventional one by factors of order 1.
where $\Lambda$ is called the chiral symmetry breaking scale, and $f_\pi$ is the pion decay constant. Eq. (1.5) now reads

$$S = \int \! d^4x \, \Lambda^2 f_\pi^2 \hat{L}_\Lambda \left( \frac{\pi^a}{f_\pi}, \frac{\Psi}{\Lambda f_\pi^2}, \frac{\partial}{\Lambda} \right),$$

where $\pi^a$ are the pions; $\Psi$ might be light fermions such as chiral quarks, or heavy nucleons for which there are sources. The result (1.7) agrees with the results of ref. [5].

### 1.2 Weakly coupled light fields

Besides the pions, the low energy theory may contain light, weakly interacting fields, such as the photon. For the photon we are guided by gauge invariance, and make the replacement $\partial \to (\partial - ieA)$ in the action (1.5):

$$S_\Lambda = \int \! d^4x \, \frac{\Lambda^4}{g^2} \hat{L}_\Lambda \left( \frac{\Phi'}{\Lambda}, \frac{\Psi'}{\Lambda^{3/2}}, \frac{eA}{\Lambda}, \frac{\partial}{\Lambda} \right) + \mathcal{L}_w(A).$$

Note that we must include an independent Lagrangian to account for interactions among the weakly coupled fields (e.g., the kinetic term for the photons). For a weakly coupled field, the factor of $g \simeq 4\pi$ is replaced by a perturbative coupling, $e$ in this case. This suggests a more natural form for (1.8):

$$S_\Lambda = \int \! d^4x \, \frac{\Lambda^4}{g^2} \left[ \hat{L}_\Lambda \left( \frac{\Phi'}{\Lambda}, \frac{\Psi'}{\Lambda^{3/2}}, \frac{\hat{e}A'}{\Lambda}, \frac{\partial}{\Lambda} \right) + \frac{1}{\hat{e}^2} \hat{L}_w \left( \frac{\hat{e}A'}{\Lambda} \right) \right].$$

where $A' = gA$ and $\hat{e} \equiv e/g \simeq e/(4\pi)$. Weak coupling now corresponds to $\hat{e} \ll 1$.

While the form of photon interactions is dictated by gauge invariance, the power counting is clearly the same for any weakly coupled field. While strongly coupled fields appear in the combination $g\Phi/\Lambda \simeq 4\pi\Phi/\Lambda$, a weakly coupled field $\phi$ appears as $\hat{e}g\phi/\Lambda$, that is with an extra factor of the weak coupling $\hat{e}$. This procedure works for nonrenormalizable interactions as well: a weakly interacting field which couples to $m$ strongly interacting fields via a dimension $4+n$ operator with coefficient $(4\pi)^{(m-1)}/M^n$ also appears as $\hat{e}g\phi/\Lambda$ in the effective theory, with a dimensionless weak coupling of

$$\hat{e} \to \Lambda^n/M^n.$$  

### 1.3 Matching operators

In QCD one needs to match operators involving quarks and gluons (e.g., four quark operators from the weak interactions, $GG, GG\tilde{G},$ etc.) onto operators in the effective theory. In order to estimate the size of these operators in the effective theory, we
continue in the spirit of naïve dimensional analysis and assume that extra loops in a diagram at the matching scale do not change the characteristic size of an amplitude in both the UV and the IR descriptions of the theory, meaning that the strongly coupled particles in either description couple with strength $g \sim 4\pi$. The computation is simplest in the primed normalization of Eq. (1.1)—operators constructed out of quark and gluon fields $q', G'$ match onto operators with the same symmetry properties constructed out of composite fields $\Phi', \Psi'$ with dimensions matched by powers of $\Lambda$, and no dimensionless coefficients other than $O(1)$. For example, consider how various QCD operators constructed of quarks and gluons map onto operators in the chiral Lagrangian constructed of pions $\pi'$ or nucleons $N'$:

\begin{align*}
\bar{q}' q' &\rightarrow c_1 \Lambda \pi' \pi' + c_2 \bar{N}' N' + \cdots , \\
(\bar{u}'_L \gamma^\mu d'_L)(\bar{d}'_L \gamma_\mu u'_L) &\rightarrow c_3 \Lambda^2 \partial_\mu \pi' \partial^\mu \pi' + c_4 \Lambda^3 \bar{N}' N' + \cdots \\
G'G' &\rightarrow c_5 \partial \pi' \partial \pi' + c_6 \Lambda \bar{N}' N' + \cdots .
\end{align*}

(1.11)

where the dots represent all other operators consistent with the symmetries, and the $c$'s are dimensionless numbers of order one. To express this mapping in terms of conventionally normalized fields, we need only rescale all of the fields by a power of $g \approx 4\pi$:

\begin{align*}
\bar{q} q &\rightarrow c_1 \Lambda \pi \pi + c_2 \bar{N} N , \\
(\bar{u}_L \gamma^\mu d_L)(\bar{d}_L \gamma_\mu u_L) &\rightarrow c_3 \frac{\Lambda^2}{16\pi^2} \partial_\mu \pi \partial^\mu \pi + c_4 \frac{\Lambda^3}{16\pi^2} \bar{N} N + \cdots \\
GG &\rightarrow c_5 \partial \pi \partial \pi + c_6 \Lambda \bar{N} N + \cdots .
\end{align*}

(1.12)

In summary, matching an operator with $n$ strongly interacting fields in the UV to operators with $m$ composite fields in the IR entails the appropriate power of $\Lambda$ to match the dimensions and an explicit factor of $(4\pi)^{n-m}$. It should be noted that the $\Delta I = 1/2$ rule is a notorious failure of such power counting arguments, since some of the NDA estimates for the matching of weak four quark operators are off by a factor of $\sim 10$, except when analyzed within the context of the chiral quark model [8].

2 Naïve dimensional analysis for supersymmetric theories

The above analysis carries over to supersymmetric theories with little modification. The main difference is that one needs to extend the power counting scheme to include auxiliary $F$ and $D$ fields. The supersymmetric generalization of eq. (1.2) is (ignoring gauge interactions for now)

\begin{equation}
S_\Lambda = \frac{1}{g^2} \int d^4 x \, \Lambda^4 \left[ \int d^2 \theta \, d^2 \bar{\theta} \, \Lambda^{-2} \bar{K} + \int d^2 \theta \, \Lambda^{-1} \bar{W} + \int d^2 \bar{\theta} \, \Lambda^{-1} \bar{W}^* \right] .
\end{equation}

(2.1)
where the dimensionless Kähler potential $\hat{K}$ and superpotential $\hat{W}$ are functions constructed out of the dimensionless superfields

$$\frac{1}{\Lambda} \Phi'(x, \theta/\sqrt{\Lambda}) = \frac{1}{\Lambda} \left( A' + \frac{\theta \Psi'}{\sqrt{\Lambda}} + \frac{\theta^2 F'}{\Lambda} \right), \quad (2.2)$$

and supersymmetric derivative

$$\frac{D}{\Lambda} \sim \frac{\partial_\theta + i \bar{\theta} \sigma \cdot \partial_x}{\Lambda}, \quad (2.3)$$

with coefficients of $\mathcal{O}(1)$. To regain a canonical normalization, we make the substitution

$$\frac{1}{\Lambda} \Phi'(x, \theta/\sqrt{\Lambda}) = g \frac{1}{\Lambda} \Phi(x, \theta/\sqrt{\Lambda}), \quad (2.4)$$

with $g \simeq 4\pi$.

As in the non-SUSY case, weakly coupled chiral superfields $\phi$ interact with composite superfields in the combination

$$\hat{\lambda} \frac{g}{\Lambda} \phi(x, \theta/\sqrt{\Lambda}), \quad (2.5)$$

where $\hat{\lambda}$ is the weak coupling. Canonically normalized weak gauge superfields $V$ and spinor superfields $W_\alpha$ couple as

$$\hat{e} \frac{g}{\Lambda} V(x, \theta/\sqrt{\Lambda}) \quad \text{and} \quad \frac{\hat{e} g}{\Lambda^{3/2}} W_\alpha(x, \theta/\sqrt{\Lambda}), \quad (2.6)$$

where $e = \hat{e} g$ is a perturbative gauge coupling. For composite gauge superfields, such as occur in a free magnetic phase [9], the factor $\hat{e}$ is of order one at the scale $\Lambda$.

### 3 Examples

We give two examples of the power counting described above. The first example is an asymptotically free supersymmetric gauge theory of the “$s$-confining” type discussed in [10]. The second example is the Effective SUSY theory introduced in [3], which involves analysis of an effective action for which the UV description is unknown.

#### 3.1 A model with composites and a calculable superpotential

The first example we consider is a theory which in the UV is an $Sp(4)$ gauge theory with a single antisymmetric tensor field $A$ and six fundamental fields $Q$ [11]; it has much in common with the phenomenological models discussed in [2]. The theory
confines, and the composites relevant for the IR description are (with non-canonical normalization):

\[ T_2' = \frac{1}{8\Lambda} (A'A') = 1_{-6,0}, \quad M_0' = \frac{(Q'Q')}{\Lambda} = \mathbb{Z}_{2/3}, \quad M_1' = \frac{(Q' A' Q')}{\Lambda^2} = \mathbb{Z}_{-1/2/3} \]

(3.1)

where we have listed the quantum numbers of the moduli under the $SU(6) \times U(1) \times U(1)_R$ global symmetry of the model. The dynamically generated superpotential is

\[ W_{\text{dyn}} \propto \left( \frac{1}{3\Lambda} T_2'^2 M_0'^3 + \frac{1}{2} M_0' M_1'^2 \right). \]

(3.2)

The relative factor of the operators in (3.2) is fixed by requiring the correct constraints on the moduli.

### 3.1.1 The effective strong interactions

To rewrite $W_{\text{dyn}}$ in terms of canonically normalized fields, we perform the operator matching as in §1.3

\[ T_2 = \frac{1}{8} (AA) \left( \frac{4\pi}{\Lambda} \right), \quad M_0 = (QQ) \left( \frac{4\pi}{\Lambda} \right), \quad M_1 = (QAQ) \left( \frac{4\pi}{\Lambda} \right)^2, \]

(3.3)

Following the NDA prescription of Eqs. (2.1-2.4), the dynamically generated superpotential (3.2) takes the form

\[ W_{\text{dyn}} \simeq 4\pi \left( \frac{4\pi}{3\Lambda} T_2 M_0^3 + \frac{1}{2} M_0 M_1^2 \right). \]

(3.4)

In particular, note that the dynamically generated Yukawa coupling among the $M_0$ and $M_1$ component fields is of order $4\pi$.

The relative factor between the two terms, reflecting the constraints on the moduli, is preserved by this rescaling. However note that the kinetic terms for these fields (from the Kähler potential) may still contain unknown order one coefficients.

Kähler potential terms must be consistent with the global symmetries of the theory; this includes terms such as

\[ K = \sum_i a_i |\Phi_i|^2 + \left( \frac{1}{\Lambda} \right)^2 \sum_i c_i \Phi_i^* D^2 \Phi_i + \left( \frac{4\pi}{\Lambda} \right)^2 \sum_{ij} c_{ij} |\Phi_i|^2 |\Phi_j|^2 + \left( \frac{4\pi}{\Lambda} \right)^3 \bar{c} T_2 M_0^2 (M_1^*)^2 + \text{h.c.} + \ldots \]

(3.5)

where $\Phi_i = \{ T_2, M_0, M_1 \}$, and the $a_i, c, \bar{c}$ coefficients are $O(1)$. Note that while each new field brings with it a factor of $4\pi/\Lambda$, the momentum expansion is in powers of $p/\Lambda$. This is consistent with having integrated out heavy fields with masses $M = \Lambda$ and couplings $g = 4\pi$. 

6
3.1.2 Perturbative interactions and spurion analysis

We now consider how to construct the effective theory when a perturbative superpotential in the UV description of the theory is included. Following the discussion of Eqs. (2.1-2.4) we include superpotential perturbations of the form

$$W_{\text{pert}} = \epsilon_1 g^2 \Lambda^3 \hat{W}(\Phi_i/\Lambda),$$

(3.6)

with $g = 4\pi$. Weak coupling then corresponds to $\epsilon \ll 1$. As an example we may take

$$W_{\text{pert}} = \frac{1}{16\pi^2} \left[ \epsilon_1 \phi'(A'A') + \epsilon_2 (Q'A'Q') + \frac{\epsilon_3}{\Lambda^2} (Q'Q')(Q'A'Q') \right],$$

(3.7)

where $\phi'$ is a superfield which is neutral under the strong $Sp(4)$ gauge group, and for simplicity, we have suppressed $SU(6)$ indices. In terms of more conventionally normalized fields this is

$$W_{\text{pert}} = 4\pi \epsilon_1 \phi (AA) + 4\pi \epsilon_2 (QAQ) + \epsilon_3 \frac{(4\pi)^3}{\Lambda^2} (QQ)(QAQ).$$

(3.8)

The perturbative parameters $\epsilon_i$ may be treated as spurions, each carrying $SU(6) \times U(1) \times U(1)_R$ quantum numbers

$$\epsilon_1 \phi = 1_{6,2}, \quad \epsilon_2 = \mathbb{1}_{1,4/3}, \quad \epsilon_3 = \mathbb{1}_{-1,2/3}.$$

(3.9)

The quantum numbers of the spurions, along with holomorphy, constrain the induced superpotential to consist of only three terms. Using the operator mapping (3.3) we find

$$W_{\text{pert}}^{\text{eff}} \approx a_1 \epsilon_1 \Lambda \phi T_2 + a_2 \frac{\epsilon_2}{4\pi} \Lambda^2 M_1 + a_3 \epsilon_3 \Lambda M_0 M_1,$$

(3.10)

while the Kähler potential will include such terms as

$$K_{\text{pert}} = \sum_{ij} a_{ij} |\Phi_i|^2 |\epsilon_j|^2 + \left( \frac{4\pi}{\Lambda} \right)^2 \sum_i b_i |\Phi_i|^2 |\epsilon_1 \phi|^2 + \ldots$$

(3.11)

where $a, b$ are coefficients of order one.

Note that the superfields receive perturbative corrections to the leading term in the Kähler potential. We must avoid an $\epsilon$ dependent rescaling of the fields, however, if we wish to maintain holomorphy in the superpotential.

3.2 Scalar and Gaugino masses in strongly coupled theories with Supersymmetry Breaking

We now use our $4\pi$ counting scheme to analyze the superpartner masses in theories with a strongly coupled sector in which supersymmetry is broken. In order to
systematically discuss supersymmetry breaking effects, we assume that the supersymmetry breaking scale in the low energy effective theory is below the scale \( \Lambda \), so that supersymmetry is linearly realized in the low energy effective theory; this allows a weakly coupled description of supersymmetry breaking in terms of a nonzero vacuum expectation value for the \( F \)-term of some “composite” superfield \( \Phi \). Such theories have been considered in refs. \([1, 3, 12]\). We assume the mechanism for ordinary gaugino masses involves conventional \( SU(3) \times SU(2) \times U(1) \) gauge interactions, and that these interactions are weakly coupled at the matching scale. A generic theory where strongly interacting particles carrying \( SU(3) \times SU(2) \times U(1) \) interactions are integrated out at the scale \( \Lambda \) may induce terms in the low energy effective theory of the form

\[
\int d^2 \theta \left[ c \hat{e}^2 \frac{(\Phi')^n}{\Lambda^n} W_\alpha W^\alpha \right] F,
\]

where \( \hat{e} \) is an \( SU(3) \times SU(2) \times U(1) \) gauge coupling \( e \) divided by \( 4\pi \), \( W \) is an \( SU(3) \times SU(2) \times U(1) \) gauge spinor superfield, \( \Phi' = 4\pi \Phi \), and we expect the coefficient \( c \) to be of order one. If we have the correct degrees of freedom to describe the true ground state, then \( \langle \Phi' \rangle < \Lambda \) for all \( \Phi \). Thus the maximum size for the light gaugino masses \( \tilde{m}_i \) occurs when there is a term of the form eq. (3.12) with \( n = 1 \) for a composite field \( \Phi \) with a nonzero \( F \)-term, so that at the scale \( \Lambda \)

\[
\tilde{m}_{\text{gaugino}} = 4\pi c \frac{e^2}{16\pi^2} \frac{F_\Phi}{\Lambda}.
\]

For the squark and slepton masses, two options have been considered. The first “gauge mediated” \([13]\) solution has the squarks and sleptons communicate with the supersymmetry breaking sector only via ordinary gauge interactions. Then the squark and slepton masses squared will arise from terms which are induced only by graphs which involve at least one weak gauge loop (and an arbitrary number of strong loops). Integrating out strongly interacting fields results in couplings of quarks and leptons to the light composite fields: in general such loop effects can only appear in the Kähler potential and will be proportional to factors of \( \hat{e}^4 \phi^\dagger \phi' / \Lambda^2 \) where \( \phi \) is a quark or lepton superfield, and \( \phi' = 4\pi \phi \). Squark and slepton masses squared may be obtained from the induced operator

\[
\int d^4 \theta c' \frac{\hat{e}^4}{16\pi^2} \frac{\Phi'^\dagger \Phi'}{\Lambda^2} \phi^\dagger \phi' = \int d^4 \theta c' \left( \frac{e^2}{16\pi^2} \right)^2 \frac{16\pi^2 \Phi'^\dagger \Phi}{\Lambda^2} \phi^\dagger \phi ,
\]

where \( c' \) is of order one, leading to squark and slepton masses squared of order

\[
\tilde{m}_{sfermion}^2 = c' \left( \frac{4\pi}{16\pi^2} \frac{F_\Phi}{\Lambda} \right)^2.
\]

Thus the phenomenologically desirable relation that squark and gluino masses are comparable in magnitude is obtained.
Alternatively, one could follow the “effective supersymmetry” approach of ref. [3] and allow some of the quark and lepton superfields to be composites of strongly interacting fields. The natural size of the composite squark and slepton masses is then much larger than that of the $SU(3) \times SU(2) \times U(1)$ gauginos, and is of order

$$m_{\text{comp sfermion}}^2 \sim \left(4\pi \frac{F_F}{\Lambda}\right)^2,$$

(3.16)

which corresponds to ((3.14)) with $\hat{e} \to 1$. Such a theory with phenomenologically acceptable gaugino masses, $m_{\text{gaugino}} \gtrsim 100$ GeV, cannot give rise to natural electroweak symmetry breaking unless at least one Higgs superfield and its “brothers” (i.e., those superfields with $O(1)$ couplings to the Higgs), are elementary, weakly coupled fields. It is however possible in a natural theory to have the first two generations of quarks and leptons carry the strong supersymmetry breaking interactions. The corresponding squarks and sleptons will have masses larger than those of the other superpartners by a factor of $\sim 16\pi^2/e^2$ [3].

4 Conclusions

The simple arguments presented here allow the construction of a natural effective field theory with only minimal understanding of the underlying dynamics, provided the strong dynamics of the underlying theory is characterized by a single scale. We have given simple power counting rules for the factors of $4\pi$ in the coefficients of dynamically generated superpotentials and Kähler potentials in a strongly coupled supersymmetric theory. We have also given a simple algorithm for counting the $4\pi$ factors in operators involving both weakly coupled superfields and composite superfields, and in symmetry breaking “spurion” factors.

All these applications may be summarized in a simple and general principle, embodied in eqs. (1.8) and (2.1): when the theory is characterized by a single dimensionful parameter $\Lambda$, all dimensionless operator coefficients are naturally of order 1 in a normalization in which the effective action has an overall factor of $1/g^2 \approx 1/(16\pi^2)$. Provided this same normalization is used, weak couplings or spurions (that is, operators with coefficients parametrically smaller than $1/(16\pi^2)$) may be directly included according to the same rule, with the same parametric factor in the UV and IR. Matrix elements of operators follow the same rule: operators with no explicit factors of $4\pi$ match onto operators also with no explicit factors of $4\pi$.

What if the strongly coupled theory is characterized by more than one scale? If these scales are widely separated, a sequence of effective field theories following the above rules may still be constructed, starting with the highest scale first. When these

3 Note that the quantity which we called $\Lambda$ in ref. [3] is equivalent to what we are calling $\Lambda/g = \Lambda/(4\pi)$ in this work.
scales are not widely separated, or when the strong dynamics is spread out over a large energy range (such as near an approximate IR fixed point) NDA is likely to fail.

Why is the assumption $g \sim 4\pi$ reasonable? Generically we expect that, given a weakly coupled theory in the UV, as we scale down in energy irrelevant operators become weaker, while the effective couplings of relevant operators become stronger. When these couplings become of order $4\pi$, the theory is strongly coupled and will typically undergo a phase transition; we then construct the effective action in terms of new degrees of freedom, which will again be weakly coupled, with some relevant and some irrelevant operators. As we scale down in energy the process continues, until we reach a phase in which there are no relevant operators, and hence the theory remains weakly coupled. For example, in QCD the derivative coupling of the pions means all interactions are irrelevant, and the pions are weakly coupled at low energies. For the supersymmetric examples presented here the quartic and Yukawa couplings, marginal at tree level, become irrelevant when radiative corrections are included.

The simple power counting rules given here have not considered factors that may be associated with numbers of fields, for example the number of flavors $N_f$, the number of preons in a composite superfield, or the number of colors $N_c$—inclusion of such factors is straightforward, and should improve the accuracy of NDA.

As this paper was being completed, ref. [15] appeared which deals with similar issues.

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