Effect of magnetic order on the superfluid response of single-crystal ErNi$_2$B$_2$C: A penetration depth study

Elbert E. M. Chia, W. Cheong, Tuson Park, and M. B. Salamon
Department of Physics, University of Illinois at Urbana-Champaign,
1110 W. Green Street, Urbana Illinois 61801

Eun-Mi Choi and Sung-Ik Lee
National Creative Research Initiative Center for
Superconductivity and Department of Physics,
Pohang University of Science and Technology,
Pohang 790-784, Republic of Korea
(Dated: March 23, 2022)

Abstract

We report measurements of the in-plane magnetic penetration depth $\Delta \lambda (T)$ in single crystals of ErNi$_2$B$_2$C down to $\sim 0.1$ K using a tunnel-diode based, self-inductive technique at 21 MHz. We observe four features: (1) a slight dip in $\Delta \lambda (T)$ at the Néel temperature $T_N = 6.0$ K, (2) a peak at $T_{WFM} = 2.3$ K, where a weak ferromagnetic component sets in, (3) another maximum at 0.45 K, and (4) a final broad drop down to 0.1 K. Converting to superfluid density $\rho_s$, we see that the antiferromagnetic order at 6 K only slightly depresses superconductivity. We seek to explain some of the above features in the context of antiferromagnetic superconductors, where competition between the antiferromagnetic molecular field and spin fluctuation scattering determines increased or decreased pairbreaking. Superfluid density data show only a slight decrease in pair density in the vicinity of the 2.3 K feature, thus supporting other evidences against bulk ferromagnetism in this temperature range.
The magnetic members of the rare-earth (RE) nickel borocarbide family, RENi$_2$B$_2$C (RE = Ho, Er, Dy, etc) have attracted much interest due to the interplay between magnetism and superconductivity. ErNi$_2$B$_2$C, in particular, is a good candidate for study: superconductivity starts at $T_c \approx 11$ K, before antiferromagnetic (AF) order sets in at $T_N \approx 6$ K. In the AF state the Er spins are directed along the $b$-axis, forming a transversely polarized, incommensurate spin-density-wave (SDW) state, with modulation vector $\delta = 0.553a^* \ (a^* = 2\pi/a)$, before squaring up at lower temperatures$^3$. Below $T_{WFM}=2.3$ K a net magnetization appears, superposed on a modulation with a periodicity of $20a^*$, confirming the microscopic existence of spontaneous weak ferromagnetism (WFM) with superconductivity$^3$$^4$.

The fact that $T_N < T_c$ enables us to study the influence of magnetism on superconductivity. In particular, in this paper we study, via the penetration depth, the pairbreaking effects of the various magnetic orders on the superfluid response of this material. There have been several previous penetration depth measurements on ErNi$_2$B$_2$C. Jacobs et al.$^5$ measured the microwave surface impedance of single-crystal ErNi$_2$B$_2$C from $T_c$ down to 4 K, but did not see the AF transition at 6 K. They concluded that the AF transition is not accompanied by changes in pairbreaking in zero field. Andreone et al. measured the microwave properties of ErNi$_2$B$_2$C thin films — microwave surface resistance$^6$ down to 2 K, and the change in penetration depth$^7$ from 2–5 K. They too, saw no feature at $T_N$, and attributed its absence to the smearing of the susceptibility $\chi(T)$. Gammel et al. performed small-angle neutron measurements (SANS)$^8$ on single-crystal ErNi$_2$B$_2$C down to 4 K, finding a decrease of $\lambda$ below $T_N$ that they could not account for quantitatively. In this paper we present high-precision measurements of the in-plane magnetic penetration depth of single-crystal ErNi$_2$B$_2$C down to 0.1 K. We see features at $T_N$ and $T_{WFM}$, and ascribe these to the pairbreaking effects of AF order at $T_N = 6$ K and the weak ferromagnetic ordering at $T_{WFM} = 2.3$ K. We also observe a peak at 0.45 K, which we attribute to the presence of a spontaneous vortex phase$^9$ (SVP), expected to occur in superconductors where ferromagnetism and superconductivity coexist$^{10,11,12}$. The superfluid density graph indicates that these three magnetic orderings coexist with superconductivity, i.e. they do not destroy superconductivity in this material.

Various theories of antiferromagnetic superconductors have been proposed$^{13,14,15,16,17,18,19}$. We shall follow that of Chi and Nagi$^{19}$, which is an extension of the mean-field model by Nass et al.$^{17,18}$ to the regime where the superconducting gap $\Delta$ is finite, and it includes the effects
of spin-fluctuations, molecular field and impurities. In the Chi-Nagi (CN) model, which applies specifically to superconductors with $T_N < T_c$, two temperature regimes are separately considered. First, in the paramagnetic regime ($T_N < T < T_c$), the depression of $T_c$ with respect to the non-magnetic counterparts, LuNi$_2$B$_2$C or YNi$_2$B$_2$C, is due to the exchange scattering of the conduction electrons from the spins of the RE Er ions. Assuming that the exchange interaction is weak, this paramagnetic phase of ErNi$_2$B$_2$C can be accounted for by the Abrikosov-Gorkov (AG) pairbreaking theory. Second, in the AF phase ($T < T_N$), the effect of pairbreaking depends on the competition between the temperature-dependent AF molecular field (with parameter $H_Q(T)$) and spin-fluctuation scattering of the conduction electrons, the latter by both magnetic RE ions (parameter $1/\tau_2^{eff}$) and non-magnetic impurities (parameter $1/\tau_1$). The molecular field opens AF gaps on parts of the Fermi surface (FS), hence destroying the superconducting gap in those areas. The non-magnetic impurities do not affect the BCS state for an $s$-wave superconductor, but weaken the effect of the AF field by destroying the pairing state for charge density waves or spin density waves — thus non-magnetic impurities promote the recovery of superconductivity. Moreover, the effect of the molecular field and spin fluctuations is governed by a sum rule, and the competition between them determines whether the AF phase gives increased or decreased pairbreaking below $T_N$. The total electronic effective magnetic scattering rate $1/\tau_2^{eff}$ is temperature-dependent and decreases with decreasing temperature (as the magnetic moments become more and more frozen). The assumptions of the CN model are: (1) the effect of inelastic scattering, which is relevant only for $T \ll T_N$, can be ignored; (2) BCS $s$-wave pairing and (3) a one-dimensional (1-D) electron band that satisfies the nesting condition $\epsilon_k = -\epsilon_{k+Q}$.

The following equations of the CN model were used. The temperature-dependence of the superconducting gap is determined from

$$\ln \left( \frac{T_c}{T_{c0}} \right) = \psi \left( \frac{1}{2} \right) - \psi \left( \frac{1}{2} + \frac{1}{2\pi T_c \tau_2^{eff}} \right)$$

(AG equation) \hspace{1cm} (1)

$$\tilde{\omega}_{n\pm} = \omega_n + Y_{\pm} \frac{\tilde{\omega}_{n\pm}}{2\lambda_{\pm}} + X_{\pm} \frac{\tilde{\omega}_{n\pm}}{2\lambda_{\pm}}$$

(Renormalized frequency) \hspace{1cm} (2)

$$\tilde{\Delta}_{n\pm} = \Delta \pm H_Q(T) + X_{\pm} \frac{\tilde{\Delta}_{n\pm}}{2\lambda_{\pm}} + X_{\pm} \frac{\tilde{\Delta}_{n\pm}}{2\lambda_{\pm}}$$

(Renormalized gap) \hspace{1cm} (3)
(Gap equation) \[
\ln \frac{T}{T_{c0}} = \pi T \sum \left\{ \frac{1}{\Delta} \left[ \frac{1}{(U_{n+}^2 + 1)^{1/2}} + \frac{\text{sgn}(U_{n-})}{(U_{n-}^2 + 1)^{1/2}} \right] - \frac{2}{\omega_n} \right\}, \tag{4}
\]
where \(T_{c0}\) is the superconducting transition temperature of the non-magnetic member of the borocarbide family LuNi\(_2\)B\(_2\)C or YNi\(_2\)B\(_2\)C, \(\psi\) is the digamma function, \(X_\pm\) and \(Y_\pm\) are linear combinations of the magnetic \((1/\tau_{\text{eff}}^2)\), non-magnetic \((1/\tau_1)\) and spin-orbit \((1/\tau_{\text{so}})\), scattering rates

\[
Y_\pm = \frac{1}{2} \left( \frac{1}{\tau_1} + \frac{1}{\tau_{\text{eff}}^2} + \frac{1}{\tau_{\text{so}}} \right) \pm \frac{1}{2} \left( \frac{1}{\tau_1} + \frac{1}{3\tau_{\text{eff}}^2} + \frac{1}{3\tau_{\text{so}}} \right), \tag{5}
\]
\[
X_\pm = \frac{1}{2} \left( \frac{1}{\tau_1} - \frac{1}{\tau_{\text{eff}}^2} + \frac{1}{\tau_{\text{so}}} \right) \pm \frac{1}{2} \left( \frac{1}{\tau_1} + \frac{1}{3\tau_{\text{eff}}^2} - \frac{1}{3\tau_{\text{so}}} \right). \tag{6}
\]

\[\lambda_\pm = [\tilde{\omega}_{n\pm}^2 + \tilde{\Delta}_{n\pm}^2]^{1/2}; \ U_{n\pm} = \tilde{\omega}_{n\pm}/\tilde{\Delta}_{n\pm}; \ \omega_n = \pi T(2n+1)\] is the Matsubara frequency. In the paramagnetic phase the distinction between + and − is lost, giving \(\tilde{\omega}_{n\pm} \equiv \tilde{\omega}_n, \ \tilde{\Delta}_{n\pm} \equiv \tilde{\Delta}_n\) and so \(U_n = \tilde{\omega}_n/\tilde{\Delta}_n\).

The temperature-dependence of the superfluid density \(\rho_s\) is given by

\[
\rho_s(T) \equiv \left[ \frac{\lambda^2(0)}{\lambda^2(T)} \right]^2 = \left[ \pi T \sum_{n \geq 0} A(\omega_n) \right], \tag{7}
\]
where

\[
A(\omega_n) = \frac{\tilde{\Delta}_{n+}^2 - \tilde{\omega}_{n+}^2}{4\varepsilon_1^2} + \frac{\tilde{\Delta}_{n-}^2 - \tilde{\omega}_{n-}^2}{4\varepsilon_2^2} + \frac{1}{4\varepsilon_1} + \frac{1}{4\varepsilon_2} + \frac{1}{\varepsilon_1 + \varepsilon_2} + \frac{\tilde{\Delta}_{n+} \tilde{\Delta}_{n-} - \tilde{\omega}_{n+} \tilde{\omega}_{n-}}{\varepsilon_1 \varepsilon_2 (\varepsilon_1 + \varepsilon_2)}, \tag{8}
\]
\[
\varepsilon_1 = |(\tilde{\omega}_{n+}^2 + \tilde{\Delta}_{n+}^2)^{1/2}|, \ \varepsilon_2 = |(\tilde{\omega}_{n-}^2 + \tilde{\Delta}_{n-}^2)^{1/2}|. \tag{9}
\]

In the paramagnetic phase \((T_N < T < T_c)\), \(\rho_s\) is given by \((\text{P: paramagnetic})\)

\[
\rho_s^P(T) = \left[ 2\pi T \sum_{n \geq 0} \frac{1}{\varepsilon(1 + U_n^2)} \right], \tag{10}
\]
where \(\varepsilon = |(\tilde{\omega}_{n}^2 + \tilde{\Delta}_{n}^2)^{1/2}|\). Note that this expression for the superfluid density is for materials in the dirty limit. This is consistent with the Er ions in ErNi\(_2\)B\(_2\)C being the "impurity" ion when compared to either LuNi\(_2\)B\(_2\)C or YNi\(_2\)B\(_2\)C.
We turn next to the parameters of the model. The effective magnetic scattering rate 
\(1/\tau_{\text{eff}}^2\) from RE ions \((1/\tau_R^2)\) and magnetic impurities \((1/\tau_i^2)\) is given by

\[
\frac{1}{\tau_{\text{eff}}^2} = \begin{cases} 
\frac{1}{\tau_i^2} + \frac{1}{\tau_R^2} & (T > T_N) \\
\frac{1}{\tau_i^2} + \frac{1}{\tau_R^2}(1 - F^2(T)) & (T \leq T_N) 
\end{cases},
\]

(11)

where \(\frac{1}{\tau_R^2} = 2\pi n_R N(0) J(J+1)(g_J-1)^2 I^2\).

(12)

The AF molecular field is given by

\[H_Q(T) = H_Q(0) F(T),\]

(13)

where \(H_Q(0) = n_R I \left| g_J - 1 \right| \sqrt{J(J+1)}\).

(14)

\(n_R\) is the concentration of RE ions, \(I\) is the exchange energy, \(g_J\) is the Landé factor, and \(J\) is the total angular momentum of the RE ion. The function \(F(T)\) can be approximated by the empirical relation

\[F(T) = 1 - \left( \frac{T}{T_N} \right)^\nu,\]

(15)

where \(\nu\) is a parameter obtained by fitting \(F(T)\) to sublattice magnetization data.

The values of the renormalized frequencies \(\tilde{\omega}_{n\pm}\) and gaps \(\tilde{\Delta}_{n\pm}\) are determined self-consistently: for a fixed temperature \(T\) and Matsubara index \(n\), one determines \(\tilde{\omega}_{n\pm}\) and \(\tilde{\Delta}_{n\pm}\) from Equations 2 and 3 such that they also satisfy Eqn. 4. After computing the \(\tilde{\omega}_{n\pm}\)’s and \(\tilde{\Delta}_{n\pm}\)’s for a fixed \(T\), one then substitutes these values into Eqn. 7 or 10 to obtain the superfluid density \(\rho_s\) at that temperature \(T\).

Details of sample growth and characterization are described in Ref. 1. The samples were then annealed according to conditions described in Ref. 24. The superconducting transitions of our sample were measured by low-field \((H=5 \text{ G})\) magnetization, zero-field resistivity and zero-field specific-heat measurements. From magnetization data, the onset of superconducting diamagnetism appears at \(T=11.0 \text{ K}\) and 90% of the full diamagnetic magnetization is reached at \(T=9.6 \text{ K}\). Resistivity data show a superconducting onset at a higher temperature of 11.3 K and zero resistivity at 9.6 K. The mid-point of the specific-heat jump\(^\text{25}\) yields a \(T_c\) of 10.1 K. A comparison of the three measurements show that bulk superconductivity occurs at \(T_c \approx 10 \text{ K}\), whereas the initial decrease of resistivity at \(\sim 11 \text{ K}\) may be due to some sort of filamentary superconductivity.
The parameters of this model are determined as follows. We denote $\Delta_0$ and $\Delta(0)$ to be the zero-temperature superconducting gap amplitude of YNi$_2$B$_2$C and ErNi$_2$B$_2$C, respectively. Tunneling measurements$^{26}$ yield $\Delta_0 = 1.83T_{c0}$. From the experimental values of $T_c$ (10.1 K for ErNi$_2$B$_2$C and $T_{c0}$ (15.5 K for YNi$_2$B$_2$C), Eqn. 1 gives $1/\tau^{eff}_2 \Delta_0 = 0.227$. We assume $1/\tau^i_2 = 0$, in which case Eqn. 11 gives $1/\tau^{R}_2 \Delta_0 = 0.227$. For ErNi$_2$B$_2$C, using the values $n_R=1/6$, $J=7.5$, $g_J=1.2$, $N(0)=0.36$ states/eV-atom-spin$^{27}$, we obtain $I=0.024$ eV from Eqn. 12 which is comparable with the experimental value of 0.031 eV$^{28}$. This justifies our assuming $1/\tau^i_2 = 0$, as any finite $\tau^i_2$ would make $I$ even smaller than the experimental value. Equation 14 then gives $H_Q(0)/\Delta_0 = 2.6$. From the temperature-dependence of the magnetic Bragg peak intensity$^{3,29}$ below $T_N$ we obtain $\nu = 4.8$ in Eqn. 15. The only remaining free parameter of the theory is $1/\tau$, the non-magnetic scattering rate, defined to be $1/\tau = 1/\tau_1 + 2/3\tau_{so}$. In an AFSC, usually $1/\tau \gg 1/\tau_{so}$, so for the present study we assume $1/\tau_{so} = 0$, such that $1/\tau = 1/\tau_1$.

To see the pairbreaking effects of the various magnetic orders we need to convert $\Delta\lambda(T)$ to $\rho_s(T)$, the superfluid density. To determine $\rho_s(T)$ we need the value of $\lambda(0)$, which has been reported over a range from 700 Å to 1150 Å. We take $\lambda(0)$ to be a parameter in our model, keeping in mind that it has to be in the vicinity of the above two values.

Figure 1 shows the temperature-dependence of the in-plane penetration depth $\Delta\lambda(T)$. The onset of superconductivity, $T^*_c$, is 11.3 K, showing that this is a high-quality single crystal. We also see the following features: (1) a slight dip in $\Delta\lambda(T)$ at $T_N=6.0$ K, (2) a peak at $T_{WF}=2.3$ K, (3) another maximum at 0.45 K, and (4) a final broad drop down to 0.1 K. We attribute the last two features to the presence of the SVP. For dirty AF superconductors, the penetration depth is expected to decrease below $T_N$ by both$^{30}$ the susceptibility ($\chi$) and mean free path ($l$) as $\lambda \sim \lambda'_L/\sqrt{1 + 4\pi\chi}$, where $\lambda'_L \approx \lambda_L(1 + \xi_0/l)$. Neither effect, however, explains our data: First, using the mean-field expression for $\chi$, in order to reproduce the experimental dip, the peak in $\chi$ at $T_c$ has to be at least an order of magnitude larger than that suggested by magnetization measurements. Second, from our resistivity data we obtained $H_{c2}(T)$, and hence we calculated $\xi_0(T)$, $l(T)$, and lastly, $\lambda'_L$. Our values of $\lambda'_L$ also are unable to explain the magnitude of the drop of $\lambda$ below $T_N$ — a conclusion also shared by Gammel et al. from their SANS data$^8$.

Since the CN model does not take into account the effect of the SVP on the superfluid density, we neglect it when we convert to superfluid density $\rho_s$: First we assume $\Delta\lambda(T)$
The superfluid data leads to some important observations. First, the data in the paramagnetic phase \( T > T_N \) fit the theoretical curve based on an isotropic superconducting gap (solid squares), and not that based on nodes. The solid line shows a superfluid calculation based on a \( d_{x^2-y^2} \) order parameter. Second, the superconductivity is only slightly depressed in the AF phase below \( T_N \). The best fit to data above \( T_N \) (solid squares) is obtained when
FIG. 2: (○) Experimental superfluid density $\rho_s(T) = [\lambda^2(T)/\lambda^2(0)]$ calculated from $\Delta \lambda(T)$ data in Fig. 1 from 0.1 K to $T_c$. Solid squares = Calculated $\rho_s(T)$ assuming paramagnetic phase from $T=0$ to $T_c$. Note that $1/\tau \Delta_0=24$, 26 or 28 give virtually the same theoretical curve. Solid line: Theoretical curve for $d_{x^2-y^2}$ order parameter. The arrow denotes Néel temperature at 6 K.

$\lambda(0) = 740 \text{ Å} —$ here we assume paramagnetism from $T=0$ to $T_c$, neglecting AF order, with parameter $1/\tau \Delta_0 = 24$. The paramagnetic curve is almost unchanged if one uses $1/\tau \Delta_0=26$ or 28. We see that the paramagnetic curve fits the data above $T_N$, and overestimates the data in the AF phase. We will show below that, as one crosses $T_N$ from above, the AF phase leads to increased or decreased superfluid density depending on the combined effects of the following three factors: (1) the AF molecular field, which decreases the magnitude of the superconducting gap and hence decreases the superfluid density, (2) freezing out of spin fluctuations, leading to decreased pairbreaking and hence increased superfluid density, and
FIG. 3: Chi-Nagi model calculation for the superconducting gap $\Delta$. Solid line = BCS temperature-dependence. (\(\bigcirc\)) Paramagnetic gap. Solid squares = Incorporating AF phase below $T_N$, for different values of $1/\tau\Delta_0$. Note that when $1/\tau\Delta_0 = 28$, superconductivity is fully recovered.

(3) scattering from non-magnetic impurities, which reduces the suppression of the gap by the molecular field. The parameters of ErNi$_2$B$_2$C are such that these three effects result in a slight decrease in superfluid density below $T_N$.

Fig. 3 shows the calculated superconducting gap amplitude $\Delta(T)$, in the presence of $H_Q(T)$, for various values of $1/\tau\Delta_0$, as well as the paramagnetic curve. The normalized paramagnetic gap (open circles) agrees excellently with the BCS gap (solid line). Tunneling measurements$^{31}$ also show that $\Delta(T)$ follows the BCS curve above $T_N$. Next, as shown in Fig. 3 in the AF phase, as $1/\tau\Delta_0$ increases, superconductivity is gradually recovered, as evidenced by the increase of $\Delta(T)$ (solid squares). Fig. 4 shows $\rho_s(T)$ for various values of
FIG. 4: Superfluid density $\rho_s(T) = [\lambda^2(0)/\lambda^2(T)]$ from 0.1 K to $T_c$. Solid circles = data. AF-phase fitting curves for $1/\tau \Delta_0 = 24$ (□), 26 (△), 28 (×).

$1/\tau \Delta_0$. Notice that when $1/\tau \Delta_0 = 26$, (1) $\rho_s$ decreases only slightly at $T_N$, and (2) $\Delta$ is only slightly depressed below the BCS value, in agreement with tunneling data$^{31,32}$. This value of $1/\tau \Delta_0$ corresponds to a mean free path (mfp) of 45 Å. Though this curve still overestimates the experimental data below $T_N$, it at least fits the data better than the paramagnetic curve.

To see if this value of $1/\tau \Delta_0$ is reasonable, we take $\Delta(0) = 1.83T_c$ for ErNi$_2$B$_2$C$^{26}$, for which the BCS coherence length $\xi_{\Delta}^{\Delta_{BCS}} = \hbar v_F/\pi \Delta(0) = 470$ Å, where $v_F = 3.6 \times 10^5$ m/s is taken from band-structure calculations$^{33}$ for LuNi$_2$B$_2$C and YNi$_2$B$_2$C. Using the relation

$$H_{c2}(0) = 0.693T_c\left(\frac{dH_{c2}}{dT}\right)|_{T_c},$$

(16)
and \( \frac{dH_{c2}}{dT}_{T_c} (H \parallel c) = -2.67 \text{ kOe/K} \), we obtain the coherence length

\[
\xi_{H_{c2}}^{H_{c2}} = \sqrt{\frac{\phi_0}{2\pi H_{c2}(0)}} = 130 \text{ Å.} \tag{17}
\]

Finally, using the relation\(^{34}\)

\[
\xi_{H_{c2}}^{H_{c2}} = 0.85(\xi_{\Delta_{BCS}}^{\Delta_{BCS}})^{1/2}, \tag{18}
\]

we obtain the mfp \( l = 42 \text{ Å} \). On the other hand, from the resistivity value just above \( T_c \), \( \rho(T^*_c) = 5.8 \mu\Omega\text{-cm} \), we get \( l = 56 \text{ Å} \). These two values agree well with the value of \( 45 \text{ Å} \) calculated from \( 1/\tau\Delta_0 = 26 \) obtained earlier, implying that this particular value of the nonmagnetic scattering rate needed to explain our \( \rho_s \) data is consistent with other measurements. Note that this value of mfp calculated from \( H_{c2} \) data does not depend on the exact value of \( T_c \). Also, the prefactors 0.693 and 0.85 in the above relations are for materials in the dirty limit. Here \( l < \xi_0 \), so our ErNi\textsubscript{2}B\textsubscript{2}C sample may be considered as “quasidirty”. It is puzzling that our sample has a high \( T_c \) and be considered quasi-dirty, yet this is consistent with the results of other papers. Also, in this sample the nonmagnetic scattering rate (\( 1/\tau \)) is at least two orders of magnitude larger than the effective magnetic scattering rate (\( 1/\tau_{eff}^2 \)), thus the mfp value is largely determined by \( 1/\tau \). Our mfp value, however, is smaller than the \( 90 \text{ Å} \) obtained from resistivity measurements just above \( T_c \) in Ref. 35. The CN model is thus able to explain our superfluid density data, both qualitatively and quantitatively. Our data, in agreement with others, also shows that AF order coexists with superconductivity below \( T_N \).

It is also interesting to note that according to Ref. 21, a near-exact cancellation of spin-fluctuation and molecular-field effects occur at a critical value of \( N(0)J^{cf} \sim 1.0 \times 10^{-3} \), where \( J^{cf} = I | g_J - 1 | \) is the conduction-electron local \( (f) \) spin exchange. For the case of ErNi\textsubscript{2}B\textsubscript{2}C, we obtain \( N(0)J^{cf} = 1.7 \times 10^{-3} \), which explains the small change in pairbreaking at \( T_N \).

We turn next to an alternative explanation for the change of \( \rho_s \) at \( T_N \). Ramakrishnan and Varma\(^{16}\) predicted that for materials with a nested FS, since the peak in susceptibility and the joint density of states (defined as the difference between the susceptibility in the superconducting state and the normal state) occur at the same \( Q \)-value, one should expect an increase in pairbreaking at \( T_N \). Conversely, a non-nested FS will give rise to decreased pairbreaking at \( T_N \). Two-dimensional angular correlation of electron-positron annihilation radiation measurements show that only one out of the three FS sheets in LuNi\textsubscript{2}B\textsubscript{2}C possesses
nesting properties, thereby accounting for the propensity for magnetic ordering found in the other magnetic members of the RE nickel borocarbides. Also, Dugdale et al. estimated that the fraction of the FS that would be able to participate in nesting is only 4.4%. Contrast this with CN model, which assumes perfect 1-D nesting. Hence the increased pairbreaking due to partial nesting on one FS sheet is partially compensated by decreased pairbreaking by the other two sheets, resulting in only a slight increase in pairbreaking at $T_N$.

As temperature further reduces below $T_N$, the theoretical curve in Fig. overestimates the experimental curve below 3 K ($\sim 0.3T_c$). This is due to additional pairbreaking effect of the ferromagnetic moments in the WFM phase, which shows up as a small peak near 2.3 K (see Fig. 1). The small dip in superfluid density shows that this WFM slightly depresses, but

![Graph](image-url)

FIG. 5: $\rho_s(T)$ from 0.1 K to $T_c$. (○) Data. (△) Calculated AF-phase curve for $1/\tau\Delta_0=26$, (□) Calculated AF-phase curve incorporating WFM. Note that $T_{WF&M}$ here is chosen to be 3 K.
does not completely destroy, superconductivity, demonstrating the coexistence of WFM and superconductivity. We model this WFM by including a temperature-dependent magnetic impurity scattering rate $1/\tau_{WFM}^2 = 1/20(g_J - 1)J(J + 1)(1 - T/T_{WFM})^{\nu'}$ (with the same value of $g_J$ and $J$ as before), and adding this to the previous effective magnetic scattering rate, i.e. $1/\tau_{total}^2 = 1/\tau_{eff}^2 + 1/\tau_{WFM}^2$ when $T < T_{WFM}$. The pre-factor $1/20$ arises from the fact that one out of every 20 spins contributes to the WFM, giving rise to a weak magnetization. The temperature dependence $(1 - T/T_{WFM})^{\nu'}$ is analogous to the molecular field formulation. We obtain $\nu' \approx 2$ from Jensen’s calculation or Choi and Canfield’s data. Note that $\rho_s$ already starts to flatten out at 3 K, consistent with neutron-scattering data, which shows that this weak ferromagnetic component already shows up at 3 K. Hence we choose $T_{WFM} = 3$ K in this WFM calculation. Fig. 5 shows $\rho_s$ when one accounts for WFM. The calculated $\rho_s$ does flatten out below 3 K, but does not increase below 2.3 K as the data did. There is as yet no direct measurement of superconducting gap amplitude at this temperature range, though our model predicts a drop in $\Delta$ there. One may need to include the effect of inelastic spin-fluctuation scattering in this low temperature region, which was ignored by the CN model.

In conclusion, we present in-plane penetration depth data of single-crystal ErNi$_2$B$_2$C down to 0.1 K. The small increase in pairbreaking at $T_N$ can be attributed to the interplay between the effects of the AF molecular field and spin-fluctuation scattering. It could also be due to the combined effects of non-perfect nesting on one piece of the FS and non-nesting on other pieces of the FS. The increased pairbreaking at $T_{WFM}$ is modelled by a magnetic impurity scattering parameter, and both magnetic orders coexist with superconductivity.

E.E.M.C wishes to thank Y. C. Chang for teaching the computational aspects of the Chi-Nagi model, as well as C. Varma and J. Thompson for encouraging discussions on interpretation of data. This material is based upon work supported by the U.S. Department of Energy, Division of Materials Sciences under Award No. DEFG02-91ER45439, through the Frederick Seitz Materials Research Laboratory at the University of Illinois at Urbana-Champaign. Research for this publication was carried out in the Center for Microanalysis of Materials, University of Illinois at Urbana-Champaign. This work in Korea was supported by the Ministry of Science and Technology of Korea through the Creative Research Initiative.
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