Fine structure of alpha decay in odd nuclei

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Abstract

Using an α decay level scheme, the fine structure in odd nuclei is explained by taking into account the radial and rotational couplings between the unpaired valence nucleon and the core of the decaying system. It is shown that the experimental behavior of the α decay fine structure phenomenon is governed by the dynamical characteristics of the system.

Keywords: alpha decay fine structure

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The $\alpha$ decay fine structure was discovered by Rosenblum [1] since 1929 by measuring the ranges of the emitted particle in the air. Usually, theoretical attempts to investigate this phenomenon are based on the calculation of the overlaps between, on one hand, the ground–state wave function of the parent and, on the other hand, the antisymmetric product between the wave functions of the nascent fragments in different configurations after the scission [2,3]. However, quantitatively this phenomenon was not explained rigorously.

It was evidenced [4], at least formally, that the significance of the emitted particle preformation and that of its penetrability through the barrier calculated from the ground–state configuration of the parent up to the scission point are equivalent. This equivalence gives a support in the attempt to investigate the $\alpha$ decay process using fission theories.

Recently, a theory based on the Landau–Zener effect was developed [5–7] intending to describe quantitatively the cluster decay fine structure phenomenon. This new formalism succeeds to reproduce the values of the fine structure hindrance factors for the $^{14}$C emission from $^{223}$Ra being in a better agreement than other microscopic theories [8] with the experiment. In this alternative description, the cluster decay fine structure is caused by the promotion of the valence unpaired nucleon on some excited daughter levels during the disintegration process. It was claimed that the same promotion effect can also govern the fine structure in the case of $\alpha$ decay. The first step in such a treatment is the elaboration of a two–center realistic level diagram during the whole disintegration process, that means, starting from the parent single–particle energy distribution and following the variations of the level energies up to their asymptotic configuration attributed to the separated daughter nucleus and the alpha particle. In this picture, it is only intended to treat the $\alpha$ cluster with a smooth potential in order to estimate its influence upon the levels of the daughter during the disintegration process, like a polarization effect. The particle–core couplings are produced merely between levels belonging to the nascent heavy nucleus, so that the influence of the potential attributed to the $\alpha$ particle needs only to be simulated. Of course, in this context, it is not assumed that the oscillator well is an appropriate tool to describe an alpha nucleus. So, to attain this purpose in a realistic manner, the supersymmetric two–center shell model (STCSM) described in Ref. [9] and improved in Ref. [5] was modified in order to reproduce the single–particle levels assumed to describe an $\alpha$ cluster in the final stage of the disintegration. Consequently, the alpha oscillator stiffness $\hbar \omega_{2\alpha}$ was forced to vary gradually from the usual STCSM value $\hbar \omega_2 = 41A_2^{1/3}$ when the normalized coordinate $R_n=0$, to the value extracted from Ref. [10] $\hbar \omega_{\alpha}=21.8$ MeV when the scission point is reached, that means $R_n = 1$. The nuclear shape parametrization being defined by two intersected spheres of different radii, $R_1$ for the daughter and $R_2$ for the emitted fragment, the single generalized coordinate remains the elongation $R$, denoting the distance between the centers of the nuclei, or the normalized coordinate $R_n = (R - R_i)/(R_f - R_i)$, where $R_i = R_0 - R_2$, $R_f = R_1 + R_2$ and $R_0$ denotes the radius of the parent. Subsequently, another three modifications were introduced in the present version of the STCSM in order to obtain good $\alpha$–level energies associated with the value $\hbar \omega_{\alpha}=21.8$ MeV. The first term characterizing the mass-asymmetry along the $\rho$–axis of Eq. (43) of Ref. [9] (proportional to $\hbar \omega_2 - \hbar \omega_1$) was multiplied by the ratio $(\omega_{2\alpha} - \omega_{1\alpha})/(\omega_{2\alpha} - \omega_{1\alpha})$. Another mass-asymmetry term along the $z$–axis was diagonalized using the eigenvalues of the asymmetric two–center potential:
In this way, energy shifts in single-particle levels associated to the change from \( \omega_2 \) to \( \omega_{2a} \) are produced, so that a good two-center oscillator with \( \hbar \omega_2 \) associated to the light fragment were multiplied by the ratio \( \omega \). With the formal definitions of these interaction terms which assume proportionality to \( \hbar \omega_2 \), the signification of the notations can be found in Ref. [9].

The spin–orbit and \( L^2 \) coefficients, together with the depths of the potentials for the \( ^{211}\text{Po} \) and \( ^{207}\text{Pb} \) used in this work are \( \kappa = 5.75 \times 10^{-2} \), \( \eta = 0.43 \) and \( V_c = 55.80 \text{ MeV} \), respectively. These values for the smooth spherical oscillator potentials were deduced from a fit of the \( ^{208}\text{Pb} \) single–particle energies given in Ref. [11]: \( E_{1i1/2} = -3.16 \text{ MeV} \), \( E_{2g9/2} = -3.94 \text{ MeV} \), \( E_{3p1/2} = -7.37 \text{ MeV} \), \( E_{2f5/2} = -7.94 \text{ MeV} \), \( E_{3p3/2} = -8.36 \text{ MeV} \) and \( E_{1i3/2} = -9.00 \text{ MeV} \). The single–particle energies of \( ^4\text{He} \) are: \(-12 \text{ MeV} \), \( 1.38 \text{ MeV} \), \( 1.68 \text{ MeV} \), \( 5.34 \text{ MeV} \), \( 10.43 \text{ MeV} \) and \( 11.86 \text{ MeV} \) for the levels \( 1s_{1/2} \), \( 1p_{3/2} \), \( 1p_{1/2} \), \( 2s_{1/2} \), \( 2d_{5/2} \) and \( 1d_{3/2} \), respectively. A good fit for the lower energies in the frame of the oscillator model with \( \hbar \omega_2 = 21.8 \text{ MeV} \) was found to be: \( \kappa = 4.51 \times 10^{-3} \), \( \eta = 0 \) and \( V_c = 52.91 \text{ MeV} \).

The STCSM level scheme is plotted in Fig. [4]. In the following, the condition of consistency is also achieved, the same shape for the microscopical and phenomenological models involved. The parent and the daughter do not have pronounced deformations, so that their ground state nuclear shapes can be approximated with spheres. The ground state configuration of \( ^{211}\text{Po} \) is \( (\pi (h_{9/2})^2 \nu (g_{9/2})^1)_{9/2}^+ \) [13]. For clarity, the levels of the parent will be labeled with the superscript P, while the superscripts D and \( \alpha \) will be used for the daughter and the \( \alpha \) nuclei, respectively. Up to now, the model evidences the variation of the levels for a modification of the nuclear shape, and indicates the origin of the nucleons belonging to the \( \alpha \)–particle. As an interesting feature, the STCSM predicts that the linked level \( 1s_{1/2}^\alpha \) of the \( \alpha \) particle emerges from the orbital \( 1g_{9/2}^P \) of the spherical \( ^{211}\text{Po} \), which is deeply located in the parent potential well. The present formalism intends to explain the fine structure by considering single–particle transitions due to the radial and the rotational couplings. The levels with the same good quantum numbers associated to some symmetries of the system cannot in general intersect, but exhibit quasi–crossings, or pseudo–crossings, or avoided level crossings. The system is characterized by an axial symmetry, therefore the good quantum numbers are the projections of the nucleon spin \( \Omega \). The radial coupling causes transitions of the unpaired nucleon near the avoided level crossings. True crossings can also be obtained between levels characterized by different quantum numbers. Generally, the rotational coupling has a maximum strength in the vicinity of the true crossings. Transitions due to both couplings are taken into account in order to explain the excitations of the unpaired nucleon. It can be considered that the last unpaired neutron, initially located in the orbital \( 2g_{9/2}^P \) has the same chance to choose one of the levels with the projection \( \Omega \) included between \( 1/2 \) and \( 9/2 \) when the nucleus starts to deform, therefore the occupation probabilities are the same. Comparing the diagrams (a)–(d) of Fig. [4] it is clear that only the level emerging from \( 2g_{9/2}^P \) with the spin projection \( \Omega = 1/2 \) finally reaches adiabatically the \( 3p_{1/2}^D \) ground–state of the daughter, which means, a disintegration performed without excitations. Due
to the rotational coupling, a nucleon initially in the state $2g_{9/2}^P \Omega=3/2$ can jump in the state $\Omega=1/2$ during the disintegration, contributing in a smaller measure to obtain finally the daughter ground–state. Even by taking into account the rotational coupling, the other levels with $\Omega > 3/2$ emerging from $2g_{9/2}^P$ have a negligible contribution in the ground–state channel. Moreover, if the Coriolis coupling is not taken into account (using only the radial coupling), the levels with $\Omega \neq 1/2$ emerging from $2g_{9/2}^P$ attain finally higher daughter levels with at least 3 MeV excess in energy (for example the orbital $2g_{9/2}^D$ of the daughter). As the penetrability decreases dramatically with the height of the barrier, it becomes clear that the processes characterized by a nucleon emerging from $2g_{9/2}^P \Omega > 3/2$ in the initial moment of the disintegration are unlikely. This discussion allows us to fix the initial conditions for our process: initially, the valence nucleon of the decaying system can be considered in the state $2g_{9/2}^P$ with $\Omega=1/2$ and $3/2$. To show how it is possible that the unpaired nucleon arrives on allowed excited states of the daughter due only to the radial effect, an arrow in Fig. 2 (a) indicates some avoided level crossings between the levels which reach adiabatically the $3p_{1/2}^D$, $2f_{5/2}^D$ and $3p_{3/2}^D$ daughter orbitals. The transition probabilities are strongly enhanced in the avoided level crossing regions in accordance with the Landau–Zener effect. The transition probability between two adiabatic single–particle states strongly depends on the velocity of passage through the avoided crossing regions $v_{tun}$, and, implicitly on the tunneling time of the barrier. When the velocity increases, the transition probability is enhanced and the nucleon follows with a larger probability the so called diabatic state. Using the above description, it is intended to reproduce the fine structure [13] exhibited by the $\alpha$ decay of $^{211}$Po: 98.9 % transitions to the $1/2^-$ (level $3p_{1/2}^D$) ground state of the daughter, 0.55% transitions to the $5/2^-$ (level $2f_{5/2}^D$) first single–particle excited state and 0.54% transitions to the $3/2^-$ (level $3p_{3/2}^D$) second single–particle excited state.

As briefly mentioned, both radial and rotational couplings caused by the relative motion between the nascent fragments [14,15] are taken into account in order to calculate the occupation probabilities of several levels of the daughter by the unpaired nucleon. For simplicity, the effect due to the radial coupling is considered to be well reproduced by the Landau–Zener promotion mechanism [16–18] in the avoided crossing regions. Therefore, the fine structure of the process is strongly related to the dynamic characteristics of the system. The rotational or Coriolis coupling causes transitions between two levels for which the value of $\Omega$ differs by one unit and it is proportional to the angular momentum operator of the single–particle $j_{\pm}$ matrix element. The STCSM provides the ingredients for calculating the single–particle transitions probabilities due to the Landau–Zener effect and to the Coriolis couplings: the interaction energies between the diabatic states $\epsilon_{ij}^{\Omega_k}$ in the avoided crossing regions, the diabatic level energies $\epsilon_i^{\Omega_k}$ (using spline interpolations) and the wave functions required to compute $\langle \Omega_k|j_{\pm}^z|\Omega_k \mp 1 \rangle$ as described in Refs. [14,15]. The behavior of these ingredients are plotted in Fig. 3. The relative velocity between the nascent fragments can also be calculated quantum mechanically using a method similar to that of the variation of constants [19], but in the following it will be considered as a fit parameter as in Refs. [5–7]. To obtain the final occupation probabilities of the daughter levels by the unpaired nucleon, a system of differential coupled equations must be solved:
\[ c_i^\Omega_k = \frac{1}{\hbar} \sum_{j \neq i} \epsilon_j^\Omega_k \exp \left( i \alpha_{ij}^\Omega_k \right) c_j^\Omega_k + \]
\[ \frac{1}{\hbar} \int \frac{\hbar^2}{2B_R R^2} \sqrt{I(I+1)-\Omega_k(\Omega_k+1)} \mid \langle i, \Omega_k | j_+ | l, \Omega_k - 1 \rangle \mid \exp \left( i \alpha_{il}^\Omega_k \right) c_l^{\Omega_k-1} + \]
\[ \frac{1}{\hbar} \int \frac{\hbar^2}{2B_R R^2} \sqrt{I(I+1)-\Omega_k(\Omega_k+1)} \mid \langle i, \Omega_k | j_- | l, \Omega_k + 1 \rangle \mid \exp \left( i \alpha_{il}^\Omega_k \right) c_l^{\Omega_k+1} \]

(0.2)

with \( \alpha_{ij}^\Omega = \int_0^t (\epsilon_i^\Omega - \epsilon_j^\Omega) dt / \hbar \), \( B_R \) is the effective mass along the elongation \( R \) which was taken approximatively equal to the reduced mass of the system and \( I=9/2 \) is the total spin of the system. The time dependence in the above equation can be removed by using the relations \( c_i^\Omega_k = v_{\text{tun}} \partial c_i^\Omega_k / \partial R \) and \( R = v_{\text{tun}} t \). The coefficients \( \left( c_i^\Omega_k \right)^2 \) give the occupation probabilities of the diabatic levels \( \lbrace j, \Omega_k \rbrace \). To solve this system, following the above discussion and inspecting the Fig. 3, it was considered that it is sufficient to choose the initial conditions so that the levels with \( \Omega=1/2 \) and \( 3/2 \) emerging from \( 2g_{9/2}^P \) have the same initial occupation probabilities, which means that the equality \( \left( c_{g_{9/2}^P}^{1/2} \right)^2 + \left( c_{g_{9/2}^P}^{3/2} \right)^2 = 1 \) is fulfilled. Also, by solving the system (0.2), it is satisfactory to take into account the levels with \( \Omega=1/2 \) emerging from \( 1p_{11/2}^P \), \( 2g_{9/2}^P \), \( 3p_{1/2}^P \), \( 2f_{5/2}^P \), \( 3p_{3/2}^P \), those with \( \Omega=3/2 \) emerging from \( 2g_{9/2}^P \), \( 2f_{5/2}^P \), \( 3p_{3/2}^P \), and those with \( \Omega=5/2 \) emerging from \( 2g_{9/2}^P \), \( 1p_{13/2}^P \). All the avoided crossing levels between the selected adiabatic states are taken into account. Levels with \( \Omega > 5/2 \) do not reach the final channels we are interested in.

For each channel, the penetrability \( P_{\Omega_{lim_i}}(Q_i) \) of the barrier was obtained using the numerical supersymmetric fission model [20], the nuclear part being given by the Yukawa–plus–exponential approximation. This penetrability depends on the \( Q_i \)–value of the channel \( i \) (\( i \) labels here the single–particle state of the daughter) and of the relative motion orbital momentum \( L_{lim_i} \). In the final channel \( 3p_{1/2}^P \), due to the conservation laws, \( L_{lim_i} \) has the value \( 5 \) \((m_i=1)\), in the final channel \( 2f_{5/2}^P \), \( L_{lim_i} \) can be either \( 3, 5 \) or \( 7 \) \((m_i=1,2,3)\) and in the final channel \( 3p_{3/2}^P \), \( L_{lim_i} \) can be either \( 3 \) or \( 5 \) \((m_i=1,2)\). For a specific final single–particle state, for example \( 2f_{5/2}^P \), it is not possible to discriminate between the possible values of the relative motion orbital momentum \( L_{lim_i} = 3, 5 \) and \( 7 \) in order to compute only one barrier penetrability. In these circumstances, by analogy to the Mang’s formulae of Refs. [3,4] concerning the radial motion and the associated angular momentum, we consider that the angular momentum \( L_{lim_i} \), used in calculating the penetrabilities, has a probability to be obtained in the final channel directly proportional to the square of the Clebsh–Gordon coefficient \( \left( jI\Omega - \Omega | L_{lim_i} \right)^2 \). So that, the spectroscopic amplitude in the channel \( i \) associated to the spin \( \Omega_k \) and the momentum \( L_{lim_i} \) will be \( p_i^{\Omega_k L_{lim_i}} = \left( c_i^{\Omega_k} \right)^2 \left( j_i I \Omega_k - \Omega_k | L_{lim_i} \right)^2 / \sum_{m_i} \left( j_i I \Omega_k - \Omega_k | L_{lim_i} \right)^2 \) where the summation on \( m_i \) is done on the allowed values of \( L_{lim_i} \). The partial half–life \( T_i^{\Omega_k} \) for the channel \( \{ i, \Omega_k \} \) becomes proportional to the quantity:

\[ T_i^{\Omega_k} \propto \frac{1}{\sum_{j} p_j^{\Omega_k L_{lim_i}} P_{L_{lim_i}}(Q_j)}, \]

(0.3)

the proportionality factor being given by the barrier assault frequency. The partial half–lives for the transitions to the ground–state \( T_{3p_{3/2}^P} \), to the first excited state \( T_{2f_{5/2}^P} \) and to the second excited state \( T_{3p_{3/2}^P} \) are:
\[
\frac{1}{T_{3p_{1/2}}} = \frac{1}{T_{3p_{1/2}}} + \frac{1}{T_{2f_{5/2}}} + \frac{1}{T_{3p_{3/2}}}
\]

The barrier assault frequency being the same for all the channels, the relative intensities \(T_{3p_{1/2}}/T_{2f_{5/2}}\) and \(T_{3p_{1/2}}/T_{3p_{3/2}}\) for the fine structure can be obtained. Several tunneling velocities, considered here as a fit parameter, have been tried. For a tunneling velocity of \(9 \times 10^6\) fm/fs, the ratio between the intensity for transitions to the first excited state and to the ground state was found to be 0.0071 and the obtained ratio of the same parameter between the second excited state and the ground state was 0.0062. These results are in good agreement with the experimental values presented before. Moreover, calculations of Ref. [19] show that in the quantum time–dependent approach, the tunneling velocity is of the order of \(1 \times 10^7\) fm/fs. These calculations suggest that the \(\alpha\) decay fine structure phenomenon can be explained quantitatively by describing the decaying system with molecular models, and it can be stated that the quantitative characteristics of this phenomenon are ruled by dynamical effects. In an avoided crossing region, the two eigenfunctions of the adiabatic levels exchange their characteristics. If the relative distance \(R\) change infinitely slow, the unpaired nucleon will remain in the same adiabatic single particle state after the passage through a quasi–crossing, any other available single particle excited state being unfavoured. For a large tunneling velocity, the unpaired nucleon will follow the diabatic state after the passage through a quasi–crossing, all the other states being unfavoured. For a finite tunneling velocity, an intermediate situation arises, some single particle states being favoured and other single particle states being unfavoured. The model propose here, as the usual picture that consists of calculating the overlaps between the parent and the channel wave functions, is also based on the existence of favoured and unfavoured transitions in odd nuclei. The proposed formalism offers a competitive description of the alpha decay mechanism, by investigating for the first time the modality in which the levels initially bunched in shells are reorganized during the disintegration to realize the final energy configuration.
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FIGURES

FIG. 1. Neutron level scheme for the $\alpha$ decay of $^{211}$Po as function of the normalized elongation. The spectroscopic notations are used to describe the levels of the parent nucleus in the left side of the figure while in the right side of the figure the levels of the daughter and the alpha particle are labeled in the first and second column, respectively.

FIG. 2. Detailed part of the level scheme. The levels with $\Omega = 1/2$ (a), $\Omega = 3/2$ (b), $\Omega = 5/2$ (c), $\Omega = 7/2$ (d), $\Omega = 9/2$ (e) and $\Omega = 13/2$ (f) are plotted with thick lines. The levels with $\Omega = 11/2$ (e) and $\Omega = 15/2$ (f) are plotted with thick dotted lines.

FIG. 3. Differences between the adiabatic levels with spin $\Omega=1/2$ are presented in the plots (a) to (f) suggesting the possible avoided crossing regions, that means, points of nearest approach between two adiabatic levels. These regions are marked with arrows and the spectroscopic notations are displayed on each plot. The adiabatic $E_i$ and the diabatic $\epsilon_i$ levels are presented in the picture (g) with full and dotted lines respectively only in the case $\Omega=1/2$. The interaction energies $\epsilon_{ij}$ between the diabatic levels of the plot (g) are presented in (h). The levels with spin $\Omega=1/2$ emerging from $2f_{5/2}^P$, $3p_{3/2}^P$ and $1i_{13/2}^P$ are presented as full lines while the level with spin 3/2 emerging from $1i_{13/2}^P$ is plotted as dotted line in picture (i). For the levels presented in (i), the matrix elements $\langle 3/2|j_+|1/2 \rangle = \langle 1/2|j_-|3/2 \rangle$ are drawn in (j). Asymptotically, for $R_n=0$, the matrix element between $\Omega=1/2$ and $3/2$ of the levels belonging to the subshell $1i_{13/2}^P$ (full line) has the value $\langle \Omega \pm 1|j_\pm|\Omega \rangle = \hbar \sqrt{(j \mp \Omega)(j \pm \Omega + 1)}$ while for $R_n \to \infty$ the same value is obtained within the matrix element for the level $\Omega=1/2$ emerging from $3p_{3/2}^P$ and the level $1i_{13/2}^P \Omega=3/2$ (dashed line). Otherwise, asymptotically the values are zero (dotted line). A pronounced maximum of the matrix elements is obtained when the levels with different spin projections intersect.
