Cosmological Constraints on New Agegraphic Dark Energy

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ABSTRACT

In this work, we consider the cosmological constraints on the new agegraphic dark energy (NADE) proposed in [arXiv:0708.0884] by using the observational data of type Ia supernovae (SNIa), the shift parameter from cosmic microwave background (CMB) and the baryon acoustic oscillation (BAO) peak from large scale structures (LSS). Thanks to its special analytic features in the radiation-dominated and matter-dominated epochs, NADE is a single-parameter model in practice because once the single model parameter $n$ is given, all other physical quantities of NADE can be determined correspondingly. The joint analysis gives the best-fit value (with $1\sigma$ uncertainty) $n = 2.716^{+0.111}_{-0.109}$, and the derived $\Omega_{m0}$, $\Omega_{q0}$ and $w_{q0}$ (with $1\sigma$ uncertainties) are $0.295^{+0.020}_{-0.020}$, $0.705^{+0.020}_{-0.020}$ and $-0.794^{+0.006}_{-0.005}$, respectively. In addition, we find that the coincidence problem could be solved naturally in the NADE model provided that $n$ is of order unity.

PACS numbers: 95.36.+x, 98.80.Es, 98.80.-k
I. INTRODUCTION

In general relativity, one can measure the spacetime without any limit of accuracy. In quantum mechanics, however, the well-known Heisenberg uncertainty relation puts a limit of accuracy in these measurements. Following the line of quantum fluctuations of spacetime, Károlyházy and his collaborators \[1\] (see also \[2\]) made an interesting observation concerning the distance measurement for Minkowski spacetime through a light-clock Gedanken experiment, namely, the distance \(t\) in Minkowski spacetime cannot be known to a better accuracy than

\[
\delta t = \lambda t_p^{2/3} t^{1/3},
\]

where \(\lambda\) is a dimensionless constant of order unity \[2, 3\]. We use the units \(\hbar = c = k_B = 1\) throughout this work. Thus, one can use the terms like length and time interchangeably, whereas \(l_p, t_p\) and \(m_p\) being the reduced Planck length, time and mass, respectively.

The Károlyházy relation \(1\) together with the time-energy uncertainty relation enables one to estimate an energy density of the metric quantum fluctuations of Minkowski spacetime \[2, 3\]. Following \[2, 3\], with respect to Eq. \(1\) a length scale \(\delta t\) can be known with a maximum precision \(\delta t\) determining thereby a minimal detectable cell \(\delta t^3 \sim t^2\) over a spatial region \(t^3\). Such a cell represents a minimal detectable unit of spacetime over a given length scale \(t\). If the age of the Minkowski spacetime is \(t\), then over a spatial region with linear size \(t\) (determining the maximal observable patch) there exists a minimal cell \(\delta t^3\) the energy of which due to time-energy uncertainty relation can not be smaller than \[2, 3\]

\[
E_{\delta t^3} \sim t^{-11/2}.
\]

Therefore, the energy density of metric fluctuations of Minkowski spacetime is given by \[2, 3\]

\[
\rho_q \sim \frac{E_{\delta t^3}}{\delta t^3} \sim \frac{1}{t_p^2 t^2} \sim \frac{m_p^2}{t^2}.
\]

We refer to the original papers \[2, 3\] for more details. It is worth noting that in fact, the Károlyházy relation \(1\) and the corresponding energy density \(3\) have been independently rediscovered later for many times in the literature (see e.g. \[4, 5, 6\]).

Based on the energy density \(3\), a so-called agegraphic dark energy (ADE) model was proposed in \[7\]. There, as the most natural choice, the time scale \(t\) in Eq. \(3\) is chosen to be the age of the universe

\[
T = \int_0^a \frac{da}{Ha}.
\]

where \(a\) is the scale factor of our universe; \(H \equiv \dot{a}/a\) is the Hubble parameter; an over dot denotes the derivative with respect to cosmic time. Then, the ADE model was extended to include the interaction between ADE and background matter \[8\]. The statefinder diagnostic and \(w - w'\) analysis for the ADE models without and with interaction were performed in \[9\]. The ADE was constrained by using some old high redshift objects \[10\] and type Ia supernovae (SNIa), the shift parameter from cosmic microwave background (CMB) and the baryon acoustic oscillation (BAO) peak from large scale structures (LSS) \[11\].

On the other hand, a new model of ADE was proposed in \[12\], where the time scale in Eq. \(3\) is chosen to be the conformal time \(\eta\) instead of the age of the universe. This new agegraphic dark energy (NADE) has some new features different from the ADE proposed in \[7\]. We will briefly review the main points of NADE model in the next section.

Thanks to its special analytic features in the radiation-dominated and matter-dominated epochs, NADE is a single-parameter model in practice, unlike the two-parameters ADE model \[11\] (this point will be explained below). If the single model parameter \(n\) is given, all other physical quantities of NADE are determined correspondingly. In this work, we find that the coincidence problem could be solved naturally in the NADE model provided that \(n\) is of order unity. In addition, we constrain the NADE by using the cosmological observations of SNIa, CMB and LSS. Different from the cosmological constraints on ADE \[11\], the single parameter \(n\) of NADE model can be strictly constrained. Then, the other physical quantities of NADE can also be determined tightly.
II. THE MODEL OF NEW AGEGRAPHIC DARK ENERGY

A. Main points of the NADE model

In the NADE model [12], we choose the time scale in Eq. (3) to be the conformal time \( \eta \) instead, which is defined by \( dt = a d\eta \) [where \( t \) is the cosmic time, do not confuse it with the \( t \) in Eq. (3)]. Therefore, the energy density of NADE reads

\[
\rho_q = \frac{3n^2 m_p^2}{\eta^2},
\]

(5)

where the numerical factor \( 3n^2 \) is introduced to parameterize some uncertainties, such as the species of quantum fields in the universe, the effect of curved spacetime [since the energy density in Eq. (3) is derived for Minkowski spacetime], and so on. Since both the numerical factor \( \lambda \) in Eq. (1) and the coefficient in time-energy uncertainty relation Eq. (2) are of order unity, it is anticipated that the parameter \( n \) in Eq. (5) which comes from Eqs. (1) and (2) is also of order unity. The conformal time \( \eta \) is given by

\[
\eta = \int \frac{dt}{a} = \int \frac{da}{a^2 H}.
\]

(6)

If we write \( \eta \) to be a definite integral, there will be an integration constant in addition. Note that \( \dot{\eta} = 1/a \).

In this work, we consider a flat Friedmann-Robertson-Walker (FRW) universe containing NADE and pressureless matter. Note that the assumption of flatness is motivated by the inflation scenario and is also consistent with the observation of CMB [26]. The corresponding Friedmann equation reads

\[
H^2 = \frac{1}{3m_p^2} (\rho_m + \rho_q),
\]

(7)

where \( \rho_m \) is the energy density of pressureless matter. It is convenient to introduce the fractional energy densities \( \Omega_i \equiv \rho_i / (3m_p^2 H^2) \) for \( i = m \) and \( q \). From Eq. (5), the corresponding fractional energy density of NADE is given by

\[
\Omega_q = \frac{n^2}{H^2 \eta^2}.
\]

(8)

By using Eqs. (5)–(8) and the energy conservation equation \( \dot{\rho}_m + 3H \rho_m = 0 \), we find that the equation of motion for \( \Omega_q \) is given by [12]

\[
\frac{d\Omega_q}{da} = \frac{\Omega_q}{a} \left( 1 - \Omega_q \right) \left( 3 - 2 \sqrt{\Omega_q} \right).
\]

(9)

From the energy conservation equation \( \dot{\rho}_q + 3H (\rho_q + p_q) = 0 \), as well as Eqs. (8) and (3), it is easy to find that the equation-of-state parameter (EoS) of NADE \( w_q \equiv p_q / \rho_q \) is given by [12]

\[
w_q = -1 + 2 \frac{\sqrt{\Omega_q}}{n}.
\]

(10)

Obviously, the scale factor \( a \) enters Eqs. (9) and (10) explicitly. When \( a \to \infty, \Omega_q \to 0 \), thus \( w_q \to -1 \) in the late time. When \( a \to 0, \Omega_q \to 0 \), we cannot obtain \( w_q \) from Eq. (10) directly. Let us consider the matter-dominated epoch, \( H^2 \propto \rho_m \propto a^{-3} \). Thus, \( a^{1/2} da \propto dt = a d\eta \). Therefore, \( \eta \propto a^{1/2} \). From Eq. (5), \( \rho_q \propto a^{-1} \). From the energy conservation equation \( \dot{\rho}_q + 3H \rho_q (1 + w_q) = 0 \), we obtain that \( w_q = -2/3 \) in the matter-dominated epoch. Since \( \rho_m \propto a^{-3} \), it is expected that \( \Omega_q \propto a^2 \). Comparing \( w_q = -2/3 \) with Eq. (10), we find that \( \Omega_q = n^2 a^2 / 4 \) in the matter-dominated epoch as expected. For \( a \ll 1 \), if \( n \) is of order unity, \( \Omega_q \ll 1 \) naturally. On the other hand, one can check that \( \Omega_q = n^2 a^2 / 4 \) satisfies

\[
\frac{d\Omega_q}{da} = \frac{\Omega_q}{a} \left( 3 - 2 \sqrt{\Omega_q} \right),
\]

which is the approximation of Eq. (9) for \( 1 - \Omega_q \simeq 1 \). Therefore, all things are consistent.
In fact, we can further extend our discussion to include the radiation-dominated epoch [12]. However, since our main aim of this work is to constrain NADE by using the cosmological observations of SNIa, CMB and LSS which only concern the matter-dominated epoch, here we skip the radiation-dominated epoch. Instead, we only give a brief summary of the results obtained in [12], namely, in the radiation-dominated epoch, \( w_q = -\frac{1}{3} \) whereas \( \Omega_q = n^2a^2 \); in the matter-dominated epoch, \( w_q = -\frac{2}{3} \) whereas \( \Omega_q = n^2a^2/4 \); eventually, the NADE dominates; in the late time \( w_q \rightarrow -1 \) when \( a \rightarrow \infty \), the NADE mimics a cosmological constant. We refer to the original paper [12] for more details.

FIG. 1: The resulting \( \Omega_q(z) \), \( \Omega_m(z) = 1 - \Omega_q(z) \) and \( w_q(z) \) obtained from Eqs. (11) and (10) with the initial condition \( \Omega_q(z_{ini}) = n^2(1 + z_{ini})^{-2}/4 \) at \( z_{ini} = 2000 \) for different \( n \).
B. NADE and the coincidence problem

It is worth noting that both $\Omega_q = n^2a^2$ and $n^2a^2/4 \lesssim 1$ naturally in radiation-dominated and matter-dominated epochs where $a \ll 1$. These results are obtained without any additional assumption. It is natural to consider the coincidence problem. In many dark energy models, especially the $\Lambda$CDM model, $\Omega_{de}$ should be fine-tuned to be extremely small in the early time, in order to interpret the fact that the energy densities of dark energy and matter are comparable today. In some dynamical dark energy models, the tracker behavior or attractors in the interacting dark energy models are used to alleviate the coincidence problem.

Here, we see that the NADE model could shed new light on the coincidence problem in a different way. The key point is that as mentioned above $\Omega_q$ of NADE can be obtained analytically in both radiation-dominated and matter-dominated epochs. In particular, $\Omega_q = n^2a^2/4 = n^2(1+z)^{-2}/4$ in the matter-dominated epoch, where $z = a^{-1} - 1$ is the redshift (we set $a_0 = 1$ throughout; the subscript “0” indicates the present value of the corresponding quantity). Therefore, we can use $\Omega_q(z_{ini}) = n^2(1+z_{ini})^{-2}/4$ at any $z_{ini}$ which is deep enough into the matter-dominated epoch to be the initial condition to solve the differential equation of $\Omega_q$. We rewrite Eq. (9) as

$$\frac{d\Omega_q}{dz} = -\Omega_q (1 - \Omega_q) \left[ 3(1+z)^{-1} - \frac{2}{n} \sqrt{\Omega_q} \right].$$

As well-known, the energy densities of matter and radiation is equal at redshift $z_{eq}$ which is given by $1 + z_{eq} \approx 2.32 \times 10^4 \Omega_{m0} h^2$ [13], where $h$ is defined by the Hubble constant $H_0 = 100 \, h \, \text{km/s/Mpc}$. For instance, $z_{eq} \approx 3409$ for $\Omega_{m0} = 0.3$ and $h = 0.7$ [14], whereas $z_{eq} \approx 2505$ for $\Omega_{m0} = 0.3$ and $h = 0.6$ [15]. Therefore, we choose $z_{ini} = 2000$ which is deep enough into the matter-dominated epoch. In fact, one can check that if we use initial condition $\Omega_q(z_{ini}) = n^2(1+z_{ini})^{-2}/4$ at $z_{ini} = 2000$, the resulting $\Omega_q(z)$ from Eq. (11) for $n = 3$ relatively deviates from $n^2(1+z)^{-2}/4$ less than approximately 1% in the large interval $10 \leq z \leq 2000$. Thus, if one uses other $z_{ini}$, the results deviate from the ones of $z_{ini} = 2000$ negligibly. This also justifies the choice of initial condition $\Omega_q(z_{ini}) = n^2(1+z_{ini})^{-2}/4$ at $z_{ini} = 2000$. In Fig. 1 we present the resulting $\Omega_q(z)$, $\Omega_m(z) = 1 - \Omega_q(z)$ and $w_q(z)$ from Eqs. (11) and (10) for different $n$. We only plot the redshift range $0 \leq z \leq 20$ since $\Omega_q \approx 0$ and $w_q \approx -2/3$ closely in the range $20 \leq z \leq 2000$. From Fig. 1, it is easy to see that $\Omega_{q0}$ and $\Omega_{m0} = 1 - \Omega_{q0}$ are comparable provided that $n$ is of order unity. We stress that this result is obtained very naturally, without any additional assumption. Therefore, the coincidence problem could be solved naturally in the NADE model.

III. CONSTRAINTS ON NADE FROM SNIA

In this section, we constrain NADE by using the latest 182 SNIa Gold dataset [16]. The latest 182 SNIa Gold dataset compiled in [16] provides the apparent magnitude $m(z)$ of the supernovae at peak brightness after implementing corrections for galactic extinction, K-correction, and light curve width-luminosity correction. The resulting apparent magnitude $m(z)$ is related to the luminosity distance $d_L(z)$ through (see e.g. [17])

$$m(z) = \widetilde{M}(M, H_0) + 5 \log_{10} D_L(z),$$

where

$$D_L(z) = (1+z) \int_0^z \frac{dz'}{E(z'; \mathbf{p})}$$

is the Hubble-free luminosity distance $H_0 d_L/c$ in a spatially flat FRW universe ($c$ is the speed of light); $\mathbf{p}$ denotes the model parameters; $E(z) = H(z)/H_0$; and

$$\widetilde{M} = M + 5 \log_{10} \left( \frac{cH_0^{-1}}{\text{Mpc}} \right) + 25 = M - 5 \log_{10} h + 42.38$$

[13]
is the magnitude zero offset; $h$ is $H_0$ in units of 100 km/s/Mpc; the absolute magnitude $M$ is assumed to be constant after the corrections mentioned above. The data points of the latest 182 SNIa Gold dataset compiled in $^{16}$ are given in terms of the distance modulus

$$
\mu_{\text{obs}}(z_i) \equiv m_{\text{obs}}(z_i) - M.
$$

(15)

On the other hand, the theoretical distance modulus is defined as

$$
\mu_{\text{th}}(z_i) \equiv m_{\text{th}}(z_i) - M = 5 \log_{10} D_L(z_i) + \mu_0,
$$

where

$$
\mu_0 \equiv 42.38 - 5 \log_{10} h.
$$

(17)

The theoretical model parameters are determined by minimizing

$$
\chi^2_{SN}(p) = \sum_{i=1}^{182} \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i)]^2}{\sigma^2(z_i)}.
$$

(18)

where $\sigma$ is the corresponding 1σ error; $p$ denotes the model parameters. The parameter $\mu_0$ is a nuisance parameter but it is independent of the data points. One can perform an uniform marginalization over $\mu_0$. However, there is an alternative way. Following $^{17, 18, 19}$, the minimization with respect to $\mu_0$ can be made by expanding the $\chi^2_{SN}$ of Eq. (18) with respect to $\mu_0$ as

$$
\chi^2_{SN}(p) = A - 2\mu_0 B + \mu_0^2 C,
$$

(19)

where

$$
A(p) = \sum_{i=1}^{182} \frac{[m_{\text{obs}}(z_i) - m_{\text{th}}(z_i; \mu_0 = 0, p)]^2}{\sigma^2_{m_{\text{obs}}}(z_i)},
$$

$$
B(p) = \sum_{i=1}^{182} \frac{m_{\text{obs}}(z_i) - m_{\text{th}}(z_i; \mu_0 = 0, p)}{\sigma^2_{m_{\text{obs}}}(z_i)},
$$

$$
C = \sum_{i=1}^{182} \frac{1}{\sigma^2_{m_{\text{obs}}}(z_i)}.
$$

Eq. (19) has a minimum for $\mu_0 = B/C$ at

$$
\chi^2_{SN}(p) = A(p) - \frac{B(p)^2}{C}.
$$

(20)

Therefore, we can instead minimize $\chi^2_{SN}$ which is independent of $\mu_0$, since $\chi^2_{SN, \text{min}} = \chi^2_{SN, \text{min}}$ obviously. It is worth noting that the corresponding $h$ can be determined by $\mu_0 = B/C$ for the best-fit parameters.

In the ADE model $^{11}$, there are two independent model parameters, namely, $n$ and $\Omega_m$. To obtain $\Omega_q(z)$ which is critical in $E(z)$, one should solve the corresponding differential equation of $\Omega_q$ with the initial condition $\Omega_q(z = 0) = \Omega_{q0} = 1 - \Omega_{m0}$, similar to the case of holographic dark energy model (see e.g. $^{20}$). In the NADE model, however, the situation is different. As mentioned in Sec. III B if $n$ is given, we can obtain $\Omega_q(z)$ from Eq. (11) with the initial condition $\Omega_q(z_{ini}) = n^2(1+z_{ini})^{-2}/4$ at $z_{ini} = 2000$ (or any $z_{ini}$ which is deep enough into the matter-dominated epoch), instead of $\Omega_q(z = 0) = \Omega_{q0} = 1 - \Omega_{m0}$. Then, all other physical quantities, such as $\Omega_m(z) = 1 - \Omega_q(z)$ and $w_q(z)$ in Eq. (10), can be obtained correspondingly. So, $\Omega_{m0} = \Omega_m(z = 0)$ and $\Omega_{q0} = \Omega_q(z = 0)$ are not independent model parameters. The only model parameter is $n$. Therefore, the NADE model is a single-parameter model in practice, unlike
the two-parameters ADE model [11]. To our knowledge, it is the third single-parameter cosmological model besides the well-known ΛCDM model and the DGP braneworld model [33]. From Eq. (7), we have

\[ E(z) = \left[ \frac{\Omega_m(1 + z)^{3n}}{1 - \Omega_q(z)} \right]^{1/2}. \] (21)

If the single model parameter \( n \) is given, we can obtain \( \Omega_q(z) \) from Eq. (11). And then, we get \( \Omega_m = 1 - \Omega_q(z = 0) \). Therefore, \( E(z) \) is in hand. So, we can find the corresponding \( \tilde{\chi}^2_{\text{SN}} \). In Fig. 2 we present the \( \chi^2 = \tilde{\chi}^2_{\text{SN}} \) and the corresponding likelihood \( L \propto e^{-\chi^2/2} \) for \( 0 < n \leq 10 \). We find that the best-fit model parameter is \( n = 2.954 \) while \( \chi^2_{\text{min}} = 160.255 \). The corresponding \( h = 0.623 \). In Table I we present the best-fit value of \( n \) and the derived \( \Omega_m, \Omega_q \) and \( w_q \) with 1σ and 2σ uncertainties. On the other hand, we also fit the ΛCDM model to the same SNIa dataset and find that \( \chi^2_{\text{min}, \Lambda} = 158.749 \) for the best-fit parameter \( \Omega_{\Lambda m} = 0.342 \) (the corresponding \( h = 0.626 \)). So, fitting to the latest 182 SNIa Gold dataset [16], the ΛCDM model is slightly better than the NADE model.

![Figure 2](image)

**FIG. 2:** The \( \chi^2 = \tilde{\chi}^2_{\text{SN}} \) and the corresponding likelihood for \( 0 < n \leq 10 \). These results are obtained from the latest 182 SNIa Gold dataset compiled in [16].

| Uncertainty | \( n \) | \( \Omega_m \) | \( \Omega_q \) | \( w_q \) |
|------------|------|--------|--------|--------|
| 1σ         | 2.954±0.264                                   | 0.255±0.038 | 0.745±0.038 | −0.805±0.012 |
| 2σ         | 2.954±0.555                                   | 0.255±0.086 | 0.745±0.072 | −0.805±0.023 |

**TABLE I:** The best-fit value of \( n \) and the derived \( \Omega_m, \Omega_q \) and \( w_q \) with 1σ and 2σ uncertainties. These results are obtained from the latest 182 SNIa Gold dataset compiled in [16].
FIG. 3: The $\chi^2$ and the corresponding likelihood for $0 < n \leq 10$. These results are obtained from the combined SNIa, CMB and LSS data.

IV. CONSTRAINTS ON NADE FROM SNIa, CMB AND LSS

Although the constraints on NADE from SNIa alone are fairly tight as shown in Table I, we can further make the constraints tighter by combining SNIa with other complementary observations. In the literature, the shift parameter $R$ from CMB [20, 21] and the parameter $A$ of BAO measurement from LSS [22] are used extensively, see [11, 16, 23] for examples. It is commonly believed that both $R$ and $A$ are model-independent and contain the essential information of the full CMB and LSS BAO data (however, see also e.g. [24] and [25]). Notice that both $R$ and $A$ are independent of Hubble constant $H_0$. In fact, the shift parameter $R$ is defined by [20, 21]

$$R \equiv \Omega_{m0}^{1/2} \int_0^{z_{\text{rec}}} \frac{d\tilde{z}}{E(\tilde{z})},$$

where $z_{\text{rec}} = 1089$ is the redshift of recombination. The shift parameter $R$ relates the angular diameter distance to the last scattering surface, the comoving size of the sound horizon at $z_{\text{rec}}$ and the angular scale of the first acoustic peak in CMB power spectrum of temperature fluctuations [20, 21]. The value of $R$ is determined to be $1.70 \pm 0.03$ [21] from the WMAP 3-year (WMAP3) data [26]. On the other hand, the parameter $A$ of the measurement of BAO peak in the distribution of SDSS luminous red galaxies [27] is given by [22]

$$A \equiv \Omega_{m0}^{1/2} E(z_b)^{-1/3} \left[ \frac{1}{z_b} \int_0^{z_b} \frac{d\tilde{z}}{E(\tilde{z})} \right]^{2/3},$$

where $z_b = 0.35$. In [22], the value of $A$ is determined to be $0.469 (n_s/0.98)^{-0.35} \pm 0.017$, here the scalar spectral index $n_s$ is taken to be 0.95 from the WMAP3 data [26].

We perform a joint analysis of SNIa, the shift parameter $R$ from CMB and the BAO peak measurement $A$ from LSS data to constrain the NADE model. The total $\chi^2$ is given by

$$\chi^2 = \chi^2_{SN} + \chi^2_{CMB} + \chi^2_{LSS},$$

where $\chi^2_{SN}$ is given in Eq. (20), $\chi^2_{CMB} = (R - R_{\text{obs}})^2/\sigma_R^2$ and $\chi^2_{LSS} = (A - A_{\text{obs}})^2/\sigma_A^2$. In Fig. 3 we present the $\chi^2$ and the corresponding likelihood $\mathcal{L} \propto e^{-\chi^2/2}$ for $0 < n \leq 10$. We find that the best-fit
model parameter is $n = 2.716$ while $\chi^2_{\text{min}} = 161.467$. The corresponding $h = 0.616$. We present the best-fit value of $n$ and the derived $\Omega_{m0}$, $\Omega_{q0}$ and $w_{q0}$ with $1\sigma$ and $2\sigma$ uncertainties in Table II. Obviously, these constraints in Table II are strict enough. On the other hand, we also fit the $\Lambda$CDM model to the same combined SNIa, CMB and LSS data and find that $\chi^2_{\text{min}, \Lambda} = 162.886$ for the best-fit parameter $\Omega_{m0} = 0.288$ (the corresponding $h = 0.637$). So, fitting to the combined SNIa, CMB and LSS data, the NADE model is slightly better than the $\Lambda$CDM model. This makes the NADE model more attractive.

| Uncertainty | $n$       | $\Omega_{m0}$ | $\Omega_{q0}$ | $w_{q0}$ |
|-------------|-----------|---------------|---------------|----------|
| $1\sigma$   | $2.716^{+0.111}_{-0.109}$ | $0.295^{+0.020}_{-0.020}$ | $0.705^{+0.020}_{-0.020}$ | $-0.794^{+0.006}_{-0.005}$ |
| $2\sigma$   | $2.716^{+0.224}_{-0.212}$ | $0.295^{+0.041}_{-0.038}$ | $0.705^{+0.038}_{-0.041}$ | $-0.794^{+0.011}_{-0.010}$ |

TABLE II: The best-fit value of $n$ and the derived $\Omega_{m0}$, $\Omega_{q0}$ and $w_{q0}$ with $1\sigma$ and $2\sigma$ uncertainties. These results are obtained from the combined SNIa, CMB and LSS data.

In this work, we consider the NADE model proposed in [12], which might have some physical motivations connected to the quantum fluctuations of spacetime. Thanks to its special analytic features in the radiation-dominated and matter-dominated epochs, NADE is a single-parameter model in practice, unlike the two-parameters ADE model [11]. If the single model parameter $n$ is given, all other physical quantities of NADE can be determined correspondingly. To our knowledge, it is the third single-parameter cosmological model besides the well-known $\Lambda$CDM model and the DGP braneworld model [33]. We find that the coincidence problem could be solved naturally in the NADE model provided that the single model parameter $n$ is of order unity. In addition, we constrain NADE by using the cosmological observations of SNIa, CMB and LSS. The joint analysis gives the best-fit parameter (with $1\sigma$ uncertainty) $n = 2.716^{+0.111}_{-0.109}$. The derived $\Omega_{m0}$, $\Omega_{q0}$ and $w_{q0}$ (with $1\sigma$ uncertainties) are $0.295^{+0.020}_{-0.020}$, $0.705^{+0.020}_{-0.020}$ and $-0.794^{+0.006}_{-0.005}$, respectively. These constraints are strict enough. On the other hand, fitting to the combined SNIa, CMB and LSS data, we find that the NADE model is slightly better than the $\Lambda$CDM model. This makes the NADE model more attractive.

If we add other observations such as Chandra X-ray observations [28], observational $H(z)$ data [29], the lookback time data compiled in [30] and so on, it is expected that the constraints on NADE model will be very tight and the resulting parameters can be used to make some exact predictions of NADE. Of course, the other SNIa datasets such as SNLS [31] and ESSENCE [32] are also useful to constrain the NADE model. It is worth noting that in this work we only used the parameters $R$ and $A$ from WMAP3 and SDSS data, the resulting constraint on $n$ is primary in some sense, and therefore it is of great interest to constrain the NADE model by using the global fitting to the full CMB and LSS data via Markov Chain Monte Carlo (MCMC) analysis. This is an issue which deserves further investigation in the future.

ACKNOWLEDGMENTS

We thank the anonymous referees for quite useful comments and suggestions, which help us to improve this work. We are grateful to Prof. Shuang Nan Zhang for helpful discussions. We also thank Minzi Feng, as well as Hui Li, Yi Zhang, Xing Wu, Li-Ming Cao, Xin Zhang, Jian Wang and Bin Hu, for kind help and discussions. This work was supported in part by a grant from China Postdoctoral Science Foundation, a grant from Chinese Academy of Sciences (No. KJCX3-SYW-N2), and by NSFC under grants No. 10325525, No. 10525060 and No. 90403029.
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