Radiative Higgs Boson Decays $H \to f\bar{f}\gamma$ Beyond the Standard Model

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Abstract

Neutral Higgs boson radiative decays of the form $h_0, H, A \to f\bar{f}\gamma$, in the light fermion limit $m_f \to 0$, are calculated in the two Higgs doublet model at one-loop level. Comparisons with the calculation within the standard model are given, which indicates that these two models are distinguishable in the decay mode fermion-antifermion-photon. Our results show that the concerned process may stand as an implement to identify the Higgs belongings in case there is an intermediate mass Higgs detected.

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I. INTRODUCTION

The discovery of the Higgs boson is one of the most important goals for future high energy physics. Although the Higgs mass cannot be precisely predicted in the Standard Model (SM), it can be constrained and deduced from detecting some processes it involves in. The direct search in the LEP experiments via the $e^+e^- \rightarrow Z^*H$ yields a lower bound of $\sim 77.1$ GeV on the Higgs mass [1]. This search is being extended at present LEP2 experiments, which will probe Higgs masses up to about 95 GeV [2]. After LEP2 the search for the Higgs particles will be continued at the LHC for Higgs masses up to the theoretical upper limit. The detection of the Higgs boson at the LHC will be divided into two mass region: $M_W \leq M_H \leq 130$ GeV, the so-called intermediate mass range, and $130$ GeV $\leq M_H \leq 800$ GeV. For searching the intermediate mass Higgs boson, the decay $H \rightarrow \gamma\gamma$ is still one of the important discovery mode [3], although techniques of secondary vertex detection in experiment is greatly improved in recent years, which allow the detection of secondary vertices from the decay of $b$ quarks in the decay of $H \rightarrow b\bar{b}$ at hadron colliders [4]. For $M_H > 2M_W, 2M_Z$, Higgs decays to real weak bosons become dominant.

Recent investigation [5] indicates that the radiative process $H \rightarrow f\bar{f}\gamma$ also has some unique characters and could be used to supplement Higgs boson searches for the intermediate mass Higgs boson, where $f$ is a light fermion. However, if the Higgs boson should be detected, to determining whether it is a Higgs boson of the SM or one of its extensions is also necessary. Many extensions of the SM contain more than one Higgs doublet. The two Higgs doublet model(2HDM) is one of the extensions [6], which has drawn much attentions these years, because in the minimal supersymmetric extension of the SM (MSSM) [6,7] two Higgs doublets have to be introduced [8]. In the 2HDM, there are three neutral and two charged Higgs bosons, $h_0, H, A, H^\pm$, of which $h_0$ and $H$ are CP-even and $A$ is CP-odd.

In this paper we study the $H \rightarrow f\bar{f}\gamma$ process in the context of the MSSM Higgs sector, where $H$ denotes $h_0, H$, and $A$. We present the decay widths versus Higgs mass changing in intermediate mass region, and compare with the results of the same process in the SM in the case of different parameter choices. In the section II, we present expressions for the decay amplitudes. In the section III, we give our numerical results and discussions.

II. FORMALISM OF HIGGS BOSONS DECAY WIDTHS

The Higgs bosons, $h_0, H, A$ couple to fermions proportionally to their masses. Hence, the lowest order diagrams of the processes $h_0, H, A \rightarrow f\bar{f}\gamma$ are those of loop diagrams in the $m_f \rightarrow 0$ limit. We perform the calculations in the Feynman–’t Hooft gauge and generally set $m_f = 0$ except for phase space intergrations.
The relevant Feynman diagrams for those processes are shown in Fig. 1. The amplitudes for \( A \to f \bar{f} \gamma \) can be expressed as

\[
M_A = M_A^\gamma + M_A^Z,
\]

where

\[
M_A^\gamma = f_A^\gamma(k_1 \cdot k_3 - k_2 \cdot k_3) \bar{u}(k_1) \not\! \gamma_5 v(k_2)(k_2 - k_1) \cdot \epsilon \bar{u}(k_1) \not\! \gamma_5 v(k_2) \tag{2}
\]

and

\[
M_A^Z = 2f_A^Z[a_f(k_2 \cdot k_3 - k_1 \cdot k_3) \bar{u} \gamma v(k_2) - v_f(k_2 \cdot k_3 - k_1 \cdot k_3) \bar{u}(k_1) \not\! \gamma_5 v(k_2) \\
+ a_f \bar{u}(k_1) \not\! \gamma_5 v(k_2)(k_1 - k_2) \cdot \epsilon - v_f \bar{u}(k_1) \not\! \gamma_5 v(k_2)(k_1 - k_2) \cdot \epsilon] \tag{3}
\]

with

\[
f_A^\gamma = \frac{-ie^3gQ_f}{24(k_1 \cdot k_2 + m_f^2)m_W^2\pi^2}[\tan \beta \tan^2 \frac{m_b^2}{m_W^2}C_0(2k_1 \cdot k_2, 0, m_A^2, m_b^2, m_b^2, m_b^2) \\
+ 4 \cot \beta \tan^2 \frac{m_b^2}{m_W^2}C_0(2k_1 \cdot k_2, 0, m_A^2, m_b^2, m_b^2, m_b^2)], \tag{4}
\]

\[
f_A^Z = \frac{-ie^3g}{16\pi^2\sin \theta_w \cos \theta_w m_W(2k_1 \cdot k_2 - m_b^2 + i\Gamma_Z m_Z)} \\
[(-1/2 + 2/3 \sin^2 \theta_w)m_b^2 \tan \beta C_0(2k_1 \cdot k_2, 0, m_A^2, m_b^2, m_b^2, m_b^2) \\
+ 2(-1/2 + 4/3 \sin^2 \theta_w) \cot \beta \tan^2 \frac{m_b^2}{m_W^2}C_0(2k_1 \cdot k_2, 0, m_A^2, m_b^2, m_b^2, m_b^2)]. \tag{5}
\]

Here and below, \( v_f = \frac{I^3_{w,f}}{2\sin \theta_w \cos \theta_w}, a_f = \frac{I^3_{w,f}}{2\sin \theta_w \cos \theta_w} \), \( k_1, k_2, \) and \( k_3 \) denote momentums of light fermions and photon, respectively, \( \tan \beta = v_2/v_1 \), i.e., the ratio of the two vacuum expectation values, and \( C_0, C_{ij} \) and \( D_0, D_{ij} \) are the three-point and four-point Feynman integrals [3].

The amplitude for \( h_0 \to f \bar{f} \gamma \) is given by

\[
M_{h_0} = M_{h_0}^{tri} + M_{h_0}^{box}, \tag{6}
\]

where

\[
M_{h_0}^{tri} = M_{h_0}^{tri,\gamma} + M_{h_0}^{tri,Z}, \quad M_{h_0}^{box} = M_{h_0}^{box,W} + M_{h_0}^{box,Z}, \tag{7}
\]

with

\[
M_{h_0}^{tri,\gamma} = M_{h_0}^{tri,\gamma,fermions} + M_{h_0}^{tri,\gamma,H^\pm} + M_{h_0}^{tri,\gamma,X}, \tag{8}
\]

\[
M_{h_0}^{tri,Z} = M_{h_0}^{tri,Z,fermions} + M_{h_0}^{tri,Z,X} + M_{h_0}^{tri,Z,H^\pm}, \tag{9}
\]

\[
M_{h_0}^{box,W} = \bar{u}(k_1) \not\! \gamma_5 \not\! f_1^{box,W} + \not\! k_3 f_2^{box,W} \cdot \epsilon \cdot k_1 + \not\! k_3 f_3^{box,W} \cdot \epsilon \cdot k_2 (1 - \gamma_5) v(k_2), \tag{10}
\]

\[
M_{h_0}^{box,Z} = \bar{u}(k_1) \not\! \gamma_5 \not\! f_1^{box,Z} + \not\! k_3 f_2^{box,Z} \cdot \epsilon \cdot k_1 + \not\! k_3 f_3^{box,Z} \cdot \epsilon \cdot k_2 [4a_f - \frac{2I^3_{w,f} Q_f}{\cos^2 \theta_w}] \\
+ 2(\tan \theta_w Q_f)^2 - 4a_f (a_f - \tan \theta_w Q_f) \gamma_5] v(k_2). \tag{11}
\]
Here, $X$ denotes $W^\pm, G^\pm, \eta^\pm$, and

\[
M_{h_0}^{\text{tri, fermions}} = \frac{e^3 g Q_f}{24(k_1 \cdot k_2 + m_f^2) m_W \pi^2} \bar{u}(k_1)[-k_3 \cdot (k_1 + k_2) \not \! k + \not \! k_3 \epsilon \cdot (k_1 + k_2)]v(k_2)
\]
\[
\left[ (-C_0 + 4C_{12})(2k_1 \cdot k_2, 0, m_{h_0}, m_b, m_b, m_b) \sin \alpha \sec \beta \\
- 4(-C_0 + 4C_{12})(2k_1 \cdot k_2, 0, m_{h_0}, m_b, m_b, m_b) \cos \alpha \csc \beta, \right]
\] (12)

\[
M_{h_0}^{\text{tri, } H^\pm} = \frac{e^3 g Q_f}{8(k_1 \cdot k_2 + m_f^2) m_W \cos \theta_w \pi^2} \bar{u}(k_1)[-k_3 \cdot (k_1 + k_2) \not \! k + \not \! k_3 \epsilon \cdot (k_1 + k_2)]v(k_2)
\]
\[
[-2 \cos \theta_w m_W \sin(\alpha - \beta) + m_Z \cos(2\beta) \sin(\alpha + \beta)] \times
\]
\[
C_{12}(2k_1 \cdot k_2, 0, m_{H^\pm}, m_{H^\pm}, m_{H^\pm}),
\] (13)

\[
M_{h_0}^{\text{tri, } X} = \frac{-e^3 g Q_f}{8(k_1 \cdot k_2 + m_f^2) \cos \theta_w \pi^2} \bar{u}(k_1)[-k_3 \cdot (k_1 + k_2) \not \! k + \not \! k_3 \epsilon \cdot (k_1 + k_2)]v(k_2)
\] (14)

\[
M_{h_0}^{\text{tri, } Z, \text{ fermions}} = \frac{-e^3 g}{96 \cos \theta_w m_W (2k_1 \cdot k_2 - m_Z^2 + i\Gamma_z m_Z) \pi^2} \sin \theta_w
\]
\[
\bar{u}(k_1)\{2v_f[-k_3 \cdot (k_1 + k_2) \not \! k + \not \! k_3 \epsilon \cdot (k_1 + k_2)]
\]
\[
- 2a_f[-k_3 \cdot (k_1 + k_2) \not \! k + \not \! k_3 \epsilon \cdot (k_1 + k_2)]\}v(k_2)
\]
\[
[\sec \beta \sin \alpha m_0^2(-3 + 4 \sin^2 \theta_w)(C_0 - 4C_{12})(2k_1 \cdot k_2, 0, m_{h_0}, m_b, m_b, m_b)
\]
\[
- 2 \csc \beta \cos \alpha m_0^2(-3 + 8 \sin^2 \theta_w)(C_0 - 4C_{12})(2k_1 \cdot k_2, 0, m_{h_0}, m_b, m_b, m_b)],
\] (15)

\[
M_{h_0}^{\text{tri, } Z, H^\pm} = \frac{-e^2 g^2 \cos(2\theta_w)}{8 \cos \theta_w (2k_1 \cdot k_2 - m_Z^2 + i\Gamma_z m_Z) \pi^2}
\]
\[
\bar{u}(k_1)\{2v_f[-k_3 \cdot (k_1 + k_2) \not \! k + \not \! k_3 \epsilon \cdot (k_1 + k_2)]
\]
\[
- 2a_f[-k_3 \cdot (k_1 + k_2) \not \! k + \not \! k_3 \epsilon \cdot (k_1 + k_2)]\}v(k_2)
\]
\[
[- \sin(\alpha - \beta) m_W + \frac{m_Z \cos(2\beta) \sin(\alpha + \beta)}{2 \cos \theta_w}]C_{12}(2k_1 \cdot k_2, 0, m_{H^\pm}, m_{H^\pm}, m_{H^\pm}),
\] (16)

\[
M_{h_0}^{\text{tri, } Z, X} = \frac{-e^2 g^2}{32 \cos^2 \theta_w (2k_1 \cdot k_2 - m_Z^2 + i\Gamma_z m_Z) \pi^2}
\]
\[
\bar{u}(k_1)\{2v_f[-k_3 \cdot (k_1 + k_2) f_{h_0, z}^2 \not \! k + \not \! k_3 \epsilon \cdot (k_1 + k_2)]\}f_{h_0, z}^1
\]
\[
- 2a_f[-k_3 \cdot (k_1 + k_2) f_{h_0, z}^2 \not \! k + \not \! k_3 \epsilon \cdot (k_1 + k_2)]\}v(k_2).
\] (17)

In above eqs. $f_{h_0, \gamma}^i$, $f_{h_0, z}^i$, $f_{box, W}^i$, and $f_{box, Z}^i$ are form factors, and their explicit expressions are given by

\[
f_{h_0, \gamma}^1 = -4 \cos \theta_w m_W \sin(\alpha - \beta)C_0(2k_1 \cdot k_2, 0, m_W^2, m_W^2, m_W^2) + [6 \cos \theta_w m_W \sin(\alpha - \beta)
\]
\[
+ m_Z \cos(2\beta) \sin(\alpha + \beta)]C_{12}(2k_1 \cdot k_2, 0, m_W^2, m_W^2, m_W^2),
\] (18)
\[ f_{h_0,\gamma}^2 = f_{h_0,\gamma}^1 - \frac{\cos \theta_w m_W}{4} \left[ 6 \sin(\alpha - \beta) - \frac{2m_{h_0}^2 \sin(\alpha - \beta) - m_Z^2 \cos(2\beta) \sin(\alpha + \beta)}{k_1 \cdot k_2 + k_2 \cdot k_3} \right] C_0(2k_1 \cdot k_2, 0, m_{W_1}^2, m_{W_2}^2, m_{W_3}^2), \]  

(19)

\[ f_{h_0,Z}^1 = 4 \cos^2 \theta_w \sin(\alpha - \beta) \left\{ (3 \cos \theta_w m_W - m_Z \sin^2 \theta_w) C_0(2k_1 \cdot k_2, 0, m_{W_1}^2, m_{W_2}^2, m_{W_3}^2) ight. \\
\left. + \left[ -20 \cos^3 \theta_w m_W \sin(\alpha - \beta) + \cos \theta_w \sin^2 \theta_w m_W \sin(\alpha - \beta) + 3 \cos^2 \theta_w \sin^2 \theta_w m_Z \\
- \cos(2\theta_w) m_Z \cos(2\beta) \sin(\alpha + \beta) \right] C_{12}(2k_1 \cdot k_2, 0, m_{W_1}^2, m_{W_2}^2, m_{W_3}^2) \right\}, \]  

(20)

\[ f_{h_0,Z}^2 = f_{h_0,Z}^1 - \frac{\cos \theta_w m_W}{4} C_0(2k_1 \cdot k_2, 0, m_{W_1}^2, m_{W_2}^2, m_{W_3}^2) \left\{ 6 \sin(\alpha - \beta) \cos^2 \theta_w \\
- \left[ (2 \cos^2 \theta_w + 1)m_{h_0}^2 - 3 \cos^2 \theta_w m_Z^2 \right] \sin(\alpha - \beta) - m_Z^2 \sin^2 \theta_w \cos(2\beta) \sin(\alpha + \beta) \right\}, \]  

(21)

\[ f_{\text{box},W}^1 = \frac{eg^3 m_W \sin(\alpha - \beta)}{128\pi^2} \left\{ 3C_0(0, 2k_2 \cdot k_3, m_{h_0}^2, m_{W_1}^2, 0, m_{W_2}^2) \\
+ (3m_{W_1}^2 - 8k_2 \cdot k_3) D_0(0, 0, m_{h_0}^2, 0, 2k_1 \cdot k_2, 2k_2 \cdot k_3, m_{W_2}^2, 0, m_{W_3}^2) \\
- 8k_2 \cdot k_3 D_1(0, 0, m_{h_0}^2, 0, 2k_1 \cdot k_2, 2k_2 \cdot k_3, m_{W_2}^2, 0, m_{W_3}^2) \\
+ 2(3k_1 \cdot k_2 + 4k_1 \cdot k_3 - 4k_2 \cdot k_3) D_2(0, 0, m_{h_0}^2, 0, 2k_1 \cdot k_2, 2k_2 \cdot k_3, m_{W_2}^2, 0, m_{W_3}^2) \\
- 6k_2 \cdot k_3 D_3(0, 0, m_{h_0}^2, 0, 2k_1 \cdot k_2, 2k_2 \cdot k_3, m_{W_2}^2, 0, m_{W_3}^2) \\
+ 8D_{00}(0, 0, m_{h_0}^2, 0, 2k_1 \cdot k_2, 2k_2 \cdot k_3, m_{W_2}^2, 0, m_{W_3}^2) \right\} \\
+ (k_1 \cdot k_3 \leftrightarrow k_2 \cdot k_3), \]  

(22)

\[ f_{\text{box},W}^2 = -\frac{eg^3 m_W \sin(\alpha - \beta)}{16\pi^2} (D_1 + D_{22})(0, 0, m_{h_0}^2, 0, 2k_1 \cdot k_2, 2k_2 \cdot k_3, m_{W_2}^2, 0, m_{W_3}^2) \\
+ (k_1 \cdot k_3 \leftrightarrow k_2 \cdot k_3), \]  

(23)

\[ f_{\text{box},W}^3 = \frac{eg^3 m_W \sin(\alpha - \beta)}{16\pi^2} (D_0 + D_2 + D_2 - D_{13} - D_{23})(0, 0, m_{h_0}^2, 0, 2k_1 \cdot k_2, 2k_2 \cdot k_3, m_{W_2}^2, 0, m_{W_3}^2) \\
+ (k_1 \cdot k_3 \leftrightarrow k_2 \cdot k_3), \]  

(24)

\[ f_{\text{box},Z}^1 = \frac{eg m_Z \sin(\alpha - \beta) Q_f}{16\pi^2 \cos \theta_w} \left\{ -C_0(0, k_2 \cdot k_3, m_{h_0}^2, m_{Z_1}^2, m_{Z_2}^2) \\
+ 2(k_2 \cdot k_3 D_1 + D_{00})(0, 0, m_{h_0}^2, 0, 2k_1 \cdot k_2, 2k_2 \cdot k_3, m_{Z_1}^2, m_{Z_2}^2, m_{Z_3}^2) \right\}, \]  

(25)

\[ f_{\text{box},Z}^2 = \frac{eg m_Z \sin(\alpha - \beta) Q_f}{8\pi^2 \cos \theta_w} (D_{22} + D_{23})(0, 0, m_{h_0}^2, 0, 2k_1 \cdot k_2, 2k_2 \cdot k_3, m_{Z_2}^2, m_{Z_3}^2, m_{Z_4}^2), \]  

(26)

\[ f_{\text{box},Z}^3 = -\frac{eg m_Z \sin(\alpha - \beta) Q_f}{8\pi^2 \cos \theta_w} (D_1 + D_{12} + D_{13})(0, 0, m_{h_0}^2, 0, 2k_1 \cdot k_2, 2k_2 \cdot k_3, m_{Z_3}^2, m_{Z_4}^2). \]  

(27)
The amplitude of the process $H \to f \bar{f} \gamma$ can be simply obtained by substituting $\alpha \to \frac{3\pi}{2} + \alpha$ and $m_{h_0} \to m_H$ in the amplitude of $h_0 \to f \bar{f} \gamma$.

For the simplicity of calculating the amplitude squares, we can parameterize the amplitudes of the process $(h_0, H, A) \to f \bar{f} \gamma$ in a general form

$$M = \bar{u}(k_1)(g_1 f + g_2 \phi \gamma_5 + g_3 k_3 \epsilon \cdot k_1 + g_4 k_3 \gamma_5 \epsilon \cdot k_1 + g_5 k_3 \epsilon \cdot k_2 + g_6 k_3 \gamma_5 \epsilon \cdot k_2)\nu(k_2). \quad (28)$$

Therefore, the amplitude square is given by

$$\sum_{\text{spins}} |M|^2 = 8[(g_1^2 + g_2^2)(k_1 \cdot k_2) + 2 \text{Re}(g_3 g_5^\dagger + g_4 g_6^\dagger)(k_1 \cdot k_2 k_2 \cdot k_3 k_1 \cdot k_3)]. \quad (29)$$

Here $g_i$ are form factors, which can be expressed as the combinations of the form factors given above. Their tedious expressions are not shown here.

The differential decay widths can be written as

$$\frac{d\Gamma(h_0, H, A \to f \bar{f} \gamma)}{d(k_1 \cdot k_2)} = \frac{1}{256\pi^3 m_{h_0, H, A}^2} \int_{(k_2 \cdot k_3)_{\text{min}}}^{(k_2 \cdot k_3)_{\text{max}}} d(k_2 \cdot k_3) \sum_{\text{spin}} |M|^2$$

with

$$(k_2 \cdot k_3)_{\text{min}} = \frac{1}{4}[m_{h_0, H, A}^2 - 2(m_f^2 + k_1 \cdot k_2)](1 - \sqrt{1 - \frac{2m_f^2}{m_f^2 + k_1 \cdot k_2}}),$$

$$(k_2 \cdot k_3)_{\text{max}} = \frac{1}{4}[m_{h_0, H, A}^2 - 2(m_f^2 + k_1 \cdot k_2)](1 + \sqrt{1 - \frac{2m_f^2}{m_f^2 + k_1 \cdot k_2}}).$$

### III. NUMERICAL RESULTS AND DISCUSSIONS

In our numerical calculation the relevant parameters are chosen as

$$m_t = 176 \text{ GeV}, \ m_b = 4.5 \text{ GeV}, \alpha(M_z) = 1/128,$$

$$M_z = 91.2 \text{ GeV}, \ M_w = 80.3 \text{ GeV}, \ \Gamma_z = 2.5 \text{ GeV}, \quad (30)$$

The Higgs boson masses $m_{h_0}$, $m_H$, and $m_{H^\pm}$ are determined by $m_A$ and $\tan \beta$ as follows

$$m_{h_0}^2 = \frac{1}{2} \left[ m_A^2 + M_z^2 + \epsilon - \sqrt{(m_A^2 + M_z^2 + \epsilon)^2 - 4m_A^2 M_z^2 \cos^2 \beta - 4\epsilon (m_A^2 \sin^2 \beta + M_z^2 \cos^2 \beta)} \right],$$

$$m_H^2 = m_A^2 + M_z^2 - m_{h_0}^2 + \epsilon, \quad (31)$$

and

$$m_{h_0}^2 = \frac{1}{2} \left[ m_A^2 + M_z^2 + \epsilon - \sqrt{(m_A^2 + M_z^2 + \epsilon)^2 - 4m_A^2 M_z^2 \cos^2 \beta - 4\epsilon (m_A^2 \sin^2 \beta + M_z^2 \cos^2 \beta)} \right],$$

$$m_{h_0}^2 = \frac{1}{2} \left[ m_A^2 + M_z^2 + \epsilon - \sqrt{(m_A^2 + M_z^2 + \epsilon)^2 - 4m_A^2 M_z^2 \cos^2 \beta - 4\epsilon (m_A^2 \sin^2 \beta + M_z^2 \cos^2 \beta)} \right],$$

$$m_H^2 = m_A^2 + M_z^2 - m_{h_0}^2 + \epsilon, \quad (32)$$

and
\[ m_{H^\pm}^2 = m_A^2 + m_W^2 \]  
(33)

with
\[
\epsilon = \frac{3G_F m_t^4}{\sqrt{2}\pi^2 \sin^2 \beta} \log(1 + \frac{m_S^2}{m_t^2}). \]  
(34)

Here the \( m_S \) is a common squark mass which is equal to 1 TeV in our numerical calculations. The mixing angle \( \alpha \) is fixed by \( \tan \beta \) and the Higgs boson mass \( m_A \).

\[
\tan 2\alpha = \tan 2\beta \frac{m_A^2 + M_z^2}{m_A^2 - M_z^2 + \epsilon/cos^2\beta}, \]  
(35)

where \(-\frac{\pi}{2} < \alpha < 0\).

Fig. 2 and Fig.3 show the total decay widths of the processes \( h_0, H, A \to f\bar{f}\gamma \) versus the Higgs mass varying in the intermediate range for two values \( \tan \beta = 1.5 \) and \( \tan \beta = 30 \). As can be seen from these figures, the total decay widths for \( h_0, H, A \to f\bar{f}\gamma \), where the neutrino, electron, muon, and light quarks contributions are included, have some obvious characters comparing with the two-photon widths of the MSSM Higgs bosons decays \[^{[1],[2]}\]. First, in the case of \( \tan \beta = 1.5 \) the width \( \Gamma_{H\to f\bar{f}\gamma} \) can exceed the width \( \Gamma_{H\to \gamma\gamma} \) for \( 140 \text{GeV} \leq m_H \leq 200 \text{Gev} \). However, in the same Higgs-mass range \( \Gamma_{h_0\to f\bar{f}\gamma} \) and \( \Gamma_{A\to f\bar{f}\gamma} \) are smaller than \( \Gamma_{h_0\to \gamma\gamma} \) and \( \Gamma_{A\to \gamma\gamma} \), respectively. And, the width \( \Gamma_{H\to f\bar{f}\gamma} \) less than that of the the width \( \Gamma_{H\to \gamma\gamma} \) for \( m_H < 140 \text{Gev} \).

Second, in the case of \( \tan \beta = 30 \) the width \( \Gamma_{H\to f\bar{f}\gamma} \) is still larger than \( \Gamma_{H\to \gamma\gamma} \) for \( 140 \text{GeV} \leq m_H < 200 \text{GeV} \) and \( \Gamma_{h_0\to f\bar{f}\gamma} \) is smaller than \( \Gamma_{h_0\to \gamma\gamma} \). Only in the vicinity of \( M_{h_0,H} \approx 130 \text{ GeV} \) the decay widths \( \Gamma_{h_0\to f\bar{f}\gamma} \) and \( \Gamma_{H\to f\bar{f}\gamma} \) are about the same as \( \Gamma_{h_0\to \gamma\gamma} \) and \( \Gamma_{H\to \gamma\gamma} \), respectively. Again, the width for the radiative decay of the pseudoscalar \( \Gamma_{A\to f\bar{f}\gamma} \) is smaller than \( \Gamma_{A\to \gamma\gamma} \) for \( M_A < 200 \text{ GeV} \).

Comparing with the same process of the SM \[^{[3]}\], we find that in general the widths in the MSSM case are less than that in the SM case, except Higgs mass is around 130 Gev, where the predictions of the SM and MSSM on the \( f\bar{f}\gamma \) widths are almost the same and indistinguishable.

In conclusion, in addition to be a supplement in searching the Higgs boson through the Higgs decay to two photons, the radiative decay of Higgs boson \( H \to f\bar{f}\gamma \) would be an more observable channel in searching the Higgs boson in the future experiment since the radiative decay widths can be larger than the two-photon decay mode for some favorable parameter space. Besides, our calculation also shows that the process \( h_0, H, A \to f\bar{f}\gamma \) may play an important role in identifying the Higgs boson being of the SM or the MSSM on the basis of the size of decay widths if the Higgs boson is discovered via this process.

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Figure Captions

Fig.1. The generic Feynman diagrams of $h_0, H, A \to f \bar{f} \gamma$ processes.

Fig.2. The neutral Higgs decay widths versus Higgs masses of $h_0, H, A \to f \bar{f} \gamma$ processes in MSSM with tan$\beta = 1.5$.

Fig.3. The neutral Higgs decay widths versus Higgs masses of $h_0, H, A \to f \bar{f} \gamma$ processes in MSSM with tan$\beta = 30$.
FIGURES

FIG. 1.
FIG. 3.