Guided discovery: an alternative teaching method to reduce students’ rote learning behavior in studying geometric transformation

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Abstract. In Indonesia, geometry transformation is taught from elementary to high school students, but at the high school level geometric transformation material already has a high level of complexity that is already involved in the concept of matrices in the formula, especially the number of formulas that must be understood is very large, giving rise to the tendency of students to learn by memorization (rote learning). Of course, this matter cannot be underestimated by mathematics teachers so it needs a solution that can reduce the tendency of students to learn using the rote-learning method. Therefore, learning methods Guided Discovery can be seen as an alternative solution to be able to reduce the tendency of students to learn by memorization. This learning method can involve students actively to find and reconstruct formulas and concepts in the topic of transformation of geometry independently. Moreover, because they have studied the topic of geometry transformation at a previous level, they will use their prior knowledge to study it so that learning will be more meaningful. This research is a literature study where it aims to describe the possibility of applying Guided Discovery methods to reduce the tendency of students to learn geometry transformation topics in rote learning (rote learning behaviour). The results of this study are that theoretically by using Guided Discovery learning methods is possible to reduce the tendency of students to learn by memorizing in learning the topic of Geometry Transformation, namely the steps of Giving Problems, Data Developing, Data Arrangement, Extra Data, Verification, and Exercise.

Keywords: guided discovery, rote-learning behaviour, geometric transformation

1. Introduction
Geometry Transformation is one branch of geometry that examines changes in the position of points in a Cartesians plane. From the set point of view, geometry transformation is a function whose domain and range are sets of points [1]. Along with the development of science, the concepts of transformation of geometry developed rapidly and the concept helped scientists to develop many technologies as in quantum image processing [2], in the innovation of camera technology to produce panoramic images [3], in the technology of biometrics [4], and many more applications based on this branch of geometry. Considering the many applications of geometric transformations in everyday life especially in the development of technology, it is considered important to be mastered as early as possible. In terms of education in Indonesia based on 2013 Curriculum, geometric transformation is taught at the level of Senior High School (SMA) in grade XII semester 2.
At that level, students are taught four concepts in the material of geometric transformation, namely reflection, translation, rotation, and dilation where all the concepts have a lot of concepts that must be understood by students. Of course, students are not the first to recognize geometry transformation only at the high school level, but this topic has been introduced since elementary and junior high school. At the elementary school level, students are introduced to the topic of geometry transformation without using the Cartesian diagram so that the emphasis is only on the distance of the starting point and end point, as well as the congruence between the initial plane and the final plane. At the junior high school level, the teacher deepens students' understanding of the topic of geometry transformation using the Cartesian diagram. At this stage, students already know the shadow coordinate points of a transformed point. While at the high school level, the topic of Geometry Transformation is even deeper, namely about the process of moving point objects from a matrix point of view. At this stage students must learn many concepts related to the displacement of points by reflection, rotation, translation and dilation. For example, as in the concept of reflection where students must understand the matrix formula of point displacement reflected by a line $k$, point $P (a, b)$, the origin point, $x$-axis, and $y$-axis. Therefore, the number of formulas and concepts that must be understood by students will provide a cognitive burden for students which will eventually make students have a tendency to learn by memorizing them (rote-learning behavior) because this is the easiest way to do [5].

Rote learning is one learning method where students learn information by memorizing that information. This method is very good when applied to study information whose level of urgency is temporary such as remembering phone numbers or house numbers. In mathematics learning, especially the topic of geometry transformation, the rote learning method is very ineffective to be applied because mathematics has abstract work-objects so it requires a very deep understanding to learn it. Mathematics requires mastery of concepts, not more about memorizing concepts [6]. The result of applying the rote learning method is that students formally know a concept, but because they do not understand it, they will not be able to use it to solve their daily problems [7]. In addition, the rote learning method also does not involve cognitive activities at all. This is in line with Ausable's statement (in Bell [8]), “learning is meaningful only when it can be related to concepts that already exist in a person’s cognitive structure. Rote learning (behaviorism-based), on the other hand, does not become linked to a person’s cognitive structure and hence is easily forgotten”.

Apart from that, the rote learning method is not very suitable with the existing curriculum in Indonesia, namely Curriculum of 2013 where the learning process must get to know not to be told [5]. Therefore to be able to overcome it, the learning method that must be used is a method of meaningful learning where students learn in full a concept including how the concept is formed, or in other words students understand what they are learning [9]-[10].

The tendency of students to learn using the rote learning method can be caused by three factors, namely the factors of teachers, students, and learning resources that they have. The teacher factor will be related to the way to teach and how to convey the material. Next, the student factor which is in accordance with what has been stated before is due to the cognitive load that is owned by students so that they tend to take the easy way, namely by memorizing it. As for learning resource factors such as student handbooks, for example, there are many books that only give steps to formulas or steps to how a concept can emerge. The books do not provide space for students to be involved in the formation of concepts or formulas that they want to learn. Not only for books produced by private publishers, but also for the books that published by government. Indonesian Mathematics Electronic School Books is online based books that is free to be downloaded and is provided by the Government of Indonesia to facilitate students to study mathematics. Moreover, some schools use the e-books as student’s textbook in the learning process, and sometimes that books become the only reference for the students to study. The following is an example of one page of activity in an electronic book on the topic of geometry transformation with a concept of reflection that does not involve students actively in the process of concept discovery:
Figure 1. A process to find the coordinate of a reflected point

Figure 1. is an example of one of the pages in the high school grade XII Indonesian Mathematics Electronic School Book which contains related to the explanation of the concept of reflection from a point to the y-axis, lines $y = x$ and lines $y = -x$. On the page, you can see the concepts of reflecting points on the y-axis, lines $y = x$, and $y = -x$ explained in their entirety and ending with a "formula box". The first thing to note is that, as the appropriate textbook in general, this book does not involve students actively in the discovery of the concept of reflecting the point in question. The next thing to consider is the affirmation of the formula that has been found by squaring the founded formula. If this case is considered in detail, then there will be a tendency for students to immediately pay attention to the formulas that have been found without understanding how the process of getting the formula learned. Thus, if students intend to use the rote learning method in their learning process, then this book is enough to "facilitate" it because the formulas they want to learn have been shown clearly. From those concerns, it can be seen that the available books still allow students to be able to learn the concept of geometry transformation by memorizing which certainly will not have a positive impact on students.

Therefore we need a teaching method that can be used to reduce the tendency of students to learn the concept of geometry transformation in memorization, so that students can learn more meaningfully. In addition, the method must also be able to actively involve students in the process of constructing an understanding of a concept that they want to learn. Thus, students can have experiences that they can use to rebuild the concepts they have learned when they forget the concept. One learning method that
can be an alternative solution is Guided Discovery learning methods. Guided Discovery is a teaching method that can involve the students actively to find new knowledge based on their prior knowledge under the guidance of the teacher [5]. Guided Discovery is also a method that has long been seen as an effective method for introducing understanding concepts from theories and principles [11]. Previous studies show that the use of this learning method can improve students' understanding of the concepts they want to learn [12, 13, 14, 15, 16]. Thus, by employing this teaching method, students’ involvement in reconstructing the concept of geometric transformation will be accommodated. Based on the description, this study’s aim is to describe theoretically of the possibility of applying Guided Discovery methods to reduce the tendency of students to learn geometry transformation topics in rote learning (rote learning behavior).

2. Method
The type of this research is literature review research where studying references related to the application of Guided Discovery learning methods in reducing the tendency of students to learn memorization in learning Geometric Transformation. Apart from articles, journals and books, this study also examines the experience of researchers in the development and use of Guided-Discovery-based students’ worksheet in reducing the tendency of students rote learning behaviour in studying the concepts of Geometric Transformation.

3. Finding and Discussion
Guided Discovery
In the very first time, Discovery was a method promoted by Plato when he told a story about the dialogue between Socrates and a young slave where by the end his method is well-known as Socratic method [17]. Bruner says that discovery is a process, a way of approaching problems rather than a product or particular item of knowledge [17]. This means that discovery learning is a learning to discover. Abruscato stated that Discovery learning is hands-on, experiential learning that requires a teacher’s full knowledge of content, pedagogy, and child development to create an environment in which new learnings are related to what has come before and to that which will follow [18]. While Moore defined discovery learning as an intentional learning through supervised problem solving following the scientific method of investigation [19]. Thus, by compiling the definitions of discovery learning by Bruner, Abruscato and Moore, it can be summarized that discovery learning is a learning process which can actively involve students to rediscover a particular concept or skill through investigative steps.

Moore divided discovery learning into three types based on the level of difficulty, namely Guided Discovery, Modified Discovery, and Open Discovery [19]. First, Guided Discovery is a level where students are fully guided by the teacher in rediscovering a concept. In this level, the teacher guides, gives direction, and provides exercises to students through the form of investigation as a way of constructing the concepts that they want to learn. Second, Modified Discovery is the level where the teacher's role in guiding students begins to decrease, and guidance is only given as needed. Third, Open Discovery is the level where teachers no longer provide guidance and students are considered adults to be able to find concepts that they want to learn independently. Of the three types of discovery learning, Open Discovery is a level for professional scientists who can independently discover new concepts or knowledge independently. Modified Discovery is a suitable level for university students who are studying to switch to professional researchers who do not need full teacher guidance. Whereas Guided Discovery is the level for students where they still need full guidance from the teacher. Thus, Guided Discovery the most appropriate learning method to reduce students’ rote learning behavior in studying geometric transformation.

There are some definitions about Guided Discovery such as what is stated by Hammerman that Guided Discovery was the name to hand-on activities and laboratory investigation that led the learner to a predetermined or a predictable data set or response [20]. While Bruner said that Guided Discovery is a teaching method based on inquiry, a constructive teaching theory within a problem solving situation where the student use their prior knowledge and experiences to find facts, correlations, and truths to be
learned [21]. Therefore, according to those two statements, it can be drawn that Guided Discovery method is a method to direct students to construct their knowledge through a discovery of new concepts and knowledge under teacher guidance.

Naturally a learning method, the Guided Discovery teaching method also has several steps that can be done during the learning process. Some experts have formulated learning steps using the Guided Discovery method. Kartawisstra et.al. states there are nine steps in conducting learning with Guided Discovery methods [22], those are:

a. The problem that will be solved is stated in various "statements" or "questions".
b. It is clearly stated the level / class of students who will take part in learning.
c. The concepts or principles that students must find are clearly written.
d. It is necessary to provide tools / materials according to the needs of students in carrying out the discovery activities.
e. Briefing discussions are conducted in the form of question and answer between students and teachers before students carry out discovery activities.
f. Discovery learning activities can be in the form of investigations / experiments to find the concepts or principles that have been set.
g. The critical thinking process needs to be explained to show the existence of students' mental operation that is expected in the activity.
h. Questions that lead to the development of student inquiry activities need to be given.
i. Teacher records include an explanation of the difficult parts of the lesson and the factors that can influence their success, especially if the investigation activity fails or does not go as planned.

Soedjadi stated that Guided Discovery learning methods have six steps that can be done during the learning process, namely [23]:

a. Giving Problem
   Students required to understand the given problem.
b. Data Development
   Students required to seek or pointing the possibilities other data as the continuity of the known data.
c. Data Arrangement
   Students required to arrange the data obtained for the first step and second steps in a list.
d. Extra Data
   Students required to add extra data as the continuity of the known data if the expected pattern is not obtained yet.
e. Answer The Problem
   Students answer the problem in the first step.
f. Checking The Answer
   Students required to see the truth of the general pattern that is obtained with some available data.

While Hirdjan in [24] states that there are 10 steps that can be done in implementing Guided Discovery teaching method in class, those are:

a. Teacher determines the task criteria such as giving problems. Then the students find solutions for the problems. The problem that has been given should contain clues about things students need to do, namely students find the solutions of the problems by themselves.
b. Smart students will have possibilities to find the answer by themselves without teacher’s guidance. Otherwise, students who incapable to find the answer need to have their first guidance by the teacher. This guidance should be in form of simple questions.
c. Having first guidance, the students who able finish the problem should check their answer by using the provided data. While for the student who cannot find the answer, they get second guidance by the teacher. The guidance is in form of questions to arrange the data that already available. The aim of this guidance is to get samples of the answer from some of the provided problems.
d. After the second guidance has already given, the students who succeed getting the answer of the provided problems should check their answer using the data that already exist. The students who do not able to find their answer get the third guidance from the teacher which in form of additional
data that is arranged into a list. The expected goal of this third guidance is to make students can find the answer. If by giving this additional data the student do not find the answer yet, the teacher have to give another short guidance verbally for the students to get the expected answer right away.

e. Students are required to check their answer after they get their third guidance.

f. After they check their answer, they use it to finish the task criteria.

h. The answer from the task criteria is still conjectural, so a proof is needed to verify it. Thus, the teacher must give the answer a confirmation whether the students are correct or not.

i. Teacher gives enrichment to the students in form of applicable problems. It is expected in solving the problems, they can use the concept they just have got.

j. If the student can answer the problems correctly, so it can be said that they succeed constructing new knowledge about a concept that they have learned.

On the cognitive aspect, Guided Discovery has some benefits that can be acquired by the students after this method is implemented [25], those are:

a. It encourages analytic learning.

b. It exploits learners’ cognitive skills.

c. It improves critical thinking skills.

d. It involves students in problem solving tasks.

e. It helps learners become aware of and articulate their mental process.

f. Learner actively in the learning process.

g. Learners understand and remember better what they have work out for themselves.

As in general, the application of the Guided Discovery method also has advantages and disadvantages in its application. Marzano stated that the application of the Guided Discovery method has the following disadvantages and advantages [26]:

| Table 1. Advantages and Disadvantages of Guided Discovery Method |
|---------------------------------|---------------------------------|
| **Advantages**                  | **Disadvantages**               |
| a. Students will actively participate during the learning process. | a. Not all of the students can follow the lessons in this way since some of them are still familiar and easily understood by the lecturing method. |
| b. Instill well as foster an attitude of inquiry. | b. Not all topics are suitable delivered with this method. Generally, topics related to the principles of the model can be developed with Guided Discovery. |
| c. Support students’ problem-solving skill. | |
| d. Provide a space for interaction among the students, as well as students to teachers, in order to train the students to communicate their idea properly and correctly. | |
| e. The subject being studied can achieve a high level of capability and longer lasting because the students are involved to find new knowledge. | |

**How To Use Guided Discovery Teaching Method in Teaching Geometric Transformation?**

Geometric transformation has four concepts that have to be taught to students i.e. reflection, translation, rotation, and dilation. Followings are the definitions of those concepts [27]:

| Table 2. The Definition of Translation, Reflection, Rotation and Dilation |
|---------------------------------|---------------------------------|
| **Concept** | **Definition** |
| Translation | A translation, denoted by $T_{(u,v)}$, is a function that moves every point a given distance $u$ in the $x$-direction and a given distance $v$ in the $y$-direction. The matrix for translation can be represented as follows: |
Reflection The reflection across a line ℓ, denoted by F_ℓ, is the function that maps a point P to a point F_ℓP such that:
(1) If P is on ℓ, then F_ℓP = P.
(2) If P is not on ℓ, then ℓ is the perpendicular bisector of the segment connecting P and F_ℓP.

Rotation A rotation of θ degrees about the origin, denoted by R_θ, is a function that maps a point P to a point R_θP such that:
(1) The points P and R_θP are equidistant from the origin.
(2) An angle of θ degrees is formed by P, the origin, and R_θP.

Dilation A dilation is a transformation of the plane with center O, while scale factor r (r>0) is a rule that assigns to each point P of the plane a point Dilation(P) so that:
1. Dilation(O)=O, (i.e., a dilation does not move the center of dilation.)
2. If P≠O, then the point dilation(P), (to be denoted more simply by P') is the point on the ray \( \overrightarrow{OP} \) so that |OP'|=r|OP|.

From Table 1., these concepts can be combined with the steps of the Guided Discovery method. Of the three steps of applying the Guided Discovery method which have been stated by the three experts, the steps of Guided Discovery is preferred from two others' steps as Hirdjan's version is more flexible because this version provides some loops. For example, in learning translation concepts, for smart students or anyone who can complete the first step, it does not have to complete the second step, but can jump directly to the steps above. So that learning becomes more effective and efficient. Therefore, the Guided Discovery steps using in this paper is an adoption from Hirdjan’s steps of Guided Discovery teaching method. As a result of the modifications are as follows:

a. Giving Problems
The teacher gives a problem, and learners seek resolution of the problem. The given problem should contain clues to the direction and the objectives about what they have to do. Such as they find the solution by themselves from the given problem.

b. Data Developing
In this phase, students are required to find/pointing to the possibility of other data as a continuation of the data that is already known. Students who are smart enough will finish the problem without guidance. Otherwise, they will get a guidance in the form of developed questions from the simplest way.

c. Data Arrangement
In this phase, teacher guides the students by giving them a more specific ways to find the formula using the data in step 1 and step 2. This way is in form of the steps to find the formula, but not in a general way.

d. Extra Data
Teacher gives students extra data that will direct them to the targeted formula. It is expected that with this guidance, students can determine the formula.

e. Verification
In this step, the students are required to verify the formula they have found by themselves. If the verification is correct, so they can continue to the next step. Otherwise, they need to recheck their work in the previous step, or they can consult with the teacher of their friends who already finish their work correctly.

f. Exercises
Teacher gives the students some problems, and it is expected that they do the problems use the formulas they found.

The following is the use of the Guided Discovery method, which uses modified steps, on the topic of geometry transformation on the concept of reflection specifically to find the transformation matrix on the reflection of a point $P(x,y)$ to $x$-axis:

a. Giving Problems
   At this stage, students are asked to determine the reflection matrix from the point $P(x,y)$ to the $x$-axis which produces an image point that is $P'(x,y)$. If they can declare the intended transfer matrix and are sure of the answer, students can proceed to the Verification stage provided that if their answers are wrong, they must proceed to the second stage, Data Developing. But if students do not find a problem, then they can proceed to the final step.

b. Data Developing
   Students get guidance in the form of new data by specifying the coordinates of point $P$ into numbers such as $P(2,3)$ and $P'(2,-3)$. If with this help students feel they can determine the reflection matrix in question, then students can proceed to the Verification stage. Otherwise, they have to continue to the third steps.

c. Data Arrangement
   The sample of this step can be seen in the following illustration:

   ![Illustration](image)

   Based on the illustration, it is known that the reflected point $P(2,3)$ is $P'(2,-3)$. Thus, if it is changed in the form of a matrix to be the following:

   $2 = 2$
   $3 = -3$

   Then,

   \[
   \begin{pmatrix}
   2 \\
   -3
   \end{pmatrix}
   =
   \begin{pmatrix}
   \cdots & \cdots & 2 \\
   \cdots & \cdots & 3
   \end{pmatrix}
   \]

   Have you been able to determine the matrix of reflected point for the $x$-axis? If yes, then please proceed to stage - e, if not, please proceed to stage - d.
d. **Extra Data**

The sample for this step can be seen in the following illustration:

Let \( x = 2, \ y = 3, \ x' = 2, \) dan \( y' = -3, \) so take a look at the following illustration:

Based on the illustration, it is known that the shadow of the point \( P(x, y) \) is \( P'(x, -y). \) Thus, if it is changed in the form of a matrix to be the following:

\[
\begin{bmatrix}
  x' \\
  y'
\end{bmatrix}
= \begin{bmatrix}
  x \\
  y
\end{bmatrix}
\]

Of course, you have found the matrix of reflected point \( P(x, y) \) to the x-axis, then please go to stage - e.

e. **Verification**

At this stage, students are asked to rewrite the reflection matrix point \( P(x, y) \) on the x-axis they have found. Then they are asked to use the matrix to find the image point coordinates from the point \( P \) if the coordinates are known \((1.6)\) so they can determine whether their answers are correct or incorrect seen from their answers which if true then the answer is \( P'(1,-6). \) If their answer is correct, so they can continue with the sixth step, otherwise they can discuss their friend until they get the right answer.

f. **Exercise**

In this stage, teacher can provide some enrichment by giving them problems new problems related to reflection.

Of the six steps, students can know and understand the process of forming a P image point against the x-axis. In general, these steps can also be applied to other geometric transformation concepts. If it is concluded in general, the Guided Discovery method provides space for students to be able to play an active role in the formation of a concept and avoid students from the tendency to learn using the rote learning method. So that learning becomes more meaningful.

**4. Conclusion**

According to the discussion section, in a nutshell, the implementation of Guided Discovery teaching method has a huge possibility in reducing students' tendency in studying Geometric Transformation by using memorizing method. Therefore, by having the six modified-Guided-Discovery-steps, the learning process will become more meaningful.
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