Multiple velocity estimation by maximum likelihood from MRI complex signals

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Abstract. The measurement of fluid velocity by encoding it in the phase of an MRI signal could allow the discrimination of the stationary spins signals from those of moving spins. This results in a wide variety of applications i.e. in medicine, in order to obtain more than angiograms, blood velocity images of veins, arteries and other vessels without having static tissue perturbing the signal of fluid in motion. This paper presents a novel theoretical and experimental method for the estimation of multiple velocities from MRI complex signals in the presence of stationary spins.

1. Introduction

The presence of surrounding stationary structures, such as soft tissue and bones, around a vessel introduces a conspicuity problem [1] in the image of the vessel and has been one of the major problems in flow measurements by NMR.

When static and moving spins are present in the same voxel, the phase of the MRI signal, which is a function of the velocity and number of flowing spins, is modified by the number of static spins. A novel method for multiple velocity estimation of moving spins from an MRI signal without the perturbance of stationary spins is presented in this paper and is compared to other conventional methods for estimation of multiple velocity from those spins.

2. Background

The velocity can then be estimated by solving the equation \( \Phi_v = \gamma m_1 v(x, y, z) \), for one value of the first moment \( m_1 \) of the velocity encoding gradient. The problem with using just one value of \( m_1 \) is the presence of a phase error caused by eddy currents, field inhomogeneities and measurement random noise. These errors can be reduced either by hardware modifications to the gradient coils [2] or by the estimation method.

The estimation method can use two or more velocity phase encoded images with different values of \( m_1 \) in order to cancel the effects of eddy currents that are independent of \( m_1 \) and three or more images in order to reduce the effects of random noise as it will be shown later.

Also, if the flow imaging experiment is done for several values of \( m_1 \), an inverse Fourier transform of the NMR image signal with respect to a variable that is function of \( m_1 \) yields the velocity density function at each voxel.

In a similar manner, the velocity can be estimated by a maximum-likelihood method which in the case of one velocity per voxel proves to be equivalent to the Fourier transform method, as it will be shown later.
3. Multiple velocity estimation by maximum-likelihood

3.1 General considerations

When stationary and moving spins are present in the same voxel, the phase of the measured signal is a non-linear function of the velocity of the moving spins and the ratio of the number of static to moving spins. In these circumstances the methods based on a one velocity model, underestimate the true velocity. In order to make more comprehensive this situation, the drawings and vector diagrams of Figure 1 will be used together with a basic mathematical description.

Figure 1. Schematic diagrams of a flow phantom with: (a) only moving spins, (c) stationary and moving spins. The corresponding phasor diagrams are given by (b) and (d). A₀ and A₁ are proportional to the amount of static and moving spins respectively. The phase of the NMR image signal, at a given voxel, is given by θ for A₀=0 and by θₓ for A₀>0.

The NMR image signal from a voxel that contains static spins together with spins moving with velocity v (Figures 1c and 1d) can be represented (ignoring phase errors and noise) by

\[ x = A_0 + A_1 e^{-j\Phi_v} \]  \hspace{1cm} (1)

where A₀ and A₁ are proportional to the number of static and moving spins respectively and Φₓ is given by

\[ \Phi_x = \lambda v(x) \]  \hspace{1cm} (2)

The phase of x is then given by Φₓ = arctan \( \frac{A_1 \sin \Phi_v}{A_0 + A_1 \cos \Phi_v} \) and is non-linearly related to the velocity v of the moving spins and to the ratio of the static to moving spins. For nonzero A₀, Φₓ<Φ_v and consequently, the velocity of the moving spins is underestimated. The higher the ratio of static to moving spins, the greater the error in the velocity estimate.

If only moving spins are present (Figures 1a, and 1b) in the observed voxel (A₀=0), then Φₓ=Φ_v is a linear function of the velocity of the moving spins and is given by (1) with A₀=0.

A solution to the problem just described is presented in the following sections. It consists in first encoding the velocity v of the moving spins into the phase of the NMR signal for several different values of m₁ [3]. Then, this velocity is estimated using a parametric approach which represents the NMR image signal with a non-linear model containing both static and moving spins and estimates the velocity of the moving spins by the maximum-likelihood (ML) method [4,5].

Two models are actually considered in order to represent the NMR image signal. The one velocity model [4] contains only the component due to moving spins and is a reduced version of the two velocities model, which contains both (static and moving spins) components and will be explained in the next section.

Each model generates a different expression for the maximum-likelihood velocity estimation (MLVE) function.

The maximum-likelihood velocity estimation (MLVE) method based on the one velocity model (ML1VE) was developed before and proved to be equivalent to the Fourier transform velocity estimation (FTVE) method [4]. The velocity estimates obtained with this method are affected by the small number of samples (generated by the different values of m₁) and by the ratio of static to moving spins. The estimation function of ML1VE method is given by
3.2 Estimation of two velocities per voxel by maximum-likelihood

The presence of stationary and moving spins within the same voxel implies the need of two velocity parameters in the model of the signal. The model, in this case, can also be represented by \( x_k = s_k + w_k \), \( k = 1, 2, \ldots, K \), but with \( s_k \) given by \( s_k = A_0 e^{-j(v_0 \lambda k + \theta_0)} + A_1 e^{-j(v \lambda k + \theta_1)} \) where \( v_0 \) is the velocity of the stationary spins and is equal to zero, \( v \) is the mean velocity of the moving spins in the voxel, \( \theta_0 \) is a constant phase error of the stationary spins due to magnetic field inhomogeneities, \( \theta_1 \) is a constant phase error due to magnetic field inhomogeneities and \( \lambda_k \) is as given by \( \lambda = \gamma m_1 \).

For the two velocities case, the function to maximize is, after simplification, given by

\[
L_2(v) = \frac{K[|\alpha_0|^2 + |\alpha_1(v)|^2] - 2Re[\alpha_0^* \alpha_1(v) \beta(v)]}{K^2 - |\beta(v)|^2}
\]

where \( \alpha_0 = e_0^H x = \sum_{k=1}^{K} x_k, \alpha_1(v) = e_1^H x = \sum_{k=1}^{K} x_k e^{-j \phi_k} \) and \( \beta(v) = e_0^H e_1 = \sum_{k=1}^{K} e^{j \phi_k}, * \) means complex conjugate and \( \Phi_k \) is as given by (2).

4. Simulation

Several NMR image signals were simulated for different values of \( v, A_0, A_1, K \) and noise to signal ratio, \( R_s = \sigma_0^2 / A_1^2 \), with the maximum velocity expected \( V_{max} = 128 \text{ cm/ sec} \) and \( N = 256 \). The velocity was estimated by three methods, Fourier transform (FTVE), linear regression (LRDVE) and maximum-likelihood (ML1VE), based on the one velocity model and by the maximum-likelihood method (ML2VE) based on the two velocities model. The ML1VE method is equivalent to the FTVE method. The ML1VE estimation function \( L(v) \) for the one velocity model implemented with an FFT should ideally result in two peaks when static \( (A_0) \) and moving \( (A_1) \) spins are present. One peak should be at \( v = 0 \text{ cm/sec} \) and the second one at the velocity \( V \) of the moving spins. However, since the sample velocity resolution \( \delta_v \) depends on \( K \), the number of values of \( \lambda_k \), for low values of \( V (V << \delta_v) \) the estimation function \( L(v) \) consists of one principal peak, result of the merge of the two peaks mentioned before. Furthermore, the location of this peak will be determined by the weighted average of the velocities of the static \( (v = V_0 = 0) \) and moving \( (v = V) \) spins, that is

\[
\tilde{v}_{ft} = \left( \frac{1}{1 + R_s} \right) V
\]

where \( R_s = A_0 / A_1 \) is the ratio of static to moving spins.

For high velocities \( (V >> \delta_v) \), the estimation function \( L(v) \) splits in two principal peaks. However, for small \( K \) (which is our case), the locations of the two peaks are shifted slightly away from zero and \( V \) in opposite directions. The transition of \( L(v) \) from one to two peaks as the velocity \( V \) increases is shown in Figure 2 for \( K = 7, R_s = 1 \text{ and } R_c = 0 \).

The ML2VE estimation function \( L_2(v) \) for the two velocity model contains only one principal peak located at the velocity being estimated. The waveforms of the estimation functions \( L_1(v) \) and \( L_2(v) \) are shown in Figures 3 and 4 for \( K = 1, R_s = 1, R_c = 0 \) and two values of \( V \).

For \( v = 70 \text{ cm/sec} \), \( L_2(v) \) has two peaks (Fig. 3a), one close to zero and the other one at \( v = 76 \text{ cm/sec} \), while \( L_2(v) \) has one peak (Fig. 3b) located exactly at \( v = 70 \text{ cm/sec} \), the velocity being estimated.

For \( v = 20 \text{ cm/sec} \), \( L_2(v) \) could not resolve the two peaks (Fig. 4a) and gives an estimated velocity of \( v = 10 \text{ cm/sec} \) while \( L_2(v) \) with one peak (Fig. 4b) gives an exact velocity estimate of \( v = 20 \text{ cm/sec} \).

The effect of the presence of static spins on the estimate of the velocity of moving spins is tested for \( K = 7, R_c = 0 \text{ and several values of the ratio } R_s \).
Figure 2. Plots of $L_1(v)$ and $L_2(v)$ for $V=70$ cm/sec, $K=7$, $R_a=1$ and $R_z=0$.

Figure 3. Plots of $L_1(v)$ for four values of $V$ and for $K=7$, $R_a=1$ and $R_z=0$.

Figure 4. Plots of $L_1(v)$ and $L_2(v)$ for $V=20$ cm/sec, $K=7$, $R_a=1$ and $R_z=0$.

For low velocities ($V<<\delta_v$) the ML1VE ($v$), obtained with $L_1(v)$, decreases proportionally to the increase of the ratio $R_a$ (Fig. 5a), as given by (5). A similar behavior is noticed on the velocity estimates obtained with the LRVE method, except that they wrap after $V \equiv \delta_v$ and its multiples (Fig. 5b). For high velocities ($V>>\delta_v$), this velocity estimate approaches the true velocities as $V$ increases (Fig. 5a), provided $A_0<A_1$. If $A_0>A_1$, and the way to select the location of the peak is only by the value of $v$ at which $L_1(v)$ is maximum, the selected peak will be the one close to zero velocity (Fig. 6b) resulting on the velocity estimate to be in great error.

If $L_1(v)$ (ML2VE) is used, $\hat{v}$ is consistently close to the true velocity $v$ for the same values of the ratio $R_a$ and for a wide range of velocities (Fig. 5c) except in a small range around zero velocity where the velocities have an error whose magnitude also depends on other parameters (i.e. $R_z$). In this case, as the ratio $R_a$ increases, the range of unstable velocity estimates around zero increases.

The effect caused by the number of samples $K$ on the velocity estimates (Fig. 6) is tested for $R_a=1$, $R_z=0$ and several values of $K$.

If $L_1(v)$ (ML1VE or FTVE) is used, the range where the velocity estimate is averaged with the static spins increases as $K$ decreases, where $\delta_v = V_s/K$.

This can be seen in figure 6a for $A_0<A_1$ and in figure 6b for $A_0>A_1$. The velocity estimates obtained with the LRVE method are also averaged with the static spins for the same range except that, as mentioned in the previous paragraph, they wrap for $v$ equal to each multiple of $\delta_v$ (Fig. 6c).

If $L_1(v)$ (ML2VE) is used (Fig. 7), as $K$ decreases, then the range of unstable velocity estimates near zero increases.

The effect that noise has over the velocity estimates is tested for $K=5$, $R_a=1$ and several levels of noise to signal ratio $R_s$. The mean and standard deviation of the estimates were calculated for a range of velocities $|V|<96$ cm/sec and 50 repetitions.

The velocity estimates obtained with the ML1VE (FTVE) method are the resultant average as given by (5) and explained in the previous paragraphs. These estimates are especially sensitive to noise at high velocities ($V>>\delta_v$) where their standard deviation is higher.
The velocity estimates obtained with the ML2VE method are particularly sensitive to noise at velocities close to zero, where their standard deviation is higher. This effect can be reduced as K increases but it remains significantly high within the range close to zero. Figures 8a and 8b shows plots of the mean and standard deviation of the velocity estimates using the ML1VE (FTVE) and the ML2VE methods.

As the noise to signal ratio $R_z$ is increased from 0.01 to 0.05 (Fig. 9), the velocity estimates of the ML2VE method within the small sensitive region around zero velocity observe an increase in their magnitude error as well as in the range of velocity estimates with error. The increase in the error of the velocity estimates obtained with the ML1VE (FTVE) method is at high velocities, increasing slightly the range of velocities with error. The velocity estimates obtained with the LRVE method have a high variance where the phase (velocity) wraps. When the noise to signal ratio is increased, the standard deviation of the velocity estimates increases proportionally (Fig. 10b).

It is convenient to notice that in the presence of static and moving spins, noise and a small number of samples, both models can be complementary. That is, in the range of velocities near zero where $L_2(v)$ becomes unstable, $L_1(v)$ still has some error but much less than $L_2(v)$. However, outside that range $L_2(v)$ provides much more accurate velocity estimates than $L_1(v)$.

5. Experiments

The phantom experiments were done on a GE SIGNA 1.5 T whole body imaging system with a 55 cm bore diameter and using an extremity coil. A gradient-echo sequence with multiple encoding of velocity [3] was used to obtain K coronal velocity encoded images.

A flow phantom consisting of water flowing at 950 ml/min through a tube with i.d. = 0.86 cm ($\bar{v}$ = 27 cm/sec) was used to represent the moving spins. A 20 cm long by 10 cm wide and approximately 10 cm height latex balloon filled with water was used to represent the stationary spins.

The pulse sequence parameters were: $T_E$=17 ms, $T_R$=800 ms, $\alpha$=90º, FOV=18 cm, Slthk=20 mm, NEX=1, MTX=128x256 and $V_{\text{max}}$=128 cm/sec.

Two cases were implemented. The first one contained only the tube with flowing water and the second one included also the balloon filled with water and placed on top of the tube.

For the first case, nine velocity encoded images (K=9) were generated, corresponding to nine values of lambda. The three methods that are based on the one velocity model (LRVE, ML1VE and FTVE) and also the method based on the two velocity model (ML2VE) were used to estimate the velocity. The velocity images obtained with the first three methods are shown in Figure 11 and serve as reference to the ones obtained with stationary spins.
For the second case, the same number of velocity encoded images were generated with the same pulse sequence parameters. A magnitude image is shown in Figure 12a as a morphological reference for the object being imaged. The velocity image obtained with the LRVE method is shown in Figure 12b. The velocity images obtained with the ML1VE (FTVE) and the ML2VE methods are shown in Figures 13a and 13b respectively. The values of the pixels of all the velocity images are in cm/sec.

6. Discussion

Visually, it can be noticed that most of the stationary spins that are not in the same voxel where moving spins are, do not appear in the image. This is expected since it is a velocity image. However, the width of the tube with flow is thinner in the velocity images obtained with the LRVE and ML1VE (FTVE) methods. This is because, towards the edges of the tube, the ratio of static to moving spins increases due to the curvature of the tube. This introduces more error in the velocity estimates of these two methods due to the averaging effect already mentioned in the previous sections. In contrast, the velocity image obtained with the ML2VE method (Fig. 13b), shows the real thickness of the tube with flow, as it can be verified by comparing it with the velocity images without static spins (Fig. 11).

Quantitatively, velocity profiles from one column of pixels (perpendicular to the direction of flow) of the velocity images obtained with all the methods mentioned in the previous paragraph are shown in the plot of Figure 14.

The velocity profiles obtained with the LRVE and ML1VE (FTVE) methods are zero or very low towards the edges of the plot. This corroborates the visual observations made in the previous paragraph. The values of the velocity estimates obtained with LRVE and ML1VE (FTVE) can be in more error when the amount of stationary spins per voxel increases. This is the case when a thicker slice is taken or when the vessel is curvilinear such that a thicker slice is necessary to include a vessel length close to
the field of view of the image. The correlation coefficients of the LRVE and ML1VE (FTVE) methods will also decrease and the image can be irregular in the velocity regions that is able to show.

7. Conclusions
When stationary and moving spins are present in the same voxel, the maximum-likelihood velocity estimation method based in the two velocities model (ML2VE) yields more accurate velocity estimates than those obtained with the single phase method, linear regression (LRVE) method and Fourier transform (FTVE) method.

When the velocity to be estimated is near zero cm/sec, the estimates obtained with the ML2VE method are erroneous due to the limitation in sample velocity resolution caused by the small amount \( K \) of observations (\( m_1 \) or \( \lambda_k \)). This limitation makes the velocity estimates, within a small range around zero velocity, very sensitive to noise. This sensitive region can be reduced by increasing the number of \( \lambda_k \) (or \( m_1 \)) steps.

In-vitro flow phantom experiments validated the velocity estimation methods which behave according to the simulated results.

The computational algorithm for the ML2VE method was not optimized for speed of calculations and takes longer to compute a 256x256 velocity image than the FTVE method and that the LRVE method.

When stationary spins are present in the same volume of excitation but in different voxels where moving spins are, the ML2VE, ML1VE, FTVE and LRVE methods yield accurate velocity estimates and the regions with stationary spins do not appear in the resultant velocity images. This improves the visualization of the tube with moving spins.

8. References
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