Ratio of strange to $u/d$ momentum fraction in disconnected insertions

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The ratio of the strange momentum fraction $\langle x \rangle_{s+i}$ to that of $u/d$ in the disconnected insertion is calculated on the lattice with overlap fermions on four domain wall fermion ensembles. These ensembles cover three lattice spacings, three volumes and several pion masses including the physical one, from which a global fitting is carried out where a complete nonperturbative renormalization and mixing between the quark and glue operators are taken into account. We find the ratio to be (x)_{s+i}/(x)_{u+d}(DI) = 0.795(79)(53) at $\mu = 2$ GeV in the $\overline{MS}$ scheme. This ratio can be used as a constraint to better determine the strange parton distribution in the global fitting of PDF’s when the connected and disconnected sea are fitted and evolved separately. We also compare this momentum fraction ratio with several recent global analyses of the PDF ratio $(s(x)+\bar{s}(x))/(\bar{u}(x)+\bar{d}(x))$ at the same $\mu$ and discuss its consequences.

**Introduction:** Understanding the structure of the nucleon in terms of quarks and gluons from QCD is one of the most challenging aspects of modern nuclear and particle physics [1] and is of great importance in learning about how the visible Universe is built. Parton distribution functions (PDF’s), which describe the number density of a parton with a certain longitudinal momentum fraction $x$ and at a particular energy scale $Q^2$ inside a nucleon, reveal a lot of pertinent and essential information about the nucleon structure. In general, PDF’s are determined by the global analysis of deep inelastic scattering and Drell-Yan experiments under the framework of QCD factorization theorems. In order to gain more details about the quark spin, gluon helicity, and the 3D tomography of the nucleon, a proposal has been made to construct an Electron-Ion Collider [1] to extend the study of the PDF’s and transverse momentum dependent distributions (TMD’s) to the needed gluon-dominated small $x$ region. For the extensively studied unpolarized PDF’s, recent attention is focused on the less-known flavor structure which is believed to implicate the non-perturbative nature of the parton distributions due to confinement. A typical example is to understand the origin of the Gottfried sum violation [2,3] which reveals that $\bar{u}(x) \neq \bar{d}(x)$. Another is the strange distribution which is the most uncertain among the unpolarized PDF’s. Three recent global fittings [4-6] with NNLO analysis show that $x(s(x)+\bar{s}(x))$ has large errors, $\sim 50\%$ or more at $x = 10^{-3}$ and the central values of the three fits differ by $\sim 30\%$, at $Q^2 = 4$ GeV$^2$. From the neutrino deep inelastic scattering (DIS), it is learned that the momentum fraction (the second moment of the unpolarized PDF) of the strange is about half of that of $\bar{u}$ and $\bar{d}$, i.e. $R_s \equiv \langle x \rangle_{s+i}/\langle x \rangle_{u+d} \sim 0.5$. This has been incorporated in the global analyses. The three recent global fittings give (the values are calculated using LHAPDF [7]) 0.56(26) [4], 0.54(17) [5] and 0.51(16) [6].

On the other hand, the strange-to-down sea-quark ratio $r_s(x) = (s(x)+\bar{s}(x))/2d(x)$ has been determined from an ATLAS analysis of inclusive W and Z boson production in $pp$ collisions at LHC to be $1.00^{+0.25}_{-0.26}$ at $x = 0.023$ and $Q^2 = 1.9$ GeV$^2$ [8], and $0.96^{+0.26}_{-0.30}$ from the measurement of the associated W+$c$ production [9]. It is suggested in Ref. [10] that this apparent dilemma between the ratio of momentum fractions $R_s$ and the ratio $r_s(x)$ at small $x$ can be understood in terms of the fact that the strange parton involves only the disconnected sea, while the $\bar{u}$ and $\bar{d}$ partons have, in addition, connected sea components whose distributions are less singular than that of the disconnected sea at small $x$. In view of this, an attempt has been made [10] to separate the connected sea (CS) from the disconnected sea (DS) $\bar{u}$ and $\bar{d}$ partons by using $s(x)+\bar{s}(x)$ data from HERMES, $\bar{u}(x)+\bar{d}(x)$ from CT10 global fit, and the preliminary lattice calculation [11] of the ratio

$$ R = \langle x \rangle_{s+i}/\langle x \rangle_{u+d} (DI). \tag{1} $$

In this manuscript, we report a complete lattice calculation of this ratio $R$ with non-perturbative renormalization and mixing with the gluon momentum fraction at several pion masses including the physical one. This can be used as a constraint in the global fitting of PDF’s to have a more precise determination of the strange parton distribution, up to the systematic uncertainty coming from the similar ratios of higher moments of PDF’s.

**Theoretical background:** The connected sea (CS) and disconnected sea (DS) partons are revealed and classified in the path-integral formulation of the Euclidean
hadronic tensor $\tilde{W}_{\mu\nu}(\vec{q},\vec{p},\tau)$ [12,14].

$$\tilde{W}_{\mu\nu}(\vec{q},\vec{p},\tau) = \frac{E_N}{M_N} \langle p | \int \frac{d^3x}{2\pi} e^{-iq\tau} J_\mu(\vec{x},t_2) J_\nu(\vec{0},t_1) | p \rangle,$$

where $|p\rangle$ is the nucleon state with momentum $p$, $E_N$ and $m_N$ are the energy and mass of the nucleon, and $\tau = t_2 - t_1$ is the time difference between the two currents $J_\mu$ and $J_\nu$. The hadronic tensor $W_{\mu\nu}$ in Minkowski space is the inverse Laplace transform of $\tilde{W}_{\mu\nu}(\vec{q},\vec{p},\tau)$, i.e. $W_{\mu\nu}(\vec{q},\vec{p},\nu) = 1/i \int_{\nu-i\infty}^{\nu+i\infty} e^{\nu\tau} \tilde{W}_{\mu\nu}(\vec{q},\vec{p},\tau)$. From the three gauge invariant and topologically distinct classes of path-integral diagrams in Fig. 1 which entail leading twist contributions, one finds that there are two sources for the antipartons where the quark propagates backward in $\tau$ between the currents. One is connected sea (CS) antipartons $\vec{q}^cs$ with $q = u, d$ in the generalized Z graph in Fig. 1a where all quark lines are connected between the currents. The other one is disconnected sea (DS) antipartons $\vec{q}^ds$ with $q = u, d, s, c$ in the disconnected graph in Fig. 1c where the quark lines connecting the currents are disjoint from those in the nucleon propagator, resulting in a vacuum polarization. Fig. 1c also contains $\vec{q}^ds$ which in most global fittings is taken to be the same as $\vec{q}^cs$, such as $s(x) = \bar{s}(x)$. In the isospin symmetric limit $\vec{q}^ds = \vec{d}^ds$, the Gottfried sum rule violation which indicates that $\vec{u} \neq \vec{d}$ comes exclusively from Fig. 1b as shown in [12]. In contrast, the time forward propagating quarks in Fig. 1c correspond to valence and CS partons $u^{v+cs}$ and $d^{v+cs}$, where valence is defined as $q^v \equiv q^{v+cs} - \vec{q}^cs$.

Upon short distance expansion, it is shown [14] that hadronic tensor from Fig. 1a and Fig. 1b becomes the connected insertions (CI) in Fig. 2a for a series of local operators $\sum_n O^p_n$ in the three-point functions from which the nucleon matrix elements for the moments of the CI are obtained. By the same token, the disconnected four-point functions in Fig. 2b become the disconnected insertions (DI) in Fig. 2b for the three-point functions to obtain the DI moments. Here $q = u, d, s, c$ are the DS flavors in the DI. One advantage of the path-integral formalism over the canonical formalism is that the parton degrees of freedom are tied to the topology of the quark skeleton diagrams in Figs. 1a, 1b and 1c so that the CS and the DS can be separated. Lattice QCD can access these three-point functions for the CI and DI which separately contain the CS and DS and calculations of the low moments of the unpolarized and polarized PDF’s for the quarks [14–17] and glue [15, 17, 18] have been carried out.

Since global analyses so far have not separated out CS from the DS, a suggestion [10] is made to extract the CS from the following approximation relation

$$\vec{u}^{cs}(x) + \vec{d}^{cs}(x) = \vec{u}(x) + \vec{d}(x) - \frac{1}{R} (s(x) + \bar{s}(x)),$$

based on the assumption that $u^{ds} + \bar{u}^{ds}$ and $s + \bar{s}$ have the same distribution, modulo a constant factor of $R$ as defined in Eq. (1). In Ref. [10], $\vec{u}(x) + \vec{d}(x)$ is the CT10 result, $s(x) + \bar{s}(x)$ is the result from the HERMES SIDIS analysis at $Q^2 = 2.5\, \text{GeV}^2$, and the value of $R$ is from an earlier lattice calculation on one lattice spacing with large pion mass [11]. In the present work, $R$ will be updated by a complete calculation with non-perturbative renormalization and all the systematic errors under control.

**Numerical Details:** The numerical setup of this study is the same as in our previous work [19]. We use overlap fermions [20] as valence quarks on four $2 + 1$-flavor RBC/UKQCD gauge ensembles with domain wall fermions [21, 22]. The parameters of the ensembles are listed in Table I. We have three different lattice spacings and lattice volumes respectively, and four values of the sea pion mass with one at the physical point. For the valence section, multiple partially-quenched valence quark masses are used, owing to the multi-mass algorithm. We choose four valence quark masses ranging from $\sim 250$ to $\sim 400$ MeV on the 24I and 32I ensembles and 7/6 quark masses in the range $[130, 400]$ MeV on the 48I/32ID ensembles. Combining these ensembles and valence pion masses in a global analysis helps to control the lattice systematic uncertainties and lead to our final result at the physical limit.

The quark and glue momentum fractions in the nucleon can be defined by the matrix element of the traceless diagonal part of the energy-momentum tensor (EMT) in the rest frame [23].

$$\langle x \rangle_{q,g} \equiv \frac{\langle N| \frac{1}{2} \tau_{\mu\nu}^{q,g} | N \rangle}{M_N \langle N|N \rangle},$$

where

![Figure 1](image1.png)

Figure 1. Three topologically distinct diagrams in the Euclidean path-integral formalism of the nucleon hadronic tensor.

![Figure 2](image2.png)

Figure 2. The three-point functions after the short-distance expansion of the hadronic tensor from Fig. 1. CI (a) is derived from Fig. 1a and Fig. 1b DI (b) originates from Fig. 1c.
with \( T_{44}^t = \int d^3x \bar{\psi}(x) \frac{1}{2} \left( \gamma_4 \bar{D} \gamma_4 - \frac{1}{4} \sum_{i=0,1,2,3} \gamma_i \bar{D_i} \right) \psi(x) \)

and \( T_{44}^q = \int d^3x \frac{1}{2} [E(x)^2 - B(x)^2] \). Here \( \psi = (1 - \frac{1}{4}D_{ov})\psi \) is for giving rise to the effective quark propagator \( (D_c + m)^{-1} \) where \( D_c \) satisfying \( \{D_c, \gamma_i\} = 0 \) is exactly chiral and can be defined from the original overlap operator \( D_{ov} \) as \( D_c = \frac{\gamma_5D_{ov}}{1+\gamma_5D_{ov}} \) \cite{24}. More details regarding the calculation of the overlap operator and eigenmodes deflation in the inversion of the fermion matrix can be found in \cite{25}. To calculate the matrix elements, we need first construct 3-point correlation functions

\[
C_{3}^{q,g}(t_f, \tau) = \sum_{\bar{x},\bar{g}} \langle \chi(t_f, \bar{g})T_{44}^{q,g}(\tau, \bar{x})\chi(0, \bar{g}) \rangle, \tag{5}
\]

where \( \chi \) is the nucleon interpolation field and \( \bar{G} \) denotes the source grid. Then, we make a ratio of the 3-point correlation function to the nucleon 2-point function

\[
\Pi^{q,g}(t_f, \tau) = \frac{\text{Tr} [\Gamma_c C_{3}^{q,g}(t_f, \tau)]}{\text{Tr} [\Gamma_c C_2(t_f)]}, \tag{6}
\]

such that \( \langle N | T_{44}^{q,g} | N \rangle = \Pi^{q,g}(t_f, \tau, \tau, \tau \gg 0) \). Here \( \Gamma_c \) is the non-polarized projector and \( C_2(t_f) = \sum_{\bar{x}} \langle \chi(t_f, \bar{x})\chi(0, \bar{g}) \rangle \). At finite \( t_f \) and \( \tau \), the excited states will contribute to the matrix element and we need to extract it by fitting the ratio by the so-called two-state fit form

\[
\Pi^{q,g}(t_f, \tau) = c_0^{q,g} e^{-\delta m(t_f-\tau)} + c_1^{q,g} e^{-\delta m \tau} + c_2^{q,g} e^{-\delta m t_f}, \tag{7}
\]

where \( c_0^{q,g} = \langle N | T_{44}^{q,g} | N \rangle \) and \( \delta m \) is the effective energy difference between the ground state and the excited states. To better use this formula, multiple source-sink separations \( t_f \) ranging from \( \sim 0.7 \) fm to \( \sim 1.5 \) fm are constructed for \( \Pi(t_f, \tau) \) on each ensemble for all the current positions \( \tau \) between the source and sink. More detailed examples of two-state fit can be found in our previous works, e.g. \cite{16, 18}.

As mentioned above, the 3-point correlation functions have two kinds of current insertions, CI and DI, as illustrated in Fig. 2. Since both the CI and the glue matrix elements mix to the DI ones through the renormalization of the bare quantities under the lattice regularization \cite{19}, the calculation of the ratio \( R \) under the \( \overline{\text{MS}} \) scheme will involve the CI and glue contributions. For the CI calculation, we use the stochastic sandwich method (SSM) with low-mode substitution (LMS) \cite{20} to better control the statistical uncertainty. Technical details regarding the LMS of random \( Z_t \) grid source and the SSM with LMS for constructing 3-point functions can be found in Refs. \cite{20, 28}. For the DI calculations, we use the low-mode average (LMA) technique to calculate the quark loops which helps to improve statistics. We make multiple measurements by shifting the source time-slice to improve the signal of the nucleon propagator. References \cite{27, 29, 30} contain more details regarding the DI calculation. The bare strange quark mass is determined in our previous study \cite{31} and the nonperturbative mass renormalization constants are calculated in \cite{32}. For the glue momentum fraction \( \langle x \rangle_g \), the cluster-decomposition error reduction (CDER) technique is applied to improve the signal \cite{17, 33}.

Renormalization: The comprehensive nonperturbative renormalization used in this work for both the quark and glue sections is formulated and implemented in \cite{17, 19}. As demonstrated in \cite{14}, the renormalization can be processed separately for CI and DI and we will focus on the DI part in this work. The renormalized momentum fractions \( \langle x \rangle^{R,DI} \) in the \( \overline{\text{MS}} \) scheme at scale \( \mu \) are

\[
\langle x \rangle^{R,DI}_{u,d,s} = Z_{QQ}^{MS}(\mu) \langle x \rangle^{DI}_{u,d,s} + \delta Z_{QQ}^{MS}(\mu) \sum_q \langle x \rangle^{CI+DI}_{q} + Z_{GG}^{MS}(\mu) \langle x \rangle_g, \tag{8}
\]

where \( \langle x \rangle^{DI/CI}_{u,d,s} \) is the bare momentum fraction in the DI/CI sector under lattice regularization, and the renormalization constants in the \( \overline{\text{MS}} \) scheme at scale \( \mu \) are determined through the RI/MOM scheme as
and $Z_{QQ}^{MS}(\mu) = \left( [Z_{QQ}(\mu R) R_{QQ}(\mu/\mu R)] x^2 \mu_R^{-2} \right)^{-1}$.

Here we have the 3-loop result for the iso-vector matching coefficient $R_{QQ}(\mu/\mu R)$ [34] but only the 1-loop results of the other $R$'s [35]. The values of the renormalization constants and more details can be found in [19] and its supplemental materials.

**Results:** The two bare matrix elements of the strange and light quarks of each valence pion mass on each ensemble are fitted using the two-state fit formula (Eq. (1)) in a joint correlated fit, such that the correlation between the two matrix elements is properly kept. This ensures the cancellation of the fluctuations of the two matrix elements in the ratio and leads to statistically more stable results.

![Figure 3](image)

**Figure 3.** The global interpolation/extrapolation on the four ensembles. The blue and cyan bands show the statistical and total uncertainties from the chiral, continuum, and infinite volume dependence. The red, blue, and green bands show the systematic uncertainties from the CS, DS, and infinite volume dependence.

After renormalization, the final ratios are fitted by the following form to track the pion mass, lattice spacing and volume dependence

$$R(m^v, m^s, a, L) = R(m^0, m^0, 0, \infty) + C_1 (m^v)^2 - (m^s)^2) + C_2 ((m^v)^2 - (m^s)^2) + C_3 a^2 + C_4 e^{-m^s L},$$

where the $C$'s are free parameters, $m^v/m^s$ is the valence/sea pion mass, and $m^0$ is the physical pion mass. The third term is accounted for the partial quenching effect. The globally fitted renormalized values with the partial quenching effect subtracted on each ensemble are plotted in Fig. 2 as a function of the valence $m^v$. The final result at the physical pion mass is $R^{MS}(2\text{GeV}) = \langle \sigma \rangle_{s+\bar{s}}^{L} / \langle \sigma \rangle_{u+\bar{u}}^{L} (\text{DI}) = 0.795(79)(53)$ under the $MS$ scheme, where the first error is the statistical one and the second error includes the systematic uncertainties from the chiral, continuum, and infinite volume interpolation/extrapolations.

**Summary and Discussion:** To see how the lattice $R$ value can help the global PDF analysis, we show in Fig. 4 the ratio of the distribution $(s(x) + \bar{s}(x)) / (\bar{u}(x) + \bar{d}(x))$ from 3 global fittings at NNLO [16] at $Q^2 = 4\text{ GeV}^2$. We see that the errors of the ratios are large at small $x < 10^{-2}$. Furthermore, it is conspicuous that they all have a characteristic shoulder with a fall off around $x \sim 10^{-2}$ toward larger $x$. This reflects the fact that the small $x$ behavior of $q^v(x)$ is more singular than that of $q^s$ and $\bar{q}^s$ (e.g. $q^s_{\text{MS}}, \bar{q}^s_{\text{MS}}$ $x^{-1}$ and $q^v, \bar{q}^v_{\text{MS}}$ $x^{-1/2}$ in Regge theory and for the small $x$ region $10^{-4}$ to $10^{-2}$) of the global fittings of PDF [24] at $Q^2 = 4\text{ GeV}^2$, we find the power of the small $x$ behavior (i.e. $\alpha \propto \alpha^x$) for $\bar{q}^s(x)$ in the range $[-1.22, -1.15]$, and $q^v(x)$ in the range $[0.6, -0.2]$ which are close to those prescribed in Regge theory) so that at $x < 10^{-2}$, where the DS dominates, the ratio stays more or less constant. When $x$ approaches $10^{-2}$ from below, the CS $\bar{u}^s(x) + \bar{d}^s(x)$ component in $\bar{u}(x) + \bar{d}(x)$ (N.B. $\bar{u}(x) + \bar{d}(x) = \bar{u}^s(x) + \bar{d}^s(x) + \bar{u}^d(x) + \bar{d}^d(x)$) sets in to make the ratio smaller. This is an indirect manifestation of the existence of the CS degrees of freedom besides the explicit evidence from the Gottfried sum rule violation. With the moment ratio $R$ available from a complete lattice calculation as given in this manuscript, it is advocated [36] to separate the CS and DS partons with corresponding evolutions to carry out a global fitting with $R$ from the lattice as a constraint. This should give a strange PDF with less statistical uncertainty than the present results from global analyses of experiments. To the extent that $s(x) + \bar{s}(x)$ and $\bar{u}^s(x) + \bar{d}^s(x)$ are largely proportional, the ratio $R$ would represent the ratio $(s(x) + \bar{s}(x)) / (\bar{u}(x) + \bar{d}(x))$ in the region $x < 10^{-2}$. To this end, we plot $R$ from the lattice in Fig. 4 up to $x < 10^{-2}$ to suggest that, if it is used as a constraint in the global analysis, it can reduce the statistical uncertainty of the strange PDF as indicated by its error as compared to those from the NNLO analyses. So as a first trial, one can carry out a global analysis by constraining $(s(x) + \bar{s}(x)) / (\bar{u}(x) + \bar{d}(x))$ to $R$ in the range $x < 10^{-2}$ to see how much it can improve the global fit, while further lattice calculation of the fourth moment ($x^3$) of the DI will serve to gauge the validity of this approach and suggest possible modification of the fitting function.

![Figure 4](image)

**Figure 4.** The global fitting results of $(s(x) + \bar{s}(x)) / (\bar{u}(x) + \bar{d}(x))$ at $Q^2 = 4\text{ GeV}^2$. The green band shows our result under the assumption that the ratio is a constant for small $x$ up to $x = 10^{-2}$. 

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Besides the hadronic tensor [14–39], recent formalisms [40–42] have been developed to calculate the explicit $x$-dependent PDF on the lattice. It is still a challenge for these approaches to have all the statistics and systematics under control at this stage. In the meantime, the lattice calculations with low quark and glue moments are getting mature and complete with non-perturbative renormalization and mixing to serve as meaningful constraints for the global analysis of PDF's. The present result for the ratio $\langle x_1^{+3}/x_2^{+3} \rangle_{\nu=\bar{u}}(\text{DI}) = 0.795(79)/(53)$ at $\mu = 2$ GeV in the $\overline{\text{MS}}$ scheme is the first such calculation to constrain the global fittings in the small $x$ region.

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