The use of reduced models in the reliability analysis of deformable systems with dynamic vibration dampers

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Abstract. The transformations of the initial design models of complex constructions and structures as the multi-degrees of freedom (DOF) systems is advantageous to use in order to reduce the amount of computational procedures in their dynamic analysis. This becomes particularly relevant in the reliability analysis requiring multiple recalculations for taking into account the stochastic properties of system parameters and impacts. The paper presents a technique based on known approaches of reduction the original multi-DOF system to an equivalent generalized design model with one degree of freedom. Various combinations of the equivalence conditions for the original system and the generalized model, including equalities of their natural frequencies, characteristic displacements or kinetic energies, were used to determine the modelling lumped mass and the corresponding rigidity of the design model. An algorithm for the probabilistic properties evaluation of the design model characteristics has been developed. The verification of the correctness of the proposed model in the area of tuning in to the workload frequency is performed. The quantitative results of solving the model problems in reliability calculations of multi-DOF structures with dynamic vibration dampers (DVD) are presented. The proposed approach allows us to simplify the process of selecting a single-mass DVD taking into account reliability requirements.

1. Introduction

Dynamic loads are one of the most dangerous types of impacts on construction systems (i.e. structures, buildings etc.). The problem of ensuring the necessary level of reliability and safety of dynamically deformable systems is significant.

It’s known that the dynamic vibration dampers are the effective means of counteracting vibrations caused by harmonic influences of a different nature [1, 2].

The selection of the DVD parameters (mass and stiffness) with taking into account the probabilistic properties of the system and the damper, satisfying the required level of reliability according to the accepted criteria of serviceability, is an iterative process as a rule. One of the fundamentally important questions in this case is the description of the system’s probabilistic characteristics which determination requires multiple solutions of the motion equations, especially when using the statistical modelling method (SMM)) [3]. This creates computational problems for complex systems with a large number of DOF, especially when using the finite-element method [4].

Initial dynamic system reduction to the design model with fewer DOF and design parameters with different methods of transformation [5] taking into account the conditions of equivalence is rationally.
Hereinafter, the term “initial (or original) system” is applied to a definition of a system (structure, construction) for which vibration dampers are used to improve its dynamic state.

This approach allows us to simplify significantly the process of selecting the damper parameters and evaluation the influence of its probabilistic properties on the system reliability as a whole.

**The purposes of the work are:** development of a methodology for the formation of rational models for analysis of the harmonically loaded linearly deformable systems with dynamic vibration dampers and estimation of their reliability, which can significantly reduce the amount of computational procedures while ensuring sufficient accuracy of the results; verification and comparison of variants of determining the design models parameters using various equivalence criteria; testing the effectiveness of applying the methodology to determine the probability of failure in model problems.

2. The main part

The number of masses in the design model depends on the structural features of the initial system, the type of the load and method of its application. In the case of a one-point load, it is rationally to transform the initial system (Figure 1, a) to a 1-DOF design model (Figure 1, b).

![Figure 1](image-url)

Figure 1. The initial system (a), the design model with 1-DOF (b) and their amplitude-frequency characteristics (c, d)

$A, [A]$ – the amplitude of the stress-strain state parameter and its permissible value;

$\omega_F$ – workload frequency

The following requirements are used as the conditions ensuring the equivalence of the original system and the generalized design model:

- the equality of a certain natural frequency $\omega_j$ of the initial system, to which the given frequency $\omega_F$ of the workload is close, and the natural frequency $\omega_0$ of the generalized model:

  \[ \omega_j = \omega_0; \]  

- the equality of displacements at the place of vibratory point load $F(t)$ in the initial system $y_{i(F)}$ and the lumped mass of the generalized model $y_0$:

  \[ y_{i(F)} = y_0. \]  

The natural frequency and mass displacement for the generalized design model (Figure 1, b) are found from the expressions [6, 7]

\[ \omega_0^2 = c_0 m_0^{-1}; \quad y_0 = \frac{F}{c_0 - m_0 \omega_F}, \]  

and for the initial system (Figure 1, a) respectively.
\[ \omega_j^2 = \frac{1}{\sum_{k=1}^{n} \delta_{m} m_{ik} \beta_{yk}}; \quad y_{i(F)} = \frac{F \delta_{ii}}{1 - \omega_j^2 \sum_{k=1}^{n} \delta_{m} m_{ik} \beta_{yk(f)}}, \]

where \( c_0 \) – the generalized stiffness of design model; \( m_0 \) – equivalent lumped mass of the design model; \( i \) – number of mass to which the point load is applied; \( \beta_{yk} = y_k / y_i \) (\( k = 1, 2, \ldots, n \)) – elements of the displacements eigenvector for frequency \( \omega_j \) (notice, that \( \beta_{yi} = 1 \)); \( \beta_{yk(f)} = y_k(f) / y_i(f) \) (\( k = 1, 2, \ldots, n \)) – elements of the relative displacements vector during steady-state forced oscillations with a frequency \( \omega_F \); \( \delta_{ik} \) (\( k = 1, 2, \ldots, n \)) – elements of the external elastic pliability matrix of the initial system; \( m_k \) (\( k = 1, 2, \ldots, n \)) – masses of the initial system.

If a workload frequency is close to the resonant frequency (\( \omega_F \rightarrow \omega_j \)), the vector of relative displacements \( \beta_{yk(f)} \) tends to its own displacements vector \( \beta_{yk} \). After substituting (3), (4) in (1), (2) and mathematical transformations, we obtain

\[ m_{q(n,y)} = 1 - \frac{k_a^2 \cdot k_b}{1 - k_a^2} \]

or

\[ m_{q(n,y)} = m_{q(n) \cdot k_F}, \]

where \( m_{q(n,y)} \) – modelling lumped mass determined from the conditions of equal natural frequencies and displacements at the load point of the initial and design models; \( m_{q(n)} = 1 - \frac{k_a^2 \cdot k_b}{1 - k_a^2} \) – correction factor taking into account the equality of displacements;

\[ k_b = \left( \frac{\sum_{k=1}^{n} a_{m_k} \beta_{yk}}{\sum_{k=1}^{n} a_{m_k} \beta_{yk}} \right) \left( \frac{\sum_{k=1}^{n} a_{m_k} \beta_{yk}}{\sum_{k=1}^{n} a_{m_k} \beta_{yk}} \right)^{-1} ; \quad k_a = \frac{\omega_F}{\omega_0} ; \quad a_{m_k} = m_k \cdot k_i \]

\( k_a, k_b \) – relative masses; \( \beta_{yk} = \delta_{m} \delta_{m}^{-1} \) (\( k = 1, 2, \ldots, n \)) – elements of the matrix of relative displacements.

For reduced stiffness we have:

\[ c_{q(n,y)} = 1 - \frac{k_a^2 \cdot k_b}{1 - k_a^2} \]

or

\[ c_{q(n,y)} = c_{q(n)} \cdot k_F, \]

where \( c_{q(n,y)} \) – reduced stiffness according to the conditions of equal natural frequencies and displacements at the load point of the initial and design models; \( c_{q(n)} = \delta_{m}^{-1} \) – the reduced stiffness from the condition that the natural frequencies of the initial and design models are equal.

Thus, the use of a generalized design model with mass and stiffness parameters determined by (5) and (6) respectively, allows us to simulate the movement of a point in the system in the place of application the harmonic load while maintaining the values of the leading parameters of the original system – frequency \( \omega_j \) and displacement \( y_{i(F)} \). A feature of the model is that it is tuned to a frequency \( \omega_F \) of external influence, taking into account the choice of the point of mass reduction.

The use of the proposed model is advantageous in the case when the frequency at which the model is tuned is sufficiently remote from two adjacent natural frequencies \( \omega_{j-1} \) and \( \omega_{j+1} \) of the original system. Otherwise, a significant influence of neighboring frequencies on the parameters of the
dynamic stress-strain state (SSS) of the system is possible, the amplitudes of which can be characterized by strong nonlinearity [8] in the considered areas.

It’s needed to note that the model works correctly only for a single one-point load. Besides the kinetic energies of the original system and the design model do not coincide.

If, for the problem to be solved, the significant criterion is the equality of the kinetic energies of the original system and the generalized design model, then we can use instead of (2) the equivalence condition in the form of equality of the maximum kinetic energies in the initial ($E_K$) and generalized ($E_{K,0}$) models ($E_K$ and $E_{K,0}$ respectively):

$$E_K = E_{K,0}.$$  \hspace{1cm} (7)

These energies are defined as

$$E_K = \sum_{k=1}^{n} \frac{m_k \dot{y}_k^2}{2} \quad \text{and} \quad E_{K,0} = \frac{m_{0(\omega,E_k)} \dot{y}_0^2}{2},$$  \hspace{1cm} (8)

where $m_k$ and $\dot{y}_k$ ($k = 1,...,n$) – masses and mass velocities of the original system; $m_{0(\omega,E_k)}$ and $\dot{y}_0$ – mass and mass velocity of the generalized model according to the conditions of the frequencies and kinetic energy equivalences.

Velocities $\dot{y}_0$ and $\dot{y}_k$ in the stable forced oscillations depend on the corresponding displacement amplitudes:

$$y_0 = \frac{F}{c_{0(\omega,E_k)} - m_{0(\omega,E_k)} \omega_F^2}; \quad y_k = \frac{F \delta_{si}}{1 - \omega_F^2 \sum_{k=1}^{n} \delta_{si} m_k \dot{y}_k^{(s)}}.$$  \hspace{1cm} (9)

As a result of replacing the second equations in (3) and (4) by (8), we obtain

$$m_{0(\omega,E_k)} = \left\{ \frac{m_k \delta_{si}^2}{\left( \sum_{i=1}^{n} \frac{\delta_{si} m_i \beta_{sk} - k_{si}^2 \delta_{si} m_i \beta_{sk}^{(s)}}{\sum_{k=1}^{n} \delta_{si} m_i \beta_{sk}^{(s)}} \right)^2} \left[ 1 - k_{si}^2 \right]^2 \right\}^{-1};$$  \hspace{1cm} (10)

$$c_{0(\omega,E_k)} = \left\{ \sum_{i=1}^{n} \frac{m_i \delta_{si}^2}{\left( \sum_{i=1}^{n} \sum_{k=1}^{n} \delta_{si} m_i \beta_{sk} - k_{si}^2 \delta_{si} m_i \beta_{sk}^{(s)} \right)^2} \left[ 1 - k_{si}^2 \right]^2 \right\}^{-1},$$  \hspace{1cm} (11)

where $s = 1,...,n$ – mass number; $i$ – number of mass at the point of loading; $\beta_{sk}^{(s)} = y_{ik(F)} / y_{ik(F)}$ – ($k = 1,2,...,n$) – elements of the relative displacements vector in forced vibrations with a frequency $\omega_F$, superscript (s) indicates the number of mass, relative to which displacement the vector is calculated; $c_{0(\omega,E_k)}$ – generalized stiffness of the design model from the conditions of the frequency and kinetic energy equivalences.

Let us evaluate the quality of the proposed variants for generalized models on the example of a beam with two DOF loaded with a one-point load $F(t)$ (Figure 2).
The calculations were performed with the following parameters’ values: 
\( l = 1000 \text{ mm} \); 
\( m_1 = 1000 \text{ kg} \); 
\( m_2 = 1000 \text{ kg} \); 
\( E = 2 \cdot 10^8 \text{ kPa} \); 
\( b = 50 \text{ mm} \); 
\( h = 200 \text{ mm} \); 
\( F = 20 \text{ kN} \).

Pre-calculated natural frequencies: 
\( \omega_1 = 70.71068 \text{ s}^{-1} \), 
\( \omega_2 = 200 \text{ s}^{-1} \); 
in the case of forced oscillations at 
\( \omega_f = 0.9 \omega_1 \); 
\( y_1 = 10.8045 \text{ mm} \); 
\( y_2 = 10.2 \text{ mm} \).

The results of calculating the generalized model parameters in the 
\( k_{\omega_y} \in [0.85; 0.95] \) interval are presented in tabular form – according to the first variant of the equivalence conditions combination – in Table 1, and according to the second – in Table 2.

**Table 1.** The results of the calculation of the generalized design model parameters from conditions (1) and (2)

| \( k_{\omega_y} \) | \( k_0 \) | \( k_F \) | \( m_{y_{(a,y)}} \), kg | \( c_{y_{(a,y)}} \), N/mm | \( \omega_{y_{(a,y)}} \), s\(^{-1} \) | \( y_1 = y_{0_{(a,y)}} \), mm |
|-----------------|-----------|-------------|-----------------|-----------------|-----------------|-----------------|
| 0.85            | 0.9679    | 1.084       | 1926.5          | 9632.6935       |                 | 7.4820          |
| 0.87            | 0.9716    | 1.088       | 1935.1          | 9675.2691       |                 | 8.5032          |
| 0.89            | 0.9755    | 1.093       | 1943.9          | 9719.6526       |                 | 9.8974          |
| **0.9**         | **0.9775**| **1.096**   | **1948.5**      | **9742.5474**   | **70.7107**     | **10.8045**     |
| 0.91            | 0.9795    | 1.099       | 1953.2          | 9765.9249       |                 | 11.9135         |
| 0.93            | 0.9837    | 1.104       | 1962.8          | 9814.1729       |                 | 15.0841         |
| 0.95            | 0.9881    | 1.110       | 1972.8          | 9864.4892       |                 | 20.7946         |

**Table 2.** The results of the calculation of the generalized design model parameters from conditions (1) and (7)

| \( k_{\omega_y} \) | \( m_{y_{(a,E)}} \), kg | \( c_{y_{(a,E)}} \), N/mm | \( \omega_{y_{(a,E)}} \), s\(^{-1} \) | \( E_K = E_{K,0} \), kJ | \( y_{0_{(a,E)}} \), mm | \( y_1 \), mm | \( \text{Error of } y, \% \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.85            | 1997.0          | 9985.4812       |                 | 0.2600          | 7.2176          | 7.4820          | 3.53            |
| 0.87            | 1997.0          | 9988.7479       |                 | 0.3388          | 8.2363          | 8.5032          | 3.13            |
| 0.89            | 1998.3          | 9991.6875       |                 | 0.4631          | 9.6280          | 9.8974          | 2.72            |
| **0.9**         | **1998.6**      | **9993.0217**   | **70.7107**     | **0.5544**      | **10.5336**     | **10.8045**     | **2.50**        |
| 0.91            | 1998.8          | 9994.2583       |                 | 0.6772          | 11.6413         | 11.9135         | 2.28            |
| 0.93            | 1999.2          | 9996.4161       |                 | 1.0961          | 14.8091         | 15.0841         | 1.82            |
| 0.95            | 1999.6          | 9998.1132       |                 | 2.1042          | 20.5166         | 20.7946         | 1.33            |

As Table 1 shows, each value of the workload frequency corresponds with own parameters of the generalized model, while the required conditions for the equality of frequencies and displacements are fully satisfied. The model works correctly in any natural frequency area of the system. A model selected according to the second variant of equivalence conditions (equal frequencies and kinetic...
energies) is also correct. The parameters of the generalized masses and stiffnesses for both models differ slightly. The displacements of the model \( y_0(x_{m,Ex}) \) and the first mass of the initial system \( y_1 \) also differ insignificantly, and the discrepancy decreases when the frequency of the external action approaches to the natural frequency of the system. Consequently, the error of the first model with respect to kinetic energy will also be insignificant.

We consider only the first model further on. Figure 3 shows graphs of dynamic magnification factor for the original system’s mass displacements (Figure 2, a) and for the generalized model (Figure 1, b) in case when \( \omega_F = 0.9 \omega_i \); model parameters – see Table 1.

The curves \( \mu_1 \) and \( \mu_0 \) in the area of the first resonant frequency almost coincide. The difference (in %) of the values \( \mu_1 \) and \( \mu_0 \) when the workload frequency is changing is shown in Figure 4. Thus, the proposed model describes the loading point displacement of the initial system in the workload frequency area rather well.

![Figure 3](image1.png)

**Figure 3.** Dependency graph of displacements’ dynamic magnification factor from the load frequency coefficient for the masses of the initial system (\( \mu_1, \mu_2 \)) and for the design model \( \mu_0 = y_0/y_L \)

![Figure 4](image2.png)

**Figure 4.** Difference of displacement’s dynamic magnification factor values for the point of loading at the generalized model (\( \mu_0 \)) and the initial system (\( \mu_1 \)), depending on the frequency

Note that the generalized mass and stiffness of design model are functions depending on random arguments which probabilistic properties are known. Then expressions (5) and (6) are written as

\[
\bar{m}_{0(x,\omega)} = \bar{\mu}_{0(\omega)} \cdot \bar{k}_F; \quad \bar{c}_{0(x,\omega)} = \bar{\mu}_{0(\omega)} \cdot \bar{k}_F,
\]  

(12)
where $\tilde{m}(a) = \frac{1}{\delta_u} \sum_{k=1}^{n} \tilde{m}_k \delta_u \beta_{sk}$; $\tilde{c}(a) = \frac{1}{\delta_u} \tilde{c} \beta_{sk}$; $\tilde{k}_f = \frac{1 - \tilde{k}_a^2}{1 - \tilde{k}_a^2} \tilde{k}_a = \tilde{\omega}_f / \tilde{\omega}_0$.

To determine the probabilistic characteristics of a generalized model one can use the statistical linearization method (SLM) [3] – due to small changes in their derivatives in the considered area. Elements of the displacements eigenvector $\beta_{sk}$ and also the relative displacements vector of forced oscillations $\beta_{sk(F)}$, as well as the coefficient $k_\omega$ are considered deterministic.

After selecting the parameters and determining the stochastic properties of the generalized design model with 1-DOF, the model is used in further calculations instead of the original system. In case of using a single-mass DVD (Figure 5, a), a two-DOF system is obtained. As a result probabilistic calculations greatly simplify.

![Figure 5. Generalized model with single-mass DVD (a) and its design scheme (b)](image)

If the damper is set in the direction of the dynamic load (Figure 5, b) it’s possible to completely eliminate the dynamic displacement $y_{1F}(t)$ of the load application point. In the case of neglecting energy dissipation in the system and damping in the damper the mean of the DVD parameters should be determined by the formula $\bar{c}_d \bar{m}_d = \bar{\omega}_f^2$, where $\bar{c}_d$ and $\bar{m}_d$ are the mean stiffness and damper mass; $\bar{\omega}_f$ – mean frequency of external exposure [1]. The probabilistic characteristics of the parameters of the DVD depend on its design, manufacturing technology and material properties.

The system of equations for stable forced vibrations in the amplitudes of inertial forces for a system with DVD (Figure 5, a):

$$
\begin{bmatrix}
\tilde{\delta} \tilde{\omega}
\end{bmatrix}
\begin{bmatrix}
\tilde{J}
\end{bmatrix}
+ \begin{bmatrix}
\tilde{\Lambda}
\end{bmatrix}
= 
\begin{bmatrix}
\tilde{\delta}_{11} & \tilde{\delta}_{12} & \tilde{J}_{1}
\tilde{\delta}_{21} & \tilde{\delta}_{22} & \tilde{J}_{2}
\end{bmatrix}
+ \begin{bmatrix}
\tilde{\Lambda}_{1F}
\tilde{\Lambda}_{2F}
\end{bmatrix}
= 0,
$$

where $\begin{bmatrix}
\tilde{\delta} \tilde{\omega}
\end{bmatrix}$ – system’s dynamic pliability matrix; $\begin{bmatrix}
\tilde{J}
\end{bmatrix}$ – inertia forces vector; $\begin{bmatrix}
\tilde{\Lambda}
\end{bmatrix}$ – load caused amplitude displacements vector; $\tilde{\delta}_{11} = \tilde{\delta}_{11} - (\tilde{m}_2 \tilde{\omega}_2)^{-1}$; $\tilde{\delta}_{22} = \tilde{\delta}_{22} - (\tilde{m}_2 \tilde{\omega}_2)^{-1}$; $\tilde{\delta}_{11} = \tilde{\delta}_{12} = \tilde{\delta}_{21} = \tilde{\omega}_0^{-1}$; $\tilde{\delta}_{22} = \tilde{\omega}_0^{-1} + \tilde{\omega}_d^2$; $\tilde{\Lambda}_{1F} = \tilde{\Lambda}_{2F} = \tilde{\delta}_{11} \tilde{F} = \tilde{F} \tilde{c}_0^{-1}$.

The amplitude values of the mass displacements of the system and the damper are defined as

$$
\begin{align*}
\tilde{y}_{1F} &= \tilde{\delta}_{11} \tilde{F} + \tilde{\delta}_{11} \tilde{J}_1 + \tilde{\delta}_{12} \tilde{J}_2, \\
\tilde{y}_{2F} &= \tilde{\delta}_{21} \tilde{F} + \tilde{\delta}_{21} \tilde{J}_1 + \tilde{\delta}_{22} \tilde{J}_2.
\end{align*}
$$

To the reliability estimation, the technique described in [3] is used. As to the criteria of serviceability for a given system with a damper, the restrictions on the mass displacements of the system and damper can be used.

If the failure probability of the total system or some of its elements reaches unacceptable values, then the parameters of the damper are adjusted, and the calculation is repeated.

The described method of analysis can be implemented using the algorithm presented in Figure 6. The advantage of the proposed reliability calculation approach for a system with a DVD as compared to the standard solution when the original system is repeatedly calculated is the size of the problem is reduced. For example, when using the SMM to obtain qualitative results in determining the
probabilistic characteristics of a multi-element system, the number of recalculations should usually be \( > 10^4 \). Moreover, reducing the size of the equations set gives a decrease in the counting time.

We use both versions of the algorithms shown in Figure 6 to solve the problem of determining the reliability of a structure whose original model (without DVD) is shown in Figure 2, a. All parameters are random variables with known probabilistic characteristics: \( l = 1000 \text{ mm} \); \( m_1 = 1000 \text{ kg} \); \( A_{s1} = 0.05 \); \( m_2 = 1000 \text{ kg} \); \( A_{s2} = 0.06 \); \( E = 2 \times 10^6 \text{ kPa} \); \( A_{e} = 0.05 \); \( b = 50 \text{ mm} \); \( A_{b} = 0.02 \); \( h = 200 \text{ mm} \); \( A_{h} = 0.03 \); \( F = 20 \text{ kN} \); \( A_{F} = 0.02 \); \( \omega_{0} = 0.9 \omega_{1} \); \( A_{\omega_{0}} = 0.05 \). The length \( l \) is considered as deterministic due to its small stochastic variability and insignificant effect on the probability of failure [9].

![Figure 6. Standard and proposed calculation algorithms](image)

Applying SLM to determine the mean and standard deviations (SD) of mass, stiffness and natural frequency of the generalized design model, we obtain: \( \bar{m} = 1948.51 \text{ kg} \); \( \bar{m} = 77.65 \text{ kg} \); \( \bar{c} = 9742.55 \text{ N/mm} \); \( \bar{c} = 505.58 \text{ N/mm} \); \( \bar{\omega}_{0} = 70.71068 \text{ s}^{-1} \); \( \bar{\omega}_{0} = 2.31329 \text{ s}^{-1} \). Note that for the initial model (Figure 2, a) the standard of the first eigenfrequency \( \omega_{1} = 2.59251 \text{ s}^{-1} \) (the SD error is 10.77%).
The following damper parameters are accepted: $m_d = 250\text{kg}$; $A_{my} = 0.01$; $c_d = 1012.5\text{ N/mm}$; $A_c = 0.02$. The scheme of the generalized design model with a damper is presented in the Figure 5, and the initial system with a damper is shown in Figure 7, a.

![Figure 7](image)

**Figure 7.** Two-mass system, protected by single-mass DVD (a), and its design scheme (b)

Natural frequencies for a generalized model with a damper, calculated with mentioned above parameters: $\omega_{01} = 56.20177\text{ s}^{-1}$; $\omega_{02} = 80.06865\text{ s}^{-1}$, and for the original two-mass system with a damper: $\omega_{01}^{\text{sys}} = 56.09808\text{ s}^{-1}$; $\omega_{02}^{\text{sys}} = 79.64974\text{ s}^{-1}$; $\omega_{03}^{\text{sys}} = 201.42349\text{ s}^{-1}$. It can be seen that the first and second eigenfrequencies for the generalized model and the system with the damper differ slightly. The graphs of the mass $m_0$ displacement of the generalized model with DVD and the mass $m_1$ of the system with the DVD are shown in Figure 8, a, and the dampers masses displacements are shown in Figure 8, b. The displacement curves in the frequency area under consideration differ insignificantly, and they practically coincide in the area of the workload frequency.

We take the restrictions on the mass displacements of the system as the conditions of serviceability: for $m_1$ (at load point) $[y_1] = 4.5\text{mm}$ and for damper mass $[y_d] = 45\text{mm}$.

![Figure 8](image)

**Figure 8.** The dependence of the displacement module on the workload frequency
(a) – load application points; (b) – damper mass
Figure 9. Dependency graph of the probability of failure from the workload frequency (a) – for a generalized model with a damper; (b) – for the initial system with a damper

Figure 9, a shows dependency graphs of the failure probability from the workload frequency according to the set limits for a generalized model with a damper, and Figure 9, b – for the initial system with a damper.

The corresponding curves in Figure 9, a, b have a fundamentally similar shape. The probability of failure for a generalized model is greater than for a system. This is due to errors in the determination of the probability characteristics of the generalized model due to its simplification. But the failure probabilities practically coincide at the tuning frequency of the generalized model and the damper ($\bar{\omega}_F = 0.9\bar{\omega}_1$ in the considered example).

3. Conclusion

1. Generalized models with characteristics determined by the considered combinations of equivalence conditions in a deterministic formulation adequately describe the dynamic SSS parameters of the original system in the area of the model tuning frequency.

2. The qualitative and quantitative results of dynamic calculations with the use of generalized models with DVD for forced harmonic and natural vibrations are in good agreement with the theory of vibrations damping.

3. Failure probability graphs for the generalized model with a damper and for the original system with a damper are similar, with acceptable quantitative discrepancies. The proposed approach can be used to select damper parameters taking into account the reliability requirements, as well as for a preliminary simplified estimation of the failure probability.

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