A Unified Framework of Deep Neural Networks by Capsules

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Abstract

With the growth of deep learning, how to describe deep neural networks unifiedly is becoming an important issue. We first formalize neural networks mathematically with their directed graph representations, and prove a generation theorem about the induced networks of connected directed acyclic graphs. Then, we set up a unified framework for deep learning with capsule networks. This capsule framework could simplify the description of existing deep neural networks, and provide a theoretical basis of graphic designing and programming techniques for deep learning models, thus would be of great significance to the advancement of deep learning.

1 Introduction

Deep learning has made a great deal of success in processing images, audios, and natural languages [1-3], influencing academia and industry dramatically. It is essentially a collection of various methods for effectively training neural networks with deep structures. A neural network is usually regarded as a hierarchical system composed of many nonlinear computing units (or neurons, nodes). The most popular neural network was once multilayer perceptron (MLP) [4]. A MLP consists of an input layer, a number of hidden layers and an output layer, as shown in Figure 1. The depth of it is the number of layers excluding the input layer. If the depth is greater than 2, a neural network is now called “deep”. For training MLPs, backpropagation (BP) is certainly the most well-known algorithm in common use [4], but it seemed to work only for shallow networks. In 1991, Hochreiter indicated that typical deep neural networks (DNNs) suffer from the problem of vanishing or exploding gradients [5]. To overcome training difficulties in DNNs, Hinton et al. started the new field of deep learning in 2006 [6, 7].

Besides deep MLPs, DNNs also include convolutional neural networks (CNNs) and recurrent neural networks (RNNs). Here, we omit RNNs for saving space. Theoretically, a CNN can be regarded as a special MLP or feedforward neural network. It generally consists of an input layer, alternating convolutional and pooling layers, a fully connected layer, and an output layer, as shown in Figure 2. Note that “convolutional layers” are also called “detection layers”, and “pooling layers” are also called “downsampling layers”. There have been a large number of CNN variants, for example, LeNet [8], AlexNet [1], VGGNet [9], GoogLeNet [10], ResNet [11], Faster R-CNN [12], DenseNet [13], Mask R-CNN [14], YOLO [15], SSD [16], and so on. They not only take the lead in competitions of image classification and recognition as well as object localization and detection [9-12], but also in other applications such as deep Q-networks [17], AlphaGo [18], speech recognition [2], and machine translation [3]. To cope with the disadvantages of CNNs, in 2017 Hinton et al. further proposed a capsule network [19], which is more convincing from the neurobiological point of view. So many deep models are dazzling with different structures. Some of them have added shortcut connections,
parallel connections, and even nested structures to traditional layered structures. How to establish a unified framework for DNNs is becoming a progressively important issue in theory. We are motivated to address it.

This paper is organized as follows. In Section 2, we propose a mathematical definition to formalize neural networks, give their directed graph representations, and prove a generation theorem about the induced networks of connected directed acyclic graphs. In Section 3, we use the concept of capsule to extend neural networks, define an induced model for capsule networks, and establish a unified framework for deep learning with a universal backpropagation algorithm. Finally, in Section 4 we make a few conclusions to summarize the significance of the capsule framework to advance deep learning in theory and application.

2 Formalization of Neural networks

2.1 Mathematical definition

A neural network is a computational model composed of nodes and connections. Nodes are divided into input nodes and neuron nodes. Input nodes can be represented by real variables, e.g. $x_1, x_2, \cdots, x_n$. The set of input nodes is denoted as $X = \{x_1, x_2, \cdots, x_n\}$. A neuron node can receive signals through connections both from input nodes and the outputs of other neuron nodes, and perform a weighted sum of these signals for a nonlinear transformation. Note that the weight measures the strength of a connection, and the nonlinear transformation is the effect of an activation function. Let $F$ be a set of activation functions, such as sigmoid, tanh, ReLU, and so on.

On $X$ and $F$, a neural network can be formally defined as a 4-tuple $net = (S, H, W, Y)$, where $S$ is a set of input nodes, $H$ is a set of neuron nodes, $W$ is a set of weighting connections, and $Y$ is a set of outputs. The neural network is recursively generated by four basic rules as follows:

1) Rule of variable. For any $z \in X$, let $y_z = z$. If $S = \{z\}$, $H = \emptyset$, $W = \emptyset$, $Y = \{y_z\}$, then the 4-tuple $net = (S, H, W, Y)$ is a neural network.

2) Rule of neuron. For any nonempty subset $S \subseteq X$, $\forall f \in F$, $\forall b \in \mathbb{R}$, construct a node $h \not\in S \cup H$ that depends on $(f, b)$ and select a set of weighting connections $w_{x_i \rightarrow h}(x_i \in S)$. Let $y_h = f(\sum_{x_i \in S} w_{x_i \rightarrow h}x_i + b)$ be the output of node $h$. If $H = \{h\}$, $W = \{w_{x_i \rightarrow h} | x_i \in S\}$, and $Y = \{y_h\}$, then $net = (S, H, W, Y)$ is a neural network.

3) Rule of growth. Suppose $net = (S, H, W, Y)$ is a neural network. For any nonempty subset $N \subseteq S \cup H$, $\forall f \in F$, $\forall b \in \mathbb{R}$, construct a node $h \not\in S \cup H$ that depends on $(f, b)$ and select a
Among the four generation rules, it should be noted that the rule of neuron is not independent. This rule can be totally generated further. Seven out of them for Figure 4(a) are displayed in Figures 5(a-g).

Using the rule of growth on the network, three new neural networks with different structures can be generated. For example, if \( S = \{ x_1 \}, W = \emptyset, Y = \{ y_1 \}, \) and \( Y' = Y \cup \{ y_1 \} \), then \( net' = (S', H', W', Y') \) is also a neural network.

### 4) Rule of convergence
Suppose \( net_k = (S_k, H_k, W_k, Y_k)(1 \leq k \leq K) \) are \( K \) neural networks, satisfying that \( \forall 1 \leq i \neq j \leq K, (S_i \cup H_j) \cap (S_j \cup H_i) = \emptyset \). For any nonempty subsets \( \mathcal{A}_k \subseteq S_k \cup H_k(1 \leq k \leq K), N = \bigcup_{k=1}^{K} \mathcal{A}_k, \forall f \in F, \forall b \in \mathbb{R}, \) construct a node \( h \notin \bigcup_{k=1}^{K} (S_k \cup H_k) \) that depends on \((f, b)\), select a set of weighting connections \( w_{z \rightarrow h}(z \in N) \). Let \( y_h = f(\sum_{z \in N} w_{z \rightarrow h} y_z + b) \) be the output of the node \( h \). If \( S = \bigcup_{k=1}^{K} S_k, H = (\bigcup_{k=1}^{K} H_k) \cup \{ h \}, W = (\bigcup_{k=1}^{K} W_k) \cup \{ w_{z \rightarrow h} \mid z \in N \}, \) and \( Y = (\bigcup_{k=1}^{K} Y_k) \cup \{ y_h \} \), then \( net = (S, H, W, Y) \) is also a neural network.

Among the four generation rules, it should be noted that the rule of neuron is not independent. This rule can be derived from the rule of variable and the rule of convergence. Moreover, the weighting connection \( w_{z \rightarrow h} \) should be taken as a combination of the weight and the connection, rather than just the weight. Additionally, if a node \( h \) depends on \((f, b), f \) is called the activation function of \( h \), and \( b \) is called the bias of \( h \).

### 2.2 Directed graph representation

Let \( X \) be a set of real variables and \( F \) be a set of activation functions. For any neural network \( net = (S, H, W, Y) \) on \( X \) and \( F \), a directed acyclic graph \( G_{net} = (V, E) \) can be constructed with the vertex set \( V = S \cup H \) and the directed edge set \( E = \{ z \rightarrow h \mid w_{z \rightarrow h} \in W \} \). \( G_{net} = (V, E) \) is called the directed graph representation of \( net = (S, H, W, Y) \). Two cases of the representation generation are discussed in the following.

**1) The case of \( X = \{ x_1 \} \)**

Using the rule of variable, for \( x_1 \in X \), let \( y_{x_1} = x_1 \). If \( S = \{ x_1 \}, \) \( H = \emptyset, W = \emptyset, \) and \( Y = \{ y_{x_1} \} \), then \( net = (S, H, W, Y) \) is a neural network. Since this network has only one input node without any function for nonlinear transformation, it is also called a trivial network, as shown in Figure 3(a). Using the rule of neuron, for a nonempty subset \( S = \{ x_1 \} \subseteq X, \forall f \in F, \forall b \in \mathbb{R}, \) construct a node \( h_1 \notin S \) that depends on \((f, b)\), select a weighting connection \( w_{x_1 \rightarrow h_1} \), and let \( y_{h_1} = f(w_{x_1 \rightarrow h_1} x_1 + b) \).

If \( H = \{ h_1 \}, W = \{ w_{x_1 \rightarrow h_1} \}, \) and \( Y = \{ y_{h_1} \}, \) then \( net = (S, H, W, Y) \) is a neural network, which has one input and one neuron. It is also called a 1-input-1-neuron network, as shown in Figure 3(b). Using the rule of growth on the network, three new neural networks with different structures can be generated, as shown in Figures 4(a-c). Likewise, they are called 1-input-2-neuron networks.

**2) The case of \( X = \{ x_1, x_2 \} \)**

Using the rule of variable, for \( x_1, x_2 \in X \), let \( y_{x_1} = x_1 \) and \( y_{x_2} = x_2 \). If \( S_1 = \{ x_1 \}, S_2 = \{ x_2 \}, H_1 = H_2 = \emptyset, W_1 = W_2 = \emptyset, Y_1 = \{ y_{x_1} \}, \) and \( Y_2 = \{ y_{x_2} \}, \) then \( net_1 = (\{ x_1 \}, \emptyset, \emptyset, \{ y_{x_1} \}) \) and \( net_2 = (\{ x_2 \}, \emptyset, \emptyset, \{ y_{x_2} \}) \) are neural networks. Obviously, both of them are trivial networks. Using
the rule of neuron, for a nonempty subset $S \subseteq X$, if $S = \{x_1\}$ or $S = \{x_2\}$, the neural network can be similarly constructed with the case of $X = \{x_1\}$.

If $X = \{x_1, x_2\}$, $\forall f \in F$, $\forall b \in \mathbb{R}$, construct a node $h_1 \notin S$ that depends on $(f, b)$, select a set of weighting connections $w_{x_i \rightarrow h_1}$ ($x_i \in S$) and let $y_{h_1} = f(\sum_{x_i \in S} w_{x_i \rightarrow h_1} x_i + b)$. If $H = \{h_1\}$, $W = \{w_{x_1 \rightarrow h_1}, w_{x_2 \rightarrow h_1}\}$, and $Y = \{y_{h_1}\}$, then $net = (S, H, W, Y)$ is a neural network. This is a 2-input-1-neuron network, as depicted in Figure 6. Using the rule of growth on this network, seven 2-input-2-neuron networks with different structures can be generated, as shown in Figures 7(a-g).

Finally, the rule of convergence is necessary. In fact, it cannot generate all neural networks only using the three rules of variable, neuron and growth. For example, the network in Figure 8(c) cannot be generated without using the rule of convergence on the two in Figures 8(a-b).

### 2.3 Induced network and its generation theorem

Suppose $G = (V, E)$ is a connected directed acyclic graph, where $V$ denotes the vertex set and $E$ denotes the directed edge set. For any vertex $h \in V$, let $IN_h = \{z \mid z \in V, z \rightarrow h \in E\}$ be the set of vertices each with a directed edge to $h$, and $OUT_h = \{z \mid z \in V, h \rightarrow z \in E\}$ be the set of vertices for $h$ to have directed edges each to. If $IN_h = \emptyset$, then $h$ is called an input node of $G$. If $OUT_h = \emptyset$, then $h$ is called an output node of $G$. Otherwise, $h$ is called a hidden node of $G$. Let $X$ stand for...
Then, a computational model of graph $G$. From the viewpoint of mathematical models, a capsule is essentially an extension of the capsule framework of Deep learning. In 2017, Hinton et al. pioneered the idea of capsules and considered a nonlinear “squashing” capsule $G\text{h}$ has at least one output node $y_h$ be the output of node $h$, and $w_{z\rightarrow h}$ be the weighting connection from $z$ to $h$. Then, a computational model of graph $G$ can be defined as follows:

1) $\forall z \in X, y_z = z$.

2) $\forall h \in M \cup O$, select $f \in F$ and $b \in \mathbb{R}$ to compute $y_h = f(\sum_{z \in IN_h} w_{z\rightarrow h}y_z + b)$.

If $S = X, H = M \cup O, W = \{w_{z\rightarrow h}| z \rightarrow h \in E\}$, and $Y = \{y_h| h \in V\}$, then $net_G = (S, H, W, Y)$ is called an induced network of graph $G$. The following generation theorem holds on the induced network.

**Generation Theorem:** For any connected directed acyclic graph $G = (V, E)$, its induced network $net_G$ is a neural network that can be recursively generated by the rules of variable, neuron, growth, and convergence.

**Proof:** By induction on $|V|$ (i.e. number of vertices), we prove the theorem as follows.

1) When $|V| = 1$, we have $|X| = 1$ and $|O| = 0$, so the induced network $net_G$ is a neural network that can be generated directly by the rule of variable.

2) When $|V| = 2$, we have $|X| = 1$ and $|O| = 1$, so the induced network $net_G$ is a neural network that can be generated directly by the rule of growth.

3) Assume that the theorem holds for $|V| \leq n$. When $|V| = n + 1 \geq 3$, the induced network $net_G$ has at least one output node $h \in O$. Let $E_h = \{z \rightarrow h \in E\}$ denote the set of edges heading to the node $h$. Moreover, let $V' = V - \{h\}$ and $E' = E - E_h$. Based on the connectedness of $G' = (V', E')$, we have two cases to discuss in the following:

i) If $G' = (V', E')$ is connected, then applying the induction assumption for $|V'| \leq n$, the induced network $net_{G'} = (S', H', W', Y')$ can be recursively generated by the rules of variable, neuron, growth, and convergence. Let $N = IN_h$. In $net_G = (S, H, W, Y)$, we use $f \in F$ and $b \in \mathbb{R}$ to stand for the activation function and bias of node $h$, and $w_{z\rightarrow h}(z \in N)$ for the weighting connection from node $z$ to the node $h$. Then, $net_G$ can be obtained by using the rule of growth on $net_{G'}$, to generate the node $h$ and its output $y_h = f(\sum_{z \in N} w_{z\rightarrow h}y_z + b)$.

ii) Otherwise, $G'$ comprises a number of disjoint connected components $G_k = (V_k, E_k)(1 \leq k \leq K)$. Using the induction assumption for $|V_k| \leq n(1 \leq k \leq K)$, the induced network $net_{G_k} = (S_k, H_k, W_k, Y_k)$ can be recursively generated by the rules of variable, neuron, growth, and convergence. Let $A_k = (S_k \cup H_k) \cap IN_h$, and $N = \bigcup_{k=1}^K A_k$. In $net_G = (S, H, W, Y)$, we use $f \in F$ and $b \in \mathbb{R}$ to stand for the activation function and bias of the node $h$, and $w_{z\rightarrow h}(z \in N)$ for the weighting connection from node $z$ to node $h$. Then, $net_G$ can be obtained by using the rule of convergence on $net_{G_k}(1 \leq k \leq K)$, to generate the node $h$ and its output $y_h = f(\sum_{z \in N} w_{z\rightarrow h}y_z + b)$.

As a result, the theorem always holds.

3 Capsule framework of Deep learning

3.1 Mathematical definition of capsules

In 2017, Hinton et al. pioneered the idea of capsules and considered a nonlinear “squashing” capsule [19]. From the viewpoint of mathematical models, a capsule is essentially an extension of the
weighting operation may be taken as an identity transfer, a scalar multiplication, a vector dot product, as shown in Figure 9, a general capsule may have \( n \) input tensors \( X_1, X_2, \ldots, X_n \), \( n \) weight tensors \( W_1, W_2, \ldots, W_n \), and a capsule bias \( B \), and \( n \) weighting operations \( \otimes_1, \otimes_2, \ldots, \otimes_n \). Note that a weighting operation may be taken as an identity transfer, a scalar multiplication, a vector dot product, a matrix multiplication, a convolution operation, and so on. Meanwhile, \( W_i \otimes_i X_i (1 \leq i \leq n) \) and \( B \) must be tensors with the same dimension. The total input of the capsule is \( U = \sum_i W_i \otimes_i X_i + B \), and the output \( Y \) is a tensor computed by a nonlinear capsule function \( \text{cap} \), namely,

\[
Y = \text{cap}(U) = \text{cap}(\sum_i W_i \otimes_i X_i + B). \tag{1}
\]

For convenience, we use \( \mathcal{F} \) to stand for a nonempty set of capsule functions, and \( \mathbb{T} \) for the set of all tensors.

### 3.2 Capsule Networks

Suppose \( \mathcal{G} = (\mathcal{V}, \mathcal{E}) \) is a connected directed acyclic graph, where \( \mathcal{V} \) denotes the vertex set and \( \mathcal{E} \) denotes the directed edge set. For any vertex \( H \in \mathcal{V} \), let \( IN_H \) be the set of vertices each with a directed edge to \( H \), and \( OUT_H \) be the set of vertices for \( H \) to have a directed edge each to. If \( IN_H = \emptyset \), then \( H \) is called an input node of \( \mathcal{G} \). If \( OUT_H = \emptyset \), then \( H \) is called an output node of \( \mathcal{G} \). Otherwise, \( H \) is called a hidden node of \( \mathcal{G} \). Let \( \mathcal{X} \) stand for the set of all input nodes, \( \mathcal{O} \) for the set of all output nodes, and \( \mathcal{M} \) for the set of all hidden nodes. Obviously, \( \mathcal{V} = \mathcal{X} \cup \mathcal{M} \cup \mathcal{O} \), and \( \mathcal{M} = \mathcal{V} - \mathcal{X} \cup \mathcal{O} \).

Furthermore, let \( Y_H \) be the output of node \( H \), and \( (W_{Z \rightarrow H}, \otimes_{Z \rightarrow H}) \) be the tensor-weighting connection from \( Z \) to \( H \). If \( \forall H \in \mathcal{M} \cup \mathcal{O}, \forall Z \in IN_H, W_{Z \rightarrow H} \otimes_{Z \rightarrow H} Y_Z \) and \( B \) are tensors with the same dimension, then a tensor-computational model of graph \( \mathcal{G} \) can be defined as follows:

1. \( \forall Z \in \mathcal{X}, Y_Z = Z \).

2. \( \forall H \in \mathcal{M} \cup \mathcal{O} \), select \( \text{cap} \in \mathcal{F} \) and \( B \in \mathbb{T} \) to compute \( Y_H = \text{cap}(\sum_{Z \in IN_H} W_{Z \rightarrow H} \otimes_{Z \rightarrow H} Y_Z + B) \).

If \( S = \mathcal{X}, \mathcal{H} = \mathcal{M} \cup \mathcal{O}, \mathcal{W} = \{(W_{Z \rightarrow H}, \otimes_{Z \rightarrow H})| Z \rightarrow H \in \mathcal{E} \} \), and \( \mathcal{Y} = \{Y_H| H \in \mathcal{V} \} \), then \( \text{net}_G = (S, \mathcal{H}, \mathcal{W}, \mathcal{Y}) \) is called a tensor-induced network of graph \( \mathcal{G} \). This network is also called a capsule network.

Using a capsule network, a MLP can be simplified as a directed acyclic path of capsules. For example, the MLP in Figure 1 has five layers: an input layer, three hidden layers, and an output layer. On the whole, each layer could be thought of as a capsule. Let \( X = (x_1, x_2, \ldots, x_5)^T \) stand for the input capsule node, \( H_1 = (cap_1, B_1) (i = 1, 2, 3) \) for the hidden capsule nodes, and \( O = (cap_4, B_4) \) for the output capsule node. Note that capsule function \( \text{cap}_i \) and capsule bias \( B_i \) are defined by the elementwise activation function and the bias vector respectively of the corresponding layer in the MLP. If the weighting operations \( \otimes \mathcal{X} \rightarrow H_1, \otimes H_1 \rightarrow H_2, \otimes H_2 \rightarrow H_3, \) and \( \otimes H_3 \rightarrow O \) are
3.3 Universal backpropagation of capsule networks

Suppose $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is a connected directed acyclic graph. Let $X = \{X_1, X_2, \ldots, X_n\}$ stand for the set of all input nodes, $\mathcal{O} = \{O_1, O_2, \ldots, O_m\}$ for the set of all output nodes, and $\mathcal{M} = \mathcal{V} - X \cup \mathcal{O} = \{H_1, H_2, \ldots, H_t\}$ for the set of all hidden nodes. $net_\mathcal{G} = (S, \mathcal{H}, W, Y)$ is a tensor-induced network of graph $\mathcal{G}$. This is also a capsule network. If the number of nodes $|S \cup \mathcal{H}| \geq 2$, then for $\forall H \in \mathcal{H}$,

$$
\begin{align*}
U_H &= \sum_{Z \in 1N_H} W_{Z \rightarrow H} \otimes_{Z \rightarrow H} Y_Z + B_H, \\
Y_H &= cap_H(U_H) = cap_H(\sum_{Z \in 1N_H} W_{Z \rightarrow H} \otimes_{Z \rightarrow H} Y_Z + B_H).
\end{align*}
$$

For any output node $H \in \mathcal{O}$, let $Y_H$ and $T_H$ be its actual output and expected output for input $X$, respectively. The loss function between them is defined as $L_H = Loss(Y_H, T_H)$. Accordingly, we
Algorithm 1: One iteration of the universal backpropagation algorithm.

1) Select a learning rate $\eta > 0$.
2) $\forall H \in M \cup O, \forall Z \in IN_H$, initialize $W_{Z \rightarrow H}$ and $B_H$.
3) $\forall H \in O$, compute $\delta_H = \frac{\partial \text{Loss}(Y_H, T_H)}{\partial U_H} \cdot \frac{\partial \text{cap}_H}{\partial U_H}$.
4) $\forall H \in M$, compute $\delta_H = \sum_{P \in OUT_H} \delta_P \cdot \frac{\partial Y_H}{\partial U_P} \cdot \frac{\partial \text{cap}_H}{\partial U_H}$.
5) Compute $\Delta W_{Z \rightarrow H} = \delta_H \cdot \frac{\partial U_H}{\partial W_{Z \rightarrow H}}$ and $\Delta B_H = \delta_H$.
6) Update $W_{Z \rightarrow H} \leftarrow W_{Z \rightarrow H} - \eta \cdot \Delta W_{Z \rightarrow H}$, $B_H \leftarrow B_H - \eta \cdot \Delta B_H$.

have the total loss function $L = \sum_{H \in O} L_H$. Let $\delta_H = \frac{\partial L}{\partial U_H}$ denote the backpropagated error signal (or sensitivity) for capsule node $H$. By the chain rule, we further obtain:

$$\forall H \in O, \left\{ \begin{array}{l}
\frac{\partial L}{\partial U_H} = \frac{\partial \text{Loss}(Y_H, T_H)}{\partial Y_H} \cdot \frac{\partial \text{cap}_H}{\partial U_H} \cdot \frac{\partial U_H}{\partial W_{Z \rightarrow H}} \cdot \frac{\partial W_{Z \rightarrow H}}{\partial U_H} = \delta_H \cdot \frac{\partial \text{cap}_H}{\partial U_H} \cdot \frac{\partial U_H}{\partial W_{Z \rightarrow H}}.
\end{array} \right. \quad (4)$$

$$\forall H \in M, \left\{ \begin{array}{l}
\frac{\partial L}{\partial U_H} = \frac{\partial \text{Loss}(Y_H, T_H)}{\partial Y_H} \cdot \frac{\partial \text{cap}_H}{\partial U_H} \cdot \frac{\partial U_H}{\partial W_{Z \rightarrow H}} \cdot \frac{\partial W_{Z \rightarrow H}}{\partial U_H} = \delta_H \cdot \frac{\partial \text{cap}_H}{\partial U_H} \cdot \frac{\partial U_H}{\partial W_{Z \rightarrow H}}.
\end{array} \right. \quad (5)$$

Note that in formulae (4)-(5), $\frac{\partial \text{cap}_H}{\partial U_H}$ depends on the specific form of capsule function cap_H. For example, when cap_H is an elementwise sigmoid function, the result is $\frac{\partial \text{cap}_H}{\partial U_H} = \text{sigmoid}(U_H)(1 - \text{sigmoid}(U_H))$. Meanwhile, $\frac{\partial U_H}{\partial W_{Z \rightarrow H}}$ and $\frac{\partial U_H}{\partial Y_H}$ also depend on the specific choice of the weighting operation $\otimes$. $Z \rightarrow H$.

Based on formulae (4)-(5), a universal backpropagation algorithm can be designed theoretically for capsule networks, with one iteration detailed in Algorithm 1. In practice, this algorithm should be changed to one of many variants with training data [20].

4 Conclusions

Based on the formalization of neural networks, we have developed capsule networks to establish a unified framework for deep learning. This capsule framework could not only simplify the description of existing DNNs, but also provide a theoretical basis of graphical designing and programming for new deep learning models. As future work, we will try to define an industrial standard and implement a graphic platform for the advancement of deep learning with capsule networks, and even with a similar extension to recurrent neural networks.
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