Z boson decay to photon plus Kaluza-Klein graviton: large extra dimensional bounds

Benjamin C. Allanach and Jordan P. Skittrall

Department of Applied Mathematics and Theoretical Physics, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Road, Cambridge, CB3 0WA, United Kingdom e-mail: B.C.Allanach@damtp.cam.ac.uk, J.P.Skittrall@damtp.cam.ac.uk

Abstract. We consider the phenomenology of the decay of a Z boson into a photon and a Kaluza-Klein excitation of the graviton in the ADD model. Using LEP data, we obtain an upper bound on the branching ratio corresponding to this process of \( \sim 10^{-11} \). We also investigate energy profiles of the process.

Key words. Large Extra Dimensions – Beyond Standard Model

1 Introduction

Nieves and Pal \([1]\) have constructed a theoretical argument to predict the decay width of a Z boson to a photon and a graviton. The ADD model \([2,3,4,5]\) predicts a “tower” of massive Kaluza-Klein excitations of the graviton (both massive spin-2 gravitons and massive spin-0 gravi-scalars) when the model is viewed from a four-dimensional perspective \([6,7]\). In a previous paper with K. Sridhar \([8]\), we extended the theoretical argument of Nieves and Pal to predict the decay width of a Z boson to a photon and a Kaluza-Klein graviton state in an ADD model where the extra dimensions are toroidally compactified with a common compactification radius. We now extend that work by using data from the L3 experiment at LEP1 \([9]\) to estimate the bounds on the size of such extra dimensions that can be achieved by considering this process. The bounds we obtain are comparable with, but not stronger than, those obtained by considering the tree-level processes \( e^+e^- \rightarrow \gamma G \) \([10]\) and \( p\bar{p} \rightarrow G + \text{jet} \) \([11,12]\). (In this paper, we use \( G \) to denote any Kaluza-Klein excitation of the graviton – massive graviton or gravi-scalar – in the tree-level processes the gravi-scalar contribution is negligible, but in the one-loop Z decay process it dominates.) By using the bounds on the size of such extra dimensions obtained from the processes \( e^+e^- \rightarrow \gamma G \) and \( p\bar{p} \rightarrow G + \text{jet} \), and, for the case of two extra dimensions, from inverse square law experiments \([13,14]\), it is therefore possible to predict stronger bounds than those measured by experiment on the contribution from the process \( Z \rightarrow \gamma G \). The weakest such bound constrains the branching ratio to around the \( 10^{-11} \) level.

We also investigate energy spectra of events from this decay channel. These spectra may prove useful in model discrimination, possibly indicating an experiment to be performed were ADD discovered. It is particularly notable that the shape of the spectra for \( n = 2 \) extra dimensions differs from those for \( n > 2 \) extra dimensions.

2 Existing bounds and bounds from LEP1 data on Z decay

Existing particle physics bounds on the size of extra dimensions come from consideration of the tree-level processes \( e^+e^- \rightarrow \gamma G \) \([10]\) and \( p\bar{p} \rightarrow G + \text{jet} \) \([11,12]\). In our
paper with K. Sridhar [8], we showed that

\[
\Gamma_{\text{tot}} = \frac{\alpha^2 G M_z^{3+n} R^n \pi^{n/2}}{72 \pi^2} \times \\
\times \left[ 0.0088 \left( \frac{7 \cdot 5!}{T(\frac{5}{7} + 6)} + \frac{3 \cdot \frac{7}{5} \cdot 5!}{T(\frac{5}{7} + 7)} \right) + \\
+ 0.27 \left\{ \sum_{j=0}^{\infty} \frac{1}{(j+2)(j+3)(j+4)} \left( \frac{7 \cdot (j+5)!}{T(\frac{5}{7} + j + 6)} + \frac{3 \cdot \frac{7}{5} \cdot (j+5)!}{T(\frac{5}{7} + j + 7)} \right) \right\} + \\
+ 21 \left\{ \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \frac{1}{(i+2)(i+3)(i+4)(j+2)(j+3)(j+4)} \times \left( \frac{7 \cdot (i+5)!}{T(\frac{5}{7} + i + j + 6)} + \frac{3 \cdot \frac{7}{5} \cdot (i+5)!}{T(\frac{5}{7} + i + j + 7)} \right) \right\} + \\
+ \frac{3}{2} \left( \frac{330}{n+4} - \frac{303}{n+5} + \frac{13}{n+6} - \frac{0.97 \cdot \frac{7}{5} \cdot (n+1) \cdot (n+2)}{T(\frac{5}{7} + 7)} - \frac{0.078 \cdot \frac{7}{5} \cdot (n+1) \cdot (n+2) \cdot (n+3)}{T(\frac{5}{7} + 8)} \right) \right],
\]

where \(\Gamma_{\text{tot}}\) is the total decay width of the Z boson to a photon and any single Kaluza-Klein excitation of the graviton, \(n\) is the number of extra dimensions, all of which are toroidally compactified with common radius \(R\), \(G\) is Newton’s constant in four dimensions, \(\alpha\) is the fine-structure constant and \(M_z\) is the mass of the Z boson. The width has contributions from decay to a photon and a spin-0 graviscalar (the section with the \((3/2)(n-1)/(n+2)\) coefficient) and from decay to a photon and a spin-2 Kaluza-Klein graviton exciton (the remainder).

Instead of being expressed in terms of a compactification radius \(R\), it is equivalently possible to express the decay width (and compactification scale) in terms of a mass \(M_D\), which is defined such that

\[
8 \pi R^n M_D^{n+2} G = 1,
\]

where again \(G\) is Newton’s constant in four dimensions. We present bounds using both measures of compactification scale.

It is possible to obtain experimental bounds on the width of the decay \(Z \to \gamma + m\) either by direct event selection, or by subtracting from the total Z width the sum of the widths of the “visible” Z decays. The method of direct event selection leads to much stronger bounds with current data, and so it is the method we use in this paper. With both methods (and in the case of the process \(e^+ e^- \to \gamma \bar{\nu} \nu\)) there is a Standard Model background to experimental data from the process \(e^+ e^- \to \gamma \bar{\nu} \nu\) [15][16][17][18]. The experimental analyses take this background at tree-level into account to produce 95% confidence bounds, which we use. Of the one-loop corrections to the background, those involving the process \(Z \to \gamma \nu \bar{\nu}\) are of particular relevance to the Z decay case, but the branching ratio for the process \(Z \to \gamma \nu \bar{\nu}\) [19] is (at

\[
7.16 \times 10^{-10}
\]

about four orders of magnitude smaller than the L3 experimental bound on the process \(Z \to \gamma + m\) [9], and so the process \(Z \to \gamma \nu \bar{\nu}\) is negligible when analysing current experimental bounds on the process \(Z \to \gamma + m\) [9].

In order to consider experimental data to derive a bound on the magnitude of the radius of the extra dimensions \(R\), we need to take into account that there will be a cut specifying the minimum photon energy, \(E_{\text{min}}\). We can put this into our summation/integration over the Kaluza-Klein states when deriving the overall width, to get

\[
\Gamma_{\text{tot}} = \frac{\alpha^2 G M_z^{3+n} R^n \pi^{n/2}}{72 \pi^2} \times \\
\times \left[ 0.0088 \times \\
\times \left\{ 10 \sum_{p=0}^{5} \left( \frac{5!}{(5-p)! T(\frac{5}{7} + p + 1)} \cdot \hat{E}_{\text{min}}^{5-p} \hat{E}_{\text{rem}}^{p} \right) - \\
-3 \sum_{p=0}^{6} \left( \frac{6!}{(6-p)! T(\frac{5}{7} + p + 1)} \cdot \hat{E}_{\text{min}}^{6-p} \hat{E}_{\text{rem}}^{p} \right) \right\} + \\
+ 0.27 \sum_{j=0}^{\infty} \frac{1}{(j+2)(j+3)(j+4)} \times \\
\times \left\{ 10 \sum_{p=0}^{5+j} \left( \frac{(5+j)!}{(5-j-p)! T(\frac{5}{7} + p + 1)} \cdot \hat{E}_{\text{min}}^{5+j-p} \hat{E}_{\text{rem}}^{p} \right) - \\
-3 \sum_{p=0}^{6+j} \left( \frac{(6+j)!}{(6-j-p)! T(\frac{5}{7} + p + 1)} \cdot \hat{E}_{\text{min}}^{6+j-p} \hat{E}_{\text{rem}}^{p} \right) \right\} \right] + \\
+ \frac{n-1}{4 (n+2) T(\frac{n}{2})} \times \\
\times \left\{ 330 \cdot \hat{E}_{\text{rem}}^{\frac{n}{2}} - 910 \cdot \hat{E}_{\text{rem}}^{\frac{n}{2}+1} + 800 \cdot \hat{E}_{\text{rem}}^{\frac{n}{2}+2} - \\
- 180 \cdot \hat{E}_{\text{rem}}^{\frac{n}{2}+3} + 26 \cdot \hat{E}_{\text{rem}}^{\frac{n}{2}+4} - 10 \cdot \hat{E}_{\text{rem}}^{\frac{n}{2}+5} - \\
- 0.74 \cdot \hat{E}_{\text{rem}}^{\frac{n}{2}+6} + 0.078 \cdot \hat{E}_{\text{rem}}^{\frac{n}{2}+7} \right\},
\]

where we have defined \(\hat{E}_{\text{min}} \equiv 2E_{\text{min}}/M_Z\) and \(\hat{E}_{\text{rem}} \equiv 1 - \hat{E}_{\text{min}}\) for notational brevity. (This reduces to equation (1) in the case \(E_{\text{min}} = 0\).)

We can apply this formula to the data of reference [9], to obtain the bounds on the size of the extra dimensions given in the second and third columns of table [11].

1 Reference [9] plots upper limits on the branching ratio for a range of values of \(E_{\text{min}}\). The bounds on the size of the extra dimensions we obtain are the strongest that arise from applying
In addition to particle physics bounds, a stronger experimental bound can be obtained for \( n = 2 \) extra dimensions from inverse square law experiments [13]. (Bounds cannot be directly taken from reference [13] for \( n > 2 \) extra dimensions as the bounds derived require the extra dimensions to be asymmetrically sized.) There are also estimates of astrophysical bounds using the temperature profile of the observed collapse of SN1987A (an upper bound is set on the amount of graviton emission as this would affect the resultant temperature) [20, 21, 22, 23], which depending on the assumptions made give estimates of \( M_D \gtrsim 30 \text{ TeV} \) to \( M_D \gtrsim 10^{10} \text{ TeV} \) for \( n = 2 \), and \( M_D \gtrsim 2.0 \text{ TeV} \) to \( M_D \gtrsim 9.3 \text{ TeV} \) for \( n = 3 \) (the bounds are comparable with or weaker than the experimental bounds for \( n > 3 \)). There are also cosmological arguments that lead to estimated bounds on the size of extra dimensions [24, 25].

We see that the bounds on the scale of the extra dimensions derived from the Z decay process are weaker than other bounds. It is possible therefore for us to use the stronger bounds on the scale to estimate upper bounds on the branching ratio of the Z boson to a photon and a Kaluza-Klein graviton/gravi-scalar at the strongest of each of the 95% confidence bounds on the mass scale \( M_D \) from table [1] (i.e. the bounds given in bold).

![Figure 2](image_url)

**Fig. 2.** \( \Gamma(Z \rightarrow \gamma G) \) and \( \text{BR}(Z \rightarrow \gamma G) \) as functions of of the mass scale \( M_D \) of the extra dimensions, for \( n = 2 \) to \( n = 6 \) extra dimensions (consecutive values of \( n \) correspond to adjacent lines in the plot). The crosses plotted show the bounds on the decay width and branching ratio obtained by considering the strongest experimental 95% confidence bounds on \( M_D \) (i.e. the bold bounds from table [1]).

### 3 Energy profiles of the process

Extra dimensional models are often distinguishable from other “new physics” models by noting that the multiple Kaluza-Klein states predicted by the extra dimensional model give a different, usually softer, energy distribution from those corresponding to models predicting single new states. With this in mind, we investigate the energy distribution of the process \( Z \rightarrow \gamma G \). It is necessary to recall the caveat that the low branching ratio means a very high Z luminosity will be required to see such a distribution. The likely main use of such profiles would therefore be in model discrimination subsequent to a signal seen elsewhere. An analysis of experimental data would also need to take into account the energy profile of the Standard Model background [17, 18], in addition to any other event selection considerations.

The differential decay width \( d\Gamma/dE \), where \( E \) is the energy of the photon in the centre-of-mass frame, is equal to \( -d\Gamma(E_{\text{min}})/dE_{\text{min}} \), where \( \Gamma(E_{\text{min}}) \) is as given in equation [3] (the minus sign comes from the minimum energy...
Figures 3 and 4 illustrate the energy profiles of the decay processes for \( n = 2 \) to \( n = 6 \) extra dimensions.

It is most obvious from the general profile plots that if such energy profiles could be obtained from an experiment and there were an indication of a toroidally-compactified ADD model, it should be possible to distinguish between \( n = 2 \) and \( n > 2 \) extra dimensions because of the non-zero energy derivative as \( E \to M_Z/2 \) in the \( n = 2 \) case.

It is also possible to see a distinction between the profiles for other numbers of extra dimensions, if we observe that taking an energy derivative of equation (3) keeps a factor of \( R^n \), so that one can obtain a parameter independent of the scale of the extra dimensions by considering \( (1/\Gamma) \cdot d\Gamma/dE \). This is illustrated in figure 5. A detailed investigation of the number of events required for distinction between the numbers of extra dimensions is beyond the scope of this paper. However, this does show that given a signal of the ADD model, it may be possible to distinguish between possible numbers of extra dimensions if it were possible to obtain a sufficiently high number of \( Z \) decays.

Although the decay we are investigating is of a real \( Z \) boson in the centre-of-mass frame, which has a uniform angular distribution, the angular distribution does not remain uniform if one takes into account the production process \( e^+e^- \to Z \), and considers the process \( e^+e^- \to Z \to \gamma G \) at the \( Z \) resonance. In this case, the de-
Fig. 4. Differential decay widths $d\Gamma/dE$ for the process $Z \to \gamma G$, where $E$ is the energy in the centre-of-mass frame of the photon produced, for mass scales $M_D = 0.5$ TeV to $M_D = 2.5$ TeV of the extra dimensions, in $n = 2$ to $n = 6$ extra dimensions.

Fig. 5. Differential energy profiles $d\Gamma/dE$, normalised by $1/\Gamma$ to be independent of the extra dimensions scale, for $n = 3$ to $n = 6$ extra dimensions.
regions near the beam, meaning that taking the angular distribution into account would result in our expecting a smaller number of events (by a factor of about 0.9) than we should expect assuming a uniform distribution. This means that the bounds we have derived are stronger than those that would be derived by taking the angular distribution into account. (However, even the slightly strong bounds we have derived are still weaker than bounds from other processes.)

The bounds derived on the branching ratio of $Z \to \gamma G$ do not have an angular dependence, but it may be necessary if using such bounds to test or exclude this ADD scenario to take into account the $Z$ production process in a particular experiment, to determine whether it is necessary to consider angular distributions.

4 Conclusions

We have shown that for a toroidally-compactified ADD model with a common radius for the extra dimensions, the branching ratio for the process $Z \to \gamma G$ is not more than $10^{-11}$ – sufficiently small that the process would not be observed in a Giga-Z collider without a significant luminosity upgrade. No such decays of on-shell $Z$ bosons should be expected at the LHC. This suggests a possible experimental search strategy in the event that it became necessary to distinguish ADD from another suspected beyond Standard Model scenario (namely an investigation of this relatively “clean” experimental decay channel – a significant excess of events would rule out this ADD model). This branching ratio also constitutes a test that ADD would have to fulfil were there some indication of ADD from one of the other search channels.

We should like to thank K. Sridhar, David Ward, Bryan Webber, Graham Wilson, and the members of the Cambridge Supersymmetry Working Group for helpful comments made whilst this paper was in preparation. This work has been partially supported by the United Kingdom’s Science and Technology Facilities Council.

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