Analysis of a COVID-19 Epidemic Model with Seasonality

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Abstract

The statistics of COVID-19 cases exhibits seasonal fluctuations in many countries. In this paper, we propose a COVID-19 epidemic model with seasonality and define the basic reproduction number $R_0$ for the disease transmission. It is proved that the disease-free equilibrium is globally asymptotically stable when $R_0 < 1$, while the disease is uniformly persistent and there exists at least one positive periodic solution when $R_0 > 1$. Numerically, we observe that there is a globally asymptotically stable positive periodic solution in the case of $R_0 > 1$. Further, we conduct a case study of the COVID-19 transmission in the USA by using statistical data.

Keywords COVID-19 · Seasonal pattern · Basic reproduction number · Effective reproduction number · Threshold dynamics

1 Introduction

Coronavirus disease (COVID-19) is caused by infection with a newly discovered corona virus named the severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) virus strain. The virus is mainly transmitted by contacting saliva drops or secretions of infected people. The reported data from the World Health Organization (WHO) reported that the cumulative number of COVID-19 new cases by the end of June 2022 was over 547 million, including over 6.3 million deaths worldwide [15]. For control and prevention of the spread of COVID-19, many governments have implemented very strict epidemic prevention and control policies, such as keeping social distance, contact tracing, travel restrictions, self-isolation, medical quarantine, and even lockdown of living places.
There are many mathematical models of COVID-19 with the compartmental structure describing the size of populations within different classes (see Xue et al. 2020; Yan et al. 2020; Liu et al. 2021; Musa et al. 2022; Wang et al. 2022; Zou et al. 2022; Zhou et al. 2022; Zhang and Li 2021; Li et al. 2021; Tang et al. 2020a, b; Yang et al. 2020; Zhang et al. 2020; Munayco et al. 2020; Liu et al. 2020; Lauer et al. 2020; Kuniya 2020; Hu et al. 2020; Gatto et al. 2020). Xue et al. (2020) presented a data-driven network model to capture the contact heterogeneity between individuals. They used the Markov Chain Monte Carlo (MCMC) optimization algorithm to estimate the values of parameters and further applied the model to analyze the transmission potential and mitigation strategies of the COVID-19 epidemic in Wuhan, China, and Toronto, Canada, and the Republic of Italy. In Yan et al. (2020), Yan et al. proposed an epidemic model to show that media reports play an increasingly important role in the COVID-19 outbreak and analyzed the impact of media reports on the epidemic in some regions of China via statistical data. They also pointed out that media reports may provide the public with epidemic information and effective control measures to guide people’s behavior changes to control the spread of the epidemic.

COVID-19 has lasted for nearly three years, and the case statistics indicates seasonal fluctuations in many countries. Liu et al. (2021) numerically studied the role of seasonality in the spread of COVID-19 pandemic by using a compartmental model and early statistical data. In this paper, we propose a time-periodic compartmental model to study the seasonality of COVID-19 based on the disease with incubation period and isolation policies. Since the most relevant parameter with seasonality is the transmission rate, we introduce a periodic transmission rate $\beta(t)$ between a susceptible and a symptomatic individual. We divide the infected population into symptomatic $I(t)$ and asymptomatic $A(t)$. During the epidemic, the potential contact population needs to be quarantined. Part of the quarantined population and infected population need to be isolated, so we introduce quarantined $Q(t)$ and isolated $J(t)$ classes into the model. Then, we define the basic reproduction number $R_0$ to analyze the global dynamics of the model. In particular, we prove that as long as one of $A(t)$ and $I(t)$ is greater than 0 at some time $t$, the epidemic will break out, which extends the previous conclusion that the disease will be persistent only if both are greater than 0. Numerically, we study the COVID-19 transmission in the USA and investigate the seasonal spread of the disease by using the statistical data from the beginning of 2020 to the end of May 2022. Moreover, we introduce the effective reproduction number $R_t$ to illustrate the scale of the epidemic over time via numerical simulations.

The rest of the paper is organized as follows. In Sect. 2, we present the model and study its well-posedness. In Sect. 3, we derive the basic reproduction number $R_0$ and the effective reproduction number $R_t$. We then establish a threshold type result on the global dynamics in terms of $R_0$. In Sect. 4, we conduct a case study for COVID-19 transmission in USA. A brief discussion then concludes the paper.

2 The Model

In this section, we formulate a time-periodic COVID-19 epidemic model based on the possible fact that there is a seasonal trend for new COVID-19 cases. In view of the
Fig. 1 Schematic flow diagram for the COVID-19 model. The model consists of seven sub-populations: susceptible $S(t)$, exposed $E(t)$, quarantined $Q(t)$, infectious with symptoms $I(t)$, asymptomatic infection $A(t)$, isolated $J(t)$ and recovered $R(t)$ individuals in a population of $N(t) = S(t) + E(t) + Q(t) + I(t) + A(t) + R(t)$ individuals.

disease with incubation period and isolation policies, the total population is divided into seven sub-classes as follows:

- Susceptible individuals $S(t)$;
- Exposed individuals $E(t)$ who are exposed to the virus but not diagnosed positive for COVID-19 yet;
- Quarantined individuals $Q(t)$ refer to the separation of COVID-19 exposed individuals from the general population before the COVID-19 positive stage, for example, medical observation, self-quarantine, etc.;
- Infectious individuals with symptoms $I(t)$ who are confirmed COVID-19 positive patients and have clinical symptoms;
- Asymptomatic infection $A(t)$ who are confirmed COVID-19 positive patients and do not have clinical symptoms;
- Isolated individuals $J(t)$ who are confirmed COVID-19 positive patients and have been isolated, for example, by hospitalization, medical isolation, etc.;
- Recovered individuals $R(t)$.

The total population size is $N(t) = S(t) + E(t) + Q(t) + I(t) + A(t) + R(t)$. From (Wang et al. 2022), the COVID-19 virus is infectious in the incubation period. Based on the above population classification, the detailed transmission diagram is given in Fig. 1, and the model parameters and their definitions are shown in Table 1. In particular, we emphasize the relationship between compartments $E(t)$, $I(t)$, $A(t)$ and $Q(t)$, as shown in Fig. 2.

Therefore, the transmission process of COVID-19 is described by the following seven differential equations:
Fig. 2 The relationship between compartments $E(t)$, $I(t)$, $A(t)$ and $Q(t)$. For compartment $E(t)$, its $pmE$ will become $I(t)$, $(1 - p)mE$ will become $A(t)$, $pmE + (1 - p)mE = mE$ will become COVID-19 positive and confirmed, and $\kappa E$ will become $Q(t)$. In particular, $mE$ and $\kappa E$ may intersect (see shaded areas); that is, some individuals in compartment $Q(t)$ may be COVID-19 positive, and similarly, COVID-19 positive individuals may also belong to $Q(t)$.

\[
\begin{align*}
\frac{dS}{dt} &= \Lambda - \beta(t)S(\sigma_1 E + I + \sigma_2 A) - \mu S, \\
\frac{dE}{dt} &= \beta(t)S(\sigma_1 E + I + \sigma_2 A) - (\mu + \kappa + m)E, \\
\frac{dI}{dt} &= pmE - (\mu + \alpha + \theta_1 + \gamma_1)I, \\
\frac{dA}{dt} &= (1 - p)mE - (\mu + \theta_2 + \gamma_2)A, \\
\frac{dQ}{dt} &= \kappa E - (\mu + \theta_3)Q, \\
\frac{dJ}{dt} &= \theta_1 I + \theta_2 A + \theta_3 Q - (\mu + s + \gamma_3)J, \\
\frac{dR}{dt} &= \gamma_1 I + \gamma_2 A + \gamma_3 J - \mu R, \\
\end{align*}
\]

where all parameters are positive constant.

Here, we assume that $\beta(t)$ is a continuous positive periodic function in $t$ with period $\omega$ for some $\omega > 0$ ($\omega$ can usually be chosen as 365-days, 52-weeks, 12-months according to actual scenario or case reported data structure, etc.). In fact, due to seasonal influence, $\beta$ usually takes the form of $\beta(t) = \beta_0(1 + b \cos(\frac{2\pi}{\omega} t + \phi))(see \ Zhang \ and \ Zhao \ 2007)$ to describe the transmission rate from a symptomatic patient to a susceptible individual, where positive constants $\beta_0$, $b$ and $\phi$ represent the COVID-19 baseline transmission rate, its magnitude of forcing and the initial phase, respectively. Such seasonal function is also adopted in Liu et al. (2021).

In view of the biological interpretations, we denote by $\Gamma$ the set

\[
\Gamma = \left\{ (S, E, I, A, Q, J, R) \in \mathbb{R}_+^7 : S + E + I + A + Q + J + R \leq \frac{\Lambda}{\mu} \right\}.
\]

By the expressions in system (2.1) and the comparison principle, it is easy to prove the following result.
Table 1  Descriptions of parameters in model (2.1)

| Parameters | Description |
|------------|-------------|
| $\Lambda$  | Recruitment rate |
| $\mu$      | Natural mortality rate |
| $\beta(t)$ | Basic transmission rate between a susceptible and a symptomatic individual |
| $\sigma_1$ | Modification factor of transmission rate for exposed individuals |
| $\sigma_2$ | Modification factor of transmission rate for asymptomatic individuals |
| $m$        | Transition rate from exposed to infectious |
| $p$        | Proportion of the exposed developing infected with symptoms |
| $\kappa$   | Quarantined rate for exposed individuals |
| $\alpha$   | Disease-induced mortality rate for $I$ |
| $s$        | Disease-induced mortality rate for $J$ |
| $\theta_1$ | Transition rate from symptomatic infected to isolated |
| $\theta_2$ | Transition rate from asymptomatic infected to isolated |
| $\theta_3$ | Transition rate from quarantined to isolated |
| $\gamma_1$ | Recovery rate of symptomatic infected individuals |
| $\gamma_2$ | Recovery rate of asymptomatic infected individuals |
| $\gamma_3$ | Recovery rate of isolated individuals |

**Theorem 2.1** For any $t_0 \in \mathbb{R}$ and each initial value $u_0 = (S_0, E_0, I_0, A_0, Q_0, J_0, R_0) \in \Gamma$, system (2.1) admits a unique nonnegative solution 

$$u(t, u_0) = (S(t, u_0), E(t, u_0), I(t, u_0), A(t, u_0), Q(t, u_0), J(t, u_0), R(t, u_0))$$

through $(t_0, u_0)$ such that $u(t, u_0) \in \Gamma$ for all $t \geq t_0$. Moreover, all solutions are ultimately bounded.

**3 Threshold Dynamics**

In this section, we first introduce the basic reproduction number $R_0$ for system (2.1) and then study the extinction and uniform persistence of the disease.

It is easy to see that system (2.1) has exactly one disease-free equilibrium $M_0 = (\frac{\Lambda}{\mu}, 0, 0, 0, 0, 0, 0)$. Linearizing system (2.1) at the disease-free equilibrium $M_0$, we then obtain the following linear periodic subsystem for the infective classes

$$\begin{cases}
\frac{dE}{dt} = \beta(t) \frac{\Lambda}{\mu} (\sigma_1 E + I + \sigma_2 A) - (\mu + \kappa + m) E, \\
\frac{dI}{dt} = pm E - (\mu + \alpha + \theta_1 + \gamma_1) I, \\
\frac{dA}{dt} = (1 - p)m E - (\mu + \theta_2 + \gamma_2) A.
\end{cases}$$  (2)
We introduce
\[
F(t) = \begin{pmatrix}
\sigma_1 \beta(t) \frac{\Delta}{\mu} & \beta(t) \frac{\Delta}{\mu} & \sigma_2 \beta(t) \frac{\Delta}{\mu} \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\]
and
\[
V(t) = \begin{pmatrix}
\mu + \kappa + m & 0 & 0 \\
-pm & \mu + \alpha + \theta_1 + \gamma_1 & 0 \\
-(1 - p)m & 0 & \mu + \theta_2 + \gamma_2
\end{pmatrix}.
\]

Let \( \Phi_V(t) \) and \( \rho(\Phi_V(\omega)) \) be the monodromy matrix of the linear \( \omega \)-periodic system \( x_t = V(t)x \) and the spectral radius of \( \Phi_V(\omega) \), respectively. Assume that \( Z(t, s), \ t \geq s, \) is a \( 3 \times 3 \) matrix-valued solution of the system as follows:
\[
\frac{\partial Z(t, s)}{\partial t} = -V(t)Z(t, s), \ \forall t \geq s, \ Z(s, s) = E
\]
where \( E \) is the \( 3 \times 3 \) identity matrix.

Let \( C_\omega \) be the ordered Banach space of all \( \omega \)-periodic functions from \( \mathbb{R} \to \mathbb{R}^3 \) with maximum norm \( \| \cdot \| \). Let \( C_\omega^+ \) be the positive cone \( \{ \varphi \in C_\omega : \varphi(t) \geq 0 \text{ for all } t \in \mathbb{R} \} \). Following (Wang and Zhao 2008), we define a linear operator \( \mathcal{L} : C_\omega \to C_\omega \) as follows:
\[
(\mathcal{L}\varphi)(t) = \int_{-\infty}^{\infty} Z(t, s)F(s)\varphi(s)ds, \ \forall t \in \mathbb{R}, \varphi \in C_\omega.
\]

Naturally,
\[
(\mathcal{L}\varphi)(t) = \int_{0}^{\infty} Z(t, t - a)F(t - a)\varphi(t - a)da, \ \forall t \in \mathbb{R}, \varphi \in C_\omega.
\]

The operator \( \mathcal{L} \) can be called the next infection operator and its spectral radius \( \rho(\mathcal{L}) \) can be defined as the basic reproduction number for system (2.1), that is
\[
R_0 = \rho(\mathcal{L}).
\]

In order to estimate \( R_0 \) in the periodic case, following (Wang and Zhao 2008), we let \( U(t, \lambda) \) be the monodromy matrix of the following linear \( \omega \)-periodic system
\[
\frac{dU}{dt} = \left[ \frac{F(t)}{\lambda} - V(t) \right] U(t)
\]
with parameter \( \lambda \in (0, \infty) \). Thus, we have the following results.
Lemma 3.1 (Wang and Zhao 2008, Theorem 2.1) The following statements are valid.

(1) If $\rho(U(\omega, \lambda)) = 1$ has a positive root $\lambda_0$, then $\lambda_0$ is an eigenvalue of $L$, and hence, $R_0 = 0$.
(2) If $R_0 > 0$, then $\lambda = R_0$ is the unique root of $\rho(U(\omega, \lambda)) = 1$.
(3) $R_0 = 0$ if and only if $\rho(U(\omega, \lambda)) < 1$ for all $\lambda > 0$.

Using Lemma 3.1, we know $R_0$ is the unique solution of $\rho(U(\omega, \lambda)) = 1$. Hence, the basic reproduction number $R_0$ can be estimated by the numerical solution of the equation. On the local asymptotic stability of the disease-free periodic solution $P_0(t)$, the following results can be deduced from Theorem 2.2 in Wang and Zhao (2008).

Lemma 3.2 The following statement are valid.

(i) $R_0 = 1$ if and only if $\rho(\Phi F - V(\omega)) = 1$;
(ii) $R_0 > 1$ if and only if $\rho(\Phi F - V(\omega)) > 1$;
(iii) $R_0 < 1$ if and only if $\rho(\Phi F - V(\omega)) < 1$.

Therefore, the disease-free equilibrium $M_0$ is locally asymptotically stable when $R_0 < 1$ and unstable when $R_0 > 1$, where $\Phi F - V(t)$ is the monodromy matrix of the linear periodic system (3.2).

In the special case of $\beta(t) \equiv \beta$ for $\forall t \geq 0$, we obtain $F(t) \equiv F$. From (van den Driessche and Watmough 2002) and (Wang and Zhao 2008, Lemma 2.2(ii)), we can obtain the expression of the basic reproduction number $[R_0]$ for the autonomous system of (2.1) as follows

$$[R_0] = \beta \Lambda \left( \sigma_1 + \frac{pm}{\mu + \alpha + \theta_1 + \gamma_1} + \frac{(1-p)m}{\mu + \theta_2 + \gamma_2} \right).$$

Following (Wu et al. 2020), we then define the effective reproduction number of (2.1).

Definition 3.1 The effective reproduction number at time $t$ is

$$R_t = \beta(t) \Lambda \left( \sigma_1 + \frac{pm}{\mu + \alpha + \theta_1 + \gamma_1} + \frac{(1-p)m}{\mu + \theta_2 + \gamma_2} \right).$$

Theorem 3.1 If $R_0 < 1$, then the disease-free equilibrium $M_0 = (\frac{\Lambda}{\mu}, 0, 0, 0, 0, 0, 0)$ is globally asymptotically stable.

Proof It is enough to verify that $M_0$ is globally attractive as $R_0 < 1$. Let $B_\epsilon(t) = F_\epsilon(t) - V(t)$ with

$$F_\epsilon(t) = \begin{pmatrix}
\sigma_1 \beta(t)(\frac{\Lambda}{\mu} + \epsilon) & \beta(t)(\frac{\Lambda}{\mu} + \epsilon) & \sigma_2 \beta(t)(\frac{\Lambda}{\mu} + \epsilon) \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}.$$
Since the total population $N(t)$ satisfies the following differential system
\begin{equation}
\frac{dN}{dt} = \Lambda - \mu N(t) - \alpha I(t) \leq \Lambda - \mu N(t), \tag{4}
\end{equation}
the comparison principle implies that there exist a $T_1 > 0$ such that $N(t) \leq \frac{\Lambda}{\mu} + \epsilon$ for all $t \geq T_1$. Thus, when $t \geq T_1$, one has
\begin{equation}
\begin{cases}
\frac{dE}{dt} \leq \beta(t)\left(\frac{\Lambda}{\mu} + \epsilon\right)(\sigma_1 E + I + \sigma_2 A) - \left(\mu + \kappa + m\right)E, \\
\frac{dI}{dt} \leq p m E - \left(\mu + \alpha + \theta_1 + \gamma_1\right)I, \\
\frac{dA}{dt} \leq (1 - p)m E - \left(\mu + \theta_2 + \gamma_2\right)A.
\end{cases} \tag{5}
\end{equation}

By (Zhang and Zhao 2007, Lemma 2.1), the system $\frac{du}{dt} = B_\epsilon(t)u$ admits a positive $\omega$-periodic solution $u(t) = e^{pt}v(t)$, where $v(t)$ is a vector-valued $\omega$-periodic function and $p = \frac{1}{\omega} \ln \rho(\Phi_{B_\epsilon}(\omega)) < 0$. It is immediate that $u(t) \to 0$ as $t \to \infty$. According to the standard comparison principle, we deduce that
\begin{equation}
\lim_{t \to \infty} (E(t), I(t), A(t)) = (0, 0, 0).
\end{equation}

By the theory of asymptotically periodic semi-flow (see (Zhao 2017, Theorem 3.2.1)), it can be concluded that
\begin{equation}
\lim_{t \to \infty} \left(S(t) - \frac{\Lambda}{\mu}, Q(t), J(t), R(t)\right) = (0, 0, 0, 0).
\end{equation}

The proof is completed. \qed

**Lemma 3.3** Let $u = (u_i(t, u_0))$ ($i = 1, 2, \cdots, 7$) be the solution of system (2.1) with $u_0 \in \Gamma$. If there exists some $t_0 \geq 0$ such that $u_i(t_0) > 0$ for some $i \in \{2, 3, 4\}$, then $u_i(t) > 0$ for both $i = 2, 3, 4$ with $t > t_0$.

**Proof** If $I(t_0) > 0$ for some $t_0 \geq 0$, then $I(t)$ satisfies
\begin{equation}
\frac{dI}{dt} \geq -\left(\mu + \alpha + \theta_1 + \gamma_1\right)I,
\end{equation}
and hence, $I(t) > 0$ for $\forall t > t_0$. It then follows the expression of $E$ in system (2.1) and the positivity of $S$ that $E(t) > 0$ for $\forall t > t_0$. By virtue of the expression of $A$ in system (2.1), it is easy to see that $A(t) > 0$ for $\forall t > t_0$.

If $E(t_0) > 0$ for some $t_0 \geq 0$, then $E(t)$ satisfies
\begin{equation}
\frac{dE}{dt} \geq -\left(\mu + \kappa + m\right)E,
\end{equation}
therefore, \( E(t) > 0 \) for \( \forall t > t_0 \). It then follows the expressions of \( I \) and \( A \) in system (2.1) that \( I(t) > 0 \) and \( A(t) > 0 \) for \( \forall t > t_0 \). It remains to show that when \( A(t_0) > 0 \) uses similar arguments in the first case. This proof is completed. \( \square \)

**Theorem 3.2** If \( R_0 > 1 \), then the disease is uniformly persistent, i.e., there exists a positive constant \( \eta > 0 \) such that any solution \((S(t), E(t), I(t), A(t), Q(t), J(t), R(t))\) in \( \Gamma \) of system (2.1) with \( E(0) > 0 \) or \( I(0) > 0 \) or \( A(0) > 0 \) satisfies

\[
\liminf_{t \to \infty} E(t) \geq \eta, \quad \liminf_{t \to \infty} I(t) \geq \eta, \quad \liminf_{t \to \infty} A(t) \geq \eta.
\]

Moreover, system (2.1) admits at least one positive periodic solution as \( R_0 > 1 \).

**Proof** Define

\[
X := \Gamma, \\
X_0 := \{(S, E, I, A, Q, J, R) \in X : E > 0, I > 0 \text{ and } A > 0\}, \\
\partial X_0 := X \setminus X_0.
\]

Let \( P(t) : \Gamma \to \Gamma \) be the solution map associated with system (2.1), and let \( P := P(\omega) \) be the Poincaré map for this system, i.e.,

\[
P(u_0) = u(\omega, u_0),
\]

where \( u(t, u_0) \) is the unique solution of system (2.1) through \((0, u_0)\). Next, we will show that system (2.1) is uniformly persistent. It is clear that both \( X \) and \( X_0 \) are positively invariant. The set \( \partial X_0 \) is relatively closed in \( X \).

Write \( u_0 = (S_0, E_0, I_0, A_0, Q_0, J_0, R_0) \in X_0 \). Let

\[
u(t, u_0) = (S(t, u_0), E(t, u_0), I(t, u_0), A(t, u_0), Q(t, u_0), J(t, u_0), R(t, y_0))
\]

be the solution of system (2.1) starting from \((0, u_0)\).

By the continuity of solutions with respect to initial values, we have

\[
\lim_{u_0 \to M_0} \|u(t, u_0) - M_0\| = 0
\]

uniformly on \([0, \omega]\). Here, the symbol \( \| \cdot \| \) denotes Euclidean distance on \( \mathbb{R}^7 \). Thus, for any \( \epsilon > 0 \), there exists a \( \delta = \delta(\epsilon) > 0 \), only dependent on \( \epsilon \), such that

\[
\|u(t, u_0) - M_0\| < \epsilon, \quad \forall t \in [0, \omega]
\]

whenever \( \|u_0 - M_0\| < \delta \). \( \square \)

**Claim 1.** \( \limsup_{n \to \infty} \|P^n(u_0) - M_0\| \geq \delta \) for each \( u_0 \in X_0 \).

If the claim is not true, then there is a \( \bar{u}_0 \in X_0 \) such that

\[
\limsup_{n \to \infty} \|P^n(\bar{u}_0) - M_0\| < \delta.
\]
Then, there exists a \( n \in \mathbb{N} \) such that \( \| P^n(\bar{u}_0) - M_0 \| < \delta \) for all \( n \geq n_0 \). Therefore,

\[
\| u(t, P^n(\bar{u}_0)) - M_0 \| < \epsilon, \quad \forall t \in [0, \omega]
\]

provided \( n \geq n_0 \). For any \( t \geq n_0 \omega \), letting \( m = \lceil \frac{t}{\omega} \rceil \) which is the greatest integer less than or equal to \( \frac{t}{\omega} \), we have \( t = \bar{t} + m \omega \) with \( \bar{t} \in [0, \omega) \) and \( m \geq n_0 \). It follows that

\[
\| u(t, \bar{u}_0) - M_0 \| = \| u(\bar{t}, P^m(\bar{u}_0)) - M_0 \| < \epsilon
\] (7)

for all \( t \geq n_0 \omega \). This implies that

\[
| S(t, \bar{u}_0) - \frac{A}{\mu} | < \epsilon, \quad 0 < E(t, \bar{u}_0) < \epsilon,
\]

\[
0 < I(t, \bar{u}_0) < \epsilon, \quad 0 < A(t, \bar{u}_0) < \epsilon
\] (8)

for all \( t \geq n_0 \omega \).

From the second, third and fourth equations of system (2.1), when \( t \geq n_0 \omega \), \((E(t, \bar{u}_0), I(t, \bar{u}_0), A(t, \bar{u}_0))\) satisfies the differential inequality as follows:

\[
\begin{align*}
\frac{dE}{dt} &\geq \beta(t)\left(\frac{A}{\mu} - \epsilon\right)(\sigma_1 E + I + \sigma_2 A) - (\mu + \kappa + m)E, \\
\frac{dI}{dt} &\geq pm E - (\mu + \alpha + \theta_1 + \gamma_1)I, \\
\frac{dA}{dt} &\geq (1 - p)m E - (\mu + \theta_2 + \gamma_2)A.
\end{align*}
\]

Consider the following auxiliary perturbed system

\[
\begin{align*}
\frac{d\hat{E}}{dt} &= \beta(t)\left(\frac{A}{\mu} - \epsilon\right)(\sigma_1 \hat{E} + \hat{I} + \sigma_2 \hat{A}) - (\mu + \kappa + m)\hat{E}, \\
\frac{d\hat{I}}{dt} &= pm \hat{E} - (\mu + \alpha + \theta_1 + \gamma_1)\hat{I}, \\
\frac{d\hat{A}}{dt} &= (1 - p)m \hat{E} - (\mu + \theta_2 + \gamma_2)\hat{A}.
\end{align*}
\] (9)

Let

\[
C_{\epsilon}(t) = \begin{pmatrix}
\sigma_1 \beta(t)\left(\frac{A}{\mu} - \epsilon\right) - (\mu + \kappa + m) & \beta(t)\left(\frac{A}{\mu} - \epsilon\right) & \sigma_2 \beta(t)\left(\frac{A}{\mu} - \epsilon\right) \\
-\left(\mu + \alpha + \theta_1 + \gamma_1\right) & 0 & 0 \\
\left(1 - p\right)m & 0 & -(\mu + \theta_2 + \gamma_2)
\end{pmatrix}
\]

Since \( R_0 > 1 \), one has \( \rho(\Phi_{F-V}(\omega)) > 1 \). Choose a sufficiently small \( \epsilon > 0 \) such that \( \rho(\Phi_{C_{\epsilon}}(\omega)) > 1 \). By (Zhang and Zhao 2007, Lemma 2.1), we know that there exists a positive, \( \omega \)-periodic function \((E^*(t), I^*(t), A^*(t))\) such that \((\hat{E}(t), \hat{I}(t), \hat{A}(t)) = e^{q t}(E^*(t), I^*(t), A^*(t))\) is a solution of system (3.8) where \( q = \frac{1}{\omega} \ln \rho(\Phi_{C_{\epsilon}}(\omega)) > 0 \). It is convenient to choose \( \xi > 0 \) satisfying
\((E(n_0\omega, \tilde{u}_0), I(n_0\omega, \tilde{u}_0), A(n_0\omega, \tilde{u}_0)) \geq \xi \cdot (\tilde{E}(n_0\omega), \tilde{I}(n_0\omega), \tilde{A}(n_0\omega))\). By the comparison principle, it follows that \((E(t, \tilde{u}_0), I(t, \tilde{u}_0), A(t, \tilde{u}_0)) \geq \xi \cdot (\tilde{E}(t), \tilde{I}(t), \tilde{A}(t))\) for all \(t \geq n_0\omega\). We thus get

\[E(n_0\omega, \tilde{u}_0) \rightarrow \infty, \quad I(n_0\omega, \tilde{u}_0) \rightarrow \infty, \quad A(n_0\omega, \tilde{u}_0) \rightarrow \infty\]

as \(n \rightarrow \infty\). This contradicts the boundedness of solutions of system (2.1). In this way, this claim is valid.

Claim 1 implies that \(M_0\) is an isolated invariant set for Poincaré mapping \(P\) in \(X\), and \(W^s(M_0) \cap X_0 = \emptyset\), where \(W^s(M_0)\) is the stable set of \(M_0\) for \(P\). Define

\[M_\delta := \{u_0 \in \partial X_0 : P^n(u_0) \in \partial X_0, \ \forall n \in \mathbb{N}\},\]

and \(\omega(u_0)\) be the omega limit set of the forward orbit \(\gamma^+(u_0) = \{P^n(u_0) : \forall n \in \mathbb{N}\}\) of (2.1). Then, we prove the following claim.

**Claim 2** \(M_0\) is globally stable for \(P\) in \(M_\delta\).

Let \(\overline{M}_\delta := \{u_0 \in X : E = I = A = 0\}\). We first show that \(M_\beta = \overline{M}_\delta\). Clearly, \(\overline{M}_\delta \subset M_\beta\), that is, it suffices to prove that for any \(u_0 \in \overline{M}_\delta\), the solution \(u(t, u_0) = (S(t, u_0), E(t, u_0), I(t, u_0), A(t, u_0), Q(t, u_0), J(t, u_0), R(t, u_0))\) through \(u_0\) satisfies \(E(t, u_0) = I(t, u_0) = A(t, u_0) = 0\) for each \(t \geq 0\). If it is not true, there exists a \(t^* > 0\) such that \(E(t^*, u_0) > 0\) or \(I(t^*, u_0) > 0\) or \(A(t^*, u_0) > 0\). We give the proof only for the case \(I(t^*, u_0) > 0\), the other case can be handled in the same way. The inequality \(I'(t) \geq -(\mu + \alpha + \theta_1 + \gamma_1)I(t)\) implies that \(I(t, u_0) > 0\) for all \(t \geq t^*\).

From the first equation of system (2.1), we have

\[S(t) = S(0)e^{-\int_0^t b(\xi)\,d\xi} + \Lambda e^{-\int_0^t b(\xi)\,d\xi} \int_0^t e^{\int_0^\xi b(\rho)\,d\rho} \,d\xi \geq \Lambda e^{-\int_0^t b(\xi)\,d\xi} \int_0^t e^{\int_0^\xi b(\rho)\,d\rho} \,d\xi > 0\]  \hspace{1cm} (10)

for all \(t > 0\), where \(b(t) = \mu + \beta(t)[\sigma_1 E(t) + I(t) + \sigma_2 A(t)]\). Hence, from the second and third equations of (2.1), we obtain

\[E(t, u_0) \geq E(t^*, u_0)e^{-(\mu + \kappa + m)t} + \int_{t^*}^t \sigma_1 \beta(\xi) I(\xi, u_0)e^{-(\mu + \kappa + m)(t-\xi)} \,d\xi > 0\]

and

\[A(t, u_0) \geq A(t^*, u_0)e^{-(\mu + \theta_2 + \gamma_2)t} + \int_{t^*}^t (1 - p) m E(\xi, u_0)e^{-(\mu + \theta_2 + \gamma_2)(t-\xi)} \,d\xi > 0\]

for all \(t > t^*\). This gives \(u(t, u_0) \in X_0\) for each \(t > t^*\), contrary to \(u_0 \in M_\beta\).

Next we prove that the omega limit set \(\omega(u_0) = M_0\) for any \(u_0 \in M_\beta\). Since \(M_\delta = \overline{M}_\delta\), we have \(E(t, u_0) = I(t, u_0) = A(t, u_0) = 0\) for all \(u_0 \in M_\beta\) and \(t \geq 0\).
In view of system (2.1), it follows that $S$, $Q$, $J$ and $R$ satisfy the following system:

\[
\begin{aligned}
\frac{dS}{dt} &= \Lambda - \mu S, \\
\frac{dQ}{dt} &= -((\mu + \theta_3)Q, \\
\frac{dJ}{dt} &= \theta_3 Q - (\mu + \gamma_3)J, \\
\frac{dR}{dt} &= \gamma_3 J - \mu R,
\end{aligned}
\]

and hence, $\lim_{t \to \infty} \left( S(t) - \frac{\Lambda}{\mu} , Q(t), J(t), R(t) \right) = (0, 0, 0, 0)$ uniformly. Therefore, $\omega(u_0) = M_0$ for any $u_0 \in M_0$. This implies that $M_0$ is globally attractive for $P$ in $M_0$. Since system (3.10) is cooperative, it follows from (Zhao 2017, Lemma 2.2.1) that $M_0$ is locally Lyapunov stable for $P$ in $M_0$. This proves Claim 2 above.

By virtue of Claim 2, we obtain that $\bigcup_{u_0 \in M_0} \omega(u_0) = M_0$ and $M_0$ cannot form a cycle for $P$ in $M_0$ (and hence in $\partial X_0$). Since $P$ admits a global attractor on $X$ due to all solutions are ultimately bounded from Theorem 2.1, it then follows from the acyclicity theorem on uniform persistence for maps [see (Zhao 2017, Theorem 1.3.1 and Remark 1.3.1)] that $P : X \to X$ is uniformly persistent with respect to $(X_0, \partial X_0)$. Thus, (Zhao 2017, Theorem 3.1.1) implies that the solution of (2.1) is uniformly persistent under $E(0) > 0$, $I(0) > 0$ and $A(0) > 0$.

If $E(0) > 0$ or $I(0) > 0$ or $A(0) > 0$, it follows from Lemma 3.3 that there exists an integer $n_0 > 0$ such that $P^{n_0}(u_0) \in X_0$. Since $P(t)u_0 = P(t - n_0\omega) (P^{n_0}(u_0)), \forall t \geq n_0\omega$, the uniform persistence also holds.

By (Zhao 2017, Theorem 1.3.10), the Poincaré map $P$ has a fixed point $(S^*(0), E^*(0), I^*(0), A^*(0), Q^*(0), J^*(0), R^*(0)) \in X_0$. The corresponding periodic solution is denoted by $(S^*(t), E^*(t), I^*(t), A^*(t), Q^*(t), J^*(t), R^*(t))$. By $E^*(0) > 0$, $I^*(0) > 0$ and $A^*(0) > 0$, it is obvious that $E^*(t) > 0$, $I^*(t) > 0$, $A^*(t) > 0$ for all $t \geq 0$. Furthermore, integrating the fifth equation of (2.1) yields

$$Q^*(t) = Q^*(0)e^{-((\mu + \theta_3)t} + \int_0^t k E^*(\xi)e^{-(\mu + \theta_3)(t-\xi)}d\xi > 0$$

for all $t > 0$. The periodicity of $Q^*(t)$ implies $Q^*(t) > 0$ for all $t \geq 0$. In the same manner, we can see that $S^*(t) > 0$, $J^*(t) > 0$, $R^*(t) > 0$ for all $t \geq 0$. Consequently, $(S^*(t), E^*(t), I^*(t), A^*(t), Q^*(t), J^*(t), R^*(t))$ is a positive $\omega$-periodic solution of (2.1).

### 4 A Case Study

In this section, we first present some numerical simulations for the model and then use the reported data to investigate the impact of seasonality on the spread of the COVID-19 epidemic in the USA.
Fig. 3 (color figure online) The long-term behaviors of \( S(t) \), \( E(t) \), \( I(t) \) and \( A(t) \) population when \( R_0 = 11.4 > 1 \)

Fig. 4 (color figure online) The global asymptotic stability of the disease-free equilibrium \( M_0 \) as \( R_0 = 0.01725 < 1 \)

4.1 Numerical Simulations of Dynamic Behavior

By virtue of Theorems 3.1 and 3.2, it is easy to see that \( R_0 \) is a threshold parameter to determine whether or not COVID-19 persists in the population. Let \( \beta_0 = 0.01 \), \( \Lambda = 10 \), \( \mu = 0.005 \), \( \sigma_1 = 0.1 \), \( \sigma_2 = 0.1 \), \( m = 0.1 \), \( p = 0.1 \), \( \alpha = 0.1 \), \( \theta_1 = 0.1 \), \( \theta_2 = 0.1 \), \( \gamma_1 = 0.1 \), \( \gamma_2 = 0.1 \), \( b_0 = 1 \) and \( \phi = \pi/6 \), we get \( R_0 = 11.4 \). The numerical results are illustrated in Fig. 3, and these are the same as Theorem 3.2, that is, the disease is persistent. Moreover, the numerical results also show that there exists a unique globally attractive positive periodic solution as \( R_0 = 11.4 > 1 \). Then, we give \( \Lambda = 1 \), \( \mu = 0.2 \), \( b_0 = 0.1 \) and keep the values of other parameters unchanged to get \( R_0 = 0.01725 \). This shows that the disease is extinct. As described in Theorem
There is a unique disease-free equilibrium $M_0$ which is globally asymptotically stable when $R_0 = 0.01725 < 1$, (see Fig. 4).

4.2 A Case Study of the COVID-19 in the USA

Since the end of 2019 when the COVID-19 epidemic broke out, the USA has the largest number of existing infection cases. The epidemic has lasted for nearly three years, according to statistical data analysis, the data of new infection cases show seasonal periodicity to some extent. To further explore the seasonality of the epidemic transmission, we collect the data [the data come from (https://www.arcgis.com/apps/opsdashboard/index.html)] of new COVID-19 cases from the outbreak of the epidemic in the USA in early 2020 to the end of May 2022, which are summarized in Table 2.

The new cases in Table 2 include symptomatic and asymptomatic infected individuals. Considering system (2.1), we easily obtain that the number of new COVID-19 cases corresponds to the term $pmE + (1 - p)mE = mE$. Since variables and parameters in system (2.1) are continuous functions of $t$, we use trigonometric functions to fit $mE$ as a periodic function with period $\frac{365}{4 \times 7} = 13$. This is because Table 2 is cumulative data for each week, so in the numerical simulation, the cumulative cases of four weeks are taken as a point; that is, $\frac{365}{4 \times 7}$ is a period. We use MATLAB software to fit $mE$ by the data in Table 2 to obtain the coefficients of trigonometric functions. The comparison of the data with the curve of $mE$ is shown in Fig. 5. Obviously, these two match well, and the expression is obtained as follows:

$$mE = 2.40025 \times 10^6 \times \left[ 1 + 0.684 \cos \left( \frac{2\pi t}{13} + \frac{\pi}{6} \right) + 0.241 \sin \left( \frac{2\pi t}{13} + \frac{\pi}{6} \right) \\ + 0.122 \cos \left( \frac{4\pi t}{13} + \frac{\pi}{6} \right) + 0.399 \sin \left( \frac{4\pi t}{13} + \frac{\pi}{6} \right) \\ + 0.122 \cos \left( \frac{6\pi t}{13} + \frac{\pi}{6} \right) + 0.403 \sin \left( \frac{6\pi t}{13} + \frac{\pi}{6} \right) \\ - 0.228 \cos \left( \frac{8\pi t}{13} + \frac{\pi}{6} \right) + 0.141 \sin \left( \frac{8\pi t}{13} + \frac{\pi}{6} \right) \\ - 0.268 \cos \left( \frac{10\pi t}{13} + \frac{\pi}{6} \right) + 0.0064 \sin \left( \frac{10\pi t}{13} + \frac{\pi}{6} \right) \right]$$
Table 2  New COVID-19 cases from the outbreak of the epidemic in the USA in early 2020 to the end of May 2022

| Date       | Cases  | Date    | Cases   | Date    | Cases  | Date       | Cases  |
|------------|--------|---------|---------|---------|--------|------------|--------|
| 2020/3/8   | 499    | 2020/9/27 | 284.778k | 2021/4/18 | 480.084k | 2021/11/7 | 517.581k |
| 2020/3/15  | 2.696k | 2020/10/4 | 300.503k | 2021/4/25 | 404.688k | 2021/11/14 | 561.197k |
| 2020/3/22  | 31.71k | 2020/10/11 | 346.277k | 2021/5/2  | 346.015k | 2021/11/21 | 656.049k |
| 2020/3/29  | 108.6k | 2020/10/18 | 391.358k | 2021/5/9  | 286.679k | 2021/11/28 | 509.969k |
| 2020/4/5   | 205.472k | 2020/10/25 | 490.957k | 2021/5/16 | 233.408k | 2021/12/5  | 859.505k |
| 2020/4/12  | 218.006k | 2020/11/1  | 584.577k | 2021/5/23 | 177.801k | 2021/12/12 | 832.487k |
| 2020/4/19  | 196.864k | 2020/11/8  | 808.71k  | 2021/5/30 | 140.807k | 2021/12/19 | 949.492k |
| 2020/4/26  | 207.182k | 2020/11/15 | 1.043m   | 2021/6/6  | 101.721k | 2021/12/26 | 1.447m |
| 2020/5/3   | 191.166k | 2020/11/22 | 1.21m    | 2021/6/13 | 100.036k | 2022/1/2   | 2.944m |
| 2020/5/10  | 170.426k | 2020/11/29 | 1.176m   | 2021/6/20 | 81.341k  | 2022/1/9   | 5.107m |
| 2020/5/17  | 155.994k | 2020/12/6  | 1.405m   | 2021/6/27 | 87.083k  | 2022/1/16  | 5.649m |
| 2020/5/24  | 154.981k | 2020/12/13 | 1.53m    | 2021/7/4  | 91.024k  | 2022/1/23  | 4.944m |
| 2020/5/31  | 144.932k | 2020/12/20 | 1.552m   | 2021/7/11 | 141.683k | 2022/1/30  | 3.536m |
| 2020/6/7   | 149.555k | 2020/12/27 | 1.318m   | 2021/7/18 | 231.58k  | 2022/2/6   | 1.997m |
| 2020/6/14  | 149.833k | 2021/1/3   | 1.567m   | 2021/7/25 | 370.348k | 2022/2/13  | 1.24m  |
| 2020/6/21  | 189.541k | 2021/1/10  | 1.744m   | 2021/8/1  | 570.405k | 2022/2/20  | 698.779k |
| 2020/6/28  | 276.805k | 2021/1/17  | 1.53m    | 2021/8/8  | 768.025k | 2022/2/27  | 464.446k |
| 2020/7/5   | 346.363k | 2021/1/24  | 1.88m    | 2021/8/15 | 922.787k | 2022/3/6   | 334.064k |
| 2020/7/12  | 409.787k | 2021/1/31  | 1.03m    | 2021/8/22 | 1.043m   | 2022/3/13  | 241.234k |
| 2020/7/19  | 451.592k | 2021/2/7   | 829.15k  | 2021/8/29 | 1.095m   | 2022/3/20  | 212.742k |
| 2020/7/26  | 459.474k | 2021/2/14  | 639.883k | 2021/9/5  | 1.164m   | 2022/3/27  | 218.515k |
| 2020/8/2   | 427.186k | 2021/2/21  | 460.301k | 2021/9/12 | 1.026m   | 2022/4/3   | 196.742k |
| 2020/8/9   | 396.656k | 2021/2/28  | 480.121k | 2021/9/19 | 1.019m   | 2022/4/10  | 244.031k |
Table 2 continued

| Date    | Cases     | Date    | Cases     | Date    | Cases     | Date    | Cases     |
|---------|-----------|---------|-----------|---------|-----------|---------|-----------|
| 2020/8/16 | 372.655k  | 2021/3/7 | 415.374k  | 2021/9/26 | 843.642k  | 2022/4/17 | 237.433k  |
| 2020/8/23 | 298.938k  | 2021/3/14 | 377.859k  | 2021/10/3 | 748.276k  | 2022/4/24 | 354.567k  |
| 2020/8/30 | 292.89k   | 2021/3/21 | 382.643k  | 2021/10/10 | 661.767k  | 2022/5/1  | 384.809k  |
| 2020/9/6  | 281.518k  | 2021/3/28 | 442.412k  | 2021/10/17 | 583.005k  | 2022/5/8  | 507.185k  |
| 2020/9/13 | 254.89k   | 2021/4/4  | 450.775k  | 2021/10/24 | 498.109k  | 2022/5/15 | 603.725k  |
| 2020/9/20 | 290.926k  | 2021/4/11 | 488.5k    | 2021/10/31 | 518.611k  | 2022/5/22 | 799.759k  |

The data come from (https://www.arcgis.com/apps/opsdashboard/index.html)
Table 3  The values of some model parameters and the initial values of compartment variables

| Symbol | Value | Source |
|--------|-------|--------|
| $\Lambda$ | $290664 \text{ (28 days)}^{-1}$ | See text |
| $\mu$ | $\frac{1}{78.6 \times 13} \text{ (28 days)}^{-1}$ | See text |
| $m$ | 0.513 | Estimated |
| $\alpha$ | 0.0263 | Khan et al. (2020) |
| $\gamma_1$ | 0.9937 | Khan et al. (2020) |
| $\gamma_2$ | 0.9937 | Khan et al. (2020) |
| $\sigma_1$ | 0.1974 | Estimated |
| $\sigma_2$ | 0.6304 | Estimated |
| $k$ | 0.9955 | Estimated |
| $p$ | 0.6884 | Estimated |
| $\theta_1$ | 0.9955 | Estimated |
| $\theta_2$ | 0.9955 | Estimated |
| $S(0)$ | $2.97 \times 10^8$ | http://www.who.int/ |
| $E(0)$ | 100 | Estimated |
| $I(0)$ | 4 | https://www.arcgis.com/apps/opsdashboard/index.html |
| $A(0)$ | 0 | https://www.arcgis.com/apps/opsdashboard/index.html |

$$-0.278 \cos \left( \frac{12 \pi t}{13} + \frac{\pi}{6} \right) - 0.0855 \sin \left( \frac{12 \pi t}{13} + \frac{\pi}{6} \right).$$

The average life expectancy of the American was 78.6 years in 2019 (see http://www.who.int/). We take this number as the current average life expectancy, therefore, the natural mortality rate $\mu = \frac{1}{78.6 \times 13} \text{ (28 days)}^{-1}$. Note that, the recruitment rate is the product of the birth rate (equals to the natural death rate) and the total number of the whole population, the total population is $2.97 \times 10^8$ from [15], and hence, $\Lambda = \frac{2.97 \times 10^8}{78.6 \times 13} = 290664 \text{ (28 days)}^{-1}$. Based on the obtained values of parameters and the initial values of compartment variables (see Table 3), applying them to model (2.1), and fitting the summarized data in Table 2 with the least square method, we get the fitted values of parameters with a high degree of agreement. These results are shown in Table 3. In particular, we get the expression of $\beta(t)$ as follows:

$$\beta(t) = 4.9794 \times 10^{-8} \times \left[ 1 + 0.09977 \cos \left( \frac{2 \pi t}{13} + \frac{\pi}{6} \right) - 0.114 \sin \left( \frac{2 \pi t}{13} + \frac{\pi}{6} \right) \right.$$

$$-0.0671 \cos \left( \frac{4 \pi t}{13} + \frac{\pi}{6} \right) + 0.0742 \sin \left( \frac{4 \pi t}{13} + \frac{\pi}{6} \right),$$

$$-0.0671 \cos \left( \frac{6 \pi t}{13} + \frac{\pi}{6} \right) + 0.1 \sin \left( \frac{6 \pi t}{13} + \frac{\pi}{6} \right),$$

$$-0.2597 \cos \left( \frac{8 \pi t}{13} + \frac{\pi}{6} \right) + 0.1144 \sin \left( \frac{8 \pi t}{13} + \frac{\pi}{6} \right)\right].$$
The basic reproduction number is an important indicator to describe the outbreak degree of the epidemic. From the values in Table 3, we get $R_0 = 6.69$ of the COVID-19 in the USA by numerical calculation. It then follows from those parameter values and $\beta(t)$ that some simulations of the epidemic in the USA. We take March 8th, 2020 as the initial time of the simulation. Each unit time length represents 4 weeks. From the fitting of the transmission for the disease in the USA, the evolution of $I(t)$ and $A(t)$ over time is shown in Fig. 6. In particular, if no control strategy is adopted, the epidemic will continue to occur seasonally, and the scale of new COVID-19 cases in the USA is illustrated in Fig. 7.

It is worth noting that in the above numerical simulations, since the size of the initial infected population is very small, we assume that the total population is susceptible and consider $S(0) = \frac{\Lambda}{\mu}$, and then calculate the basic reproduction number $R_0$. However, with the continuous outbreak of COVID-19, the size of the susceptible population will decrease significantly due to the increase in the number of infected people. The value of the basic reproduction number cannot accurately describe the outbreak scale of the epidemic. Therefore, in the early stage of the outbreak of the disease, the basic reproduction number $R_0$ can be used to describe the outbreak scale. If the complete epidemic data are used to estimate other parameters to calculate the basic reproduction number, it may not be accurate and cannot describe the risk at a certain time $t$. So, we
introduce the effective reproduction number $R_t$ to describe the epidemic scale over time. In virtue of the obtained values of parameters and the expression of $\beta(t)$, it is easy to see that the effective reproduction number $R_t$ as shown in Fig. 8.

5 Discussion

The COVID-19 pandemic puts epidemic modeling at the forefront of global public policy-making. Nevertheless, it is still very challenging to model and predict the spread of COVID-19. Especially for the specific model, the parameters of the model are fitted by using statistical data, so that the model with known parameters can be applied to describe the actual spread of the epidemic, which can give some scientific evaluation and prediction. In this paper, we use a time-periodic compartmental model to describe the COVID-19 seasonal transmission rate. We find that there is a seasonal pattern of the new COVID-19 cases, and the numbers of the peak value of new cases are from November of the current year to February of the following year, and reach a nadir from May to June in the USA (see Fig. 5). This seasonal pattern may be related to new year’s day and Christmas. During the festivals, people travel and frequent social activities, which lead to the virus’s rapid spread. Furthermore, winter and spring are the peaks of influenza outbreaks, reducing people’s immunity, which may also be the reason for the COVID-19 peak.

We illustrate that the basic reproduction number $R_0$ serves as a threshold value for the extinction and persistent of the disease. More precisely, if $R_0 < 1$, then the unique disease-free equilibrium is globally asymptotically stable (see Theorem 3.1 and Fig. 4); while the disease is uniformly persistent and there exists at least one positive periodic solution if $R_0 > 1$ (see Theorem 3.2 and Fig. 3). Numerical simulations show that there is only one positive periodic solution that is globally asymptotically stable as $R_0 > 1$ (see Fig. 3). Figure 5 indicates that the fitted curve of new COVID-19 cases matches the statistical data very well. The expression of $\beta(t)$ and other parameters are further estimated by the least square method, and $R_0 = 6.69$ is obtained via numerical calculation. The numerical results illustrate that if no measures are taken, the epidemic in the USA will continue to break out, as shown in Fig. 7. It is not always reasonable to use $R_0$ to describe the outbreak of the whole epidemic. In order to describe the change of the epidemic situation over time, we introduce the effective reproduction number $R_t$, its change with time as shown in Fig. 8.
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