Synchronization and chaos in spin-transfer-torque nano-oscillators coupled via a high-speed operational amplifier

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Abstract
We propose a system of two coupled spin-torque nano-oscillators (STNOs), one driver and another response, and demonstrate using numerical studies the synchronization of the response system to the frequency of the driver system. To this end we use a high-speed operational amplifier in the form of a voltage follower, which essentially isolates the drive system from the response system. We find the occurrence of 1 : 1 as well as 2 : 1 synchronization in the system, wherein the oscillators show limit cycle dynamics. An increase in power output is noticed when the two oscillators are locked in 1 : 1 synchronization. Moreover in the crossover region between these two synchronization dynamics we show the existence of chaotic dynamics in the slave system. The coupled dynamics under periodic forcing, using a small ac input current in addition to that of the dc part, is also studied. The slave oscillator is seen to retain its qualitative identity in the parameter space in spite of being fed in, at times, a chaotic signal. Such electrically coupled STNOs will be highly useful in fabricating commercial spin-valve oscillators with high power output, when integrated with other spintronic devices.

Keywords: spin valve pillars, spin-transfer torque, spin-torque nano-oscillators, synchronization

(Some figures may appear in colour only in the online journal)

1. Introduction

Extensive theoretical and experimental studies on spin-valve geometries following the discovery of spin-transfer torques in magnetic multilayer structures [1–4] unmasked two important phenomena relevant to the spintronics industry—current induced magnetization switching and self-sustained microwave oscillations in nanopillar devices [5–9]. These were observed in F1/NM/F2 standard trilayers in which F1 is the ferromagnetic pinned layer, which spin polarizes the input current, and F2 is the ferromagnetic free layer whose dynamics is studied in most of the cases. NM is a non-magnetic spacer layer. The self-sustained oscillations in nanopillar devices can be understood in terms of the balance between the torque generated by the damping forces and the spin-transfer torque which acts in opposite direction to the former. These spin-torque nano-oscillators (STNOs), whose oscillations are in microwave range (frequency in GHz), are excellent candidates for oscillators to be integrated into a spintronics motivated architecture. But their appeal is marred by the feeble output power from a single oscillator.

One way of improving the output power is to synchronize several such non-linear spin-torque oscillators. Two different schemes of synchronizing the STNOs are often considered. In an experiment using electrical nano-contacts at close proximity on the same mesa, Kaka et al [10] showed that a direct spin-wave coupling can synchronize two STNOs. This scheme has proven to be very fruitful and is replicated in
various experiments [11, 12]. Recently attempts have been made to theoretically explain the spin-wave-induced coupling, predominantly using linear spin-wave theory [13, 14]. Another effective coupling scheme uses electrically connected STNOs to get them phase-locked to the ac generated by themselves. Following the experimental demonstration of injection locking of STNOs to applied ac current by Rippard et al. [15], it was numerically shown that an array of oscillators electrically connected in series mutually synchronize in frequency as well as in phase [16]. The coupling was due to the microwave component of the common current flowing through the oscillators. This and similar coupling schemes have been explored extensively in the literature ever since [17–23]. This way of augmenting power by an array of electrically connected phase coherent oscillators, once realized, may prove to be a great milestone towards a nano-scale oscillator with useful power output. Analytical as well as numerical studies of the synchronization effects in STNOs subjected to microwave magnetic fields also appear in the literature [24, 25].

We propose a novel way of electrically coupling STNOs, in a drive-response scenario, which we believe will be of substantial interest in the background of the aforementioned developments.

In this work we study the various types of synchronization as well as chaotic dynamics a drive-response coupling of two STNOs can bring about. To this end, we propose a coupling using a high-speed operational amplifier (op amp), which acts like a voltage follower. It essentially isolates the driver (master) oscillator from any feedback from the response (slave) system. The intention here is to study the dynamical response of a slave STNO to the signal input from another identical oscillator, once realized, may well be subject to a constant Oersted field also along the $e_x$ direction. The dynamics of the macrospin magnetization of the free layer is governed by the Landau–Lifshitz–Gilbert–Slonczewski (LLGS) equation [9].

$$\frac{\partial m}{\partial t} = -\alpha m \times \frac{\partial m}{\partial t} - m \times (H_{eff} - \beta m \times e_x), \quad (1)$$

where $m(= [m_x, m_y, m_z])$ is the normalized magnetization vector of the free layer. The effective field consisting of an external magnetic field ($H_{ext}$), anisotropy field (both in the $e_x$ direction, with the thin film assumed to have a uni-axial anisotropy whose easy axis is aligned along the direction of the applied field), and demagnetization field perpendicular to the layer, is given by

$$H_{eff} = H_{ext} e_x + \kappa m_x e_x - 4\pi M_s m_z e_z. \quad (2)$$

The parameter $\beta$ is proportional to the spin current density (for a given pillar geometry, and is roughly of the order $200 \text{ Oe}$ with typical current densities of the order of $10^8 \text{ A cm}^{-2}$). The rescaled applied dc current, $\tilde{i}_{dc}$, is the same as $\beta$ in what follows which has the dimensions of field intensity, frequently expressed in the literature in the cgs unit Oersted. The expression for $\beta$ is [26]

$$\beta = \frac{h A j}{2 M_s V e g(P)}, \quad (3)$$

where $A$ is the area of the cross section, $j$ is the current density and $V$ is the volume of the pinned layer. $g(P)$ is a dimensionless function of the degree of spin polarization of the pinned layer ($0 \leq P \leq 1$), with typical numerical value $\sim 0.3$. The sample parameters appearing in (1) and (2) are given values similar to that of permalloy film. So, damping constant $\alpha = 0.02$, anisotropy constant $\kappa = 45 \text{ Oe}$, demagnetization field constant $4\pi M_s = 8400 \text{ Oe}$ and the gyromagnetic ratio $\gamma = 1.7 \times 10^7 \text{ Oe}^{-1} \text{s}^{-1}$.

We investigate the effect of coupling on the dynamical regions of the phase space of second STNO. Our coupling scheme using a high-speed op amp is shown in figure 1. The op amp acts as voltage follower and effectively isolates the drive circuit from that of the response circuit. The voltage appearing across its non-inverting terminal is that of the STNO1 generated by virtue of the GMR effect. By the property of the op amp in buffer configuration essentially the same voltage appears across STNO2 and the coupling resistor $R_C$. Denoting the free layer magnetization of STNO1 as $m_1$ and

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that of STNO2 as \( m_2 \) we derive the following pair of equations governing the dynamics of the above drive-response system:

\[
\frac{\partial m_1}{\partial t} = \alpha m_1 \times \frac{\partial m_1}{\partial t} - \gamma m_1 \times (H_{\text{eff}_1} - \beta m_1 \times e), \quad (4)
\]

\[
\frac{\partial m_2}{\partial t} = \alpha m_2 \times \frac{\partial m_2}{\partial t} - \gamma m_2 \times (H_{\text{eff}_2} - \beta'(t) m_2 \times e), \quad (5)
\]

where

\[
\beta'(t) = \beta \left(1 + \frac{R_1(t)}{R_C + R_2(t)}\right), \quad (6)
\]

\[
R_i = R_0 - \Delta R \cos(\theta_i). \quad (7)
\]

The resistances of the two STNOs, \( R_1 \) and \( R_2 \), depend on the dynamical state of the free layer and is modelled using the standard equation (7), where \( \theta \) is the angle between the free layer and the pinned layer magnetizations [16]. If \( R_P \) and \( R_{AP} \) are the resistances of the spin valve in parallel and anti-parallel configurations, respectively, then \( R_0 = (R_0 + R_{AP})/2 \) and \( \Delta R = (R_{AP} - R_0) / 2 \). The right-hand side of equation (6) comprises the contribution from coupling as well as the bias voltage of the slave STNO.

3. Coupled dynamics—synchronization and chaos

3.1. Synchronization

The STNOs are given different initial conditions and are given 10% mismatch in the anisotropy field and about 1% mismatch in the demagnetization field. The coupled LLGS equation, (4) and (5), is simulated using a fourth order Runge–Kutta algorithm with a time step of 0.5 ps. The inclusion of time delay (due to op amp action) turned out to be of no significance to the results we are presenting here and hence omitted from the discussions that follow until section 4.

When the GMR values are chosen to be \( R_P = 10 \Omega \) and \( R_{AP} = 11 \Omega \), we see the occurrence of 1:1 as well as 2:1 synchronization as plateaus in figure 2. In the 1:1 synchronization regime, the master and slave STNOs precess with the same frequency, whereas in 2:1 synchronization the master STNO has double the frequency of precession as compared to the slave STNO. As the coupling resistance \( R_C \) is increased the limit cycle frequency of the slave decreases in the OOP regime and then cross over to the IP regime. After this, increasing \( R_C \) causes the frequency to slowly go up. This also matches with the general response of a STNO to spin current, as increasing \( R_C \) effectively reduces the strength of coupling [27]. Upon close inspection, evidence for 1:2 synchronizations can also be found in the figure. This is discussed in some detail later in this section. The nature of free layer magnetization dynamics in these regions are further elucidated in figure 3. We see that there is a definite phase-locking happening between the STNOs, though the phase of one lags the other (figures 3(b) and (d)). While 1:1 mode-locking, when STNO1 is undergoing IP oscillations STNO2 goes to OOP oscillation. During 2:1 mode-locking both STNO1 as well as STNO2 executes IP oscillations.

In order to see the power gain at the synchronization frequency we plot the Fourier spectrum of both the STNOs in a single figure (figure 4(b)). For comparison the scenario during desynchronization is also given at the top of the
Figure 3. The phase space trajectory (limit cycles) and time trace of free layer magnetization dynamics at 1:1 as well as 2:1 synchronization phases. Solid red lines (lower trajectory in (a) and (c)) denote the master whereas dashed blue lines (upper trajectory in (a) and (c)) denote the slave dynamics. To avoid overlap of the figures, in (a) and (c), the trajectory of the slave oscillator (dashed blue lines) has been shifted up by 1 unit along the \( m_z \) axis. (a), (b) Phase space trajectory and time trace of \( m_z \) respectively, at the 1:1 synchronization region. The coupling resistance \( R_C = 60 \Omega \) and the other parameter values are as in figure 2. It is clear that when the master is executing IP oscillations, the slave is executing OOP oscillations. (c), (d) Phase space trajectory and time trace of \( m_z \) respectively, at the 2:1 synchronization region. The coupling resistance \( R_C = 63 \Omega \). It can be seen that both the master and the slave are now executing IP oscillations.

Figure 4. The power spectrum for the synchronized as well as the desynchronized phase. Parameter values are the same as that in figure 2. Synchronized precession is at 11 GHz. Desynchronized precession is at 11 GHz for the master and 7.5 GHz for the slave. At synchronization \( R_C = 60 \Omega \) and at desynchronization \( R_C = 80 \Omega \).

Figure 5. The phase portrait in the \( a_{dk} - R_C \) plane at the GMR value \( R_{AP} = 11 \Omega \). \( h_{ext} \) is fixed at 200 Oe. We see a well delimited synchronization region (red asterisks) surrounded by desynchronization regions (blank). Chaos is observed only at isolated points (blue circles).
The occurrence of chaos in coupled STNOs at the GMR value $R_{AP} = 12 \Omega$. (a) At $R_C = 60 \Omega$, which showed synchronization earlier, we see the limit cycle frequency approaching zero. This is due to irregular switching of STNO2 dynamics among the available OOP and IP modes which, at these parameter values, is the same as 1 : 1 and 2 : 1 synchronization modes, respectively. The red line is the frequency of STNO1. (b) The time trace of $m_z$ displaying the random jump between IP and OOP modes. (c) The power spectrum of STNO2 showing the vanishing of the well-defined peaks. The scale of power is the same as that in figure 4.

Figure 6. The occurrence of chaos in coupled STNOs at the GMR value $R_{AP} = 12 \Omega$. (a) At $R_C = 60 \Omega$, which showed synchronization earlier, we see the limit cycle frequency approaching zero. This is due to irregular switching of STNO2 dynamics among the available OOP and IP modes which, at these parameter values, is the same as 1 : 1 and 2 : 1 synchronization modes, respectively. The red line is the frequency of STNO1. (b) The time trace of $m_z$ displaying the random jump between IP and OOP modes. (c) The power spectrum of STNO2 showing the vanishing of the well-defined peaks. The scale of power is the same as that in figure 4.

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3.2. Chaos

When the GMR values are chosen to be $R_P = 10 \Omega$ and $R_{AP} = 12 \Omega$, as shown in figure 6, we see the occurrence of chaos at the boundary between 1 : 1 and 2 : 1 synchronization regions. This is because the system switches between these modes of oscillations in a random manner. In figure 6 we have shown the time trace as well as the power spectrum during this phase. This is interesting because it can be used to estimate the GMR ratio itself in conjunction with other experimental techniques. During chaos, the power spectrum gets noisy and there is no useful power to be derived out of the system. Notwithstanding the commercial problems chaotic dynamics can bring about, from a dynamical systems point of view, they are still extremely important and interesting. The effect brought about by increasing $R_{AP}$ can be understood in the following way: increasing $R_{AP}$ essentially implies a direct increase in the GMR value which has a direct impact on the electrical coupling and can sometimes enhance the synchronization regimes [16]. In our case the chaotic region seems to be sensitive to the GMR value, and the greater the GMR value, the stronger the chaotic dynamics.

For gaining a better understanding of chaotic dynamics we turn our attention to the control space dynamics in the $R_C - a_{dc}$ plane (figure 7). We see the onset of chaotic dynamics within the synchronization region itself as expected. As in the previous case, here also the dynamics turns into the multi-periodic regime for some parameter values but is included in the desynchronization region in phase portraits. Thus we see that in these coupled systems where various $m:n$ synchronizations happen in close by parameter ranges, chaotic dynamics tends to happen at the boundary between these regions. This is also crucial in noisy systems, because noise invariably makes the system randomly switch between the available states and can result in the resonance peak vanishing even at synchronization [23].

The phase picture in the $h_{ext} - R_C$ space also shows the embedding of the chaos region within the synchronization region (figure 8). Notice that chaos regions also appear outside of the synchronization regions in figure 7 as well as in figure 8. This is because in the simulations we have only looked for 1 : 1 and 2 : 1 mode-locking where as other $m:n$ synchronizations are also possible in the system. We see evidence of such a locking in figure 2, where a small plateau appears at the frequency appropriate for 1 : 2 mode-locking.
Figure 7. The phase portrait in the $a_{dc}-R_C$ plane at the GMR value $R_{AP}=12\,\Omega$. All other parameter values are the same as in figure 5. We see chaotic dynamics (blue circles) encapsulated by the synchronization regions (red asterisks). Blank regions correspond to desynchronization dynamics.

Figure 8. The phase portrait in the $h_{ext}-R_C$ plane. $a_{dc}$ is fixed at 200 Oe. Other parameter values and colour codings are as in figure 7. Here again chaos is closely tied to synchronization dynamics.

Arguably chaotic dynamics is expected to be found associated with such higher order mode-locking as well. Here it is worth pointing out that fractional synchronization in coupled STNOs are also experimentally observed [21].

3.3. Robustness under noise

Real world experiments are seldom free from external noise. This can affect the reliability of our synchronization as well as chaotic regimes. In order to address the issue of robustness, we studied numerically the effect of incorporating Gaussian white noise to the spin current, which is a good numerical approximation to thermal noise. The result of such a numerical experiment incorporating noise is shown in figure 9. We notice that when Gaussian white noise with standard deviation 0.3 was used, introducing an equivalent error of $\pm 1$ Oe in the spin current, our synchronization and chaotic regions remain more or less intact.

We even pushed the system with an error of $\pm 5$ Oe in spin current and still found the synchronization regions intact, though more and more desynchronization regions turned to chaotic regions. We believe this suffices to state that the system under consideration is indeed robust to thermal fluctuations.

4. Coupled dynamics with periodic forcing

In order to incorporate the full richness of spin-valve dynamics into our study, we let both of our STNOs be susceptible to dynamical chaos. We use a small ac input current, of frequency $\omega$, in addition to the dc part to generate dynamical chaos. A time varying current is imperative to witness chaos in an isolated STNO, whose phase space is otherwise just two dimensional (under the macrospin assumption). Such a system displays three distinct dynamical regimes, namely synchronization, modifications and chaos in the $a_{dc}-\omega$ parameter space [28]. Qualitatively, similar dynamical behaviour is noticed even with a periodically alternating Oersted field instead of the alternating spin current [29]. Figure 2 in section 2 is applicable here with the modification that apart from the dc biasing voltage both the STNOs are driven by ac current sources with tunable frequency as well. We have a small ac current, in addition to dc current, flowing through both of the STNOs.

It should be noted that this scenario is qualitatively different from the previous case in various important aspects. Here the master and slave oscillators are driven using a periodic signal, whereas in the unforced scenario only the slave STNO experiences a time varying signal (fed from the output of STNO1) in the form of coupling signal. Also, here the master STNO can go chaotic, feeding the slave with a chaotic signal as shown later in this section, whereas in the unforced case the slave is at best fed a periodic signal. Moreover, the meaning of synchronization itself differs considerably from the earlier case. In the unforced case, the frequency of the slave STNO synchronize with that of the master STNO. In the forced case...
it is the synchronization of the slave STNO with that of the external forcing which is considered as synchronization.

Again the op amp in voltage follower mode replicates the voltage being applied to its non-inverting terminal on its output terminal which acts as the coupling signal. In the present analysis we take into account the time delay, \( \tau \), introduced by the op amp action between the two oscillators. Since this is due to the internal switching delay of the op amp, it is taken to be a constant in the simulations (\( \tau = 0.05 \) ns). For the sake of numerical calculations, the delay coupled oscillator pair is approximated as an array of \( N \) coupled oscillators, each having a coupling delay of \( \Delta = \tau/N \) with its previous member [30, 31]. It is noticed that time delay has no effect on the dynamics of the system and is included here for the sake of completeness of the analysis. Our effort to introduce phase synchronization via tuning time delay has also been futile as yet.

The modified coupled LLGS equations are given below (see section 2 for details):

\[
\frac{\partial m_1}{\partial t} = \alpha m_1 \times \frac{\partial m_1}{\partial t} = -\gamma m_1 \times (H_{\text{eff}1} - a(t) m_1 \times e_z),
\]

(8)

\[
\frac{\partial m_2}{\partial t} = \alpha m_2 \times \frac{\partial m_2}{\partial t} = -\gamma m_2 \times (H_{\text{eff}2} - \beta(t-\tau) m_2 \times e_z),
\]

(9)

where

\[
a(t) = (a_{dc} + a_{ac} \cos \omega t),
\]

(10)

\[
\beta(t-\tau) = a(t) + \frac{a(t-\tau) \times R_1(t-\tau)}{R_C + R_2(t)}.
\]

(11)

The \( \omega-a_{dc} \) phase diagram for the drive system, STNO1 (figure 10), features the synchronization branches with a chaotic stem, as expected (see figure 1 in [28]). Interestingly, the response system, STNO2, too shows synchronization branches and a chaotic stem (red crosses and blue stars, respectively, in figure 10) identical to that of the drive system, but with a prominent shift of the entire phase diagram towards a lower value of spin current, \( a_{dc} \), with the shift determined only by the coupling resistor \( R_C \). An important observation is that the qualitative picture of the phase diagram is preserved by the response STNO, in spite of being fed in, at times, a chaotic signal. One may speculate that for an extended system of N-STNOs, coupled in the manner discussed here, the individual STNOs will continue to preserve their qualitative phase (tree) structures, albeit shifted. Although the phase diagram of STNO1, the chaotic stem and synchronization branches, appears shifted compared to that of STNO2, it has to be noted that upon a careful reading the two ‘trees’ are not exactly identical in their detail. For instance, there are points on the stem region of STNO1 that correspond to chaotic motion, but whose counterparts in the stem region of STNO2 do not.

![Figure 10. Phase diagram of the free layer magnetization dynamics in the \( a_{dc}-\omega \) plane for the slave STNO. The delay time \( \tau = 0.05 \) ns. The parameter values are \( a_{dc} = 20 \) Oe, \( \kappa = 0 \), \( 4\pi M_s = 8400 \) Oe, \( R_1 = 10 \) \( \Omega \), \( R_{xy} = 11 \) \( \Omega \), \( R_C = 20 \) \( \Omega \). The three dynamical regions are synchronization (red asterisks), modification (blank) and chaos (blue circles). The phase diagram for the master, STNO1, shown shaded for reference, also has similar dynamic regimes.](image)

An important parameter in the set of coupled equations (8) and (9), is the coupling resistance in the slave circuit, \( R_C \). For a coupling resistance of 20 \( \Omega \), the shift in \( a_{dc} \) is noticed to be nearly 60 Oe. The shift in the value of \( a_{dc} \) as a function of \( R_C \) is shown in figure 11(b). Agreeably, the shift in the value of \( a_{dc} \) approaches zero for large values of \( R_C \), when \( \beta(t) \) approaches \( a(t) \) and the signal from STNO1 is effectively nullified.

We rewrite here the expression for the coefficient \( \beta \), equation (11), to gain a heuristic understanding of the contribution due to coupling.

\[
\beta = a_{dc} \left(1 + \frac{R_1(t-\tau)}{R_C + R_2(t)}\right) + a_{ac} \left(\cos \omega t + \cos \omega(t-\tau)\right) \frac{R_1(t-\tau)}{R_C + R_2(t)}
\]

\[
= a_{dc} + a_{ac} \times f(t)
\]

(12)

For some sample values of the parameters \( \omega \) and \( a_{dc} \) we study the temporal behaviour of the term \( R_1(t-\tau)/(R_C + R_2(t)) \) (see figure 11(a)). It is noticed that this ratio shows sharp fluctuations over a period, but varies smoothly in between. For the sample values we studied, the time period of fluctuations are comparable (~0.4 ns) to the time period of the ac part of the spin current (~0.25 ns). However, the magnitude of these fluctuations are bounded in the range of 0.04, but with a significant average value compared to 1. Thus, allowing for small fluctuations, the effective value of the dc current increases \( (a'_{dc} \) in (12)), consequently reducing the critical value of \( a_{dc} \) at which chaotic dynamics sets in. For the same reason, the time periodic part of \( \beta, f(t) \) in (12), remains periodic with the same frequency \( \omega \) as the applied spin current.

5. Discussion and conclusion

In summary, we have proposed a system of two coupled spin-torque nano-oscillators—a drive system and a response system—and studied its behaviour numerically. The occurrence of 1:1 as well as 2:1 synchronization in the system are examined in detail. In the crossover region between these two synchronization dynamics we have shown the existence of chaotic dynamics and how it depends upon system parameters. We have demonstrated the power...
augmentation in the synchronization regimes which is of great practical importance in the current spintronics industry. We extended the study to the coupled dynamics under the periodic forcing scenario and demonstrated the interesting possibility of controlling the nature of dynamics of the response oscillator—periodic oscillations synchronized to the applied ac spin current, or chaotic. Our simulations show a prominent shift of the chaos regions towards the low spin current side due to coupling, the shift being determined by the coupling resistor. The pivotal role played by the coupling resistor in unforced as well as forced scenarios, as an experimentally tunable parameter for the response system, is demonstrated.

Commercially available ultra-high-speed op amps (frequency >1 GHz) have frequency ranges up to 2 GHz (for example, the model LMH6702 from Texas Instruments is a 1.7 GHz, ultra-low distortion, wide-band op amp). Although frequency of limit cycles in STNOs usually shoots above this range, making the immediate experimental realization of the coupled system impractical, we nevertheless believe higher frequency op amps would be available commercially in the near future. Moreover, from our results it is apparent that it is the average value of fluctuations which is responsible for dynamical effects. Hence, minor distortions in the high-frequency coupling signal due to op amp will not alter the results presented here.

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