Pressure-Regulated, Feedback-Modulated Star Formation In Disk Galaxies

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ABSTRACT

The star formation rate (SFR) in galactic disks depends on both the quantity of available interstellar medium (ISM) gas and its physical state. Conversely, the ISM’s physical state depends on the SFR, because the “feedback” energy and momentum injected by recently-formed massive stars is crucial to offsetting losses from turbulent dissipation and radiative cooling. The ISM’s physical state also responds to the gravitational field that confines it, with increased weight driving higher pressure. In a quasi-steady state, it is expected that the mean total pressure of different thermal phases will match each other, that the component pressures and total pressure will satisfy thermal and dynamical equilibrium requirements, and that the SFR will adjust as needed to provide the requisite stellar radiation and supernova feedback. The pressure-regulated, feedback-modulated (PRFM) theory of the star-forming ISM formalizes these ideas, leading to a prediction that the SFR per unit area, $\Sigma_{\text{SFR}}$, will scale nearly linearly with ISM weight $W$. In terms of large-scale gas surface density $\Sigma_{\text{gas}}$, stellar plus dark matter density $\rho_{\text{sd}}$, and effective ISM velocity dispersion $\sigma_{\text{eff}}$, an observable weight estimator is $W \approx P_{\text{DE}} = \pi G \Sigma_{\text{gas}}^2 / 2 + \Sigma_{\text{gas}} (2 G \rho_{\text{sd}})^{1/2} \sigma_{\text{eff}}$, and this is predicted to match the total midplane pressure $P_{\text{tot}}$. Using a suite of multiphase magnetohydrodynamic simulations run with the TIGRESS computational framework, we test the principles of the PRFM model and calibrate the total feedback yield $\Upsilon_{\text{tot}} = P_{\text{tot}} / \Sigma_{\text{SFR}} \sim 1000 \text{ km s}^{-1}$, as well as its components. We compare results from TIGRESS to theory, previous numerical simulations, and observations, finding excellent agreement.

Keywords: Star formation (1569), Interstellar medium (847), Stellar feedback (1602), Magnetohydrodynamical simulations (1966)

1. INTRODUCTION

1.1. Feedback and Star Formation/ISM Regulation

The importance of star formation “feedback” to energetics of the interstellar medium (ISM) has been appreciated throughout the modern history of astronomy (see e.g. Spitzer’s chapter in Middlehurst & Aller 1968, for a mid-20th-century view), with the idea of star formation self-regulation a corollary: the energy returned by stars to their surroundings may prevent or limit gravitational collapse and further star formation. From observations, it is evident that young, massive stars impart large amounts of energy to their near and far environ-
confined (“blister”) H II regions also provided estimates of evaporation rates and outflow velocities of ionized gas (Whitworth 1979). Radiation pressure on dust leads to gradients of radiation and gas pressure within H II regions and increases the net force on the surrounding shell (Draine 2011), and spherical solutions for H II region expansion accounting for this have been developed (e.g. Martínez-González et al. 2014; Kim et al. 2016; Rahner et al. 2017; Akimkin et al. 2017). These idealized spherical and hemispherical H II region solutions have also been used to estimate giant molecular cloud (GMC) lifetimes and star formation efficiencies as limited by feedback (e.g. Elmegreen 1983; Franco et al. 1994; Matzner 2002; Krumholz & Matzner 2009; Full et al. 2010; Murray et al. 2010; Kim et al. 2016). In recent years, numerical simulations have been applied to model more realistically effects of stellar energy inputs in turbulent, self-gravitating models of individual GMCs that are highly inhomogeneous. Under most circumstances photoionization and radiation pressure effects dominate on GMC scales, and radiation hydrodynamic simulations (Kim et al. 2018, 2021b; He et al. 2019; Fukushima et al. 2020; Fukushima & Yajima 2021) have demonstrated that clouds are dispersed on realistic timescales and yield realistic lifetime star formation efficiencies, when compared to empirical estimates based on spatial correlations of molecular gas and star formation tracers (e.g. Chevance et al. 2020; Kim et al. 2021a). A recent comprehensive review of feedback effects in GMCs is given in Chevance et al. (2022).

In galaxies with significant gas content (spirals and dwarfs), the interstellar medium (ISM) takes on a disk configuration on large scales. These disk galaxies are clearly long-lived systems with ongoing star formation. Thus, unlike the situation for individual star-forming molecular clouds, feedback from star formation does not disperse the whole of the ISM disk on a dynamical timescale (although significant ISM material can be carried away in galactic winds – see e.g. reviews of Veilleux et al. 2005, 2020). Rather, the feedback returned from young, massive stars contributes to the overall energetic state of the ISM, with important consequences for dynamics on a range of scales.

For ISM disks in rotationally-supported galaxies, a major focus over the years has been on consideration of large-scale (exceeding the ISM scale height $h_{\text{gas}}$) gravitational instabilities and how they are limited. Starting with Goldreich & Lynden-Bell (1965), many theoretical investigations have framed star formation regulation in terms of processes that maintain a minimum effective velocity dispersion and therefore keep the Toomre parameter $Q \equiv \kappa c_{\text{eff}}/(\pi G \Sigma_{\text{gas}})$ close to unity; here $\kappa$ is the epicyclic frequency, $c_{\text{eff}}$ is the effective sound speed, and $\Sigma_{\text{gas}}$ is the large-scale gas surface density (e.g. Binney & Tremaine 2008) Chapter 6; see also Toomre (1964) for the corresponding analysis for a stellar disk). If $Q$ is close to unity, large-scale self-gravitating instability is expected to be marginal; see Elmegreen (2002); McKee & Ostriker (2007) for reviews of earlier work, and below for some more recent contributions. However, large-scale gravitational instability is only one among several mechanisms that could collect mass into GMCs within a realistic turbulent, magnetized, multiphase ISM (see e.g. Dobbs et al. 2014), and there does not appear to be a link between observed star formation efficiency and the Toomre parameter (Leroy et al. 2008). In the Milky Way, observed GMC spin axes appear to be randomly oriented rather than aligning with the direction of galactic angular momentum (Koda et al. 2006), which would argue against GMC formation being driven solely by large-scale gravitational instability.

Whether disks are susceptible to large-scale (essentially two-dimensional) instabilities or not, global evolutionary timescales ($\sim 10^9\text{yr}$) are much larger than the turbulent crossing time (comparable to the vertical dynamical time, $\sim 10^7\text{yr}$) or the cooling time of warm or cold ISM gas ($\sim 10^6$ or $10^4\text{yr}$, respectively). It is therefore imperative to understand what sustains the turbulent and thermal pressure in the face of rapid dissipation and cooling, and what implications this may have for star formation. Since a local deficit of thermal and turbulent pressure in a region smaller than the disk scale height $h_{\text{gas}}$ could lead to gravitationally-driven contraction even if the disk is gravitationally stable on scales $h_{\text{gas}}$, regulation of the ISM pressure and star formation in disks may be primarily a local process that depends on energy inputs from recently-formed (massive) stars, regardless of larger-scale galactic dynamics.

In the two-phase ISM model of Field et al. (1969) as updated by Wolfire et al. (1995), most of the ISM mass is divided between cold ($T \sim 10^2$ K) and warm

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1 Connected to this, a suggestion of long standing is that self-gravitating instabilities lead to conversion of potential energy to turbulence while driving secular gaseous inflow (e.g. von Weizsäcker 1951; Goldreich & Lynden-Bell 1965; Fleck 1981; Elmegreen et al. 2003; Agertz et al. 2009; Krumholz et al. 2018). Self-gravitating instabilities (as well as magnetorotational instabilities – see e.g. Kim et al. (2003); Piontek & Ostriker (2005)) can certainly contribute to motions within the ISM. The idea that self-gravitating instabilities are the main source of ISM turbulence is however challenged by simulations that show gravity-driven turbulence is transonic only when $Q$ is small and runaway collapse is occurring (e.g. Kim & Ostriker 2007), and that self-gravitating ISM models with weak or no feedback have unrealistically high rates of collapse and star formation (e.g. Shetty & Ostriker 2008; Hopkins et al. 2011).
(\(T \sim 10^4\) K) phases at the same pressure (see Heiles & Troland 2003; Murray et al. 2018, for H I observations), with photoelectric heating by stellar FUV incident on small grains responsible for maintaining the thermal pressure. Based on empirical estimates of the heating rate in the Milky Way, models show that the thermal pressure of two atomic phases could be sustained at all radii (Wolfire et al. 2003; see also Dickey et al. 2009 for observations). The three-phase ISM model of McKee & Ostriker (1977) argues that for the observed SN rate in the Milky Way, hot ISM gas with pressure consistent with observations could be maintained (see also Cox & Smith 1974; McCray & Snow 1979). McKee & Ostriker (1977) also note that balancing collisional losses in a warm-cold cloud distribution with kinetic energy inputs from SNe, the velocity dispersion would be roughly in agreement with observations. Thompson et al. (2005) argued that both the radiation pressure and the turbulent pressure in cold gas driven by SNe should increase linearly with the star formation rate per unit area, and that if disks are self-regulated such that \(Q \sim 1\), the star formation rate in normal disks would scale as the square of the gas surface density (see also Faucher-Giguère et al. 2013; Hayward & Hopkins 2017; Orr et al. 2018, for related work). In an approach to large-scale self-regulation that does not presuppose \(Q \sim 1\), Franco & Shore (1984) argued that the star formation rate can be estimated by assuming stellar winds and SNe supply the ISM’s energy, with shells expanding until their kinetic and self-gravitational energies are comparable; under this hypothesis they found that the star formation rate per unit gas mass would be proportional to the square root of the gas density. Other star formation self-regulation models consider the porosity of hot gas produced by SN feedback as the controlling lever (e.g. Silk 2001; Dekel et al. 2019).

1.2. Thermal and Dynamical Equilibrium Model of Star Formation/ISM Co-Regulation

Adopting the assumptions that the main source of energy in the ISM disk is young stars and that the conversion from gas to stars is slow, to obtain a general theoretical model it is conceptually useful to think of a local patch in the star-forming ISM as a quasi-equilibrium ecosystem that has approximately plane-parallel vertical structure (in a time-averaged sense). Predictions for the star formation rate as well as the volumetric ISM quantities (pressure and density) then emerge from considering the requirements for equilibrium to be maintained self-consistently (Ostriker et al. 2010; Ostriker & Shetty 2011, hereafter OML10 and OS11). In particular, treating the ISM disk as a fluid system with source terms and spatially averaging horizontally, time-independent versions of the energy equation and vertical momentum equation should be satisfied. The pressure plays a special role as it appears in the energy density, energy flux, and momentum flux. As we shall describe in more detail in Section 2, from the vertical momentum equation the total pressure at the midplane (see e.g. Boulares & Cox 1990; Piontek & Ostriker 2007, for consideration of magnetic terms) must balance the weight of the ISM, which depends on gas surface density and the stellar and dark matter potentials. The thermal portion of the pressure must also satisfy balance between heating and cooling, where the heating is proportional to the star formation rate if the main source is stellar radiation. At the same time, the midplane turbulent pressure is sourced by the vertical momentum flux from expanding individual SN remnants or superbubbles, which is proportional to the star formation rate. Satisfying the energy equation and vertical momentum equation simultaneously then requires the star formation rate per unit area in the disk, \(\Sigma_{SFR}\), to be proportional to the weight of the ISM per unit area, \(W\). The latter is often referred to as the dynamical equilibrium pressure since these must balance in equilibrium. Here, we shall use the expression “\(P_{DE}\)” to refer to a commonly-adopted estimate for the weight \(W\) (see Section 2.1).

Based on the above physical considerations, OML10 and OS11 formulated the theory of pressure-regulated, feedback-modulated (PRFM)\(^2\) star formation, with OML10 addressing heating-cooling balance and comparing the prediction of \(\Sigma_{SFR}\) to observations of atomic-dominated regions of galaxies (where thermal pressure is significant), and OS11 focusing on turbulent pressure driving and comparing the prediction of \(\Sigma_{SFR}\) to observations of molecule-dominated galactic centers and starburst galaxies (where thermal pressure is negligible). Kim et al. (2011, hereafter KKO11) and Kim et al. (2013, hereafter KOK13) developed hydrodynamic simulations within the shearing-box local disk framework to test the PRFM theory, with star formation (following self-gravitating collapse) setting the rate of photoelectric heating and SN momentum injection to warm-cold ISM gas. These simulations covered a range of gas surface density and stellar potential such that the resulting \(\Sigma_{SFR}\) varied over two orders of magnitude, and

\(^2\) We here introduce the “PRFM” appellation to reflect the fact that pressure is regulated – i.e. dictated – by the laws of momentum conservation and gravity in the vertical direction, while feedback from star formation modulates – i.e. tunes – the component energy densities of the ISM gas to match the required total pressure.
successfully validated the hypothesis that a quasi-steady state is reached with pressure simultaneously satisfying the equilibrium requirements from the energy and vertical momentum equations. KKO11 and KOK13 demonstrated that $\Sigma_{\text{SFR}}$ is approximately proportional to either the ISM weight or midplane total pressure (also showing these are equal) and computed the feedback yields (ratios of pressure components to $\Sigma_{\text{SFR}}$; see Section 2.2), showing that the thermal yield is consistent with the prediction of OML10 and the turbulent yield is consistent with the prediction of OS11. Magnetic fields were included in the numerical simulations of Kim & Ostriker (2015a), validating the magnetohydrodynamic (MHD) generalization of the PRFM theory and demonstrating that magnetic pressure and tension terms are comparable in magnitude to thermal and turbulent ones.

1.3. Observational Tests

Several observational studies have combined tracers of molecular and/or atomic gas with tracers of star formation to test several aspects of the model predictions from OML10; OS11; KOK13 for the relationships among pressure, weight, and star formation rate. In OML10, the theoretical prediction for $\Sigma_{\text{SFR}}$ was compared to observations of a set of 11 nearby spiral galaxies, using azimuthally averaged radial profiles of molecular and atomic gas (based on CO $J = 2 - 1$ and $1 - 0$ and H I 21 cm maps, respectively), the stellar surface density and dark matter density (based on 3.6 $\mu$m maps and gas rotation curves, respectively), and star formation surface densities (based on FUV and 24 $\mu$m maps), all drawn from Leroy et al. (2008). In OS11, the theoretical prediction for $\Sigma_{\text{SFR}}$ was compared to observations for a set of starbursts (both local and higher redshift) collected by Genzel et al. (2010). Good agreement was demonstrated for both “normal” and “starburst” conditions.

Herrera-Camus et al. (2017) used Herschel observations of the C II 158$\mu$m line in a set of 31 galaxies to confirm that the thermal pressure is consistent with the requirement for two-phase equilibrium of atomic gas (Wolfire et al. 2003; see also Section 2) for the expected radiation field, given the star formation rate as measured from a combination of 24 $\mu$m and H$\alpha$ observations. Herrera-Camus et al. (2017) also showed that the relationship between $P_{\text{DE}}$ and $\Sigma_{\text{SFR}}$ is consistent with the numerical results of KOK13. Using data from 28 PHANGS galaxies, Sun et al. (2020a) showed that the turbulent pressure in molecular gas, as estimated at $\sim$100pc scale from ALMA observations of CO $J = 2 - 1$ (Leroy et al. 2021), increases approximately linearly with the $\sim$kpc-scale $\Sigma_{\text{SFR}}$ as estimated from a combination of near-UV and 12$\mu$m data (Leroy et al. 2019), consistent with OS11. Sun et al. (2020a) also found a relationship between $P_{\text{DE}}$ and $\Sigma_{\text{SFR}}$ quantitatively similar to the PRFM theory prediction, but with slightly shallower slope than found in the simulations of KOK13 (0.8 vs. 1.1). Barrera-Ballesteros et al. (2021), using observations of 96 galaxies mapped at $\sim$ kpc scale in CO for the EDGE survey (Bolatto et al. 2017) and in optical emission for the CALIFA IFU survey (Sánchez et al. 2012), found good agreement with both the slope and coefficient of the $P_{\text{DE}}$ vs. $\Sigma_{\text{SFR}}$ relation from KOK13 when using an $\alpha_{\text{CO}}(Z, \Sigma_*)$ relation based on Bolatto et al. (2013), or a slightly shallower slope (0.96) when using constant $\alpha_{\text{CO}}$.

The above studies of normal galaxies cover the range $\Sigma_{\text{SFR}} \sim 10^{-3} - 10^{-1}$ $M_\odot$ pc$^{-2}$ Myr$^{-1}$, but similar results have also been found in the regime of higher $\Sigma_{\text{SFR}}$. Using ALMA and SINFONI observations with $\sim$ kpc resolution of two disk-like starbursts at $z \sim 0.1$, Molina et al. (2020) found that the $\Sigma_{\text{SFR}}$ and estimated $P_{\text{DE}}$ are intermediate between those of PHANGS and a set of local ULIRGs with high-resolution ALMA observations of CO (1-0) and radio continuum (tracing free-free emission) as collected by Wilson et al. (2019); they noted however that the $z \sim 0.1$ starburst $P_{\text{DE}}$ may be slightly overestimated because they assumed the stellar and gas scale heights are the same (rather than the former being larger). Molina et al. (2020) found that combining all data (over 6 orders of magnitude in pressure), $\Sigma_{\text{SFR}}$ and $P_{\text{DE}}$ are related by a power law with slope 0.8. The true slope may be closer to unity, however, because they adopted an assumption of constant $\alpha_{\text{CO}}$ rather than $\alpha_{\text{CO}}$ decreasing at higher $W_{\text{CO}}$ (cf. Ostriker et al. 2010; Narayanan et al. 2012; Gong et al. 2020); constant $\alpha_{\text{CO}}$ tends to overestimate $\Sigma_{\text{mol}}$ and $P_{\text{DE}}$ at high $W_{\text{CO}}$. From the $z \sim 0.1$ DYNAMO sample in a similar regime of $\Sigma_{\text{SFR}}$ and $P_{\text{DE}}$ to the Molina et al. (2020) sample, Fisher et al. (2019) and Girard et al. (2021) also found results consistent with a power-law slope 0.8.

1.4. Testing Theory with TIGRESS and other Star-Forming, Multiphase ISM Simulations

The computational studies of KKO11; KOK13, Kim & Ostriker (2015a) (and OS11, Shetty & Ostriker (2012) for the starburst regime) validated the basic principles of the PRFM theory for the warm and cold gas that represents the majority of the ISM mass. However, these were not complete models of the three-phase ISM since momentum from SNe was injected “by hand” in warm-cold gas (a similar approach has been adopted in many galaxy formation simulations). In reality, in a given SN remnant the momentum deposited in ambient gas in-
creases in time as the remnant initially expands adiabatically, and after the onset of cooling the leftover hot gas joins the hot phase of the ISM. The combined hot gas from many SN remnants fills a significant fraction of the ISM volume. Kim & Ostriker (2017, hereafter KO17) introduced the TIGRESS\(^3\) computational framework built on the Athena MHD code (Stone et al. 2008; Stone & Gardiner 2009), in which warm and cold gas are treated similarly to our previous work, but SNe are modeled via energy rather than momentum injection and expanding individual remnants form cool outer shells when the leading shock front drops below \(\sim 200\) \(\text{km s}^{-1}\) (Kim & Ostriker 2015b). With this approach, the Sedov-Taylor stage of evolution is resolved, thereby producing a hot ISM while also allowing momentum injection to adapt to the local environment.

In a fiducial TIGRESS MHD simulation with background conditions similar to the solar neighborhood, KO17 showed that a self-regulated, quasi-equilibrium state is reached with mean thermal, turbulent, and magnetic pressure as well as \(\Sigma_{\text{SFR}} \approx 0.005\) \(\text{M}_{\odot}\) \(\text{pc}^{-2}\) \(\text{Myr}^{-1}\) in agreement with observed values\(^4\), and comparable warm and hot volumes and pressures in the three-phase ISM near the midplane. In addition, the solar neighborhood TIGRESS simulation demonstrated that correlated SNe produce superbubbles (see also Kim et al. 2017a). These superbubbles drive a warm-cold fountain flow reaching several kpc from the midplane, and their breakout leads to a hot, fast galactic wind (Kim & Ostriker 2018; Vijayan et al. 2020). Confirming the previous finding by Kim & Ostriker (2015a) that weight and total pressure balance as a function of height \(z\) in warm-cold ISM MHD simulations, Vijayan et al. (2020) found that the same balance is satisfied for the three-phase TIGRESS solar neighborhood simulation. At the midplane, the turbulent, thermal, and magnetic terms are all comparable in the warm gas, while thermal pressure dominates in the hot phase; total pressures are comparable across thermal phases. The majority of the volume is filled with warm gas below \(|z| \sim 1\) kpc, while hot wind occupies the majority of the volume at higher altitudes where warm fountain flow clouds turn around (see also Kado-Fong et al. 2020, for analysis of the distribution and properties of photoionized gas).

Using the same TIGRESS computational framework, we have recently explored a range of conditions in galactic disks. As we shall describe further in Section 2 and Section 3, the two essential “background” galactic disk properties that may be varied as independent parameters are the total surface density of gas, \(\Sigma_{\text{gas}}\) and the midplane density of stars plus dark matter, \(\rho_{\text{sd}}\). The star formation rate per unit area and the distributions of ISM density, pressure, velocity, and magnetic fields then emerge self-consistently. Based on seven different TIGRESS disk models with emergent \(\Sigma_{\text{SFR}}\) varying by four orders of magnitude, Kim et al. (2020a) presented a detailed investigation of the dependence of multiphase outflow properties on the star formation rate and ISM properties. There, the outflow mass, momentum, energy, and metal loading factors, as well as velocities, were separately measured for warm-cold gas and hot gas, with scaling relations calculated as a function of \(\Sigma_{\text{SFR}}\) and midplane pressure or ISM weight. In Kim et al. (2020b), these overall loading factors were combined with measurements of the mass-loading PDFs to derive analytic joint probability distributions (in velocity and sound speed) of mass, momentum, energy, and metal loading as a function of \(\Sigma_{\text{SFR}}\).

The TIGRESS implementation has also been used to study effects of spiral arms on star formation rates and ISM dynamical equilibrium (Kim et al. 2020c), demonstrating the local validity of the PRFM model, and showing that spurs can form downstream from arms due to correlated feedback. Extending to galactic centers, Moon et al. (2021, 2022) applied the TIGRESS computational framework to model star-forming nuclear rings created by bar-driven inflows and showed that the PRFM theory is also satisfied in these more extreme environments, with the interesting twist that the ring mass adjusts as needed for the star formation rate to match the inflow rate.

Simulations of the large-scale star-forming ISM with feedback have also been conducted by a number of other groups. Those most similar to the TIGRESS models focus on a local ISM patch at \(\sim\) kpc scale (see Kannan et al. 2020; Brucy et al. 2020; Rathjen et al. 2021, and references therein), as this affords sufficient resolution to follow both gravitational collapse in cold gas and the evolution and interaction of the hot gas (produced by SN shocks) with other thermal phases. One difference from the TIGRESS models is that other simulations have generally been run for shorter timescales, typically \(\sim 100\) Myr; this may be compared to 700 Myr for the TIGRESS solar neighborhood model presented in KO17. Because there may be a strong early burst of star formation (depending on initial conditions

\(^3\) Three-phase Interstellar medium in Galaxies Resolving Evolution with Star formation and Supernova feedback

\(^4\) Observational estimates employing a variety of methods indicate a mean value \(\Sigma_{\text{SFR}} \sim 0.003 – 0.005\) \(\text{M}_{\odot}\) \(\text{pc}^{-2}\) \(\text{Myr}^{-1}\) for the solar neighborhood, but there is also evidence of significant bursts (Bertelli & Nasi 2001; Vergely et al. 2002; Fuchs et al. 2009; Tremblay et al. 2014; Mor et al. 2019; Ruiz-Lara et al. 2020; Zari et al. 2022).
and feedback ingredients), simulations run for relatively short periods may not have reached a quasi-steady equilibrium state. For solar neighborhood (unmagnetized) simulations with AREPO-RT (Springel 2010; Kannan et al. 2019) that include just SNe in comparison to simulations with SNe plus radiation, Kannan et al. (2020) found a factor 3-10 higher $\Sigma_{\text{SFR}}$ than in KO17. The initial set of solar neighborhood SILCC MHD simulations with FLASH (Fryxell et al. 2000) reported in Gatto et al. (2017) had similarly high $\Sigma_{\text{SFR}}$, while SILCC simulations that included radiation and stellar wind feedback as well as SNe reported by Peters et al. (2017) found comparable $\Sigma_{\text{SFR}}$ to that of KO17 (and observations), and the additional inclusion of cosmic ray feedback in Rathjen et al. (2021) reduced $\Sigma_{\text{SFR}}$ even below this level. In an initial set of RAMSES MHD (Teyssier 2002) simulations including SNe plus constant background heating, Hennebelle & Iffrig (2014) found $\Sigma_{\text{SFR}} \sim 0.01 - 0.1 \, M_\odot \, \text{pc}^{-2} \, \text{Myr}^{-1}$, significantly higher than in KO17. The simulations from other groups mentioned above did not include background shear, and as a result magnetic fields (if they were included) decayed over time. In RAMSES MHD simulations with initial $\Sigma_{\text{gas}} = 19 \, M_\odot \, \text{pc}^{-2}$ that also included ionizing radiation, Colling et al. (2018) studied the effects of shear, finding $\Sigma_{\text{SFR}} \sim 0.03 - 0.01 \, M_\odot \, \text{pc}^{-2} \, \text{Myr}^{-1}$ for shear varying between the solar neighborhood value and twice that. Brucy et al. (2020) considered a wider range of initial gas surface density, $\Sigma_{\text{gas}} = 13 - 160 \, M_\odot \, \text{pc}^{-2}$, finding $\Sigma_{\text{SFR}} \sim 0.003 - 1 \, M_\odot \, \text{pc}^{-2} \, \text{Myr}^{-1}$ with SN and radiative heating feedback, and somewhat lower values at high $\Sigma_{\text{gas}}$ when additional large-scale turbulent driving is included.

While none of the above kpc-scale “ISM patch” studies have directly investigated whether the PRFM theory predictions hold in their simulations, there have been a few studies using lower resolution simulations that have explored the issues of the pressure-weight balance and the pressure-$\Sigma_{\text{SFR}}$ relation. Because these global galaxy and cosmological zoom simulations are not able to follow the Sedov-Taylor expansion and subsequent radiative stages of individual SN remnants or resolve the hot ISM more generally, subgrid models are adopted for the SN feedback that drives turbulence. Benincasa et al. (2016) employed isolated global-galaxy simulations with a Milky-Way like model to study pressure balance and the role of pressure in star formation regulation. They found that the total midplane pressure (azimuthally averaged and averaged over 100 Myr) agrees well with the total vertical weight of the ISM, and this is insensitive to the adopted star formation efficiency per dynamical time or density threshold for star formation. Using FIRE-2 cosmological zoom simulations of disk galaxies, Gurvich et al. (2020) conducted a detailed analysis of the pressure and weight in vertical columns decomposed in $z$ into slabs, separately measuring thermal and turbulent pressures and the partial contributions within a slab from distinct temperature bins. They found that the median over time of the ratio between total pressure and vertical weight is generally within $\sim 20\%$ of unity, with departures attributed to the approximate treatment of “long-range” radiation and to the fact that the spatial region for SN momentum deposition (at their numerical resolution) is an appreciable fraction of the gas scale height. They also found that the total pressures in different temperature bins agree within a factor of $\sim 2$, and that near the midplane the turbulent pressure is the largest contribution for $T < 10^5$ K gas while the thermal pressure dominates for hotter gas. Gurvich et al. (2020) confirmed as well that the total pressure is roughly linearly proportional to $\Sigma_{\text{SFR}}$ with a coefficient compatible with theoretical predictions.

In this paper, we return to the PRFM theory, comparing predictions with numerical results based on the same set of TIGRESS MHD simulations used to analyze outflow properties in Kim et al. (2020a). We shall show that the key elements of the theory are validated: namely, a state of quasi-equilibrium is reached in which (a) there is both warm and cold gas at the midplane with thermal pressure at a level predicted by the photoelectric heating rate, and turbulent pressure at a level predicted by the SN rate; (b) the total pressures of hot and warm-cold gas are comparable, and this matches the vertical weight of the ISM. Consistent with our previous theoretical and computational results, we shall show that as a consequence of (a) and (b), $\Sigma_{\text{SFR}}$ has a nearly linear dependence on $W$ (or $P_{\text{DE}}$) over four decades. We shall also provide quantitative analysis of other measures related to pressure, including the relative proportions of different components (thermal, turbulent, magnetic), the component and total feedback yields (ratios of pressures to $\Sigma_{\text{SFR}}$), and the effective equation of state (pressure-density relation).

The plan of the paper is as follows: In Section 2 we review and update key elements of the PRFM theory and define necessary terminology. Section 3 briefly summarizes the numerical methods and model inputs used in our TIGRESS simulations, while Section 4 presents results from analysis of our simulation suite. In Section 5 we summarize and discuss our conclusions.
2. PRESSURE-REGULATED, FEEDBACK-MODULATED THEORY OF THE EQUILIBRIUM STAR-FORMING ISM

2.1. Pressure Requirement: Dynamical Equilibrium

ISM Weight and Gas Scale Height

In a disk system, the weight (per unit area) of the ISM at a given midplane location is obtained by a vertical integral of the product of the gas density $\rho$ and the vertical component of the combined gravitational field,

$$ W = \int_0^{z_{\rm max}} dz \rho(g_{\rm gas} + g_{\rm ext}) = \frac{\Sigma_{\rm gas}}{2} \langle \rho z \rangle $$

$$ = W_{\rm gas} + W_{\rm ext}, $$

(1)

where $g_{\rm ext} = \partial \Phi_{\rm ext} / \partial z$ and the external gravitational potential $\Phi_{\rm ext}$ includes that of the old stellar disk as well as dark matter (the vertical gravity from the former is dominant within the actively star-forming disk of normal galaxies). For plane-parallel (slab) geometry (i.e. if the density and the gravitational field are functions only of $z$, e.g. based on a horizontal average), $\rho = (4\pi G)^{-1} \partial \tilde{\rho}_{\rm gas} / \partial z$, and we can define $\tilde{g} = g(z)/g(z_{\rm max})$ for either the gas or external component of the potential. In slab geometry $g_{\rm gas}(z_{\rm max}) = 2\pi G \Sigma_{\rm gas}$ for the total gas surface density $\Sigma_{\rm gas}$ and similarly for an external potential component with $\Sigma_{\rm ext}$ the total equivalent surface density.$^5$ We therefore have the contribution to the weight from the gas gravity

$$ W_{\rm gas} = \pi G \Sigma_{\rm gas}^2 \int_0^{z_{\rm max}} dz \frac{\partial \tilde{g}_{\rm gas}}{\partial z} \tilde{g}_{\rm gas} = \frac{\pi G \Sigma_{\rm gas}^2}{2} $$

(2)

and from the external gravity

$$ W_{\rm ext} = \pi G \Sigma_{\rm gas} \Sigma_{\rm ext} \int_0^{z_{\rm max}} dz \frac{\partial \tilde{g}_{\rm gas}}{\partial z} \tilde{g}_{\rm ext}. $$

(3)

Since $\tilde{g}_{\rm gas}(z_{\rm max}) = 1 = \tilde{g}_{\rm ext}(z_{\rm max})$, the dimensionless integral in Equation 3 has an upper limit unity; the value depends on the vertical profile shapes of $\tilde{g}_{\rm gas}$ and $\tilde{g}_{\rm ext}$ (with a value 1/2 when the profiles are the same).

In the case that both density profiles follow exponentials $\rho \propto \exp(-z/h)$, $\tilde{g} = 1 - \exp(-z/h)$, the integral is

$$ h_{\rm gas} / (h_{\rm gas} + h_{\rm ext}). $$

In most circumstances the thickness of the mass-containing portion of the gas disk is smaller than that of the stellar disk and dark matter, so that within the gas layer $\Sigma_{\rm ext} \frac{\partial \tilde{g}_{\rm ext}}{\partial z} \approx 2\rho_{\rm mid}$ for $\rho_{\rm mid} = \rho_{\ast} + \rho_{\rm dm}$ the midplane density of stars plus dark matter.$^6$ Equation 3 can then be expressed as $W_{\rm ext} = 2\pi G \rho_{\ast} \Sigma_{\rm gas} / \rho_0$ for $\rho_0$ the midplane gas density and $\zeta_d \approx 1/3$ (see Equation 6 of OML10). The exact value of $\zeta_d$ depends on the functional form of the vertical density profile $\rho \propto \partial \tilde{g}_{\rm gas}/\partial z$. If gas gravity dominates the potential, the resulting sech$^2$ density profile yields $\zeta_d = \ln(2)/2 \approx 0.35$, while a potential dominated by external gravity leads to a Gaussian density profile and $\zeta_d = 1/\pi = 0.32$.

We can define the half-thickness of the gas disk as $h_{\rm gas} = \Sigma_{\rm gas} / (2\rho_0)$; with this definition,

$$ W = \frac{\pi G \Sigma_{\rm gas}^2}{2} + 4\pi \zeta_d G \Sigma_{\rm gas} \rho_{\ast} h_{\rm gas}. $$

(4)

We note that Equation 4 holds quite generally, given a value of $h_{\rm gas}$. Proceeding further to obtain an estimate of $h_{\rm gas}$ in terms of large-scale disk properties, one must assume that the disk structure represents a (quasi) equilibrium state, in the sense that an average over a few vertical dynamical times (typically a few tens of Myr) is well defined, evolving only over a longer timescale.

In vertical dynamical equilibrium, the ISM weight $W$ must be equal to $\Delta P_{\rm tot}$, the difference in the total vertical momentum flux across the ISM layer. Here we will consider the terms in $P_{\rm tot}$ associated with thermal, turbulent, and magnetic stresses, although in principle $P_{\rm tot}$ may also contain terms associated with radiation and cosmic ray pressure (see OS11). Writing $\Delta P_{\rm tot} \approx P_{\rm tot}(z = 0) \equiv \sigma_{\rm eff} \rho_0 = \sigma_{\rm eff} \Sigma_{\rm gas} / (2h_{\rm gas})$ for a total effective velocity dispersion $\sigma_{\rm eff}$ (assumed constant in $z$) and equating $\Delta P_{\rm tot} = W$, we can solve a quadratic to obtain

$$ h_{\rm gas} = \frac{2\sigma_{\rm eff}^2}{\pi G \Sigma_{\rm gas} + (\pi G \Sigma_{\rm gas})^2 + 32\pi \zeta_d G \rho_{\ast} \sigma_{\rm eff}^2 1/2}. $$

(5)

and

$$ W = \frac{\pi G \Sigma_{\rm gas}^2}{4} \left\{ 1 + \left[ 1 + \frac{32\zeta_d \rho_{\ast} \sigma_{\rm eff}^2}{\pi G \Sigma_{\rm gas}^2} \right]^{1/2} \right\}. $$

(6)

Equation 5 reduces to the familiar limits $h_{\rm gas} \rightarrow \sigma_{\rm eff}^2 / (\pi G \Sigma_{\rm gas})$ or $h_{\rm gas} \rightarrow \sigma_{\rm eff} / (8G \rho_{\ast})^{1/2}$ in the gas-gravity and external-gravity dominated limits, respectively, where for the latter case we also take $\zeta_d \rightarrow 1/\pi$.

$^5$ For a stellar disk, the contribution is $g_{\ast}(z_{\rm max}) = 2\pi G \Sigma_\ast$ for $\Sigma_\ast$ the stellar surface density. The dark matter potential is presumably approximately spherical rather than disk-like, and the same would be true for a stellar bulge potential. In the (typical) case that the gas scale height is smaller than the gradient scale of the spherical dark matter or bulge potential, however, the local vertical gravity is still just a linear function of height, $g_{\ast} \approx 4\pi G \rho_{\ast} z$ for a dark matter potential with a flat rotation curve and local density $\rho_{\ast}$, or $g_{\ast} \approx 4\pi G \rho_{\ast} z / 3$ for a stellar bulge with uniform density $\rho_{\ast}$ (see respectively OML10; OS11). The total equivalent “external” surface density is then $\Sigma_{\rm ext} \equiv \Sigma_\ast + 2\zeta_{\rm max} (\rho_{\rm dm} + \rho_{\ast} / 3)$.

$^6$ In galactic center regions the external potential is dominated by a stellar bulge rather than the combination of a stellar disk plus dark matter halo, and the appropriate substitution in all of the following formulae is $\rho_{\ast} \rightarrow \rho_{\ast} / 3$. 
If we consider limiting forms for the square root in Equation 6, we obtain a simplified (good within 20%) expression for the weight,

\[ W \approx P_{DE} = \frac{\pi G \Sigma_{gas}^2}{2} + \Sigma_{gas}(2G\rho_{sd})^{1/2}\sigma_{eff}, \]  

(7)

which has been adopted in many observational studies (e.g. Blitz & Rosolowsky 2004; Herrera-Camus et al. 2017; Sun et al. 2020a; Barrera-Ballesteros et al. 2021). Comparing Equation 7 to Equation 1, this also implies that under equilibrium conditions, the average gravity in the vertical direction is given by \( g_{z} = \pi G \Sigma_{gas} + 2(2G\rho_{sd})^{1/2}\sigma_{eff} \). We note that \( \sigma_{eff} \) in Equation 5 - Equation 7 represents the square root of the ratio of total vertical stresses to density, and therefore should include a magnetic contribution (see Section 2.3 and Section 4.1).

Equation 5 and Equation 6 give the ISM scale height and weight under the assumption that it consists of a volume-filling medium with an effective velocity dispersion \( \sigma_{eff} \). OML10 provided generalized expressions in the case that the gas is divided into a diffuse component of surface density \( \Sigma_{diff} \) and velocity dispersion \( \sigma_{eff} \) that is volume-filling, and a gravitationally bound cloud (GBC) component of surface density \( \Sigma_{GBC} = \Sigma_{gas} - \Sigma_{diff} \) with negligible volume and scale height. The resulting expression for the midplane weight or pressure in equilibrium is

\[
W = \frac{\pi G \Sigma_{GBC}^2}{4} \left( 1 + 2 \frac{\Sigma_{GBC}}{\Sigma_{diff}} + \left[ \left( 1 + 2 \frac{\Sigma_{GBC}}{\Sigma_{diff}} \right)^2 + \frac{32 \zeta_{d} \rho_{sd} \sigma_{eff}^2}{\pi G \Sigma_{diff}} \right]^{1/2} \right) \]  

(8)

(see Eq. 11 of OML10), with diffuse-gas scale height \( h_{diff} = \sigma_{eff}^{2/3} \Sigma_{diff} / (2W) \). These expressions reduce to Equation 5 and Equation 6 when \( \Sigma_{GBC} \ll \Sigma_{diff} \approx \Sigma_{gas} \).

Although the traditional view has been that GMCs are gravitationally bound entities and therefore might be considered as collectively comprising a “GBC” component of the ISM, with the atomic gas comprising the “diffuse” component, this has been called into question by recent work. On the theoretical side, numerical simulations of star formation in turbulent clouds (with feedback) show that both the star formation rate per free-fall time and the lifetime SFE exceed observational estimates unless the cloud-scale virial parameter is at least \( \sim 2 - 4 \) (e.g. Padoan et al. 2012; Kim et al. 2021b; Evans et al. 2022). The analysis of dense structures in TIGRESS simulations also shows that only a small fraction of the mass is gravitationally bound, and that many structures that would be classified as gravitationally bound based on typical observational estimates in fact contain only a small fraction of bound gas when detailed internal structure and tidal effects are taken into account (Mao et al. 2020). On the observational side, meta-analysis of several Galactic and extragalactic surveys suggests that the fraction of molecular gas that is in bound structures may be well below unity (Evans et al. 2021), and the mean value of the virial parameter for molecular gas at \( \sim 100 \) pc scale in a large sample of PHANGS galaxies is closer to 4 than to 1 (Sun et al. 2020b). GMCs are more likely to be gravitationally bound in atomic-dominated (rather than molecular-dominated) regions of galaxies because they correspond to higher overdensities in those environments, but since by definition \( \Sigma_{H_2} \ll \Sigma_{HI} \) when atomic gas dominates it would correspond to the limit \( \Sigma_{GBC} \ll \Sigma_{diff} \) so that Equation 6 holds. In any case, the short lifetime and transient nature of GMCs (e.g. Leisawitz et al. 1989; Kawanura et al. 2009; Kruijssen et al. 2019) implies that regardless of the exact values of their virial parameters it may be most appropriate to consider GMCs as temporary condensations within the dynamic ISM.

Thus, we adopt the assumption that regardless of its chemical state, the ISM may be treated as a volume-filling diffuse medium so that in statistical equilibrium the weight and total midplane pressure are equal and may be expressed in terms of the disk parameters by Equation 6 or Equation 7.

We emphasize that because \( \sigma_{eff} \) enters the expressions (Equation 4 and Equation 6) for ISM weight (or equilibrium midplane pressure) by way of representing the vertical extent \( h_{gas} \) of the mass-containing component of the ISM, any observational estimate of \( \sigma_{eff} \) must be mass-weighted and correspond to an average over the vertical direction. In face-on galaxies, the thermal and turbulent motions in the vertical direction combine in quadrature to produce the observed linewidth; the mass-weighted large-scale average of this (based on proxies such as H I 21 cm and CO) is the thermal and turbulent contribution to \( \sigma_{eff}^2 \). The azimuthal component of the magnetic field is usually the largest, so that the magnetic contribution to \( \sigma_{eff}^2 \) is dominated by the mass-weighted large-scale average of \( B^2_{\phi} / (8\pi \rho) \).
2.2. Pressure Response: Feedback Modulation and Yields

We next consider how pressures are modulated by feedback from young, massive stars. The two most important direct feedback mechanisms for setting the large-scale pressure in the warm and cold gas that makes up the ISM’s mass reservoir are the driving of kinetic turbulence by expanding SN remnants and the photoelectric heating of atomic and low-A$\lambda$ molecular gas induced by far-UV stellar photons impinging on small grains. The resulting turbulent pressure scales with the momentum injection rate from SNe, while the thermal pressure scales with the FUV intensity (see below).

We note that within young ($\lesssim 10$ Myr) star-forming molecular clouds, photoionization produces high-pressure gas that, in combination with radiation pressure on dust, is quite important for dispersing dense gas clouds. However, the momentum injection from dusty H II region expansion over the lifetime of the massive-star population (averaging over the IMF) is much less than the momentum injection produced when the same stars die as SNe (e.g. Kim et al. 2018). While idealized spherical solutions for H II region expansion around star clusters can produce momentum per unit stellar mass approaching that produced by SNe (see e.g. Eq. 14 of Chevance et al. 2022), the actual momentum in simulations of cluster-forming turbulent clouds that use ray tracing to follow radiation is lower by a factor $\sim 5 - 10$. This reduction is in part because distributed star formation and fractal ionization fronts lead to force cancellation, and in part because a significant fraction of photons are absorbed by dust or escape from inhomogeneous clouds through low-density channels (Kim et al. 2019b). Stellar winds also inject considerable energy in their environments, but most is radiated away and the momentum injection is comparable to that from radiation pressure (Lancaster et al. 2021a,b). In the extremely high-column environments of nascent super star clusters, the pressure of infrared radiation reprocessed by dust can exceed the direct radiation pressure and even the pressure of photoionized gas (e.g. Murray et al. 2010). However, the gravitational force in these environments is also extremely high. Gravity exceeds the reprocessed radiation force unless the IMF is top-heavy or the dust abundance is significantly enhanced; numerical radiation hydrodynamic simulations show that reprocessed radiation is therefore ineffective at limiting star formation unless these exceptions apply (Skinner & Ostriker 2015).

In high-A$\lambda$ regions where FUV photons are shielded, low-energy CRs are the main mechanism for heating gas and therefore setting the thermal pressure. However, the low temperature produced by CR heating (e.g. Gong et al. 2017) generally implies the thermal pressure is lower than kinetic turbulent pressure (as well as magnetic pressure) in molecular gas (e.g. Heyer & Dame 2015).

In the Milky Way, the midplane energy density of $\sim$GeV CRs is $\sim 1$ eV cm$^{-3}$ (e.g. Grenier et al. 2015), comparable to the thermal, turbulent, and magnetic energy densities. However, the ion-neutral damping of Alfvén waves in the primarily-neutral gas leads to strong CR diffusion and is expected to limit the support against gravity by CRs, as confirmed by low CR pressure gradients near the midplane in recent numerical simulations of CR transport in TIGRESS simulations (Armillotta et al. 2021, 2022).

We shall define $m_*$ as the total mass in stars formed for every high mass star that undergoes a SN; $m_* \approx 100$ M$\odot$ for a Kroupa (2001) IMF. In quasi steady state, the rate per unit area in the disk of core collapse SNe from high mass stellar death will then be given by $\Sigma_{\text{SFR}}/m_*$. Taking the spherical momentum injected per SN as $p_*$, and assuming that the turbulent pressure is equal to the vertical momentum flux injected to each side of the disk, OS11 predicted that the turbulent pressure will follow

$$P_{\text{turb}} = \frac{1}{4} \frac{p_*}{m_*} \Sigma_{\text{SFR}}.$$  (9)

OS11 allowed for an additional multiplicative factor $f_p$ between unity (for strong dissipation) and 2 (for weak dissipation); since the numerical simulations of KOK13 found $f_p = 1.2(\Sigma_{\text{SFR}}/0.001$ M$\odot$ pc$^{-2}$ Myr$^{-1})^{-0.11}$, here we adopt $f_p = 1$ as a good approximation. The scaling of turbulent pressure with $\Sigma_{\text{SFR}}$ can also be obtained by equating the turbulent energy driving rate with the turbulent energy dissipation rate (Thompson et al. 2005; Ostriker & Shetty 2011; Hennebelle & Hollifield 2014). Per unit area in the disk, the kinetic energy input rate from SNe is an order-unity factor times $\sigma_v (p_*/m_*) \Sigma_{\text{SFR}}$ for $\sigma_v$ the velocity dispersion on the energy-containing scale. Assuming the dissipation timescale is comparable to the vertical crossing time $h_{\text{gas}}/\sigma_v$ (Stone et al. 1998; Mac Low et al. 1998), the kinetic energy dissipation rate per unit area is then equal to $(1/2) \sigma_v^2 h_{\text{gas}} = \sigma_v P_{\text{turb}}$ times an order-unity factor. Equating leads to $P_{\text{turb}} \sim (p_*/m_*) \Sigma_{\text{SFR}}$, but without a specific prediction for the numerical coefficient.

The momentum injection per SN $p_*$ is insensitive to the ambient environment because it is primarily set by the condition for the expanding blast wave to become radiative when the shock velocity drops to $v_{\text{cool}} \sim 200$ km s$^{-1}$ (for solar metallicity; see Eq. 9 of Kim & Ostriker 2015b or Eq. 39.22 of Draine 2011). This yields
mumon momentum $\sim 0.6E_{SN}/v_{cool}$ at the shell formation time since the kinetic energy in the Sedov-Taylor stage is $E_{kin} = 0.283E_{SN}$. The numerical simulations of Kim & Ostriker (2015b) for single SN explosions at a range of ambient (uniform) density of hydrogen nuclei $n_H = 0.1 - 100$ cm$^{-3}$ found that the momentum increases by $\approx 50\%$ after cooling and shell formation, reaching a level $p_* \approx 2.95 \times 10^5$ M$_{\odot}$ km s$^{-1} (n_H/1$ cm$^{-3})^{-0.16}$, consistent with theoretical expectations (these and other simulations assume $E_{SN} = 10^{51}$ erg; e.g., Cioffi et al. 1988; Thornton et al. 1998). For inhomogeneous conditions corresponding to a two-phase ISM the result is quite similar, $p_* \approx 2.8 \times 10^5$ M$_{\odot}$ km s$^{-1} (n_H/1$ cm$^{-3})^{-0.17}$ (see also Iffrig & Hennebelle 2015; Martizzi et al. 2015; Walch & Naab 2015); here $n_H$ is the density of hydrogen nuclei in the ambient medium averaged over both phases. These results are also consistent with the radial shell momentum of seven radiative SN remnants inferred from H I observations (Koo et al. 2020). The momentum at shell formation also shows a weak dependence on the metallicity $\propto Z^{-0.17}$ at $Z \geq 0.01Z_{\odot}$ and nearly constant otherwise (J.-G. Kim et al. 2022, submitted; see also Thornton et al. 1998; Oku et al. 2022).

While correlation of SN originating from a star cluster (or neighboring clusters) could in principle enhance the momentum per event if most of the injected SN energy is retained by the superbubble (McCray & Kafatos 1987; Gentry et al. 2017), this requires a contact discontinuity to be maintained at the interface between the hot bubble interior and the surrounding gas. However, this is unlikely in a realistic, clumpy ambient medium. Instead, Kelvin-Helmholz and other instabilities at interfaces excite turbulence that drives mixing between hot gas and surrounding higher-density (warm or cold) gas with subsequent rapid cooling (Fielding et al. 2020; Lancaster et al. 2021a), so that the energy in the interior of the hot bubble drops in between SN events. Kim et al. (2017a) reported on numerical simulations of multiple SN events in a cloudy ambient medium, considering a range of ambient mean density ($n_H = 0.1 - 10$ cm$^{-3}$) and SN intervals (0.01 $- 1$ Myr), finding quite similar momentum per event to the single-SN case, $p_* \sim 0.7 - 3 \times 10^5$ M$_{\odot}$ km s$^{-1}$. The lower (upper) value is for an interval between SNe of 0.01 (1) Myr, corresponding to a total clustered stellar mass of $4 \times 10^5$ ($4 \times 10^3$) M$_{\odot}$. Realistic clustered stellar masses (which may include several individual star clusters that are born within a few tens of Myr of each other) are likely in the middle of this range, implying $p_* \sim 2 \times 10^5$ M$_{\odot}$ km s$^{-1}$.

The turbulent feedback yield is given by the ratio between $P_{turb}$ and $\Sigma_{SFR}$, which from Equation 9 and the idealized supernova simulations described above is expected to be

$$\Upsilon_{turb} \equiv \frac{P_{turb}}{\Sigma_{SFR}} = \frac{250 \text{ km s}^{-1}}{1000 \text{ km s}^{-1}} \frac{p_*/m_*}{1000 \text{ km s}^{-1}} \sim 500 \text{ km s}^{-1}. \quad (10)$$

The SN momentum injection is relatively insensitive to ambient conditions so we expect $\Upsilon_{turb}$ to decrease only weakly in the higher-density, inner regions of galaxies.

For a given FUV radiation field, a wide range of thermal pressures are possible in the atomic ISM, with the lowest pressures associated with warm, low-density gas (cooled mostly by Ly$\alpha$ and recombination on grains), and the highest pressures associated with cold, high-density gas (cooled mostly by C II and O I fine structure lines). Which among these possible pressures does the ISM select? OML10 hypothesized that the equilibrium midplane pressure should be in the “two-phase” range where both cold and warm phases are available, adopting the geometric mean $P_{two} \equiv (P_{max, warm}P_{min, cold})^{1/2}$ between the largest possible pressure for a purely warm medium and the smallest possible pressure at which a cold medium becomes possible. The resulting thermal pressure is given in Equation 15 of OML10, which uses the Wolfire et al. (2003) analytic fit to their thermal equilibrium curve (their Eq. 33), also assuming that the mean pressure is a factor 1.4 above the minimum possible pressure of the cold medium.

The primary scaling of the equilibrium thermal pressure is with the FUV radiation field mean intensity, $J_{FUV}$. Since the FUV originates from young stars, we expect $J_{FUV} \propto \Sigma_{SFR}$, with an additional attenuation factor that depends on radiative transfer through the ISM. For the simplest possible case of a slab containing a uniform distribution of dusty gas producing extinction and uniformly-distributed stars producing emission, the radiation field will be $J_{FUV} = \Sigma_{FUV}f_r/(4\pi)$ where $\Sigma_{FUV} \propto \Sigma_{SFR}$ is the energy per unit area per unit time in FUV produced by recently formed stars, and

$$f_r = \frac{1 - E_2(\tau_\perp/2)}{\tau_\perp} \quad (11)$$

(OML10). Here, $E_2$ is the second exponential integral and $\tau_\perp = \kappa_{FUV}\Sigma_{gas}$ is the mean optical depth to FUV vertically through the disk; $E_2(x)/x$ is logarithmic at small argument and decreases exponentially at large argument. Taking $\kappa_{FUV} = 10^3$ cm$^2$ g$^{-1}$ and total surface density $\Sigma_{gas} = 10$ M$_{\odot}$ pc$^{-2}$ in the solar neighborhood, the local value would be $f_{r,0} \approx 0.41$. From Equation (15) of OML10, we can then write the predicted yield
for thermal pressure as

\[ \Upsilon_{\text{th}} \equiv \frac{P_{\text{th}}}{\Sigma_{\text{SFR}}} = \frac{240 \text{ km s}^{-1}}{1 + 3.1 \left( \frac{\Sigma_{\text{gas}} Z_f/Z_g}{10^5 \text{ M}_\odot \text{ pc}^{-2}} \right)^{0.4}} \]  

(12)

where \( Z_f' \) is the dust abundance relative to the solar neighborhood value and the dust abundance is assumed to scale with gas metallicity (\( Z_f' = Z_g' \)). In Equation 12 we explicitly show the dependence on \( f_r \), which was omitted for simplicity in Equation 18 of OML10 for the relationship between \( P_{\text{th}} \) and \( \Sigma_{\text{SFR}} \). We note that in contrast to the relatively weak variation of \( \Upsilon_{\text{turb}} \) with environment, attenuation of FUV radiation is expected to strongly reduce \( \Upsilon_{\text{th}} \) under higher-density galactic conditions. In particular, \( f_r \sim 1/\tau_\perp \) at high optical depth, with \( \tau_\perp \approx \Sigma_{\text{gas}}/5 \text{ M}_\odot \text{ pc}^{-2} \).

In addition to turbulent kinetic and thermal pressure, magnetic pressure also helps to support against gravity. Magnetic fields are driven by dynamo activity, with kinetic turbulence and large-scale galactic shear both contributing to amplification via folding and stretching of field lines, and buoyancy and superbubble expansion combining with Coriolis forces to create poloidal from toroidal fields (e.g. Kulsrud 2005). Many aspects of dynamo theory – including how mean fields are generated – remain however poorly understood, even in idealized situations that lack the complexity of the multiphase ISM. Nevertheless, the previous numerical simulations of Kim & Ostriker (2015a) do show amplification of both the turbulent and mean magnetic field, with the saturation level of turbulent magnetic pressure about one-third of the saturation level of turbulent kinetic pressure, and the pressure in the mean magnetic field (which is primarily azimuthal) comparable to that in the turbulent magnetic field. Since the turbulent magnetic pressure scales with turbulent kinetic pressure, from Equation 9 and Equation 10 we can expect the turbulent portion of the magnetic pressure to scale nearly linearly with \( \Sigma_{\text{SFR}} \). It is less clear whether the mean component of the magnetic pressure would also scale with \( \Sigma_{\text{SFR}} \). In summary, uncertainties related to dynamo theory preclude making a definitive prediction for the magnetic pressure or the magnetic yield at this time, but it is reasonable to expect \( \Upsilon_{\text{mag}} \) to be \( \sim 0.5 - 1 \times \Upsilon_{\text{turb}} \). This is consistent with observed estimates of magnetic field strength in the Milky Way and other spiral galaxies, which based on Zeeman splitting, rotation measure, and synchrotron emission find comparable magnetic pressure to the turbulent pressure (e.g. Heiles & Troland 2005; Beck et al. 2019), but are subject to significant systematic uncertainties.

Taking the sum of the turbulent kinetic, magnetic, and thermal yield terms discussed above, the theoretically predicted value of the feedback yield is \( \Upsilon_{\text{tot}} \sim 1000 \text{ km s}^{-1} \). One of the main goals of the present work is to evaluate \( \Upsilon_{\text{tot}} \) from realistic MHD simulations with a range of conditions. This will provide a numerical test of the theory.

Finally, it is worth noting that the drop in \( \Upsilon_{\text{th}} \) compared to \( \Upsilon_{\text{turb}} \) is expected to lead to a reduction in the fraction of gas mass in the warm phase in high-density environments. The warm mass fraction is given by \( f_{m,w} = (P_{\text{th}}/P_{\text{tot}})(\sigma_{\text{w}}^2/c_w^2) \) where \( c_w \) is the thermal speed in warm gas (\( \sim 10 \text{ km s}^{-1} \), insensitive to environment). With \( P_{\text{th}}/P_{\text{tot}} = \Upsilon_{\text{th}}/\Upsilon_{\text{tot}} \) decreasing \( \propto 1/\Sigma_{\text{gas}} \) at high surface density due to the \( 1/\tau_\perp \) dependence of \( f_r \) in Equation 12, \( f_{m,w} \) is expected to become small in these dense environments. While an increase in \( \sigma_{\text{eff}} \) (dominated by turbulence) could offset this to some extent, empirically the effective velocity dispersion does not increase as rapidly as the gas surface density (e.g. Sun et al. 2018; Wilson et al. 2019).

### 2.3. Predicting Large-Scale Galactic SFRs

From Section 2.2, if the energy and pressure in the ISM are sustained by feedback from star formation, we can relate the pressure and star formation rate via \( P_{\text{tot}} = \Upsilon_{\text{tot}} \Sigma_{\text{SFR}} \), where \( \Upsilon_{\text{tot}} \) is the total feedback yield, including thermal, turbulent (kinetic), and magnetic terms. For current purposes, the \( P_{\text{tot}} \) we are interested in is the total midplane pressure as it would appear in the vertical component of the vertical momentum equation, consisting of the sum \( P_{\text{tot}} = P_{\text{th}} + P_{\text{turb}} + \Pi_{\text{mag}} \) for \( P_{\text{th}} \equiv \langle \rho c_s^2 \rangle \) (thermal pressure), \( P_{\text{turb}} \equiv \langle \rho v_z^2 \rangle \) (vertical Reynolds stress, or turbulent pressure), and \( \Pi_{\text{mag}} \equiv \langle |\mathbf{B}|^2 - 2B_z^2 \rangle/(8\pi) = \langle B_x^2 + B_y^2 - B_z^2 \rangle/(8\pi) \) (vertical Maxwell stress, combining magnetic pressure and tension); here angle brackets denote horizontal averages at the midplane. In the case of isotropic velocity and magnetic fields, the turbulent and magnetic terms would be equivalent to arbitrary one-dimensional projections of the vector velocity and magnetic field. In reality, however, both the turbulent velocity and magnetic field are generally anisotropic.

Under the assumption that vertical dynamical equilibrium is satisfied (at least as a quasi-steady state), and also assuming the midplane pressure is much larger than the pressure above the mass-containing portion of the disk, the weight of the ISM calculated in Section 2.1 must be balanced by the midplane pressure, \( \mathcal{W} = P_{\text{tot}} \). The star formation rate per unit area may then be pre-
dicted as a function of large-scale ISM properties as

$$\Sigma_{\text{SFR}} = \frac{\mathcal{W}}{\gamma_{\text{tot}}} \approx \frac{P_{\text{DE}}}{\theta_{\text{tot}}}, \quad (13)$$

where Equation 6 and Equation 7 express $\mathcal{W}$ and $P_{\text{DE}}$, respectively, in terms of $\Sigma_{\text{gas}}, \rho_{\text{ad}}, \sigma_{\text{eff}}$. We expect $\gamma_{\text{tot}}$ to decrease slightly in regions of high $\Sigma_{\text{gas}}$. In particular, when $\Sigma_{\text{gas}}$ increases, $\gamma_{\text{th}}$ decreases due to greater radiation extinction, reducing heating; and $\gamma_{\text{turb}}$ decreases due to the reduction in SN momentum injection at higher ambient density. As a consequence, since higher $\Sigma_{\text{gas}}$ is associated with higher pressure, this is expected to yield a dependence of $\Sigma_{\text{SFR}}$ on $P_{\text{DE}}$ that is slightly superlinear. We note that in the above, we implicitly assume other parameters, including metallicity and the IMF, are held fixed; a lower metallicity or top-heavy IMF would tend to increase $\gamma_{\text{tot}}$.

With $P_{\text{tot}} = \sigma_{\text{eff}}^2 \Sigma_{\text{gas}}/(2h_{\text{gas}})$ for $\sigma_{\text{eff}}$ the effective total velocity dispersion and $h_{\text{gas}}$ the semi-thickness of the mass-containing disk, Equation 13 can also be expressed as

$$\Sigma_{\text{SFR}} = \frac{\sigma_{\text{eff}} \Sigma_{\text{gas}}}{\theta_{\text{tot}}} \equiv \frac{\epsilon_{\text{ver}} \Sigma_{\text{gas}}}{\gamma_{\text{ver}}} \quad (14)$$

Here, for convenience we have incorporated a factor of two in defining the vertical dynamical time $t_{\text{ver}} = 2h_{\text{gas}}/\sigma_{\text{eff}}$; an explicit formula for $t_{\text{ver}}$ (in terms of $\Sigma_{\text{gas}}, \rho_{\text{ad}}, \sigma_{\text{eff}}$) can be obtained by substituting for $h_{\text{gas}}$ from Equation 5. In this formulation, the ratio $\sigma_{\text{eff}}/\gamma_{\text{tot}} \equiv \epsilon_{\text{ver}}$ represents the star formation efficiency per vertical dynamical time; this efficiency $\epsilon_{\text{ver}} \sim 1\%$ since $\sigma_{\text{eff}} \sim 10 \text{ km s}^{-1}$ and $\gamma_{\text{tot}} \sim 1000 \text{ km s}^{-1}$. Star formation is also commonly quantified in terms of the gas depletion time, $t_{\text{dep}} \equiv \Sigma_{\text{gas}}/\Sigma_{\text{SFR}} = M_{\text{gas}}/M_\star$. Using Equation 14 and Equation 5,

$$t_{\text{dep}} = \frac{1}{\epsilon_{\text{ver}}} t_{\text{ver}} = \frac{\gamma_{\text{tot}}}{\sigma_{\text{eff}}} t_{\text{ver}} \approx \frac{\theta_{\text{tot}}}{2\gamma_{\text{tot}}} \frac{1}{\epsilon_{\text{ver}} \sigma_{\text{eff}}} \approx \frac{\pi G \Sigma_{\text{gas}} + 2\sigma_{\text{eff}} (2\rho_{\text{ad}})^{1/2}}{\epsilon_{\text{ver}} \sigma_{\text{eff}}}, \quad (15)$$

implying a depletion time two orders of magnitude longer than the vertical dynamical time. Because $\gamma_{\text{tot}}$ tends to decrease (modestly) and $\sigma_{\text{eff}}$ to increase (modestly) in galactic centers and other high-$\Sigma_{\text{gas}}$ environments, it is expected that the efficiency of star formation will (modestly) increase under these conditions.

Given $\gamma_{\text{tot}}$, quantitative predictions of star formation are obtained through Equation 13 (or equivalently Equation 15). Theoretical estimates of feedback yields in Section 2.2 predict $\gamma_{\text{tot}} \sim 1000 \text{ km s}^{-1}$; our numerical results for $\gamma_{\text{tot}}$ will be presented in Section 4.4.

We point out that Equation 13 is equivalent to Equation (22) of OML10. To make the connection, note that in OML10 $\alpha \equiv P_{\text{tot}}/P_{\text{th}}$. Taking the ratio of the right-hand side of Equation (18) in OML10 to $P_{\text{th}}$ to obtain the adopted value of $1/\gamma_{\text{th}}$ in that work, and substituting in $\alpha \rightarrow \gamma_{\text{tot}}/\gamma_{\text{th}}$ in Equation (22) of OML10, one can obtain Equation 13 above. We further note that for the reasons discussed at the end of Section 2.1 (see also Section 2.4, where we argue that the star formation timescale in gravitationally bound gas is likely to be quite small), Equation (22) of OML10 (rather than Equation 23) is expected to apply in general. Since the ratio $\gamma_{\text{tot}}/\gamma_{\text{th}}$ is not constant, Equation 13 combined with values of $\gamma_{\text{tot}}$ that calibrate for varying ISM conditions is preferred over Equation (22) of OML10.

2.4. Connection to Small-Scale Star Formation

It is worth remarking on the connection between the PRFM theory of star formation regulation that is motivated by maintaining the average conditions in the ISM, and the localized process of star formation in highly overdense structures. Since star formation involves collapse, it presumably takes place within individual gravitationally bound structures, which comprise a (small) fraction $f_{gb}$ of the total ISM mass. If the free-fall time for these structures is $t_{ff,gb}$ and their star formation efficiency per free-fall time is $\epsilon_{ff,gb}$, we can set $\epsilon_{ff,gb}$ equal to $\epsilon_{ff,gb} f_{gb} \Sigma_{gas}/t_{ff,gb}$ to obtain

$$\langle f_{gb} \rangle = \frac{\epsilon_{ff,gb}}{\epsilon_{ff,gb}} \frac{t_{ff,gb}}{t_{ff}}, \quad (16)$$

where the angle brackets denote a time average. The value of $\epsilon_{ff,gb}$ is set by small-scale gravoturbulent fragmentation processes (e.g. Dobbs et al. 2014; Padoan et al. 2014). Physically, we can understand Equation 16 as saying that given the small-scale $\epsilon_{ff,gb}$ together with the large-scale $\epsilon_{ver}$ (as derived from PRFM theory), the fraction of material that is contained in bound clouds will adjust to satisfy both constraints (in a time-averaged sense). Quantitatively, numerical simulations show that for gravitationally-bound systems, $\epsilon_{ff,gb} \gtrsim 0.1$ (e.g. Padoan et al. 2012; Raskutti et al. 2016; Kim et al. 2021b). Then with $\epsilon_{ver} \sim 0.01$ and $t_{ff,gb} < t_{ver}$, the expected $\langle f_{gb} \rangle \lesssim 0.1$, i.e. only a small fraction of the ISM will be in bound structures. The Mao et al. (2020) analysis of the TIGRESS solar neighborhood simulation indeed shows only a very small fraction of gas (0.01–0.1) is gravitationally bound.

In both the real ISM and in numerical simulations, star formation takes place within massive clouds that are overdense and overpressured with respect to average conditions at the ISM midplane. These massive clouds are typically observed as GMCs in CO lines, although at low metallicity CO emission may be weak, and if self-shielding is low enough they would primarily consist of
H I rather than $H_2$ (e.g. Bialy & Sternberg 2016; Gong et al. 2017). Analogously to Equation 16, we may write the expected time-averaged fraction of ISM material in GMCs as $f_{\text{GMC}} = (\varepsilon_{\text{ver}}/\varepsilon_{\text{GMC}})(t_{\text{H,GMC}}/t_{\text{ver}})$. While GMCs are self-gravitating, recent work suggests that especially in molecule-dominated regions of galaxies their typical virial parameters are closer to $\sim 4$ rather than $1$ (Sun et al. 2020b), and they would therefore have $\varepsilon_{\text{H,GMC}} \lesssim 0.01$ since the efficiency decreases exponentially with increasing virial parameter (e.g. Krumholz & McKee 2005; Padoan et al. 2012; Federrath & Klessen 2012; Kim et al. 2021b). The fraction of the ISM’s mass in GMCs can thus be significant.

We emphasize that the above $f_{\text{GMC}}$ is what must hold in a time-averaged sense, while individual GMCs continually form and disperse. Formation is subject to the level of turbulence in the diffuse ISM (primarily driven by SNe), while dispersal of the denser clouds that have formed massive stars is likely due to $H_\text{II}$ regions (see Section 1). The instantaneous $f_{\text{GMC}}$ may be above or below the equilibrium value. If $f_{\text{GMC}}$ and $f_{\text{gb}}$ are above the equilibrium level, the “excess” star formation and feedback that ensue will drive greater-than-equilibrium heating and momentum injection on large scales (after dispersing existing GMCs), temporarily limiting contraction of diffuse gas into denser, star-forming clouds. If $f_{\text{GMC}}$ and $f_{\text{gb}}$ are below equilibrium, the level of feedback will be low enough that new overdense clouds readily form. Within these GMCs, gravitationally bound regions will form and star formation will commence. The cyclic formation and dispersal of overdense and star-forming structures is evident in the time series correlation analysis of the TIGRESS solar neighborhood simulation by Mao et al. (2020) (see also Semenov et al. 2017; Orr et al. 2019; Moon et al. 2022).

2.5. Testing the theory

The remainder of this paper will largely focus on testing the key elements of the theory laid out in this section. The tests will make use of a set of numerical simulations that sample a range of parameters representing normal star-forming disk galaxies. We first describe the numerical methods employed in these simulations and the model parameter set (Section 3). Then we proceed to present results from our simulation analysis that (1) confirm the prediction of vertical dynamical equilibrium from Section 2.1, i.e. that midplane $P_{\text{tot}}$ values (in both hot and warm/cold phases) agree with $W$ and $P_{\text{DE}}$ (see Section 4.2); (2) characterize the individual pressure components, and compare to the theoretical predictions of Section 2.2 for thermal and turbulent feedback yields (see Section 4.3, Section 4.4); (3) for comparison to the prediction for large-scale star formation described in Section 2.3, quantify the relationship between midplane weight ($W$ or $P_{\text{DE}}$) or pressure ($P_{\text{tot}}$) and $\Sigma_{\text{SFR}}$ as measured in the simulations, and compare the simulation and theory results to observations (Section 4.5, Section 4.7).

3. NUMERICAL METHODS AND MODELS

3.1. TIGRESS Implementation

We use the TIGRESS numerical framework to simulate the three-phase ISM with self-consistent star formation and feedback (KO17); TIGRESS is built on the Athena finite-volume code for magnetohydrodynamics (Stone et al. 2008; Stone & Gardiner 2009). To focus on local patches within a differentially rotating galactic disk, we employ shearing-periodic boundary conditions in the local radial direction ($\hat{x}$), periodic boundary conditions in the local azimuthal direction ($\hat{y}$) (Stone & Gardiner 2010), and open boundary conditions in the vertical direction ($\hat{z}$). We use piecewise-linear reconstruction with the Roe Riemann solver. The gravity of the gas and of star particles (representing stellar clusters) is obtained via a fast Fourier transform solution of the Poisson equation (Gammie 2001; Koyama & Ostriker 2009), with the mass of each particle mapped onto the grid using a triangle-shaped cloud kernel (Hockney & Eastwood 1981).

Star particles are initially created as sinks for mass and momentum on the grid when gravitational collapse causes the numerical solution to be unresolved, adopting the criteria and methods of Gong & Ostriker (2013) with modifications as described in KO17, Kim et al. (2020a). Star particle positions and velocities are advanced using a symplectic kick-drift-kick leapfrog integrator for Hill’s equations (Quinn et al. 2010). Star particles may accrete further gas over time and merge with other sinks up until the point when the first SN occurs (typically after 3 - 4 Myr). Each star particle is surrounded by a $3^3$-cell control volume which is treated as ghost zones for actively-accreting particles, with the accretion rate determined by fluxes of mass and momentum returned by the Riemann solver at the surfaces of the control volume. The mass and momentum accreted in this way is shared between the sink particle and the cells in the control volume. After its first SN event, and up until its lifetime of 40 Myr, a star particle will no longer accrete or merge. Throughout their lifetimes, star particles are sources of FUV radiation.

We employ simple cooling functions suitable for the warm-cold ISM at $T < 10^{4.2}$ K (Koyama & Inutsuka 2002).
warm or cold gas is set to efficient from the photoelectric effect in cells containing correction perpendicular to the disk. The heating rate co-

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Table 1. TIGRESS Simulation Model Parameters

| Model | $$\Sigma_{\text{gas}}$$ | $$\Sigma_{\text{gas,0}}$$ | $$\Sigma_{\ast}$$ | $$\rho_{\ast}$$ | $$\rho_{\text{dm}}$$ | $$t_{\text{orb}}$$ | $$L_x, L_y$$ | $$L_z$$ | $$\Delta x$$ |
|-------|------------------|------------------|------------------|---------------|------------------|-----------------|-----------|--------|---------|
| R2    | 70               | 150              | 450              | 0.92          | 0.08             | 61.4            | 512       | 3584   | 2       |
| R4    | 30               | 50               | 208              | 0.42          | 0.024            | 114             | 512       | 3584   | 2       |
| R8    | 10               | 12               | 42               | 0.086         | 0.0064           | 219             | 1024      | 7168   | 4       |
| R16   | 2.5              | 2.5              | 1.71             | 0.0035        | 0.0014           | 518             | 2048      | 14336  | 8       |
| LGR2  | 70               | 150              | 110              | 0.11          | 0.015            | 123             | 512       | 3584   | 2       |
| LGR4  | 40               | 60               | 50               | 0.05          | 0.005            | 205             | 512       | 3584   | 2       |
| LGR8  | 11               | 12               | 10               | 0.01          | 0.0016           | 410             | 1024      | 7168   | 4       |

Note— Model parameters listed are: (1) model name, (2) target gas surface density, (3) initial gas surface density, (4) stellar surface density, (5) stellar midplane volume density, (6) dark matter midplane volume density, (7) galactocentric orbit time, (8) horizontal box dimensions, (9) vertical box dimension, (10) spatial resolution of simulation.

2002, see Kim et al. (2008) for form with correction of a typographical error), and for the ionized and hot ISM at T > 10^{4.2} K (Sutherland & Dopita 1993). The adopted cooling functions are appropriate for ISM gas at solar neighborhood abundances (we do not follow changes in metallicity in the current simulations).

The star cluster particle attributes that lead to feedback are assigned based on their mass and age using the STARBURST99 population synthesis package (Leitherer et al. 1999), assuming a Kroupa (2001) IMF and the Geneva evolutionary tracks for non-rotating, solar metallicity stars. The FUV intensity is taken to be

$$J_{\text{FUV}}(t) = \frac{\Sigma_{\text{FUV}}(t)}{4\pi} f_\tau,$$

with the age-dependent luminosity-to-mass ratio in FUV from STARBURST99 used to obtain the FUV luminosity per unit area $$\Sigma_{\text{FUV}}$$ (averaged over the whole domain) by summing over star particles. The (time-dependent) factor $$f_\tau$$ gives in Equation 11 takes into account diminution in an approximate manner, based on the solution of the equation of radiation transfer in a slab for uniform emissivity (OML10), with $$\tau_\perp(t) = \kappa_{\text{FUV}} \Sigma_{\text{gas}}(t)$$ the mean optical depth to FUV in the direction perpendicular to the disk. The heating rate coefficient from the photoelectric effect in cells containing warm or cold gas is set to

$$\Gamma = \Gamma_0 \left( \frac{J_{\text{FUV}}}{J_{\text{FUV,0}}} + 0.0024 \right),$$

where $$\Gamma_0 = 2 \times 10^{-26} \text{ erg s}^{-1}$$ and $$J_{\text{FUV,0}} = 6.8 L_\odot \text{ pc}^{-2}/(4\pi) = 2.2 \times 10^{-4} \text{ erg s}^{-1}\text{cm}^{-2}\text{sr}^{-1}$$ are adopted as reference values for the heating rate coefficient and mean FUV intensity in the solar neighborhood. Photoelectric heating is not applied to gas at T > 10^{5} K. As we do not explicitly track ionization in warm and cold gas in the present simulations, our photoelectric heating efficiency is effectively constant, rather than depending on a grain charging parameter that is sensitive to $$n_e$$ (Wolfire et al. 1995, 2003). We note that with our adopted heating and cooling functions, the geometric mean $$P_{\text{two}} = (P_{\text{max,warm}} P_{\text{min,cold}})^{1/2}$$ between the maximum warm and minimum cold pressure in thermal equilibrium is given by

$$P_{\text{two}}/k_B = 3.1 \times 10^3 \text{ cm}^{-3} \text{ K} \left( \frac{J_{\text{FUV}}}{J_{\text{FUV,0}}} + 0.0024 \right).$$

Thus, if $$P_{\text{th}} = P_{\text{two}}$$ the value of $$\Upsilon_{\text{th}}$$ would be 172$$f_\tau/f_{\tau,\odot}$$ km s^{-1} with our adopted heating and cooling functions, which is slightly lower than it would be with the Wolfire et al. (2003) heating and cooling functions (see Equation 12). The above assumes $$J_{\text{FUV}}/J_{\text{FUV,0}} \gg 1$$, as is generally the case.

The treatment of SNe is as described in KO17, where full details and tests of the method are presented. The SN event rate from any given star particle is set by its mass and age, and we allow for runaways by ejecting a massless test particle with 50% probability for each event. We turn off runaways in the R2 model, however, for the sake of computational efficiency. Different treatments of SN events are applied depending on the density in ambient gas, which is used to compute the ratio $$R_M$$ between the mass in the feedback region and the mass that the remnant would have (at that density).
when it becomes radiative, calibrated by Kim & Ostriker (2015b) to be $M_{\text{sf}} = 1540 M_\odot (n_\text{H}/\text{cm}^{-3})^{-0.33}$. For the majority of cases, where a feedback region of radius at least $R_{\text{min}} = 3\Delta x$ has $R_M < 1$, the Sedov-Taylor stage is resolved and we deposit $10^{51}$ ergs of energy within the feedback region (72% thermal and 28% kinetic). For a minority of cases, the ambient density may be high enough so that $R_M > 1$ even for the smallest allowed feedback region radius, in which case the Sedov-Taylor stage is unresolved and we deposit momentum equal to $p_{\text{final}} = 2.8 \times 10^5 M_\odot \text{ km s}^{-1} (n_\text{H}/\text{cm}^{-3})^{-0.17}$ (calibrated by Kim & Ostriker 2015b) within the feedback region. For the simulations presented here, more than 90% of the SN are well resolved, with $R_M < 0.1$, and are treated with energy rather than momentum deposition.

### 3.2. Model Parameters

The MHD simulations we analyze here are the same as those used for analysis of outflow properties in Kim et al. (2020a,b). Physical and numerical parameters for the seven simulations are listed in Table 1. These simulations cover a range of background states for the gas and gravitational potential (stellar and dark matter) that would be encountered in nearby disk galaxies. Although galactocentric radius does not directly enter in the equations for a local shearing-box model, galaxies generally have gas and stellar surface densities that decline with increasing radius, so we label our models based on a nominal galactocentric radius (in kpc). There are two sets of models, R2 to R16 (nominal radius $R = 2$ to 16 kpc with higher external gravity), and LGR2 to LGR8 ($R = 2$ to 8 kpc with lower external gravity).

Table 1 lists the (constant in time) parameters for the stellar disk and dark matter halo. The scale height of the stellar disk is $z_\ast = 245$ pc (R2, R4, R8, R16) or $z_\ast = 500$ pc (LGR2, LGR4, LGR8), with stellar surface density $\Sigma_\ast$ related to midplane stellar volume density by $\rho_\ast = \Sigma_\ast/(2z_\ast)$ based on Equation 6 of KO17. The Table lists both the initial gas surface density $\Sigma_{\text{gas},0}$ at the time the simulation is begun, and the target surface density $\Sigma_{\text{gas}}$ after a transient stage of evolution; ISM and star formation properties are measured when the mean gas surface density of the disk is near this target value. The orbit time for the simulation domain about the galactic center is listed, together with the physical box size. The number of zones in the domain is $N_x \times N_y \times N_z = 256 \times 256 \times 1792$ for all models.

All models are initiated with a horizontal magnetic field aligned in the $\hat{y}$ (i.e. azimuthal) direction, with plasma $\beta = 8\pi P_{\text{th}}/B^2$ everywhere equal to $\beta = 10$ for all models except R2, which has initial $\beta = 2$. We set the initial magnetic field higher (lower $\beta$) for this model because the orbital time is significantly shorter for model R2 than other models, so that the background magnetic field would not have time to grow.

The R8 model is the most similar in its parameters to conditions in the solar neighborhood. A simulation with the same parameters (but slightly different treatment of star particles and initial turbulence) was previously presented as the fiducial TIGRESS model in KO17, with outflow properties analyzed in Kim & Ostriker (2018); Vijayan et al. (2020). The LGR4 model is the most similar to the mean (weighted by molecular mass) properties in the PHANGS survey of nearby star-forming galaxies.

We note that in addition to the standard runs listed in Table 1, additional simulations with the same physical parameters but different numerical resolution and computational domain size have been run to confirm convergence; see Section 4 in KO17 and Appendix A in Kim et al. (2020a).
4. NUMERICAL RESULTS

4.1. Temporal evolution and ISM structure

Evolution of the gas surface density $\Sigma_{\text{gas}} \equiv M_{\text{gas}}/(L_L L_y)$ for all TIGRESS models is shown in Figure 1. For each model, the temporal range used for computing the ISM and star formation properties we tabulate is marked as a heavy curve. This interval is chosen to have a range within $\sim 10\%$ of the target $\Sigma_{\text{gas}}$, so that tabulated results represent well-defined conditions in the disk. The median value of $\Sigma_{\text{gas}}$ over this sampling interval for each model is listed in Table 2. Evolution of the star formation rate surface density $\Sigma_{\text{SFR}} \equiv \dot{M}_*/(L_L L_y)$ for all models is shown in Figure 2, with the median $\Sigma_{\text{SFR}}$ over the sampling interval as well as the depletion time defined by $t_{\text{dep}} = \Sigma_{\text{gas}}/\Sigma_{\text{SFR}}$ listed in Table 2.

As previously discussed in Mao et al. (2020); Vijayan et al. (2020); Kim et al. (2020a), the values of $\Sigma_{\text{SFR}}$ fluctuate in time as the gas cycles between phases where there is a relatively large range of density and star formation, and phases where feedback has dispersed much of the dense gas, reducing $\Sigma_{\text{SFR}}$. Feedback also causes the gas scale height $H$ to oscillate in time (Kim et al. 2020a), but for both $\Sigma_{\text{SFR}}$ and $H$ (and other box-averaged variables) there are well-defined quasi-steady mean values subsequent to the initial transients. Table 2 includes the median value of the root mean square scale height $H \equiv (\sum_{ijk} \rho z^2 / \sum_{ijk} \rho)^{1/2}$ for the warm-cold ($T < 2 \times 10^4$ K) “two-phase” gas. Hereafter we shall use the subscript “2p” to denote quantities computed based on selecting only zones with gas in this warm-cold range.

Contributions to the total pressure in the TIGRESS simulations include thermal, turbulent, and magnetic terms. For the purpose of considering overall force balance in the direction perpendicular to the disk, the relevant component pressures are horizontal averages at the midplane of the corresponding stress terms in the vertical momentum equation. The individual terms are: $P_{\text{th}} \equiv \rho c_{\text{th}}^2$ (thermal pressure), $P_{\text{turb}} \equiv \rho c_{\text{turb}}^2$ (vertical Reynolds stress, i.e. effective turbulent pressure), and $\Pi_{\text{mag}} \equiv (\langle B^2 \rangle - 2B_z^2)/(8\pi) = (B_x^2 + B_y^2 - B_z^2)/(8\pi)$ (vertical Maxwell stress, combining magnetic pressure and tension). Figure 3 shows the temporal evolution for all models of these individual terms in the two-phase gas at the midplane, as well as the total of these terms, $P_{\text{tot,2p}} = P_{\text{th,2p}} + P_{\text{turb,2p}} + \Pi_{\text{mag,2p}}$. The evolution of the total pressure $P_{\text{tot,hot}}$ for the hot gas ($T > 5 \times 10^5$ K) is also shown, together with the evolution of the commonly-used estimator for the midplane pressure based on dynamical equilibrium, $P_{\text{DE}}$ (Equation 7). For each model, median values over the sampling interval of individual pressure components (2p only) and totals (2p and hot) are listed in Table 2. In Table 2 and elsewhere, pressure values with subscript “2p” or “hot” denote horizontal averages made at the midplane in the simulations. As we shall discuss in more detail below, the total pressure for the warm-cold and hot phases are comparable, and while these fluctuate in time they remain close to the expected dynamical equilibrium pressure, $P_{\text{DE}}$.

In any zone, the effective vertical velocity dispersion is computed using $\sigma_{\text{eff}}^2 \equiv P_{\text{tot}}/\rho$, and we define the mass-weighted average of the effective vertical velocity dispersion as $\sigma_{\text{eff}} \equiv (\sum_{ijk} P_{\text{tot}} / \sum_{ijk} \rho)^{1/2}$. Table 2 lists the time average of this RMS velocity dispersion computed from all (not just midplane) two-phase gas. We use this $\sigma_{\text{eff,2p}}$ in $P_{\text{DE}}$. Table 2 also lists, for the two-phase gas, the median midplane values of the gas hydrogen number density, $n_{\text{H,2p}} \equiv \rho_{\text{2p}}/(1.4\mu_{\text{H}})$.

### Table 2. Measured Star Formation and ISM Properties

| Model | $\Sigma_{\text{gas}}$ | $\Sigma_{\text{SFR}}$ | $t_{\text{dep}}$ | $P_{\text{DE}}$ | $P_{\text{tot,hot}}$ | $P_{\text{tot,2p}}$ | $P_{\text{th,2p}}$ | $P_{\text{turb,2p}}$ | $\Pi_{\text{mag,2p}}$ | $n_{\text{H,2p}}$ | $\sigma_{\text{eff,2p}}$ | $H_{2p}$ |
|-------|-----------------|-----------------|-------------|-------------|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|---------|
| R2    | 7.09            | 1.10            | 6.45 $\times$ 10$^4$ | 2.00 $\times$ 10$^6$ | 1.36 $\times$ 10$^6$ | 1.92 $\times$ 10$^6$ | 1.13 $\times$ 10$^5$ | 1.26 $\times$ 10$^6$ | 5.37 $\times$ 10$^5$ | 26.1 | 43.4 | 282 |
| R4    | 3.02            | 5.37 $\times$ 10$^{-2}$ | 5.63 $\times$ 10$^2$ | 3.57 $\times$ 10$^5$ | 1.44 $\times$ 10$^5$ | 2.41 $\times$ 10$^5$ | 1.76 $\times$ 10$^4$ | 1.95 $\times$ 10$^5$ | 2.22 $\times$ 10$^4$ | 2.52 | 30.3 | 294 |
| R8    | 9.87            | 2.67 $\times$ 10$^{-3}$ | 3.70 $\times$ 10$^3$ | 2.34 $\times$ 10$^4$ | 1.70 $\times$ 10$^4$ | 1.91 $\times$ 10$^4$ | 5.02 $\times$ 10$^3$ | 5.71 $\times$ 10$^3$ | 7.86 $\times$ 10$^3$ | 1.18 | 13.9 | 351 |
| R16   | 2.45            | 6.21 $\times$ 10$^{-5}$ | 3.94 $\times$ 10$^4$ | 1.03 $\times$ 10$^3$ | 1.03 $\times$ 10$^3$ | 7.77 $\times$ 10$^2$ | 3.39 $\times$ 10$^2$ | 1.88 $\times$ 10$^2$ | 1.67 $\times$ 10$^2$ | 0.0919 | 10.7 | 679 |
| LGR2  | 65.9            | 1.17 $\times$ 10$^{-1}$ | 5.65 $\times$ 10$^2$ | 9.14 $\times$ 10$^5$ | 6.66 $\times$ 10$^5$ | 1.01 $\times$ 10$^6$ | 6.60 $\times$ 10$^4$ | 6.41 $\times$ 10$^5$ | 2.77 $\times$ 10$^5$ | 15.3 | 34.3 | 460 |
| LGR4  | 39.6            | 5.41 $\times$ 10$^{-2}$ | 7.32 $\times$ 10$^2$ | 1.80 $\times$ 10$^5$ | 1.01 $\times$ 10$^5$ | 1.38 $\times$ 10$^5$ | 1.34 $\times$ 10$^4$ | 1.01 $\times$ 10$^5$ | 1.92 $\times$ 10$^4$ | 3.49 | 19.4 | 307 |
| LGR8  | 11.2            | 2.16 $\times$ 10$^{-3}$ | 5.17 $\times$ 10$^4$ | 1.09 $\times$ 10$^4$ | 7.39 $\times$ 10$^3$ | 9.65 $\times$ 10$^3$ | 2.36 $\times$ 10$^3$ | 4.78 $\times$ 10$^3$ | 1.91 $\times$ 10$^3$ | 0.738 | 11.3 | 438 |

Note: Numerically measured quantities in each model. Reported values are medians of the distribution for the sampling period indicated in Figure 1. Pressures and density are based on horizontal averages at the midplane. The “2p” subscript indicates that only warm-cold gas ($T < 2 \times 10^4$ K) is included in the measurement. Effective velocity dispersion includes all pressure components (see text).
Figure 3. Evolution of midplane pressures in all models. Solid curves show total pressure and component pressures (thermal, turbulent, magnetic) for the two-phase (warm-cold) gas, dashed curves show total pressure for hot gas, and dotted curves show estimated dynamical equilibrium pressure. For reference, $\Sigma_{SFR}$ is also shown, in units $10^{-5} M_\odot$ pc$^{-2}$ Myr$^{-1}$ (gray shading).

Snapshots of density and temperature slices (Figure 4) show that the gas is highly structured. The warm and cold gas is concentrated in the midplane, but SNe drive fountain flows in the warm gas extending to several kpc, together with hot winds that escape from the disk. Temporally-averaged vertical profiles of density (total and individual phases) as well as pressure (total and individual components) are smooth (see Kim & Ostriker 2018; Vijayan et al. 2020; Kado-Fong et al. 2020; Kim et al. 2020a), but the instantaneous snapshots show a highly inhomogeneous medium, and except for model R16 there is comparable volume near the midplane occupied by hot and warm gas (see KO17).
While there are orders of magnitude difference between density and temperature of the different phases, pressures are much more similar. Figure 5 shows that hot bubbles and outflow “chimneys” have thermal pressure slightly larger than that of the warm-cold gas. Turbulent pressure is similar in magnitude between the hot gas and warm/cold gas, although the latter has fluctuations on smaller spatial scales. The magnetic pressure $P_{\text{mag}} \equiv |B|^2/(8\pi)$ is, however, quite small within the volume occupied by the low-density hot gas.

4.2. Pressure equilibrium

As noted above, the pressures in warm/cold gas and in hot gas for individual snapshots are generally similar. In more detail, from Figure 3 the hot gas and two-phase gas do not track each other’s fluctuations in the midplane average pressure (hot gas pressure is largest during SN feedback episodes; two-phase gas pressure variations are more complex), but nevertheless the median value of the total vertical pressure at the midplane is only slightly lower ($\sim 30\%$) for hot gas than for warm/cold gas (see Table 2). Figure 6 shows midplane values of the total pressure in the two-phase and the hot gas at intervals of 1 Myr, along with median values over the sampling interval. From a theoretical point of view, in any quasi-steady non-self-gravitating system the time-averaged pressures must be similar in different thermal phases, because otherwise the component with higher pressure would have expanded to occupy a larger (average) fraction of the volume. Of course, any component that is strongly self-gravitating would be expected to have significantly larger pressure, but this is not the case for the two-phase gas overall; only a small fraction of the (cold, dense) gas in the TIGRESS simulations is gravitationally bound (Mao et al. 2020).

In the limit of very short cooling time compared to the dynamical time, the density and thermal pressure distribution would follow the thermal equilibrium curve obtained by balancing radiative heating and cooling, $n\Lambda = \Gamma$, for $\Gamma \propto J_{\text{FUV}} \propto \Sigma_{\text{SFR}}$ when photoelectric heating dominates (see Equation 18). Figure 7 shows, for model R8 (top row) and model LGR4 (bottom row) the joint pdfs of density and thermal pressure at $t = t_{\text{orb}}$. The mass-weighted distributions (right panels) show that the cold gas (which has the shortest cooling time) is very close to the thermal equilibrium curve set by the instantaneous heating rate. At higher (lower) heating rate, the equilibrium curve would shift diagonally upward/rightward (downward/leftward) with the same characteristic warm gas temperature, which is set by Ly$\alpha$ cooling.

For the volume-weighted distributions (left panels), the hot gas locus is evident at high temperature. Since the cooling time in the hot gas is very long, its pressure adjusts only by dynamical means, and at any given time can be either higher or lower than that of the warm gas (see also Figure 3); in the particular snapshots shown, hot gas is slightly overpressured for LGR4 and slightly underpressured for R8.

In addition to thermal equilibrium, a second aspect of equilibrium involving pressure in galactic disks is dynamical equilibrium. As described in Section 2, vertical dynamical equilibrium is satisfied if the difference between midplane pressure and the pressure above the main gas layer is equal to the weight of that gas in the total gravitational potential, $W$. 

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**Figure 4.** Vertical slice showing hydrogen number density $n_H$ and gas temperature $T$ in all models at $t = t_{\text{orb}}$. Axes are in units of the respective scale height $H_{2p}$ for each model (see Column (13) in Table 2).
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Figure 5. Vertical slices showing thermal pressure $P_{\text{th}}$ (top), turbulent pressure $P_{\text{turb}} \equiv \rho v_z^2$ (middle), and magnetic pressure $P_{\text{mag}} \equiv |\mathbf{B}|^2/(8\pi)$ (middle) in all models (labeled at top) at $t = t_{\text{orb}}$. Axes are in units of the respective scale height for each model.

Figure 6. Total vertical pressure $P_{\text{tot}} \equiv P_{\text{th}} + P_{\text{turb}} + P_{\text{mag}}$ in hot gas vs. two-phase gas for all models. Midplane-averaged values at intervals of 1 Myr are shown with individual small circles, and medians over the temporal domain shown in Figure 1 are shown as large points with 25th and 75th percentiles indicated. For reference the dotted line shows the identity $P_{\text{tot,hot}} = P_{\text{tot},2p}$.

In our simulations, we can measure the weight at any point $(x, y)$ in the midplane by vertical integration, following the definition in Equation 1, and average horizontally. This directly-measured weight can be compared to the commonly-adopted estimator $P_{\text{DE}}$ for the weight given in Equation 7, using $\sigma_{\text{eff},2p}$. When we make this comparison the weight is for the two-phase gas; in the simulation it is almost the same for all gas since the hot gas mass is quite low. Figure 8 shows that $P_{\text{DE}}$ is indeed an almost linear estimator for the ISM weight,

$$\log(W_{2p}/k_B) = 1.03 \log(P_{\text{DE}}/k_B) - 0.267,$$

although for most models $P_{\text{DE}}$ is $\sim 30\%$ higher than $W$ (the difference is greater for model R16, driving the larger offset in Equation 20). Here and in other relations based on fits to the simulations, pressures and weights are in units of $k_B \text{ cm}^{-3} \text{ K}$, i.e. we report $P/k_B$ or $W/k_B$ in cgs units. The result of Equation 20 demonstrates the validity of adopting $P_{\text{DE}}$ as a simple estimator, although at the same time shows that it cannot be expected to recover the true weight to better than a few tens of percent.

With our simulations, we can compare the midplane value of the total pressure in the two-phase gas to either the true ISM weight $W$ or the estimator $P_{\text{DE}}$, with the
results shown in Figure 9. Best-fit relations,
\[
\log(P_{\text{tot},2p}/k_B) = 0.99 \log(W_{2p}/k_B) + 0.083 \quad (21a)
\]
\[
\log(P_{\text{tot},2p}/k_B) = 1.03 \log(P_{\text{DE},2p}/k_B) - 0.199, \quad (21b)
\]
show that vertical dynamical equilibrium is satisfied within a few tens of percent.

4.3. Pressure components

In the solar neighborhood of the Milky Way, all pressure components are observed to be roughly similar (e.g. Boulares & Cox 1990), and Figure 3 shows that this is also true for our R8 model, which adopts similar background conditions of gas surface density and external gravitational potential to the solar neighborhood. Since the momentum input per SN is relatively insensitive to the ambient density, we expect from Equation 9 that the turbulent pressure will scale nearly linearly with $\Sigma_{\text{SFR}}$. With the higher shielding (lower $f_\tau$) in regions of higher surface density (implemented in TIGRESS via Equation 11), the radiative heating rate for a given SFR is reduced, and this is expected to decrease the thermal pressure relative to the turbulent pressure. With magnetic pressure driven by the dynamo, it is not expected to exceed the turbulent pressure, so that overall $P_{\text{turb},2p}$ will be the largest single contributor to $P_{\text{tot},2p}$. As a consequence, we expect $P_{\text{th},2p}/P_{\text{tot},2p}$ will decrease and $P_{\text{turb},2p}/P_{\text{tot},2p}$ will increase slightly for the models of higher surface density, which correspond to higher pressure. In Figure 10 we show for all models the fractional contributions to the total midplane pressure as a function of $P_{\text{DE}}$, along with best-fit power law relations for the thermal and turbulent fractions:
\[
\log\left(\frac{P_{\text{th},2p}}{P_{\text{tot},2p}}\right) = -0.275 \log(P_{\text{DE},2p}/k_B) + 0.517 \quad (22a)
\]
\[
\log\left(\frac{P_{\text{turb},2p}}{P_{\text{tot},2p}}\right) = 0.129 \log(P_{\text{DE},2p}/k_B) - 0.918. \quad (22b)
\]
The expected trends in the fractional contributions to the median pressure are evident in these fits. In addition, Figure 3 makes clear the overall decrease in the time-dependent $P_{\text{th},2p}/P_{\text{tot},2p}$ from R8 to R4 to R2, or LGR8 to LGR4 to LGR2.

Magnetic fields grow due to the combination of sheared rotation, turbulence, and buoyancy in our simulations. Figure 10 shows, for all models, the separate contribution to the total pressure from the mean mag-

![Figure 7](image-url)
Figure 8. Measured weight of warm-cold (two-phase) gas, $W_{2p}$, vs. estimated dynamical equilibrium weight, $P_{DE}$, for all models. Individual points at intervals 1 Myr are plotted for each model, as well as medians with 25th and 75th percentiles indicated. For reference the dotted line shows the identity $W_{2p} = P_{DE}$ while the solid line shows the best fit (see text).

Magnetic component $\Pi_{T,2p}$, which is computed using the mean magnetic field based on horizontal averages, and the fluctuation component $\Pi_{B,2p}$, which is computed using the difference between the horizontally-averaged magnetic field and the total. As with other quantities, the “$2p$” subscript indicates the average includes only zones containing warm and cold gas. In general, we find that the magnetic field requires some time to grow, such that it has not necessarily reached saturation for our simulations. The exception is model R8, which has a longer duration (in orbit times) than other models, showing saturation after $t > 2\tau_{\text{orb}}$ (see Kim et al. 2019a). As previously found in the simulations of Kim & Osstriker (2015a) with sheared rotation, turbulence, and buoyancy but only two-phase gas, the level of turbulent magnetic pressure remains below the level of the turbulent kinetic pressure in our simulations. Since there are no clear relationships evident for relative importance of magnetic pressure in different models (potentially for numerical reasons, if magnetic growth is not saturated), we do not attempt to obtain fits for magnetic contributions.

We have demonstrated above that the ISM pressure is regulated in disk systems as it obeys certain “rules” that follow from conservation laws of energy and momentum, in particular the thermal pressure must be consistent with a balance between heating and cooling (for short cooling time), and the total pressure consistent with balancing the weight of the gas (in an average sense), with comparable total midplane pressures in hot and two-phase gas. At the same time, pressure responds to the star formation rate. The input FUV flux $\Sigma_{\text{FUV}}$ responsible for heating atomic gas scales linearly with $\Sigma_{\text{SFR}}$, but as noted above the increasing attenuation of FUV in higher surface density regions (smaller $f_\tau$ in Equation 11) implies a relative reduction in the photoelectric heating rate. With the equilibrium thermal pressure following $P_{th}/k_B = T T/\Lambda$ for $T/\Lambda$ roughly constant and $\Gamma \propto \Sigma_{\text{SFR}} f_\tau$, the thermal pressure is expected to scale sublinearly with $\Sigma_{\text{SFR}}$. As discussed in Section 2, since the momentum injection per SN $p_*$ is relatively insensitive to ambient conditions, the turbulent pressure driven by SNe is expected to scale approximately linearly with the rate of SNe per unit area per unit time in the disk, leading to an approximately linear relation between $P_{\text{turb}}$ and $\Sigma_{\text{SFR}}$ (Equation 9). Since the turbulent pressure is the largest single component of $P_{\text{tot}}$, the total pressure is also expected to be roughly linear in $\Sigma_{\text{SFR}}$.

Figure 11 shows the relation between the measured midplane $P_{\text{th},2p}$, $P_{\text{turb},2p}$, and $P_{\text{tot},2p}$ with $\Sigma_{\text{SFR}}$. The best fit power law relations are:

$$\log(P_{\text{th},2p}/k_B) = 0.603 \log(\Sigma_{\text{SFR}}) + 4.99 \quad (23a)$$
$$\log(P_{\text{turb},2p}/k_B) = 0.960 \log(\Sigma_{\text{SFR}}) + 6.17 \quad (23b)$$
$$\log(P_{\text{tot},2p}/k_B) = 0.840 \log(\Sigma_{\text{SFR}}) + 6.26 \quad (23c)$$

with $\Sigma_{\text{SFR}}$ in $M_\odot$ pc$^{-2}$ Myr$^{-1}$; we overlay these fits as solid lines.

Given the mean value of $\Sigma_{\text{SFR}}$ and attenuation factor, we can obtain $J_{\text{FUV}}$, and for our adopted heating and cooling functions this then leads to a characteristic value for the equilibrium thermal pressure $P_{\text{two}} = (P_{\text{max, warm}} P_{\text{min, cold}})^{1/2}$ given in Equation 19. For each model, we show these reference equilibrium values as black triangles in Figure 11. The reference values are slightly above the mean values in the simulation for the models with higher $\Sigma_{\text{gas}}$ and $\Sigma_{\text{SFR}}$, which is not surprising given the shorter cooling times in these high density models. Similarly, the black triangles in the turbulent pressure panel show the prediction of Equation 9 assuming a characteristic value $p_* = 10^6 M_\odot$ km s$^{-1}$ for each SN (with the value $m_* = 95.5 M_\odot$ adopted by TIGRESS). The numerical results from TIGRESS follow this prediction quite well overall, with values consistent with mean $p_* \approx 1.3 \times 10^6 M_\odot$ km s$^{-1}$. The implied values of $p_*$ increase slightly at low $\Sigma_{\text{SFR}}$ presumably due to the slight increase in $p_*$ in conditions of lower ambient density (as predicted by theory and idealized sim-
Figure 9. Measured total midplane pressure of warm-cold (two-phase) gas, $P_{\text{tot,2p}}$, vs. measured and estimated weight of gas, $W_{\text{2p}}$, and $P_{\text{DE}}$, for all models. Individual points at intervals 1 Myr are plotted for each model, as well as medians with 25th and 75th percentiles indicated, with dotted lines showing the identity relation in each panel and the solid lines showing the best fits (see text).

Figure 10. Fractional contributions to the total pressure in warm-cold ("2p") gas from thermal, turbulent, and magnetic (mean and perturbation) terms, as a function of $P_{\text{DE}}$. Individual points are plotted for each model at intervals 1 Myr, as well as medians with 25th and 75th percentiles indicated. Best fit relations are also shown for the thermal and turbulent fractions (see text).

The quantitative modulation of individual pressure components by feedback can be characterized by the yield parameters $\Upsilon \equiv P/\Sigma_{\text{SFR}}$. Since pressure has units of momentum/time/area and $\Sigma_{\text{SFR}}$ has units of mass/time/area, the natural unit for the feedback yield is a velocity. Due to the shielding of radiation in regions...
Figure 11. Thermal, turbulent, and total pressure as a function of $\Sigma_{\text{SFR}}$. Here, $\Sigma_{\text{SFR}}$ for each point is based on an average over the past 40 Myr in the simulation, and the pressures represent horizontal midplane averages. Median values and 25th and 75th percentiles are also shown for each model. Solid lines show best-fit power laws, dotted lines show results from KOK13, and black triangles show the analytic predictions for thermal and turbulent pressure (see text).

Figure 12. Thermal, turbulent, and total feedback yield as a function of $P_{\text{DE}}$. Median values and 25th and 75th percentiles are also shown for each model. Solid lines show best-fit power laws (see text).

of high surface density, $Y_{\text{th}}$ decreases with increasing $\Sigma_{\text{SFR}}$, whereas $Y_{\text{turb}}$ is relatively flat because SN momentum input is insensitive to environment. Since turbulent and magnetic terms are at least as large as thermal terms and are relatively insensitive to ambient conditions, the total yield only decreases slightly at higher star formation rate. Fits to the TIGRESS simulations give

$$Y_{\text{th}} = 110 \text{ km s}^{-1} \left( \frac{\Sigma_{\text{SFR}}}{0.01 \text{ M}_\odot \text{ pc}^{-2} \text{ Myr}^{-1}} \right)^{-0.4} \quad (24a)$$

$$Y_{\text{turb}} = 330 \text{ km s}^{-1} \left( \frac{\Sigma_{\text{SFR}}}{0.01 \text{ M}_\odot \text{ pc}^{-2} \text{ Myr}^{-1}} \right)^{-0.05} \quad (24b)$$

$$Y_{\text{tot}} = 740 \text{ km s}^{-1} \left( \frac{\Sigma_{\text{SFR}}}{0.01 \text{ M}_\odot \text{ pc}^{-2} \text{ Myr}^{-1}} \right)^{-0.18} \quad (24c)$$

where for convenience we normalize here relative to typical areal star formation rates in nearby galaxies. Comparing Equation 24b to Equation 10 implies that the effective value of momentum/mass from SN injection in TIGRESS is $(p_*/m_*)_{\text{eff}} = 1300 \text{ km s}^{-1} (\Sigma_{\text{SFR}}/0.01 \text{ M}_\odot \text{ pc}^{-2} \text{ Myr}^{-1})^{-0.05}$.

We may compare the yields for TIGRESS to the corresponding relations reported in KOK13: $Y_{\text{th}} = 200 \text{ km s}^{-1} (\Sigma_{\text{SFR}}/0.01 \text{ M}_\odot \text{ pc}^{-2} \text{ Myr}^{-1})^{-0.14}$, $Y_{\text{turb}} = 700 \text{ km s}^{-1} (\Sigma_{\text{SFR}}/0.01 \text{ M}_\odot \text{ pc}^{-2} \text{ Myr}^{-1})^{-0.11}$, and $Y_{\text{tot}} = 770 \text{ km s}^{-1} (\Sigma_{\text{SFR}}/0.01 \text{ M}_\odot \text{ pc}^{-2} \text{ Myr}^{-1})^{-0.15}$; we have converted units for most direct comparison (in addition to different units for yield, KOK13 use the notation $\eta$ instead of $Y$). The weaker dependence of $Y_{\text{th}}$ in KOK13 is because shielding of radiation was not included, while the larger coefficient for $Y_{\text{turb}}$ is because a (constant) value $p_* = 3 \times 10^5 \text{ M}_\odot \text{ km s}^{-1}$ was adopted, which is larger than $(p_*/m_*)_{\text{eff}}$ in TIGRESS. In spite of these differences, the relation between $Y_{\text{tot}}$ and $\Sigma_{\text{SFR}}$...
from TIGRESS is almost the same as that from KOK13, due to the inclusion of magnetic fields in TIGRESS. We note that while the KOK13 models were unmagnetized, the Kim & Ostriker (2015a) MHD simulations for a model representing the solar neighborhood (similar to R8) found similar thermal and turbulent pressure to those in KOK13, with $\Pi_{\text{mag}} \sim 0.7P_{\text{turb}}$. Thus, inclusion of magnetic fields happens to compensate for the lower $(p_\ast/m_\ast)_{\text{eff}}$ from SNe in TIGRESS compared to the value imposed in KOK13.

Since $P_{\text{DE}}$ can be obtained relatively easily from observable large-scale parameters in star-forming galaxies (see Equation 7), it is also useful to see how the feedback yields depend on $P_{\text{DE}}$. Figure 12 shows both individual points at 1 Myr intervals, and medians from the distribution over the sampling interval. Here, we use a trailing 40 Myr average for $\Sigma_{\text{SFR}}$ in the denominator of each computed value of $\Upsilon$. Since midplane pressures have considerable fluctuations, there is significant scatter in the $\Upsilon$ values for each model, but clear correlations are evident. The best-fit power law relations shown as solid lines in Figure 12 are:

$$
\log(\Upsilon_{\text{th}}) = -0.506 \log(P_{\text{DE}}/k_B) + 4.45 \quad (25a)
$$

$$
\log(\Upsilon_{\text{turb}}) = -0.060 \log(P_{\text{DE}}/k_B) + 2.81 \quad (25b)
$$

$$
\log(\Upsilon_{\text{tot}}) = -0.212 \log(P_{\text{DE}}/k_B) + 3.86 \quad (25c)
$$

where $P_{\text{DE}}/k_B$ is in cm$^{-3}$ K as before, and $\Upsilon$ is in km s$^{-1}$. Evidently, $\Upsilon_{\text{tot}}$ is expected to range between a few thousand km s$^{-1}$ in low-pressure, far outer-galaxy regions (or low surface brightness diffuse disks or dwarfs) to a few hundred km s$^{-1}$ in the high pressure regions surrounding galactic centers (or starburst regions elsewhere). This decrease in $\Upsilon$ reflects the increasing efficiency of radiative losses in higher pressure interstellar environments to the energy that has been deposited in them by stars.

4.5. Star formation-pressure relations

The two main theoretical principles invoked by OML10 and OS11 are (1) the star-forming ISM constantly adjusts to keep its pressure in equilibrium with its weight, and (2) in an equilibrium state, the supply of energy from massive, luminous stars must match the demand for energy to resupply the ISM’s continual losses. Under the simplifying assumption of constant feedback yield this principle leads to a prediction that $\Sigma_{\text{SFR}}$ varies linearly with the ISM weight $W \approx P_{\text{DE}}$. The TIGRESS simulations have validated the theoretical principles of pressure balancing weight and energy supply matching demand, but have also demonstrated that the feedback yield is not constant. We may still express the equilibrium star formation rate in terms of the equilibrium weight using the feedback yield as $\Sigma_{\text{SFR}} = P_{\text{DE}}/\Upsilon_{\text{tot}}$, but because $\Upsilon_{\text{tot}}$ decreases with increasing $\Sigma_{\text{SFR}}$ or $P_{\text{DE}}$ (see Equation 24c or Equation 25c), the prediction for pressure-regulated, feedback-modulated star formation is that $\Sigma_{\text{SFR}}$ will increase superlinearly with pressure.

Figure 13 shows $\Sigma_{\text{SFR}}$ vs. the measured midplane pressure in the warm-cold gas ($P_{\text{tot,2p}}$), the measured ISM weight ($W_{\text{2p}}$), and the estimated ISM weight ($P_{\text{DE}}$). The best-fit power law relations overlaid in the figure are

$$
\log(\Sigma_{\text{SFR}}) = 1.18 \log(P_{\text{tot,2p}}/k_B) - 7.43 \quad (26a)
$$

$$
\log(\Sigma_{\text{SFR}}) = 1.17 \log(W_{\text{2p}}/k_B) - 7.32 \quad (26b)
$$

$$
\log(\Sigma_{\text{SFR}}) = 1.21 \log(P_{\text{DE}}/k_B) - 7.66. \quad (26c)
$$

As expected, these relations are slightly superlinear, and almost the same for the three versions of the pressure. In the figure, we also show for reference the fits reported in KOK13, respectively $\log(\Sigma_{\text{SFR}}) = 1.18 \log(P_{\text{tot,2p}}/k_B) - 7.4$ (left panel) and $\log(\Sigma_{\text{SFR}}) = 1.13 \log(P_{\text{DE}}/k_B) - 7.3$ (center and right panels). In spite of having a far more complex model of the ISM than in KOK13, the TIGRESS simulations show quite similar results for the relationship between star formation and pressure as the earlier simulations. This validates the conceptual and practical simplifications in modeling the ISM adopted in KOK13, in particular the assumption that the main role of SNe in modulating star formation is driving turbulence in the warm-cold gas via momentum injection during radiative SNR stages.

4.6. Effective equation of state and velocity dispersion

In situations where it is either not necessary or not possible to follow the detailed thermodynamics of a gaseous system, an effective equation of state is often adopted that relates the gas pressure and density. We can use the results from our TIGRESS simulations to propose an effective equation of state for star-forming interstellar gas. We base this effective equation of state on the fitted relationship between midplane averages of pressure and density from individual snapshots, median values for each model, and the best-fit power-law relationship

$$
\log(P_{\text{tot,2p}}/k_B) = 1.43 \log(n_{\text{H,2p}}) + 4.30. \quad (27)
$$

Interestingly, the exponent 1.43 in this pressure-density relation exceeds the minimal value (4/3) required for
Pressure-Regulated, Feedback-Modulated Star Formation

Figure 13. $\Sigma_{SFR}$ (40Myr average) as a function of measured total midplane pressure $P_{tot,2p}$, measured ISM weight $W_{2p}$, and estimated weight $P_{DE}$. Individual points from 1 Myr intervals as well as median values and 25th and 75th percentiles from the sampling interval are also shown for each model. Solid lines show best-fit power laws and dotted and dashed lines show results from KOK13 (see text).

Figure 14. Total pressure $P_{tot} \equiv P_{th} + P_{turb} + \Pi_{mag}$ vs. hydrogen density $n_H$ in two-phase gas for all models. Midplane-averaged values at intervals of 1 Myr are shown with individual small circles, together with medians and 25th and 75th percentiles from the sampling interval shown as large points. The best-fit power law with slope 1.43 is shown as a dashed line. Dotted lines indicate isotherms of $T_{eff} = P_{tot,2p} / (n_{H,2p} k_B) = 1.7 \times 10^4 K (\sigma_{eff,2p} / 10 \text{ km s}^{-1})^2$. Spherical polytropes to be stable (e.g. Bonnor 1958). Isotherms of $T_{eff} = P_{tot,2p} / (n_{H,2p} k_B)$ are also shown in the figure; $T_{eff} \sim 10^4 - 10^5 K$ for midplane density between $n_{H,2p} \sim 0.1 - 10 \text{ cm}^{-3}$ for our suite of TIGRESS simulations.

As discussed in Section 4.1, the ISM gas is characterized by an effective vertical velocity dispersion $\sigma_{eff}^2 = P_{tot} / \rho$, which takes into account turbulent, thermal, and magnetic stress terms. If we consider just the two-phase gas at the midplane, the fit in Equation 27 corresponds to an effective midplane velocity dispersion $\sigma_{eff,2p} \equiv [P_{tot,2p} / (1.4 n_H n_{H,2p})]^{1/2} = 9.8 \text{ km s}^{-1} [P_{tot,2p} / (10^4 k_B \text{ cm}^{-3} \text{ K})]^{0.15}$, ranging over $\sim 7 - 20 \text{ km s}^{-1}$ for our set of models. If instead we consider contributions from warm-cold gas over the whole volume, the result is $\sim 30\%$ higher. These values range from $\sim 10 - 40 \text{ km s}^{-1}$ for individual models, listed as $\sigma_{eff,2p}$ in Table 2. This mass-weighted mean effective velocity dispersion increases with higher pressure following $\sigma_{eff,2p} = 12 \text{ km s}^{-1} [P_{DE} / (10^4 k_B \text{ cm}^{-3} \text{ K})]^{0.22}$ for $P_{DE} / k_B > 10^4 k_B \text{ cm}^{-3} \text{ K}$.

4.7. Observational comparisons

As discussed in Section 1.3, there have been several previous studies comparing the predictions of PRFM star formation to observations, starting with the OML10 and OS11 papers. In particular, observational surveys with $\sim \text{kpc-scale}$ resolution (or in the case of PHANGS, higher resolution averaged over $\sim \text{kpc}$ scales) have shown that there is a near-linear relationship between $P_{DE}$ and $\Sigma_{SFR}$, with coefficient consistent with theory and numerical simulations. A compendium of observational results based on $\sim \text{kpc}$ patches from Leroy et al. (2008); Herrera-Camus et al. (2017); Sun et al. (2020a); Barrera-Ballesteros et al. (2021) is shown in Figure 15. For all of these works, $P_{DE}$ is computed as in Equation 7; readers are referred to the original publications for details on the assumptions made in obtaining estimates of $\Sigma_{gas}$, $\rho_{sd}$, $\sigma_{eff}$, and $\Sigma_{SFR}$ from observables. Overall, the different surveys show quite similar results, although there do appear to be some systematic differences.
Figure 15. $\Sigma_{\text{SFR}}$ as a function of estimated weight $P_{DE}$, comparing TIGRESS numerical results (Equation 26c, solid, as shown in Figure 13) to observations from several recent surveys of galaxies resolved at $\sim$ kpc scale. Observational results shown are from Leroy et al. (2008); Herrera-Camus et al. (2017); Sun et al. (2020a); Barrera-Ballesteros et al. (2021). The previous numerical result from KOK13, Eq. 27 is also shown (dashed). Also overlaid for reference is Equation 28 with constant $\Upsilon_{\text{tot}} = 200, 1000, 5000 \text{ km s}^{-1}$ (light gray lines, top to bottom).

From Equation 13, the theoretical prediction is that the mean SFR per unit area in the disk will be related to the dynamical equilibrium midplane pressure (Equation 7) via the total feedback yield $\Upsilon_{\text{tot}}$; this may be written in commonly adopted units as

$$\frac{\Sigma_{\text{SFR}}}{M_\odot \text{ pc}^{-2} \text{ Myr}^{-1}} = 2.07 \times 10^{-4} \frac{P_{DE}/k_B}{\text{cm}^{-3} \text{ K}} \frac{P_{DE}/k_B}{\Upsilon_{\text{tot}} \text{ [km s}^{-1}]}.$$  

(28)

The theoretical expectation (see Section 2.2) is that $\Upsilon_{\text{tot}} \sim 1000 \text{ km s}^{-1}$ from a combination of thermal, turbulent kinetic, and magnetic contributions for solar neighborhood conditions, decreasing a few tens of percent in inner disks where shielding of radiation reduces the thermal pressure contribution. In Figure 15 we show Equation 28 with $\Upsilon_{\text{tot}} = 200, 1000, 5000 \text{ km s}^{-1}$; evidently, $\Upsilon_{\text{tot}} = 1000 \text{ km s}^{-1}$ characterizes the center of the observed distribution well, consistent with the theoretical expectation.

Figure 15 overlays on the data the best-fit power-law relationship between $P_{DE}$ and $\Sigma_{\text{SFR}}$ from our suite of TIGRESS multiphase ISM simulations (see Equation 26c and Figure 13). Additionally, we show the fit previously obtained by Kim et al. (2013), based on simulations of warm-cold gas in which effects of SNe were treated by fixed momentum injection. Since the two sets of simulations have only slight differences in $\Upsilon_{\text{tot}}$, the resulting predicted $\Sigma_{\text{SFR}}$ is also quite similar, and both are in good agreement with the observations. The decrease in the feedback yield in higher-density, higher-pressure conditions makes the numerical relations superlinear, with the numerical fits crossing the $\Upsilon_{\text{tot}} = 1000 \text{ km s}^{-1}$ line near the center of the observed distribution. This corresponds to slightly higher-pressure, higher-$\Sigma_{\text{SFR}}$ conditions than the solar neighborhood.

5. SUMMARY AND DISCUSSION

In this paper, we investigate the co-regulation of star formation and ISM properties in disk galaxies for a range of conditions representative of the nearby Universe. Our goal is to test the PRFM theory first enunciated in OML10 and OS11, which posits that the star formation rate and mean pressure in the multiphase ISM are intimately linked through the energetic feedback provided by high mass stars, and both can be quantitatively predicted via simple considerations of thermal and dynamical equilibrium. The key physical concept in the PRFM theory is that the same midplane pressure that balances the vertical weight of the ISM “atmosphere” must be equal to the sum of individual pressures derived from considerations of balance between energy and/or momentum injection by stellar feedback into the ISM, and losses from the ISM. Previously, in KKO11; KOK13; and Kim & Ostriker (2015a), we used numerical simulations focused just on the two-phase ISM to test these ideas and to compute feedback yields, defined as $\Upsilon_i = P_i/\Sigma_{\text{SFR}}$ for $P_i$ representing thermal, turbulent, or magnetic pressure.

In the present work, we use a set of seven TIGRESS MHD simulations to provide further numerical tests of the PRFM principles – now with numerical models that include a hot ISM component produced by correlated SNe, and to measure the pressure components and feedback yields. The simulations we employ represent horizontal patches ranging in size from $(512 \text{ pc})^2$ to $(2048 \text{ pc})^2$, with a vertical dimension 7 times as large, and minimum physical resolution in the range $2 - 8 \text{ pc}$. Each simulation is run for at least 1.5 orbits at the corresponding galactic radius.

5.1. Summary of key numerical results

The main conclusions from analysis of our simulations are as follows:
Quasi-steady state—In all simulations, a quasi-steady state is reached after a few tenths of a galactic orbital time (Figure 3). In this quasi-steady state, the SFR fluctuates temporally, leading to fluctuations of feedback and pressure. Because feedback is extended in time over the lifetimes of massive stars (several tens of Myr), temporal pressure fluctuations have lower amplitudes compared to those in $\Sigma_{\text{SFR}}$. In general, the ISM includes all three phases of gas (hot, warm, and cold). The hot gas is produced by repeated shocks from correlated SN, with the resulting superbubbles expanding preferentially in the vertical direction to create chimneys where hot galactic winds are vented. In the warm and cold phases, the loci of highest occupation in the pressure-density phase plane (Figure 7) follow thermal equilibrium curve set by the instantaneous heating rate, but there is non-negligible occupation of the out-of-equilibrium regime due to dynamical effects.

Multiphase pressure equilibrium—Hot gas and two-phase (warm+cold) gas reach a state of approximate mutual pressure equilibrium. Medians of $P_{\text{tot,hot}}$ and $P_{\text{tot,2p}}$ are within 50% of each other at the midplane of the disk, in all models (Figure 6). The models with “inner-galaxy” conditions (higher $\Sigma_{\text{gas}}$ and $P_\gamma$) have systematically higher emergent $P_{\text{tot}}$ and $\Sigma_{\text{SFR}}$. For the two-phase gas, the ratio $P_{\text{th}}/P_{\text{tot}}$ declines and $P_{\text{turb}}/P_{\text{tot}}$ increases in the higher-pressure, inner galaxy models (Equation 22a, Equation 22b), with the former due to the increased shielding applied for photoelectric heating at higher $\Sigma_{\text{gas}}$. Magnetic pressure can become comparable to turbulent pressure in the warm-cold gas (at the midplane), but is negligible in the hot gas. In the hot phase, thermal pressure and Reynolds stress are comparable overall, although thermal exceeds turbulent pressure at the midplane.

Vertical force balance—The midplane total pressure is in vertical dynamical equilibrium with the weight of the ISM (Figure 9), as an immediate consequence of quasi-steady state for the vertical component of the momentum equation. While the total pressure $P_{\text{tot}} = P_{\text{th}} + P_{\text{turb}} + \Pi_{\text{mag}}$ has larger variations in time than the weight $W$ (defined in Equation 1), the mean values are within 15% of each other (except for model R16, where the difference in means is within 30%). The simple form $P_{\text{DE}}$ given in Equation 7, commonly used in observations, provides an excellent estimate of $W$. Although $P_{\text{DE}}$ is consistently larger than $W$, the difference is within 20 – 30%, except for model R16. Thus, the mean value of $P_{\text{DE}}$ is within 10 – 40% of the mean midplane total pressure $P_{\text{tot}}$. It is important to recognize that $P_{\text{DE}}$ should not be thought of as an “external” force/area that acts on clouds. Rather, $P_{\text{DE}}$ is equal to the statistical average of the total pressure (the vertical component of the momentum flux) over the gas at the midplane. Since the nonthermal (turbulent, magnetic) stresses are dominated by large scales that may exceed the sizes of individual clouds, these stresses cannot be thought of as “surface” terms on individual clouds.

Feedback yields—Numerical results for the thermal, turbulent, and total feedback yields $\Upsilon_{\text{th}}, \Upsilon_{\text{turb}},$ and $\Upsilon_{\text{tot}}$ are given as a function of $\Sigma_{\text{SFR}}$ in Equation 24a–Equation 24c, or as a function of $P_{\text{DE}}$ in Equation 25a–Equation 25c. Consistent with the hypothesis of OML10, the set of measured thermal yields $\Upsilon_{\text{th}} = P_{\text{th}}/\Sigma_{\text{SFR}}$ for our simulations is close to expected values based on thermal equilibrium with pressure equal to the “two-phase” value ($P_{\text{th}} = P_{\text{two}} \propto \Sigma_{\text{SFR}} f_\gamma$ from Equation 19 and Equation 17) for our adopted photoelectric heating rate and cooling functions; this is equivalent to Equation 12 for the Wolfire et al. (2003) heating and cooling functions for atomic gas. The turbulent yield $\Upsilon_{\text{turb}} = P_{\text{turb}}/\Sigma_{\text{SFR}}$ measured in the simulations is consistent with the prediction of OS11 as given in Equation 10 with $P_\gamma/m_\gamma \sim 1300$ km s$^{-1}$ averaged over models, and less than a factor 2 decrease in $\Upsilon_{\text{turb}}$ from outer-to-inner disk models. The weak variation in $\Upsilon_{\text{tot}} = P_{\text{tot}}/\Sigma_{\text{SFR}}$ with galactic conditions ($\Upsilon_{\text{tot}} \propto \Sigma_{\text{SFR}}^{-0.2} \propto P_{\text{DE}}^{-0.2}$) reflects the fact that the largest component of the pressure in the ISM is $P_{\text{turb}}$, and the net momentum injection per SN event is insensitive to environment (as previously demonstrated in idealized simulations). We note that the total yield $\Upsilon_{\text{tot}}$ agrees to better than 10% with that previously reported in KOK13.

$\Sigma_{\text{SFR}}$ - pressure relation—There is a nearly linear relationship between $\Sigma_{\text{SFR}}$ and midplane pressure: Equation 26a, Equation 26b, Equation 26c respectively give the best-fit power law relations between $\Sigma_{\text{SFR}}$ and $P_{\text{tot}}, W,$ and $P_{\text{DE}},$ which follow the same scalings ($\Sigma_{\text{SFR}} \propto P^{1.2}$) and have very similar coefficients. The relationship between $\Sigma_{\text{SFR}}$ and $P_{\text{tot}}$ reflects the role of feedback in setting physical ISM pressures (through the yields), while the relationship between $\Sigma_{\text{SFR}}$ and $W$ (or $P_{\text{DE}}$) reflects both the role of feedback and vertical dynamical equilibrium. The weak decrease of total feedback yield in higher-pressure (higher density) environments explains why $\Sigma_{\text{SFR}} = P_{\text{DE}}/\Sigma_{\text{tot}}$ is slightly superlinear. Quantitatively, we find essentially the same relation between $\Sigma_{\text{SFR}}$ and $P_{\text{DE}}$ as previously reported in KOK13 (see comparison in Figure 13). The relation between $\Sigma_{\text{SFR}}$ and $P_{\text{DE}}$ is both the most important physical concept and the most useful practical result of the PRFM.
theory, because it provides a quantitative prediction for star formation given the basic gas and stellar properties of a galactic disk (see Figure 15 for theory/observation comparison).

**Effective equation of state.**—From the measured pressure and density averaged over the two-phase gas, we obtain an effective equation of state for star-forming gas as given in Equation 27, which has \( P \propto \rho^{1.43} \). The measured relationship encodes the total effective velocity dispersion \( \sigma^2_{\text{eff}} = P_{\text{tot}}/\rho \) for the warm-cold gas. Based on our fit to the set of TIGRESS simulations presented here, this corresponds to \( \sigma_{\text{eff,}2p} = 10 \text{ km s}^{-1}[P/(10^4 k_B \text{ cm}^{-3} \text{ K})]^{0.2} \) where \( P \) is either the measured midplane total pressure \( P_{\text{tot}} \) or the estimated gas weight \( P_{\text{DE}} \). This (or similar) effective equation of state relation can be combined with Equation 7 to obtain \( P_{\text{DE}} \) and \( \sigma_{\text{eff}} \) solely as a function of \( \Sigma_{\text{gas}} \) and \( \rho_{\text{sd}} \).

### 5.2. Discussion and prospects

**Application to galaxy formation modeling**—As recapitulated above, the individual elements of the PRFM theory are clearly validated by the numerical results obtained with our suite of TIGRESS simulations. We regard this as a success, and on the basis of this we encourage use of the theory in modeling where the detailed properties of the ISM and of star formation on \( \sim\text{pc} \) scales cannot be directly resolved, but the gaseous and stellar content on larger (\( \sim 10^2 - 10^3 \text{pc} \)) scales are known. Given the gas surface density \( \Sigma_{\text{gas}} \) and the stellar plus dark matter volume density \( \rho_{\text{sd}} \), the predicted star formation rate is obtained using Equation 13 for \( \Sigma_{\text{SFR}} \) (or Equation 15 for \( t_{\text{dep}} = M_{\text{gas}}/\dot{M}_{\ast} \)) with Equation 7 for \( P_{\text{DE}} \) and Equation 25c for \( \gamma_{\text{tot}} \).

Cosmological galaxy formation simulations and semi-analytic models are an obvious use case for application of the PRFM model and of our numerical calibrations of \( \gamma_{\text{tot}} \) and \( \sigma_{\text{eff}} \). In particular, while zoom simulations may resolve the vertical scale height and therefore the mean density of the ISM, this is generally not true in large-box cosmological simulations. Nevertheless, the local gas surface density \( \Sigma_{\text{gas}} \) can be computed by integrating through the disk, and this may be combined with (resolved) stellar plus dark matter density in the disk to obtain the local \( P_{\text{DE}} \). With \( P_{\text{DE}} \) in hand, our results provide a prediction for \( t_{\text{dep}} \) that can be used in setting cell-by-cell values of the SFR, given the gas mass.

Potentially, generalized versions of the effective equation of state we derive from our simulations (as reported in Equation 27), could be adopted for star-forming gas in cosmological simulations. The pressure-density relation we report here quantitatively lies between the one that has been adopted for the Illustris(TNG) simulations (Springel & Hernquist 2003; Robertson et al. 2004; Vogelsberger et al. 2013; Pillepich et al. 2018) and the one adopted for the EAGLE simulations (Schaye & Dalla Vecchia 2008; Schaye et al. 2015). Adoption of an equation of state derived from TIGRESS would therefore be unlikely to introduce significant differences in overall disk stability if it were adopted for cosmological simulations, but would have the merit of being calibrated from resolved multiphase MHD simulations. To obtain an effective equation of state that is suitable for modeling galaxy formation over all of cosmic time, however, it will be necessary to generalize the results obtained here with lower metallicity TIGRESS simulations.

**Parameter dependence and functional form of \( t_{\text{dep}} \) and \( \Sigma_{\text{SFR}} \)**—The local star formation rate in a galaxy can be broken down into available gaseous “fuel” and a timescale at which that fuel is converted to stars; these combine to make \( \Sigma_{\text{SFR}} = \Sigma_{\text{gas}}/t_{\text{dep}} \) for a disk system. We emphasize that the gas and stars are equally important in setting \( t_{\text{dep}} \), as given in Equation 15. That is, \( \Sigma_{\text{gas}} \) and \( \rho_{\text{sd}} = \rho_{\ast} + \rho_{\text{dm}} \) (dominated by the stars) enter mathematically and physically on an equal footing in \( t_{\text{dep}} \) via the vertical dynamical time \( t_{\text{ver}} \). This can be contrasted with the commonly adopted assumption that the relevant timescale for star formation is the free-fall time \( t_{\text{ff}} \), proportional to the inverse square root of gas density.\(^8\)

In practice, the gas and stars are usually of comparable importance in determining the vertical gravitational field on \( \sim\text{kpc} \) scales within the main ISM layer; even if the global galactic gas fraction is high, the stellar and gas vertical gravity may be comparable. Since the combined gravitational field from gas, stars, and dark matter controls the ISM pressure (and scale height), it determines the necessary star formation and feedback required to maintain equilibrium.

It is worth noting that \( \Sigma_{\text{SFR}} = P_{\text{DE}}/\gamma_{\text{tot}} \) does not functionally correspond to a power-law dependence of \( \Sigma_{\text{SFR}} \) on \( \Sigma_{\text{gas}} \), or even a product of power laws in \( \Sigma_{\text{gas}} \) and \( \Sigma_{\ast} \), unless one or the other of the two terms in \( P_{\text{DE}} \) (Equation 7) dominates. Moreover, the thickness of the stellar disk \( h_{\ast} \) is just as important as \( \Sigma_{\ast} \), since from Section 2.1 it is the midplane stellar volume density \( \rho_{\ast} = \Sigma_{\ast}/(2h_{\ast}) \) rather than \( \Sigma_{\ast} \) that controls the gas pressure when \( h_{\text{gas}} < h_{\ast} \) (the usual case). The effective vertical gas velocity dispersion \( \sigma_{\text{eff}} \) is similarly im-

\(^8\) Only in the case where the gas dominates the vertical gravity are \( t_{\text{ff}} \) and \( t_{\text{ver}} \) nearly the same, both \( \sim 0.5\sigma_{\text{eff}}/(G\Sigma_{\text{gas}}) \). In the limit where stars dominate, \( t_{\text{ver}} \sim (2G\rho_{\ast})^{-1/2} \) while \( t_{\text{ff}} \sim 0.5G^{-1/4}(\sigma_{\text{eff}}/\Sigma_{\text{gas}})^{1/2}\rho_{\ast}^{-1/4} \).
important because it enters in setting the gas disk’s half-
thickness $h_{\text{gas}}$ (see Equation 5) and therefore controls the
gas pressure (see Equation 4 or Equation 7). While several different properties are required to fully describe the local galactic environment, the specific parameter combination embodied by the equilibrium pressure has a special physical significance. By referring explicitly to pressure in the “PRFM” moniker, we underscore this point.

Even though the dependence of $\Sigma_{\text{SFR}}$ is on the combination of variables in $P_{\text{DE}}$ rather than as a power law $\Sigma_{\text{SFR}} \propto \Sigma_{\text{gas}}^{1+p}$, an apparent power law relation can arise observationally for a number of reasons. For example, if the vertical gravity is dominated by the gas and kinetic turbulence dominates the pressure, as may be the case in starburst regions, $\Sigma_{\text{SFR}} = P_{\text{DE}}/T_{\text{tot}} \rightarrow \pi G \Sigma_{\text{gas}}^2/(2T_{\text{turb}})$ for $T_{\text{turb}}$ of a few 100 km s$^{-1}$ (center panel of Figure 12). As previously noted in OS11 and Narayanan et al. (2012), the approximate relation $\Sigma_{\text{SFR}} \propto \Sigma_{\text{gas}}^2$ in this regime could appear as a slope between 1 and 2 in log $\Sigma_{\text{SFR}}$ vs. log $W_{\text{CO}}$, since the decrease of $\alpha_{\text{CO}} = \Sigma_{\text{mol}}/W_{\text{CO}}$ in higher-excitation gas (see Gong et al. 2020, and references therein) means that $\Sigma_{\text{gas}}$ increases sublinearly with $W_{\text{CO}}$. This may explain the power law with $p = 0.4$ identified by Kennicutt (1998), which adopted constant $\alpha_{\text{CO}}$ (see also Kennicutt & De Los Reyes 2021). Similarly, a power law dependence $\Sigma_{\text{SFR}} \propto \Sigma_{\text{gas}}^{2.0}$ (e.g. Shi et al. 2011, 2018) could describe normal galaxies if their vertical gravity is dominated by the stellar component and there is limited variation in $h_*$.

Enhanced observational testing—Observational tests to date at $\sim$ kpc-scale resolution, as summarized from previous work in Section 1 and directly compared with our new simulation results in Section 4.7, show good agreement with the PRFM model. For the future, it will be especially valuable to refine the empirical measurements of parameters that enter in $P_{\text{DE}}$. As noted above, the midplane density of the old stellar disk, $\rho_*$, is needed to obtain $P_{\text{DE}}$. This requires knowledge of both the total surface density of old stars, $\Sigma_*$, and the effective half-thickness $h_*$ of the stellar layer. For face-on galaxies, $h_*$ cannot be directly measured, and a common practice for normal spiral galaxies (following van der Kruit & Searle 1982) has been to assume a constant stellar disk thickness proportional to the radial exponential scale length $R_*$ of the old stellar disk, with $\rho_* = \Sigma_*/(0.54 R_*)$ (e.g. Leroy et al. 2008; Ostriker et al. 2010; Herrera-Camus et al. 2017; Sun et al. 2020a; Barrera-Ballesteros et al. 2021). However, as noted in OML10, the choice $h_* = 0.27 R_*$ may in fact overestimate $h_*$, leading to an underestimate for $\rho_*$. Also, stellar disks may flare with radius (de Grijs & Peletier 1997; Narayan & Jog 2002; Momany et al. 2006; LÓPEZ-CORREDOIRA & Molgó 2014; López-Corredoira et al. 2020). An alternative to adopting an empirical relationship between $h_*$ and $R_*$ is to estimate $h_*$ from dynamical considerations. For example, a very simple estimate adopts $h_* = \sigma_{z,*}^2/(\pi G \Sigma_*)$ for $\sigma_{z,*}$ the effective vertical velocity dispersion of the old stellar disk, based on the Spitzer (1942) solution for an isothermal plane-parallel system; in this case (and assuming the dark matter contribution is negligible) the second term in Equation 7 becomes $\sqrt{\pi G \Sigma_{\text{gas}} \Sigma_* \sigma_{\text{eff}}}/\sigma_{z,*}$. However, improved estimates for the vertical stellar distribution can be obtained by including the influence of gas and dark matter gravity in confining the stellar disk and solving the coupled system of Jeans equation and hydrostatic equation for stars and gas, respectively, and allowing for radial and vertical gradients in velocity dispersions (e.g. Sarkar & Jog 2018, 2020a,b). A path forward to more accurate values of $h_*$ would be to seek statistical relationships between observed measures of the stellar disk thickness and other stellar properties from edge-on disk galaxies, while simultaneously using synthetic observations of simulated edge-on disks to calibrate the true $h_*$ in terms of observables (including testing sensitivity to dust extinction).

Empirical measurements of $P_{\text{DE}}$ require the total gas surface density $\Sigma_{\text{gas}}$ and the effective vertical velocity dispersion $\sigma_{\text{eff}}$. Especially in the central regions of galaxies, improved calibrations of $\alpha_{\text{CO}}$, e.g. making use of two or more rotational lines to allow for varying excitation (Gong et al. 2020), will aid in obtaining more accurate $\Sigma_{\text{gas}}$ from CO emission. The effective vertical velocity dispersion $\sigma_{\text{eff}}$ includes magnetic contributions that are difficult to measure empirically; since these are likely to scale with the (more easily observable) kinetic turbulence, however, this can be accounted for at lowest order with a simple multiplicative factor. It is important to note that the closest observable proxy of the kinetic contribution to $\sigma_{\text{eff}}$ is a mass-weighted value. Thus, it must be derived from observations of the atomic and molecular gas that together comprise most of the ISM’s mass. While ionized gas velocity dispersions are sometimes more readily available, especially for high-redshift galaxies, these sample expanding H II regions and diffuse ionized gas; since the motions of ionized gas are not in general representative of the neutral ISM (ion linewidth are typically larger, e.g. Girard et al. 2021), linewidths of H$\alpha$ or other tracers of ionized gas should not be used as a proxy to estimate the kinetic contribution to $\sigma_{\text{eff}}$. Even when CO and H I lines are spectrally resolved, it is difficult to correct for spiral-arm streaming and other in-plane motions that can contaminate mea-
measurement of the vertical velocity dispersion, so face-on systems provide the most reliable targets.

Further model development — The suite of TIGRESS simulations analyzed here represents a significant advance in resolved modeling of the star-forming, multiphase, magnetized ISM. Nevertheless, the TIGRESS implementation employed for this suite (as described in Kim & Ostriker 2017) has limitations that could affect our results quantitatively, if not qualitatively. First, a simple fitted cooling function and fixed FUV heating efficiency are adopted. Rather than directly following the FUV photons responsible for photoelectric heating via radiative transfer, we also adopt a simple analytic attenuation formula. In addition, we do not follow ionizing radiation (or other “early feedback”) from young clusters, which is known to strongly affect the immediate environment of forming stars. Extending beyond these simplifying assumptions is an important direction for future work, and efforts along these lines have already begun. Namely, we have extended our TIGRESS framework to include adaptive ray tracing radiative transfer (as implemented by Kim et al. 2017b) in order to follow FUV as well as ionizing radiation produced by the star cluster particles that form, and we have also implemented more sophisticated heating and cooling functions based on nonequilibrium photochemistry (J.-G. Kim et al. 2022, submitted). Initial simulations with this new framework produce results that are generally quite consistent with those reported here, and in particular turning photoionizing radiation on or off changes $\Sigma_{\text{SFR}}$ by $\lesssim 30\%$ (C.-G. Kim et al 2022, in preparation). Another potential concern is that with our current implementation of sink particles – representing stellar clusters with a coeval stellar population, star formation may be somewhat more correlated in space and time than is realistic, which could quantitatively affect certain results (e.g. for galactic wind power, as seen in simulations by Smith et al. 2021). Testing sensitivity to this correlation, as well as exploring alternatives to the current sink particle approach, are also important directions for future work.

The TIGRESS simulations analyzed here assume solar metallicity, but it is of much interest to investigate how higher or lower metallicity would affect the results. We have every expectation that the PRFM theory will hold in some generality, but quantitative calibration of feedback yields at low metallicity are needed for realistic application to high-redshift galaxies. By extending the range of simulated systems, not just in metallicity but to environments with much higher and lower gas surface density, with deeper and shallower stellar potentials, with global Galactic as well as local frameworks (while still resolving all phases of the ISM and feedback effects), it will be possible to test the general validity of the PRFM theory. In strongly disturbed systems (tidal encounter and merger systems), as well as galaxies at early stages before disk rotation dominates other kinetic and thermal energy (e.g. Gurvich et al. 2022), it remains to be seen whether a modified version of the PRFM theory (generalizing to quasi-spherical rather than disk geometry) can be used to predict the SFR. This could only be expected if feedback is the main source resupplying losses of the forms of energy that combine to support against gravity. From dimensional analysis, the expected gas depletion time would be $t_{\text{dep}} \sim T_{\text{rot}} / g$ for $g$ the relevant gravitational field that is supported by terms with rapid losses ($g_r$ for a disk, or $g_z$ for a quasi-spherical system). Our own analyses of TIGRESS model extensions – with spiral arm potential perturbations in mid-disk environments (Kim et al. 2020c), and of bar-fed star-forming rings in galactic center regions (Moon et al. 2021, 2022) – have already corroborated the PRFM principles. It would be straightforward for other groups to apply the same kind of analysis to their own simulations with physics implementations similar to TIGRESS and a fully resolved multiphase ISM, in order to test these principles further.

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