Quantum gravitational contributions to the beta function of quantum electrodynamics

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We show in a diagrammatic and regularization independent analysis that the quadratic contribution to the beta function which has been conjectured to render quantum electrodynamics asymptotically free near the Planck scale has its origin in a surface term. Such surface term is intrinsically arbitrarily valued and it is argued to vanish in a consistent treatment of the model.

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Because of the negative mass dimension of the coupling constant perturbative Einstein quantum gravity (EQG) is nonrenormalizable \(^{[1,2]}\). However one can still make sense of EQG if it is interpreted as an effective quantum field theory within a low energy expansion of a more fundamental theory. In an effective field theory all interactions compatible with its essential symmetry content are in principle allowed into the Lagrangian \(^{[3]}\) and thus it establishes a systematic framework to calculate quantum gravitational effects \(^{[4]}\).

This approach has been used to study the asymptotic behavior at high energies of quantum field theories that incorporate the gravitational field. Robinson and Wilczek suggest that the gravitational field improve the asymptotic freedom of pure Yang-Mills near the Planck scale \(^{[5]}\). However, a similar calculation in the Maxwell-Einstein theory suggests that such conclusion is gauge dependent \(^{[6]}\). In a contribution \(^{[7]}\) in which the effective action is calculated in a gauge-condition independent version of the background field method using dimensional regularization it is argued that the gravitational field plays no role in the beta function of the Yang-Mills coupling. Another calculation using conventional diagrammatic methods confirms this conclusion \(^{[8]}\).

In a recent publication, D. Toms \(^{[9]}\) claimed that quadratic divergent contributions were responsible to improve asymptotic freedom of fine structure constant by quantum gravity effects by using proper time cutoff regularization and effective action methods. However, the physical reality of the result in \(^{[3]}\) has been questioned \(^{[10,11]}\).

The purpose of this contribution is to shed light on the origin of such controversies using only a diagrammatic analysis. As an effective model EQG is intrinsically regularization dependent and consequently regularization becomes part of the model. We show however that the quadratic contributions to the beta function stem from ambiguous, arbitrarily valued, regularization dependent surface terms. We present the one loop calculation of the vacuum polarization tensor of the Maxwell-Einstein theory, both with and without matter, in the Feynman and harmonic gauges for the photon and graviton, respectively. We carry out calculations such that regularization ambiguities are isolated from divergent integrals and compare with the results found in the literature showing explicitly the origin of the ambiguities. We evaluate arbitrary parameters in both cutoff and dimensional regularization. Finally we argue that such ambiguities can be fixed on physical grounds demanding transversality of the vacuum polarization tensor in the limit of weak gravity. Our analysis is based on the point of view discussed by Jackiw in \(^{[12]}\). He argues that it can happen that radiative corrections can give rise to arbitrary finite quantities which must be fixed either by symmetries of the underlying theory and/or, just as for infinite radiative corrections, by experimental data.

We start with the Maxwell-Einstein Lagrangian

\[
S_{ME} = \int d^4x \sqrt{-g} \left[ \frac{2}{\kappa^2} R - \frac{1}{4} g^{\mu\nu} g^{\alpha\beta} F_{\alpha\mu} F_{\beta\nu} \right].
\]

As usual, \(F_{\mu\nu}\) is the electromagnetic field strength...
while in the Feynman gauge the photon propagator reads

$$\Delta^{\mu\nu}(p) = \frac{iP^{\alpha\lambda\sigma\beta}}{(p^2 - \mu^2 + i\epsilon)},$$

with

$$P^{\alpha\lambda\sigma\beta} = \frac{1}{2} \left( \eta^{\beta\lambda} \eta^{\sigma\alpha} + \eta^{\beta\sigma} \eta^{\lambda\alpha} - \eta^{\alpha\lambda} \eta^{\sigma\beta} \right),$$

while in the Feynman gauge the photon propagator is

$$\Delta^{\mu\nu}(p) = \frac{-i\eta^{\mu\nu}}{(p^2 - \mu^2 + i\epsilon)}.$$  

The one-loop contribution corresponding to the diagram in figure 1a is given by

$$\Pi^{\mu\nu}(p) = -\kappa^2 \int_k \frac{\eta_{\beta\sigma} P_{\gamma\lambda\beta\sigma}}{(k^2 - \mu^2)(k^2 - p^2 - \mu^2)} \times \tau^{\gamma\lambda\mu\delta}(p, p - k)\tau^{\beta\sigma\nu\alpha}(p - k, p),$$

where in $\tau^{\mu\nu\rho\sigma}$ above the momenta flow towards the vertex of the Feynman diagram.

We isolate the divergent content of the amplitude above as basic divergent integrals following [13] as a convenient method to evaluate the extent to which the final result depends of a particular choice of regularization. We begin by using in [7] the identity

$$\frac{1}{(k^2 + p^2 - \mu^2)} = \frac{1}{k^2 - \mu^2} - \frac{2k \cdot p + p^2}{(k^2 - \mu^2)(k^2 + p^2 - \mu^2)}$$

in order to eliminate the external momentum $p$ from the basic divergent integrals which will be expressed as

$$I_{\text{log}}(\mu^2) = \int_k \frac{1}{(k^2 - \mu^2)^2}$$

and

$$I_{\text{quad}}(\mu^2) = \int_k \frac{1}{(k^2 - \mu^2)^2}. $$

We adopt the abbreviation $\int_k \equiv \int d^4k/(2\pi)^4$. After some tensorial algebra, the one-loop photon vacuum polarization reads

$$\Pi^{\mu\nu}_{\text{grav}}(p) = -\kappa^2 \left[ \frac{5}{12} F(p^2) \left( p^2 \eta^{\mu\nu} - p^\mu p^\nu \right) p^2 - I_{\text{quad}}(\mu^2) \left( p^2 \eta^{\mu\nu} - p^\mu p^\nu \right) + \Upsilon^{\mu\nu} \right].$$

The quadratic divergent term $I_{\text{quad}}(\mu^2)$ will be canceled by the tadpole diagram in figure 1b, whereas $F(p^2)$ stands for

$$F(p^2) = I_{\text{log}}(\mu^2) - \frac{i}{16\pi^2} \ln \left( -\frac{p^2}{\mu^2} \right).$$

The apparent infrared divergence is eliminated by using the regularization independent identity

$$I_{\text{log}}(\mu^2) - I_{\text{log}}(\lambda^2) = -\frac{i}{16\pi^2} \ln \left( \frac{\mu^2}{\lambda^2} \right), \lambda \neq 0,$$
in which \( \lambda \) plays the role of renormalization group constant. Thus
\[
F(p^2) = I_{\log}(\lambda^2) - \frac{i}{16\pi^2} \ln \left( \frac{p^2}{\lambda^2} \right),
\]
with \( \Lambda \to \infty \). It has been shown that setting such surface terms to zero amounts to allowing shifts in the integration variable in the Feynman amplitudes. Gauge invariance of Green’s functions are automatically satisfied within perturbation theory by setting \( c_i = 0 \) and their generalizations to higher loops. Moreover this leads to momentum routing invariance in the Feynman diagram.

To make contact with other results in the literature let us evaluate the expression (11) for \( \Pi^{\mu\nu}(p) \) in both dimensional and cutoff regularizations. For this purpose we use the followings straightforward result
\[
I_{\log}^D(\lambda^2) = \frac{i}{16\pi^2} \left[ \frac{2}{d-4} + \ln \left( \frac{\lambda^2}{\Lambda^2} \right) \right] + \mathcal{O}(d-4)
\]
and, in momentum cutoff regularization,
\[
I_{\log}^A(\lambda^2) = \frac{i}{16\pi^2} \left[ 1 + \ln \left( \frac{\lambda^2}{\Lambda^2} \right) \right] + \mathcal{O} \left( \frac{\lambda^2}{\Lambda^2} \right),
\]
recalling that \( \Lambda \to \infty \) can play the rôle of effective upper energy limit. Finally, using (18) and (19) in (11) yields
\[
\Pi^{\mu\nu}_{D\text{Reg}}(p) = \frac{5k^2 \Lambda}{192\pi^2} \left[ \frac{2}{d-4} + \ln \left( \frac{p^2}{\mu^2} \right) \right] + \mathcal{O}(d-4)
\]
and
\[
\Pi^{\mu\nu}_{A\text{Reg}}(p) = \frac{5k^2 \Lambda}{192\pi^2} \left[ \frac{2}{d-4} + \ln \left( \frac{p^2}{\mu^2} \right) \right] + \mathcal{O}(d-4)
\]
whereas
\[
\Pi^{\mu\nu}_{A}(p) = \frac{5k^2 \Lambda}{192\pi^2} \left[ \frac{2}{d-4} + \ln \left( \frac{p^2}{\mu^2} \right) \right] + \mathcal{O}(d-4)
\]
Some comments are in order. Firstly the coefficient of \( \Lambda^2 \) is the same as the one obtained by D. Toms in [8] where it is claimed to contribute to asymptotic freedom of the structure constant near the Planck scale. Secondly the polarization tensor is not transverse in cutoff regularization whereas it is transverse in dimensional regularization. And last but not least notice that the term \( \Lambda^2 \) in (21) stem from the arbitrarily valued surface term \( c_2 \). For a renormalizable model such surface terms are completely fixed by gauge invariance. Consider the vacuum polarization tensor of QED evaluated in this framework [13] as an illustration. We have
\[
\Pi^{QED}_{\mu\nu} = \int \frac{k}{2} \{ \gamma_\mu S(k+p) \gamma_\nu S(k) \},
\]
where $S(k)$ is the fermion propagator. It can be written as

$$
\Pi^{QED}_{\mu\nu} = \tilde{\Pi}_{\mu\nu} + 4 \left[ c_2 g_{\mu\nu} + \left( \frac{c_3}{3} - c_1 \right) p^2 \eta_{\mu\nu} - \left( c_1 - \frac{2c_3}{3} \right) p_{\mu} p_{\nu} \right]
$$

(23)

where

$$
\tilde{\Pi}_{\mu\nu} = \frac{4}{3} \left( p^2 g_{\mu\nu} - p_{\mu} p_{\nu} \right) \times \left[ I_{log}(m^2) - \frac{i}{(4\pi)^{2}} \left( \frac{1}{3} + \frac{(2m^2 + p^2)}{p^2} F(p^2, m^2) \right) \right].
$$

(24)

$F(p^2, m^2)$ is defined by

$$
F(p^2, m^2) = \int_{0}^{1} dz \ln \left[ \frac{p^2 z(1-z) - m^2}{-m^2} \right]
$$

(25)

and the arbitrary parameters $c_i$’s are defined as before. Notice that in this case gauge invariance fixes their values as $c_1 = c_2 = c_3 = 0$, which is the result we would have obtained should we had evaluated these parameters in dimensional regularization. Moreover a second possibility also renders a transverse vacuum polarization tensor for QED, namely $\eta_{\mu\nu}$. The full one-loop photon vacuum polarization tensor take the form

$$
\Pi^{\mu\nu}(p) = - \left[ F(p^2) \left( \frac{e^2}{3} + \frac{5\kappa^2}{12} p^2 \right) + \frac{i}{12} \left( Z_3 - 1 \right) + Z_4 p^2 \right] (\eta^{\mu\nu} p^2 - p^{\mu} p^{\nu})
$$

$$
- \kappa^2 \Upsilon_1^{\mu\nu} - 4e^2 \Upsilon_2^{\mu\nu}
$$

(26)

where

$$
\Upsilon_2^{\mu\nu} = c_2 \eta^{\mu\nu} - \left( \frac{1}{6} c_3 \right) \left( \eta^{\mu\nu} p^2 + 2p^\mu p^\nu \right)
$$

(27)

and $\Upsilon_1^{\mu\nu}$ is given by (23). Recall that the quadratic contribution comes from the surface term $c_2$ contained in both $\Upsilon_1^{\mu\nu}$ and $\Upsilon_2^{\mu\nu}$. Just as in the case of pure QED, $c_2$ breaks gauge invariance in the matter sector of (24). Hence we must set it to zero in the matter sector on gauge invariance grounds or equivalently one has to use dimensional regularization which automatically evaluates such surface terms to zero. For consistency with the limit where $\kappa \to 0$, the $c_2$ term which would originate a quadratic contribution to the fine structure beta function rendering the theory asymptotically free does not exist.

A final comment is in order. It is well known that a naive cutoff in the three or four momenta in the loop integral violates gauge invariance. However some variations of this method in conjunction with Pauli-Villars or proper time regularization have been used in effective field theories because it is advantageous to introduce an explicit cutoff in such models. The proper time approach introduced by Schwinger is not free of ambiguities. Consider for instance the quadratically divergent integrals discussed in (16):

$$
A = \int_{k} \frac{k^2}{(k^2 - m^2)^2}
$$

and

$$
B = I_{quad}(m^2) + m^2 I_{log}(m^2).
$$
Using the proper time approach via the identity

\[ \Gamma(n) \frac{1}{(k^2 + m^2)^n} = \int_0^\infty d\tau \tau^{n-1} e^{-\tau(k^2 + m^2)} \]

yields for the divergent structure of \( A \) and \( B \) the results

\[ A = \frac{i}{8\pi^2}(\Lambda^2 - m^2 \ln \Lambda^2) \]

and

\[ B = \frac{i}{16\pi^2}(\Lambda^2 - 2m^2 \ln \Lambda^2) \]

instead of the expected equality \( A = B \). In the approach we have discussed here the equality \( A = B \) is built in our framework \(^{[13]}\).

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