On the properties of superconducting planar resonators at mK temperatures

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Planar superconducting resonators are now being increasingly used at mK temperatures in a number of novel applications. They are also interesting devices in their own right since they allow us to probe the properties of both the superconductor and its environment. We have experimentally investigated three types of niobium resonators - including a lumped element design - fabricated on sapphire and SiO₂/Si substrates. They all exhibit a non-trivial temperature dependence of their centre frequency and quality factor. Our results shed new light on the interaction between the electromagnetic waves in the resonator and two-level fluctuators in the substrate.

I. INTRODUCTION

The superconducting microwave resonator is an ubiquitous device with uses ranging from the very practical - such as filters for telecommunications - to rather exotic such as tests of cavity quantum electrodynamics. Over recent years there has been a resurgence in the interest in on-chip resonators; the primary reason being their use as very sensitive photon and much of the work to date has been motivated by the need to understand noise properties of kinetic inductance detectors (KID). Resonators are also being used as elements in circuitry for quantum information processing. The latter has triggered a wave of investigations into the properties of resonators when operated at mK temperatures and very low (ideally single photon) microwave powers. Of particular interest has been the effects of two-level fluctuators (see e.g. and references therein). Superconducting resonators at microwave frequencies differ from their normal metal counterparts primarily because of their high quality factor (Q) and a relatively large kinetic inductance. For example, for an ideal high-Q LC resonator we can write the centre frequency as the temperature is reduced. This has been shown to explain the so-called 'back-bending' effect in superconducting resonators. However, at temperatures ~ Tc/10, these mechanisms saturate (in Nb and its compounds around 1 K) and other effects become prominent, most notably a decrease in the centre frequency as the temperature is reduced. This 'back-bending' is now widely believed to be due to the presence of two-level fluctuators (TLF) in the substrate.

Here we will discuss measurements of the centre frequency and losses in three types of Nb resonators: conventional λ/2 and λ/4 geometric resonators and a recently developed type of lumped element resonator.

II. THEORY

A generic expression for the transmittance, S₂₁, of a transmission line shunted by a resonator (e.g. the lumped element and λ/4 resonators considered here) can be written

\[ S_{21} = 2 \left[ 1 + \frac{g}{1 + 2jQ_u \delta f} \right]^{-1} \]

where \( Q_u \) is the unloaded quality factor (Q in the absence of coupling to the feedline), \( g \) is a coupling parameter \( \propto Q_u \) - the exact expression for which will depend on the type of resonator used - and \( \delta f = (f - f_0)/f_0 \) is the fractional shift from resonance. The value of \( Q_u \) is in general given by summing over all loss mechanisms \( \sum_k Q_k^{-1} \) but losses in the dielectric and the superconducting film itself usually dominate in planar resonators and radiation losses etc. can be neglected. Microwave losses are usually quantified by the loss tangent \( \tan \delta = 1/Q_u \), which in a dielectric is proportional to the sum of the various absorption mechanisms \( \sum_k \alpha_k \).

At sub-Kelvin temperatures one would expect the properties of a dielectric to stay essentially constant. However, experimentally one finds a variation in both the effective dielectric constant and losses. Much of this can be explained if one assumes the presence of a bath of TLF that couple to the resonator via their electric dipole moment. Many of the details of the corresponding theory were worked out over 30 years ago when the effects of TLF in glasses at low temperatures were being investigated. The theory is based on the application of the Bloch equations to an ensemble of spin systems and while it is simple it nevertheless captures most of the essential physics. For resonant absorption due to an ensemble of TLF \( \alpha_r \) has the form

\[ \alpha_r = \frac{\pi \omega n d^2}{3 c_0 e_0 e} \left( 1 + \frac{P}{P_c} \right)^{-1/2} \tanh \left( \frac{\hbar \omega}{2k_B T} \right) \]  (1)

where \( \omega \) is the measurement frequency, \( d \) is the dipole moment, \( c_0 \) is the speed of light in vacuum and \( n \) is the density of states of the TLF that couple to the stray field of the resonator. Note that this density should in principle depend on frequency, but is experimentally of-
ten found to be constant. The temperature dependence of the absorption reflects the population difference of the two states of the fluctuators, as the temperature increases the TLF will therefore absorb less power. Note that Eq. 1 only accounts for resonant absorption, non-resonant relaxation processes due the TLF bath are not taken into account.

The relation above predicts a power dependence of the absorption \( \propto P^{-1/2} \) above some critical power \( P_c \), although in practice one may expect another high-power roll-off once all the TLF are saturated and other loss mechanisms start to dominate. The critical intensity is given by \( P_c = 3h^2 c_0 \epsilon_0 \epsilon_r / 2d^2 T_1 T_2 \) where \( T_1 \) and \( T_2 \) are the relaxation and the dephasing times of the TLF, respectively.

The variation in permittivity is dominated by resonant processes and can therefore be found by applying the Kramers-Kronig relations to Eq. 1

\[
\frac{\epsilon(T) - \epsilon(T_0)}{\epsilon(T_0)} = -\frac{2\mu d^2}{3\epsilon} \left[ \ln \left( \frac{T}{T_0} \right) - (g(T, \omega) - g(T_0, \omega)) \right]
\]

where \( g(T, \omega) = \text{Re} \Psi(\frac{1}{2} + \omega + \omega^2/2\pi i k_B T) \), \( T_0 \) is a reference temperature and \( \Psi \) the complex digamma function significant only for \( kT \lesssim \hbar \omega / 2 \). From cavity perturbation theory we then have that \( \Delta f_0 / f_0 = -F/2\Delta \epsilon / \epsilon, F \) being a filling factor which depends on the geometry and the electric field distribution. The substrate filling factor varies slightly between the three types of resonator discussed here; but is over 0.9 in all cases. For samples on SiO2/Si substrates the contribution of the SiO2 layer to \( F \) is about 0.2. Eq. 1 predicts an upward turn in the temperature dependence of the centre frequency at mK temperatures, where the \( g(T, \omega) \) term is important. In the experiments by Gao et al (1) Eq. 2 was used to fit the temperature dependence of the resonance frequency, but the low-temperature upturn was not clearly observed.

The aim of this experimental study is to verify whether the theory developed for a spin solution in glass can accurately describe the temperature and power dependence of the resonance frequency and losses in superconducting resonators at mK temperatures.

### III. EXPERIMENTAL

We have fabricated a wide variety of \( \lambda / 2, \lambda / 4 \) and lumped element (LE) niobium resonators (see fig. 1) on R-cut sapphire and thermally oxidized high-resistivity (> 1 kOhm/cm²) silicon substrates with 400 nm of SiO2. The simplest design is the in-line \( \lambda / 2 \) resonator. Its advantages is that it can be made with loaded quality factors of > 10⁶ and is easy to measure (due to the high S/N ratio on resonance), there is however only one resonator per chip and a calibrated measurement is needed in order to quantify its unloaded quality factor (which is difficult at mK temperatures). \( \lambda / 4 \) resonators have the advantage that their parameters can be extracted relatively easily (using numerical fitting) from uncalibrated measurements since the transmittance far from resonance can be used as a reference level. Several \( \lambda / 4 \) resonators can be coupled to a common feedline, making it possible to measure in a frequency-multiplexed configuration. The lumped element resonator shares these advantages and is much more compact (typical size is about 200x200 \( \mu \)m²) than geometric resonators. Our LE resonators fabricated in Nb on sapphire exhibit loaded \( Q \) close to \( 10^5 \) with an unloaded \( Q \) that is about 3 times higher; i.e the resonators are by design overcoupled.

The structures were patterned in 200 nm thick sputtered Nb films using ion-beam etching and were designed to have their centre frequencies in the c-band (4-8 GHz). Table I shows a summary of the samples discussed in this paper. Other resonators measured to-date showed similar behaviour. Each chip is 5x10 mm² in size and incorporates a central 50 \( \Omega \) coplanar feed-line and one or more resonators. For the measurement the chip is glued to a gold-plated alumina carrier inside a superconducting box. All measurements (with the exception for the data above

| Table I: Summary of the resonators used in these experiments presented in this paper |
|-------------------------------------------------|--------------|----------------|-----------------|
| **Type** | **Chip** | **Substrate** | \( f_0 \) at 50 mK (MHz) |
|---------|---------|---------------|-----------------|
| R1      | \( \lambda / 2^a \) | A SiO2/Si     | 6037            |
| R2      | \( \lambda / 4 \)    | B SiO2/Si     | 5040            |
| R3      | \( \lambda / 4 \)    | B SiO2/Si     | 5795            |
| R4      | \( \lambda / 4 \)    | B SiO2/Si     | 8100            |
| R5      | LE      | C SiO2/Si     | 6805            |
| R6      | LE      | C SiO2/Si     | 7250            |
| R7      | LE      | C SiO2/Si     | 7813            |
| R8      | LE      | D Sapphire    | 6046            |
| R9      | LE      | D Sapphire    | 6458            |
| R10     | LE      | D Sapphire    | 6959            |

\(^a\)Single resonator on the chip, measured in-line
1 K in fig. 4 were carried out in a dilution refrigerator with heavily filtered microwave lines. The transmitted signal was amplified using an InP HEMT amplifier held at 1 K with a noise temperature of about 4 K, before further amplification stages at room temperature.

IV. RESULTS AND DISCUSSION

Figure 2 shows how the loss tangent depends on the energy in the resonator on resonance $W = 2Q_u(1 - S_{21})P_{in}/\omega_0$ for three different lumped element resonators on SiO$_2$/Si coupled to the same on-chip transmission line and measured simultaneously. Absolute power metrology at mK temperatures is very problematic so only relative measurements were possible; we estimate that the maximum power reaching the chip during the measurements is about -70 dBm, meaning $W_0$ is of the order of $10^{-16}$ Ws. Shown is also R8, a sapphire resonator of the same design. For the resonators fabricated on SiO$_2$/Si the curves have a slope of approximately -0.4 in this range of applied powers. Eq. 1 predicts dielectric losses changing with a slope of -0.5 for powers above $P_c$ and power-independent behavior below $P_c$, it is therefore possible that we are in the intermediate regime. However, it is also possible that losses in the superconductor not captured by Eq. 1 play a role. Indeed, the current distribution in superconducting resonators is such that its local value can be significant even at our lowest powers leading to higher conductor losses as the power is increased. It is also interesting to note that the power entering the resonator depends on $Q$ and $Q$ due to the TLF in turn depends on the power, $Q \propto P(Q)$. Hence, the system is non-linear and in principle weakly hysteretic.

The main conclusion from fig. 2 is that the loss tangent is strongly power dependent for SiO$_2$/Si whereas a weak dependence is observed for identical structures on sapphire; in the latter case $Q$ only changes by about a factor of two even when the power is changed by over 40 dB.

Experimentally, the temperature dependence of the resonance frequency of superconducting resonators is quite complicated since it is affected both by the properties of the superconducting film (e.g. the kinetic inductance) and changes in the dielectric and it is in general difficult to separate these two effects. However, at temperatures $\ll T_c$, which is the case here, we can assume that the thermal effects on the kinetic inductance are small. Figure 3 shows the relative shift of the resonance frequency as a function of $h\omega/k_B T$ together with a fit to Eq. 2 All data from the resonators on a particular substrate can be fitted using a single parameter; we find $F_e^2/3\epsilon = 6 \cdot 10^{-5}$ and $2.8 \cdot 10^{-6}$ for the SiO$_2$/Si and sapphire respectively; a difference of about a factor of twenty; consistent with the loss tangent measurements. Note that the curves shown come from several resonators of two different designs (on separate chips), with each resonator measured at several different power levels, but they can all be fitted using a single parameter. We note that the values for $F_e^2/3\epsilon$ we obtain from samples fabricated on sapphire are consistently lower than those reported by Gao et al.

In order to verify that the temperature dependence does not depend on the geometry or our data analysis, we have in addition to lumped-element and $\lambda/4$ (in shunt configuration) resonators also measured undercoupled inline $\lambda/2$ coplanar resonators. Fig. 4 shows the measured temperature dependence of $1/Q_l - Q_l$ being the loaded quality factor and $f_0$. Again we can fit all the $f_0$ curves
using a single parameter, although in this case the fitting parameter $F d^2 n / 3 e$ is $4.95 \cdot 10^{-5}$, slightly lower than for the other samples. Some of this difference is likely to be due to somewhat different filling factors.

At temperatures above 1K the losses are dominated by the conductor losses described by the Mattis-Bardeen theory but at low temperatures and low microwave power conductor losses are no longer important; resonant absorption dominates and the losses increase as the temperature is reduced as captured by Eq. 1. High power on the other hand saturates the TLS thus suppressing the resonant absorption. Fig. 1 (bottom) illustrates the regime of decreasing losses at temperatures below about 300 mK, which can only be observed in a very narrow range of microwave power. This regime is obviously not described by resonant absorption Eq. 1. The likely explanation is that the relative significance of relaxation absorption processes increases for temperatures below $\approx \hbar \omega / k_B$ and intermediate power levels, just enough to suppress the resonant absorption. As the power is increased, the crossover to the relaxation losses is pushed to higher temperatures.

Whilst the presence of TLF is known to affect resonators it is nevertheless surprising how well our data can be fitted using a single parameter independent of both the frequency (over a range of several GHz) and the location on the chip. While our data do not give us a reliable way to identify the nature of these TLF, one can with some certainty conclude that their origin must somehow be intrinsic to the materials used. The bath of TLF appears to have a very wide distribution of both the tunnel splittings and the symmetry parameter of the (effective) double-well. The large $nd^2$ factor in SiO$_2$/Si comes as no surprise given the large density of TLF one would expect in an amorphous oxide material. The $nd^2$ factor in sapphire is significantly smaller, but still measurable. This is in itself quite interesting as it is well known that the only two-levels systems seen in very high-quality dielectric sapphire resonators are known to be due to relatively narrow electron spin resonances on incidental paramagnetic ions, such as Fe or Cr; if these were the origin of the TLF in our samples one would not expect the data to agree with the theory used here. This suggests that the TLF seen here are located in the interface between the film and the substrate, as opposed to the bulk material.

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