Diffractive microlensing – I. Flickering planetesimals at the edge of the Solar system

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ABSTRACT

Microlensing and occultation are generally studied in the geometric optics limit. However, diffraction may be important when recently discovered Kuiper Belt objects (KBOs) occult distant stars. In particular, the effects of diffraction become more important as the wavelength of the observation and the distance to the KBO increase. For sufficiently distant and massive KBOs or Oort cloud objects not only is diffraction important but so is gravitational lensing. For an object similar to Eris but located in the Oort cloud, the signature of gravitational lensing would be detected easily during an occultation and would give constraints on the mass and radius of the object.

Key words: gravitational lensing – Kuiper Belt – minor planets, asteroids – Oort Cloud.

1 INTRODUCTION

Bailey (1976) first argued that small bodies in the distant Solar system could be detected through stellar occultations. Roques, Moncuquet & Sicardy (1987) developed the treatment of occultation by irregular bodies including diffraction. More recently with the discovery of the population of Kuiper Belt objects (KBOs) (Kuiper 1951; Jewitt 1999), several research groups have begun searching for more distant and smaller bodies through occultations (Roques & Moncuquet 2000; Bickerton, Kavelaars & Welch 2008; Roques et al. 2009). Because KBOs typically subtend small angles, the diffraction of radiation around the objects may be important even at visible wavelengths and naturally more important at longer wavelengths (Roques, Moncuquet & Sicardy 1996). The discovery of more distant and more massive KBOs such as Eris (Brown, Trujillo & Rabinowitz 2005) begs the question of whether gravitational lensing of background stars by large KBOs and objects in the Oort cloud (Oort 1950) is important. Cooray (2002) argued that for distant massive KBOs and Oort cloud objects lensing may be important; furthermore, Gaudi & Bloom (2005) argued that GAIA could measure the astrometric displacement from microlensing by an planet more massive than a few Jupiters within $10^4$ au regardless of its location on the sky. The technique in this paper probes much lower mass objects, but also exploits lensing to provide constraints on the properties of the asteroid.

Generally the diffractive effects of microlensing are neglected because the variation in the time delays across the lens is usually much larger than the coherence time of the observation, $1/\Delta \nu$ where $\Delta \nu$ is the bandwidth of the observation. However, near a caustic crossing, diffraction may be important as argued by Jaroszynski & Paczynski (1995) to account for rapid variations in the light from Q2237+0305 (The Einstein Cross). Typically, the differential time delay of highly magnified images for a point lens is about $2GM/c^2$, the crossing time over the Schwarzschild radius of the lens; consequently, for the diffractive effects of lensing to be observable, the Schwarzschild radius of the lens should be comparable or larger than the wavelength of the radiation. The largest of the KBOs, Eris, has a Schwarzschild radius $R_S = 2GM/c^2 \approx 25 \mu m$. Therefore, quite naturally for observations of large KBOs diffractive microlensing may be important for observations in the near- and mid-infrared whenever the gravitational effects are important. This paper focuses on just this regime.

The first section, Section 2, outlines microlensing in the diffraction regime and generalizes the earlier results to include occultation. This yields an expression for the transmission that is nearly identical to the unlensed result. The next section, Section 3, outlines the types of objects for which diffractive lensing may be important, examines several interesting cases and connects the diffraction patterns to the geometric limit (Section 2.2) in the limit where the lensing is weak. The final section (Section 4) outlines how diffractive lensing could constrain the properties of known objects and speculates on the probability of such lensing events.

2 DIFRACTIVE MICROLENSING

The following definitions will prove useful throughout the paper. The source lies a distance $v$ in the plane of the sky from the centre of the lensing occulter. The variable $u$ is a radial variable over the plane containing the lensing occulter and $\varphi$ gives the polar angle in this plane with $\varphi = 0$ pointing towards the projection of the source on to this plane. The magnification is given by squared modulus of the integral over the phases in the lens plane (Schneider, Ehlers &
vides a mapping to convert an unlensed diffraction pattern to an
following section will show, the geometric optics approximation
To compare with the standard, high-frequency results for lensing
2.2 Geometric optics
The lower bound accounts for the effects of occultation
The size of the Einstein radius in these units is simply
The limit where the gravitational field of the lens is negligible is
...f u/2)} μω
du ≡ μω = \int_{u_d}^{\infty} u^{1-f} e^{u^2/2} \int_{0}^{2\pi} d\phi e^{-iuv\cos\phi} = \frac{1}{2} \left(1 - i \frac{f}{2}\right)

The result for f = 0 is simply i exp (−i v^2/2). These analytic results are used to calculate the integral in equation (4), both with and without lensing. Rather than performing the integration to u → ∞, the result of the integration from u = 0 to u = u_d is subtracted from equation (5).

2.2 Geometric optics
To compare with the standard, high-frequency results for lensing and occultation (e.g. Agol 2002), the geometric limit is useful. In the geometric limit, the images are located at those places in the image plane u where the phase in the integral for μω is stationary, so v = u − f/u, yielding two image positions:

u_± = 1 \left( v ± \sqrt{v^2 + 4f} \right), \phi = 0

with the magnifications

μ_± = \frac{u_±}{v} \left| \frac{du_±}{dv} \right| = \frac{1}{2} \left( \frac{v^2 + 2f}{v \sqrt{v^2 + 4f}} \pm 1 \right).

The size of the Einstein radius in these units is simply √T, so if √T > u_d one expects that lensing will be important. To account for the occultation in the limit of a point source, only the magnification for images with u_+ > u_d is included (Agol 2002). As the following section will show, the geometric optics approximation not only provides an estimate of the magnification, but it also provides a mapping to convert an unlensed diffraction pattern to an approximation of the lensed pattern through the locations of images in equation (6).

3 RESULTS
Because KBOs provide the inspiration for this work, it is natural to take the largest KBO, Eris, as an exemplar. Eris is a dwarf planet orbiting the Sun at a distance of up to 100 au. Its radius is about 1200 km (Brown et al. 2006; Stansberry et al. 2008), yielding a value of u_d = 770 at 1 μm and a Fresnel number of about 10^6. Its mass is about 1.7 × 10^22 g (Brown & Schaller 2007), so its Schwarzschild radius is about 25 μm so f ≈ 310 at 1 μm. At 100 au, the diffraction effects are not important over a reasonable bandwidth. Furthermore, u_d ≫ √T so microlensing is not important either.

On the other hand, Eris is likely to be one of the closest and biggest members of a large population of KBOs and Oort cloud objects. Eris provides a fiducial density of about 2.3 g cm\(^{-3}\) (about one-half that of the Earth, 5.5 g cm\(^{-3}\)) to examine a variety of objects at various distances and sizes and determine the possible regimes where microlensing and diffraction could both be important. Fig. 1 examines the various regimes using the fiducial density of Eris and a wavelength of 1 μm. The values of f and u_d^2 at 1 μm are depicted on the figure, and they both scale inversely with wavelength. In particular, to exploit the point-source approximation in the previous section, the angular size of the object or the angular size of the object’s Einstein radius should be greater than the angular size of the typical source (e.g. a solar-type star in the bulge taken to be 7.6 kpc away). In the figure, the point-source approximation is valid for objects that lie either to the right of the green line or the blue lines (the left-hand blue line uses Earth’s mass density). Microlensing becomes important for objects near or above the red lines (the lower red line uses Earth’s mass density).

The figure indicates that observations of Eris at the distance of 100 au give u_d^2 ≈ 10^6 at 1 μm and f ≈ 10^6, so u_d^2 ≫ f ≫ 1 so neither lensing nor diffraction are important for Eris, as discussed earlier (at least at 1 μm – Eris will diffract for λ ≈ 1 m). On the other hand, if there were an Eris-like object at 10^2 au (upper cross)
and it was observed at 10 μm, \( u_d = 7.8 \) and \( f \approx 31.4 \), yielding interesting diffractive effects over a reasonable bandwidth. At this distance Eris would subtend about \( 1.7 \times 10^{-15} \) radians, larger than any Sun-like star beyond a few hundred parsecs, so the star can be taken to be a point source. At 10^6 au, the asteroid is essentially beyond the realm of the Solar system as passing molecular clouds would unbind objects beyond \( 4 \times 10^6 \) au (Bailey 1983). Because this is a statistical process, some objects would remain but they would be quite rare.

Regardless of the paucity of objects at this distance, an Eris-like object at 10^5 au provides an excellent illustration of diffractive microlensing and occultation. Beyond a distance of one parsec, Eris would no longer fully occult a star because even when fully aligned the lensing would be sufficient to bend the light around the limb of Eris and towards Earth. Even at this distance solar-like stars beyond a kiloparsec would be essentially point sources for the purposes of the lensing signature. Furthermore, the diffraction pattern depends on the values of \( u_d \) and \( f \), so it would also apply to more massive and nearer objects but at larger wavelengths — e.g. observations of an object like Earth at 85 GHz from a distance of 8000 au give the same set of values.

Although such objects would be difficult to detect and the possibility of an occultation event for a given object would be small, the properties of an occultation and microlensing event for an Eris-like object at 10^5 and 10^6 au are extremely illustrative. At 10^5 au, diffraction is important even at 1 μm. Fig. 2 gives the diffraction and lensing pattern for an Eris-like asteroid at 10^6 au at 1 and 10 μm. The complete diffraction pattern including the effects of gravitational lensing is given in black. The diffraction pattern with \( f = 0 \) (no lensing) is given in red. The diffraction pattern including lensing oscillates about the geometric-optics value of the microlens magnification (green curve) from equation (7). One can obtain an approximate idea of the diffraction pattern including lensing by using the lensing mapping, i.e. \( \nu \rightarrow \nu - f/\nu \), and taking the product of the geometric magnification with the unlensed diffraction pattern resulting in the blue curve. When the lensing effect is weak as in Fig. 4, this a posteriori lensing correction works quite well, so occultation diffraction patterns can be corrected for a modest amount of lensing without calculating the full lensing diffraction pattern. However, this mapping fails to reproduce the central fringing that occurs when both diffraction and lensing are important.

Averaging the magnification over a finite bandwidth smooths out much of the small-scale oscillation in the light curves as shown in Fig. 3. However, even over a moderate bandwidth of 20 per cent the microlensed diffraction patterns are easily distinguished from the unlensed patterns. Again the lensing mapping does a reasonable job of reproducing the pattern. This is important to emphasize. Microlensing does not simply contract the diffraction pattern, so it is not covariant with varying the relative velocities of the lens, source and detector nor is it covariant with varying the impact parameter of the occultation (how close to exactly aligned the source, lens and detector become). The lensing both increases the amplitude of the diffractive oscillations and shifts the locations of the peaks and troughs in a non-trivial manner, following the lensing mapping if the lensing is weak.

If the Eris-like asteroid were closer to Earth at 10^4 au, Fig. 1 indicates that the lensing would be weaker (the lower cross is well below the red lines). The results depicted in Fig. 4 bear this out. The location of the first peak moves inwards from 1.09\( u_d \) to 1.04\( u_d \) because the light is slightly bent about the limb of the asteroid, decreasing the duration of the occultation by about 4.8 per cent, approximately the value of \( f/\nu_3^2 \approx 5.2 \) per cent.

**4 CONCLUSIONS**

Microlensing is important for large asteroids (similar to Eris) in the Oort cloud and beyond; furthermore, in the near-infrared and
redward diffraction is important to understand the light curves from the combined microlensing and occultation of background stars by such objects. The effects of microlensing are not covariant with variations in the velocity of source, lens and observer nor with variations in the impact parameter; therefore, observations of diffractive microlensing combined with a measured distance to the asteroid constrain the mass and radius of the asteroid, or equivalently with an assumed value of the density of the asteroid, such observations would yield a mass, radius and distance. Observations of diffractive microlensing may provide the only way of estimating the masses of such objects unless they have a satellite as Eris does (Brown & Schaller 2007).

The angular size of even a large asteroid such as Eris is extremely small at the distance of the Oort cloud, so one would expect that the chance for an occultation would be tiny. Over the course of a year, the asteroid will sweep out a region of the sky. The chance for an occultation would be tiny. Over the course of that year. If one assumes that the Oort cloud contains about 10 Earth masses of objects similar to Eris at about 10^4 au, one would have to monitor about 4 × 10^8 stars for 12 yr (24 h per day) to find a single microlensing occultation event by an object at 10^4 au. The motion of the Earth in its orbit determines the duration of the event of about a minute.

Currently the Optical Gravitational Lensing Experiment (OGLE) collaboration monitors about 4 × 10^8 stars – the present state of the art. OGLE-III ran from 2001 to 2009 and made nearly 2 × 10^11 photometric measurements (Udalski et al. 2008). The key is not only the sensitivity of the current slew of experiments, although monitoring additional stars or using several telescopes separated by at least the diameter of the asteroid would increase the detection rate, but the cadence of OGLE-III at about 5 min would simply undersample and miss such an event. OGLE-IV will provide both a higher sensitivity and a higher cadence, but probably not a rapid enough cadence to distinguish such an event. The best approach would be a difficult hybrid of the high cadence of observations such as Roques et al. (2006) on a large telescope and the monitoring of many stars as OGLE. Because the goal is not small bodies as in Roques et al. (2006), a 21 ms cadence is overkill; an 1 s cadence would be sufficient allowing observations of much fainter stars. Furthermore, stars with especially small angular sizes are not necessary. Both of these factors along with the dedicated use of a large telescope could make probing the largest and presumably the rarest objects in the Oort cloud possible.

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