Permeability impact on electromagnetohydrodynamic flow through corrugated walls of microchannel with variable viscosity

Madhia Rashid, Sohail Nadeem and Iqra Shahzadi

Abstract
This investigation based on electromagnetohydrodynamic flow in microchannels through lightly corrugated walls effects is reported in the presence of variable liquid properties. In microparallel plates, we consider incompressible and electrically conducting viscous fluid. With small amplitudes, the wall corrugations are described by periodic sin waves. The governing equations are rendered dimensionless and solved with the help of the perturbation technique. The analytical solutions for velocity are obtained and analyzed graphically. A connection between flow rate and roughness is acquired by perturbation solutions of the stream function. By utilizing numerical computations, we analyzed the corrugation consequences on the velocity for electromagnetohydrodynamic flow. We graphically clarified the velocity and temperature profiles and their dependencies on all parameters. The three-dimensional velocity and contour distributions shown that the wall roughness can cause changes in the velocity distribution. For in phase the phase difference among the two corrugated walls is equals to 0, and for out of phase the phase difference is equal to 180 between the two walls. The wave phenomenon of the flow shape becomes obvious with the expansion of the corrugation. The electromagnetohydrodynamic velocities first grow and then reduce. The electromagnetohydrodynamic velocity increases for Reynolds number, Hartmann number, and Darcy parameter. Velocity profile decreases for variable viscosity, velocity slip parameter.

Keywords
Electromagnetohydrodynamic, corrugated walls, perturbation method and variable viscosity

Date received: 4 February 2020; accepted: 24 June 2020

Handling Editor: James Baldwin

Introduction
Microfluidics is most significant in small-scale electromechanical framework. It is used for blending, stream control, division, discovery, and contemplating basic biochemical and physical procedures. Microfluidics assumes significant job in innovative procedures including productive plan of the transfer of mass and heat. In recent years, numerous microfluidic gadgets created, for example, the electro-assimilation micropumps, and electromagnetohydrodynamic (EMHD) micropumps. The significant microfluidic framework is EMHD micropump which produces the ceaseless stream design. In EMHD micropumps, the siphoning source is Lorentz power, because of the communication among

1Department of Mathematics, Quaid-i-Azam University, Islamabad, Pakistan
2Mathematics and Its Applications in Life Sciences Research Group, Ton Duc Thang University, Ho Chi Minh City, Vietnam
3Faculty of Mathematics and Statistics, Ton Duc Thang University, Ho Chi Minh City, Vietnam
4Department of Mathematics, Air University, Islamabad, Pakistan

Corresponding author:
Sohail Nadeem, Mathematics and Its Applications in Life Sciences Research Group, Ton Duc Thang University, Ho Chi Minh City 7000, Vietnam.
Email: sohail.nadeem@tdtu.edu.vn
electric and attractive fields. The EMHD micropump in microfluidic frameworks has significant research, for example, stream control in the fluidic systems, siphoning, mixing and blending, microcoolers, and thermal reactors.\textsuperscript{3-7} Utilizations of EMHD gadgets, for example, liquid siphoning, in fluidic systems the stream control and liquid blending, fluid chromatography and mixing\textsuperscript{8} in microchannels, a lot of the attention is paid toward the numerical and analytical models of EMHD stream.\textsuperscript{9,10} The impact of electromagnetic fields superficially pressure between parallel plate microchannel is examined by Tso and Sundaravadi\textsuperscript{11} et al.\textsuperscript{21} Chakraborty and Paul\textsuperscript{12} explore the EMHD powers impact on liquid stream for parallel plate microchannel. Nagaraju et al.\textsuperscript{13} discussed the impact of second law analysis of flow in a circular pipe with magnetic field effects.

The past investigations dependent on smooth channels. Harshness consistently exists on the outside of channel because of adsorption of different species. Unpleasantness applied in mechanical assembling and biomedical territories.\textsuperscript{14,15} The impact of roughness together with corrugated walls in microparallel channel for EMHD Newtonian fluid is investigated by Bure et al.\textsuperscript{16} The electroosmotic Jeffrey fluid flow is considered between the slit of microchannel by Liu et al.\textsuperscript{17} Nagaraju and M Garvandha\textsuperscript{18} explored the effect of magnetohydrodynamic (MHD) viscous fluid flow in a circular pipe. In microchannel, Shojaeian and Shojaeian\textsuperscript{2} explored analytical solution of mixed electromagnetic gaseous flows. In micropumps systematic and numerical computations of electromagnetic fields investigated by Rivero and Cuevas.\textsuperscript{8} Reddy et al.\textsuperscript{19} researched the EMHD flow instabilities in a two-phase. The magnetic and electric field impacts are considered on corrugated walls of microchannels in the presence of EMHD effect by D Si and Jian.\textsuperscript{20} The Stokes problem for the viscous fluid is examined by Phan-Thien\textsuperscript{21} by considering corrugated pipes. The impact of roughness among corrugated plates is investigated by Wang.\textsuperscript{22} The method of perturbation is applied by Chu\textsuperscript{23} to see the impact of surface roughness for circular microtube having corrugated walls. Gajjela et al.\textsuperscript{24} examined the impact of mass transfer in a horizontal pipe with magnetic Newtonian flow. C-O Ng and Wang\textsuperscript{25} explored the Darcy–Brinkman flow in the presence of corrugation. The flow depends on the orientation of corrugations as well as the phase difference of corrugations. Govardhan et al.\textsuperscript{26} explored the effect of MHD and radiation on mixed convection unsteady flow over the stretching sheet.

Currently, investigation of fluid flows and heat transfer over porous medium has engrossed much attention. It is a fact that porous medium has many real-world examples. Examples of natural porous media phenomena are beach sand, bile duct, sandstone, limestone, rye bread, and wood. In human beings, natural porous mediums are the human lung, gall bladder with stones and in small blood vessels. Some of pathological states, for example, the distribution of full of fat cholesterol and in the lumen of coronary artery’s clogging blood can be measured as alike to a porous medium. Darcy’s law is very much essential in order to study the fluid flow problems in porous medium. Flow of fluid in permeable medium is driven by Darcy’s law, while the liquid in free stream locale is executed by Navier Stokes condition. Different viable applications experience the course through a permeable medium especially in geophysical liquid elements. Hasan et al.\textsuperscript{27} deliberated the convective radiative flow of nanofluid through porous medium. Shirvan and Colleagues presented the investigation and sensitivity analysis of effective parameters on combined heat transfer performance in a porous solar cavity receiver by response surface methodology. The peristaltic flow in a porous medium through an annulus is inspected by Mekheimer and Elmaboud.\textsuperscript{29} Rapits et al.\textsuperscript{30} and Varshney\textsuperscript{31} have tackled issues of the progression of a viscous fluid through a permeable medium bounded by a vertical surface. Mekheimer and Al-Arabi\textsuperscript{32} examined nonlinear peristaltic transport of MHD move through a permeable medium. El-Sayed\textsuperscript{33} examined the electrohydrodynamic instability of two superposed viscous and streaming fluids through permeable medium. The stenosed arteries having permeable walls for nanofluid flow are examined by N Akbar et al.\textsuperscript{34} The impact of metallic nanoparticles on the flow of blood through stenosed artery having permeable walls is described by Nadeem and Ijaz.\textsuperscript{35} Transfer of heat through convective marvels is a fundamental part in specific techniques, for example, material drying, and transpiration cooling process and warm vitality stockpiling. So, it appears to be sensible to consider the convective limit condition in its place of isoflux or isothermal conditions and mixed convection impacts.\textsuperscript{36} The impacts of convective boundary for asymmetric channel for the model of peristaltic flows is investigated by Munir et al.\textsuperscript{37} Moreover, the Rabinowitsch fluid model along with convective boundary is explored by Sadaf and Nadeem.\textsuperscript{38} The principle motivation of this investigation is to deliberate the flow of EMHD viscous fluid with variable viscosity through corrugated microparallel plates having permeable walls. The framework is considered under the impact of Lorentz power which is produced by electrical and attractive field connection. The EMHD equations for viscous fluid under wavy and permeable condition are derived and then analytical solutions for velocity are calculated by applying perturbation. The sundry parameter impacts are analyzed through graphs.
Figure 1. Geometrical sketch of EMHD flow in microchannel.

Mathematical formulation

We consider the EMHD flow of viscous, incompressible, and electrically conducting Newtonian fluids between corrugated walls separated by 2H distance. We assumed that the length L of channel in z-direction and in x-direction width W and the flow is taken opposite corrugation due to Lorenz force. The flow is taken opposite to corrugations of the walls. The wavy upper and lower walls are described by

\[ y_u^* = H + \varepsilon H \sin(\lambda^* x^*) \quad \text{and} \quad y_l^* = -H - \varepsilon H \sin(\lambda^* x^*) \]  

(1)

The periodic sinusoidal describes the corrugations of wavy walls where \( \lambda^* \) is the wave number and \( \varepsilon \) is the amplitude. We can apply electric field \( \vec{E}^* \) in x-direction and magnetic field \( \vec{B}^* \) in y-direction, \( \vec{J} \times \vec{B}^* \) is Lorenz force taken along the z-direction and created by electric and magnetic field interaction, where current density is symbolized by \( \vec{J} \) (see Figure 1).

The conservation of mass and momentum equations is expressed as

\[ \nabla^* \vec{u}^* = 0 \]  

(2)

\[ \rho \frac{\partial \vec{u}^*}{\partial t^*} + \rho (\vec{u}^* \cdot \nabla^*) \vec{u}^* = -\nabla^* p + \nabla^* (\mu \nabla^* \vec{u}^*) + \vec{J} \times \vec{B}^* \]  

(3)

We consider incompressible fluid between microparallel plates which are taken along z-direction. Pressure gradient in microchannel is neglected because adopting open channel in the z-direction, thus the momentum equation becomes

\[ \rho \frac{\partial \vec{w}^*}{\partial t^*} = \mu^* (y^*) \left( \frac{\partial^2 \vec{w}^*}{\partial x^2} + \frac{\partial^2 \vec{w}^*}{\partial y^2} \right) + \frac{\partial}{\partial y^*} \left( \mu^* (y^*) \frac{\partial \vec{w}^*}{\partial y^*} \right) + \alpha B^* (E^* - B^* \vec{w}^*) \]  

(4)

Velocity and electric field in periodical forms are expressed as

\[ \vec{w}^* = R \{ \vec{w}(x^*, y^*) e^{i \omega t^*} \}, \quad E^* = R \{ E_0 e^{i \omega t^*} \} \]  

(5)

where real part is denoted by \( R \{ \} \); \( \vec{w} \), \( \omega^* \), and \( E_0 \) are amplitude of velocity, angular frequency, and electric field; and imaginary unit is \( i \). Using equation (5) into equation (4), we get

\[ i \rho \omega \vec{w} = \mu^* (y^*) \left( \frac{\partial^2 \vec{w}}{\partial x^2} + \frac{\partial^2 \vec{w}}{\partial y^2} \right) + \frac{\partial}{\partial y^*} \left( \mu^* (y^*) \frac{\partial \vec{w}}{\partial y^*} \right) \]

\[ + \alpha B^* E_0 - \alpha B^* \vec{w} \]  

(6)

Non-dimensional variables are defined as

\[ (x, y) = \frac{(x^*, y^*)}{H}, \quad w = \frac{\vec{w}}{H \omega}, \quad \lambda^* = \lambda H \]  

(7)

Using equation (7) into equation (6), we get

\[ \mu(y) \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) + \frac{\partial}{\partial y^*} \left( \mu(y) \frac{\partial w}{\partial y^*} \right) \]

\[ - (Ha^2 + Re)i \omega w + Ha \beta = 0 \]  

(8)

where

\[ Re = \frac{\rho o H^2}{\mu_0}, \quad Ha = B^* H \left( \frac{\sigma}{\mu_0} \right)^{\frac{1}{2}}, \quad \beta = \left( E_0 \left( \frac{\sigma}{\mu_0} \right)^{\frac{1}{2}} \right) / \omega, \quad \mu(y) = \frac{\mu^* (y^*)}{\mu_0} \]  

(9)

where \( Re \), \( Ha \), \( \beta \), and \( \mu(y) \) represent the Reynolds number, Hartmann number, non-dimensional parameter, and variable viscosity. The corresponding non-dimensional boundary conditions are given by Wen et al.\(^{41}\) and Akbar et al.\(^{42}\)

\[ w = 1 - \sqrt{\frac{Da}{\delta}} \left( \frac{\partial w}{\partial y} \right) \text{at } y = y_u \]  

(10)

Perturbation analysis

In order to solve equation (10), we consider the perturbation expansion by writing

\[ w(x, y) = w_0(y) + \epsilon w_1(x, y) + \epsilon^2 w_2(x, y) + \cdots \]  

(11)

Now using equation (11) into equation (9) and the boundary conditions (equation (10)) are expand by Taylor series about upper and lower wavy wall positions at \( y = 1 \) and \( y = -1 \) expanded by Taylor series...
and collecting the coefficients of like powers of \( \varepsilon \), one gets the zeroth-order equation as

\[
\mu(y) \frac{d^2 w_0}{dy^2} + \frac{d}{dy} \left( \mu(y) \frac{dw_0}{dy} \right) - (Ha^2 + \text{Re}i)w_0 = 0 \quad (12)
\]

along with the corresponding boundary conditions

\[
w_0[y] = -1 - \frac{\sqrt{Da}}{\delta} \left( \frac{dw_0}{dy} \right) \text{ at } y = 1
\]

\[
\frac{dw_0}{dy} = 0 \text{ at } y = -1 \quad (13)
\]

\[
w_0(y) = \begin{cases} 
HaB/(Ha^2 + \text{Re}i) + \left( \sqrt{-2/\alpha + 2} \right) \left( \text{BesselI} \left( 0, 2 \sqrt{(-1/\alpha + y) (-Ha^2 - \text{Re}i)} \right) \right) \\
A_1 + \text{BesselK} \left( 0, 2 \sqrt{(-1/\alpha + y) (-Ha^2 - \text{Re}i)} \right) A_2 \bigg/ \sqrt{2 - 2\alpha y} 
\end{cases} 
\]

The first-order perturbation equation is found in the form

\[
\mu(y) \left( \frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2} \right) + \frac{\partial}{\partial y} \left( \mu(y) \frac{\partial w_1}{\partial y} \right) - (Ha^2 + \text{Re}i)w_1 = 0 \quad (14)
\]

The corresponding boundary conditions are

\[
w_1 + \sin (\lambda x) \frac{\partial w}{\partial y} + \sqrt{Da} \left( \frac{\partial w}{\partial y} \right) + \sin (\lambda x) \frac{\partial^2 w_0}{\partial y^2} = 0 \text{ at } y = 1
\]

\[
\frac{\partial w_1}{\partial y} + \sin (\lambda x) \frac{\partial^2 w_0}{\partial y^2} = 0 \text{ at } y = -1 \quad (15)
\]

The second-order perturbation equation is found in the form

\[
\mu(y) \left( \frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2} \right) + \frac{\partial}{\partial y} \left( \mu(y) \frac{\partial w_2}{\partial y} \right) - (Ha^2 + \text{Re}i)w_2 = 0 \quad (16)
\]

The corresponding boundary conditions are

\[
w_2 + \sin (\lambda x) \frac{\partial w_1}{\partial y} + \frac{1}{2} \sin^2 (\lambda x) \frac{\partial^2 w_0}{\partial y^2} + \frac{\sqrt{Da}}{\delta} \left( \frac{\partial w}{\partial y} \right) + \sin (\lambda x) \frac{\partial^2 w_0}{\partial y^2} + \frac{1}{2} \sin^2 (\lambda x) \frac{\partial^2 w_0}{\partial y^2} = 0 \text{ at } y = 1
\]

\[
\frac{\partial w_2}{\partial y} + \sin (\lambda x) \frac{\partial w_1}{\partial y} + \frac{1}{2} \sin^2 (\lambda x) \frac{\partial^2 w_0}{\partial y^2} = 0 \text{ at } y = -1 \quad (17)
\]

The effect of variable viscosity on flow can be investigated for any given function \( \mu(y) \). For the present investigation, we assume the viscosity variation in the dimensionless form as

\[
\mu(y) = e^{-\alpha y} \text{ or } \mu(y) = 1 - \alpha y \text{ for } \alpha = 1 \quad (18)
\]

**Solution of the problem**

By solving zero-order system with the corresponding boundary conditions (equation (13)), we obtained

\[
\mu(y) = e^{-\alpha y} \text{ or } \mu(y) = 1 - \alpha y \text{ for } \alpha = 1 \quad (18)
\]

The boundary conditions are transformed in the following form

\[
f(y) + \frac{d w_0}{dy} + \sqrt{Da} \left( \frac{d f(y)}{dy} + \frac{d^2 w_0}{dy^2} \right) = 0 \text{ at } y = 1
\]

\[
\frac{df(y)}{dy} + \frac{d^2 w_0}{dy^2} = 0 \text{ at } y = -1 \quad (22)
\]

Based upon boundary conditions, the solutions of equation (21) are

\[
f(y) = \begin{cases} 
e^{-\lambda B_1} \text{Hypergeometric } U \left[ \frac{Ha^2 + \text{Re}i - \alpha \lambda}{2a \lambda}, 1, -\frac{2a}{\alpha} + 2\lambda \right] \\
e^{-\lambda B_2} \text{Laguerre } L \left[ \frac{Ha^2 + \text{Re}i - \alpha \lambda}{2a \lambda}, -\frac{2a}{\alpha} + 2\lambda \right] \\
e^{-\lambda B_1'} \text{Hypergeometric } U \left[ -\frac{Ha^2 + \text{Re}i - \alpha \lambda}{2a \lambda}, 1, -\frac{2a}{\alpha} + 2\lambda \right] \\
e^{-\lambda B_2'} \text{Laguerre } L \left[ \frac{Ha^2 + \text{Re}i - \alpha \lambda}{2a \lambda}, -\frac{2a}{\alpha} + 2\lambda \right] 
\end{cases} 
\]

\[
(23)
\]
First-order problem solution can be expressed as



\[
\begin{align*}
& w_1(x, y) = \sin(\lambda x) \\
& + e^{-\gamma h} B_1 \text{Hypergeometric U} \left[ \frac{-Ha^2 + \text{Re}i - \alpha \lambda}{2 \alpha \lambda}, 1, - \frac{2 \lambda}{\alpha} + 2 y \alpha \right] \\
& + e^{-\gamma h} B_2 \text{Laguerre L} \left[ \frac{-Ha^2 + \text{Re}i - \alpha \lambda}{2 \alpha \lambda}, - \frac{2 \lambda}{\alpha} + 2 y \alpha \right] \\
& + e^{-\gamma h} B'_1 \text{Hypergeometric U} \left[ \frac{-Ha^2 + \text{Re}i - \alpha \lambda}{2 \alpha \lambda}, 1, - \frac{2 \lambda}{\alpha} + 2 y \alpha \right] \\
& + e^{-\gamma h} B'_2 \text{Laguerre L} \left[ \frac{-Ha^2 + \text{Re}i - \alpha \lambda}{2 \alpha \lambda}, - \frac{2 \lambda}{\alpha} + 2 y \alpha \right]
\end{align*}
\]

The boundary conditions (equation (17)) of the second-order system can be simplified using the solution (equations (19) and (24)). Based on boundary conditions, the second-order system solution can be chosen as

\[
w_2(x, y) = g(y) + \cos(2\lambda x) h(y)
\]

By utilizing equation (25) into equations (16) and (17), we get the following forms

\[
(1 - \alpha y) \frac{d^2 g(y)}{dy^2} - \alpha \frac{dg(y)}{dy} - (Ha^2 + \text{Re}i) g(y) = 0
\]

\[
(1 - \alpha y) \left( \frac{d^2 h(y)}{dy^2} - 4 \alpha^2 y h(y) \right) - \alpha \frac{dh(y)}{dy} - (Ha^2 + \text{Re}i) h(y) = 0
\]

The boundary conditions are transformed into the following form

\[
g(y) + \frac{1}{2} \left( \frac{df(y)}{dy} + \frac{1}{2} \frac{d^2 w_0}{dy^2} \right)
\]

\[
+ \frac{\sqrt{Da}}{\delta} \left( \frac{dg(y)}{dy} + \frac{1}{2} \left( \frac{d^2 f(y)}{dy^2} + \frac{1}{2} \frac{d^3 w_0}{dy^3} \right) \right) = 0 \text{ at } y = 1
\]

\[
\frac{dg(y)}{dy} + \frac{1}{2} \left( \frac{d^2 f(y)}{dy^2} + \frac{1}{2} \frac{d^3 w_0}{dy^3} \right) = 0 \text{ at } y = -1
\]

By utilizing the above boundary conditions, the exact solutions of \(g(y)\) and \(h(y)\) can be expressed as

\[
g(y) = \left\{ \begin{array}{l}
\sqrt{-2/\alpha + 2y} \left( \text{Bessel} 0, 2 \sqrt{-1/\alpha + y} \left( -Ha^2 - \text{Re}i \right) \right) C_1 \\
+ \text{BesselK} 0, 2 \sqrt{-1/\alpha + y} \left( -Ha^2 - \text{Re}i \right) C_2 \end{array} \right\} \sqrt{-2 - 2\alpha y}
\]

\[
h(y) = \left\{ \begin{array}{l}
\sqrt{-2/\alpha + 2y} \left( \text{Bessel} 0, 2 \sqrt{-1/\alpha + y} \left( -Ha^2 - \text{Re}i \right) \right) C'_1 \\
+ \text{BesselK} 0, 2 \sqrt{-1/\alpha + y} \left( -Ha^2 - \text{Re}i \right) C'_2 \end{array} \right\} \sqrt{-2 - 2\alpha y}
\]

Thus, we obtained solution of the second-order system.
The mean velocity on average over one wavelength (0, $2\pi/\lambda$) of the corrugations can be expressed as

$$w_m^\pm = \frac{1 + \varepsilon \sin(\lambda x)}{2\pi/\lambda} \int_{-\lambda}^{\lambda} w^\pm(x, y) dx$$

Inserting equation (35) into equation (36), the mean velocity takes the form

$$w_m^\pm = \frac{1 + \varepsilon \sin(\lambda x)}{2\pi/\lambda} \int_{-\lambda}^{\lambda} w^\pm(x, y) dx$$

$$= w_{0m}[1 + \varepsilon^2 \phi^\pm + O(\varepsilon^4)]$$

where $w_{0m}$ indicates the mean velocity for the perfectly smooth walls and $\phi^\pm$ indicates the leading order perturbations to a mean velocity due to the corrugations. When $\phi^\pm$ is positive then mean velocity increases, while when $\phi^\pm$ negative then mean velocity decreases.

Results and discussion

In previous sections, we obtained analytical solutions using perturbation method for velocity and volume flow rate of EMHD fluids bounded by microparallel plates with corrugated walls. For general microfluidic analysis, consider $H \sim 100 \mu m$ is half height of channel, the conditions of domain on density of water set with physical properties is $\rho \sim 10^3$ kg m$^{-3}$, the electrical conductivity $\sigma \sim 2.2 \times 10^{-4} \sim 10^6$ S m$^{-1}$ and the viscosity $\mu \sim 10^{-3}$ kg m$^{-1}$ s$^{-1}$. If range of magnetic field is the $O(B^\prime) \sim 0.018 \sim 0.447$. The electric field frequency $O(\omega)$ is changed from the 50 to 500 s$^{-1}$, and range of the frequency is $0 \sim 1 \times 10^6$ s$^{-1}$. The order of Reynolds number $O(Re)$ is changed between 0.5 and 5, and the dimensionless parameter is fixed value, that is, $\beta = 5$. 
The three-dimensional velocity and contour distributions for various values of variable viscosity are shown in Figures 2 and 3. In microchannel, the wall roughness can cause changes in the velocity distribution. In Figure 2, the phase difference among the two corrugated walls equals $0^\circ$. In Figure 3, the phase difference is equal to $180^\circ$ between the two walls. As shown in Figures 2 and 3, the wave phenomenon of the flow shape becomes obvious with the expansion of the corrugation. The wavy pattern increases by increase in the value of parameters, and we find that the velocity distribution is dependent on the shape of channel.

The two-dimensional (2D) variations of the EMHD velocity $w$ for various estimations of Reynolds number, Hartmann number, Darcy number, variable viscosity, and velocity slip parameter are shown in Figures 4–8. The 2D variations of these parameters are taken at $x = 0.5$, when we take $\varepsilon = 0.1$ and $\beta = 5$. From these Figures, the velocities first grow and then reduce by increasing values of $y$. Figure 4 shows that the variation of the velocity for different values of Reynolds number $Re$, with increasing Reynolds number $Re$ the velocity increases. Figure 5 shows that the velocity $w$ increases with different values of Hartmann number $Ha$. The effect of the porous parameter on the 2D velocity profile is investigated in Figure 5. It is examined that velocity profile increases with increasing values of Darcy parameter. Figure 7 is plotted to examine the behavior of variable viscosity on the velocity profile. Velocity profile decreases with different values of variable viscosity $\alpha$. Figure 8 illustrated the impact of velocity slip parameter on the velocity profile. It is observed that the velocity decreases with the increasing velocity slip parameter $\delta$.

**Tables description**

The impact of mean velocity $\phi^z$ on EMHD flow of variable viscosity discussed in the microchannel through corrugated walls, Tables 1 and 2 expressed the behavior of variable viscosity $\alpha$ on the mean velocity $\phi^z$. Table 1 demonstrates that the mean velocity $\phi^z$ increases with the increasing value of $x$ and the mean velocity $\phi^z$ increases with the raise of the variable

![Figure 2. 3D velocity distribution and contour for $\delta = 0.1$ and $\delta = 0.4$ in phase.](image-url)
viscosity $\alpha$. Table 2 shows that the mean velocity $\phi^+_{\text{m}}$ decreases with the increasing value of $x$ and the mean velocity $\phi^+_{\text{m}}$ increases with the raise of the variable viscosity $\alpha$.

**Deductions**

The present study analyzes the behavior of variable liquid properties on EMHD flow between corrugated

---

**Figure 3.** 3D velocity distribution and contour for $\delta = 0.1$ and $\delta = 0.4$ out of phase.

**Figure 4.** 2D velocity profile for distinct values of $\text{Re}$.

**Figure 5.** 2D velocity profile for distinct values of $\text{Ha}$.
walls through microchannel in the presence of porous medium. Under consideration, the liquid is electrically conducting and incompressible. We achieved the analytical solution of velocity $w$. Graphs for the velocity $w$ are plotted. The main findings from the present model are:

- The unobvious effects of wave on the velocity and temperature reduce by small value of $\varepsilon$ parameter.
- When amplitude $\varepsilon$ approach to 0, the profile of velocity distributions of flow through the corrugated walls along a smooth channel.
- The profile of velocity distribution depends upon the shape of the channel.

### Table 1. Effect of variable viscosity parameter $\alpha$ on mean velocity $\phi^+$.

| $x$ | $\alpha = 0.5$ | $\alpha = 1.5$ | $\alpha = 2.0$ |
|-----|----------------|----------------|----------------|
| 0   | -0.4599        | -0.2565        | -0.1824        |
| 0.1 | -0.3889        | -0.2124        | -0.1714        |
| 0.2 | -0.3179        | -0.1688        | -0.1613        |
| 0.3 | -0.2469        | -0.1252        | -0.1522        |
| 0.4 | -0.1759        | -0.0816        | -0.1470        |
| 0.5 | -0.1049        | -0.0380        | -0.1339        |
| 0.6 | -0.0339        | -0.0056        | -0.1227        |
| 0.7 | -0.0371        | -0.0051        | -0.1119        |
| 0.8 | -0.0339        | -0.0047        | -0.1016        |
| 0.9 | -0.0371        | -0.0043        | -0.0913        |
| 1.0 | -0.0339        | -0.0038        | -0.0814        |

### Table 2. Effect of variable viscosity parameter $\alpha$ on mean velocity $\phi^-$.

| $x$ | $\alpha = 0.5$ | $\alpha = 1.5$ | $\alpha = 2.0$ |
|-----|----------------|----------------|----------------|
| 0   | 1.6596         | 3.7681         | 4.7597         |
| 0.1 | 1.6524         | 3.7671         | 4.7589         |
| 0.2 | 1.6546         | 3.7665         | 4.7577         |
| 0.3 | 1.6561         | 3.7659         | 4.7567         |
| 0.4 | 1.6570         | 3.7648         | 4.7555         |
| 0.5 | 1.6572         | 3.7639         | 4.7544         |
| 0.6 | 1.6568         | 3.7627         | 4.7532         |
| 0.7 | 1.6557         | 3.7618         | 4.7521         |
| 0.8 | 1.6540         | 3.7611         | 4.7514         |
| 0.9 | 1.6516         | 3.7608         | 4.7506         |
| 1.0 | 1.6485         | 3.7589         | 4.7495         |
• With increasing values of the Reynolds number Re, velocity profile increases.
• The EMHD velocity increases with the increasing values of Darcy number.
• The EMHD velocity increases with the increasing values of Hartmann number.
• The impacts of variable viscosity diminish the velocity profile.
• Velocity profile decreases for various values of velocity slip parameter $\delta$.
• Wave phenomenon of velocity becomes obvious with increases in the corrugation.
• Velocity profile is more prominent in the center of the channel and lesser near the sides of walls in all cases.

Declaration of conflicting interests
The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding
The author(s) received no financial support for the research, authorship, and/or publication of this article.

ORCID iD
Madhia Rashid https://orcid.org/0000-0001-6293-7569

References
1. Laser DJ and Santiago JG. A review of micropumps. J Micro mech Microeng 2004; 14: R35–R64.
2. Masliyah JH and Bhattacharjee S. Electrokinetic and colloid transport phenomena. Hoboken, NJ: John Wiley, 2006.
3. Jang J and Lee SS. Theoretial and experimental study of MHD (magnetohydrodynamic) micropump. Sens Actuators A 2000; 80: 84–89.
4. Shojaeian M and Shojaeian M. Analytical solution of mixed electromagnetic/pressure driven gaseous flows in microchannels. Microfluid Nanofluid 2012; 12: 553–564.
5. Bau HH, Zhu J, Qian S, et al. A magnetohydrodynamically controlled fluidic network. Sens Actuators B 2003; 88: 205–216.
6. Gao Y, Wong TN, Yang C, et al. Two-fluid electroosmotic flow in microchannels. J Colloid Interface Sci 2005; 284: 306–314.
7. Gleeson JP, Roche OM, West J, et al. Modeling annular micromixers. SIAM J Appl Math 2004; 64: 1294–1310.
8. Rivero M and Cuevas S. Analysis of the slip condition in magnetohydrodynamic (MHD) micropumps. Sens Actuators B 2012; 166: 884–892.
9. Shojaeian M and Shojaeian M. Viscous dissipation effect on heat transfer characteristics of mixed electromagnetic/pressure driven liquid flows inside micropumps. Korean J Chem Eng 2013; 30: 823–830.
10. Abdullah M and Duwairi HM. Thermal and flow analysis of two-dimensional fully developed flow in an AC magneto-hydrodynamic micropump. Microsyst Technol 2008; 14: 1117–1123.
11. Tso CP and Sundaravadivelu K. Capillary flow between parallel plates in the presence of an electromagnetic field. J Phys D: Appl Phys 2001; 34: 3522.
12. Chakraborty S and Paul D. Microchannel flow control through a combine electromagnetohydrodynamic transport. J Phys D: Appl Phys 2006; 39: 5364.
13. Nagaraju G, Jangili S, Ramana Murthy JV, et al. Second law analysis of flow in a circular pipe with uniform suction and magnetic field effects. J Heat Transf 2019; 141: 012004.
14. Tsougeni K, Petrou PS, Papageorgiou DP, et al. Controlled protein adsorption on microfluidic channels with engineered roughness and wettability. Sens Actuators B 2012; 161: 216–222.
15. Huang Y, Liu S, Yang W, et al. Surface roughness analysis and improvement of PMMA-based microfluidic chip chambers by CO2 laser cutting. Appl Surf Sci 2010; 256: 1675–1678.
16. Buren M, Jian Y and Chang L. Electromagnetohydrodynamic flow through a microparallel channel with corrugated walls. J Phys D: Appl Phys 2014; 47: 425501.
17. Liu QS, Jian YJ and Yang LG. Alternating current electroosmotic flow of the Jeffreys fluids through a slit microchannel. Phys Fluids 2011; 23: 102001.
18. Nagaraju G and Garvandha M. Magnetohydrodynamic viscous fluid flow and heat transfer in a circular pipe under an externally applied constant suction. Heliyon 2019; 5: e01281.
19. Reddy PD, Bandypadhyay D, Joo SW, et al. Parametric study on instabilities in a two-layer electromagnetohydrodynamic channel flow confined between two parallel electrodes. Phys Rev E 2011; 83: 036313.
20. Si D and Jian YJ. Electromagnetohydrodynamic (EMHD) micropump of Jeffrey fluids through two paral-llel microchannels with corrugated walls. J Phys D: Appl Phys 2015; 48: 085501.
21. Phan-Thien N. On the stokes flow of viscous fluids through corrugated pipes. J Appl Mech 1980; 47: 961–963.
22. Wang CY. On stokes flow between corrugated plates. J Appl Mech 1979; 46: 462–464.
23. Chu ZKH. Slip flow in an annulus with corrugated walls. J Phys D: Appl Phys 2000; 33: 627.
24. Gijbels A, Garvandha M and Matta A. Effect of mass transfer in a horizontal pipe with suction and chemical reaction on magnetic Newtonian flow. Int Inf Eng Technol Assoc 2019; 6: 527–534.
25. Ng C-O and Wang CY. Darcy–Brinkman flow through a corrugated channel. Transport Porous Med 2010; 85: 605–618.
26. Govardhan K, Nagaraju G, Kaladhar K, et al. MHD and radiation effects on mixed convection unsteady flow of micropolar fluid over a stretching sheet. Procedia Com-put Sci 2015; 57: 65–76.
27. Hassan M, Marin M, Alsharif A, et al. Convective heat transfer flow of nanofluid in a porous medium over wavy surface. Phys Lett A 2018; 382: 2749–2753.
28. Shirvan KM, Mamourian M, Mirzakhanlari S, et al. Numerical investigation and sensitivity analysis of
effective parameters on combined heat transfer performance in a porous solar cavity receiver by response surface methodology. *Int J Heat Mass Tran* 2017; 105: 811–825.

29. Mekheimer KS and Elmaboud YA. Peristaltic flow through a porous medium in an annulus: application of an endoscope. *Appl Math Inform Sci* 2008; 2: 103–121.

30. Raptis AA, Kafousias NG and Massalas CV. Free convection and mass transfer flow through a porous medium bounded by an infinite vertical porous plate with constant heat flux. *ZAMM: J Appl Math Mech/Z Angew Math Mech* 1982; 62: 489–491.

31. Varshney CL. Fluctuating flow of viscous fluid through a porous medium bounded by a porous plate. *Indian J Pure Appl Math* 1979; 10: 1558–1564.

32. Mekheimer KS and Al-Arabi TH. Nonlinear peristaltic transport of MHD flow through a porous medium. *Int J Math Math Sci* 2003; 2003: 537431.

33. EI-Sayed MF. Electrohydrodynamic instability of two superposed viscous streaming fluids through porous media. *Can J Phys* 1997; 75: 499–508.

34. Akbar NS, Rahman SU, Ellahi R, et al. Nano fluid flow in tapering stenose arteries with permeable walls. *Int J Therm Sci* 2014; 85: 54–61.

35. Nadeem S and Ijaz S. Theoretical analysis of metallic nanoparticles on blood flow through stenosed artery with permeable walls. *Phys Lett A* 2015; 379: 542–554.

36. Selimefendigil F and Öztöp HF. MHD mixed convection and entropy generation of power law fluids in a cavity with a partial heater under the effect of a rotating cylinder. *Int J Heat Mass Tran* 2016; 98: 40–51.

37. Munir AF, Tasawar H and Bashir A. Peristaltic flow in an asymmetric channel with convective boundary conditions and Joule heating. *J Cent South Univ* 2014; 21: 1411–1416.

38. Sadaf H and Nadeem S. Analysis of combined convective and viscous dissipation effects for peristaltic flow of Rabinowitsch fluid model. *J Bionic Eng* 2017; 14: 182–190.

39. Malvandi A, Hedayati F and Ganji DD. Thermodynamic optimization of fluid flow over an isothermal moving plate. *Alex Eng J* 2013; 52: 277–283.

40. Rashid M, Shahzadi I and Nadeem S. Corrugated walls analysis in microchannels through porous medium under electromagnetohydrodynamic (EMHD) effects. *Results Phys* 2018; 9: 171–182.

41. Wen CY, Yeh CP, Tsai CH, et al. Rapid magnetic microfluidic mixer utilizing AC electromagnetic field. *Electrophoresis* 2009; 30: 4179–4186.

42. Akbar NS, Butt AW, Tripathi D, et al. Physical hydrodynamic propulsion model study on creeping viscous flow through a ciliated porous tube. *Pramana* 2017; 88: 52.

**Appendix I**

**Notation**

- $B$: magnetic field
- $Da$: Darcy number
- $E$: electric field
- $H$: distance between corrugated walls
- $Ha$: Hartmann number
- $j$: current density
- $L$: length of channel
- $p$: pressure
- $Re$: Reynolds number
- $t$: time
- $w$: velocity component $x$, $y$, $z$-axis, $y$-axis, and $z$-axis directions
- $\beta$: strength of electric field
- $\delta$: velocity slip
- $\varepsilon$: amplitude parameter
- $\lambda^* \mu(y)$: wave number, variable viscosity
- $\phi$: mean velocity
- $\omega$: angular frequency