Efficient quantum repeater based on deterministic Rydberg gates

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We propose an efficient quantum repeater architecture with mesoscopic atomic ensembles, where the Rydberg blockade is employed for deterministic local entanglement generation, entanglement swapping and entanglement purification. Compared with conventional atomic-ensemble-based quantum repeater, the entanglement distribution rate is improved by up to two orders of magnitude with the help of the deterministic Rydberg gate. This new quantum repeater scheme is robust and fast, and thus opens up a new way for practical long-distance quantum communication.

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Quantum information can be transmitted directly over distances above some hundred kilometers only at unpractically low rates due to loss and decoherence. In order to remedy this limitation the concept of a quantum repeater has been introduced [1], where quantum entanglement is distributed over small distances, stored in quantum memories, purified, and swapped in a nested architecture [2]. A quantum repeater can in principle be implemented with atomic ensembles and linear optics only [3]. However, despite significant progress during the last years on both the theoretical [4–6] and experimental side [7, 8], – see [9, 10] for recent reviews – the entanglement distribution rate achievable in such an architecture is still much too inefficient to be of practical interest, even under ideal conditions. This is predominantly due to the fact that linear optical methods only allow for a probabilistic entanglement manipulation, posing severe limitations on the overall success probability, and therefore on the rate of entanglement distribution.

In this letter we introduce a deterministic quantum repeater protocol using quantum gates for entanglement swapping and purification. The quantum gates rely on the Rydberg blockade effect in mesoscopic atomic ensembles [11], and the remarkable recent advances in exploiting this effect for quantum information processing [12–16]. Deterministic operation provides an enhancement of two orders of magnitude in the rate of entanglement distribution as compared with the best quantum repeater based on linear optics [3]. For realistic local errors around $10^{-3}–10^{-2}$ the new quantum repeater architecture yields a rate of about 10 ebits per sec. We thus show that deterministic quantum repeaters based on Rydberg gates open up a new avenue for high-rate, long-distance quantum communication.

In our protocol, mesoscopic atomic ensembles of the size of a few micrometers are exploited as a quantum memory. If the atoms in such ensembles are laser-excited to high-lying Rydberg states, strong and long-range van-der-Waals or dipole-dipole interactions give rise to the Rydberg blockade, which prevents the excitation of more than one Rydberg atom within a volume, which is smaller than the blockade radius [11, 16]. Based on the large nonlinearity associated with the blockade effect deterministic entangling quantum gates can be performed between collective excited states in one or different atomic ensembles by applying a series of collective and single atom laser pulses [12]. Our protocol starts by local and deterministic entanglement generation in one atomic ensemble with the help of a collective Rydberg gate. The
entanglement is then linked between neighboring sites by linear optical methods, where two photon interference is explored. Further entanglement swapping and entanglement purification are implemented based on Rydberg gates between two nearest memory atomic ensembles at one site. The protocol presented here is improved in three respects compared with conventional schemes: (i) local entanglement manipulation is performed deterministically, (ii) the number of times required to convert atomic states into photons is reduced to a minimum, (iii) the detection step in entanglement swapping and entanglement purification can be performed with the help of field ionization, thereby significantly increasing the detection efficiency.

We envision a setup with mesoscopic cold atomic ensembles with a diameter of several microns. The relevant energy levels are shown Fig. 1a and comprise an electronic ground state manifold with five sublevels $|g\rangle, |s\rangle, |s'\rangle, |t\rangle, |t'\rangle$, and two Rydberg states which we denote by $|r\rangle$ and $|r'\rangle$. Initially all the atoms are prepared in the ground state $|g\rangle$. We assume these sublevels can be addressed individually, and that atoms in the two Rydberg states experience strong interactions.

In our scheme, we first generate a qubit-type entanglement in one atomic ensemble, which can be done as follows (see Fig. 1a). i) A collective excitation in one atomic ensemble, which can be done as a Rydberg states experience strong interactions. Vert the collective excitations in sites, say, A and B, read light pulses are applied to connect sites are linked using methods from linear optics pulses to bring $\pi$ in the ground state $|s\rangle$, and two memory qubits at neighboring sites, described by $|\phi\rangle_{AB} = (|s_A\rangle|s_B\rangle + |t_A\rangle|t_B\rangle)/\sqrt{2}$. This process is heralded, with a success probability of $p = \frac{1}{2} \eta_r \eta_d \eta_{dtt}$, where $\eta_r$ is the retrieval efficiency, $\eta_d$ is the photon detection efficiency, and $\eta_{dtt}$ denotes the loss in the photonic channel with $L_{att}$ the attenuation length. If no coincidence is registered, the local entanglement generation and linking steps are repeated until success.

Finally, after neighboring communication sites are linked, we can connect them by entanglement swapping. Suppose we have generated entanglement $|\phi\rangle_{AB}$ and $|\phi\rangle_{BC}$ between atomic ensembles A and B, B, and C, as shown in Fig. 1c. The two atomic ensembles at site B are placed close to each other within the blockade radius, so that we can perform a two-qubit gate between them. To implement entanglement swapping, we first apply a CNOT gate between the memory qubits stored in atomic ensembles B, B, and C, which can be done by a series of single atom $\pi$ pulses [21]: i) a $\pi$ pulse excites $|s_{B_u}\rangle$ to $|r_{B_u}\rangle$, ii) a $\pi$ pulse brings $|s_{B_u}\rangle$ to $|r_{B_u}\rangle$, iii) a $\pi$ pulse transfers $|r_{B_u}\rangle$ to $|s_{B_s}\rangle$, iv) a $\pi$ pulse transfers $|r_{B_u}\rangle$ to $|s_{B_s}\rangle$, and v) a final $\pi$ pulse returns $|r_{B_s}\rangle$ to $|s_{B_s}\rangle$. The corresponding truth table is shown in Table 1.

![Table 1: Truth table of the CNOT gate operation between two ensembles located at the same communication site, required for entanglement swapping. The steps involving the Rydberg blockade mechanism are denoted by $\Rightarrow$.](image)

After applying the CNOT gate we measure the memory qubits in the ensembles B, B, and D in four states $|+_{B_u}\rangle|s_{B_r}\rangle, |+_{B_u}\rangle|+_{B_s}\rangle|t_{B_u}\rangle$, and $|+_{B_u}\rangle|+_{B_s}\rangle|t_{B_r}\rangle$, where $|\pm_{B_u}\rangle = (|s_{B_u}\rangle \pm |t_{B_u}\rangle)/\sqrt{2}$, in order to project the memory qubits at sites A and C into the desired entangled state. In contrast to conventional schemes where the collective excitations are converted into photons for the detection, we suggest to measure the quantum state by transferring the excitation to a Rydberg state, field-ionizing the atom and detecting the ions. Detection of single Rydberg atoms has been demonstrated in photon counting experiment with near-unity detection efficiencies $\eta_d$ [19]. After the detection, the states are projected into $|s_{A_s}\rangle|s_{C_s}\rangle + |t_{A_s}\rangle|t_{C_s}\rangle)/\sqrt{2}$, up to a local unitary transformation.

The communication distance can be extended further by entanglement swapping. Since entanglement swapping is deterministic, the entanglement distribution rate is similar to the one of a quantum repeater based on trapped ions [17]. For $L = 2^n L_0$, the total time needed
can be approximated by

\[ T_{tot} \approx \prod_{i=0}^{n} \frac{T_{cc}}{p} \approx \prod_{i=1}^{n} \frac{\alpha_i}{2^{n-1} \eta_{dd}^2 \eta_{pd}^2 \epsilon^{-L/(2a L_{att})}} \]

where \( T_{cc} = \frac{L}{c} \) is the classical communication time with \( c \) the light speed, \( p = \frac{1}{2} \eta_{dd}^2 \eta_{pd}^2 \epsilon^2 \) and \( \alpha_0/p \) are the average of times one has to repeat before entanglement is successfully linked over the entire distance, with a numerical result of \( \alpha_0 \approx 3 \) for \( p \ll 1 \) [20]. The coefficients \( \alpha_{i=0} > 1 \) denote the average number of attempts needed to implement entanglement swapping due to non-uniform detection efficiency.

Let us now take into account local manipulation errors, which have been neglected in the discussion so far. The intrinsic errors in local manipulation are mainly induced by decay of the atoms when they are excited to Rydberg states, the imperfect Rydberg blockade induced by finite dipole-dipole shifts [21], and the imprecision of the collective pulses caused by an uncertainty of the atom number \( N \). The decay of the Rydberg states causes decoherence errors proportional to the Rydberg states decay rate \( \gamma \) and inversely proportional to the Rabi frequency of the collective pulses \( \Omega_N \) or single atom pulses \( \Omega_j \). The finite value of the Rydberg interaction energy shift \( \Delta_{dd} \) (imperfect blockade) will cause several kinds of errors. The first one is that two excitations may be generated in the atomic ensembles and thus causes losses. Secondly, the adiabatic elimination of the doubly excited Rydberg states will cause an ac stark shift on the one atom excitation states, and thus cause dephasing errors. These errors in the local entanglement generation step are of the order of \( \Omega_N^2 N/s^3 \Delta_{dd}^2 \) and can be estimated as

\[ E_{loc} = 1 - F_{loc} = 2 \frac{2 \pi}{\Omega_N^2} + \frac{2 \pi}{\Omega_j} + 2 \frac{\Omega_N^2}{\Delta_{dd}^2} + 2 \frac{\Omega_j^2}{\Delta_{dd}^2}, \]

with \( F_{loc} \) the fidelity of the locally achieved entanglement. The imprecision of the collective pulses is on the order of \( 1/N \) for an uncertainty of the atom numbers \( \sqrt{N} \). For \( N > 100 \), this error is less than 1% and can be safely neglected.

In Fig. 2a we plot the optimized local errors \( E_{loc} \) versus \( \Delta_{dd} \) for \( \tau = 1/(2\pi \gamma) = 200 \) and 300 \( \mu s \) (and \( \Omega_N = \Omega_j \)). One can see the local error is only a few percent for a dipole shift of \( \Delta_{dd} = 20 - 100 \text{ MHz} \).

Local imperfections are mainly decoherence, dephasing and loss errors. We can thus neglect spin flip errors and describe the local entanglement by a mixed entangled state \( \rho = (1 - (p_1 + p_0))p_2 + p_1p_1 + p_0p_0 \), where

\[ \rho_2 = F_{loc}[\phi^+](\phi^+| + (1 - F_{loc})|\phi^\prime\rangle|\phi^\prime\rangle, \]

with

\[ |\phi^\prime\rangle = (|A\rangle|B\rangle \pm |t_A\rangle|t_B\rangle)\sqrt{2}, \]

\( p_1 \sim (\Omega_N^2/\Delta_{dd}^2) \) and \( p_0 \sim (\Omega_j/\Delta_{dd}^2) \) are respectively the small probabilities to generate erroneously a single excitation and vacancy contribution \( p_1 \) and \( p_0 \), which are created due to double excitations (imperfect Rydberg blockade).

After linking the neighboring sites, we obtain a density matrix \( \rho^0 = (1 - (p_1 + p_0))p_2 + (p_1p_1 + p_0p_0) \), with the success probability of \( p = \frac{1}{2} \eta_{dd}^2 \eta_{pd}^2 e^{-L_{att}/(L - (2a L_{att}))} \), where \( \rho_2^0 = F_{0b}(|\phi^+\rangle|\phi^+\rangle + (1 - F_{0b})|\phi^\prime\rangle|\phi^\prime\rangle \) up to a local unitary transformation, with \( F_0 = F_{loc}^2 + (1 - F_{loc})^2 \) The errors in the photonic channel are neglected since two photon interference is used. The errors in the subsequent entanglement swapping step are similar to the ones of the local entanglement generation, and can be estimated using the average error of the CNOT gate \( E_{cnot} = 1 - F_{cnot} = \frac{2\pi}{4\pi} + \frac{3\pi^2}{2\Delta_{dd}^2} \). Fig. 2a shows the optimized swapping errors \( E_{cnot} \) versus \( \Delta_{dd} \) for \( \tau = 1/(2\pi \gamma) = 200 \) and 300 \( \mu s \). The entanglement swapping errors are smaller than for the generation of local entanglement since no collective pulses with associated generation of collective excitations are required.

After \( n \)-step entanglement swapping, the mixed entangled state reads

\[ \rho^n = (1 - (p_1p_1)p_2 + (p_1p_1)) \rho_2^0 \]

where \( \rho_2^0 = F_{n|d}(|\phi^+\rangle|\phi^+\rangle + (1 - F_{n|d})|\phi^-\rangle|\phi^-\rangle \). The fidelity can be approximately by \( F_n = (F_{n-1}^2 + (1 - F_{n-1})^2)F_{cnot} \), for \( F_{cnot} \) close to one. Note that the probability of obtaining the two excitations is independent of \( n \), thanks to the use of qubit-type entanglement and the detection of two excitations in each step [3]. The success probability of each entanglement swapping step is \( p \approx (1 - (2p_1)p_2) \eta_{dd}^2 \), where we have assumed for simplicity that the probability to obtain a double Rydberg excitation during the entanglement swapping (\( \Omega_j/\Delta_{dd}^2 \)) is on the order of \( O(p_1) \).

The local errors will accumulate during entanglement connection [1], and thus entanglement purification has to be performed which can be achieved by employing two CNOT gates [22]. Assume we have generated two pairs of mixed entangled states between \( A_n \) and \( C_n \), and \( A_d \) and \( C_d \), described by \( \rho^i \). We first apply a \( \pi/2 \) Ra-
man pulse coupling $|s\rangle$ and $|t\rangle$ to change the two excitation component to $\rho^2_\alpha = F_\alpha |\phi^-\rangle\langle\phi^-| + (1 - F_\alpha)|\psi^+\rangle\langle\psi^+|$, with $|\psi^+\rangle = \{ (s_{\alpha,d})|t_{\alpha,d}\rangle + (t_{\alpha,d})s_{\alpha,d}\}/\sqrt{2}$. We then perform two local CNOT gates with $A_\alpha$ and $C_\alpha$ the control qubit and $A_d$ and $C_d$ the target qubits, where we have assumed the two atomic ensembles at one site are located within the blockade radius. After the CNOT gates, we measure the target qubits in ensembles $A_d$ and $C_d$ in the $|s\rangle$ and $|t\rangle$ basis. If both qubits are in the $|s\rangle$ or the $|t\rangle$ state, the memory qubits in $A_\alpha$ and $C_\alpha$ are kept, otherwise the results are discarded. After entanglement purification, we obtain a mixed state $\rho^2_\alpha = F^2|\phi^-\rangle\langle\phi^-| + (1 - F^2)|\psi^+\rangle\langle\psi^+|$, where the leakage to other states is neglected for large values of $F_{\text{ent}}$, and the achieved fidelity can be estimated as $F^2 = 1/(1 + 2F_{\text{ent}}^2 - F_{\text{ent}}^4)$. The success propagability of purification is $p \approx (F^2_s + (1 - F^2_s)F_{\text{ent}}^2)$ after entanglement purification, the total density matrix can be described by $\rho = (1 - O(p_1/F^2_o)\rho^2_\alpha + O(p_1/F^2_o)p_1)$ where we have assumed that the one excitation term only contributes a false signal.

The main result of our work is illustrated in Fig. 2b, where the performance of the quantum repeater is plotted as a function of the communication distance for $n = 4$, $\eta_r = \eta_{sd} = 0.9, \eta_{cd} = 0.95, \lambda_{\text{att}} = 22 \text{ km}$ and $c = 2 \times 10^8 \text{ cm/s}$ in fibers. For comparison, we also show the performance of the best known atomic-ensemble-based repeater protocol without purification [5]. It can be seen that the entanglement distribution rate is enhanced by up to two orders of magnitude. For $L = 1000 \text{ km}$, the total time needed is on the order of a few hundred milliseconds.

The presented quantum repeater can be implemented using cold alkali atoms. Individual addressing of different sublevels can be achieved by choosing suitable laser polarization and applying a constant magnetic field. In our protocol, we suggest to use the isotropic repulsive van der Waals interactions by exciting the atoms to Rydberg to s-states with a principal quantum number $n$ around 70. In this case, the interaction energy between two atoms at a distance of $r$ can be approximated by $V = -c_1 \frac{n^2}{r^2} + c_1' \frac{n^2}{r^{10}}$ [24], with $c_1 < 0$ and $c_1' > 0$, where interactions proportional to $1/r^{10}$ are neglected. The interactions are repulsive for large and attractive for small $r$, yielding a critical distance $r_c$ where the repulsive shift is maximal. We use $r_c$ to estimate the minimum distance required to assure repulsive interatomic interactions, and find for Rb, $c_1 = -0.85$ and $c_1' = 0.8$, and $n = 70$, a critical distance $r_c = 0.3 \mu m$, corresponding to a density of $1/(r_c^3) = 3.7 \times 10^{13} \text{ cm}^3$. For a fixed density, one is interested in maximizing the number $N$ of atoms within the blockade radius, for high photon retrieval efficiencies and a uncertainty in the atom number. As illustrated in Fig. 2a, an interaction energy shift $\Delta_{sd} > 20 \text{ MHz}$ allows for local errors of less than $2\%$. This yields a maximum Rydberg blockade radius $R_b \approx (\frac{c_1'n^2}{\Delta_{sd}})^{1/6}$; a more accurate rate calculation using the interaction energy in [24] gives $R_b < 6 \mu m$ for $n = 70$. Thereby, a diameter of 2 to $3 \mu m$ is sufficient for achieving high fidelity local operations (a density of $3 \times 10^{13} \text{ cm}^3$ and a volume of $(2 \mu m)^3$ would allow for about $N = 240$ atoms per ensemble). With the help of a bad cavity, the retrieval efficiency can be estimated as $\eta_r = \frac{C}{r^2},$ where $C = N_c^2 \frac{24F}{\pi k}$, with $k$ the wave number of the emitted photon, and $c_r$ the transition coefficient [24]. For a finesse of $F = 100$, cavity mode width $\omega_0 = 5 \mu m$, $c_r = \frac{1}{3}$ and $k = 2\pi/\mu m$, we can obtain a high retrieval efficiency of 0.91.

Finally, to implement long distance quantum communication over 1000 km, the coherence times of the quantum memory have to be on the order of a few hundred milliseconds. This should be achievable for an atomic memory with cold atoms, where a storage time of about one second for classical light has been achieved [25].

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Bernoulli distribution with a success probability of $p$. We then generate $m'$ numbers according to a Bernoulli distribution, where $m' = m - k$ with $k$ the number of "1" events in the last step. This procedure is repeated until finally $m' = 0$. The average number of times needed gives $\alpha_0/p$. The coefficients $\alpha_{i\neq 0}$ are determined similarly.

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