The standard model (SM) is a very successful but only effective theory, which has to be extended at higher energies. Grand Unified Theories provide a beautiful framework for a more fundamental theory because the additional symmetries of the underlying group $G_{\text{GUT}}$ restrict some of the arbitrary features, in particular via relations between Yukawa couplings. Unfortunately, these relations are troublesome in simple GUT models like minimal SU(5). The equality $m_d = m_e$ is correct for the third but fails for the first and second generation. It was, however, noticed early that since the GUT scale $M_{\text{GUT}}$ is close to the Planck scale $M_P$, it is natural to expect corrections to the fermion mass matrices, which do not respect this equality. Moreover, these corrections are important for supersymmetric GUT models because they can significantly reduce the proton decay rate via dimension-five operators. For minimal supersymmetric SU(5), they are sufficient to make it consistent with the present experimental bound.

Since these additional contributions are important, it is reasonable to study them in more detail. In particular, we would like to know, whether they are totally arbitrary from the GUT model’s point of view, or whether there is any mechanism which would naturally lead to the required relations among Yukawa couplings. We can think of two possibilities, the first of which is to start with two operators in supersymmetric SO(10) GUT models like minimal SU(5) $\Sigma$:

$$W = \frac{1}{4} \epsilon_{abcd} \left( Y_{ij} 10^{ab} 10^{ij} H^e + f_{ij} 10^{ab} \frac{\sigma^c}{M_P} H^f + f_{ij} 10^{ab} \frac{\sigma^c}{M_P} H^f \frac{\sigma^f}{M_P} \right) + \sqrt{2} \left( Y_{ij} \Pi_a 10^{ab} 5^b + h_{ij} \Pi_a \frac{\sigma^a}{M_P} 10^{ab} 5^b + h_{ij} \Pi_a \frac{\sigma^a}{M_P} 5^b \right),$$

thus

$$Y_u = Y_1 + 3 \frac{\sigma}{M_P} f^S + 1 \frac{\sigma}{M_P} \left( 3 f^S + 5 f^A \right),$$

$$Y_d = Y_2 - 3 \frac{\sigma}{M_P} h_1 + 2 \frac{\sigma}{M_P} h_2,$$

$$Y_e = Y_3 - 3 \frac{\sigma}{M_P} h_1 - 3 \frac{\sigma}{M_P} h_2.$$

Here $\sigma/M_P \sim O\left(10^{-2}\right)$, and $S$ and $A$ denote the symmetric and antisymmetric parts of the matrices. From Eqs. one reads off,

$$Y_d - Y_e = 5 \frac{\sigma}{M_P} h_2,$$

hence the failure of Yukawa unification is naturally accounted for by the presence of $h_2$.

The dimension-five operators that lead to proton decay arise from the couplings $H_c$ and $\Sigma$, which acquire masses $O\left(M_{\text{GUT}}\right)$. If we integrate them out, two operators remain:

$$W_5 = \frac{1}{M_{H_c}} \left[ \frac{1}{2} Y_{qq} Y_{ql} QQQL + Y_{ae} Y_{ad} u^c e^c d^c \right],$$

Operator Analysis for Proton Decay in SUSY SO(10) GUT Models

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Non-renormalizable operators both account for the failure of down quark and charged lepton Yukawa couplings to unify and reduce the proton decay rate via dimension-five operators in minimal SUSY SU(5) GUT. We extend the analysis to SUSY SO(10) GUT models.

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where

$Y_{qq} = Y_{qq}^S = Y_{ue} = Y_u^S = \frac{5\sigma}{M_p} \left( f_1^S + \frac{1}{4} f_2^S \right)$,

$Y_{ue} = Y_u^A = \frac{5\sigma}{2 M_p} f^A$,

$Y_{ql} = Y_c = \frac{5\sigma}{M_p} h_1$

$Y_{ud} = Y_d = \frac{5\sigma}{M_p} h_1$.

The entries in $f_j$ and $h_j$ can lead to a simple pattern of these Wilson coefficients with small entries only [3].

In SO(10), the analogous five-dimensional operator is $16 \times 16 = 45$. Here, we use the scenario, where SO(10) is broken to SU(5) by a pair $16_H \oplus 16_R^*$ (cf. the discussion at the end of this letter). SU(5) is broken to G$_{33}$ by the adjoint $45_H$ which includes the $\Sigma(24)$ of SU(5); finally the breaking of G$_{33}$ is achieved by $10_H$ which includes both $H(5)$ and $\overline{H}(5)$ of SU(5). This breaking pattern guarantees that the SO(10) gauge coupling constant remains perturbative up to $M_p$ [11].

In order express the SO(10) in SU(5) fields, we consider a set of operators $b_j$ ($j = 1, \ldots, 5$) plus their Hermitian conjugates, $b^\dagger_j$, satisfying [14]

$$\{b_i, b_j\} = \{b_i^\dagger, b_j^\dagger\} = 0, \quad \{b_i, b_j^\dagger\} = 0$$.  

With $\Gamma$ matrices, defined as

$$\Gamma_{2j-1} = -i \left( b_j - b_j^\dagger \right), \quad \Gamma_{2j} = \left( b_j + b_j^\dagger \right),$$

we can construct the generators of SO(10), $\Sigma_{\mu\nu}$, as

$$\Sigma_{\mu\nu} = \frac{1}{2i} [\Gamma_\mu, \Gamma_\nu].$$

The spinor representation can be split into two 16-dimensional representations $\Psi_\pm$ by chiral projection, $\frac{1}{\sqrt{2}}(1 \pm \Gamma_0)$, $\Gamma_0 = i \prod_j \Gamma_j$. We define an SU(5) invariant vacuum state $|0\rangle$ and expand the spinors in terms of SU(5) fields. The SM fermions are assigned to 16.

$$|16\rangle = |\Psi_+\rangle = |0\rangle \psi_0 + \frac{1}{2\sqrt{15}} b_1^\dagger b_2^\dagger |0\rangle \psi^{12}$$

$$+ \frac{1}{4\sqrt{15}} \epsilon^{ijklm} b_j^\dagger b_k^\dagger b_l^\dagger b_m^\dagger |0\rangle \overline{\psi}_i$$,

where we identify $\overline{\psi}_i$ and $\psi^{ij}$ with 5$^*$ and 10 of SU(5), respectively. The singlet $\psi_0$ denotes the left-handed antineutrino.

The fundamental representation can be written in SU(5) fields as

$$\phi_\mu = \begin{cases} \phi_{2j} = \frac{1}{2} (\phi_{e_j} + \phi_{\bar{e}_j}) \\ \phi_{2j-1} = \frac{1}{2i} (\phi_{e_j} - \phi_{\bar{e}_j}) \end{cases}.$$  

where $\phi_{e_j}$ and $\phi_{\bar{e}_j}$ transform like SU(5) representations. Thus we are able to compute the SO(10) in SU(5) fields which then only have to be reduced to irreducible representations. We obtain

$$\Gamma_\mu \phi_\mu = - b_j \phi_{e_j} + b_j \phi_{\bar{e}_j}.$$  

To have a canonical kinetic term for the SU(5) Higgs fields, we normalize the fields by

$$\phi_{e_j} = \sqrt{2} H_j, \quad \phi_{\bar{e}_j} = \sqrt{2} H_j^*.$$  

Now we are able to express the basic Yukawa couplings, $16 \times 16 = 45$. It appears in four different invariants,

$$(16 \times 16)_{10} = (16 \times 45)_H = (16 \times 45)_R, \quad (16 \times 10)_{16*}, \quad (16 \times 45)_{16*}, \quad (16 \times 45)_{16*}.$$  

Note that in Ref. [11], only the second term is studied. To calculate the different couplings, we generalize Eqn. [10] so that

$$45 : \quad \Sigma_{\mu\nu} \phi_{10} = - i \left( b_1^\dagger b_1^\dagger \phi_{e_1 e_1} + b_2^\dagger b_2^\dagger \phi_{e_2 e_2} \right)$$

$$45 : \quad \Sigma_{\mu\nu} \phi_{10} = - i \left( b_1^\dagger b_1^\dagger \phi_{e_1 e_1} + b_2^\dagger b_2^\dagger \phi_{e_2 e_2} \right)$$

The tensors of $\phi_{10}$ can be decomposed into their irreducible forms as

$$\phi_{e_1 e_1} = h, \quad \phi_{e_2 e_2} = h,$$

$$\phi_{e_1 e_2} = h_{ij}, \quad \phi_{e_2 e_1} = h_{ij} + \frac{1}{3} \delta_{ij} h.$$  

with the $1, 10^*$, 10 and 24-plet, normalized as

$$h = \sqrt{10} H, \quad h_{ij} = \sqrt{2} H_{ij},$$

$$h_{ij} = \sqrt{2} H_{ij}, \quad h_{ij} = \sqrt{2} H_{ij}.$$

(18)
Analogously, we have for $\phi_{\mu \nu \lambda}$

$$\phi_{ci, cj, \bar{e}_k} = f_{ik} + \frac{i}{4} \left( \delta_k f^i - \delta_i f^k \right),$$
$$\phi_{ci, \bar{e}_j, \bar{e}_k} = f_{jk} - \frac{i}{4} \left( \delta_j f^i - \delta_k f^i \right),$$
$$\phi_{ci, cj, \bar{e}_k} = \epsilon_{ijklm} f_{lm} ,$$
$$\phi_{ci, \bar{e}_j, \bar{e}_k} = \epsilon_{ijklm} f^{lm} ,$$

(20)

We identify the 5, 10, 45, 5, 10* and 45*-plet of 120, which are normalized as

$$f^i = \frac{4}{\sqrt{3}} h^i , \quad f^{ij} = \frac{1}{\sqrt{3}} h^{ij} ,$$
$$f^i = \frac{4}{\sqrt{3}} h^i , \quad f^{ij} = \frac{1}{\sqrt{3}} h^{ij} , \quad f^{ij} = \frac{2}{\sqrt{3}} h^{ij} .$$

(21)

For the first invariant (15a) we need the coupling 10 − 10 − 45, which can be decomposed as

$$\sqrt{2} \left[ (5_{10} 5_{10}^* 10_{10} 145) + 5_{10} 5_{10} 10_{145} + 5_{10}^* 5_{10} 10_{145} + (5_{10} 5_{10}^* 10_{145}) \right] .$$

(22)

Since the vev of the 45$_H$ is taken in the 24-direction of SU(5), only the last two terms are relevant. Now we integrate out the heavy field 10 in Eqs. (12) by means of $W_{10}^{120} = 2 M_{10} 5 5^*$ and obtain the coupling given in Eqn. (20a).

The calculation for the second term (15b) is straightforward. We compute

$$W_{120}^{120} = \frac{i}{\sqrt{3}} f_{ab} \left[ (1_{10} 1_{10} + 1_{10} 1_{10} 10_{10}) 10^H + 2 \cdot 5_{10}^* 5_{10}^* 10_H ight.$$  
$$+ 2 \left( 1_{10} 5_{10}^* - 5_{10} 1_{10} 5_{10}^* \right) 5_{10}^* + (5_{10} 10_{10} - 10_{10} 5_{10}^*) 45_{H145} \right]$$

(23)

and calculate the relevant terms of the coupling 10 − 45 − 120,

$$\sqrt{3} \left[ (5_{10} 24_{145} 45_{120} + 5_{10} 24_{145} 45_{120}) ight.$$  
$$- 5_{10} 24_{145} 5_{120} - 5_{10} 24_{145} 5_{120} + \ldots .$$

(24)

With the mass term

$$W_{120}^{120} = M_{120} \left( \frac{1}{2} 10^{10*} + 45^{45*} - 2 \cdot 5^{5*} \right) ,$$

(25)

we then get the result of Eqn. (20a).

The remaining two operators read

$$(16 \ 10^H)_{16*} (16 \ 45^H)_{16} = \tilde{\Psi} B \Gamma_{\mu} \phi^\mu \Sigma_{\nu\rho} \Psi \phi^{\nu\rho} ,$$

(26)

$$(16 \ 10^H)_{144*} (16 \ 45^H)_{144} = \tilde{\Psi} B \phi^\mu \Gamma_{\nu} \Psi \phi^{\mu\nu} - (\text{Eqn. (20)}) .$$

(27)

The first expression in Eqn. (26) describes the reducible 160 representation. Since the 144 requires

$$\Gamma_{\mu} \phi^\mu = 0 ,$$

(28)

we add $\Gamma_{\mu} = 1$ to project out the 16 contribution which is already calculated in Eqn. (20). Then we get the 144 contribution just by the difference of the two terms.

Altogether, the couplings of the four operators read

$$\hat{Y}_{10} = \frac{h_{10}}{M_{10}} \left\{ \frac{1}{2} \epsilon_{abde} 10_{ab} 10_{cd} \Sigma_{ef} H_f ight.$$  
$$- 2 \mathcal{P}_a \Sigma_b \left( 10_{bc}^* 5_{jc} + 10_{jc}^* 5_{bc} \right) \right\}$$

(29a)

$$\hat{Y}_{120} = \frac{h_{120}}{M_{120}} \left\{ -2 \epsilon_{abde} 10_{ab} 10_{cd}^f \Sigma_d \Sigma_f ight.$$  
$$- 4 \mathcal{P}_a \Sigma_b \left( 10_{bc}^* 5_{jc} - 10_{jc}^* 5_{bc} \right) \right\}$$

(29b)

$$\hat{Y}_{16} = \frac{h_{16}}{M_{16}} \left\{ \frac{1}{2} \epsilon_{abde} 10_{ab} 10_{cd}^f \Sigma_d \Sigma_f ight.$$  
$$+ 2 \mathcal{P}_a \Sigma_b \left( 10_{bc}^* 5_{jc} - \mathcal{P}_a \Sigma_b \left( 10_{ab}^* 5_{jc} \right) \right) \right\}$$

(29c)

$$\hat{Y}_{144} = \frac{h_{144}}{M_{144}} \left\{ \epsilon_{abde} 10_{ab} 10_{cd}^f \Sigma_d \Sigma_f ight.$$  
$$- \frac{1}{2} \epsilon_{abde} 10_{ab} 10_{cd}^f \Sigma_d \Sigma_f ight\}$$

(29d)

where we only list the SU(5) relevant terms. Without loss of generality, we can assume that the heavy particles all have the same mass and can compare the couplings with those of SU(5) (cf. Eqn. (10)).

$$f_1 = \frac{1}{4} h^{10} + h^{144} ,$$
$$f_2 = -2 h^{120} + \frac{1}{2} h^{16} - \frac{1}{2} h^{144} ,$$
$$h_1 = -2 h^{10} - h^{120} + 2 h^{16} + 2 h^{144} ,$$
$$h_2 = -4 h^{120} - h^{16} + h^{144} .$$

(30)

Here, $h^{10}$ is symmetric whereas $h^{120}$ is antisymmetric; $h^{16}$ and $h^{144}$, are not constrained by symmetry requirements. We see that SO(10) does not restrict the contributions from the higher-dimensional operators, contrary to the basic Yukawa couplings (cf. Eqn. (13)). With these equations, we reduce the SO(10) case down to SU(5), so the implications of the higher-dimensional operators for proton decay in SO(10) are the same as in SU(5).$

If we consider the complete breaking of SO(10) to G_{345}, more five-dimensional operators can appear. SO(10) can be broken via a pair $16_H \oplus 16_H^*$, where the SU(5) singlet component obtains a vev $O(M_{GUT})$. Then the two new dimension-five operators, $16 \ 16_H \ 16_H$ and $16 \ 16_H \ 16_H^*$, generate Majorana masses for the right-handed neutrinos. If, moreover, the $5_{16}$ and $5_{16}^*$ acquire vevs as well (as in Refs. [2]), these operators allow additional contributions to Eqn. (20). The second coupling
was partially worked out in Refs. [12]. Alternatively, if we use a $210_H$ to break $\text{SO}(10)$, additional terms can arise, since the $210$ includes a $24$ of $\text{SU}(5)$ [15].

We extended the analysis of higher-dimensional operators in $\text{SU}(5)$ to $\text{SO}(10)$. In contrast to the basic Yukawa couplings, these operators are not restricted compared to $\text{SU}(5)$. In the simple case, where $\text{SO}(10)$ is broken to $G_{\text{SM}}$ via $16_H^*, 16_H^*$ and $45_H^*$ and the former have only vevs in the $\text{SU}(5)$ singlet direction, these represent all possible operators of dimension five. Hence, it would be interesting to study if this model, with only the Yukawa couplings $16 \times 16 \times 10_H$ and the dimension-five operators studied in this paper (including those which generate Majorana masses for the right-handed neutrinos), both reproduces the fermion masses and mixing angles and is consistent with the experimental limit on proton decay.

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