The neutron charge form factor in helium-3

D.H. Lu, K. Tsushima, A.W. Thomas and A.G. Williams

Department of Physics and Mathematical Physics
and Special Research Centre for the Subatomic Structure of Matter,
University of Adelaide, Australia 5005

K. Saito

Physics Division, Tohoku College of Pharmacy,
Sendai 981-8558, Japan

Abstract

In order to measure the charge form factor of the neutron, $G_n^E(Q^2)$, one needs to use a neutron bound in a nuclear target. We calculate the change in the form factor for a neutron bound in $^3$He, with respect to the free case, using several versions of the quark meson coupling model. It is found that the form factor may be suppressed by as much as 12% at $Q^2 = 0.5 \text{ GeV}^2$ with respect to that of the free neutron.

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There are currently a number of very sophisticated experiments underway at Bonn, Mainz, MIT Bates and TJNAF, in which it is proposed to measure the elusive, electric form factor of the neutron, $G^n_E(Q^2)$ [1–3]. This quantity is of great interest in QCD because of the subtle interplay of quark and meson degrees of freedom [4,5]. In the absence of a free neutron target, the neutron form factors have to be extracted from electron-nucleus scattering. However, because it has zero charge the neutron charge form factor vanishes at $Q^2 = 0$ and is substantially smaller than the proton charge form factor for $Q^2 < 1 \text{ GeV}^2$. As a consequence, the extracted neutron charge form factor has typically been associated with large systematic errors – except for its charge radius, which can be measured accurately by the scattering of ultra-cold neutrons by electrons [3]. By using a polarized beam and target the new experiments aim to produce results which should be much more direct and quite insensitive to two-body exchange currents [7].

Our concern is that the binding energy of $^3\text{He}$ (and the central density) is significantly higher than for the deuteron and therefore the possible medium modification of the internal structure of the bound “nucleon” could be significant. We therefore investigate the modification of the bound neutron charge form factor in $^3\text{He}$ within the quark-meson coupling (QMC) model [3,14]. This effect has been estimated before. For example, it has been shown that the neutron charge form factor is altered dramatically at small momentum transfer in the chiral $\pi\rho\omega$ Lagrangian [11], the linear sigma model [12] and the chiral bag model [13]. In Ref. [14], we showed that the in-medium nucleon electromagnetic form factors in the standard QMC model are somewhat reduced, and they are consistent with constraints deduced from $y$-scaling data [15].

Recently, extensions of QMC have been proposed in which, in order to simulate the eventual quark-gluon deconfinement at high density, the bag constant is allowed to decrease as the bag density (or mean scalar field) increases [16–18]. Here we examine the variation of $G^n_E(Q^2)$ for a neutron bound in $^3\text{He}$ for several of these variations that are consistent with other constraints [15], as well as for the standard QMC model.

For convenience we briefly review the standard version of QMC in which the bag constant,
B, is fixed. The additional complications when B varies with density are explained in Ref. [18]. At the present stage of development of the QMC model, a nuclear system is viewed as a collection of non-overlapping MIT bags [19]. The medium effects arise through the self-consistent coupling of phenomenological scalar ($\sigma$) and vector ($\omega$, $\rho$) meson fields to the confined valence quarks — rather than to the nucleons, as in Quantum Hadrodynamics (QHD) [20]. As a result, the internal structure of the bound “nucleon” is modified by the medium with respect to the free case.

In mean field approximation for the meson fields, the lowest state of the quark inside a bag with a radius $R$ is given by [19]

$$q_m(t, r) = \frac{N_0}{\sqrt{4\pi}} e^{-i\epsilon_q t/R} \left( \frac{j_0(xr/R)}{i\beta_q \sigma \cdot \hat{r} j_1(xr/R)} \right) \theta(R - r) \chi_m,$$

where

$$\epsilon_q \left( \begin{array}{c} u \\ d \end{array} \right) = \Omega_q + g_\omega^2 \sigma R \pm \frac{1}{2} g_\rho^2 \rho, \quad \beta_q = \frac{\Omega_q - m_q^* R}{\Omega_q + m_q^* R},$$

$$N_0^{-2} = 2R^3 j_0^2(x)[\Omega_q (\Omega_q - 1) + m_q^* R/2]/x^2,$$

with $\Omega_q \equiv \sqrt{x^2 + (m_q^* R)^2}$, $m_q^* \equiv m_q - g_\sigma^2 \sigma$, and $\chi_m$ the quark Pauli spinor. The parameters in this expression ($x$, $m_q^*$ and $R$) are determined by the boundary condition at the bag surface, $j_0(x) = \beta_q j_1(x)$, together with the mean values of the scalar ($\sigma$) and the time components of the vector ($\sigma$ and $\rho$) fields ($\bar{b}$ stands for the neutral $\rho$ meson mean field), which are governed by the equations of motion,

$$\bar{\omega} = \frac{g_\omega \rho}{m_\omega^2},$$

$$\bar{\sigma} = \frac{g_\sigma}{m_\sigma^2} C(\sigma) \frac{2}{(2\pi)^3} \sum_{i=p, n} \int_{k_F^{(i)}}^\infty d^3k \frac{m_N^*(\sigma)}{\sqrt{m_N^2(\sigma) + k^2}},$$

$$\bar{\rho} = \frac{g_\rho}{2m_\rho^2} (\rho_p - \rho_n).$$

Here $\rho = \rho_p + \rho_n$ is the baryon density, $k_F^{(p,n)}$ is the proton(neutron) Fermi momentum, the bound nucleon effective mass is $m_N^*(\sigma) = 3\Omega_q/R - z_0/R + (4\pi/3)R^3B_0$, and the function $C(\sigma)$
is \( S(\pi)/S(0) \), with \( S(\pi) = [\Omega_q/2 + m_q^* R(\Omega_q - 1)]/[\Omega_q(\Omega_q - 1) + m_q^* R/2] \). The meson-nucleon coupling constants are related to the meson-quark coupling constants by \( g_\sigma = 3g_\sigma^q S(0) \), \( g_\omega = 3g_\omega^q \), and \( g_\rho = g_\rho^q \).

In practice, the quantities \( z_0 \) and \( B_0 \) are first determined by requiring the free nucleon mass to be \( m_N^*(\sigma = 0) = 939 \) MeV and by the stability condition, \( \partial m_N^*(\sigma)/\partial R = 0 \), for a given bag radius of the free nucleon, \( R_0 \) (treated as an input parameter). Then the coupling constants, \( g_\sigma \), \( g_\omega \) and \( g_\rho \), are chosen to fit the saturation properties of nuclear matter (saturation energy and bulk symmetry energy) at normal nuclear density. Note that the \( \rho \) meson is now explicitly included in addition to the original \( \sigma \) and \( \omega \) meson fields in order to describe asymmetric nuclear matter. These coupled, nonlinear equations can be solved self-consistently for an arbitrary baryon density, where the solution contains a medium-modified quark wavefunction.

The neutron charge form factor extracted from the electron–nucleus scattering experiment should be interpreted as an average value of the neutron contribution over the finite nucleus. In QMC, the neutron substructure is modified by its surrounding nuclear medium. In a local density approximation, the neutron charge form factor in a finite nucleus can be written as

\[
G_E^n(Q^2) = \int G_{E(n,m)}(Q^2, \rho(\vec{r})) \rho_n(\vec{r}) d\vec{r},
\]

where \( G_{E(n,m)}(Q^2, \rho(\vec{r})) \) denotes the density-dependent charge form factor of a neutron immersed in a uniform density of protons (\( \rho_p \)) and neutrons (\( \rho_n \)).

We neglect other residual off-shell effects of the bound nucleon [21], and so the neutron charge form factor can be conveniently evaluated in the Breit frame,

\[
G_E^n(Q^2) = \langle n(\vec{q}/2)|j^0(0)|n(-\vec{q}/2)\rangle,
\]

\[
j^\mu(x) = \sum_f e_f \bar{q}_f(x)\gamma^\mu q_f(x) - ie[\pi^\dagger(x)\partial^\mu\pi(x) - \pi(x)\partial^\mu\pi^\dagger(x)],
\]

\[
|n\rangle = \sqrt{Z_n^f}[1 + (m_n - H_0 - \Lambda H_I\Lambda)^{-1}H_I]|n_0\rangle,
\]

where \( Q^2 = -q^2 = \vec{q}^2 \), \( q_f(x) \) is the quark field operator of the flavor \( f \), \( e_f \) is its charge.
operator, \( \pi(x) \) either destroys a negatively charged pion or creates a positively charged one, \( \Lambda \) is a projection operator which projects out all the components of \(|n\rangle\) with at least one pion, \( H_0 \) is the Hamiltonian for the bare baryon and free pion, and \( H_I \) is the interaction Hamiltonian which describes the process of emission and absorption of pions. The probability of finding a bare neutron bag, \(|n_0\rangle\), in the physical neutron, \( Z_n^2 \), is determined by the probability conservation condition [5].

To calculate the neutron form factor, we use the Peierls-Thouless projection method [22] combined with a Lorentz contraction for the nucleon internal wave function in the Breit frame [23,24]. These techniques for implementing the center-of-mass and recoil corrections have been quite successful in the case of the electromagnetic form factors for free nucleons [23]. Detailed calculations give

\[
G_E^{\mathrm{quark}}(Q^2) = \left( \frac{2}{3} P_{NN} - \frac{1}{3} P_{\Delta\Delta} \right) G_E^{\mathrm{core}}(Q^2),
\]

\[
G_E^{\mathrm{pion}}(Q^2) = G_E^{(\pi)}(Q^2; N) + G_E^{(\pi)}(Q^2; \Delta),
\]

where

\[
P_{BC} = \frac{f_{NB} f_{NC}}{12\pi^2 m_\pi^2} \int_0^\infty \frac{dk}{k} \frac{k^4 u^2(kR)}{\omega_{BN} + \omega_k (\omega_{CN} + \omega_k) \omega_k},
\]

\[
G_E^{(\pi)}(Q^2; N) = -\frac{1}{36\pi^3} \left( \frac{f_{NN}^*}{m_\pi} \right)^2 \int d^3 k \frac{u(kR) u(k'R) \vec{k} \cdot \vec{k}'}{\omega_k \omega_{k'} (\omega_k + \omega_{k'})},
\]

\[
G_E^{(\pi)}(Q^2; \Delta) = \frac{1}{72\pi^3} \left( \frac{f_{N\Delta}^*}{m_\pi} \right)^2 \int d^3 k \frac{u(kR) u(k'R) \vec{k} \cdot \vec{k}'}{(\omega_{\Delta N} + \omega_k)(\omega_{\Delta N} + \omega_{k'})(\omega_k + \omega_{k'})}.
\]

Here \( u(kR) = 3j_1(kR)/kR, \omega_k = \sqrt{m_B^2 + \vec{k}^2}, \omega_{BN} \simeq m_B - m_N, \vec{k}' = \vec{k} + \vec{q} \), and \( f_{NB}^* \) is the renormalized \( \pi NB \) coupling constant in medium. The charge form factor for the quark core of the proton is

\[
G_E^{\mathrm{core}}(Q^2) = \int d^3 r j_0(Qr) f_q(r) K(r)/D_{PT},
\]

\[
D_{PT} = \int d^3 r f_q(r) K(r),
\]

where \( f_q(r) \equiv j_0^2(xr/R) + \beta_q^2 j_1^2(xr/R) \) and \( K(r) \equiv \int d^3 z f_q(z) f_q(-\vec{z} - \vec{r}) \) is the recoil function to account for the correlation of the two spectator quarks. The effect of the Lorentz
contraction for the quark core is taken into account by a simple rescaling formula \[23\],

\[ G_E(Q^2) = (m_N^*/E^*)^2 G_{E,sph}^{\text{m}}(Q^2 m_N^*/E^*) \],

where \( E^* = \sqrt{m_N^* + Q^2/4} \) and the superscript “sph” refers to the form factor calculated with the spherical bag wave function.

The pion cloud plays a vital role in this calculation. For the simplest case where the valence quarks in the bare neutron bag, \(|n_0\rangle\), occupy the 1s state in the bag, the final small but non-vanishing \(G_{E,n}^{\text{m}}(Q^2)\) comes primarily from the photon interaction with either the proton or the pion in the virtual process, \(n \rightarrow p\pi^-\).

In principle, the existence of the \(\pi\) and \(\Delta\) inside the nuclear medium will also lead to some modifications of their properties. As the \(\Delta\) is treated on the same footing as the nucleon, its mass should vary in a similar manner as the nucleon. Thus, as a first approximation, we assume that the in-medium and free space \(N - \Delta\) mass splittings are approximately equal, i.e., we take \(m_\Delta^* - m_N^* = m_\Delta - m_N\). As the pion is a nearly perfect Goldstone boson, we also take \(m_\pi^* = m_\pi\) in this work. The \(\pi NN\) coupling constant in free space is taken to be the empirical value, i.e. \(f_{NN}^2 \simeq 3.03\), which corresponds to the usual \(\pi NN\) coupling constant, \(f_{\pi NN}^2 \simeq 0.081\). In the medium, the \(\pi NN\) coupling constant might be expected to decrease slightly because of the enhancement of the lower component of the quark wave function. It can be shown that this reduction is about one-third of that of the \(\sigma NN\) coupling constant, \(g_\sigma(\sigma)\), which behaves as \(g_\sigma(\sigma)/g_\sigma \simeq 1 - a(g_\sigma(\sigma))/2\). Hence, \(f_{\pi NN}^2/f_{NN}^2 \simeq 1 - a(g_\sigma(\sigma))/6\), where \(a \simeq (8.8, 11) \times 10^{-4} \text{ MeV}^{-1}\) for \(R_0 = (0.8, 1.0) \text{ fm}\) and \(g_\sigma(\sigma) = s_1(\rho/\rho_0) + s_2(\rho/\rho_0)^2 + s_3(\rho/\rho_0)^3\) for QMC-II — c.f. Ref \[25\], where the coefficients \(s_i\) may be found.

There is no direct experimental data for the neutron density distribution in \(^3\text{He}\). We identify the neutron density distribution in \(^3\text{He}\) with the proton density in \(^3\text{H}\), which is certainly reasonable provided that the charge symmetry breaking is small. The baryon density distribution in \(^3\text{He}\) is calculated from the charge densities of \(^3\text{He}\) and \(^3\text{H}\). Fig. [4] shows the neutron charge form factor with \(R_0 = 1.0\ \text{fm}\), using two different experimental fits to the charge distribution (FB refers to a Fourier-Bessel fit \[26\] and SOG to a fit using a Sum-of-Gaussians \[27\]). The \(\pi NN\) coupling constant is taken to have its free space value
in these calculations. The standard QMC gives roughly a 5% reduction with respect to the free case at $Q^2 = 0.5 \text{ GeV}^2$. The long-dashed and the dot-dashed curves in Fig. 4 are for variations of the original QMC involving possible reduction of the bag constant \[16–18\]. The SOG fit is used for these two curves. DC-II refers to a QMC variant where the bag constant is directly related to $\pi$ through $B/B_0 = e^{-4g_B^2\pi/m_N}$ and SCALE to a model where $B/B_0 = (m_{N^*}/m_N)^\kappa$. According to $y$-scaling constraints \[15\], the maximum values allowed for the parameters $g_{\sigma}^B$ and $\kappa$ would be 1.2 \[18\]. It is clear that the possible reduction of the neutron charge form factor is significant in QMC once the bag constant is allowed to decrease — i.e., the reduction is nearly 12% at $Q^2 = 0.5 \text{ GeV}^2$, for both parametrizations of the bag constant.

Fig. 4 presents the dependence of the neutron charge form factor on the $\pi NN$ coupling constant with two different bag radii. Note that the absolute value of the neutron charge form factor is clearly quite sensitive to the bag radius, however, the really important issue here is the relative change with respect to the free case. For a typical reduction of the $\pi NN$ coupling constant in medium (3% at $\rho_0$ in QMC) with a fixed bag constant, the neutron charge form factor decreases by less than 2% at $Q^2 = 0.5 \text{ GeV}^2$. If we allow variation in the $\pi NN$ coupling constant and allow the bag constant to decrease at the same time, the combined possible reduction for the neutron charge form factor is also about 12% for $R_0 = 1.0 \text{ fm}$ (solid line) and 8% for $R_0 = 0.8 \text{ fm}$ (dot-dashed line) at $Q^2 = 0.5 \text{ GeV}^2$.

At present the only experimental information on the modification of $G_{E}^{n}(Q^2)$ comes from the comparison of measurements of the asymmetry in the scattering of polarized electrons from polarized $^3\text{He}$ and deuteron targets. At $Q^2 \simeq 0.35 \text{ GeV}^2$ the deuteron measurement is significantly larger than the $^3\text{He}$ measurement — the ratio being almost a factor of two. While the experimental errors do not allow firm conclusions to be drawn yet, the situation will improve dramatically in the next few years.

In conclusion, we have calculated the neutron charge form factor in $^3\text{He}$ using QMC and a number of its variants. The in-medium quenching of the neutron charge form factor is insensitive to the details of the $^3\text{He}$ matter density chosen. The neutron charge form factor
is the result of a cancellation between the contributions of a $\pi^-$ cloud and a proton-like core. Because the long-range pion cloud is not greatly altered in medium the charge radius decreases by only a few percent (3.3% and 3.8% for $R_0 = 1.0$ and 0.8 fm, respectively). However, the expansion of the core (especially when the bag pressure is allowed to decrease) does lead to a significant decrease of $G^n_L(Q^2)$ above 0.2 GeV$^2$. The maximal allowed reduction of the neutron charge charge form factor at $Q^2 = 0.5$ GeV$^2$ may reach 12% for a reasonable bag radius, such as $R_0 = 1.0$ fm. The present results are comparable to the previous results in the generalized Skyrme model [11], the chiral sigma model [12], and the chiral bag model [13]. It is worthwhile to point out that while the picture of the nucleon in these models is quite different, the consistency of the predictions implies a certain degree of model-independence of the results. We would, however, like to emphasize that our model describes the nucleon and nuclear system consistently. It is able to reproduce the nuclear charge distributions for the closed shell nuclei, as well as the saturation energy, density and compressibility of nuclear matter. The medium modification implied by the model is consistent with the current experimental limits, and it will be very interesting if it is confirmed in future, more precise measurements.

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FIG. 1. The neutron charge form factor in $^3$He using the standard QMC model and two different density parametrizations (FB and SOG stand for a Fourier-Bessel fit and a Sum-of-Gaussians fit). The effect of a possible reduction of the bag constant is also shown. The $\pi NN$ coupling constant is maintained at its free space value. Here DC-II and SCALE refer to two ways of implementing a reduction of the bag constant (see text). The corresponding free space neutron charge form factor is also shown for ease of comparison. (The free bag radius was taken to be $R_0 = 1.0$ fm.)
FIG. 2. The neutron charge form factor in $^3$He with a possible reduction of the $\pi NN$ coupling constant in medium. The numbers in the legends indicate the free bag radius (in fm) used in each calculation. The dashed and long-dashed curves refer to the standard QMC with density-dependent $\pi NN$ coupling constant; and the dot-dashed and solid lines refer to a QMC with the density-dependent $\pi NN$ coupling constant, together with the maximum allowed reduction of the bag constant in the direct coupling model. The dotted curves are for the corresponding form factors in free space for both bag radii.