Casimir Force Acting on a Multilayer Graphene Sheet with Strong Diamagnetism

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Received: 8 June, 2019; Accepted: 11 August, 2019; Published: 31 August, 2019

We calculate the Casimir force acting on randomly stacked graphene layers and investigate the impact of diamagnetism on the Casimir force. The randomly stacked graphene layers are predicted to have a much smaller permeability than regularly stacked graphene layers such as graphite. We show that the Casimir force between randomly stacked graphene layers and a conducting plate is enhanced as the permeability of the randomly stacked graphene layers decreases from 1 to 0, especially for large separations. Conversely, the magnitude of the Casimir force between the randomly stacked graphene layers and a magnetic-dielectric plate such as yttrium-iron-garnet decreases as the permeability of the randomly stacked graphene layers decreases, and the force can be repulsive if the permittivity of the magnetic-dielectric plate contains a permeability much smaller than its permeability.

Keywords Construction and use of effective interatomic interactions; Atom–solid interactions; Carbon; Silicon; Silicon oxides

I. INTRODUCTION

Much attention has been paid to differences in the material properties between graphene and graphite. For instance, Cao et al. recently demonstrated that twisted bilayer graphene with a ‘magic’ angle induces superconductivity at 1.7 K [1]. The important features of superconductivity are zero electrical resistance and the expulsion of magnetic flux fields. From the latter property the superconductor is often referred to as a perfect diamagnetic material, whose relative magnetic permeability $\mu$ is zero. The magnetic permeability of almost all existing diamagnetic materials, except for superconductors, is nearly equal to that of vacuum, i.e., $\mu \approx 1$. However, there is room for realizing materials with small magnetic permeabilities without superconductivity. Ominato and Koshino predicted that a random-stacked multilayer graphene sheet, which is abbreviated as a multilayer graphene sheet below, could significantly screen the magnetic field inside the sample [2, 3].

If a multilayer graphene sheet has a small permeability, its diamagnetism must affect various physical phenomena. We focus on the Casimir force [4] acting on a multilayer graphene sheet based on the Lifshitz theory [5]. The Lifshitz theory has succeed in explaining the Casimir force between various materials. In particular, the theoretical values of the Casimir force between metallic objects obtained through the Lifshitz theory are in a good agreement with measured experimental values with a torsion pendulum [6] and an atomic microscope [7]. Hereafter, the Lifshitz theory is verified for more complicated materials such as magnetic materials [8], superconductors [9–11], and metamaterials [12]. Both graphite and graphene are very important materials for studying the Casimir force. Graphite is a good candidate to verify whether the Lifshitz theory can correctly predict the Casimir force between uniaxial materials [13]. Graphene allows the study on unique properties of the Casimir force between two-dimensional conductive materials [14–17] because the Casimir force acts on graphene layers in different manner than on metallic films. It has been demonstrated that the Casimir force between a gold sphere and a single graphene sheet on a SiO$_2$ substrate is modified by the presence of a single graphene sheet [18]. The Casimir force between metallic films hardly depends on temperature but the force acting on graphene layers may depend strongly on tempera-
ture [17]. Furthermore, the graphene sheet is an important component of micro-electro-mechanical systems [19–21]; the study of the Casimir force between a graphene sheet and various materials provides a valuable contribution to engineering.

Although multilayer graphene, considered by Ominato and Koshino, is not yet fabricated, it would be a good material to use to verify that the Lifshitz theory can be applied to diamagnetic materials [22]. Detailed physical properties of multilayer graphene have not been determined; however, it is probably a uniaxial conductive-magnetic material. Thus, this study deals with the impact of magnetic properties on the Casimir force acting on thin uniaxial conductive-magnetic plates. The Lifshitz theory requires knowledge of the permittivity and permeability at an arbitrary frequency of the electromagnetic field. However, if the distance between objects is large, the Casimir force can be approximately determined only by the static permittivity and permeability [4]. This means that the Casimir force acting on multilayer graphene can be predicted by a few physical quantities without knowledge of the permittivity or permeability for an oscillating electromagnetic field if the distance between a multilayer graphene sheet and a facing object is large.

This paper is structured as follows. In Section II, the permittivity of graphite is expressed using the Lorentz dispersion model, and the Casimir energy between a graphite sheet and a perfectly conductive plate, which reflects light of any frequency is calculated. In Section III, we calculate the Casimir energy between a perfectly conducting plate and a thin conductive diamagnetic uniaxial plate with the same permittivity but different permeability for graphite, and present the impact of diamagnetism of a multilayer graphene sheet on the Casimir force. In Section IV, we consider the Casimir energy between a multilayer graphene sheet and a diamagnetic-magnetic plate, and show that the Casimir force on a multilayer graphene sheet can be repulsive if the substrate is diamagnetic-magnetic material and the permeability of the substrate is much larger than the permittivity. Finally, in Section V, we summarize the unique aspects of the Casimir force acting on a multilayer graphene sheet with diamagnetism and discuss diamagnetic levitation.

II. CASIMIR ENERGY BETWEEN GRAPHITE AND A PERFECTLY CONDUCTIVE PLATE

A. Lifshitz formula

The material dependence of the Casimir energy between two objects mainly depends on their electromagnetic properties. Thus, the dielectric and magnetic properties of multilayer graphene are important to consider in determining the Casimir force acting on multilayer graphene. The dielectric and magnetic properties of multilayer graphene significantly depend on the relative rotation angles between the graphene layers. The interactions between regularly stacked graphene sheets that do not exhibit moiré patterns such as a hexagonal crystal structure [23, 24] are strong. The dispersion relation of multilayer graphene is different from that of a single graphene sheet. For instance, the Dirac point disappears and the band gap changes, depending on the rotation angle [25]. Meanwhile, if the rotating angle is not too small, the interlayer coupling can be weakened; the dielectric and magnetic properties of a graphene sheet remain, and the diamagnetic effect is enhanced by stacking graphene sheets [2]. For the dielectric properties, although there may be quantitative differences between multilayer graphene and graphite, it is reasonable to suppose that the dielectric properties of multilayer graphene are qualitatively the same as those of graphite. The static permittivity along the surface of multilayer graphene probably diverges as well as that along the surface of graphite. In addition, the vertical permittivity toward the surface may be much smaller than the parallel permittivity for both graphite and multilayer graphene. Thus, it is helpful to consider the Casimir energy between graphite and a perfectly conductive plate to understand the Casimir force acting on a multilayer graphene sheet.

The Casimir force between two objects also depends on their shapes. A pair of parallel plates is easy to calculate, but the combination of a sphere and a plate is often used in experiments. Thus, in this study, we consider the Casimir force between a flat multilayer graphene sheet and a spherical object. Using the proximity force approximation [4], the Casimir force $F(a)$ between a sphere of radius $R$ and a plate is given by

$$F(a) = 2\pi R E(a), \quad a \ll R,$$

where $a$ is the separation distance between a sphere and a plate, and $E(a)$ is the Casimir energy between the plates with separation distance $a$ per unit area.

We consider the Casimir energy between a uniaxial dielectric-magnetic body of thickness $d$ (plate 1) and a semi-infinite plate (plate 2), whose permittivity and permeability at angular frequency $\omega$ of the electromagnetic field are given by $\varepsilon(\omega)$ and $\mu(\omega)$, respectively (see Figure 1). The

![Figure 1: Configuration of a multilayer graphene sheet (upper plate), which is a uniaxial conductive-magnetic plate with thickness $d$ and a semi-infinite dielectric-magnetic plate (lower plate).](image-url)
The permittivity of the uniaxial conductive-magnetic body along the x-axis and the z-axis are denoted by \( \varepsilon_x(\omega) \) and \( \varepsilon_z(\omega) \), respectively. Similarly, the permeability along the x-axis and the z-axis is denoted by \( \mu_x(\omega) \) and \( \mu_z(\omega) \), respectively. The permittivity and permeability along the y-axis are same as those along the x-axis. The multilayer graphene sheet and the plate facing it correspond to plates 1 and 2, respectively.

The Casimir energy between the plates with a gap \( a \) is defined by the difference in the vacuum energy of the electromagnetic field between the plates with the gaps \( a \) and \( \infty \), and it is expressed as the summation of the contributions from the electromagnetic field with a transverse magnetic (TM) mode and a transverse electric (TE) mode. The electric magnetic field in a vacuum is characterized by frequency \( \xi \) and wave vector \( k = (k_x, k_y, k_z) \). According to the Lifshitz theory \([4, 5]\), the Casimir energy at temperature \( T \) can be expressed as

\[
E(a) = \frac{k_B T}{2\pi} \sum_{l=0}^{\infty} \int_0^\infty k_z \, dk_z \left[ G_{TM}(\xi, k_L, a) + G_{TE}(\xi, k_L, a) \right],
\]

where the prime near the summation symbol indicates the multiples of the \( l = 0 \) term by a factor of \( 1/2 \). The functions \( G_{TM} \) and \( G_{TE} \) are the contributions to the Casimir energy characterized by the Matsubara frequency \( \xi_l \equiv 2\pi Tq_lT/h \) and \( k_L \equiv (k_x, k_y, k_z) \) in the TM mode and the TE mode, respectively. The functions \( G_{TM} \) and \( G_{TE} \) are expressed by

\[
G_{\sigma}(\xi, k_L) = \ln \left[ 1 - r_{\sigma}^{(1)} r_{\sigma}^{(2)} e^{-2aq|\xi|} \right].
\]

Similarly, the permeability along the \( \xi \)-axis is denoted by \( \mu(\xi) \). The functions \( \mu_{TM} \) and \( \mu_{TE} \) are expressed as

\[
r_{TM}(i\xi, k_L) = \frac{\varepsilon_x(\xi) k_L^2 - k^2}{\varepsilon_x(\xi) k_L^2 + k^2 + 2\varepsilon_x(\xi) k_L k x \coth(k x d)},
\]

\[
r_{TE}(i\xi, k_L) = \frac{\mu_x(\xi) k_L^2 - k^2}{\mu_x(\xi) k_L^2 + k^2 + 2\mu_x(\xi) k_L k x \coth(k x d)},
\]

where

\[
k_x(i\xi_l, k_L) = \sqrt{k_1^2 + \varepsilon_x(i\xi_l) \frac{\mu_x(i\xi_l)}{c^2}},
\]

\[
k_z(i\xi_l, k_L) = \sqrt{k_1^2 + \varepsilon_x(i\xi_l) \frac{\mu_x(i\xi_l)}{c^2}},
\]

In Eqs. (4) and (5), \( \varepsilon_x \) and \( \varepsilon_z \) denote \( \varepsilon_x(i\xi_l) \) and \( \varepsilon_z(i\xi_l) \), respectively. Similarly, \( \mu_x \) and \( \mu_z \) denote \( \mu_x(i\xi_l) \) and \( \mu_z(i\xi_l) \), respectively. The permittivity and permeability along the imaginary frequency, \( \varepsilon(i\xi_l) \) and \( \mu(i\xi_l) \), respectively, are calculated by the Kramers-Kronig relation \([4]\):

\[
\varepsilon(i\xi_l) = 1 + \frac{2}{\pi} \int_0^\infty \frac{\omega \varepsilon_x(\omega)}{\omega^2 + \xi_l^2} \, d\omega,
\]

\[
\mu(i\xi_l) = 1 + \frac{2}{\pi} \int_0^\infty \frac{\omega \mu_x(\omega)}{\omega^2 + \xi_l^2} \, d\omega,
\]

Summarizing the above, the Casimir force between the plates is determined from the imaginary part of the permittivity and permeability of the plates. As a special case, the reflection coefficients of a perfectly conductive plate are given by \( r_{TM} = 1 \) and \( r_{TE} = -1 \). The Casimir energy between perfectly conductive plates \([26]\) at \( T = 0 \) is

\[
E_{\omega}(a) = -\frac{\pi^2 \hbar c}{720 a^3}.
\]

The Casimir energy between perfectly conducting plates at \( T = 0 \) is smaller than that between arbitrary plates at \( T = 0 \). Thus, \( E_{\omega}(a) \) gives a lower bound for the Casimir energy between the plates. We note that the Casimir energy at finite temperatures can be larger than \( E_{\omega}(a) \).

B. Casimir energy between non-magnetic graphite and a perfectly conductive plate

Determining the permittivity and permeability of multilayer graphite is indispensable in the following calculations. To express the permittivity and permeability of graphite as a function of frequency, Blagov et al. used the experimentally obtained optical data and asymptotic forms for high and low frequencies \([4, 13]\). The permittivity for high frequencies is given by \( \varepsilon_{\omega,\infty}(a) \propto \omega^{-3} \). Meanwhile, the permittivity for low frequencies is significantly different, depending on the direction. The permittivity along the \( x \)-axis diverges in the limit of \( \omega \to 0 \) and is asymptotically expressed by the Drude model. On the other hand, the permittivity along the \( z \)-axis converges to a finite value in the limit of \( \omega \to 0 \). To calculate the Casimir force between graphene sheets, Drosdoff and Woods employed the following Drude and Lorentz model \([15]\):

\[
\varepsilon_{\omega}(a) = 1 - \frac{\omega_p^2}{\omega(\omega + i\Gamma_{0,a})} \sum_{j=1}^{k} \frac{F_{j,a}}{(\omega^2 - \omega_{j,a}^2 + i\omega\Gamma_{j,a})},
\]

where \( \omega_{p,a} \) is a plasma frequency along the \( a \)-axis \((a \in \{x, z\})\). In this study, we use this expression, where parameters \( \omega_p, \omega_{p,a}, \Gamma_{0,a}, F_{j,a} \) and \( \Gamma_{j,a} \) are described in Refs. 27 and 28. The plasma frequencies for \( \alpha = x \) and \( y \) are 27 eV and 19 eV, respectively.

Combining Eqs. (8) and (11), we obtain the permittivity along the imaginary axis. Figure 2 shows the permittivity graphite along the imaginary axis \( \varepsilon_x(i\xi) \) and \( \varepsilon_z(i\xi) \) in log-log scale. We calculate the Casimir energy between
graphite and a perfectly conductive plate with infinite thickness, \(d = \infty\). Here the diamagnetic property of graphite is neglected, and it is considered in Section II.C. The reflection coefficient of the plate with infinite thickness is given by

\[
r_{TM}(i\xi l, k_{\perp}) = \frac{\varepsilon_x l \varepsilon_z l q_l - k_{xz}}{\varepsilon_x l \varepsilon_z l q_l + k_{xz}}, \tag{12}
\]

\[
r_{TE}(i\xi l, k_{\perp}) = \frac{q_l - k_{xl}}{q_l + k_{xl}}, \tag{13}
\]

Substituting Eqs. (12) and (13) into Eq. (3) and computing the integral in Eq. (2), we obtain the Casimir energy between graphite and the perfectly conductive plate. The solid line in Figure 3 shows the ratio of the Casimir energy between graphite and the perfectly conductive plate at room temperature \(T = 300\) K to the Casimir energy between perfectly conductive plates, i.e., \(\eta(a) \equiv \frac{R(a)}{R_{\infty}(a)}\).

We consider the asymptotic behavior of \(\eta(a)\) for large \(a\). The minimum nonzero-Matsubara frequency at room temperature is \(\xi_1 = 2.47 \times 10^{14}\) rad s\(^{-1}\), and the corresponding wavelength is 7.6 μm. Since \(q_l = \sqrt{k_{\perp}^2 + \xi_1^2/c^2}\) is always larger than \(\xi_l/c\), the exponent of the exponential function in the left-hand side of Eq. (3), \(aq_l\), is greater than 0.82μa at room temperature, where the unit of \(a\) is μm. This implies that \(\exp(-aq_l)\) rapidly decreases as \(l\) increases, especially for large \(a\). Thus, the asymptotic behavior of the Casimir energy for large separation distances is mainly determined only by the terms with \(l = 0\). The reflection coefficient of graphite at \(l = 0\) is given by \(r_{TE} = 0\) and \(r_{TM} = 1\). As a result, the Casimir energy between graphite and the perfectly conductive plate for large separation distance is expressed by

\[
E(a) \approx -\frac{k_B T \zeta(3)}{16\pi a^2}, \quad a \gg \frac{\xi_1}{c}, \tag{14}
\]

where \(\zeta(\cdot)\) is the Riemann zeta function. The broken line in Figure 3 shows the Riemann zeta function.

### C. Casimir energy for graphite with weak diamagnetism

Graphite is a diamagnetic material, and the mass magnetic susceptibility along the z-axis and the x-axis at \(\omega = 0\) is reported as \(\chi_z \approx -22 \times 10^{-6}\) emu g\(^{-1}\) and \(\chi_x \approx -0.5 \times 10^{-6}\) emu g\(^{-1}\), respectively [29–31]. Using the density of graphite \(\rho = 2.2 \times 10^3\) kg m\(^{-3}\), the mass magnetic susceptibilities in SI units are \(\chi_z \approx -6.1 \times 10^{-4}\) and \(\chi_x \approx -0.1 \times 10^{-5}\), respectively [32]. The reflection coefficient for magnetic materials in the TE mode is given by

\[
r_{TE}(i\xi l, k_{\perp}) = \frac{\mu_x l \mu_z l q_l - k_{xl}}{\mu_x l \mu_z l q_l + k_{xl}}, \tag{15}
\]

If the permittivity of graphite obeys the Drude model, \(\varepsilon(i\xi)^2\) converges to zero as \(\xi\) tends to 0. Thus, the reflection coefficient for \(l = 0\) is given by \(r_{TM} = 1\) and

Figure 2: Permittivity of graphite along the imaginary axis in log-log scale.

Figure 3: Normalized Casimir energy between graphite and a perfectly conductive plate at \(T = 300\) K by the Casimir energy between perfectly conductive plates. The broken line shows the asymptotic behavior for large separations.

Figure 4: Difference between the normalized Casimir energy of graphite with \(\sqrt{\mu_x l(0)\mu_z l(0)} = 0.9997\) and that without diamagnetism.
Using $\mu_x = 1 + \chi_x$ and $\mu_z = 1 + \chi_z$, the reflection coefficient in the TE mode for $l = 0$ is $-1.6 \times 10^{-4}$. The oscillating magnetic susceptibility of graphite and the graphene multilayer is very small for $\chi \geq \chi_l$. Thus, we assume that $\mu_{xl} = \mu_{zl} = 1$ for $l \geq 1$ in the following calculations.

Since the reflection coefficient is $1$ in the TM mode, and $r_{TE}$ can be expressed as a function of $\mu_g \equiv \mu_{x0}\mu_{z0}$, the magnetic dependence of the Casimir energy is determined by $\mu_g$.

Figure 4 shows the change in the normalized Casimir energy at room temperature by considering the diamagnetism of graphite as a function of the separation distance $a$. The impact on the Casimir energy increases as the separation distance increases, making it too small to detect with current technology.

III. CASIMIR ENERGY BETWEEN A MULTILAYER GRAPHENE SHEET AND PERFECTLY CONDUCTIVE PLATE

Now, we consider the Casimir energy between a multilayer graphene sheet and a perfectly conducting plate. We assume that a multilayer graphene sheet is a thin uniaxial conductive-magnetic plate with a permeability less than $1$ and the same permittivity as graphite. The Casimir energy also depends on the thickness of the plate and the permeability of multilayer graphene. To consider separately the impact of the Casimir energy on these values, we initially set the thickness of the multilayer graphene sheet to infinity. 

Figure 5 shows the Casimir energy between a multilayer graphene sheet and a perfectly conducting plate for $\mu_g = 0$ (perfectly diamagnetic case) and $1$ (nonmagnetic case) at room temperature. The broken lines show the asymptotic behavior at large separations. The Casimir energy is enhanced due to the diamagnetic effect. From Eq. (16), the reflection coefficient of a perfectly diamagnetic plate of $l = 0$ in TE mode is $-1$, which is equal to the reflection coefficient of a perfectly conducting plate. The slope of the dashed line in Figure 5 with $\mu_g = 0$ is double that with $\mu_g = 1$. Recalling Eq. (14), the asymptotic behavior of the Casimir energy is expressed as

$$E(a) \approx -\frac{k_B T \zeta(3)}{8\pi a^2}, \quad a \gg \frac{\xi_1}{c},$$

where $\zeta(3)$ is a polylogarithm of the third order. Figure 6 shows the limit value of the normalized Casimir energy by $-k_B T/(16\pi a^2)$ as the separation distance tends to infinity as a function of $\mu_g$, taking on the value of maximum $2\zeta(3)$ at $\mu_g = 0$.

We assumed that the permittivity of multilayer graphene at low frequencies is described by the Drude model to this point. However, this is not clear [33]. Here, we assume that the permittivity along the $x$-axis is described by a plasma model. The permittivity along the imaginary axis is expressed as follows:

$$\epsilon_p(i\xi) = 1 + \frac{a_p^2}{\xi^2}.$$  

The significant difference between the plasma model and the Drude model appears in the limit value of $\xi^2\epsilon(i\xi)$ as $\xi$ tends to $0$. In the Drude model, the limit value takes 0. On
the other hand, the limit value in the plasma model takes a positive value, \( \alpha_p^2 \). This difference causes the change in the expression of the permittivity of multilayer graphene. Figure 7 shows the dependence of the Casimir energy between a multilayer graphene sheet and perfectly conducting plate on the thickness. Thus, the dependence of the Casimir energy on the thickness originates in the contributions of the terms in the summation in Eq. (2).

The plasma frequency of graphite for the permittivity along the \( x \)-axis is 27 eV and the Matsubara frequency at room temperature is 0.16 eV. Accordingly, the value of \( \gamma \) for graphite with \( \mu_{x0} = 1 \) is 166. The reflection constant of the material that obeys the Drude model in the TE mode is obtained by setting \( \gamma \) to zero in Eq. (21) as a special case. If \( \mu_{x1} = \mu_{z1} = 1 \), (i.e., non-magnetic case) and \( \gamma = 0 \), the reflection constant in the TE mode with \( l = 0 \) is zero. Thus, the term with \( l = 0 \) in the TE mode of the summation in Eq. (20) does not contribute to the Casimir energy. Meanwhile, if multilayer graphene obeys the plasma model at low frequencies, the term with \( l = 0 \) in the TE mode must be considered.

According to the recently derived representation for the reflection coefficient based on the Dirac model, the reflection coefficient of graphene for the TE mode is not zero [14, 34]. Thus, the plasma model may be more suitable to the expression of the permittivity of multilayer graphene. Figure 7 shows the dependence of the Casimir energy between a multilayer graphene sheet whose permittivity obeys the plasma model near \( \xi = 0 \) and a perfectly conducting plate at \( a = 1 \) \( \mu\)m on the plasma frequency for \( \mu_{x0} = 1 \) and different \( \mu_{z0} \). In the calculation, the permittivity of multilayer graphene for \( l \geq 1 \) is described by the Lorentz model [see Eq. (11)], and the permittivity at low frequencies \( \xi \ll \xi_1 \) is described by the plasma model. The Casimir energy increases as the plasma frequency increases except at \( \mu_{z0} = 0 \). Since the reflection coefficient is \(-1\) at \( \mu_{z0} = 0 \), independently of the plasma frequency, the Casimir energy is also independent of the plasma frequency. The increase in the magnitude of the Casimir energy by the enhancement of diamagnetism is suppressed for large plasma frequencies.

Advanced technologies are required to fabricate a thick random-stacked multilayer graphene sheet. Thus, it is important to consider the dependence of the Casimir energy on the thickness of the multilayer graphene. Figure 8 shows the normalized Casimir energies at \( a = 1 \) \( \mu\)m for \( \mu_{x0} = 1 \) and \( \mu_{z0} = 0 \) as a function of the thickness \( d \). The permeability along the \( x \)-axis is 1 for both cases. The Casimir energy increases as the thickness increases, and its increasing rate is almost independently of the permeability. The impact of the magnetic property on the Casimir energy at large separations is determined by the term with \( l = 0 \) in the TE mode. The thickness dependence of the reflection coefficient is determined through the product of \( \sqrt{\mu_{x1}\mu_{z1}} \) and \( \coth(k_{x1}d) \), as shown in Eq. (5). As a special case, if \( \mu_{x1} = 0 \), the reflection coefficient of \( l = 0 \) in the TE mode is independent of the thickness. Thus, the dependence of the Casimir energy on the thickness originates in the contributions of the terms with \( l \geq 1 \) in the summation in Eq. (2).

\[ r_{TE}(0, k_\perp) = \frac{\mu_g^2 y^2 - (y^2 + \gamma^2)}{\mu_g y + \sqrt{y^2 + \gamma^2}}, \]

where \( y = \epsilon k_\perp / \xi_1 \) and \( \gamma = \sqrt{\mu_{z0} \omega_p / \xi_1} \). In particular, if the thickness \( d \) is infinity, the reflection constant is written as

\[ r_{TE}(0, k_\perp) = \frac{\mu_g y - \sqrt{y^2 + \gamma^2}}{\mu_g y + \sqrt{y^2 + \gamma^2}}. \]
IV. CASIMIR ENERGY BETWEEN A MULTILAYER GRAPHENE SHEET AND A DIELECTRIC-MAGNETIC PLATE

The diamagnetic effect on the Casimir energy depends on

\[
E(a) \approx -\frac{k_b T}{16\pi a^2} \left\{ \text{Li}_3 \left[ \frac{\epsilon^{(2)}(0) - 1}{\epsilon^{(2)}(0) + 1} \right] + \text{Li}_3 \left[ \left( \sqrt{|\mu_g - 1|} \frac{\mu^{(2)}(0) - 1}{\mu^{(2)}(0) + 1} \right) \right] \right\}.
\]

If the permeability of the dielectric-magnetic material (plate 2) is larger than 1, the argument of the polylogarithm is negative. Since the polylogarithm is a monotonically increasing function, the second term increases the Casimir energy as \( \mu_g \) decreases from 1 to 0. To observe this increase by experiment, it is desirable to choose a material with smaller permittivity than permeability such as yttrium-iron-garnet (YIG). We use \( \mu^{(2)}(0) = 150 \) as the static permeability of YIG \([35, 36]\). The permittivity of YIG is in the range of 4 to 12 \([37–39]\). We calculate the Casimir energy between a diamagnetic multilayer graphene sheet with \( \mu^{(2)}_0 = 0 \) and a dielectric-magnetic material with static permittivity \( \epsilon^{(2)}(0) \) and permeability \( \mu^{(2)}(0) \). The oscillating permittivity and oscillating permeability of the dielectric-magnetic plate are assumed to be \( \infty \) and 1, respectively. If the oscillating permittivity is finite, the change in the Casimir energy due to the non-zero permeability is more enhanced, because the contribution of term with the \( l = 0 \) becomes more important than the remaining terms. Figure 9 shows the dependence of the Casimir energy on the separation distances for different static permittivities \( \epsilon^{(2)}(0) = 4 \) and 12. The permeability is 150 for both cases. The dashed lines show the asymptotic behavior of the Casimir energy for large separation distances given by Eq. (22). In contrast to Figure 5, the normalized Casimir energy decreases as the separation increases near 1 \( \mu \). Interestingly, the normalized Casimir energy for \( \epsilon^{(2)}(0) = 4 \) becomes negative. Recalling that the Casimir force between the perfectly conducting plates is always negative, the negative sign of the normalized Casimir energy means that the Casimir energy is positive. It follows that the Casimir force between a dielectric-magnetic sphere and a diamagnetic multilayer graphene sheet is repulsive from the proximity force approximation Eq. (1).

V. CONCLUSIONS

We have investigated the impact of diamagnetism induced in a multilayer graphene sheet on the Casimir energy. The change in the Casimir energy due to the decrease in the permeability at large separations is summarized as follows. The Casimir energy between a diamagnetic multilayer graphene sheet and a nonmagnetic plate decreases as the static permeability of a multilayer graphene sheet decreases from 1 to 0. Conversely, the Casimir energy between a diamagnetic multilayer graphene sheet and a magnetic plate with permeability greater than 1 increases as the permeability of multilayer graphene decreases. The Casimir energy between two diamagnetic multilayer graphene sheets decreases as the permeability decreases. We assumed that the permittivity of multilayer graphene along the \( x \)-axis diverges as the frequency tends to zero and showed that the Casimir energy at large separations is mainly governed by this property and the static permeability. If the permittivity has a quadratic divergence near \( \xi = 0 \), the Casimir energy at large separations depends on the plasma frequency. As shown in Figure 7, the increase in the Casimir energy due to diamagnetism is more evident for small \( \omega_p \). The absolute value of the Casimir energy decreases as the thickness of a multilayer graphene sheet increases. In addition, it is also expected that the permeability decreases as the thickness of a multilayer graphene sheet increases, and a strong diamagnetic material is realized \([2]\). Thus, it is desirable to develop a method for producing a thick multilayer graphene sheet to observe the impact of diamagnetism on the Casimir force.

Combining the proximity force approximation and the obtained results, it is conjectured that the attractive force between a multilayer graphene sheet and a dielec-
tric-magnetic sphere weakens as the permeability of the multilayer graphene sheet decreases. In particular, if the permittivity of the dielectric-magnetic plate is much smaller than the permeability of the plate, the Casimir force may be repulsive. This implies that a multilayer graphene sheet is levitated above the dielectric-magnetic plate. It is well known that a thin pyrolytic graphite sheet can be levitated above a sufficiently strong neodymium magnet [32]. The significant difference between them is that the levitation by the repulsive Casimir force can occur even if the magnetic moment of the dielectric-magnetic plate is zero. A superconductor is an excellent diamagnetic material and may enable quantum levitation. However, superconductor diamagnetism disappears above the superconductive transition temperature. At room temperature, graphite has a smaller net magnetism disappears above the superconductive transition temperature. Similarly, quantum levitation may be optically controlled as well [41].

Acknowledgments

This research was supported by the Ministry of Education, Culture, Sports, Science and Technology (MEXT), through a Grant-in-Aid for Scientific Research (C), MEXT KAKENHI Grant Number 25390117.

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