Bankrupting DoS Attackers Despite Uncertainty*

Trisha Chakraborty1, Abir Islam2, Valerie King3,
Daniel Rayborn4, Jared Saia5, and Maxwell Young6

1,4,6Department of Computer Science and Engineering, Mississippi State University, MS, USA
tc2006@msstate.edu,dcr101@msstate.edu,myoung@cse.msstate.edu

2,5Department of Computer Science, University of New Mexico, NM, USA
abir@cs.unm.edu,saia@cs.unm.edu

3Department of Computer Science, University of Victoria, BC, Canada,
val@cs.uvic.ca

Abstract

On-demand provisioning in the cloud allows for services to remain available despite massive
denial-of-service (DoS) attacks. Unfortunately, on-demand provisioning is expensive and must
be weighed against the costs incurred by an adversary. This leads to a recent threat known as
economic denial-of-sustainability (EDoS), where the cost for defending a service is higher than
that of attacking.

A natural tool for combating EDoS is to impose costs via resource burning (RB). Here, a
client must verifiably consume resources—for example, by solving a computational challenge—
before service is rendered. However, prior RB-based defenses with security guarantees do not
account for the cost of on-demand provisioning.

Another common approach is the use of heuristics—such as a client’s reputation score or the
geographical location—to identify and discard spurious job requests. However, these heuristics
may err and existing approaches do not provide security guarantees when this occurs.

Here, we propose an EDoS defense, LCharge, that uses resource burning while accounting
for on-demand provisioning. LCharge leverages an estimate of the number of job requests
from honest clients (i.e., good jobs) in any set S of requests to within an O(α)-factor, for any
unknown α > 0, but retains a strong security guarantee despite the uncertainty of this estimate.
Specifically, against an adversary that expends B resources to attack, the total cost for defending
is O(α5/2 √B(g + 1) + α3(g + α)) where g is the number of good jobs. Notably, for large B
relative to g and α, the adversary has higher cost, implying that the algorithm has an economic
advantage. Finally, we prove a lower bound for our problem of Ω(√αBg), showing that the cost
of LCharge is asymptotically tight for α = Θ(1).

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“Because that’s where the money is.”
– Willie Sutton’s response when asked why he robbed banks.

1 Introduction

Money is the key to understanding modern denial-of-service (DoS) attacks. On one side, DoS attackers are now primarily motivated by money obtained via extortion; and so they invest money to rent botnets to launch attacks. On the other side, service providers can now defend by paying money to provision additional cloud resources, thereby ensuring legitimate clients do not suffer service disruptions [59]. But provisioning comes at a cost.

Thus, modern DoS attacks have shifted from traditional resource bottlenecks—such as computational power—to monetary cost. Attackers can now exploit economic asymmetries, provided that they can spend less to launch an attack than is required to keep a service available. These new types of DoS attacks are commonly referred to as economic denial-of-sustainability (EDoS) [15,30,47,48]. A major challenge for defenders is setting the correct amount of provisioning, in order to manage costs, while continuing to provide service to legitimate clients [59,60,64].

In this paper, we consider the problem of defending against EDoS attacks. Our algorithm uses two key tools. The first is resource burning (RB)—the verifiable expenditure of a network resource—which has been used to mitigate attacks by an adversary who is financially motivated [23,49]. Examples of resources are computational power, computer memory, or bandwidth [23,25,26], and burning such resources ultimately translates into a monetary cost. To be as general our approach is agnostic to the specific resource used.

The second tool is an estimator, which for any set of requests, estimates the number originating from legitimate clients; however, we do not assume the accuracy of the estimate is known. Estimators may be constructed using machine learning techniques for classification [12,45,50]. But, to be as general as possible, our approach is agnostic to any specific implementation of the estimator

Our Goal. Design an algorithm that guarantees that the total amount of resource burning performed by legitimate clients is asymptotically less than that performed by the attacker, with respect to the size of the EDoS attack.

1.1 Formal Problem Statement

A server receives a stream of jobs one after another. Each job is either good: issued by a legitimate client; or bad: issued by the adversary. Costs are paid by either the algorithm or the adversary, as described below.

The following steps occur for each new job:

1. The algorithm sets an RB cost for the next job equal to some positive value.

2. The adversary decides if the next job will be good or bad.

3. If the next job is good, the RB cost is charged to the algorithm. If the job is bad, the adversary pays the RB cost.
4. The job is serviced. If the job is bad, the algorithm pays an additional cost of 1, which we call the **provisioning cost**.

5. The algorithm is able to call an estimator on any continuous subsequence of jobs that have been seen so far. The estimator provides an estimate of the number of good jobs in that subsequence (see Section 1.1.1).

### 1.1.1 The Estimator

Our algorithm uses an *estimator* that estimates the number of good jobs in some set of jobs. Let \( \alpha \geq 1 \) be some fixed integer that is *unknown to the algorithm in advance*. For a set of jobs \( S \), let \( g(S) \) be the number of good jobs in that set and \( \hat{g}(S) \) be the value returned by the estimator in that set. We make two key assumptions about the function \( \hat{g} \).

First, our approximation guarantee:

\[
g(S)/\alpha - \alpha \leq \hat{g}(S) \leq \alpha g(S) + \alpha
\]

Second, we assume \( \hat{g} \) is an additive function. In particular, for any two disjoint sets of jobs \( S \) and \( T \):

\[
\hat{g}(S \cup T) = \hat{g}(S) + \hat{g}(T)
\]

We note that our assumption for the approximation guarantee is similar to that used by Scully, Grosof and Mitzenmacher [46] for an estimator that estimates the lengths of jobs.

### 1.1.2 Resource Burning

Our model is purposefully agnostic about the type of resource burned. We will refer generically to an *RB challenge*, which imposes the desired RB cost on a client who solves this challenge; to specify the RB cost \( h \), we will refer to an *\( h \)-hard* RB challenge. The theoretical guarantees of our defense do not rely on a specific method for issuing an RB challenge or verifying its solution. We expand on these aspects in Section 2 and address some related practical issues later in Section 7. In our formal problem, all jobs both pay the RB cost and are serviced given the ability to provision on-demand.

It is worth addressing the practical issue of a legitimate client having insufficient resources to meet the RB cost set by the algorithm. Here, we make two points:

- First, as justification for our model, consider that for every good job, there is either (1) the RB cost to obtain service, if the corresponding client decides to pay the RB cost; or (2) the notion of an opportunity cost of not being serviced, if the client decides not to pay the RB cost. In the case where RB challenges become so hard that good jobs simply give up, then the opportunity cost is going to be less than the RB cost. Thus, in our model, we are pessimistically upper bounding the cost to the job by saying this cost is the RB cost.

- Second, all prior RB-based defenses [6, 28, 11, 43, 52, 54] have a point where they fail against a sufficiently powerful adversary. This is also true of more traditional defenses, such as those based on over-provisioning and replication. This is unsurprising, since DDoS attacks are notoriously
challenging and no single technique offers a complete solution. Our approach raises the relative
cost to the adversary for attacking, and it offers a new tool that can be used in combination
with other mitigation techniques.

1.1.3 The Adversary

Our adversary is omniscient: it knows the algorithm before the problem begins. Moreover the adver-
sary dynamically controls all aspects of the problem. In particular, it sets (1) the distribution of good
and bad jobs in an online manner; and (2) sets the values returned by the estimator dynamically,
subject to the constraints in Section 1.1.1.

These assumptions pessimistically model an adversary that has a deep knowledge of the tools
used by the defense algorithm.

1.2 Our Result

Let \( g \) be the number of good jobs and let \( B \) be the total cost to the adversary for attacking. In
Section 4, we prove the following upper bound on the total cost of our DoS defense algorithm,
\( \text{LCharge} \).

**Theorem 1.1.** \( \text{LCharge} \) has total cost \( O(\alpha^{5/2} \sqrt{B(g+1)} + \alpha^2(g + \alpha)) \).

In Section 5 we also prove the following lower bound for our problem.

**Theorem 1.2.** Let \( \alpha \) be any positive integer. Let \( g \) be any multiple of \( \alpha \) in the range \( [\alpha, n/\alpha] \), where
\( n \), the number of jobs grows large. Then, any randomized algorithm has expected cost \( \Omega(\sqrt{\alpha T g}) \),
where \( T \) is the expected cost to the adversary.

2 Related Work

2.1 DoS Attacks and Defenses

DoS attacks are a persistent threat, with current trends pointing to an increase in their occurrence\textsuperscript{31}; consequently, the associated literature is vast. Here, we discuss and compare against prior results
most pertinent to our approach.

**Resource Burning for DoS.** Several DoS defenses use resource-burning, requiring good clients
to solve computational challenges. Early work by Juels and Brainard \textsuperscript{27} examines cryptographic
puzzles to mitigate connection depletion attacks, such as TCP SYN flooding. Work by Aura et al. \textsuperscript{6}
followed soon after, looking to apply computational puzzles to more general DoS attacks. In contrast
to computational resources, Walfish et al. \textsuperscript{52} propose a defense that instead burns communication
capacity.

Some such defenses rely on routers to participate in the defense. For example, Parno et al. \textsuperscript{43}
look at how capabilities (informally, where routers prioritize specified traffic flows to the server that
have been given permission) can be defended against DoS attacks via computational puzzles. Work
by Chen et al. [14] also makes use of routers to filter traffic flows, but with a focus on minimizing modifications to the existing network architecture.

Methods have been explored for appropriately tuning the hardness of RB challenges. Mankins et al. [36] explore methods of pricing access to the server resources based on client behavior, where the costs imposed can be either monetary or computational. Along similar lines, Wang and Reiter [54] explore ways that clients can bid on puzzles via auctions. More recently, Noureddine et al. [41] apply game theory to determine the difficulty of the computational challenges used in the context of DoS attacks at the transport layer.

Other work has focused on specific practical aspects related to DDoS defense. User experience is addressed by Kaiser and Feng [28], who provide a browser plugin that makes a computational puzzle-based DoS defense largely transparent to the user. Another issue is the wasted work inherent in solving puzzles, and Abliz and Znati [2] propose the use of puzzles whose solutions provide useful work on real applications/services.

Generally, all of the above-mentioned approaches have clients quickly increase the amount of resource burning until service is received. However, these approaches do not account for on-demand provisioning, nor do they provide an asymptotic advantage to good clients over the adversary in terms of cost.

**Generating Challenges and Verifying Solutions.** Challenge generation and validation are efficient operations, and in our model they have zero cost. However, this functionality is itself subject to attack [5] and must itself be protected. Fortunately, unlike the range of services offered by the server, the narrow functionality of generating and validating challenges can be easily outsourced and scaled, while being resistant to attack (see [53]). In practice, a dedicated service can provide this narrow functionality and, under our model, jobs may contact this service to receive a challenge and have their solutions verified.

**Provisioning Costs.** Many servers are equipped with sufficient resources to handle good jobs. However, during a DoS attack, additional resources must be provisioned on-demand. This imposes an economic cost on the system that grows as a function of the traffic load. In practice, the cost for DoS protection can be significant and is a factor in how defenses are designed [7,60,64].

**Classification and Filtering** Perfectly differentiating good and bad jobs can be challenging, since the clientele often enjoy a large degree of anonymity, and face little-to-no admission control for using a service. Nonetheless, classifiers are a common method for mitigating DoS attacks. A classification accuracy above 95% has been demonstrated [17,40,61]. Despite such accuracy, we face uncertainty when using classifiers, and some bad jobs will still consume resources due to classification error. Thus, classification alone is not a silver bullet, but rather part of a more comprehensive defense.

Many DoS defenses rely on a range of techniques for filtering out malicious traffic [63], including IP profiling [35,58]; and capability-based schemes [5,56]. These techniques are complimentary to our proposed approach and can further reduce the volume of malicious traffic. However, again, they are not a solution by themselves, as they may err. Even if the error rate is a small constant, classification and filtering alone can not ensure that algorithmic provisioning cost will be asymptotically smaller than the attacker’s cost.
LCharge

When the server receives a job, do:

- The job is issued an $|S|$-hard challenge, where $S$ is the set of all jobs in the current iteration.
- If $\hat{g}(S) \geq 1$, then end the current iteration and start a new one.

Figure 1: Pseudocode for LCharge.

2.2 Resource Burning

There is significant research, spanning multiple decades, using resource burning to address general security problems; see the surveys [4, 23]. Security algorithms that use resource burning arise in the domains of wireless networks [21], peer-to-peer systems [10, 33], blockchains [34], and e-commerce [23]. Our algorithm is purposefully agnostic about the specific resource burned, which can include computational power [54], bandwidth [53], computer memory [1], or human effort [42, 51].

2.3 Algorithms with Predictions

Algorithms with predictions is a new research area which seeks to use predictions to achieve better algorithmic performance. In general, the goal is to design an algorithm that performs well when prediction accuracy is high, and where performance drops off gently with decreased prediction accuracy. Critically, the algorithm is not assumed to have any a priori knowledge about the accuracy of the predictor.

The use of predictions offers a promising approach for improving algorithmic performance. This approach has found success in the context of contention resolution [19]; bloom filters [37]; online problems such as job scheduling [32, 44, 46]; and many others (see surveys by Mitzenmacher and Vassilvitskii [38, 39]). To the best of our knowledge, our result is the first in this area that addresses a problem in computer security.

2.4 Resource-Competitive Algorithms

There is a growing body of research on security algorithms whose costs are parameterized by the adversary’s cost. Such results are broadly referred to as resource-competitive, and are discussed in survey [9]. Specific recent results in this area include algorithms to address: malicious interference on a broadcast channel [13, 18, 20, 29], contention resolution [8], interactive communication [3, 16], the Sybil attack [22, 24], and bridge assignment in Tor [62]. Notably, our result differs from these prior works by using an estimator and incorporating the approximation guarantee of the estimator into the cost.
3 Our Algorithm: LCharge

Our algorithm, LCharge, makes critical use of an estimator. It partitions the jobs into iterations, which are ended when \( \hat{g}(S) \geq 1 \), where \( S \) is the set of jobs in the current iteration. The cost of the \( i \)-th job in any iteration is \( i \). LCharge, is given in Figure 1.

In the next section, we analyze a general class of algorithms that charge the \( i \)-th job in the iteration a RB cost of \( i^\gamma \), for some positive \( \gamma \). Our analysis shows that LCharge, which sets \( \gamma = 1 \), is asymptotically optimal across this class of algorithms.

4 Analysis

Let \( g \) be the total number of good jobs and \( b \) be the total number of bad jobs.

Lemma 1. The number of iterations is no more than \( \alpha(g + 1) \).

Proof. Let \( S_i \) be the set of jobs in iteration \( i \) for \( 1 \leq i \leq \ell \), where \( \ell \) is the total number of iterations. By the algorithm, \( \hat{g}(S_i) \geq 1 \) for all \( i \). Let \( S = \cup_{1 \leq i \leq \ell} S_i \). Using the additive property of \( \hat{g} \), we get \( \hat{g}(S) = \sum_{1 \leq i \leq \ell} \hat{g}(S_i) \geq \ell \). But by the estimator approximation guarantee, \( \hat{g}(S) \leq \alpha(g(S) + 1) \). Thus, \( \ell \leq \alpha(g(S) + 1) = \alpha(g + 1) \).

Lemma 2. The number of good jobs in any iteration is no more than \( \alpha(\alpha + 1) \).

Proof. Fix some iteration \( i \). Let \( S \) be the set of jobs in that iteration, and let \( S' \) be the set of jobs in the iteration except for the last job. By the algorithm rule for ending an iteration, \( \hat{g}(S') < 1 \). Using the approximation guarantee for \( \hat{g} \), we know that \( g(S')/\alpha - \alpha \leq \hat{g}(S') < 1 \), which implies that \( g(S') < \alpha(\alpha + 1) \). This implies that \( g(S) \leq \alpha(\alpha + 1) \) since \( \alpha \) is an integer.

Lemma 3. The number of iterations is at least \( \max\{1, \frac{g-\alpha^2}{(\alpha+\alpha^2)}\} \).

Proof. Let \( S_i \) be the set of jobs in iteration \( i \) for \( 1 \leq i \leq \ell \), where \( \ell \) is the total number of iterations. By Lemma 2, \( \alpha(\alpha + 1) \geq g(S_i) \geq \hat{g}(S_i) \) for all \( i \).

Let \( S = \cup_{1 \leq i \leq \ell} S_i \). Using the additive property of \( \hat{g} \), we get

\[
\hat{g}(S) = \sum_{1 \leq i \leq \ell} \hat{g}(S_i) \leq (\alpha^2 + \alpha)\ell
\]

But by the estimator approximation guarantee, \( \hat{g}(S) \geq (g(S)/\alpha) - \alpha \). Thus,

\[
\ell \geq (g/\alpha - \alpha)/(\alpha^2 + \alpha) = (g - \alpha^2)/(\alpha(\alpha^2 + \alpha))
\]

Recall that \( B \) is the cost to the adversary.

Lemma 4. For any positive \( \gamma \), \( B \geq \frac{(b-\alpha(g+1))^{\gamma+1}}{(\gamma+1)(\alpha(g+1))^{\gamma}} \).
Proof. The cost to the adversary is minimized when the bad jobs in each iteration occur before the good jobs. Furthermore, this cost is minimized when the bad jobs are distributed as uniformly as possible across iterations. To see this, assume $j$ jobs that are distributed non-uniformly among $\ell$ iterations. Thus, there exists at least one iteration $i_1$ with at least $\lceil j/\ell \rceil + 1$ bad jobs, and also at least one iteration $i_2$ with at most $\lfloor j/\ell \rfloor - 1$. Since the cost function $i^\gamma$ is monotonically increasing, moving one bad job from iteration $i_1$ to iteration $i_2$.

To bound the adversarial cost in an iteration, we use an integral lower bound. Namely, for any $x$ and $\gamma$, $\sum_{i=1}^{x} i^\gamma \geq 1 + \int_{i=1}^{x} i^\gamma di \geq \frac{x^{\gamma+1}}{\gamma+1}$, where the last inequality holds for $\gamma \geq 0$. If the number of iterations is $\ell > 0$, we let $x = \lfloor b/\ell \rfloor$ in the above and bound the total cost of the adversary as:

$$B \geq \frac{\ell}{\gamma + 1} \left\lfloor \frac{b}{\ell} \right\rfloor^{\gamma+1} \geq \frac{\ell}{\gamma + 1} \left( \frac{b}{\ell} - 1 \right)^{\gamma+1} \geq \frac{b - \ell}{\gamma + 1} \left( \frac{b}{\ell} - 1 \right)^{\gamma} \geq \frac{b - \alpha(g + 1)}{\gamma + 1} \left( \frac{b}{\alpha(g + 1)} - 1 \right)^{\gamma} \geq \frac{b - \alpha(g + 1)}{\gamma + 1} \left( b - \alpha(g + 1) \right)^{\gamma} \geq \frac{b - \alpha(g + 1)}{\gamma + 1} \left( (\gamma + 1) \alpha(g + 1) \right)^{\gamma}$$

The second step holds since $(b/\ell) - 1 = (b - \ell)/\ell$. The fourth step holds since, by Lemma 1, $\ell \leq \alpha(g + 1)$.

Recall that the cost to the total algorithm is the sum of its RB cost and the provisioning cost. In the following, let $A_{RB}$ denote the RB cost to the algorithm.

Lemma 5. For $\gamma \geq 1$,

$$A_{RB} = O \left( \alpha^{2\gamma} g + \alpha^{2b\gamma} + \alpha^{2(\gamma+1)} \right).$$

For $0 \leq \gamma < 1$,

$$A_{RB} = O \left( \alpha^{2\gamma+3} g + \frac{\alpha^{3\gamma} b^{\gamma+1}}{\max\{1, (g - \alpha^2)^\gamma\}} \right).$$

Proof. Let $\ell$ be the number of iterations, and for $i \in [1, \ell]$ let $g_i$ and $b_i$ be the number of good and bad jobs in iteration $i$, respectively. Then the RB cost to the algorithm in an iteration is maximized when all good jobs come at the end of the iteration. For a fixed iteration $i$, the cost is: $\sum_{j=1}^{g_i} (b_i + j)^\gamma$. Then, the total RB cost of the algorithm, $A_{RB}$ is:

$$\sum_{i=1}^{\ell} \sum_{j=1}^{g_i} (b_i + j)^\gamma$$
Note that for all $1 \leq i \leq \ell$, $g_i \leq \alpha^2 + \alpha$, by Lemma 2. For simplicity of notation, in this proof, we let $\beta = \alpha^2 + \alpha$; and note that for all $i \in [1, \ell]$, $g_i \leq \beta$, by our estimator property.

We use exchange arguments to determine the settings for which $A_{RB}$ is maximized subject to our constraints. Consider any initial setting of $\vec{g}, \vec{b}$. We assume, without loss of generality, that the $b_i$ values are sorted in decreasing order, since if this is not the case, we can swap iteration indices to make it so, without changing the function output.

**Good jobs always packed left.** We first show that $A_{RB}$ is maximized when the good jobs ‘are ‘packed left”: $\beta$ good jobs in each of the first $\lfloor g/\beta \rfloor$ iterations, and $g \mod \beta$ good jobs in the last iteration.

To see this, we first claim that, without decreasing $A_{RB}$, we can rearrange good jobs so that the $g_i$ values are non-increasing in $i$. To see this, consider any two iterations $j, k$ such that $1 \leq j < k \leq \ell$, where $g_j < g_k$. We move the last $g_k - g_j$ good jobs in iteration $k$ to the end of iteration $j$. This will not decrease $A_{RB}$ since the cost incurred by each of these moved jobs can only increase, since $b_j \geq b_k$, and our RB-cost function can only increase with the job number in the iteration. Repeating this exchange establishes the claim.

Next, we argue that, without decreasing $A_{RB}$, we can rearrange good jobs so that the $g_i$ values are $\beta$ for all $1 \leq i < \ell$. To see this, consider the case where there is any iteration before the last that has less than $\beta$ good jobs, and let $j$ be the leftmost such iteration with $g_j < \beta$. Since $j < \ell$, there must be some iteration $k$, $j < k$ such that $g_k > 0$. We move the last good job from iteration $k$ to the end of iteration $j$. This will not decrease $A_{RB}$ since the cost incurred by this moved job can only increase, since our RB-cost function can only increase with the job number in the iteration. Repeating this exchange establishes the claim.

**Analyzing $\Delta_i$.** Next, we set up exchange arguments for the bad jobs. For any $i \in [1, \ell]$, define,

$$\Delta_i = \sum_{j=1}^{\beta} (b_i + 1 + j)\gamma - \sum_{j=1}^{\beta} (b_i + j)\gamma$$

If we consider swapping a single bad job from iteration $k$ into iteration $i$ where $1 \leq i < k \leq \ell$, this results in the following change in the overall cost: $\Delta_i - \Delta_{k-1}$.

Let $f : \mathbb{R} \to \mathbb{R}$ such that, $f(x) = \sum_{j=1}^{\beta} (x + j)^\gamma$. For any $i \in [1, \ell]$, this function is continuous when $x \in [b_i, b_i + 1]$ and differentiable when $x \in (b_i, b_i + 1)$, since it is the sum of a finite number of continuous and differentiable functions respectively. For any $i \in [1, \ell]$, by applying the Mean Value Theorem for the interval $[b_i, b_i + 1]$, we have:

$$\Delta_i = \frac{f(b_i + 1) - f(b_i)}{(b_i + 1) - b_i} = f'(x_i) \text{ for some } x_i \in [b_i, b_i + 1].$$

**Bad jobs packed left when $\gamma \geq 1$.** When $\gamma \geq 1$, $f$ is convex. Hence, $f'(x)$ is non-decreasing in $x$. Using this fact and the Mean Value Theorem analysis above, we get that $\Delta_1 \geq f'(b_1)$; and, for any $1 < k \leq \ell$, $\Delta_{k-1} \leq f'(b_k)$. In addition, $f'(b_1) \geq f'(b_k)$ since $b_1 \geq b_k$, by our initial assumption that the $b_i$ values are non-increasing in $i$. Thus, $\Delta_1 - \Delta_{k-1} \geq 0$. 

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This implies that $A_{RB}$ does not decrease whenever we move a bad job from some iteration with index $k > 1$ to iteration 1. Hence, $A_{RB}$ is maximized when all bad jobs occur in the first iteration.

**Bad jobs evenly spread when $\gamma < 1$.** When $\gamma < 1$, $f$ is concave. Hence, $f'(x)$ is non-increasing in $x$. Using this fact and the Mean Value Theorem analysis above, we get that $\Delta_i \leq f'(b_i)$; and, for any $1 < k \leq \ell$, $\Delta_{k-1} \geq f'(b_k)$. In addition, $f'(b_k) \geq f'(b_i)$ since $b_i \geq b_k$, by our initial assumption that the $b_i$ values are non-increasing in $i$. Thus, $\Delta_i - \Delta_{k-1} \leq 0$.

This implies that $A_{RB}$ increases whenever we swap bad jobs from an iteration with a larger number of bad jobs to an iteration with a smaller number. Hence, $A_{RB}$ is maximized when bad jobs are distributed as evenly as possible across iterations.

**Upper bound on $A_{RB}$.** We can now bound $A_{RB}$ via a case analysis.

**Case $\gamma \geq 1$:** With the above exchange argument in hand, we have:

\[
A_{RB} \leq \left( \frac{g}{\beta} \right) \sum_{j=1}^{\beta} j^\gamma + \sum_{j=1}^{\beta} (b + j)^\gamma
\]

\[
\leq \left( \frac{g}{\beta} \right) \sum_{j=1}^{\beta} j^\gamma + \beta (b + \beta)^\gamma
\]

\[
\leq 2 \left( \frac{g}{\beta} \right) \frac{\beta^{\gamma+1}}{\gamma + 1} + \beta (b + \beta)^\gamma
\]

\[
\leq 2 \left( \frac{g}{\beta} \right) \frac{\beta^{\gamma+1}}{\gamma + 1} + \beta (2b)^\gamma + \beta (2\beta)^\gamma
\]

\[
= O \left( \beta^\gamma g + \beta b^\gamma + \beta^{\gamma+1} \right)
\]

where the third line follows by upper bounding the sum by an integral, and the fourth line follows from the inequality $(x + y)^d \leq (2x)^d + (2y)^d$ for any positive $x, y$, and $d$, which holds because $x + y \leq 2x$ or $x + y \leq 2y$. Finally, note that replacing $\beta$ with $\alpha^2 + \alpha$ gives the same asymptotic upper bound in the lemma statement.

**Case $\gamma < 1$:** With the above exchange argument in hand, we have:

\[
A_{RB} \leq \ell \sum_{j=1}^{\beta} (b/\ell + j)^\gamma
\]

\[
\leq \ell \sum_{j=[1,\beta+b/\ell+1]} j^\gamma
\]

\[
\leq 2\ell \left( \beta + b/\ell + 1 \right)^{\gamma+1}
\]

\[
\leq 2\ell \left( 2\beta + 2 \right)^{\gamma+1} + (2b/\ell)^{\gamma+1}
\]

\[
= O \left( \alpha^{2\gamma+3} g + \frac{\alpha^{2\gamma+3} b^\gamma + 1}{\max\{1, (g - \alpha^2 \gamma)\}} \right)
\]
In the above, the last line follows by Lemma \[\text{3}\] and noting that \(1/\gamma\) is a fixed constant. \[\square\]

Note from Lemma \[\text{5}\] that for \(b \geq \alpha^2\), \(A_{RB}\) is minimized when \(\gamma \geq 1\). Using this fact, we show \(\gamma = 1\) minimizes the total cost of the algorithm as a function of \(B\) and \(g\). The following theorem bounds the RB cost of the algorithm for that case.

\[\textbf{Theorem 1.1.}\] The total cost to \(LCharge\) is \(O\left(\alpha^{5/2} \sqrt{T(g+1)} + \alpha^3(g + \alpha)\right)\).

\[\textbf{Proof.}\] To bound the cost of \(A\), we perform a case analysis.

\textbf{Case: } \(b \geq 2\alpha(g + 1)\). In this case, \(\alpha(g + 1) \leq b/2\). By Lemma \[\text{4}\] we have:

\[B \geq \frac{(b - \alpha(g + 1))^{\gamma+1}}{(\gamma + 1)(\alpha(g + 1))^{\gamma}}\]

\[\geq \frac{(b/2)^{\gamma+1}}{(\gamma + 1)(\alpha(g + 1))^{\gamma}}\]

\[= \frac{b^{\gamma+1}}{2^{\gamma+1}(\gamma + 1)\alpha^{\gamma}(g + 1)^{\gamma}}\]

Note that the provisioning cost to the algorithm is exactly \(b\). Let \(A\) denote the total cost to the algorithm. We now use Lemma \[\text{5}\] specifically, the case for \(\gamma \geq 1\), which gives the smallest worst case RB cost to the algorithm, across all values of \(g\) and \(b\) and \(\alpha\). This yields:

\[A \leq b + c(\alpha^2g + \alpha^2b^\gamma + \alpha^{2\gamma+2})\]

where \(c > 0\) is a sufficiently large constant. Thus we have:

\[A \leq b + c(\alpha^2g + \alpha^2b^\gamma + \alpha^{2\gamma+2})\]

\[\leq b + c(\alpha^{2\gamma}(b/2\alpha) + \alpha^2b^\gamma + \alpha^{2\gamma+2})\]

\[\leq b + c(\alpha^{2\gamma}b + \alpha^2b^\gamma + \alpha^{2\gamma+2})\]

Where the second line holds since \(\alpha(g + 1) \leq b/2\) implies that \(\alpha g \leq b/2\), or \(g \leq (b/2\alpha)\). Note that the above is minimized over the range \(\gamma \geq 1\), when \(\gamma = 1\). In this case, we have:

\[A \leq b + c(ab + \alpha^2b + \alpha^4)\]

\[\leq (c\alpha + \alpha^2 + 1)b + c\alpha^4\]

and we also have:

\[B \geq \frac{b^2}{8\alpha(g + 1)}\]

So, in this case, we have

\[O(\sqrt{T(g + 1)}) = b/\sqrt{8\alpha}\.\]

Thus,

\[A = O(\alpha^{5/2}\sqrt{T(g + 1)} + \alpha^4).\]
Case: $b < 2\alpha(g + 1)$. In this case, when $\gamma = 1$, we have

$$A \leq b + c(\alpha^2 g + \alpha^2 b + \alpha^4)$$

$$\leq 2\alpha(g + 1) + c(\alpha^2 g + \alpha^2 (2\alpha(g + 1) + \alpha^4))$$

$$= O(\alpha^3 g + \alpha^4).$$

which completes the argument.

\[\square\]

5 Lower Bounds

In this section we give lower bounds for our problem. Notably, while L\textsc{Charg}e is deterministic, our lower bounds hold even for randomized algorithms. They show that for $\alpha = \Theta(1)$, L\textsc{Charg}e is asymptotically optimal.

The following theorem gives a lower bound on the expected cost of any randomized algorithm for any positive $\alpha$ and $g$ that is a multiple of $\alpha$, and any input size $n$ that is a multiple of $\alpha g$. The proof of it makes use of Yao’s Minimax principle [57].

\textbf{Theorem 1.2}. Let $\alpha$ be any positive integer. Let $g$ be any multiple of $\alpha$ in the range $[\alpha, n/\alpha]$, where $n$, the number of jobs grows large. Then, any randomized algorithm has expected cost $\Omega(\sqrt{\alpha Tg})$, where $T$ is the expected cost to the adversary.

\textit{Proof.} Fix any positive integers $\alpha$ and $g$, where $g$ is a multiple of $\alpha$. Let $n$ be any multiple of $\alpha g$.

Now consider two distributions of $n$ jobs. In the first distribution, the number of good jobs will be $g_1 = \alpha g$, in the second it will be $g_2 = g/\alpha$. For $i \in \{1, 2\}$, partition from left to right, the sequence of jobs into contiguous subsequences of $n/g_i$ jobs. In each partition, set one job selected uniformly at random to be good, and the remaining jobs to be bad.

\textbf{Estimator}. Define a single estimator, which when given any set $S$ of consecutive jobs, always returns $\hat{g}(S) = (g/n)|S|$. We now show for both distributions that for any set $S$, this estimator guarantees the following,

$$g(S)/\alpha - \alpha \leq \hat{g}(S) \leq \alpha g(S) + \alpha$$

For any $S$, let $g_{\text{max}}(S) = \max\{g_1, g_2\} = \alpha g$ and $g_{\text{min}}(S) = \min\{g_1, g_2\} = g/\alpha$ denote the maximum and minimum number of good jobs in $S$ for the two distributions respectively.

For each distribution we have the following based on the observation that there is one good job in every partition of size $n/g_i$ for $i \in \{1, 2\}$

$$g(S) \leq \lceil |S|/(n/g_i) \rceil \leq \left( \frac{g_{\text{max}}(S)}{n} \right) |S| + 1 \leq \left( \frac{\alpha g}{n} \right) |S| + 1. \quad (1)$$

\[\text{12}\]
Thus:

\[
\left( \frac{1}{\alpha} \right) g(S) - \alpha \leq \left( \frac{1}{\alpha} \right) \left( \frac{\alpha g}{n} |S| + 1 \right) - \alpha \\
\leq \left( \frac{g}{n} \right) |S| + \frac{1}{\alpha} - \alpha \\
\leq \hat{g}(S)
\]

In the above, the first line follows from Equation [1] and the last line follows for \( \alpha \geq 1 \) and by the definition of the estimator.

Next, note that for any \( S \), we also have:

\[
g(S) \geq \left\lfloor \frac{|S|}{(n/g_{\min}(S))} \right\rfloor \geq \left( \frac{g_{\min}(S)}{n} \right) |S| - 1 \geq \left( \frac{g}{\alpha n} \right) |S| - 1.
\]

Thus:

\[
\alpha g(S) + \alpha \geq \alpha \left( \frac{g}{\alpha n} |S| - 1 \right) + \alpha \\
\geq \left( \frac{g}{n} \right) |S| \\
= \hat{g}(S)
\]

In the above, the first line follows from Equation [2] and the last follows by the definition of the estimator.

By the above analysis, our estimator has the \( \alpha \)-approximation property for both distributions.

**Cost Analysis.** Note that, for both distributions, the estimator is the same, so the algorithm sets the same cost \( c_i \) for every job \( i \in [1,n] \). Thus, \( C = \sum_{i=1}^{n} c_i \) is the same for both distributions. Now let \( A_1, T_1 \) \((A_2, T_2)\) be the expected algorithmic cost and expected adversarial cost for distribution 1 (distribution 2). Then, we have:

\[
A_1 = C\alpha g/n + (n - \alpha g) \\
T_1 = \frac{n - \alpha g}{n} C.
\]

Similarly,

\[
A_2 = C\alpha g/(\alpha n) + (n - g/\alpha) \\
T_2 = \frac{n - g/\alpha}{n} C.
\]

Hence, we have:

\[
\frac{A_1}{\sqrt{T_1 g_1}} = \sqrt{\frac{C\alpha g}{n(n - \alpha g)}} + \sqrt{\frac{n(n - \alpha g)}{C\alpha g}} \\
= V_1 C^{1/2} + 1/(C^{1/2} V_1).
\]
Where \( V_1 = \sqrt{\frac{\alpha g}{n(n - \alpha g)}} \) and

\[
A_2 \sqrt{T_2g_2} = \sqrt{\frac{Cg/\alpha}{n(n - g/\alpha)}} + \sqrt{\frac{n(n - g/\alpha)}{Cg/\alpha}} = V_2 C^{1/2} + 1/(V_2 C^{1/2})
\]

Where \( V_2 = \sqrt{\frac{g/\alpha}{n(n - g/\alpha)}} \).

Let the adversary choose each distribution with probability \( 1/2 \). Then let \( f(C) \) be the expected value of the ratio of \( A/\sqrt{Tg} \) as a function of \( C \). Then we have:

\[
f(C) = \frac{1}{2} \left( V_1 C^{1/2} + 1/(C^{1/2} V_1) + V_2 C^{1/2} + 1/(V_2 C^{1/2}) \right)
\]

\[
= \frac{1}{2} \left( (V_1 + V_2)C^{1/2} + (1/V_1 + 1/V_2)C^{-1/2} \right).
\]

Setting to zero the first derivative with respect to \( C \) of \( f(C) \), we find that the minimum of this function occurs when \( C = \sqrt{V_1V_2} \). In which case,

\[
f(C) = \frac{1}{2} \left( \frac{V_1 + V_2}{\sqrt{V_1V_2}} \right)
\]

\[
= \frac{1}{2} \left( \sqrt{V_1/V_2} + \sqrt{V_2/V_1} \right)
\]

\[
\geq \frac{1}{2} \sqrt{V_1/V_2}
\]

\[
\geq \left( \frac{1}{2} \right) \left( \frac{\alpha g}{n(n - \alpha g)} \cdot \frac{n(n - g/\alpha)}{g/\alpha} \right)^{1/4}
\]

\[
\geq \alpha^{1/2}/2.
\]

We have shown that, for a specific adversarial probability distribution over inputs, any deterministic algorithm will have expected cost \( \Omega(\sqrt{\alpha Tg}) \). Thus, by Yao’s minimax principle \([57]\), we know that any randomized algorithm will have expected cost \( \Omega(\sqrt{\alpha Tg}) \) for some worst-case input.

### 6 Preliminary Simulation Results

We present preliminary simulation results to explore the behavior predicted by our upper-bound analysis (Section 4). These results offer promising indications that LCHARGE performs well.

#### 6.1 Empirical Setup

For every sequence of \( n \) jobs, we partition it into contiguous subsequences of \( k \) jobs each. Denote each job by \( J_i \) where \( i \in [1, n] \).

**Distribution.** For each partition, the last \( \alpha \) jobs are good and all others are bad.
Estimator. For each partition, $J_i, \ldots, J_{i+k}$, we have $\hat{g}(J_m) = 1$ for $m \in [i, i + \alpha - 1]$ and $\hat{g}(J_m') = 0$ for all $m' \in [i + \alpha, i + k]$. For any disjoint sets of jobs $A$ and $B$, we set $\hat{g}(A \cup B) = \hat{g}(A) + \hat{g}(B)$. Our construction for the estimator satisfies the approximation and additive guarantees for any value $\alpha$.

With the distribution and estimator established for each solitary job, we can evaluate $L_{\text{Charge}}$. The estimator delineates the iterations over the execution, while the distribution allows us to tally the cost to both the algorithm and the adversary. Our code is available online on GitHub \[11\].

6.2 Specific Experiments

For our first experiment, we fix $\alpha = 1$ and investigate the impact of different values of $\gamma$ (recall Section 3); specifically, we let $\gamma = 0, 1/2, 1$ and 2. The effects of varying $\gamma$ should become apparent when a large attack occurs (i.e., when $B$ is large). To this end, we let $k = 2^m$ where $m \in [1, 18]$, which means that we have many bad jobs in each partition. For each value $m$, the experiment consists of 10 iterations, and the total number of jobs is $10k$. We highlight that by varying $k$ (and, thus, the number of jobs) over this large range, we capture cost values over several orders of magnitude.

For our second experiment, we fix $\gamma = 1$ and investigate the impact of different values of $\alpha$; specifically, we let $\alpha = 1, 2, 4, 8, 16$ and 32. Similarly, we set $k = 2^m$ where $m \in [6, 18]$. Note that this setting still corresponds to a large attack, since $\alpha$ out of each consecutive $k$ jobs are good. We initiate our experiment at a slightly higher value of $m$ (in comparison to our first experiment) for the purposes of placing many bad jobs in each partition, even when $k$ is small.

6.3 Results

The results of our first experiment are plotted in Figure 2 (Left). The results show scaling consistent with our theoretical analysis. Notably, setting $\gamma = 1$ achieves the most advantageous cost for algo-
algorithm versus the adversary. We observe that for $\gamma = 0, 2$ and $1/2$, the algorithm cost increases by up to a factor of approximately $3 \times 10^3$, $12$, and $10$, respectively, relative to $\text{LCharge}$ (i.e., $\gamma = 1$). We include the line $A = B$ as a reference point, and this closely corresponds to the case for $\gamma = 0$, since the provisioning cost incurred by the algorithm is close to the number of bad jobs. We also include the equation $A = 2.75\sqrt{B(g+1)} + g$, which closely fits the line for $\gamma = 1$.

The results of our second experiment are plotted in Figure 2 (Right). As expected, when $\alpha$ increases—and, thus, the accuracy of our estimator worsens—we observe that the cost ratio of the algorithm to the adversary increases. Specifically, for a fixed bad cost value $2 \times 10^7$, we see that $\alpha = 2, 4, 8, 16$ and $32$ yield a ratio that is, respectively, a factor of $2.0, 3.3, 6.7, 11.3$, and $23.3$ larger than than for $\alpha = 1$. We note that, despite this effect, the cost of the algorithm versus the adversary is still far below the $A = B$ lines; thus, $\text{LCharge}$ still achieves a significant advantage.

7 Notes on RB Challenges in Practice

We highlight two practical issues regarding the hardness of RB challenges. First, rather than issuing a challenge of hardness $i$ to the $i^{\text{th}}$ job in the current window, we can instead use a hardness of $2^\left\lceil \log_2 i \right\rceil$. This yields the same asymptotic costs, but now the hardness value changes only $O(\log x)$ times rather than $x$ times, where $x$ is the number of jobs in the current window.

Why is this useful? Imagine a system where the current RB challenge hardness is published to an online bulletin board or otherwise broadcast to participants. Then, a job can solve an RB challenge of the currently required difficulty when they need service. In this way, communication overhead related to RB challenge hardness is significantly reduced.

Second, in practice, both Alice and Bob might start solving an RB challenge of cost, say, $h$. If Bob returns a solution first, then the current RB cost increments to $h + 1$, and if Alice later solves her original challenge, it is insufficient. The above modification of increasing the RB hardness by powers of two would mitigate this issue. However, even without this modification, we could imagine a setup for RB challenges where the resource burned for an $h$-hard challenge could count towards an $h + 1$ challenge. For example, consider an RB challenge where the goal is to find an output to a secure hash function that generates an output with a prefix of $h$ zeros. The resources expended in search of such an input would also be expended in search of an input that generates an output with a prefix of $h + 1$ zeros. We could imagine a similar property for RB challenges based on CAPTCHAS.

8 Conclusion

We have presented an algorithm $\text{LCharge}$ for defending against EDoS, which is a recent variant of DoS attack. $\text{LCharge}$ dynamically adjusts the difficulty of RB challenges based on feedback from an estimator which estimates the number of good jobs seen in any subsequence. Critically, the error of this estimator, $\alpha$, is unknown to $\text{LCharge}$ at any point during the algorithm.

We showed that, with $\text{LCharge}$, the cost to the algorithm grows slowly with the (1) cost payed by an adversary during a significant attack; (2) the error of the estimator; and (3) the number of good jobs. Notably, during a significant attack the cost to the algorithm is asymptotically less than the
cost to the adversary. We believe this is an important property for deterring economically-motivated DoS attacks.

Also, we provided lower bounds showing that these costs of LCharge is asymptotically tight when $\alpha = \Theta(1)$. While LCharge is deterministic, our lower bounds hold even for randomized algorithms.

Finally, we gave preliminary empirical results that indicate that LCharge may perform well in practice.

**Future Work.** We view our result as an encouraging step towards developing DoS defenses for a range of domains. However, many open problems remain including including the following:

First, can we handle heterogeneous jobs? In particular, in this paper, the provisioning cost to service any job is always 1. Our results extend to the case where the effort for any two jobs does not differ by more than a constant factor. This captures settings where jobs are roughly homogeneous, or where the server terminates a job after a threshold of service is provided. However, addressing heterogeneous jobs, where the effort required to service jobs varies by more than a constant factor, is a natural next step. The results of Scully, Grosos and Mitzenmacher [46] may be particularly useful for addressing this problem.

Second, can we handle non-linear provisioning costs? In this paper, we consider a provisioning cost that scales linearly in the number of jobs. While this is a natural model, it may be interesting to consider different functions for the provisioning cost.

Third, can we obtain asymptotically tight results for all values of $\alpha$? In this paper, our results are asymptotically tight when $\alpha = \Theta(1)$. But, it would be nice to tighten the asymptotic results for arbitrary values of $\alpha$.

Finally, while our preliminary simulation results are encouraging, we would like to perform a comprehensive evaluation of LCharge. This would be challenging, requiring significant effort to: (1) implement client and server processes; (2) determine an appropriate estimator, or build and train one for our specific purposes; and (3) generate network traffic that is faithful to real-world DDoS attacks.

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