Two-Step mmWave Positioning Scheme with RIS-Part II: Position Estimation and Error Analysis

Tuo Wu, Cunhua Pan, Yijin Pan, Sheng Hong, Hong Ren, Maged Elkashlan, Feng Shu and Jiangzhou Wang, Fellow, IEEE

Abstract

In this series of work, we propose a comprehensive two-step three-dimensional (3D) positioning scheme in a millimeter wave (mmWave) system, where the reconfigurable intelligent surface (RIS) is leveraged to enhance the positioning performance of mobile users (MUs). Specifically, the first step is the estimation error modeling and analysis, while the second step is the corresponding positioning algorithm design and bias analysis. The first step is introduced in Part I of this series of work, and the second step is investigated in this paper. Our aim in this series of work is to obtain the closed-form solution of the MU’s position through a two-stage weight least square (TSWLS) algorithm. In the first stage, we construct the pseudolinear equations based on the angle of arrival (AOA) and the time difference of arrival (TDOA) estimation at the RISs, then we obtain a preliminary estimation by solving these equations using the weight least square (WLS) method. Based on the preliminary estimation in the first stage, a new set of pseudolinear equations are obtained, and a finer estimation is obtained by solving the equations using the WLS method in the second stage. By combining the estimation of both stages, the final estimation of the MU’s position is obtained. Further, we study the theoretical bias of the proposed algorithm by considering the estimation error in both stages. Simulation results demonstrate the superiority of the proposed positioning scheme.

(Corresponding author: Cunhua Pan).
T. Wu and M. Elkashlan are with the School of Electronic Engineering and Computer Science at Queen Mary University of London, London E1 4NS, U.K. (Email:{tuo.wu, maged.elkashlan}@qmul.ac.uk). C. Pan, Y. Pan and H. Ren are with the National Mobile Communications Research Laboratory, Southeast University, Nanjing 210096, China. (Email: {cpan, panyj, hren}@seu.edu.cn). S. Hong is with Information Engineering School of Nanchang University, Nanchang 330031, China. (Email: shenghong@ncu.edu.cn). F. Shu is with the School of Electronic and Optical Engineering, Nanjing University of Science and Technology, Nanjing 210094, China, and also with the School of Information and Communication Engineering, Hainan University, Haikou 570228, China. (E-mail: shufeng0101@163.com). J. Wang is with the School of Engineering, University of Kent, UK. (e-mail: J.Z.Wang@kent.ac.uk).
Index Terms

Reconfigurable intelligent surface (RIS), intelligent reflecting surface (IRS), positioning, radio localization.

I. INTRODUCTION

With the upcoming of the sixth generation (6G) wireless network, a high positioning accuracy is urgently required by many position-related services [1], such as smart factories [2], autonomous vessels [3], automated vehicles [4] and mobile user (MU) sensing [5]. Hence, a wealth of wireless positioning algorithms that are widely deployed in millimeter wave (mmWave) of the fifth generation (5G) network, are now being re-imagined in a 6G smart environment setting to achieve a higher positioning accuracy [5].

By deploying positioning algorithms in mmWave [6]–[9], the positioning accuracy can be higher as the positioning parameters (e.g., time of arrival (TOA), time difference of arrival (TDOA) and angle of arrival (AOA)) can be estimated more accurately with narrow beams. However, since wireless signals in the mmWave band are more vulnerable to blockages [7], the reliability of high-precision positioning capability faces great challenges [10]. Reconfigurable intelligent surface (RIS) is a promising technique that is capable of constructing new line-of-sight (LoS) communication links [11]–[15], which can be deployed in existing radio positioning systems with various benefits. First, when deploying a wireless positioning reference, RIS is more preferred than the base station (BS) in terms of hardware cost [16]. Second, as a passive panel, RIS provides reliable and high-precision estimation with low energy consumption [17]. Finally, the large size of the RIS panel enables high accuracy radio positioning parameter estimation [18]. Therefore, it is meaningful to investigate RIS-aided positioning.

In general, two-step positioning operates in the following steps. First, the radio positioning parameters are estimated through some estimation methods [19]–[21]. Second, based on these estimated radio positioning parameters, the positioning algorithms are designed to estimate the position of MU. Specifically, the expression for the complex non-linear geometric relationship between the radio positioning parameters and the position coordinates should first be derived. The non-linear equations are then formulated by a combination of the parameter estimation error and the non-linear geometric relationship. By solving these equations using iterative or non-iterative algorithms, the location of the mobile user (MU) can be determined. Therefore,
the estimation error of the radio positioning parameters should be carefully considered in the RIS-aided positioning algorithm design [22]–[25].

Currently, RIS-aided positioning is extensively studied, with special attention to performance analysis [26]–[30] and algorithm design [31]–[33]. For performance analysis, the authors in [26] derived the theoretical positioning performance bounds, such as Cramér-Rao lower bound (CRLB), to provide analytical performance validation. In [27], Liu et al. derived the CRLB of a multiple-RIS-aided mmWave positioning system. In [28], the authors provided CRLB expressions and the effective achievable data rate (EADR), in which the phase shifts of the RIS were optimized to maximize the EADR. The authors in [29] derived the CRLB in terms of the position parameters of the target node, and provided the theoretical relationship between the optimal power and the phase shifts of the RIS. To assess the performance of synchronous and asynchronous signaling schemes, Alouini et al. derived the CRLB in [30]. For algorithm design, [31] proposed a received signal strength (RSS) based positioning scheme enabled by an RIS and designed an iterative algorithm to obtain the optimal phase shifts. An optimization method was proposed in [32] to design the phase shift for RIS-aided positioning and communication systems. In [33], a reduced-complexity maximum likelihood based estimation procedure was designed to jointly recover the user position and the synchronization offset.

However, the above-mentioned contributions mainly focused on performance analysis, such as CRLB, and did not provide positioning algorithms. For contributions with algorithm design, the focus was on deriving the optimal phase shifts rather than the position estimation. Moreover, the positioning parameters were assumed to be known and their estimation error followed the Gaussian distribution. However, the Gaussian distribution may not hold if the positioning parameters are estimated using the practical methods as detailed in Part I of this series work [34]. Therefore, practical positioning algorithms based on radio positioning parameters at the RISs and actual estimation error analysis are needed. In addition, to evaluate the positioning performance, existing work commonly utilized the CRLB and the root mean square error (RMSE). However, since the estimation error in practice does not follow the Gaussian distribution, the estimated position parameters generally are not biased. As shown in [35], the bias is considerably high when the signal-noise ratio (SNR) is low. Thus, it is necessary to conduct a bias analysis for the non-Gaussian estimation error.

In this paper, we propose a comprehensive two-step positioning scheme in RIS-aided mmWave systems. In the first step, the angle estimation error is modeled and analyzed in terms of the
probability density functions (PDF), the details of which can be found in Part I of this series work [34]. In the second step, pseudolinear equations are first derived based on the AOA and TDOA parameters at multiple RISs. Then, by applying the two-stage weight least square (TSWLS) algorithm, we derive the closed-form expression of the position of the MU. Finally, bias analysis is provided for the proposed positioning algorithm.

In this paper, we consider the second step. Our contributions are summarized as follows.

1) A two-step three-dimensional (3D) positioning system aided by multiple RISs is proposed. As multiple RISs are deployed in the positioning system, we utilize the AOAs and TDOAs associated with all the RISs for positioning design. With this in mind, pseudolinear equations involving the AOAs and TDOAs are formulated based on the geometry relationship and the estimation error.

2) The TSWLS algorithm is derived to obtain the closed-form expression of the MU’s 3D position. To be specific, in the first stage, we construct the pseudolinear equations using the AOAs and TDOAs parameters at the RISs. Then, preliminary estimation is derived by solving these pseudolinear equations using the WLS method. In the second stage, based on the preliminary estimation in the first stage, a new set of pseudolinear equations are formulated and the finer estimation is obtained by solving the equations. Finally, using the estimation of both stages, the final estimation of the MU’s position is obtained.

3) To evaluate the performance of the proposed positioning scheme, we investigate the theoretical estimation bias. More specifically, by decomposing the matrices containing the estimation information into true values and estimation error, the estimation error of both stages can be further derived, and the theoretical bias of both stages is derived accordingly. Finally, the bias of the final solution is derived using the bias of both stages.

4) Simulation results are provided to evaluate the performance of the proposed positioning scheme. The derived results are in good agreement with the simulation results, which confirms the accuracy of the derived results. With the practical non-Gaussian estimation error, the proposed positioning algorithm demonstrates promising potential.

The remainder of the paper is organized as follows. The system model for the two-step positioning aided by multiple RISs is described in Section II. The closed-form solution of the MU’s position is given in Section III. Section IV derives the bias analysis of the proposed positioning scheme. Simulation results is given in Section V. Section VI concludes this work.
Consider a RIS-aided positioning system, where a MU sends pilot signals to the BS to locate the MU with the assistance of the multiple RISs. The BS is equipped with a uniform linear array (ULA) of $N_b$ antennas, and the MU is equipped with a single antenna. Moreover, there are $M$ RISs, and each RIS is a uniform planar array (UPA).

The BS is placed parallel to the x-axis with the center located at $p = [x_p, y_p, z_p]^T$. The $i$th RIS is placed parallel to the y-o-z plane with its center located at $s_i = [x_i, y_i, z_i]^T$, $i = 1, 2, \cdots, M$. The UPA-based RIS has $N_{y,z} = N_y \times N_z$ reflecting elements, where $N_y$ and $N_z$ denote the numbers of reflecting elements along the y-axis and z-axis, respectively. The true position of the MU is $q = [x_q, y_q, z_q]^T$ and is assumed to be placed parallel to the x-o-y plane. The estimated location of the MU is $\hat{q} = [\hat{x}_q, \hat{y}_q, \hat{z}_q]^T$. Generally, once the RISs and BS have been deployed, the coordinates $s_i$ and $p$ are known and invariant. In order to locate the MU, we need to obtain the estimated $\hat{q}$.

In this paper, multiple estimated radio positioning parameters (AOAs and TDOAs) are utilized for the wireless positioning. As shown in Fig. 1, there are two kinds of line-of-sight (LoS) communication links containing the radio positioning parameters$^1$, which are the direct links

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**Fig. 1: RIS-aided positioning system model.**

II. SYSTEM MODEL

1. For mmWave signals, channel measurement campaigns$^{[36],[37]}$ reveal that the signal power of the LoS component is about 13 dB higher than the sum of power of non-line-of-sight (NLoS) components, so the NLoS is ignored here.
from the MU to the BS, and the reflecting links through the RISs.

A. AOA information

The AOA information in the LoS links can be estimated using the angle estimation algorithm mentioned in our first paper [34]. For the MU-RIS-BS link, the available angle information includes the AOAs (azimuth and elevation angles) at the RISs, the AODs (azimuth and elevation angles) at the RISs, and the AOAs at the BS. For the MU-BS link, the angle information for positioning is the AOA at the BS. However, since we can only estimate the azimuths of the AOAs at the BS due to the assumption of ULAs, the 3D coordinates of the MU cannot be estimated without the elevation of the AOAs. Therefore, we use the AOAs at the RISs for the positioning estimation in this paper.

In Fig. 1, the azimuth AOA $\theta_{MR,i}$ at the RIS is the angle between the projection of the wave vector on the x-o-y plane and the y-axis as

$$\theta_{MR,i} = \arctan \frac{x_q - x_i}{y_q - y_i}, \quad (1)$$

where $\theta_{MR,i} \in (0, \pi)$.

The elevation AoA $\phi_{MR,i}$ at the RIS of the MU-RIS link is

$$\phi_{MR,i} = \arctan \frac{z_q - z_i}{\sin \theta_{MR,i} (x_q - x_i) + \cos \theta_{MR,i} (y_q - y_i)}, \quad (2)$$

where $\phi_{MR,i} \in (-\pi/2, \pi/2)$.

B. TDOA information

The TDOA information (time delays) can be extracted from the received pilot signals. Then, the difference of link distances can be readily obtained by multiplying the light speed with TDOAs. Thus, in the following, we focus on the difference of distances, as it is equivalent to TDOAs.

Let $R_{BU}$ denote the true distance of the MU-BS link, which can be calculated as

$$R_{BU} = \sqrt{(x_q - x_p)^2 + (y_q - y_p)^2 + (z_q - z_p)^2}. \quad (3)$$

Additionally, the true distance from the MU to the $i$th RIS can be expressed as

$$R_{RU,i} = \sqrt{(x_q - x_i)^2 + (y_q - y_i)^2 + (z_q - z_i)^2}. \quad (4)$$

Using $R_{BU}$ as the reference distance, the true distance difference between $R_{RU,i}$ and $R_{BU}$ can be written as

$$R_{B,i} = R_{RU,i} - R_{BU}. \quad (5)$$

The true distance difference $R_{B,i}$ will be used in the following positioning algorithm design.
C. Radio positioning parameters estimation

The estimated radio positioning parameters can be modeled as

\[ \hat{\theta}_{MR,i} = \theta_{MR,i} + n_i, \quad \hat{\phi}_{MR,i} = \phi_{MR,i} + \omega_i, \quad \hat{R}_{B,i} = R_{B,i} + \nu_i, \]

where \( \{\hat{\theta}_{MR,i}, \hat{\phi}_{MR,i}, \hat{R}_{B,i}\} \) denote the estimation of \( \{\theta_{MR,i}, \phi_{MR,i}, R_{B,i}\} \) and \( \{n_i, \omega_i, \nu_i\} \) denote the estimation error. For the sake of illustration, we collect all the estimation positioning parameters in the following vectors

\[ \hat{\theta}_r = \theta_r + n, \quad \hat{\phi}_r = \phi_r + \omega, \quad \hat{R}_B = R_B + \nu, \]

where

\[ \hat{\theta}_r = [\hat{\theta}_{MR,1}, \hat{\theta}_{MR,2}, ..., \hat{\theta}_{MR,M}]^T, \quad \theta_r = [\theta_{MR,1}, \theta_{MR,2}, ..., \theta_{MR,M}]^T, \quad \hat{\phi}_r = [\hat{\phi}_{MR,1}, \hat{\phi}_{MR,2}, ..., \hat{\phi}_{MR,M}]^T, \]
\[ \phi_r = [\phi_{MR,1}, \phi_{MR,2}, ..., \phi_{MR,M}]^T, \quad \hat{R}_B = [\hat{R}_{B,1}, \hat{R}_{B,2}, ..., \hat{R}_{B,M}]^T, \quad R_B = [R_{B,1}, R_{B,2}, ..., R_{B,M}]^T, \]
\[ n = [n_1, n_2, ..., n_M]^T, \quad \omega = [\omega_1, \omega_2, ..., \omega_M]^T, \quad \nu = [\nu_1, \nu_2, ..., \nu_M]^T. \]

(8)

In the existing works, for tractability, \( n, \omega \) and \( \nu \) are assumed to be the additive zero-mean complex Gaussian noise. However, according to the error analysis of our previous work Part I \[34\], the PDF of elevation estimation error should be modeled as (29), while the PDF of azimuth estimation error should be modeled as (57) or (58) in \[34\]. Besides, as the TDOA information can be extracted from the received pilot signals \[20\], we assume that \( \nu_i \) follows the Gaussian distribution. Furthermore, the covariance matrices can be written as

\[ Q_n = \begin{bmatrix} \sigma_{n_1}^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{n_M}^2 \end{bmatrix}, \quad Q_\omega = \begin{bmatrix} \sigma_{\omega_1}^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{\omega_M}^2 \end{bmatrix}, \quad Q_\nu = \begin{bmatrix} \sigma_{\nu_1}^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{\nu_M}^2 \end{bmatrix}, \]

(9)

where \( \sigma_{n_i}^2, \sigma_{\omega_i}^2 \) and \( \sigma_{\nu_i}^2 \) denote the variance of \( n_i, \omega_i \) and \( \nu_i \), respectively. \( \sigma_{\nu_i}^2 \) and \( \sigma_{\omega_i}^2 \) can be calculated according to the algorithms provided in our previous work \[34\], and \( \sigma_{\nu_i}^2 \) can be derived according to the Gaussian distribution.

This paper focuses on the position estimation algorithm design, and the phase shift matrices of the RISs are set to a unit matrix for the sake of analysis.

III. CLOSED-FORM SOLUTION

In this section, we propose an algorithm to derive a closed-form solution to the MU’s position. First, the pseudolinear equations based on the AOA and TDOA information of the RISs are derived. Then, an algorithm is proposed to estimate the position of the MU based the pseudolinear equations.
Table 1 Symbols description of TSWLS algorithm

| Symbol | Description |
|--------|-------------|
| q      | true position of the MU |
| s_i    | position of the i\(th\) RIS |
| S      | matrix consisting of s_i |
| \(\theta_{MR,i}\), \(\phi_{MR,i}\) | true AOAs at the RIS |
| \(R_{BU}\) | true distance of the MU-BS link |
| \(R_{BU,i}\) | true distance from the MU to the i\(th\) RIS |
| \(R_{B,i}\) | true distance difference between \(R_{BU}\) and \(R_{BU,i}\) |
| \(Q_n\), \(Q_\omega\), \(Q_\nu\) | covariance matrices |
| \(\hat{\theta}_{MR,i}\), \(\hat{\phi}_{MR,i}\) | vector consisting the trigonometric functions of \(\hat{\theta}_{MR,i}\), \(\hat{\phi}_{MR,i}\) |
| \(\hat{\theta}_i\), \(\hat{\phi}_i\) | matrix consisting of \(\hat{\theta}_{MR,i}\), \(\hat{\phi}_{MR,i}\) |
| u      | true vector including \(x_q\), \(y_q\), \(z_q\), \(R_{BU}\) |
| \(\hat{h}_i\) | vector consisting of \(\hat{h}_i\) |
| \(\hat{G}_{\phi_i}, \hat{G}_{\theta_i}, \hat{G}_i\) | matrix consisting of \(\hat{g}_{\phi_{MR,i}}, \hat{g}_{\theta_{MR,i}}, \hat{g}_{t,i}\) |
| \(z_r\) | error vector at Stage 1 |
| \(B_r\) | coefficient matrix at Stage 1 |
| \(\hat{G}_r\) | matrix collecting the vectors of estimated parameters |
| \(h_r\) | vector collecting the scalars of estimated parameters |
| \(W_r\) | weight matrix of the WLS expression at Stage 1 |
| \(\Omega_r\) | covariance matrix at Stage 1 |
| \(h_1\) | vector of the relationship between MU’s estimated position and BS’s postion |
| \(G_1\) | coefficient matrix relating true MU’s position to BS’s position |
| \(\xi\) | square of position difference between the MU and the BS |
| \(B_r\) | coefficient matrix at Stage 2 |
| \(z_1\) | error vector at Stage 2 |
| \(W_1\) | weight matrix of the WLS expression at Stage 2 |
| \(\Omega_1\) | covariance matrix at Stage 2 |

A. Pseudolinear equations based on the AOA estimation

In this subsection, we derive the pseudolinear equations based on the geometry relationship of AOAs at the i\(th\) RIS of the MU-RIS-BS link.

First, from (1), we have

$$\cos \theta_{MR,i}(x_q - x_i) - \sin \theta_{MR,i}(y_q - y_i) = 0.$$  \hspace{1cm} (10)

By replacing the true value \(\theta_{MR,i}\) with the estimated \(\hat{\theta}_{MR,i}\), we have

$$\eta_{\theta_{MR,i}} = \cos \hat{\theta}_{MR,i}(x_q - x_i) - \sin \hat{\theta}_{MR,i}(y_q - y_i),$$  \hspace{1cm} (11)

where \(\eta_{\theta_{MR,i}}\) denotes the residual error of \(\theta_{MR,i}\) due to the estimation error. For the sake of
analysis, let \( \hat{g}_{MR,i} = [-\cos \hat{\theta}_{MR,i}, \sin \hat{\theta}_{MR,i}, 0]^T \). Using the definitions of \( q \) and \( s_i \), (11) can be rewritten as

\[
\eta_{\theta_{MR,i}} = \hat{g}_{\theta_{MR,i}}^T s_i - \hat{g}_{\theta_{MR,i}}^T q. \quad (12)
\]

Assuming that the angle estimation algorithms have good performance \cite{19}, thus it is reasonable to assume that the estimation error is very small. Hence, we have

\[
\sin(n_i) \approx n_i \quad \text{and} \quad \cos(n_i) \approx 1,
\]

where \( n_i \) is the estimation error of \( \theta_{MR,i} \). Consequently, we have the following approximations

\[
\begin{align*}
\sin \hat{\theta}_{MR,i} &= \sin(\theta_{MR,i} + n_i) \approx \sin \theta_{MR,i} + n_i \cos \theta_{MR,i}, \\
\cos \hat{\theta}_{MR,i} &= \cos(\theta_{MR,i} + n_i) \approx \cos \theta_{MR,i} - n_i \sin \theta_{MR,i}.
\end{align*}
\quad (13)
\]

By substituting (13) into (11), we have

\[
\eta_{\theta_{MR,i}} \approx (\cos \theta_{MR,i} - n_i \sin \theta_{MR,i})(x_q - x_i) - (\sin \theta_{MR,i} + n_i \cos \theta_{MR,i})(y_q - y_i)
\]

\[
= \cos \theta_{MR,i}(x_q - x_i) - \sin \theta_{MR,i}(y_q - y_i) - n_i[(x_q - x_i) \sin \theta_{MR,i} + (y_q - y_i) \cos \theta_{MR,i}]
\]

\[
= -n_i R_{RU,i} \cos \phi_{MR,i}, \quad (14)
\]

In (14), (10) and \((x_q - x_i) \sin \theta_{MR,i} + (y_q - y_i) \cos \theta_{MR,i} = l_{i1} + l_{i2} = R_{RU,i} \cos \phi_{MR,i} \) shown in Fig. 2-(a) are used.

![Diagram](a) The azimuth AOA at \( i \)th RIS of the MU-RIS-BS link.

![Diagram](b) The elevation AOA at the \( i \)th RIS of the MU-RIS-BS link.

Fig. 2: The AOAs at the \( i \)th RIS of the MU-RIS-BS link.

Next, using (2), we have

\[
0 = \cos \phi_{MR,i}(z_q - z_i) - \sin \phi_{MR,i} (\sin \theta_{MR,i}(x_q - x_i) + \cos \theta_{MR,i}(y_q - y_i)). \quad (15)
\]
By replacing \( \{\phi_{MR,i}, \theta_{MR,i}\} \) with \( \{\hat{\phi}_{MR,i}, \hat{\theta}_{MR,i}\} \), we have
\[
\eta_{\phi_{MR,i}} = \cos \hat{\phi}_{MR,i}(z_q - z_i) - \sin \hat{\phi}_{MR,i} \sin \hat{\theta}_{MR,i}(x_q - x_i) - \sin \hat{\phi}_{MR,i} \cos \hat{\theta}_{MR,i}(y_q - y_i),
\] (16)
where \( \eta_{\phi_{MR,i}} \) is the residual error of \( \phi_{MR,i} \). For simplicity, we define \( \tilde{\phi}_{\phi_{MR,i}} = [\sin \hat{\phi}_{MR,i} \sin \hat{\theta}_{MR,i}, \sin \hat{\phi}_{MR,i} \cos \hat{\theta}_{MR,i} - \cos \hat{\phi}_{MR,i}]^T \). Then, by utilizing the definitions of \( q \) and \( s_i \), (16) can be expressed as
\[
\eta_{\phi_{MR,i}} = \tilde{g}_{\phi_{MR,i}}^T s_i - \tilde{g}_{\phi_{MR,i}}^T q.
\] (17)

It is assumed that the estimation error is very small \[19\], we have \( \sin(\omega_i) \approx \omega_i \) and \( \cos(\omega_i) \approx 1 \), where \( \omega_i \) is the estimation error of \( \phi_{MR,i} \). As a result, we have the following approximations as
\[
\cos \hat{\phi}_{MR,i} = \cos(\phi_{MR,i} + \omega_i) \approx \sin \phi_{MR,i},
\]
\[
\sin \hat{\theta}_{MR,i} = \sin(\theta_{MR,i} + \omega_i) \approx \cos \phi_{MR,i} - \omega_i \sin \phi_{MR,i}.
\] (18)

Then, by substituting (18) into (16), and performing some mathematical manipulations, we have
\[
\eta_{\phi_{MR,i}} \approx (\cos \phi_{MR,i} - \omega_i \sin \phi_{MR,i})(z_q - z_i)
- (\sin \phi_{MR,i} + \omega_i \cos \phi_{MR,i})(x_q - x_i) \sin \theta_{MR,i} + (y_q - y_i) \cos \theta_{MR,i}
= -\omega_i R_{RU,i}.
\] (19)

To derive (19), we have used (15) and \( ((z_q - z_i) \sin \phi_{MR,i} + ((x_q - x_i) \sin \theta_{MR,i} + (y_q - y_i) \cos \theta_{MR,i}) \cos \phi_{MR,i}) = d_1 + d_2 = R_{RU,i} \), which can be verified using Fig. 2-(b).

Finally, using (12), (14), (17) and (19), we have
\[
\tilde{g}_{\phi_{MR,i}}^T s_i - \tilde{g}_{\phi_{MR,i}}^T q \approx -n_i R_{RU,i} \cos \phi_{MR,i}, \quad \tilde{g}_{\phi_{MR,i}}^T s_i - \tilde{g}_{\phi_{MR,i}}^T q \approx -\omega_i R_{RU,i}.
\] (20)

B. Pseudolinear equations based on the TDOA estimation

TDOA estimation is used in this subsection to derive the pseudolinear equations. First, by taking the square of both sides of (3), we have
\[
R_{BU}^2 = K_q^2 + K_p^2 - 2x_qx_p - 2y_qy_p - 2z_qz_p,
\] (21)
where \( K_q^2 = x_q^2 + y_q^2 + z_q^2 \) and \( K_p^2 = x_p^2 + y_p^2 + z_p^2 \). Similarly, by taking the square of both sides of (4), we have
\[
R_{RU,i}^2 = K_q^2 + K_i^2 - 2x_qx_i - 2y_qy_i - 2z_qz_i,
\] (22)
where \( K_i^2 = x_i^2 + y_i^2 + z_i^2 \). Then, using (5), we can obtain \( R_{RU,i} = R_{B,i} + R_{BU} \), thus we have
\[
R_{RU,i}^2 = (R_{B,i} + R_{BU})^2.
\] (23)
By substituting (22) into (23) and expanding the right hand side of (23), (23) is rewritten as
\[ R_{B,i}^2 + 2R_{B,i}R_{BU} + R_{BU}^2 = K_q^2 + K_i^2 - 2x_q x_i - 2y_q y_i - 2z_q z_i. \]  
(24)

Moreover, using (21) and (24), we have the following expression
\[ R_{B,i}^2 + 2R_{B,i}R_{BU} = K_i^2 - K_p^2 - 2x_i p x_q - 2y_i p y_q - 2z_i p z_q, \]  
(25)

where \( x_{i,p} = (x_i - x_p) \), \( y_{i,p} = (y_i - y_p) \) and \( z_{i,p} = (z_i - z_p) \). In order to derive the pseudolinear equations related to TDOA, (25) can be transformed into
\[ -x_{i,p} x_q - y_{i,p} y_q - z_{i,p} z_q - R_{B,i} R_{BU} = -\frac{1}{2} K_i^2 + \frac{1}{2} K_p^2 + \frac{1}{2} R_{B,i}^2. \]  
(26)

Additionally, for the sake of analysis, we define
\[ \mathbf{g}_{t,i} = -[x_{i,p}, y_{i,p}, z_{i,p}, R_{B,i}]^T, \quad \mathbf{u} = [x_q, y_q, z_q, R_{BU}]^T, \quad h_i = -\frac{1}{2} K_i^2 + \frac{1}{2} K_p^2 + \frac{1}{2} R_{B,i}^2. \]  
(27)

Accordingly, we can derive the pseudolinear equation with respect to the TDOA as
\[ h_i - \mathbf{g}_{t,i}^T \mathbf{u} = 0. \]  
(28)

As \( R_{B,i} \) is estimated as \( \hat{R}_{B,i} \) by applying the TDOA estimation, by replacing \( R_{B,i} \) with \( \hat{R}_{B,i} \), \( \mathbf{g}_{t,i} \) and \( h_i \) in (28) are re-interpreted as \( \hat{\mathbf{g}}_{t,i} \) and \( \hat{h}_i \), given by
\[ \hat{\mathbf{g}}_{t,i} = -[x_{i,p}, y_{i,p}, z_{i,p}, \hat{R}_{B,i}]^T, \quad \hat{h}_i = -\frac{1}{2} K_i^2 + \frac{1}{2} K_p^2 + \frac{1}{2} \hat{R}_{B,i}^2. \]  
(29)

Thus, according to (24), (28) can be derived as
\[ \eta_t = \hat{h}_i - \hat{\mathbf{g}}_{t,i}^T \mathbf{u} = R_{RU,i} \nu_i + \frac{1}{2} \nu_i^2, \]  
(30)

where \( \eta_t \) and \( \nu_i \) denote the residual error and the TDOA estimation error of \( R_{B,i} \) in (6), respectively. As \( \nu_i \ll R_{RU,i} \) is always satisfied in practice, the second order term on the right hand side of (30) can be neglected, and (30) can be approximated as
\[ \hat{h}_i - \hat{\mathbf{g}}_{t,i}^T \mathbf{u} \approx R_{RU,i} \nu_i. \]  
(31)

C. Closed-form solution of the position of MU

In this subsection, we derive the compact form of the pseudolinear equations with respect to the AOA and TDOA estimation. We also derive the closed-form solution of the MU’s position using the TSWLS algorithm [35], the details are summarized in Algorithm [1] and the flowchart of proposed algorithm is shown in Fig[3] First, we define
\[ \hat{\mathbf{g}}_{\theta_{MR,i}} = [\mathbf{g}_{\theta_{MR,i}}^T, 0]^T, \quad \hat{\mathbf{g}}_{\phi_{MR,i}} = [\mathbf{g}_{\phi_{MR,i}}^T, 0]^T. \]  
(32)
Algorithm 1 Two-Stage Weighted Least Squares (TSWLS) Algorithm

1: Construct the pseudolinear equations based on the AOA and TDOA estimation;
2: Calculate the preliminary estimation $\hat{u}$ at Stage 1 using Algorithm 2;
3: Construct the pseudolinear equations based on the preliminary estimation at Stage 1;
4: Calculate the estimation of $\xi = (q - p) \odot (q - p)$, denoted by $\hat{\xi}$;
5: Derive the final estimation of $q$ using $\hat{\xi}$ and $\hat{u}$.

A better estimation performance can be obtained using (20) together with (31), both of which contain the unknown location $q$. Thus, we can derive the following compact form of equations as

$$
\hat{h}_r - \hat{G}_r u = B_r z_r,
$$

where $z_r = [n^T, \omega^T, \nu^T]^T$ and its covariance matrix is written as

$$
Q_r = \begin{bmatrix}
Q_n & O & O \\
O & Q_\omega & O \\
O & O & Q_\nu
\end{bmatrix}.
$$

On the left hand side of (33), we have

$$
\hat{h}_r = [1^T (\hat{G}_{\theta_1} \odot S)^T, 1^T (\hat{G}_{\phi_1} \odot S)^T, \hat{h}_t^T]^T,
$$

$$
\hat{G}_r = [\hat{G}_{\theta_1}^T, \hat{G}_{\phi_1}^T, \hat{G}_t^T]^T,
$$

where

$$
\hat{G}_{\theta_1} = [\hat{g}_{\theta_{MR,1}}, \hat{g}_{\theta_{MR,2}}, \ldots, \hat{g}_{\theta_{MR,M}}]^T, \quad \hat{G}_{\phi_1} = [\hat{g}_{\phi_{MR,1}}, \hat{g}_{\phi_{MR,2}}, \ldots, \hat{g}_{\phi_{MR,M}}]^T;
$$

$$
\hat{G}_t = [\hat{G}_{t_1}, \hat{G}_{t_2}, \ldots, \hat{G}_{t_M}]^T, \quad S = [s_1, s_2, \ldots, s_M]^T, \quad \hat{G}_t = [\hat{g}_{t_1}, \hat{g}_{t_2}, \ldots, \hat{g}_{t_M}]^T.
$$
On the right hand side of (33), we have

\[ B_t = [B_{t,n}, B_{t,\omega}, B_{t,\nu}]^T, \quad (37) \]

where

\[ B_{t,n} = [B_{\theta MR,n}, O, O]^T, \quad B_{t,\omega} = [O, B_{\phi MR,\omega}, O]^T, \quad B_{t,\nu} = [O, O, B_t]^T, \]

\[ B_{\theta MR,n} = -\text{diag}[R_{RU,1}\cos\phi_{MR,1}, \ldots, R_{RU,1}\cos\phi_{MR,M}], \]

\[ B_{\phi MR,\omega} = -\text{diag}[R_{RU,1}, \ldots, R_{RU,M}], \quad B_t = \text{diag}[R_{RU,1}, \ldots, R_{RU,M}]. \quad (38) \]

Using the compact pseudolinear equations (33), we propose the TSWLS algorithm to estimate the position of the MU as follows.

1) Stage 1: In the first stage, we aim to derive the estimation of \( u \) using the WLS method [30]. Based on (33), the WLS cost function can be formulated as

\[ f(u) = (\hat{h}_r - \hat{\mathbf{G}}_r u)^T W_r (\hat{h}_r - \hat{\mathbf{G}}_r u) = \hat{h}_r^T W_r \hat{h}_r - 2\hat{h}_r^T W_r \hat{\mathbf{G}}_r u + u^T \hat{\mathbf{G}}_r^T W_r \hat{\mathbf{G}}_r u, \quad (39) \]

where \( W_r \) denotes the weight matrix of Stage 1.

To derive the estimation of \( u \), the cost function \( f(u) \) in (39) should be minimized. By setting the first-order derivative of \( f(u) \) equal to zero, we have

\[ \frac{\partial f(u)}{\partial u} = -2(\hat{\mathbf{G}}_r^T W_r \hat{h}_r) + 2(\hat{\mathbf{G}}_r^T W_r \hat{\mathbf{G}}_r u) = 0. \]

Hence, \( u \) is estimated as

\[ \hat{u} = [\hat{x}_q, \hat{y}_q, \hat{z}_q, \hat{R}_{BU}]^T = (\hat{\mathbf{G}}_r^T W_r \hat{\mathbf{G}}_r)^{-1} \hat{\mathbf{G}}_r^T W_r \hat{h}_r. \quad (40) \]

As the estimation error is correlated, the weight matrix \( W_r \) should be equal to the inverse of the covariance matrix of the estimation error as

\[ W_r = \Omega_r^{-1}, \quad \Omega_r = \mathbb{E}[B_r z_t z_t^T B_r^T] = B_r Q_r B_r^T, \quad (41) \]

where \( \Omega_r \) denotes the covariance matrix of the estimation error. However, as shown in (37) and (38), matrix \( B_r \) contains the true distances \( \{R_{RU,1}, R_{RU,2}, \ldots, R_{RU,M}\} \), which remain unknown. Therefore, the weight matrix is initialized by taking the estimated distances as the approximation of true values. Then, the weight matrix can be iteratively updated by the new estimations. The detailed initialization algorithm is provided in Algorithm 2.

2) Stage 2: As we can see from (3), \( R_{BU} \) and \( q = [x_q, y_q, z_q]^T \) are related, which is ignored in the first stage. Therefore, we aim to utilize this relationship to construct a new set of pseudolinear equations in this stage.

Considering the estimation error, the relationship between the MU’s preliminary estimated position and the BS’s position is

\[ \left[ (\hat{x}_q - x_p)^2, (\hat{y}_q - y_p)^2, (\hat{z}_q - z_p)^2, \hat{R}_{BU}^2 \right]^T \overset{\Delta}{=} \hat{h}_1. \quad (42) \]
Algorithm 2 Initial Location Estimation

1: Set $\hat{W}_r = I$;
2: Derive the initial estimation $\hat{u}$ in (40) by replacing $W_r$ with $\hat{W}_r$;
3: Calculate $\{\hat{R}_{RU,1}, \hat{R}_{RU,2}, \ldots, \hat{R}_{RU,M}\}$ to replace $\{R_{RU,1}, R_{RU,2}, \ldots, R_{RU,M}\}$ using the initial estimation $\hat{u}$;
4: Derive $\hat{B}_r$ to replace $B_r$ in (41) using the definition of $B_r$ in (37) and $\{\hat{R}_{RU,1}, \hat{R}_{RU,2}, \ldots, \hat{R}_{RU,M}\}$;
5: Derive $\hat{W}_r$ to replace $W_r$ in (40) with $\hat{B}_r$ based on (41);
6: Repeat.

Without the estimation error, the relationship between the MU’s position and the BS’s position is represented by

$$\begin{bmatrix} (x_q - x_p)^2, (y_q - y_p)^2, (z_q - z_p)^2, R_{BU}^2 \end{bmatrix}^T \triangleq G_1 \xi,$$

where

$$G_1 = \begin{bmatrix} I_{3 \times 3} \\ 1_{1 \times 3} \end{bmatrix}, \quad \xi = \begin{bmatrix} (x_q - x_p)^2, (y_q - y_p)^2, (z_q - z_p)^2 \end{bmatrix}^T = (q - p) \odot (q - p).$$

Using (42) and (43), we can derive the compact form of the pseudolinear equations in Stage 2 as

$$\hat{h}_1 - G_1 \xi = z_1,$$

where $z_1 = [z_11, z_{12}, z_{13}, z_{14}]^T = [(\hat{x}_q - x_p)^2, (\hat{y}_q - y_p)^2, (\hat{z}_q - z_p)^2, \hat{R}_{BU}^2]^T - [(x_q - x_p)^2, (y_q - y_p)^2, (z_q - z_p)^2, R_{BU}^2]^T$ denotes the residual vector due to the estimation error. Denote the positioning estimation error of the first stage as $\hat{u} = [e_1, e_2, e_3, e_4]^T$, i.e., $\hat{u} = u + \hat{u}$. Then, the elements of $z_1$ can be represented as the functions of the elements of error $\hat{u}$, which can be expressed as

$$z_{11} = (\hat{x}_q - x_p)^2 - (x_q - x_p)^2 = 2(x_q - x_p)e_1 + e_1^2 \approx 2(x_q - x_p)e_1,$$

$$z_{12} = (\hat{y}_q - y_p)^2 - (y_q - y_p)^2 = 2(y_q - y_p)e_2 + e_2^2 \approx 2(y_q - y_p)e_2,$$

$$z_{13} = (\hat{z}_q - z_p)^2 - (z_q - z_p)^2 = 2(z_q - z_p)e_3 + e_3^2 \approx 2(z_q - z_p)e_3,$$

$$z_{14} = \hat{R}_{BU}^2 - R_{BU}^2 = 2R_{BU}e_4 + e_4^2 \approx 2R_{BU}e_4.$$

Based on (45), we can again apply the WLS method to estimate $\xi$, thus the WLS cost function can be written as

$$f(\xi) = (\hat{h}_1 - G_1 \xi)^T W_1 (\hat{h}_1 - G_1 \xi) = \hat{h}_1^T W_1 \hat{h}_1 - 2\hat{h}_1^T W_1 G_1 \xi + \xi^T G_1^T W_1 G_1 \xi,$$

where $W_1$ denotes the weight matrix at Stage 2. To derive the estimation of $\xi$, $f(\xi)$ in (47) should be minimized, leading to $\frac{\partial f(\xi)}{\partial \xi} = -2(G_1^T W_1 \hat{h}_1) + 2(G_1^T W_1 G_1 \xi) = 0$. Therefore, the
estimation of $\xi$ at Stage 2 is given by
\[ \hat{\xi} = \left( \mathbf{G}_1^T \mathbf{W}_1 \mathbf{G}_1 \right)^{-1} \mathbf{G}_1^T \mathbf{W}_1 \hat{\mathbf{h}}_1. \] (48)

By defining $\Omega_1 = \mathbb{E}[\mathbf{z}_1 \mathbf{z}_1^T]$ as the covariance matrix of $\mathbf{z}_1$, we have $\mathbf{W}_1 = \Omega_1^{-1}$. Moreover, $\Omega_1$ can be further derived as
\[ \Omega_1 = \mathbf{B}_1 (\hat{\mathbf{G}}_r^T \mathbf{W}_r \hat{\mathbf{G}}_r)^{-1} \mathbf{B}_1, \quad \mathbf{B}_1 = 2 \text{diag}\left\{ \mathbf{u} - \begin{bmatrix} \mathbf{p} \\ 0 \end{bmatrix} \right\}, \] (49)
where the proof of (49) is given in Appendix A. Although $\mathbf{B}_1$ contains the true values $\{x_q, y_q, z_q, R_{BU}\}$, it can be approximated as
\[ \hat{\mathbf{B}}_1 = 2 \text{diag}\left\{ \hat{\mathbf{u}} - \begin{bmatrix} \mathbf{p} \\ 0 \end{bmatrix} \right\}. \] (50)

Then, $\Omega_1$ can be approximated as $\hat{\Omega}_1 = \hat{\mathbf{B}}_1 (\hat{\mathbf{G}}_r^T \mathbf{W}_r \hat{\mathbf{G}}_r)^{-1} \hat{\mathbf{B}}_1$. Hence, $\mathbf{W}_1$ can be estimated as $\hat{\mathbf{W}}_1 = \hat{\Omega}_1^{-1} = \hat{\mathbf{B}}_1^{-1} (\hat{\mathbf{G}}_r^T \mathbf{W}_r \hat{\mathbf{G}}_r) \hat{\mathbf{B}}_1^{-1}$, and $\hat{\xi}$ is approximated as
\[ \hat{\xi} \approx \left( \mathbf{G}_1^T \mathbf{W}_1 \mathbf{G}_1 \right)^{-1} \mathbf{G}_1^T \mathbf{W}_1 \hat{\mathbf{h}}_1. \] (51)

3) Final Solution: Based on the estimations in Stages 1 and 2, the final closed-form expression of the position of MU can be expressed as:
\[ \hat{\mathbf{q}} = \Pi \sqrt{\hat{\xi}} + \mathbf{p}, \quad \Pi = \text{diag}\{\text{sgn}(\hat{\mathbf{u}}(1 : 3) - \mathbf{p})\}, \] (52)
where $\text{sgn}(x)$ denotes the signum function$^2$.

IV. BIAS ANALYSIS

Modern positioning applications are susceptible to positioning bias, which can significantly restrict their performance$^{[35]}$. Hence, it is necessary to perform bias analysis to theoretically evaluate the performance of the proposed positioning scheme. To analyze the bias here, we use the second-order statistics of the estimation error. In addition, it is assumed that the error level is not large, which enables us to ignore the error terms of the third order or larger. Furthermore, we need to derive the bias in both stages so that the bias for the final estimation in the previous section can be determined.

As the estimation error is inevitable, $\hat{\mathbf{G}}_r$ defined in (35) consists of two parts: the true matrix $\mathbf{G}_r$ and the error matrix $\tilde{\mathbf{G}}_r$, i.e., $\hat{\mathbf{G}}_r = \mathbf{G}_r + \tilde{\mathbf{G}}_r$. According to (35) and (36), to derive the expressions of $\mathbf{G}_r$ and $\tilde{\mathbf{G}}_r$, we need to decompose $\hat{\mathbf{g}}_{\theta_{MR,i}}$, $\hat{\mathbf{g}}_{\phi_{MR,i}}$, and $\hat{\mathbf{g}}_{t,i}$ into the true values and the estimation error at first. And the flowchart of bias analysis is shown in Fig.4.

$^2$The signum function $\text{sgn}(x)$ is a piecewise function which is defined as: $\text{sgn}(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0. \end{cases}$
First, let us decompose $\hat{g}_{\theta_{MR,i}}$ into the true value and the estimation error. Using the approximations in (13) and the definition of $\hat{g}_{\theta_{MR,i}}$ in (32), $\hat{g}_{\theta_{MR,i}}$ is approximated as

$$\hat{g}_{\theta_{MR,i}} \approx \left[\cos \theta_{MR,i}, \sin \theta_{MR,i}, 0, 0\right]^T + n_i \left[\sin \theta_{MR,i}, \cos \theta_{MR,i}, 0, 0\right]^T,$$

where $n_i g_{1n_i}$ denotes the error term.

Similarly, by utilizing the approximations in (13), (18) and the definition of $\hat{g}_{\phi_{MR,i}}$ in (32), $\hat{g}_{\phi_{MR,i}}$ is approximated as

$$\hat{g}_{\phi_{MR,i}} \approx \left[(\sin \phi_{MR,i} + \omega_i \cos \phi_{MR,i}) (\sin \theta_{MR,i} + n_i \cos \theta_{MR,i}),
(\sin \phi_{MR,i} + \omega_i \cos \phi_{MR,i}) (\cos \theta_{MR,i} - n_i \sin \theta_{MR,i}),
-(\cos \phi_{MR,i} - \omega_i \sin \phi_{MR,i})\right]^T.$$

By ignoring the terms $\omega_i n_i \cos \phi_{MR,i} \cos \theta_{MR,i}$ and $-\omega_i n_i \sin \phi_{MR,i} \sin \theta_{MR,i}$ [38] and defining $\hat{g}_{\phi_{MR,i}} = \left[\sin \phi_{MR,i} \sin \theta_{MR,i}, \sin \phi_{MR,i} \cos \theta_{MR,i}, -\cos \phi_{MR,i}, 0\right]^T$, $\omega_i g_{1n_i} = \left[\cos \phi_{MR,i} \sin \theta_{MR,i}, \cos \phi_{MR,i} \cos \theta_{MR,i}, -\sin \phi_{MR,i} \sin \theta_{MR,i}, 0, 0\right]^T$, and $g_{2n_i} = \left[\sin \phi_{MR,i} \cos \theta_{MR,i}, -\sin \phi_{MR,i} \sin \theta_{MR,i}, 0\right]^T$, we have

$$\hat{g}_{\phi_{MR,i}} \approx \hat{g}_{\phi_{MR,i}} + \omega_1 g_{\omega_i} + n_i g_{2n_i},$$

where $\omega_1 g_{\omega_i}$ and $n_i g_{2n_i}$ denote the error terms, respectively.

**Table 2** Symbols description of bias analysis

| Symbol | Description |
|--------|-------------|
| $\hat{g}_{\theta_{MR,i}}$, $\hat{g}_{\phi_{MR,i}}$, $g_{1n_i}$ | true value vectors of $\hat{g}_{\theta_{MR,i}}$, $\hat{g}_{\phi_{MR,i}}$, $g_{1n_i}$ |
| $G_{\theta}$, $G_{\phi}$, $G_t$ | true value matrices consisting of $\hat{g}_{\theta_{MR,i}}$, $\hat{g}_{\phi_{MR,i}}$, $g_{1n_i}$ |
| $g_{1n_i}, \{g_{2n_i}, g_{\omega_i}\}$, $g_{1n_i}$ | error coefficient vectors of $\hat{g}_{\theta_{MR,i}}$, $\hat{g}_{\phi_{MR,i}}$, $g_{1n_i}$ |
| $G_{1n_i}, G_{2n_i}, G_{\omega_i}, G_{\nu}$ | error coefficient matrices consisting of $g_{1n_i}, g_{2n_i}, g_{\omega_i}, g_{\nu}$ |
| $\hat{G}_t$ | true value matrix of $G_t$ |
| $\hat{G}_r$ | error coefficient matrix of $\hat{G}_t$ |
| $P_t$ | true value of $P_t$ |
| $\hat{P}_t$ | error of $P_t$ |
| $\bar{u}$ | estimation error of $\bar{u}$ |
| $\bar{\xi}$ | estimation error of $\bar{\xi}$ |
| $W_1$ | true value of $W_1$ |
| $\hat{W}_1$ | estimation error of $\hat{W}_1$ |
| $P_1$ | true value of $P_1$ |
| $\hat{P}_1$ | estimation error of $\hat{P}_1$ |
Similarly, we can decompose $\tilde{g}_{t,i}$ into the true value and the estimation error. Based on the definition of $g_{t,i}$ in (27) and $\tilde{g}_{t,i}$ in (29), we can rewrite $\tilde{g}_{t,i}$ as $\tilde{g}_{t,i} = -[x_{i,p}, y_{i,p}, z_{i,p}, R_{B,i}]^T - [0, 0, 0, \nu_i]^T = g_{t,i} + \nu_i[0, 0, 0, -1]^T$. Letting $g_{\nu} = [0, 0, 0, -1]^T$, we have

$$g_{t,i} = \tilde{g}_{t,i} + \nu_i g_{\nu},$$

where $\nu_i g_{\nu}$ denotes the error term.

Therefore, using (53), (54) and (55), $\hat{G}_r$ can be written as

$$\hat{G}_r = G_r + \hat{G}_r.$$  

Moreover, using the definition of $n$, $\omega$ and $\nu$ in (3), $\hat{G}_r$ can be written as

$$\hat{G}_r = \hat{G}_n \odot [(n1_{4 \times 4})^T, (n1_{4 \times 4})^T] + \hat{G}_\omega \odot [(\omega 1_{4 \times 4})^T, (\omega 1_{4 \times 4})^T]^T + \hat{G}_\nu \odot [(\nu 1_{4 \times 4})^T, (\nu 1_{4 \times 4})^T]^T,$$

where

$$\hat{G}_n = [G_{1n}^T, G_{2n}^T, O_{4 \times M}]^T, \quad \hat{G}_\omega = [O_{4 \times M}, G_{1\omega}^T, O_{4 \times M}]^T, \quad \hat{G}_\nu = [O_{4 \times M}, O_{4 \times M}, G_{\nu}^T]^T,$$

$$G_{1n} = [g_{1n_1}, g_{1n_2}, \ldots, g_{1n_M}]^T, \quad G_{2n} = [g_{2n_1}, g_{2n_2}, \ldots, g_{2n_M}]^T,$$

$$G_{\omega} = [g_{\omega_1}, g_{\omega_2}, \ldots, g_{\omega_M}]^T, \quad G_{\nu} = [g_{\nu}, g_{\nu}, \ldots, g_{\nu}]^T.$$

In addition, $G_r$ is given by

$$G_r = [G_{\theta r}^T, G_{\varphi r}^T, G_{t}^T]^T,$$

where

$$G_{\theta r} = [g_{\theta_{MR,1}}, g_{\theta_{MR,2}}, \ldots, g_{\theta_{MR,M}}]^T, \quad G_{\varphi r} = [g_{\phi_{MR,1}}, g_{\phi_{MR,R,2}}, \ldots, g_{\phi_{MR,M}}]^T,$$

$$G_{t} = [g_{t,1}, g_{t,2}, \ldots, g_{t,M}]^T.$$  

A. Bias in Stage 1

In this subsection, we aim to derive the bias of $\hat{u}$ obtained in Stage 1. As we can see from the flowchart in Fig. 4, to derive the bias of $\hat{u}$, the expression of estimation error of Stage 1 should be derived, which is given by $\tilde{u} = \hat{u} - u$. Then, the bias of $\hat{u}$ is given by taking the expectation of $\tilde{u}$, which is written as $E(\tilde{u})$. Using (40) and the definition of $u$, $\tilde{u}$ can be further derived as

$$\tilde{u} = \hat{u} - u = (\hat{G}_r^T W_r \hat{G}_r)^{-1} \hat{G}_r^T W_r (\hat{h}_r - \hat{G}_r u).$$

Since we have $\hat{h}_r - \hat{G}_r u = B_r z_r$ in (33), $\tilde{u}$ can be derived as $\tilde{u} = (\hat{G}_r^T W_r \hat{G}_r)^{-1} \hat{G}_r^T W_r B_r z_r$. As we have mentioned above, it is necessary to consider the second order term when analyzing the bias (35). However, we have used (31) to derive (33) rather than (30), which ignores the second
order term $\frac{1}{2}\nu r^T r$. Therefore, to obtain the expression of $\tilde{u}$ and derive the bias of Stage 1, $B_r z_r$ should be extended to $B_r z_r + [0^T, \frac{1}{\sqrt{2}} \nu^T]^T \odot [0^T, \frac{1}{\sqrt{2}} \nu^T]^T$, where $0$ is a $2M \times 1$ column vector. Then, $\tilde{u}$ can be further derived as $\tilde{u} = (\hat{G}_r^T W_r \hat{G}_r)^{-1} \hat{G}_r^T W_r \left( B_r z_r + [0^T, \frac{1}{\sqrt{2}} \nu^T]^T \odot [0^T, \frac{1}{\sqrt{2}} \nu^T]^T \right)$.

Letting $\eta = [0^T, \frac{1}{\sqrt{2}} \nu^T]^T$ and $\hat{P}_r = \hat{G}_r^T W_r \hat{G}_r$, $\tilde{u}$ can be rewritten as

$$\tilde{u} = \hat{P}_r^{-1} \hat{G}_r^T W_r (B_r z_r + \eta \odot \eta).$$

(62)

According to the definition of $\hat{G}_r$ in (56), $\hat{P}_r$ can be further derived as

$$\hat{P}_r = (G_r + \hat{G}_r)^T W_r (G_r + \hat{G}_r) = G_r^T W_r G_r + \hat{G}_r^T W_r \hat{G}_r + G_r^T W_r \hat{G}_r + \hat{G}_r^T W_r G_r.$$

(63)

Here, $\hat{G}_r^T W_r \hat{G}_r$ can be neglected, thus $\hat{P}_r$ can be approximated as

$$\hat{P}_r \approx G_r^T W_r G_r + \hat{G}_r^T W_r \hat{G}_r + G_r^T W_r \hat{G}_r = P_r + \hat{P}_r,$$

(64)

where $P_r = G_r^T W_r G_r$ and $\hat{P}_r = \hat{G}_r^T W_r \hat{G}_r + G_r^T W_r \hat{G}_r$, respectively. However, if we use the definition of $\hat{P}_r$ in (64) to derive (62), the inverse matrix $\hat{P}_r^{-1} = (P_r + \hat{P}_r)^{-1}$ is complex and challenging to derive. Fortunately, we can derive the approximation of $\hat{P}_r^{-1}$ by utilizing the Newmann expansion [35] when the error level is small, which is written as

$$\hat{P}_r^{-1} \approx \left( I - P_r^{-1} \hat{P}_r \right) P_r^{-1}.$$

(65)

By substituting (65) and (56) into (62), $\tilde{u}$ is derived as

$$\tilde{u} \approx \left( I - P_r^{-1} \hat{P}_r \right) P_r^{-1} (G_r + \hat{G}_r)^T W_r (B_r z_r + \eta \odot \eta).$$

(66)

For the sake of illustration, using $P_r$ in (64), let us define $H_r = (G_r^T W_r G_r)^{-1} G_r^T W_r = P_r^{-1} G_r^T W_r$. Then, $\tilde{u}$ can be represented as

\[ \tilde{u} \approx \left( I - P_r^{-1} \hat{P}_r \right) P_r^{-1} G_r^T W_r (B_r z_r + \eta \odot \eta). \]

Fig. 4: Flowchart of bias analysis.

If we consider $\hat{G}_r^T W_r \hat{G}_r$ when deriving the expression of $\tilde{u}$, it will be multiplied by $B_r z_r$, and the terms including $\hat{G}_r^T W_r \hat{G}_r$ in the final expression of $\tilde{u}$ is higher than second order, thus we neglect this term here.
\[ \begin{align*}
\tilde{u} &\approx H_r B_r z_t + H_r (\eta \odot \eta) - P^{-1}_r \tilde{P}_r H_r B_r z_t - P^{-1}_r \tilde{P}_r H_r (\eta \odot \eta) + P^{-1}_r \tilde{G}_r^T W_1 B_r z_t \\
&\quad + P^{-1}_r \tilde{G}_r^T W_1 (\eta \odot \eta) - P^{-1}_r \tilde{P}_r P^{-1}_r \tilde{G}_r^T W_1 B_r z_t - P^{-1}_r \tilde{P}_r P^{-1}_r \tilde{G}_r^T W_1 (\eta \odot \eta),
\end{align*} \] (67)

For simplicity, we can ignore the error terms in (67) which are higher than second order. As a result, using \( \tilde{P}_r \) in (64), \( \tilde{u} \) is approximated as

\[ \tilde{u} \approx H_r B_r z_t + H_r (\eta \odot \eta) - P^{-1}_r \tilde{G}_r^T W_1 G_r H_r B_r z_t - P^{-1}_r \tilde{G}_r^T W_1 \tilde{G}_r H_r B_r z_t + P^{-1}_r \tilde{G}_r^T W_1 B_r z_r. \] (68)

As a result, the bias in Stage 1 is written as

\[ E[\tilde{u}] = E[H_r B_r z_t + H_r (\eta \odot \eta) - P^{-1}_r \tilde{G}_r^T W_1 G_r H_r B_r z_t - P^{-1}_r \tilde{G}_r^T W_1 \tilde{G}_r H_r B_r z_r + P^{-1}_r \tilde{G}_r^T W_1 B_r z_r] \]

\[ = E_1 + E_2 + E_3, \] (69)

where \( E_1 = E[H_r B_r z_t] \), \( E_2 = E[H_r (\eta \odot \eta)] \) and \( E_3 = E[-P^{-1}_r \tilde{G}_r^T W_1 G_r H_r B_r z_t - P^{-1}_r \tilde{G}_r^T W_1 \tilde{G}_r H_r B_r z_r] \), the detailed derivations of which are given in Appendix B.

B. Bias in Stage 2

In this subsection, we aim to derive the bias of \( \tilde{\xi} \) according to the solution obtained in Stage 2. Similar to the derivations of the bias in Stage 1, we need to derive the expression of estimation error of \( \xi \), and the bias of \( \xi \) can be derived by taking the expectation of the estimation error of \( \xi \). First, denote \( \tilde{\xi} \) as the estimation error of \( \xi \), e.g., \( \tilde{\xi} = \tilde{\xi} - \xi \). Then, using the definition of \( \tilde{\xi} \) in (51), \( \tilde{\xi} \) can be further derived as

\[ \tilde{\xi} = (G_1^T \tilde{W}_1 G_1)^{-1} G_1^T \tilde{W}_1 (\tilde{h}_1 - G_1 \xi). \] (70)

According to (45), \( \tilde{\xi} \) is derived as

\[ \tilde{\xi} = (G_1^T \tilde{W}_1 G_1)^{-1} G_1^T \tilde{W}_1 z_1. \] (71)

Using the derivations of (46), the definition of \( B_1 \) in (49) and the definition of \( \tilde{u} \) below (45), we have \( z_1 = B_1 \tilde{u} + \tilde{u} \odot \tilde{u} \). Then, (71) can be derived as

\[ \tilde{\xi} = (G_1^T \tilde{W}_1 G_1)^{-1} G_1^T \tilde{W}_1 (B_1 \tilde{u} + \tilde{u} \odot \tilde{u}). \] (72)

By defining \( \tilde{P}_1 = G_1^T \tilde{W}_1 G_1 \), we have \( \tilde{\xi} = \tilde{P}_1^{-1} G_1^T \tilde{W}_1 (B_1 \tilde{u} + \tilde{u} \odot \tilde{u}) \). According to the definition of \( \tilde{W}_1 \) below (50), \( \tilde{W}_1 \) is composed of the true value and the estimation error, which are denoted by \( W_1 \) and \( \tilde{W}_1 \), respectively. To further derive the expression of \( \tilde{\xi} \), the expressions of \( W_1 \) and \( \tilde{W}_1 \) should be derived, which are given by

\[ \tilde{W}_1 = W_1 + \tilde{W}_1, \]

\[ W_1 = B_1^{-1} P_1 B_1^{-1}, \]

\[ \tilde{W}_1 = B_1^{-1} \tilde{P}_1 B_1^{-1} - W_1 \tilde{B}_1 B_1^{-1} - \tilde{B}_1 B_1^{-1} W_1, \] (73)

where \( \tilde{B}_1 \) is given in (93). The details of deriving \( W_1 \) and \( \tilde{W}_1 \) are given in Appendix C.

Similarly, based on the definition of \( \tilde{P}_1 \) below (72), \( \tilde{P}_1 \) can be obtained as a summation of the
true value and the estimation error, which are denoted by $P_1$ and $\hat{P}_1$, respectively. Hence, we have

$$\hat{P}_1 = P_1 + \tilde{P}_1, \quad P_1 = G_1^T W_1 G_1, \quad \tilde{P}_1 = G_1^T \tilde{W}_1 G_1. \quad (74)$$

Similar to (65), we can derive the approximation of $\hat{P}_1^{-1}$ as

$$\hat{P}_1^{-1} \approx (I - P_1^{-1} \hat{P}_1)P_1^{-1} = P_1^{-1} - P_1^{-1} \hat{P}_1 P_1^{-1}. \quad (75)$$

Therefore, using the approximation of $\hat{P}_1^{-1}$ in (75) and the definition of $\tilde{W}_1$ in (73), (72) can be derived as

$$\tilde{\xi} \approx (P_1^{-1} - P_1^{-1} \hat{P}_1 P_1^{-1}) G_1^T \tilde{W}_1 (B_1 \hat{u} + \hat{u} \odot \hat{u}). \quad (76)$$

For notation simplicity, let $H_1 = P_1^{-1} G_1^T W_1$, we have

$$\tilde{\xi} \approx H_1 (B_1 \hat{u} + \hat{u} \odot \hat{u}) + P_1^{-1} G_1^T \tilde{W}_1 (B_1 \hat{u} + \hat{u} \odot \hat{u}) - P_1^{-1} \hat{P}_1 H_1 (B_1 \hat{u} + \hat{u} \odot \hat{u})$$

$$- P_1^{-1} \hat{P}_1^{-1} G_1 \tilde{W}_1 (B_1 \hat{u} + \hat{u} \odot \hat{u}). \quad (77)$$

By ignoring the error terms higher than the second order and substituting $\hat{P}_1$ in (74) into (77), we have

$$\tilde{\xi} \approx H_1 (B_1 \hat{u} + \hat{u} \odot \hat{u}) + P_1^{-1} G_1^T \tilde{W}_1 B_1 \hat{u} - P_1^{-1} \hat{P}_1 H_1 B_1 \hat{u}. \quad (78)$$

By assuming that $P_2 = I_{4 \times 4} - G_1 H_1$, we have

$$\tilde{\xi} \approx H_1 (B_1 \hat{u} + \hat{u} \odot \hat{u}) + P_1^{-1} G_1^T \tilde{W}_1 P_2 B_1 \hat{u}. \quad (79)$$

Finally, by taking expectation of $\tilde{\xi}$, the bias of Stage 2 is given by

$$E[\tilde{\xi}] = H_1 B_1 E[\hat{u}] + H_1 E[\hat{u} \odot \hat{u}] + P_1^{-1} G_1^T E[\tilde{W}_1 P_2 B_1 \hat{u}]. \quad (80)$$

To further derive the bias of Stage 2, let us introduce Proposition 1 as follows.

**Proposition 1.** For the vector $\tilde{a} \in \mathbb{C}^{M \times 1}$, the expectation of $\tilde{a} \odot \tilde{a}$ can be expressed as a column vector $c_a$ containing the diagonal elements of $\Omega_a$, which is the expectation of the second order moment of $\tilde{a}$.

**Proof:** Please see Appendix D. \[■\]

Using Proposition 1, $E[\hat{u} \odot \hat{u}]$ can be derived as $c_a$ containing the diagonal elements of $\Omega_a$. Furthermore, the details of deriving $\Omega_a$ is given by (91) at Appendix A. Therefore, the bias in Stage 2 can be further derived as

$$E[\xi] = H_1 (B_1 E[\hat{u}] + c_a) + P_1^{-1} G_1^T E[\tilde{W}_1 P_2 B_1 \hat{u}], \quad (81)$$

where $E[\hat{u}]$ is given in (69), and the derivations of $E[\tilde{W}_1 P_2 B_1 \hat{u}]$ are given in Appendix E.
C. Bias of Final Solution

First, let us define the estimation error of MU’s position as $\tilde{q} = \hat{q} - q$. Then using the definition of $\xi$ in (44), $\tilde{\xi}$ can be derived as

$$
\tilde{\xi} = (\hat{q} - p) \odot (\hat{q} - p) = \hat{q} \odot \hat{q} + 2\hat{q} \odot (q - p) + \xi.
$$

By utilizing $\tilde{\xi} = \hat{\xi} - \xi$, we have $\hat{\xi} - \tilde{q} \odot \tilde{q} = 2\hat{q} \odot (q - p)$. Then, by assuming that $B_q\tilde{q} = 2\hat{q} \odot (q - p)$, where $B_q = 2\text{diag}\{(q - p)\}$, we have the expression of $\hat{q}$ given as

$$
\hat{q} = B_q^{-1}(\hat{\xi} - \tilde{q} \odot \tilde{q}).
$$

Then, we can derive the bias of the final solution as

$$
E[\tilde{q}] = E[B_q^{-1}(\hat{\xi} - \hat{q} \odot \hat{q})].
$$

Using Proposition 1, we have $E[\tilde{q} \odot \tilde{q}] = c_q$, where $c_q$ is a column vector formed by the diagonal elements of $\Omega_q$, and the details of deriving $\Omega_q$ can be seen from Appendix F. Therefore, (84) can be further derive as

$$
E[\tilde{q}] = B_q^{-1}(E[\hat{\xi}] - c_q),
$$

where $E[\hat{\xi}]$ is given in (81).

V. Simulation Results

This section presents simulation results to evaluate the performance of the proposed positioning scheme aided by multiple RISs. Moreover, the MU, the BS and the RISs are assumed to be placed in a 3D area. The location of the BS is $p = [0, 0, 40]^T$, while the locations of three RISs are $s_1 = [0, 5, 39]^T$, $s_2 = [1, 2, 38]^T$ and $s_3 = [0, 0, 34]^T$. We consider a mm-wave massive MISO channel from the MU to the RIS. The phase shift matrix of the RIS is set to a unit matrix. Furthermore, it is assumed that the inter-element spacing of UPA at the RIS is $d_2 = \lambda/2$ and the pass loss exponent is set as 2.5. Besides, we assume that the carrier frequency is 4.9 GHz and the number of subcarriers is 128. The noise power spectrum density is $N_0 = -174$ dBm/Hz, bandwidth is $B_w = 20$ MHz. The following results are obtained by averaging over 10,000 random estimation error realizations. Unless otherwise stated, we assume that the SNR is 10 dB and the RIS size is $N_y = N_z = 16$.

Fig. 5 evaluates the impact of the radio positioning parameters’ estimation error and the number of the RISs on the performance of the proposed positioning scheme by comparing the

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4The BSs are generally installed on high places, and the MU’s location is unknown and impossible to overlap with the BS, hence $B_q$ is a full-rank matrix.
RMSE. To be specific, we consider two types of variance, which are the variance of AOA estimation error $\sigma_a^2$ and the variance of TDOA estimation error $\sigma_t^2$, respectively. As shown in Fig. 5, when $\sigma_a^2$ decreases from $1$ rad$^2$ to $10^{-8}$ rad$^2$, the RMSE decreases, which means that the accuracy of angle estimation obtained in our first paper [34] affects the positioning accuracy of the proposed scheme. Besides, when $\sigma_t^2$ becomes larger, the RMSE of the proposed positioning scheme becomes larger as well. It means that the positioning accuracy deteriorates as the TDOA estimation error increases. Furthermore, we also compare the positioning scheme aided by 2 RISs with that aided by 3 RISs. It can be seen from Fig. 5 that the RMSE of the proposed positioning scheme that employs 3 RISs is much less than that of the proposed scheme that utilizes only 2 RISs. This implies that the positioning accuracy of the proposed scheme utilizing 3 RISs is much better.

Fig. 6 illustrates the impact of the number of the RIS elements on the performance of the proposed positioning scheme. As seen, RMSE decreases with the number of the RIS’s reflecting elements. This implies that the positioning accuracy is better with a larger number of RIS reflecting elements. Furthermore, when $\sigma_t^2 = 10^{-10}$ and $N_y = N_z = 64$, the RMSE is $10^{-2}$ in the case of 2 RISs, while the RMSE achieves $10^{-4}$ in the case of 3 RISs, which means that more RISs provide higher positioning accuracy. However, in the case of 3 RISs, even when the size of the RIS increases from $N_y = N_z = 64$ to $N_y = N_z = 128$, the RMSE remains at the same level. It means that the size of the RIS has little impact on the positioning accuracy in the case of 3 RISs when $N_y$ and $N_z$ are larger than 128.

Fig. 7 compares the RMSE of the proposed positioning scheme aided by 2 RISs with that aided by 3 RISs when the signal-noise ratio (SNR) increases from $-10$ dB to $30$ dB. As shown in
the figure, the RMSE decreases with SNR as expected. Furthermore, it is shown that the RMSE of 2 RISs is about $10^2$ times larger than that of 3 RISs. It implies that increasing the number of RISs can significantly improve the positioning accuracy of the proposed scheme. Besides, the RMSE reduces from $8.0 \times 10^{-2}$ to $10^{-3}$ in the case of 3 RISs and $N_y = N_z = 16$, while the RMSE stays at almost the same level in the case of 3 RISs and $N_y = N_z = 64$. It means that the impact of SNR becomes smaller as the size of the RIS becomes large.

Fig. 8 confirms the accuracy of the theoretical analysis on the bias of the proposed positioning scheme. In the case of 2 RISs, the proposed algorithm achieves almost the same as the theoretical bias when the variance of the angle estimation error $\sigma_a^2$ is smaller than $10^{-6}$. In the case of 3 RISs, the theoretical bias matches very well with simulation results when $\sigma_a^2$ is not larger than $10^{-4}$. Therefore, the proposed positioning scheme aided by 3 RISs performs better than that aided by 2 RISs.

VI. CONCLUSION

In this series of work, we proposed a comprehensive two-step 3D positioning scheme aided by multiple RISs. We investigated the second step including position estimation and bias analysis. We derived the pseudolinear equations based on the radio positioning parameters estimation at the RISs. We obtained the closed-form solution of the position of the MU through a TSWLS algorithm. In the first stage, the preliminary estimation of the position of the MU was obtained using an approximation algorithm. In the second stage, a more accurate estimation was derived using geometric relationships. Finally, we investigated the bias analysis to evaluate the perfor-
mance of the proposed algorithm. Simulation results affirmed the accuracy of the derived results under the realistic case of non-Gaussian estimation error.

**APPENDIX A**

First, using the definition of $z_1$ in (46), $\Omega_1$ is derived as

$$
\Omega_1 = \text{diag} \left[ 2(x_q - x_p), 2(y_q - y_p), 2(z_q - z_p), 2R_{BU} \right]^T.
$$

Using the expression of $\Omega$, which is denoted as $\tilde{\Omega}$ and taking expectation of $\tilde{u}\tilde{u}^T$, $\Omega_u$ is given by

$$
\Omega_u = \mathbb{E} \left[ [e_1, e_2, e_3, e_4]^T [e_1, e_2, e_3, e_4] \right] \text{diag} \left[ 2(x_q - x_p), 2(y_q - y_p), 2(z_q - z_p), 2R_{BU} \right]^T.
$$

Then, we need to derive the expression of $\Omega_u$ as follows. First, we will derive the error of $u$, which is denoted as $\hat{u}$. According to (40) and the definition of $u$, $\hat{u}$ is formulated as

$$
\hat{u} = \tilde{u} - u = (\hat{G}_r^TW_r\hat{G}_r)^{-1}\hat{G}_r^TW_r(\hat{h}_r - \hat{G}_ru).
$$

According to (33), we have

$$
\tilde{u} = (\hat{G}_r^TW_r\hat{G}_r)^{-1}\hat{G}_r^TW_rB_rz_r.
$$

Using the expression of $\tilde{u}$ and taking expectation of $\tilde{u}\tilde{u}^T$, $\Omega_u$ is given by

$$
\Omega_u = (\hat{G}_r^TW_r\hat{G}_r)^{-1}\hat{G}_r^TW_rB_rQ_rB_r^TW_r^T\hat{G}_r((\hat{G}_r^TW_r\hat{G}_r)^{-1})^T.
$$

Note that $W_r = (B_rQ_rB_r^T)^{-1}$ according to (41) and $W_r$ is a diagonal matrix. Thus, we have $W_r^T = W_r$. As a result, (90) can be derived as

$$
\Omega_u = (\hat{G}_r^TW_r\hat{G}_r)^{-1}.
$$

As a result, $\Omega_1$ is derived as

$$
\Omega_1 = B_1(\hat{G}_r^TW_r\hat{G}_r)^{-1}B_1.
$$

**APPENDIX B**

By utilizing (37), recall that $z_r = [n^T, \omega^T, \nu^T]^T$, $B_rz_r$ can be expanded as $(B_{r,n}n + B_{r,\omega}\omega + B_{r,\nu}\nu)$. Then, we have

$$
E_1 = \mathbb{E}[H_rB_rz_r] = H_r\mathbb{E}[B_{r,n}n + B_{r,\omega}\omega + B_{r,\nu}\nu] = H_rB_{r,n}\mathbb{E}[n] + H_rB_{r,\omega}\mathbb{E}[\omega],
$$

where $\mathbb{E}(n)$ and $\mathbb{E}(\omega)$ denote the expectation of $n$ and $\omega$, respectively. $\mathbb{E}(n)$ and $\mathbb{E}(\omega)$ can be obtained based on our first paper [34].

For $E_2$, we have
\[ E_2 = \mathbb{E}[H_r(\eta \odot \eta)] = \frac{1}{2} H_r \mathbb{E}[[0, \ldots , 0, \nu_1^2, \ldots , \nu_M^2]^T] = H_r q_u, \]

where

\[ q_u = \frac{1}{2} [0, \ldots , 0, \sigma_{\nu_1}^2, \ldots , \sigma_{\nu_M}^2]^T. \]

Then, \( E_3 \) can be derived as

\[ E_3 = \mathbb{E}[P_r^{-1} \tilde{G}_r^T W_r B_r z_r] - \mathbb{E}[P_r^{-1} \tilde{G}_r^T W_r \tilde{G}_r H_r B_r z_r] - \mathbb{E}[P_r^{-1} \tilde{G}_r^T W_r \tilde{G}_r H_r B_r z_r] = E_{31} - E_{32} - E_{33}, \]

where the detailed derivations of \( E_{31}, E_{32} \) and \( E_{33} \) are given as follows.

For \( E_{31} \), as we have \( \tilde{G}_r \) in \( [57] \) and \( B_r z_r = (B_{r,n} n + B_{r,\omega} \omega + B_{r,\nu} \nu) \), \( E_{31} \) can be derived as

\[ E_{31} = \mathbb{E}[P_r^{-1}(\tilde{G}_r^T \odot [(n1_{1 \times 4})^T, (n1_{1 \times 4})^T])W_r B_{r,n} \text{diag}(n)1_{M \times 1}] + \mathbb{E}[P_r^{-1}(\tilde{G}_r^T \odot [(\omega1_{1 \times 4})^T, (\nu1_{1 \times 4})^T])W_r B_{r,\omega} \text{diag}(\omega)1_{M \times 1}] + \mathbb{E}[P_r^{-1}(\tilde{G}_r^T \odot [(\nu1_{1 \times 4})^T, (\nu1_{1 \times 4})^T])W_r B_{r,\nu} \text{diag}(\nu)1_{M \times 1}]. \]

To further derive the expression of \( E_{31} \), we introduce Proposition 2 and Proposition 3 as follows.

**Proposition 2.** For vectors \( \alpha \in \mathbb{C}^{M \times 1}, \beta \in \mathbb{C}^{N \times 1} \) and matrix \( G \in \mathbb{C}^{M \times N} \), and corresponding diagonal matrices \( \text{diag}(\alpha) \) and \( \text{diag}(\beta) \) with these vectors as their main diagonals, we have \( (\alpha^T \beta) \odot G = \text{diag}(\alpha) G \text{diag}(\beta). \)

**Proof:** Please see reference [39].

**Proposition 3.** For a vector \( x \in \mathbb{C}^{M \times 1} \) with its covariance matrix given by \( Q_x \), we have

\[ \mathbb{E}[(A \odot [(x1_{1 \times N})^T, (x1_{1 \times N})^T, (x1_{1 \times N})^T])B \text{diag}(x)] = A(B \odot [Q_x, Q_x, Q_x]^T), \]

where \( A \in \mathbb{C}^{N \times 3M} \) and \( B \in \mathbb{C}^{3M \times M}. \)

**Proof:** First, the expression on the left hand side of the equation can be derived as

\[ \mathbb{E}[(A \odot [(x1_{1 \times N})^T, (x1_{1 \times N})^T, (x1_{1 \times N})^T])B \text{diag}(x)] = \mathbb{E}[(A \odot (1_{N \times 1}(1_{1 \times 3} \otimes x^T)))B \text{diag}(x)] = \mathbb{E}[(1_{N \times 1}(1_{1 \times 3} \otimes x^T)) \odot A]B \text{diag}(x) = \mathbb{E}[(1_{N \times 1}(1_{3 \times 1} \otimes x)^T) \odot A]B \text{diag}(x). \]

Then, using Proposition 2, we have
\[ \mathbb{E}[(1_{N \times 1}(1_{3 \times 1} \otimes x)^T) \otimes A]\mathbb{B}\text{diag}(x)] \\
= \mathbb{E}[\text{diag}(1_{N \times 1})A\text{diag}(1_{3 \times 1} \otimes x)\mathbb{B}\text{diag}(x)] \\
= \mathbb{E}[A\text{diag}(1_{3 \times 1} \otimes x)\mathbb{B}\text{diag}(x)] \\
= \mathbb{E}[A((1_{3 \times 1} \otimes x)x^T) \otimes B] \\
= \mathbb{E}[A((1_{1 \times 3} \otimes xx^T)^T \otimes B)].
\]

Then, by calculating the expectation of \( xx^T \), the expression can be rewritten as
\[
\mathbb{E}[A((1_{1 \times 3} \otimes xx^T)^T \otimes B)] = A((1_{1 \times 3} \otimes Q_z)^T \otimes B) = A(B \otimes [Q_z, Q_x, Q_z]^T),
\]
which completes the proof of Proposition 3.

Consequently, using Proposition 3, \( E_{31} \) can be derived as
\[
E_{31} = P_r^{-1}G_n^T((W_rB_{r,n}) \otimes [Q_n, Q_n, Q_n])1_{M \times 1} + P_r^{-1}G_\omega^T((W_rB_{r,\omega}) \otimes [Q_\omega, Q_\omega, Q_\omega])1_{M \times 1} \\
+ P_r^{-1}G_\nu^T((W_rB_{r,\nu}) \otimes [Q_\nu, Q_\nu, Q_\nu])1_{M \times 1} \\
= P_r^{-1}(G_n^T((W_rB_{r,n}) \otimes Q_n)1_{M \times 1} + G_\omega^T((W_rB_{r,\omega}) \otimes Q_\omega)1_{M \times 1} + G_\nu^T((W_rB_{r,\nu}) \otimes Q_\nu)1_{M \times 1}),
\]
where
\[
\bar{Q}_n = [1, 1, 1]^T \otimes Q_n, \quad \bar{Q}_\omega = [1, 1, 1]^T \otimes Q_\omega, \quad \bar{Q}_\nu = [1, 1, 1]^T \otimes Q_\nu.
\]

Similarly, by applying Proposition 2 and Proposition 3, \( E_{32} \) can be derived as
\[
E_{32} = \mathbb{E}[P_r^{-1}(G_n^T \otimes [(n1_{1 \times 4})^T, (n1_{1 \times 4})^T, (n1_{1 \times 4})^T] + G_\omega^T \otimes [(\omega1_{1 \times 4})^T, (\omega1_{1 \times 4})^T, (\omega1_{1 \times 4})^T] \\
+ G_\nu^T \otimes [(\nu1_{1 \times 4})^T, (\nu1_{1 \times 4})^T, (\nu1_{1 \times 4})^T])W_rG_rH_r(B_r,n + B_r,\omega + B_r,\nu)] \\
= P_r^{-1}(G_n^T((W_rG_rH_rB_{r,n}) \otimes Q_n)1_{M \times 1} + G_\omega^T((W_rG_rH_rB_{r,\omega}) \otimes Q_\omega)1_{M \times 1} \\
+ G_\nu^T((W_rG_rH_rB_{r,\nu}) \otimes Q_\nu)1_{M \times 1}).
\]

Similarly, using Proposition 2 and Proposition 3, \( E_{33} \) is formulated as
\[
E_{33} = \mathbb{E}[P_r^{-1}G_r^TW_r(G_r \otimes [(n1_{1 \times 4})^T, (n1_{1 \times 4})^T, (n1_{1 \times 4})^T]^T + G_\omega^T \otimes [(\omega1_{1 \times 4})^T, (\omega1_{1 \times 4})^T, (\omega1_{1 \times 4})^T]^T \\
+ G_\nu^T \otimes [(\nu1_{1 \times 4})^T, (\nu1_{1 \times 4})^T, (\nu1_{1 \times 4})^T]^T]H_r(B_r,n + B_r,\omega + B_r,\nu) \\
= P_r^{-1}G_r^T(((W_rG_rH_rB_{r,n}) \otimes \bar{Q}_n)1_{M \times 1} + ((W_rG_rH_rB_{r,\omega}) \otimes \bar{Q}_\omega)1_{M \times 1} \\
+ ((W_rG_rH_rB_{r,\nu}) \otimes \bar{Q}_\nu)1_{M \times 1}).
\]
APPENDIX C

First, based on the definition of \( \hat{W}_1 \) below (50) and the definition of \( \hat{P}_r \) above (62), we have \( \hat{W}_1 = \hat{B}_1^{-1}\hat{P}_r\hat{B}_1^{-1} \). Then, according to the definition of \( B_1 \) in (49) and \( \hat{B}_1 \) in (50), we have

\[
\hat{B}_1 = B_1 + \hat{B}_1, \quad \tilde{B}_1 = 2\text{diag}\{\tilde{u}\}. \tag{93}
\]

If we use \( \tilde{B}_1 = B_1 + \hat{B}_1 \) to derive \( \hat{W}_1 \), the inverse matrix \( \hat{B}_1^{-1} \sim (B_1 + \hat{B}_1)^{-1} \) would be complex and challenging. Fortunately, using the Newmann expansion [35], we can derive the approximation of \( \hat{B}_1^{-1} \approx B_1^{-1} - B_1^{-1}\hat{B}_1 B_1^{-1} \). As a result, \( \hat{W}_1 \) is given by

\[
\hat{W}_1 = (B_1^{-1} - B_1^{-1}\hat{B}_1 B_1^{-1})(\hat{P}_r + \hat{P}_1)(B_1^{-1} - B_1^{-1}\hat{B}_1 B_1^{-1}). \tag{94}
\]

By neglecting the error terms that are higher than the first order, \( \hat{W}_1 \) can be approximated as

\[
\hat{W}_1 \approx B_1^{-1}\hat{P}_r B_1^{-1} + B_1^{-1}\hat{P}_1 B_1^{-1} - B_1^{-1}\hat{P}_r B_1^{-1}\hat{B}_1 B_1^{-1} - B_1^{-1}\hat{B}_1 B_1^{-1} B_1^{-1} P_1 B_1^{-1} - B_1^{-1}\hat{B}_1 B_1^{-1} P_1 B_1^{-1} = W_1 + \tilde{W}_1, \tag{95}
\]

where \( W_1 \) and \( \tilde{W}_1 \) are defined as

\[
W_1 = B_1^{-1}P_1 B_1^{-1}, \quad \tilde{W}_1 = B_1^{-1}\hat{P}_r B_1^{-1} - W_1\hat{B}_1 B_1^{-1} - \hat{B}_1 B_1^{-1} W_1. \tag{96}
\]

APPENDIX D

Before proving Proposition 1, let us introduce Proposition 4 as follows.

**Proposition 4.** The Hadamard product of two vectors \( a \in \mathbb{C}^{N \times 1} \) and \( b \in \mathbb{C}^{N \times 1} \), is the same as matrix multiplication of one vector by the corresponding diagonal matrix of the other vector:

\[
a \odot b = \text{diag}(a)b.
\]

**Proof:** please see reference [39].

Let \( E[\hat{a} \odot \hat{a}] \) denotes the expectation of \( \hat{a} \odot \hat{a} \), using Proposition 4, we have \( E[\hat{a} \odot \hat{a}] = E[\text{diag}(\hat{a})\hat{a}] = E[\text{diag}(\hat{a})\text{diag}(\hat{a})]M_{1 \times 1} \). Using Proposition 2, \( E[\hat{a} \odot \hat{a}] \) can be rewritten as

\[
E[\hat{a} \odot \hat{a}] = E[((\hat{a}\hat{a}^T) \odot I_{M \times M})I_{M \times 1}] = \{\Omega_a \odot I_{M \times M}\}I_{M \times 1} = c_a,
\]

where \( c_a \) denotes the vector containing the diagonal elements of \( \Omega_a \). Here, we complete the proof of Proposition 1.

APPENDIX E

Using the definition of \( \tilde{W}_1 \) in (96), \( E[\tilde{W}_1 P_2 B_1 \tilde{u}] \) can be derived as

\[
E[\tilde{W}_1 P_2 B_1 \tilde{u}] = E[(B_1^{-1}\hat{P}_r B_1^{-1} - W_1\hat{B}_1 B_1^{-1} - \hat{B}_1 B_1^{-1} W_1)P_2 B_1 \tilde{u}]
\]

\[
= E[B_1^{-1}\hat{P}_r B_1^{-1} P_2 B_1 \tilde{u}] - E[W_1 B_1^{-1} P_2 B_1 \tilde{u}] - E[\hat{B}_1 B_1^{-1} W_1 P_2 B_1 \tilde{u}],
\]

let \( a_1 = E[B_1^{-1}\hat{P}_r B_1^{-1} P_2 B_1 \tilde{u}] \), \( a_2 = -E[W_1 B_1^{-1} P_2 B_1 \tilde{u}] \) and \( a_3 = -E[\hat{B}_1 B_1^{-1} W_1 P_2 B_1 \tilde{u}] \), we have

\[
E[\tilde{W}_1 P_2 B_1 \tilde{u}] = a_1 + a_2 + a_3,
\]

while \( a_1, a_2, a_3 \) are given as follows.
Using the definition of $\tilde{P}_i$ below (64), $a_1$ can be derived as
\[ a_1 = \mathbb{E}[B_1^{-1} \tilde{P}_i B_1^{-1} P_2 B_1 \tilde{u}] = \mathbb{E}[B_1^{-1} (\tilde{G}_r^T W_r G_r + G_r^T W_r \tilde{G}_r) B_1^{-1} P_2 B_1 \tilde{u}]\]
Using the definition of $\tilde{u}$ in (63) and ignoring the error terms that are higher than second order, $a_1$ can be approximated as $\mathbb{E}[B_1^{-1} (\tilde{G}_r^T W_r G_r + G_r^T W_r \tilde{G}_r) B_1^{-1} P_2 B_1 H_i B_i z_i]$. For notation simplicity, we assume that $B_1^{-1} P_2 B_1 H_i = P_3$. Then, by applying Proposition 2 and Proposition 3, $a_1$ can be derived as
\[ a_1 = B_1^{-1} (G_n^T((W_r G_r P_3 B_{r,n}) \odot Q_n)1_{M \times 1} + G_w^T((W_r G_r P_3 B_{r,w}) \odot Q_w)1_{M \times 1} \]
\[+ \tilde{G}_r^T((W_r G_r P_3 B_{r,n}) \odot \tilde{Q}_n)1_{M \times 1} + \tilde{G}_r^T((W_r G_r P_3 B_{r,w}) \odot \tilde{Q}_w)1_{M \times 1}) \]
Moreover, for $a_2$, using the definition of $\tilde{B}_1$ in (63), we have
\[ a_2 = -\mathbb{E}[W_1 B_1^{-1} \tilde{B}_1 P_2 B_1 \tilde{u}] = -2W_1 B_1^{-1} \mathbb{E}[(\tilde{u}u^T) \odot (P_2 B_1)]1_{4 \times 1} = -2W_1 B_1^{-1} (\Omega_u \odot (P_2 B_1))1_{4 \times 1}. \]
Using Proposition 2, we have
\[ a_2 = -2W_1 B_1^{-1} \mathbb{E}[(\tilde{u}u^T) \odot (P_2 B_1)]1_{4 \times 1} = -2W_1 B_1^{-1} (\Omega_u \odot (P_2 B_1))1_{4 \times 1}. \]
As $\Omega_u$ is a diagonal matrix, $a_2$ can be further derived as
\[ a_2 = -2W_1 B_1^{-1} \text{diag}(\Omega_u(P_2 B_1))1_{4 \times 1} = -2W_1 B_1^{-1} \rho_4, \]
where $\rho_4$ represents column vector formed by the diagonal elements of $P_4 = P_2 B_1 \Omega_u$.
Similar to the derivations of $a_2$, $a_3$ it can be derived as
\[ a_3 = -\mathbb{E}[B_1^{-1} \tilde{B}_1 W_1 P_2 B_1 \tilde{u}] = -2B_1^{-1} \rho_{4,W1}, \]
where $\rho_{4,W1}$ represents column vector formed by the diagonal elements of $W_1 P_4$.

APPENDIX F

First, let us derive the covariance matrix of $\xi$, which is denoted by $\Omega_\xi$. Using the definition of $\tilde{\xi}$ in (71), its covariance matrix $\Omega_\xi$ is given by
\[ \Omega_\xi = \mathbb{E}[\xi \xi^T] \approx G_1^T \tilde{W}_1 G_1^{-1} G_1^T \tilde{W}_1 \tilde{W}_1^{-1} \tilde{W}_1^T G_1 ((G_1^T \tilde{W}_1 G_1^{-1})^T = (G_1^T \tilde{W}_1 G_1)^{-1}, \]
where $\mathbb{E}(z_i z_i^T) \approx \Omega_1 = W_1^{-1}$ is given below (50).
Furthermore, by neglecting the second order term in (83), the $\tilde{q}$ can be approximated as $B_q^{-1} \tilde{\xi}$, therefore, the covariance matrix of $\tilde{q}$ can be derived as
\[ \Omega_q = \mathbb{E}[\tilde{q} \tilde{q}^T] = B_q^{-1} (G_1^T \tilde{W}_1 G_1)^{-1} B_q^{-1}. \]

REFERENCES

[1] C. L. Nguyen, O. Georgiou, G. Gradoni, and M. Di Renzo, “Wireless fingerprinting localization in smart environments using reconfigurable intelligent surfaces,” IEEE Access, vol. 9, pp. 135 526–135 541, 2021.
[2] H. Ren, K. Wang, and C. Pan, “Intelligent reflecting surface-aided URLLC in a factory automation scenario,” IEEE Transactions on Communications, vol. 70, no. 1, pp. 707–723, 2022.

[3] S. Han, C.-L. I., T. Xie, S. Wang, Y. Huang, L. Dai, Q. Sun, and C. Cui, “Achieving high spectrum efficiency on high speed train for 5G new radio and beyond,” IEEE Wireless Communications, vol. 26, no. 5, pp. 62–69, 2019.

[4] R. Liu, Q. Wu, M. Di Renzo, and Y. Yuan, “A path to smart radio environments: An industrial viewpoint on reconfigurable intelligent surfaces,” IEEE Wireless Communications, vol. 29, no. 1, pp. 202–208, 2022.

[5] M. Di Renzo, A. Zappone, M. Debbah, M.-S. Alouini, C. Yuen, J. de Rosny, and S. Tretyakov, “Smart radio environments empowered by reconfigurable intelligent surfaces: How it works, state of research, and the road ahead,” IEEE Journal on Selected Areas in Communications, vol. 38, no. 11, pp. 2450–2525, 2020.

[6] C. Pan, H. Ren, K. Wang, J. F. Kolb, M. Elkashlan, M. Chen, M. Di Renzo, Y. Hao, J. Wang, A. L. Swindlehurst, X. You, and L. Hanzo, “Reconfigurable intelligent surfaces for 6G systems: Principles, applications, and research directions,” IEEE Communications Magazine, vol. 59, no. 6, pp. 14–20, 2021.

[7] Y. Pan, K. Wang, C. Pan, H. Zhu, and J. Wang, “Self-sustainable reconfigurable intelligent surface aided simultaneous terahertz information and power transfer (STIPT),” IEEE Transactions on Wireless Communications, pp. 1–1, 2022.

[8] H. Ren, K. Wang, and C. Pan, “Intelligent reflecting surface-aided URLLC in a factory automation scenario,” IEEE Transactions on Communications, pp. 1–1, 2021.

[9] F. Jiang and A. L. Swindlehurst, “Optimization of uav heading for the ground-to-air uplink,” IEEE Journal on Selected Areas in Communications, vol. 30, no. 5, pp. 993–1005, 2012.

[10] K. Zhi, C. Pan, H. Ren, and K. Wang, “Statistical CSI-based design for reconfigurable intelligent surface-aided massive MIMO systems with direct links,” IEEE Wireless Communications Letters, vol. 10, no. 5, pp. 1128–1132, 2021.

[11] M. Renzo, M. Debbah, D. T. Phan-Huy, A. Zappone, M. S. Alouini, C. Yuen, V. Sciancalepore, G. C. Alexandropoulos, J. Hoydis, and H. Gacanin, “Smart radio environments empowered by ai reconfigurable meta-surfaces: An idea whose time has come,” EURASIP Journal on Wireless Communications and Networking, vol. 2019, no. 1, 2019.

[12] C. Pan, H. Ren, K. Wang, W. Xu, M. Elkashlan, A. Nallanathan, and L. Hanzo, “Multicell MIMO communications relying on intelligent reflecting surfaces,” IEEE Transactions on Wireless Communications, vol. 19, no. 8, pp. 5218–5233, 2020.

[13] G. Zhou, C. Pan, H. Ren, K. Wang, and Z. Peng, “Secure wireless communication in RIS-aided MISO system with hardware impairments,” IEEE Wireless Communications Letters, vol. 10, no. 6, pp. 1309–1313, 2021.

[14] G. Zhou, C. Pan, H. Ren, K. Wang, and A. Nallanathan, “A framework of robust transmission design for IRS-aided MISO communications with imperfect cascaded channels,” IEEE Transactions on Signal Processing, vol. 68, pp. 5092–5106, 2020.

[15] K. Zhi, C. Pan, H. Ren, and K. Wang, “Uplink achievable rate of intelligent reflecting surface-aided millimeter-wave communications with low-resolution adc and phase noise,” IEEE Wireless Communications Letters, vol. 10, no. 3, pp. 654–658, 2021.

[16] Z. Zhang and L. Dai, “A joint precoding framework for wideband reconfigurable intelligent surface-aided cell-free network,” IEEE Transactions on Signal Processing, vol. 69, pp. 4085–4101, 2021.

[17] D. Fan, F. Gao, Y. Liu, Y. Deng, G. Wang, Z. Zhong, and A. Nallanathan, “Angle domain channel estimation in hybrid millimeter wave massive MIMO systems,” IEEE Transactions on Wireless Communications, vol. 17, no. 12, pp. 8165–8179, 2018.

[18] G. Zhou, C. Pan, H. Ren, P. Popovski, and A. L. Swindlehurst, “Channel estimation for RIS-aided multiuser millimeter-wave systems,” 2021.

[19] C. Hu, L. Dai, S. Han, and X. Wang, “Two-timescale channel estimation for reconfigurable intelligent surface aided wireless communications,” IEEE Transactions on Communications, vol. 69, no. 11, pp. 7736–7747, 2021.
[20] D. Dardari, P. Closas, and P. M. Djurić, “Indoor tracking: Theory, methods, and technologies,” *IEEE Transactions on Vehicular Technology*, vol. 64, no. 4, pp. 1263–1278, 2015.

[21] Q. Li, B. Chen, and M. Yang, “Improved two-step constrained total least-squares TDOA localization algorithm based on the alternating direction method of multipliers,” *IEEE Sensors Journal*, vol. 20, no. 22, pp. 13666–13673, 2020.

[22] S. Rao, A. Mezghani, and A. L. Swindlehurst, “Channel estimation in one-bit massive MIMO systems: Angular versus unstructured models,” *IEEE Journal of Selected Topics in Signal Processing*, vol. 13, no. 5, pp. 1017–1031, 2019.

[23] W. H. Foy, “Position-location solutions by Taylor-series estimation,” *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-12, no. 2, pp. 187–194, 2007.

[24] Y. T. Chan and K. C. Ho, “A simple and efficient estimator for hyperbolic location,” *IEEE Transactions on Signal Processing*, vol. 42, no. 8, pp. 1905–1915, 2002.

[25] X. Yang, C.-K. Wen, S. Jin, A. L. Swindlehurst, and J. Zhang, “Joint channel estimation and localization for cooperative millimeter wave systems,” in *2020 IEEE 21st International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, 2020, pp. 1–5.

[26] J. He, H. Wymeersch, L. Kong, O. Silvén, and M. Juntti, “Large intelligent surface for positioning in millimeter wave MIMO systems,” in *2020 IEEE 91st Vehicular Technology Conference*, 2020, pp. 1–5.

[27] Y. Liu, S. Hong, C. Pan, Y. Wang, Y. Pan, and M. Chen, “Optimization of RIS configurations for multiple-RIS-aided mmWave positioning systems based on crlb analysis,” 2021.

[28] R. Wang, Z. Xing, and E. Liu, “Joint location and communication study for intelligent reflecting surface aided wireless communication system,” 2021.

[29] Z. Feng, B. Wang, M. Luan, and F. Hu, “Power optimization for target localization with reconfigurable intelligent surfaces,” *Signal Processing*, vol. 189, no. 5, p. 108252, 2021.

[30] A. Elzanaty, A. Guerra, F. Guidi, and M.-S. Alouini, “Reconfigurable intelligent surfaces for localization: Position and orientation error bounds,” *IEEE Transactions on Signal Processing*, vol. 69, pp. 5386–5402, 2021.

[31] H. Zhang, H. Zhang, B. Di, K. Bian, Z. Han, and L. Song, “Towards ubiquitous positioning by leveraging reconfigurable intelligent surface,” *IEEE Communications Letters*, vol. 25, no. 1, pp. 284–288, 2021.

[32] J. He, H. Wymeersch, T. Sanguanpuak, O. Silven, and M. Juntti, “Adaptive beamforming design for mmWave RIS-aided joint localization and communication,” in *2020 IEEE Wireless Communications and Networking Conference Workshops (WCNCW)*, 2020, pp. 1–6.

[33] A. Fascista, M. F. Keskin, A. Coluccia, H. Wymeersch, and G. Seco-Granados, “RIS-aided joint localization and synchronization with a single-antenna receiver: Beamforming design and low-complexity estimation,” 2022.

[34] T. Wu, C. Pan, Y. Pan, S. Hong, H. Ren, and M. Elkashlan, “Two-step mmWave positioning scheme with RIS-Part I: Angle estimation and analysis,” *To appear*.

[35] K. C. Ho, “Bias reduction for an explicit solution of source localization using TDOA,” *Signal Processing, IEEE Transactions on*, vol. 60, no. 5, pp. 2101–2114, 2012.

[36] E. Čišija, A. M. Ahmed, A. Serzin, and H. Wymeersch, “RIS-aided mmWave MIMO radar system for adaptive multi-target localization,” in *2021 IEEE Statistical Signal Processing Workshop (SSP)*, 2021, pp. 196–200.

[37] C. Chaccour, M. N. Soorki, W. Saad, M. Bennis, and P. Popovski, “Can terahertz provide high-rate reliable low latency communications for wireless vr?” 2020.

[38] Y. Wang and K. C. Ho, “An asymptotically efficient estimator in closed-form for 3D AOA localization using a sensor network,” *IEEE Transactions on Wireless Communications*, vol. 14, no. 12, pp. 6524–6535, 2015.

[39] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge University Press, 1985.