1 Overview

Steganography is the task of concealing a message within a medium such that the presence of the hidden message cannot be detected. Though the prospect of steganography is conceivably interesting in many contexts, and though work has been done both towards formalizing steganographic security and providing provably secure constructions, little work exists attempting to provide efficient and provably secure steganographic schemes in specific, useful domains.

Beginning from the starting point of the initial definition of steganographic security, I have engaged in an exploration which has developed to include two primary tasks, both pointing towards the realization of efficient and secure steganographic systems in practice: (a) investigating the syntactic and semantic applicability of the current formalism of steganographic security to a broader range of potentially interesting domains and (b) constructing and implementing provably secure (symmetric-key) steganographic schemes in domains which are well-suited to the current formalism.

In the remainder of this document, I provide a high-level overview of existing work in the area of provably secure steganography, and I describe the progress I have made in the tasks stated above.¹

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¹This work is the report authored in conjunction with research performed with the Columbia University cryptography laboratory.
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2 Paper: Understanding Provably Secure Steganography

In 2002, Hopper et. al. published “Provably Secure Steganography,” a work studying steganography from a complexity-theoretic perspective which set forth the current formal definition of provable steganographic security [2]. As such, the content of this work is vital to any exploration seeking the realization of provably secure steganography in practice. What follows in this section is a summary of this work, including the formal definition of steganographic security which it defines.

In this work, the authors formulate steganography as a game involving three parties: Alice, Bob, and Ward. In this game, Alice attempts to pass a secret message to Bob, and Ward is attempting to determine whether or not Alice is sending a secret message.

Towards formally defining security in this setting, the authors abstract communication between Alice and Bob as occurring over a channel. A channel $C$, formally, is a statistical distribution on bit sequences where each bit is marked with monotonically increasing time values: a statistical distribution with support $\{(0,1), (t_1), (0,1), (t_2), \ldots\}$ where $\forall t_i, t_{i+1} \geq t_i$.

Communication over a channel is an action which requires one or more parties to draw from a channel. Specifically, the authors assume the existence of an oracle capable of drawing fixed-length bit sequences from the channel conditioned on a history $h$ of bits already drawn. The authors denote by $C_h$ the channel distribution conditioned on history $h$, and the authors denote by $C_h^b$ the conditional distribution over the next $b$ bits (a block) drawn from the channel, again conditioned on $h$. For any steganographic construction under this formalism, the authors state that $b$ must be fixed, and the authors require that, for all potential histories $h$, the minimum entropy of $C_h^b$ be greater than $1$.

The authors define a stegosystem as a pair of probabilistic algorithms $S = (SE, SD)$. $SE$ takes as input a key $K \in \{0,1\}^k$, a hiddentext $m \in \{0,1\}^*$, a message history $h$, and an oracle $M(h)$ which samples blocks according to $C_h^b$. The result of $SE^{M(h)}(K, m, h)$ should be a sequence of blocks $c_1 // c_2 // \ldots // c_t$, called the stegotext, from the support of $C_h^{b_1}$. $SD$ takes as input a key $K$, a stegotext $s$, a message history $h$, and returns as a result a hiddentext $m$.

At this point in the work, the authors discuss some of their choices in their formulation of the definition of a stegosystem. Namely, the authors note that this formalism is constructed under the assumption that stegosystems do not take advantage of additional knowledge of the channel, resulting in (among other things) dependence on the use of the channel oracle. The authors also note that this dependence may not be justified in all cases, as there are reasonable circumstances under which (a) the oracle draws are not independent or are not efficient (e.g. a human oracle) and/or (b) potentially $SD$ may not practically have access to an oracle.

Correctness of a stegosystem The authors say that a stegosystem $S = (SE, SD)$ is correct if the following holds:

$$\forall m, | m | < p(k) : Pr[SD^M(K, SE^M(k, m, h), h) = m] \geq \frac{2}{3}$$

Steganographic secrecy with passive wardens The setting for steganographic secrecy against passive wardens is the following game played by a warden $W$:

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2 This definition of a channel is chosen specifically by the authors so as to be able to incorporate the notion of time; this allows for timing attacks to be included within the formal definition of security.

3 Equivalently stated, the block $B$ with maximum probability over the support of $C_h^b$ must have $Pr[B] < 0.5$
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Aubrey Alston (ada2145@columbia.edu)

1. $W$ has access to an oracle $M(h)$ which is able to sample blocks from the distribution $C_h^k$ for arbitrary histories $h$. $W$ is allowed to make as many draws from $M(h)$ as desired.

2. $W$ is given access to a second oracle which is either $SE(K, m, h)$ or a generic oracle $O(m, h)$ which samples from $C_h^{SE(K,m,h)}$. $W$ makes at most $q$ queries to this oracle, totaling $l$ bits of hiddden text.

3. $W$ outputs a bit which signals whether the output of the second oracle contains concealed messages.

In this setting, the advantage of an adversary (warden) against a stegosystem $S = (SE, SD)$ defined for a particular channel $C$ is

$$Adv_{S,C}(W) = |Pr_{K,r}[W^M,SE(K,m,h) = 1] - Pr_r[W^M,O(m,h) = 1]|$$

We would then say that a stegosystem is steganographically secret against $(t(k), q(k), l(k))$—wardens if the maximum advantage of any warden making $q(k)$ oracle queries totalling at most $l(k)$ bits of hidden text and running in time at most $t(k)$ is negligible in $k$, where $k$ is the security parameter of the system. Here, the authors attempt to define a notion of universal steganographic secrecy: a stegosystem $S = (SE, SD)$ is universally steganographically secret if, for every channel distribution $C$ having minimum entropy greater than one and for every probabilistic polynomial time warden $W$, $Adv_{S,C}(W)$ is negligible in $k$.

An example construction Hopper et. al. provide multiple constructions which they prove to be secure in the passive warden model. The first they provide is a stateful scheme which they prove to be universally steganographically secret conditioned upon the use of a pseudo-random function (PRF).

The scheme provided assumes that Alice and Bob both have access to a channel oracle $M(h)$ and a synchronized $d$-bit counter and that Alice and Bob have access to a PRF $F: \{0,1\}^k \times \{0,1\}^d \times (\{0,1\}, t)^b \Rightarrow \{0,1\}$. Further, the scheme assumes the existence of an error-correcting code able to correct a $3/8 + \text{negl}(n)$ fraction of errors, the procedures of which are denoted by $Enc$ and $Dec$. The exact functionality of $SE$ and $SD$ are given below:

**Algorithm 1** Steganographic encoding procedure

1. **procedure** $SE(K \in \{0,1\}^k, m', h)$
2. $m = Enc(m')$
3. Split $m$ into bits $m_1 \parallel m_2 \parallel ... m_l$
4. **for** $i$ from 1 to $l$ **do**
5. Sample $M(h)$ at most two times until $F_K(N, M(h)) = m_i$. Set $c_i = M(h)$ for the final sample.
6. Set $h = h \parallel c_i$.
7. Increment counter $N$.
8. **end for**
9. **return** $c_1 \parallel ... \parallel c_l$
10. **end procedure**

**Algorithm 2** Steganographic decoding procedure

1. **procedure** $SD(K \in \{0,1\}^k, c, h)$
2. Split $c$ into $b - \text{bit}$ blocks $c_1 \parallel ... \parallel c_l$.
3. **for** $i$ from 1 to $l$ **do**
4. Set $m_i = F_K(N, c_i)$.
5. Increment $N$.
6. **end for**
7. Set $m = m_1 \parallel ... \parallel m_l$.
8. **return** $Dec(m)$
9. **end procedure**

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Given the minimal entropy assumption of the channel, the authors are able to succinctly derive an upper bound on the failure probability (per message) bit of approximately $\frac{3}{8}$, allowing the scheme to be correct given the use of a satisfactory error correcting code. The authors then show that the steganographic secrecy of this construction under the passive warden model directly reduces to the PRF security of $F$, thereby allowing them to conclude that this scheme $S$ is both correct and universally steganographically secret.

**Other notions** The authors additionally define additional formal notions for steganography, including the notion of robust steganography. This definition is an attempt to incorporate the idea of an active warden who modifies messages as they are sent; however, as the authors admit, an unbounded adversary may choose to simply replace messages completely from the channel distribution, destroying concealed messages. Their specific solution is to bound the warden by a relation of permissible modifications, but the practical relevance of these limitations is not directly clear.

### 2.1 Initial Thoughts and Questions

“Provably Secure Steganography,” aside from presenting a useful formal framework for defining secure steganography, coerced from me an initial set of thoughts and questions surrounding the intersection of provably secure steganography and its realization in practice. In this section, I summarize and discuss these thoughts and questions.

#### 2.1.1 The Notion of Universal Steganography

The formal model presented by Hopper et. al. explicitly and implicitly portrays some desire to encapsulate as many potential channels and domains as possible. On this topic, I am left with two questions; namely, (1) in attempting to achieve such generality, are some domains left out or made more difficult to manipulate within the framework? and (2) are universal steganographic constructions under this model the most valuable contribution in practice?

Regarding the first of these questions, clearly the definition of ‘universal’ disqualifies some channels, given the minimum entropy assumption enforced. Additionally, one result of the generality of the formalism is the assumption that the specific characterization of the channel is unknown but that sampling is feasible: could there potentially be a useful channel where the characterization is known but sampling many times is not feasible? The reliance on fixed block sizes also seems to add difficulty in domains where variable-length messages are common: a proof of security for a steganographic scheme within such a domain would require the added difficulty of proving minimum entropy while dividing messages along fixed-size block boundaries.

Regarding the second of these questions, consider the simple stateful universal steganographic scheme given in the original paper. This scheme remains sufficiently general to apply to any channel which, under the correct choice of $b$, follows the minimum entropy assumption. In using this scheme, I may achieve a rate of secrecy of one bit per block. Intuitively, however, it seems that there should exist an optimal scheme given an arbitrary channel $C$, even for fixed $b$, that uses channel-specific properties to achieve a much better rate.

To illustrate this intuition, say we model the output of Alice as a stochastic process which obeys the distribution $C^b_h$; when Alice is preparing to send symbol $i$, there exists some number $r_i \leq 2^b$ of potential symbols she may output while obeying the channel distribution. In theory, then, it seems reasonable to believe that Alice should be able to hide $\log_2(r_i)$, potentially greater than 1, bits of hidden text in that symbol. In the same manner, this specific example provides some motivation to potentially support the idea that fixing $b$ may not lead to optimal schemes in arbitrary domains, especially ones involving channels of high entropy which also portray predictable structural properties.$^{4}$

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$^4$Consider the contrived case where the last $c < b$ bits of symbol $i$ may restrict the degree of freedom of symbol $i+1$. 

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2.1.2 Potential Extensions of the Model

While considering the formalism of the channel in the context of provably secure steganography, a simple potential generalization occurred to me which I can best describe as \textit{n-out-of-k-channel steganography}. There exist heuristic methods of making steganography harder to detect by distributing concealed messages among multiple mediums. Using this same line of thought, would it be worthwhile to also formalize a definition of steganographic security (and also constructions) where access to \( n \) out of \( k \) channels is required to detect and retrieve concealed messages?

In parallel to the idea of proving the security of steganographic schemes, it also occurred to me that we might be able to formalize a technique for identifying optimality in secrecy rate for general schemes.

3 Exploring Appliciability of the Formalism of Provably Secure Steganography

Having considered the current formal framework of provably secure steganography as given by Hopper et. al., I then moved to consider how well this framework directly admits construction of efficient, high-rate, and provably secure stegosystems in general domains. This section contains a summary of work I encountered attempting to achieve such a construction, as well as a presentation of my own work to attempt to modify the formal framework to be more accommodating to such constructions.

3.1 Paper: Variable-length \( P \)-Codes

In [3], Tri Van Le attempts to give a somewhat modified paradigm for provably secure steganography, and he also attempts to introduce a new steganographic primitive, called a \( P \)-code, which would allow him to construct what he calls “essentially optimal” steganographic systems. What follows in this section is a high-level summary of the components of this work which diverge from [2].

Following some motivation and preliminaries, the author introduces his modified formal framework for provably secure steganography which is equivalent to that in [2], up to two minor modifications. The first of these modifications is trivial, in that he introduces new names for the constructs already present in the framework of Hopper et al. (e.g., “sampler” for oracle, the symbol \( P \) to represent the channel, “chosen hiddentext security” for steganographic secrecy in the presence of passive wardens). The second of these modifications is more interesting: he models the channel as a statistical distribution over the support of a finite message space (as opposed to fixed-length blocks of bits). While his definition of advantage does not incorporate a complexity-theoretic attempt to classify wardens as in [2], this modification seems to be promising.

Within the bounds of this framework, Le then defines what he calls \( P \)-codes. A \( P \)-code is a uniquely decodable, variable-length decoding scheme \( \Gamma = (\Gamma_{enc}, \Gamma_{dec}) \) where the domain of \( \Gamma_{enc} \) is \( \{0, 1\}^n \), the range is the message space of the channel \( \mathcal{P} \), and the quantity

\[
\sum_{c \in \{\Gamma_{dec}(x) : x \in \{0, 1\}^n\}} |Pr_{x \sim \{0, 1\}^n}[\Gamma_{enc}(x) = c] - Pr[c]|\]

is negligible in \( n \).

We see through later constructions given by Le that the existence of \( P \)-codes admits extremely simple steganographic schemes. Unfortunately, however, though Le attempts to give a valid \( P \)-code derived

\[\text{Page 5}\]
from an arithmetic coding scheme, there seems to be an error in his construction which could introduce vulnerability in stegosystems which use it. In the first two steps of his $\Gamma_{enc}$ procedure, he does the following:

1. Initialize a set of variables to be used and set the initial history to be empty.
2. Sample $C_{ho}$ $l$ times into $c_1, \ldots, c_l$

This sequence of messages $c_1, \ldots, c_l$ is then later output as the first $l$ messages of $\Gamma_{enc}$. If Alice uses $\Gamma_{enc}$ as in the example construction of Le, Ward will see $l$ messages which obey the distribution $C_{ho}$, but there is no guarantee that the sequence $c_j+1, \ldots, c_l$ obeys $C_{ho}[c_1||\ldots||c_j]$ for all $j$, and so the use of steganography might be easily detected.

The remainder of [3] concerns itself with applying $P$-codes to construct public- and private-key steganographic schemes, as well as proving that $P$-codes admit “essentially optimal” rates in many cases.

3.2 An Alternative Formalism for Channels with Variable-length Messages

Since the modified framework of provable steganographic security in [3] seemed to address some concerns I found with the notion of universal steganography from [2], I thought it appropriate to attempt to reconcile the two frameworks with the goal of obtaining a formalism which is potentially more natural for a wider variety of domains. This section contains the result of my attempt.

Definition of a channel A channel $C$ is a distribution on messages from a finite message space $\mathcal{M}$, parameterized by an associated set of labels $\tau$. In other words, $C$ is a statistical distribution over support $(c_1 \in \mathcal{M}, \tau_1), \ldots, (c_{|\mathcal{M}|} \in \mathcal{M}, \tau_n)$. Note that this definition is a superset of the previous definition, as we may choose $\mathcal{M} = \{0,1\}$ and define $\tau$ such that each $\tau_i$ must contain a time label which is monotonically increasing among all $\tau_i, \tau_j, \tau_k, i < j < k$.

Denote by $C_h$ the channel distribution conditioned on a history $h$ of drawn and parameterized messages. Further denote by $C_h^{\rightarrow(i)}$ the conditional distribution of the next $i$ messages in the channel given the drawn and parameterized history $h$. Introduce in addition the notion of the view of a channel: define the $q(n)$-view of the next $i$ messages of a channel $C$ conditioned on history $h$ to be the distribution $C_h^{\rightarrow(i)}$ with support restricted to only those $i$-message sequences whose length in binary representation is less than $q(n)$ bits; denote such a channel view by $C_h^{\rightarrow(i)}|q(n)$.

Definition of a (symmetric-key) stegosystem A stegosystem $S$ is a pair of probabilistic algorithms $(SE, SD)$. $SE$ takes as input a key $K \in \{0,1\}^n$, a hiddentext $m \in \{0,1\}^*$, a history $h$, and uses an oracle $M$ capable of sampling from $C_h^{\rightarrow(i)}|s(n)$ (for some $i$, $s(n)$ defined by the system) to return as output a sequence of messages $c_1 || \ldots || c_l$ from the support of $C_h^{\rightarrow(i)}|p(n)$ (where $l$ may vary between hiddentexts $m$). $SD$ takes as input a key $K$, a sequence of messages $c_1 || \ldots || c_l$, a history $h$, and an oracle $M$, and returns a hiddentext $m$. Define correctness in a manner similar to the framework of [2].

Steganographic secrecy against passive wardens A passive warden $W$ is a warden which plays the following game:

1. $W$ is given access to an oracle $M(h)$ which is capable of sampling from $C_h^{\rightarrow(i)}|v(n)$ given arbitrary histories $h$ and for some function $v(n)$.
2. $W$ is then given access to one of two oracles:

   (a) $O_q(m, h) \Leftarrow SE(K, m, h)$

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$\tau$ may yet contain other parameters, e.g. signal strength, such that said parameters may be incorporated into the definition of a stegosystem.
(b) or $O_1(m, h) \leftarrow C_h^{\oplus(1)}|v(n)$

3. $W$ then outputs a bit, representing whether or not he was given $O_0$ or $O_1$.

Denote by $(t, q, v(n))$-warden a warden which takes at most $t$ steps and $q$ queries having at most a $v(n)$-view of the channel while playing this game. In this setting, the advantage of $W$, $\text{Adv}(W)$, is the quantity

$$|Pr_{K,r}[[W^{M,O_1} \Rightarrow 1] - Pr_{r}[[W^{M,O_0} \Rightarrow 1]]$$

Say that a stegosystem $S$ is **steganographically secret** if the advantage of any probabilistic polynomial-time warden is negligible in $n$.

This alternative framework seems to offer three primary benefits over the previous: (1) it is clearly more natural for modeling channels in domains involving variable-length messages, (2) it offers the capacity to incorporate elements other than time into stegosystems, and (3) it partially decouples the number of bits viewed by a warden from the number of messages produced by $SE$.

### 4 Constructing and Implementing Provably Secure Steganography in Practice

Though it may be possible to improve upon the formal framework given in [2], the fact remains that, in reality, the framework it provides is applicable to many domains. As such, I spent time exploring current applications of provably secure steganography in practice.

There are potentially many motivating reasons to pursue the creation of secure steganographic systems; consider as a motivating scenario the world of Alice, Bob, and Ward, where Ward has a large degree of control of Alice and Bob. What if Ward is, say, a nation-state government who has outlawed encryption? What if Ward institutes a strict key disclosure law, where all keys used by Alice and Bob to communicate must be disclosed upon request? Steganography immediately becomes relevant in these situations as a means for Alice and Bob to communicate confidentially.

Of course, in such a situation, the utility of steganographic systems is linked to the channels over which they operate. In a scenario akin to the ones given, Ward will have a vested interest in disallowing steganography: even if a system exhibits a provable 0% detection rate, Ward may simply outlaw the medium used. For this reason, we would reasonably conclude that a steganographic system operating over an outdated landline phone protocol would be significantly less useful than a system which uses the English language as a medium; more generally, one might say that the value of a steganographic system directly correlates with the indispensability of the underlying channel.

With this idea in mind, I began to consider what vital technologies might also serve as suitable steganographic mediums. In beginning this search, I noted two things: (1) that computer networking is ubiquitous and fundamental to the function of the modern world, and (2) that secure computer networking is ubiquitous and fundamental to the function of the modern world. While exploring existing work attempting to use networking protocols as steganographic channels, I found that nearly no work attempts to provide constructions within any framework of provably secure steganography. In considering (2), I realized that there is one common aspect to virtually every secure networking protocol which also lends itself very well towards the formal steganographic model of [2]: cryptographic primitives and cryptographic protocols.

This section summarizes existing work I encountered while considering the use of networking protocols as a steganographic medium and presents my own work in using randomness in cryptography to design and implement practical steganographic schemes.

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10 In practice, such a coupling could result in the proof derived within a framework not being sound in that the system proved secure actually has a vulnerability.

11 Specifically the uniform randomness used by cryptographic primitives and protocols.
4.1 Paper: HICCUPS, Network Steganography at the Link Layer

HICCUPS is a system presented by Szczypiorski [5] which attempts to use the data link layer to implement a steganographic system. This system is included in this report as an example of a typical steganographic system in practice which provides no rigorous guarantee of secrecy: the entire premise of the system is the use of intentionally incorrect values in integrity fields (e.g. checksum fields), a technique whose use may be trivially detected by simply verifying the correctness of included checksums.

There may be some use in the method in highly unreliable networks; however, as the transmission probability decreases, so does the plausibility of faking transmission errors, but, in the case of this system, there is yet no proof that the distribution of errors induced in the checksum will match the distribution of natural errors induced within the network. Further, the use of this method arbitrarily in a network may lead to ‘hubs’ of slightly increased transmission error probability centered around the nodes employing it, providing yet another possibility for detection which has not been addressed.

4.2 Paper: Murdoch and Lewis, Network Steganography at the Internet/Transport Layer

Diverging somewhat from the methods of the previously discussed paper, in “Embedding Covert Channels in TCP/IP” [4], Steven Murdoch and Stephen Lewis present an overview of common approaches to achieving steganography in the TCP/IP networking layer, concluding that most existing approaches are vulnerable in practice. While their approach does not make use of the formal framework of provable steganography, they raise multiple concerns which relate closely.

The authors begin with an overview of current approaches in TCP/IP steganography, stating that most existing proposals result in output distributions different from the distribution expected from ordinary TCP/IP. As is the case for steganography at the link layer, TCP/IP steganography proposals attempt to hide information in the header fields of packets as they are constructed. The authors discuss the fields that are most commonly used, listed below:

**IP Header Fields** IP is a common Internet-layer protocol responsible for addressing and host-to-host routing of packets. The following headers are commonly incorporated into IP steganography proposals:

1. **Type of Service** The ToS field is an 8-bit field which is commonly unused. Most platforms will, by default, set the ToS field to zero; thus, any use will be easily detected.

2. **IP Identification (IPID)** The IPID is a 16-bit identifier for datagrams used during the fragmentation process. Though meant to be unpredictable, the IPID field is not random on most platforms, instead being a number generated by a predictable deterministic process depending on the host platform. There exist steganographic proposals for this field which replace the IPID with pseudo-random numbers; however, the authors indicate that such a method is easily detected, as the IPID is not random.

3. **IP Flags** The (two) flags used in IP packets have a well-defined meaning; steganographic use would, aside from potentially inducing undefined network behavior, be easily detected.

4. **IP Fragment Offset** When packets are fragmented, this field is used to reconstruct the original packet. The authors indicate that there exist steganographic proposals which convey information by modulating packet sizes; however, this is detectable within many networks where such fragmentation is unusual.

**TCP Header Fields** TCP is a common transport-layer protocol responsible for connection-oriented, reliable host-to-host communication on top of the IP protocol. The following headers are commonly incorporated into TCP steganography proposals:
1. **Sequence Number** The sequence number of a TCP packet is a 32-bit number which numbers the packets sent in a TCP connection. Since the relationship between subsequent sequence numbers is determined, there only exists a degree of freedom in the choice of the *Initial Sequence Number* (ISN). These numbers are chosen carefully to enforce uniqueness and overlap constraints; however, existing proposals either replace the ISN with the hidden text or encrypt the hidden text (using e.g. DES) and include it as the ISN. In practice, however, such a choice of ISN does not follow the platform-dependent ISN selection method and is therefore easily detected.

2. **Timestamp** The timestamp field is a header field (technically two 32-bit header fields) used by some platforms (in some situations) to measure round-trip latency. Use of this field for steganographic purposes is detectable in many cases by nature of the fact that many platforms no longer use the field; additionally, use of the field in existing proposals differs from the expected distribution.

In the remainder of the paper, the authors explore in detail the platform-dependent mechanisms which determine the previously discussed fields. Further, they develop a suite of exhaustive tests which are used to detect uses of these fields for steganographic purposes (noting the previous proposals which failed these tests). Using these tests, the authors construct a steganographic suite called *Lathra* which is able to avoid detection in this model on OpenBSD and Linux platforms.

This work by Murdoch and Lewis emphasizes the trend in practical work in steganography to neglect to provide rigorous guarantees of security; even more, this work reveals the extreme difficulty in constructing useful, secure steganographic schemes in the face of platforms which do not always obey protocol specifications.

### 4.3 Embedding Steganography within Cryptographic Primitives and Protocols

[5] and [4] illustrated the difficulty which exists in using base networking protocols as steganographic channels: the varied (yet predictable) tendencies of their implementations to diverge from their specifications. As a result, the underlying channel distribution is less easily characterized and less amenable to the framework and constructions of [2], all the while not admitting much prospect for high-rate steganographic schemes.

Not all network functionality is as forgiving of breaches of specification; among such functionalities are secure networking protocols, used ubiquitously to guarantee the confidentiality and safety of systems, people, and governments alike. Since their removal would cause modern businesses to halt, banks to fail, and potentially lives to be lost, secure networking protocols certainly seem to meet the threshold of indispensability. Even more promising, these protocols also share a common denominator which must be implemented properly and which frequently makes use of randomness: cryptography.

In this section, I present my work on the use of cryptographic constructions used in secure networking protocols to create steganographic channels. I first give and prove the security of a simple stegosystem which operates over the initialization vectors of ciphers operating in CBC mode; I then present a system which implements this scheme and my work to integrate it into OpenSSL as a real-world steganographic application; finally, I present initial work I have performed to design stegosystems utilizing primitives in elliptic curve cryptography as steganographic channels.

#### 4.3.1 Steganography among Initialization Vectors in TLS 1.2/AES-CBC

TLS (Transport Layer Security) is one of the most pervasive modern cryptographic protocols, used to provide confidentiality, integrity, and source verification in addition to reliable transport. One of the most well-known applications of TLS is the HTTPS protocol, used to secure standard web traffic.

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12 These methods tend to rely on header fields, usually less than 6% of total packet size and consisting primarily of heavily determined, low-entropy fields.
The TLS protocol specification exhibits a high degree of structure, itself being composed of a set of sub-protocols: the handshaking protocol, the cipher agreement protocol, and the application data protocol [1]. It is in this third sub-protocol, the application data protocol, that secured data transfer takes place; owing also to the structure of TLS, all data transferred through the application data protocol must pass through a single well-defined set of functionalities, the record layer, which determines how confidentiality and integrity guarantees are enforced [1].

As one might imagine, TLS guarantees confidentiality of data at the record layer through the use of encryption. However, as the length of data handled by TLS is not guaranteed to fit into a single block for a given cipher specification, TLS necessarily employs ciphers in a variable-length mode of operation. Conveniently, the latest non-draft specification for TLS, TLS 1.2, prefers the use of AES in CBC mode, and the specification states that initialization vectors (IVs) must be uniformly random and unpredictable per-IV [1].

Given that IVs must be random, the channel distribution that would correspond to the sequence of IVs seen during a sequence of TLS 1.2 connections is both well-defined and well-suited to the development of a stegosystem that achieves a rate of secrecy greater than one bit per block. In this section, I provide a simple stateful symmetric-key stegosystem for the IVs of TLS 1.2/AES-CBC and a proof of its steganographic secrecy within the framework of [2].

A stegosystem for TLS 1.2/AES-CBC Let $F(K,x)$ be a secure pseudo-random permutation $F : \{0,1\}^n \times \{0,1\}^b \Rightarrow \{0,1\}^b$. Let $N$ be a synchronized counter shared between Alice (the holder of the secret) and Bob (the receiver of the secret), initialized to 0. Take $b$ as both the block size for $S$ and the IV size of the cipher. Because initialization vectors are chosen uniformly at random in TLS 1.2/AES-CBC, we have that the conditional channel distribution $C^b_h$ is the uniform distribution over support $\{0,1\}^b$ for all histories $h$. We define the following stegosystem $S = (SE,SD)$:

Algorithm 3 Steganographic encoding procedure
1: procedure $SE(K \in \{0,1\}^n, m, h)$
2: Split $m$ into $r$-bit blocks $m_1, ..., m_k$.
3: for $i$ from 1 to $k$ do
4: Increment $N$.  
5: yield $F(K,N) \oplus m_i$
6: end for
7: end procedure

Algorithm 4 Steganographic decoding procedure
1: procedure $SD(K \in \{0,1\}^n, c_1 \| ... \| c_l, h)$
2: for $i$ from 1 to $l$ do
3: Increment $N$.  
4: yield $F(K,N) \oplus c_i$
5: end for
6: end procedure

Note in the above scheme that I use the term $yield$ rather than $return$. This is simply done to induce the timestamp parameter of the bits in any stegotext block $c$ to match the timestamp of the would-be innocent initialization vector.

Proof of Steganographic Secrecy This proof makes use of the well-known result that the advantage of a secure PRP $F(K \in \{0,1\}^n, x \in X = \{0,1\}^b)$ is negligible in the PRF setting with $q$ queries if $\frac{q^2}{|X|}$

This was at least the case until recently; due to timing attacks on CBC, recent proposals have proposed to drop support for CBC entirely in TLS 1.3, and most TLS 1.2 implementations default to GCM.
is negligible.

**Claim** \( S \) is steganographically secret against any probabilistic polynomial-time warden when \( n = b \).

Say that there exists a \((t(n), q(n), l(n))\)-warden \( W \) which achieves non-negligible advantage in the game for steganographic security given in \([2]\) where \( t(n), q(n), \) and \( l(n) \) are polynomials in \( n \). In other words, that the quantity

\[
\text{Adv}_{S,C}(W) = | Pr_{K,r}[W_r^{M,SE(K,m,h)} = 1] - Pr_{r}[W_r^{M,O(m,h)} = 1] |
\]

is non-negligible for this warden. In the steganographic secrecy setting, consider a party \( A \) which simulates either \( SE(K,m,h) \) or \( O(m,h) \) by playing the PRF security game as follows:

1. The broker flips a single coin \( r \). If \( r = 1 \), the broker chooses a key \( K \in \{0,1\}^n \) for \( F \). Else, the broker chooses a truly random function \( F' \).
2. \( A \) initializes a counter \( N \) to 1.
3. \( A \) then waits for a queries from \( W \). On query \( m \), \( A \) queries the broker with \( N \) and obtains \( v = F(K,N) \) or \( v = F'(N) \). \( A \) then responds to \( W \) with \( v \oplus m \) and then increments \( N \).
4. Once \( W \) outputs a guess, \( A \) forwards this as its guess to the broker.

Note that, in the third step, if \( r = 1 \), the response to \( A \) is precisely the value of \( SE(K,m,h) \); if \( r = 0 \), the response to \( A \) is random, since \( v = F'(N) \) is random, and therefore a simulation of \( O(m,h) \) given our characterization of \( C_h \). Since \( A \) makes as many queries as \( W \) and as much time as \( W \) to output a guess, we know that \( A \) is polynomial both in time taken and queries made.

Consider the quantity \(| Pr_{r}[A \Rightarrow 1 \mid r = 1] - Pr_{r}[A \Rightarrow 1 \mid r = 0] |\); this is the advantage of \( A \) in the PRF security game against \( F \). The value \( Pr_{r}[A \Rightarrow 1 \mid r = 1] \) is precisely the probability that the warden outputs 1 given that \( SE(K,m,h) \) is being simulated; the value \( Pr_{r}[A \Rightarrow 1 \mid r = 0] \) is precisely the value that \( O(m,h) \) is being simulated. Therefore, the advantage of \( A \) in the PRF security game is exactly the advantage of \( W \) in the setting for steganographic secrecy; thus, if \( W \) has non-negligible advantage against \( S = (SE,SD) \), then \( A \) has non-negligible advantage against \( F \) in the PRF security game. This directly contradicts our choice of \( F \); by the previously stated result, we know that the advantage of any probabilistic polynomial-time adversary is \(< \frac{2(n)^2}{2^n} \) when \( b = n \), therefore negligible.

Taking \( n = b = 128 \), the IV size for AES, we conclude that \( S \) is steganographically secret against all probabilistic polynomial-time wardens in our application (and that therefore the probability of detection is less than \( \frac{\text{poly}(128)}{2^{128}} \)).

### 4.3.1.1 Steg-MQ, a Steganographic Message Queue

Having designed and proved a stegosystem suitable for use in TLS 1.2, I next endeavored to integrate it into a real-world application. I completed this task in two steps: (1) implementation of a steganographic message queue and (2) modification of the OpenSSL implementation of TLS (version 1.1.1) to integrate the change.

Towards (1), I implemented a system called *Steg-MQ*. Steg-MQ is a steganographic message queue: in other words, it’s a process which runs on a machine and provides the following functionality to other processes:

1. The ability to *publish* data to be hidden according to a channel distribution and steganographic scheme. Published stegotexts are stored into one of multiple stegotext queues, each designated for use by a specific application which handles the transmission of the stegotext.

\[\text{poly}(128)\]

\[^{14}\text{In practice, then, we would probably want to switch keys after many uses of } SE.\]
2. The ability to *consume* and retrieve channel messages that have been published into an application’s stegotext queue (ideally to then be sent by the application).

3. The ability to *decode* incoming stegotexts and publish them into one of multiple application-specific hiddentext queues.

4. The ability to *retrieve* and consume decoded hiddentexts from an application’s hiddentext queue.

Systems like Steg-MQ are necessary for the practical implementation in practice. While stegosystems may be sound in theoretical formulation, real-time transmission of the output of $SE$ requires available cover (e.g., a packet being sent via TLS). Another practical area of concern is duplication of implementation; by centralizing the implementation of steganographic operations to a single service, we can reduce inevitable error that would occur from multiple applications independently implementing them.

The current rudimentary form of Steg-MQ (implementing only *publish* and *consume* for a single global queue and for only the channel distribution and stegosystem defined in this section) may be found on GitHub (https://github.com/ad-alston/steg-mq).

After implementing the proposed stegosystem within Steg-MQ, and after implementing a simple dynamic library that may be used by any application to interface with Steg-MQ, integration required only a minimal change to the OpenSSL implementation of TLS (which may also be found on GitHub: https://github.com/ad-alston/openssl-steganography). The interaction between Steg-MQ and OpenSSL’s implementation of TLS 1.2 is depicted in the following sequence diagram:

![Figure 1: Sequence Diagram for Steg-MQ and OpenSSL](image)

4.3.2 Steganography among Elliptic Curves

After designing and implementing a steganographic system that operates over channels of initialization vectors, I began to explore the use of other cryptographic primitives as steganographic channels. Most
recently, I have been exploring the possible use of ephemeral-key elliptic curve Diffie-Hellman exchanges and the elliptic curve digital signature algorithm to this end. This section presents a high-level overview of the approaches I have been considering.

### 4.3.2.1 Ephemeral-key Elliptic Curve Diffie-Hellman and Elliptic Curve Digital Signature Algorithm

Say that Alice and Bob are preparing to perform an ephemeral-key elliptic curve Diffie-Hellman exchange for a curve whose domain parameters include a generator \( G \) of order \( n \). I present a stateful stegosystem \( S = (SE, SD) \) which operates on a channel distribution \( C \) over ephemeral keys generated within this domain. Let \( F(K, x) \) be a PRP \( \{0, 1\}^k \times \{0, 1\}^n \Rightarrow \{0, 1\}^n \) and \( H_K(K, x) \) be a PRP \( \{0, 1\}^k \times \{0, 1\}^r \Rightarrow \{0, 1\}^r \). Let \( N \) be a synchronized counter set to 0.

#### Algorithm 5 Steganographic encoding procedure for ECDH

1. procedure \( SE(K \in \{0, 1\}^n, m, h) \)
2. Split \( m \) into \( r \)-bit blocks \( m_1, ..., m_k \).
3. for \( i \) from 1 to \( k \) do
4. Set \( d = d(F(K, m_i)) \), where \( d(j) = F(K, N + j) \).
5. yield \( (d, dG) \)
6. Increment \( N \).
7. end for
8. end procedure

#### Algorithm 6 Steganographic decoding procedure for ECDH

1. procedure \( SD(K \in \{0, 1\}^n, c, h) \)
2. Enumerate \( d^{(j)} = F(K, N + j) \) until \( d^{(j)}G = c \).
3. If no such \( j \) found, quit.
4. return \( H^{-1}(K, j) \).
5. Increment \( N \).
6. end procedure

Note that the above procedure allows the transfer of \( r \) bits of transformation per elliptic curve communicated. In the context of ECDH, the scheme would be applied as follows:

1. Alice, wanting to communicate an \( r \)-bit message, computes her ephemeral key as \( SE(K, m) = (d_A, Q_A) \).
2. Bob generates his ephemeral key as usual (or uses \( SE \) himself with another key and counter).
3. Alice sends Bob her ephemeral public key \( Q_A \).
4. Bob runs \( SD(K, Q_A) \) and obtains the message.
5. Alice and Bob complete ECDH as usual.

No direct proof of secrecy is given (for the sake of the length of this document); however, the intuition behind such a proof would be that, as \( d_A \) should be pseudo-random, the distribution over the public component of the public key should be indistinguishable from the innocent case.

Since an explicit enumeration of \( 2^r \) values is required, in practice, such a scheme may only ever be implemented with \( r \) of 10-15 (with precomputation and other clever implementation tricks). This may still prove useful, however: consider a generic web service which runs on HTTPS and experiences 10-20 requests per second. Such a web service may be able to covertly communicate megabytes of information per day by means of ECDH alone. Such a rate of secrecy would additionally be more than enough for two parties to establish a covert key for future communication in, say, 3-4 connections.
I also note that the above approach is also relevant to the elliptic curve digital signature algorithm (ECDSA) by a similar application.

**A note on ECDSA.** If it can be shown that the distribution of the x-coordinates of points \( P = dG \) where \( d \) is chosen uniformly at random is itself uniformly random\(^{15}\), one may obtain a higher rate of secrecy by simply obtaining random curve points as follows:

1. Apply a PRP to hide message \( m \) (whose length in bits is equal to that of the x-coordinate of points on this curve).

2. Obtain the y-coordinate by solving the proper quadratic equation. (If there is no point for the chosen \( x \), modify the scheme by using the last \( k \) bits of the message as padding and increment through all \( 2^k \) padding values until the point is valid.)

## 5 Conclusion and Next Steps

Over the course of this project, I have explored publications relating to the semantic definition of provably secure steganography, and I have surveyed available work on implementing efficient provably secure steganography in practice. As a result of this investigation, I myself devised and presented a natural modification to the formal framework of provably secure steganography, and I have designed and implemented provably secure steganographic constructions in a highly relevant real-world context to address the lack of such systems in practice.

The continuation of this work would rest in considering all of the points considered in greater depth, in so doing addressing one or all of the following ideas:

**The formal framework of provably secure steganography:** Is there a natural and useful domain where all of the frameworks given in this paper are not sound or are exceedingly cumbersome? Is there some domain where one is and one is not? Is there some way to formalize a measure of optimality of rate based upon a stochastic model of communication?

**Extensions of steganography:** Where might a construction of \( n\text{-out-of-}k\)-channel steganography be useful? An ambitious goal may be to give a construction of \( n\text{-}k\) steganography in this domain.

**Steganography in practice:** Further substantiate the implementation of Steg-MQ. Implement the scheme for elliptic curve steganography within Steg-MQ and augment OpenSSL to use it. Use Steg-MQ to integrate one of these schemes (potentially the scheme for ECDSA) in another application (potentially a blockchain technology).

**Cryptography as a channel:** Investigate other opportunities to take advantage of randomness in cryptographic primitives for the sake of steganography; explore the truth behind relevant questions (such as the one regarding the distribution of x-coordinates: is this true in some curves?)

**More ambitious channels for steganography:** Find more ambitious channels that don’t rely so much on uniform randomness. Attempt to develop and implement a provably secure stegosystem which uses something like web technologies, mark-up languages, program execution patterns, or even natural language as a steganographic channel.

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\(^{15}\)I admit that this may simply not be the case for some or any curves.
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