Damage identification in beam structure based on thresholded variance of normalized wavelet scalogram

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Abstract. In this paper, a damage identification algorithm based on continuous wavelet transform of one-dimensional structures exploiting mode shapes is presented. Numerical models of aluminium and carbon composite beams, containing a mill-cut and impact damage, respectively are considered for this study. Wavelet scalogram is used to obtain the transform coefficients at different wavelet scales and is subsequently normalized in order to emphasize locations with largest coefficients. Variance of normalized wavelet scalogram is computed along the axis of the beam yielding sharp peaks in the zones corresponding to damage in beams. This operation excludes wavelet scale factors as variables for damage localization problems. The universal threshold is applied to filter out lower amplitude peaks that do not indicate damage. These results are summed up for all nodes of beams and all wavelet functions that are analysed in this paper in order to also exclude the number of different wavelet functions as another variable for damage localization. The universal threshold is applied the second time to yield the final result on the locations of damage. Results suggest that the proposed damage localization method is a fast and reliable tool for damage detection in one-dimensional metal and composite beam structures.

1. Introduction

Wavelet Transform (WT) is a mathematical technique that is widely used in engineering for signal processing purposes. The main applications of WT include signal de-noising, image compression and different feature extraction from signals. The later of these applications can be divided in discontinuity detection and energetic feature identification.

WT is also a promising tool for structural damage identification which mainly employs two categories of WT – Discrete Wavelet Transform (DWT) where the scale parameter has values of powers of 2 and Continuous Wavelet Transform (CWT) where scale parameter has continuous values. While DWT is less redundant of the two, it has three main disadvantages. First of all, it is sensitive to shifting of input signals which generates unpredictable DWT results. Secondly, it has poor directional sensitivity and, thirdly, it lacks the phase information for analysing real signals [1]. Also, some important signal features might be missed due to discrete scaling of signal. Thus CWT is a suggested method for damage identification from signals with transient nature.

Identification of faults is one of top priorities in the field of machinery – one-dimensional wavelet transform with different modifications supplemented with auxiliary algorithms has been used to detect flaws in bearings [2-6], gears [7] and gearboxes [8]. Numerous studies have exploited one-dimensional wavelet transform for damage identification in beam structures [9-13] and real industrial objects, such
as wind turbines [14, 15]. Detection of the most energetic features in the signal at different scale and time or space is of particular interest in damage identification. Such three-dimensional plots of coefficients of CWT are known as wavelet scalograms [16, 17]. More beneficial, however, is a normalized wavelet scalogram (NWS). Regions of maxima in NWS are called wavelet transform ridges – these are used for determination of instantaneous frequency of a signal [17]. These ridges also correspond to time or, in case of spatial WT, - a coordinate with the most energetic features of the signal. It is well known that zones of damage attain large values of WT coefficients, therefore ridges in a NWS denote the location of damage for a spatial CWT. In [18] it is stated that wavelet ridges might be used for monitoring of damage development in the structure.

In this paper, a damage identification methodology, based on variance of normalized wavelet scalogram of Continuous Wavelet Transform is applied on four specimens – two aluminium beams (containing one and two sites of mill-cut damage) and two carbon fibre reinforced polymer composite beams containing a single impact damage. Numerically simulated vibration mode shape signals are used as an input for the algorithm in order to develop the damage localization model. Damage is assessed through calculation of variance of normalized wavelet scalogram. Overall, 78 wavelet functions available in MATLAB Wavelet Toolbox are tested. Results suggest a reliable damage localization, while also avoiding the problem of making a right choice of two variables that are of high importance in damage detection, namely, appropriate wavelet function and corresponding scale parameter.

2. Materials and methods

2.1 Damage identification algorithm

Wavelet Transform (WT) is a mathematical transform of a signal to a reciprocal domain, for example, from time to frequency to give more options for signal analysis. Unlike Fourier Transform, WT offers an advantage of adaptive time and frequency windows through operation of dilation. Although it is a common practice to perform WT on signals in time domain, it can also be successfully applied to spatial signals $f(x)$ [19]. $\psi(x)$ is called a mother wavelet function, whereas a set of function $\psi_{a, s}(x)$ (child wavelets) is created by translating (parameter $a$) and dilating (scale parameter $s$) the $\psi(x)$ as:

$$\psi_{a, s}(x) = \frac{1}{\sqrt{|s|}} \psi \left(\frac{x-a}{s}\right)$$

If $0 < s < 1$, the function is expanded, if $s > 1$, it is compressed. CWT is given by

$$W_{a, s}(x) = \int_{-\infty}^{\infty} f(x) \frac{1}{\sqrt{|s|}} \psi^* \left(\frac{x-a}{s}\right) dx = \int_{-\infty}^{\infty} f(x) \psi_{a, s}^*(x) dx$$

where asterisk denotes complex conjugation. Using equation (2) CWT coefficients are calculated. Any visual inspection of structural mode shapes does not reveal the location of damage. CWT coefficients, on the other hand, are extremely sensitive to any discontinuities and singularities of signal $f(x)$, caused by local loss of stiffness, for example. Hence, the damage can be detected in mode shapes that have large amplitude wavelet coefficients.

The methodology of damage localization in beam structures is explained in the following steps.

- If signal is one-dimensional mode shapes of beams, the damage index for each mode is

$$W(i, n, s) = \int_{L} w(i, n) \psi_{a, s}^*(x) dx$$

where $L$ is the length of the beam, $w$ is transverse displacement of the beam, $n$ is a mode number and $i$ is number of integration points in longitudinal direction of the beam ($x$ direction).

- The WT coefficients can be analysed for each mode individually. Instead, it is proposed to summarize the results from all modes and express as a summarized damage index. It is defined as the average summation of damage indices for all modes $N$, normalized with respect to the largest value of each mode

$$W(i, s) = \frac{1}{\sum_{n=1}^{N} W(i, n, s)}$$

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In most cases, the presence of damage modifies the vibration signal in a way that the portion of signal affected by damage has relatively larger energy than other parts of this signal. Hence, detection of these energetic portions of a signal can reveal the location of damage. One of the most effective techniques is the construction of wavelet scalograms which are 3-D plots consisting of coefficients of CWT with respect to wavelet scale and dimension of the structure. Wavelet scalogram is essentially a squared magnitude of WT coefficients.

\[
WS(i, s) = |W(i, s)|^2
\]  

(5)

More beneficial in localization of a fault, however is a normalized wavelet scalogram (NWS), where WT coefficients are first normalized

\[
NWS(i, s) = \left| \frac{W(i, s)}{s^2} \right|^2
\]  

(6)

In present study, damage metric is defined as a variance of a normalized wavelet scalogram with respect to scale parameter and expressed as follows:

\[
S(i) = \sigma^2 \left( \left| \frac{W(i, s)}{s^2} \right|^2 , s \right)
\]  

(7)

Equation (7) is used to calculate variance distributions for all 78 wavelet functions, found in MATLAB Wavelet Toolbox. These wavelets are grouped in families, like daubechies, gaus, coiflet, symlet, complex morlet, shannon, biorthogonal, reverse biorthogonal, etc. Also, individual wavelets like haar, mexican hat, morlet, meyer and discrete meyer are considered. The reason for testing such a large number of wavelets lies in the fact that the performance of each individual wavelet function is not known prior to actually testing it. The shape of analysed signal is different in every situation; thus it is recommended to test all available wavelets.

Universal threshold is applied to variance distribution for every wavelet function to filter out the insignificant values of \( S(i) \). Universal threshold is defined as

\[
T = \sigma \sqrt{2 \ln(I)}
\]  

(8)

where \( I \) in our case is the total number of data points and \( \sigma \) is the standard deviation of \( S(i) \). Originally, this threshold was adapted in image noise reduction routine by using wavelets [20-22].

The \( S(i) \) values that do not pass the threshold value \( T \), are assigned a value of zero, otherwise these value are assigned a value of 1, giving thresholded variance of normalized wavelet scalogram

\[
if(S(i) \geq T) \rightarrow TS(i) = 1 \\
else \rightarrow TS(i) = 0
\]  

(9)

Values of TS\((i)\) are summed up over all 78 wavelet functions for each \( i \), yielding summarized thresholded variance of normalized wavelet scalogram

\[
\Lambda(i) = \sum_{j=1}^{78} TS(i, j)
\]  

(10)

All \( i \) values of \( \Lambda(i) \) are summed together to yield total thresholded variance of normalized wavelet scalogram

\[
\Theta = \sum_{i=1}^{I} \sum_{j=1}^{78} TS(i, j) = \sum_{i=1}^{I} \Lambda(i)
\]  

(11)

where \( I \) is the number of nodes of beams (refer to subsection 2.2 Numerical simulations).

\( \Lambda(i) \) is expressed in percents of \( \Theta \) for each \( i \), giving fractional thresholded variance of normalized wavelet scalogram

\[
Z(i) = 100 \cdot \Lambda(i) / \Theta
\]  

(12)
The universal threshold \( T_2 \) to distinguish from threshold applied to \( S(i) \) is applied to \( Z(i) \) so just the most significant peaks remain and the logical decision as the one stated in equation (9) is applied once more

\[
if (Z(i) \geq T_2) \rightarrow T(Z(i)) = 1 \\
else \rightarrow T(Z(i)) = 0
\]

The final decision on the location of damage is based on those coordinates \( x \) (related to \( i \)) that correspond to \( Z(i) \) values that have passed the universal threshold \( T_2 \).

Damage localization is considered for four cases of beams, all at clamped-clamped boundary conditions:

a) aluminium beam with length of 1250 mm and containing a mill-cut damage.

b) aluminium beam with length of 1250 mm and containing 2 sites of mill-cut damage.

c) composite beam with length of 350 mm and containing an impact damage.

d) composite beam with length of 550 mm and containing an impact damage.

2.2 Numerical simulations

Commercial finite element program ANSYS is used to build numerical models of cases a)-d).

Geometrical configuration of beams for all cases is shown in figure 1.

FE model of aluminium beams (cases a) and b)) consists of 2D beam elements. Each node has 3 degrees of freedom, namely translations along the X and Y axes and rotation along the Z axis. Beams are constructed by means of 148 equal length elements (\( i = 149 \) nodes). The elastic material properties are taken as follows: \( E = 69 \text{ GPa}, \nu = 0.31, \rho = 2708 \text{ kg/m}^3 \).

Laminated carbon/epoxy composite beams are considered for cases c) and d). The laminate lay-up is \([0/90/+45/-45]_s\) with a ply thickness of 0.3 mm. FE element model also consists of 2D beam elements. The beam in case c) is assembled of 35 elements (\( i = 36 \) nodes), while the beam in case d) – of 55 elements (\( i = 56 \) nodes). The elastic material properties determined from an experiment are as follows: \( E_x = 54.5 \text{ GPa}, \ E_y = 31.04 \text{ GPa}, \ G_{xy} = 7.09 \text{ GPa}, \ G_{yz} = 6.5 \text{ GPa}, \ \nu_{xy} = 0.3, \ \rho = 1364.9 \text{ kg/m}^3 \).

The damage in aluminium beams is modelled by reducing the flexural stiffness \( EI \) of the selected elements, which is achieved by decreasing the thickness of elements in the damaged region of the beam. As for composite beams, reduction of stiffness is achieved by decreasing the elastic modulus of elements in the damaged region of the beam. Afterwards, a numerical modal analysis is conducted. In total, 14 modal frequencies and corresponding mode shapes are extracted for cases a) and b) and 15 modal frequencies and mode shapes for cases c) and d). Although usually only the first few mode shapes are considered because of the lower amplitudes and greater noise susceptibility, this study considers contribution of all mode shapes.
3. Results and discussion

Wavelet scalograms are computed for scale parameters 1 till 32. This choice of scales is justified by the findings that signal discontinuities generally correspond to finer scale parameters [23]. The resulting normalized wavelet scalograms are shown in figure 2. Significant amplitudes of WT coefficients are associated to lower scales which is why a logarithmic scale is chosen for the representation of scalogram plots. The portion of nodes between two parallel magenta lines is the zone of damage. As one can see, the WT coefficients with the highest energy mostly fall in the zone of damage for all of the cases corresponding to finer scales. In cases with composite beams, there is a more pronounced effect of smearing of the total WT energy along the longitudinal coordinate of beams. However, the highest fraction of this energy is still in the zone of damage.

Distributions of variance of normalized wavelet scalograms $S(i)$ are shown in figure 3. These distributions are shown only for wavelets with the best performance, namely, gaus of order 6 for cases a) and b), gaus of order 3 for case c) and reverse biorthogonal of order 2.4 for case d). Also, the respective
universal threshold values calculated from equation (8) are depicted. The peaks of the largest magnitude are in the zone of damage and have passed the universal threshold for all of the cases. However, smaller peaks not exceeding the threshold are present for cases c) and d). These peaks are filtered out by calculation of thresholded variance of normalized wavelet scalogram (refer to equation (9)).

The results for fractional thresholded variance of normalized wavelet scalogram $Z(i)$ for all of the coordinates are shown in bar charts in figure 4 where values of application of second threshold are also highlighted. Bars crossing the 2nd threshold $T_2$ are threshold crossings and the corresponding coordinates $x_i^*$ are interpreted as the locations of damage. For all cases, except for case c), a total of 3 threshold crossings are found.

Figure 3. $S(i)$ distribution along the beams for cases a), b) c) and d).
Figure 4. $Z(i)$ for cases a), b) c) and d).

The results of damage localization are depicted in table 1, where values of application of second threshold are also highlighted. Values $T_2C_i$ are $i^{th}$ threshold crossings.

| Case | $\Theta$ | $T_2$ | $T_2C_1$ | $x_1^*$ (mm) | $T_2C_2$ | $x_2^*$ (mm) | $T_2C_3$ | $x_3^*$ (mm) | True location |
|------|--------|-------|----------|-------------|--------|-------------|--------|-------------|---------------|
| a)   | 180    | 16.07 | 16.11    | 751.69      | 18.89  | 785.47      | 17.78  | 793.92      | [750; 800]    |
| b)   | 259    | 12.85 | 13.51    | 456.08      | 13.13  | 481.42      | 14.67  | 768.58      | [450; 500]U   [750; 800] |
| c)   | 163    | 12.74 | 17.79    | 175.00      | 15.95  | 195.59      |        |             | [150; 200]    |
| d)   | 162    | 14.95 | 16.67    | 327.89      | 17.90  | 338.46      | 17.90  | 359.62      | [320; 370]    |

By comparing the results of damage localization with the true positions of damage, it is clear that the algorithm has successfully identified the locations of damage for all of the cases. On the other hand, the damage is modelled as a decrease of flexural stiffness for finite elements continuously in the coordinate range specified in table 1, whereas the algorithm has managed to point out only several discrete coordinates. Nevertheless, these coordinates still fall in the range of true damage, thus the localization of damage is successful. The robustness of the proposed algorithm with respect to noise is not studied in this work, although it is known that the usual mode shape transformations like mode shape curvature
are susceptible to noise. Studies of Cao et al. [9, 24] show that the effect of noise can be suppressed by fusing complex wavelets with modal curvature technique. This approach was adopted in finding single as well as multiple damages in beams and plates.

4. Conclusions
The present study focuses on damage localisation in numerically simulated aluminium and composite beam structures using spatial Continuous Wavelet Transform of mode shapes. The algorithm involves the calculation of normalized wavelet scalogram for each of wavelet functions and a subsequent calculation of variance of this scalogram with respect to scale parameter. Then, an application of universal threshold, widely used in signal de-noising is utilized to suppress non-significant peaks of calculated variance. These results are summed up for all of the wavelet functions over all nodes of the beams. After the application of universal threshold for the second time the location of damage is pointed out. It is concluded that those values of damage metrics, introduced in this study, that exceed the values of universal threshold correspond to the nodes or coordinates of the beams that lie in true range of damage.

The proposed algorithm has several advantages as compared to conventional damage localization techniques, namely, there is no need of baseline data of healthy structure and a complex problem of selecting a specific wavelet function is avoided as the algorithm employs the contribution of all wavelet functions available in MATLAB Wavelet Toolbox and, finally, no need to select the most appropriate scale parameter of a wavelet function. However, more studies have to be conducted, namely, experimental validation on beam and plate structures and numerical experiments on noise susceptibility.

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