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THE RIBBED-ANNULAR DOME'S UPPER TIER MODEL STABILITY EXPERIMENTAL STUDIES

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The work’s aim is to check the stability loss hypothesis with the snap-through effect of the ribbed-annular dome's upper tier on a full-scale model by experimental tests, and confirming the nonlinear tier's work under external load. An equivalent von-Mises truss with elastic supports mockup was tested, which was 1/4 of the dome's upper tier with eight ribs connected by upper and lower support rings. Horizontal elastic supports were performed as twin steel puffs and simulated the dome's upper tier lower support ring's deformations. Primary and secondary processing of the obtained data were made, as well as the full-scale experimental data results analysis. The equivalent von-Mises truss with elastic horizontal supports deformation dependencies from external concentrated load were determined. The experimental model's behavior study results were compared with the theoretical studies' results. The analysis of the experimental and theoretical studies results confirmed the experimental data reliability and confirmed the using analytical expressions feasibility for the von-Mises truss with elastic horizontal supports stability preliminary assessment.

Keywords: ribbed-annular dome, von-Mises truss, stability loss, elastic horizontal supports, support ring, puff, upper tier.

1. Introduction

Topicality. In recent years in Ukraine is quite popular the construction of low-pitched domes coverings with ribbed-annular type with a dome's height to span ratio - 1/5..1/4 and spans up to 30m, with the main load-carrying ribs which made of the single closed bent-welded steel profiles square or rectangular cross-section with wall thicknesses from 3 to 10 mm. With this domes' configuration [1, 2] the one of the dome's reliable operation important factors, along with ensuring the dome rods' buckling resistance is to ensure the dome's upper tier stability. The dome's upper tier stability study is devoted to [3, 4, 5], where von-Mises truss [6] was proposed as a calculation model, which simulates the equivalent arch work, according to the ribbed domes calculating method [7]. In work [5] the stability loss problem of the dome's upper tier by six possible computational cases was considered. In works [3] and [4] the von-Mises trusses' stability theoretical methods studies with rigid nodes and taking into account the initial imperfections when applied an inclined external concentrated load to the truss' ridge node were considered, and proposed the criteria for determining critical loads' values in the ridge node. The von-Mises trusses' stability was also considered in [8, 9, 10, 11, 12]. In the work [14] the stability loss problem according to the asymmetric scheme

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for high three-hinged two-rod systems was considered. The von-Mises trusses stability theoretical studies in the elastic supports' presence in the base [11] were provided, which simulate the tiers joints flexibility of the ribbed dome. In addition, to confirm the theoretical research reliability, scientists have performed the classical three-hinged two-rod system's stability full-scale experimental studies [13].

2. Physical modeling

As a dome's upper tier model, it was decided to use the von-Mises truss as a popular model for two-rod inclined systems' theoretical stability studies, which allows modeling these systems nonlinear behavior. To confirm the von-Mises trusses' stability loss predicting theory [4, 11], full-scale experimental tests were conducted, which aimed to analyze the truss' performance under load for both groups of limit states. As an experimental model was designed and tested the steel ribbed-annular dome's upper tier model. The upper tier of the ribbed-ring dome with a diameter of 18.0 m with a rise of 5.195 m was taken as a basis. The dome ribs' number - 8, the dome tiers' number in height - 6. The upper tier rods' angle to the horizon - 5 degrees (85 degrees from vertical axis).

The upper tier is a spatial rod system consisting of two support rings connected by ribs. For the testing convenience, it was decided to use a two-rod equivalent truss, which is 1/4 of the upper tier. The geometric scale of the model is 1:1. The paired puffs provided truss' stability in the horizontal direction. The support frame provided the truss' stability outside the plane. Puffs in the model act as the lower support ring of the dome's upper tier. Nodal joints on the supports were made as a sliding hinge in which the support angle rests on the support frame through a fluoroplastic gasket. The connection in the ridge node is made using steel gussets connected to the model's rods by welding and connected with a class 8.5 bolt with a diameter of 24 mm, to prevent friction between the gussets surfaces between them is a fluoroplastic gasket. The model's rods are designed from a closed bent-welded profile with a cross section of 80x80x3 mm. Puffs are made of square steel with a cross section of 16x16 mm. An adopted steel for constructions - S235.

The upper tier's model was arranged on a special supporting steel frame, which was pre-designed and manufactured for providing experiments. The special supporting frame, in turn, rests on the power floor, which was arranged in the research laboratory for metal structures long-term testing, which belongs to the Department of Metal and Wooden Structures of the Kyiv National University of Construction and Architecture.

The test bench photo and the model elements layout scheme see Fig. 1.

The initial geometric and physical parameters of the designed equivalent von-Mises truss experimental model described in table 1.

The support frame on which the truss was tested, is made of: a support beam - 2 paired I-beams №27, interconnected by steel plates 220x100x8mm with a step of 600 mm; end supports - channel bars 20U, with upwards-oriented web and welded to the support beam; the loading frame's support which is made of channel bar 20U to which on both sides are welded racks of channel 10U between which is arranged a screw mechanical jack.
The initial experimental model parameters

| Parameter’s name, dimension | Value           |
|-----------------------------|-----------------|
| The distance between the supports - $2a_0$, m | 3,85            |
| Truss’ rise - $f$, m        | 0,168           |
| Steel construction class    | S235            |
| Steel elasticity modulus - $E_0$, N/mm$^2$ | 2,060E+05       |
| Estimated rods' cross-sectional area - $A_{cal}$, m$^2$ | 9,240E-04       |
| Estimated elasticity - $E_0A_{cal}$, N    | 1,903E+08       |
| The rods' angle from the vertical axis - $a_0$, degrees. | 85              |
| The angle inclination sine - $\sin a_0$ | 9,962E-01       |
| The angle inclination tangent - $\tan a_0$ | 1,143E+01       |
The model was loaded using a hydraulic jack with a capacity of 50 tons. Clock-type indicators carried out the model supports’ horizontal movements’ registration: I1 — on the right support, I2 — on the left support. The vertical ridge node displacements were recorded using a deflectometer I3 and additionally with a caliper.

To determine the internal forces and stresses in the model's rods, strain gauges were glued to each rod, which were connected by a wire system to the strain gauge station SIIT3.

The models tests was performed sometime after fixing the measuring instruments to ensure the stability of their readings.

Prior to each test, the experimental model was loaded with a load of 10% out the maximum load value for the model element inclusion (joints and puffs) to the work.

Each experimental test was divided into a number of consecutive loadings and each loading in its turn was divided into several stages:

- the load application to the model;
- the model left under the load for a while (15 minutes);
- taking readings of indicators I1-I3;
- taking readings of a strain gauge station (5 attempts on each load step);
- the next load step with repeating the above steps.

Gradual model loading took place in steps of 200 kilogram-forces in the first loading stages until the load reaches a value of 1000 kilogram-forces. After loading the model with a load of 1000 kilogram-forces, the step was reduced to 100 kilogram-forces. When the load reached the value at which the model lost their stability, the load has been increased until the ridge node vertical displacement reached a value of 10 mm.

After each loading stage, the model was kept under load for 15 minutes, after which the measuring instruments' readings were taken, a visual inspection was performed and changes in the model elements were recorded.

3. Tests results analysis

The model stability loss under step load was characterized by a decrease in the load on the model with increasing force in the hydraulic jack and with increasing deflection of the model. An equivalent truss' experimental model under step load lost their stability when the load reached the value of 1302.8 kilogram-forces.

For the research convenience, the load on the von-Mises truss was further considered as the reduced external load per rod’s design stiffness unit according to the formula:

$$ P_{rel} = \frac{P}{E_0 \cdot A_{cal}}, $$

where: $P$ — external vertical concentrated load, which is applied to the von-Mises truss’ ridge; $E_0$ — elasticity modulus; $A_{cal}$ — the truss rods estimated cross-sectional area.

Similarly, the vertical model displacements were further considered as the reduced vertical displacements values to the initial truss' rise by the formula:
\[ v_{\text{rel}} = \frac{v_p}{f}, \]  

where: \( v_p \) – the truss’ ridge node vertical displacement under the external force \( P \); \( f \) – the von-Mises truss' rise before loading.

The truss' supports horizontal displacements, respectively, were subsequently considered as the reduced displacement values to the truss' half-span according to the formula:

\[ u_{\text{rel}} = \frac{u_0}{a_0}, \]  

where: \( u_0 \) – the von-Mises truss’ elastic support horizontal displacement; \( a_0 \) – an initial von-Mises truss' half-span value.

The experimental tests primary and secondary analysis results data were processed according to known processing methods of scientific researches results [15, 16]. The displacements determining results for the model's end supports and the ridge node's displacements are shown on the displacements graphs see Fig. 2.

In the graphs of Fig. 2: zone 1 - the truss' elastic work zone under the load, zone 2 (highlighted in gray color) - nodal displacements zone, which corresponds to the plastic deformations zone of the equivalent von Mises truss after stability loss. As can be seen from the graph of Fig. 2(a), the model's deformation occurred asymmetrically, and after the stability loss on the left support there was a truss rod's pinching on the support's limiting device, as evidenced by the left support's decrease in horizontal displacements with increasing the external load value.

Due to the pinching effect at the left support, it was decided to consider only the tests results for the equivalent von-Mises truss' further analysis in the subcritical stage inclusive.

According to the experimental researches results secondary data processing, the internal stresses and internal forces in the model's puffs were determined.
Taking into account that at each load stage, the obtained supports' displacements had different values, it was made a statistical correlation for the deformations' values and for the horizontal forces' values for both supports, and it was obtained linear dependences of these values from the corresponding loads' values with a provision of $R^2 = 99\%$. According to these dependences, the internal forces' values in puffs and their horizontal displacements were determined.

The secondary processing results of the strain gauges readings shown on the graphs of Fig. 3.

\[ k_s = \frac{N_x}{d_x}, \]  \hspace{1cm} (4)
where: $N_x$ – an internal force in a puff determined using the tensometry methods, $d_x$ – horizontal displacement of the support, to which connected the puff, obtained using the clock-type indicators readings.

The stiffness value obtained by expression (4) for each load stage had a value: $k_s = 5.570E+07$ N/m.

To verify the test results reliability, it was decided to use the analytical dependences given in the study [11].

Where the dimensionless parameter of vertical and horizontal displacements was determined by the formula:

$$
\psi_{uv} = \frac{f}{a_0} \cdot \frac{1 - (u_p/f)}{1 + (u_0/a_0)}.
$$

Combining the expressions (2), (3) and (5), was obtained the expression:

$$
\psi_{uv} = \frac{f}{a_0} \cdot \frac{1 - u_{[rel]}}{1 + u_{[rel]}}.
$$

where: $f$ – truss’ rise value before experimental tests start; $a_0$ – truss’ half-span value before experimental tests start; $u_{[rel]}$ – the ridge node relative vertical displacement, determined using expression (2); $u_{[rel]}$ – truss supports’ relative horizontal displacement, determined using expression (3).

Deformation criteria of two-rod system with elastic supports with a symmetrical nonlinear deformation:

$$
P = \frac{E_0 \cdot A_{cal}}{2} \frac{1}{1 + \psi_{uv}^2} \left(1 - \frac{1}{\sqrt{1 + \psi_{uv}^2}} \sin \alpha_0\right) \cdot \sin \alpha_0.
$$

Combining the formulas (1) and (7) was obtained the next expression:

$$
P_{[rel]} = \frac{E_0 \cdot A_{cal}}{2} \frac{1}{1 + \psi_{uv}^2} \left(1 - \frac{1}{\sqrt{1 + \psi_{uv}^2}} \frac{k_{s0} \cdot a_0}{E_0 \cdot A_{cal}} + \sin \alpha_0\right) \cdot \sin \alpha_0.
$$

where: $\psi_{uv}$ – dimensionless parameter of horizontal and vertical displacements, determined using formulae (6); $k_{s0}$ – an initial truss elastic supports’ stiffness; $a_0$ – truss’ half-span value before applying the load; $\alpha_0$ – an initial truss rod’s inclination angle; $E_0$ – the rod’s material elasticity module; $A_{cal}$ – estimated truss rod’s cross-sectional area.

The horizontal elastic support’s reduced stiffness has a value:

$$
k_{s0} \cdot a_0 = \frac{5.570E + 07 \text{ N/m} \cdot 1.925 \text{m}}{1.903E + 08 \text{ N}} = 0.563.
$$
The results of the relative vertical and horizontal displacements values and the truss' deformation dimensionless parameters values and also the relative concentrated external forces values which were obtained theoretically and by full-scale experiment are shown in Table 2.

### Table 2

| Step | $\nu_{[rel]}$ | $u_{[rel]}$ | $\psi_{uv}$ | $P_{[rel]} \exp.$ | $P_{[rel]} \text{ theory}$ |
|------|--------------|-------------|-------------|-------------------|---------------------------|
| 1    | 0,000E+00    | 0,000E+00   | 8,727E-02  | 0,000E+00       | 1,176E-06                |
| 2    | 1,762E-02    | 5,688E-05   | 8,573E-02  | 1,031E-05       | 9,328E-06                |
| 3    | 5,631E-02    | 1,455E-04   | 8,235E-02  | 2,577E-05       | 2,570E-05                |
| 4    | 1,036E-01    | 2,117E-04   | 7,822E-02  | 4,123E-05       | 4,296E-05                |
| 5    | 1,382E-01    | 2,649E-04   | 7,519E-02  | 5,154E-05       | 5,380E-05                |
| 6    | 1,588E-01    | 2,935E-04   | 7,340E-02  | 5,669E-05       | 5,953E-05                |
| 7    | 1,820E-01    | 3,195E-04   | 7,136E-02  | 6,185E-05       | 6,540E-05                |
| 8    | 2,132E-01    | 3,558E-04   | 6,865E-02  | 6,714E-05       | 7,228E-05                |

The deformations graphs of the equivalent and the theoretical von-Mises trusses, which was described in [11] are shown in Fig. 4

![Fig. 4. The dependences' graphs of the relative vertical deformations $\nu_{[rel]}$ for experimental and theoretical models from the relative concentrated force in the ridge node $P_{[rel]}$.](image)

After statistical processing data in Table 2, the difference between the full-scale experiment results for the equivalent von-Mises truss with horizontal elastic supports and the theoretical stability studies results for the von-Mises truss according to expression (8) was - 5.254%.
4. Conclusions

1. The equivalent von-Mises truss' with elastic horizontal supports full-scale experimental tests were performed, which simulates the ribbed-annular dome upper tier's operation with a span of 18 m with rise of 5.195 m with the ribs' number - 8 and with the tiers' number - 6.

2. The full-scale experimental tests' primary and secondary data processing according to known methods of scientific data processing were carried out.

3. An experimental studies results analysis showed that the equivalent von-Mises truss with elastic horizontal supports deformed nonlinearly along the asymmetric scheme and lost stability at an external concentrated load value of 1302.8 kilogram-forces, with vertical truss' deformations of 35.81 mm and horizontal supports displacements - 0.68 mm each. The reduced stiffness of the puffs simulating the elastic horizontal supports was 0.563.

4. The experimental studies results were used to verify the analytical expression (8), which is given in [11]. Experimental and theoretical results analysis has shown a good convergence of these results and allowed to draw conclusions not only about a full-scale experiment validity, but also about using analytical expressions feasibility for determining von-Mises trusses stability criterion with horizontal elastic supports, which loaded by vertical concentrated load in the ridge. This in turn means that due to analytical expressions it is possible to predict the three-hinged trusses with elastic horizontal supports stability loss, which in turn allows predicting the ribbed-annular dome upper tier's overall stability loss.

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ЕКСПЕРИМЕНТАЛЬНІ ДОСЛІДЖЕННЯ СТІЙКОСТІ МОДЕЛІ ВЕРХНЬОГО ЯРУСУ РЕБРИСТО-КІЛЬЦЕВОГО КУПОЛУ.

Анотація. Мета. Ціль проведення даного дослідження - перевірка гіпотези втрати стійкості з ефектом проклачування верхнім ярусом ребристо-кільцевого куполу на натурній моделі експериментальних випробуваннях, та підтвердження нелінійної роботи ярусу під дією зовнішнього навантаження. Методика. За аналог куполу для моделювання виконані зі сталевих замкнутих гнутозварних профілів прямокутного перерізу. У якості опорного кільця верхнього ярусу куполу було прийнято рішення використати ферму Мізеса, як популярну модель для теоретичних досліджень стійкості з ефектом проклачування. Результати. Проведена первинна та вторинна обработка отриманих даних, та проведен аналіз результатів натурного випробування.

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Abstract. Purpose. The work’s aim is to check the stability loss hypothesis with the snap-through effect of the ribbed-annular dome's upper tier on a full-scale model by experimental tests, and confirming the nonlinear tier's work under external load. Methodology. The ribbed-ring dome circumscribe in the plan as a circle with a diameter of 18 m with the dome rise ratio to the span - 1/4, which consisted of 8 ribs and had 6 tiers in height, was taken as an dome-model analog. The upper tier of the dome is bounded by the lower ring, which is the upper ring for the tier below, and the dome's upper support ring. Tier rings and ribs are made of steel closed bent welded profiles with rectangular cross section. As a dome's upper tier model, it was decided to use the von-Mises truss as a popular model for two-rod inclined systems' theoretical stability studies, which allows modeling these systems nonlinear behavior. The classic von-Mises truss using in modeling the dome's upper tier behavior is associated with a number of problems. First, the tier is a three-dimensional system with eight rods, so it was decided to model the tier with an equivalent low-pitched truss, which is 1/4 of the upper tier. Secondly, the lower tier ring has limited rigidity and can be deformed, while the classic truss has fixed supports, which is why it was decided to add elastic horizontal supports to the classic von-Mises truss model. Horizontal elastic supports were performed as steel pair puffs and were simulated the dome's upper tier lower support ring deformations. Findings. The obtained data primary and secondary processing, and the full-scale experiment results analysis were carried out. The ribbed-annular dome's upper tier nonlinear deformations nature under the external concentrated vertical load action in the ridge node was confirmed. It was established that in the ridge joint the upper tier's stability loss nature has the snap-through effect. Scientific innovation. The deformation dependencies for the equivalent von-Mises truss with elastic supports with the help of full-scale experiment were obtained. A comparison of the equivalent truss' behavior experimental studies results with the existing theoretical studies' results was made. The experimental and theoretical studies results analysis confirmed the experimental data results reliability and analytical expressions feasibility use for the preliminary assessment of von-Mises truss' with elastic supports stability. Practical value. The obtained results of experimental research allow creating tools for designers to increase the dome structures reliability.

Keywords: ribbed-annular dome, Mises' truss, stability loss, elastic horizontal supports, support ring, puff, upper tier.
The ribbed-annular dome's upper tier model stability experimental studies were carried out. Fig. 4. Ref. 16.