Periodic relativity: the theory of gravity in flat space time

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Abstract

In periodic relativity (PR), the curved space time of general relativity are eliminated by making use of an alternative flat metric without weak field approximation. PR satisfies Einstein’s field equations. Theory allows every two body system to deviate differently from the flat Minkowski metric. PR differs from general relativity (GR) in predictions of the proper time intervals of distant objects. PR proposes a definite connection between the proper time interval of an object and Doppler frequency shift of its constituent particles as the object makes a relative motion with respect to the rest frame of the coordinate time. This is because fundamentally time is periodic in nature. Coordinate and proper time in GR are linear time. Periodic time of PR is the key parameter in development of quantum gravity theory in which the universe begins with a quantum fluctuation in the fundamental substance of the universe which is infinite, motionless and indivisible. PR is based on the dynamic weak equivalence principle which equates the gravitational mass with the relativistic mass. PR provides accurate solutions for the rotation curves of galaxies and the energy levels of the Hydrogen spectra including Lamb shift using common formalism. Flat space time with Lorentz invariant acceleration presented here makes it possible to unite PR with quantum mechanics. PR satisfies Einstein’s field equations with respect to the three major GR tests within the solar system and with respect to the derivation of Friedmann equation in cosmology. PR predicts limiting radius of the event horizon of M87 black hole to be \(3R_g\) and the range of prograde and retrograde spin \(a_*\) between \(±0.385\) and \(±0.73\). Mathematical proof of periodic nature of time is presented by way of introducing gravity into the electromagnetic wave formalism. Theory shows that the electromagnetic wave is held together by gravitational forces. Theory explains the mechanism of gravitational redshift predicted by general relativity and predicts powerful gravitational radiation at Planck epoch. Gravitational-wave strain amplitude is derived using quantum mechanical formalism.

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1. INTRODUCTION

Periodic relativity (PR) is a theory of accelerations in the flat space time. This article is a revised, improved and extended version of the previous work [1, 2]. In PR, the deviation to the flat Minkowski metric in the presence of the gravitational field gets introduced in the form,

$$\left( \frac{d\tau}{dt} \right)^2 = (1 - n\beta^2),$$  \hspace{2cm} (1.1)

where the deviation factor $n$ is directly introduced in the Lorentz transformation equation. Here $t$ is the coordinate time, $\tau$ the proper time of the orbiting body, $n$ is a unitless real number and $\beta = v/c$. The corresponding line element in polar coordinates is,

$$ds^2 = c^2dt^2 - ndr^2 - nr^2d\theta^2 - n(r^2\sin^2 \theta)d\phi^2.$$  \hspace{1cm} (1.2)

In this theory the weak field approximation of general relativity (GR) is eliminated. The unitless real number $n$ can be defined as a ratio of the theoretical acceleration to the experimentaly observed acceleration in a special case of an orbital motion. One of the simplest form of actually observed acceleration is $v^2/r$. Complex form of Lorentz invariant acceleration is discussed later. Here any constant value of $n$ gives a flat metric. This gives,

$$d\tau = dt \left( 1 - \frac{nv^2}{2c^2} \right),$$  \hspace{2cm} (1.3)

to the first order accuracy for small values of $v$ and $n$. As discussed later, the line element of Eq. (1.2) satisfies Einstein’s field equation for the empty space $R_{\mu\nu} = 0$ for any constant value of $n$. In PR it is proposed that the proper time $d\tau$ of a massive object such as a planet has a definite connection with the Doppler shift of the massive particles of which the massive object is composed. This causes every two body system to deviate differently from the flat Minkowski metric. Weak field approximation of general relativity (GR) [3–5] does not allow such freedom.

In PR the factor $(\cos \psi + \sin \psi)$ associated with elliptical trajectory and ignored by both Newton and Einstein introduces geodesic like trajectories. Angle $\psi$ is shown in Figs. 1 and 3. The field equations in the presence of matter are derived using the energy conservation law and proper use of the relativistic mass. This can supplant the Riemannian geometry. The weak equivalence principle (WEP) is replaced by the dynamic WEP which states that the
gravitational mass is equal to the relativistic mass. The main effect of having different deviation factors for different two body systems is that the proper time interval predictions of GR and PR are different, specially for distant objects. This is where the two theories could be tested for the soundness of their underlying logic of the meaning of time. Unfortunately, this test capability does not exist at present time.

Similarly, De Broglie pilot wave theory \[6,7\] can also explain Hafele-Keating experiment \[8\]. When Cesium atom in an atomic clock (in aircraft) moves relative to the observer on the surface of the earth, it develops a resultant pilot wave with respect to the observer and in the direction of the velocity of the aircraft. The resultant pilot wave is the sum total of all the individual pilot waves of the constituent particles of the cesium atom. The resultant pilot wave suffers kinematic and gravitational frequency shift as per the known science. In atomic clocks the time is measured by measuring the frequency of the hyperfine splitting of the 6s electron in the outer shell of the Cesium atom. This is measuring of frequency of light spectra in simple language. The world time standard relates 1 second with 9192631770 cycles/sec. or Hertz frequency of the light spectra in atomic clock. The frequency shift of the resultant pilot wave of the Cesium atom also includes contributions from the change in frequency of the hyperfine splitting of the 6s electron in the outer shell responsible for causing time dialation. Thus time is periodic in nature.

A. Relativistic invariant

The relativistic invariant \(s^2\) presented by Minkowski, Lorentz and Einstein relates two points in space-time by the expression,

\[
s^2 = x^2 + y^2 + z^2 - c^2 t^2.
\]

(1.4)

Here \(x^2 + y^2 + z^2\) represents three dimensional space, \(c\) is the velocity of light and \(t\) the ordinary linear time as we generally know. Einstein’s relativity theory is founded upon this simple equation and a hypothesis based on the Eötvös experiment which showed that the gravitational mass of a body is equal to its inertial mass. It is also well known that if we replace \(c\) with velocity \(v < c\) for other massive particles, Eq. (1.4) no longer remain meaningful. One of the fundamental proposal of the present theory is to recognize continuity between the electromagnetic wave spectrum and the massive particle wave spectrum. Such
argument can be supported if we can supplant Eq. (1.4) by an equation which is not only applicable to velocity of light but also to velocity of all the other particles which travel at speeds less than that of light.

B. Periodic invariant

It is possible to propose one such equation which I call the periodic invariant. It can be written as

\[ s^2 = \lambda^2 - V^2 T^2, \]

where \( \lambda = \frac{h}{p} \) is the associated de Broglie wavelength[6,7], \( V = \frac{c^2}{v} \) the phase velocity, \( v \) the particle velocity, and \( T \) the period of the wave. One can see that Eq. (1.5) does satisfy light particles as well as other massive particles. If we replace ordinary particle velocity with that of light, we get \( v = c, V = c \) and

\[ s^2 = \lambda^2 - c^2 T^2, \]

If we multiply Eq. (1.6) for light with a real number \( n^2 \) and set \( (n\lambda)^2 \) equal to the cartesian distance \( x^2 + y^2 + z^2 \) and \( (nT)^2 \) equal to the linear time \( t^2 \), Eq. (1.6) becomes equivalent to Eq. (1.4). Therefore Eq. (1.4) is a special case of Eq. (1.5). Eq. (1.6) implies that space-time is wavy and that time does not flow in one direction but is strictly a periodic or cyclic phenomenon. Eqs. (1.4) and (1.6) both behave identically in the absence of gravitational field, but in the presence of \( g \) field, for astronomical distances, Eq. (1.6) remains null and Eq. (1.4) yields time like geodesics. In PR, both light as well as massive particles always travel along null paths. This makes it difficult to solve mundane problems of macroscopic proportions. This is why it becomes necessary to introduce approximations in the form of linear time and linear euclidean distance in Eq. (1.5) which permits time like and space like geodesics for addressing the problems involving complex structures. However for certain fundamental measurements such as gravitational redshift and deflection of light, Eq. (1.6) should yield more accurate results than that given by Eq. (1.4) because the reality does not get compromised.

We can also say that Eqs. (1.4) and (1.6) both behave identically even in the presence of gravitational field when the space-time interval involves atomic and sub-atomic
distances. Thus the validity of the algebraic structure (Clifford and Lie Algebra) associated with Eq. (1.4) and the related gauge and spinor groups of particle physics is maintained. It is only at astronomical distances that the difference between two equations become perceptible and can affect any local symmetry formalism based on diffeomorphism. Therefore the validity of Dirac equation \[9\] is maintained with respect to Eqs. (1.5) and (1.6). Same is true for the algebra of Lorentz transformation when the space-time interval involves atomic and sub-atomic distances.

We can write Eq. (1.5) as

\[s^2 = \left(\frac{ch}{cp}\right)^2 - \left(\frac{c^2}{v\nu}\right)^2,\]  

where \(h\) is Planck’s constant, \(p\) the particle momentum and \(\nu = 1/T\) is the frequency of the associated de Broglie wave. This period-frequency relation is the only fundamental and basic equation that relates the concept of time to the physical world in an objectively real manner. The relativistic invariant relates the space and time continuum on a macrocosmic scale. The periodic invariant does the same on a microcosmic scale. If we introduce the energy-momentum invariant

\[E^2 = E_0^2 + (cp)^2.\]  

in Eq. (1.7), we get,

\[s^2 = \left(\frac{hc}{(E^2 - E_0^2)}\right) - \left(\frac{c^2}{v\nu}\right)^2,\]  

where \(E\) = total energy of the particle and \(E_0 = m_0c^2\) is the rest energy of particle. Relativistic mass is little used by modern physicists. Notwithstanding the modern usage we will use \(m\) for relativistic mass and \(m_0\) for rest mass throughout the article.

2. QUANTUM INVARIANT

The invariant Eq. (1.9) has a general form applicable to all de Broglie particles. In relativity, the vanishing of the invariant \(s^2\) given by Eq. (1.4) does not mean that the distance between two space-time points gets obliterated. It simply means that the two space-time points can be connected by a light signal in vacuum. The new invariant Eq. (1.9), however, can vanish in two different ways. First, in the characteristic relativistic sense implying that
two points in space-time can be connected by an energy signal (which can be a light signal or a massive particle signal), and secondly in an absolute sense where both terms on the right also vanish individually like the Euclidean invariant. In the first case, we get the relation,

\[ \frac{E^2 - E_0^2}{\nu^2} = \frac{h^2 v^2}{c^2}. \] (2.1)

Substituting the photon parameters \( E_0 = 0 \) and \( v = c \) into Eq. (2.1) gives the quantum hypothesis of Max Planck, \( E = h\nu \). This provides sufficient reason to declare that Eq. (2.1) is a general form of Max Planck’s quantum hypothesis applicable to both massless as well massive particles.

Essentially there is no difference between the relativistic invariant Eq. (1.4) and the invariant Eq. (1.9), other than the fact that the former defines the space-time continuum and the latter defines the energy-vibration continuum. The equivalence of both these continuums will become clear when we define the quantum invariant with the assumptions that, given sufficient energy, all particles having rest masses can disintegrate into particles with zero rest masses; and that all particles having zero rest masses will have a constant velocity in space regardless of the inertial frames of reference and equal to the velocity of light. These two assumptions would allow us to adopt the hypothesis that the creation begins with a vibration of the primal energy. We can introduce the photon parameters \( E_0 = 0 \) and \( v = c \) in Eq. (1.9) to simulate the initial state of the universe. This gives,

\[ s^2 = \left( \frac{hc}{E} \right)^2 - \left( \frac{1}{\nu} \right)^2. \] (2.2)

And since the path of a massless particle is a null geodesic, for \( s^2 = 0 \), Eq. (2.2) can be further simplified to a form which is independent of the law of propagation of light. We shall call this form the Quantum Invariant.

\[ s^2 = \left( \frac{h}{E} \right)^2 - \left( \frac{1}{\nu} \right)^2. \] (2.3)

The quantum invariant can vanish in an absolute sense when \( E > E_p \) and \( \nu > \nu_p \), where \( E_p \) and \( \nu_p \) are Planck energy and Planck frequency respectively. In this case when the particle tries to acquire more energy than the Planck energy, it will violate the Compton limit on the wavelength and thus the particle wave will collapse and will become perfectly motionless leaving no mass gap. Thus the space-time continuum connecting two points gets completely obliterated and the resulting sub-quantic medium resembles a singularity.
characterised by a perfectly motionless indivisible field which is infinite like empty space. This is the fundamental substance of the universe from which a specific finite excitation had arisen as a wave particle duality. Such a singularity suggests a motionless field devoid of ripples capable of giving birth to energy which is always in motion.

The singular motionless field can not be described as energy because there are no oscillations in it. Since the singular motionless field is not the energy, it does not gravitate. The aether of the earlier theories is ”energy in waves” which can interact with the motion of the planet but the motionless singular field is undetectable because it does not interact with any form of energy. Therefore in PR the accelerated expansion of the universe takes place within the infinitude of the singular motionless field. With the vibration in the small fraction of the singular motionless field comes the periodic phenomenon. Therefore time begins with the first vibration. Concept of proper time assumes linear time and distance scales whereas the true nature of reality is founded upon non-linear periods and wavelengths of the subatomic particles. Nevertheless to deal with a compound wave of a massive object such as a planet is not as simple as analyzing an individual particle. Thus the concept of proper time is useful in such cases as an approximation.

3. QUANTUM ENERGY EQUATION

General form of Max Planck’s quantum hypothesis (2.1) can be written in various alternate forms of which Eq. (3.3) is the most familiar.

\[ E = \left\{ E_0^2 + h^2 \nu^2 \beta^2 \right\}^{1/2}, \]  
\[ E = m_0 c^2 \{1 + \gamma^2 \beta^2 \}^{1/2}, \]  
\[ E = m_0 c^2 (1 - \beta^2)^{-1/2}, \]  
\[ E = \pm \left\{ (m_0 c^2)^2 + (mc^2)(mv^2) \right\}^{1/2}, \]

where \( m \) is the relativistic mass, \( \beta = v/c \) and \( \gamma = m/m_0 = (1 - \beta^2)^{-1/2} \). In PR, \( \beta \) and \( \gamma \) need not be constants, however, their instantaneous values are related with each other and with other parameters in the same manner as in special relativity.
A. True force

In order to come up with a truly invariant relationship between force and energy, we shall differentiate Eq. (3.1) w.r.t. time.

\[
\frac{dE}{dt} = \frac{d}{dt} \left\{ E_0^2 + h^2 \nu^2 \beta^2 \right\}^{1/2} = vF, \tag{3.5}
\]

\[
vF = \frac{1}{2E} \left( 2h^2 \nu^2 \beta \frac{d\beta}{dt} + 2h^2 \beta^2 \nu \frac{d\nu}{dt} \right), \tag{3.6}
\]

\[
vF = \frac{1}{2E} \left( 2E^2 \frac{v}{c^2} \frac{dv}{dt} + 2Eh^2 \frac{v^2}{c^2} \frac{d\nu}{dt} \right), \tag{3.7}
\]

where \( F \) is a new form of Lorentz force which we shall call the true force and \( v \) the velocity vector. Eq. (3.5) reduces to

\[
vF = v \left( ma + \frac{hv}{c^2} \frac{dv}{dt} \right), \tag{3.8}
\]

where \( a = dv/dt \) is the acceleration of the particle. From Eq. (3.8) we can deduce that the change in the energy of the particle is associated with two different changes in the state of the particle.

- The change in the velocity of the particle.
- The change in the frequency of the associated de Broglie wave.

When the second aspect is neglected, the invariant relationship between the force and the energy is lost. With respect to the massive particles, this second term on the right is comparable to the Doppler effect. Hence we will call it the de Broglie effect; and since this second term also has the units of force, we shall call this new force the de Broglie force. This shows that the true force consists of a sum of two forces, the classical force and the de Broglie force.

Even though there are equations in Einstein’s relativity for relating force and energy, it remains a fact that the relativity theory has failed to provide satisfactory quantification of force and energy. The principle reason in my opinion is the concept of time as adopted by the relativity theory which assumes that the time is linear and flow in one direction from past to present and from present to future. This prevailing concept of time moving in one direction is a self-imposed illusion of the mind, just like imagining a blue sky which in reality is colorless, or riding a marry-go-round while all the time thinking that we are...
moving forward. Other authors have arrived at similar conclusion by analyzing the block universe concept \[10\]. Relativity no doubt unifies the space and the time continuum, but because of the adoption of the linear time scales, it becomes very convenient to compromise the invariance of force and energy.

Both the classical as well as the relativistic mechanics are founded upon the assumption that \( \frac{d\nu}{dt} \) is always zero for calculations involving force and energy. This is the very reason for which general relativity has failed to provide satisfactory quantification of force and energy. So whether one should hold on to the concept of proper time and linearity of time scales or adopt the view that the reality is based on non-linear periods? The answer to this question may not be very simple, but one thing is certain that if we assume \( \frac{d\nu}{dt} = 0 \), then the derivation of gravitational redshift discussed in sec.4.D.1 would not be possible.

4. TWO-BODY PROBLEM AND THE GRAVITATIONAL FIELD

A. Dynamic weak equivalence principle

In order to deal with the static spherically symmetric gravitational field produced by a spherically symmetric body at rest, we work in a single plane and base our formalism on following postulate which basically means that the orbital energy is constant and consists of sum of kinetic energy and gravitational potential energy.

In empty space, the rate of change of kinetic energy of a particle is equal to the rate of change of potential energy influencing the particle.

In classical two-body problem, the gravitational field gets introduced as a single central potential acting radially, while the transverse component is assumed absent. In PR we will introduce gravitational field in terms of two components of a force acting normal and tangential to the particle trajectory, rather than as a resultant central force acting radially. This will allow us to account for the curvature of the trajectory. Moreover PR is not opposed to many of the conclusions of special and general relativity such as mass energy equivalence, distinction between rest mass and the relativistic mass, perihelic precession formalism, quadrupole formalism etc. Hence we will introduce another modification to Newton’s inverse square law
on following grounds. It is well recognized that the light is affected by the central gravitational potential due to rest mass of a body just like any other massive orbiting body. Making use of this observation we conclude that the mass of the orbiting body in the inverse square law should be relativistic mass and not the rest mass. This is because photon does not have any rest mass but only kinetic energy which can be correctly represented in terms of relativistic mass. And this relativistic mass is a variable parameter in gravitational field and not a constant. We shall adhere to this principle even while discussing massive bodies in orbit which also have some kinetic energy. Proof of the correctness of this assumption is evident in the derivation of the orbital period derivative of a binary star discussed in [11]. These two changes introduces two more variables in addition to the radial distance in the formalism of central potential. Hence it is not very straightforward to introduce the classical theories of gravitational potentials in PR. However, in PR also the central potential acts radially and the transverse component is assumed absent. Therefore when the second body has only radial motion as in the case of gravitational redshift of light, the two additional variables disappear and the potential reduces to classical Newtonian potential. However, this does not happen in case of bending of light.

How does the assumption that the gravitational attraction exists between the relativistic masses reflect on the equivalence principle? It appears that this assumption does not violate any of the three equivalence principles, the weak (WEP), Einstein (EEP) and the strong (SEP). "Universality of free fall" is based on Newton's weak equivalence principle (WEP) which states: "the property of a body called mass is proportional to the weight." In WEP Newton did not specify whether the mass is the rest mass or relativistic mass and no discussion of motion. In Einstein's notion, free fall indicates inertial mass as well as relativistic mass but again in consideration of Eötvös experiment only inertial mass is equated to gravitational mass. Besides, Eötvös experiment is a static experiment which does not involve moving masses. The present theory conforms with Eötvös experiment when two gravitating bodies are in equilibrium and at rest in the same coordinate system. For example, if an object is thrown radially upwards in the coordinate system of the earth, it will attain a maximum height and then freely fall back to the earth. Momentarily when the object is at the maximum height, both the object and the earth are perfectly at rest in the same coordinate system. At this moment, the relativistic mass of the object is the same as its rest mass. Therefore at this moment of equilibrium, the gravitational mass is equal to inertial
mass. Rest of the time it is the relativistic mass which is equal to the gravitational mass. This is the dynamic version of WEP we have introduced in this theory which states that the gravitational mass of a body is equal to its relativistic mass.

Whether one is working with the coordinate time of the central potential or with the proper time of the orbiting body, one of the two masses would certainly be the relativistic mass. So this factor needs to be considered. The effects of dynamic WEP gets absorbed in what is later defined as the deviation factor "n" and eventually shows up as a deviation in the proper time interval of the body. And proper time interval of the planet is not a part of the present day ephemerids [12–15]. As long as the proper time interval is not included as one of the observables in ephemerids, there is no way to compare the GR predictions with the PR theory. The present day ephemerids are a three dimensional ephemerids. Introduction of proper time interval as a variable orbital parameter would introduce the fourth dimension to the ephemerids.

Fig. 1 shows the radial vector $r$ connecting the central mass $M_0$ with the particle in motion having rest mass $m_0$. $\theta$ is the polar angle and $\psi$ is the angle between the radial vector $r$ and the tangent vector $\hat{T}$. Here we are dealing with two coordinate systems, one centered on the central mass with polar parameters and another centered on the particle in motion with axes along the tangent and the normal to the trajectory. Both these coordinate systems are oriented w.r.t. each other in such a way that the normal vector and the radial vector make an angle equal to $((\pi/2) - \psi)$ between them, and $\psi$ is a variable. Therefore in
PR the transverse component is absent w.r.t. polar coordinate system of the central mass but not w.r.t. the coordinate system of the particle in motion. Hence, the rate of change of potential energy influencing the particle can be given by

$$\frac{dE_p}{dt} = F \cdot v = ma \cdot v =, \quad (4.1)$$

$$-\frac{\mu m}{r^2} (\cos \psi + \sin \psi) \hat{r} \cdot v = -\frac{\mu m}{r^2} \frac{dr}{dt} \left(\cos \psi + \sin \psi\right). \quad (4.2)$$

where $\mu = GM_0$, $G$ = gravitational constant, $m$ = relativistic mass and following relations hold as usual.

$$\hat{r} \cdot \frac{dr}{dt} = \hat{r} \cdot v = \left(\frac{dr}{dt} \hat{r} \cdot \hat{r} + \frac{r d\theta}{dt} \hat{\theta} \cdot \hat{r}\right) = \frac{dr}{dt}. \quad (4.2)$$

Gravitational potential can be deduced from Eq. (4.2) as

$$-\int \frac{\mu \gamma}{r^2} (\cos \psi + \sin \psi) dr, \quad (4.3)$$

where $\gamma = m/m_0$ is a variable. $\gamma$ and $\psi$ both are functions of $r$.

**B. Massless particles in gravitational field**

For massless particles, the rate of change of kinetic energy can be given by Eq. (3.8) as described below. Following is applicable to all massless particles.

- The particles will have velocity equal to $c$. The particles cannot be accelerated along the direction of motion in a conventional sense. The wavefront can however be accelerated normal to the direction of motion. The wave can be subjected to Doppler shift along the direction of motion.

1. **Electromagnetic wave can be accelerated at constant velocity of photon**

Physicists usually argue that photon travel at a constant speed of light and cannot be accelerated but this is not true. Einstein’s theory show that light can be subjected to gravitational redshift. In this, light is acted upon by gravitational acceleration at a constant
velocity. In Newton’s standard definition of force \( F = ma \), the component of force causing acceleration of light is ignored. And with that additional component of acceleration causing the de Broglie force given in Eq. \( (3.8) \), the gravitational redshift of light can be calculated without using general relativity. Physicists usually define speed of light as distance travelled divided by time taken to travel the distance. But this is not very accurate definition. The correct definition is the wavelength divided by period of the electromagnetic wave. This ratio of wavelength to period is constant even when light experiences gravitational frequency shift (redshift) because with the change in the frequency (or period), the wavelength also change simultaneously in such a way that the velocity remains constant. In Periodic relativity, time is periodic so light can accelerate at constant velocity but in general relativity time is linear so light cannot accelerate at constant velocity. In general relativity if you accelerate the light, the velocity of light must increase, it cannot remain constant. This is because time dilation and length contraction does not occur simultaneously in general relativity, like the period and wavelength described above. Therefore gravitational redshift of light in general relativity itself is an indicator of failure of the equivalence principle. In periodic Relativity Newtonian force \( F = ma \) is modified as \( F = ma + \) de Broglie force. With this additional de Broglie force, how can you say gravitational mass is equal to inertial (rest) mass? This de Broglie force term shown in Eq. \( (4.20) \) enables the explicit derivation of gravitation redshift of light. In PR, gravitational mass is equal to relativistic mass which solves this problem but relativistic mass is abandoned by general relativity theorists because it is not convenient to introduce weak field approximation in the flat Minkowski metric. This is the conflict between linear time and periodic time. At the quantum level, things don’t work too well with the linear time. Therefore in general relativity the other three forces, electromagnetic, strong and weak can not cause time dilation and length contractions. But in periodic relativity they do. In PR we can also derive deflection of light without using general relativity. In this, we make use of angle \( \psi \) shown in Eq. \( (4.29) \) associated with the geometry of ellipse, which was ignored by both Newton and Einstein. Both these derivations satisfy Einstein’s field equations for \( n = 1 \).
C. Curvilinear Gravity

Newton realized the equality of gravitational force and the centrifugal force acting on the orbiting body and then introduced Kepler’s third law of orbital periods to arrive at the inverse square law of gravitation given by

\[ m_0 \frac{d^2 \mathbf{r}}{dt^2} = -\frac{GM_0 m_0}{r^2} \hat{r}. \]  

(4.4)

where \( GM_0 = \mu \). Here we introduce the dynamic weak equivalence principle which states that the gravitational mass is equal to the relativistic mass. Therefore Eq. (4.4) becomes

\[ m \frac{d^2 \mathbf{r}}{dt^2} = -\frac{\mu m}{r^2} \hat{r}. \]  

(4.5)

In classical mechanics, we have two different expressions for the acceleration acting on a body in motion. One is a general expression \( \frac{d\mathbf{v}}{dt} \) in cartesian coordinates which include the curvature term, and another is for Newtonian gravity in polar coordinates \( \frac{d^2 \mathbf{r}}{dt^2} \) based on the angular momentum vector \( \mathbf{h} \), which is supposed to be a constant in order to satisfy Kepler’s third law of equal areas in equal times. In Newton’s theory the normal component of \( \frac{d\mathbf{v}}{dt} \) containing curvature term is equated with radial acceleration \( \frac{d^2 \mathbf{r}}{dt^2} \). In PR we will show that these two accelerations are not equal. The reason for this is that the Newtonian gravity ignores the variation of angle \( \psi \) along the trajectory by assuming constant \( \mathbf{h} \). At the same time we maintain that the velocity vectors in both coordinate systems are equal, \( \mathbf{v} = \frac{d\mathbf{r}}{dt} \). This is because \( \frac{d\mathbf{r}}{dt} \) is not a radial vector. As shown in Figs.1, 2 and 3, \( \psi \) is the angle between the radial vector and the tangential velocity vector. Explanation given below makes it clear that the theory is Lorentz invariant and factor \( (\cos \psi + \sin \psi) \) in Eq. (4.3) introduces geodesic like trajectories. The details are as follows.

As shown in Fig. 2 this angle \( \psi \) is related to curvature through the expression

\[ \phi = \theta + \psi. \]  

(4.6)

where \( \frac{d\phi}{ds} = \kappa \) is the curvature. Newtonian gravity ignores this curvature term by assuming constant \( \psi = \pi/2 \). This can be verified from following arguments.

\[ \mathbf{h} = \frac{\mathbf{L}}{m} = \frac{\mathbf{p} \times \mathbf{r}}{m} \equiv \frac{\sqrt{\mathbf{p} \cdot \mathbf{r}}}{m} \sin \psi \hat{\mathbf{h}} = r^2 \frac{d\theta}{dt} \sin \psi \hat{\mathbf{h}}. \]  

(4.7)

From Eq. (4.7) we can see that \( \mathbf{h} \) can be the desired constant only if \( \sin \psi = 1 \). This shows that the Newtonian gravity approximates the curvature of the trajectory of the orbiting body.
Hence in periodic relativity it is considered unreasonable to equate the normal component of the cartesian acceleration $dv/dt$ with the Newtonian polar acceleration $d^2r/dt^2$.

In order to account for the variation of angle $\psi$ along the trajectory, we propose that the absolute sum of vector and scalar products of $(\mu/r^2)\hat{r}$ and $\hat{a}$ is equal to magnitude of $dv/dt$. The relation of these vectors to angle $\psi$ is shown in Fig. 2.

\[
\left| \frac{dv}{dt} \right| = \left| -|\hat{a}| \times \frac{\mu}{r^2} \hat{r} - \frac{\mu}{r^2} \hat{r} \cdot \hat{a} \right|. \tag{4.8}
\]

\[
\left| \frac{dv}{dt} \right| = \left| |\hat{a}| \frac{\mu}{r^2} \sin (\beta + \gamma) \hat{h} + \frac{\mu}{r^2} |\hat{a}| \cos (\beta + \gamma) \right|. \tag{4.9}
\]

where

\[
\beta = \left( \frac{\pi}{2} - \psi \right). \tag{4.10}
\]

\[
\gamma = \tan^{-1} \left( \frac{a_t}{a_n} \right). \tag{4.11}
\]

Various magnitudes of the parameters shown in Fig. 1 are as follows.

\[
a_t = \frac{dv}{dt}. \tag{4.12}
\]

\[
a_t = \left( \frac{d^2s}{dt^2} + \frac{v}{\nu} \frac{d\nu}{dt} \right). \tag{4.13}
\]
\( a_n = \kappa \left( \frac{ds}{dt} \right)^2 \). \hspace{1cm} (4.14)

\( a_r = -\frac{\mu}{r^2} = \frac{|d^2\mathbf{r}|}{|dt^2|} \). \hspace{1cm} (4.15)

\( \mathbf{v} = \frac{d\mathbf{r}}{dt} \). \hspace{1cm} (4.16)

Substitution of Eq. (4.10) in Eq. (4.9) gives

\[ \left| \frac{d\mathbf{v}}{dt} \right| = \frac{\mu}{r^2} (\cos (\psi - \gamma) + \sin (\psi - \gamma)). \hspace{1cm} (4.17) \]

When the tangential component of the acceleration is absent then we have \( a_t \hat{T} = 0 \). This gives \( \gamma = 0 \) and Eq. (4.17) reduces to

\[ \left| \frac{d\mathbf{v}}{dt} \right| = \frac{\mu}{r^2} (\cos \psi + \sin \psi). \hspace{1cm} (4.18) \]

Similarly we can show that

\[ \left| \frac{d\mathbf{v}}{dt} \right| = \left| \frac{d^2\mathbf{r}}{dt^2} \right| (\cos (\psi - \gamma) + \sin (\psi - \gamma)). \hspace{1cm} (4.19) \]

The first term on the right of Eq. (4.17) can be interpreted as an angular acceleration vector with its axis perpendicular to the plane of motion. This could be the additional acceleration quantity responsible for the rotation of the velocity vector \( \mathbf{v} \) about the coordinate origin \( o \), causing the curvature of the trajectory.

**D. Lorentz invariant acceleration**

In relativity we can either write our equations in terms of proper time or alternatively we can write them in terms of relativistic mass. Eq. (1.1) can be written as

\[ \left( \frac{d\tau}{dt} \right)^2 = (1 - n\beta^2) = \left( \frac{m_0}{m} \right)^2 = \left( \frac{E_0}{E} \right)^2, \hspace{1cm} (4.20) \]

where \( E = mc^2 = h\nu \). This gives

\[ E = (E_0^2 + nE^2 \beta^2)^{1/2}. \hspace{1cm} (4.21) \]
If we introduce photon parameters \( m_0 = 0 \) and \( v = c \) in Eq. (4.21), we can recover photon energy \( E = mc^2 = h\nu \) by putting \( n = 1 \) for flat Minkowski space time. Further we can write Eq. (1.11) for photon as

\[
\left( \frac{cd\tau}{cdt} \right)^2 = (1 - n),
\]

(4.22)

\[
\left( \frac{ds}{cdt} \right)^2 = (1 - n),
\]

(4.23)

For \( n = 1 \) we get \( ds^2 = 0 \) signifying null trajectory for light.

Differentiating Eq. (4.21) w.r.t. time for \( n = 1 \) we get,

\[
\left( \frac{1}{v} \right) \frac{dE}{dt} = \vec{F} = \left( ma + \frac{h\nu dv}{c^2 dt} \right),
\]

(4.24)

Here we arrive at the same relation that we described as true force in Eq. (3.8) except that now we have introduced the deviation factor \( n \). I like to point out that this true force is same as the Lorentz force. Here we have used the relation \( E = mc^2 = h\nu \).

Therefore

\[
F = \frac{dp}{dt} = \left( ma + \frac{h\nu dv}{c^2 dt} \right),
\]

(4.25)

where \( F \) is the Lorentz force and \( \mathbf{v} \) the velocity vector and \( a \) is the classical acceleration of the particle given by

\[
a = \left( \frac{d^2s}{dt^2} \mathbf{T} + \kappa \left( \frac{ds}{dt} \right)^2 \mathbf{N} \right).
\]

(4.26)

Therefore,

\[
\text{Lorentz force} = \text{Classical force} + \text{de Broglie force}.
\]

From Eq. (4.25) we can define Lorentz invariant acceleration \( a_t \) as

\[
a_t = \left( \left( \frac{d^2s}{dt^2} + \frac{v dv}{\nu dt} \right) \mathbf{T} + \kappa \left( \frac{ds}{dt} \right)^2 \mathbf{N} \right).
\]

(4.27)

The de Broglie force acts along the tangent vector. Now we equate Lorentz force with the gravitational force given by Eq. (4.17)

\[
ma_t = \frac{dp}{dt} = m \left( \left( \frac{d^2s}{dt^2} + \frac{v dv}{\nu dt} \right) \mathbf{T} + \kappa \left( \frac{ds}{dt} \right)^2 \mathbf{N} \right)
\]

(4.28)

\[
a_t = \frac{dv}{dt} = \left( \left( \frac{d^2s}{dt^2} + \frac{v dv}{\nu dt} \right) \mathbf{T} + \kappa \left( \frac{ds}{dt} \right)^2 \mathbf{N} \right)
\]

(4.29)
1. Gravitational redshift in periodic relativity

In PR the light is a wave and does not have any inertial mass, only the relativistic mass which is equivalent to its kinetic energy. We can apply Eq. (4.29) to the gravitational redshift problem \[16–19\] involving a photon travelling in a straight line from the sun to the earth along a path connecting their centers. In this case we have \(d^2s/dt^2 = 0, \kappa = 0\) and \(\psi = 0, \gamma = 0\) and both, the gravitational force and the de Broglie force act along the radial direction, thus

\[ \frac{\mu m}{r^2} \hat{r} = \frac{mv}{\nu} d\nu \hat{r}. \]

(4.30)

\[ \frac{\mu m}{r^2} = \frac{h}{c} d\nu = \frac{h}{c} d\nu dr = \frac{h}{c} d\nu, \]

\[ \frac{\mu}{r^2} = \frac{hc^2}{mc^2} d\nu, \]

\[ \frac{\mu}{r^2} dr = \frac{h}{E} c^2 d\nu = c^2 \frac{1}{\nu} d\nu. \]

Integration over the entire trajectory gives

\[ \int_{\Delta}^{\Delta+l} \frac{\mu}{r^2} dr = c^2 \int_{\nu_s}^{\nu_{\infty}} \frac{1}{\nu} d\nu, \]

(4.31)

where \(\Delta\) is the solar radius, \(l\) the distance traveled by photon, \(\nu_s\) the frequency of light on the surface of the sun and \(\nu_{\infty}\) the frequency of light on earth. This gives

\[ \frac{\mu}{c^2} \frac{l}{(\Delta^2 + \Delta l)} = \frac{\varphi_1 - \varphi_2}{c^2} = \ln \left( 1 - \frac{\delta\nu}{\nu_s} \right), \]

(4.32)

\[ -\frac{\delta\nu}{\nu_s} = \frac{1}{c^2} (\varphi_1 - \varphi_2) + \frac{1}{c^4} \left( \frac{1}{2} \varphi_1^2 - \varphi_1 \varphi_2 + \frac{1}{2} \varphi_2^2 \right) + O \left( \frac{1}{c^6} \right). \]

(4.33)

The first order term is exactly the same value predicted by general relativity in terms of gravitational potentials \(\varphi_1\) and \(\varphi_2\) at locations separated by distance \(l\), and verified experimentally \[16–19\] with a high level of accuracy. The second order term for general relativity is slightly different,

\[ + \frac{1}{c^4} \left( -\frac{1}{2} \varphi_1^2 - \varphi_1 \varphi_2 + \frac{3}{2} \varphi_2^2 \right), \]

(4.34)

and below the accuracy \[3, 17, 20, 22\] in the measurement of gravitational redshift. For \(l > 100\Delta\), Eq. (4.32) approaches

\[ -\frac{\delta\nu}{\nu_s} = \frac{\mu}{c^2 \Delta}. \]

(4.35)
For $l < 0.01\Delta$, Eq. (4.32) approaches the formula for the Doppler effect $[16]$, given by

$$\frac{\delta \nu}{\nu_s} = \frac{\mu l}{c^2 \Delta^2} = \frac{gl}{c^2} = \frac{\vartheta}{c}.$$  \hspace{1cm} (4.36)

It should be noted here that if Eqs. (1.5) and (1.6) had nothing to do with Eq. (1.4), then it would be impossible to derive the gravitational redshift Eq. (4.32). It is mainly due to the periodic representation of Eq. (1.5) that the frequency term appears in Eqs. (2.1), (3.1), (3.8), and (4.30). This makes it possible to derive Eq. (4.32) without mentioning linear time or linear distance and without utilizing Riemannian geometry and geodesic trajectories. Hence unlike GR, the gravitational frequency shift in PR is invariant.

The special theory of relativity assumes global coordinate systems and global invariance of speed of light as well as the equivalence of mass and energy. Einstein had to abandon the global coordinates and global invariance of speed of light while formulating general relativity because they were in conflict with the principle of equivalence of inertial mass and gravitational mass. However, he permitted the existence of a local system of inertial coordinates in a small region around any event. In PR we have gone one step further and restricted the local system of inertial coordinates to have only instantaneous existence. This makes the proper time a continuously variable phenomenon, which could now be identified with the continuously variable period of the body associated with the inertial coordinate. This has significant effect if the body is a fundamental particle such as a photon. When such instantaneous coordinate system is fixed on a massive body such as a planet, it acts exactly like the coordinate system of general relativity because the period of the associated wave of the planet does not change significantly from instance to instance and such is also the case with its proper time which remains practically constant. Similarly any fundamental particle traveling along a constant radial distance from a central massive body shall also have constant period and constant proper time. This is consistent with the general relativity definitions of tangential and radial velocities of light in a gravitational field $[3]$ given in geometrical units ($c_0 = 1$) by

$$c_t = 1 + \varphi = \sqrt{1 + 2\varphi} = \sqrt{c_r}.$$  \hspace{1cm} (4.37)

In general relativity, the rate of proper time at a fixed radial position in a gravitational field relative to the coordinate time can be obtained from general form of the metrical space time
line element for a spherically symmetrical static field in polar coordinate and is given by
\[ \frac{d\tau(r)}{dt} = \sqrt{g_{tt}(r)}. \] (4.38)

From Schwarzschild metric we have \( g_{tt}(r) = 1 + 2\varphi \). Now in general relativity, there is no explicit derivation of formula for gravitational redshift, but it is implicitly deduced from Eq. (4.38) and given by
\[ \frac{\nu_2}{\nu_1} = \sqrt{1 + 2\varphi_1 \over 1 + 2\varphi_2}. \] (4.39)

The only support this formula has is the experimental verification of the first order term. Hence we would not be violating any scientific law if we propose that the correct implication of Eq. (4.38) is
\[ \frac{\nu_2}{\nu_1} = e^{\sqrt{1+2\varphi_1}} e^{-\sqrt{1+2\varphi_2}}. \] (4.40)

Eq. (4.40) also yields the same first order term, besides it can also be explicitly derived and is exactly the same formula (in geometrical units) given by Eq. (4.32). It is to be noted here that Eq. (1.5) and Eq. (1.6) has this unique property that they remain null even in a gravitational field. This is because equations contain only periodic time and wavelengths which are not independent of each other. This is not the case with Eq. (1.4) which uses linear time and linear distance, so their interdependence is lost which is then restored with a deviation factor. In general relativity this deviation factor is used for all massive particles but is dismissed for photon on the ground that they travel along null geodesics. So computing proper time of photon is out of question in GR. But in PR we can use deviation factor \( n \) for photon as well and compute proper time of photon which can be negligibly small value close to null. For this specific case we define the deviation factor \( n \) as a ratio of Newtonian gravitational acceleration to the de Broglie acceleration given in Eq. (4.30).
\[ n = \frac{GM}{r^2} \left( \frac{h}{mc} \frac{dv}{dt} \right)^{-1}. \] (4.41)

If we look at Eq. (4.18), we find that Eq. (4.41) is valid for both cases \( \psi = 0 \) and \( \psi = \pi/2 \). When this ratio is \( n = 1 \), we get null geodesics but for any radial motion to take place, this ratio has to deviate from 1, otherwise the particle will make only circular motion due to equal and opposite radial forces acting on it. So for radial motion to take place, we must have \( n = (1 \pm \epsilon) \) where \( \epsilon \) can be a very small value close to zero and \( \pm \) sign depends on
whether the trajectory is time like or space like. In Eq. (1.1), if we substitute \( v = c \) and 
\( n = (1 \pm \epsilon) \), we get

\[
\left( \frac{d\tau}{dt} \right)^2 = \pm \epsilon. \tag{4.42}
\]

The positive sign on the right should give time like geodesic applicable to electromagnetic waves and negative sign gives space like geodesic applicable to gravitational waves. Both travel at speed of light.

2. Bending of light in periodic relativity

For the bending of light around the sun \([17, 23, 24]\), we introduce light parameters \( v = ds/dt = c \), \( d^2 s/dt^2 = 0 \) and \( cdt = ds \), along with \( \kappa = d\phi/ds \) for the curvature of the trajectory in Eq. (4.29). In this case we will have \( dv/dt = 0 \) because the ray is equally blue shifted and then red shifted, and the frequency shift is 0 at the limb of the sun. This gives,

\[
\left| \frac{c^2 dv}{\nu ds} \hat{T} + c^2 d\phi ds \hat{N} \right| = \left| \frac{\mu}{r^2} (\cos (\psi - \gamma) + \sin (\psi - \gamma)) \hat{N} \right|. \tag{4.43}
\]

Multiplying both sides by \( d\psi \), we get

\[
\left| \frac{1}{\nu} dv d\psi \hat{T} + d\phi d\psi \hat{N} \right| = \frac{\mu}{c^2 r^2} (\cos (\psi - \gamma) + \sin (\psi - \gamma)) dsd\psi. \tag{4.44}
\]
We integrate both sides with proper limits. For the star light approaching the sun we get,
\[
\left| \int_{\nu_1}^{\nu_2} \frac{1}{\nu} d\nu d\psi \hat{T} + \int_{-\phi}^{\phi} \int_{\pi/2}^{\pi} d\phi d\psi \hat{N} \right| = \frac{\mu c^2}{\Delta} \int_{-\infty}^{0} \int_{\pi}^{0} \frac{1}{r^2} (\cos(\psi - \gamma) + \sin(\psi - \gamma)) d\psi ds.
\]
(4.45)

For the star light approaching earth from the limb of the sun we get,
\[
\left| \int_{\nu_1}^{\nu_2} \frac{1}{\nu} d\nu d\psi \hat{T} + \int_{0}^{\phi} \int_{\pi/2}^{\pi} d\phi d\psi \hat{N} \right| = \frac{\mu c^2}{\Delta} \int_{0}^{\infty} \int_{\pi/2}^{0} \frac{1}{r^2} (\cos(\psi - \gamma) + \sin(\psi - \gamma)) d\psi ds.
\]
(4.46)

\[
\left| (\ln \nu_2 - \ln \nu_1) \hat{T} + \phi \hat{N} \right| = \frac{\mu c^2}{\Delta} \int_{-\infty}^{0} \int_{\pi}^{0} \frac{1}{r^2} (\cos(\psi - \gamma) + \sin(\psi - \gamma)) d\psi ds.
\]
(4.47)

\[
\left| (\ln \nu_1 - \ln \nu_2) \hat{T} + \phi \hat{N} \right| = \frac{\mu c^2}{\Delta} \int_{0}^{\infty} \int_{\pi/2}^{0} \frac{1}{r^2} (\cos(\psi - \gamma) + \sin(\psi - \gamma)) d\psi ds.
\]
(4.48)

If we add l.h.s. of Eqs. (4.47) and (4.48) we get,
\[
l.h.s. = \left| 0 \hat{T} + 2\phi \hat{N} \right|.
\]
(4.49)

From Eq. (4.49) we see that the magnitude of the tangential component is zero. Therefore \(\gamma = 0\). Hence substituting \(r^2 = s^2 + \Delta^2\) in Eqs. (4.47) and (4.48) we get
\[
2\phi = \frac{4\mu}{c^2\Delta}.
\]
(4.50)

We have used Eqs. (4.24) and (4.26) to explain both, the bending of light and the gravitational frequency shift, which correspond to the flat Minkowski metric with \(n = 1\). As can be seen, the higher order terms does not exist in PR theory. The second order term in general relativity is below the accuracy in the measurement of deflection of light [3, 17, 20, 23], and is given by
\[
+ \left(\frac{15\pi}{4} - 4\right) \left(\frac{\mu}{c^2\Delta}\right)^2.
\]
(4.51)

Experimental verification of the second order effect given by Eq. (4.51) was the principal goal of LATOR mission [25] which is now abandoned. If this theory is correct, the experiment can yield null result. If the fundamental postulates of a theory, physical or mathematical, are built upon approximations, then there is a chance of appearance of pseudo terms resembling
higher order terms in the end results. We have a reason to believe that the orbital energy equations (4.24) and (4.26) in PR are exact in nature, and that is not the case with general relativity. In general relativity, Newtonian potential gets introduced into metric component $g_{00}$ as a deviation to flat Minkowski metric. This constitutes the weak-field approximation. For this very reason Schwarzschild metric does not remain null for light in the gravitational field as we have already discussed. Other competing theories modify Newtonian potential by way of Poisson’s equation and multipole expansion. Another significant approximation in Schwarzschild solution is the assumption that the angle of deflection is subtended at the center of the sun. In PR, even though the schematic diagram shows the same arrangement, the calculations give us actual angle $\phi$ measured between the line $\theta = 0$ and the line normal to the velocity vector at the end of the trajectory. These factors add up to give different second order terms in these two theories. It is interesting to note that the higher order terms are in higher powers of Newtonian potential. It should also be noted that we have utilized Eq. (1.5) for the slow moving accelerating particle for determining the bending of light.

E. Curvic and conic gravity

Newtonian gravity is based on the constant vector $\mathbf{h}$ which yields the conic sections. Therefore we can distinguish the gravity that uses the Lorentz invariant acceleration as the curvilinear (or curvic) gravity and the Newtonian gravity with constant $\mathbf{h}$ as the conic gravity. Accelerations of the curvic and conic gravity are related by Eq. (4.19).

It also needs to be understood that $d^2\mathbf{r}/dt^2$ is a radial vector but $d\mathbf{r}/dt$ is not a radial vector which acts along the velocity vector $\mathbf{v}$. Moreover, the constant vector $\mathbf{h}$ does not play any role in defining the velocity vector $\mathbf{v}$. Therefore factor $(\cos \psi + \sin \psi)$ does not appear in this expression of velocity $\mathbf{v} = d\mathbf{r}/dt$ which remains unaltered. This can be verified from following analysis. By definition we have

$$\cos \psi = \frac{dr}{ds}, \text{ and } \sin \psi = \frac{r d\theta}{ds}. \quad (4.52)$$

$$\frac{d\mathbf{r}}{dt} = \left( \frac{dr}{dt} \hat{\mathbf{r}} + \frac{r d\theta}{dt} \hat{\mathbf{\theta}} \right). \quad (4.53)$$

$$\frac{d\mathbf{r}}{dt} = \frac{ds}{dt} (\cos (\psi + \theta) \hat{\mathbf{i}} + \sin (\psi + \theta) \hat{\mathbf{j}}). \quad (4.54)$$
Substitution of Eq. (1.7) gives

$$\frac{d\mathbf{r}}{dt} = \frac{ds}{dt} \sqrt{(\cos^2 \phi + \sin^2 \phi)} \mathbf{T} = \frac{ds}{dt} \hat{T} = \mathbf{v}. \quad (4.55)$$

From Fig. 1 we can verify that the unit vector acting at an angle $\phi$ is $\hat{T}$. Therefore Eq. (4.55) is not influenced by the constant $h$ assumption.

**F. Massive particles in gravitational field**

From Eqs. (4.29) and (3.8) we get,

$$a(-\hat{r}) = \left( \frac{d^2s}{dt^2} \hat{T} + \kappa \left( \frac{ds}{dt} \right)^2 \hat{N} \right). \quad (4.56)$$

$$\frac{d\mathbf{v}}{dt} = \left( \kappa \left( \frac{ds}{dt} \right)^2 \hat{N} \right) = \frac{d^2\mathbf{r}}{dt^2} (\cos \psi + \sin \psi) \hat{N}. \quad (4.57)$$

So any conversion of acceleration between radial direction and normal to tangential velocity vector is accompanied by the conversion factor $(\cos \psi + \sin \psi)$. This factor acts as a single scalar quantity and does not get split into normal and tangential vector components.

1. **Perihelic precession of planets**

We assume that the general relativity theory as applicable to solar system planets is valid in weak-field approximation. We also declare however, that the theory fails to predict accurate higher order terms for gravitational red-shift and deflection of light, because in introducing the weak-field approximation, it compromises the global invariance of speed of light in gravitational field. The weak-field approximation also leads to the value of limiting radius of the event horizon of a black hole which is at variance with the measured value for M87 black hole. While Schwarzschild solution is sufficiently accurate in predicting the perihelic precession [3, 26, 27] of the planets of the solar system, it may not be dependable for describing the photon trajectories in strong gravitational fields. Not only so, even trajectories of massive bodies in a strong gravitational field or extremely weak-gravitational field compared to Sun can deviate significantly from the Schwarzschild solution. In PR we can present a very simple and accurate derivation for perihelic precession of planets as given
below, compared to which the general relativity derivation \[28–30\] is very complicated. In PR from Eqs. (4.17) and (4.19) we get,

\[
\frac{dv}{dt} = \left| \frac{d^2 \mathbf{r}}{dt^2} \right| \left( \cos (\psi - \gamma) + \sin (\psi - \gamma) \right) \hat{\mathbf{N}}
\]

\[
= - \left| \frac{\mu}{r^2} \hat{\mathbf{r}} \right| \left( \cos (\psi - \gamma) + \sin (\psi - \gamma) \right) \hat{\mathbf{N}}.
\] (4.58)

Eq. (4.58) contains Newton’s inverse square law of gravity given by

\[
\frac{d^2 \mathbf{r}}{dt^2} = - \frac{\mu}{r^2} \hat{\mathbf{r}} = - \frac{GM_0}{r^2} \hat{\mathbf{r}}.
\] (4.59)

Now we introduce the line element of periodic relativity in Eq. (4.59). The line element is obtained from,

\[
\left( \frac{d\tau}{dt} \right)^2 = (1 - n\beta^2),
\] (4.60)

where \( \beta = v/c \), and \( n \) is a constant real number which keeps the Minkowski metric flat. With this we replace the coordinate time interval \( dt \) of Eq. (4.59) with the proper time interval \( d\tau \).

\[
\frac{d^2 \mathbf{r}}{d\tau^2} (1 - n\beta^2) = - \left( \frac{\mu}{r^2} \right) \hat{\mathbf{r}}.
\] (4.61)

\[
\frac{d^2 \mathbf{r}}{d\tau^2} = - \left( \frac{\mu}{r^2} \right) (1 - n\beta^2)^{-1} \hat{\mathbf{r}}.
\] (4.62)

\[
\frac{d^2 \mathbf{r}}{d\tau^2} = - \left( \frac{\mu}{r^2} \right) (1 + n\beta^2) \hat{\mathbf{r}}.
\] (4.63)

We will write \( \nu^2 \) on r.h.s. in polar coordinates.

\[
\frac{d^2 \mathbf{r}}{d\tau^2} = - \left( \frac{\mu}{r^2} \right) \hat{\mathbf{r}} - \left( \frac{n\mu}{r^2c^2} \right) \left[ \left( \frac{dr}{dt} \right)^2 + r^2 \left( \frac{d\theta}{dt} \right)^2 + r^2 \sin^2 \theta \left( \frac{d\phi}{dt} \right)^2 \right] \hat{\mathbf{r}}.
\] (4.64)

Working in \( (r, \theta) \) plane we can put \( d\phi = 0 \) and on r.h.s. substitute

\[
r^2 \left( \frac{d\theta}{dt} \right)^2 = \frac{h^2}{r^2}.
\] (4.65)

Hence Eq. (4.64) reduces to

\[
\frac{d^2 \mathbf{r}}{d\tau^2} = - \left( \frac{\mu}{r^2} \right) \hat{\mathbf{r}} - \left( \frac{n\mu}{r^2c^2} \right) \left[ \left( \frac{dr}{dt} \right)^2 + \frac{h^2}{r^2} \right] \hat{\mathbf{r}}.
\] (4.66)
On l.h.s. we substitute
\[ \frac{d^2 r}{d\tau^2} = \left( \frac{d^2 r}{d\tau^2} - \frac{h^2}{r^3} \right) \hat{r}. \]  
(4.67)

\[ \left( \frac{d^2 r}{d\tau^2} - \frac{h^2}{r^3} \right) \hat{r} = - \left[ \left( \frac{\mu}{r^2} \right) + \left( \frac{n\mu}{r^2c^2} \right) \left( \frac{dr}{dt} \right)^2 + \left( \frac{n\mu h^2}{r^4c^2} \right) \right] \hat{r}. \]  
(4.68)

\[ \frac{d^2 r}{d\tau^2} = - \left( \frac{\mu}{r^2} \right) - \left( \frac{n\mu}{r^2c^2} \right) \left( \frac{dr}{dt} \right)^2 - \left( \frac{n\mu h^2}{r^4c^2} \right) + \frac{h^2}{r^3}. \]  
(4.69)

On the l.h.s. of Eq. (4.69) we have the proper time and on the r.h.s. we have the coordinate time. To obtain the time independent solution of the equation of motion we make the following substitution which gives a second order non-homogeneous, non-linear differential equation.

\[ u = \frac{1}{r}, \quad \text{and} \quad \frac{d}{d\tau} = \frac{d}{dt} = h u^2 \frac{d}{d\theta}. \]  
(4.70)

\[ -h^2 u^2 \frac{d^2 u}{d\theta^2} = - \mu u^2 - \left( \frac{n\mu h^2}{c^2} \right) u^4 + h^2 u^3 - \left( \frac{n\mu}{c^2} \right) u^2 \left( -u \frac{du}{d\theta} \right)^2. \]  
(4.71)

\[ \frac{d^2 u}{d\theta^2} = \frac{\mu}{h^2} + \left( \frac{n\mu}{c^2} \right) u^2 - u + \left( \frac{n\mu}{c^2 h^2} \right) \left( -u \frac{du}{d\theta} \right)^2. \]  
(4.72)

Angle \( \psi \) between the radial vector and the velocity vector is defined as

\[ \psi = \tan^{-1} \frac{r}{\dot{r}}, \quad \text{where} \quad \dot{r} = \frac{dr}{d\theta}. \]  
(4.73)

\[ \sin \psi = \frac{(r/\dot{r})}{\sqrt{(r/\dot{r})^2 + 1}}. \]  
(4.74)

\[ \sin^2 \psi = \frac{(r/\dot{r})}{\sqrt{(r/\dot{r})^2 + 1}}. \]  
(4.75)

\[ \sin^2 \psi = \frac{r^2}{r^2 + \dot{r}^2} = \left[ 1 + \frac{1}{u^2} \left( \frac{du}{d\theta} \right)^2 \right]^{-1}. \]  
(4.76)

Using Eq. (4.76) we can write,

\[ \left( \frac{du}{d\theta} \right)^2 = \frac{u^2}{\sin^2 \psi} - u^2. \]  
(4.77)
General relativity has ignored angle $\psi$ by assuming $\psi = \pi/2$. This gives,

$$\left( \frac{du}{d\theta} \right)^2 = 0. \quad (4.78)$$

Substituting Eq. (4.78) in Eq. (4.72) gives,

$$\frac{d^2u}{d\theta^2} + u = \frac{\mu h^2}{\mu c^2} + \frac{n\mu u^2}{c^2}. \quad (4.79)$$

Eq. (4.79) is exactly the same equation obtained in general relativity for $n = 3$. So rest of the derivation for perihelic precession of planets is same as in GR. Later we will show that line element of PR given by Eq. (1.2) satisfies Einstein’s field equations for all constant values of $n$.

2. **Proper time of a planet**

   We substitute value of $n = 3$ from Eq. (4.79) in Eq. (1.3) for proper time and for second order velocity term on the right we will introduce classical vis-viva equation for planetary velocity

   $$v^2 = \mu \left( \frac{2}{r} - \frac{1}{a} \right). \quad (4.80)$$

   $$d\tau = dt \left( 1 - \frac{n\mu}{2c^2a} \left( \frac{2}{r} - \frac{1}{a} \right) \right). \quad (4.81)$$

   Substituting $r = h^2/\mu(1 + e \cos \theta)$ to obtain

   $$d\tau = dt \left( 1 - \frac{n\mu(1 + 2e \cos \theta + e^2)}{2c^2a(1 - e^2)} \right). \quad (4.82)$$

   For $n = 3$ and for circular orbits $a = r$ and $e = 0$. This gives

   $$d\tau = dt \left( 1 - \frac{3\mu}{2c^2r} \right). \quad (4.83)$$

   GR does not have a convenient way of expressing proper time equation such as Eq. (4.82) but it does provide expression for proper time in equatorial Keplerian circular orbits which is exactly same as Eq. (4.83). Unlike GR, in PR value of $n$ is locked by measurement of proper time.
5. BLACK HOLE

A. Non rotating black hole

The line element of the periodic relativity Eq. (1.1) introduces deviation to the flat Minkowski metric without affecting flatness of the metric. We can write velocity \( v \) in polar coordinates as given below.

\[
\left( \frac{d\tau}{dt} \right)^2 = \left[ 1 - \left( \frac{n}{c^2} \right) \left\{ \left( \frac{dr}{dt} \right)^2 + r^2 \left( \frac{d\theta}{dt} \right)^2 + r^2 \sin^2 \theta \left( \frac{d\phi}{dt} \right)^2 \right\} \right].
\] (5.1)

Working in \((r, \theta)\) plane we can put \( d\phi = 0 \) and substitute

\[
r^2 \left( \frac{d\theta}{dt} \right)^2 = \omega^2 r^2 = \frac{h^2}{r^2},
\] (5.2)

\[
\left( \frac{d\tau}{dt} \right)^2 = \left[ 1 - \left( \frac{n}{c^2} \right) \left( \frac{dr}{dt} \right)^2 + \frac{h^2}{r^2} \right].
\] (5.3)

where \( h^2 = c^2 r^2 \sin^2 \psi \) for orbiting light. \( \psi \) can vary from \( \pi/2 \) to \( (\pi/2 - \kappa) \) where \( \kappa \) is a very small angle. For circular orbits, \( \psi = \pi/2 = constant \) and \( dr/dt = 0 \). General relativity ignores angle \( \psi \). So we have

\[
v^2 = c^2 = \left\{ \left( \frac{dr}{dt} \right)^2 + c^2 \sin^2 \psi \right\}
\] (5.4)

\[
\left( \frac{dr}{dt} \right)^2 = c^2(1 - \sin^2 \psi) = c^2 \cos^2 \psi.
\] (5.5)

\[
\left( \frac{dv}{dt} \right) = v_r = c \cos \psi.
\] (5.6)

From Eq. (5.6) we can see that for \( \psi > \pi/2 \) the radial velocity vector \( v_r \) becomes negative implying beginning of the inspiraling of the light wave. This effect is not detectable in general relativity with the Schwarzschild radius. This effect can also be used in analyzing the inspiraling of the binary black holes.

\[
v^2 = c^2 = c^2(\cos^2 \psi + \sin^2 \psi)
\] (5.7)
Latus rectum of the elliptical orbit is given by
\[
\frac{h^2}{\mu} = a(1 - \epsilon^2), \tag{5.8}
\]
where \(a\) is the semi-major axis of ellipse and \(\epsilon\) is the eccentricity. \(\mu = GM_0\). Substitution of Eq. (5.8) in Eq. (5.3) gives,
\[
\left(\frac{d\tau}{dt}\right)^2 = \left[1 - \left(\frac{n}{c^2}\right)\left(\frac{dr}{dt} + \frac{\mu a(1 - \epsilon^2)}{r^2}\right)^2\right]. \tag{5.9}
\]
As an approximation we can ignore \(dr/dt\) term as negligibly small. Thus we get,
\[
\left(\frac{d\tau}{dt}\right)^2 = \left[1 - \left(\frac{n}{c^2}\right)\frac{\mu a(1 - \epsilon^2)}{r^2}\right]. \tag{5.10}
\]
For evaluating inspiraling binary blackholes, deviation factor \(n\) in PR line element given by Eq. (5.1) can be defined as a ratio of accelerations given below, which is a unitless constant and satisfies Einstein’s field equations.
\[
n = \frac{Gm_1}{r^2} (\cos\psi + \sin\psi) \left(\frac{v^2}{r}\right)^{-1}, \tag{5.11}
\]
where \(m_1, m_2\) are black hole masses, coordinate system fixed on center of mass \(m_1\), \(v\) is the orbital velocity of \(m_2\) and \(r\) is the separation between two. Factor \((\cos\psi + \sin\psi)\) gives geodesic like trajectories in PR.

Event Horizon Telescope (EHT) data for M87 black hole \cite{31, 32} suggest an assymetrical photon ring which our theory can associate with the elliptical parameters of Eq. (5.10). Here we will assume a circular orbit for the photon which reduces Eq. (5.10) to
\[
\left(\frac{d\tau}{dt}\right)^2 = \left[1 - \left(\frac{n\mu}{rc^2}\right)\right]. \tag{5.12}
\]
Eq. (5.12) does not become singular for \(r = r_l = (n\mu/c^2) = nR_g\), where \(r_l\) is the limiting radius of the event horizon of a non rotating black hole. This is because for photon, \(ds^2 = (cd\tau)^2 = 0\). Light travels along a null geodesic. In PR, deviation factor \(n\) can only be established experimentally because it is associated with the natural frequency of the orbiting body which depends on its material constitution, relative velocity, kind of trajectory, spin and gravitational acceleration acting on it. Event Horizon Telescope (EHT) data for M87 black hole \cite{31, 32} does not give us limiting radius of the event horizon. What we get from EHT is the photon capture radius \(R_c\) corresponding to the impact parameter of the light ray approaching black hole from infinity. Therefore we defer our conclusion on the value of \(n = n_e\) and \(r_l = n_eR_g\) in Eq. (5.12) till we have analyzed the impact parameter calculations as per the PR theory.
1. Impact parameter of light ray

In order to calculate the impact parameter of light ray approaching black hole from infinity, we have to consider the case when $dr/dt \neq 0$ in Eq. (5.3). For $d\phi = 0$, we can write,

$$c^2 d\tau^2 = c^2 dt^2 - n dt^2 \left\{ \left( \frac{dr}{dt} \right)^2 + \frac{\hbar^2}{r^2} \right\}. \quad (5.13)$$

We have the quantity in bracket equal to $v^2$ and

$$\frac{d\tau}{dt} = \frac{m_0}{m} = \text{Rest mass} \div \text{Relativistic mass}. \quad (5.14)$$

Substitution in Eq. (5.13) gives,

$$c^2 m_0^2 = m^2 c^2 - n (mv)^2. \quad (5.15)$$

For photon $m_0 = 0$ and $v = c$. This gives

$$0 = \frac{m^2 c^4}{c^2} - n (mc)^2. \quad (5.16)$$

$$0 = \frac{E^2}{c^2} - np^2, \quad (5.17)$$

where $E$ and $p$ are energy and momentum of photon. This gives,

$$\frac{c^2 p^2}{E^2} = \frac{1}{n} \quad (5.18)$$

If we introduce angular momentum $L = p \times r$ in Eq. (5.18), we get the impact parameter $b$ given by,

$$\frac{c^2 L^2}{E^2} = b^2 = \frac{r^2}{n} = \frac{r}{n} \left( \frac{rc^2}{c^2} \right). \quad (5.19)$$

Here we will introduce the boundary condition corresponding to $dr/dt = 0$, when $r = r_t = n_e R_g$ and $c^2/r = \mu/r^2$. This gives,

$$\frac{c^2 L^2}{E^2} = b^2 = \frac{r}{n} \left( \frac{rc^2}{\mu} \right) = \frac{r^3}{nR_g} = \frac{(n_e R_g)^3}{nR_g}. \quad (5.20)$$

As per EHT data the angular radius of the observed photon ring of M87 black hole is approximately $20\mu as$ which is close to the photon capture radius $b = R_c = \sqrt{27} R_g$ of a non rotating Schwarzschild black hole. Hence we can write Eq. (5.20) as,

$$b^2 = \frac{(n_e R_g)^3}{nR_g} = 27 R_g^2. \quad (5.21)$$
For flat Minkowski metric we have \( n = 1 \). This gives,

\[
n_e = 3. \tag{5.22}
\]

Thus we are now able to predict the limiting radius of the event horizon of the M87 Black hole from Eq. \((5.12)\) to be,

\[
r_l = n_e R_g = 3R_g. \tag{5.23}
\]

**B. Spin of the M87 black hole**

We can introduce a constant parameter \( z \) in Eq. \((5.12)\) to account for the mass equivalent of the rotation energy of the black hole. This will add an additional mass to parameter \( \mu \) given by

\[
\mu(1 + z) = GM_0(1 + z). \tag{5.24}
\]

Eq. \((5.24)\) eliminates the need for having a special line element to account for spin like the Kerr metric \[33\]. This is because our theory is an energy based theory and not a geometrical theory. For analyzing spin of the black hole, here we will assume circular orbits at the photon capture radius. Then Eq. \((5.12)\) assumes the form,

\[
\left(\frac{d\tau}{dt}\right)^2 = 1 - \left(\frac{\sqrt{27}(1 + z)\mu}{rc^2}\right). \tag{5.25}
\]

Thus the photon ring radius can be given by

\[
r_p = \sqrt{27}(1 + z)R_g. \tag{5.26}
\]

EHT data \[31, 32\] provides M87 photon ring diameter with a unitless parameter \( \alpha \) having a range

\[
10.7 \leq \alpha \leq 11.5. \tag{5.27}
\]

This gives

\[
r_p = \frac{\alpha}{2}R_g. \tag{5.28}
\]

Combining Eqs. \((5.26)\) and \((5.28)\) gives

\[
z = \left(\frac{\alpha}{2\sqrt{27}} - 1\right). \tag{5.29}
\]
1. Spin energy of black hole

Energy of rotation is given by

$$W_{\text{rot}} = \left( \frac{J \omega^2}{2} \right), \quad (5.30)$$

where angular velocity $\omega = 2\pi N$. $N$ in sec$^{-1}$. $J$ is the moment of inertia of the spherical core of the black hole with a radius $r = x r_p$ given by

$$J = \left( \frac{2}{5} M_0 x^2 r_p^2 \right), \quad (5.31)$$

where $x \leq 1$ is a constant. Here we select $x = 1$. Substitution of Eq. (5.31) in Eq. (5.30) gives

$$W_{\text{rot}} = \left( \frac{M_0}{5} \right) (2\pi r_p N)^2. \quad (5.32)$$

Energy of rotation in Eq. (5.32) can be represented by equivalent mass $M_s$.

$$W_{\text{rot}} = M_s c^2 = z M_0 c^2. \quad (5.33)$$

Combining Eqs. (5.32) and (5.33) gives

$$2\pi r_p N = c\sqrt{5z}. \quad (5.34)$$

Condition that nothing exceeds speed of light constrains Eq. (5.34) to

$$2\pi r_p N = c\sqrt{5z} \leq c. \quad (5.35)$$

This introduces the spin parameter $a_*$ in Eq. (5.33) as

$$2\pi r_p N = c\sqrt{5z} = a_* c. \quad (5.36)$$

where $-1 \leq a_* \leq +1$. This gives us two relations.

$$a_* = \pm \sqrt{5z}. \quad (5.37)$$

$$N = \frac{a_* c}{2\pi r_p}. \quad (5.38)$$

Substitution of Eq. (5.29) in Eq. (5.37) gives

$$a_* = \pm \sqrt{5} \left( \frac{\alpha}{2\sqrt{27}} - 1 \right)^{1/2}. \quad (5.39)$$
Inserting values of $\alpha$ from Eq. (5.27) gives the range of prograde and retrograde spin $a_*$ between $\pm 0.385$ and $\pm 0.73$. The lower bound on spin given here is very close to the same proposed by Nemmen with his jet power model [34].

Schwarzschild singularity is similar to one presented here by Eq. (5.12), but in GR the value of $n$ is fixed at 2 through weak field approximation which results in curved space time with linear time. In PR value of $n$ is not constrained to a fixed value but is dependent on natural frequency of the orbiting body which can be different at event horizon and at photon capture radius. Besides time is considered periodic, related to periods of the oscillations at the fundamental level. Oscillations are real and physical but time is not. The factor of 2 in the Schwarzschild radius is also a cause for the accurate prediction of perihelic precession of Mercury in general relativity. But the same factor has constrained GR to local invariance of speed of light. Because of this fixed value of 2 in GR, it is not possible in GR to come up with the spin solution presented here. The value of $\sqrt{27}$ comes from the photon ring observation data and the photon impact parameter analysis which depends on the energy level of the approaching light ray. In PR the metric does not become singular at event horizon for Minkowski flat space time corresponding to $n = 1$. A greater accuracy in measurement of photon ring diameter can increase the accuracy in predicting the spin. Probably this is the only theory that puts an upper bound on the spin.

6. EINSTEIN’S FIELD EQUATIONS

Now we are in a position to write Eq. (1.2) in a metric form as follows.

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu.$$  

(6.1)

$$g_{\mu\nu} = \begin{pmatrix} c^2 & 0 & 0 & 0 \\ 0 & -n & 0 & 0 \\ 0 & 0 & -nr^2 & 0 \\ 0 & 0 & 0 & -n(r^2 \sin^2 \theta) \end{pmatrix}.$$  

(6.2)

We can adjust the value of deviation factor $n$ to match the observed value of the perihelic precession for individual planets. As a matter of fact all future strong field variations in general relativity could be explained by adjusting this parameter $n$. This kind of adjustment
is not possible in general relativity and other metric theories because that will affect the predicted values of deflection of light, gravitational redshift and the limiting radius of event horizon. This factor \( n \) may have an internal structure dependent on the natural frequency and composition of the orbiting body (gravitational frequency shift of the constituent massive particles of the body). If we alter this factor \( n \) in Eq. (4.79) with any suitable constant then it will always satisfy Einstein’s field equations.

Here we verify to see that Eq. (1.2) does satisfy Einstein’s field equations. For this purpose it will be necessary to calculate Christoffel symbols \( \Gamma^\sigma_{\mu\nu} \). At the same time the proper time interval should be experimentally verified because all deviations and variations get accumulated in the expression for proper time and any error in the theory would show up there as well.

The metric (6.2) is diagonal, so the non-zero components of the contravariant metric tensor are \( g^{\sigma\sigma} = 1/g_{\sigma\sigma} \). Hence the diagonality of the metric allows us to simplify the definition of the Christoffel symbols to

\[
\Gamma^\sigma_{\mu\nu} = \frac{1}{2} g^{\sigma\sigma} \left( \frac{\partial g_{\sigma\mu}}{\partial x^\nu} + \frac{\partial g_{\sigma\nu}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\sigma} \right),
\]

(6.3)

where the suffixes assume four values 0, 1, 2, 3 and no summations are implied. We consider the case of static spherically symmetric field produced by a spherically symmetric body at rest. Line element given by Eq. (1.2) is compatible with spherical symmetry. Coordinate \( x^0 \) is taken to be time \( t \), and the spatial coordinates may be taken to be spherical polar coordinates \( x^1 = r, x^2 = \theta, x^3 = \phi \). We can determine the values of \( g_{\mu\nu} \) from metric (6.2),

\[
g_{00} = c^2, \quad g_{11} = -n, \quad g_{22} = -nr^2, \quad g_{33} = -nr^2 \sin^2 \theta,
\]

\[
g^{\mu\nu} = 1/g_{\mu\nu} \quad \text{and} \quad g_{\mu\nu} = 0 \quad \text{for} \quad \mu \neq \nu.
\]

(6.4)

Inserting these values into Eq. (6.3) we find that the only non-vanishing Christoffel symbols are

\[
\Gamma^1_{11} = \frac{1}{2n} \frac{\partial n}{\partial r}, \quad \Gamma^2_{33} = -\sin \theta \cos \theta
\]

\[
\Gamma^1_{22} = -r - \frac{r^2}{2n} \frac{\partial n}{\partial r}, \quad \Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{r} + \frac{1}{2n} \frac{\partial n}{\partial r}
\]

\[
\Gamma^1_{33} = -r \sin^2 \theta \left( 1 + \frac{r}{2n} \frac{\partial n}{\partial r} \right), \quad \Gamma^3_{23} = \Gamma^3_{32} = \cot \theta.
\]

(6.5)
The expression for the Ricci tensor is
\[ R_{\mu\nu} = \Gamma_{\mu\alpha,\nu}^\alpha - \Gamma_{\mu\nu,\alpha}^\alpha - \Gamma_{\mu\nu}^\alpha \Gamma_{\alpha\beta}^\beta + \Gamma_{\mu\beta}^\alpha \Gamma_{\nu\alpha}^\beta. \] (6.6)

Einstein’s law of gravitation requires the Ricci tensor to vanish \( R_{\mu\nu} = 0 \) in empty space. We can now write the components of the Ricci tensor, each of which must vanish in order for the field equations to be satisfied. From symmetry arguments we can expect all the non-diagonal components to be zero. Hence the only components of interest in case of our line element are the diagonal elements. Substitution of Eq. (6.5) in Eq. (6.6) gives
\[ R_{00} = 0, \] (6.7)
\[ R_{11} = \frac{1}{nr} \frac{\partial n}{\partial r} - \frac{1}{n} \left( \frac{\partial n}{\partial r} \right)^2 + \frac{1}{n} \frac{\partial^2 n}{\partial r^2}, \] (6.8)
\[ R_{22} = \frac{3r}{2n} \frac{\partial n}{\partial r} - \frac{r^2}{4n^2} \left( \frac{\partial n}{\partial r} \right)^2 + \frac{r^2}{2n} \frac{\partial^2 n}{\partial r^2}, \] (6.9)
\[ R_{33} = R_{22} \sin^2 \theta. \] (6.10)

The vanishing of Eq. (6.8) leads to
\[ \frac{\partial^2 n}{\partial r^2} = \frac{1}{n} \left( \frac{\partial n}{\partial r} \right)^2 - \frac{1}{r} \frac{\partial n}{\partial r}. \] (6.11)

Substituting of Eq. (6.11) in Eq. (6.9) and equating it to zero gives the condition for vanishing of the Ricci tensor Eq. (6.6).
\[ \left( \frac{r}{n} \frac{\partial n}{\partial r} \right)^2 + 4 \left( \frac{r}{n} \frac{\partial n}{\partial r} \right) = 0. \] (6.12)

This quadratic equation has two solutions.
\[ \left( \frac{r}{n} \frac{\partial n}{\partial r} \right) = 0 \quad \text{and} \quad \left( \frac{r}{n} \frac{\partial n}{\partial r} \right) = -4. \] (6.13)

This shows that any constant value of \( n \) will satisfy the first solution. This means that our derivation of gravitational redshift, deflection of light, perihelic precession of planets and the limiting radius of event horizon of a black hole are exact solutions of Einstein’s field equations. These solutions however are at variance with the Schwarzschild solution.

7. **ROTATION CURVES OF GALAXIES**

In sec.6 we obtained two solutions to Einstein’s field equations,
\[ \left( \frac{r}{n} \frac{\partial n}{\partial r} \right) = 0 \quad \text{and} \quad \left( \frac{r}{n} \frac{\partial n}{\partial r} \right) = -4. \] (7.1)
So far we have seen the application of the first solution which requires \( n \) to be any real number constant. Now we look at the second solution which we can write as

\[
\int \frac{\partial n}{n} = -4 \int \frac{\partial r}{r}. \quad (7.2)
\]

\[
\ln(nr^4) = C. \quad (7.3)
\]

where \( C \) is a constant of integration. This gives

\[
n = \frac{e^C}{r^4} = \frac{k}{r^4}. \quad (7.4)
\]

where \( k = e^C = \text{constant}. \quad (7.5)\]

In this second solution \( n \) need not be a constant. We make use of Eq. (4.29) in order to apply the second solution to rotation curves of a galaxies. Assuming circular orbit we substitute \( \psi = \pi/2 \) and \( \gamma = 0 \). This gives

\[
|a| = \kappa \left( \frac{ds}{dt} \right)^2 = \frac{v^2}{r} = \frac{\mu}{r^2}. \quad (7.6)
\]

Then we define unitless deviation factor \( n \) as a ratio of Newtonian acceleration to the observed acceleration given by

\[
n = \frac{\mu/r^2}{v^2/r}. \quad (7.7)
\]

This ratio is \( n = 1 \) for circular orbits in flat Minkowski space time. Equating Eq. (7.7) with Eq. (7.4) we get,

\[
n = \frac{\mu}{v^2r} = \frac{k}{r^4}. \quad (7.8)
\]

This gives,

\[
k = \frac{\mu r^3}{v^2}. \quad (7.9)
\]

Value of \( k \) given by Eq. (7.9) is a constant of orbit. This means that every star orbit in a galaxy will have its own constant \( k \). We can write Eq. (7.8) as

\[
v^2 = 4\pi^2 r^2 = \frac{\mu}{nr}. \quad (7.10)
\]
\[ P = \frac{2\pi r}{v}. \]  
\[ (7.11) \]

\[ P^2 = \frac{4\pi^2 r^3 n}{\mu}. \]  
\[ (7.12) \]

For \( n = 1 \), Eq. (7.12) reduces to Kepler’s third law, where \( P \) is the orbital period. By substituting Eq. (7.8) in Eq. (1.1) we can compute the ratio \( d\tau/dt \). We can apply these equations of stellar motion to Blue Horizontal-Branch (BHB) halo stars of the Milky Way [35]. The circular velocity estimates are based on Naab’s simulation [36]. To this data, one additional data point for solar radius of 8\( kpc \) [37] is added and the results obtained from Eqs. (7.9), (7.4) and (1.1) are shown in Table I. Computed values are based on the stellar mass at the galactic center, which is \( 5.0924 \times 10^{10} M_\odot \) [38, 39]. Observed values of \( r \) and circular velocities constrain the integration constant \( k \) which provides a measure of non-uniform distribution of the galactic matter and the cold dark matter at a given radius. Hence it is appropriate to describe \( k \) as a galactic matter distribution constant. We also find that Eqs. (7.11) and (7.12) both yield exactly the same orbital period when velocity and deviation \( n \) along with the galactic stellar mass are used from the Tables. For the Sun, both yield 223.4 million years.

Table II shows solar system data from NASA planet fact sheets. Radial distance equal to semi major axis and mean orbital velocity are used. \( k \) and \( n \) are computed using Eqs. (7.9) and (7.4). \( 1 - d\tau/dt \) are of order \( 10^{-8} \) to \( 10^{-12} \) and not shown in the table. In case of moon, earth mass \( 5.9736 \times 10^{24} \) Kg. is used. Value of \( n \) for Mercury shown in Table II should not be compared with that used in the Eq. (1.79) for the perihelic precession of planets because here we have used second solution of Einstein’s field equations with constant \( k \), where as perihelic precession is derived from the first solution of Einstein’s field equations with constant \( n \). These two solutions are derived from two roots of a quadratic equation. The purpose of presenting the solar system data is only to show that there is no discontinuity like the MOND function. One should not look for precision in Table II because it is based on circular orbit approximation. It is sufficient to note that \( n = 1 \) for flat Minkowski metric is recovered at small distances.

We can also apply these equations of stellar motion to rotation curves of M31 [40] and NGC3198 [41]. The results obtained from Eqs. (7.9), (7.4) and (1.1) are shown in Tables III and IV. Computed values are based on the stellar mass at the galactic center, which is
### TABLE I: Milky Way rotation curve based on proper time.

| $r$ (kpc) | $v$ (km/s) | $k \times 10^{-8}$ | $n$ | $d\tau/dt$ |
|-----------|-----------|-------------------|-----|------------|
| 7.5       | 216       | 1.79546           | 0.62593 | $1 - 1.6246 \times 10^{-7}$ |
| 8.0       | 220       | 2.10050           | 0.56566 | $1 - 1.5231 \times 10^{-7}$ |
| 12.5      | 227       | 7.52624           | 0.34004 | $1 - 9.748 \times 10^{-8}$ |
| 17.5      | 179       | 33.2129           | 0.39061 | $1 - 6.9628 \times 10^{-8}$ |
| 22.5      | 168       | 80.1362           | 0.34490 | $1 - 5.4155 \times 10^{-8}$ |
| 27.5      | 183       | 123.309           | 0.23782 | $1 - 4.43091 \times 10^{-8}$ |
| 32.5      | 143       | 333.332           | 0.32956 | $1 - 3.7492 \times 10^{-8}$ |
| 37.5      | 170       | 362.322           | 0.20210 | $1 - 3.2493 \times 10^{-8}$ |
| 42.5      | 183       | 455.160           | 0.15388 | $1 - 2.8670 \times 10^{-8}$ |
| 47.5      | 165       | 781.650           | 0.16936 | $1 - 2.5652 \times 10^{-8}$ |
| 55        | 183       | 986.474           | 0.11891 | $1 - 2.2154 \times 10^{-8}$ |

1.4 x 10$^{11}$ $M_\odot$ for M31 and 5.0 x 10$^9$ $M_\odot$ for NGC3198.

**A. Perihelic precession of electron in hydrogen atom**

Periodic relativity is a theory of accelerations. Whether the acceleration is due to gravity or Coulomb force, it does not matter. In case of rotation curves of galaxies, we defined deviation factor $n$ as a unitless ratio of accelerations given by Eq. (7.7). We used the same method to define the ratio of Coulomb acceleration and radial acceleration of electron to obtained energy levels of hydrogen atom [42] without the use of potential energy term of the Schrodinger and Dirac wave equations. Here we apply the same formalism of sec. 4.F.1 for perihelic precession of planets to analyze the perihelic precession of electron in hydrogen atom.

We have defined $\bar{n}$ for Hydrogen atom as the ratio of acceleration due to Coulomb force and the centrifugal acceleration which is indirectly observed in the form of spectra [42].

$$\bar{n} = \frac{-e^2 kZ/m_0 r^2}{v^2/r} = \frac{-e^2 kZ}{m_0 v^2 r},$$

(7.13)

where $e$ is the electric charge, $m_0$ the electron rest mass, $k$ Coulomb’s constant, $Z$ atomic number (1 for Hydrogen), $v$ is the orbital velocity of electron, and $r$ the radial distance. We
also discovered that in our relativistic formalism the orbital velocity of electron to the first order accuracy is about 30% less than that in the Bohr model and is given by,

\[ v \approx \left( \frac{e^2 k Z}{\sqrt{2} \hbar n} \right), \quad (7.14) \]

where \( \hbar \) is the Planck constant and \( n \) the principal quantum number. If we substitute Eq. (7.14) and \( r = a_0 n^2 \) in Eq. (7.13), we get \( \bar{n} = 2 \). But if we substitute Bohr velocity then we get \( \bar{n} = 1 \). For this reason we cannot start our formalism like Eq. (4.59), but we have
TABLE IV: NGC3198 rotation curve.  $k$ in m$^4$, $P$ in yrs.

| $r$ (kpc) | $v$ (km/s) | $k \times 10^{-79}$ | $n$ | $d\tau/dt$ | $P \times 10^{-8}$ |
|-----------|------------|-------------------|-----|-------------|-----------------|
| 0.68      | 55         | 0.202             | 10.45 | $1 - 1.76 \times 10^{-7}$ | 0.759           |
| 1.36      | 92         | 0.579             | 1.868 | $1 - 8.79 \times 10^{-8}$ | 0.908           |
| 2.72      | 123        | 2.593             | 0.522 | $1 - 4.39 \times 10^{-8}$ | 1.358           |
| 5.44      | 147        | 14.52             | 0.183 | $1 - 2.2 \times 10^{-8}$  | 2.273           |
| 8.16      | 156        | 43.52             | 0.108 | $1 - 1.466 \times 10^{-8}$ | 3.213           |
| 13.6      | 154        | 206.78            | 0.066 | $1 - 8.79 \times 10^{-9}$ | 5.425           |
| 19.04     | 148        | 614.36            | 0.0515 | $1 - 6.28 \times 10^{-9}$ | 7.903           |
| 24.48     | 148        | 1305.7            | 0.040 | $1 - 4.88 \times 10^{-9}$ | 10.16           |
| 29.92     | 149        | 2352.1            | 0.0323 | $1 - 3.99 \times 10^{-9}$ | 12.33           |

To start from fundamentals by first establishing the principle of equivalence as applicable to hydrogen atom. Equivalence of gravitational mass and inertial mass in Einstein’s theory follows from equivalence of gravitational acceleration and inertial acceleration. In a specific case of orbital motion of planets, we can restrict this definition of equivalence of accelerations to following statement.

- In Keplerian circular orbit, the gravitational acceleration is equal and opposite to the centrifugal acceleration acting on the body.

Newton started out with this equality and introduced Kepler’s third law of orbital periods in it and thus arrived at the inverse square law of gravitation. From this we can define the principle of equivalence as applicable to Hydrogen atom.

- In Keplerian circular orbit, the electromagnetic acceleration is equal and opposite to the centrifugal acceleration acting on the body.

This definition of equivalence of accelerations is not possible in Schrodinger or Dirac theory because they have abandoned the velocity parameter. Thus we can write,

$$\frac{d^2\mathbf{r}}{dt^2} = -\frac{v^2}{r}\mathbf{\hat{r}} = -\frac{ke^2Z}{2m_0r^2}\mathbf{\hat{r}}. \quad (7.15)$$

$$m_0\frac{d^2\mathbf{r}}{dt^2} = -\frac{ke^2Z}{2r^2}\mathbf{\hat{r}}. \quad (7.16)$$

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Eq. (7.16) gives inverse square law of the electromagnetic attraction which is not same as the Coulomb’s law of electrostatic attraction. Now we introduce the line element of periodic relativity Eq. (4.60) in Eq. (7.15). With this we replace the coordinate time interval $dt$ of Eq. (7.15) with the proper time interval $d\tau$.

\[ \frac{d^2 \mathbf{r}}{d\tau^2} (1 - \bar{n}^2 \beta^2) = -\frac{ke^2 Z}{2m_0 r^2} \hat{r}. \] (7.17)

\[ \frac{d^2 \mathbf{r}}{d\tau^2} = - \left( \frac{ke^2 Z}{2m_0 r^2} \right) (1 - \bar{n}^2 \beta^2)^{-1} \hat{r}. \] (7.18)

\[ \frac{d^2 \mathbf{r}}{d\tau^2} = - \left( \frac{ke^2 Z}{2m_0 r^2} \right) (1 + \bar{n}^2 \beta^2) \hat{r}. \] (7.19)

We will write $v^2$ on r.h.s. in polar coordinates.

\[ \frac{d^2 \mathbf{r}}{d\tau^2} = - \left( \frac{ke^2 Z}{2m_0 r^2} \right) \hat{r} - \left( \frac{\bar{n}ke^2 Z}{2m_0 r^2 c^2} \right) \left[ \left( \frac{dr}{dt} \right)^2 + r^2 \left( \frac{d\theta}{dt} \right)^2 + r^2 \sin^2 \theta \left( \frac{d\phi}{dt} \right)^2 \right] \hat{r}. \] (7.20)

Working in $(r, \theta)$ plane we can put $d\phi = 0$ and on r.h.s. substitute

\[ r^2 \left( \frac{d\theta}{dt} \right)^2 = \frac{H^2}{r^2}. \] (7.21)

Hence Eq. (7.20) reduces to

\[ \frac{d^2 \mathbf{r}}{d\tau^2} = - \left( \frac{ke^2 Z}{2m_0 r^2} \right) \hat{r} - \left( \frac{\bar{n}ke^2 Z}{2m_0 r^2 c^2} \right) \left[ \left( \frac{dr}{dt} \right)^2 + \frac{H^2}{r^2} \right] \hat{r}. \] (7.22)

On l.h.s. we substitute

\[ \frac{d^2 \mathbf{r}}{d\tau^2} = \left( \frac{d^2 \mathbf{r}}{d\tau^2} - \frac{H^2}{r^3} \right) \hat{r}. \] (7.23)

\[ \left( \frac{d^2 \mathbf{r}}{d\tau^2} - \frac{H^2}{r^3} \right) = - \left( \frac{ke^2 Z}{2m_0 r^2} \right) - \left( \frac{\bar{n}ke^2 Z}{2m_0 r^2 c^2} \right) \left( \frac{dr}{dt} \right)^2 - \left( \frac{\bar{n}ke^2 Z H^2}{2m_0 r^4 c^2} \right). \] (7.24)

\[ \frac{d^2 \mathbf{r}}{d\tau^2} = - \left( \frac{ke^2 Z}{2m_0 r^2} \right) - \left( \frac{\bar{n}ke^2 Z}{2m_0 r^2 c^2} \right) \left( \frac{dr}{dt} \right)^2 - \left( \frac{\bar{n}ke^2 Z H^2}{2m_0 r^4 c^2} \right) + \frac{H^2}{r^3}. \] (7.25)

On the l.h.s. of Eq. (7.25) we have the proper time and on the r.h.s. we have the coordinate time. To obtain the time independent solution of the equation of motion we make the
following substitution which gives a second order non-homogeneous, non-linear differential equation.

\[ u = \frac{1}{r}, \quad \text{and} \quad \frac{d}{d\tau} = \frac{d}{dt} = Hu \frac{d}{d\theta}, \quad (7.26) \]

\[ -H^2 u^2 \frac{d^2 u}{d\theta^2} = -\left( \frac{ke^2 Z}{2m_0} \right) u^2 - \left( \frac{\bar{n}ke^2 Z H^2}{2m_0c^2} \right) u^4 + H^2 u^3 - \left( \frac{\bar{n}ke^2 Z}{2m_0c^2} \right) u^2 \left( -u \frac{du}{d\theta} \right)^2. \quad (7.27) \]

\[ \frac{d^2 u}{d\theta^2} + u = \left( \frac{ke^2 Z}{2m_0 H^2} \right) + \left( \frac{\bar{n}ke^2 Z}{2m_0c^2} \right) u^2. \quad (7.28) \]

Using the argument of Eq. (4.78) we can substitute,

\[ \left( \frac{du}{d\theta} \right)^2 = 0. \quad (7.29) \]

This gives,

\[ \frac{d^2 u}{d\theta^2} + u = \left( \frac{ke^2 Z}{2m_0 H^2} \right) + \left( \frac{\bar{n}ke^2 Z}{2m_0c^2} \right) u^2. \quad (7.30) \]

We define the constant,

\[ \eta = \left( \frac{ke^2 Z}{2m_0} \right). \quad (7.31) \]

\[ \frac{d^2 u}{d\theta^2} + u = \left( \frac{\eta}{H^2} \right) + \left( \frac{\bar{n}\eta}{c^2} \right) u^2. \quad (7.32) \]

Eq. (7.32) is similar to Eq. (4.79) and can be solved as in general relativity. Here we have \( \bar{n} = 2 \). Solution of Eq. (7.32) in the non-relativistic limit can be obtained by putting \( \bar{n} = 0 \) and is given by

\[ u^{(0)} = \frac{\eta}{H^2} [1 + \epsilon \cos(\theta - \theta_0)]. \quad (7.33) \]

We can identify \( H^2/\eta \) with the semi latus rectum of the ellipse and \( \epsilon \) with the eccentricity. For circular orbits Eq. (7.33) reduces to classical Bohr orbits.

\[ u^{(0)} = \frac{1}{r} = \frac{1}{a_0 n^2}. \quad (7.34) \]

Substituting \( u^{(0)} \) back into the last term of Eq. (7.32) gives an approximate differential equation which can be solved to give following approximate relativistic solution with terms significant to perihelic precession.

\[ u = \frac{\eta}{H^2} [1 + \epsilon \cos(\theta - \theta_0)] + \left( \frac{\bar{n}\eta c}{c^2} \right) \left( \frac{\eta^2}{H^4} \right) \theta \sin(\theta - \theta_0). \quad (7.35) \]
By ignoring higher order terms in $c$, Eq. (7.35) becomes,

$$u = \frac{\eta}{H^2}[1 + \epsilon \cos(\theta - \theta_0 - k\theta)]$$  \hfill (7.36)

where,

$$k\theta = \left(\frac{\tilde{n}\eta^2}{c^2}\right) \left(\frac{\eta}{H^2}\right) = \left(\frac{\tilde{n}\eta^2}{c^2 H^2}\right).$$  \hfill (7.37)

Therefore in relativity, the electron orbit undergoes angular precession given by Eq. (7.37). This value of angular precession for $\tilde{n} = 2$, $\theta = 2\pi$ and $Z = 1$ is given per orbital period $T$ by,

$$\Delta \omega = \left(\frac{4\pi}{c^2 a(1 - \epsilon^2) T}\right) \left(\frac{ke^2}{2m_0}\right).$$  \hfill (7.38)

For near circular orbit, we can put $\epsilon = 0$ and $a = a_0n^2$ which gives,

$$\Delta \omega = \left(\frac{2\pi}{c^2 a_0 n^2 T}\right) \left(\frac{ke^2}{m_0}\right) = \left(\frac{2\pi \alpha^2}{n^2 T}\right).$$  \hfill (7.39)

where $\alpha$ is the fine structure constant.

**B. Orbital period of electron in hydrogen atom**

We use Eq. (7.11) to derive the orbital period of electron in the hydrogen atom. Substitution of velocity from Eq. (7.14) and Bohr orbits from Eq. (7.34) gives,

$$P = \left(\frac{2\sqrt{2}\pi (hn)^3}{m_0 (ke^2 Z)^2}\right).$$  \hfill (7.40)

$$P = \left(\frac{\sqrt{2}hn^3}{m_0 c^2 \alpha^2}\right),$$  \hfill (7.41)

where $\alpha$ is the fine structure constant. Rydberg constant is given by,

$$R_\infty = \left(\frac{m_0 c^2 \alpha^2}{2hc}\right).$$  \hfill (7.42)

Substitution in Eq. (7.41) gives orbital period of electron in hydrogen atom,

$$P = \left(\frac{n^3}{\sqrt{2}c R_\infty}\right),$$  \hfill (7.43)

where $n$ is the principle quantum number. Here periodic nature of time becomes evident. Time makes a quantum jump proportional to cube of the principle quantum number. Now $n$ is not just an integer constant but a function of orbital period of electron and therefore time dependent. And space dependent through relation $r = a_0 n^2$.  

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We have from Eqs. (4.3) and (4.48),
\[ (E - m_0c^2) = \Phi m_0 = - \int \frac{\mu m}{r^2} (\cos \psi + \sin \psi) \, dr. \] \hspace{1cm} (8.1)

For cosmological application we are only interested in radial motions hence we take \( \psi = 0 \).

Secondly for small radial motions we assume \( \gamma \approx \text{const.} \) which gives
\[ mc^2 - m_0c^2 = \frac{\mu}{r} \gamma m_0, \] \hspace{1cm} (8.2)
\[ \{1 - (1/\gamma)\}c^2 = \frac{\mu}{r}, \] \hspace{1cm} (8.3)

The energy-momentum invariant Eq. (1.8) gives
\[ \gamma = (m/m_0) = \pm \{1 - (v^2/c^2)\}^{-1/2}, \] \hspace{1cm} (8.4)

Here the \( \pm \) sign is due to the positive and negative energies of Dirac’s theory. Introduction of Eq. (8.4) in Eq. (8.3) gives
\[ c^2(1 \mp \{1 - (v^2/c^2)\}^{1/2}) = \frac{\mu}{r}, \] \hspace{1cm} (8.5)
\[ c^2 - v^2 = \left[ \frac{\mu}{rc} - c \right]^2, \] \hspace{1cm} (8.6)
\[ c^2dt^2 - (dx^2 + dy^2 + dz^2) = \left[ \left( \frac{\mu}{rc} \right)^2 + c^2 - \frac{2\mu}{r} \right] \, dt^2 = ds^2, \] \hspace{1cm} (8.7)

Eq. (8.7) is simply the flat Minkowski metric given by Eq. (1.2) when \( n = 1 \), and this equation is based on the conservation of energy equation (8.1). For application in cosmology we can introduce deviation factor \( n \) in Eq. (8.7) and then assuming \((\mu/rc)^2\) to be negligibly small, the general line element satisfying the Weyl postulate and the cosmological principle can be given by
\[ ds^2 = c^2dt^2 - na^2 \left( \frac{dr^2}{1 - Kr^2} + r^2d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) = \left[ c^2 - \frac{2\mu}{ar} \right] \, dt^2, \] \hspace{1cm} (8.8)

where \( a(t) \) is the scale factor and parameter \( K \) is equal to +1 or 0 or -1 as in Friedmann model and decides the curvature of 3-surfaces. All the observable evidence indicate that
the universe is near flat corresponding to $K = 0$, so we introduce this value in Eq. (8.8) at
the outset. This and the fact that each galaxy has a constant set of coordinates $(r, \theta, \phi)$,
will considerably simplify the mathematics required for analyzing the model. For small and
constant values of $n$, line element Eq. (8.8) does satisfy Einstein’s field equation $R_{\mu\nu} = 0$.
We can write this equation as
\[ \frac{2\mu}{ar} dt^2 - na^2 \left( dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) = 0. \] (8.9)
This can be transformed to metric form as
\[
g_{\mu\nu} dx^\mu dx^\nu = 0, \quad \text{where} \quad g_{\mu\nu} = \begin{pmatrix}
2\mu/ar & 0 & 0 & 0 \\
0 & -a^2n & 0 & 0 \\
0 & 0 & -a^2nr^2 & 0 \\
0 & 0 & 0 & -a^2n(r^2 \sin^2 \theta)
\end{pmatrix}. \] (8.11)
where $dx^0 = dt$, $dx^1 = dr$, $dx^2 = d\theta$, $dx^3 = d\phi$. The metric Eq. (8.10) yields
\[
\nabla^2 [g_{\mu\nu} dx^\mu dx^\nu] = 0, \quad (8.12)
\]
where
\[
\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right). \] (8.13)
In order to analyze the expanding universe scenario, we can use Eq. (8.9) for a small radial
motion of the galaxy keeping $\theta$ and $\phi$ constant. This gives
\[
\frac{\mu}{ar} = \frac{na^2}{2} \left( \frac{dr}{dt} \right)^2 = \frac{na^2v^2}{2} \equiv \frac{n(Hr)^2}{2}. \] (8.14)
where $H = \dot{a}/a$ is Hubble parameter and deviation factor $n$ associated with this system can
conform to GR provided we select
\[
n = -\frac{1}{2} \left( 1 - \frac{\Lambda}{3H^2} \right). \] (8.15)
Here dark energy [43–48] associated with the cosmological constant $\Lambda$ is presumed to cause
deviation in the flat Minkowski metric. In GR $\Lambda$ gets introduced through the action principle.
Here $n$ is a unitless number. By introducing this deviation factor we are proposing that the
presence of uniformly distributed dark energy on a cosmological scale can cause redshift of all the constituent particles of a galaxy. This is because the dark energy causes accelerated expansion of the universe which is bound to affect the redshift of every galaxy. This factor is not accounted by the weak field approximation and the corresponding deviation to the flat Minkowski metric in GR. Since PR relates the proper time of a body with the frequency shift of all the constituent particles of a body, we are justified in proposing the deviation factor Eq. (8.15) which only alters the proper time interval of a galaxy without introducing any curvature. This deviation factor \( n \) remains constant for any given epoch but varies from epoch to epoch because it is a function of the Hubble parameter. Therefore \( n \) satisfies Einstein’s field equation \( R_{\mu \nu} = 0 \).

For the field point within the source of gravitation, in accordance with Poisson’s equation we get from Eq. (8.14) and (8.13),

\[
H^2 - \frac{\Lambda}{3} = \frac{8}{3} \pi G \rho.
\]

This is same as the Friedmann equation for flat universe [45, 46, 48]. Hence the critical density in this model when \( \Lambda = 0 \) comes out to be same as the Friedmann model

\[
\rho_c = \frac{3H^2}{8\pi G}.
\]

If we substitute \( t = 1/H \) and

\[
c^2 \rho = \frac{1}{2} g \sigma T^4,
\]

we get the time temperature relation

\[
t = \left( \frac{3c^2}{16\pi G g \sigma} \right)^{1/2} T^{-2},
\]

where \( t \) is the time of the epoch, \( g \) the \( g \) factor, \( \sigma \) radiation constant, \( T \) the temperature.

If we take time derivative of Eq. (8.14), we get for constant \( \Lambda \), the acceleration equation

\[
\left( \frac{\ddot{a}}{a} \right) - \frac{\Lambda}{3} = \frac{2GM_0}{r^3},
\]

For a point particle on a homogeneous sphere of radius \( r \) and energy density \( \rho \) Eq. (8.20) reduces to

\[
\left( \frac{\ddot{a}}{a} \right) - \frac{\Lambda}{3} = \frac{8}{3} \pi G \rho.
\]
Positive sign on the right imply accelerated expansion. Here we can introduce the equation of state \( w = p/\rho \) through the relation

\[
\rho \propto a^{-3(1+w)}.
\]

(8.22)

which yields the relations

\[
\dot{\rho} = -3H(\rho + p) \quad \text{and} \quad \dot{H} = -4\pi G(\rho + p).
\]

(8.23)

If we compare Eqs. (8.21) and (8.16), we find that \( \dot{H} = 0 \), which means that Eq. (8.21) is valid for \( w = -1 \). Therefore for other values of \( w \), we can introduce Eq. (8.23) in Eq. (8.21) which gives

\[
\left( \frac{\ddot{a}}{a} \right) - \frac{\Lambda}{3} = H^2 + \dot{H} - \frac{\Lambda}{3} = -\frac{4}{3} \pi G(\rho + 3p).
\]

(8.24)

Therefore accelerated expansion occurs for \( (\rho + 3p) < 0 \). Since \( H \) is constant for \( w = -1 \), we get the inflationary exponential expansion.

\[
a \propto e^{Ht}.
\]

(8.25)

Looking at the above results we find that the theory is in conformance with the GR cosmology and the \( \Lambda CDM \) model. For obtaining proper time interval of a galaxy we substitute Eq. (8.15) for constant \( n \) in Eq. (133) for the proper time interval where \( v \) is to be replaced by \( av = Hr \). This gives

\[
d\tau = dt \left( 1 + \frac{r^2}{4c^2} \left( H^2 - \frac{\Lambda}{3} \right) \right),
\]

(8.26)

\[
d\tau = dt \left( 1 + \frac{2\pi G\rho r^2}{3c^2} \right).
\]

(8.27)

Eq. (8.27) is valid for small values of \( v \) and this is where PR will differ from GR.

9. QUANTUM GRAVITY

I have discussed the quantum gravity theory (PQGC) [49] based on the periodic nature of time which has striking similarity to Friedmann equations. In this theory Hubble parameter is equated to Planck frequency at Planck epoch. Both have the units of frequency. So the
universe begins with the first oscillations when Hubble parameter begins to roll down from a very high energy value which is Planck energy. This generates field of particles with Planck energy. There is no separate equation for scale factor and exponential expansion, but the exponential term of the wave equation itself induces the exponential expansion which is continuous till the present epoch. The term equivalent to the scale factor is provided by the familiar separated time dependent function $f(t)$ of the wave equation. The superscript of the exponential term can be written in terms of energy $E$, or Hubble parameter $H$ for curvature $K = 0$. PQGC uses a modified Laplacian which includes particle spin parameter \cite{50}. The theory generates all the particle fields of the standard model from a single formula. The theory is based on unification of all particle fields into a single field at Planck epoch. Scale factor $a$ and cosmological constant $\Lambda$ are included in separated time dependent function $f(t)$ which can be seen through following relation. In PQGC \cite{49} we have derived expression,

$$\ddot{f} = \frac{B_1}{\hbar c^2 C^1} \bigg[ \frac{8\pi G \rho}{3} \bigg].$$

\text{(9.1)}

If we substitute Eq. (8.21) in Eq. (9.1) we get,

$$\ddot{f} = \frac{B_1}{\hbar c^2 C^1} \left[ \frac{\Lambda}{3} - \left( \frac{\ddot{a}}{a} \right) \right].$$

\text{(9.2)}

From Eq. (9.2) we can see that the expansion of universe is due to quantum field expanding and not due to the space expanding as in the GR. Dark energy (68.3\%) is the energy of quantum field which is not manifest as the particles, or manifest as hard to detect ground state particles. Quantum fields are expanding within the singular motionless fundamental substance of the universe which is infinite in extent, but the quantum fields are not infinite. So the zero point energy of the quantum fields can be restricted to match the cosmological constant as in Eq. (9.2) and this should solve the cosmological constant problem. The presence of the motionless infinite fundamental substance of the universe sets the same initial conditions everywhere including causally disconnected regions of space which explain the cosmological Horizon Problem. The theory works with or without the dark energy. James Web Space Telescope (JWST) has discovered large density of galaxies at high redshifts which puts tight constraints on the expansion history of the universe and rule out the major portion of the parameter space of the dark energy models \cite{51}.
A. Gravitation within electromagnetic wave

General relativity derivation of gravitational frequency shift of light does not provide the quantum mechanism of the frequency shift. However, with periodic time in PR it is possible to explain this quantum mechanism as follows. In an electromagnetic wave there is a stream of successive wavelets traveling at constant velocity of light. Each wavelet having energy equal to a quanta called photon. The mass equivalent of this energy is given by $E/c^2$. If we consider the gravitational attraction between two successive photons within the electromagnetic wave then we can write the expression for gravitational force between them as

$$F = \frac{G}{\lambda^2} \left( \frac{E}{c^2} \right)^2 \bar{v} = \left( ma + \frac{h\nu}{c^2} \frac{d\nu}{dt} \right), \quad (9.3)$$

where $G$ is the gravitational constant and $\lambda$ the distance between two successive photons. For light the classical acceleration $a = 0$ and $v = c$, but the de Broglie force is not zero. Eq. (9.3) reduces to

$$F = \frac{G}{\lambda^2} \left( \frac{E}{c^2} \right)^2 = \left( \frac{h\nu}{c} \frac{d\nu}{dt} \right), \quad (9.4)$$

Here the gravitational force $F$ on the left is always attractive but r.h.s. can be positive or negative depending on whether light is red shifted or blue shifted. This is like two binary stars ejecting lot of energy and moving away from each other or absorbing lot of energy and coming close to each other. In both cases gravity is always attractive. Substituting $\lambda = cT = c/\nu$ gives,

$$\frac{Gh}{c^5} = \frac{1}{\nu^4} \frac{d\nu}{dt}. \quad (9.5)$$

Now we can safely replace linear time increment $dt$ with the periodic time increment $dT = 1/d\nu$.

$$\frac{Gh}{c^5} = \frac{1}{\nu^4} (d\nu)^2. \quad (9.6)$$

Substituting $h = 2\pi\hbar$ we get,

$$\nu = \frac{1}{\sqrt{2\pi}} \left( \frac{d\nu}{\nu} \right) \sqrt{\frac{c^5}{G\hbar}} \quad (9.7)$$
where quantity in the bracket is just the redshift factor $z$ as defined in the standard model of big bang \[45\]. However, what we have here is the gravitational redshift and not the Doppler shift of Hubble’s law. Hence

$$2\pi\nu = \sqrt{2\pi z} \sqrt{\frac{c^5}{G\hbar}}. \quad (9.8)$$

In our earlier work \[49\], we defined quantity $\alpha$ as the inverse of the Planck length $l_p$ and the Hubble parameter $H$ in terms of the Planck angular frequency $\omega_p$

$$\alpha = \sqrt{\frac{c^3}{G\hbar}} = \frac{1}{l_p}, \quad (9.9)$$

$$H = \kappa\omega_p = \kappa\alpha c = \kappa\sqrt{\frac{c^5}{G\hbar}}, \quad (9.10)$$

where we defined $\kappa$ with an ansatz that it is a unitless real number. Hubble parameter has the units of frequency. Earlier \[49\] we had defined Planck energy as,

$$E_p = \hbar \alpha c = \hbar \omega_p. \quad (9.11)$$

As a final solution to the wave equation, particle energy levels were given by

$$E = E_p \sqrt{\frac{B1}{C1}}, \quad (9.12)$$

where $B1$ and $C1$ are unitless numbers. In case of electromagnetic wave, $E = h\nu$. Inserting this in Eq. (9.12) we get,

$$H = 2\pi\kappa\nu \sqrt{\frac{C1}{B1}} \quad (9.13)$$

$$\frac{d\nu}{\nu} = \sqrt{\frac{B1}{2\pi C1}} = \sqrt{\frac{G\hbar}{c^5}}\nu. \quad (9.14)$$

From Eq. (9.13) we see that the Hubble parameter is a function of particle frequency and from Eq. (9.14) we see that the gravitational frequency shift is a function of unitless variables $B1$ and $C1$ and proportional to frequency of light at any given time. So here we have the robust relativistic derivation of the variable gravitational redshift as the Hubble parameter rolls down from Planck epoch to the present epoch. This is done using de Broglie force, periodic time and gravitational attraction between two successive photons within the
electromagnetic wave which merge with each other at about $10^{-26}$ sec. before the big crunch in PQGC terminating the motion of the wave meaning that the wave will collapse. In PQGC it is not possible to generate photon parameters before this time. This is where gravity unites with the electromagnetic force. Same phenomenon occurs with Savitons at Planck energy when the Saviton waves collapse during big crunch. Gravitational constant $G$ in Eqs. (9.9) and (9.10) does not come from a quantum gravity theory, but it comes through Einstein’s field equations. So what we have here is the first theoretical evidence of existence of gravitational forces within the electromagnetic wave.

As we change the value of $\kappa$ by hand, it changes the Hubble parameter and we know from general relativity that the energy density of the universe is proportional to square of the Hubble parameter. Energy density of the universe is continuously changing due to the expansion of the universe. In the lower energy density epochs, particles also have lower energies and so does photons of electromagnetic waves. Gravitational redshift discussed in this formalism is intrinsic to electromagnetic waves. Lower the energy of the wave, lower is the intrinsic gravitational redshift. Therefore at present epoch this gravitational redshift is very small. This is due to very small value of the Hubble parameter at present epoch which is comparable to the small value of the cosmological constant.

From these calculations we can conclude that the electromagnetic wave is held together by gravitational forces. Not only that, even the seven monochromatic wave frequencies are held together by gravitational forces as a packet of white light. When these gravitational forces are overcome by processes like refraction, diffraction or polarization, then the electromagnetic wave decomposes into components like photon particles or seven colors of the rainbow etc. Mechanism of gravitational frequency shift of light can thus be explained as due to absorption or ejection of quanta of energy from or to the surroundings. When photons absorb gravitational energy, two successive photons come closer due to greater gravitational attraction as per Newton’s inverse square law and thus light gets blue shifted. When photon loses energy to the surroundings, the gravitational attraction between two successive photons is reduced which stretches the wavelength and the light gets red shifted. All this happens at a constant velocity of light.

Whenever GR physicists use Lorentz force as the rate of change of momentum, Eq. (4.25), they are ignoring the de Broglie force and the periodic time. This is because GR has abandoned the relativistic mass and thus further differentiation of momentum is not possible.
This eliminates the de Broglie force and saves the principle of equivalence. As a consequence, both the principle of equivalence and the gravitational redshift are not invariant in GR. But in PR, relativistic mass is part of the theory and the gravitational mass is equal to relativistic mass. This makes both, the principle of equivalence as well as the gravitational redshift invariant.

One should be careful in using Eq. (9.5) because it is partly based on quantum distance $\lambda$ and partly on linear time $t$. They don’t go together. One may get tempted to write,

$$\int dt = \frac{c^5}{G\hbar} \int \frac{1}{\nu^4} d\nu,$$

(9.15)

but it will not give you any meaningful results. It has to be either linear time and linear distance which we used earlier in Eq. (4.31) for deriving gravitational redshift of light coming from the surface of the sun or the periodic time and periodic distance as we used above in Eq. (9.6).

If we plug in gamma ray photon data in Eq. (9.14) we get gravitational redshift of about $1.13 \times 10^{-22}$ at $10^{-26}$ sec. after big bang which is very small. If we plug in Saviton data which is a massless boson having planck energy then we get gravitational redshift of 0.4 at Planck time which is significant. This gravitational redshift occurs over a distance of single wavelength. This is not an astronomical distance. This is the kind of gravitational radiation coming out from about $10^{62}$ Savitons at Planck time which created the entire universe. So all the known fundamental particles of physics are products of this gravitational radiation.

**B. Quartic law of quantum gravity**

We can write Eq. (9.4) as

$$F = \frac{G}{\lambda^2} \left( \frac{E}{c^2} \right)^2.$$

(9.16)

Substituting $\lambda = cT = c/\nu$ gives,

$$F = \frac{G\hbar^2 \nu^4}{c^6} = \frac{GE^4}{\hbar^2 c^5}.$$

(9.17)

Eq. (9.17) shows that the gravitational force is a quartic function of energy or frequency and $F \propto E^4$ or $F \propto \nu^4$. 
We can write Eq. (9.8) as

\[ E = z \sqrt{\frac{c^5 h}{G}} \]  

(9.18)

Here energy \( E \) can be positive or negative depending on redshift or blueshift. Positive only means light wave is throwing out energy to the surrounding and negative means energy is absorbed from the surrounding. Substitution of Eq. (9.18) in Eq. (9.17) gives,

\[ F = z^4 \sqrt{\frac{c^4}{G}} \]  

(9.19)

In Eq. (9.19), regardless of redshift or blueshift, the gravitational force \( F \) is always positive and attractive. The formalism developed for the electromagnetic wave above can be extended to the first massless bosons created at Planck epoch, namely savitons in PQGC theory [49].

After introduction of gravity in electromagnetic wave, the main difference between photons and savitons is the amount of energy they carry. Savitons have planck energy and contains energy required to create strong force. The maximum amount of energy that savitons can carry is limited by its smallest possible wavelength which is the Planck length. This corresponds to a specific particle frequency limited by the relation \( \nu_{max} = c/\lambda_{min} \). When saviton acquire more energy then allowable, the particle wave will collapse. Two successive savitons in a wave will unite with each other and frquency \( \nu \) will drop to zero. Both the energy and gravity will disappear and saviton will become motionless and unite with the fundamental motionless substance of the universe which is the singularity. The process is reversible.

1. How elementary particles acquire mass and charge

In PQGC [49], mass is acquired by a particle when part of its kinetic energy gets transformed into potential energy so the velocity of the particle drops and inertia sets in. Higgs boson also acquire its mass in this way. The first particles created in this theory were bosons called Savitons having Planck energy and massless like photons and having velocity of light. In this, energy gets condensed into mass like water getting condensed into ice. This is phase transformation of energy into mass. So the particles acquire their mass through phase transformation of their own energy. Gravitational redshift described in Eq. (9.18) plays the principle role in creation of new particles. All the matter of the universe is created
through this red shifting. In a reverse process of blue shifting, all the matter of the universe is absorbed by savitons in a massless state which then turns around to acquire Planck energy and ultimately unite with the fundamental motionless substance of the universe.

Charge is acquired as follows. We know that electron positron pair can annihilate into two photons. This process is reversible. When photons transform into electron positron pair, they acquire their mass as described above and the electric field associated with the electromagnetic wave, condenses into charge. So this is also a phase transformation of the energy of the electric field. The process is similar to Bose Einstein Condensate.

2. Predominance of matter over anti-matter

In PQGC, universe begins with the creation of the first particle ($10^{62}$ particles) we call Saviton which is a massless boson having Planck energy at Planck time. Like photon, saviton is its own anti-particle. So the universe was matter dominated right from the beginning. Savitons decayed into other particles through gravitational red shifting. A small fraction of these decayed particles developed matter anti-matter division with opposite spin and opposite charge. All quarks and anti-quarks except top quark remained confined in pairs due to strong force of gluons. Few like electron and positron remained free. It is possible that the concept of dipole gravity in conjunction with the relativistic mass along with the radial component of Lense-Thirring force can explain creation of particle anti-particle pairs having Planck energy at Planck time from a single particle of Planck length rotating at a very high speed? This can naturally cause inflation and the expansion of the universe. In this, rotational kinetic energy of dipole gravity particle is transformed into rectilinear kinetic energy of monople gravity particles.

10. GRAVITATIONAL WAVES

The orbital period derivative of a binary pulsar is calculated and verified with great accuracy. The most popular explanation for the resulting energy loss of the binary system is given in terms of the emission of gravitational waves. Recently LIGO scientists detected the gravitational waves arising from binary black hole merger. But no gravitational waves arising from the binary pulsars have yet been detected.
the circumstances, it would be useful to look at the alternative explanation for the cause of the orbital period decay of binary pulsars. PR relates the orbital period of the pulsar with the period of the phase of the constituent particles of the pulsar. The idea is that, when small amount of gravitational energy is released, it may not generate gravitational waves, but this energy can alter the period of the phase of the constituent particles of the pulsar. Constituent particles of the pulsar are in bound states in atoms and molecules. They have their orbital motions around their nucleus. At the same time, each particle also has a somewhat rectilinear motion along the direction of the orbital velocity of the pulsar. This later motion has the de Broglie phase wave associated with it. Decay of the orbital period of the pulsar causes decay of the period of this phase of the constituent particles of the pulsar. So the gravitational energy lost by the pulsar is equal to the energy gained by the constituent particles of the pulsar. This results in increased kinetic energy of the pulsar. So the potential (gravitational) energy is converted into kinetic energy. When very large amount of gravitational energy is released, then only it will generate gravitational waves, like in case of binary black holes. This is because the large amount of energy released destroys the bonds between the constituent particles of the orbiting body first, and then the excess energy finds its way as the gravitational waves.

A. Gravitational redshift of gravitational Waves

Xian et. al. [59] have discussed mass-redshift degeneracy with respect to cosmological, Doppler and gravitational redshift of gravitational waves arising from merger of binary black holes (BBHs) in the vicinity of a supermassive black hole (SMBH). Here the authors have used the standard gravitational redshift formula of GR which is not invariant.

\[ 1 + Z_g = \left(1 - \frac{R_s}{r}\right)^{-\frac{1}{2}}. \]  

(10.1)

where \( R_s \) is the Schwarzschild radius and \( r = \Delta \) is the radius of the body. Eq. (10.1) reduces to

\[ Z_g^{GR} = \left(\frac{GM}{c^2 \Delta}\right). \]  

(10.2)
Earlier we have derived gravitational redshift formula in Sec. 4D.1 which is invariant and given by

$$\frac{\mu}{c^2 (\Delta^2 + \Delta l)} = \frac{\phi_1 - \phi_2}{c^2} = \ln \left( 1 - \frac{\delta \nu}{\nu_s} \right), \quad (10.3)$$

For very large value of $l$ Eq. (4.32) reduces to

$$Z^\text{PR}_g = \exp \left( \frac{GM}{c^2 \Delta} \right) - 1, \quad (10.4)$$

and for a very small value of the quantity in the bracket like in case of gravitational redshift of light coming from the sun, Eq. (10.4) reduces to Eq. (10.2). But in case of gravitational redshift of gravitational waves arriving from BBH merger, Eq. (10.4) cannot be ignored because it gives greater redshift in strong field than the GR formula which is based on weak field approximation. In case of SMBHs, the difference will be even more striking. If we use the GW150914 data [57]; $m_1 = 36M_\odot$, $m_2 = 29M_\odot$, we get the chirp mass $M = 28.55M_\odot$. We use the Schwarzschild radius for the sum of the component masses $M = (m_1 + m_2)$ to be $R_s = \Delta = 192.7$ Km. This gives $Z^\text{GR}_g = 0.2195$ and $Z^\text{PR}_g = 0.2455$. If we compute the emitted frequency $f_e$ at source based on the observed frequency at peak amplitude $f_o = 150$ Hz, we get $f_e^\text{GR} = 192.2$ Hz and $f_e^\text{PR} = 198.8$ Hz. Hence prediction of PR is 6.6 Hz higher than GR in the strong field regime of GW150914. This difference can become even more striking in case of SMBHs. This can change the orbital frequency of BBH to 99.4 Hz.

**B. Prediction of graviton mass in periodic relativity**

For a particle having only relativistic mass $m$, traveling radially in the gravitational field of mass $M$ at velocity of light, we defined the invariant relation between Newtonian force and the de Broglie force in Sec. 4D.1 as given below. Using this equality we derived the formula for gravitational redshift of light given by Eq. (10.3).

$$\frac{\mu m}{r^2} = \frac{h}{c} \frac{d\nu}{dt}, \quad (10.5)$$

where $\mu = GM$, $m$ is the relativistic mass traveling at speed of light in the gravitational field of mass $M$, $h$ is Planck constant, $c$ is the velocity of light. $\nu$ is the de Broglie frequency of the particle wave. Using this equality of Eq. (10.5) we will define the deviation factor $n$ as a ratio of Newtonian acceleration to de Broglie acceleration for the PR line element.
given by Eq. (1.1) which can now become a wave equation because of the presence of the frequency term and Compton wavelength. For null geodesic we have \( v = c \) and \( n = 1 \).

\[
n = 1 = \frac{GM}{r^2} \left( \frac{h \, dv}{mc \, dt} \right)^{-1},
\]

(10.6)

We can relate Eq. (10.6) with GW150914 by substituting \( M = (m_1 + m_2) \), \( r = R_s = 192.7 \) Km, and \( \dot{\nu} = \dot{f} \). Then for mass of graviton in the source frame we get

\[
m = m_g = \frac{R_s^2 h \dot{f}}{cG(m_1 + m_2)},
\]

(10.7)

where \( \dot{f} \) is given by

\[
\frac{df}{dt} = \frac{96}{5} \pi \frac{8}{3} \left( \frac{GM}{c^3} \right)^{\frac{5}{3}} f_{11}^{\frac{11}{3}},
\]

(10.8)

where chirp mass \( \mathcal{M} = 28.55M_\odot \) and \( f = 150 \) Hz. This gives \( m_g = 7.86 \times 10^{-14} \) eV/c² and the corresponding Compton wavelength to be \( \lambda_g = 1.57 \times 10^4 \) Km. This graviton mass is heavier than what is measured for photon recently [60].

C. Gravitational wave equation in periodic relativity

From Eq. (4.41) we can write for \( n = 1, \)

\[
\frac{d(h\nu)}{dt} = \frac{GM}{r^2} (mc).
\]

(10.9)

\[
\frac{dE}{dt} = \frac{GM}{r^2} (p^2)^{1/2}.
\]

(10.10)

We replace \( E \) by the differential operator \( i h \partial/\partial t \), and \( p \) by \( -i h \nabla_j \) that act on the wave function \( \psi \). The spin dependent \( \nabla_j \) operator is discussed at length in [50].

\[
\frac{d}{dt} \left( i h \frac{\partial \psi}{\partial t} \right) = \frac{GM}{r^2} [(-i h \nabla_j)^2]^{1/2} \psi.
\]

(10.11)

Eq. (10.11) is valid for both orbital and radial motion of gravitational waves. Assuming spherically symmetric potential, we define operator \( \nabla^2_j \) in spherical polar coordinates as

\[
\nabla^2_j = \frac{1}{r^2} \left[ \frac{\partial}{\partial r} \left( r^2 \frac{\partial \psi}{\partial r} \right) - \frac{1}{\hbar^2} (L + S)^2 \right],
\]

(10.12)
where $L$ is the orbital angular momentum operator, and $S$ is the spin angular momentum operator. The wave equation reduces to

$$\frac{d}{dt} \left( \frac{\partial \psi}{\partial t} \right) = \frac{GM}{r^2} (\nabla_j^2)^{1/2} \psi.$$  \hspace{1cm} (10.13)

Note that the use of ratio of accelerations eliminates the need for potential energy term commonly used in quantum mechanics. We consider a particular solution of Eq. (10.13) that can be written as a product $\psi(r, t) = z(r) f(t)$. A general solution can be written as a sum of such separated solutions. If we substitute the above product in Eq. (10.13) and divide thru by the product, we get

$$\frac{d}{dt} \left( \frac{1}{f} \frac{df}{dt} \right) = \frac{GM}{z} \frac{1}{r^2} (\nabla_j^2)^{1/2} \frac{1}{z}.$$  \hspace{1cm} (10.14)

If we define another function $u(r)$ such that

$$\frac{1}{z} \frac{GM}{r^2} (\nabla_j^2)^{1/2} \frac{1}{z} = \frac{GM}{r^2} \left( \frac{1}{u} \nabla_j^2 u \right)^{1/2},$$  \hspace{1cm} (10.15)

then we can write Eq. (10.14) as

$$\frac{d}{dt} \left( i \frac{1}{f} \frac{df}{dt} \right) = i \hbar \frac{GM}{r^2} \left( \frac{1}{u} \nabla_j^2 u \right)^{1/2}.$$  \hspace{1cm} (10.16)

Since the left side depends only on $t$ and right side only on $r$, both side must be equal to a same separation constant which in this case is $dE/dt$. Then equation for $f$ can be easily integrated to give,

$$f(t) = C \exp(-iEt/\hbar),$$  \hspace{1cm} (10.17)

where $C$ is an arbitrary constant and the equation for $u$ becomes,

$$\frac{dE}{dt} = i \hbar \frac{GM}{r^2} \left( \frac{1}{u} \nabla_j^2 u \right)^{1/2},$$  \hspace{1cm} (10.18)

$$\dot{\omega}^2 u(r) = - \left( \frac{GM}{r^2} \right)^2 \nabla_j^2 u(r).$$  \hspace{1cm} (10.19)

where $E = h\nu$ and $\dot{\omega} = 2\pi\dot{\nu}$. Substitution of Eq. (10.12) in Eq. (10.19) gives,

$$\left[ \left( \frac{\dot{\omega}^2}{GM} \right)^2 + \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) \right] u(r) = \left[ i \left( \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \right)^{1/2} + \frac{S}{\hbar} \right]^2 u(r).$$  \hspace{1cm} (10.20)
The radial and the angular parts can then be separated by substituting

\[ u(r, \theta, \phi) = R(r)Y(\theta, \phi) \]  

in Eq. (10.20) and dividing thru by \( RY \).

\[ \left( \frac{\dot{\omega} r^3}{GM} \right)^2 + \frac{1}{R \frac{\partial}{\partial r}} \left( r^2 \frac{\partial R}{\partial r} \right) = \frac{1}{Y} \left[ i \left( \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial Y}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2 Y}{\partial \phi^2} \right) \right]^{\frac{1}{2}} + \frac{S}{\hbar} \sqrt{Y} \right]^2. \]  

(10.22)

Since the left side of Eq. (10.22) depends only on \( r \) and the right side depends only on \( \theta \) and \( \phi \), both sides must be equal to a constant that we call \( \Gamma \). Thus Eq. (10.22) gives us a radial equation

\[ \frac{\partial}{\partial r} \left( r^2 \frac{\partial R}{\partial r} \right) + \left( \frac{\dot{\omega} r^3}{GM} \right)^2 R - \Gamma R = 0, \]  

(10.23)

and an angular equation

\[ i \left( \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial Y}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2 Y}{\partial \phi^2} \right) = \left( \sqrt{\Gamma Y} - \frac{S}{\hbar} \sqrt{Y} \right) = \sqrt{\lambda Y}. \]  

(10.24)

From [50] we know that \( \Gamma = j(j + 1) \), \( S/\hbar = \sqrt{s(s + 1)} \) and \( \sqrt{\lambda} = \sqrt{l(l + 1)} \). Hence,

\[ \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial Y}{\partial \theta} \right) + \lambda Y = - \frac{1}{\sin^2 \theta} \frac{\partial^2 Y}{\partial \phi^2}. \]  

(10.25)

The angular equation can be further separated by substituting

\[ Y(\theta, \phi) = \Theta(\theta)\Phi(\phi) \]  

(10.26)

and dividing by \( \Theta \Phi \).

\[ \left[ \frac{1}{\Theta \sin \theta} \frac{d}{d\theta} \left( \sin \theta \frac{d \Theta}{d\theta} \right) + \lambda \right] \sin^2 \theta = - \frac{1}{\Phi \sin^2 \theta} \frac{d^2 \Phi}{d\phi^2} = \eta. \]  

(10.27)

Hence we end up with two equations of Schrödinger theory.

\[ \frac{d^2 \Phi}{d\phi^2} + \eta \Phi = 0, \]  

(10.28)

\[ \frac{1}{\sin \theta} \frac{d}{d\theta} \left( \sin \theta \frac{d \Theta}{d\theta} \right) + \left( \lambda - \frac{\eta}{\sin^2 \theta} \right) \Theta = 0. \]  

(10.29)
Eq. (10.28) have the same solution as that given by the Schrödinger theory where $\eta$ is chosen to be equal to square of an integer $m$ which takes on positive or negative integer values or zero. Therefore,

$$\Phi_m(\phi) = (2\pi)^{-\frac{1}{2}} \exp(\text{i}m\phi).$$  \hspace{1cm} (10.30)

We confine the motion of gravitational waves in $z - x$ plane by putting $\phi = 0$. Substituting $\phi = 0$ in Eq. (10.30) gives,

$$\Phi = \frac{1}{\sqrt{2\pi}}.$$  \hspace{1cm} (10.31)

Therefore Eq. (10.28) can be satisfied only if we put $\eta = 0$. This reduces Eq. (10.29) to

$$\frac{1}{\sin \theta} \frac{d}{d\theta} \left( \sin \theta \frac{d\Theta}{d\theta} \right) + \lambda \Theta = 0.$$  \hspace{1cm} (10.32)

Differentiating the first term brings it to the form,

$$\tan \theta \frac{d^2 \Theta}{d\theta^2} + \frac{d\Theta}{d\theta} + \lambda \tan \theta \Theta = 0.$$  \hspace{1cm} (10.33)

1. Gravitational-wave strain amplitude using quantum mechanical formalism

Eq. (10.33) is a second order homogeneous linear equation. We seek a particular solution of Eq. (10.33) by introducing the initial condition $\theta = 0$. This gives

$$\frac{d\Theta}{d\theta} = 0.$$  \hspace{1cm} (10.34)

Therefore we get the first particular solution

$$\Theta_1 = C_3 = \text{Constant}.$$  \hspace{1cm} (10.35)

If we can find another particular solution $\Theta_2$ of Eq. (10.33) so that both solutions are linearly independent, then the general solution can be expressed as

$$\Theta = C_1 \Theta_1 + C_2 \Theta_2,$$  \hspace{1cm} (10.36)

where $C_1$ and $C_2$ are arbitrary constants. The coefficients of three terms on the left of Eq. (10.33) can be designated $a_0$, $a_1$ and $a_2$. In order to use Liouville’s formula for finding the second particular solution, it is required that $a_0 = 1$ and also, the solution can be very
simplified if we have \( a_1 = 1 \), which we already have. The first condition can be met only for those values of \( \theta \) for which \( \tan \theta = 1 \). So we proceed with the derivation with a remark that our general solution of Eq. (10.33) will be valid only for \( \theta = 45^\circ \) and the like. However, it should be possible to derive general solution applicable to all values of \( \theta \).

By virtue of Liouville’s formula, we can write

\[
\Theta_2' \Theta_1 - \Theta_2 \Theta_1' = C \exp(- \int a_1 d\theta). \tag{10.37}
\]

Dividing both sides by \( \Theta_1^2 \) we get

\[
\frac{\Theta_2' \Theta_1 - \Theta_2 \Theta_1'}{\Theta_1^2} = \frac{1}{\Theta_1} C \exp(- \int a_1 d\theta). \tag{10.38}
\]

For \( a_1 = 1 \) and \( \Theta_1 = C_3 \) we get

\[
\frac{d}{d\theta} \left( \Theta_2 \Theta_1 \right) = \frac{1}{C_3} C \exp(- \theta), \tag{10.39}
\]

\[
\frac{\Theta_2}{\Theta_1} = \int \frac{1}{C_3} C \exp(- \theta) d\theta + C'. \tag{10.40}
\]

For getting a particular solution, we can put \( C' = 0 \) and \( C = 1 \).

\[
\Theta_2 = \int \frac{1}{C_3} \exp(- \theta) d\theta, \tag{10.41}
\]

\[
\Theta_2 = - \frac{1}{C_3} \exp(- \theta). \tag{10.42}
\]

Comparing Eq. (10.32) with Eq. (10.35) we can see that \( \Theta_1 \) and \( \Theta_2 \) are linearly independent solutions. Hence, the general solution of Eq. (10.33) is given by

\[
\Theta = C_1 C_3 - \frac{C_2}{C_3} \exp(- \theta). \tag{10.43}
\]

By putting \( C_4 = C_1 C_3 \) and \( C_5 = C_2 / C_3 \) we can write

\[
\Theta = C_4 - C_5 \exp(- \theta). \tag{10.44}
\]

Substitution of Eq. (10.44) in Eq. (10.33) gives

\[
\left( \frac{C_4}{C_5} \right) = \exp(- \theta) \left( \frac{1}{\lambda} + 1 - \frac{1}{\lambda \tan \theta} \right). \tag{10.45}
\]
Remembering that our solution is valid only for \( \tan \theta = 1 \), we get
\[
\left( \frac{C_4}{C_5} \right) = e^{\exp(-45^\circ)}.
\] (10.46)

The normalization condition for the angular wave function \( \Theta \) in Eq. (10.44) is given by
\[
\int |\Theta|^2 d\theta = \int (C_4 - C_5 e^{\exp(-\theta)})^2 d\theta = 1.
\] (10.47)

Integration of Eq. (10.47) gives
\[
\frac{C_4^2 \theta - 2C_5^2 \exp(-2\theta) + C' + 2C_4C_5 \exp(-\theta) + C''}{C_5^2} = 1,
\] (10.48)
where we put constants of integration \( C' = 0, C'' = 0 \) and divide thru by \( C_5^2 \).

Substituting Eq. (10.46) in Eq. (10.49) and introducing initial condition as \( \theta = 0 \), we get
\[
C_5 = \frac{1}{\sqrt{2}} (\exp(-45) - 1)^{-1/2}.
\] (10.50)

We can write Eq. (10.44) as
\[
\Theta = C_5 \left( \frac{C_4}{C_5} - \exp(-\theta) \right).
\] (10.51)

Substituting Eq. (10.46) and Eq. (10.50) in Eq. (10.51) we get
\[
\Theta = \frac{1}{\sqrt{2}} (\exp(-45) - 1)^{-1/2} (\exp(-45) - \exp(-\theta)).
\] (10.52)

\[
\Theta = \frac{i}{\sqrt{2}} \left[ (\exp(-45) - \exp(-\theta) + \frac{1}{2} \exp(-45)^2 - \frac{1}{2} \exp(-45) \exp(-\theta) \right].
\] (10.53)

Last two terms in the bracket are very small in around \( \theta = 45 \) and can be ignored. Hence
\[
\Theta = \frac{i}{\sqrt{2}} (\exp(-\theta) - \exp(-45)).
\] (10.54)

What we have here is the amplitude of the gravitational wave propagating along the radial vector at \( 45^\circ \) to the z-axis which is lined up with the tangent to an imaginary circular orbit of radius \( r_j \). We will discuss this circular orbit little later. The x-axis can be projected to pass through the mass center of the binary black holes and the z-axis can also be parallely shifted to pass through this mass center. It appears that the gravitational waves spiral out
from within the circle mentioned above and exit the circle at \( \theta = 45^\circ \) to the z-axis. This makes the gravitational waves independent of the angular momentum eigen values \( \lambda \). This we can see in Eq. (10.45) when constants on the left become free from \( \lambda \). When \( \theta = 45 \) in Eq. (10.54), the amplitude is zero. Small variation in \( \theta = 45 \pm 0.05 \) gives amplitude variation of \( \Theta = \pm 1 \times 10^{-21} \). These are the experimentally measured values of the gravitational-wave strain amplitude for GW150914 [57].

2. Radial vector and graviton spin

We will rewrite the radial equation (10.23) in dimensionless form by introducing a unitless independent variable \( \rho \) so that

\[
\rho = \left( \frac{\dot{\omega}_r}{GM} \right)^2 = \left( \frac{\dot{\omega}_r^{5/2}}{GM} \right)^2 r = \alpha r. \tag{10.55}
\]

\[
\partial \rho = 6 \alpha \partial r. \tag{10.56}
\]

Substitution of Eq. (10.55) in Eq. (10.23) gives

\[
36 \frac{\partial}{\partial \rho} \left( \rho^2 \frac{\partial R}{\partial \rho} \right) + \rho R - \Gamma R = 0. \tag{10.57}
\]

Dividing Eq. (10.57) by \( \rho^2 \) we get

\[
\frac{36}{\rho^2} \frac{\partial}{\partial \rho} \left( \rho^2 \frac{\partial R}{\partial \rho} \right) + \left( \frac{1}{\rho} - \frac{\Gamma}{\rho^2} \right) R = 0. \tag{10.58}
\]

As far as the leading terms are concerned, for sufficiently large \( \rho \) it is apparent that \( R(\rho) = \rho^n e^{\pm \frac{\Gamma}{4}} \) satisfies Eq. (10.58) when \( n \) has any finite value. This suggests that we look for an exact solution of Eq. (10.58) of the form

\[
R(\rho) = F(\rho) e^{-\frac{\Gamma}{4}} \tag{10.59}
\]

where \( F(\rho) \) is a polynomial of finite order in \( \rho \). Substitution of Eq. (10.59) into Eq. (10.58) gives equation for \( F(\rho) \) as

\[
F'' + \left( \frac{2}{\rho} - 1 \right) F' + \left[ \frac{1}{4} - \frac{1}{\rho} + \frac{1}{36 \rho^2} - \frac{\Gamma}{36 \rho^2} \right] F = 0. \tag{10.60}
\]

\[
F'' + \left( \frac{2}{\rho} - 1 \right) F' + \left[ \frac{1}{4} - \frac{35}{36 \rho} - \frac{\Gamma}{36 \rho^2} \right] F = 0. \tag{10.61}
\]
Now we find a solution for \( F \) in the form

\[
F(\rho) = \rho^s(a_0 + a_1\rho + a_2\rho^2 + \cdots) = \rho^sL(\rho), \quad a_0 \neq 0, \ s \geq 0. \tag{10.62}
\]

Substitution of Eq. (10.62) into Eq. (10.61) and dividing thru by \( \rho^{(s-2)} \) gives us the equation for \( L \).

\[
\rho^2L'' + \rho(2(s+1) - \rho)L' + \left[ \left( s^2 + s - \frac{\Gamma}{36} \right) - \left( \frac{35}{36} + s \right) \rho + \frac{1}{4}\rho^2 \right] L = 0. \tag{10.63}
\]

If we set \( \rho = 0 \) in Eq. (10.63), it follows from Eq. (10.62) that

\[
s^2 + s - \frac{\Gamma}{36} = 0. \tag{10.64}
\]

This quadratic equation in \( s \) has two solutions,

\[
s = -\frac{1}{2} \pm \frac{1}{6}(\Gamma + 9)^{\frac{1}{2}}. \tag{10.65}
\]

The boundary condition that \( R(\rho) \) be finite at \( \rho = 0 \) requires that we choose upper sign for \( s \). With this, Eq. (10.63) reduces to

\[
\rho L'' + \{ 2(s+1) - \rho \} L' + \left( \frac{1}{4}\rho - \frac{35}{36} - s \right) L = 0. \tag{10.66}
\]

Equation (10.66) can be solved by substituting Eq. (10.62). The recursion relation between the coefficients of successive terms of the series is observed to be

\[
a_{\nu+1} = \frac{(35/36) + s - (\rho/4)}{\nu(\nu+1) + 2(\nu+1)(s+1) - (\nu+1)\rho} a_{\nu}. \tag{10.67}
\]

We can see from Eq. (10.67) that the series can terminate when the following condition is satisfied.

\[
\frac{1}{4}\rho - \frac{35}{36} - s = 0. \tag{10.68}
\]

Substituting \( \rho \) from Eq. (10.55) and \( s \) from Eq. (10.65) we get

\[
\Gamma = \left\{ \left[ \frac{3}{2} \left( \frac{\dot{\omega} r^3}{GM} \right)^2 - \frac{17}{6} \right]^2 - 9 \right\}. \tag{10.69}
\]

\[
r = \left( \frac{2GM}{\dot{\omega}} \right)^{1/3} \left[ \frac{17}{36} + \frac{1}{6}(\Gamma + 9)^{1/2} \right]^{1/6}. \tag{10.70}
\]
where from Eq. \(10.24\) we have \(\Gamma = j(j + 1)\). Due to the note following Eq. \(10.36\) the angular position of the radial vector is restricted to a fixed value resulting in zero angular velocity which allows only one angular momentum eigen value \(l = 0\), giving \(\lambda = 0\). Substitution of \(\lambda = 0\) in Eq. \(10.35\) does not affect the outcome in Eq. \(10.46\) because \(+\infty\) and \(-\infty\) cancel out. Hence the value of \(\Gamma = j(j + 1)\) in Eq. \(10.70\) is exclusively decided by the spin of the graviton. The corresponding radial vector for \(j = 0, 1, 2\) are as given below. I have used following GW150914 data \([57]\) for computing radial distance \(r_j\) for given \(j\). These are the radial distances at which the series terminates for the given value of \(j\). We have \(f = 150\) Hz., \(m_1 = 36M_{\odot}, m_2 = 29M_{\odot}, M = (m_1 + m_2)\), chirp mass \(M = 5.588 \times 10^{31}\) Kg., \(\dot{\nu} = \dot{f} = 14344.907\). We get \(r_{j=0} = 5.7366 \times 10^5\) m, \(r_{j=1} = 5.7873 \times 10^5\) m and \(r_{j=2} = 5.8715 \times 10^5\) m. These radii are greater than the photon capture radius \(\sqrt{27}R_g\). They give the point of origin of gravitational wave which is spin dependent. If you can measure this radius, you know the spin of the graviton.

11. CONCLUSION

We have presented derivation for the deflection of light from fundamentals by introducing parameter \(\psi\) ignored by general relativity and by introducing vectors. Here we can relate the additional component of acceleration with the rotation of the velocity vector which causes the curvature of the trajectory. We have distinguished the Cartesian curvilinear acceleration from the polar conic acceleration and explained why they are not equal even though they are derived from the same velocity vector. We have derived expression for the Lorentz invariant acceleration. Michelson Morley set out to detect ether with an assumption that if ether existed it would interact with the light waves in their interferometer. The very nature of this experiment imply that ether has properties of waves and therefore must be in a state of vibration. In PR we have proposed the singular motionless state of the primal energy which is devoid of any vibrations or motion and hence does not interact with any form of manifest energy. The primary prediction of Periodic Relativity (PR) is the existence of the Fundamental Substance of the universe which is perfectly motionless (not like particles having mass gap), formless (having no boundary) and therefore infinite in extent. It is a unified field of all forms of energies (known and unknown to physics) and therefore indivisible. This motionless Fundamental Substance is not the energy but becomes energy when a small
portion of it begins to move and apparently divide into particle waves having wavelengths and periods. This is how motion, energy, wavelengths (space) and periods (time) are simultaneously created and the infinite becomes limited with various forms and the Absolute One becomes many; thus laws of relativity becomes operational. Gravity and entanglement comes into existence between different forms of created energies due to indivisibility of the Fundamental Substance which is one without a second. If second something were to exist, then there would be a boundary between the two and that would make both finite and limited. In periodic relativity (PR), the Fundamental Substance replaces the space-time fabric of general relativity (GR). The Fundamental Substance has all the properties of empty space and cannot be detected by Interferometer because there are no waves or oscillations or motion in it. The entire universe exists within it, and it permeates everything in the universe. All other relativistic discussions in PR are in support of this primary prediction.

Periodic nature of time is fundamental to quantum gravity theory in which the universe begins with a vibration in the fundamental substance of the universe which is singular motionless and infinite. This is how energy, space (wavelength), and time (period) are simultaneously created. This becomes possible when we can equate Hubble parameter with Planck frequency at Planck epoch. This solves the problem of time in quantum gravity theory. Periodic time is possible only in flat space time so we must have a theory of gravity in flat space time. What we have presented here is an alternative to Schwarzschild solution which solves the problem of rotation curves of galaxies. We have presented a theory of rotation curves of galaxies which is based on the second solution of Einstein’s field equations. Deviation factor \( n \) appears in the expression for the modified Kepler’s third law which now yields correct orbital periods for the stars of galaxies. Deviation factor \( n \) plays the same role as the MOND function in the expression for acceleration. This kind of solution cannot be obtained in general relativity because of the weak field approximation. We were able to predict the limiting radius of the event horizon and the spin of M87 black hole for the measured value of the photon ring diameter. Theory of gravity in flat space time is required to unify the quantum physics with periodic relativity. This allowed us to introduce Einstein’s field equations in hydrogen atom model and eliminate the potential energy term from Schrodinger equation and replace it with deviation factor as a ratio of accelerations. Rotation curves of galaxies uses the same approach where deviation factor is introduced as a ratio of accelerations. Here we have shown that we can introduce gravity into electromag-
netic wave formalism by working with wavelengths and periods of the wave for eliminating linear distance and linear time. Working with wavelengths and periods is a natural way of quantizing space and time. But this requires that we first develop a theory of gravity in flat space time, which we already did. We showed that the electromagnetic wave is held together by gravitational forces just as a planet or solar system is held together by gravity. The theory provides mathematical proof for the periodic nature of time and existence of very powerful gravitational radiation at Planck epoch. All this is possible due to the introduction of relativistic mass in the formalism. Theory points out the existence of de Broglie force which gives equation of propagation of massless bosons. Ratio of Newtonian acceleration and de Broglie acceleration is utilized in quantum mechanical formalism for deriving formula for gravitational-wave strain amplitude. A method of determining graviton spin is discussed. This theory provides reasonably accurate way of introducing gravity in quantum mechanics.

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