Impurity-induced Dicke quantum phase transition in an impurity-doped cavity-Bose-Einstein condensate

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We present a new generalized Dicke model, an impurity-doped Dicke model (IDDM), by the use of an impurity-doped cavity-Bose-Einstein condensate. It is shown that the impurity atom can induce Dicke quantum phase transition (QPT) from the normal phase to superradiant phase at a critical value of the impurity population. It is found that the IDDM exhibits continuous Dicke QPT with an infinite number of critical points, which is significantly different from that observed in the standard Dicke model with only one critical point. It is revealed that the impurity-induced Dicke QPT can happen in an arbitrary coupling regime of the cavity field and atoms while the Dicke QPT in the standard Dicke model occurs only in the strong coupling regime of the cavity field and atoms. This opens a way to observe the Dicke QPT in the intermediate and even weak coupling regime of the cavity field and atoms.

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I. INTRODUCTION

In recent years ultracold atoms in optical cavities have revealed themselves as attractive new systems for studying strongly-interacting quantum many-body theories. Their high degree of tunability makes them especially attractive for this purpose. One example, which has been extensively studied theoretically and experimentally, is the Dicke quantum phase transition (QPT) from the normal phase to the superradiant phase with a Bose-Einstein condensate (BEC) in an optical cavity [1–15]. The Dicke model [16] describes a large number of two-level atoms interacting with a single cavity field mode, and predicts the existence of the Dicke QPT [17, 18] from the normal phase to the superradiant phase. However, it is very hard to observe the Dicke QPT in the standard Dicke model, since the critical collective atom-field coupling strength needs to be of the same order as the energy separation between the two atomic levels. Fortunately, strong collective atom-field coupling has realized experimentally in a BEC coupling with a ultrahigh-finesse cavity filed [19, 20]. Employing the cavity-BEC system, the Dicke QPT has been observed experimentally through an atom-field coupling between a motional degree of freedom of the BEC and the cavity field [2]. The Dicke QPT corresponds to the process of self-organization of atoms [21]. In the experimental realization of the Dicke QPT based on the cavity-BEC system [2], the normal phase corresponds to the BEC being in the ground state associated with vacuum cavity field state while both the BEC and cavity field have collective excitations in the super-radiant phase. A few extended Dicke models [9, 10, 12, 22, 23] have been proposed to reveal rich phase diagrams and exotic QPTs, which are different from those in the original Dicke model.

In this paper, motivated by the recent experimental progress of cavity-BEC and impurity-doped BEC systems [24, 25] we propose a generalized Dicke model, an impurity-doped Dicke model (IDDM), by the use of an impurity-doped cavity-BEC. In our model, the impurity atom is an internal two-level system. The impurity-BEC interaction is tunable by an external magnetic field in the vicinity of Feshbach resonances [26, 27]. The cavity-BEC system adopted in our scheme is the same as that in the Dicke QPT experiment [2]. The IDDM can reduce the IDDM to the original Dicke model when the impurity-BEC interaction is switched off. We discuss how the presence of an impurity atom modifies the results of the original Dicke model. We show that the impurity atom can induce the Dicke QPT from the normal phase to the superradiant phase with the impurity population being the QPT parameter. It is found that the IDDM exhibits continuous Dicke QPT with an infinite number of critical points, which is different from that observed in the standard Dicke model with only one critical point. It is predicted that the impurity-induced Dicke QPT can happen in an arbitrary coupling regime of the cavity field and atoms while the Dicke QPT in the standard Dicke model occurs only in the strong coupling regime of the cavity field and atoms. This opens a possibility to observe the Dicke QPT in the intermediate and even weak coupling regime of the cavity field and atoms. We propose a scheme to control the impurity population in the cavity-BEC through making quantum measurements on an auxiliary atom outside the cavity, which is correlated to the impurity atom in the cavity-BEC.

The rest of the paper is organized as follows. In Sec. II, we present the IDDM by the use of an impurity-doped cavity-BEC system. In Sec. III, we investigate QPT properties of the IDDM and show the presence of continuous Dicke QPT with an infinite number of critical points in the IDDM. We also analyze the impurity-population dependence of the Dicke QPT, and show how to manipulate the impurity-population in the cavity-BEC system. Finally, we shall conclude our paper with discussions and remarks in the last section.

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II. THE IMPURITY-DOPED DICKE MODEL

In this section, we establish the IDDM through combining cavity-BEC and impurity-doped BEC techniques. Our proposed experimental setup is indicated in Fig. 1. A two-level impurity atom (qubit) with energy splitting \( \omega_Q \) is doped into an atomic BEC in a ultrahigh-finesse cavity. Both the impurity and BEC couple to a single cavity field and a transverse pump field.

In the absence of the impurity atom, the cavity-BEC system under consideration is the same at that employed in the experiments to observe the Dicke QPT \[2\]. The cavity contains \( N \) \( ^{87} \)Rb condensed atoms interacting with a single cavity model of frequency \( \omega_c \) and a transverse pump field of frequency \( \omega_p \). The excited atoms may remit photons either along or transverse to the cavity axis. This process couples the zero momentum atomic ground state to the symmetric superposition states of the \( k \)-momentum states. This yields an effective two-level system. Suppose that the frequency \( \omega_c \) and \( \omega_p \) are detuned far from the atomic resonance frequency \( \omega_\text{a} \), the excited atomic state can be adiabatically eliminated. In this case, the single atom Hamiltonian of the system under consideration can be written as

\[
\hat{H}_{(1)} = \frac{\hat{p}_z^2 + \hat{p}_r^2}{2m} + (U \cos^2 k \hat{z} - \Delta_c) \hat{a}^\dagger \hat{a} + V \cos^2 (k \hat{z}) \eta (\hat{a}^\dagger + \hat{a}) \cos (k \hat{z}).
\]

Here the first term is the kinetic energy of the atom with momentum operators \( \hat{p}_{x,z} \). The second term describes the cavity field, where \( \hat{a}^\dagger (\hat{a}) \) is the creation (annihilation) operator of the cavity field, which satisfy the bosonic commutation \( \{ \hat{a}, \hat{a}^\dagger \} = 1 \), \( U = \frac{\eta}{2} \) is the light shift induced by the atom where \( g_0 \) is the atom-cavity coupling strength, \( \Delta_c = \omega_c - \omega_\text{a} \) and \( \Delta = \omega_p - \omega_\text{a} \), \( k \) is the wave-vector, which is approximated to be equal on the cavity and pump fields. The third term describe the potential along the \( z \)-axis created by the pump field, the depth of the potential \( V = \frac{\Delta z}{\Delta_p} \) controlled by the maximum pump Rabi frequency \( \Omega_p \).

Next we consider interactions between the impurity qubit and the cavity-BEC. The impurity simultaneously interacts with the BEC, the cavity field, and the pump field. Firstly, we consider the interaction between the impurity and the BEC. We assume that the impurity interacts with the condensates via coherent collisions and only the upper state \( |0 \rangle \) interacts with the condensate considering its state-dependent trapped potential. Similar treatment can also be found in the Ref. \[28\]. Neglecting the constant term, The impurity-BEC coupling Hamiltonian has the form

\[
\hat{H}_{QB} = \kappa (\hat{\sigma}_z + 1) \hat{J}_r,
\]

where \( \hat{\sigma}_z \) is the pauli operator of the impurity qubit and \( \kappa = \kappa_{0,0} - \kappa_{1,0} \), where \( \kappa_{1,0} = (4\pi a_{01}(m) / \Omega_0) \int d^3 \vec{r} |\Phi_0(\vec{r})|^2 |\Phi_1(\vec{r})|^2 \) is the coupling strength between the impurity and zero( \( k \)-
momentum component BEC with $\varphi_0(\hat{r})$ being the wave function of the impurity in the upper state and $a_{\varphi(1),0}$ being the s-wave scattering length. Secondly, we consider the interactions between the impurity qubit and the cavity field and the pump field. The Hamiltonian of impurity qubit interacting with the cavity field and the pump field reads as

$$\hat{H}_Q = \omega \hat{a}^\dagger \hat{a} + \frac{\omega_Q}{2} \hat{\sigma}_z + g_Q \left( \hat{a}^\dagger \hat{\sigma}_- + \hat{a} \hat{\sigma}_+ \right) + \Omega_Q (\hat{\sigma}_+ + \chi) \hat{\sigma}_-, \hspace{1cm} (5)$$

where $\sigma_+$ ($\sigma_-$) is the raising (lowering) operator of the impurity qubit, $g_Q$ the coupling strength between the impurity qubit and the cavity field, $\Omega_Q$ the pump Rabi frequency. Here we have made a rotating wave approximation. In the far-detuning regime, one can use the Fröhlich-Nakajima transformation $\{24, 31\}$ to make the Hamiltonian in Eq. (5) become the following expression

$$\hat{H}_Q = \omega \hat{a}^\dagger \hat{a} + \frac{\omega_Q}{2} \hat{\sigma}_z + \xi_0 \hat{\sigma}_z \hat{\sigma}_-^2 + \frac{\xi_1}{2} \hat{\sigma}_z \hat{\sigma}_+ (\hat{a}^\dagger + \hat{a}), \hspace{1cm} (6)$$

where $\xi_1 = g_Q^2 / \Delta_Q$ and $\xi_2 = g_Q \Omega_Q / \Delta_Q$ with $\Delta_Q = \omega_Q - \omega_p$.

Combining Eq. (3) with Eqs. (4) and (6), we arrive at the total Hamiltonian of the IDDM

$$\hat{H} = (\omega + \xi_1 \hat{\sigma}_z) \hat{a}^\dagger \hat{a} + [\omega_0 + \kappa (\hat{\sigma}_z + 1)] \hat{J}_z + \frac{\kappa}{2} \hat{J}_z^2 + \frac{\omega_Q}{2} \hat{\sigma}_z$$

$$+ \frac{\lambda}{\sqrt{N}} \left( \hat{a} + \hat{a}^\dagger \right) (\hat{J}_+ + \hat{J}_-) + \xi_2 \hat{\sigma}_z (\hat{a} + \hat{a}^\dagger). \hspace{1cm} (7)$$

It is obvious that the Hamiltonian of the impurity-doped Dicke model reduces to that of the original Dicke model when the impurity-cavity-BEC interactions are switched off (i.e., $\kappa = 0, \xi_1 = \xi_2 = 0$) and the atomic nonlinear interaction in the BEC vanishes (i.e., $\chi = 0$).

### III. IMPURITY-INDUCED DICKE QUANTUM PHASE TRANSITION

In this section, we study quantum phases and QPTs in the IDDM proposed in the previous section. In order to understand QPTs, it is necessary to investigate the ground-state properties for a many-body system under consideration $\{18\}$. For the impurity-doped Dicke model in our proposal, its ground-state properties can be analyzed in terms of Holstein-Primakoff transformation $\{31\}$ due to the large number of atoms in the BEC. From the Hamiltonian (7), we can see that the properties of the cavity-BEC system is related to the initial state of the impurity qubit. We consider the impurity qubit as a control tool over the cavity-BEC system which is the controlled target system. In what follows, we will neglect the nonlinear interaction among condensed atoms to focus our attention on the influence of the impurity on QPT. Namely, we will take $\chi = 0$ in the following studies, which can be realized by Feshbach resonance techniques. Let the impurity population $\delta = \langle \sigma_z \rangle$, and make use of Holstein-Primakoff transformation to represent the angular momentum operators as single-mode bosonic operators ($[\hat{c}, \hat{c}^\dagger] = 1$)

$$\hat{J}_+ = \hat{c} \sqrt{N - \hat{c}^\dagger \hat{c}}, \hspace{0.5cm} \hat{J}_- = \sqrt{N - \hat{c}^\dagger \hat{c}},$$

$$\hat{J}_z = \hat{c} \hat{c}^\dagger - N/2, \hspace{1cm} (8)$$

after taking the mean value over a quantum state of the impurity atom we can rewrite the Hamiltonian (7) as the following form

$$\hat{H} = f_1 \hat{a}^\dagger \hat{a} + f_2 \hat{c} \hat{c}^\dagger + \xi_0 \delta (\hat{a} + \hat{a}^\dagger)$$

$$+ \frac{\lambda}{\sqrt{N}} \left( \hat{a} + \hat{a}^\dagger \right) \left( \hat{c} \sqrt{N - \hat{c}^\dagger \hat{c}} + \sqrt{N - \hat{c}^\dagger \hat{c}} \right), \hspace{1cm} (9)$$

where we have neglected a constant term, and effective frequencies of the two coupled oscillator modes are given by

$$f_1 = \omega + \xi_1 \delta, \hspace{1cm} f_2 = \omega_0 + \kappa (1 + \delta). \hspace{1cm} (10)$$

In order to describe the collective behaviors of the condensed atoms and the photon, one can introduce new bosonic operators $\hat{d} = \hat{a} + \sqrt{N} \hat{a}$ and $\hat{b} = \hat{c} - \sqrt{N} \hat{c}$ $\{32\}$, where $\alpha$ and $\beta$ are real numbers. Substituting bosonic operators $\hat{d}$ and $\hat{b}$ into the Hamiltonian (9) and neglecting terms with $N$ in the denominator, the Hamiltonian (9) can be expanded by

$$\hat{H} = N E_0 + \sqrt{N} \hat{H}_1 + \hat{H}_2, \hspace{1cm} (11)$$

where $E_0, \hat{H}_1$ and $\hat{H}_2$ are defined by

$$E_0 = f_1 \alpha^2 + f_2 \beta^2 - 4 \lambda K \alpha \beta,$$

$$\hat{H}_1 = \left[ 2 \lambda \alpha \left( K - \frac{\beta^2}{K} \right) - f_2 \beta \right] \hat{b} \hat{b}^\dagger$$

$$+ (f_1 \alpha - 2 \lambda K \beta) \left( \hat{d} + \hat{d}^\dagger \right) - 2 \xi_2 \delta \alpha, \hspace{1cm} (13)$$

$$\hat{H}_2 = f_1 \hat{a}^\dagger \hat{a} + \left( f_2 + \frac{2 \lambda \beta}{K} \right) \hat{b} \hat{b}^\dagger$$

$$+ \frac{\lambda}{2} \left( K - \frac{\beta^2}{K} \right) \left( \hat{d} + \hat{d}^\dagger \right) (\hat{b} + \hat{b}^\dagger)$$

$$+ \frac{\beta^3}{2 K^3} (\hat{b} + \hat{b}^\dagger)^2 + \xi_2 \delta (\hat{d} + \hat{d}^\dagger), \hspace{1cm} (14)$$

where we have introduced the parameter $K = \sqrt{1 - \beta^2}$. The collective excitation parameters $\alpha$ and $\beta$ can be determined from the equilibrium conditions $\partial E_0 / \partial \alpha = 0$ and $\partial E_0 / \partial \beta = 0$, which leads to the following two equations

$$f_1 \alpha - 2 \lambda K \beta = 0, \hspace{1cm} 2 \lambda \alpha \left( K - \frac{\beta^2}{K} \right) - f_2 \beta = 0, \hspace{1cm} (15)$$

from which we can obtain an equation governing the fundamental features of the QPT in the IDDM

$$\beta \left[ 8 \lambda^2 \beta^2 + 4 f_1 f_2 - 4 \lambda^2 \right] = 0. \hspace{1cm} (16)$$

Now we discuss quantum phases and QPT in the impurity-doped Dicke model. When $f_1 f_2 \geq 4 \lambda^2$, from Eq. (16) we can find $\alpha = \beta = 0$ due to $\lambda^2 > 0$. This means that both the condensed atoms and the photon have not collective excitations.
Hence the cavity-BEC system is in the normal phase. However, when \( f_1 f_2 < 4 \lambda^2 \), from Eqs. (15) and (16) we can obtain the two nonzero collective excitation parameters

\[
\alpha^2 = \frac{\lambda^2}{f_1} (1 - \nu^2), \quad \beta^2 = \frac{1}{2} (1 - \nu), \tag{17}
\]

where we have let \( \nu = f_1 f_2 / (4 \lambda^2) \). Eq. (17) implies that there exist macroscopic quantum population of the collective excitations of the condensed atoms and the photon in the IDDM. In this case, the cavity-BEC system is in the superradiant phase. The Dicke QPT is the QPT from the normal phase to the superradiant phase.

From the QPT equation (16) we can see that there exist two independent QPT parameters. One is the cavity-field-atom coupling strength \( \lambda \), another is the impurity population parameter \( \delta \). This is one important difference between the IDDM and the original Dicke model in which there is only one QPT parameter, the coupling strength \( \lambda \). Through the analysis below, we can see that it is the introduction of the new QPT parameter \( \delta \) that makes the IDDM to reveal new QPT characteristics which do not appear in the original Dicke model. In the following, under the condition \( f_1 \approx \omega \) due to \( \omega \gg \xi \delta \), we investigate the QPT in the IDDM for the three cases: (1) \( \delta \) is the QPT parameter with \( \lambda \) being an arbitrary fixed parameter; (2) \( \lambda \) is the QPT parameter with \( \delta \) being an arbitrary fixed parameter; (3) Both \( \lambda \) and \( \delta \) are independent QPT parameters.

In the first case, the impurity population \( \delta \) is the QPT parameter while the cavity-field-atom coupling strength \( \lambda \) is an arbitrary fixed parameter. So we can understand the QPT as the impurity induced QPT. From the QPT equation (16) we can find that the critical parameter \( \delta_c \) at the QPT point satisfies the following equation

\[
\delta_c = \frac{4 \lambda^2 - \omega \omega_0}{\omega \kappa} - 1, \tag{18}
\]

which indicates that there does always exist a critical impurity population \( \delta_c \) for an arbitrary value of the cavity-field-atom coupling strength \( \lambda \). From Eqs. (15) and (16), we can find the two quantum phases of the normal phase and the superradiant phase. The normal phase is in the regime of \( \delta < \delta_c \) \( (\delta > \delta_c) \) when \( \kappa < 0 \) \( (\kappa > 0) \), and we have \( \alpha^2 = \beta^2 = 0 \). In the superradiant-phase regime, we have nonzero collective excitations given by

\[
\alpha^2 = \frac{\lambda^2}{\omega^2} \frac{(\omega_0 + \kappa + \delta \omega)^2}{16 \lambda^2}, \quad \beta^2 = \frac{1}{2} \frac{\omega_0 (\omega_0 + \kappa + \delta \omega)}{4 \lambda^2}. \tag{19}
\]

It is interesting to note that in Eq. (18) the critical parameter \( \delta_c \) at the QPT point can vary continuously since the cavity-field-atom coupling strength \( \lambda \) can be manipulated continuously. This implies that the impurity-induced QPT in the IDDM is a continuous QPT in which a quantum system can undergo a continuous phase transition at the absolute zero of temperature as some parameter entering its Hamiltonian is varied continuously [35]. From the critical-point equation (18), we can see that the impurity-induced Dicke QPT happens even in the weak coupling regime of the cavity field and atoms. This is one of important differences between the IDDM and the original Dicke model in which the Dicke QPT appears only in the strong coupling regime of the cavity field and atoms.

We can determine the type of QPTs which happen in the impurity-doped Dicke model through investigating the non-analyticity of the scaled energy \( E_0 \) at the critical point in the thermodynamic limit \( N \rightarrow \infty \). If the \( n \)th derivative shows nonanalytic behavior then it is an \( n \)th order QPT. This has led researchers to examine the behavior of different correlations near the critical point, especially their analyticity properties as revealed by differentiation. In Figure 2 we have plotted the scaled ground-state energy \( E_0 \) and its second derivative \( \partial^2 E_0 / \partial \delta^2 \) with respect to the QPT parameter \( \delta \). It is easy to know that the first derivative of the scaled ground-state energy \( E_0 \) is continuous. From Fig. 2 we can see that the second derivative \( \partial^2 E_0 / \partial \delta^2 \) is discontinuous at the quantum critical point \( \delta = \delta_c \). Therefore, we can conclude that the QPT induced by the impurity is the second-order QPT.

In the second case, the cavity-field-atom coupling strength \( \lambda \) is the QPT parameter while the impurity population \( \delta \) is an arbitrary fixed parameter. So we can understand the QPT as the cavity-field-atom coupling induced QPT. From the QPT equation (16) we can find that the critical parameter \( \lambda_c \) at the QPT point satisfies the following equation

\[
4 \lambda_c^2 - f_1 f_2 = 0. \tag{20}
\]

Obviously, the cavity-field-atom coupling induced QPT in the IDDM is also a continuous QPT in which the QPT parameter can vary continuously by changing the impurity population. From equation (20), we can find the critical coupling strength to be

\[
\lambda_c = \frac{1}{2} \sqrt{\omega \omega_0 + \omega (1 + \delta)}, \tag{21}
\]

which indicates that the critical coupling strength \( \lambda_c \) can continuously vary with the impurity population \( \delta \) \( (-1 \leq \delta \leq 1) \). This means that the atom-field-coupling induced QPT in the IDDM is a continuous QPT in which the critical coupling strength \( \lambda_c \) can take continuously an infinite number of values when the impurity population varies in the regime \(-1 \leq \delta \leq 1 \). Hence, there are an infinite number of critical points of the QPT in the impurity-doped Dicke model. This is an important difference between the impurity-doped Dicke model and the original Dicke model in which there is only one critical point of the QPT with the coupling strength being \( \lambda_c = \sqrt{\omega \omega_0} / 2 \), which can be recovered from Eq. (21) when we take \( \kappa = 0 \). From Eq. (21) we can also see that the attractive (repulsive) interaction \( \kappa < 0 \) \( (\kappa > 0) \) between the impurity and the condensed atoms can decrease (increase) the critical coupling strength \( \lambda_c \). Therefore, we can realize the Dicke QPT in a broad range of the coupling strength \( \lambda \) through preparing various states of the impurity atom. This provides a wide window to observe experimentally the Dicke QPT.

The third case is a general situation in which two QPT parameters \( \delta \) and \( \lambda \) vary independently. In this case, nonzero
collective excitations are given by Eq. (21). In the thermodynamic limit \( N \to \infty \) we can obtain the scaled population inversion of BEC \( \langle J_z \rangle /N \) and the scaled intracavity intensity \( I/N \) as

\[
\langle J_z \rangle /N = \beta^2 - 1/2, \quad I/N = \alpha^2.
\] (22)

We have plotted the phase diagrams of the IDDM for the general case in Figure 3, which are described by the scaled population inversion of BEC \( \langle J_z \rangle /N \) and the scaled intracavity intensity \( I/N \) with respect to the impurity population \( \delta \) and the coupling strength \( \lambda \). The related parameters are taken as \( \omega = 400 \) and \( \kappa = -1/2 \) in unit of \( \omega_0 \). From Figure 3 we can see that the normal phase is in the region of \( \langle J_z \rangle /N = 0 \) and \( I/N = 0 \) while the superradiant phase is in the nonzero region of \( \langle J_z \rangle /N \) and \( I/N \). The Dicke QPT happens at the critical curve \( AB \) in the phase diagrams indicated in Figure 3. The critical curve in the phase diagrams appears as the intersection of the two phase regimes for the normal and superradiant phases, and it can be described by the equation

\[
\lambda^2 + 50\delta - 50 = 0.
\] (23)

The cavity-BEC is in normal-phase in the regime of \( \lambda^2 + 50\delta - 50 < 0 \) and in superradiant phase when \( \lambda^2 + 50\delta - 50 > 0 \). In superradiant phase, the collective excitations increase with the QPT parameters \( \delta \) and \( \lambda \).

Finally, we show how to manipulate the impurity population, which is the key point to observe the Dicke QPT induced by the impurity atom. In order to do this, we introduce an auxiliary atom outside the cavity, which is correlated with the impurity atom. We indicate that the impurity population can be controlled by making projective measurements upon the auxiliary atom. As an example, we consider the case of the impurity atom \( A \) and the auxiliary atom \( B \) initially being in the well-known Werner state

\[
\rho = \frac{1 - z}{4} I + z |\Psi\rangle \langle \Psi |, \quad 0 \leq z \leq 1,
\] (24)

where \( I \) is the unit operator, \( |\Psi\rangle \) is Bell state \( |\Psi\rangle = (|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B) / \sqrt{2} \). In this state, if one dose not measure the auxiliary atom, the impurity population is zero, i.e., \( \delta = \text{Tr}_{AB} [\rho \delta^2 z] = 0 \).

We now introduce two orthogonal complete projection operators

\[
\hat{\Pi}_\pm (\theta) = |\psi(\theta)\rangle \langle \psi(\theta) |_z \pm |\phi(\theta)\rangle \langle \phi(\theta) |_z,
\] (25)

where \( |\psi(\theta)\rangle \) and \( |\phi(\theta)\rangle \) are two orthogonal quantum states of the auxi-
the impurity population vanishes when quantum states of the auxiliary atom. Eq. (28) indicates that for different directions upon the auxiliary atom. Therefore, we can manipulate the impurity population through making projective measurements along different directions upon quantum states of the auxiliary atom. Eq. (28) indicates that the impurity population vanishes when \( z = 0 \), in which there does not exist quantum correlation between the impurity atom and the auxiliary atom. In this sense, the nonzero impurity population is induced by quantum correlation between the impurity atom and the auxiliary atom.

### IV. CONCLUSIONS

In conclusion, we have presented a generalized Dicke model, i.e., the IDDM, by the use of an impurity-doped cavity-Bose-Einstein condensate. The original Dicke mode can be recovered under certain conditions as a special case of the IDDM. We have shown that the impurity atom can induce the Dicke QPT from the normal phase to superradiant phase at a critical value of the impurity population. The impurity-induced Dicke QPT can be manipulated by the use of the impurity population. We have proposed a scheme to control the impurity population in the BEC through making quantum measurements on an auxiliary atom outside the cavity, which is correlated to the impurity atom in the BEC. We have found that the IDDM exhibits the continuous Dicke QPT with an infinite number of critical points. This multi-critical-point Dicke QPT is very different from the Dicke QPT observed in the standard Dicke model with only one critical point. In the IDDM, both the impurity atom and condensed atoms can induce the Dicke QPT. It is the interaction between the impurity-induced Dicke QPT and the cavity-field-BEC coupling induced Dicke QPT that leads the appearance of multi-critical points in the IDDM. These multi-critical points may be used as a resource for processing quantum information [14, 34]. We have predicted that the impurity-induced Dicke QPT can happen in an arbitrary coupling regime of the cavity field and atoms while the Dicke QPT in the standard Dicke model occurs only in the strong coupling regime of the cavity field and atoms. Hence, the IDDM reveals new regions of the Dicke QPT. This opens a way to observe the Dicke QPT in the intermediate and even weak coupling regime of the cavity field and atoms. Based on current experimental developments, we believe that it is possible to observe experimentally the impurity-induced Dicke QPT by measuring the atomic population or the mean photon number of the cavity field. The experimental realization of the scheme proposed in the present paper deserves further investigation.

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