A composite Higgs model with minimal fine-tuning: the large-N and weak-technicolor limit

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A Composite Higgs Model with Minimal Fine-Tuning

I. The Large-\(N\) and Weak-technicolor Limit

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Abstract

We suggest a criterion to minimize the amount of fine-tuning in a composite Higgs model. The paradigm of this type of model is the top-condensate model of Bardeen, Hill and Lindner (BHL). Although “minimally fine-tuned”, this model failed to account correctly for the masses of the top quark and the 125 GeV Higgs boson. We propose a generalization of the BHL model that employs finely-tuned extended technicolor (ETC) plus technicolor (TC) interactions. The additional freedom of this model may accommodate both \(m_t(173)\) and \(M_H(125)\). This paper studies the large-\(N_{TC}\) and \(N_C\) limit of this model in which technicolor is weak and does not contribute to electroweak symmetry breaking. Refinements including walking-TC dynamics and a renormalization group analysis of \(m_t\) and \(M_H\) will appear in a subsequent paper. A likely generic signal of this model is enhanced production of longitudinally-polarized weak bosons, alone and in association with \(H(125)\)

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1. Introduction and Plan

The concept of naturalness in particle physics is over 40 years old \(1, 2\) and the quest for a natural theory of electroweak symmetry breaking (EWSB) has been a dominant theme for almost that long \(3, 4\). This is because the elementary Higgs boson \(5, 6, 7\) introduced to trigger gauge symmetry breaking and give mass to the \(W\) and \(Z\) in the standard electroweak model \(8, 9\) is so very unnatural. There is no cut-off to the quadratically divergent corrections to its squared mass this side of the Planck scale. The discovery at the CERN LHC of a 125 GeV Higgs boson, \(H(125)\) \(10, 11\), possibly the lone Higgs boson of the standard model, has left supersymmetric and composite models of a light Higgs boson as the only remaining approaches to naturalness. Both involve a new energy scale \(\Lambda\) — either the scale of supersymmetry breaking or the scale of the new strong dynamics binding the composite Higgs — that serves to cut off the corrections to \(M_H^2\) at \(\mathcal{O}(\Lambda^2/16\pi^2)\) or, perhaps, \(\mathcal{O}(\Lambda^2/(16\pi^2)^2)\). Thus, \(\Lambda\) (or \(\Lambda/4\pi\)) must not be larger than about 1 TeV in order that the theory is natural. Generally, this is achieved by having the standard quadratic divergence in \(M_H^2\) from the top quark (and weak bosons) canceled by contributions from partners of the top (and \(W, Z\)). The failure, so far, to find these partners at masses below 1-2 TeV\(^1\) has put considerable stress on both supersymmetric and composite Higgs models. All such models and, in particular, composite Higgs ones — the subject of this paper — require a degree of fine-tuning of parameters that calls their “naturalness” into serious question \(12, 13\).

Therefore, in order to maintain the hypothesis that \(H(125)\) is a fermion-antifermion composite, I will provisionally adopt a “principle of least unnaturalness”: the least unnatural description of a composite \(H(125)\) is one that involves the smallest fine-tuning for \(M_H^2\) and the fewest number of free parameters that must be fine-tuned to achieve this.

The paradigm of this sort of light composite Higgs description is the topcolor model of Bardeen, Hill and Lindner (BHL) \(14\). In their model, \(q_L = (t,b)_L\) and \(t_R\) are assumed to have a new strong (presumably broken gauge) interaction at some high scale \(\Lambda\), giving rise to the \(SU(2) \otimes U(1)\)-invariant four-fermion interaction \(\mathcal{L}_{tt}\) at energies below \(\Lambda\),

\[
\mathcal{L}_{tt} = G \bar{q}_L^{ia} t_{Ra} \bar{t}_R^{ib} q_L^{Lib}.
\]

Here, the \(SU(2)_{EW}\) and color-\(SU(3)_C\) indices, \(i\) and \(a, b\), are summed over; the coupling \(G = \mathcal{O}(1/\Lambda^2)\). This Nambu–Jona-Lasinio (NJL) interaction produces the top-quark mass \(m_t\) and a \(\bar{q}_Lt_R\) composite scalar doublet \(\phi\) if \(G\) satisfies

\[
\frac{G N_C}{8\pi^2} \left( \Lambda^2 - m_t^2 \ln \frac{\Lambda^2}{m_t^2} \right) = 1, \text{ i.e., } G > G_c = \frac{8\pi^2}{N_C \Lambda^2}.
\]

\(^1\)https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsB2G, https://twiki.cern.ch/twiki/bin/view/CMSPublic/ResultsSUS, https://twiki.cern.ch/twiki/bin/view/ATLASPublic/ExoticsPublicResults, https://twiki.cern.ch/twiki/bin/view/ATLASPublic/SupersymmetricPublicResults.
Here, $\Lambda$ is also the cutoff of the momentum integral defining $m_t$ in the NJL-bubble, or large-$N_C$, approximation to its gap equation, while $N_C = 3$ is the number of ordinary colors. The composite scalar is a complex doublet under $SU(2)_{EW}$. It consists of three massless Goldstone bosons, eaten by $W$ and $Z$, and a Higgs boson $H$ of mass $M_H = 2m_t$.

It is clear from Eq. (2) that $m_t$ and $M_H$ can be nonzero but very much less than $\Lambda$ if and only if $G$ is greater than but very close to $G_c$. This is the fine-tuning of the BHL model, but it is the model’s only fine-tuning. Once it is imposed, all other $\Lambda$-dependence is logarithmic. Thus, even though $\Lambda$ is very large in BHL, the model exemplifies our notion of being least unnatural.

The low-energy Lagrangian describing $H$ interactions with $q_L$, $t_R$ and the EW gauge bosons is just the standard-model Lagrangian $[14]$. In that formulation, the negative $\mathcal{O}(\Lambda^2)$ contribution to $M_H^2$ and the $\ln\Lambda^2$ contribution to its quartic self-interaction are induced by the Yukawa interaction $\Gamma_t \bar{t}tH$, where $\Gamma_t$ is obtained from the residue of the Higgs pole in the $\bar{t}t \rightarrow \bar{t}t$ amplitude in the $0^\pm$ channel. Then, the Higgs vacuum expectation value (vev) $v$ is determined in the usual way from the quartic scalar coupling $\lambda \sim \Gamma_t^4 N_C \ln\Lambda^2/16\pi^2$ and the negative $M_H^2$. The value of $v$ is set by $M_W = \frac{1}{2}gv$, and then $m_t = \Gamma_t v/\sqrt{2}$. Thus, $m_t$, $M_H$ and $M_{W,Z}$ are all closely related in the BHL model. It is the most minimal dynamical model of electroweak symmetry breaking.

The renormalization group equations for the Yukawa coupling $\Gamma_t$ of $\bar{t}tH$, the quartic coupling $\lambda$, and the SM gauge couplings $g_{1,2,3}$ result in a significant reduction of $m_t$ and $M_H/m_t$, with smaller values obtained for larger $\Lambda$. Unfortunately, even for $\Lambda = 10^{15}$ GeV, BHL obtained $m_t = 229$ GeV and $M_H = 256$ GeV. Still, the importance of the BHL model is that it suggests a connection between the relatively large value of the top-quark mass and the lightness of the Higgs boson.

The purpose of this work is to ameliorate the fine-tuning of the BHL model and to obtain masses for the Higgs boson and top quark that are closer to their measured values. I demonstrate this in a simple model of technicolor (TC) plus strong extended technicolor (ETC). Technicolor with weakly-coupled extended technicolor, cannot account for the large value of $m_t$. But, if ETC is strong with its four-fermion coupling $g_{ETC}^2/M_{ETC}^2$ finely-tuned, it can produce a large $m_t$ that is much smaller than the mass scale $M_{ETC}$ of the ETC boson giving rise to $L_{\bar{t}t} [15,16]$. This is similar to the BHL model, but now the relevant scale, $\Lambda \simeq M_{ETC}$, is expected to be $\mathcal{O}(10)-\mathcal{O}(100)$ TeV, much lower than the BHL scale.\footnote{Relatively low masses for ETC bosons generating third-generation masses need not conflict with limits on flavor-changing neutral current interactions.} Furthermore, as shown in Ref. [17], the symmetry breaking phase transition must be second order. But, then, it implies the existence of a composite complex-scalar doublet $\phi$ with three Goldstone bosons and a massive but light scalar that couples strongly to the top quark, \textit{exactly as in the BHL model}. This scalar will be our candidate for $H(125)$. In our model, \footnote{In the UV-complete model with Lagrangian $L_{\bar{t}t}$ there is no quadratic divergence from the $HHWW$ vertex with a $W$-loop. Quadratic divergences involving weak-boson exchange do occur in subleading order in $1/N_C$.}
the Higgs boson is a composite of $\bar{t}t$ and $\bar{U}U$, where $U$ is the up-component of a technifermion doublet $T = (U, D)$.

In this paper, we treat this TC-ETC model in the large-$N$ ($N_C, N_{TC}$) limit. In a realistic model of this type, we expect TC to be strong enough to participate in EWSB. That is, its coupling $\alpha_{TC}$ is near an infrared fixed point of its $\beta$-function \cite{18, 19} and, so, it evolves slowly \cite{20, 21, 22, 23} and reaches the critical value $\alpha_c$ for chiral symmetry breaking at a scale $\Lambda_{TC}$ of order several hundred GeV. This seems necessary to account for light quark and lepton masses induced by ETC bosons with $M_{ETC}$ of many 100’s of TeV \cite{24}, for then the relevant technifermion condensates at $M_{ETC}$ are enhanced by a large anomalous dimension, $\gamma_m \simeq 1$ \cite{25}.

Including a walking $\alpha_{TC}$ in our analysis is a complication we will defer to a subsequent paper. The renormalization group running down from $\Lambda$ of fermion and Higgs masses referred to above will also be deferred. A further simplification is that light quark and lepton masses are left out; their inclusion is not technically difficult. The phenomenology of this TC-ETC model will be developed in a third paper (see Sec. 6 for a brief foretaste).

This model has features that might allow it to account better for the Higgs and top masses than the BHL model does. First, the technifermion contribution to the composite Higgs loosens the tight connection among $m_t$, $M_W$ and $M_H$. Second, there are now two large Yukawa couplings of the composite Higgs to fermions, $\Gamma_t$ to $\bar{t}t$ and $\Gamma_U$ to $\bar{U}U$. The renormalization group equations for $\Gamma_U$ will involve the strong walking gauge coupling of technicolor when that is included in the model. Third — and this is a point we are uncertain about — in addition to the light scalar $H$ induced by fine-tuning, there may be a lightest $0^{++}$ technihadron bound state. This state could mix with $H$ and drive down its mass, and certainly otherwise complicate the model’s phenomenology. This possibility will be considered in the third paper of this series.

In the remainder of this paper, then, we discuss the composite model in the large-$N$, weak-TC limit. We assume that all of EWSB comes from the single composite Higgs boson of the model, i.e., its vev is $v = 246$ GeV. Our development follows that in BHL. In Sec. 2 a model Lagrangian is presented and used to calculate the dynamical masses $m_t$ and $m_U$ at the scale $\Lambda$. The $\bar{q}q$ and $\bar{T}T$ scattering amplitudes are computed in Sec. 3 and their scalar and pseudoscalar (Goldstone) poles are revealed. We find that $M_H$ (at scale $\Lambda$) generically lies between $2m_t$ and $2m_U$. The electroweak gauge boson propagators in $O(g^2_{1,2})$ and large-$N$ approximations, including their Goldstone pole contributions, are computed in Sec. 4. A numerical study of the $2 \rightarrow 2$ scattering amplitudes in the scalar channel and the value of $M_H$ in one fitting scheme are presented in Sec. 5. We also calculate $\Gamma_t$ and the Higgs vev $v$ from the residue of the Higgs pole in the $\bar{t}t \rightarrow \bar{t}t$ amplitude. Section 6 includes preliminary comments on the model’s bound-state spectrum that may be of use to experimentalists. In particular, the possibility that weak boson production is enhanced by $\rho_T$ and $\omega_T$ states is discussed. We summarize the large-$N$, weak-TC results and what remains to be done in Sec. 7.

There has been much previous work using the NJL mechanism to describe the Higgs
boson, including Refs. [26, 27, 28] which preceded BHL in involving a new strong interaction of top quarks as the dynamics of EWSB. Topcolor led to the so-called top-seesaw models of Dobrescu and Hill [29] and Chivukula, et al. [30] and, more recently, Refs. [31, 32]. Bar-Shalom and collaborators proposed a “hybrid model” with a dynamical Higgs-like scalar plus an elementary scalar to describe $H(125)$ [33, 34]. They used an NJL Lagrangian with fourth generation quarks interacting via a topcolor interaction with scale $\Lambda \sim 1$ TeV to generate the dynamical scalar. Apart from the use of NJL in the bubble approximation, these models do not resemble ours, and the use of fourth generation quarks is reminiscent of the top-seesaw mechanism. Finally, as this paper was being written, there appeared one by Di Chiara, et al., who proposed a model of $H(125)$ based on TC and ETC, using a Lagrangian which is a truncated version of that introduced in Sec. 2 [35]. This model bears no further resemblance to ours; in particular, and among other things, strong, fine-tuned ETC is not employed in their paper to make the Higgs boson much lighter than the TC and ETC scales.

2. The TC-ETC Model in the Large-$N$ Approximation.

The fermions in this model transform under electroweak ($SU(2) \otimes U(1))_{EW}$, ordinary color $SU(3)_C$ and technicolor $SU(N_{TC})$, and they are

$$q_L = \left( \begin{array}{c} t \\ b \end{array} \right)_L \in (2, \frac{1}{6}, 3, 1), \quad t_R \in (1, \frac{2}{3}, 3, 1), \quad b_R \in (1, -\frac{1}{3}, 3, 1),$$

$$T_L = \left( \begin{array}{c} U \\ D \end{array} \right)_L \in (2, 0, 1, R_{TC}), \quad U_R \in (1, \frac{1}{2}, 1, R_{TC}), \quad D_R \in (1, -\frac{1}{2}, 1, R_{TC}),$$

where $R_{TC}$ is a complex representation of $SU(N_{TC})$. As explained above, light quarks and leptons are not dealt with in this paper. Likewise, additional technifermions are not included here.

The hard masses of $t$ and $U$ are generated by ETC interactions at a scale $\Lambda \simeq M_{ETC} = O(10) - O(100)$ TeV. At energies below $\Lambda$, the effective interaction is taken to be a sum of terms similar to the BHL Lagrangian, $\mathcal{L}_{tt}$, in Eq. (1):

$$\mathcal{L}_{ETC} = G_1 \bar{q}_L^i a t_{Ra} \bar{t}_R^b q_{Li} + G_2 \left( \bar{q}_L^i a t_{Ra} \bar{U}_R^\alpha T_{Li} + \text{h.c.} \right) + G_3 \bar{T}_L^\alpha U_{Ra} \bar{U}^\beta R T_{Li} \beta,$$

where the $SU(2)_{EW}$ and color-$SU(3)_C$ and $SU(N_{TC})$ indices, $i$ and $a, b$, and $\alpha, \beta$ are summed over. This interaction is to be thought of as having been Fierzed from an ETC interaction involving left times right-handed currents. The $SU(3)_C$ and $SU(N_{TC})$ indices appearing here therefore cannot correspond to exchange of ordinary massless color and TC gluons. The couplings $G_{1,2,3}$ are nominally positive and of $O(1/\Lambda^2)$. In this simplest form of our TC-ETC model, the $D$-technifermion is assumed to get no, or at least negligible, hard mass

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4The last two papers contain a large bibliography of related work.
from ETC. Then in the neglect of EW interactions, this model has an \((SU(2)_L \otimes U(1)_R)_q \otimes (SU(2)_L \otimes U(1)_R)_T\) flavor symmetry which is explicitly broken to \(SU(2) \otimes U(1)\) by the \(G_2\) term in \(\mathcal{L}_{ETC}\). If \(\mathcal{L}_{ETC}\) generates both \(\bar{t}t\) and \(\bar{U}U\) condensates and \(G_2 \neq 0\), this flavor symmetry is spontaneously broken to \(U(1)\) and just three Goldstone bosons appear. We shall see in Sec. 6 that all three \(G_i\) are comparable and that \(G_2\) is not weak. Therefore, there are not three relatively light pseudo-Goldstone bosons.

It is not difficult to add terms that generate \(m_D \neq 0\), but not so easy to maintain \(m_D = m_U\) in this model\(^5\). For example, adding

\[ G_3 \bar{T}_L^a D_R a \bar{D}_R^b T_L^b \]  \hspace{1cm} (5)

to \(\mathcal{L}_{ETC}\) gives an \((SU(2)_L \otimes SU(2)_R)_T\) invariant interaction and \(m_D = m_U\) only if \(G_2 = 0\). But, then, there is an unacceptable triplet of very light pseudo-Goldstone bosons (see Sec. 6). Adding instead

\[ G_2 \left( \bar{q}_L^a b_R a \bar{D}_R^b T_L^a + \text{h.c.} \right) + G_3 \bar{T}_L^a D_R a \bar{D}_R^b T_L^b \]  \hspace{1cm} (6)
generates \(m_b \neq 0\) as well as \(m_D \neq 0\). These masses will differ from \(m_t\) and \(m_U\), respectively. But that does not necessarily upset the observed closeness of the \(\rho\)-parameter to one. Further analysis of such a model is beyond our scope in this paper.

Following BHL, the gap equations for \(m_t\) and \(m_U\) renormalized at the scale \(\Lambda\) are (see the Appendix)\(^7\)

\[
m_t = -\frac{1}{2} G_1 \langle \bar{t}t \rangle - \frac{1}{2} G_2 \langle \bar{U}U \rangle \\
= \frac{G_1 N_{TC} m_t}{8\pi^2} \left( \Lambda^2 - m_t^2 \ln \frac{\Lambda^2}{m_t^2} \right) + \frac{G_2 N_{TC} m_U}{8\pi^2} \left( \Lambda^2 - m_U^2 \ln \frac{\Lambda^2}{m_U^2} \right) ; \hspace{1cm} (7)
\]

\[
m_U = -\frac{1}{2} G_2 \langle \bar{U}U \rangle - \frac{1}{2} G_3 \langle \bar{U}U \rangle \\
= \frac{G_2 N_{TC} m_t}{8\pi^2} \left( \Lambda^2 - m_t^2 \ln \frac{\Lambda^2}{m_t^2} \right) + \frac{G_3 N_{TC} m_U}{8\pi^2} \left( \Lambda^2 - m_U^2 \ln \frac{\Lambda^2}{m_U^2} \right) . \hspace{1cm} (8)
\]

Here, \(N_{TC}\) is the dimensionality of the \(T\)-representation \(R_{TC}\). So long as \(G_2 \neq 0\) — which we assume throughout this paper — the independence of the \(N_C\) and \(N_{TC}\) imply that (just multiply Eq. (7) by \(m_U\) and Eq. (8) by \(m_t\))

\[ G_2 = G_1 \frac{m_U}{m_t} = G_3 \frac{m_t}{m_U} . \hspace{1cm} (9) \]

\(^5\)I thank Sekhar Chivukula for the conversation that led to this paragraph.

\(^6\)As did BHL, we assume that the condensates \(\langle \bar{t}\gamma_5 t \rangle = \langle \bar{U}\gamma_5 U \rangle = 0\).

\(^7\)The gap equations approximated in Eqs. (7,8) are integrals over an ETC boson propagator with mass \(\Lambda\) times the mass term in the \(t\) or \(U\) propagator. In a walking-\(\sigma_{TC}\) model, the \(U\)-mass term has both a dynamical piece, falling off roughly as \(1/p\) above the technicolor scale \(\Lambda_{TC}\) and a hard piece \(m_U(p)\) that is constant up to \(p \approx \Lambda \gg \Lambda_{TC}\). In accord with our weak-TC assumption, the dynamical piece is ignored. In any case, the hard-mass term will dominate the integral unless \(m_U \ll \Lambda_{TC}^2/\Lambda\).

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6
Then, Eqs. [10] imply the following generalization of the "fine-tuning condition in Eq. [2]:

\[
\frac{G_1 N_C}{8\pi^2} \left( \Lambda^2 - m_t^2 \ln \frac{\Lambda^2}{m_t^2} \right) + \frac{G_3 N_{TC}}{8\pi^2} \left( \Lambda^2 - m_U^2 \ln \frac{\Lambda^2}{m_U^2} \right) = 1.
\] (10)

The mass parameters in this weak-TC model, \( m_t, m_U, M_W, M_H \) and \( \Lambda \) are not independent. If \( m_t \) and \( m_U \) are nonzero, only one of the three \( G_i \) is an independent parameter.

As in BHL, Eqs. (7,8) contain this model's only quadratic divergences and, for nonzero \( m_t, m_U \ll \Lambda \), its only fine-tuning of parameters. Once Eq. (10) is enforced, all other \( \Lambda \)-dependence is logarithmic.

3. The \( 2 \rightarrow 2 \) Amplitudes in the Scalar and Goldstone Boson Channels

We again follow BHL to calculate the \( 2 \rightarrow 2 \) amplitudes in the scalar and pseudoscalar channels. For the neutral scalar channel, there are three amplitudes to calculate: \( \bar{t}t \rightarrow \bar{t}t, \bar{U}U \rightarrow \bar{U}U \) and \( \bar{t}t \leftrightarrow \bar{U}U \). The effective Hamiltonian for the bubble is

\[
\mathcal{H}_{0^+} = -\frac{1}{4} G_1\bar{t}^a t_a \bar{b}^b t_b - \frac{1}{2} G_2 \bar{t}^a t_a \bar{U}^\alpha U_\alpha - \frac{1}{4} G_3 \bar{U}^\alpha U_\alpha \bar{U}^\beta U_\beta .
\] (11)

The \( \bar{t}t \rightarrow \bar{t}t \) amplitude is

\[
\Gamma_{0^+}^{\bar{t}t}(p) = -\frac{1}{2} G_1 - (-\frac{1}{2} G_1)^2 i \int d^4 x \ e^{ipx} \langle \Omega | T(\bar{t}^a t_a(x) \bar{b}^b t_b(0)) | \Omega \rangle - (-\frac{1}{2} G_2)^2 i \int d^4 x \ e^{ipx} \langle \Omega | T(\bar{t}^a U_\alpha(x) \bar{U}^\alpha t_a(0)) | \Omega \rangle + \cdots
\] (12)

The integrals are cut off at \( \Lambda \) and evaluated in the Appendix. The relation \( G_2^2/G_1 = G_3 \) makes this sum a geometric series,

\[
\Gamma_{0^+}^{\bar{t}t}(p) = -\frac{1}{2} G_1 \left[ 1 - \frac{G_1 N_C}{8\pi^2} \left( \Lambda^2 - m_t^2 \ln \frac{\Lambda^2}{m_t^2} \right) \right] - \frac{G_1 N_C (p^2 - 4m_t^2)}{16\pi^2} \int_0^1 dx \ln \left( \frac{\Lambda^2}{m_t^2 - p^2 x(1-x)} \right) - \frac{G_3 N_{TC} (p^2 - 4m_U^2)}{16\pi^2} \int_0^1 dx \ln \left( \frac{\Lambda^2}{m_U^2 - p^2 x(1-x)} \right) \right]^{-1}.
\] (13)

By Eq. (10), the first line on the left is zero and \( \Gamma_{0^+}^{\bar{t}t} \) becomes

\[
\Gamma_{0^+}^{\bar{t}t}(p) = m_t^2 \left[ \frac{N_C m_t^2 (p^2 - 4m_t^2)}{8\pi^2} \int_0^1 dx \ln \left( \frac{\Lambda^2}{m_t^2 - p^2 x(1-x)} \right) \right] + \frac{N_{TC} m_U^2 (p^2 - 4m_U^2)}{8\pi^2} \int_0^1 dx \ln \left( \frac{\Lambda^2}{m_U^2 - p^2 x(1-x)} \right) \right]^{-1}.
\] (14)
The scalar-channel amplitudes for $UU \to UU$ and $t\bar{t} \leftrightarrow UU$ are $\Gamma_{0+}^{UU} = (m_U/m_t)^2 \Gamma_{0+}^{t\bar{t}}$ and $\Gamma_{0+}^{tt\leftrightarrow UU} = (m_U/m_t)^2 \Gamma_{0+}^{t\bar{t}}$. Then the sum of the four $2 \to 2$ amplitudes in the neutral scalar channel is

$$\Gamma_{0+}^0(p) = (m_t + m_U)^2 \left[ \frac{N_cm_t^2(p^2 - 4m_t^2)}{8\pi^2} \int_0^1 dx \ln \left( \frac{\Lambda^2}{m_t^2 - p^2x(1-x)} \right) \right. + \left. \frac{N_{TC}m_U^2(p^2 - 4m_U^2)}{8\pi^2} \int_0^1 dx \ln \left( \frac{\Lambda^2}{m_U^2 - p^2x(1-x)} \right) \right]^{-1}. \quad (15)$$

The scalar amplitude has a pole at $p^2 = M_H^2$, the solution of

$$N_cm_t^2(M_H^2 - 4m_t^2) \int_0^1 dx \ln \left( \frac{\Lambda^2}{m_t^2 - M_H^2x(1-x)} \right) + N_{TC}m_U^2(M_H^2 - 4m_U^2) \int_0^1 dx \ln \left( \frac{\Lambda^2}{m_U^2 - M_H^2x(1-x)} \right) = 0. \quad (16)$$

This result, the Higgs mass at scale $\Lambda$ in the large-$N$ approximation, will be modified by renormalization-group running from $\Lambda$ down to the $H$-pole. The BHL mass, $M_H(\Lambda) = 2m_t$, is obtained by setting $m_U = 0$ in Eq. (16). A very good approximation to the solution to Eq. (16) is

$$M_H = 2\sqrt{\frac{N_cm_t^4 + N_{TC}m_U^4}{N_cm_t^4 + N_{TC}m_U^4}}. \quad (17)$$

The effective Hamiltonian for the neutral and charged Goldstone poles in $2 \to 2$ scattering is

$$H_{0-} = \frac{1}{4} G_1 \bar{t}^a \gamma_5 t_a \bar{b}^b \gamma_5 b_b + \frac{1}{2} G_2 \bar{t}^a \gamma_5 t_a \bar{U}^\alpha \gamma_5 U_\alpha + \frac{1}{4} G_3 \bar{U}^\alpha \gamma_5 U_\alpha \bar{U}^\beta \gamma_5 U_\beta - G_1 \bar{b}^b L_{t,a} \bar{b}^b R_{b,b} - G_2 \left( \bar{b}^b L_{t,a} \bar{U}^\alpha R_{b,b} D_{La} + \text{h.c.} \right) - G_3 D^\alpha L_{t,a} \bar{U}^\alpha R_{b,b} D_{Lb}. \quad (18)$$

The $t\bar{t} \to t\bar{t}$ amplitude is (note the $i$'s in $i\gamma_5$, left out in BHL):

$$\Gamma_{0-}^{t\bar{t}}(p) = -\frac{1}{4} G_1 - \left( \frac{1}{4} G_1 \right)^2 i \int d^4 x e^{ip\cdot x} \langle \Omega | T(\bar{t}^a i\gamma_5 t_a(x) \bar{b}^b i\gamma_5 b_b(0)) | \Omega \rangle + \cdots. \quad (19)$$

Proceeding as in the scalar case, the sum of the $2 \to 2$ amplitudes in the neutral channel is

$$\Gamma_{0-}^0(p) = \frac{8\pi^2(m_t + m_U)^2}{p^2} \left[ N_cm_t^2 \int_0^1 dx \ln \left( \frac{\Lambda^2}{m_t^2 - p^2x(1-x)} \right) + N_{TC}m_U^2 \int_0^1 dx \ln \left( \frac{\Lambda^2}{m_U^2 - p^2x(1-x)} \right) \right]^{-1}. \quad (20)$$

The corresponding amplitude in the charged $t\bar{b} \to t\bar{b}$ channel is

$$\Gamma_{0-}^{t\bar{b}}(p) = -\frac{1}{4} G_1 - \left( G_1 \right)^2 i \int d^4 x e^{ip\cdot x} \langle \Omega | T(\bar{t}^a R_{t,a} \bar{b}^b R_{b,b}(x) \bar{t}^b L_{t,a}(0)) | \Omega \rangle + \cdots. \quad (21)$$
Including the other channels, these sum up to

\[
\Gamma_{0^-}^\pm (p) = \frac{2\pi^2 (m_t + m_U)^2}{p^2} \left[ N_C m_t^2 \int_0^1 dx \ln \left( \frac{\Lambda^2}{m_t^2 x - p^2 x (1 - x)} \right) \right]^{-1} + N_{TC} m_U^2 \int_0^1 dx \ln \left( \frac{\Lambda^2}{m_U^2 x - p^2 x (1 - x)} \right) .
\] (22)

The manipulations [14] used to obtain these results are given in the Appendix.

As noted above, there may be an isotriplet of pseudo-Goldstone bosons that acquire mass from the \( G_2 \)-interaction. This is discussed briefly in Sec. 6 and considered in more detail in a later paper.

4. The Electroweak Gauge Boson Propagators

In this section we compute the EW propagators in the NJL-bubble approximation, neglecting EW gauge-boson radiative corrections. As in BHL, the EW fields are rescaled to bring the gauge coupling into their kinetic terms, i.e., \((1/4g^2) F^2_{\mu\nu}\). The \((SU(2) \otimes U(1))_{EW}\) currents are

\[
j_{\mu}^A = \bar{q} L \gamma_{\mu} \frac{\tau_A}{2} q_L + \bar{T} L \gamma_{\mu} \frac{\tau_A}{2} T_L \quad (A = 1, 2, 3) ; \\
j_{\mu}^0 = \frac{1}{6} (\bar{q} L \gamma_{\mu} q_L + \bar{q} R \gamma_{\mu} q_R) + \bar{q} R \gamma_{\mu} \frac{\tau_3}{2} q_R + \bar{T} R \gamma_{\mu} \frac{\tau_3}{2} T_R .
\] (23)

The inverse \(W\)-propagator is

\[
\frac{1}{g_2^2} (D^\pm (p))_{\mu\nu}^{-1} = \frac{1}{g_2^2} (p_\mu p_\nu - p^2 g_{\mu\nu}) + \frac{i}{2} \int d^4 x \ e^{ipx} \langle \Omega | T(j^{(1+i2)}_{L\mu}(x) j^{(1-i2)}_{L\nu}(0)) | \Omega \rangle \\
= (p_\mu p_\nu - p^2 g_{\mu\nu}) \left( \frac{1}{g^2_{2W}(p^2)} - \frac{f^2_W(p^2)}{p^2} \right) .
\] (24)

In the second line of Eq. (24), \(g_{2W}^{-2}(p^2)\) is computed from the bare inverse \(W\)-propagator plus the one-loop correlator \(\Pi_{\mu\nu}^{\pm}(p)\) of a pair of charged weak currents. It is given in the TC-ETC model by (see the Appendix for details)

\[
g_{2W}^{-2}(p^2) = g_2^{-2} + \frac{1}{16\pi^2} \int_0^1 dx \ 2x (1 - x) \left[ N_C \ln \left( \frac{\Lambda^2}{m_t^2 x - p^2 x (1 - x)} \right) + N_{TC} \ln \left( \frac{\Lambda^2}{m_U^2 x - p^2 x (1 - x)} \right) \right] .
\] (25)

The contribution to the \(W\)-propagator from the massless pole in \(\Gamma_{0^-}^\pm\) in Eq. (22) is

\[
f^2_W(p^2) = \frac{1}{16\pi^2} \int_0^1 dx \ x \left[ N_C m_t^2 \ln \left( \frac{\Lambda^2}{m_t^2 x - p^2 x (1 - x)} \right) + N_{TC} m_U^2 \ln \left( \frac{\Lambda^2}{m_U^2 x - p^2 x (1 - x)} \right) \right] .
\] (26)
A comment is in order here: The fermion masses in the one-loop-EW and fermionbubble-sum contributions to the weak-boson propagators must be the hard masses generated
by \( \mathcal{L}_{ETC} \). It is these masses that satisfy the gap Eqs. (7,8), and those relations are used to
remove the \( \Lambda^2 \)-dependence from the bubble sums. Furthermore, the masses in the \( m^2 g_{\mu \nu} \) part
of the EW loop must be the same as those in the \( m^2 p_\mu p_\nu / p^2 \) terms coming from the bubble
sum in order that Ward-Takahashi (WT) identities are maintained and the propagators are transverse. Therefore, in accord with the model defined by \( \mathcal{L}_{ETC} \), \( m_b = M_D = 0 \)
in \( g_{2W}(p^2) \) and \( f^2_W(p^2) \). A more complete treatment of the propagators will include the
dynamical strong-TC contributions to the Goldstone poles and, to satisfy the WT identities, the fermion masses. These do not have ln \( \Lambda^2 \) dependence as they are cut-off by TC dynamics
(mainly) at scale \( \Lambda_{TC} \). This more complicated analysis is deferred to a later paper. The
upshot of all this is that in the weak-TC limit all of EWSB comes from \( \mathcal{L}_{ETC} \) and, since
there is just one complex Higgs doublet,

\[ f^2_W(0) = 1/(4\sqrt{2}G_F) = (123 \text{ GeV})^2 \cong M_W^2/g_{2W}^2(0), \quad (27) \]

The inverse neutral propagator matrix is

\[
\frac{1}{g_i g_j} (D^0(p))^{-1}_{\mu \nu} = \begin{pmatrix}
1/g_2^2 & 0 \\
0 & 1/g_1^2
\end{pmatrix} (p_\mu p_\nu - p^2 g_{\mu \nu}) \\
+ i \int d^4 x e^{ip \cdot x} \langle \Omega | \begin{pmatrix}
T(j^3_\mu(x)j^3_\nu(0)) & T(j^3_\mu(x)j^3_\nu(0)) \\
T(j^0_\mu(x)j^3_\nu(0)) & T(j^0_\mu(x)j^0_\nu(0))
\end{pmatrix} | \Omega \rangle. \quad (28)
\]

As for the \( W \)-propagator, this is calculated from the bare inverse propagators, the one-loop
neutral-current correlators, and the large-\( N \) bubble sums for \( \Gamma^0_{\mu \nu} \). This gives

\[
\frac{1}{g_i g_j} (D^0(p))^{-1}_{\mu \nu} = (p_\mu p_\nu - p^2 g_{\mu \nu}) \begin{pmatrix}
1/g_2^2(p^2) & 0 \\
0 & 1/g_1^2(p^2)
\end{pmatrix} - \begin{pmatrix}
1 & -1 \\
-1 & 1
\end{pmatrix} \begin{pmatrix}
f^2_Z(p^2) \\
0
\end{pmatrix} / p^2, \quad (29)
\]

where \( g_2^2(p^2), g_1^2(p^2), \) and \( f^2_Z(p^2) \) have been defined to give a massless photon pole in
the diagonalized neutral propagator. Reading off \( f^2_Z \) from the massless pole term in the
\[ \langle j_{\mu}^a j_{\nu}^b \rangle \text{-correlator, we obtain (see the Appendix)} \]
\[
f_2^Z(p^2) = \frac{1}{32\pi^2} \int_0^1 dx \left[ N_C m_t^2 \ln \left( \frac{\Lambda^2}{m_t^2 - p^2 x(1-x)} \right) + N_{TC} m_U^2 \ln \left( \frac{\Lambda^2}{m_U^2 - p^2 x(1-x)} \right) \right] + \frac{N_C p_t^2}{16\pi^2} \int_0^1 dx \frac{1}{x(1-x)} \ln \left( \frac{-p^2 x(1-x)}{m_t^2 - p^2 x(1-x)} \right); \]
\[
g_{2Z}(p^2) = g_t^2 + \frac{1}{16\pi^2} \int_0^1 dx \left\{ N_C \left[ \frac{4}{3} \ln \left( \frac{\Lambda^2}{m_t^2 - p^2 x(1-x)} \right) + \frac{2}{3} \ln \left( \frac{\Lambda^2}{-p^2 x(1-x)} \right) \right] \right\} \]
\[*\]
\[
g_{1Z}(p^2) = g_t^2 + \frac{1}{16\pi^2} \int_0^1 dx \left\{ N_C \left[ \frac{20}{9} \ln \left( \frac{\Lambda^2}{m_t^2 - p^2 x(1-x)} \right) + \frac{2}{3} \ln \left( \frac{\Lambda^2}{-p^2 x(1-x)} \right) \right] \right\}. \]

The \((-p^2 x(1-x))\) arguments of the logarithms come from a b or D-fermion loop with \(m_b = m_D = 0\).

The \(\rho\)-parameter,
\[
\rho \approx \frac{f_W^2(0)}{f_2^Z(0)} = \frac{(32\pi^2)^{-1} \left[ N_C m_t^2 \left( \ln(\Lambda^2/m_t^2) + \frac{1}{2} \right) + N_{TC} m_U^2 \left( \ln(\Lambda^2/m_U^2) + \frac{1}{2} \right) \right]}{(32\pi^2)^{-1} \left[ N_C m_t^2 \ln(\Lambda^2/m_t^2) + N_{TC} m_U^2 \ln(\Lambda^2/m_U^2) \right]}, \]

the running of the EW gauge couplings, and the W and Z pole masses, solutions of
\[
M_W^2 = g_{2W}^2(M_W^2) f_{2W}^2(M_W),
M_Z^2 = (g_{1Z}^2(M_Z^2) + g_{2Z}^2(M_Z)) f_{2Z}^2(M_Z), \]

will be discussed in the numerical calculations next, in Sec. 5.

There has been much discussion over the years of the constraint on technicolor theories from the S-parameter [33, 37, 38, 39]. However, as emphasized in Refs. [40, 41], all of these calculations of S assume that TC dynamics is QCD-like, with asymptotic freedom setting in rather quickly above \(\Lambda_{TC}\). But TC dynamics cannot be QCD-like. As noted earlier, \(\alpha_{TC}\) must be a walking gauge coupling to avoid unwanted large flavor-changing neutral current interactions, and this invalidates the assumptions made to calculate \(S\). Calculating \(S\) in the strong dynamics of walking technicolor is now the object of a number of groups using lattice gauge theoretic techniques; see, e.g., Refs. [42, 43, 44].

5. Numerical Calculations in the Large-\(N\) Approximation

Here we present outcomes of the large-\(N\), weak-TC results of Secs. 3 and 4 for a simple numerical scheme. In this scheme we fix \(\Lambda\) and then calculate \(m_t(\Lambda)\) using one-loop QCD
running with six flavors from $\Lambda$ down to $2m_t$ and with five flavors down to $m_t = 173$ GeV \cite{45}. Fixing $f_W(0) = (4\sqrt{2}G_F)^{-1/2} \approx 123$ GeV in Eq. (33) then determines $N_{TC}m_U^2$. Note that, in leading-log approximation, this same combination, $N_{TC}m_U^2$, appears in the formula for $M_H^2$, Eq. (16). Finally, we choose $N_{TC} = 15$ (which corresponds to an $SU(6)$ TC gauge group with $T = (U, D)$ in the antisymmetric second-rank tensor representation) and calculate $M_H(\Lambda)$. As a check on our calculation, we obtain the Higgs vev $v$ from $m_t(\Lambda) = \Gamma_t(\Lambda) v/\sqrt{2}$, where

$$
\left(\frac{\Gamma_t(\Lambda)}{\sqrt{2}}\right)^2 = \lim_{p^2 \to M_H^2} (p^2 - M_H^2) \Gamma^{\mu\nu}_t(p),
$$

and compare the result to the input $2f_W(0) = 246$ GeV.

We consider two cases, $\Lambda = 20$ TeV and $\Lambda = 500$ TeV. There is no obvious reason not to have such a high scale for generating $m_t$ since the only price is more fine tuning of $G_1$ in Eq. (4). In a more complete TC-ETC model, such a large ETC mass for the third generation may also suffice to produce masses for the lighter quarks while suppressing their flavor-changing neutral current interactions. This would eliminate the need for a “tumbled” spectrum of ETC masses. The difference in the two cases we consider is greater than one might have anticipated given the merely logarithmic dependence on $\Lambda$ of the $2 \to 2$ amplitudes calculated in Sec. 3.

The results are in Table 1. The EW couplings $g_{1,2}$ used in Eqs. (25,31,32) to calculate the $W$ and $Z$ pole-masses were determined by requiring that $g_{1,2}^2$ at $p = M_Z = 91.18$ GeV give $\sin^2 \theta_W(M_Z) = g_1^2/(g_1^2 + g_2^2) = 0.23116$ and $\alpha(M_Z) = 1/128$ \cite{45}. All the results in the lower half of the table are very good except for a slightly high $Z$-pole mass in the second case.

It is worth recalling that the Higgs mass will be renormalized from $\Lambda$ down to the electroweak scale. In BHL \cite{14}, $M_H(\Lambda = 10^{15}$ GeV) = 330 GeV decreased to 256 GeV. We expect our values of $M_H$ to decrease as well but, of course, we cannot guess by how much — especially since the effect of a walking TC coupling has to be included in the running of $m_U$. Another thing is that, since we are assuming there is only one Higgs boson, its coupling to the top quark at the top mass will be $v/(\sqrt{2}m_t) \cong 1$, so that the $ggH$ coupling

| $\Lambda$ | $m_t$  | $m_U$  | $M_H$  | $\Gamma_t$ | $v = \sqrt{2}m_t/\Gamma_t$ |
|----------|--------|--------|--------|------------|--------------------------|
| 20 TeV   | 134 GeV| 167 GeV| 330 GeV| 0.783      | 242 GeV                  |
| 500 TeV  | 118 GeV| 126 GeV| 250 GeV| 0.685      | 244 GeV                  |

Table 1: The Higgs mass, $\rho$-parameter and the $W$, $Z$-pole masses calculated for ETC scales $\Lambda = 20$ and 500 TeV. The calculation scheme adopted is described in the text.
of the Higgs to QCD gluons will have its standard-model strength and the production rates \( \sigma(gg \to \gamma\gamma, ZZ^*, WW^*) \) will be as in the standard model.

6. Preliminary Remarks on the Model’s Phenomenology

The low-lying bound-state spectrum of this model depends on the magnitude of the ETC couplings \( G_i \) and the TC gauge coupling \( \alpha_{TC} \). The chiral-flavor symmetry of our model is \((SU(2)_L \otimes U(1)_R)_q \otimes (SU(2)_L \otimes U(1)_R)_T\) explicitly broken to \( SU(2) \otimes U(1) \) by \( G_2 \).

Suppose first, then, that \( G_2 = 0 \). There are three possibilities: (1) \( G_1 \) and \( G_3 \) are supercritical, i.e., \( G_1 > 8\pi^2/N_C\Lambda^2, G_3 > 8\pi^2/N_{TC}\Lambda^2 \) and \( m_t, m_U \neq 0 \) are solutions of the gap Eqs. \((7,8)\); (2) \( G_3 \) is supercritical \( (m_U \neq 0) \), but \( G_1 \) is not \( (m_t = 0) \); and (3) vice-versa. Possibilities (2) and (3) are excluded because (2) \( m_t \) cannot be zero and (3) TC would likely generate a dynamical mass for \( U \) and \( D \) spontaneously breaking their chiral symmetry, now \( SU(2)_L \otimes SU(2)_R \), giving rise to an additional triplet of massless Goldstone bosons. They could acquire mass only from the EW gauge interactions and they would be very light \( [24] \), hence excluded (e.g., by production of the charged pair in \( e^+e^- \) annihilation).

In possibility (1), we would have two light composite scalar doublet bound states, hence two light Higgs bosons and two massless Goldstone triplets. One combination of the Goldstone triplets would be eaten by the \( W \) and \( Z \), but the orthogonal triplet is again very light and excluded.

Once \( G_2 \neq 0 \), its magnitude is fixed by Eqs. \((9,10)\). Both \( m_t \) and \( m_U \) are nonzero. Now, both terms in Eq. \((10)\) must be less than one. We saw in Sec. 5 that a very small \( m_U/m_t \) or \( m_t/m_U \) is unlikely to be compatible with \( f_W(0) \approx 123 \text{ GeV} \). Thus, \( G_2 \) is not much different from \( G_1 \) and/or \( G_3 \) and it cooperates with them to make the model just barely critical, producing \( 0 < m_t, m_U \ll \Lambda \) — our model’s fine-tuning.

What does this mean for the spectrum of relatively low-lying bound states? So long as TC is present and confining, we expect isovector \( \rho_T \) and isoscalar \( \omega_T \) which are \( \bar{T}T \) states. It is not clear how heavy the lightest \( \rho_T \) and \( \omega_T \) are. If their binding is due mainly to TC, we would guess their masses are in the range 500 GeV to 2 TeV. If the strong ETC interactions \( \mathcal{L}_{ETC} \) contribute to their mass other than through the hard mass \( m_U \), they might be much heavier. Because the hard technifermion mass is an \( I = 1 \) operator, the neutral and charged \( \rho_T \) and the \( \omega_T \) should all have nearly the same mass. It is also possible that the mass-eigenstate vectors are ideally mixed \( \bar{U}U \) and \( \bar{D}D \) states.

Assuming they are lighter than \( \sim 2 \text{ TeV} \), the \( \rho_T \) and \( \omega_T \) will be produced at the LHC at observable rates by the Drell-Yan process, \( q\bar{q} \to \gamma, Z, W \to \rho_T \) or \( \omega_T \) \([10]\) and, if they are heavy enough, via weak vector boson fusion (see Delgado, Grojean, Maina and Rosenfeld in

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\(^8\)Presumably, once its effects are included, \( \alpha_{TC} \) becomes strong enough to confine technicolor at a distance scale of \( \mathcal{O}(1 \text{ TeV})^{-1} \).

\(^9\)If we relax this condition on \( f_W(0) \), then there must be at least two Higgs doublets and, therefore, two Higgs bosons in a TC-ETC model. This is an interesting complication that we do not pursue in this paper.
Ref. [47].

How do $\rho_T$ and $\omega_T$ decay? There may be a triplet of lightest “pseudoscalars”, induced by the criticality of $\mathcal{L}_{ETC}$ and by TC. This triplet would be an admixture of $\bar{q}t$ and $\bar{T}U$ states that is orthogonal to the three Goldstone bosons eaten by $W^\pm$ and $Z^0$. They are not light pseudo-Goldstone bosons, for they get a large mass from the near-critical $G_2$ interaction. In fact, there is no obvious reason that they are much lighter than $\rho_T$ and $\omega_T$. Thus, we expect the vectors’ dominant decay modes to involve longitudinally-polarized weak bosons, the erstwhile “pions” absorbed in the Higgs mechanism, alone and possibly in association with $H(125)$:

$$\rho_T^{\pm,0} \to W_L^\pm Z_L, W_L^+ W_L^- \text{ and } W_L^\mp H, Z_L H;$$
$$\omega_T \to W_L^+ W_L^- Z_L \text{ and } Z_L H.$$  

These decays are strong (TC) interactions. Thus, heavier $\rho_T$ and $\omega_T$ are unlikely to be narrow resonances. In that case, the presence of the $\rho_T$ and $\omega_T$ will be signaled by increases in the rates of the above processes at higher invariant masses.

7. Summary and Plans

In this paper we presented a simple model of a light composite Higgs boson. It is inspired by the top-condensate model of Bardeen, Hill and Lindner [14] and, in our view, its paradigmatic position as a dynamical model embodying our notion of “least unnaturalness”. Our model combines technicolor with strong extended technicolor to jointly account for electroweak symmetry breaking, the light Higgs boson discovered at the LHC, and the mass of the top quark. The strong ETC interaction with finely-tuned couplings is essential for these to occur at energies much less than the ETC scale $\Lambda$. This mechanism was anticipated in Ref. [17].

Our simple model employs one technifermion doublet $T = (U, D)$ interacting with the third generation quarks $q = (t, b)$ via three ETC interactions with strengths $G_1, G_2, G_3 = \mathcal{O}(1/\Lambda^2)$, where $\Lambda \sim 10–500$ TeV. These interactions were treated in the NJL approximation of large $(N_C, N_{TC})$. While the TC interaction of $T$ is expected to be an important part of the model, it is also a significant complication. We neglected TC in this paper.

The solution of the model in this large-$N$, weak-TC limit then closely followed BHL: The gap equations in Sec. 2 for the hard masses $m_t$ and $m_U$ are quadratically divergent, and requiring $m_t, m_U \ll \Lambda$ is a fine tuning of a part in $\mathcal{O}(\Lambda^2/m^2)$. These gap equations also imply the relation $G_2 = G_1(m_U/m_t) = G_3(m_t/m_U)$ among the model’s ETC couplings. This relation was essential for turning the complicated NJL bubble sums for the $2 \to 2$ scattering amplitudes in Sec. 3 into simple geometric series. As in BHL, all $\Lambda^2$-dependence in these amplitudes was removed by applying the condition (10) for nontrivial solutions to the gap equations. In the scalar channel, the Higgs boson pole occurs at $M_H^2 \approx 4(N_C m_t^2 + N_{TC} m_U^2)/(N_C m_t^2 + N_{TC} m_U^2)$. This is the model’s only Higgs boson (necessarily, since $G_2$ is not weak), so its vev is $v = 246$ GeV. There are three Goldstone boson channels. Their
massless poles disappear from the physical spectrum, producing the massive \( W \) and \( Z \)-boson poles in their propagators (Sec. 4). Integrals used in Secs. 3 and 4 are in the Appendix.

In Sec. 5 we carried out a simple numerical analysis of our model by (1) fixing \( \Lambda \) and then \( m_t(\Lambda) \) so that \( m_t = 173 \text{ GeV} \) at the weak scale and (2) determining \( N_{TC} m_U^2(\Lambda) \) from the residue of the Goldstone pole in the \( W \)-propagator, \( f_W(0) = 123 \text{ GeV} \). Fixing \( N_{TC} \) then determined \( M_H(\Lambda) \). The results are in Table 1 for the two choices \( \Lambda = 20 \text{ TeV} \) and 500 TeV. As in BHL, these values of the Higgs mass are expected to decrease when run down to the weak scale. However, the effect of TC on the running is unknown and, like the inclusion of TC dynamics, is deferred to the next paper. The \( \rho \)-parameter and the \( W \) and \( Z \)-pole masses were also calculated and in quite good agreement with experiment. Of course, more elaborate numerical schemes are possible, e.g., a “best fit” to \( m_t \) and \( M_H \) with fixed values of \( \Lambda \) and the Higgs vev \( v = 2f_W(0) = 246 \text{ GeV} \), or even a scan over \( \Lambda \) for a best fit to \( m_t \) and \( M_H \).

Finally, in Sec. 6 we speculated briefly on the model’s phenomenology. We noted that the model with \( G_2 = 0 \) is excluded because it has a triplet of nearly massless pseudo-Goldstone bosons; the charged ones would have been discovered decades ago in \( e^+e^- \) annihilation. Further, the constraint \( f_W(0) = 123 \text{ GeV} \) implies that \( m_t \) and \( m_U \) are likely to be comparable and this, in turn, implies that all three \( G_i \) are comparable and nearly critical, i.e., nearly large enough to induce nonzero \( m_t, m_U \) by themselves. Given this, it is difficult to see what this model’s phenomenology is because it will be controlled by two strong interactions, TC and ETC, with very different energy scales. One possibility that suggested itself deals with the model’s lowest lying spin-one, isovector and isoscalar \( \bar T T \) states. If their binding is determined by TC, not ETC, dynamics, the masses of these \( \rho_T \) and \( \omega_T \) should be \( \frac{1}{2} - 2 \text{ TeV} \) and possibly within reach of the LHC. Their spin-zero \( \pi_T \) partners are not pseudo-Goldstone bosons and, so, are likely to be as heavy as they are. Then, the principal observational modes of the vectors are their strong decays to longitudinally polarized weak bosons, either in diboson and triboson combinations or in association with \( H(125) \). Beyond this, understanding the phenomenology of this model, or any model like it, requires a much better understanding of its dynamics. This and the phenomenology are the subjects of planned papers.
Appendix: Integrals used in the text

The calculations presented here come from Ref. [14] and C. T. Hill (private communication). Momentum integrals are in Minkowski space until Wick-rotated and then cut off at momentum $\Lambda$.

Sections 2 and 3:

The fermion condensates at scale $\Lambda$ in Eq. (7):

$$
\langle \bar{t}t \rangle_{\Lambda} = \sum_a \langle \bar{t}^a(0)t_a(0) \rangle_{\Lambda} = -iN_C (\text{Tr} S_t(0))_{\Lambda} \\
\equiv -4iN_C \int^{\Lambda^2} \frac{d^4k}{(2\pi)^4} \frac{k}{k^2 - m_t^2} = -\frac{N_C m_t}{4\pi^2} \int^\Lambda d^2 k \frac{k^2}{k^2 + m_t^2} \\
\approx -\frac{N_C m_t}{4\pi^2} [\Lambda^2 - m_t^2 \ln(\Lambda^2/m_t^2)] ,
$$

(38)

for $\Lambda^2 \gg m_t^2$.

The scalar $\bar{t}t \rightarrow \bar{t}t$ integral in Eq. (12):

$$
i \int d^4x e^{ip \cdot x} \langle \Omega | T(\bar{t}^a t_a(x) \bar{t}^b t_b(0)) | \Omega \rangle \\
= 4iN_C \int^{\Lambda^2} \frac{d^4k}{(2\pi)^4} \frac{k \cdot (k + p) + m_t^2}{((k + p)^2 - m_t^2)(k^2 - m_t^2)} \\
= 2iN_C \int^{\Lambda^2} \frac{d^4k}{(2\pi)^4} \frac{((k + p)^2 - m_t^2) + (k^2 - m_t^2) - (p^2 - 4m_t^2)}{((k + p)^2 - m_t^2)(k^2 - m_t^2)} \\
\approx \frac{N_C}{4\pi^2} [\Lambda^2 - m_t^2 \ln(\Lambda^2/m_t^2)] + \frac{N_C (p^2 - 4m_t^2)}{8\pi^2} \int_0^1 dx \ln \left( \frac{\Lambda^2}{m_t^2 - p^2 x (1 - x)} \right) .
$$

(39)

The Goldstone boson $\bar{t}t \rightarrow \bar{t}t$ integral in Eq. (19):

$$
i \int d^4x e^{ip \cdot x} \langle \Omega | T(\bar{t}^a i\gamma_5 t_a(x) \bar{t}^b i\gamma_5 t_b(0)) | \Omega \rangle \\
= 4iN_C \int^{\Lambda^2} \frac{d^4k}{(2\pi)^4} \frac{k \cdot (k + p) - m_t^2}{((k + p)^2 - m_t^2)(k^2 - m_t^2)} \\
= 2iN_C \int^{\Lambda^2} \frac{d^4k}{(2\pi)^4} \frac{((k + p)^2 - m_t^2) + (k^2 - m_t^2) - p^2}{((k + p)^2 - m_t^2)(k^2 - m_t^2)} \\
\approx \frac{N_C}{4\pi^2} [\Lambda^2 - m_t^2 \ln(\Lambda^2/m_t^2)] + \frac{N_C p^2}{8\pi^2} \int_0^1 dx \ln \left( \frac{\Lambda^2}{m_t^2 - p^2 x (1 - x)} \right) .
$$

(40)
The Goldstone boson $t \bar{b} \rightarrow t \bar{b}$ integral in Eq. (22):

\[
i \int d^4x \ e^{ip \cdot x} \langle \Omega | T(\bar{t}^a_R b_L \alpha(x) \bar{t}^b_L t_R b(0)) | \Omega \rangle
\]
\[
= \frac{N_c}{\pi^2} \left( \frac{g_2}{\sqrt{2}} \right)^2 \int d^4x \ e^{ip \cdot x} \langle \Omega | T(\bar{t}^a_R \gamma_\mu b_L(x) \bar{t}^b_L \gamma_\mu t_L b(0)) | \Omega \rangle
\]
\[
= \frac{dN_c \Gamma(2-d/2)}{4(4\pi)^{d/2}} \int_0^1 dx \left[ \frac{2(p_\mu p_\nu - p^2 g_{\mu\nu})(1 - x) + m_t^2 g_{\mu\nu}}{(m_t^2 x - p^2 x(1 - x))^{(2-d/2)}} \right]. \tag{42}
\]

Using

\[
\frac{d \Gamma(2-d/2)(\Delta^2)^{(d/2-2)}}{4(4\pi)^{d/2}} = \frac{2}{16\pi^2} \left[ \epsilon^{-1} - \frac{1}{2} \gamma + \frac{1}{2} \ln 4\pi - \frac{1}{4} - \frac{1}{2} \ln \Delta^2 + \mathcal{O}(\epsilon) \right] \longrightarrow \frac{1}{16\pi^2} \ln(\Lambda^2/\Delta^2),
\]

we get for the sum of the quark and technifermion loops:

\[
\Pi_{\mu\nu}^\pm(p) = N_c \frac{16\pi^2}{16\pi^2} \int_0^1 dx \left[ \frac{2(p_\mu p_\nu - p^2 g_{\mu\nu})(1 - x) + m_t^2 g_{\mu\nu}}{(m_t^2 x - p^2 x(1 - x))} \right] \ln \left( \frac{\Lambda^2}{m_t^2 x - p^2 x(1 - x)} \right)
\]
\[+ (N_c, m_t \rightarrow N_{TC}, m_U). \tag{44}
\]
The charged Goldstone-boson pole contribution to $\Pi^{\pm}_{\mu\nu}$ is

$$\Pi^{\pm}_{\mu\nu,\text{GB}} (p) = \frac{i}{g_2^2} \left( \frac{g_2}{\sqrt{2}} \right)^2 \int d^4x d^4y e^{ip(x-y)} \langle \Omega | T \left[ j^{(1+i2)} \left( x \right) \left( iL_{ETC} (0) + \cdots \right) j^{(1-i2)} \left( 0 \right) \right] | \Omega \rangle$$

$$= \frac{2p_\mu p_\nu}{(16\pi^2)^2} \left\{ \int_0^1 dx x N_C m_t \ln \left( \frac{\Lambda^2}{m_t^2 x - p^2 x (1 - x)} \right)^2 \left( -4\Gamma_0^{-} (p) \right) \right\}$$

$$+ (t\bar{b} \leftrightarrow U \bar{D}) + (U \bar{d} \rightarrow U \bar{D}) \text{ terms} \right\}$$

$$= - \frac{p_\mu p_\nu}{16\pi^2 p^2} \int_0^1 dx x \left[ N_C m_t^2 \ln \left( \frac{\Lambda^2}{m_t^2 x - p^2 x (1 - x)} \right) + (N_C, m_t \rightarrow N_T, m_U) \right].$$

The factor of $-4\Gamma_0^{-}$ comes from the first term on the right in Eq. (21), which indicates that $G_1 + \cdots$ sums to this. These GB-pole terms combine with the $m^2 g_{\mu\nu}$-terms in Eq. (44) to make a transverse massless-GB pole term. Then, with Eq. (24), $g_{2W}^2 (p^2)$ and $f_W^2 (p^2)$ are easily read off, and are given in Eqs. (25,26).

The calculations for the neutral EW propagator matrix are similar, if more tedious. The important thing there is to arrange the terms so that there is a massless photon pole.

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