Spin currents in a normal two-dimensional electron gas in contact with a spin–orbit interaction region

Aleksei A Sukhanov, Vladimir A Sablikov and Yurii Ya Tkach

Kotel’nikov Institute of Radio Engineering and Electronics, Russian Academy of Sciences, Fryazino, Moscow District, 141190, Russia

Received 18 June 2009, in final form 28 July 2009
Published 13 August 2009
Online at stacks.iop.org/JPhysCM/21/375801

Abstract
Spin effects in a normal two-dimensional (2D) electron gas in lateral contact with a 2D region with spin–orbit interaction are studied. The peculiarity of this system is the presence of spin-dependent scattering of electrons from the interface. This results in an equilibrium edge spin current and nontrivial spin responses to a particle current. We investigate the spatial distribution of the spin currents and spin density under non-equilibrium conditions caused by a ballistic electron current flowing normal or parallel to the interface. The parallel electron current is found to generate a spin density near the interface and to change the edge spin current. The perpendicular electron current changes the edge spin current proportionally to the electron current and produces a bulk spin current penetrating deep into the normal region. This spin current has two components, one of which is directed normal to the interface and polarized parallel to it, and the second is parallel to the interface and is polarized in the plane perpendicular to the contact line. Both spin currents have a high degree of polarization (∼40–60%).

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Spin transport in semiconductor nanostructures with spin–orbit interaction (SOI) attracts much attention, stimulated by the interesting spin dynamics in systems with non-conserving spin and possible applications in creating and manipulating spin polarization in nonmagnetic systems by purely electric methods using SOI [1–4]. Spin currents are widely studied in cases where there is a spin Hall effect, either intrinsic [5–9] or extrinsic [10, 11, 7]. The basic experimentally observable effect is the spin accumulation near the lateral edges of the sample if the electric current is driven along it. The spin density is accumulated because the transverse spin current does not pass through the edges.

A somewhat different effect appears due to electron scattering on the sample boundary if the particle current flows parallel to it. In this case a spin polarization is formed near the boundary and the spin density distribution has a rather complicated form [12–15].

In this paper another system is studied, in which electrons can pass through the boundary of the SOI region. We consider a 2D system composed of an electron gas with SOI and normal (N) 2D electron gas without SOI, lying in one plane. This system is designated as SOI/N contact. The main purpose is to find out whether the spin currents, which exist in the SOI region due to the coupling between spin and orbital degrees of freedom, penetrate into the N region and how the spin density is distributed there under the non-equilibrium conditions when an electron current flows.

This system is also interesting because it reveals a new aspect of the problem of equilibrium spin currents. The existence of background spin currents in thermodynamic equilibrium was pointed out by Rashba [16] for an infinite 2D electron gas. Usaj et al [12] showed that equilibrium spin current exists also in a bounded 2D electron system with SOI, the spin current flowing near the edges of the sample.

The phenomenon of equilibrium spin currents has provoked an intense discussion on the discerning of spin currents and their definition. The question arose since spin currents in the SOI medium are not unambiguously defined [16–18]. Sun et al [18] found that the equilibrium spin current exists in a quantum ring even if the SOI is absent in...
some segment. In this connection, of more interest is a system in which a 2D SOI medium is in contact with a bulky normal 2D electron gas where the spin current is well defined. The question is whether the equilibrium spin current exceeds the boundary of the SOI medium, entering into the N electron gas and what spin effects appear there.

This question was raised in our previous papers [19, 20]. We have established that the equilibrium spin current does not pass through the boundary, but an equilibrium edge spin current flows near the boundary in both SOI and N regions. Its density considerably exceeds the density of the bulk equilibrium spin current. An important point is that the spin current is well defined in the N electron gas and its definition is beyond doubt. In the present work we study spin effects resulting from the interface scattering under non-equilibrium conditions appearing when an electron current flows in the system. This question is important since only non-equilibrium transport produces a spin accumulation that can be detected experimentally [7, 21].

The calculations are carried out for a weak deviation from the equilibrium caused by a ballistic electron current flowing normal or parallel to the interface. The electron current is supposed to be created by a difference of chemical potentials \( \Delta \mu \) of electrons moving in opposite directions. Two main effects are found. The electron current parallel to interface leads to spin accumulation near the interface and changes in the edge spin current. The accumulated spin density is proportional to \( \Delta \mu \), while the augmentation of the edge spin current is proportional to \( \Delta \mu^2 \). This means that the non-equilibrium spin current does not depend on the polarity of the electron current. If the electron current is perpendicular to the interface, the edge spin currents vary linearly with \( \Delta \mu \). However, the main effect consists in the appearance of the bulk spin current in N region. It has two components. One component is directed parallel to the interface and polarized in the plane perpendicular to it. The second component is directed normal to the boundary and polarized parallel to it. These spin currents penetrate deep into the normal region and have considerable polarization.

2. The model

Consider a 2D SOI/N contact. The energy diagram is shown in figure 1. The SOI and N regions are located correspondingly at \( x < 0 \) and \( x > 0 \). To be specific we suppose that the potential of the SOI region is higher than that of the N region. This arrangement allows one to consider an interesting case where free electrons are present only in the N region, though they penetrate into the SOI barrier. We assume also that the external electric field is too weak to change the equilibrium electron states and the electron current is caused by a non-equilibrium occupation of unperturbed electron states. Such a situation can be realized, for example, if electrons move ballistically and their distribution function is formed by adiabatic contacts, with the applied voltage dropping across the near-contact regions.

The wavefunctions are determined by the Hamiltonian

\[
H = \frac{p_x^2 + p_y^2}{2m} + \frac{\alpha}{\hbar}(p_z \sigma_z - p_x \sigma_y) + U(x),
\]

where \( p_{x,y} \) are electron momentum components; \( m \) is the effective mass, which is supposed to be constant all over the structure; \( \sigma_{x,y} \) are the Pauli matrices; \( \alpha \) is the SOI parameter \( \alpha = 0 \) for \( x > 0 \) and \( \alpha = \text{const} \) for \( x < 0 \); \( U(x) \) is the potential energy, which is also a step function: \( U(x) = -U_0 \Theta(x) \).

The wavefunctions of electrons moving to the right (R) and to the left (L) are [19]

\[
\psi_{\lambda,\lambda}(x) = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \lambda, -q_x, k_y \sum_{\lambda'} t^R_{\lambda',\lambda}(\lambda', -q_x, k_y), \\ \lambda, q_x, k_y \sum_{\lambda'} t^L_{\lambda',\lambda}(\lambda', q_x, k_y), \end{array} \right),
\]

and

\[
\psi_{\lambda,\lambda}(x) = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \lambda, -q_x, k_y \sum_{\lambda'} t^L_{\lambda',\lambda}(\lambda', q_x, k_y), \\ \lambda, q_x, k_y \sum_{\lambda'} t^R_{\lambda',\lambda}(\lambda', -q_x, k_y), \end{array} \right),
\]

\( k_{x,y} \) and \( q_x \) are components of the electron wavevectors in the SOI and N regions; the index \( \lambda \) numbers the branches of the energy spectrum and spin states.

The reflection and transmission matrices, \( t^R_{\lambda',\lambda}(k, q_x, k_y) \) and \( t^L_{\lambda',\lambda}(k, q_x, k_y) \), are found using the following boundary conditions [22]:

\[
\psi^+_{\lambda,\lambda} = 0, \quad \frac{\partial \psi}{\partial x} \bigg|_{x = 0} = \left[ \frac{\partial \psi}{\partial x} - (ik_w \sigma_z - \beta k_x \sigma_x) \psi \right]_{x = 0},
\]

where \( k_w = ma / \hbar^2 \) is the characteristic wavevector of the SOI. The parameter \( \beta \) is introduced to take into account the SOI resulting from a potential gradient at the interface. This parameter is usually small. Its value is estimated as [22, 20] \( \beta = 2k_w U/eF_z \), with \( F_z \) being the electric field normal to the 2D layer. For a typical experimental situation of InGaAs quantum wells [23] with \( k_w = 10^5 \) cm\(^{-1} \), \( U = 5 \times 10^{-2} \) eV and \( F_z = 10^6 \) V cm\(^{-1} \), one finds \( \beta = 0.01 \).

The basic states \( |\lambda, k_x, k_y \rangle \) are characterized by the energy \( E \), the wavevector component \( k_x \), which is invariant since the structure is uniform along the \( y \) axis, and the spin index \( \lambda \). These states were described in detail previously [19, 22]. Therefore, we only mention that there are three kinds of states. For \( E < -E_w \), where \( E_w = ma^2 / (2\hbar^2) \) is the characteristic energy of the SOI, the electron states are evanescent. They are characterized by the complex wavevector \( k_x \). For \( E > -E_w \),
all states are either propagating, with real \( k_x \), or entirely decaying, with imaginary \( k_x \). Each of these states exists in a corresponding interval of the tangential momentum \( k_y \). If \( E > 0 \), the index \( \lambda = \pm \) stands for the chirality. In an interval \(-E_{so} < E < 0\), the electron states have only negative chirality, but there are two kinds of states with a given energy. They differ in the wavevector \( k_y \) and for convenience are also labeled by the same index \( \lambda = \pm \). Of course, all these states are used in calculating the spin density and the currents.

Using the wavefunctions (2) and (3), we find the spin current density \( J_x^\lambda(r, \lambda, \vec{k}; x) \) and the spin density \( S_x^\lambda(r, \lambda, \vec{k}; x) \) for a state \( |\psi^{(r)}_{\lambda, \vec{k}}\rangle \), with \( r = (R, L) \) denoting the right- or left-moving states:

\[
J_x^\lambda(r, \lambda, \vec{k}; x) = \frac{\hbar}{4} \left( \psi^{(r)}_{\lambda, \vec{k}} \right| \sigma_x v_i + v_i \sigma_y | \psi^{(r)}_{\lambda, \vec{k}} \rangle, \quad (5)
\]

\[
S_x^\lambda(r, \lambda, \vec{k}; x) = \frac{\hbar}{2} \left( \psi^{(r)}_{\lambda, \vec{k}} \right| \sigma_y | \psi^{(r)}_{\lambda, \vec{k}} \rangle, \quad (6)
\]

where \( \gamma \) labels the spin components, \( \vec{v} \) is the velocity.

The total spin current \( J \) and spin density \( S \) are found by summing equations (5) and (6) over all states occupied according to the distribution function.

In the presence of a charge current \( J \), electrons have an asymmetric momentum distribution function \( f_\gamma(k) \). We simulate this asymmetry using two semicircles in \( \vec{k} \)-space with different Fermi momenta and, accordingly, with different chemical potentials \( \mu_+ \) and \( \mu_- \) for electrons moving in opposite directions. This distribution function can be realized when electrons move ballistically between two contacts with the potential difference \( \Delta \mu = \mu_+ - \mu_- \). In what follows we consider two cases where the electron current is normal or parallel to the boundary. Accordingly, all quantities (\( \Delta \mu \), \( S \) and \( J \)) are labeled with index \( j = (x, y) \) specifying the direction of the electron current.

As a result we obtain the following expressions for the spin current and spin density:

\[
J_x^\gamma(\Delta \mu_j, x) = \sum_{r=R,L} \frac{\delta \mu_i}{\hbar} \int_{-U_0}^\infty dE \int d\vec{k}_y \enspace f_\gamma(k_y, \Delta \mu_j) D_{\gamma}^{(r)}(E, k_y) J_x^\gamma(r, \lambda, \vec{k}), \quad (7)
\]

\[
S_x^\gamma(\Delta \mu_j, x) = \sum_{r=R,L} \frac{\delta \mu_i}{\hbar} \int_{-U_0}^\infty dE \int d\vec{k}_y \enspace f_\gamma(k_y, \Delta \mu_j) D_{\gamma}^{(r)}(E, k_y) S_x^\gamma(r, \lambda, \vec{k}), \quad (8)
\]

with \( D_{\gamma}^{(r)}(E, k_y) \) the density of states with chirality \( \lambda \).

For small \( \Delta \mu_j \ll E_{so}, U_0 \), the spin current and density can be expanded in \( \Delta \mu_j \):

\[
J_x^\gamma(x) = J_x^{\gamma(0)}(1 - \delta_{\gamma j})(1 - \delta_{i_x} \delta_{y z}) + \chi_{\gamma j}^\gamma(1 - \delta_{\gamma j}) \Delta \mu_j + \chi_{\gamma}^{\gamma(2)}(1 - \delta_{\gamma j}) \Delta \mu_j^2, \quad (9)
\]

\[
S_x^\gamma(x) = \chi_{\gamma}^\gamma(1 - \delta_{\gamma j})(1 - \delta_{i_x} \delta_{y z}) \Delta \mu_j, \quad (10)
\]

Here the expansion of \( J \) includes the equilibrium spin current \( J_x^{\gamma(0)} \). A quadratic term is also important because some linear terms are equal to zero. To be exact, the linear response of the tangential spin current to the parallel current of particles is absent.

In section 3 we study in detail the response functions of the spin density, \( \chi_{\gamma}^\gamma \), and the spin current, \( \chi_{\gamma}^{\gamma(2)} \). They are functions of the coordinate \( x \). The calculations are carried out using the following distribution function:

\[
f_{\gamma}(\vec{k}, \Delta \mu_j) = \left\{ \begin{array}{ll}
\Theta(\mu + \Delta \mu_j/2 - E), & k_j > 0, \\
\Theta(\mu - \Delta \mu_j/2 - E), & k_j < 0.
\end{array} \right. \quad (11)
\]

Thus, the calculation scheme is the following. First, the matrices of reflection and transmission coefficients, \( r_{R,L}^{\gamma} \) and \( l_{R,L}^{\gamma} \), are found from equations (2)–(4). Then the partial spin current and spin density components, \( J_x^\gamma \) and \( S_x^\gamma \), are determined using equations (5) and (6). Finally, the total spin current and spin density, \( J_x^{\gamma}(x) \) and \( S_x^{\gamma}(x) \), as functions of \( x \), are obtained from equations (7) and (8) using numerical integration.

3. Results and discussion

Below, the results of calculations of the spin density and spin currents are presented for two directions of the electron current: normal and parallel to the SOI/N interface.

But we begin with recalling the main results concerning the equilibrium spin current.

In unrestricted 2D electron gas with SOI the equilibrium spin current has two nonzero components: \( J_x^{\gamma(0)} = -J_y^{\gamma(0)} = (2/3\pi)J_N \), where \( J_N = \hbar^2 k_0^3/4m \) [16]. Near the SOI/N interface the perpendicular component of the spin current vanishes and it is absent in the N region: \( J_y^{\gamma(0)}(x > 0) = 0 \). However, near to the interface there is an edge equilibrium spin current, which flows parallel to the boundary in both regions [19]. The density of this current is considerably higher than the bulk density \((2/3\pi)J_N\), since the edge current is proportional to the SOI constant \( \alpha \), while the bulk spin current is proportional to \( \alpha^2 \). The spin of the edge spin current is polarized in a plane perpendicular to the interface and turns in this plane with increasing distance \( x \). Therefore, only two components of the spin current, \( J_x^{\gamma(0)}(x) \) and \( J_y^{\gamma(0)}(x) \), are presented in figure 2.

The signs of the spin currents \( J_x^{\gamma}(x) \) in SOI and N regions are seen to be opposite, but the total current is nonzero. It is interesting that the equilibrium edge spin current exists at \( \mu < -E_{so}, \) i.e. even if free electrons are absent in the bulk of the SOI region. The current is caused by electrons penetrating under the barrier into the SOI region. On increasing the Fermi level above the conduction band bottom in the SOI region, \( \mu > -E_{so} \), the spin current appears in the bulk of the SOI region. It increases with \( \mu \) to reach a limiting value \((2/3\pi)J_N\) at \( \mu = 0 \) and then does not change. In contrast, the edge spin current increases monotonically with \( \mu \), approximately as \((\mu + U_0)^{1/2}\).
3.1. Parallel electron current

If the electron current flows parallel to the SOI/N interface, the picture of the distribution of edge spin currents changes quite a bit. The point is that in this case the linear response of the spin current is absent, $G^z_\parallel = G^z_\perp = 0$. This occurs for the following reason. The edge spin current is created equally by electrons moving up and down the $y$ axis. If the distribution function $f(k_y)$ is symmetric, the charge currents of the up- and down-moving electrons cancel each other, while the spin currents, in contrast, are summed. Therefore, in the thermodynamic equilibrium the particle current is absent but the spin currents, in contrast, are summed. Therefore, in the bulk of both SOI and N regions. As a function of the difference in occupation between electron states with velocities directed along $y$ axis and opposite to it.

The spin susceptibility component $\chi^z_\parallel$ oscillates with a period $\sim \pi/k_{so}$ and slowly goes to zero with the distance in the bulk of both SOI and N regions. As a function of the Fermi energy at a given $\Delta \mu$, the spin susceptibilities $\chi^z_\parallel$ and $\chi^z_\perp$ increase in the range $\mu < -E_{so}$ and decrease at $\mu > -E_{so}$.

Numerical estimation of the spin density accumulated near the boundary in the InGaAs quantum well for $U_0 = 6E_{so}$, $\mu = E_{so}$ and the electric current density $1 \text{ mA cm}^{-1}$ give a value of the order of $2S/\hbar \sim 1 \mu \text{m}^{-2}$. Recent experiments have shown that such a value of spin density can be detected by Kerr rotation microscopy [7, 21].

3.2. Normal electron current

The electron current can flow perpendicularly through the interface only if $\mu > -E_{so}$, i.e. when free electrons are present in the bulk of the SOI region. First, note that in the bulk of the SOI region there is an equilibrium spin current $J^{z0}_x$ [16] flowing normal to the interface, but this current does not pass into the N region [19]. In the presence of a non-equilibrium electron current, this spin current is unchanged if $\mu > 0$, and slightly diminishes proportionally to $[eV/(\mu + E_{so})]^2 \ll 1$.
otherwise. Thus spin effects in the N region originate solely from the spin-dependent scattering of electrons from the interface. The main effect is the appearance of the spin current in the N region. There are two components of the non-equilibrium spin current: normal and tangential.

The spin current component directed normally and polarized in the y direction, \( J_{z}^{\perp} \), arises in both SOI and N regions as a result of the interface scattering. This is a longitudinal spin current effect. Its dependence on the distance \( x \) is shown in figure 4 for two opposed directions of the electron current (\( \Delta \mu > 0 \) and \( \Delta \mu < 0 \)). In the SOI region the spatial dependence of \( J_{z}^{\perp} \) reflects the interference pattern of the incident and reflected electron flows. In contrast, in the N region the spin current does not depend on the distance, since the spin is a conserved quantity there. The spin density is constant as well. The spin polarization (defined as \( 2J_{z}^{\perp}/(hJ) \)) depends on the chemical potential reaching a maximum (about 50%) when \( \mu \) lies close to the conduction band bottom in the SOI region.

The tangential spin current has two components of spin polarization lying in the plane perpendicular to the interface, \( J_{x}^{\parallel} \) and \( J_{y}^{\parallel} \). This is a transverse spin current effect. The dependence of the transverse currents on the distance is shown in figure 5 for opposed polarities of the electron current. Changes of the transverse spin current in the N region under the action of an electron current \( J_{x} \) are, of course, a manifestation of the spin Hall effect caused by the spin-dependent scattering from the interface.

The mechanism of the non-equilibrium spin effects in the bulk of the N region can be understood in a way similar to that suggested in [19] to explain the equilibrium edge spin currents. Consider an unpolarized electron flow falling on the interface from the N region. During the scattering the electrons partially penetrate into the SOI region where an effective magnetic field forces their spin to precess. In the case of the Rashba SOI, this field is perpendicular to the wavevector and lies in the plane. As a result of the spin precession, the spin polarization of reflected electrons changes. Spin components \( S_{x} \) and \( S_{y} \) of electrons with equal \( k_{x} \) but opposite \( k_{y} \) turn out to be of opposite sign and cancel each other. Therefore under the equilibrium conditions the spin density is absent, but the edge spin current exists since the partial spin currents of electrons with opposed \( k_{y} \) are summed. However, in the bulk of the N region, the spin current is absent because the spin currents of electrons with opposed \( k_{y} \) cancel each other. This compensation is disturbed when an electron current flows through the interface resulting in the net total spin current in the bulk.

The fact that this current exists in the N region far away from the interface, where no SOI is present, is caused by the infinite size of the interface scatterer in the y direction. It is obvious that the transverse spin currents should decay with distance deep into the N region because of scattering processes not considered here.

The spin currents \( J_{x}^{\parallel} \) and \( J_{y}^{\parallel} \) increase linearly with \( \Delta \mu \) and therefore can be characterized by spin Hall conductivities \( \bar{G}_{x}^{\parallel} \) and \( \bar{G}_{y}^{\parallel} \). As distinct from the data of figure 5, they describe only the non-equilibrium part of the spin currents. The spatial distributions of the spin Hall conductivities are shown in figure 6.

It is worth emphasizing that the spin Hall current in the bulk of the N region increases with the Fermi energy even if \( \mu \gg E_{\text{so}} \) approximately as \( \mu^{1/2} \), in contrast to the case for the normal spin current \( J_{x}^{\parallel} \) and the spin density \( S_{y}^{\parallel} \). Numerically
is

velocity

by electrons moving nearly parallel to the interface with

explained taking into account that this current is created mainly

mechanism of increasing the transverse spin current with

that it could be attractive for experimental observation. The

scattering of electrons from the interface.

the same plane. The spin effects are caused by the spin-dependent

gas in contact with a 2D medium with SOI, which lies in the

We have found substantial spin effects in a normal 2D electron

4. Conclusion

for the data of figure 6 the density of the transverse spin current

is \( \sim 0.45(\hbar/2e)J \) at \( \mu = 0.5E_{so} \) and exceeds \( 0.6(\hbar/2e)J \) at

\( \mu = 2.5E_{so} \). Thus this effect is more robust and we believe

that it could be attractive for experimental observation. The

mechanism of increasing the transverse spin current with \( \mu \) is

explained taking into account that this current is created mainly

by electrons moving nearly parallel to the interface with the

velocity \( \sim \hbar k_F/m \).

The transverse spin current in the N region could be

investigated experimentally by measuring the spin density

accumulated near the lateral edges of the sample. The density

of spins near the edge is estimated as follows:

\[
\gamma_s = \frac{2L_{str}}{hD_s} G_{\mu\mu}^s \Delta \mu_s
\]

with \( D_s \) the spin diffusion coefficient, \( L_{str} \) the spin relaxation

length. For \( G_{xx}^s = 0.05k_{so}, \ k_{so} = 10^5 \text{ cm}^{-1}, \ L_{str} = 2 \times 10^{-3} \text{ cm}, \ D_s = 10^2 \text{ cm}^2 \text{ s}^{-1} \) and \( \Delta \mu = 10^{-5} \text{ eV} \) one finds

\( \gamma_s = 3 \times 10^5 \text{ cm}^{-2} = 30 \mu\text{m}^{-2} \). This value of spin density is

readily measurable in experiments [7, 21].

Acknowledgments

This work was supported by the Russian Foundation for Basic

Research (project No 08-02-00777) and Russian Academy of

Sciences (programs ‘Quantum Nanostructures’ and ‘Strongly

Correlated Electrons in Solids and Structures’).

References

[1] Awschalom D D, Loss D and Samarth N (ed) 2002

Semiconductor Spintronics and Quantum Computation

(Nanoscience and Technology) (Berlin: Springer)

[2] Zutic J, Fabian J and Sarma S D 2004 Rev. Mod. Phys. 76 323

[3] Fabian J, Matos-Abiague A, Ertler C, Stano P and Zutic I 2007

Acta Phys. Slovaca 57 565

[4] Awschalom D D and Flate E 2007 Nat. Phys. 3 153

[5] Murakami S, Nagaosa N and Zhang S C 2003 Science 301 1348

Murakami S, Nagaosa N and Zhang S C 2004 Phys. Rev. B

69 235206

[6] Sinova J, Culeer D, Niu Q, Sinitsyn N A, Jungwirth T and

MacDonald A H 2004 Phys. Rev. Lett. 92 126603

[7] Kato Y K, Myers R C, Gossard A C and Awschalom D D 2004

Science 306 1910

[8] Wunderlich J, Kaestner B, Sinova J and Jungwirth T 2005

Phys. Rev. Lett. 94 047204

[9] Engel H A, Rashba E I and Halperin B I 2007 Handbook of

Magnetcism and Advanced Magnetic Materials vol 5, ed H Kronm¨uller and S Parkin (Chichester: Wiley)

[10] D’yakonov M I and Perel V I 1971 JETP Lett. 13 467

D’yakonov M I and Perel V I 1971 Phys. Lett. A 35 459

[11] Hirsch J E 1999 Phys. Rev. Lett. 83 1834
[12] Usaj G and Balseiro C A 2005 *Europhys. Lett.* **72** 631
[13] Reynoso A, Usaj G and Balseiro C A 2006 *Phys. Rev.* B **73** 115342
[14] Xing Y, Sun Q F, Tang L and Hu J P 2006 *Phys. Rev.* B **74** 155313
[15] Silvestrov P G, Zyuzin V A and Mishchenko E G 2009 *Phys. Rev. Lett.* **102** 196802
[16] Rashba E I 2003 *Phys. Rev.* B **68** 241315
[17] Sonin E B 2007 *Phys. Rev.* B **76** 033306
[18] Sun Q F and Wang X C 2008 *Phys. Rev.* B **77** 035327
[19] Sablikov V A, Sukhanov A A and Tkach Yu Ya 2008 *Phys. Rev.* B **78** 153302
[20] Tkach Yu Ya, Sablikov V A and Sukhanov A A 2009 *J. Phys.: Condens. Matter* **21** 125801
[21] Crooker S A and Smith D L 2005 *Phys. Rev. Lett.* **94** 236601
[22] Sablikov V A and Tkach Yu Ya 2007 *Phys. Rev.* B **76** 245321
[23] Nitta J, Akazaki T, Takayanagi H and Enoki T 1997 *Phys. Rev. Lett.* **78** 1335