A DYNAMICALLY COLLAPSING CORE AND A PRECURSOR OF A CORE IN A FILAMENT SUPPORTED BY TURBULENT AND MAGNETIC PRESSURES

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ABSTRACT

To study physical properties of the natal filament gas around the cloud core harboring an exceptionally young low-mass protostar GF 9-2, we carried out $J = 1\rightarrow 0$ line observations of $^{12}$CO, $^{13}$CO, and $^{18}$O molecules using the Nobeyama 45 m telescope. The mapping area covers $\sim$ one-fifth of the whole filament. Our $^{13}$CO and $^{18}$O maps clearly demonstrate that the core formed at the local density maxima of the filament, and the internal motions of the filament gas are totally governed by turbulence with Mach number of $\sim$2. We estimated the scale height of the filament to be $H = 0.3–0.7$ pc, yielding the central density of $n_c = 800–4200$ cm$^{-3}$. Our analysis adopting an isothermal cylinder model shows that the filament is supported by the turbulent and magnetic pressures against the radial and axial collapse due to self-gravity. Since both the dissipation timescales of the turbulence and the transverse magnetic fields can be comparable to the free-fall time of the filament gas of $10^6$ yr, we conclude that the local decay of the supersonic turbulence and magnetic fields made the filament gas locally unstable, hence making the core collapse. Furthermore, we newly detected a gas condensation with velocity width enhancement to $\sim$0.3 pc southwest of the GF 9-2 core. The condensation has a radius of $\sim$0.15 pc and an LTE mass of $\sim$5 $M_\odot$. Its internal motion is turbulent with Mach number of $\sim$3, suggesting a gravitationally unbound state. Considering the uncertainties in our estimates, however, we propose that the condensation is a precursor of a core cloud, which would have been produced by the collision of the two gas components identified in the filament.

Key words: evolution – ISM: clouds – ISM: individual objects (GF9-2, L1082C, PSC20503+6006) – turbulence

Online-only material: color figures

1. INTRODUCTION

Filamentary molecular clouds—filaments—are now considered to be one of the evolutionary stages for the interstellar medium (ISM) to evolve from molecular clouds to stars. Analyzing dark globular filaments identified in optical images, Schneider & Elmegreen (1979) pointed out that a filament would fragment into equally spaced condensations of gas and dust due to its gravity. Indeed, molecular line observations in the 1980s showed that filaments contain such condensations—dense molecular cloud cores—which are now known to be the immediate sites of star formation. This hierarchical structure is typically seen in nearby low-mass star-forming regions such as the Taurus molecular cloud (e.g., Palla & Stahler 2002, and references therein). Recently, our knowledge about filaments is significantly improved by a systematic far-infrared (FIR) imaging survey of the thermal emission from the interstellar dust using Herschel Space Observatory (André et al. 2010, and references therein). FIR images taken with Herschel demonstrated that molecular clouds have ubiquitous networks of filaments that are highly likely to have been produced through the interaction among gravity, interstellar turbulence, and magnetic field. These observations clearly suggest that, in general, dense cores originate in the fragmentation process of filaments. Another important result from the Herschel observations is that filaments are omnipresent even in non-star-forming cloud complexes (e.g., Men’shchikov et al. 2010).

On the other hand, theoretical studies have long discussed that molecular clouds collapse into sheetlike clouds, which are formed through a compression process such as cloud–cloud collision, and that the sheetlike clouds fragment to filamentary clouds, which will form dense cloud cores (e.g., Miyama et al. 1984). Considering the balance between self-gravity and pressure gradient in the radial direction of a cylinder, Stodólkiewicz (1963) and Ostriker (1964) found that a filament radially collapses if the line mass (the mass per unit length) of the filament exceeds the critical value of $2c_s^2 / G$, where $c_s$ is the isothermal sound speed and $G$ is the gravitational constant. The stability of a cylinder in equilibrium against axial perturbations was studied for the isothermal incompressible cylinder composed of polytropic gas by Chandrasekhar & Fermi (1953b): they found critical wave numbers against the axial fragmentation, but such an unstable cylinder can be stabilized by the axial magnetic fields of $(0, 0, B_z)$ in the cylindrical coordinates. Stodólkiewicz (1963) studied how stability of the compressible gas depends on the magnetic fields: the critical wavelength becomes longer for $(0, B_\phi, 0)$, whereas it becomes shorter for $(0, 0, B_z)$. In contrast, Nagasawa (1987) showed that critical wavelengths do not change with a uniform $(0, 0, B_z)$, but the growth rates of the unstable modes are suppressed, which differs from the conclusion drawn from the Stodólkiewicz’s calculations. As summarized in Larson (1985), all the studies showed that a filament breaks into “clumps” with a separation of about 4 times the diameter of the cylinder. Inutsuka & Miyama (1997) investigated the axial fragmentation and radial collapse of a cylinder in more detail; they showed that merging and clustering of newly formed “clumps” occur within a timescale of the fragmentation process of the natal filaments.

Although details remain a matter of debate, several unstable modes for the filament fragmentation are proposed by modern numerical simulations (see reviews by, e.g., Mac Low & Klessen 2004; André et al. 2009, and references therein). These simulations may be categorized by the roles of magnetic fields and turbulence being considered. In principal, complex interplay
between magnetic fields plus turbulence and self-gravity of the cloud is believed to control the duration of core formation. One scenario is the “fast mode” with a weak magnetic field (e.g., Padoan & Nordlund 1999; Padoan et al. 2001; Hartmann et al. 2001; Klessen et al. 2000, 2005; Krumholz & McKee 2005; Krumholz & Tan 2007). The opposite is the “slow mode” with a strong magnetic field (e.g., Allen & Shu 2000; Elmegreen 2007; Nakamura & Li 2005, 2008). The difference between the “fast” and “slow” modes is whether or not a core forms within the free-fall time of the natal gas. All the three-dimensional (3D) simulations predict the formation of a network of filaments, but their complexity in velocity fields differs significantly (see review by André et al. 2009). In general, the “slow mode” expects a quiescent ambient velocity field controlled by the magnetic field, whereas the “fast mode” predicts a large-scale supersonic velocity field. In other words, physical properties of the low density gas in the filament are considered to determine the formation mechanism and evolution of the dense cores.

To link physical properties of such low density gas to our knowledge about collapse of a cloud core, we performed a detailed study of the dense cloud core GF 9-2, which is also known as L1082C (e.g., Bontemps et al. 1996; Caselli et al. 2002, and references therein). This core contains an extremely young low-mass protostar (Furuya et al. 2006, hereafter Paper I) whose circumstellar materials are probably responsible for the IRAS point source PSC 20503+6006 (e.g., Ciardi et al. 1998). The core has been identified by several molecular lines such as CS (2–1) (Ciardi et al. 2000), N2H+ (1–0) (Caselli et al. 2002; Paper I), H13CO+ (1–0), NH3 (1,1), and CCS 43–32 lines (Paper I). The core is located in the GF 9 filament (d = 200 pc), which is the ninth filament cataloged in Schneider & Elmegreen (1979). Using the optical image, Schneider & Elmegreen (1979) estimated a total length of the filament to be 1’25, corresponding to 4.4 pc, although the filament curves as a portion of a shell. The filament contains seven dense cores identified in the NH3(1,1) lines (Furuya et al. 2008, hereafter Paper II). The physical properties of the filament were subsequently studied by near-infrared (NIR) extinction (Ciardi et al. 2000) and optical and NIR polarization observations (Poidevin & Bastien 2006). In particular, Poidevin & Bastien (2006) revealed that there exist well-aligned large-scale magnetic fields and claimed that the fields are almost perpendicular to the axis of the filament as the first order approximation.

The protostar harbored in the GF 9-2 core has not developed an extensive molecular outflow (Bontemps et al. 1996; Paper I). This fact yields a rare opportunity to investigate core collapse conditions free from the disturbance by the outflow. Our previous studies toward the core using the Nobeyama 45 m telescope, the CSO 10.4 m telescope, and the OVRO millimeter-array showed that the GF 9-2 core has a radial density profile of ρ(r) ∝ r−2 (Paper I). Furthermore, we detected blueskewed profiles in optically thick lines of HCO+ (3–2), (1–0), and HCN (1–0), suggesting gas infall motions all over the core (Furuya et al. 2009, hereafter Paper III). Modeling the infall spectra, we pointed out that the core has undergone its gravitational collapse for ~2 × 10^5 yr from an initially unstable state in Paper III. This is because the observed radial density profile was between those predicted in the runaway collapse scenario (Larson 1969; Penston 1969; Hunter 1977; the Larson-Penston-Hunter (LPH) solution) and the quasi-static inside-out collapse scenario (Shu 1977) and because the observed velocity field inside the core has a reasonable consistency with that expected from the former scenario. However, it was impossible to investigate what had triggered the dynamical collapse of the core based on the data tracing only high density molecular gas (10^5 < n(H2)/cm^−3 < 10^6). Clearly it required unveiling a close link between the initial conditions of the collapse and the physical properties of the low density ambient gas in the filament. We therefore carried out wide-field spectroscopic observations of the low density (10^3 < n(H2)/cm^−3 < 10^4) gas in the filament using the Nobeyama 45 m telescope.

Contrary to the previous papers that dealt with the dense core gas, this paper presents a detailed observational study of the low density natal filament gas surrounding the dense core GF 9-2. The organization of this paper is as follows: Section 2 describes the observations; Section 3 summarizes the results that were directly derived from the obtained spectra and maps of the CO emission; Section 4 describes our analysis of the excitation conditions of the CO lines, the velocity structure of the filament gas, and the calculations of column density; Section 5 discusses the physical properties of the natal filament gas and a scenario of dense core formation; and Section 6 gives a summary of this work. Finally, details of our analysis presented in Section 4 and the associated error calculations are described in the Appendix.

2. OBSERVATIONS

Using the Nobeyama Radio Observatory 45 m telescope, we carried out On-The-Fly (OTF) mapping observations (Sawada et al. 2008) of the J = 1–0 transitions of 12C16O (rest frequency 2.345 GHz), 13CO (2.262 GHz), 12CO (2.330 GHz), and C18O (2.295 GHz) in the GF 9 filament with a beam size (HPBW) of 24″. The vertical scale of the plot is the main-beam brightness temperature (TMB) in K units. The velocity resolutions of the spectra are 0.1 km s^−1. The RMS noise levels are 760 mK for 12CO, 360 mK for 13CO, and 130 mK for C18O. The systemic velocity of the GF 9-2 dense core is VLSR = −2.48 km s^−1 (Paper I). The negative dip seen in the 12CO spectrum around VLSR = −0.8 km s^−1 is an artifact caused by the presence of emission at the off position (see Section 2).

(A color version of this figure is available in the online journal.)
an effective velocity resolution of 0.10 km s^{-1}.

The 13CO and C18O lines were observed toward a portion of the filamentary cloud toward the dense cloud core GF 9-2. The green contours represent the total map of the H13CO+ (1–0) emission (Paper I). The upper panels show the spectra starting from the 3σ levels with the 3 mm continuum position (R.A. = 20°51′29″.827, Decl. = 60°18′38″.06 in J2000; Paper I) designated by the yellow star. The white boxes in the upper panels indicate the area shown in the lower panels. The two circles in the upper panels are the areas where the emission was integrated to obtain the spectra shown in Figure 5. The hatched circle at the bottom left corner of each panel indicates the effective spatial resolution of 24″. The numbers in the upper right corners of the panels show the corresponding velocity ranges for 13CO and C18O lines.

The RMS noise (the 1σ levels) of the 25 beams was estimated to be 14% at 115 GHz. All the spectra were calibrated by the mean main-beam efficiencies (η_sys) of the GF 9-2 dense core (V_{LSR} = −2.48 km s^{-1}; Paper I). Averaging the 12CO and 13CO spectra in a circle with a 50″ diameter toward the core, we verified consistency with those taken with the FCRAO 14 m pointing was checked every 1 hr by observing the SiO J = 1–0 v = 1, and v = 2 maser lines and was found to be accurate to less than 3″. The data reduction was done using the NOSTAR package (Sawada et al. 2008). We produced 3D data cubes for the lines with an effective spatial resolution in FWHM of 24″, and a velocity resolution of 0.10 km s^{-1}.

### 3. RESULTS

Figure 1 shows J = 1–0 transition spectra of 12CO, 13CO, and C18O molecules in T_{mb} obtained by averaging all the data. The 12CO line is the brightest with the peak T_{mb} of ~5 K and seems to have a self-absorption feature over the LSR-velocity (V_{LSR}) range of −2.6 to −1.8 km s^{-1}. The 12CO and 13CO lines show single-peaked profiles. These peaks fall in the V_{LSR} range where the 12CO line shows the self-absorption. The mean-beam efficiencies (η_{mb}) for the 25 beams were 40% ± 2% at 110 GHz and 32% ± 2% at 115 GHz. The spectra were calibrated by the standard chopper wheel method and were converted into main-beam brightness temperature (T_{mb}) by dividing by η_{mb}. The uncertainty in our intensity calibration is ~15%. The telescope pointing was checked every 1 hr by observing the SiO J = 1–0 v = 1, and v = 2 maser lines and was found to be accurate to less than 3″. The data reduction was done using the NOSTAR package (Sawada et al. 2008). We produced 3D data cubes for the lines with an effective spatial resolution in FWHM of 24″, and an effective velocity resolution of 0.10 km s^{-1}. For producing the 3D cubes, we adopted a pixel size of 12″, and spatially smoothed the original data by convolving a Gaussian function with FWHM = 18.9″ to obtain the final effective resolution of 24″, while we applied no smoothing along the velocity axis.

### 4. DISCUSSION

The 12CO line is the brightest with the peak T_{mb} of ~5 K and seems to have a self-absorption feature over the LSR-velocity (V_{LSR}) range of −2.6 to −1.8 km s^{-1}. The 12CO and C13O lines show single-peaked profiles. These peaks fall in the V_{LSR} range where the 12CO line shows the self-absorption. The mean-beam efficiencies (η_{mb}) for the 25 beams were 40% ± 2% at 110 GHz and 32% ± 2% at 115 GHz. The spectra were calibrated by the standard chopper wheel method and were converted into main-beam brightness temperature (T_{mb}) by dividing by η_{mb}. The uncertainty in our intensity calibration is ~15%. The telescope pointing was checked every 1 hr by observing the SiO J = 1–0 v = 1, and v = 2 maser lines and was found to be accurate to less than 3″. The data reduction was done using the NOSTAR package (Sawada et al. 2008). We produced 3D data cubes for the lines with an effective spatial resolution in FWHM of 24″, and an effective velocity resolution of 0.10 km s^{-1}. For producing the 3D cubes, we adopted a pixel size of 12″, and spatially smoothed the original data by convolving a Gaussian function with FWHM = 18.9″ to obtain the final effective resolution of 24″, while we applied no smoothing along the velocity axis.

### 5. CONCLUSION

The 12CO line is the brightest with the peak T_{mb} of ~5 K and seems to have a self-absorption feature over the LSR-velocity (V_{LSR}) range of −2.6 to −1.8 km s^{-1}. The 12CO and C13O lines show single-peaked profiles. These peaks fall in the V_{LSR} range where the 12CO line shows the self-absorption. The mean-beam efficiencies (η_{mb}) for the 25 beams were 40% ± 2% at 110 GHz and 32% ± 2% at 115 GHz. The spectra were calibrated by the standard chopper wheel method and were converted into main-beam brightness temperature (T_{mb}) by dividing by η_{mb}. The uncertainty in our intensity calibration is ~15%. The telescope pointing was checked every 1 hr by observing the SiO J = 1–0 v = 1, and v = 2 maser lines and was found to be accurate to less than 3″. The data reduction was done using the NOSTAR package (Sawada et al. 2008). We produced 3D data cubes for the lines with an effective spatial resolution in FWHM of 24″, and an effective velocity resolution of 0.10 km s^{-1}. For producing the 3D cubes, we adopted a pixel size of 12″, and spatially smoothed the original data by convolving a Gaussian function with FWHM = 18.9″ to obtain the final effective resolution of 24″, while we applied no smoothing along the velocity axis.
telescope (Ciardi et al. 1998) in terms of the peak intensities and the spectral shapes.

We present total integrated intensity maps of the three CO isotopologues in Figure 2, where the total map of a dense gas tracer, H13CO+ (1–0) emission (Paper I), is overlaid to assess spatial relations with the GF 9-2 dense cloud core. The 13CO emission is detected all over the mapped area and becomes intense to the southwest of the core. On the other hand, the 13CO emission clearly represents a portion of the GF 9 filamentary dark cloud (Schneider & Elmegreen 1979), which is elongated along the east–west direction. The 13CO emission becomes intense toward the southwest, as seen in the 12CO map. In the central region, the C18O emission is elongated along northeast–southwest (Figure 2(f)), which agrees with the elongation of the core traced by, e.g., the H13CO+ (1–0), N2H+ (1–0) (Paper I), and 13CO (1–0) (Ciardi et al. 1998) lines. These results clearly indicate that the C18O line probes the gas with a medium density between those traced by the 13CO and H13CO+ lines. Consequently, we have a seamless data set over a volume density range of $10^2 \lesssim n(H_2)/cm^{-3} \lesssim 10^6$.

Velocity channel maps with a resolution of 0.2 km s$^{-1}$ are shown in Figures 3 and 4, where we overlaid the channel maps for the higher-density gas tracer with green contours on those for the lower-density gas tracer with black contours. First, the large-scale east–west structure is mainly seen in the 13CO panels of $-3.3 \lesssim V_{\text{LSR}}/\text{km s}^{-1} \lesssim -2.5$. This velocity range includes the $V_{\text{sys}}$ of the GF 9-2 dense core ($-2.48 \text{ km s}^{-1}$). Hereafter, we refer to the east–west structure seen in the velocity range as “Component 1.” Second, Component 1 is likely to have substructures with scales of $\lesssim 0.1 \text{ pc}$ (100″ corresponds to 0.097 pc at $d = 200 \text{ pc}$), one of which is the GF 9-2 core. Third, one finds an overall agreement between the spatial distributions of

![Figure 3](image-url)
the $^{13}$CO and C$^{18}$O emission and between the C$^{18}$O and H$^{13}$CO$^+$ emission in each velocity channel. Note that the peak position of the C$^{18}$O emission does not coincide with the H$^{13}$CO$^+$ peak in the GF 9-2 core (see $V_{LSR} = -2.7$ and $-2.5$ km s$^{-1}$ in panels of Figure 4). Fourth, the $^{13}$CO channel maps show that, in addition to Component 1, there exists another spatially coherent structure in the southern parts of the velocity panels of $-2.1 \lesssim V_{LSR}/$km s$^{-1} \lesssim -0.7$. This structure, “Component 2,” is elongated along the southeast–northwest direction and shows intense $^{13}$CO emission toward the southeast. Its local maxima is found at the bottom left corners of the velocity channels of $V_{LSR} = -2.1$ and $-1.9$ km s$^{-1}$ (Figure 3). For the C$^{18}$O line, Component 2 is mainly identified in the panels of $-2.3 \lesssim V_{LSR}/$km s$^{-1} \lesssim -1.7$. Last, the gas seen in the $V_{LSR} = -2.3$ km s$^{-1}$ panel is highly likely emanated from the two components. Despite the fact that the two components are not well separable in the velocity space, we define the boundary velocity between the dual components as $-2.2$ km s$^{-1}$ in $V_{LSR}$. This is because the C$^{18}$O emission associated with the GF 9-2 core is not recognized in the channel maps redward of $V_{LSR} \gtrsim -2.1$ km s$^{-1}$ (Figure 4(a)).

Since we detected significant $^{13}$CO emission to the southwest of the core (e.g., Figure 2), we compared two spectra obtained toward the core and the southwestern region (Figure 5). These spectra were obtained by averaging the emission inside the circles shown in Figures 2(a)–(c). Figure 5 shows the following features: (1) the $^{12}$CO emission in the southwest is stronger than in the center, (2) a comparison between the two $^{13}$CO spectra suggests that dividing the filament gas into the two components by $V_{LSR} = -2.2$ km s$^{-1}$ seems to be reasonable, (3) the C$^{18}$O emission toward the core is approximately twice as intense as the emission toward the southwest, and (4) toward the core center, the LSR-velocity of the C$^{18}$O peak agrees with those of the central dips seen in the $^{12}$CO and $^{13}$CO spectra (Figure 5). However, these LSR-velocities are shifted by $\sim -0.2$ km s$^{-1}$ with respect to the $V_{sys}$.

We show the two integrated intensity maps of Components 1 and 2 in Figure 6 by adopting $V_{LSR} = -2.2$ km s$^{-1}$ as the boundary velocity. The $^{12}$CO emission in the velocity range of $-4.0 \lesssim V_{LSR}/$km s$^{-1} \lesssim -2.2$ (Figure 6(a)) is clearly associated with the dense core gas, which reconciles with the fact that the C$^{18}$O emission seen in each panel center of $-3.2 \lesssim V_{LSR}/$km s$^{-1} \lesssim -2.2$ (Figure 4(a)) is predominantly associated with the core. In contrast, the $^{13}$CO gas in Figure 6(b) is not associated with the dense core. As seen in the $V_{LSR} = -2.3$ km s$^{-1}$ panels of Figures 3 and 4(a) as well as the spectrum in Figure 5(b), it is impossible to completely separate the two components. We consider that they are partially overlapping in the line of sight and are partially interacting with each other. In Section 4 we analyze the data by showing their boundary instead of separating them.

4. ANALYSIS

4.1. Producing Opacity-corrected $^{13}$CO $J = 1$–0 Spectra

In order to discuss the velocity structure of the filament gas, we corrected the cube data for the line broadening due to optical thickness (e.g., Phillips et al. 1979), and made opacity-corrected $^{13}$CO spectra. This required us to estimate the velocity width of the $^{13}$CO line in the limit of $\tau \rightarrow 0$, $\Delta \nu(\tau \rightarrow 0)$, where $\tau$ denotes the $^{13}$CO optical depth. The choice of the $^{13}$CO is due to the large optical depth of the $^{12}$CO emission and due to the absence of the C$^{18}$O emission in the low density region of the filament. As described in Appendices A.1 and A.2, we utilized the three CO isotopologues to estimate the optical depth and excitation temperature ($T_{ex}$) of the $^{13}$CO line. We multiplied a factor of $\tau/(1 - e^{-\tau})$ by the observed $T_{mb}$ of the $^{13}$CO line to obtain the opacity-corrected $T_{mb}$ at each LSR-velocity (see Equation (A10) in Appendix A.4). After obtaining the opacity-free spectrum such as shown in Figure 18(e), we generated a 3D data cube by keeping the effective spatial resolution of 24′′ and the velocity resolution of 0.1 km s$^{-1}$ (Section 2). The opacity-free $^{13}$CO cube data were analyzed as described in the subsequent subsections, and our usage of the $^{13}$CO and C$^{18}$O line data are limited to estimating $\tau$ and $T_{ex}$ of the $^{13}$CO emission.

4.2. Maps of Optical Depth and Excitation Temperature

Figures 7 and 8 present velocity channel maps of the $v_{CO}$ and $T_{ex}$ of the $^{13}$CO (1–0) line emission, respectively. Because of
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Figure 5. Spectra of the $J = 1$--0 lines of the $^{12}$CO (green), $^{13}$CO (black), and C$^{18}$O (magenta) obtained by averaging the emission over the two circles with a $100''$ radius shown in Figures 2(a)–(c). The vertical scale is the $T_{mb}$ in K, the same as in Figure 1. Velocity resolutions of the spectra are 0.1 km s$^{-1}$. Panel (a) represents the spectra toward the core center, while panel (b) represents those at $(\Delta \alpha, \Delta \delta) = (-250'', -150'')$ with respect to the 3 mm continuum position. The vertical dashed line at $V_{LSR} = -2.48$ km s$^{-1}$ in panel (a) shows the systemic velocity of the cloud ($V_{sys}$; Paper I), and the vertical dotted lines in the two panels indicate $V_{LSR} = -2.2$ km s$^{-1}$ which is the velocity boundary between the two gas components (see Section 4.3). (A color version of this figure is available in the online journal.)

Figure 6. Integrated intensity maps of the $^{13}$CO $J = 1$--0 emission in units of K km s$^{-1}$. The maps in panels (a) and (b) show the integrated emission in $T_{mb}$ over the LSR-velocity ranges of $-4.0 \leq V_{LSR}/\text{km s}^{-1} \leq -2.2$ and $-2.2 \leq V_{LSR}/\text{km s}^{-1} \leq -0.6$, respectively (see Figure 5). Note that the intensity ranges in the two maps are the same. The white contours have intervals of 1.0 K km s$^{-1}$ starting from the 1.0 K km s$^{-1}$ level, which corresponds to the $12\sigma$ and $8\sigma$ levels in maps (a) and (b), respectively. The peak intensity of 5.2 K km s$^{-1}$ in panel (a) is found at $(\Delta \alpha, \Delta \delta) = (-76'', -78'')$ with respect to the 3 mm continuum source position, while that of (b) (3.4 K km s$^{-1}$) is measured at $(\Delta \alpha, \Delta \delta) = (-86'', -326'')$. The green contours show the H$^{13}$CO$^+$ total map in the same fashion as for Figure 2. The magenta dashed circles show the area where the CO isotopologue emission are integrated to obtain the spectra shown in Figure 5(b). (A color version of this figure is available in the online journal.)

our 3$\sigma$ level detection threshold (Appendix A.2) and the quality assessment of the results (Appendix D), we have no data points blueward of the $V_{LSR} = -3.7$ km s$^{-1}$ channel and redward of the $V_{LSR} = -1.1$ km s$^{-1}$ channel where the weak $^{12}$CO emission is seen in Figure 3. Figure 7 shows that the $^{13}$CO emission is moderately opaque with $\tau_{^{13}\text{CO}} \sim 1$ in the ambient regions and becomes optically thick toward the core. We measure a mean $\tau_{^{13}\text{CO}}$ of 0.78 $\pm$ 0.32 and a mean $T_{ex}$ of 8.4 $\pm$ 1.0 K for all the data shown in Figures 7 and 8, respectively.

Toward the position of the dense core, we have $\tau$ and $T_{ex}$ estimates blueward of $V_{LSR} = -2.3$ km s$^{-1}$, but no estimates redward of $V_{LSR} = -2.1$ km s$^{-1}$. The highest optical depth in the GF 9-2 core region is measured at the local peak of 5.8 $\pm$ 0.9 at $V_{LSR} = -2.7$ km s$^{-1}$, leading to the C$^{18}$O optical depth of 1.1 with the solar abundance ratio of $^{13}$CO/[C$^{18}$O] = 5.5. We measure a mean $\tau_{^{13}\text{CO}}$ of 1.31 $\pm$ 0.30 and a mean $T_{ex}$ of 8.0 $\pm$ 0.7 K for the region enclosed by the 3$\sigma$ level contour of the H$^{13}$CO$^+$ emission. Inside the 50% intensity contour with respect to the peak intensity for the $N_2H^+$ (1--0) emission (Paper I), a mean $T_{ex}$ of 8.0 $\pm$ 0.7 K for the region enclosed by the 3$\sigma$ level contour of the H$^{13}$CO$^+$ emission. Inside the 50% intensity contour with respect to the peak intensity for the $N_2H^+$ (1--0) emission (Paper I), a mean $T_{ex}$ is calculated to be 7.9 $\pm$ 0.6 K, which agrees with the $T_{ex} = 9.5 \pm 1.9$ K derived from the $N_2H^+$ line in the previous work within the uncertainties. In addition, for a circular region...
Figure 7. Velocity channel maps of the optical depth of the \(^{13}\)CO (1–0) emission with velocity intervals of 0.2 km s\(^{-1}\). The central LSR-velocity of each channel in km s\(^{-1}\) is shown at the top left corner of each panel. The optical thickness is shown in logarithmic scale (see the color bar). The maximum optical depth of 10.0 ± 3.2 is measured near the southern edge of the \(V_{\text{LSR}} = -2.1\) km s\(^{-1}\) panel, while the second highest value of 5.8 ± 0.9 is measured toward the GF 9-2 core at \(V_{\text{LSR}} = -2.7\) km s\(^{-1}\).

(A color version of this figure is available in the online journal.)

with a radius of 0.1 pc centered on the GF 9-2 core, we measure a mean \(T_{\text{ex}}\) of 8.0 ± 0.1 K, which agrees with the \(T_{\text{ex}}\) value expected from the Monte Carlo simulation by Ciardi et al. (2000, see their Figure 9). In conclusion, the calculated \(T_{\text{ex}}\) velocity channel maps have reasonable consistency with the previous results. Assuming that the gas is in LTE, we consider that the \(T_{\text{ex}}\) represents the kinetic temperature \(T_{\text{kin}}\) of the gas.

In Figure 8, we found regions where \(T_{\text{ex}}\) is elevated up to \(\sim 14\) K to the southwest of the core in \(V_{\text{LSR}} = -2.9\) to \(-2.5\) km s\(^{-1}\) panels and the \(-1.7\) km s\(^{-1}\) one, which belong to Components 1 and 2, respectively. It is also interesting that the velocity channels between them (i.e., \(-2.3 \leq V_{\text{LSR}}/\text{km s}\(^{-1}\) ≤ -1.9) do not have such high temperature regions. These features will be discussed in Section 5.5.2.

In order to calculate the column density of the \(^{13}\)CO molecules, we subsequently produced a mean \(T_{\text{ex}}\), \(\langle T_{\text{ex}} \rangle\), map from the \(T_{\text{ex}}\) data cube. The \(\langle T_{\text{ex}} \rangle\) map is shown in Figure 9. We averaged \(T_{\text{ex}}(v)\) values along the velocity axis where the \(T_{\text{ex}}^\text{mb}(v)\) intensities exceed our detection threshold of the \(3\sigma\) level (see Figures 18(d) and (e)). We obtained the mean value for \(T_{\text{ex}}\) of 7.5 ± 1.0 K over the presented region and adopted this value as the representative \(T_{\text{kin}}\) for the region.

4.3. Maps of Total Intensity, Centroid Velocity, and Velocity Width

The color images in Figure 10 represent total integrated intensity, intensity-weighted mean velocity, i.e., centroid velocity (\(v_{\text{cent}}\)), and velocity width (FWHM) maps produced from the opacity-corrected \(^{13}\)CO spectra. After calculating the zeroth, first, and second order moments as well as their errors at each spatial position, we produced the maps without any smoothing (Appendices B and D). Since we are interested in the natal cloud of the dense core, we indicate the boundary between Components 1 and 2 by black and white contours that correspond to \(v_{\text{cent}} = -2.2\) km s\(^{-1}\) in Figure 10(b).

Figure 10(a) shows that the morphology of the \(^{13}\)CO bright region is principally similar to the distribution of the \(^{13}\)C\(^{18}\)O emission (Figures 2(c) and (f)) and that the dense core traced by the \(^{13}\)C\(^{18}\)O\(^+\) emission has formed in the inner densest part of the filamentary cloud. Figure 10(b) shows that Component 1 gas is confined in a narrow velocity range of \(\sim 0.6\) km s\(^{-1}\) (\(-2.8 \leq v_{\text{cent}}/\text{km s}\(^{-1}\) ≤ -2.2), which is about twice the isothermal sound velocity \((c_s^2 = 8\ln 2(kT_{\text{kin}}/\mu m_{\text{H}}))\), where \(\mu\) denotes the mean molecular weight of 2.33 for [He] = 0.1
Figure 8. Velocity channel maps of the excitation temperature of the $^{13}$CO (1–0) emission in K with a velocity resolution of 0.2 km s$^{-1}$. The central LSR-velocity of each velocity channel in km s$^{-1}$ is shown at the top left corner of each panel. The two numbers in the parenthesis with the +/- sign indicate the mean and standard deviation of the excitation temperature in each panel in units of K. Notice that the excitation temperature is shown in linear scale (see the color bar). (A color version of this figure is available in the online journal.)

Figure 9. Plot of the mean excitation temperature obtained by averaging the $T_{\text{ex}}(v)$ spectra at each pixel position across the velocity axis (see Figure 8). The contours represent the total map of the H$^{13}$CO$^+$ (1–0) emission in the same manner as for Figure 2. The 0.1 pc scale is indicated at the top right corner, and the color bar is on the right-hand side of the panel. (A color version of this figure is available in the online journal.)

The measured $v_{\text{cent}}$ of the immediate surroundings of the dense core shown in blue-to-dark magenta (corresponding to $-2.8 \lesssim v_{\text{cent}}/\text{km s}^{-1} \lesssim -2.6$) has reasonable consistency with the velocity structure of the dense core traced by the H$^{13}$CO$^+$ (1–0) and N$_2$H$^+$ (1–0) line observations (see Figure 12 in Paper I). As we argued in Section 3 with Figures 5 and 6, the two gas components cannot be separated completely. Figure 10(b) demonstrates that the main body of Component 2 gas is seen in $V_{\text{LSR}} \gtrsim -2$ km s$^{-1}$. Another important result from Figure 10(b) is that neither systematic motions of the gas caused by any activities of YSOs, such as a well-collimated typical molecular outflow, nor any sharp discontinuity of the line-of-sight velocity, e.g., due to shock fronts, can be recognized. These velocity features strongly suggest that both Components 1 and 2 belong to the GF 9 filament (Schneider & Elmegreen 1979), where seven dense cores and three possible candidates are located at regular intervals of $\sim 0.9$ pc (Paper II).

Figure 10(c) shows that the velocity width is enhanced up to $\sim 1.4$ km s$^{-1}$ in the fourth quadrant of the map. The extent of this region is almost the same as the southwestern intensity enhancement seen in Figure 10(a). The mean velocity width for the fourth quadrant is $\Delta v_{\text{FWHM}} = 0.96 \pm 0.24$ km s$^{-1}$, while the mean value for the first to third quadrants is $0.69 \pm 0.12$ km s$^{-1}$. Both the mean values exceed the isothermal sound velocity width in FWHM of $0.38 \pm 0.13$ km s$^{-1}$ for $T_{\text{kin}} = 7.5 \pm 1.0$ K (Section 4.2).
4.4. Column Density Map and LTE Mass

After obtaining the maps of the total integrated intensity, ∫ T_{rot}^{int}(v)dv (Figure 10(a)), and the mean excitation temperature (T_{ex}) (Figure 9), one can readily calculate the 13CO column density, N_{13CO} (Appendix C). Here we used the (T_{ex}) map (Figure 9) instead of the T_{ex} channel maps (Figure 8). Considering the T_{ex} range of 5.6 ≤ T_{ex}/K ≤ 13.6 (see Figure 8), the dependency of T_{ex} in the N_{13CO} calculations is smaller than the other errors (see Equations (C1) and (C2) in Appendix C). Furthermore, the T_{ex}(v) spectra are generally “flat” with respect to the LSR-velocity (see Figure 18(d)), justifying the usage of the (T_{ex}) value as the representative T_{ex} at each map pixel.

Figure 11 presents a map of the molecular hydrogen column density (N_{H_2}) that is converted from the 13CO column density by adopting the 13CO fractional abundance of X(13CO) = 2 × 10^{-6} (e.g., Dickman 1978; Frerking et al. 1982; Lequeux 2005). The N_{H_2} map clearly demonstrates that the filament has a high-column density region in the center where the H^{13}CO^{+} core is observed. In addition, the map also shows that the filament gas has a column density range of 20.7 ≤ log N_{H_2}/cm^{-2} ≤ 21.8. We measure a mean N_{H_2} of (2.7 ± 0.9) × 10^{21} cm^{-2} for Component 1 and (2.4 ± 0.7) × 10^{21} cm^{-2} for Component 2. We calculate LTE masses (M_{LTE}) of 24 ± 10 M_⊙ and 17 ± 7 M_⊙ for Components 1 and 2, respectively, with d = 200 pc.

We checked for consistency with the previous work by Ciardi et al. (2000). We measured the maximum N_{13CO} of 1.2 × 10^{16} cm^{-2} at the center of the GF 9-2 core. This agrees with the value that can be read from Figure 6(a) in Ciardi et al. (2000); 1.4 × 10^{16} cm^{-2}). It should be noted that Ciardi et al. (2000) used d = 440 pc and estimated M_{LTE} of 53 ± 8 M_⊙ for a rectangular region of 480′′ × 600′′ centered on the GF 9-2 core. Toward the same rectangle, we measured a mean N_{13CO} of (5.3 ± 2.2) × 10^{15} cm^{-2}. With d = 440 pc, we recalculated M_{LTE} of 55 ± 23 M_⊙ from the mean N_{13CO}. Therefore, our analysis with d = 200 pc has good consistency with the previous one with d = 440 pc.

A hint regarding the origin and nature of the filament may be obtained from a histogram of the H_2 column density and its cumulative distribution shown in Figure 12. The histogram is well approximated by a log-normal probability distribution function with a standard deviation of 0.15, indicating that the turbulence formed the density structure of the filament gas (e.g., Ostriker et al. 2001; Padoan & Nordlund 2002; Ossenkopf & Mac Low 2002; Nakamura & Li 2007; Kainulainen et al. 2009). In addition, Figure 12(b) shows that 50% of the filament materials exist in the higher column density region of N_{H_2} ≥ 2.5 × 10^{21} cm^{-2}, which is similar to the results obtained in Taurus (Goldsmith et al. 2008). The order of the derived column density (N_{H_2} ∼ 10^{21} cm^{-2}) corresponds to the visual extinction of A_v ∼ 1, which is comparable to those measured from the optical images (Schneider & Elmegreen 1979; Poidevin & Bastien 2006).

5. DISCUSSION

In this section, we discuss the nature of the filament gas to shed light on the initial conditions of the dynamical collapse of the GF 9-2 cloud core, which harbors the exceptionally young low-mass protostar (Papers I and III). On the basis of all the results and analysis, we conclude that the core is physically associated with the Component 1 filament (Sections 3 and 4.3). This is because the dense core is located at the local density maxima of Component 1 and the LSR-velocity of the core falls...
in the velocity range of Component 1. Although it is impossible to separate the two components completely, Component 2 is not the natal gas of the GF 9-2 core.

5.1. Turbulent Motions of the Gas in Component 1

We present a map of the ratio between the non-thermal velocity dispersion \( \sigma_{\text{nth}} \) and the sound velocity \( c_s \) in Figure 13 to examine the degree of turbulence in Component 1. The ratio is obtained from,

\[
\left( \frac{\sigma_{\text{nth}}}{c_s} \right)^2 = \left\{ \frac{\Delta v(\tau \to 0)}{c_s \sqrt{8 \ln 2}} \right\}^2 - \left( \frac{\sigma_{\text{thm}}}{c_s} \right)^2,
\]

where \( \sigma_{\text{thm}}^2 \) is given by \( k T_{\text{kin}}/m_{13\text{CO}} \) and \( m_{13\text{CO}} \) denotes the molecular mass of \(^{13}\text{CO}\). For calculations of \( \sigma_{\text{thm}} \), we used the \( \langle T_{\text{ex}} \rangle \) map (Figure 9). Over Component 1, we obtained a mean ratio of \( \langle \sigma_{\text{thm}}/c_s \rangle = 2.1 \pm 0.50 \), corresponding to \( \langle \sigma_{\text{thm}} \rangle = 0.34 \pm 0.80 \) km s\(^{-1}\) s\(^{-1}\). The observed non-thermal contribution must be simply attributed to the random turbulent motions of the gas because we did not identify any systematic motions of the filament gas (Section 4.3).

In the following, we deal with Component 1 as a single filament, and we do not consider an interpretation proposed by Hacar & Tafalla (2011) that a supersonic turbulent filament is mimicked by “twisted” subsonic filaments. In summary, the internal motion of the filament gas is governed by the supersonic turbulence. Note that our conclusion differs from that of Poidevin & Bastien (2006), who inferred the absence of turbulence in the filament based on their polarization maps.

5.2. Is Component 1 Radially Supported by the Supersonic Turbulence?

Next we assess whether or not the filament is dynamically supported by the supersonic turbulence against radial collapse. This yields the first step to discuss how the gravitational collapse of the GF 9-2 core (Paper I) was triggered in the supersonic turbulent filament. For this purpose, we examine the stability of the Component 1 gas based on a model for an isothermal cylinder in hydrostatic equilibrium (Stodol’skiwicz 1963; Ostriker 1964). Although the mean \( T_{\text{ex}} \) map shows that the gas temperature in the southwestern region is enhanced up to \( \sim 10 \) K (Figure 9), we assume that the filament is isothermal with 7.5 K (Section 4.3) as the first order of approximation. The model gives the radial density distribution of

\[
\rho(\tilde{\omega}) = \rho_c \left\{ 1 + \left( \frac{\tilde{\omega}}{H} \right)^2 \right\}^{-2}.
\]
Here $\bar{\omega}$ is the radial distance from the cylinder axis, and $H$ is the scale height given by

$$H = \sqrt{\frac{2c_s^2}{\pi G \rho_c}} \sim \lambda_1,$$  

(3)

where $\rho_c$ is the central density of the filament and $\lambda_1$ the Jeans length at $\rho_c$. Therefore, the critical line mass ($m_{\text{line,crit}}$), above which the cylinder radially collapses, is calculated by

$$m_{\text{line,crit}} = \int_0^{\infty} 2\pi \bar{\omega} \rho(\bar{\omega}) d\bar{\omega} = \frac{2c_s^2}{G},$$

(4)

which does not depend on $\rho_c$. To apply this model to Component 1 where the supersonic turbulence dominates the gas kinematics, we replace $c_s^2$ with $c_s^2 + \sigma_{\text{nth}}^2$ and define an effective critical line mass by

$$m_{\text{line,crit}}^{\text{eff}} = \frac{2 k T_{\text{kin}}}{G \mu m_H} \left[ 1 + \left( \frac{\sigma_{\text{nth}}}{c_s} \right)^2 \right],$$

(5)

This equation represents the maximum line mass that can be supported by the total internal pressure of the filament. We stress that the $\sigma_{\text{nth}}$ term represents the non-thermal pressures due to predominantly the supersonic turbulence (Section 5.1) and probably the magnetic field as well (described in Section 5.4), although it is impossible to separate them. Using $(T_{\text{kin}}) = 7.5 \pm 1.0 \text{ K (Section 4.3)}$ and $(\sigma_{\text{nth}}/c_s) = 2.1 \pm 0.50$ (Section 5.1), we calculated an effective critical mass ($M_{\text{crit}}^{\text{eff}}$) of $51^{+3}_{-2} M_\odot$ for Component 1 by multiplying $m_{\text{line,crit}}^{\text{eff}}$ of $41^{+35}_{-26} M_\odot \text{pc}^{-1}$ by the observed length of the filament (0.77 pc; see Figure 2). Comparing the $M_{\text{crit}}^{\text{eff}}$ value to the $M_{\text{LTE}}$ of $24 \pm 10 M_\odot$ (Section 4.4), we suggest that the filament is highly likely gravitationally stable with respect to radial collapse owing to the turbulent support. Note that the filament cannot be supported only by the thermal pressure because the LTE mass is larger than $m_{\text{line,crit}} \times 0.77 \text{ pc} \sim 9 M_\odot$ (see Equation (4)).

5.3. Does Component 1 Fragment Axially?

5.3.1. Estimate of the Filament Scale Height from the Column Density Map

To discuss the axial fragmentation of Component 1, we estimated the scale height of the filament from the column density map (Figure 11) using the Stodłókowski–Ostriker model (Section 5.2). A column density profile of an isothermal cylinder supported by the thermal and turbulent pressures can be written from Equation (2) as

$$N_{\text{H}_2}(r) = \frac{c_s^2 + \sigma_{\text{nth}}^2}{\mu m_H G H} \left[ 1 + \left( \frac{r}{H} \right)^2 \right]^{-1},$$

(6)

where $r$ is the projected distance from the central axis of the cylinder in the plane of the sky. We applied this equation to the Component 1 gas in the column density map (Figure 11). In the model fitting we adopted P.A. = $-90^\circ$ for the cylinder axis, which was forced to pass the position of the 3 mm continuum source, and $(\sigma_{\text{nth}}) = 2.1 c_s = 0.34 \text{ km s}^{-1}$, setting only $H$ as a free parameter. This is because we could not well determine the P.A. value and the location of the central axis as free parameters owing to the insufficient spatial coverage of the observations.

Figure 14 presents maps of the column density, the best-fit model, and the residual for Component 1. Figure 15 shows the radial profile of the column density averaged along the R.A. direction, and we obtained the best-fit value of $H = 0.68 \pm 0.04 \text{ pc}$ by considering the uncertainty in the column density (see Appendices C and E). We notice that the derived width of $2H = 1.4 \text{ pc}$ is significantly larger than the observed width of the filament of $\sim 600''$ (see, e.g., Figure 10), corresponding to 0.58 pc. Our analysis may be also affected by the small dynamic range of the column density of $20.9 \lesssim \log N_{\text{H}_2}/\text{cm}^2 \lesssim 20.8$ (Figures 11 and 12) and possibly by the asymmetric structure of the filament (see Figure 15), which exists all over the observed region. If we underestimated the peak column density, then the $H$ value would become small according to Equation (6).

The best-fit $H$ value leads to the filament width in FWHM to be $1.5H = 1.0 \pm 0.03 \text{ pc}$, which is significantly larger than the typical width of $\sim 0.1 \text{ pc}$ derived from the Herschel survey toward the IC 5146 (Arzoumanian et al. 2011) and B211/213 (Palmeirim et al. 2013) regions. In order to assess whether or not the difference is caused by that between the adopted models, we reanalyzed our column density map using the Plummer-like cylinder model (see, e.g., Equation (B1) in Palmeirim et al. 2013, and references therein) instead of using Equation (6). Notice that the Plummer-like function (Plummer 1911; Nutter et al. 2008) has three free parameters, while the Stodłókowski–Ostriker-like cylinder that we adopted has only one, i.e., $H$. With the Plummer-like function we estimated the filament width in FWHM to be $0.38 \pm 0.06 \text{ pc}$, which is still significantly larger than those by Herschel. We therefore conclude that the difference between the Herschel filaments and the GF 9 one is real. One possible cause is that the CO lines in our study could not trace the higher density regions detected by the Herschel survey with the dust continuum emission. An alternative cause is that there might exist an intrinsic difference between the GF 9 and the regions that Herschel surveyed.

Considering all the results as well as their limitations, we adopt fiducial values of $H = 0.3-0.7 \text{ pc}$ for further discussion (Table 1). Here the lower limit of 0.3 pc is obtained from the map ($2H = 0.58 \text{ pc}$), while the upper limit is from the best-fit value. The estimated range of the scale height leads to the $\rho_c$ value in Equation (2) of $(3-16) \times 10^{-21} \text{ g cm}^{-3}$ using the relation in our model of

$$\rho_c = \frac{2}{\pi G H^2} (c_s^2 + \sigma_{\text{nth}}^2).$$  

(7)
bein the N2H+ (1–0), H13CO+ (1–0), CCS 43–32, and NH3 (1,1) core” agree fairly well with those of the dense core observed (Paper III). The LTE mass of the “tenuous core” is calculated to be 0.1 pc surrounding the dynamically infalling dense core GF 9-2 (not the dense core itself).

The resultant ρc value corresponds to n(H2) = 800–4200 cm−3, which has reasonable consistency with the critical density required to excite the 13CO J = 1–0 transition. It should be noted that the above central density characterizes the tenuous filament gas surrounding the dense core GF 9-2 (not the dense core itself).

If the cylinder model is appropriate to the description of our data, a tenuous corelike structure is identified in the residual NH3 map (Figure 14(c)). The position and extent of the “tenuous core” agree fairly well with those of the dense core observed in the N2H+ (1–0), H13CO+ (1–0), CCS 43–32, and NH3 (1,1) lines (Paper I). We therefore speculate that it is a low-density envelope surrounding the dynamically infalling dense core (Paper III). The LTE mass of the “tenuous core” is calculated to be \( M_{\text{LTE}} \sim 0.8 \, M_\odot \) where TEC denotes the tenuous envelope of the core. Furthermore, we can estimate a “core formation efficiency” for the GF 9-2 dense cloud core of \( \eta_{\text{CF}} = M_{\text{LTE}} / (M_{\text{LTE}} + M_{\text{TEC}}) = 1.3 \, M_\odot / (1.3 \, M_\odot + 0.8 \, M_\odot) \gtrsim 60\% \), where

\( M_{\text{LTE}} \) is the mass of the dense core traced by the N2H+ and H13CO+ lines (Paper I).

5.3.2. Gravitational Stability of Component 1 against Axial Perturbations

Theoretical studies by, e.g., Nagasawa (1987) and Inutsuka & Miyama (1997) showed that the minimum wavelength (\( \lambda_{\text{min}} \)) that makes an infinite filament unstable against perturbations along the major axis is given by \( \lambda_{\text{min}} \sim 4H \), regardless of the presence of the magnetic field along the axis (the definition of H in Nagasawa (1987) differs by a factor of 1/\( \sqrt{8} \) times of that in, e.g., Inutsuka & Miyama 1992). The line mass of the filament can be calculated from the LTE mass (Section 4.4) and the observed length as 24 \( M_\odot / 0.77 \, pc = 31 \, M_\odot \, pc^{-1} \). Therefore, the critical “clump mass” produced by the axial perturbation may be 31 \( M_\odot \, pc^{-1} \times 4H = 37–87 \, M_\odot \). It is clear that the estimated range of the clump masses is one order of magnitude larger than the LTE masses of the NH3 cores (\( M_{\text{LTE}} = 2–8 \, M_\odot \); Paper II). We therefore consider that the filament is gravitationally stable against the axial perturbations probably owing to the transverse magnetic field (Poidevin & Bastien 2006), which is reanalyzed in Section 5.4.

Alternatively, the GF 9 filament may have narrow subfilaments (Hacar & Tafalla 2011; Hacar et al. 2013) with \( H \sim 0.1 \, pc \) and \( n_c \sim 10^4 \, cm^{-3} \). In fact, such a sub-structure may be recognized in the C18O total intensity map of Figure 2(c). If this is the case, the axial fragmentation of the sub filament explains the observed spatial intervals of ~0.9 pc between the NH3 cores (Paper II) because of \( (4–8)H = 0.4–0.8 \, pc \). However, this scenario has a caveat that the expected “clump masses” given by \( m_{\text{line, crit}} \times (4–8)H = 16–40 \, M_\odot \) are larger than the LTE-masses of the NH3 cores.

In summary, our analysis suggests that the natal gas of the core is in dynamical equilibrium with respect to both the radial and axial collapses. Naively speaking, this inference does not seem to be reconciled with the presence of the dynamically collapsing core (Papers I and III). Before resolving this issue, we assess the role of the large-scale magnetic fields existing in the filament.

5.4. Role of the Magnetic Fields in the Gas Surrounding the GF 9-2 Cloud Core

Figure 16 shows a comparison between the optical polarization map taken from Poidevin & Bastien (2006) and the distribution of the dense cores, including the candidates, seen in
the NH₃ emission (Paper II). Since the directions of the optical polarization angles are thought to be parallel to the magnetic fields, Poidevin & Bastien (2006) pointed out that the magnetic fields are almost perpendicular to the filamentary dark cloud (Schneider & Elmegreen 1979). Such a large-scale configuration is also found in the Taurus molecular cloud (see, e.g., Chapman et al. 2011, and references therein) suggesting that the filament formed through compression of the diffuse gas by external pressure along the magnetic fields or/and through gas accretion due to self-gravity along the magnetic field. However, this scenario should be reexamined in more detail because the optical polarization angles in the vicinity of the GF 9-2 core are inclined by ∼45° with respect to the Component 1 filament axis. It is also interesting that the polarization angles are almost parallel to the elongation of Component 2 gas.

An alternative mechanism to produce such a large-scale configuration is that the observed fields are the projected component of a toroidal field. Because of no firm evidence, we do not discuss this interpretation here. On the other hand, the magnetic fields in the filament should be far from good alignment at small scales owing to micro-turbulence (e.g., White 1977; Heyer et al. 2008). However, we cannot discuss this issue because the angular resolution of the polarization map is too coarse to assess the role of the magnetic fields at scales smaller than the core size of ∼0.1 pc.

It is well known that not only supersonic turbulence but also the magnetic fields prevent interstellar gas from collapsing due to its self-gravity. Applying the Chandrasekhar & Fermi method (Chandrasekhar & Fermi 1953a), Poidevin & Bastien (2006) estimated the magnetic field strength in the plane of the sky, \( |\mathbf{B}_{\text{pos}}| \). They derived \( |\mathbf{B}_{\text{pos}}| \) of 170 ± 56 \( \mu \)G for the "core," which was observed by the CS (2−1) emission (Ciardi et al. 2000). To calculate \( |\mathbf{B}_{\text{pos}}| \), they used Equation (4) in Crutcher (2004) of \( |\mathbf{B}_{\text{pos}}| \approx 9.3 \frac{\sqrt{\pi}}{3} (A_{\text{FWHM}}/\delta \phi) \mu \text{G} \), where \( \delta \phi \) is the dispersion in polarization angles in degrees. Since we have the revised mean velocity width of \( \Delta v_{\text{FWHM}} = 0.69 \pm 0.12 \text{ km s}^{-1} \) (Section 4.3) and the number density of \( n_\text{e}(\text{H}_2) = 800–4200 \text{ cm}^{-3} \) (Section 5.2), we recalculated \( |\mathbf{B}_{\text{pos}}| \) of 55 ± 30 \( \mu \)G. Here we assumed that the \( \delta \phi \) value of 5:9 derived from the infrared data toward the CS (2−1) core (Poidevin & Bastien 2006) holds in the region we observed.

With the magnetic field strength, we estimate the magnetic pressure of \( \log P_{\text{mag}} = \log (B^2/8\pi) = -10.6 \) to ∼9.5, which is larger than the effective internal pressure of the filament given by \( \log P_{\text{eff}} = \log (\rho c_s^2 + 2\pi B^2/8\pi) = -11.3 \) to ∼10.6. Considering the ratio of \( P_{\text{eff}}/P_{\text{mag}} = 0.03–0.2 \) (Table 1), we speculate that the significant magnetic pressure can support the filament against the axial fragmentation.

Subsequently, we examine whether or not the large-scale magnetic fields can support the GF 9-2 core through a comparison of

\[
\frac{M}{\Phi} \text{ [GF 9-2]} \geq \frac{M_{\text{LTE}}}{\Phi_{\text{LTE}}} = \frac{M_0}{R_0} \frac{R_0}{R_0} \left( \frac{0.34}{10^{-9} G} \right) \left( \frac{100 \text{ pc}}{d} \right)^{1/2} \approx \left( 4.8 \times 10^9 \right)^{3/2} \frac{2900}{d} \sim 2^{-3},
\]

where the denominator is the critical mass-to-magnetic-flux ratio for a uniform spherical cloud, and the numerator is the ratio for the GF 9-2 dense core traced by the \( \text{H_2}CO^+ \) and \( \text{N}_2\text{H}^+ \) lines (the \( M_{\text{LTE}} \) and \( R_{\text{LTE}} \) values are taken from Table 8 in Paper I), which exists in the large-scale magnetic fields (Figure 16). Although the non-dimensional coefficient in the relation of \( |M/\Phi| \propto \sqrt{1/G} \) differs in various models (see, e.g., Lang 1980; Shu 1992; Lequeux 2005; Bodenheimer 2011, and references therein), such a difference is negligible compared to the uncertainty in the estimate of the magnetic field strength (Section 5.4) and that in the \( M_{\text{LTE}} \) estimate due to the uncertain fractional abundances of the tracers. The above comparison suggests that the GF 9-2 core is in a magnetically super-critical state, which does not contradict the fact that the core is dynamically collapsing because no magnetic fields can halt such collapse once it has begun.

5.5. Core Formation in the Filament

The discussion so far indicates that the natal gas of the GF 9-2 core is supported by the turbulent and magnetic pressures against its self-gravity. In Table 1, we summarize the derived physical properties of the natal filament gas. Recall that the central 30′ region of the dense core, corresponding to a diameter of ∼0.06 pc at \( d = 200 \text{ pc} \), is dynamically infalling onto the protostar (Papers I and III). These facts reinforce our assertion that a dynamically collapsing core formed in a gravitationally stable filament. Such a scenario immediately raises the following questions: How have the turbulence and the magnetic fields decayed locally at the spatial scale of the core? Can such dissipation determine the initial conditions of the core collapse? The answer to the former may be that such a spatial scale was set by the Jeans length of the tenuous gas. As for the latter, we require that the dissipation timescales of the supersonic turbulence (\( t_{\text{disp}} \)) and the magnetic fields (\( t_{\text{mag}} \)) are less than the free-fall time (\( t_f \)) of the gas traced by the \( \text{^{13}CO} \) line. After addressing these issues, we discuss the nature of the southwestern condensation.
5.5.1. The GF 9-2 Core: An Unstable Core in the Stable Filament

Theoretical studies have predicted the presence of gravitationally unstable cores in a stable filament. For instance, 3D simulations by Klessen et al. (2000, 2005) showed that density enhancements caused by strong shocks can produce gravitationally unstable dense cores, despite the fact that parental clouds are being prevented from the global collapse because of the turbulent support. In other words, a local collapse can occur even in the turbulent-supported self-gravitating global medium (Klessen et al. 2000). Although their calculations did not include the effect of magnetic fields, their results are reconciled with the previous low-resolution two-dimensional calculations considering magnetic fields (Vázquez-Semadeni et al. 1996). Subsequent numerical simulations (e.g., Mac Low 1999; Ostriker et al. 2001; Vázquez-Semadeni et al. 2005; Heitsch et al. 2009) with magnetic fields claim that the core formation must have completed within the global free-fall time of the natal cloud gas. A recent theoretical study by Leão et al. (2013) demonstrated that magnetic flux can be quickly removed from a cloud core by magnetic reconnection diffusion, which removes the magnetic flux through reconnection of the field lines by the co-existing turbulence (not by ambiglobal diffusion) with a timescale as short as \( t_{ff} \).

Contrary to these theoretical predictions, we propose a simple scenario that the supersonic turbulence and the magnetic fields locally decay at the spatial scale comparable to the Jeans length of the filament gas: \( \lambda_J \sim 0.3–0.6 \) pc for \( t_{ff} = 7.5 \) K and \( n_{H_2} = 800–4200 \) cm\(^{-3}\) (Section 5.3.1). Once the dissipation of the turbulence and the magnetic fields had occurred locally over a region with a size scale of the Jeans length, the region lost the support against self-gravity and contracted into a compact gas clump with 0.1 pc size, i.e., a typical size of a low-mass star-forming cloud core, by increasing its density as high as \( \sim (800–4200) \) cm\(^{-3}\) \( \times \lambda_J / 0.1 \) pc \( \sim (2–91) \times 10^4 \) cm\(^{-3}\). With this number density, the NH\(_3\) (1,1) lines (critical density of \( n_{crit} \sim 10^4 \) cm\(^{-3}\)) and the H\(^{13}\)CO\(^+\) (1–0) and N\(_2\)H\(^+\) (1–0) transitions (\( n_{crit} \sim 10^5 \) cm\(^{-3}\)) can be collisionally excited.

Because the latter higher density tracers were detected toward the GF 9-2 core, the above argument may favor the larger side of \( \lambda_J \sim 0.6 \) pc. An alternative idea is that the core formed as a result of internal shock in the filament. However, this is unlikely because the shock compression with a Mach number (\( M \)) of \( \sim 2 \) (Section 5.1) would increase the gas density by a factor of \( M^2 \) in the case of the J-type shock (Spitzer 1978), i.e., up to at most \( \sim (800–4200) \) cm\(^{-3}\) \( \times 2^2 < 2 \times 10^4 \) cm\(^{-3}\). Clearly this is lower than the critical densities of the N\(_2\)H\(^+\) and H\(^{13}\)CO\(^+\) lines.

Next, we examine whether the relation of \( t_{turb} \lesssim t_{ff} \) holds or not. Assuming that the core formed through the local dissipation of the supersonic turbulence with a spatial scale of \( \lambda_J \sim 0.3–0.6 \) pc, the dissipation timescale is given by

\[
t_{turb} \sim \frac{\lambda_J}{\sigma_{n_{crit}}} \sim 10^6 \left( \frac{\lambda_J}{0.3–0.6 \text{ pc}} \right) \left( \frac{\sigma_{n_{crit}}}{0.34 \text{ km s}^{-1}} \right)^{-1} \text{yr.} \tag{9}
\]

for the natal Component 1 gas. The other timescale to assess the core formation process is the free-fall time of

\[
t_{ff} = \sqrt{\frac{3\pi}{32G\rho}} \sim 10^6 \left( \frac{n_{H_2}}{800–4200 \text{ cm}^{-3}} \right)^{-1/2} \text{yr.} \tag{10}
\]

The two timescales are comparable to each other within large uncertainties, hence we conclude that \( t_{disp} \sim t_{ff} \) holds.

Last, we attempt to give some constraints on the dissipation timescale of the magnetic fields \( t_{disp} \). The upper limit of \( t_{disp} \) may be estimated by the ambiglobal diffusion timescale \( t_{AD} \) for a spherical cloud with a radius of \( R \sim \lambda_J/2 \) assuming a typical ionization degree in the ISM (see, e.g., Equation (10.5) in Stahler & Palla 2005 and Equation (2.51) in Bodenheimer 2011) as

\[
t_{disp} \lesssim t_{AD} \sim 10^5–10^6 \left( \frac{n_{H_2}}{800–4200 \text{ cm}^{-3}} \right)^{1/2} \times \left( \frac{B}{55 \pm 30 \mu G} \right)^{-2} \left( \frac{R}{0.2–0.3 \text{ pc}} \right)^2 \text{yr.} \tag{11}
\]

The lower limit of \( t_{disp} \) is given by the timescale of the magnetic reconnection diffusion \( t_{MRD} \), which should be comparable to or less than \( t_{ff} \) (Leão et al. 2013). Hence, we may have a very robust constraint of \( t_{ff} \lesssim t_{disp} \lesssim 100 t_{ff} \), which does include the “slow mode” of star formation (Section 1).

We therefore discuss that the formation of the GF 9-2 core was triggered by the local dissipation(s) of the supersonic turbulence and probably the magnetic fields with the timescale comparable to \( t_{ff} \), i.e., the “fast mode” of star formation (Section 1). Namely, this scenario naturally produces an unstable core collapsing in a runaway fashion.

5.5.2. The Southwestern Condensation with Velocity Width Enhancement: A “Protocore” Prior To a Dense Core?

We discuss the nature of another well-defined object discovered by our observations, i.e., the southwestern condensation where both the temperature and velocity width of the gas are enhanced. Neither indirect evidence for star formation, such as the presence of a pointlike infrared source, masers, radio continuum emission, and a molecular outflow, nor emission of dense gas tracers, such as NH\(_3\) (1,1) lines, has been reported toward the region (Ciardi et al. 1998, 2000; Poidevin & Bastien 2006; Paper II). Hence, the C\(^{18}\)O (1–0) emission shown in Figure 2(c) is the densest gas tracer detected toward the region. As mentioned above, we reject the possibility that the condensation represents a molecular outflow lobe driven by the protostar embedded in the GF 9-2 core for the following reasons. First, we did not detect any high velocity wing emission in the \(^{12}\)CO spectrum toward the condensation (see Figure 5(b)), which is evidence for a well-developed outflow. Second, our recent interferometric observations of the \(^{13}\)CO (3–2) line with the SMA clearly demonstrated the presence of a compact \((\sim 5 \times 10^{-3} \) pc\) molecular outflow driven by the 3 mm continuum source in the GF 9-2 core (Furuya et al. 2014). Therefore, we argue that the southwestern condensation may be a precursor of a dense cloud core, i.e., protocore.

In order to verify such an interpretation, we need to assess the dynamical state of the condensation, hence we estimated the spatial extent of the condensation using the results from Figures 11 and 12. With the 90% percentile of \( N_{H_2} = 3.9 \times 10^{21} \) cm\(^{-2}\) in Figure 12 a closed contour is uniquely defined for the condensation. In Figure 11, the 70% percentile of \( N_{H_2} = 3.6 \times 10^{21} \) cm\(^{-2}\) encloses both the GF 9-2 core and the condensation by a single contour. We therefore judged that these contours should give us a robust size range, which is presented by an effective radius of \( r_{eff} = 0.09–0.14 \) pc. Here we used \( r_{eff} = \sqrt{A/\pi} \), where \( A \) is the area enclosed by the contour. We obtained a column density range of \((3.6–3.9) \times 10^{21} \) cm\(^{-2}\), leading to an \( M_{LTE} \) range of \( 1.9–4.3 \) M\(_{\odot}\) with the fixed \( \lambda(13\text{CO}) \).
value. We also estimated that $\Delta v_{\text{FWHM}}$ ranges 1.1–1.2 km s$^{-1}$ from Figure 10(c) (corresponding to the velocity dispersion range of 0.47–0.50 km s$^{-1}$), yielding $\sigma_{\text{nth}}/(c_s) \sim 3$. These quantities lead to an energy balance based on the Virial theorem between the supersonic turbulence and the self-gravity for a uniform spherical cloud of

$$\frac{E_{\text{turb}}}{|E_{\text{grav}}|} = \frac{3}{5} \frac{M_{\text{LTE}} \sigma^2}{c_s^2} = 1–13, \quad (12)$$

suggesting that the condensation may be dispersed by the turbulence. It should be noted that if we include the uncertainty in the X$^{13}$CO (a factor of three) for the $M_{\text{LTE}}$ estimate (Section 4.4), the possibility that the condensation is gravitationally bound is not completely ruled out. On the other hand, the southwestern condensation seems to be in a magnetically subcritical state because of

$$\frac{|\vec{M}|_{\text{protocore}}}{|\vec{M}|_{\text{critical}}} = \frac{400–5000}{2900} = 0.1–2, \quad (13)$$

using Equation (8) for the above $M_{\text{LTE}}$ and $r_{\text{eff}}$ ranges.

If the southwestern condensation is not gravitationally bound, it must be a transient object that will eventually disperse by the turbulence with the timescale of $2r_{\text{eff}}/\sigma \sim 10^5$ yr. We speculate that it might be a “failed core” in Vázquez-Semadeni et al. (2005), a by-product from core formation. Alternatively, if the magnetic pressure overcomes the turbulent one, the magnetic fields should sustain the condensation.

If the condensation is gravitationally bound, it may be a precursor of a low-mass star-forming dense core, protocore, which is at an evolutionary phase before the dissipation of the supersonic turbulence (see Figure 17). More specifically, we consider that the evolutionary phase of this object is prior to the coherent core phase, which is characterized by $\sigma_{\text{nth}}/(c_s) \lesssim 1$ (Goodman et al. 1998; Barranco & Goodman 1998; Caselli et al. 2002; Pineda et al. 2010). Such a protocore interpretation is consistent with the following two facts. First, the object is two times larger than that of a typical low-mass star-forming core. If the object contracts to half its size, its mean density would increase up to $\sim 10^4$ cm$^{-3}$ which can be traced by, e.g., collisionally excited NH$_3$ (1,1) emission. Second, the object is massive ($\sim 5 M_\odot$) enough to evolve toward a typical cloud core, regardless of an $\eta_{\text{LTE}}$ value (Section 5.3.2). Since the GF 9 filament is likely gravitationally stable (Sections 5.2 and 5.3), we argue that the formation of the protocore was triggered by a collision between Components 1 and 2. In this scenario, the velocity width enhancement $\Delta v_{\text{FWHM}}$ is interpreted as the sum of the turbulent motions in the two components and the radial velocity difference between them. The difference is estimated to $\Delta v_{\text{FWHM}} \sim 1$ km s$^{-1}$ from Figures 5(b) and 10(b), suggesting a weak collision with $M \sim 2$–3. It is a non-trivial issue to estimate analytically how much the gas temperature is raised (Section 4.2) due to the weak collision (it is possibly a C-type shock). Nevertheless, we believe that the cloud–cloud collision scenario reasonably explains the observed enhancements of the gas temperature and velocity width. Similar to the case of the GF 9-2 core (Section 5.5.1), an unstable core is anticipated to form under the conditions with a collision timescale of $t_{\text{coll}} \sim 2r_{\text{eff}}/\Delta v_{\text{FWHM}} \sim 10^5$ yr, which is comparable to $t_{\text{disp}} \sim t_{\text{mag}}$ of $10^5$ yr for the condensation gas with $n$(H$_2$) of $2 \times 10^3–10^4$ cm$^{-3}$.

6. SUMMARY

Analyzing the CO isotopologue data taken with the Nobeyama 45 m telescope, we have studied the physical properties of the low density ($\sim 10^3$ cm$^{-3}$) filament gas surrounding the GF 9-2 dense cloud core, which harbors the exceptionally young protostar. Our main results are summarized as follows.

1. The $^{13}$CO map covering a $\sim 0.78 \times 0.78$ pc$^2$ square field centered on the core clearly shows the filamentary morphology. The C$^{18}$O map demonstrates that the GF 9-2 core formed at the local intensity maxima of the filament. Solving the optical depths and excitation temperatures of the CO isotopologue lines, we obtained the opacity-free $^{13}$CO spectra that were used to produce spectral momentum maps.

2. On the basis of the spatial and velocity structures of the filament gas, we identified two gas components, one of which is the natal filament of the GF 9-2 dense core (Component 1). The centroid velocity map shows that Component 1 is confined in a rather narrow LSR-velocity range of $\sim 0.6$ km s$^{-1}$ and does not show any systematic motions. In contrast, the velocity width is enhanced up to $\Delta v_{\text{FWHM}} = 1.4$ km s$^{-1}$ to the southwest of the core. A mean velocity dispersion of the southwestern region is $\sigma = 0.47$ km s$^{-1}$.

3. Using the optical depth, excitation temperature, and velocity width maps, we calculated the H$_2$ column density. The column density map clearly traces the natal filament of the GF 9-2 core, and the core is located at the local column density peak in the filament. Furthermore, the column density histogram is well described by a log-normal function, suggesting that the supersonic turbulence governs the density structure of the filament.

4. The mean temperature of the gas in the observed area is 7.5 K with a standard deviation of 1.0 K. Subtracting the contributions of thermal gas motions from the $\Delta v_{\text{FWHM}}$ map, we made a ratio map between the non-thermal velocity dispersion and the local sound speed $\sigma_{\text{nth}}/(c_s)$. The map clearly demonstrates that the natal tenuous filament is in a supersonic turbulent state whose Mach number ranges from 0.96 to 3.4 with a mean of 2.1.

5. Considering the turbulent pressure into a model of an isothermal cylinder in hydrostatic equilibrium, we assessed the dynamical stability of the filament. The maximum mass that can be radially supported by the internal pressures (predominantly the turbulent pressure) is estimated to be $M_{\text{eff}} = 51^{+32}_{-12} M_\odot$. On the other hand, we obtained the LTE mass of 24 $\pm 10$ $M_\odot$ for the filament, suggestive of a gravitationally stable state against radial collapse. Furthermore, analyzing the column density map on the basis of the isothermal cylinder model, we estimated a scale height of the filament to be $H = 0.3$–0.7 pc, yielding the central number density of $n_1$(H$_2$) = 800–4200 cm$^{-3}$. Since the NH$_3$ core masses in the filament are smaller than those expected in the axial unstable modes, the filament is likely to be gravitationally stable against axial fragmentation as well.

6. With the $\langle \sigma_{\text{nth}} \rangle$ of 0.34 $\pm$ 0.80 km s$^{-1}$ and the $n_1$(H$_2$) = 800–4200 cm$^{-3}$, we recalculated the strength of the large-scale well-aligned magnetic fields in the natal filament gas to be $|B| = 55 \pm 30 \mu$G by following the Chandrasekhar & Fermi method in Poidievyn & Bastien (2006).

7. The transverse magnetic field can support the filament against the axial collapse, because the magnetic pressure
appears to exceed the internal (thermal and turbulent) gas pressure. In contrast, the GF 9-2 core is in a magnetically super critical state through a comparison of the mass-to-magnetic-flux ratio of the core with the theoretical critical value. This inference agrees with the previous results that the core is dynamically collapsing.

8. The dissipation time of the supersonic turbulence is comparable to the free-fall time of the natal tenuous gas of $10^6$ yr within the uncertainties. Although the dissipation time of the magnetic fields has large uncertainties, we consider that the local dissipation(s) of the turbulence and the magnetic fields made a part of the filament gas unstable, resulting in the formation of the gravitationally unstable GF 9-2 core.

9. The southwestern condensation, where the gas temperature and velocity width are enhanced, may be a precursor of a low-mass star-forming cloud core, protocore, if it is gravitationally bound. The formation of the southwestern protocore must have been triggered by a collision between the two gas components. Because of $\langle \sigma_{nth}/c_s \rangle \sim 3$, the evolutionary stage of the southwestern protocore is highly likely before the phase of the coherent dense core, which is characterized by $\langle \sigma_{nth}/c_s \rangle \lesssim 1$ (Goodman et al. 1998).

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Facility: No: 45 m

APPENDIX A

OPTICAL DEPTH AND EXCITATION TEMPERATURE OF THE $^{13}$CO $J = 1-0$ LINE EMISSION

A.1. Radiative Transfer Equation

The main beam brightness temperature ($T_{mb}$) of molecular line emission emanated from a homogeneous gas cloud is written by

$$ T_{mb}(v) = f [J_v(T_{ex}) - J_v(T_{bg})] [1 - \exp(-\tau(v))], $$

(A1)

where $\tau(v)$ is the optical depth of the line as a function of radial velocity, $v$, $f$ is the beam filling factor, $T_{ex}$ is the excitation temperature of the line, and $T_{bg}$ is the temperature of the cosmic background radiation. The function $J_v(T)$ is the radiation temperature defined by $(h \nu/k)(1/\exp(h \nu/kT) - 1)$, where $\nu$ is the frequency of the line, $T$ is the gas temperature, $k$ is the Boltzmann constant, and $h$ is the Planck constant.

A.2. Estimate of Optical Depth and Excitation Temperature

The optical depth and excitation temperature of the $^{13}$CO (1–0) line can be estimated by applying Equation (A1) to a set of CO isotopologue spectra, assuming the abundance ratios between $^{12}$CO and $^{13}$CO and between $^{12}$CO and $^{13}$CO are explicitly assumed. For simplicity, we assumed that the $^{12}$CO (1–0) line at a rest frequency of 115.271202 GHz, the $^{13}$CO (1–0) line at 110.201353 GHz, and the $^{13}$CO (1–0) line at 109.782173 GHz are excited with a common $T_{ex}$ at each spatial position; we also assumed $J_{115GHz}(T) = J_{110GHz}(T)$ and $f = 1.0$. Adopting the solar abundance ratios of $\alpha \equiv [^{12}$CO]/[$^{13}$CO] = 89 and
\[ \beta \equiv \frac{^{13}\text{CO}}{\text{C}^{18}\text{O}} = 5.5 \text{ (e.g., Lang 1980; Garden et al. 1991; Lequeux 2005)} , \]

we solved the following equations for \( \tau \) and a quantity of \( A \) defined by \( J_{110\text{GHz}}(T_{\text{ex}}) - J_{110\text{GHz}}(T_{\text{bg}}) \) at each pixel in the 3D space of right ascension offset (\( \Delta \alpha \)), declination offset (\( \Delta \delta \)), and LSR-velocity (\( V_{\text{LSR}} \)):

\[
\begin{align*}
T_{12} &= A(1 - e^{-\alpha \tau}), \\
T_{13} &= A(1 - e^{-\beta \tau}), \\
T_{18} &= A(1 - e^{-\tau/\beta}),
\end{align*}
\] (A2)

where \( T_{12}, T_{13}, \) and \( T_{18} \) are the main beam brightness temperatures of the \(^{12}\text{CO},^{13}\text{CO}, \) and \( \text{C}^{18}\text{O} \) isotopologue lines, respectively. As seen above, we adopted detection threshold of 3 \( \sigma \), where \( \sigma \) denotes the rms noise levels in \( T_{\text{mb}} \) for the \(^{12}\text{CO} \) and \( \text{C}^{18}\text{O} \) lines, respectively. As seen above, we adopted detection threshold of 3 \( \sigma \). We did not consider the case that only the \(^{12}\text{CO} \) line is detected, as we cannot define an intensity ratio.

Case (1) is the category where both the \(^{12}\text{CO} \) and \(^{13}\text{CO} \) lines are detected with \( T_{12} > T_{13} \), but the \( \text{C}^{18}\text{O} \) line does not show significant emission. Case (2) corresponds to the category that both the \(^{13}\text{CO} \) and \( \text{C}^{18}\text{O} \) lines are detected with \( T_{13} > T_{18} \), but the \(^{12}\text{CO} \) line is too strong or is not detected. Case (2) is often found near the velocity channels where the \(^{12}\text{CO} \) line suffers self-absorption and the \(^{13}\text{CO} \) one shows intense emission (see Figure 18(a)). For Cases (1) and (2), we numerically solved the following equations for \( \tau \):

\[
\begin{align*}
R_{12/13} &= \frac{1 - e^{-\alpha \tau}}{1 - e^{-\tau/\beta}}, \text{ for Case (1)}, \\
R_{13/18} &= \frac{1 - e^{-\tau/\beta}}{1 - e^{-\tau/\beta}}, \text{ for Case (2)},
\end{align*}
\] (A4)

We employed the bisection method with an accuracy of 0.001 and performed about 10 repetitions for many cases, as expected from 1/2^{10} to 0.001. After obtaining \( \tau \), we calculated \( T_{\text{ex}} \) using Equation (A2). In the 3D space of \( (\Delta \alpha, \Delta \delta, V_{\text{LSR}}) \) where we have a total of 72 \( \times \) 72 \( \times \) 150 = 7,775 \( \times \) 10^5 points, we obtained solutions at 34668 points for Case (1) and 784 points for Case (2), which correspond to 4155 and 632 spatial positions in the \( (\Delta \alpha, \Delta \delta) \) coordinates, respectively.

Case (3) is a category where all three lines are detected with an intensity order of \( T_{12} > T_{13} > T_{18} \), allowing us to utilize the maximum likelihood method on the basis of \( \chi^2 \). We searched the best-fit values for \( \tau \) and A by minimizing the \( \chi^2 \) value defined by

\[
\chi^2 = \sum_{i = 12, 13, 18} \left( \frac{T_i - T_{\text{model}}}{\Delta T_{\text{rms}}} \right)^2,
\] (A5)

where \( T_{\text{model}} = A(1 - e^{-\alpha \tau}), T_{13} = A(1 - e^{-\tau/\beta}), \) and \( T_{18} = A(1 - e^{-\tau/\beta}) \). In order to give better initial guesses for the likelihood method analysis, we solved Equation (A4) for \( \tau \) and A by means of the bisection method before performing \( \chi^2 \)-fitting. We calculated the \(^{13}\text{CO} \) optical depths from both \( R_{12/13} \) and \( R_{13/18} \), and adopted the mean value between them as the center of the “searching area” in the \( \Lambda - \tau \) plane (see Figure 19) for finding the minimum \( \chi^2 \) value, \( \chi_{\text{min}}^2 \). In the \( (\Delta \alpha, \Delta \delta, V_{\text{LSR}}) \) space, there are 449 points that satisfy the criteria for Case (3), corresponding to 328 positions in the \( (\Delta \alpha, \Delta \delta) \) space. For the 449 points we calculated that the \( \chi_{\text{min}}^2 \) values have a minimum of 0.10, a maximum of 8.0, a mean of 0.69, a standard deviation of 1.0, and a median of 0.29. Since the obtained \( \chi_{\text{min}}^2 \) is distributed around unity, our estimates of \( \tau \) and A are considered to be reasonable.

Case (4) corresponds to the data sets where either the \(^{12}\text{CO} \) or/and \(^{13}\text{CO} \) line shows self-absorption or their intensity ratios are inconsistent with those expected from the given abundance ratios. We identified 20 points in the 3D space for such a
case, which may be negligible compared to the numbers of the solutions obtained in the above three cases because it corresponds to 1.7% of all the analyzed data points. Eleven out of the 20 points are found either at the boundary LSR-velocity between Components 1 and 2, $V_{\text{LSR}} = -2.2$ km s$^{-1}$ defined in Section 3, or at the adjacent channels. Since such anomaly ratios do not allow us to estimate $\tau$ and $T_{\text{ex}}$, we estimated $\tau$ from the observed $T_{13}$ value using an empirical relation between $\tau$ and $T_{13}$ obtained from the Case (3) analysis: $T_{13} = (3.95 \pm 0.02)[1 - \exp(-1.68 \pm 0.03)\tau]$ for $\tau \geq 0.1$. After obtaining $\tau$ by this way, we calculated $T_{\text{ex}}$ using Equation (A2).

In summary, we obtained a set of ($\tau$, $T_{\text{ex}}$) values at a total of 36,622 points in the ($\Delta\tau$, $\Delta T_{\text{ex}}$, $V_{\text{LSR}}$) space; these are 34,668 points from Case (1), 784 points from Case (2), 1,154 points from Case (3), and 20 points from Case (4).

A.3. Error Estimates for the Optical Depth and Excitation Temperature

In this subsection, we describe error estimates for the optical depth and excitation temperature of the $^{13}$CO emission. Figures 20 and 21 show observed velocity channel maps of the uncertainties in the $^{13}$CO optical depth and excitation temperature, respectively. Comparing Figure 7 with Figure 20 and comparing Figure 8 with Figure 21, we found that the resultant uncertainties in the $\tau$ and $T_{\text{ex}}$ are 15% with respect to their values.

A.3.1. Error Estimates for Cases (1) and (2)

Since the $^{13}$CO optical depth in Equation (A4) cannot be explicitly solved, we estimated the uncertainty in $\tau$ of $\Delta\tau_{\text{rms}}$ as follows. In Case (2), for example, the ratio $R_{13/18}$ is considered to be a function of $\tau$, and thus the following equation holds:

$$(\Delta R_{13/18}^{\text{rms}})^2 = \left(\frac{\partial R_{13/18}}{\partial \tau}\right)^2 (\Delta\tau_{\text{rms}})^2.$$  \hspace{1cm} (A6)

Furthermore, recall that the ratio $R_{13/18}$ is defined by $R_{13/18} = T_{13}/T_{18}$, where the two temperatures of $T_{13}$ and $T_{18}$ have the uncertainties of $\Delta T_{13}^{\text{rms}}$ and $\Delta T_{18}^{\text{rms}}$, respectively. Therefore, we have

$$(\Delta R_{13/18}^{\text{rms}})^2 = \left(\frac{\partial R_{13/18}}{\partial T_{13}}\right)^2 (\Delta T_{13}^{\text{rms}})^2 + \left(\frac{\partial R_{13/18}}{\partial T_{18}}\right)^2 (\Delta T_{18}^{\text{rms}})^2.$$  \hspace{1cm} (A7)

Here the covariance, $(\Delta T_{13}^{\text{rms}} \cdot \Delta T_{18}^{\text{rms}})$, was set to be zero because the two variables are independent of each other. Combining Equations (A6) and (A7), we can write the desired $\Delta\tau_{\text{rms}}$ by

$$(\Delta\tau_{\text{rms}})^2 = \left(\frac{\partial R_{13/18}}{\partial \tau}\right)^2 \left(\left(\frac{\partial R_{13/18}}{\partial T_{13}}\right)^2 (\Delta T_{13}^{\text{rms}})^2 + \left(\frac{\partial R_{13/18}}{\partial T_{18}}\right)^2 (\Delta T_{18}^{\text{rms}})^2\right),$$  \hspace{1cm} (A8)

where $(\partial/\partial\tau)R_{13/18} = (e^{-\tau}/1 - \exp(-\tau/\beta)) - (e^{-\tau}/\beta(1 - \exp(-\tau/\beta)))$, $(\partial/\partial T_{13})R_{13/18} = (1/T_{18})$, and $(\partial/\partial T_{18})R_{13/18} = -(T_{13}/T_{18}^2)$. After obtaining $\Delta\tau_{\text{rms}}$, we subsequently calculated the uncertainty in $T_{\text{ex}}$ of $\Delta T_{\text{ex}}^{\text{rms}}$ through the relationship from Equation (A2) as follows:

$$(T_{13} - T_{\text{ex}}^{\text{model}})^2 = \left(\frac{\partial T_{13}}{\partial \tau}\right)^2 (\Delta\tau_{\text{rms}})^2 + \left(\frac{\partial T_{13}}{\partial T_{\text{ex}}}\right)^2 (\Delta T_{\text{ex}}^{\text{rms}})^2,$$  \hspace{1cm} (A9)

where $(\partial/\partial\tau)T_{13} = A e^{-\tau}$ and $(\partial/\partial T_{\text{ex}})T_{13} = (h/v/kT_{\text{ex}}) \exp(h/v/kT_{\text{ex}})[1 - \exp(-\tau)]/[\exp(h/v/kT_{\text{ex}}) - 1]^2$. Replacing $\Delta\tau_{\text{rms}}$ with Equation (A8), the desired $\Delta T_{\text{ex}}^{\text{rms}}$ can be explicitly written as a function of $T_{13}$, $T_{18}$, $\Delta T_{13}^{\text{rms}}$, $\Delta T_{18}^{\text{rms}}$, $T_{\text{ex}}$, and $\tau$.

A.3.2. Error Estimates for Case (3)

For Case (3) where we performed the maximum likelihood analysis, the uncertainties of the best-fit parameters, i.e., the 68.3% confidence intervals for the two “parameters of interest,” are given by the projections onto the $\tau$ and $A$ axes of the “confidence region ellipses”, where the function $\chi^2(\tau, A)$ takes a value of $\chi^2_{\text{min}} + 2.30$ (Press et al. 2007, p. 815). Figure 19 presents an example showing such an error analysis at the $V_{\text{LSR}} = -2.6$ km s$^{-1}$ channel of the spectra shown in Figure 18(a). After numerically obtaining $\Delta A_{\text{rms}}$, we calculated $\Delta T_{\text{ex}}^{\text{rms}}$ through $(\Delta A_{\text{rms}})^2 = (\partial A/\partial T_{\text{ex}})^2 (\Delta T_{\text{ex}}^{\text{rms}})^2$, where $(\partial A/\partial T_{\text{ex}}) = (h/v/kT_{\text{ex}}) \exp(h/v/kT_{\text{ex}})[1 - \exp(-\tau)]/[\exp(h/v/kT_{\text{ex}}) - 1]^2$.

A.4. Error Estimates for Optical-depth-corrected Brightness Temperature

As described in Section 4, the optical-depth-corrected main beam brightness temperature, $T_{\text{mb}}^{\text{corr}}$, is given by

$$T_{\text{mb}}^{\text{corr}} = \frac{\tau T_{\text{mb}}}{1 - e^{-\tau}}.$$  \hspace{1cm} (A10)

The uncertainty, $\Delta T_{\text{mb}}^{\text{corr}}$, at each velocity channel is calculated by

$$\Delta T_{\text{mb}}^{\text{corr}} = \left(\frac{\partial T_{\text{mb}}^{\text{corr}}}{\partial \tau}\right) \Delta\tau_{\text{rms}} + \left(\frac{\partial T_{\text{mb}}^{\text{corr}}}{\partial \tau}\right) \Delta T_{\text{rms}}.$$  \hspace{1cm} (A11)
Figure 20. Velocity channel maps of the errors of the $^{13}$CO optical depth maps shown in Figure 7. The central LSR-velocity of each velocity channel in km s$^{-1}$ is shown at the top left corner of each panel. See Appendix A.3 for details.

(A color version of this figure is available in the online journal.)

Figure 21. Velocity channel maps of the errors of the $^{13}$CO excitation temperature maps shown in Figure 8. Notice that the error is shown in linear scale. See Appendix A.3 for details.

(A color version of this figure is available in the online journal.)
where \((\partial T_{\text{corr}}/\partial \tau) = (1/1 - e^{-\tau}) + (r e^{-r}/(1 - e^{-r}))\) and \((\partial T_{\text{corr}}/\partial T_1) = (\tau/1 - e^{-\tau})\). Notice that the optical depth error in Equation (A11), \(\Delta \tau_{\text{rms}}\), is given by Equation (A8) for Cases (1) and (2), and it is given through the error analysis described in Appendix A.3.2 for Case (3). One can see that the mean \(\Delta T_{\text{corr}}\) values, which are obtained by averaging \(\Delta T_{\text{corr}}(v)\) values along the velocity axis at each map position, are enhanced toward the GF 9-2 dense cloud core. This is most likely because \(\Delta \tau_{\text{rms}}\) values showed relatively large ones toward the core (see Figure 20). The mean \(\langle \Delta T_{\text{corr}} \rangle \) all over the observed area was 1.5 K (median = 1.4) with standard deviation of 0.66 K.

APPENDIX B

ERROR ESTIMATES FOR SPECTRAL MOMENTA

In the following subsections, we present our error estimate for the spectral moment calculations described in Section 4.3. The resultant error maps are shown in Figure 22.

B.1. Error Estimates for the Zeroth Moment

The zeroth spectral moment along the velocity axis, corresponding to the integrated intensity in units of K km s\(^{-1}\), is defined by \(I = \int T(v)dv\) and is calculated by

\[
I = \sum_{i=1}^{N} T_{\text{mb},i} \Delta v
\]

(B1)

over \(N\) velocity channels where the emission exceeds the detection threshold. Here \(T_{\text{mb},i}\) is given by Equation (A10), and \(\Delta v\) denotes the width of each velocity channel. Given the definition, the error of the zeroth moment is written by

\[
\Delta I = \sqrt{\sum_{i=1}^{N} \left( \frac{\partial I}{\partial T_{\text{mb},i}} \right)^2 \langle \Delta T_{\text{mb},i} \rangle^2}
\]

(B2)

and is calculated as

\[
\Delta I = \sqrt{\langle \Delta v \rangle^2 \sum_{i=1}^{N} \langle \Delta T_{\text{mb},i} \rangle^2}. \]  

(B3)

Figure 22(a) presents the error map of the zeroth moment, which is similar to the total integrated intensity map (Figure 2). The mean \(\Delta I\) calculated over whole the region is 3.5 K km s\(^{-1}\) (median = 3.5 K km s\(^{-1}\)) with a standard deviation of 0.88 K km s\(^{-1}\). Comparing Figure 10(a) with Figure 22(a), the uncertainties are about 60% with respect to the total intensities.

B.2. Error Estimates for the First Moment

The first moment, which gives an intensity-weighted mean velocity, i.e., centroid velocity \((v_{\text{cent}})\), is defined by \(v_{\text{cent}} = (\int T(v)dv/\int T(v)dv)\). This should be calculated by

\[
v_{\text{cent}} = \frac{\sum_{i=1}^{N} v_i T_{\text{mb},i}}{\sum_{i=1}^{N} T_{\text{mb},i}}. \]  

(B4)

Therefore, the uncertainty of \(v_{\text{cent}}\) is given by

\[
\Delta v_{\text{cent}} = \left[ \sum_{i=1}^{N} \left( \frac{\partial v_{\text{cent}}}{\partial v_i} \right)^2 \Delta v_i \right] + \sum_{i=1}^{N} \left( \frac{\partial v_{\text{cent}}}{\partial T_{\text{mb},i}} \right)^2 \Delta T_{\text{mb},i}\right]. \]  

(B5)

Here \(\Delta v_i\) is the uncertainty in the LSR-velocity at the \(i\)th channel, which can be replaced by the velocity resolution of the spectrometer, \(\Delta v_{\text{res}}\). Since \(\Delta v_{\text{res}}\) is generally represented by FWHM of the window function of the spectrometer (\(\Delta v_{\text{FWHM}}\)), one has to divide it by \(\sqrt{8\ln2}\) to obtain the uncertainty in velocity, i.e., standard deviation. We thus obtained the equation below for computing the error associated with the centroid velocity as

\[
\Delta v_{\text{cent}} = \sqrt{\sum_{i=1}^{N} \left( \frac{T_{\text{mb}}}{I} \right)^2 \left[ \left( \frac{\Delta v_{\text{res}}}{\sqrt{8\ln2}} \right)^2 + \sum_{i=1}^{N} \left( \frac{\partial v_{\text{res}}}{I^2} \right)^2 \right] \langle \Delta T_{\text{corr}} \rangle^2}. \]  

(B6)

Figure 22(b) shows the error map of \(\Delta v_{\text{cent}}\); the mean value \(\langle \Delta v_{\text{cent}} \rangle\) over the observed region is 0.27 km s\(^{-1}\) (median = 0.26 km s\(^{-1}\)) with a standard deviation of 0.07 km s\(^{-1}\). The \(\Delta v_{\text{cent}}\) map seems fairly “flat” compared with that of the \(\Delta I\). This is probably because estimating velocity width is principally sensitive to the dual terminal LSR-velocities at each spectrum, whereas \(\Delta I\) is generally sensitive to the peak value of the spectrum.

B.3. Error Estimates for the Second Moment

The second moment, which corresponds to the intensity-weighted velocity dispersion, is defined by \(\sigma = \)
\[ \sqrt{\int T(v)(v - v_{\text{cen}})^2 dv} / \int T(v) dv. \]

In practice, we adopt a definition of
\[ \sigma = \sqrt{\frac{\sum_{i=1}^{N} (v_i - v_{\text{cen}})^2 T_{\text{mb},i}}{\sum_{i=1}^{N} T_{\text{mb},i}}}. \]  

(B7)

This leads to the uncertainty of the velocity dispersion as
\[ \Delta \sigma = \sqrt{\sum_{i=1}^{N} \left( \frac{\partial \sigma}{\partial v_i} \right)^2 \left( \frac{\Delta v_{\text{whm}}}{8 \ln 2} \right)^2 + \sum_{i=1}^{N} \left( \frac{\partial \sigma}{\partial T_{\text{mb},i}} \right)^2 \Delta T_{\text{mb},i}^2}, \]  

where \( \frac{\partial \sigma}{\partial v_i} = (1/\sigma I^2)(v_i - v_{\text{cen}})T_{\text{mb},i}^{-1} - \sum_{i=1}^{N} (v_i - v_{\text{cen}})(T_{\text{mb},i}^{-1})^2 \) and \( \frac{\partial \sigma}{\partial T_{\text{mb},i}} = (1/2\sigma I)(v_i - v_{\text{cen}})^2 - 3\sigma^2 \).

Figure 22(c) shows the \( \Delta \sigma \) map whose mean over the observed area, \( \langle \Delta \sigma \rangle \), is 0.44 km s\(^{-1} \) (median = 0.42 km s\(^{-1} \)) with a standard deviation of 0.13 km s\(^{-1} \). The \( \Delta \sigma \) map appears to be insensitive to the uncertainty in determination of the terminal LSR-velocities. This is probably because velocity dispersions are determined mainly over the velocity ranges having the 50% level intensity with respect to the peak.

APPENDIX C

CALCULATION OF THE \(^{13}\text{CO} \) COLUMN DENSITY AND ERROR ESTIMATE

The column density of \(^{13}\text{CO} \) molecules \( (N_{^{13}\text{CO}}) \) can be calculated from the total integrated intensity, i.e., the zeroth spectral moment, of the transition from the \( J + 1 \) to \( J \) state, where \( J \) is the rotational quantum number, by assuming LTE:
\[ N_{^{13}\text{CO}} = \frac{3k}{8\pi^3 \mu^2 B} \left( \frac{\langle T_{\text{ex}} \rangle + hB/3k}{J + 1} \right) \exp[hB(J + 1)/k(T_{\text{ex}})] \times \int T_{\text{mb}}(v) dv \left( \frac{J_{\text{ex}}(T_{\text{bg}})}{J_{\text{ex}}(T_{\text{ex}})} - J_{\text{bg}}(T_{\text{bg}}) \right). \]  

(C1)

where \( \mu \) is the dipole moment of \(^{13}\text{CO} \) (0.1101 D), \( B \) the rotational constant (55101.0138 MHz; see, e.g., Appendix B of Paper I, and references therein), and \( \langle T_{\text{ex}} \rangle \) is the mean excitation temperature averaged along the LSR-velocity axis at each pixel position (Figure 8).

Since Equation (C1) is a function of \( \langle T_{\text{ex}} \rangle \) and \( I(= \int T_{\text{mb}}(v) dv) \), its uncertainty is given by
\[ \Delta N_{^{13}\text{CO}} = \sqrt{\left( \frac{\partial N_{^{13}\text{CO}}}{\partial \langle T_{\text{ex}} \rangle} \right)^2 \Delta \langle T_{\text{ex}} \rangle^2 + \left( \frac{\partial N_{^{13}\text{CO}}}{\partial I} \right)^2 \Delta I^2}. \]  

(C2)

In Appendices A.2 and B.1, we showed that \( T_{\text{ex}} \) has a typical uncertainty of 15%, while \( I \) has a much larger uncertainty of 60%. Therefore, ignoring the first term, Equation (C2) can be approximated by
\[ \Delta N_{^{13}\text{CO}} \approx \frac{\partial N_{^{13}\text{CO}}}{\partial I} \Delta I. \]  

(C3)

Another method of estimating \( \Delta N_{^{13}\text{CO}} \) is as follows. Since Equation (C2) becomes a rather complicated formula given by a function of \( \langle T_{\text{ex}} \rangle \) and \( I \), we numerically estimated \( \Delta N_{^{13}\text{CO}} \) by changing \( \langle T_{\text{ex}} \rangle \) values by \( \pm \Delta \langle T_{\text{ex}} \rangle \) and \( I \) values by \( \pm \Delta I \) from the upper \( N_{^{13}\text{CO}}^{\text{upper}} \) and lower \( N_{^{13}\text{CO}}^{\text{lower}} \) values that were obtained by changing \( \langle T_{\text{ex}} \rangle \) values by \( \pm \Delta \langle T_{\text{ex}} \rangle \) and \( I \) values by \( \pm \Delta I \) in Equation (C1). We thus adopted
\[ \Delta N_{^{13}\text{CO}} \approx \frac{1}{2}(N_{^{13}\text{CO}}^{\text{upper}} - N_{^{13}\text{CO}}^{\text{lower}}). \]  

(C4)

The resultant \( \Delta N_{^{13}\text{CO}} \) maps produced from the two methods expressed by Equations (C3) and (C4) agree with each other within an uncertainty of ~2%. We therefore arbitrary adopted the map calculated by Equation (C4) (Figure 23), which is further used in the analysis discussed in Section 5.5. Figure 23 presents a summary of our uncertainty estimate. The resultant \( \Delta N_{H_2}/N_{H_2} \) map has a mean uncertainty of 68% with a standard deviation of 12%.

APPENDIX D

DATA SELECTION CRITERIA FOR PRODUCING THE MAPS OF MEAN EXCITATION TEMPERATURE, SPECTRAL MOMENTA, AND COLUMN DENSITY

As summarized in the end of Appendix A.2, we obtained the solutions at a total of 36,622 positions in the 3D space of \( (\Delta \alpha, \Delta \delta, V_{\text{LSR}}) \). Using the results from error calculations in Appendices A.3 and A.4, we checked whether or not the 36,622 solutions satisfy the following three conditions:
\[ \tau > \Delta \tau, \]
\[ T_{\text{ex}} > \Delta T_{\text{ex}}, \]  

(D1)

and
\[ T_{\text{mb}} > \Delta T_{\text{mb}}. \]  

Another method of estimating \( \Delta N_{^{13}\text{CO}} \) is as follows. Since Equation (C2) becomes a rather complicated formula given by a function of \( \langle T_{\text{ex}} \rangle \) and \( I \), we numerically estimated \( \Delta N_{^{13}\text{CO}} \) by changing \( \langle T_{\text{ex}} \rangle \) values by \( \pm \Delta \langle T_{\text{ex}} \rangle \) and \( I \) values by \( \pm \Delta I \) from the upper \( N_{^{13}\text{CO}}^{\text{upper}} \) and lower \( N_{^{13}\text{CO}}^{\text{lower}} \) values that were obtained by changing \( \langle T_{\text{ex}} \rangle \) values by \( \pm \Delta \langle T_{\text{ex}} \rangle \) and \( I \) values by \( \pm \Delta I \) in Equation (C1). We thus adopted
\[ \Delta N_{^{13}\text{CO}} \approx \frac{1}{2}(N_{^{13}\text{CO}}^{\text{upper}} - N_{^{13}\text{CO}}^{\text{lower}}). \]  

(C4)

The resultant \( \Delta N_{^{13}\text{CO}} \) maps produced from the two methods expressed by Equations (C3) and (C4) agree with each other within an uncertainty of ~2%. We therefore arbitrary adopted the map calculated by Equation (C4) (Figure 23), which is further used in the analysis discussed in Section 5.5. Figure 23 presents a summary of our uncertainty estimate. The resultant \( \Delta N_{H_2}/N_{H_2} \) map has a mean uncertainty of 68% with a standard deviation of 12%.

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\[ \tau > \Delta \tau, \]
\[ T_{\text{ex}} > \Delta T_{\text{ex}}, \]  

(D1)

and
\[ T_{\text{mb}} > \Delta T_{\text{mb}}. \]  

A total of 35,916 points in the 3D space (98.1%) passed the above test: 34,664 points from Case (1), 783 points from Case (2), and 449 points from Case (3), corresponding to 2923, 304, and 328 positions in the \( (\Delta \alpha, \Delta \delta) \) space. Furthermore, we discarded 21 points that do not satisfy a set of empirical
Here we accepted the 20 points obtained from Case (4), which are required to pass only the first and third conditions in Equation (D2). This is because we were unable to estimate \(\Delta\tau\) and \(\Delta T_{\text{ex}}\), which are required to pass only the first and third conditions in Case (4) (see Appendix A.2). Toward the central regions of the \(V_{1\text{SR}}\) space, we could not obtain reliable \(\tau\) and \(T_{\text{ex}}\) solutions owing to the three conditions in Equation (D2). Eventually, we obtained reliable solutions at a total of 3555 optical-depth-corrected \(^{13}\text{CO}\) spectra in the \(72 \times 72 = 5184\) observed positions. Here we accepted the 20 points obtained from Case (4), which are required to pass only the first and third conditions in Equation (D2). This is because we were unable to estimate the uncertainties of \(\Delta\tau\), \(\Delta T_{\text{ex}}\), and \(\Delta T_{\text{mb}}\) in Case (4) (see Appendix A.2). Toward the central regions of the \(V_{1\text{SR}} = -2.1\) and \(-1.9\ \text{km}\ \text{s}^{-1}\) panels in Figures 7 and 8, we could not obtain reliable \(\tau\) and \(T_{\text{ex}}\) solutions owing to the three conditions in Equation (D2).

Subsequently, we performed momentum analysis of the 3555 spectra, as described in Appendix B. For this purpose, one needs to define an LSR-velocity range over which the spectrum moments are calculated. We defined the range by two terminal velocities of \(v_t,\text{blue}\) and \(v_t,\text{red}\), where the \(T_{\text{mb}}^{\text{cent}}(v)\) spectra first drop below the \(3\sigma\) levels in searching from the peak LSR-velocity toward the blueward and redward directions, respectively. After several iterations, we found that the conditions of

\[
|v_t,\text{blue} - v_t,\text{red}| > \sigma > \Delta\sigma > \frac{\Delta v_{\text{res}}}{\sqrt{8\ln 2}}, \quad I > \Delta I,
\]

yielding \(\Delta H = 0.02\) for the number of the data points of \(N = 28\).

### APPENDIX E

#### ERROR ESTIMATE OF THE FILAMENT SCALE HEIGHT

In our model analysis based on the Stodolśkiewicz–Ostriker cylinder (Section 5.3.1), the averaged radial column density profile shown in Figure 15 was analyzed considering the uncertainty of each column density, \(\Delta N_{H_2}\), obtained from Figure 23 (see also Appendix C). Then the uncertainty in the best-fit scale height \(H\) of \(\Delta H = 0.02\) was determined from the interval between the minimum \(\chi^2\) value \(\chi_{\text{min}}^2\) (20.8) and \(\chi_{\text{min}}^2 + 1\), as shown in Figure 24. Since the degree of freedom is 27, the reduced \(\chi_{\text{min}}^2\) value becomes 0.77.

![Figure 24. Plot of the scale height \(H\) vs. \(\chi^2\) values in the radial column density profile analysis shown in Figure 15 (see also Section 5.3.1); the degree of freedom (dof) is 27. The lower and upper horizontal dashed lines present the minimum \(\chi^2\) value of \(\chi_{\text{min}}^2\) and the \(\chi_{\text{min}}^2 + 1\) value, respectively. The interval between the two vertical dashed lines gives the 1σ uncertainty in \(H\) of \(\Delta H = 0.04\) pc. See Appendix E.](image)

In addition, we can analytically derive the uncertainty of \(H\) from the following approximate equation,

\[
\Delta H \sim \frac{\Delta\chi^2}{2N} \sim \frac{1}{N} = \frac{H}{N} \quad \text{for} \quad r < H,
\]

yielding \(\Delta H = 0.02\) for the number of the data points of \(N = 28\).

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