On a Possible Stationary Point in High-Energy Scattering

V.A. Petrov and A.P. Samokhin
Institute for High Energy Physics,
“NRC Kurchatov Institute, Protvino”, RF

Abstract

We discuss a curious observation: at energies from the ISR and up to the LHC, inclusively, the differential cross-section of elastic proton-proton scattering remains almost energy-independent at the transferred momentum $t \approx -0.21\text{GeV}^2$ at the level of $\approx 7.5 \text{mb/GeV}^2$. The latter value can be considered as a prediction for $d\sigma/dt$ at 13 TeV. We also obtain a lower bound for the forward $pp$ slope at 13 GeV.

1 Hints at the Stationary Point

It is well and long ago known that at $-t < 0.15 \text{ GeV}^2$ the $pp$ differential cross-section grows with energy. On the contrary, at $-t > 0.3 \text{ GeV}^2$ they clearly decrease[1]. Recent LHC data confirm these both trends.

This circumstance may impose a speculation that there could exist between these two regions of $t$ a point $t_*$ where differential cross-section doesn’t change with energy at all. The stationary points of the differential cross-sections were discussed en passant many years ago in Refs. [2],[3] but didn’t find any conceptual development. We, thus, assume that there exists a fixed (energy independent) point $t_*$ where

$$d[(d\sigma/dt)(s,t_*)]/ds \approx 0$$
Figure 1: Energy evolution of the differential cross-section of elastic $pp$ scattering at fixed values of transferred momenta. The values of $-t$ go down as: $0.1 \text{ GeV}^2$, $0.15 \text{ GeV}^2$, $0.21 \text{ GeV}^2$, $0.25 \text{ GeV}^2$, $0.3 \text{ GeV}^2$.

at "high enough energies". The thorough inspection of the available data shows that such a point seems to exist indeed. At Fig.1 the energy evolution of $d\sigma_{pp}/dt$ at various fixed values of $t$ is presented.

Fig.1 bears an evidently qualitative character. We believe, however, that it demonstrates our point quite well. Certainly, the exact value of the stationary point $t_*$, which we believe at the moment is near $-0.21 \text{ GeV}^2$, may slightly change after more detailed analysis. As we can see below, the effect in question is possible only for the processes with asymptotically growing cross-sections. That is why we have considered the data starting from the ISR where $\sigma_{pp}^{\text{tot}}$ begin to grow.

Formally, it is easy to invent a model amplitude which exhibit such a property. Let us consider a single Regge-pole amplitude of the form $\beta(t)s^{\alpha(t)}$. If the Regge-trajectory $\alpha(t)$ intersect the line $\alpha = 1$ at some negative $t_*$ then the differential cross-section defined by such an amplitude is stationary at
\[ t = t_*. \] This is possible for the Pomeron trajectory only with \( \alpha(0) > 1. \) However, there are arguments that the Pomeron trajectory is non-linear and never intersects the level \( \alpha = 1 \) at negative \( t \) [4]. Sure, the genuine scattering amplitude is much more complicated than this simplistic model. Notwithstanding the plethora of "realistic" models which describe the data with varied success, we estimate the differential cross-section at the stationary point with help of a simple quasi-classical expression

\[ d\sigma_{pp} = 2\pi r dr. \]

Uncertainty relations imply that

\[ r \approx 1/2q, \quad q = \sqrt{-t}. \]

Hence, we get the estimate of the differential cross-section at the stationary point

\[ d\sigma_{pp}/dt |_{t=t_*} = \pi/4t_*^2. \]

After having inspected the available data we choose, as a trial value of \( t_* \), the value \(-0.21 \text{ GeV}^2\). Then

\[ (d\sigma_{pp}/dt)(s, t_*) \approx 7 \text{ mb}/\text{GeV}^2. \]

It is bizarre that such a primitive evaluation lies so close to the data which we estimate at the level \( \approx 7,5 \text{ mb}/\text{GeV}^2 \) [5]-[8] (c.f. Fig.1).

2 Estimate of the Forward Slope at 13 TeV

Let us rewrite the differential cross-section in the form

\[ (d\sigma_{pp}/dt)(s, t) = (d\sigma_{pp}/dt)(s, t = 0) \exp[\beta(s, t)], \]

where

\[ \beta(s, t) = \int_0^t dt' B(s, t') \]

and

\[ B(s, t) = d(\ln([d\sigma_{pp}/dt](s, t)]))/dt \]
is the local slope. Existence of the stationary point means that at \( t = t_* \),
which we take as before at \(-0.21 \text{ GeV}^2\)
\[
(d\sigma_{pp}/dt)(s, t_*) = (d\sigma_{pp}/dt)(s, t = 0) \exp[\beta(s, t_*)] = 7.5 \text{ mb/GeV}^2.
\]

This quantity is, by our assumption, energy independent therefore the
growth of \((d\sigma_{pp}/dt)(s, t = 0)\) is to be compensated by the decrease of \(\beta(s, t_*)\).
Taking use of the mean value theorem we have
\[
\beta(s, t_*) = t_* B(s, \tilde{t}), \tilde{t} \in [t_* , 0].
\]
We obtain
\[
B_{pp}(s, \tilde{t}) = \ln[(d\sigma_{pp}/dt|_{t=0})/(d\sigma_{pp}/dt|_{t=t_*})]/(-t_*),
\]
or
\[
B_{pp}(s, \tilde{t}) = 9.5 \ln[0.08 \sigma_{tot}^{pp} \sqrt{1 + \rho^2}],
\]
where \( \rho = \text{Re}T_{pp}(s, 0)/\text{Im}T_{pp}(s, 0) \).

To estimate \(B_{pp}(s = (13 \text{ TeV})^2, 0)\) we take use of the parametrization
suggested in [9] which gives
\[
\sigma_{tot}^{pp} \approx 110.0 \text{ mb}
\]
at \( \sqrt{s} = 13 \text{ TeV} \) and \( \rho \approx 0.14 \).

From these values we can envisage that
\[
B_{pp}(s = (13 \text{ TeV})^2, 0) \geq B_{pp}(s = (13 \text{ TeV})^2, \tilde{t}) \approx 20.8 \text{ GeV}^{-2}.
\]

3 Discussion

We have shown, at a tentative level, that it is quite probable that there exists
a stationary, energy independent value \( t = t_* \approx -0.21 \text{ GeV}^2 \) where the value
of \((d\sigma_{pp}/dt)(s, t_*)\) remains constant with the energy growth. We have some
reasons to believe that one another stationary point lies near \(-2 \text{ GeV}^2\). It
seems natural to investigate other channels: \( pp, \pi p, Kp \). At the moment we
have no physical explanation of the phenomenon considered above.
4 Acknowledgements

We are grateful to V.V. Ezhela and A.V. Kisselev for help and fruitful discussions.

References

[1] G. Giacomelli. Phys. Rep. **23C** (1976) 191.

[2] V. Barger, J. Luthe and R.J.N. Phillips. Nucl. Phys. **B88** (1975) 237.

[3] G.G. Arushanov, E.I. Ismatov, V.G. Arushanov, M.G. Sattarov and A. Yulchiev. Ukr. Phys. J. (in Russian) **28** (1983) 498.

[4] A.A. Godizov and V.A. Petrov. JHEP **0707** (2007) 083.

[5] E. Nagy *et al.* Nucl. Phys. **B150** (1979) 221.

[6] U. Amaldi and K.R. Schubert. Nucl. Phys. **B166** (1980) 301.

[7] A. Breakstone *et al.* Nucl. Phys. **B248** (1984) 253.

[8] G. Antchev *et al.* (TOTEM Collaboration). EPL **101** (2013) 21002.

[9] J.R. Cudell *et al.* (COMPETE Collaboration) Phys. Rev. Lett. **89** (2002) 201801.