Area Spectrum of the Large AdS Black Hole from Quasinormal Modes

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Abstract

Using the new physical interpretation of quasinormal modes proposed by Maggiore, we calculate the area and entropy spectra for the 3-dimensional and 5-dimensional large AdS black holes. The spectra are obtained by imposing the Bohr-Sommerfeld quantization condition to the adiabatic invariant quantity. With this semiclassical method, we find that the spacings of the area and entropy spectra are equidistant and independent of the AdS radius of the black hole for both the cases. However, the spacings of the spectra are not the same for different dimension of space-time. The equidistant area spectra will be broken when the black hole has other parameters (i.e., charge and angular momentum) or in a non-Einstein’s gravity theory.

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I. INTRODUCTION

In 1974, Bekenstein presented the famous conjecture [1], namely, in a quantum theory, the black hole area should be represented by a quantum operator with a discrete spectrum of eigenvalues. Since then, the quantization of the black hole horizon area has become a more interesting investigation. Proving that the black hole horizon area is an adiabatic invariant, Bekenstein obtained an equidistant area spectrum \( A_n = \epsilon \hbar n (n = 0, 1, 2, \ldots) \). While the spacing \( \epsilon \) has been somewhat controversial.

In this direction, an important step was made by Hod about ten years ago. He suggested that the spacing of the area spectrum \( \epsilon \) can be determined by utilizing the quasi-normal mode frequencies of an oscillating black hole [2]. Kunstatter pointed that, for a system with energy \( E \) and vibrational frequency \( \Delta \omega(E) \), the ratio \( \frac{E}{\Delta \omega(E)} \) is an adiabatic invariant [3]. Substituting \( M \) for \( E \) and identifying \( \Delta \omega(E) \) as the most appropriate choice for the frequency, the area spectrum \( A_n = 4n\ell_p^2 \ln 3 \) was obtained by way of the Bohr-sommerfeld quantization condition. This rejuvenated the interest of investigation for the quantization of black hole area [2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. Recently, Maggiore argued that, in high damping limit, the proper frequency of the equivalent harmonic oscillator, which is interpreted as the quasi-normal mode frequencies \( \omega(E) \), should be of the form \( \omega(E) = \sqrt{\omega_R^2 + \omega_I^2} \) rather than the real part \( \omega_R \) [18]. It is easy to see that when \( \omega_I \ll \omega_R \), one could get \( \omega(E) = \omega_R \) approximately which was adopted extensively, in [2, 4, 6]. However, at high excited quasi-normal modes (i.e., \( \omega_R \ll \omega_I \)), thus, one should get \( \omega(E) = |\omega_I| \).

Motivated by this idea, the area spectrum was obtained by Medved and Vagenas [9, 10], where they took the choice that the vibrational frequency \( \Delta \omega(E) = |\omega_I|_m - |\omega_I|_{m-1} \). They found the area spectrum is non-equidistant and depends on the angular momentum \( J \) of Kerr black hole. It can also be seen that, when the angular momentum \( J \to 0 \), the area spectrum is equidistant. Especially, for the 5-dimensional Gauss-Bonnet(GB) black hole, Kothawala et.al. first pointed out that the area spectrum is non-equidistant, but the entropy spectrum is equidistant [11]. However, when one setting the Gauss-Bonnet coupling constant \( \alpha_{GB} \to 0 \), the area spectrum would be equidistant. The reason of this may be that the relationship \( S = \frac{A}{4\hbar} \) between the horizon area and associated entropy does not hold anymore. In [19], we conjecture that, for the non-rotating black holes with no charge, the entropy spectrum
is equidistant and is independent of the dimension of space-time. However, the spacing of the area spectrum depends on gravity theory. In Einstein’s gravity, it is equally spaced, otherwise it is non-equidistant. For a charging or rotating black hole (e.g., the Kerr black hole), the spacing of area spectrum will not be equal because of the existence of the angular momentum $J$ or the charge $Q$.

In this paper, we will calculate the area and entropy spectra of large Ads black hole using the asymptotic quasinormal modes of obtained from [20]. The quasinormal modes for Schwarzschild-Ads black hole were also obtained in [21, 22], the numerical result can be found in [23]. The Stability of simply rotating Myers-Perry-AdS black holes and higher dimensional R-N-Ads black hole were discussed in [24]. Especially, in [22], the highly real quasinormal modes has a logarithm term in the imaginary part rather than the real part, which is different from the Schwarzschild case. However, it seems has no affect on obtaining a equidistant spectra.

The paper is organized as follows. In Section II, we briefly review the thermodynamics and the quasinormal modes of the large AdS black hole. In Section III, we will apply the semiclassical method to the 3-dimensional lager AdS black hole and obtain an equidistant area spectrum. The area spectrum of the 5-dimensional lager AdS black hole is obtained in Section IV, which is also equally spaced. Finally, the paper ends with a brief Summary.

II. QUASINORMAL MODES OF THE LARGE ADS BLACK HOLE

In this section, we give a brief review of the AdS black hole. For a $d$-dimensional AdS black hole, the metric can be written as

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2d\Omega_{d-2}^2.$$  \hspace{1cm} (1)

The metric function is

$$f(r) = 1 + \frac{r^2}{l^2} - \frac{\omega_{d-1}M}{r^{d-3}},$$  \hspace{1cm} (2)

where $l$ and $M$ are the AdS radius and mass of the black hole, respectively. For a large black hole, the metric function $f(r)$ is simplified to

$$f(r) = \frac{r^2}{l^2} - \frac{\omega_{d-1}M}{r^{d-3}}.$$  \hspace{1cm} (3)

The event horizon is located at $f(r_h) = 0$ and the radius $r_h$ satisfies

$$r_h = (l^2\omega_{d-1}M)^{\frac{1}{d-3}}.$$  \hspace{1cm} (4)
The Hawking temperature is
\[ T = \frac{d - 1}{4\pi} \frac{r_h}{l^2}. \] (5)

The first law of black hole thermodynamics is
\[ dM = TdS. \] (6)

Performing the partial wave decomposition \( \Phi = e^{i(\omega t - \vec{p} \cdot \vec{x})} \Psi(r) \) and solving the massless scalar wave equation with proper boundary conditions, the quasi-normal modes can be obtained \[20\]. For the \( d = 3 \) case, the quasi-normal modes are read as
\[ \frac{\omega}{4\pi T} = \pm \hat{p} - im, \quad m = 1, 2, ..., \] (7)
where \( \hat{p} \) is a dimensionless variable and given by
\[ \hat{p}^2 = \frac{p^2}{4\pi T}. \] (8)

For the case \( d = 5 \), the quasi-normal modes are
\[ \frac{\omega}{\pi T} = 2m(\pm 1 - i), \quad m = 1, 2, .... \] (9)

It is easy to see that, for both the cases, the quasi-normal modes \( \omega \) are complex frequencies. In fact, this is a universal character of quasi-normal modes. For the quasi-normal modes \( \omega \) appeared in the wave function as \( e^{i\omega t} \), the image part will decrease to zero at infinite. The quasi-normal modes are also generally believed to depend only on the black hole parameters (i.e., the mass \( M \), charge \( Q \) and angular momentum \( J \)). It can be proved that the quasi-normal modes (7) and (9) only depend on the mass \( M \) of the large AdS black hole.

### III. AREA SPECTRUM OF THE 3-DIMENSIONAL LARGE ADS BLACK HOLE

In this section, we first give a general statement of the method we used. Then we apply it to the 3-dimensional large AdS black hole and obtain the area and entropy spectra, respectively.

Several years ago, Kunstatter proposed that given a system with energy \( E \) and vibrational frequency \( \Delta \omega(E) \), a nature adiabatic invariant quantity is \[3\]:
\[ I = \int \frac{dE}{\Delta \omega(E)}. \] (10)
At the large $n$ limit, the Bohr-Sommerfeld quantization can be expressed as

$$I \approx nh.$$ (11)

One may argue, the quantum number $n$ should replace by $(n + \frac{1}{2})$ like harmonic oscillator. Then, one could discuss the remnant of black hole. However, we need keep in mind that the number $n$ is a large number and $n \sim (n + \frac{1}{2})$. So, it is not proper to discuss the remnant of black hole under this case. Adiabatic invariant quantity $I$ is

$$I = \int \frac{dE}{\Delta \omega} = \int \frac{dM}{\Delta \omega},$$ (12)

where we have identified the energy $E$ of this black hole system as it’s mass $M$ in the second step. Through the interpretation that identifying the vibrational frequency $\Delta \omega(E)$ as the quasinormal modes, much work has been done (e.g., [2, 4, 6]), where the vibrational frequency $\Delta \omega(E)$ was regarded as the real part of the quasinormal modes. Recently, Maggiore refined Hod’s treatment by arguing that the physically relevant frequency would actually be [18]

$$\omega(E) = \sqrt{|\omega_R|^2 + |\omega_I|^2},$$ (13)

where $\omega_R$ and $\omega_I$ are the real and imaginary parts of the quasinormal modes frequency respectively. When $\omega_I \to 0$, one could get $\omega(E) = |\omega_R|$ approximately. However, at the case of large $m$ or highly excited quasinormal modes for which $\omega_R \ll \omega_I$, the frequency of the harmonic oscillator becomes $\omega(E) = |\omega_I|$. Under this supposition, work has been applied to different black holes. The results show that, for the non-rotating black holes with no charge, the area and entropy spectra are both equally spaced in Einstein’s gravity [19]. For other gravity theory, the result is partially hold. Take the Gauss Bonnet gravity as an example, it is first pointed in [11] that the area spectrum is not equally spaced, but the entropy is equidistant still. If the black hole has other parameters, i.e., the angular momentum $J$, the equidistant area spectrum will be broken [9, 10]. However, if setting the angular momentum $J \to 0$, the area spectrum will tend to an equally spaced spectrum.

For the case $d=3$, the radius of event horizon, the area and the Hawking temperature are given, respectively

$$r_h = l(\bar{\omega}_2 M)^\frac{1}{2},$$ (14)

$$A = 2\pi r_h,$$ (15)

$$T = \frac{r_h}{2\pi l^2}.$$ (16)
At the large $m$ limit, the vibrational frequency $\Delta \omega$ is

$$\Delta \omega = |\omega_I|_m - |\omega_I|_{m-1} = \frac{2r_h}{l^2}. \quad (17)$$

Substituting $\Delta \omega$ into (12), we obtain the adiabatic invariant quantity

$$I = \int \frac{dM}{\Delta \omega} = \int \frac{1}{dM} \frac{dr_h}{\Delta \omega} = \frac{r_h}{\omega_2}. \quad (18)$$

Using the Bohr-Sommerfeld quantization condition (10), at the large $n$ limit, we obtain

$$\frac{r_h}{\omega_2} = n\hbar. \quad (19)$$

Recalling the area from (15), the area spectrum of this black hole is given

$$\mathcal{A}_n = 2\pi \hbar \omega_2 \cdot n. \quad (20)$$

It is clear that this area spectrum is equally spaced with equidistant $\Delta \mathcal{A} = 2\pi \hbar \omega_2$. The area spectrum is independent of the AdS radius $l$ of the black hole. Recalling the relationship $S = \frac{A}{4\hbar}$ between horizon area and associated entropy, one could get the entropy spectrum

$$S_n = \frac{\pi \omega_2}{2} \cdot n, \quad (21)$$

with the spacing

$$\Delta S = S_{n+1} - S_n = \frac{\pi \omega_2}{2}. \quad (22)$$

The entropy is also equidistant and the spacing is independent of the AdS radius $l$ of the black hole.

**IV. AREA SPECTRUM OF THE 5-DIMENSIONAL LARGE ADS BLACK HOLE**

In this section, we would like to focus on the 5-dimensional large AdS black hole and obtain the area and entropy spectra.

For the case $d=5$, the radius of event horizon, the area and the Hawking temperature are given, respectively

$$r_h = (l^2 \omega_{d-1} M)^{\frac{1}{d}}, \quad (23)$$

$$\mathcal{A} = 2\pi^2 r_h^3, \quad (24)$$

$$T = \frac{r_h}{\pi l^2}. \quad (25)$$
For a general $m$, the vibrational frequency $\Delta \omega$ can be obtained

$$\Delta \omega = \sqrt{\left(\omega_R^m\right)^2 + \left(\omega_I^m\right)^2} - \sqrt{\left(\omega_R^{m-1}\right)^2 + \left(\omega_I^{m-1}\right)^2} = \frac{2\sqrt{2}r_h}{l^2}.$$  \quad (26)

Substituting $\Delta \omega$ into (12), we obtain the adiabatic invariant quantity

$$I = \int \frac{dM}{\Delta \omega} = \int \frac{1}{dM} \frac{dr_h}{\Delta \omega} = \frac{\sqrt{2}r_h^3}{3\omega_4}. \quad (27)$$

Using the Bohr-Sommerfeld quantization condition (10), at the large $n$ limit, we obtain

$$\frac{\sqrt{2}r_h^3}{3\omega_4} = n\hbar. \quad (28)$$

Recalling the area from (24), the area spectrum are

$$A_n = 3\sqrt{2}\pi^2 \omega_4 \hbar \cdot n. \quad (29)$$

This area spectrum is also equally spaced and with equidistant spacing $\Delta A = 3\sqrt{2}\pi^2 \omega_4 \hbar$. For this case, it shares the same character with the case $d=3$ that the area spectrum is independent of the AdS radius $l$ of the black hole. However, the spacings of the area spectra are not equal, which is due to the different dimension of space-time. Then the entropy spectrum can also be obtained

$$S_n = \frac{3\sqrt{2}\pi^2 \omega_4}{4} \cdot n, \quad (30)$$

with the spacing

$$\Delta S = S_{n+1} - S_n = \frac{3\sqrt{2}\pi^2 \omega_4}{4}. \quad (31)$$

The entropy is also equidistant and the spacing is independent of the AdS radius $l$ of the black hole.

In fact, the spacing of area and entropy spectra for d-dimensional Schwarzschild-AdS black hole are also obtained in [22]

$$\Delta A = 16\pi \sin\left(\frac{\pi}{D-1}\right),$$

$$\Delta S = 4\pi \sin\left(\frac{\pi}{D-1}\right). \quad (32)$$
TABLE I: The spacings of the area spectra $A_n$ and entropy spectra $S_n$ for various black holes. √ and × stand for that the spectrum is equidistant and non-equidistant, respectively.

If we choose the proper value of $\omega$, i.e. $\omega_2 = 8$ and $\omega_4 = \frac{4}{3\pi}$, it will reduce to our cases. Using the asymptotic quasinormal modes given in [21], one could see that the result is the same.

V. SUMMARY AND OUTLOOK

In this paper, we succeed in utilizing a new physical interpretation of quasinormal modes to the large Ads black hole following the Kunstatters method. By modifying the frequency $\omega(E)$ appeared in the adiabatic invariant of black hole and using the Bohr-Sommerfeld quantization at the large $n$ limit, we investigate the area and entropy spectra of 3-dimensional and 5-dimensional large AdS black holes. For both the cases, the spectra are equidistant and independent of the AdS radius $l$. However, the spacings of the spectra are different, which is because of the different dimension of space-time. Although our results are still somewhat speculative, they certainly propose a reasonable physical interpretation on the spectrum of the large AdS black hole quasinormal modes. Our results still suppose the conjecture that, for the non-rotating black holes with no charge, the spacings of the area and entropy spectra are equidistant and are independent of the dimension of space-time. From the Table I, it easy to see that the extra parameter (angular momentum $J$) of black hole will broken the equidistant spectra. So, it is our conjecture that, for a Reissner-Norström (or Kerr-Newmann) black hole, the area spectrum will not equidistant. But if setting the charge $Q$
(and angular momentum $J \to 0$, the equidistant area and entropy spectra will reoccur. This needs to be proved in future.

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