Positive and Negative Drag, Dynamic Phases, and Commensurability in Coupled One-Dimensional Channels of Particles with Yukawa Interactions

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We introduce a simple model consisting of two or three coupled one-dimensional channels of particles with Yukawa interactions. For the two channel system, when an external drive is applied only to the top or primary channel, we find a transition from locked flow where particles in both channels move together to decoupled flow where the particles in the secondary or undriven channel move at a slower velocity than the particles in the primary or driven channel. Pronounced commensurability effects in the decoupling transition occur when the ratio of the number of particles in the top and bottom channels is varied, and the coupling of the two channels is enhanced when this ratio is an integer or a rational fraction. Near the commensurate fillings, we find additional features in the velocity-force curves caused by the slipping of individual vacancies or incommensurations in the secondary channels. For three coupled channels, when only the top channel is driven we find a remarkably rich variety of distinct dynamic phases, including multiple decoupling and recoupling transitions. These transitions produce pronounced signatures in the velocity response of each channel. We also find regimes where a negative drag effect can be induced in one of the non-driven channels. The particles in this channel move in the opposite direction from the particles in the driven channel due to the mixing of the two different periodic frequencies produced by the discrete motion of the particles in the two other channels. In the two channel system, we also demonstrate a ratchet effect for the particles in the secondary channel when an asymmetric drive is applied to the primary channel. This ratchet effect is similar to that observed in superconducting vortex systems when there is a coupling between two different species of vortices.

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I. INTRODUCTION

There are many systems composed of repulsively interacting particles with one dimensional (1D) or quasi-1D motion, including colloids in narrow channels \cite{1,2,3,4,5,6,7,8,9,10}, Wigner crystal states in wires \cite{11,12,13,14,15,16,17,18,19,20}, dusty plasmas in grooves \cite{21}, macroscopic charged ball bearings in channels \cite{22}, and vortices in type-II superconductors confined within narrow strips or channels \cite{23,24,25}. In many of these systems, interesting structural transitions from 1D lines of particles to zig-zag or buckled states can occur \cite{4,17,19}. There can also be higher order transitions from 2 rows to 3 rows of particles or transitions to disordered states \cite{5,23,24,27}. Under an external drive, this type of system also exhibits a variety of dynamical behavior such as ordered or disordered motion through constrictions \cite{1,20,27} or dynamic commensurability effects \cite{23,24}.

Here we propose a simple system consisting of particles in two or three coupled 1D channels. The particles in each channel interact with the other particles in the same channel as well as with particles in adjacent channels via a Yukawa potential. An external drive applied to only one channel produces drag effects on the particles in the undriven channels, causing them to move. Our system is illustrated in Fig. 1. The two channel system is similar to the transformer geometry studied for vortices in two superconducting layers where an external drive is applied to one (primary) layer and the response of the nondriven (secondary) layer is measured \cite{28,29}. If the vortices in the two layers are fully coupled, the response of the secondary layer is exactly the same as the response of the primary layer. If the vortices are only partially coupled, the response in the secondary layer is smaller than that of the primary layer. The transformer geometry has also been studied for vortex systems with multiple layers, such as vortices in the strongly layered high-temperature superconductors \cite{31}. Drag effects have also been predicted for two coupled 1D wires containing classical 1D Wigner crystals when only one of the wires is driven \cite{13}. In this case the interaction between particles in neighboring wires is repulsive, unlike the attractive interaction between vortices in neighboring layers. For the coupled 1D Wigner crystals there is also a transition from a completely locked state, where the response in both wires is identical, to a partially locked state, where the response of the secondary wire is reduced. Drag effects in coupled wire experiments have been interpreted as arising from the formation of Wigner crystal states \cite{32}.

In this work we consider the effect of changing the ratio of the number of particles in each channel on the locking or coupling between the channels, with a particular focus on ratios that are integers or rational fractions. Commensuration effects \cite{33} occur when the spacing between particles in one channel is a simple rational fraction of the spacing between particles in another channel, while incommensurations such as vacancies or interstitials appear when the two spacings are incommensurable. Commensuration effects have been studied extensively for systems in which a varied number of particles interacts with
Two channels with $R_s$ distinct from Frenkel-Kontrova systems. Additionally, formability of the substrate makes our proposed model each channel is an integer or a rational fraction. The de-

FIG. 1: Image of the sample geometry. The locations of the three channels are indicated by dotted lines. Black dots are particles within the channels. The arrow denotes the driving force which is applied only to particles within the top channel, termed the primary channel $p$. The bottom undriven channels are the secondary channels $s_1$ and $s_2$. The ratio of the number of particles in each channel is $R_{s1,p} = N_{s1}/N_p$ and $R_{s2,p} = N_{s2}/N_p$, where $N_{s1}$ and $N_{s2}$ are the number of particles in the secondary channels and $N_p = 16$ is the number of particles in the primary channel. The spacing between channels $d$ is marked in panel (a). (a) Two channels with $R_{s1,p} = 1.0$ (b) Two channels with $R_{s1,p} = 0.5$. (c) Three channels with $R_{s1,p} = 1.0$ and $R_{s2,p} = 1.0$. (d) Three channels with $R_{s1,p} = 1.0$ and $R_{s2,p} = 1.5$.

a rigid periodic substrate, such as atoms and molecules on surfaces [34], vortices in superconductors with periodic pinning arrays [35, 37], and colloids interacting with 1D [38] or two-dimensional (2D) optical trap arrays [39], all of which can be viewed as physical realizations of the Frenkel-Kontrova model. These studies find that the coupling to the substrate or the effective pinning of the particles by the substrate is strongly enhanced when the ratio of the number of particles to the number of substrate minima is an integer or a rational fraction, as indicated by the appearance of peaks in the critical depinning force or enhanced ordering of the particles at the commensurate fillings. In our system, for the two channel geometry illustrated in Fig. 1 the particles in the secondary channel can be regarded as a distortable or moveable periodic pinning substrate for the particles in the primary channel, suggesting that enhanced drag or coupling could occur when the ratio of the number of particles in each channel is an integer or a rational fraction. The de-

The paper is organized as follows: In Section II, we describe our simulation method and sample geometry.
We consider two channels of particles in Section III and illustrate a drive-induced decoupling transition for commensurate channels in Section III A. In Section III B we describe the two step decoupling transition that occurs for incommensurate channels which contain vacancy or interstitial sites that can act like a second species of particle. The effects of finite temperature and finite size appear in Section III C. Section III D shows that the nonlinear response of the system can be exploited to create a ratchet effect, where ac motion in the driven channel induces dc transport in the drag channel. In Section IV we turn to samples with three channels. We show in Section IV A that when the driven channel is commensurate with the neighboring drag channel, four different types of coupled and decoupled flow can occur as the occupancy of the second drag channel is varied, including regimes of intermittent coupling. In Section IV B, the driven channel is incommensurate with the neighboring drag channel and we find a complex series of coupling-decoupling transitions that produce a significant amount of structure in the velocity-force curves. In Section IV C, we consider in detail the negative drag that can occur at incommensurate fillings when the particles in one of the drag channels move in the direction opposite to the particles in the driven channel. It is also possible for the outer channels to remain coupled while the central channel is decoupled, as described in Section IV D. In Section IV E we summarize all five of the dynamical phases and the negative drag by showing that they can be achieved in a single system. The paper concludes in Section V with a discussion and summary.

II. SIMULATION

We model interacting Yukawa particles confined to move along 1D channels as illustrated in Fig. [I] Each particle interacts with other particles in the same channel and with particles in adjacent channels. The separation between channels is $d = 2$ and, unless otherwise noted, there are $N_p = 16$ particles in the driven or primary channel with a lattice spacing $a$, where $L$ is the length of the channel. The particles in the primary channel $p$ are coupled to an applied external driving force $F_D$. For a two channel system, the drag or secondary channel $s_1$ contains $N_{s_1}$ particles, and the commensurability ratio is $R_{s_1,p} = N_{s_1}/N_p$. In a three channel system such as that shown in Fig. [I](c,d), the additional secondary channel $s_2$ is adjacent to $s_1$ but not to the primary channel $p$, and it contains $N_{s_2}$ particles, giving a commensurability ratio of $R_{s_2,p} = N_{s_2}/N_p$.

The particle motions evolve under overdamped dynamics where the colloids obey the following equation of motion:

$$\frac{d \mathbf{R}_i}{dt} = \mathbf{F}_i^{pp} + \mathbf{F}_p^D$$

Here $\eta$ is the damping constant, $\mathbf{R}_i$ is the location of particle $i$, and the repulsive particle-particle interaction force is $\mathbf{F}_i^{pp} = \sum_{j \neq i}^{N_p} -\nabla V(R_{ij})$. The potential has a Yukawa or screened Coulomb form of

$$V(R_{ij}) = \frac{E_0}{R_{ij}} e^{-\kappa R_{ij}}$$

with $E_0 = Z^* a/4\pi\epsilon \epsilon_0 a_0$, where $\epsilon$ is the solvent dielectric constant, $Z^*$ is the effective charge, and $1/\kappa$ is the screening length. For colloidal systems, the length scale $a_0$ is on the order of a micron. We measure forces in units of $F_0 = E_0/a_0$ and time in units of $\tau = \eta/E_0$. In a typical case, the distance between particles in channel $p$ is $a = 2.25$ while the distance between adjacent channels is $d = 1.125a$ and the screening length is $1/\kappa = 2d$, which is long enough to ensure strong coupling between the particles in all three channels. The driving force $F_p^D = F_D \mathbf{x}$ is applied only to all the particles in the primary channel. We increase $F_D$ from zero in small increments of $\delta F_D$, holding the drive at constant values for a fixed time interval during which we measure the velocity of the particles in each channel. We have carefully checked that our waiting times are long enough to eliminate transient effects. We use $\delta F_D = 0.001$ and a wait time of $10^5$ simulation time steps. We impose periodic boundary conditions in the $x$-direction along the length of the channels. The velocity of the particles in the primary channel is given by $V_p$ and the particle velocity in the secondary channels is given by $V_{s_1}$ and $V_{s_2}$. We normalize all the velocities by the number of particles in each channel, $V_p = N_p^{-1} \sum_i v_i$, $V_{s_1} = N_{s_1}^{-1} \sum_i v_i$, and $V_{s_2} = N_{s_2}^{-1} \sum_i v_i$.

III. TWO CHANNEL SYSTEMS

A. Coupling-Decoupling Transitions for Commensurate Channels

We first focus on the two channel system at commensurate filling with a particle ratio of $R_{s_1,p} = 1.0$ and with $d/a = 0.67$. In Fig. [2](a) we plot $V_p$ and $V_{s_1}$ together versus $F_D$. Both $V_p$ and $V_{s_1}$ increase linearly with $F_D$ for low $F_D$ and have identical values, indicating that the motion in the two channels is locked. At $F_D = 2.125$, we find a transition to a partially decoupled state where $V_{s_1}$ monotonically decreases with increasing $F_D$ while $V_p$ continues to increase with $F_D$ at a rate faster than the linear increase that occurred below the transition. The particles in $s_1$ are not completely decoupled from the primary channel since they still exhibit a nonzero velocity; since the particles in $s_1$ do not experience a driving force, they can move only due to interactions with the particles in $p$. Just above the decoupling transition, $V_p$ increases with a square root form. The general shape of the velocity force curves in Fig. [2](a) is the same as that of the current-voltage curves obtained for superconducting transformer geometries [23-31], where the vortex velocities are proportional to the voltage and the applied
as the particle density increases. (c) The result is plotted in Fig. 2(b), where we fix $a$ and vary $N_s$. For the Yukawa system this effect can be attributed to the reduced size of the periodic potential that the particles in $s_1$ experience from the particles in $p$. Once the primary channel is in motion, the particles in $s_1$ shift to positions that are slightly behind the driven particles. As the drive increases, the size of this shift increases until the particles in $p$ slip more than $0.5a$ ahead of the particles in $s_1$, producing the partial decoupling. When the particle density increases, the amount of shift required to pass the position $0.5a$ decreases since $a$ decreases with increasing particle density in both channels.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2}
\caption{In Fig. 2(c) we plot $V_p$ and $V_{s_1}$ for a two channel system with $R_{s_1,p} = 0.92$, where the number of particles in $s_1$ is smaller than the number of particles in $p$. Here $V_{s_1} = N_{p}^{-1} \sum_{i} N_{s_1} v_i$ is the velocity of the particles in $s_1$ normalized by $N_p$, the number of particles in $p$. For low $F_D$ the channels are locked and all of the particles in the system move at the same velocity. The slope of $V_{s_1}$ versus $F_D$ is slightly smaller than the slope of $V_p$ versus $F_D$ in Fig. 2(c) due to the fact that $N_{s_1} < N_p$. At $F_D = 1.04$ we observe a transition to a partially coupled state; however, this transition occurs at a drive well below the decoupling transition $F_D = 2.125$ shown for the commensurate system in Fig. 2(a). Additionally, just above $F_D = 1.04$, Fig. 2(c) indicates that the velocity-force curve does not have the characteristic square root shape found close to $F_c$ in Fig. 2(a). For $F_D > 1.04$, $V_{s_1}$ continues to increase with increasing $F_D$ but with a smaller slope than in the locked regime. A second decoupling transition appears at $F_D = 2.25$. For $F_D > 2.25$, $V_{s_1}$ decreases with increasing $F_D$ and there is also a corresponding increase in the slope of $V_p$. The second decoupling transition occurs at a drive close to the value $F_c = 2.125$ where decoupling of the commensurate system occurs, as shown in Fig. 2(a). This indicates that the second decoupling transition for the incommensurate system is the same as the sole decoupling transition found in the commensurate system, where all the particles in $s_1$ begin to slip with respect to the particles in $p$. The two step decoupling transition for the incommensurate system appears due to the presence of vacancies in $s_1$. At commensuration, all of the particles in $s_1$ are located within potential minima created by the spacing of the particles in $p$. Below commensuration, a fraction of the sites in this periodic potential are empty, producing effective vacancies in $s_1$. In the locked phase at $F_D < 1.04$, all the particles in $s_1$ move at the same velocity as the particles in $p$. At the first decoupling transition, the vacancies in $s_1$ begin to slip with respect to the particles in $p$. This can be viewed as a depinning transition. Every time a vacancy slips, only one of the particles in $s_1$ slips with respect to $p$ while the remaining particles in $s_1$ stay locked with $p$. As a result, most of the particles in $s_1$ continue to increase in velocity with increasing $F_D$. For drives above the second decoupling transition, all of the particles in $s_1$ slip with respect to $p$.

\section{Dynamics and Commensurability}

We measure the decoupling or unlocking force $F_c$ as a function of $d/a$ in a two channel system with $R_{s_1,p} = 1.0$. The result is plotted in Fig. 2(b), where we fix $d = 2.0$ and vary $a$ by changing $N_p$ and $N_{s_1}$. Here $F_c$ decreases with increasing particle density. A similar effect occurs in layered vortex systems, where for higher fields or higher vortex densities the coupling between the layers is gradually reduced [31]. For the Yukawa system this effect can
and the slipping is no longer dominated by the motion of vacancies.

In Fig. 2(d) we plot $V_p$ and $V_{s1}'$ versus $F_D$ for the same system in Fig. 2(c) but with $R_{s1,p} = 1.08$, where $N_{s1} > N_p$, so that a few incommensurate particles appear in $s_1$. The overall shape of the velocity-force curve in this case is very similar to that for $R_{s1,p} = 0.92$ shown in Fig. 2(c), with a first decoupling occurring at a lower drive of $F_D = 0.6$ than that for $R_{s1,p} = 0.92$, and a second decoupling transition occurring close to $F_D = 2.0$. Here the incommensurations in $s_1$ form doubly occupied sites in the periodic potential created by the particles in $p$. At $R_{s1,p} = 0.92$ when there are vacancies in $s_1$, slipping of a particle adjacent to a vacancy occurs because the particle is able to move closer to the barrier separating two minima in the periodic potential. This is because the force the particle experiences on one side from a neighboring particle in $s_1$ is not compensated due to the missing particle at the vacancy site. As a result, there is an extra force of the order of $F_{pp}(a)$ on the slipping particle, where $a$ is the lattice constant of the particles in $p$. For the doubly occupied sites at $R_{s1,p} = 1.08$, a similar situation occurs; however, the slipping particle in $s_1$ is located at a doubly occupied site and feels an uncompensated force from the other particle located within the same site. The extra force in this case is $F_{pp}(a')$, where $a' < a$ in order for the site to be doubly occupied. Thus, $F_{pp}(a') < F_{pp}(a)$, so the initial decoupling transition occurs at a lower value of $F_D$ for samples with $R_{s1,p} > 1$ that have incommensurations than for samples with $R_{s1,p} < 1$ that contain vacancies.

The appearance of multiple decoupling transitions just below and above commensuration and only one transition at commensuration is similar to the single and multiple depinning transitions observed in vortex systems [10] and colloidal systems [11] with periodic potentials at and near commensuration. In these 2D systems, at the matching filling of 1 particle per substrate minimum there is a single transition from a pinned state to a flowing state, while at fillings slightly away from commensuration, well-defined vacancies or interstitial particles appear which are highly mobile and depin at a lower external drive than the commensurate particles. The 2D systems are generally more complicated and allow for more than two depinning transitions near but not at commensuration [10]; however, in our 1D case, it is the presence of two types of particles, the commensurate particles and the interstitial or vacancy sites, that produce the multiple depinning transitions. There are some important differences between our two channel system and the 2D vortex and colloidal systems. Our system contains no fixed periodic substrate so there is no pinned phase; however, there is a moving fully coupled state which is analogous to the pinned state. The regime in which the vacancies or incommensurations slip is then analogous to the depinning transitions of interstitials or vacancies, and the high driving phases at which all the particles are slipping corresponds to the completely depinned regime in the 2D systems.

In order to show more clearly that the particles in $s_1$ are experiencing a periodic potential produced by the particles in $p$ and that there are two effective types of particles in $s_1$ away from commensuration, in Fig. 3(a) we plot the time trace of the velocity $v(t)$ of a single particle in $s_1$ for the system in Fig. 2(a) at $R_{s1,p} = 1.0$ in the locked phase at $F_D = 2.25$. The value of $v(t)$ is nearly constant except during the periodic slip events, during which $v$ drops briefly below zero indicating that the particle temporarily moves backwards. In Fig. 3(b) we show the Fourier transform $S(f)$ of $v(t)$ highlighting the presence of a single characteristic frequency determined by the slipping events. In the locked phase, there is no high-frequency oscillation of the velocity of any of the particles. In Fig. 3(c) we plot $v(t)$ for the system with $R_{s1,p} = 1.08$ from Fig. 2(d) at $F_D = 0.65$ where the channels are not completely locked but where $V_{s1}'$ is still increasing with increasing $F_D$, and in Fig. 3(d) we show $S(f)$ for the same data. There are now two frequencies present. The lower frequency is produced by the same slipping events that occurred for the commensurate system in Fig. 3(a,b), while the higher frequency originates from the motion of the incommensuration through $s_1$. In Fig. 3(e,f) we plot $v(t)$ and $S(f)$ for a sample with $R_{s1,p} = 0.92$ at $F_D = 1.1$. Again, there are two characteristic frequencies. The lower frequency is associated with the same slipping events shown previously, while the higher frequency is produced by the motion of a vacancy through $s_1$, rather than by the motion of an incommen-
There is a single transition from the completely coupled state $R$ is no locked phase, but there is still a local maximum in single transition out of the locked phase. (d) At $R$ in the incommensurations begin to slip while the other particles in $s_1$ remain locked with $p$. (c) At $R_{s1,p} = 2.0$ there is a single transition out of the locked phase. (d) At $R_{s1,p} = 1.92$ (upper $V_s$ curve) and $R_{s1,p} = 2.08$ (lower $V_s$ curve) there is no locked phase, but there is still a local maximum in $V_s$ which is higher for $R_{s1,p} = 1.92$.

The two stage decoupling process is most pronounced for fillings close to $R_{s1,p} = 1.0$, but the same effects appear near certain fractional ratios. For example, in Fig. 4(a) we plot $V_p$ and $V_{s1}$ versus $F_D$ for a sample with $R_{s1,p} = 0.5$ which has a single sharp decoupling transition at $F_D = 1.9$. Just above this filling at $R_{s1,p} = 0.58$, shown in Fig. 4(b), there is an initial decoupling transition of the incommensurations near $F_D = 1.5$ into a state where $V_{s1}$ still increases with increasing $F_D$ but with a greatly reduced slope. A second decoupling transition appears at $F_D = 1.8$, and above this drive $V_{s1}$ decreases with increasing $F_D$. In this case, incommensurations appear with respect to the particle configuration that occurs at $R_{s1,p} = 0.5$. There is a single frequency associated with the motion of the particles in $s_1$ for $R_{s1,p} = 0.5$, while for the incommensurate case of $R_{s1,p} = 0.58$, two frequencies are present. This trend persists for higher values of $R_{s1,p}$ as shown in Fig. 4(c) at $R_{s1,p} = 2.0$. Here there is a single sharp decoupling transition, while just below and just above this filling at $R_{s1,p} = 1.92$ and $R_{s1,p} = 2.08$, Fig. 4(d) shows that the locking phase is absent but that a strong local maximum in $V_{s1}$ appears at $F_D = 0.19$ for $R_{s1,p} = 1.92$ and at $F_D = 0.125$ for $R_{s1,p} = 2.08$.

By performing a series of simulations for varied $R_{s1,p}$, we determine the location $F_c$ of the transition from complete locking to a decoupled state and map out where the commensuration effects occur. In Fig. 5(a), we plot the decoupling force $F_c$ versus $R_{s1,p}$ for a two channel sample with $d/a = 0.44$, which falls in the strong coupling regime in Fig. 2(b). There are peaks in $F_c$ at $R_{s1,p} = 1.0$, 2.0, and 3.0, along with submatching peaks at $R_{s1,p} = 1/3$, 1/2, and 2/3. Additionally, weaker anomalies appear at $R_{s1,p} = 1.5$ and 2.5. In Fig. 5(b) we show $F_c$ versus $R_{s1,p}$ for a sample with $d/a = 0.67$. The value of $F_c$ at the commensurate filling of $R_{s1,p} = 1.0$ is lower for the $d/a = 0.67$ sample than for the $d/a = 0.44$ sample. Figure 5(b) also has clear peaks in $F_c$ at $R_{s1,p} = 2.0$, 1.5, and 0.5, while above $R_{s1,p} = 2.0$ within our resolution there are no peaks or regions where the system is locked. In the regions with $F_c = 0$ where the locked phase is absent, the second decoupling transition still appears at higher drives and can be detected as the point at which $V_{s1}$ changes from increasing to decreasing with increasing $F_D$. For higher particle densities and fixed $d$, the commensurability effects still persist as shown in Fig. 5(c) for $d/a = 1.0$. Here, peaks in $F_c$ occur at $R_{s1,p} = 0.5$, 1.0, and 2.0.

The appearance of the commensuration effects at integer and fractional fillings suggests that this system ex-
hibits the same behavior found for the depinning of repulsively interacting particles on a 1D fixed periodic potential; however, there are several differences between the two systems. For particles on a fixed periodic potential, the depinning force \( F_c \) at fields where the particle-particle interactions cancel due to symmetry equals the maximum value of the pinning force \( F_p \) so that \( F_c = F_p \) at fillings 1/2, 1/4, 1/6, 1/4/1/2, and 1.0. For the drag system shown in Fig. 5 this does not occur and there is even a trend for \( F_c \) to increase at the lowest fillings. This is because the substrate potential created by the particles in \( p \) is not fixed but can distort since the particles in either channel can slide. At \( R_{s1,p} = 1.0 \), the periodic potential is fairly rigid due to the matching of the particle positions in \( p \) and \( s1 \), and any distorion of the particles in \( p \) is energetically unfavorable. In contrast, at very low fillings such as \( R_{s1,p} = 0.125 \), the particles in \( p \) distort near the locations of the particles in \( s1 \) in order to create a localized lowering of the density in \( p \) above each particle in \( s1 \). As a result, the particles in \( s1 \) no longer experience the same periodic potential from \( p \) that was present for the commensurate case of \( R_{s1,p} = 1.0 \). Even at \( R_{s1,p} = 0.5 \), the particles in \( p \) can distort, reducing the strength of the coupling to the particles in \( s1 \).

In order to better understand the changes in dynamics at the different fillings, in Fig. 6 we plot \( V_{s1} \) as a function of \( F_D \) for varied \( R_{s1,p} \) in a system with \( d/a = 0.67 \). At \( R_{s1,p} = 0.5 \), a single decoupling transition occurs and \( V_{s1} \) is a monotonically decreasing function. For \( R_{s1,p} = 0.562 \) and 0.625, shown in Fig. 7(a), there is a clear double peak structure in \( V_{s1} \) with one peak falling at the depinning of the incommensurations and the second peak appearing at the unlocking transition. At \( R_{s1,p} = 0.6875 \) in Fig. 7(a), there is now a three peak structure in \( V_{s1} \). The first peak, shown in the inset of Fig. 7(a), falls at the transition out of the completely locked phase at \( F_D = 0.11 \). The second and largest peak is at \( F_D = 0.3 \), while a third broad peak also appears that is centered at \( F_D = 1.45 \). The broad peak is the remnant of the second peak in \( V_{s1} \) found for \( R_{s1,p} = 0.562 \) and 0.625; with increasing \( R_{s1,p} \), this peak broadens and the center shifts to higher values of \( F_D \). For \( 0.11 < F_D < 0.3 \), the particles in \( s1 \) are almost completely locked but there is a single incommensuration which has begun to slip. For \( R_{s1,p} = 0.75 \), the initial peak is lost and the decoupling transition peak now falls at \( F_D = 0.11 \). There is also a very broad maximum centered at \( F_D = 4.0 \). Another interesting feature is that at higher \( F_D \) such as at \( F_D = 6.0 \), \( V_{s1} \) for \( R_{s1,p} = 0.75 \) is higher than \( V_{s1} \) at the lower values of \( R_{s1,p} \), even though at low \( F_D \) \( R_{s1,p} \) showed the lowest value of \( V_{s1} \). This suggests that at high values of \( F_D \), additional drag is produced by the interaction between the incommensurations in \( s1 \) and the particles in \( p \).

In Fig. 7(b) we plot \( V_{s1} \) versus \( F_D \) for \( R_{s1,p} = 0.8125 \), 0.875, 0.9375, and 1.0, from bottom to top. (c) \( R_{s1,p} = 1.0625, 1.125, 1.1875, \) and 1.25, from bottom to top. In vortex systems with 2D periodic pinning arrays, experiments have shown that the pinning is enhanced at the matching fields in the highly
driven system, the vortices form a very ordered moving commensurate state, while at the incommensurate fields the moving state is not as well ordered and thus the effectiveness of the pinning increases away from commensuration at high drives. Although the disorder in the incommensurate state causes the system to begin slipping at a lower drive for incommensurate fields, at high drives the disordered state experiences more fluctuations than the ordered state which induces some additional drag.

In Fig. 7(b) we plot $V^\text{max}_{s1}$ versus $R_{s1,p}$. Here the measurement of $V^\text{max}_{s1}$ is performed not at a fixed $F_D$ but at the $F_D$ where $V_s$ reaches its maximum for each value of $R_{s1,p}$. In this case, a strong peak in $V^\text{max}_{s1}$ appears at $R_{s1,p} = 1.0$. This peak is wider than the peak in $F_c$ at $R_{s1,p} = 1.0$ in Fig. 5(b) due to the fact that the maximum value of $V_s$ increases as $R_{s1,p} = 1.0$ is approached, as shown in Fig. 8(b,c).

C. Finite Size and Temperature Effects

To determine whether further higher order submatching effects in $R_{s1,p}$ can be resolved for larger systems and whether the values of $F_D$ at which the unlocking transitions occur change with system size, we consider the system at $d/a = 0.67$ from Fig. 5(b) and analyze $F_c$ for samples of size $2L$ and $4L$. Here we hold $d$ fixed by increasing $N_p$ to 32 and 64, respectively. In Fig. 8(a) we plot $F_c$ versus $R_{s1,p}$ for samples of size $L$, $2L$, and $4L$, with the curves shifted vertically for clarity. In the larger samples, there are clearly fractional peaks falling at $R_{s1,p} = 1/4$, $1/3$, $1/2$, $2/3$, $3/4$, and $3/2$. The $4L$ sample even shows some evidence of a peak at $R_{s1,p} = 1/8$. We expect that for even larger systems, even more fractional peaks will appear but that the higher order peaks will be increasingly weak in size, similar to the behavior of fractional peaks observed in other systems such as vortices on periodic substrates [33, 34]. In Fig. 8(b) we plot the same data without vertical shifts to show that the depinning thresholds for the three systems overlap exactly; only the resolution is changed by the system size. We find no changes in the velocity-force curves as the size of the sample is increased, indicating that the system sizes we are studying capture the essential behavior. We also find a similar lack of dependence on sample size for the three layer systems that are described in Section IV. We note that for commensurate-incommensurate systems such as the Frenkel-Kontrova model [33], submatching effects theoretically occur for all rational values of $m/n$, where $m$ is the number of particles and $n$ is the number of substrate minima. In the Frenkel-Kontrova model, true incommensurate behavior occurs only for systems of infinite size at irrational filling ratios. In our system the higher order submatching effects are destroyed due to the fact that we do not have a fixed substrate; in-
versus $F$, what are termed Yukawa chains [21]. If more than one plasmas interacting with a one-dimensional groove to create effective incommensurations are more mobile and hence decreases until for $T > T_F$ at lower temperatures. For $T = 0.2$, $0.22$, $0.88$, $2.0$, and $4.5$. As the temperature increases, the value of $F_c$ decreases until for $T > 2.0$ the locked phase has almost completely vanished; however, drag effects on the secondary channel continue to persist up to much higher temperatures. For $R_{s1,p} = 0.896$, the locking phase is lost at lower $T$ than for $R_{s1,p} = 1.0$ due to the fact that the effective incommensurations are more mobile and hence require a smaller level of thermal fluctuations to escape from the potential minima. These results show that the drag and locking features described for the zero temperature system should persist under finite temperature provided that the thermal fluctuations are not excessively strong.

Possible experimental realizations of the two channel system include modified versions of the colloidal experiments which have already been performed on coupled one-dimensional channels [9]. Colloidal systems are subject to thermal fluctuations and hydrodynamic interactions which can arise in the surrounding fluid. We showed above that the dynamic phases are robust against moderate thermal fluctuations. There is ongoing discussion regarding how the inclusion of hydrodynamic effects would impact the dynamics of driven colloidal systems. Recent two-dimensional simulations of an electrophoretically driven charged colloidal system similar to the one we consider showed that when the charge on the colloids are sufficiently strong, the dynamical behavior of the system is not altered by the addition of hydrodynamic interactions [49]. Due to the good agreement that has been found between numerous simulations of driven colloid systems in which hydrodynamic effects are neglected and the actual behavior of driven colloids in experiment, we expect that at least some of the features that we describe should be observable in a colloidal realization of this system. Another possible realization would be to generalize the recent experiments performed with dusty plasmas interacting with a one-dimensional groove to create what are termed Yukawa chains [21]. If more than one groove were created in the substrate, it should be possible to couple two or more of the Yukawa chains, to use a laser to drive one of the chains, and then to analyze the response of the secondary chain. In this case inertial effects could modify the behavior since dusty plasma systems are generally not in the overdamped limit.

**D. Ratchet Effect With ac Drives**

We next show that when the particles in $p$ are driven with an ac drive, it is possible to generate a net dc motion of the particles in $s_1$ or a ratchet effect. Ratchet effects produced by applied ac drives have been studied extensively in systems of particles interacting with asymmetric substrates[50]; however, it is also possible to create a ratchet effect in the absence of an asymmetric substrate when the ac drive has certain asymmetries and when the response of the system is nonlinear [42, 44, 51]. This type of ratchet has been realized in systems with two interacting species of superconducting vortices such as when Josephson vortices couple to pancake vortices [42, 44], as well as in interacting binary colloidal systems where only one colloid species couples to an external driving field and produces a rectification of the other colloid species [51].

Here we consider an ac square drive applied only to $p$. The period $\tau$ of the square drive is divided unevenly
into two parts as illustrated in the inset of Fig. 9. In part A, we apply a force $\mathbf{F}_D = F_D \hat{x}$ in the positive direction for a duration $\tau_A$, while in part B we apply a force $\mathbf{F}_D = -F_D \hat{x}$ in the negative direction for a duration $\tau_B = \tau - \tau_A$. In selecting $F_A$ and $F_B$, we impose the condition $F_A \tau_A - F_B \tau_B = 0$ so that there is no net dc drive. If the response of the system is perfectly linear, this drive will not generate a net dc motion of the particles in either channel. On the other hand, if the coupling between $s_1$ and $p$ is nonlinear, it is possible to induce a dc motion of the particles in $s_1$ by applying this ac drive to the particles in $p$. If both $F_A$ and $F_B$ are below the first decoupling transition $F_c$, the motion of the particles in both channels is completely locked, the response is perfectly linear, and there is no ratchet effect. If $F_A < F_c$ and $F_B > F_c$, a net dc drift of the particles in $s_1$ will occur since the particles in $s_1$ remain completely locked with the particles in $p$ during part A of the drive cycle, but during part B of the cycle the particles in $s_1$ are partially decoupled and do not move all the way back to their starting position by the end of the cycle.

In Fig. 9 we illustrate the ratchet effect which produces a finite positive value of $V_{s1}$ under the ac drive described above. We fix $F_B/F_A = 4.0$ and $d/a = 0.67$, and plot the time-averaged $V_{s1}$ versus $F_A$ for $R_{s1,p} = 0.75, 1.0, and 1.25$. For low $F_A$, $V_{s1}$ starts small but rapidly grows with increasing $F_A$, reaching a sharp peak for $R_{s1,p} = 0.75$ and $R_{s1,p} = 1.0$. As $F_A$ increases above this peak, $V_{s1}$ gradually decreases with increasing $F_A$ since the drag effect becomes smaller for higher drives as shown in Fig. 2(a). For $R_{s1,p} = 1.25$, the ratchet effect is strongly reduced but still persists, indicating that the ratchet effect should be a robust feature for all fillings. In all cases there is no induced dc flow of the particles in $p$. Our system can be regarded as containing two species of particles: the directly driven particles in $p$, and the undriven particles in $s_1$ that experience a drag from the particles in $p$. It would be very interesting to look for a similar ratchet effect in coupled quantum wires in the regime where Wigner crystallization may be occurring. This could be achieved by applying an ac drive of the type illustrated in the inset of Fig. 9 to one wire and determining whether a dc response is induced in the second wire.

IV. THREE CHANNEL SYSTEMS

A. Coupling-Decoupling Transitions for Partial Commensuration

We next consider a system with three channels of particles where only the top channel is subjected to a driving force. We measure the velocities in each channel, denoted by $V_p$, $V_{s1}$, and $V_{s2}$, for particle ratios of $R_{s1,p} = N_{s1}/N_p$, $R_{s2,p} = N_{s2}/N_p$, and $R_{s2,s1} = N_{s2}/N_{s1}$. For the commensurate case when all channels contain the same number of particles, $R_{s1,p} = R_{s2,p} = R_{s2,s1} = 1.0$, the behavior is the same as in the two channel case at commensuration. There is a single decoupling transition from region I, the completely locked phase, to region II, where the particles in $s_1$ and $s_2$ remain locked with each other but are partially decoupled from the particles in $p$. This is illustrated in the plot of $V_p$, $V_{s1}$, and $V_{s2}$ versus $F_D$ in Fig. 10(a) for a sample with $d/a = 0.67$, $R_{s1,p} = 1.0$, and $R_{s2,s1} = 1.0$. The decoupling between the primary and the secondary channels occurs at $F_D = 1.75$. The value of $F_D$ at decoupling is lower than for a sample containing only two channels since the primary channel must now drag twice as many secondary particles.

In Fig. 10(b) we plot the channel velocities versus $F_D$ for a sample with $R_{s1,p} = 1.0$ but with more particles in $s_2$, $R_{s2,s1} = 1.16$. For $F_D < 0.37$ the system is in the...
completely locked region I, while for $0.37 \leq F_D < 1.625$ the particles in $p$ and $s_1$ remain locked but the particles in $s_2$ partially decouple. We term this range of $F_D$ region III, and in this region $V_{s_2}$ still increases with increasing $F_D$. For $1.625 \leq F_D < 3.56$, all three of the channels are unlocked; we call this region IV. Within region IV, the velocity curves contain numerous small steps associated with the intermittent coupling of the particles in $s_1$ and $s_2$. At the low $F_D$ end of region IV, $V_{s_1}$ and $V_{s_2}$ both decrease with increasing $F_D$, but for $2.7 < F_D < 3.56$, $V_{s_2}$ begins to increase with increasing $F_D$ until $V_{s_1}$ and $V_{s_2}$ join at the recoupling transition into region II. Once the system is in region II, both $V_{s_1}$ and $V_{s_2}$ decrease monotonically with increasing $F_D$.

For samples with $R_{s_1,p} = 1.0$ but with increasing $R_{s_2,s_1}$, the general features of the velocity force curves are the same as Fig. 10(b), but the transition into region II is pushed to higher $F_D$. This is illustrated in Fig. 11(c) where we plot $V_p$, $V_{s_1}$, and $V_{s_2}$ versus $F_D$ for a system with $R_{s_1,p} = 1.0$ and $R_{s_2,s_1} = 1.5$. Here region II does not appear until $F_D = 12.5$. Fig. 11(c) also shows more clearly the increase in $V_{s_2}$ just below the onset of region II. For samples with $R_{s_1,p} = 1.0$ and $R_{s_2,s_1} < 1.0$, only regions I and II occur and the velocity force curves have the same form as the curves illustrated in Fig. 10(a).

In Fig. 11 we map out the dynamic phase diagram for a three channel system with $R_{s_1,p} = 1.0$ and varied $R_{s_2,s_1}$. The value of $F_D$ at which a transition out of region I occurs shows commensurate peaks at $R_{s_2,s_1} = 1.0$ and $R_{s_2,s_1} = 2.0$, while the region III-region IV transition falls at a roughly constant value of $F_D = 2.1$. The region II-region IV transition line shifts to slightly higher $F_D$ with increasing $R_{s_2,s_1}$. This trend continues for $F_D$ values higher than those shown in Fig. 11 until at $R_{s_2,s_1} = 2.0$ the II-IV transition drops to a value of $F_D = 7.5$ (not shown in the figure). These results indicate that commensurability effects also occur in the moving phases at high $F_D$.

**B. Dynamics for Increased Incommensuration**

In Figs. 12 and 13 we plot $V_p$, $V_{s_1}$, and $V_{s_2}$ versus $F_D$ for a three channel system with $R_{s_1,p} = 0.75$ and varied $R_{s_2,p}$. (a) At $R_{s_2,p} = 1.0$ there is a single transition from region I to region II. (b) At $R_{s_2,p} = 0.833$, the single region I-region II transition is accompanied by an additional secondary maximum in $V_{s_1}$ and $V_{s_2}$ centered at $F_D = 1.75$.
ondary channels. In Fig. 13(a) we divide region IV into two subregions. Just above the III-IV transition we have region IV, in which \( V_{s1} \) increases with increasing \( F_D \), and \( V_{s2} \) decreases with increasing \( F_D \). In region IV, \( V_{s1} \) decreases with increasing \( F_D \) and \( V_{s2} \) increases with increasing \( F_D \). There is a transition to region II at high \( F_D \). (b) At \( R_{s2,p} = 1.583 \), there is a small window of region I at low \( F_D \) which is not highlighted on the figure. There is a transition directly from region III to region IV, with region IV absent.

FIG. 13: \( V_p, V_{s1}, \) and \( V_{s2} \) versus \( F_D \) for a three channel system with \( R_{s1,p} = 0.75 \) and varied \( R_{s2,p} \). (a) At \( R_{s2,p} = 1.25 \), region I is followed by a transition into region III. In region IV, all the channels are unlocked, \( V_{s1} \) increases with increasing \( F_D \), and \( V_{s2} \) decreases with increasing \( F_D \). In region IV, \( V_{s1} \) decreases with increasing \( F_D \) and \( V_{s2} \) increases with increasing \( F_D \). There is a transition to region II at high \( F_D \). (b) At \( R_{s2,p} = 1.583 \), there is a small window of region I at low \( F_D \) which is not highlighted on the figure. There is a transition directly from region III to region IV, with region IV absent.

FIG. 14: The dynamic phase diagram of \( F_D \) vs \( R_{s2,p} \) for the system in Fig. 12 with \( R_{s1,p} = 0.75 \).

FIG. 15: \( V_p, V_{s1}, \) and \( V_{s2} \) vs \( F_D \) for the three channel system with \( R_{s1,p} = 1.25 \) and \( d/a = 0.67 \). (a) At \( R_{s2,s1} = 0.6 \) only regions I and II are present. (b) At \( R_{s2,s1} = 0.86 \) the system enters region II more than once.

aries at \( R_{s2,p} = 1.5 \), which is the \( R_{s2,s1} = 2.0 \) filling. The value of \( F_D \) at which the transition from region II to region IV occurs increases with increasing \( R_{s2,p} \), except at \( R_{s2,p} = 1.5 \) where the II-IV transition suddenly drops to a lower value of \( F_D \).

In Fig. 15(a) we plot \( V_p, V_{s1}, \) and \( V_{s2} \) versus \( F_D \) for a three layer system with \( R_{s1,p} = 1.25 \) at \( R_{s2,s1} = 0.6 \).
Only regions I and II are present, and there is an additional second broad maximum in $V_{s1}$ and $V_{s2}$ centered near $F_D = 1.5$. In general, for $R_{s1,p} = 1.25$ and $R_{s2,s1} < 0.8$ or $R_{s2,s1} > 1.0$, only regions I and II appear. For $0.86 \leq R_{s2,s1} < 1.0$, region II is broken into two sections by an intermediate transition to region IV, as shown in Fig. 16(b) for $R_{s2,s1} = 0.86$. The system passes from region I to region II, then enters region IV and finally returns to region II at high $F_D$.

The dynamic phase diagrams presented here give a concise description of the velocity-force curves as the system parameters are varied. Such dynamic phase diagrams have been widely used in studies of driven particle systems such as vortices in type-II superconductors [14]; however, they have no connection with equilibrium phase diagrams obtained from systems in the thermodynamic limit. Having more than three phase transition lines meet in an equilibrium phase diagram would be highly unusual; however, in the nonequilibrium dynamic phase diagram, having more than three lines meet has no special implications since the lines do not represent true phase transition lines. Whether nonequilibrium systems can undergo true phase transitions that resemble equilibrium phase transitions is currently a topic of active study and is beyond the scope of this manuscript to address.

The appearance of multiple phases typically occurs when the $s_1$ and $s_2$ channels can become unlocked with each other. For $R_{s1,s2} < 1.0$, incommensurations in the form of holes are present in one channel; however, the mobility of the holes is less than that of the interstitials which arise when $R_{s1,s2} > 1.0$.

### C. Negative Drag

For $1.0 < R_{s2,s1} \leq 1.6$ and fixed $R_{s1,p} = 1.25$, we show that a negative drag effect can occur for the particles in $s_2$. During negative drag, the particles in $s_2$ move in the direction opposite to the direction in which the particles in $p$ are being driven. Negative drag has been observed in coupled 1D wires where Wigner crystallization is expected to occur [52]. In Fig. 16(a) we plot $V_p$, $V_{s1}$, and $V_{s2}$ for a three channel system with $R_{s2,s1} = 1.067$. Here the sample is in the locked region I for $F_D < 0.1$. For $0.1 < F_D < 1.4$, region IV$^A$ appears with all three channels decoupled, $V_{s1}$ increasing with increasing $F_D$, and $V_{s2}$ decreasing with increasing $F_D$. At $F_D = 1.4$ there is a cusp in both $V_{s1}$ and $V_{s2}$ at the onset of region IV$^B$. The cusp also marks the point at which $V_{s2}$ reaches its maximum negative value. In Fig. 16(a) this is labeled ND for the negative drag region, which extends from $1.0 < F_D < 1.6$. In Fig. 16(b) we plot $V_{s2}$ alone versus $F_D$ for the system in Fig. 16(a) showing the negative drag effect more clearly and also showing the presence of a local maximum in $V_{s2}$ at $F_D = 4.5$. Above this drive, $V_{s2}$ decreases with increasing $F_D$ but remains positive.

In Fig. 17 we plot the time dependent velocity $v(t)$ of a single particle in each of the three channels for the system in Fig. 16 at $F_D = 1.36$ where the particles in $s_2$ undergo negative drag. The velocity of the particle in $p$ is always positive and is composed of two frequencies. The velocity of the particle in $s_1$ again shows two frequencies and drops below zero for a portion of each cycle; however, the overall time average of the velocity remains positive. The particle in $s_2$ also experiences a combination of positive and negative velocities; however, the negative velocity portion of each cycle is greater than the positive velocity portion, and the particle takes a step backwards at the negative cusp in each cycle. It was previously demonstrated that a system driven by two external ac drives can exhibit a ratchet effect in the absence of an asymmetric substrate [52]. In our three channel system, when $N_p$, $N_{s1}$, and $N_{s2}$ are all different, the dynamical potential produced by the particles in $p$ and $s_1$ acts effectively like two ac driving signals for the particles in $s_2$. In some cases, the interfering frequencies of these ac drives can create a local potential maximum in $s_2$ that is moving in a direction opposite to $F_D$. As $F_D$ is further increased, the different ac frequencies shift, increasing or decreasing the ratchet effect until for high enough $F_D$ the coupling between $s_2$ and the particles in the other channels becomes so weak that a ratchet effect can no longer occur.
always positive. The velocity of the particle in each of the channels for the system in Fig. 15 at $F = 1.36$. Upper curve: $p$; middle curve: $s_1$; lower curve: $s_2$. The velocity of the particle in $p$ exhibits two frequencies and is always positive. The velocity of the particle in $s_1$ also shows two frequencies and passes below zero for a portion of each cycle, but the time averaged velocity remains positive. The particle in $s_2$ spends a larger fraction of each cycle moving in the negative direction, producing a negative time averaged velocity.

In Fig. 18 we plot only the normalized velocities $V_{s_2}$ versus $F_D$ for three channel samples with $R_{s_1,p} = 1.25$ and $R_{s_2,s_1} = 1.13, 1.2, 1.26, 1.33$, as labeled. The largest negative maximum occurs for $R_{s_2,s_1} = 1.2$.

FIG. 17: The time dependent velocity $v$ of a single particle in each of the channels for the system in Fig. 15 at $F_D = 1.36$. Upper curve: $p$; middle curve: $s_1$; lower curve: $s_2$. The velocity of the particle in $p$ exhibits two frequencies and is always positive. The velocity of the particle in $s_1$ also shows two frequencies and passes below zero for a portion of each cycle, but the time averaged velocity remains positive. The particle in $s_2$ spends a larger fraction of each cycle moving in the negative direction, producing a negative time averaged velocity.

In Fig. 18 we plot only the normalized velocities $V_{s_2}$ versus $F_D$ for three channel samples with $R_{s_1,p} = 1.25$ and $R_{s_2,s_1} = 1.13, 1.2, 1.26, 1.33$, as labeled. The largest negative maximum occurs for $R_{s_2,s_1} = 1.2$.

FIG. 18: $V_{s_2}$ vs $F_D$ for the system in Fig. 15 at $R_{s_2,s_1} = 1.13, 1.2, 1.26, 1.33$, as labeled. The largest negative maximum occurs for $R_{s_2,s_1} = 1.2$.

In Fig. 19(a) we show $|V_{s_2}|$ taken at the IV$^A$-IV$^B$ transition as a function of $R_{s_1,s_2}$ showing that the overall maximum negative value of $V_{s_2}$ occurs at $R_{s_1,s_2} = 1.2$. In Fig. 19(b) we show the dynamic phase diagram of $F_D$ versus $R_{s_2,s_1}$ for the three channel system with $R_{s_1,p} = 1.25$ and $d/a = 0.67$. There are peaks in the transition out of region $I$ at $R_{s_2,s_1} = 0.4, 0.6, 0.8, 1.2$, and $1.8$. These peaks correspond to $R_{s_2,p} = 0.5, 0.75, 1.0, 1.5$, and $2.25$, with the most prominent peak appearing at $R_{s_2,p} = 1.0$. For $R_{s_2,s_1} < 0.8$ the system exhibits only regions $I$ and $II$, while for $0.8 < R_{s_2,s_1} \leq 1.0$, the transition from region $I$ to region $II$ is followed by a transition into region IV$^B$ at higher $F_D$. At even higher $F_D > 7.7$, not shown in the figure, the line marking the transition from region $II$ to region IV$^B$ changes curvature and approaches $R_{s_2,s_1} = 1.0$ with increasing $F_D$. As a result, for $0.8 < R_{s_1,s_2} \leq 1.0$ there is a high-drive transition from region IV$^B$ back to region $II$ (not shown) when the particles in $s_1$ and $s_2$ recouple, similar to the region IV$^A$-region $II$ transition illustrated at high $F_D$ in Fig. 15(b). At $R_{s_2,s_1} = 1.0$, the dashed line indicates the transition from region IV$^B$ to region IV$^A$. For $R_{s_2,s_1} > 1.5$, the upper region IV$^A$-region IV$^B$ transition saturates to the line $F_D = 0.18$. Near $R_{s_2,s_1} = 1.0$, the upper IV$^A$-IV$^B$ tran-
FIG. 20: The dynamic phase diagram of $F_D$ vs $R_{s2,s1}$ for the three channel system with $R_{s1,p} = 1.25$ and $d/a = 0.67$. The prominent commensurate peak in the region I-region II transition at $R_{s2,s1} = 0.8$ also corresponds to the commensurability condition of $R_{s2,p} = 1.0$. The dashed line indicates that at $R_{s2,s1} = 1.0$, the system crosses from region IV$^B$ to region IV$^A$. At higher $F_D$ (not shown), the line marking the end of region IV$^A$ approaches $R_{s2,s1}$ from above, and once it reaches $R_{s2,s1}$, region IV$^A$ disappears. Also at higher $F_D$ (not shown), the line marking the beginning of region IV$^B$ approaches $R_{s2,s1}$ from below, producing a transition from region IV$^B$ to region II with increasing $F_D$.

FIG. 21: $V_p$, $V_{s1}$, and $V_{s2}$ vs $F_D$ for a three channel system with $R_{s1,p} = R_{s2,s1} = 1.13$, $R_{s2,p} = 1.0$, and $d/a = 0.94$. Here we observe a transition from region I to region V, where the particles in $p$ and $s_2$ remain locked to each other but the particles in $s_1$ are unlocked. This is followed by region IV, when the particles in $s_2$ unlock from the particles in $p$ and $V_{s1}$ increases with increasing $F_D$. (b) The same data plotted over a larger range of $F_D$ shows that $V_{s1}$ reaches a plateau at $F_D = 1.9$ and then decreases with increasing $F_D$.

D. Unlocking of the Central Channel

Another possible dynamic phase has the particles in $p$ and $s_2$ locked with each other while the particles in $s_1$ are unlocked. We term this region V, and expect it to occur when the average interaction between the particles in $p$ and $s_2$ is greater than the interaction between the particles in $p$ and $s_1$ even though the distance between $s_1$ and $p$ is shorter than the distance between $s_2$ and $p$. In Fig. 21(a) we show an example of the occurrence of region V in a system with $R_{s1,p} = 1.133$, $R_{s2,s1} = 1.133$, $R_{s2,p} = 1.0$, and $d/a = 1.06$. In this case the $p$ and $s_2$ channels are commensurate. At low $F_D$, the system is in the locked phase I. As $F_D$ increases, the particles in $s_1$ decouple from the particles in $s_2$ and $p$, which remain locked to each other. This is indicated by the region in which $V_{s1}$ splits away from $V_p$ and $V_{s2}$ and increases at a diminished rate with increasing $F_D$. At $F_D = 0.14$, the particles in $s_2$ also decouple from $p$ and the system enters region IV, in which $V_{s2}$ monotonically decreases with increasing $F_D$. After the particles in $s_2$ decouple from the particles in $p$, the coupling between the particles in $p$ and $s_1$ is increased, as indicated by the increase in the slope of $V_{s1}$ at the onset of region IV. $V_{s1}$ continues to increase with increasing $F_D$ throughout region IV and even rises above $V_{s2}$ for $F_D > 0.2$. In Fig. 21(b) we plot the same data over a larger range of $F_D$ to show that $V_{s2}$ reaches a maximum value near $F_D = 1.9$ before turning over and beginning to decrease with increasing $F_D$.

The results in Fig. 21 show that it is possible to achieve region V in certain situations, such as when the particles in $p$ and $s_2$ are commensurate. In general it is very difficult to obtain region V behavior in our system. The coupling between the particles in $p$ and those in $s_2$ is relatively weak since the distance between $p$ and $s_2$ is equal to the screening length. As a result, particles in $s_2$ experience a weak interaction only with those particles in $p$ that lie directly above their positions, and interact much more weakly still with the other particles in
$p$. (Note that we do not cut off the interaction at the screening length, but continue to compute the weak interaction out to longer distances.) This suggests that for different screening lengths $1/\kappa$ and interchannel distances $d$ the coupling between particles in $p$ and particles in $s_2$ could be enhanced, producing a more widespread occurrence of region V and leading to additional commensuration effects. The densities of the particles in the channels, and not merely the ratio of their numbers, also plays an important role in determining which dynamical regions will appear. For higher particle density (smaller $a$), the couplings between the particles in all the channels are reduced, as demonstrated for the two channel case in Fig. 2(b). Even if the effective coupling between the particles in $p$ and those in $s_2$ is strengthened by altering the density of the particles in the channel, this coupling must still be stronger than the coupling between the particles in $p$ and those in $s_1$ in order for region V to appear.

In order to understand where region V occurs as a function of the coupling between the channels, in Fig. 22(a) we plot the dynamic phase diagram of $F_D$ versus $d/a$ for a system with fixed $R_{s_1,p} = 1.133$, $R_{s_2,p} = 1.0$, and $R_{s_1,s_2} = 0.883$. For small $d/a$ the system passes directly from region I to region II. At $d/a = 0.5$ a window of region IV opens between regions I and II. Region V first appears at $d/a = 0.75$, and gradually disappears for increasing $R_{s_1,s_2}$. We next consider the case of $d/a = 1.06$ and $R_{s_2,p} = 1.0$ for varied $R_{s_2,s_1}$, as shown in Fig. 22(b). Here, there is a pronounced commensurability peak in the transition out of region I at $R_{s_2,s_1} = 1.0$, where all the channels contain the same number of particles. At $R_{s_2,s_1} = 1.0$ the system passes directly from region I to region II. In windows just below and just above $R_{s_2,s_1} = 1.0$ we find that region V appears and is accompanied by a transition to region IV with increasing $F_D$. The width of region V grows as $R_{s_1,s_2}$ is increased. For $R_{s_1,s_2} > 1.0$, region V gradually decreases in size with increasing $R_{s_2,s_1}$; while for $R_{s_2,s_1} < 0.75$, region V vanishes completely. For $0.5 < R_{s_2,s_1} < 0.75$ the system transitions from region I into region IV with increasing $F_D$ and eventually enters region II at high $F_D$ (not shown). For $R_{s_2,s_1} < 0.5$ there is only a single transition from region I to region II. A second commensurate peak in the transition out of region I appears at $R_{s_2,s_1} = 0.5$. 
E. Five Dynamical Phases

In Fig. 23 we plot the dynamic phase diagram of $F_D$ versus $R_{p,s^2} = N_p/N_{s^2}$ for a system which exhibits all five phases as well as several regions where a negative drag effect occurs. Here we vary $N_p$ and fix $R_{s^2,s^1} = 1.133$ and $d/a_{s^1} = 0.833$, where $a_{s^1}$ is the spacing of the particles in $s_1$. For this choice of parameters, we observe region V only at $R_{p,s^2} = 1.0$, the value shown in Fig. 24. At $R_{p,s^2} = 0.882$, which also corresponds to $R_{s^1,p} = 1.0$, there is a single transition from region I to region III, indicated by the dashed line. Here the particles in $p$ and $s_1$ are locked because they are commensurate. The dynamics for $0.6 < R_{p,s^2} < 1.75$ is dominated by region IV. For $1.75 < R_{p,s^2} < 1.95$, a transition from region IV to region II occurs at higher $F_D$. The location of this transition shifts to higher values of $F_D$ as $R_{p,s^2}$ drops below $R_{p,s^2} = 1.95$. For $R_{p,s^2} > 1.95$, the system goes directly into region II for finite $F_D$. Region II appears for high $R_{p,s^2}$ since as $N_p$ increases, the effectiveness of the coupling between the particles in $p$ and the particles in $s_1$ and $s_2$ decreases. As a result, even though the particles in $s_1$ and $s_2$ are incommensurate, the coupling between the primary and secondary channels eventually becomes so weak that the particles in $s_1$ and $s_2$ couple with each other and decouple from the particles in $p$. At $R_{p,s^2} = 0.58825$ there is another peak in the transition out of region I produced by the commensurability condition of $R_{s^1,p} = 2/3$ at this filling. For $R_{p,s^2} < 0.6$, region I grows in extent and there is a window of region III which separates region I at low drives and region IV at higher drives. For $R_{p,s^2} < 0.15$, there is a single transition from region I directly to region IV.

In Fig. 24 we indicate the regions in the $F_D$ versus $R_{p,s^2}$ plot where negative drag of the particles in $s_2$ occurs for the system in Fig. 23 and in Fig. 25 we show representative velocity force curves for the three different negative drag regions. In Fig. 24 the largest region of negative drag occurs for $0.52 < R_{p,s^2} < 0.82$. There is a small negative drag window near $R_{p,s^2} = 0.3$. We illustrate a typical velocity force curve from this window in Fig. 25(a) where we plot $V_p$, $V_{s^1}$, and $V_{s^2}$ for $R_{p,s^2} = 0.35$. Here the negative drag occurs in region IV. There are also a number of slip events which appear as sharp changes in $V_{s^2}$ near $F_D = 1.25$. For higher $F_D$ beyond what is shown in the figure, $V_{s^2}$ continues to increase back above zero, passes through a broad peak, and then slowly decreases back toward zero at high $F_D$. In Fig. 25(b) we plot the velocity force curves at $R_{s^2,p} = 0.823$ where the system exhibits only region IV flow. Here the maximum negative value of $V_{s^2}$ occurs at $F_D = 0.95$ in the form of a cusp which is accompanied by a cusplike peak in $V_{s^1}$. For $0 < F_D < 0.96$, $V_{s^1}$ increases linearly with increasing $F_D$ but the particles in $s_1$ are not completely locked with the particles in $p$. This corresponds to region IV$^A$ as was discussed earlier; however, in the phase diagram of Fig. 26 we omit the distinction between regions IV$^A$ and IV$^B$ for clarity. In Fig. 25(c) we plot the velocity force curves at $R_{p,s^2} = 1.53$ in the third region of negative drag. Here we find that the magnitude of the maximum negative velocity in $V_{s^2}$ is reduced compared to the other
two negative drag regions.

The general features of the phases outlined so far also occur for other parameters of density and filling, indicating that they are robust features of the system. We have not observed negative drag of the particles in \( s_1 \) or \( p \).

V. DISCUSSION AND SUMMARY

We investigated a simple system consisting of two or three coupled 1D channels of particles interacting via a repulsive Yukawa potential where only one of the channels is driven. For two channel systems with an equal number of particles in each channel, we find a single transition from a completely locked state to a partially decoupled state where particles in the secondary channel slip with respect to particles in the driven channel. In the decoupled state, the velocity of the particles in the secondary channel gradually decreases with increasing drive while the velocity of the driven particles increases linearly with increasing drive. When the number of particles in the secondary channel is slightly away from commensuration with the number of particles in the primary channels, a two stage decoupling transition occurs where the first decoupling is associated with individual slips of the incommensurations or vacancies in the secondary channel. The velocity of the particles in the secondary channel continues to increase with increasing drive until the second decoupling transition is reached, whereupon all the particles in the secondary channel begin to slip. The driving force at which the transition from the completely locked to the decoupled flow occurs has peaks at integer incommensurate ratios of the number of particles in the two channels as well as at certain fractional ratios such as 1/2 or 3/2; however, there are no peaks for low filling ratios since the particles in the driven channel are effectively moving not over a fixed substrate but over a distortable substrate. We also observe a ratchet effect in the two channel system where the particles in the secondary channel can be rectified by an asymmetric ac drive applied to the primary channel. This ratchet effect is similar to the ratchet effect found for coupled binary particle species where only one species is driven.

For three channels we find that a remarkably rich variety of dynamical phases such as coupling and decoupling transitions are possible and produce a variety of commensuration effects as well as pronounced signatures in the velocity force curves. The commensuration effects occur whenever the ratio of the number of particles in at least two of the channels is an integer or rational fraction. We also observe a negative drag effect for the secondary channel which is furthest from the driven channel. Here, the particles in the secondary channel move in the direction opposite to the driving direction of the primary channel. When the negative drag occurs, all three channels have incommensurate fillings. The resulting multiple periodic forces experienced by the particle in the furthest secondary channel create a bi-harmonic ratchet effect of a type that has been observed in systems driven with multiple ac drives.

Our results could be tested for colloidal particles confined to two or three channels where one of the channels is driven by optical means or via microfluidics. Since the motion of physical colloids is never perfectly one-dimensional, some smearing of the effects we observe might occur, but the general features we describe should be observable. A similar experiment could be performed in a dusty plasma system with the dust particles confined in grooves and driven in one dimension with a laser focused in a single plane. Some of the effects we observe could be relevant for certain superconducting vortex systems in which two different types of vortices are coupled and one of the two vortex types is driven with an external current. Additionally, these effects could also be realized using coupled wires in which one-dimensional Wigner crystal states occur. The velocity-force responses that we predict could be a potentially powerful method for determining whether Wigner crystals are actually present in the wires. It would also be interesting to study ratchet effects with asymmetric ac drives for three or more channels. Here, it may be possible to induce dc currents flowing in different directions for different channels. Although the system we consider appears very simple, we have shown that it exhibits a rich variety of behaviors even without substrates or other complications. If a periodic substrate were introduced in one or more of the channels, we expect that an even greater variety of commensuration effects and coupling between excitations in the channels could occur. It would also be interesting to consider cases where the channels are not strictly one-dimensional but have a finite width to allow for transitions to buckled or zig-zag states. Even for the incommensurate fillings, such buckling transitions could produce interesting new features in the drag behavior.

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