A measurement method of transverse light-shift in atomic spin co-magnetometer

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Received: 10 October 2021 / Accepted: 22 February 2022 / Published online: 11 March 2022
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Abstract
We disclose a method to obtain the transverse light-shift along the probe light of a single-axis alkali metal-noble gas co-magnetometer. The relationship between transverse compensating field and light-shift is deduced through the steady-state solution of Bloch equations. The variety of probe light intensity is used to obtain the residual magnetic field, and step modulation tests are applied to acquire the total spin-relaxation rate of electron spins and self-compensation point. The transverse light-shift can be calculated by these parameters, and it is reduced from −0.115 to −0.039 nT by optimizing the probe light wavelength. Finally, long-term static test for a K-Rb-21Ne co-magnetometer has been performed. The bias instability of about 0.009 deg/h and rate ramp coefficient of 0.096 (deg/h)/h have been achieved to prove the improvement of stability.

1 Introduction

An alkali metal-noble gas co-magnetometer has found a wide range of applications in sensing rotation rate [1–3], testing Lorentz and CPT violation [4, 5], and searching for anomalous spin-dependent forces [6, 7]. In atomic spin co-magnetometers, the electron spins of alkali-metal are operated in spin-exchange relaxation free (SERF) regime [8], so that the electron spins are ultra-high sensitive to magnetic field [9]. The nuclear spins of noble gas can be hyperpolarized by spin-exchange collisions with the polarized electron spins [10]. The polarized nuclear spins have ability of self-compensation to the external magnetic fields, and meanwhile, the ultra-high sensitivity of rotation rate can be maintained [2]. However, the transverse light-shift (ac-Stark) [11–13] arising from the non-ideal linearly polarized probe beam with detuning wavelength can affect the direction and stability of electron spins [14, 15]. Thus, the research on measurement and optimization of the transverse light-shift is essential for atomic spin co-magnetometers.

Recently, the influence of vector light-shift caused by the circularly polarized pump light of atomic spin co-magnetometer in SERF regime has been widely studied [16–18]. The light-shift interaction appears as a fictitious magnetic field, so that the atomic spins process around the total effective magnetic field [19]. Therefore, the light-shift should be minimized as far as possible. Moreover, the principle of light-shift caused by the pump light of a K-Rb-21Ne co-magnetometer has been analyzed [20], and the light-shift of Rb atoms can be reduced by collision mixing [11]. Furthermore, the longitudinal light-shift would lead to cross-talk effect for a dual-axis co-magnetometer, which limits the measurement accuracy of the two sensitive axes [2, 21]. However, there are few studies on transverse light-shift caused by the probe light of SERF co-magnetometer, which has usually been ignored in the steady-state solution of Bloch equations [22, 23]. The light-shift caused by a linearly polarized probe light in nuclear magnetic resonance (NMR) gyroscope [24] and atomic clock [25] have also been discussed. These researches most consider the probe light as an ideal linearly polarized light, which only causes tensor light-shift. Therefore, the existence of the transverse light-shift cannot be neglected to improve the accuracy.

In this paper, we propose a method to measure the transverse light-shift based on the steady-state solution of Bloch
equations through the variety of probe light intensity and transverse compensation magnetic field. The relationship between transverse light-shift and wavelength of the probe light has been studied, and we propose an optimization method for reducing the light-shift. Finally, the transverse light-shift has been reduced from $-0.115$ to $-0.039$ nT and the light-shift related term has been effectively suppressed. Meanwhile, the calibration coefficient and long-term stability of co-magnetometer have been concurrently improved.

2 Theory of the measurement method

The behavior of K-Rb-21Ne co-magnetometer can be represented by a set of Bloch equations. The evolutions of electron spin polarization $P^e$ and nuclear spin polarization $P^n$ can be described as below [1, 2],

$$\frac{dP^e}{dt} = \frac{\gamma_e}{Q(P^e)} \left( B + B_n + L \right) \times P^e - \Omega \times P^e + \left( R_p P^e + R_{se} P^n + R_m P^n - R_{ot} P^e \right)$$

$$\frac{dP^n}{dt} = \gamma_n \left( B + B_x \right) \times P^e - \Omega \times P^n + R_{se} (P^e - P^n) - R_{ot} P^n,$$

where $\Omega$ is the rotation velocity. $Q(P^e)$ is the slowing-down factor of electron. $\gamma_e$ and $\gamma_n$ are the gyromagnetic ratios of electron and nuclear spins, respectively. $R_p$ and $R_m$ are the pumping rates produced by the pump and probe lights. $R_{se}$ and $R_{ot}$ are the spin-exchange rates from electron to nucleus and from nucleus to electron, respectively. $R_{tot}$ is the total spin-relaxation rate of electron spins, which is equal to $R_p + R_m + R_{se} + R_{ot}$. $R_p$, $R_{se}$, and $R_{ot}$ are the electron and nuclear spin-destruction rate, respectively. $B_y$ and $B_x$ are the magnetic fields generated by the magnetizations of electron and nuclear spins. The electron spins process in the residual magnetic field $B$. $L$ is the light-shift field, which is a magnetic-like field coupling to electrons generated by the pump and probe lights. When the probe beam is along $x$-direction, the steady-state solution of Bloch equations about $L$ caused by the probe light can be simplified as follows,

$$P^e(L) \approx \frac{\gamma_e P^e R_{tot}^2}{\gamma_e^2 (L + \delta B_z)^2} \left[ -\frac{\delta B_z L \gamma_e}{R_{tot} B_n} + \frac{\gamma_e L \gamma_n}{R_{tot}} + o(10^{-6}) \right],$$

where $\delta B_z = B_z - B_x$ is residual magnetic field along $z$-axis after the coil compensation. $B_x = -B_p - B_n$ is self-compensation point of the co-magnetometer to cancel the fields from nuclear and electron magnetization. $B_z$ is the compensation magnetic field applied by $z$-axis coil. The transverse light-shift $L_x$ are mainly related to $\delta B_x$, and the longitudinal light-shift $L_z$, which are approximated to constants with the parameters of temperature and pump light invariant. Thus, the $\delta B_z$ related terms of transverse electron spin polarization $P^e_x$ in a K-Rb-21Ne co-magnetometer can be deduced as,

$$P^e_x(\delta B_z) \approx \frac{\gamma_e P^e_x R_{tot}^2}{\gamma_e^2 (L + \delta B_z)^2} \left[ -\frac{\delta B_z L \gamma_e}{B_n} + \frac{L \gamma_e}{R_{tot}} + o(10^{-6}) \right].$$

(3)

Once the compensation field of $B_x$ has been found, the residual magnetic field $B_x$ can be compensated by the coils to create a zero field environment for atomic spins. In order to accurately zero the magnetic field along $y$-axis, a smoothed square wave modulation of $\delta B_z$ at a low frequency and small amplitude should be used. In effect, the modulation takes a derivative of $P^e$ with respect to $\delta B_z$,

$$\frac{\partial P^e_x(\delta B_z)}{\partial (\delta B_z)} \propto \left( B_y + L_x \gamma_e B_x / R_{tot} \right) / B_n.$$
y-axis is zero. It is obvious that the output signal is insensitive to the small various of \( \delta B_{yc} \) when the value of transverse compensating magnetic field \( B_{yc} \) is equal to \(-B_y - \frac{I_0 L_x}{R_{tot}} B_c \).

However, the atomic spins are still sensitive to the variety of magnetic field \( \delta B_{yc} \) when \( B_{yc} \) only compensates for residual magnetic field \( B_y \). Thus, the relationship of \( B_{yc} = -B_y - \frac{I_0 L_x}{R_{tot}} B_c \) should be achieved during the zero procedure in order to reduce the effects of a drifting magnetic field.

In summary, when the residual magnetic field \( B_y \) is determined, the light-shift \( L_x \) presented as fictitious magnetic field can be obtained. Considering depolarization of probe light caused by optical devices and glass cell, the probe light has a certain degree of elliptical polarization. Therefore, \( L_x \) consists of vector light-shift \( L_{x-v} \) and tensor light-shift \( L_{x-t} \).

Considering total energy level splitting effect of the probe light on D1 and D2 lines of Rb atoms, the \( L_{x-v} \) and \( L_{x-t} \) in \( L_x \) can be expressed by [11, 26],

\[
L_{x-v} = \frac{I_0 r_c c}{\gamma_e} \left( -f_{D1} \frac{(v_{probe} - v_{D1})}{(v_{probe} - v_{D1})^2 + (\Gamma_{D1}/2)^2} + f_{D2} \frac{(v_{probe} - v_{D2})}{(v_{probe} - v_{D2})^2 + (\Gamma_{D2}/2)^2} \right) s_x,
\]

\[
L_{x-t} = \frac{I_0 A(v)}{\gamma_e} \left( 3m_F^2 - F(F + 1) \right) (3 \cos^2 \alpha - 1),
\]

where \( r_c \) is classical electron radius and \( c \) is light velocity. \( f_{D1} \) and \( f_{D2} \) are oscillator strength of D1 and D2 lines, respectively. \( s_x \) is ellipticity of the non-ideal probe light. \( v_{D1,D2} \) are the frequency of probe beam, \( \Gamma_{D1,D2} \) are broadening width under D1 and D2 resonance lines, respectively. \( I_0 \) is probe light intensity. \( A(v_{probe}) \) is coefficient related to probe light frequency, whose variation trend with \( v_{probe} \) is Lorentz linear. \( m_F \) is the magnetic sublevel of the total angular momentum \( F \) and \( \alpha \) is a magic angle [26]. The value of total transverse light-shift \( L_x \) is equal to the sum of \( L_{x-v} \) and \( L_{x-t} \). The \( |L_x| \) increases linearly with the increasing of probe intensity, so that the relationship between transverse compensating field \( B_{yc} \) and probe intensity are linear. When the light intensity is zero, the light-shift \( L_x \) can be zeroed at the same time and \( B_{yc} = -B_y \). Therefore, the residual magnetic field \( B_y \) can be obtained by varying the intensity of probe light and deducing \( B_{yc} \) at the point of zero intensity light. Finally, the value of \( L_x \gamma_e B_c/R_{tot}^2 \) can be acquired by adding \(-B_{yc} \) and \(-B_y \). Moreover, the total relaxation rate \( R_{tot}^2 \) and the compensation point \( B_c \) can be calculated by fitting the dispersion curves generated by \( B_y \) modulation as a function of \( \delta B_y \) [14]. The step modulation output signals \( \Delta S \) can be expressed as follows,

\[
\frac{\Delta S}{\Delta B_y} = k \frac{P_c R_{tot}^2}{\gamma_e B_c} \cdot \frac{\delta B_y}{((\delta B_y + L_x)^2 + (R_{tot}^2/\gamma_e^2))^2},
\]

where \( k \) is a constant relating to the light intensity and amplifier magnification. Therefore, the transverse light-shift \( L_x \) can be calculated by,

\[
L_x = - \frac{R_{tot}^2 (B_{yc} + B_y)}{\gamma_e B_c}.
\]

According to Eqs. (5) and (6), the relationship between probe light wavelength and transverse light-shift \( L_x \) presents a Lorentz linear with other parameters unchanged. Thus, the \( L_x \) can be reduced by optimizing the detuning of probe light wavelength.

### 3 Experimental setup and results

The schematic diagram of the K-Rb-\(^{21}\)Ne co-magnetometer is shown in Fig. 2. The sensitive core is a 10-mm-diameter spherical cell, which contains a drop of K and Rb alkali metals, 3 atm of \(^{21}\)Ne and 40 Torr of \( \text{N}_2 \) for quenching. The vapor cell is placed in an oven, which can heat the temperature of the cell to 180 °C. Three layers of \( \mu- \) metal magnetic shields surround the oven to provide a weak magnetic
environment for atoms. A set of three-axis magnetic field coils is used to compensate residual fields in the shields, which makes the electron spins operate in near zero-field environment.

A circularly polarized pump light along $z$-axis tuned on the K D1 resonance line is used to polarize the K atoms, and the Rb atoms are polarized through the spin-exchange collision among K atoms. The polarized alkali-metal atoms ultimately hyperpolarize the $^{21}$Ne atoms [27]. A potassium vacuum cell is used to present the absorption spectrum, and the wavelength of pump light can be stabilized to the order of megahertz by saturated absorption. Thus, its effect of light wavelength fluctuation on light-shift drift can be ignored in our setup. Moreover, the pump light only produces longitudinal light-shift and influences the value of compensation field along $z$-axis. A non-ideal linearly polarized probe beam detuned to the red side of Rb D1 line, which is orthogonal to the pump beam, is utilized to detect the variety of the transverse polarization of Rb atoms along $x$-axis. The probe beam contains elliptical polarization component due to the limitation of polarization performance of the polarizer and the depolarization of glass cell, which can cause transverse light-shift along the probe beam. The relationship between probe light intensity and compensating field $B_c$ is shown in Fig. 3. According to the linear fitting curves, the residual magnetic field $B_y$ can be obtained by the intercept. The insert map shows measurement results of $B_y$ under different light wavelength and the average value is 3.379 nT in this setup.

The $B_z$ step modulation output signals under different wavelength of the probe light have been tested in Fig. 4. The $R_{\text{tot}}^c/\gamma_e$ and $B_c$ can be obtained by fitting the dispersion curves with Eq. (7), and the fitting results under different wavelength are shown in the insert map of Figs. 4 and 5, respectively. It is obvious that the values of $R_{\text{tot}}^c/\gamma_e$ and $B_c$ are slightly influenced by the wavelength of probe light due to transverse pumping effect on the electron spins. According to the values of $R_{\text{tot}}^c/\gamma_e$ and $B_c$, the light-shift $L_x$ can be calculated by Eq. (8). The values of $L_x$ and the related term $L_x/R_e$ and $B_c$ under different wavelength of probe light are shown in Fig. 5. The relationship between $L_x$ and the wavelength of probe light is a Lorentz linear, and the effect of $L_x$ can be magnified by the terms of $R_{\text{tot}}^c/\gamma_e$ and $B_c$ for about one order of magnitude. $L_x$ has the most serious effect near the wavelength of 795 nm, which is at the transition frequency of Rb D1 line. When the wavelength is detuned to the red side of Rb D1 line, $L_x$ decays exponentially and it is basically unchanged with the light wavelength larger than 795.6 nm.
At 3 atm of $^{21}$Ne, the calibration coefficient of co-magnetometer firstly increases and then decreases with the increase of the light wavelength. The performance of the co-magnetometer can be optimized by reducing the influence of transverse light-shift $L_x$ and meanwhile increasing the calibration coefficient. In our experiments, the $L_x$ can be reduced from $-0.115$ to $-0.039$ nT by optimizing the wavelength of probe light to $795.68$ nm. Meanwhile, the calibration coefficient is $26.19$ V/$°$/s. When the wavelength is greater than $795.68$ nm, the value of calibration coefficient gradually decreases and the value of $L_x$ is almost invariant. Thus, the optimal wavelength of probe light is $795.68$ nm for our setup, and the related term $L_x/\rho eB_c$ can be reduced from $1.113$ to $0.431$ nT during the optimization routine.

The transverse light-shift can be suppressed and meanwhile the calibration coefficient can be increased by optimizing the probe light wavelength. Moreover, the sensitivity of $L_x$ with wavelength drift obviously decreases, which is conducive to improve the stability of output signals. To clarify the performance on long time scales, the analysis results of Allan deviation [28] of 10-h static test data are shown in Fig. 6. It can be known that the bias instability and rate ramp can be suppressed by optimization of probe light wavelength, and the values are shown in Table 1. Since the calibration coefficient is almost unchanged at $795.68$ nm and $795.86$ nm, the reduction of low-frequency drift at $795.68$ nm proves that the suppression of $L_x$ improves the long-term stability of the K-Rb-$^{21}$Ne co-magnetometer. Comparing the test results of wavelength at $795.35$ nm and $795.68$ nm, the low-frequency noise becomes worse at $795.35$ nm, because the effect of $L_x$ increases and the calibration coefficient decreases.

### 4 Conclusion

In conclusion, a method for measuring the transverse light-shift has been examined in a compact K-Rb-$^{21}$Ne co-magnetometer. The value of the transverse light-shift $L_x$ could be calculated by Eq. (8). The residual magnetic field $B_y$ could be obtained by measuring the relationship between transverse compensating field $B_{yc}$ and the intensity of probe light. In addition, other related parameters $\rho eB_c/\rho e$ and $B_y$ could be obtained by fitting the $B_y$ step modulation output signal $\Delta S$. Finally, the transverse light-shift $L_x$ has been reduced from $-0.115$ to $-0.039$ nT by optimizing wavelength of the probe light to $795.68$ nm. Meanwhile, the related term $L_x/\rho eB_c/\rho e$ has been reduced from $1.113$ to $0.431$ nT. Moreover, the long-term stability for output signal of K-Rb-$^{21}$Ne co-magnetometer has been tested under different wavelength. Allan deviation analysis has been used to prove that low-frequency drift can be suppressed by reducing the effect of transverse light-shift. Finally, the bias instability of about $0.009$ deg/h and rate ramp coefficient of $0.096$ (deg/h)/h have been achieved by optimizing probe light wavelength. Therefore, the influence of transverse light-shift could be effectively restrained, which is beneficial to improve the accuracy of rotation rate measurement. Further improvement may be realized by properly increasing the pressure of $^{21}$Ne in the vapor cell, which can directly inhibit the generation of transverse light-shift.

**Acknowledgements** This work was supported in part by the National Natural Science Foundation of China under Grant 61803015, Grant 61901431. China Postdoctoral Science Foundation under 2021M703049. Basic Scientific Research Fund of NIM under AKYJJ1906.

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