Current noise spectrum of a quantum shuttle

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Abstract

We present a method for calculating the full current noise spectrum $S(\omega)$ for the class of nano-electromechanical systems (NEMS) that can be described by a Markovian generalized master equation. As a specific example we apply the method to a quantum shuttle. The noise spectrum of the shuttle has peaks at integer multiples of the mechanical frequency, which is slightly renormalized. The renormalization explains a previously observed small deviation of the shuttle current compared to the expected value given by the product of the natural mechanical frequency and the electron charge. For a certain parameter range the quantum shuttle exhibits a coexistence regime, where the charges are transported by two different mechanisms: shuttling and sequential tunneling. In our previous studies we showed that characteristic features in the zero-frequency noise could be quantitatively understood as a slow switching process between the two current channels, and the present study shows that this interpretation holds also qualitatively at finite frequency.

Key words: NEMS, Quantum shuttles, Current noise
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1. Introduction

A decade of advances in microfabrication technology has pushed the typical length scales of electromechanical systems to the limit, where quantum mechanical effects of the mechanical motion must be taken into account \cite{1}. Such nano-electromechanical systems (NEMS) exhibit a strong interplay between mechanical and electronic (or magnetic) degrees of freedom, and their electronic transport properties reflect this interplay in an intricate manner.

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A modern trend in transport studies of mesoscopic systems has been to not only consider the current-voltage characteristics of a given device, but also to examine the noise properties, or even the higher cumulants (i.e. the full counting statistics (FCS)) of the current distribution \cite{2,3}. The current noise, either its zero-frequency component or the whole frequency spectrum, provides more information than just the mean current and can be used to discern among different possible mechanisms resulting in the same mean current. While the noise spectra in generic mesoscopic systems have been studied well over a decade, it is only very recently that the study of noise spectra of NEMS has been initiated \cite{4,5,6}.
The aforementioned three studies deal with the noise spectra of a classical shuttle, a classical nanomechanical resonator coupled to a single electron transistor (SET), and the C_{60}-SET in a strong electromechanical coupling regime, respectively. The first two studies [4,5] found peaks in the current noise spectra at the first two multiples of the mechanical frequency. For low bias voltages and strong electromechanical coupling the third study [6] found a power-law frequency dependence of the noise spectrum attributed to scale-free avalanche charge transfer processes. In all three cases the noise spectra revealed interesting details about the systems. From the technical point of view, two of the studies [4,6] used Monte Carlo simulations, whereas [5] used a model-specific numerical evaluation of the MacDonald formula (see below).

In this work, we present a study of the full frequency spectrum of the current noise of a quantum shuttle [7,8,9]. We extend the general formalism developed for the zero-frequency noise [10,11] and the FCS [12] calculations for NEMS described by a Markovian generalized master equation. The presented formalism applies not only to the shuttle studied here but could equally well be used for all three systems from the previous studies [4,5,6] for the determination of the noise spectra.

We apply the developed theory to compute numerically the noise spectrum of the shuttle in the deep quantum regime as function of the damping coefficient. The spectrum has peaks at integer multiples of the slightly renormalized mechanical frequency. The renormalization of the bare oscillator frequency as read off from the current spectrum explains a small but observable deviation from the expected value of the current in the shuttling regime $I_{\text{shut}} = e\omega_0/2\pi$ [8]. It turns out that it is the renormalized oscillator frequency $\tilde{\omega}_0$ which should enter this relation. Finally, we focus on the low-frequency part of the spectrum when approaching the semi-classical regime for intermediate values of the damping, i.e. in the coexistence regime, where both shutting and tunneling are effective. We use the frequency dependence of the spectrum for $\omega \ll \omega_0$ to identify additional qualitative evidence for the bistable behavior of the shuttle in this regime described by a simple analytical theory of a slow switching between two current channels (compare with Refs. [12,13]).

2. Model

We consider the model of a quantum shuttle described in [7,8,9,10]. The shuttle consists of a mechanically oscillating nanoscale grain situated between two leads (see Fig. 1). In the strong Coulomb blockade regime the grain can be treated as having a single electronic level only. A high bias between the leads drives electrons through the grain and exerts an electrostatic force on the grain, when charged. The grain is assumed to move in a harmonic potential, and the oscillations of the grain are treated fully quantum mechanically. Damping of the oscillations is described by interactions with a surrounding heat bath.

From the Hamiltonian of the model one can derive a generalized master equation (GME) resolved with respect to the number of electrons $n$ that have been collected in the right lead during the time span $0$ to $t$. The $n$-resolved GME describes the time evolution of the $n$-resolved system density matrix $\hat{\rho}^{(n)}(t)$, where the 'system' consists of the electronic level of the grain and the quantized oscillations. In the following we only need the $n$-resolved GME for the part of $\hat{\rho}^{(n)}(t)$ that is diagonal in the electronic components, which reads [10]...
\[ \dot{\rho}_{00}^{(n)}(t) = \frac{i}{\hbar} [\hat{H}_{\text{ext}}, \rho_{00}^{(n)}(t)] + \mathcal{L}_{\text{damp}} \rho_{00}^{(n)}(t) - \frac{\Gamma L}{2} \left( e^{-\frac{\hbar}{i\omega} \hat{x}}, \rho_{00}^{(n)}(t) \right) + \Gamma R e^{-\frac{\hbar}{i\omega} \hat{p}} \hat{\rho}_{11}^{(n)}(t) e^{\frac{\hbar}{i\omega} \hat{x}}, \]

\[ \dot{\rho}_{11}^{(n)}(t) = \frac{i}{\hbar} [\hat{H}_{\text{ext}}, \rho_{11}^{(n)}(t)] + \mathcal{L}_{\text{damp}} \rho_{11}^{(n)}(t) - \frac{\Gamma R}{2} \left( e^{-\frac{\hbar}{i\omega} \hat{x}}, \rho_{11}^{(n)}(t) \right) + \Gamma L e^{-\frac{\hbar}{i\omega} \hat{p}} \rho_{00}^{(n)}(t) e^{\frac{\hbar}{i\omega} \hat{x}}, \]

with \( n = 0, 1, 2, \ldots \) and \( \rho_{11}^{(-1)}(t) \equiv 0 \). The commutators describe the coherent evolution of the charged \((\rho_{11}^{(n)}(t) \equiv (1|\hat{p}^{(n)}|1))\) or empty \((\rho_{00}^{(-1)}(t) \equiv (0|\hat{p}^{(n)}|0))\) shuttle with mass \( m \) and natural frequency \( \omega_0 \). The electric field \(^3\) between the leads is denoted \( E \). The terms proportional to \( \Gamma_{L(R)} \) describe transfer processes from the left (to the right) lead with hopping amplitudes that depend exponentially on the ratio between position \( \hat{x} \) and the electron tunneling length \( \lambda \). The mechanical damping of the oscillator is described by the damping kernel (here at zero temperature) \( \mathcal{L}_{\text{damp}} \rho_{jj}^{(n)} = -\frac{i\hbar}{2m} [\hat{x}, [\hat{p}, \rho_{jj}^{(n)}]] - \frac{m \omega_0}{2\hbar} [\hat{x}, [\hat{x}, \rho_{jj}^{(n)}]], j = 0, 1, 8, 10, \)

The \( n \)-resolved GME can be recast into the compact form \(^1\)

\[ \dot{\rho}^{(n)} = (\mathcal{L} - \mathcal{I}_R) \rho^{(n)} + \mathcal{I}_R \rho^{(n-1)}, \rho^{(-1)} \equiv 0, \]

where we have introduced the Liouvillean \( \mathcal{L} \), describing the evolution of the system density matrix \( \rho(t) = \sum_n \rho^{(n)}(t) \), i.e. \( \dot{\rho}(t) = \mathcal{L}\rho(t) \), and the superoperator for the tunnel current through the right junction (taking \( e = 1 \)), defined by its action on the density operator \( \mathcal{I}_R \rho = \Gamma \hbar e^{\frac{\hbar}{i\omega} \hat{x}} |0\rangle \langle 1| \hat{p} |1\rangle \langle 0| e^{-\frac{\hbar}{i\omega} \hat{x}}. \)

Assuming that the system tends exponentially to a stationary state \( \rho^{\text{stat}} \) the Liouvillean has a single eigenvalue equal to zero with \( \rho^{\text{stat}} \) being the (unique and normalized) right eigenvector which we denote by \( |0\rangle \) \(^2\).

The corresponding left eigenvector is the identity operator \( \hat{1} \) which we denote by \( \langle 0| \), and from the definition of the inner product \(^4\) we have \( \langle 0| 0 \rangle = \text{Tr}(\hat{1} \rho^{\text{stat}}) = 1 \).

1. In terms of \( \mathcal{I}_R \) the average tunnel current in the stationary state can be expressed as
   \[ \langle \hat{I}_R \rangle = \text{Tr}(\mathcal{I}_R \rho^{\text{stat}}) = \langle 0| \mathcal{I}_R |0 \rangle. \]

We define the projectors \( P = |0\rangle \langle 0| \) and \( Q = 1 - P \) obeying the relations \( P\mathcal{L} = P\mathcal{L}P = 0 \) and \( Q\mathcal{L}Q = \mathcal{L} \).

In terms of the two projectors we can express the resolvent of the Liouvillean \( \mathcal{G}(-i\omega) = (-i\omega - \mathcal{L})^{-1} \) as

\[ \mathcal{G}(-i\omega) = -\frac{1}{i\omega} P - Q \frac{1}{i\omega + \mathcal{L}} Q = -\frac{1}{i\omega} P - \mathcal{R}(\omega), \]

where we have introduced the frequency dependent superoperator \( \mathcal{R}(\omega) \), which is well-defined even for \( \omega = 0 \), since the inversion in that case is performed only in the subspace where \( \mathcal{L} \) is regular.

3. Theory

We consider the current autocorrelation function defined as

\[ C_{II}(t', t'') = \frac{1}{2} \langle \{ \Delta \hat{I}(t), \Delta \hat{I}(t'') \} \rangle, \]

where \( \Delta \hat{I}(t) = \hat{I}(t) - \langle \hat{I}(t) \rangle \). In the stationary state \( C_{II}(t', t'') \) can only be a function of the time difference \( t = t' - t'' \), and we thus write

\[ C_{II}(t) = \frac{1}{2} \langle \{ \Delta \hat{I}(t), \Delta \hat{I}(0) \} \rangle. \]

The current noise spectrum is the Fourier transform of \( C_{II}(t) \), i.e.

\[ S_{II}(\omega) \equiv \int_{-\infty}^{\infty} dt C_{II}(t) e^{i\omega t}. \]

In order to calculate the current noise measurable in, say, the right lead one must recognize that the current running in the lead is a sum of two contributions, namely the tunnel current through the right junction and a displacement current induced by electrons tunneling between leads and grain. This is reflected in the Ramo-Shockley theorem \(^2\)

\[ \hat{I} = c_L \hat{I}_L + c_R \hat{I}_R. \]

Here \( \hat{I} \) is the current operator for the current running in the lead, whereas \( \hat{I}_{L(R)} \) is the operator for the tunnel current through the left (right) junction, and \( c_{L(R)} \) is the relative capacitance of the left (right) junction in the sense \( c_L + c_R = 1 \). Combining the Ramo-Shockley theorem...
Theorem with charge conservation leads to an expression for the current noise measured in the lead [14]
\[ S_{11}(\omega) = c_L S_{1R} I_R(\omega) + c_R S_{1L} I_L(\omega) - c_L c_R \omega^2 S_{NN}(\omega), \]
where \( N = |1\rangle\langle 1 | \) is the occupation number operator of the electronic level of the grain. In the following we neglect any dependence of the capacitances on the position of the grain and consider the symmetric case \( c_L = c_R = \frac{1}{2} \).

The two first terms of Eq. (10) can be evaluated using the methods developed by MacDonald [15]. The starting point of the derivation is the property \( C_{11}(t) = C_{11}(-t) \), which immediately leads to
\[ S_{1R} I_R(\omega) = \int_0^\infty dt C_{1R} I_R(t)(e^{i\omega t} + e^{-i\omega t}). \]  
(11)

Let us consider the first term
\[ S_{1R}^R I_R(\omega) = \int_0^\infty dt C_{1R}^R I_R(t)e^{i\omega t}, \]  
(12)
the second term, \( S_{1R}^L I_R(\omega) \), follows analogously. Defining \( \hat{Q}_R(t) \) as the operator of charge collected in the right lead in the time span \( 0 \) to \( t \) we have
\[ \Delta \hat{Q}_R(t) = \hat{Q}_R(t) - \langle \hat{Q}_R(t) \rangle = \int_0^t dt' \Delta \hat{I}_R(t'), \]  
(13)
and we can express the current autocorrelation function as
\[ C_{1R} I_R(t) = \frac{1}{2i} \frac{d}{dt} \langle \Delta \hat{Q}_R(t), \Delta \hat{I}_R(0) \rangle. \]  
(14)
Introducing the convergence factor \( \varepsilon \rightarrow 0^+ \) and performing the integration by parts in Eq. (12) we get
\[ S_{1R}^R I_R(\omega) = \int_0^\infty dt \frac{d}{dt} \langle \Delta \hat{Q}_R(t), \Delta \hat{I}_R(0) \rangle e^{i\varepsilon t}, \]  
(15)
and consequently
\[ S_{1R} I_R(\omega) = \int_0^\infty dt \frac{d}{dt} \langle \Delta \hat{Q}_R^2(t) \rangle (\omega \sin \omega t + \varepsilon \cos \omega t)e^{-\varepsilon t}. \]  
(18)

We now make use of the fact that
\[ \langle \Delta \hat{Q}_R^2(t) \rangle = \langle \hat{Q}_R^2(t) \rangle - \langle \hat{Q}_R(t) \rangle^2 = \langle n^2(t) \rangle - \langle n(t) \rangle^2, \]  
(19)
with \( \langle n^\alpha(t) \rangle \equiv \sum_{n=0}^\infty n^\alpha P_n(t), \alpha = 1, 2, \) where \( P_n(t) \) is the probability of having collected \( n = 0, 1, 2, \ldots \) electrons in the right lead during the time span \( 0 \) to \( t \). This finally leads us to the commonly used form of the MacDonald formula (cf. [5,14,15])
\[ S_{1R} I_R(\omega) = \omega \int_0^\infty dt \sin(\omega t) \frac{d}{dt} \left[ \langle n^2(t) \rangle - \langle n(t) \rangle^2 \right], \]  
(20)
where the regularization
\[ \omega \sin(\omega t) \rightarrow \frac{1}{2i} \left[ (\omega + i\varepsilon)e^{(\omega + i\varepsilon)t} - (\omega - i\varepsilon)e^{-(\omega - i\varepsilon)t} \right] = (\omega \sin \omega t + \varepsilon \cos \omega t)e^{-\varepsilon t}, \varepsilon \rightarrow 0^+ \]  
(21)
is implied. Only the proper treatment of the regularization ensures correct results including the \( \omega = 0 \) case where the zero-frequency noise MacDonald formula is recovered (by using the Laplace transform identity for \( \varepsilon \rightarrow 0^+ \))
\[ S_{1R} I_R(0) = \varepsilon \int_0^\infty dt e^{-\varepsilon t} \frac{d}{dt} \left[ \langle n^2(t) \rangle - \langle n(t) \rangle^2 \right] \]  
\[ = \frac{d}{dt} \left[ \langle n^2(t) \rangle - \langle n(t) \rangle^2 \right] \bigg|_{t \rightarrow \infty}. \]  
(22)

In order to evaluate the current noise spectrum we now introduce the quantity
\[ \tilde{S}(\omega) = \omega \int_0^\infty dt e^{i\omega t} \frac{d}{dt} \left[ \langle n^2(t) \rangle - 2\langle n(t) \rangle \frac{d}{dt} \langle n(t) \rangle \right], \]  
(23)
with either \( S(\omega) = \text{Im}\tilde{S}(\omega) \) or \( S(\omega) = \frac{1}{2i} \left( \tilde{S}(\omega) + \tilde{S}(-\omega) \right) \) and evaluate it along the lines of [11]. Since \( P_n(t) = \text{Tr}(\hat{\rho}(n)(t)) \), Eq. (2) leads to (keeping in mind that \( \text{Tr}(\hat{\mathbf{0}}) = 0 \))
\[ \hat{P}_n(t) = \text{Tr}[\hat{I}_R(\hat{\rho}^{(n-1)}(t) - \hat{\rho}^{(n)}(t))], \]  
(24)
and as shown in [11]
\[ \frac{d}{dt} \langle n(t) \rangle = \text{Tr}(\hat{I}_R \hat{\rho}(t)) = \langle \hat{0}|\hat{I}_R|0 \rangle, \]  
(25)
\[ \frac{d}{dt} \langle n^2(t) \rangle = 2\text{Tr}[\hat{I}_R \sum_n n \hat{\rho}^{(n)}(t)] + \langle \hat{0}|\hat{I}_R|0 \rangle, \]  
(26)
where we have used \( \dot{\rho}(t) = \dot{\rho}^{\text{stat}} \), since we are considering the stationary limit. The sum entering Eq. (26) is evaluated by introducing an operator-valued generating function defined as \( \tilde{F}(t, z) = \sum_{n=0}^{\infty} \hat{\rho}^{(n)}(t) z^n \), from which we get
\[
\frac{\partial}{\partial z} \tilde{F}(t, z)|_{z = 1} = \sum_{n} n \hat{\rho}^{(n)}(t). \tag{27}
\]

From the definition of the Laplace transform, \( \tilde{F}(s, z) = \int_{0}^{\infty} dt \tilde{F}(t, z) e^{-st} \), we see that the integration in Eq. (23) can be considered as a Laplace transform evaluated at \( s = -i\omega + \varepsilon \) (remember the proper regularization; from now on we skip explicitly mentioning the \( \varepsilon \)-factors). In [11] it was shown that
\[
\frac{\partial}{\partial z} \tilde{F}(s = -i\omega, z)|_{z = 1} = \hat{G}(-i\omega) \hat{I}_R \hat{G}(-i\omega) \hat{\rho}(0) + \hat{G}(-i\omega) \sum_{n} n \hat{\rho}^{(n)}(0). \tag{28}
\]

Again, we have \( \hat{\rho}(0) = \hat{\rho}^{\text{stat}} \), and moreover we assume the factorized initial condition [14,16] \( \hat{\rho}^{(n)}(0) = \delta_{0n} \hat{\rho}^{\text{stat}} \), i.e. we start counting the charge passing through the right junction at \( t = 0 \) and the system is in its stationary state. Now, combining Eqs. (5, 23-28) and having in mind that \( \mathcal{P} \hat{\rho}^{\text{stat}} = \hat{\rho}^{\text{stat}}, \mathcal{Q} \hat{\rho}^{\text{stat}} = 0 \), straightforward algebra leads to
\[
\hat{S}(\omega) = \iota \left( \langle \langle \hat{0} | \hat{I}_R | 0 \rangle \rangle - 2 \langle \langle \hat{0} | \hat{I}_R \hat{R}(\omega) | \hat{I}_R | 0 \rangle \rangle \right). \tag{29}
\]

We thus arrive at
\[
\hat{S}_{L,R}(\omega) = \langle \langle \hat{0} | \hat{I}_R | 0 \rangle \rangle - 2 \text{Re} \left[ \langle \langle \hat{0} | \hat{I}_R \hat{R}(\omega) | \hat{I}_R | 0 \rangle \rangle \right] = \langle \langle \hat{0} | \hat{I}_R | 0 \rangle \rangle - 2 \langle \langle \hat{0} | \hat{I}_R | \hat{L} \left( \frac{\hat{L}}{\hat{L}^2 + \omega^2} \right) \hat{I}_R | 0 \rangle \rangle. \tag{30}
\]

For the left junction one similarly finds
\[
\hat{S}_{L,L}(\omega) = \langle \langle \hat{0} | \hat{I}_L | 0 \rangle \rangle - 2 \text{Re} \left[ \langle \langle \hat{0} | \hat{I}_L \hat{R}(\omega) | \hat{I}_L | 0 \rangle \rangle \right] \tag{31}
\]
with
\[
\hat{I}_L \hat{\rho} = \Gamma_L e^{-\frac{\hat{z}}{2}} | 0 \rangle \langle 0 | \hat{\rho} | 0 \rangle e^{-\frac{\hat{z}}{2}}. \tag{32}
\]

For the evaluation of the charge-charge correlation function \( S_{NN}(\omega) \) we note that \( \hat{N} \) is a system operator, and the quantum regression theorem thus applies [17]. Following [11] we immediately get
\[
S_{NN}(\omega) = -2 \text{Re} \left[ \langle \langle \hat{0} | \hat{N} \hat{R}(\omega) | \hat{N} | 0 \rangle \rangle \right], \tag{33}
\]

having introduced the superoperator \( \hat{N} \) corresponding to \( \hat{N} \), defined as
\[
\hat{N} \hat{\rho} = | 1 \rangle \langle 1 | \rho \langle 1 | 1 \rangle. \tag{34}
\]

Collecting all terms in Eq. (10) we finally obtain the expression for the current noise measured in the leads for the symmetric setup (\( c_L = c_R = \frac{1}{2} \))
\[
S_{II}(\omega) = \langle \langle \hat{0} | \hat{I}_R | 0 \rangle \rangle + \frac{\omega^2}{2} \text{Re} \left[ \langle \langle \hat{0} | \hat{N} \hat{R}(\omega) | \hat{N} | 0 \rangle \rangle \right] - \text{Re} \left[ \langle \langle \hat{0} | \hat{I}_R \hat{R}(\omega) | \hat{I}_R + \hat{I}_L \hat{R}(\omega) | \hat{I}_L | 0 \rangle \rangle \right]. \tag{35}
\]

We notice that for \( \omega = 0 \) we get the previous result [10,11,12]
\[
S_{II}(0) = \langle \langle \hat{0} | \hat{I}_R | 0 \rangle \rangle - 2 \langle \langle \hat{0} | \hat{I}_R \hat{R}(0) | \hat{I}_R | 0 \rangle \rangle, \tag{36}
\]

since the zero-frequency tunnel current noise is the same at both junctions and the second term is real [11].

The numerical evaluation of Eq. (35) is only possible by truncating the number of oscillator states. As in previous studies [8,10,12] we retain the 100 lowest oscillator states, which however still leaves us with the task of dealing numerically with the matrix representations of the relevant superoperators, which are of size \( 20000 \times 20000 \). As explained in [11] the stationary density matrix \( \hat{\rho}^{\text{stat}} \) (or \( | 0 \rangle \langle 0 | \)) can be found using the Arnoldi iteration scheme, and \( \hat{R}(\omega) \) can be evaluated using the generalized minimum residual method (GMRes). Both methods are iterative and rely crucially on an appropriate choice of preconditioner to ensure that the iterations converge and to speed up the computation. It should be stressed that finding a suitable preconditioner for a given problem is by no means simple. For finding \( \hat{\rho}^{\text{stat}} \) and \( \hat{R}(0) \) it turns out that the Sylvester part of the Liouvillean, which is the part that can be written \( \hat{L} \hat{\rho}^{\text{stat}} = \hat{A} \hat{\rho} + \hat{\rho} \hat{A}^\dagger \), is well-suited for preconditioning [11]. This preconditioner separates the zero eigenvalue from the rest of the spectrum of \( \hat{L} \) leading to a decrease in computation time.

For finding \( \hat{R}(\omega) \) the original preconditioner must be modified in order to separate the relevant eigenvalue from the rest of the spectrum. A reasonable choice is the superoperator \( \hat{M} \) defined as
\[
\hat{M} \hat{\rho} = (\hat{A} + \frac{i\omega}{2}) \hat{\rho} + \hat{\rho} (\hat{A} - \frac{i\omega}{2})^\dagger. \tag{37}
\]

For the range of parameters discussed in the present paper this choice of preconditioner was sufficient for convergence, however the obtained computational speedup is considerably smaller than the speedup provided by \( \hat{L} \hat{\rho}^{\text{stat}} \), when computing \( \hat{R}(0) \). This observation combined with the fact that GMRes fails to converge for certain parameters in the semi-classical
Peaks are seen at $\omega_F = 0 = F(\gamma)$, where $\gamma$ is the renormalization of the mechanical frequency with the damping $\gamma$. A close look at the spectrum reveals a slight enhancement of the zero-frequency noise in the deep quantum shuttling regime. The other parameters are $\Gamma = 0.5$, $\lambda = 0.06$, and $F(\gamma) = \gamma$. In Fig. 2 we show results for the current noise spectrum of a large class of nano-shuttle approaches the semiclassical regime can be understood in terms of a simple model of a bistable system switching slowly between two current channels (shuttling and tunneling). Denoting the currents of the two channels $I_S$ and $I_T$, respectively, and the switching rates $\Gamma_S$ and $\Gamma_T$, one can show (following [13]) that the ratio between the current noise spectrum and the current (in the zero-frequency limit known as the Fano factor) $F(\omega) = S(\omega)/I$ for the bistable system has the Lorentzian form

$$F(\omega) = \frac{2 \Gamma_S \Gamma_T}{\Gamma_S + \Gamma_T} \frac{(I_S - I_T)^2}{(\Gamma_S + \Gamma_T)^2 + \omega^2}, \quad (38)$$

where $I = I_S \Gamma_S + I_T \Gamma_T$. The two switching rates, $\Gamma_S$ and $\Gamma_T$, can be extracted from the numerical values of current and zero-frequency noise, and by comparing Eq. (38) with the noise spectrum obtained numerically, one can perform another independent test of the hypothesis that the shuttle behaves as a bistable system in the coexistence regime.

In Fig. 3 we show numerical results for the low-frequency current noise of the quantum shuttle in the coexistence regime. Together with the numerical results we show the analytic expression for the current noise of the bistable system (Eq. (38)) with rates extracted from the numerical values of current and zero-frequency noise. It should be noted that the agreement between the numerical and analytic results could only be obtained by assuming that the shuttling current for the given values of the damping is not fully saturated to the value $I_{\text{shut}} = \hat{\omega}_0/2\pi$. The current noise spectrum thus provides us with qualitative evidence for the shuttle behaving as a bistable system in the coexistence regime, while it, however, leaves us with an open question concerning the saturation of the shuttling current.

5. Conclusion

We have presented a theory for the calculation of the current noise spectrum of a large class of nano-shuttles.
Fig. 3. The ratio between the current noise and the current $F(\omega) = S_{ii}(\omega)/\langle I \rangle$ for low frequencies ($\omega \ll \omega_0$). The parameters are $\gamma = 0.035\omega_0$ (lowest curve), $0.04\omega_0, 0.045\omega_0, 0.05\omega_0$ (topmost curve), $\Gamma_L = \Gamma_R = 0.01\omega_0$, $\lambda = 1.5x_0$, $d \equiv eE/m\omega_0^2 = 0.5x_0$, where $x_0 = \sqrt{\hbar/m\omega_0}$. The circles indicate numerical results, while the full lines indicate the analytic results for the current noise spectrum of a bistable system. It should be noted that in order to obtain the agreement between the numerical and analytic results it is necessary to assume that the shuttling current for the given values of $\gamma$ is not fully saturated to the value $I_{\text{shut}} = 1.03\omega_0/2\pi$. Corresponding to the different values of $\gamma$ we have used $I_{\text{shut}} = 1.01\omega_0/2\pi$ (for $\gamma = 0.03\omega_0$), $1.00\omega_0/2\pi$, $0.98\omega_0/2\pi$, $0.94\omega_0/2\pi$ (for $\gamma = 0.05\omega_0$), respectively.

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