Application of the novel fractional grey model FAGMO(1,1,k) to predict China’s nuclear energy consumption ♠

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Abstract

At present, the energy structure of China is shifting towards cleaner and lower amounts of carbon fuel, driven by environmental needs and technological advances. Nuclear energy, which is one of the major low-carbon resources, plays a key role in China’s clean energy development. To formulate appropriate energy policies, it is necessary to conduct reliable forecasts. This paper discusses the nuclear energy consumption of China by means of a novel fractional grey model FAGMO(1,1,k). The fractional accumulated generating matrix is introduced to analyse the fractional grey model properties. Thereafter, the modelling procedures of the FAGMO(1,1,k) are presented in detail, along with the transforms of its optimal parameters. A stochastic testing scheme is provided to validate the accuracy and properties of the optimal parameters of the FAGMO(1,1,k). Finally, this model is used to forecast China’s nuclear energy consumption and the results demonstrate that the FAGMO(1,1,k) model provides accurate prediction, outperforming other grey models.

Keywords: Nuclear energy consumption, Grey system, Fractional order

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1. Introduction

Energy is the most important strategic resource and provides a key material basis for economic development and social progress. Energy consumption prediction constitutes an important aspect of energy policies for countries globally, particularly developing countries such as China, where the energy consumption structure is changing at a rapid speed. Numerous models have been introduced for forecasting energy consumption, such as dynamic causality analysis, nonlinear and asymmetric analysis, time-series analysis, machine learning models, the coupling mathematical model, autoregressive distributed lag model, hybrid forecasting system, machining system, fuzzy systems, LEAP model, TIMES model, NEMS model, and grey model. Among these prevalent methods, simple linear regression, multivariate linear regression, and time-series analysis are often significant in accurately demonstrating the phenomena of long-term trends. However, these exhibit the limitations of requiring a large amount of observed data, at least 50 or more sets, to construct models. The computational intelligence method requires a substantial amount of training data to derive the optimised parameters. However, in many practical situations, it is very difficult and sometimes even impossible to obtain complete information. Therefore, it is important to identify a favourable method for forecasting the trend of an analysed system using scarce information with less errors.

The grey forecasting theory, proposed by Professor Deng, offers a feasible and efficient method for dealing with uncertain problems containing poor information. The main advantage of this theory is that only four or more samples are required to describe the behaviour and evolution of the analysed system. In Deng’s pioneering work, the first-order one variable grey model GM(1,1) was discussed in detail. Over three decades of development, the classical contin-
uous GM(1,1) model has been studied extensively; for example, by Xie et al. [30, 31, 32], Wang et al. [33, 34, 35], Ma et al. [36, 37, 38, 39], and others. However, we note that these generalised grey models all include integer-order accumulation, which results in less flexibility in time-series forecasting. Thus, the fractional-order accumulation grey model is considered in this paper.

By extending the integer accumulated generating operation into the fractional accumulated generating operation, Wu et al. [40] first proposed the fractional accumulation GM(1,1) model known as the FAGM(1,1) model. The computational results demonstrated that the novel model outperformed the conventional GM(1,1) model. Later, Wu and his peers successfully applied fractional accumulation to the fuel production of China [41], tourism demand [42] and electricity consumption [43]. Subsequently, Xiao et al. [44] studied the GM(1,1) model, in which they regarded the fractional accumulated generator matrix as a type of generalised accumulated generating operation. Gao et al. [45] presented a new discrete fractional accumulation GM(1,1) model known as FAGM(1,1,D) and applied it to China’s CO₂ emissions. Mao et al. [46] investigated a novel fractional grey model FGM(q,1). Interested readers can refer to [47, 48, 49, 50] for further details on fractional accumulation grey models.

A further significant issue in grey system theory is that the solution applied for prediction does not match the grey difference equation. In 2009, Kong and Wei [51] proposed a parameter optimisation technique to study the DGM(2,1) model. Later, Chen et al. used a similar technique to improve the GM(1,1) [52] and ONGM(1,1) models [53], in which the basic structure of the original models remain in the optimised ones. Recently, Ma and Liu [54] studied the exact non-homogeneous grey prediction model (ENGM) with an exact basic equation and background value. Thereafter, Ma and Liu [55] considered the GMC(1,n) model with optimised parameters and applied it to forecasting the urban consumption per capita and industrial power consumption of China. Following the concept of fractional accumulation and the parameter optimisation method, we propose a novel FAGMO(1,1,k) model.

In this paper, we study the nuclear energy consumption of China by means
of the FAGMO(1,1,\(k\)) model. The computational results indicate that the proposed grey model outperforms the existing ENGM model, optimised non-homogeneous grey model abbreviated as the ONGM(1,1,\(k\)) model, FAGM(1,1) model and FAGM(1,1,\(k\)) model. The main contributions of our paper are listed below. 1) A fractional accumulation grey model with optimised parameters is developed. 2) Detailed properties of optimised parameters are studied according to two theorems. These indicate that the first parameter is the most important factor affecting the accuracy of the FAGM(1,1,\(k\)) model. 3) Simulation results and two practical cases are considered to assess the effectiveness of the FAGMO(1,1,\(k\)) model compared to other models. 4) The FAGMO(1,1,\(k\)) grey forecasting model is implemented to forecast the nuclear energy consumption of China. It is demonstrated in the results that the newly proposed model offers higher precision than other grey models.

The remainder of this paper is organised as follows. Section 2 provides a compendium of China’s energy consumption. Section 3 discusses several preliminaries. A detailed discussion of the FAGM(1,1,\(k\)) model is provided in section 4. Section 5 discusses the optimised parameters. Modelling evaluation criteria and detailed steps are provided in section 6. Section 7 discusses the validation of the FAGMO(1,1,\(k\)) model. Applications are explained in section 8 and conclusions are drawn in the final section.

2. Brief overview of China’s energy consumption

This section presents a systematic and comprehensive investigation of China’s energy consumption using five fuels, namely coal, oil, natural gas, nuclear energy and renewables. In China, renewables include hydroelectricity, wind, solar, geothermal, biomass and others. According to the statistical data of British Petroleum (BP) *Statistical Review of World Energy 2018* (www.bp.com/statisticalreview), the International Energy Agency (IEA) *World Energy Outlook 2017* (www.iea.org/weo2017), Asia-Pacific Economic Cooperation (APEC) *Energy Overview 2017* (www.apec.org/Publications), and National Bureau of
China’s primary energy consumption increased from 142.9 million tonnes oil equivalent (Mtoe) in the first year of the third Five-Year Plan (1966 to 1970) to 3132.2 Mtoe in the second year of the 13th Five-Year Plan (2016 to 2020), and increased dramatically since the turn of the millennium owing to continuous economic growth. According to the statistical data of BP, China’s primary energy consumption from 1966 to 2017 is plotted in Fig. 1.

Fig. 1: Total primary energy consumption of China from 1966 to 2017

It is well known that China is the world’s largest energy consumer, accounting for 23% and 23.2% of the global energy consumption in 2016 and 2017, respectively. While coal remains the dominant fuel, its share of total energy consumption was 62% in 2016 and 60.4% in 2017. China’s 13th Five-Year Plan set an ambitious target for adjusting the primary energy consumption structure. The energy plan set by China for the 13th Five-Year Plan can met the adjustment target of the primary energy consumption structure. A brief overview of China’s primary energy consumption from the perspective of five fuel types is provided below.
2.1. Coal

Since the foundation of the People’s Republic of China, coal has always been the primary energy fuel, owing to abundant domestic reserves and its low cost\cite{15}. From Figs. 2 and 3 it can be observed that coal soared from 122.4 Mtoe in 1966 to 1892.6 Mtoe in 2017, although the percentage of coal in the total primary energy consumption decreased from 85.7% in 1966 to 60.4% in 2017. Specifically, despite a continuous increase in coal consumption during the third and fourth Five-Year Plan periods, the proportion of coal in the total primary energy consumption has decreased from 85.7% to 72.5%. Following China’s Reform and Opening-Up Policy in 1978, the coal consumption has expanded rapidly from 282.8 Mtoe in 1978 to 1685.8 Mtoe in 2010, while the share of coal in the total primary energy consumption is around at 73.8%. At the beginning of the 12th Five-Year Plan (2011 to 2015), the Chinese government has been stepping up its efforts to reduce coal consumption to deal with air pollution and climate change. The “supply-side reform” removes unnecessary and out-dated production capacity to avoid supply overcapacity in the coal mining industry. During this period, the consumption of coal decreased from 1903.9 Mtoe in 2011 to 1892.6 Mtoe in 2017, while the share of coal decreased from 70.8% to 60.4%.

2.2. Oil

Oil is a major component of primary energy resources globally and plays a strategic role in economic growth. It can be observed in Figs. 2 and 3 that the oil consumption in China increased from 14.3 Mtoe in 1966 to 608.4 Mtoe in 2017, with an average annual growth rate of 7.5%. Owing to China’s oil reserves accounting for only 2% of the global amount, China is highly dependent on overseas oil imports of more than 60%. China became a net importer of crude oil in 1993 and the world’s second largest oil consumer in 2002\cite{15}. In April 2015, China surpassed the US as the world’s largest oil importer, with imports of 7.4 million barrels per day (Mbbl/D), thereby exceeding the US imports of 7.2 Mbbl/D. Following the 11th Five-Year Plan, the oil consumption increased rapidly owing to economic growth and the improved quality of life.
Figure 2: China’s primary energy consumption under fuel types

Figure 3: Percentage of China’s primary energy consumption under fuel types
2.3. Natural gas

Natural gas is a fossil fuel for electricity generation, chemical feedstock, heating and cooking, among others. Chinese organisations have estimated that the technically and ultimately recoverable resources of natural gas are 6.1 trillion cubic meters (tcm) and 37 tcm \[56\], respectively. However, natural gas has not become a major energy resource in China because the domestic natural gas industry has developed slowly. In recent years, the Chinese government has set the stable natural gas supply as one of the country’s energy strategies and encourages gas transportation from areas with significant resources to East China. The National Development and Reform Commission constructed three west-east gas pipelines in 2004, 2007 and 2015, respectively. Furthermore, the “shifting from coal to gas” policy has a significant impact on the natural gas market. The natural gas consumption has increased from 116.2 Mtoe in 2011 to 206.7 Mtoe in 2017, with an average annual growth rate of 8.6%.

2.4. Nuclear energy

Nuclear energy is almost always used to generate electricity. To reduce the air pollution from coal-fired power plants, nuclear energy is an inevitable strategic option for China. In fact, China began to develop nuclear energy in the 1980s and the Qinshan Nuclear Power Plant began operating in 1991. In 2012, the State Council set a goal of 58 GW nuclear capacity by 2020. At the beginning of the 13th Five-Year Plan, 38 nuclear power reactors were in operation with a production of 213.3 TWh, while 19 nuclear power reactors were under construction. At present, the Chinese government focuses on fourth-generation reactors with increased safety. From the 11th Five-Year Plan (2006 to2010), nuclear energy consumption has soared rapidly from 12.4 Mtoe in 2006 to 56.2 Mtoe in 2017, with an average annual growth rate of 13.4%.

2.5. Renewables

China’s renewable energy has been expanding rapidly in recent decades, owing to the development of the modern renewable energy industry. In 2017,
China’s renewables consumption accounted for 21.9% of the total global amount, increasing by 31% and accounting for 36% of the global renewables consumption growth. Meanwhile, the renewables consumption increased from 101.1 Mtoe in 2006 to 368.3 Mtoe in 2017; the share has increased from 5.1% to 11.8% with an average annual growth rate of 11.4%. The 13th Five-Year Plan set targets for an installed wind power generation capacity of 250 GW, solar power generation capacity of 110 GW, and hydropower generation capacity of 350 GW by 2020.

In summary, China’s primary energy consumption using five fuels for the period of 1966 to 2017 can be provided below. The coal consumption has gradually declined, the oil consumption has gradually increased, and the natural gas, nuclear energy and renewables have rapidly increased. China’s primary energy consumption structure exhibits a diversified trend, and the clean energy has increased yearly.

3. Definitions and properties of fractional accumulation

This section provides the fractional accumulated generating operation (AGO), which can reduce the randomness of raw data in grey theory. Correspondingly, the inverse operation of accumulated generation is known as the inverse accumulated generating operation (IAGO). The $r$th AGO and $r$th IAGO are provided below, which can be found in paper [40, 46].

**Definition 1.** Let $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\}$ be an original sequence and $X^{(r)} (r > 0)$ be the $r$th accumulated generating operation ($r$-AGO) sequence of $X^{(0)}$, where $x^{(r)}(k) = \sum_{i=1}^{k} x^{(r-1)}(i), k = 1, 2, \ldots, n$. Denote by $A^r$ the $r$-AGO matrix that satisfies $X^{(r)} = X^{(0)} A^r$, and

$$A^r = \begin{bmatrix}
[0] & [1] & [2] & \cdots & [\frac{r}{n-1}] \\
0 & [1] & [2] & \cdots & [\frac{r}{n-2}] \\
0 & 0 & [1] & \cdots & [\frac{r}{n-3}] \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & [1]
\end{bmatrix}_{n \times n}.$$
with \( [r] = \frac{x^{(r+1)} - x^{(i-1)}}{i!} = \binom{r+i-1}{i} \), \( [0] = 0 \), \( [0] = (0)! = 1 \).

Obviously, the 1-AGO sequence \( x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 1, 2, \ldots, n \), namely

\[
X^{(1)} = X^{(0)} A \quad \text{with} \quad A = \begin{pmatrix}
1 & 1 & 1 & \cdots & 1 \\
0 & 1 & 1 & \cdots & 1 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1
\end{pmatrix}_{n \times n}
\]

**Definition 2.** The inverse accumulated generation is defined as \( x^{(r-1)}(k) = x^{(r)}(k) - x^{(r)}(k-1), k = 1, 2, \ldots, n \). Denote by \( D^r \) the \( r \)-th inverse accumulated generating operation (r-IAGO) matrix, which satisfies \( X^{(0)} = X^{(r)} D^r \), and

\[
D^r = \begin{pmatrix}
\binom{r}{0} & \binom{r}{1} & \binom{r}{2} & \cdots & \binom{r}{n-1} \\
0 & \binom{r}{0} & \binom{r}{1} & \cdots & \binom{r}{n-2} \\
0 & 0 & \binom{r}{0} & \cdots & \binom{r}{n-3} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & \binom{r}{0}
\end{pmatrix}_{n \times n}
\]

with \( \binom{r}{0} = \frac{(-1)^{r-i} \cdot (r-i)}{i!} = (-1)^{i} r^{(r-1)} - r^{(i+1)} = -1 \), \( \binom{r}{i} = 0, i > r \).

Similarly, the 1-IAGO sequence \( x^{(0)}(k) = x^{(1)}(k) - x^{(1)}(k-1), k = 1, 2, \ldots, n \);

that is, \( X^{(0)} = X^{(1)} D \), with \( D = \begin{pmatrix}
1 & -1 & 0 & \cdots & 0 \\
0 & 1 & -1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1
\end{pmatrix}_{n \times n}
\)

**Theorem 1.** The expression \( \binom{r}{i}, r \in \mathbb{R}^+, i \in \mathbb{N}^+ \) is a function of \( r \) and \( i \); for any value \( i \),

- \( r \in (0, 1), \binom{r}{i} \) is a monotonically decreasing function of \( i \);
- \( r = 1, \binom{r}{i} = 1 \); and
- \( r \in (1, +\infty), \binom{r}{i} \) is a monotonically increasing function of \( i \).
Proof 1. We consider the difference

\[
[r]_i - [r]_{i-1} = (r^i_{i-1}) - (r^{i-1}_{i-1}) = \frac{(r + i - 1)!}{i! (r - 1)!} \left( \frac{r + i - 1}{i} - 1 \right) = \frac{(r + i - 2)!}{(i - 1)! (r - 1)!} \frac{r - 1}{i} = \frac{(r + i - 2)!}{i! (r - 1)!} (r - 1).
\]

From the difference results, we complete the proof.

To gain an improved understanding of Theorem 1, two figures are displayed in the following Fig. 4.

Figure 4: Function \([r]_i\) versus values \(r\) and \(i\): left \(r \in (0, 1)\), right \(r \in (1, +\infty)\)

It follows from \(X^{(r)} = X^{(0)} A^r\) that

\[
x^{(r)}(k) = \sum_{i=1}^{k} \left[ r \right]_{i-1} x^{(0)}(i) = \sum_{i=0}^{k-1} \left[ r \right]_i x^{(0)}(k - i),
\]

which means that \(x^{(r)}(k)\) is the weight of \(x^{(0)}(i), i = 1, 2, \ldots, k\).

From Theorem 1, when \(r \in (0, 1)\), the weight of the old data is smaller than that of the new data. When \(r = 1\), the weights of the old and new data are all 1. When \(r \in (1, +\infty)\), the weight of the old data is larger than that of the new data.
Theorem 2. The values of r-AGO $A^r$ and r-IAGO $D^r$ satisfy $(A^r)^{-1} = D^r$.

Proof 2. From the definition of $A^r$, it is easy to calculate the determinant $\det (A^r) = 1$, which means that $A^r$ is reversible.

Employing mathematical induction, when $r = 1$, we obtain

$$AD = \begin{pmatrix}
1 & 1 & 1 & \cdots & 1 \\
0 & 1 & 1 & \cdots & 1 \\
0 & 0 & 1 & \cdots & 1 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1
\end{pmatrix} \begin{pmatrix}
1 & -1 & 0 & \cdots & 0 \\
0 & 1 & -1 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1
\end{pmatrix} = I.$$

Assuming that the properties hold true when $r = m$, this means that

$$\begin{pmatrix}
[m_0] & [m_1] & [m_2] & \cdots & [m] \\
0 & [m_0] & [m_1] & \cdots & [m] \\
0 & 0 & [m_0] & \cdots & [m] \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & [m]
\end{pmatrix} \begin{pmatrix}
[-m] & [-m] & [-m] & \cdots & [-m] \\
0 & [-m] & [-m] & \cdots & [-m] \\
0 & 0 & [-m] & \cdots & [-m] \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & [-m]
\end{pmatrix} = I.$$

Then, when $r = m + 1$, we obtain

$$A^{m+1}D^{m+1} = A^m(AD)D^m = A^mID^m = A^mD^m = I,$$

so the result $(A^r)^{-1} = D^r$ is proven.

4. Fractional grey FAGM(1,1,k) model

Definition 3. The first-order differential equation

$$\frac{dx^{(r)}(t)}{dt} + ax^{(r)}(t) = bt + c, \quad r > 0$$

(2)

is known as the whitening differential equation of the FAGM(1,1,k) model. The parameter $a$ is a development coefficient, while $bt + c$ is the grey action quantity.

The discrete differential equation

$$x^{(r-1)}(k) + az^{(r)}(k) = b\frac{2k - 1}{2} + c$$

(3)
is referred to as the basic equation of the FAGM(1,1,k). \( x^{(r-1)}(k) = x^{(r)}(k) - x^{(r)}(k-1), \)
\( z^{(r)}(k) = 0.5 \left( x^{(r)}(k-1) + x^{(r)}(k) \right) \).

The least-squares estimation for \( \phi = (a, b, c) \) of the FAGM(1,1,k) model satisfies
\[
\phi = (B^T B)^{-1} B^T Y, \tag{4}
\]
where
\[
B = \begin{pmatrix}
-z^{(r)}(2) & \frac{3}{2} & 1 \\
-z^{(r)}(3) & \frac{5}{2} & 1 \\
\vdots & \vdots & \vdots \\
-z^{(r)}(\nu) & \frac{2\nu-1}{2} & 1
\end{pmatrix}, \quad Y = \begin{pmatrix}
x^{(r-1)}(2) \\
x^{(r-1)}(3) \\
\vdots \\
x^{(r-1)}(\nu)
\end{pmatrix},
\]
in which \( \nu \) is the number of samples used to construct the model.

**Theorem 3.** The time response function of the FAGM(1,1,k) model is
\[
\hat{x}^{(r)}(k) = \left[ x^{(0)}(1) - \frac{b}{a} + \frac{b}{a^2} - \frac{c}{a} \right] e^{-a(k-1)} + \frac{b}{a} k - \frac{b}{a^2} + \frac{c}{a}, \quad k = 2, 3, \ldots, n, \tag{5}
\]
and the restored value of \( \hat{x}^{(0)}(k) \) \( k = 2, 3, \ldots, n \) can be expressed by
\[
\hat{X}^{(0)} = \hat{X}^{(r)} D^r. \tag{6}
\]

**Proof 3.** From Eq. (2), we have
\[
\frac{dx^{(r)}(t)}{dt} = -ax^{(r)}(t) + bt + c. \tag{7}
\]
Let \( u(t) = -ax^{(r)}(t) + bt + c \); then, Eq. (7) is transformed into
\[
\frac{du(t)}{dt} = -a \frac{dx^{(r)}(t)}{dt} + b = -au(t) + b. \tag{8}
\]
To perform the indefinite integral on Eq. (8) and reduce it, we obtain
\[
-a \left( -ax^{(r)}(t) + bt + c \right) + b = e^{-at}e^c, \tag{9}
\]
where \( e^c \) is a constant to be determined.
Substituting $t = 1$ and $x^{(r)}(t)|_{t=1} = x^{(0)}(1)$ into Eq. (9), we obtain

$$e^\kappa = e^a \left( a^2 x^{(0)}(1) - ab - ac + b \right).$$ (10)

It follows from Eqs. (9) and (10) that

$$x^{(r)}(t) = x^{(0)}(1) - \frac{b}{a} + \frac{b}{a^2} - \frac{c}{a} e^{-a(t-1)} + \frac{b}{a} t - \frac{b}{a^2} + \frac{c}{a}.$$ (11)

Thus, the time response function of the FAGM(1,1,k) model is

$$\hat{x}^{(r)}(k) = \left[ x^{(0)}(1) - \frac{b}{a} + \frac{b}{a^2} - \frac{c}{a} \right] e^{-a(k-1)} + \frac{b}{a} k - \frac{b}{a^2} + \frac{c}{a}, k = 2, 3, \ldots, n,$$

and the restored value of $\hat{x}^{(0)}(k)$ $k = 2, 3, \ldots, n$ can be expressed by

$$\left( \hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \ldots, \hat{x}^{(0)}(n) \right) = \left( \hat{x}^{(r)}(1), \hat{x}^{(r)}(2), \ldots, \hat{x}^{(r)}(n) \right) D^r.$$

Setting $b = 0$ in Eq. (2), the fractional FAGM(1,1,k) model is reduced to the fractional FAGM(1,1) model with the form

$$\frac{d^{x^{(r)}(t)}}{dt} + aX^{(r)}(t) = c.$$ (12)

Setting $r = 1$ in Eq. (2), the fractional FAGM(1,1,k) model is reduced to the GM(1,1,k,c) model with the form

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = bt + c.$$ (13)

Setting $r = 1, c = 0$ in Eq. (2), the fractional FAGM(1,1,k) model is reduced to the GM(1,1,k) model with the form

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = bt.$$ (14)

Setting $r = 1, b = 0, c = 1$ in Eq. (2), the fractional FAGM(1,1,k) model is reduced to the GM(1,1) model with the form

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = c.$$ (15)

Thereafter, the flaw of the FAGM(1,1,k) model is provided. Integrating both sides of Eq. (2) in the interval $[k-1, k]$, we obtain

$$\int_{k-1}^{k} \frac{d^{x^{(r)}(t)}}{dt} + \int_{k-1}^{k} ax^{(r)}(t) dt = \int_{k-1}^{k} bt dt + \int_{k-1}^{k} c dt.$$ (16)
With the knowledge of $\int_{k-1}^{k} dx^{(r)}(t) = x^{(r)}(k) - x^{(r)}(k - 1) = x^{(r-1)}(k)$, $\int_{k-1}^{k} t dt = (2k - 1)/2$ and $\int_{k-1}^{k} dt = 1$, the exact discrete differential equation is expressed by
\[
x^{(r-1)}(k) + a \int_{k-1}^{k} x^{(r)}(t) dt = \frac{(2k - 1)b}{2} + c. \tag{17}
\]

A comparison between Eq. (17) and the basic Eq. (3) indicates that differences exist in the background value $\hat{x}^{(r)}(k) = 0.5 \left( x^{(r-1)}(k-1) + x^{(r)}(k) \right)$ and $\int_{k-1}^{k} x^{(r)}(t) dt$. It is highly inaccurate to compute the integration utilising the trapezoid formula if $x^{(r)}(t)$ is not a linear function. Thus, the basic form and whitenisation differential equation of the FAGM(1,1,k) model do not strictly match.

5. Parameter optimisation of FAGM(1,1,k) model

It can easily be verified that the parameters $\phi = (a, b, c)$, derived by the least-squares estimation in Eq. (3) and the parameters of the time response function $\hat{x}^{(r)}(k), k = 2, 3, \ldots, n$ derived by Eq. (2), have different meanings. When the response function does not satisfy the basic equation, large errors may arise. To match the basic Eq. (3) and response function (5), the system parameters are optimised in this system.

Setting the optimised parameters of the grey system as $(\alpha, \beta, \gamma)$ and replacing the parameters $\phi = (a, b, c)$ in Eq. (2), the whitening differential equation is rewritten as
\[
\frac{dx^{(r)}(t)}{dt} + \alpha x^{(r)}(t) = \beta t + \gamma, \quad r > 0. \tag{18}
\]

Similarly, the general solution of Eq. (18) is given by
\[
x^{(r)}(t) = \left[ x^{(0)}(1) - \frac{\beta}{\alpha} + \frac{\beta}{\alpha^2} - \frac{\gamma}{\alpha} \right] e^{-\alpha(t-1)} + \frac{\beta}{\alpha}t - \frac{\beta}{\alpha^2} + \frac{\gamma}{\alpha}. \tag{19}
\]

Furthermore, we have
\[
\hat{x}^{(r)}(k) = \left[ x^{(0)}(1) - \frac{\beta}{\alpha} + \frac{\beta}{\alpha^2} - \frac{\gamma}{\alpha} \right] e^{-\alpha(k-1)} + \frac{\beta}{\alpha}k - \frac{\beta}{\alpha^2} + \frac{\gamma}{\alpha}, \quad k = 2, 3, \ldots, n. \tag{20}
\]
Substituting Eq. (20) into the left side of Eq. (3), we obtain

\[
L(t) = \left( x^{(r)}(k) - x^{(r-1)}(k-1) \right) + \frac{a}{2} \left( x^{(r-1)}(k-1) + x^{(r)}(k) \right) \\
= \left( 1 + \frac{a}{2} \right) x^{(r)}(k) - \left( 1 - \frac{a}{2} \right) x^{(r-1)}(k-1) \\
= \left( 1 + \frac{a}{2} \right) \left[ \left( x^{(0)}(1) - \frac{\beta}{\alpha} + \frac{\beta}{\alpha^2} - \frac{\gamma}{\alpha} \right) e^{-\alpha(k-1)} + \frac{\beta}{\alpha} k - \frac{\beta}{\alpha^2} + \frac{\gamma}{\alpha} \right] \\
- \left( 1 - \frac{a}{2} \right) \left[ \left( x^{(0)}(1) - \frac{\beta}{\alpha} + \frac{\beta}{\alpha^2} - \frac{\gamma}{\alpha} \right) e^{-\alpha(k-2)} + \frac{\beta}{\alpha} (k-1) - \frac{\beta}{\alpha^2} + \frac{\gamma}{\alpha} \right] \\
= \left[ \left( 1 + \frac{a}{2} \right) - \left( 1 - \frac{a}{2} \right) e^{\alpha} \right] \left( x^{(0)}(1) - \frac{\beta}{\alpha} + \frac{\beta}{\alpha^2} - \frac{\gamma}{\alpha} \right) e^{-\alpha(k-1)} \\
+ \frac{\beta}{\alpha} a k + \left( 1 - \frac{a}{2} \right) \frac{\beta}{\alpha} - \left( \frac{\beta}{\alpha^2} - \frac{\gamma}{\alpha} \right) a. \\
\]  

(21)

Owing to the left side \( L(t) \) and right side \( R(t) \) equivalence, namely \( L(t) - R(t) = 0 \), it is implied that

\[
\left( 1 + \frac{a}{2} \right) - \left( 1 - \frac{a}{2} \right) e^{\alpha} = 0, \\
\left( 1 - \frac{a}{2} \right) \frac{\beta}{\alpha} - \left( \frac{\beta}{\alpha^2} - \frac{\gamma}{\alpha} \right) a = c - \frac{b}{2}. \\
\]  

(22)  

(23)  

(24)

It follows from Eqs. (22) to (24) that

\[
\alpha = \ln \frac{2 + a}{2 - a}, \\
\beta = \frac{b}{a} \ln \frac{2 + a}{2 - a}, \\
\gamma = \frac{ac}{a} - \frac{ab}{2a} + \frac{\beta}{\alpha} + \frac{\beta}{2} - \frac{\beta}{a}. \\
\]  

(25)  

(26)  

(27)

Thus, the optimised parameters \((\alpha, \beta, \gamma)\) are obtained by Eqs. (25) to (27), and they also indicate that the parameters \((a, b, c)\) derived by the least-squares estimation satisfy the relationship in Eqs. (24) to (27). In this paper, the FAGM(1,1,k) model with optimised parameters is referred to as the FAGMO(1,1,k) model.

**Theorem 4.** Assuming that the original data \((x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))\) satisfy Eq. (24) with the given parameters \((\hat{\alpha}, \hat{\beta}, \hat{\gamma})\), the parameters \((\alpha, \beta, \gamma)\) of
FAGMO(1,1,k) obtained by Eqs. (4) and (25) to (27) satisfy the relationship \( \hat{\alpha} = \alpha, \hat{\beta} = \beta, \hat{\gamma} = \gamma \), and the predicted values \((\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \ldots, \hat{x}^{(0)}(n))\) are equal to the given data \((x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))\).

**Proof 4.** Substituting the original data \(x^{(0)}(k), k = 1, 2, \ldots, n\) into Eq. (4), the parameters \((\hat{\alpha}, \hat{\beta}, \hat{\gamma})\) can be derived. The parameters \((\alpha, \beta, \gamma)\) of FAGMO(1,1,k) can be obtained from Eqs. (25) to (27). Obviously, \(\hat{\alpha} = \alpha, \hat{\beta} = \beta, \hat{\gamma} = \gamma\). Therefore, the predicted values \(\hat{x}^{(0)}(k), k = 1, 2, \ldots, n\) of the FAGMO(1,1,k) are equal to the given data \(x^{(0)}(k), k = 1, 2, \ldots, n\).

Theorem 4 demonstrates that the FAGMO(1,1,k) model is accurate for predicting arbitrary sequences that can be modelled by Eq. (20), while the FAGM(1,1,k) model cannot describe the sequences accurately owing to there always being a non-zero difference between the real parameters and the parameters \((a, b, c)\).

**Theorem 5.** The optimised parameters \((\alpha, \beta, \gamma)\) are approximately equivalent to the parameters \((a, b, c)\) when the value of \(|a|\) is very small; that is,

\[
\alpha \approx a, \beta \approx b, \gamma \approx c.
\]

**Proof 5.** We first consider the difference between parameter \(\alpha\) and \(a\), which is

\[
\varepsilon_1 (a) = \alpha - a = \ln \frac{2 + a}{2 - a} - a.
\]

It is known that \(\varepsilon_1 (a)|_{a=0} = 0\) and the first-order derivative is

\[
\frac{d \varepsilon_1 (a)}{da} = \frac{d}{da} \left( \ln \frac{2 + a}{2 - a} - a \right) = \frac{4}{4 - a^2} - 1 = \frac{a^2}{4 - a^2}.
\]

When \(|a| < 2\), the derivative of \(\varepsilon_1 (a)\) is positive, which indicates that the function \(\varepsilon_1 (a)\) is a monotonically increasing function in the interval \([-2, 2]\). Thus, the value \(\varepsilon_1 (a)\) approaches zero as \(|a|\) decreases. Therefore, \(\alpha \approx a\) when \(|a|\) is very small.

Secondly, the difference between \(\beta\) and \(b\) is expressed as

\[
\varepsilon_2 (a) = \beta - b = \frac{b}{a} \ln \frac{2 + a}{2 - a} - b = \frac{b}{a} \varepsilon_1 (a).
\]
Owing to \( \lim_{a \to 0} \frac{1}{a} \ln \frac{2+a}{a} = \lim_{a \to 0} \frac{2-a-\alpha a+2a^2}{2-a} = \lim_{a \to 0} \frac{4}{a^2} = 1 \), we know that \( \varepsilon_2(a) \to 0 \) when \( a \to 0 \).

The first-order derivative of \( \varepsilon_2(a) \) is

\[
\frac{d\varepsilon_2(a)}{da} = b \left( \frac{4a}{4-a^2} - \ln \frac{2+a}{2-a} \right),
\]

which is also positive when \( |a| < 2 \). Thus, \( \varepsilon_2(a) \) decreases when the value of \( |a| \) decreases and \( \beta \approx b \) when \( |a| \) is very small.

Thirdly, the difference between \( \gamma \) and \( c \) is expressed as

\[
\varepsilon_3(a) = \gamma - c = \frac{\alpha c}{a} + \frac{\alpha b}{2a} + \frac{\beta}{\alpha} + \frac{\beta}{2} - c = \frac{\varepsilon_1(a)}{a} \left( c - \frac{b}{a} \right). \tag{33}
\]

It follows from \( \alpha \approx a \) and \( \beta \approx b \) that \( \gamma \approx c \) when \( |a| \) is very small.

From Theorem 6, we know that the differences between the parameters \( (\alpha, \beta, \gamma) \) and \( (a, b, c) \) are decrease along with smaller \( |a| \). Table 1 provides the values of \( \varepsilon_1(a) \) and \( \varepsilon_1(a)/a \) under different values of \( |a| \).

| \( |a| \) | 0.1 | 0.2 | 0.3 | 0.5 | 0.7 | 1.0 | 1.3 | 1.6 | 1.9 |
|---|---|---|---|---|---|---|---|---|---|
| \( \varepsilon_1(a) \) | 0.0001 | 0.0007 | 0.0023 | 0.0108 | 0.0309 | 0.0986 | 0.2506 | 0.5972 | 1.7636 |
| \( \varepsilon_1(a)/a \) | 0.0008 | 0.0034 | 0.0076 | 0.0217 | 0.0441 | 0.0986 | 0.1928 | 0.3733 | 0.9282 |

6. Modelling evaluation criteria and detailed modelling steps

To evaluate forecasting accuracy of the FAGMO(1,1,k) model, the root mean squared percentage error (RMSPE) is applied to the prior-sample period (RMSPEPR) and post-sample period (RMSPEPO). In general, the RMSPEPR, RMSPEPO and RMSPE are defined as

\[
\text{RMSPEPR} = \sqrt{\frac{1}{\nu} \sum_{k=1}^{\nu} \left( \frac{\hat{x}_1^{(0)}(k) - x_1^{(0)}(k)}{x_1^{(0)}(k)} \right)^2} \times 100%, \tag{34}
\]

\[
\text{RMSPEPO} = \sqrt{\frac{1}{n-\nu} \sum_{k=\nu+1}^{n} \left( \frac{\hat{x}_1^{(0)}(k) - x_1^{(0)}(k)}{x_1^{(0)}(k)} \right)^2} \times 100%, \tag{35}
\]

\[
\text{RMSPE} = \sqrt{\frac{1}{n} \sum_{k=1}^{n} \left( \frac{\hat{x}_1^{(0)}(k) - x_1^{(0)}(k)}{x_1^{(0)}(k)} \right)^2} \times 100%, \tag{36}
\]
where $\nu$ is the number of samples used to construct the model and $n$ is the total number of samples.

The index of agreement of the forecasting results is defined as

$$IA = 1 - \frac{\sum_{k=1}^{n} (\hat{x}^{(0)}(k) - x^{(0)}(k))^2}{\sum_{k=1}^{n} (|\hat{x}^{(0)}(k) - \overline{x}| + |x^{(0)}(k) - \overline{x}|)^2},$$

(37)

which is also a useful performance measure for sensitivity to differences in the observed and predicted data, where $\overline{x}$ is the average sample value.

The average forecasting error (AE) and the mean absolute forecasting error (MAE) are

$$AE = \frac{1}{n} \sum_{k=1}^{n} (\hat{x}^{(0)}(k) - x^{(0)}(k)), \quad (38)$$

$$MAE = \frac{1}{n} \sum_{k=1}^{n} |\hat{x}^{(0)}(k) - x^{(0)}(k)|, \quad (39)$$

where AE reflects the positive and negative errors between the predicted and observed values, while MAE is applied for estimating the change in the forecasting model.

The detailed modelling steps of the fractional FAGMO(1,1,$k$) are provided below.

**Step 1:** Determine the original data series $x^{(0)}(i), i = 1, 2, \ldots, n,$ and r-AGO series $X^{(r)} = X^{(0)} A^r$.

**Step 2:** Calculate the matrices $B$ and $Y$ to determine $(a, b, c)$ using Eq. 4.

**Step 3:** Compute the parameters $(\alpha, \beta, \gamma)$ by employing Eqs. (25) to (27).

**Step 4:** Substitute the values of $x^{(0)}(1)$ and $(\alpha, \beta, \gamma)$ into Eq. (20) to compute the predicted values $\hat{X}^{(r)}$.

**Step 5:** Apply the $r$-IAGO matrix to obtain the restored values $\hat{X}^{(0)} = \hat{X}^{(r)} D^r$.

7. Validation of FAGMO(1,1,$k$) model

This section provides numerical examples to validate the accuracy of the FAGMO(1,1,$k$) model compared to the FAGM(1,1,$k$) model and others.
7.1. Validation of FAGMO(1,1,k) and FAGM(1,1,k) models

This subsection presents a numerical example to validate the accuracy of the FAGMO(1,1,k) and FAGM(1,1,k) models. The values \( r \) and \( \alpha \) are provided in the interval \([0.01, 2]\) and \([-1.99, 1.99]\), respectively. The initial point \( x^{(r)}(1) \) is randomly generated in the interval \([1, 2]\) by the uniform distribution, while the parameters \( \beta \) and \( \gamma \) are randomly generated in the intervals \([0, 5]\) and \([0, 100]\), respectively, by the uniform distribution. The other \( x^{(r)}(i) (i > 1) \) are generated with the aid of Eq. (20). All data used for the example are explained in Fig. 5.

\[
\frac{dx^{(r)}(t)}{dt} + \alpha x^{(r)}(t) = \beta t + \gamma
\]

given in an interval randomly generated
\( x^{(r)}(1) \) randomly generated
\( x^{(r)}(i) \) calculated by Eq.(20)

Figure 5: Diagram of data for validation

We define the notation in the following analysis

\[
\varepsilon_{\text{params}} = (p - \alpha)^2 + (l - \beta)^2 + (q - \gamma)^2,
\] (40)

where \((\alpha, \beta, \gamma)\) are the provided parameters of Eq. (20) and \((p, l, q)\) are the estimated parameters of the FAGM(1,1,k) or FAGMO(1,1,k) model.

When applying the above parameters, the graphs are displayed in Figs. 6 and 7. We observe from Fig. 6 that the maximum \( \varepsilon_{\text{params}} \) of FAGMO(1,1,k) and FAGM(1,1,k) are \(5.4228 \times 10^{-5}\) and \(489.9434\), respectively, where the magnitude is approximately 9034932. Furthermore, the \( \varepsilon_{\text{params}} \) of the FAGM(1,1,k) model is very small when \( \alpha \) is near zero, which is coincident with Theorem 5. From Fig.
the maximum RMSPEs of FAGMO(1,1,k) and FAGM(1,1,k) are 0.0103% and 814.3864%, respectively, where the magnitude is approximately 79100.

It is known that the parameters $\beta$, $\gamma$, and initial points $x^{(r)}(1)$ are all randomly generated, which implies that the values of parameters $\beta$, $\gamma$ and $x^{(r)}(1)$ have no influence on the output series. Here, the values $r$ and $\alpha$ are the most important factors affecting the accuracy of the grey models.

$$\begin{align*}
\text{Figure 6: Values of } \varepsilon_{\text{params}} \text{ of FAGMO(1,1,k) (left) and FAGM(1,1,k) (right) models}
\end{align*}$$

$$\begin{align*}
\text{Figure 7: Values of RMSPE of FAGMO(1,1,k) (left) and FAGM(1,1,k) (right) models}
\end{align*}$$

7.2. Validation of FAGMO(1,1,k) model and other grey models

This subsection further demonstrates the advantage of the FAGMO(1,1,k) model using two real cases.
Case 1: (Predicting cumulative oil field production). We consider an example from the paper [54] that provides sample data. The data from 1999 to 2009 are applied to construct the grey model, while the data from 2010 to 2012 are used for prediction. The values are listed in Table 2, indicating that the FAGMO(1,1,k) model outperforms the other models in this case.

| Year | Data   | ENGM  | FAGM(1,1) | FAGM(1,1,k) | FAGMO(1,1,k) |
|------|--------|-------|-----------|-------------|--------------|
|      |        | r = 1 | r = 0.1106 | r = 0.4073  | r = 0.4052   |
| 1999 | 73.8217| 73.8217| 73.8217    | 73.8217     | 73.8217      |
| 2000 | 136.8817| 138.4900| 138.1621   | 137.1758    | 136.4573     |
| 2001 | 195.0590| 195.4541| 195.5377   | 196.1598    | 195.7633     |
| 2002 | 247.8547| 247.9776| 247.7638   | 249.3183    | 249.1781     |
| 2003 | 297.0902| 296.4067| 295.7629   | 297.2895    | 297.2750     |
| 2004 | 342.6394| 341.0604| 340.1238   | 341.0008    | 341.0322     |
| 2005 | 382.4312| 382.2332| 381.2700   | 381.2882    | 381.3320     |
| 2006 | 420.0399| 420.1964| 419.5291   | 418.8204    | 418.8699     |
| 2007 | 454.0430| 455.2001| 455.1670   | 454.1099    | 454.1712     |
| 2008 | 485.1171| 487.4752| 488.4068   | 487.5452    | 487.6290     |
| 2009 | 519.8508| 517.2342| 519.4402   | 519.4217    | 519.5393     |
| 2010 | 552.6569| 544.6734| 548.4350   | 549.9665    | 550.1281     |
| 2011 | 581.6092| 569.9736| 575.5400   | 579.3572    | 579.5714     |
| 2012 | 608.1863| 593.3015| 600.8887   | 607.7346    | 608.0086     |
| RMSPEPR | 0.4521% | 0.4582% | 0.3539% | 0.3259% | 0.3332% |
| RMSPEPO | 2.0066% | 1.0185% | 0.3617% | 0.332% | 0.3332% |

Case 2: (Predicting foundation settlement close neighbouring Yangtze River). We consider an example from the paper [53], which provides sample data to construct the grey model. The values are presented in Table 3, indicating that the FAGMO(1,1,k) model outperforms the other models in this case.
Table 3: Results of ONGM(1,1,k,c), FAGM(1,1), FAGM(1,1,k) and FAGMO(1,1,k) models

| Day | Data | ONGM(1,1,k,c) | FAGM(1,1) | FAGM(1,1,k) | FAGMO(1,1,k) |
|-----|------|--------------|-----------|-------------|--------------|
| 10  | 23.36| 23.3600      | 23.3600   | 23.3600     | 23.3600      |
| 20  | 43.19| 42.1779      | 43.3517   | 43.0586     | 43.0644      |
| 30  | 58.73| 59.2549      | 59.4403   | 58.8205     | 58.8124      |
| 40  | 70.87| 72.8374      | 72.4009   | 71.9763     | 71.9545      |
| 50  | 83.71| 83.6405      | 82.8451   | 83.0247     | 82.9932      |
| 60  | 92.91| 92.2330      | 91.2620   | 92.2158     | 92.1789      |
| 70  | 99.73| 99.0672      | 98.0442   | 99.6885     | 99.6491      |
| 80  | 105.08| 104.5030    | 103.5079  | 105.5215    | 105.4805     |
| 90  | 109.73| 108.8264    | 107.9079  | 109.7568    | 109.7127     |
| 100 | 112.19| 112.2652    | 111.4497  | 112.4117    | 112.3598     |
| 110 | 113.45| 115.0002    | 114.2991  | 113.4857    | 113.4181     |

RMSPE 1.2730% 1.3257% 0.6030% 0.6011%

8. Applications

In this section, the FAGMO(1,1,k) model is applied to forecast the nuclear energy consumption of China. The computational results of the FAGMO(1,1,k) model are compared to the ENGM [54], ONGM(1,1,k) [53], FAGM(1,1) [40] and FAGM(1,1,k) models.

8.1. Raw data

Raw data of the nuclear energy consumption of China were collected from the report of the BP Statistical Review of World Energy 2018. The first 10 samples belonging to the 11th and the 12th Five-Year Plans are applied to construct the prediction model, while the remaining samples of the 13th Five-Year Plan are used to validate and compare the forecasting results (see Table 4).

Table 4: Raw data of nuclear energy consumption of China, Mtoe

| Year | Data | Year | Data | Year | Data |
|------|------|------|------|------|------|
| 2006 | 12.4 | 2011 | 19.5 | 2016 | 48.2 |
| 2007 | 14.1 | 2012 | 22.0 | 2017 | 56.2 |
| 2008 | 15.5 | 2013 | 25.3 |      |      |
| 2009 | 15.9 | 2014 | 30.0 |      |      |
| 2010 | 16.7 | 2015 | 38.6 |      |      |
8.2. Simulation and prediction results

The simulation and prediction results are listed in Table 5 and Fig. 8 while the errors are listed in Table 6 and Fig. 9.

The nuclear energy consumption of China from 2016 to 2017 is predicted according to the established grey models. It can be observed in Table 5 and Fig. 8 that five grey models, namely ENGM, ONGM(1,1,k), FAGM(1,1), FAGM(1,1,k) and FAGMO(1,1,k), successfully identify the trend of China’s nuclear energy consumption. However, these grey models differ from one another in terms of the prediction values from 2016 to 2020. From Fig. 8, China’s nuclear energy consumption is overestimated by the ENGM, ONGM(1,1,k) and FAGM(1,1,k) models, and underestimated by the FAGM(1,1) model. The values predicted by FAGMO(1,1,k) are substantially closer to the raw data than those predicted by the other models.

We can observe from Table 6 and Fig. 9 that the RMSPEPR, RMSPEPO and RMSPE of FAGMO(1,1,k) are 3.1409%, 4.1502% and 3.3304%, respectively. The RMSPEPR, RMSPEPO and RMSPE of ENGM are as high as 8.3788%, 30.3663% and 14.5667%, those of ONGM(1,1,k) are 2.0494%, 12.0510% and 5.2635%, those of FAGM(1,1) are 4.8680%, 11.7968% and 6.5529%, and those of FAGM(1,1,k) are 2.3299%, 6.3828% and 3.3636%, respectively. The IA, AE and MAE of FAGMO(1,1,k) are 0.9985, 0.2526 and 0.7513, those of ENGM are 0.9538, 4.0536 and 4.0536, those of ONGM(1,1,k) are 0.9911, 1.0225 and 1.1896, those of FAGM(1,1) are 0.9887, -1.1818 and 1.7105, and those of FAGM(1,1,k) are 0.9971, 0.2736 and 0.8043, respectively. The computational results indicate that the FAGMO(1,1,k) model outperforms ENGM, ONGM(1,1,k), FAGM(1,1) and FAGM(1,1,k), while ENGM exhibits the most inferior performance.
Table 5: Simulation and prediction results of nuclear energy consumption by grey models

| Year | Data | ENGM | ONGM(1,1,k) | FAGM(1,1) | FAGM(1,1,k) | FAGMO(1,1,k) |
|------|------|------|-------------|-----------|-------------|--------------|
| 2006 | 12.4 | 12.4000 | 12.4000 | 12.4000 | 12.4000 | 12.4000 |
| 2007 | 14.1 | 14.9788 | 14.4788 | 15.0242 | 14.7054 | 15.0891 |
| 2008 | 15.5 | 15.6744 | 15.1057 | 13.9808 | 15.0121 | 14.8608 |
| 2009 | 15.9 | 16.6846 | 16.0032 | 15.0566 | 15.8012 | 15.5886 |
| 2010 | 16.7 | 18.1520 | 17.2884 | 16.9219 | 17.0700 | 16.9760 |
| 2011 | 19.5 | 20.2831 | 19.1286 | 19.3953 | 18.9344 | 19.0534 |
| 2012 | 22.0 | 23.3785 | 21.7635 | 22.4687 | 21.5861 | 21.9432 |
| 2013 | 25.3 | 27.8741 | 25.5363 | 26.1951 | 25.3029 | 25.8633 |
| 2014 | 30.0 | 34.036 | 30.9383 | 30.6625 | 30.4740 | 31.1013 |
| 2015 | 38.6 | 43.8871 | 38.6732 | 35.9872 | 37.6390 | 38.0473 |
| 2016 | 48.2 | 57.6610 | 49.7483 | 42.3129 | 47.5433 | 47.2178 |
| 2017 | 56.2 | 77.6662 | 65.6063 | 49.8133 | 61.2149 | 59.2933 |
| 2018 | – | 106.7219 | 88.3125 | 58.6959 | 80.0704 | 75.1679 |
| 2019 | – | 148.9226 | 120.8244 | 69.2074 | 106.0614 | 96.0147 |
| 2020 | – | 210.2150 | 167.3766 | 81.6403 | 141.8758 | 123.3723 |

Figure 8: Compare the five grey models for nuclear energy consumption
Table 6: Relative error values of nuclear energy consumption by five grey models

| Year | ENG | ONGM(1,1,k) | FAGM(1,1) | FAGM(1,1,k) | FAGMO(1,1,k) |
|------|-----|-------------|-----------|-------------|--------------|
| 2006 | 0   | 0           | 0         | 0           | 0            |
| 2007 | 0.0623 | 0.0269     | 0.0655    | 0.0429      | 0.0701       |
| 2008 | 0.0112 | 0.0254     | 0.0980    | 0.0315      | 0.0412       |
| 2009 | 0.0493 | 0.0065     | 0.0530    | 0.0062      | 0.0196       |
| 2010 | 0.0869 | 0.0352     | 0.0133    | 0.0222      | 0.0165       |
| 2011 | 0.0402 | 0.0190     | 0.0054    | 0.0290      | 0.0231       |
| 2012 | 0.0627 | 0.0108     | 0.0213    | 0.0188      | 0.0026       |
| 2013 | 0.1017 | 0.0093     | 0.0354    | 0.0001      | 0.0222       |
| 2014 | 0.1468 | 0.0313     | 0.0221    | 0.0158      | 0.0367       |
| 2015 | 0.1370 | 0.0019     | 0.0677    | 0.0249      | 0.0143       |
| 2016 | 0.1963 | 0.0321     | 0.1221    | 0.0136      | 0.0204       |
| 2017 | 0.3820 | 0.1674     | 0.1136    | 0.0892      | 0.0550       |

RMSPEPR 8.3788% 2.0494% 4.8680% 2.3299% 3.1409%
RMSPEPO 30.3663% 12.0510% 11.7968% 6.3828% 4.1502%
RMSPE 14.5667% 5.2635% 6.5529% 3.3636% 3.3304%
IA 0.9538 0.9911 0.9887 0.9971 0.9985
AE 4.0536 1.0225 -1.1818 0.2736 0.2526
MAE 4.0536 1.1896 1.7105 0.8043 0.7513

Figure 9: Errors among five grey models for nuclear energy consumption
8.3. Further discussions

As demonstrated by the case study, the novel FAGMO(1,1,k) model outperforms other grey models. Moreover, it should be noted that in this paper we only conduct short-term forecasting, while it is well known that several existing energy models can perform long-term forecasting, such as LEAP, TIMES and NEMS. We will discuss the difference between our model and these models further, following a very brief introduction to such models.

- **LEAP** (long-range energy alternatives planning system) \[14, 15\] is a scenario-based energy environment modelling tool for climate change mitigation and energy policy analysis. It can be applied to examine energy production and consumption, as well as resource extraction in all sectors. The model studies the effects of various factors on energy consumption under different scenarios given an objective. LEAP is generally used for forecasting studies of between 20 and 50 years.

- **TIMES** (The Integrated MARKAL-EFOM System) \[16, 17\] is an evolution of MARKAL, which was developed by the Energy Technology Systems Analysis Programme of the IEA. It combines technical engineering and economic approaches, and uses linear programming to produce a least-cost energy system under numerous user-specified constraints. The software is used to analyse energy, economic and environmental issues at different levels over several decades.

- **NEMS** (National Energy Modeling System) \[18, 19\] is a long-standing US government policy model, which computes equilibrium fuel prices and quantities for the US energy sector. NEMS is used to model the demand side explicitly; in particular, to determine consumer technology choices in the residential and commercial building sectors.

These models can perform long-term energy consumption projections. However, they may require a large amount of data, such as population growth, GDP, urbanisation, energy policies and energy strategies. In numerous practical situations, it is very difficult to obtain complete information because of time and cost limitations. The grey prediction model is an efficient method for conducting accurate forecasting with at least four samples. Compared to the
major energy models, the grey model is an effective choice for predicting China’s nuclear energy consumption.

This paper collected 12 samples of China’s nuclear energy consumption from the *BP Statistical Review of World Energy 2018*. Thus, the LEAP, TIMES and NEMS models are all inapplicable owing to poor information. By employing the grey system theory and actual data from the 11th and the 12th Five-Year Plans, the FAGMO(1,1,k) model was constructed. It can be observed in Table 5 that the prediction value of FAGMO(1,1,k) is 123.3723 Mtoe in 2020, which is larger than the 84.6318 Mtoe provided in the *BP energy outlook 2018*. The main reasons for this are as follows.

i) It is infeasible to consider factors such as energy policies and China’s energy strategies, which affect the current situation of China’s nuclear energy consumption, in our proposed model because the FAGMO(1,1,k) model is univariate. However, the forecasting models of institutions including BP, the IEA and APEC are based on widely collected data. Furthermore, grey models are mainly used for short-term forecasting in the calculation process, such as [22, 23, 24, 27, 57]. Therefore, the forecasting results are relatively acceptable, reflecting the growth trend of future nuclear energy consumption in China.

ii) In China’s nuclear energy market, 38 nuclear power reactors are in operation, 19 nuclear power reactors are under construction and more are to be constructed by the end of 2016. This is the reason for the increase in nuclear energy consumption in recent years. However, no new nuclear projects have been approved for construction in 2016. Moreover, the State Council approved new safety rules and a nuclear power development plan following Japan’s Fukushima Daiichi crisis in 2011. These factors have also resulted in a slight slowdown in China’s nuclear energy consumption.

In the future, nuclear energy could provide an important alternative to fossil fuels such as coal and oil, and its proportion of the total primary energy consumption will increase yearly. Based on our forecasting results using the FAGMO(1,1,k) model, the future nuclear energy consumption of China will increase rapidly if no certain restrictions are placed thereon. This implies that
higher management and technical levels are necessary to meet the safety and quality requirements. Therefore, China’s government and policy makers should pay additional attention to the safety and quality issues of nuclear energy to achieve long-term, environmentally friendly and low-carbon energy goals and lay the foundation for the sustainable development of China’s energy and economy.

9. Conclusions

By applying the grey modelling technique and parameter optimisation method, the fractional FAGMO(1,1,k) model was proposed to predict China’s nuclear energy consumption of the 13th Five-Year Plan, based on the updated data from 2006 to 2015. The forecasting results provide the growth trend of the future nuclear energy consumption of China, and also offer a guideline for policymaking and project planning.

It can be observed that FAGMO(1,1,k) is quite easy to use, with satisfactory accuracy in short-term nuclear consumption forecasting. For long-term prediction, its error will be larger because only 10 samples are used for modelling. This study is expected to be able to forecast the energy consumption of other countries that share similar patterns of economic development and energy consumption structures, among others. Furthermore, the optimised method applied to improve the FAGM(1,1,k) model can be used to improve other first-order grey models, such as NGBM(1,1), GMC(1,n) and RDGM(1,n). These are possible extensions and suggested directions for our future research.

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