Constraints on Left-Right Symmetric and $E_6$ Superstring Models from the Number of Light Neutrino Species

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Abstract

Using the experimental value for the rate $R_{\text{exp}}^\text{LEP} = \Gamma_{\text{inv}}/\Gamma_{\bar{l}l} = 5.942\pm0.016$, we derive constraints on the number of light neutrino species with the invisible width method in the framework of a left-right symmetric model (LRSM) and a $E_6$ superstring model as a function of their respective mixing angles for the neutral vector gauge bosons. Using the above LEP result we get the constraints $-1.6 \times 10^{-3} \leq \phi \leq 1.1 \times 10^{-3}$ and $-1.3 \times 10^{-2} \leq \phi_{E_6} \leq 0.4 \times 10^{-2}$, which are stronger than those obtained in previous studies of these models.

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I. INTRODUCTION

The number of fermion generations, which is associated to the number of light neutrinos, is one of the most important predictions of the Standard Model of the electroweak interactions (SM) [1, 2, 3]. In the SM the decay width of the $Z_1$ boson into each neutrino family is calculated to be $\Gamma_{\nu\bar{\nu}} = 166.3 \pm 1.5$ MeV [4]. Additional generations, or other new weakly interacting particles with masses below $M_{Z_1}/2$, would lead to a decay width of the $Z_1$ into invisible channels larger than the SM prediction for three families while a smaller value could be produced, for example, by the presence of one or more right-handed neutrinos mixed with the left-handed ones [5]. Thus the number of light neutrino generations $N_\nu$, defined as the ratio between the measured invisible decay width of the $Z_1$, $\Gamma_{\text{inv}}$, and the SM expectation $\Gamma_{\nu\bar{\nu}}$ for each neutrino family, need not be an integer number and has to be measured with the highest possible accuracy.

The most precise measurement of the number of light ($m_\nu \ll 1$ GeV) active neutrino types, and therefore the number of associated fermion families, comes from the invisible $Z_1$ width $\Gamma_{\text{inv}}$, obtained by subtracting the observed width into quarks and charged leptons from the total width obtained from the lineshape. The number of effective neutrinos $N_\nu$ is given by [6]

$$N_\nu = \frac{\Gamma_{\text{inv}}}{\Gamma_{l\bar{l}}} \left( \frac{\Gamma_{l\bar{l}}}{\Gamma_{\nu\bar{\nu}}} \right)_{\text{SM}},$$

where $\left( \frac{\Gamma_{l\bar{l}}}{\Gamma_{\nu\bar{\nu}}} \right)_{\text{SM}}$, the SM expression for the ratio of widths into a charged lepton and a single active neutrino, is introduced to reduce the model dependence. The experimental value from the four LEP experiments is $N_\nu = 2.9841 \pm 0.0083$ [4, 7, 8, 9], excluding the possibility of a fourth family unless the neutrino is very heavy. This result is in agreement with cosmological constraints on the number of relativistic species around the time of Big Bang nucleosynthesis, which seems to indicate the existence of three very light neutrino species [10, 11, 12, 13]. On the other hand, the LEP result is about two sigmas away from the SM expectation, $N_\nu = 3$. While not statistically significant, this result suggests that the $Z\nu\bar{\nu}$-couplings might be suppressed with respect to the SM value [6, 14, 15].

Using the experimental value for $R_{\text{exp}} = \frac{\Gamma_{\text{inv}}}{\Gamma_{l\bar{l}}} = 5.942 \pm 0.016$ [7, 8, 9], we will determine the allowed region for $N_\nu$ as a function of $\phi$ and $\phi_{E_6}$, the mixing angles between the light and the heavy neutral gauge bosons $Z_1$ and $Z_2$ of the LRSM [16, 17, 18, 19, 20] and $E_6$ superstring
models \([21, 22, 23, 24, 25, 26]\). In this way we will be able to get bounds on these mixing angles using the LEP data on the number of light neutrino species. On the other hand, if one assumes that the results for \(\Gamma_{inv}\) and \(\Gamma_{ll}\) for a future TESLA-like Giga-\(Z_1\) experiment agree with the central values obtained at LEP, one would get \((\frac{\Gamma_{inv}}{\Gamma_{ll}})_{Giga-Z_1} = 5.942 \pm 0.012\) (most conservative) or \((\frac{\Gamma_{inv}}{\Gamma_{ll}})_{Giga-Z_1} = 5.942 \pm 0.006\) (most optimistic) \([6]\). In this case we estimate also a limit for these mixing angles.

This paper is organized as follows: In Sect. II we present the numerical computation and, finally, we summarize our results in Sec. III.

II. RESULTS

We will take advantage of our previous work on the LRSM \([16, 17, 18, 19]\) and the \(E_6\) superstring model \([27, 28]\) and we will calculate the partial widths for \(Z_1 \rightarrow l\bar{l}\) and \(Z_1 \rightarrow \nu\bar{\nu}\) using the transition amplitudes given in Eqs. (21) and (22) of Ref. \([16, 17, 18, 19]\) and the couplings of the two neutral gauge bosons involved in the \(E_6\) model to fermions given in Refs. \([22, 26]\). In order to compare the respective expressions with the experimental result for the number of light neutrinos species \(N_\nu\), we will use the definition for \(N_\nu\) in a SM analysis \([29]\) given in Eq. (1) and the LEP result for the \(Z_1\) invisible width \([4, 7, 8, 9]\),

\[
R_{\text{lep}} = \left( \frac{\Gamma_{inv}}{\Gamma_{ll}} \right) = 5.942 \pm 0.016.
\]

This definition replaces the expression \(N_\nu = \frac{\Gamma_{inv}}{\Gamma_{\nu\nu}}\) since (2) reduces the influence of the top quarks mass. To get information on the meaning of \(N_\nu\) in the LRSM model we should define the corresponding expression \([14, 15]\),

\[
(N_\nu)_{LRSM} = R_{\text{exp}}(\frac{\Gamma_{ll}}{\Gamma_{\nu\nu}})_{LRSM}.
\]

This new expression is a function of the mixing angle \(\phi\), and in this case the quantity defined as the number of light neutrinos species is not a constant and not necessarily an integer. Also, \((N_\nu)_{LRSM}\) in formula (3) is independent from the \(Z_2\) mass and therefore depends only of the mixing angle \(\phi\) of the LRSM. Experimental values for \(\Gamma_{inv}\) and for \(\Gamma_{ll}\) are reported in literature which, in our case, can give a bound for the angle \(\phi\). However, we can look to those experimental numbers in another way. The partial widths \(\Gamma_{inv} = 499.0 \pm 1.5\)
$MeV$ and $\Gamma_{l\bar{l}} = 83.984 \pm 0.086 \ MeV$ were reported recently [4], but we use the value given by (3) for the $R_{exp}$ rate of Ref. [7, 8, 9]. All these measurements are independent of any model and can be fitted with the LRSM parameter $(N_{\nu})_{LRSM}$ in terms of $\phi$.

The same argument applies to the case of the $E_6$ superstring model and we will be able to get also a limit on its respective mixing angle $\phi_{E_6}$ between the light $Z_1$ and heavy $Z_{\theta}$ gauge bosons involved in this model [21, 22, 23, 24, 25, 26, 28].

In order to estimate a limit for the number of light neutrino species $(N_{\nu})_{LRSM}$ in the framework of a left-right symmetric model, we plot the expression (3) in Fig. 1. In this figure we show the allowed region for $(N_{\nu})_{LRSM}$ as a function of $\phi$ with 90% C.L. The allowed region is the inclined band that is a result of both factors in Eq. (3). In this figure $(\Gamma_{l\bar{l}}\nu\bar{\nu})_{LRSM}$ gives the inclination while $R_{exp}$ gives the broading. This analysis was done using the experimental value given in Eq. (2) for $R_{exp}$ reported by [7, 8, 9] with a 90% C.L. In the same figure we show the SM ($\phi = 0$) result at 90% C.L. with the dashed horizontal lines. The allowed region in the LRSM (dotted line) for $(N_{\nu})_{LRSM}$ is wider that the one for the SM.

If now we reverse the arguments, that is to say we fix the number of neutrinos in the LRSM to be three then the theoretical expression for $R$ will be given by

$$R_{LRSM} = \frac{3\Gamma_{\nu\phi}}{\Gamma_{l\bar{l}}}.$$  

(4)

The plot of this quantity as function of the mixing angle $\phi$ is shows in Fig. 2. The horizontal lines give the experimental region at 90% C.L. From the figure we observe that the constraint for the angle $\phi$ is:

$$-1.6 \times 10^{-3} \leq \phi \leq 1.1 \times 10^{-3},$$  

(5)

which is about one order of magnitude stronger than the one obtained in previous studies of the LRSM [14, 15, 30, 31] and it is consistent with our results obtained recently with the LEP data obtained for the process $e^+e^- \rightarrow \tau^+\tau^-\gamma$ [28].

In the case of a future TESLA-like Giga-$Z_1$ experiment [6] we obtain the following bounds for the mixing angle $\phi$:

$$-1.1 \times 10^{-3} \leq \phi \leq 0.9 \times 10^{-3}, \text{ (most conservative)},$$  

(6)
−0.8 × 10^{-3} \leq \phi \leq 0.33 \times 10^{-3}, \text{ (most optimistic).} \quad (7)

Finally, in Figs. 3 and 4 we present the respective plots for $(N_\nu)_{E_6}$ and $R_{E_6}$ as a function of $\phi_{E_6}$. In this case, we get the following constraints on this mixing angle,

\[-1.3 \times 10^{-2} \leq \phi_{E_6} \leq 0.4 \times 10^{-2}, \quad (8)\]

if use is made of the LEP results and

\[-1.1 \times 10^{-2} \leq \phi_{E_6} \leq 0.2 \times 10^{-2} \text{ (most conservative)}, \quad (9)\]
\[-0.75 \times 10^{-2} \leq \phi_{E_6} \leq 0.1 \times 10^{-2} \text{ (most optimistic)}, \quad (10)\]

for the case of the expected parameters of the future TESLA-like Giga-Z\(_1\) experiment.

The constraints obtain in the Eqs. (8), (9) and (10) for the mixing angle $\phi_{E_6}$ are about one order of magnitude stronger than the one obtained in previous studies of the $E_6$ superstring models [32, 33, 34, 35, 36].

### III. CONCLUSIONS

We have determined a bound on the number of light neutrinos species in the frame work of a left-right symmetric model and a $E_6$ superstring model as a function of the mixing angles $\phi$ and $\phi_{E_6}$. Using these results and the LEP values obtained for $N_\nu$, we were able to put limits on the LRSM and $E_6$ superstring mixing angles $\phi$ and $\phi_{E_6}$ which are stronger than those obtained in previous studies of these models. If new precision measurements find small deviations from three for $N_\nu$, these models may explain these deviations with small values of their mixing angles $\phi$ and $\phi_{E_6}$. We have found that the new data on $R_{\exp} = \frac{\Gamma_{\text{inv}}}{\Gamma_{l\bar{l}}}$ can shrink appreciably the allowed regions for $N_\nu$ in the LRSM and the $E_6$ superstring model. As expected, in the limit of vanishing $\phi$, $\phi_{E_6}$ and $M_{Z} \rightarrow \infty$ we recover the bound on $N_\nu$ for the SM previously obtained in the literature [4, 6, 7, 8, 9].
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FIG. 1: Allowed region for $(N_\nu)_{LRSM}$ as a function of the mixing angle $\phi$ with the experimental value $R_{\text{exp}}^{\text{LEP}}$. The dashed lines shows the SM allowed region for $N_\nu$ at 90% C.L., while the dotted lines shows the same result for the LRSM.

FIG. 2: The curve shows the shape for $R_{LRSM}$ as a function of the mixing angle $\phi$. The dashed lines shows the experimental region for $R_{\text{exp}}^{\text{LEP}}$ at 90% C.L. and the dashed vertical region indicates the allowed region for the mixing angle.
FIG. 3: The same as in Fig. 1, but for $E_6$, with $\theta_{E_6} = 37.8^\circ$ and $M_{Z_0} = 7M_{Z_1}$.

FIG. 4: The same as in Fig. 2, but for $E_6$, with $\theta_{E_6} = 37.8^\circ$ and $M_{Z_0} = 7M_{Z_1}$.