Relativistic Kinetic Equations for Finite Domains and Freeze-out Problem

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The relativistic kinetic equations for the two domains separated by the hypersurface with both space- and time-like parts are derived. The particle exchange between the domains separated by the time-like boundaries generates source terms and modifies the collision term of the kinetic equation. The correct hydrodynamic equations for the “hydro+cascade” models are obtained and their differences from existing freeze-out models of the hadronic matter are discussed.

Key words: freeze-out, kinetic equations with source terms, “hydro+cascade” equations

I. INTRODUCTION

In recent years an essential progress has been achieved in our understanding of the freeze-out problem in relativistic hydrodynamics, i.e. how to convert hydrodynamic solution into free streaming particles. Thus, in works [1] the correct generalization of the famous Cooper-Frye formula [2] for the time-like hypersurfaces, the cut-off formula, was derived for the first time. This was a necessary, but yet not sufficient step to formulate the relativistic hydrodynamics without causal paradoxes. The main problem was to formulate the energy-momentum and particle conservation not only for the expanding fluid alone, but to extend it onto the system consisting of the fluid and the gas of free streaming particles which are emitted from the freeze-out hypersurface. The principal solution of this task was given in [3] and its further analysis is presented in [4]. The advantage of this approach is the absence of the logical and causal paradoxes usually arising, if emission of particles happens from the time-like hypersurfaces [5]. The disadvantage of this approach lies in its tremendous numerical complexity.

Numerous attempts to improve and to develop this approach further using primitive kinetic models [5,6] were not very successful so far. Very recently a more fundamental treatment [4] based on the analysis of the Boltzmann equation was suggested. This approach, however, did not overcome the usual difficulty of transport equations in describing the phase transition phenomenon. This difficulty has been overcome naturally within the “hydro + cascade” models suggested in Ref. [7] (BD) and further developed in [8] (TLS). The latter models assume that the nucleus-nucleus collisions proceed in three stages: hydrodynamic expansion (“hydro”) of quark gluon plasma (QGP), phase transition from QGP to hadron gas (HG) and the stage of hadronic rescattering and resonance decays (“cascade”). The switch from hydro to cascade modeling takes place at the boundary between the mixed and hadronic phases. The spectrum of hadrons leaving this hypersurface of the QGP–HG transition is taken as input for the cascade.

Evidently, such an approach incorporates the most attractive features of both hydrodynamics, which describes the QGP–HG phase transition very well, and cascade, which works better during hadronic rescattering. However, both the BD and TLS models face some principal difficulties which cannot be ignored. Thus, within the BD approach the initial distribution for cascade is found by the Cooper-Frye formula [2], which takes into account particles with all possible velocities, whereas in the TLS model the initial cascade distribution is given by the cut-off formula [10], which accounts for only those particles that can leave the phase boundary. As shown below the Cooper-Frye formula will lead to causal and mathematical problems in the present version of BD model because the QGP–HG phase boundary inevitably has time-like parts. On the other hand the TLS model from the beginning does not conserve energy, momentum and number of charges and this, as demonstrated later, is due to the fact that the equations of motion used in [8] are not complete and, hence, should be modified.

The main difficulty of the “hydro + cascade” approach looks very similar to the freeze-out problem in relativistic hydrodynamics. In both cases the finite domains (subsystems) have time-like boundaries through which the exchange of particles is occurring and this should be taken into account. In relativistic hydrodynamics the problem was solved by the constraints which appeared on the freeze-out hypersurface and provided the global energy-momentum and charge conservation [11]. Similarly, in kinetic theory one has to modify the transport equations by the source terms which describe the exchange of the particles on the time-like parts of the boundary between domains. Therefore, we shall consider the two semi-infinite domains which are separated by the hypersurface Σ∗ of general type and derive the kinetic equations for this case in Sect. II. In Sect. III the modification of the collision terms is found and the relation between the system obtained and the Boltzmann equation is discussed. The correct equations of motion for the “hydro + cascade” approach are analyzed in Sect. IV.

II. DRIFT TERM FOR SEMI-FINITE DOMAIN

Let us consider the two semi-finite domains, “in” and “out”, separated by the hypersurface Σ∗ which in (3+1) dimensions will be parameterized as \( t = t^*(\vec{x}) = x_0^*(\vec{x}) \). The distribution function \( \phi_{in}(x, p) \) for \( t \leq t^*(\vec{x}) \) belongs
to the “in” domain, whereas \( \phi_{\text{out}}(x, p) \) denotes the
distribution function of the “out” domain for \( t \geq t^*(\bar{x}) \). Throughout this work it is assumed that the initial condi-
tions for \( \phi_{\text{in}}(x, p) \) are given, whereas the initial condi-
tions for \( \phi_{\text{out}}(x, p) \) are not specified yet and will be the
theme of the subsequent discussion. For simplicity we con-
sider a classical gas of point-like Boltzmann particles.

Similarly to Ref. [3] we derive the kinetic equations for
\( \phi_{\text{in}}(x, p) \) and \( \phi_{\text{out}}(x, p) \) from the requirement of par-
ticle number conservation. Therefore, the particles leaving
one domain (and crossing hypersurface \( \Sigma^* \)) should be
subtracted from the corresponding distribution function
and added to the other one. Now we consider the closed
hypersurface of the “in” domain, \( \Delta x^3 \), which consists of
two semi-planes \( \Sigma_1 \) and \( \Sigma_2 \) of constant time \( t_1 \) and \( t_2 \),
respectively, that are connected from \( t_1 < t_2 < t_1 \) by the
part of the boundary \( \Sigma^*(t_1, t_2) \). The original number of
particles on the hypersurface \( \sigma_{11} \) is given by the standard
expression

\[
N_1 = - \int d\sigma_{11} \frac{d^3p}{p^0} p^\mu \phi_{\text{in}}(x, p), \tag{1}
\]

where \( d\sigma_{11} \) is the external normal vector to \( \sigma_{11} \) and, hence, the product \( p^\mu d\sigma_{11} \leq 0 \) is non-positive. It is clear
that without collisions these particles can cross either hyp-
ersurface \( \sigma_{12} \) or \( \Sigma^*(t_1, t_2) \). The corresponding numbers of
particles are as follows

\[
N_2 = \int d\sigma_{12} \frac{d^3p}{p^0} p^\mu \phi_{\text{in}}(x, p), \tag{2}
\]

\[
N_{\text{loss}}^* = \int d\sigma_{12} \frac{d^3p}{p^0} p^\mu \phi_{\text{in}}(x, p) \Theta(p^\mu d\sigma_{11}). \tag{3}
\]

The \( \Theta \)-function in the \( \text{loss} \) term [3] is very important be-
cause it accounts for the particles leaving the “in” domain
(see also discussion in [3]). For the space-like parts of
the hypersurface \( \Sigma^*(t_1, t_2) \) which are defined by negative
sign \( ds^2 < 0 \) of the element square, \( ds^2 = dt^*(\bar{x})^2 - dx^2 \),
the product \( p^\mu d\sigma_{11} > 0 \) is always positive and, therefore,
particles with all possible momenta can leave the “in”
domain through the \( \Sigma^*(t_1, t_2) \). For the time-like parts of
\( \Sigma^*(t_1, t_2) \) (with sign \( ds^2 > 0 \)) the product \( p^\mu d\sigma_{11} \) can
have either sign, and the \( \Theta \)-function cuts off those parti-
cles which return to the “in” domain.

Similarly one has to consider the particles coming to the
“in” domain from outside. This is possible through the time-like parts of the hypersurface \( \Sigma^*(t_1, t_2) \), if par-
ticle momentum satisfies the inequality \( -p^\mu d\sigma_{11} > 0 \). In
terms of the external normal \( d\sigma_{11} \) with respect to the
“in” domain (the same as in [3]) the number of gained
particles

\[
N_{\text{gain}}^* = - \int d\sigma_{11} \frac{d^3p}{p^0} p^\mu \phi_{\text{out}}(x, p) \Theta(-p^\mu d\sigma_{11}) \tag{4}
\]
is, evidently, non-negative (compare it with contribution of
feed-back particles in [3]). Since the total number of
particles is conserved, i.e. \( N_2 = N_1 - N_{\text{loss}}^* + N_{\text{gain}}^* \), one
can use the Gauss theorem to rewrite the obtained in-
tegral over the closed hypersurface \( \Delta x^3 \) as the integral
over 4-volume \( \Delta x^4 \) surrounded by \( \Delta x^3 \)

\[
\int d^4x \frac{d^3p}{p^0} p^\mu \partial_\mu \phi_{\text{in}}(x, p) = \int d^4x \frac{d^3p}{p^0} p^\mu \left[ \phi_{\text{in}}(x, p) - \phi_{\text{out}}(x, p) \right] \Theta(-p^\mu d\sigma_{11}). \tag{5}
\]

Note that in contrast to the usual case [4], i.e. in the
absence of boundary \( \Sigma^* \), the r.h.s of Eq. (5) does not
vanish identically.

The r.h.s of Eq. (5) can be transformed further to the
4-volume integral in the following sequence of steps. First
we express the integration element \( d\sigma_{11} \) via the normal
vector \( n_\mu^* \) as follows \( (dx^j > 0, \text{for } j = 1, 2, 3) \)

\[
d\sigma_{11} = n^*_\mu d\sigma_{11}^j dx^j dx^3; \quad n^*_\mu \equiv \delta_{\mu 0} - \frac{\partial t^*(\bar{x})}{\partial x^\mu} (1 - \delta_{\mu 0}), \tag{6}
\]

where \( \delta_{\mu \nu} \) denotes the Kronecker \( \delta \)-function. Then, using
identity \( \int dt \delta(t - t_3) = 1 \) for the Dirac \( \delta \)-function with
\( t_1 \leq t \leq t_2 \), we rewrite the r.h.s. integral in [3] as

\[
\int d\sigma_{11} \cdots \equiv \int d^4x \delta(t - t^*(\bar{x})) n^*_\mu \cdots, \tag{7}
\]

where the 4-dimensional volume \( V_4^* \) is a direct prod-
uct of the 3- and 1-dimensional volumes \( V_3^*(t_1, t_2) \) and
\( (t_2 - t_1) \), respectively. Evidently, the Dirac \( \delta \)-function
allows us to extend integration in [3] to the unified 4-
volume \( V_4^* \equiv \Delta x^4 \cup V_4^*_2 \) of \( \Delta x^4 \) and \( V_4^*_2 \). Finally, with the help of
notations

\[
\Theta_+ \equiv \Theta(t - t^*(\bar{x})); \quad \Theta_- \equiv 1 - \Theta_+ \tag{8}
\]

it is possible to extend the l.h.s. integral in Eq. (5) from
\( \Delta x^3 \) to \( V_4^* \). Collecting all the above results, from Eq. (5)
one obtains

\[
\int d^4x \frac{d^3p}{p^0} \Theta_- p^\mu \partial_\mu \phi_{\text{in}} = V_4^* \int \theta d^4x \frac{d^3p}{p^0} p^\mu \left[ \phi_{\text{in}} - \phi_{\text{out}} \right] \Theta(-p^\mu n^*_\mu) \delta(t - t^*(\bar{x})). \tag{9}
\]

Since volumes \( \Delta x^4 \) and \( V_4^* \) are arbitrary, one obtains the
collisionless kinetic equation for the distribution function of
the “in” domain

\[
\Theta_+ p^\mu \partial_\mu \phi_{\text{in}}(x, p) = p^\mu n^*_\mu \left[ \phi_{\text{in}}(x, p) - \phi_{\text{out}}(x, p) \right] \Theta(-p^\mu n^*_\mu) \delta(t - t^*(\bar{x})). \tag{10}
\]

Similarly one can obtain the equation for the distribution
function of the “out” domain

\[
\Theta_- p^\mu \partial_\mu \phi_{\text{out}}(x, p) = p^\mu n^*_\mu \left[ \phi_{\text{in}}(x, p) - \phi_{\text{out}}(x, p) \right] \Theta(p^\mu n^*_\mu) \delta(t - t^*(\bar{x})). \tag{11}
\]
with the same normal vector \( n^* \) as in Eq. (10). Note the asymmetry between the r.h.s. of Eqs. (10) and (11): for the space-like parts of hypersurface \( \Sigma^* \) the source term with \( \Theta(t - p^* n^*) \) vanishes identically because \( p^* n^* > 0 \). This reflects the causal properties of the equations above: propagation of particles faster than light is forbidden, and hence no particle can (re)enter the “in” domain.

### III. COLLISION TERM FOR SEMI-FINITE DOMAIN

Since in the general case \( \phi_{in}(x,p) \neq \phi_{out}(x,p) \) on \( \Sigma^* \), the r.h.s. of Eqs. (10) and (11) cannot vanish simultaneously on this hypersurface. Therefore, functions \( \Theta_\Sigma^* \equiv \Theta_{\Sigma^*} \neq 0 \) and \( \Theta^*_\Sigma \equiv \Theta_{\Sigma^*} \neq 0 \) do not vanish simultaneously on \( \Sigma^* \) as well. For definiteness it is assumed that

\[
\Theta_\Sigma^* = \Theta_{\Sigma^*} = \Theta(0) = \frac{1}{2}, \tag{12}
\]

but the final results are independent of this choice.

Now the collision terms for Eqs. (10) and (11) can be readily obtained. Adopting the usual assumptions on the distribution functions \( \phi \), one can repeat the standard derivation of the collision terms \( \Theta \) and get the desired expressions. We shall not recapitulate this standard part, but only discuss how to modify the derivation for our purpose. First of all, one has to start the derivation in the \( \Delta x^4 \) volume of the “in” domain and then extend it onto the unified 4-volume \( \mathcal{V}^4 = \Delta x^4 \cup \mathcal{V}_3^4 \) similarly to the preceding section. Then the first part of the collision term for Eq. (10) reads as

\[
C^{\Sigma}_{\Sigma^*}(x,p) = \Theta_\Sigma^* \left( I_G[\phi_{in}, \phi_{in}] - I_L[\phi_{in}, \phi_{in}] \right), \tag{13}
\]

\[
I_G[\phi_A, \phi_B] = \frac{1}{2} \int D^0 P \phi_A(p'_1) \phi_B(p'_1) W_{p,p'_1}, \tag{14}
\]

\[
I_L[\phi_A, \phi_B] = \frac{1}{2} \int D^0 P \phi_A(p) \phi_B(p) W_{p,p'} \tag{15}
\]

where the invariant measure of integration is denoted as \( D^0 P \equiv d^4p_1 d^4p' \) and \( W_{p,p'} \) is the transition rate in the elementary reaction with energy-momentum conservation given in the form \( p^0 + p^1 = p^0 + p^1 \). The r.h.s. of (13) contains the square of \( \Theta_\Sigma^* \)-function because the additional \( \Theta_\Sigma^* \) accounts for the fact that on the boundary hypersurface \( \Sigma^* \) one has to take only one half of the traditional collision term (due to Eq. (12) only one half of \( \Sigma^* \) belongs to the “in” domain). It is easy to understand that on \( \Sigma^* \) the second part of the collision term (according to Eq. (12)) is defined by the collisions between particles of “in” and “out” domains

\[
C_{\Sigma^*}(x,p) = \Theta_{\Sigma^*} \left( I_G[\phi_{in}, \phi_{out}] - I_L[\phi_{in}, \phi_{out}] \right) \tag{16}
\]

Combining results (12), (13) and (16), we obtain the kinetic equation for the semi-finite “in” domain

\[
\Theta \equiv p^\mu \partial_\mu \phi_{in}(x,p) = C^{\Sigma}_{\Sigma^*}(x,p) + C_{\Sigma^*}(x,p) + p^\mu n^\mu \left[ \phi_{in}(x,p) - \phi_{out}(x,p) \right] \Theta(-p^\nu n^\nu) \delta(t - t^*(x)) \tag{17}
\]

The corresponding equation for the “out” domain

\[
\Theta > p^\mu \partial_\mu \phi_{out}(x,p) = C^I_{\Sigma^*}(x,p) + C^I_{\Sigma^*}(x,p) + p^\mu n^\mu \left[ \phi_{in}(x,p) - \phi_{out}(x,p) \right] \Theta(-p^\nu n^\nu) \delta(t - t^*(x)) \tag{18}
\]

can be derived similarly. In (13) we used the evident notations \( C^{\Sigma}_{\Sigma^*} = \Theta_\Sigma^* \left( I_G[\phi_{in}, \phi_{in}] - I_L[\phi_{in}, \phi_{in}] \right) \) and \( C^{\Sigma}_{\Sigma^*} = \Theta_{\Sigma^*} \left( I_G[\phi_{in}, \phi_{in}] - I_L[\phi_{in}, \phi_{in}] \right) \).

For the continuous distribution functions on \( \Sigma^* \), i.e., \( \phi_{out}[\Sigma^*] = \phi_{in}[\Sigma^*] \), the source terms in r.h.s. of Eqs. (17) and (18) vanish and one recovers the Boltzmann equations. With the help of the evident relations

\[
-\partial_\mu \Theta_\Sigma^* = \partial_\mu \Theta_{\Sigma^*} = n^\mu \delta(t - t^*(x)), \tag{19}
\]

\[
C^{\Sigma}_{\Sigma^*} + C_{\Sigma^*} + C^{\Sigma}_{\Sigma^*} = I_G[\Phi, \Phi] - I_L[\Phi, \Phi], \tag{20}
\]

where the notation \( \Phi(x,p) \equiv \Theta \phi_{in}(x,p) + \Theta \phi_{out}(x,p) \) is used, one can get the following result for the sum of Eqs. (17) and (18)

\[
p^\mu \partial_\mu \Phi(x,p) = I_G[\Phi, \Phi] - I_L[\Phi, \Phi]. \tag{21}
\]

In other words, the usual Boltzmann equation follows from the system of Eqs. (17) and (18) automatically without any assumption about the behavior of \( \phi_{in} \) and \( \phi_{out} \) on the boundary hypersurface \( \Sigma^* \). In fact the system (17)-(18) generalizes the relativistic kinetic equation to the case of the strong temporal and spatial inhomogeneity, i.e., if \( \phi_{in}(x,p) \neq \phi_{out}(x,p) \) on \( \Sigma^* \). Of course, one has to be extremely careful while discussing the strong temporal inhomogeneity (or discontinuity on the space-like parts of \( \Sigma^* \)) such as the so called time-like shocks \( \Sigma^* \) because their existence may contradict to the usual assumptions adopted for distribution function. Therefore, in what follows we shall discuss exclusively the spatial inhomogeneities or discontinuities on the time-like parts of \( \Sigma^* \) which are less restrictive because in some sense the equations above are delocalized in space.

From the system (17)-(18) it is possible to derive the macroscopic equations of motion by multiplying the corresponding equation with \( p^\mu \) and integrating it over the invariant measure. Thus Eq. (17) generates the following expression

\[
\Theta \equiv \partial_\mu T^{\mu\nu}_{in} = \int \frac{d^3p}{p^0} p^\nu C^{\Sigma}_{\Sigma^*}(x,p) + \int \frac{d^3p}{p^0} p^\nu n^\mu \left[ \phi_{in} - \phi_{out} \right] \Theta(-p^\nu n^\nu) \delta(t - t^*(x)). \tag{22}
\]

Similarly to the usual Boltzmann equation the momentum integral of the collision term \( C^{\Sigma}_{\Sigma^*} \) vanishes due to its symmetries, but it can be shown that the integral of the second collision term \( C^{\Sigma}_{\Sigma^*} \) does not vanish because it involves two different (and not identical) distribution functions. The corresponding equation for the “out” domain follows similarly

\[
\Theta > \partial_\mu T^{\mu\nu}_{out} = \int \frac{d^3p}{p^0} p^\nu C^{\Sigma}_{\Sigma^*}(x,p) + \int \frac{d^3p}{p^0} p^\nu n^\mu \left[ \phi_{in} - \phi_{out} \right] \Theta(p^\nu n^\nu) \delta(t - t^*(x)). \tag{23}
\]

Note that similar equations (with \( \delta \)-like term) first were obtained within the relativistic hydrodynamics in \( \Sigma \).
IV. DISCUSSION

It is clear that Eqs. (13), (18), (22) and (23) remain valid for the finite domains as well. With their help we are ready to analyze the “hydro+cascade” models. In the TLS model the cut-off formula relates \( \phi_{\text{in}} \) (≡ hydro, Eq. (23)) and \( \phi_{\text{out}} \) (≡ cascade, Eq. (18)) on \( \Sigma^* \) as follows

\[
\phi_{\text{out}} \bigg|_{\Sigma^*} = \Theta(p^\mu n^\nu_\mu) \phi_{\text{in}} \bigg|_{\Sigma^*} = \Theta(p^\mu n^\nu_\mu) \phi_{\text{in}} \bigg|_{\Sigma^*}, \quad (24)
\]

i.e., for the space-like parts of hypersurface \( \Sigma^* \) these functions are identical, whereas for the time-like parts of \( \Sigma^* \) there are no returning particles to the “in” domain. In this case the source term in cascade Eq. (18) is zero, while the source term in hydro Eq. (22) does not vanish on the time-like parts of the boundary \( \Sigma^* \). Therefore, the main defect of the TLS model is not even the energy-momentum non-conservation, but the incorrect hydrodynamic equations. The absence of the \( \delta \)-like source term in Eq. (22) breaks the conservation laws (evidently, the system obeys the conservation laws), but its inclusion into consideration will inevitably change the hydrodynamic solution of Ref. [8]. The full analysis of the possible solutions of the systems \( (17,22) \) and \( (23,18) \) requires a special consideration. We only mention that from the negative sign of the TLS source term in the r.h.s. of (22) for equal indices \( \nu = \mu \) one immediately can deduce that such a correction to the hydro equations should increase the degree of the fluid rarefaction in comparison with the standard hydrodynamic expansion. It is, therefore, quite possible that such a source term will generate a discontinuity between “in” and “out” domains. In the thermodynamically normal media the rarefaction shocks are mechanically unstable. However, it is well known that on the phase transition boundary between QGP and HG the properties of the mixed phase are thermodynamically anomalous [1] and the usual rarefaction shocks are possible. Another possibility is the occurrence of the new type of the discontinuity, the freeze-out shock suggested in Refs. [13], where the post freeze-out state is described by the cut-off distribution and, hence, is very similar to the TLS ansatz. It is clear that in both cases the additional rarefaction will reduce the mean transverse size and the life-time of the hadronizing QGP.

Let us consider briefly the BD approach. Since in the BD model the hydro and cascade distributions on \( \Sigma^* \) are equal \( \phi_{\text{out}} \big|_{\Sigma^*} = \phi_{\text{in}} \big|_{\Sigma^*} \), the corresponding source terms vanish in all equations. Therefore, at first glance the BD approach correctly conjugates the hydro and cascade solutions on the arbitrary hypersurface. For the over-simplified kinetics considered above it is so. However, the real situation differs essentially from our consideration. Thus, the hydro part in both the BD and TLS models is assumed to be in the local thermodynamic equilibrium, whereas the matter in the cascade domain can be far from equilibrium (this was, actually, the main reason why both groups decided to use the cascade). Consequently, the BD transport equations for all hadrons are homogeneous in the hydro domain, whereas for the cascade domain they are inhomogeneous. Since the initial BD cascade distribution on the time-like parts of \( \Sigma^* \) contains the particles returning to the fluid domain \( (p^\nu n^\nu_\mu < 0) \), then these particles will move towards the space-like parts of the hypersurface \( \Sigma^* \) which are located inside of the light cone originated at each point of the time-like part of \( \Sigma^* \). Then for each hadron the inhomogeneous BD cascade equation will generate a different distribution function than the one already obtained from the hydro equations (or homogeneous transport ones) on these space-like parts of \( \Sigma^* \). Thus, one arrives at a causal paradox, the recoil problem [4], and at a mathematical inconsistency.

Evidently, the inclusion of the viscosity into the hydro equations (apart from its tremendous complexity for the mixture of about hundred hadrons) will not solve the problem because for the small deviations from equilibrium (and, hence, a small viscosity effect) the influence of the returning particles may remain essential. If, on the contrary, the matter in the hydro domain is far from equilibrium, then the usage of the hydro equations becomes problematic. Therefore, a more realistic way is to find the boundary conditions for \( \phi_{\text{in}} \) and \( \phi_{\text{out}} \) on the separating hypersurface \( \Sigma^* \) form the system of kinetic equations \( (17,18) \), and then to apply these boundary conditions to the system \( (11,22) \) which ensures the correct treatment of the relativistic nuclear collisions within the frame of the “hydro+cascade” model.

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[1] K. A. Bugaev, Nucl. Phys. A 606 (1996) 559; and K. A. Bugaev and M. I. Gorenstein, nucl-th/9903072 (1999) 70 p.
[2] F. Cooper and G. Frye, Phys. Rev. D 10 (1974) 186.
[3] K. A. Bugaev, M. I. Gorenstein and W. Greiner, J. of Phys. G 25 (1999) 2147; and Heavy Ion Physics 10 (1999) 333.
[4] V. K. Magas et al., Heavy Ion Physics 9 (1999) 193.
[5] V. K. Magas et al., Nucl. Phys. A 661 (1999) 596.
[6] Yu. M. Sinyukov, S. V. Akkelin and Y. Hama, Phys. Rev. Lett. 89 (2002) 052301.
[7] S. Bass and A. Dumitru, Phys. Rev. C61 (2000) 064909; 86 (2001) 4783; and nucl-th/0110037 (2001) 28 p.
[8] D. Teaney, J. Lauret and E. V. Shuryak, Phys. Rev. Lett. 86 (2001) 4783; and nucl-th/0110037 (2001) 28 p.
[9] S. R. de Groot, W. A. van Leeuwen and Ch. G. and Weert, “Relativistic Kinetic Theory”, North-Holland Publishing, Amsterdam, (1980).
[10] L. P. Csernai, Zh. Eksp. Fiz. (Russ.) 92 (1987) 379; and Sov. Phys. JETP 65 (1987) 216.
[11] K. A. Bugaev, M. I. Gorenstein and V. I. Zhdanov, Z. Phys. C39 (1988) 365; K. A. Bugaev and M. I. Gorenstein, Z. Phys. C43 (1989) 261.