High resolution bremsstrahlung and fast electron characterization in ultrafast intense laser–solid interactions

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Abstract. The scaling of the intensity, angular and material dependence of bremsstrahlung radiation from an intense ($I > 10^{18}$ W cm$^{-2}$) laser–solid interaction has been characterized at energies between 100 keV and 1 MeV. These are the first high resolution ($E/\Delta E > 200$) measurements of bremsstrahlung photons from a relativistic laser–plasma interaction. The measurement was performed using a high purity germanium detector at the high-repetition rate (500 Hz) $\lambda^3$ laser facility. The bremsstrahlung spectra were observed to have a two effective temperature energy distribution which ranged between 80 ($\pm 10$) and 550 ($\pm 60$) keV depending on laser intensity and observation angle. The two temperatures were determined to result from separate populations of accelerated electrons. One population was isotropic and produced the lower effective bremsstrahlung temperature. The higher bremsstrahlung temperature was produced by an energetic electron beam directed out of the front of the target in the direction of the specular laser reflection, which was also the direction the bremsstrahlung effective temperature peaked. Both effective bremsstrahlung temperatures scaled consistently with a previously measured experimental electron temperature scaling on $\lambda^3$. The electron populations and bremsstrahlung temperatures were modeled in the particle-in-cell code OSIRIS.
and the Monte Carlo code MCNPX and were in good agreement with the experimental results. The observed directionality and intensity scaling suggest a significant difference between picosecond and femtosecond duration pulse interactions.

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1. Introduction

The generation of bremsstrahlung radiation from the deceleration of an energetic electron beam in matter provides a photon source with applications in medicine [1], homeland security [2] and nuclear physics [3–5]. High intensity ultrafast lasers allow the direct acceleration of electrons to MeV energies in laser–solid target interactions [6] providing an MeV source of electrons and bremsstrahlung photons. Previous efforts to characterize laser driven bremsstrahlung sources over 100 keV have relied on experimental measurements using low resolution absorption filters [7] and nuclear activation stacks [8], as well as simulations [9]. Additionally, high-resolution measurements have been performed using crystal spectrometers at energies of hundreds of keV [10], but in a relatively narrow spectral range. As an alternative, this work presents the first high-resolution bremsstrahlung measurements using a high purity germanium (HPGe) detector, which was capable of measuring photons from 30 keV to 3 MeV with a resolution of $E/\Delta E > 200$, with a 500 Hz repetition rate, high power laser system. The bremsstrahlung spectra was acquired by operating the HPGe detector in a single-hit regime and integrating up to 1.9 million shots. This novel application of an HPGe detector had not been practical previously due to the low repetition rates of typical high intensity lasers.

Previous work to characterize the hot electrons generated in a femtosecond laser–solid interaction found that a beam of energetic electrons escapes from the front, or laser side, of a bulk target near the direction of the specular light reflection [11]. This beam was also observed during the present work and was observed to have a $7^\circ$ full-width-at-half-maximum (FWHM). It was postulated in reference [11] that a significant fraction of the electrons were also accelerated into the target, however the target thickness made a direct measurement of the forward accelerated electrons impossible. In this work, the accumulated high resolution photon spectra provide a measurement of the bremsstrahlung as well as insight into the behavior of the electrons, both inside and outside of the target, through modeling of the bremsstrahlung process.

The spectral and angular emission characteristics of bremsstrahlung are tied to the electron kinematics. The angular component of the differential bremsstrahlung cross section for an
electron in a solid can be modeled as a Gaussian [12]

$$\frac{d\sigma}{d\Omega} = \frac{1}{\pi (b\theta_b)^2} \exp \left(-\frac{\theta^2}{(b\theta_b)^2}\right)$$

(1)

with a FWHM angle given by

$$\theta_b = \frac{m_e c^2}{E_e + m_e c^2},$$

(2)

where $E_e$ is the electron energy, $\bar{b}$ is a fitting constant $\approx 1$ and the angle $\theta$ is the angle between the electron and bremsstrahlung photon. The angularly integrated cross section can be described with [12]

$$\frac{d\sigma}{dE} = \frac{a Z^2}{E}(1 - b E / E_e),$$

(3)

where $E$ is the energy, $a \approx 11$ mb, $b$ is a fitting constant $\approx 0.83$ based on the results of Seltzer and Berger [13], and $E_e$ is still the electron energy. As a result, the most energetic electrons produce the highest energy bremsstrahlung photons with a well-defined directionality. This enables an indirect measurement of the electron energy distribution and direction through the correlation between the direction of the high energy electrons and high energy photons. For distributions of electrons the bremsstrahlung spectra is a summation of the bremsstrahlung spectra from electrons of each energy which has no compact analytical form. In this work, bremsstrahlung modeling of the full three-dimensional experiment was accomplished using the Monte Carlo code MNCP, and will be discussed in the simulation section.

2. Experimental setup

The experiment was performed using the $\lambda^3$ laser facility at the Center for Ultrafast Optical Science at the University of Michigan. $\lambda^3$ is a 0.5 kHz Ti:sapphire system ($\lambda \approx 800$ nm) producing laser pulses with $\tau = 30$ fs FWHM pulse duration and an amplified spontaneous emission-to-peak intensity contrast of $10^{-8}$. The laser was focused to a 1.3 $\mu$m FWHM focal spot by an $f/1.2$ off-axis parabolic mirror in conjunction with a deformable mirror (Xinetics Inc.) optimized by a genetic algorithm [14]. Each laser pulse contained between 2 and 10 mJ producing focused intensities of $2.5 \times 10^{18} - 1.2 \times 10^{19}$ W cm$^{-2}$. This corresponds to a peak normalized vector potential $a_0 = \frac{e E}{m_e c \omega_0}$ of 1.1–2.4. P-polarized light was incident at 45° after passing through a 2 $\mu$m thick nitrocellulose pellicle used to protect the paraboloid from the target ablation debris. The pellicle was necessary to prevent the ablated material from accumulating on the paraboloid as a result of the large number of shots. The high repetition rate required the target surface to be rapidly refreshed which was accomplished by rotating and translating the target so that each shot interacted with a fresh area in an inward spiral pattern. Each target was a 10 cm diameter bulk Eu$_2$O$_3$, SiO$_2$ or Mo disc with a thickness of 1.2 ± 0.2 cm. Special care was taken to align the target surface with a Mitutoyo 513 – 405T dial gauge to maintain the surface peak to valley deviation over a full rotation of ±2 $\mu$m, significantly smaller than the Rayleigh range ($2z_r = 11 \mu$m). Target focus was optimized by maximizing the x-ray signal on a Si x-ray diode. The spot separation of 120 $\mu$m allowed $5(\pm 1.5) \times 10^5$ shots per target. The 7° FWHM half angle specular electron beam was measured experimentally by
imaging the electron distribution with Fujifilm BAS-MS image plate. The electron distribution was observed to be much lower in energy and more uniform at other angles in front of the target, and energetic electrons were not observable through the target due to the thickness.

The HPGe detector was an Ortec GMX55P4–83 PopTop detector with a 70 mm diameter by 60.2 mm long crystal which was cryogenically cooled to 77 °K to reduce leakage current. The detector was calibrated for energy and efficiency using 133Ba, 60Co, 157Cs and 22Na calibration sources which provided an absolute total efficiency. The detector was experimentally measured to have a resolution of 1.7 keV at 661.33 keV, typical for HPGe detectors, which results in a resolution $E/\Delta E > 200$ over the entire range of the measurement. Most bremsstrahlung spectrum were accumulated over 90 000 or 120 000 shots with a count rate of 75–125 counts s$^{-1}$, or equivalently, a detection event probability of 15–25% of the shot rate. The HPGe detector was located 7.7 m from the interaction and the detector element was shielded with >5 cm of lead, except for a variable aperture limiting the detector solid angle to $6 \times 10^{-5}$–$9 \times 10^{-7}$ sr. The low detection probability and the large detector distance were required to prevent photon pile-up. Photon pile-up is the collection of multiple photons in the detection volume in a time period shorter than a μs, which records the sum of the photon energies as a single, more energetic, photon. The HPGe line of sight passed through the 3.5 mm stainless steel chamber wall which blocked photons with energies lower than 30 keV and allowed 66–85% of the photons in the energy range of interest to reach the detector, which was accounted for in the measured spectra. The removal of the low energy photons was critical to prevent pile-up from photons below the energy range of interest. At the experimental detection probability 8–14% of the detected photons were due to pile-up. The appendix contains the details of how this pile-up affects the measured photon spectrum, and how the combined effects of detector efficiency and photon pile-up were accounted for.

The bremsstrahlung spectrum was recorded at four angles relative to the target as shown in figure 1. This was accomplished by rotating the target stage and steering the beam inside the chamber so as to maintain the line of sight through, and outside of, the chamber. No significant attenuation of the bremsstrahlung signal as a result of the target stage was observed for the observation angles through the target.

3. Bremsstrahlung spectrum

Bremsstrahlung spectra for all angles and materials demonstrated a consistent shape as seen in figure 2. The high resolution of the measurement was evident in figure 2(a) which resolves Pb $K_{\alpha,\beta}$ lines as well as a 511 keV positron annihilation line. The spectral shape was fit as a two effective temperature distribution with exponential effective temperatures of the form $dN/dE = a \times \exp(-E/T_{b})$ as demonstrated in figure 2(b). The lower effective temperature, $T_{b1}$ (blue), was fit using a least squares approach between the energies of 120 and 500 keV, while the higher effective temperature, $T_{b2}$ (red), was fit using the same approach between 500 keV and 1 MeV. The highest effective temperatures were observed on an Eu$_2$O$_3$ target in the $\theta = -45^\circ$ specular direction with a normalized vector potential of $a_0 = 2.3$ yielding $T_{b1} = 300 (\pm 30)$ keV and $T_{b2} = 550 (\pm 60)$ keV.

 Photon pile-up was one of the sources of error in this measurement. A Monte Carlo code was used to model the effect of the detection of multiple photons at the experimental detection rate, and it was shown that the two temperature spectral shape was not a result of pile-up.
Figure 1. Experimental setup of the target chamber. The bremsstrahlung observation angles are shown.

Figure 2. (a) Eu$_2$O$_3$ spectrum accumulated over 1.9 million shots. The resolution is sufficient to resolve nuclear peaks (511 keV) and Pb K$_{\alpha,\beta}$ peaks. (b) Bremsstrahlung spectrum showing the characteristic two effective temperature spectral shape for SiO$_2$ along the specular direction. $T_{b1} = 286$ (±9) keV (blue) and $T_{b2} = 480$ (±31) keV (red) are shown along with the 95% confidence fits (dashed lines). The data was taken with $a_0 = 2.3$ over 180 k shots. Data is summed into ten channel bins for clarity.

The details of the effects of photon pile-up on the measurement are provided in the appendix. The error bars were primarily a result of the spectra noise and the effect of photon pile-up. The background signal comprised less than 4% of the total counts in a typical spectrum, and was uniformly distributed across the energy range of 120 keV–1 MeV. The variation in the signal at...
Figure 3. (a) The $T_{b1}$ (black) and $T_{b2}$ (green) experimental bremsstrahlung temperatures, observed from $\theta = 0^\circ$ on Eu$_2$O$_3$, with the power law fits for $T_{b1}$ (blue) and $T_{b2}$ (red). (b) The calculated $T_{e1}$ (blue) and $T_{e2}$ (red) experimental electron temperatures from the bremsstrahlung temperatures in (a) with the Beg (black), $\lambda^3$ (red) and ponderomotive (green) theoretical scalings (dashed). The calculated scalings use the relationship between $T_e$ and $T_b$ determined from MCNPX.

high energies was a product of the decreasing signal and the decreasing collection efficiency of the HPGe germanium crystal leading to low count numbers per channel.

The bremsstrahlung temperatures were observed to scale with laser intensity as shown in figure 3(a) with the two temperatures exhibiting similar scalings as a function of the normalized laser vector potential. The temperature scalings exhibited least squares power law fits of $T_{b1} = 101 \times a_0^{1.16}$ and $T_{b2} = 149 \times a_0^{1.31}$ keV. The scalings were determined using a linear regression of the measured data in the logarithmic domain. The error bars on each point indicate the standard deviation of the measured temperatures of multiple experimental runs with the same laser parameters. A weighted linear regression, using the error bars on each data point, agreed with the standard linear regression within 3%. Using a relationship found with MCNPX simulations, $T_e = 0.73 \times T_b^{0.09}$, the experimental electron temperature scaling was calculated from the bremsstrahlung scaling, $T_{e1} = 110 \times a_0^{1.26}$ and $T_{e2} = 168 \times a_0^{1.42}$ keV, and compared to existing electron temperature scaling laws as shown in figure 3(b). The slope of the electron scalings was closest to the $T_e = 145 \times a_0^{1.28}$ scaling observed previously on $\lambda^3$ [11]. In comparison, Wilks’ ponderomotive scaling has a steeper growth rate in this region of intensity. Alternatively, Beg scaling [15], which was experimentally derived from a variety of picosecond duration laser systems, was also plotted and exhibited a growth rate that was lower than our femtosecond results.

The angular distribution of the bremsstrahlung temperature was observed to peak in the specular direction, $\theta = -45^\circ$, and fall off to a minimum temperature behind the target at $\theta = 90^\circ$, as shown in figure 4. This directionality indicates the specular beam of electrons contained the most energetic electrons, as discussed previously with equation (2). The bremsstrahlung and electron directionality was also observed in particle-in-cell simulations and MCNPX modeling, as discussed in the simulation section. This angular distribution is
Figure 4. Experimental (data points) bremsstrahlung temperatures as a function of observed angle compared to MCNPX predictions (dotted lines). $T_{t2}$ (red) and $T_{t1}$ (blue) are shown, along with the target geometry and laser direction for clarity. The heavy dashed line (black) shows the average electron energy as a function of angle, PIC $\langle E_e \rangle$ taken from a particle-in-cell simulation.

Table 1. Bremsstrahlung temperature along the $\theta = 0^\circ$ direction compared to MCNPX simulation results for $T_e = 200$ keV. Note the Z dependence in the MCNPX simulation is not observed experimentally.

| Material | $T_{t1}$ (keV) | $T_{t2}$ (keV) | MCNPX (keV) |
|----------|----------------|----------------|-------------|
| SiO$_2$  | 140 ($\pm$25)  | 200 ($\pm$30)  | 101         |
| Mo       | 165 ($\pm$25)  | 235 ($\pm$25)  | 121         |
| Eu$_2$O$_3$ | 150 ($\pm$25) | 218 ($\pm$25)  | 144         |

in contrast to previous results with picosecond lasers which have shown highly directional bremsstrahlung signals between $\theta = 45^\circ$ and $90^\circ$ [8].

The experimental bremsstrahlung temperature was observed to be independent of the target material as demonstrated in table 1. The measurements were made along the surface ($\theta = 0^\circ$) direction with $a_0 = 1.6$, and the temperatures from each target material were similar, within the error bars. The temperatures did not exhibit the weak linear scaling with target material observed in MCNPX simulations.

4. Simulations

To simulate the absorption of laser energy and generation of the hot electron distribution, two-dimensional (2D) particle-in-cell simulations were run using the osiris 2.0 framework [16] under conditions similar to the experiment. The charge density profile was constructed
shows the electron average energy as a function of angle, $(\text{PIC})$

The peak electron density was $n_0 = 30n_c$, where $n_c = \omega_0^2m_e\epsilon_0/e^2$ is the critical density for the laser of frequency $\omega_0$, the target thickness was $L = 20c/\omega_0$ and the scalelength of the plasma density was $\lambda_{pp} = 6c/\omega_0$. The target was at a $45^\circ$ angle with respect to the simulation box. A gaussian laser pulse with $a_0 = 1$ was initiated propagating in the $x_1$ direction, linearly polarized in the $x_2$ direction with a waist of $w_0 = 8c/\omega_0$ (1 $\mu$m) and a fifth order polynomial temporal shape with a duration of $\omega_0\Delta t = 65$. Two particle species were used; species 1 with charge to mass ratio $q/m = -|e|/m_e$ initiated with a thermal velocity of $v_{th} = 0.01c$ and species 2 with $q/m = +10|e|/28m_p$ initiated at rest, where $m_p$ is the proton mass. 64 particles-per-cell (PPC) with cubic weighting were used for species 1 and 4 PPC with linear weighting were used for species 2. The domain, of dimensions $x_1 \times x_2 = 30 \times 15\mu m^2$ was divided into $5000 \times 2496$ grid cells, yielding cell sizes $\Delta x_1, \Delta x_2 = 0.048c/\omega_0$. The simulation was run for $\omega_0\Delta t = 1000$ in steps of $\omega_0\Delta t = 0.033$. Compensated binomial smoothing was applied to fields and currents on the grid.

Figure 4 shows the electron average energy as a function of angle, $(\text{PIC} \langle E_\theta \rangle)$ taken from the particle-in-cell simulation. For a 2D velocity distribution, the average energy and temperature are equivalent for a Maxwell–Boltzmann velocity distribution. The mean energy was used to allow a quantitive comparison between the simulation electron energies and the experimental bremsstrahlung temperature because the electron distribution was non-Maxwellian. The average energy was calculated by taking the average of electron energies $E$ for each angle above a cut-off energy $E_{lower}$ from the momentum phase space, i.e. the numerical equivalent of

$$\int_{E_{lower}}^{\infty} \int_{\theta - \Delta \theta/2}^{\theta + \Delta \theta/2} f(\theta, E)E \, d\theta \, dE,$$

where $\Delta \theta$ is the bin width. The figure shows the average energy of electrons above a 100 keV cut-off. The simulated electron distribution shows two populations of electrons, a near isotropic component of heated electrons and a population with higher average energy that is accelerated in the near specular direction in a chaotic direct laser acceleration mechanism by the reflected/incident waves interacting with the surface plasma, as previously observed at lower intensity [11].

The Monte Carlo code MCNPX [17] was used to simulate the relationship between the electron and bremsstrahlung spectra. MCNPX uses tabulated bremsstrahlung production tables based on the Bethe–Heitler Born-approximation [18] and applicable models have been developed in the literature [19, 20]. While the model emphasizes the MeV region, subsequent improvements have been made for the keV region [21]. The MCNPX simulations modeled the target, chamber wall, air and detector locations with the appropriate geometry. A multiplication factor of 100 was used for bremsstrahlung production in the electron physics model to reduce the simulation run time by increasing the photon production per simulated electron. The accuracy of this method was validated by a run without multiplication which showed no discernable difference in effective bremsstrahlung temperatures. Photons were recorded with surface current tallies at the detector positions, which were located at the same distance and angle as the experimental detectors. All runs used $1.4 \times 10^8$ electrons with standard Maxwellian energy distributions which were directed in isotropic spheres, $7^\circ$ half angle cones, or both. The cone source contained a uniform flux through all angles encompassed by the $7^\circ$ half angle. Half angle beams were modeled in the specular ($-45^\circ$), laser ($45^\circ$) and target normal ($90^\circ$) directions. MCNPX simulations confirmed the correlation of the bremsstrahlung directionality with the electron direction, as predicted by the analytical model. This reinforced the conclusion that the highest energy electrons were contained the specular beam.

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The two electron populations observed in the particle-in-cell simulations were investigated and compared to the experimental results. Figure 5 shows the calculated bremsstrahlung signal from separate sources of an isotropic sphere ($T_e = 200$ keV) and a cone ($T_e = 900$ keV) from a Eu$_2$O$_3$ target. The combination of the two signals can accurately reproduce the experimentally observed two temperature spectrum for Eu$_2$O$_3$ at $a_0 = 2.4$ by providing different effective temperatures from the two electron populations. This demonstrates the importance of measuring the electrons accelerated in all directions, both into and out of the target, in order to fully characterize the electron behavior.

The angular characteristics of the bremsstrahlung radiation shown in figure 4 were reproduced in MCNPX using the particle-in-cell electron distribution. The electron source was modeled as a combination of an isotropic source with a Maxwellian temperature of 200 keV and a beamed source with a temperature of 700 keV directed in the $-45^\circ$ direction. It was observed that the result was very sensitive to the beamed source temperature, but only weakly dependent on the isotropic distribution. This can be attributed to the larger number of measurable photons from the higher energy source, whereas the photons from the isotropic source tended to be absorbed on the way to the detector.

In order to compare the scalings of the bremsstrahlung and electron temperatures, the relationship between the Maxwellian electron temperature and the effective bremsstrahlung temperature was determined for electron temperatures spanning the range 200 keV–5 MeV. The relationship was observed to scale as $T_b = 1.34 \times T_e^{0.92}$ keV, as shown in figure 6, for an electron beam directed into a 3.5 mm steel slab and observed through the slab. MCNPX was also used model the effect of different target materials on the bremsstrahlung temperature. A weak linear material scaling of the temperature with target material atomic number was observed when electron beams were propagated through a slab of material. This further reinforces the fact that
the bremsstrahlung signal temperature was predominantly influenced by the specular electron beam, which did not interact with the target material, leading to no observation of material scaling.

5. Conclusion

The results of a systematic high resolution measurement of bremsstrahlung scaling in ultrafast laser–solid interactions with thick targets have been presented. The high resolution HPGe detector measurement was shown to be able to resolve nuclear linewidths which may allow the detection technique to be used to measure short lived isomers and isotopes. The bremsstrahlung spectrum exhibited a characteristic two temperature shape. The two temperatures, $T_{b1}$ and $T_{b2}$, were observed to scale with the $\lambda^3$ scaling. The effective bremsstrahlung temperature was not observed to depend on the target material, which was corroborated with MCNPX simulations.

The MCNPX and PIC results confirm the overall population of electrons was well characterized by an isotropic source with a high energy beam directed in the specular direction. The importance of accounting for both the electrons that escape the target and the electrons that propagate into the target, as well as the sensitivity of this measurement technique to both populations, was shown through MCNPX simulations. These results lead to the conclusion that the isotropic population produces the $T_{b1}$ temperature through the ponderomotive expulsion of electrons from the focal spot of the laser, while the specularly beamed electrons experience chaotic acceleration, as previously mentioned, and produce the $T_{b2}$ temperature. The scaling was also observed to be slower than Wilks ponderomotive scaling, which is consistent with theoretical works that postulate lower temperatures [11, 22–24].

The electron directionality, along with the higher observed intensity scaling than Beg scaling, suggest a significant difference between the coupling of laser energy into electron
motion for picosecond and femtosecond duration interactions. This also suggests that the discrepancy between femtosecond and picosecond electron directions can be attributed, in part, to the specular acceleration mechanism, which can not be supported during picosecond duration interactions due to the distortion of the critical surface. Additionally, the presence of a higher temperature scaling, consistent with previous femtosecond laser interactions, suggests further studies of the bremsstrahlung scaling at higher intensities may yield higher temperatures than picosecond lasers.

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Appendix

In the single-hit regime photon pile-up is governed by Poisson statistics which predict the probability of the detector recording \( n \) events is

\[
P(n) = \frac{\bar{x}^n e^{-\bar{x}}}{n!},
\]

where \( \bar{x} \) is the theoretical photon interaction probability. This probability, which is the first moment of the distribution, or \( \bar{x} = \sum_{n=0}^{\infty} n P(n) \), was higher than the experimental detection probability \( x \). The experimental detection probability, which was defined previously as the percentage of laser shots that resulted in a detection event on the HPGe detector, was described by \( x = \sum_{n=1}^{\infty} P(n) \) due to photon pile-up. The percentage of detection events observed by the detector that were the result of piled-up photons was calculated by summing the probabilities of the detection of more than one photon, or \( \sum_{n=2}^{\infty} P(n) \). This predicted 8–13% of the detected photons were due to pile-up for the detection event probabilities used during this experiment.

In order to evaluate the effect of the piled-up photons on the temperature measurement, a Monte Carlo code was written to simulate the observed photon spectrum. A dual exponential spectrum was used to model a true bremsstrahlung spectrum, \( f(E) \), of the form

\[
f(E) = (1 - e^{-E/F_e / A}) \times e^{-E/E_b},
\]

where \( E_b \) was the bremsstrahlung temperature, \( F_e = 50 \) was selected to match the e-folding of the steel transmission curve above 50 keV, and \( A = e^{-1} \) was chosen to eliminate photons below 50 keV, such that the low energy (<100 keV) photon attenuation was accurately modeled. Using this distribution, a spectrum was numerically compiled by modeling the probability of the photon producing a ‘hit’ on the detector using the probabilities \( P(n) \). Single photons hits, \( P(1) \), were simply recorded in the correct detector channel. Pile-up hits, \( P(2,3) \), were combined with one or two additional photons, repetitively, which were separately generated with the same bremsstrahlung spectrum, and then recorded in the channel.
corresponding to the sum of their energies. This effect alone caused the effective temperature to be overestimated by \( \leq 15\% \).

However, the HPGe detector has a detection efficiency that strongly depends on the absorption process, which becomes dominated by the Compton effect over 200 keV \([25]\), and drops in efficiency. In the Compton range, the HPGe detector was measured to have an intrinsic collection efficiency of \( 0.05 \times E (\text{MeV})^{-0.9} \). In order to recover the true photon spectrum, the intrinsic efficiency of the detector was used to correct the signal. Specifically, the measured spectra were divided by the intrinsic efficiency of each channel, which increased the number of counts in the high energy channels by up to eight times more than the 100 keV channel. This efficiency correction, when combined with the pile-up effect, amplifies the effect of the piled-up photons, as they are collected by the detector with high efficiency, and then recorded in high energy channels which are corrected for low efficiency.

In order to correctly model this process, each detected photon was multiplied by the detection efficiency. Specifically, each detection event incremented the corresponding channel by

\[
C(E_{\gamma_1} + \cdots + E_{\gamma_n}) = \frac{\sum_{z=1}^{z=n} D(E_{\gamma_z})}{n},
\]

where \( E_{\gamma_z} \) is the energy of the photon \( \gamma_z \), \( C(E) \) is the channel corresponding to the summed energies of the detected photons and \( D(E) \) is the intrinsic detection efficiency of the detector for a photon of energy \( E \). After the spectrum was compiled in this fashion, the intrinsic efficiency was used to correct the signal, so that it could be compared to the experimental spectrum. As a result of this numerical modeling it was observed that the experimentally measured temperature overestimated the true temperature by 35–70% over the true temperature range of 100 keV–1 MeV. This relationship was found to follow \( T = 1.1 \times T_{\text{exp}}^{0.92} \) where \( T \) was the true temperature and \( T_{\text{exp}} \) was the experimentally observed temperature. This relationship was used to correct the observed temperatures used for the scaling and directionality analysis.

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