THE FUTURE OF K PHYSICS∗†

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Abstract

We discuss the opportunities for experiments at the frontier of physics using K-meson beams after the current round of precision experiments looking for CP violation in the K meson decay amplitude and for flavor-changing neutral currents are completed and the B-factories at KEK and SLAC are running. We emphasize those experiments that will give complementary information on the parameters of the Standard Model, especially the Cabibbo-Kobayashi-Maskawa matrix elements, and on possible physics beyond the Standard Model.

1 Introduction

In the late 1960s and early 1970s experiments on the properties and decays of K mesons reached a peak of activity, much of it sparked by the discovery in 1964 of CP violation in the neutral K system. Many beautiful experiments were done that pinned down the properties of the short and long lived neutral K mesons, all consistent with CP violation being present in the mass matrix and hence manifest by small admixtures (summarized in the parameter $\epsilon$) of the “wrong” CP state being present in the $K_S$ and $K_L$.

There was a rebirth of K physics in the early 1980s. Gauge theories of the strong and electroweak interactions had finally provided a well-defined basis for calculations. The phase present in the three generation weak mixing matrix, the Cabibbo-Kobayashi-Maskawa (CKM) matrix, provided an origin for CP violation. Furthermore, it was understood that this phase would not only enter the mass matrix through diagrams involving virtual heavy quarks and W bosons, but would enter weak decay amplitudes as well. In particular, loop diagrams involving W bosons and top quarks would give detectable CP-violating contributions to neutral K decay to two pions (summarized by the parameter $\epsilon'$) as well as to the mass matrix, setting off a series of

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experiments to measure \( \epsilon' \). In addition, it was possible take meaningful, systematic account of the strong interaction (QCD) corrections and these calculations were done for a number of processes in leading order. Indeed, for \( \epsilon' \) the strong interactions were not just corrections, but an essential part of the effect in lowest order!

On the experimental side, high flux beams became available and the corresponding high rate data acquisition systems developed, along with increasingly ‘smart’ triggers. Also important were improved detectors, especially those for photons through major advances in calorimetry. This was already the situation in 1989 when I reviewed the situation in a talk with a title very similar to the present one at the Fermilab Workshop on the Main Injector. At that time another important development was in progress and already noted: “the rise of the top quark.” Through the 1980s and early 1990s the experimental lower limit on the mass of the top quark rose monotonically. Calculations of many amplitudes for CP-violating processes gave rapidly increasing and eventually dominant contributions from loop diagrams with top quarks. The QCD corrections for interesting processes were soon redone for the case where the top mass was comparable to or greater than the W mass, first in leading order (LO) in the late 1980s, and then in next-to-leading order (NLO) in the 1990s.

Meanwhile the Standard Model was checked and rechecked with increasing precision. Our confidence in \( SU(3) \times SU(2) \times U(1) \) as the gauge theory of strong and electroweak interactions has been immensely reinforced through verification of its predictions at the one-loop level and beyond. But the parade of beautiful results confirming the Standard Model of three generations of quarks and leptons and the interactions between them has made even more insistent the search for physics that lies beyond the Standard Model. Some of the theoretical reasons we look for new physics have become standard in themselves: Why are there three generations? What is the connection between quarks and leptons? Is there not a further unification of interactions? How do we solve the naturalness problem of keeping masses at the weak scale (rather than running up to the Planck scale) without incredible adjustments of initial parameters? How can we eliminate or relate the many parameters that we have in the Standard Model?

Aside from theory and aesthetics, we have some hints from experiment as well. Neutrino masses and oscillations seem to be the preferred explanation of the data on solar neutrinos. Running the three gauge couplings to higher mass scales indicates that they will converge, perhaps to a common intersection. In addition, the Standard Model does not appear able to explain baryogenesis. Baryogenesis at a very high (unification) scale would get wiped out by inflation and then require reheating to a level that violates other aspects of big bang cosmology. On the other hand, CP violation at the electroweak scale in the Standard
Model is orders of magnitude too small to give the observed excess of baryons over antibaryons within the Standard Model.

Many of these problems could be remedied in extensions of the Standard Model such as supersymmetry. Such new physics, however, generally leads both to flavor-changing-neutral-currents (FCNC) and to CP violation. We want the new heavy particles to be coupled to those of the Standard Model (otherwise, their existence would solve few, if any, of the problems while adding new parameters) and the new sector typically will have its own own phases. New CP-violating, FCNC will then occur either at tree level or through loops, but with a different (than the Standard Model) weighting process-by-process. Examples of such new physics are flavor-changing, horizontal gauge bosons, where the FCNC may enter at tree level (no loops), and supersymmetry, where a new set (but not the only ones) of flavor-changing one-loop diagrams is obtained immediately by taking the Standard Model diagrams and replacing the internal (virtual) particles by their supersymmetric partners.

So, with this background, why do K physics? It is because K mesons remain a system where we can probe with extremely high precision either to obtain important results that pin down Standard Model parameters or to uncover new physics, especially as it relates to CP-violating phenomena. More specifically, as one-loop amplitudes depend on heavy particles and their couplings, we obtain precision measurements of the CKM matrix elements within the Standard Model and/or see the effects of FCNC and CP violation due to new heavy particles and the associated phases. This is generally complementary to the CP violation studies that will be done at the B-factories and the direct search for new physics at the energy frontier.

In the remainder of this talk, I will be looking at K physics at the beginning of the next century, after the round of $\epsilon'/\epsilon$ experiments now underway is finished and after the KEK and SLAC B-factories are in operation. I will emphasize a few processes that I find particularly interesting to investigate, necessarily omitting many other ones that may manifest new physics as well. Those that I stress have the property of being CP-violating and having a non-zero rate predicted in the Standard Model. They are not easy experimentally. In fact, they are surely very difficult, but that is the level one must reach to make very significant contributions to this physics in the next century.

2 The $K^0 - \bar{K}^0$ Mass Matrix

The neutral K mass matrix is the archetype of the flavor-changing-neutral-current transition. Both long-distance and short-distance contributions are important in the real part of the mass matrix that is (primarily) responsible for $\Delta M_K = M_{K_L} - M_{K_S}$. The short-distance
contribution to the real part is dominated by the box diagram with W bosons and charm quarks. On the other hand, the parameter $\epsilon$, which represents CP violation in the mass matrix, arises from the imaginary part and receives important contributions from the box diagrams with both charm and top quarks:

$$|\epsilon| = \frac{1}{\sqrt{2}} \frac{G_F}{12\pi^2} \frac{M_K}{\Delta M_K} (B_K f_K^2) \cdot$$

$$\left\{ \eta_1 m_c^2 Im(V_{cd}^*V_{cs})^2 + \eta_2 m_t^2 f_2(x_t) Im(V_{td}^*V_{ts})^2 + \eta_3 m_c^2 f_3(x_c, x_t) Im(V_{cd}^*V_{cs}V_{td}^*V_{ts}) \right\},$$

where $f_2$ and $f_3$ are slowly varying functions of $x_t = m_t^2/M_W^2$ and of $x_c = m_c^2/M_W^2$. The factors $\eta_i$ are QCD correction factors that were calculated in leading order many years ago, while the next-to-leading-order (NLO) values have only just recently been fully calculated to be $1.3 \pm 0.2$, $0.57 \pm 0.01$, and $0.47 \pm 0.04$, respectively. The change in going from LO to NLO values is quite significant for $\eta_1$ (from charm), less so for $\eta_3$ (from charm-top), and rather small for $\eta_2$ (purely from top, which lives at the weak scale). Since the latter is the dominant (roughly 70 percent) contribution, the overall effect of including the NLO calculation is less than might have been expected at the outset.

$\Delta M_K$ and $\epsilon$ illustrate well the tight restrictions that are imposed on extensions of the Standard Model by measurements of FCNC and CP-violation in the K system. A case in point is supersymmetry, where for some time information from the neutral K mass matrix has been built into models. Potentially very large contributions arise through box diagrams with squarks and gluinos and strong (rather than weak) interaction couplings. One is forced to “universality” (degeneracy of the squark masses at the Planck scale) or to “alignment” (of the squark and quark mass matrices) to avoid a value of $\epsilon$ that is orders of magnitude too large. As noted before, the Standard Model diagrams with all internal particles in the loop replaced by their supersymmetric partners can contribute as well. The situation for supersymmetric grand unified theories with a large top mass have been re-examined in the last few years, and additional constraints have been found on such theories.

In another extension of the Standard Model, left-right symmetric gauge theories, it has been known for some time that the neutral K mass difference and $\epsilon$ greatly restrict mixing between the left and righthanded sectors and push the mass of the right-handed gauge bosons above a TeV. It has recently been noted that with current parameters and masses, FCNC Higgs bosons that occur in the theory must be pushed up to many tens of TeV in mass.
3 CP Violation in the Decay Amplitude for $K \rightarrow \pi\pi$

In the Standard Model, it is natural to expect that there will be CP-violating contributions to $K$ decay amplitudes that also carry the phase found in the quark mixing matrix. Indeed, so-called “penguin diagrams” involving a virtual $W$ and a charm or top quark, with gluons connecting the virtual heavy quark to light quarks, give rise to amplitudes with the CKM phase, and these were predicted to produce a measurable CP-violating effect in the decay of a $K$ to two pions.\[4\]

A CP-violating difference in rates comes about through the interference of two (or more) amplitudes with different weak (and strong) phases. In the case of the decay of a $K$ to two pions, the two relevant amplitudes correspond to final isospin zero and two. Since the isospin zero amplitude, $A_0$, has a magnitude more than twenty times the isospin two amplitude, $A_2$, the resulting CP-violating interference as summarized in the parameter $\epsilon'$,

$$\epsilon' = \frac{i}{\sqrt{2}} e^{i(\delta_2 - \delta_0)} \text{Im} \left( \frac{A_2}{A_0} \right), \quad (2)$$

is unfortunately very much suppressed by the ratio $|A_2/A_0|$ and the presence of the small imaginary part due to gluonic penguin diagrams in $A_0$.

The actual prediction of $\epsilon'$ requires a systematic analysis of the $\Delta S = 1$ transition with a full account of heavy quark loops and QCD corrections.\[5\], \[19\] In addition to those from gluonic penguins, there are CP-violating contributions coming from “electroweak penguins” (penguin diagrams where one gluon is replaced by a photon or $Z$ boson) and from box diagrams containing heavy quarks and $W$ bosons.\[20\], \[21\] While electroweak rather than strong couplings enter, these contributions gain a factor of $A_0/A_2$ relative to the gluonic penguins and they grow roughly like $m_t^2$. While not of much significance for small $m_t$, as the top mass increases the relative strength of the electroweak penguin and box diagram contributions increase rapidly. Even more importantly, they enter with opposite sign to the contribution from the gluonic penguin, leading to a cancellation that decreases the predicted value $[21]$ of $\epsilon'$. In the last few years all of these contributions have been put together into full NLO calculations of the $\Delta S = 1$ effective non-leptonic Hamiltonian.\[22\] However, even combined with improved calculations of hadronic matrix elements using the lattice and our experimental knowledge now of $m_t$, theoretical predictions of $\epsilon/\epsilon'$ remain with large uncertainties $[23] - [26]$ because of the cancellation between contributions of comparable magnitude from gluonic penguins and electroweak penguins. For now, including the uncertainty in the matrix
element of the operator containing the dominant contribution from gluonic penguin diagrams (due to a potential change in the effective strange quark mass in lattice calculations), I would put the present theoretically plausible range for $\epsilon'/\epsilon$ as

$$-5 \times 10^{-4} < \epsilon'/\epsilon < 30 \times 10^{-4}.$$  

(3)

This is to be compared to:

$$\epsilon'/\epsilon = 23 \pm 3.5 \pm 6 \times 10^{-4}$$  

(4)

and

$$\epsilon'/\epsilon = 7.4 \pm 5.2 \pm 2.9 \times 10^{-4}$$  

(5)

from the NA31 experiment at CERN and the E731 experiment at Fermilab, respectively. These results hint that the value is non-zero, but for such an important measurement, the fact that even the combined result is only about three standard deviations from zero is not a satisfactory situation. Hence, another round of measurements is underway and will be taking data over the next few years: NA48 at CERN and KTeV at Fermilab, together with with the CHLOE detector at the DAPHNE phi factory. All these aim at a precision in the neighborhood of $10^{-4}$ for $\epsilon'/\epsilon$. Barring a cruel cancellation, they should finally determine a non-zero value. As witness the level of effort to carry out these experiments, this remains an extremely important measurement.

4 $K_L \to \pi^0 \ell^+\ell^-$

The process $K_2 \to \pi^0 \ell^+\ell^-$ with one photon coupling to the charged leptons is CP-violating, and it was realized even before the third generation quarks were found that there would be a CP-violating contribution to $K_L \to \pi^0 e^+e^-$ from heavy quark loops. Not long afterward, analysis of this decay that included important QCD corrections showed that the situation was very much unlike $K \to \pi\pi$ in that the interfering, CP-violating amplitudes from the mass matrix and the decay amplitude should be comparable in magnitude. Both experiment and theory for this process have been refined since then, although not as much as we might have hoped.

CP violation in the decay amplitude. As we are dealing with charged leptons in the final state, gluonic penguin diagrams, so important in $K \to \pi\pi$, are irrelevant, and the interesting CP violating contributions to the amplitude come from electroweak penguin and box (with W bosons, a heavy quark and a neutrino) diagrams. With the known top mass, about half the decay rate comes from non-interfering amplitudes due to the Z penguin and box graphs that involve an axial-vector coupling to the charged leptons. Both the leading order and
the NLO QCD corrections \cite{34} have been calculated for large $m_t$. With the known top mass and NLO corrections, a recent prediction \cite{27} is:

$$BR(K_L \rightarrow \pi^0 e^+ e^-) \big|_{\text{decay amplitude}} = 4.5 \pm 2.6 \times 10^{-12},$$

(6)

where the uncertainty in CKM parameters is responsible for most of the range and interference with the CP-violating amplitude from the mass matrix is not included.

**CP violation in the mass matrix.** The amplitude for this contribution is equal to $|\epsilon|$ times the amplitude for $K_S \rightarrow \pi^0 \ell^+ \ell^-$, the latter being CP-allowed. A direct measurement of $K_S \rightarrow \pi^0 \ell^+ \ell^-$ would therefore nail down this contribution. In its absence we resort to theory, and in particular to chiral perturbation theory. Much work has been done in this area, and a recent review \cite{35} gives an optimistically small value for the branching ratio coming solely from CP violation in the mass matrix:

$$BR(K_L \rightarrow \pi^0 e^+ e^-) \big|_{\text{mass matrix}} \leq 1.5 \times 10^{-12},$$

(7)

based on an assumed $SU(3)_F$ octet amplitude. Other estimates \cite{27} range up to about $5 \times 10^{-12}$. In any case, the interfering amplitudes from the decay and the mass matrix do indeed seem to be comparable; perhaps that from the decay amplitude is even dominant.

**CP-conserving amplitude.** There is a CP-conserving amplitude for this process that is higher order in $\alpha$ and proceeds through a two photon intermediate state. The helicity-conserving electromagnetic interaction forbids the process $\gamma \gamma \rightarrow \ell^+ \ell^-$ when the total angular momentum is zero and the leptons massless. Consequently, if the $\ell^+ \ell^-$ in $K_L \rightarrow \pi^0 \gamma \gamma \rightarrow \pi^0 \ell^+ \ell^-$ has total angular momentum zero, the absorptive part of the amplitude (corresponding to an on-shell $\gamma \gamma$ intermediate state) must have a factor of $m_\ell$. For $K_L \rightarrow \pi^0 e^+ e^-$, the factor of $m_e^2$ in the branching ratio reduces it to a level that is completely negligible compared to the other contributions we are considering, even after account of the off-shell and dispersive contributions. \cite{35} Furthermore, chiral perturbation theory (carried out to order $p^4$, as needed to get the $J_{\gamma \gamma} = 0$ amplitude) gives a $\gamma \gamma$ mass spectrum in $K_L \rightarrow \pi^0 \gamma \gamma$ that agrees with experimental observations, \cite{36} although the predicted rate is off by about a factor of two. \cite{35} Since total angular momentum one is forbidden for two real photons, the next intermediate state of relevance has angular momentum two (calculated at order $p^6$ in chiral perturbation theory and much more uncertain theoretically) for the $\gamma \gamma$ or $\ell^+ \ell^-$ system. Most, but not all, estimates would have this contribution small. \cite{35} The branching ratio due to the CP-conserving part of $K_L \rightarrow \pi^0 e^+ e^-$ is then plausibly in the range \cite{35}

$$BR(K_L \rightarrow \pi^0 e^+ e^-) \big|_{\text{CP-conserving}} \leq 2 \times 10^{-12}$$

(8)

While there is no interference in the overall rate, the CP-conserving and CP-violating amplitudes do interfere to produce a charge asymmetry in
the lepton spectrum \[37\] that could be useful in sorting out the strength of the various contributions.

The present experimental limit \[38\] of $4.3 \times 10^{-9}$ is well above these expectations, but it is likely to be improved substantially in the future. It is important to observe this decay where the discussion above indicates that the contribution from CP-violation in the amplitude is at least comparable to the other contribution. A fully convincing demonstration of this will ultimately require a measurement of the branching ratio for $K_S \rightarrow \pi^0 \ell^+\ell^-$, eliminating the need for relying on theoretical estimates for the CP-violating contribution from the mass matrix. Measurement of $K_L \rightarrow \pi^0 \mu^+\mu^-$, while suppressed by phase space, should help sort out the contribution from the CP-conserving, two photon intermediate state, aside from having different experimental backgrounds.

5 $K_L \rightarrow \pi^0 \nu \bar{\nu}$

The process $K_L \rightarrow \pi^0 \nu \bar{\nu}$ should be almost purely CP violating,\[39\] and the contribution from the $K^0 - \bar{K}^0$ mass matrix is negligible compared to that from the decay amplitude. The neutral leptons in the final state ensure that there is no contribution from gluonic or electromagnetic penguins. That leaves the Z penguin and box diagrams, which are dominated by the contribution from top quarks, so that the amplitude is proportional to $\text{Im}[V_{td}V_{ts}^*]$. The leading order QCD corrections for large top mass were carried out \[10\] several years ago and the NLO corrections \[11\] more recently. Although with the top quark contribution totally dominating, the change in going from leading order to next-to-leading-order is not large, the lack of other uncertainties (the matrix element is fixed by charged current semileptonic decays) makes it important to calculate the NLO QCD corrections and reduce the renormalization scale dependence of the resulting amplitude. The calculated branching ratio is \[11\]

$$BR(K_L \rightarrow \pi^0 \nu \bar{\nu}) = 2.8 \pm 1.7 \times 10^{-11},$$ \[9\]

where we gain a factor of three from summing over the three types of neutrinos and the uncertainty comes primarily from the CKM matrix elements that enter.

As we have already noted, this process is especially clean theoretically, with CP-violation from the decay amplitude completely dominating over that from the mass matrix in the Standard Model and the matrix element known from semileptonic decays. In principle it offers us a process with which to measure \[24\], \[12\] $\text{Im}[V_{td}V_{ts}^*]$ with high precision. It is correspondingly an excellent place to look for physics beyond the Standard Model, with $K_L \rightarrow \pi^0 \nu \bar{\nu}$ similar to the CP-violating asymmetries that are to be measured in B decays as a probe of CP violation.
As just one example of new physics that could enter, one can get significantly different predictions for the rate in multi-Higgs models. The present upper limit on the branching ratio of $5.8 \times 10^{-5}$ is many orders of magnitude greater than what we must aim for. A big jump in sensitivity should come from the KTeV experiment, while a KEK proposal and experiments being considered at FNAL hope to get to the level needed to see the decay in the Standard Model.

6 Conclusion

When the round of $\epsilon'/\epsilon$ experiments now underway is completed, that issue should be settled; if nature hasn’t been unusually cruel in giving cancelling effects, we should have a non-zero measurement. Important as this is, I don’t see doing another round since we will not be able to translate the measurement, not matter how precise, back to precision information on the underlying theory.

Rather, our focus should move on to other K decays. The process $K_L \to \pi^0 \ell^+ \ell^-$ is of special interest because it offers a first example of a K decay where the CP-violation originating from the decay amplitudes will apparently be at least as important as either CP-violation from the mass matrix or CP-conserving contributions that are higher order in $\alpha$. This is all the more interesting if $\epsilon'$ is not firmly measured to be non-zero, or something has turned up that suggests CP-violating effects from outside the Standard Model.

Other measurements complement those from the B factories, especially $K_L \to \pi^0 \nu \bar{\nu}$. In the Standard Model, this process provides an independent measure of $\text{Im}[V_{td}V_{ts}^*]$, or since $V_{ts}$ is directly related to $V_{cb}$ with three generations, of $\text{Im}V_{td}$, the height of the unitarity triangle. This process serves as well as a sensitive and clean probe of new physics. In any case, if either experiments with K or B mesons show evidence for physics beyond the Standard Model, one will want to push a number of other experiments, and some of the ones I left out, for example those involving lepton flavor violation or muon polarization, may come to the fore.
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