The phase diagram of QCD with two and four flavors: results with HYP fermions

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We study the finite temperature phase transition of four and two flavor staggered fermions with hypercubic smeared link actions. These preliminary studies suggest that the improved flavor symmetry of the fermionic action can have a significant effect on the finite temperature phase diagram.

1. INTRODUCTION

In this talk we present preliminary results of the two and four flavor QCD finite temperature phase diagram on $N_T = 4$ temporal lattices using a hypercubic blocked (HYP) staggered fermion action. Because of the large lattice spacing these simulations are more relevant to illustrate the effect of the HYP action than to extract physical values. Our goal here is to show how the improved flavor symmetry of the HYP fermions influences the phase diagram.

In any fermionic action the thin link gauge connections can be replaced by some sort of smeared links. Smearing partially removes short range vacuum fluctuations (dislocations) improving the topological properties of the gauge field configuration and improving the chiral properties of the fermionic action. At the same time smearing can distort the short range properties of the configuration introducing new type of lattice artifacts. An optimal smearing is a transformation where short range distortions are minimal while most of the dislocations are removed. The hypercubic blocking was designed to be very compact, and its parameters are optimized non-perturbatively to remove most of the vacuum fluctuations. Interestingly the non-perturbatively optimized parameters remove the perturbative tree level flavor symmetry violating terms of the action.

The HYP blocking can be combined with any fermionic action. Here we consider staggered fermions. In [1] we showed in the quenched approximation that flavor symmetry with HYP staggered fermions is improved by about an order of magnitude relative to thin link staggered fermions. The improved flavor symmetry is especially important for physical quantities that are sensitive to chiral symmetry. The finite temperature phase transition restores chiral symmetry at vanishing quark mass. Theoretical predictions for this transition are based on the full chiral and flavor symmetry of the fermionic action. Lattice simulations studying the nature of the finite temperature phase transition at small quark masses could therefore be very sensitive to chiral symmetry violations of the lattice action. Since the finite temperature phase transition occurs at large lattice spacing, $a \approx 0.3 - 0.15 \text{fm}$ on typical temporal lattices $N_T = 4 - 8$, the use of chirally improved actions could be essential there.

Before presenting our numerical results we mention a puzzle about the flavor dependence of the finite temperature phase transition. Phenomenological instanton model calculations predict a strong flavor dependence for the chiral transition temperatures [4]. For $N_f = 2$ flavors the chiral transition is expected to be second order at $T_c \approx 150 \text{MeV}$ with crossover at finite quark mass values. For $N_f = 3$ flavors the transition is predicted to be first order, $T_c \approx 100 \text{MeV}$ in the chiral limit while for $N_f = 4$ flavors the chiral transition, if it exists at all, is at a very small tem-
perature. At finite quark mass both the $N_f = 3$ and 4 flavor cases show first order phase transition that persists up to some critical quark mass value and the phase transition temperature depends strongly on the quark mass. Lattice simulations observe only a weak flavor dependence, especially for $N_f = 4$ flavors. Instanton models rely on plausible but unproven assumptions yet they predict the zero temperature chiral behavior of QCD very successfully. It is puzzling why their prediction is different from lattice calculations in the finite temperature case.

2. $N_f = 4$ FLAVOR SIMULATIONS

We performed simulations on $N_T = 4$ temporal lattices with $N_S = 8, 10$ and 16 spatial size. The details of the simulations can be found in [5]. To see the effect of smearing first we compare thin link, one level of APE and HYP smeared actions at approximately matched quark masses. In quenched simulations we found that the three actions are matched at quark masses $am_{\text{HYP}} = 0.1$, $am_{\text{APE1}} = 0.08$, and $am_{\text{thin}} = 0.06$. Dynamical simulations show that once the quark masses are matched, the lattice spacing at fixed gauge coupling $\beta$ remains approximately matched also. As it can be seen in figure 1 the strong first order transition observed with thin link action weakens and disappears as smearing is introduced at constant physical quark mass. One might wonder if the HYP action might show a first order transition at some smaller gauge coupling. We cannot exclude this possibility without actual simulations, but even if there is a first order phase transition, it occurs at a much smaller temperature than the phase transition of the thin link action suggesting that it is not a physically relevant transition.

We did not find first order signals even at smaller quark masses close to the physical light quark mass indicating that our simulations were all above the end point of the first order transition line. To identify the crossover it is traditional to look at the susceptibility of the quark condensate. We found only very broad peaks in the susceptibility. However the peak of the susceptibility does not necessarily identify the crossover transition.

![Figure 1. The chiral condensate $\langle \bar{\psi}\psi \rangle$ for the thin link, APE1 and HYP actions at approximately matched quark masses ($N_f = 4$).](image)

It is more sensitive to nearby critical points like the end point of the first order line. If the transition line has a strong quark mass-temperature dependence, this could be very different from the actual crossover region. Because of that concern we decided to use the chiral condensate at fixed gauge coupling to identify the crossover temperature. At small quark masses we expect the condensate to depend linearly on the quark mass,

$$\frac{\langle \bar{\psi}\psi \rangle}{m_q} = \frac{\Sigma}{m_q} + c$$

where $\Sigma$ and $c$ are constants independent of the quark mass. We distinguish three different cases. If all quark masses are in the chirally symmetric phase, $\Sigma = 0$. If all quark masses are in the chirally broken phase, $\Sigma > 0$ is the chiral condensate. If at fixed gauge coupling the smaller quark mass points are in the chirally symmetric while the larger quark mass points are in the chirally broken phase, the formers predict $\Sigma = 0$ while the latter ones predict a non-zero condensate at the transition point. $\Sigma$ however is not this value but its linear extrapolation to zero quark mass and could take any value, even negative ones. Figure
Figure 2. $\langle \bar{\psi} \psi \rangle / am_q$ as the function of $1/am_q$ at three different gauge coupling values ($N_f = 4$).

Figure 3. The predicted crossover line for the $N_f = 4$ flavor system on $N_T = 4$ temporal lattices.

Figure 4. $\langle \bar{\psi} \psi \rangle / am_q$ as the function of the gauge coupling for $N_f = 2$ flavors at two different quark mass values.

3. $N_f = 2$ FLAVOR SIMULATIONS

We have preliminary results of the two flavor system on $8^3 \times 4$ lattices at several quark masses. In the simulations we approximate the square root of the fermionic determinant with a finite polynomial. Some of the details of the algorithm are discussed in [5]. The phase diagram does not show significant deviation from the thin link action results. Figure 4 shows $\langle \bar{\psi} \psi \rangle / am_q$ as the function of the gauge coupling for $am_q = 0.01$ and $am_q = 0.04$. The figure suggests a phase transition around $\beta = 5.2$. Further data at quark mass $am_q = 0.02, 0.03$ and $0.06$ support this conclusion. The phase transition for the two flavor system on $N_T = 4$ is a crossover, it occurs around $\beta = 5.2$ and depends only weakly on the quark mass. The susceptibility of the chiral condensate predicts the same value.

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