Passivity-based Trajectory-tracking for Marine Craft with Disturbance Rejection

Alejandro Donaire * Jose Guadalupe Romero ** Tristan Perez ***

* PRISMA Lab, Dep. of Elec. Eng. and Inf. Tech., University of Naples Federico II, and School of Engineering, The University of Newcastle, Australia. (e-mail: Alejandro.Donaire@newcastle.edu.au)
** Laboratoire d’Informatique, de Robotique et de Microelectronique de Montpellier, France. (e-mail: Jose.Romero-Velazquez@lirmm.fr)
*** School of Electrical Eng. and Comp. Sc., Queensland University of Technology, Australia. (e-mail: Tristan.Perez@qut.edu.au)

Abstract: This paper presents a Hamiltonian model of marine vehicle dynamics in six degrees of freedom in both body-fixed and inertial momentum coordinates. The model in body-fixed coordinates presents a particular structure of the mass matrix that allows the adaptation and application of passivity-based control interconnection and damping assignment design methodologies developed for robust stabilisation of mechanical systems in terms of generalised coordinates. As an example of application, we follow this methodology to design a passivity-based tracking controller with integral action for fully actuated vehicles in six degrees of freedom. We also describe a momentum transformation that allows an alternative model representation that resembles general port-Hamiltonian mechanical systems with a coordinate dependent mass matrix. This can be seen as an enabling step towards the adaptation of the theory of control of port-Hamiltonian systems developed in robotic manipulators and multi-body mechanical systems to the case of marine craft dynamics.

Keywords: Marine systems, Port-Hamiltonian Systems, Nonlinear control.

1. INTRODUCTION

Interconnection and damping assignment passivity-based control (IDA-PBC) is an attractive technique for designing motion-control strategies related to physical systems. This technique uses the control action to transform the open-loop system into a closed-loop system in port-Hamiltonian form (Ortega et al., 2002; van der Schaft, 2000). The closed-loop potential energy is chosen such that it attains a minimum at the desired configuration of the system—this determines the closed-loop equilibrium. Under certain conditions on different terms of the model, stability can be proven using the closed-loop energy as a Lyapunov function. The design also presents passivity properties with respect to force inputs and velocity outputs.

The passivity properties of the hydrodynamic and rigid-body models have been exploited for design of control systems for craft. For example, Fossen and Berge (1997) (see also the summary in Fossen (2011)) use the concept of vectorial integrator backstepping for the design of dynamic positioning for ships—a technique that uses control-Lyapunov functions and can be related to feedback passivation (Ortega et al., 1998). Sorensen and Egeland (1995) use passivity-based techniques to design a ride controller (reduction of roll and pitch) for a surface-effect ship. Woolsey and Leonard (2002) consider the dynamics of fully-actuated underwater vehicles as a Hamiltonian system, and address the problem of stabilisation of underwater vehicles using internal rotors as actuators. This control approach involves shaping the kinetic energy of the system preserving the Hamiltonian structure and adding dissipation to ensure asymptotic stability of the closed-loop. The use of IDA-PBC for positioning with integral action of open-frame fully-actuated underwater vehicles is addressed by Donaire and Perez (2010) and this work is extended to tracking by Donaire et al. (2011) in three degrees of freedom. Considerations of actuator saturation and the addition of anti-windup is addressed by Donaire and Perez (2012) for the problem of dynamic positioning of offshore vessels. Astolfi et al. (2002) consider the problem of stabilisation of the under-actuated Kirchhoff equations for an underwater vehicle moving in an ideal fluid, that is, neglecting hydrodynamic dissipative forces. They apply IDA-PBC to deal with the stabilisation problems of the steady longitudinal motion and the steady rising/diving with forward/reverse motion. Valentinis et al. (2013) solve the attitude and speed regulation problem for an slender underwater vehicle with a full hydrodynamic model (potential plus viscous effects) and focus on both forward speed and attitude (roll, pitch, and yaw) tracking based on energy shaping and damping assignment such that the closed-loop system retains a port-Hamiltonian form. The unactuated channels of the system are left in open loop, and a suppression control is used to completely remove the uncontrolled behaviour from the target dynamics.

In this paper, we present a Hamiltonian model of marine vehicle dynamics in six degrees of freedom in both body-
fixed and inertial momentum coordinates. The model in body-fixed coordinates presents a particular structure of the mass matrix that allows the adaptation and application of a change of coordinates to assign a full-rank dissipation matrix first proposed by Doraire and Perez (2012), and then generalised for mechanical systems by Romero et al. (2013b,a). We follow this methodology to design a passivity-based tracking controller with integral action for fully actuated vehicles in six degrees of freedom. This extends the work in Doraire and Perez (2012) to tracking in all degrees of freedom. We also describe a momentum transformation that allows an alternative model representation that resembles general port-Hamiltonian mechanical systems with a coordinate dependent mass matrix. This can be seen as an enabling step towards the adaptation of the theory of control of port-Hamiltonian systems developed in robotic manipulators and multi-body mechanical systems to the case of marine craft dynamics.

2. PORT-HAMILTONIAN MODELS

The dynamics of mechanical systems in generalised coordinates can be described using the Euler-Lagrange equation (Lanczos, 1986):

$$\frac{d}{dt} [\nabla_q L(q, \dot{q})] - \nabla_q L(q, \dot{q}) = \tau, \quad (1)$$

where \( q \) and \( \dot{q} \) are the \( n \)-dimensional vectors of generalised coordinates and velocities respectively, and \( \tau \) is the vector of generalised forces. The Lagrangian \( L(q, \dot{q}) \) is the difference between the kinetic co-energy and the potential energy of the system. For systems within the realm of classical mechanics, the Lagrangian takes the form

$$L(q, \dot{q}) = \frac{1}{2} \dot{q}^T M(q) \dot{q} - V(q), \quad (2)$$

where the generalised mass matrix \( M(q) > 0 \) is symmetric for all \( q \).

In classical mechanics, the conjugate generalised momentum is \( p = \nabla_q L(q, \dot{q}) = M(q) \dot{q} \). Using the momentum and the generalised coordinate vector, the set of \( n \) second-order differential equations arising from (1) can be transformed, using the Legendre’s transformation, into a set of \( 2n \) first-order differential equations (Lanczos, 1986):

$$\dot{q} = \nabla_p H(p, q), \quad (3)$$

$$\dot{p} = -\nabla_q H(p, q) + \tau, \quad (4)$$

where the Hamiltonian \( H(p, q) \) is the sum of the kinetic energy and the potential energy:

$$H(p, q) = \frac{1}{2} p^T M^{-1}(q)p + V(q). \quad (5)$$

This function represents the total energy of the system. The equations (3)-(4) are called the Hamilton’s canonical equations of motion, which can be seen as a particular state-space representation for the Euler-Lagrange model (1). We should note, however, the Hamiltonian models are more general in the sense that there are systems that admit Hamiltonian but not Lagrangian representations (Lanczos, 1986).

In the control literature, the Hamiltonian model (3)-(4) has been generalised to what is known as a port-Hamiltonian (pH) system (van der Schaft, 2006):

$$\dot{x} = [J(x) - R(x)] \nabla H(x) + G(x) u, \quad (6)$$

$$y = G^T(x) \nabla H(x), \quad (7)$$

where \( x \) is the state vector and the pair \( u, y \) are the input and output \( m \)-dimensional vectors. These are conjugate variables; that is, their inner product represents (or is akin to) the power exchanged between the system and its environment. The function \( J(x) = -J^T(x) \) describes the power preserving interconnection structure through which the components of the system exchange energy. The symmetric function \( R(x) \geq 0 \) captures dissipative phenomena in the system. The function \( G(x) \) weighs the action of the input on the system and defines the conjugate output. From (6)-(7), it follows that

$$\dot{H}(x) = y^T u - [\nabla H(x)]^T R(x) \nabla H(x) \leq y^T u, \quad (8)$$

which shows passivity of the pH model (van der Schaft, 2000).

3. DYNAMICS OF MARINE CRAFT IN PORT-HAMILTONIAN FORM

The classical equations of motion used for marine craft can be written as follows (Fossen, 2002):

$$\dot{\eta} = J(\eta) \nu, \quad (9)$$

$$M \ddot{\nu} + C(\nu) \nu + D(\nu) \nu + g(\eta) = \tau_c + \tau_d, \quad (10)$$

where \( \eta \) describe the pose of the vehicle (position and orientation) (North, East, Down, roll, pitch, yaw), \( \nu \) is the body-fixed linear-angular velocity (surge, sway, heave, roll, pitch, yaw). The vector \( \tau_c \) represents the force and torque control inputs, and \( \tau_d \) represent the force and torque disturbance inputs. The constant matrix \( M = M^T > 0 \) is the total generalised mass matrix due to rigid-body mass distribution and fluid added mass, \( C(\nu) = -C^T(\nu) \) is the total Coriolis-centripetal matrix, and \( D(\nu) = D^T(\nu) > 0 \) is the total hydrodynamic damping matrix, and \( g(\eta) \) is the vector of hydrostatic forces and torques due to gravity and buoyancy. The function \( J(\eta) \) is a \( 6 \times 6 \) kinematic transformation matrix, which is well-defined if the pitch angle \( \theta \neq \pm \frac{\pi}{2} \). See, for example, Fossen (2011) for further details on this model.

Following on Doraire and Perez (2012), we will write the dynamics (9)-(10) in pH form. We first make the following assumption:

Assumption 1 (A1). There exist function \( V(\eta) : \mathbb{R}^3 \times \mathcal{S}^3 \to \mathbb{R} \) that satisfies

$$J^T(\eta) \nabla_\eta V(\eta) = g(\eta). \quad (11)$$

Note that this equation is satisfied for example for neutrally buoyant underwater vehicles, in which case the function \( V \) has the form

$$V(\eta) = -W \sin(\theta) X + W \cos(\theta) \sin \phi Y + +W \cos(\theta) \cos \phi Z, \quad (12)$$

where \( W = m g \) is the submerged weigh of the vehicle, and \((X, Y, Z)\) are the cartesian coordinates of the centre of buoyancy relative to the centre of gravity.

3.1 pH Model in Body-fixed Coordinates

The following proposition establishes the pH model in terms of a transformation of the body-fixed velocity.
Proposition 1. Consider the dynamics (9)-(10). Then under assumption A1, the dynamics of the marine craft can be written in port-Hamiltonian form as follows
\[
\begin{bmatrix}
\dot{p} \\
\dot{\eta}
\end{bmatrix} = \begin{bmatrix}
0 & J(y) \\
-J^T(\eta) & -J_2(p)
\end{bmatrix} \nabla H + \begin{bmatrix}
0 \\
I_n
\end{bmatrix} (\tau_c + \tau_d),
\] (13)\]
where
\[
H(\eta, p) = \frac{1}{2} p^T M^{-1} p + V(\eta),
\] (14)
the momentum is defined through the following transformation of the body-fixed velocities \(\eta\):\[
p = M \dot{\nu},
\] (15)
and
\[
J_2(p) = C(\nu) + D(\nu)\bigg|_{\nu = M^{-1} p},
\] (16)
which satisfies \(J_2(p) + J^T_2(p) > 0\).

**Proof** The proof follows from construction the state equations for \(\eta\) and \(p\). First, we note that
\[
\dot{\eta} = J(\eta) \nu = J(\eta) M^{-1} p
\]
(17)
which is the first row of (13). Then, from (10) we obtain
\[
\dot{p} = M \dot{\nu}
= -g(\eta) - C(\nu) \nu - D(\nu) \nu + \tau_c + \tau_d
= -J^T(\eta) \nabla_\eta V(\eta) - J_2(p) M^{-1} p + \tau_c + \tau_d,
\] (18)
which is the second row of (13). The fact that \(J_2(p)\) is positive definite follows readily from the properties of \(C(\nu) = -C^T(\nu)\) and \(D(\nu) = D^T(\nu) > 0\).

### 3.2 pH Model in Inertial Coordinates

An alternative port-Hamiltonian model, still under assumption A1, can be built by defining a new momentum vector as follows\footnote{Note that the momentum (15) is not the conjugate momentum of the generalised coordinate vector \(\eta\) since \(\nu\) has as components the body-fixed angular velocity (quasi-coordinates) (Greenwood, 2003, p193). These do not equate to the time-derivative of the Euler angles—which are part of \(\eta\).}
\[
l = J^{-T}(\eta) M \nu.
\] (19)
where
\[
H_n(\eta, l) = \frac{1}{2} J^T(\eta) M^{-1} J(\eta) l + V(\eta),
\] (21)
and
\[
L(\eta, l) = \left(\sum_{j=1}^n \nabla_{\eta_j} [J^{-T} M \nu]^T \right) T - \sum_{j=1}^n \nabla_{\eta_j} [J^{-T} M \nu]^T + J^{-T} C(\nu) J^{-1} + J^{-T} D(\nu) J^{-1},
\] (23)

\[\begin{bmatrix}
\dot{\eta} \\
\dot{p}
\end{bmatrix} = \begin{bmatrix}
S_{11} & S_{12} & S_{13} \\
-S_{12} & S_{22} & S_{23} \\
-S_{13} & -S_{23} & S_{33}
\end{bmatrix} \nabla H_n + \begin{bmatrix}
0 \\
0
\end{bmatrix} d(t),
\] (27)

where \(\epsilon_i \in \mathbb{R}^n\) is the \(i\)-th vector of the Euclidean basis. The first three terms of the matrix \(L(\eta, l)\) in (23) determine the skewsymmetric component of the matrix and accounts for the gyroscopic forces, whilst the last term describes the damping and implies that \(L(\eta, l) + L^T(\eta, l) > 0\). Note that the expression of Hamiltonian function (21) and (22) are equivalent, but (21) uses a factorisation of the mass matrix in terms of \(J\). This inspires a change of momenta to obtain a pH model with constant mass matrix as in (13). This type of change of coordinates thus leads to an identity mass matrix, which has been exploited to design controllers and observers for general mechanical systems, see for example Romero et al. (2013b); Venkatraman et al. (2010).

While the pH model (13) is related to the so-called body-fixed vector representation (9)-(10), the pH model (20) is related to the NED vector representation presented in (Fossen, 2011, p171):
\[M_\eta(\eta) \ddot{\eta} + C_\eta(\eta, \dot{\eta}) \dot{\eta} + D_\eta(\eta, \dot{\eta}) \dot{\eta} + g_\eta(\eta)
= J^{-T}(\eta)(\tau_c + \tau_d).
\] (24)

### 4. TRACKING CONTROL OF FULLY-ACTUATED MARINE CRAFT WITH INTEGRAL ACTION

We consider the marine craft dynamics (13) and a time-varying reference \(\eta^*(t)\) together with its time derivatives \(\dot{\eta}^*(t)\) and \(\ddot{\eta}^*(t)\). In this section, we propose a robust PBC tracking controller that ensures
\[
\lim_{t \to \infty} \eta(t) = \eta^*(t).
\]

We consider that the disturbance vector \(\tau_d\) has a constant and a time-varying component, i.e. \(\tau_d(t) = d + d(t)\). Therefore, the controller should ensure robust properties with respect to both components. Specifically, it is desirable that the controller ensures tracking in spite of constant unknown disturbances and that the state trajectories are bounded if there are bounded time-varying disturbances.

First, we define the tracking errors
\[
\begin{bmatrix}
\dot{\bar{\eta}} \\
\dot{\bar{p}}
\end{bmatrix} = \begin{bmatrix}
\eta - \eta^* \\
p - p^*
\end{bmatrix},
\] (25)
where \(\eta^*(t)\) is the position reference and \(p^*\) is a function to be selected. We will also extend the state vector with a new state \(\zeta\), which is the state of the integrator that compensates the constant disturbance.

We will design a control law such that the closed-loop dynamics has the desired pH form
\[
\begin{bmatrix}
\dot{\bar{\eta}} \\
\dot{\bar{p}} \\
\dot{\zeta}
\end{bmatrix} = \begin{bmatrix}
S_{11} & S_{12} & S_{13} \\
-S_{12} & S_{22} & S_{23} \\
-S_{13} & -S_{23} & S_{33}
\end{bmatrix} \nabla H_d + \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix} d(t),
\] (27)

with
\[
H_d(\bar{\eta}, \bar{p}, \zeta) = \frac{1}{2} \bar{p}^T M^{-1} \bar{p} + V_d(\bar{\eta}) + \frac{1}{2} (\zeta - \alpha)^T K_I (\zeta - \alpha),
\] (28)
The matrices \(S_{ij}\) with \(i, j = 1, 2, 3\) are functions to be selected. The constant vector \(\alpha\) will be properly defined during the design, and the constant matrix \(K_I\) is symmetric and positive definite. The function \(V_d\) should have a
minimum at $\tilde{\eta} = 0$. In addition, the matrices $S_{11}$, $S_{22}$ and $S_{33}$ should satisfy

$$S_{11} + \frac{S_{12}^T}{S_{22}} < -\epsilon_1 I_n < 0$$  \hfill (29)
$$S_{22} + \frac{S_{23}^T}{S_{33}} < -\epsilon_2 I_n < 0$$  \hfill (30)
$$S_{33} + \frac{S_{32}^T}{S_{22}} < -\epsilon_3 I_n < 0$$  \hfill (31)

with $\epsilon_1, \epsilon_2, \epsilon_3 \in \mathbb{R}_{>0}$ and $I_n$ is the $n \times n$-identity matrix. As we will show in this section, the closed loop (27) has the desirable stability features to achieve the control objective. We concentrate first on the control law. We start the design by writing the dynamics of the position error (25), and we substitute the derivatives of the states $\eta$ and $\tilde{\eta}$ in (13) and (27) as follows

$$\dot{\tilde{\eta}} = \eta - \eta^* = J(\eta)M^{-1}p - \eta^*$$
$$\equiv S_{11}\nabla V_d + S_{12}M^{-1}\tilde{p} + S_{13}K_I(\zeta - \alpha)$$  \hfill (32)

where $\zeta$ and $\alpha$ are chosen $S_{12} = S_{13} = J(\eta)^3$. In the second step, we need to ensure that the dynamics of $\tilde{p}$ is as the desired dynamics in (27). We compute the time derivative of $\tilde{p}$ as follow

$$\dot{\tilde{p}} = \tilde{p} - \tilde{p}^*$$
$$= -J^T\nabla V - J_2(p)M^{-1}p + \tau_c + d + dt(t) -$$
$$-\frac{d}{dt}[MJ^{-1}S_{11}]\nabla V_d - MJ^{-1}S_{11}\nabla^2 V_d \tilde{\eta} -$$
$$MK_I \dot{\zeta} - \frac{d}{dt}[MJ^{-1}] \dot{\eta}^* - MJ^{-1} \dot{\eta}^*$$
$$= -J^T\nabla V_d + S_{22}M^{-1}\tilde{p} + S_{23}K_I(\zeta - \alpha) + d(t).$$  \hfill (34)

Then, we derive the control law from (34) as follows

$$\tau_c = J^T\nabla V + J_2M^{-1}p + \frac{d}{dt}[MJ^{-1}S_{11}]\nabla V_d +$$
$$+MJ^{-1}S_{11}\nabla^2 V_d(JM^{-1}p - \eta^*) + MK_I \dot{\zeta} +$$
$$+\frac{d}{dt}[MJ^{-1}] \dot{\eta}^* + MJ^{-1} \dot{\eta}^* - J^T\nabla V_d +$$
$$+S_{22}M^{-1}p - S_{23}J^{-1}S_{11}\nabla V_d - S_{23}J^{-1} \dot{\eta}^* -$$
$$-S_{23}K_I(\zeta - \alpha) + S_{23}K_I(\zeta - \alpha) - d.$$  \hfill (35)

The control law (35) is independent of the disturbance $d$ if $\dot{\zeta}$ does not depend on $d$ and if $\alpha$ is chosen as

$$\alpha = K_I^{-1}(S_{22} - S_{23})^{-1}d.$$  \hfill (36)

The dynamics of the integral action is given by

$$\dot{\tilde{\eta}} = -S_{13}^T\nabla V_d - \frac{S_{12}^T}{S_{22}}M^{-1}\tilde{p} + S_{33}K_I(\zeta - \alpha)$$
$$= -J^T\nabla V_d - \frac{S_{12}^T}{S_{22}}M^{-1}[p - MJ^{-1}S_{11}\nabla V_d -$$
$$-MK_I(\zeta - \alpha) - MJ^{-1} \dot{\eta}^* + S_{33}K_I(\zeta - \alpha)$$
$$= -J^T\nabla V_d - \frac{S_{12}^T}{S_{22}}M^{-1}p + \frac{S_{23}^T}{S_{22}}J^{-1}S_{11}\nabla V_d +$$
$$+S_{23}J^{-1} \dot{\eta}^* + (S_{13}^T + S_{33})K_I(\zeta - \alpha),$$  \hfill (37)

which is independent on the disturbance $d$—see (36)—if

$$S_{23} = -S_{13}^T.$$  \hfill (38)

Now, we can formalise the controller design and the stability properties in the following proposition.

**Proposition 2.** Consider the marine craft dynamics (9)-(10) and assume that A1 holds. Consider also the control law

$$\tau_c = J^T\nabla V + J_2M^{-1}p + \frac{d}{dt}[MJ^{-1}S_{11}]\nabla V_d +$$
$$+MJ^{-1}S_{11}\nabla^2 V_d(JM^{-1}p - \eta^*) + MK_I \dot{\zeta} +$$
$$+\frac{d}{dt}[MJ^{-1}] \dot{\eta}^* + MJ^{-1} \dot{\eta}^* - J^T\nabla V_d +$$
$$+S_{22}M^{-1}p - S_{23}J^{-1}S_{11}\nabla V_d - S_{23}J^{-1} \dot{\eta}^* -$$
$$-(S_{22} + S_{23}K_I)\dot{\zeta},$$  \hfill (39)

and

$$\zeta = -J^T\nabla V_d + S_{33}M^{-1}p - J^{-1}S_{11}\nabla V_d - J^{-1} \dot{\eta}^*,$$  \hfill (40)

where function $V_d$ and the matrices $S_{11}$, $S_{22}$, $S_{33}$ and $K_I$ should be chosen to satisfy (29)-(31), $K_I > K_I^T > 0$, and $\arg \min V_d(\tilde{\eta}) = 0$. Then, the closed loop has the following properties:

**Property 1 (P1).** The closed-loop error dynamics can be written in port-Hamiltonian form

$$\begin{bmatrix} \dot{\tilde{\eta}} \\ \dot{\zeta} \end{bmatrix} = \begin{bmatrix} S_{11} & J(\eta) \\ -J^T(\eta) & S_{33} \end{bmatrix} \nabla H_d + \begin{bmatrix} d(t) \\ 0 \end{bmatrix},$$  \hfill (41)

with

$$H_d(\tilde{\eta}, \tilde{p}, \zeta) = \frac{1}{2} \tilde{p}^T M^{-1} \tilde{p} + V_d(\tilde{\eta}) + \frac{1}{2} (\zeta - \alpha)^T K_I (\zeta - \alpha).$$  \hfill (42)

**Property 2 (P2).** Under the assumptions that there is constant unknown disturbance $d$, that time-varying disturbance is zero, namely $d(t) = 0$, and that $V_d$ is selected such that

$$\kappa_1 \|\tilde{\eta}\|^2 \leq V_d(\tilde{\eta}) \leq \kappa_2 \|\tilde{\eta}\|^2,$$  \hfill (43)
$$\kappa_4 \|\eta\|^2 \leq \|V_d(\tilde{\eta})\|^2 \leq \kappa_3 \|\eta\|^2,$$  \hfill (44)

with $\kappa_1, \kappa_2, \kappa_3, \kappa_4 \in \mathbb{R}_{>0}$, then, the tracking error $\tilde{\eta}(t)$ converges exponentially to zero. Therefore the control objective is achieved, namely

$$\lim_{t \to \infty} \tilde{\eta}(t) = \eta^*(t).$$

**Property 3 (P3).** Under the action of constant unknown disturbance $d$ and a bounded time-varying disturbance $d(t)$, the controller (39)-(40) ensures bounded states provided that the trajectories do not reach the singularity of the model (the pitch angle $\theta$ satisfies uniformly $|\theta| \leq \frac{\pi}{2}$).

**Proof** The claim in P1 follows readily from the development in section 4. Indeed, the dynamics (41) and the control law (39)-(40) is obtained respectively from (27) and (35)-(37) using (36) and (38).

The exponential convergency of the marine craft position vector $\eta$ to the time-varying reference $\eta^*$ follows from the exponential stability of the tracking errors to zero. To show that, we will study the (almost) global exponential
stability of the equilibrium \((\bar{\eta}, \bar{\rho}, \zeta, \epsilon) = (0, 0, \alpha)\) of the error dynamics (41). The Hamiltonian \(H_d\) has a minimum at the equilibrium, and since \(M^{-1}\) and \(K_I\) are positive definite and \(V_0\) satisfies (43), then \(H_d\) qualify as a Lyapunov candidate function and can be bounded as follows

\[
\begin{align*}
&c_1 |(\bar{\eta}, \bar{\rho}, \zeta - \alpha)|^2 \leq H_d(\bar{\eta}, \bar{\rho}, \zeta) \leq c_2 |(\bar{\zeta}, \bar{\rho}, \zeta - \alpha)|^2 \quad (45)
\end{align*}
\]

with \(c_1, c_2 \in \mathbb{R}_+, c_2 \neq 0\). We, then compute the derivative of \(H_d\) respect to time along the trajectories of the dynamics (41) as follows

\[
\dot{H}_d = \left[\nabla_\bar{\eta} H_d \right]^T \left(\nabla_\bar{\rho} H_d \right)^T \left(\nabla_\zeta H_d \right)^T \left[\frac{\dot{\bar{\eta}}}{\dot{\bar{\rho}} \dot{\zeta}}\right] =
\]

\[
\begin{align*}
&= \left(\nabla_\bar{\eta} H_d \right)^T S_{11} \nabla_\bar{\eta} H_d + \left(\nabla_\bar{\rho} H_d \right)^T J \nabla_\bar{\rho} H_d + \left(\nabla_\zeta H_d \right)^T J \nabla_\zeta H_d -
\end{align*}
\]

\[
\begin{align*}
&- \left(\nabla_\bar{\rho} H_d \right)^T J^T \nabla_\bar{\rho} H_d + S_{22} \nabla_\bar{\rho} H_d - \left(\nabla_\bar{\rho} H_d \right)^T S_{12} \nabla_\zeta H_d +
\end{align*}
\]

\[
\begin{align*}
&+ \left(\nabla_\zeta H_d \right)^T S_{23} \nabla_\bar{\rho} H_d + \left(\nabla_\zeta H_d \right)^T S_{33} \nabla_\zeta H_d + \left(\nabla_\zeta H_d \right)^T d(t)
\end{align*}
\]

\[
= -\frac{1}{2} \left|\nabla_\bar{\eta} V_d \right|^2 - \frac{\epsilon_2}{2} \left|\nabla_\bar{\rho} H_d \right|^2 - \frac{\epsilon_3}{2} \left|\nabla_\zeta H_d \right|^2 +
\]

\[
\begin{align*}
&+ \epsilon_1 \left|\nabla_\bar{\eta} V_d \right|^2 \left|\nabla_\bar{\rho} H_d \right|^2 - \frac{\epsilon_3 \epsilon_1 K_I}{2} \left|\zeta - \alpha\right|^2 +
\end{align*}
\]

\[
\begin{align*}
&+ \left(\nabla_\bar{\rho} H_d \right)^T d(t)
\end{align*}
\]

\[
\leq -\frac{\epsilon_1 K_3}{2} \left|\nabla_\bar{\eta} V_d \right|^2 - \frac{\epsilon_2}{2} \left|\nabla_\bar{\rho} H_d \right|^2 - \frac{\epsilon_3 k_3}{2} \left|\zeta - \alpha\right|^2 +
\]

\[
\begin{align*}
&+ \epsilon_1 \left|\nabla_\bar{\rho} H_d \right|^2 \left|\nabla_\bar{\eta} V_d \right|^2 - \frac{\epsilon_3 \epsilon_1 K_I}{2} \left|\zeta - \alpha\right|^2 +
\end{align*}
\]

\[
\begin{align*}
&+ \left(\nabla_\bar{\rho} H_d \right)^T d(t)
\end{align*}
\]

\[
\leq -\frac{\epsilon_1 K_3}{2} \left|\nabla_\bar{\eta} V_d \right|^2 - \frac{\epsilon_2}{4} \left|\nabla_\bar{\rho} H_d \right|^2 - \frac{\epsilon_3 k_3}{2} \left|\zeta - \alpha\right|^2 +
\]

\[
\begin{align*}
&+ \frac{1}{\epsilon_2} \left|d(t)\right|^2
\end{align*}
\]

\[
\leq -\frac{\epsilon_1 K_3}{2} \left|\nabla_\bar{\eta} V_d \right|^2 - \frac{\epsilon_2}{4} \left|\nabla_\bar{\rho} H_d \right|^2 - \frac{\epsilon_3 k_3}{2} \left|\zeta - \alpha\right|^2 +
\]

\[
\begin{align*}
&+ \frac{1}{\epsilon_2} \left|d(t)\right|^2
\end{align*}
\]

\[
\leq -\frac{\gamma}{\epsilon_2} H_d(\bar{\eta}, \bar{\rho}, \zeta) + \frac{1}{\epsilon_2} \left|d(t)\right|^2,
\]

where \(\gamma = \min\{2\epsilon_1 k_3, 2\epsilon_2 k_2, 2\epsilon_3 k_3\}\), \(k_2 = |M^{-1}|^2\) and \(k_3 = |K_I|^2\).

Exponential stability of the closed loop with constant disturbances \(d\) and without time-varying disturbances \(d(t) = 0\) follows directly from (46). Indeed, the inequality

\[
\dot{H}_d(\bar{\eta}, \bar{\rho}, \zeta) \leq -\frac{1}{\epsilon_2} H_d(\bar{\eta}, \bar{\rho}, \zeta)
\]

and the bound in the Lyapunov function (45) ensure (almost global) exponential stability of the equilibrium \((\bar{\eta}, \bar{\rho}, \zeta, \epsilon) = (0, 0, \alpha)\) (see e.g. Khalil (2000)). Exponential stability of the equilibrium implies that \(\bar{\eta}(t)\) exponential converge to the reference \(\eta^*(t)\), in spite of the presence of unknown constant disturbances, which shows P2.

![Reference and actual motion variables (surge).](image)

The bounded-input-bounded-state property follow from (46) and assuming \(d(t) \neq 0\), which yields

\[
\dot{H}_d(\bar{\eta}, \bar{\rho}, \zeta) \leq -\frac{c_1 \gamma}{\epsilon_2} \left|\bar{\eta}, \bar{\rho}, \zeta - \alpha\right|^2 + \frac{1}{\epsilon_2} \left|d(t)\right|^2,
\]

\[
\leq -\frac{c_1 \gamma (1 - \rho)}{\epsilon_2} \left|\bar{\eta}, \bar{\rho}, \zeta - \alpha\right|^2 < 0
\]

for all \(|d(t)|^2 < \rho \epsilon_2 c_2\) \((\bar{\eta}, \bar{\rho}, \zeta - \alpha)^2\) and \(\rho \in (0, 1)\), which shows P3 (Khalil, 2000). Note that the proof is valid for every trajectory that remains away from the Euler-angle singularity of the model \((\theta \neq \pm \frac{\pi}{2})\).

5. CASE STUDY - OPEN-FRAME UUV

We consider an open-frame underwater vehicle with a mass of 140kg in closed loop with the control law (39). The vehicle has four thrusters in an x-type configuration, which provides actuation in all the degrees of freedom of interest on the horizontal plane. The mass, damping and Coriolis matrices of the model are

\[
M = \begin{bmatrix} 290 & 0 & 0 & 0 \\ 0 & 404 & 50 & 0 \\ 0 & 50 & 132 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},
\]

\[
D = \begin{bmatrix} 95 + 268|v| & 0 & 0 \\ 0 & 613 + 164|u| & 0 \\ 0 & 0 & 105 \end{bmatrix},
\]

\[
C = \begin{bmatrix} 0 & 0 & -404 v - 50 r \\ 0 & 0 & 290 u \\ 404 v + 50 r - 290 u & 0 \end{bmatrix}.
\]

The controller parameters are \(S_{11} = -\text{diag}(3, 3, 1), S_{22} = -\text{diag}(60, 80, 80), S_{33} = -\text{diag}(5, 5, 1), K_I = \text{diag}(5, 5, 5, 5, 5),\) and \(V_d = \bar{\eta}^* K_d \bar{\eta}_d\), with \(K_d = \text{diag}(3, 3, 5, 1, 5, 1, 5, 1)\).

In the simulation, the vehicle have to follow a desired circular trajectory. Figures 1 to 3 show the displacements and velocities in the degrees of freedom of interest. As we can see, the actual position and velocities of the vehicle track their reference. The controller recover the trajectory tracking even under the action of a constant disturbance \(d = (50, 100, 10)\), which acts on the vehicle from \(t = 20s\) until the end of the simulation. The control forces are shown in Figure 4. Also, the bottom-right plot of Figure 4 shows the \(xy\)-plane trajectory of the underwater vehicle. As we can see from these figures, the designed controllers perform satisfactorily both trajectory tracking and disturbance rejection tasks.

6. CONCLUSIONS

We present a Hamiltonian model of marine vehicle dynamics in six degrees of freedom in both body-fixed and inertial momentum coordinates. The latter opens the possibility of considering PBC strategies developed for robotic
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