Rip cosmologies, Wormhole Solutions and Big Trip in the $f(T,T)$ theory of gravity

Maxime Z. Arouko

Département de Physique, Université d’Abomey-Calavi, BP 526 Calavi, Benin

Ines G. Salako

École de Génie Rural (EGR), 01 BP 55 Kétou, Benin and
Institut de Mathématiques et de Sciences Physiques (IMSP), 01 BP 613 Porto-Novo, Benin

A. D. Kanfon

Département de Physique, Université d’Abomey-Calavi, BP 526 Calavi, Benin and
Faculté des Sciences et Techniques de Natitingou, BP 72, Natitingou, Benin

M. J. S. Houndjo and Etienne Baffou

Faculté des Sciences et Techniques de Natitingou(FAST), BP 72, Natitingou, Benin and
Institut de Mathématiques et de Sciences Physiques (IMSP), 01 BP 613 Porto-Novo, Benin

Rip cosmological models have been investigated in the framework of $f(T,T)$ theory of gravity, where $T$ denotes the torsion and $T$ is the trace of the energy-momentum tensor. These phantom cosmological models revealed that at initial epoch a EoS parameter $\omega < -1$ and tends asymptotically at late phase to $-1$ ($\omega \rightarrow -1$). On the other hand, Wormhole Solutions and Big Trip have been subject of an investigation. The wormhole throat radius $R(t)$ and the conditions to be satisfied so that produces the Big Trip phenomenon have been discussed.

PACS numbers:

Contents

I. Introduction

II. A brief review of the $f(T,T)$ theory of gravity

III. Physical parameters
   A. Anisotropic case
   B. Isotropic case

IV. Rip cosmologies
   A. Little Rip
   B. Pseudo Rip
   C. Emergent Little Rip
   D. Bouncing with Little Rip

V. Wormhole Solutions and Big Trip
   A. I-Little Rip case
   B. II-Pseudo Rip case
   C. III-Emergent Little Rip
   D. IV-Bouncing with Little Rip case

VI. Conclusion

*Electronic address: maximearouko55@gmail.com
†Electronic address: inessalako@gmail.com
‡Electronic address: kanfon@yahoo.fr
§Electronic address: Sthoundjo@yahoo.fr
¶Electronic address: baffouh. etienne@yahoo.fr
I. INTRODUCTION

Some irrefutable results such as [1–6] show that the universe is experiencing an accelerated expansion. A possible candidate responsible for this current behavior of the universe is a mysterious energy with negative pressure which always stays unelucidated. One of the approaches to better understand this mysterious energy is to modify the gravity. For this purpose, several modified theories of gravitation such as $f(R)$ gravity, $f(T)$ gravity, $f(R, T)$ gravity have been proposed from time to time. Similarly, starting from Tele-parallel Theory equivalent of GR (TEGR) but not from GTR, one can consider the matter-coupled modified gravity theory. One of such theories, namely $f(T, T)$ gravity, has been first proposed by Harko et al [7]. In this modified theory the part of the gravitational Lagrangian is taken as an arbitrary function of torsion scalar $T$ and the trace of the energy-momentum tensor $T$. Comparing with the other theories based on the formalism of curvature or torsion, $f(T, T)$ seems to be a completely different modification for describing the gravity.

Wormholes (WHs) are tunnels or passages that connect two different regions of space-time (or even two distinct universes). They have been firstly proposed as a tool for teaching General Relativity (GR) [8]. Observational evidences for such a GR solution have been searched [9]-[15] and make optimistic the possibility of soon confirming the existence of WHs. Moreover, the existence of binary systems containing a WH and a (neutron) star has also been proposed [16, 17].

GR WHs are expected to be filled by exotic matter, that is, matter that does not respect the energy conditions [8, 18], and may present negative mass (density). An alternative to obtain WH solutions in accordance with the energy conditions is to search for Morris-Thorne metric [8] solutions in extended theories of gravity.

Extended theories of gravity are firstly motivated by the lack of theoretical explanation for some observational/experimental effects attained when considering GR as the underlying theory of gravity, such as dark energy [19], dark matter [20, 21], missing satellites [22, 23], massive pulsars [24, 25], super-Chandrasekhar white dwarfs [26, 27], hierarchy problem [28, 29], among others. Attempts to solve or evade these shortcomings have been developed in different gravitational theories, as one can check Refs.[28]-[36].

Indeed, WHs with non-exotic matter have already been obtained in a multimetric repulsive gravity model [37], in higher-order curvature gravity [38] and in a trace of the energy-momentum tensor squared gravity [39], for instance. The difficulty in constructing non-exotic matter WHs have led some authors to obtain WHs with arbitrarily small quantities of exotic matter [40]. WHs particularly satisfying the weak and null energy conditions (WEC and NEC) were obtained, respectively, in [41, 42] and [43, 44].

Our intention in the present article is to investigate some Rip cosmological models without any finite time future singularity in the framework of $f(T, T)$ theory of gravity, where $T$ denotes the torsion and $T$ is the trace of the energy-momentum tensor. It will be question in the first section II a brief review of the $f(T, T)$ theory of gravity. In section III we will discuss the physical parameters where anisotropic and isotropic cases will be addressed. We present in the section IV the four different phantom models and Wormhole Solutions and Big Trip will be discussed in the section V where the wormhole throat radius, the Big Trip time and the conditions to be satisfied so that the Big Trip phenomenon occurs will presented. Section VI is devoted to a summary and conclusion.

II. A BRIEF REVIEW OF THE $f(T, T)$ THEORY OF GRAVITY

In GR framework, the metric contains the gravitational potentials responsible for the curvature of space-time. Those potentials can also be represented by the torsion tensor, as in the teleparallel gravity framework (check, for instance, [51]).

The extension of teleparallel gravity is attained when the scalar torsion of teleparallel action is substituted by an arbitrary function of it, namely $f(T)$ [52, 53]. As it is done in teleparallel gravity, the extended versions are also described by the orthonormal tetrads, and their components are defined on the tangent space of each point of the manifold.

In these theories, the line element is written as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \eta_{ij} \theta^i \theta^j,$$  (1)
such that

\[ dx^\mu = e_i^\mu \theta^i; \quad \theta^i = e^i_\mu dx^\mu, \]

with \( \eta_{ij} = \text{diag}(1, -1, -1, -1) \) being the Minkowskian metric and \( \{e^i_\mu\} \) are the components of the tetrad, which satisfy the following identity:

\[ e_i^\mu e^i_\nu = \delta^\mu_\nu, \quad e^i_\mu e^\mu_j = \delta^i_j. \]

(3)

In GR, one assumes the Levi-Civita’s connection,

\[ \Gamma^\rho_{\mu \nu} = \frac{1}{2} \eta^{\sigma\rho} (\partial_\nu g_{\sigma\mu} + \partial_\mu g_{\sigma\nu} - \partial_\sigma g_{\mu\nu}), \]

(4)

which preserves the curvature whereas the torsion vanishes. In the teleparallel theory and its extended versions, one keeps the scalar torsion by using the Weitzenböck’s connection, defined as:

\[ \Gamma^\lambda_{\mu \nu} = e_i^\lambda \partial_\mu e^i_\nu = -e_i^\mu \partial_\nu e^i_\lambda. \]

(5)

From the above connection, one obtains the geometric objects of the formalism. The torsion is defined by

\[ T^\lambda_{\mu \nu} = \Gamma^\lambda_{\mu \nu} - \Gamma^\lambda_{\nu \mu}, \]

(6)

from which we define the contorsion as

\[ K^\lambda_{\mu \nu} = \Gamma^\lambda_{\mu \nu} - \Gamma^\lambda_{\nu \mu} = \frac{1}{2} (T^\lambda_{\mu \nu} + T^\lambda_{\nu \mu} - T^\mu_{\lambda \nu}). \]

(7)

Then, we can write

\[ K^\mu_{\nu \lambda} = -\frac{1}{2} (T^\mu_{\nu \lambda} - T^\nu_{\mu \lambda} + T^\lambda_{\mu \nu}). \]

(8)

The torsion and contorsion tensors are used to define another tensor, as

\[ S^\lambda_{\mu \nu} = \frac{1}{2} (K^\mu_{\nu \lambda} + \delta^\mu_\nu T^\alpha_{\lambda \alpha} - \delta^\lambda_\nu T^\alpha_{\mu \alpha}), \]

(9)

such that the torsion scalar can be constructed from torsion and contorsion as

\[ T = S^\mu_{\nu} T^\sigma_{\mu \nu}. \]

(10)

In the present article, we will consider an extension of the \( f(T) \) theories which also considers terms proportional to the trace of the energy-momentum tensor \( \mathcal{T} \) in the action, namely, the \( f(T, \mathcal{T}) \) theory \([7]\). The \( f(T, \mathcal{T}) \) theory action, for geometrized units, which shall be assumed throughout this work, can be written as

\[ S = \int d^4x \ e \left[ \frac{T + f(T, \mathcal{T})}{16\pi} + \mathcal{L}_m \right], \]

(11)

with \( \mathcal{L}_m \) being the matter lagrangian.

Varying the action with respect to the tetrad, one obtains the equations of motion \([7]\)

\[ \partial_\xi (ee^\sigma \mathcal{S}_{\rho}^{\sigma \xi}) - ee^\lambda_{\alpha} S^{\xi \sigma} T_{\xi \rho} \lambda (1 + f_T) + ee^\sigma_\alpha (\partial_\xi T) S_{\rho}^{\sigma \xi} f_{TT} + \frac{1}{4} e^e_{\alpha} T = -\frac{1}{4} e^e_{\alpha} f(T) - ee^\sigma_\alpha (\partial_\xi T) S_{\rho}^{\sigma \xi} f_{TT} + \frac{f_T}{2} (e T + ee^\sigma_\alpha p) + 4\pi e T^\sigma_{\alpha}, \]

(12)

with \( f_T = \partial f/\partial T, f_T = \partial f/\partial T, f_{TT} = \partial^2 f/\partial T^2, \mathcal{T}^\sigma_{\alpha} \) is the energy-momentum tensor of the matter field and \( p \) its pressure.
By using some transformations, we can establish the following relations:

\[ e^a e^{-1} \partial_i (e^\xi S_{\rho \sigma} \xi) - S^\rho \sigma T_{\rho \xi \nu} = -\nabla^\xi S_{\nu \xi} - S^{\xi \rho \sigma} K_{\rho \xi \nu}, \]  

\[ G_{\mu \nu} - \frac{1}{2} g_{\mu \nu} T = -\nabla^\rho S_{\nu \rho \mu} - S^{\sigma \rho \mu} K_{\rho \sigma \nu}, \]  

(13)

(14)

with \( G_{\mu \nu} \) being the Einstein tensor.

Hence, from the combination of Eqs. (13) and (14), the field equations (12) can be written as

\[ G_{\mu \nu} = \kappa_T \mathcal{T}_{\mu \nu}^{\text{eff}}, \]  

(15)

where

\[ \kappa_T = \frac{2}{(1 + f_T)}. \]  

(16)

and

\[ \mathcal{T}_{\mu \nu}^{\text{eff}} = -S^\rho \mu f_{\rho \nu} \partial_\rho T - S^{\sigma \rho \mu} f_{\sigma \rho \nu} \partial_\rho T - \frac{1}{4} g_{\mu \nu} f + \frac{1}{4} T g_{\mu \nu} f + \frac{f_T}{2} (\mathcal{T}_{\mu \nu} + g_{\mu \nu} p) + 4\pi \mathcal{T}_{\mu \nu}. \]  

(17)

This additive term represents the effective energy-momentum tensor and comes from a minimal coupling with matter. This quantity will vanish ds qu’on aurait supprimer la contribution de \( T \) from algebraic function \( f(T, \mathcal{T}) \). In other words, this type of coupling (matter-geometry) will generate an additional field that will be perceived as responsible for the acceleration of the universe. Consequently, we observe the non conservation of the energy impulse tensor. In order to develop viable cosmological models in according to observational data, an appropriate choice is required.

In the current work, we are focused to study some little rip cosmological models in the extended teleparallel gravity and for this purpose we user an algebraic function according to observational data \( f(T, \mathcal{T}) = \alpha T^n - 2\Lambda \), where \( \alpha, n = 0 \) and \( \Lambda \) are arbitrary constants. Thus, the equations (15) and (16) yields to

\[ G_{\mu \nu} = \kappa_T \mathcal{T}_{\mu \nu}^{\text{eff}}, \]  

(18)

where

\[ \mathcal{T}_{\mu \nu}^{\text{eff}} = g_{\mu \nu} \left( -\frac{\alpha T - 2\Lambda}{4} + \frac{\alpha p}{2} \right) + \frac{\kappa_T}{2} (\frac{\alpha}{2} + 4\pi). \]  

(19)

For \( \alpha = 0 \), the usual known in the literature ΛCDM model is recovered. We consider the anisotropic metric known as Locally Rotationally Symmetric Bianchi Type-I model

\[ ds^2 = dt^2 - A^2 dx_1^2 - B^2 (dy^2 + dz^2), \]  

(20)

where \( A = A(t) \) and \( B = B(t) \) are cosmic scale factors. Note that the flat FRW model is recovered by setting \( A(t) = B(t) = a(t) \).

In the present investigation, we assume the content of the universe is a cloud of one dimensional cosmic strings with string tension density \( \xi \) flowing along \( x \)-axis.

Thus, the energy-momentum tensor presents itself as following

\[ \mathcal{T}_{\mu \nu} = (p + \rho)u_\mu u_\nu - pg_{\mu \nu} - \xi x_\mu x_\nu, \]  

(21)

with

\[ u^\mu u_\mu = -x^\mu x_\mu = 1 \]  

(22)

and

\[ u^\mu x_\mu = 0. \]  

(23)
Thus, we can consider \( \rho \) as being the contribution of particle energy density \( \rho_p \) and string tension density \( \xi \). Note that the contribution of string tension density \( \xi \) is vanished when by setting \( A(t) = B(t) \).

The field equations reads

\[
6(k + 2) \dot{H} + 27H^2 = (k + 2)^2 \kappa_T \left\{ \left( -\frac{\alpha T - 2\Lambda}{4} + \frac{\alpha p}{2} \right) + (-p + \xi) \left( \frac{\alpha}{2} + 4\pi \right) \right\},
\]

(24)

\[
3(k^2 + 3k + 2) \dot{H} + 9(k^2 + k + 1)H^2 = (k + 2)^2 \kappa_T \left\{ \left( -\frac{\alpha T - 2\Lambda}{4} + \frac{\alpha p}{2} \right) - p \left( \frac{\alpha}{2} + 4\pi \right) \right\},
\]

(25)

\[
9(2k + 1)H^2 = (k + 2)^2 \kappa_T \left\{ \left( -\frac{\alpha T - 2\Lambda}{4} + \frac{\alpha p}{2} \right) + \rho \left( \frac{\alpha}{2} + 4\pi \right) \right\}.
\]

(26)

where

\[
T = -2 \left( 2 \frac{\dot{A} \dot{B}}{AB} + \frac{\dot{B}^2}{B^2} \right)
\]

(27)

\[
\mathcal{T} = \rho + \xi - 2p
\]

(28)

The parameter \( k \) provides information about the anisotropic behaviour of the model in the time. Note that by considering the isotropic model is recovered. In order to investigate on the isotropization phenomenon, we define the Hubble parameters in the direction of \([x, y, z]\)

\[
H_x = \frac{\dot{A}}{A}, \quad H_y = \frac{\dot{B}}{B}, \quad H_z = \frac{\dot{B}}{B}.
\]

(29)

The generalized mean Hubble parameter \( H \) is given in the form

\[
H = \frac{1}{3} \frac{\dot{V}}{V} = \frac{1}{3} \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{B}}{B} \right),
\]

(30)

where

\[
V = A B^2 = a^3,
\]

(31)

is the spatial volume of the universe and \( a \) is the scale factor of the universe. The rate of expansion will evaluated by anisotropy parameter given as

\[
\Delta = \frac{1}{3} \sum_{i=1}^{3} \left( \frac{H_i - H}{H} \right)^2,
\]

(32)

where \( i = (x, y, z) \).

Other parameters such that expansion scalar, deceleration parameter and Jerk parameter whose enter online in the processus of isotropization are defined respectively as

Expansion scalar: \( \theta = u^i_j = \left( \frac{\dot{A}}{A} + 2 \frac{\dot{B}}{B} \right) \),

Deceleration parameter: \( q = -1 + \frac{d}{dt} \left( \frac{1}{H} \right) \),

Jerk parameter: \( j = \frac{\ddot{a}}{aH^3} = \frac{\ddot{H}}{H^3} - (2 + 3q) \).

(33)

(34)

(35)

**III. PHYSICAL PARAMETERS**

Current data show us that the Universe is supposed to be homogeneous and isotropic at large scales this is not the case when a local analysis is done. In such a situation, the anisotropic effects can not be explained by making use of the usual flat FRW model. In the paper, the parameter \( k \) express this anisotropic universe and we can note that a flat FRW model is recovered when \( k = 1 \).
A. Anisotropic case

In order to explain the phenomenon of isotropization we can establish respectively some Physical parameters such as pressure, energy density and string tension density from the field equations \((23)-(26)\) dependent of Hubble parameter and anisotropic parameter \(k\)

\[
P = \frac{9H^2 \left( -16 \left( 1 + k + k^2 \right) \pi + (-3 + (-1 + k)k)\alpha \right) + (2 + k)^2 \left( 16\pi + \alpha \right) \Lambda - 3(2 + k)(16(1 + k)\pi + \alpha - k\alpha)H}{2(2 + k)^2(4\pi - \alpha)(16\pi + \alpha)}
\]

\[
\rho = \frac{9H^2(16(1 + 2k)\pi + 3(1 + (-1 + k)k)\alpha) - (2 + k)^2(16\pi + \alpha)\Lambda + 3(2 + k)(5 + 3k)\alpha H}{2(2 + k)^2(4\pi - \alpha)(16\pi + \alpha)}
\]

\[
\xi = -\frac{6(-1 + k)}{(2 + k)(16\pi + \alpha)} \left( 3H^2 + \dot{H} \right)
\]

Consequently, the state parameter (EoS), \(\omega = \frac{P}{\rho}\) yields to

\[
\omega = -1 + \frac{12(4\pi - \alpha) \left( -3H^2(-1 + k)k - (1 + k)(2 + k)\dot{H} \right)}{9H^2(16(1 + 2k)\pi + 3(1 + (-1 + k)k)\alpha) - (2 + k)^2(16\pi + \alpha)\Lambda + 3(2 + k)(5 + 3k)\alpha H}
\]

By considering the previous expressions of the pressure and energy density we can evaluat the following quantity as

\[
\rho + p = -\frac{6 \left( 3H^2(-1 + k)k + (1 + k)(2 + k)\dot{H} \right)}{(2 + k)^2(16\pi + \alpha)}
\]

we can remark for \(\alpha \to 0\), the usual \(\Lambda\)CDM model is recovered, consequently Consequently the pressure and EoS parameter becomes \(p = -\rho\) and \(\omega = -1\). A model that describes an accelerating expanded phantom-like meets the following requirements as \(\dot{H} > 0, t > 0\) and hence the weak energy condition \(\rho + p \geq 0; \rho \geq 0\) is not satisfied. In view of equation \((38)\), the satisfaction of these conditions requires an appropriate choice of parameters \(k\) and \(\alpha\) in order to preserve the anisotropic nature of the universe.

By considering the General Relativity limit case \(\alpha \to 0\), the EoS parameter reads

\[
\omega = -1 + \frac{-9H^2(-1 + k)k - 3(1 + k)(2 + k)\dot{H}}{9H^2(1 + 2k) - (2 + k)^2\Lambda}.
\]

In the absence of a cosmological constant it yields

\[
\omega = -1 + \frac{3H^2(-1 + k)k + (1 + k)(2 + k)\dot{H}}{3H^2(1 + 2k)}.
\]

B. Isotropic case

We develop in this section, the isotropic model for \(k = 1\). Therefore, the EoS parameter takes the form

\[
\omega = -1 + \frac{8(-4\pi + \alpha)\dot{H}}{(16\pi + \alpha)(3H^2 - \Lambda) + 8\alpha\dot{H}}
\]

In particular case i.e \(\alpha \to 0\) and \(\Lambda_0 \to 0\), the EoS parameter yields to the FRW model

\[
\omega = -1 - \frac{2}{3} \frac{\dot{H}}{H^2}.
\]

Consequently, the weak energy condition in this case becomes

\[
\rho + p = -\frac{4\dot{H}}{16\pi + \alpha}.
\]

This model responds perfectly to a model that describes an accelerating expanded phantom-like universe \(\dot{H} > 0\) and \(w < -1\).
IV. RIP COSMOLOGIES

A. Little Rip

Little Rip model has been subject a renewed interest and interesting results have been obtained \[65, 66\]. We define respectively the Hubble parameter and the scale factor as follows

\[ H = H_0 e^{\lambda t}, \quad H_0 > 0, \quad \lambda > 0 \]  

(44)

\[ a = a_0 \exp \left[ \frac{H_0}{\lambda} (e^{\lambda t} - e^{\lambda t_0}) \right]. \]  

(45)

where \( a_0 \) is evaluated scale factor at the current epoch \( t_0 \).

By considering (31) and (27), we can deduce

\[ A = a_0^2 e^{2H_0 t} (e^{\lambda t} - e^{\lambda t_0}) \]  

(46)

\[ B = a_0^2 \frac{H_0}{2} e^{\frac{1}{2}H_0 t} (e^{\lambda t} - e^{\lambda t_0}) \]  

(47)

\[ T = \frac{9}{2} e^{2\lambda H_0^2}. \]  

(48)

Thus, we can see that a inertial force is produced taking into account that the Hubble rate parameter increases exponentially with time. A particle with mass \( m \) at a given point will be subject to an inertial force is given by Frampton et al. (2012) of the form

\[ F_i = ml (H^2 + \dot{H}) = ml (H_0^2 e^{2\lambda t} + H_0 \lambda e^{\lambda t}) \]  

(49)

We can remark the principal characteristic of the Little Rip model is the fact that for the time \( t \to \infty \), the inertial force \( F_i \to \infty \). From (41), we can deduce respectively for the Little Rip model the deceleration parameter and the jerk parameter

\[ q = -1 - \frac{\lambda}{H_0} e^{-\lambda t}, \]  

(50)

\[ j = 1 + \frac{3\lambda}{H_0} e^{-\lambda t} + \left( \frac{\lambda}{H_0} \right)^2 e^{-2\lambda t}. \]  

(51)

Its remark that this two parameters previous defined tend towards respectively to \(-1\) and \(1\). The deceleration parameter \( q \) evaluated at current epoch yields to

\[ q_0 = -1 - \frac{\lambda}{H_0} e^{-\lambda t_0}, \]  

(52)

which shows that \( q_0 < -1 \). By considering the \( \Lambda \)CDM model, we remark that the jerk parameter evaluated at current epoch yields to \( j_0 = 1 \). However, by considering the Little Rip model, the jerk parameter evaluated at current epoch yields to

\[ j_0 = 1 + \frac{3\lambda}{H_0} e^{-\lambda t_0} + \left( \frac{\lambda}{H_0} \right)^2 e^{-2\lambda t_0}. \]  

(53)

From \[53\], we can see \( j_0^{\text{RP}} \) evaluated for Little Rip model greater than that \( j_0^{\text{CDM}} \) evaluated for \( \Lambda \)CDM model.

By considering \( \dot{H} = \lambda H > 0 \), we determine EoS parameter \( \omega_{LR} \) for the LR model by replacing (23) into (33)

\[ \omega_{LR} = -1 - \frac{12 e^{4\lambda} (4\pi - \alpha) H_0 ((1 + k)(2 + k)\lambda + 3 e^{4\lambda} (-1 + k) k H_0)}{-(2 + k)^2 (16\pi + \alpha) \Lambda + 3 e^{4\lambda} (2 + k) (5 + 3k) \alpha \lambda + 3 e^{4\lambda} (16 (1 + 2k) \pi + 3 (1 + (-1 + k) \alpha) H_0)} \]  

(54)

From \[54\], its can see that the function \( \omega_{LR} \) depends on the anisotropic parameter \( k \), the coupling constant \( \alpha \), the parameters of the scale factors \( \lambda \) and \( H_0 \). We evaluat this function respectively at an initial and epoch \((t \to 0)\) and at a late phase \((t \to \infty)\) as

\[ W_{LR}(t \to 0) = -1 - \frac{12 (4\pi - \alpha) H_0 ((1 + k)(2 + k)\lambda + 3 (-1 + k) k H_0)}{-(2 + k)^2 (16\pi + \alpha) \Lambda + 3 H_0 ((2 + k) (5 + 3k) \alpha \lambda + (48 (1 + 2k) \pi + 9 (1 + (-1 + k) \alpha) H_0))} \]  

(55)
\[ \omega_{LR}(t \rightarrow \infty) = -1 - \frac{4(4\pi - \alpha)(-1 + k)k}{16\pi(1 + 2k) + 3\alpha^2(-1 + k)k} \]  

(56)

The analysis EoS parameter for Little Rip model i.e (55) and (56) at an initial epoch reveal a phantom phase with \( \omega_{LR} < -1 \) However, The analysis at late phase shows that \( \omega_{LR} \rightarrow -1 \).

Now, we can concern ourselves to isotropic case by looking at the behavior of \( \omega_{LR} \) for the little Rip model. Thus, the EoS parameter becomes

\[ \omega_{LR}^i = -1 - \frac{8e^{t\lambda}(4\pi - \alpha)\lambda H_0}{-(16\pi + \alpha)\Lambda + e^{t\lambda}H_0 (8\alpha \lambda + 3e^{t\lambda}(16\pi + \alpha)H_0)} , \]

(57)

which tends to \(-1\) for \( t \rightarrow \infty \). By considering the General Relativity case i.e \( \alpha \rightarrow 0 \), the EoS parameter becomes

\[ \omega_{LR}^{(GR)} = -1 + \frac{2e^{t\lambda}\lambda H_0}{\Lambda - 3e^{2t\lambda}H_0^2} , \]

(58)

and for \( \Lambda \simeq 0 \), we obtain

\[ \omega_{LR}^{(GR)} = -1 - \frac{2e^{-t\lambda}\lambda}{3H_0} . \]

(59)

B. Pseudo Rip

Pseudo rip known through its phantom behaviour with no finite time singularity is characterized by a hubble parameter defined by

\[ H = H_0 - H_1 e^{-\lambda t}, \]

(60)

where \( H_0, H_1 \) and \( \lambda \) are positive constants with \( H_0 > H_1 \).

We can remark the principal characteristic of the Pseudo Rip model is the fact that for the time \( t \rightarrow \infty \), a de Sitter universe is recovered i.e \( H \rightarrow H_0 \). Furthermore, this model presents phantom behaviour i.e

\[ \dot{H} = \lambda H_1 e^{-\lambda t} = \lambda(H_0 - H) > 0 \]

(61)

From (60), we can determine the scale factor as following

\[ a = a_0 e^{\frac{H_0(t - t_0)}{\lambda} + \frac{H_1}{\lambda}(e^{-\lambda t} - e^{-\lambda t_0})} . \]

(62)

By make using (61), we obtain

\[ A = a_0^2 e^{\frac{H_1}{\lambda}(e^{\lambda t} - e^{-\lambda t_0}) + 2H_0(t - t_0)} \]

(63)

\[ B = a_0^2 e^{\frac{H_1}{\lambda}(e^{\lambda t} - e^{-\lambda t_0}) + \frac{1}{2}H_0(t - t_0)} \]

(64)

\[ T = -\frac{9}{2}e^{-2t\lambda}(e^{-\lambda t}H_0 + H_1)^2 \]

(65)

By make using (61), the inertial force for Pseudo Rip model is equal to

\[ F_i = ml \left( \lambda H_1 e^{-\lambda t} + (H_0 - H_1 e^{-\lambda t})^2 \right) \]

(66)

In this case the inertial force is limited, i.e for the time \( t \rightarrow \infty \), \( F_i \rightarrow ml H_0^2 \). From (61), we can deduce respectively for the Pseudo Rip model the deceleration parameter and the jerk parameter

\[ q = -1 - \frac{\lambda H_1 e^{-\lambda t}}{(H_0 - H_1 e^{-\lambda t})^2} , \]

(67)

\[ j = 1 - \frac{\lambda H_1 e^{-\lambda t} [\lambda + 3(H_0 - H_1 e^{-\lambda t})]}{(H_0 - H_1 e^{-\lambda t})^3}. \]

(68)
Its remark easily at late epoch that this two parameters previous defined tend towards respectively to $-1$ and $1$ but these parameters evaluated at initial epoch yields respectively

$$q(t \to 0) = -1 - \frac{\lambda H_1}{(H_0 - H_1)^2}$$

$$j(t \to 0) = 1 - \frac{\lambda H_1 [\lambda + 3(H_0 - H_1)]}{(H_0 - H_1)^3}$$

(69)

On the other hand, we observe the singularities at

$$t = \ln \left( \frac{H_1}{H_0} \right)^{\frac{1}{2}}.$$ 

(70)

By make using (61), we determine EoS parameter $\omega_{PR}$ for the PR model by replacing ((60)) into ( (37)) The EoS parameter for the PR model can be obtained as

$$W_{PR}^0(t) = -1 + \frac{12(4\pi - \alpha)(1 + k)kH_0^2 - 6(-1 + k)kH_0H_1 + H_1((1 + k)(2 + k)\lambda + 3(-1 + k)kH_1))}{144(1 + 2k)^2H_0^2 + 27[1 + (-1 + k)k\lambda H_0^2 - (2 + k)^2(16\pi - \alpha)]}.$$ 

(72)

and at a late phase ($t \to \infty$)

$$\omega_{PR}^0(t \to \infty) = -1 - \frac{36(4\pi - \alpha)(-1 + k)kH_0^2}{144(1 + 2k)^2H_0^2 + 27[1 + (-1 + k)k\lambda H_0^2 - (2 + k)^2(16\pi - \alpha)].}$$ 

(73)

The analysis EoS parameter for Pseudo Rip model i.e (72) and (73) at an initial epoch reveal a phantom phase with $\omega_{PR} < -1$ However The analysis at late phase shows that $\omega_{PR} \to -1$. we can concern ourselves to isotropic case by looking at the behavior of $\omega_{PR}$ for the Pseudo Rip model. Thus, the EoS parameter becomes

$$W_{PR}^j(t) = -1 - \frac{72e^{-\lambda k}(4\pi - \alpha)\lambda H_1}{-9(16\pi + \alpha)\Lambda + 72e^{-\lambda k}\lambda H_1 + 27(16\pi + \alpha)(H_0 - e^{-\lambda k}H_1)^2}.$$ 

(74)

which asymptotically tends to $-1$ for $t \to \infty$. By considering the General Relativity case i.e $\alpha \to 0$, the EoS parameter reads

$$\omega_{PR}^{(GR)} = -1 - \frac{2e^{-\lambda k}H_1}{\Lambda + 3(H_0 - e^{-\lambda k}H_1)^2}.$$ 

(75)

and $\Lambda \simeq 0$, we have

$$\omega_{PR}^{(GR)} = -1 - \frac{2e^{-\lambda k}H_1}{3(-e^{-\lambda k}H_0 + H_1)^2}.$$ 

(76)

Easily, we remark that this model presents a $\omega$-singularity for $t = t_\omega = \ln \left( \frac{H_0}{H_0} \right)^{\frac{1}{2}}$ when we consider General Relativity case.

C. Emergent Little Rip

Nous presentons dans cette sous section un autre model propos par Mukherjee et al. [67] presentant un comportement phantomique. Ce modle est caracteris par un facteur d'echelle presentant une solution mergante dfini par
The EoS parameter for the ELR model can be obtained as

\[ a(t) = a_i \left( \nu + e^{\mu t} \right)^\gamma, \]  

with \( a_i, \mu, \nu \) and \( \gamma \) are positive constants. Thus, we can deduce the Hubble parameter as

\[ H(t) = \frac{\mu \gamma e^{\mu t}}{\nu + e^{\mu t}}. \]  

Note that for \( t \to \infty \), the scale factor \( a \to \infty \) and the Hubble parameter \( H \to \mu \gamma \). Thus, the de Sitter universe is recovered. This model also presents phantom behaviour i.e.

\[ \dot{H} = \frac{\mu \gamma e^{\mu t}}{\nu + e^{\mu t}} \left[ \mu - \frac{1}{\gamma} \frac{\mu \gamma e^{\mu t}}{\nu + e^{\mu t}} \right] = H \left( \mu - \frac{H}{\gamma} \right) > 0 \]  

for values well-prcised of positive constants \( \mu \) and \( \gamma \). By make using (31), we obtain

\[ A = a_0^2 \left( \nu + e^{\mu t} \right)^{2\gamma} \]  

\[ B = a_0^2 \left( \nu + e^{\mu t} \right)^2 \]  

\[ T = -\frac{9e^{2\mu t} \mu^2}{2 \left( e^{\mu t} + \nu \right)^2} \]  

By make using (61) and (78), we can determine the inertial force for the emergent little rip model as following

\[ F_i = ml \left( \frac{\mu \gamma e^{\mu t}}{\nu + e^{\mu t}} \left[ \mu - \frac{1}{\gamma} \frac{\mu \gamma e^{\mu t}}{\nu + e^{\mu t}} \right] + \frac{\mu^2 \gamma^2 e^{2\mu t}}{(\nu + e^{\mu t})^2} \right) \]  

The deceleration parameter and the jerk parameter for this emergent little rip (ELR) model are obtained as

\[ q = -1 - \frac{\nu}{\gamma}, \]  

\[ j = \left( 1 - \frac{3}{\gamma} + \frac{2}{\gamma^2} \right) + \frac{\mu}{H} + \frac{\mu \left( \mu - 2/\gamma \right)}{H^2}. \]  

Its remark easily at late epoch that the deceleration parameter and the jerk parameter for this emergent little rip model tend towards respectively to \( q \to -1 \) and \( j \to 1 - \frac{3}{\gamma} + \frac{2}{\gamma^2} + 2 + \frac{(\mu - 2/\gamma)}{\mu} \) but these parameters evaluated at initial epoch yields respectively

\[ q(t \to 0) = -1 - \frac{\lambda H_1}{(H_0 - H_1)^2}, \]  

\[ j(t \to 0) = 1 + \frac{\nu - 2}{\gamma} + \frac{1}{\gamma^2} \left[ 2 + \frac{(\mu + 1)^2 (\mu - 2/\gamma)}{\mu} \right]. \]  

By make using (61), we determine EoS parameter \( \omega_{ELR} \) for the Emergent Little Rip model by replacing (61) into (77). The EoS parameter for the ELR model can be obtained as

\[ W_{ELR}^a(t) = -1 + \frac{12e^{4\mu}(4\pi - \alpha)\mu^2 \left( 3e^{4\mu}(-1 + k)k\gamma + (1 + k)(2 + k)\nu \right)}{e^{2\mu} \left( (2 + k)^2(16\pi + \alpha)\Lambda - 9(16(1 + 2k)\pi + 3(1 + (-1 + k)k)\alpha)\gamma^2 \mu^2 \right) + e^{4\mu}(2 + k) \left( (2 + k)(16\pi + \alpha)\Lambda - 3(5 + 3k) \right)\mu^2}, \]  

From (87) we can evaluate the EoS parameter at a late epoch as following

\[ \omega_{ELR}(t \to \infty) = -1 + \frac{12\gamma\mu^2(4\pi - \alpha)[3(-1 + k)k\gamma]}{(2 + k)^2(16\pi + \alpha)\Lambda - 9[16(1 + 2k)\pi + 3(1 + (-1 + k)k)\alpha]\gamma^2 \mu^2} \]  

Now, from (87), we can evaluate the EoS parameter at a initial epoch as following
\( W_{ELR}^{iso}(t \to 0) = -1 - \frac{12(4\pi - \alpha)\gamma \mu^2 (3(-1 + k)k\gamma + (1 + k)(2 + k)v)}{(1 + v)^2 \left[ -(2 + k)(16\pi + \alpha)\Lambda + \frac{9(16(1+2k)\pi + (1(-1 + k)k)\alpha)\gamma^2 \mu^2 + (3(2+k)(5+3\alpha)\gamma^2 \mu^2 v}{(1 + v)^2} \right]} \) (89)

By make using of previous expressions, we obtain the EoS parameter for isotropic universe

\( W_{ELR}^{iso}(t) = -1 + \frac{8e^{\mu}(4\pi - \alpha)\gamma \mu^2 v}{e^{2\mu}(16\pi + \alpha)(\Lambda - 3\gamma^2 \mu^2) + 2e^{\mu}(16\pi \Lambda + \alpha(\Lambda - 4\gamma^2 \mu^2))v + (16\pi + \alpha)\Lambda \mu^2} \) which asymptotically approaches to \(-1\) as \(t \to \infty\).

By considering the limit of General Relativistic for \(\alpha \to 0\), we obtain

\( \omega_{ELR}^{iso(\mathcal{GR})} = -1 + \frac{2e^{-\mu} \gamma \mu^2 v}{e^{2\mu}(\Lambda - 3\gamma^2 \mu^2) + 2e^{\mu} \Lambda v + \Lambda \mu^2} \) (91)

In absence of cosmological constant \(\Lambda \simeq 0\), we obtain

\( \omega_{ELR}^{iso(\mathcal{GR})} = -1 - \frac{2e^{-\mu} v}{3\gamma}. \) (92)

D. Bouncing with Little Rip

Bouncing with Little Rip has been subject an investigation by Myrzakulov and Sebastini. The scale factor for this model is given by

\( a(t) = a_0 e^{(t-t_0)2n}, \) (93)

where \(a_0 > 0\) is the scale factor evaluated at the today time \(t_0\). This model is governed by a constant parameter \(n \neq 0\) which shows the bouncing behaviour.

Thus, we can deduce the Hubble parameter for this model as following

\( H(t) = 2n(t - t_0)^{2n-1}. \) (94)

It is clear that this model exhibits a behavior similar to little rip at late epoch. Taking the first derivative of we obtain

\( \dot{H} = 2n(2n - 1)(t - t_0)^{2n-2}. \) (95)

which yields \(\dot{H} > 0\) for \(n > \frac{1}{2}\). For a exponent \(n\) assumes positive integral numbers, This model present a phantom behaviour, condition is verified when the exponent \(n\) assumes positive integral numbers. Likewise, it note that this model present a bouncing at \(t = t_0\) when the bouncing scale factor becomes \(a_0\). It is easily to remark for positive integral values of \(n\) when \(t \to \infty\), we obtain \(a \to \infty\) and \(H \to \infty\). By make using \(\mathcal{H}\), we obtain

\( A = a_0^2 e^{2(t-t_0)2n} \) (96)

\( B = a_0^2 e^{\frac{1}{3}(t-t_0)^2n} \) (97)

\( T = -18n^2(-t_0 + t)^{-2+4n} \) (98)

By make using \(\mathcal{H}\), the inertial force for the Bouncing with Little Rip model is equal to

\( F_i = ml \left( 4n^2(t - t_0)^{2(2n-1)} + 2n(2n - 1)(t - t_0)^{2n-2} \right) \) (99)

From \(\mathcal{H}\), we can deduce respectively for the Bouncing with Little Rip model the deceleration parameter and the jerk parameter

\( q = -1 - \frac{2n - 1}{2n(t - t_0)^{2n}}, \) (100)

\( j = 1 + \frac{3(2n - 1)}{2n(t - t_0)^{2n}} + \frac{(n - 1)(2n - 1)}{2n^2(t - t_0)^{2n}}. \) (101)
The deceleration parameter is a negative quantity for $n > \frac{1}{2}$ and evolves to an asymptotic value of $q = -1$. The jerk parameter evolves to $j = 1$ at late times.

For the BLR model we can calculate the EoS parameter as

$$W_{BLR}^{iso}(t) = -1 + \frac{24n (1 + k)(2 + k)(1 + 2n) - 6(1 + k)kn(t - t_0)^{2n}(t - t_0)^{-2 + 2n}(4\pi - \alpha)}{6(2 + k)(5 + 3k)n(-1 + 2n)(t - t_0)^{-2 + 2n} \alpha + 36n^2(t - t_0)^{-2 + 4n}(16(1 + 2k)\pi + 3(1 + (-1 + k)\alpha) - (2 + k)^2(16\pi + \alpha)\Lambda}$$

which asymptotically reduces to

$$\omega_{BLR}(t \to \infty) = -1 + \frac{24n(4\pi - \alpha)[-6(1 + k)kn]}{36n^2[16(1 + 2k)\pi + 3(1 + (-1 + k)\alpha) - (2 + k)^2(16\pi + \alpha)\Lambda]}$$

$$W_{BLR}^{0}(t \to 0) = -1 + \frac{24n (1 + k)(2 + k)(1 + 2n) - 6(1 + k)kn(-t_0)^{2n}(t - t_0)^{2n}(-1 + n)(4\pi - \alpha)}{6(2 + k)(5 + 3k)n(-1 + 2n)(t - t_0)^{2n}(-1 + n) \alpha + 36n^2(-t_0)^{-2 + 4n}(16(1 + 2k)\pi + 3(1 + (-1 + k)\alpha) - (2 + k)^2(16\pi + \alpha)\Lambda}$$

The EoS parameter for this BLR model in an isotropic universe can be expressed as,

$$W_{BLR}(t) = -1 - \frac{16n(-1 + 2n)(t - t_0)^{2n}(4\pi - \alpha)}{16n(-1 + 2n)(t - t_0)^{2n} \alpha + 12n^2(t - t_0)^{4n}(16\pi + \alpha) - (t - t_0)^{2n}(16\pi + \alpha)\Lambda}$$

which asymptotically approaches to $-1$ as $t \to \infty$. In the limit of GR with $\alpha \to 0$

$$\omega_{BLR}^{iso(GR)} = -1 - \frac{4n(-1 + 2n)(t - t_0)^{2n}}{12n^2(t - t_0)^{4n} - (t - t_0)^2\Lambda}.$$ (106)

and $\Lambda \approx 0$, we have

$$\omega_{BLR}^{iso(GR)} = -1 + \frac{(1 - 2n)(t - t_0)^{-2n}}{3n}.$$ (107)

V. WORMHOLE SOLUTIONS AND BIG TRIP

The phenomenon called Big Trip is often observed when the size of the wormhole throat takes volume. This is due to the fact a strong phantom energy buildup on the wormhole which can lead to an absorption of the whole universe before the appearance of the rip phenomenon. Now, we will be interested in the determination of the wormhole throat radius and his behaviour under the phantom energy accumulation. This wormhole throat radius is determined via the following differential equation for the isotropic case with vanishing cosmological constant [68, 70]

$$\dot{R} = -C_0 R^2 (\rho + p).$$ (108)

when $C_0$ is a positive dimensionless constant.

A. I-Little Rip case

We determine the wormhole throat radius $R(t)$ by combining (103) and (104) into (108)

$$\frac{1}{R_{LR}(t)} = C_1 + \frac{4C_0 e^{\lambda t} H_0}{16\pi + \alpha},$$

where $C_1$ is an integration constant which is determined at big trip time as

$$C_1 = \frac{2C_0}{16\pi + \alpha} H_0 e^{\lambda t_{big}}.$$ (109)
Thus, we can easily the wormhole throat radius

\[ R_{LR}(t) = \frac{16\pi + \alpha}{2C_0H_0} \left[ e^{\lambda t} - e^{-\lambda t} \right]^{-1}. \]  

We can also determine the Big Trip assuming that \( R(t_0) = R_0 \) through

\[ t_{\text{Big}} = \ln \left[ e^{\lambda t_0} + \frac{16\pi + \alpha}{2C_0H_0R_0} \right]^{\frac{1}{\lambda}}, \]  

which leads asymptotically to in General relativity \( \alpha \to 0 \)

\[ t_{\text{Big}}^{GR} = \ln \left[ e^{\lambda t_0} + \frac{8\pi}{C_0H_0R_0} \right]^{\frac{1}{\lambda}}. \]

The difference between (111) and (112) reveal the importance of the coefficient \( \alpha \) but especially the contribution of the modified gravity theory in the process of Big Trip phenomenon.

For this Little Rip case, it is necessary to remark that the Big Trip phenomenon must realize when the following condition is verified \( t = t_0 \)

\[ R_0 > \frac{16\pi + \alpha}{2C_0H_0} e^{\lambda t_0}. \]  

B. II-Pseudo Rip case

By analogy to the same previous method, we determine the wormhole throat radius for the Pseudo Rip case by combining (43) and (60) into (108)

\[ R_{PR}(t) = \frac{16\pi + \alpha}{2C_0H_1} \left[ e^{-\lambda t} - e^{-\lambda t} \right]^{-1}. \]

Thus, we can deduce the Big Trip time for the Pseudo Rip case as

\[ t_{\text{Big}}^{PR} = \ln \left[ e^{-\lambda t_0} - \frac{16\pi + \alpha}{2C_0H_1R_0} \right]^{\frac{1}{\lambda}}. \]

As previously, the following condition will must verified at \( t = t_0 \) for the realization of the Big Trip phenomenon

\[ R_0 > \frac{16\pi + \alpha}{2C_0H_1} e^{\lambda t_0}. \]  

In the limit of General Relativity, the Big Trip time becomes

\[ t_{\text{Big}}^{GR} = \ln \left[ e^{-\lambda t_0} - \frac{8\pi}{C_0H_1R_0} \right]^{\frac{1}{\lambda}}. \]

It note the effect of the coupling constant \( \alpha \) through the equations (115) and (117).

C. III-Emergent Little Rip

We determine the wormhole throat radius for the Emergent Little Rip case by combining (43) and (78) into (108) as following

\[ R_{ELR}(t) = \frac{16\pi + \alpha}{2C_0H_0} \left[ \frac{1}{\nu + e^{\mu t}} - \frac{1}{\nu + e^{\mu t}} \right]^{-1}. \]
Thus, we can determine the Big Trip time for the Emergent Little Rip case

\[
t_{\text{Big}} = \ln \left[ \left( \frac{1}{\nu + e^{\mu t_0}} - \frac{16\pi + \alpha}{2C_0 \mu \gamma R_0} \right)^{-1} - \nu \right]^\frac{1}{\mu},
\]

which we deduce at the limit of General Relativity, the Big Trip time

\[
t_{\text{Big}}^{\text{GR}} = \ln \left[ \left( \frac{1}{\nu + e^{\mu t_0}} - \frac{8\pi}{C_0 \mu \gamma R_0} \right)^{-1} - \nu \right]^\frac{1}{\mu},
\]

and the condition to be satisfied so that produces the Big Trip phenomenon

\[
R_0 > \frac{(16\pi + \alpha)(\nu + e^{\mu t_0})}{2C_0 \mu \gamma},
\]

and

\[
R_0 < \frac{\nu(16\pi + \alpha)}{2C_0 \mu \gamma [\nu - (\nu + e^{\mu t_0})]}.
\]

\[\text{D. IV-Bouncing with Little Rip case}\]

By combining (43) and (94) into (108), We determine respectively the wormhole throat radius and the Big Trip time for the Bouncing with Little Rip case as following

\[
R_{BLR}(t) = \frac{16\pi + \alpha}{4C_0 n} \left[ (t_B - t_0)^{2n-1} - (t - t_0)^{2n-1} \right]^{-1},
\]

and

\[
t_B = t_0 + \left[ (t_1 - t_0)^{2n-1} + \frac{16\pi + \alpha}{4C_0 n R_1} \right]^{\frac{1}{2n-1}}.
\]

We deduce at the limit of General Relativity, the Big Trip time

\[
t_B = t_0 + \left[ (t_1 - t_0)^{2n-1} + \frac{16\pi}{4C_0 n R_1} \right]^{\frac{1}{2n-1}}.
\]

where \( R(t_1) = R_1 \)

\[\text{VI. CONCLUSION}\]

In this paper, some phantom models without any Big Rip singularity at finite time have been subject of an investigation in the context of \( f(T, \mathcal{T}) \) theory of gravity, where \( T \) denotes the torsion and \( \mathcal{T} \) is the trace of the energy-momentum tensor. These phantom cosmological models revealed that at initial epoch a EoS parameter \( \omega < -1 \) and tends asymptotically at late phase to \( -1 \) (\( \omega \to -1 \)). These models are seems to that of Little Rip models where it remark that for the time \( t \to \infty \), a de Sitter universe is recovered i.e \( H \to H_0 \). Four different phantom models have been investigated where we focused on anisotropic and isotropic universe. We found that the coupling constant change dynamically the behaviour of EoS parameter.

In addition to his studies, Some wormhole solutions have been obtained for these Four different phantom models. We have also determined for these phantom models, the wormhole throat radius, the Big Trip time and we have discussed we discussed about the conditions to be satisfied so that the Big Trip phenomenon occurs. It should be
noted, as previously that the coupling constant of $f(T, T)$ theory of gravity affect the Big Trip time when passing the boundary of the General Relativity.

[1] S. Perlmutter et al. [SNCP Collaboration], Astrophys. J. 517, 565 (1999); A. G. Riess et al.[SNST Collaboration], Astron. J. 116, 1009 (1998).

[2] D. N. Spergel et al. [WMAP Collaboration], Astrophys. J. Suppl. 148, 175 (2003); ibid. 170, 377 (2007); E. Komatsu et al. [WMAP Collaboration], ibid. 180, 330 (2009).

[3] E. Komatsu et al. [WMAP Collaboration], Astrophys. J. Suppl. 192, 18 (2011).

[4] M. Tegmark et al., Phys. Rev. D 69, 103501 (2004); U. Seljak et al. [SDSS Collaboration], Phys. Rev. D 71, 103515 (2005).

[5] D. J. Eisenstein et al., Astrophys. J. 633, 560 (2005).

[6] B. Jain and A. Taylor, Phys. Rev. Lett. 91, 141302 (2003).

[7] Harko T et al. 2014 $f(T, T)$ gravity and cosmology J. Cosm. Astrop. Phys. 12 021.

[8] Morris M S and Thorne K S 1988 Wormholes in spacetime and their use for interstellar travel: A tool for teaching general relativity Amer. J. Phys. 56 395

[9] Rahaman F et al. 2014 Possible existence of wormholes in the galactic halo region Eur. Phys. J. C 74 2750

[10] Tsukamoto N et al. 2012 Can we distinguish between black holes and wormholes by their Einstein-ring systems? Class. Quant. Grav. 86 104062

[11] Kuhfittig P K F 2014 Gravitational lensing of wormholes in the galactic halo region Eur. Phys. J. C 74 2818

[12] Lukmanova R et al. 2016 Gravitational Microlensing by Ellis Wormhole: Second Order Effects Int. J. Theor. Phys. 55 4723

[13] Li Z and Bambi C 2014 Distinguishing black holes and wormholes with orbiting hot spots Phys. Rev. D 90 024071

[14] Afe B 2010 Gravitational Microlensing by the Ellis Wormhole Astrophys. J. 725 787

[15] Toki Y et al. 2011 Astrometric Image Centroid Displacements due to Gravitational Microlensing by the Ellis Wormhole Astrophys. J. 740 121

[16] Dzhunushaliev V et al. 2013 Mixed neutron-star-plus-wormhole systems: Linear stability analysis Phys. Rev. D 87 104036

[17] Dzhunushaliev V et al. 2014 Hiding a neutron star inside a wormhole Phys. Rev. D 89 084018

[18] Visser M 1996 Lorentzian Wormholes - From Einstein to Hawking (New York: Springer)

[19] Abdalla M C B et al. LETTER TO THE EDITOR: Consistent modified gravity: dark energy, acceleration and the absence of cosmic doomsday Class. Quant. Grav. 23 L235

[20] Akerib D S et al. Results from a Search for Dark Matter in the Complete LUX Exposure Phys. Rev. Lett. 118 021303

[21] Mambrini Y et al. The LHC diphoton resonance and dark matter Phys. Lett. B 755 426

[22] Klypin A et al. 1999 Where Are the Missing Galactic Satellites? Astrophys. J. 522 82

[23] Kravtsov A V et al. 2004 The Tumultuous Lives of Galactic Dwarfs and the Missing Satellites Problem Astrophys. J. 609 482

[24] Antoniadis J et al. 2013 A Massive Pulsar in a Compact Relativistic Binary Science 340 448

[25] Demorest P B et al. 2010 A two-solar-mass neutron star measured using Shapiro delay Nature 467 1081.

[26] Howell D A et al. 2006 The type Ia supernova SNLS-03D3bb from a super-Chandrasekhar-mass white dwarf star Nature 443 308.

[27] Silverman J M et al. 2011 Fourteenth months of observations of the possible super-Chandrasekhar mass Type Ia Supernova 2009dc Month. Not. Roy. Astron. Soc. 410 585.

[28] Randall L and Sundrum R 1999 Large Mass Hierarchy from a Small Extra Dimension Phys. Rev. Lett. 83 3370

[29] Arkani-Hamed N et al. 1998 The hierarchy problem and new dimensions at a millimeter Phys. Rept. 322 199

[30] Nojiri S and Odintsov S D 2011 Unified cosmic history in modified gravity: From F(R) theory to Lorentz non-invariant models Phys. Rep. 505 59

[31] Sakstein J et al. 2016 Testing gravity using galaxy clusters: new constraints on beyond Horndeski theories J. Cosm. Astrop. Phys. 07 019

[32] Zhang P 2007 Behavior of $f(R)$ gravity in the solar system, galaxies, and clusters Phys. Rev. D 76 024007

[33] Moraes P H R S and Sahoo P K 2017 The simplest non-minimal matter-geometry coupling in the $f(R,T)$ cosmology Eur. Phys. J. C 77 480

[34] Moraes P H R S, Arbañil J D V and Malheiro M 2016 Stellar equilibrium configurations of compact stars in $f(R,T)$ theory of gravity J. Cosm. Astrop. Phys. 06 005

[35] Moraes P H R S, Correa R A C and Ribeiro G 2018 Evading the non-continuity equation in the $f(R, T)$ cosmology Eur. Phys. J. C 78 192

[36] Carvalho G A, Lobato R V, Moraes P H R S, Arbañil J D V, Otoniel E, Marinho R M and Malheiro M 2017 Stellar equilibrium configurations of white dwarfs in the $f(R, T)$ gravity Eur. Phys. J. C 77 871

[37] Hohmann M 2014 Traversable wormholes without exotic matter in multiminimum repulsive gravity Phys. Rev. D 89 087503

[38] Harko T et al. 2013 Modified-gravity wormholes without exotic matter Phys. Rev. D 87 067504

[39] Moraes P H S and Sahoo P K 2018 Nonexotic matter wormholes in a trace of the energy-momentum tensor squared gravity Phys. Rev. D 97 024007

[40] Fewster C J and Roman T A 2005 On wormholes with arbitrarily small quantities of exotic matter Phys. Rev. D 72 044023
[41] Mehdizadeh M R et al. 2015 Einstein-Gauss-Bonnet traversable wormholes satisfying the weak energy condition Phys. Rev. D 91 084004
[42] Zangeneh M K et al. 2015 Traversable wormholes satisfying the weak energy condition in third-order Lovelock gravity Phys. Rev. D 92 124049
[43] Garcia N M and Lobo F S N 2011 Nonminimal curvature-matter coupled wormholes with matter satisfying the null energy condition Class. Quant. Grav. 28 085018
[44] Zangeneh M K et al. 2014 Higher-dimensional evolving wormholes satisfying the null energy condition Phys. Rev. D 90 024072
[45] Pace M and Said J L 2017 Quark stars in f(T, T)-gravity Eur. Phys. J. C 77 62
[46] Farrugia G and Said J L 2016 Growth factor in f (T, T ) gravity Phys. Rev. D 94 124004
[47] Sánchez-Gómez D et al. 2016 Constraining f (T, T ) gravity models using type Ia supernovae Phys. Rev. D 94 024034
[48] Junior E L B et al. 2016 Reconstruction, thermodynamics and stability of the ΛCDM model in f(T,T) gravity Class. Quant. Grav. 28 085018
[49] Rezaei T M and Amani A 2017 Stability and interacting f(T, T ) gravity with modified Chaplygin gas Can. J. Phys. 95 1068
[50] Nassur S B et al. 2015 From the early to the late time universe within f(T,T) gravity Astrophys. Space Sci. 360 60
[51] Aldrovandi R and Pereira J G 2013 Teleparallel Gravity (Springer)
[52] Cai Y-F et al. 2016 f(T) teleparallel gravity and cosmology Rep. Prog. Phys. 79 106901
[53] Li B et al. 2011 f(T) gravity and local Lorentz invariance Phys. Rev. D 83 064035
[54] Bahamonde S et al. 2015 Modified teleparallel theories of gravity Phys. Rev. D 92 104042
[55] Bamba K et al. 2012 Reconstruction of f(T) gravity: Rip cosmology, finite-time future singularities, and thermodynamics Phys. Rev. D 85 104036
[56] Cai Y-F et al. 2011 Matter bounce cosmology with the f(T) gravity Class. Quant. Grav. 28 215011
[57] Lobo F S N 2006 Chaplygin traversable wormholes Phys. Rev. D 73 064028
[58] Lemos J P and Lobo F S N 2004 Plane symmetric traversable wormholes in an anti de Sitter background Phys. Rev. D 69 104007
[59] Wang D and Meng X-h 2016 Traversable geometric dark energy wormholes constrained by astrophysical observations Eur. Phys. J. C 76 484
[60] Jamil M et al. 2010 Wormholes supported by polytropic phantom energy Eur. Phys. J. C 67 513
[61] Mehdizadeh M R and Ziaie A H 2017 Einstein-Cartan wormhole solutions Phys. Rev. D 95 064049
[62] Sharif M and Jawad A 2014 Phantom-like generalized cosmic chaplygin gas and traversable wormhole solutions Eur. Phys. J. Plus 129 15
[63] N. M. García, T. Harko, F. S. N. Lobo, J. P. Mimoso, J. Phys. Conf. Ser. 314, 012060 (2011).
[64] Harko T et al. 2011 f(R,T) gravity Phys. Rev. D 84 024020.
[65] P. H. Frampton, K. J. Ludwick and R. J. Scherrer, Phys. Rev. D, 84, 063003 (2011). arxiv:1106.4996
[66] P. H. Frampton, K. J. Ludwick, S. Nojiri, S. D. Odintsov and R. J. Scherrer, Phys. Lett. B, 708, 204 (2012). arxiv:1108.0067
[67] S. Mukherjee, B.C. Paul, N. K. Dadhich, S. D. Maharaj and A. Beesham, Class.Quant. Gravit., 23, 6927 (2006).
[68] R. Myrzakulov and L. Sebastiani, Astrophys. Space Sci., 352, 281 (2014).
[69] A. V. Astashenok, S. Nojiri, S. D. Odintsov, A. V. Yurov, Phys. Lett. B, 709, 396 (2012).
[70] E. Babichev, V. Dokuchaev and Y. Eroshenko, Phys. Rev. Lett., 93, 021102 (2004).