Gauge dependence of vacuum expectation values of gauge invariant operators from soft breaking of BRST symmetry. Example of Gribov-Zwanziger action

P. M. Lavrov\textsuperscript{a,*}, A. A. Reshetnyak\textsuperscript{a,b†}

\textsuperscript{a}Tomsk State Pedagogical University, Tomsk 634041, Russia
\textsuperscript{b}Institute of Strength Physics and Materials Science, 634021 Tomsk, Russia

Abstract

We review the study of influence of the so-called soft BRST symmetry breaking within the Batalin-Vilkovisky (BV) formalism introduced in our papers [JHEP 1110 (2011) 043, arXiv:1108.4820 [hep-th], MPLA 27 (2012) 1250067, arXiv:1201.4720 [hep-th]] on gauge dependence of the effective action and vacuum expectation values of gauge invariant operators. We derive the Ward identities for generating functionals of Green’s functions for a given theory with soft BRST symmetry breaking term being added to the quantum action and investigate theirs gauge dependence. It is strongly argued that gauge theories with a soft breaking of BRST symmetry are inconsistent within the BV formalism because of the gauge-dependence of S-matrix. The application to the Gribov-Zwanziger action (enlarging SU(\(N\)) Yang-Mills gauge theory by means of not gauge-invariant horizon functional) for the one-parameter family of \(R_\xi\) gauges with use of the new form of the Hermitian augmented Faddeev-Popov operator (being by Faddeev-Popov operator for transverse components of Yang–Mills fields) is considered.

1 Introduction

The BRST symmetry concept, equivalently expressing a gauge invariance via a special one-parameter global supersymmetry [1], not only appears as a defining tool within quantum gauge theory, because of all known fundamental interactions are described in terms of gauge theories [2], but provides the success of perturbative calculations at high energy and of numerical studies in lattice gauge theory [3] as well as strong evidence that the interactions of quarks and gluons are correctly described by the non-Abelian gauge theory known as QCD.

Not long ago, the Gribov-Zwanziger (GZ) theory [7, 9, 10] has been intensively studied in a series of the papers [4], [5], [6]. The GZ theory is characterized by breakdown of BRST symmetry due to the Gribov copies being nothing else that gauge-equivalent configurations that satisfy the Landau gauge condition. The analytical proof [7] of the presence of Gribov copies in physical spectrum was confirmed by the lattice simulations in some kinds of QCD models, like SU(2) gluodynamics, (see e.g. [8] and references therein), being not unexpected result due to finding the field configurations within the same Landau gauge condition. The resolution of this problem can be realized by an addition to the quantum action constructed by Faddeev-Popov receipt of the special horizon functional [9, 10], which is not however BRST invariant one.

\textsuperscript{*}e-mail: lavrov@tspu.edu.ru
\textsuperscript{†}e-mail: reshet@ispms.tsc.ru
Note, that practically all the research [3], [4], [5], [9], [10], [8] of the Gribov horizon in the YM theories have been performed in the Landau gauge only, with except for the some kind of covariant gauges in [6] and Coulomb gauge [11] in the space-time with dimensions \(d = 2, 3, 4\). In spite of that fact, there is a significant arbitrariness in the choice of the admissible gauges, which, in part, related to the choice of the reference frame, see e.g. [20]. It is well known the Green’s functions depend on the choice of the gauge however this dependence is highly structured so that it should be canceled for the physical quantities like \(S\)-matrix. Modern considerations of the gauge independence of the \(S\)-matrix for the YM theories essentially use the BRST symmetry [12]. Therefore, any violation of the BRST symmetry principle, as well as any ideas of the elaboration the gauge theories with breakdown of this symmetry leads to serious problems with consistency of the final field-theoretical model.

In the work, we, first, present the results of gauge dependence investigations for the general gauge theories, being more general (like the supergravity, superstring models with open algebras and higher-spin fields as reducible gauge theories, see e.g. [17], [18], [19]) than the YM type theories with the so-called soft breaking of BRST symmetry, which are based on the research made in [15], [16] in the BV method [13], [14]. Second, we suggest the horizon functional for the family of \(R_\xi\) gauge, to be constructed for non-Hermitian Faddeev-Popov operator having for vanishing gauge parameter \(\xi\) the same form as in case of Landau gauge [9], [10].

The paper is organized as follows. In Section 2, we suggest the definition of the soft breaking of BRST symmetry in the BV formalism. In Section 3 we study impossibility to introduce BRST-like transformations and derive the Ward identities for the generating functionals of Green’s functions. Investigation of the dependence of the effective action on gauges is presented in Section 4. A generalization of the GZ action for the one-parameter family of \(R_\xi\) gauges is considered in Section 5. We summarize the results and throw light on perspectives in Section 6.

We use the condensed notations of DeWitt [20] and Refs. [15], [16]. Derivatives with respect to sources and antifields are taken from the left, while those with respect to fields are taken from the right. Left (right) derivatives with respect to fields (antifields) are labeled by a subscript \(l\) (\(r\)). The Grassmann parity of any homogeneous quantity \(A\) is denoted as \(\varepsilon(A)\).

## 2 Soft breaking of BRST symmetry in the BV formalism

Let us consider a theory of gauge fields \(A^i\), \(i = 1, 2, \ldots, n\), \((\varepsilon(A^i) = \varepsilon_i)\), with an initial action \(S_0 = S_0(A)\) to be invariant under the gauge transformations \(\delta A^i = R_\alpha^i(A)\xi^\alpha\) with arbitrary functions of the space-time coordinates \(\xi^\alpha\) \((\varepsilon(\xi^\alpha) = \varepsilon_\alpha)\), that means the presence of the Noether’s identities

\[
S_{0,i}(A)R_\alpha^i(A) = 0 \quad \text{for} \quad \alpha = 1, 2, \ldots, m, \quad 0 < m < n.
\]

among the classical equations of motion, \(S_{0,i} = 0\). Here, the functions \(R_\alpha^i(A)\) \((\varepsilon(R_\alpha^i) = \varepsilon_i + \varepsilon_\alpha)\) are the generators of the gauge transformations and we have used the DeWitt’s notation \(S_{0,i} \equiv \delta S_0 / \delta A^i\). The structure of configuration space \(\{\Phi^A\}\) in the BV formalism depends on the type of given classical gauge theory (for details, on reducible or (and) gauge theories with open algebras see [13], [14]). Explicit contents of \(\{\Phi^A\}\) is not critical for our aims. We need in the fact of existence of the configuration space \(\mathcal{M}\) parameterized by the fields \(\Phi \equiv \{\Phi^A\} = \{A^i, C^\alpha, \overline{C}^\alpha, B^\alpha, \ldots\}\) with \(\varepsilon(\Phi^A) = \varepsilon_A\), where the dots indicate the full set of additional to the classical \(A^i\), ghost \(C^\alpha\), antighost \(\overline{C}^\alpha\), Nakanishi-Lautrup \(B^\alpha\) fields in the BV method. The BV method implies an introduction to each field \(\Phi^A\) of the total configuration space the respective antifield \(\Phi^*_A\) with opposite Grassmann parities to that of the corresponding field \(\Phi^A\), \(\Phi^*_A \equiv \{\Phi^*_A\} = \{A^*_i, C^*_\alpha, \overline{C}^*_\alpha, B^*_\alpha, \ldots\}\), with \(\varepsilon(\Phi^*_A) = \varepsilon_A + 1\).

On the field-antifield space of \((\Phi^A, \Phi^*_A)\), (in particular, being by odd cotangent bundle \(\Pi T^*\mathcal{M}\)) one defines the main object of the BV quantization, i.e. the bosonic functional \(S =\)
\[ S(\Phi, \Phi^*) \] obeying the master equation
\[ \frac{1}{\hbar}(\tilde{S}, \tilde{S}) = i\hbar \Delta \tilde{S} \] (2)
with the boundary condition
\[ \tilde{S}|_{\mathcal{M}, \hbar=0} = \mathcal{S}_0(A) . \] (3)
to be compatible with the Eq. (2).

In (2) we used the notation of odd second order nilpotent Laplacian operator \( \Delta \),
\[ \Delta \equiv (-1)^{\varepsilon_A} \frac{\delta}{\delta \Phi^A} \frac{\delta}{\delta \Phi_A}, \quad \varepsilon(\Delta) = 1, \] (4)
and the antibracket, \((H, G)\), which may be reproduced by \( \Delta \) acting on the product of two functionals \( H \) and \( G \) on field-antifield space:
\[ \Delta (H \cdot G) = (\Delta H) \cdot G + (-1)^{\varepsilon(H)} H \cdot \Delta G + (H, G)\varepsilon(H). \] (5)

The action \( \tilde{S} \) should be modified by means of corresponding fermionic gauge fixing functional
\[ \Psi = \Psi(\Phi) \] in such a way that we able to construct the non-degenerate (on space \( \mathcal{M} \)) action \( S_{ext} \) by the rule
\[ S_{ext}(\Phi, \Phi^*) = \tilde{S}(\Phi, \Phi^* + \delta \Psi \delta \Phi) . \] (6)
The quantum action \( S_{ext} \) obeys the same master equation (2) as the functional \( \tilde{S} \),
\[ \frac{1}{\hbar}(S_{ext}, S_{ext}) = i\hbar \Delta S_{ext} \] (7)
and should be used to construct the generating functional of Green’s functions in the BV formalism [13], [14].

Following to Refs. [9], [10] and our research [15], [16], we deform the action \( S_{ext} \) by adding
a functional \( M = M(\Phi, \Phi^*) \), defining now the full action \( S \) as
\[ S = S_{ext} + M \quad \varepsilon(M) = 0. \] (8)
We speak on a soft breaking of BRST symmetry in the BV formalism if the condition holds
\[ \frac{1}{\hbar}(M, M) = -i\hbar \Delta M. \] (9)
Note, that in classical limit, \( \hbar \to 0 \), we assume that \( M = M_0 + O(\hbar) \), Eq. (9) is reduced to
\[ (M_0, M_0) = 0. \] (10)
used, in fact in Ref. [15]. The reason to use the notation of ”a soft breaking of BRST symmetry“ in BV formalism may be explained as follows. The master equation (7) in the BV formalism may be equivalently presented in the form
\[ \Delta \exp \left\{ \frac{i}{\hbar} S_{ext} \right\} = 0. \] (11)
Using the action \( S_{ext} \) as a solution to this equation in order to construct Green’s functions for general gauge theories one can derive the BRST symmetry transformations [13], [14]. Modifying the action \( S_{ext} \) by a special functional \( M \) (it allows us to speak of ”soft“) which satisfies the equation
\[ \Delta \exp \left\{ -\frac{i}{\hbar} M \right\} = 0, \] (12)
and is not a BRST invariant, i.e. $(S_{ext}, M) - i\hbar M \neq 0$, we get the action $S$ from Eq. (8) not obeying the equation like Eq.(11)
\[ \Delta \exp \left\{ \frac{i}{\hbar} S \right\} \neq 0. \] (13)

The BRST symmetry will be broken if we shall construct Green’s functions in the BV formalism using this action (see beginning of the Section 3 for details). From (7) and (9) it follows that the basic equation of our approach to the soft breaking of BRST symmetry reads
\[ \frac{1}{2} (S, S) - i\hbar \Delta S = (S, M). \] (14)

In classical limit, for $S = S_0 + \sum_{n \geq 1} \hbar^n S_n$, it follows from (14) the equation,
\[ \frac{1}{2} (S_0, S_0) = (S_0, M_0), \] (15)

coinciding in classical limit, $\hbar \to 0$, with the basic equation to the soft breaking of BRST symmetry considered in Ref.[15] when a regularization like dimensional one for the local functional $S$ implies that $\Delta S \sim \delta(0) = 0$.

It should be noted that the condition (9) will be automatically satisfied in case when the soft breaking of BRST symmetry originates from a modification of the integration measure in the path integral. In this case $M$ will be a functional of the field variables $\Phi^A$ only, i.e. $M = M(\Phi)$.

As we have already mentioned in Ref. [15], this is exactly the situation for Yang-Mills theory in the Landau gauge, when one takes into account the Gribov horizon [9], [10], [5]. We consider the more general situation of $M = M(\Phi, \Phi^*)$ not restricting ourselves to this special case.

Doing so, we suppose, first, that Gribov horizon may exist for general gauge theories. Second, Gribov region of the fields $A^i$ can be singled out by an addition of the functional $M$ to full action of a given gauge system, but it violates BRST symmetry.

It is interesting to show that the right-hand side of the basic equation (14) can be presented in the form
\[ (S, M) = \hat{s}M - i\hbar \Delta M, \text{ for } \hat{s} = (S_{ext}, \bullet) - i\hbar \Delta \] (16)
being the quantum Slavnov-Taylor operator defined as $\hbar$-deformation of its classical analog.

Because of the master equation for $S_{ext}$ (7) this operator is nilpotent,
\[ \hat{s}^2 = 0. \] (17)

However, as compared to the consideration in [15], the presence of the additional term to $\hat{s}$ in the right-hand side of the relation (16) leads to the inequality
\[ \hat{s} \left\{ \frac{1}{2} (S, S) - i\hbar \Delta S \right\} \neq 0. \] (18)

which being written for the action $S_{ext}$ should be the identity for general gauge theories without a (soft) breaking of BRST symmetry.

3 Modifications of BRST transformations and Ward identities

Here we consider some quantum consequences of the equations (7), (9) and (14). To do it we introduce the generating functional of Green’s functions,
\[ Z(J, \Phi^*) = \int D\Phi \exp \left\{ \frac{i}{\hbar} (S(\Phi, \Phi^*) + J_A \Phi^A) \right\}, \] (19)
with $J_A (\varepsilon(J_A) = \varepsilon_A)$ being the usual sources for the fields $\Phi^A$ and $S(\Phi, \Phi^*)$ being by a solution of the basic quantum equation (14) and having the form (8).
The integrand of the vacuum functional \( Z(0, \Phi^*) = Z(\Phi^*) \) looks as

\[
\mathcal{N} = \mathcal{N}(\Phi, \Phi^*) = D\Phi \exp \left\{ \frac{i}{\hbar} S(\Phi, \Phi^*) \right\}.
\]  

(20)

Remind that in the BV formalism the BRST symmetry appears as invariance of the integrand \( \mathcal{N} \) of vacuum functional under the change of variables, \( \Phi^A \rightarrow \Phi'^A = \Phi^A + \delta_B \Phi^A \) determined with the help of the non-BRST broken action \( S_{ext}(\Phi, \Phi^*) \),

\[
\delta_B \Phi^A = \frac{\delta S_{ext}}{\delta \Phi^*_A} \theta, \quad \delta_B \Phi^*_A = 0,
\]

(21)

where \( \theta \) is a nilpotent constant odd parameter. Carrying out the change of variables (21) in (20) we obtain

\[
\mathcal{N}' = \mathcal{N} \left( 1 - \frac{i}{\hbar} \theta \frac{\delta M}{\delta \Phi^*_A} \frac{\delta S_{ext}}{\delta \Phi^*_A} \right).
\]

(22)

Non-invariance of the integrand means violation of the standard BRST symmetry. Of course, one may to restore the invariance of \( \mathcal{N} \) by modifying of the definition of BRST transformations in case of the theory under consideration. A general way for its modification may be achieved with help of following one-parameter functional \( S_\kappa \)

\[
S_\kappa = S_{ext} + \kappa M, \quad \kappa \in \mathbb{R},
\]

(23)

to define the BRST trasformations

\[
\delta_{B_\kappa} \Phi^A = \frac{\delta S_\kappa}{\delta \Phi^*_A} \theta, \quad \delta_{B_\kappa} \Phi^*_A = 0.
\]

(24)

Note, we have the standard BRST transformations (21) for \( \kappa = 0 \). Performing the change of variables (26) in the integrand \( \mathcal{N} \) (20) one has

\[
\mathcal{N}' = \mathcal{N} \left( 1 - \frac{i}{\hbar} \theta \left[ \kappa \frac{\delta M \delta S_{ext}}{\delta \Phi^*_A \delta \Phi^*_A} \right] - 2i \theta \kappa \frac{\delta M \delta S_{ext}}{\delta \Phi^*_A \delta \Phi^*_A} + (1 - \kappa) \frac{\delta M \delta S_{ext}}{\delta \Phi^*_A \delta \Phi^*_A} \right).
\]

(25)

So that we may state, that the non-invariance of the integrand exists for any choice of the parameter \( \kappa \). For instance, for \( \kappa = 1 \) we have the modified BRST-like transformations

\[
\delta_{B_1} \Phi^A = \frac{\delta S}{\delta \Phi^*_A} \theta, \quad \delta_{B_1} \Phi^*_A = 0,
\]

(26)

which is not stay invariant the integrand \( \mathcal{N} \).

Let us turn to the properties of the generating functional \( Z(J, \Phi^*) \). First, from the averaging of the Eq. (11) over the total configuration space \( M \) with measure \( \exp \left\{ \frac{i}{\hbar} \left( S + J_A \Phi^A \right) \right\} \)

\[
0 = \int D\Phi \Delta \exp \left\{ \frac{i}{\hbar} S_{ext} \right\} \exp \left\{ \frac{i}{\hbar} \left( S + J_A \Phi^A \right) \right\}
\]

it follows after integrating by parts in the functional integral the following identity for the generating functional \( Z \),

\[
\left( J_A + M_A \left( \frac{h}{\delta J_A}, \Phi^* \right) \right) \left( \frac{h}{\delta \Phi^*_A} \right) - M^{*}(\frac{h}{\delta J_A}, \Phi^*) \right) Z(J, \Phi^*) = 0.
\]

(27)

In the Eq. (27) the notations

\[
M_A \left( \frac{h}{\delta J_A}, \Phi^* \right) \equiv \frac{\delta M(\Phi, \Phi^*)}{\delta \Phi_A} \bigg|_{\Phi \rightarrow \frac{h}{\delta J_A}} \quad \text{and} \quad M^{*} \left( \frac{h}{\delta J_A}, \Phi^* \right) \equiv \frac{\delta M(\Phi, \Phi^*)}{\delta \Phi^*_A} \bigg|_{\Phi \rightarrow \frac{h}{\delta J_A}}
\]
have been used. In case of $M = 0$, the identity (27) is reduced to the usual Ward identity for the generating functional of Green’s functions in the BV formalism. Therefore, we refer to (27) as the Ward identity for $Z$ in a gauge theory with softly broken BRST symmetry. Note, that for the regularization scheme likes dimensional one, we will have $\Delta M = 0$ and, therefore the equation (9) is reduced to $(M, M) = 0$, which leads to the vanishing of the combination, $M_A \left( \frac{\delta}{\delta \Phi^*}, \Phi^* \right) M^{A*} \left( \frac{\delta}{\delta \Phi^*}, \Phi^* \right)$ in the Ward identity (27) as it was firstly derived in [15].

Second, after introducing the generating functional of connected Green’s functions,

$$W(J, \Phi^*) = -i \hbar \ln Z(J, \Phi^*),$$

the Ward identity (27) can be rewritten for $W$ as

$$\left( J_A + M_A \left( \frac{\delta W}{\delta J_A}, \Phi^* \right) \right) \left( \frac{\delta W}{\delta \Phi^*_A} - M^{A*} \left( \frac{\delta W}{\delta J_A}, \Phi^* \right) \right) = 0. \quad (29)$$

In turn, the generating functional of the vertex functions (effective action) is obtained by Legendre transforming of $W$,

$$\Gamma(\Phi, \Phi^*) = W(J, \Phi^*) - J_A \Phi^A, \quad \text{where} \quad \Phi^A = \frac{\delta W}{\delta J_A}, \quad \frac{\delta \Gamma}{\delta \Phi^A} = -J_A. \quad (30)$$

Taking into account the equality, $\frac{\delta \Gamma}{\delta \Phi^A} = \frac{\delta W}{\delta \Phi^A}$, which follows from the Legendre transformation, we can present the identity (29) in terms of $\Gamma$ as

$$\frac{1}{2}(\Gamma, \Gamma) = \delta \Gamma M^{A*} + \delta M^{A*} - \delta \Gamma \Phi^A \Phi^A. \quad (31)$$

In the Eq. (31) we have used the notations

$$\widehat{M}_A = \frac{\delta M(\Phi, \Phi^*)}{\delta \Phi^A} \big|_{\Phi \rightarrow \widehat{\Phi}} \quad \text{and} \quad \widehat{M}^{A*} = \frac{\delta M(\Phi, \Phi^*)}{\delta \Phi^*_A} \big|_{\Phi \rightarrow \widehat{\Phi}}, \quad (32)$$

where the sign $\widehat{\Phi}^A$ means the field $\Phi^A$ enlarged by the special derivatives $i \hbar (\Gamma'')^{-1}_{AB} \frac{\delta}{\delta \Phi^B}$,

$$\widehat{\Phi}^A = \Phi^A + i \hbar (\Gamma'')^{-1}_{AB} \frac{\delta}{\delta \Phi^B} \quad (33)$$

and the matrix $(\Gamma'')^{-1}$ is inverse to the matrix $\Gamma''$ with elements

$$(\Gamma'')_{AB} = \frac{\delta}{\delta \Phi^A} \left( \frac{\delta \Gamma}{\delta \Phi^B} \right) : (\Gamma'')^{-1}_{AC} (\Gamma'')_{CB} = \delta^A_B. \quad (34)$$

We see again, in case of vanishing $M$ the identity (31) coincides with the Ward identity for the effective action in the BV formalism. Emphasize that the identity (31) is compatible with the classical equation (14), since $\hbar \rightarrow 0$ yields $\Gamma = S_0, \widehat{M} = M_0$, and (31) is reduced to (15).

In similar manner we can derive the Ward identity which follows from (12). To this end, we average the equation (12) over the configuration space of the fields $\Phi^A$ with measure exp $\left\{ \frac{i}{\hbar} (S_{ext} + J_A \Phi^A) \right\}$,

$$0 = \int D\Phi \Delta \exp \left\{ -\frac{i}{\hbar} M \right\} \exp \left\{ \frac{i}{\hbar} (S_{ext} + J_A \Phi^A) \right\}. \quad (35)$$

and derive after usual manipulations with functional integral the identity in terms of mean fields $\widehat{\Phi}^A$ (30)

$$\widehat{M}_A \widehat{M}^{A*} = -i \hbar \widehat{M}^{A*} \quad \text{where} \quad \widehat{M}_A^{A*} = \frac{\delta^2 M}{\delta \Phi^*_A \delta \Phi^A} \big|_{\Phi \rightarrow \widehat{\Phi}}. \quad (35)$$

The identity (35) is reduced to the identity, $\widehat{M}_A \widehat{M}^{A*} = 0$, derived in [15], when the regularization scheme likes dimensional one is applied.
4 Gauge dependence of the effective action

In this Section we present our research [15], [16] devoted to the gauge dependence of the generating functionals \( Z, W \) and \( \Gamma \) for general gauge theories with a soft breaking of BRST symmetry as it was defined in the previous section. The derivation of this dependence is based on the fact that any variation of the gauge-fixing functional, \( \Psi(\Phi) \to \Psi(\Phi) + \delta \Psi(\Phi) \), leads to a variation both the action \( S_{ext} \text{(6)} \), the functional \( Z \text{[21]} \) and the functional \( M \). The variation of \( S_{ext} \) can be presented in the form

\[
\delta S_{ext} = \frac{\delta \delta \Psi \delta S_{ext}}{\delta \Phi_A} \quad \text{or as} \quad \delta S_{ext} = -(S_{ext}, \delta \Psi) = -\delta \delta \Psi , \tag{36}
\]

whereas we denote as \( \delta M(\Phi, \Phi^* ) \) the variation of \( M \) corresponding to the variation \( \delta \Psi \). From (19), (36) and the variation of \( M \) we obtain the gauge variation of \( Z \),

\[
\delta Z(J, \Phi^*) = \frac{i}{\hbar} \int D\Phi \left( \frac{\delta \delta \Psi \delta S_{ext}}{\delta \Phi_A} + \delta M \right) \exp \left\{ \frac{i}{\hbar} (S(\Phi, \Phi^*) + J_A \Phi^A) \right\} . \tag{37}
\]

By means of the equality

\[
0 = \int D\Phi \left[ \frac{\delta \delta \Psi \delta S_{ext}}{\delta \Phi_A} \right] = \int D\Phi \left[ \frac{\delta \delta \Psi \delta S_{ext}}{\delta \Phi_A} - \frac{i}{\hbar} (J_A + \delta S/\delta \Phi_A) \frac{\delta S_{ext}}{\delta \Phi_A} \delta \Psi \right] \exp \left\{ \frac{i}{\hbar} (S(\Phi, \Phi^*) + J_A \Phi^A) \right\} ,
\]

where the equation (7) was used, we can rewrite the variation (37) as

\[
\delta Z(J, \Phi^*) = \frac{i}{\hbar} \left[ (J_A + M_A(\frac{\hbar}{i} \frac{\delta}{\delta J}, \Phi^*)) \left( \frac{\delta}{\delta \Phi_A} \delta S_{ext} \right) \delta \Psi(\frac{\hbar}{i} \frac{\delta}{\delta J}) \right.
\]

\[
\left. + \delta M \left( \frac{\hbar}{i} \frac{\delta}{\delta J}, \Phi^* \right) \right] Z(J, \Phi^*) . \tag{38}
\]

Now, it is easy to get the corresponding variation of the generating functional of connected Green’s functions, \( \delta W(J, \Phi^*) = \frac{\hbar}{i} Z^{-1} \delta Z \), in the form

\[
\delta W(J, \Phi^*) = \left( J_A + M_A(\frac{\hbar}{i} \frac{\delta}{\delta J}, \Phi^*) \right) \frac{\delta}{\delta \Phi_A} \delta S_{ext} \delta \Psi(\frac{\hbar}{i} \frac{\delta}{\delta J}) + i \delta M \left( \frac{\hbar}{i} \frac{\delta}{\delta J}, \Phi^* \right) , \tag{39}
\]

with use of the Ward identity (29).

Now, we are able to reach our final purpose concerning the derivation of the gauge variation of the effective action. Doing so, we repeat our derivation from Ref. [16].

First, we note that \( \delta \Gamma = \delta W \). Second, we observe that the change of the variables \( (J_A, \Phi_A^*) \to (\Phi^A, \Phi_A^*) \) from the Legendre transformation (30) implies that

\[
\left. \frac{\delta}{\delta \Phi_A^*} \right|_J = \left. \frac{\delta}{\delta \Phi_A^*} \right|_\Phi + \left. \frac{\delta \Phi_i}{\delta \Phi_A^*} \frac{\delta \Phi_j}{\delta \Phi_A^*} \right|_{\Phi^*} . \tag{40}
\]

Third, differentiating the Ward identities for \( Z \text{(27)} \) with respect to the sources \( J_B \), we get

\[
\frac{\hbar}{i} \frac{\delta Z}{\delta \Phi_B^*} + \frac{\hbar}{i} \left( J_A + M_A(\frac{\hbar}{i} \frac{\delta}{\delta J}, \Phi^*) \right) \frac{\delta^2 Z}{\delta J_B \delta \Phi_A^*} ( -1)^{\varepsilon \alpha \beta B} - M_B^A(\frac{\hbar}{i} \frac{\delta}{\delta J}, \Phi^*) Z -
\]

\[
- ( -1 )^{\varepsilon \mu} J_A M_A^A(\frac{\hbar}{i} \frac{\delta}{\delta J}, \Phi^*) \frac{\delta Z}{\delta J_B} - M_A^A(\frac{\hbar}{i} \frac{\delta}{\delta J}, \Phi^*) M_A^A(\frac{\hbar}{i} \frac{\delta}{\delta J}, \Phi^*) \frac{\delta Z}{\delta J_B} = 0 . \tag{41}
\]

Next, using the interrelation of the derivatives for \( Z \) and \( W \),

\[
\left( \frac{\delta Z}{\delta J_A} \frac{\delta Z}{\delta \Phi_A^*} \right) = \frac{i}{\hbar} \exp \left( \frac{\hbar}{i} W \right) \left( \frac{\delta W}{\delta J_A} \frac{\delta W}{\delta \Phi_A^*} \right) , \tag{42}
\]

\[
\frac{\delta^2 Z}{\delta \Phi_B^* \delta J_A} = \exp \left( \frac{\hbar}{i} W \right) \left[ \left( \frac{\hbar}{i} \right)^2 \frac{\delta W}{\delta \Phi_B^*} \frac{\delta W}{\delta J_A} + \frac{i}{\hbar} \frac{\delta^2 W}{\delta \Phi_B^* \delta J_A} \right] , \tag{43}
\]
Eqs. (41) may be presented in terms of functional $W$ as,

$$
\frac{\delta W(J, \Phi^*)}{\delta \Phi_B^*} = (-1)^{\varepsilon_B} \left( J_A + M_A \left( \frac{\hbar}{\delta J} + \frac{\delta W}{\delta J}, \Phi^* \right) \right) \frac{\delta^2 W(J, \Phi^*)}{\delta \Phi_A^* \delta J_B} - M^{B*} \left( \frac{\hbar}{\delta J} + \frac{\delta W}{\delta J}, \Phi^* \right) = -\frac{i}{\hbar} (-1)^{\varepsilon_B} \left( J_A + M_A \left( \frac{\hbar}{\delta J} + \frac{\delta W}{\delta J}, \Phi^* \right) \right) \left( \frac{\delta W}{\delta \Phi_A^*} - M^{A*} \left( \frac{\hbar}{\delta J} + \frac{\delta W}{\delta J}, \Phi^* \right) \right) \frac{\delta W}{\delta J_B} \quad (44).
$$

Then, from (44) it follows

$$
\frac{\delta \Gamma}{\delta \Phi_A^*} - M^{B*} - \left( \frac{\delta \Gamma}{\delta \Phi^*} - M_A \right) \frac{\delta \Phi^*}{\delta \Phi_A^*} = -\left( \frac{\delta \Gamma}{\delta \Phi^*} - M^{B*} \right) (-1)^{\varepsilon_B} + \frac{i}{\hbar} \left[ -M_A \frac{\delta \Gamma}{\delta \Phi^*} - \frac{\delta \Gamma}{\delta \Phi^*} M^{A*} + M_A \hat{M}^{A*}, \Phi^* \right], \quad (45)
$$

where the brackets $[ \ , \ ]$ denote the supercommutator.

From (39), (40) and (46) the variation of the effective action can be presented in the "local-like" form,

$$
\delta \Gamma = -\langle \Gamma, \langle \delta \Psi \rangle \rangle + \left( M_A \frac{\delta}{\delta \Phi^*} + (-1)^{\varepsilon_A} M^{A*} \frac{\delta l}{\delta \Phi} \right) \langle \delta \Psi \rangle - \frac{i}{\hbar} \left[ M_A \frac{\delta \Gamma}{\delta \Phi^*} + \frac{\delta \Gamma}{\delta \Phi^*} M^{A*} - M_A \hat{M}^{A*}, \Phi^* \right] \langle \delta \Psi \rangle + \langle \delta M \rangle, \quad (47)
$$

with local (for $M = 0$) operator acting on the functional $\langle \delta \Psi \rangle$. Here we imply the notations

$$
\langle \delta \Psi \rangle = \delta \Psi(\tilde{\Phi}) \cdot 1 \quad \text{and} \quad \langle \delta M \rangle = \delta M(\tilde{\Phi}, \Phi^*) \cdot 1. \quad (48)
$$

Then, using the identities,

$$
\frac{\delta \Phi^*}{\delta \Phi^*} = (-1)^{\varepsilon_B(\varepsilon_A+1)} \frac{\delta}{\delta J_B} \frac{\delta W}{\delta \Phi_A^*} = -(-1)^{\varepsilon_B(\varepsilon_A+1)} (\Gamma''^{-1})^{BC} \frac{\delta l}{\delta \Phi^*} \frac{\delta \Gamma}{\delta \Phi^*}, \quad (49)
$$

following from the Legendre transformation (30) we can present the variation of the effective action in the equivalent, so-called non-local (due to explicit presence of the quantities $(\Gamma''^{-1})^{BC}$) form,

$$
\delta \Gamma = \frac{\delta \Gamma}{\delta \Phi} \hat{F}^A \langle \delta \Psi \rangle - M_A \hat{F}^A \langle \delta \Psi \rangle + \langle \delta M \rangle, \quad (50)
$$

where the operator $\hat{F}^A$ is derived from the Eqs. (40), (46), (47) as follows

$$
\hat{F}^A = -\frac{\delta}{\delta \Phi^*} + (-1)^{\varepsilon_B(\varepsilon_A+1)} (\Gamma''^{-1})^{BC} \left( \frac{\delta l}{\delta \Phi^*} \frac{\delta \Gamma}{\delta \Phi^*} \right) \frac{\delta l}{\delta \Phi_B^*}. \quad (51)
$$

From the variation (50) it follows that on shell the effective action is generally gauge dependent because of

$$
\frac{\delta \Gamma}{\delta \Phi^*} = 0 \quad \rightarrow \quad \delta \Gamma \neq 0. \quad (52)
$$
This fact does not permit to formulate consistently of a soft breaking of BRST symmetry within the field-antifield formalism, if only two last terms in (50) cancel each other,

\[ \langle \delta M \rangle = \widehat{M}_A \widehat{F}^A \langle \delta \Psi \rangle . \] (53)

However, this is rather a strong restriction on the BRST-breaking functional \( M \) for the effective action to be gauge independent on-shell. The same statement is valid for the physical S-matrix. Really, Eq. (53) fixes the gauge variation of \( M = M(\Phi, \Phi^*) \) under a change of the gauge-fixing functional \( \Psi \) to be

\[ \delta M = \frac{\delta M}{\delta \Phi^a} \widehat{F}^A_0 \delta \Psi \text{ where } \widehat{F}^A_0 = (-1)^{\varepsilon_B(\varepsilon_{A}+1)(S^{0})_{-1}}BC\left( \frac{\delta l}{\delta \Phi^C} \frac{\delta S}{\delta \Phi^*_A} \right) \frac{\delta l}{\delta \Phi^B}. \] (54)

It was shown in [15] that already in the case of Yang-Mills theories in linear \( R_\xi \) gauge which includes the Landau gauge the relation (54) does not satisfy. We are forced to claim that a consistent quantization of general gauge theories when restriction on the domain of integration in functional integral is taken as an addition to the full action of a given gauge system violating the BRST symmetry does not exist. As a consequence, the last fact implies that the vacuum expectation values of gauge invariant operators calculated for the theory with soft breaking of the BRST symmetry is gauge dependent.

5 Gribov-Zwanziger action in one-parameter \( R_\xi \)-gauges

In this section we shall apply our above-described general consideration of a soft BRST breaking to the important case of Yang-Mills theories, since those had been the subject of recent investigations [4]–[6]. The initial classical action \( S_0 \) of Yang-Mills fields \( A^a_{\mu}(x) \), which take values in the adjoint representation of \( su(N) \) so that, \( a = 1, \ldots, N^2-1 \), has the standard form

\[ S_0(A) = -\frac{1}{4} \int d^Dx \, F^a_{\mu\nu} F^{\mu\nu a} \quad \text{with} \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu, \] (55)

where \( \mu, \nu = 0, 1, \ldots, D-1 \), the Minkowski space has mostly "+" signature, \((-,-,\ldots,+)\), and \( f^{abc} \) denote the (totally antisymmetric) structure constants of the Lie algebra \( su(N) \). The action (55) is invariant under the gauge transformations

\[ \delta A^a_\mu = D^{ab}_\mu \xi^b \quad \text{with} \quad D^{ab}_\mu = \delta^{ab} \partial_\mu + f^{abc} A^c_\mu. \] (56)

The total field configuration space of Yang-Mills theory,\n
\[ \{ \Phi^A \} = \{ A^a_{\mu}, B^a, C^a, \bar{C}^a \} \quad \text{with} \quad \varepsilon(C^a) = \varepsilon(\bar{C}^a) = 1, \quad \varepsilon(A^a_{\mu}) = \varepsilon(B^a) = 0, \] (57)

includes the (scalar) Faddeev-Popov ghost and antighost fields \( C^a \) and \( \bar{C}^a \), respectively, as well as the Nakanishi-Lautrup auxiliary fields \( B^a \). The corresponding set of antifields is

\[ \{ \Phi^*_A \} = \{ A^{*a\mu}, B^{*a}, C^{*a}, \bar{C}^{*a} \} \quad \text{with} \quad \varepsilon(A^{*a\mu}) = \varepsilon(B^{*a}) = 1, \quad \varepsilon(C^{*a}) = \varepsilon(\bar{C}^{*a}) = 0. \] (58)

A solution to the classical master equation can be presented in the form

\[ S(\Phi, \Phi^*) = S_0(A) + A^{*a\mu} D^{ab}_\mu C^b + \frac{1}{2} C^{*a} f^{abc} C^c + \bar{C}^{*a} B^a. \] (59)

The gauge-fixing functional can be chosen as

\[ \Psi(\Phi) = \bar{C}^a \chi^a(A, B) \] (60)
with free bosonic functions $\chi^a$, so that the non-degenerate action $S_{ext}$ (6) becomes

$$S_{ext}(\Phi, \Phi^*) = S_0(A) + \left( A^{*a}_\mu + \bar{C}^a_c \frac{\delta \chi^c}{\delta A^a_\mu} D^c_\mu C^b + \frac{1}{2} C^{*a}_b f^{abc} C^c + (\bar{C}^{*a} + \chi^a) B^a \right)$$

$$= S_{FP}(\Phi) + A^{*a}_\mu D^c_\mu C^b + \frac{1}{2} C^{*a}_b f^{abc} C^c + \bar{C}^{*a} B^a ,$$

(61)

where $S_{FP}(\Phi)$ is the Faddeev-Popov action

$$S_{FP}(\Phi) = S_0(A) + \bar{C}^a K^{ab} C^b + \chi^a B^a$$

(62)

The actions (62) and (61) are invariant under the BRST transformation

$$\delta_B A^a_\mu = D^a_\mu C^b \theta , \quad \delta_B \bar{C}^a = B^a \theta , \quad \delta_B B^a = 0 , \quad \delta_B C^a = \frac{1}{2} f^{abc} C^c C^b \theta$$

(63)

where $\theta$ is a constant Grassmann parameter.

In [9, 10] it has been shown that the Gribov horizon [7] in Yang-Mills theory (55) in the Landau gauge,

$$\chi^a(A, B) = \partial^\mu A^a_\mu \rightarrow K^{ab} = \partial^\mu D^a_\mu ,$$

(64)

can be taken into account by adding to the Faddeev-Popov action (62) the non-local functional $^1$

$$M(A) = \gamma^2 \left( f^{abc} A^b_\mu (K^{-1})^{ad} f^{dec} A^e_\mu + D(N^2-1) \right) ,$$

(65)

where $K^{-1}$ is the matrix inverse to the Faddeev-Popov operator $K^{ab}$ in (64). The so-called thermodynamic or Gribov parameter $\gamma$ is determined in a self-consistent way by the gap equation [9, 10]

$$\frac{\partial E_{vac}}{\partial \gamma} = 0 ,$$

(66)

where $E_{vac}$ is the vacuum energy given by

$$\exp \left\{ \frac{i}{\hbar} E_{vac} \right\} = \int D\Phi \exp \left\{ \frac{i}{\hbar} S_{GZ}(\Phi) \right\}$$

(67)

pertaining to the Gribov-Zwanziger action [4, 5, 6]

$$S_{GZ}(\Phi) = S_{FP}(\Phi) + M(A) .$$

(68)

Note that the functional $M(A)$ in (65) is not invariant under the BRST transformation (63) but trivially satisfies the condition (9) of soft BRST breaking because of its independence on antifields.

The Gribov-Zwanziger action was intensively investigated in a series of papers [4, 5] where various quantum properties of gauge models with this action have been studied. We stress however that it was impossible in principle to establish the gauge independence of physical quantities in these theories because they were formulated practically in the Landau gauge (64) only, with except for covariant gauges in [6]. Here, we are going to clarify this crucial issue.

To this end, we discuss the Gribov-Zwanziger action (68) for the one-parameter family of $R_\xi$ gauges,

$$\chi^a(A, B, \xi) = \partial^\mu A^a_\mu + \frac{\xi}{2} B^a$$

(69)

$^1$The choice of [4, 5, 6] agrees with ours after Wick rotation, integrating out auxiliary fields and renaming $\gamma^4 \rightarrow \gamma^2$. 
with a real parameter $\xi$ interpolating between the Landau gauge ($\xi=0$) and the Feynman gauge ($\xi=1$). The Faddeev-Popov action is then written as

$$S_{FP}(\Phi, \xi) = S_0(A) + C^a \partial^\mu D^a_{\mu} C^b + (\partial^\mu A^a_\mu) B^a + \frac{\xi}{2} B^a B^a.$$  

(70)

The Faddeev-Popov operator $K^{ab}$ is formally independent of $\xi$ if it is considered in acting of the space of the Yang–Mills fields $A^a_\mu$, but the functional $M$ must be modified away from $\xi=0$, already because $K^{ab}$ ceases to be hermitian [4, 5, 6].

To solve the problem of Gribov horizon definition here, we consider Hermitian augmented Faddeev-Popov operator for $R_{\xi}$ gauge,

$$\bar{K}^{ab}(\xi, A, B) = \partial^\mu D^{ab}_\mu + f^{abc}\xi \frac{B^c}{2}, \quad (\bar{K}^{ab}(\xi))^+ = K^{ab}(\xi),$$  

(71)

whose eigen-values in the equation $\bar{K}^{ab}(\xi)u^b_n = \lambda^a_n u^a_n$ should be real and determine the Gribov region $\Omega(\xi)$ as

$$\Omega(\xi) \equiv \{ A^a_\mu, \partial^\mu A^a_\mu = -\frac{\xi}{2} B^a, K^{ab} > 0 \}$$

We suppose that the proper eigen-values of the $\bar{K}^{ab}(\xi)$ operator completely control the ones of non-Hermitian $K^{ab}(\xi)$ in such way, that the Gribov-Zwanziger $R$-valued functional,

$$M(A, B, \xi) = \gamma^2(\xi) \left( f^{abc} \frac{1}{2} K^{-1} a d (A, B, \xi) f^{dec} A^{e\mu} + D(N^2 - 1) \right)$$  

(72)

really determine Gribov region $\tilde{\Omega}(\xi)$. One should be noted, first, that thermodynamic Gribov parameter, $\gamma^2(\xi)$ should depend on gauge parameter, $\xi$, to be determined in self-consistent way from the Eqs. (66), (67) with $S_{GZ}(\Phi, \xi)$ and vacuum energy, $E_{\text{vac}}(\xi)$. Second, we stress that so suggested introduction of Gribov-Zwanziger horizon functional is based on the representation of the Yang-Mills connection via transverse, $A^{aT}_\mu$, and longitudinal, $A^{aL}_\mu$, parts,

$$A^{aT}_\mu = \left( \delta^\nu - \frac{\partial^\nu}{\partial^2} \right) A^a_\mu = A^a_\mu + \frac{\xi}{2} \frac{\partial^\mu}{\partial^2} B^a,$$

(73)

$$A^{aL}_\mu = \partial^\nu A^a_\mu = -\frac{\xi}{2} \frac{\partial^\mu}{\partial^2} B^a,$$

(74)

introduced in [6], so that, $R_{\xi}$-gauge (69) is equivalent to the pairs of conditions,

$$\partial^\mu A^{aT}_\mu = 0, \quad \partial^\mu A^{aL}_\mu = -\frac{\xi}{2} B^a,$$

(75)

As the consequence, the operator, $\bar{K}^{ab}(\xi, A, B)$, is nothing else, Faddeev-Popov operator for the transverse components of Yang-Mills field, $A^{aT}_\mu$,

$$\bar{K}^{ab}(\xi, A, B) = \partial^\mu (\partial^\mu + f^{acb} A^{cT}_\mu) = K^{ab}(A^{T}).$$  

(76)

Now, we may state, following to [6], that the domain of integration with respect to the fields, $A^a_\mu$, in the path integral should be restricted to the region, $\Omega(\xi)$,

$$\tilde{\Omega}(\xi) = \{ A^a_\mu | A^a_\mu = A^{aT}_\mu + A^{aL}_\mu, A^{aT}_\mu \in \Omega(\xi) \}$$

(77)

with $A^{aT}_\mu$ components of connections from only Gribov region, $\Omega(\xi)$. Of course, at least for small $\xi$ we suggest on smooth character of dependence of $\tilde{\Omega}(\xi)$ with Gribov region, $\Omega(0)$, for Landau gauge,

$$\lim_{\xi \rightarrow 0} \tilde{\Omega}(\xi) = \Omega(0).$$  

(78)
Now, we propose the Gribov-Zwanziger action for Yang-Mills theories (55) in the $R_\xi$ gauge family (69) as

$$S_{GZ}(\Phi, \xi) = S_{FP}(\Phi, \xi) + M(A, B, \xi) .$$

(79)

Because the BRST transformation (63) does not depend on the gauge fixing, from (72) by continuity we can conclude that

$$\delta_B M(A, B, \xi) \neq 0 \quad \rightarrow \quad \delta_B S_{GZ}(\Phi, \xi) \neq 0 .$$

(80)

Let us recall our consistency condition (54), which takes the form

$$\delta M(A, B, \xi) = \frac{1}{2} \frac{\delta M(A, B, \xi)}{\delta \Phi^A} \tilde{F}_0^A \tilde{C}^a B^a \delta \xi .$$

(81)

Since the right-hand side necessarily depends on the ghost, antighost or auxiliary fields, it cannot match the left-hand side for our suggestion for $M$ in Eq. (72). Therefore, soft breaking of BRST symmetry is not consistent in $R_\xi$ gauges for Yang-Mills theory.

6 Conclusions

In the present paper we have considered a definition of soft breaking of BRST symmetry in the field-antifield formalism using any regularization scheme respecting gauge invariance. To this purpose, we added a BRST ‘breaking functional’ $M$ to the gauge-fixed action $S_{\text{ext}}$ which, in turn, is constructed from an arbitrary classical gauge-invariant action $S_0$ by the BV method rules. The soft breaking of BRST symmetry was determined by the analog of the quantum master equation, $(M, M) = -2i\hbar \Delta M$. It was proved the non-invariance of the integrand of vacuum functional under the BRST transformations determined by means of the functional, $(S_{\text{ext}} + \kappa M)$, for any value of the real parameter $\kappa$. We have obtained all Ward identities for the generating functional of Green’s functions $Z$, of connected Green’s functions $W$ and of vertex functions $\Gamma$ both for dimensional-like regularization and for more general one, when $\delta(0) \neq 0$. The Ward identities were used to investigate the gauge dependence of those functionals. It was argued that effective action $\Gamma$ as well as the S-matrix are on-shell gauge dependent. We were forced to claim that a consistent quantization of gauge systems in the BV formalism with the soft breaking of BRST symmetry does not exist.

We discussed the Gribov-Zwanziger action for the one-parameter family of $R_\xi$ gauges. To this aim we suggest the new form of the Gribov-Zwanziger horizon functional given in (72) which is given on a base of Hermitian augmented Faddeev-Popov operator $\tilde{K}_{\alpha\beta}(\xi)$ in (71) to be coinciding with the Faddeev-Popov operator constructed with respect only transverse component of Yang-Mills fields, suggested firstly in [6]. Already in this simple case, the functional $\Gamma$ turned out to depend on the gauge even on shell. We are forced to conclude that a consistent quantization of gauge theories with a soft breaking of BRST symmetry does not exist.

Our basic aim on this stage of finding a consistent quantum prescription to treat the theory with soft breaking of the BRST symmetry is to apply for Yang–Mills and more general gauge theories with Gribov copies the BV formalism with composite fields. This perspective is now under our intensive consideration.

Acknowledgments

A.R. is grateful to V.Rubakov, S.Konstein and to the participants of the International Seminar QUARKS’2012 for valuable comments and discussion. The work is supported by the LRSS grant 224.2012.2 as well as by the RFBR grant 12-02-00121 and by the RFBR-Ukraine grant 11-02-90445.
References

[1] I.V. Tyutin, *Gauge invariance in field theory and statistical physics in operator formalism*, Lebedev Inst. preprint N 39 (1975), arXiv:0812.0580[hep-th].

C. Becchi, A. Rouet and R. Stora, *Renormalization of the abelian Higgs-Kibble model*, Commun. Math. Phys. 42 (1975) 127;

[2] L.D. Faddeev and A.A. Slavnov, *Gauge fields: Introduction to quantum theory*, Benjamin/Cummings, 1980;

M. Henneaux and C. Teitelboim, *Quantization of gauge systems*, Princeton University Press, 1992;

S. Weinberg, *The quantum theory of fields, Vol. II*, Cambridge University Press, 1996;

D.M. Gitman and I.V. Tyutin, *Quantization of fields with constraints*, Springer, 1990.

[3] I. L. Bogolubsky, E. M. Ilgenfritz, M. Muller-Preussker, and A. Sternbeck, *Lattice gluodynamics computation of Landau gauge Green’s functions in the deep infrared*, Phys. Lett. B676 (2009) 69, arXiv:0901.0736[hep-lat];

V. Bornyakov, V. Mitrujshkin, and M. Muller-Preussker, *SU(2) lattice gluon propagator: Continuum limit, finite-volume effects and infrared mass scale m(IR)*, Phys. Rev. D81 (2010) 054503, arXiv:0912.4475[hep-lat].

[4] M.A.L. Capri, A.J. Gómes, M.S. Guimaraes, V.E.R. Lemes, S.P. Sorella and D.G. Tedesko, *A remark on the BRST symmetry in the Gribov-Zwanziger theory*, Phys. Rev. D82 (2010) 105019, arXiv:1009.4135 [hep-th];

L. Baulieu, M.A.L. Capri, A.J. Gomes, et all *Renormalizability of a quark-gluon model with soft BRST breaking in the infrared region*, Eur. Phys. J. C66 (2010) 451, arXiv:0901.3158 [hep-th];

D. Dudal, S.P. Sorella, N. Vandersickel and H. Verschelde, *Gribov no-pole condition, Zwanziger horizon function, Kugo-Ojima confinement criterion, boundary conditions, BRST breaking and all that*, Phys. Rev. D79 (2009) 121701, arXiv:0904.0641 [hep-th].

[5] L. Baulieu and S.P. Sorella, *Soft breaking of BRST invariance for introducing non-perturbative infrared effects in a local and renormalizable way*, Phys. Lett. B671 (2009) 481, arXiv:0808.1356 [hep-th];

M.A.L. Capri, A.J. Gómes, M.S. Guimaraes, V.E.R. Lemes, S.P. Sorella and D.G. Tedesko, *Renormalizability of the linearly broken formulation of the BRST symmetry in presence of the Gribov horizon in Landau gauge Euclidean Yang-Mills theories*, arXiv:1102.5695 [hep-th].

D. Dudal, S.P. Sorella and N. Vandersickel, *The dynamical origin of the refinement of the Gribov-Zwanziger theory*, arXiv:1105.3371 [hep-th].

[6] R.F. Sobreiro and S.P. Sorella, *A study of the Gribov copies in linear covariant gauges in Euclidean Yang-Mills theories*, JHEP 0506 (2005) 054, arXiv:hep-th/0506165.

[7] V.N. Gribov, *Quantization of nonabelian gauge theories*, Nucl. Phys. B139 (1978) 1.

[8] V.G. Bornyakov, V.K. Mitrujshkin and R.N. Rogalyov, *Gluon propagators in 3D SU(2) theory and effects of Gribov copies*, arXiv:1112.4975[hep-lat].

[9] D. Zwanziger, *Action from the Gribov horizon*, Nucl. Phys. B321 (1989) 591.

[10] D. Zwanziger, *Local and renormalizable action from the Gribov horizon*, Nucl. Phys. B323 (1989) 513.

[11] D. Zwanziger, *Some exact properties of the gluon propagator*, arXiv:1209.1974[hep-th].
[12] P.M. Lavrov and I.V. Tyutin. *On the structure of renormalization in gauge theories*, Sov. J. Nucl. Phys. 34 (1981) 156;

[13] I.A. Batalin and G.A. Vilkovisky, *Gauge algebra and quantization*, Phys. Lett. 102B (1981) 27.

[14] I.A. Batalin and G.A. Vilkovisky, *Quantization of gauge theories with linearly dependent generators*, Phys. Rev. D28 (1983) 2567.

[15] P. Lavrov, O. Lechtenfeld and A. Reshetnyak, *Is soft breaking of BRST symmetry consistent?*, JHEP 1110 (2011) 043, arXiv:1108.4820 [hep-th].

[16] P. Lavrov, O. Radchenko and A. Reshetnyak, *Soft breaking of BRST symmetry and gauge dependence*, MPLA ?1110 (2012) 043, arXiv:1201.4720 [hep-th].

[17] M. Vasiliev, *Higher spin gauge theories in various dimensions*, Fortsch. Phys. 52 (2004) 702–717, [arXiv:hep-th/0401177];

[18] R.R. Metsaev, *Cubic interaction vertices of massive and massless higher spin fields*, Nucl. Phys. B759 (2006) 147–201, [arXiv:hep-th/0512342]; R.R. Metsaev, *Gauge invariant formulation of massive totally symmetric fermionic fields in (A)dS space*, Phys. Lett. B643 (2006) 205–212, [arXiv:hep-th/0609029];

[19] I.L. Buchbinder, V.A. Krykhtin, P.M. Lavrov, *Gauge invariant Lagrangian formulation of higher massive bosonic field theory in AdS space*, Nucl. Phys. B762 (2007) 344–376, [arXiv:hep-th/0608005];

C. Burdik, A. Reshetnyak, *On representations of Higher Spin symmetry algebras for mixed-symmetry HS fields on AdS-spaces. Lagrangian formulation*, J. Phys. Conf. Ser. 343 (2012) 012102, [arXiv:1111.5516[hep-th]];

I.L. Buchbinder, V.A. Krykhtin, A.A. Reshetnyak, *BRST approach to Lagrangian construction for fermionic higher spin fields in AdS space*, Nucl. Phys. B787 (2007) 211, [arXiv:hep-th/0703049];
I.L. Buchbinder and A. Reshetnyak, General Lagrangian Formulation for Higher Spin Fields with Arbitrary Index Symmetry. I. Bosonic fields, Nucl. Phys. B 862 (2012) 270, [arXiv:1110.5044[hep-th]].

[20] B.S. DeWitt, *Dynamical theory of groups and fields*, Gordon and Breach, 1965.

[21] B.L. Voronov, P.M. Lavrov and I.V. Tyutin, *Canonical transformations and gauge dependence in general gauge theories*, Sov. J. Nucl. Phys. 36 (1982) 292.