Analysis of clustering phenomena in ab initio approaches

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Abstract

An approach for explicit consideration of cluster effects in nuclear systems and accurate ab initio calculations of cluster characteristics of nuclei is devised. The essential block of the approach is a construction of a basis which incorporates both conventional No-Core Shell Model wave functions and translationally-invariant wave functions of various cluster channels. The results of computations of the total binding energies of \(^{8}\)Be nucleus as well as the spectroscopic factors of cluster channels (amount of clustering) are presented. An unexpected fresh result of the rigorous study is that the contribution of “non-clustered” components of the basis to the total binding energy is great even in the typical cluster systems such as the discussed nucleus.

Keywords: clustering, nuclear structure, light nuclei, ab initio computation

One of the fundamental properties of light nuclei is the clustering displaying itself in a certain degree of separation of a nucleus into two or more multi-nucleon substructures. A great body of experimental information has been accumulated over many years of studies of the clustering phenomena. Theoretical studies of clustering originated in Ref. \cite{1}. A microscopic, i.e. starting from a certain NN-potential and considering a nucleus or a two-fragment collision channel as an A-nucleon or an \((A_1 + A_2)\)-nucleon system – so-called Resonating Group Model (RGM) – has been put forward in it. Forty-year evolution of these studies has been summarized in Ref. \cite{2}. A large number of microscopic approaches taking cluster properties into account was discussed in the monograph. In the view of the authors one of the main lines of nuclear theory to construct a unified theory of nuclear structure and nuclear collision dynamics in the framework of microscopic approaches. The "dynamic" clustering is peculiar to the collision processes therefore the nuclear reaction theory involve cluster concepts almost without exceptions. In succeeding years a variety of theoretical techniques have been developed to study nuclear clustering. Within modern microscopic models such as the Generator Coordinate Method (GCM) \cite{3, 4}, Microscopic Cluster Model (MCM) \cite{5, 6}, THSR-approach \cite{7}, Antisymmetrized Molecular Dynamics (AMD) \cite{8} and Fermionic Molecular Dynamics (FMD) \cite{9, 10}, clustering in various nuclear states has been confirmed to emerge directly from NN-interactions. A detail discussion of these approaches is presented in review \cite{11}. Large-scale calculations of cluster characteristics of nuclear states: cluster spectroscopic factors and form factors have been studied in the framework of advanced shell-model method – so-called Cluster-Nucleon Configuration Interaction Model \cite{12, 13, 14, 15}.

The supercomputing era provides new possibilities for building the unified theory. Due to that the development of ab initio approaches to description of nuclear structure and dynamics is recently one of the basic lines of the advancement of nuclear science. Such approaches are based on Hamiltonians involving universal (common for wide range of objects under study), realistic NN-, NNN-, etc. potentials. No-Core Shell Model (NCSM) is one of the most advanced among these approaches. A typical basis of this model contains all possible nucleon configurations on equal terms up to a certain truncation level. Obviously in the most cases huge basis is required to achieve convergence of the results. Recently various versions of NCSM \cite{16, 17, 18, 19, 20, 21} occupy a prominent place in nuclear structure calculations.

Microscopic calculations of light nuclei properties demonstrate "non-equivalence" of different nucleon configurations. For example many of Slater determinants play a negligible role in computations of nuclear total binding energy in NCSM. For this reason, methods of selection of essential components of a certain nature, a priori or after preliminary analysis, are recently popular \cite{22, 23, 24, 25, 26}.

The studying of mentioned non-equivalence originated by clustering phenomena is, in our opinion, an intriguing issue. In the current paper we carry out the analysis of the role of cluster components in solutions of the A-nucleon Schrödinger equation with ab initio NN-potentials. For these purposes we combine the wave functions (WFs) of various cluster channels and standard NCSM components into unified basis. Another vital issue is concerning the realistic numerical values of cluster characteristics – spectroscopic factors (SFs) of cluster channels. These values carry information on "nuclear geometry".

\footnotesize
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as well as the information necessary for calculation of the reaction cross sections. In nuclear structure studies SFs play a role of "amount of clustering", according to the terminology of Ref. [27]. Ab initio approaches are required to compute these values. In the framework of our studies we explore both these problems. We have performed computations of the total binding energies (TBEs) of clustered nuclei as well as the SFs of their fragmentation channels in the framework of different models in which the clustering is taken into account in a number of ways. We also introduce a definition of aggregate amount of clustering (AoC) useful in the case when the multi-channel problem is considered. This mathematical object is non-trivial owing to the strong non-orthogonality of different cluster channels.

To design the desired formalism we build up partial bases of translationally-invariant A-nucleon WFs of channels manifesting two-fragment separation \( A = A_1 + A_2 \). Various channels are distinguished by the internal state WFs of the fragments \( A_1 \) and \( A_2 \) as well as the channel spin and relative motion angular momentum. These WFs are ab initio calculated. The next problem is to assemble the non-orthogonal partial bases corresponding to these channels into a unified orthogonal basis and to add a number of eigenvectors obtained in ordinary NCSM calculations (called "polarization terms" in Ref. [2]) to this assembled cluster basis. For this purpose the WFs of channels are transformed to the shell-model form and undergo the orthonormalization procedure together with polarization terms. In such a way we build the universal basis suitable to describe arbitrary states taking one- or multi-channel clustering into account. This potentiality is topical especially for the states manifesting pronounced cluster properties.

It should be noted that an approach aimed at an accurate ab initio description of cluster reactions induced by light nuclei collisions has been developed in Refs. [28, 29]. This approach exploits the microscopic Hamiltonians together with RGM approach (so accounting for the "dynamic" clustering). It was called NCSM/RGM. The "polarization terms" were introduced into NCSM/RGM in Refs. [30, 31, 32, 33, 34]. The new model received the name No-Core Shell Model with Continuum (NCSMC).

By contrast in the current paper we concentrate on manifestation of cluster structure in bound and resonance states. Another difference between the NCSM and our approach is technical one. The technique of so-called cluster coefficients (CCs), presented in Refs. [35, 36, 37, 38], is used in our work for transformation of the cluster WFs to the superpositions of the Slater determinants. It provides a general approach to work with a broad variety of rather heavily and excited fragments. Besides that applying this formalism we obtain pure algebraic approach which seems to be of a great convenience.

Our calculations are carried out with the use of Hamiltonians containing high-precision modern NN-potentials JISP16 [39] and Daejeon16 [40]. The first one is constructed using the J-matrix inverse scattering method. The latter one is built using the N3LO limitation of Chiral Effective Field Theory [41] softened by similarity renormalization group (SRG) transformation. SRG transformation had been proposed in Refs. [42]. Both these potentials are well-tested in broad spectral calculations of \( A \leq 16 \) nuclei.

Notice that a NCSM approach was already used in computation of the \( \alpha \)-cluster SFs in \( ^{4} \)He nucleus where SRG-softened N3LO-based NN-interaction was used. Results of these studies were presented in Ref. [43]. We involve these values of SFs in our analysis.

Let us consider a two-fragment system \( A_1 + A_2 \). The oscillator-basis terms of the cluster channel \( c \) are built in the translationally-invariant form:

\[
\Psi_{A,\mu}^{c,TL} = \tilde{A} [\Psi_{A_1}^{(k)}(\tilde{r}_1) \Psi_{A_2}^{(l)}(\tilde{r}_2)]_{JM,T},
\]

where \( A = A_1 + A_2 \), \( \tilde{A} \) is the antisymmetrizer, \( \Psi_{A_i}^{(k)} \) is a translationally-invariant internal WF of the fragment labelled by a set of quantum numbers \( \{k_i\} \); \( \varphi_\mu(\tilde{r}) \) is the WF of the relative motion. A channel WF as a whole is labelled by the set of quantum numbers \( c_\mu \) which includes \( \{k_1, k_2\}, J, M, T \). The goal of the subsequent transformations is to present function (1) as a linear combination of the Slater determinants (SDs) containing the one-nucleon WFs of the oscillator basis. The reason of it was mentioned above. For these purposes function (1) is presented in the form of the linear combination of the WFs with fixed magnetic quantum numbers \( m, M \), and multiplied by the function of the center of mass (CM) zero vibrations \( \Phi_{000}(\tilde{R}) \).

Then the transformation of WFs caused by changing from \( \tilde{R} \), \( \tilde{\rho} \) to \( \tilde{R}_1, \tilde{R}_2 \) coordinates – different-mass Talmi-Moshinsky transformation defined in [44]

\[
\Psi_{A,\mu}^{c,TL} = \Phi_{000}(\tilde{R})\Psi_{A,\mu}^{c,TL} = \sum_{N_1, L_1, M_1, m, M_1} \begin{pmatrix} 000 \\ N_1, L_1, M_1 \\ N_2, L_2, M_2 \end{pmatrix} A_1(N_1, L_1, M_1, \tilde{R}_1) \Phi_{A_1}^{(k_1)} \Phi_{A_2}^{(k_2)}(\tilde{R}_2)_{JM,T} = \sum_{N_1, L_1, M_1, m, M_1} \begin{pmatrix} 000 \\ N_1, L_1, M_1 \\ N_2, L_2, M_2 \end{pmatrix} A(N_1, L_1, M_1, \tilde{R}_1) \Phi_{A_1}^{(k_1)} \Phi_{A_2}^{(k_2)}(\tilde{R}_2)_{JM,T}.
\]

is performed.

The main procedure of the method is to transform internal WFs corresponding to each fragment with none-zero center of mass vibrations into a superposition of SDs

\[
\Phi_{N_1, L_1, M_1}^{A}(\tilde{R}) \Psi_{n, m}^{(k_1)}(\tilde{R}_1) = \sum_k X_{N_1, L_1, M_1}^{A(k)} \Psi_{A(k)}^{D}.
\]

Quantity \( X_{N_1, L_1, M_1}^{A(k)} \) is called a cluster coefficient. There is a large number of methods elaborated for the calculations of CCs (see Refs. [35, 36, 37, 38]). The most general scheme is based on the method of the second quantization of the oscillator quanta. In this scheme the WF of the CM motion is presented as

\[
\Phi_{N_1, L_1, M_1}^{A}(\tilde{R}) = N_{N_1, L_1}(\tilde{\mu})^{N_1-L_1} Y_{L_1, M_1}(\tilde{\mu}) \Phi_{000}(\tilde{R}),
\]

where \( \tilde{\mu} \) is the creation operator of the oscillator quantum, and \( N_{N_1, L_1} \) is the norm constant. Thus the CC turns out to be reduced to a matrix element of the tensor operator \( \tilde{\mu} \):

\[
< \Psi_{A(k)}^{D} | \phi_{N_1, L_1}^{A}(\tilde{R}_1) \Psi_{A_1}^{(k_1)} | > \equiv N_{N_1, L_1}(\tilde{\mu})^{N_1-L_1} Y_{L_1, M_1}(\tilde{\mu}) \Phi_{000}(\tilde{R}) \Phi_{A(k)}^{D}.
\]
Contrary to pioneering work \[45\] in which translationally-invariant WFs were written in terms of Jacobi coordinates, the formula

$$\Psi^{c,T}_{A,i} = \Psi^{c}_{A,i}/\Phi^{A}_{0 0 0}(\vec{R})$$ (6)

is considered here as the definition of these functions.

It should be noted that in the general case WFs of cluster-channel terms \([\kB]\) of one and the same channel \(c_k\) characterized by the pair of internal functions \(\Psi^{(1)}_{A_1}, \Psi^{(2)}_{A_2}\) and different values of relative motion quantum numbers \(n, l\) are non-orthogonal. The same is true for the terms of different channels. Moreover all these WFs are non-orthogonal with the polarization terms - eigenvectors obtained in ordinary NCSM calculations. So the next step is to build a basis of orthonormalized WFs which includes both the polarization terms and the terms of several cluster channels. The basis is obtained by the diagonalization of matrix

$$\begin{bmatrix}
\langle \Psi^{(f)}_{pol} | \Psi^{(b)}_{pol} \rangle & \langle \Psi^{(f)}_{pol} | \Psi^{c}_{A,i} \rangle \\
\langle \Psi^{(f)}_{pol} | \Psi^{c}_{A,i} \rangle & \langle \Psi^{c}_{A,i} | \Psi^{c}_{A,i} \rangle
\end{bmatrix}$$ (7)

in which the square brackets denote the sub-matrices. The cluster-channel terms \(\Psi^{c}_{A,i}\) are, in fact, expressed in the form of superpositions of SDs with the help of formula \([\kB]\).

The eigenvectors of the matrix normalized by its eigenvalues give the desirable basis. Each term of the basis takes the form of a SDs linear combination. Therefore the computation of the matrix elements of both the kinetic and the potential energy in the discussed basis is identical to the ordinary shell-model computation. An arbitrary microscopic \(ab\) \(initio\) or effective, including two-, three-, etc. nucleon forces Hamiltonian may be utilized. The calculations of the matrix elements of operators, the estimates of error bars etc. are also analogous to those in the shell model. The only difference is the list of the basis vectors. This list is considerably shorter compared to the NCSM one. The limitations on the use of the approach are imposed by the dimensionality of the basis vectors.

This approach as a whole is very adaptable due to the possibilities to vary: a number of cluster channels and polarization terms; \(n_\text{max}; A_1, A_2\)-nucleon shell-model spaces determined by the corresponding truncation level parameters \((N^{(\ell)}_{\text{max}})\) which are the maximal values of the total number of the oscillator quanta in each subsystem. This gives a way to take into account various halo, cluster and other properties of a system.

The formalism presented above is convenient for the calculation of the SFs of arbitrary solutions of \(A\)-nucleon Schrödinger equations \(\Psi_A\). The SF of a certain cluster channel \(c_k\), which may be called in the context of the current paper the one-channel amount of clustering, is defined as the sum of squared overlaps of the wave function \(\Psi_A\) with the normalized antisymmetric WFs \(\Psi^{c}_{\text{pol}}\). WFs \(\Psi^{c}_{\text{pol}}\) are obtained by the diagonalization of sub-matrix contained in right-lower quadrant of the matrix \((7)\) which is reduced by additional condition \(\kappa = \kappa'\). The just presented definition is completely equivalent to the one proposed in Ref. \[46\] (so-called "new SF"). This definition plays an important role in the theory of nuclear reactions. Detailed discussion of various aspects of this concept is presented in Refs. \[27 47 44 45\].

A treatment of the multi-channel problem is a more delicate problem. Obviously the analysis of statistical weights of components contained in a WF is possible only with the proviso that these components are orthogonal one to another. The basis of cluster-channel terms \([\kB]\) incorporating all channels \(c_k\) of fragmentation \(A_1 + A_2\) (all internal states of these clusters) is complete and what is more linear dependent. Orthonormalization of the basis mixes the terms of different channels and, because of the linear dependency, the result of the procedure is ambiguous. Therefore a possibility to estimate contributions of individual cluster channels correctly is doubtful in principle. Nevertheless it is possible to perform the analysis of aggregate AoC for chosen list of channels. For this purpose the definition of the AoC of a channel can be generalized to the aggregate AoC of a group of channels. This value is defined similarly to the value of one-channel AoC but the sum of squared overlaps of the wave function \(\Psi_A\) with the WFs \(\Psi^{c}_{\text{pol}}\) is over all terms of a chosen number of cluster channels \([c_k]\). WFs of the terms are obtained by the orthonormalization of the WFs \(\Psi^{c}_{A,i}\) corresponding to all these channels. The definition allows one to determine the measure of clustering depending upon the choice of the set of cluster channels.

Notice that the choice of the maximal value of \(n\) and \(A_1-, A_2\)-nucleon truncation level parameters \((N^{(\ell)}_{\text{max}})\) may be different for calculation of the WFs of nuclear states and for the evaluation of the SFs and the aggregate AoC.

Characteristics of several clustered systems were analyzed in our work. In this paper the cluster properties of \(^{8}\text{Be}\) nucleus as \(a + a\) system are presented to demonstrate some unexpected aspects of clustering typical for all the systems. We demonstrate here the values of the total binding energy (TBE), the one-channel and aggregate AoC. Widely-used code Antoine re-arranged by us for performing NCSM computations is exploited to calculate the WFs of the clusters and the polarization terms.

The TBEs of \(^{8}\text{Be}\) nucleus have been computed with the use of variously constructed bases. In all instances the maximal total number of the oscillator quanta \(N^{(\ell)}_{\text{max}}\) is considered as a basic characteristic parameter together with \(\text{lus}\) one.

The bases are the following. First, conventional basis of NCSM is used. Let us denote this version as \(mod1\) bellow. Second, two types of pure cluster bases are exploited. One of them contains the WFs of the ground states of both \(a\)-clusters \(\Psi_a\) in their lowest shell-model configurations. The other one incorporates the realistic WFs of \(0^+_1\) and \(0^+_2\) states of \(^{4}\text{He}\) calculated using the code Antoine with truncation level \(N_a^{(\ell)} = 2\). This basis is three-channel. These versions are denoted as \(mod2\) and \(mod3\). Third, two hybrid bases \(mod4\) and \(mod5\) are built. Each of them, being restricted by the maximal total number of the oscillator quanta \(N^{(\ell)}_{\text{max}}\) involves the complete set of the NCSM WFs limited by inequality \(N^{(\ell)}_{\text{pol}} \leq N^{(\ell)}_{\text{max}}\). The sole additional cluster component with \(N_{\text{pol}} = N^{(\ell)}_{\text{max}}\) corresponding to \(mod2\) version is incorporated in \(mod4\). For \(mod5\) version all cluster components with \(N_{\text{pol}} = N^{(\ell)}_{\text{max}}\) corresponding to \(mod3\) are incorporated. Evidently both \(mod4\) and \(mod5\) bases are in-
complete in the space of the WFs with $N_{\text{tot}} = N_{\text{max}}^\text{mod}$. For each NN-potential the parameter $\hbar \omega$ demonstrating the best convergence of the energy value of $^8$Be nucleus in NCSM calculations is chosen. Internal cluster WFs contained in the cluster terms are calculated in the framework of NCSM using the same NN-potential and the same $\hbar \omega$ parameter as the ones used for NCSM terms of eight-nucleon WFs.

The TBE of $^8$Be nucleus calculated in the framework of mod1, mod2, and mod3 models as a function of total number of the oscillator quanta $N_{\text{tot}}$ is presented in figures 12. The former one have been computed using the JISP16 potential with the oscillation parameter $\hbar \omega = 22.5$ MeV, the latter one – using the Daejeon16 potential with the parameter $\hbar \omega = 15.0$ MeV.

In spite of significant difference between two considered potentials, computations demonstrate very similar qualitative pattern. The cluster components corresponding to lowest value of $N_{\text{tot}}$ contribute dominatingly compared to non-clustered ones with the same truncation level both for one-channel mod2 and three-channel mod3 models. However even in such a trivial basis the role of non-clustered components is not negligible. The extension of the basis makes the pattern drastic – the relative contribution of components of such a type increases much rapidly than the clustered ones. Thus the use of a basis consists from cluster WFs in the ab initio approaches results in very large underestimation of the binding energy even though a system under study is strongly clustered. This result is irrespective of whether the lowest-configuration cluster model mod2 or realistic cluster model mod3 is considered.

Properties similar to the just demonstrated were pointed out in recent published paper [14]. TBE of this systems, computed in the cluster approach is underestimated by 1 MeV in comparison with the results of accurate NCSMC calculation. It should be noted, however, that the effect detected in the current work for $^8$Be nucleus is much greater.

It makes sense to compare the quantitative results obtained by application of different potentials. The Daejeon16 version being essentially more “soft” than JISP16 brings about larger contribution of the cluster components to the TBE compared to JISP16. This trend is confirmed in the case that the results of Ref. [43], in which “supersoft” version of N3LO-based potential [42] is exploited, to take into account. Nevertheless the contribution of non-clustered components to $^8$Be TBE remains distinct although it is noticeably smaller. A confirmation of the trend for a lighter cluster system may be found in just mentioned Ref. [42].

To perform more detail analysis of relationship between the contributions of cluster and non-clustered components into the TBE we have carried out the calculation of its values in mod4 and mod5 versions. This study allows one to compare the contributions of these two types of components at fixed values $N_{\text{tot}}$. To be brief we present here the results of calculations in which Daejeon16 potential has been used. The corresponding results obtained by use of JISP16 potential are qualitatively analogous.

The results are presented in Tab. 1. The analysis of difference in the TBE values between the ones computed in the mixed bases mod4 or mod5 with the truncation level $N_{\text{tot}}$ and ones computed with the use of of NCSM mod1 characterized by the truncation level values $N_{\text{tot}} - 2$ as well as $N_{\text{tot}}$ is the most informative. In all cases extension of the NCSM basis restricted at the level $N_{\text{tot}} - 2$ by including the complete set of the NCSM WFs with $N_{\text{tot}}$ changes the TBE value much stronger than the addition of only the extra cluster components with the same $N_{\text{tot}}$. For large values of $N_{\text{tot}}$ the cluster components increase the TBE by few hundreds keV while involving the complete set of the components corresponding to $N_{\text{tot}}$ adds several MeV. Thus the aggregate contribution of non-clustered components to TBE in this area of $N_{\text{tot}}$ values is dominating. Moreover the

\begin{figure}[h]
\centering
\includegraphics[width=0.49\textwidth]{figure1.png}
\caption{TBE of $^8$Be nucleus calculated using JISP16 potential. Black columns – mod1, dark-grey columns – mod2, pale-grey columns – mod3. The horizontal line shows the experimental TBE.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.49\textwidth]{figure2.png}
\caption{TBE of $^8$Be calculated using Daejeon16 potential. Notations are the same as in figure 1.}
\end{figure}
trend is that their relative contribution grows with the increasing of $N_{tot}$.

The AoC, being one-channel or aggregate, serves as a direct measure of cluster content of nuclear states. The computations of this values have been performed in the current paper starting from the following points. The WF of initial nucleus $^8$Be ground state was calculated in the framework of NCSM. The terms of the cluster channels (1) are written in the form containing internal WFs of α clusters which in turn are calculated in the basis limited by the condition $N_{\text{max}}^\alpha = 2$. The channels corresponding to the ground states of both clusters comprising a nucleus are considered together with the channels which contain one or both α-particles in $0^+_1$ state. Higher excitations of $^4$He nucleus have very large values of the decay width to be considered as realistic clusters. In Tab. 2 we present the values of one-channel (ground-ground) and aggregate two-channel (plus ground-excited), and all three-channel (plus excited-excited) AoC.

The table demonstrates a rapid convergency of the AoC values. The saturation is achieved at the level $N_{tot}=8$ with reasonable precision. The small value of one-channel AoC for $N_{tot}=4$ is an artefact of the chosen basis. Indeed, the quantum number $n=0$ characterising the relative motion wave function is contained for this choice of $N_{\text{max}}$. The numerical results of the one-channel $ab\ initio$ calculations of SFs with realistic WFs of the clusters as well as the results of Ref. [43] favour the view that the system under study is strongly clustered. All these statements based on the results of $ab\ initio$ calculations provide a support for a variety of microscopic models of "nuclear cluster geometry", based on clustered probe WFs and the dynamics describing by effective nucleon-nucleon Hamiltonians, such as AMD [8], THSR-approach [7], etc. (this "geometry" manifests itself in a specific density distribution of a clustered system in the internal coordinate system). Indeed, the clustered components constitutes the major portion of the realistic WF. At the same time another implication of this analysis is that the use of realistic NN-potentials would result in significant disagreement between TBE values computed in the clustered models and the experimental ones. Effective NN-potentials turn out to be necessary to compensate this disagreement. Besides that it is preferable realistic internal cluster WFs to be used in such approaches.

The results of the current study contained some other unexpected points.

Let us consider, first of all, relationship between the values of AoC in one-channel and multi-channel cases which are illustrated by Tab. 1. In the region of saturation of the results the aggregate AoC value is ever so slightly greater than the one-channel AoC. Consequently the extremely strong non-orthogonality of the WFs of the discussed channels takes place and turns out to be the reason of that. Thus the procedure of extension of cluster-channel basis is slow in affecting the convergency of both eigenvalues and eigenvectors of $ab\ initio$ Hamiltonians in spite of the completeness of the basis.

Another point have been detected in the simultaneous analysis of TBE and AoC. We stress, that taking into account that the terms of the cluster basis are non-orthogonal to NCSM wave functions, non-clustered components are defined as all components of the basis obtained by the orthonormalization procedure [7] besides the considered cluster ones. Denoting aggregate AoC as $B$ one can define the statistical weight of non-clustered components as $\bar{B} = 1 - B$. Introducing the ratio $\delta E = (E^{\text{mod} 1} - E^{\text{mod} 2(3)})/E^{\text{mod} 1}$ it is possible to define the measure of contribution of non-clustered components into the TBE.

A comparison between these values deduced from the respective tables demonstrate that $\delta E > \bar{B}$ for larger values of $N_{tot}$. Thus the contribution of non-clustered components to the TBE related to their statistical weight is greater than the corresponding relative contribution of the cluster ones and this trend is the most expressive at larger values of $N_{tot}$.

Contrastingly lighter system $^6$He manifests other properties. According to Ref. [14] AoC of the ground-state α-particle channels being approximately equal to the ones presented in Tab. 2 contributes, as it has been noted above, only 1 MeV to TBE of the system.

Summing all the presented up a brief review of the basic points should be made.

1. A formalism convenient to construct a basis that takes into consideration the cluster properties of nuclear systems in $ab\ initio$ calculations of their characteristics is built. The keystone of the formalism is the technique of the cluster coefficients. The developed approach is universal. Various versions of the basis can be applied to conform to the cluster properties of systems under study.

2. An additive and normalized to unity measure of cluster content – amount of clustering [27] – is extended to be used for studies of the multi-channel problem.

3. The total binding energies of $^8$Be nucleus as well as the values of one- and multi-channel amount of clustering are computed. As the result of the combined analysis of these values a fuller picture of a realistic clustered state as a pure cluster configuration immersed into a "sea" of diffuse non-clustered components is beginning to emerge. The statistical weight of these components is shown to be moderate but their contribution to the TBE is great.

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