Basis independent study of Supersymmetry without R–parity and the tau neutrino mass

Javier Ferrandis

Institut de Física Corpuscular - C.S.I.C.
Departament de Física Teòrica, Universitat de València
46100 Burjassot, València, Spain
email: ferrandis@bigfoot.com

Abstract

In the most general R–parity violating model, including both bilinear and trilinear terms, the sneutrino receives a vacuum expectation value, $<\tilde{\nu}>$. We investigate the constraints on $<\tilde{\nu}>$ following a basis independent approach, highlighting the relations between the three most popular basis. We study the prediction for the tau neutrino mass which follows from the minimization of the scalar potential in a SUGRA model with universality of the soft parameters at the GUT scale. Finally we show that the tau neutrino mass controls the R-parity violating effects both in the fermionic and scalar sectors.
I. INTRODUCTION

It is well known that if the conservation of the discrete symmetry known as R–parity is imposed the presence of terms in the superpotential which violate leptonic and baryonic number is not allowed. The existing phenomenological studies of explicit R–parity breaking have so far concentrated on exploring the effects of the trilinear and the bilinear terms separately. In this paper we study both together following a basis independent approach to parametrize the R–parity violating effects [1–3]. It has been pointed out that it is not possible to eliminate completely the effects of the bilinear terms by simply redefining fields [4–7], and so any complete study should include all possible R–parity violating terms. The studies which have concentrated solely on the effects of the trilinear terms [8] have claimed the elimination of the bilinear terms considering them proportional to the neutrino masses; in contrast one can find published papers which point out [11–17] that the effects of the bilinear terms cannot generally be considered negligible. One of the sources of this apparent contradiction is that we can find papers which quote strong bounds on the sneutrino vevs and other papers which tell us that these vevs can be very large. Hence there is some confusion in the literature with regard to limits on sneutrino vevs. In this work we show, in the one generation model, that the best way to understand the R–parity violating effects lies in following an approach which defines a set of basis independent variables which clearly parametrize R–parity violating effects. Therefore any limit obtained on these variables holds in any basis. We shall see that sneutrino vevs are not suitable to parametrize R–parity violation unless we clearly specify the basis in which we are working. Following this approach too we show, motivated by the recent experimental results [19,20] and phenomenological studies [24–30] on neutrino masses, that the $\nu_\tau$ mass controls R–parity violating effects both in the fermionic sector and in the scalar sector [3]. To get this result we need to take into account the minimization of the scalar potential which allows us to connect the parameters which control the R-parity violating effects in the scalar sector with the parameters which control the effects in the fermionic sector.

II. ONE GENERATION $\mathbb{R}$–MSSM MODEL

The one generation model is a useful toy model which has been used on various occasions [11,18,31] to explore R–parity violating effects since it contains the main ingredients of the complete model with three generations. The superpotential, $W$, of the model appears as:

$$W = W_{MSSM} + W_R,$$

where $W_{MSSM}$ is the familiar superpotential of the MSSM. Note that we denote $\hat{L}_0 = \hat{H}_d$, with $v_0 = v_d$ and we keep the $h_b$ for the case where $\lambda_0^D \propto M_b$:

$$W_{MSSM} = \left[h_u \hat{Q}_3 \hat{H}_u \hat{U}_3 + \lambda_0^D \hat{L}_0 \hat{Q}_3 \hat{D}_3 + h_{\tau} \hat{L}_0 \hat{L}_3 \hat{R}_3 - \mu_0 \hat{L}_0 \hat{H}_u \right]$$

*Some correlations between neutrino masses and lepton number violating processes have been studied in previous works [13].
$W_R$ breaks R–parity and is given by:

$$W_R = -\mu_3 \tilde{L}_3 \tilde{H}_u + \lambda_3^D \tilde{L}_3 \tilde{Q}_3 \tilde{D}_3$$

(1)

The scalar potential contains some relevant terms for the following discussion:

$$V_{\text{soft}} = (\tilde{L}_0 \tilde{L}_3) \left( M_{\tilde{L}_0}^2 M_{\tilde{L}_3}^2 \right) \left( \tilde{L}_0 \tilde{L}_3 \right) - (E_\alpha \tilde{L}_\alpha H_u - A_{\alpha\beta} \lambda_3^D \tilde{L}_\alpha \tilde{Q}_3 \tilde{D}_3 + h.c.)$$

(2)

We assume that $E_\alpha = B_{\alpha\beta} \mu_\alpha$. Then $B_{\alpha\beta}$ and $A_{\alpha\beta}$ will have covariant properties of transformation under the $SU(2)$ symmetry of the $L_\alpha$ fields ($SU(4)$ symmetry in the three generation model). The three minimization equations are [7]

$$
\begin{align*}
(m_u^2 - g_Z^2 v_Y^2) v_u &- E_0 v_0 - E_3 v_3 = 0 \\
(m_{00}^2 + g_Z^2 v_Y^2) v_0 - v_u E_0 + m_{03}^2 v_3 &= 0 \\
(m_{33}^2 + g_Z^2 v_Y^2) v_3 - v_u E_3 + m_{30}^2 v_0 &= 0 ,
\end{align*}
$$

(3)

where:

$$m_u^2 = \left( M_{H_u}^2 + \sum_{\alpha=0}^{3} |\mu_\alpha|^2 \right)$$

$$m_{\alpha\beta} = \left( M_{L_{\alpha\beta}}^2 + \mu_\alpha \mu_\beta^* \right)$$

$$v_Y^2 = \left( (v_0^2 + v_3^2) - v_u^2 \right)$$

Under what conditions can we rotate the $\mu_i$–terms without affecting the scalar potential? The following two conditions are sufficient [7,13,34,35]:

- $\mu_\alpha$ is an eigenvector of $M_{\tilde{L}}^2$.
- $E_\alpha$ is proportional to $\mu_\alpha$: $E_\alpha = B\mu_\alpha$.

These conditions are not satisfied in general at the electroweak scale, although it may be possible to satisfy them at some unification scale where soft breaking parameters are universal. Therefore it is not possible in general to eliminate the effects of the bilinear terms and the main consequence will be the appearance of the tau neutrino mass at tree level.

†We assume there is no charge or color breaking minima [3,10]
III. BASIS FOR R–PARITY. INVARIANTS

It is useful to work in other basis to understand better the magnitude of the R–parity violating phenomenological effects. We define the following rotation of the Higgs down and leptonic fields, $L_\alpha$, which in turn induces a rotation of the $\mu$–terms:

$$\mathcal{O} = \begin{pmatrix} s_\theta & c_\theta \\ -c_\theta & s_\theta \end{pmatrix}$$  \hspace{1cm} (5)

Under this change of basis in the Lagrangian we find that the bilinear soft parameters and the trilinear couplings are rotated in the same way, in matricial form:

$$E' = \mathcal{O}E$$
$$B' = \mathcal{O}B\mathcal{O}$$
$$(A_D)' = \mathcal{O}A_D\mathcal{O}$$  \hspace{1cm} (6)

The $2 \times 2$ mass matrix for the soft masses changes to $\mathcal{O}^j M_\mu^\nu \mathcal{O}$:

$$\begin{pmatrix} M'_0^2 \\ M'_3^2 \end{pmatrix} = \begin{pmatrix} s^2_\theta M^2_{L_0} + c^2_\theta M^2_{L_3} + s_\theta c_\theta (M^2_{L_{03}} + M^2_{L_{30}}) \\ M^2_{L_0} s^2_\theta + M^2_{L_3} c^2_\theta + s_\theta c_\theta \Delta M^2 \end{pmatrix} \begin{pmatrix} s^2_\theta M^2_{L_0} + M^2_{L_3} c^2_\theta + s_\theta c_\theta \Delta M^2 \\ M^2_{L_{03}} + c^2_\theta M^2_{L_0} + s_\theta c_\theta (M^2_{L_{03}} - M^2_{L_{30}}) \end{pmatrix}$$  \hspace{1cm} (7)

Where $\Delta M^2 = M^2_{L_3} - M^2_{L_0}$. From the phenomenological point of view it will be convenient to define the non covariant parameters $B_0$ and $B_3$ as $E_0 = B_0 \mu_0$ and $E_3 = B_3 \mu_3$ because as we will see the combinations $\Delta B = B_3 - B_0$ and $\Delta M^2$ appear frequently and their RGE's are simpler to analyze. Now it is obvious that: $E'_\alpha \neq \mu'_\alpha B'_\alpha$. If we write $E'_\alpha$ as a function of $B_\alpha$ we obtain two useful equations:

$$E'_0 = (s_\theta \mu_0 + c_\theta \mu_3) \left( s^2_\theta B_0 + c^2_\theta B_3 \right) + s_\theta c_\theta (s_\theta \mu_3 - c_\theta \mu_0) \Delta B$$

$$E'_3 = (s_\theta \mu_3 - c_\theta \mu_0) \left( s^2_\theta B_3 + c^2_\theta B_0 \right) + s_\theta c_\theta (s_\theta \mu_0 + c_\theta \mu_3) \Delta B$$  \hspace{1cm} (8)

The most useful basis are those which involve some simplification of the phenomenological studies. Assuming that our initial basis is arbitrary, we give below the rotation angle to three basis of particular interest:

- I) The basis where the trilinear coupling is zero:

$$s_\theta = \frac{\lambda_0^D}{\sqrt{(\lambda_0^D)^2 + (\lambda_3^D)^2}} \hspace{1cm} c_\theta = \frac{\lambda_3^D}{\sqrt{(\lambda_0^D)^2 + (\lambda_3^D)^2}}$$  \hspace{1cm} (9)

- II) The basis where $\mu'_3 = 0$:

$$s_\theta = \frac{\mu_0}{\sqrt{\mu_0^2 + \mu_3^2}} \hspace{1cm} c_\theta = \frac{\mu_3}{\sqrt{\mu_0^2 + \mu_3^2}}$$  \hspace{1cm} (10)
III) The basis where $v'_3 = 0$:

\[ s_\theta = \frac{v_0}{\sqrt{v_0^2 + v_3^2}} \quad c_\theta = \frac{v_3}{\sqrt{v_0^2 + v_3^2}} \]  

(11)

We notice that it is impossible to get at the same time $\mu'_3 = 0$ and $v'_3 = 0$. Basis I) is favoured if we wish to use the renormalization group equations. From the phenomenological point of view basis III) is convenient because in this basis the Higgs down, $\tilde{L}_0$, is the field which gives mass to the bottom quark; in the general case this is not true since we have an additional contribution from a non-zero $v'_3$. We notice that it is easy to find basis independent parameters, for instance:

\[ v_d = \sqrt{v_0^2 + v_3^2} \Rightarrow v = \sqrt{v_u^2 + v_d^2} \quad \mu = \sqrt{\mu_0^2 + \mu_3^2} \]  

(12)

\[ \lambda^D = \sqrt{\left(\lambda^D_0\right)^2 + \left(\lambda^D_3\right)^2} \]

From the above we can deduce that the generalization of the MSSM definition of $\tan \beta$, given by

\[ \tan \beta = \frac{v_u}{v_d}, \]  

(13)

is also a basis invariant. There are other invariants which turn out to be very useful, and are defined as:

\[ \cos \zeta = \frac{\mu_3 v_\alpha}{\mu v_d} \Rightarrow \sin \zeta = \frac{\left(\mu_0 v_3 - \mu_3 v_0\right)}{\mu v_d} \]  

(14)

\[ \cos \gamma = \frac{\lambda^D_0 \mu_3}{\lambda^D \mu} \Rightarrow \sin \gamma = \frac{\left(\lambda^D_0 \mu_3 - \lambda^D_3 \mu_0\right)}{\lambda^D \mu} = \left(\frac{\mu_3}{\mu}\right)^I \]  

(15)

\[ \cos \chi = \frac{\lambda^D_3 v_\alpha}{\lambda^D v_d} \Rightarrow \sin \chi = \frac{\left(\lambda^D_0 v_3 - \lambda^D_3 v_0\right)}{\lambda^D v_d} = \left(\frac{v_3}{v_d}\right)^I \]  

(16)

Here the index I) indicates that the expression has to be evaluated in the basis I) where $\lambda^D_3 = 0$. In ref. [3] it was shown that some invariants are useful in obtaining cosmological bounds on R–parity violating effects. We will show that the R–parity violating invariant variables $\sin \zeta$ and $\sin \gamma$ are very useful to study the $\nu_\tau$ mass and the R–parity violating effects in general in the fermionic sector, while $\sin \chi$ is appropriate for studying for instance the phenomenology of the process $Z \Rightarrow b\bar{b}$. We will extract new experimental bounds on the $\tilde{\nu}_\tau$ vev from the latter and other processes. In this model the masses of the vector bosons and the quark masses are given by:

\[ \text{\textsuperscript{4}We follow the same notation as ref. [11] in defining sin \zeta and sin \gamma.} \]
\[ M_W^2 = \frac{g^2}{4} (v_u^2 + v_d^2) = \frac{g^2}{4} v^2 \]  

(17)

\[ M_t = \frac{h_t}{\sqrt{2}} v_u = \frac{h_t}{\sqrt{2}} \frac{2M_W}{g} \frac{t_\beta}{\sqrt{1 + t_\beta^2}} \]  

(18)

\[ M_b = \frac{1}{\sqrt{2}} (\lambda^D_0 v_0 + \lambda^D_3 v_3) = \frac{c_\chi \lambda^D}{\sqrt{2}} \frac{2M_W}{g} \frac{1}{\sqrt{1 + t_\beta^2}} \]  

(19)

We can see that the expressions on the right hand side are basis independent because the masses are physical observables. We keep the expression \( h_b \) for the case where \( \lambda^D_0 \propto M_b \).

**Basis in the three generation model.** Basis II) and III) can be straightforwardly generalized to more generations. An arbitrary basis is not useful unless we can define it unambiguously. We can see that in the three generation model the only useful generalization of basis I) is the basis linked through the renormalization group equations to the high energy basis (SUGRA basis with universality) where the \( \mu_i \) terms have been rotated away. This basis can be useful for finding relations with the SUGRA parameter space. We must point out that in a three generation model there is no basis which simplifies the RGE’s as in the one generation model.

**IV. ONE GENERATION SUGRA \( R - \text{MSSM} \)**

\( \lambda^D_3 \)-approach. The one generation \( R - \text{MSSM} \) model is completely fixed if we rotate at the GUT scale to some basis where, if there is universality of the soft parameters, we eliminate the bilinear terms completely both in the superpotential and in the soft sector. In this basis all \( R \)-parity violating effects are fixed if \( \lambda^D_3(GUT) \) is known. Then we use the RGE’s to compute the parameters at the electroweak scale. We can see that the presence of trilinear parameters induces the appearance through RGE’s evolution of a vev because at the electroweak scale the parameters \( \mu_3, M^2_{L_{03}} \) and \( M^2_{L_{30}} \) will be non-zero.

\[
\mu_3(GUT) = 0 \quad \lambda^D_3(GUT) \neq 0 \quad \Rightarrow \quad \begin{cases} 
\mu_3 \neq 0 \\
E_3 \neq 0 \\
M^2_{L_{03}} \neq 0 \quad \Rightarrow \quad v_3 \neq 0 \\
M^2_{L_{30}} \neq 0 \\
\lambda^D_3 \neq 0 
\end{cases}
\]  

(20)

\( \mu_3 \)-approach. Another possibility is to rotate the fields at the GUT scale to a basis where \( \lambda^D_3(GUT) = 0 \), and make use of the fact that the RGE’s satisfy the following properties:

\[
\frac{d\lambda^D_3}{dt} \propto \lambda^D_3 \quad \frac{dM^2_{L_{03}}}{dt} \propto (M^2_{L_{03}}, M^2_{L_{30}}, \lambda^D_3) 
\]  

(21)

There is also an analogous equation for \( M^2_{L_{30}} \). The Lagrangian parameters at the electroweak scale can then be computed, and one finds:
TABLE I. Most useful basis for R–parity at the electroweak scale

| I           | II          | III          |
|-------------|-------------|--------------|
| µ₃ ≠ 0      | µ₃ = 0      | µ₃ ≠ 0      |
| E₃ ≠ 0      | E₃ ≠ 0      | E₃ ≠ 0      |
| M₁²₉₀₃ = 0  | M₁²₉₀₃ ≠ 0  | M₂²₉₀₃ ≠ 0  |
| M₂²ₙ₀ = 0   | M₂²ₙ₀ ≠ 0   | M₂²ₙ₀ ≠ 0   |
| λ₃⁰ = 0     | λ₃⁰ ≠ 0     | λ₃⁰ ≠ 0     |
| v₃ ≠ 0      | v₃ ≠ 0      | v₃ = 0      |

Following this approach we get at the electroweak scale the parameters in the basis I). As we can see all the R-parity violating effects are completely fixed by µ₃(GUT). It does not matter which approach we are following because both are equivalent. The main point is that if we do not neglect any contributions in the superpotential we can rotate from one basis to another and thus compare calculations which have been performed in different basis. These results have to be the same. To build a SUGRA extension of the one generation R–MSSM model we need to look for basis independent parameters. In any case the most correct parameters are:

\[
\begin{align*}
\mu_3(GUT) &\neq 0 \\
\lambda_3^D(GUT) &\neq 0
\end{align*}
\]

Following this approach we get at the electroweak scale the parameters in the basis I). As we can see all the R-parity violating effects are completely fixed by µ₃(GUT). It does not matter which approach we are following because both are equivalent. The main point is that if we do not neglect any contributions in the superpotential we can rotate from one basis to another and thus compare calculations which have been performed in different basis. These results have to be the same. To build a SUGRA extension of the one generation R–MSSM model we need to look for basis independent parameters. In any case the most correct parameters are:

\[
\left( A_0, M_0, M_{1/2}, t_\beta, \mu_3^G \right)
\]

in the basis I) where \( \lambda_3^D(GUT) = 0 \). Or:

\[
\left( A_0, M_0, M_{1/2}, t_\beta, (\lambda_3^D)^G \right)
\]

in the basis II) where \( \mu_3(GUT) = 0 \). We notice that the soft masses \( M_{1/2} \) and \( M_{L_{3/2}} \) are zero in any basis at the GUT scale if there is universality. In the MSSM we need in addition to the first four parameters the sign of \( \mu_0 \) because the CP–odd scalar mass satisfies at tree level the following relation \( M_A^2 = t_\beta \mu_0 B_0 \). Then the sign of \( B_0 \) is fixed and we can compute it through the minimization equations. In this model, as we will see later, the relation between the CP–odd mass and the minimization equations is more complex because there are additional contributions and there is mixing between neutral Higgses and sneutrinos. Now we need to specify \( \mu_3^G \) including its sign, and this will allow us to compute \( \mu_0, B_0 \) and \( B_3 \) through the minimization equations. We have complemented the analysis of this work with a numerical study following the \( \mu_3 \)–approach which simplifies the RGE’s substantially but we have expressed our results in terms of basis independent parameters.

Notation. From now on we will denote the parameters in basis I) as \( \mu_3, v_3 \) and \( \lambda_3^D \) and the parameters in the other basis using an index II) or III). In addition we will use \( v_3' \) to denote the vev in the basis II) and \( \mu_3' \) to denote the \( \mu_3 \)-term in the basis III) to avoid confusion.
V. CONSTRAINTS ON SNEUTRINO VEVS

Suppose we have obtained a bound on $\langle \tilde{\nu}_\tau \rangle$ in the basis I (where $\lambda_3^D = 0$). If we wish to translate this bound to the basis II we need to use the equations (5). Then we obtain:

$$v_3' = v_3 \left(1 - \left(\frac{\mu_3}{\mu}\right)^2\right)^{1/2} - \frac{\mu_3}{\mu} \left(\frac{v^2}{1 + t_\beta} - v_3^2\right)^{1/2}$$

(23)

We notice that to fix $v_3'$ we need only $t_\beta$ and $\mu_3/\mu$. In figure (1) we can see the relation between $v_3$ and $v_3'$ for two different values of $t_\beta$ ($t_\beta = 2$ in continuous line and $t_\beta = 10$ in dashed-------

\footnote{We recall that $(\mu_3/\mu)^T = s_\gamma$ is an invariant.}
line) and three different values of the ratio $\mu_3/\mu$ for each value of $t_\beta$. As we can observe in figure (1) even if $v_3'$ is small the vev in the basis $I$ can be very large depending on the values of $\mu_3/\mu$ and $t_\beta$. Then we can see that as $v_3' = v s_\beta s_\zeta$, the smaller the value of $v_3'$ the smaller $s_\zeta$ will be and more fixed is $v_3$, or in other words, greater is the adjustment between $\mu_3/\mu$ and $v_3/v_d$ to get a small value for $s_\zeta$. When we study the $\nu_\tau$ mass we will see to what extent this adjustment is possible.

The trilinear couplings ($\lambda_0^D, \lambda_3^D$) transform in the same way as the vevs and this leads us to the question of the experimental bounds on the coupling $\lambda_3^D$ because we could generate after a rotation a trilinear coupling that is experimentally constrained. In this way we could derive experimental bounds on the sneutrino vevs and lepton number violating $\mu-$terms from the known bounds on trilinear couplings. It is possible to derive an experimental bound on $\lambda_3^D$ from the LEP measurements of the process $Z \to b\bar{b}$ [36,37] where the trilinear coupling $L_3Q_3D_3$ contributes radiatively. This bound has been derived in the limit where

![Diagram](image_url)
the radiative contributions coming from the bilinear terms are neglected. The bilinear terms contribute through new loops originating from the mixing between charged higgses and sleptons [17,18]. The results of the refs [36,37] concluded that one can derive the bound
\[ \lambda D^3 \leq 0.26(0.45) \] at 1(2) \( \sigma \) respectively (if the squarks are \( \approx 100 \text{ GeV} \)). The basis where the radiative contributions coming from the bilinear terms and the sneutrino vevs is minimal is the basis III) (where the vev is zero). The relation between the sneutrino vev in the basis I) and the R-parity violating trilinear terms in the basis III) is obtained from the invariant

\[ \sin \chi = \left( \frac{v_3}{v_d} \right)^I = \left( \frac{\lambda^D_3}{\lambda^D} \right)^{III} \] (24)

Since the R-parity violating contribution to the process \( Z \to b\bar{b} \) is a basis independent we can compute the parameters at the electroweak scale from the GUT scale parameters following the \( \mu_3 \)-approach, and afterwards rotate to the basis III) (where \( v'_3 = 0 \)). Then we can make use of the identity \( \lambda^D = (\lambda^D_0)^I = h_b \) to obtain:

\[ (\lambda^D_0) \frac{v_3}{\sqrt{v_0^2 + v_3^2}} = \frac{\sqrt{2m_b}}{v_0} \frac{v_3}{\sqrt{v_0^2 + v_3^2}} \leq \lambda_M \] (25)

Here \( \lambda_M \) denotes the experimental bound on \( \lambda^D_3 \). Then we get a bound on \( v_3^I \):

\[ v_3^2 \leq \frac{\lambda_M^2 v^4}{(2m_b^2(1 + t_\beta^2) + \lambda_M^2 v^2)} \frac{1}{(1 + t_\beta^2)} \] (26)

From this inequality we notice that making use of the experimental bound, \( \lambda_M \), allows us to get a bound on \( v_3 \). This bound depends on \( t_\beta \) with the maximum allowed value being achieved with the minimum allowed value for \( t_\beta \). The minimum value for \( t_\beta \) comes from the perturbativity bound on the top quark yukawa coupling, or in other words, \( h_t \) grows with the energy scale and the maximum value at the GUT scale is \( \sqrt{4\pi} \) which is related to the minimum of \( t_\beta (\approx 1.8) \) as we can see from the formula (18). In figure (2) we can see that the experimental bound on \( v_3 \) coming from the minimum value for \( t_\beta \) is 115 GeV approximately. Moreover the larger \( t_\beta \) is the stronger the bound on \( v_3 \) will be. If \( t_\beta = 10 \) the bound on \( v_3 \) is approx. 25 GeV. To translate this bound to basis II) we need to know the tau neutrino mass: However since \( v'_3 \) is generally less than \( v_3 \) any bound on \( v_3 \) will be stronger for \( v'_3 \).

**Constraints on sneutrino vevs in the three generation model.** This method can be generalized to obtain constraints on sneutrino vevs in the three generation model. Since we are interested in obtaining upper bounds on the vevs we can simplify the calculation assuming that in the basis where the vevs are maximum the R–parity violating trilinear couplings are zero. Then if we rotate to basis III) (where sneutrino vevs are zero) we generate R–parity violating trilinears:

\[ (\lambda^D_{1ij})' = -\sqrt{\frac{v_d^2 + v_2^2 + v_3^2}{v_d^2}} \lambda^D_{0ij} \] (27)
From this we obtain a bound on sneutrino vevs assuming that \( \lambda_{D_{ii}} \) is proportional to the mass of the down quark of the ith-generation:

\[
(v_1^2 + v_2^2 + v_3^2) \leq \frac{\lambda^2_{D_{ii}} v^4}{(2m_{q_i}^2(1 + t_{3\beta}^2) + \lambda^2_{D_{ii}} v^2)} \cdot \frac{1}{(1 + t_{3\beta}^2)}
\]

Finally we can use the bounds on \( (\lambda_{D_{ii}})' \) from double beta decay and electron neutrino mass \[8\]. The strongest upper bound comes from \( (\lambda_{D_{133}})' \) (contribution to the radiative electron neutrino mass):

\[
\sqrt{v_1^2 + v_2^2 + v_3^2} \leq 5 \text{ GeV} \left( \frac{M}{100 \text{ GeV}} \right)^{1/2}
\]

Where \( M \) is some specific scalar mass. All these constraints are plotted in figure (2) as a function of \( t_{3\beta} \). In the general case these bounds are valid if we assume that there are no cancellations between the different contributions to \( (\lambda_{D_{ij}})' \).

VI. TAU NEUTRINO MASS AND R-PARITY VIOLATING EFFECTS

To make clearer the relation between the tau neutrino mass and R-parity violating effects in the fermionic sector it is useful to rotate to the basis III) \( (v_3 = 0) \) where R-parity violating effects are more evident. In this basis we have the following: \( v'_0 = v_d, \mu'_3 = \mu \sin \zeta, \mu'_0 = \mu \cos \zeta \). Written in the basis III) the neutralino-neutrino Majorana mass matrix takes the form:

\[
M_N = \begin{pmatrix}
M_1 & 0 & \frac{1}{2}g'v_u & -\frac{1}{2}g'v_d & 0 \\
0 & M_2 & -\frac{1}{2}gv_u & \frac{1}{2}gv_d & 0 \\
\frac{1}{2}g'v_u & -\frac{1}{2}gv_u & 0 & -\mu \cos \zeta & -\mu \sin \zeta \\
-\frac{1}{2}g'v_d & \frac{1}{2}gv_d & -\mu \cos \zeta & 0 & 0 \\
0 & 0 & -\mu \sin \zeta & 0 & 0
\end{pmatrix}
\]

There are several contributions to the tau neutrino mass: the bilinear term in the superpotential \( \mu_\alpha \tilde{L}_\alpha \tilde{H}_u \) which provides a mass for the fermionic component of one combination of the doublets \( L_\alpha \) (the Higgsino); the vacuum expectation values, \( \langle L_\alpha \rangle = v_\alpha / \sqrt{2} \), which contribute to the spontaneous breaking of the electroweak symmetry, induce a mixing between the neutral members of the fermion doublets, \( L_\alpha \), and the neutral gauginos. Therefore the neutrino gets a mass and under the experimental assumption \( M_{\tilde{\chi}_1^0} \gg M_{\nu_\tau} \) we can derive an approximate formula for the \( \nu_\tau \) mass:

\[
M^{\text{tree}}_{\nu_\tau} = \frac{M_2^2 M_2^2 \mu c_{3\beta}^2}{(M_2^2 M_2^2 s_{2\beta}^2 c_{3\beta} - M_1 M_2 \mu)}
\]

On the other hand in an arbitrary basis the trilinear term contributes radiatively \[34,39,41\] through a quark-squark loop. This contribution, \( M^{\lambda_{D}}_{\nu_\tau} \), is proportional to \( (\lambda_{3}^D)^2 \). The bilinear terms induce a mixing between different fields which contribute to the same effective couplings giving new radiative contributions \[12\] which we denote as \( M^{\text{Sp}}_{\nu_\tau} \) (We will not analyze
here how these different radiative contributions depend on the SUGRA parameter space). To sum up, the tau neutrino mass at one loop depends on three components:

\[ M_{\nu_{\tau}} = M_{\nu_{\tau}}^{\text{tree}} + \left( M_{\nu_{\tau}}^{\lambda_3^D} + M_{\nu_{\tau}}^{S_p} \right)^{1-\text{loop}} \]

The contribution to the tau neutrino mass at each order of perturbation theory is a basis independent. At tree level this fact is obvious through the formula (30) which depends only on basis independent parameters. At the one loop level the basis independence is manifested if we consider all the radiative contributions, both bilinear and trilinear. If we rotate to the basis I) \((\lambda_3^D = 0)\) the contribution coming from the trilinear terms is transferred to the radiative contribution coming from the bilinear terms because as is obvious the tau neutrino mass is a basis independent. We can notice too that the MSSM limit is recovered when the \(\nu_{\tau}\) mass tends to zero because this implies that \(\sin \zeta\) and \(\lambda_3^D\) tend to zero.

\[ M_{\nu_{\tau}} \to 0 \implies \begin{cases} \sin \zeta \to 0 \\ \lambda_3^D \to 0 \end{cases} \implies \text{MSSM} \]

It is also apparent in the basis III) that the R–parity violating effects in the charged lepton sector depend on the tau neutrino mass. The smaller the tau neutrino mass, the smaller will be these effects since the charged lepton Dirac mass matrix is:

\[
\begin{bmatrix}
M_2 & \frac{1}{\sqrt{2}} g v_d & 0 \\
\frac{1}{\sqrt{2}} g v_d & \mu \cos \zeta & 0 \\
0 & \mu \sin \zeta & \frac{1}{\sqrt{2}} h_{\tau} v_d
\end{bmatrix}
\]

The tau Yukawa coupling is a basis independent which is related to the tau neutrino mass through the exact tree level formula (18):

\[
h_{\tau}^2 = \frac{2 M_{\tau}^2}{v_d^2} \left\{ 1 - s_{\zeta}^2 f(M_2, \mu, t_\beta, c_\zeta) \right\}
\]

The chargino masses are, in the limit of \(s_\zeta\) tending to zero, approximately given by:

\[
M_{\tilde{\chi}^\pm} = M_{\tilde{\chi}^\pm}^{\text{MSSM}} - s_{\zeta}^2 \frac{\mu}{2} \left\{ 1 + \frac{(M_2 - \mu)}{\sqrt{(M_2 - \mu)^2 + 4 M_W^2 s_{2\beta}^2}} \right\}
\]

(31)

As we can see the invariant magnitude \(\sin \zeta\) is a key parameter of the model. Now it is clear that to understand the reach of the bilinear R–parity violating effects in the fermionic sector it is crucial to know the attainable values of \(\sin \zeta\). We will study in a SUGRA scenario how the tau neutrino mass (through \(\sin \zeta\)) is related with the SUGRA parameter space.

**Minimization equations and universality of soft parameters.** It is possible to derive a formula for \(\sin \zeta\) using the minimization equations of the scalar potential. This formula will be useful for studying the attainable values of \(\sin \zeta\) in a SUGRA inspired scenario. Moreover, \(\sin \zeta\) is a basis independent, therefore it does not matter which basis has been used to derive the relation between \(\sin \zeta\) and the SUGRA parameter space. We
will assume that we have computed all the parameters at the electroweak scale in the basis I) following the $\mu_3$-approach to R-parity through the RGE’s from GUT to weak scale. Then we rotate to basis II) (where $\mu'_3 = 0$) and we get directly a formula for $\sin \zeta$ (because $\sin \zeta = v'_3/v_d$). The relevant minimization equation is:

$$\left(M^2_{L_3} + g_Z^2 v_Y^2 - \frac{\mu_3^2}{\mu^2} \Delta M^2\right) v_d \sin \zeta - v_u \frac{\mu_0 \mu_3}{\mu} \Delta B + \frac{\mu_0 \mu_3}{\mu^2} \Delta M^2 v_d \cos \zeta = 0$$  (32)

From the above one obtains a quadratic equation for $s_\zeta$. We find that the invariant $s_\zeta$ which appeared in the fermionic mass matrices is proportional to another invariant $s_\gamma = \mu_3/\mu$ which will be useful for studying the $\nu_\tau$ mass. At first order in $\mu_3/\mu$ the expression for $\sin \zeta$ reduces to:

$$\sin \zeta = \frac{\mu_0 \mu_3}{\mu^2} (\delta_B t_\beta \pm \delta_M) = \frac{1}{2} \sin(2\gamma) (\delta_B t_\beta \pm \delta_M)$$  (33)

FIG. 3. Soft terms which contribute to the invariant $s_\zeta$
where we define \( M_{\nu_3}^2 = M_{L_3}^2 + g_Z^2 v_Y^2 \) and:

\[
\delta_B = \frac{\mu \Delta B}{(M_{\nu_3}^2 - \frac{\mu_2^2}{\mu^2} \Delta M^2)} \quad \delta_M = \frac{\Delta M^2}{(M_{\nu_3}^2 - \frac{\mu_2^2}{\mu^2} \Delta M^2)},
\]

We can see that the parameters which appear on the right hand side \( \Delta B, \Delta M, \mu_3 \) and \( \mu \) have to be computed in the basis I), although the important point is that these parameters are fixed if we know the SUGRA parameters (basis independent). There are some limiting cases: if \( \delta_M \approx 0 \) then \( \sin \zeta = \frac{\mu_3 \mu_2}{\mu^2} \delta_B t_\beta \) or if \( \delta_B t_B \approx 0 \) then \( \sin \zeta = \frac{\mu_3 \mu_2}{\mu^2} \delta_M (1 + \frac{\mu_3 \mu_2}{\mu^2} \delta_M)^{-1/2} \) and in principle it would also be possible to get a very small \( s_\lambda \) through a fine tuning between \( \delta_B t_\beta \) and \( \delta_M \); in the latter case there could be large R-parity violating effects in the scalar sector but small ones in the fermionic sector. However we will see that this scenario is practically impossible because \( \Delta B \) is one order of magnitude less than \( \Delta M \). In order to predict the tau neutrino mass in a universality scenario we need to study the attainable values of \( \Delta M \) and \( \Delta B \). Both parameters are zero at the GUT scale and they are generated radiatively. The main question is if they could be small enough to explain a very light neutrino. Using the renormalization group equations (see appendix) we find (We recall that in the \( \mu_3 \)-approach one can take \( \lambda^D_3 = 0 \) and \( h_b = \lambda^D_0 \)):

\[
8\pi^2 \frac{d\Delta M^2}{dt} = -3h_b^2 (M_{L_0}^2 + M_Q^2 + M_D^2 + A_b^2) - 16\pi^2 \frac{d\Delta B}{dt} = -6h_b^2 A_b
\]

We can perform an approximate analytical integration to get:

\[
\Delta M^2(Z) \approx \frac{t_\beta}{8\pi^2} \frac{3h_b^2}{2} (3M_6^2 + A_b^2) \quad \Delta B(Z) \approx \left( \frac{t_\beta}{8\pi^2} \right) A_b
\]

Expanding the common term in the previous equations and making use of the formula (19) we get:

\[
(\Delta M^2(Z))^{approx} = \frac{3g^2}{16\pi^2} \frac{m_s^2}{M^2_W} \ln \left( \frac{M_G}{M_Z} \right) (3M_6^2 + A_b^2) \left( 1 + t_\beta^2 \right)
\]

We can see that the term \( \Delta B \) is generally less than \( \Delta M \):

\[
\frac{\Delta B}{\sqrt{\Delta M^2}} \approx \left[ \frac{3g^2}{16\pi^2} \frac{m_s^2}{M^2_W} \ln \left( \frac{M_G}{M_Z} \right) \left( 1 + t_\beta^2 \right) \right]^{1/2} \left[ 1 + 3 \frac{M_3^2}{A_b^2} \right]^{-1/2} \leq \frac{5}{100} (1 + t_\beta^2)^{1/2}
\]

Here the term 5/100 comes from rounding off the constant term. The exact numerical calculations of \( \Delta B \) and \( \Delta M \) confirm this analytical estimation as can be seen in figure (3) where the numerical calculations are shown as a continous line and the analytical approximation to \( \Delta M \) is shown as a dotted line. We notice too that there is a minimum value for \( \delta_M \) since there is a minimum value for \( t_\beta \) which comes from the perturbativity bound on the top quark Yukawa coupling. Then we can see that one can find the dominant relation between
the tau neutrino mass and the SUGRA space if we use the formula (37) inside the formula for \( s_\gamma \) where we are neglecting the contribution coming from \( \delta_B \). Then we find:

\[
M_{\nu_\tau} \approx \delta_M^2 \left( \frac{\mu_0 \mu_3}{\mu^2} \right)^2 \frac{M_Z^2 M_\gamma \mu_3^2}{(M_Z^2 M_\gamma s_{2\beta} - M_1 M_2 \mu)}
\]

If we insert the expression (37) in the above expression we find:

\[
M_{\nu_\tau} \approx \left[ \frac{3 g^2 m_b^2}{16 \pi^2 M_W^2} \ln \left( \frac{M_G}{M_Z} \right) \left( 3 + \left( \frac{A_b}{M_0} \right)^2 \right) \right]^2 \left( 1 + t_{\beta}^2 \right) \left( \frac{\mu_0 \mu_3}{\mu^2} \right)^2 \frac{M_Z^2 M_\gamma \mu}{(M_Z^2 M_\gamma s_{2\beta} - M_1 M_2 \mu)} \tag{39}
\]

This approximate formula for the \( \nu_\tau \) mass allows to understand qualitatively the dominant dependence on the SUGRA parameter space. As we can see, the \( \nu_\tau \) mass is proportional

\[
\text{FIG. 4. } \nu_\tau \text{ mass as a function of the invariant } s_\gamma = (\mu_3/\mu)^t \text{ for two different values of } t_{\beta}
\]
to \((1 + t_3^2)\) and \(\mu_3/\mu\). Once we have fixed the SUGRA parameters \(M_0, A_0\) and \(M_{1/2}\) (as is indicated in figure (4)) we observe that the larger the value of \(t_3\), the larger the \(\nu_\tau\) mass will be and the smaller the value of \(\mu_3/\mu\), the smaller the \(\nu_\tau\) will be, and if \(\mu_3/\mu\) tends to zero the \(\nu_\tau\) mass tends to zero. In figure (4) we compare for two different values of \(t_3\) the numerical solution (continous line), the approximate formula (30) (dashed line) (where we make use of the definition of \(s_\zeta\)) and our approximate formula (33) (dotted line). We can see that the first approximation (30) is not perfect and we have to sum up the second approximation which is to neglect the contribution to \(s_\zeta\) coming from \(\delta_B t_3\). After fixing the SUGRA parameters as is indicated in figure (4) we notice that the \(\nu_\tau\) mass grows with \(t_3\). Apparently this is contradictory with one of the results of ref. [11] which states that the tau neutrino decreases with increasing \(t_3\). However both results are consistent because we have included the implicit dependence of \(\sin \zeta\) on \(t_3\) while in ref. [11] \(s_\zeta\) has been fixed. The dependence on \(t_3\) is partially cancelled by \(c_\beta^2\) and we obtain the \((1 + t_3^2)\) factor. It was claimed in the same paper [11] that the ratio \(\mu_3/\mu\) need not be supressed in order to supress the tau neutrino mass. This afirmation depends on what one means by a supressed tau neutrino mass, because as we can see from the minimization equation \(s_\zeta\) is proportional to \(\mu_3/\mu\). If we assume a tau neutrino mass of the order of the laboratory bound, 18 MeV [21,22], we can see in figure (4) that the ratio does not need to be supressed and it can be of the order 1. If we assume a tau neutrino mass of the order of the cosmological bound, \(\approx 100\) eV, for a small value of \(t_3 (t_3 = 2)\) a ratio \(\mu_3/\mu\) of order \(10^{-1}\) would be allowed. However if we assume a tau neutrino mass of the order of 1 eV or less (as can be infered from combinations of different experimental constraints [23]) we find for \(t_3 = 2\) a ratio \(\mu_3/\mu\) of the order \(10^{-3}\) or less. Therefore \(\mu_3/\mu\) has to be supressed to get neutrino masses of the eV order. We stress that the tau neutrino mass is proportional to the misalignement between the vectors \(\mu_3\) and \(v_3\) and this misalignement is shown through the basis invariant \(s_\zeta\). The unavoidable misalignement is the result of the non–universality of the soft parameters at the electroweak scale. Even though the non–universality is generated radiatively from the GUT scale to the electroweak scale the parameter \(s_\zeta\) can be of order 1, that is to say, the misalignement can be maximum at the electroweak scale. In spite of this one can get a light tau neutrino mass, from 100 eV to 18 MeV, whithout a suppression on the ratio \(\mu_3/\mu\), but if we require a lighter neutrino the ratio \(\mu_3/\mu\) needs to be supressed. Therefore if the tau neutrino is so light the ratio \(\mu_3/\mu\) at high energy has to be supressed too, thus giving us information about the correct GUT model. Finally we must point out that we can obtain a tau neutrino mass one order of magnitude less if we set a larger value for \(M_{1/2}\).

**Calculation of \(\sin \zeta\) in the three generation model.** We must add some comments about to what extent the results would still be valid in a three generation model. Since the trilinear terms can be very “large” we cannot ignore them because they contribute radiatively to the tau neutrino mass. If we assume that there is no cancellation between the radiative and tree level contributions to the tau neutrino mass we can constrain the R–parity violating effects of the bilinear terms in the same way as in the one generation model, i.e., through the parameter \(\sin \zeta\). A crucial point is if the relation between the tau neutrino mass and the SUGRA parameter space deduced from the minimization equations is modified. In a three generation model we can write four of the five minimization equations in matricial form:

\[
M^2 \bar{\nu} + (\bar{\mu} \cdot \bar{\nu}) \bar{\mu} - v_u E^2 + g_2^2 v^2 \bar{\nu} = 0
\]
where $\vec{\mu}, \vec{v}$ and $\vec{E}$ represent four vectors under rotation of the leptonic fields and $M^2_L$ is the $4 \times 4$ soft mass matrix. Rotating to the basis II) (where $\mu_i$ terms are zero) we can see that we obtain a system of four linear equations for the sneutrino vevs in the basis II) and from this we can find an expression for $\sin(\zeta)$ in a three generation model. This relation is more complex analytically but of the same form as the relation in the one generation model, i.e., this relation contains terms $\Delta B_i = B_i - B_0$, terms $\Delta M^2_i = M^2_{L_i} - M^2_{L_0}$ and the ratios $\mu_i/\mu$. Therefore the reasoning is similar to that in the one generation model: the parameters $\Delta B_i$ and $\Delta M^2_i$ are not very small at the weak scale because of the perturbative bound on $h_{top}$, thus we need to constrain the ratios $\mu_i/\mu$ if we require the tree level contribution to tau neutrino mass be small.

VII. R–PARITY VIOLATING EFFECTS IN THE SCALAR SECTOR

FIG. 5. Possible Values for $\delta_A$ as a function of the invariant $s_\gamma = (\mu_3/\mu)^I$
R–parity violation induces mixing effects between neutral higgses and sneutrinos [16,56] and between charged higgses and sleptons [17,18]. We have shown in the previous section that the tau neutrino mass is proportional to \( \sin \zeta \) and \( \lambda_3^D \). One may question if this also implies that the effects in the scalar sector tend to zero as the tau neutrino mass tends to zero, or in other words, we wish to establish the relation between the tau neutrino mass and the R–parity violating effects in the scalar sector. In order to find this relation we rotate the \( 3 \times 3 \) CP-odd neutral scalar mass matrix to the basis \( \Pi \) (where \( \mu_3^I = 0 \)) and we find:

\[
M_{A^0}^2 = \begin{bmatrix}
    m_{u}^2 - g_Z^2 v_Y^2 & \left( \frac{\mu_3^2}{\mu} B_0 + \frac{\mu_2^2}{\mu} B_3 \right) & \frac{\mu_0 \mu_3}{\mu} \Delta B \\
    \left( \frac{\mu_2^2}{\mu} B_0 + \frac{\mu_3^2}{\mu} B_3 \right) & m_{00}^2 + g_Z^2 v_Y^2 & \frac{\mu_0 \mu_3}{\mu} \Delta M^2 \\
    \frac{\mu_0 \mu_3}{\mu} \Delta B & \frac{\mu_0 \mu_3}{\mu} \Delta M^2 & m_{33}^2 + g_Z^2 v_Y^2
\end{bmatrix},
\]

where:

\[
m_{00}^2 + g_Z^2 v_Y^2 = \frac{\mu_0^2}{\mu^2} (M_L^{00})^2 + \frac{\mu_3^2}{\mu^2} (M_L^{33})^2 + \mu^2 + g_Z^2 v_Y^2 = (M_A^0) + \frac{\mu_3^2}{\mu^2} \Delta M^2
\]

\[
m_{33}^2 + g_Z^2 v_Y^2 = \frac{\mu_3^2}{\mu^2} (M_L^{33})^2 + \frac{\mu_3^2}{\mu^2} (M_L^{00})^2 + g_Z^2 v_Y^2 = (M_{\nu}^0) - \frac{\mu_3^2}{\mu^2} \Delta M^2 \tag{40}
\]

Here \( M_{\nu}^0 \) and \( M_A^0 \) coincide with the MSSM expressions. One can easily show for the CP–even mass matrix and the charged scalar mass matrix that the R–parity violating mixing terms are of the form: \( \mu_3 \mu_0 \Delta B / \mu, \mu_3 \mu_0 \Delta M^2 / \mu^2 \). In addition there are new terms proportional to \( g_Z^2 v^2 s_\zeta \). We prefer to use the CP–odd scalar mass matrix for the sake of simplicity although the conclusions would be the same if we were to study the other scalar mass matrices. We find the following tree level formula for the mass of the CP-odd and for the imaginary part of the stau neutrino field in the R–parity violating model:

\[
M_{A^0,\nu}^R = \frac{1}{2} \left\{ (M_A^0 + M_{\nu}^0) \pm \left( M_A^0 - M_{\nu}^0 + 2 \frac{\mu_3^2}{\mu^2} \Delta M^2 \right) \left( 1 + \frac{2 \left( \frac{\mu_3 \mu_0}{\mu^2} \Delta M^2 \right)^2}{(M_A^0 - M_{\nu}^0 + 2 \frac{\mu_3^2}{\mu^2} \Delta M^2)} \right)^{1/2} \right\} \tag{41}
\]

We define two useful parameters \( \delta_A \) and \( \delta_{\nu} \) as:

\[
\delta_{\nu} = \frac{\mu_3^2}{\mu^2} \frac{\Delta M^2}{M_{\nu_3}^0}, \quad \delta_A = \frac{\mu_3^2}{\mu^2} \frac{\Delta M^2}{M_A^0}
\]

In the limit \( \delta_{A,\nu} \to 0 \) we find approximate formulas which give us the mass shifts from their MSSM counterparts:

\[
(M_{A^0,\nu}^0)^R = (M_{A,\nu_3}^0) \left\{ 1 \pm \delta_A \nu_3 \left( 1 + \frac{\mu_0^2}{\mu^2} \frac{\Delta M^2}{(\mu^2 - \Delta M^2)} \right) \right\} \tag{42}
\]

Our main task is to find the relation between the parameters \( \delta_{A,\nu} \) and the tau neutrino mass. As is obvious these parameters are very similar to the soft parameter \( \delta_M \) which appeared in the calculation of \( s_\zeta \) through the minimization equations. To get a more exact idea of
the values the parameters $\delta_{A\tilde{\nu}}$ can reach we make use of the numerical solutions. In figure (3) we show the value of $\delta_A$ for three different values of $t_\beta$, fixing the SUGRA parameters $M_0$, $A_0$ and $M_{1/2}$ as is indicated in the figure. In figure (3) the maximum values of $\delta_A$ are obtained with the laboratory bound on the $\nu_\tau$ mass. As we can see, the R–parity violating effects can be large if we assume both $t_\beta$ and the ratio $\mu_3/\mu$ to be large. Therefore large R–parity violating effects are related to neutrino masses of the order 18 MeV. In contrast, if we assume the minimum value for $t_\beta$ ($t_\beta = 2$) the maximum values for $\delta_A$ are of the order $10^{-3}$, even setting $\mu_3/\mu = 1$. If we assume a neutrino mass of the order 10 eV R–parity violating effects in the scalar sector are in any case negligible, $\delta_A \leq 10^{-6}$.

VIII. CONCLUSIONS

The phenomenology of supersymmetric R–parity violating models has recently attracted a lot of attention [45–55]. These studies make use of different basis to parametrize R–parity violating effects. Some of the above references get bounds on sneutrino vevs or lepton number violating $\mu$–terms while the others put bounds on the trilinear couplings. In this work we have shown that these bounds are misleading unless we indicate the basis in which we are working. We advocated, following [3], the use of basis independent parameters which clearly show the magnitude of R–parity violating effects. As an application we have derived experimental constraints on $\tilde{\nu}_\tau$ vev from $Z \rightarrow b\bar{b}$, double beta decay and the bounds on $\nu_e$ mass. We have shown that R–parity violating effects in both scalar and fermionic sectors are controlled by the tau neutrino mass. By making use of the minimization equations we find a relation between R–parity violating effects in the scalar sector and the tau neutrino mass. We have studied the prediction for the tau neutrino mass in a SUGRA inspired scenario with universality of soft parameters at the GUT scale and we have found that if we demand a very light tau neutrino ($\leq$eV) the ratio $\mu_3/\mu$ has to be suppressed, although if one allows masses up to the laboratory bound limit the ratio $\mu_3/\mu$ is unsuppressed. We have complemented our work with a numerical study making use of the renormalization group equations of this model and we have compared the numerical and the analytical predictions.

IX. ACKNOWLEDGEMENTS

I would like to thank Diego Restrepo for the verification of all the numerical results in this paper. I would like to thank M.A.Diaz, A.Akeroyd and C.Savoy for beneficial discussions. This work was supported by DGICYT under grants PB95-1077 and by the TMR network grant ERBFMRXCT960090 of the European Union. The author is supported by a Spanish fellowship FPI of Ministerio de Educaci´on y Ciencia.

X. APPENDIX.

In this appendix we give the most relevant renormalization group equations for this work. We make use of a condensed notation, where $\alpha = 0, 3$. The RGE’s for trilinear couplings
can be found in [14] and the RGE’s for soft parameters can be found in [42]. For the trilinear interactions (Yukawas) of the superpotential one has:

\[
16\pi^2 \frac{dh_t}{dt} = h_t \left( 6h_t^2 + (\lambda^D_0)^2 + (\lambda^D_3)^2 - \left( \frac{13}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2 \right) \right) \quad (43)
\]

\[
16\pi^2 \frac{d\lambda^D}{dt} = \lambda^D \left( h_t^2 + h^2_\tau + \sum_{\alpha=0,3} 6(\lambda^D_\alpha)^2 - \left( \frac{7}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2 \right) \right) \quad (44)
\]

\[
16\pi^2 \frac{dh^\tau}{dt} = h^\tau \left( 3(\lambda^D_0)^2 + 4h^2_\tau + 3(\lambda^D_3)^2 - \left( \frac{9}{5}g_1^2 + 3g_2^2 \right) \right) \quad (45)
\]

The RGE’s for the soft masses relevant for our work are (if we make use of two greek indices at the same time we assume they are different)

\[
8\pi^2 \frac{dM^2_{\alpha\beta}}{dt} = h^2_t \left( \sum_{\beta=0,3} M^2_{\alpha\beta} + M^2_R + A_{\tau}^2 \right) + 3(\lambda^D_\alpha)^2(M^2_{\alpha\alpha} + M^2_Q + M^2_D + A_{\beta}^2) +
\]

\[
3\lambda^D_\alpha \lambda^D_\beta M^2_{\alpha\beta} - \left( \frac{3}{5}g_1^2M_1^2 + 3g_2^2M_2^2 \right) - \frac{3}{10}g_3^2S \quad (46)
\]

\[
16\pi^2 \frac{dM^2_{\alpha\beta}}{dt} = (h^2_t + 3\lambda^2_\beta M^2_{\alpha\beta} + (3(\lambda^D_\alpha)^2 - h^2_\tau)M^2_{\alpha\alpha} +
\]

\[
3\lambda^D_\alpha \lambda^D_\beta (M^2_{\alpha\alpha} + M^2_Q + 2M^2_D + 2M^2_Q + 2A_{\alpha}^D A_{\beta}^D) \quad (47)
\]

Where \( S = (M^2_{\alpha\alpha} - M^2_{L_0} - M^2_{L_3} + M^2_Q - 2M^2_D + M^2_R + M^2_R) \). For the bilinear terms in the superpotential:

\[
16\pi^2 \frac{d\mu_\alpha}{dt} = \mu_\alpha \left( 3h^2_t + 3(\lambda^D)^2 + h^2_\tau - \left( \frac{3}{5}g_1^2 + 3g_2^2 \right) \right) + 3\lambda^D_\alpha \lambda^D_\beta \mu_\beta \quad (48)
\]

And for the soft bilinear terms:

\[
16\pi^2 \frac{dE_\alpha}{dt} = E_\alpha \left( 3h^2_t + 3(\lambda^D)^2 + h^2_\tau - \frac{3}{5}g_1^2 - 3g_2^2 \right) +
\]

\[
\mu_\alpha \left( 6h^2_t A_t + 6(\lambda^D_\alpha)^2 A_{\alpha}^D + 2h^2_\tau A_\tau + \frac{6}{5}g_1^2 M_1 + 6g_2^2 M_2 \right) + 3\lambda^D_\alpha \lambda^D_\beta \left( E_\beta + 2\mu_\beta A_{\alpha}^D \right) \quad (49)
\]

As usual \( g_i \) denote the gauge couplings \( SU(3)_C \times SU(2)_L \times U(1)_Y \) (we use unification normalization: \( g_1 = \frac{2}{3}g_Y \)) and \( M_i \) are gaugino mass parameters. The rest of RGE’s for: \( A_t, A_0^D, A_\tau, A_\alpha^D \) and \( M^2_Q, M^2_U, M^2_D, M^2_R \) and \( M^2_{\alpha\alpha} \) can be computed directly from the general formulas [42].
REFERENCES

[1] Sacha Davidson and John Ellis, Phys.Lett. B390 (1997) 210-220.
[2] Sacha Davidson and John Ellis, Phys.Rev. D56 (1997) 4182-4193.
[3] Sacha Davidson. [hep-ph/9808425]
[4] C.S. Aulakh and R.N. Mohapatra, Phys.Lett. 119B (1982) 136.
[5] L.J. Hall and M. Suzuki, Nucl.Phys. B231 (1984) 419.
[6] I.-H. Lee, Phys. Lett. 138B (1984) 121; Nucl. Phys. B246 (1984) 120.
[7] E. Nardi, Phys. Rev. D55 (1997) 5772-5779.
[8] Herbi Dreiner. Published in Perspectives on Supersymmetry (G.Kane ed. World Scientific 1998). [hep-ph/9707435]
[9] J. Alberto Casas. Published in Perspectives on Supersymmetry (G.Kane ed. World Scientific 1998). [hep-ph/9707473]
[10] S. A. Abel and C. A. Savoy. [hep-ph/9803218]
[11] H.P. Nilles and N. Polonsky Nucl. Phys. B484 (1997) 33-62.
[12] R. Hempfling, Nucl. Phys. B478 (1996) 3-30.
[13] Marek Nowakowski and Apostolos Pilaftsis. Nucl. Phys. B461 (1996) 19-49.
[14] Anjan S. Joshipura and Marek Nowakowski. Phys. Rev. D51 (1995) 2421.
[15] M. A. Diaz, Jorge C. Romao, Jose W.F. Valle, Nucl.Phys.B524:23-40,1998
[16] F.de Campos, M.A. Garcia-Jareño, A.S. Joshipura, J. Rosiek and J.W.F. Valle, Nucl.Phys.B451 (1995) 3-15
[17] Sourov Roy and Biswarup Mukhopadhyaya. Phys. Rev. D55 (1997) 7020-7029.
[18] A. Akeroyd and Marco A. Diaz and J. Ferrandis and M. A. Garcia-Jareño and Jose W. F. Valle. Nucl. Phys. B529 (1998) 3-22. [hep-ph/9707395]
[19] Fukuda Y. et al. Phys. Rev. Lett. 81 (1998) 1562-1567.
[20] Fukuda Y. et al. Phys. Rev. Lett. 81 (1998) 1158-1162.
[21] Barate R. et al., ALEPH Collaboration, Eur.Phys.J.C2 (1998) 395-406.
[22] Ackerstaff K. et al., OPAL Collaboration, Eur.Phys.J.C5 (1998)2, 229-237.
[23] V. Barger and T. J. Weiler and K. Whisnant. [hep-ph/9808367]
[24] Otto C.W. Kong [hep-ph/9808304]
[25] Biswarup Mukhopadhyaya and Sourov Roy and Francesco Vissani. [hep-ph/9808268]
[26] Vadim Bednyakov and Amand Faessler and Sergey Kovalenko. [hep-ph/9808224]
[27] E. J. Chun and S. K. Kang and C. W. Kim and U. W. Lee. [hep-ph/9807327]
[28] Amand Faessler and Sergey Kovalenko and Fedor Simkovic, Phys. Rev. D58 (1998).
[29] Anjan S. Joshipura and Sudhir K. Vempati. [hep-ph/9808232]
[30] Anjan S. Joshipura and V. Ravindran and Sudhir K. Vempati. [hep-ph/9706482]
[31] M. Bisset , O.C.W. Kong , C. Macesanu and L.H. Orr, Phys. Lett. B430 (1998) 274-280.
[32] M. A. Diaz, Talk given at International Workshop on Physics Beyond the Standard Model: From Theory to Experiment (Valencia 97), Valencia, Spain, 13-17 Oct 1997, [hep-ph/9802407]
[33] M. A. Diaz, Talk given at International Europhysics Conference on High-Energy Physics (HEP 97), Jerusalem, Israel, 19-26 Aug 1997. [hep-ph/9712213]
[34] F.M. Borzumati, Y. Grossman, E. Nardi and Y. Nir, Phys. Lett. B384 (1996) 123.
[35] T. Banks, Y. Grossman, E. Nardi, and Y. Nir, Phys. Rev. D52 (1995) 5319.
[36] G. Bhattacharyya, J. Ellis and K. Sridhar, Mod. Phys. Lett. A10 (1995) 1583.
[37] G. Bhattacharyya, D. Choudhury and K. Sridhar, Phys. Lett B355 (1995) 193-198.
[38] S. Dimopoulos and L.J. Hall, Phys. Lett. B207 (1987) 210.
[39] K. S. Babu and R. N. Mohapatra. Phys. Rev. Lett. 75 (1995) 2276-2279.
[40] R. Barbieri, M.M. Guzzo, A. Masiero, and D. Tommasini, Phys. Lett. B252 (1990) 251.
[41] E. Roulet and D. Tommasini, Phys. Lett. B256 (1991) 218.
[42] B. de Carlos and P.L. White, Phys. Rev. D54 (1996) 3427.
[43] V. Barger, M.S. Berger, R.J.N. Phillips and T. Wohrmann Phys. Rev. D53 (1996) 6407-6415.
[44] H. Dreiner and H. Pois. [hep-ph/9511444]
[45] Tai-fu Feng. [hep-ph/9806505]
[46] Tai-fu Feng. [hep-ph/9808379]
[47] Marco A. Diaz and E. Torrente-Lujan and J. W. F. Valle [hep-ph/9808412]
[48] S. Bar-Shalom and G. Eilam and J. Wudka and A. Soni [hep-ph/9809253]
[49] Anjan S. Joshipura and A. Yu. Smirnov. [hep-ph/9806376]
[50] Dafne Guetta and Jesus M. Mira and Enrico Nardi, [hep-ph/9806359]
[51] A. Wodecki and Wieslaw A. Kaminski [hep-ph/9806288]
[52] J. Hisano [hep-ph/9806222]
[53] S. Bar-Shalom and G. Eilam and A. Soni [hep-ph/9804339]
[54] M. Carena and D. Choudhury and S. Lola and C. Quigg. [hep-ph/9804380]
[55] M. Carena and S. Pokorski and C. E. M. Wagner. Phys. Lett. B430 (1998) 281-289.
[56] Yuval Grossman and Howard E. Haber. Phys. Rev. Lett. 78 (1997) 3438-3441