The Study on Mathematical Model and Simulation of Asynchronous Motor Considering Iron Loss

Zhen GUO ¹,² and Qing-wei ZHANG²
¹College of Information and Control Engineering, China University of Petroleum(East China), Qingdao, 266580, China
²College of Automation Engineering, Qingdao University of Technology, Qingdao, 266520, China
qs2004b@163.com

Abstract. When the dynamic mathematical model of asynchronous motor is established, it usually ignores the motor iron loss. Actually, the motor iron loss exists and it has a great influence on the performance of the motor, especially on the efficiency. A motor model considering iron loss is built in this paper and compared with the motor model ignoring iron loss by Simulink. The paper provides a practical model that has high-performance control and it is closer to the actual physical behaviour of asynchronous motors. The simulation results verify the correctness, validity and accuracy of the model and the conclusions provide a theoretical reference for the research of asynchronous motor and the efficiency optimal design of its control system.

1. Introduction
With the development of AC motor control theory and power electronics technology, motor control system is becoming more and more complex [1,2]. Computer simulation has increasingly become an indispensable means of control system research. The simulation model of the control object is very important to the simulation research of the control system, and the accuracy of the model directly affects the reliability of the simulation results. MATLAB software is a popular powerful simulation tool at present. It also provides great convenience for modeling and simulation of electrical system [3]. However, the simulation model of asynchronous motor provided by the module library ignores iron loss of the motor. When it is used to study the performance of the motor related to efficiency, it is bound to reduce the simulation precision and it can not be used for the accurate simulation of asynchronous motor vector control [4-6]. Therefore, it is necessary to establish a more accurate model of asynchronous motor. The structure of the model should be simple, easy to implement and has strong stability for simulation research. In the text, the dynamic mathematical model of asynchronous motor considering iron loss in two phase stationary alpha and beta coordinate system is derived based on mathematical analysis. Finally, two models are compared with the simulation examples to verify the feasibility of the asynchronous motor model considering the iron loss.

2. Mathematical model of asynchronous motor ignoring iron loss
In the two-phase stationary α, β coordinate system, the voltage equations of asynchronous motor are written as
The flux equations are given as
\[
\begin{align*}
\psi_{\alpha} &= L_s i_{\alpha} + L_m i_{\beta} \\
\psi_{\beta} &= L_s i_{\beta} + L_m i_{\alpha} \\
\psi_{\alpha} &= L_r i_{\alpha} + L_m i_{\alpha} \\
\psi_{\beta} &= L_r i_{\beta} + L_m i_{\beta}
\end{align*}
\] (2)

The torque equation is given as
\[
T_e = n_p L_m (i_{\beta} i_{\alpha} - i_{\alpha} i_{\beta}) = n_p (\psi_{\alpha} i_{\beta} - \psi_{\beta} i_{\alpha})
\] (3)

The equation of motion is given as
\[
p \omega_r = \frac{n_p}{J} (T_e - T_L)
\] (4)

Where the \( U_{s\alpha} \) and \( U_{s\beta} \) are the stator \( \alpha \)-axes and \( \beta \)-axes input voltages; \( i_{s\alpha} \) and \( i_{s\beta} \) are the \( \alpha \)-axes and \( \beta \)-axes stator current; \( i_{\alpha} \) and \( i_{\beta} \) are the \( \alpha \)-axes and \( \beta \)-axes rotor current; \( \psi_{s\alpha} \) and \( \psi_{s\beta} \) are the \( \alpha \)-axes and \( \beta \)-axes stator fluxes; \( \psi_{\alpha} \) and \( \psi_{\beta} \) are the \( \alpha \)-axes and \( \beta \)-axes rotor fluxes; \( R_s \) is the stator resistance; \( R_r \) is the rotor resistance; \( L_m \) is the magnetizing inductance, \( L_s \) is the stator inductance, and \( L_r \) is the rotor inductance; \( \omega_r \) is the rotor angular frequency; \( n_p \) is the number of pole-pairs; \( J \) is the moment of inertia; \( T_L \) is the load torque and \( T_e \) is the electromagnetic torque; \( p \) is the differential operator. The dynamic mathematical model of asynchronous motor ignoring iron loss in \( \alpha \beta \) stationary coordinate can be composed from equation (1) to equation (4).

3. Mathematical model and equivalent circuit of asynchronous motor considering iron loss

The iron loss of motor is related to core structure parameters, voltage and frequency and flux density. In the dynamic model, if the iron loss is to be considered, the loss of the motor iron loss can be expressed by the loss of an equivalent pure resistance winding. Usually, the stator iron loss is the main factor in the asynchronous motor, so the equivalent winding of the stator is assumed only on the stator. In the \( \alpha \beta \) stationary coordinate system, two equivalent windings of iron loss are added to the stator side except the four windings on the original stator and rotor axis. The voltage equations are written as equation (5).
\[
\begin{align*}
U_{s\alpha} &= R_s i_{s\alpha} + p \psi_{s\alpha} \\
U_{s\beta} &= R_s i_{s\beta} + p \psi_{s\beta} \\
0 &= R_r i_{\alpha} + \omega_r \psi_{\beta} + p \psi_{\alpha} \\
0 &= R_r i_{\beta} - \omega_r \psi_{\alpha} + p \psi_{\beta} \\
0 &= R_m i_{\alpha} - p \psi_{m\alpha} \\
0 &= R_m i_{\beta} - p \psi_{m\beta}
\end{align*}
\] (5)

The motor excitation current equations are given as
\[
\begin{align*}
i_{m\alpha} &= i_{s\alpha} + i_{\alpha} - i_{Rm\alpha} \\
i_{m\beta} &= i_{s\beta} + i_{\beta} - i_{Rm\beta}
\end{align*}
\] (6)

The relationships between magnetic flux and current are as follows
The torque equation is given as
\[
T_e = \frac{L_m}{L_r} \left[ \psi_{ra} \left( i_{\beta} - i_{Rm\beta} \right) - \psi_{r\beta} \left( i_{\alpha} - i_{Rm\alpha} \right) \right]
\] (8)

The equation of motion is given as
\[
p\omega_r = \frac{1}{J} \left( T_e - T_L \right)
\] (9)

Where \(i_{Rm\alpha}\) and \(i_{Rm\beta}\) are the \(\alpha\)-axes and \(\beta\)-axes equivalent iron loss resistance current; \(i_{l_{ma}}\) and \(i_{l_{mb}}\) are the \(\alpha\)-axes and \(\beta\)-axes excitation current; \(R_m\) is the equivalent iron loss resistance; \(\psi_{ma}\) and \(\psi_{mb}\) are the \(\alpha\)-axes and \(\beta\)-axes main flux.

The dynamic mathematical model of asynchronous motor considering iron loss in \(\alpha\beta\) stationary coordinate can be composed from equation (5) to equation (9) and the equivalent circuit as shown in Figure 1.

**Figure 1.** The equivalent circuit of asynchronous motor considering iron loss in \(\alpha\beta\) stationary coordinate system

4. Simulation Results and Analysis

The simulation model of asynchronous motor considering iron loss according the equation (5) to equation (9) is established and it is shown in Figure 2.

Simulation experiment 1: set asynchronous motor to start directly under rated voltage and rated load. The electromagnetic torque waveform, the angular velocity waveform of the rotor and the \(\alpha\) component waveform of the stator current are shown in Figure 3.

Simulation experiment 2: set asynchronous motor to start under rated voltage and rated load at 0.2s. The electromagnetic torque waveform of the starting process, the rotor angular velocity waveform and the \(\alpha\) component waveform of the stator current are shown in Figure 4.

It can be seen that the motor model considering iron loss is more stable and less oscillation than the motor model without considering iron loss. The inertia of the system is larger and the starting speed is slower. The stator current amplitude is larger in the steady state than that without considering iron loss.
Figure 2. The simulation model of asynchronous motor considering iron loss. A three-phase squirrel cage asynchronous motor with a triangular connection is used in simulation. The parameters of the motor are shown in Table 1.

Table 1. Test motor parameters

| parameters                          | values          |
|-------------------------------------|-----------------|
| Rated voltage                       | 220V            |
| Rated shaft power                   | 80W             |
| Rated frequency                     | 50Hz            |
| Stator resistance                   | 72Ω             |
| Stator leakage inductance           | 0.26H           |
| Magnetizing inductance              | 1.64H           |
| Stator inductance                   | 1.9H            |
| Rated load                          | 0.5N · m        |
| Rated current                       | 0.78A           |
| Rated speed                         | 1400r/min       |
| Equivalent iron loss resistance     | 2400Ω           |
| Rotor resistance                    | 70Ω             |
| Rotor leakage inductance            | 0.26H           |
| Number of the pole pairs            | 2               |
| Rotor inductance                    | 1.9H            |
| Rotor inertia                       | 0.0005kg · m²   |
Figure 3. Simulation waves of setting asynchronous motor start directly under rated condition.
The rotor angular velocity waveform

(c) The $\alpha$ component waveform of the stator current

**Figure 4.** Simulation waves of asynchronous motors load applied at 0.2s

5. Conclusion
The rated operation efficiency is relatively low and the iron loss is great for such small power asynchronous motor. The traditional model which ignores the iron loss will bring more inaccuracy. The difference between the two models is compared by the simulation of the MATLAB/Simulink and simulation results provide that the mathematical model of the motor considering iron loss is more accurate and valid than the mathematical model ignoring the iron loss.

References
[1] Shiyi Shao, Abdi. E, and McMahon. R., Vector Control of The BDFM for Wind Power Generation, IEEE International Conference on Sustainable Energy Technologies, 2008, pp. 322-327.
[2] Christian Kral, and Anton Haumer, Consistent equivalent circuit parameters of induction motors for the calculation of partial load efficiencies, IEEE Industrial Electronics, 2008, pp. 698-705.
[3] Yao Jun, and Ma Songhui, Simulink modeling and simulation, Xi'an Electronic and Science University press, Xi'an, 2002.
[4] Mohan P.R., Reddy T.B., Kumar M.V. A Simple Generalized Pulse Width Modulation Algorithm for Vector Control based Voltage Source Inverter fed Induction Motor Drives [C]//Emerging Research Areas: Magnetics, Machines and Drives, 2014 Annual International Conference on, IEEE, 2014, pp 1-5.
[5] Mark A.P., Irudayaraj G.C.R., Vairamani R., et al. Dynamic Performance Analysis for Different Vector-Controlled CSI- Fed Induction Motor Drives, Journal of Power Electronics, 2014, 14(5), pp1-11.
[6] Baby B., Shajilal A.S. An Improved Indirect Vector Controlled Current Source Inverter Fed Induction Motor Drive With Rotor Resistance Adaptation, Emerging Research Areas: Magnetics, Machines and Drives, 2014 Annual International Conference on, IEEE, 2014, pp 1-6.