Anomalous $AV^*V$ Green’s function in soft-wall AdS/QCD

Juan José Sanz-Cillero

Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Italy E-mail: juan.sanzcillero@ba.infn.it

In this talk we study the Green’s function of two vector and one axial-vector currents within the soft-wall anti-de-Sitter (AdS) model of Quantum Chromodynamics (QCD), with a quadratic dilaton and chiral symmetry broken through a field $X$ which gains a vacuum expectation value. We compare our predictions at high energies with the Operator Product Expansion both in the massless quark limit and for $m_q \neq 0$. The soft-wall model yields a zero magnetic susceptibility $\chi = 0$ and some problems are found in the case with $m_q \neq 0$. We also discuss the relation proposed by Son and Yamamoto between the $AV^*V$ and $VV - AA$ correlators, which is not obeyed at high energies in soft wall AdS/QCD.

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1. Introduction: $AV^V$ Green’s function

The $AV^V$ Green’s function was recently studied in the framework of soft-wall anti-de-Sitter (AdS) theories [1]. This analysis was motivated by a previous work by Son and Yamamoto [2] for holographic theories where chiral symmetry is broken through boundary condition [3]. In Ref. [2], the authors found an interesting relation between the $VV - AA$ correlator and the Green’s function involving two vector currents $J_\mu = \bar{q} \gamma_\mu q$ and $J^e_{\sigma} = \bar{q} Q_\sigma q$ and an axial-vector current $J^5_v = \bar{q} A_v \gamma_5 q$, with $V$ and $A$ diagonal matrices and the electric charge matrix $Q$:

$$T_{\mu\nu}(q,k) = i \int d^4 x e^{iq\cdot x} \langle 0 | T[J_\mu(x)J^5_v(0)] | \gamma(k,\varepsilon) \rangle = -\frac{iQ^2}{4\pi} \text{Tr}[QVA] P^{\alpha\beta}_\mu(q) \left\{ P^{T\beta}_\nu(q) w_T(Q^2) + P^{L\beta}_\nu(q) w_L(Q^2) \right\} \left[ \gamma_{\alpha\beta} \right],$$

with $k \to 0$ and related to the three-point Green’s function $\langle 0 | T[J_\mu(x)J^5_v(y)J^e_{\sigma}(z)] | 0 \rangle$. We use the notation $Q^2 \equiv -q^2$, $f_\mu = \frac{1}{2} \epsilon_{\mu
u\alpha\beta} f^{\alpha\beta}$ and $f^{\alpha\beta} = k^\alpha \epsilon^\beta - k^\beta \epsilon^\alpha$, and the transverse and longitudinal projectors, respectively, $P^{T\beta}_\nu(q) = \eta_{\nu\alpha} - q_{\nu}q_\alpha/q^2$ and $P^{L\beta}_\nu(q) = q_{\nu}q_\alpha/q^2$.

At short-distance it is possible to use the Operator Product Expansion (OPE) for $m_q = 0$ [4, 5, 6]:

$$w_L(Q^2) = \frac{2N_C}{Q^2}, \quad w_T(Q^2) = \frac{NC}{Q^2} + \frac{128\pi^2\alpha_s \chi(q)^2}{9Q^6} + \mathcal{O}\left(\frac{\Lambda^4}{Q^8}\right).$$

where the longitudinal component is completely fixed by the anomaly and does not receive any correction [4, 5, 6] and $\chi$ is defined by the condensate $\langle 0 | \bar{q} \sigma^{a\beta} q | \gamma \rangle = i \epsilon \chi \langle 0 | \bar{q} q | 0 \rangle f^{a\beta}$.

If we allow $m_q \neq 0$, the OPE yields corrections proportional to the quark mass at one loop [6]:

$$w_L(Q^2) - 2w_T(Q^2) = \mathcal{O}\left(\frac{\Lambda^4}{Q^8}\right), \quad w_T(Q^2) = \frac{NC}{Q^2} \left[ 1 + \frac{m_q^2}{Q^2} \ln \frac{m_q^2}{Q^2} - \frac{8\pi^2m_q \langle \bar{q} q \rangle \chi}{NCQ^2} + \mathcal{O}\left(\frac{\Lambda^4}{Q^4}\right) \right].$$

(1.3)

2. The holographic setup in AdS/QCD

We will consider a gauged $U(n_f)_R \otimes U(n_f)_L$ chiral symmetry and the AdS line element $ds^2 = g_{MN} dx^M dx^N = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$, with the coordinate indices $M, N = 0, 1, 2, 3, 5, \eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1, -1)$, $R$ the AdS curvature radius (set to unity from now on) and the 5D coordinate being in the range $0^+ < z < +\infty$. The 5D Yang-Mills action describing the fields $A^M_{L,R}$ dual to the left and right currents $J^\mu_{L,R}$, as well as the scalar-pseudoscalar field $X$, is given by

$$S_{YM} = \frac{1}{kYM} \int d^5 x \sqrt{g} e^{-\Phi} Tr \left\{ |DX|^2 - \mathcal{Y}(X) - \frac{1}{4g_s^2} (F^2_L + F^2_R) \right\},$$

(2.1)

with the field strength tensors $F^M_{L,R} = F^{MN\alpha} T^\alpha$, $T^a$ the $U(n_f)$ group generators, the $X$ field potential $\mathcal{Y}(X)$ and $g$ the determinant of the metric tensor $g_{MN}$. We take the quadratic dilaton background $\Phi(z) = (cz)^2$, chosen in order to recover linear Regge trajectories for vector resonances, and $kYM$ is a parameter included to provide canonical 4d dimensions for the fields. The covariant derivative acting on $X$ is defined as $D^M X = \partial^M X - iA^M_{L,R} X + iX A^M_{L,R}$. The gauge fields $A^M_{L,R}$ are usually
combined into a vector field \( V^M = \frac{\partial^M}{\partial y} + \frac{\partial^M}{\partial x} \) and an axial-vector field \( A^M = \frac{\partial^M}{\partial y} - \frac{\partial^M}{\partial x} \). The study of the vector and scalar correlators at high energies allows one to fix the constants in the Yang-Mills action: \( k_{YM} = \frac{16\pi^2}{N_c} \) and \( g_s^2 = \frac{3}{4} \). In this kind of approaches \([7]\), one introduces a spinless field \( X \) which is dual to the quark bifundamental operator \( \bar{q}^\alpha R q^\beta L \). This field gains the v.e.v. \( X = \frac{v(y)}{2} e^{2\pi} \) \([7]\). Chiral symmetry becomes broken when \( v(y) \neq 0 \), as the left and right sectors of the theory get connected to each other. Moreover, a phase-shift \( \pi \) is induced for the v.e.v. in the bulk when the parallel axial-vector source is switched on: \( \pi \) gets coupled to \( A^\parallel \) in the equations of motion (EoM). Thus, for the bulk to boundary (B-to-b) propagators one finds the EoM, within the gauge

\[
\partial_y \left( \frac{e^{-y^2}}{y} \partial_y V_\perp \right) - \bar{Q}^2 e^{-y^2} V_\perp = 0, \quad \partial_y \left( \frac{e^{-y^2}}{y} \partial_y A_\perp \right) - \bar{Q}^2 e^{-y^2} A_\perp - \frac{g_s^2 y^2}{y^3} (\pi - A_\perp) = 0
\]

with \( y \equiv cz \) and \( \bar{Q}^2 = Q^2/c^2 \). In momentum space the 5D fields \( \bar{q}^\alpha (q,y) = -i \frac{\partial}{\partial y} A_\mu^\parallel(q,y) \) and \( \pi(q,y) \) are respectively related to the B-to-b propagators \( A_\parallel(q,y) \) and \( \pi(q,y) \) \([7]\).

The vector EoM can be analytically solved \([7]\), but for the remaining EoM one needs to specify the v.e.v. \( v(y) \). Its asymptotic behaviour close to the UV brane \( y \rightarrow 0 \) in our choice of coordinates is related to the explicit (quark mass \( m_q \)) and spontaneous chiral symmetry breaking (quark condensate \( \sigma \propto \langle \bar{q}q \rangle \) in massless QCD):

\[
v(y) \approx 0 \frac{m_q}{c} + \frac{\sigma}{c^3} y^3 + \mathcal{O}(y^4), \quad (2.3)
\]

where the first terms of its power expansion in \( y \) determine the behaviour of \( w_{TL} \) at high-energies \([7]\).

The QCD chiral anomaly will be provided by the Chern-Simons action and, more precisely, the \( AV^3 V \) amplitude studied here will be provided by the piece \([7]\)

\[
S_{CS} \big|_{AV^3 V} = 3 \kappa_{CS} \varepsilon_{ABCDE} \int d^5 x \ Tr \left[ A_\nu^A \left( \frac{\partial}{\partial (y)} F_\nu^{BC} F_\nu^{DE} \right) \right] = 48 \kappa_{CS} d^{ab} F_{em}^\nu \int d^5 x A_\nu^a \partial_\mu^b V_\mu^e, \quad (2.4)
\]

with the group factor \( d^{ab} = Tr[Q(T^a, T^b)] \). This yields the structure functions

\[
w_{L(T)}(Q^2) = -\frac{2N_c}{Q^2} \int_0^\infty dy A_\parallel(\perp)(Q^2, y) \partial_y V_\perp(Q^2, y), \quad (2.5)
\]

The global normalization is fixed \textit{a posteriori} through \( \kappa_{CS} = -\frac{N_c}{96\pi} \) in the case with \( m_q = 0 \).

3. \( w_{TL} \) results for \( m_q = 0 \) and \( m_q \neq 0 \)

In the massless quark limit one can demonstrate that \( A_\| (Q^2, y) = 1 \) \([7]\). The perpendicular B-to-b propagators can be solved perturbatively in \( 1/\bar{Q}^2 \) in the form \( A_\| (Q^2, y) = \sum_{n=0}^\infty A_n^\| (1/\bar{Q}^2)^n \) and \( V_\perp (Q^2, y) = \sum_{n=0}^\infty V_n^\perp (1/\bar{Q}^2)^n \), with \( t \equiv yQ/c \). This yields the high-energy expansion

\[
w_L(Q^2) = \frac{2N_c}{Q^2}, \quad w_T(Q^2) = \frac{N_c}{Q^2} \left[ 1 - \frac{3\tau^2}{2Q^2} + \mathcal{O} \left( \frac{Q^8}{Q^8} \right) \right], \quad (3.1)
\]
with $\tau \simeq 2.7$ defined by the integral of Bessel functions provided in Ref. \[1\]. The parallel B-to-b propagator $A_{||} = 1$ ensures the recovery of the OPE prediction for $w_L$, which becomes fully determined by the boundary conditions. Conversely, the QCD dynamics is contained in $w_T$. The comparison with the OPE (1.3) leads to a vanishing prediction for the magnetic susceptibility $\chi = 0$.

In the case with $m_q \neq 0$, all the B-to-b propagators can be solved perturbatively in the way we did for Eq. (3.1), gaining corrections proportional to the quark mass and leading to the amplitudes

$$w_L(Q^2) = \frac{2N_c}{Q^2} \left[ 1 - (1 - \pi(Q^2, 0)) \frac{3m_q^2}{8Q^2} + \mathcal{O}\left( \frac{m_q^3}{Q^4} \right) \right],$$

$$w_T(Q^2) = \frac{N_c}{2Q^2} \left[ 1 - \frac{m_q^2}{4Q^2} + \mathcal{O}\left( \frac{m_q^3}{Q^4} \right) + \mathcal{O}\left( \frac{e^4}{Q^6} \right) \right]. \quad (3.2)$$

As $A_{||}$ and $\pi$ EoMs are coupled, the perturbative solutions for $Q^2 \to \infty$ depend on the UV boundary condition $\pi(Q^2, 0)$. The comparison of the NLO term proportional to $m_q$ with the OPE (1.3) yields again a vanishing magnetic susceptibility $\chi = 0$. The $m_q^2$ terms are more cumbersome since the recovery of the finite OPE log $m_q^2 \ln \frac{m_q^2}{Q^2}$ in $w_L(Q^2)$ requires a logarithmic dependence on $Q^2$ of the UV boundary condition $\pi(Q^2, 0)$. The transverse component of the amplitude is even more problematic as the holographic model generates an $m_q^2/Q^2$ term without logs and it is impossible to recover the finite logarithms from the OPE without including any further ingredient to the theory.

4. Checking the Son-Yamamoto relation

This work was motivated by the relation proposed by Son and Yamamoto for $m_q = 0$ \[3\] in the kind of model where chiral symmetry is broken through boundary conditions \[3\]:

$$w_T(Q^2) - \frac{N_c}{Q^2} = \frac{N_c}{F^2} \Pi_{VV- AA}(Q^2). \quad (4.1)$$

Actually, although this kind of models fulfills this relation for any energy, the left-hand and right-hand sides of (4.1) do not obey the expected OPE short distance behaviour \[3\]: $w_T(Q^2) - \frac{N_c}{Q^2} = \mathcal{O}(e^{-Q^2})$, $\Pi_{VV- AA}(Q^2) = \mathcal{O}(e^{-Q^2})$.

In the type of models where chiral symmetry is broken through a scalar-pseudoscalar field $X$ that gains a v.e.v. \[7\], one gets the right $1/Q^6$ behaviour for the $VV - AA$ correlator but the subleading corrections in the $AV^+ V$ Green’s function do not start at the expected orders \[3, 11\]:

$$w_T(Q^2) - \frac{N_c}{Q^2} = -\frac{3N_c\sigma^2\tau}{2Q^8} + \mathcal{O}(\frac{e^8}{Q^{10}}), \quad \Pi_{VV- AA}(Q^2) = -\frac{N_c\sigma^2}{10\pi^2Q^8} + \mathcal{O}(\frac{e^8}{Q^{10}}). \quad (4.2)$$

Hence, Son-Yamamoto relation (4.1) is not fulfilled in this kind of models at high energies \[1, 11\].

It is worthy to mention an interesting result: if we saturate the two Weinberg sum-rules for $w_T(Q^2) - N_c/Q^2$ stemming from the OPE \[4, 5\] through the lightest multiplet of vector and axial-vector resonances one gets the minimal hadronical approximation (MHA) \[8\],

$$w_T(Q^2) \bigg|_{\text{MHA}} - \frac{N_c}{Q^2} = -\frac{N_cM_V^2M_A^2}{Q^2(M_V^2 + Q^2)(M_A^2 + Q^2)} = \frac{N_c}{F^2} \Pi_{VV- AA}(Q^2) \bigg|_{\text{MHA}}, \quad (4.3)$$
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which fulfills the Son-Yamamoto relation (4.1). Although the MHA may lead to inaccurate short-distance determinations it provides a fair estimate of the low-energy constants [9]. This may explain the reasonable agreement for the low-energy relation $C_{\text{W}22}^W = -\frac{N_C}{32\pi^2 F L_{10}}$ [10].

5. Conclusions

We have studied the AV*V Green’s function in the soft-wall [1]. When $m_q = 0$ one has the B-to-b propagators $\pi = A_{\parallel} = 1$. This ensures the exact recovery of the longitudinal structure amplitude $w_L(Q^2) = 2N_C/Q^2$ prescribed by QCD [3, 4]. On the other hand, the transverse component corrections predicted in the soft-wall model start at $O(1/Q^8)$, producing a zero magnetic susceptibility $\chi$. This hints the need for further ingredients in our holographic description like, e.g., the inclusion of a five-dimensional field $B^{MN}$ dual to the tensor operator $\bar{q}\sigma^{\alpha\beta}q$ [11].

The case $m_q \neq 0$ brings further problems. One needs to specify the value of $\pi(Q^2, y)$ at $y \to 0$ and the study of the subleading terms in the OPE proportional to $m_q \sigma$ yields again $\chi = 0$. Thus, the problem of the $m_q$ corrections needs further understanding which might be obtained from the longitudinal part of the $\Pi_{AA}(Q^2)$ correlator.

We have also tested the Son-Yamamoto relation between the AV*V Green’s function and the $VV - AA$ correlator [2]. The hard and soft-wall models show problems at high energies and the OPE is not well recovered [1, 2]. However, the low-energy relation between even and odd-sector low-energy constants $C_{\text{W}22}^W = -\frac{N_C}{32\pi^2 F L_{10}}$ seems to be reasonably well satisfied [10].

References

[1] P. Colangelo et al., Phys. Rev. D 85 (2012) 035013.
[2] D. T. Son and N. Yamamoto, [arXiv:1010.0718 [hep-ph]].
[3] D. T. Son and M.A. Stephanov, Phys. Rev. D 69 (2004) 065020; T. Sakai and S. Sugimoto, Prog. Theor. Phys. 113 (2005) 843; 114 (2005) 1083; J. Hirn, V. Sanz, JHEP 0512, 030 (2005).
[4] A. Vainshtein, Phys. Lett. B 569 (2003) 187; A. Czarnecki, W. J. Marciano, A. Vainshtein, Phys. Rev. D 67 (2003) 073006 [Erratum-ibid. D 73 (2006) 119901].
[5] M. Knecht et al., JHEP 0403, 035 (2004).
[6] S. L. Adler, Phys. Rev. 177 (1969) 2426; J. S. Bell and R. Jackiw, Nuovo Cim. A 60 (1969) 47; S. L. Adler and W. A. Bardeen, Phys. Rev. 182 (1969) 1517.
[7] J. Erlich et al., Phys. Rev. Lett. 95 (2005) 261602; L. Da Rold and A. Pomarol, Nucl. Phys. B 721, 79 (2005); A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. D 74, 015005 (2006); P. Colangelo et al., Phys. Rev. D 78 (2008) 055009.
[8] M. Knecht and E. de Rafael, Phys. Lett. B 424 (1998) 335.
[9] P. Masjuan and S. Peris, JHEP 0705 (2007) 040 [arXiv:0704.1247 [hep-ph]].
[10] M. Knecht, S. Peris, E. de Rafael, JHEP 1110, 048 (2011).
[11] L. Cappiello, O. Cata and G. D’Ambrosio, Phys. Rev. D82, 095008 (2010); S. K. Domokos, J. A. Harvey and A. B. Royston, JHEP 1105, 107 (2011); R. Alvares, C. Hoyos and A. Karch, [arXiv:1108.1191 [hep-ph]]; A. Gorsky et al., Phys. Rev. D 85 (2012) 086006.