Parameterized Repair of Concurrent Systems

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\textbf{Abstract.} We present an algorithm for the repair of parameterized systems. The repair problem is, for a given process implementation, to find a refinement such that a given property is satisfied by the resulting parameterized system, and deadlocks are avoided. Our algorithm employs a constraint-based search for candidate repairs, and uses a parameterized model checker to determine their correctness and update the constraint system in case errors are reachable. We apply this algorithm on systems that can be represented as well-structured transition systems (WSTS), including disjunctive systems, pairwise rendezvous systems, and broadcast protocols. Moreover, we show that parameterized deadlock detection can be decided in NEXPTIME for disjunctive systems, vastly improving on the best known lower bound, and that it is in general undecidable for broadcast protocols.

\section{Introduction}

Concurrent systems are hard to get correct, and are therefore a promising application area for formal methods. For systems that are composed of an arbitrary number of processes $n$, methods such as parameterized model checking can provide correctness guarantees that hold regardless of $n$. While the parameterized model checking problem (PMCP) is undecidable even if we restrict systems to uniform finite-state processes \cite{1}, there exist several approaches that decide the problem for specific classes of systems and properties \cite{2,3,4,5,15,20,22,23,24,33}.

However, if parameterized model checking detects a fault in the given system, it does not tell us how to repair the system such that it satisfies the specification. To repair the system, the user has to find out which behavior of the system causes the fault, and how it can be corrected. Both tasks may be nontrivial.

For faults in the internal behavior of a process, the approach we propose is similar to existing repair approaches \cite{5,37}: we start with a non-deterministic implementation, and restrict non-determinism to obtain a correct implementation. This non-determinism may have been added by a designer to “propose” possible repairs for a system that is known or suspected to be faulty.

For concurrent systems, the large number of possible interactions between processes is known to be the main challenge in guaranteeing their correctness. Therefore, we need to go beyond these existing repair approaches, and also repair the communication between processes by choosing the right options out of a set of possible interactions, a problem that is related to the idea of synchronization.
synthesis [7, 9, 38]. Moreover, we aim for an approach that avoids introducing deadlocks, which is particularly important for a repair algorithm, since often the easiest way to “repair” a system is to let it run into a deadlock as quickly as possible. In contrast to repairs of internal behavior, we introduce non-determinism for repairing communication automatically.

Regardless of whether faults are fixed in the internal behavior or in the communication of processes, we aim for a parameterized correctness guarantee, i.e., the repaired implementation should be correct in a system with any number of processes. We show how to achieve this by integrating techniques from parameterized model checking into our repair approach.

**Overview of the approach.** Let us illustrate our approach with an example of pairwise synchronous communication between the two processes depicted in Fig. 1 and 2. Pairwise synchronous means send actions (e.g. `write!`) proceed only if another executes a corresponding receive action (e.g. `write?`). Consider the parameterized system composed of one scheduler (Fig. 1) and an arbitrary number of reader-writer processes (Fig. 2) running concurrently. Note that this system can reach a global state in which multiple processes are in the `writing` state at the same time! We want to repair the system by restricting communication.

Our counterexample-guided automatic repair approach builds on a constraint-based search for candidate repairs and a parameterized model checker. The parameterized model checker searches for error sequences, for example two sequential `write!` transitions by different reader-writer processes, such that they both occupy the `writing` state at the same time. To avoid these errors, error sequences are encoded into constraints on the behavior of processes, which restrict non-determinism and communication. In our example, all errors can be avoided by simply removing all outgoing transitions of state $q_{A,0}$ of the scheduler. However, then the system will deadlock immediately! Therefore, we also check reachability of deadlocks and discard candidate repairs where this is the case. Fig. 3 shows a repair that is safe and does not introduce deadlocks.
Contributions. Our main contribution is a counterexample-guided parameterized repair approach, based on model checking of well-structured transition systems (WSTS) [1, 30]. We investigate which information a parameterized model checker needs to provide to guide the search for candidate repairs, and how this information can be encoded into propositional constraints. Moreover, we investigate how to systematically avoid deadlocks in the repaired system. Our repair algorithm supports many classes of systems, including guarded protocols with disjunctive guards [20], rendezvous systems [33] and broadcast protocols [24].

Since existing model checking algorithms for WSTS do not support deadlock detection, our approach has a subprocedure to solve this problem, which relies on new theoretical results: for disjunctive systems, we provide a novel deadlock detection algorithm that vastly improves on the complexity of the best known solution, while for broadcast protocols we prove that deadlock detection is in general undecidable, so approximate methods have to be used. We evaluate an implementation of our algorithm on benchmarks from different application domains, including a distributed lock service and a robot-flocking protocol.

This paper is organized as follows. In Sect. 2 we present our basic system model for disjunctive systems. In Sect. 3 we define the parameterized repair problem, propose a solution, and highlight three research challenges. In Sect. 4 we develop algorithms for parameterized model checking and deadlock detection that provide information on error paths for our repair approach. In Sect. 5 we introduce our parameterized repair algorithm, and in Sect. 6 we extend it to more general properties and systems. We provide our experimental evaluation in Sect. 7, followed by a discussion of related work in Sect. 8, and our conclusion and discussion of future work in Sect. 9.

2 System Model

In the following, let $Q$ be a finite set of states.

Processes. A process template is a transition system $U = (Q_U, \text{init}_U, G_U, \delta_U)$, where $Q_U \subseteq Q$ is a finite set of states including the initial state $\text{init}_U$, $G_U \subseteq \mathcal{P}(Q)$ is a set of guards, and $\delta_U : Q_U \times G_U \times Q_U$ is a guarded transition relation.

We denote by $t_U$ a transition of $U$, i.e., $t_U \in \delta_U$, and by $\delta_U(q_U)$ the set of all outgoing transitions of $q_U \in Q_U$. We assume that $\delta_U$ is total, i.e., for every $q_U \in Q_U$, $\delta_U(q_U) \neq \emptyset$. Define the size of $U$ as $|U| = |Q_U|$. An instance of template $U$ will be called a $U$-process.

Disjunctive Systems. Fix process templates $A$ and $B$ with $Q = Q_A \cup Q_B$, and let $G = G_A \cup G_B$ and $\delta = \delta_A \cup \delta_B$. We consider systems $A \parallel B^n$, consisting of one $A$-process and $n$ $B$-processes in an interleaving parallel composition.\(^3\)

The systems we consider are called “disjunctive” since guards are interpreted disjunctively, i.e., a transition with a guard $g$ is enabled if there exists another process that is currently in one of the states in $g$. Figures 4 and 5 give examples

\(^3\) The form $A \parallel B^n$ is only assumed for simplicity of presentation. Our results extend to systems with an arbitrary number of process templates.
of process templates. An example disjunctive system is $A\parallel B^n$, where $A$ is the writer and $B$ the reader, and the guards determine which transition can be taken by a process, depending on its own state and the state of other processes in the system. Transitions with the trivial guard $g = Q$ are displayed without a guard. We formalize the semantics of disjunctive systems in the following.

**Counter System.** A configuration of a system $A\parallel B^n$ is a tuple $(q_A, c)$, where $q_A \in Q_A$, and $c : Q_B \rightarrow \mathbb{N}_0$. We identify $c$ with the vector $(c(q_0), \ldots, c(q_{|B|-1})) \in \mathbb{N}_0^{|B|}$, and also use $c(i)$ to refer to $c(q_i)$. Intuitively, $c(i)$ indicates how many processes are in state $q_i$. We denote by $u_i$ the unit vector with $u_i(i) = 1$ and $u_i(j) = 0$ for $j \neq i$.

Given a configuration $s = (q_A, c)$, we say that the guard $g$ of a local transition $(q_U, g, q_U') \in \delta_U$ is satisfied in $s$, denoted $s \models_{q_U} g$, if one of the following conditions holds:

1. $q_U = q_A$, and $\exists q_i \in Q_B$ with $q_i \in g$ and $c(i) \geq 1$ (A takes the transition, a $B$-process is in $g$)
2. $q_U \neq q_A$, $c(q_U) \geq 1$, and $q_A \in g$ (B-process takes the transition, $A$ is in $g$)
3. $q_U \neq q_A$, $c(q_U) \geq 1$, and $\exists q_i \in Q_B$ with $q_i \in g$, $q_i \neq q_U$, and $c(i) \geq 1$ (B-process takes the transition, another $B$-process is in different state in $g$)
4. $q_U \neq q_A$, $q_U \in g$, and $c(q_U) \geq 2$ (B-process takes the transition, another $B$-process is in same state in $g$)

We also say that the local transition $(q_U, g, q_U')$ is enabled in $s$.

Then the configuration space of all systems $A\parallel B^n$, for fixed $A, B$ but arbitrary $n \in \mathbb{N}$, is the transition system $M = (S, S_0, \Delta)$ where:

- $S \subseteq Q_A \times \mathbb{N}_0^{|B|}$ is the set of states,
- $S_0 = \{(\text{init}_A, c) \mid c(q) = 0 \text{ if } q \neq \text{init}_B\}$ is the set of initial states,
- $\Delta$ is the set of transitions $((q_A, c), (q_A', c'))$ s.t. one of the following holds:
  1. $c = c' \land \exists q_A, g, q_A' \in \delta_A : (q_A, c) \models_{q_A, g}$ (transition of $A$)
  2. $q_A = q_A', \exists q_i, g, q_i \in \delta_B : c(i) \geq 1 \land c' = c - u_i + u_j \land (q_A, c) \models_{q_i, g}$ (transition of a $B$-process)

We will also call $M$ the counter system (of $A$ and $B$), and will call configurations states of $M$, or global states.

Let $s, s' \in S$ be states of $M$, and $U \in \{A \cup B\}$. For a transition $(s, s') \in \Delta$ we also write $s \rightarrow s'$. If the transition is based on the local transition $t_U = (q_U, g, q_U') \in \delta_U$, we also write $s \xrightarrow{t_U} s'$ or $s \xrightarrow{U} s'$. Let $\Delta_{\text{local}}(s) = \{t_U \mid s \xrightarrow{U} s'\}$, i.e., the set of all enabled outgoing local transitions from $s$, and let $\Delta(s, t_U) = s'$ if $s \xrightarrow{t_U} s'$. From now on we assume wlog. that each guard $g \in G$ is a singleton.\footnote{This is not a restriction as any local transition $(q_U, g, q_U')$ with a guard $g \in G$ and $|g| > 1$ can be split into $|g|$ transitions $(q_U, g_i, q_U')$ where for all $i \leq |g| : g_i \in g$ is a singleton guard.}
3 Parameterized Repair

In the following, we define the parameterized repair problem, provide a high-level algorithm to solve it, and discuss the challenges in implementing the algorithm.

The Parameterized Repair Problem. Let $M = (S, S_0, \Delta)$ be the counter system for process templates $A = (Q_A, \text{init}_A, G_A, \delta_A)$, $B = (Q_B, \text{init}_B, G_B, \delta_B)$, and $ERR \subseteq Q_A \times \mathbb{N}[|B|]$ a set of error states. The parameterized repair problem is to decide if there exist process templates $A' = (Q_A, \text{init}_A, G_A, \delta'_A)$ with $\delta'_A \subseteq \delta_A$ and $B' = (Q_B, \text{init}_B, G_B, \delta'_B)$ with $\delta'_B \subseteq \delta_B$ such that the counter system $M'$ for $A'$ and $B'$ does not reach any state in $ERR$.

If they exist, we call $\delta' = \delta'_A \cup \delta'_B$ a repair for $A$ and $B$. We call $M'$ the restriction of $M$ to $\delta'$, also denoted $\text{Restrict}(M, \delta')$.

Note that by our assumption that the local transition relations are total, a trivial repair that disables all transitions from some state is not allowed.

High-Level Parameterized Repair Algorithm. Figure 6 sketches the basic idea of our parameterized repair algorithm. The algorithm starts with a counter system $M$ (based on non-deterministic process templates $A$ and $B$) and checks if error states are reachable in $M$. If not, $A$ and $B$ are already correct. Otherwise, the parameterized model checker returns an error sequence $E$, i.e., one or more concrete error paths (possibly in systems of different sizes). The constraints on $A$ and $B$ are then refined to avoid any error paths in $E$, and a SAT solver is used to find subsets $\delta'_A \subseteq \delta_A$ and $\delta'_B \subseteq \delta_B$ of the non-deterministic transition relations of $A$ and $B$, such that $\delta' = \delta'_A \cup \delta'_B$ satisfies the constraints. Then, $\delta'$ is
used to restrict \( M \). To guarantee that \( \delta' \) does not introduce deadlocks, the next step is a parameterized deadlock detection. This works in a similar way as the model checker, and is used to refine the constraints if deadlocks are reachable. Otherwise, \( M' \) is sent to the model checker for the next iteration.

**Research Challenges.** Parameterized model checking of disjunctive systems is known to be decidable, either directly via cutoffs [20] or via simulation by rendezvous systems [33], and this simulation also allows to decide the parameterized deadlock detection problem. However, these theoretical solutions do not provide practical algorithms that allow us to extract the information that is needed for our repair approach. Therefore, the following challenges need to be overcome to obtain an effective parameterized repair algorithm:

- **C1** The parameterized model checker needs to provide information about error paths in the current candidate model that allow us to refine the candidate processes such that these error paths will be avoided in systems of any size.
- **C2** We need an effective approach for parameterized deadlock detection, preferably supplying similar information as the model checker.
- **C3** The information obtained from model checking and deadlock detection needs to be encoded into constraints such that their solution represents a restriction of \( M \) that avoids any error paths that have already been found, and possibly satisfies additional properties, such as keeping certain states reachable.

Research challenges **C1** and **C2** will be addressed in Sect. 4, and **C3** will be addressed in Sect. 5, where we also present our parameterized repair algorithm.

4 Parameterized Model Checking of Disjunctive Systems

In this section, we address research challenges **C1** and **C2**: after establishing that counter systems can be framed as well-structured transition systems (WSTS) (Sect. 4.1), we introduce a parameterized model checking algorithm for disjunctive systems that suits our needs (Sect. 4.2), and finally show how the algorithm can be modified to also check for the reachability of deadlocked states (Sect. 4.3). Full proofs for the lemmas and theorems in this section can be found in App. A.

4.1 Counter Systems as WSTS

**Well-quasi-order.** Given a set of states \( S \), a binary relation \( \preceq \subseteq S \times S \) is a well-quasi-order (wqo) if \( \preceq \) is reflexive, transitive, and if any infinite sequence \( s_0, s_1, \ldots \in S^\omega \) contains a pair \( s_i \preceq s_j \) with \( i < j \). A subset \( R \subseteq S \) is an antichain if any two distinct elements of \( R \) are incomparable wrt. \( \preceq \). Therefore, \( \preceq \) is a wqo on \( S \) if and only if it is well-founded and has no infinite antichains.

**Upward-closed Sets.** Let \( \preceq \) be a wqo on \( S \). The upward closure of a set \( R \subseteq S \), denoted \( \uparrow R \), is the set \( \{ s \in S \mid \exists s' \in R : s' \preceq s \} \). We say that \( R \) is upward-closed if \( \uparrow R = R \). If \( R \) is upward-closed, then we call \( B \subseteq S \) a basis of \( R \) if \( \uparrow B = R \).
If \(\preceq\) is also antisymmetric, then any basis of \(R\) has a unique subset of minimal elements. We call this set the minimal basis of \(R\), denoted \(\text{minBasis}(R)\).

**Compatibility.** Given a counter system \(M = (S, S_0, \Delta)\), we say that a wqo \(\preceq \subseteq S \times S\) is compatible with \(\Delta\) if the following holds:

\[
\forall s, s', r \in S: \text{ if } s \rightarrow s' \text{ and } s \preceq r \text{ then } \exists r' \text{ with } s' \preceq r' \text{ and } r \rightarrow^* r'.
\]

We say \(\preceq\) is strongly compatible with \(\Delta\) if the above holds with \(r \rightarrow^* r'\) instead of \(r \rightarrow r'\).

**WSTS [1].** We say that \((M, \preceq)\) with \(M = (S, S_0, \Delta)\) is a well-structured transition system if \(\preceq\) is a wqo on \(S\) that is compatible with \(\Delta\).

**Lemma 1.** Let \(M = (S, S_0, \Delta)\) be a counter system for process templates \(A, B\), and let \(\preceq \subseteq S \times S\) be the binary relation defined by:

\[
(q_A, c) \preceq (q'_A, d) \iff (q_A = q'_A \land c \preceq d),
\]

where \(\preceq\) is the component-wise ordering of vectors. Then \((M, \preceq)\) is a WSTS.

**Predecessor, Effective pred-basis [30].** Let \(M = (S, S_0, \Delta)\) be a counter system and let \(R \subseteq S\). Then the set of immediate predecessors of \(R\) is

\[
\text{pred}(R) = \{ s \in S | \exists r \in R : s \rightarrow r \}.
\]

A WSTS \((M, \preceq)\) has effective pred-basis if there exists an algorithm that takes as input any finite set \(R \subseteq S\) and returns a finite basis of \(\text{pred}(\uparrow R)\). Note that, for a given set \(R \subseteq S\) that is upward-closed with respect to \(\preceq\), \(\text{pred}(R)\) is upward-closed if \(\preceq\) is strongly compatible with \(\Delta\).

For backward reachability analysis, we want to compute \(\text{pred}^*(R)\) as the limit of the sequence \(R_0 \subseteq R_1 \subseteq \ldots\) where \(R_0 = R\) and \(R_{i+1} = R_i \cup \text{pred}(R_i)\). Note that if we have strong compatibility and effective pred-basis, we can compute \(\text{pred}^*(R)\) for any upward-closed set \(R\). If we can furthermore check intersection of upward-closed sets with initial states (which is easy for counter systems), then reachability of arbitrary upward-closed sets is decidable.

The following lemma, like Lemma 1, can be considered folklore. We present it here mainly to show how we can effectively compute the predecessors, which is an important ingredient of our model checking algorithm.

**Lemma 2.** Let \(M = (S, S_0, \Delta)\) be a counter system for guarded process templates \(A, B\). Then \((M, \preceq)\) has effective pred-basis.

**Proof.** Let \(R \subseteq S\) be finite, \(f = (\{ t = j \land c(j) = 1 \} \lor (c'(t) \geq 1 \land c(j) = 0))\). Then one can prove that \(\uparrow \text{CBasis} = \uparrow \text{pred}(\uparrow R)\), for the following set \(\text{CBasis}:\)

\[
\text{CBasis} = \{ (q_A, c) \in S \mid \exists (q'_A, c') \in R : \\
( (q_A, g, q'_A) \in \delta_A \land (q_A, c) \models g \land (c = c') \lor (c'(t) = 0 \land c = c' + u_i) ) ] \}
\]

\[
\lor ( (g \land c = c' + u_i) \lor (c'(t) = 0 \land c'(j) \geq 1 \land c = c' + u_j - u_j + u_i) )
\]

\[
\lor ( f \land c = c' + u_i ) \lor (c'(t) = 0 \land c'(j) = 0 \land c = c + u_i + u_i ) ) \} \}. \]
4.2 Model Checking Algorithm

Our model checking algorithm is a variant of the known backwards reachability algorithm for WSTS [1]. We present it in detail to show how it stores intermediate results to return an error sequence, from which we derive concrete error paths.

**Algorithm 1.** Given a counter system $M$ and a finite basis $ERR$ of the set of error states, the algorithm iteratively computes the set of predecessors until it reaches an initial state, or a fixed point. The procedure returns either True, i.e. the system is safe, or an error sequence $E_0, \ldots, E_k$, where $E_0 = ERR$, $\forall 0 < i < k : E_i = \text{minBasis}(\text{pred}(\uparrow E_{i-1}))$, and $E_k = \text{minBasis}(\text{pred}(\uparrow E_{k-1})) \cap S_0$. That is, every $E_i$ is the minimal basis of the states that can reach $ERR$ in $i$ steps.

**Algorithm 1 Parameterized Model Checking**

1: procedure MODELCheck(Counter System $M, ERR$)
2: temporarySet $\leftarrow$ ERR, $E_0 \leftarrow ERR$, $i \leftarrow 1$, visitedSet $\leftarrow \emptyset$
3: //If visitedSet $=$ temporarySet then a fixed point is reached
4: while temporarySet $\neq$ visitedSet do
5: visitedSet $\leftarrow$ temporarySet
6: $E_i \leftarrow \text{minBasis}(\text{pred}(\uparrow E_{i-1}))$
7: //pred is computed as described in the proof of Lemma 2
8: if $E_i \cap S_0 \neq \emptyset$ then //check intersection with initial states
9: return False, $\{E_0, \ldots, E_i \cap S_0\}$
10: temporarySet $\leftarrow$ minBasis(visitedSet $\cup$ $E_i$)
11: $i \leftarrow i + 1$
12: return True, $\emptyset$

**Example.** Consider the reader-writer system in Figures 4 and 5. Suppose the error states are all states where the writer is in $w$ while a reader is in $r$. In other words, the error set of the corresponding counter system $M$ is $\uparrow E_0$ where $E_0 = \{(w, (0, 1))\}$ and $(0, 1)$ means zero reader-processes are in $nr$ and one in $r$. Note that $\uparrow E_0 = \{(w, (i_0, i_1)) \mid (w, (0, 1)) \preceq (w, (i_0, i_1))\}$, i.e. all elements with the same $w$, $i_0 \geq 0$ and $i_1 \geq 1$. If we run Algorithm 1 with the parameters $M, \{(w, (0, 1))\}$, we get the following error sequence: $E_0 = \{(w, (0, 1))\}$, $E_1 = \{(nw, (0, 1))\}$, $E_2 = \{(nw, (1, 0))\}$, with $E_2 \cap S_0 \neq \emptyset$, i.e., the error is reachable.

**Properties of Algorithm 1.** Correctness of the algorithm follows from the correctness of the algorithm by Abdulla et al. [1], and from Lemma 2. Termination follows from the fact that a non-terminating run would produce an infinite minimal basis, which is impossible since a minimal basis is an antichain.

4.3 Deadlock Detection in Disjunctive Systems

The repair of concurrent systems is much harder than fixing monolithic systems. One of the sources of complexity is that a repair might introduce a deadlock, which is usually an unwanted behavior. In this section we show how to extend our parameterized model checking algorithm to the detection of deadlocks.
To this end, we need to refine the wqo $\preceq$ introduced in Sect. 4.1, since a set of deadlocked states is in general not upward-closed under $\preceq$: let $s = (q_A, c), r = (q_A, d)$ be global states with $s \preceq r$. If $s$ is deadlocked, then $c(i) = 0$ for every $q_i$ that appears in a guard of an outgoing local transition from $s$. Now if $d(i) > 0$ for one of these $q_i$, then some transition is enabled in $r$, which is therefore not deadlocked. In the following we assume wlog. that $\delta_B$ does not contain transitions of the form $(q_i, \{q_i\}, q_j)$, i.e., a transition from $q_i$ is guarded by $q_i$.\footnote{A system that does not satisfy this assumption can easily be transformed into one that does, with a linear blowup in the number of states, and preserving reachability properties including reachability of deadlocks.}

**Refined wqo for Deadlock Detection.** Consider $\preceq_0 \subseteq \mathbb{N}_0^{|B|} \times \mathbb{N}_0^{|B|}$ where

$$c \preceq_0 d \iff (c \preceq d \land \forall i \leq |B| : (c(i) = 0 \Leftrightarrow d(i) = 0)),$$

and $\preceq_0 \subseteq S \times S$ where $(q_A, c) \preceq_0 (q'_A, d) \Leftrightarrow (q_A = q'_A \land c \preceq_0 d)$.\footnote{A system that does not satisfy this assumption can easily be transformed into one that does, with a linear blowup in the number of states, and preserving reachability properties including reachability of deadlocks.}

**Lemma 3.** Let $M = (S, S_0, \Delta)$ be a counter system for process templates $A, B$. Then $(M, \preceq_0)$ is a WSTS.

Note however that while $\preceq_0$ is compatible with $\Delta$, it is not strongly compatible. As noted before, this implies that for a set of states $R$ that is upward-closed (wrt. $\preceq_0$), $\text{pred}(R)$ is not upward-closed, and we can therefore not use upward-closed sets to compute $\text{pred}^*(R)$ when checking reachability of $R$. Therefore, we now introduce an overapproximation of $\text{pred}(R)$ that is upward-closed wrt. $\preceq_0$ and is safe in the sense that every state in the overapproximation is backwards reachable (in a number of steps) from $R$.

**O-Predecessor.** Let $M = (S, S_0, \Delta)$ be a counter system and let $R \subseteq S$. Then the set of $O$-predecessors of $R$ is

$$\text{opred}(R) = \text{pred}(R) \cup \{ (q_A, c) \in S \mid \exists (q'_A, c') \in R, t_B = (q_i, g, q_j) \in \delta_B : (q_A, c) \xrightarrow{t_B^+} (q'_A, c') \land (c(i) = 0 \lor c'(i) = 0) \},$$

where $(q_A, c) \xrightarrow{t_B^+} (q'_A, c')$ denotes that $(q'_A, c')$ is reachable from $(q_A, c)$ by executing the local transition $t_B$ one or more times.

**Lemma 4.** Let $R \subseteq S$ be upward-closed with respect to $\preceq_0$. Then $\text{opred}(R)$ is upward-closed with respect to $\preceq_0$.

**Effective opred-basis.** Similar to what we had before, we need to have effective opred-basis, i.e., to be able to compute a basis of $\text{opred}(R)$ from a basis of $R$.

**Lemma 5.** Let $M = (S, S_0, \Delta)$ be a counter system for process templates $A, B$. Then $(M, \preceq_0)$ is a WSTS with effective opred-basis.
Proof. Let \( R \subseteq S \) be finite. Then it suffices to prove that a finite basis of \( \text{opred}(\uparrow R) \) can be computed from \( R \) (where the upward-closure is wrt. \( \preceq_0 \)). Consider the following set of states:

\[
\text{DB-Pred}(R) = \{ (q_A, c) \in S \mid \exists (q'_A, c') \in R: \\
\quad \exists g : (q_A, c) \models g \land (q_A, c) \models g' \land (q'_A, c') \models g' \\
\quad \land \left( \begin{array}{l}
q_A = q'_A \\
\forall (q, g, q) : (q, g) \in \delta_B (q_A, c) \models g, q \\
\land \left( (c = c' + u_i - u_j) \lor (c'(j) = 1 \land c = c' + u_i) \right) \\
\lor (c'(j) = k \land c = c' + k \cdot u_i - k \cdot u_j) \right) \}
\]

Obviously, \( \text{DB-Pred}(R) \subseteq \text{opred}(\uparrow R) \). By definitions of \( \text{DB-Pred} \), \( \preceq_0 \), and counter systems, we can also prove that \( \text{DB-Pred}(R) \subseteq \text{minBasis}(\text{opred}(\uparrow R)) \).

Let \( \text{opred}^* \) and \( \text{DB-Pred}^* \) be defined similarly to \( \text{pred}^* \). Then, note that \( \text{pred}(R) \subseteq \text{opred}(R) \subseteq \text{pred}^*(R) \) for any \( R \subseteq S \), and therefore \( \text{opred}^*(R) = \text{pred}^*(R) \).

**Corollary 1** Let \( M = (S, S_0, \Delta) \) be a counter system for process templates \( A, B \), and \( \text{ERR} \) a finite set of deadlocked states. Then \( \uparrow (\text{DB-Pred}^*(\text{ERR})) = \text{opred}^*(\uparrow \text{ERR}) = \text{pred}^*(\uparrow \text{ERR}) \).

Based on these results, we can show the following.

**Theorem 1.** Deadlock detection in disjunctive systems is decidable in NEXP-TIME.

**Proof idea.** If a deadlocked run exists, then we can construct a configuration sequence witnessing the deadlock of length at most \( 2^{|B|} \). This can be done by (i) separating the original run into segments that agree for all \( i \) on whether \( c[i] = 0 \), (ii) removing “loops” between repetitions of segments, and (iii) replacing each segment with a single configuration, such that we always move from one configuration to the next by a single application of \( \text{opred} \), i.e., by moving arbitrarily many processes into a position \( j \) that was empty, or from a position \( i \) such that it becomes empty. Since every step in the resulting sequence can be checked against \( \text{DB-Pred} \) in time linear in \( |\delta| \), we obtain NEXPTIME complexity.

**An Algorithm for Deadlock Detection.** Now we can modify Algorithm 1 to detect deadlocks in a counter system \( M \): instead of passing a basis of the set of errors in the parameter \( \text{ERR} \), we pass a basis of the deadlocked states. Furthermore, in Line 6 we now compute \( \text{opred} \) instead of \( \text{pred} \), using \( \text{DB-Pred} \) from the proof of Lemma 5, and the computation of a minimal basis in Lines 6 and 10 needs to be done wrt. the refined wqo \( \preceq_0 \). Finally, by the proof idea of Thm. 1 we can make the algorithm run in 2EXPTIME by declaring the absence of deadlocks if a fixed point is not reached after \( 2^{|B|} \) applications of \( \text{DB-Pred} \).

## 5 Parameterized Repair Algorithm

Now, we can introduce a parameterized repair algorithm that interleaves the backwards model checking algorithm (Algorithm 1) with a forward reachability analysis and the computation of candidate repairs.
Forward Reachability Analysis. In the following, for a set $R \subseteq S$, let $\text{Succ}(R) = \{s' \in S \mid \exists s \in R : s \rightarrow s'\}$. Furthermore, for $s \in S$, let $\Delta^{\text{local}}(s, R) = \{t_U \in \delta \mid t_U \in \Delta^{\text{local}}(s) \land \Delta(s, t_U) \in R\}$.

Given an error sequence $E_0, \ldots, E_k$, let the reachable error sequence $\mathcal{RE} = R E_0, \ldots, R E_k$ be defined by $R E_k = E_k$ (which by definition only contains initial states), and $R E_{i-1} = \text{Succ}(R E_i) \cap \uparrow E_{i-1}$ for $1 \leq i \leq k$. That is, each $R E_i$ contains a set of states that can reach $\uparrow \text{ERR}$ in $i$ steps, and are reachable from $S_0$ in $k - i$ steps. Thus, it represents a set of concrete error paths of length $k$.

Constraint Solving for Candidate Repairs. The generation of candidate repairs is guided by constraints over the local transitions $\delta$ as atomic propositions, such that a satisfying assignment of the constraints corresponds to the candidate repair, where only transitions that are assigned $\text{true}$ remain in $\delta'$. During an execution of the algorithm, these constraints ensure that all error paths discovered so far will be avoided, and include a set of fixed constraints that express additional desired properties of the system, as explained in the following.

Initial Constraints. To avoid the construction of repairs that violate the totality assumption on the transition relations of the process templates, every repair for disjunctive systems has to satisfy the following constraint:

$$
\text{TRConstr}_{\text{Disj}} = \bigwedge_{q_A \in Q_A} t_A \in \delta_A(q_A) \land \bigwedge_{q_B \in Q_B} t_B \in \delta_B(q_B)
$$

Informally, $\text{TRConstr}_{\text{Disj}}$ guarantees that a candidate repair returned by the SAT solver never removes all local transitions of a local state in $Q_A \cup Q_B$. Furthermore a designer can add constraints that are needed to obtain a repair that conforms with their requirements, for example to ensure that certain states remain reachable in the repair (see Appendix D.1 and D.2 for more examples).

Algorithm 2. Given a counter system $M$, a basis $\text{ERR}$ of the error states, and initial Boolean constraints $\text{initConstr}$ on the transition relation (including at least $\text{TRConstr}_{\text{Disj}}$), the algorithm returns either a repair $\delta'$ or the string $\text{Unrealizable}$ to denote that no repair exists.

Properties of Algorithm 2.

Theorem 2 (Soundness). For every repair $\delta'$ returned by Algorithm 2:

- $\text{Restrict}(M, \delta')$ is safe, i.e., $\uparrow \text{ERR}$ is not reachable, and
- the transition relation of $\text{Restrict}(M, \delta')$ is total in the first two arguments.

Proof. The parameterized model checker guarantees that the transition relation is safe, i.e., $\uparrow \text{ERR}$ is not reachable. Moreover, the transition relation constraint $\text{TRConstr}$ is part of $\text{initConstr}$ and guarantees that, for any candidate repair returned by the SAT solver, the transition relation is total. □

Theorem 3 (Completeness). If Algorithm 2 returns “Unrealizable”, then the parameterized system has no repair.
Algorithm 2 Parameterized Repair

1: procedure PARAMREPAIR($M$, $ERR$, $InitConstr$)
2:  $M' ← M$, $accumConstr ← True$, $isCorrect ← False$
3:  while $isCorrect = False$ do //loop until a repair is found or unrealizability
4:    $isCorrect = [E_0, . . . , E_k] ← ModelCheck(M', ERR)$
5:    if $isCorrect = False$ then
6:      $RE_k ← E_k$ // $E_k$ contains only initial states
7:      $RE_{k−1} ← Succ(RE_k)∩ □E_{k−1}, . . . , RE_0 ← Succ(RE_0)∩ □E_0$
8:      //for every initial state in $RE_k$ compute the corresponding constraints
9:      $newConstr ← \bigwedge_{s \in RE_k} BuildConstr(s, [RE_{k−1}, . . . , RE_0])$
10:     //append current iteration’s constraints to previous iterations’ constraints
11:     $accumConstr ← newConstr \land accumConstr$
12:     $δ' = SAT \leftarrow SAT(accumConstr \land initConstr)$
13:     if $isSAT = False$ then
14:         return Unrealizable
15:     $M' = Restrict(M, δ')$ //compute a new candidate using the repair $δ'$
16:     else return $δ'$ //a repair is found once $isCorrect = True$

1: procedure BUILDCONSTR(State $s$, $RE$) //$s$ is an initial state
2:  //RE[1:] is a list obtained by removing the first element from $RE$
3:  if $RE[1:]$ is empty then
4:    return $\Delta_{local} \bigtriangleup RE[0]$ //if $t_U \in \Delta_{local}(s)$ leads to $RE[0]$, delete it
5:  else //if $t_U$ leads to $RE[0]$, delete $t_U$ or all transitions leading to $RE[1]$
6:    return $\bigwedge_{s \in \Delta_{local}(s, RE[0])} (\neg t_U \lor BuildConstr(\Delta(s, t_U), RE[1:]))$

Proof. Algorithm 2 returns "Unrealizable" if $accumConstr \land initConstr$ has become unsatisfiable. We consider an arbitrary $δ' \subseteq δ$ and show that it cannot be a repair. Note that for the given run of the algorithm, there is an iteration $i$ of the loop such that $δ'$, seen as an assignment of truth values to atomic propositions $δ$, was a satisfying assignment of $accumConstr \land initConstr$ up to this point, and is not anymore after this iteration.

If $i = 0$, i.e., $δ'$ was never a satisfying assignment, then $δ'$ does not satisfy $initConstr$ and can clearly not be a repair. If $i > 0$, then $δ'$ is a satisfying assignment for $initConstr$ and all constraints added before round $i$, but not for the constraints $\bigwedge_{s \in RE_k} BuildConstr(s, [RE_{k−1}, . . . , RE_0])$ added in this iteration of the loop, based on a reachable error sequence $RE = RE_k, . . . , RE_0$. By construction of $BuildConstr$, this means we can construct out of $δ'$ and $RE$ a concrete error path in $Restrict(M, δ')$, and $δ'$ can also not be a repair.

Note also that a reachable error sequence $RE$ of length $k$ represents all possible error paths of length $k$, in systems of any size, and therefore these error paths will be avoided in all these systems. This property is formalized in Appendix B.

Theorem 4 (Termination). Algorithm 2 always terminates.

Proof. For a counter system based on $A$ and $B$, the number of possible repairs is bounded by $2^{||A||}$. In every iteration of the algorithm, either the algorithm
terminates, or it adds constraints that exclude at least the repair that is currently under consideration. Therefore, the algorithm will always terminate.

**Parameterized Repair with Deadlock Detection.** Note that Algorithm 2 does not include any measures that prevent it from producing a repair with deadlocked runs. However, it can be extended with a subprocedure for deadlock detection based on the approach explained in Sect. 4.3, called in an interleaving way with the model checker as depicted in Fig. 6.

**What can be done if a repair doesn’t exist?** If Algorithm 2 returns “unrealizable”, then there is no repair for the given input. To still obtain a repair, a designer can add more non-determinism and/or allow for more communication between processes, and then run the algorithm again on the new instance of the system. Moreover, unlike in monolithic systems, even if the result is “unrealizable”, it may still be possible to obtain a solution that is good enough in practice. For instance, we can change our algorithm slightly as follows: When the SAT solver returns “UNSAT” after adding the constraints for an error sequence, instead of terminating we can continue computing the error sequence until a fixed point is reached. Then, we can determine the minimal number of processes \( m_e \) that is needed for the last candidate repair to reach an error, and conclude that this candidate is safe for any system up to size \( m_e - 1 \).

### 6 Extensions

**Beyond Reachability.** Algorithm 2 can also be used for repair with respect to general safety properties, based on the automata-theoretic approach to model checking. That is, a safety property \( \varphi \) (on the behavior of a fixed number of processes, e.g. process \( A \) and \( k \) copies of \( B \)) is encoded into a finite-state automaton \( A_\varphi \) that reads the run of these processes in a system \( A\parallel B^n \) and accepts all runs that violate \( \varphi \). Then, we can consider the product of the original system \( M \) with the automaton \( A_\varphi \) and explicit copies of \( B \) that appear in \( \varphi \), and define a wqo on this product that allows us to use Algorithm 1 to check if there are any executions that violate \( \varphi \), and accordingly modify Algorithm 2 to find a repair such that \( \varphi \) is satisfied. Details can be found in App. C.

**Beyond Disjunctive Systems.** Furthermore, we have extended Algorithm 2 to other systems that can be framed as WSTS, in particular rendezvous systems [33] and systems based on broadcasts or other global synchronizations [24,35]. All of these are known to be WSTS, and there are two remaining challenges:

1. how to find suitable constraints to determine a restriction \( \delta' \), and
2. how to exclude deadlocks.

The first is relatively easy, but the constraints become more complicated because we now have synchronous transitions of multiple processes. Deadlock detection is decidable for rendezvous systems, but the best known method is by reduction to reachability in VASS [33], which has recently been shown to be TOWER-hard [17]. For broadcast protocols we can show that the situation is even worse:
Theorem 5. Deadlock detection in broadcast protocols is undecidable.

Proof idea. Reachability in affine VASS has recently been shown to be undecidable in almost all cases, including the case where all transitions use broadcast matrices [11]. We can reduce this undecidable problem to deadlock detection in broadcast protocols by modifying the construction of German and Sistla [33] for reducing reachability in (non-affine) VASS to deadlock detection in rendezvous systems. A full proof with the modified construction is given in App. D.2

Since solving the problem exactly is impractical or impossible in general, we propose to use approximate methods. For rendezvous systems, a possible overapproximation is a system that simulates pairwise transitions by a pair of disjunctive transitions, and for broadcast protocols we can use lossy broadcast systems, for which the problem is decidable [18].

7 Implementation & Evaluation

We have implemented a prototype of our parameterized repair algorithm that supports the three types of systems (disjunctive, pairwise and broadcast), and safety and reachability properties. For disjunctive and rendezvous systems, we have evaluated it on different variants of reader-writer-protocols, based on the one given in Sect. 2. A detailed treatment of a more complex version, including an explanation of the whole repair process is given in Appendix E. All of these reader-writer-protocols have been solved successfully within less than 2s.

For broadcast systems, we have evaluated our algorithm on a range of more complex benchmarks, inspired by different applications from the literature:

- the cache coherence protocol MESI [21] (Appendix F includes details of this benchmark and its repair process),
- a distributed Smoke Detector [35],
- a sensor network implementing a Two-Object Tracker [14],
- a distributed Robot Flocking protocol [13], and
- a distributed Lock Service inspired by the Chubby protocol [12].

Typical desired safety properties are mutual exclusion and similar properties.

On these benchmarks, we have compared the performance of three different algorithms: (i) Algorithm 2, (ii) a more naive version where the parameterized verification algorithm only returns a single error path, and (iii) a completely naive version where the parameterized verification algorithm only returns a Yes/No answer. Table 1 summarizes the experimental results we obtained.

We note that even examples with 10 or 12 states and more than a hundred local transitions can be solved in very reasonable time by (i) and (ii), but (iii) often requires significantly more iterations and time. Moreover, there seems to be no significant overhead in (i) compared to (ii), whereas in only one case variant

Note that in the terminology of Delzanno et al., deadlock detection is a special case of the Target problem.
Table 1: Running time and number of iterations for the different algorithms. Each benchmark is listed with its number of local states, actions, and edges (i.e., local transitions). We evaluated the algorithms on a complete set of errors $C$, and for most benchmarks also on two partial sets of errors with $P_1 \cup P_2 = C$. Smallest number of iterations and runtime per benchmark highlighted in boldface.

| Benchmark         | Size | Errors | Iterations | Time(s) |
|-------------------|------|--------|------------|---------|
|                   | States | Actions | Edges | (i) | (ii) | (iii) | (i) | (ii) | (iii) |
| MESI              | 4     | 4      | 26       | $C$   | 4    | 4     | 78   | 2.3  | 2.4  | 126   |
| MESI              | 4     | 4      | 26       | $P_1$ | 2    | 2     | 2    | 1.2  | 1.2  | 1.3   |
| MESI              | 4     | 4      | 26       | $P_2$ | 4    | 4     | 78   | 2.3  | 2.4  | 125   |
| Smoke Detector    | 6     | 5      | 39       | $C$   | 4    | 4     | 66   | 2.4  | 2.4  | 96.8  |
| Two-Object Tracker| 12    | 8      | 128      | $C$   | 7    | 7     | -    | 10.8 | 10.4 | TO    |
| Two-Object Tracker| 12    | 8      | 128      | $P_1$ | 7    | 7     | -    | 10.3 | 10.4 | TO    |
| Two-Object Tracker| 12    | 8      | 128      | $P_2$ | 7    | 7     | -    | 10   | 10.8 | TO    |
| Robot Flocking    | 10    | 10     | 147      | $C$   | 1    | 1     | 87   | 1    | 1    | 988   |
| Robot Flocking    | 10    | 10     | 147      | $P_1$ | 1    | 1     | 79   | 1    | 1    | 962   |
| Robot Flocking    | 10    | 10     | 147      | $P_2$ | 1    | 1     | 87   | 1    | 1    | 795   |
| Lock Service      | 10    | 8      | 95       | $C$   | 7    | 8     | 3    | 4.7  | 4.9  | 2.4   |
| Lock Service      | 10    | 8      | 95       | $P_1$ | 3    | 3     | 3    | 1.8  | 1.8  | 2.3   |
| Lock Service      | 10    | 8      | 95       | $P_2$ | 6    | 6     | 3    | 3.8  | 3.7  | 2.3   |

(i) requires fewer iterations than (ii). We conjecture that these examples are still too small to show significant differences between these two variants.

Finally, note that for the Lock Service benchmark the naive variant (iii) required fewer iterations than (i) or (ii). This seems to be simply a coincidence, caused by heuristics of the SAT solver that prefers certain solutions over others. Since such a behavior could also have negative effects, we believe that this points to the necessity of making our approach more robust against such coincidences by giving less freedom to the SAT solver.

8 Related Work

Many automatic repair approaches have been considered in the literature, most of them restricted to monolithic systems [5,19,31,34,37,39]. Additionally, there are several approaches for synchronization synthesis and repair of concurrent systems. Some of them differ from ours in the underlying approach, e.g., being based on automata-theoretic synthesis [7,28]. Others are based on a similar underlying counterexample-guided synthesis/repair principle, but differ in other aspects from ours. For instance, there are approaches that repair the program by adding atomic sections, which forbid the interruption of a sequence of program statements by other processes [9,41]. Assume-Guarantee-Repair [32] combines verification and repair, and uses a learning-based algorithm to find counterexamples and restrict transition guards to avoid errors. In contrast to ours, this algorithm is not guaranteed to terminate. From lazy synthesis [27] we borrow the idea to construct the set of all error paths of a given length instead of a single
concrete error path, but this approach only supports systems with a fixed number of components. Some of these existing approaches are more general than ours in that they support certain infinite-state processes [9,32,41], or more expressive specifications and other features like partial information [7,28].

The most important difference between our approach and all of the existing repair approaches is that, to the best of our knowledge, none of them provide correctness guarantees for systems with a parametric number of components. This includes also the approach of McClurg et al. [38] for the synthesis of synchronizations in a software-defined network. Although they use a variant of Petri nets as a system model, which would be suitable to express parameterized systems, their restrictions are such that the approach is restricted to a fixed number of components. In contrast, we include a parameterized model checker in our repair algorithm, and can therefore provide parameterized correctness guarantees. There exists a wealth of results on parameterized model checking, collected in several good surveys recently [10,16,25].

9 Conclusion and Future Work

We have investigated the parameterized repair problem for systems of the form $A \parallel B^n$ with an arbitrary $n \in \mathbb{N}$. We introduced a general parameterized repair algorithm, based on interleaving the generation of candidate repairs with parameterized model checking and deadlock detection, and instantiated this approach to different classes of systems: disjunctive systems, pairwise rendezvous systems, and broadcast protocols. Since deadlock detection is an important part of our method, we investigated the problem in detail for these classes of systems, and found that the problem can be decided in NEXPTIME for disjunctive systems, and is undecidable for broadcast protocols. To the best of our knowledge, the best known solution for deadlock detection in disjunctive systems is based on deadlock detection in pairwise rendezvous systems by a reduction to reachability in VASS [33], which has recently been shown to be Tower-hard [17].

Besides reachability properties and the absence of deadlocks, our algorithm can guarantee general safety properties, based on the automata-theoretic approach to model checking. A prototype implementation of the algorithm shows that it can effectively repair non-deterministic overapproximations of many examples from the literature. However, the current algorithm cannot guarantee any liveness properties, like termination or the absence of undesired loops.

In future work, we will look at larger classes of systems and properties, e.g., liveness properties, and investigate the inclusion of approaches that automatically suggest specific relaxations (adding non-determinism and synchronization options) to the system, in case the repair for the given input is unrealizable. Also, in order to improve the practicality of our approach we would like to examine the inclusion of symbolic techniques for counter abstraction [8], and advanced parameterized model checking techniques, e.g., cutoff results for disjunctive systems [6,22,36], or recent pruning results for immediate observation Petri nets, which model exactly the class of disjunctive systems [26].
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A  Full Proofs of Theorems and Lemmas from Section 4

Lemma 1. Let $M = (S, S_0, \Delta)$ be a counter system for process templates $A, B$, and let $\preceq \subseteq S \times S$ be the binary relation defined by:

$$(q_A, c) \preceq (q'_A, d) \iff (q_A = q'_A \land c \preceq d),$$

where $\preceq$ is the component-wise ordering of vectors. Then $(M, \preceq)$ is a WSTS.

Proof. The partial order $\preceq$ is a wqo due to the fact that $\subseteq$ is a wqo. Moreover, we show that $\preceq$ is strongly compatible with $\Delta$. Let $s = (q_A, c), s' = (q'_A, c'), r = (q_A, d) \in S$ such that $s \xrightarrow{t_u} s' \in \Delta$ and $s \preceq r$. Since the transition $t_U$ is enabled in $s$, it is also enabled in $r$ and $\exists r' = (q'_A, d') \in S$ with $r \xrightarrow{t_u} r' \in \Delta$. Then it is easy to see that $s' \preceq r'$: either $t_U$ is a transition of $A$, then we have $c = c'$ and $d = d'$, or $t_U$ is a transition of $B$ with $t_U = (q_i, g, q_j)$, then $q_A = q'_A$ and $c' = c - c_i + c_j \preceq d - c_i + c_j = d'$. \hfill \Box

Lemma 2. Let $M = (S, S_0, \Delta)$ be a counter system for guarded process templates $A, B$. Then $(M, \preceq)$ has effective pred-basis.

Proof. Let $R \subseteq S$ be finite. Since $\text{pred}(\uparrow R)$ will be upward-closed with respect to $\preceq$, it is sufficient to prove that a basis of $\text{pred}(\uparrow R)$ can be computed from $R$. Let $g = \{ q_i \}$, $f = \{ (t = j \land c(j) = 1) \lor (c(t) \geq 1 \land c^t(j) = 0) \}$. Consider the following set of states:

$$CBasis = \{ (q_A, c) \in S \mid \exists (q'_A, c') \in R : [ (q_A, g, q'_A) \in \delta_A \land (q_A, c) \models q_A \land (c = c') \lor (c(t) = 0 \land c = c' + u_i) ] \}
\land \bigvee \left\{ (q_A, g, q'_A) \in \delta_B \land (q_A, c) \models q_A \land (q'_A = q^A) \right. \
\land \left. (c = c' + u_i - u_j) \lor (c(t) = 0 \land c^t(j) \geq 1 \land c = c' + u_i - u_j + u_i) \right\}. \hfill \Box
$$

Clearly, $CBasis \subseteq \text{pred}(\uparrow R)$, and $CBasis$ is finite. We claim that also $CBasis \supseteq \text{minBasis}(\text{pred}(\uparrow R))$. For the purpose of reaching a contradiction, assume $CBasis \not\supseteq \text{minBasis}(\text{pred}(\uparrow R))$, which implies that there exists a $(q_A, c) \in (\text{minBasis}(\text{pred}(\uparrow R)) \cap \non-CBasis)$. Since $(q_A, c) \not\in CBasis$, there exists $(q'_A, c') \not\in R$ with $(q_A, c) \rightarrow (q'_A, c')$ and since $(q_A, c) \in \text{minBasis}(\text{pred}(\uparrow R))$, there is a $(q'_A, d') \in R$ with $(q'_A, d') \preceq (q'_A, c')$. We differentiate between two cases:

- Case 1: Suppose $(q_A, c) \xrightarrow{t_A} (q'_A, c')$ with $t_A = (q_A, g, q'_A) \in \delta_A$ and $(q_A, c) \models q_A \land g$. Then $c = c'$, and by definition of CBasis there exists $(q_A, d) \in CBasis$ with $[(q_A, d) \rightarrow (q'_A, d')] \land d = d' \land d^t(t) \geq 1$ or $[(q_A, d) \rightarrow (q'_A, d' + u_i) \land d = d' + u_i \land d^t(t) = 0]$. Furthermore, we have $d' \preceq c'$, which implies $(q_A, d) \preceq (q_A, c)$ with $(q'_A, d') \in R$. Contradiction.

- Case 2: Suppose $(q_A, c) \xrightarrow{t_B} (q'_A, c')$ with $t_B = (q_i, g, q_j) \in \delta_B$ and $(q_A, c) \models q_i \land g$. Then $q_A = q'_A \land c^t = c' + u_i - u_j$. By definition of CBasis there exists $(q_A, d) \in CBasis$ such that one of the following holds:

  * $(q_A, d) \rightarrow (q'_A, d') \land d' = d - u_i + u_j$
Lemma 3. Let \( M = (S, S_0, \Delta) \) be a counter system for process templates \( A, B \). Then \( (M, \preceq_0) \) is a WSTS.

Proof. The partial order \( \preceq_0 \) is obviously reflexive and transitive. Additionally \( \preceq_0 \) is well-founded and it has no infinite antichains due to the fact that \( \preceq \) is a wqo and the comparison of vector positions against zero only introduces a finite case distinction. Thus, \( \preceq_0 \) is a wqo. Moreover we show that \( \preceq_0 \) is compatible with \( \Delta \). Let \( s = (q_A, c), s' = (q_A', c'), r = (q_A, d) \in S \) such that \( s \xrightarrow{\Delta} s' \) and \( s \preceq_0 r \). We show that there exists \( r' = (q_A', d') \in S \) and a sequence of transitions \( r \xrightarrow{s} r' \) such that \( s' \preceq r' \). First, note that since transition \( t_U \) is enabled in \( s \), it is also enabled in \( r \). If \( t_U \) is a transition of \( A \) then we have \( c = d \) and \( c' = d' \), and we are done. If \( t_U \) is a transition of \( B \) with \( t_U = (q_i, g, q_j) \) and \( (c(i) > 1 \lor c(i) = d(i)) \) then \( q_A = q_A' \) and \( c = c - u_i + u_j \).\( \preceq_0 d - u_i + u_j = d' \). However if \( c(i) = 1 \land d(i) = b > 1 \), then \( r' \) is reachable from \( r \) by executing \( t_U \) \( b \) times. In this case, we get \( c' = c - u_i + u_j \preceq_0 d - b \cdot u_i + b \cdot u_j = d' \). \( \square \)

Lemma 4. Let \( R \subseteq S \) be upward-closed with respect to \( \preceq_0 \). Then \( \text{opred}(R) \) is upward-closed with respect to \( \preceq_0 \).

Proof. Assume \( \text{opred}(R) \) is not upward-closed. Let \( (q_A, c) \preceq_0 (q_A, d) \) such that \( (q_A, c) \in \text{opred}(R) \) and \( (q_A, d) \notin \text{opred}(R) \). We write \( s \xrightarrow{t_U} r \) if \( r \) is reachable from \( s \) by executing the local transition \( t_U \) exactly \( c \) times. For a transition \( t_U = (q_i, g, q_j) \) and a state \( (q_A', c') \in R \), we distinguish two cases:

- Case 1: \( (q_A, c) \xrightarrow{t_U} (q_A', c') \) and \( c'(i) > 0 \). Since \( (q_A, c) \preceq_0 (q_A, d) \), then \( t_U \) is enabled in \( (q_A, d) \) and there exists \( (q_A', d') \in R \) such that \( (q_A, d) \xrightarrow{t_U} (q_A', d') \) with \( d'(i) > 0 \).

- Case 2: \( (q_A, c) \xrightarrow{t_U} (q_A', c') \) with \( c \in N_0, c = c(i) \) and \( c'(i) = 0 \). Since \( (q_A, c) \preceq_0 (q_A, d) \), then \( t_U \) is enabled in \( (q_A, d) \) and there exists \( c' \in N_0 \), and \( (q_A', d') \in R \) such that \( c' = d(i) \) and \( (q_A, d) \xrightarrow{t_U} (q_A', d') \) with \( d'(i) = 0 \).

In both cases we have \( (q_A', c') \preceq_0 (q_A', d') \), then \( (q_A, d) \in \text{opred}(R) \), which contradicts our assumption. \( \square \)

Lemma 5. Let \( M = (S, S_0, \Delta) \) be a counter system for process templates \( A, B \). Then \( (M, \preceq_0) \) is a WSTS with effective \( \text{opred-basis} \).
Proof. Let $R \subseteq S$ be finite. Then it suffices to prove that a finite basis of $\text{opred}(\uparrow R)$ can be computed from $R$ (where the upward-closure is wrt. $\subseteq_0$).
Consider the following set of states:

$$DB-Pred(R) = \{ (q_A, c) \in S \mid \exists (q_A', c') \in R : (q_A, g, q_A') \in \delta_A \land (q_A, c) \models q_A g \land c = c' \}
\bigwedge \{(q_A = q_A' \land (q_A, g, q_A') \in \delta_B \land (q_A, c) \models q_A g) \land (c = c' + u_i - u_j) \lor (c' \downarrow (j) = 1 \land c = c' + u_j) \lor (c' \downarrow (j) = k \land c = c' + k \cdot u_i - k \cdot u_j) \}$$

That is, $DB-Pred$ contains all states that map with a single transition into $R$, as well as states $(q_A, c)$ with $(q_A, c) \rightarrow (q_A, c' + u_j)$ for $(q_A, c') \in R$ with $c'(j) = 1$.

Obviously, $DB-Pred(R) \subseteq \text{opred}(\uparrow R)$. We claim that also $DB-Pred(R) \supseteq \text{minBasis}(\text{opred}(\uparrow R))$. For the purpose of reaching a contradiction, assume $DB-Pred(R) \not\supseteq \text{minBasis}(\text{opred}(\uparrow R))$, and let $(q_A, c) \in (\text{minBasis}(\text{opred}(\uparrow R)) \cap \neg DB-Pred(R))$. Since $(q_A, c) \not\in DB-Pred(R)$, there exist $(q_A', c') \in R$ with $(q_A, c) \rightarrow (q_A', c')$, $k \in \mathbb{N}_0$, and $(q_A', c') \in R$ with $(q_A, c) \not\subseteq_0 (q_A', c')$. We differentiate between two cases:

- Case 1: $c = c'$, and $\exists g$ with $(q_A, g, q_A') \in \delta_A$ and $(q_A, c) \models q_A g$. By definition of $\subseteq_0$ we have $(q_A, d') \models q_A g$, and therefore $(q_A, d') \rightarrow (q_A', c')$ with $(q_A, d') \not\subseteq_0 (q_A, c)$. Contradiction.

- Case 2: $k = 1$, $q_A = q_A'$, and $\exists g$ with $(q_A, g, q_A) \in \delta_B$, $(q_A, c) \models q_A g$, and $(c' = c - u_i + u_j)$. Then $(q_A, c) \models q_A g$ implies $(q_A', (c' + u_i)) \models q_A g$. However, in this case there is also a transition $(q_A', d' + u_i) \rightarrow (q_A, d' + u_i)$, and by definition of $DB-Pred$, then $(q_A', d' + u_i) \in DB-Pred(R)$, with $(d' + u_i)(j) = 1 \leq c(j)$, $d' + u_i \not\subseteq_0 c$, and $(q_A, d' + u_i) \not\subseteq_0 (q_A, c)$. Contradiction.

- Case 3: $k > 1$, $q_A = q_A'$, $c(j) = 0$. Then $c'(j) > 1$ and $d'(j) \geq 1$. For $d = d' + d'(j) \cdot (u_i - u_j)$ we have $(q_A, d) \models q_A g$, $(q_A, d) \rightarrow (q_A', c')$, and $(q_A, d) \not\subseteq_0 (q_A, c)$. Contradiction.

- Case 4: $k > 1$, $q_A = q_A'$, $c(j) > 0$, and $c'(i) = 0$. Then $c(j) > 1$, and either $d'(j) = 1$ or $d'(j) = 1$.

- If $d'(j) > 1$. Let $k' = \min\{k, (d'(j) - 1)\}$, then for $d = d' + k' \cdot (u_i - u_j)$ we have $(q_A, d) \models q_A g$, $d(j) > 0$, $(q_A, d) \rightarrow (q_A', c')$, and $(q_A, d) \not\subseteq_0 (q_A, c)$. Contradiction.

- If $d'(j) = 1$. Then, for $d = d' + u_i$ we have $(q_A, d) \models q_A g$, $d(j) > 0$, $(q_A, d) \rightarrow (q_A', d' + u_i)$, and $(q_A, d) \not\subseteq_0 (q_A, c)$. Contradiction.

$\square$
Theorem 1. Deadlock detection in disjunctive systems is decidable in NEXP-TIME.

Proof. We show that if a deadlocked run exists, then we can construct a sequence of configurations that witnesses the deadlock and can be traversed by at most $2^{|B|}$ applications of $DB$-Pred.

Let $\pi$ be a deadlocked run. We first separate $\pi$ into segments that agree for all $i$ on whether $c[i] = 0$, and then remove “loops” between repetitions of segments, i.e., if two different segments agree on which positions $i$ have $c[i] = 0$, then we remove everything between the two segments. The result is a sequence of at most $2^{|B|}$ segments, which however may not respect the semantics of counter systems, i.e., is not necessarily a run.

To obtain something that conforms to the semantics of counter systems again, we replace each segment with a single configuration, where we first keep the values at positions $i$ with $c[i] > 0$ symbolic. Since $\pi$ was a run, the difference in 0-positions between two segments can be at most one position that moves from $c[i] = 0$ to $c[i] > 0$, and one position that moves from $c[i] > 0$ to $c[i] = 0$, possibly both. In any case, and even if more than one process has to be moved from one position to obtain $c[i] = 0$, the transition between segments can be done with a single application of $opred$, and therefore of $DB$-Pred.

What remains is to instantiate the symbolic values such that the sequence conforms to the semantics of counter systems (modulo summarizing multiple applications of the same local transition). Since the time needed for applications of $DB$-Pred is independent of the size of these values, this can simply be done by starting with sufficiently large values in the deadlocked state.

Since every step in the resulting sequence of length at most $2^{|B|}$ can be checked against $DB$-Pred in time linear in $|\delta|$, we obtain NEXP-TIME complexity for the problem of deadlock detection. \hfill \Box

B Additonal Properties of Algorithm 2

Local Witnesses are Upward-closed. We show another property of our algorithm: even though for the reachable error sequence $RE$ we do not consider the upward closure, the error paths we discover are in a sense upward-closed. This implies that an $RE$ of length $k$ represents all possible error paths of length $k$. We formalize this in the following.

Given a reachable error sequence $RE = RE_k, \ldots, RE_0$, we denote by $UE$ the sequence $\uparrow RE_k, \ldots, \uparrow RE_0$. Furthermore, let a local witness of $RE$ be a sequence $T_{RE} = t_{V_k} \ldots t_{V_0}$ where for all $i \in \{1, \ldots, k\}$ there exists $s \in RE_i, s' \in RE_{i-1}$ with $s \xrightarrow{t_{V_i}} s'$. We define similarly the local witness $T_{UE}$ of $UE$.

Lemma 6. Let $RE$ be a reachable error sequence. Then every local witness $T_{UE}$ of $UE$ is also a local witness of $RE$. 

C Beyond Reachability

Algorithm 2 can also be used for repair with respect to general safety properties, based on the automata-theoretic approach to model checking. We define a safety property as a set of runs that lead to an erroneous behavior of the system. This set of runs is encoded in a finite-state automaton.

A finite-state automaton is a tuple $\mathcal{A} = (Q^A, q_0^A, \Sigma, \delta^A, F)$ where:

- $Q^A$ is a finite set of states,
- $q_0^A$ is the initial state,
- $\Sigma$ is an input alphabet,
- $\delta^A \subseteq Q^A \times \Sigma \times Q^A$ is a transition relation, and
- $F \subseteq Q^A$ is a set of accepting states.

A run of the automaton is a finite sequence $q_0^A a_0 q_1^A a_1 q_2^A a_2 \ldots q_n^A$ where $\forall i' : (q_i^A, a_i, q_{i+1}^A) \in \delta^A$. A run of the automaton is accepting if it visits a state in $F$.

C.1 Checking Safety Properties

Let $M = (Q_A \times \mathbb{N}_0, S_0, G, \Delta)$ be a counter system of process templates $A$ and $B$ that violates a safety property $\varphi$ over the states of $A$, and let $\mathcal{A} = (Q^A, q_0^A, Q_A, \delta^A, F)$ be the automaton that accepts all words over $Q_A$ that violate $\varphi$. To repair $M$, the composition $M \times \mathcal{A}$ and the set of error states $\mathcal{E} = \{((q_A, c), q^A) \mid (q_A, c) \in S \land q^A \in F\}$ can be given as inputs to the procedure ParamRepair. This technique is correct due to the following property that holds on the composition $M \times \mathcal{A}$.

Corollary 2 Let $\preceq_A \subseteq (M \times \mathcal{A}) \times (M \times \mathcal{A})$ be a binary relation defined by:

$$((q_A, c), q^A) \preceq_A ((q'_A, c'), q'^A) \iff c \preceq c' \land q_A = q'_A \land q^A = q'^A$$

then $(M \times \mathcal{A}, \preceq_A)$ is a WSTS with effective pred-basis.

Similarly, the algorithm can be used for any safety property $\varphi(A, B^{(k)})$ over the states of $A$, and of $k$ $B$-processes. To this end, we consider the composition
and we find out that the error sequence can be avoided if we remove finally assures that the new system is safe. Note that some states q

2

4

7

is not restricted to disjunctive systems. In principle, it can be used may return the following error se-

quences where we only consider states that were omitted from error sequences in order to keep the presenta-

tion simple.

Example. Consider again the simple reader-writer system in Figures 5 and 4, and assume that instead of local transition (nr, {nw}, r) we have an unguarded transition (nr, Q, r). We want to repair the system with respect to the safety property \( \varphi = G[(w \land nr_1) \implies (nr_1 \land nw)] \) where G, W are the temporal operators always and weak until, respectively. Figure 7 depicts the automaton equivalent to \( \neg \varphi \). To repair the system we first need to split the guards as mentioned in Section 2, i.e., (nr, Q, r) is split into (nr, {nr}r), (nr, {r}, r), (nr, {nw}, r), and (nr, {w}, r). Then we consider the composition \( C = M \times B \times A \) and we run Algorithm 2 on the parameters \( C, (\neg, -, (*, *), q^d_{n}) \) where (\neg, -) means any writer state and any reader state, and * means 0 or 1. The model checker in Line 4 may return the following error sequences where we only consider states that didn’t occur before:

\[
E_0 = \{((\neg, -, (*, *)), q^d_{n})\},
E_1 = \{((w, r_1, (0, 0)), q^d_{s})\},
E_2 = \{((w, nr_1, (0, 0), q^d_{0}), ((w, nr_1, (0, 1), q^d_{0}), ((w, nr_1, (1, 0), q^d_{0})\},
E_3 = \{((nw, nr_1, (0, 0)), q^d_{0}), ((nw, nr_1, (0, 1)), q^d_{0}), ((w, r_1, (0, 0)), q^d_{0}),
((w, r_1, (0, 1), q^d_{0}), (w, r_1, (1, 0), q^d_{0})\}
\]

In Line 12 we find out that the error sequence can be avoided if we remove the transitions \{nr, {nr}, r\}, (nr, {r}, r), (nr, {nw}, r). Another call to the model checker in Line 4 finally assures that the new system is safe. Note that some states were omitted from error sequences in order to keep the presentation simple.

D Beyond Disjunctive Systems

Algorithm 2 is not restricted to disjunctive systems. In principle, it can be used for any system that can be modeled as a WSTS with effective _pred_-basis, as long as we can construct the transition relation constraint (TRConstr) for the corresponding system. In this section we show two other classes of systems that can be modeled in this framework: pairwise rendezvous (PR) and broadcast (BC) systems. We introduce transition relation constraints for these systems, as well as a procedure _BuildSyncConstr_ that must be used instead of _BuildConstr_ when a transition relation comprises synchronous actions.

Since these two classes of systems require processes to synchronize on certain actions, we first introduce a different notion of process templates.

Processes. A _synchronizing process template_ is a transition system \( U = (Q_U, init_U, \Sigma, \delta_U) \) with

\footnote{By symmetry, property \( \varphi(A, B^{(k)}) \) can be violated by these \( k \) explicitly modeled processes if it can be violated by any combination of \( k \) processes in the system.}
\[ Q_U \subseteq Q \text{ is a finite set of states including the initial state } \text{init}_U, \]

\[ \Sigma = \Sigma_{sync} \times \{?, !, ??, !!\} \cup \{\tau\} \text{ where } \Sigma_{sync} \text{ is a set of synchronizing actions,} \]

and \( \tau \) is an internal action,

\[ \delta_U : Q_U \times \Sigma \times Q_U \text{ is a transition relation.} \]

Synchronizing actions like \( (a, !) \) or \( (b, ?) \) are shortened to \( a! \) and \( b?. \) Intuitively actions of the form \( a! \) and \( a? \) are PR send and receive actions, respectively, and

\( a!!, a?!! \) are BC send and receive actions, respectively.

All processes mentioned in the following are based on a synchronizing process template. We will define global systems based on either PR or BC synchronization in the following subsections.

D.1 Pairwise Rendezvous Systems

A PR system [33] consists of a finite number of processes running concurrently. As before, we consider systems of the form \( A \parallel B^n \). The semantics is interleaving, except for actions where two processes synchronize. That is, at every time step, either exactly one process makes an internal transition \( \tau \), or exactly two processes synchronize on a single action \( a \in \Sigma_{sync} \). For a synchronizing action \( a \in \Sigma_{sync} \), the initiator process locally executes the \( a! \) action and the recipient process executes the \( a? \) action.

Similar to what we defined for disjunctive systems, the configuration space of all systems \( A \parallel B^n \), for fixed \( A, B \) but arbitrary \( n \in \mathbb{N} \), is the counter system \( M^{PR} = (S, S_0, \Delta) \), where:

\[ S \subseteq Q_A \times \mathbb{N}_0^{|B|} \text{ is the set of states,} \]

\[ S_0 = \{ (\text{init}_A, c) \mid \forall q_B \in Q_B : c(q_B) = 0 \text{ if } q_B \neq \text{init}_B \} \]

is the set of initial states,

\[ \Delta \text{ is the set of transitions } ((q_A, c), (q'_A, c')) \text{ such that one of the following holds:} \]

1. \( (q_A, \tau, q'_A) \in \delta_A \land c = c' \) (internal transition \( A \))
2. \( \exists q_i, q_j : (q_i, \tau, q_j) \in \delta_B \land c(i) \geq 1 \land c' = c - u_i + u_j \land q_A = q'_A \) (internal transition \( B \))
3. \( a \in \Sigma_{sync} \land (q_A, a!, q'_A) \in \delta_A \land \exists q_i, q_j : (q_i, a?, q_j) \in \delta_B \land c(i) \geq 1, c' = c - u_i + u_j \) (synchronizing transition \( A, B \))
4. \( a \in \Sigma_{sync} \land (q_A, a?, q'_A) \in \delta_A \land \exists q_i, q_j : (q_i, a!, q_j) \in \delta_B \land c(i) \geq 1, c' = c - u_i + u_j \) (synchronizing transition \( B, A \))
5. \( \exists q_i, q_j : (q_i, a!, q_j) \in \delta_B \land \exists q_m : (q_i, a?, q_m) \in \delta_B \land c(i) \geq 1 \land c(l) \geq 1 \land c' = c - u_i + u_j - u_m \) (synchronizing transition \( B, B \))

The following result can be considered folklore, a proof can be found in the survey by Bloem et al. [10].

Lemma 7. Let \( M^{PR} = (S, S_0, \Delta) \) be a counter system for process templates \( A, B \) with PR synchronization. Then \( (M^{PR}, \leq) \) is a WSTS with effective pred-basis.

Initial Constraints. The constraint \( TRConstr_{PR} \), ensuring that not all local transitions from any given local state are removed, is constructed in a similar way as \( TRConstr_{Disj} \).
Furthermore, the user may want to ensure that in the returned repair, either (a) for all \( a \in \Sigma_{\text{sync}} \), \( t_a! \) is deleted if and only if all \( t_a? \) are deleted, or (b) that synchronized actions are deterministic, i.e., for every state \( q_U \) and every synchronized action \( a \), there is exactly one transition on \( a? \) from \( q_U \). We give user constraints that ensure such behavior.

Denote by \( t_a? \), \( t_a! \) synchronous local transitions based on an action \( a \). Then, the constraint ensuring property (a) is

\[
\bigwedge_{a \in \Sigma_{\text{sync}}} \left[ (t_a! \land \bigvee_{t_a? \in \delta} \neg t_a?) \lor (\neg t_a! \land \bigwedge_{t_a? \in \delta} \neg t_a?) \right]
\]

To encode property (b), for \( U \in \{A, B\} \) and \( a \in \Sigma_{\text{sync}} \), let \( \{t_{q_U}^a1?, \ldots, t_{q_U}^am?\} \) be the set of all \( a? \) transitions from state \( q_U \in Q_U \). Additionally, let \( \text{one}(t_{q_U}^a?) = \bigvee_{j \in \{1, \ldots, m\}} [t_{q_U}^a j? \land \neg t_{q_U}^a l?] \). Then, (b) is ensured by

\[
\bigwedge_{a \in \Sigma_{\text{sync}}} \bigwedge_{q_U \in Q} \text{one}(t_{q_U}^a?)
\]

**Deadlock Detection for PR Systems.** German and Sistla \([33]\) have shown that deadlock detection in PR systems can be reduced to reachability in VASS, and vice versa. Thus, at least a rudimentary version of repair including deadlock detection is possible, where the deadlock detection only excludes the current candidate repair, but may not be able to provide constraints on candidates that may be considered in the future. Moreover, the reachability problem in VASS has recently been shown to be Tower-hard, so a practical solution is unlikely to be based on an exact approach.

### D.2 Broadcast Systems

In broadcast systems, the semantics is interleaving, except for actions where all processes synchronize, with one process “broadcasting” a message to all other processes. Via such a broadcast synchronization, a special process can be selected while the system is running, so we can restrict our model to systems that only contain an arbitrary number of user processes with identical template \( B \). Formally, at every time step either exactly one process makes an internal transition \( \tau \), or all processes synchronize on a single action \( a \in \Sigma_{\text{sync}} \). For a synchronized action \( a \in \Sigma_{\text{sync}} \), we say that the initiator process executes the \( a! \) action and all recipient processes execute the \( a?? \) action. For every action \( a \in \Sigma_{\text{sync}} \) and every state \( q_B \in Q_B \), there exists a state \( q_B^a \in Q_B \) such that \( (q_B, a??, q_B^a) \in \delta_B \). Like Esparza et al. \([24]\), we assume w.l.o.g. that the transitions of recipients are deterministic for any given action, which implies that the effect of a broadcast message on the recipients can be modeled by multiplication of a broadcast matrix. We denote by \( M_a \) the broadcast matrix for action \( a \).

Then, the configuration space of all broadcast systems \( B^n \), for fixed \( B \) but arbitrary \( n \in \mathbb{N} \), is the counter system \( M^{BC} = (S, S_0, \Delta) \) where:
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- $S \subseteq \mathbb{N}_0^{|B|}$ is the set of states,
- $S_0 = \{c \mid \forall q_B \in Q_B : c(q_B) = 0 \text{ iff } q_B \neq \text{init}_B\}$ is the set of initial states,
- $\Delta$ is the set of transitions $(c, c')$ such that one of the following holds:
  1. $\exists q_i, q_j \in Q_B : (q_i, \tau, q_j) \in \delta_B \wedge c' = c - u_i + u_j$ (internal transition)
  2. $\exists a \in \Sigma_{\text{sync}} : c' = M_a \cdot (c - u_i) + u_j$ (broadcast)

Lemma 8. [24] Let $M^{BC} = (S, S_0, \Delta)$ be a counter system for process template $B$ with BC synchronization. Then $(M^{BC}, \preceq)$ is a WSTS with effective pred-basis.

Initial Constraints. $TRConstr_{BC}$ is defined similarly to $TRConstr_{PR}$, except that we do not have process $A$ and can omit transitions of $A$. We denote by $t_{a??}$, $t_{a!!}$ synchronous transitions based on an action $a$. To ensure that in any repair and for all $a \in \Sigma_{\text{sync}}$, $t_{a!!}$ is deleted if and only if all $t_{a??}$ are deleted, the designer can use the following constraint:

$$\bigwedge_{a \in \Sigma_{\text{sync}}} \left[(t_{a!!} \land (\bigvee_{t_{a??} \in \delta_B} t_{a??})) \lor (\neg t_{a!!} \land (\bigwedge_{t_{a??} \in \delta_B} \neg t_{a??}))\right]$$

Deadlock Detection for BC Systems.

Theorem 5. Deadlock detection in broadcast protocols is undecidable.

The main ingredient of the proof is the following lemma:

Lemma 9. There is a polynomial-time reduction from the reachability problem of affine VASS with broadcast matrices to the deadlock detection problem in broadcast protocols.

Proof. We modify the construction from the proofs of Theorems 3.17 and 3.18 from German and Sistla [33], using affine VASS instead of VASS and broadcast protocols instead of pairwise rendezvous systems.

Starting from an arbitrary affine VASS $G$ that only uses broadcast matrices and where we want to check if configuration $(q_2, c_2)$ is reachable from $(q_1, c_1)$, we first transform it to an affine VASS $G^*$ with the following properties

- each transition only changes the vector $c$ in one of the following ways: (i) it adds to or subtracts from $c$ a unit vector, or (ii) it multiplies $c$ with a broadcast matrix $M$ (this allows us to simulate every transition with a single transition in the broadcast system), and
- some configuration $(q'_2, 0)$ is reachable from some configuration $(q'_1, 0)$ in $G^*$ if and only if $(q_2, c_2)$ is reachable from $(q_1, c_1)$ in $G$.

The transformation is straightforward by splitting more complex transitions and adding auxiliary states. Now, based on $G^*$ we define process templates $A$ and $B$ such that $A \parallel B^a$ can reach a deadlock iff $(q'_2, 0)$ is reachable from $(q'_1, 0)$ in $G^*$.

The states of $A$ are the discrete states of $G^*$, plus additional states $q', q''$. If the state vector of $G^*$ is $m$-dimensional, then $B$ has states $q_1, \ldots, q_m$, plus states init, $v$. Then, corresponding to every transition in $G^*$ that changes the state from
Algorithm 3 Synchronous Constraint Computation

1: procedure BSC(State $s$, $\mathcal{RE}$)  
2: if $\mathcal{RE}[1:]$ is empty then 
3: return $\bigwedge_{t_U \in \Delta_{local}(s,\mathcal{RE}[0])} \neg t_U \bigwedge_{a \in \Sigma_{sync} \land \Delta(s,a) \in \mathcal{RE}[0]} T(s,a)$  
4: else 
5: return $\bigwedge_{t_U \in \Delta_{local}(s,\mathcal{RE}[0])} (\neg t_U \lor BSC(\Delta(s,t_U), \mathcal{RE}[1:]))$  
6: $\bigwedge_{a \in \Sigma_{sync} \land \Delta(s,a) \in \mathcal{RE}[0]} [T(s,a) \lor BSC(\Delta(s,t_a), \mathcal{RE}[1:])]$  

$q$ to $q'$ and either adds or subtracts unit vector $u_i$, we have a rendezvous sending transition from $q$ to $q'$ in $A$, and a corresponding receiving transition in $B$ from init to $q_i$ (if $u_i$ was added), or from $q_i$ to init (if $u_i$ was subtracted). For every transition that changes the state from $q$ to $q'$ and multiplies $c$ with a matrix $M$, $A$ has a broadcast sending transition from $q$ to $q'$, and receiving transitions between the states $q_1, \ldots, q_m$ that correspond to the effect of $M$.

The additional states $q', q''$ of $A$ are used to connect reachability of $(q'_2, 0)$ to a deadlock in $A||B^n$ in the following way: (i) there are self-loops on all states of $A$ except on $q'$, i.e., the system can only deadlock if $A$ is in $q'$, (ii) there is a broadcast sending transition from $q'_2$ to $q'$ in $A$, which sends all $B$-processes that are in $q_1, \ldots, q_m$ to special state $v$, and (iii) from $v$ there is a broadcast sending transition to init in $B$, and a corresponding receiving transition from $q'$ to $q''$ in $A$. Thus, $A||B^n$ can only deadlock in a configuration where $A$ is in $q'$ and there are no $B$-processes in $v$, which is only reachable through a transition from a configuration where $A$ is in $q_2$ and no $B$-processes are in $q_1, \ldots, q_m$. Letting $q_1$ be the initial state of $A$ and init the initial state of $B$, such a configuration is reachable in $A||B^n$ if and only if $(q'_2, 0)$ is reachable from $(q'_1, 0)$ in $G^*$. \[\square\]

D.3 Synchronous Systems Constraints

The procedure BuildConstr in Algorithm 2 does not take into consideration synchronous actions. Hence, we need a new procedure that offers special treatment for synchronization. To simplify presentation we assume w.l.o.g. that each $a+,$ with $+ \in \{!\ldots., ??\ldots., !\}$, appears on exactly one local transition. We denote by $\Delta_{sync}(s,a)$ the state obtained by executing action $a$ in state $s$. Additionally, let $\Delta_{local}(s,a) = \{(q_t : a, q'_t) \in \delta \mid * \in \{?, !, ??, !!\}\}$, and $a$ is enabled in $s$, and let $T(s,a) = \bigvee_{t_a \in \Delta_{local}(s,a)} t_a$. In a Broadcast system we say that an action $a$ is enabled in a global state $c$ if $\exists i, j < |B|$ s.t. $c(i) > 0$ and $(q_{B_i}, a!, q_{B_j}) \in \delta_B$. In a Pairwise rendezvous system we say that an action $a$ is enabled in a global state $c$ if $\exists i, j < |B|$ s.t. $c(i) > 0, c(j) > 0$ and $(q_{B_i}, a!, q_{B_j}), (q_{B_j}, a?, q_{B_i}) \in \delta_B$.

Given a synchronous system $M^X = (S, S_0, \Sigma, \Delta)$ with $X \in \{BR, PR\}$, a state $s$, and a reachable error sequence $\mathcal{RE}$, Algorithm 3 computes a propositional formula over the set of local transitions that encodes all possible ways for a state $s$ to avoid reaching an error.
E  Example: Reader-Writer

Consider the parameterized pairwise system that consists of one scheduler (Figure 1) and a parameterized number of instances of the reader-writer process template (Figure 2). The scheduler process template has all possible receive actions from every state. In such system, the scheduler can not guarantee that, at any moment, there is at most one process in the writing state $q_1$ (Figure 2). Let $t_{U_1} = [q_0, (\text{write})], t_{U_2} = [q_{A,0}, (\text{write})], q_{A,1}, t_{U_3} = [q_{A,1}, (\text{write})], q_{A,0}, t_{U_4} = [q_0, (\text{read})], q_2, t_{U_5} = [q_{A,0}, (\text{read})], q_{A,1}, t_{U_6} = [q_{A,1}, (\text{read})], q_{A,0}, t_{U_7} = [q_1, (\text{done}_w)], q_0, t_{U_8} = [q_{A,1}, (\text{done}_w)], q_{A,0}, t_{U_9} = [q_{A,0}, (\text{done}_w)], q_{A,1}, t_{U_{10}} = [q_2, (\text{done}_r)], q_0, t_{U_{11}} = [q_{A,1}, (\text{done}_r)], q_{A,0}, t_{U_{12}} = [q_{A,0}, (\text{done}_r)], q_{A,1}.

Let $ERR = \uparrow((q_{A,0}, (0, 2, 0))(q_{A,1}, (0, 2, 0)))$.

Let $UserConstr_{PR} = (t_{U_1} \land (t_{U_2} \lor t_{U_3})) \land (t_{U_4} \lor t_{U_5}) \land (t_{U_6} \land (t_{U_7} \lor t_{U_8})) \land (t_{U_9} \lor (t_{U_{10}} \lor t_{U_{11}}))$. Then running our repair algorithm will produce the following results:

First call to model checker returns:
$RE_0 = \{(q_{A,0}, (0, 2, 0))\}$, $RE_1 = \{(q_{A,1}, (1, 1, 0))\}$, $RE_2 = \{(q_{A,0}, (2, 0, 0))\}$.

Constraints for SAT: $accConstr_1 = TRConstr_{PR} \land UserConstr_{PR} \land (\neg t_{U_1} \lor \neg t_{U_2} \lor \neg t_{U_3})$.

SAT solvers solution 1:
$\neg t_{U_2} \land \neg t_{U_3} \land \neg t_{U_5} \land \neg t_{U_6}$.

Second call to model checker returns:
$RE_0 = \{(q_{A,0}, (0, 2, 0))\}$, $RE_1 = \{(q_{A,1}, (1, 1, 0))\}$, $RE_2 = \{(q_{A,0}, (2, 1, 0))\}$, $RE_3 = \{(q_{A,1}, (3, 0, 0))\}$, $RE_4 = \{(q_{A,0}, (4, 0, 0))\}$. Constraints for SAT:
$accConstr_2 = accConstr_1 \land (\neg t_{U_1} \lor \neg t_{U_2} \lor \neg t_{U_3} \lor \neg t_{U_5} \lor \neg t_{U_6})$.

SAT solvers solution 2:
$\neg t_{U_5} \land \neg t_{U_1} \land \neg t_{U_6} \land \neg t_{U_7}$.

Third call to model checker returns:
$RE_0 = \{(q_{A,0}, (0, 2, 0))\}$, $RE_1 = \{(q_{A,1}, (1, 1, 0))\}$, $RE_2 = \{(q_{A,0}, (2, 1, 0))\}$, $RE_3 = \{(q_{A,1}, (3, 0, 0))\}$, $RE_4 = \{(q_{A,0}, (3, 0, 0))\}$. Constraints for SAT:
$accConstr_3 = accConstr_2 \land (\neg t_{U_1} \lor \neg t_{U_2} \lor \neg t_{U_4} \lor \neg t_{U_6})$.

SAT solvers solution 3:
$\neg t_{U_6} \land \neg t_{U_1} \land \neg t_{U_7} \land \neg t_{U_8}$.

The fourth call of the model checker returns true and we obtain the correct scheduler in Figure 3.
Consider the cache coherence protocol MESI in Figure 8 where:

- \( M \) stands for \textit{modified} and indicates that the cache has been changed.
- \( E \) stands for \textit{exclusive} and indicates that no other process seizes this cache line.
- \( S \) stands for \textit{shared} and indicates that more than one process hold this cache line.
- \( I \) stands for \textit{invalid} and indicates that the cache's content is not guaranteed to be valid as it might have been changed by some process.

Initially all processes are in \( I \) and let a state vector be as follows: \((M, E, S, I)\). An important property for MESI protocol is that a cache line should not be modified by one process (in state \( M \)) and in shared state for another process (in state \( S \)). In such case the set of error states is: \( \uparrow(1, 0, 1, 0) \). We can run Algorithm 2 on \( M, \uparrow(1, 0, 1, 0), TRConstr_{BC} \land \bigwedge_{a \in \Sigma_{sync}} \bigwedge_{q_B \in Q_B} one(t_{q_B}^a) \). The model checker will return the following error sequence (nonessential states are omitted):

\[
E_0 = \{(1, 0, 1, 0)\}, \quad E_1 = \{(0, 1, 1, 0)\}, \quad E_2 = \{(0, 1, 0, 1)\}, \quad E_3 = \{(0, 0, 1, 1)\}, \quad E_4 = \{(0, 0, 0, 2)\}. \]

Running the procedure \textsc{BuildSyncConstr} (Algorithm 3) in Line 9 will return the following Boolean formula \( newConstr = \)

\[
\neg(I, \text{read!!}, S) \lor \neg(I, \text{read??}, I) \lor \neg(S, \text{write-inv!!}, E) \lor \neg(I, \text{write-inv??}, I) \\
\lor \neg(E, \text{read??}, E) \lor \neg(I, \text{read!!}, S) \lor \neg(E, \text{write}, S) \\
\]

Running the SAT solve in Line 12 on

\[
newConstr \land TRConstr'_{BC} \land \bigwedge_{a \in \Sigma_{sync}} \bigwedge_{q_B \in Q_B} one(t_{q_B}^a) \land t_U \]

will return the only solution \( \neg(E, \text{read??}, E) \) which clearly fixes the system.