Optimizing the selection of combination of alternative functions of ergatic system multifunctional elements

G I Zverev and V V Menshikh
Voronezh Institute of the Ministry of the Interior of Russia, Voronezh, Russia

E-mail: georgiyzverev@gmail.com, menshikh@list.ru

Abstract. Ergatic systems have found their application in those areas where operator's participation is an integral part of the effective functioning of these systems, that is, it has the ability to quickly solve problems and contingencies that arise during work process. Critical application systems are one of the varieties of ergatic systems. For their successful functioning, it is necessary to choose the optimal set of alternative functions of multifunctional elements. The relevance of this problem lies in the wide distribution of this type systems, as well as in their special importance. So this article provides a description of the structural and parametric models and functional capabilities of ergatic systems. As a model, it is proposed to use weighted hypergraphs. The statement of the problem of optimizing the choice of a combination of alternative functions of multifunctional elements of such systems is given. The description of the developed numerical method based on the use of the scheme of branches and boundaries is also given. The final part of the article presents a numerical example, divided into initial, intermediate and final stages of optimization, for demonstration of the described numerical method operation.

1. Introduction
The functioning efficiency of a wide class of systems is achieved due to a human operator's participation in the process that organizes the operation of various technical elements in these systems. The systems that are unable to function without a human operator's intervention are commonly referred to as ergatic [1] ones. Among the examples of such systems are aircraft control systems, and airports and railway station traffic control services [2].

Critical application systems, whose elements are capable of switching to alternative tasks when the operating conditions of the system are changed, occupy a special place among ergatic systems. Among the examples of critical applications are systems for managing the resources of law enforcement agencies in the destructive emergency situation conditions [3, 4 and 5].

The described ergatic systems comprise multifunctional elements that is the elements with a set of alternative functions [6]. Realization of each function involves interconnections between the elements (subsets of elements) within the system. And in critical application systems,
high-level interconnections may arise, involving not just two but also a greater number of elements [7].

Ergatic system efficiency may reduce due to various reasons. An example of such a reduction would be an internal or external destructive influence [8] caused by the emergency situation consequences.

2. Description of structural and parametric models and functional capabilities of ergatic systems

The functioning of the ergatic systems under consideration depends on the choice of one of the sets of alternative functions of \( f \) system elements. It is assumed that each function of the element determines, among other things, the totality of its connections with other elements. Thus, the sets of alternative functions must be coordinated: the choice of function for one element limits the possibilities to choose functions for other elements so that their connections with this element are preserved.

In this case, as structural and parametric models of the said systems it is appropriate to use weighted hypergraphs [7] of the following form:

\[
\Gamma(f) = (S, E(f), P, K(f)),
\]

(1)

whose vertices \( S \) correspond to the system elements;
edges \( E(f) \) characterize the interconnection between the elements (subsets of elements) for the selected set of functions \( f \);
weights of vertices \( P \) reflect the elements' functioning efficiency degree in accordance with the selected functions;
weights of edges \( K(f) \) reflect the degree of efficiency of interconnection between the elements (subsets of elements).

We will assume that:
1) the weights of the vertices and edges do not depend on the choice of the element function sets;
2) in order to ensure the system's functioning with any selection of set of functions \( f \), all elements are used, while the edges shall correspond to the given set of functions only.

Denote:
\( n = |S| \) – number of the system elements;
\( m = \bigcup_j E(f) \) – number of all potential interconnections between the system elements;
\( K = \bigcup_f K(f) \) – weights of all potential interconnections between the system elements;
\( q_{p\in f} = \begin{cases} 1, & \text{if execution of } f \text{ implies presence in } \Gamma(f) \text{ of } e_p \in E(f); \\ 0, & \text{if otherwise.} \end{cases} \)

For convenience, henceforth we will assume that:
\( k_j(f) = \begin{cases} k_j \in K, & \text{if } e_j \in E(f); \\ 0, & \text{if otherwise.} \end{cases} \)
The system functioning efficiency is characterized by the importance (contribution to the overall system efficiency) of certain elements of system $s_i$ depending on the selected function $f_{i}$. 

In this case, the functionality of the system on the whole can be estimated by the expression:

$$H\left(\Gamma(f),f,A(f)\right),$$

(3)

where

$$f = (f_{i_1}, \ldots, f_{i_m})$$

– set of selected functions of the system elements;

$$A(f) = \left\{ \alpha_l(f_{i_l}), \ldots, \alpha_n(f_{i_n}) \right\}$$

– importance (contribution to the overall efficiency) of the system elements depending on the functions selected for them.

This function monotonically increases with each of its arguments.

A special form of the said function is described by the authors in [7], and for the numerically specified weight values it has the form:

$$H\left(\Gamma(f),f,A(f)\right) = \sum_{i=1}^{n} \alpha_i(f_{i})p_i + \sum_{j=1}^{m} k_j(f)$$

(4)

and when fuzzy representation of the arguments [9] is used, from (3), the above expression (4) takes the form:

$$H\left(\Gamma(f),f,A(f)\right) = \mathbb{1} \left[ \frac{1}{m} \left( \text{T} \left( \mu_{i_l}(f_{i_l}), \mu_{p_l} \right) \right), \frac{1}{m} \left( \mu_{i_j}(f_{i_j}) \right) \right]$$

(5)

Here, we need to explain that the arguments from (4) are assigned their membership functions $\mu_{i_l}(f_{i_l}), \mu_{p_l}, \mu_{i_j}$, and the aggregation operations are presented in the form of T-norm $\text{T}$ and T-conorm $\perp$ by the known methods [10 and 11].

3. Setting the problem of the elements functions selection optimization

As stated before, various destructive influences may cause the efficiency of ergatic system functioning to decrease. In this connection, a problem of choosing the optimal combination of the system element functions arises, which combination maximizes the efficiency of its functioning. For this purpose, let us introduce variables $x_{i_l}$, responsible for the selection of certain alternative functions for the system elements:

$$x_{i_l} = \begin{cases} 
1, & \text{if } f_{i_l} \text{ is selected;} \\
0, & \text{if otherwise.}
\end{cases}$$

(6)

Denote $X = \left(x_{i_1}, \ldots, x_{i_m}\right)$.

Thus, the problem of optimizing the selection of the system element functions takes the form of finding

$$X^* = \arg \max_x \sum_{i=1}^{n} p_i \sum_{i=1}^{n} x_{i_l} \left( \alpha_i(f_{i_l}) + \sum_{j=1}^{m} \frac{k_j(f)}{\lambda_j} n_{i_j} \right),$$

(7)

subject to constraints:
\[ \sum_{i=1}^{n} x_{i} = 1; \quad \forall p \quad q_{p_{ij}} x_{i} = q_{p_{ij}} x_{p_{j}}. \]  

Here, we should explain that:

- \( n_{i} \) – number of system \( s_{i} \) element functions;
- \( n_{u_{i}} = \begin{cases} 1, & \text{if } f_{u_{i}} \text{ implies availability of connection } e_{j}; \\ 0, & \text{if otherwise}; \end{cases} \)
- \( \lambda_{j} = \sum_{i=1}^{n} \sum_{i=1}^{n} n_{u_{ij}} x_{i} \) – number of elements whose elected functions imply availability of connection \( e_{j} \).

It should be noted that (8) shows that one of the alternative functions is selected for each element, while (9) reflects availability of consistency of the selected system elements in terms of interconnection.

For solution of problems (7) – (9), numeric method has been developed based on using a branch-and-bound algorithms.

4. Description of the method based on using a branch-and-bound algorithms

The root of the partial solutions tree is the vertex \( \omega \) corresponding to the absence of function choice. The tree vertices correspond to tuples of the form \( \langle f_{i_{1}}, \ldots, f_{i_{n}} \rangle \), where \( l \in \{1, \ldots, n\} \).

Branching is performed by the rule:
- descendants of the vertex \( \omega \) are the vertices \( \langle f_{i_{1}}, \ldots, f_{i_{n}} \rangle \);
- descendants of the vertices \( \langle f_{i_{1}}, \ldots, f_{i_{n}} \rangle \) are the vertices \( \langle f_{i_{1}}, \ldots, f_{i_{n}}, f_{l(i+1)} \rangle, \ldots, \langle f_{i_{1}}, \ldots, f_{i_{n}}, f_{l(i+1)m+1} \rangle \).

For estimation of the vertex \( \omega \), we suggest:

\[ \sum_{i=1}^{n} \alpha_{\max} p_{i} + \sum_{j=1}^{m} k_{j}, \quad (10) \]

where

\[ \alpha_{\max} = \max_{j} \{ \alpha_{j}(f_{i_{1}}), \ldots, \alpha_{j}(f_{i_{n}}) \}. \quad (11) \]

For estimation of the vertices \( \langle f_{i_{1}}, \ldots, f_{i_{n}} \rangle \), we suggest:

\[ \sum_{i=1}^{n} \alpha_{j}(f_{i_{j}}) p_{i} + \sum_{i=1}^{n} \alpha_{\max} p_{i} + \sum_{j=1}^{m} k_{j}(f) \lambda_{j} n_{u_{ij}}, \quad (12) \]

where
\[ p_{i,j} = \begin{cases} n_{i,j} \text{, for the selected functions } \{ f_{i_1}, \ldots, f_{i_l} \}; \\ 1 \text{, for other functions.} \end{cases} \]  

(13)

Correctness of the choice of the mentioned estimations is supported by the fact that they are maximal and monotonically decreasing when descending the partial solutions tree in accordance with the general principle of branch-and-bound algorithm.

The partial solutions tree traversal is performed in two stages:

1) finding the initial solution using directed descent of the partial solutions tree;
2) optimizing the initial solution using restricted traversal of the partial solutions tree vertices.

Realization of the first stage is actually the realization of a greedy algorithm that makes the locally best choice at every step, so that the final solution be optimal. The obtained solution is referred to as the record.

The second stage of the tree traversal consists in traversing the solutions tree in accordance with the following rules:

1) if, at any step of the tree viewing, the estimation of the partial solution (12) appears to be no better than the record one, then no further descent is performed, and one level up ascent is performed and an alternative path is selected;
2) if, at any step of the tree viewing, a new solution is found that improves the previous one, it is referred to as the record, and all the subsequent comparisons are made with it.

5. Numerical example for demonstration of the method operation

Let there be an ergatic system whose structural and parametric model looks as follows:

**Figure 1.** Structural and parametric model of a conventional ergatic system.
A table of alternative functions is specified for each system element. The weights of the elements and connections are also specified, as well as the importance of each alternative function.

The above-mentioned data are presented in tables 1 – 3.

**Table 1.** Alternative functions of the system elements.

|   | S1 | f11 | f12 |
|---|----|-----|-----|
| e1 | 0  | 1   | 0   |
| e2 | 1  | 0   | 1   |

|   | S2 | f21 | f22 |
|---|----|-----|-----|
| e1 | 1  | 1   | 1   |
| e3 | 1  | 0   | 0   |
| e5 | 0  | 1   | 1   |

|   | S3 | f31 | f32 |
|---|----|-----|-----|
| e2 | 1  | 0   | 0   |
| e4 | 1  | 1   | 1   |

|   | S5 | f51 |
|---|----|-----|
| e3 | 1   |     |
| e4 | 1   |     |

|   | S6 | f61 | f62 | f63 |
|---|----|-----|-----|-----|
| e5 | 0  | 1   | 1   | 1   |
| e6 | 1  | 1   | 1   | 0   |
Table 2. Weights of the system vertices and connections.

|   |   |   |   |
|---|---|---|---|
| $S_1$ | 4 | $e_1$ | 4 |
| $S_2$ | 6 | $e_2$ | 3 |
| $S_3$ | 2 | $e_3$ | 3 |
| $S_4$ | 7 | $e_4$ | 7 |
| $S_5$ | 4 | $e_5$ | 8 |
| $S_6$ | 8 | $e_6$ | 4 |

Table 3. Importance of the alternative functions of the system elements.

|   |   |
|---|---|
| $f_{11}$ | 0.6 |
| $f_{12}$ | 0.4 |
| $f_{21}$ | 0.2 |
| $f_{22}$ | 0.7 |
| $f_{31}$ | 0.6 |
| $f_{32}$ | 0.3 |
| $f_{41}$ | 0.6 |
| $f_{42}$ | 0.4 |
| $f_{51}$ | 0.7 |
| $f_{61}$ | 0.8 |
| $f_{62}$ | 0.5 |
| $f_{63}$ | 0.1 |
In order to build the solution tree, we need to find the value of the vertex $\omega$. Thus, $\alpha_{\text{max}}(f_{\eta_i})$ is found from the alternative functions table and corresponds to the maximally important $f_{\eta_i}$ for each of the system elements.

As one can see from the values presented in table 2, for this example the value of $\omega = 50.2$. The finding of the initial solution is shown in figure 2.

**Figure 2.** Initial optimization stage.

The intermediate and final optimization results are shown in figures 3 and 4.

**Figure 3.** Intermediate optimization stage.
6. Conclusion

Thus, the paper presents a mathematical model and an optimization algorithm for selecting the optimal combination of alternative functions for multifunctional elements of ergatic systems, based on branch-and-bound algorithms, as well as a numerical optimization example that illustrates the operation of the suggested method.

Further evolution of the ideas set forth in this article will consist in the development of methods for estimating the survivability of ergatic systems being considered, under various destructive influences on them.

References

[1] Malafeev S I and Kopeikin A I 2012 Reliability of technical systems. Examples and problems (Moscow: Gornaya Kniga) p 299

[2] Zverev G I and Menshikh V V 2019 An approach to assessment of ergatic systems survivability Informatics: problems, methodology, technology: collection materials of the XIX int. scientific and methodological conf. ed D N Borisov (Voronezh: Scientific Research Publications (Welborn LLC)) pp. 1138-1140

[3] Menshikh V V, Samorokovskiy A F, Sereda E N and Gorlov V V 2017 Modeling the collective actions of law enforcement officers: monograph (Voronezh: Voronezh Institute of the Ministry of the Interior of Russia) p 236

[4] Menshikh V V and Pyankov O V 2018 Interval estimation of a system balance based on the conflict theory Journal of Siberian Federal University. Series «Mathematics and Physics» vol 11 (Krasnoyarsk: Siberian Federal University) 2 pp 249-257

[5] Menshikh V V and Pyankov O V 2016 Structural Parametric Modelling of an Information Analytical System Bulletin of the South Ural State University. Series «Mathematical Modeling, Programming & Computer Software» (Chelyabinsk: South Ural State University (National Research University)) 1 pp 105-113

[6] Menshikh V V and Spiridonova N E 2019 Structural and parametric modeling of heterogeneous systems for ensuring their safety Complex systems: control and modeling problems. Proc. of the XXI Int. Conf. (Samara : Ofort LLC) pp 289-292

[7] Menshikh V V and Zverev G I 2019 Weighted hypergraph-based modeling of the ergatic

![Figure 4. Final optimization stage.](image-url)
systems survivability estimation Complex systems: control and modeling problems. Proc. of the XXI Int. Conf. (Samara: Ofort LLC) pp 388-391

[8] Zverev G I 2019 Evaluation of ergatic system functioning efficiency under destructive influence All-Russian Scientific and Practical Conference «Topical issues of operation of security systems and protected telecommunication systems»: collection of materials (Voronezh: Voronezh Institute of the Ministry of the Interior of Russia) pp 142-143

[9] Zverev G I 2019 The question of describing ergatic system structural and parametric model / Public safety, law and order in the III millennium: collection of articles (Voronezh: Voronezh Institute of the Ministry of the Interior of Russia) 5 pp 145-147

[10] Zak Iu A 2013 Decision-making with fuzzy and diffuse data: Fuzzy-technology (Moscow: Book House «LIBROCOM») p 352

[11] Averkin A N [et al.] 2013 Fuzzy sets in control and artificial intelligence models (Moscow: Book on Request) p 312