Compatibility between the scalar field models of tachyon, \(k\)-essence and quintessence in \(f(R,T)\) gravity

Vinod Kumar Bhardwaj\(^1\), Anirudh Pradhan\(^2\), Archana Dixit\(^3\)

\(^{1,2,3}\)Department of Mathematics, Institute of Applied Sciences and Humanities, G L A University
Mathura-281 406, Uttar Pradesh, India
\(^1\)E-mail: dr.vinodbhardwaj@gmail.com
\(^2\)E-mail: pradhan.anirudh@gmail.com
\(^3\)E-mail: archana.dixit@gla.ac.in

Abstract

We investigate the behavior of the scalar field in \(f(R,T)\) gravity (Harko et al., Phys. Rev. D 84, 024020, 2011) inside the structure of a flat FRW cosmological model, where \(R\) and \(T\) have their usual meaning. The deterministic solution of FEs has been settled by considering the scale factor \(S(t) = t^m e^{kt}\), where \(m\) and \(k\) are positive constant. Here we utilize three ongoing imperatives \((H_0 = 73.8\) and \(q_0 = -0.73)\) from SNe Ia Union observation (Cunha, Phys. Rev. D 79, 047301, 2009); \((H_0 = 73.8\) and \(q_0 = -0.55)\) from Type Ia Supernova (SNIa) observation in joined with (BAO) and (CMB) observation (Giostri et al., J. Cosmol. Astropart. Phys. 03, 027, 2012); and \((H_0 = 69.2\) and \(q_0 = -0.52)\) from (OHD+JLA) observation (Yu et al., Astrophys. J. 856, 3, 2018). We have measured and graphed several cosmological parameters and found that these findings are appropriate in comparison with the universe’s physical and kinematic properties and also compatible with ongoing observations. The correspondence between the scalar field models of tachyon, \(k\)-essence, and quintessence is investigated by us. Our model explains the potentials and the dynamics of the scalar fields in the FRW universe. The reconstructed potential is very sensible and has a scaling arrangement. Subsequently, our study supports recent observations.

Keywords: \(F(R,T)\)-gravity; Tachyon field; \(k\)-essence; Quintessence

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1 Introduction

The Scientific cosmology is intended to determine the broad-scale structure of the universe. The cosmic perceptions of type-Ia supernovae tests \([1\)–\(7]\) propose that the observable universe is going through rapid growth. The ongoing observational information unequivocally shows that the models of the universe, obtained in this paper, are prevailed by a segment with negative pressure, named as dark energy (DE), which comprises three fourth of the basic critical density. In order to explain the concept of dark energy and accelerated expansion, the decisions of certain hypothetical models must be unique to quintessence scalar field models \([8\)–\(9]\), \(k\)-essence \([10\)–\(11]\), tachyon field \([12\)–\(13]\), phantom field \([14\)–\(15]\), quintom \([16\)–\(17]\) as well as dark energy models like chaplygin gas \([18\)–\(19]\). Many cosmologists were studying cosmological theories of dark energy in alternate gravitational speculations.

Regarding their cosmological implications, altered theories of gravitation have been broadly concentrated. Numerous authors \([20\)–\(23]\) were studying cosmological theories of dark energy in modified gravitational speculations. In recent years there has been a growing interest in modified gravity theories despite the clear evidence of the late-time acceleration of the Universe and the presence of dark matter and energy.

Several ideas had been developed and reviewed in the literature. Among these, \(F(R)\) gravity theory is an appropriate candidate theory that analyzed Einstein’s (GR) theory by supplanting the gravitational action \(R\) with the arbitrary function \(f(R)\). This theory gives more perplexing conditions than general relativity and gives a larger range of arrangements. Utilizing this theory, Starobinsky \([24]\) initially proposed an accelerated inflation plan. Sotiriou \([25]\) addressed the conditions for the \(f(R)\) and scalar-tensor theories to be comparable.
Various modified theories are discussed in [20] their structures have been examined by analyzing Big Rip and future singularities. Huang [27] explored an inflation model of $f(R)$. Likewise several cosmologists have also researched $f(T)$ gravity theory in different ways. Modified gravity models additionally include $f(T)$ gravity [28, 29], $f(R)$ gravity [30, 31], Gauss-Bonnet gravity [32], scalar-tensor speculations [33, 34], Galileon gravity [35], Braneworld models [36] and so on.

In the same context, several authors have extensively researched the astrophysical and cosmological implications of $f(R, T)$ gravity. Many researchers are recently involved in confronting cosmological string models in the adjusted scenario for $f(R, T)$ gravity theories [37, 38]. Authors [39, 40, 41] have studied the physical aspects of cosmological models in updated $f(R, T)$ gravity in various contexts. Sharif et al. [42] examined the energy conditions for FRW Universe with perfect fluid in $f(R, T)$ gravity. Houndjo [43] has arranged a cosmological rehabilitation of $f(R, T)$ gravity which handles the transition from a matter-dominated phase to an accelerated phase. Ahmed and Pradhan [44] discussed the accelerating cosmological model of Bianchi type-V, with $\Lambda(T)$ gravity. In the modified $f(R, T)$ theory of gravity, Singh and Kumar [45] have shown the effects of bulk viscosity and explained the early and late acceleration of the Universe. Sahoo et al. [46] addressed the Kaluza Klein cosmological model in $f(R, T)$ gravity where $\Lambda$ is a stress-energy tensor depending on $T$. The future advancement of the dark energy universe was explored by Bamba et al. [47] in modified gravities incorporating a perfect fluid with the inhomogeneous equation of the state in $f(R)$ gravity. There are a large number of works which are impossible to be mentioned here but we can refer to some latest references in $f(R, T)$ gravity [48-65] (and references therein).

On the other hand, the tachyon field has been suggested as a possible candidate for dark energy. A rolling tachyon has an interesting equation of state whose parameter smoothly/swimmingly estimates the values between -1 and 0 [66, 67]. Thus, tachyon can be realized as a suitable candidate for inflation at high energy [68]. This is also treated as a source of DE depending on the form of tachyon potential [69]. The tachyon has been intensively studied in the last few years also in application to cosmology [70-74]. Sheykhi et al. [75] has reconstructed the potential and the dynamics of the tachyon field according to the evolutionary nature of ghost dark energy. Setare et al. [76] investigated the spherical collapse and the evolution of over-densities in the context of the tachyon scalar field model. In this article, the authors have also compared the results with the results of Einstein-de Sitter and $\Lambda$-cold dark matter models. Recently, Dimitrijevic et al. [77] has discussed a class of tachyon models in a braneworld cosmology scenario based on the second Randall-Sundrum model extended to more general warp factors. A model of tachyon inflation in the framework of holographic cosmology has been recently studied in [78].

Another possibility to explain the mystery of dark energy is $k$-essence. It is to work with the theme that the unknown DE constituent is due to minimally coupled scalar field $\phi$ with non-canonical kinetic energy [79]. In the present time, $k$-essence scenario has received much attention. Originally, it was purported as a model of inflation [80] and then a model of DE [79]. Many authors [81-88] have studied DE cosmological models in the framework of $k$-essence in various contexts. Sebastiani et al. [89] has focused on $f(T)$ gravity and $k$-essence cosmology. Recently, Srivastava et al. [90] has discussed Bianchi-III universe with $k$-essence for holographic dark energy. Dubey et al. [91] has obtained Tsallis HDE in the Bianchi-I universe using hybrid expansion law with $k$-essence.

Guendelman et al. [92] identified a quintessence, unified dark energy and dark matter, and process of containment where the effective vacuum energy density and dynamically induced dust-like matter appear as dynamically produced, correspondingly. Aktas, [93] has explored the tachyon, $k$-essence, and quintessence of DE in (FRW) universe models with varying $G$ and $\Lambda$ in the theory of $f(R, T)$ gravitation. They used a linearly varying deceleration parameter (LVDP) for the solution of the field equation. An advanced cosmological study of $G$ and $\Lambda$ play a significant role because it may be responsible for accelerating the Universe. Several authors have thoroughly covered the cosmological constant as a function of time in different variable $G$ theories in various contexts. In general relativity (GR), lots of modifications were suggested to allow for a variable $G$ based on various arguments. In a flat FRW universe, Norman Cruz [94] consider a relation between the holographic dark energy density and the kinetic $k$-essence energy density. They defined if $c > 1$, the holographic dark energy is described as a kinetic $k$-essence scalar field that certain way.

Granda and Olivere [95] introduced and worked on the relationship between quintessence, tachyon, $k$-essence, and dilation energy density with HDE in the flat FRW universe, adding a new infrared cut-off for the HDE. This
relationship allows the models of the scalar fields to recreate the potentials and dynamics, which characterize accelerated expansion. The work in [96] showed the reconstruction of scalar-field energy models for general Lagrangian density $p(\phi, X)$, where $\phi$ and $X$ is the scalar field and kinematic term. Sharif and Jawad [97, 98] worked on the reconstruction of scalar field dark energy models in the flat Kaluza-Klein universe. They are also developing their correspondence with certain dark energy simulations of scalar fields. One of the most fascinating and challenging problems in cosmology is the discovery of the accelerated expansion of the Universe [99]-[105]. There are many ways to resolve this problem, including: the cosmological constant, the quintessence field [99]-[105], a brane cosmology scenarios [106]-[108] and models of k-essence [109]-[111]. The cosmological constant is the easiest option among the earlier proposals which requires a fine-tuning value.

Considering the scalar field dark energy models as an effective explanation of the underlying dark energy theory, it is fascinating to concentrate on how the scalar field models can be utilized to depict the energy density and this section can be divided into three parts tachyon, k-essence, and quintessence fields with redshift. Finally, the results of the models are summarized in Sect. 5.

2 Dynamics of the Field Equations

Harko et al. [119] proposed the modified theory of gravity in the form of $f(R, T)$ theory to describe the accelerated expansion of universe. The altered conditions are likewise comprehended for fluctuating $\Lambda$ and $G$. To give the arrangement of $f(R, T)$ gravity recommended by Harko et al. [119] following three $f(R, T)$ models

$$f(R, T) = \begin{cases} R + 2\lambda_1(T), \\ f_1(R) + f_2(T), \\ f_R + f_2(R)f_3(T). \end{cases}$$

(1)

The adjusted field conditions in $f(R, T)$ theory with variable $\Lambda$ and $G$ are given by Tiwari et al. [74] as

$$G_{\alpha\beta} = [8\pi G(t) + 2\lambda'_1(T)]T_{\alpha\beta} + [2p\lambda'_1(T) + \lambda_1(T) + \Lambda(t)]g_{\alpha\beta},$$

(2)

where ‘dash’ implies differentiation. We select $\lambda_1(T) = \mu T$, Eq. (2) can be inferred as Tiwari et al. [120], here ($\mu \rightarrow$ constant).

$$G_{\alpha\beta} = [8\pi G(t) + 2\mu]T_{\alpha\beta} + [\mu p - \mu p + \Lambda(t)]g_{\alpha\beta},$$

(3)

In the current study, we will discuss about the different dark energy models in FRW space-time with variable $\Lambda$ and $G$ in adjusted $f(R, T)$ gravity theory. To acquire GRT arrangements, we get $\mu = 0$ in Eq. (3). The flat FRW universe is given by

$$ds^2 = dt^2 - S^2 \left[dr^2 + r^2(d\theta^2 + \sin^2 \theta \, d\phi^2)\right].$$

(4)

the Ricci scalar is defined for flat FRW universe as:

$$R = -6\left(\ddot{S} \frac{\dot{S}^2}{S^2} + \dddot{S}ight).$$

(5)

Here we defined energy-momentum tensor as:

$$T_{\alpha\beta} = -pG_{\alpha\beta} + (\rho + p)u_{\alpha}u_{\beta}$$

(6)

where $\rho$ and $p$ represents the energy density and pressure respectively. $u^i = (0, 0, 0, 1)$ indicates the four-velocity vector components. $T^{DF} = \rho - 3p$ is the trace of energy-momentum tensor.

The field equations of a dark energy model for flat FRW universe in $f(R, T)$ theory of gravity are expressed as:

$$\frac{\dot{X}}{\dot{\phi}} = \frac{\lambda_1(T) + \Lambda(t)}{\mu T},$$

$$\frac{\dot{\phi}}{\dot{\phi}} = \frac{f_2(R)}{f_2(R) + f_3(T)},$$

$$\frac{\dot{\phi}}{\dot{\phi}} = \frac{f_2(R)}{f_2(R) + f_3(T)}.$$
\[ 3H^2 + 2\dot{H} = -8\pi G\rho + \mu \rho - 3\mu p + \Lambda \]  
\[ 3H^2 = 8\pi G\rho - \mu p + 3\mu \rho + \Lambda \]

Here, \( H = \frac{\dot{S}}{S} \) is the Hubble parameter.

3 Solution of the Field Equations

We have two field equations as mentioned by (7) and (8), which involves five unknowns as \( H, \rho, p, G, \Lambda \). Thus, for deterministic solution, we need three more assumptions as

(i) we take the ratio between \( H^2 \) and \( \Lambda \), i.e. \( \Lambda = \xi H^2 \), Schutzhold [121] here \( \xi \) is a constant. The cosmological term is a variable that depends on the values of \( G, \rho, H^2 \) and \( \dot{H} \). These equations are related to \( H \) for a known value of \( \Lambda \) and many authors have been worked on this assumption [122, 123].

(ii) Expression of EoS parameter between energy density and the pressure as \( \omega = \frac{p}{\rho} \).

(iii) We assume a deceleration parameter as:

\[ q = -\left( \frac{\dot{H} + H^2}{H^2} \right) = \frac{m}{(m + kt)^2} - 1 \]

From Eq. (9), we observed that \( q > 0 \) for \( m(1 - m) > 2mkt + k^2t^2 \) and \( q = -1 \) for \( m = 0 \). It is observed that for \( k = 0 \), \( q = \frac{m}{m} - 1 \), in this case \( q > 0 \) or \( q < 0 \) according as \( m < 1 \) or \( m > 1 \) respectively.

On solving Eq. (9), we get metric potential as

\[ S = t^m \exp(kt) \]

where \( m \) and \( k \) are constants.

Eq. (9) defines a relationship between constants \( k \) and \( m \) for the present universe (\( t_0 = 13.8 \text{Gyr} \)) with \( q_0 = -0.73 \) (Cunha [124]).

\[ k = \frac{1}{13.8} \left[ \sqrt{\frac{m}{0.27}} - m \right] \]

From Eq. (11), it is obvious that model for the present universe is true for \( m > 0.27 \). It is observed that our model is in the accelerating phase for \( m > 1 \), and model is in the transition phase for \( 0.27 < m \leq 1 \).

From Eq. (9), we obtain a relation between \( k \) and \( m \) for the present universe (\( t_0 = 13.8 \text{Gyr} \)) with \( q_0 = -0.55 \) (Giostri et al. [125]), we again a relation as:

\[ k = \frac{1}{13.8} \left[ \sqrt{\frac{m}{0.45}} - m \right] \]

Eq. 12 explains that the model is appropriate for \( m \geq 0.45 \). It is also clear from the relation that our model is in the transition phase for \( 0.45 < m \leq 1 \) and the accelerating phase for \( m > 1 \).

Similarly, from Eq. (9), we find a relation between \( k \) and \( m \) for the present universe \( t_0 = 12.36 \) and \( q_0 = -0.52 \) (Amirhaschi & Amirhaschi [126]; Yu et al. [127]) as:

\[ k = \frac{1}{12.36} \left[ \sqrt{\frac{m}{0.48}} - m \right] \]

The relation expresses that the model is valid for \( m \geq 0.48 \) and also represent the accelerated expansion of the universe for \( m > 1 \).

The various estimations of pair \( (k, m) \) are obtained from Eqs. (11)-(13) as per three observational imperatives [124, 125, 126, 127] and are given in Table-1. These experimental estimations of \( k \) and \( m \) are utilized for plotting and validation of the derived models.
Table 1: Table of Values of $m$ and $k$

| $m$ | $k$ (Cunha et al. 2009) | $k$ (Giostri et al. 2012) | $k$ (Yu et al. 2018) |
|-----|------------------------|-------------------------|---------------------|
| 0.3 | 0.0546443879           | 0.0374278847            | 0.03969008212       |
| 0.5 | 0.06237881413          | 0.04015163428           | 0.04212141796       |
| 0.8 | 0.06676274870          | 0.03864734297           | 0.0397244699        |
| 1.0 | 0.06699281858          | 0.0355883948            | 0.0358718123        |
| 1.5 | 0.0621030872           | 0.0236044824            | 0.0216639930        |
| 2.0 | 0.0522938602           | 0.0078395005            | 0.0033366871        |

Figure 1: Variation of Energy density $\rho$ versus cosmic time $t$ with $m = 2.0, \mu = -6, \xi = 3.75, \omega = -2/3$.

By using Eq. (7), Eq. (8) and Eq. (10), EoS parameter and $\Lambda = \xi H^2$, for this model, the dynamics of the universe are determined as:

\[
\rho = \frac{(\xi - 3)(1 + \omega)(m + kt)^2 + 2m}{\mu(\omega^2 - 1)t^2} \tag{14}
\]

\[
p = \frac{\omega [(\xi - 3)(1 + \omega)(m + kt)^2 + 2m]}{\mu(\omega^2 - 1)t^2} \tag{15}
\]

\[
G = \frac{\mu [(3 - \xi)(1 + \omega)(m + kt)^2 + 2m(\omega - 3)]}{4\pi [(\xi - 3)(1 + \omega)(m + kt)^2 + 2m]} \tag{16}
\]

\[
\Lambda = \xi (m + kt)^2 t^{-2} \tag{17}
\]

By using Eq. (5), Eq. (6) and Eq. (10), for $f(R, T) = R + 2\mu T$ model, we obtained trace Ricci scalar of dark energy matter distribution as:

\[
R = 6 \left( \frac{m - 2m^2 - 2k^2 t^2 - 4mkt}{t^2} \right) \tag{18}
\]

\[
T^{DF} = (1 - 3\omega) \left( \frac{(\xi - 3)(1 + \omega)(m + kt)^2 + 2m}{\mu(\omega^2 - 1)t^2} \right) \tag{19}
\]

Using Eq. (18) and Eq. (19), we get $f(R, T) = R + 2\mu T$ as

\[
f(R, T) = 6 \left( \frac{m - 2m^2 - 2k^2 t^2 - 4mkt}{t^2} \right) + 2(1 - 3\omega) \left( \frac{(\xi - 3)(1 + \omega)(m + kt)^2 + 2m}{(\omega^2 - 1)t^2} \right) \tag{20}
\]

Figure 1, referring to Eq. (14) delineates the variety of the energy density $\rho$ versus cosmic time $t$ for three observational data. From the figure, we see that $\rho$ is a positive diminishing function of time $t$ and it, converges to zero as $t \to \infty$. Our obtained results harmonize with the most recent observations. It is worth mentioning...
that $\rho$ in case I (Cunha [124]) decreases quickly in comparison with cases II (Giostee et al. [125]) and case III (Yu et al. [127]).

Figure 2 corresponding to Eq. (15) exhibits a variety of fluid pressure $p$ versus cosmic time $t$ for all three

![Figure 2](image2.png)

Figure 2: Variation of pressure $p$ versus time $t$ with $m = 2.0, \mu = -6, \xi = 3.75, \omega = -2/3$.

![Figure 3](image3.png)

Figure 3: Variation of gravitational term $G$ versus time $t$ with $m = 2.0, \mu = -6, \xi = 3.75, \omega = -2/3$.

![Figure 4](image4.png)

Figure 4: Variation of cosmological term $\Lambda$ versus time $t$ with $m = 2.0, \mu = -6, \xi = 3.75, \omega = -2/3$. 
observations. Here we found that the isotropic pressure is negative throughout the development. From the figure, we have seen that pressure is a decreasing function of cosmic time $t$ and it tends to zero at an early stage. The graph shows that pressure $p$ is high at the beginning time and it decreases as time will increase. This negative pressure actually causes the accelerated expansion of the universe for all three recent observations.

Figure 3, corresponding to Eq. (16) depicts the gravitational term $G(t)$ is zero initially and increases slowly, and leads to infinity. In the late time $m > 0$, it is easy to see that the gravitational term increases with time, for all three observations. These results are consistent with observations. The graph shows that gravitational term $G$ increases adversely with the expansion of cosmic time $t$ and gets zero for some late epochs in all instances.

Here in this model the cosmological constant $\Lambda$ determines the nature of the universe. Fig. 4 explain the cosmological term $\Lambda$ vs. cosmic time $t$. We see that in the present epoch $\Lambda$ is a decreasing function of time $t$, and it has acquired a small positive value. Recent cosmological findings [128]-[129] indicate that our universe could accelerating one. Thus, the nature of $\Lambda$ in our derived models is supported by observations. However, as compared to radiating dominated and rigid fluid, it decreases more sharply with cosmic time an empty universe. In the radiating dominated universe, $\Lambda$ term often decreases.

4 Correspondence with scalar field models in $f(R, T)$ gravity

The DE models in the scalar field are already part of the family of dynamic of dark energy models that describe the phenomenon of DE. The literature contains a wide variety of such models as tachyon, $k$-essence, quintessence, phantom, ghost dilation, and condensate, etc.

Reconstituting the capacity models of DE would be necessary to clarify the cosmological nature of quantum gravity via scalar fields. Such models depict the universe’s quintessence behavior and also provide an effective overview of DE. In this section, we explore the field of tachyon, $k$-essence and, quintessence DE models in $f(R, T)$ theory with variable $G$ and $\Lambda$. Here we consider the value of EoS parameter is $\omega = -1/3$ and $-2/3$ with reference to redshift. Now we plot kinetic energy and scalar potential with respect to $z$ by using $a = a_0(1 + z)^{-1}$ in all cases, here we assume $a_0 = 1$ used many authors [130, 131].

4.1 Tachyon field with redshift

The tachyon field was suggested to be the source of dark energy [69, 87] and may be described by effective field theory corresponding to some kind of tachyon condensate with an effective Lagrangian density given by [88, 67]. This field is one of the DE components which describes the accelerated expansion of the universe. The EoS parameter of the tachyon DE matter distribution lies between $-1$ and 0 Gibbon [66]. The energy density $\rho$ and pressure $p$ in flat FRW background for tachyon matter distribution related to SF ($\Phi$) and scalar potential $V(\Phi)$ are given as:

$$p_{TF} = -T^i_i = V(\Phi)\sqrt{1 - \dot{\Phi}^2}$$

(21)

$$\rho_{TF} = T^4_4 = V(\Phi) \left(1 - \dot{\Phi}^2\right)^{-1/2}$$

(22)

Here, $\dot{\Phi}^2$ is the kinetic energy (KE) and $V(\Phi)$ is the scalar potential for a given scalar field. In redshift cut-off $\Phi$ and $V(\Phi)$ obtained as:

$$\Phi_{TF} = \frac{m\sqrt{\omega + 1}W\left(\frac{k(\pm\tau)^{1/m}}{m}\right)}{k} + c_1$$

(23)

$$V(z)_{TF} = \frac{k^2\sqrt{\omega - 3(\omega - 1)\left(mW\left(\frac{k(\pm\tau)^{1/m}}{m}\right) + m\right)^2 + 2m}}{\mu m^2(\omega^2 - 1)W\left(\frac{k(\pm\tau)^{1/m}}{m}\right)^2}$$

(24)
Figure 5: Variation of SF for tachyon versus Redshift $z$ with $m = 2, \mu = -6, \xi = 3.75$ and $\epsilon_1 = 0$.

Figure 5, shows the kinetic tachyon energy with respect to redshift $z$ for three recent observations [124, 125, 126] by considering the $\omega = -1/3 \& -2/3$. Graph represents that kinetic energy is decreases as $z$ increases. We find if $z \to -1$, $\Phi$ is very high and $z \to 1$ $\Phi$ is very low. Similarly Figure 6(a,b,c) represents the scalar field effect with redshift $(z)$. For all three observations $V(z)$ increases with redshift for $\xi > 3$. Obtain the following expression for the tachyon potential in terms of the redshift we found that the tachyon field decreases as the universe expands.
The k-essence scalar field (SF) model is used to describe the universe’s late-time acceleration, described by the string theory Born-Infeld action \([134, 135]\). k-essence scenarios are well known attractor-like dynamics, and thus ignore the fine-tuning of the initial scalar field conditions\([136]\). In a flat FRW context, the values of pressure \(p\) and energy density \(\rho\) are given as, for the kinetic and scalar potential distribution of k-essence \([137]\).

\[
p_{ke} = -T_i^i = V(\Phi) \left( \frac{\dot{\Phi}^4}{4} - \frac{\dot{\Phi}^2}{2} \right)
\]

(25)

\[
\rho_{ke} = T_4^4 = V(\Phi) \left( \frac{3\dot{\Phi}^4}{4} - \frac{\dot{\Phi}^2}{2} \right)
\]

(26)

In redshift cut-off \(\Phi\) and \(V(\Phi)\) of the k-essence is obtained as:

\[
\Phi_{ke} = m \sqrt{\omega + 1} W \left( \frac{k (\frac{\dot{\Phi}^4}{\omega + 1})^{1/m}}{m} \right) + c_2
\]

(27)

\[
V(z)_{ke} = \frac{k^2 (3\omega - 1)^2 (3 - \xi)(\omega + 1) \left( mW \left( \frac{k (\frac{\dot{\Phi}^4}{\omega + 1})^{1/m}}{m} \right) + m \right)^2 - 2m}{2\mu m^2 (\omega - 1) (\omega^2 - 1) W \left( \frac{k (\frac{\dot{\Phi}^4}{\omega + 1})^{1/m}}{m} \right)^2}
\]

(28)

Figure 7, shows the kinetic k-essence energy with respect to redshift \(z\) for three recent observations \([124, 125, 126]\) by considering the \(\omega = -1/3 \& -2/3\). The behavior of k-essence field is similar to tachyon field. Our graph represents the kinetic energy is decreases as \(z\) increases. Similarly Figure 8(a,b,c), represents the scalar potential effect with redshift \((z)\). For all three observations it increases with redshift for \(\xi > 3\).

### 4.3 Quintessence field with redshift

Quintessence is a hypothetical form of dark energy, which is described by homogeneous and time-dependent SF, as well as the scalar potential which leads to universe acceleration \([135]\). EoS quintessence parameter indicates the accelerated universe expansion within the \(-1 \leq \omega \leq -\frac{1}{3}\) \([135]\). In flat FRW universe, the relations of energy density and the pressure respectively, in terms of quintessence SF and scalar potential is given as \([107]\):

\[
p_Q = -T_i^i = \frac{\dot{\Phi}^2}{2} - V(\Phi)
\]

(29)

\[
\rho_Q = T_4^4 = \frac{\dot{\Phi}^2}{2} + V(\Phi)
\]

(30)
In redshift cut-off $\Phi$ and $V(\Phi)$ of the quintessence is obtained as:

\[
\dot{\Phi}_Q = \sqrt{\frac{k^2 \left( (\xi - 3)(\omega + 1) \left( mW \left( \frac{k(\frac{1}{m})^{1/m}}{m} + m \right)^2 + 2m \right) \right)}{\mu m^2(\omega - 1)W \left( \frac{k(\frac{1}{m})^{1/m}}{m} \right)^2}}
\]  

(31)
Figure 10: Variation of scalar potential for quintessence versus $z$ with $m = 2, \mu = -6, \xi = 3.75$.

On integration Eq. (31), we get

$$
\Phi_Q = a_1 \log(m) + a_2 + a_3 \text{log}
\left( m \frac{a_1}{m} \frac{1}{m} \right)
$$

where,

$$
a_1 = W \left( \frac{k \left( \frac{1}{\omega+1} \right)^{1/m}}{m} \right)
$$

$$
a_2 = \sqrt{k^2 \left( (\xi - 3)(\omega + 1) \left( mW \left( \frac{k \left( \frac{1}{\omega+1} \right)^{1/m}}{m} \right) + m \right)^2 + 2m \right)}
$$

$$
a_3 = \sqrt{2m + m^2(1 + \omega)(\xi - 3)}
$$

$$
a_4 = \sqrt{\omega + 1)(\xi - 3)}
$$

$$
V(z)_Q = \frac{k^2 \left( (3 - \xi)(\omega + 1) \left( mW \left( \frac{k \left( \frac{1}{\omega+1} \right)^{1/m}}{m} \right) + m \right)^2 - 2m \right)}{2\mu m^2(\omega + 1)W \left( \frac{k \left( \frac{1}{\omega+1} \right)^{1/m}}{m} \right)^2}
$$

(a) $k = 0.0522938602$, Cunha (2009)

(b) $k = 0.0078395005$, Giostri et al.(2012)

(c) $k = 0.0033366871$, Yu et al.(OHD+JLA Data 2018)
Figure 9, shows the kinetic quintessence energy with respect to redshift $z$ for three recent observations \cite{78, 79, 80} by considering the $\omega = -1/3 \ & -2/3$. The behaviour of the quintessence field is similar to k-essence and tachyon field. Our graph represents the kinetic energy is decreases as $z$ increases. Similarly Figure 10(a,b,c) represents the Scalar potential effect with redshift $(z)$. For all three observations it increases with redshift for $\xi > 3$.

5 Conclusion

In the present manuscript, the scalar fields are playing a very important role in cosmological consequence. Particularly cosmological inflation, the late-time acceleration of the universe, or dark matter, and its properties can be clarified in the sense of unique scalar field models. In this model, we considered a generalized gravity model with an arbitrary coupling between matter (described by the trace of the stress-energy tensor) and geometry, with the Lagrangian given by $T$ and the Ricci scalar arbitrary function. Here we derived the corresponding gravitational field equations and analyzed some particular cases that may be important to understand some open problems of cosmology and astrophysics. In this paper, we have investigated various DE candidates in $f(R, T)$ gravitation theory for flat FRW universe with varying $G$ and $\Lambda$ by using the latest findings.

The main outcomes of the derived models are summarized as:

- In our model Fig. 1 depicts the energy density $\rho$ is infinitely large at the initial stage and tends to zero as $t \to \infty$, which is consistent with the Big Bang theory.

- Fig. 2 illustrates that pressure $p$ is a decreasing function of time. This negative pressure actually causes the accelerated expansion of the universe for all the three recent observations. Also $t \to \infty$, $\xi = 3$, we get $G \to \frac{4(\omega-3)}{4\pi}$ in $f(R, T)$ theory.

- Gravitational term is initially zero and it increases as $t$ increases (Fig. 3). Different kinds of tests, experiments, and measurements were involved to verify the variation of the gravitational constant with time.

- Schmidt et al. \cite{94} propose that $\Lambda$ with a magnitude of $\Lambda(Gh/c^3) = 10^{-23}$ has positive cosmological constants. Such findings on the magnitude and the redshift of type Ia supernova indicates that through the cosmological $\Lambda - \text{term}$ our universe can be accelerating with induced cosmological density.

- Finally, we studied the relationship between scalar field DE models in Figs. 5-10 involving quintessence, tachyon, and k-essence. Here we have subsequently explored the scalar fields and given particular attention to dark energy models by using a constant EoS parameter $\omega = -1/3, -2/3$

- Figs. 9-10 displays the evolutionary trajectories of the $\Phi$ kinetic energy and the related scalar potential concerning the redshift $z$. We notice that the quintessence field increases with a high redshift. Similarly, the evolution of tachyon and k-essence fields is very much similar to quintessence.

- We may conclude that our model begins with Big Bang and terminate with Big Rip. This outcome is a consequence of the significant shift in the evolution of the transition between matter and dark energy dominated epochs. The positive values of scalar potential are obtained for negative values of $\mu$. In addition, for $\mu = 0$, we get $G = 0$ GRT solutions with FRW universe the tachyon field, k-essence, and quintessence matter distributions are obtained. Such DE solutions minimize the importance of GRT. We wish to point out that now the results are the current state of the universe is consistent with the scalar field and corresponding potential. We have formulated the potential and the dynamics of these scalar field models representing tachyon, k-essence, and quintessence cosmology.

Hence, the solutions exhibited in this work can be helpful for better comprehension of the features of FRW models in the Universe evolution in $f(R, T)$ gravity for tachyon field, k-essence field, and quintessence field with redshift.
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