Cosmological Entropy and Black Holes in Galactic Halos

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Abstract

In this talk we discuss intermediate mass black holes (IMBHs) by their amplification of distant sources; MACHO searches have studied event times $2h \lesssim t_0 \lesssim 2y$ corresponding masses in the range $10^{-6}M_\odot \lesssim M \lesssim 100M_\odot$. We suggest that larger masses up to $10^6M_\odot$ are also of considerable interest by arguments about the entropy of the universe. One percent by mass of dark energy can provide ninety-nine percent of total entropy.

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Introduction.

Attempts to unify gravity with the other interactions of Nature may be guided by the holographic [1] principle which provides an upper limit on the amount of information which can be contained is a given three-dimensional volume in terms of its two-dimensional surface area. Although the principle is not proven rigorously, it provides the best available guide to estimates of cosmological entropy and hence to suggest which future observations can help to quantify where the entropy lies. It may well be objects not yet detected for want of a motivation to make the requisite observations. One such set of observations is the subject of the present talk.

Our present aim is not to explain all or even most of the dark matter which could be e.g. WIMPS but rather to suggest that a small fraction (say, 1%) may account for a large fraction (say, 90%) of the entropy. Thus, if a “pie chart” were drawn for entropy, rather than energy, the appearance would be dramatically different.

Here we assume throughout the holographic principle is physically correct that the upper limit of entropy, taking into account all degrees of freedom, both gravitational and non-gravitational, is the area of the surface surrounding a volume in units of the Planck area. Taking a standard estimate for the volume and area of the visible universe, this gives an upper limit for the entropy of the universe. The value is \((S_U)^{\text{max}} \sim 10^{123}\).

The conventional wisdom (see e.g. [2]) is that the present entropy of the universe is overwhelmingly dominated by the supermassive black holes (SMBHs) at the cores of galaxies. This provides a lower limit on the cosmological entropy which, taking for simplicity \(10^{12}\) galaxies each containing a SMBH of mass \(10^7 M_\odot\), gives \((S_U)^{\text{min}} \sim 10^{103}\) since the entropy of a black hole with \(M_{BH} = \eta M_\odot\) is \(S_{BH}(\eta M_\odot) \sim 10^{77} \eta^2\). If we further acknowledge that the galaxies are receding from one another at an accelerated rate such that coalescence is, in general, unlikely and they can be regarded as totally segregated and disentangled then the upper limit on \(S_U\) is refined to \((S_U)^{\text{max}} \sim 10^{111}\). This diminution from \(10^{123}\) to \(10^{111}\) arises from dividing out the number \(10^{12}\) of galaxies since the maximum total entropy of the universe becomes the sum of the maximum possible entropies of the separate galaxies.

This provides the cosmological entropy range\(^\#2\)

\[
103 \lesssim \log_{10} S_U \lesssim 111 \tag{1}
\]

the first of two interesting windows which are the subject for this Letter. Conventional wisdom is \(S_U \sim (S_U)^{\text{min}} = 10^{103}\).

\(^\#2\)These limits apply to the visible universe not to a single galaxy.
Intermediate Mass Black Holes

If we consider normal baryonic matter, other than black holes, contributions to the entropy are far smaller. The background radiation and relic neutrinos each provide $\sim 10^{88}$. We have learned in the last decade about the dark side of the universe. WMAP [3] suggests that the pie slices for the overall energy include 24% dark matter and 72% dark energy. Dark energy has no known microstructure, and especially if it is characterized only by a cosmological constant, may be assumed to have zero entropy. As already mentioned, the baryonic matter other than the SMBHs contributes far less than $(S_U)^{\text{min}}$.

This leaves the dark matter which is concentrated in halos of galaxies and clusters.

It is counter to the second law of thermodynamics, if higher entropy configurations are available, that essentially all the entropy of the universe is concentrated in only the known supermassive black holes (SMBH). The Schwarzschild radius for a $10^7M_\odot$ SMBH is $\sim 3 \times 10^7$ km and so $10^{12}$ of them occupy only $\sim 10^{-36}$ of the volume of the visible universe.

Several years ago important work by Xu and Ostriker [4] showed by numerical simulations that IMBHs with masses above $10^6M_\odot$ would have the property of disrupting the dynamics of a galactic halo leading to runaway spiral into the center. This provides an upper limit $(M_{\text{IMBH}})^{\text{max}} \sim 10^6M_\odot$.

Gravitational lensing observations are amongst the most useful for determining the mass distributions of dark matter. Weak lensing by, for example, the HST shows the strong distortion of radiation from more distant galaxies by the mass of the dark matter and leads to astonishing three-dimensional maps of the dark matter trapped within clusters. At the scales we consider $\sim 3 \times 10^7$ km, however, weak lensing has no realistic possibility of detecting IMBHs in the foreseeable future.

Gravitational microlensing presents a much more optimistic possibility. This technique which exploits the amplification of a distant source was first emphasized in modern times (Einstein considered it in 1912 unpublished

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#3 With intermediate mass black holes (IMBH) this fraction is a few times $10^{-35}$ so the present proposal makes only a tiny change to the surprising compression of the total entropy but does suggest what can constitute a far bigger fraction of entropy than the SMBHs.

#4 These authors made the assumption, as here, that there are many such objects in the halo, not just one or a few which could have a higher mass.
work) by Paczynski [5]. Subsequent observations [6, 7] found many examples of MACHOs, yet insufficient to account for all of the halo by an order of magnitude. These MACHO searches looked for masses in the range $10^{-6}M_\odot \lesssim M \lesssim 10^2M_\odot$.

According to [5] the time $t_0$ of a microlensing event is given by

$$t_0 \equiv \frac{r_E}{v}$$

where $r_E$ is the Einstein radius and $v$ is the lens velocity usually taken as $v = 200$ km/s. The radius $r_E$ is proportional to the square root of the lens mass and numerically one finds

$$t_0 \simeq 0.2y \left( \frac{M}{M_\odot} \right)^{1/2}$$

so that, for the MACHO masses considered, $2h \lesssim t_0 \lesssim 2y$. Although some of the already observed MACHOs may be IMBHs, they do not saturate the possible mass or entropy for dark matter so let us set as definition $(M_{IMBH})_{\text{min}} \sim 10^2M_\odot$. This provides the range for IMBH mass

$$2 \lesssim \log_{10} \eta = \log_{10}(M_{IMBH}/M_\odot) \lesssim 6$$

which, after Eq. (1), provides a second window of interest. It corresponds to $2y \lesssim t_0 \lesssim 200y$. Ranges (1) and (4) are related in the next section.

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#5See previous footnote
Cosmological entropy considerations

As mentioned already, the key guide will be the holographic principle [1] which informs us that the cosmological entropy is in the window (II). It cannot be at the absolute maximum value because that is possible only if every halo has already completely collapsed into a single black hole.

Also, the absolute minimum although not excluded seems intuitively implausible because all the entropy is compressed into $10^{-36} V_U$.

The natural suggestion is that there exist IMBHs in the mass region (II). The number is limited by the total halo mass $10^{12} M_\odot$. The total entropy is higher for higher IMBH mass because $S \propto M^2$. Let $n$ be the number of IMBHs per halo, $\eta$ be the ratio $(M_{IMBH}/M_\odot)$, $S_U$ be the total entropy for $10^{12}$ halos and $t_0$ be the microlensing longevity. The Table shows five possibilities. Each corresponds to the intermediate mass black holes contributing an average density $\rho_{IMBH} \sim 1\% \rho_{DM}$ where $\rho_{DM}$ is the mean dark matter density.

### Intermediate Mass Black Holes and Microlensing Longevity

| $\log_{10} n$ | $\log_{10} \eta$ | $\log_{10} S_{halo}$ | $\log_{10} S_U$ | $t_0$ (years) |
|--------------|-----------------|---------------------|-----------------|---------------|
| 8            | 2               | 88                  | 100             | 2             |
| 7            | 3               | 89                  | 101             | 6             |
| 6            | 4               | 90                  | 102             | 20            |
| 5            | 5               | 91                  | 103             | 60            |
| 4            | 6               | 92                  | 104             | 200           |
Observation of Intermediate Mass Black Holes

Since microlensing observations [6, 7] already impinge on the lower end of the range [1] and the Table, it is likely that observations which look at longer time periods, have higher statistics or sensitivity to the period of maximum amplification can detect heavier mass IMBHs in the halo.

If this can be achieved, and it seems a worthwhile enterprise, then the known entropy of the universe could be increased by an order of magnitude. There exists an interesting analysis [8] of wide binaries which places a weak limit on IMBHs which perhaps can be strengthened? To my knowledge, the IMBHs as listed in the Table, and contributing $\sim 1\% \rho_{DM}$ are not excluded by existing observations.

A previous analysis [8] assigned an upper limit on the fraction ($f$) of the halo mass that can be constituted by IMBHs. This was $f = 0.2$. We have no reason to suggest that all of the dark matter halo mass is from IMBH and the fraction $f$ could be small. Yet IMBHs can still provide a very large fraction of the entropy of the universe. For example, taking $f = 0.01$ and $10^6 M_\odot$ as mass allows $10^4$ IMBHs per halo, a total of $\sim 10^{16}$ Mega-$M_\odot$ black holes in the universe and the fraction of the total entropy of the universe provided by intermediate mass black holes is $\sim 90\%$.

It is this entropy argument based on holography and the second law of thermodynamics which is the most compelling supportive argument for IMBHs. If each galaxy halo asymptotes to a black hole the final entropy of the universe will be $\sim 10^{111}$ as in Eq.(1) and the universe will contain just $\sim 10^{12}$ supersgigantic black holes. Conventional wisdom is that the present entropy due entirely to SMBHs is $\sim 10^{-8}$ of this asymptotic value. IMBHs increase the fraction up to $\sim 10^{-7}$, closer to asymptopia and therefore more probable according to the second law of thermodynamics. Note that a present fraction greater than $\sim 10^{-7}$ is not possible consistent with the present observational data.

The entropy arguments are new and provide additional motivation to tighten these upper bounds or discover the halo black holes. One observational method is high longevity microlensing events. It is up to the ingenuity of observers to identify other, possibly more fruitful, methods some of which have already been explored in a preliminary way.
Acknowledgements

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