WHICH IS THE BEST INFLATION MODEL?

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Reasonable-looking models of inflation are compared, taking into account the possibility that the curvature perturbation might originate from some ‘curvaton’ field different from the inflaton.

1. Introduction

After twenty years of trial and error, we are in possession of several reasonable-looking models of inflation. In this talk I have a stab at comparing the pros and cons of these models. I should make it clear at the outset that I am looking only for a model of the last few tens of \( e \)-folds of inflation, enough that the observable Universe starts out well inside the horizon. This ‘observable’ inflation takes place with \( H \) at least five orders of magnitude below the Planck scale, and wipes out practically all memory of earlier epochs. Understanding it will be enough to be going on with!

The task of assessing a model of inflation has recently been complicated, by the realisation that the curvature perturbation might not originate from the vacuum fluctuation of the inflaton (the inflaton paradigm). Instead it might originate from the vacuum of some other ‘curvaton’ field \(^1,2,3,4,5\) (curvaton paradigm). The inflaton paradigm \(^6\) strongly constrains the inflationary potential through the CMB normalization of the curvature perturbation; also, with the advent of PLANCK, the inflaton paradigm will rule out many models through their prediction for the spectral index. In contrast, the curvaton paradigm imposes only weak constraints on the inflationary potential \(^7\).

Nearly all inflation models invoke a slowly-rolling inflaton field \( \phi \), whose
potential $V$ must satisfy the flatness conditions $|V'| \ll V/M_P$ and $|V''| \ll V/M_P^2$. This has several implications. First, the inflaton mass during inflation must satisfy
\[ m^2 \ll V/M_P^2 \simeq 3H^2 \text{ flatness} \quad . \tag{1} \]
where $H$ is the Hubble parameter. Second, the quartic self-coupling $\lambda$ must very tiny. Third, to keep control of the infinite number of non-renormalizable terms that are generically expected in the potential, inflation should presumably take place with $\phi \ll M_P$. (As mentioned later, there is a proposal for eliminating such terms so as to allow $\phi \gg M_P$.)

To achieve the flatness, supersymmetry is usually invoked with the potential generated by an $F$ term, $F \equiv \tilde{M}_S^2$, giving
\[ V = \tilde{M}_S^4 - 3M_P^2 m_{3/2}(t) v_{f \text{pot}} \quad , \tag{2} \]
where $m_{3/2}(t)$ is the field-dependent gravitino mass evaluated during inflation. However, the $F$ term breaks supersymmetry which typically generates a soft mass term at least of order $\tilde{M}_S^2/M_P$. In order to satisfy the flatness condition Eq. (1), the actual mass must be suppressed below the generic value. The suppression would be severe if the terms of Eq. (2) canceled during inflation, which should therefore be avoided. Given that, the generic soft mass is of order $H$, requiring only mild suppression to achieve the flatness condition.

2. Low-scale inflation

I consider first ‘low-scale’ inflation models, in which the scale $\tilde{M}_S$ of SUSY breaking during inflation is the same as the scale $M_S$ of SUSY breaking in the vacuum. A typical value of $M_S$, corresponding to gravity-mediated SUSY breaking in the vacuum, is $M_S \sim 10^{10}$ GeV. The value is an order of magnitude or two bigger for anomaly-mediated SUSY breaking, and some orders of magnitude smaller for gauge- or gaugino-mediated SUSY breaking. The lowest possible scale $M_S \sim$ TeV is rarely invoked, because of the difficulty of satisfying collider constraints within a reasonably simple framework.

Non-hybrid modular inflation. The potential of a modulus is usually supposed to come entirely from supersymmetry breaking, with a potential of the form
\[ V(\phi) = M_S^4 f(\phi/M_P) , \tag{3} \]
where $M_S$ is the scale of supersymmetry breaking in the vacuum and $f \sim |f'| \sim |f''| \cdots \sim 1$. (In the case of gauge- or gaugino mediation, where scales other than $M_S$ and $M_P$ are relevant, it is not quite clear that this form should hold but it seems to be assumed in the literature.)

Expanding around the VEV we learn that the modulus mass is of order $M_S^2/M_P$, hence of order the gravitino mass. Expanding instead around a maximum, chosen as the origin of $\phi$, the potential becomes

$$V(\phi) = V_0 - \frac{1}{2} m^2 \phi^2 + \cdots,$$

with again $m \sim M_S^2/M_P$. But $V_0 \sim M_S^4$, hence $m \sim V_0^2/M_P$ which marginally violates the flatness condition Eq. (1). It might happen that the modulus potential is a bit flatter than the above estimate so that Eq. (1) is satisfied. Otherwise there will be ‘fast-roll’ inflation $^8,^9$ which may not last for enough $e$-folds.

Even if it leads to slow roll, this type of modular inflation cannot satisfy the CMB constraint, which requires $^6 M_S \sim 10^{15}$ GeV. Two fixes have been proposed. Kadota and Stewart $^10$ use the complex field $\Phi$ consisting of a modulus and its axionic partner. (They also have an alternative scheme using two complex moduli.) The tree-level potential is of the form Eq. (3), approximated as

$$V = V_0 + m^2 \left( -|\Phi|^2 + \frac{1}{M_P} (\Phi^3 + \Phi^*^3) + \frac{1}{M_P^4} |\Phi|^4 \right)$$

The maximum of the potential is supposed to be a point of enhanced symmetry (the fixed point of a symmetry group), so that the 1-loop correction can drive the maximum away from the origin to form the rim of a crater. With a finely-tuned choice of the initial condition, this two-field inflation model can generate a curvature perturbation with the required magnitude even with the generic (fast roll) mass.

Banks $^11$ instead identifies the modulus with a bulk field in the Horava-Witten setup, so that in Eq. (3) $M_S$ is replaced by $M^2/\Lambda^2$ with $M \sim 10^{16}$ the GUT scale. If the potential is flat enough for slow roll, the CMB normalization can then be satisfied. This is the only modular inflation model which invokes a departure from Eq. (3).

**Hybrid modular inflation.** In the models considered so far, the inflationary potential is a function only of the inflaton field. The alternative hybrid inflation paradigm $^12$ supposes that the potential is a function of
the inflation field $\phi$ and a ‘waterfall’ field $\chi$, with a tree-level potential

$$V = V_0 - \frac{1}{2} m_\chi^2 \chi^2 + \lambda \chi^2 \phi^2 + \frac{1}{2} m^2 \phi^2 + \cdots.$$  \hspace{1cm} (6)$$

Inflation takes place in the regime $\phi > \phi_c \equiv m_\chi / \sqrt{2\lambda}$, with $\chi = 0$. By ‘hybrid modular’ inflation, I mean hybrid inflation in which (i) the waterfall field is a modulus and (ii) the inflaton mass comes from gravity mediated supersymmetry breaking. Gravity mediated supersymmetry breaking is also assumed for the MSSM, so that roughly $V_0^{1/4} \sim M_S \sim 10^{10}$ GeV and $m \sim m_\chi \sim m_{3/2} \sim 100$ GeV. To be precise though, the model requires that the actual masses satisfy a mild hierarchy $m \ll m_{3/2} \ll m_\chi$.

In the original tree-level version \cite{9} this hierarchy is supposed to be entirely accidental. However, the maximum of the modulus potential must represent a point of enhanced symmetry, to ensure the absence of a linear term $\chi \phi^2$ \cite{13}. It is therefore reasonable to suppose \cite{14} that radiative corrections lead to a running mass $m(\phi)$. With strong running, $m(\phi)$ can pass through zero, permitting slow-roll inflation even though its generic value is of order $m_{3/2}$. Such strong running implies a strong running of the spectral index (going in the opposite direction from the one proposed by the WMAP group) which may rule out the model in the future \cite{15}.

**Inflation without slow roll.** Two ways are known, by which the modular potential Eq. (3) can inflate without slow roll. **Thermal modular inflation** \cite{16} invokes a finite temperature contribution $\sim T^2 \phi^2$, leading to inflation in the regime $m \lesssim T \lesssim M_S$ and $M_P/M_S$ $e$-folds of inflation. **Locked inflation** \cite{17} invokes the same setup as hybrid modular inflation, but with the alternative hierarchy $m \sim m_\chi \gg m_{3/2}$; then $\phi$ oscillates until its amplitude is of order $\phi_c$, leading to $\sim \frac{3}{2} M_P/M_S$ $e$-folds of inflation. By themselves, these schemes hardly generate enough inflation, but extra fast-roll inflation can occur later while the modulus descends to the vacuum \cite{7,18}, and there may also be ordinary (late-time) thermal inflation \cite{19}. More seriously, these schemes must be married to a curvaton scenario, since they involve no slow-rolling inflaton. Still, it is interesting that inflation can take place without any kind of slow roll.

**The inflaton as a $\mu$ field.** It is commonly supposed that the $\mu$ parameter of the MSSM is generated by the VEV of some field which I will call the ‘$\mu$ field’. This has led to two models of inflation. In the model of Dine and Riotto \cite{20} $\mu \sim \langle S^3 \rangle / M_P^2$ and $|S|$ is the inflaton in a non-hybrid model. Gauge-mediated SUSY breaking is assumed for the MSSM, with the inflaton potential generated by a tachyonic soft mass, and by terms $S^3 / M_P^2$.
and $-X S^4/M_S^2$ in the superpotential. CMB normalization for this model requires $V \ll \tilde{M}_S^4$ implying unacceptable fine-tuning of the soft mass. This would be avoided if the model could be married to some version of the curvaton paradigm.

In the model of King and collaborators $^{21,4}$, $\mu = \lambda \langle N \rangle$ with $|N|$ the waterfall field for a hybrid inflation. The mass of $N$ is $\kappa \langle S \rangle$ with $|S|$ the inflaton. The VEVs of $N$ and $S$ break Peccei-Quinn symmetry so that the model contains the QCD axion. CMB normalization for this model again requires unacceptable fine tuning for the inflaton soft mass, which will disappear if the model can be married to some version of the curvaton paradigm. Preliminary results $^5$ suggest that instead the Higgs flat direction may act as a curvaton, generating the curvature perturbation through a preheating mechanism.

3. Other models using ordinary field theory

To have an inflation scale lower than $M_S$, one would need either a cancellation of the two terms of Eq. (2), or a model in which the SUSY breaking scale $\tilde{M}_S$ is lower than $M_S$. Neither has been attempted, but there are proposals for a higher inflation scale.

**GUT inflation.** Many models of inflation, starting with the original one $^{22}$, use a GUT Higgs field to generate a potential $V \sim 10^{15}$ GeV or so. The archetypal proposal (with several variants) uses the superpotential $^{23}$ $W = gS(M^2 - \Phi \bar{\Phi})$, with $|S|$ the inflaton and $|\Phi|$ a Higgs field which is the waterfall field, and with $M \sim 10^{16}$ GeV the GUT scale. This gives $\tilde{M}_S = \sqrt{g}M$. Assuming that the slope is dominated by the 1-loop correction, the complete potential is $^{24}$

$$V = g^2 M^4 \left( 1 + c g^2 \ln (\phi/Q) \right), \quad (7)$$

with $c$ a loop suppression factor and $Q$ the renormalization scale. For reasonable $c$, the CMB normalization is satisfied independently of $g$. By choosing $g \sim 10^{-2}$ or so, inflation takes place at $\phi \ll M_P$, which may justify the neglect of non-renormalizable terms. Subject to the proviso that GUT theories tend to be complicated, this type of model seems to be quite attractive, though it does not explain why the inflaton mass is significantly less than $H$.

**D-term inflation.** Inflation with a Fayet-Iliopoulos $D$ term at the string scale was proposed, in the context of the weakly coupled heterotic string $^{25,26}$. Ignoring non-renormalizable terms it gives the potential
Eq. (7), but now with $g$ a gauge coupling and $M \sim 10^{17}$ GeV the string scale. However, $g$ presumably cannot now be small, which means that the inflaton field value of order $M_P$ so that non-renormalizable terms probably spoil the flatness. Also, the high string scale gives the wrong CMB normalization. Replacing the string scale by an ad hoc scale also leads to problems. On the whole, I would say that $D$-term inflation was a promising idea that didn’t really work.

**D-to-F term inflation.** Instead of using the string-scale FI $D$-term directly, one may suppose that driving it to zero generates an $F$ term which can give inflation at a somewhat lower scale. It has been shown how to construct models of this type in the framework of the weakly coupled heterotic string. The construction suppresses the mass of order $H$, by making the inflaton the pseudo-Nambu-Goldstone-boson (PNGB) of a (rather complicated, stringy) symmetry. However, a working example has not yet been written down, nor any connection made with a known (say GUT) extension of the Standard Model.

**Models invoking an ordinary global symmetry.** One may also suppress the mass of order $H$ by using a PNGB associated with an ordinary symmetry like $SU(N)$. One proposal invokes non-hybrid inflation, with the inflaton mass canceled by the contribution of a PNGB, and another makes the inflaton itself a PNGB in a hybrid model. There has not been any definite proposal for the origin of the inflation scale in either of these schemes, and they seem to be attractive only to the extent that one takes seriously the mass of order $H$ problem. Much more radically, it has been proposed to use global symmetry without supersymmetry. This type of inflation model is inspired by ‘little higgs’ extensions of the Standard Model, but have not been unified with one of them. Such unification would require a high-energy completion of a ‘little higgs’ model, since inflation presumably takes place far above the scale $10 \text{ TeV}$ of the ‘little hierarchy’ which the little higgs takes care of.

4. **Models going beyond ordinary field theory**

All of the above models inflation (save the last) assume some effective field theory, which is constructed along the same conservative lines as are customary when considering other extensions of the Standard Model such as

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bThe mass of order $H$ is in contrast absent since we are not dealing with an $F$ term; this was the main motivation for the model.
the MSSM, models of the axion or models for the origin of neutrino mass. Given that a conservative setup seems to work, it is not quite clear why one should look any further. Still, I end by considering some models which introduce radical new features.

The oldest of them, usually called chaotic inflation, invokes a potential \( V = \frac{1}{2}m^2 \phi^2 \) or \( V = \lambda \phi^4 \), which is supposed to be valid right up to \( \phi \sim 10M_P \) or so. This is indeed radical, because one usually supposes that the tree-level potential will have an infinity of non-renormalizable terms of order \( \phi^d/M_P^{d-4} \). By virtue of the large field value, these models in contrast to the others give a significant tensor perturbation. Because of that, and the predicted tilt \( n - 1 \), the quartic model is in danger of being ruled out by observation.

Gauge inflation makes the inflaton the fifth component of a five-dimensional gauge field with Wilson Line symmetry breaking. The effective field theory of this proposal contains an infinite number of fields, a Kaluza-Klein tower. The sinusoidal inflaton potential is obtained by summing the Coleman-Weinberg potentials of the entire tower, and there has been much debate about the manner in which the sum converges. Leaving such worries aside, gauge inflation can justify the ‘chaotic’ \( \frac{1}{2}m^2 \phi^2 \) proposal, though according to there is no known string-theoretic realisation.

Colliding brane inflation identifies the inflation of the 4-D effective theory, with the distance between colliding D-branes in extra dimensions. The proposal has received enormous attention, because the calculation of the potential within various setups apparently involves deep issues of string theory. Taking these issues seriously, it is not easy to get a flat potential which satisfies the CMB constraint. But, as the reheating process presumably cannot be described within the same effective 4-D theory as the inflation, it may be difficult to marry this type of inflation to a curvaton scenario which requires that the curvaton field exists both during and after inflation.

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