One-loop renormalisation of general $\mathcal{N} = \frac{1}{2}$ supersymmetric gauge theory

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We investigate the one-loop renormalisability of a general $\mathcal{N} = \frac{1}{2}$ supersymmetric gauge theory coupled to chiral matter, and show the existence of an $\mathcal{N} = \frac{1}{2}$ supersymmetric $SU(N) \times U(1)$ theory which is renormalisable at one loop.
1. Introduction

There has recently been much interest in theories defined on non-anti-commutative superspace [1, 2]. Such theories are non-hermitian and turn out to have only half the supersymmetry of the corresponding ordinary supersymmetric theory–hence the term “$\mathcal{N} = \frac{1}{2}$ supersymmetry”. These theories are not power-counting renormalisable but it has been argued [3]–[7] that they are in fact nevertheless renormalisable, in the sense that only a finite number of additional terms need to be added to the lagrangian to absorb divergences to all orders. This is primarily because although the theory contains operators of dimension five and higher, they are not accompanied by their hermitian conjugates which would be required to generate divergent diagrams. This argument does not of course guarantee that the precise form of the lagrangian will be preserved by renormalisation; nor does the $\mathcal{N} = \frac{1}{2}$ supersymmetry, since some terms in the lagrangian are inert under this symmetry. Moreover, the argument also requires (in the gauged case) the assumption of gauge invariance to rule out some classes of divergent structure. As we showed in Ref. [8], there are problems with this assumption; even at one loop, at least in the standard class of gauges, divergent non-gauge-invariant terms are generated. However, in the case of pure $\mathcal{N} = \frac{1}{2}$ supersymmetry (i.e. no chiral matter) we displayed a divergent field redefinition which miraculously removed the non-gauge-invariant terms and restored gauge invariance. Moreover, we displayed a slightly modified (but still $\mathcal{N} = \frac{1}{2}$ supersymmetric) version of the original pure $\mathcal{N} = \frac{1}{2}$ lagrangian which had a form preserved under renormalisation. The authors of Ref. [9] obtained the one loop effective action for pure $\mathcal{N} = \frac{1}{2}$ supersymmetry using a superfield formalism. Although they found divergent contributions which broke supergauge invariance, their final result was gauge-invariant without the need for any redefinition. On the other hand it is hard to make any inferences about renormalisability from their superfield form of the one-loop result. In the present work we consider the $\mathcal{N} = \frac{1}{2}$ supersymmetric action coupled to chiral matter. The original non-anticommutative theory defined in superfields appears to require a $U(N)$ gauge group [2, 3]. In Ref. [8] we considered the component form of the pure $\mathcal{N} = \frac{1}{2}$ supersymmetric action adapted to $SU(N)$. We argued that it was only for $SU(N)$ that a form-invariant lagrangian could be defined; indeed the $U(N)$ gauge symmetry is not preserved under renormalisation. In the case with chiral matter it turns out that the lagrangian is no longer form-invariant in the $SU(N)$ case either. In fact, a general $\mathcal{N} = \frac{1}{2}$ supersymmetric $SU(N)$ invariant action cannot be defined. However, we shall demonstrate the existence of a new $\mathcal{N} = \frac{1}{2}$ supersymmetric
$SU(N) \times U(1)$ action which is renormalisable and preserves $N = \frac{1}{2}$ supersymmetry at one loop.

The action for an $N = \frac{1}{2}$ supersymmetric $U(N)$ gauge theory coupled to chiral matter is given by

$$S = \int d^4x \left[ \text{tr}\{-\frac{1}{2} F^{\mu\nu} F_{\mu\nu} - 2i \bar{\lambda} \sigma^\mu (D_\mu \lambda) + D^2 \} ight.$$ 
$$- 2ig C^{\mu\nu} \text{tr}\{F_{\mu\nu} \bar{\lambda} \lambda \} + g^2 |C|^2 \text{tr}\{ (\bar{\lambda} \lambda)^2 \}$$
$$+ \left\{ \bar{F} F - i \bar{\psi} \sigma^\mu D_\mu \psi - D^\mu \tilde{\phi} D_\mu \phi \ight.$$ 
$$+ g \bar{\phi} D \phi + i \sqrt{2} g (\bar{\phi} \lambda \psi - \bar{\psi} \lambda \phi)$$
$$+ \sqrt{2} g C^{\mu\nu} D_\mu \bar{\phi} \lambda \sigma_\nu \psi + ig C^{\mu\nu} \bar{\phi} F_{\mu\nu} F + \frac{1}{4} |C|^2 g^2 \bar{\phi} \lambda \lambda F$$
$$+ (\phi \rightarrow \tilde{\phi}, \psi \rightarrow \tilde{\psi}, F \rightarrow \tilde{F}, C^{\mu\nu} \rightarrow -C^{\mu\nu}) \right\},$$

(1.1)

where we include a multiplet $\{\phi, \psi, F\}$ transforming according to the fundamental representation and, to ensure anomaly cancellation, a multiplet $\{\tilde{\phi}, \tilde{\psi}, \tilde{F}\}$ transforming according to its conjugate. We define

$$D_\mu \phi = \partial_\mu \phi + ig A_\mu \phi, \quad D_\mu \lambda = \partial_\mu \lambda + ig [A_\mu, \lambda], \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig [A_\mu, A_\nu],$$

(1.2)

(with a similar expression for $D_\mu \tilde{\phi}$) where

$$A_\mu = A^A_\mu R^A, \quad \lambda = \lambda^A R^A, \quad D = D^A R^A,$$

(1.3)

with $R^A$ being the group matrices for $U(N)$ in the fundamental representation. These satisfy

$$[R^A, R^B] = i f^{ABC} R^C, \quad \{R^A, R^B\} = d^{ABC} R^C,$$

(1.4)

where $d^{ABC}$ is totally symmetric. If one decomposes $U(N)$ as $SU(N) \times U(1)$ then our convention is that $R^a$ are the $SU(N)$ generators and $R^0$ the $U(1)$ generator. Of course then $f^{ABC} = 0$ unless all indices are $SU(N)$. The matrices are normalised so that $\text{Tr}[R^A R^B] = \frac{1}{2} \delta^{AB}$. In particular, $R^0 = \sqrt{\frac{1}{2N} 1}$. We note that $d^{ab0} = \sqrt{\frac{2}{N} \delta^{ab}}$, $d^{000} = \sqrt{\frac{2}{N}}$. In Eq. (1.1), $C^{\mu\nu}$ is related to the non-anti-commutativity parameter $C^{\alpha\beta}$ by

$$C^{\mu\nu} = C^{\alpha\beta} \epsilon_{\beta\gamma} \sigma^\mu_{\alpha\gamma},$$

(1.5)

where

$$\sigma^{\mu\nu} = \frac{1}{4} (\sigma^\mu \sigma^\nu - \sigma^\nu \overline{\sigma}^\mu),$$

$$\overline{\sigma}^{\mu\nu} = \frac{1}{4} (\overline{\sigma}^\mu \sigma^\nu - \overline{\sigma}^\nu \sigma^\mu),$$

(1.6)
and
\[ |C|^2 = C^{\mu\nu} C_{\mu\nu}. \] (1.7)

Our conventions are in accord with [1]; in particular,
\[ \sigma^\mu \bar{\sigma}^\nu = -\eta^{\mu\nu} + 2 \sigma^{\mu\nu}. \] (1.8)

Properties of \( C \) which follow from Eq. (1.5) are
\[ C^{\alpha\beta} = \frac{1}{2} \epsilon^{\alpha\gamma} (\sigma^{\mu\nu})_\gamma^\beta C_{\mu\nu}, \] (1.9a)
\[ C^{\mu\nu} \sigma_{\nu\alpha} = C^{\alpha\mu} \sigma^{\nu\beta}, \] (1.9b)
\[ C^{\mu\nu} \bar{\sigma}^{\nu\alpha} = -C^{\alpha\beta} \sigma^{\mu\gamma}. \] (1.9c)

Upon substituting Eq. (1.3) into Eq. (1.1) and using Eq. (1.4), we obtain the action in the \( U(N) \) case in the form:
\[
S = \int d^4 x \left[ -\frac{1}{4} F^{\mu\nu A} F_{\mu\nu}^A - i \bar{\lambda}^A \bar{\sigma}^\mu (D_\mu \lambda)^A + \frac{1}{2} D^A D^A \\
- \frac{1}{2} ig C^{\mu\nu} d^{ABC} F^{A}_{\mu\nu} \bar{\lambda}^B \lambda^C + \frac{1}{8} g^2 |C|^2 d^{ABE} d^{CDE} (\bar{\lambda}^A \bar{\lambda}^B)(\lambda^C \lambda^D) \\
+ \left\{ \bar{F} F - i \bar{\psi} \sigma^\mu D_\mu \psi - D^\mu \bar{\phi} D_\mu \phi \\
+ g \bar{\phi} D \phi + i \sqrt{2} g (\bar{\phi} \lambda \psi - \bar{\psi} \lambda \phi) \\
+ \sqrt{2} g C^{\mu\nu} D_\mu \bar{\phi} \bar{\sigma}^\nu \psi + ig C^{\mu\nu} \bar{\phi} F_{\mu\nu} F + \frac{1}{8} |C|^2 g^2 d^{ABE} \bar{\phi} R^A \bar{\lambda}^B \lambda^C F \\
+ (\phi \rightarrow \bar{\phi}, \psi \rightarrow \bar{\psi}, F \rightarrow \bar{F}, C^{\mu\nu} \rightarrow -C^{\mu\nu}) \right\} \right].
\] (1.10)

with gauge coupling \( g \), gauge field \( A_\mu \), gaugino \( \lambda \) and with
\[
F^{A}_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu - g f^{ABC} A^B_\mu A^C_\nu, \\
D_\mu \lambda^A = \partial_\mu \lambda^A - g f^{ABC} A^B_\mu \lambda^C.
\] (1.11)

However, it is clear that the \( U(N) \) action cannot be renormalisable, since for any \( U(N) \) gauge theory the gauge couplings for the \( SU(N) \) and \( U(1) \) parts of the theory renormalise differently. To obtain a renormalisable theory one must introduce different couplings for the \( SU(N) \) and \( U(1) \) parts of the gauge group and then the \( U(N) \) gauge-invariance is lost. This is a trivial point but one which does not seem to have been made in other discussions of the renormalisation of \( \mathcal{N} = \frac{1}{2} \) supersymmetric gauge theory. Remarkably, we shall see that by a judicious introduction of different couplings for the \( SU(N) \) and \( U(1) \) parts of the
gauge group, we can obtain an $SU(N) \times U(1)$ theory which still has $\mathcal{N} = \frac{1}{2}$ supersymmetry which is preserved under renormalisation. We propose replacing Eq. (1.10) by

$$S = \int d^4 x \left[ -\frac{1}{4} F^{\mu \nu A} F^{A}_{\mu \nu} - i \bar{\lambda}^A \sigma^\mu (D_\mu \lambda)^A + \frac{1}{2} D^A D^A 
- \frac{i}{2} C^{\mu \nu} d^{ABC} e^{ABC} F^{A}_{\mu \nu} \bar{\lambda}^B \lambda^C 
+ \frac{g^2}{8} |C|^2 d^{abc} d^{cde} \bar{\lambda}^a (\bar{\lambda}^c \lambda^d) + \frac{1}{4N} g_0^2 |C|^2 (\bar{\lambda}^a \lambda^a) (\bar{\lambda}^b \lambda^b) 
\right.$$  

$$+ \left\{ \bar{F} F - i \bar{\psi} \sigma^\mu D_\mu \psi - D^\mu \ddot{\phi} D_\mu \phi 
+ \delta \bar{D} \phi + i \sqrt{2} (\ddot{\phi} \lambda \psi - \bar{\psi} \lambda \phi) 
\right. 

\left. + \sqrt{2} C^{\mu \nu} D_\mu \bar{\phi} \lambda \bar{\sigma}_\nu \psi + i C^{\mu \nu} \bar{D}_\mu \psi F + \frac{1}{8N} |C|^2 d^{ABC} \phi R^A \bar{\lambda}^B \lambda^C 
+ \frac{1}{N} \gamma_1 g_0^2 |C|^2 (\bar{\lambda}^a \lambda^a) (\bar{\lambda}^0 \lambda^0) 
- \gamma_2 C^{\mu \nu} g \left( \sqrt{2} D_\mu \bar{\phi} \lambda^a \lambda^a \bar{\sigma}_\nu \psi + \sqrt{2} \bar{\phi} \lambda^a \lambda^a \bar{\sigma}_\nu D_\mu \psi + i \bar{\phi} F^{a}_{\mu \nu} R^a F 
+ (\phi \rightarrow \bar{\phi}, \psi \rightarrow \bar{\psi}, F \rightarrow \bar{F}, C^{\mu \nu} \rightarrow -C^{\mu \nu}) \right) \right\}, \tag{1.12}$$

where $\gamma_1, \gamma_2$ are constants,

$$\hat{A}_\mu = \hat{A}^A_{\mu} R^A = g A^A_{\mu} R^a + g_0 A^0_{\mu} R^0, \tag{1.13}$$

with similar definitions for $\hat{\lambda}, \hat{D}, \hat{F}_{\mu \nu}$, and now

$$D_\mu \phi = (\partial_\mu + i \hat{A}_\mu) \phi. \tag{1.14}$$

We also have

$$e^{abc} = g, \quad e^{a0b} = e^{a00} = e^{000} = g_0, \quad e^{0ab} = \frac{g^2}{g_0}. \tag{1.15}$$

It is easy to show that Eq. (1.12) is invariant under

$$\delta A^A_{\mu} = - i \bar{\lambda}^A \sigma_\mu \epsilon,$$

$$\delta \lambda_\alpha = i \epsilon_\alpha D^A + (\sigma^{\mu \nu} \epsilon)_\alpha \left[ F^{A}_{\mu \nu} + \frac{1}{2} g C^{\mu \nu} e^{ABC} d^{ABC} \bar{\lambda}^B \lambda^C \right], \quad \delta \bar{\lambda}^A_\dot{\alpha} = 0,$$

$$\delta D^A = - \epsilon \sigma^\mu D_\mu \bar{\lambda}^A,$$

$$\delta \phi = \sqrt{2} \epsilon \psi, \quad \delta \bar{\phi} = 0,$$

$$\delta \psi^a = \sqrt{2} \epsilon^a F, \quad \delta \bar{\psi}^\dot{a} = - i \sqrt{2} (D_\mu \bar{\phi})(\epsilon \sigma^\mu)_\dot{\alpha},$$

$$\delta F = 0, \quad \delta \bar{F} = - i \sqrt{2} D_\mu \bar{\psi} \sigma^\mu \epsilon - 2 i \bar{\phi} \epsilon \lambda + 2 C^{\mu \nu} D_\mu (\bar{\psi} \epsilon \sigma_\mu \lambda). \tag{1.16}$$
Apart from the term with the coefficient $\gamma_1$ and the group of terms with coefficient $\gamma_2$, Eq. (1.12) reduces to the original $U(N)$ lagrangian Eq. (1.10) derived from nonanticommuting superspace upon setting $g_0 = g$. These remaining terms are separately invariant under $\mathcal{N} = \frac{1}{2}$ supersymmetry and must be included to obtain a renormalisable lagrangian, as we shall see.

In Ref. [8] we gave an $SU(N)$-invariant theory with $\mathcal{N} = \frac{1}{2}$ supersymmetry in the pure gauge case. The supersymmetry transformations in that case were essentially obtained by striking out any 0 index in the $U(N)$ transformations. However in the general case these transformations do not close, since the gauge-field part of $\sqrt{2}gC^{\mu\nu}D_\mu\bar{\phi}\lambda\bar{\sigma}_\nu\psi$ term produces a $C^{\mu\nu}\bar{\phi}^a\bar{\lambda}^a\bar{\lambda}\bar{\sigma}_\nu\psi$ term which in the $U(N)$ case is cancelled by the variation of $\bar{\phi}\lambda^0\psi$, a term which is absent for $SU(N)$. Of course because of the $\frac{g^2}{g_0}$ terms, one cannot obtain the $SU(N)$ theory simply by setting $g_0 = 0$ in the $SU(N) \times U(1)$ theory.

We use the standard gauge-fixing term

$$S_{gf} = \frac{1}{2\alpha} \int d^4x (\partial \cdot A)^2$$

(1.17)

with its associated ghost terms. The gauge propagators for $SU(N)$ and $U(1)$ are both given by

$$\Delta_{\mu\nu} = -\frac{1}{p^2} \left( \eta_{\mu\nu} + (\alpha - 1) \frac{p_\mu p_\nu}{p^2} \right)$$

(1.18)

(omitting group factors) and the gaugino propagator is

$$\Delta_{\alpha\dot{\alpha}} = \frac{p_\mu \sigma^\mu_{\alpha\dot{\alpha}}}{p^2},$$

(1.19)

where the momentum enters at the end of the propagator with the undotted index. The one-loop graphs contributing to the “standard” terms in the lagrangian (those without a $C^{\mu\nu}$) are the same as in the ordinary $\mathcal{N} = 1$ case, so anomalous dimensions and gauge $\beta$-functions are as for $\mathcal{N} = 1$. Since our gauge-fixing term in Eq. (1.17) does not preserve supersymmetry, the anomalous dimensions for $A_\mu$ and $\lambda$ are different (and moreover gauge-parameter dependent), as are those for $\phi$ and $\psi$. However, the gauge $\beta$-functions are of course gauge-independent. The one-loop one-particle-irreducible (1PI) graphs contributing to the new terms (those containing $C$) are depicted in Figs. 1–8.
2. Renormalisation of the $SU(N) \times U(1)$ action

Ordinarily the divergences in one-loop diagrams should be cancelled by the one-loop divergences in $S_B$, obtained by replacing the fields and couplings in Eq. (1.12) with bare fields and couplings given by

\[
\begin{align*}
\lambda_B^a &= Z_1^{\lambda} \lambda^a, \quad \lambda_B^0 = Z_0^{\lambda} \lambda^0, \quad A_{\mu B}^a = Z_1^{A} A_{\mu}^a, \quad A_{\mu B}^0 = Z_0^{A} A_{\mu}^0, \\
\phi_B &= Z_1^{\phi} \phi, \quad \psi_B = Z_1^{\psi} \psi, \quad g_B = Z_g g, \quad g_0B = Z_{g0} g, \\
\gamma_{1B} &= Z_1, \quad \gamma_{2B} = Z_2, \quad C_{\mu\nu B}^{\mu\nu} = Z_C C_{\mu\nu}^{\mu\nu}, \quad |C|_{B}^2 = Z_{|C|^2} |C|^2.
\end{align*}
\]

In Eq. (2.1), $Z_1$ and $Z_2$ are divergent contributions, in other words we have set the renormalised couplings $\gamma_1$ and $\gamma_2$ to zero for simplicity. The other renormalisation constants start with tree-level values of 1. As we mentioned before, the renormalisation constants for the fields and for the gauge couplings $g$, $g_0$ are the same as in the ordinary $\mathcal{N} = 1$ supersymmetric theory and are therefore given up to one loop by:

\[
\begin{align*}
Z_{\lambda} &= 1 - g^2 L(2\alpha N + 2), \\
Z_{A} &= 1 + g^2 L[(3 - \alpha)N - 2] \\
Z_{g} &= 1 + g^2 L(1 - 3N), \\
Z_{\phi} &= 1 + 2(1 - \alpha)L\hat{C}_2, \\
Z_{\psi} &= 1 - 2(1 + \alpha)L\hat{C}_2,
\end{align*}
\]

where (using dimensional regularisation with $d = 4 - \epsilon$) $L = \frac{1}{16\pi^2\epsilon}$ and

\[
\hat{C}_2 = g^2 R^a R^a + g_0^2 R^0 R^0 = \frac{1}{2} (N g^2 + \frac{1}{N} \Delta)
\]

with

\[
\Delta = g_0^2 - g^2.
\]

(We have given here the renormalisation constants corresponding to the $SU(N)$ sector of the $U(N)$ theory; those for the $U(1)$ sector, namely $Z_{\lambda_0}$, $Z_{A_0}$ and $Z_{g_0}$, are given by omitting the terms in $N$ and replacing $g$ by $g_0$.)
Upon inserting Eq. (2.2) into Eq. (1.12) we obtain the one-loop contributions from $S_B$ as

\[
S_B^{(1)} = L \int d^4 x \left( iC^{\mu\nu} \left[ \frac{1}{7} (3 + 5\alpha) N + 2 \right] g^3 d^{abc} \partial_\mu A^a_\nu \bar{\lambda}^b \lambda^c 
+ [3(\alpha - 1)N + 4] g^2 g_0 d^{ab0} \partial_\mu A^a_\nu \bar{\lambda}^b \lambda^0 
+ 2[(3 + \alpha)N g^2 + g_0^2 g_0 d^{b0c} \partial_\mu A^0_\nu \bar{\lambda}^b \lambda^c + 2g_0^2 d^{000} \partial_\mu A^0_\nu \bar{\lambda}^0 \lambda^0] 
- \frac{2}{1} (1 + \alpha) N) + 1 \right] i d^{abef} f^{cde} g^4 C^{\mu\nu} A^c_\mu A^d_\nu \bar{\sigma}^a \bar{\lambda}^b 
- 2i(\alpha N + 1) d^{0be} f^{cde} g^3 g_0 C^{\mu\nu} A^c_\mu A^d_\nu \bar{\lambda}^0 \lambda^b 
- g^2 |C|^2 \left[ \frac{1}{4} [(3 + 2\alpha) N + 1] g^2 d^{ab} d^{cde} (\bar{\lambda}^a \bar{\lambda}^b) (\bar{\lambda}^c \bar{\lambda}^d) 
+ \left[ (3 + \alpha) \frac{9}{9g_0^2} + \frac{2}{2N} \right] (\bar{\lambda}^a \bar{\lambda}^b) (\bar{\lambda}^c \bar{\lambda}^b) - \frac{2}{N} g_0^2 (\bar{\lambda}^a \bar{\lambda}^b) (\bar{\lambda}^0 \lambda^0) \right] 
- \frac{1}{2} i Z_C^{(1)} C^{\mu\nu} d^{ABC} e^{A\mu B\lambda C} F_{\mu\nu} \bar{\lambda} \lambda C 
+ Z_{|C|^2} \left[ \frac{1}{8} g^2 |C|^2 d^{ab} d^{cde} (\bar{\lambda}^a \bar{\lambda}^b) (\bar{\lambda}^c \bar{\lambda}^d) + \frac{1}{4N} g_0^4 |C|^2 (\bar{\lambda}^a \bar{\lambda}^b) (\bar{\lambda}^b \bar{\lambda}^d) \right] 
+ \left\{ \sqrt{2} C^{\mu\nu} \left[ - \left( (3 + \alpha) N g^2 + Z_2^{(1)} \right) g_0 \phi \bar{\phi} \lambda^a \lambda^b \sigma_\nu \sigma_\lambda \right] - 2(1 - \alpha) \hat{C}_2 g_0 \partial_\mu (\bar{\phi} \lambda^0 R^0 \sigma_\nu \psi) 
- 2 \alpha \hat{C}_2 g_0 \partial_\mu (\bar{\phi} \lambda^0 R^0 \sigma_\nu \psi) 
+ i \sqrt{2} C^{\mu\nu} \left[ g^2 A^b_\mu \bar{\phi} \lambda^b \left[ - Z_2^{(1)} R^a R^b + \left( \frac{1}{2} \alpha N g^2 (3 + \alpha) + Z_2^{(1)} \right) R^a R^b + 2\alpha \hat{C}_2 \right] \sigma_\nu \psi 
+ g_0^2 \left( \frac{1}{2} g_0 N g^2 (3 + \alpha) + 2\alpha \hat{C}_2 \right) A^b_\mu \bar{\phi} \lambda^0 R^b \sigma_\nu \psi 
+ g_0^2 \left( (3 + \alpha) N g^2 + 2\alpha \hat{C}_2 \right) A^0_\mu \bar{\phi} \lambda^a R^0 \sigma_\nu \psi + 2g_0^2 \alpha \hat{C}_2 A^0_\mu \bar{\phi} \lambda^0 (R^0)^2 \sigma_\nu \psi \right] 
+ i C^{\mu\nu} \left[ 2(1 - \alpha) \hat{C}_2 - (3 + \alpha) N g^2 - 2Z_2^{(1)} \right] g_0 \partial_\mu (\bar{\phi} \lambda^a R^a + 2(1 - \alpha) \hat{C}_2 g_0 \partial_\mu (\bar{\phi} \lambda^0 R^0) 
+ \left( (\alpha - 1) \hat{C}_2 + (3 + \alpha) N g^2 + Z_2^{(1)} \right) g^2 f^{abc} A^a_\mu A^b_\nu R^c \left[ F \right] 
+ \frac{1}{8} |C|^2 \left[ (1 - \alpha) \hat{C}_2 - (6 + 2\alpha) N g^2 \right] g^2 A^{A\mu B\lambda C} \phi^a R^a \bar{\lambda} \lambda^a \left[ F \right] 
+ 2 \left( (1 - \alpha) \hat{C}_2 - (3 + \alpha) N g^2 \right) g^2 d^{00c} \bar{\phi} R^a \bar{\lambda}^0 \lambda^c + (1 - \alpha) \hat{C}_2 d^{000} \bar{\phi} R^0 \lambda^0 \lambda^0 \left[ F \right] 
+ Z_C^{(1)} C^{\mu\nu} \left[ \sqrt{2} D_\mu \bar{\phi} \lambda^a \sigma_\nu \psi + i \phi \bar{F}_\mu \psi \right] + \frac{1}{8} |C|^2 d^{ABC} \bar{\phi} R^A \bar{\lambda} \lambda^C \left[ F \right] 
+ \frac{1}{X} g_0^2 |C|^2 (\bar{\lambda}^a \bar{\lambda}^a) (\bar{\lambda}^0 \lambda^0) 
- \frac{1}{\gamma} C^{\mu\nu} g \left( \sqrt{2} D_\mu \bar{\phi} \lambda^a R^a \sigma_\nu \psi + \sqrt{2} \phi \bar{\lambda}^a R^a \sigma_\nu \psi + i \phi \bar{F}_\mu \psi + i \phi \bar{F}_\mu R^a \left[ F \right] \right) 
+ (\phi \rightarrow \bar{\phi}, \psi \rightarrow \bar{\psi}, \bar{F} \rightarrow F, C^{\mu\nu} \rightarrow -C^{\mu\nu} \left[ \right] \right) \right). 
\]

(2.5)
The results $\Gamma^{(1)\text{pole}}_{1\text{PI}}$, $i = 1\ldots8$ for the one-loop divergences from the 1PI graphs in Figs. 1–8 respectively are given in Appendix A. It is clear that they cannot be cancelled by Eq. (2.4), in particular since they contain contributions involving $\bar{\sigma}^{\mu\nu}$ which do not appear in Eq. (2.3). As we showed in Ref. [8], this can be remedied by field redefinitions, or, to put it another way, additional non-linear field renormalisations. We find that a field redefinition
\[
\delta \lambda^A = -\frac{1}{2} NLg^2 C^{\mu\nu} e^{BAC} d^{ABC} c^A c^B d^C \sigma_\mu \bar{\lambda}^C A^B, \tag{2.6}
\]
where $c^A = 1 - \delta^{A0}$, $d^A = 1 + \delta^{A0}$, results in a change in the action
\[
\delta S_{\lambda} = NLg^2 \int d^4x \left[ -\frac{1}{4} i C^{\mu\nu}(d^{abc} g \partial_\mu A^a_\nu \bar{\lambda}^b \bar{\lambda}^c - d^{abc} f^{cde} g^2 A^d_\mu A^a_\nu \bar{\lambda}^c \bar{\lambda}^b) 
+ id^{abc} g C^\mu_\nu A^a_\mu \bar{\lambda}^b \bar{\lambda}^c - \frac{1}{2} i d^{cde} f^{abc} g^2 C^{\mu\rho} A^c_\mu A^d_\nu \bar{\lambda}^a \bar{\lambda}^b
+ i C^{\rho\sigma} d^{abc} g_0 A^{a}_\sigma \bar{\lambda}^0 \delta^{\mu \rho} + 2 \bar{\sigma}^{\mu \rho} \partial_\mu \bar{\lambda}^c 
+ i C^{\mu\nu} f^{abc} d^{cde} g_0 A^b_\mu A^c_\nu \bar{\lambda}^0 \bar{\lambda}^0 
- \left\{ \frac{1}{2} i \sqrt{g} C^{\mu\nu} d^{abc} A^b_\mu \bar{\phi} R^c \bar{\lambda}^a \bar{\lambda}^b \psi + i \sqrt{g} g_0 C^{\mu\nu} d^{abc} A^b_\mu \bar{\phi} R^c \bar{\lambda}^0 \bar{\sigma}_\nu \psi 
+ (\phi \to \bar{\phi}, \psi \to \bar{\psi}, F \to \bar{F}, C^{\mu\nu} \to -C^{\mu\nu}) \right\} \right], \tag{2.7}
\]
which miraculously casts all the $C$-dependent terms apart from those linear in $F$, $\bar{F}$ into the correct form. Then finally redefinitions of $\bar{F}$, $\bar{F}$ can be used to deal with the terms linear in $F$, $\bar{F}$. Explicitly, we need
\[
\delta \bar{F} = L \left\{ \left( 5Ng^2 + 2(1 + \alpha) \hat{C}_2 \right) g_0 \partial_\mu A^0_\mu 
+ \left( \frac{1}{4} Ng^2 + (1 + \alpha) \hat{C}_2 \right) g^2 f^{abc} A^b_\mu A^c_\nu \right\} i C^{\mu\nu} \bar{\phi} R^a 
+ 2(\alpha + 1) \hat{C}_2 g_0 \partial_\mu A^0_\mu i C^{\mu\nu} \bar{\phi} R^0 
+ \frac{1}{8} |C|^2 \left[ \left. -37Ng^2 + (63 + \alpha) \hat{C}_2 \right] g^2 d^{abc} \bar{\phi} R^c \bar{\lambda}^a \bar{\lambda}^b 
+ \left. -32Ng^2 + (31 + \alpha) \hat{C}_2 \right] g g_0 d^{abc} \bar{\phi} R^0 \bar{\lambda}^a \bar{\lambda}^b 
+ (31 + \alpha) \hat{C}_2 g_0 d^{abc} \bar{\phi} R^0 \bar{\lambda}^a \bar{\lambda}^b 
+ 6Ng^2 + (\alpha - 1) \hat{C}_2 \right] g^2 d^{abc} \bar{\phi} R^0 \bar{\lambda}^a \bar{\lambda}^b \right\} \tag{2.8}
\]
(with a similar redefinition of $\bar{\bar{F}}$) which produce a change in the action
\[
\delta S_f = \int d^4x \left( \delta \bar{F} F + \delta \bar{F} \bar{F} \right). \tag{2.9}
\]
We now find that with
\[
Z_C^{(1)} = Z_{|C|^2}^{(1)} = 0, \quad Z_1^{(1)} = -3Ng^2L, \quad Z_2^{(1)} = -Ng^2L, \tag{2.10}
\]
we have
\[ \Gamma^{(1)\text{pole}' } = \sum_{i=1}^{8} \Gamma^{(1)\text{pole}}_{i1\Pi} + \delta S_{\lambda} + \delta S_{\bar{F}} + S^{(1)}_{B} = 0, \]  
(2.11)
i.e. $\Gamma^{(1)'}$ is finite.

This demonstrates that our theory is renormalisable and that the $\mathcal{N} = \frac{1}{2}$ supersymmetry is preserved. However we find that to obtain a renormalisable lagrangian it is vital (since $Z_{1}^{(1)}, Z_{2}^{(1)} \neq 0$) to include the terms involving $\gamma_{1}, \gamma_{2}$ in Eq. (1.12), which were not in the original formulation of the theory[2] though they are independently $\mathcal{N} = \frac{1}{2}$ supersymmetric. This is not unexpected since in general any terms which are not forbidden by a symmetry will be generated under renormalisation. It is therefore all the more remarkable that we do not need to renormalise the nonanticommutativity parameter $C$ and that the other $\bar{\lambda}^{4}$ terms (which are also separately $\mathcal{N} = \frac{1}{2}$ supersymmetric) do not require any counterterms. On the other hand our renormalised lagrangian is no longer of the form derived from nonanticommutative superspace. Of course this was also found in the case of the $\mathcal{N} = \frac{1}{2}$ Wess-Zumino model[4].

We note here that the requirement to make a divergent redefinition of $\bar{F}$ is not as surprising as it may first appear (if calculating in components with a conventional covariant gauge). In fact, if one renormalises the ordinary $\mathcal{N} = 1$ theory in its uneliminated component form, i.e. before eliminating the auxiliary fields, one is compelled to make a similar non-linear renormalisation of $F$ to render the theory finite. This has not to our knowledge previously been discussed, and we give the details in a forthcoming publication[11].

3. Conclusions

We have studied the renormalisability of a general $\mathcal{N} = \frac{1}{2}$ supersymmetric theory coupled to chiral matter. The non-renormalisability of the standard $U(N)$ version was apparent from the outset, and it appeared impossible to define a general $SU(N)$ invariant $\mathcal{N} = \frac{1}{2}$ supersymmetric theory; however we were able to define an $SU(N) \times U(1)$ invariant action which still possessed $\mathcal{N} = \frac{1}{2}$ supersymmetry, which as we showed was preserved under renormalisation. Moreover we find that the non-anticommutativity parameter $C$ is unrenormalised (at least at one loop).

We have restored gauge invariance by a somewhat unconventional expedient which works rather miraculously. One could speculate to what extent the $\mathcal{N} = \frac{1}{2}$ supersymmetry and the identities Eq. (1.3) were required to make this trick work. If one treats the
action (1.1) as primordial, ignoring its derivation from non-anticommuting superspace, the identities Eq. (1.9) can be regarded as a consequence of the self-duality of $C^{\mu\nu}$ (with $C^{\alpha\beta}$ now defined by Eq. (1.9a)). It would be interesting to examine a theory of the same form but in which $C^{\mu\nu}$ was replaced by a general antisymmetric tensor. Moreover, suppose one considered a theory with an action based on Eq. (1.1) but including all the hermitian conjugate terms which are missing. The only new diagrams would simply be the “hermitian conjugates” of those in Figs. 1–8. Eq. (2.6) would now need to be supplemented by its hermitian conjugate. However, the variation of the action would now include additional unwanted non-gauge-invariant terms since it is now not only the gaugino kinetic term which varies. This raises the possibility of a theory (albeit non-renormalisable) with ineradicable non-gauge-invariant divergences.

An interesting feature of our results is the redefinition (or non-linear renormalisation) of the gaugino field. As we have mentioned, the attendant non-linear redefinition of the auxiliary field $F$ has its counterpart even in the $N = 1$ theory, so that non-linear field redefinitions may be an unavoidable consequence of working in the uneliminated component formalism with conventional gauge-fixing; as we mentioned, no such field redefinition was required in the $N = \frac{1}{2}$ superfield calculation of Ref. [9].

Appendix A. Results for One-Loop Diagrams

The divergent contributions to the effective action from the graphs in Fig. 1 are of the form:

$$ig^2 L d^{ABC} e^{ABC} C^{\mu\nu} \left[ \partial_\mu A^A_{\rho} \bar{\lambda}^B \left( T_1^{ABC} \delta_\nu^\rho + \bar{A}_1^{ABC} \bar{\sigma}_\nu^\rho \right) \bar{\lambda}^C \right]$$

$$+ A^A_{\rho} \bar{\lambda}^B \left( \tilde{T}_1^{ABC} \delta_\nu^\rho + A_1^{ABC} \bar{\sigma}_\nu^\rho \right) \partial_\mu \bar{\lambda}^C \right], \quad (A.1)$$

where the contributions to $T_1$, $\tilde{T}_1$, $A_1$, $\bar{A}_1$ from the individual graphs are given in Table 1:
| Graph | $T_1$ | $\bar{T}_1$ | $\bar{A}_1$ | $A_1$ |
|-------|-------|-------------|-------------|--------|
| 1a    | $-(3 + \alpha)Nd^Ac^Bc^C$ | $0$         | $0$         | $0$    |
| 1b    | $-NC^Ad^Bc^C$              | $0$         | $-\frac{2}{3}NC^Ad^Bc^C - \frac{2}{3}NC^Ad^Bc^C$ |
| 1c    | $-2\alpha NC^Ad^Bc^C$     | $\frac{1}{2}(2 - \alpha)NC^Ad^Bc^C$ | $\frac{2}{3}NC^Ad^Bc^C - \frac{1}{2}(2 + 3\alpha)NC^Ad^Bc^C$ |
| 1d    | $\frac{1}{2}(5 + \alpha)NC^A$ | $0$         | $0$         | $0$    |
| 1e    | $0$                          | $-\frac{1}{2}(3 - \alpha)NC^Ad^Bc^C$ | $0$         | $(1 + \alpha)NC^Ad^Bc^C$ |
| 1f    | $-g^{ABC}/e^{ABC}$         | $0$         | $0$         | $-\frac{4}{3}g^{ABC}/e^{ABC}$ |
| 1g    | $-\frac{1}{2}g^{ABC}/e^{ABC}$ | $0$         | $0$         | $-\frac{2}{3}g^{ABC}/e^{ABC}$ |
| 1h    | $-\frac{1}{2}g^{ABC}/e^{ABC}$ | $0$         | $0$         | $2g^{ABC}/e^{ABC}$ |

**Table 1**: Contributions to $T_1$, $\bar{T}_1$, $A_1$, $\bar{A}_1$ from Fig. 1

In Table 1, $g^{ab0} = g^{a0b} = g^{0ab} = g^{000} = g_0$ and $g^{abc} = g$.

We note here that Figs. 1f-1h correspond to both $\phi$, $\psi$ and $\tilde{\phi}$, $\tilde{\psi}$ loops, which contribute identically due to the change in sign $C^{\mu\nu} \rightarrow -C^{\mu\nu}$ between the $\phi$, $\psi$ and $\tilde{\phi}$, $\tilde{\psi}$ interactions in the lagrangian. Possible contributions of the form $gLf^{ABC}C^{\mu\nu} \partial_\mu A^A_\rho \bar{\lambda}_B \bar{\sigma}_\nu \bar{\lambda}_C$, $gLf^{ABC}C^{\mu\nu} A^A_\rho \bar{\lambda}_B \bar{\sigma}_\nu \partial_\mu \bar{\lambda}_C$ cancel between $\phi$, $\psi$ and $\tilde{\phi}$, $\tilde{\psi}$ loops.

The divergent contributions to the effective action from the graphs in Fig. 1 are given by

$$
\Gamma^{(1)}_{\text{1PI}} = i g^2 LC^{\mu\nu}d^{ABC}e^{ABC}\left[\partial_\mu A^A_\rho \bar{\lambda}_B \left([- (1 + 2\alpha)NC^Ad^Bc^C + \frac{1}{2}(5 + \alpha)NC^A - (3 + \alpha)Nd^Ac^Bc^C - 2g^{ABC}/e^{ABC})\delta_\nu^\rho + \frac{2}{3}N [c^A d^B c^C - c^A c^B d^C] \bar{\sigma}_\nu^\rho \right) \bar{\lambda}_C + A^A_\rho \bar{\lambda}_B \left(- \frac{1}{2}NC^Ad^Bc^C \delta_\nu^\rho + \frac{1}{3}N [c^A d^B c^C - 4c^A c^B d^C] \bar{\sigma}_\nu^\rho \right) \partial_\mu \bar{\lambda}_C \right] 
$$

$$
= i LC^{\mu\nu}\left[-\left\{ \frac{5}{4}(1 + 2\alpha)N + 2 \right\} g^3 d^{abc} \partial_\mu A^a_\nu \bar{\lambda}_b \bar{\lambda}_c + [3(1 - \alpha)N - 4]g^2 g_0d^{ab0} \partial_\mu A^a_\nu \bar{\lambda}_b \bar{\lambda}_0 
- 2[(3 + \alpha)Ng^2 + g_0^22g^2g_0d^{abc} \partial_\mu A^0_\nu \bar{\lambda}_b \bar{\lambda}_c - 2g^2g_0d^{000} \partial_\mu A^0_\nu \bar{\lambda}_0 \bar{\lambda}_0 
- Ng^2g_0d^{abc} A^a_\nu \bar{\lambda}_0 \partial_\mu \bar{\lambda}_c 
- Ng^3d^{abc} A^a_\nu \bar{\lambda}_b \bar{\sigma}_\nu^\rho \partial_\mu \bar{\lambda}_c - 2Ng^2g_0d^{0ac} A^a_\nu \bar{\lambda}_0 \bar{\sigma}_\nu^\rho \partial_\mu \bar{\lambda}_c \right] \right].
$$

The divergent contributions to the effective action from the graphs in Fig. 2 are of the
form:

\[ ig^3 L e^{EAB} F^{CDE} C^{\mu\nu} T_2^{ABCD} A_\mu^C A_\nu^D \tilde{\lambda}^A \tilde{\lambda}^B + d^{CDE} f^{ABE} C^{\mu\rho} A_2^{ABCD} A_\mu^C A_\nu^D \tilde{\lambda}^A \tilde{\sigma}^{\nu \rho} \tilde{\lambda}^B \]  

(A.3)

where the contributions to \( T_2, A_2 \) from the individual graphs are given in Table 2:

| Graph | \( T_2 \) | \( A_2 \) |
|-------|-------------|-------------|
| 2a    | \( \frac{1}{2} N d^A c^B \) | \( \frac{1}{3} N d^A c^B c^C c^D \) |
| 2b    | \( -\frac{1}{2} (3 - \alpha) N \delta^{A0} \) | |
| 2c    | \( \frac{1}{2} (3 + \alpha) N c^A c^B \) | 0 |
| 2d    | \( \frac{1}{2} (2 - \alpha) N \delta^{A0} \) | |
| 2e    | \( -\frac{1}{2} \alpha N d^A c^B \) | \( \frac{1}{6} (4 + 3 \alpha) N d^A c^B c^C c^D \) |
| 2f    | \( \frac{3}{4} \alpha N d^A c^B \) | \( -\frac{1}{2} (2 + \alpha) N d^A c^B c^C c^D \) |
| 2g    | \( \frac{3}{4} \alpha N d^A c^B \) | \( \frac{1}{2} N d^A c^B c^C c^D \) |
| 2h    | \( -\frac{3}{4} (1 + \alpha) N \) | 0 |
| 2i    | \( \frac{3}{4} \alpha N \) | 0 |
| 2j    | \( \frac{1}{2} \) | \( \frac{1}{3} \) |
| 2k    | 0 | \( \frac{2}{3} \) |
| 2l    | \( \frac{1}{2} \) | -1 |

Table 2: Contributions from Fig. 2

The contributions from Figs. 2(m)–(o) are zero. The graphs in Fig. 2 add to

\[ \Gamma^{(1)\text{pole}}_{21\text{PI}} = \frac{1}{4} i g^3 L \left[ (2 + 2 \alpha) N d^A c^B + 2(3 + \alpha) N c^A c^B - 3N - 2 N \delta^{A0} + 4 \right] e^{EAB} d^{ABE} F^{CDE} C^{\mu\nu} A_\mu^C A_\nu^D \tilde{\lambda}^A \tilde{\lambda}^B \]

\[ + \frac{1}{2} i g^3 L d^A c^B c^C d^C EAB d^{CDE} f^{ABE} C^{\mu\rho} A_\mu^C A_\nu^D \tilde{\lambda}^A \tilde{\sigma}^{\nu \rho} \tilde{\lambda}^B \]

\[ = \left[ \frac{1}{4} (5 + 6 \alpha) N + 1 \right] i L d^{a_1 b_1} f^{c_1 d_1} g^4 C^{\mu\nu} A_\mu^{c_1} A_\nu^{d_1} \tilde{\lambda}^{a_1} \tilde{\lambda}^{b_1} \]

\[ + \frac{1}{2} i N L d^{c_1 d_1} f^{a_1 b_1} g^4 C^{\mu\rho} A_\mu^{c_1} A_\nu^{d_1} \tilde{\lambda}^{a_1} \tilde{\sigma}^{\nu \rho} \tilde{\lambda}^{b_1} \]

\[ - i L [(1 - 2 \alpha) N - 2] d^{a_1 b_1} f^{c_1 d_1} g^4 g_0 C^{\mu\nu} A_\mu^{c_1} A_\nu^{d_1} \tilde{\lambda}^{a_1} \tilde{\lambda}^{b_1} \].

The results for Fig. 3 are of the form:

\[ g^2 L |C|^2 \left[ X_1^{abcd} (\tilde{\lambda}^a \tilde{\lambda}^b)(\tilde{\lambda}^c \tilde{\lambda}^d) + X_2 (\tilde{\lambda}^a \tilde{\lambda}^a)(\tilde{\lambda}^b \tilde{\lambda}^b) + X_3 (\tilde{\lambda}^a \tilde{\lambda}^a)(\tilde{\lambda}^0 \tilde{\lambda}^0) \right] \]  

(A.5)
where the contributions to $X_{1-3}$ are given in Table 3:

| Graph | $X_1^{abcd}$ | $X_2$ | $X_3$ |
|-------|--------------|-------|-------|
| 3a    | $\frac{1}{4}(3+\alpha)Ng^2d^{abe}d^{cde} + 2g^2d^{abcd} + (1-\alpha)g^2d^{adcb} - \frac{4}{N}\frac{g^4}{g_0^4}f^{eac}f^{ebd}$ | $(3+\alpha)\frac{g^2}{g_0^2}$ | 0     |
| 3b    | $\frac{1}{2}\alpha Ng^2d^{abe}d^{cde}$ | 0     | $-2\alpha g_0^2$ |
| 3c    | $-\frac{1}{4}(1+\alpha)Ng^2d^{abe}d^{cde}$ | 0     | $(1+\alpha)g_0^2$ |
| 3d    | $g^2\left[-2d^{abcd} + (\alpha - 1)d^{adcb} + \frac{4}{N}\frac{g^2}{g_0^4}f^{eac}f^{ebd}\right]$ | 0     | $(3+\alpha)g_0^2$ |
| 3e    | $\frac{1}{3}g^2(\tilde{d}^{abcd} - \tilde{d}^{acdb})$ | 0     | $-g_0^2$ |
| 3f    | $\frac{1}{4}g^2d^{abe}d^{cde}$ | $\frac{1}{2N}g^2$ | 0     |

Table 3: Contributions from Fig. 3

In Table 3,

$$d^{abcd} = \text{Tr}[F^a F^b D^c D^d], \quad \tilde{d}^{abcd} = \text{Tr}[F^a D^c F^b D^d],$$

(A.6)

where the matrices $F_a$ and $D_a$ are defined in Appendix B. These results add to

$$\Gamma^{(1)\text{pole}}_{31\Pi} = g^2L|C|^2\left[\frac{1}{4}((3+2\alpha)N + 1)g^2d^{abe}d^{cde}(\bar{\lambda}^a\bar{\lambda}^b)(\bar{\lambda}^c\bar{\lambda}^d)\right.\
+ \frac{1}{2N}\left[2(3+\alpha)Ng^4 + g^2\right](\bar{\lambda}^a\bar{\lambda}^a)(\bar{\lambda}^b\bar{\lambda}^b) + 3g_0^2(\bar{\lambda}^a\bar{\lambda}^a)(\bar{\lambda}^0\bar{\lambda}^0)\right].$$

(A.7)

In obtaining these results we have made frequent use of the Fierz identity

$$(\bar{\lambda}^a\bar{\lambda}^b)(\bar{\lambda}^c\bar{\lambda}^d) + (\bar{\lambda}^a\bar{\lambda}^c)(\bar{\lambda}^b\bar{\lambda}^d) + (\bar{\lambda}^a\bar{\lambda}^d)(\bar{\lambda}^b\bar{\lambda}^c) = 0$$

(A.8)

The contributions from the graphs shown in Fig. 4 are of the form

$$\sqrt{2}g_A LC^{\mu\nu}\partial_{\mu}\bar{\phi}\lambda^A X^A\sigma_{\nu}\psi + \sqrt{2}g_A LC^{\mu\nu}\bar{\phi}\lambda^A Y^A\sigma_{\nu}\partial_{\mu}\psi$$

(A.9)

where $g_a \equiv g$ and $X^A$ and $Y^A$ are as given in Table 4. (There are analogous diagrams with $\bar{\phi}, \bar{\psi}$ external legs which we do not show explicitly; their contributions may easily be read off using $\phi \rightarrow \bar{\phi}, \psi \rightarrow \bar{\psi}, F \rightarrow \bar{F}, C^{\mu\nu} \rightarrow -C^{\mu\nu}$.) The contributions to $X^A, Y^A$ are shown in Table 4:
Table 4: Contributions to $X^A$ and $Y^A$ from Fig. 4

| Graph | $X^A$ | $Y^A$ |
|-------|-------|-------|
| 4a    | $\frac{3}{2}Ng^2c^AR^A$ | $-\alpha Nc^AR^A$ |
| 4b    | $\alpha Ng^2c^AR^A$ | $\alpha Nc^AR^A$ |
| 4c    | $\alpha Ng^2c^AR^A$ | 0 |
| 4d    | $-2[\hat{C}_2 - \frac{1}{2}Ng^2c^AR^A]$ | $2[\hat{C}_2 - \frac{1}{2}Ng^2c^AR^A]$ |
| 4e    | $-2[\hat{C}_2 - \frac{1}{2}Ng^2c^AR^A]$ | 0 |
| 4f    | $-(1-2\alpha)[\hat{C}_2 - \frac{1}{2}Ng^2c^AR^A]$ | 0 |
| 4g    | $2[2\hat{C}_2 - \frac{1}{2}Ng^2c^AR^A]$ | 0 |
| 4h    | $2[2\hat{C}_2 - \frac{1}{2}Ng^2c^AR^A]$ | $-2[2\hat{C}_2 - \frac{1}{2}Ng^2c^AR^A]$ |
| 4i    | 0 | $2[\hat{C}_2 - \frac{1}{2}Ng^2c^AR^A]$ |
| 4j    | $-3\hat{C}_2 R^A$ | 0 |

These graphs add to

$$\frac{1}{4\pi} \Gamma^{(1)\text{pole}} = \sqrt{2} g_A LC^{\mu\nu} \partial_\mu \bar{\phi} \bar{\lambda}^A \left[ 2\alpha \hat{C}_2 R^A + (2 + \alpha) Ng^2c^A R^A \right] \bar{\sigma}_\nu \psi$$

$$- \sqrt{2} Ng_A g^2c^A C^{\mu\nu} \partial_\mu \bar{\phi} \bar{\lambda}^A R^A \bar{\sigma}_\nu \partial_\mu \psi$$

$$= L \left\{ \left[ 2\alpha \hat{C}_2 + (2 + \alpha) Ng^2 \right] \sqrt{2} g C^{\mu\nu} \partial_\mu \bar{\phi} \bar{\lambda}^A R^A \bar{\sigma}_\nu \psi \right. \right.$$  

$$+ 2\alpha \hat{C}_2 \sqrt{2} g_0 C^{\mu\nu} \partial_\mu \bar{\phi} \bar{\lambda}^0 R^0 \bar{\sigma}_\nu \psi$$

$$- N \sqrt{2} g^3 C^{\mu\nu} \partial_\mu \bar{\phi} \bar{\lambda}^A R^A \bar{\sigma}_\nu \partial_\mu \psi \right\}. \quad (A.10)$$

The contributions from the graphs shown in Fig. 5 are of the form

$$\sqrt{2} ig_A g_B LC^{\mu\nu} A_\mu^F \bar{\phi} \bar{\lambda}^A Z^{AB} \bar{\sigma}_\nu \psi \quad (A.11)$$

where in the case of Figs. 5(a)–5(v), $Z^{AB}$ contains the contributions shown in Table 5:
| Graph | $\hat{C}_2 R^A R^B$ | $\hat{C}_2 R^B R^A$ | $Ng^2 R^A R^B$ | $Ng^2 R^B R^A$ | $g^2 f^{ACE} f^{BDE} R^C R^D$ |
|-------|-----------------|-----------------|----------------|----------------|------------------|
| 5a    | 0               | 2               | $-c^A$         | $-c^B$         | $2c^A c^B$       |
| 5b    | -2              | 0               | $c^A + c^B$    | 0              | $-2c^A c^B$      |
| 5c    | 0               | 0               | $2c^B$         | 0              | $-4c^A c^B$      |
| 5d    | 4               | 0               | $-2(c^A + c^B)$| 0              | $4c^A c^B$       |
| 5e    | 0               | 2               | 0              | $-c^A$         | 0                |
| 5f    | 0               | 0               | $\frac{1}{2}(1 - \alpha)c^B$ | 0 | $(\alpha - 1)c^A c^B$ |
| 5g    | 2               | 0               | $-(c^A + c^B)$ | 0              | $2c^A c^B$       |
| 5h    | 0               | $-\alpha$       | 0              | $\frac{1}{2}\alpha c^B$ | 0                |
| 5i    | 0               | 0               | 0              | $-\frac{3}{4}\alpha c^B$ | 0                |
| 5j    | 0               | $3 + \alpha$    | 0              | $-\frac{1}{4}(3 + \alpha)c^B$ | 0                |
| 5k    | 0               | 0               | $-\frac{1}{4}\alpha c^B$ | 0 | $\frac{1}{2}\alpha c^A c^B$ |
| 5l    | 0               | $1 - \alpha$    | $\frac{1}{4}(\alpha - 1)c^A$ | $\frac{1}{4}(\alpha - 1)(c^A + c^B)$ | $\frac{1}{2}(1 - \alpha)c^A c^B$ |
| 5m    | 0               | $-2\alpha$      | $\alpha c^A$  | $\alpha c^B$  | $-2\alpha c^A c^B$ |
| 5n    | 0               | 0               | $-\alpha c^A$ | 0              | $2\alpha c^A c^B$ |
| 5o    | 0               | $\alpha$        | $-\frac{1}{2}\alpha c^A$ | $-\frac{1}{2}\alpha c^B$ | $\alpha c^A c^B$ |
| 5p    | 0               | 0               | $-\frac{1}{4}(3 + \alpha)c^A$ | $-\frac{1}{4}(3 + \alpha)c^A$ | $\frac{1}{2}(3 + \alpha)c^A c^B$ |
| 5q    | 0               | 0               | 0              | $-\alpha c^A$ | 0                |
| 5r    | 0               | 0               | $\frac{1}{2}\alpha c^A$ | 0 | $-\alpha c^A c^B$ |
| 5s    | 0               | 0               | $\frac{3}{2}(1 + \alpha)c^B$ | $-\frac{3}{2}(1 + \alpha)c^B$ | $-3(1 + \alpha)c^A c^B$ |
| 5t    | 0               | 0               | 0              | 0              | $-2\alpha c^A c^B$ |
| 5u    | 0               | 0               | 0              | 0              | $2\alpha c^A c^B$ |
| 5v    | 0               | 0               | $-\frac{3}{4}\alpha c^B$ | $\frac{3}{4}\alpha c^B$ | $\frac{3}{2}\alpha c^A c^B$ |

*Table 5a: Contributions to $Z^{AB}$ from Figs. 5(a)–5(v)*

The contributions from Table 5a add to

$$i\sqrt{2}g_A g_B L C^{\mu\nu} A_\mu^B \tilde{\phi} \tilde{A}^A \left[ 4\hat{C}_2 R^A R^B + (8 - 2\alpha)\hat{C}_2 R^B R^A \\
+ Ng^2 (-4R^A R^B c^A + 2R^A R^B c^B) - (2 + \alpha)R^B R^A c^A - \frac{1}{2}(7 + \alpha)R^B R^A c^B \right] \bar{\sigma}_\nu \psi. \quad (A.12)$$

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In the case of Figs. 5(w)–5(cc), the contributions to \(Z_{ab}\) are shown in Table 5b:

| Graph | \(Ng^2R^aR^b\) | \(Ng^2R^bR^a\) | \(\frac{1}{N}\Delta R^aR^b\) | \(\frac{1}{N}\Delta R^bR^a\) | \(g^2\delta_{ab}\) |
|-------|-----------------|-----------------|-----------------|-----------------|------------------|
| 5w    | \(-\frac{1}{2}(3 + \alpha)\) | 0               | \(-(3 + \alpha)\) | \(3 + \alpha\) | \(\frac{1}{4}(3 + \alpha)\) |
| 5x    | 0               | -1              | 0               | 0               | \(\frac{1}{2}\) |
| 5y    | 0               | 0               | 0               | -4              | -1               |
| 5z    | \(\frac{1}{2}(2 + \alpha)\) | 0               | \(2 + \alpha\)  | \(-(2 + \alpha)\) | \(-\frac{1}{4}(2 + \alpha)\) |
| 5aa   | \(-\frac{1}{2}\alpha\) | 0               | \(\alpha\)      | \(-\alpha\)     | \(\frac{1}{4}\alpha\) |
| 5bb   | 0               | \(-\frac{1}{2}\) | -1              | -1              | \(-\frac{1}{4}\) |
| 5cc   | \(\frac{1}{2}\alpha\) | 0               | \(-\alpha\)     | \(\alpha\)      | \(-\frac{1}{4}\alpha\) |

*Table 5b: Contributions to \(Z_{ab}\) from Fig. 5(w)–5(cc)*

The contributions to \(Z^{0b}\) from Figs. 5(w)–5(cc) are shown in Table 5c:

| Graph | \(Ng^2R^bR^0\) | \(\frac{1}{N}\Delta R^0R^b\) |
|-------|-----------------|-----------------|
| 5w    | \(-3 + \alpha\) | 0               |
| 5x    | \(-2\)          | 0               |
| 5y    | 0               | -4              |
| 5z    | \(2 + \alpha\)  | 0               |
| 5aa   | \(-\alpha\)     | 0               |
| 5bb   | \(-1\)          | -2              |
| 5cc   | \(\alpha\)      | 0               |

*Table 5c: Contributions to \(Z^{0b}\) from Fig. 5(w)–5(cc)*

| Graph | \((a0)\) | \((00)\) |
|-------|----------|----------|
| \(g^2R^aR^0 + 2\frac{1}{N}\Delta R^aR^b\) | \(g^2 + \frac{1}{N}\Delta\) |
| 5y    | \(-2\)   | \(-4\)   |
| 5bb   | \(-1\)   | \(-2\)   |

*Table 5d: Contributions to \(Z^{a0}\) and \(Z^{00}\) from Fig. 5(y), 5(bb)*
The contributions to $Z_{a0}$ and $Z_{00}^0$ from Figs. 5(w)–5(cc) are shown in Table 5d (those not shown explicitly are zero). Adding the results from Table 5a in Eq. (A.12) to those from Tables 5b–5d, we obtain

$$\Gamma_{\text{pole}}^{(1)} = i\sqrt{2}NLC_{\mu\nu} \left[ g^A A^b_{\mu} \overline{\phi} \lambda^a \left[ \frac{1}{2} d^{abc} R^c - R^a R^b - \frac{1}{2} (7 + 3\alpha) R^b R^a \right] \bar{\sigma}_{\nu} \psi 
+ g^3 g_0 \left[ d^{0bc} R^c - \frac{1}{2} (3 + \alpha) \right] A^b_{\mu} \overline{\phi} \lambda^0 R^0 R^b \bar{\sigma}_{\nu} \psi 
- (3 + \alpha) g^3 g_0 A^0_{\mu} \overline{\phi} \lambda^a R^a R^0 \bar{\sigma}_{\nu} \psi 
- 2\frac{\alpha}{N^2} \hat{C}_2 A^A_{\mu} \overline{\phi} \lambda^B g_A g_B R^B R^A \bar{\sigma}_{\nu} \psi \right]. \quad (A.13)$$

The contributions from Fig. 6 are of the form

$$iLC_{\mu\nu} (g_A \partial_{\mu} A^A_{\nu} \overline{\phi} X R^A F + g_A A^A_{\mu} \partial_{\mu} \overline{\phi} Y R^A F) \quad (A.14)$$

where $X$ and $Y$ are given in Table 6:

| Graph | $X$ | $Y$ |
|-------|-----|-----|
| 6a    | 0   | $3N g^2 c^A$ |
| 6b    | 0   | $2[2\hat{C}_2 - Ng^2 c^A]$ |
| 6c    | $-4\hat{C}_2 - Ng^2 c^A$ | $-4\hat{C}_2 - Ng^2 c^A$ |
| 6d    | $-(5 + \alpha) Ng^2 c^A$ | 0 |
| 6e    | $2\alpha Ng^2 c^A$ | $-2Ng^2 c^A$ |

*Table 6: Contributions from Fig. 6*

The contributions in Table 6 add to

$$\Gamma_{\text{pole}}^{(1)} (g_A) \partial_{\mu} A^A_{\nu} \left[ 4\hat{C}_2 + (4 - \alpha) Ng^2 c^A \right] R^A F \quad (A.15)$$

The contributions from Fig. 7 are of the form

$$ig^2 LC_{\mu\nu} A^a_{\mu} A^b_{\nu} \overline{\phi} Z f^{abc} R^c F \quad (A.16)$$

where $Z$ is given in Table 7:

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| Graph | 7a | $-\frac{3}{4}\alpha Ng^2$ |
|-------|----|---------------------------|
| 7b    | 0  |                           |
| 7c    | 0  |                           |
| 7d    | 0  |                           |
| 7e    | $-\frac{1}{4}(2 + \alpha)Ng^2$ |
| 7f    | $2\hat{C}_2 - Ng^2$ |
| 7g    | $-\frac{3}{2}\alpha Ng^2$ |
| 7h    | $\frac{3}{2}(1 + \alpha)Ng^2$ |
| 7i    | $\frac{1}{4}(3 + \alpha)Ng^2$ |
| 7j    | $\frac{1}{2}\alpha Ng^2$ |
| 7k    | $-\frac{3}{4}\alpha Ng^2$ |
| 7l    | 0  |                           |

**Table 7:** Contributions from Fig. 7

| Graph | ab | a0 | 00 |
|-------|----|----|----|
| 8a    | 0  | 0  | 0  |
| 8b    | $-g^2\delta^{ab} - \frac{4}{N}\Delta R^a R^b$ | $-\sqrt{\frac{N}{2}} \left[g^2 + \frac{4}{N\pi}\Delta\right] R^a$ | $-2g^2 - \frac{2}{N\pi}\Delta$ |
| 8c    | $\frac{1}{2}g^2\delta^{ab} + \frac{1}{N}\Delta R^a R^b$ | $\frac{1}{4} \left(\frac{2}{N}\right)^\frac{3}{2} \Delta R^a$ | $\frac{1}{4}g^2 + \frac{1}{2N\pi}\Delta$ |
| 8d    | $-\alpha g^2 Nd^{abc} R^c$ | $-\alpha g^2 \sqrt{2N}$ | 0 |
| 8e    | $(1 + \alpha)g^2 Nd^{abc} R^c$ | $(1 + \alpha)g^2 \sqrt{2N}$ | 0 |
| 8f    | $-\frac{1}{2}\alpha g^2 Nd^{abc} R^c$ | $-\frac{1}{2}\alpha g^2 \sqrt{2N}$ | 0 |
| 8g    | 0  | 0  | 0  |
| 8h    | $\frac{1}{2}\alpha g^2 Nd^{abc} R^c$ | $\frac{1}{2}\alpha g^2 \sqrt{2N} R^a$ | 0 |
| 8i    | $\frac{1}{4}(3 + \alpha)g^2[\frac{1}{2}Nd^{abc} R^c + \delta^{ab}]$ | 0 | 0 |
| 8j    | $\frac{1}{5}\alpha g^2 Nd^{abc} R^c$ | $\frac{1}{5}\alpha g^2 \sqrt{2N} R^a$ | 0 |
| 8k    | $-\frac{1}{4}g^2\delta^{ab} - \frac{1}{N}\Delta R^a R^b$ | $-\frac{1}{4} \sqrt{\frac{N}{2}} \left[g^2 + \frac{4}{N\pi}\Delta\right] R^a$ | $-\frac{1}{2}g^2 - \frac{1}{2N\pi}\Delta$ |

**Table 8:** Contributions from Fig. 8
The contributions in Table 7 add to

$$\Gamma^{(1)\text{pole}}_{71\Pi} = ig^2 LC^{\mu\nu} A^a_\mu A^b_\nu \bar{\phi} \left[ 2\hat{C}_2 + \frac{1}{4}(3 - 4\alpha)Ng^2 \right] f^{abc} R^c F$$  \hspace{1cm} (A.17)$$

The contributions from Fig. 8 are of the form

$$L_{\alpha\beta\gamma\delta} |C|^2 \bar{\chi}^A \bar{\chi}^B \phi Z_{AB} F$$  \hspace{1cm} (A.18)$$

where the contributions to $Z$ are given in Table 8. The contributions in Table 8 add to

$$\Gamma^{(1)\text{pole}}_{81\Pi} = L |C|^2 \bar{\phi} \left[ g^2 \left[ \frac{1}{8}(43 + 2\alpha)Ng^2 - 8\hat{C}_2 \right] \bar{\chi}^a \bar{\chi}^b d^{abc} R^c \right.$$

$$+ gg_0 \left[ \frac{1}{4}(19 + \alpha)Ng^2 - 8\hat{C}_2 \right] \bar{\chi}^0 \bar{\chi}^b d^{0bc} R^c \right.$$  \hspace{1cm} (A.19)

$$+ \frac{1}{4}\alpha Ng^4 d^{0bc} \bar{\chi}^b \bar{\chi}^c - 4Ng_0^2 \hat{C}_2 d^{000} \bar{\chi}^0 \bar{\chi}^0 1 \} F.$$  \hspace{1cm} (A.19)$$

Appendix B. Group identities for $SU(N)$

The basic commutation relations for $SU(N)$ are (for the fundamental representation):

$$[R^a, R^b] = i f^{abc} R^c, \quad \{ R^a, R^b \} = d^{abc} R^c + \frac{1}{N} \delta^{ab},$$  \hspace{1cm} (B.1)$$

where $d^{abc}$ is totally symmetric. Defining matrices $F^a, D^a$ by $(F^a)^{bc} = f^{bac}, \ (D^a)^{bc} = d^{bac}$, useful identities for $SU(N)$ are

$$\text{Tr}[F^a F^b] = -N\delta^{ab}, \quad \text{Tr}[D^a D^b] = \frac{N^2 - 4}{N} \delta^{ab},$$

$$\text{Tr}[F^a F^b D^c] = -\frac{N}{2} d^{abc}, \quad \text{Tr}[F^a D^b D^c] = \frac{N^2 - 4}{2N} f^{abc},$$

$$C_2(R) = \frac{N^2 - 1}{2N},$$

$$\text{Tr}[F^a D^b F^c D^d] = \frac{N}{4}(d^{acx} d^{bdx} - d^{abx} d^{cdx} - d^{adx} d^{bcx}).$$

$$d^{acd} R^b R^c R^d = \frac{N^2 - 4}{2N} R^b R^a,$$

$$d^{ace} f^{bde} R^c R^d = i \left[ -\frac{1}{2}NR^a R^b + \frac{1}{N}[R^a, R^b] + \frac{1}{4}\delta^{ab} \right],$$

$$d^{ace} f^{bde} R^d R^c = i \left[ \frac{1}{2}NR^b R^a + \frac{1}{N}[R^a, R^b] - \frac{1}{4}\delta^{ab} \right],$$

$$d^{acd} R^c R^b R^d = -\frac{1}{N}[R^a, R^b] + \frac{1}{4}\delta^{ab},$$

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Fig. 1: Diagrams with one gauge, two gaugino lines; the dot represents the position of a $C$. 
Fig. 2: Diagrams with two gauge and two gaugino lines; the dot represents the position of a $C$. 
Fig. 3: Diagrams with four gaugino lines; the dot represents the position of a $C$ or $|C|^2$. 
Fig. 4: Diagrams with one gaugino, one scalar and one chiral fermion line; the dot represents the position of a $C$. 
Fig. 5: Diagrams with one gaugino, one scalar, one chiral fermion and one gauge line; the dot represents the position of a $C$. 
Fig. 5(ctd): Diagrams with one gaugino, one scalar, one chiral fermion and one gauge line; the dot represents the position of a $C$. 
**Fig. 5b**: Diagrams with one gaugino, one scalar, one chiral fermion and one gauge line contributing an explicit $d^{abc}$; the dot represents the position of a $C$.

**Fig. 6**: Diagrams with one gauge, one scalar and one auxiliary line; the dot represents the position of a $C$. 
Fig. 7: Diagrams with two gauge, one scalar and one auxiliary line; the dot represents the position of a C.
Fig. 8: Diagrams with two gaugino, one scalar and one auxiliary line; the dot represents the position of a $C$ or a $|C|^2$.

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