Analysis and Suppression of Phase Unbalance Induced Force Ripples in Permanent Magnet Linear Motors

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ABSTRACT Low force ripples from permanent magnet linear motors (PMLMs) are typically essential requirements in a wide range of demanding control applications with high motion quality. Apart from the sources present in rotary PM machines, phase unbalance due to the end effect is another major contributor of the thrust force ripples in the PMLMs. In this paper, the phase unbalance and induced force ripples are investigated in the case of a planar PMLM typically equipped with single-layer concentrated windings and quasi-Halbach magnets. It demonstrates that although the PMLM is quite carefully deigned to minimize the end effect, the phase unbalance still exists, including both the amplitude and phase position unbalances. And the phase unbalances give rise to even order force ripple harmonics, in which the 2\textsuperscript{nd}-order force ripple is particularly significant. In order to compensate the end effect and the unbalance between the three phases, a novel PMLM topology is presented, in which the width of the two end teeth (instead of the edge teeth) is adjusted to reduce the force ripple. And the impact of the end teeth width on the ripple force magnitude is further revealed. It is shown that by proper design of the end teeth width, the 2\textsuperscript{nd}-order force ripple of the PMLM can be minimized to quite a low extent. In comparison with the original design, the 2\textsuperscript{nd}-order force ripple factor of the novel topology has been suppressed from 0.4\% to less than 0.03\% in the case study. Such a technique has provided a new way of force ripple minimization, and is applicable to other types of linear motors or actuators with appropriate changes.

INDEX TERMS End effect, force ripple, permanent magnet linear motor, phase unbalance.

I. INTRODUCTION

Permanent magnet linear motors (PMLMs) have increasingly been used in a wide variety of applications due mainly to their excellent dynamic characteristics and accurate positioning capability [1]–[5]. The most notable feature is the absence of mechanical transmission components, which makes a linear motion system simpler and more reliable, since frictional wear occurs only in linear bearings and the risk to jamming in rotary to linear transmission is greatly eliminated. However, in comparison with their rotary counterparts, the PMLMs generally suffer from considerably serious force ripples, which deteriorate their controllability and compromise their position- and speed-control accuracy. Since force smoothness is an essential requirement in a wide range of high-performance motion control applications, the minimization of force ripples in PMLMs is of great importance and remains one of the major design challenges, especially for applications in which force smoothness is crucial [6]–[10].

In general, various causes are responsible for force ripple generation in PMLMs, including cogging force, EMF harmonics under balanced condition, phase unbalance, magnetic saturation, and controller imperfection. Besides the sources that are also present in rotary PM machines, the phase unbalance due to the end effect is an important contributor of force ripples in PMLMs. Given the fact that the techniques for reducing the force ripples due to other sources have been well addressed [11]–[15], the scope of this paper is restricted...
within the analysis and suppression of the phase unbalance induced thrust force ripples.

In literature, few techniques and topologies have been presented for minimizing the force ripples due to phase unbalance in PMLMs [16]–[18]. Although certain active-control based strategies can be employed by means of suitable control of current waveforms at the expense of increased control complexity, it is generally desirable to eliminate the force ripples from the source, i.e. by appropriate design of the PMLM itself. In [16], two auxiliary teeth are introduced to minimize the phase unbalance. A new design of winding and mover arrangement is proposed in a doubly salient PMLM in [17] to reduce the magnetic unbalance and force ripple. However, it should be noted that both the designs in [16] and [17] still suffer from considerable asymmetries between the three phases, and furthermore, if not properly dimensioned, they would result in deteriorative end-effect cogging forces. In [18], dispersed armature-cores are adopted in the primary, and thus increased number of ends are exhibited to overcome the longitudinal end effect, but at the cost of decreased force density and more complex mechanical construction.

On the whole, almost all methods in literature are focusing on the edge-teeth structure optimization to deal with the end effect and phase unbalance. In these methods, either the specific dimensions are adjusted to achieve the appropriate armature length (so as to minimize the end-effect cogging force), or the shape of the edge teeth is optimized to slow down the change of the magnetic reluctance at the cut-off edges (so as to reduce the end effect). However, it should be noted that as the end effect cogging force is highly sensitive to the edge teeth (both the dimension and the shape), those methods in literature would makes the two negative impacts coupled. Particularly for the first case (i.e., adjustment of edge teeth dimensions), it is quite hard to balance the end-effect cogging force and force ripple.

This paper analyzes the force ripples due to phase unbalance in PMLMs, with particular reference to the 2nd-order force ripple that is caused by the unbalances of the fundamental and 3rd-order electromotive forces (EMFs) in the three phase windings. A novel armature-core magnetic design with unequal tooth width is presented, and the influence of the tooth width adjustment on the 2nd-order force ripple is also investigated.

The innovation of the paper lies that a novel topology of the PMLM has been proposed to elimination the 2nd-order force ripple due to phase unbalance. The topology has focused on the end teeth, instead of the edge teeth in literature. By proper adjustment of the end teeth, the 2nd-order force ripple has been minimized to quite a low level. Since it does not change the armature axial length and slot-open width, the cogging force (both the end-effect component and tooth-ripple component) remains the same. In this sense, we can say that the reduction of force ripple and cogging force is decoupled, and they can be minimized with separate methods and will not affect each other.

II. PHASE UNBALANCE AND FORCE RIPPLE: GENERAL ANALYSIS

Fig. 1 illustrates the typical schematic structure of a PMLM with cogging-force-minimized design. It employs fractional-slot-per-pole (typically in this case, 14-pole/12-slot combination) and single-layer concentrated winding design to reduce the tooth-ripple cogging force, and adopts appropriately armature-core length to suppress the end-effect cogging force, and selects quasi-Halbach magnetized magnets to achieve sinusoidal air-gap flux density distribution. By the combinations of these techniques, the resultant cogging force of the PMLM can be suppressed to quite a low level, and could be neglected in most cases [19]. However, as an inherent characteristic in the PMLM, due to the end effect associated with the finite length of the armature, the magnetic circuit seen by each phase in the PMLM is not symmetric. This leads to unbalance in the EMF waveform, and in turn, would give rise to unexpected thrust force ripples.

Without loss of generality, under unbalanced condition, the EMFs in the three phases can be expressed as the followings by taking phase C as reference

$$
e_A = \sum_{n=1,3,5,...}^\infty \sqrt{2}(E_n + \Delta E_{na}) \cdot \cos \left[ n \left( \frac{\pi v t}{p} - \frac{2\pi}{3} \right) + \phi_n + \Delta \phi_{na} \right]
$$

$$
e_B = \sum_{n=1,3,5,...}^\infty \sqrt{2}(E_n + \Delta E_{nb}) \cdot \cos \left[ n \left( \frac{\pi v t}{p} + \frac{2\pi}{3} \right) + \phi_n + \Delta \phi_{nb} \right]
$$

$$
e_C = \sum_{n=1,3,5,...}^\infty \sqrt{2}E_n \cdot \cos \left[ n \left( \frac{\pi v t}{p} \right) + \phi_n \right]
$$

where $p$ is the pole pitch, $v$ is the velocity, $E_n$ and $\phi_n$ are the RMS value and phase position of the $n^{th}$-order EMF harmonics of phase C, respectively, and $\Delta E_{na}$, $\Delta E_{nb}$, and $\Delta \phi_{na}$, $\Delta \phi_{nb}$ are the RMS unbalances and phase position unbalances in the $n^{th}$-order EMF harmonics of phases A and B, respectively.
Under balanced sinusoidal excitations, i.e.,

\[
\begin{align*}
i_a &= \sqrt{2}I \cdot \cos \left( \frac{\pi}{\tau_p}vt + \varphi_1 - \frac{2\pi}{3} \right) \\
i_b &= \sqrt{2}I \cdot \cos \left( \frac{\pi}{\tau_p}vt + \varphi_1 + \frac{2\pi}{3} \right) \\
i_c &= \sqrt{2}I \cdot \cos \left( \frac{\pi}{\tau_p}vt + \varphi_1 \right)
\end{align*}
\]  

where \( I \) is the RMS value of the current, the thrust force can be calculated by

\[
F_{em}(t) = \frac{1}{V} (e_a i_a + e_b i_b + e_c i_c) = F_0 + F_2 + F_4 + F_6 + \cdots
\]

where \( F_0, F_2, F_4 \) and \( F_6 \) are the DC, 2\(^{nd}\)-, 4\(^{th}\)- and 6\(^{th}\)-order components of the thrust force \( F_{em} \), respectively. More specifically, \( F_0 \) and \( F_{2k} \) are expressed by

\[
F_0 = \frac{I}{V} \sum_{i=a,b,c} (E_i + \Delta E_i) \cdot \cos \Delta \varphi_i
\]

\[
F_{2k} = \frac{I}{V} \left[ (E_{2k-1} + \Delta E_{2k-1}) \cdot \cos \left( \frac{2k \pi}{\tau_p}vt + \varphi_{2k-1} + \Delta \varphi_{2k-1} - \frac{4k \pi}{3} \right) + (E_{2k-1} + \Delta E_{2k-1}) \cdot \cos \left( \frac{2k \pi}{\tau_p}vt + \varphi_{2k-1} + \Delta \varphi_{2k-1} + \varphi_1 - \frac{4k \pi}{3} \right) \\
+ E_{2k-1} \cdot \cos \left( \frac{2k \pi}{\tau_p}vt + \varphi_{2k-1} + \varphi_1 \right) + (E_{2k+1} + \Delta E_{2k+1}) \cdot \cos \left( \frac{2k \pi}{\tau_p}vt + \varphi_{2k+1} + \Delta \varphi_{2k+1} - \varphi_1 - \frac{4k \pi}{3} \right) + (E_{2k+1} + \Delta E_{2k+1}) \cdot \cos \left( \frac{2k \pi}{\tau_p}vt + \varphi_{2k+1} + \Delta \varphi_{2k+1} + \varphi_1 + \frac{4k \pi}{3} \right) \\
+ E_{2k+1} \cdot \cos \left( \frac{2k \pi}{\tau_p}vt + \varphi_{2k+1} - \varphi_1 \right) \right] \\
for k = 1, 2, 3, \ldots.
\]

It shows that both the magnitude and phase position unbalances in each order of the EMF harmonics will give rises to even order force ripples, which are absent in rotary PM machines. The interaction of the fundamental excitation currents with the unbalanced \((2k - 1)^{th}\) and \((2k + 1)^{th}\) EMF harmonics \((k = 1, 2, 3, \ldots)\) will produce a \((2k)^{th}\)-order force ripple harmonic.

By way of example, Fig. 2 shows the measured ripple force of a planar PMLM (with the structure shown in Fig. 1) under sinusoidal excitations that is designed for specifically active vibration control applications, where force-ripple-free operation is highly appreciated.

The prototype is shown in Fig. 3, and it has integrated the proof-mass and rubber springs in it so as to achieve the reaction force actuation [20]. Its primary design parameters are listed in Table 1. Further, Table 2 gives the magnitude and phase position unbalances of the fundamental, 3\(^{rd}\)-, and 5\(^{th}\)-order EMFs of the three phases.

It can be seen that in the PMLM, the ripple force harmonics of 2\(^{nd}\)-, 6\(^{th}\)- and 12\(^{th}\)-orders have relatively higher magnitudes. The 6\(^{th}\)- and 12\(^{th}\)-order harmonics are mainly due to the 5\(^{th}\)-, 7\(^{th}\)-, 11\(^{th}\) and 13\(^{th}\) EMF harmonics, which are the same as in rotary PM motors. In particular, due to the unbalance of the fundamental and 3\(^{rd}\)-order harmonic components of the EMFs, the 2\(^{nd}\)-order force ripple in the PMLM becomes quite higher. This is of particularly importance, as the 3\(^{rd}\) EMF harmonic does not contribute to force ripples in a balanced 3-phase system. Furthermore, as the frequency of the force ripple becomes lower, the filtering effect of the mover mass on the force ripples tends to be weaker, which may lead to increased velocity fluctuations. Therefore, additional counter-measures must be taken in PMLMs to minimize the phase unbalance and induced 2\(^{nd}\)-order force ripples.

III. ANALYSIS OF 2\(^{nd}\)-ORDER FORCE RIPPLE

As highlighted, the 2\(^{nd}\)-order force ripple is unique in the PMLM. And this section focuses on the analysis of the
production mechanism of the 2\textsuperscript{nd}-order force ripple, giving a simple expression of the function between the 2\textsuperscript{nd}-order force ripple and the magnitude and phase unbalances in the fundamental and 3\textsuperscript{rd}-order EMFs of the three phases.

From (4) and (5), the normalized 2\textsuperscript{nd}-order force ripple with respect to the DC component can be expressed as

\[ k = \frac{F_2}{F_0} = k_1 + k_2 + k_3 + k_4 \]  

(6)

where \( k_1, k_2, k_3, \) and \( k_4 \) are the 2\textsuperscript{nd}-order force ripple components caused by the magnitude unbalance of the fundamental EMF, phase unbalance of the fundamental EMF, magnitude unbalance of the 3\textsuperscript{rd}-order EMF, and phase unbalance of the 3\textsuperscript{rd}-order EMF, respectively. The derivations of the expressions of \( k_1 \) and \( k_2 \) are shown in the Appendix C, and the expressions of \( k_3 \) and \( k_4 \) can be obtained in the similar way.

They are given by

\[
k_1 = \frac{\Delta E_{1a} \cos \left( 2\frac{\pi}{t_p} \cdot \tau_v + 2\varphi_1 - \frac{4}{3}\pi \right)}{3E_1} + \frac{\Delta E_{1b} \cos \left( 2\frac{\pi}{t_p} \cdot \tau_v + 2\varphi_1 + \frac{4}{3}\pi \right)}{3E_1}
\]

(7)

\[
k_2 = -\frac{\sin \Delta \varphi_{1a} \sin \left( 2\frac{\pi}{t_p} \cdot \tau_v + 2\varphi_1 - \frac{4}{3}\pi \right)}{3} - \frac{\sin \Delta \varphi_{1b} \sin \left( 2\frac{\pi}{t_p} \cdot \tau_v + 2\varphi_1 + \frac{4}{3}\pi \right)}{3}
\]

(8)

\[
k_3 = \frac{\Delta E_{3a} \cos \left( 2\frac{\pi}{t_p} \cdot \tau_v + \varphi_3 - \varphi_1 + \frac{2}{3}\pi \right)}{3E_1} + \frac{\Delta E_{3b} \cos \left( 2\frac{\pi}{t_p} \cdot \tau_v + \varphi_3 - \varphi_1 - \frac{2}{3}\pi \right)}{3E_1}
\]

(9)

\[
k_4 = -\frac{E_3 \sin \Delta \varphi_{3a} \sin \left( 2\frac{\pi}{t_p} \cdot \tau_v + \varphi_3 - \varphi_1 + \frac{2}{3}\pi \right)}{3E_1} - \frac{E_3 \sin \Delta \varphi_{3b} \sin \left( 2\frac{\pi}{t_p} \cdot \tau_v + \varphi_3 - \varphi_1 - \frac{2}{3}\pi \right)}{3E_1}
\]

(10)

It should be noted that apart from the four 2\textsuperscript{nd}-order force ripple components above, there exists other terms that are generated by the magnitude unbalances and phase unbalances in the fundamental and 3\textsuperscript{rd}-order EMF harmonics. However, as they are much smaller in comparison to the peak values of \( k_1, k_2, k_3 \) and \( k_4 \), the force ripple factor caused by them are quite small, and thus neglected in the subsequent analysis.

For the PMLM shown in Fig. 1, because of its symmetric structure, the following relationships exist.

\[
\Delta E_{1a} = \Delta E_{1b} \text{ def} \Delta E_1 \tag{11}
\]

\[
\Delta \varphi_{1a} = -\Delta \varphi_{1b} \text{ def} \Delta \varphi_1 \tag{12}
\]

\[
\Delta E_{3a} = \Delta E_{3b} \text{ def} \Delta E_3 \tag{13}
\]

\[
\Delta \varphi_{3a} = -\Delta \varphi_{3b} \text{ def} \Delta \varphi_3 \tag{14}
\]

Further, from the fundamental and 3\textsuperscript{rd}-order EMF harmonics of phase C, it can be shown that the initial phase angle of the 3\textsuperscript{rd}-order harmonic is related to that of the fundamental by:

\[
\varphi_3 - 3\varphi_1 = (2k + 1) \pi \text{ for } k \text{ is an integer} \tag{15}
\]

Therefore, the expression of the 2\textsuperscript{nd}-order force ripple factors can be simplified as

\[
k_1 = -\frac{\Delta E_1}{3E_1} \cdot \cos \left( 2\frac{\pi}{t_p} \cdot \tau_v + 2\varphi_1 \right)
\]

\[
= k'_1 \cos \left( 2\frac{\pi}{t_p} \cdot \tau_v + 2\varphi_1 \right) \tag{16}
\]

\[
k_2 = -\frac{\sin \Delta \varphi_1}{\sqrt{3}} \cdot \cos \left( 2\frac{\pi}{t_p} \cdot \tau_v + 2\varphi_1 \right)
\]

\[
= k'_2 \cos \left( 2\frac{\pi}{t_p} \cdot \tau_v + 2\varphi_1 \right) \tag{17}
\]
TABLE 2. Amplitude and phase position unbalances of the fundamental, 3rd-, and 5th-order EMFs in three phases (taking phase c as reference).

|             | Fundamental EMF | 3rd-order EMF | 5th-order EMF | 7th-order EMF |
|-------------|-----------------|---------------|---------------|---------------|
| Reference   | RMS (V)         | RMS (V)       | RMS (V)       | RMS (V)       |
| Phase C     | E₁             | E₁           | E₁            | E₁            |
| Unbalances  | 133.86         | -48.48       | 5.92          | 34.54         |
| Phase A     | ∆E₁a, ∆φ₁a     | ∆E₁b, ∆φ₁b   | ∆E₁a, ∆φ₁a   | ∆E₁a, ∆φ₁a   |
| Unbalances  | 0.15           | 0.06         | -0.21         | -5.11         |
| Phase B     | ∆E₁b, ∆φ₁b     | ∆E₁b, ∆φ₁b   | ∆E₁b, ∆φ₁b   | ∆E₁b, ∆φ₁b   |
| Unbalances  | 0.15           | -0.06        | -0.22         | 5.12          |

TABLE 3. Peak values of the 2nd-order force ripple factors.

| k₁p | k₂p | k₃p | k₄p | k₅p |
|-----|-----|-----|-----|-----|
| 0.04% | 0.06% | 0.05% | 0.23% | 0.38% |

FIGURE 4. Optimization of edge-tooth structure to deal with end effect.

\[
k_3 = \frac{\Delta E_3}{3E_1} \cdot \cos(2\pi vt + 2\varphi_1)
\]

\[
k_4 = \frac{\sin \Delta \varphi_3 \cdot E_3 / E_1}{\sqrt{3}} \cdot \cos\left(\frac{2\pi vt + 2\varphi_1}{\tau_p}\right)
\]

\[
k = k_1 + k_2 + k_3 + k_4
\]

Using the results in Table 2, the peak value of the resultant 2nd-order force ripple factor, as well as its four components, can be obtained, and listed in Table 3.

It can be seen that for the design in Fig. 1 and Fig. 3, the phase unbalance in the 3rd-order EMFs has contributed to the most significant component in the 2nd-order force ripple.

IV. MINIMIZATION OF 2ND-ORDER FORCE RIPPLE

By examining the sources of 2nd-order force ripple in PMLMs, this section proposes a design technique for minimization of the 2nd-order force ripple due to phase unbalance. As highlighted above, the unbalance in the fundamental and 3rd harmonic components of the EMFs has contributed to large 2nd-order thrust force ripples in the PMLM. The most significant source is the phase unbalance in the 3rd-order EMFs. In order to deal with the end effect, the edge tooth structure is generally optimized in literature, as shown in Fig. 4. However, as the end-effect cogging force is highly sensitive to the edge teeth (both the dimension and the shape), these methods make the end-effect cogging force and force ripple coupled in some way. Particularly for the case when the edge-tooth dimensions are adjusted, it is quite hard to balance the two negative impacts.

Based on the facts above, the physical nature in the PMLM gives us the inspiration that why don’t we optimize the end teeth, instead of the edge teeth. Therefore, a novel armature-core magnetic design is proposed, as shown in Fig. 5. In this topology, the width of the two end teeth has been adjusted to reduce the phase unbalance, and hence the force ripple, where \( w_t \) is the tooth width in usual design and \( \Delta \) refers to the adjustment.

By adjusting the end-tooth width, both the magnitudes and phase positions of the EMFs in phase A and B would be changed accordingly. And it is hopeful that with proper end-tooth width values, the three phase can achieve a new “quasi-balance”, thus leading to minimized force ripple.

Fig. 6 shows the variations of \( k'_1, k'_2, k'_3 \) and \( k'_4 \) with the tooth width adjustment \( \Delta \), with the primary machine parameters listed in Table 1. As will be seen, although the motive of this design is to reduce the phase unbalance in the 3rd-order EMFs, the change of \( \Delta \) would lead to changes in all the four force ripple factors. And among them, \( k'_2 \), i.e., the 2nd-order force ripple factor due to the phase unbalance in the fundamental EMFs, is the most sensitive to \( \Delta \), while \( k'_3 \), i.e., the factor due to the magnitude unbalance in the 3rd-order EMFs, is the least sensitive to \( \Delta \).

Fig. 7 shows the variation of the resultant 2nd-order force ripple factor with the tooth width adjustment \( \Delta \).
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It shows that by optimizing the end teeth width, the resultant 2nd-order force ripple due to the phase unbalance can be minimized to almost 0. In this case, in comparison with the original design, the 2nd-order force ripple factor of the novel topology has been suppressed from 0.4% to less than 0.03%. It should be noted that since the optimal Δ is very small, it will not affect the tooth tip design and hence the cogging force.

V. ADDITIONAL CONSIDERATIONS

The aforementioned analysis is mainly focused on the PMLM with the topology of fractional-slot-per-pole and single-layer concentrated winding design. One may wonder if this method, i.e., the novel topology with end-teeth width adjustment, apply to other linear motors with different winding types or slot-pole combinations.

As a linear electrical machine, it is all the same that there exists the cut-off, either at the primary or the second side, no matter the machine is of planar or tubular type, and no matter what the slot-pole combination or the winding type are. And therefore, there are always one or two phase windings which lie at the longitudinal ends, and the corresponding windings would be affected more seriously by end effect than other windings. In the same way, we can properly adjust the width of the end teeth which correspond to those phase windings. And the performance can also be improved in a similar manner, although the effectiveness may not be as good as that in this manuscript.

Here, we take a linear motor with double-layer concentrated windings for example, and the schematic structure is illustrated in Fig. 8. The main difference of this motor topology from the one in the manuscript (shown in Fig. 1) lies at the structure of the primary armature core. In order to cope with the force ripple related to the end effect, we can adjust the dimensions of the two end teeth in the same manner, as that with single-layer windings. And we have also investigated the topology, and have confirmed the effectiveness.

In this sense, this method can be regarded of a general strategy in some way, to eliminate the force-unbalance-induced 2nd-order force ripples, and be extended to other types of linear motors or actuators with different slot-pole combination the winding types, with appropriate changes.

VI. CONCLUSION

This paper has focused on the analysis and minimization of force ripples due to phase unbalance in PMLMs, with particular reference to the 2nd-order force ripple. The following key observations can be drawn:

(1) Phase unbalance is another major contributor of the force ripples in PMLMs besides those present in rotary machines. Particularly, the unbalance in the fundamental and 3rd-order EMFs leads to large 2nd-order force ripples, which do not exist in rotary machines.

(2) In the PMLM with single-layer windings and equal tooth width, the phase unbalance in the 3rd-order EMFs contributes to the most significant component in the 2nd-order force ripple.

(3) The 2nd-order force ripple due to phase unbalance can be effectively suppressed by appropriate design of the end teeth width.

APPENDIX A

Definition of $F_0$

$$F_0 = \frac{3E_1I}{v}$$
APPENDIX B

Definition of $F_{2-1}$

$$F_{2-1} = \frac{1}{V} \left[ (E_1 + \Delta E_{1a}) \cdot \cos \left( \frac{2\pi}{\tau_p} vt + 2\varphi_1 + \Delta \varphi_{1a} - \frac{4\pi}{3} \right) ight. 
+ (E_1 + \Delta E_{1b}) \cdot \cos \left( \frac{2\pi}{\tau_p} vt + 2\varphi_1 + \Delta \varphi_{1b} + \frac{4\pi}{3} \right) 
\left. + E_1 \cdot \cos \left( \frac{2\pi}{\tau_p} vt + 2\varphi_1 \right) \right]$$

$$\approx \frac{1}{V} \left[ \Delta E_{1a} \cos \left( \frac{2\pi}{\tau_p} vt + 2\varphi_1 + \frac{4\pi}{3} \right) 
+ \Delta E_{1b} \cos \left( \frac{2\pi}{\tau_p} vt + 2\varphi_1 + \frac{4\pi}{3} \right) 
- E_1 \sin \Delta \varphi_{1a} \sin \left( \frac{2\pi}{\tau_p} vt + 2\varphi_1 - \frac{4\pi}{3} \right) 
- E_1 \sin \Delta \varphi_{1b} \sin \left( \frac{2\pi}{\tau_p} vt + 2\varphi_1 + \frac{4\pi}{3} \right) 
- \Delta E_{1a} \sin \Delta \varphi_{1a} \sin \left( \frac{2\pi}{\tau_p} vt + 2\varphi_1 - \frac{4\pi}{3} \right) 
- \Delta E_{1b} \sin \Delta \varphi_{1b} \sin \left( \frac{2\pi}{\tau_p} vt + 2\varphi_1 + \frac{4\pi}{3} \right) \right]$$

where

$$k_1 = \frac{\Delta E_{1a} \cos \left( \frac{2\pi}{\tau_p} vt + 2\varphi_1 - \frac{4\pi}{3} \right)}{3E_1} + \frac{\Delta E_{1b} \cos \left( \frac{2\pi}{\tau_p} vt + 2\varphi_1 + \frac{4\pi}{3} \right)}{3E_1}$$

$$k_2 = \frac{-\sin \Delta \varphi_{1a} \sin \left( \frac{2\pi}{\tau_p} vt + 2\varphi_1 - \frac{4\pi}{3} \right)}{3} - \frac{\sin \Delta \varphi_{1b} \sin \left( \frac{2\pi}{\tau_p} vt + 2\varphi_1 + \frac{4\pi}{3} \right)}{3}$$

APPENDIX C

Definition of $\frac{F_{2-1}}{F_0}$

$$\frac{F_{2-1}}{F_0} = k_1 + k_2$$

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