Sensor positioning in the context of wave-based damage identification in dams

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Dams and their adjoining reservoirs play an important role in the drinking water supply as well as in the flood defense. Their world-wide use and possible massive devastation due to dam failures make the monitoring of the structural health of these huge structures imperative. Therefore, it is necessary to identify faults/damages in the dams’ structure non-invasively in order to proceed with the current use. Additionally, monitoring of these huge structures is very complex, time-consuming and costly. To circumvent the aforementioned challenges, an inverse analysis, here an enhanced, regularized, cyclic full-waveform inversion is proposed based on wavelengths between seismic and ultrasonic waves. Parameters of the dam material, such as stiffness, density and permeability, are estimated in synthetic experiments. Having this, an optimal design of experiment approach is used to identify the most informative sensor positions regarding the reconstruction quality. An optimized sensor layout may further improve the efficiency of the solution of the inverse problem.

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1 Introduction

For the identification of damages in dams the full waveform inversion (FWI) became an advantageous tool. The implementation of frequency cycles for the probing waves is increasing the quality of the results by performing a regularization. There is more potential for enhancement when looking at the optimal sensor positions. Therefore, the FWI is improved by calculating the optimal positions for sources and receivers.

In the scope of this paper a simplified dam is modeled and the FWI is used on artificially generated measurement data. Additionally, the sensor positions are optimized by evaluating the root-mean-squared error between the data from the numerical model and the synthetic measurement data.

2 Methods

2.1 Full Waveform Inversion (FWI)

The FWI is gaining information about material parameters by probing the material with seismic waves. In this case the body waves, divided into primary wave (P-wave velocity $v_p$) and secondary wave (S-wave velocity $v_s$), are considered in the two-dimensional case ($x$-$z$-plane) with the elastic wave equation [2]

\begin{equation}
\rho(x,z) \frac{\partial v_i}{\partial t} = \frac{\partial \sigma_{ij}}{\partial x_j} + f_i, \quad i = x, z, \quad j = x, z + \text{boundary conditions and initial conditions},
\end{equation}

where $\rho(x,z)$ is the density, $v_i$ are the particle velocities, $t$ stands for time, $\sigma_{ij}$ denotes the stress tensor components, $f_i$ are the directed body forces, $\lambda(x,z)$ and $\mu(x,z)$ are the Lamé parameters and $\delta_{ij}$ is known as Kronecker’s delta. The FWI is solving an inverse problem by minimizing the residual energy between the experimental and the numerical data [6]. Finite differences are used to solve the problem on a discretized domain. The three parameters $\rho$, $v_p$, and $v_s$ form the FWI model parameters $m \in (\mathbb{R}^n)^3$, where $n$ is the number of grid points for which the parameters are found in a discretized manner.

2.2 Design of Experiments (DoE)

Since the model parameters identified by the FWI depend on the positions of the sources and receivers, which were used during the process, the optimal experimental design can be calculated based on the FWI results [1, 4, 5]. Therefore, the root-mean-squared error (RMSE)

\begin{equation}
\text{RMSE}(x) = \|m(x) - m^*\|_2
\end{equation}

is used to solve the problem on a discretized domain. The three parameters $\rho$, $\lambda$, and $\mu$ are used to solve the problem on a discretized domain. The three parameters $\rho$, $\lambda$, and $\mu$ are used to solve the problem on a discretized domain. The three parameters $\rho$, $\lambda$, and $\mu$ are used to solve the problem on a discretized domain. The three parameters $\rho$, $\lambda$, and $\mu$ are used to solve the problem on a discretized domain. The three parameters $\rho$, $\lambda$, and $\mu$ are used to solve the problem on a discretized domain. The three parameters $\rho$, $\lambda$, and $\mu$ are used to solve the problem on a discretized domain. The three parameters $\rho$, $\lambda$, and $\mu$ are used to solve the problem on a discretized domain. The three parameters $\rho$, $\lambda$, and $\mu$ are used to solve the problem on a discretized domain. The three parameters $\rho$, $\lambda$, and $\mu$ are used to solve the problem on a discretized domain. 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between the reconstructed parameters \( m(x) \) depending on the sensor positions \( x \) and the reference parameter values \( m^* \) from the artificial true model is used as the objective function. When the RMSE is minimal the optimal sensor positions for the sources and receivers are found.

### 3 Application

A simplified model of a dam structure as shown in Fig. 1 serves as the application example. The FWI problem is solved in a two-dimensional time domain finite-differences scheme. The grid size is 21 times 16, which results in 1008 degrees of freedom considering the three model parameters \( \rho, v_p, \) and \( v_s \). For the time stepping an interval of \( 1 \times 10^{-7} \) s is used.

The FWI was performed using the program DENISE [2,3] and for the OED the genetic algorithm incorporated in MATLAB was used. Additionally, the \( x \)-coordinates of the sources \( x_S = 70 \text{ mm} \) and the receivers \( x_R = 170 \text{ mm} \) were fixed. The \( \text{RMSE}(v_s) \) was calculated for the area of interest as depicted in Fig. 1 and used for the objective function. The initial sensor setup for the OED is depicted in the third column of Table 1. The results for the parameter \( v_s \) and the respective values of \( \text{RMSE}(v_s) \) are summarized in Table 1.

**Table 1:** FWI results for S-wave velocity \( v_s \) depending on the initial model and the sensor positions (sources \( \ast \) and receivers \( \odot \))

| True model                                      | Initial model with intact dam |
|------------------------------------------------|-----------------------------|
| Receivers at top                               | Initial sensor positions    |
| RMSE\((v_s) = 2.22\)                            | RMSE\((v_s) = 3.52\)        |
| RMSE\((v_s) = 1.12\)                            |                             |

### 4 Discussion and Conclusion

In the shown application example the quality of the results is most sensitive to the S-wave velocity, s.t. \( \text{RMSE}(v_s) \) was used for the objective function. With the optimal sensor positions the fault area can be detected, but for non-optimal sensor placement, e.g. receivers at top, false damage detection might occur, which needs to be prevented. Optimal experimental design is improving the quality of the results of the FWI. Open topics that still need to be addressed are the sensor dependency on the location of the damaged area, consideration of more complex damaged forms and sensor clustering.

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