Time Evolution of Hadronization and Grey Tracks in DIS off Nuclei

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Abstract

The analysis of the grey tracks produced in Deep Inelastic Scattering (DIS) off nuclei provides important information on the space-time development of hadronization in nuclear medium. This method is complementary to the measurement of nuclear attenuation of leading inclusive hadrons. While the latter is focused on the hadronization dynamics for the quite rare process of leading hadrons production, the former covers the main bulk of inelastic events, and its $Q^2$ dependence is a very sensitive tool to discriminate between different models of hadronization. Employing the model of perturbative hadronization developed earlier, we calculate the $Q^2$ and $x_{Bj}$ dependences of the number of collisions and relate it to the mean number of grey tracks, using an empirical relation obtained from the analysis of data from the Fermilab E665 experiment on DIS of muons off the $Xe$ nucleus. We found the number of grey tracks to rise steeply with $Q^2$ in good agreement with the experimental data.
Nuclear targets serve as a natural and unique analyzer of the space-time development of strong interactions at high energies \[1, 2\]. Due to Lorentz time dilation, the projectile partons may keep their coherence for some time, but once they become incoherent, the cross section of the final state interaction (FSI) increases and the nucleus may act as a filter of hadronization mechanisms, provided quantities which are sensitive to the nuclear modifications of FSI are measured.

One way for experimental testing of various theoretical models which have been proposed to investigate the space-time evolution of hadronization, is the measurement of the nuclear modification factor in inclusive production of leading hadrons \[3, 4, 5\]. Recent experimental data from the HERMES experiment at HERA \[5\] have confirmed the predictions made in \[1\], based on the perturbative gluon radiation approach. Attenuation of the leading hadron is partially ascribed to the FSI of the pre-hadron (a colorless $\bar{q}q$ dipole prior formation of the hadronic wave function) produced in the color neutralization of the radiating quark which was created in the hard $\gamma^*\text{-nucleon}$ interaction. However, other approaches for in-medium hadronization have recently been employed, which are also able to provide a reasonable explanation of the HERMES data. We mean, in particular, the approach of Ref. \[6\], in which nuclear attenuation is explained in terms of the induced energy loss scenario, in which the hit parton loses energy by hard multiple scattering with other nucleon’s partons, and the approaches of Ref. \[7, 8, 9\], based upon the color string model. Thus it appears that other, more selective phenomena which could discriminate various models of hadronization are required. In this respect, the investigation of the $Q^2$ dependence of nuclear effects would probably be one of the best candidates \[^1\]. As a matter of fact, recent data from HERMES are in a good agreement with calculations performed in \[2\], but in strict contradiction with the $Q^2$ dependence predicted in \[6\]. Unfortunately the HERMES data exhibit only a mild $Q^2$ dependence and therefore could not be very selective for other hadronization models.

Note that production of leading hadrons with large fraction $z_h$ of the initial parton momentum is a rare, nearly exclusive process, which has quite a specific time development. In the main bulk of events, the jet energy is shared by many hadrons and therefore in this case it is a difficult task to find observables which are sensitive to the time development of hadronization. In Ref. \[10\] it has been proposed as a possible candidate the DIS exclusive process $A(e, e'B)X$, where $B$ is a detected recoil nucleus with a less number of nucleons than the target one \[11\], and $X$, as usual, denotes the unobserved jets of hadrons resulting from hadronization. It has indeed been shown \[10\] that the survival probability of the recoil nucleus $B$ in such a process is rather sensitive to the time evolution of hadronization and, particularly, to its $Q^2$ dependence. This quantity, which is quite difficult to access experimentally in case of heavy nuclei, could be investigate using few-body nuclei. Indeed, first experimental data were recently obtained at Jefferson Lab \[12\] for a deuteron target, i.e. by measuring the recoiling protons in the DIS process $D(e, e'p)X$ in a wide range of kinematics. Preliminary results of calculations \[13\] show a good agreement with the predictions of the hadronization model of Ref. \[10\].

\[^1\]In this note $Q^2 = -q^2 = q^2 - \nu^2$ denotes the squared four-momentum transfer
distribution in $Q^2$ and other kinematic variables could also shed light on the time evolution of the jet in the nuclear matter. A typical process of this type is the production of the so called "grey tracks" (GT) in DIS semi-inclusive processes. GT are those hadrons, predominantly protons, whose momenta are in a few hundred MeV/c range. They have been observed in various DIS processes induced by different projectiles; in this note we will consider the GT production in the Fermilab E665 experiment [14] on $\mu - Xe$ and $\mu - D$ processes at 490 GeV beam energy; in this experiment the GT have been associated to protons with momentum in the interval 200 – 600 MeV/c.

The investigation of the $Q^2$-dependence of the GT production may bring forth precious information about multiple FSI of the jet originated from the DIS in the nuclear medium. The mean number of GT, to be denoted henceforth as $\langle n_{gr} \rangle$, correlates with the intensity of the FSI: this is why the production of GT has been traditionally used as a probe for the dynamics of the interaction and the centrality of collisions. The latter is usually characterized by the mean number of collisions, $\langle \nu_c \rangle$, i.e. by the number of bound nucleons which took part in the interaction $^2$.

The dependence of $\langle n_{gr} \rangle$ upon $Q^2$, $\nu$ and $x_{Bj}$ has been measured in the E665 experiment and it is the aim of this note to present a theoretical interpretation of such a dependence resulting from a specific model of hadronization. To this end we will proceed in the following way: our hadronization model allows one to calculate the average number of collisions $\langle \nu_c \rangle$ which is related to $\langle n_{gr} \rangle$. The relation between the observed $\langle n_{gr} \rangle$ and models for $\langle \nu_c \rangle$ can be obtained with various Monte-Carlo generators for hadron cascading in nuclei. The results of these calculations however do not seem to be very accurate and a more reliable and model independent approach has been adopted in [14], where the following relation has been found,

$$\langle \nu_c \rangle = (2.08 \pm 0.13) + (3.72 \pm 0.14) \langle n_{gr} \rangle ,$$  

(1)

basing on the measured average total hadronic net charge and its relation to $\langle \nu_c \rangle$.

We have used the same relation Eq. (1) to obtain $\langle n_{gr} \rangle$, after calculating the average number of collisions $\langle \nu_c \rangle$ occurring during the evolution of a jet produced in DIS and propagating through nuclear matter. For minimum bias DIS events we obtain,

$$\langle \nu_c \rangle = \int d^2b \int_{-\infty}^{\infty} dz \rho_A(b, z) \int_{z}^{\infty} dz' \rho_A(b, z') \sigma_{eff}(z - z') + 1 .$$  

(2)

This equation results from the following description of the DIS process: the hard interaction of the lepton takes place on a bound nucleon at impact parameter $b$ and longitudinal coordinate $z$; then the knocked out quark hadronizes and initiates a jet which propagates through the nucleus interacting with other bound nucleons with the effective cross section $\sigma_{eff}(z - z')$ depending on the distance (or time, provided that the quark propagates with the speed of light). The first term in Eq. (2) gives the amount of new nucleons involved in the process, i.e. the number of collisions, whereas the second term represents the contribution from the recoil nucleon formed in the initial, hard collision between $\gamma^*$ and one of the bound nucleons.

$^2$Note that this number is different from the expansion parameter, $\sigma_{in} T_A(b)$ in the Glauber formula which is also frequently called number of collisions (see the discussion in [15]).
Evaluation of Eq. (2) requires an explicit form for the time-dependent effective cross section $\sigma_{\text{eff}}$, which can only be obtained within a model for hadronization. To this end, we employ the model suggested in [10]: it combines the soft part of the hadronization dynamics, described in terms of the string model, with the hard part, described within perturbative QCD. There are many experimental evidences showing that gluons are located within small clouds around the valence quarks [17, 18]. Therefore, if $Q$ is less than the mass scale $\lambda = 0.65\text{GeV}$ controlling the transverse quark-gluon separation, they can hardly be shaken off; in this case the string model is a proper description of hadronization. If, however, $Q > \lambda$, perturbative gluon radiation should be taken into account. Employing $\lambda$ as the bottom limit for the integration over the gluon transverse momenta (see below), double counting is excluded when the string and pQCD contributions are added to describe hadron production. This model of hadronization is close to the one used in [1, 2], where the string contribution was mocked up by the soft part of gluon spectrum, and the predictions of the two versions of the model are rather similar.

It was found in [10] that the multiplicity of the produced hadrons, or better to say the pre-hadrons which are colorless dipoles, rises with time as

$$\langle n_h(t) \rangle = n_M(t) + n_G(t),$$  \hspace{1cm} (3)

where $n_M$ is the amount of the pre-hadrons produced due to decays of the string, and $n_G(t)$ the one produced by gluon radiation.

The string contribution to $\langle n_h(t) \rangle$ has been found in [10] employing the standard dynamics of string decay [19, 20]. The pre-hadron multiplicity as function of time, $n_M(t)$, was found in the following form [10],

$$n_M(t) = \frac{\ln(1 + t/\Delta t)}{\ln 2}.$$  \hspace{1cm} (4)

Here the time scale $\Delta t$ is related to the probability $w$ of a $\bar{q}q$ pair creation in the color field of the string, per unit of time and per unit of string length. This parameter evaluated either in the Schwinger model or from the phenomenology turns out to be $w \approx 2\text{fm}^{-2}$. Correspondingly, $\Delta t = \sqrt{2/\omega} = 1\text{fm}$.

Note that the logarithmic dependence on time in Eq. (4) is rather obvious. It is related to the fact that hadrons produced via string decays build a plateau in rapidity. Since the momenta acquired by pre-hadrons are proportional to the time taken by the string decay, the number of decays rises linear in the $\ln(t)$ scale.

Concerning the gluon contribution to the hadron multiplicity Eq. (3), we employ the large $N_c$ approximation and replace each radiated gluon by a color octet $\bar{q}q$ pair, and then combine the quarks and antiquarks into colorless dipoles (pre-hadrons). This is the origin of the second term $n_G(t)$ in (3).

It is well known that radiation does not happen instantaneously, but takes time called coherence time, $t_c = 2\omega/k_T^2$, where $\omega$ and $k_T$ are the energy and transverse momentum of the radiated gluon. At shorter times the quark-gluon system is still in coherence with the initial quark. Integrating the perturbative gluon radiation spectrum over $\omega$ and $k_T$ with a weight factor $\Theta(t - t_c)$ one gets the amount of gluons which lost coherence, i.e. were radiated
over time interval $t$ after the DIS interaction. The time dependence of the gluon radiation was found in \cite{10} to be controlled by the parameter $t_0 = (m_N x_{Bj})^{-1} = 0.2 \text{ fm}/x_{Bj}$, and the number of perturbatively radiated gluons was evaluated in the following form

$$n_G(t) = \frac{16}{27} \left\{ \ln \left( \frac{Q}{\lambda} \right) + \ln \left( \frac{t \Lambda_{QCD}}{2} \right) \ln \left( \frac{\ln(Q/\Lambda_{QCD})}{\ln(\lambda/\Lambda_{QCD})} \right) \right\}, \tag{5}$$

for $t < t_0$. At longer times, $t > t_0$, the $t$-dependence starts leveling off,

$$n_G(t) = \frac{16}{27} \left\{ \ln \left( \frac{Q}{\lambda} t_0 \right) + \ln \left( \frac{t \Lambda_{QCD}}{2} \right) \ln \left( \frac{\ln(Q/\Lambda_{QCD} \sqrt{t_0/t})}{\ln(\lambda/\Lambda_{QCD})} \right) \right\} + \ln \left( \frac{Q^2 t_0}{2 \Lambda_{QCD}} \right) \ln \left( \frac{\ln(Q/\Lambda_{QCD})}{\ln(Q/\Lambda_{QCD} \sqrt{t_0/t})} \right), \tag{6}$$

and saturates at $t > t_0 Q^2/\lambda^2 = 2\nu/\lambda^2$, which is a very long time interval, substantially exceeding the nuclear size for the energies $\nu = 50 - 400 \text{ GeV}$ covered in the E665 experiment.

Having obtained the explicit forms of the string and perturbative radiation contributions in Eq. (3), let us now discuss the interaction cross section of the produced colorless dipoles. These, as we mentioned, should be treated as pre-hadrons, until they form hadronic wave functions and get definite masses. As for their interaction cross section, what matters is their transverse size, rather than their mass. Since the decay of a string is a soft process, it is natural to assume that the decay products have a size of the order of the hadronic size and therefore interact with about the same cross section.

Less obvious is the situation with pre-hadrons produced from perturbative gluons. Since the mean transverse momenta of the gluons follow the photon virtuality $Q^2$, the initial size of the produced pre-hadrons should be small, of the order of $1/Q$. Then one may expect the color transparency effect to be at work \cite{21, 22}. Indeed, this happens for leading hadrons, and the evolution of the pre-hadron size has a strong impact on nuclear transparency \cite{1, 2}. In the present case, however, we deal with the lowest part of the hadron spectrum which is independent of the photon energy $\nu$, provided that it is sufficiently high. Only those pre-hadrons are produced within the nuclear volume whose energy does not exceed $E_h \lesssim R_A (Q^2 - \lambda^2)/2 \ln(Q^2/\lambda^2)$. The size of such a pre-hadron evolves so fast that the largest part of the effect of color transparency is washed out. Even for the most energetic exclusively produced hadrons in the E665 experiment, the effect of color transparency is rather moderate \cite{23, 24}; it is much weaker and hardly detectable at lower energies \cite{25}. Therefore, we neglect these corrections and assume that pre-hadrons interact with the hadronic cross sections.

Eventually, we are in the position to evaluate the time dependent effective absorption cross section in (2), to be used in the description of GT production; by treating all the $\bar{q}q$ colorless dipoles as mesons (M), we obtain

$$\sigma_{\text{eff}}(t) = \sigma_{in}^{MN} \left[ n_M(t) + n_G(t) \right], \tag{7}$$

Note that this expression is different from the one used in \cite{10}. Firstly, we excluded the term $\sigma_{tot}^{NN}$ corresponding to interaction of the nucleon originated from the first DIS event,
since we treat this nucleon as a participant giving rise to the last term in (2). Secondly, we replaced the total meson-nucleon (MN) cross section by the inelastic one. This is because most of recoiling protons in elastic and diffractive scattering have too small momenta to contribute to the production of GT. Therefore, we use

\[
\sigma_{\text{MN}}^{\text{in}} = \sigma_{\text{tot}}^{\pi N} - \sigma_{\text{el}}^{\pi N} - \sigma_{\text{sd}}^{\pi N} = 17.7 \text{ mb.}
\]

Before illustrating the calculation of the \( Q^2 \) dependence of the mean number of GT \( n_{\text{gr}}(Q^2) \), let us briefly discuss the expected dependence of \( n_{\text{gr}} \) upon the energy transfer \( \nu \). Both the string model and perturbative radiation lead to the mean multiplicity which rises logarithmically with energy. Naively, one may conclude that the amount of GT follows the multiplicity and should rise with energy as well. However, as already discussed, it takes time to produce hadrons. The observed multiplicity does not emerge in an instantaneous explosion-like particle production, but is a result of time development of the hadronization process. The full time of jet production is proportional to its energy and exceeds the nuclear size at the E665 energies. However, only those gluons which are radiated inside the nucleus can contribute to the production of GT. With the condition \( t_c < R_A \) one should integrate the radiation spectrum from \( \omega_{\text{min}} = k_T \) up to \( \omega_{\text{max}} = k_T^2 R_A \). The result is independent of the jet energy. Indeed, this is confirmed by the results of the E665 experiment (see Figs. 2, 17 in [14]).

At the same time, one should expect a \( Q^2 \) dependence of the mean number of GT. Indeed, the stronger kick gets the quark, the more gluons it shakes off. The mean transverse momentum of radiated gluons follows \( Q^2 \) which plays the role of the upper cut-off for the integration \( d^2 k_T/k_T^2 \). Correspondingly, the multiplicity of the radiated gluons rises logarithmically with \( Q^2 \).

Let us now quantitatively analyze the behavior of \( \langle n_{\text{gr}}(Q^2) \rangle \). To this end we have calculated the mean number of collisions by using Eqs. (2) and (7). Then, applying \( \text{[11]} \), we have obtained the number of GT, \( \langle n_{\text{gr}}(Q^2) \rangle \). We fixed \( x_{Bj} \) at the mean value \( \langle x_{Bj} \rangle = 0.068 \) [14]. The result shown by dashed curve is compared with the experimental data from the E665 experiment [14] in Fig. 1.

If the time interval \( t_0 \) is of the order or smaller than the nuclear size, the chosen value of \( x_{Bj} \) affects hadronization and is important. The E665 data are subject to quite a strong correlation between \( Q^2 \) and \( x_{Bj} \), which may change the results depicted in Fig. 1. We parametrize this correlation as \( Q^2 = A + B x_{Bj} \), where the parameters \( A = 2.2 \text{ GeV}^2 \) and \( B = 178 \text{ GeV}^2 \) are found using the values of \( \langle Q^2 \rangle \) and \( \langle x_{Bj} \rangle \) for two regions called in [14] ”shadowing” and ”non-shadowing”. At very small \( x_{Bj} < 0.01 \) we introduce an additional \( x_{Bj} \) dependence of parameter \( A \) which is, however, unimportant for the kinematic region under discussion. Our results corrected for the \( Q^2 - x_{Bj} \) correlation are depicted by the solid curve in Fig. 1. In spite of the difference between the two curves, both agree with the data.

In order to see the dependence of \( \langle n_{\text{gr}} \rangle \) on \( x_{Bj} \) explicitly we calculated it either fixing \( Q^2 \) at the mean value \( \langle Q^2 \rangle = 14.3 \text{ GeV}^2 \), or applying the \( Q^2 - x_{Bj} \) correlation introduced above. The results are depicted in Fig. 2 by the dashed and solid curves, respectively. Although both curves slightly overestimate the data, the solid one better reproduces the measured slope.

In closing, the following comments are in order:

• In our calculations we employed the same hadronization model as in [11 2 10] with
Figure 1: The mean number of grey tracks $\langle n_{gr} \rangle$ produced in the $\mu Xe$ DIS as function of $Q^2$. The theoretical predictions based upon Eqs. (1), (2) and (7), are compared with the experimental data from Ref. [14], which correspond to non-shadowing ($x_{Bj} > 0.02$) region. The dashed curve corresponds to fixed $x_{Bj} = \langle x_{Bj} \rangle = 0.068$. The solid curve includes the correlation between $Q^2$ and $x_{Bj}$ (see text).

no readjustment of the parameters, and the empirical relation between the number of grey tracks and the number of collisions given by Eq. (1). The model successfully passed the new test;

- the results of calculations may be subject to further corrections. In particular, we neglected resonance decays, which is justified by their rather high energies. On the other hand, the formation time of hadronic wave function is much shorter than the resonance life time and is rather short for pre-hadrons produced within the nucleus. This is the reason why we neglected the effects of color transparency which are important for leading hadrons $[1, 2]$. This may explain why our results exhibited in Fig. 1 somewhat overestimate the data;

- the $Q^2$ dependence of the mean number of grey tracks serves as a sensitive tool for testing theoretical models of hadronization. It can be seen that the amount of grey tracks doubles within the range of $Q^2$ covered by the kinematics of the E665 experiment, and our calculations well reproduce such a steep variation of $\langle n_{gr}(Q^2) \rangle$;

- whereas nuclear attenuation of leading hadrons carries information of space-time devel-
Figure 2: The mean number of grey tracks $\langle n_{gr} \rangle$ produced in the $\mu Xe$ DIS as function of $x_{Bj}$. The experimental data are from Ref. [14]. The dashed and solid curves show the results of calculations with fixed $Q^2 = 14.3 \text{ GeV}^2$ and corrected for the $Q^2 - x_{Bj}$ correlation respectively.

development of hadronization in rather rare events, grey tracks provide precious information about hadronization dynamics for the main bulk of DIS events;

- We found a rather flat dependence of $\langle n_{gr} \rangle$ on $x_{Bj}$ at fixed $Q^2$. However, the $Q^2 - x_{Bj}$ correlation existing in the E665 experiment leads to a growth of the number of GT with $x_{Bj}$. Our calculations corrected for this effect well describe the measured $x_{Bj}$ dependence of GT;

- in our calculations we assumed that the value of Bjorken scaling variable $x_{Bj}$ is sufficiently large to neglect the sea, and this is the reason why we compared our results with the E665 data at large values of $x_{Bj}$. At small values of $x_{Bj}$ one should take into account the production of two jets with different momenta, with only one of them producing a grey track corresponding to the target nucleon on which the DIS process occurred. On the other hand, the E665 experiment detected no substantial change in behavior of different observables in the shadowed compared to the nonshadowed regions. This was explained by a large contribution from diffraction [14].

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References

[1] B.Z. Kopeliovich, J. Nemchik and E. Predazzi, in *Future Physics at HERA*, Proceedings of the Workshop 1995/96, edited by G. Ingelman, A. De Roeck and R. Klanner, DESY, 1995/1996, vol.2, p. 1038 (nucl-th/9607036); in Proceedings of the *ELFE Summer School on Confinement Physics*, edited by S.D. Bass and P.A.M. Guichon, Editions Frontieres, 1995, p. 391, Gif-sur-Yvette (hep-ph/9511214).

[2] B.Z. Kopeliovich, J. Nemchik and E. Predazzi, A. Hayashigaki, Nucl. Phys. A740 (2004) 211.

[3] EMC Collaboration, J. Ashman et al., Z. Phys. C52 (1991) 1.

[4] L.S. Osborne et al., Phys. Rev. Lett. 40 (1978) 1624.

[5] HERMES Collaboration, A. Airapetian et al., Eur. Phys. J. C20 (2001) 479; Phys. Lett. B577 (2003) 37.

[6] E. Wang and X.-N. Wang, Phys. Rev. Lett. 89 (2002) 162301.

[7] A. Accardi, V. Muccifora and H.J. Pirner, Nucl. Phys. A720 (2003) 131.

[8] T. Falter, W. Cassing, K. Gallmeister, U. Mosel, nucl-th/0303011 nucl-th/0406023

[9] N. Z. Akopov, G. M. Elbakian and L. A. Grigorian, hep-ph/0205123.

[10] C. Ciofi degli Atti and B.Z. Kopeliovich, Eur. Phys. J. A17 (2003) 133.

[11] C. Ciofi degli Atti, S. Scopetta and L. P. Kaptari, Eur. Phys. J. A5 (1999) 191.

[12] C. Keppel, S. Kuhn, and W. Melnitchouk, spokespersons *The structure of the free neutron via spectator tagging*, TJNAF BoNuS Collaboration, E03-012.

[13] C. Ciofi degli Atti, L.P. Kaptari, and B.Z. Kopeliovich, Eur. Phys. J. A19 (2004) 145; and in preparation.

[14] E665 Collaboration, M.R. Adams et al., Z. Phys. C65 (1995) 225.

[15] B.Z. Kopeliovich, Phys. Rev. C68 (2003) 044906.

[16] J.F. Gunion and G. Bertsch, Phys. Rev. D25 (1982) 746.

[17] B.Z. Kopeliovich, A. Schäfer and A.V. Tarasov, Phys. Rev. D62 (2000) 054022 (hep-ph/9908245).
[18] B.Z. Kopeliovich and B. Povh, J. Phys. G30 (2004) S999.

[19] A. Casher, H. Neubereger and S. Nussinov, Phys. Rev. D20 (1979) 179.

[20] B.Z. Kopeliovich, Phys. Lett. B243 (1990) 141.

[21] A.B. Zamolodchikov, B.Z. Kopeliovich and L.I. Lapidus, Sov. Phys. JETP Lett. 33 (1981) 612.

[22] S.J. Brodsky and A. Mueller, Phys. Lett. B206 (1988) 685.

[23] E665 Collaboration, M.R. Adams et al., Phys. Rev. Lett. 74 (1995) 1525.

[24] B.Z. Kopeliovich, J. Nemchik, N.N. Nikolaev, and B.G. Zakharov, Phys. Lett. B324 (1994) 469.

[25] B.Z. Kopeliovich, J. Nemchik, A. Schäfer, A. Tarasov, Phys. Rev. C65 (2002) 035201.