Instantaneous versus non-instantaneous approach to relativistic ionization of atomic hydrogen by electron impact

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We present a theoretical model for atomic hydrogen ionization by electron impact in the instantaneous approximation and the more accurate non-instantaneous approach using the methods of Quantum Electrodynamics, for the binary coplanar and the coplanar asymmetric geometries. All electrons are described by plane wave functions in the coplanar binary geometry but in the asymmetric geometry the ejected electron is described by a Sommerfeld-Maue wave function. It is shown that the two models give the same results in the non relativistic limit for the binary coplanar geometry where the interactions can be treated as instantaneous. However, this is no longer true for the relativistic case where one has to take into account both the instantaneous interaction and the radiation interaction. These results are obtained in the first order of perturbation theory.

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I. INTRODUCTION

The interaction of electrons with atoms is the field that most deeply probes both the structure and reaction dynamics of a many-body system [1]. Electron-atom collisions that ionize the target provide a large and interesting diversity of phenomena. The reason for this is that a three-body final state allows a wide range of kinematic regions to be investigated. These different kinetic regions depend sensitively on different aspects of the description of the collision. Up to now, there has been no calculation of differential cross-sections by a method that is generally valid. The understanding of the ionization by electron impact has advanced by an iterative process involving experiments and calculations that emphasize different aspects of the reaction. Kinematic regions have been found that are completely understood in the sense that absolute differential cross-sections in detailed agreement with experiment can be calculated.

These form the basis of a structure probe, electron momentum spectroscopy, that is extremely sensitive to one-electron and electron-correlation properties of the target ground states of the residual ion in the case of heavy atoms. Other kinematic regions require a complete description of the collision which may be facilitated by including the boundary condition for the three charged particles in the final states. This point is not trivial at all because there is no separation distance at which the Coulomb forces in the three-body system are strictly negligible. The pioneering experiments of Ehrhardt et al [2] are of this type. Electron-impact ionization has been studied experimentally using relativistic electrons by Dangerfield and Spicer [3], Hoffman et all (1979) [4] and Anholt (1979) [5]. The measurements with relativistic electrons have all been of total cross-sections typically for the K and L shells of heavy atoms. Theoretical models for total ionization cross-sections have been developed by a number of authors including Scofield (1978) [6], Mitroy and Stockmann (1980) [7]. This field has been reviewed by Moiseiwitsh (1980) [8]. Fuss, Mitroy and Spicer (1982) [9] have developed a theoretical model for the binary (e,2e) reaction. This (e,2e) reaction is that were the outgoing electrons have equal energy. The theoretical model they have developed uses the impulse and relativistic plane wave Born approximations (RP-WBA). Nakel and Whelan (1999) [10] have reviewed the experimental and theoretical developments in the study of relativistic (e,2e). They argued that the fully relativistic distorted wave approximation (RDWBA) proposed by Walters et al [11] was the simple possible approximation that allows to gain understanding of the relativistic (e,2e) processes.

As for electron impact ionization for a hydrogen atomic target, the first work that relied on the model developed by Fuss et al [9], was presented by Attaourti et al [12] who studied the importance of the relativistic electronic dressing during the process of laser-assisted ionization of atomic hydrogen by electron impact. Taking a zero laser electric field, one recovers the binary (e,2e) process in the absence of the laser field in the instantaneous approximation where a direct non relativistic interaction potential was used. Later on, Attaourti et al (2005) [13] developed a simple semi-relativistic model using a Sommerfeld-Maue wave function to describe the ejected electron and the Darwin semi-relativistic wave function to describe the hydrogen atomic target in its
ground state also in the instantaneous interaction. This simple model allows to investigate both the relativistic binary (e,2e) reaction process (RPWBA) and the semi-relativistic (e,2e) process in the coplanar asymmetric geometry (SRCBA).

The purpose of the present work is to go beyond the instantaneous approximation and use the propagator approach of QED to include both the instantaneous and radiation contributions necessary to describe more accurately the (e,2e) processes in the relativistic domain. Indeed, it is well known that non relativistically, interactions are instantaneous but this is no longer true in the relativistic case. This important feature of relativistic collision processes is explicitly contained in the formalism of QED. This important point is clearly explained for example by F. Gross (1999) and many others. To what extent the contribution of both the instantaneous and radiation parts to these (e,2e) processes affect and modify the non polarized differential cross section is the question we want to address in our work. Through calculations. In section 4, we discuss some relevant approximations and give theoretical results using full QED hamiltonian.

In section 3, the unpolarized triple differential cross section (TDCS) is then given by:

\[ \frac{d \sigma}{dE_B d\Omega_B d\Omega_f} = \frac{|p_f| |p_2| (2E_i E_f/c^2 - p_i, p_f + c^2)}{|p_i|^4} \times 4E_B \Phi_{1/2,1/2}(q = \Delta - p_B) \]

where \( \Phi_{1/2,1/2}(q) \) are the Fourier transforms of the relativistic atomic hydrogen wave functions \[ \|^1 2 \], and the quantity \( \Delta = p_i - p_f \) is the momentum transfer.

II. BINARY COPLANAR GEOMETRY

A. Theoretical model within the frame work of the instantaneous approximation

The transition matrix element for the direct channel (we do not consider exchange effects) is given by

\[ S_{fi} = -i \int dt < \psi_f(x_1) | \phi_f(x_2) | V_d | \psi_p(x_1) \phi_i(x_2) > \]

where \( V_d = 1/r_{12} - 1/r_1 \) is the direct interaction potential \( (t_1 = t_2 = t \Rightarrow x_1^0 = x_2^0 = x^0) \) and in the RPWBA, \( \psi_{p_f}(x_1) \) is the wave function describing the scattered electron

\[ \psi_{p_f}(x_1) = \frac{u(p_f, s_f)}{\sqrt{2E_f V}} e^{-i p_f, x_1} \]

(2)

given by a free Dirac solution normalized to the volume \( V \). For the incident electron, we use

\[ \psi_{p_i}(x_1) = \frac{u(p_i, s_i)}{\sqrt{2E_i V}} e^{-i p_i, x_1} \]

(3)

For the atomic target, \( \phi_i(x_2) = \phi_i(t, r_2) \) is the relativistic wave function of atomic hydrogen in its ground state. For the ejected electron, we use again a free Dirac solution normalized to the volume \( V \) and \( \phi_f(x_2) \) is given by:

\[ \phi_f(x_2) = \psi_{p_2}(x_2) = \frac{u(p_2, s_B)}{\sqrt{2E_B V}} e^{-i p_B, r_2} \]

(4)

The unpolarized triple differential cross section (TDCS) is then given by:

\[ \frac{d \sigma}{dE_B d\Omega_B d\Omega_f} = \frac{|p_f| |p_2| (2E_i E_f/c^2 - p_i, p_f + c^2)}{|p_i|^4} \times 4E_B \Phi_{1/2,1/2}(q = \Delta - p_B) \]

(5)

The functions \( \Phi_{1/2,1/2}(q) \) are the Fourier transforms of the relativistic atomic hydrogen wave functions \[ \|^1 2 \], and the quantity \( \Delta = p_i - p_f \) is the momentum transfer.

B. Theoretical model with the inclusion of the photon propagator

The transition matrix element for the direct channel in the non-instantaneous approximation is given by:

\[ S_{fi} = -i \int_{-\infty}^{+\infty} dt < \phi_f(x) | A(x) | \phi_i(x) > \]

(6)

where \( A(x) = \gamma_\mu A^\mu = A_0(x) \gamma_0 - A(x).\gamma \). Contravariant four vectors are written \( x^\mu = (t, x) \). \( \phi_f(x) = \psi_{p_f}(x) \) of the incident electron is defined in Eq. 4, and \( \phi_i(t, x) = \psi_{p_i}(x) \) is the relativistic wave function of atomic hydrogen in its ground state. The electromagnetic potential \( A^\mu \) due to the scattered electron is given by

\[ A^\mu(x) = -4\pi \int G(x-y) J^\mu(y) d^4 y \]

(7)

where \( J^\mu(y) \) is the four-vector current for the electron

\[ J^\mu(y) = \overline{\psi}_{p_f}(y) \gamma^\mu \psi_{p_f}(y), \quad \overline{\psi}_{p_f}(y) = \psi_{p_f}(y)^\dagger \gamma^0 \]

where \( \psi_{p_f}(y) \) and \( \psi_{p_f}(y) \) are defined respectively in Eq. 4 and Eq. 4. Using the Fourier decomposition of the Green’s function

\[ G(x-y) = -\frac{1}{(2\pi)^4} \int \frac{e^{i \kappa(x-y)}}{\kappa^2 + i\varepsilon} d^4 \kappa \]

(9)

the scattering matrix element becomes

\[ S_{fi} = \int \overline{\psi}_{p_f}(x) \gamma_\mu \psi_{p_f}(x) \int \frac{e^{i \kappa(x-y)}}{\kappa^2 + i\varepsilon} d^4 \kappa \]

\[ \times \overline{\psi}_{p_f}(y) \gamma^\mu \psi_{p_f}(y) d^4 y d^4 x \]

(10)
The $y$-integration can be performed easily yielding
\[\int d^4y \exp[i(\kappa + p_f - p_i)y] = (2\pi)^4 \delta[\kappa - (p_i - p_f)] \] (11)

Now the $\kappa$-integration is done, and the $S_{fi}$ reads with the usual normalization
\[S_{fi} = \frac{\overline{u}(p_i, s_f)\gamma^\mu u(p_i, s_i)}{\sqrt{4E_fE_iV^2(p_i - p_f)^2}} \sqrt{2E_BV} \times \Phi_i(q)(2\pi)\delta(E_i - E_f - E_B + |\varepsilon_b|) \] (12)

We note
\[\mathcal{F}_\mu = \overline{u}(p_B, s_B)\gamma_\mu \] (13)

Using the standard procedures of QED [16], one obtains for the spin unpolarized triple differential cross section (TDCS)
\[\frac{d\sigma}{dE_Bd\Omega_f d\Omega_B} = \frac{1}{|p_f|c^3} \left( \frac{1}{2(1 - p_i - p_f)^2} \sum_{s_i, s_f} |M_{fi}|^2 \right) \] (14)

with
\[\frac{1}{2} \sum_{s_i, s_f, s_B} |M_{fi}|^2 = \frac{2c^2}{\left(\mathcal{F}_f\overline{u}(p_f, s_f)\gamma^\mu u(p_i, s_i)\right)^2} = 2c^2 \left\{ (\mathcal{F}_f p_f) (\mathcal{F}^* s_f) + (\mathcal{F}_f s_f) (\mathcal{F}^* p_f) \right\} \] (15)

Using the spinor normalization condition $u^+(p, s)u(p, s') = 2E\delta_{ss'}$, we find for the TDCS
\[\frac{d\sigma}{dE_Bd\Omega_f d\Omega_B} = \frac{1}{|p_f|c^3} \left( \frac{1}{2(1 - p_i - p_f)^2} \sum_{s_i, s_f} |M_{fi}|^2 \right) \times [E_f(p_B, p_i) + E_i(p_B, p_f) + E_B(2(p_i - p_f) - c^2)] \times \Phi_{1/2, 1/2}(q = \Delta - \mathbf{p}_B). \] (16)

Note that
\[(p_i - p_f) = (E_i - E_f)/c - (p_i - p_f) \]
\[(p_B, p_i) = E_BE_i/c^2 - p_Bp_i \]
\[(p_B, p_f) = E_BE_f/c^2 - p_Bp_f \]
\[(p_i, p_f) = E_iE_f/c^2 - p_ip_f \] (17)

III. COPLANAR ASYMMETRIC GEOMETRY

A. The SRCBA theoretical model in the instantaneous approximation

In this section, the same computation is done, the main difference lying in the description of the ejected electron where now, a Sommerfeld-Maue wave function accurate to the order $Z/c$ in the relativistic corrections, is used.

We have $\phi_f(t, x) = \exp(-iE_Bt)\psi_B(-)(x)$ and $\psi_B(-)(x)$ is given by:
\[\psi_B(-)(x) = \exp(\pi\eta_B/2)\Gamma(1 + i\eta_B) \exp(i\mathbf{p}_B\cdot\mathbf{x}) \times \{1 - i\eta_B + 1, i(p_Bx + \mathbf{p}_B\cdot\mathbf{x})\} \exp(-i\mathbf{p}_B\cdot\mathbf{x}) \times \frac{u(p_B, s_B)}{\sqrt{2E_BV}} \] (18)

normalized to the volume $V$. The Sommerfeld parameter is given by
\[\eta_B = \frac{E_B}{c^2 p_B} \] (19)

The transition matrix element for the direct channel is given by
\[S_{fi} = -i \int d\mathbf{x} \frac{\overline{\mathcal{F}}(p_f, s_f)\gamma^\mu u(p_i, s_i)}{\sqrt{2E_fV}} \gamma^\mu \frac{\mathcal{F}(p_B, s_B)}{\sqrt{2E_BV}} \times \left\{ 1 - i\eta_B + 1, i(p_Bx + \mathbf{p}_B\cdot\mathbf{x}) \right\} \exp(-i\mathbf{p}_B\cdot\mathbf{x}) \times \frac{u(p_B, s_B)}{\sqrt{2E_BV}} \] (20)

for the description of the quantities $\overline{S}_{fi}^{(1)}, \overline{S}_{fi}^{(2)}, \overline{S}_{fi}^{(2)}$ and for more details see [13].

B. The SRCBA theoretical model with the inclusion of the photon propagator

The transition matrix element $S_{fi}$ for direct channel in the non-instantaneous approximation is given by
\[S_{fi} = -i \int_{-\infty}^{+\infty} dt < \phi_f(x)|A(x)|\phi_i(x) > \] (22)

We replace all wave functions and $A(x)$ in $S_{fi}$ and we get :
\[S_{fi} = -i \frac{4\pi 2\pi \delta(E_i - E_f - E_B + |\varepsilon_b|)}{\sqrt{8V^4E_iE_f(p_B - p_f)^2}} \left( \overline{\mathcal{F}}(p_B, s_B)\gamma^\mu \overline{u}(p_f, s_f)\gamma^\mu u(p_i, s_i)H_1(q) + \overline{\mathcal{F}}(p_B, s_B)\gamma^\mu \overline{u}(p_f, s_f)\gamma^\mu u(p_i, s_i)H_2(q) \right) \] (23)
The quantities $H_1(q = \Delta - p_B)$, $H_2(q = \Delta - p_B)$, $\bar{H}_2(q = \Delta - p_B)$ are given in \[3\]. This transition matrix element contains three terms, denoted respectively by $S_{fi}^{(1)}$, $S_{fi}^{(2,1)}$, $S_{fi}^{(2,2)}$. 

$$S_{fi}^{(1)} = -i [\mathbf{p}(PB, s_B) \gamma_\mu \mathbf{p}(p_f, s_f) \gamma_\mu u(p_i, s_i)] H_1(q)$$

$$S_{fi}^{(2,1)} = -i [\mathbf{p}(PB, s_B) \gamma_\mu \mathbf{p}(p_f, s_f) \gamma_\mu u(p_i, s_i)] H_2(q)$$

$$S_{fi}^{(2,2)} = -i [\mathbf{p}(PB, s_B) \gamma_\mu \mathbf{p}(p_f, s_f) \gamma_\mu u(p_i, s_i)] \bar{H}_2(q)$$

We note

$$S_{fi}^{(1)} = -i A(s_B, s_f, s_i) H_1(q)$$

$$S_{fi}^{(2,1)} = -i B(s_B, s_f, s_i) H_2(q)$$

$$S_{fi}^{(2,2)} = -i C(s_B, s_f, s_i) \bar{H}_2(q)$$

We calculate the square of $S_{fi}$. We have 9 terms: $|S_{fi}^{(1)}|^2$, $|S_{fi}^{(2,1)}|^2$, $|S_{fi}^{(2,1)}|^2$, and $S_{fi}^{(1)} S_{fi}^{(2,1)} S_{fi}^{(2,1)} S_{fi}^{(1)}$, $S_{fi}^{(2,2)} S_{fi}^{(2,2)} S_{fi}^{(1)}$, $S_{fi}^{(2,1)} S_{fi}^{(2,2)} S_{fi}^{(1)}$, as well as $S_{fi}^{(1)} S_{fi}^{(2,2)}$. The different sums over spin states are given by :

$$\frac{1}{2} \sum_{s_f, s_i, s_B} \sum A(s_B, s_f, s_i) C(s_B, s_f, s_i)$$

$$= \frac{1}{2} \sum_{s_f, s_i, s_B} \sum C(s_B, s_f, s_i) A(s_B, s_f, s_i)$$

$$= \frac{1}{2} \sum_{s_f, s_i, s_B} \sum |A(s_B, s_f, s_i)|^2$$

$$= \frac{1}{2} \sum_{s_f, s_i, s_B} \sum |C(s_B, s_f, s_i)|^2$$

$$= 8 c^2 \left[ E_f(p_B, p_i) + E_i(p_B, p_f) - E_B c^2 \right]$$

$$\frac{1}{2} \sum_{s_f, s_i, s_B} \sum A(s_B, s_f, s_i) B(s_B, s_f, s_i)$$

$$= \frac{1}{2} \sum_{s_f, s_i, s_B} \sum B(s_B, s_f, s_i) A(s_B, s_f, s_i)$$

$$= \frac{1}{2} \sum_{s_f, s_i, s_B} \sum |A(s_B, s_f, s_i)|^2$$

$$= \frac{1}{2} \sum_{s_f, s_i, s_B} \sum |B(s_B, s_f, s_i)|^2$$

$$= 8 \left[ E_f^2 - c^4 \right] \left[ E_f(p_B, p_i) + E_i(p_B, p_f) - E_B c^2 \right]$$

We calculate the polarization of TDAS in the non-instantaneous approximation. The unpolarized TDAS is given by

$$\frac{d \sigma}{dE dB d\Omega_B d\Omega_f} = \frac{1}{24 \pi^2 \rho^2} \frac{|p_f| |p_B| \exp(\pi \eta_B)}{(|p_i| - |p_f|)^2} \Gamma(1 - i \eta_B)^2 \left[ S_{fi}^{(1)} + S_{fi}^{(2,1)} + S_{fi}^{(2,2)} \right]^2$$

IV. RESULTS AND DISCUSSION

Before beginning the discussion of the results obtained, it is worthwhile to recall the meaning of some abbreviations that will appear throughout this section. The NRPWBA stands for the non relativistic plane wave Born approximation where non relativistic plane waves are used to describe the incident, scattered and ejected electrons. This approximation is valid only in the non relativistic regime and particularly in the case of the coplanar symmetric geometry. The NRCBA approximation stands for the non relativistic Coulomb Born approximation which is valid only in the non relativistic regime and in the case of the asymmetric coplanar geometry of Ehrhardt \[2\]. The RPWBAMP stands for the relativistic plane wave Born approximation without propagator, which is valid for the case of a coplanar binary geometry and uses Dirac free plane wave solutions as well as a direct interaction potential. Its results take only into account the instantaneous part of the interaction potential. The RPWBAWP stands for the relativistic plane wave Born approximation with the inclusion of the photon propagator. It is valid for the case of a coplanar binary geometry, uses Dirac free plane wave solutions and its results take into account the instantaneous part of the interaction as well as the radiation part of the interaction. The SRCBANP stands for the semi relativistic Coulomb Born approximation which is valid both for the coplanar asymmetric geometry and the binary coplanar geometry. In this approximation, the incident and scattered electrons are described by free Dirac plane waves while the ejected
electron is described by a Sommerfeld-Maue wave function and the atomic hydrogen target is described by a Darwin wave function \cite{14}. However, it uses a direct interaction potential in the instantaneous approximation. Finally, the SRCBAWP stands for the semi relativistic Coulomb Born approximation with the full inclusion of the photon propagator. It is similar to the previous approximation except that it includes both the instantaneous and radiation parts of the interaction.

Due to this number of approximations that may be confusing, we will focus in our discussion on the most important features of our model and restrain ourselves to consider in the non relativistic regime, the comparison of all the TDCSs in the case of the binary coplanar geometry. In the case of the coplanar asymmetric geometry, we will compare the three relevant TDCSs, the SRCBAWP, the SRCBANP and the NRCBA.

A. The non relativistic regime

1. The binary coplanar geometry

We begin our discussion by the kinematics of the process. For the binary coplanar geometry, we choose the following angular situation where \( p_i \) is along the \( O_z \) axis and \( \theta_i = \phi_i = 0^\circ \). For the scattered electron, we choose \( (\theta_f = 45^\circ, \phi_f = 0^\circ) \) and for the ejected electron we choose \( \phi_B = 180^\circ \) and the angle \( \theta_B \) varies from 30° to 60°. The energies of the incident and ejected electrons are respectively \( E_i = 2700 \text{ eV} \) and \( E_B = 1349.5 \text{ eV} \). It is expected that for such a regime and choice of geometry, relativistic effects will be small and it is indeed the case as this can be seen in Fig.1 where the six approaches give nearly the same results. Even if this is not apparent on this figure, we have plotted the results of six approaches and obtained close results. In the binary coplanar geometry, the TDCSs are well peaked around the specific angle and energy of the ejected electron being non relativistic (it corresponds to a relativistic parameter \( \gamma_i = (1 - (\beta_i)^2)^{-1/2} \)), the description of the interaction potential by a direct non relativistic potential or the use of the photon propagator is not important in that case.

2. The coplanar asymmetric geometry

In this geometry, the use of any approach involving plane waves will lead to irrelevant results since the Coulomb description of the ejected electron is needed. The angular choice is as follows : \( p_i \) is along the \( O_z \) axis and \( \theta_i = \phi_i = 0^\circ \). For the scattered electron, we choose \( (\theta_f = 3^\circ, \phi_f = 0^\circ) \), for the ejected electron we choose \( \phi_B = 180^\circ \) and the angle \( \theta_B \) varies from \(-180^\circ \) to \( 180^\circ \). The energy of the incident electron is \( E_i = 2700 \text{ eV} \) and the energy of the ejected electron is \( E_B = 54. \text{ eV} \). The range of the angular variable \( \theta_B \) is due to the fact that in the coplanar asymmetric geometry, one expects to see two peaks, a binary peak which is, in our case, located in the vicinity of \( \theta_B = 77^\circ \) and a recoil peak which is located in the vicinity of \( \theta_B = -105^\circ \). As a validity check of our calculations, we have also reproduced the results of Byron and Joachain \cite{17} for the energies \( E_i = 250 \text{ eV} \) and \( E_B = 5 \text{ eV} \). The three approaches : NRCBA, SRCBAWP and SRCBANP are shown in Fig. 2 where the spread of the three TDCSs covers the whole range that the angular variable \( \theta_B \) can take and small differences begin to appear particularly at the recoil peak. However and as might be expected for such energies, the SRCBAWP and the SRCBANP remain very close because for such a regime the use of the instantaneous or the non instantaneous approximation amounts to nearly the same. Radiation effects are not important enough for such non relativistic energies.

- FIG. 1: TDCSs scaled in \( 10^{-3} \) for \( E_i = 2700 \text{ eV} \) and \( E_B = 1349.5 \text{ eV} \), and the angles \( \theta_f = 45^\circ, \theta_i = \phi_i = \phi_f = 0^\circ \). The angle \( \phi_B \) is such that \( \phi_B = 180^\circ \). The curves of the six approaches overlap.
FIG. 2: TDCS for $E_i = 2700$ eV and $E_B = 54$ eV, and the angles $\theta_f = 3^\circ$, $\theta_i = \phi_i = \phi_f = 0^\circ$. The angle $\phi_B$ is such that $\phi_B = 180^\circ$. The curves overlap.

B. The relativistic regime

1. The binary coplanar geometry

For this regime, the energy of the incident electron is $E_i = 511002$ eV and the energy of the ejected electron is $E_B = 225501$ eV. This value of the energy $E_i$ corresponds to a relativistic parameter $\gamma_i = 2$. We choose the following angular situation where $p_i$ is along the $Oz$ axis ($\theta_i = \phi_i = 0^\circ$). For the scattered electron, we choose ($\theta_f = 55^\circ$, $\phi_f = 0^\circ$) and for the ejected electron we choose $\phi_B = 180^\circ$ and the angle $\theta_B$ varies from $30^\circ$ to $60^\circ$. Relativistic effects can no longer be neglected and also the use of the instantaneous approximation becomes itself questionable. Let us explain what is contained in Fig 3. The first important point that has to be mentioned is the following: in such a regime, the use of non relativistic approaches is not physically founded. So, we will focus mainly on the four relativistic approximations aforementioned. What appears clearly is that the four relativistic models give different results with the importance of radiation effects clearly shown. To summarize Fig 3, we can say that as regards to the SRCBAWP and SRCBANP, the instantaneous approximations is no longer valid. We have a situation where the TDCS(SRCBAWP) is higher than the TDCS(SRCBANP) and also where the TDCS(RPWBAWP) is higher than the TDCS(RPWBANP). This is a general rule and we have made many simulations to assert that it is a valid rule in the relativistic regime. However, there is no general rule as to the comparison between the TDCS(SRCBAWP) and the TDCS(RPWBAWP) as well as the comparison between the TDCS(RPWBANP) and the TDCS(SRCBANP) because another choice of $(\theta_f, \text{e.g} \theta_f = 39^\circ$ will give rise to a reverse situation where $TDCS(^{\text{RPWBAWP}}) > TDCS(^{\text{SRCBAWP}})$ whereas $TDCS(^{\text{RPWBANP}}) > TDCS(^{\text{SRCBAWP}})$.

In Fig. 4, we only compare the two SRCBAWP and SRCBANP to assess the importance of radiation effects and as this behavior never changes, one can easily see that the use of the instantaneous approximation underestimates the value of the TDCS using the photon propagator by a factor two. The same behavior is observed with the RPWBAWP and the RPWBANP.
2. The coplanar asymmetric geometry

In this geometry, the angular choice is as follows: $p_i$ is along the $Oz$ axis and $\theta_i = 3^\circ, \phi_i = 0^\circ$. For the scattered electron, we choose $(\theta_f = 3^\circ, \phi_f = 0^\circ)$, for the ejected electron we choose $\phi_B = 180^\circ$ and the angle $\theta_B$ varies from $0^\circ$ to $180^\circ$. The energy of the incident electron is $E_i = 511002$ eV and the energy of the ejected electron is $E_B = 10220.04$ eV. In the relativistic regime, there is no occurrence of a recoil peak and the spread of the angular variable $\theta_B$ of the ejected electron is reduced. As expected, radiation effects are indeed important and in Fig. 5, we see that the binary peak is located in the vicinity of $\theta_B = 75^\circ$ whereas the TDCS(SRCBANP) underestimates the TDCS(SRCBAWP) by a factor 2. The comparison with the two other approaches using relativistic plane waves is not relevant for this geometry.

V. CONCLUSION

In this work, we have studied the two theoretical models (instantaneous and non-instantaneous approximations) for the relativistic ionization of atomic hydrogen by electron impact using non relativistic or relativistic plane wave functions to describe all electrons in the binary coplanar geometry and using the Sommerfeld-Maue wave function to describe the ejected electron in the case of the coplanar asymmetric geometry. In all the approaches using relativistic plane waves, the use of the exact relativistic wave function for atomic hydrogen in its ground state is tractable, whereas in the asymmetric case, one can only use the semi relativistic Darwin wave function \[14\] to describe the atomic target. The general conclusions that can be drawn from this work are summarized in what follows. In the non relativistic regime, the results obtained in the binary coplanar geometry give the same results for an appropriate choice of the angular parameters whereas in the coplanar asymmetric geometry, on can only compare the three approaches NRCBA, SRCBAWP and SRCBANP. This comparison has been made and it was shown that though relatively small, the presence of a recoil peak together with a visible binary peak even for an incident electron energy of 2700 eV reproduces the qualitative features of the Ehrhardt geometry \[2\].

In the relativistic regime radiation effects are important and the general rule that can be firmly asserted is the...
following: the instantaneous approach is always smaller than the non instantaneous approach because radiation effects can no longer be ignored. The use of the photon propagator as well as the QED formalism becomes necessary.

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