The well-known Bogoliubov expression for the spectrum of a weakly interacting dilute Bose gas becomes inadequate when the density or interactions strength are increased. The corrections to the spectrum due to stronger interactions were first considered by Beliaev (S. T. Beliaev, Sov. Phys.-JETP, 7:289, (1958)). We revisit Beliaev’s theory and consider its application to a dilute gas with van der Waals interactions, where the scattering length may be tuned via a Fano-Feshbach resonance. We numerically evaluate Beliaev’s expression for the excitation spectrum in the intermediate momentum regime, and we also examine the consequences of the momentum dependence of the two-body scattering amplitude. These results are relevant to the interpretation of a recent Bragg spectroscopy experiment of a strongly interacting Bose gas.

**I. INTRODUCTION**

A dilute Bose Einstein condensate with sufficiently weak interactions is a many body system which can be described to a great degree of success by mean field theory. On the other hand, condensed-matter superfluids such as $^4$He are much more complex systems and difficult to describe from first principles due to their high density and stronger interactions, leading to, in superfluid Helium, a condensate fraction of only about 15% [1]. Superfluid Helium also exhibits the interesting property of a roton minimum in its excitation spectrum, which is absent in a weakly interacting dilute alkali gas. It is therefore of theoretical and experimental interest to try to bridge the two regimes by studying a Bose gas with stronger interactions.

Dilute gases with stronger interactions can be studied by considering perturbation theory corrections to the mean field theory. Lee, Huang and Yang (LHY), and Brueckner and Sawada [2, 3, 4], first determined the quantum depletion and the correction to the chemical potential of the ground state of a homogeneous dilute Bose gas. It was found that the leading corrections are proportional to the dimensionless parameter $\sqrt{n\alpha^3}$ where $n$ is the density and $\alpha$ the scattering length of the atoms. Beliaev [5, 6] first applied the methods of quantum field theory, specifically Green’s functions, to the description of a system of bosons. We shall distinguish between the first order Beliaev’s theory, which is equivalent to the Bogoliubov spectrum for the case of contact interactions, and second order theory, which gives $\sqrt{n\alpha^3}$ corrections to the excitation spectrum. These two levels correspond roughly to first and second order expansions of the Green’s functions of the system in perturbation theory. A description of the quantum field theory of bosons and the Green’s functions technique is available in textbooks [7, 8]. Beliaev’s work thus extends the LHY results for the ground state to the excitation spectrum.

A dilute Bose gas with tunable interactions can be realized experimentally with the utilization of a Fano-Feshbach resonance. A difficulty arises in that the gas becomes more unstable with increasing interaction strength due to 3-body collisions. Nevertheless, in the shorter lifetime of a more strongly interacting gas, it is still possible to observe relatively high frequency excitations (which require short observation time, according to Heisenberg’s uncertainty principle). Such an experiment has been recently performed in JILA [9].

Most studies of a dilute Bose gas assume an effective potential in the form of contact interactions. This model is usually very successful, but a correction is needed to describe excitations with higher momenta, due to the momentum dependence of the scattering amplitude of two colliding atoms. In section II we shall examine how this correction can be incorporated in the first order theory of Beliaev [3].

In section III we revisit Beliaev’s second order theory, now assuming, for that purpose, a momentum independent scattering amplitude. Under this condition, Beliaev derived a closed form integral expression for the excitation spectrum. However, the integrals could only be evaluated analytically in the low and high momentum regimes. It is found that for low momenta (compared to $1/\xi$, where $\xi$ is the healing length), the effect of stronger interactions is an upward shift in the excitation frequency (compared to the Bogoliubov spectrum), while at high momenta, the effect is to lower the excitation frequency. These results were also re-derived using the pseudopotential method [10]. The interesting question that arises is: what happens in the intermediate regime? where is the transition point from upward shift to downward shift? This question is of particular relevance to the experiment [9] which probed the spectrum at this intermediate regime. In this paper, we answer this question by evaluating numerically the relevant expressions. To the best of our knowledge (perhaps somewhat surprisingly), this is the first time that Beliaev’s theory’s prediction for intermediate momenta is examined.

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II. EFFECT OF A MOMENTUM DEPENDENT SCATTERING AMPLITUDE

Beliaev’s original first order theory already incorporated, in general, the effect of a momentum dependent scattering amplitude. Here, we shall take a few more steps to specialize it to the description of a dilute gas with van der Waals interactions. We shall begin with some definitions. We use units $\hbar = 1$ and also let the mass of the bosons be $m = 1$.

Let $n$ be the homogeneous total density of the gas, and $n_0$ the density of the condensed part. In the original paper, Beliaev formulated his theory in terms of $n_0$. At the first order of theory, which we discuss in this section, the two are equal. Quantum depletion, giving rise to $n_0 < n$, arise at the second order of the theory. Let $\Psi_p^\mp(r)$ be a wave-function solution of the Schrödinger equation of the two-body scattering problem with interaction potential $U(r)$, which behaves at infinity like a plane wave $e^{ip \cdot r}$ and an outgoing spherical wave. The scattering amplitude $\tilde{f}(p', p)$ is related to the $\Psi^+$-function by

$$\tilde{f}(p', p) = \int e^{-ip' \cdot r} U(r) \Psi_p^{(+)}(r) dr.$$  \hspace{1cm} (1)

Notice that this definition differs by a numerical factor of $-4\pi$ from the usual one. The usual scattering amplitude is the value of $\tilde{f}(p', p)$ at $|p'| = |p|$ (on the energy shell). Eq. (1) thus generalizes the usual scattering amplitude to an ‘off the energy shell’ amplitude. We shall also need a symmetrized amplitude defined as:

$$\tilde{f}_s(p', p) = \frac{\tilde{f}(p', p) + \tilde{f}(-p', p)}{2}.$$  \hspace{1cm} (2)

According to Beliaev’s first order theory, the excitation spectrum of a dilute Bose gas is given by the dispersion relation:

$$\epsilon_p^{(1)} = \sqrt{\epsilon_p^0 + 2n_0 \tilde{f}_s(p, 0)} - n_0 \tilde{f}(0, 0)^2 - n_0^2 |\tilde{f}(p, 0)|^2.$$  \hspace{1cm} (3)

where $\epsilon_p^0 \equiv p^2/2$ is the free particle kinetic energy. The superscript (1) in $\epsilon_p^{(1)}$ indicates first order. In the case of a gas with van der Waals interactions, it is possible to derive analytic expressions for the scattering amplitude. At low energies, s-wave scattering is the dominant process so that for on shell scattering $p = p'$ we may define an isotropic amplitude $\tilde{f}(p) = \tilde{f}(p', p)$. From the usual definition of the scattering length $a$ we have, $\tilde{f}(0) = f_0 = 4\pi a$.

The scattering amplitude is related to the s-wave phase shift $\delta_0(p)$ as

$$\tilde{f}(p) = \frac{-4\pi}{p \cot \delta_0(p) - i}.$$  \hspace{1cm} (4)

From the standard effective-range expansion, $p \cot \delta_0(p) = -\frac{1}{2} + \frac{a}{2} r_e p^2 + \ldots$, where $r_e$ is the effective range. Thus

$$\tilde{f}(p) = \frac{4\pi a}{1 - \frac{a}{2} r_e p^2 + iap}.$$  \hspace{1cm} (5)

For neutral atoms with van der Waals interactions, the following useful relationship exists between the scattering length $a$, the effective range $r_e$, and the $C_6$ coefficient:

$$r_e/\beta_6 = \left( \frac{2}{3x_e} \right) \frac{1}{(a/\beta_6)^2} \left[ 1 + \left( 1 - x_e(a/\beta_6)^2 \right) \right],$$  \hspace{1cm} (6)

where $\beta_6 = (mC_6/\hbar^2)^{1/4}$ is a length scale associated with the van der Waals interaction, and $x_e = [\Gamma(1/4)^2 / (2\pi)]$, with $\Gamma$ being the usual Gamma function. For clarity, we have written here explicitly factors of $\hbar$ and $m$. Typical values of $\beta_6$ for alkali atoms are about 100 bohr.

By extending the ideas of [11], we find that the off-shell amplitude $\tilde{f}(p, 0)$ which appears in Eq. (3), is given, for a van der Waals interaction, by:

$$\tilde{f}(p, 0) = 4\pi a (1 + \frac{1}{2} p^2 + \ldots),$$  \hspace{1cm} (7)

with

$$b = -\frac{1}{30} \beta_6^2 \sqrt{\pi} \left( -5\sqrt{2}/\beta_6/a + 6 \frac{\Gamma(9/4)}{\Gamma(7/4)} \right).$$  \hspace{1cm} (8)

We have checked this analytic expression for the off shell amplitude with numerical calculations using a model potential of $^{85}$Rb, with good agreement.

A few notes are in place. First, the appearance of the factor $\frac{1}{2}$ in the expression $\tilde{f}_s(p, p)$ is due to moving to the center of mass system of an excited particle with momentum $p$ and a condensate particle with zero momentum. Second, in using Eq. (3) with Eq. (4), one should note that the scattering amplitude has an imaginary part. This gives rise to an imaginary part of the energy describing correctly, for high momentum, the decay of a particle excitation due to the scattering of the particle off the rest of the condensate’s atoms. However at low momenta (the quasi-particle phonon regime), the decay is actually suppressed, and is only seen in the next level of theory (Beliaev’s second order theory). At first order of the theory it is sufficient to take the real part of the scattering amplitude when using it to calculate the spectrum. The off shell scattering amplitude $\tilde{f}(p, 0)$ is always real.

for $pa \ll 1$, eq. (3) reduces to the familiar Bogoliubov spectrum:

$$\epsilon_p^{(1)} = \sqrt{(\epsilon_p^0)^2 + 2n_0 f_0 \epsilon_p^0}.$$  \hspace{1cm} (9)

By increasing $pa$ one enters the regime where finite-momentum corrections are necessary. For a fixed $p$, this could happen due to increasing $a$. If the scattering length is larger than the van der Waals length scale, i.e. $a > \beta_6$, then according to Eq. (6), $r_e \approx 1.4 \beta_6$, while according to Eq. (8), $b \approx 0.44 \beta_6^2$. If the condition $p \beta_6 \ll 1$ is still satisfied (which is normally the case), then the correction to the off shell amplitude $\tilde{f}(p, 0)$ is very small and one can take it to be equal to $a$. Furthermore, in Eq. (5), the
effective range contribution due to non-zero $r_e$ is smaller by a factor $p/\beta_0$ than the universal universal factor $ipa$. Therefore, we arrive in a final universal momentum dependent dispersion relation, valid for $p/\beta_0 \ll 1$:

$$\epsilon_p^{(1)} = \sqrt{\left[ \frac{\epsilon_p^0}{1 + (pa)^2/4} - 4\pi na \right]^2 - n_G^2(4\pi a)^2}.$$  

(10)

At this level of the theory, we may replace $n_0$ in Eq. (10) with $n$. We define an energy shift $E = \epsilon_p^{(1)} - \epsilon_p^0$ which is the difference between the excitation energy of a quasi-particle with momentum $p$ in the condensate, and the kinetic energy of a free particle with the same momentum. In Fig. 1 we plot the energy shift as a function of the dimensionless parameter $p/\xi$ where $\xi = \frac{1}{\sqrt{4\pi na}}$ is the healing length. The shift is shown for different values of $\frac{a}{\xi} = \sqrt{8\pi na^3}$. The curve corresponding to $a/\xi = 0$ should be interpreted as taking the limit of $\sqrt{4\pi na} \to 0$ (notice also that the vertical axis is proportional to $na$), and gives the usual Bogoliubov spectrum, with the asymptotic shift value of $4\pi na$ for large momentum. Increasing $a/\xi$ gives rise to observable deviations at high momentum. At small momenta the shift is linear in momentum and the slope gives the speed of sound in the condensate. Since the change in the real part of the scattering amplitude is proportional to $p^2$, the slope of all curves is the same at small momenta, and the speed of sound is unchanged by this effect.

III. EFFECT OF QUANTUM DEPLETION

The second order of Beliaev’s theory is associated with the LHY quantum depletion, but is concerned mainly with the excitation spectrum rather than the ground state, and includes other second order processes such as Beliaev’s damping (the decay of a quasi-particle into two lower energy quasi-particles). Beliaev summed the relevant Feynman diagrams which contribute to the Green’s function of the system up to second order. The excitation spectrum is then found by the poles of the Green’s function in the $(p, p^0)$ space, with $p$ momentum and $p^0$ energy. Mohling and Sirlin [10] arrived at the same excitation spectrum using a more straightforward application of the LHY pseudopotential method. We shall briefly state here the results in the language of Green’s functions [7, 8]. In this section we use the 4-momentum notation $p = (p, p^0)$. In a Bose condensate there are two Green’s functions, a regular Green’s function $G_{11}(p)$, and an anomalous Green’s function $G_{12}(p)$. Correspondingly, there are two self-energies, a regular self-energy $\Sigma_{11}(p)$ and an anomalous self-energy $\Sigma_{12}(p)$. Also, we have the free particle Green’s function $G^0(p)$. The Dyson’s equations of this system can written in the matrix form:

$$G(p) = G^0(p) + G^0(p)\Sigma(p)G(p),$$

(11)

with

$$G(p) = \begin{bmatrix} G_{11}(p) & G_{12}(p) \\ G_{12}(-p) & G_{11}(-p) \end{bmatrix},$$

(12)

$$G^0(p) = \begin{bmatrix} G^0(p) & 0 \\ 0 & G^0(-p) \end{bmatrix},$$

(13)

$$\Sigma(p) = \begin{bmatrix} \Sigma_{11}(p) & \Sigma_{12}(p) \\ \Sigma_{12}(-p) & \Sigma_{11}(-p) \end{bmatrix}.$$  

(14)

The self-energies can be calculated in perturbation theory using Feynman’s diagrams. Then, the Green’s functions can be obtained by solving the algebraic matrix equation (11), and their poles give the excitation spectrum (both kinds of Green’s functions have the same poles). Following the original papers, we assume in this section (in contrast to the previous one) a momentum independent scattering amplitude, valid for $pa \ll 1$, i.e, $f_0 = 4\pi a$. The first order of theory gives:

$$\Sigma_{11}(p) = 2n_0f_0$$

(15)

$$\Sigma_{12}(p) = n_0f_0.$$  

(16)

When these expressions are used to calculate the excitation spectrum via the poles of the corresponding Green’s functions, one retrieves the ordinary Bogoliubov spectrum, Eq. (9).

The second order contributions (additive to the first order), written $\Sigma_{11}^{(2)}$ and $\Sigma_{12}^{(2)}$, are:
\[ \Sigma^{(2)}_{12} (p) = \frac{1}{2} n_0 f_0^2 \frac{1}{(2\pi)^3} \int \frac{dq}{\epsilon_q} \epsilon_k^{(1)} R(q,k) \times \]

\[ \left[ \begin{array}{c}
\frac{1}{p^0 - \epsilon_q^{(1)} + i\delta} - \frac{1}{p^0 + \epsilon_q^{(1)} - i\delta} \\
\frac{1}{\pi^2 \sqrt{n_0 f_0^2 n_0 f_0}}
\end{array} \right] + \]

\[ \frac{1}{\pi^2 \sqrt{n_0 f_0^2 n_0 f_0}}, \]

\[ \Sigma^{(2)}_{11} (p) = \frac{1}{2} n_0 f_0^2 \frac{1}{(2\pi)^3} \int \frac{dq}{\epsilon_q} \epsilon_k^{(1)} \times \]

\[ \left[ \begin{array}{c}
\frac{Q^- (q,k)}{p^0 - \epsilon_q^{(1)} + i\delta} - \frac{Q^+ (q,k)}{p^0 + \epsilon_q^{(1)} - i\delta} + \epsilon_q^{(1)} + \epsilon_k^{(1)} \\
\frac{8}{3\pi^2 \sqrt{n_0 f_0^2 n_0 f_0}}
\end{array} \right], \]  

(17)

where \( \mathbf{k} = \mathbf{p} - \mathbf{q} \), and the functions \( R, Q^\pm \) are:

\[ R(q,k) = 2 \epsilon_q^{(1)} \epsilon_k^{(1)} + n_0 f_0^2, \]

(18)

\[ Q^\pm (q,k) = 3 \epsilon_q^{(1)} \epsilon_k^{(1)} + n_0 f_0 (\epsilon_q^{(1)} + \epsilon_k^{(1)}) + n_0 f_0^2 \pm \left[ n_0 f_0 (\epsilon_q^{(1)} + \epsilon_k^{(1)}) - \epsilon_q^{(1)} \epsilon_k^{(1)} \right]. \]

The \( i\delta \) factors in Eq. (17) are needed for convergence and should be understood in the usual sense of the limit \( \delta \to 0^+ \). By solving the Dyson’s matrix equation with these self-energies, the Green function \( G_{11} \) can be expressed in the convenient form (showing its poles):

\[ G_{11} (p) = \frac{A_p}{p^0 - \epsilon_p^{(1)} - \Lambda_-(p)} - \frac{B_p}{p^0 + \epsilon_p^{(1)} + \Lambda_+(p)}, \]

(19)

where \( A_p \) and \( B_p \) are certain functions of \( p \) (with no poles), and \( \Lambda^\pm (p) \) are second-order corrections

\[ \Lambda^\pm (p) = \frac{\epsilon_p^{(1)} - (\Sigma_{11}(p) + \Sigma_{11}(-p) - 2\mu)^{(2)}}{2\epsilon_p^{(1)}} + \frac{n_0 f_0}{2\epsilon_p^{(1)}} (\Sigma_{11}(p) + \Sigma_{11}(-p) - 2\mu - 2\Sigma_{12}(p))^{(2)} + \]

\[ \pm \frac{1}{2} (\Sigma_{11}(p) - \Sigma_{11}(-p))^{(2)}, \]

(20)

with \( \mu^{(2)} \) the second order correction to the chemical potential,

\[ \mu^{(2)} = (5/3\pi^2) \sqrt{n_0 f_0^2 n_0 f_0}. \]

(21)

Inspecting the poles of the Green’s function, the excitation spectrum in this approximation is then given by:

\[ \epsilon_p = \epsilon_p^{(1)} + \Lambda^- (p; \epsilon_p^{(1)}). \]

(22)

In general, there is no known analytic solution to the integrals in Eq. (17). Beliaev derived an analytic solution in the limit of small momenta \( p \xi \ll 1 \):

\[ \epsilon_p = p \sqrt{n_0 f_0} \left( 1 + \frac{1}{\pi^2} \sqrt{n_0 f_0^2} \right) - \frac{i}{640\pi n} \left( 3p^5 \right) \]

(23)

This result is expressed here in terms of the total density \( n \) rather than the condensate density \( n_0 \). The relation between \( n \) and \( n_0 \) is:

\[ n - n_0 = n_0 \sqrt{n_0 f_0^3/(3\pi^2)}. \]

(24)

According to Eq. (23), the second order effects give rise to an upward shift in the excitation energy with linear dependence in momentum, and thus a shift in the velocity of sound:

\[ c = \sqrt{f_0 n} \left[ 1 + \frac{1}{\pi^2} (n f_0^3)^{1/2} \right]. \]

(25)

This result for the speed of sound, derived above from the slope of the one-particle excitation spectrum at low momentum, is identical to that obtained by considering only the LHY expression for the ground state energy:

\[ E = \frac{V}{2m} \left[ 1 + \frac{1}{15\pi^2} (n f_0^3)^{1/2} \right], \]

(26)

where we explicitly included factors of \( m \) and \( h \), and using the thermodynamic relations \( P = -\left( \frac{\partial E}{\partial V} \right)_N \) for the pressure \( P \), and \( mc^2 = \frac{\partial E}{\partial m} \). Thus, from the macroscopic thermodynamic point of view, the increase in speed of
sound is traced to the equation of state becoming stiffer due to quantum depletion. Eq. (23) also has an imaginary part which is due to the decay of one quasi-particle into two other quasi-particles.

On the other hand, Mohling and Sirlin [10] give an expression for the shift at high momenta, with \( p\xi \gg 1 \), (but, at the same time, \( pa \ll 1 \)). The result may be written in the form:

\[
\epsilon_p = \epsilon_p^0 + 2n\tilde{f}_0 - \mu_{LHY} - \frac{i}{2} \mu_{LHY} \sigma \left( \frac{p}{2} \right), \quad (p\xi \gg 1),
\]

with the LHY chemical potential:

\[
\mu_{LHY} = n\tilde{f}_0 \left( 1 + \frac{4}{3\pi^2} \sqrt{n\tilde{f}_0^3} \right),
\]

and the cross section for two identical bosons

\[
\sigma = 8\pi a^2.
\]

We may understand the result of Eq. (27) as follows: the excitation energy is the difference between final state and initial state energies. The initial state energy of the (quasi-)particle is \( \mu \), and the final state energy at high momentum is the free kinetic energy \( \epsilon_p^0 \) plus the sum of direct and exchange interactions of the excited particle with all other, low momentum particles, \( 2n\tilde{f}_0 \). In addition, the excitation decays due to collision of the high momentum particle with all other particles, with cross section \( \sigma \) and center of mass momentum \( p/2 \). Note, that Eq. (27) only considers the high momentum (\( p\xi \gg 1 \)) effect due the many body interactions. The correction due to the momentum-dependent scattering amplitude, as described by Eq. (10) can be neglected here as long as \( pa \ll 1 \). The required conditions, \( 1/\xi \ll p \ll 1/a \), can in principle be satisfied simultaneously for a sufficiently dilute gas.

What happens in the intermediate range, \( p\xi \approx 1 \)? We have calculated numerically the excitation energy Eq. (22). This was done by deforming the path of integration from the real line into the complex plane around the singularities indicated by the \( i\delta \) factors. The result is shown in Fig. (2). The plot shows the quantity \( \Re(\epsilon_p - \epsilon_p^{(1)}) \), i.e the difference between the real part of Beliaev’s excitation energy and the Bogoliubov energy. At low momenta there is an upward linear shift consistent with Eq. (23). This linear shift is consistent with the correction to the speed of sound derived by LHY theory. The linear behavior holds up to \( p\xi \approx 0.6 \). At high momentum there is a down shift with very slow \( 1/p \) convergence towards the asymptotic value \( \frac{4}{3} \tilde{f}_0 n \sqrt{\frac{\tilde{f}_0^3 n}{\pi^2}} \) consistent with Eq. (27). In between, we find the transition point at \( p \approx 6.18 \xi^{-1} \), where the second order contribution is zero, so that at this point the excitation energy to second order is identical with the first order Bogoliubov’s theory.

The imaginary part of the excitation energy gives rise to quasi-particle damping with an effective cross-section \( \sigma_p = -2\Im(\tilde{f}_0) \) ranging from 0 at \( p = 0 \) to \( 8\pi a^2 \) at \( p\xi \gg 1 \). It is shown in Fig. (3). This damping cross-section has been calculated before using another method and measured in experiment [12]. The two methods of calculation give identical results.

### A. Discussion and Conclusions

In this paper we have revisited Beliaev’s theory with emphasis on two mechanisms for a shift in the excitation spectrum of a Bose gas from the usual Bogoliubov theory. One is due to the momentum dependence of scattering amplitude, when the condition \( ka \ll 1 \) is not valid, which generally causes a downward shift in the excitation frequency at higher momentum, as demonstrated in
Fig. (1). The other is a purely many body effect, occurring when the condition $\sqrt{8\pi \hbar a^3} \ll 1$ is not obeyed. This effect is associated with quantum depletion due to interactions as well as with quasi-particle decay. As shown in Fig. (2). It gives rise to upward shift at low momentum and downward shift at high momentum.

We note that the two separate effects were calculated under exclusive conditions. The effect due to momentum dependent scattering amplitude ($pa \approx 1$) was calculated in first order of the interactions, valid for $\sqrt{8\pi \hbar a^3} \ll 1$. On the other, the second order effects describing corrections assumed $pa \ll 1$. Currently, there is no known result that holds in the regime where both the momentum dependence of the scattering amplitude and quantum depletion due to strong interactions are important. The physical interpretation that follows Eq. (27) suggests that at high enough momentum the shift due to these two effects should be additive and combine to give

$$
\epsilon_p = c_p^0 + 2n\Re \left[ f_s(p/2) \right] - \mu_{LHY} - \text{in} \frac{|p|}{2} (p\xi \gg 1).
$$

This expression assumes the asymptotic value of the energy shown in of Fig. (2). Because of the slow convergence, even at $p\xi = 20$ the energy is still only 75% of the asymptotic value. The Bragg spectroscopy experiment in Ref. [9] explored the regime where both effects discussed above are important, and showed evidence for a downward shift in the excitation energy of a strongly interacting gas. But for the measurements with the strongest interactions, which showed the largest shifts, the experiment had $p\xi \approx 2$, where Fig. (2) shows upward rather than downward shift. On the other hand, the prediction of down shift due to the 2-body momentum dependence of the scattering length seems to go some way towards explaining the observations.

Finally, we note that the dynamic structure factor is a quantity more directly related to the actual measurement process of a Bragg experiment [14, 15]. A partial attempt at incorporating second order effects due to strong interactions in the dynamic structure factor is found in Ref. [16]. It would be of interest to complete this program.

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