Knowledge of damage identification about tensegrities via flexibility disassembly

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Abstract. Tensegrity structures composing of continuous cables and discrete struts are under tension and compression, respectively. In order to determine the damage extents of tensegrity structures, a new method for tensegrity structural damage identification is presented based on flexibility disassembly. To decompose a tensegrity structural flexibility matrix into the matrix representation of the connectivity between degrees-of-freedoms and the diagonal matrix comprising of magnitude informations. Step 1: Calculate perturbation flexibility; Step 2: Compute the flexibility connectivity matrix and perturbation flexibility parameters; Step 3: Calculate the perturbation stiffness parameters. The efficiency of the proposed method is demonstrated by a numerical example comprising of 12 cables and 4 struts with pretensioned. Accurate identification of local damage depends on the availability of good measured data, an accurate and reasonable algorithm.

1. Introduction
As we know that tensegrity structures consists of continuous cables and discrete struts for almost half a century [1-2]. Most tensegrity structures continuously accumulate loads under services [3]. Fabbrocino et al. via tensegrity prisms researched the wave dynamics of highly nonlinear tensegrity metamaterials [4].

In order to assure safety, via directly the incomplete modal parameters Wu et al. researched structural damage [5]. Based on the optimization solution method Wong et al. proposed an iterative method for structural damage detection [6]. Lam et al. took into consideration the eigenparameters sensitivity analysis to analyze the damage situation [7]. Yang made use of flexibility sensitivity analysis to evaluate the extents of the damage elements [8].

This paper presents a novel via flexibility disassembly theory for structural damage identification. The proposed method decompose structural flexibility matrix into the connectivity matrix and the diagonal matrix. The proposed method can precisely reckon the element perturbation stiffness parameter only making use of a few of lower-frequency incomplete modes without any higher-order approximation or iteration.

Technological process as follows. In Part Two, the structural stiffness and flexibility matrix disassembly is simply referred. In Part Three, a novel damage identification method is recommended particularly. In Part Four, applying the example to prove the effectiveness and accuracy. In Part Five, the conclusions of the operations are depicted.
2. Theoretical development

In this chapter, via the structural mechanism a novel flexibility disassembly method is proposed.

\[ K = MPMT \] (1)

in which matrix \( M \) knowed as stiffness connectivity matrix and the matrix \( P \) knowed as property matrix. The number of degrees-of-freedoms are defined as \( n \) and the number of element stiffness parameters are defined as \( N \). The \( i \) th element stiffness perturbation parameter is supposed as \( \alpha_i (0 \leq \alpha_i \leq 1) \). If the value of \( \alpha_i \) shows less than or equal to one, it means \( i \) th element is partially or completely damaged.

\[ K_d = MP_dM^T \] (2)

\[ \Delta K = M\Delta PM^T \] (3)

\[ \Delta P = \text{diag}(p_1\alpha_1, p_2\alpha_2, \cdots, p_N\alpha_N) \] (4)

The two different cases need to be discussed in the following parts.

2.1. Case 1 \( n = N \)

The \( F \) and \( F_d \) can be calculated as

\[ F = (M^{-1})^TP^{-1}M^{-1} \] (5)

\[ F_d = (M^{-1})^TP_d^{-1}M^{-1} \] (6)

That is \( F \cdot K = F_d \cdot K_d = I_{n,n} \). Letting \( Q = P^{-1} \), \( Q_d = P_d^{-1} \).

\[ P = (M^{-1})^T \] (7)

There are different expressive styles for Eqs. (5) and (6)

\[ F = PQP^T \] (8)

\[ F_d = PQ_dP^T \] (9)

According to presents, the \( b_i \) is knowed as the \( i \) th flexibility parameter \( (b_i = p_i^{-1}) \). Subtracting Eq. (8) from Eq. (9).
\[ \Delta F = PAQ^T \]  

\[ \Delta Q = Q_d \cdot Q = [Q_1, Q_2, \ldots, Q_n] \]  

where \( Q_i = \left[ \cdots, b_1 \left( \frac{\alpha_i}{1-\alpha_i} \right), \cdots, \right]^T \).

Define the \( i \)th element flexibility perturbation parameter as \( \beta_i \).

\[ \beta_i = \frac{\alpha_i}{1-\alpha_i} \]  

From Eq. (12), the relations of \( \alpha_i \) and \( \beta_i \) as follows:

\[ \beta_i = \frac{1}{1-\alpha_i} - 1 \]  

or,

\[ \alpha_i = \frac{\beta_i}{1 + \beta_i} \]  

Correspondently, only the exact \( \Delta F \) is completed, can we gain the \( \alpha_i \). The proposed methods for calculating the perturbation parameters are precise without any higher-order sensitivity analysis or iteration.

2.2. Case 2 \( n > N \)

\[ \Delta F = G AQ^T \]  

Considering that the rank deficiency of matrix \( M \) is zero.

\[ G = (MM^T)^{-1} M \]  

3. Damage identification

*Step One:* Calculate \( \Delta F \). Via the first few low-frequency modes, the exact \( \Delta F \) can be calculated approximately [9].

\[ \Delta F = \sum_{j=1}^{m} \frac{1}{\lambda_{ij}} f_{ij} f_{ij}^T - \sum_{j=1}^{m} \frac{1}{\lambda_{ij}} f_{ij} f_{ij}^T \]
Where $m$ are defined as the number of measured modes in modal survey, the eigenparameters of the undamaged and damaged structures are defined as $\lambda_{uj}(f_{uj})$ and $\lambda_{dj}(f_{dj})$, respectively.

**Step Two:** Making use of Eq. (7) to calculate the $P$ (for the Case 1) or by Eq. (16) (for the Case 2); Making use of Eq. (10) to calculate the $\beta_i$ (for the Case 1) or by Eq. (15) (for the Case 2).

**Step Three:** Making use of Eq. (14) to calculate the stiffness perturbation parameters $\alpha_i$.

### 4. The numerical example

In this part, based on the Matlab (2014b) a numerical example is carried out to demonstrate the effectiveness and accuracy of the proposed novel approach. **Figure 1.** shows that the model used in this application comprises of two modules, with twelve cables and four struts, the parts of this quadruplex unit nodes are fixed and pined, respectively. Assume that only the nine modes are used to calculate the flexibility matrix change $\Delta F$. Two damage scenarios were considered. In the first scenario, only cable 3 is assumed to be damaged with a stiffness loss of 50%. The second scenario has two damages in which cables 3 and 9 have 50% and 50% reduction in stiffness, respectively.

Here, stiffness matrix and connectivity matrix are showed partially.

\[
K = 10^6 \begin{bmatrix}
2.0119 & 0 & 0 & \cdots & 1.7727 & -0.8754 & 0.8864 \\
\vdots & 3.2678 & \vdots & \vdots & \vdots & 0 & 0 & 0 \\
\vdots & 1.6728 & 0 & -0.7975 & -1.7727 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & 0.8864 & -0.7975 & \vdots & \vdots \\
0 & 0 & \vdots & \vdots & -0.7865 & \vdots & \vdots \\
1.7727 & 0 & 0 & \vdots & \vdots & \vdots & \vdots & -1.7727 \\
-0.8754 & 0 & 0 & \vdots & \vdots & \vdots & \vdots & 3.2678 & -1.6728 \\
0.8864 & 0 & 0 & \vdots & \vdots & \vdots & \vdots & -1.6728 & 1.6948 \\
\end{bmatrix}
\]

**Figure 1.** Quadruplex unit (a) Top view, (b) Perspective view

For the first damage case, the exact element damage extent of elements is shown in **Figure 2.** By using Eq. (14), the algebraic solution of the damage extent can be obtained as 0.483542 (3.29%). The value in bracket denotes the comparative error between the calculated value and the assumed value. According to the above results of the based on flexibility disassembly methods, conclusion can be
easily drawn that the proposed based on flexibility disassembly method remarkably well for damaged structural survey.

Table 1. Properties of the quadruplex unit

| Parameter                              | Value         |
|----------------------------------------|---------------|
| Cross section of struts, $A_s$         | $0.325 \times 10^{-4} m^2$ |
| Cross section of cable, $A_c$          | $0.28 \times 10^{-4} m^2$ |
| Young’s modulus of struts, $E_s$       | 200 GPa       |
| Young’s modulus of cables, $E_c$       | 40 GPa        |
| Pre-stress in struts ($-2q$)           | -13.4722 KN   |
| Pre-stress in lower cables ($q$)       | 5500 N        |
| Pre-stress in upper cables ($2q$)      | 7.7781 KN     |
| Pre-stress in bracing cables ($2q$)    | 7.7781 KN     |

Note that $q$ is the self-stress coefficient, $q = 5.5 \text{ KN/m}$.  

\[
M = \begin{bmatrix}
0 & 0 & \cdots & -1 & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
-0.4082 & 0 & \cdots & 0 & \cdots & 0 & \cdots \\
0 & 0 & \cdots & 0 & \cdots & 0 & \cdots \\
0 & 0 & \cdots & 0 & 0 & 0.7071 & 0 \\
0 & 0 & \cdots & 0 & 0 & 0.7071 & 0 \\
\end{bmatrix}
\]  

(19)

Figure 2. Single element damage extent
For the second damage case, the exact element damage extent of elements is shown in Figure 3. By using Eq. (14), the algebraic solution of the damage extent can be obtained as 0.484836 (3.03%) and 0.482533 (3.49%), respectively. The value in bracket denotes the comparative error between the calculated value and the assumed value. According to the above results of the based on flexibility disassembly methods, conclusion can be easily drawn that the the proposed based on flexibility disassembly method remarkably well for damaged structural survey.

The example indicates the flexibility disassembly method is avail for damaged structural analysis and research well. Accurate identification of local damage depends on the availability of good measured data, an accurate and reasonable algorithm.

5. Conclusion

Based on a novel flexibility disassembly technique for identifying structural damage has been researched in this study. The proposed procedure has two significant advantages. The first is that making use of incomplete modal parameters without any eigenvector expansion or model reduction. The second is that without any higher-order approximation or iteration. The proposed procedure may be a promising method in structural damage identification. Larger occupation of prestress and experimentally measured data will be took into account to demonstrate the procedure in the futural technical research.

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