On Weak Dress Codes for Cloud Storage

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Abstract—In a distributed storage network, reliability and bandwidth optimization can be provided by regenerating codes. Recently table based regenerating codes viz. DRESS (Distributed Replication-based Exact Simple Storage) codes have been proposed which also optimizes the disk I/O. Dress codes consists of an outer MDS code with an inner fractional repetition (FR) code with replication degree $\rho$. Several constructions of FR codes based on regular graphs, resolvable designs and bipartite graphs are known. This paper presents a simple modular construction of FR codes. We also generalize the concept of FR codes to weak fractional repetition (WFR) codes where each node has different number of packets. We present a construction of WFR codes based on partial regular graph. Finally we present a simple generalized ring construction of both strong and weak fractional repetition codes.

I. INTRODUCTION

Cloud computing has emerged as a fascinating area of computing providing various services such as on demand videos, music, computing etc. to end users from any computing device. Usually data of the user is stored on different data centers in a distributed fashion. Many such commercial cloud services are available from Amazon, Google and Apple etc. [1], [2], [3]. To provide seamless end user experience for different applications such as Gmail etc. one has to optimize various parameters of the storage system such as reliability, scalability, storage, bandwidth, disk I/O, etc. In such a distributed storage system (DSS) data is stored on a set of nodes in cloud network, which are not reliable independently. Redundancy is added in network to increase the reliability of the system. Google File System (GFS) is one such system which uses replication of data into various chunk servers (large data chunk files) together with replication of logs and master index for reliability [4]. Although this scheme is simple but inefficient in terms of utilization of various storage system parameters like bandwidth. One can provide reliability to such systems using erasure coding which uses an MDS (Maximum Distance Separable) code [5] however these codes fails to optimize bandwidth, complexity and disk I/O. To optimize these parameters recently regenerating codes were introduced and studied in detail by many researchers [6], [7], [8], [9], [10], [11].

Consider a distributed storage system (DSS) consisting of $n$ nodes, where each node can store $\alpha$ symbols from $\mathbb{F}_q$ (finite field with $q$ number of elements). A file size of $B$ symbols can be stored in such a DSS either using simply a replication (as it is done currently in many cloud storage system) of packets or by using an MDS code such that the file can be retrieved using erasure codes by contacting any $k$ nodes out of those $n$ nodes in network ($k < n$) [12]. When a node fails, a new code can be generated (known as repair of a failed node). In a simple replication system one can download it from any other node having a copy of the data. This is good bandwidth wise and disk I/O wise but it can tolerates only few errors as compare to an erasure code system (where a replacement node can contact to $k$ nodes and extract the data of that failed node) which can tolerates more failures of nodes but it is inefficient for bandwidth and disk I/O. To minimize the bandwidth, the concept of regenerating code was introduced by Dimakis et al. [6]. In regenerating codes instead of downloading whole file $B$ (by contacting all $k$ nodes), a node can be repaired by contacting $d$ nodes ($d$ is known as the repair degree) and downloading $\beta$ packets from each of them out of $\alpha$ packets which are stored on each node. This new generated data is functionally equivalent to lost data. Thus repair bandwidth becomes $d\beta$. Regenerating codes over $\mathbb{F}_q$ are characterized by parameters: $[\mathbb{F}_q, [n, k, d], [\alpha, \beta, B]]$. Bandwidth can be reduced further if each node is willing to store more [7]. Thus there is a tradeoff between storage ($\alpha$) and repair bandwidth ($d\beta$). For internet applications one can search for minimum bandwidth repair (MBR) regenerating codes and for archival applications one can search for minimum storage repair (MSR) codes. These are two extremal points on the storage-repair bandwidth tradeoff curve [13], [7]. For regenerating codes it is well known that

$$k \leq d \leq (n - 1).$$

(1)

Cut-set bound of network coding gives the following relation between the parameters of regenerating codes.

$$B \leq \sum_{i=0}^{k-1} \min\{\alpha, (d-i)\beta\}.$$ 

(2)

Clearly we want to minimize both $\alpha$ as well as $\beta$ since minimizing $\alpha$ leads to minimum storage solution and minimizing $\beta$ leads to a solution which minimizes repair bandwidth. There is a trade off between choices of $\alpha$ and $\beta$ as shown in [14].

For regenerating codes to be optimal, equality holds in (2) and for the MBR (Minimum Bandwidth Regeneration) point, the parameter of the code must satisfy (3) (see [14]).

$$B = \frac{kd - (k(k - 1))}{2}\beta.$$ 

(3)

For $\beta = 1$, the storage capacity (the limit on the maximum file size that can be delivered to any user contacting any $k$ out
of $n$ nodes) in a DSS is given by [15]

$$C_{MBR}(n, k, d) = kd - \binom{k}{2}.$$  

Constructions of regenerating codes for $\beta = 1$ achieving storage capacity [4] are of special interest. The problem of regeneration becomes harder if one wants to have an exact repair of the failed node. Rashmi et al. presented an explicit construction of exact-Minimum Bandwidth Repair (MBR) codes for the parameter set $[n, k = n - 2, d = n - 1]$ by fully connected graph [15]. An explicit MBR code is also given for all feasible values of the system parameters $[n, k, d]$ and MSR codes for all $2k - 2 \leq d \leq n - 1$ based on a common product matrix framework [15]. El Rouayheb et al. [12] has shown the construction of optimal regenerating codes (called Fractional Repetition Codes) by using table-based repair model, which is basically a generalization of the codes given in [15]. El Rouayheb et al. constructed the regenerating codes for single node failure based on regular graph and multiple node failure based on Steiner systems. These codes are fast and low-complexity repair codes. It simply reads only one of its stored packet and forward it to the replacement node with no additional processing. This property is referred as uncoded repair. By combining Fractional Repetition code with an MDS code a concept of DRESS (Distributed Replication-based Exact Simple Storage) codes has been introduced in [6]. It was an open problem to construct FR codes (and hence dress codes) beyond Steiner systems. Several constructions of Fractional Repetition Codes (and hence dress codes) are known based on bipartite graph [16], resolvable designs [17] and regular graphs [12, 18]. In this paper, we present the extension of the table based method presented in [12] for single node failure. We call it weak fractional repetition codes (and hence weak dress codes). The construction in this paper gives the regenerating code for the values of $n$ and $d$ for which regular graph does not exist. We also present some general constructions of weak dress codes.

The rest of the paper is organized as follows. In Section 2, we summarize popular FR code generation given by El Rouayheb et al. [12] for single node failure. In Section 3, we provide a simple construction of FR code for $\rho = d$ and $n = \theta$. A construction of weak fractional repetition code from partial regular graph is given in Section 4. A ring construction is given in Section 5 which provides both weak and strong fractional repetition codes. Finally Section 6 concludes with general remarks.

II. PREVIOUS WORK

El Rouayheb et al. [12] presented the table based regenerating codes to reduce complexity. This construction consists of containing an outer MDS code with an inner fractional repetition code with repetition degree $\rho$ that can tolerate up to $\rho - 1$ nodes failing together. In this section, we summarize the results of [12] required for our purpose and also give an example of FR code based on regular graph.

**Definition 1.** (Fractional Repetition Code) : A Fractional Repetition (FR) code $C$, with repetition degree $\rho$, for an $(n, k, d)$ DSS, is a collection $C$ of $n$ subsets $U_1, U_2, \ldots, U_n$ of a set $\Omega = \{1, \ldots, \theta\}$, each having size $d$, i.e., $|U_i| = d$, satisfying the condition that each element of $\Omega$ belongs to exactly $\rho$ sets in the collection.

As MDS code exists for any parameters, the challenging part is to get an FR code. In [12], FR codes of repetition degree 2 are designed with an uncoded repair based on regular graph which can tolerate at most one node failure. For all feasible values of $n$ and $d$, these codes achieves the capacity $C_{MBR}$. Either of $n$ and $d$ must be even to get these codes. In fact this is the necessary and sufficient condition for constructing codes from this method [12].

**Proposition 1.** The parameter $\theta$ in Def. [7] of an FR code of degree $\rho$ for an $(n, k, d)$ DSS is given by,

$$\theta = nd.$$  

**Theorem 1.** A projective plane of order $m$ gives dress codes with $n = m^2 + m + 1$ and $\rho = d = m + 1$ by taking points as packets and lines as storage nodes.

**Theorem 2.** A Steiner system $S(2, \alpha, \nu)$ gives dress codes with parameters $\rho = \frac{(\nu - 1)}{r - 1}$ and $n = \frac{m^2}{\alpha}$ using lines as storage nodes and points as packets.

In order to understand our work we now describe briefly the construction of FR codes based on regular graph [12].

**Definition 2.** (Regular Graph) : A regular graph is a graph where each vertex has the same number of neighbors; i.e. every vertex has the same degree. A $d$-regular graph $R_{n,d}$ of $n$ vertices is the regular graph where all vertices have the same degree $d$. Thus graph $R_{n,d}$ has $\frac{nd}{2}$ vertices.

**Construction 1.** To construct a FR code with repetition degree $\rho = 2$ and with nd even, Algorithm [7] is well known [12].

**Algorithm 1** FR code construction with $\rho = 2$ for $(n, k, d)$ DSS, where nd is even

1) Generate a $d$-regular graph $R_{n,d}$ on n vertices $U_1, U_2, \ldots, U_n.$
2) Index the edges of $R_{n,d}$ from 1 to $\frac{nd}{2}$.
3) Store on node $U_i$ in the DSS the packets indexed by the edges that are adjacent to vertex $U_i$ in the graph.

**Example 1.** Consider a DSS $(n = 6, k = 4, d = 5)$ whose storage capacity is 14 according to [4]. Let $F_q$ be the finite field of size $q$. File $x = (x_1, x_2, \ldots, x_{14}) \in F_{14}$ has 14 packets, which we want to store on the system. Figure [7] shows a regenerating code which can save the file of 14 packets. Regenerating code in Figure [7] is made of two components--an outer MDS code (15, 14) and a repetition code based on a regular graph shown in Figure [2]. When we give file $x$ as a input to the outer MDS code, we get $y = (y_1, y_2, \ldots, y_{15})$ as output, where $y_1, y_2, \ldots, y_{14}$ are same as $x_1, x_2, \ldots, x_{14}$ and $y_{15}$ is the parity packet. The coded data packets will be saved on 6 different nodes available in the system, according
III. MODULAR CONSTRUCTION OF FRACTIONAL REPEITION CODES FOR $\rho = d$ AND $n = \theta$

In [12], El Rouayheb et al. have shown the construction of FR codes for repetition degree 3 or more based on Stainer system. In this section we give a different approach to get the fractional repetition code for $\rho = d$ and $n = \theta$.

**Proposition 2.** Let $n$ and $t(> 1)$ be positive integers, For $0 \leq j \leq n-1$, let $C_j^n = \{t^{i-1} + j \pmod{n+1}|1 \leq i \leq \rho\}$, where $\rho$ is a positive integer such that $t^{\rho-1} < n$. Then $|C_j^n| = \rho$ and $\mathcal{C} = \{C_j^n|0 \leq j \leq n-1\}$ is a FRC over $\Omega = \{1, 2, \ldots, n\}$ with $\rho = d$ and $n = \theta$.

**Proof:** By the construction of $C_j^n$ it is clear that each such set will have size $\rho = d$ and since $\theta = n$, the necessary and sufficient condition for FRC (equation (5)) is satisfied. $\blacksquare$

A fractional repetition code based on Proposition 2 for $n = 8, t = 2$ and $\rho = 3$ is given in Figure 3.

![Fractional replication code](image)

**IV. CONSTRUCTION OF WEAK FRACTIONAL REPEITION CODES FOR $\rho = 2$**

**Definition 3.** (Weak Fractional Repetition Code) : A Weak Fractional Repetition (WFR) code $\mathcal{C}$ for a $(n, k, d)$ DSS, with repetition degree $\rho$ for every packet, is a collection of $n$ subsets $U_1, U_2, \ldots, U_n$ of a set $\Omega = \{1, \ldots, \theta\}$ such that $|U_i| = d_i$, for $1 \leq i \leq n$, where $d = \max\{d_i\}$, $d - d_i = \delta_i$ is the order of weakness of node $U_i$ and $\sum_{i=1}^{n} |U_i| = nd - \delta$, where $\delta = \sum_{i=1}^{n} \delta_i$ is defined as total order of weakness of $\mathcal{C}$.

**Remark 1.** WFR code with weakness order $\delta = 0$ gives fractional repetition codes of Def. 1.

Clearly, for WFR Codes, (6) $\rho d = nd - \delta$.

FR codes for $\rho = 2$ (WFR code with weakness order 0) has been constructed using regular graph by Ramachandran.

![Regular graph](image)
et al. [12]. Regular graph does not exist for cases where $n$ and $d$ both are odd. Motivated by this we define a Partial Regular Graph.

**Definition 4. (Partial Regular Graph)**: A graph $G(n, d)$ with total $n$ vertices such that the degree of some $(n - 1)$ vertices is $d$ and one vertex has degree $d - 1$. We call these graphs as Partial Regular Graph denoted by $PRG(n, d)$.

**Remark 2.** With the abuse of notations, we will use the same symbols for graph and its adjacency matrix.

**A. Properties of Partial Regular Graph**

Let a partial regular graph be denoted by $PRG(n, d)$. The rows of its matrix are denoted by $R_i$ and columns are denoted by $C_i$ where $i \in \{1, 2, \ldots, n\}$ such that degree of every vertex is $d$ except one vertex $U_k$ for some $k \in \{1, 2, \ldots, n\}$ which has degree $d - 1$. This adjacency matrix of $PRG(n, d)$ has the following properties.

1. $PRG(n, d)$ is a partial regular graph iff both $n$ and $d$ are odd.
2. $wt(R_i) = d$ for all $i$ such that $1 \leq i \leq n$ except for $i = k$ and $wt(R_k) = d - 1$.
3. $wt(C_i) = d$ for all $i$ such that $1 \leq i \leq n$ except for $i = k$ and $wt(C_k) = d - 1$.

So the total degree of the graph becomes even. In fact it is the necessary condition for the existence of an undirected graph.

**B. Construction of the Partial Regular Graph from a Circulant Graph**

**Definition 5. (Circulant Graph)**: A graph is called as a circulant graph $C_n(d)$ of degree $d$ if its adjacency matrix is a circulant, i.e. $(a_{i,j}) = (a_{j-1})$ for $i, j = 0, \ldots, n-1$ (subscripts of adjacency matrix are taken modulo $n$), $a_{i,j} \in \mathbb{F}_q$ and weight of each row is $d$.

Circulant matrix is characterized by its $1^{st}$ row. If the first row of the circulant matrix is $(a_0, a_1, \ldots, a_{n-1}) \in \mathbb{F}_q^n$ one can form a polynomial $q(t) = a_0 + a_1t + a_2t^2 + \ldots + a_{n-1}t^{n-1}$. The algebra of circulant matrix has one-to-one correspondence with polynomial algebra in $\mathbb{F}_q[t]/(t^n - 1)$.

**Remark 3.** Each circulant graph is a regular graph.

Let $P(\pi) = \begin{pmatrix} 1 & \pi(1) & \pi(2) & \cdots & \pi(n) \end{pmatrix}$ be the permutation function on $n$ symbols then the corresponding permutation matrix $P_n$ is the $n \times n$ matrix whose entries are all 0 except that in row $i$, the entry $\pi(i)$ equals 1. We know from elementary algebra that to get a permutation matrix $P_n$ from a permutation $P(\pi)$ on the identity matrix $I_n$. Algorithm 2 constructs a Partial Regular Graph $PRG(n, d)$ from a circulant graph $C_n(d - 1)$ whose $1^{st}$ row is defined by the polynomial $q(t) = t + t^2 + \ldots + t^{(d-1)/2} + t^{n-(d-1)/2} + \ldots + t^{n-1}$, where weight of $q(t) = d - 1$.

**C. Construction of the WFR Code**

WFR code from the partial regular graph can be constructed in the same way as in the previous case of regular graph. Algorithm 2 can be used to construct a partial regular graph which can be used together with Algorithm 3 to construct a WFR code.

**Algorithm 2** Construction of Partial Regular Graph $PRG(n, d)$ from Circulant Matrix $C_n(d - 1)$

1. Pick a circulant matrix $C_n(d - 1)$ whose first row is defined by $q(t) = t + t^2 + \ldots + t^{(d-1)/2} + t^{n-(d-1)/2} + \ldots + t^{n-1}$.
2. Get a permutation matrix $P_{n-1}$ whose permutation function $P(\pi)$ on $n - 1$ symbols is given by
   $\begin{pmatrix} 1 & (n+1)/2 & \cdots & (n-1)/2 & n \end{pmatrix}$.
3. Expand matrix $P_{n-1}$ to make it $n \times n$ matrix $S_n$ by adding 0 at $n^{th}$ row and $n^{th}$ column such that $S_n = \begin{pmatrix} P_{n-1} & 0^T \\ 0 & 0 \end{pmatrix}$, where $0 = [0 \ 0 \ldots 0] \in \mathbb{Z}_2^{n-1}$.
4. Add above matrix $S_n$ to $C_n(d - 1)$ to get the final matrix $PRG(n, d)$.

$PRG(n, d) = S_n + C_n(d - 1)$ (mod 2).

**Algorithm 3** Construction of the Weak FR code from the Partial Regular Graph $PRG(n, d)$ where $n$ and $d$ both are odd.

1. Generate the above graph $PRG(n, d)$ on $n$ vertices $U_1, U_2, \ldots, U_n$.
2. Index the edges of $G_{n,d}$ from 1 to $\frac{nd-1}{2}$.
3. Store on node $U_i$ in the DSS the packets indexed by the edges that are adjacent to vertex $U_i$ in the graph.

**Example 2.** To generate the Weak Fractional Repetition Code for $n = 9$ and $d = 7$, note that $q(t) = 0 + t + t^2 + t^3 + 0t^4 + \ldots + t^{d-1}$.
Using step 4 of Algorithm 2, thus by step 3 of Algorithm 2, we get the following adjacency matrix for partial regular graph shown in Figure 4. By taking file \( x \) as an input to the outer MDS code we will get \( y = (y_1, y_2, \ldots, y_{31}) \) as output where \( y_1 = x_1, y_2 = x_2, \ldots, y_{30} = x_{30} \) and \( y_{31} \) is the parity packet. The coded packets will be saved on 9 different nodes in the system according to partial regular graph shown in Figure 4. The whole file can be retrieved by recovering just 30 distinct packets out of these 31, due to the property of outer MDS code. As 30 distinct packets can be recovered by contacting to any 7 nodes out of these 9 nodes so whole file can be retrieved by contacting to any 7 nodes. In case of failure of a single node, its data can be recovered by contacting exactly 7 nodes and downloading 1 packet from each of these nodes, except in case of node \( U_9 \) which has to contact only 6 nodes in case of failure. For instance, when node \( U_1 \) fails, it contacts node \( U_2, U_3, U_4, U_5, U_7, U_8, U_9 \) to get the packets \( y_1, y_{10}, y_{23}, y_{19}, y_{25}, y_{18}, y_9 \) respectively. Node \( U_9 \) saves only 6 data packets. So we put a default null packet as the 7th packet. To recover this node on failure, user needs to contact only 6 other nodes.

\[
0t^5 + t^6 + t^7 + t^8 \quad \text{and hence}
\]
\[
C_9(6) = \begin{pmatrix}
0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\
1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\
1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\
1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\
\end{pmatrix}. 
\]

For the permutation function \( P(\pi) \)
\[
P(\pi) = \begin{pmatrix}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
5 & 6 & 7 & 8 & 1 & 2 & 3 & 4 \\
\end{pmatrix}
\]

the permutation matrix \( P_S \) is given by
\[
P_S = \begin{pmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
\end{pmatrix}. 
\]

Thus by step 3 of Algorithm 2, \( S_9 \) is given by
\[
S_9 = \begin{pmatrix}
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
\end{pmatrix}. 
\]

Using step 4 of Algorithm 2, one gets the following adjacency matrix for partial regular graph \( PRG(9, 7) \) with \( n = 9 \) and \( d = 7 \).
\[
PRG(9, 7) = \begin{pmatrix}
0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\
1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\
1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\
1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 0 \\
\end{pmatrix}. 
\]

Adjacency matrix \( PRG(9, 7) \) will give us a graph shown in Figure 4. It is a partial regular graph which satisfies the properties given in subsection A. This graph will give us the Weak Fractional Repetition Code as shown in the Figure 5.

In this example, suppose we want to store a file \( x = (x_1, x_2, x_3, \ldots, x_{30}) \in \mathbb{F}_q \) on a DSS \( (n = 9, k = 7, d = 7) \). There is no regular graph which can give us appropriate fractional repetition codes. However Figure 4 shows a partial regular graph which gives us weak fractional repetition codes to save the file of 30 packets on DSS \( (n = 9, k = 7, d = 7) \). Here regenerating code is made of two components - an outer MDS code \((31, 30)\) and a weak fractional repetition code based on partial regular graph.

V. CONSTRUCTION OF GENERALIZED RING CODE WITH \( \rho = 2 \)

The construction of Section IV can be generalized to give rise both strong and weak fractional repetition codes. Suppose we have \( \theta \) packets from set \( \Omega = \{1, 2, \ldots, \theta\} \) to be stored on \( n \) nodes \( U_1, U_2, \ldots, U_n \) such that \( \rho = 2 \). We first place \( n \) nodes on a circle (See Figure 4 for \( n = 9 \) and \( \theta = 31 \) packets) and then starting from first node start placing the packets between successive nodes until all packets are exhausted. For \( n = 9 \) and \( \theta = 31 \) this gives the WFR code as shown in Figure 4. Here every packet has \( qn + r \) for \( 0 \leq r < n - 1 \) this simple construction gives FR code for \( r = 0 \) and WFR code for \( r > 0 \). This simple construction can be modified in different ways to obtain a general class of generalized ring codes. This will be included in the extended version of the paper.

VI. CONCLUSION

Motivated by the emergence of fractional repetition codes which were based on regular graphs, we presented weak fractional repetition codes which are based on partial regular graph. WFR code gives regenerating codes for system parameters for which regular graphs does not exist. In this process, we used the properties of generalized ring constructions.

Fig. 5. Weak Fractional Repetition code based on generalized ring construction for \( \theta = 31, \rho = 2 \) and \( n = 9 \).
constructed WFR codes of repetition degree 2. As FR code exists where \( nd = \text{even} \) and we presented WFR codes for \( nd = \text{odd} \), by combining these two, we get regenerating codes for single node failure for all feasible values of system parameters for any DSS. However WFR codes of higher repetition degree can be used for multiple node failure i.e. \( \rho > 2 \) as it gives more flexibility to a DSS. In future, weak fractional repetition codes could be constructed by Steiner system but distribution of default null packets remains an open problem. We also presented a simple modular construction of fractional repetition codes for \( \rho = d \) and \( n = \theta \) and a generalized ring construction that gives both strong and weak fractional repetition codes. It would be an interesting future task to generalize some of these constructions for \( \rho > 2 \). WFR codes could be useful for heterogeneous distributed storage systems [20].

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