Neutron-skin values and matter and neutron radii determined from reaction cross sections of proton scattering on $^{12}\text{C}$, $^{40,48}\text{Ca}$, $^{58}\text{Ni}$, $^{208}\text{Pb}$

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**Background:** Very lately, the PREX and the CREX collaboration present skin values, $r_{\text{skin}}^{208}$ (newPREX2) = 0.278 ± 0.078 (exp) ± 0.012 (theor.) fm and $r_{\text{skin}}^{48}$ = 0.121 ± 0.026 (exp) ± 0.024 (model), respectively. We recently determined a neutron-skin value $r_{\text{skin}}^{208} = 0.278 ± 0.035$ fm from measured reaction cross sections $\sigma_R^{}(\text{exp})$ of $p^+208\text{Pb}$ scattering in a range of incident energies $10 \leq E_{\text{in}} \leq 100$ MeV where the chiral (Kyushu) $g$-matrix folding model is reliable for $^{12}\text{C}$+$^{12}\text{C}$ scattering. The data $\sigma_R^{}(\text{exp})$ are available for proton scattering on $^{58}\text{Ni}$, $^{40,48}\text{Ca}$, $^{12}\text{C}$ targets.

**Purpose:** Our first aim is to test the Kyushu g-matrix folding model for $p^+208\text{Pb}$ scattering in $20 \leq E_{\text{in}} \leq 180$ MeV. Our second aim is to determine skin values $r_{\text{skin}}$ and matter and neutron radii, $r_m$ and $r_n$, for $^{208}\text{Pb}$, $^{58}\text{Ni}$, $^{40,48}\text{Ca}$, $^{12}\text{C}$ from the $\sigma_R^{}(\text{exp})$.

**Methods:** Our method is the Kyushu $g$-matrix folding model with the densities scaled from the D1S-GHFB+AMP densities, where D1S-GHFB+AMP stands for Gogny-D1S HFB (GHFB) with the angular momentum projection (AMP).

**Results:** As for proton scattering, we find that our model is reliable in $20 \leq E_{\text{in}} \leq 180$ MeV. For $^{208}\text{Pb}$, the skin value deduced from $\sigma_R^{}(\text{exp})$ in $20 \leq E_{\text{in}} \leq 180$ MeV is $r_{\text{skin}}^{208}(\sigma_R^{}(\text{exp})) = 0.299 ± 0.020$ fm. Our results on $r_{\text{skin}}$ are compared with the previous works.

**Conclusion:** Our result $r_{\text{skin}}^{208}(\sigma_R^{}(\text{exp})) = 0.299 ± 0.020$ fm agrees with $r_{\text{skin}}^{208}$ (PREX2) = 0.283 ± 0.071 fm. In addition, our result $r_{\text{skin}}^{48}$ = 0.103 ± 0.022 fm is consistent with the CREX value.

**I. INTRODUCTION**

Many theoretical predictions on the symmetry energy $S_{\text{sym}}(\rho)$ have been made so far by taking several experimental and observational constraints on $S_{\text{sym}}(\rho)$ and their combinations. In neutron star (NS), the $S_{\text{sym}}(\rho)$ and its density $\rho$ dependence influence strongly the nature within the star. The symmetry energy $S_{\text{sym}}(\rho)$ cannot be measured by experiment directly. In place of $S_{\text{sym}}(\rho)$, the neutron-skin thickness $r_{\text{skin}}$ is measured to determine the slope parameter $L$, since a strong correlation between $r_{\text{skin}}^{208}$ and $L$ is well known [1].

Horowitz et al. [2] proposed a direct measurement for neutron skin thickness $r_{\text{skin}} = r_n - r_p$, where $r_n$ and $r_p$ are the root-mean-square radii of neutrons and protons, respectively.

The PREX collaboration has reported a new value,

$$r_{\text{skin}}^{208}(\text{PREX2}) = 0.283 ± 0.071\text{ fm}, \quad (1)$$

combining the original Lead Radius EXperiment (PREX) result [3] with the updated PREX2 result [5]. Very lately, the PREX collaboration has presented an accurate value[? ,]

$$r_{\text{skin}}^{208}(\text{newPREX2}) = 0.278±0.078 \pm 0.012 \text{ fm,} \quad (2)$$

The value is most reliable for $^{208}\text{Pb}$. The $r_{\text{skin}}^{208}$ (PREX2) value is considerably larger than the other experimental values that are significantly model dependent [6–9]. As an exceptional case, a nonlocal dispersive-optical-model (DOM) analysis of $^{208}\text{Pb}$ deduces $r_{\text{skin}}^{208,\text{DOM}} = 0.25 ± 0.05 \text{ fm}$ [10] consistent with $r_{\text{skin}}^{208}$ (PREX2).

Very recently, the CREX group has presented [11]

$$r_{\text{skin}}^{48}(\text{CREX}) = 0.121 ± 0.026 \text{ (exp) ± 0.024 (model) fm.} \quad \text{(\ref{eq:CREX})}$$

The CREX value is most reliable for $^{48}\text{Ca}$.

The $r_{\text{skin}}^{208}$ (PREX2) provides crucial tests for the equation of state (EoS) of nuclear matter [12–16]. For example, Reed et al. [17] report a value of the slope parameter $L$ and examine the impact of such a stiff symmetry energy on some critical NS observables. They deduce

$$L = 106 ± 37 = 69–143 \text{ MeV} \quad (4)$$

from $r_{\text{skin}}^{208}$ (PREX2).

In Ref. [18], we accumulated the 206 EoSs from Refs. [1] [19] [33] in which $r_{\text{skin}}^{208}$ and/or $L$ is presented. The correlation

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between $r_{\text{skin}}^{208}$ and $L$ is more reliable when the number of EoSs is larger. The resulting relation

$$L = 620.39 \, r_{\text{skin}}^{208} - 57.963 \, \text{MeV}$$

(5)

has a strong correlation with the correlation coefficient $R = 0.99$, as shown in Fig. 1.

![FIG. 1. $r_{\text{skin}}^{208}$ dependence of $J$, $L$, $K_{\text{sym}}$, where $J$, $L$, $K_{\text{sym}}$ are a constant term, the first-derivative and the second-derivative term of the symmetry energy. The dots show 206 EoSs taken from Table of Ref. [18]. Obviously, the correlation between $r_{\text{skin}}^{208}$ and $L$ is linear.](image)

The relation (5) allows us to deduce a constraint on $L$ from the PREX2 value. The resulting range of $L$ are $L = 76–165$ MeV, while equation shown in Ref. [1] yields $L = 76–172$ MeV. These values and $L = 69–143$ MeV support stiffer EoSs. The stiffer EoSs allow us to consider the phase transition such as QCD transition in NS. The following EoSs satisfy $L = 76–172$ MeV: SkO, FKVV, Rs, SY-sym34, es325, TFA, NL4, BSR6, Ro, Sk-Rs, E0009, Gs, Z271, GM3, PKDD, E0008(TMA), SK272, GM1, Sr, T-S4, SK255, SV, es35, S271, SkI3, rG2, PC-PK1, SkI2, E0025, PC-LA, rNL, E0036, rTM1, TM1, NL4, rNL-SH, NL-SH, rNL,R1A, PC-F2, PK1, PC-F1, NL3, rNL3, PC-F3, PC-F4, NL*3, TFB, rNL*3, SkI5, NL2, rNL-Z, TFC, NL1, rNL1, SkI1 in Table I of Ref. [18].

As an indirect measurement, meanwhile, the high-resolution $E1$ polarization experiment ($E1pE$) yields

$$r_{\text{skin}}^{208}(E1pE) = 0.156^{+0.025}_{-0.021} = 0.135–0.181 \, \text{fm}$$

(6)

for $^{208}\text{Pb}$ [44]

$$r_{\text{skin}}^{48}(E1pE) = 0.17 \pm 0.03 = 0.14–0.20 \, \text{fm}$$

(7)

for $^{48}\text{Ca}$ [45].

There is no overlap between $r_{\text{skin}}^{208}(\text{PREX2})$ and $r_{\text{skin}}^{208}(E1pE)$ in one $\sigma$ level. However, we determined a value of $r_{\text{skin}}^{208}(\exp)$ from measured reaction cross sections $\sigma_{\text{R}}(\exp)$ of $p^+\text{Pb}$ scattering in a range of incident energies, $30 \leq E_{\text{in}} \leq 100$ MeV [46]; the value is $r_{\text{skin}}^{208}(\exp) = 0.278 \pm 0.035 \, \text{fm}$. Our result agrees with $R_{\text{skin}}^{208}(\text{PREX2})$. We also deduced $r_{\text{p}}(\exp) = 5.722 \pm 0.035 \, \text{fm}$ and $r_{\text{n}}(\exp) = 5.614 \pm 0.022 \, \text{fm}$ in addition to $r_{\text{kin}}^{208}(\exp)$. As for He$^+\text{Pb}$ scattering, we determine $r_{\text{kin}}^{208}(\exp) = 0.416 \pm 0.146 \, \text{fm}$ [47]. Our results are consistent with PREX II and therefore supports larger slope parameter $L$.

Our model is the chiral (Kyushu) $g$-matrix folding model with the densities calculated with Gogny-D1S HFB (DIS-GHFB) with the angular momentum projection (AMP) [48, 49]. For $p^+\text{Pb}$ scattering, the neutron density is scaled so that the $r_n$ of the scaled neutron density can reproduce the data [50, 52] on $\sigma_R$, since the $r_p$ of DIS-GHFB+AMP proton density agrees with the $r_p(\exp)$ [53] determined from electron scattering. For $^{12}$C scattering on $^{4}$He, $^{12}$C, $^{27}$Al targets, we tested reliability of the Kyushu $g$-matrix folding model and found that the Kyushu $g$-matrix folding model is reliable in $30 \leq E_{\text{in}} \leq 100$ MeV and $250 \leq E_{\text{in}} \leq 400$ MeV [49]. This is the reason why we took $30 \leq E_{\text{in}} \leq 100$ MeV in the analyses [46] of $p^+\text{Pb}$ scattering. After the analyses, we find that the Kyushu $g$-matrix folding model reproduces the lower bound of the data on $\sigma_R$ for $^{12}$C+$^{12}$C scattering at $E_{\text{in}} = 10.4$ MeV per nucleon.

The $g$-matrix folding model is a standard way of deriving the microscopic optical potential for proton scattering and nucleus-nucleus scattering [49, 55–64]. The folding model is composed of the single folding model for proton scattering and the double folding model for nucleus-nucleus scattering. The relation between the single and the double folding model is clearly shown in Ref. [60]. Applying the double-folding model based on Melbourne $g$-matrix [58] for the data [65] on interaction cross sections, we found that $^{31}$Ne is a halo nucleus with large deformation [64], and deduced the matter radii $r_m$ for Ne isotopes [66]. Also for Mg isotopes, we determined the $r_m$ from $\sigma_R(\exp)$ for scattering of Mg isotopes on a $^{12}$C target [67].

Now, we consider proton scattering on $^{208}\text{Pb}$, $^{58}$Ni, $^{40,48}$Ca, $^{12}$C targets, since there is no interaction cross section for proton scattering. In fact, good data on $\sigma_R$ are available in Refs. [50, 52] for $^{208}\text{Pb}$, Refs. [51, 52] for $^{58}$Ni, Ref. [71] for $^{48}$Ca, Refs. [50, 52] for $^{44}$Ca, and Refs. [51, 52, 72] for $^{12}$C. We have already shown that for $p^+\text{Pb}$ scattering the $\sigma_R$ calculated with $r_{\text{kin}}^{208}(\text{PREX2})$ and $r_{\text{p}}(\exp) = 5.444\, fm$ [53] of electron scattering reproduce the data at $E_{\text{lab}} = 534.1, 549, 806$ MeV [73].

In this paper, we first test the Kyushu $g$-matrix single folding model for $p^+\text{Pb}$ scattering, since the PREX2 data is available. We find that the present model is reliable in $20 \leq E_{\text{in}} \leq 180$ MeV, as shown in Sec. III A. After the testing, we determine $r_{\text{n}}(\exp)$, $r_{\text{p}}(\exp)$, $r_{\text{kin}}^{208}(\exp)$ for $^{208}\text{Pb}$, $^{58}$Ni, $^{40,48}$Ca, $^{12}$C from the $\sigma_R(\exp)$ in $20 \leq E_{\text{in}} \leq 180$ MeV, as shown in Sec. III B. For each nucleus, the DIS-GHFB+AMP proton and neutron densities are scaled so as to reproduce $\sigma_R(\exp)$ under that condition that the $r_p(\text{scaling})$ of the scaled proton density agrees with $r_p(\exp)$ of electron scattering.

We explain our model in Sec. II and our results in Sec. III. Section IV is devoted to a summary.
II. MODEL

Our model is the Kyushu $g$-matrix folding model \cite{48, 49} with the proton and neutron densities scaled from the DIS-GHFB+AMP densities.

A. The Kyushu $g$-matrix folding model

Kohno calculated the $g$ matrix for the symmetric nuclear matter, using the Brueckner-Hartree-Fock method with chiral N$^3$LO 2NFs and NNLO 3NFs \cite{74}, where N$^3$LO 3NF is abbreviation of next-to-next-to-next-to-leading-order three-body force and NNLO 2NFs is of next-to-next-to-leading-order two-body force. He set $c_D = -2.5$ and $c_E = 0.25$ so that the energy per nucleon can become minimum at $\rho = \rho_0$; see Fig. 2 for the definition of $c_D$ and $c_E$. Toyokawa \textit{et al.} localized the non-local chiral $g$ matrix into three-range Gaussian forms \cite{48}, using the localization method proposed by the Melbourne group \cite{58, 73, 76}. The resulting local $g$ matrix is referred to as “Kyushu $g$-matrix”. The Kyushu $g$-matrix is constructed from chiral interaction with the cutoff 550 MeV.

The Kyushu $g$-matrix folding model is successful in reproducing $\sigma_R$, differential cross sections $d\sigma/d\Omega$, vector analyzing powers $A_V$ for $^4$He scattering at $E_{\text{in}} = 30$–200 MeV per nucleon \cite{48}. The success is true for proton scattering at $E_{\text{in}} = 65$ MeV \cite{78}.

In Ref. \cite{79}, we tested the Kyushu $g$-matrix folding model \cite{48} for $^{12}$C scattering on $^8$Be, $^{12}$C, $^{27}$Al targets in $30 \leq E_{\text{in}} \leq 400$ MeV. We found that the Kyushu $g$-matrix folding model is reliable for $\sigma_R$ in $30 \leq E_{\text{in}} \leq 100$ MeV and $250 \leq E_{\text{in}} \leq 400$ MeV. This indicates that the Kyushu $g$-matrix folding model is applicable in the $E_{\text{in}}$ range. After the test, we found that our model reproduces the lower bound of measured reaction cross section $\sigma_R$ \cite{54} at $E_{\text{in}} = 10.4$ MeV. Our model is reliable for $10 \leq E_{\text{in}} \leq 100$ MeV and $250 \leq E_{\text{in}} \leq 400$ MeV.

We recapitulate the single folding model for nucleon-nucleus scattering. The potential $U(\mathbf{R})$ consists of the direct and exchange parts \cite{60}, $U^{\text{DR}}(\mathbf{R})$ and $U^{\text{EX}}(\mathbf{R})$, defined by

$$U^{\text{DR}}(\mathbf{R}) = \sum_{\mu, \nu} \int \rho^c_\mu(\mathbf{r}_T) g^{\text{DR}}_{\mu \nu}(s; \rho_{\mu \nu}) d\mathbf{r}_T , \quad (8a)$$

$$U^{\text{EX}}(\mathbf{R}) = \sum_{\mu, \nu} \int \rho^c_\mu(\mathbf{r}_T, \mathbf{r}_T + s) \times g^{\text{EX}}_{\mu \nu}(s; \rho_{\mu \nu}) \exp [-i\mathbf{K}(s) \cdot s/M] d\mathbf{r}_T \quad (8b)$$

where $\mathbf{R}$ is the relative coordinate between a projectile (P) and a target (T), $s = -\mathbf{r}_T + \mathbf{R}$, and $\mathbf{r}_T$ is the coordinate of the interacting nucleon from the center-of-mass of T. Each of $\mu$ and $\nu$ denotes the $z$-component of isospin, i.e., $(1/2, -1/2)$ corresponds to (neutron, proton). The nonlocal $U^{\text{EX}}(\mathbf{R})$ has been localized in Eq. (8b) with the local semi-classical approximation \cite{55}, where $K(s)$ is the local momentum between P and T, and $M = A/(1 + A)$ for the target mass number A; see Ref. \cite{80} for the validity of the localization. The direct and exchange parts, $g^{\text{DR}}_{\mu \nu}$ and $g^{\text{EX}}_{\mu \nu}$, of the $g$-matrix depend on the local density

$$\rho_{\mu \nu} = \sigma^\mu \rho^c_{\mu \nu}(\mathbf{r}_T + s/2) \quad (9)$$

at the midpoint of the interacting nucleon pair, where $\sigma^\mu$ having $\mu = -1/2$ is the Pauli matrix of an incident proton. As a way of taking the center-of-mass correction to the DIS-GHFB+AMP densities, we use the method of Ref. \cite{66}, since the procedure is quite simple.

The direct and exchange parts, $g^{\text{DR}}_{\mu \nu}$ and $g^{\text{EX}}_{\mu \nu}$, of the $g$-matrix, are described by \cite{66}

$$g^{\text{DR}}_{\mu \nu}(s; \rho_{\mu \nu}) = \begin{cases} 
\frac{1}{4} \sum_S \hat{S}^2 g^{S1}_{\mu \nu}(s; \rho_{\mu \nu}) & \text{for } \mu + \nu = \pm 1 \\
\frac{1}{8} \sum_{S,T} \hat{S}^2 g^{ST}_{\mu \nu}(s; \rho_{\mu \nu}) & \text{for } \mu + \nu = 0 
\end{cases} \quad (10)$$

$$g^{\text{EX}}_{\mu \nu}(s; \rho_{\mu \nu}) = \begin{cases} 
\frac{1}{4} \sum_S (-1)^{S+1} \hat{S}^2 g^{S1}_{\mu \nu}(s; \rho_{\mu \nu}) & \text{for } \mu + \nu = \pm 1 \\
\frac{1}{8} \sum_{S,T} (-1)^{S+T} \hat{S}^2 g^{ST}_{\mu \nu}(s; \rho_{\mu \nu}) & \text{for } \mu + \nu = 0 
\end{cases} \quad (11)$$

where $\hat{S} = \sqrt{2S + 1}$ and $g^{ST}_{\mu \nu}$ are the spin-isospin components of the $g$-matrix; see Ref. \cite{63} for the explicit form of $g^{\text{DR}}_{\mu \nu}$ and $g^{\text{EX}}_{\mu \nu}$.

The potential $U(\mathbf{R})$ thus obtained has the form of $U(\mathbf{R}) = U_{\text{cent}}(\mathbf{R}) + \ell \cdot \alpha \cdot U_{\text{spin-orbit}}(\mathbf{R})$, where $U_{\text{cent}}(\mathbf{R})$ and $U_{\text{spin-orbit}}(\mathbf{R})$ are the central and the spin-orbit part of $U(\mathbf{R})$, respectively, and $\ell$ is the orbital angular momentum of the proton scattering; see Eq. (28) in Ref. \cite{81} for the derivation. The relative wave function $\psi$ between P and T can be decomposed into partial waves $\chi_\ell$, each with different $\ell$. The $\chi_\ell$ is obtained by solving the Schrödinger equation having $U(\mathbf{R})$. The elastic $S$-matrix elements $S_\ell$ are obtained from the asymptotic
form of $\chi_t$. The total reaction cross section $\sigma_R$ is calculable from the $S_t$ as

$$\sigma_R = \frac{\pi}{K^2} \sum_\ell (2\ell + 1)(1 - |S_t|)^2.$$  \hfill (12)

B. Scaling procedure of proton and neutron densities

The proton and neutron densities, $\rho_p(r)$ and $\rho_n(r)$, are scaled from the D1S-GHFB+AMP densities. We can obtain the scaled density $\rho_{\text{scaling}}(r)$ from the original density $\rho(r)$ as

$$\rho_{\text{scaling}}(r) = \frac{1}{\alpha^3} \rho(r/\alpha)$$  \hfill (13)

with a scaling factor

$$\alpha = \sqrt{\frac{\langle r^2 \rangle_{\text{scaling}}}{\langle r^2 \rangle}}.$$  \hfill (14)

For later convenience, we refer to the proton (neutron) radius of the scaled proton (neutron) density $\rho_p^{\text{scaling}}(r)$, as $r_p(\text{scaling})$ ($r_n(\text{scaling})$).

Table I shows the scaling factors, $\alpha_p$ and $\alpha_n$, from the D1S-GHFB+AMP densities to the scaled densities that reproduce data $\sigma_R(\text{exp})$ and $r_p(\text{exp})$ of electron scattering.

| Symbol | Value |
|--------|-------|
| $\alpha_p$ | 1.015 |
| $\alpha_n$ | 1.003 |

| Mass Number | $\alpha_p$ | $\alpha_n$ |
|-------------|-----------|-----------|
| $^{208}\text{Pb}$ | 1.015 | 1.000 |
| $^{58}\text{Ni}$ | 1.003 | 0.994 |
| $^{48}\text{Ca}$ | 0.973 | 0.982 |
| $^{40}\text{Ca}$ | 1.000 | 0.995 |
| $^{12}\text{C}$ | 0.942 | 0.957 |

C. Effective nucleon-nucleon interaction for targets

For a change of the proton and neutron distributions in targets, a microscopic approach is to modify D1S. For the Gogny EoSs, the effective nucleon-nucleon interaction can be described as

$$V(\vec{r}) = \sum_{i=1,2} t_0^i (1 + x_0^i P_\sigma) \rho^{\sigma i} \delta(\vec{r})$$

$$+ \sum_{i=1,2} (W_i + B_i P_\sigma - H_i P_\tau - M_i P_\pi P_\tau) e^{-\vec{r}^2/\sigma^2}$$

$$+ iW_0(\sigma_1 + \sigma_2)[\vec{k} \times \delta(\vec{r})\vec{k}],$$

where $\sigma$ and $\tau$ are the Pauli spin and isospin operators, respectively, and the corresponding exchange operators $P_\sigma$ and $P_\tau$ are defined as usual.

For $^{208}\text{Pb}$, we have changed all parameters of D1S, but cannot find the NN interaction that reproduce $r_{\text{skin}}(\text{PREX2}) = 0.283 \pm 0.071$ fm. The best fitting is the D1PK2-GHFB+AMP with $r_{\text{skin}}(\text{D1PK2}) = 0.185$ fm; note that $r_{\text{skin}}(\text{D1S}) = 0.137$ fm. The parameters of D1PK2 are shown in Table II.

III. RESULTS

First of all, we regard reliable $r_m$ and $r_n$ as reference values, $r_m(\text{ref})$ and $r_n(\text{ref})$, in order to determine $r_m(\text{exp})$ from $\sigma_R(\text{exp})$. The reference values are shown below; see Table III for the numerical values. Whenever we calculate $\sigma_R$, we use the Kyushu $g$-matrix model.

$^{58}\text{Ni}$: The reference values are the $r_m(\text{AMP})$ and $r_n(\text{AMP})$ calculated with D1S-GHFB+AMP, since the $\sigma_R(\text{AMP})$ are near the upper bound of the data, as shown in Fig. 5. $E_m$ dependence of $\sigma_R(\text{AMP})$ is similar to that of the data. We then define the ratio $F/E_m = \sigma_R(\text{exp})/\sigma_R(\text{ref}) = \sigma_R(\text{exp})/\sigma_R(\text{AMP})$, and introduce the average value of $F/E_m$ as a fine-tuning factor $F$. The factor is $F = 0.96473$ close to 1. The $F\sigma_R(\text{AMP})$ almost reproduce the central values of the data, as shown in Fig. 5. The scaling procedure is not made to get the reference values, i.e., $\alpha = 1$.

$^{208}\text{Pb}$: The reference values are the $r_m(\text{PREX2})$ and $r_n(\text{PREX2})$ evaluated from $r_{\text{skin}}(\text{PREX2})$ and $r_p(\text{exp})$ of electron scattering. In this case, the $\sigma_R(\text{exp})$ based on the densities scaled to $r_n(\text{PREX2})$ and $r_p(\text{exp})$ reproduce the data within error-bar, as shown in Fig. 5. For this reason, $F$ is 1.

$^{48}\text{Ca}$: We can obtain $r_m(\text{CREX})$ and $r_n(\text{CREX})$ from the CREX value of Eq. 3 and $r_p(\text{exp}) = 3.385$ fm of electron scattering. In this case, the $\sigma_R(\text{CREX})$ based on $r_n(\text{CREX})$ and $r_p(\text{exp})$ are near the central values of the data, as shown in Fig. 4. The fine-tuning factor is $F = 0.9810$ close to 1. The $F\sigma_R(\text{E1pE})$ almost reproduce the central values of the data, as shown in Fig. 6.

$^{40,48}\text{Ca}$: As for $^{40,48}\text{Ca}$, Zenihiro et al. measured the differential cross section and the analyzing powers for $\text{p}^+\text{p}^{40,48}\text{Ca}$ scattering in RCNP, and determined $r_{\text{skin}}(\text{RCNP})$ for $^{48}\text{Ca}$. For $^{48}\text{Ca}$, the value $r_{\text{skin}}(\text{RCNP}) = 0.168 \pm 0.025$ fm is consistent with $r_{\text{skin}}(\text{E1pE})$. For $^{40}\text{Ca}$, their values are shown in Table III as reference values. Since $\sigma_R(\text{RCNP}) = \sigma_R(\text{AMP})$, the $\sigma_R(\text{AMP})$ are reliable. In this case, the $\sigma_R(\text{AMP})$ overshoot the data, as shown in Fig. 7. The fine-tuning factor is $F = 0.92716$ close to 1. The $F\sigma_R(\text{AMP})$ almost reproduce the central values of the data, as shown in Fig. 4.
TABLE III. Reference values of $r_m$ (ref), $r_n$ (ref), $r_{\text{skin}}$ (ref) together with $r_p$ (exp) of electron scattering. The $r_p$ (exp) are deduced from the electron scattering [53, 82]. In actual calculations, the central values are taken as reference values. The radii are shown in units of fm.

| Ref. | $r_p$ (exp) | $r_m$ (ref) | $r_n$ (ref) | $r_{\text{skin}}$ (ref) |
|------|-------------|-------------|-------------|------------------------|
| $^{208}\text{Pb}$ | PREX2 | 5.444 | 5.617 ± 0.044 | 5.727 ± 0.071 | 0.283 ± 0.071 |
| $^{58}\text{Ni}$ | D1S AMP | 3.727 | 3.721 | 3.715 | −0.013 |
| $^{48}\text{Ca}$ | CREX | 3.385 | 3.456 ± 0.050 | 3.506 ± 0.050 | 0.121 ± 0.050 |
| $^{40}\text{Ca}$ | [53] | 3.385 | 3.380 ± 0.022 | 3.375 ± 0.023 | −0.010 ± 0.023 |
| $^{12}\text{C}$ | [53] | 2.327 | 2.35 ± 0.02 | 2.37 ± 0.02 | 0.05 ± 0.02 |

TABLE IV. Fine-tuning factor $F$.

| $^{208}\text{Pb}$ | 1 |
| $^{58}\text{Ni}$ | 0.96473 |
| $^{48}\text{Ca}$ | 0.9810 |
| $^{40}\text{Ca}$ | 0.92716 |
| $^{12}\text{C}$ | 0.93077 |

When we determine $r_m$ (exp) from data $\sigma_R$ (exp), we scale the D1S-GHFB+AMP proton and neutron densities so as to $F \sigma_R$ (ref) = $\sigma_R$ (exp) and $r_p$ (scaling) = $r_p$ (exp). Next, we deduce $r_m$ (exp) from $r_n$ (scaling) and $r_p$ (scaling) for each $E_{\text{in}}$. The resulting $r_m$ (exp) depends on $E_{\text{in}}$. For all the $r_m$ (exp), we take the weighted mean and its error. Finally, we evaluate $r_{\text{skin}}$ (exp) and $r_n$ (exp) from the resulting $r_m$ (exp) and the $r_p$ (exp) [53, 52] of the electron scattering. For later convenience, we refer to this procedure as “experimental scaling procedure with $F$ (ESP-F)".

A. Test of the Kyushu $g$-matrix folding model for $p + ^{208}\text{Pb}$ scattering

Now we test the Kyushu $g$-matrix model for proton scattering. As shown in Fig. 3, the $\sigma_R$ (squares with error bar) based on $r_m$ (PRESX2) and $r_p$ (exp) are consistent with data [50, 52] in $20 \leq E_{\text{in}} \leq 180$ MeV; see Table III for $r_{\text{skin}}$ (PRESX2) and $r_p$ (exp). This indicates that our model is good in $20 \leq E_{\text{in}} \leq 180$ MeV for proton scattering.

Figure 3 shows reaction cross sections $\sigma_R$ of $p + ^{208}\text{Pb}$ scattering as a function of $E_{\text{in}}$. The results of the D1S-GHFB+AMP densities reproduce the data [50, 52] with 4% errors. This is true for the neutron density scaled to the central value of PREX2 and the D1S-GHFB+AMP density. In the results of the Woods-Saxon type neutron density ($r_{WS} = 6.59$ fm, $a_{WS} = 0.7$ fm) fitted to the central value of PREX2, we use the D1S-GHFB+AMP proton density. The results of the Woods-Saxon type neutron density ($r_{WS} = 6.59$ fm, $a_{WS} = 0.7$ fm) and the D1S-GHFB+AMP proton density are close to those of the neutron density scaled to the central value of PREX2 and the D1S-GHFB+AMP proton density. The Woods-Saxon type neutron density ($r_{WS} = 6.81$ fm, $a_{WS} = 0.6$ fm) yields almost the same results as the case of $r_{WS} = 6.59$ fm, $a_{WS} = 0.7$ fm, that is, the former under-shoots the latter by 0.974. We then do not show the former results in Fig. 4.

The results of ESP-F are $r_{\text{skin}}$ (exp) = 0.299 ± 0.020 fm, $r_n$ (exp) = 5.743 ± 0.020 fm, $r_m$ (exp) = 5.627 ± 0.020 fm. The present skin value 0.299 ± 0.020 fm almost agrees with our previous value $r_{\text{skin}}$ (exp) = 0.278 ± 0.035 fm of Ref. [46].
2. $^{58}\text{Ni}$

Figure 5 shows $\sigma_R$ as a function of $E_{\text{in}}$ for $p^{+^{58}\text{Ni}}$ scattering. The results $\sigma_R(\text{AMP})$ of the Kyushu $g$-matrix folding model with the D1S-GHFB+AMP densities (closed circles) almost reproduce data $\sigma_R(\text{exp})$ [51, 52, 68–70] in $10 \leq E_{\text{in}} \leq 81\text{MeV}$; note that the data has high accuracy of 2.7%.

The result of ESP-F is $r_{\text{skin}}(\text{exp}) = 3.711 \pm 0.010$ fm and $r_n = 3.488 \pm 0.022$ fm. Our skin value agrees with $r_{\text{skin}}^{^{48}\text{Ca}}(\text{CREX}).$

A novel method for measuring nuclear reactions in inverse kinematics with stored ion beams was successfully used to extract the matter radius of $^{58}\text{Ni}$ [85]. The experiment was performed at the experimental heavy-ion storage ring at the GSI facility. Their results determined from the differential cross section for $^{58}\text{Ni}+^4\text{He}$ scattering are $r_m(\text{GSI}) = 3.70(7)$ fm, $r_p(\text{GSI}) = 3.68$ fm, $r_n(\text{GSI}) = 3.71(12)$ fm, $r_{\text{skin}}(\text{GSI}) = 0.03(12)$ fm.

3. $^{48}\text{Ca}$

Figure 6 shows $\sigma_R$ as a function of $E_{\text{in}}$ for $p^{+^{48}\text{Ca}}$ scattering. The $\sigma_R(\text{AMP})$ almost reproduce the data [71]. The results $\sigma_R(\text{CREX})$ based on $r_n(\text{CREX})$ and $r_p(\text{exp})$ [82] are near the central values of the data [71]. $E_{\text{in}}$ dependence of $\sigma_R(E1pE)$ is similar to that of the data [71]. The $F\sigma_R(\text{CREX})$ almost reproduce the central values of the data.

The results of ESP-F are $r_{\text{skin}} = 0.103 \pm 0.022$ fm and $r_n = 3.488 \pm 0.022$ fm. Our skin value agrees with $r_{\text{skin}}^{^{48}\text{Ca}}(\text{CREX}).$
4. $^{40}$Ca

Figure 7 shows $\sigma_R$ as a function of $E_{\text{in}}$ for $p+^{40}$Ca scattering. The Kyushu $\gamma$-matrix folding model with the D1S-GHFB+AMP densities overestimates $\sigma_R(\text{exp})$ [50–52]; note that the data has high accuracy of 2.7%. Note that $\sigma_R(\text{AMP}) = \sigma_R(\text{PCNP})$, since $r_m(\text{AMP})$ is very close to $r_m(\text{RCNP})$. $E_{\text{in}}$ dependence of $\sigma_R(\text{AMP})$ is similar to that of the data [50–52].

The result of ESP-F is $r_m(\text{exp}) = 3.372 \pm 0.011$ fm. Using the $r_m(\text{exp})$ and $r_p(\text{exp}) = 3.378$ fm of electron scattering, we can obtain $r_{\text{skin}}(\text{exp}) = -0.011 \pm 0.011$ fm and $r_n(\text{exp}) = 3.367 \pm 0.011$ fm. Our results are close to those of shown in Ref. [83]; see Table III for the values of Ref. [83].

![Figure 7](image.png)

**FIG. 7.** $E_{\text{in}}$ dependence of reaction cross sections $\sigma_R$ for $p+^{40}$Ca scattering. Closed circles denote results of the original (D1S-GHFB+AMP) densities and the scaled ones, Open circles correspond to $F\sigma_R(\text{AMP})$. The data (crosses) are taken from Refs. [50–52].

5. $^{12}$C

Figure 8 shows $\sigma_R$ as a function of $E_{\text{in}}$ for $p+^{12}$C scattering. The results $\sigma_R(\text{AMP})$ of D1S-GHFB+AMP overshoot data $\sigma_R(\text{exp})$ [51, 52, 72].

![Figure 8](image.png)

**FIG. 8.** $E_{\text{in}}$ dependence of reaction cross sections $\sigma_R$ for $p+^{12}$C scattering. Closed circles denote results $F\sigma_R(\text{AMP})$. The data (crosses) are taken from Refs. [51, 52, 72].

Figure 9 shows $\sigma_R(\text{GSI})$ based on $r_m(\text{GSI})$ and $r_p(\text{exp}) = 2.327$ fm of electron scattering for $p+^{12}$C scattering. The results $\sigma_R(\text{GSI})$ are near the upper bound of $\sigma_R(\text{exp})$ [51, 52, 72]. The $\sigma_R(\text{GSI})$ (open circles) are near the central values of $\sigma_R(\text{exp})$.

The result of ESP-F is $r_m(\text{exp}) = 2.340 \pm 0.009$ fm. Using the $r_m(\text{exp})$ and $r_p(\text{exp}) = 2.327$ fm, we can obtain $r_{\text{skin}}(\text{exp}) = 0.026 \pm 0.009$ fm and $r_n(\text{exp}) = 2.354 \pm 0.009$ fm.

![Figure 9](image.png)

**FIG. 9.** $E_{\text{in}}$ dependence of reaction cross sections $\sigma_R$ for $p+^{12}$C scattering. Closed circles denote results $\sigma_R(\text{GSI})$, while open circles correspond to $F\sigma_R(\text{GSI})$. The data (crosses) are taken from Refs. [51, 52, 72].

Tanihata et al. determined $r_m$ from interaction cross sections for He, Li, Be, B isotopes [54]. In Ref. [86], the experimental values on $r_m$ are accumulated from $^4$He to $^{32}$Mg. Our result $r_m(\text{exp}) = 2.340 \pm 0.009$ fm is slightly smaller to $r_m(\text{GSI}) = 2.35(2)$ fm. As for neutron radius, this is the
case, because \( r_n(\text{exp}) = 2.354 \pm 0.009 \text{ fm} \) and \( r_n(\text{GSI}) = 2.37(2) \text{ fm} \).

### IV. SUMMARY

In this paper, we consider the \(^{208}\text{Pb}, ^{58}\text{Ni}, ^{40,48}\text{Ca}, ^{12}\text{C}\), as stable nuclei and determine \( r_{\text{skin}}(\sigma_R), r_m(\sigma_R), r_n(\sigma_R) \) from measured \( \sigma_R \). Our results on \( r_{\text{skin}}(\sigma_R), r_m(\sigma_R), r_n(\sigma_R) \) are summarized in Table V. Comparing Table V with Table III, we find that our results are close to the reference values.

| \( r_m(\sigma_R) \) | \( r_n(\sigma_R) \) | \( r_{\text{skin}}(\sigma_R) \) |
|-------------------|-------------------|-------------------|
| \(^{208}\text{Pb}\) 5.627 ± 0.020 | 5.743 ± 0.020 | 0.299 ± 0.020 |
| \(^{58}\text{Ni}\) 3.711 ± 0.010 | 3.740 ± 0.010 | 0.055 ± 0.010 |
| \(^{48}\text{Ca}\) 3.445 ± 0.022 | 3.488 ± 0.022 | 0.103 ± 0.022 |
| \(^{40}\text{Ca}\) 3.372 ± 0.011 | 3.367 ± 0.011 | -0.011 ± 0.011 |
| \(^{12}\text{C}\) 2.340 ± 0.009 | 2.354 ± 0.009 | 0.026 ± 0.009 |

We show mass-number \((A)\) dependence of stable nuclei in Figs. 10, 11, 12. Figure 10 shows skin values as a function of \( S_p - S_n \), where \( S_p \) \((S_n)\) is the proton (neutron) separation energy. The skin values \( r_{\text{skin}}(\sigma_R) \) determined from measured \( \sigma_R \) for \(^{208}\text{Pb}, ^{58}\text{Ni}, ^{40,48}\text{Ca}\) are compared with the data of PREX2 \([5]\), \(^{116,118,120,122,124}\text{Sn} \) \([87,88]\), \(^{48}\text{Ca} \) \([45]\). Our results are consistent with the previous experimental skin-values.

**Figure 11** shows matter radii \( r_m \) as a function of \( A^{1/3} \). For \(^{208}\text{Pb}, ^{116,118,120,122,124}\text{Sn}, ^{48}\text{Ca}\), the \( r_m \) are derived from the corresponding skin values \([5,45,87,88]\) and the corresponding \( r_p \) of electron scattering. For \(^{12}\text{C}, ^{40,48}\text{Ca}, ^{58}\text{Ni}, ^{208}\text{Pb}\), our results are added. Our results are consistent with the previous works.

**Figure 12** shows neutron radii \( r_n \) as a function of mass number \( A^{1/3} \). For \(^{208}\text{Pb}, ^{116,118,120,122,124}\text{Sn}, ^{48}\text{Ca}\), the \( r_n \) are derived from the corresponding skin values \([5,45,87,88]\) and the corresponding \( r_p \) of electron scattering. For \(^{12}\text{C}, ^{40,48}\text{Ca}, ^{58}\text{Ni}, ^{208}\text{Pb}\), our results are added. Our results are consistent with the previous works.
FIG. 12. Neutron radii $r_n$ as a function of mass number $A^{1/3}$. The symbol " $\alpha_D$ " means the results of the $E1$ polarizability experiment ($E1pE$) for $^{120}\text{Sn}$ [88] and $^{48}\text{Ca}$ [45]. The symbol "PREX" stands for the result deuced from $r_{\text{skin}}^{\text{PREX2}} = 0.283 \pm 0.071 \text{fm}$. The symbol "SDR" shows the results [87] of the measurement based on the isovector spin-dipole resonances (SDR) in the Sb isotopes. Open circles stand for the results of this work. The dashed line is a guide for the eyes. The data (closed circles with error bar) are taken from Refs. [5, 45, 87, 88].

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