Non-standard interactions and bimagic baseline for neutrino oscillations

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For standard interactions of neutrinos with matter bimagic baseline of length about 2540 Km is known to be suitable for getting good discovery limits of neutrino mass hierarchy, $\sin^2 \theta_{13}$ and $CP$ violation in the $\nu_e \rightarrow \nu_\mu$ oscillation channel. We discuss how even in presence of non-standard interactions (NSIs) of neutrinos with matter this baseline is found to be suitable for getting these discovery limits. This is because even in presence of NSIs one could get the $\nu_e \rightarrow \nu_\mu$ oscillation probability to be almost independent of $CP$ violating phase $\delta$ and $\theta_{13}$ for one hierarchy and highly dependent on these two for the other hierarchy over certain parts of neutrino energy range. For another certain part of the energy range the reverse of this happens with respect to the hierarchies. We present the discovery limits of NSIs also in the same neutrino energy range. However, as with the increase of neutrino energy the NSI effect in the above oscillation probability gets relatively more pronounced in comparison to the vacuum oscillation parameters, so we consider higher neutrino energy range also for getting better discovery limits of NSIs. Analysis presented here for 2540 Km could also be implemented for longer bimagic baseline $> 6000$ Km.

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I. INTRODUCTION

The present experiments on neutrino oscillations confirms that there is mixing between different flavours of neutrinos (νe, νμ, ντ). The probability of neutrino oscillations depends on various parameters of the neutrino mixing matrix—the PMNS matrix [1]. The current experiments tells us about two of the angles θ_{23} and θ_{12} [2] with some accuracy but for θ_{13} only the upper bound is given [2] and the CP violating phase δ is totally unknown. Although the mass squared difference of the different neutrinos (∆m^2_{ij} = m_i^2 - m_j^2) are known to us but the sign of ∆m^2_{31} (which is related to mass hierarchy) is still unknown. Due to the correlations among these unknowns there are ambiguities [3] in analysing neutrino oscillation datas. To reduce these ambiguities one may consider neutrino oscillation experiments in long baseline [4] - particulaly in magic baseline [5]. The magic baseline satisfies certain condition on its length from the detector and is found to be about 7500 Km where the perturbative expression of probability P(νe → νμ) becomes independent of δ up to order α^2 (where α = ∆m^2_{21}/∆m^2_{31}). Although this could result in finding out the other unknown oscillation parameters conveniently but for measurement of δ this baseline is not suitable. To circumvent this problem, conditions on neutrino energy has been considered [6, 7] in νe → νμ channel for which also the perturbative expression of probability becomes independent of δ but only on a part of the neutrino energy spectrum. But the other part of the spectrum will be sensitive to CP violating phase δ. As pointed out in [7], one may consider satisfying two different energy conditions simultaneously for two hierarchies in the same baseline which results in fixing the length of bimagic baseline to about 2540 Km. Unlike magic baseline, here the baseline is shorter so the neutrino flux for such baseline is reduced by lesser amount at the detector. Also in this oscillation channel νe → νμ which has been considered in this work, the detection of muon is easier in comparison to some other channels where the detection of electron is required.

We have studied the effect of non-standard interactions (NSIs) of neutrinos with matter in bimagic baseline. At first there is discussion on how to obtain the perturbative expression of the probability of oscillation up to order α^2 in νe → νμ channel in presence of NSIs. The NSIs present in the νe → νμ oscillation probability are ǫ_{ee}, ǫ_{eμ} and ǫ_{eτ} among which ǫ_{ee}, ǫ_{eμ} are ∝ α but ǫ_{eτ} has no such constraints in considering perturbation. In our numerical analysis we have considered the experimentally allowed range which covered the perturbative regime and also has gone beyond that. We have also presented the δ and θ_{13} independent perturbative expression of the oscillation probability in presence of the NSIs under two different magic energy conditions corresponding to two different hierarchies. In presence of NSIs different discovery limits for hierarchy of neutrino masses, for sin^2 θ_{13} and also for CP violation have been shown in figures. Discovery limits of NSIs particularly ǫ_{ee}, ǫ_{eμ} and ǫ_{eτ} for specific values of θ_{13} and δ in their allowed range have also been presented. One may note that to satisfy the bimagic conditions one requires lower neutrino energy within 5 GeV. However, the perturbative expression of oscillation probability shows that the NSI effect will relatively increase in comparison to other neutrino oscillation parameters in vacuum if the neutrino energy is higher. For this reason we have considered higher neutrino energy of 50 GeV also to study the discovery limits of NSIs in the same baseline of 2540 Km for which better limits are obtained.
II. $\nu_e \rightarrow \nu_\mu$ Oscillation Probability in Presence of NSI

The fermion-neutrino interaction in matter is defined by the Lagrangian:

$$L_{NSI}^M = \frac{G_F}{\sqrt{2}} \epsilon_{\alpha\beta}^{fP} [\bar{\nu}_\beta \gamma^\mu L \nu_\alpha] [\bar{f} \gamma_\mu Pf] + \frac{G_F}{\sqrt{2}} \left( \epsilon_{\alpha\beta}^{fP} \right)^* [\bar{\nu}_\alpha \gamma^\mu L \nu_\beta] [\bar{f} \gamma_\mu Pf]$$  \hspace{1cm} (1)

where $P \in L, R$, $L = 1 - \gamma^5$, $R = 1 + \gamma^5$, $f = e, u, d$ and $\epsilon_{\alpha\beta}^{fP}$ is the deviation from standard interactions. There are model dependent bounds on these NSI parameters [8, 9]. In $R$-parity violating Supersymmetric models these NSI parameters could be related to trilinear lepton number violating couplings [10]. Also such parameters could be sizable [11] in unified supersymmetric models [12]. The model independent bounds have been discussed in [13]. The above NSI parameters can be reduced to the effective parameters as:

$$\epsilon_{\alpha\beta} = \sum_{f,P} \frac{\epsilon_{\alpha\beta}^{fP} n_f}{n_e}$$  \hspace{1cm} (2)

where $n_f$ is the number density of fermion, $n_e$ is the electron number density. In neutrino oscillation experiments this effective parameter ($\epsilon_{\alpha\beta}$) corresponds to the replacement in the matter interaction part of the evolution of flavoured neutrinos. This change can be seen as below:

$$H_{matter} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} \epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon_{e\mu}^* & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ \epsilon_{e\tau}^* & \epsilon_{\mu\tau}^* & \epsilon_{\tau\tau} \end{pmatrix}$$  \hspace{1cm} (3)

In general, $\epsilon_{e\mu}$, $\epsilon_{e\tau}$ and $\epsilon_{\mu\tau}$ could be complex. However, for our numerical analysis we have considered $\epsilon_{e\mu}$ and $\epsilon_{e\tau}$ to be real. If we assume uncorrelated errors, the bounds on $\epsilon_{\alpha\beta}$ can be approximately written as [13]

$$\epsilon_{\alpha\beta} \lesssim \left[ \sum_P (\epsilon_{\alpha\beta}^{eP})^2 + (3\epsilon_{\alpha\beta}^{\mu P})^2 + (3\epsilon_{\alpha\beta}^{\nu P})^2 \right]^{1/2}$$  \hspace{1cm} (4)

The NSIs $\epsilon_{ee}$, $\epsilon_{e\mu}$ and $\epsilon_{e\tau}$ play significant role in $\nu_e \rightarrow \nu_\mu$ oscillation channel which we have considered. The bounds for these [13] in the context of neutrino oscillation for neutrinos passing through neutral earth like matter is $\epsilon_{ee} < 4$, $\epsilon_{e\mu} < 0.33$ and $\epsilon_{e\tau} < 3$.

In vacuum, flavor eigenstates $\nu_\alpha$ may be related to mass eigenstates of neutrinos $\nu_i$ as

$$|\nu_\alpha> = \sum_i U_{\alpha i} |\nu_i> \hspace{0.5cm}, \hspace{0.5cm} U = R_{23} R_{13}(\delta) R_{12} \hspace{0.5cm} \text{and} \hspace{0.5cm} i = 1, 2, 3,$$  \hspace{1cm} (5)

where $U$ is PMNS matrix [1] and $R_{ij}$ are the rotation matrices and $R_{13}(\delta)$ contains the $CP$ violating phase $\delta$ signifying the complex rotation [11]. General probability expression for oscillation of neutrino of flavor $l$ to neutrino flavor $m$ in matter (satisfying adiabatic condition for the density of matter) is given by

$$P(\nu_l \rightarrow \nu_m) = \delta_{lm} - 4 \sum_{i>j} \text{Re}[J_{ij}^m] \sin^2 \Delta_{ij} + \sum_{i>j} \text{Im}[J_{ij}^m] \sin 2\Delta_{ij}$$  \hspace{1cm} (6)

where

$$J_{ij}^m = U_{li} U_{mj}^* U_{mi} U_{lj}^*.$$  \hspace{1cm} (7)
\[ \Delta'_{ij} = \frac{\Delta'm^2_{ij} L}{4E}. \]  

(8)

Here

\[ \Delta'_{i} = m_{i}^{2} - m_{j}^{2} \]  

(9)

and label (') indicates the neutrino matter interaction induced quantities corresponding to those quantities in vacuum.

We discuss in brief the perturbative approach for evaluating the induced quantities and for obtaining the probability of oscillation \( \nu_e \rightarrow \nu_\mu \) for neutrinos passing through earth matter. The diagonal neutrino mass matrix is approximately given by

\[ m \approx \Delta m^3_{21} \text{diag}(0, \alpha, 1). \]  

(10)

The effective Hamiltonian induced by interaction of matter with neutrinos is written in weak interaction basis as

\[ H \approx R_{23} M R_{23}^\dagger \]  

(11)

where

\[ M = \frac{\Delta m^2_{31}}{2E} R_{13}(\delta) R_{12} \frac{m}{\Delta m^2_{31}} R_{12}^\dagger R_{13}(\delta)^\dagger + \frac{\Delta m^2_{31}}{2E} \text{diag}(A, 0, 0) + \frac{\Delta m^2_{31}}{2E} R_{23}^\dagger \begin{pmatrix} Z & X & Y \\ X^* & B & C \\ Y^* & C^* & D \end{pmatrix} R_{23}. \]  

(12)

In equation (12)

\[ A = \frac{2E\sqrt{3}G_F n_e}{\Delta m^2_{31}}, \ X = A \epsilon_{e\mu}, \ Y = A \epsilon_{e\tau}, \ Z = A \epsilon_{e\nu}, \ B = A \epsilon_{\mu\mu}, \ C = A \epsilon_{\mu\tau}, \ D = A \epsilon_{\tau\tau}, \]  

(13)

where \( A \) is considered due to Standard model interaction of neutrinos with electron and \( G_F \) is the Fermi constant and \( n_e \) is the electron number density written as \( n_e = (0.5N_A)\rho \) and \( N_A \) is Avogadro’s number and \( \rho \) is the matter density in gm/cc. \( \epsilon_{e\nu}, \epsilon_{e\mu}, \epsilon_{e\tau}, \epsilon_{\mu\mu}, \epsilon_{\mu\tau}, \epsilon_{\tau\tau} \) are considered due to NSI of neutrinos with matter. We consider magnitude of these NSI parameters except \( \epsilon_{e\nu} \) not higher than \( \alpha \) in using perturbation method. As \( \epsilon_{e\nu} \) has been considered in the leading part of the Hamiltonian, our perturbative result is fine even for its highest experimentally allowed value. In our numerical analysis we have considered even the uppermost allowed values of other NSI parameters [13]. In equation (12), (‘\( \ast \)’) is denoted for complex conjugation.

The mixing matrix \( U' \) can be found out as \( U' = R_{23} \ W \). Here \( W \) is the normalized eigenvectors of \( \Delta m^2_{31} \ M/(2E) \) calculated through perturbative technique similar to the one adopted in [14]. We have taken into account only the non-degenerate perturbative approach. Let us consider the case where NSIs are present and where \( \sin^2 \theta_{13} \) is small and of the order of \( \alpha \) or less. \( M \) in equation (12) can be written as \( M = M^{(0)} + M^{(1)} + M^{(2)} \) where \( M \) contains terms of the order of \( \alpha \). Then we can write

\[ M^{(0)} = \frac{\Delta m^2_{31}}{2E} \text{diag}(A', 0, 1), \quad M^{(1)} = \frac{\Delta m^2_{31}}{2E} \begin{pmatrix} \alpha s^2_1 & b \\ b^* & \alpha c^2_1 + c_2 s_3 - s_2 c_3 \\ a^* & e_{23} c - s_{23} f \end{pmatrix}, \]  

\[ M^{(2)} = \frac{\Delta m^2_{31}}{2E} \begin{pmatrix} s^2_3 & 0 & -e^{i\delta} \alpha c_{13}s^2_{12}s_{13} \\ 0 & s^2_{23} & -e^{-i\delta} \alpha c_{13}s_{12}s_{13} \\ -e^{i\delta} \alpha c_{13}s^2_{12}s_{13} & -e^{-i\delta} \alpha c_{12}s_{12}s_{13} & -s^2_{13} \end{pmatrix}. \]  

(14)
where

\[ A' = A(1 + \epsilon_{ee}), \quad a = c_{23} Y + e^{-i\delta} s_{13} + X s_{23}, \quad b = c_{23} X + c_{12} a s_{12} - Y s_{23}, \]

\[ c = B c_{23} - C^* s_{23}, \quad d = C c_{23} - D s_{23}, \quad e = C^* c_{23} + B s_{23}, \quad f = D c_{23} + C s_{23} \tag{15} \]

The eigenvalues of \( H \) upto second order in \( \alpha \) are

\[ \frac{m^2_{1}}{2E} \approx \frac{\Delta m^2_{31}}{2E} \left[ A' + \alpha s^2_{12} + s^2_{13} + \frac{|b|^2}{A'} + \frac{|a|^2}{(-1 + A')} \right], \]

\[ \frac{m^2_{2}}{2E} \approx \frac{\Delta m^2_{31}}{2E} \left[ \alpha c^2_{12} - \frac{|b|^2}{A'} + c c^2_{23} - (d c^2_{23} + c s^2_{23})^2 \right], \]

\[ \frac{m^2_{3}}{2E} \approx \frac{\Delta m^2_{31}}{2E} \left[ 1 - s^2_{13} + \frac{|a|^2}{(1 - A')} + f c^2_{23} + \epsilon s_{23} + (\epsilon c_{23} - f s_{23})^2 \right] \tag{16} \]

Using the eigenvalues of \( H \) given in equation (16) to equation (9), the term \( \Delta'_{ij} \) in equation (6) can be calculated (\( \Delta'_{ij} \) is defined in equation (8)). From equation (16), it is seen that NSIs are present in the eigenvalues of \( H \) but those are of the order of \( \alpha^2 \). The oscillation probability \( P(\nu_e \to \nu_\mu) \) is calculated up to order \( \alpha^2 \).

As the non-zero terms in \( A^{lm}_{ij} \) in equation (6) calculated using equation (7) is already at order \( \alpha^2 \), only terms which are zeroth order in \( \alpha \) is considered in calculating \( \Delta'_{ij} \). Hence, the oscillation probability, \( P(\nu_e \to \nu_\mu) \), in the presence of NSI up to order \( \alpha^2 \) can be written as:

\[ P(\nu_e \to \nu_\mu) \approx \frac{4c_{23}}{A^2} |b|^2 \sin^2 \left( \frac{\Delta m^2_{31} A'L}{4E} \right) + \frac{4s^2_{23}}{(1 - A')^2} |a|^2 \sin^2 \left( \frac{\Delta m^2_{31} (1 - A')L}{4E} \right) \]

\[ + \frac{8s_{23} c_{23}}{A'(1 - A')} Re(b^*a) \sin \left( \frac{\Delta m^2_{31} A'L}{4E} \right) \cos \left( \frac{\Delta m^2_{31} L}{4E} \right) \sin \left( \frac{\Delta m^2_{31} (1 - A')L}{4E} \right) \]

\[ + \frac{8s_{23} c_{23}}{A'(1 - A')} Im(b^*a) \sin \left( \frac{\Delta m^2_{31} A'L}{4E} \right) \sin \left( \frac{\Delta m^2_{31} L}{4E} \right) \sin \left( \frac{\Delta m^2_{31} (1 - A')L}{4E} \right) \tag{17} \]

For NSI terms \( X = Y = Z = 0 \) in equation (17), the probability expression reduces to that for the standard model interaction of neutrinos with matter.

### III. Bimagic Conditions on Neutrino Energy

If we want the probability, \( P(\nu_e \to \nu_\mu) \) to be independent of the \( CP \) violating phase \( \delta \) and \( \theta_{13} \) up to order \( \alpha^2 \) then we have to use the condition:

\[ \sin \frac{\Delta m^2_{31} (1 - A') L}{4E} = 0 \tag{18} \]

One may note here that this corresponds to two different conditions for two different hierarchies of neutrino masses. Considering \( \epsilon_{ee} = 0 \) in \( A' \) and keeping \( \epsilon_{31} \) and \( \epsilon_{e\tau} \) less than \( \alpha \) as required by the perturbation theory one can see that this condition on neutrino energy is the same one as discussed earlier without NSIs [7]. For a given length \( L \) of the baseline above condition constrains the neutrino energy \( E \) as

\[ E = \Delta m^2_{31} / \left( \pm 4n\pi/L + 2\sqrt{2} G_F n_e (1 + \epsilon_{ee}) \right) . \tag{19} \]
As long as $\epsilon_{ee}$ is unknown it seems that this magic energy $E$ cannot be known. However, what is important in our work is to know the possible range of this magic energy depending on the presently allowed range of $\epsilon_{ee}$ which is less than 4. Using eq.(18) in eq.(17) we get

$$P(\nu_e \rightarrow \nu_\mu) \approx \frac{4c_{23}^2}{A^2}(c_{23}X + c_{12}s_{12} - Ys_{23})^2\sin^2\left(\frac{\Delta m_{31}^2 A'}{4E}\right).$$

which is independent of $\theta_{13}$ and $\delta$.

![Plots for probability of oscillation $\nu_e \rightarrow \nu_\mu$ versus neutrino energy ($E$) varying $\delta$ over the entire range of 0 to 2\pi and $\theta_{13}$ in the entire allowed range 0 to 12\degree.]

FIG. 1: Plots for probability of oscillation $\nu_e \rightarrow \nu_\mu$ versus neutrino energy ($E$) varying $\delta$ over the entire range of 0 to 2\pi and $\theta_{13}$ in the entire allowed range 0 to 12\degree.

In figure 1 we have shown the probability of oscillation $\nu_e \rightarrow \nu_\mu$ versus energy after varying $\theta_{13}$ in the entire allowed range of 0$^\circ$–12$^\circ$ and also varying $\delta$ in the full range of 0–2\pi for $L = 2540$ Km for both normal (NH) and inverted hierarchies (IH). We have made the plots for two NSIs ($\epsilon_{\mu\mu}$ and $\epsilon_{e\tau}$) but considering one at a time. The value of the NSI considered is $\epsilon_{e\mu} = 0.024$ and $\epsilon_{e\tau} = 0.024$. From the plots we can see that the probability becomes almost independent of $\delta$ and $\theta_{13}$ at the magic energies. As for example, for $n = 1$ in eq.(19) these energies are ($E = 1.9(3.3)$ GeV for NH(IH) respectively. Magic energies for higher $n$ values gets smaller as seen from eq.(19) and the figure.

Next we discuss the bimagic conditions on neutrino energy. As discussed in [7] the sensitivity of the hierarchy is maximum if one of the hierarchies (say NH or IH) obey the condition in eq.(18) for which probability is independent of $\theta_{13}$ and $\delta$ whereas for the other hierarchy the probability has maximum dependence with $\theta_{13}$ and $\delta$ which can be achieved by imposing the condition

$$\sin\frac{\Delta m_{31}^2 (1 - A') L}{4E} = \pm 1.$$  

These two conditions mentioned in (18 ) and (21) can be rewritten in two different ways: (a) conditions for IH with $\delta$ and $\theta_{13}$ independence and NH with maximum dependence to those - which can be written as:

$$\frac{\Delta m_{31}^2 (1 + |A'|) L}{4E h c} = n\pi$$  for IH

(22)
\[
\frac{|\Delta m^2_{31}|(1 - |A'|)L}{4E\hbar c} = (m - 1/2)\pi \text{ for NH}
\] (23)

where \(n, m\) are integers and \(n > 0\).

(b) conditions for NH with \(\delta\) and \(\theta_{13}\) independence and IH with maximum dependence to those - which can be written as:

\[
\frac{|\Delta m^2_{31}|(1 - |A'|)L}{4E\hbar c} = n' \pi \text{ for NH}
\] (24)

\[
\frac{|\Delta m^2_{31}|(1 + |A'|)L}{4E\hbar c} = (m' - 1/2)\pi \text{ for IH}
\] (25)

where \(n'\) and \(m'\) are integers and \(n' \neq 0\) and \(m' > 0\).

Solving the equations (22) and (23) one gets the length of the baseline \(L\) as

\[
L(\text{Km}) = \frac{(n - m + 1/2)\pi\hbar c}{\sqrt{2}G_F n_e (1 + \epsilon_{ee})} \approx 16260.5 \times \frac{(n - m + 1/2)}{\rho (\text{gm/cc})(1 + \epsilon_{ee})}
\]

which implies

\[
\rho L(\text{Km gm/cc}) \approx 16260.5 \times (n - m + 1/2)/(1 + \epsilon_{ee})
\] (26)

and for the inverted hierarchy the energy \(E_{IH}\) with \(\delta\) and \(\theta_{13}\) independence as

\[
E_{IH}(\text{GeV}) = \frac{1}{2\pi\hbar c (\text{GeV Km}) \sqrt{|\Delta m^2_{31}|(\text{GeV}^2) L(\text{Km})}} \frac{|\Delta m^2_{31}|(\text{GeV}^2)L(\text{Km})}{(n + m - 1/2)}
\] (27)

Similarly, solving the equations (24) and (25) one gets the length of the baseline \(L'\) as

\[
\rho L'(\text{Km gm/cc}) \approx 16260.5 \times (m' - n' - 1/2)/(1 + \epsilon_{ee})
\] (28)

and for the normal hierarchy the energy \(E_{NH}\) with \(\delta\) and \(\theta_{13}\) independence as

\[
E_{NH}(\text{GeV}) = \frac{1}{2\pi\hbar c (\text{GeV Km}) \sqrt{|\Delta m^2_{31}|(\text{GeV}^2) L'(\text{Km})}} \frac{|\Delta m^2_{31}|(\text{GeV}^2)L'(\text{Km})}{(n' + m' - 1/2)}
\] (29)

Firstly, for \(\epsilon_{ee} = 0\) one can get one possible solution for common baseline i.e., \(L = L'\) to be about 2540 Km if the choices are made as follows: \(n = 1, m = 1\) and \(n' = 1\) and \(m' = 2\). The neutrino energy \(E_{IH} \approx 3.3\) GeV and \(E_{NH} \approx 1.9\) Gev. However, one could get more common solutions for bimagic baseline with \(L = L'\) for length by considering suitable choices of \(m, n, m'\) and \(n'\) for which \(n - m = m' - n' - 1; n - m\) could be 1 or 4 resulting in \(L = L' > 6000\) Km. As for example, considering \(\rho \approx 4\) gm/cc with \(n - m = 1\) the length is about 6100 Km.

As the present upper bound of some of the NSIs could be quite large \(\gtrsim \alpha [13]\) which is not considered in our perturbative approach it is natural to ask what happen to such magic energies in the same 2540 Km baseline in those cases. We have checked numerically that even for highest allowed values of NSIs as for
example, for $\epsilon_{ee} = 0.33$ the $E_{NH}(E_{IH}) \approx 2.03(3.26)$ GeV; for $\epsilon_{ee} = 4.0$ the $E_{NH}(E_{IH}) \approx 1.18(3.18)$ GeV. However, for $\epsilon_{ee} \gtrsim 0.5$ it is difficult to get bimagic energies although we have presented numerical analysis of discovery limits of various oscillation parameters for that also. For $\epsilon_{ee} = 0.5$ the bimagic energies are the $E_{NH}(E_{IH}) \approx 2.02(3.57)$ GeV. It is important to note that all these bimagic energies are within 1-5 GeV which is the full neutrino energy range in our analysis. Interestingly, one can check here that the perturbative results for bimagic baseline length and the energies hold good even for higher values of $\epsilon_{ee} \gg \alpha$. Even for such large $\epsilon_{ee}$ it is possible to obtain bimagic energies within 1-5 GeV for the same baseline length of 2540 Km. As for example, for $\epsilon_{ee} = 4$ considering $n = 3$, $m = 1$, $n' = 4$ and $n'' = 1$ gives $L = L' \approx 2540$ Km and $E_{IH}$ and $E_{NH}$ obtained from perturbative approach are very near to the numerical values for bimagic energies mentioned above. As the bimagic energies are within 1-5 GeV, by considering this as the neutrino energy range in our numerical analysis, it may be expected to get better discovery limits to hierarchy, $\theta_{13}$ and CP violation for various choices of NSIs.

Now, we can write down the probabilities at the particular energies $E_{IH}$ and $E_{NH}$ according to the conditions discussed above. At $E \approx E_{IH}$ for condition satisfying eq. (22) the probability is given by:

$$P(\nu_e \to \nu_\mu)(IH) \approx \frac{4e_{23}^2}{|A'|^2} |b|^2 \sin^2 \left( \frac{\pi}{2} \left( n + m - \frac{1}{2} \right) \right)$$

and for the condition satisfying eq. (23) the probability is given by:

$$P(\nu_e \to \nu_\mu)(NH) \approx \frac{4e_{23}^2}{|A'|^2} |b|^2 \cos^2 \left( \frac{\pi}{2} \left( n + m - \frac{1}{2} \right) \right) + \frac{4e_{23}^2}{(1 - |A'|)^2} |a|^2$$

$$- \frac{8s_{23}a_{23}}{|A'|^2(1 - |A'|)} \left[ Re \left( b_2^* \right) \cos^2 \left( \frac{\pi}{2} \left( n + m - \frac{1}{2} \right) \right) + \frac{1}{2} Im \left( b_2^* \right) \sin \left( \pi \left( n + m - \frac{1}{2} \right) \right) \right]$$

At $E \approx E_{NH}$ for condition satisfying eq. (24) the probability is given by:

$$P(\nu_e \to \nu_\mu)(NH) \approx \frac{4e_{23}^2}{|A'|^2} |b|^2 \sin^2 \left( \frac{\pi}{2} \left( n' + m' - \frac{1}{2} \right) \right)$$

and for the condition satisfying eq. (25) the probability is given by

$$P(\nu_e \to \nu_\mu)(IH) \approx \frac{4e_{23}^2}{|A'|^2} |b|^2 \cos^2 \left( \frac{\pi}{2} \left( n' + m' - \frac{1}{2} \right) \right) + \frac{4e_{23}^2}{(1 + |A'|)^2} |a|^2$$

$$+ \frac{8s_{23}a_{23}}{|A'|^2(1 + |A'|)} \left[ Re \left( b_2^* \right) \cos^2 \left( \frac{\pi}{2} \left( n' + m' - \frac{1}{2} \right) \right) - \frac{1}{2} Im \left( b_2^* \right) \sin \left( \pi \left( n' + m' - \frac{1}{2} \right) \right) \right]$$

One can see that at $E_{IH} = 3.3$ GeV from eqs. (30) and (31) in the case of $\epsilon_{ee} = \epsilon_{e\mu} = 0$ case i.e., $X = Y = 0$ if $\theta_{13}$ also vanishes then $a = 0$ and there is no difference in $P_{IH}$ and $P_{NH}$. Same thing happens at $E_{NH} = 1.9$ GeV as seen in eqs. (32) and (33). This means that for $\epsilon_{ee} = \epsilon_{e\mu} = 0$ case it is not so likely to get the hierarchy discovery limit at $\theta_{13} = 0$. However, on the contrary in presence of these NSIs there is difference in $P_{IH}$ and $P_{NH}$ even for $\theta_{13} = 0$. So in presence of NSIs like $\epsilon_{ee}$ and $\epsilon_{e\mu}$ one could get discovery limit of hierarchy even at $\theta_{13} = 0$. However, in case of $\epsilon_{ee}$ if $\theta_{13} = 0$ then there is no difference between $P_{IH}$ and $P_{NH}$ and so it is not likely to get discovery limit at $\theta_{13} = 0$. These features are found in our numerical analysis.
In probing other NSIs like $\epsilon_{\mu\tau}$, $\epsilon_{\mu\nu}$ and $\epsilon_{\tau\tau}$, oscillation channel $\nu_e \rightarrow \nu_{\mu}$ is not appropriate. This follows from the probability in equation (17). For those NSIs considering the disappearance channel ($\nu_{\mu} \rightarrow \nu_{\mu}$) is appropriate. One cannot get any condition on neutrino energy in general to remove the dependence on $\delta$ in the oscillation probability for this channel. However, it is found that upto the order $\alpha$ without imposing any condition on neutrino energy this oscillation probability is already independent of $\delta$ as shown below [15].

$$P(\nu_{\mu} \rightarrow \nu_{\mu}) = 1 - 4c^2_{23}s^2_{23}\sin^2 \frac{\Delta m^2_{31}L}{4E} + 4c^2_{12}c^2_{23}s^2_{23}\alpha \frac{\Delta m^2_{31}L}{4E} \sin \frac{\Delta m^2_{31}L}{2E}$$
$$+ 2c^2_{23}s^2_{23} \left[ (c^2_{23} - s^2_{23}) (\epsilon_{\mu\mu} - \epsilon_{\tau\tau}) - 4c^2_{23}s^2_{23}Re(\epsilon_{\mu\tau}) \right] \frac{\Delta m^2_{31}A'L}{2E} \sin \frac{\Delta m^2_{31}L}{2E}$$
$$- 8c^2_{23}s^3_{23} (c^2_{23} - s^2_{23}) \left[ c^2_{23}s^2_{23} (\epsilon_{\mu\mu} - \epsilon_{\tau\tau}) + (c^2_{23} - s^2_{23}) Re(\epsilon_{\mu\tau}) \right] A' \sin^2 \frac{\Delta m^2_{31}L}{4E} \quad (34)$$

To get the sensitivity of NSIs like $\epsilon_{\mu\mu}$, $\epsilon_{\mu\tau}$ and $\epsilon_{\tau\tau}$, this disappearance channel is appropriate but it is not much suitable for finding sensitivity to $\delta$ or discovery limits for $CP$ violation. As the probability upto order $\alpha$ is already independent of $\delta$ here one does not require the bimagic conditions and as such there is no restriction on the length of the baseline and neutrino energy. We have not done this analysis separately but sensitivity of some of the above-mentioned NSI parameters have been discussed in the disappearance channel ($\nu_{\mu} \rightarrow \nu_{\mu}$) in [16].

IV. NUMERICAL SIMULATION

As an outcome of the bimagic energy conditions in presence of NSIs the length of the baseline is $\approx 2540$ Km. To study the oscillation of $\nu_e \rightarrow \nu_{\mu}$ we have considered the experimental set-up and the detector characteristics as discussed in [7] for a running time of 2.5 year. We consider the neutrino factory having $5 \times 10^{21}$ muon decays per year with parent muon of energy 5 GeV and the magnetized totally active scintillator detector of 25 kt mass with energy threshold of 1 GeV. The numerical simulation has been done by using GLoBES [17]. In presenting the discovery limits of hierarchy, $sin^2 \theta_{13}$ and $CP$ violation in bimagic baseline, highest possible values for the NSIs have been considered. However, when we observe that the discovery limits are either covering the entire allowed region or not at all obtainable then we refrain from presenting those figures and instead we present discovery limits for some lower values of NSIs.

![FIG. 2: The 3σ contours showing discovery limit of hierarchy for different NSIs $\epsilon_{\tau\tau}$, $\epsilon_{\mu\mu}$ and $\epsilon_{ee}$.](image)

In figure 2 we have shown for what value of $\delta$ and $\theta_{13}$ at 3 $\sigma$ confidence level one can identify the specific hierarchy which could be either normal or inverted. For $\epsilon_{\tau\tau} = 3$ and $\epsilon_{\mu\mu} = 0.33$ (one at a time) NH could
be discovered at any value of \( \theta_{13} \) irrespective of any specific value of \( \delta \) whereas for IH nowhere it is found to be discovered. So in our figures we have chosen some lower values of these two NSIs. Considering \( \epsilon_{e\mu} = 0.1 \) and \( \epsilon_{e\tau} = 0.1 \) (one at a time) we have shown in figure 2 the discovery limit of hierarchy. From these figures for favorable values of \( \delta \) one could identify the inverted hierarchy of nature for \( \epsilon_{e\tau} \) at \( \sin^2 \theta_{13} \) as small as \( 4 \times 10^{-4} \) and for \( \epsilon_{e\mu} \) at \( \sin^2 \theta_{13} \) as small as about \( 2.5 \times 10^{-4} \). For normal hierarchy however, it is found from the figures that for \( \epsilon_{e\tau} = 0.1 \), only for \( \delta \) in the range of \( 3\pi/4 \) to \( 5\pi/4 \) and \( \sin^2 \theta_{13} \gtrsim 5 \times 10^{-4} \) one could reach the discovery limit. For other values of \( \delta \) normal hierarchy can be identified for any value of \( \theta_{13} \) including the zero value. Similarly, for \( \epsilon_{e\mu} = 0.1 \), only for \( \delta \) in the range of about \( \pi/4 \) to \( 5\pi/4 \) and \( \sin^2 \theta_{13} \gtrsim 2 \times 10^{-4} \) one could reach the discovery limit. Here also for other values of \( \delta \) normal hierarchy can be identified for any values of \( \theta_{13} \) including the zero value. For \( \epsilon_{ee} = 4 \) the normal hierarchy can be identified at \( \sin^2 \theta_{13} \gtrsim 10^{-4} \) and the inverted hierarchy can be identified at \( \sin^2 \theta_{13} \gtrsim 10^{-2} \).

![FIG. 3: The 3σ contours showing discovery limits of \( \theta_{13} \) for different NSI’s \( \epsilon_{e\tau}, \epsilon_{e\mu} \) and \( \epsilon_{ee} \).](image)

![FIG. 4: The 3σ discovery limits for the \( CP \) violating phase \( \delta \) for different NSI’s \( \epsilon_{e\tau}, \epsilon_{e\mu} \) and \( \epsilon_{ee} \).](image)

We have shown the discovery limits of \( \theta_{13} \) in figure 3 at 3σ confidence level. From figure 3 it can be seen that for NSI \( \epsilon_{e\mu} \sim 0.33 \) the discovery limits for \( \sin^2 \theta_{13} \) could be as low as \( \sin^2 \theta_{13} \sim 5.0 \times 10^{-4} \) and for \( \epsilon_{e\tau} \sim 3.0 \) the limit could be as low as \( 2.0 \times 10^{-3} \) for both the hierarchies. However, for \( \epsilon_{ee} \sim 4 \) this limit improves for NH and can be as low as \( \sin^2 \theta_{13} \sim 2 \times 10^{-5} \), but for IH it could be as low as \( \sin^2 \theta_{13} \sim 1.5 \times 10^{-3} \). In figure 4 we have shown the discovery limit of \( CP \) violation for different NSI at 3σ confidence level. The discovery limit for \( CP \) violating region is possible for \( \epsilon_{e\tau} = 3.0 \) at \( \sin^2 \theta_{13} \geq 2.5 \times 10^{-2} \) for \( 3\pi/8 \leq \delta \leq 3\pi/4 \) for NH. But for IH it is very difficult to obtain any discovery limit. However, for lower values of \( \epsilon_{e\tau} \) discovery limits could be easily obtained. For \( \epsilon_{e\mu} = 0.33 \) the discovery limit of \( CP \) violating region for NH is found for \( \sin^2 \theta_{13} \gtrsim 2 \times 10^{-3} \) with \( \pi/8 \leq \delta \leq 7\pi/8 \) and also for \( \sin^2 \theta_{13} \gtrsim 8 \times 10^{-4} \) with \( 9\pi/8 \leq \delta \leq 15\pi/8 \). In the case of IH for the same value of \( \epsilon_{e\mu} \) the discovery limits are found for \( \sin^2 \theta_{13} \gtrsim 4 \times 10^{-3} \) with \( \pi/8 \leq \delta \leq 3\pi/4 \).
and also for $\sin^2 \theta_{13} \gtrsim 4 \times 10^{-3}$ with $5\pi/4 \lesssim \delta \lesssim 15\pi/8$. For higher value of $\epsilon_{ee} = 4$ it is not possible to get discovery limit for $CP$ violating region. For $\epsilon_{ee} = 0.4$ the discovery limit of $CP$ violating region for NH is found for $\sin^2 \theta_{13} \gtrsim 7 \times 10^{-5}$ with $\pi/4 \lesssim \delta \lesssim \pi/2$ and also for $\sin^2 \theta_{13} \gtrsim 6 \times 10^{-5}$ with $5\pi/4 \lesssim \delta \lesssim 7\pi/4$. In the case of IH, for the same value of $\epsilon_{ee}$ the discovery limit are found for $\sin^2 \theta_{13} \gtrsim 9 \times 10^{-3}$ with $\pi/8 \lesssim \delta \lesssim \pi/2$ and also for $\sin^2 \theta_{13} \gtrsim 2 \times 10^{-3}$ with $5\pi/4 \lesssim \delta \lesssim 7\pi/4$.

![Graphs](image)

**FIG. 5:** Discovery limits of NSI ($\epsilon_{e\tau}$) for different fixed values of $\theta_{13}$ and $\delta$ considering muon energy 5 GeV.

Although we find good discovery limits for hierarchy, $\theta_{13}$ and $CP$ violation in bimagic baseline even in presence of NSI, however, to get good discovery limits of NSI, the neutrino energy around 5 GeV (as required by magic energy conditions) is not appropriate. One can see from the expression of $P(\nu_e \rightarrow \nu_\mu)$ in Eq. (17) that the NSI terms are energy independent whereas the terms containing only vacuum mixing parameters are suppressed by neutrino energy. This feature is present irrespective of specific channel for neutrino oscillation. Naturally for higher energy the relative effect of NSI parameters are enhanced with respect to vacuum neutrino mixing parameter and one might expect to get better discovery limits of NSIs.

Now considering 5 GeV as the maximum neutrino energy, we have presented the discovery limits of some NSIs - $\epsilon_{e\tau}$, $\epsilon_{e\mu}$ and $\epsilon_{ee}$ for various fixed values of $\sin^2 \theta_{13}$ and $\delta$ in figures 5, 6 and 7 respectively. We can see from these figures that the discovery limits of $\epsilon_{e\tau}$ and $\epsilon_{ee}$ are as low as $(\approx 0.015)$ for either of the hierarchy at $3\sigma$ confidence level. For $\epsilon_{ee}$ the limit is as low as at the order of $(\approx 10^{-1})$. But at higher neutrino energy say for 50 GeV from figures 8 and 9 one can see that for IH the discovery limit of $\epsilon_{e\tau}$ can be as low as $3 \times 10^{-3}$ and that of $\epsilon_{e\mu}$ could be as low as $7 \times 10^{-4}$. Similarly, for NH the discovery limit of $\epsilon_{e\tau}$ is as low as 0.01 and for $\epsilon_{e\mu}$ is as low as 0.002. For the case of $\epsilon_{ee}$, from figure 10 we can see that the discovery limit of the NSI ($\epsilon_{ee}$) is not so good and could be as low as of the order of $10^{-1}$. However, the overall probability of oscillation is suppressed with the increase in neutrino energy. Naturally it is expected that just increasing energy one may not keep getting better NSI discovery limits. In fact, we have checked at neutrino energy above 60 GeV there is insignificant improvement in discovery limits of $\epsilon_{e\mu}$ and $\epsilon_{e\tau}$ in 2540 Km baseline.
FIG. 6: Discovery limits of NSI ($\epsilon_{\mu\mu}$) for different fixed values of $\theta_{13}$ and $\delta$ considering muon energy to be 5 GeV.

FIG. 7: Discovery limits of NSI ($\epsilon_{ee}$) for different fixed values of $\theta_{13}$ and $\delta$ considering muon energy 5 GeV.
FIG. 8: Discovery limits of NSI ($\epsilon_{ee}$) for different fixed values of $\theta_{13}$ and $\delta$ considering muon energy 50 GeV.

FIG. 9: Discovery limits of NSI ($\epsilon_{e\mu}$) for different fixed values of $\theta_{13}$ and $\delta$ considering muon energy 50 GeV.
V. CONCLUSION

It is found that for getting good discovery limits for hierarchy, \( \sin^2 \theta_{13} \) and \( CP \) violation particularly in the \( \nu_e \to \nu_\mu \) oscillation channel, 2540 Km baseline is suitable even when NSI of neutrinos with matter are present. This is because the bimagic energies \( E_{IH} \) and \( E_{NH} \) lie within specific energy range, which is 1-5 GeV for this baseline even in presence of NSIs with their lower or higher allowed values (except for \( \epsilon_{e\tau} \gtrsim 0.5 \)) and this neutrino energy range has been chosen in our analysis with NSIs. It is important to note that this energy range is also suitable for no NSIs as in that case also bimagic energies are within 1-5 GeV [7].

To show what could be the utmost effect to the discovery limits corresponding to no-NSI case, we have considered highest possible values as obtained in the model independent case [13]. However, for model dependent cases [8–10] these bounds are in general, more stringent.

The discovery limits of hierarchy actually improves in presence of NSIs. Even one could get discovery limits at \( \theta_{13} = 0 \) for \( \epsilon_{e\mu} \) and \( \epsilon_{e\tau} \) which in absence of those NSIs are not expected. This is due to the fact that at bimagic energies the \( P_{IH} \) and \( P_{NH} \) are unequal even at \( \theta_{13} = 0 \) in presence of those NSIs. This does not occur for \( \epsilon_{ee} \). In this case, as for example, for \( \epsilon_{ee} = 4 \) the hierarchy discovery limits could be obtained at as low as \( \sin^2 \theta_{13} \gtrsim 10^{-4} \). Considering highest possible allowed values of \( \epsilon_{e\tau}, \epsilon_{e\mu} \) and \( \epsilon_{ee} \) we find that the discovery limits of \( \sin^2 \theta_{13} \) could be as low as \( 2 \times 10^{-3}, 6 \times 10^{-4} \) and \( 2 \times 10^{-5} \) respectively for normal hierarchy and as low as \( 2.8 \times 10^{-3}, 7 \times 10^{-4} \) and \( 1.5 \times 10^{-3} \) respectively for inverted hierarchy. Considering favorable values of \( \delta \) the discovery limits of \( CP \) violation are possible at following \( \sin^2 \theta_{13} \) values.

For \( \epsilon_{e\tau} = 3 \) the discovery limits of \( CP \) violation could be possible for high value of \( \sin^2 \theta_{13} \) at about 0.025 for normal hierarchy only. For inverted hierarchy it is not possible. For \( \epsilon_{e\mu} = 0.33 \) the discovery limits of \( CP \) violation could be obtained for \( \sin^2 \theta_{13} \) as low as \( 10^{-3} \) for normal hierarchy and at about \( 4 \times 10^{-3} \) for inverted hierarchy. For \( \epsilon_{ee} = 4.0 \) discovery limits of \( CP \) violation cannot be obtained. However, for
lower values of both $\epsilon_{\tau\tau}$ and $\epsilon_{ee}$ one could get discovery limits of $CP$ violation at some $\sin^2 \theta_{13}$ values. The discovery limits of NSIs could be improved if we consider neutrino energy up to 50 GeV and it could be as small as $10^{-3}$ for $\epsilon_{\mu\mu}$ and $\epsilon_{\tau\tau}$ and could be as small as $10^{-1}$ for $\epsilon_{ee}$. These NSI discovery limits essentially would give the upper bound on the respective parameters if they are not discovered.

It is interesting to note that there are other bimagic baselines with length greater than 6000 Km apart from 2540 Km as discussed before. One may explore the discovery limits of various vacuum neutrino oscillation parameters using those baselines also.

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