A distributed network-based economic model predictive control for networked smart energy systems considering communication network constraints

Alireza Nazmadini | Alimorad Khajeh Zadeh | Mehdi Jafari Shahbaz Zadeh

Abstract

Generally, the modern networked energy systems are constructed from some coupled sub-systems interconnected via their states and share their information through a communication network featuring time delays. This article suggests an efficient multi-agent based cooperative-distributed networked economic model predictive control scheme for energy consumption optimisation and reducing the cost of consumed electricity in network-based smart energy systems considering communication network delays. In this regard, a buffer-based moving horizon strategy with estimator is suggested to estimate the own states of every sub-system and the coupled states, that is, the states in which their information exchange among sub-systems and their values are not accessible due to communication time delays. Moreover, the boundedness of the estimation error as well as the stability of the closed-loop system are established. The usability and proficiency of the suggested scheme are proved by applying the developed approach on two case studies.

1 INTRODUCTION

The smart energy systems are identified as the larger wide area systems which are built up by humans. Nowadays, the improvements of information and communication technologies make the possibility to use the network-based approaches for the designing of networked wide-area controllers for networked smart energy systems. The network-based systems include a number of coupled sub-systems which exchange data via the communication channels. However, because of the communication limitations, time delay and packet loss phenomenon are inevitable in practical usages. So, the motivating aforementioned issues in the network-based systems occur in the control of widely distributed network-based smart energy systems with strong interacted sub-systems.

The distributed scheme of model-based predictive control, the so-called DMPC, is recently employed in the control of wide-area network-based plants such as water distribution networks, smart energy systems, chemical plants, etc. [1–11]. Most of the available results on DMPC-based algorithms are evolved considering the assumption that the entire systems’ states are accessible and the communication channels between sub-systems are ideal with no communication inherent imperfections, for instance time delay and packet loss [1–6]. However, these assumptions may not hold in practice.

Nevertheless, some DMPC-based schemes are used in the handling of the above-mentioned communication network imperfections owing to their ability to predict the system behaviour during a period of time [7–11]. Among different DMPC frameworks, cooperative DMPC is an important class of DMPC-based strategies, wherein a global performance criterion is optimised employing a local controller. This scheme is suitable for the stabilisation of wide-area plants with intensely interacted sub-systems such as smart energy systems [7].

On the other hand, with the increase in the political and commercial interests of energy systems, the complex control methods are required for the responsibility of generating, transmitting and delivering the energy [12]. However, designing the appropriate control schemes to operate the wide-area modern energy systems in a reliable and economical manner is a serious challenge. As a consequence, an important objective in developing new control schemes for modern network based energy systems is to include the economic aims such as cost of consumed energy, profitability, efficiency, sustainability and capacity...
This article suggests a new distributed network-based economic model predictive control, the so-called DNEMPC scheme for optimisation of the energy consumption and reduction of the cost of consumed electricity in the network-based smart energy systems. The proposed strategy is based on moving horizon estimation (MHE) strategy which can handle the system constraints and works as an estimator and a predictor simultaneously. To this end, the states which are coupled between sub-systems and their values are unknown, due to time delays, are considered as unknown disturbances. Then each local MHE estimates these coupled states and also the local state of each sub-system by solving an optimisation problem. Moreover, the performance function of the control problem is considered to have the capability for exchanging the information through network non-ideal behaviours which is the main motivation of the research in the current study.

The remainder of this article is presented as follows. In Section 2, the transmission network between sub-systems is explained and the formulation of problem is provided. Furthermore, some primary assumptions are represented. The proposed MHE-based cooperative DNEMPC scheme is explained in Section 3. Section 4 is assigned to the convergence analysis of the proposed estimator as well as the stability analysis of the closed-loop system. The simulation studies to verify the obtained theoretical results are given in Section 5 and eventually, some conclusions are drawn in Section 6.

Notations. Throughout the current article, symbol $∥∥$ stands for the induced 2-norm and Euclidean norm for matrices and vectors, respectively. Superscripts “−1” and “T” indicate the inverse of square matrices and the transpose operation, respectively. $\mathbb{R}^n$ and $\mathbb{R}^{m \times m}$, respectively, stand for Euclidean space with $n$ dimension and the collection of every $n \times m$ real matrices. Furthermore, $\mathbb{I}_M$ refers to the collection of $1, 2, \ldots, M$ integers. Considering $A_i$ is a vector or matrix, the notation $\text{col}(A_i)$ demonstrates $[A_{i1}^T, \ldots, A_{in}^T]^T$. Moreover, $\text{blkdiag}(A_i)$ for $A_i$, $i \in \mathbb{I}_M$ refers to a block-diagonal matrix, where the elements of its main diagonal are $A_i$, $i \in \mathbb{I}_M$ and its other components are 0. Also, row$(A_i)$ indicates the matrix $[A_1, \ldots, A_M]$, and $A > 0$ means that $A$ is a symmetric positive definite matrix, and $\oplus$ shows the Minkowski sum. Considering that $\mathbb{X}^i$ is a sequence set, we define $\prod_{i \in \mathbb{I}_M} \mathbb{X}^i = \mathbb{X}^1 \times \cdots \times \mathbb{X}^M$. The symbol $\emptyset$ stands for a closed ball of radius $r > 0$ created at origin. In addition, the interior of set $\mathbb{X}$ is represented by $\text{int}(\mathbb{X})$. Furthermore, $\mathbb{X}^+$ predicates as a successor state for state vector $\mathbb{X}$. Moreover, if $\mathbb{X} \subseteq \mathbb{R}^n$ is a convex and compact set in which the origin will be in its interior, then $\mathbb{X}$ is a C-set. Considering $\mathbb{X} \subseteq \mathbb{R}^n$, if $\mathbb{X} + w \subseteq \mathbb{X}$ for all $w \in \mathbb{W}$, then $\mathbb{X}$ is called a robust positively invariant (RPI) for $x^+ = Ax + w$; $w \in \mathbb{W} \subseteq \mathbb{R}^n$ and all $x \in \mathbb{X}$.

2 | NETWORK AND SYSTEM DESCRIPTION

This article considers a networked large-scale smart energy system containing $M$ linear discrete-time sub-systems (Figure 1). The sub-systems are interacted through their states. Sub-system $i \in \mathbb{I}_M$ can exchange the data with its neighbouring sub-systems $N_i$, where $N_i \subseteq \mathbb{I}_M$ and $N_i \neq \emptyset$. The sub-systems are supposed to have the capability for exchanging the information through the communication channels featuring induced time-varying delays. For sub-system $i$, the time delay of the exchanged data from its neighbouring sub-system $j$ is presented by $\tau^{ij}_k$ at time step $k$.

Assumption 1. The time-varying communication delays $\tau^{ij}_k$ are bounded by $d_{min} < \tau^{ij}_k < d_{max}$, where $d_{min}$ and $d_{max}$ are certain positive integers displaying lower and upper bounds of time delays, respectively.

Remark 1. Owing to [14, 15], if the properties of the communication network are known, then the upper bound of delays can be approximated. Thus, all considered bounds of delays are supposed to be known and are constant integers. Moreover, the time-varying communication delays $\tau^{ij}_k$ could be replaced with...
the upper bounds of delays. This does not affect the computation of the controller law.

The current article considers, for delay compensation, a buffer-based strategy described in [7]. According to [7] all DNEMPC controllers accumulate their predictive input sequence with length \( N_c \) into unique packages with a time stamp and transfer them to their neighbouring sub-systems via the communication channels. It should be noted that, in order to ensure that the time-labelled data is accurate and precise, it is necessary that the clocks of all sub-systems’ controllers and sensors are synchronised. For this, some real-time clocks and appropriate synchronisation algorithm can be employed [16].

On the other hand, every local controller includes at most \((M - 1)\) buffers where each buffer is assigned to a neighbouring sub-system and the related packet is saved till the entrance of the next buffer. Once a newer packet data is received, its time label is compared to the time label of the existing stored packet data in the buffer. If the newly received packet data is newer than the stored packet, the buffer will be updated, otherwise the newly received packet data will be ignored and the buffer contents are moved to the left, and afterwards a zero is placed to the right side of the buffer. The buffer output data will be employed for the prediction of interacting states which are not accessible at the current time step, owing to the communication employed for the prediction of interacting states which are not to the right side of the buffer. The buffer output data will be employed for the prediction of interacting states which are not accessible at the current time step, owing to the communication channels time delays.

For instance, assume that at time step \((k - 1)\) the obtained predictive control sequence packet of the \(j\)th local controller, that is, \{\(u_j(k - 1), u_j(k), \ldots, u_j(k + N_c - 2)\}\), is written on the right side of the \(j\)th local controller. If at time step \(k\), a non-credible packet with time delay \(\tau^\mu_j\) is received after the buffer is shifted to the left and zero is added to the right of the sequence. Now \{\(u_j(k), \ldots, u_j(k + N_c - 2), 0\)\} is arranged into the buffer. In the worst case, this stage is repeated \(d_{\text{max}} - 1\) times until the next credible control packet is received. Thus, if \(d_{\text{max}} \leq N_c\), the existence of at least one predicted control data is guaranteed on the output side of the buffer.

**Assumption 2.** For each sub-system, the control horizon \(N_c\) is equal to or greater than \(d_{\text{max}} - d_{\text{min}} + 1\).

**Remark 2.** It is worth mentioning that this article can be extended for the case of simultaneously occurring of communication delays and packet dropout. In some studies, packet dropouts are considered as the extended delays [6, 7]. In other words, when the time delay in a packet is greater than a pre-specified upper bound of delays (here, \(d_{\text{max}}\)), it is treated as a loss packet. In the case of packet dropout, an upper bound \((T_p\) on the number of successive time steps with packet dropouts is considered. In this model, in the worst case, a sub-system will receive a new valid packet from another sub-system within \(d_{\text{max}} + T_p\) time steps. This implies that we have a network with a maximum delay of \(d_{\text{max}} + T_p\). Generally, the communication channels induced time delays that include the influence of packet dropouts, data received as disordering and network communication time delays.

The dynamic model for sub-system \(i\) is described as the linear discrete-time formulation as follows [9]:

\[
\begin{align*}
\dot{x}_{k+1}^i &= A^i x_k^i + B^i u_k^i + \sum_{j\in\mathbb{N}_i} A_j^i x_j^i + D^i w_k^i \quad (1a) \\
\dot{y}_k^i &= C^i x_k^i + v_k 
\end{align*}
\]

in which \(u' \in U' \subseteq \mathbb{R}^{n_u}\), \(x' \in \mathbb{X} \subseteq \mathbb{R}^{n_x}\) and \(y' \in \mathbb{R}^{n_y}\) are the control input, state vector and measured system output, respectively. Furthermore, \(W'\) and \(V'\) are \(C\)-sets and matrices \(A^i, B^i, A_j^i, D^i\) and \(C^i\) are known and constant with appropriate dimensions. Also, \(U'\) and \(X'\) are polytopic and polyhedral constraint sets, respectively, which the origin is considered an interior point in them. Moreover, \(v_k^i \in V' \subseteq \mathbb{R}^{n_v}\) and \(u_k^i \in U' \subseteq \mathbb{R}^{n_u}\) stand for the unknown output and state disturbances, respectively. Also, \(\mathbb{X}'\) is assumed as a set of all state variables where there exists a feasible control command existing in \(U'\). Besides, \(x_k^i\) shows the state trajectory of the \(i\)th sub-system which is predicted and computed in \(\alpha\)th sub-system, when the induced communication time delay occurs.

**Assumption 3.** For \(i \in \mathbb{I}_M\), the pairs \((A^i, C^i)\) and \((A^i, B^i)\) are supposed to be detectable, stabilisable, respectively.

The whole regular system, ignoring the communication-induced time delays and disturbances, can be written as:

\[
\begin{align*}
\ddot{x}(k + 1) &= A \ddot{x}(k) + Bu(k), \quad (2a) \\
\ddot{y}(k) &= C \ddot{x}(k), \quad (2b)
\end{align*}
\]

where state \(\ddot{x} = \text{col}(\ddot{x}') \in \mathbb{X} \subseteq \mathbb{R}^{n_x}\), control input \(u = \text{col}(u') \in U \subseteq \mathbb{R}^{n_u}\) and measured output \(\ddot{y} = \text{col}(\ddot{y}') \in \mathbb{R}^{n_y}\). Furthermore, \(U = \text{mathop} \prod_{i \in \mathbb{I}_M} U'\) and \(\mathbb{X} = \text{mathop} \prod_{i \in \mathbb{I}_M} \mathbb{X}'\) are input and state constraint sets, respectively. The \(A\) matrix is introduced as follows [17]:

\[
A \triangleq \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1M} \\ \vdots & \ddots & \vdots & \vdots \\ A_{M1} & A_{M2} & \cdots & A_{MM} \end{bmatrix}.
\]

Also, \(B \triangleq \text{blkdiag}(B')\) and \(C \triangleq \text{blkdiag}(C')\).

### 3 THE SUGGESTED COOPERATIVE DNEMPC WITH INDUCED COMMUNICATION CHANNEL DELAYS

In this section, firstly an efficient distributed moving horizon estimator–predictor is proposed. Then using the estimated and predicted states, a cooperative DNEMPC is designed such that
the states of the closed-loop system converge to a neighbourhood of the origin.

For reducing the estimation error, a pre-estimator is developed for every \( i \)th sub-system \( i \) as follows:

\[
\begin{align*}
    \dot{x}_{k+1}^i &= A^i x_k^i + B_i u_k^i + \sum_{j \in \mathbb{N}_i} A^{ij} \dot{x}_{k}^{pj} + L^i (y_k^i - \hat{y}_k^i), \\
    \dot{y}_k^i &= C_i x_k^i,
\end{align*}
\]  

(3a)

where \( x_k^i \) and \( \hat{y}_k^i \) are the current predicted states of \( x_k^i \) and current output of pre-estimator, respectively. Also, \( L^i \) shows the gain matrix of pre-estimator which should be computed in such a manner that \( A^i L^i = A^i - L^i C_i \) is Schur. In (3), \( x_0^i = x_{k,0}^i \) and \( \dot{x}_{k,0}^{pj} = \dot{x}_{k,0}^{pj} \) are the initial states. Moreover, \( \dot{x}_k^i \) denotes the current estimation of \( i \)th sub-systems.

Next for sub-system \( i \), a local constrained MHE is designed via using the constrained optimisation problem as follows:

\[
\begin{align*}
    \min_{\dot{x}_k^{N_1}, \ldots, \dot{x}_k^{N_k}, \hat{y}_k^{J_1}, \ldots, \hat{y}_k^{J_i}} & \quad 1/2 \sum_{j=0}^{k} \left( \sum_{l=k-N_l}^{k} \| y_l^i - C_i x_l^{j} \|^2 + \mu_i \| x_l^{j} - x_l^{N_l} \|^2 \right) \\
    \text{subject to} & \quad \dot{x}_{l+1}^{j} = A^i x_l^{j} + B_i u_l^i + \sum_{j \in \mathbb{N}_i} A^{ij} \dot{x}_{l}^{pj} + L^i (y_l^i - \hat{y}_l^i), \\
    & \quad l = k - N_l, \ldots, k - 1, \\
    & \quad \dot{y}_l^i = C_i x_l^{j}, l = k - N_l, \ldots, k
\end{align*}
\]  

(4a)

In sub-system \( i \), the current \( j \)th predicted state is computed employing (6):

\[
\begin{align*}
    \dot{x}_k^{pj} &= A^i \dot{x}_{k-1}^{pj} + B_i u_k^{pj} + \sum_{j \in \mathbb{N}_i} A^{ij} \dot{x}_{k-1}^{pj} + L^i (y_k^i - \hat{y}_k^i), \\
    \hat{y}_k^i &= C_i \dot{x}_k^{pj}, j \in \mathbb{N}_j.
\end{align*}
\]  

(5)

where \( \hat{y}_k^j \) shows the \( j \)th buffer output, \( j \in \mathbb{N}_j \) installed in the sub-system at time step \( k \) and \( \mathbb{N}_j \) presents the neighbouring sub-systems of the \( j \)th sub-system without considering the \( i \)th sub-system. It should be noted that, the \( u_k^{pj} \) input trajectory is equivalent to \( u_k \), when no communication channels time delay occurs. Moreover, when delays occur, \( y_{k-1}^j \) is considered as 0 in (4e) and (6).

In the following, the NDMPC is developed for system (1). In this regard, for every \( i \)th sub-system, at the time step \( k \), the cooperative control criterion function is considered as

\[
V(k) = \sum_{l=0}^{N_l-1} \left[ \sum_{j \in \mathbb{N}_j} \| \dot{x}_{k+l}^{j} \|^2_{Q_j} + C_{k+l} \| \dot{y}_{k+l}^{j} \|^2_{R_j} \right] + \| \dot{x}_{k+N_l} \|^2_{P_j}
\]  

(7)

in which \( \dot{y}_{k}^{j} \) is the control trajectory which should be designed. Moreover, the prices of electricity consumption considered in the optimisation problem as the cost coefficients \( C_{k} \) and \( C_{k}^{j} \). For all \( i \in \mathbb{N}_i, Q_j > 0, R_l > 0 \) and \( P_j > 0 \) are weighting matrices. It is noteworthy that if \( A^i, Q_j \) are detectable, then \( Q_j \) \( \geq 0 \). Parameter \( N_l \) stands for both control and prediction horizons which meet \( \delta \leq N_l \). Also, terms \( \dot{x}_{k}^{j}(\cdot) \) and \( \dot{x}_{k+N_l}^{j}(\cdot) \) are estimated local state trajectory \( x_{k}^{j}(\cdot) \) and the predicted interacted state between the \( i \)th and \( j \)th sub-systems which are computed through the \( i \)th local MHE.

For \( i \)th sub-system, at time-step \( k \) and every iteration \( \rho \), the optimal control command is obtained through solving (7), which is an optimal regulating control problem with constraints
\[
\min_{\mathbf{u}_k^i} V(k)
\]
\[
\text{s.t. } \mu_{\text{min}} \leq \mathbf{u}_k^i \in \mathcal{U}, \quad l = 0, \ldots, k + N_r - 1
\]

(7b)

After \( \rho \) iterations, at time step \( k \), the obtained optimal solution of mentioned problem is illustrated by \( (\mathbf{u}_k^i)^T = [(\mathbf{u}_k^i)^T, \ldots, (\mathbf{u}_{k+N_r-1}^i)^T] \). The final solution is resulted from a convex combination between the current and last optimal solutions of the economic model predictive control problem (7), that is, \( \mathbf{u}_k^i = \mathbf{u}_k^i + (1 - \alpha) \mathbf{u}_{k-1}^i \) where \( \alpha \) is the \( \alpha \)th sub-systems’ weighting factor which meets \( \sum_{\alpha \in \mathcal{G}} \alpha = 1 \). Now, \( \mathbf{u}_k^i \) is accumulated into one packet data and transmitted to every interconnected sub-system \( j \neq i \), through a communication network featuring time delays. Also, the first value of \( \mathbf{u}_k^i \) is exerted to the \( \alpha \)th sub-system.

4 THE ANALYSIS OF ESTIMATION ERROR

To derive an analytic expression for the estimation error, (4) is represented as a convex quadratic program (QP) as follows:

\[
\min_{\mathbf{z}_k} \frac{1}{2} \mathbf{z}_k^T H_i \mathbf{z}_k + F_i \mathbf{z}_k + r_i
\]

(8)

in which constant matrices \( G_i \) and \( \xi_i \), with appropriate dimensions, present the constraints of (4b)–(4e). \( r_i \) is a constant term, and \( \mathbf{z}_k = \text{col}(\hat{\mathbf{x}}_{k-N_r}, \ldots, \hat{\mathbf{x}}_{k-1}) \) shows an uncertain vector of optimisation in which \( X = \text{col}(\mathbf{x}_{k-N_r}, \ldots, \mathbf{x}_{k-1}) \). The corresponding matrices \( H_i \) and \( F_i \) in (8) are

\[
H_i = \begin{bmatrix} (A_i^N)^T & \mathbf{A}_i^L & \mathbf{A}_i^L & \mathbf{A}_i^L & \mathbf{A}_i^L \\ \mathbf{A}_i^L & \mathbf{A}_i^L & \mathbf{A}_i^L & \mathbf{A}_i^L & \mathbf{A}_i^L \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{A}_i^L & \mathbf{A}_i^L & \mathbf{A}_i^L & \mathbf{A}_i^L & \mathbf{A}_i^L \end{bmatrix}
\]

(9a)

\[
F_i = \begin{bmatrix} - (A_i^N)^T Q_{N_r}^i y^i & (A_i^L)^T B_{N_r}^i y^i & - \mu_{N_r} \mathbf{z}_{k-N_r}^i \\ - (A_i^N)^T Q_{N_r}^i y^i & -(A_i^L)^T B_{N_r}^i y^i & - \mu_{N_r} \mathbf{z}_{k-N_r}^i \\ \vdots & \vdots & \vdots \\ - (A_i^N)^T Q_{N_r}^i y^i & -(A_i^L)^T B_{N_r}^i y^i & - \mu_{N_r} \mathbf{z}_{k-N_r}^i \end{bmatrix}
\]

(9b)

where

\[
A_i^N \Delta \equiv \begin{bmatrix} C^i \\ C_i A_i^i \\ \vdots \\ C_i^N \end{bmatrix}, \quad A_i^L \Delta \equiv \begin{bmatrix} C_i A_i^i \\ C_i A_i^i \\ \vdots \\ C_i^N \end{bmatrix}
\]

In (9b), to update matrix \( F_i \), we need the past inputs, current and past measured outputs, predicted state and a priori estimate of computed state \( \hat{x}_{k-N_r} \). For the \( \alpha \)th MHE, the estimation error is introduced as

\[
\hat{e}_{k-N_r}^i \Delta \equiv \begin{bmatrix} e_{k-N_r}^i \Delta \equiv \text{col}(y_{k-N_r}, \ldots, y_{k-1}) \\ \mathbf{z}_{k-N_r}^i \Delta \equiv \text{col}(\hat{x}_{k-N_r}, \ldots, \hat{x}_{k-1}) \end{bmatrix}
\]

(10)

where \( X^e \Delta \equiv \text{col}(\hat{x}_{k-N_r}^e, \ldots, \hat{x}_{k-1}^e) \). Note that a sequence of the estimation error is obtained using (4b) and (4c) in (10) where the current estimation error is denoted by \( \hat{e}_{k-N_r}^i \).

The boundedness of \( \hat{e}_{k-N_r}^i \) will be established with the following theorem.

Theorem 1. In sub-system (1) with pre-estimator (3), consider the constrained distributed MHE problem described in (4) and the constrained DNEMPC problem presented in (7). If Assumptions 1–3 hold, and if \( \mu_i \geq 0 \) and \( A_i^1 \) is Schur, then the current estimation error \( \hat{e}_{k-N_r}^i \) will be...
bounded and there exists a $C$-net $E^i$ so that for all $k \geq 0$ if $\epsilon^i_{j,k} \in E^i$ then $\epsilon^i_{j,k} \in E^i$.

The proof is given in the Appendix.

For the $i$th sub-system, the prediction error is defined as

$$\hat{e}^i_{j,k} = x^i_{j,k} - \tilde{x}^i_{j,k}$$

and $\hat{e}^i_{j,k} = x^i_{j,k} - \tilde{x}^i_{j,k}$. Let $\hat{e}^i$ and $\hat{\tilde{x}}^i$ are vectors that involve whole prediction errors and predicted states, at time step $k$, respectively. The next corollary is implied from Theorem 1, which is needed to guarantee the feasibility of all sub-systems’ obtained input trajectory.

**Corollary 1.** With the existence of conditions, considered in Theorem 1, the prediction errors $\hat{e}^i_{j,k}$ and $\hat{\tilde{x}}^i_{j,k}$ are bounded for all $j \in \mathbb{N}$ and $i \in I_M$. Also, there are two sets $\mathcal{X}^p$ and $\mathcal{E}^p$, both including the origin so that $\mathcal{X}^p \oplus \mathcal{E}^p \subseteq \mathcal{X}$ and for every $\hat{e}^i_{j,k} \in \mathcal{X}^p$ and $\hat{\tilde{x}}^i_{j,k} \in \mathcal{E}^p$, $\hat{e}^i_{j,k} \in \mathcal{X}$ for all $k > 0$. Furthermore, there are two sets $\mathcal{X}$ and $\mathcal{E}$, both including the origin so that $\mathcal{X} \oplus \mathcal{E} \subseteq \mathcal{X}$ and for every $\hat{\tilde{x}}^i_{j,k} \in \mathcal{X}$ and $e^i_{j,k} \in \mathcal{E}$, $\hat{\tilde{x}}^i_{j,k} \in \mathcal{X}$ for all $k > 0$.

The convexity of $U_i$, Corollary 1 and the initialisation procedure employed in the suggested cooperative guarantee that if, for every sub-system, there is a feasible input trajectory, then there are feasible input trajectories for all sub-systems, at future times.

## 5 | SIMULATION STUDIES

In this section to verify the usability and proficiency of the proposed DNEMPC strategy, it is employed for design of distributed controller for smart energy systems. For this, the following two cases are considered.

### 5.1 | Example 1

In the first simulation study, the proposed DNEMPC method is exerted to control the heat pump for heating the smart residential buildings with the aim of optimising energy consumption and reducing the cost of consumed electricity. In fact, the objective of the economic forecasting control presented in this case study is to control the heat pump in the heating apparatus of the networked distributed buildings with floor heating apparatus by considering the perturbations of radiation of solar and ambient temperature so that by transmitting the pump power consumption time, at times when electricity prices are lower, the demands of the system are met (reaching room temperature to the optimum temperature) as well as the cost of consuming the pump is also minimised.

In this regard, firstly, a dynamic model of two-building floor-heating system joined to a heat pump placed in the ground source is presented. The current article adopts the developed method in [18] to construct the model of considered networked building climate control system. Commonly, for a building with $N$ area zone, the model of the $i$th sub-system ($i = 1, 2, \ldots, N$) can be represented as follows.

$$x^i_{k+1} = A^i x^i_k + B^i u^i_k + \sum_{j \in \mathbb{N}} A^{ji} x^j_k + D^i w^i_k$$

$$f^i_k = C^i x^i_k + d^i_k,$$

where $x^i = [T^i, T^i, T^i]^T$ is the state vector, where $T_0$, $T_1$ and $T_2$ stand for building air temperature, water temperature in the heating pipes placed in the floor and floor temperature, respectively. Furthermore, $u$ shows the control input which is the power employed via the compressor connected to the heat pump and $W^i = [T_0, \phi_i]^T$ stands for the disturbances which are the sun radiation $\varphi$, and ambient temperature $T_a$. Furthermore, the indoor temperature is considered as the output variable, $y^i = T^i$. Moreover, matrices in (11) are as follows:

$$A^i = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix}, \quad B^i = \begin{bmatrix} 0 & 0 & \eta \\ 0 & C_{p,r} & \frac{1}{p} \end{bmatrix}^T,$$

$$A^{ji} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \tilde{z}_{33} \end{bmatrix}, \quad i \neq j, \quad C^i = [1 \ 0 \ 0],$$

$$D^i = \begin{bmatrix} \frac{(UA)_{fr}}{C_{p,r}} & \frac{1}{p} \\ C_{p,r} & \frac{C_{p,w}}{p} \\ 0 & 0 \end{bmatrix}, \quad a_{11} = \frac{(-l(UA)_{fr} - (UA)_{ra})}{C_{p,r}}, \quad a_{12} = \frac{(UA)_{fr}}{C_{p,r}},$$

$$a_{21} = \frac{(UA)_{fr}}{C_{p,r}}, \quad a_{22} = \frac{(-l(UA)_{fr} - (UA)_{ra})}{C_{p,r}},$$

$$a_{23} = \frac{(UA)_{fr}}{C_{p,w}}, \quad a_{32} = \frac{(UA)_{fr}}{C_{p,f}},$$

$$a_{33} = \frac{(-l(UA)_{fr} - (UA)_{ra})}{C_{p,r}},$$

where $Q_{ra}$ and $Q_{fr}$ denote the heat which is transferred from the buildings’ air to the environment and from the floor to the buildings’ air, respectively. Moreover, $Q_{ra}$ shows the heat exchanged between the floor and water which is flowing in heating pipes, under the floor. Finally, the term $UA$ stands for a product of the surface area of the layer between two heat-exchanging media and the heat conductivity. The nomenclatures of systems’ parameters are provided in Table 1 [18].

Here we consider two building climate control system (two sub-systems) with the same dynamic which interacted through
TABLE 1 Parameters of the system model

| Symbol | Value | Description |
|--------|-------|-------------|
| \(C_{p,r}\) (kJ/°C) | 810 | Room air heat capacity |
| \(C_{p,w}\) (kJ/°C) | 836 | Heat capacity of water in floor-heating pipes |
| \(C_{p,f}\) (kJ/°C) | 3315 | Floor heat capacity |
| \((UA)_{fr}\) (kJ/(°C s)) | 624 | Coefficient of transferring heat between floor and room air |
| \((UA)_{ra}\) (kJ/(°C s)) | 28 | Coefficient of transferring heat between room air and ambient |
| \((UA)_{wf}\) (kJ/(°C s)) | 28 | Coefficient of transferring heat between water and floor |

The states are considered as \(T_{i}^f\) and \(T_{i}^w\) for \(i = 1, 2\). It is supposed that, for every sub-system, the measured output is the indoor temperature, that is, \(T_{ir}\). The inputs to sub-systems are the power employed via the compressor in the heat pump [19]. Moreover, the data is exchanged between two sub-systems among the non-ideal communication channels featuring random delays varying between 1 and 4 s.

To show the potential of the proposed DNEMPC, we need to know the electricity prices for a specific time period. This work considers the day-ahead prices of electricity consumption using the Nordic power exchange market presented in [20], as depicted in Figure 2. Furthermore, the disturbances \(\phi\) and \(T_{\alpha}\) are considered as diurnal cycles featuring added noise [20], as illustrated in Figure 3.

First a local pre-estimator is designed for each sub-system in the form of (4) using (19). In the context of DMHE design, \(N_{e} = 20\). For the DMPC control problem for both sub-systems, the control and prediction horizons are equal, and \(N_{c} = 30\).

The main objective is to minimise the total cost of electricity consumption in a predefined time period as well as holding the indoor temperature, \(T_{ir}\), in a specified limit in the presence of communication network time delay. The data is transmitted among the non-ideal communication channels with random delays varying between 1 and 12 s.

Figure 4 shows time-varying communication delay sequences induced by the communication network in a day. Moreover, to have a better perceptiveness, a conventional economic model predictive (CEMPC) approach, presented in [21], is adopted and applied for comparison.

The predicted indoor temperature and obtained optimal schedule of compressor, for a 5-day horizon and over the predefined time-varying constraints are provided in Figure 5. The constraints show that the temperature can be lower during the night-time compared to daytime.

Furthermore, Figure 6 illustrates the actual electricity prices and the obtained optimal heat pump power input. Figures 5 and 6 verify that the consumption of power is shifted to periods when the electricity is cheap and show that the house floor thermal capacity has the ability to save enough energy so that, during the daytime, the heat pump can be turned off. This issue implies that the slow heat dynamics of the floor can be employed to move the consumption of energy to the periods where the price of electricity is low as well as the indoor temperature is kept in the desired value. Furthermore, Figure 6 reveals that the proposed DNEMPC method has better performance than conventional CEMPC method in shifting the consumption of power to periods when the electricity is cheaper.

5.2 Example 2

To verify the usability and proficiency of the proposed DNEMPC strategy, it is applied for the design of load frequency controller for a four-area interconnected power system over communication network. The main aim of the load frequency controller in the modern multi-area power system is to keep the uniform frequency at each area and to maintain the tie-line power exchanges in a pre-defined tolerance in the presence of load disturbances or sudden changes in load demands. In this regard, a four-area network-based power system model, as depicted in Figure 7, is given for the design of network-based
FIGURE 3  The solar radiation and ambient temperature curves

FIGURE 4  Time delay sequence

FIGURE 5  The indoor temperature
FIGURE 6  The optimal schedule for the heat pump and the spot price of electricity

![Diagram](image.png)

FIGURE 7  Four-area interconnected power system

![Diagram](image.png)

Table 2: The value of parameters of four area power grid

| Symbol | Area 1 | Area 2 | Area 3 | Area 4 |
|--------|--------|--------|--------|--------|
| $M_i^c$ | 3      | 3.5    | 3.25   | 3.75   |
| $R_i^f$ | 0.07   | 0.04   | 0.06   | 0.05   |
| $D_i$  | 0.275  | 2.75   | 2      | 3      |
| $T_{CH_i}$ | 20 | 10 | 15 | 12 |
| $T_{Gi}$ | 15 | 25 | 20 | 15 |

For all areas, the nominal value of operating frequency, static load and the set point of load reference are considered same as presented in [22], where $f = 60$ Hz, $\Delta P_{ref,i} = 0$ and $-0.3 \leq \Delta P_{ref,i} \leq 0.3$ ($i = 1, 2, 3, 4$). Also, the time-varying induced delays of communication channels are considered as $\tau_{12}(k), \tau_{13}(k), \tau_{14}(k), \tau_{23}(k), \tau_{24}(k)$ and $\tau_{34}(k)$. Parameters are presented in Table 2:

$$A_i = \begin{bmatrix} -\frac{D_i}{M_i^c} & \frac{1}{M_i^c} & 0 & -\frac{1}{M_i^c} \\ 0 & -1 & \frac{1}{T_{CH_i}} & 0 \\ -\frac{1}{R_i^f} & 0 & -\frac{1}{T_{Gi}} & 0 \\ \sum_{j=1}^{N} T_{ij} & 0 & 0 & 0 \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{T_{Gi}} \end{bmatrix}.$$

(12)

$$A_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\sum_{j=1, j \neq i}^{N} T_{ij} & 0 & 0 \end{bmatrix}, \quad C_i = \begin{bmatrix} \beta_i & 0 & 0 & 1 \end{bmatrix}.$$

(14)

Moreover, $A_{ii}, A_{ij}$ and $B_i$ stand for real and constant matrices which are adopted from [22] and the values of entire the parameters are presented in Table 2:

$$x'_{k+1} = A^i x'_k + B^i u'_k + \sum_{\in N_i} A'^i x'^{i_{ij}}_{k},$$

$$y'_k = C^i x'_k,$$

(12)

where $x' \in \mathbb{R}^{n_i}, u' \in \mathbb{R}^{m_i}$ and $y' \in \mathbb{R}^{l_i}$ stand for the state, control and output vectors of the $i$th control area, respectively. Also, $x'^{i_{ij}} \in \mathbb{R}^{n_{ij}}$ is the state vector of the neighbouring control area.

The output and state vectors for the $i$th area are presented as follows [22]:

$$x' = [\Delta \omega, \Delta P_{mech}, \Delta P_{ref, i}],$$

$$y' = \beta_i \Delta \omega + \Delta P_{ref, i},$$

(13)
FIGURE 8 The system output and control commands in Case 1: DNEMPC (solid lines), DNMPC (dashed lines) and CEMPC (dotted lines)

FIGURE 9 The system output and control commands in Case 2: DNEMPC (solid lines), DNMPC (dashed lines) and CEMPC (dotted lines)

To prove the superiority of the suggested method, a distributed networked model predictive control (DNMPC) and CEMPC control approach are adopted, respectively, from the [9, 21] and applied for comparison. Furthermore, the efficiency of the developed network-based control technique is evaluated through the following scenarios:

**Plan 1.** The data is transferred via the communication channels with random delays changing between 1 and 5 s (i.e. $d_{\text{max}}$ is chosen as 5 s).
method. The proposed strategy can be developed and employed easily to other wide-area systems.

REFERENCES

1. Zheng, Y., et al.: Distributed model predictive control for on-connected microgrid power management. IEEE Trans. Control Syst. Technol. 26(3), 1028–1039 (2018)
2. Santander, O., et al.: Robust economic model predictive control: disturbance rejection, robustness and periodic operation in chemical reactors. Eng. Optim. 51(5), 896–914 (2019)
3. Zheng, Y., et al.: A distributed model predictive control based load frequency control scheme for multi-area interconnected power system using discrete-time Laguerre functions. ISA Trans. 68, 127–140 (2017)
4. Pourkargar, D.B., et al.: Distributed model predictive control of process networks: impact of control architecture. IFAC – PapersOnLine 50(1), 12452–12457 (2017)
5. Farina, M., et al.: Distributed moving horizon estimation for nonlinear constrained systems. Int. J. Robust Nonlinear Control 22(14), 123–143 (2012)
6. Bijami, E., Farsangi, M.M.: Robust hierarchical damping controller for uncertain wide-area power systems. IET Gener. Transm. Distrib. 12(22), 5958–5967 (2018)
7. Razavinasab, Z., et al.: State estimation-based distributed model predictive control of large-scale networked systems with communication delays. IET Control Theory Appl. 11(15), 2497–2505 (2017)
8. Song, Y., et al.: Distributed output feedback MPC with randomly occurring actuator saturation and packet loss. Int. J. Robust Nonlinear Control 26, 3036–3057 (2016)
9. Razavinasab, Z., et al.: Robust output feedback distributed model predictive control of networked systems with communication delays in the presence of disturbance. ISA Trans. 80, 12–21 (2018)
10. Pin, G., Parisini, T.: Networked predictive control of uncertain constrained nonlinear systems: recursive feasibility and input-to-state stability analysis. IEE Trans. Automat. Control 56, 72–87 (2011)
11. Quevedo, D.E., Nesic, D.: Input-to-state stability of packetized predictive control over unreliable networks affected by packet-dropouts. IEEE Trans. Automat. Control 56, 370–375 (2011)
12. Studler, M., et al.: Modelling and evaluation of control schemes for enhancing load shift of electricity demand for cooling devices. Environ. Modell. Software 24, 285–295 (2009)
13. Patiño, J., et al.: An economic MPC approach for a microgrid energy management system. 2014 IEEE PES transmission & distribution conference and exposition – Latin America, Medellin, pp. 1–6 (2014)
14. Cruz, R.: A Calculus for network delay, part I: network elements in isolation. IEEE Trans. Inf. Theory 37, 114–131 (1991)
15. Vatanski, N., et al.: Networked control with delay measurement and estimation. Control Eng. Pract. 17(2), 231–244 (2007)
16. Wang, F., et al.: Dual time synchronization method for wireless sensor networks. Electron. Lett. 51(2), 179–181 (2015)
17. Rakocevic, S.V., et al.: Invariant approximations of the robust positive invariant set. IEEE Trans. Automat. Control 50(3), 408–410 (2005)
18. Halvgaard, R.: Model predictive control for smart energy systems. Ph.D Thesis, Technical University of Denmark (2014)
19. Privara, S., et al.: Model predictive control of a building heating system: the first experience. Energy Build. 43, 564–572 (2011)
20. Halvgaard, R., et al.: Madsen Henrik Model Predictive Control for a Smart Solar Tank Based on Weather and Consumption Forecasts. Energy Procedia 30, 270–278 (2012)
21. Halvgaard, R., et al.: Economic model predictive control for building climate control in a smart grid. 2012 IEEE PES Innovative Smart Grid Technologies (ISGT), Berlin University of Technology, Berlin (2012)
22. Bijami, E., et al.: Distributed control of networked wide-area systems: a power system application. IEEE Trans. Smart Grid 11(4), 3334–3345 (2010)
23. Bijami, E., Farsangi, M.M.: A distributed control framework and delay-dependent stability analysis for large-scale networked control systems with non-ideal communication network. Trans. Inst. Meas. Control 41(5), 768–779 (2019)
APPENDIX A

In order to prove Theorem 1, firstly, a dynamic model is pursued for the estimation error (10), based on the QP active set strategy. The Karush–Kuhn–Tucker conditions for (8) are as follows:

\[ \zeta^i_k = -H^{-1}_k \left( F^i_k + G_{eA}^T \lambda_{eA}^k \right), \]

(A.1a)

\[ \lambda_{eA}^k = -(G_{eA}^i H_{eA}^{-1} G_{eA}^{T})^{-1} (G_{eA}^i H_{eA}^{-1} F^i_k + \xi_{eA}^k), \]

(A.1b)

\[ \lambda_{eA}^k, \lambda_{vA}^k > 0, \]

(A.1c)

where \( \zeta^i_k = \text{col}(\tilde{\zeta}_{eA}^i, \lambda_{eA}^k, G_{eA}^i) \) and \( \lambda_{eA}^k \) stand for the active Lagrange multipliers and the relating active inequality matrices of (7), respectively.

Substituting (9a) and (9b) into (15a) and (15b), using (10) and rearranging the terms yield that

\[ \dot{e}_{k-N|k} = A_i^i e_{k-N|k-1} + D_i^i w_{k-N|k-1} + D_i^i v_{k-N|k-1} \]

\[ + H_{eA}^{-1} G_{eA}^T \lambda_{eA}^k \]

(A.1a)

\[ \lambda_{eA}^k = (G_{eA}^i H_{eA}^{-1} G_{eA}^{T})^{-1} (G_{eA}^i H_{eA}^{-1} A_i^i e_{k-N|k-1} + D_i^i w_{k-N|k-1} + D_i^i v_{k-N|k-1} \]

\[ + D_i^i v_{k-N|k-1} - \hat{e}_{k-N|k} \]

(A.1b)

in which

\[ A_i^i = H_{eA}^{-1} \left[ \mu_{A_i^i} \text{row}_{j \in \mathbb{N}_i} \left( \left[ \mu_{A_i^j} 0 0 \right] \right) \right], \]

\[ D_i^i = H_{eA}^{-1} \left[ \mu_{D_i^j} \left( -A_i^j \right)^T D_{Ne}^j \right], \]

\[ D_i^j = -H_{eA}^{-1} \left[ \mu_{L_i^j} \left( A_i^j \right)^T Q_{Ne} \right], \]

where

\[ w_{k-N|k-1}^j = \text{col} \left( w_{k-N|k-1}^i, \ldots, w_{k-1}^i \right), \]

\[ v_{k-N|k-1}^j = \text{col} \left( v_{k-N|k-1}^i, \ldots, v_{k}^i \right) \]

and \( D_{Ne}^j \) is defined as

\[ D_{Ne}^j \Delta = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ C^j D^j & 0 & \cdots & 0 \\ C^j A_i^j D^j & C^j D^j & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C^j (A_i^j)^{N_i-1} D^j & C^j (A_i^j)^{N_i-2} D^j & \cdots & C^j D^j \end{bmatrix}, \]

where

\[ w_{k-N|k-1}^j = \text{col} \left( w_{k-N|k-1}^j, \ldots, w_{k-1}^j \right), \]

\[ v_{k-N|k-1}^j = \text{col} \left( v_{k-N|k-1}^j, \ldots, v_{k}^j \right), \]

\[ A_i^j = \left[ \mu_{A_i^j} \text{row}_{j \in \mathbb{N}_i} \left( \left[ \mu_{A_i^j} 0 0 \right] \right) \right], \]

\[ D_i^j = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ C^j D^j & 0 & \cdots & 0 \\ C^j A_i^j D^j & C^j D^j & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C^j (A_i^j)^{N_i-1} D^j & C^j (A_i^j)^{N_i-2} D^j & \cdots & C^j D^j \end{bmatrix}, \]

Substituting (12b) into (12a) follows that

\[ \dot{e}_{k-N|k} = A_i^i e_{k-N|k-1} + D_i^i w_{k-N|k-1} + D_i^i v_{k-N|k-1}. \]

Rewriting (4b) for \( l = k - N_i \) gives

\[ \zeta_{k-N|l}^j = A^j \zeta_{k-N|l}^j + B^j \dot{u}_{k-N} + \sum_{j \in \mathbb{N}_j} A^j \tilde{Z}_{k-N}^{p_j} \]

\[ + L^j (\dot{y}_{k-N|k} - \dot{y}'_{k-N|k}), \]

(A.17)

Also (1a) is rewritten in the following form:

\[ \dot{x}_{k-N+1}^j = A^j x_{k-N}^j + B^j \dot{u}_{k-N} + \sum_{j \in \mathbb{N}_j} A^j x_{k-N}^{p_j} + D^j w_{k-N}. \]

(A.18)
Subtracting (17) from (18) and \( N_e \) iterations, it is obtained that
\[
\epsilon_{k+1|k}^j = (A_L^j)^N_e \epsilon_{k-N|k}^j + G^{ij} \epsilon_{k-N|k}^j + G_{ij} \epsilon_{k-N|k}^j - G_{ij} \epsilon_{k-N|k}^j - G_{ij} \epsilon_{k-N|k}^j
\]
(19)
in which \( G_e \triangleq \left[ D^j (A_L^j)^N_e \right], G_i \triangleq \left[ L^j (A_L^j)^N_e \right], \) and \( G_{N_e} \triangleq \left[ A^{ij} (A_L^j)^N_e \right]. \) Equation (19) can be rewritten as
\[
\epsilon_{k|k}^j = M^j \epsilon_{k-N|k}^j + G^{ij} \epsilon_{k-N|k}^j + G_{ij} \epsilon_{k-N|k}^j - G_{ij} \epsilon_{k-N|k}^j - G_{ij} \epsilon_{k-N|k}^j
\]
(20)
where \( M^j = \left[ (A_L^j)^N_e \ 0 \right]. \) Multiplying \((M^j)^{-1}\) at both sides of (20) yields that
\[
\epsilon_{k-N|k}^j = (M^j)^{-1} \epsilon_{k-N|k}^j + G^{ij} \epsilon_{k-N|k}^j + G_{ij} \epsilon_{k-N|k}^j - G_{ij} \epsilon_{k-N|k}^j - G_{ij} \epsilon_{k-N|k}^j
\]
(21)

By combining (21) and (16) and rearranging the terms, the following equation is obtained:
\[
\epsilon_{k+1|k}^j = M^j A' (M^j)^{-1} \epsilon_{k|k}^j + \tilde{w}_i^j,
\]
(22)
in which \( \tilde{w}_i^j = (I - M^j A' (M^j)^{-1}) G_{ij} \epsilon_{k-N|k}^j + M^j D_u \epsilon_{k-N|k}^j + M^j D_x \epsilon_{k-N|k}^j \) and \( \tilde{w}_i^j \) are reflected as a disturbance that lies in the \( \mathbb{C} \)-set \( \tilde{W}_i^j \) introduced by
\[
\tilde{W}_i^j = \left( I - M^j A' (M^j)^{-1} \right) \times G_{ij} \tilde{W}_i^j + M^j D_u \tilde{W}_i^j + M^j D_x \tilde{W}_i^j + \tilde{W}_i^j + \tilde{W}_i^j + \tilde{W}_i^j,
\]
where
\[
\tilde{W}_i^j \triangleq \left( \tilde{W} \times \cdots \times \tilde{W} \right)_{N_e}, \quad \tilde{W}_i^j \triangleq \left( \tilde{W} \times \cdots \times \tilde{W} \right)_{N_e + 1}
\]
and
\[
\tilde{W}_i^j \triangleq \left( \tilde{W} \times \cdots \times \tilde{W} \right)_{N_e + 2}
\]
The fact \( A'_{ij} \) is Schur implies that \( A'_{ij} \) and \( M^j A' (M^j)^{-1} \) are Schur matrices and there is a \( \mathbb{C} \)-set \( E' \) which is robust positively invariant for (22) \[17\]. It follows that \( M^j A' (M^j)^{-1} E' \oplus \tilde{W}_i^j \subset E' \) and if \( \epsilon_{k=0}^j \in E' \), then \( \epsilon_k^j \in E' \), \( \forall k \geq 0. \)