Instability of a Kerr black hole in f(R) gravity

Yun Soo Myung

Institute of Basic Science and Department of Computer Simulation,
Inje University, Gimhae 621-749, Korea

Abstract

We study the stability of a rotating (Kerr) black hole in the viable \( f(R) \) gravity. The linearized-Ricci scalar equation shows the superradiant instability, leading to the instability of the Kerr black hole in \( f(R) \) gravity.

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ysmyung@inje.ac.kr
1 Introduction

One of modified gravity theories, $f(R)$ gravity \cite{1, 2, 3, 4} has much attentions as a strong candidate for explaining the current and future accelerating phases in the evolution of universe \cite{5, 6}. On the other hand, the Schwarzschild-de Sitter black hole was firstly obtained for a constant curvature scalar from $f(R)$ gravity \cite{7}. A Schwarzschild-anti de Sitter black hole solution was obtained from $f(R)$ gravity by requiring a negatively constant curvature scalar \cite{8}. The trace of stress-energy tensor should be zero to obtain a constant curvature black hole when $f(R)$ gravity couples with matter of the Maxwell field \cite{8}, the Yang-Mills field \cite{9}, and a nonlinear Maxwell field \cite{10}.

Most of astrophysical black holes including supermassive black holes are considered to be a rotating black hole. A rotating black hole solution \cite{11} should be stable against the external perturbations because it stands as a realistic object in the sky \cite{12}. The stability analysis of the Kerr black hole is not as straightforward as one has performed the stability analysis of a spherically symmetric Schwarzschild black hole \cite{13, 14, 15} because it is an axis-symmetric black hole. Here we would like to mention that the stability analysis is based on the linearized equations and thus, it does not guarantee the stability of black holes at the nonlinear level. The Kerr black hole has been proven to be stable against a massless graviton \cite{16, 17, 18} and a massless scalar \cite{19}. However, there exist the superradiant instability (the black-hole bomb) when one chooses a massive scalar \cite{20, 21, 22, 23, 24, 25} and a massive vector \cite{26}. For example of $f(R) = R + hR^2$, the Kerr black hole is unstable because it could be transformed into a massive scalar-tensor theory \cite{27}.

It is known that the Kerr solution could be obtained from a limited form \cite{6} of $f(R)$ gravity \cite{28}. Interestingly, it was shown that a perturbed Kerr black hole could distinguish Einstein gravity from $f(R)$ gravity \cite{29}. However, the stability analysis of $f(R)$-rotating black hole is a formidable task because $f(R)$ gravity contains fourth-order derivative terms in the linearized equation. Transforming the limited form of $f(R)$ gravity into the scalar-tensor theory might resolve difficulty, which leads to the fact that the $f(R)$-rotating black hole is unstable against a massive scalar perturbation when one used the black-hole bomb idea \cite{30}.

In this work, we examine the stability of a rotating black hole in the viable $f(R)$ gravity. We consider the linearized Ricci scalar as a truly massive spin-0 graviton propagating on the Kerr black hole spacetimes. Solving its linearized equation shows a superradiant instability, which dictates the instability of the Kerr black hole in $f(R)$ gravity. This will be compared to the Gregory-
Laflamme instability of the massive spin-2 graviton in the dRGT massive gravity \[31, 32\] and the fourth-order gravity \[33\].

2 \( f(R) \)-rotating black holes

We start with the \( f(R) \) gravity action
\[
S_f = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} f(R)
\]
(1)
with \( \kappa^2 = 8\pi G \). The Einstein equation takes the form
\[
R_{\mu\nu} f'(R) - \frac{1}{2} g_{\mu\nu} f(R) + \left( g_{\mu\nu} \nabla^2 - \nabla_{\mu} \nabla_{\nu} \right) f'(R) = 0,
\]
(2)
where the prime (‘) denotes the differentiation with respect to its argument. It is well-known that Eq. (2) provides a solution with constant curvature scalar \( R = \bar{R} \). In this case, Eq. (2) reduces to
\[
\bar{R}_{\mu\nu} f'(\bar{R}) - \frac{1}{2} \bar{g}_{\mu\nu} f(\bar{R}) = 0
\]
(3)
and thus, the trace of (3) determines the constant curvature scalar to be
\[
\bar{R} = \frac{2f(\bar{R})}{f'(\bar{R})} \equiv 4\Lambda_f
\]
(4)
with \( \Lambda_f \) the cosmological constant. The subscript ‘f’ denotes that the \( \Lambda_f \) arose from the \( f(R) \) gravity. Substituting this expression into (3) leads to the Ricci tensor
\[
\bar{R}_{\mu\nu} = \frac{f'(\bar{R})}{2f(\bar{R})} \bar{g}_{\mu\nu} = \Lambda_f \bar{g}_{\mu\nu}.
\]
(5)

To find the Kerr black hole solution with \( \Lambda_f = 0 \) \( (\bar{R}_{\mu\nu} = \bar{R} = 0) \), one requires \( f(0) = 0 \) with \( f'(0) \neq 0 \). To this end, one has to choose a specific form of \( f(R) \) as \[28\]
\[
f(R) = a_1 R + a_2 R^2 + a_3 R^3 + \cdots.
\]
(6)
In Table 1, we list viable models of \( f(R) \) gravity which provide the form (6). Hence, a model of \( f(R) = R - \mu^4/R \) could not provide a rotating black hole \[35, 36\] because \( f(0) \to -\infty \) and \( f'(0) \to \infty \). Also, the form of \( f(R) = \alpha \sqrt{R} + \beta \) \[37, 38\] is excluded because \( f(0) = \alpha \sqrt{\beta} \neq 0 \). By the same token, the two models of \( f(R) = R^p e^{q/R} \) and \( f(R) = R^p (\ln[\alpha R])^q \) \[39\] are not suitable for seeking the Kerr black hole solution.
viable $f(R)$ gravity & $f(0)$ & $f'(0)$ & $f''(0)$ & $m_f^2 = \frac{f''(0)}{f'(0)}$
---
$f_Q = R + hR^2$ & 0 & 1 & 2h/

$f_{pE} = R^p e^{qR}$ & 0 & 1 & 2q/

$f_S = R + \lambda R_s[(1 + R^2/R_s^2)^{n} - 1]$ & 0 & 1 & $\frac{2\lambda}{R_s}$ & $\frac{-m_s^2}{6\lambda}$

$f_{hS}^{(1)} = R - m^2 \frac{c_1(R/M^2)^n}{1 + c_2(R/M^2)^n}$ & 0 & 1 & $\frac{2c_1}{m}$ & $\frac{m^2}{6c_2}$

$f_{hS}^{(2)} = R - m^2 \frac{c_1(R/M^2)^n}{1 + c_2(R/M^2)^n}$ & 0 & 1 & $\frac{-2c_1}{m}$ & $\frac{-m^2}{6c_1}$

$f_{hS}^{(>)} = R - m^2 \frac{c_1(R/M^2)^n}{1 + c_2(R/M^2)^n}$ & 0 & 1 & 0 & N/A

$f_{nE} = R - 2\Lambda(1 - e^{-R/\Lambda})$ & 0 & 1 & $\frac{1}{\Lambda}$ & $\frac{1}{\Lambda}$

Table 1: Viable models of $f(R)$ gravity to provide the Kerr black hole as a solution. The condition of $m_f^2 > 0$ might be different from that of a viable $f(R)$ gravity to explain the accelerating universe [3].

In this work, we use the Boyer-Lindquist coordinates to represent an axisymmetric Kerr black hole solution with mass $M$ and angular momentum $J$ [11]

$$ds_{Kerr}^2 = g_{\mu\nu}dx^\mu dx^\nu = -\left(1 - \frac{2Mr}{\rho^2}\right)dt^2 - \frac{2Mra\sin^2\theta}{\rho^2}2dtd\phi + \frac{\rho^2}{\Delta}dr^2 + \rho^2d\theta^2 + \left(r^2 + a^2 + \frac{2Mra^2\sin^2\theta}{\rho^2}\right)\sin^2\theta d\phi^2$$

(7)

with

$$\Delta = r^2 + a^2 - 2Mr, \quad \rho^2 = r^2 + a^2 \cos^2\theta, \quad a = \frac{J}{M}. \quad (8)$$

Here we use Planck units of $G = c = \hbar = 1$ and thus, the mass $M$ has a length scale. In the nonrotating limit of $a \to 0$, (7) recovers the Schwarzschild black hole, while the limit of $a \to 1$ corresponds to the extremal Kerr black hole. From the condition of $\Delta = 0(g^{rr} = 0)$, we determine two horizons which are located at

$$r_{\pm} = M \pm \sqrt{M^2 - a^2}. \quad (9)$$

The angular velocity at the event horizon takes the form

$$\Omega = \frac{a}{2Mr_+} = \frac{a}{r_+^2 + a^2}. \quad (10)$$

3
In general, one introduces the metric perturbation around the Kerr black hole to study the stability of the black hole

\[ g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}. \]  

(11)

Hereafter we denote the background quantities with the “overbar” (\( \bar{R}_{\mu\nu} = 0, \bar{R} = 0 \)). The Taylor expansions around the zero curvature scalar background is employed to define the linearized Ricci scalar [34] as

\[ f(R) = f(\bar{R}) + f'(\bar{R})\delta R(h) + \cdots, \]  

(12)

\[ f'(R) = f'(\bar{R}) + f''(\bar{R})\delta R(h) + \cdots. \]  

(13)

The linearized equation to (2) is given by

\[ \delta R_{\mu\nu}(h) + \frac{f''(0)}{f'(0)} \left[ -\frac{f'(0)}{2f''(0)} \bar{g}_{\mu\nu} + \bar{g}_{\mu\nu} \nabla^2 - \nabla_\mu \nabla_\nu \right] \delta R(h) = 0, \]  

(14)

where the linearized Ricci tensor and scalar could be expressed in terms of \( h_{\mu\nu} \) as

\[ \delta R_{\mu\nu}(h) = \frac{1}{2} \left[ \nabla^\rho \nabla_\mu h_{\rho\nu} + \nabla^\rho \nabla_\nu h_{\mu\rho} - \nabla^2 h_{\mu\nu} - \nabla_\mu \nabla_\nu h \right], \]  

(15)

\[ \delta R(h) = \nabla^\rho \nabla_\sigma h_{\rho\sigma} - \nabla^2 h. \]  

(16)

Considering (15) and (16), the linearized equation (14) is a fourth-order differential equation with respect to the metric perturbation \( h_{\mu\nu} \), which is not a tractable equation to be solved. Choosing the Lorentz gauge of \( \nabla_\nu h^{\mu\nu} = \nabla^\mu h/2 \) and using the trace-reversed perturbation of \( \tilde{h}_{\mu\nu} = h_{\mu\nu} - h\bar{g}_{\mu\nu}/2 \), equation (14) takes a relatively simple from [29]

\[ \nabla^2 \tilde{h}_{\mu\nu} + 2\tilde{R}_{\mu\nu\rho\sigma} \tilde{h}^{\rho\sigma} + \frac{1}{3m_f^2} \left( \bar{g}_{\mu\nu} \nabla^2 - \nabla_\mu \nabla_\nu \right) \nabla^2 \tilde{h} = 0, \]  

(17)

where the mass squared \( m_f^2 \) is defined by

\[ m_f^2 = \frac{f'(0)}{3f''(0)} > 0. \]  

(18)

In case of Einstein gravity [\( f(R) = R, f'(0) = 1, f''(0) = 0 \)], Eq. (17) leads to a well-known second-order equation for \( \tilde{h}_{\mu\nu} \) since the last fourth-order term is decoupled from (17). However, we could not solve (17) directly for \( m_f^2 \neq \infty \) because it is a coupled fourth-order equation for \( \tilde{h}_{\mu\nu} \) and \( \tilde{h} \).
3 Superradiant instability of Ricci scalar

It is well-known that \( f(R) \) gravity has 3 degrees of freedom (DOF) without ghost in Minkowski spacetimes: 2 DOF for a massless spin-2 graviton and 1 DOF for a massive spin-0 graviton. The massive spin-0 graviton is usually described by the trace \( h \) of \( h_{\mu \nu} \), but it could be represented by the linearized Ricci scalar \( \delta R \) because \( \delta R = -\nabla^2 h/2 = \nabla^2 \tilde{h}/2 \) under the Lorentz gauge.

For this purpose, we may take the trace of (14) with \( \bar{g}^{\mu \nu} \). Then, we have a massive equation for \( \delta R \)

\[
\left( \nabla^2 - m_f^2 \right) \delta R = 0 \tag{19}
\]

which is considered as a second-order equation that describes the linearized Ricci scalar propagating on the background of Kerr black hole. In the previous work \([30]\), we have replaced \( \delta R \) by a scalaron \( \delta A \) which could be interpreted to be a massive scalar in the scalar-tensor theory. This result is meaningful only if the scalaron approach (the scalar-tensor theory) represents \( f(R) \) gravity truly. However, it is noted that the linearized Ricci scalar by itself is regarded as a physically propagating scalar because the \( f(R) \) gravity includes a massive scalar graviton with single DOF. In Table 1, we list \( m_f^2 \) for viable \( f(R) \) models. In order to not have a tachyonic scalar, it should be positive \( (m_f^2 > 0) \) which implies that \( f'(0) > 0 \) and \( f''(0) > 0 \). Thus, one requires either \( \lambda < 0 \) or \( n < 0 \) for the Starobinsky model \( (f_S) \) \([40]\). Also, \( 0 < c_1 < 1 \) is required for the \( n = 1 \) Hu-Sawicki model \( (f_{HS}^{n=1}) \) and \( c_1 < 0 \) for the \( n = 2 \) Hu-Sawicki model \( (f_{HS}^{n=2}) \) \([41]\). However, these are not mandatory to explain the accelerating universe when one uses viable \( f(R) \) gravity \([3]\).

Reminding the axis-symmetric background \((7)\), it is convenient to separate the linearized Ricci scalar into \([43]\)

\[
\delta R(t, r, \theta, \phi) = e^{-i\omega t + im\phi} S_l^m(\theta) u(r), \tag{20}
\]

where \( S_l^m(\theta) \) are spheroidal angular functions with \( l \) the spheroidal harmonic index and \( m \) the azimuthal harmonic number. Also, we choose a positive frequency \( \omega \) of the mode here. Plugging (20) into the linearized massive equation (19), one has the angular and radial equations for \( S_l^m(\theta) \) and \( u(r) \)
\[
\frac{1}{\sin \theta} \partial_{\theta} (\sin \theta \partial_{\theta} S_l^m) \\
+ \left[ a^2(\omega^2 - m_f^2) \cos^2 \theta - \frac{m^2}{\sin^2 \theta} + A_{lm} \right] S_l^m = 0, \quad (21)
\]

\[
\Delta \partial_r (\Delta \partial_r u) + \left[ \omega^2(r^2 + a^2)^2 - 4Mam\omega r + a^2m^2 \right.
\]
\[
- \Delta(a^2\omega^2 + m_f^2r^2 + A_{lm}) \big] u = 0, \quad (22)
\]

where $A_{lm}$ is the separation constant whose form is given by \[44, 22\]
\[
A_{lm} = l(l+1) + \sum_{k=1}^{\infty} c_k a^{2k}(m_f^2 - \omega^2)^k \quad (23)
\]
for $\omega \simeq m_f$.

The radial Teukolsky equation takes the Schrödinger form
\[
- \frac{d^2 \psi}{dy^2} + V(r, \omega) \psi = \omega^2 \psi, \quad \psi(r) = \sqrt{r^2 + a^2} u, \quad (24)
\]
where the tortoise coordinate $y$ is defined by $dy = \frac{r^2 + a^2}{\Delta} dr$ and a $\omega$-dependent potential $V_\omega(r)$ is given by
\[
V_\omega(r) = \frac{\Delta m_f^2}{r^2 + a^2} + \frac{4Mram\omega - a^2m^2 + \Delta[A_{lm} + (\omega^2 - m_f^2)a^2]}{(r^2 + a^2)^2} \\
+ \frac{\Delta(3r^2 - 4Mr + a^2)}{(r^2 + a^2)^3} - \frac{3\Delta^2 r^2}{(r^2 + a^2)^4}. \quad (25)
\]

Its asymptotic form is given by
\[
V_\omega \rightarrow \omega^2 - m_f^2, \quad y \rightarrow \infty \quad (r \rightarrow \infty), \quad (26)
\]
and its form near the event horizon is
\[
V_\omega \rightarrow (\omega - m\Omega)^2, \quad y \rightarrow -\infty \quad (r \rightarrow r_+). \quad (27)
\]

Here we impose the two boundary conditions of purely ingoing waves near the horizon and a decaying (bounded) solution at spatial infinity. These are known to be boundary conditions for quasibound states \[20\]. Near the horizon and at the spatial infinity, the linearized Ricci scalar takes the form
\[
\psi \sim e^{-i(\omega - m\Omega)y}, \quad y \rightarrow -\infty \quad (28)
\]
\[
\psi = e^{-\sqrt{m_f^2 - \omega^2}y}, \quad y \rightarrow \infty. \quad (29)
\]
Then, we may choose an ingoing mode near the horizon

\[ [e^{-i\omega t}\psi]_\text{in} \sim e^{-i\omega t} e^{-i(\omega-m\Omega)y}. \]

From (29), a bound state of exponentially decaying mode at spatial infinity is characterized by the condition

\[ \omega^2 < m_f^2. \]  

(30)

The three boundary conditions (28)-(30) imply a discrete set of resonances \( \{\omega_n\} \) which corresponds to bound states of the linearized Ricci scalar.

In addition, let us consider a wave of \( e^{-i\omega t} e^{im\phi} \) with \( m > 0 \) and real \( \omega \) which is propagating into a rotating black hole with angular velocity \( \Omega \). If the frequency of the incident wave satisfies the condition \( 16 \)

\[ \omega < m\Omega, \]  

(31)

then the scattered wave is amplified. This is called the superradiance condition for a bosonic field \( 14 \).

The existence of superradiant modes can be converted into an instability of the black hole background if a mechanism to trap these modes in a vicinity of the black hole is provided. There are two mechanisms to achieve it. If one surrounds the black hole by putting a reflecting mirror, the wave will bounce back and forth between black hole and mirror, amplifying itself each time and eventually producing a nonnegligible backreaction on the black hole background. This yields an exponentially growing mode which can be no longer considered as a perturbation, demonstrating the instability of the black hole. Secondly, the nature may provide its own mirror when one introduces a massive scalar. Press and Teukolsky have suggested to use this mechanism to define the black-hole bomb \( 16 \) by introducing a massive scalar with mass \( M \) propagating around the Kerr black hole with mass \( M \). For \( \omega < M(\omega^2 < M^2) \), the mass term works as a mirror effectively. The maximum growth rate for the instability is associated with modes with \( \omega = \omega_R + i\omega_I \). The sign of \( \omega_I \) usually determines whether the solution is decaying (\( \omega_I < 0 \)) or growing (\( \omega_I > 0 \)) in the time evolution. It was shown that \( \omega_I M \sim 6 \times 10^{-5} \) for mirrornike boundary conditions \( 21 \) and \( \omega_I M \sim 1.72 \times 10^{-7} \) for massive scalars \( 25 \). Here the growth time scale is given by \( \tau = 1/\omega_I \).

More explicitly, according to the Hod’s argument \( 45 \), two ingredients are necessary to trigger the instability of the Kerr black hole when one uses a massive scalar perturbation: 1) The existence of an ergoregion where superradiant amplification of the waves takes place. 2) The existence of a trapping
potential well (∼) for quasibound states is between the potential barrier from ergoregion and potential barrier from the mass (see Fig. 15 of Ref. [12] and Fig. 7 of Ref. [46]). The first ingredient is usually implemented by the super-radiance condition (31). The second ingredient is supplied by the condition of the bound states for modes in the regime

$$\frac{m_j^2}{2} < \omega^2 < m_j^2. \tag{32}$$

Combining (31) with (32), one finds a restricted regime for the mass

$$m_f < \sqrt{2}\omega < \sqrt{2}m\Omega \tag{33}$$

which implies an inequality between mass $m_f$ of the Ricci scalar and the angular velocity $\Omega$ of the rotating black hole

$$m_f < \sqrt{2}m\Omega \tag{34}$$

which is the main result of our work.

The bound (34) is reminiscent of the Gregory-Laflamme s-mode instability [47] for a massive spin-2 graviton with mass $\mu$ propagating on the spherically symmetric Schwarzschild black hole spacetimes (mass $2M_S = r_0$). Choosing the transverse-traceless gauge of $\bar{\nabla}\mu h_{\mu\nu} = 0$ and $h = 0$, its linearized equation takes the form

$$\bar{\nabla}^2 h_{\mu\nu} + 2\bar{R}_{\alpha\mu\beta\nu}h^{\alpha\beta} - \mu^2 h_{\mu\nu} = 0. \tag{35}$$

which describes 5 DOF of a massive spin-2 propagating on the Schwarzschild black hole spacetimes. To this end, we would like to mention that the stability of the Schwarzschild black hole in four-dimensional massive gravity is determined by using the Gregory-Laflamme instability of a five-dimensional black string. It turned out that the small Schwarzschild black holes in the dRGT massive gravity [31, 32] and fourth-order gravity [33] are unstable against the metric and Ricci tensor perturbations because the inequality is satisfied as

$$\mu \leq \frac{0.438}{M_S}. \tag{36}$$

On the other hand, the dynamics of Ricci scalar with mass $m_f$ is expected to be stable in the complementary regime

$$m_f \geq \sqrt{2}m\Omega. \tag{37}$$

Similarly, the massive graviton is stable if it propagates around the large Schwarzschild black hole which satisfies the bound [48]

$$\mu > \frac{0.438}{M_S}. \tag{38}$$
4 Discussions

We have investigated the stability of a rotating black hole in the viable $f(R)$ gravity explicitly. Even though viable $f(R)$ gravity is promising to describe the current accelerating universe, it does not have a room to accommodate a rotating black hole because the Kerr black is unstable against the Ricci scalar perturbation. This superradiant instability (the black-hole bomb) arose from the nature of $f(R)$ gravity which provides a massive scalar graviton with single DOF, in addition to a massless spin-2 graviton with 2 DOF. This implies strongly that the Kerr black holes do not exist in $f(R)$ gravity and/or they do not form in the process of the $f(R)$ gravitational collapse \[31\]. On the other hand, we expect from the scalar-tensor theory \[34\] that the Schwarzschild black hole is stable against the Ricci scalar perturbation in viable $f(R)$ gravity because it is a nonrotating black hole.

We summarize the type of black hole instabilities found in the dRGT massive gravity, fourth-order gravity, and $f(R)$ gravity in Table 2. Let us compare the instability condition of Kerr black hole in $f(R)$ gravity with the instability condition of Schwarzschild black hole in dRGT massive gravity and fourth-order gravity. The instability of the Schwarzschild black hole in four-dimensional massive gravity is determined by using the Gregory-Laflamme instability of a five-dimensional black string. The two conditions of $\mu \leq \frac{0.438}{M_S}$ and $m_s \leq \frac{1}{2M_S}$ imply that the small Schwarzschild black holes in the dRGT massive gravity \[31, 32\] and fourth-order gravity \[33\] are unstable against the $s$-mode metric and Ricci tensor perturbations. These instabilities arose from the massiveness of $s$-mode spin-2 graviton propagating on the non-rotating small black hole with mass $M_S$. On the other hand, the condition of $m_f < \frac{\sqrt{7}m\Omega}{2}$ arose from the massiveness of spin-0 graviton with azimuthal number $m$ propagating on the rotating black hole with angular velocity $\Omega$. Even though the massiveness is a common factor for both instabilities, the phenomena of the instability are different: GL black string instability and black hole bomb.

Finally, we conclude that the massive graviton instabilities are quite different from the Regge-Wheeler-Zerilli stability for a massless graviton \[13, 14, 17\]. It suggests that a massive gravity is hard to possess the black hole.

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Table 2: Type of black hole instabilities in the dRGT massive gravity (MG), fourth-order gravity (FOG), and $f(R)$ gravity. TT denotes the transverse-traceless gauge and GL represents the Gregory-Laflamme black string.

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