Decoherence in a superconducting flux qubit with a π-junction

T. Kato1, A. A. Golubov2, and Y. Nakamura3,4 *

1 Institute for Solid State Physics, The University of Tokyo, Kashiwa, Chiba 277-8581, Japan
2 Faculty of Science and Technology, University of Twente, 7500 AE Enschede, The Netherlands
3 Nano Electronics Research Laboratories, NEC Corporation, Tsukuba, Ibaraki 305-8501, Japan
4 Frontier Research System, The Institute of Physical and Chemical Research (RIKEN), Wako, Saitama 351-0198, Japan

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We consider the use of a π-junction for flux qubits to realize degenerate quantum levels without external magnetic field. On the basis of the Caldeira-Leggett model, we derive an effective spin-Boson model, and study decoherence of this type of qubits. We estimate the dephasing time by using parameters from recent experiments of SIFS junctions, and show that high critical current and large subgap resistance are required for the π-junction to realize a long coherent time.

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It is now well established that in addition to conventional Josephson junctions having an energy minimum at zero phase difference across the junction, there exist the so-called π-junctions which provide the phase shift of π in the ground state. The intrinsic π-shifts were first realized in grain boundary Josephson junctions in $d$-wave superconductors [1, 2]. Subsequently, π-junctions have been realized in hybrid structures between high-$T_c$ and low-$T_c$ superconductors [2, 3] and by injection of quasi-particles [4]. Recent development in fabrication of superconductor-ferromagnet-superconductor (SFS) junction made it possible to obtain a π-junction with high critical current density [5]. An advantage of SFS junctions is the possibility to combine them with usual low-$T_c$ superconductive circuits using conventional fabrication technique.

The use of π-junctions provides several new applications. For example, the application of π-junctions as complementary devices in SFQ logic was recently proposed [6] and realized in high-$T_c$-low-$T_c$ junctions [7]. It is interesting that before this ‘classical’ application of the π-junction, the use of π-junctions for realization of quantum two-state systems was considered [8]. In this qubit system, the π-junction was used as a π phase shifter along the loop instead of current biasing or external magnetic flux. After this proposal, remarkable progress in fabrication, coherent control of one qubit, controllable coupling between qubits, and readout with high fidelity has been achieved in superconducting qubits [9, 10, 11, 12, 13]. Nevertheless, up to now, the use of π-junctions to qubits has not been studied experimentally. One of difficulties for realization may lie on the original proposal in which a qubit consists of complicated circuits with many Josephson junctions [8]. Another serious difficulty comes from dissipation due to quasi-particle excitation, which is unavoidable in many realizations of π-junctions. Generally, qubits suffer strong decoherence by excitation in the environment.

In this paper, we consider the use of π-junction for phase bias of flux qubits. The circuit we study is shown in Fig. 1. In this circuit, we need no external flux to realize degenerate quantum levels, because the phase drop across the three Josephson junction is adjusted as π by the π-junction with a large Josephson energy. This type of phase bias can avoid dephasing due to noise in external flux, and is frequently called as a ‘quiet qubit’. In actual experiments, however, damping at the π-junction may cause severe decoherence on the qubit. The purpose of this paper is to derive the effective spin-Boson model describing the flux qubit with a damped π-junction, and to estimate the dephasing time by using realistic experimental parameters. We clarify the condition for long coherence time in this qubit system, and discuss the possibility of the use of π-junctions for qubits by referring recent experiments on SIFS junctions.

In order to describe damped dynamics, we introduce the RSJ model for the π-junction as shown in Fig. 1 where dissipation is expressed by a resistance $R$ shunted in parallel to the junction. We expect that this phenomenological model may give a qualitative estimate of decoherence effects by π-junctions. We introduce the charging energy $E_{C,\pi} = e^2/(2C_\pi)$ and damping fre-

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FIG. 1: Flux qubit circuits with a π-junction. Shunt resistance at the π-junction is introduced for estimate of damping effects.
quency $\gamma = 1/(RC_\pi)$ of the $\pi$-junction.

The Hamiltonian consists of three parts as $H = H_{\text{qubit}} + H_L + H_{\text{coup}}$. The first part $H_{\text{qubit}}$ describes a flux qubit, and is given as $[12]$

$$H_{\text{qubit}} = -E_J (\cos \phi_1 + \cos \phi_2 + \alpha \cos \phi_3) + 4E_C (n_1^2 + n_2^2 + \alpha^{-1} n_3^2),$$

(1)

where $E_J$ is a Josephson energy, and $E_C$ is a charging energy. Here, $\phi_i$ and $n_i$ are a phase difference and induced charge at the $i$-th junction, respectively. The area of one junction is reduced by the factor $\alpha$, which is typically taken as 0.8 $[12]$. The second part of the Hamiltonian describes the inductance energy of the loop, and is given as

$$H_L = \frac{1}{2L} \left( \frac{\Phi_0}{2\pi} \right)^2 (\phi_1 + \phi_2 + \phi_3 + \phi_\pi - \phi_{\text{ext}})^2,$$

(2)

where $\Phi_0 = h/(2e)$. Here, $\phi_\pi$ is a phase of the $\pi$ junction, and $\phi_{\text{ext}} = 2\pi(\Phi_{\text{ext}}/\Phi_0)$ is a phase induced by the external flux through the loop. By assuming small inductance $L$, the inductive part of the Hamiltonian can be treated as a constraint condition

$$\phi_1 + \phi_2 + \phi_3 + \phi_\pi = \phi_{\text{ext}}.$$

(3)

The third term describes the damped $\pi$-junctions, and is expressed by the Caldeira-Leggett Hamiltonian

$$H_\pi = +E_{J,\pi} \cos \phi_\pi + 4E_{C,\pi} n_\pi^2 + \sum_\alpha \left\{ \frac{P_\alpha^2}{2m_\alpha} + \frac{1}{2} m_\alpha \omega_\alpha^2 \left( x_\alpha - \frac{C_\alpha}{m_\alpha \omega_\alpha^2} \phi_\pi \right)^2 \right\}$$

(4)

The damping property is determined by the spectral function

$$J(\omega) = \frac{\pi}{2} \sum_\alpha \frac{C_\alpha^2}{m_\alpha \omega_\alpha^2} \delta(\omega - \omega_\alpha).$$

(5)

In the RSJ model, the spectral function is given as

$$J(\omega)/M = \gamma \omega e^{-\omega/\omega_c},$$

(6)

where $M = 1/(8E_{C,\pi})$ is a mass of the $\pi$-junction, and $\omega_c$ is a high-frequency cutoff.

In this paper, we focus on the ‘passive’ use of the $\pi$-junction. For this use, the Josephson energy of the $\pi$-junction should be taken as sufficiently large. Hence, we assume $E_{J,\pi} \gg E_C, E_J$, and approximate the Josephson energy of the $\pi$-junction as $E_{J,\pi}(\phi_\pi - \pi)^2/2$. Within this approximation, the phase of the $\pi$-junction is kept almost $\pi$. The remaining dynamics around the potential minima is described by a damped oscillator with a eigenfrequency $\omega_\pi = (8E_{J,\pi}E_{C,\pi})^{1/2}/h$. In the following discussion, we set $h = 1$.

Under the condition $E_J \gg E_C$, which is taken for usual flux qubits, we can truncate the Hamiltonian $H_{\text{qubit}}$ into the two-level Hamiltonian as $H_{\text{qubit}} = H_{\text{two-state}} + H_{\text{coup}}$. The first part $H_{\text{two-state}} = (\Delta/2)\sigma_z + (\epsilon/2)\sigma_x$ describes the qubit system, where $\Delta$ is a tunneling splitting, and $\epsilon$ is a bias proportional to the external flux $\Phi_{\text{ext}}$. The second part, which describes the coupling between the qubit and the $\pi$-junction, is given as

$$H_{\text{coup}} = -E_{J,\text{eff}}(\alpha)\Delta \phi_\pi \sigma_z,$$

(7)

where $E_{J,\text{eff}} = (1 - 1/(4\omega_c^2))^{1/2} E_J$, and $\Delta \phi_\pi = \phi_\pi - \pi$.

To simplify the Hamiltonian $H_{\pi}$, we change the variables as $x = M^{1/2} \Delta \phi_\pi$, $p = M^{-1/2} \epsilon_\pi$. We further replace the sum in the Hamiltonian of the harmonic oscillators by the integral. This can be performed by replacing the variables as $X_\alpha = m^{1/2} x_\alpha/\omega_\alpha$, $P_\alpha = m^{-1/2} \epsilon_\pi/(\omega_\alpha)^{1/2}$, and $C_\omega = (Mm_\alpha)^{-1/2} C_\alpha/(\Delta \omega)^{1/2}$, where $\Delta \omega$ is a length of one slice in the $\omega$-direction. In the limit $\Delta \omega \to 0$, we obtain

$$H_\pi = \frac{P_\alpha^2}{2} + \frac{1}{2} \omega_\alpha^2 x_\alpha^2 + \int_0^\infty d\omega \left\{ \frac{P_\alpha^2}{2} + \frac{1}{2} \omega^2 \omega_{\text{ext}}^2 - C_\omega X_\omega \dot{x} + \frac{C_\omega}{2 \omega^2} \dot{x}^2 \right\}.$$

(8)

The coefficient $C_\omega$ can be related to the spectral function as

$$J(\omega)/M = \frac{\pi C_\omega^2}{2 \omega},$$

(9)

The Hamiltonian of the $\pi$-junction describing a damped oscillator can be diagonalized exactly $[14, 15]$. In order to express the eigenmodes with the energy $\omega$, we introduce a canonical transformation for the operators as

$$\hat{X}_\omega = a(\omega) \hat{x} + \int_0^\infty d\omega' b_{\omega'}(\omega) \hat{X}_{\omega'},$$

(10)

where the coefficients, $a(\omega)$ and $b_{\omega'}(\omega)$ are assumed to be real. The coefficients are chosen to satisfy the eigenmode equations

$$\dot{\omega}_0^2 a(\omega) + \int_0^\infty d\omega' C_{\omega''} b_{\omega''}(\omega) = \omega^2 a(\omega),$$

(11)

$$C_{\omega'} a(\omega) + \omega^2 b_{\omega'}(\omega) = \omega_{\omega'}^2 b_{\omega'}(\omega),$$

(12)

where $\omega_0^2 = \omega_\pi^2 + \int d\omega' \omega_{\omega'}^2/\omega^2$. Then, the Hamiltonian of the $\pi$-junction can be diagonalized as

$$H_\pi = \int_0^\infty d\omega \left( \frac{P_\alpha^2}{2} + \frac{1}{2} \omega^2 \hat{X}_\omega^2 \right).$$

(13)

In order to solve the eigenmode equations, eqs. $[11]$ and $[12]$, we may follow the calculation in Fano’s paper $[14]$. We only give the result for $a(\omega)$ as

$$|a(\omega)|^2 = \frac{C_{\omega}^2}{(\pi^2 C_{\omega}^2/4\omega^2) + (\omega^2 - \omega_0^2 - F(\omega))^2}$$

(14)

$$F(\omega) = \frac{P_\alpha^2}{2} \frac{C_{\omega'}^2}{(\omega^2 - \omega'_{\omega'}^2)}.$$
The part of the energy renormalization is modified as
\[ \tilde{\omega}^2 + F(\omega) = \omega_0^2 + P \int_0^\infty d\omega' \frac{\omega'^2 C_{\omega'}^2}{\omega'^2 (\omega'^2 - \omega^2)}. \tag{16} \]

Here, the second term in r.h.s. can be neglected, because it can be shown to be \( O(\Delta/\omega_c) \).

Thus, we obtain the new expression for the \( \pi \)-junction as \( \[13\] \), while the coupling term \( \[17\] \) is rewritten by the relation
\[ x = \int_0^\infty d\omega' a(\omega') \bar{X}_{\omega'}. \tag{17} \]

As a result, we obtain the total Hamiltonian as
\[ H = \frac{\Delta}{2} \sigma_x + \bar{\epsilon} \sigma_z - E_E \text{eff} (8E_{C,\pi})^{1/2} \sigma_x \int_0^\infty d\omega a(\omega) \bar{X}_{\omega} + \int_0^\infty d\omega \left( \frac{P^2}{\omega^2} + \frac{1}{2} \bar{X}_\omega^2 \right). \tag{18} \]

In this modified spin-Boson model, the effective spectral function is given by
\[ J_{\text{eff}}(\omega) = 8 E_{E,\text{eff}}^2 E_{C,\pi} \times \frac{\pi |a(\omega)|^2}{\omega} \left[ \frac{\omega}{(\pi C_{\omega^2}/2)\omega^2} + (\omega^2 - \omega_0^2)^2 \right]. \tag{19} \]

For the RSJ model, by substituting \( J(\omega)/M = \pi C_\omega^2/(2\omega) = \gamma \omega \), the effective spectral function is obtained for \( \omega \ll \omega_c \) as
\[ J_{\text{eff}}(\omega) = 8 E_{E,\text{eff}}^2 E_{C,\pi} \times \frac{\pi \gamma \omega}{\omega^2 (2\omega_0^2 + (\omega^2 - \omega_0^2)^2)^2}. \tag{20} \]

Note that the form factor of the damped oscillator (the factor in the bracket) appears in the effective spectral function.

By using the effective spectral function \( J_{\text{eff}}(\omega) \), we estimate the dephasing time of the qubit at the optimal point (\( \bar{\epsilon} = 0 \)), where long coherence time is realized by suppressing a linear coupling to the heat-bath. The dephasing time is evaluated within the spin-Boson model in the form \[14\] \[17\]
\[ \tau_\varphi^{-1} = \frac{1}{2} \tau_{\text{relax}}^{-1} + \frac{1}{T_2^*}. \tag{21} \]

The relaxation rate \( \tau_{\text{relax}} \) is calculated as
\[ \tau_{\text{relax}}^{-1} = 2 J_{\text{eff}}(\Delta) \coth \left( \frac{\Delta}{2kBT} \right). \tag{22} \]

On the other hand, \( 1/T_2^* \), which is a pure dephasing rate due to a quadratic coupling to the heat-bath at the

![FIG. 2: The estimated relaxation time \( \tau_{\text{relax}} \) and pure dephasing time \( T_2^* \). The result for a 10 \( \mu \text{m} \times 10 \mu \text{m} \) (1 \( \mu \text{m} \times 1 \mu \text{m} \)) \( \pi \)-junction is shown by the solid (dashed) line.](image)

We estimate the dephasing time in the present flux qubit by using the parameters in Ref. \[13\]. In the experiment, the parameters are chosen as \( E_F/k_B = 12 \text{ K} \), \( E_C/k_B = 350 \text{ mK} \), \( \Delta/k_B = 160 \text{ mK} \), \( T = 25 \text{ mK} \), and \( \alpha = 0.8 \). There are several candidates of \( \pi \)-junctions for phase bias. We have estimated dephasing time for several \( \pi \)-junction systems, and found that only underdamped \( \pi \)-junctions may give a sufficiently long dephasing time.

Here, we discuss underdamped SIFS junctions by using the parameters in Ref. \[10\]. We choose a capacitance and a subgap resistance for unit area as \( c = 0.08 \text{ F/m}^2 \) and \( r_n = 3.0 \times 10^{-7} \text{ fhm}^2 \) \[24\], respectively. In Ref. \[10\], the measured critical current density is \( j_c = 5.0 \times 10^4 \text{ A/m}^2 \). Here, we take the critical current density as a parameter, and discuss its dependence keeping \( r_n \) constant. In Fig. \[2\], we show the relaxation time \( \tau_{\text{relax}} \) and the pure dephasing time \( T_2^* \) as a function of the critical current density for a 1 \( \mu \text{m} \times 1 \mu \text{m} \) and 10 \( \mu \text{m} \times 10 \mu \text{m} \) junction. In this estimate, the relaxation process is always dominant (\( \tau_{\text{relax}} \ll T_2^* \)), and therefore the dephasing time \( \tau_\varphi \) is determined by \( \tau_{\text{relax}} \). We find that at the critical current density of Ref. \[19\] the dephasing time is very short, while long coherence time is obtained for junction with larger area and higher critical current density. The relaxation time has a resonant structure at a low critical current \( j_c = j_c^* \), where the resonant condition \( \omega_0 = \Delta \) is satisfied. For \( j_c \gg j_c^* \), the relaxation time and the pure dephasing time depend on the junction area A and crit-
ical current density \( j_c \) as \( \tau_{\text{relax}} \propto j_c^2 A \) and \( T_2^* \propto j_c^4 A^2 \), respectively. As seen in Fig.4 if we use underdamped \( \pi \)-junctions with large critical current \( j_c \sim 10^{7} \text{ A/m}^2 \) and large junction area \( A \sim 10 \mu \text{m} \times 10 \mu \text{m} \), coherence time becomes of order of 1 ns, which is sufficiently long comparing to the decoherence time limited by other sources. We note that when the relaxation process is dominant, the dephasing time is proportional to \( \tau_{\text{relax}} \propto j_c^2 A \) for realization of long coherence time we need to increase both the Josephson energy \( E_{J,\pi} \) and the subgap resistance \( R \) of the \( \pi \)-junction.

Thus, for long-time coherent operations, one has to improve the quality factor by changing experimental parameters of \( \pi \)-junctions. Especially important parameter is the critical current density in the \( \pi \)-state of the junction. In the SIFS junctions described in Ref.19 the critical current density \( j_c \approx 5 \times 10^4 \text{ A/m}^2 \) in the \( \pi \)-state was still rather low, three orders of magnitude less than \( j_c \approx 4 \times 10^7 \text{ A/m}^2 \) in a SIS junction having the same tunnel barrier. Possible reason for the strong suppression of the critical current is the use of diluted alloy \( \text{Ni}_x\text{Cu}_{1-x} \) which has rather strong disorder leading to fast decay of the supercurrent with increasing F-layer thickness. Since 0-\( \pi \) transition occurs at certain critical thickness of the F-layer, the supercurrent in the \( \pi \)-state is much smaller than in the 0-state. However, smallness of \( j_c \) is not an intrinsic property of SIFS junctions. In a clean homogeneous ferromagnet the decay length may become much longer than the 0-\( \pi \) transition thickness. Recent experiments [21] using \( \text{Ni}_3\text{Al} \) have demonstrated multiple 0-\( \pi \) transitions with only modest decay of \( j_c \) as a function of the thickness of \( \text{Ni}_3\text{Al} \). Therefore, choosing different materials for a ferromagnet layer may finally lead to increasing \( j_c \) and thus to an increase of the dephasing time of qubits with an SIFS junctions.

Finally, we discuss the advantage of the present phase bias. In usual flux qubits, external magnetic flux is an attractive option for individual phase biasing on qubits.

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