Different load types modeling using MatLab

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Abstract. Schemes with balanced, imbalanced and non-sinusoidal loads were modelled in MatLab in this paper. Verification (comparison of calculated values and measured) were done for active power (with difference less than 5%). There was a power phases inequality before transformer and after it.

1. General description of scheme

Engineers need to care about electricity quality from the first step of schemes creation. There are numerous of factors, which describe the condition of any scheme in power quality case. In a real schemes different loads (e.g. balanced, imbalanced and non-sinusoidal) can work simultaneously. But for modeling it is useful to model them separately for more accuracy.

General scheme for modeling is shown in the Pic.1. In this scheme: G-generator, Line – transmission line, T-transformer.

MatLab model for this scheme is shown in the Pic.2.

Picture 1. General scheme.
On this picture Load1 is represented as balanced load, Load2 – as imbalanced.
Location of measurement devices is shown in Pic.3.

2. Balanced load case
For balanced load case scheme, which is shown in Pic.1, consisted of only one load, in which impedances in phases are equal.

Parameters of scheme are next: nominal voltage: 110 kV, transmission line length: 15 km, wire type: AC 240/32 (more detailed parameters in [1, Table 3.5, p. 83]), type of transformer: ТДН 40000/110 (parameters are in [1, Table 5.18, p. 250]). Active power of load: 10 MW, reactive power of load: 4 Mvar.

Parameters for Matlab model was calculated using [2].

Measured values are represented in Table 1.

| Number of measurement device/ power | $P_{nom} = 10$ MW | $Q_{nom} = 4$ Mvar |
|------------------------------------|------------------|------------------|
| P1, MW                             | 9.641            |                  |
| P2, MW                             | 9.641            |                  |
| P3, MW                             | 9.627            |                  |
| P4, MW                             | 9.627            |                  |
| P5, MW                             | 9.553            |                  |

For verification power losses were calculated and compared with measured values. Difference between them should not be more than 5%:

$$\varepsilon = \frac{\Delta P_{net\, meas} - \Delta P_{net\, calc}}{\Delta P_{net\, meas}} \cdot 100\% \quad (1)$$

Equations for calculations are next:
\[ \Delta P_{\text{net}} = \Delta P_{\text{Line}} + \Delta P_{T\,oc} + \Delta P_T \]  
(2)

Where
\[ \Delta P_{\text{Line}} = 3 \cdot I^2_{\text{Line}} \cdot R_{\text{Line}} \]  
(3)
\[ \Delta P_T = \Delta P_{T\,HV} + \Delta P_{T\,LV}. \]  
(4)
\[ \Delta P_{T\,HV} = 3 \cdot I^2_{T\,HV} \cdot R_{T\,HV}. \]  
(5)
\[ \Delta P_{T\,LV} = 3 \cdot I^2_{T\,LV} \cdot R_{T\,LV}. \]  
(6)

Current \( I^2_{T\,LV} \) can be calculated using transformation ratio:
\[ k = \frac{U_{T\,HV}}{U_{T\,LV}} \]  
(7)
\[ I'_{T\,LV} = \frac{I_{T\,LV}}{k} \]  
(8)

Results of calculation are shown in Table 2:

| Calculated power losses \( \Delta P \), kW |
|------------------------------------------|
| \( \Delta P_{\text{Line}} \) | 0.013 |
| \( \Delta P_{T\,HV} \) | 0.011 |
| \( \Delta P_{T\,LV} \) | 0.011 |
| \( \Delta P_{\text{net}} \) | 0.085 |

Error is 3.4%, and that is quite accurate.

3. Imbalanced load case

For imbalanced loads case the same scheme with the same parameters was used, except parameters of load. Imbalanced load can be modelled as a 3 different loads connected in separate phases. Parameters of load are next: \( P_a = 5 \text{ MW}, Q_a = 2 \text{ Mvar}, P_b = 3 \text{ MW}, Q_b = 1.2 \text{ Mvar}, P_c = 2 \text{ MW}, Q_c = 0.8 \text{ Mvar}. \)

Procedure of verification is the same in general, but imbalanced load leads to different power floes in phases, and measurements should be made in each phase separately, as well as calculations. Measurement values are shown in Table 3.

| Number of measurement device/ power | Power in phase a \( P_a \), MW | Power in phase b \( P_b \), MW | Power in phase c \( P_c \), MW |
|-----------------------------------|-------------------------------|-------------------------------|-------------------------------|
| P1, MW                            | 6.456                         | 6.266                         | 5.169                         |
| P2, MW                            | 6.456                         | 6.266                         | 5.169                         |
| P3, MW                            | 6.438                         | 6.247                         | 5.156                         |
| P4, MW                            | 6.438                         | 6.247                         | 5.156                         |
| P5, MW                            | 5.746                         | 5.941                         | 6.028                         |
| P6, MW                            | 2.298                         | 3.127                         | 3.767                         |
| P7, MW                            | 3.448                         | 2.814                         | 2.26                          |
Calculated values are shown in Table 4.

| Measured quantity/Value       | Phase | Phase | Phase | Total |
|-------------------------------|-------|-------|-------|-------|
| $\Delta P_{\text{Line}}$, kW  | 0.017 | 0.019 | 0.013 | 0.049 |
| $\Delta P_{\text{T HV}}$, kW  | 0.014 | 0.016 | 0.01  | 0.04  |
| $\Delta P_{\text{T LV}}$, kW  | 0.012 | 0.012 | 0.013 | 0.037 |

There is no difference between calculated and measured values.

From the Table 3 it can be seen that power in phases before transformer (P4) and after it (P5) are different. It is connected with current transformation in transformer. In model transformer has so-called Star/Delta connection (Y/Δ) (Pic.4).

![Transformer winding connections: left – Star connection, right – Delta connection.](image)

Current in primary winding are transferred in currents in secondary winding:

\[
I_{ab} = k_T(I_B) \quad (9) \\
I_{bc} = k_T(I_C) \quad (10) \\
I_{ca} = k_T(I_A) \quad (11)
\]

From picture next equations can be written:

\[
I_a = I_{ab} - I_{ca} \quad (12) \\
I_b = I_{bc} - I_{ab} \quad (13) \\
I_c = I_{ca} - I_{bc} \quad (14)
\]

or, uniting (9) and (12), (10) and (23), (11) and (14), it can be written as:

\[
I_a = \frac{k_T(I_B) - k_T(I_A)}{\sqrt{3}} = \frac{k_T(I_B - I_A)}{\sqrt{3}}; \quad (15) \\
I_b = \frac{k_T(I_C) - I_B}{\sqrt{3}}; \quad (16) \\
I_c = \frac{k_T(I_A) - I_C}{\sqrt{3}} \quad (17)
\]

There will be different currents in phases because of different phase loads. If current in phase a after transformer is biggest, current in phase c is smallest and current in phase b is medium between them, then before transformer current in phase B will be biggest one proportional to current ab), current in
phase A will be approximately medium (proportional to current ca) and current in phase C will be smaller one (proportional to current bc).

Also there is a voltage transformation. In Delta connection there is no neutral point, and that’s why it is impossible to calculate phase-to-ground voltages directly. For this purpose artificial neutrals or neutraler devices are used [3].

Voltages and currents also change the angle (transformer connection group). In modeled case, this group is 11 (angle between primary and secondary voltages is 330 deg).

Therefore, power is multiplication of current and voltage, and according to above, power in phase c after transformer should be bigger than power in phase b, and power in phase a should be the smallest one. Table 3 proves this.

4. Scheme with non-sinusoidal load

Non-sinusoidal load is a load, that can create a distortion in a sinusoidal waveform of voltage. Usually sources of this distortions are AC loads, that apply a power from DC sources (invertors).

Any non-sinusiodal signal (voltage or current) can be represented as a sum of the numerous of harmonics (Fourier decomposition):

\[
X(\omega t) = A_0 + \sum_{n=1}^{\infty} (A_n \cos(n\omega t) + B_n \sin(n\omega t))
\]  

(18)

In this paper this load is a 6-pulse inverter (Pic. 5).

![Picture 5. Scheme for 6-pulse inverter](image)

For this device, current can be written (using Fourier decomposition) as:

\[
i(\omega t) = \frac{2\sqrt{3}I_d}{\pi} \sin(\omega t) - \frac{1}{5} \sin(5\omega t) + \frac{1}{7} \sin(7\omega t) - \cdots
\]  

(19)

It can be seen that there are no even harmonics and harmonics, multiplied for 3:

\[n = 6k \pm 1,\]  

(20)

where n – number of harmonic, k=1,2,3...

In this work harmonics, which are close to fundamental, will be modelled (5 and 7), as well as harmonic, quite far for fundamental (23), will be modelled. Each of this harmonics exists only in certain frequency:

\[n = 5; \quad f_5 = 50 \text{ Hz} \cdot 5 = 250 \text{ Hz},\]
\[n = 7; \quad f_7 = 50 \text{ Hz} \cdot 7 = 350 \text{ Hz},\]
\[n = 23; \quad f_{23} = 50 \text{ Hz} \cdot 23 = 1150 \text{ Hz}\]

Scheme for this case is shown in the Pic. 6.
Picture 6. Scheme for non-sinusoidal load case

On this scheme: G – generator, Line – Transmission Line, T-Transformer, CL1 and CL2 – Cable lines, Load NS – non-sinusoidal load, Load1 and Load2 - balanced loads.

Parameters of the scheme are next: nominal voltage: 110 kV, line length: 50 km, type of wires in line: AC 240/32 (more detailed parameters are in [1, Table 3.5, p. 83], transformer: ТДН 40000/110 (see general description, chapter 2).

Parameters of the Cable line 1: length: 50 km, type of wires: ААГУ 120 (more detailed parameters are in [1, Table.3.29, p.106]), voltage – 10 kV.

Parameters of the Cable Line 2: length: 60 km, type of wires: ААГУ 120 (more detailed parameters are in [1, Table.3.29, p.106]), voltage – 10 kV.

Load parameters: Load1: P=8 MW, Q=3Mvar; Load2: P=10 MW, Q=4 Mvar.

Non-sinusoidal load is modeled by the current sources with different frequencies and magnitudes. By the way, current magnitude is decreasing with increasing of harmonic order (Pic.7).

Parameters of this model are next (Table 5):

| Number of harmonic/ parameter | n = 5 | n = 7 | n = 23 |
|------------------------------|-------|-------|--------|
| Current amplitude in phase a, A | 120 | 60 | 1.875 |
| Angle of phase a, deg | 0 | 5 | 0 |
| Current amplitude in phase b, A | 120 | 60 | 1.875 |
| Angle of phase b, deg | -120 | -115 | -120 |
| Current amplitude in phase c, A | 120 | 60 | 1.875 |
| Angle of phase c, deg | 120 | 125 | 120 |

Picture 7. Model of non-sinusoidal load.
Verification was done with the same manner, but for each harmonic. measured values are shown in Table 6.

| Number of measurement device/ power | Power in 1st harmonic, \( P_1 \), MW | Power in 5th harmonic, \( P_5 \), kW | Power in 7th harmonic, \( P_7 \), kW | Power in 23th harmonic, \( P_{23} \), kW |
|-------------------------------------|--------------------------------------|-----------------------------------|-----------------------------------|--------------------------------------|
| P1 17.19                            | 1.492·10^{-4}                        | -3.672·10^{-5}                    | 1.746·10^{-5}                     |
| P2 17.19                            | 0.149·10^{-3}                        | -3.673·10^{-5}                    | 1.746·10^{-5}                     |
| P3 17.05                            | -0.522                               | -0.1014                           | -9.772·10^{-5}                   |
| P4 17.04                            | -0.5203                              | -0.1033                           | -0.119·10^{-3}                   |
| P5 16.92                            | -0.79                                | -0.1647                           | -0.132·10^{-3}                   |
| P6 7.794                            | 11.39                                | 4.019                             | 5.26·10^{-3}                     |
| P7 6.911                            | -3.06                                | -8.409                            | -7.596·10^{-3}                   |
| P8 6.911                            | 36.08                                | 13.41                             | 0.036                             |
| P9 0                                | 66.74                                | 21.83                             | 0.043                             |

Calculated values and error between it and measured values are shown in Table 7.

| Measured quantity/value | 1st harmonic, kW | 5th harmonic, kW | 7th harmonic, W | 23rd harmonic, W |
|-------------------------|-----------------|-----------------|----------------|-----------------|
| \( \Delta P_{\text{Line}} \) | 144.327         | 520.53          | 102.974        | 0.113           |
| \( \Delta P_{T \text{HV}} \) | 37.338          | 119.851         | 22.075         | 0.001752        |
| \( \Delta P_{T \text{LV}} \) | 33.880          | 2414.739        | 460.13         | 0.00632         |
| \( \Delta P_{T \text{CL1}} \) | 813.915         | 16694.647       | 3614.245       | 3.641           |
| \( \Delta P_{T \text{CL2}} \) | 1399.706        | 1773.133        | 623.819        | 0.78            |
| \( \sum \Delta P \) | 265.465         | 215.219         | 4823.268       | 4.534           |
| \( \varepsilon, \% \) | 0.03            | 1.44            | 1.92           | 4.98            |

Error is raise with increasing of harmonic order. It is connected with so-called skin-effect, where distribution of current on wire is non-linear.

**Conclusion**

1. Schemes with different load types were modeled;
2. In this schemes error between calculated and measured values are in normal limits (no more than 5%);
3. Difference in phase power before transformer and after it was explained.

**References**

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