Minimal radiative Dirac neutrino mass models

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Neutrinos may be Dirac particles whose masses arise radiatively at one-loop, naturally explaining their small values. In this work we show that all the one-loop realizations of the dimension-five operator to effectively generate Dirac neutrino masses can be implemented by using a single local symmetry: $U(1)_{B-L}$. Since this symmetry is anomalous, new chiral fermions, charged under $B-L$, are required. The minimal model consistent with neutrino data includes three chiral fermions, two of them with the same lepton number. The next minimal models contain five chiral fermions and their $B-L$ charges can be fixed by requiring a dark matter candidate in the spectrum. We list the full particle content as well as the relevant Lagrangian terms for each of these models. They are new and simple models that can simultaneously accommodate Dirac neutrino masses (at one-loop) and dark matter without invoking any discrete symmetries.

I. INTRODUCTION

The interpretation of neutrino experimental data in terms of neutrino oscillations is compatible with both Majorana or Dirac neutrino masses [1]. The former possibility has received the most attention but, given the lack of signals in neutrinoless double beta decay experiments [2–7], the latter cannot be dismissed. If neutrinos are Dirac particles, the Standard Model (SM) particle content must be extended with right-handed neutrinos, and some symmetry must be imposed to prevent their Majorana mass terms. At least a $Z_3$ symmetry is required to guarantee the neutrino Diracness through

$$\mathcal{L}_\nu = y_D (\nu_R)^\dagger L \cdot H + \text{h.c.},$$

with $L \cdot H = \epsilon_{ab} L^a H^b$, where $L$ is the lepton doublet, $H$ is the SM Higgs doublet with hypercharge $Y = 1$, and $y_D$ is the matrix of neutrino Yukawa couplings. To be compatible with neutrino oscillation data [9], $y_D$ should be at least of order $2 \times 3$. A possible assignment for the set of SM fields that transform non-trivially under $Z_3$ is: $L \sim \omega$, $(e_R)^\dagger \sim \omega^2$ and $(\nu_R)^\dagger \sim \omega^3$, with $\omega^3 = 1$. At this level, the neutrino mass problem is not longer a phenomenological issue but a theoretical one, in which it is necessary to explain the smallness of the Yukawa couplings in $y_D$, which must be of order $10^{-11}$.

To do so, we assume that the symmetry allows for the 5-dimensional operator with total lepton number conservation [10]

$$\mathcal{L}_5 = \frac{h}{\Lambda} (\nu_R)^\dagger L \cdot H S^* + \text{h.c.},$$

where $\Lambda$ is the new physics scale, and that this operator is first realized at one-loop level [11] (see [11–18] for the tree-level realizations).

Regarding the symmetry, we follow the usual approach of promoting baryon number ($B$) minus lepton number ($L$) from an accidental global symmetry of the SM, to a local Abelian symmetry, $U(1)_{B-L}$, which is spontaneously broken. One of the main novelties of our work is that we do not impose any other symmetries, discrete or otherwise. Thus, the charges of the right-handed neutrinos under $U(1)_{B-L}$ should be such that the tree-level Dirac mass

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1 Throughout the text we will follow the convention of defining only left-handed Weyl spinors [8], and we will use the SU(2) metric to build scalar products.
term (1) is forbidden. This requirement automatically excludes the usual assignment where the three right-handed neutrinos have $B - L$ charges equal to $-1$.

The classification of all the topologies at one-loop level that realize the effective operator (2) has been presented in [11]. There, in addition to the $U(1)_{B - L}$ symmetry, at least one additional $Z_2$ symmetry was imposed to avoid the Majorana mass terms for the right-handed neutrinos, and a further $Z_2'$ was required to avoid i) the appearance of tree-level realizations in the cases when $U(1)_{B - L}$ is not able to do it, and ii) to have a dark matter candidate in the particle spectrum – one of the new particles needed to realize (2). Here, we focus instead on the simplest realizations of each topology that can be realized with a single symmetry, $U(1)_{B - L}$. This same symmetry would be responsible for the stability of possible dark matter candidates appearing in the different realizations. Let us stress that until now in the literature there is not a simple realization of operator (2) at one-loop invoking only a single symmetry. There were some efforts in this direction but either some Majorana terms were left out, which would need to be forbidden with an extra $Z_2$-symmetry [19, 20], or the found models require many extra fields [19].

Using only $U(1)_{B - L}$, we find, for each topology, the realization with the minimum number of fields. The minimal model requires three chiral fields, two of them sharing the same lepton number, so that the spectrum contains two massive Dirac neutrinos. The next minimal models include five chiral fields. Interestingly, their $B - L$ charges are fixed once the requirement to have a dark matter particle is imposed. Hence, these new models may account for dark matter and Dirac neutrino masses (at one-loop) without invoking any discrete symmetries.

The rest of the paper is organized as follows. In the next section we introduce the notation and derive the conditions necessary to realize the different topologies that give rise to one-loop Dirac neutrino masses. Our main results are presented in section III. There, the particle content of the minimal models, with three and five chiral fields, is spelled out. Finally, in section IV our conclusions are drawn.

II. GENERAL SETUP

We use the notation for the topologies defined in [11], which are displayed in figure 1. There the flux of lepton number is illustrated by the wide colored arrows. The green arrow represents the flux of the doublet lepton number, $L(L_i) = -1$, the yellow is for $L(\nu_R \beta) = -\nu$ with $\beta$ at least 1, 2, the blue for $L(S) = s$, and the red for some internal circulating $L$ charge associated with a chiral fermion, which we choose as a free parameter in our setup.

We do not consider the T1-1 or T4 topologies of [11] because the former is already included in topology T3-1 when the $X_3$ scalar field is decoupled, whereas the latter requires further symmetries to forbid the tree-level contribution\(^2\). In [11] it was assumed that the internal fermion lines were already vectorlike fermions, which allow them to use the $\nu = 1$ solution for anomaly cancellation conditions, but they needed to impose additional $Z_2$ symmetries to forbid the tree-level contribution to neutrino masses and to stabilize the dark matter. Here, we instead assume chiral fields for the fermions that are singlets under the SM gauge group. This allows us to search for new minimal realizations of the topologies with a single extra symmetry beyond the SM. It is clear that our solution with three chiral states, corresponding to the three right-handed neutrinos, can be easily extrapolated to all the solutions found in [11], with the advantage that not further discrete symmetries are imposed. We explicitly illustrate this for the case of topology T3-1-A, whose minimal solution involves just a Dirac fermion.

In our setup, the only extra symmetry beyond the SM, $U(1)_{B - L}$, must forbid the tree-level terms

$$\mathcal{L}_{\nu} = y^\nu_{Rk} (\nu_{Rk})^\dagger L \cdot H + M^\nu_{Rk} (\nu_{Rk})^\dagger (\nu_{R\gamma})^\dagger S + h.c.,$$

as well as allowing for the dimension five operator (2). This is accomplished by the $L$ charge assignments in Table I. There, $\nu$ is the lepton number of the left-handed antineutrino, which is common to at least the two right-handed neutrinos required to explain the neutrino oscillation data. As already mentioned, the solution with $\nu = 1$, studied in [11], is no longer considered in this work.

To find the new possible solutions, we explore the anomaly cancellation conditions with five chiral fields by checking that their charges do not generate direct or induced Majorana mass terms for them. In addition to the chiral fields for $\beta = 1, 2$, we introduce $(\nu_{Rk})^\dagger$ with $L(\nu_{Rk}) = -\nu_k$, and a heavy Dirac fermion field with Weyl

\(^2\) The exception cases are the IV and V solutions of T4-3-I. However, they require mixing between the charged leptons and the new fermion fields, thus leading to charged lepton flavor violation processes which are quite constrained.
FIG. 1. Topologies (in notation of [11]) leading to one-loop Dirac neutrino masses.

TABLE I. General assignment of lepton number for external legs of the one-loop topologies in figure 1

| Fields | $H$ | $L_i$ | $(\nu_R^3)^\dagger$ | $S$ |
|--------|-----|-------|----------------|-----|
| $L$    | 0   | −1    | $\nu \neq 1$     | $\nu - 1 \neq -2\nu$ |

components $\psi_L$ and $(\psi_R)^\dagger$, such that $L(\psi_L) = l$ and $L(\psi_R) = -r$ respectively. The linear and cubic anomaly cancellation conditions are [21, 22]

$$2\nu + \nu_k + l + r = 3, \quad 2\nu^3 + \nu_k^3 + r^3 + l^3 = 3.$$  (4)

From the linear equation

$$\nu_k = 3 - 2\nu - l - r.$$  (5)

To further proceed, we separate the topologies in two types: The set (A) with T1-3-D and T3-1-A, corresponding to the ones with the yellow $\nu$ flux in figure 1, and where $S$ is in a vertex only involving scalars; and the set (B)
with T1-3-E and T1-2-(A/B), with the blue s flux in figure 1, and where S is in a Yukawa-type vertex. For each case we have:

(A) The heavy Dirac fermion has a vectorlike mass, such that

\[ l = -r . \]  

Replacing back in eq. (5) we obtain

\[ \nu_k = 3 - 2 \nu . \]  

Solving the cubic equation for \( \nu \) gives rise to two different roots: 1 and 4. Thus, we choose \( \nu = 4 \) which leads to \( \nu_k = -5 . \)

On the other hand, the fact that the new fermion field \( \psi \) does not contribute to the anomalies implies that the only possible realization within the T1-3-D topology are the solutions I and II with \( \psi_L = L_i \) and \( \psi_R = e_Ri \), since in these cases the corresponding contributions to the anomalies are already taken into account.

(B) The two SM-singlet chiral fields can acquire a Dirac mass (after the spontaneous symmetry breaking (SSB) of U(1)\( _{B-L} \)) through

\[ \mathcal{L}_\psi = h_S (\psi_R)\dagger \psi_L S + \text{h.c.} . \]  

If we choose \( r \) as the free charge circulating in the loop, and since from Table I we have \( s = \nu - 1 \), then from the condition in eq. (8): \( r + l + s = 0 \), we get

\[ l = 1 - \nu - r , \]  

and replacing back in eq. (5)

\[ \nu_k = 2 - \nu . \]  

Using (9) and (10) in the cubic condition for anomaly cancellation, eq. (4), we end up with the one-parameter solution

\[ \nu = \frac{r^2 - r + 2}{3 - r} . \]  

If \( \nu_k = 0 \) we would have a solution with four chiral fields when \( \nu = 2 \), however the required \( r \) charge is irrational and will not be further considered here.

To label the solutions we use the conventions of [11], as Ta-n-b-I \( \alpha \), where “a” refers to the topology itself, “n” indicates the different choices of the fermion and scalar lines in a given topology, “b” denotes the field assignments for external fields, “I” denotes the simplest solution with standard model singlet scalars or fermions, and \( \alpha \) is the parameter which fixes the hypercharges of the \( X_i \) fields inside the loop.

It is worth mentioning that when both \( \psi_L \) and \( \psi_R \) are SM singlets (in the T3-1-A-I with \( \alpha = 0 \), T1-3-E-I with \( \alpha = 0 \), T1-2-A-I with \( \alpha = 0 \) and T1-2-B-II with \( \alpha = +1 \) models), the condition \( r \neq 1 \) or \( l \neq -1 \) must be imposed in order to avoid the tree-level realization of (2) that involves a SM singlet fermion mediator –the so-called type I Dirac seesaw. In this way one of the two required terms in that realization (\( \psi_R^i L_i \cdot H \) and \( \nu_R^i \psi_L^i S^* \)) is forbidden.

The solutions of the two sets are displayed in Table II. Solution (A) is the well known one studied in [15] for tree-level realization of the 5-dimensional operator. In this solution \( r \) is quite free, in fact \( r = \pm 1/2, \pm 1/3, \ldots \). The solution (B) was obtained after exploring all the solutions of eq. (11) for rational values of \( r \leq 10 \), and with both the numerator and denominator less or equal than 10. In the analysis we also exclude direct or induced Dirac fermion terms between the right handed neutrinos and \( \psi_L \).

Higher SU(2)\(_L\) fermion representations, as required in T1-2 topologies, need to be introduced as vectorlike fermions to not spoil the anomaly cancellation conditions of the standard model. We will denote vectorlike doublet Weyl fermions fields with \( Y = -1 \) (\( Y = +1 \)) as \( \Psi_{L,R} (\Upsilon_{L,R}) \).

Regarding the scalars circulating in the loop, we will use \( \sigma \) and \( \eta \) to represent SU(2)\(_L\) scalar singlets and scalar doublets respectively, and we will denote their non-zero lepton number with the same symbols.
### TABLE II. Solutions for Dirac neutrino masses for $i \neq j \neq k$.

| Fields | $(\nu_R^i)^\dagger$ | $(\nu_R^j)^\dagger$ | $(\nu_R^k)^\dagger$ | $\psi_L$ | $(\psi_R)^\dagger$ | $S$ |
|--------|---------------------|---------------------|---------------------|----------|---------------------|-----|
| $L$ (A) | +4                 | +4                 | −5                 | −$r$     | $r$                 | +3  |
| (B)    | $\frac{8}{5}$      | $\frac{8}{5}$      | $\frac{2}{5}$     | $\frac{7}{5}$ | $-\frac{10}{5}$ | $\frac{3}{5}$ |

The new solutions correspond to the case in which some of the $X_i$ fermion fields in figure 1 can be chosen as chiral fields. We explore the solutions with the minimal number of fermion fields beyond the standard model. We are interested, therefore, in the solutions in which at least two right handed neutrinos have the same $U(1)_{B-L}$ charge, because in such a case both of them can couple to the same set of extra chiral fermions. All the solutions for the $B-L$ charges presented below have been chosen in such a way the DM particle does not decay.

#### A. Chiral T1-3-D-I ($\alpha = -2$)

We will start our analysis with the case in which an internal fermion line in figure 1 can be interpreted as a standard model field. Therefore, only the right handed neutrinos contribute to the anomaly cancellation conditions of the SM with $U(1)_{B-L}$. There are three well known solutions with three chiral fields [15, 23]. However, solution (A) in Table II, is the only one that satisfies our constraints.

In fact, with the additions of two charged scalars, $\sigma^\pm_{1,2}$, which are singlet under $SU(2)_L$, we can build the Dirac version of the Zee mechanism to generate neutrino masses [24]. The couplings required to build the diagram displayed in figure 2 are

$$
\mathcal{L} \subset \left[ f_{ij} L_i \cdot L_j \sigma^+_i + h_c^{ij} (e_{Ri})^\dagger L_j \cdot \tilde{H} + h_R^{ij} e_{Ri} \nu_{R\beta} + h.c \right] + V(\sigma^+_1, \sigma^+_2, H) \quad (12)
$$

where $L_i$ are the SM lepton doublets, $H = (H^+, H^0)^T$, $\tilde{H} = i\sigma_2 H^*$ and $V(\sigma^+_1, \sigma^+_2, H)$ is the scalar potential.

Therefore, $\psi_L (\psi_R)$ in solution (A) of Table II corresponds to three lepton doublets $L_i$ (right-handed electrons $e_{Ri})$ with $l = -1$ ($r = 1$) as the usual lepton number. Since we use one set of charged scalar fields, $\sigma^+_1, \sigma^+_2$, the model has two massless chiral fields, one of them, $\nu_{Rk}$, contributing to effective number of relativistic degrees of freedom, $N_{\text{eff}} [20, 23, 25]$. From the Lepton number flux in figure 2, we have

$$
L(\sigma^+_1) = -2, \quad L(\sigma^+_2) = -5. \quad (13)
$$
The full solution is presented in Table III. This is by far the minimal model for Dirac neutrino masses with a gauged $U(1)_{B-L}$. The Dirac Zee model with $\nu = \nu_k = 1$ and extra discrete symmetries has been studied in [24].

It is worth noticing that the restrictions from $N_{\text{eff}}$ are expected to be stronger in our model because of the larger lepton number assignment for the right-handed neutrinos. However, since they do not couple directly to any SM particles, we can simply assume that their interaction with the extra gauge boson and scalars are sufficiently suppressed that they decouple early enough from the thermal bath.

Regarding $\nu_{Rk}$, it can give rise to either a third Dirac neutrino mass if we extend the scalar sector with $S'$ and $\sigma'^{\pm}_2$ of $L$ charges $-6$ and $+8$ respectively, or a Majorana dark matter candidate if we extend the scalar sector with a $S'$ of $L$ charge $+10$ [23]. In both cases we end up with a physical Goldstone boson (GB) which could contribute to $N_{\text{eff}}$ through interactions with the Higgs [26]. The conditions that GB decouples from the bath in the early universe are analyzed in [23] and require couplings of GB with SM Higgs not larger than $10^{-3}$. This discussion can be easily extended to the other one-loop realizations below.

We have implemented the model with three non-zero Dirac neutrino masses in SARAH [27]. We use the method in [24, 28] to express $f_{13}, f_{23}, h_{11}, h_{22}, h_{R33}^R, h_{R31}^R, h_{R32}^R$ and $h_{R23}^R$ as a function of the neutrino masses and mixings, by using $f_{12}^R, h_{12}^R, h_{13}^R, h_{21}^R$ as free parameters. As an example of the consistency of the model, we show in figure 3 the observable $\text{Br}(\mu^+ \to e^+ \gamma) < 4.2 \times 10^{-13}$ [29] as function a $f_{12}$. The other parameters were fixed as $\theta = 0.1$, $M_{\sigma_1} = 500$ GeV, $M_{\sigma_2} = 750$ GeV, $h_{12}^R, h_{13}^R, h_{21}^R = 10^{-4}$, where $\theta$ is the mixing angle between the charged mass eigenstates $\sigma_1$ and $\sigma_2$. We can see that the value for the parameter $f_{12}$ is restricted to values lower than 0.02.

### B. Chiral T1-3-E-I ($\alpha = 0$)

We consider now the topologies where two SM-singlet chiral fields acquire a Dirac mass after the SSB of $U(1)_{B-L}$ from the term $L_\psi$ in eq. (8). For them we will use the solution (B). The relevant terms in the Lagrangian include

$$\mathcal{L} \supset \mathcal{L}_\psi + \mathcal{L}_{\sigma \psi} + \mathcal{L}_{\eta \psi} + V(\sigma_a, \eta_a, S, H),$$

(14)
FIG. 4. T1-3-E-I ($\alpha = 0$)

where $L_\psi$ was given in eq. (8),

\[
L_\sigma \psi = h_1^{1a} (\nu_R \beta)^\dagger \psi_L \sigma_a^* + h.c
\]

\[
L_\eta \psi = h_1^{\eta a} (\psi_R)^\dagger L_i \cdot \eta_a + h.c,
\]

and $V(\sigma_a, \eta_a, S, H)$ is the scalar potential.

After the spontaneous breaking of the $U(1)_{B-L}$ symmetry, this topology is reduced to the well known Dirac radiative seesaw model, but with different Lepton number assignments. In fact, the model with $\nu = \nu_k = r = 1$ and two extra $Z_2$ discrete symmetries was first introduced in [30, 31], while the case with $r \neq 1$ which requires only one extra $Z_2$ symmetry was studied in [32]. The minimal set of fermion fields is achieved when the two non-zero Dirac neutrino masses are generated with two set of SM singlet and doublet scalars: $\eta_a, \sigma_a$ [32].

From the figure 4, the charges of the scalars are

\[
\eta = 1 - r,
\]

\[
\sigma = 1 - r.
\]

The solution compatible with the fields in figure 4 is displayed in Table IV. The phenomenology of the radiative seesaw model with $\nu = 1$ has been already studied in the literature [19, 20, 30–32]. Because of the similar charges associated to solution (B), we do not expect significant differences with respect to those works.

C. Chiral T1-2-A-I $\alpha = 0$

The solution compatible with the fields in figure 5 (left), requires at least the following terms in the Lagrangian

\[
L \supset L_\psi + L_\sigma \psi + \left[ M_\Psi (\Psi_R) \cdot \Psi_L + h_2^{\eta a} (\Psi_R) \cdot L_i \sigma_a + y_1 (\psi_R)^\dagger \Psi_L \cdot H + h.c \right] + V(\sigma_a, S, H),
\]

where $\Psi_L = (\Psi_0^L, \Psi^T_L), (\Psi_R) = ((\Psi_R^-))^T$ and $V(\sigma_a, S, H)$ is the scalar potential. A set of at least two SM-singlet scalars is required to generate a rank-2 neutrino mass matrix.

From the figure 5 (left)

\[
\sigma = 1 - r.
\]

The corresponding charges for the solution (B) are shown in Table IV.

D. Chiral T1-2-B-I $\alpha = 0$

The solution compatible with the fields in figure 5 (right), requires at least the following terms in the Lagrangian

\[
L \supset L_\psi + L_\eta \psi + \left[ M_T (\tilde{\gamma}_R) \cdot \tilde{\gamma}_L + h_2^{\eta a} (\tilde{\gamma}_L) \cdot \eta_a \nu_R \beta + y'_1 \tilde{\gamma}_R \cdot H \psi_L + h.c \right] + V(\eta_a, S, H),
\]
where $\Upsilon_L = (\Upsilon_L^+ , \Upsilon_L^0)^T$, $\Upsilon_R = ((\Upsilon_R^0)^T, - (\Upsilon_R^+)^T)^T$ and $V(\eta, S, H)$ is the scalar potential. A set of at least two scalar doublets is required to generate a rank-2 neutrino mass matrix.

From the figure

$$\eta = 1 - r .$$  \hspace{1cm} (20)

The solutions compatible with the fields in figure 5 (right) correspond to the ones displayed in Table IV.

The phenomenological analysis of the chiral realizations of topologies T1-2 will be done elsewhere. It is worth noting, however, that a model independent analysis of the effect of the right handed neutrinos on $N_{\text{eff}}$ can be inferred from [25]. The ratio $M_{\nu g_{BL}}/g_{BL}$, where $g_{BL}$ is the U(1)$_{B-L}$ gauge coupling, must be larger than 7–8 TeV in order to be in agreement with the cosmological constraints.

E. vectorlike solutions

In [11], all the obtained solutions for the topologies in figure 1 have internal vectorlike fermions. We want to stress that it is possible to realize all of them with a single extra symmetry U(1)$_{B-L}$. For that, we can use the solution (A) with a proper choice of the circulating free charge $r$.

For example, in the simplest case of just one circulating SM-singlet Dirac fermion line as in T3-1-A, we have the diagram displayed in figure 6.

From the flux of lepton number in figure 6, we have.

$$\eta = 1 - r , \quad \sigma = \nu - r .$$  \hspace{1cm} (21)

The corresponding charges for solution (A) are shown in Table V.
Since the charges of the right-handed neutrinos are now bigger than those in solution (B) used in section III B, the constrains from $N_{\text{eff}}$ are expected to be stronger. In fact, a recent detailed phenomenological analysis of the Dirac radiative seesaw [20] includes the case of a right-handed neutrino with $\nu = 4$ and can be fully applied here. In particular, the restrictions from $N_{\text{eff}}$ for both scalar and Dirac fermion dark matter cases are more important than the restriction from $Z_{BL}$ searches at the LHC. We remit the reader there for further details.

It is clear that after SSB of $U(1)_{B-L}$ both solutions T1-3-E-1 ($\alpha = 0$) in section III B and T3-1-A ($\alpha = 0$) reduce to the Dirac radiative seesaw. Moreover, we can add extra circulating charges in the loop. In fact, when all the circulating particles in the loop are color octets [32], we can have a bound state dark matter candidate formed by two Dirac color-octet fermions [33]. With our solutions, the extra $Z_2$ symmetry in [32] is not longer required.

### IV. CONCLUSIONS

We found new and simple models for Dirac neutrino masses within an extension of the Standard Model by an spontaneously broken $U(1)_{B-L}$ gauge symmetry. Specifically, we studied the minimal chiral realizations, at one-loop, of the dimension-5 total lepton number conserving operator that gives rise to Dirac neutrino masses without imposing extra symmetries. The minimal models contain three or five chiral fields, two of them with the same charges under $B-L$. In the latter case, their charges can be fixed by the requirement to have a dark matter particle in the spectrum. The full particle content as well as the relevant Lagrangian terms were given for each of these models. We also showed that known solutions with vectorlike fermions can be obtained with just the single symmetry $U(1)_{B-L}$. These new models, therefore, can simultaneously accommodate one-loop Dirac neutrino masses and dark matter without invoking any discrete symmetries.

NOTE: During the completion of this work, a related study [34] has appeared. They also present the solution in section III E (Table V) by fixing $r = 1/2$.

### ACKNOWLEDGMENTS

Work supported by Sostenibilidad-UdeA, and by COLCIENCIAS through the Grants 111565842691 and 111577657253. D.R thanks Martin Hirsch for illuminating discussions. O.Z. acknowledges the ICTP Simons...
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