Addressing the Multi-Channel Inverse Problem at High Energy Colliders: A Model Independent Approach to the Search for New Physics with Trileptons

Sourabh Dube, Julian Glatzer, Sunil Somalwar, Alex Sood, and Scott Thomas
Department of Physics and Astronomy, Rutgers University, Piscataway, New Jersey 08854

We describe a method for interpreting trilepton searches at high energy colliders in a model-independent fashion and apply it to the recent searches at the Tevatron. The key step is to recognize that the trilepton signature is comprised of four experimentally very different channels defined by the number of \( \tau \) leptons in the trilepton state. Contributions from these multiple channels to the overall experimental sensitivity (cross section times branching ratio) are model-independent and can be parametrized in terms of relevant new particle masses. Given the trileptonic branching ratios of a specific model, these experimentally obtained multichannel sensitivities can be combined to obtain a cross section measurement that can be used to confront the model with data. Our model-independent results are more widely applicable than the current Tevatron trilepton results which are stated exclusively in terms of mSUGRA parameters of supersymmetry. The technique presented here can be expanded beyond trilepton searches to the more general “inverse problem” of experimentally discriminating between competing models that seek to explain new physics discovered in multiple channels.

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I. INTRODUCTION

The canonical method to search for signatures of new physics at high energy colliders is through the use of high transverse momentum objects such as jets, isolated photons, electrons and muons, often accompanied by transverse momentum imbalance. Isolated electrons and muons are easier to identify and suffer smaller backgrounds than jets. A classic new physics search that makes use of these relatively clean objects is inclusive trileptons plus missing transverse energy.

Standard Model background for trileptons can be mostly estimated using data-driven techniques with minimal reliance on monte carlo simulations. This signature also covers a wide range of new physics scenarios.

The most widely discussed possibility for new physics that could give rise to the trilepton signature is supersymmetry (SUSY) [1]. In this case trileptons can arise from chargino-neutralino production followed by their cascade decays. The chargino, \( \tilde{\chi}^\pm \), decays to the lowest neutralino, \( \tilde{\chi}^0 \), which is the stable lightest supersymmetric particle (LSP), yielding a charged lepton and neutrino, while the neutralino, \( \tilde{\chi}^0 \), yields two charged leptons and the LSP. The stable neutralinos and neutrino escape undetected and thus carry away transverse energy. This gives the three leptons with missing transverse energy “trilepton” signature [2], as depicted schematically in Figure 1.

Although the trilepton signature can arise over a fairly wide region of SUSY parameter space, it is often discussed within the context of specific model assumptions or special subspaces of the general parameter space. The most widely abused subspace in this regard is one with a universal mass for all scalars, \( m_0 \), and another for all gauginos, \( m_{1/2} \), both defined at the unification scale. For obscure historical reasons, this SUSY subspace is referred to as minimal supergravity (mSUGRA).

The Tevatron trilepton searches have traditionally been interpreted within the context of mSUGRA parameters, or experimentally convenient variations thereof. Both CDF [3] and DØ [4, 5] collaborations show results as a function of \( m_0 \) and \( m_{1/2} \), while holding other parameters fixed (Section II). Experimentists resort to mSUGRA for conveying trilepton results not because of its theoretical merit, but for the practical reason that it has fewer parameters than other choices. Even then, not all of its five parameters are covered. This model-specific restrictive approach makes it difficult to interpret search results for more general superpartner mass spectra and mixings or for other new physics models besides supersymmetry.

Below, we describe an economical way to state the

Fig. 1: Supersymmetric cascade transitions relevant to the trilepton plus missing transverse energy signature. Solid lines with sparticle names to the left are the minimal spectrum that yields trileptons, while dotted lines with names to the right are additional states that may be present.
trilepton results in a model-independent fashion. Section III lays out three main principles that allow experimentalists to distance themselves from specific models while interpreting search results. In there, we also identify the multiple channels that constitute the trilepton signature and formulate a scheme for parametrizing their experimental sensitivities as a function of relevant phenomenological parameters.

Section IV has the complete scheme for presenting search sensitivities in a model-independent fashion. Also included is the derivation of an equation for combining these sensitivities using the particle masses and branching ratios predicted by a model under consideration to obtain the experimental cross section measurement specifically for that model. If this measured value agrees with the cross section predicted by the model, the model is consistent with the data.

In section V we take on the task of converting the Tevatron trilepton results into a model-independent format. We formulate a suitable scheme for parametrizing the trilepton multichannel sensitivities as a function of the mass parameters and then evaluate the coefficients of parametrization using CDF results in section VI. Section VII has the resultant model-independent formulation of the CDF trilepton results and a simple example to show how to use these results. In the following sections VIII and IX we use these results to recover CDF’s mSUGRA trilepton interpretation, to project sensitivities to a significantly larger data sample, and to address a couple of non-mSUGRA scenarios.

Finally, in section X we briefly describe how the technique we use here for trileptons may be used more generally in the context of deciphering the nature of new physics that manifests itself in multiple channels.

The model-independent experimental parametrizations presented in this article are electronically available in the form of a spreadsheet utility at Rutgers University’s Department of Physics publication archive website [5].

II. A BRIEF REVIEW OF CDF AND DØ TRILEPTON SEARCHES

At the Tevatron, CDF [3] and DØ [4] have conducted similar trilepton analyses using around 2-3 fb$^{-1}$ data each. The final state consists of at least two electrons or muons, and both experiments allow an isolated track in lieu of the third lepton in order to add sensitivity for the $\tau$-lepton decays.

• CDF: CDF interprets its results in the context of the canonical mSUGRA scenario which has five parameters and considerably simplifies the large MSSM parameter space. CDF conducts an exclusive multichannel analysis [3] where events are sorted based on the expected signal purity, and the results combined in the end. The analysis is split into channels with three electrons and/or muons, and those where the third object is an isolated track. The published cross section results, presented for 2 fb$^{-1}$ of data, are shown as a function of two of the mSUGRA parameters $(m_0, m_{1/2})$ or as a function of the $\tilde{\chi}_1^+$ mass while keeping the other mSUGRA parameters fixed at some suitable values $(\tan(\beta)=3, A_0=0$ and $\mu > 0)$. This intractable choice immediately begs the question of search sensitivity for other models as well as for other values of the mSUGRA parameters.

• DØ: DØ interprets its results in the context of the mSUGRA scenario, as well as an MSSM scenario that follows all the mSUGRA mass constraints. The DØ analysis has channels with two electrons and/or muons where the third object is an isolated track, and a channel with one electron or muon, one hadronically reconstructed tau-lepton, and one isolated track. The final results are presented in terms of the mSUGRA parameters $m_0$, $m_{1/2}$ and $\tan(\beta)$. In the same way as CDF, the DØ results cannot be reinterpreted easily when the theory parameters are different from those assumed in the standard result.

III. THREE ORGANIZING PRINCIPLES

There are three major hurdles in stating trilepton results in a model-independent fashion. First, there are several channels that make up the trilepton signature. Second, it is not clear exactly what experimental information to give out as a measure of experimental sensitivity since these channels have different acceptances and Standard Model backgrounds. Finally, the experimental acceptance depends on the nature of signal in a phenomenological model of interest.

A brute force approach of publishing experimental acceptance and background on a channel-by-channel basis for different models is not only prohibitively cumbersome, but would also require the reader to delve deeply into experimental details. However, volumes of data need not be published to achieve model-independence if experimental sensitivity can be quantified concisely and stated separately for a small set of phenomenologically important channels and parameters of universal interest. We now discuss these three requirements one by one.

A. Experimental Sensitivity \{cB\}

The reach of an experimental search (or measurement) depends on the amount of collected data (integrated luminosity), detector’s acceptance for the signal, and the extent of Standard Model background. A concise and commonly used experimentally accessible quantity that
characterizes the overall search sensitivity is the product of production cross-section and the branching ratio, \( \{\sigma B\} \). To measure it, the number of standard model background is subtracted from the observed number of events and then the detector acceptance and the integrated luminosity are divided out. \( \{\sigma B\} \) is enhanced by optimizing the selection criteria (“cuts”) to increase acceptance while keeping the backgrounds in check. A model is successfully confronted by the experiment if the measured \( \{\sigma B\} \) is comparable to the model’s value for the signal. If an experiment fails to find the signal, the \( \{\sigma B\} \) sensitivity is typically expressed as a 95% confidence level upper limit.

If it is to be used in a model-independent context, a way must be found to measure and tabulate it as generically as possible so that it can be used to reconstruct the sensitivity for other models.

**B. Identifying Relevant Multichannels**

Experimental search is often carried out in multiple channels to cover as much signal as possible. As described above, CDF devotes an analysis channel for its highest quality electrons and muons, and a separate one for leptons that are not as well reconstructed. Further, in order to include the short-lived \( \tau \) lepton as one of the trileptons, there is another higher-background trilepton channel with two leptons and an isolated track which serves as a proxy for the \( \tau \) lepton. These channels suffer from different amount of Standard Model backgrounds and have varying detector acceptance.

These experimental search channels described in terms of electron and muon quality or the presence of a track do not have direct relevance from a phenomenological point of view. However, their overall ability to detect the \( \tau \) lepton is of interest because the \( \tau \) flavor content of the trilepton state is an important clue to the nature of new physics. Another important reason to focus on the \( \tau \) lepton is that the experimental acceptance depends drastically on the number of \( \tau \) leptons in the trilepton state because the detection of hadronic decays of the \( \tau \) lepton draws a substantial background. Detecting trileptons when none of the leptons are \( \tau \)'s is straightforward and detecting one \( \tau \) via its hadronic decays is manageable. However, detection of trileptons with two (three) \( \tau \)'s requires at least one (two) \( \tau \) leptons decay leptonically. Since the leptonic decay of the \( \tau \) lepton takes place approximately one-third of the time, even a trilepton search with only electrons and muons in its search channels will still have indirect sensitivity to the three physics channels above that contain \( \tau \) leptons.

The presence of \( \tau \) leptons in the trilepton final state greatly affects the search sensitivity and is thus a major source of model-dependence. Therefore, the combined \( \{\sigma B\} \) sensitivity gleaned from the multiple experimental channels should be mapped onto \( \{\sigma B\} \) sensitivities tabulated in terms of the \( \tau \) content of the signal. Accordingly, we classify the trilepton signature into four exclusive channels based on their \( \tau \)-content:

- \( 0\tau \)
- \( 1\tau \) (In SUSY language, e.g., because \( \tilde{\chi}_{1}^{\pm} \to \tau \nu \tilde{\chi}_{1}^{0} \)).
- \( 2\tau \) (\( \chi_{2}^{0} \to \tau \tau \tilde{\chi}_{1}^{0} \)).
- \( 3\tau \) (Both).

We will denote the measured experimental sensitivity for a channel that contains \( i \) \( \tau \) leptons by \( \{\sigma B\}_{i} \). Each \( \{\sigma B\}_{i} \) is measured by assuming a 100% branching fraction for the channel with \( i \) \( \tau \) leptons, i.e., that the trilepton signal contains exactly \( i \) \( \tau \) leptons. Each of the four \( \{\sigma B\}_{i} \)'s receives varied contributions from the underlying experimental search channels which are characterized by how many electrons, muons and tracks they contain, how well reconstructed these objects are, etc.

**C. Identifying Universal Parameters**

The final principle in achieving model-independence is to avoid the parameters of specific models and express the results as a function of the underlying parameters such as particle masses and mass differences that characterize the signature decay. In case of trileptons and supersymmetry, the chargino, neutralino, and slepton masses control the production and cascade decays responsible for the trilepton signature. Their masses and mass differences are the appropriate parameters for expressing the experimental results as opposed to the \( m_{0} \) and \( m_{1/2} \) parameters of mSUGRA. Therefore, the \( \{\sigma B\} \) search sensitivity can be expressed in a model-independent fashion if it is given as a function of these universal mass parameters.

For trileptons, we pick three mass parameters for this purpose. The scale for missing energy is set by the mass \( M \) of the undetected new particle (lower state). The mass difference \( \Delta M_{1} \) between the upper state and a possible intermediate state plays a role in the distribution of trilepton momenta. Finally, the total energy available to the decay products is given by \( \Delta M_{2} \), the mass difference between the upper and the lower state. For the specific instance of trileptons from supersymmetric chargino-neutralino decays, \( M \) is the LSP mass, \( \Delta M_{1} \) is the mass difference between the chargino and the intermediate right-handed slepton and \( \Delta M_{2} \) is the chargino-LSP mass difference.

In supersymmetric theories that have direct production of Wino-like states, the mass of the lightest chargino is approximately equal to one of the neutralinos. We have thus made a concession to the supersymmetric origin of the trilepton signature in assuming that the two
upper states have the same mass. We also allow at most one intermediate state particle. The most general treatment would require additional mass parameters beyond the three we employ.

IV. A RECIPE FOR MODEL-INDEPENDENT INTERPRETATION

At this point, we claim that model-independence can be achieved, i.e., the trilepton search results can be used for confronting an arbitrary model that predicts trilepton signal, provided (a) the search results are tabulated in terms of separate $\{\sigma B\}_i$ measurements for the four trilepton $\tau$ subchannels, and (b) the $\{\sigma B\}_i$'s are stated as a function of the mass parameters described above. We substantiate this claim by showing how to deduce the experimental cross section measurement for a model under consideration, using the masses and the four $\tau$ branching ratios predicted by the model.

Consider a model with cross section $\sigma$ for producing the parent states such as chargino-neutralino that lead to the trilepton signature. The model also predicts branching ratio $B_i$'s for trilepton channels with $i$ $\tau$'s. On the experimental side, let us assume the detector acceptance (efficiency) for these channels to be $A_i$ and the collected luminosity to be $L$. Then the number of observed trilepton events, $N$, is given by $N = \sum_{i=0}^{3} L \sigma B_i A_i$.

Now recall that the very same $N$ observed events gave the individual $\{\sigma B\}_i$ sensitivity for a channel with 100% branching ratio for $i$ $\tau$ leptons, as though evaluating a model that predicts trileptons with only $i$ $\tau$'s. Therefore, it also follows that $N = L \sigma B_i A_i$. Equating the two $N$'s gives the desired equation to calculate the experimental cross section measurement for a model:

$$\frac{1}{\sigma_{XM}} = \sum_{i=0}^{3} \frac{B_i}{\{\sigma B\}_i}.$$  (1)

We have added a subscript $XM$ on the cross section to indicate that $\sigma_{XM}$ is the cross section as measured by the experiment for the model. Note that the equation is free of experimental details such as acceptance, number of events and luminosity. $\sigma_{XM}$ is an aggregate of the $\{\sigma B\}_i$ multichannel cross section measurements.

To recapitulate, the experiment provides its multichannel $\{\sigma B\}_i$ search sensitivities for each of the four trilepton $\tau$ channels as a function of the three mass parameters. The model provides the (three) relevant sparticle mass parameters and the (four) multichannel trilepton branching ratios, $B_i$'s. Equation 1 then blends the phenomenological and experimental information to give the experimental reach, quantified as cross section for the model under consideration, $\sigma_{XM}$. The model is confronted experimentally if the trilepton production cross section it predicts exceeds $\sigma_{XM}$. We reiterate that the experiment need not separately provide the detector acceptance or the Standard Model background information as this information is already incorporated in the $\{\sigma B\}_i$ sensitivity measurements. Also note that $\sigma_{XM}$ is not a pure experimental quantity since models with differing trilepton branching ratios would yield different $\sigma_{XM}$'s from the same experimental data.

We now describe how to express the trilepton $\{\sigma B\}_i$ sensitivities as a function of the three mass parameters described above and then determine the parametrization using Tevatron results. Resulting model-independent version of the Tevatron trilepton search results follows in section V.

V. $\{\sigma B\}$, SEARCH SENSITIVITIES AS FUNCTION OF GENERIC MASS PARAMETERS

As described above, the three mass parameters in our scheme are $M$, $\Delta M_1$, and $\Delta M_2$. We empirically find that the dependence of $\{\sigma B\}_i$ on $M$ conveniently factors out from the $\Delta M_1$ and $\Delta M_2$ dependence, giving

$$\{\sigma B\}_i^{-1} = f_i(M) \times h_i(\Delta M_1, \Delta M_2)$$  (2)

with $i = 0, 1, 2, 3$ indicating the $\tau$ content and $f_i$ and $h_i$ are parametric functions to be determined from data.

In the absence of an intermediate particle (say when the $\tilde{\chi}^{\pm}$ is heavier than the $\chi^0_1$), we can disregard $\Delta M_1$, giving

$$\{\sigma B\}_i^{-1} = f_i(M) \times g_i(\Delta M_2)$$  (3)

where $g_i$ is another parametric function. These functions can be written as Taylor expansions:

$$f(M) = 1 + a_1(M) + a_2(M)^2$$  (4)

$$g(\Delta M_2) = b_0 + b_1(\Delta M_2) + b_2(\Delta M_2)^2$$  (5)

$$h(\Delta M_1, \Delta M_2) = e_0 + c_1(\Delta M_2) + d_1(\Delta M_1) + c_2(\Delta M_2)^2 + d_2(\Delta M_1)^2 + e_2(\Delta M_1 \times \Delta M_2).$$  (6)

Note that the subscripts $i$ denoting the $\tau$ content are implicit for the functions $f, g$ and $h$ on the left hand side as well as for the coefficients on the right hand side of these equations.

The values of the expansion coefficients are determined from experimental data. The coefficients in turn define the $f, g$ and $h$ functions and thereby the $\{\sigma B\}_i$ sensitivities as a function of the masses. Since the Tevatron trilepton results are confined to the mSUGRA scenario, we carry out these determinations with the aid of extensive standalone simulation and using public information from the CDF collaboration. We describe this procedure in the next section before giving the results.
the similar to CDF’s. The charged particle selection catches isolated $E_T$ by CDF. We calculate the missing energy to approximate the isolated track selection used to generate numerous samples of simulated trilepton events for different choices of the mass difference between $\chi_{1}^{\pm}$ and $\chi_{0}^{0}$, and $\chi_{1}^{\pm}$ and $\tilde{l}^{\pm}$. See 7 for further details of sample generation. Subsamples for each of the four $\tau$ channels undergo identical analysis. We select events with three electrons or muons, or events with two electrons or muons and any other charged particle, with $p_T$ thresholds similar to CDF’s. The charged particle selection catches the $\tau$ lepton single-prong decays and it is required to be isolated1 to approximate the isolated track selection used by CDF. We calculate the missing energy $E_T$ by taking the vector sum of the transverse momenta of all neutrinos and LSP’s present in the event.

VI. DETERMINATION OF $\{\sigma B\}$, SENSITIVITY PARAMETRIZATION USING CDF RESULTS

We mimic CDF analysis using pythia v6.409 6 to generate numerous samples of simulated trilepton events for different choices of the mass difference between $\chi_{1}^{+}$ and $\chi_{0}^{0}$, and $\chi_{1}^{+}$ and $\tilde{l}^{\pm}$. See 7 for further details of sample generation. Subsamples for each of the four $\tau$ channels undergo identical analysis. We select events with three electrons or muons, or events with two electrons or muons and any other charged particle, with $p_T$ thresholds similar to CDF’s. The charged particle selection catches the $\tau$ lepton single-prong decays and it is required to be isolated1 to approximate the isolated track selection used by CDF. We calculate the missing energy $E_T$ by taking the vector sum of the transverse momenta of all neutrinos and LSP’s present in the event.

Table I: Selections for Pythia based simulations to mimic the Tevatron trilepton searches. CDF and DØ selection criteria tend to be fairly similar. (OS=opposite-sign)

| Variable        | Selection                        |
|-----------------|----------------------------------|
| $p_T^{1,2,3}$   | > 15, 5, 5 GeV/c                 |
| $|\eta^{1,2,3}|$ | < 1.1                            |
| $E_T$           | > 20 GeV                         |
| max OS Mass     | > 20 GeV/c^2, $\notin [76, 106]$ GeV/c^2 |
| next OS Mass    | > 13 GeV/c^2, $\notin [76, 106]$ GeV/c^2 |

Other selections listed in Table II are made and the final $\{\sigma B\}_i$ sensitivities for each subsample are calculated. The selections follow those of the CDF analysis described in Ref. 8 and are meant to reproduce the CDF analysis 8. We then systematically scan the $\Delta M_1$-$\Delta M_2$ space while maintaining the following mass relations:

1. The sum of $p_T$’s of other charged particles within an $\eta - \phi$ cone of 0.4 is required to be less than 10% of the $p_T$ of the charged particle.

M($\tilde{\chi}_{1}^{\pm}$) = M($\chi_{0}^{0}$), M($\tilde{\nu}_R$) = M($\tilde{\nu}_L$) = M($\tilde{\tau}_1$). The decay of $\chi_{1}^{\pm}$ and $\chi_{0}^{0}$ to $\tilde{\nu}$’s is turned off and thus the $\tilde{\nu}$’s play no further part in the analysis. We set M($\chi_{1}^{0}$) = 70 GeV/c^2 and vary $\Delta M_2$ from 40 to 90 GeV/c^2 in steps of 5 GeV/c^2.

The two cases, positive $\Delta M_1$, and negative $\Delta M_1$ are treated separately. In the case of negative $\Delta M_1$, the $\{\sigma B\}_i$ sensitivities depend only on $\Delta M_2$. When $\Delta M_1$ is positive, i.e., there is an intermediate state, we vary its mass M($\tilde{l}^{\pm}$) from a value 5 GeV/c^2 higher than M($\chi_{1}^{0}$), to 5 GeV/c^2 less than M($\chi_{1}^{0}$) in steps of 5 GeV/c^2. In all cases, the events are split into four subsamples with 0, 1, 2 and 3$\tau$’s in the final state.

We then parametrize the $\{\sigma B\}_i$ sensitivities (for each subsample) as a function of $\Delta M_1$, $\Delta M_2$ and $M$. First we determine the term which depends on the overall mass scale by fixing the relative masses of M($\chi_{1}^{0}$), M($\tilde{l}^{\pm}$) and M($\chi_{1}^{0}$). To do this we fix $\Delta M_1$ = 25 GeV/c^2, and $\Delta M_2$ = 50 GeV/c^2 and fit the $\{\sigma B\}_i$ sensitivities as a function of M($\chi_{1}^{0}$) (=M). This dependence is given by $f(M)$, and illustrated in Figure 2 for the 0$\tau$ and 1$\tau$ cases. The fit parameters are given in Table II.

We now proceed to determine the term which depends on the mass differences between M($\chi_{1}^{0}$), M($\tilde{l}^{\pm}$) and M($\chi_{1}^{0}$). We fix M($\chi_{1}^{0}$) at 70 GeV/c^2, and fit the $\{\sigma B\}_i$ sensitivities as a function of a) $\Delta M_2$, b) M($\tilde{l}^{\pm}$)>M($\chi_{1}^{0}$), and given by $g(\Delta M_2)$, or the final $\{\sigma B\}_i$ sensitivities are then calculated by multiplying the f and g (or h) functions. The $\{\sigma B\}_i$ sensitivities and their fits are shown for the 0$\tau$ and 1$\tau$ case in Figures 3 and 4 respectively for the case where $\Delta M_1$ is positive. For the negative $\Delta M_1$ case, the same are shown in Figure 5.

VII. MODEL-INDEPENDENT TRILEPTON RESULTS

The values of the parameters for the $f$, $g$ and $h$ functions that determine the $\{\sigma B\}_i$ sensitivities (See Eqs. 3-5) obtained from our analysis of the CDF 2 fb$^{-1}$ trilepton search are given in Table II. Several comments are in order. These results are obtained with 2 fb$^{-1}$ integrated...
luminosity. To apply them to different amount of data requires scaling by the square-root of the luminosity. Also note that these results describe a null finding in terms of 95% confidence level upper limits. When searching for new physics, it is customary to express the sensitivities in units of standard deviations, e.g., a 3σ or 5σ discovery potential. (95% C.L. upper limit sensitivity amounts to a 1.64σ measurement sensitivity.) Finally, we do not explicitly write the units of the coefficients in the table. They are such that when the mass parameters are in units of GeV/c², the resulting cross-section values are in picobarns.

The parametrization is valid when the lower state mass \( M \) is less than 150 GeV/c² and the upper-lower state mass difference \( \Delta M_2 \) is less than 90 GeV/c². If \( \Delta M_1 > 0 \) because an intermediate state exists, we further require that \( \Delta M_1 > 5 \text{ GeV/c}^2 \) and \( (\Delta M_2 - \Delta M_1) > 5 \text{ GeV/c}^2 \), i.e., the intermediate state mass should also be at least 5 GeV/c² away from the upper and the lower states. The results are accurate to 20-30% in general and to 30-40% in regions closer to the extreme edges of the applicability range.

A fairly extensive amount of simulation is required to obtain the full parametrization from data as above, and it may be difficult to carry out the detailed (full) experimental simulation for each grid point in the parameter space. A convenient experimental strategy would then be to use a hybrid simulation scheme consisting of a fully simulated sparse grid that is filled with a finer grid of points generated with faster standalone simulation as we do above.

### A. Example: A Simple Toy Model

Consider a supersymmetric scenario with the following mass spectrum: \( M(\tilde{\chi}_1^0)=M(\tilde{\chi}_2^0)=150 \text{ GeV/c}^2, \quad M(\tilde{t})=130 \text{ GeV/c}^2, \quad \text{and } M(\tilde{\chi}_2^0)(=M)=110 \text{ GeV/c}^2 \). Suppose that the \( \chi_2^0 \)'s always contribute 2τ's to the trilepton state because it decays via τ's, but the \( \chi_1^± \)'s leptonic decays occur democratically to electron, muon and τ lepton via the three slepton flavors. This would imply branching fractions of zero for trileptons with 0 and 1 τ's \((B_0 = B_1 = 0)\) and that the 2τ's occur twice as frequently as 3 τ's. Arbitrarily, we pick \( B_2 = 0.2 \) and \( B_3 = 0.1 \), implying that 70% of the decays do not yield trileptons.

The three mass parameters then are \( M_0(=M(\tilde{\chi}_1^0)) = 110 \text{ GeV/c}^2, \quad M_1 = 20 \text{ GeV/c}^2, \quad \text{and } M_2 = 40 \text{ GeV/c}^2 \). Using the Taylor expansion results from table [II] we get the upper limit experimental sensitivities at 95% confidence level for this mass spectrum with 2 fb⁻¹ data to be \( \{aB\}_0=8.02 \text{ pb}⁻¹, \{aB\}_1=3.87 \text{ pb}⁻¹, \{aB\}_2=0.49 \text{ pb}⁻¹ \) and \( \{aB\}_3=0.11 \text{ pb}⁻¹ \). The next step is to fold in the model’s τ channel branching fractions \((B_0 = B_1 = 0, B_2 = 0.2, \text{ and } B_3 = 0.1)\) using equation [II] to get the upper limit \( \sigma_{XM} \) of 9.2 pb. If the model’s chargino-neutralino production cross section is

| \( \Delta M_1, \Delta M_2 \) | \( f(\tau) \) | \( 1 \tau^s \) | \( 2 \tau^s \) | \( 3 \tau^s \) |
|-----------------------------|----------|----------|----------|----------|
| \( a_1 \) | 2.70 × 10⁻² | 4.48 × 10⁻² | 5.61 × 10⁻² | 4.27 × 10⁻² |
| \( a_2 \) | -9.48 × 10⁻⁵ | -1.69 × 10⁻⁴ | -2.29 × 10⁻⁴ | -1.59 × 10⁻⁴ |
| \( g(\Delta M_2) \) | | | | |
| \( b_0 \) | -4.39 | -3.59 | -3.71 × 10⁻² | 2.11 × 10⁻¹ |
| \( b_1 \) | 3.28 × 10⁻¹ | 1.72 × 10⁻¹ | 6.60 × 10⁻⁴ | -8.20 × 10⁻³ |
| \( b_2 \) | -2.08 × 10⁻³ | -9.41 × 10⁻⁴ | 1.51 × 10⁻⁴ | 1.13 × 10⁻⁵ |
| \( h(\Delta M_1, \Delta M_2) \) | | | | |
| \( c_0 \) | -2.84 | -1.73 | -2.67 × 10⁻¹ | 1.22 × 10⁻² |
| \( c_1 \) | 1.92 × 10⁻¹ | 9.66 × 10⁻² | 8.71 × 10⁻³ | -9.86 × 10⁻⁴ |
| \( d_1 \) | -3.60 × 10⁻² | -3.74 × 10⁻² | -4.36 × 10⁻³ | -1.01 × 10⁻³ |
| \( d_2 \) | -1.56 × 10⁻³ | -7.36 × 10⁻⁴ | -7.91 × 10⁻⁵ | 8.02 × 10⁻⁶ |
| \( d_3 \) | -2.40 × 10⁻³ | -1.49 × 10⁻³ | -5.25 × 10⁻⁴ | -1.58 × 10⁻⁴ |
| \( e_2 \) | 2.72 × 10⁻³ | 1.73 × 10⁻³ | 5.66 × 10⁻⁴ | 1.59 × 10⁻⁴ |
higher than this value, then the model is ruled by this CDF result at more than 95% confidence level.

A spreadsheet utility to carry out the procedure demonstrated in this example is available electronically [9].

VIII. RECOVERING CDF’S MSUGRA RESULT

The trilepton search results from the Tevatron have been restricted to the mSUGRA scenario. We used the framework described above to generalize the CDF 2 $fb^{-1}$ result. In order to establish the veracity of our scheme, we apply the parametrized formulation above to the very region of mSUGRA parameter space addressed by CDF ($\tan(\beta)=3$, $A_0=0$ and $\mu > 0$). The result is shown in Fig. 6 as two exclusion lobes. The dashed line on the right indicates where the chargino and the intermediate slepton masses are equal and the one on the left is where the chargino mass equals that of the intermediate sneutrino. Thus, the $m_0-m_{1/2}$ parameter space shown in the figure is split into three regions: no intermediate state for the rightmost region, one for the middle region and two for the leftmost region. Our exclusion curves compare well with CDF’s (Fig. 2 in [3]; also see Fig. 8 in [3]), but the left exclusion lobe is somewhat smaller than CDF’s in the region where two intermediate particles play a role in the trilepton kinematics. Since our parametrization scheme allows at most one intermediate particle between the chargino and the LSP, we maintain consistency by ignoring the signal decays that occur via the sneutrino, thereby lowering our estimate of the sensitivity in that region.

IX. OTHER SUPERSYMMETRY SCENARIOS AND PROJECTIONS

We now examine the Tevatron trilepton search reach with $20 \text{ fb}^{-1}$ data for three different supersymmetry scenarios. The results are shown in Fig. 7 as 95% confidence level upper limit sensitivities, or equivalently, as 1.64 $\sigma$ measurement potentials.

The green curve (inverted triangles) depicts the normal mSUGRA scenario used by the Tevatron experiments. The chargino mass upper limit of approximately $180 \text{ GeV}/c^2$ comes from the intersection (at the upright triangle) of the green curve with the solid black curve representing cross section for wino-like production, of which mSUGRA is a special case. The particle masses and branching ratios are all dictated by the parameters of mSUGRA. For example, the total trilepton branching ratio for the chargino mass of $150 \text{ GeV}/c^2$ is 92.4%. This curve for experimental sensitivity is obtained by using our parametrization for the mSUGRA mass values and then using Equation (1) to blend in the mSUGRA branching ratios. A multiplicative factor takes care of the ten-fold increase in luminosity over the CDF’s.

The figure also shows sensitivity as a function of the upper state (chargino) mass for two other scenarios for which the mass difference between the upper state and the intermediate state, $\Delta M_1$, is held fixed at $30 \text{ GeV}/c^2$ and the upper and lower state mass difference, $\Delta M_2$, at $60 \text{ GeV}/c^2$. Additionally, for the blue curve (squares), all three leptons are forced to be $\tau$’s, i.e., $B_0 = B_1 = B_2 = 0$, and $B_3 = 1$. As expected, this all-$\tau$ scenario shows a poor

![Fig. 6: mSUGRA exclusion region with our generalization of the CDF result. It compares favorably with CDF’s original exclusion. The dashed line on the right indicates equal chargino and intermediate slepton masses and similar line on the left indicates equal chargino and intermediate sneutrino masses.](image)

![Fig. 7: Tevatron trilepton reach for 20 $fb^{-1}$ for three scenarios described in the text.](image)
sensitivity due to the experimental difficulty in detecting the $\tau$ lepton.

Finally, for the red curve (circles), all lepton flavors are treated democratically, giving the trilepton branching ratio values of $B_0 = 4/9$, $B_1 = B_2 = 2/9$ and $B_3 = 1/9$. The experimental sensitivity for this scenario is somewhat better than the same for mSUGRA because of a smaller fraction of $\tau$’s.

These scenarios demonstrate how the experimental data presented in our model-independent scheme can be used with relative ease. Other models can similarly be confronted by the Tevatron data with the spreadsheet tool we provide [3].

X. ADDRESSING THE INVERSE PROBLEM

The phrase “inverse problem” is used in the context of anticipated discoveries at the Large Hadron Collider. It refers to the challenge of identifying the correct theory behind discoveries that crop up in several experimental channels such as multileptons, photons and jets. It could be a difficult problem to solve given the limited experimental channels in contrast to preponderence of theories and their vast parametric spaces.

The multichannel cross section equation 1 presented here in the context of generalizing trilepton supersymmetry search results, is of a more general use in addressing the “inverse” problem of new physics. Let us say that there is simultaneous evidence for new physics in several channels that have varying degree of experimental sensitivities for detecting the signal. It is quite likely that several theories such as supersymmetry and technicolor will be put forth as candidates for explaining the observation. In addition, these theories have several sub-models and span a large parameter space.

Equation 1 should be useful for confronting competing models with the available data. For each model, the sum in equation 1 can be carried out in an extended fashion over all sub-channels of all experimental signatures corresponding to various hypothesized parent states in a theory. The grand $\sigma_{X,M}$ experimental sensitivity thus obtained can be compared to the model’s entire cross section for new physics production.

For example, one could sum over the $\{aB\}$, measurements from signatures such as trilepton, like-sign leptons, diphoton, N-jets, etc., using a particular SUSY model’s branching ratios for chargino-neutralino, squark-gluino and other experimentally accessible supersymmetric production processes. The sum can be further extended to include $\{aB\}$, measurements from multiple experiments such as CMS and ATLAS at the LHC. The total $\sigma_{X,M}$ thus obtained serves as the grand experimental measurement of the supersymmetry cross section predicted by the model under consideration. The very same underlying experimental $\{aB\}$ measurements can be simultaneously used to confront another model of new physics that may not involve supersymmetry.

XI. LAST WORDS

In this paper, we have formulated a recipe for the experimentalists to present trilepton search results in a model-independent way. The inherent problem of a vast parameter space in models of SUSY can be mitigated by categorizing the experimental sensitivity according to the $\tau$-lepton content and by expressing it as a function of the few mass parameters that decide the kinematics of the decay.

Using this method, we showed how to extend the applicability of the Tevatron trilepton results from their very limited mSUGRA-based focus. In doing so, we also attempt to bring uniformity to the disparate methods used by the experiments to interpret their SUSY trilepton searches. Our scheme may also be useful in addressing the broader “inverse problem” of pinpointing new physics if it is discovered in multiple channels.

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