Majorana Neutrinos in Muon Decay

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Abstract

The normal muon decay $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$ is studied as a tool to discriminate between the Dirac and Majorana type neutrinos. For the purpose to do this, we propose a new parameterization in place of the Michel parameters for the energy spectrum of $e^+$. The $\chi^2$-fitting is used by noticing different energy spectra between the Dirac and Majorana neutrino cases. We assume the interaction Hamiltonian which consists of the $V - A$ and $V + A$ charged currents.

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I. INTRODUCTION

Neutrinoless double beta decay \[1\] is a best subject for investigating whether the neutrino is of the Dirac or Majorana type. However, it needs to improve the present sensitivity of the detector for observing this decay mode, because of the tiny neutrino masses and/or the small contributions from the right-handed weak current. Under this circumstances, it seems to be meaningful as a complementary study to survey whether muon decay can be used as a tool to determine the type of the neutrino. Recently, we have proposed a new parameterization of the $e\pm$ energy spectrum of the muon decay that is suitable for discriminating between the types of neutrino \[2\]. It is shown that there is a possibility to take advantage of the different energy dependences for their energy spectra. We propose a method in which the $\chi^2$ value determined experimentally by assuming the Dirac-type neutrino is compared with the one determined for the Majorana-type neutrino. This may provide a test to determine the type of neutrino, although it is indirect.

II. GENERAL FRAMEWORK

We assume the following effective weak interaction Hamiltonian for the $\mu\pm$ decay \[3\]:

$$
\mathcal{H}_W(x) = \frac{G_F}{\sqrt{2}} \left\{ j^{\dagger}_e e_L^\alpha j^{\alpha}_{\mu L} + \lambda j^{\dagger}_{e R} j^{\alpha}_{\mu R} + \eta j^{\dagger}_{e R} j^{\alpha}_{e L} + \kappa j^{\dagger}_{e L} j^{\alpha}_{\mu R} \right\} + \text{H.c.},
$$

(2.1)

where $G_F$ is the Fermi coupling constant. This weak interaction is naturally expected from the gauge models that contain both $V - A$ and $V + A$ currents with the left- and right-handed weak gauge bosons. The appearance of the coupling constant $\lambda$ is mainly due to the right-handed weak gauge bosons $W_R$, while terms with $\eta$ and $\kappa$ come from the possible mixing between the left-handed and right-handed weak gauge bosons, $W_L$ and $W_R$. In the $SU(2)_L \times SU(2)_R \times U(1)$ gauge model, the coupling constants $\kappa$ and $\eta$ become identical. However, they are treated as independent constants in this note in order to allow comparison with the more general case without a restriction from the gauge theory (see, e.g., \[4\]).

The left-handed and right-handed charged weak leptonic currents, $j_{\ell L}$ and $j_{\ell R}$ with flavor $\ell = e$ and $\mu$, are expressed in terms of the mass eigenstates of charged leptons $E_\ell$ and neutrinos $N_j$ with mass $m_j$ as

$$
j_{\ell L} = \sum_{j=1}^{2n} E_\ell \gamma_5 (1 - \gamma_5) U_{\ell j} N_j, \quad j_{\ell R} = \sum_{j=1}^{2n} E_\ell \gamma_5 (1 + \gamma_5) V_{\ell j} N_j,
$$

(2.2)
for the case of the $n$ generations. Here $U_{\ell j}$ and $V_{\ell j}$ stand for the left-handed and right-handed lepton mixing matrices, respectively.

III. DIFFERENTIAL DECAY RATE FOR NORMAL MUON DECAY

The $\mu^\pm$ decay takes place as

$$\mu^\pm \rightarrow e^\pm + N_j + \overline{N}_k,$$

(3.1)

where $\overline{N}_k$ represents an antineutrino for the Dirac neutrino case, but it should be understood as $N_k$ for the Majorana neutrino case.

If the radiative corrections are not included, the differential decay rate for emitted positron in the rest frame of polarized $\mu^+$ is expressed as

$$\frac{d^2 \Gamma(\mu^+ \rightarrow e^+\nu\overline{\nu})}{dx d\cos \theta} = \left(\frac{m_{\mu} G_F^2 W^4}{6 \cdot 4 \pi^3}\right) A \sqrt{x^2 - x_0^2} D(x, \theta),$$

(3.2)

where

$$x = \frac{E}{W}, \quad x_0 = \frac{m_\mu}{W} = 9.65 \times 10^{-3}, \quad W = \frac{m_{\mu}^2 + m_e^2}{2 m_{\mu}} = 52.8 \text{ MeV}.$$  

(3.3)

Here $m_{\mu}$ and $m_e$ are the muon and electron masses, respectively, and $E$ is the energy of $e^+$. The angle $\theta$ represents the direction of the emitted $e^+$ with respect to the muon polarization vector $\vec{P}_\mu$ at the instant of the $\mu^+$ decay. The allowed range of $x$ is limited kinematically as

$$x_0 \leq x \leq x_{\text{max}} = (1 - r_{jk}^2) \simeq 1 \quad \text{with} \quad r_{jk}^2 = \frac{(m_j + m_k)^2}{2 m_{\mu} W}.$$  

(3.4)

Here $m_j$ and $m_k$ are masses of neutrinos emitted in the $\mu^+$ decay.

The constant $A$ in Eq. (3.2) is introduced to simplify the expression for the energy spectrum by taking the arbitrariness of its normalization. It is referred to as a normalization factor. There are various possibilities for the choice of $A$, when experimental data are analyzed, although these choices differ only by rearrangements of the terms in the theoretical expression. See Appendix A.

IV. ENERGY SPECTRUM OF POSITRON

We ignore some small terms proportional to both $m_e$ and neutrino masses ($m_j$ and $m_k$) in this note, for simplicity. Then, the $e^+$ energy spectrum part, $x D(x, \theta)$, is expressed as
\[ xD(x, \theta) = x[N(x) + P_\mu \cos \theta P(x)], \]  

where \( P_\mu = |\vec{P}_\mu| \) is the rate of muon polarization, and the isotropic part \( N(x) \) and anisotropic part \( P(x) \) are

\[
N(x) = \frac{1}{A} \left[ a_+(3x - 2x^2) + 12( k_{+c} + \varepsilon_m k_{+m}) x (1 - x) \right], \tag{4.2}
\]

\[
P(x) = \frac{1}{A} \left[ a_-( -x + 2x^2) + 12(k_{-c} + \varepsilon_m k_{-m}) x (1 - x) \right]. \tag{4.3}
\]

Here, the decay formulae for the Dirac and Majorana neutrinos are obtained by setting \( \varepsilon_m = 0 \) and \( \varepsilon_m = 1 \), respectively. The first terms with setting \( A = a_+ = a_- = 1 \) in these \( N(x) \) and \( P(x) \) correspond to the theoretical predictions from the standard model. The well-known Michel parameterization \[4\] is obtained if we choose the normalization factor \( A = A_{10} = a_+ + 2k_{+c} \).

V. COEFFICIENTS

In the Dirac neutrino case, the coefficients in Eqs. (4.2) and (4.3) are expressed as follows:

\[
a_\pm = (1 \pm \lambda^2) \quad \text{and} \quad k_{\pm c} = \left( \kappa^2 \pm \eta^2 \right)/2. \quad (5.1)
\]

Here it is assumed that all neutrinos can be emitted in the muon decay. Then, the unitarity relations for the lepton mixing matrices \( U \) and \( V \), namely, \( \Sigma_j |U_{\ell j}|^2 = \Sigma_j |V_{\ell j}|^2 = 1 \) have been used.

On the other hand, in the Majorana neutrino case, the coefficients are:

\[
a_\pm = \left[ (1 - \overline{u}_e^2) (1 - \overline{u}_\mu^2) \pm \lambda^2 \overline{v}_e^2 \overline{v}_\mu^2 \right],
\]

\[
k_{\pm c} = \left[ \kappa^2 (1 - \overline{u}_e^2) \overline{v}_\mu^2 \pm \eta^2 \overline{v}_e^2 (1 - \overline{u}_\mu^2) \right]/2,
\]

\[
k_{\pm m} = \left[ \kappa^2 |\overline{w}_{e\mu}|^2 \pm \eta^2 |\overline{w}_{e\mu h}|^2 \right]/2,
\]

where \( \overline{u}_\ell^2, \overline{v}_\ell^2, \overline{w}_{e\mu} \) and \( \overline{w}_{e\mu h} \) are small quantities. Note that \( k_{\pm c} \) has the same order of magnitude as \( k_{\pm m} \), in contrast to the Dirac neutrino case. In the Majorana neutrino case, we assume the existence of heavy Majorana neutrinos, which are not emitted in the muon decay. Then, we have \( \Sigma'_j |U_{\ell j}|^2 = 1 - \overline{u}_\ell^2 \) and \( \Sigma'_j |V_{\ell j}|^2 = \overline{v}_\ell^2 \) where the primed sums are taken over only the light neutrinos. In addition, the following products of \( U \) and \( V \) appear:

\[ \overline{w}_{e\mu} \equiv \Sigma'_j U_{e j} V_{\mu j} \quad \text{and} \quad \overline{w}_{e\mu h} \equiv \Sigma'_k V_{e k} U_{\mu k}. \]
VI. ENERGY SPECTRUM OF $e^+$ IN THE $\mu^+$ DECAY

We propose a new parameterization that directly represents deviations from the standard model. Namely, if we assume the $SU(2)_L \times SU(2)_R \times U(1)$ model for simplicity, our expression for the energy spectrum of $e^+$ becomes

$$xD(x, \theta) = x^2[(3 - 2x) + 2\rho_c(3 - 4x) + 12\varepsilon_m\rho_m(1 - x)]$$
$$+ x^2\mu\xi \cos \theta(-1 + 2x),$$ \hspace{1cm} (6.1)

where the parameters $\rho_c$, $\rho_m$, and $\xi$ are, respectively, given by

$$\rho_c = \frac{k_c}{A_{10}} > 0, \quad \rho_m = \frac{k_m}{A_{10}} > 0, \quad \text{and} \quad \xi = \frac{a_+ + 6k_c}{A_{10}},$$ \hspace{1cm} (6.2)

with the choice of normalization factor $A_{10} = a_+ + 2k_c$. Note that we have

$$\xi = \frac{1 - \lambda^2}{1 + \lambda^2 + 2\eta^2} \quad \text{for the Dirac neutrino case},$$ \hspace{1cm} (6.3)

$$\xi \simeq 1 \quad \text{for the Majorana neutrino case},$$ \hspace{1cm} (6.4)

$$\xi = 1 \quad \text{for the standard model}.$$ \hspace{1cm} (6.5)

A. Dirac type neutrino case

Since the well-known Michel parameter $\rho_M$ is related to our $\rho_c$ as $2\rho_c = (1 - \frac{4}{9}\rho_M)$, we have the following expression:

$$xD(x, \theta) = x^2[(3 - 2x) + 2\rho_c(3 - 4x) + P_\mu \xi \cos \theta(-1 + 2x)]$$
$$= 6x^2[(1 - x) + \frac{2}{9}\rho_M(4x - 3)] + P_\mu \xi \cos \theta x^2(-1 + 2x).$$ \hspace{1cm} (6.6)

The TWIST group [6] reported a precise experimental result

$$\rho_M = 0.75080 \pm 0.00032(\text{stat}) \pm 0.00097(\text{syst}) \pm 0.00023,$$ \hspace{1cm} (6.7)

by using angle-integrated spectrum derived from Eq.(6.7). The mean value is $\rho_M = 0.75080$, although $\rho_M < 0.75$ is satisfied within experimental uncertainty. Note that theoretical prediction from our model is $\rho_M = 0.75(1 - 2\rho_c) < 0.75$, because $\rho_c = \eta^2/(1 + \lambda^2 + 2\eta^2) > 0$. They determined this $\rho_M$ by minimizing the $\chi^2$ value.
B. Majorana type neutrino case

The following expression is derived for the Majorana type neutrino case:

\[ xD(x, \theta) = x^2 [(3 - 2x) + 2\rho_c(3 - 4x) + 12\rho_m(1 - x) + P_\mu \xi \cos \theta(-1 + 2x)]. \quad (6.9) \]

Theoretically we predict

\[ \rho_c \simeq \frac{1}{2} \left( \kappa^2 \bar{\nu}_\mu^2 + \eta^2 \bar{\nu}_e^2 \right) > 0, \quad \rho_m \simeq \frac{1}{2} \left( \kappa^2 |\omega_{e\mu}|^2 + \eta^2 |\omega_{ee\mu h}|^2 \right) > 0. \quad (6.10) \]

Therefore, there is a possibility to take advantage of the different \( x \) dependences of the terms including the parameters \( \rho_c \) and \( \rho_m \) in the energy spectrum, when \( \chi^2 \) value for the Dirac-type neutrino case is compared with the one for the Majorana-type neutrino case. This may provide a test to determine the type of neutrino, although it is indirect.

VII. SUMMARY: HOW TO DETERMINE THE TYPE OF NEUTRINO

There is a method that might make it possible to distinguish between two neutrino types. This method makes use of the different \( x \) dependences of the terms including the parameters \( \rho_c \) and \( \rho_m \) in the energy spectrum by comparing the \( \chi^2 \) values for the Dirac-type neutrino with those for the Majorana-type neutrino. Here we use the data with \( \theta = \pi/2 \), so that it may need new data-taking.

For example, suppose we analyze experimental data with \( \theta = \pi/2 \) by using

\[ xD(x, \theta = \pi/2) = x^2 [(3 - 2x) + 2\rho_c(3 - 4x) + 12\rho_m(1 - x)], \quad (7.1) \]
and obtain some \( \chi^2 \) value, say, \( \chi^2_m \) for the Majorana neutrino case. Then, suppose we repeat a similar analysis using

\[ xD(x, \theta = \pi/2) = x^2 [(3 - 2x) + 2\rho_c(3 - 4x)], \quad (7.2) \]
and thereby determine \( \chi^2_d \) for the Dirac neutrino case.

If \( \chi^2_m \) is much smaller than \( \chi^2_d \), we can conclude that there is a higher probability that neutrinos are of the Majorana type.

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APPENDIX A: NORMALIZATION FACTOR $A$ AND THE $\chi^2$-FITTING

We summarize the meaning of the freedom for the normalization factor $A$ in the $\chi^2$-fitting of the $e^+$ energy spectrum $xD(x)$.

First let us consider the case where $xD(x)$ consists of the leading term $f(x)$ and its small deviation term $g(x)$ with weight $w$, namely,

$$xD(x) = \frac{1}{A} [f(x) + w g(x)]. \quad (A1)$$

Then the unknown constant $w$ is determined experimentally by minimizing the $\chi^2_A$-value \[7\], that is,

$$\chi^2_A = \sum_k \left[ \frac{d(x_k) - y_{A,k}}{\sigma_k} \right]^2, \quad (A2)$$

where $x_k$ stands for the energy of the $k$-th positron observed by experiment, and $d(x_k)$ and $\sigma_k$ mean, respectively, the corresponding experimental value of spectrum and its experimental error. The theoretical expression is $y_{A,k} = f(x_k) + w g(x_k)$ in this case. The summation is taken over all observed data. This case is referred to as analysis A, which corresponds to the choice of the normalization factor $A = 1$.

Since we have $\chi^2_A = a_A w^2 - 2b_A w + c_A$ in this analysis A, the minimal value of $\chi^2_A$ and the corresponding value of $w$, respectively, are determined by

$$\chi^2_{A,\text{min}} = \left( c_A - \frac{b_A^2}{a_A} \right) \quad \text{and} \quad w_A = \left( \frac{b_A}{a_A} \right). \quad (A3)$$

Here $a_A$, $b_A$, and $c_A$ are defined by

$$a_A = \sum_k \left( \frac{g(x_k)}{\sigma_k} \right)^2, \quad b_A = \sum_k \left( \frac{g(x_k)}{\sigma_k} \right) \left( \frac{d(x_k) - f(x_k)}{\sigma_k} \right),$$

$$c_A = \sum_k \left( \frac{d(x_k) - f(x_k)}{\sigma_k} \right)^2. \quad (A4)$$

Next, let us consider the case where terms in $xD(x)$ are rearranged as follows,

$$xD(x) = \frac{1}{A} [f(x) + w g(x)] = \left( \frac{1 + w}{A} \right) \left\{ f(x) + \frac{w}{1 + w} [g(x) - f(x)] \right\}. \quad (A5)$$

Then we can determine $\left( \frac{w}{1 + w} \right)$ experimentally by minimizing the $\chi^2_B$-value,

$$\chi^2_B = \sum_k \left[ \frac{d(x_k) - y_{B,k}}{\sigma_k} \right]^2, \quad (A6)$$
where the theoretical expression is defined by

$$y_{B,k} = f(x_k) + \frac{w}{1+w} [g(x_k) - f(x_k)].$$  \hspace{1cm} (A7)

This case is referred to as analysis B, which corresponds to the choice of the normalization factor $A = (1+w)$. The minimal value of $\chi^2_B$ and the unknown constant $w$ in this analysis B become as follows

$$\chi^2_{B,\text{min}} = \left( c_B - \frac{b_B^2}{a_B} \right), \quad \text{and} \quad w_B = \left( \frac{b_B}{a_B - b_B} \right). \hspace{1cm} (A8)$$

Here $a_B, b_B$, and $c_B$ are defined by

$$a_B = \sum_k \left( \frac{g(x_k) - f(x_k)}{\sigma_k} \right)^2, \quad b_B = \sum_k \left( \frac{g(x_k) - f(x_k)}{\sigma_k} \right) \left( \frac{d(x_k) - f(x_k)}{\sigma_k} \right), \quad c_B = c_A. \hspace{1cm} (A9)$$

In general, the value of $w_B$ is different from $w_A$. Therefore, the final value of $w$ determined experimentally should be chosen as the $w_B$, if $\chi^2_{A,\text{min}} > \chi^2_{B,\text{min}}$, or vice versa. So, this means that there is freedom in the $\chi^2$-fitting such as the choice of the normalization factor $A$ in the analysis of $xD(x)$.

It is worthwhile to note that the $x$ dependence of $[d(x) - f(x)]$ plays an important role in the above procedure. That is, in the muon decay, it means the $x$ dependence of the difference between the experimental data and the prediction from the standard model. Let us consider the following imaginative example: If the $x$ dependence of $[d(x) - f(x)]$ happens to be proportional to that of $g(x)$, that is, $[d(x) - f(x)] = L_A g(x)$, then we have $\chi^2_{A,\text{min}} = 0$ and correspondingly $w_A = L_A$ in analysis A. In other words, we have the relation, $\chi^2_{A,\text{min}} \ll \chi^2_{B,\text{min}}$. While, if $[d(x) - f(x)] = L_B [g(x) - f(x)]$, then we have $\chi^2_{B,\text{min}} = 0$ and correspondingly $w_B = L_B/(1-L_B)$ in analysis B. Namely, we have the opposite conclusion, $\chi^2_{A,\text{min}} \gg \chi^2_{B,\text{min}}$.

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