Comment on the numerical solutions of a new coupled MKdV system (2008 Phys. Scr. 78 045008)

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Abstract

In this comment we point out some wrong statements in the paper by Inc and Cavlak (2008 Phys. Scr. 78 045008).

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In a recent paper appearing in this journal Inc and Cavlak [1] applied the Adomian decomposition method (ADM) and the variational iteration method (VIM) to a new coupled modified Korteweg–de Vries (MKdV) system. The authors state that ‘The methods provide the solution in a convergent series with components that are elegantly computed. The VIM and the decomposition method avoid the complexity provided by other pure numerical methods’. In what follows, we analyse the methods proposed by the authors and determine if this claim is true.

Inc and Cavlak [1] studied nonlinear partial differential equations of the form

$$u_t = f(u, u_x, u_{xx}, u_{xxx}, u_{xxxx}, \ldots),$$

(1)

where $u$ is a vector of components $u_1(x, t), u_2(x, t), \ldots, u_n(x, t)$, $f$ is a vector of nonlinear functions $f_1, f_2, \ldots, f_n$ and the subscripts $t$ and $x$ indicate differentiation with respect to these variables. Inc and Cavlak [1] chose a problem with an exact solution that is sufficiently simple to facilitate the application of both the ADM and VIM. It is the kind of tailor-made toy problem that is always selected for the application of such approaches.

Inc and Cavlak [1] applied the ADM and VIM in such a way that they merely obtained the time-power series for the solutions:

$$u(x, t) = \sum_{j=0}^{\infty} u_j(x) t^j.$$

(2)

To be precise, the ADM yielded the pure Taylor expansion at about $t = 0$ term by term and the VIM gave it in a rather mixed way, but it is expected that cancellation of terms in the summation of the contributions would give exactly the same series. In any case, it is most striking that the authors had resorted to more or less complicated methods to obtain a time series that one derives more easily and straightforwardly by simply substituting equation (2) into equation (1) and equating the coefficients of the polynomials in the lhs and rhs. In this way, one obtains a recurrence relation that completely determines the coefficients of the time series (2) provided that one knows $u(x, 0) = u_0(x)$. The authors did not indicate any advantage of those methods with respect to the well-known Taylor series. Since working harder to obtain the same results is just a matter of taste, we will not discuss this point any further. We just wanted to call the reader’s attention to it.

Inc and Cavlak [1] compared their time-power series with the exact solution for some values of $t$ and $x$ and concluded that ‘numerical approximations show a high degree of accuracy, and in most cases of $\phi_n$, the $n$-term approximation is accurate for quite low values of $n$. The proofs of the convergence were investigated by Cherruault and co-operator’ (and gave some references that are unnecessary for our purposes as we shall see below). Later they also stated that ‘The errors obtained by using the approximate solution are given by using only two iterations of the decomposition method. The error is smaller for values of $t$ close to the initial point 0. For values of $t$ away from 0, the error is decreasing (we believe that the authors meant increasing). However, the overall errors can be made even smaller by adding more iterates. The convergence is rapid’. Of course, the reader will not doubt that the accuracy of the Taylor expansion of the
solutions about $t = 0$ will decrease as we move away from the time of origin.

The exact solutions to the problem chosen by Inc and Cavlak [1] are:

\begin{align*}
u(x, t) &= 1 + \frac{1}{2} \tanh \left( x - \frac{11 t}{2} \right), \\
z(x, t) &= 2 - \tanh \left( x - \frac{11 t}{2} \right).
\end{align*}

(3)

Everybody knows that the \tanh(\theta) is singular at $\theta = (2j + 1)\pi i/2$, $j = 0, 1, \ldots$. From the singular point closest to the origin we determine that the convergence radius of the time-power series expansion will be $R(x) = (2/11)\sqrt{x^2 + \pi^2/4}$. Since this series will not converge for $t > R(x)$ we conclude that the authors’ statements quoted above cannot be true. Inc and Cavlak [1] showed results for $x = -15, -10, -5, 5, 10$ and $t = 0.1, 0.2, 0.3, 0.4, 0.5$. Notice that the smallest convergence radius $R(\pm 5) = 0.9528972974$ is considerably larger than the largest $t$-value in the authors’ tables. In other words, the pairs of $x, t$ values are conveniently chosen to support the authors’ conclusions. Obviously, the most unfavourable case is $R(0) = \pi/11 = 0.2855993321$. In our opinion there is no necessity for a numerical verification of present arguments. However, we have decided to add a graphical exemplification of them because of a negative experience with a referee regarding a similar criticism about a paper in another journal (see below).

Figure 1 shows $\tanh(11 t/2)$ (the relevant term when $x = 0$) and the Taylor series of degree 5 (the greatest order chosen by Inc and Cavlak [1]). We clearly appreciate how the accuracy of the series deteriorates as time approaches $R(0)$. Figure 1 also shows that increasing the degree of the Taylor series to order 15 does not do much to improve this behaviour. Clearly, increasing the order of the ADM or VIM will not correct the essential limitation of the approximations that in the end produce a time series. This obvious fact also contradicts the authors’ statements quoted above.

Throughout this comment we have tried to prove two points. Firstly, that the well-known Taylor-series expansion provides the same kind of results that Inc and Cavlak [1] obtained by the more complicated ADM and VIM. The reader may decide if the application of any of those elegant approximate methods is worth the extra effort. Secondly, the resulting series are suitable only in a neighbourhood of $t = 0$. The reason is that nonlinear equations spontaneously generate singular points of the type described above.

In the study of nonlinear systems one is primarily interested in their overall picture, namely the qualitative and quantitative long-time behaviour of their solutions. A power series can never provide such information. It is unlikely that one may be interested in what happens in the early times of the phenomenon. If that were the case one may try the Taylor expansion and improve it by means of, for example, Padé approximants. In fact, Padé approximants overcome the problem of the singular points discussed above.

We have raised this kind of criticism before [2–7] but some journals are unwilling to publish comments on some of the papers they publish. This journal seems to exhibit a different policy in this regard [8].

References

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