A POSSIBLE EXPLANATION WHY $\tau_{B^\pm} \sim \tau_{B^0}$ BUT $\tau_{D^\pm} \sim 2\tau_{D^0}$

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Abstract

Data show that $\tau_{B^\pm} \sim \tau_{B^0}$, but $\tau_{D^\pm} \sim 2\tau_{D^0}$. The naive interpretation which attributes $\tau_{D^\pm} \sim 2\tau_{D^0}$ to a destructive interference between two quark diagrams for $D^\pm$ decays, definitely fails in the $B$-case. We investigate Close and Lipkin’s suggestion that the phases for producing radially excited states $\psi_{2s}$ in the decay products of $B$-mesons can possess an opposite sign to the integrals for $\psi_{1s}$ decay products. Their contributions can partially compensate each other to result in $\tau_{B^\pm} \sim \tau_{B^0}$. Since $D$-mesons are much lighter than $B$-mesons, such possibilities do not exist in $D$-decays.

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I. Introduction.

The naive explanation for $\tau_{D^\pm} \sim 2\tau_{D^0}$ is that a destructive interference between two quark diagrams for $D^\pm$ reduces the strength of decay amplitudes and thereby elongates the life of $D^\pm$. More explicitly, if the lifetime of a meson is mainly determined by the Cabibbo favored decay modes, for $D^+$ there is only one topology $D^+ \rightarrow \bar{K}^0 M^+$, (where $M$ generically refers to $\pi, \rho$ etc and $K$ to strange mesons) whereas for $D^0$ there are two channels $D^0 \rightarrow \bar{K}^0 M^0$ and $D^0 \rightarrow K^- M^+$. For the $D^+$ decays, the two quark diagrams shown in Fig.1 (a) and (b) interfere, while for $D^0$, the two diagrams (c) and (d) correspond to two different modes, so do not interfere. For the $B$ decays, similar diagrams exist and there could be also destructive interference in $B^-$ decays. However, the experimental data show that $\tau_{B^\pm} \sim \tau_{B^0}$.

The explanation for the lifetime differences in D and B cases involves nonperturbative QCD phenomena. Actually some authors proposed the so-called Pauli Interference (PI) mechanism as a correction to “pure” spectator mechanism for taking into account of the light degrees of freedom. The PI effects only exist in $D^\pm$ and $B^\pm$ decays but not in $D^0$ and $B^0$ decays. Based on QCD, Bigi et al. introduced a virtual gluon so that one of the quark produced by the weak decay of the heavy quark interferes with the spectator quark. In this mechanism, the PI term modifies the “pure” spectator diagram and it is found that such interference is destructive and is proportional to $\Gamma_0/m_Q^2$ (Q=b or c). This mechanism partly explains why $\tau_{D^\pm} \sim 2\tau_{D^0}$ and $\tau_{B^\pm} \sim \tau_{B^0}$.

In the present work we try to investigate the lifetime differences in another way which is based on the idea of Close and Lipkin. Recently Close and Lipkin have analysed the data on low lying exclusive quasi-two body final states in both $D$ and $B$ decays. They noted that in $D$ decays the sign of interference in exclusive channels is still ambiguous while in $B$ decays there is a clear and uniform tendency towards constructive interference between the color favoured and colour suppressed exclusive channels where all final state mesons have nodeless wave functions. They noted that in $B$ decays, in order that $\tau_{B^\pm} \sim \tau_{B^0}$, this interference must be compensated in as
yet unmeasured channels. They suggested that the sign of interference may be changed in channels
where excited states of the decay products, whose wavefunctions contain nodes, are involved. It
is the motivation of the present paper to include the contributions from the excited states of the
B decay products so that constructive interference is obtained in $B^\pm$ decays. Such excited states
only exist in B decays but not in D decays because of phase space requirement. It will be shown
that in our model the lifetime differences in B and D mesons can also be explained.

The effective Hamiltonian of non-leptonic decays in the D-case \[1\] is
\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} [c_1 \bar{s}\gamma_\mu (1 - \gamma_5) c\bar{u}\gamma^\mu (1 - \gamma_5) d + c_2 \bar{s}\gamma_\mu (1 - \gamma_5) d\bar{u}\gamma_\mu (1 - \gamma_5) c],
\]
where $c_1 = \frac{c_1 + c_2}{2}$ and $c_2 = \frac{c_1 + -c_2}{2}$. By the renormalization group equation (RGE) we have
\[
c_- = \left( \frac{\alpha_s(m_b^2)}{\alpha_s(m_b^2)} \right)^{12/23} \left( \frac{\alpha_s(m_b^2)}{\alpha_s(M_W^2)} \right)^{12/23}, \quad c_+ = \frac{1}{\sqrt{c_-}}.
\]
With the Fiertz transformation, the coefficients $c_1$ and $c_2$ in eq. (1) should be replaced by $a_1$ and $a_2$ with
\[
a_1 = c_1 + \xi c_2, \quad \text{and} \quad a_2 = c_2 + \xi c_1,
\]
where $\xi$ is $1/N_c$ if the factorization assumption holds perfectly, otherwise $\xi = (1 + \delta)/N_c$ where $\delta$
denotes a color-octet contribution proportional to $<\lambda^a \lambda^a>$. Recently, Blok and Shifman
gave a more theoretical estimation \[10\], but they also pointed out that the obtained value is not
accurate for practical calculations. Generally, $\delta$ is a negative number ranged between 0 to $-1$, so
that $\xi$ takes values between 0 to $1/N_c$. Later, in our numerical calculations we will take $\delta$ as 0,
$-0.5$ and $-1$ respectively.

For the B-case, we have a similar Hamiltonian as eq. (1)
\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} [c_1^{(B)} \bar{c}\gamma_\mu (1 - \gamma_5) b\bar{d}\gamma^\mu (1 - \gamma_5) u + c_2^{(B)} \bar{c}\gamma_\mu (1 - \gamma_5) u\bar{d}\gamma_\mu (1 - \gamma_5) b] \tag{4}
\]
and coefficients $c_1^{(B)}$ and $c_2^{(B)}$
\[
c_1^{(B)} = \left( \frac{\alpha_s(m_b^2)}{\alpha_s(M_W^2)} \right)^{12/23}, \quad c_2^{(B)} = \frac{1}{\sqrt{c_1^{(B)}}}
\]
whereas $a_1^{(B)}$, $a_2^{(B)}$ have similar forms in analog to that for the charm case.

It is noted that in the case of D meson decays $a_1$ is positive and and $a_2$ is negative. From
the data of D-physics the value of $a_2$ is about $-0.5$ \[1\]. In $D^+$ decays, $a_1$ term corresponds to
the external W-emission while \( a_2 \) to the internal W-emission, naturally a destructive interference would occur between the two quark diagrams.

In the following we will express the corresponding transition amplitudes as \( A_1 \) and \( A_2 \) which are proportional to \( a_1 \) and \( a_2 \) respectively, thus \( A_1 = \kappa_1 a_1 \) and \( A_2 = \kappa_2 a_2 \) where \( \kappa_1 \) and \( \kappa_2 \) are the hadronic transition matrix elements.

Then we have the amplitude square as,

\[
|<\bar{K}^0\pi^+|H_{eff}|D^+>|^2 = |A_1|^2 + |A_2|^2 + 2\text{Re}(A_1 A_2^*). \tag{6}
\]

Beside a common phase factor such as the Cabibbo-Kabayashi-Maskawa phase, both \( A_1 \) and \( A_2 \) are real. Thus if \( A_1 \cdot A_2 \) is negative, this is a destructive interference. Otherwise we have constructive interference.

In contrast, for \( D^0 \) decays,

\[
< K^-\pi^+|H_{eff}|D^0> \propto a_1 \quad \text{and} \quad < \bar{K}^0\pi^0|H_{eff}|D^0> \propto a_2.
\]

We can roughly assume

\[
< K^-\pi^+|H_{eff}|D^0> \approx A_1 \quad \text{and} \quad < \bar{K}^0\pi^0|H_{eff}|D^0> \approx A_2.
\]

Thus if we only consider the C-K-M favored channels which dominate the lifetime of D-mesons, we have

\[
\Gamma(D^+) = (|A_1|^2 + |A_2|^2 + 2\text{Re}(A_1 A_2^*)) \times \text{LIPS} \tag{7}
\]
\[
\Gamma(D^0) = (|A_1|^2 + |A_2|^2) \times \text{LIPS}, \tag{8}
\]

where LIPS is the Lorentz-invariant-phase-space of the final products. If \( A_2 \sim -0.26 A_1 \), one can numerically obtain \( \Gamma(D^0) \sim 2\Gamma(D^+) \) (or \( \tau(D^+) \sim 2\tau(D^0) \)). Of course, the other channels (Cabibbo-suppressed) and semi-leptonic decays all contribute to the lifetime, so this obtained number is not rigorous. However, since the Cabibbo favored channels dominate, one can expect that a solution for \( A_1 \) and \( A_2 \) does not deviate much from the aforementioned value.

Taking \( \alpha_s(M_Z^2) = 0.118 \) [1], one can obtain a ratio of \( A_1/A_2 \) for D-decays to be roughly consistent with the required value. By our recent knowledge, the hadronic matrix elements can be evaluated more easily in terms of the heavy quark effective theory (HQET) [2].
In the same scenario and by eqs. (4), (5) $a_1$ and $a_2$ are still of opposite signs in B-decays. It is a consequence of renormalization group equation which is proved to be valid for perturbative QCD. If so, one could expect a result similar to the D-case that $\tau_{B^-} \sim 2\tau_{B^0}$. However, this does not coincide with the data for B-decays.

The $B^{(\pm)}$ lifetime is very close to that of $B^0$ as $\tau_{B^{(\pm)}} \sim (1.62 \pm 0.06) \times 10^{-12}$ s and $\tau_{B^0} \sim (1.56 \pm 0.06) \times 10^{-12}$ s [1]. There could be small measurement uncertainty as $\tau_{(B^{\pm})} \sim 1.47 \times 10^{-12}$ s, $\tau_{(B^0)} \sim 1.25 \times 10^{-12}$ s, by the ALEPH collaboration [13][15] and $\tau_{(B^{\pm})} \sim 1.72 \times 10^{-12}$ s, $\tau_{(B^0)} \sim 1.63 \times 10^{-12}$ s, by the DELPHI collaboration [14].

Similar quark diagrams exist in B-decays, namely there are both external and internal W-emissions for $B^- \rightarrow D^0\pi^-$ which destructively interfere, but for $B^0, B^0 \rightarrow D^+\pi^-$ and $B^0 \rightarrow D^0\pi^0$ corresponding to external and internal W-emissions respectively do not interfere. Thus if that is the case, one would wonder why $\tau_{B^{\pm}}$ is so close to $\tau_{B^0}$.

To fit the data of B decays, one needs to take a positive value for $a_2$ [11]. This contradicts to the result of RGE which is obviously correct by the perturbative QCD theory and there is no doubt of application of perturbative QCD at the $m_b$ energy region.

However, one can notice that even though $A_1, A_2$ are proportional to $a_1, a_2$ respectively, they also possess certain factors corresponding to the hadronic matrix elements. These hadronic matrix elements involve some overlapping integrations of the decay parent and daughter wavefunctions. If the integrations can contribute a negative sign, the interference between two diagrams would turn over to be constructive and it may be equivalent to an "effective" positive $a_2$ value.

The hadronization process is very non-perturbative and we cannot evaluate it accurately, so that we attribute the non-perturbative effects into the parameters of meson wavefunctions which exist in the overlapping integration. To evaluate such overlapping integrations, one needs to invoke some concrete models and later we employ the non-relativistic quark model. Since the decaying B-meson is a pseudoscalar at $\psi_{1s}$ radial ground state, if the decay product is at $\psi_{1s}$ state, the overlapping integration would certainly be positive, however, if the decay products can be radially excited states $\psi_{2s}$, the integration can turn sign (see next section for details). Because D-meson is much lighter than B-meson, it does not seem to exist $\psi_{2s}$ states as decay products of D, but
definitely there should be $\psi_{2s}$ excited states showing up as decay products of B-meson. This change may modify the whole picture and finally leads to a consequence that $\tau_{B^{(')}s}} \sim \tau_{B^0}$. Later our numerical results will show that the involvement of the $\psi_{2s}$ decay products can indeed do the job.

In next section, we give the formulation in every detail and in Sec.III, we present our numerical results while the last section is devoted to conclusion and discussion.

II. Formulation

(i) The transition amplitudes.

As usual, we ignore the W-exchange and annihilation diagrams because the two fast quarks would pick up a quark-pair from vacuum and speed them up [8]. Even though the factorization approach is not very reliable to evaluate the internal W-emission diagrams, we may use a phenomenological parameter $\delta$ to compensate it. Therefore by the vacuum saturation

$$\langle K^-\pi^+|a_1(\bar{s}c)(\bar{u}d) + a_2(\bar{s}d)(\bar{u}c)|D^0 \rangle = a_1 \langle \pi^+|(\bar{u}d)|0 > + a_2 \langle K^-|(\bar{s}d)|0 > \rangle$$

$$a_1 f_\pi p_\pi^{\mu} < K^-|(\bar{s}c)|D^0 > + a_2 f_D p_D^{\mu} < K^-|(\bar{s}d)|0 >$$

(9)

where $(\bar{q}q') \equiv \bar{q}\gamma_\mu(1 - \gamma_5)q'$. The second term corresponds to an W-annihilation diagram and obviously is much smaller than the first one as it is proportional to $f_D(m_K^2 - m_\pi^2)$. As argued in literatures this term is negligible and we will omit such contributions in later calculations. Then we also have

$$\langle K^0\pi^0|a_1(\bar{s}c)(\bar{u}d) + a_2(\bar{s}d)(\bar{u}c)|D^0 \rangle = a_2 f_K p_K^{\mu} < \pi^0|(\bar{u}c)|D^0 >,$$  \hspace{1cm} (10)

and

$$\langle K^0\pi^+|a_1(\bar{s}c)(\bar{u}d) + a_2(\bar{s}d)(\bar{u}c)|D^+ \rangle = a_1 f_\pi p_\pi^{\mu} < K^0|\bar{s}c)|D^+ > + a_2 f_K p_K^{\mu} < \pi^+|(\bar{u}c)|D^+ >.$$

(11)

Instead, for $P \rightarrow PV$

$$\langle K^-\rho^+|a_1(\bar{s}c)(\bar{u}d) + a_2(\bar{s}d)(\bar{u}c)|D^0 \rangle = a_1 f_\rho m_\rho \epsilon^{\ast \mu} < K^-|(\bar{s}c)|D^0 >,$$

(12)
The above formulae indicate that the external and internal W-emissions in $D^+$ decays interfere.

For the B-case, we can have similar expressions with an effective hamiltonian eq.(4) and corresponding coefficients $a_1^B$, $a_2^B$ in eq.(3).

(ii) The matrix elements.

It is noted that in the scenario of factorization, the hadronic matrix elements are related to a weak transition [16], for $P \to P$

$$< K^{-}\pi^+|a_1(\bar{s}c)(\bar{u}d) + a_2(\bar{s}d)(\bar{u}c)|D^0 > = a_1 f_\pi p_\pi^0 < K^{-}|(\bar{s}c)|D^0 >, \quad (13)$$

$$< \bar{K}^0\pi^0|a_1(\bar{s}c)(\bar{u}d) + a_2(\bar{s}d)(\bar{u}c)|D^0 > = a_2 f_{K^*} m_{K^*} \epsilon^* \mu < \pi^0|(\bar{u}c)|D^0 >, \quad (14)$$

$$< \bar{K}^0\rho^0|a_1(\bar{s}c)(\bar{u}d) + a_2(\bar{s}d)(\bar{u}c)|D^0 > = a_2 f_{K\rho} p_{K\rho}^0 < \rho^0|(\bar{u}c)|D^0 >, \quad (15)$$

and

$$< \bar{K}^0\pi^+|a_1(\bar{s}c)(\bar{u}d) + a_2(\bar{s}d)(\bar{u}c)|D^+ > = a_1 f_\pi p_\pi^0 < \bar{K}^0\pi^+|(\bar{s}c)|D^+ > + \quad (16)$$

$$a_2 f_{K^*} \epsilon^* \mu \epsilon_{K^*} < \pi^+|(\bar{u}c)|D^+ >,$$

$$< \bar{K}^0\rho^+|a_1(\bar{s}c)(\bar{u}d) + a_2(\bar{s}d)(\bar{u}c)|D^+ > = a_1 f_{\rho} m_{\rho} \epsilon^* \mu < \bar{K}^0\rho^+|(\bar{s}c)|D^+ > + \quad (17)$$

$$a_2 f_{K\rho} p_{K\rho}^0 < \rho^+|(\bar{u}c)|D^+ >.$$
\[ i\{\epsilon^*_\mu(M_I + M_{X\ast})A_1(q^2) - (\frac{\epsilon^* \cdot q}{M_I + M_{X\ast}})(P_I + P_{X\ast}) \}_{\mu}A_2(q^2) - \]
\[ \frac{\epsilon^* \cdot q}{q^2} 2M_{X\ast} q_{\mu} A_3(q^2) \}, \]  
(22)

with \( A_3(0) = A_0(0) \) and here

\[ A_3(q^2) = \frac{M_I + M_{X\ast}}{2M_{X\ast}} A_1(q^2) - \frac{M_I - M_{X\ast}}{2M_{X\ast}} A_2(q^2). \]  
(23)

So our task is to calculate the form factors. Taking the nearest pole approximation

\[ F_1(q^2) \approx \frac{h_1}{1 - q^2/M_1^2} \quad \text{for} \quad P_I \rightarrow P_X, \]  
(24)

\[ V(q^2) \approx \frac{h_V}{1 - q^2/M_2^2}, \quad A_0(q^2) = \frac{h_{A_0}}{1 - q^2/M_3^2} \quad \text{for} \quad P_I \rightarrow P_{X\ast}, \]  
(25)

where \( M_1, M_2, M_3 \) are masses of mesons corresponding to the nearest poles which can be found in the data book. With this approximation, to evaluate the form factors, one only needs to calculate the constant parameters \( h_0 = h_1, h_V, h_{A_1}, h_{A_2} \) and \( h_{A_3} = h_{A_0} \), which turn out to be the values of the form factors at the unphysical kinematic region \( q^2 = 0 \) and we will use the non-relativistic quark model to calculate them. Moreover, for the case of a pseudoscalar B or D meson transiting to a vector meson, we use the helicity amplitude method \[17\] which can much simplify our calculations.

The parameters are related to an overlapping integral over the wavefunctions of initial pseudoscalar and final pseudoscalar or vector mesons. To carry out the integration, one needs to invoke concrete models and the most popular one is to take the wavefunction of harmonic oscillation potential as the orbital part. In the Bauer-Stech-Wirbel approach \[16\] the following wavefunction model is employed

\[ R_m(p_T, x) = N_m \sqrt{x(1-x)} \exp(-p_T^2/2\omega^2) \cdot \exp(-m_2^2/(2a^2)(x - 1)) \exp(-m_2^2/(2a^2)(x - 2m_2^2/(2m_2^2))) \]  
(26)

where \( N_m \) is the normalization factor while Guo and Huang \[16\] used the following wavefunction form in the light-cone formalism

\[ R_m(x, k_{pert}) = A \exp(-b^2(\frac{k_2^2 + m_2^2}{x_1} + \frac{k_1^2 + m_1^2}{x_2})). \]  
(27)

These wavefunctions apply in the infinite-momentum frame. Here instead, we choose the wavefunction at the rest frame of the decaying meson \[18\]. Everything in the picture is non-relativistic, but it is accurate enough for the qualitative conclusion and we will discuss it in the final section.
Here we only list the radial wavefunctions of $\psi_{1s}$ and $\psi_{2s}$ and the others can be found in ref. [15]

$$\psi_{1s} = \left(\frac{4\beta^3}{\sqrt{\pi}}\right)^{1/2}\exp\left(-\frac{1}{2}\beta^2 r^2\right)\sqrt{m}Y_{00}(\theta, \phi), \quad (28)$$

where adding a factor $\sqrt{m}$ is for proper normalization, and

$$\psi_{2s} = \left(\frac{4\beta^3}{6\sqrt{\pi}}\right)^{1/2}(3 - 2\beta^2 r^2)\exp\left(-\frac{1}{2}\beta^2 r^2\right)Y_{00}(\theta, \phi)\sqrt{m}, \quad (29)$$

where $\beta$ is the only free parameter to be fixed by data and here $r \equiv |\vec{r}_1 - \vec{r}_2|$ in the potential picture. To convert into the momentum-space, we have

$$h_1 = h_0 = \frac{2m_I}{m_I - M_X^2} \cdot \int d^3p_{||} \phi_X^*(\vec{p}_{||})\phi_I(\vec{p}_I)\left(\frac{p_{I0}}{p_{I0}^0 + p_{I1}} + \frac{p_{I1}}{p_{I1}^0 + p_{I1}}\right)\sqrt{(p_{I0}^0 + m_I)(p_{I0}^0 + m_{I1})} \quad (30)$$

and

$$h_V = \frac{i}{m_I + m_X^*} \int d^3p_{||} \phi_X^*(\vec{p}_{||})\phi_I(\vec{p}_I)\left(\frac{p_{I0}^2}{p_{I0}^2 + m_{I1}} - \frac{p_{I0}^2}{p_{I0}^2 + m_{I1}}\right)\sqrt{(p_{I0}^2 + m_{I1})(p_{I0}^2 + m_{I1})}, \quad (31)$$

$$h_{A_1} = \frac{i}{m_I + m_X^*} \int d^3p_{||} \phi_X^*(\vec{p}_{||})\phi_I(\vec{p}_I) \times \left(1 - \frac{p_{I0}^3 p_{I1}^3}{(p_{I0}^2 + m_{I1})(p_{I0}^2 + m_{I1})}\right)\sqrt{(p_{I0}^2 + m_{I1})(p_{I0}^2 + m_{I1})}, \quad (32)$$

$$h_{A_2} = \frac{2(m_I + m_X^*)^2}{3m_{I0}^2 + m_{X^*}^2} h_{A_1} - \frac{i4m_I m_X^*}{m_I - m_{X^*} + (m_{I0}^2 + m_{X^*}^2)} \int d^3p_{||} \phi_X^*(\vec{p}_{||})\phi_I(\vec{p}_I)\left(\frac{p_{I0}^2}{p_{I0}^0 + m_{I1}} + \frac{p_{I1}^3}{p_{I1}^0 + m_{I1}}\right)\sqrt{(p_{I0}^2 + m_{I1})(p_{I0}^2 + m_{I1})}, \quad (33)$$

where the $\phi_{(X,X^*)}$ are wavefunctions of $\psi_{(1s,2s)}$ in the momentum space, i.e. the Fourier transformed (28) and (29), in the expressions, $\vec{p}_I$ and $\vec{p}_{I0}$ denote the 3-momenta of the quarks which take part in the reaction in the initial and final mesons, while $m_{I1}$, $m_{I1}$ are their masses respectively. $p_{I0}$ and $p_{I1}$ correspond to the third and the zero-th components of the concerned 4-momenta. In the helicity-coupling picture, all momenta of the mesons are along $\hat{z}$, so

$$p_{I} \equiv |\vec{p}_{I}|, \quad p_{X(X^*)}^3 \equiv \pm |\vec{p}_{X(X^*)}|,$$

but the quark momenta can be along any directions. In the CM frame of the decaying meson $\vec{p}_I = 0$ and $|\vec{p}_{X(X^*)}| = \frac{m_{I}^2 - m_{X(X^*)}^2}{2m_I}$ as $q^2 = 0$, thus one has

$$p_1 + p_2 \equiv p_I = (M, \vec{0}), \quad \text{and} \quad p_{I0} + p_{I1} \equiv p_{X(X^*)} = (p_{X(X^*)}^0, 0, p_{X(X^*)}^3).$$
The resultant formulae look quite different from that given in ref. [16], but as a matter of fact, as $|\vec{p}| \gg M$, they coincide with each other.

Substituting all the information into eqs. (21) and (22), we can have the final numerical results.

III. The numerical results

In the whole calculations, only $\beta$ is a free parameter and one can fix it by the energy-minimum condition

$$\frac{\partial E}{\partial \beta} = \frac{\partial <H>}{\beta} = 0.$$ \hspace{1cm} (34)

Then one obtains

$$\beta_{1s} = \left(\frac{4\mu}{3\sqrt{\pi a^2}}\right)^{1/3} \hspace{1cm} (34)$$
$$\beta_{2s} = \left(\frac{6\mu}{7\sqrt{\pi a^2}}\right)^{1/3}, \hspace{1cm} (35)$$

where $\mu$ is the reduced mass and $a$ is an average radius of the meson. There can be an uncertainty for $a$ and $\mu$, it does not affect our qualitative conclusion even though indeed the numerical results can be declined by a few ten percents. (see below).

Even though $f_D$ is not well measured yet, there are some reasonable estimated values, so we take $f_D = 0.15 \text{ GeV}$ and $f_B = 0.125 \text{ GeV}$. Numerically we use

$$f_\pi = 0.132, \ f_K = 0.161, \ f_\rho = 0.212, \ f_{K^*} = 0.221$$

in GeV.

By the well-measured value $\alpha_s(M_Z^2) = 0.118$, we have $\alpha_s(m_b = 5 \text{ GeV}) = 0.203$, $\alpha_s(m_c = 1.5 \text{ GeV}) = 0.265$, and

$$c_1^{(D)} = 1.26, \ c_2^{(D)} = -0.51$$
$$c_1^{(B)} = 1.10, \ c_2^{(B)} = -0.23.$$ 

Our result is fully consistent with ref. [23] obtained in terms of RGE.

It is also noted that since $m_c$ is not very large, one can expect, the real values of $c_{1,2}^{(D)}$ may deviate from that predicted by the perturbative QCD calculation, for example, it is claimed that
a set of $c_1^{(D)} = 1.26 \pm 0.04$ and $c_2^{(D)} = -0.51 \pm 0.05$ can fit data better. However, in below we will rely on the perturbative QCD and use the values obtained by RGE.

The corresponding $a_1^{(B,D)}$ and $a_2^{(B,D)}$ would depend on $\xi$ of eq.(3).

For the radially excited $\psi_{2s}$ states, we will take $M_D(2s) \approx 2.4$ GeV and $M_{\pi}(2s) = 1.0$ GeV, $M_K(2s) = 1.4$ GeV. By eq.(34), we fix

$$\beta_B = 0.5, \beta_D(1s) = 0.45, \beta_D(2s) = 0.39, \beta_{\pi}(1s) = 0.3, \beta_{\pi}(2s) = 0.26, \beta_K(1s) = 0.4, \beta_K(2s) = 0.34$$
in GeV. All the parameters are obtained according to eq.(34) and (35).

Numerically, we have

$$\frac{\Gamma_{D^+}}{\Gamma_{D^0}} = \begin{cases} 
0.9, & \delta = 0 \\
0.70, & \delta = -0.5 \\
0.56, & \delta = -1.0 
\end{cases}$$

(36)

It seems that the $\delta = -1$ solution suits the data on D-decays better than other $\delta$ values and this conclusion was also predicted by Stech et al. a long while ago [24].

For the B-case, without considering the $\psi_{2s}$ excited state contribution, we have

$$\frac{\Gamma_{B^-}}{\Gamma_{B^0}} = \begin{cases} 
1.28, & \delta = 0 \\
0.90, & \delta = -0.5 \\
0.59, & \delta = -1.0 
\end{cases}$$

(37)

If one looks at $\delta = -1$ which is consistent with that obtained in D-decays, the ratio is close to 0.5 as expected (see the introduction). When we take into account the contributions from the $\psi_{2s}$ excited states, the whole result is modified as

$$\frac{\Gamma_{B^-}}{\Gamma_{B^0}} = \begin{cases} 
1.02, & \delta = 0 \\
0.99, & \delta = -0.5 \\
0.98 & \delta = -1.0 
\end{cases}$$

(38)

this result is very consistent with the data on the lifetimes of both D and B-mesons. We will discuss this result in next section.

**IV. Conclusion and discussion**

B and D mesons all contain a heavy quark and a light one, we have every reason to believe that they have similar characteristics. Indeed a symmetry between b and c quarks (B and D mesons) [12] is confirmed by phenomenology. However, one obvious discrepancy that $\tau_{D^\pm} \sim 2\tau_{D^0}$ while $\tau_{B^\pm} \sim \tau_{B^0}$ implies some distinction between B and D mesons.
There have been alternative ways to interpret the lifetime difference of B and D. For example, Bander, Silverman and Soni [21] suggested that the reaction $D^0 \rightarrow s + \bar{d} + \text{gluon}$ as a source for the difference in the lifetimes of $D^0$ and $D^\pm$ and in other way, one can suppose that the factorization factor $\delta$ can be different for B and D or the signs of $a_2$ can change etc. However, if we consider similarities between B and D, it is natural to accept an assumption that $\delta$ would not be too declined in B and D cases. In literature [24], in D-physics, $\delta$ is very close to $-1$ and our results confirm this allegation. Cheng found [22] that $r_2 = -0.67, -(0.9 - 1.1)$ for $D \rightarrow \bar{K}\pi, \bar{K}^*\pi$ respectively, where our $\delta = (N_c/2)r_2$, it indicates that $\delta \sim -1$. But to fit B-decay data, Cheng concluded $r_2 = +0.36$ which drastically deviates from the parameter for D-decays, so one would ask how it could be so?

Instead, we accept the assumption that a symmetry between $b$ and $c$ holds and $c_1^{(D,B)}, c_2^{(D,B)}$ can be derived with the RGE. Meanwhile we also notice that since B-mesons are much heavier than D-mesons, there can be radially excited states $\psi_{2s}^D$ and $\psi_{2s}^\pi$ as decay products in B-decays, but not for D-decays. The $\psi_{2s}$ states may cause the hadronic matrix elements to be in opposite sign to the $\psi_{1s}$ final states and it would result in a change to make $\tau_{B^\pm} \sim \tau_{B^0}$. Obviously, it is determined by an overlapping integral between wavefunctions of the final and initial mesons. Our numerical results show that the integrals for $\psi_{2s}$ and $\psi_{1s}$ can have opposite signs depending on the parameter $\beta$. Our $\beta$-values are reasonably determined by data, even though not very accurate. We show that as $\delta \sim -1$, as taking into account the contribution from $\psi_{2s}^{D,\pi}$ as well as $\psi_{1s}^{D,\pi}$, approximately

$$\tau_{B^\pm} \sim \tau_{B^0}, \quad \tau_{D^\pm} \sim 2\tau_{D^0}.$$ 

Our mechanism is in parallel to the PI effects discussed by some authors [3,4]. It is based on the common knowledge that as long as all the exclusive channels (in fact, the main ones) are summed up, the total width should be obtained, i.e. equivalent to the inclusive evaluation. Thus in our picture an interference between the decay products of the $b$ ($c$) quark and the light one is automatically considered via the $a_1$ and $a_2$ interference.

Since, indeed, we only consider the most Cabibbo favorable channels to estimate the lifetimes, there can be contributions from the rare decays and the numerical results can deviate a bit, but in general the same mechanism proposed by Close and Lipkin can apply. Hence the rule is the same to all channels, namely $\psi_{2s}$ always contributes as well as $\psi_{1s}$, our results seem sufficiently
convincing. As a matter of fact, the $\psi_{2s}$ is still light enough and there is large phase space available for B, but on contrary, not for D-meson.

For evaluating the hadronic matrix elements, we use the non-relativistic quark model. Even though the model is approximate, our qualitative conclusion does not change.

Surely, we can make the ratios of lifetimes for D and B mesons perfectly coincide with the data by carefully adjusting the $\beta$ values in the wavefunctions. However, since there are many uncertain factors such as the contributions of the rare decays, non-relativistic form of the wavefunctions and the factorization factor $\delta$ etc, which make a very accurate evaluation impossible, so only adjusting $\beta$ value to fit data seems not necessary. In fact, as the most important point, one can draw a qualitative conclusion confidently that the the contribution of $\psi_{2s}$ is important to B-decays, namely the puzzle of the lifetimes of B and D mesons can be reasonably explained away by its participation.

It is important to notice that not only the lifetimes of B-meson is in contrary to our knowledge based on the perturbative QCD and D-physics if the $\psi_{2s}$ contributions is not taken into account, but also similar puzzles exist at many channels of B-meson decays. It is that the value of $a_2$ is not universal and its sign is also uncertain. It is hard to understand. So we hope that by taking into account of the $\psi_{2s}$ contributions, all the discrepancies may get a reasonable explanation. Because the relatively heavy $\psi_{2s}$ is still light to B-meson and does not affect its phase space integration very much, so maybe in measurements of exclusive channels, certain $\psi_{2s}$ with the same quantum numbers as $\psi_{1s}$ gets mixed in and is not well tagged out. It causes the superficial discrepancy. To carefully and thoroughly investigate the influence and effects of possible $\psi_{2s}$ decay products in B-decays is the goal of our next works.

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**Figure Captions**

Fig.1(a)-(d). The quark diagrams for the non-leptonic decays of B and D mesons (here we take \( D \rightarrow K\pi \) as an example).

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Fig. 1(a)
Fig. 1(b)
Fig. 1(c)
Fig. 1(d)