Superconducting fluctuations in the Luther-Emery liquid

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The single-particle superconducting Green’s functions of a Luther-Emery liquid is computed by bosonization techniques. Using a formulation introduced by Poilblanc and Scalapino [Phys. Rev. B 66, 052513 (2002)], an asymptotic expression of the superconducting gap is deduced in the long wavelength and small frequency limit. Due to superconducting phase fluctuations, the gap exhibits as a function of size $L$ a $(1/L)^{1/2K_p}$ power-law decay as well as an interesting singularity at the spectral gap energy. Similarities and differences with the 2-leg t-J ladder are outlined.

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relevant and opens a spin gap $\mathcal{M}$. The excitations above the ground state are known \cite{14} to consist only of massive solitons (spinons) carrying the spin $\pm 1/2$ and charge zero, in agreement with the exact solution of the lattice Hubbard model \cite{14}. It is important to note that with the usual definition of the spin gap $\Delta$, the as the gap between the ground state and the lowest triplet excited state, $\Delta \approx 2\mathcal{M}$. The fermion operators can be decomposed as $e^{i\mathbf{p}\cdot \mathbf{r}}\hat{\psi}_x(x) = e^{i\mathbf{k}\cdot \mathbf{r}}\hat{\phi}_x(x)$, where $\phi_x(x)$ is the single particle gap. These expressions (8), the momentum and Matsubara-frequency dependence of the gap is given by

$$\Delta_{SC}(q, \omega_n) = C\mathcal{M}(\frac{2\pi \xi}{L})^{2\alpha} \frac{2F_1(\frac{3}{2} - \eta', \frac{3}{2} - \eta'; 1; y)}{2F_1(1 - \eta, \frac{3}{2} - \eta; 2; y)},$$

where $\eta = -(Q\xi)^2$, $Q^2 = (q - k_F)^2 + (\omega_n)^2/v^2$, $2F_1$ is the hypergeometric function \cite{21} and $C = \Gamma(\frac{3}{2} - 2\eta)/\Gamma(2 - 2\eta)$. The approximate expression \cite{9} although a long-wavelength limit still applies up to momentum $Q \sim 2\pi/\xi$. Note that $(Q\alpha)^2$ terms are dropped since $\xi \gg \alpha$. $\Delta_{SC}$ is maximum (minimum) at $Q = 0$ (i.e. at the Fermi momentum) for $k_F < 1.57$ ($k_F > 1.57$) and shows only a very moderate dependence in $Q$ as shown in Fig. 1. For $Q \to 0$, $\Delta_{SC}(q, \omega_n) \approx \Delta_{SC}(0,0) (1 - C'(Q\xi)^2)$, where $C' = \frac{1}{\xi^2} \left[ \frac{1}{2} K^2 - (1/3L) \right]$ decreases for increasing SC correlations i.e. for increasing $K_F$, e.g. $C' \approx 0.674, 0.188$ and 0.016 for $k_F = 0.5$ (charge density wave regime), 1 and 1.5 (superconducting regime), respectively.

The real-frequency expression of the gap is obtained by the analytic continuation $(\omega_n)^2 \to -(\omega + i\varepsilon)^2$ in Eq. 9. For $|\omega| > \mathcal{M}$, the hypergeometric functions have branch cuts, leading to a nonzero imaginary part in $\Delta_{SC}(0, \omega)$ as can be seen on Fig. 2(b). Above the threshold, the gap function presents a singular behavior: $\Delta(0, \omega) \sim \Delta(0, 0)(1 - (\omega + i\varepsilon)^2)^{-\frac{1}{2\pi \rho}}$. The divergence at $\omega = \mathcal{M}$ is in fact cut-off once $|\omega - \mathcal{M}| < 1/L$ since we are really dealing with a system of finite size, so that $\Delta_{SC}(q, \omega \to \mathcal{M}) = \mathcal{M}$. The full $\omega$-dependence of $\Delta_{SC}$ plotted in Fig. 2(a) indeed reveals a strong singularity at $\omega = \mathcal{M}$ increasingly pronounced as system size is increased.

In the absence of data for a single attractive Hubbard chain or a single $t - J$ chain, we compare our results with those of Ref. \cite{12}. In our case, we need form factors in a system of finite size. However, if the system size $L$ is much larger than the correlation length $\xi = v/\mathcal{M}$, since the dominant contribution to $\Delta_{SC}(q, \omega)$ comes from integration on the region $x, v \tau \ll \xi$, it is a good approximation to replace finite volume form factors by infinite volume ones. Keeping this in mind, we define the diagonal and off-diagonal Green’s functions as:

$$G(x, \tau) = -(T_{x,\tau} \hat{\psi}_x(x, \tau) \hat{\psi}_x^\dagger(0, 0)), \quad (6)$$

$$F(x, \tau) = -(T_{x,\tau} \hat{\psi}_x(x, \tau) \hat{\psi}_{-x}(0, 0)). \quad (7)$$

Following \cite{12}, the asymptotic (long-distance) behaviors of the diagonal and off-diagonal Green’s functions is then obtained as,

$$G(x, \tau) = \frac{Z_1}{2\pi^2} \left( \frac{\pi v}{2\alpha\mathcal{M}} \right)^{\frac{1}{2}} \frac{1}{i\mathbf{x} - \mathbf{v} \mathbf{v}^{\dagger} \mathbf{r}^{\dagger} \mathbf{r}} e^{2\alpha_\rho} e^{-\frac{\alpha}{\rho} L}, \quad (8)$$

$$F(x, \tau) \approx -\frac{Z_1}{2\pi^2\alpha} \left( \frac{\pi v}{2\alpha\mathcal{M}} \right)^{\frac{1}{2}} \frac{1}{i\mathbf{x} - \mathbf{v} \mathbf{v}^{\dagger} \mathbf{r}^{\dagger} \mathbf{r}} e^{-\frac{\alpha}{\rho} L},$$

where $\rho = \sqrt{x^2 + (\mathbf{v}^{\dagger})^2}$, $\eta = \frac{1}{2}\left(K_F + 1/K_F - 2\right)$, $\eta' = \frac{1}{2}\left(K_F - 1/K_F\right)$, $Z_1$ is a dimensionless constant calculated in \cite{22} and $\mathcal{M}$ the single particle gap. These expressions show that $G$ and $F$ have a similar exponential decay at large distances above the characteristic length-scale $\xi = v/\mathcal{M}$. More rapidly decaying terms like $e^{-3\alpha\rho\rho/v}$ and higher that involve more than one spinon in the intermediate state have been neglected. Note that the SC Green’s function decays with system size like as $(1/L)^{\frac{1}{2\pi \rho}}$ due to SC phase fluctuations. However, apart from this overall power-law scaling factor, we expect the SC gap to remain finite and bear interesting $q$ and $\omega$ dependence. Using the Fourier transforms of the above expressions \cite{8}, the momentum and Matsubara-frequency dependence of the gap is given by

FIG. 1: Superconducting gap (normalized to its $Q = 0$ value) vs $Q\xi$. 

\[
\Delta_{SC}(q, \omega_n) = C\mathcal{M}(\frac{2\pi \xi}{L})^{2\alpha} \frac{2F_1(\frac{3}{2} - \eta', \frac{3}{2} - \eta'; 1; y)}{2F_1(1 - \eta, \frac{3}{2} - \eta; 2; y)},
\]
results to numerical calculations on the t-J ladder model at 1/8-doping and $J = 0.4$ [9]. The numerical calculations on $2 \times 12$ ladder show that there are two spinon gaps, $\mathcal{M}(q_y = 0) = 0.08t$ and $\mathcal{M}(q_y = \pi) = 0.12t$ such that $F(q_y = 0, \pi), G(q_y = 0, \pi)$ and $\Delta(q_y = 0, \pi)$ develop an imaginary part for $\omega > \Delta(q_y)$. The comparison of these spinon gaps with the actual spin gap of a $2 \times 24$ ladder [24] (defined as the gap between the lowest triplet excitation and the singlet ground state) gives $\Delta(q_y = 0) = 0.11t \approx 2\mathcal{M}(q_y = 0)$ and $\Delta(q_y = \pi) = 0.17t \approx \mathcal{M}(q_y = \pi) + \mathcal{M}(q_y = 0)$. The discrepancies could result from a spinon-spinon attraction or from the overestimation of the spinon gaps in the $2 \times 12$ ladder. It is also important to note that in [2], the correlation length is of the order of magnitude of the system size. We note that no sharp peak is present in the imaginary part at the threshold, in contrast to the prediction of [10], but the prediction of a rather constant behavior of $\Delta$ below the threshold is in agreement with [10].

A more detailed comparison between analytic and numerical result is possible. In the case of the ladder system, away from half-filling, the gapped modes is expected to present an approximate $SO(6)$ symmetry [22]. This allows a description of the gapped modes by the $SO(6)$ Gross-Neveu (GN) model [23] and a form factor calculation of the superconducting gap $\Delta(q_50, \omega_n)$ along the lines of the present paper [24]. The novelty in the case of the $SO(6)$ GN model is that on top of the spinon excitations of mass $\mathcal{M}$ (known as kinks in the literature on the GN model), there are also massive fermion excitations (bound states) with a mass $\mathcal{M}\sqrt{2}$. This implies a second threshold in $\Delta(q_50, \omega_n)$ at the energy $\omega = \mathcal{M}(1 + \sqrt{2})$ besides the threshold at energy $\omega = \mathcal{M}$. Whether this could be related to some higher energy features seen in numerics [2] needs further clarifications. This will be discussed in more details in a separate publication.

In conclusion, we have computed the fluctuating SC gap of the LE chain. A simple form is obtained with a factorization into a power-law factor accounting for SC suppression due to quantum phase fluctuations multiplied by a function containing the dynamics of the pairing interaction. We point out some differences with the case of the 2-leg t-J ladder and suggest that the gapped sectors of the latter could be better described by the $SO(6)$ GN model.

FIG. 2: Real (a)) and imaginary (b) parts of the superconducting gap vs $\omega$ obtained by bosonization results for a 1D LE liquid (energies are in units of the spectral (solitonic) gap $\mathcal{M}$) for $K_\rho = 1.3$ and $q = k_F$. Data for two lengths are shown.

[1] J. R. Schrieffer, in Theory of Superconductivity (Addison-Wesley, Reading, MA, 1964), Chap. 7.
[2] D. Poilblanc and D. J. Scalapino, Phys. Rev. B 66, 052513 (2002).
[3] D. Poilblanc, D. J. Scalapino and S. Capponi (unpublished).
[4] E. Dagotto and T. M. Rice, Science 271, 618 (1996), and references therein.
[5] H. J. Schulz, in Correlated fermions and transport in mesoscopic systems, edited by T. Martin, G. Montambaux, and J. Tran Thanh Van (Editions frontières, Gif sur Yvette, France, 1996), p. 81.
[6] E. Dagotto, Rep. Prog. Phys. 62, 1525 (1999).
[7] H. J. Schulz, in Mesoscopic quantum physics, Les Houches LXI, edited by E. Akkermans, G. Montambaux, J. L. Pichard, and J. Zinn-Justin (Elsevier, Amsterdam, 1995), p. 533.
[8] For numerical studies see e.g. C. A. Hayward et al., Phys. Rev. Lett. 75, 926 (1995); D. Poilblanc, D. J. Scalapino, and W. Hanke, Phys. Rev. B 52, 6796 (1995).
[9] H. J. Mikeska and H. Schmidt, J. Low Temp. Phys 2, 371 (1970).
[10] N. D. Mermin and H. Wagner, Phys. Rev. Lett. 17, 1133 (1967).
[11] J. Voit, Eur. Phys. J. B 5, 505 (1998).
[12] A. M. Tsvelik and F. H. L. Essler, cond-mat/0205294 (unpublished).
[13] A. Luther and V. J. Emery, Phys. Rev. Lett. 33, 589 (1974).
[14] N. Andrei, in Low-Dimensional Quantum Field Theories For Condensed Matter Physicists, edited by S. Lundqvist, G. Morandi, and L. Yu (World Scientific, Singapore, 1993), and references therein.
[15] M. Nakamura, K. Nomura, and A. Kitazawa, Phys. Rev. Lett. 79, 3214 (1997). cond-mat/9708204
[16] N. Andrei and J. H. Lowenstein, Phys. Rev. Lett. 43, 1698 (1979).
[17] M. Karowski and P. Wiesl, Nucl. Phys. B 139, 455 (1978).
[18] D. Controzzi, F. H. L. Essler, and A. M. Tsvelik, in New Theoretical approaches to strongly correlated sys-
tems, Vol. 23 of NATO Science Series II. Mathematics, Physics and Chemistry, edited by A. M. Tsvelik (Kluwer Academic Publishers, Dordrecht, 2001), p. 25.

[19] F. H. Essler and V. E. Korepin, cond-mat/9808018 (unpublished).

[20] S. Lukyanov and A. B. Zamolodchikov, Nucl. Phys. B 607, 437 (2001).

[21] M. Abramowitz and I. Stegun, Handbook of mathematical functions (Dover, New York, 1972).

[22] H. Schulz, cond-mat/9808167 (unpublished).

[23] D. J. Gross and A. Neveu, Phys. Rev. D 10, 3235 (1974).

[24] Data obtained using an approximate Contractor Renormalisation (CORE) method; see S. Capponi and D. Poilblanc, Phys. Rev. B 66, 180503 (2002) and references therein.

[25] Numerical investigations of bound states are given in D. Poilblanc, O. Chiappa, S. White and D. J. Scalapino, Phys. Rev. B 62, R14633 (2000).

[26] A calculation of the spectral functions in the ladder using the SO(6) GN model has been reported in C. Ahn and E. Paik, J. Korean Phys. Soc. 39, 965 (2001).