Low Dimensional Manifold Regularization Based Blind Image Inpainting and Non-Uniform Impulse Noise Recovery

MEI GAO1,2, BAOSHENG KANG1, XIANGCHU FENG3, LIXIA CAO2, AND WENJUAN ZHANG2
1School of Information Science and Technology, Northwest University, Xi’an 710127, China
2School of Science, Xi’an Technological University, Xi’an 710021, China
3Department of Mathematics, Xidian University, Xi’an 710071, China
Corresponding author: Baosheng Kang (bskang@163.com)

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ABSTRACT Blind image inpainting is a challenging task in image processing. Motivated by the excellent performance of low dimensional manifold model (LDMM) in image inpainting for large-scale pixels missing, we introduce a novel blind inpainting model to repair images with missing pixels or damaged with impulse noise, in spite of the unknown locations of the corrupted pixels. We applied logarithmic transformation to separate the image and binary mask. LDMM regularization and $l_0$ norm regularization were applied respectively to simultaneously estimate the image and the mask. The resulted minimization problem was then solved by the split Bregman algorithm. The simulation results showed that the proposed model, compared with the existing ones, can effectively restore the image with large uniform and non-uniform missing rate.

INDEX TERMS Blind image inpainting, low dimensional manifold model, split Bregman algorithm, total variation.

I. INTRODUCTION

Image inpainting is a very important problem in image processing. It has been widely studied and applied in many fields, such as digital restoration [1], art conservation [2], and video inpainting [3]. Therefore, image inpainting has attracted much attention. A variety of image inpainting methods have been developed to restore the image in past decades. Up to now, image inpainting methods can be roughly classified into two categories, the knowledge-driven methods and the data-driven methods. The knowledge-driven methods and the data-driven methods are two different approaches, and they are all being studied in parallel.

The knowledge-driven methods include the partial differential equation (PDE)-based methods [4], [5], the exemplar-based methods [6]–[9], and the sparse-representation (SR)-based methods [10]–[12].

The PDE-based methods are the most fundamental methods of all image inpainting methods. The basic idea is to extend effective information in the area surrounding the corrupted region so as to achieve the purpose of restoration [13]. In [13], Bertalmio et al. first proposed the concept of digital image inpainting and put forward an inpainting model based on third-order PDE (BSCB model). Chan et al. proposed the total variation (TV) model [14]. As a further improvement of the TV method, the curvature-driven diffusions (CDD) inpainting model was developed by Chan et al. [15]. Typical models of the PDE-based methods also include Mumford-Shah model [16], Euler’s elastica model [17], Mumford-Shah-Euler model [18], etc.. These methods can achieve good results for image inpainting of small scale damage, e.g. removing scratches, removing text coverage, filling holes, and so on. However, they often result in blurring of structure or texture for image inpainting of large scale damage, such as target removal.

The basic idea of the exemplar-based methods is to fill in the corrupted region by copying the patch from the source regions. The most representative method is proposed by Criminisi et al. [19], which guides the inpainting order by defining the priority of patch, so that texture and structure information can be propagated simultaneously. Compared with the PDE-based methods, the exemplar-based methods...
are not only used to effectively repair large-scale damage, but also to improve the restoration efficiency and guarantee the integrity and continuity of edge structure. However, some limitations can still be found through these methods, such as block mismatch, and unreasonable repair order.

The basic idea of the SR-based methods is to restore the damaged image using the over-complete dictionary and sparse coding. In 2005, Elad et al. [20] proposed an inpainting method based on Morphological Component Analysis (MCA), which can simultaneously realize the restoration of cartoon layer and texture layer. In 2006, Elad et al. [21] proposed a K-SVD algorithm, and then used it to repair the missing pixels in images. Although good visual effects for smooth images can be found through these methods, they still have some problems, e.g. time-consuming learning, high computational complexity, very limited prior knowledge, and poor adaptive ability.

The second category is the data-driven methods, i.e. deep learning-based methods [22]–[24]. The main idea is to use large amount of real images for training and learning, so as to automatically repair the damaged regions of the image and achieve the purpose of image restoration. Köhler et al. [25] established a deep learning framework based on the multi-layer perception (MLP) architecture for image inpainting. Zhu et al. [26] proposed a new technique depending on CNN, which assisted in detecting the patch-oriented inpainting process. Zeng et al. [27] designed a Pyramid-context Encoder Network (PEN-Net) for image inpainting. Jiang et al. [27] introduced LDMM briefly. In Section II, we introduce LDMM briefly. In Section III, we propose the blind image inpainting model based on LDMM. The split Bregman iteration based algorithms are applied to solve the optimization problems resulted from the proposed model. Numerical experiments are conducted in Section IV for three image restoration tasks:

1. blind image inpainting with different percentage of uniform pixels missing,
2. impulse noise removal with different percentage of non-uniform pixels corrupted.
3. blind inpainting for RGB color image.

Finally, a brief conclusion will be given in Section V.

II. LOW DIMENSIONAL MANIFOLD MODEL

As for LDMM, it uses the patch manifold dimension associated with the image as a regularization to restore the image in that the patch manifold dimension associated with many natural images have low dimension.

Consider an \( m \times n \) size discrete grayscale image \( I' \in R^{m\times n} \). For any pixel \( x = (i,j) \), where \( 1 \leq i \leq m, 1 \leq j \leq n \), let this pixel constitute the left-top pixel in an 2D rectangular patch of size \( s_1 \times s_2 \) of the image \( I' \), and denote this patch as \( (P'I')(x) \). Define \( S(P'I') \) as the collection of all such patches, i.e.:

\[
S(P'I') = \{ (P'I')(x) | x \in \Theta \subset [1, 2, \cdots, m] \times [1, 2, \cdots, n] \} \subset R^d,
\]

\[d = s_1 \times s_2.\]

(1)

where \( \Theta \) is an index set to make the union of the patch set \( S(P'I') \) cover the whole image, the boundary points are using reflection extension in this paper.

The patch set \( S(P'I') \) can be seen as a point cloud in \( R^d \), where \( d = s_1 \times s_2 \). It is observed that \( S(P'I') \) samples a low dimensional smooth manifold \( M(I') \) embedded in \( R^d \), which is called the patch manifold associated with image \( I' \). Based on the assumption that the patch manifold of many natural images has low dimensional structure, Osher et al. [32] proposed LDMM in which the dimension of the patch manifold was used as a regularization to recover an image:

\[
\min_{I' \in R^{m\times n}, M \subset R^d} \dim(M),
\]

subject to \( I_0' = AI' + \epsilon \), \( S(P'I') \subset M \),

(2)

where \( \dim(M) \) is the dimension of the manifold \( M \).
In LDMM [32], the dimension of the patch manifold $M$ is computed as follows:

$$\dim(M) = \sum_{i=1}^{d} \|\nabla_M \alpha_i(x)\|^2_{L^2(M)},$$

where $\alpha_i(i = 1, 2, \cdots, d)$ are the coordinate functions on $M$, i.e.,

$$\alpha_i(x) = x_i \quad \forall x = (x_1, x_2, \cdots, x_d) \in M \subseteq \mathbb{R}^d.$$  

Using the above formula, the problem (4) can be rewritten as

$$\min_{I' \in \mathbb{R}^{m \times n}, M \subseteq \mathbb{R}^d} \left\{ \sum_{i=1}^{d} \|\nabla_M \alpha_i(x)\|^2_{L^2(M)} + \gamma \|l_0 - AI'\|^2_2, \right\}$$

subject to $S(PI') \subset M,$

where $\gamma > 0$ in the penalty term is a parameter.

The above optimization problem (5) of highly nonlinear and non-convex is solved by iterative method. The most difficult part of iterative computing is to solve the following type of optimization problem:

$$\min_{r \in H^1(M)} \|\nabla_M r\|^2_{L^2(M)} + \beta \sum_{y \in S} (r(y) - s(y)),$$

where $r$ can be any $\alpha_i$, $s(y)$ is a given function on $S$, and $\beta > 0$ is a penalty parameter.

For (6), with a standard variational approach, the key step problem is to solve a Laplace-Beltrami equation on a point cloud:

$$\begin{cases}
-\Delta_M r(x) + \beta \sum_{y \in S} \tau(x - y)(r(y) - s(y)) = 0, & x \in M, \\
\frac{\partial r}{\partial n}(x) = 0, & x \in \partial M,
\end{cases}$$

where $\partial M$ is the boundary of $M$ and $n$ is the out normal of $\partial M$. If $M$ has no boundary, $\partial M = \varnothing$.

Equation (7) is solved by the point integral method (PIM) [36], [37]. It is discretized as follows:

$$\sum_{y \in S} R_t(x, y)(r(x) - r(y)) + \delta t \sum_{y \in S} \tilde{R}_t(x, y)(r(y) - s(y)) = 0,$$

where $\delta > 0$ is a parameter, $s(y)$ is the given value. For a parameter $t > 0$,

$$R_t(x, y) = C_t \frac{|x - y|^2}{4t},$$

where $C_t$ is the normalizing factor,

$$\tilde{R}(z) = \int_{-\infty}^{\infty} R(u)du \quad \text{and} \quad \tilde{R}_t(x, y) = C_t \tilde{R}(\frac{|x - y|^2}{4t}).$$

If we set $R(z) = \exp(-z)$, then $\tilde{R}_t = R_t$ are Gaussians. The LDMM, with the PIM used, has been shown to achieve good performances, especially in non-blind image inpainting.

## III. PROPOSED MODEL AND NUMERICAL ALGORITHM

### A. OUR PROPOSED MODEL

In this subsection, we apply LDMM to blind image inpainting. We first give a random pixel missing model under additive Gaussian noise, and it can also represent the problem of multiplicative impulse noise under additive Gaussian noise.

In our problem, we need to simultaneously estimate the image $I \in \mathbb{R}^d$ and the mask $A \in \mathbb{R}^{b \times b}$. The diagonal element of the mask matrix $A$ is 1, when a pixel is observed, or the diagonal element of the mask matrix $A$ is 0, when a pixel is corrupted or missing pixel.

The problem of estimating $I$ and $A$, reported in [31], is

$$\hat{(I, A)} = \arg \min_{I, A} \|I\|_1 + \frac{\theta_1}{2} \|l_0 - AI\|^2_2 + \frac{\theta_2}{2} \|\text{diag}(E - A)\|_0,$$

where $E$ is identity matrix, $\theta_1, \theta_2 > 0$ are the regularization parameters. If the element of the $\text{diag}(E - A)$ is 0, it means that the pixel is observed, and if it is 1, it means that pixel is the noise pixel. In this paper, our goal is to estimate the image $I$ from the partial observations $l_0$ without prior knowledge of the observation mask $A$. Since $I$ and $A$ are multiplied, the equation (11) is difficult to solve.

There are a large number of methods to remove additive Gaussian noise, but we ignore the influence of additive noise to simplify the model and only discuss the problem of random pixel loss or multiplicative impulse noise. In order to separate $I$ and $A$, We use a logarithmic transform on both. Firstly, we consider diagonal matrix $A = \text{diag}(a)$ with $a \in \{0, 1\}^b$.

When a pixel with index $k$ is observed, the corresponding mask element $a_k = 1$, and when pixel $k$ is lost, $a_k = 0$. Suppose that we don’t take the noise case into account, a pixel $k$ in image $I$ is defined as the scalar product,

$$i_k = j_k \times a_k.$$  

Secondly, defining $g_k = \log a_k$, we have

$$g_k = \begin{cases} 0, & \text{if } k \text{ is observed}, \\
-K, & \text{otherwise},
\end{cases}$$

where $K$ is a positive integer and $K > \log 255$. To avoid taking a logarithm of 0, we make such an approximation. We take the $l_0$ norm constraint, so the effect of different $K$ values on the result is the same.

Then, assuming that $I, A$ are always positive, we apply a logarithmic transform on (12),

$$\log i_k = \log(j_k \times a_k),$$

$$\log i_k = \log j_k + \log a_k.$$  

where $\log f' = \log f \circ g' \circ \log f$ is $f'$. A small positive bias term $\xi > 0$ is added to $y'$ to guarantee positivity.

Reorganize the data in $y', f', g' \in \mathbb{R}^d$, and return it to $y, f, g \in \mathbb{R}^{m \times n}$, where $b = m \times n$, i.e. $y = f + g$.

Thus, our task is estimating $f$ and $g$, given the log transformed observation $y$. Previously, TV regularization on log
transformed images has been used [38], and synthesis models which provide enhanced sparse representations in transform domains have also been used for image denoising and restoration [21]. We assume that logarithmic transformation of our image \( f \) is piece-wise smooth. Meanwhile, the negative elements of \( g \) correspond to the non-observed pixels, and the zero elements of \( g \) correspond to the observed pixels. Therefore, we take the dimension of the patch manifold as a regularization on the log transformed image \( f \), and the \( l_0 \) norm as a regularization on the log transformed mask \( g \). We propose the following minimization model:

\[
(f, g) = \arg \min_{f, g \in \mathbb{R}^{m \times n}, M \subset \mathbb{R}^d} \dim(M) + \lambda_1 \| y - f - g \|_2^2 + \lambda_2 \| g \|_0,
\]

subject to \( S(Pf) \subset M \),

where \( \lambda_1, \lambda_2 > 0 \) are the regularization parameters respectively. The dimension of the patch manifold \( M \) is computed as \( \dim(M) = \sum_{i=1}^{d} \| \nabla_M \alpha_i(x) \|_2^2 \), where \( \alpha_i \) (\( i = 1, 2, \ldots, d \)) are the coordinate functions on the manifold \( M \). The optimization problem (17) can be reformulated as:

\[
(f, g) = \arg \min_{f, g \in \mathbb{R}^{m \times n}, M \subset \mathbb{R}^d} \sum_{i=1}^{d} \| \nabla_M \alpha_i(x) \|_2^2 + \lambda_1 \| y - f - g \|_2^2 + \lambda_2 \| g \|_0,
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\[
\begin{align*}
\hat{f} = \arg \min_{f \in \mathbb{R}^{m \times n}, M \subset \mathbb{R}^d} & \| y - f \|_2^2 \\
& + \mu \| \nabla_M \alpha(x) \|_2^2 \quad \text{subject to } S(Pf) \subset M,
\end{align*}
\]

\[
\hat{g} = \arg \min_{g \in \mathbb{R}^{m \times n}} \| g \|_0 \\
& + \mu \| \nabla M \alpha(x) \|_2^2 \quad \text{subject to } S(Pf) \subset M.
\]

B. NUMERICAL ALGORITHM

In this subsection numerical algorithm is presented to solve the proposed model (18). Since the problem (18) is highly nonlinear and non-convex, our algorithm is built upon the split Bregman iteration [39]. To solve the model (18), we adopt the iterative schemes as follows:

First, the manifold is fixed, \( f \), \( g \), and the coordinate functions are computed. With a guess of the manifold \( M \), a guess of \( g \) and a guess of \( f \) satisfying \( S(Pf) \subset M \), we use the idea of the split Bregman iteration to update \( f \), \( g \), and \( \alpha_i \) sequentially.

- \( \alpha_i \) are computed by inverting the logarithmic transformation, i.e.

\[
\alpha_i = \frac{1}{\lambda_i} \arg \min_{M \subset \mathbb{R}^d} \| M \alpha(x) \|_2^2 \\
& + \mu \| \nabla M \alpha(x) \|_2^2 \\
& \quad \text{subject to } S(Pf) \subset M.
\]

- Update \( f \) as follows:

\[
f_{n+1} = \arg \min_{f \in \mathbb{R}^{m \times n}} \| y - f - g_{n+1} \|_2^2 + \mu \| \nabla M \alpha_i(x) \|_2^2 \quad \text{subject to } S(Pf) \subset M.
\]

- Update \( g \) as follows:

\[
g_{n+1} = \arg \min_{g \in \mathbb{R}^{m \times n}} \| g \|_0 + \| y - f_{n+1} - g \|_2^2.
\]

- Update \( M \) by setting

\[
M_{n+1} = \left\{ \alpha_i(x) : x \in M \right\}.
\]

- Repeat above two steps until convergence.

The equation (19) can be solved using PIM, problem (20) has a solution given by solving a least-squares problem, and (21) can be computed using the hard threshold. Thus, we obtain the complete description of the algorithm for solving (18) in Algorithm 1.

\[
H_{\sqrt{\lambda_2}}(\cdot) \text{ is the hard threshold operator and is defined as,}
\]

\[
H_{\sqrt{\lambda_2}}(B, \sqrt{\lambda_2}) = \begin{cases} 0, & |B| \leq \sqrt{\lambda_2}, \\ |B|, & |B| > \sqrt{\lambda_2}. \end{cases}
\]

And finally, we get \( \hat{f} \) and \( \hat{g} \) as estimates of \( f \) and \( g \), respectively. Reorganize the data in \( \hat{f}, \hat{g} \in \mathbb{R}^{m \times n} \), and return it to \( \hat{f}', \hat{g}' \in \mathbb{R}^d \). The estimates of the image and the mask are computed by inverting the logarithmic transformation, i.e. \( \hat{I} = 10^{\hat{f}'}, \hat{A} = 10^{\hat{g}'} \).

IV. EXPERIMENTS RESULT

In this section, we carry out experiments on synthetic images to demonstrate the performance of our method, with comparison to the representative method: BITV [31] method code can be found at [https://github.com/manyafon/Blind-Inpainting-IO-TV], KALS [40] method code can be obtained
Ensure: Restore indexes, which are defined as follows:

$$\text{peak signal-to-noise ratio (PSNR)} [41] \text{ as the measurement we can be non-uniform here.}$$

The usual pixels missing recovery problem requires a laptop with an Intel(R) Pentium(R) CPU (2.16GHz) and 4GB memory. The usual pixels missing recovery problem requires the assumption that the pixels missing location is known and the assumption that the noise distribution is uniform and the stopping criterion were tuned mutually to achieve the maximal PSNR or the best SSIM for a fair comparison.

To quantify the quality of the inpainting results, we use the peak signal-to-noise ratio (PSNR) [41] as the measurement indexes, which are defined as follows:

$$\text{PSNR: } = 10 \log_{10} \frac{MN}{\parallel \max I_o - \min I_o \parallel^2_2} \quad (\text{dB}) \quad (25)$$

In addition, the structural similarity (SSIM) [42] is introduced to evaluate the similarity between $I$ and $I_o$. The high PSNR and SSIM values of the image can show good performance of the model.

During the experiment of our model, there are two important parameters ($\lambda_1, \lambda_2$) to be tuned. We found that $\lambda_1 = 0.5, \lambda_2 = 0.1$ can provide optimal results of our method in blind inpainting. For BITV and KALS methods, the parameters and the stopping criterion were tuned mutually to achieve the maximal PSNR or the best SSIM for a fair comparison.

### A. Blind Image Inpainting with Different Percentage of Uniform Pixels Missing

In general, missing is uniform, and each point determines whether it is lost or not according to a certain probability. We tested four synthetic images to restore images with different pixel uniform missing rates in the first experiment: Barbara image (256 × 256 pixels), Lena image (256 × 256 pixels), pixel uniform missing rates in the first experiment: Barbara image (256 × 256 pixels), Lena image (256 × 256 pixels). The high and the usual multiplicative impulse noise problem assumes that the noise distribution is uniform and we can be non-uniform here.

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FIGURE 3. Inpainting results of twelve images—Comparison. (a) Original image; (b) Observed image with 10% pixels missing; (c) Ours; (d) BITV; (e) KALS.
in Figure 1. The reconstructed images are shown in Figure 2. Due to the outstanding performance of LDMM in image inpainting, we first considered the worst case. The observed image was obtained with the binary mask which had only 5% pixels, i.e. 95% pixels were randomly discarded. Then, we show the images inpainting results for 70% of the pixels missing, 50% of the pixels missing and 25% of the pixels missing respectively in Figure 2. In the experiments, we set the number of iterations to 100. Visually, we can easily observe that our model is much clearer than those of BITV and KALS. Although KALS cannot restore large area pixel missing images, it has a strong ability to restore small area pixel missing images with obvious visual effects, such as the eyes of mandrill in Figure 2. BITV can restore images with the maximum pixel missing of 95%, which showed a better performance than what KALS can do, but our approach presented best results.

To reflect our model’s ability to restored images with different proportions of missing pixels and quantify the performance of our approach, Table 1 shows comparison of our method with BITV and KALS methods by the PSNR, SSIM and time values. It can be revealed that our method provides the optimal PSNR, SSIM values in all examples, which are shown in bold face in Table 1. However, that although the SSIM value of KALS is higher than that of BITV in large area pixel missing, the visual effect is significantly lower than that of BITV. Therefore, we selected the image with 10% pixel missing in the subsequent experiment in order to make a better comparison. Regardless of the calculation time, our approach showed the best blind image inpainting capability, not only in the ability to repair large-scale missing pixel areas, but also in the high PSNR and SSIM values.

In Figure 3, we demonstrate blind image inpainting with 10% pixels missing. Twelve original images and their observed images with 10% pixels missing are shown in Figures 3(a) and 3(b), respectively. The observed images are applied to conduct the comparison. The inpainted results of our proposal, BITV and KALS are presented in Figures 3(c)-(f), respectively. It shown that our proposed method is the best performer among all three methods visually. The PSNR, SSIM and time comparison of twelve images are plotted in Figure 4. It can be seen that the PSNR and SSIM values of our method reach maximal values for different images. In view of the long execution time of our method in the previous experiments, we set the iterations to 50 in this experiment, but it can be seen from Figure 4(c) that the proposed approach still takes the longest time.

### B. IMPULSE NOISE REMOVAL WITH DIFFERENT PERCENTAGE OF NON-UNIFORM PIXELS CORRUPTED

In the second experiment, we demonstrated that our method can also restore images with different non-uniform pixels corrupted rates. Here we take a non-uniform sampling depending on the polar coordinates of the pixels. The farther away they are from the center, the greater the probability of loss is. The four original synthetic images we used for the test are shown in Figure 5: Pepper image (256 × 256 pixels), Parrots image (256 × 256 pixels), Scenery image (256 × 256 pixels) and Villa image (256 × 256 pixels).

In the experiments, we set the number of iterations of our method to 100. The PSNR, SSIM and time values of the

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**TABLE 1. Comparison of PSNR, SSIM and time values on blind image inpainting.**

| Image                | PSNR (dB) | SSIM | time (second) |
|----------------------|-----------|------|---------------|
|                      | ours      | BITV | KALS          |
| Barbara-95% pixels missing | 28.78     | 18.96 | 17.31         |
| Lena-70% pixels missing   | 34.40     | 22.07 | 18.87         |
| Cameraman-50% pixels missing | 35.02     | 22.51 | 19.81         |
| Mandrill-25% pixels missing | 43.08     | 27.02 | 23.66         |

**FIGURE 4.** Performance (in SSIM, PSNR and time) plots of inpainting for twelve images of our approach, BITV and KALS.
TABLE 2. Comparison of PSNR, SSIM and time values on blind image inpainting.

| Image | pixels missing | PSNR (dB) | SSIM | time (s) |
|-------|---------------|-----------|------|---------|
|       |               | Ours | BITV | KALS    | Ours | BITV | KALS | Ours | BITV | KALS |
| Pepper | 90%          | 22.23 | 19.09 | 18.23 | 0.79 | 0.64 | 0.66 | 16699.29 | 45.97 | 1.52 |
|       | 10%          | 40.70 | 28.74 | 24.29 | 0.99 | 0.90 | 0.90 | 8993.91 | 22.34 | 1.43 |
| Parrots | 90%         | 23.81 | 19.98 | 17.87 | 0.79 | 0.73 | 0.78 | 8918.76 | 48.92 | 1.49 |
|        | 10%          | 42.03 | 27.89 | 24.95 | 0.99 | 0.89 | 0.89 | 7798.02 | 19.98 | 1.89 |
| Scenery | 90%         | 20.18 | 18.20 | 16.69 | 0.54 | 0.40 | 0.47 | 10545.99 | 44.02 | 1.44 |
|        | 10%          | 34.61 | 22.44 | 20.20 | 0.98 | 0.66 | 0.67 | 9198.52 | 26.37 | 1.44 |
| Villa  | 90%          | 22.91 | 20.55 | 19.46 | 0.64 | 0.53 | 0.61 | 10040.35 | 43.91 | 1.43 |
|        | 10%          | 36.95 | 26.99 | 23.22 | 0.97 | 0.79 | 0.76 | 9747.83 | 21.25 | 1.41 |

FIGURE 5. Original images. (a) Pepper; (b) Parrots; (c) Scenery; (d) Villa.

FIGURE 6. Inpainting results of several images comparison. (a) Observed image with 90% pixels corrupted; (b) Ours; (c) BITV; (e) KALS.

FIGURE 7. Inpainting results of several images comparison. (a) Observed image with 10% pixels corrupted; (b) Ours; (c) BITV; (e) KALS.

FIGURE 8. Original RGB color images. (a) Lena; (b) House; (c) Parrots; (d) Butterfly.

Inpainting results from all three methods are summarized in Table 2. The visual comparison of the inpainting results for 90% of the pixels corrupted and 10% of the pixels corrupted are shown in Figures 6 and 7, respectively. As can be seen from Figures 6 and 7, KALS achieves good visual inpainting effect in the structure and texture regions, especially in small area pixel corrupted images, which can be observed in the boundary of the Pepper, around the eyes of the Parrots, the reflection in the water in the Scenery, and the branches around the Villa. The inpainting effect of BITV in texture region is obviously weaker than that of KALS. Our model is the best performer among all three methods, in particular, for “Scenery” with rich textures.

C. BLIND INPAINTING FOR RGB COLOR IMAGE

Currently, most of the images displayed in the medium are color images. Therefore, in this section, we present the experimental results to illustrate the inpainting ability of the proposed method for RGB color images in the case of uniform
and non-uniform pixels missing. For the sake of brevity, we only show the visual optimization results obtained.

The images we used in the test are shown in Figure 8. The reconstructed images are shown in Figure 9. The visual results show through our method the image can be restored very well in both the cartoon part and the texture part, whether it is uniform or non-uniform pixels missing and corrupted, whether it is large-scale or small-scale pixels missing and corrupted.

V. CONCLUSION

In this paper, we proposed a low dimensional manifold approach for blind image inpainting and impulse noise recovery to simultaneously identify the image and the mask. Experiments were carried out on synthetic image. Moreover, quantitative measures were used to show the blind image inpainting ability of our model, and the blind image inpainting effect of our model is much better than those with existing methods.

As our model showed a long period of consumption in the calculation, we will continue to investigate the proposed method to obtain stronger blind image inpainting capabilities and less computation time than what the existing methods showed. Furthermore, our future work is to optimize the parameters of the model with the depth model after expanding the iterative formula of the model.

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REFERENCES

[1] B. Cornelis, T. Ružič, E. Gezels, A. Dooms, A. Půžurica, L. Platša, J. Cornelis, M. Martens, M. De Mey, and I. Daubechies, “Crack detection and inpainting for virtual restoration of paintings: The case of the ghent altarpiece,” Signal Process., vol. 93, no. 3, pp. 605–619, Mar. 2013.
[2] W. Hu, Z. Li, and Z. Liu, “Fast morphological component analysis Tangka image inpainting based on nonlocal mean filter,” J. Comput.-Aided Design Comput. Graph., vol. 26, no. 7, pp. 1067–1074, 2014.
[3] M. Ghoniem, Y. Chahir, and A. Elmoataz, “Nonlocal video denoising, simplification and inpainting using discrete regularization on graphs,” Signal Process., vol. 90, no. 8, pp. 2445–2455, Aug. 2010.
[4] G. Sridevi and S. S. Kumar, “Image inpainting and enhancement using fractional order variational model,” Defence Sci. J., vol. 67, no. 3, pp. 308–315, Jul. 2017.
[5] B. V. Rathish Kumar and A. Halim, “A linear fourth-order PDE-based gray-scale image inpainting model,” Comput. Appl. Math., vol. 38, no. 1, pp. 1–21, Mar. 2019.
[6] F. Yao, “Damaged region filling by improved criminisi image inpainting algorithm for thangka,” Cluster Comput., vol. 22, no. 6, pp. 1–9, 2018.
[7] S. Yang, H. Liang, Y. Wang, H. Cai, and X. Chen, “Image inpainting based on multi-patch match with adaptive size,” Appl. Sci., vol. 10, no. 14, pp. 4921, Jul. 2020.
[8] W. Wan and J. Liu, “Nonlocal patches based Gaussian mixture model for image inpainting,” Appl. Math. Model., vol. 87, pp. 317–331, Nov. 2020.
[9] L. Zhang and M. Chang, “Image inpainting for object removal based on adaptive two-round search strategy,” IEEE Access, vol. 8, pp. 94357–94372, 2020.
[10] L. Zhang, B. Kang, B. Liu, and F. Zhang, “Image inpainting based on exemplar and sparse representation,” Int. J. Signal Process., Image Process. Pattern Recognit., vol. 9, no. 9, pp. 177–188, Sep. 2016.
[11] J. Mo and Y. Zhou, “The research of image inpainting algorithm using self-adaptive group structure and sparse representation,” Cluster Comput., vol. 22, no. 3, pp. 7593–7601, 2019.
[12] R. Li, L. Tang, Y. Bai, Q. Wang, X. Zhang, and M. Liu, “Group-based sparse representation based on lp -Norm minimization for image inpainting,” IEEE Access, vol. 8, pp. 308–315, 2017.
[13] M. Bertalmio, G. Sapiro, V. Caselles, and C. Ballester, “Image inpainting,” in Proc. 27th Annul. Conf. Comput. Graph. Interact. Techn., 2000, pp. 417–424.
[14] J. Shen and T. F. Chan, “Mathematical models for local nontexture inpainting,” SIAM J. Appl. Math., vol. 62, no. 3, pp. 1019–1043, Jan. 2002.
[15] T. F. Chan and J. Shen, “Nontexture inpainting by curvature-driven diffusions,” J. Vis. Commun. Image Represent., vol. 12, no. 4, pp. 436–449, Dec. 2001.
[16] A. Tsai, A. Yezzi, and A. S. Willsky, “Curve evolution implementation of the Mumford-Shah functional for image segmentation, denoising, interpolation, and magnification,” IEEE Trans. Image Process., vol. 10, no. 8, pp. 1169–1186, Aug. 2001.
[17] T. F. Chan, S. H. Kang, and J. H. Shen, “Euler’s elastica and curvature based inpainting,” SIAM J. Appl. Math., vol. 63, no. 2, pp. 564–592, 2003.
[18] E. Esedoglu and J. Shen, “Digital inpainting based on the Mumford–Shah–Euler image model,” Eur. J. Appl. Math., vol. 13, no. 4, pp. 353–370, Aug. 2002.
[19] A. Criminisi, P. Perez, and K. Toyama, “Region filling and object removal by exemplar-based image inpainting,” IEEE Trans. Image Process., vol. 13, no. 9, pp. 1200–1212, Sep. 2004.
[20] M. Elad, J.-L. Starck, P. Querre, and D. L. Donoho, “Simultaneous cartoon and texture image inpainting using morphological component analysis (MCA),” Appl. Comput. Harmon. Anal., vol. 19, no. 3, pp. 340–358, Nov. 2005.
[21] M. Aharon, M. Elad, and A. Bruckstein, “rmK-SVD: An algorithm for designing overcomplete dictionaries for sparse representation,” IEEE Trans. Signal Process., vol. 54, no. 11, pp. 4311–4322, Nov. 2006.
[22] G. Liu, F. A. Reda, K. J. Shih, T. C. Wang, and B. Catanzaro, “Image inpainting for irregular holes using partial convolutions,” in Proc. Eur. Conf. Comput. Vis. (ECCV), 2018, pp. 85–100.
[23] J. Yu, Z. Lin, J. Yang, X. Shen, X. Lu, and T. S. Huang, “Generative image inpainting with contextual attention,” in Proc. IEEE/CVF Conf. Comput. Vis. Pattern Recognit., Jan. 2018, pp. 5505–5514.

[24] M.-C. Sagong, Y.-G. Shin, S.-W. Kim, S. Park, and S.-J. Ko, “PEPSI: Fast image inpainting with parallel decoding network,” in Proc. IEEE/CVF Conf. Comput. Vis. Pattern Recognit. (CVPR), Jun. 2019, pp. 11360–11368.

[25] R. Köhler, C. Schuler, B. Schölkopf, and S. Harmeling, “Mask specific inpainting with deep neural networks,” in Proc. Pattern Recognit., vol. 8753, 2014, pp. 523–534.

[26] X. Zhu, Y. Qian, X. Zhao, B. Sun, and Y. Sun, “A deep learning approach to patch-based image inpainting forensics,” Signal Process., Image Commun., vol. 67, pp. 90–99, Sep. 2018.

[27] Y. Zeng, J. Fu, H. Chao, and B. Guo, “Learning pyramid-context encoder network for high-quality image inpainting,” in Proc. IEEE/CVF Conf. Comput. Vis. Pattern Recognit. (CVPR), Jun. 2019, pp. 1486–1494.

[28] X. Zhu, Y. Qian, X. Zhao, J. Xu, and J. Zhu, “Image inpainting based on generative adversarial networks,” IEEE Access, vol. 8, pp. 22884–22892, 2020.

[29] H. Ji, Z. Shen, and Y. Xu, Wavelet Frame Based Image Restoration With Missing/Damaged Pixels. [Online]. Available: http://pdfs.semanticscholar.org/0d09/cb88989898f76deeed6fbc3d3a28eb45719d.pdf

[30] B. Dong, H. Ji, J. Li, Z. Shen, and Y. Xu, “Wavelet frame based blind image inpainting,” Appl. Comput. Harmon. Anal., vol. 32, no. 2, pp. 268–279, Mar. 2012.

[31] M. V. Afonso and J. M. R. Sanches, “Blind inpainting using ηi and total variation regularization,” IEEE Trans. Image Process., vol. 24, no. 7, pp. 2239–2253, Mar. 2015.

[32] S. Osher, Z. Shi, and W. Zhu, “Low dimensional manifold model for image processing,” SIAM J. Imag. Sci., vol. 10, no. 4, pp. 1669–1690, Jan. 2017.

[33] W. Cong, G. Wang, Q. Yang, J. Li, J. Hsieh, and R. Lai, “CT image reconstruction in a low dimensional manifold,” Inverse Problem Imago, vol. 13, no. 3, pp. 449–460, 2019.

[34] W. Zhu, Z. Shi, and S. Osher, “Low dimensional manifold model in hyperspectral image reconstruction,” in Hyperspectral Image Analysis (Advances in Computer Vision and Pattern Recognition), S. Prasad and J. Chanussot, Eds. Cham, Switzerland: Springer, 2020, pp. 295–317, doi: 10.1007/978-3-030-38617-7_1.

[35] M. V. Afonso, J. M. Bioucas-Dias, and M. A. T. Figueiredo, “Fast image recovery using variable splitting and constrained optimization,” IEEE Trans. Image Process., vol. 19, no. 9, pp. 2345–2356, Sep. 2010.

[36] Z. Li and Z. Shi, “A convergent point integral method for isotropic elliptic equations on a point cloud,” Multiscale Model. Simul., vol. 14, no. 2, pp. 874–905, Jan. 2016.

[37] Z. Shi and J. Sun, “Convergence of the point integral method for Laplace–Beltrami equation on point cloud,” Res. Math. Sci., vol. 4, no. 1, Dec. 2017.

[38] A. K. Oh, Z. T. Harmany, and R. M. Willett, “Logarithmic total variation regularization for cross-validation in photon-limited imaging,” presented at the ICHF, 2013.

[39] T. Goldstein and S. Osher, “The split Bregman method for L1-regularized problems,” SIAM J. Imag. Sci., vol. 2, no. 2, pp. 323–343, 2009.

[40] Y. Wang, A. Szlam, and G. Lerman, “Robust locally linear analysis with applications to image denoising and blind inpainting,” SIAM J. Imag. Sci., vol. 6, no. 1, pp. 526–563, 2013.

[41] S. Durand, J. Faadili, and M. Niaikova, “Multiplicative noise removal using L1 fidelity on frame coefficients,” J. Math. Imag. Vis., vol. 36, no. 3, pp. 201–226, 2010.

[42] Z. Wang, A. C. Bovik, H. R. Sheikh, and E. P. Simoncelli, “Image quality assessment: From error visibility to structural similarity,” IEEE Trans. Image Process., vol. 13, no. 4, pp. 600–612, Apr. 2004.

[43] M. Yan, “Restoration of images corrupted by impulse noise and mixed Gaussian impulse noise using blind inpainting,” SIAM J. Imag. Sci., vol. 6, no. 3, pp. 1227–1245, Jan. 2013.

[44] MEI GAO received the B.S. degree in mathematics from Shaanxi Normal University, Xi’an, China, in 2001, and the M.S. degree in computer technology from Xi’an Technological University, Xi’an, in 2009. She is currently pursuing the Ph.D. degree with the School of Information Science and Technology, Northwest University, Xi’an. She ever worked as a Visiting Scholar with the School of Information Science and Technology, Northwest University. She is currently a Lecturer with the School of Science, Xi’an Technological University. Her research interests include the applications of variation and regularization methods and partial differential equations for image processing.

[45] XIANGCHU FENG received the B.S. degree in computational mathematics from Xian Jiaotong University, Xi’an, China, in 1984, and the M.S. and Ph.D. degrees in applied mathematics from Xidian University, Xi’an, in 1989 and 1999, respectively. He is currently a Professor of applied mathematics with the School of Mathematics and Statistics, Xidian University. His research interests include numerical analysis, wavelets, low rank matrix approximation, and partial differential equations for image processing.

[46] LIXIA CAO received the B.S. degree in mathematics from Northwest University, Xi’an, China, in 1996, the M.S. degree in computer application technology from the Nanjing University of Science and Technology, Nanjing, China, in 2005, and the Ph.D. degree in information management and information system from the Xi’an University of Architecture and Technology, Xi’an, in 2016. She is currently an Associate Professor with the School of Science, Xi’an Technological University. Her research interests include rough theory, complex networks, management decision analysis, and game theory.

[47] WENJUAN ZHANG received the M.S. and Ph.D. degrees from Xidian University, Xi’an, China, in 2005 and 2013, respectively. She ever worked as a Visiting Scholar with the Department of Mathematics, University of Florida. She is currently an Associate Professor with the School of Science, Xi’an Technological University. Her research interests include the applications of variation and regularization methods and partial differential equations in image segmentation, low rank, and sparse approximation for image processing.