Improved parametrization of the growth index for dark energy and DGP models

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Abstract

We propose two improved parameterized form for the growth index of the linear matter perturbations: (I) \( \gamma(z) = \gamma_0 + \left( \gamma_\infty - \gamma_0 \right) \frac{z}{z+1} \) and (II) \( \gamma(z) = \gamma_0 + \gamma_1 \frac{z}{z+1} + \left( \gamma_\infty - \gamma_1 - \gamma_0 \right) \frac{z^\alpha}{z+1} \). With these forms of \( \gamma(z) \), we analyze the accuracy of the approximation the growth factor \( f \) by \( \Omega_m^{\gamma(z)} \) for both the \( \omega \)CDM model and the DGP model. For the first improved parameterized form, we find that the approximation accuracy is enhanced at the high redshifts for both kinds of models, but it is not at the low redshifts. For the second improved parameterized form, it is found that \( \Omega_m^{\gamma(z)} \) approximates the growth factor \( f \) very well for all redshifts. For chosen \( \alpha \), the relative error is below 0.003\% for the \( \Lambda \)CDM model and 0.028\% for the DGP model when \( \Omega_m = 0.27 \). Thus, the second improved parameterized form of \( \gamma(z) \) should be useful for the high precision constraint on the growth index of different models with the observational data. Moreover, we also show that \( \alpha \) depends on the equation of state \( \omega \) and the fractional energy density of matter \( \Omega_m \), which may help us learn more information about dark energy and DGP models.

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Recently, dark energy and modified gravity have been attracted a lot of attention because that both of them can provide a possible way to explain the accelerating expansion of our present Universe which has been strongly confirmed by many observations [1–3]. In general, dark energy is regarded as an exotic energy component with negative pressure. The modified gravity are such a kind of theories which modify Einstein’s general relativity including the scalar-tensor theory [4], the f(R) theory [5] and the Dvali-Gabadadze-Porrati (DGP) braneworld scenarios [6]. Since the dark energy and modified gravity can give rise to the current accelerated expansion, it is natural to ask which one describes correctly the real evolution of the Universe [7–12]. It is well known that in the same cosmic expansion history the growth of matter perturbations are different in the different theoretical models [13–44]. Thus the growth function of the linear matter density \( \delta(z) \equiv \delta \rho_m / \rho_m \) has been regarded as an effective tool to distinguish the dark energy and the modified gravity at present.

At scales much smaller than the Hubble radius, the growth function \( \delta(z) \) satisfies the simple equation [45]

\[
\ddot{\delta} + 2H \dot{\delta} - 4\pi G_{\text{eff}} \rho_m \delta = 0,
\]

(1)

where the dot denotes the derivative with respect to the time \( t \). \( G_{\text{eff}} \) is an effective gravity constant. After defining the growth factor \( f \equiv d \ln \delta / d \ln a \), one can find that Eq. (1) becomes

\[
\frac{d f}{d \ln a} + f^2 + \left( \frac{\dot{H}}{H^2} + 2 \right) f = \frac{3 G_{\text{eff}}}{2 G_N} \Omega_m.
\]

(2)

where \( \Omega_m = \rho_m / 3H^2 \) and \( G_N \) is the Newton gravity constant in general relativity. In Ref. [46], the growth factor \( f \) can be approximated very well as

\[
f = \Omega_m^\gamma,
\]

(3)

where \( \gamma \) is so-called the growth index. In general, it is a function of redshift \( z \). At the high redshift, one can set \( \Omega_m = 1 \) and obtain that \( \gamma_\infty = \frac{3(1-w)}{5-6w} \) for the \( w \)CDM model and \( \gamma_\infty = 11/16 \) for the DGP model [6]. However, at the low redshift, it is very difficult to obtain the analytical expression of \( \gamma \). Since the growth index varies with the redshift \( z \), the authors in Refs. [37–40] proposed a linear approximation of \( \gamma(z) \), i.e., \( \gamma(z) \approx \gamma_0 + \gamma_1 z \), and found that the sign of \( \gamma_1 \) is negative for \( w \)CDM model and is positive for the DGP model. Thus they claimed that the signs of \( \gamma_1 \) may provide another signals to discriminate the dark energy and the modified gravity [37–40]. However, in Ref. [41], the authors argued that the linear expansion is only valid at the low redshift region (\( z < 0.5 \)) and then the signs of \( \gamma_1 \) cannot discriminate different models from current observations [47–54] because that there are few growth factor data points at \( z < 0.5 \). Thus, the
authors \[41\] proposed that the growth index has a form

\[
\gamma(z) = \gamma_0 + \gamma_1 \frac{z}{z + 1}.
\] (4)

The merit of such a form \(\gamma(z)\) is that it is applicable to all the data points and can be used to distinguish

the models using observational data. Moreover, they also found \[41\] that this form of \(\gamma\) yields that \(\Omega_m^{\gamma(z)}\) approximates the growth factor \(f\) very well both for the \(\Lambda\)CDM model (the error is \(\sim 0.03\%\)) for all redshifts when \(\Omega_{m0} = 0.27\) (see fig. 1) and for the DGP model (the error is \(\sim 0.18\%\)) (see fig. 2). However, it is very easy to find from Eq. 4 that as the redshift \(z \rightarrow \infty\) the growth index \(\gamma(z)\) approaches to \(\gamma_0 + \gamma_1\) rather than \(\gamma_{\infty}\). In the evolution of the Universe the early difference may affect the behaviors of the growth factor \(f\) at the low redshifts. Thus, it is necessary to enhance the accuracy of the parametrization \[41\] at the high redshifts.
A natural improvement to the growth index (4) is
\[
\gamma(z) = \gamma_0 + (\gamma_\infty - \gamma_0) \frac{z}{z + 1}.
\] (5)

Obviously, the above growth index tends to \(\gamma_\infty\) as \(z \to \infty\). In order to check whether our improve form of parametrization Eq. (5) yields more accuracy than the form (1) in Ref. [41], we must evaluate the values of \(\gamma_0\) and \(\gamma_\infty\). The previous discussions tell us that the expressions of \(\gamma_\infty\) for the \(w\)CDM model and the DGP model are given by [26, 27, 31]. For the \(w\)CDM model and the DGP model, the Friedmann equations give

\[
\frac{\dot{H}}{H^2} = -\frac{3}{2}w(1 - \Omega_m), \quad G_{\text{eff}} = G_N,
\] (6)

and

\[
\frac{\dot{H}}{H^2} = -\frac{3\Omega_m}{1 + \Omega_m}, \quad G_{\text{eff}} = \frac{2(1 + 2\Omega_m^2)}{3(1 + \Omega_m^2)}G_N,
\] (7)
respectively. Substituting Eqs. (6) and (7) into Eq. (2), we can obtain the value of \( f \) at \( z = 0 \) for given \( \Omega_m \) by resorting to numerical methods, and then get the value of \( \gamma_0 \) by the relation \( \gamma_0 = \ln f(z = 0)/\ln \Omega_m \). Through a simple comparison, one can find that the value of \( \gamma_0 \) is the same as that in Refs. [37–41]. Make use of the values of \( \gamma_\infty \), \( \gamma_0 \) and the improved growth index (5), we plot the relative error \( \Omega_\gamma - f \) for the \( \Lambda \)CDM model in fig. (3) and for the DGP model in fig. (4).

Comparing figs. (1), (2) and (3), (4), We can obtain that for both \( \omega \)CDM and DGP models the quantity \( \Omega_\gamma \) with the improved growth index (5) approximates the growth factor \( f \) better than the growth index (4) at the high redshifts. But at the low redshifts, the relative error \( \Omega_\gamma - f \) obtained by the growth index \( \gamma(z) = \gamma_0 + (\gamma_\infty - \gamma_0) \frac{z}{z + 1} \) is larger than that by \( \gamma(z) = \gamma_0 + \gamma_1 \frac{z}{z + 1} \) in Ref. [41]. Thus, the improved growth index (5) is not a good approximation for the \( \gamma(z) \) in Eq. (3).

In order to make use of the virtues of the growth indexes (4) and (5), we propose another improved parameterized form on \( \gamma(z) \)

\[
\gamma(z) = \gamma_0 + \gamma_1 \frac{z}{z + 1} + (\gamma_\infty - \gamma_1 - \gamma_0) \left( \frac{z}{z + 1} \right)^{\alpha},
\]

where \( \alpha \) is a numerical parameter depended on the equation of state \( \omega \) and the fractional energy density of matter \( \Omega_m \). Obviously, as \( z \to 0 \) and \( z \to \infty \), \( \gamma(z) \) approach to \( \gamma_0 \) and \( \gamma_\infty \), respectively. Here we assume \( \alpha > 1 \) so that the third term in the \( \gamma(z) \) can be regarded as a higher order correction to the growth index (4). This growth index \( \gamma(z) \) contains four parameters and is more complicated than in (4). However, it will improved the accuracy of the approximation. Moreover, as the parameter \( \gamma_1 \) in (4), the coefficient \( \gamma_\infty - \gamma_1 - \gamma_0 \) and exponent \( \alpha \) in the third term can provide us more ways to understand the differences between dark energy and DGP models.

Similarly, in order to check how well \( \Omega_m^{\gamma(z)} \) with the improve form of parametrization Eq. (8) approximates the growth factor \( f \), we must obtain the values of the parameters (i.e., \( \gamma_0 \), \( \gamma_1 \) and \( \gamma_\infty \)) appeared in Eq. (8). The calculation of \( \gamma_0 \) and \( \gamma_\infty \) are similar to those in the previous discussion. As in Ref. [37–41], the value of \( \gamma_1 \) can be approximated by the derivative \( \gamma'(z) \) at redshift \( z = 0 \) because that the derivative of the third term in the \( \gamma(z) \) (8) with respect to \( z \) is equal to zero at the point \( z = 0 \) since \( \alpha > 1 \). Thus, for the \( \omega \)CDM model and the DGP model, \( \gamma_1 \) can be obtained by

\[
\gamma_1 = \frac{1}{\ln \Omega_m} \left[ \frac{3}{2} \Omega_m^{1-\gamma_0} - \Omega_m^{\gamma_0} - \frac{3}{2} \omega(2\gamma_0 - 1)(1 - \Omega_m) - \frac{1}{2} \right],
\]

and

\[
\gamma_1 = \frac{1}{\ln \Omega_m} \left[ -\Omega_m^{\gamma_0} + \frac{1 + 2\Omega_m^2}{1 + \Omega_m^2} \Omega_m^{1-\gamma_0} - \frac{1}{2} + \frac{3(1 - \Omega_m)}{1 + \Omega_m} (\gamma_0 - \frac{1}{2}) \right],
\]

(9)  (10)
respectively. Obviously, the forms of $\gamma_1$ for both models are identical to those in [37–41]. Moreover, we find that in the third term in (8) the coefficient $\gamma_\infty - \gamma_1 - \gamma_0$ is positive for dark energy and is negative for DGP model, which is plotted in Figs. (5)-(6). The exponent $\alpha$ can be estimated by using the value of $\gamma(z)$ at $z = z_0$

$$\alpha = \left[ \ln \frac{z_0}{z_0 + 1} \right]^{-1} \ln \left[ \frac{\gamma(z_0) - \gamma_0 - \gamma_1 z_0}{\gamma_\infty - \gamma_0 - \gamma_1} \right].$$

(11)

Obviously, the above expression of $\alpha$ gives the different values for different $z_0$. In the approximation of growth index $\gamma(z)$, the error is larger at the low redshift. Thus, we take the value $\alpha$ at $z_0 = 1$ for simplicity in our evaluation. In Figs. (7)-(8), we show the possible region of $\alpha$ with a given region of $\Omega_{m0}$: 0.25 ≤ $\Omega_{m0}$ ≤ 0.30. From these figures, we find that $\alpha$ depends on the equation of state $\omega$ and the fractional energy density of matter $\Omega_{m0}$. The the coefficient $\gamma_\infty - \gamma_1 - \gamma_0$ and exponent $\alpha$ in the corrected term contain more information about dark energy and DGP models, which could provide us more methods to discriminate them.
FIG. 7: The $\alpha$ for $\omega$CDM model with $0.25 \leq \Omega_{m0} \leq 0.30$. The solid, dashed and dotted curves correspond to $w = -1$, $-0.8$ and $-1.2$, respectively.

FIG. 8: The $\alpha$ for DGP model with $0.25 \leq \Omega_{m0} \leq 0.30$.

Let us now adopt the improved form of $\gamma(z)$ \cite{5} and compare the numerical result $f$ with the analytical approximation $\Omega_m^{\gamma(z)}$. The results for the $\omega$CDM and DGP models are shown in Figs. (9) and (10) respectively. For the $\Lambda$CDM model, we find that the relative error is below 0.003% for all redshifts when $\Omega_{m0} = 0.27$. This is much less than that obtained in Ref. \cite{41} where with $\gamma(z) = \gamma_0 + \gamma_1 \frac{z}{e^{\frac{z}{z_i}}}$ the error is only below 0.03%. Thus, using our improved parameterizations of growth index (8) the error enhances one order of magnitude improvement. Comparing Figs. (1) and (9), we also find that at high redshifts $\Omega_m^{\gamma(z)}$ approximates $f$ more accurately than that in Ref. \cite{41} for the dark energy models with different $\omega$.

For the DGP model, one can find from fig. (10) the largest relative error $\frac{\Omega_m^{\gamma(z)} - f}{f}$ is 0.028% for all redshifts when $\Omega_{m0} = 0.27$, which is also less than that in Ref. \cite{41} where the largest one is 0.18%. Moreover, comparing figs. (2) and (10) we find that at high redshifts the accuracy of the approximation with our improved form (8) is
FIG. 9: The relative error $\Omega_{\gamma(z)}^m f$ with redshift for the $\omega$CDM model with $\Omega_{m0} = 0.27$. Here $\gamma(z) = \gamma_0 + \gamma_1 \frac{z}{z+1} + (\gamma_\infty - \gamma_1 - \gamma_0)(\frac{1}{z+1})^\alpha$. The solid, dashed and dotted curves correspond to $w = -1, -0.8$ and $-1.2$, respectively.

FIG. 10: The relative error $\Omega_{\gamma(z)}^m f$ with redshift for the DGP model. Here $\gamma(z) = \gamma_0 + \gamma_1 \frac{z}{z+1} + (\gamma_\infty - \gamma_1 - \gamma_0)(\frac{1}{z+1})^\alpha$. The solid, dashed and dotted curves correspond to $\Omega_{m0} = 0.27$, 0.24 and 0.3, respectively.

also improved by eight times than with the old one [41]. Therefore, with the second improved parameterizations of the growth index [8], the $\Omega_{m0}^{\gamma(z)}$ approximates the grow factor $f$ more accurately than those in the previous literatures [37–41, 43] both for $\Lambda$CDM and DGP models.

In summary, we proposed two improved parameterized form for the growth index of the linear matter perturbations and analyzed the growth factor for both $\omega$CDM and DGP models. Using the first improved parameterized form, we find that $\Omega_{m0}^{\gamma(z)}$ approximates the grow factor $f$ more accurately than that in the case the growth index $\gamma(z)$ is parameterized by $\gamma(z) = \gamma_0 + \gamma_1 \frac{z}{z+1}$ at the high redshifts for both kinds of models, but it is not at the low redshifts. However, if we adopt to the second improved parameterized form, one can find that the accuracy of the approximation the growth factor $f$ by $\Omega_{m0}^{\gamma(z)}$ is enhanced evidently for all redshifts. The relative error is under 0.003% for the $\Lambda$CDM model and 0.028% for the DGP model when $\Omega_{m0} = 0.27$. Comparing with those in [41], such parameterizations improve almost the approximation accuracy by one
order of magnitude. Thus, the second improved parameterized form of $\gamma(z)$ should be useful for the high precision constraint on the growth index of different models with the observational data.

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