PHASE-TRANSIENT HIERARCHICAL TURBULENCE AS AN ENERGY CORRELATION GENERATOR OF BLAZAR LIGHT CURVES

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ABSTRACT
Hierarchical turbulent structure constituting a jet is considered to reproduce energy-dependent variability in blazars, particularly, the correlation between X- and gamma-ray light curves measured in the TeV blazar Markarian 421. The scale-invariant filaments are featured by the ordered magnetic fields that involve hydromagnetic fluctuations serving as electron scatterers for diffusive shock acceleration, and the spatial size scales are identified with the local maximum electron energies, which are reflected in the synchrotron spectral energy distribution (SED) above the near-infrared/optical break. The structural transition of filaments is found to be responsible for the observed change of spectral hysteresis.

Subject headings: BL Lacertae objects: individual (Mrk 421) — galaxies: jets — magnetic fields — radiation mechanisms: nonthermal — turbulence

1. INTRODUCTION
A noticeable feature associated with blazars is that the updated shortest variability timescale reaches a few minutes (e.g., Mrk 421: Cui 2004; Blażejowski et al. 2005), not likely to be reconciled with the light-crossing time at the black hole horizon. One possible explanation for this fact is that small-scale structure does exist in the parsec-scale jet anchored in the galactic core (Honda & Honda 2004). Indeed, in the plausible circumstance that the successive impingement of plasma blobs ejected from the core) into the jet bulk engenders collisionless shocks, electromagnetic current filamentation (characterized by the skin depth) could be prominent (Medvedev & Loeb 1999). It is known that the merging of smaller filaments leads eventually to accumulation of magnetic energy in larger scales (Honda et al. 2000a; Silva et al. 2003).

Reflecting the self-similar (power law) characteristic in the inertially cascading range, the local magnetic intensity of the self-organized filaments will obey $|B| \sim B_0 (\lambda/d)^{(\delta-1)/2}$, where $\lambda$ and $d$ reflect the transverse size scale of a filament and the maximum, respectively, $B_0 \equiv |B|_{\lambda=1}$, and $\beta$ ($>1$) corresponds to the filamentary turbulent spectral index. The value of $d$ is limited by the transverse size of jet (or blob size; $D$). Then, it is reasonable to consider that in fluid timescales, the well-developed coherent fields are sure to actually meet hydromagnetic disturbance independent of the filamentation; that is, the turbulent hierarchy is established (see Fig. 1). The spectral index of the superposed fluctuations [denoted as $\beta'$ ($>1$)] could be different from $\beta$, and the correlation length scale is presumably limited by $\sim \lambda$.

At this site, the electrons bound to the local mean fields suffer scattering by the fluctuations, to be diffusively accelerated by the collisionless shocks (see Honda & Honda 2007). When the acceleration and cooling efficiency depend on the spatial size scales, the local maximum energies of accelerated electrons will be identified by $\lambda$ ($\S$ 2), to be reflected in the synchrotron SED extending to the X-ray region. More interestingly, the spatially inhomogeneous property of particle energetics is expected to cause the energy-dependent variability of broadband SEDs. Here the naive question arises whether or not this idea is responsible for the observed elusive patterns of energy correlation of light curves (e.g., Takahashi et al. 1996; Fossati et al. 2000a, 2000b; Blażejowski et al. 2005): this is the original motivation of the current work.

In the present simplistic model, light travel time effects would still prevent the detection of variability signatures on timescales shorter than $D(c_0^2_\delta)$, where $\delta = \delta(1+z)$, and $\delta$, $z$, and $c$ are the beaming factor of the jet, redshift, and speed of light, respectively. However, if a filamented piece is isolated, having loose causal relation with the dynamics of a bulk region serving as a dominant emitter, an intrinsic rapid variability involved in the subsystem would be viable. Namely, it is inferred that the shorter timescale is at least potentially realized, and observable, unless energetic emissions from such a compact domain are crucially degraded by synchrotron self-absorption and/or $\gamma\gamma$ absorption (e.g., Aharonian 2004). As is, the basic notion of the present model seems to provide a vital clue to settle the debate as to the causality problem incidental to observed rapid variabilities.

In this Letter, I demonstrate that the hierarchical system incorporated with the synchrotron self-Compton (SSC) mechanism accurately generates the time lag of gamma-ray flaring activity behind the X-ray, confirmed in the high-frequency-peaked BL Lac object Mrk 421 (Blażejowski et al. 2005). We address that in general, both lag and lead can appear in X-ray interband correlations, accompanying the structural transition. The major transition history is argued in light of the observed spectral hysteresis patterns. We also work out ($B_m$, $d$), to provide the constraint on the field strength and $D$ that should be compared with those of previous models.

2. AN IMPROVED EMITTER MODEL WITH HIERARCHICAL STRUCTURE
We consider a circumstance in which relativistic shocks propagate through a relativistic jet with the Lorentz factor $\Gamma$, such that the shock viewed upstream (jet frame) is weakly to mildly relativistic. Note the relation of $\delta \sim \Gamma$. The overall geometry and relative size scales of the aforementioned hierarchy are sketched in Figure 1. Provided that the gyrating electrons trapped in the filament (with the size $\lambda$) are resonantly scattered by the magnetic fluctuations, the mean acceleration time upstream is approximately given by $t_{acc} = (3\eta_f c_0 \bar{c}) [r(r-1)]$, where $\eta = (3/2b)(\lambda/2r_f)^{2/3}$, $b$ is the energy density ratio of fluctuating/local mean magnetic fields (assumed to be $b \ll 1$),
Fig. 1.—Schematic of the beamed jet including the emitting blobs and top view of the transverse cut of a blob region (with the diameter $D$). A number of circle-like “bubbles” symbolically represent the transverse section of scale-invariant filaments (with size $l_\lambda$, whose maximum is $d$). Note that $d$ is limited by $D$ (for the values, see § 4). The magnified view of a small sample domain illustrates the fluctuating magnetic field that scatters gyrating electrons bound to the local mean field.

$\gamma(\gamma, |B|)$ is the electron gyroradius ($\gamma$ being the Lorentz factor), and $r$ is the shock compression ratio. In the regime in which flares saturate, $\tau_{\text{swf}}$ will be comparable to synchrotron cooling time $\tau_{\text{syn}}(\gamma, |B|)$. Balancing these timescales gives the (local)2-flares saturate, will be comparable to synchrotron cooling and $r_\lambda$. More speculatively, this scaling (for the values, see § 4). The magnified view of a small sample domain illustrates the fluctuating magnetic field that scatters gyrating electrons bound to the local mean field.

3. PROPERTIES OF ENERGY-DEPENDENT SPECTRAL VARIABILITY AND Hysteresis

3.1. X-Ray Interband Correlation

In this context, we derive the $\nu$-dependence of the flaring activity timescale (denoted as $\tau$). In $\nu_p < \nu < \nu_s$, which typically covers the X-ray band, we have $\gamma^* = (4\pi/3)(\delta\beta\times (m_eeB_0^4|B|^2)|1/2$, which is written as $\gamma^*(\lambda, \nu) = (\nu/\nu_p)^{\delta\beta}/(\lambda\nu)_{-1/3}$. Utilizing this, the expression of $\tau_{\text{swf}}(\gamma, |B|)$ is recast into $\tau_{\text{swf}}(\lambda, \nu) = (\gamma^*)^{1/2} = (\lambda\nu)_{3/2}$, which is $\tau_0 = 36\pi m_0^2 e/(B_e^2)$. The relation of $\nu$ to $\nu_p$ can be derived from the equality of $\gamma^*(\lambda, \nu) = \gamma(\lambda, \nu)$, such that $\nu = (\nu_p/\nu_s)^{\delta\beta}/(\lambda\nu)_{-1/3}$, where the new $\nu_0 = (\gamma_0/\gamma_s)^{3/2}$, and $\nu_0 = (\gamma_0/\gamma_s)^{1/2}$. Substituting this into $\tau(\nu, \nu)$, we arrive at the result $\tau(\nu, \nu_0) = (\nu/\nu_p)^{1-9/2} = (\nu/\nu_s)^{3/2}$, where $\sigma(\beta, \beta') = (3\beta - 1)/(3\beta + 1)$; that is,

$$\tau \propto \nu^{1-1-\sigma}. \quad (1)$$

The critical function $\beta(\beta')$ for the key range of ($1 < \beta < 2$), plotted in Fig. 2. Note that for the special $\beta = 2$ case, $\sigma = (3\beta - 1)/(3\beta + 1) = 1$ is always satisfied, and $\beta > \beta$ ensures $\beta > \beta$, (because of $\beta < 1$), $\beta' = 2$ and $\beta > 2$ lead to soft and hard lag, respectively, irrespective of the $\beta$ value. While the index $\beta$ is
expected to be variable (reflecting the long-term structural evolution of
filaments; see § 3.3 for details), $\beta'$ would be a constant since a
mechanism of superimposed magnetic fluctuations ($\S$ 1) perhaps
has universality. One can exclude $\beta' > 2$, which yields by no means
soft lag, which is at odds with the observational facts, whereupon
we can take the modified upper bound (indicated in Fig. 2, arrows)
into account. With these ingredients, I conjecture the preferential
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$$\Delta \tau = 1.8 \frac{\delta_5}{\delta_5^{1/2}} B_{10}^{1/2} \gamma_{14}^{1/4} \gamma_{1}^{1/14} \eta_{-1} \text{ hr} \quad (3)$$

for the structural phase $\Phi_e$, where $\gamma_{14} = \nu_{0}/10^{14}$ Hz, $\eta_{-1} = [1 - (\epsilon_0/\epsilon_{0})^{1/14}]^{1/14}$, and $\epsilon_1/1$ keV. Concerning the
validity, it has been checked that, e.g., for a soft-lag episode (in 1994 May; Takahashi et al. 1996), the measured time lag plotted
against $\epsilon_1$ could be more naturally fitted by the function (3) of
$\Delta \tau(\epsilon_1, \epsilon_0)$ (given $\epsilon_0 \simeq 4-5$ keV for ASCA), rather than the
function of $\sim \epsilon_1^{1/2}[1 - (\epsilon_0/\epsilon_{0})^{1/2}]$ for the homogeneous ($\sigma = 0$)
model.

3.2. X/Gamma-Ray Cross-Band Correlation

The interband correlation property is reflected in the cross-band
correlation between X- and gamma-rays, provided the SSC
mechanism as a dominant gamma-ray emitter (e.g., Maraschi et al. 1992; Dermer & Schlickeiser 1993). Along the
heuristic (time independent) manner, we suppose $\gamma \sim \gamma'$ for scattering electrons, and examine the correlation between an
X-ray band $\epsilon_1$ (compared to $\epsilon_0$) and gamma-ray band $\epsilon_2$,
susceptible to the inverse Comptonization of low-energy synchrotron photons (with $\epsilon_2$). Here we focus on the feasible,
Thomson regime of $(\epsilon_0 B \gamma) > m_e c^2$; note that using the expression of $\gamma'(\lambda(\epsilon_1))$ ($\S$ 3.1), this range can be written as
$\epsilon_1 < B_{10}^{1/2} (\epsilon_2^{5/3} \nu_{14}^{1/2})^{1/2} \text{ keV}$ for $\Phi_2$. The Lorentz factor
of the electrons that execute the boost of $\epsilon_2/\epsilon_1 = (\gamma')^2$ is denoted as $\gamma_1 = (\epsilon_2/\epsilon_1)^{1/2} \gamma'$ (for $\Phi_2$). Then, simply estimating
$\Delta \tau_{xy} = \tau(\epsilon_1/\gamma_1 - \epsilon_2)$ ($> 0$, for $\beta > \beta'$) would be adequate
for the present purpose. For convenience, one may eliminate $\epsilon_1$ from $\gamma_1$, [transform $\lambda(\epsilon_1)$ into $\lambda(\epsilon_2)$], and adopt the positive
soft-lag representation of $\Delta \tau_{xy} = (\Delta \tau_{xy})$, so that the negative
sign indicates gamma-ray lag. Again using $\nu_0$, we find for $\Phi_2$

$$\Delta \tau_{xy} = -1.7 \frac{\delta_5^{1/2} \delta_5^{1/3}}{\delta_5^{1/2}} B_{10}^{1/3} \gamma_{14}^{1/16} \gamma_{-1} \text{ days,} \quad (4)$$

where $\eta_{-1} = [1 - 0.79 \epsilon_1^{1/14} (\epsilon_1, \delta_5, \delta_5^{1/2} \gamma_{14}^{1/16})^{1/10}]^{-1}$, $\gamma_{14} = \nu_{0}/1 \text{ TeV}$, and $\epsilon_{25} = \epsilon_{2}/25 \text{ keV}$. The simultaneous
equations (3) and (4) contain the solutions ($\delta_5, B_{10}$), for given
observable quantities $\nu_0$ and $(\Delta \tau, \Delta \tau_{xy})$, as well as $\epsilon_0$, $\gamma_1$, $\epsilon_2$, $\epsilon_1$, $\gamma_1$ inherent in detectors.

In Figure 3 (top) for $\nu_{25} = 2, \Delta \tau = 1 \text{ hr}$, and ($\epsilon_{1}, \epsilon_1, \epsilon_1^{1/4} \gamma_{14}^{1/4}$), compared to Mrk 421 ($z = 0.031$) measurements (Takahashi et al. 1996; Blažejowski et al. 2005), the self-consistent numerical solution $\delta$ is plotted against $\Delta \tau_{xy}$, given $\epsilon_1$ that covers a gamma-ray band associated with the Whipple
observation (Catanese & Wekes 1999). For the allowed domain
of $\delta > 1$ (Piner et al. 1997), a typical TeV range of $\epsilon_{1} = 1-2$
(susceptible to the significant variation in the mid state) is found
with $\Phi_2$ to a priori restrict the domain of the observable $-\Delta \tau_{xy}$ to
$1.4-2.2$ days. Surprisingly, this quantitatively agrees with $\Delta \tau_{xy} = -1.8 \pm 0.4$ days that has been revealed by multiband
monitoring in the 2002/2003 season (Blažejowski et al. 2005).
In order to solidify the argument, the solutions for the high state
with $\Phi_2$, have also been sought. The results show that the upper
bound of $-\Delta \tau_{xy}$, at which $\delta$ diverges, shifts (from 2.2 days) to
1.7 days and the Whipple coverage $\epsilon_{1i} < 10$ restricts to $-\Delta \tau_{gi} > 0.7$ days; these combination yields $-\Delta \tau_{gi} = 0.7$–1.7 days. This is certainly compatible with the measured $\Delta \tau_{gi} = -1.2 \pm 0.5$ days (in the 2003/2004 season; for the significance, see Blażejowski et al. 2005).

3.3. Hysteresis Reversal via Structural Transition

From the view point of activity history, it is claimed that, involving the fluctuations with a common $\beta = 5/3$, the coherent structure, at least, in the dominant emission region has been in strong structural deformation, while the larger can be in-

is readily obtained from equation (3), and in parallel, one for $\Phi_1$, $\Phi_2$, $\Phi_{3i}$, $B_3$, $B_6$ must satisfy

$$B_{3i}^{3/3} = \begin{cases} 54 \nu_{1q}^{-5/2} (\Delta \tau_{1}^{-1} \epsilon_{11}^{-1/2} \eta_{1}^{-1/2})^{2/3} G, \\ 33 \nu_{1q}^{-2/3} (\Delta \tau_{2}^{-1} \epsilon_{11}^{-1/2} \eta_{2}^{-1/2})^{2/3} G, \end{cases}$$

respectively, where $\eta_{1}^{-1} = [1 - (\epsilon_{11} \epsilon_{16})^{1/3}]/10^{-3}$ and $\Delta \tau$ is in hours. In Figure 3 (bottom), we plot the self-consistent solution $B_3$ (against $\Delta \tau_{3i}$, corresponding to $\delta$ $\Delta \tau_{gi}$. Figure 3 (top)) that obeys equation (5) for $\Phi_2$ (inset) with the same parameter values as the top panel. We see that the observed $\delta > 1$ (Piner et al. 1999) provides the constraint for which local magnetic intensity $|B|$ never exceeds 47 G for $\Phi_2$ (51 G for $\Phi_{3i}$). Whereas a mean magnetic intensity $B$ is not well defined within the present framework, the obtained scaling of $B_{3i}^{3/3} = 5$ seems to be reconciled with the conventional $B^{3/2} = 0.1$–1 G derived from fitting a variety of homogeneous SSC models to the measured broadband SEDs (e.g., Ghisellini et al. 1998; Tavecchio et al. 1998; Krawczynski et al. 2001).

In turn, the quantity of $\xi = -4 d_\nu = 7.5 \nu_{1q}^{-4} \beta_{L}^{-5/3}$ (valid for $\beta = 5/3$; § 2) is self-consistently determined. Making use of equation (5) to eliminate $B_{3i}$, we have $d = 2.6 \times 10^{14} (\Delta \tau_{3i}^{-1/3})^{1/3} cm$ for $\Phi_2$ $[2.4 \times 10^{14} (\Delta \tau_{3i}^{-1/3})^{1/3} cm$ for $\Phi_{3i}]$, given the common parameter values (such as $\nu_{1q} = 2$). To estimate $\xi$, here we call for another expression, $\nu_{L} = 1.0 \times 10^{-22} \beta_{L}^{-5/3} \Delta \tau_{3i}^{-1/3}$ Hz [independent of ($\beta, \beta_{L}$); § 2]. Using this to eliminate $\Delta \tau_{gi}$ from the $d$-expression, we obtain the simple scaling of $d = 8.2 \times 10^{14} \beta_{L}^{3/2} \nu_{L}^{1/3} cm$ for $\Phi_2$ $[7.5 \times 10^{14} \beta_{L}^{3/2} \nu_{L}^{1/3} cm$ for $\Phi_{3i}]$, where $\nu_{L} = 10^{10} \Delta \tau_{3i}^{-1/3} Hz$.

The size $d$ implies the allowable minimum of $D$; e.g., $\nu_{L} = 0.1$–10 (yet involving the large observational uncertainty) provides $D_{16} \approx 10^3$ to 1 ($D_{16} = D/10^{16}$ cm), as reconciled with the previous results (e.g., Fossati et al. 2000b; Krawczynski et al. 2001; Blażejowski et al. 2005). It also turns out, from the $d$-scaling, that the range of $\nu_{L} < 10^3$ accommodates $\xi \ll 1$, and thereby the assumption of $b \ll 1$ (§ 2).

In addition, given an energy input into the jet, particle density $n$ is estimated. Assuming that electron injection operates at $\gamma_{in} \ll \gamma_{in}^{1/3} \approx \gamma_{in}^{1/3} \approx \gamma_{in}^{1/3}$ (for $p = 1.6$), to find that the steady luminosity of $10^{44}$ erg s$^{-1}$, which appears to retain a dominant portion around the $\nu_{L}$, requires $n \geq 6 \times 10^{14} \gamma_{in}^{-1} D_{16}^{2/3} \nu_{L}^{-1/3} cm^{-3}$ (when supposing a spherical emitting volume with the diameter of $D$). Recalling $B_{3i}^{3/3} \leq 5$, we thus read $n \geq 10^{14} D_{16} \nu_{L}^{2/3} cm^{-3}$ for ordinary $\gamma_{in}^{-1} \approx O(1)$; note that an upper bound can be given by imposing the conditions of, e.g., pair-plasma production ($T = 1$ MeV) and radial confinement ($nT \leq B_{3i}^{2/3}/8\pi$), such that $n \leq 10^{-6} cm^{-3}$ (suggesting $D_{16} \approx 10^{-2}$).

In conclusion, the gamma-ray lags of 1–2 days measured in Mrk 421 have been nicely reproduced by the hierarchical turbulent model of a jet. The crucial finding is that the structural transition $\Phi_2 \rightarrow \Phi_{3i}$ results in downshifting the upper bound of the observable lag [in a TeV ($\epsilon_{1i} \approx 1$) band] from 2.2 to 1.7 days, in accordance with a closer inspection from 2002 to 2004 by Blażejowski et al. (2005).

A typical 1.8 day lag (in the 2002/2003 season) suggests $\delta = 10^{-2}$–92 and $B_{3i} = 10^{12}$–22 G (Fig. 3); the latter provides an upper limit of local magnetic intensity. The present model as a possible alternative to the previous leptonic (e.g., Sikora et al. 1994; Bednarek & Protheroe 1997; Konopelko et al. 2003) and hadronic scenarios (e.g., Mücke & Protheroe 2001) will shed light on puzzling aspects of broadband spectral variability.

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