Shear Behavior of RC Beams Strengthened by External Vertical Prestressing Rebar

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Shear design is an important part of structural design. External vertical prestressing rebars (EVPRs) have proven to be an effective way to enhance structural shear resistance. The objectives of this study are to simulate EVPR strengthening of concrete beams using a nonlinear threedimensional finite element model and to explore its shear enhancement features under different EVPR stirrup ratios, vertical compressive stress degrees, and optimal arrangements of EVPRs. Concrete, common reinforced bars, and EVPRs use solid, steel, and truss elements, respectively. In addition, the total strain crack model is used to characterise the concrete. The results indicate that the EVPR stirrup ratio can reduce the diagonal crack width and improve the shear capacity, and the vertical compressive stress degree can effectively control crack development in the initial loading. A "small-area EVPR dense arrangement" is the recommended EVPR configuration method. Both experiments and numerical analyses show that EVPRs can effectively improve the shear performance of concrete.

1. Introduction

Shear failure of concrete beams is a sudden brittle failure. Once shear damage occurs, the consequences are very serious. Therefore, in order to improve the shear strength of concrete beams, scholars have performed numerous studies. Recent research has been conducted on the shear theory of reinforced concrete (RC) beams, especially critical shear crack theory (CSCT) [1–5]. From the aspect of materials, steel fibre reinforced concrete (SFRC) [6–8] and ultra-high performance concrete (UHPC) [9–11] are used for shear reinforcement research. For existing concrete structures, carbon fibre reinforced polymer (CFRP) [12–14], carbon FRP ropes [15, 16], steel plates [17, 18], and other methods for shear reinforcement are utilized, such as U-shaped mortar jackets [19].

In addition to the abovementioned shear strengthening methods, there is also an external vertical prestress shear strengthening method that arranges external vertical prestressing rebars (EVPRs) outside the web. Adhikary and Mutsuyoshi [18] compared multiple shear strengthening methods and proved that vertical prestress reinforcement is the most effective one. Aboutaha and Burns [20] used external prestressing bars to retrofit composite beams that originally lacked shear reinforcement and had a smooth interface bonded with epoxy. Before retrofitting, these beams experienced sudden horizontal shear failure. Ductile flexural failure occurred after being retrofitted by external prestressing bars. Experimental results obtained by Altin et al. [21] showed that this strengthening method is effective and that the specimens’ strength, rigidity, and ductility were improved. Clamps controlled any shear cracks and helped to improve the ductile flexural behaviour of the members. An investigation conducted by Bolger [22] showed that the shear strengthening method of concrete girders using external vertical clamping with existing shear cracks repaired with epoxy injection is effective.

Many studies have used various numerical analysis methods to study the shear behaviour of beams. Lei [23] used ABAQUS to establish a finite element model (FEM) to analyse the effect of RC beams strengthened with steel plates. A cohesive layer was established at the interface between the
steel plate and concrete, and the cohesive element used zero-
thickness COH3D8. Lampropoulo [24] conducted a nu-
meric study on full-scale beams strengthened with ultra-high
performance fibre reinforced concrete (UHPFRC) layers and
jackets, and the interface between the concrete beam and the
UHPFRC was modelled using special two-dimensional el-
ements, representing a well-roughened interface for the
analysis of reinforcement technology. Ferreira et al. [25]
extended a shear-sensitive fibre beam model to account for
the effects of unbonded vertical external prestressed rein-
forcement in the structural response of RC beams. Bahraq
et al. [10] presented a study on the shear behaviour of RC
beams strengthened by jacketing the surfaces of beams using
UHPC by experiments and numerical analysis. The bond
between normal concrete and UHPC was considered as a
perfect bond because during all experimental tests there was
no debonding observed. Yin et al. [26] proposed a novel
technique using equivalent beam elements at the interface
between UHPC and normal strength concrete (NSC) sub-
strate for the prediction of the structural behaviour of RC
members strengthened with UHPC. Compared with nu-
merical models of perfectly bonded interfaces and numerical
models of unbonded interfaces, the developed FEM was
found to effectively and efficiently predict the structural
response of composite UHPC-concrete members with good
accuracy.

The present research group has used external tensile
prestressed steel bars to strengthen the shear test using
parameters such as tensile strength and vertical prestressed
steel bar space, and the relevant results of the experimental
part have been published in this journal [27]. U-shaped
mortar jackets [19] can improve the rigidity and shear ca-
acity of the structure, but it is a passive strengthening
method. External vertical prestressing is an effective way to
enhance the strength of concrete structure, but so far the
strength efficiency and related influencing factors of this
strengthening method have not been clearly explored.

In this study, the total strain crack model, which has
many good examples for analysing concrete structures
[28–30], uses nonlinear three-dimensional finite element
analysis (FEA) to model the shear strength concrete beams
of EVPRs. The main objectives of this study are as follows:

1. To investigate the principle of enhancing the shear
   performance by EVPRs through numerical analysis.
2. To develop an effective method to simulate the effect
   of EVPRs on concrete beams.
3. Given that testing is limited by conditions and that
   the study of specimens has been insufficient, this
   study takes a closer look at the following: (i) the
   parameters, including the vertical compressive stress
degree and EVPR stirrup ratio; and (ii) the optimal
   arrangement of EVPRs.

2. Experimental Program

2.1. Geometric Features. Seven RC beams were cast using
normal concrete with the cube compressive strength of
28.2 MPa ($f_{cu}$), and the corresponding compressive
strength $f_c$ is 18.9 MPa. These beams were made of rect-
angular cross sections (250 mm × 500 mm) and spanned
over 2600 mm. The beams were reinforced with steel rein-
forcement composed of five 22 mm bars in the tensile
zone and two 22 mm bars in the compression zone. In the
longitudinal direction of the beam, the specimens were
divided into two segments: the left segment without
stirrups and the right segment with embedded stirrups, as
shown in Figure 1. Stirrups with a space of 100 mm, a
diameter of 12 mm, and a stirrup ratio of 0.905% were
embedded in the right segment to avoid shear failure. The
segment without stirrups is the expected zone to encounter
shear failure in the RC and EVPR members.

The EVPR anchoring device for reinforcing the beam is
shown in detail in Figure 2. Four EVPR anchoring devices
are arranged on the side without the stirrups, and the ar-
range ment space is shown in Figure 1. Gauges PS1, PS2, PS3,
PS4, TS1, TS2, TS3, and SS were placed on the steel bars to
measure the strain values.

In this study, the following is the main basis for the
configuration of EVPR. Studies [31] showed that a vertical
compressive stress $\sigma_z$ which was applied in the shear span
of the beam can close the cracked diagonal crack and the
vertical compressive stress $\sigma_z$ is approximately 0.04$f_c$. The
vertical compressive stress $\sigma_z$ is calculated according to
the following equation:

$$\sigma_z = \frac{F_p}{(b \cdot s)} \text{ (MPa)}$$

where $F_p$ is the sum of the pretensioning forces of the EVPRs
on the cross section (N), $b$ is the width of the beam (mm),
and $s_p$ is the space of the EVPRs (mm).

Concrete with strength grade C30 was used in this study,
and its $f_c$ is approximately 18.9 MPa. Therefore, the vertical
compressive stress $\sigma_z$ should be 0.04 × 18.9 MPa, that is,
0.76 MPa. The research range of the vertical compressive
stress $\sigma_z$ was 0.51 MPa–0.74 MPa. The specific parameters of
the specimens are shown in Table 1.

The description of the members and related parameters
are provided in Table 2. Among them, the test beams RC-1
and RC-2 are the control beams; the other members are
reinforced with EVPRs. The information of the failure model
and ultimate bearing capacity of members is also provided in
Table 2. The terms $\rho_{EP}$ and EVPR shown in Table 2 are
detailed in Section 4.

2.2. Material Property. All RC beams were cast using C30
concrete, with a cube compressive strength of 28.16 MPa.
The obtained average cube compressive strength ($f'_c$) after
28 days of casting was 30.11 MPa. The steel bars and
stirrups used to reinforce the RC beam specimens were
HRB400 bars. The EVPRs were subjected to HPB300.
Uniaxial coupon tensile tests were employed to examine
the mechanical properties of the utilized reinforcing bars.
The characteristics, including elastic modulus $E_s$, yield
strength $f_y$, and ultimate strength $f_u$, of the steel bars are
listed in Table 3. More detailed material parameters are
given in [27].
3. Numerical Simulation of Test Specimens

FEMs were developed to investigate the behaviour of EVPR strengthening beams. Midas FEA software was used to create the FEMs. The designations of the developed FEMs that are compared with the experimental results are given in Table 2. This section provides details of the element types, material properties, loading and boundary conditions, and failure criteria of the developed FEMs.

3.1. Concrete. In this study, the rotating crack model of the total strain crack model was used to simulate concrete and the advantages of which are as follows: (1) the crack distribution is convenient to show; the crack unit does not separate at the crack position and (2) the crack direction changes with the direction of the main strain, only the normal stress is generated on the crack surface, and the calculation process is simpler.

The nonlinear properties of concrete under compression were assigned to the developed FEMs by defining...
the stress-strain relation developed by the parabolic hardening softening model [32, 33], which depends on three parameters: concrete compressive strength $f_c$, concrete fracture energy $G_c$, and concrete characteristic element length $h_c$, as shown in Figure 3. Concrete compressive strength $f_c$, concrete characteristic element length $h_c$, and elasticity modulus of concrete $E_c$ can be obtained through experimental tests.

The calculation formula of $G_c$ is shown in the following equation:

$$ G_c = G_{co} f_c^{0.7} $$

In equation (2), $f_{c,mo}$ is the benchmark average compressive strength, and its value is 10 N/mm$^2$. $G_{co}$ is related to the maximum aggregate size, and the corresponding relationship is listed in Table 4.

The peak strain $\varepsilon_{c0}$ corresponding to the concrete compressive strength $f_c$ and the ultimate compression strain at the softening stage are expressed in the following equations:

$$ \varepsilon_{c0} = \frac{4}{3} \frac{f_c}{E_c} $$

$$ \varepsilon_{cu} = \varepsilon_{c0} \frac{G_c}{h_c f_c} $$

The linear softening tension model [32, 34] employed nonlinear tension material properties, whose constitutive model consists of a linear ascending segment and a linear softening segment, as shown in Figure 4.

The constitutive model of the linear softening tension model includes two main parameters: the peak tension strain $\varepsilon_{t0}$ and the ultimate tension strain $\varepsilon_{tu}$, which are given by the following equations:

$$ \varepsilon_{t0} = \frac{f_t}{E_c} $$

$$ \varepsilon_{tu} = 2 \frac{1}{f_t} \frac{G_c}{h_c} $$

### Table 2: Main parameters of beams.

| Specimen label | Diameter (mm) | Space (mm) | Tension force (kN) | $\rho_{EPR}$ | Ultimate load (kN) | Failure mode |
|----------------|---------------|------------|--------------------|-------------|-------------------|--------------|
| RC-1           | —             | —          | —                  | —           | 253               | Diagonal tension failure |
| RC-2           | —             | —          | —                  | —           | 276               | Diagonal tension failure |
| EP-P22-S1      | 14            | 340        | 22                 | 0.36%       | 505.0             | Shear compression failure |
| EP-P22-S2      | 14            | 240        | 22                 | 0.51%       | 516.8             | Shear compression failure |
| EP-P22-S3      | 14            | 280        | 22                 | 0.44%       | 527.6             | Shear compression failure |
| EP-P26-S3      | 14            | 280        | 26                 | 0.44%       | 535.2             | Shear compression failure |
| EP-P18-S3      | 14            | 280        | 18                 | 0.44%       | 517.6             | Shear compression failure |

### Table 3: Properties of steel bars [27].

| Type                        | Diameter (mm) | Yield strength $f_y$ (MPa) | Ultimate strength $f_u$ (MPa) | Elastic modulus $E_s$ (MPa) |
|-----------------------------|---------------|---------------------------|-----------------------------|-----------------------------|
| EVPR                        | 14            | 300                       | 515                         | $2.1 \times 10^5$           |
| Stirrup                     | 12            | 445                       | 630                         | $2.0 \times 10^5$           |
| Longitudinal reinforcement  | 22            | 455                       | 670                         | $2.0 \times 10^5$           |

### Table 4: Relationship between $G_{co}$ and maximum aggregate size [32].

| $D_{max}$(mm) | $G_{co}$ (J/m$^2$) |
|---------------|--------------------|
| 8             | 23                 |
| 16            | 30                 |
| 32            | 58                 |

3.2. Steel Reinforcement. In this study, the reinforcement was modelled using embedded rebar elements that added the stiffness of the reinforcement to the parent elements. The basic properties of steel bars include position information, shape information, and physical properties without degrees.
of freedom. There is no slip between the rebar elements and the parent elements, and the strain of the rebar elements is calculated by the displacement of the parent elements.

The von Mises model is adopted in the constitutive structure of steel bars, which is a good response to the mechanical properties of metal materials. For EVPR, the ideal elasto-plastic model is used, and $\varepsilon_u = 13.2\%$. The trilinear model is used for longitudinal steel bars, where $f_u = 670\text{MPa}$ and $\varepsilon_u = 13\%$. The stress-strain curve of the steel bar is shown in Figure 5, where $f_u$ is the ultimate tensile strength of the steel bar and $f_y$ is the yield strength of the steel bar. In this study, the main mechanical parameters of the steel bars were obtained through experiments.

In order to analyse the parameters of the EVPR strengthening beam in Section 4, this part evaluates the test results to verify the validity of the numerical analysis method, and the baseline FEM for parameter analysis is obtained. The FEM is introduced in Sections 4.1–4.3, respectively, and the models of the control beam and strengthened beam are established and compared.

3.3. Analysis Model. The EVPR strengthening beam consists of concrete beams, longitudinal reinforcements, and stirrups in the concrete beams, EVPRs, and anchoring systems.

The anchoring systems contain EVPRs (HPB300: diameter 14 mm and length 800 mm), anchoring beam, nut, and backing plate. The entire FEM is meshed according to a size of 50 mm.

In the numerical analysis, EVPRs and the embedded rebar elements (including longitudinal reinforcements and stirrups) in the concrete beam are simulated by truss elements and rebar elements, respectively. Eight-node solid elements are utilized to simulate concrete beams and anchoring systems. The distribution of cracks, stress, and strain of solid elements can be clearly and directly observed in the analysis results. The concrete beam adopts the concrete total strain crack model; the EVPRs and the longitudinal reinforcement and stirrups are analysed by the von Mises model.

3.4. Numerical Analysis: Control Members. The control members in the experiment were modelled and analysed. The finite element prediction data were compared with the experimental data to verify the validity of the FEA method.

Figure 6(a) shows the principal tensile stress contour of the control beam, and Figure 6(b) shows the destruction results of the test. The diagonal tensile failure loads of the experimental and finite element analyses were 255 kN and 273 kN, respectively, with a difference of only 7.06%. Both show brittle failure, as shown in Figure 6(c).

The analysis results show that the concrete adopts the total strain crack model and the steel bar adopts the von Mises model, which can well simulate the control members.
3.5. Numerical Analysis: EVPR Strengthening Members. This section presents the results of the model analysis of the EVPR strengthening beams and compares them with the experimental data. Figure 7(a) shows the principal tensile stress contour of the strengthening beam, and Figure 7(b) shows the destruction results of the test.

3.5.1. Test Results. Shear compression failure occurred in the EVPR strengthening beam, and the shearing capacity and ductility were greatly improved compared with those of the control member.

First, during the loading process, there were many flexural cracks in the beam. As the loading force increased, the flexural cracks near the support developed into diagonal cracks and further produced one or several main diagonal cracks. Subsequently, the main diagonal cracks gradually developed, resulting in specimen shears and breaks (Figure 7(b)).

When shear failure occurs, the EVPRs near the loading point reach the yield strength. At the same time, the tensile stress of the longitudinal reinforcements does not reach the yield strength, which means that the structure does not exhibit flexural failure. Sometimes, the anchoring beam in the EVPR anchoring device showed local buckling.

3.5.2. FEA Simulation. As can be seen from Figure 7, FEA can simulate the development process of concrete cracks and the final failure form. The obtained shear capacity was approximately 7.90% different from the test results. In the FEA results, the tensile stress of some EVPRs reached the yield strength, and the longitudinal tensile steel did not reach the yield strength, which is consistent with the test results. In addition, Figure 7(c) shows the load-displacement curve of the FEM and the test, which are in good agreement with the force process, bearing capacity, and development after failure.

The comparison results of other FEMs and experimental comparison analysis results are similar to those of Figure 7 and are not illustrated one by one. It can be clearly seen from Figure 7 and Table 4 that the FEM predicted the crack development process, ultimate bearing capacity, and failure form, which agree well with the various experimental members.

Table 5 shows that the difference between the experimental results and the numerical results is essentially less than 10%, which indicates that the established FEM is effective and can be used to predict the effect of the shear-strengthened beam using EVPRs for the shear behaviour. The established FEM can also be used for the study of different parameters of the design to test the influence of parameters on the shear performance of the shear-strengthened beam using EVPRs.
4. Parametric Study

It was shown in Sections 2 and 3 that the diagonal tensile failure of the control specimens and the shear failure of the strengthened specimens were well simulated. In this section, multiparameter research is carried out based on the baseline FEM. The effects of various parameters on the shear performance of EVPR beams are studied using the vertical compressive stress degree and different EVPR stirrup ratios.

4.1. Effect of Vertical Compressive Stress Degree. The pre-tensioning force of the EVPRs is transmitted to the RC beam through an anchor device that forms a vertical compressive stress on the beam. The vertical compressive stress has an inhibitory effect on the development of diagonal cracks. To characterise the magnitude of the vertical compressive stress, the concept of vertical compressive stress degree $\gamma_p$ is proposed, which is the ratio of the vertical compressive stress to the tensile strength of the concrete. The calculation formula is as follows:

![Figure 7: Results of EP-P22-S2 beam: (a) FEM failure pattern, (b) experiment failure pattern, and (c) load-deflection response.](image)

![Table 5: Results contradistinction between experimental and FEM developed models.](table)
The load and midspan displacement curves of the four different FEMs of the vertical compressive stress degree $\gamma_p$ are shown in Figure 8. It is clearly shown in Figure 8 that the four curves are substantially coincident; that is, the four types of vertical compressive stress degree models have little difference in stiffness degradation and ultimate bearing capacity, and the final bearing capacity difference is less than 2%.

Figure 9 shows the axial tensile stress of EVPRs labelled PS3. The yield strength of the vertical prestressed steel bar is 300 MPa. When the load is 580 kN, 640 kN, 720 kN, and 780 kN, the EVPRs of $\gamma_p = 0.6$, $\gamma_p = 0.4$, $\gamma_p = 0.2$, and $\gamma_p = 0$ models reached the yield strength.

Before the EVPRs reach the yield strength, the concrete of the EVPR beam gradually cracks and the stiffness of the EVPR beam decreases, but the stiffness decreases slowly. After the EVPR yield, the stiffness of the reinforced beams decreases rapidly, and the load-displacement curve develops in an approximately horizontal direction. Thereafter, the load-carrying capacity of the reinforced beams increases slowly. At this time, because the longitudinal reinforcement does not reach the yield strength, the continued load can be provided.

The crack width located in the main crack in the middle of the beam height was investigated with the development of the load. When the loading force is 85 kN, cracks begin to appear in all four models. When the load is 300 kN, the crack widths of the four EVPR models $\gamma_p = 0$, $\gamma_p = 0.2$, $\gamma_p = 0.4$, and $\gamma_p = 0.6$ are 0.56 mm, 0.17 mm, 0.09 mm, and 0.05 mm, respectively, as shown in Figure 10. When the vertical prestress was 0.2, the crack width was reduced by 69.65% compared with the vertical prestressing. When the vertical prestress was 0.4 and 0.6, the crack width was further reduced to 83.91% and 91.07%. When the beam was damaged, the crack widths of the four models were close to 2 mm. The analysis results fully demonstrate that the vertical prestressing degree plays a favourable role in suppressing the development of cracks during the use phase of the structure, and the effect is very obvious.

4.2. Effect of EVPR Stirrup Ratio. Similar to the RC structure, this study proposes a new concept similar to the stirrup ratio of ordinary concrete beams. The EVPR stirrup ratio, $\rho_{\text{SEP}}$, is used to characterise the degree of EVPR placement. Tests have shown that the EVPR ratio has a significant impact on the shear performance of the beam.

The EVPR ratio is expressed as the ratio of the area of the EVPRs to the corresponding space and beam width and is calculated as follows:

$$\rho_{\text{SEP}} = \frac{A_{\text{SEP}}}{b \cdot s_p}$$

(8)

where $A_{\text{SEP}}$ is the total area of the EVPRs in one section, $b$ is the width of the beam, and $s_p$ is the space of the EVPRs in the longitudinal direction.

Based on the EVPR beam model of $\gamma_p = 0.2$ in Section 4.1, the effects of different EVPR ratios on the shear resistance of EVPR strengthening beams under the same vertical prestressing degree are studied. The longitudinal space of the EVPR was adjusted from 200 mm to 250 mm. Three FEMs with EVPR ratios of 0.4925%, 0.6435%, and 0.8144% were established. The detailed model parameters are listed in Table 7.

From the load-displacement curve in Figure 11, it can be seen that the shear bearing capacity of the EVPR strengthening beam increases with an increase in the EVPR stirrup ratio, and $\rho_{\text{SEP}} = 0.8144\%$ will increase by 12.1% compared with $\rho_{\text{SEP}} = 0.4925\%$.

The change in the width of the crack at the high-middle position of the diagonal cracked beam with loading is shown in Figure 12. Because beams with a small EVPR coupling ratio have relatively weak constraints on cracks, before EVPR yields, the greater the EVPR stirrup ratio, the better the degree of control over crack development. At the same time, it can be seen from Figure 13 that the smaller the EVPR stirrup ratio, the earlier the EVPR yield. Therefore, the strengthened beam with a lower EVPR stirrup ratio, especially after all the stirrups in the diagonal crack area have yielded, develops cracks more quickly. For the strengthened beams with a large EVPR stirrup ratio, the cracks can be well and smoothly controlled until the bearing capacity of the structure is reached.

It can be seen from the calculation and analysis that the larger the EVPR stirrup ratio, the better the shear performance. EVPRs are similar to ordinary stirrups and can enhance the shear bearing capacity of RC beams. In addition, EVPRs can provide not only vertical compressive stress but can also function as ordinary stirrups, thereby effectively controlling the development of crack width.

4.3. Effect of Combination of Area and Space of EVPRs under the Same $\rho_{\text{SEP}}$. It can be seen from equation (8) that the EVPR stirrup ratio is also related to the area and space of the EVPRs, except for the structure width $b$. For a given structure, the width $b$ is fixed so that the EVPR stirrup ratio mainly changes with the area and space of the EVPRs. The same EVPR stirrup ratio corresponds to countless combinations of EVPR area and space. In the formula, the EVPR area and space are the numerator and denominator, respectively. Therefore, for a given EVPR stirrup ratio, there are two types of EVPR arrangements: a small EVPR area...
with a small EVPR space or a large EVPR space. Under the same conditions of vertical compressive stress degree and EVPR stirrup ratio, the combination that is more beneficial to the shear capacity of the structure needs further study.

Based on the sample vertical compressive stress degree $c_p$ in Section 5.1, the EVPR stirrup ratio is set to $0.616\%$. Three configurations of EVPRs were used in this study:

- **Combination (i):** $A_{sEP} = 308\, \text{mm}^2$ (corresponding diameter: 14 mm) and $s_p = 200\, \text{mm}$
- **Combination (ii):** $A_{sEP} = 402\, \text{mm}^2$ (corresponding diameter: 16 mm) and $s_p = 261\, \text{mm}$
- **Combination (iii):** $A_{sEP} = 509\, \text{mm}^2$ (corresponding diameter: 18 mm) and $s_p = 330\, \text{mm}$

The specific parameters of the model are listed in Table 8. As shown in Figure 14, the combination of EVPRs is a sample of a small area of EVPRs with a small space of EVPRs. The shear capacity is the highest. Combination (i) (EVPRs with a diameter of 14 mm and a space of 200 mm) has an ultimate bearing capacity of 1050 kN, which is higher

| Vertical compressive stress degree $\gamma_p$ | Diameter of EVPRs (mm) | Space of EVPRs (mm) | EVPRs stirrup ratio $\rho_{sEP}$ (%) |
|---------------------------------------------|------------------------|---------------------|-------------------------------------|
| 0                                           | 14                     | 200                 | 1.539                               |
| 0.2                                         | 14                     | 200                 | 1.539                               |
| 0.4                                         | 14                     | 200                 | 1.539                               |
| 0.6                                         | 14                     | 200                 | 1.539                               |

**Table 6: Parameter table for vertical compressive stress degree.**
than that of Combination (ii) (diameter of 16 mm, space of 261 mm) and Combination (iii) (diameter of 18 mm, space of 330 mm). At the time of final failure, the EVPRs all yielded and had good ductility. However, as can be seen from Figure 15, the EVPRs of Combination (iii) yielded first and Combination (i) yielded last, so that the stiffness of Combination (iii) structure first decreased. At the same time, for the control of diagonal cracks, the small area of the EVPR dense arrangement is far better than the large-area EVPR sparse arrangement.

Therefore, in the theoretical calculations, a scheme with a small EVPR dense arrangement is more conducive to the bearing capacity and crack control of the structure. In practical applications, the densely arranged vertical prestressing steel bars will bring more construction burden, and supplementary anchoring measures are needed, and in combination with the actual situation, a scheme with a small EVPR area and a dense layout should be used as much as possible.

### Table 7: Parameter table for EVPR stirrup ratio.

| EVPR stirrup ratio $\rho_{\text{SPP}}$ | Space of EVPRs (mm) | Vertical compressive stress degree $\gamma_p$ | Diameter of EVPRs (mm) |
|--------------------------------------|---------------------|--------------------------------------------|------------------------|
| 0.4925                               | 250                 | 0.2                                        | 14                     |
| 0.6435                               | 250                 | 0.2                                        | 16                     |
| 0.8144                               | 250                 | 0.2                                        | 18                     |
5. Summary and Conclusion

Based on experimental research, this study analysed the shear behaviour of EVPR strengthening beams without web reinforcement using a three-dimensional FEM. The main conclusions are as follows:

(1) Both experiments and numerical analyses show that EVPRs can effectively improve the shear performance of concrete. The shear capacity can be significantly improved, and crack propagation can be well restrained.

(2) EVPRs can provide not only vertical compressive stress but also function as ordinary stirrups. Therefore, in the numerical analysis, the effect of EVPRs cannot be simulated only by applying vertical compressive stress, which will cause the effect of the stirrups to not be achieved. This study uses tensioned truss units to simulate EVPRs. The comparison results show that this method is effective.

(3) Vertical compressive stress has little effect on resisting shear capacity, but it can have a good inhibitory effect on crack development in the initial loading. With an increase in the vertical compressive stress, the initial crack suppression effect is more obvious.

(4) The EVPR stirrup ratio plays an important role not only in shear capacity but also in controlling cracks. With the increase in the EVPR stirrup ratio, the shear capacity is significantly improved, and diagonal cracks can be better controlled.

(5) Under the condition of the same EVPR stirrup ratio, a “small-area EVPR dense arrangement” is more suitable than the “big area EVPR sparse arrangement” in strengthening the shear performance of the reinforced beam.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this article.

Authors’ Contributions

Li Y., Wang W.Q., and Xue X.W. performed the methodologies, supervised the study, and reviewed and edited the article. Wu M.Z. performed the methodologies and investigations and wrote the original draft.

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References

[1] M. Aurelio and S. Joseph, "Behavior of beams and punching in slabs without shear reinforcement," in Proceedings of the IABSE Colloquium, Stuttgart, Germany, 1991.

[2] F. Cavagnis, M. F. Ruiz, and A. Muttoni, "A mechanical model for failures in shear of members without transverse reinforcement based on development of a critical shear crack," Engineering Structures, vol. 157, no. 1, pp. 300–315, 2018.

[3] B. Hu and Y.-F. Wu, "Effect of shear span-to-depth ratio on shear strength components of RC beams," Engineering Structures, vol. 168, pp. 770–783, 2018.

[4] B. Hu and Y.-F. Wu, "Quantification of shear cracking in reinforced concrete beams," Engineering Structures, vol. 147, pp. 666–678, 2017.

[5] Y. Wu and B. Hu, "Shear strength components in reinforced concrete members," Engineering Structures, vol. 143, no. 9, Article ID 04017092, 2017.

[6] E. Lantsoght, "Database of shear experiments on steel fiber reinforced concrete beams without stirrups," Materials, vol. 12, no. 6, p. 917, 2019.

[7] X. Xue, X. Hua, and J. Zhou, "Test and prediction of shear strength for the steel fiber–reinforced concrete beams," Advances in Mechanical Engineering, vol. 11, no. 4, pp. 1–15, 2019.

[8] M. M. Islam, M. S. Khatun, M. R. U. Islam, J. F. Dola, M. Hussan, and A. Siddique, "Finite element analysis of steel fiber reinforced concrete (SFRC): validation of experimental shear capacities of beams," Procedia Engineering, vol. 90, pp. 89–95, 2014.

[9] R. Zagon and K. Zoltan, "Shear behaviour of UHPC concrete beams," Procedia Technology, vol. 22, pp. 122–126, 2016.

[10] A. A. Bahraq, M. A. Al-Osta, S. Ahmad, M. M. Al-Zahrani, S. O. Al-Dulaijan, and M. K. Rahman, "Experimental and numerical investigation of shear behavior of RC beams strengthened by ultra-high performance concrete," International Journal of Concrete Structures and Materials, vol. 13, no. 1, pp. 33–51, 2019.

[11] X.-L. Ma, B.-C. Chen, Y. Yang et al., "Calculation method of shear bearing capacity of R-UHPC beam," Journal of Traffic & Transportation Engineering, vol. 17, no. 5, pp. 16–26, 2017.

[12] B. Taljsten, "Strengthening concrete beams for shear with CFRP sheets," Construction and Building Materials, vol. 17, no. 1, pp. 15–26, 2003.

[13] N. A. Hoult and J. M. Lees, "Modeling of an unbonded CFRP strap shear retrofitting system for reinforced concrete beams," Journal of Composites for Construction, vol. 13, no. 4, pp. 292–301, 2009.

[14] Y. Zhou, M. Guo, L. Sui et al., "Shear strength components of adjustable hybrid bonded CFRP shear-strengthened RC beams," Composites Part B, vol. 163, pp. 36–51, 2018.

[15] E. Kaya, C. Küstan, S. Sheikh, and A. Ilki, "Flexural retrofit of support regions of reinforced concrete beams with anchored FRP ropes using NSM and ETS methods under reversed cyclic loading," ASCE’s Journal of Composites for Construction, vol. 21, pp. 1574–1582, 2017.

[16] C. Challoris, P.-M. Kosmidou, and N. Papadopoulos, "Investigation of a new strengthening technique for RC deep beams using carbon FRP ropes as transverse reinforcements," Fibers, vol. 6, no. 3, p. 52, 2018.

[17] S. Altin, Ö. Anıl, and M. E. Kara, "Improving shear capacity of existing RC beams using external bonding of steel plates," Engineering Structures, vol. 27, no. 5, pp. 781–791, 2005.

[18] B. B. Adhikary and H. Mutsuyoshi, "Shear strengthening of RC beams with web-bonded continuous steel plates," Construction and Building Materials, vol. 20, no. 5, pp. 296–307, 2006.

[19] C. Challoris, V. Kytinou, M. Voutetaki, and N. Papadopoulos, "Repair of heavily damaged RC beams failing in shear using U-shaped mortar jackets," Buildings, vol. 9, no. 6, p. 146, 2019.

[20] R. Aboutaha and N. Burns, Shear Strengthening of Prestressed Prestressed Concrete Composite Flexural Members, University of Texas at Austin, Austin, TX, USA, 1991.

[21] S. Altin, T. Tankut, Ö. Anıl, and Y. Demirel, “Response of reinforced concrete beams with clamps applied externally: an experimental study,” Engineering Structures, vol. 25, no. 9, pp. 1217–1229, 2003.

[22] P. Bolger, Strengthening of Headstocks with Vertical Clamping to Enhance Shear Capacity, University of Southern Queensland, Toowoomba, Australia, 2006.

[23] D. Lei, G. Chen, Y. Chen, and Q. Ren, “Experimental research and numerical simulation of RC beams strengthened with bonded steel plates,” Science China Technological Sciences, vol. 55, no. 12, pp. 3270–3277, 2012.

[24] A. P. Lampropoulos, S. A. Paschalis, O. T. Tsiooulo, and S. E. Dritos, "Strengthening of reinforced concrete beams using ultra high performance fibre reinforced concrete (UHPFRC)," Engineering Structures, vol. 106, pp. 370–384, 2016.

[25] D. Ferreira, J. M. Bairán, and A. Mari, “Shear strengthening of reinforced concrete beams by means of vertical prestressed reinforcement,” Structure and Infrastructure Engineering, vol. 12, no. 3, pp. 394–410, 2016.

[26] H. Yin, K. Shirai, and W. Teo, "Numerical model for predicting the structural response of composite UHPFRC-concrete members considering the bond strength at the interface," Composite Structures, vol. 215, pp. 185–197, 2019.

[27] X. Xue, X. Wang, X. Hua et al., "Experimental investigation of the shear behavior of a concrete beam without web reinforcements using externally vertical prestressing rebars," Advances in Civil Engineering, vol. 2019, Article ID 3452056, 13 pages, 2019.

[28] X. Xue, J. Zhou, X. Hua, and H. Li, "Analysis of the generating and influencing factors of vertical cracking in abutments during construction," Advances in Materials Science and Engineering, vol. 2018, Article ID 1907360, 13 pages, 2018.

[29] X. Xue, M. Wu, Z. Li, and P. Zhou, "Numerical analysis of dead load shear force distribution in webs of multiclined inclined web box-girder bridge," Advances in Civil Engineering, vol. 2020, Article ID 9670704, 10 pages, 2020.

[30] Z. Li, M. Wu, J. Wu, Y. Cui, and X. Xue, "Steel fibre reinforced concrete meso-scale numerical analysis," Advances in Civil Engineering, vol. 2020, Article ID 2084646, 16 pages, 2020.

[31] M. Shamsi, H. Sezen, and A. Khaloo, "Behavior of reinforced concrete beams post-tensioned in the critical shear region," Engineering Structures, vol. 29, no. 7, pp. 1465–1474, 2007.

[32] Midas IT, Midas FEA Analysis and Algorithm Manual, Midas IT (Beijing) Co., Ltd., 2017, http://www.MIDASUser.com.

[33] P. H. Feenstra, "Computational aspects of biaxial stress in plain and reinforced concrete," Ph.D thesis, TU Delft: Department of Civil Engineering, Delft, Netherlands, 1993.

[34] D. A. Hordijk, ”Local approach to fatigue of concrete,” Ph.D thesis, TU Delft: Department of Civil Engineering, Delft, Netherlands, 1991.