A reply to “Problems with modelling closed timelike curves as post-selected teleportation”

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In arXiv:1107.4675 Ralph uses our post-selection model of closed timelike curves (P-CTC) to construct an “unproved-theorem” paradox, and claims that this voids our argument that P-CTCs are able to resolve such types of paradoxes. Here we point out that Ralph has not accounted for all the interactions needed for his construction. A more careful analysis confirms that indeed there is no paradox, contrary to his claims.

In [1] we proposed a quantum prescription for dealing with the closed timelike curves that arise (for example) from certain solutions of Einstein’s equations of general relativity [2]. Our model, P-CTC, is based on quantum teleportation with post-selection. Closed timelike curves allow for non-causal trajectories in spacetime, also known as time machines. Such devices notoriously generate time-travel paradoxes, such as the “unproven-theorem” paradox: a time traveler reads a theorem in a book, travels back in time and narrates this theorem to a mathematician of the past that writes the theorem in a book, travels back in time and narrates this theorem to a mathematician of the past that writes the theorem in a book, the same book the traveller will consult. While not a logical contradiction, this paradox is inherently unsatisfactory [3]. In [1] we gave an argument on how P-CTC might resolve the unproven theorem paradox.

In [4] Ralph has employed our model to purportedly generate an unproved-theorem paradox, claiming that this voids our argument above. In particular, Ralph uses a well-known nonlinear mechanism of CTCs [3] to allow Bob to send information to the past using a phase flip transformation (see Fig.1a). This by itself is not sufficient to generate paradoxes, since it is well known that self-consistent time loops are possible (e.g. see [1]). However, Ralph suggests that Alice can use the information that Bob is sending back in time to her to write a theorem in a book. In Alice's future, Bob uses the same book on which Alice has written the theorem to decide which information to send her back in time. The theorem has come out of nowhere since Alice has learned it from Bob, and Bob from Alice: the unproven-theorem paradox.

A more careful analysis shows that Ralph’s argument is incomplete, since he has not analyzed the transformation that Bob must implement to read the theorem from Alice’s book and to write this information into the CTC through the phase-flip transformation (to send it back in time to Alice). When one analyzes the action of such transformation, then it becomes clear that the post-selected teleportation mechanism of the P-CTC intervenes and prevents the paradox from happening.

To read Alice’s book and to write it into the CTC with the phase-flip, Bob will use a unitary transformation: in fact, any physical transformation can always be purified into a unitary, at the expense of considering also the evolution of the environment. In other words, Bob’s phase-flip must be part of a controlled-unitary that uses as input Alice’s book, or something (e.g. some part of...
the environment) correlated to Alice’s book (see Fig. 1b).

Let us analyze two possible controlled unitaries that Bob can use to implement Ralph’s proposal. The first one is a simple C-NOT in the $+, -$ basis, where the control bit is Alice’s book (or some environmental degree of freedom correlated to Alice’s book) and the controlled bit is the one he sends through the CTC, Ralph’s phase-flip transformation. The C-NOT acts as follows on a basis:

$|++\rangle \rightarrow |+\rangle$;
$|+-\rangle \rightarrow |++\rangle$;
$|--\rangle \rightarrow |-\rangle$.

Implementing Bob’s transformation in this way, we obtain an impossibility when post-selection is introduced: the amplitude for the whole process of Fig. 1b is always zero. The second one is a “copy operation on the $+, -$ basis”, where the first bit (Alice’s book) is the source, and the second bit (Bob’s phase-flip) is the destination, namely

$|++\rangle_e \rightarrow |++\rangle$;
$|+-\rangle_e \rightarrow |++\rangle$;
$|--\rangle_e \rightarrow |--\rangle$.

Implementing Bob’s transformation in this way (initializing the environmental qubit $e$), we find a tautology: Bob can only write a single fixed state ($|++\rangle$) as his theorem, namely his theorem is vacuous. In fact, one cannot argue in this case that the state $|+\rangle$ is the theorem itself: it is true that the appearance of this state is determined by the structure itself of the P-CTC (i.e. Alice’s state and Bob’s actions). However, once this has been established, the map has a single pure-state fixed point that cannot sustain arbitrary information that could encode an arbitrary theorem (in contrast, for example, to Deutsch’s map 3). Even though both Alice and Bob can choose which will be the single fixed point of the map, this is inequivalent to the unproven-theorem paradox (the theorem appearing out of nowhere). On the contrary, if Bob is aware of Alice’s choice of initial state, he can write a theorem in the time loop with his choice of transformations. But that implies that Bob is the theorem’s author: it has not appeared out of nowhere. [Equivalently, if Alice is aware of what transformations Bob will implement, she can write the theorem in the loop with her choice of the initial state, but again this is not an unproven-theorem paradox because she definitely is the author herself.] Analogous considerations will hold for physical realizations of the P-CTC different from the one analyzed by Ralph.

Note that in 4 we give a completely different resolution to the unproven-theorem paradox.

In conclusion, Ralph’s treatment is incomplete so that he fails in obtaining an unproved-theorem paradox from his scheme, contrary to his claims.

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