On the self-consistency of evolutionary synthesis models

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Abstract. Evolutionary synthesis models have been used to study the physical properties of unresolved populations in a wide range of scenarios. Unfortunately, their self-consistency are difficult to test and there are some theoretical open questions without an answer: (1) The change of the homology relations assumed in the computation of isochrones due to the effect of stellar winds (or rotation) and the discontinuities in the stellar evolution are not considered. (2) There is no a consensus about how the isochrones must be integrated. (3) The discreteness of the stellar populations (that produce an intrinsic statistical dispersion) usually are not taken into account, and model results are interpreted in a deterministic way instead a statistical one... The objective of this contribution is to present some inconsistencies in the computation and some cautions in the application of the results of such codes.

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1. Introduction

In the last few years, the increasingly detailed observations of stellar populations in a wide variety of environments have provided a huge amount of high quality data, which have been used to constrain both stellar evolution theory and stellar models among many other variables.

In contrast, the development of increasingly complicated evolutionary synthesis codes has, in general, focused only on the use of updated physical ingredients, but some of their underlying hypotheses (track interpolation and isochrone computation in particular) have remained unchanged. Unfortunately, their self-consistency are difficult to test and there are some theoretical and practical open questions without an answer.

2. From tracks to isochrones: Equivalent evolutionary points

An homology transformation gives us a scale relation of a property $x$ of stars ($L$, $T_{\text{eff}}$, $\tau_{\text{ms,life}}$) with their Mass, $M$, mean molecular weight, $\mu$, nuclear energy production, $\epsilon_0$, opacity coefficient, $\kappa_0$, and Hydrogen abundance in the core, $X$, for the same configuration. As an example,
for the CNO burning, Kramers opacity law and radiative transport configuration the relations are:

\[ R(M) \propto (\epsilon_0 \kappa_0)^{1/20} \mu^{13/20} M^{4/5} \]
\[ L(M) \propto \kappa_0^{-1} \mu^4 M^3 \]
\[ \tau_{ms}(M) \propto \kappa_0 \mu^{-4} X M^2 \]

Other configurations means, mainly, a change in the exponents. These relations, together with \( L \propto R^2 T_{\text{eff}}^4 \), allow us to define equivalent evolutionary points: points where \( \kappa_0, \mu \) (structure) and \( \epsilon_0, X \) (burning state) are similar. Then, interpolations must be done in the \( \log M - \log x \) plane. Note that the \( M \) value used in the homology relations does not refer to the initial mass, but the current mass of the star in the equivalent evolutionary point.

This simple scheme would be a good approximation if (a) the stellar tracks are close enough, (b) there are no discontinuities in the stellar evolution (i.e. adjacent tracks “resemble” each other), and (c) stellar winds do not vary the stellar evolution. Unfortunately, these conditions are not always true. There are intrinsic discontinuities in the stellar evolution at low mass (c.f. Tinsley & Gunn 1976) and high mass (Wolf-Rayet, WR, vs. non-WR tracks). The problem is specially dramatic in high-mass stars, where the effects of stellar winds strongly affects the stellar evolution and burning lifetimes of the stars, and may produce failures in the use of homology relations when the initial mass is used. Even more, there are no homology relations to compute mass lost rates (which are, indeed, an input of the evolutionary tracks) neither the surface abundances (which are related with the internal structure and the mass lost rate and which are fundamental for obtain the WR population). It means that, at least in the case of massive stars, where the mass lost rates affects the stellar evolution, isochrones do not necessary reflect the physics of the assumed tracks if they are obtained from the interpolation of the initial mass (as it is usual in synthesis codes for starburst galaxies).

3. From isochrones to integrated properties

Assuming that the isochrone reflect the input physics used in the evolutionary tracks, the next step is to populate the isochrone and obtain the integrated properties. It can be done by a direct convolution of
from the isochrone with the Initial Mass Function (IMF) and the Star Formation Rate, together with the individual stellar properties.

This integration can be done using Monte Carlo simulations (where the number of stars used in the simulation plays a fundamental role), analytical integrations (but it is needed an analytical formulation of the isochrone and the stellar properties), or numerical integrations (where the mass intervals must be chosen in such a way that all the relevant evolutionary phases are correctly included in the computation).

Other possible solution is the use of the so-called Fuel Consumption Theorem, FCT, (Buzzoni, 1989). In this case, the relevant Post-Main Sequence stages are approximated by a single stellar track of the star with mass at the turn-off point of the Main Sequence, \( M_{TO} \). Note that it is needed to assure that the isochrone and the \( M_{TO} \) stellar track are similar enough. I refer to Girardi & Bertelli (1998, sect. 2) or Marigo & Girardi (2001) for more information.

However, the results of synthesis codes differs qualitatively, depending on the way how this integration is performed...

4. The use of synthesis models

The final question is the comparison of the results of synthesis models with real data. In this case, it must be taken into account that (1) synthesis models give us a mean value of the observed properties (that is only exact in the asymptotic limit of an infinite number of stars in the cluster), and (2) the total luminosity of the modeled cluster must be larger than the individual luminosity of any of the stars included in the model. This last constrain imposes a minimum amount of stars (or, equivalently, a minimum initial stellar cluster mass for single stellar populations or star forming rate for composed ones) where the results of synthesis models can be compared safely with real data. This limits is around \( 10^5 \) \( M_\odot \) for a single stellar population using a Salpeter IMF slope in the range 0.09–120 \( M_\odot \) (Cerviño 2002 submitted). Below this limit, the presence of ±1 star may influence the resulting properties and sampling effects must be included in the model. Even more, the resulting properties may show bimodal distributions and the results of synthesis models may be biased respect the the observed properties (Cerviño & Valls-Gabaud 2002). The relevance of these sampling effects have been illustrated in Bruzual (2002) and Cerviño et al. (2000, see also Cerviño et al. 2002 and references therein for an analytical

\footnote{Note that the IMF gives us a probability of obtain certain amount of stars in a given mass interval.}
formalism). These sampling effects may also play an important role in chemical evolution models, as it is shown in Cerviño & Mollá (2002).

5. Conclusions

It is needed a more careful (physical) study in the way how interpolations between tracks are done, specially for massive stars (WR). There are also some open questions that, at this moment, had not been solved theoretically, as example: What is the scheme needed to obtain realistic results for systems with metallicities that differs from the one tabulated in tracks and isochrones? How homology relations works in rotating stars?

From the observational point of view, any comparison of the results of synthesis models with real data must take into account that the synthesis model results are a BAND, with a intrinsic dispersion that depends on the cluster stellar mass, instead an infinitely narrow line. Evermore, there is an intrinsic limit in the cluster stellar mass below any comparison of observed data with synthesis models must be performed including sampling effects in the model.

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