We suggest that the recently observed charmed scalar mesons $D_s^0(2308)$ (BELLE) and $D_s^{0+}(2405)$ (FOCUS) are considered as different resonances. Using the QCD sum rule approach we investigate the possible four-quark structure of these mesons and also of the very narrow $D_s^{++}(2317)$, firstly observed by BABAR. We use diquark-antidiquark currents and work to the order of $1/m_s$ in full QCD, without relying on $1/m_c$ expansion. Our results indicate that a four-quark structure is acceptable for the resonances observed by BELLE and BABAR: $D_s^0(2308)$ and $D_s^{++}(2317)$ respectively, but not for the resonances observed by FOCUS: $D_s^{0+}(2405)$.

Recently the first observations of the scalar charmed mesons have been reported. The very narrow $D_s^{++}(2317)$ was first discovered in the $D_s^-\pi^0$ channel by the BABAR Collaboration [1] and its existence was confirmed by CLEO [2], BELLE [3] and FOCUS [4] Collaborations. Its mass was commonly measured as 2317 MeV, which is approximately 160 MeV below the prediction of the very successful quark model for the charmed mesons [5]. The BELLE Collaboration [6] has also reported the observation of a rather broad scalar meson $D_s^0(2308)$, and the FOCUS Collaboration[7] reported evidence for broad structures in both neutral and charged final states that, if interpreted as resonances in the $J^P = 0^+$ channel, would be the $D_s^0(2407)$ and the $D_s^{0+}(2403)$ mesons. While the mass of the scalar meson, $D_s^0(2308)$, observed by BELLE Collaboration is also below the prediction of ref. [5] (approximately 100 MeV), the masses of the states observed by FOCUS Collaboration are in complete agreement with ref. [5].

Due to its low mass, the structure of the meson $D_s^{++}(2317)$ has been extensively debated. It has been interpreted as a $c\bar{s}$ state [5, 10, 11, 12], two-meson molecular state [13, 14], $D-K$ mixing [13], four-quark states [16, 17, 18] or a mixture between two-meson and four-quark states [19]. The same analyses would also apply to the meson $D_s^0(2308)$.

In the light sector the idea that the scalar mesons could be four-quark bound states is not new [20] and, therefore, it is natural to consider analogous states in the charm sector.

We propose that the resonances observed by BELLE [6] and FOCUS [7] Collaborations be considered as two different resonances. In this work we use the method of QCD sum rules (QCDSR) [21] to study the two-point functions of the scalar mesons, $D_{sJ}(2317)$, $D_0^*(2308)$ and $D_0^*(2405)$ considered as four-quark states. The use of the QCD sum rules to study the charmed scalar mesons was already done in refs. [8, 11, 12], but in these calculations they were interpreted as two-quark states.

In a recent calculation [22], some of us have considered that the lowest lying scalar mesons are $S$-wave bound states of a diquark-antidiquark pair. As suggested in ref. [23] the diquark was taken to be a spin zero colour anti-triplet. We extend this prescription to the charm sector and, therefore, the corresponding interpolating fields containing zero, one and two strange quarks are:

\[
\begin{align*}
 j_0 & = \epsilon_{abc}\epsilon_{dec}(q_a^T C\gamma_5 c_b)(\bar{u}_d\gamma_5 C\bar{d}_c^T), \\
 j_s & = \frac{\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}} [(u_a^T C\gamma_5 c_b)(\bar{u}_d\gamma_5 C\bar{d}_c^T) + u \leftrightarrow d], \\
 j_{ss} & = \epsilon_{abc}\epsilon_{dec}(s_a^T C\gamma_5 c_b)(\bar{d}_a\gamma_5 C\bar{s}_b^T),
\end{align*}
\]

where $a$, $b$, $c$, ... are colour indices, $C$ is the charge conjugation matrix and $q$ represents the quark $u$ or $d$ according to the charge of the meson. Since $D_{sJ}$ has one $s$ quark, we choose the $j_s$ current to have the same quantum numbers of $D_{sJ}$, which is supposed to be an isoscalar. However, since we are working in the SU(2) limit, the isoscalar and isovector states are mass degenerate and, therefore, this particular choice has no relevance here.

The QCDSR for the charmed scalar mesons are constructed from the two-point correlation function

\[
\Pi(q) = i \int d^4x \, e^{iq.x} \langle 0|T[j_{sJ}(x)j_{sJ}^\dagger(0)]|0\rangle.
\]

The coupling of the scalar meson, $S$, to the scalar current, $j_S$, can be parametrized in terms of the meson decay constant $f_S$ as [22]: $\langle 0|j_S|S\rangle = \sqrt{2}f_Sm_S^2$, therefore, the phenomenological side of Eq. (2) can be written as

\[
\Pi^{phen}(q^2) = \frac{2f_S^2m_S^8}{m_S^2 - q^2} + \cdots,
\]

where the dots denote higher resonance contributions that will be parametrized, as usual, through the introduction of the continuum threshold parameter $s_0$ [24].

In the OPE side we work at leading order and consider condensates up to dimension six. We deal with the strange quark as a light one and consider the diagrams up to order $m_s$. To keep the charm quark mass finite, we use the momentum-space expression for the charm quark propagator. We follow ref. [25] and calculate the light
quark part of the correlation function in the coordinate-space, which is then Fourier transformed to the momentum space in $D$ dimensions. The resulting light-quark part is combined with the charm-quark part before it is dimensionally regularized at $D = 4$.

We can write the correlation function in the OPE side in terms of a dispersion relation:

$$\Pi^{OPE}(q^2) = \int_{m_s^2}^{\infty} ds \frac{\rho(s)}{s - q^2}, \quad (4)$$

where the spectral density is given by the imaginary part of the correlation function: $\rho(s) = \frac{1}{2i} \text{Im}[\Pi^{OPE}(s)]$. After making a Borel transform on both sides, and transferring the continuum contribution to the OPE side, the sum rule for the scalar meson $S$ can be written as

$$2 f_S^2 m_S^2 e^{-m_S^2/M^4} = \int_{s_0}^{s} ds e^{-s/M^2} \rho_S(s), \quad (5)$$

where $\rho_S(s) = \rho^{pert}(s) + \rho^{mix}(s) + \rho^{G}(s)$, with

$$\rho^{pert}(s) = \frac{1}{2\pi^3} \int_{\Lambda}^{1} d\alpha \left( \frac{1 - \alpha}{\alpha} \right)^3 (m_c^2 - s)^4 \quad (6)$$

$$\rho^{G}(s) = \frac{\langle g^2 G^2 \rangle}{2\pi^3 \pi^6} \int_{\Lambda}^{1} d\alpha \left( \frac{m_c^2 - s}{s} \right) \left[ \frac{m_c^2}{9} \left( \frac{1 - \alpha}{\alpha} \right)^3 + \left( m_c^2 - s \right) \left( \frac{1 - \alpha}{2\alpha} + \frac{(1 - \alpha)^2}{4\alpha^2} \right) \right], \quad (7)$$

$$\rho^{mix}(s) + \rho^{\langle \bar{q}q \rangle}(s) + \rho^{\langle G^2 \rangle}(s), \quad (8)$$

which are common to all three resonances and where the lower limit of the integration is given by $\Lambda = m_c^2/s$.

From $j_0$ we get: $\rho^{m_s}(s) = 0$,

$$\rho^{\langle \bar{q}q \rangle}(s) = -\frac{m_s}{2\pi^4} \int_{\Lambda}^{1} d\alpha \left( \frac{1 - \alpha}{\alpha} \right)^2 (m_c^2 - s)^2, \quad (9)$$

$$\rho^{mix}(s) = \frac{m_s \langle \bar{q}q \rangle G_q}{2\pi^4} \left[ \frac{1}{2} \int_{\Lambda}^{1} d\alpha \left( \frac{1 - \alpha}{\alpha} \right)^2 (m_c^2 - s^2) + \int_{\Lambda}^{1} d\alpha \frac{1 - \alpha}{\alpha} (m_c^2 - s) \right], \quad (10)$$

$$\rho^{\langle \bar{q}q \rangle^2}(s) = -\frac{\langle \bar{q}q \rangle^2}{12\pi^2} \int_{\Lambda}^{1} d\alpha (m_c^2 - s). \quad (11)$$

From $j_s$ we get: $\rho^{m_s}(s) = 0$,

$$\rho^{\langle \bar{q}q \rangle}(s) = \frac{1}{2\pi^4} \int_{\Lambda}^{1} d\alpha \frac{1 - \alpha}{\alpha} (m_c^2 - s)^2 \left[ -\langle \bar{q}q \rangle \left( 2m_s + m_c \frac{1 - \alpha}{\alpha} \right) + m_s \beta \right], \quad (12)$$

$$\rho^{mix}(s) = \frac{1}{2\pi^4} \int_{\Lambda}^{1} d\alpha (m_c^2 - s) \left[ -m_s \langle \bar{s}g\sigma G_s \rangle \frac{1}{6} \right. \left. + \langle \bar{q}q \rangle \left( m_s(1 - \ln(1 - \alpha)) - \frac{1 - \alpha}{2\alpha} \right) \right], \quad (13)$$

Finally from $j_{ss}$ we get

$$\rho^{m_s}(s) = \frac{m_s m_c}{2\pi^4} \int_{\Lambda}^{1} d\alpha \left( \frac{1 - \alpha}{\alpha} \right)^3 (m_c^2 - s)^3, \quad (15)$$

$$\rho^{\langle \bar{q}q \rangle}(s) = \frac{1}{2\pi^4} \int_{\Lambda}^{1} d\alpha \left( \frac{1 - \alpha}{\alpha} \right) (m_c^2 - s_0)^2 \left[ \left( 2m_s - m_c \frac{1 - \alpha}{\alpha} \right) - 2m_s \langle \bar{q}q \rangle \right], \quad (16)$$

$$\rho^{mix}(s) = \frac{1}{2\pi^4} \int_{\Lambda}^{1} d\alpha (m_c^2 - s) \left[ \langle \bar{s}g\sigma G_s \rangle \frac{m_s}{3} \right. \left. - m_s \frac{1 - \alpha}{\alpha} - m_c \frac{1 - \alpha}{2\alpha} \left( 1 - \frac{1 - \alpha}{2\alpha} \right) \right], \quad (17)$$

$$\rho^{\langle \bar{q}q \rangle^2}(s) = -\frac{\langle \bar{q}q \rangle^2}{12\pi^2} \int_{\Lambda}^{1} d\alpha (m_c^2 - s). \quad (18)$$

For the charm quark propagator with two and three gluons attached we use the momentum-space expressions given in ref. \textit{26}.

In order to get rid of the meson decay constant and extract the resonance mass, $m_S$, we first take the derivative of Eq. \textit{12} with respect to $1/M^2$ and then we divide it by Eq. \textit{15} to get

$$m_S^2 = \frac{\int_{s_0}^{s} ds e^{-s/M^2} \rho_S(s)}{\int_{m_c^2}^{s} ds e^{-s/M^2} \rho_S(s)}. \quad (19)$$

In the numerical analysis of the sum rules, the values used for the quark masses and condensates are: $m_s = 0.13 \text{ GeV}$, $m_c = 1.2 \text{ GeV}$, $\langle \bar{q}q \rangle = -(0.23)^3 \text{ GeV}^3$, $\langle \bar{s}s \rangle = 0.8 \langle \bar{q}q \rangle$, $\langle \bar{q}g\sigma G_q \rangle = m_c^3 \langle \bar{q}q \rangle$ with $m_c^2 = 0.8 \text{ GeV}^2$, $\langle g^2 G^2 \rangle = 0.5 \text{ GeV}^4$ and $\langle g^2 G^2 \rangle = 0.045 \text{ GeV}^6$. The value for the quark condensate was obtained using the Gell-Mann - Oakes - Renner relation, and the mass of the light quarks, $m_u + m_d = 14 \text{ MeV}$, at the renormalization scale of $1 \text{ GeV}$. \textit{27}. Since the charm quark mass introduces a natural scale in the problem, we chose to work at the renormalization scale of $m_c \sim 1 \text{ GeV}$. 

and the strange quark masses in the intervals: $1.1 \leq m_s \leq 1.3$ GeV and $0.11 \leq m_s \leq 0.15$ GeV, we get results for the resonance masses still between the lower and upper lines in figures 1 and 2. A bigger value for the charm quark mass makes the results more stable as a function of the Borel mass. One can also vary the value of the quark condensate. Keeping the continuum threshold and the quark masses fixed at $\sqrt{s_0} = 2.7$ GeV, $m_c = 1.2$ GeV and $m_s = 0.13$ GeV and varying the quark condensate in the interval: $(\bar{q}q) = (-0.23 \pm 0.01 \text{GeV})^3$, we get a bigger (smaller) result for the resonance masses using a smaller (bigger) value of the condensate. In Fig. 3 we show the the mass of the $D_0^{(1s)}$ state, as a function of the Borel mass, for the combination of the values of the continuum threshold and quark condensate that gives the lower and upper limits for the $D_0^{(1s)}$ mass.

In ref. [28] it was shown that the renormalization scale was an important source of uncertainty, in the analysis of the $B$ meson decay constant. To check how the change of the scale would change our results we also show, through the dashed line in Fig. 3, the result for the $D_0^{(1s)}$ resonance mass using the values of the strange quark mass and quark condensate at the scale 2 GeV: $(\bar{q}q)(2 \text{GeV}) = (-0.267 \text{GeV})^3$ and $m_s(2 \text{GeV}) = 0.10 \text{GeV}$ [28]. We see that we get a less stable result for the resonance mass, but it is still compatible with the results at the scale 1 GeV, considering the variation in the continuum threshold. Therefore, we conclude that it is the variation of the continuum threshold that causes the most significant variations in the resonance masses, and it is our most important source of uncertainty.
Comparing figures 1 and 2 we see that the $D_{1s}^{(1s)}$ and $D_{0}^{(2s)}$ resonance masses are basically degenerated, while the mass of $D_{0}^{(0s)}$ is around 100 MeV smaller than the others. While it is natural to expect that the inclusion of a strange quark would increase the resonance mass by around the strange quark mass (as was the case when one goes from $D_{0}^{(2s)}$ to $D_{0}^{(1s)}$), it is really interesting to observe that this does not happen when one goes from $D_{1s}^{(1s)}$ to $D_{0}^{(2s)}$. In terms of the OPE contributions, we can trace this behavior to the fact that the quark condensate compensates this decrease.

Considering the variations on the quark masses, the quark condensate and on the continuum threshold discussed above, in the Borel window considered here our results for the resonance masses are given in Table I.

| resonance               | $D_{0}^{(0s)}$ | $D_{0}^{(1s)}$ | $D_{0}^{(2s)}$ |
|-------------------------|---------------|---------------|---------------|
| mass (GeV)              | $2.22 \pm 0.21$ | $2.32 \pm 0.18$ | $2.30 \pm 0.20$ |

Comparing the results in Table I with the resonance masses given by BABAR, BELLE and FOCUS: $D_{sJ}^{(+)}(2317)$, $D_{sJ}^{0}(2308)$ and $D_{sJ}^{0,+}(2405)$, we see that we can identify the four-quark states represented by $D_{0}^{(1s)}$ and $D_{0}^{(2s)}$ with the BABAR and BELLE resonances respectively. However, we do not find a four-quark state whose mass is compatible with the FOCUS resonances, $D_{sJ}^{0,+}(2405)$. Therefore, we associate the FOCUS resonances, $D_{sJ}^{0,+}(2405)$, with a scalar $cq$ state, since its mass is completely in agreement with the predictions of the quark model in ref. [5]. It is also interesting to point out that a mass of about 2.4 GeV is also compatible with the QCD sum rule calculation for a $cq$ scalar meson [11].

One can still argue that while a pole approximation is justified for the very narrow BABAR resonance, this may not be the case for the rather broad BELLE and FOCUS resonances. To check if the width of the resonances could modify the pattern observed in the masses of the four-quark states, we have modified the phenomenological side of the sum rule, in Eq. [22], through the introduction of a Breit-Wigner-type resonance form:

$$\Pi^{phen}(M^2) = 2f_{S}^{2}m_{S}^{2}\int_{(m_{s}+m_{D})}^{s_{0}}ds \ e^{-s/M^{2}}\rho_{BW}(s),$$

where

$$\rho_{BW}(s) = \frac{1}{\pi}\frac{\Gamma(s)m_{S}}{(s-m_{S}^{2})^{2}+m_{S}^{2}\Gamma(s)^{2}},$$

with $\Gamma(s) = \Gamma_{0}\sqrt{\frac{\lambda(s,m_{s}^{2},m_{D}^{2})}{\lambda(m_{s},m_{D},m_{D}^{2})}}\frac{m_{S}^{2}}{s}$, and $\lambda(x,y,z) = x^{2}+y^{2}+z^{2}-2xy-2xz-2yz$.

Of course now we can not obtain an expression for the resonance mass as Eq. [19]. However, we can still use the resonance mass as a parameter to compare the compatibility between the right-hand side (RHS) and the left-hand side (LHS) of the sum rule in Eq. [22]:

$$\frac{\int_{(m_{s}+m_{D})}^{s_{0}}ds e^{-s/M^{2}}\rho_{BW}(s)}{\int_{(m_{s}+m_{D})}^{s_{0}}ds e^{-s/M^{2}}\rho_{BW}(s)} = \int_{m_{s}^{2}+m_{D}^{2}}^{s_{0}}\frac{ds}{\sqrt{s}}.$$

In Fig.4 we show the RHS (solid line) and the LHS of the sum rule in Eq. [22] for $D_{0}^{(0s)}$, for different values of the resonance mass. Dashed line: $m_{s} = 2.1$ GeV; dotted line: $m_{s} = 2.2$ GeV; dot-dashed line: $m_{s} = 2.3$ GeV.

![FIG. 4: The RHS (solid line) and the LHS of the sum rule in Eq. (22) for $D_{0}^{(0s)}$, for different values of the resonance mass. Dashed line: $m_{s} = 2.1$ GeV; dotted line: $m_{s} = 2.2$ GeV; dot-dashed line: $m_{s} = 2.3$ GeV.](image-url)
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