The GeoFlow experiment - spherical
Rayleigh-Bénard convection under the influence of
an artificial central force field

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Abstract. Spherical Rayleigh-Bénard convection under the influence of an artificial central
force field produced by the so-called dielectrophoretic effect is studied as a simplified model of
the flow in the outer earth core. The fluid motion there is most probably driving the earth’s
dynamo and the energy source for the earth’s magnetic field. Studying convective flows in
earth-like geometry could lead to a deeper understanding of the basics of these processes.
This research is a preparatory study for the experiments on the International Space Station
(ISS). A bifurcation-theoretical approach shows the existence of heteroclinic cycles between
spherical modes \((l, l + 1)\) for the non-rotating system. This behavior depends strongly on the
radius ratio of the spheres and will be hard to detect in the experiment. For slow rotations
interactions of the azimuthal modes \((m, m + 1)\) found in numerical simulations for supercritical
states are supposed to be experimentally observable.

1. Introduction
Experiments with convection caused by an artificial central force field in a microgravity envi-
ronment are on the one hand a technical challenge and on the other hand important for a better
understanding of large-scale astrophysical and geophysical motion. Only without the influence
of the axial force due to gravity in the laboratory, can the effects of the pure central field be
studied. The GeoFlow experiment chosen for running on ISS aims at simulating the spherical
Rayleigh-Bénard problem, i.e. thermal convection in the gap between two concentric spheres
with different radii \(R_2 > R_1\). The central force field is generated by using the dielectrophoretic
effect [4]. Different from gravity the dielectrophoretic effect leads to a \(1/r^5\) dependent force field.
The governing equations are used in the Boussinesq approximation for an incompressible fluid
[8]. The model depends on four parameters: the rotation rate given by the Taylor number \(Ta\),
the strength of the central force field measured by the Rayleigh number \(Ra\), the radius ratio of
inner and outer shell \(\eta = R_1/R_2\) and the material properties of the fluid, such as the viscosity
and thermal conductivity, determined by the Prandtl number \(Pr\).
2. Bifurcation analysis

For the non-rotating case, generically only a single $l$-mode becomes unstable and stationary states are expected. However, the linear stability curves show that, for some critical values of $\eta$, two consecutive $l$-modes coexist ([7], [5]). Because of experimental limitations only the $(l, l+1)$ interactions with $l \geq 2$ are possible. Thus, we are focusing on the (2, 3) and (3, 4) mode interactions. At these codimension 2 bifurcation points complex time-dependent dynamics can occur, in particular, heteroclinic cycles which connect axisymmetric steady-states [11]. The symmetry group of the problem, i.e. here the $O(3)$ group, plays a relevant role in the existence and robustness of these cycles [12] and [2]. However, the so-called "auto-adjoint" degeneracy, which is the generic case in the astrophysical framework, is also required. Unfortunately with our artificial central force field ($1/r^5$ dependence), we show that this is only the case in the neighborhood of a low critical Prandtl number $Pr < 1$ ([3], [8]). Because of the Prandtl number restrictions in the experiment, which must be greater than $Pr = 40$, we cannot observe these cycles in the GeoFlow frame.

Recently we pointed out that for the (3, 4) interaction, i.e. the critical values $\eta_c = 0.4466$ and $Ra_c = 2200$, and for $Pr > 40$, another degeneracy occurs [8] and new types of heteroclinic cycles are observed [1].

The originality of these cycles is that they connect steady-states with the cube symmetry ($l = 4$), noted $\beta$, and with the tetrahedral symmetry ($l = 3$), noted $\gamma$, contrary to the previous scenario which always connects axisymmetric solutions of the even mode. Furthermore in the phase space these connections are very complex. Indeed they involve 9 dimensional invariant subspaces while usually they occur in planes. We determine the region of occurrence of these cycles in the $(\eta, Ra)$ parameter plane (Fig. 1) using a classical bifurcation analysis: reduction on the center manifold and a third order approximation of a 16-D system of the amplitude equations [10]. The stability of the $\beta$ and $\gamma$ equilibria are determined by the eigenvalues sign of the different Jordan blocks of the Jacobian matrix, i.e. the isotypic components [10]. In order to find heteroclinic cycles, each equilibrium must be a saddle-node where stable and unstable manifolds coexist. On the figure 1, the sign change of the eigenvalues in the different isotypic

![Figure 1.](image-url)
directions are represented by a dashed line for the $\gamma$ equilibrium (the two first only) and by a plain curve for the $\beta$ equilibrium.

In the region $R_1$ of possible heteroclinic cycles, a direct simulation shows, after a transition, a convergence to the tetrahedral mode 3 solution. This result proves that the degeneracy is present, otherwise this "pure" mode 3 solution could not exist [1]. According to these numerical simulations, the cycle does not seem asymptotically stable and the presence of a stable steady-state destroys the cycle. It is this solution that we observe and because it is very close to the pure mode 3 one, it looks like this last one. Another negative prospect for the experimental observation is the strong sensitivity to the aspect ratio $\eta$: a variation of 1% has a significant influence on the existence of the cycle!

Thus we do not expect to be able to observe these dynamics in the GeoFlow experiment and that is why in the following we are focusing on the rotating case.

3. Numerical simulations

For the numerical simulation a pseudo-spectral method is used. The primary variables, separated into toroidal and poloidal parts, are decomposed into Chebyshev polynomials (radial direction), Legendre polynomials (meridional direction) and trigonometric functions in the periodic azimuthal direction. Details can be found in [5, 6] and [9]. The single Legendre and Fourier modes will be called l-modes and m-modes respectively in the following. Most of the calculations are based on a spatial resolution of $(25 \times 35 \times 25)$ for the particular directions plus additional modes during the real space $\leftrightarrow$ spectral space transformations to avoid aliasing effects. A calculation runs a few thermal time scales and is stopped if the time derivative of the kinetic energy falls below an upper bound for steady states or a (quasi)periodic behavior does not change for a huge number of period lengths. Irregular solutions are calculated also with higher resolution of $(30 \times 40 \times 30)$ as cross check.

Figure 2. Stability diagram showing different kinds of stable solutions for $\eta = 0.5$ and $\Pr = 64.64$: line --- results from linear stability analysis and - - - - roughly labels transition to irregular time dependence.

Figure 2 shows an overview of the different stable solutions found in simulations by varying the Taylor and Rayleigh number. The lower line marks the onset of convection and is the result of a linear stability analysis [5]. Above this line we find steady state solutions with only one excited m-mode as the result of a pitchfork bifurcation. A second bifurcation (not yet examined
in detail) leads to time-periodic solutions. The variations of the amplitude are mostly small compared with the absolute value and therefore hard to detect in the real experiment. Beneath that also the Rayleigh number range where time-periodicity appears is rather small and requires high accuracy with regard to the adjustability of the high voltage source. The demonstration of quasiperiodic solutions that appear only in very small parameter regions seem not to be experimentally feasible. Another increase of the Rayleigh number leads to irregular or chaotic time dependence (above dashed line in figure 2). Nevertheless the remaining spatial patterns are still dominated by large structures.

At least for slow rotation, mode interactions seem to exist. In contrast to the $l$-mode cycles found in the bifurcation analysis, here consecutive azimuthal modes $(m, m+1)$ interact with each other. In the analyzed parameter region this happens for $m = 4$. Three snapshots during one cycle are shown in figure 3. On the left and right respectively are plotted the $m = 4$ and $m = 5$ states while in the middle a configuration in between both of them is shown. The pictures illustrate results from a numerical simulation with $Ta = 30000$, $Ra = 5000$, $Pr = 43$ and a radius ratio of $\eta = 0.5$.

![Figure 3. Cycle between $m = 4$ and $m = 5$ dominated states; shown is the temperature distribution in the equatorial plane (bright color means hot and dark cold fluid)]](image)

The nature of these interactions is different from that found for the $l$-modes. First it is not a transition between pure modes, dominated (in energy) only by one wave number - here $m = 4$ and $m = 5$ (see figure 4). Furthermore the transition time between the $m = 4$ and $m = 5$ dominated configuration is of the same order or even longer than the residence time of the particular mode.

A further increase of the Rayleigh number leads to smaller and smaller structures and finally it becomes turbulent. The analysis of this transition is work in progress.

4. Conclusions
The influence of a central force field on spherical Rayleigh-Bénard convection is investigated. For fixed radius ratio and Prandtl number the strength of the central force field and the rate of rotation are varied. Two approaches, based respectively on bifurcation theory and numerical simulation of the system, are used to analyze dynamical properties of the solutions and transitions between them.

Bifurcation analysis shows that for a critical aspect ratio of $\eta = 0.4466$ not only does one stable mode appears, but two consecutive modes occur, and that interactions arise as heteroclinic cycles. Unfortunately this will be a difficult task to observe this dynamics experimentally because of the strong dependence on the radius ratio $\eta$.

Mode interactions of a different kind appear also far from the onset of convection where the system already shows a chaotic time dependence. Nevertheless the spatial structures stay
nearly unchanged and an interaction between two consecutive $m$-modes, e.g. $m = (4,5)$ for slow rotation, is found.

These results can be directly compared with measurements of an experimental setup considered for running on ISS next year.

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Figure 4. Histogram of the kinetic energy of the excited azimuthal wave modes for the three states shown in figure 3.