Roper resonance in the relativistic three-quark model

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Abstract

The relativistic three-quark equations are found in the framework of the dispersion relation technique. The approximate solutions of these equations using the method based on the extraction of leading singularities of the amplitude are obtained. The calculated mass values of two ($N = 2, 56^*$) baryonic multiplets $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ are in good agreement with the experimental ones.

In the recent papers [1,2] the Faddeev equations are represented in the form of the dispersion relation over the two-body subenergy. The behaviour of the low-energy three-body amplitude is determined by its leading singularities in the pair invariant masses. Then the purpose was to extract the singular part of the amplitude. The suggested method of approximate solution of the Faddeev equations was verified on the example of the S-wave baryonic spectroscopy. We calculated the lowest baryon masses ($J^P = \frac{1}{2}^+, \frac{3}{2}^+$) using the method based on the extraction of leading singularities of the amplitude.

In the present paper the relativistic Faddeev equations are also constructed in the form of the dispersion relation over the two-body subenergy. We calculated the mass values of two ($N = 2, 56^*$) baryon multiplets with $J^P = \frac{1}{2}^+, \frac{3}{2}^+$, which are in good agreement with the experimental ones [3].

One used the results of the bootstrap quark model [4] and determined the diquark amplitude with $J^P = 0^+, 1^+$. The integral equation systems, corresponding to the ($N = 2, 56^*$) baryonic multiplets $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ are analogous to the integral equations for the S-wave lowest baryonic multiplets $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ [1,2]. However, for the excited baryons the long-range forces, which due to the confinement, are important. Namely, the box-diagrams can be important in the formation of hadron spectra [5]. For the sake of simplicity we restrict ourselves to the introduction of the quark mass shift $\Delta$, which are defined by the contributions of the nearest production thresholds of pair mesons and baryons. We
suggested that the parameter $\Delta$ takes into account the confinement potential effectively:

\[ m_{\text{eff}} = m + \Delta \quad \text{and} \quad m^{s}_{\text{eff}} = m^s + \Delta \]

and changes the behaviour of diquark amplitude [6]. It allows to construct the excited baryonic amplitudes and calculate the mass spectrum by analogy with [6]. In the considered calculation the quark masses ($m$ and $m^s$) are analogous [1,2]: $m = 0.410$ GeV, $m^s = 0.557$ GeV. $\Delta$ is equal 0.168 GeV.

The construction of the approximate solutions of Faddeev equations is based on the extraction of the leading singularities which are close to the region of pair energy $\sim 4m^2$. First of all there are threshold square root singularities. Also possible are pole singularities which correspond to the bound states. They are situated on the first sheet of complex pair energy plane in the case of real bound state and on the second sheet in case of virtual bound state. This diagrams have only two-particle singularities. Other diagrams apart from two-particle singularities have their own specific triangle singularities. It is weaker than two-particle singularities. Such classification allows us to search the approximate solution by taking into account some definite number of leading singularities and neglecting all the weaker ones. We consider the approximation, which corresponds to the single interaction of all three particles (two-particle and triangle singularities) [1,2].

In the present paper the suggested method of approximate solution of the relativistic three-quark equations allows us to calculate the excited baryons spectrum ($N = 2, 56^*$) with $J^P = \frac{1}{2}^+, \frac{3}{2}^+$. The interactions, determined this spectrum, are similar to ones in the S-wave lowest baryons case. We use two vertex constants $g_0 = 0.702$ and $g_1 = 0.540$, which corresponds to the quark-quark interaction in $0^+$ and $1^+$ states. One introduce two dimensionless cut-off parameters $\lambda_0 = 10.5$ and $\lambda_1 = 11.5$, which can be determined by mean of fixing of excited baryon mass values ($N^*, \Sigma^*$). We decreased the colour-magnetic interaction in $0^+$ strange channels ($\lambda_0^s < \lambda_1$). Then we calculated the mass values of two excited multiplets $J^P = \frac{1}{2}^+, \frac{3}{2}^+ (N = 2, 56^*)$ (Table) in good agreement with the experimental data [3] and the other model results [7-12].

| $J^P$ | \(M(\text{GeV})\) | $J^P$ | \(M(\text{GeV})\) |
|-------|----------------|-------|----------------|
| \(N^*\) | 1.440(1.440) | \(\Delta^*\) | 1.715(1.600) |
| \(\Lambda^*\) | 1.610(1.600) | \(\Sigma^*\) | 1.865 |
| \(\Sigma^*\) | 1.655(1.660) | \(\Xi^*\) | 2.010 |
| \(\Xi^*\) | 1.785 | \(\Omega^*\) | 2.155 |

The cut-off parameters $\lambda_0^s = 10.5$ and $\lambda_1 = 11.5$. The vertex functions $g_0 = 0.702, g_1 = 0.540$. Experimental values of the baryon masses [3] are given in parentheses.

The reason of essential difference between $\Sigma^*$ and $\Lambda^*$ is the spin of the lighter diquark. The model explain both the sign and magnitude of this mass splitting.

The Roper resonance $N^*(1440)$ is the lightest member of the $J^P = \frac{1}{2}^+$ multiplet. Within the constituent quark picture [12,13] this resonance commonly assigned to a radial
excitation of the nucleon, whereas it has been argued [14-16] that it might be a hybrid state, containing an explicit excited glue-field configuration (i.e. $gqq$-state).

The model under consideration proceeds from the assumption that the quark interaction forces are the two-component ones. The long-range component of the forces is neglected. The creation of low-lying baryons is mainly due to the constituent gluon exchange. But for the excited baryons the long-range forces are important. The confinement with comparatively large energy is actually realized as the production of the new $q\bar{q}$ pairs. The long-range forces are determined by the contribution of the nearest production thresholds of pair quarks. We suggest that quark mass shift $\Delta$ takes into account the confinement potential effectively and changes the behaviour of pair quarks amplitude. It allows us to construct the excited baryon amplitudes and calculate the baryon mass spectrum by analogy with P-wave meson spectrum in the bootstrap quark model [6].

We manage with the quarks as with real particles. However, in the soft region, the quark diagrams should be treated as spectral integrals over quark masses with the spectral density $\rho(m^2)$: the integration over quark masses in the amplitude puts away the quark singularities and introduces the hadron ones. We can believe that the approximation $\rho(m^2) \to \delta(m^2 - m_{eff}^2)$ could be possible for the excited baryons (here $m_{eff}$ is the effective ”mass” of the constituent quark). We hope this approach is sufficiently good for the calculation of excited baryonic spectrum.

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