Constraint on Cosmological Model with Matter Creation Using Complementary Astronomical Observations

Yuan Qiang · Tong-Jie Zhang · Ze-Long Yi

Received: date / Accepted: date

Abstract The universe with adiabatic matter creation is considered. It is thought that the negative pressure caused by matter creation can play the role of a dark energy component, and drive the accelerating expansion of the universe. Using the Type Ia supernovae (SNe Ia) data, the observational Hubble parameter data, the Cosmic Microwave Background (CMB) data and the Baryonic Acoustic Oscillation (BAO) data, we make constraints on the cosmological parameters, assuming a spatially flat universe. Our results show that the model with matter creation is consistent with the SNe Ia data, while the joint constraints of all these observational data disfavor this model. If the cosmological constant is taken into account, a traditional model without matter creation is favored by the joint observations.

Keywords matter creation cosmology · observational constraints · supernovae

1 Introduction

A great encouraging development in modern cosmology is the discovery of the accelerating expansion of the universe through observations of distant Type Ia Supernovae [12]. The anisotropy of the cosmic microwave background (CMB) results from balloon and ground experiments [3,4,5,6,7,8] and recent WMAP [9] observation confirmed the result from SNe Ia and favored a spatially flat universe. It is well known that all known types of matter with positive pressure generate attractive forces and decelerate the expansion of the universe. The discovery from SNe Ia and CMB indicate the existence of a new component with negative pressure, which is now generally called dark energy.

The simplest one of dark energy is a cosmological constant [10,11,12]. An explanation of the cosmological constant is the vacuum energy, however, it is 120 orders of magnitude smaller than the naive expectation from quantum field theory. Bothering physicists much, other types of dark energy are proposed, such as quintessence [13,14,15], which is described in terms of a cosmic scalar field φ; or other modified cosmological models are discussed, such as the Cardassian expansion model which investigates the acceleration of the universe by a modification to the Friedmann equations [16,17], the brane world model which explain the acceleration through the fact that general relativity is formulated in 5 dimensions instead of the usual 4 [18,19,20] and so on.

All of these dark energy cosmological models are based on the Big Bang cosmology. A model with adiabatic matter creation was proposed firstly in order to interpret the cosmological entropy and solve the big-bang singularity problem [21]. The basic idea is to modify the usual energy conservation law in open system in the framework of cosmology, which adds a balance...
equation for the number density of the created particles to the dynamic equations of the universe. Nevertheless, after the discovery of the accelerating expansion of the universe, this model was reconsidered to explain it and got some unexpected results. The matter creation pressure \( p_c \), which is negative as pointed out several decades ago by Zel’dovich [22], might play the role of a dark energy component and lead to the accelerating expansion of the universe. Lima & Alcaniz tested the model without cosmological constant through the lookback time-redshift relation, luminosity distance-redshift relation, angular size-redshift relation and the galaxy number counts-redshift relation [23][24]. It was shown that this model was consistent with the observational accelerating expansion of the universe, and could also alleviate the conflict between the age of the universe and the age of the oldest globular clusters. Zimdahl et al. employed the SNe Ia data to test the matter creation scenario and also got the result of accelerating expansion [25]. Freaza et al., however, based on the observational SNe Ia data and the simulated Supernova Acceleration Probe (SNAP) data, showed the matter creation mechanism was not favored to explain the cosmic acceleration [26].

In this paper, we use recently 186 SNe Ia sample [27], combined with the observational \( H(z) \) data from the differential age measurements of galaxies [28], the CMB and BAO data [32][33], to test the cosmological model with matter creation and make constraints of the parameters. As a comparison, the model with both matter creation and cosmological constant is also examined. This paper is organized as follows: we present the basic cosmological equations of the universe with adiabatic matter creation in Sec. 2. In Sec. 3 we give a brief introduction to the observational data, and the results and discussion are given in Sec. 4.

## 2 The Cosmological Basic Equations with Adiabatic Matter Creation

The Robertson-Walker (RW) metric describing the spacetime of the universe is

\[
ds^2 = -dt^2 + a^2(t)[\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2 \sin^2 \theta d\phi^2],
\]

where \( r, \theta, \phi \) are dimensionless comoving coordinates, \( k = 0, \pm 1 \) represent the curvature of the spatial section and \( a(t) \) is the scale factor. Using the Einstein field equation, we can acquire the equations to describe the dynamic behavior of the universe, namely the Friedmann equations

\[
\begin{align*}
\frac{\dot{a}}{a} &= -\frac{4\pi G}{3} \rho + \frac{\Lambda}{3} - \frac{k}{a^2}, \\
\end{align*}
\]

(3)

where \( \rho = \rho_M + \rho_R \) is the energy density (matter and radiation), \( p \) and \( \Lambda \) are the thermal pressure and cosmological constant respectively. \( p_c \) is the matter creation pressure that takes the following form [34]

\[
p_c = -\frac{\rho + p}{3nH}\psi,
\]

(4)

where \( H \equiv \dot{a}/a \) is the Hubble parameter and \( \psi \) is the matter creation rate. In models with adiabatic creation, the balance equation for the particle number density \( n \) is [34][35]

\[
\begin{align*}
\dot{n} + \frac{3}{a} \frac{\dot{a}}{a} n &= \psi, \\
\end{align*}
\]

(5)

We take the form of matter creation rate as [23]

\[
\psi = 3\beta nH,
\]

(6)

where the parameter \( \beta \) is defined on the interval \([0, 1]\) and assumed to be constant. Matter and radiation correspond to \( \beta_M \) and \( \beta_R \) respectively. Together with the equation of state (EOS)

\[
p = w \rho,
\]

(7)

the equation system becomes complete.

From Eqs. (3) and (5), we can get

\[
p + p_c = -\frac{d}{da} (\rho a^3),
\]

(8)

Combining Eqs. (4), (7) and (8), it is easy to find

\[
\begin{align*}
w &= 0, \quad \rho_M = \rho_M^0 \left(\frac{a}{a_0}\right)^{3(1-\beta_M)} \quad \text{for matter} \\
w &= 1/3, \quad \rho_R = \rho_R^0 \left(\frac{a}{a_0}\right)^{3(1-\beta_R)} \quad \text{for radiation} \\
w &= -1, \quad \rho_A = \rho_A^0 \quad \text{for cosmological constant}
\end{align*}
\]

(9)

Using Eq. (9) and noting that \( 1 + z = a_0/a \), we can rewrite Eq. (2) in the form of the Hubble parameter

\[
H^2 = H_0^2 E^2(z),
\]

(10)

where \( H_0 \) is the Hubble constant, and

\[
E^2(z) = \Omega_M(1+z)^{3(1-\beta_M)} + \Omega_R(1+z)^{4(1-\beta_R)} + \Omega_A + \Omega_k(1+z)^2
\]

(11)

represents the expansion rate, in which \( \Omega_M, \Omega_R, \Omega_A \) and \( \Omega_k \) are the matter density, radiation density, cosmological constant and spatial curvature parameters at present.

The deceleration parameter \( q \) is defined as

\[
q = -\frac{\ddot{a}}{a \dot{a}^2}.
\]

(12)

Using Eqs. (2), (5) and (9), the deceleration parameter \( q \) reduces to

\[
q(z) = \frac{1}{E^2(z)^2} \left[ 1 - \frac{3\beta_M}{2} \Omega_M(1+z)^{3(1-\beta_M)} + (1-2\beta_R)\Omega_R(1+z)^{4(1-\beta_R)} - \Omega_A \right].
\]

(13)
In the following discussion, we consider a spatially flat universe and neglect the radiation term for its extremely small value today, that is, \( \Omega_k = \Omega_R = 0 \), so \( \Omega_M + \Omega_A = 1 \).

If there is no cosmological constant, that is, \( \Omega_A = 0, \Omega_M = 1 \), Eq. (11) can be rewritten as
\[
E^2(z) = (1 + z)^{3(1-\beta)},
\]
where \( \beta \) is \( \beta_m \), and \( q(z) \) can be simplified as
\[
q(z) = \frac{1 - 3\beta}{2},
\]
which shows that \( q \) is independent of redshift. From Eq. (15) we know that, if the universe is accelerating expanding, that is, \( q_0 < 0 \), \( \beta \) needs to be greater than 1/3.

3 The Observational Data

SNe Ia are thought to be standard candles and can be used as distance probes. The theoretical prediction for luminosity distance of an astronomical object in a spatially flat universe is
\[
d_L = \frac{c(1+z)}{H_0} \int_0^z \frac{dz}{E(z)},
\]
where \( E(z) \) is defined in Eq. (10). The distance modulus is
\[
\mu_{th} = 5 \log \frac{d_L}{Mpc} + 25 = 42.38 + 5 \log \left( \frac{1+z}{h} \right) \int_0^z \frac{dz}{E(z)},
\]
where \( h = H_0/(100 \text{ km/s/Mpc}) \). The \( \chi^2 \) parameter for SNe Ia is
\[
\chi^2_{SN} = \sum_i \frac{[\mu_{th}(z_i; h, \Omega_M, \beta) - \mu_{SN}(z_i)]^2}{\sigma_{SN}(z_i)},
\]
where \( \mu_{SN}(z_i) \) is the observed distance modulus of the SN with redshift \( z_i \) and \( \sigma_{SN}(z_i) \) is the observational error.

We also include the observational Hubble parameter \( H(z) \) in this work. The Hubble parameter depends on the differential age of the universe in this form
\[
H(z) = \frac{1}{1+z} \frac{dz}{dt},
\]
which provides a direct measurement for \( H(z) \) through a determination of \( dz/dt \). Using the differential ages of passively evolving galaxies determined from the Gemini Deep Deep Survey (GDDS), Simon et al. determined a set of observational \( H(z) \) data in the redshift range \( 0 \sim 1.8 \) [28]. These observational \( H(z) \) data have been used to constrain the dark energy potential and its redshift dependence by Simon et al. [28]. Various cosmological models were tested using this data set in recent several years. Yi & Zhang used them to analyze the holographic dark energy models and showed that the fitting results did not conflict with some other independent cosmological tests [29]. Samushia & Ratra used the data set to constrain the parameters of \( \Lambda \)CDM, XCDM and \( \phi \)CDM models. The constraints are consistent with those derived from SNe Ia [30]. Wei & Zhang tested a series of other cosmological models with interaction between dark matter and dark energy using this data set. In one word, the observational \( H(z) \) data are demonstrated to be an effective complementarity to other cosmological probes.

For the Hubble parameter \( H(z) \) data, we have
\[
\chi^2_H = \sum_k \frac{[H_{th}(z_i; h, \Omega_M, \beta) - H_{ob}(z_i)]^2}{\sigma_H(z_i)}.
\]

The model-independent shift parameter \( R \), which can be derived from CMB data, is defined as
\[
R = \sqrt{\Omega_M} \int_0^{z_r} \frac{dz}{E(z)},
\]
with \( z_r = 1089 \) the redshift of recombination. From the three-year result of WMAP, Wang & Mukherjee estimated \( R = 1.70 \pm 0.03 \) [32].

Using a large spectroscopic sample of luminous red galaxies from Sloan Digital Sky Survey (SDSS), Eisenstein et al. successfully found the acoustic peaks in the bayonic matter anisotropy power spectrum, described by the model-independent \( A \) parameter
\[
A = \sqrt{\Omega_M} \left[ \frac{1}{z_1 E^{1/2}(z_1)} \int_0^{z_1} \frac{dz}{E(z)} \right]^{2/3},
\]
with \( z_1 = 0.35 \) the redshift at which the acoustic scale has been measured [33]. Eisenstein et al. suggested the value of \( A \) parameter as \( A = 0.469 \pm 0.017 \).

The model parameters \( h, \Omega_M \) and \( \beta \) can be determined through the \( \chi^2 \) minimization method. The combined \( \chi^2 \) can be written as
\[
\chi^2 = \chi^2_{SN} + \chi^2_H + \frac{(R - 1.70)^2}{0.03^2} + \frac{(A - 0.469)^2}{0.017^2}.
\]

By minimizing \( \chi^2 \) we can get the best fitting values of the parameters.

4 Results and Discussion

Using the observations of SNe Ia, Hubble parameter \( H(z) \), CMB and BAO, we test the cosmological model with adiabatic matter creation, assuming a spatially flat universe. We show in Fig. 1 the confidence regions of parameters \( h \) and \( \beta \) of this model, for different sets of observational data. The best fitting results of parameters are listed in Table I. We can see from this figure that, for the SNe Ia data, \( \beta \) is greater that 1/3 at 3\sigma.
The negative value of the cosmological constant and the matter creation may jointly generate the acceleration of the universe. We also test the model with a cosmological constant in order to give a comparison. After marginalizing parameter \( h \), we show the confidence regions of parameters \( \Omega_M \) and \( \beta \) in Fig. 2 and the best fitting parameters in Table 2 respectively. The cosmological constant \( \Omega_A = 1 - \Omega_M \). For the SNe Ia and \( H(z) \) data, the constraints of model parameters are not very strong. The statistical uncertainties of model parameters are so large that we cannot give any convincible conclusions. While the joint constraints of these 4 kinds of observations show that \( \beta \) tends to be zero, which reduces to the familiar \( \Lambda \)CDM cosmology. The best fitting result \( \Omega_M = 0.30 \pm 0.02 \) is also consistent with other studies of the traditional \( \Lambda \)CDM model [27,9]. We also notice that our result is very similar with [20]. However, in their treatment, a small SNe Ia sample (16 low-redshift and 38 high-redshift supernovae from Perlmutter et al. [2]) could not give strong constrains on the model parameters. They drew the conclusion by adopting a Gaussian prior of \( \Omega_M = 0.27 \pm 0.06 \). In our work, we give the constraints from joint astronomical observations directly.

In summary, the idea that a negative matter creation pressure may play the role of dark energy and drive the accelerating expansion of universe, seems not favored by the combined observations. While for the model with both the adiabatic matter creation and cosmological constant, the joint constraints tend to give \( \beta \simeq 0 \), which means no matter creation, and the model becomes the traditional \( \Lambda \)CDM one.

### Acknowledgements

We would like to thank Yue Bin, Dr. Peng-Jie Zhang, Li Chen and Zong-Hong Zhu, for their friendly discussion. This work was supported by the National Science Foundation of China (Grants No.10473002 and 10273003), the Scientific

---

**Tables**

| Test               | \( h \)   | \( \beta \) | \( \chi^2/d.o.f \) |
|--------------------|-----------|-------------|---------------------|
| SNe Ia             | 0.64      | 0.54        | 239.97/184          |
| \( H(z) \)         | 0.66      | 0.33        | 8.50/7              |
| SNe Ia+\( H(z) \)  | 0.62      | 0.42        | 269.40/193          |
| SNe Ia+\( H(z) \)+CMB+BAO | 0.56     | -0.04       | 753.71/195          |

| Test               | \( \Omega_M \) | \( \beta \) | \( \chi^2/d.o.f \) |
|--------------------|-----------------|-------------|---------------------|
| SNe Ia             | 0.10            | -0.66       | 237.64/184          |
| \( H(z) \)         | 0.91            | 0.31        | 13.94/7             |
| SNe Ia+\( H(z) \)  | 0.19            | -0.26       | 250.87/193          |
| SNe Ia+\( H(z) \)+CMB+BAO | 0.30     | -0.02       | 256.42/195          |

---

**Figures**

Fig. 1 Confidence regions of 1, 2 and 3 \( \sigma \) (from inside to outside) for the two parameters \( h \) and \( \beta \), for different observational data set as labelled in the figure. The horizon lines are of \( \beta = 1/3 \) and 0.

Fig. 2 Confidence regions for the two parameters \( \Omega_M \) and \( \beta \) for the cosmological model with both the matter creation and the cosmological constant. Parameter \( h \) has been marginalized. The lines are coded as Fig. 1.
Research Foundation for the Returned Overseas Chinese Scholars, State Education Ministry and the Research Foundation for Undergraduate of Beijing Normal University.

References

1. Riess, A.G., Filippenko, A.V., Challis, P., et al.: AJ 116, 1009(1998)
2. Perlmutter, S., Aldering, G., Goldhaber, G., et al.: ApJ 517, 565(1999)
3. Miller, A.D., Caldwell, R., Devlin, M.J., et al.: ApJ 524, L1(1999)
4. de Bernardis, P., Ade, P.A.R., Bock, J.J., et al.: Nature 404, 955(2000)
5. Hanany, S., Ade, P., Balbi, A., et al.: ApJ 545, L5(2000)
6. Halverson, N.W., Litch, E.M., Pryke, C.: ApJ 568, 38(2002)
7. Mason, B.S., Pearson, T.J., Readhead, A.C.S., et al.: ApJ 591, 540(2003)
8. Benoît, A., Ade, P., Amblard, A., et al.: A&A 399, L25(2003)
9. Spergel, D.N., Verde, L., Peiris, H.V., et al.: ApJS 148, 175(2003)
10. Weinberg, S.: Rev. Mod. Phys. 61, 1(1989)
11. Carroll, S.M., Press, W.H., Turner, E.L.: ARA&A 30, 499(1992)
12. Ostriker, J.P., Steinhardt, P.J.: Nature 377, 600(1999)
13. Ratra, B., Peebles, P.J.E.: Phys. Rev. D 37, 3406(1999)
14. Coble, K., Dodelson, S., Frieman, J.A.: Phys. Rev. D 55, 1851(1997)
15. Caldwell, R.R., Dave, R., Steinhardt, P.J.: Phys. Rev. Lett. 80, 1582(1998)
16. Freese, K., Lewis, M.: Phys. Lett. B 540, 1(2002)
17. Zhu, Z., Fujimoto, M.: ApJ 581, 1(2002)
18. Randall, L., Sundrum, R.: Phys. Rev. Lett. 83, 3370(1999)
19. Deffayet, C., Dvali, G., Gabadadze, G.: Phys. Rev. D 65, 044023(2002)
20. Avelino, P.P., Martins, C.J.A.P.: ApJ 565, 661(2002)
21. Prigogine, I., Geheniau, J., Gunzig, E., Nardone, F.: Gen. Rel. Grav. 21, 767(1989)
22. Zel'dovich, Ya. B.: Journal of Experimental and Theoretical Physics 12, 307(1970)
23. Lima, J.A.S., Alcaniz, J.S.: A&A 348, 1(1999)
24. Alcaniz, J.S., Lima, J.A.S.: A&A 349, 729(1999)
25. Zimdahl, W., Schwarz, D.J., Balakin, A.B., Pavon, D.: Phys. Rev. D 64, 063501(2001)
26. Freaza, M.P., de Souza, R.S., Waga, I.: Phys. Rev. D 66, 103502(2002)
27. Riess, A.G., Strolger, L., Tonry, J., et al.: ApJ 607, 665(2004)
28. Simon, J., Verde, L., Jimenez, R.: Phys. Rev. D 71, 123001(2005)
29. Yi, Z.L., Zhang, T.J.: Mod. Phys. Lett. A 22, 41(2007)
30. Samushia, L., Ratra, B.: ApJ 650, L5(2006)
31. Wei, H., Zhang, S.N.: Phys. Lett. B 644, 7(2007)
32. Wang, Y., Mukherjee, P.: ApJ 650, 1(2006)
33. Eisenstein, D.J., Zehavi, I., Hogg, D.W., et al.: ApJ 633, 560(2005)
34. Calvao, M.O., Lima, J.A.S., Waga, I.: Phys. Lett. A 162, 233(1992)
35. Lima, J.A.S., Germano, A.S.M.: Phys. Lett. A 170, 373(1992)