Design and Fabrication of Wolter-type 4-mirror system

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Abstract. A Wolter-type 4-mirror system was designed and evaluated by ray-trace calculation. The design and fabrication process were developed. The master mandrel of oxygen-free copper was shaped by diamond turning. The Pyrex glass replica was made by a vacuum replication technique. A visible light image of a metal mesh could be obtained using this replica mirror.

1. Introduction

A Wolter type-I mirror is a grazing-incidence optics, which has been used in x-ray astronomy, x-ray microscopy, and neutron imaging. A Wolter mirror for microscopy is a 2-mirror system which is a combination of axisymmetric hyperboloidal and ellipsoidal mirrors with a common focus [1] as shown in Fig. 1(a). On the other hand, a 2-mirror system composed of two hyperboloidal mirrors with a common focus can produce a virtual image as shown in Fig. 1(b). By combining these two types of Wolter mirrors, a 4-mirror imaging optics can be designed. The first Wolter mirror forms a virtual image of an object and the second Wolter mirror forms its real image.

As shown in Fig. 1(a), the sum of aperture angle $\alpha$ of object side and aperture angle $\beta$ of image side is $2 \times \text{(number of reflection)} \times \text{grazing-incidence angle}$. Therefore, the numerical aperture of a Wolter mirror is limited to 4-times of the grazing-incidence angle. The grazing-incidence angle must be smaller than a critical angle of total reflection, which is less than 10 mrad for hard x-rays at the energy above 5 keV. Then, the numerical aperture is very small. The advantage of this 4-mirror system is that the numerical aperture can be approximately doubled compared to a 2-mirror system [2].

A disadvantage of the 4-mirror system is that the influence of surface roughness and shape error is larger than that of the 2-mirror system. It is necessary to sufficiently reduce surface roughness in the 4-mirror system. However, shape error of the second Wolter mirror is not so serious compared to the first Wolter mirror because the object has been already enlarged.

The design and fabrication process are described in this paper.

2. Design of Wolter-type 4-mirror system

A wolter-type 4-mirror system of the average grazing-incidence angle of 5.5 mrad is designed. The critical angle of x-ray total reflection of Pyrex glass is calculated to be 6.1 mrad at 5 keV from the refractive index [3]. Therefore, the mirror made of glass can be used for x-rays at the energy below about 5 keV. The reflectivities of 2-times and 4-times reflection are calculated to be 79 % and 63 % at 5 keV with no surface roughness.
Figure 2 shows design parameters of the 4-mirror system. Region surrounded by a square dashed line in Fig. 2 shows a 2-mirror system that corresponds to Fig. 1(a). First, the parameters of the 2-mirror system are determined. The ellipsoid and hyperboloid shapes can be determined by the following 5 parameters: (1) the radius of the mirror at the intersection plane of the hyperboloid and ellipsoid surfaces ($R$), (2) the grazing-incidence angle for the ellipsoid surface at the intersection ($\theta_1$), (3) the grazing-incidence angle for the hyperboloid surface at the intersection ($\theta_2$), (4) the magnification ratio of the mirror ($M$), and (5) the length of the ellipsoidal mirror along the optical axis ($L$). The length of the hyperboloidal mirror (hyperboloid 0) is determined by ray-tracing calculation from the image point so as to enter all the rays reflected by the ellipsoidal mirror. These parameters are optimized as a two mirror system. After determining the parameters of the 2-mirror system, the shape of hyperboloid 1 can be calculated from the grazing-incidence angles $\theta_3$ in Fig. 2. The shape of hyperboloid 2 can be also calculated from $\theta_4$ [2].

The grazing-incidence angle is changed by the reflecting position. By slightly changing the parameters repeatedly, the average grazing-incidence angle is made close to 5.5 mrad. The total mirror length is 38 mm. The magnification ratio is 9.9. The numerical aperture is $4.0 \times 10^{-2}$ and the theoretical resolution for Rayleigh criterion is 2.4 nm at 5 keV. On the other hand, the numerical aperture of the 2-mirror system which consists of the ellipsoid and hyperboloid mirrors of the 4-mirror system is $1.8 \times 10^{-2}$ and the theoretical resolution is calculated to be 5.2 nm.

The geometrical aberration was estimated from ray-trace calculation. The off-axis resolution was estimated from $2.35 \times$ standard deviation from the gravitation center of 1000 rays as shown in Fig. 3.

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**Figure 1.** (a) Axisymmetric hyperboloidal and ellipsoidal mirrors with a common focus can produce a real image. (b) Axisymmetric two hyperboloidal mirrors can produce a virtual image.

**Figure 2.** Design parameters of Wolter-type 4-mirror system. $M$: magnification ratio of the 2-mirror system, $R$: radius, $L$: length of the ellipsoidal mirror, $\theta_3$, $\theta_2$, $\theta_3$, $\theta_4$: the grazing-incidence angles.
Assuming that the spatial resolution is 50 nm, the off-axis distance is 20 μm for the 4-mirror system and 40 μm for the 2-mirror system. In the 4-mirror system, the two hyperboloid mirrors of object side enlarges an object image by a factor of about 2. Then, radius of field of view of the 4-mirror system is about half compared with the 2-mirror system. To obtain the 2.4 nm resolution of the Rayleigh criteria, the field of view for the 4-mirror system is calculated to be 6 μm in diameter.

3. Fabrication process of 4-mirror system

Fabrication process of the 4-mirror system was tested by using diamond turning and vacuum replication method [4] as shown in Fig. 4. An oxygen-free copper rod of 10 mm in diameter was used for the master mandrel. The position of each mirror surface of the 4-mirror system was divided into 1000 points. Using a numerically controlled lathe, the mandrel was formed by a single crystal diamond bite. Gold layer of about 100 nm thick was deposited on the mandrel surface as the parting agent. An electric furnace was used in the replication process. The mandrel was put into a Pyrex glass tube and heated up to 655 degrees Celsius in 30 min. After maintaining this condition for 75 min, the furnace was turned off and left about 5 hours to cool down. The coefficient of thermal expansion of the mandrel is larger than that of the glass tube so that the glass is separated from the mandrel after cooling down. Figure 5(a) shows the fabricated Wolter-type 4-mirror system made of Pyrex glass.

![Figure 3](image3.png)

**Figure 3.** Geometrical aberration calculated from spot diagrams of the 4-mirror and 2-mirror systems [2].

![Figure 4](image4.png)

**Figure 4.** Fabrication process of the 4-mirror system. (a) A mandrel of oxygen-free copper rod was formed by diamond turning. (b) Au layer of about 100 nm was deposited as release agent. (c) The replica mirror of Pyrex glass was made by the vacuum replication method.
It was necessary to polish the surface for x-ray imaging, so that the preliminary test was performed using visible light. Figure 5(b) is a visible light image of a metal mesh. A mesh of 250 μm pitch could be imaged by the replica mirror. As a next step, tungsten carbide was used as material of master mandrel for more precision machining. The replication process is currently under way.

The Wolter-type 4-mirror system has about two times the numerical aperture of the 2-mirror system. This also means that the focal length is about half of the 2-mirror system and the magnification ratio is about 2 times that of the 2-mirror system under the constant object-image distance. Therefore, the 4-mirror system is considered to be effective in high-energy x-ray and neutron imaging, in which the critical angle of total reflection is small.

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![Figure 5](image)

**Figure 5.** (a) Fabricated 4-mirror system made by Pyrex glass. (b) Visible light image of a metal mesh.

**References**

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