Quantum Mechanics of Spin Transfer in Ferromagnetic Multilayers

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We use a quantum mechanical treatment of a ballistic spin current to describe novel aspects of spin transfer to a ferromagnetic multilayer. We demonstrate quantum phenomena from spin transmission resonance (STR) to magnetoelectric spin echo (MESE), depending on the coupling between the magnetic moments in the ferromagnetic thin films. Our calculation reveals new channels through which the zero spin transfer occurs in multilayers: the STR and MESE. We also illustrate that counter-intuitively, a negative spin torque can act initially on the second moment in a bilayer system.

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The phenomenon of spin transfer [1, 2] has resulted in a recent wave of progress [3, 4, 5, 6, 7, 8, 9, 10, 11] to study interactions between spin-polarized electrons and a magnetic moment in a ferromagnetic film. However, the physics of spin transfer still has not been fully appreciated quantum mechanically. The underlying principle for this phenomenon is angular momentum conservation. Since the incoming electrons are spin-filtered by the magnetic moment, as a reaction to the filtering effect the magnetic moment becomes tilted to align along the direction of the incoming spin. This alignment is also known as the spin torque because of the associated moment change. However, spin is a quantum mechanical concept while torque is a classical quantity. These disparate views are reconciled since spin is transferred quantum mechanically from the incoming electrons to the magnetic moment, but the effect on the moment appears in a classical manner. In this paper, we demonstrate some quantum mechanical aspects of the spin transfer in a ferromagnetic multilayer by adopting an adiabatic approximation to describe the dual electron spin/ferromagnetic moment system.

Two very interesting quantum mechanical phenomena in these problems are the spin transmission resonance (STR) [12] and the magnetoelectric spin echo (MESE) effect [13]. The STR is due to quantum interference of the right-moving (transmitted) and left-moving (reflected) electron wave functions in the ferromagnetic film. On the other hand, the MESE is a consequence of the time reversal symmetry between the two magnetic moments. The STR and MESE have some similarities as well as differences between them. Both phenomena occur with effectively zero spin transfer to the moments. However, as we elucidate below, the physics governing the two phenomena is quite different.

First, STR can occur not only in a multilayer system but also in a single ferromagnetic film while the MESE can only take place in a multilayer system. For STR, resonance of the electron wave function is essential, while for the MESE we require an anti-symmetric configuration between the two magnetic moments all the time. In this way the spin current, once absorbed by the first moment, can be generated by the second moment. That is, the second moment plays the role of a spin battery. Consequently, a strong interaction between the two moments is necessary for the MESE while STR is more or less insensitive to the moment-moment interaction. Moreover specific values of the kinetic energy of the incoming electrons are required for STR; this is not the case for the MESE.

To account for both STR and the MESE, we consider a simple model Hamiltonian for two single-domain ferromagnetic films with all key interactions included

\[
H = \frac{\gamma^2}{2m} - 2J_H \sum_{i=1}^{2} \mathbf{M}_i \cdot \mathbf{s} - \frac{J_M}{\gamma_0} \mathbf{M}_1 \cdot \mathbf{M}_2 ,
\]

where \(m\) is the electron mass, \(J_H\) is the coupling between the magnetic moment and the incoming electron spins, \(J_M\) is the coupling between the moments, and \(\gamma_0\) is the gyromagnetic ratio. We consider the ballistic regime as in Refs. [12, 13].

The magnetic moments \(\mathbf{M}_1\) and \(\mathbf{M}_2\) in the magnetic multilayers are assumed to originate from the local spins in the same ferromagnets with a constant magnitude \(M_0\). The directions of \(\mathbf{M}_1\) and \(\mathbf{M}_2\) at a given time are \((\theta_1, \phi_1)\) and \((\theta_2, \phi_2)\), respectively, and are subject to the interactions. In the Hamiltonian Eq. (1), the second term transfers the electron spin to the magnetic moments via the spin flip process while the third term controls the relative orientation of \(\mathbf{M}_1\) and \(\mathbf{M}_2\).

Origins of the coupling between the magnetic moments are the exchange interaction and a magnetic dipole interaction. The effective coupling is determined by the competition between the two interactions depending on the distance between the magnetic moments. A simple evaluation of the magnetic interaction energy shows that the anti-parallel configuration is more stable than the parallel case when the distance between the two ferromagnetic films is larger than the atomic scale. For example, for one-dimensional uniform ferromagnets of length \(L\) separated by the distance \(d\), the magnetic dipole interaction energy is approximately \(\pm 2M_0^2/L^2d\) for paral-
earlier, a necessary condition for the MESE is that the layers be aligned (+) and anti-parallel (−) configurations. For two-dimensional uniform ferromagnets with size of \( \times \), the energy is about \( \pm 2M^2_1 \ln(L/d)/L^2 \). As we mentioned earlier, a necessary condition for the MESE is that the two moments remain anti-parallel for all times. Thus \( J_M \) must be negative with magnitude large compared to \( J_H \). Otherwise, the spin torque will align easily both moments parallel to the initial direction of the incoming spin; the MESE can no longer occur.

We show in Fig. 1 the geometry of the problem. The incident direction of the electrons is in the positive \( X \) direction, and the two films are placed perpendicular to this direction, in the \( YZ \) plane. Translational symmetry is assumed in the plane. Therefore, the problem is effectively one-dimensional as in Refs. \([4,12]\); this is a reasonable approximation in the ballistic regime. It is also assumed that regions I, III, and V are non-ferromagnetic and are the same in nature for simplicity. When we solve for the dynamics of the magnetic moments we use the adiabatic approximation. While this is applicable in most cases, it should be used with caution for the MESE because the magnetic moments will oscillate rapidly if the moment-moment coupling is large. The applicability of the adiabatic approximation requires the time scale for the electrons to be much shorter than that of the moments.

In principle, one can use any basis set to represent the wave functions in each region. For example, one could use the spin-up/down state \(|\pm\rangle\) in the lab frame for the basis. However, using the right basis makes the calculations much easier. We introduce eigenstates \(|\chi_{\sigma}\rangle\) and \(|\xi_{\sigma}\rangle\) of the interactions \(2J_H S_1 \cdot M_1\) and \(2J_H S_2 \cdot M_2\) such that \(2J_H S_1 \cdot M_1|\chi_{\sigma}\rangle = \pm J_H M_1|\chi_{\sigma}\rangle\) and \(2J_H S_2 \cdot M_2|\xi_{\sigma}\rangle = \pm J_H M_0|\xi_{\sigma}\rangle\). Using the eigenstates of the interaction, we can represent the wave function in each region. We emphasize that this procedure can be systematically generalized to a system with more ferromagnetic films.

The wave functions in regions I, II, III, IV, and V can be written as follows:

\[
|\psi_I\rangle = |+\rangle e^{ikx} + \sum_{\sigma=\uparrow,\downarrow} R_{\sigma}|\chi_{\sigma}\rangle \langle \chi_{\sigma}| e^{-ikx}
\]

\[
|\psi_{II}\rangle = \sum_{\sigma=\uparrow,\downarrow} \left( A_{\sigma} e^{ikx} + B_{\sigma} e^{-ikx} \right) \langle \chi_{\sigma}| \langle \xi_{\sigma}| e^{ikx}
\]

\[
|\psi_{III}\rangle = \sum_{\sigma=\uparrow,\downarrow} T_{\sigma}|\chi_{\sigma}\rangle \langle \chi_{\sigma}| e^{-ikx} + \sum_{\sigma=\uparrow,\downarrow} R'_{\sigma}|\xi_{\sigma}\rangle \langle \xi_{\sigma}| e^{-ikx}
\]

\[
|\psi_{IV}\rangle = \sum_{\sigma=\uparrow,\downarrow} \left( A'_{\sigma} e^{ikx} + B'_{\sigma} e^{-ikx} \right) \langle \xi_{\sigma}| \langle \xi_{\sigma}| e^{ikx}
\]

\[
|\psi_{V}\rangle = \sum_{\sigma=\uparrow,\downarrow} T'_{\sigma}|\xi_{\sigma}\rangle \langle \xi_{\sigma}| e^{-ikx}
\]

The coefficients \( R_{\sigma} \) to \( T'_{\sigma} \) are determined by the boundary conditions at \( x = 0, L_1, L_1 + d, \) and \( L_1 + d + L_2 \). In general, the coefficients depend on the directions of the moments in the magnetic multilayer. If different bases are used to expand the wave functions of each region, the coefficients will be changed. However, the wave functions remain the same. Since expressions of the coefficients are too long, we do not show them here. Instead, we explain important properties associated with the coefficients and the wave functions, below, for the extreme cases: i) the STR with no dipole interaction and ii) the MESE with a strong dipole interaction.

Under the STR condition, the ferromagnetic film becomes transparent to the incoming spins \([12]\). One set of conditions that is required is that \( L_1 = 2nL_{00} \) \((n = 1, 2, \ldots)\) for an energy ratio \( \eta = 5/4 \). Here \( L_{00} = \pi/\sqrt{2mJ_HM_0} \) is a typical length scale of the ferromagnetic film and \( \eta \) is the ratio of the incoming electron energy \( k^2/2m \) to the interaction energy \( J_H M_0 \). If we have the STR condition only for the first film in region II, \( T_{\uparrow} = T'_{\uparrow} \) so that the spin state of the forward-moving wave in region III is the same as that of the incident wave. But, unlike the single layer case, \( R_{\uparrow} \neq 0 \) because the wave function in region III is partially reflected at \( x_2 = L_1 + d \) and the reflected wave can then pass through the first film in reverse (the same STR condition applies). We point out that even in this case \( R_{\uparrow} \neq R'_{\uparrow} \) because the magnetic moments are not necessarily parallel to each other. Suppose STR takes place only in region IV where the second film resides. Then clearly \( R'_{\downarrow} = 0 \) so that no wave is reflected at \( x_2 \). Nevertheless, \( T'_{\uparrow} \neq T'_{\downarrow} \) because the forward-moving wave in region III has both \(|+\rangle\) and
spin current. The difference between region I and V. Later, we will describe region I and III (and between III and V). The net spin II(IV) is equivalent to the spin current difference between spin torque acting on the magnetic moment in region in each region while the spin current is not because the spin current, so the result is more properly thought to the spin current, so the result is more properly thought of as an STR, occurring twice, once in each film. In all cases we have discussed so far the flux is conserved in each region while the spin current is not because the spin torque acting on the magnetic moment in region II(IV) is equivalent to the spin current difference between region I and III (and between III and V). The net spin torque acting on the multilayer system is the spin current difference between region I and V. Later, we will describe the dynamics of the magnetic moments in terms of the spin current.

The equations of motion of the two magnetic moments $M_1$ and $M_2$ are

$$\frac{dM_1}{dt} = 2J_H M_1 \times \langle \psi_{II} | s | \psi_{II} \rangle + J_M M_1 \times M_2$$

$$\frac{dM_2}{dt} = 2J_H M_2 \times \langle \psi_{IV} | s | \psi_{IV} \rangle + J_M M_2 \times M_1 .$$

(3)

Note that different wave functions are used to evaluate the spin expectation values for the equations of the moments. We use dimensionless units as in Ref. 12, where $m_i = M_i / M_0$ and the time is $\tau = j \tau_0$, where $j_0 = N_e L_{00} / (m S_{local})$ is the one-dimensional current with the number of incoming electrons $N_e$ per unit length and $S_{local} = M_0 / \gamma_0$. In these units, the coupling constant due to the magnetic dipole interaction becomes $\alpha = (S_{local} / 2N_e L_{00}) J_M / J_H$. Now the equations of motion for $m_i$ become

$$\frac{dm_{ix}}{d\tau} = -\beta_i m_{ix} m_{iy} + \gamma_i m_{iy} + \alpha (m_i \times m_j)_x,$$

$$\frac{dm_{iy}}{d\tau} = -\beta_i m_{iy} m_{iz} - \gamma_i m_{iz} + \alpha (m_i \times m_j)_y,$$

$$\frac{dm_{iz}}{d\tau} = \beta_i (1 - m_{iz}^2) + \alpha (m_i \times m_j)_z,$$

where $i, j = 1, 2$ ($i \neq j$),

$$\beta_i = \frac{1}{2} \int dx \text{Im} \left[ C_i^* C_{i\uparrow} \right],$$

$$\gamma_i = \frac{1}{2} \int dx \text{Re} \left[ C_i^* C_{i\uparrow} \right].$$

FIG. 2: (Color online) The time evolution of the magnetic moments (solid curves) and the spin torque (dashed curves) on the moment for $j = 5/4$, $L_1 = 2.6L_0$, $L_2 = 2.4L_0$, and $d = L_0$. No magnetic dipole interaction is included ($\alpha = 0$). The initial value of the moments is $m_1 = -m_2 = (0, 1, 0)$. The spin torque on $m_{1x}$ is $N_{1xx} = Q_{I,xx} - Q_{III,xx}$ and the torque on $m_{2x}$ is $N_{2xx} = Q_{I,xx} - Q_{V,xx}$ with $C_{i\sigma} = A_{i\sigma} e^{ik_{x}x} + B_{i\sigma} e^{-ik_{x}x}$, $C_{2\sigma} = A_{2\sigma} e^{ik_{x}x} + B_{2\sigma} e^{-ik_{x}x}$. The integration range for $\beta_i$ and $\gamma_i$ is given by the thickness of the corresponding ferromagnetic film.

As we mentioned earlier, another way to understand the dynamics is using the spin current tensor;

$$Q_{ij} = \frac{s}{2im} \left[ \langle \psi | \sigma_i \partial_j | \psi \rangle - h.c \right],$$

(5)

where $s = 1/2$ is the electron spin. The spin current in a particular region can be calculated using the wave function in the region. The time evolution of the magnetic moment is governed by the spin torque which is the net spin flux absorbed by the moment $\mathbf{B}$. For example, the time evolution of $m_{1z}$ and $m_{2z}$ is determined by $N_{1zz}$ and $N_{2zz}$, respectively, where $N_{1zz} = Q_{I,zz} - Q_{III,zz}$ and $N_{2zz} = Q_{I,zz} - Q_{V,zz}$. For a wave function $|\psi\rangle = \left( \begin{array}{c} a_1 \\ a_2 \end{array} \right) e^{ikx} + \left( \begin{array}{c} b_1 \\ b_2 \end{array} \right) e^{-ikx}$, the spin current becomes

$$Q_{xx} = \frac{k}{m} \left\{ \text{Re} \left[ a_1^* a_2^* - b_1^* b_2 \right] \right\},$$

$$Q_{yx} = \frac{k}{m} \left\{ \text{Im} \left[ a_1^* a_2 - b_1^* b_2 \right] \right\},$$

$$Q_{zz} = \frac{k}{2m} \left\{ \left( |a_1|^2 - |a_2|^2 \right) - \left( |b_1|^2 - |b_2|^2 \right) \right\} .$$

(6)

The components $a_i$ and $b_i$ are given by the wave functions in the regions I, III, and V; namely, $a_i$ ($b_i$) is the forward-moving (backward-moving) component, respectively. For example, in region III, $a_1 = \sum_{\sigma} T_{\sigma} (|+\chi_{\sigma}|)^2$, $b_1 = \sum_{\sigma} T_{\sigma} (|-\chi_{\sigma}|)^2,$
We solved Eq. (11) for a typical case with \( \eta = 5/4 \), \( L_1/L_{00} = 2.6 \), \( L_2/L_{00} = 2.4 \), and \( d/L_{00} = 1 \). The magnetic dipole interaction is set to zero (\( \alpha = 0 \)) but a small interaction (\( |\alpha| < 1 \)) gives a similar result. The initial direction of \( m_1 = (0, 1, 0) \) and \( m_2 = (0, -1, 0) \). In Fig. 2, we plot \( m_{1z} \) and \( m_{2z} \) as the solid curves with the corresponding labels. Based on the single layer analogy, one could expect that \( m_1 \) and \( m_2 \) end up aligning with the electron spin current along the \( Z \) direction. The behavior of \( m_1 \) does follow this naive expectation except for a dip near \( \tau = 22 \). However, the initial dynamics of \( m_2 \) is unexpected; the magnetic moment initially acquires a moment in the negative \( Z \) direction, anti-parallel to the incoming spin current.

We also plot the spin torque acting on the moments as dashed curves with labels. The behavior of the torques explains the dynamics of the magnetic moments. Mostly a positive spin torque is applied to \( m_1 \) so that it aligns along the \( Z \) direction more or less monotonically (there is a small dip in the torque which corresponds to the dip in the \( m_{1z} \) curve mentioned above). On the other hand \( m_2 \) moves below the XY plane due to a negative torque up to \( \tau = 8 \). Then a positive torque begins acting on \( m_2 \) until the magnetic moment aligns to the \( Z \) direction.

A second case is one in which a strong magnetic dipole interaction exists between the two films: MESE. In Fig. 3, we plot the numerical results for \( \alpha = -500 \), \( \eta = 15 \), \( L_1 = 2L_{00} \), \( L_2 = 2L_{00} \), and \( d = L_{00} \). The dashed line is the spin current in region I and V while the dashed curve is the spin current in region III. The initial value of the moments is \( m_1 = -m_2 = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}) \).

FIG. 3: (Color online) The time evolution of \( m_{1z} \) and \( m_{2z} \) (solid curves) for MESE with \( \alpha = -500 \), \( \eta = 15 \), \( L_1 = 2L_{00} \), \( L_2 = 2L_{00} \), and \( d = L_{00} \). The dashed line is the spin current in region I, III, and V while the dashed curve is the spin current in region III. The initial value of the moments is \( m_1 = -m_2 = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}) \).

As discussed in Ref. [13], the thickness of the two films should be the same. Brataas et al. estimated the effect of the thickness mismatch. We quantify this effect by calculating \( (Q_{I,zx} - Q_{V,zx})/Q_{I,zx} \) for given parameters. For the same parameters except for \( \eta = 10 \), we found that a 2% mismatch \( (L_2/L_{00} = 2.04) \) gives 0.3% of the spin transfer on the time scale of Fig. 3 \( (\tau = 20) \) while a 5% mismatch yields 1.7% of the spin transfer on this time scale. A longer time scale of course leads to more spin transfer. We also found that the spin transfer is insensitive to \( \alpha \) (for large values), and that if \( \eta \) is increased, the spin transfer is decreased for the same time scale.

In summary we have studied quantum mechanical aspects of the spin transfer to ferromagnetic bilayers. Our formulation is readily generalized to multilayers. The physics of the spin transfer in the multilayers is generally different from that of a single layer. We demonstrated novel quantum mechanical features which can occur only in the multilayer system such as negative spin torque and magnetoelectric spin echo (MESE). The spin transmission resonance (STR) in a multilayer is also illustrated. Our calculation reveals new channels through which the zero spin transfer occurs in multilayers: the STR and MESE. In spite of irregularities in real materials, we expect to see some of the qualitative features described here.

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