PALM: An Incremental Construction of Hyperplanes for Data Stream Regression

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Abstract—Data stream has been the underlying challenge in the age of big data because it calls for real-time data processing with the absence of a retraining process and/or an iterative learning approach. In realm of fuzzy system community, data stream is handled by algorithmic development of self-adaptive neuro-fuzzy systems (SANFS) characterized by the single-pass learning mode and the open structure property which enables effective handling of fast and rapidly changing natures of data streams. The underlying bottleneck of SANFS lies in its design principle which involves a high number of free parameters (rule premise and rule consequent) to be adapted in the training process. This figure can even double in the case of type-2 fuzzy system. In this work, a novel SANFS, namely parsimonious learning machine (PALM), is proposed. PALM features utilization of a new type of fuzzy rule based on the concept of hyperplane clustering which significantly reduces the number of network parameters because it has no rule premise parameters. PALM is proposed in both type-1 and type-2 fuzzy systems where all of which characterize a fully dynamic rule-based system. That is, it is capable of automatically generating, merging and tuning the hyperplane-based fuzzy rule in the single pass manner. The efficacy of PALM has been evaluated through numerical study with six real-world synthetic data streams from public database and our own real-world project of autonomous vehicles. The proposed model showcases significant improvements in terms of computational complexity and number of required parameters against several renowned SANFSs, while attaining comparable and often better predictive accuracy.

Index Terms—data stream, fuzzy, hyperplane, incremental, learning machine, parsimonious

I. INTRODUCTION

A DVANCE in both hardware and software technologies has triggered generation of a large quantity of data in an automated way. Such applications can be exemplified by space, autonomous systems, aircraft, meteorological analysis, stock market analysis, sensors networks, users of the internet, etc., where the generated data are not only massive and possibly unbounded but also produced at a rapid rate under complex environments. Such online data are known as data stream [1], [2]. A data stream can be expressed in a more formal way [3] as \( S = \{ x^1, x^2, ..., x^i, ..., x^\infty \} \), where \( x^i \) is enormous sequence of data objects and possibly unbounded. Each of the data object can be defined by an \( n \) dimensional feature vector as \( x^i = [x^i_1, x^i_2, ..., x^i_n] \), which may belong to a continuous, categorical, or mixed feature space. In the field of data stream mining, developing a learning algorithm as a universal approximator is challenging due to the following factors: 1) the whole data to train the learning algorithm is not readily available since the data arrive continuously; 2) the size of a data stream is not bounded; 3) dealing with a huge amount of data; 4) distribution of the incoming unseen data may slide over time slowly, rapidly, abruptly, gradually, locally, globally, cyclically or otherwise. Such variations in the data distribution of data streams over time are known as concept drift [4], [5]; 5) data are discarded after being processed.

To cope with above stated challenges in data streams, the learning algorithm should be equipped with the following features: 1) capability of working in single pass mode; 2) handling various concept drifts in data streams; 3) has low memory burden and computational complexity to enable real-time deployment under resource constrained environment. In realm of fuzzy system, such learning aptitude is demonstrated by Self Adaptive Neuro-Fuzzy System (SANFS) [6]. Until now, existing SANFSs are usually constructed via hypersphere-based or hyperellipsoid-based clustering techniques (HSBC or HEBC) to automatically partition the input space into a number of fuzzy rule and rely on the assumption of normal distribution due to the use of Gaussian membership function [7], [8], [9], [10], [11]. As a result, they are always associated with rule premise parameters, the mean and width of Gaussian distribution due to the use of Gaussian membership function, which need to be continuously adjusted. Other than the HSSC or HESC, the data cloud based clustering (DCBC) concept is utilized in [12], [13] to construct the SANFS. Unlike the HSSC and HESC, the data clouds do not have any specific shape. Therefore, required parameters in DCBC are less than HSSC and HESC. However, in DCBC, parameters like mean, accumulated distance of a specific point to all other points need to be calculated. In other words, it does not offer significant reduction on the computational complexity and memory demand of SANFS. Hyperplane-Based Clustering (HPBC) provides a promising avenue to overcome this drawback because it bridges the rule premise and the rule consequent by means of the hyperplane construction.

Although the concept of HPBC already exists since the last two decades [14], [15], [16], all of them are characterized by a static structure and are not compatible for data stream analytic due to their offline characteristics. Besides, majority of these algorithms still use the Gaussian or bell-shaped Gaussian function [17] to create the rule premise and are not free of the rule premise parameters. This problem is solved in [18],
where they have proposed a new function to accommodate the hyperplanes directly in the rule premise. Nevertheless, their model also exhibit a fixed structure and operates in the batch learning node. Based on this research gap, a novel SANFS, namely parsimonious learning machine (PALM), is proposed in this work. The novelty of this work can be summarized as follows:

1) PALM is constructed using the HPBC technique and its fuzzy rule is fully characterized by a hyperplane which underpins both the rule consequent and the rule premise. This strategy reduces the rule base parameter to the level of \( C \ast (P + 1) \) where \( C, P \) are respectively the number of fuzzy rule and input dimension.

2) PALM is proposed in both type-1 and type-2 versions derived from the concept of type-1 and type-2 fuzzy systems. Type-1 version incurs less network parameters and faster training speed than the type-2 version whereas type-2 version expands the degree of freedom of the type-1 version by applying the interval-valued concept leading to be more robust against uncertainty than the type-1 version.

3) PALM features a fully open network structure where its rules can be automatically generated, merged and updated on demand in the one-pass learning fashion. The rule generation process is based on the self-constructing clustering approach \([19], [20]\) checking coherence of input and output space. The rule merging scenario is driven by the similarity analysis via the distance and orientation of two hyperplanes. The online hyperplane tuning scenario is executed using the fuzzily weighted generalized recursive least square (FWGRLS) method.

4) Two real-world problems from our own project, namely online identification of Quadcopter unmanned aerial vehicle (UAV) and helicopter UAV, are presented in this paper and exemplify real-world streaming data problems. The two datasets are collected from indoor flight tests in the UAV lab of the university of new south wales (UNSW), Canberra campus and are made publicly available in \([21]\).

The efficacy of both type-1 and type-2 PALMs have been numerically evaluated using six real-world and synthetic streaming data problems. Moreover, PALM is also compared against prominent SANFSs in the literature and demonstrates encouraging numerical results in which it generates compact and parsimonious network structure while delivering comparable and even better accuracy than other benchmarked algorithms.

The remainder of this paper is structured as follows: Section II discusses literature survey over closely related works. In Section III, the network architecture of both type-1 and type-2 PALM are elaborated. Section IV describes the online learning policy of type-1 PALM, while Section V presents online learning mechanism of type-2 PALM. In Section VI, the proposed PALM’s efficacy has been evaluated through real-world and synthetic data streams. Finally, the paper ends by drawing the concluding remarks in Section VII.

II. RELATED WORK AND RESEARCH GAP WITH THE STATE-OF-THE-ART ALGORITHMS

SANFS can be employed for data stream regression, since they can learn from scratch with no base knowledge and are embedded with the self-organizing property to adapt to the changing system dynamics \([22]\). It fully work in a single-pass learning scenario, which is efficient for online learning under limited computational resources. An early work in this domain is seen in \([6]\) where an SANFS, namely SONFIN, was proposed. Evolving clustering method (ECM) is implemented in \([23]\) to evolve fuzzy rules. Another pioneering work in this area is the development of the online evolving T-S fuzzy system namely \( \epsilon \)TS \([7]\) by Angelov. \( \epsilon \)TS has been improved in the several follow-up works: \( \epsilon \)TS+ \([24]\), Simp\( \epsilon \)TS \([8]\), AnYa \([12]\). However, \( \epsilon \)TS+, and Simp\( \epsilon \)TS generate axis parallel ellipsoidal clusters, which cannot deal effectively with non-axis parallel data distribution. To deal with the non-axis parallel data distribution, an evolving multi-variable Gaussian (eMG) function was introduced in the fuzzy system in \([25]\). Another example of SANFS exploiting the multivariable Gaussian function is found in \([10]\) where the concept of statistical contribution is implemented to grow and prune the fuzzy rules on the fly. This work has been extended in \([9]\) where the idea of statistical contribution is used as a basis of input contribution estimation for the online feature selection scenario.

The idea of SANFS was implemented in type-2 fuzzy system in \([26]\). Afterward, they have extended their concept in local recurrent architecture \([27]\), and interactive recurrent architecture \([28]\). These works utilize Karnik-Mendel (KM) type reduction technique \([29]\), which relies on an iterative approach to find left-most and right-most points. To mitigate this shortcoming, the KM type reduction technique can be replaced with \( q \) design coefficient \([30]\) introduced in \([31]\).

SANFS is also introduced under the context of metacognitive learning machine (McLM) which encompasses three fundamental pillars of human learning: what-to-learn, how-to-learn, when-to-learn. The idea of McLM was introduced in \([32]\). McLM has been modified with the use of Scaffolding theory, McSLM, which aims to realize the plug-and-play learning fashion \([33]\). To solve the problem of uncertainty, temporal system dynamics and the unknown system order McSLM was extended in recurrent interval-valued metacognitive scaffolding fuzzy neural network (RIVMcSFNN) \([11]\). The vast majority of SANFSs are developed using the concept of HSSC and HESC which impose considerable memory demand and computational burden because both rule premise and rule consequent have to be stored and evolved during the training process.

III. NETWORK ARCHITECTURE OF PALM

In this section, the network architecture of PALM is presented in details. The T-S fuzzy system is a commonly used technique to approximate complex nonlinear systems due to its universal approximation property. The rule base in the T-S fuzzy model of that multi-input single-output (MISO) system can be expressed in the following IF-THEN rule format:
Exact $Y$ indicates the number of rules, $j, b$ are consequent parameters of the sub-model belonging to the $x$ dimension of input feature, $\Gamma = 1, 70$, $\text{dst}_{ij}(\omega_j)$ denotes the distance of each sample to their corresponding hyperplane. In our work, $\text{dst}_{ij}(\omega_j)$ is defined as [18] as follows:

$$\text{dst}(j) = \frac{|X_i \omega_j|}{||\omega_j||}$$

(3)

where $X_t \in \mathbb{R}^{1 \times (n+1)}$ and $\omega_j \in \mathbb{R}^{(n+1) \times 1}$ respectively stand for the input vector of the $t \rightarrow th$ observation and the output weight vector of the $j \rightarrow th$ rule. This membership function enables the incorporation of HPBC directly into the T-S fuzzy system directly with the absence of rule parameters except the first order linear function or hyperplane. Because a point to plane distance is not unique, the compatibility measure is executed using the minimum point to plane distance. The following discusses the network structure of PALM encompassing its type-1 and type-2 versions. PALM can be modelled as a four-layered network working in tandem, where the fuzzy rule triggers a hyperplane-shaped cluster and is induced by (3). Since T-S fuzzy rules can be developed solely using a hyperplane, PALM is free from antecedent parameters which results in dramatic reduction of network parameters. Furthermore, it operates in the one-pass learning fashion where it works point by point and a data point is discarded directly once learned.

A. Network Layers of Type-1 PALM :

Type-1 PALM is built upon a four-layered network architecture where the membership function (2) is utilized to fit the hyperplane-shaped cluster in identifying type-1 T-S fuzzy model.

1) Layer 1 (Input Layer): A single data point $x_n$ is fed into this layer at the $n \rightarrow th$ observation. Without performing any specific operation, the PALM passes the data stream directly to the next layer. Therefore, the output of this layer can be expressed as: $f_{OUT1}^1 = f_{OUT1}^2(x_i) = x_i$.

2) Layer 2 (Rule Layer): In PALM, the minimum point-to-plane distance is integrated in the hyperplane membership function (3) transforming the crisp input to the fuzzy input or performing the fuzzification process. In case of type-1 T-S fuzzy system, this hyperplane-shaped membership function can be expressed as:

$$f_{OT1}^2 = \mu_{B_i}(x) = \exp \left( -\Gamma \frac{\text{dst}(j)}{\max (\text{dst}(j))} \right)$$

(4)

The distance ($\text{dst}_{ij}(\omega_j)$) in (4) is the distance between the $i$th input and $j$th hyper plane as with 3 and calculated as follows:

$$\text{dst}(j) = \frac{|y_d - (\sum_{i=1}^{n} a_{ij} x_i + b_{ij})|}{\sqrt{1 + \sum_{i=1}^{n} (a_{ij})^2}}$$

(5)

where $a_{i,j}$ and $b_{i,j}$ are the $i \rightarrow th$ coefficient of $j \rightarrow th$ rule consequent and $y_d$ is the target variable. The distance measure as applied in [36] is a plausible choice to calculate the membership degree of HPBC because it delineates the relevance of existing hyperplanes to current data concept. This fact differs the PALM’s inference engine from that of [18]. Considering a MISO system, the IF-THEN rule of type-1 PALM can be expressed as follows:

\begin{align*}
R^j : \quad & \text{If} \ x_1 \text{ is } B^j_1 \text{ and } x_2 \text{ is } B^j_2 \text{ and ... and } x_n \text{ is } B^j_n \\
& \text{Then } y_j = b_{0j} + a_{1j} x_1 + \ldots + a_{nj} x_n
\end{align*}

(1)

where $R^j$ stands for the $j$th rule, $j = 1, 2, 3, \ldots, R$, and $n$ indicates the number of rules, $i = 1, 2, \ldots, n$, $n$ denotes the number of input attributes, $a$ and $b$ are consequent parameters of the sub-model belonging to the $j$th rule, $y_j$ is the output of the $j$th sub-model. The T-S fuzzy model can approximate a nonlinear system with a combination of several piecewise linear systems by partitioning the entire input space into several fuzzy regions. It expresses each input-output space with a linear equation as presented in (1). Approximation using T-S fuzzy model leads to a nonlinear programming problem and hinders its practical use. A simple solution to the problem is the utilization of various clustering techniques to identify the rule premise parameters. Because of the generation of the linear equation in the consequent part, the HPBC can be applied to construct the T-S fuzzy system efficiently. The advantages of using HPBC in the T-S fuzzy model can be seen graphically in Fig. 1.

![Figure 1. Clustering in T-S fuzzy model using hyper planes](image-url)
\[ R^1 : \text{ IF } X_n \text{ is } f_{OT1}^2, \text{ THEN } y_j = x_e^j \omega_j \]  
where \( x_e \) is the extended input vector and is expressed by inserting the intercept to the original input vector as \( x_e = [1, x_1, x_2, \ldots, x_n] \), \( \omega_j \) is the weight vector for the \( j \)th rule, \( \omega_j \) is the consequent part of the \( j \)th rule. It is observed from (6) that the drawback of HPBC-based TS fuzzy system lies in the high level fuzzy inference scheme which degrades the transparency of fuzzy rule. The intercept of extended input vector controls the slope of hyperplane which functions to prevent the untypical gradient problem.

3) Layer 3 (Consequent Layer): The consequent layer is akin to the basic T-S fuzzy model’s rule consequent part \( (y_j = b_0 + a_1 x_1 + \ldots + a_n x_n) \). The final output of this layer for the \( j \)th hyperplane can be calculated by weighting the extended input variable \( (x_e) \) with its corresponding weight vector as follows:

\[ f_{OT1}^3 = x_e^T \omega_j \]  

It is used in (7) after updating recursively by the FWGRLS method, which ensures a smooth change in the weight value.

4) Layer 4 (Output Layer): In this layer, the rule firing strength is normalized and combined with the output of consequent layer to produce the end-output of type-1 PALM. The final crisp output of the PALM for type-1 model can be expressed as follows:

\[ f_{OT1}^4 = \frac{\sum_{j=1}^R f_{OT1}^2 f_{OT1}^3}{\sum_{i=1}^n f_{OT1}^2} \]  

The normalization term in (8) guarantees the partition of unity where the sum of normalized membership degree is unity. The T-S fuzzy system is functionally-equivalent to the radial basis function (RBF) network if the rule firing strength is directly connected to the output of the consequent layer [37]. It is also depicted that the final crisp output is produced by the weighted average defuzzification scheme.

B. Network Layers of the Type-2 PALM:

Akin to its type-1 variant, Type-2 PALM is structured as a four-layered network working in tandem. It differs from the type-1 variant in the use of interval-valued hyperplane generating the type-2 fuzzy rule. The four layers of the type-2 PALM are outlined as follows:

1) Layer 1 (Input Layer): In the type-2 model, the crisp input data stream \( x_n \) are inserted into this layer and passes directly to the next layer without performing any specific operation. Therefore, the type-2 PALM expresses similar output like the type-1 counterpart as follows: \( f_{out}^1 = f_b^1(x_i) = x_i \).

2) Layer 2 (Rule Layer): In case of interval type-2 fuzzy system, the output of this layer can be expressed as:

\[ \bar{f}_{out}^2 = \exp \left( -\Gamma \frac{\bar{d}st(j)}{\max (\bar{d}st(j))} \right) \]  

where \( \bar{f}_{out}^2 = [\bar{f}_{out}^2; \bar{f}_{out}^3] \) is the upper and lower layer output, \( \bar{d}st = [\bar{d}st; \bar{d}st] \) is interval valued distance i.e. distance from incoming input data to upper and lower hyper planes. The distance between the \( i \)th input and \( j \)th upper hyper plane (\( \bar{d}st \)), and \( i \)th input and \( j \)th lower hyper plane (\( \bar{d}st \)) are calculated as follows:

\[ \bar{d}st_{ji} = \left| y_d - (\sum_{i=1}^n \bar{a}_{ij} x_i + \bar{b}_i) \right| \]  

where \( \bar{a}_i = [\bar{a}_{ij}; \bar{a}_{ij}] \) and \( \bar{b}_i = [\bar{b}_i; \bar{b}_i] \) are the interval-valued coefficients of the rule consequent of Type-2 PALM. The use of interval-valued coefficients result in the interval-valued firing strength which forms the footprint of uncertainty (FoU). The FoU is the key component against uncertainty of data streams and sets the degree of tolerance against uncertainty.

In a MISO system, the IF-THEN rule of type-2 PALM can be expressed as:

\[ R^1 : \text{ IF } X_n \text{ is } f_{OT1}^2, \text{ THEN } y_j = x_e^j \bar{\omega}_j \]  

The type-2 fuzzy rule is similar to that of the type-1 variant except the presence of interval-valued firing strength and interval-valued weight vector.

3) Layer 3 (Consequent Layer): The consequent layer is similar to the basic T-S fuzzy model’s rule consequent part \( (y_j = b_0 + b_1 x_1 + \ldots + b_n x_n) \) produced by the weighted operation between the extended input vector \( x_e \) and the output weight vector \( \omega_i \). In case of interval type-2 fuzzy system, the final output of this layer for the \( j \)th hyper plane can be calculated by weighting the extended input variable \( x_e \) with the interval-valued output weight vectors \( \bar{\omega}_i = [\bar{\omega}_i; \bar{\omega}_i] \) as follows:

\[ \bar{T}_{out}^4 = x_e^j \bar{\omega}_j, \bar{f}_{out}^4 = x_e^j \bar{\omega}_j \]  

The lower weight (\( \omega \)) and upper weight (\( \bar{\omega} \)) are used in (12) after updating recursively by FWGRLS method, which ensures a smooth change in weight value.

4) Layer 4 (Output Layer): Before performing the defuzzification method, the type reduction mechanism is carried out to craft the type-reduced set - the transformation from the type-2 fuzzy variable to the type-1 fuzzy variable. One of the commonly used type-reduction method is the Karnik Mendel (KM) procedure [29]. However, in the KM method, there is an involvement of an iterative process due to the requirement of reordering the rule consequent first in ascending order before getting the cross-over points iteratively incurring expensive computational cost. Therefore, instead of the KM method, the \( q \) design factor [30] is utilized to orchestrate the type reduction process. The final crisp output of the type-2 PALM can be expressed as follows:

\[ \bar{f}_{out}^5 = \frac{1}{2} (y_{out} + y_{rout}) \]  

where
\[
\begin{align*}
\tilde{y}_{l_{\text{out}}} &= \frac{\sum_{j=1}^{R} q_{l} f_{l_{\text{out}}}^{3} f_{l_{\text{out}}}^{4}}{\sum_{i=1}^{R} f_{l_{\text{out}}}^{4}} + \frac{\sum_{j=1}^{R} (1 - q_{l}) f_{l_{\text{out}}}^{3} f_{l_{\text{out}}}^{4}}{\sum_{i=1}^{R} f_{l_{\text{out}}}^{3}} \quad \text{(14)} \\
\tilde{y}_{r_{\text{out}}} &= \frac{\sum_{j=1}^{R} q_{r} f_{r_{\text{out}}}^{3} f_{r_{\text{out}}}^{4}}{\sum_{i=1}^{R} f_{r_{\text{out}}}^{4}} + \frac{\sum_{j=1}^{R} (1 - q_{r}) f_{r_{\text{out}}}^{3} f_{r_{\text{out}}}^{4}}{\sum_{i=1}^{R} f_{r_{\text{out}}}^{3}} \quad \text{(15)}
\end{align*}
\]

where \(y_{l_{\text{out}}}\) and \(y_{r_{\text{out}}}\) are the left and right outputs resulted from the type reduction mechanism. \(q_{l}\) and \(q_{r}\), utilized in (14) and (15), are the design factors initialized in a way to satisfy the condition \(q_{l} < q_{r}\). In our \(q\) design factor, the \(q_{l}\) and \(q_{r}\) steers the proportion of the upper and lower rules to the final crisp outputs \(\tilde{y}_{l_{\text{out}}}\) and \(\tilde{y}_{r_{\text{out}}}\) of the PALM. The normalization process of the type-2 fuzzy inference scheme [31] was modified in [11] to prevent the generation of the invalid interval. The generation of this invalid interval as a result of the normalization process of [31] was also proved in [11]. Therefore, normalization process as adopted in [11] is applied and advanced in terms of \(q_{l}\) and \(q_{r}\) in our work. Besides, in order to improve the performance of the proposed PALM, the \(q_{l}\) and \(q_{r}\) are not left constant rather continuously adapted using gradient decent technique as explained in section IV. Notwithstanding that the type-2 PALM is supposed to handle uncertainty better than its type-1 variant, it incurs a higher number of network parameters in the level of \(2 \times R \times (n + 1)\) as a result of the use of upper and lower weight vectors \(\tilde{\omega}_{i} = [\tilde{\omega}_{i}^{l}; \tilde{\omega}_{i}^{r}]\). In addition, the implementation of \(q\)-design factor imposes extra computational cost because \(q_{l}\) and \(q_{r}\) call for a tuning procedure with the gradient descent method.

IV. ONLINE LEARNING POLICY IN TYPE-1 PALM

This section describes the online learning policy of our proposed type-1 PALM. PALM is capable of starting its learning process from scratch with an empty rule base. Its fuzzy rules can be automatically generated on the fly using the self constructive clustering (SSC) method which checks the input and output coherence. The complexity reduction mechanism is implemented using the hyperplane merging module which vots similarity of two hyperplanes using the distance and angle concept. The hyperplane-based fuzzy rule is adjusted using the fuzzily weighted generalized recursive least square (FWGRLS) method in the single-pass learning fashion.

A. Mechanism of Growing Rules

The rule growing mechanism of type-1 PALM is adopted from the self-constructive clustering (SSC) method developed in [19], [20] to adapt the number of rules. This method has been successfully applied to automatically generate interval-valued data clouds in [13] but its use for HPBC deserves an in-depth investigation. In this technique, the rule significance is measured by calculating the input and output coherence. The coherence is measured by analysing the correlation between the existing data samples and the target concept. Hereby assuming the input vector as \(X_{i} \in \mathbb{R}^{n}\), target vector as \(T_{i} \in \mathbb{R}^{n}\), hyperplane of the \(i\)th local sub-model as \(H_{i} \in \mathbb{R}^{1 \times (n + 1)}\),

the input and output coherence between \(X_{i} \in \mathbb{R}^{n}\) and each \(H_{i} \in \mathbb{R}^{1 \times (n + 1)}\) are calculated as follows:

\[
I_c(H_{i}, X_{i}) = \xi(H_{i}, X_{i}) \quad \text{(16)}
\]

\[
O_c(H_{i}, X_{i}) = \xi(X_{i}, T_{i}) - \xi(H_{i}, T_{i}) \quad \text{(17)}
\]

where \(\xi(\cdot)\) express the correlation function. There are various linear and nonlinear correlation methods for measuring correlation, which can be applied. Among them, the nonlinear methods for measuring the correlation between variables are hard to employ in the online environment since they commonly use the discretization or Parzen window method. On the other hand, Pearson correlation is a widely used method for measuring correlation between two variables. However, it suffers from some limitations: it’s insensitivity to the scaling and translation of variables and sensitivity to rotation [38]. To solve these problems, a method namely maximal information compression index (MCI) is proposed in [38], which has also been utilized in the SSC method to measure the correlation \(\xi(\cdot)\) between variables as follows:

\[
\xi(X_{i}, T_{i}) = \frac{1}{2} (\text{var}(X_{i}) + \text{var}(T_{i})) - \sqrt{(\text{var}(X_{i}) + \text{var}(T_{i}))^2 - 4 \text{var}(X_{i})(\text{var}(T_{i})(1 - \rho(X_{i}, T_{i})^2))} 
\]

\[
\rho(X_{i}, T_{i}) = \frac{\text{cov}(X_{i}, T_{i})}{\sqrt{\text{var}(X_{i}) \text{var}(T_{i})}} 
\]

where \(\text{var}(X_{i})\), \(\text{var}(T_{i})\) express the variance of \(X_{i}\) and \(T_{i}\) respectively, \(\text{cov}(X_{i}, T_{i})\) presents the covariance between two variables \(X_{i}\) and \(T_{i}\), \(\rho(X_{i}, T_{i})\) stands for Pearson correlation index of \(X_{i}\) and \(T_{i}\). In a similar way, the correlation \(\xi(H_{i}, X_{i})\) and \(\xi(H_{i}, T_{i})\) can be measured using (18) and (19). In addition, the MCI method measures the compressed information when a newly observed sample is ignored. Properties of the MCI method in our work can be expressed as follows:

1) \(0 \leq \xi(X_{i}, T_{i}) \leq \frac{1}{2} (\text{var}(X_{i}) + \text{var}(T_{i}))\).
2) A maximum possible correlation is \(\xi(X_{i}, T_{i}) = 0\).
3) Express symmetric behavior \(\xi(X_{i}, T_{i}) = \xi(T_{i}, X_{i})\).
4) Invariance against the translation of the dataset.
5) Express the robustness against rotation.

\(I_c(H_{i}, X_{i})\) is projected to explore the similarity between \(H_{i}\) and \(X_{i}\) directly, while \(O_c(H_{i}, X_{i})\) is meant to examine the dissimilarity between \(H_{i}\) and \(X_{i}\) indirectly by utilizing the target vector as a reference. In the present hypothesis, the input and output coherence need to satisfy the following conditions to add a new rule or hyperplane:

\[
I_c(H_{i}, X_{i}) > b_1 \quad \text{and} \quad O_c(H_{i}, X_{i}) < b_2 \quad \text{(20)}
\]

where \(b_1 \in [0.01, 0.04]\), and \(b_2 \in [0.01, 0.1]\) are predetermined thresholds. If the hypothesis satisfies both the conditions of (20), a new rule is added with the highest input coherence. Besides, the accommodated data points of a rule are updated as \(N_{f_{j}} = N_{f_{j}} + 1\). Also, the correlation measure functions \(\xi(\cdot)\) are updated with (18) and (19). Due to the utilization
of the local learning scenario, each rule is adapted separately and therefore covariance matrix is independent to each rule \(C_j(k) \in \mathbb{R}^{(n+1) \times (n+1)}\), here \(n\) is the number of inputs. When a new hyperplane is added by satisfying (20), the hyperplane parameters and the output covariance matrix of FWGRLS method are crafted as follows:

\[
\pi_{R+1} = \pi_R^*, \quad C_{R+1} = \Omega I
\]  

(21)

Due to the utilization of the local learning scenario, the consequent of the newly added rules can be assigned as the closest rule, since the expected trend in the local region can be portrayed easily from the nearest rule. The value of \(\Omega\) in (21) is very large \((10^9)\). The reason for initializing the C matrix with a large value is to obtain a fast convergence to the real solution [39]. The proof of such consequent parameter setting is detailed in [40]. In addition, the covariance matrix of the individual rule has no relationship with each other. Thus, when the rules are pruned in the rule merging module, the covariance matrix, and consequent parameters are deleted as it does not affect the convergence characteristics of the C matrix and consequent of remaining rules.

B. Mechanism of Merging Rules

In SANFS, the rule evolution mechanism usually generate redundant rules. These unnecessary rules create complicity in the rule base, which hinders some desirable features of fuzzy rules: transparency and tractability in their operation. Notably, in handling data streams, two overlapping clusters or rules may easily be obtained when new samples occupied the gap between the existing two clusters. Several useful methods have been employed to merge redundant rules or clusters in [24], [41], [9], [13]. However, all these techniques are appropriate for mainly hypersphere-based or ellipsoid-based clusters.

In realm of hyperplane clusters, there is a possibility of generating a higher number of hyperplanes in dealing with the same dataset than spherical or ellipsoidal clusters because of the nature of HPBC in which each hyperplane represents specific operating region of the approximation curve. This opens higher chance in generating redundant rules than HSSC and HESC. Therefore, an appropriate merging technique is vital and has to achieve tradeoff between diversity of fuzzy rules and generalization power of the rule base. To understand clearly, the merging of two hyperplanes due to the new incoming training data samples is illustrated in Fig. 2.

In [42], to merge the hyperplanes, the similarity and dissimilarity between them are obtained by measuring only the angle between the hyperplanes. This strategy is, however, not conclusive to decide the similarity between two hyperplanes because it solely considers the orientation of hyperplane without looking at the relationship of two hyperplanes in the target space.

In our work, to measure the similarity between the hyperplane-shaped fuzzy rules, the angle between them is estimated as follows [43], [9]:

\[
\theta_{hp} = \arccos \left( \frac{\omega_{acm}^T \omega_{R+1}}{|\omega_{R}||\omega_{R+1}|} \right)
\]  

(22)

where \(\theta_{hp}\) is ranged between 0 and \(\pi\) radian, \(\omega_R = [b_1, R, b_2, R, ..., b_k, R]\), \(\omega_{R+1} = [b_1, R+1, b_2, R+1, ..., b_k, R+1]\). The angle between the hyperplanes is not sufficient to decide whether the rule merging scenario should take place because it does not inform the closeness of two hyperplanes in the target space. Therefore, the spatial proximity between two hyperplanes in the hyperspace are taken into account. If we consider two hyperplanes as \(l_{R1} = a_1 + x b_1\), and \(l_{R2} = a_2 + x b_2\), then the minimum distance between them can be projected as follows:

\[
d_{R,R+1} = \left| |a_1 - a_2| \right| \frac{|b_1 \times b_2|}{|b_1|^2}
\]  

(23)

The rule merging condition is formulated as follows:

\[
\theta_{hp} \leq c_1 \text{ and } d_{R,R+1} \leq c_2
\]  

(24)

where \(c_1 \in [0.01, 0.1]\), \(c_2 \in [0.001, 0.1]\) are predefined thresholds. If (24) is satisfied, fuzzy rules are merged. It is worth noting that the merging technique is only applicable in the local learning context because, in case of global learning, the orientation and similarity of two hyperplanes have no direct correlation to their relationship.

In our merging mechanism, a dominant rule having higher support is retained, whereas a less dominant hyperplane (rule) resided by less number of samples is pruned to mitigate the structural simplification scenario of PALM. A dominant rule has a higher influence on the merged cluster because support is retained, whereas a less dominant hyperplane (rule) is removed.

In [43], [9], [13] can be used to merge the two hyperplanes. In [42], the Yager’s participatory learning-inspired merging alternative, the Yager’s participatory learning-inspired merging scenario [41] can be used to merge the two hyperplanes.

C. Adaptation of Hyperplanes

In previous work on hyperplane based T-S fuzzy system [36], recursive least square (RLS) method is employed to calculate parameters of hyperplane. As an advancement to the
RLS method, a term for decaying the consequent parameter in the cost function of the RLS method is utilized in [44] and helps to obtain a solid generalization performance - generalized recursive least square (GRLS) approach. However, their approach is formed in the context of global learning. A local learning method has some advantages over its global counterpart: interpretability and robustness over noise. The interpretability is supported by the fact that each hyperplane portrays specific operating region of approximation curve. Also, in local learning, the generation or deletion of any rule does not harm the convergence of the consequent parameters does not harm the convergence of the consequent parameters of other rules, which results in a significantly stable updating process [45].

Due to the desired features of local learning scenario, the GRLS method is extended in [9], [11]: Fuzzily Weighted Generalised Recursive Least Square (FWGRLS) method. FWGRLS can be seen also as a variation of Fuzzily Weighted Recursive Least Square (FWRLS) method [7] with insertion of weight decay term. The FWGRLS method is formed in the proposed type-1 PALM, where the cost function can be expressed as:

\[ J^p_{L_j} = (y_t - x_e \pi_j) \Lambda_j (y_t - x_e \pi_j) + 2\beta \varphi(\pi_j) + (\pi - \pi_j)(C_j x_e)^{-1}(\pi - \pi_j) \]  

(27)

\[ J^p = \sum_{j=1}^{i} J^p_{L_j} \]  

(28)

where \( \Lambda_j \) denotes a diagonal matrix with the diagonal element of \( R_j \), \( \beta \) represents a regularization parameter, \( \varphi \) is a decaying factor, \( x_e \) is the extended input vector, \( C_j \) is the covariance matrix, \( \pi_j \) is the local subsystem of the \( j \)th hyperplane. Following the similar approach as [9], the final expression of the FWGRLS approach is formed as follows:

\[ \pi_j(k) = \pi_j(k - 1) - \beta C_j(k) \nabla \varphi \pi_j(k - 1) + \Upsilon(k)(y_t(k) - x_e \pi_j(k)); \quad j = [1, 2, ..., R] \]  

(29)

\[ C_j(k) = C_j(k - 1) - \Upsilon(k)x_e C_j(k - 1) \]  

(30)

\[ \Upsilon(k) = C_j(k - 1)x_e \left( \frac{1}{\Lambda_j} + x_e C_j(k - 1)x_e^T \right)^{-1} \]  

(31)

with the initial conditions

\[ \pi_1(1) = 0 \quad \text{and} \quad C_1(1) = \Omega I \]  

(32)

where \( \Upsilon(k) \) denotes the Kalman gain, \( R \) is the number of rules, \( \Omega = 10^5 \) is a large positive constant. In this work, the regularization parameter \( \beta \) is assigned as an extremely small value (\( \beta \approx 10^{-7} \)). It can be observed that the FWGRLS method is similar to the RLS method without the term \( \beta \pi_j(k) \nabla \varphi(k) \). This term steers the value of \( \pi_j(k) \) even to update an insignificant amount of it minimizing the impact of inconsequential rules. The quadratic weight decay function is chosen in PALM written as follows:

\[ \varphi(\pi_j(k - 1)) = \frac{1}{2}(\pi_j(k - 1))^2 \]  

(33)

Its gradient can be expressed as:

\[ \nabla \varphi(\pi_j(k - 1)) = \pi_j(k - 1) \]  

(34)

By utilizing this function, the adapted-weight is shrunk to a factor proportional to the present value. It helps to intensify the generalization capability by maintaining dynamic of output weights into small values [46].

V. ONLINE LEARNING POLICY IN TYPE-2 PALM

The learning policy of the type-1 PALM is extended in the context of the type-2 fuzzy system, where the design factor is utilized to carry out the type-reduction scenario. The learning mechanisms are detailed in the following subsections.
A. Mechanism of Growing Rules

In realm of the type-2 fuzzy system, the SSC method has been extended to the type-2 SSC (T2SSC) in [13]. It has been adopted and extended in terms of the design factors $q_l$ and $q_r$, since the original work in [13] only deals with a single design factor $q$. In this T2SSC method, the rule significance is measured by calculating the input and output coherence as done in the type-1 system. By assuming $\mathcal{H}_i = [\tilde{H}_i, H_i] \in \mathbb{R}^{R \times (1+n)}$ as interval-valued hyperplane of the $i$th local sub-model, the input and output coherence for our proposed type-2 system can be extended as follows:

$$I_{cL}(\tilde{H}_i, X_t) = (1 - q_l)\xi(\tilde{H}_i, X_t) + q_l\xi(H_i, X_t)$$  \hspace{1cm} (35)

$$I_{cR}(\tilde{H}_i, X_t) = (1 - q_r)\xi(\tilde{H}_i, X_t) + q_r\xi(H_i, X_t)$$  \hspace{1cm} (36)

$$I_c(\tilde{H}_i, X_t) = \frac{(I_{cL}(\tilde{H}_i, X_t) + I_{cR}(\tilde{H}_i, X_t))}{2}$$  \hspace{1cm} (37)

$$O_c(\tilde{H}_i, X_t) = \xi(X_t, T_t) - \xi(\tilde{H}_i, T_t)$$  \hspace{1cm} (38)

where

$$\xi_L(\tilde{H}_i, T_t) = (1 - q_l)\xi(\tilde{H}_i, T_t) + q_l\xi(H_i, T_t)$$  \hspace{1cm} (39)

$$\xi_R(\tilde{H}_i, T_t) = (1 - q_r)\xi(\tilde{H}_i, T_t) + q_r\xi(H_i, T_t)$$  \hspace{1cm} (40)

$$\xi(\tilde{H}_i, T_t) = \frac{(\xi_L(\tilde{H}_i, T_t) + \xi_R(\tilde{H}_i, T_t))}{2}$$  \hspace{1cm} (41)

Unlike the direct calculation of input coherence $I_c(\cdot)$ in type-1 system, in type-2 system the $I_c(\cdot)$ is calculated using (37) based on left $I_{cL}(\cdot)$ and right $I_{cR}(\cdot)$ input coherence. By using the MCI method in the T2SSC rule growing process, the correlation is measured using (18) and (19), where $(X_t, T_t)$ are substituted with $(\tilde{H}_i, X_t), (\tilde{H}_i, X_t), (\tilde{H}_i, T_t), (H_i, T_t)$. The conditions for growing rules remain the same as expressed in (20) and is only modified to fit the type-2 fuzzy system platform. The parameter settings for the predefined thresholds are as with the type-1 fuzzy model.

B. Mechanism of Merging Rules

The merging mechanism of the type-1 PALM is extended for the type-2 fuzzy model. To merge the rules, both the angle and distance between two interval-valued hyperplanes are measured as follows:

$$\tilde{\theta}_h = \arccos \left( \frac{\tilde{\omega}_R \tilde{\omega}_{R+1}}{|\tilde{\omega}_R| |\tilde{\omega}_{R+1}|} \right)$$  \hspace{1cm} (42)

$$\tilde{d}_{R,R+1} = \left| (\tilde{b}_1 - \tilde{b}_2) \right|$$  \hspace{1cm} (43)

where $\tilde{\theta}_h = [\tilde{\theta}_h, \tilde{\theta}_hp]$, and $\tilde{d}_{R,R+1} = [\tilde{d}_{R,R+1}, \tilde{d}_{R,R+1}+1]$. This $\tilde{\theta}_h$ and $\tilde{d}_{R,R+1}$ also needs to satisfy the condition of (24) to merge the rules, where the same range of $c_1$ and $c_2$ are applied in the type-2 PALM. The formula of merged weight in (25) is extended for the interval-valued merged weight as follows:

$$\omega_{new} = \frac{\omega_{old} N_{old} + \omega_{old} N_{old+1}}{N_{old} + N_{old+1}}$$  \hspace{1cm} (44)

where $\tilde{\omega} = [\tilde{\omega} \tilde{\omega}]$. As with the type-1 PALM, the weighted average strategy is followed in the rule merging procedure of the type-2 PALM.

C. Learning of the Hyperplane Submodels Parameters

The FWGRLS method [9] is extended to adjust the upper and lower hyperplanes of the interval-type-2 PALM. The final expression of the FWGRLS method is shown as follows:

$$\pi_j(k) = \pi_j(k-1) - \beta \tilde{C}_j(k) \nabla \pi_j(k-1) + \tilde{\Upsilon}(k)(y_t(k) - x_e \pi_j(k)); \ j = [1, 2, ..., R]$$  \hspace{1cm} (45)

$$\tilde{\Upsilon}(k) = \tilde{C}_j(k-1)x_e \left( \frac{1}{\Lambda_j} + x_e \tilde{C}_j(k-1) x_e^T \right)^{-1}$$  \hspace{1cm} (46)

where

$$\tilde{\pi}_j = [\pi_j \tilde{\pi}_j], \tilde{C}_j = [C_j \tilde{C}_j], \tilde{\Upsilon}_j = [\Upsilon_j \tilde{\Upsilon}_j], \text{ and } \Lambda_j = [\Lambda_j \Lambda_j].$$

The quadratic weight decay function of FEWGRLS method remains in the type-2 PALM to provide the weight decay effect in the rule merging scenario.

D. Adaptation of $q$ Design Factors

The $q$ design factor as used in [11] is extended in terms of left $q_l$ and right $q_r$ design factors to actualize a high degree of freedom of the type-2 fuzzy model. They are initialized in such a way that the condition $q_r > q_l$ is maintained. In this adaptation process, the gradient of $q_l$ and $q_r$ with respect to error $E = \frac{1}{2}(y_d - y_{out})^2$ can be expressed as follows:

$$\frac{\partial E}{\partial q_l} = \frac{\partial E}{\partial y_{out}} \cdot \frac{\partial y_{out}}{\partial q_l} \cdot \frac{\partial y_{out}}{\partial q_l}$$

$$= -\frac{1}{2} (y_d - y_{out}) \left( \frac{f_4^3}{\sum_{i=1}^{R} f_4^3} - \frac{f_4^3}{\sum_{i=1}^{R} f_4^3} \right)$$  \hspace{1cm} (48)

$$\frac{\partial E}{\partial q_r} = \frac{\partial E}{\partial y_{out}} \cdot \frac{\partial y_{out}}{\partial q_r} \cdot \frac{\partial y_{out}}{\partial q_r}$$

$$= -\frac{1}{2} (y_d - y_{out}) \left( \frac{f_4^3}{\sum_{i=1}^{R} f_4^3} - \frac{f_4^3}{\sum_{i=1}^{R} f_4^3} \right)$$  \hspace{1cm} (49)
The objective of the BJ gas furnace problem is to model the system at a setting in the literature as follows: A total of 3000 samples between k = 5001 and k = 5500 is tested with unseen 500 samples in the range of k = 5001 – 5500 to assess the generalization capability of the PALM.

b) Mackey-Glass Chaotic Time Series Dataset: Mackey-Glass (MG) chaotic time series problem having its root in [49] is a popular benchmark problem to forecast the future value of a chaotic differential delay equation by using the past values. Many researchers have used the MG dataset to evaluate their SANFSs’ learning and generalization performance. This dataset is characterized by their nonlinear and chaotic behaviors where its nonlinear oscillations replicate most of the physiological processes. The MG dataset is initially proposed as a control model of the generation of white blood cells. The mathematical model is expressed as:

\[
\frac{dy(k)}{dt} = \frac{by(k - \delta)}{1 + y^{10}(k - \delta)} - ay(k)
\]

where \(b = 0.2, a = 0.1, \) and \(\delta = 85\). The chaotic element is primarily attributed by \(\delta \geq 17\). Data samples are generated through the fourth-order Range Kutta method and our goal is to predict the system output \(\hat{y}(k + 85)\) at \(k = 85\) using four inputs: \(y(k), y(k - 6), y(k - 12), \) and \(y(k - 18)\). This series-parallel regression model can be expressed as follows:

\[
\hat{y}(k + 85) = f(y(k), y(k - 6), y(k - 12), y(k - 18))
\]

For the training purpose, a total of 3000 samples between \(k = 201\) and \(k = 3200\) is generated with the help of the 4th-order Range-Kutta method, whereas the predictive model is tested with unseen 500 samples in the range of \(k = 5001 – 5500\) to assess the generalization capability of the PALM.

c) Non-linear System Identification Dataset: A nonlinear system identification is put forward to validate the efficacy of PALM and has frequently been used by researchers to test their SANFSs. The nonlinear dynamic of the system can be formulated by the following differential equation:

\[
y(k + 1) = \frac{y(k)}{1 + y^2(k)} + u^3(k)
\]

where \(u(k) = \sin(2\pi k/100)\). The predicted output of the system \(\hat{y}(k + 1)\) depends on the previous inputs and its own lagged outputs, which can be expressed as follows:

\[
\hat{y}(k + 1) = f(y(k), y(k - 1), \ldots, y(k - 10), u(k))
\]
The first 50000 samples are employed to build our predictive model, and other 200 samples are fed the model to test model’s generalization.

2) Real-World Streaming Datasets: Three different real-world streaming datasets from two rotary wing unmanned aerial vehicle’s (RUAV) experimental flight tests and a time-varying stock index forecasting data are exploited to study the performance of PALM.

a) Quadcopter Unmanned Aerial Vehicle Streaming Data: A real-world streaming dataset is collected from a Pixhawk autopilot framework based quadcopter RUAV’s experimental flight test. All experiments are performed in the indoor UAV laboratory at the University of New South Wales, Canberra campus. To record quadcopter flight data, the Robot Operating System (ROS), running under the Ubuntu 16.04 version of Linux is used. By using the ROS, a well-structured communication layer is introduced into the quadcopter reducing the burden of having to reinvent necessary software.

During the real-time flight testing accurate vehicle position, velocity, and orientation are the required information to identify the quadcopter online. For system identification, a flight data of quadcopter’s altitude containing approximately 9000 samples are recorded with some noise from VICON optical motion capture system. Among them, 60% of the samples are used for training and remaining 40% are for testing. In this work, our model’s output $y(k)$ is estimated as $\hat{y}(k)$ from the previous point $y(k-6)$, and the system input $u(k)$, which is the required thrust to the rotors of the quadcopter. The regression model from the quadcopter data stream can be expressed as follows:

$$\hat{y}(k) = f(y(k-6), u(k))$$  (57)

b) Helicopter Unmanned Aerial Vehicle Streaming Data: The chosen RUAV for gathering streaming dataset is a Taiwanese made Align Trex450 Pro Direct Flight Control (DFC), fly bar-less, helicopter. The high degree of non-linearity associated with the Trex450 RUAV vertical dynamics makes it challenging to build a regression model from experimental data streams. All experiments are conducted at the UAV laboratory of the UNSW Canberra campus. Flight data consists of 6000 samples collected in near hover, heave and in ground effect flight conditions to simulate non-stationary environments. First 3600 samples are used for the training data, and the rest of the data are aimed to test the model. The nonlinear dependence of the helicopter RUAV is governed by the regression model as follows:

$$\hat{y}(k+1) = f(y(k), u(k))$$  (58)

where $\hat{y}(k+1)$ is the estimated output of the helicopter system at $k = 1$.

c) Time-Varying Stock Index Forecasting Data: Our proposed PALM has been evaluated by the time-varying dataset, namely the prediction of Standard and Poor’s 500 (S&P-500 (^GSPC)) market index [50], [51]. The dataset consists of sixty years of daily index values ranging from 3 January 1950 to 12 March 2009, downloaded from [52]. This problem comprises 14893 data samples. In our work, the reversed order data points of the same 60 years indexes have amalgamated with the original dataset, forming a new dataset with 29786 index values. Among them, 14893 samples are allocated to train the model and the remainder of 14893 samples are used for the validation data. The target variable is the next day S&P-500 index $y(k+1)$ predicted using previous five consecutive days indexes: $y(k)$, $y(k-1)$, $y(k-2)$, $y(k-3)$ and $y(k-4)$. The functional relationship of the predictive model is formalized as follows:

$$\hat{y}(k+1) = f(y(k), y(k-1), y(k-2), y(k-3), y(k-4))$$  (59)

This dataset carries the sudden drift property which happens around 2008. This property corresponds to the economic recession in the US due to the housing crisis in 2009.

| Model        | Reference | RMSE using testing samples | NDEI using testing samples | Number of rules | Number of inputs | Network Parameters | Number of training samples | Execution time (sec) |
|--------------|-----------|----------------------------|----------------------------|-----------------|------------------|---------------------|--------------------------|----------------------|
| DFNN         | [37]      | 0.7800                     | 4.8619                     | 1               | 2                | 6               | 200                      | 0.0933                |
| GDFNN        | [47]      | 0.0617                     | 0.3843                     | 1               | 2                | 7               | 200                      | 0.0964                |
| FAOSPFPNN    | [48]      | 0.0716                     | 0.4466                     | 1               | 2                | 4               | 200                      | 0.0897                |
| eTS          | [7]       | 0.0604                     | 0.3763                     | 5               | 2                | 30              | 200                      | 0.0635                |
| simp_eTS     | [8]       | 0.0607                     | 0.3782                     | 3               | 2                | 18              | 200                      | 1.5255                |
| GENEREIS     | [9]       | 0.0479                     | 0.2988                     | 2               | 2                | 18              | 200                      | 0.0925                |
| PANTIS       | [10]      | 0.0672                     | 0.4191                     | 2               | 2                | 18              | 200                      | 0.3162                |
| pRVFLN       | [13]      | 0.1195                     | 0.2984                     | 2               | 2                | 10              | 200                      | 0.0614                |
| Type-1 PALM (L) | -      | 0.0457                     | 0.2854                     | 6               | 2                | 18              | 200                      | 0.1574                |
| Type-1 PALM (G) | -      | 0.0509                     | 0.3178                     | 8               | 2                | 24              | 200                      | 0.1339                |
| Type-2 PALM (L) | -      | 0.0466                     | 0.2905                     | 4               | 2                | 3               | 200                      | 0.1359                |
| Type-2 PALM (G) | -      | 0.0096                     | 0.0598                     | 13              | 2                | 39              | 200                      | 0.3207                |

B. Results and Discussion

In this work, we have developed PALM by implementing type-1 and type-2 fuzzy concept, where both of them are simulated under two parameter optimization scenarios: 1) Type-1 PALM (L); 2) Type-1 PALM (G); 3) Type-2 PALM (L); 4) Type-2 PALM (G). $L$ denotes the Local update strategy
while \( G \) stands for the Global learning mechanism. Proposed models are tested with three synthetic and three real-world streaming datasets. Furthermore, the models are compared against eight prominent variants of SANFSs, namely DFNN [37], GDFNN [47], FAOSPFNN [48], eTS [7], simp eTS [8], GENEFIS [9], PANFIS [10], and pRVFLN [13]. Proposed PALMs’ efficacy has been evaluated by measuring the root mean square error (RMSE), and nondimensional error index (NDEI) written as follows:

\[
MSE = \frac{1}{N_T} \sum_{k=1}^{N} (y_t - y_k)^2, \quad RMSE = \sqrt{MSE} \\
NDEI = \frac{RMSE}{Std(T_s)}
\]

where \( N_T \) is the total number of testing samples, and \( Std(T_s) \) denotes a standard deviation over all actual output values in the testing set. A comparison is produced under the same computational platform in Intel(R) Xeon(R) E5-1630 v4 CPU with a 3.70 GHz processor and 16.0 GB installed memory.

1) Results and Discussion on Synthetic Streaming Datasets: Table I sums up the outcomes of the Box-Jenkins time series for all benchmarked models. Among various models, our proposed Type-2 PALM (G) clearly outperforms other consolidated algorithms in terms of predictive accuracy. For instance, the measured NDEI is just 0.0598 - the lowest among all models. Type-2 PALM (G) generates thirteen (13) rules to achieve this accuracy level. Although the number of generated rules is higher than that of remaining models, this accuracy far exceeds its counterparts whose accuracy hovers around 0.29. A fair comparison is also established by utilizing very close number of rules in some benchmarked strategies namely eTS, simp eTS, PANFIS, and GENEFIS. By doing so the lowest observed NDEI among the benchmarked variations is 0.29, delivered by GENEFIS. It is substantially higher than the measured NDEI of Type-2 PALM (G). The advantage of HPBC is evidenced by the number of PALM’s network parameters, where with thirteen rules and two inputs, PALM evolves only 39 parameters, whereas the number of network parameters of other algorithm for instance GENEFIS is 117. PALM requires the fewest parameters than all the other variants of SANFS as well and affects positively to execution speed of PALM. On the other hand, with only one rule the NDEI of PALM is also lower than the benchmarked variants as observed in Type-2 PALM (L) from Table I, where it requires only 3 network parameter. It is important to note that the rule merging mechanism is active in the case of only local learning scenario. Here the number of induced rules are 6 and 1, which is lower than i.e. 8 and 13 in their global learning versions. In both cases of G and L, the NDEI is very close to each other with a very similar number of rules. In short, PALM constructs a compact regression model using the Box-Jenkins time series with the least number of network parameters while producing the most reliable prediction.

The prediction of Mackey–Glass chaotic time series is challenging due to the nonlinear and chaotic behavior. Numerical results on the Mackey–Glass chaotic time series dataset is consolidated in Table II, where 500 unseen samples are used to test all the models. Due to the highly nonlinear behavior, an NDEI lower than 0.2 was obtained from only GENEFIS [9] among other benchmarked algorithms. However, it costs 42 rules and requires a big number (1050) of network parameters. On the contrary, with only 13 rules, 65 network parameters and faster execution, the Type-2 PALM (G) attains NDEI of 0.0685, where this result is traced within 2.45 seconds due to the deployment of fewer parameters than its counterparts. The use of rule merging method in local learning mode reduces the generated rules to five (5) - Type-1 PALM (L). A comparable accuracy is obtained from Type-1 PALM (L) with only 5 rules and 25 network parameters. An accomplishment of such accuracy with few parameters decreases the computational complexity in predicting complex nonlinear system as witnessed from Type-1 PALM (L) in Table II. Due to low computational burden, the lowest execution time of 0.8316 seconds is achieved by the Type-1 PALM (L).

PALM has been utilized to estimate a high-dimensional nonlinear system with 50000 training samples. This study case depicts similar trend where PALM is capable of delivering

### Table II

| Model            | Reference | RMSE using testing samples | NDEI using testing samples | Number of rules | Number of inputs | Network Parameters | Number of training samples | Execution time (sec) |
|------------------|-----------|----------------------------|----------------------------|-----------------|-----------------|---------------------|-------------------------|----------------------|
| DFNN             | [37]      | 3.0531                     | 12.0463                    | 1               | 4               | 10                  | 3000                    | 11.1674              |
| GDFNN            | [47]      | 0.1320                     | 0.6030                     | 1               | 4               | 13                  | 3000                    | 12.1076              |
| FAOSPFNN         | [48]      | 0.2360                     | 0.9314                     | 1               | 4               | 6                   | 3000                    | 13.2213              |
| eTS              | [7]       | 0.0734                     | 0.2899                     | 48              | 4               | 480                 | 3000                    | 8.6174               |
| simp eTS        | [8]       | 0.0823                     | 0.2461                     | 75              | 4               | 750                 | 3000                    | 20.9274              |
| GENEFIS          | [9]       | 0.0303                     | 0.1198                     | 42              | 4               | 1050                | 3000                    | 4.9364               |
| PANFIS           | [10]      | 0.0721                     | 0.2847                     | 33              | 4               | 825                 | 3000                    | 4.8679               |
| pRVFLN           | [13]      | 0.1168                     | 0.4615                     | 2               | 4               | 18                  | 2993                    | 0.9236               |
| Type-1 PALM (L)  | -         | 0.0688                     | 0.2718                     | 5               | 4               | 25                  | 3000                    | 0.8316               |
| Type-1 PALM (G)  | -         | 0.0405                     | 0.1600                     | 13              | 4               | 65                  | 3000                    | 0.9645               |
| Type-2 PALM (L)  | -         | 0.0524                     | 0.2070                     | 10              | 4               | 50                  | 3000                    | 2.7798               |
| Type-2 PALM (G)  | -         | 0.0159                     | 0.0685                     | 13              | 4               | 65                  | 3000                    | 2.4502               |
### Table III
**MODELING OF THE NON-LINEAR SYSTEM USING VARIOUS SELF-ADAPTIVE NEURO-FUZZY SYSTEMS**

| Model   | Reference | RMSE using testing samples | NDEI using testing samples | Number of rules | Number of inputs | Network Parameters | Number of training samples | Execution time (sec) |
|---------|-----------|----------------------------|---------------------------|-----------------|-----------------|-------------------|--------------------------|----------------------|
| DFN    | [37]      | 0.0380                     | 0.0404                    | 2               | 2               | 12                | 50000                    | 219.246              |
| DGFNN  | [47]      | 0.0440                     | 0.0468                    | 2               | 2               | 14                | 50000                    | 2355.726             |
| FAOSPENN | [48]    | 0.0027                     | 0.0029                    | 4               | 2               | 16                | 50000                    | 387.7890             |
| eTS    | [7]       | 0.0750                     | 0.08054                   | 7               | 2               | 42                | 50000                    | 108.5791             |
| simp_eTS | [8]      | 0.0747                     | 0.07892                   | 7               | 2               | 42                | 50000                    | 129.5552             |
| GENEFIS | [9]       | 0.00041                    | 0.000413                  | 6               | 2               | 54                | 50000                    | 10.9021              |
| PANFIS | [10]      | 0.001264                   | 0.002581                  | 27              | 2               | 243               | 50000                    | 42.4945              |
| prRVFLN | [13]     | 0.06395                    | 0.06596                   | 7               | 2               | 10                | 49999                    | 12.0105              |
| Type-1 PALM (L) | -     | 0.08808                    | 0.09371                   | 5               | 2               | 15                | 50000                    | 9.9177               |
| Type-1 PALM (G) | -     | 0.07457                    | 0.07934                   | 29              | 2               | 87                | 50000                    | 37.3684              |
| Type-2 PALM (L) | -     | 0.03277                    | 0.03487                   | 3               | 2               | 9                 | 50000                    | 13.7455              |
| Type-2 PALM (G) | -     | 0.00387                    | 0.00412                   | 21              | 2               | 63                | 50000                    | 55.4863              |

### Table IV
**ONLINE MODELING OF THE QUADCOPTER UTILIZING VARIOUS SELF-ADAPTIVE NEURO-FUZZY SYSTEMS**

| Model   | Reference | RMSE using testing samples | NDEI using testing samples | Number of rules | Number of inputs | Network Parameters | Number of training samples | Execution time (sec) |
|---------|-----------|----------------------------|---------------------------|-----------------|-----------------|-------------------|--------------------------|----------------------|
| DFN    | [37]      | 0.1469                     | 0.6925                    | 1               | 2               | 6                 | 5467                     | 19.0962              |
| DGFNN  | [47]      | 0.1442                     | 0.6800                    | 1               | 2               | 7                 | 5467                     | 20.1737              |
| FAOSPENN | [48]     | 0.2141                     | 1.0097                    | 12              | 2               | 48                | 5467                     | 25.4000              |
| eTS    | [7]       | 0.1361                     | 0.6417                    | 4               | 2               | 24                | 5467                     | 3.0856               |
| simp_eTS | [8]      | 0.1282                     | 0.6048                    | 4               | 2               | 24                | 5467                     | 3.9084               |
| GENEFIS | [9]       | 0.1327                     | 0.6257                    | 1               | 2               | 9                 | 5467                     | 1.7838               |
| PANFIS | [10]      | 0.1925                     | 0.9077                    | 47              | 2               | 424               | 5467                     | 6.0244               |
| prRVFLN | [13]     | 0.1191                     | 0.5223                    | 1               | 2               | 5                 | 5467                     | 0.9485               |
| Type-1 PALM (L) | -     | 0.1318                     | 0.6216                    | 7               | 2               | 21                | 5467                     | 1.6843               |
| Type-1 PALM (G) | -     | 0.1107                     | 0.5223                    | 7               | 2               | 9                 | 5467                     | 6.7249               |
| Type-2 PALM (L) | -     | 0.0772                     | 0.3642                    | 8               | 2               | 24                | 5467                     | 2.9308               |
| Type-2 PALM (G) | -     | 0.0186                     | 0.0878                    | 7               | 2               | 21                | 5467                     | 2.6116               |

Comparable accuracy but with much less computational complexity and memory demand. The deployment of rule merging module lessens the number of rules from 29 to 5 in case of type-1 PALM, and 3 from 21 in type-2 PALM. The obtained NDEI of PALMs with such a small number of rules is also similar to other SANFS variants. To sum up, the PALM can deal with streaming examples with low computational burden due to the utilization of few network parameters, where it maintains a comparable or better predictive accuracy.

2) Results and Discussion on Real-World Data Streams:

Table IV outlines the results of online identification of a quadcopter RUAV from experimental flight test data. A total 9112 samples of quadcopter’s hovering test with a very high noise from motion capture technique namely VICON [53] is recorded. Building SANFS using the noisy streaming dataset is computationally expensive as seen from a high execution time of the benchmarked SANFSs. Contrast with these standard SANFSs, a quick execution time is seen from PALMs. It indicates that PALMs are computationally efficient and are suitable for online identification due to the utilization of few network parameters, where it maintains a comparable or better predictive accuracy. For instance, the lowest NDEI at just 0.0878 is elicited in Type-2 PALM (G). To put it plainly, due to utilizing incremental HPBC, PALM can perform better than its counterparts SANFSs driven by HSSC and HESC methods when dealing with noisy datasets.

The online identification of a helicopter RUAV (Trex450 Pro) from experimental flight data at hovering condition are tabulated in Table V. The highest identification accuracy with the NDEI of only 0.0758 is obtained from the proposed Type-2 PALM (G) with 21 rules. As with the previous experiments, the activation of rule merging scenario reduces the fuzzy rules significantly from 17 to 4 in type-1 PALM, and from 21 to 4 in type-2 PALM. The highest accuracy is produced by type-2 PALM with only 4 rules due to most likely uncertainty handling capacity of type-2 fuzzy system. PALM’s prediction on the helicopter’s hovering dynamic and its rule evolution are depicted in Fig. 3. These figures are produced by the type-2 PALM(L). Furthermore, the predictive capability, rule evolution, NDEI evolution and error of the PALM for six streaming datasets are attached in the supplementary document to keep the paper compact.

The numerical results on the time-varying Stock Index Forecasting S&P-500 (^GSPC) problem are organized in Table VI. The lowest number of network parameters is obtained from PALM, and subsequently, the fastest training speed of 1.6897 seconds is attained by Type-1 PALM (G). All consolidated algorithms generate the same level of accuracy around 0.015.
Table V
ONLINE MODELING OF THE HELICOPTER UTILIZING VARIOUS SELF-ADAPTIVE NEURO-FUZZY SYSTEMS

| Model       | Reference | RMSE using testing samples | NDEI using testing samples | Number of rules | Number of inputs | Network Parameters | Number of training samples | Execution time (sec) |
|-------------|-----------|-----------------------------|-----------------------------|----------------|------------------|---------------------|--------------------------|---------------------|
| DFNN        | [37]      | 0.0402                      | 0.6644                      | 1              | 2                | 6                   | 3600                     | 8.7760              |
| GDFNN       | [47]      | 0.0326                      | 0.5082                      | 2              | 2                | 14                  | 3600                     | 11.2705             |
| FAOSPPNN    | [48]      | 0.0368                      | 0.5733                      | 2              | 2                | 8                   | 3600                     | 2.4266              |
| eTS         | [7]       | 0.0535                      | 0.8352                      | 3              | 2                | 18                  | 3600                     | 1.3822              |
| simp_eTS    | [8]       | 0.0534                      | 0.8336                      | 3              | 2                | 18                  | 3600                     | 2.3144              |
| GENERIS     | [9]       | 0.0355                      | 0.5541                      | 2              | 2                | 18                  | 3600                     | 0.6736              |
| PANFIS      | [10]      | 0.0362                      | 0.5652                      | 2              | 2                | 81                  | 3600                     | 1.4571              |
| pRVFLN      | [13]      | 0.0329                      | 0.5137                      | 2              | 2                | 10                  | 3360                     | 1.0195              |
| Type-1 PALM (L) | -   | 0.0356                      | 0.5549                      | 4              | 2                | 12                  | 3600                     | 4.7757              |
| Type-1 PALM (G) | -   | 0.0311                      | 0.4857                      | 17             | 2                | 51                  | 3600                     | 2.4481              |
| Type-2 PALM (L) | -   | 0.0225                      | 0.3520                      | 4              | 2                | 12                  | 3600                     | 1.8247              |
| Type-2 PALM (G) | -   | 0.0048                      | 0.0758                      | 21             | 2                | 63                  | 3600                     | 5.7608              |

Table VI
MODELING OF THE TIME-VARYING STOCK INDEX FORECASTING USING VARIOUS SELF-ADAPTIVE NEURO-FUZZY SYSTEMS

| Model       | Reference | RMSE using testing samples | NDEI using testing samples | Number of rules | Number of inputs | Network Parameters | Number of training samples | Execution time (sec) |
|-------------|-----------|-----------------------------|-----------------------------|----------------|------------------|---------------------|--------------------------|---------------------|
| DFNN        | [37]      | 0.00441                     | 0.01554                     | 1              | 5                | 12                  | 14893                    | 347.7522            |
| GDFNN       | [47]      | 0.03053                     | 1.0707                      | 1              | 5                | 16                  | 14893                    | 344.4558            |
| FAOSPPNN    | [48]      | 0.20252                     | 0.71316                     | 1              | 5                | 7                   | 14893                    | 15.1439             |
| eTS         | [7]       | 0.01879                     | 0.06629                     | 3              | 5                | 36                  | 14893                    | 31.1606             |
| simp_eTS    | [8]       | 0.000042                    | 0.00149                     | 1              | 5                | 12                  | 14893                    | 12.5562             |
| GENERIS     | [9]       | 0.00454                     | 0.01603                     | 1              | 5                | 36                  | 14893                    | 2.0188              |
| PANFIS      | [10]      | 0.000441                    | 0.001555                    | 1              | 5                | 36                  | 14893                    | 4.3333              |
| pRVFLN      | [13]      | 0.000441                    | 0.001555                    | 1              | 5                | 11                  | 11170                    | 2.5104              |
| Type-1 PALM (L) | -   | 0.000042                    | 0.00149                     | 1              | 5                | 6                   | 14893                    | 1.8921              |
| Type-1 PALM (G) | -   | 0.000041                    | 0.001441                    | 1              | 5                | 6                   | 14893                    | 1.6897              |
| Type-2 PALM (L) | -   | 0.000040                    | 0.001555                    | 1              | 5                | 6                   | 14893                    | 3.4968              |
| Type-2 PALM (G) | -   | 0.000040                    | 0.001555                    | 1              | 5                | 6                   | 14893                    | 3.1460              |

C. Sensitivity Analysis of Predefined Thresholds

In the rule growing purpose, two predefined thresholds ($b_1$ and $b_2$) are utilized in our work. During various experimentations, it has been observed that the higher the value of $b_1$, the lesser the number of hyperplanes are added and vice versa. Unlike the effect of $b_1$, in case of $b_2$, the higher the value the more the hyperplanes are added and vice versa. To further validate this feature, the sensitivity of $b_1$ and $b_2$ is evaluated using the Box–Jenkins (BJ) gas furnace dataset. The same I/O relationship as described in the subsection VI-A is applied here, where the model is trained also with same 200 samples and remaining 90 unseen samples are used to test the model.

In the first test, $b_2$ is varied in the range of $[0.052, 0.053, 0.054, 0.055]$, while the value of $b_1$ is kept fixed at 0.020. On the other hand, the varied range for $b_1$ is $[0.020, 0.022, 0.024, 0.026]$, while $b_2$ is maintained at 0.055. In the second test, the altering range for $b_1$ is $[0.031, 0.033, 0.035, 0.037]$, and for $b_2$ is $[0.044, 0.046, 0.048, 0.050]$. In this test, for a varying $b_1$, the constant value of $b_2$ is 0.050, where $b_1$ is fixed at 0.035 during the change of $b_2$. To evaluate the sensitivity of these thresholds, normalized RMSE (NRMSE), NDEI, running time, and number of rules are reported in Table VII. The NRMSE formula can be expressed as:

$$NRMSE = \sqrt{\frac{MSE}{Std(T_s)}}$$

(62)

From Table VII, it has been observed that in the first test for different values of $b_1$ and $b_2$, the value of NRMSE and NDEI remains stable at 0.023 and 0.059 respectively. The execution time varies in a stable range of $[0.31, 0.35]$ seconds and the number of generated rules is 13. In the second test, the NRMSE, NDEI, and execution time are relatively constant in the range of $[0.046, 0.048]$, $[0.115, 0.121]$, $[0.26, 0.31]$ correspondingly. The value of $b_1$ increases, and $b_2$ reduces compared to test 1, and the fewer number of rules are generated across different experiments of our work.

VII. CONCLUSIONS

A novel SANFS, namely PALM, is proposed in this paper for data stream regression. The PALM is developed with the concept of HPBC which incurs very low network parameters. The reduction of network parameters bring down the execution times because only the output weight vector calls for the tuning scenario without compromise on predictive accuracy. PALM possesses a highly adaptive rule base where its fuzzy rules can be automatically added when necessary based on the SCC theory. It implements the rule merging scenario for
complexity reduction and the concept of distance and angle is introduced to coalesce similar rules. The efficiency of the PALM has been tested in six real-world and artificial data stream regression problems where PALM outperforms recently published works in terms of network parameters and running time. It also delivers state-of-the-art accuracies which happen to be comparable and often better than its counterparts. In the future, PALM will be incorporated under a deep network structure.

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