Distant source image motion due to gravitational field of the Galaxy stars

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Abstract

Gravitational field of stars of the Galaxy causes additional motions of images of extragalactic sources. For typical realization of stars this results in a small rotation of extragalactic reference frame in the direction of Galactic rotation. We estimate angular velocity of corresponding image motion and show that it is different in case of discrete and continuous matter distribution in the Galaxy.

1 Introduction

Perspectives of microarcsecond astrometry raise questions concerning motion of images of extragalactic radiation sources due to gravitational field of moving stars [1],[2],[3],[4],[5]. This gravitational image motion (GIM) may be comparable to proper motion of quasars. There is some hope that corresponding accuracy to observe such effects will be available for future space-based radio interferometers [1]. However, it will be difficult to separate GIM from real proper motions, and this puts some limit for accuracy of fundamental reference frame based on extragalactic sources [4],[5]. Indeed, we cannot take into account positions of all stars in the Galaxy; we only may work with probability distribution of Galactic stars leading to distribution of image motions of certain extragalactic object [2],[3].

It has been pointed out in [2],[3] that stochastic GIM of an extragalactic source induced by stellar motions is accompanied with a systematic component which depends upon bulk velocity of microlensing stars, including stars that are far away from the line of sight. This requires some explanation. If
we perform a statistical averaging using all possible positions of microlenses (stars), we shall obtain a value different from the most probable value of GIM that might be observed at the present epoch. There is also a difference between the statistical average and arithmetic mean over all reference sources used in extragalactic reference frame. This difference appears to be essential (an example will be given in this paper below). The reason is that in reality the number of these sources is not sufficiently large. In this paper we treat this question in more detail. We show that one can estimate probable GIM effect, without recourse to probability distributions, after some restriction of sample of observed sources. The result differs in case of continuous (e.g., dark matter) and discrete (e.g., stars) mass distributions.

2 Motion of source image microlensed by a point mass

In case of microlensing of quasars by Galaxy stars we may neglect cosmological curvature. Evidently this does not contradict to assumption that the radiation source is at the infinity.

Let unperturbed light ray moves from the infinitely distant radiation source in negative direction of $z$-axis of Cartesian coordinates $\{x, y, z\}$, the observer being at the origin. Let position of microlensing point mass $M$ be $(r, z)$, where $r = (x, y)$ is a two-dimensional vector in the transverse plane; i.e. $r = |r|$ is the impact distance of the unperturbed ray with respect to the mass. We consider the case of weak microlensing $r > > (mz)^{1/2}$, $m = GM/c^2$; therefore the light trajectory in the gravitational field of the point mass can be obtained in the post-Newtonian approximation. Note that in our problem we may have $r$ of the same order as $z$. In this case

$$\Psi = -2 \frac{r}{r^2} \left[ 1 + z(z^2 + r^2)^{-1/2} \right].$$

where $\Psi = (\Psi_1, \Psi_2)$ is a two dimensional vector describing the source image angular shift per unit value of $m$ [3,4,7]. Formula (1) may be obtained from a more general relation [7] in the case of an infinite source.

Further $V_p = (v, w)$ stands for velocity of the point microlens, $w$ is the velocity component parallel to the line of sight and $v$ represents the transversal components. In virtue of (1) this leads to the source GIM that equals to $mU$ (in radians per unit of time), where [3]
\[
U = \frac{d\Psi}{dt} = -2 \left\{ \frac{1}{r^4} \left[ vr^2 - 2r \cdot v \right] \cdot \left[ 1 + \frac{z}{\sqrt{z^2 + r^2}} \right] + \frac{zr^2}{2(z^2 + r^2)^{3/2}} \right. + \frac{wr - zv/2}{(z^2 + r^2)^{3/2}} \right\} \quad (2)
\]

Here \(U\) is a function of the microlens position \((r, z)\) and its velocity \(v\).

### 3 Motion of source image microlensed by Galaxy stars

Because of smallness of the effect, the action of all Galaxy stars will be taken into account linearly. This will be performed by integrating (2). However this requires justification. In fact we are dealing with extended sources; this is not taken into account by Eq.(2). As we shall see below, this may be important in our problem.

We are interested in some estimate of probable GIM value that may be obtained after taking arithmetic mean for as many reference sources as possible. As we pointed out in the Introduction, this is not a statistical average, because in reality the number of observed sources is limited. For example, the total number of extragalactic sources in ICRS is about 600. Moreover, this is much lesser, if we confine ourselves, e.g., to the Galaxy plane.

We deal with an ensemble of possible positions of stars in the Galaxy. Consider a remote extended source, which has impact distance \(r_i\) with respect to \(i\)-th star in this realization. Let \(p = \min\{r_i\}\), where this minimum is taken over all Galactic stars. Now we consider two type of events: (A) when \(p \gg L\), where \(L = D \cdot \alpha_S\), \(\alpha_S\) is angular size of the source, \(D\) is a typical distance to microlens (of the order of 50 kpc; (B) when \(p \sim L\) or \(p < L\) (at least one of stars is projected onto the source). We suppose that for typical value of \(L\) we have \(L < R_E = (4mD)^{1/2}\), in this case we may strengthen our arguments by considering even larger domain \(B'\), defined by condition \(p < L\) for corresponding realizations of stars. The rest of events we refer to the domain \(A'\); corresponding probability is \(P_{A'} \approx 1\). The events from \(B'\) can be separated in observations: these are strong microlensing events that are characterized by considerable brightness amplification and relatively fast...
image motions. These events might be taken into account, including the case of an extended source \cite{8,9}. Probability of these events $P_{B'} \sim 10^{-6}$ is so small that they are not practically essential. In case of limited number of reference sources it is reasonable to exclude such rare microlens realizations that would hardly occur during this century.

Nevertheless, though $P_{B'} \ll P_{A'}$, the input of $B'$ into the statistical average GIM $< U >$ is essential because the events $B'$ introduce large image velocities. To present an example, we derive below a nonzero value of GIM for a fictitious observer at the center of the Galaxy. Because of stationary rotation of the Galaxy, one must have $< U > = 0$ (no matter, for an infinite time average or for a statistical average). This shows that in this case the sets $A'$ and $B'$ compensate each other. However, for a limited time interval we typically have a nonzero GIM value, because we do not meet realizations from $B'$.

Therefore, the question is how to obtain a consistent GIM estimate. The most correct way is to obtain probability for all necessary velocity intervals. However, to have an order-of-magnitude estimate, we can avoid calculations of probability distributions, if we confine ourselves to obtain average value of GIM in the domain $A'$. Further we have in mind just this average when speaking about average GIM effect. It is important to note that this value practically does not depend upon the exact size of $B'$. This is provided by convergence of integrals that will be considered below.

Thus we return to Eq.\cite{2}, which describes contribution of a single star. We assume that a star at the point $(r, z)$ has velocity $V_p(r, z)$. Then we consider sum over all stars in the Galaxy, which must further be averaged with the Galactic mass density $\rho(r, z)$. This enables us to pass on integration yielding average GIM in the domain $A'$ for the weak microlensing events.

\begin{equation}
< U_{\text{tot}} >_{A'} = \frac{G}{c^2} \int dz \int d^2r \rho(r, z) U(r, z, V_p(r, z))
\end{equation}

where we suppose that $\rho$ vanishes outside a bounded domain.

Taking into account the explicit form \cite{2} it is easy to see that considerable contribution may be due to stars at large impact distances from the line of sight. Also one can show that the singularities in \cite{3} for small $r$ are integrable. This allows us to avoid the question about exact value of lower limit of $r$ in the definition of the domain $A'$. 

4
4 Continuous versus discrete mass distribution

It is interesting to compare GIM (3) with a corresponding expression in case of a continuous mass distribution $\rho = \rho(r, z, t)$, satisfying the continuity equation

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho \nabla_p) = 0. \quad (4)$$

In this case Eq. (1) should be integrated with the density $\rho$:

$$\Psi_{tot} = \frac{G}{c^2} \int dz \int d^2r \rho(r, z, t) \Psi(r, z).$$

Note that we have the same formula for $\Psi_{tot}$ in case of continuous and discrete matter distribution. However, this is not true for GIM.

The shift of remote source image in case of continuous matter distribution has been treated in [6]. These shifts appear to be of the order of $10^{-5} \div 10^{-6}$ radians. However, this value cannot be observed from the Solar system. In order to deal with observable (in principle) values one must derive changes of the source positions with time:

$$U^*_{tot} \equiv \frac{d\Psi_{tot}}{dt} = \frac{G}{c^2} \int dz \int d^2r \frac{\partial \rho}{\partial t} \Psi(r, z). \quad (5)$$

This will be compared to $< U_{tot} >_{A'}$. After standard elimination of singular points at $r = 0$ in $U^*_{tot}$ and taking into account Eq. (4) we represent this integral as sum of two terms

$$U^*_{tot} = U^{(0)} + U^{(1)}, \quad (6)$$

$U^{(0)}$ and $U^{(1)}$ being two-dimensional vectors, the components of the first term ($i=1,2$)

$$U^{(0)}_i = \frac{G}{c^2} \int dz \int d^2r \rho(\nabla_p \nabla) \Psi_i$$

are the same as the components of Eq. (3):

$$U^{(0)} = < U_{tot} >_{A'}.$$ \qquad (7)

The second term is
\[ U_1^i = -\frac{G}{c^2} \int dz \int d^2r \text{div}(V_p \rho \Psi_i). \]

For this latter term we have in cylindrical coordinates \( \{r, \varphi, z\} \)

\[ U_i^{(1)} = -\frac{G}{c^2} \int_{-\infty}^{\infty} dz \int_0^{2\pi} d\varphi \left\{ \frac{\partial}{\partial r} (r \rho V_r \Psi_i) + \frac{\partial}{\partial \varphi} (\rho V_\varphi \Psi_i) + r \frac{\partial}{\partial z} (\rho W \Psi_i) \right\} = \]

\[ = -\frac{G}{c^2} \int_{-\infty}^{\infty} dz \int_{0}^{\infty} dr \int_{0}^{2\pi} d\varphi \left\{ \frac{\partial}{\partial r} (r \rho V_r \Psi_i) \right\} = \]

\[ = \frac{G}{c^2} \int_{-\infty}^{\infty} dz \int_{0}^{2\pi} d\varphi \lim_{r \to 0} (r \rho V_r \Psi_i), \tag{8} \]

where \( V_r = (V_p \cdot e_r), \) \( V_\varphi = (V_p \cdot e_\varphi), \) \( e_r = \{\cos(\varphi), \sin(\varphi), 0\}, \) \( e_\varphi = \{-\sin(\varphi), \cos(\varphi), 0\}, \) \( W \) is the longitudinal component of \( V_p \) along \( z \) axis, the latter is directed along the line of sight to the source. For \( r \to 0 \) we have \( r \Psi \to -4e_r (z > 0), \) and \( r \Psi \to 0 (z < 0). \) Then simple calculations yield

\[ U^{(1)} = -\frac{4\pi G}{c^2} \int_{0}^{\infty} dz \rho(0, z) V_\perp, \tag{9} \]

where \( V_\perp \) represents transverse components of \( V_p. \) Thus \( U^{(1)} \) is the difference of GIM effects for continuous and discrete distributions defined by mass density on the line of sight. Note that \( U^{(1)} \) does not go to zero in case of microlens masses fragmentation, if we make these masses smaller with leaving \( \rho \) unchanged.

5 Average image motion in case of our Galaxy

We use a four-component model of mass distribution in the Galaxy (buldge+ disk + halo + dark corona) according to [6]. We confine ourselves to this model in view of its simplicity, though more recent and more adequate models of Galactic density are available at present. However, for order-of-magnitude estimates the model of [6] is enough. In this model the spherical components (bulge, halo and corona) have isothermal sphere mass density

\[ \rho_S(R) = \frac{3M_S R_S^2}{4\pi (R_S^2 + R^2)^2}, \]

\( R \) being distance from the Galactic center.
The parameters of spherical components are taken as follows: \( M_B = 1.5 \cdot 10^{11} \) (in Solar masses), \( R_B = 5 \text{kpc} \); \( M_H = 5 \cdot 10^{10} \), \( R_H = 25 \text{kpc} \); \( M_C = 8 \cdot 10^{11} \), \( R_C = 50 \text{kpc} \). We changed these parameters as compared to [6] to have better correspondence to the observed rotation curves.

In case of the Galactic disk (in the cylindrical coordinates \( r, \varphi, z \) with the origin at the Galactic center)

\[
\rho_D(r) = \frac{M_D r_D}{4\pi H (r_D^2 + r^2)^{\frac{3}{2}}}, \quad |z| < H; \quad \rho_D(r) = 0, \quad |z| > H;
\]

where \( H = 0.6 \text{kpc}, R_D = 15 \text{kpc} \) and \( M_D = 8 \cdot 10^{10} \).

We use this model along with corresponding rotation curves, which are recovered by means of potentials of the above mass distributions [6]. The dependence of the rotation velocity around the Galactic center in the Galaxy plane is

\[
V^2 = \frac{M_B r^2}{(R_B^2 + r^2)^{\frac{3}{2}}} + \frac{M_C r^2}{(R_C^2 + r^2)^{\frac{3}{2}}} + \frac{M_H r^2}{(R_H^2 + r^2)^{\frac{3}{2}}} + \frac{M_D 2\Omega(r)}{2H},
\]

\[
\Omega(r) = \frac{2r_D}{\sqrt{r^2 + r_D^2}} - \frac{r_D + H}{\sqrt{r^2 + (r_D + H)^2}} - \frac{r_D - H}{\sqrt{r^2 + (r_D - H)^2}}.
\]

For an observer at the Galactic center we have \( U_{tot}^* = 0 \) (see Eq.(5)); this is evident in case of a stationary mass density and the observer at rest. We assume that all the components except continuous corona consist of stars (though if the corona also consists of stars, this does not change the result significantly). Then in view of Eqs.(6), (7) we have the average GIM \( < U_{tot} >_A' = -U^{(1)} \). This yields for source in the Galactic plane \( 3 \cdot 10^{-8} \) arc-seconds per year. This also can be checked directly by means of the formula (3). When inclination increases, the effect decays strongly due to decreasing of star number on the line of sight.

If the observer is located in the Solar system, we must take into account its own velocity and the velocity of stellar rotation around the Galactic center. Note that it is more convenient to calculate first \( U_{tot}^* \) and \( U^{(1)} \), then the value of average GIM \( < U_{tot} >_A' \) is defined from Eq.(6). Dependence of GIM upon the Galactic azimuth of sources in the Galactic plane for the observer in the Solar system is shown on Fig.1.
Figure 1: Observer in the Solar system, sources near the Galactic plane: average GIM against Galactic azimuthal angle.
6 Discussion

We have shown that GIM differs for a continuous and discrete mass density distribution. In the latter case GIM performs random walks \cite{2,3,4} as distinct from a regular motion in case of continuous matter. In this paper we draw attention to the fact that the difference between these two cases remains also after averaging of the stochastic GIM (for the same mass density). For example, the observer at the Galactic center would see an additional fictitious motion of quasars around this center. However, if most of the Galactic matter would be continuous, the GIM effect will be absent.

For an observer in the Solar system we have a nonzero average GIM value leading to apparent rotation of extragalactic reference frame in the direction of the Galaxy rotation. We stress that the averaging procedure involves only those realizations of Galactic stars, which have concern with weak microlensing. For sources in the Galactic plane maximal effect amounts about \( 1.5 \cdot 10^{-7} \) arcseconds per year. This is extremely small. However, in principle, the effect can be observed, because it falls off with inclination of the line of sight to Galactic plane. Formal algorithm for such measurement must involve observations of sources at different Galactic latitudes. In this procedure the strongly microlensed images must be eliminated.

As concerned practical measurement of the average GIM effect, we note that it is far beyond modern possibilities. Moreover, observation of the average effect requires too many ”good” extragalactic sources with approximately the same Galactic latitude. This is necessary in order to separate GIM from random image motions that may be much more essential.

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