To what extent is Gluon Confinement an empirical fact?

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Abstract. Experimental verifications of Confinement in hadron physics have established the absence of charges with a fraction of the electron’s charge by studying the energy deposited in ionization tracks at high energies, and performing Millikan experiments with charged droplets at rest. These experiments test only the absence of particles with fractional charge in the asymptotic spectrum, and thus “Quark” Confinement.

However what theory suggests is that Color is confined, that is, all asymptotic particles are color singlets. Since QCD is a non-Abelian theory, the gluon force carriers (indirectly revealed in hadron jets) are colored. We empirically examine what can be said about Gluon Confinement based on the lack of detection of appropriate events, establishing an upper bound for high-energy free-gluon production.

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1 Introduction

As stated in the abstract, both Millikan oil-drop type experiments \cite{1} and ionization track measurements \cite{2} are devised to test the absence of electric charges that are fractions of the electron’s charge. No evidence has been found for free quarks and bounds have been put on their existence. There are at least four works \cite{3,4,5,6} that constrain the free quark production cross-section in proton collisions at various energies to be less than 10 fbarn.

Gluon confinement is simultaneously necessary if modern theoretical pictures of confinement make any sense \cite{7}, as flux tubes are expected to form between color sources and sinks, and cutting them produces new flux tubes, but not isolated such sources (and gluons do carry color in Quantum Chromodynamics, thus acting as sources). In fact, independently of certain details on infrared behavior that are being sorted out, the theoretical community consensus \cite{8,9,10} via lattice computations, Dyson-Schwinger and Renormalization Group equations, is that the gluon propagator shows violations of reflection positivity, showing the gluon cannot appear at distances beyond a Fermi without hadronizing.

However there are no bounds known to us that put quantitative limits to gluon production, beyond the vague statement “no gluons have been seen as free particles”. No such quantitative experimental statements are presently made about the presence or absence of colored, neutral particles. This is not a totally satisfactory state of experimental affairs, and the purpose of this note is to establish a maximum in the possible gluon production cross section based on contemporary experimental data.

In order to do so, we will consider the (unlikely) hypothesis that gluons might be being produced in accelerator reactions such as

\begin{equation}
\left\{ \begin{array}{c}
e^+ + e^- \rightarrow g + X \\
p + p \rightarrow g + Y
\end{array} \right.
\end{equation}

As a working example we will take the ALICE experiment at the LHC \cite{11}, but our arguments can easily be extended to other collaborations that have recently taken ample data.

We will try to quantify how does the information “no gluon-like event was reported in excess of background” translate in terms of a bound on the semiinclusive production cross-section $\sigma_{pp\rightarrow g+X}$ with the gluon in the asymptotic state. We will find that modern experiments can easily establish a bound

\begin{equation}
\sigma_{pp\rightarrow g+X}|_{t\leq190 \text{ MeV}} \leq 3.5 \text{ fbarn}
\end{equation}

for a large range of energies.

2 A bound on free gluon production

Let us therefore follow what would happen to a hypothetical liberated gluon after reaching out beyond a few Fermi from the primary collision point where the two beam protons produced it. The situation is depicted in figure 1.
Indeed, from elementary theory is very short and thus all putative (high energy) gluons limits on gluon production above the background. This means that there is ample room to set about 3 barn at high energies, much larger than the ap- color scaling, that the total gluon-proton cross-section is secondary vertex in the material via.

Since color is a conserved quantity in Chromodynam- nes in general would be spallated off an atomic nucleus $A$ in the material. Since the neutron itself would leave no ionization trace, we will concentrate on gluon-proton interactions, and thus the secondary re- action of choice to implement gluon non-detection is

$$g + A \rightarrow g' + p + X$$

where $X$ represents the (unmeasured) nuclear remainder, $g'$ should be the undetected, scattered gluon due to color conservation and $p$ is the secondary proton that tags the incoming neutral particle. The main background in this channel is obviously induced by neutral particles, especially neutrons, but also neutral pions and long-lived kaons, that, as the would be gluon, appear suddenly at the sec- ondary vertex in the material via

$$n + A \rightarrow p + X .$$

In section 3 we will argue, based on Regge theory and color scaling, that the total gluon-proton cross-section is about 3 barn at high energies, much larger than the approxi- mately 50 mbarn of the background nucleon-nucleon cross-section. This means that there is ample room to set limits on gluon production above the background.

First of all, let us argue that the gluon mean free-path is very short and thus all putative (high energy) gluons collide early-on in the inner material of the experiment. Indeed, from elementary theory

$$\lambda_{\text{max}} = \frac{1}{n_p \sigma_{\text{min}}} .$$

We estimate $\sigma_{\text{min}} \approx 3.2$ barn in section 3 below, and the proton density of Beryllium (commonly used in acc- elerator beampipes due, precisely, to its low density $\rho = 1.85 \times 10^{-3} \, \text{g/cm}^3$ among other features) is $n_p = 4.9 \times 10^{23} \, \text{protons/cm}^3$. Thus in beryllium we obtain $\lambda_{\text{max}} = 0.6 \, \text{cm}$. In a typical silicon composite in the inner tracker, on the other hand, $n = 7.0 \times 10^{23} \, \text{protons/cm}^3$ and $\lambda = 0.5 \, \text{cm}$. In the particular case of the ALICE experiment, the secondary proton can be identified by the track’s energy deposition $dE/dx$, but best identification advises that the proton be transferred a momentum of at least 275 MeV, so it may reach the Time of Flight (TOF) detector in the presence of ALICE’s 0.5 Tesla magnetic induction field. Slower protons have a trajectory with curvature radius that does not allow them to reach that detector. Without TOF identification, chances of misidentification increase, and since a charged pion could stem from the decay of neutral $A$ or other resonances, and we want to avoid increasing the background that needs to be controlled, we would recommend that in the secondary collision the momentum transferred to the proton exceeds those 275 MeV. Thus we will take a subtracted cross section $\sigma_{gp}(s) - \sigma_{gp}(s, t \geq t_0)$ with $t_0 = (p_p - p_p)^2 = (E' - M_N)^2 - \mathbf{p}^2$, that numeri- cally is $t_0 = -(190 \, \text{MeV})^2$. This will reduce the secondary cross-section to 2.9 barn and therefore the mean free path to $\lambda_{\text{max}} = 0.7 \, \text{cm}$ in Beryllium and $\lambda_{\text{max}} = 0.5 \, \text{cm}$ in Silicon.

So, as the material thicknesses in the innermost work- ings of the experiment are several centimeters, we can safely assume that every gluon that might be produced collides with a nucleus in the beampipe or inner tracker and produces a secondary proton (if it doesn’t otherwise interact before that point).

The number of gluons that could be reaching the material would be the product of the integrated luminosity of the accelerator during observation time and the decon- fining cross-section being tested, namely

$$N_g = L \cdot \sigma_{pp \rightarrow g+X} .$$

The number of secondary protons emitted $N_p$ by the reaction in Eq. (2) that we could expect for each incident free gluon is then readily shown to be

$$\frac{N_p}{N_g} = \frac{\mathcal{E}_f(t) \Omega}{4\pi} f_{ggp} .$$

In this formula $\mathcal{E}_f(E)$ is the energy-dependent proton de- tection efficiency, that we estimate to be perfect if the proton momentum exceeds the 275 $MeV$ threshold, but null for less energetic secondaries, $\mathcal{E}_f(E) = \theta(t_0 - t)$. Also $\frac{\Omega}{4\pi}$ is the detectors solid-angle acceptance. For AL- ICE, that detects particles in the pseudorapidity range ($-1,1$) or about 46 degrees in polar angle, with full az-imuthal coverage, $\Omega/(4\pi) \simeq 0.76$. The last factor is $0 < f_{ggp} < 1$, the fraction of collisions where the gluon actually collides with a proton instead of a neutron (as a spallated neutron would be less easily detected, we neglect such interactions). For the light ele- ments such as $Be$ and $Si$ that we are considering, $f_{ggp} \simeq \frac{1}{2}$.
If the experimental collaboration could understand the background, and in the face of negative detection of a secondary proton above that background, namely \( N_p < 1 \), we would obtain using Eqs. (5) and (6)

\[
\sigma_{pp \to g + X}(s, E_g) \leq \frac{4\pi}{L E_f(t) \Omega f_{gp}}
\]

(7)

Which is the wanted bound on the cross-section. This is a function of two arguments, the proton-proton collision energy \( s \) and the deconfined gluon’s energy \( E_g \). All parameters are determined by the experimental setup except this \( E_g \) that we will require to be larger than 12 GeV, corresponding to an approximate Mandelstam’s \( s \) for the secondary reaction \( s_{gp} > (5 \text{ GeV})^2 \) to guarantee that we can employ Regge theory to control the secondary reaction’s cross section. Our bound can probably be extended well below this cut, but perhaps this is worth a more careful analysis in the future.

The numerical value of the gluon production cross section according to Eq. (7) is 3.5 fbarn (as advanced in Eq. (1) ) using as integrated luminosity \( L \approx 760 \text{ pb}^{-1} \) (that the LHC accumulated by June 5th 2011).

### 3 Estimate of the total gluon-proton cross section

In this section we will support our statement that all potentially produced gluons would be slowed upon interacting in the inner parts of the experiment, by showing that the cross-sections \( \sigma_{gp} \) will turn out to be large.

No experiment can directly access this secondary cross-section by preparing a beam of gluons, but we can estimate it theoretically employing the concept of parton-nucleon scattering amplitude [12]. The conjecture is motivated by the fact that quarks and gluons carry the strong force, and states that if a colored parton (gluon or quark) could ever be produced in isolation, it would undergo Regge scattering off a hadron just like any other hadron pair. This idea has been employed to derive Regge behavior of Deep Inelastic Scattering at low \( x \) [13], and recently employed to discuss the limitations of the Generalized Parton Distribution factorization theorem in exclusive processes [14].

Regge theory parametrizes high-energy cross sections as simple powers of the energy. In the case of reactions where no quantum numbers are exchanged, the leading Regge pole is the so-called Pomeron

\[
\sigma_{gp}(s) = \sigma_{gp}'(s) \times (e^{bt_{min}})
\]

(8)

with

\[
\sigma_{gp}'(s) \equiv \frac{4\pi^2}{\lambda^2(s, m_p^2, m_p^2)} \times f_R^R(t) f_R^R(t) \left( \frac{s}{\Lambda^2} \right)^{\alpha(t)}
\]

(9)

\[
\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc
\]

(10)

with the Pomeron’s Regge exponent \( \alpha(0) \approx 1 \) yielding an approximately \( s \)-independent cross-section for

\[
s_{gp} \in (5 \text{ GeV})^2, (50 \text{ GeV})^2
\]

Regge exponents are universal, in the sense that they depend only on the quantum numbers exchanged, and not on the precise hadrons that exchange them. Thus we expect the gluon-proton interaction to have the same Pomeron-dominated behavior of the total proton-proton cross-section.

Therefore we will estimate the secondary cross section to be proportional to the proton-proton reaction \( \sigma_{pp} \) (well studied). To determine the proportionality constant we will notice that the non-universal part of the Regge exchange is in the \( \beta \) factors (in Feynman language, the vertex couplings of the Regge pole to the external hadron lines). These couplings have been empirically determined for various physical meson-meson, baryon-baryon, meson-baryon, and photon-hadron reactions. But we need to estimate them theoretically in the case of gluon-hadron interactions.

To extrapolate from known physics, we recall that the pomeron in Quantum Chromodynamics is supposed to arise from a tower of glueball exchanges [15,16] with positive parity and charge conjugation. At least in the BFKL [17] domain at low Bjorken-\( x \) one can derive Regge behavior from correlated two-gluon exchange in QCD. Adopting this Pomeron-glueball connection, the differences between the pomeron coupling to various hadrons can be understood from the coupling of two gluons.

Since the parton-nucleon scattering amplitude is a colored amplitude, its counting with the number of colors \( N_c \) is much enhanced respect to color singlet-color singlet interactions. Therefore we are going to bypass momentum-space or spin wavefunction suppressions and concentrate on these large color factors.

#### 3.1 Color factors for hadron-hadron scattering

As a warm-up, let’s reproduce the color factors appropriate for the Pomeron-mediated pion-pion amplitude leading to the \( \sigma_{\pi\pi} \) total cross-section.

Consider first the Feynman diagrams in figure corresponding to two-gluon exchange between two mesons.

![Fig. 2. Pomeron-mediated meson-meson interaction.](image)

We employ the standard \( \text{t'}\text{Hooft} \) \( N_c \) scaling in which \( g(N_c) = g(3) \sqrt{3/N_c} \), and denote \( g(3) \equiv g \) that we maintain in the following. The meson wavefunction is normalized by \( 1/\sqrt{N_c} \). All three diagrams in the figure carry the same color factor equal to

\[
C_{MM} = \left( \frac{1}{\sqrt{N_c}} \right)^4 \left( \frac{g}{\sqrt{N_c}} \right)^4 T_{ij} T_{kl} T_{lk} T_{ji}
\]
\[
\frac{g^4}{N_c^2} \left( \delta_{mq}\delta_{lp} - \frac{1}{N_c} \delta_{ml}\delta_{pq} \right) \frac{1}{2} \left( \delta_{gm}\delta_{pl} - \frac{1}{N_c} \delta_{gp}\delta_{ml} \right)
\]
from where
\[
C_{MM} = \frac{1}{4} \frac{g^4}{N_c} \cdot (N_c^2 - 1) \tag{11}
\]
Substituting \( N_c = 3 \) we obtain
\[
C_{MM} = \frac{2}{81} g^4 \tag{12}
\]
Turning now to the baryon-baryon Pomeron-mediated interaction, there are three possible Feynman diagrams (in the lowest order of the vertex couplings) with different color factor. We consider them separately. Observe first the diagram in figure 3:

![Diagram](image1)

Fig. 3. Baryon-baryon Pomeron exchange with double quark-two gluon coupling.

Since baryons have at least \( N_c \) quark constituents, the normalization is now \( 1/\sqrt{N_c!} \). This yields
\[
C_{BB1} = \frac{1}{(N_c!)^2} \left( \frac{g}{\sqrt{N_c}} \right)^4 T_{kl}^{(a)} T_{pq}^{(a)} T_{qr}^{(b)} T_{lm}^{(b)} \epsilon_{ijkl} \epsilon_{ijmn} \epsilon_{pmn} \epsilon_{ron} \tag{13}
\]
\[
= \frac{1}{(N_c!)^2} \left( \frac{g}{\sqrt{N_c}} \right)^4 T_{kl}^{(a)} T_{pq}^{(a)} T_{qr}^{(b)} T_{lm}^{(b)} \epsilon_{ijkl} \epsilon_{ijmn} \epsilon_{pmn} \epsilon_{ron}
\]
\[
= \frac{1}{(N_c!)^2} \frac{g^4}{N_c^2} \cdot (N_c^2 - 1) = \frac{4}{81} g^4
\]
(for \( N_c = 3 \)).

We now turn to the second possible diagram represented in figure 4:

![Diagram](image2)

Fig. 4. Baryon-baryon interaction with an exchanged Pomeron and a single quark-two gluon coupling.

We obtain straightforwardly
\[
C_{BB2} = \left( \frac{g}{\sqrt{N_c}} \right)^4 T_{pq}^{(a)} T_{il}^{(a)} T_{km}^{(b)} T_{pq}^{(b)} \epsilon_{ijhk} \epsilon_{ilmn} \epsilon_{mnop} \epsilon_{onm} \tag{14}
\]
\[
= \frac{1}{(N_c!)^4} \left( \frac{g}{\sqrt{N_c}} \right)^4 T_{pq}^{(a)} T_{il}^{(a)} T_{km}^{(b)} T_{pq}^{(b)} \epsilon_{ijhk} \epsilon_{ilmn} \epsilon_{mnop} \epsilon_{onm}
\]
\[
= -\frac{1}{2} \frac{g^4}{81} \cdot (N_c^2 - 1) = -\frac{2}{81} g^4
\]

And finally we have the diagram in figure 5:

![Diagram](image3)

Fig. 5. Feynman diagram for baryon-baryon interaction with the two-gluons coupling to different quarks.

that leads to
\[
C_{BB3} = \left( \frac{g}{\sqrt{N_c}} \right)^4 T_{il}^{(a)} T_{pr}^{(a)} T_{km}^{(b)} T_{pq}^{(b)} \epsilon_{ijhk} \epsilon_{ilmn} \epsilon_{mnop} \epsilon_{onm} \tag{15}
\]
\[
= \frac{1}{(N_c!)^2} \left( \frac{g}{\sqrt{N_c}} \right)^4 T_{il}^{(a)} T_{pr}^{(a)} T_{km}^{(b)} T_{pq}^{(b)} \epsilon_{ijhk} \epsilon_{ilmn} \epsilon_{mnop} \epsilon_{onm}
\]
\[
= \frac{1}{4} \frac{g^4}{81} \cdot (N_c^2 - 1) = \frac{1}{81} g^4
\]

Comparing the color factors of the pion-pion and proton-proton amplitudes, of the same order of magnitude for \( N_c = 3 \), one would expect that the amplitudes and, consequently, the total cross sections, to be also comparable. Will be approximately equal. Experimental data [2] bear this expectation (compare the proton-proton cross-section \( \sigma_{pp}(20 \, GeV) \approx 40 - 50 \, mb \) with the \( \pi^+\pi^- - (20 \, GeV) = 13.4 \pm 0.6 \, mb \) \( \pi \pi \) cross-section obtained in [13][19].

### 3.2 Proton and deconfined-gluon scattering

Having reviewed the traditional cases of meson-meson and baryon-baryon scattering, our next step will be to calculate the gluon-baryon color factor and scale the high energy Regge parametrization of experimental proton-proton scattering by this new color factor instead of that naturally corresponding to proton-proton. The gluon carries the index of the adjoint representation of \( SU(N_c) \) and the proton color analysis proceeds as in subsection 3.1.

There are four possible couplings of the two Pomeron-generating gluons to the gluon-proton system. The first
two, employing the three-gluon vertex, are depicted in figure 6.

Fig. 6. Gluon-baryon interaction with an exchanged Pomeron featuring the three-gluon vertex.

These two diagrams carry as color factors

\[
C_{gB1} = \left( -i \left( \frac{g^4}{\sqrt{3}} \right)^2 \left[ f_{abc}f_{de} + f_{ade}f_{bc} + f_{ace}f_{db} \right] \right)
\]

and

\[
C_{gB2} = \left( -i \left( \frac{g^4}{\sqrt{3}} \right)^2 \left[ f_{abc}f_{de} + f_{ade}f_{bc} + f_{ace}f_{db} \right] \right)
\]

respectively. The alternative two diagrams feature the four-gluon vertex and are depicted in figure 7.

Fig. 7. Gluon-baryon interaction with an exchanged Pomeron featuring the four-gluon vertex.

The two color factors can easily be seen to vanish,

\[
C_{gB3} = 0
\]

because two of the gluons out of the four-gluon vertex are forced to exit in a color singlet (as appropriate for color-singlet Pomeron exchange).

Once the color factors have been computed, we just divide the proton-proton amplitude by the typical 1/81 from Eq. (13), (14) and (15), and multiply it by the 1/9 from Eq. (16) and (17) (as usual hadron-hadron scattering is suppressed in the large-\( N_c \) limit).

Squaring, we expect the gluon-proton cross-section to be a factor of 81 times the proton-proton cross-section. In figure 5 we plot the total proton-proton cross-section and its color rescaling to give what we theorize to be the total gluon-proton cross-section.

After including the lower limit for the momentum transfer to the secondary proton in ALICE (due to the TOF identification requirement), and following [20] for the \( b \)-exponential slope, the possible gluon-proton cross section is depicted in figure 9.
4 Summary and conclusions

To summarize, we have argued that gluon confinement is only weakly established experimentally. By assuming that a gluon might have been deconfined in a high energy experiment, we have asked ourselves what the characteristic signature would be.

With minimum theoretical assumptions (Regge gluon-nucleon interaction) we think that the emission of a secondary proton from the inner detector material would be the tell-tale signal. The background to this reaction is the production of neutrons, but their color-singlet interaction is much smaller than the gluon’s, allowing to set a meaningful bound on the cross section for free gluon production, that we estimate at 3.5 fbarn for collider energies.

This we think is a reachable goal for modern experiments, and would be a very stringent limit (compare for example with the 1-50 pbarn typical of supersymmetric particle -notably gluino- searches).

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