Assuming that center vortices are the confining gauge field configurations, we argue that in gauges that are sensitive to the confining center vortex degrees of freedom, and where the latter lie on the Gribov horizon, the corresponding ghost form factor is infrared divergent. Furthermore, this infrared divergence disappears when center vortices are removed from the Yang-Mills ensemble. On the other hand, for gauge conditions which are insensitive to center vortex degrees of freedom, the ghost form factor is infrared finite and does not change (qualitatively) when center vortices are removed. Evidence for our observation is provided from lattice calculations.

PACS numbers: 11.15.Ha, 12.38.Aw, 12.38.Gc

I. CENTER DEPENDENT GAUGES

In Landau gauge the Kugo-Ojima confinement criterion requires an infrared divergent ghost form factor. Indeed, the ghost form factor in Landau gauge has been calculated on the lattice and is found to be infrared divergent [1], [2]. Furthermore, the lattice calculations also show, when the confining center vortex field configurations detected by the method of center projection [3] are removed from the Yang-Mills ensemble (by the method of ref. [4]), the temporal string tension disappears and, at the same time, the ghost form factor of Landau gauge loses its infrared divergent behaviour [1], see fig. 1, which is a necessary and sufficient condition for the Kugo-Ojima confinement criterion [5] to be realized. Thus, there seems to be an intrinsic relation between the Kugo-Ojima confinement criterion in Landau gauge and Wilson’s confinement criterion, i.e. an area law for the temporal Wilson loop. However, recent lattice calculations [6] show that the ghost form factor is infrared divergent even above the critical temperature in the deconfined region. Thus, an infrared divergent ghost form factor is a necessary but not yet sufficient condition for confinement (in the sense of Wilson) in Landau gauge.

According to the confinement mechanism proposed by Gribov [7] and further elaborated by Zwanziger [8], confinement arises due to the infrared dominance of the field configuration near the Gribov horizon, which are expected to give rise to an infrared diverging ghost form factor. This confinement scenario is expected to be realized in both Landau and Coulomb gauge. In Coulomb gauge, in particular, an infrared divergent ghost form factor is required for a confining (i.e. linearly arising) static Coulomb potential, which is a necessary but not sufficient condition [9] for confinement in the sense of Wilson’s criterion. Gribov’s confinement scenario and the center vortex picture of confinement are compatible in the sense that center vortices lie on the Gribov horizon in both Landau and Coulomb gauge [10] and the corresponding ghost form factors loose their infrared diverging behaviour, when center vortices are removed. This has been explicitly demonstrated in Landau gauge [1] (see fig. 1) and, on the basis of the results obtained in ref. [10], can be expected to be true also in Coulomb gauge.

Consider Landau and Coulomb gauge on the lattice defined by

$$\sum_{x} \sum_{\mu=1}^{d} tr U_\mu(x) \rightarrow \max ,$$ (1)
where $d = D$ for Landau gauge and $d = D - 1$ for Coulomb gauge ($D$—number of space time dimensions). Eliminating the center vortices by the method of ref. [1] implies to multiply the links $U_\mu(x)$ in the so-called maximal center gauge (see below) by $Z_\mu = \text{sign}(trU_\mu)$. This procedure changes the gauge condition (1) to

$$\sum_x \sum_{\mu=1}^d trU_\mu(x) \text{sign}(trU_\mu(x)) = \sum_x \sum_{\mu=1}^d |trU_\mu(x)| \to \text{max} \, . \quad (2)$$

The original Landau and Coulomb gauge conditions (1) and consequently also the corresponding ghost Green functions obviously feel the confining center vortex degrees of freedom. Eliminating the confining center vortices turns the gauge condition (1) in the gauge condition (2), which does no longer depend on the center vortex degrees of freedom, and, hence, the corresponding ghost form factor is insensitive to the confining center vortices. The modified gauge condition (2) is for $d = D$, in fact, equivalent to the so-called maximum center gauge condition.

II. CENTER INDEPENDENT GAUGES

The maximal center gauge is defined by the condition

$$\sum_x \sum_{\mu=1}^D (trU_\mu(x))^2 \to \text{max} \, . \quad (3)$$

This condition fixes the gauge group $SU(2)$ only up to the coset $SU(2)/Z(2) \simeq SO(3)$ and is thus insensitive to the center vortex degrees of freedom (the replacement $U_\mu \to Z_\mu U_\mu$ obviously does not change the gauge condition). In fact, using the $SU(2)$ trace identity $2(trU)^2 = tr\hat{U} + 1$, where $\hat{U}^{ab} = \frac{1}{2} tr(\tau_a U\tau_b U^\dagger)$ is the adjoint representation of $U$, eq. (3) becomes

$$\sum_x \sum_{\mu=1}^D tr\hat{U}_\mu(x) \to \text{max} \, , \quad (4)$$

which manifestly depends only on the coset variables $\hat{U}_\mu \in SU(2)/Z(2) \simeq SO(3)$. The maximal center gauge condition (3) is, in fact, equivalent to the condition (2) following from the Landau gauge by eliminating the center degrees of freedom as is also seen from eq. (4).

In the continuum theory it is explicitly seen that the maximal center gauge brings a given gauge configuration as close as possible to the “nearest” center vortex configuration, and reduces to the Landau gauge in the absence of center vortices in the gauge configuration considered. Furthermore, for gauge configurations which are pure center vortices, the maximal center gauge condition is identically fulfilled and thus does not fix the gauge of center vortex configurations at all (see eq. (76) of ref. [13]).

Since the maximal center gauge does not feel the confining center vortex degrees of freedom, one expects that its ghost form factor is infrared finite (and consequently does not change by a removal of center vortices). This, indeed, is found in lattice calculations, ref. [11], see figure 2.

Abelian dominance and evidence for the dual Meissner effect was most significantly observed on the lattice by using the method of Abelian projection in the maximal Abelian gauge. The maximal Abelian gauge is defined on the lattice by

$$\sum_{x,\mu} \frac{1}{2} tr(\tau_3 U_\mu(x) \tau_3 U_\mu^\dagger(x)) = \sum_{x,\mu} (\hat{U}_\mu(x))^{33} \to \text{max} \, (5)$$

and fixes only the coset $SU(2)/U(1)$. It was expected, that the ghost form factor in the maximal Abelian gauge behaves similar to the one in Landau gauge [12], since the maximal Abelian gauge implies Landau gauge for the Abelian projected part of the gauge field. However, like the maximal center gauge, also this gauge condition does not feel the confining center degrees of freedom. Consequently, its ghost form factor is expected to be infrared finite, too. Indeed, this is observed in recent lattice calculations [12]. Since the maximal Abelian gauge condition

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{(taken from ref.11) The ghost form factor (full symbols) in maximal center gauge as function of momentum. For sake of comparison the ghost form factor in Landau gauge and in two loop perturbation theory are also shown.}
\end{figure}
depends only on the coset degrees of freedom $SU(2)/U(1)$ and thus does not feel the center $Z(2) \subset U(1)$, we also expect that the ghost form factor does not change by removing the center vortices.

III. CONCLUSIONS

The above considered examples shows: If a gauge condition is sensitive to the confining center vortex degrees of freedom and the latter lie on the Gribov horizon (like in Landau gauge and Coulomb gauge), the corresponding ghost form factor is infrared divergent. This infrared divergence disappears, however, when the center vortices are removed from the Yang-Mills ensemble.

On the other hand, if a gauge condition is insensitive to the center vortex degrees of freedom (like maximal center gauge and maximal Abelian gauge) its ghost form factor is infrared finite and does not qualitatively change, when the center vortices are removed.

Acknowledgement:

Discussions with G. Burgio, A. Cucchieri, T. Mendes, M. Quandt and P. Watson are gratefully acknowledged. The author is grateful to A. Cucchieri and T. Mendes for providing their lattice results prior to publication. This work was supported by DFG Re 856/4-3.

[1] J. Gattnar, K. Langfeld and H. Reinhardt, Phys. Rev. Lett. 93, 061601 (2004) [arXiv:hep-lat/0403011].
[2] E. M. Ilgenfritz, M. Muller-Preussker, A. Sternbeck and A. Schiller, [arXiv:hep-lat/0601027].
[3] L. Del Debbio, M. Faber, J. Greensite and S. Olejnik, Phys. Rev. D 55, 2298 (1997) [arXiv:hep-lat/9610005].
[4] P. de Forcrand and M. D’Elia, Phys. Rev. Lett. 82, 4582 (1999) [arXiv:hep-lat/9901020].
[5] T. Kugo and I. Ojima, Prog. Theor. Phys. Suppl. 66, 1 (1979).
[6] A. Maas, talk given at “Quark confinement and Hadron Spectrum VII”, Azores Portugal, 2.-7. Sept. 2006
[7] V. N. Gribov, Nucl. Phys. B 139, 1 (1978).
[8] D. Zwanziger, Nucl. Phys. B 518, 237 (1998).
[9] D. Zwanziger, Phys. Rev. Lett. 90, 102001 (2003) [arXiv:hep-lat/0209105].
[10] J. Greensite, S. Olejnik and D. Zwanziger, Phys. Rev. D 69, 074506 (2004) [arXiv:hep-lat/0401003].
[11] K. Langfeld, G. Schulze and H. Reinhardt, Phys. Rev. Lett. 95, 221601 (2005) [arXiv:hep-lat/0508007].
[12] T. Mendes, talk given at “Quark confinement and Hadron Spectrum VII”, Azores Portugal, 2.-7. Sept. 2006
[13] M. Engelhardt and H. Reinhardt, Nucl. Phys. B 567, 249 (2000) [arXiv:hep-th/9907139].