Coherently controlled quantum features in a coupled interferometric scheme

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Over the last several decades, entangled photon pairs generated by spontaneous parametric down conversion processes in both second-order and third-order nonlinear optical materials have been intensively studied for various quantum features such as Bell inequality violation and anticorrelation. In an interferometric scheme, anticorrelation results from photon bunching based on randomness when entangled photon pairs coincidently impinge on a beam splitter. Compared with post-measurement-based probabilistic confirmation, a coherence version has been recently proposed using the wave nature of photons. Here, the origin of quantum features in a coupled interferometric scheme is investigated using pure coherence optics. In addition, a deterministic method of entangled photon-pair generation is proposed for on-demand coherence control of quantum processing.

Quantum entanglement is the heart of quantum technologies such as quantum computing, quantum communications, and quantum sensing. Although intensive research has been performed in both interferometric and noninterferometric schemes for quantum features such as the Hong–Ou–Mandel (HOM) dip, photonic de Broglie wavelength (PBW), Bell inequality violation, and Franson-type nonlocal correlation, the fundamental physics of entangled photon-pair generation itself has still been veiled in terms of probabilistic measurements via coincidence detection of coupled photon pairs. Thus, nondeterministic measurement-based quantum technologies have prevailed, resulting in extreme inefficiency compared with their classical counterparts that are deterministic and macroscopic.

Recently, a novel method of deterministic quantum correlation has been proposed and demonstrated to unveil secrets of quantum entanglement for both HOM dip and PBW using the wave nature of photons. The HOM-type correlation is due to photon bunching on a BS via destructive quantum interference between paired entangled photons, while PBW is due to higher order entangled photons such as a N00N state in a Mach–Zehnder interferometer. As a result, the fundamental physics of quantum features has been found in the phase property of a coupled system, where the coupled system does not have to be confined by the Heisenberg's uncertainty principle. Based on this wave nature of photons, collective control of coherent photons is a great benefit for macroscopic quantum technologies compatible with the classical counterparts. Here, the fundamental physics of quantum correlation is investigated using the wave nature of photons to identify the origin of quantum features demonstrated in an interferometric scheme. For typical X\(^{(2)}\)-generated entangled photon pairs, some misunderstandings regarding quantum correlation are pointed out not to criticize but to support the novelty of the wave nature of photons. Without violating quantum mechanics, a proper choice of photon property should depend on photon resources according to the wave-particle duality. Finally, a coherence version of quantum feature generation is proposed for potential applications of deterministic and macroscopic quantum information processing.

Figure 1 shows a particular scheme of HOM-type quantum correlation in a coupled interferometric scheme, where entangled photon pairs are generated from spontaneous parametric down conversion (SPDC) processes in a X\(^{(2)}\) nonlinear material. Due to the spontaneous emission decay process, an initial phase is randomly assigned to each photon pair, where each photon pair has also a random frequency detuning from the fixed half-frequency of the pump photon used for X\(^{(2)}\). To satisfy the energy conservation law in the X\(^{(2)}\) process, the random detuning of the photon pairs must be symmetric as shown in Fig. 1a. In related HOM-type experiments, a typical line shape observed by coincidence measurements shows a broad dip, whose decay is the inverse of the photon pairs' bandwidth Δf. Unlike the theory in Ref., based on the wave nature of photons, however, the λ-dependent g\(^{(1)}\) correlation has never been observed, where the g\(^{(1)}\) correlation is a typical double-slit interference fringe. In the
In other words, the initial phases \( \phi_i \) and \( \phi_f \) of the jth photon pair are random within the bandwidth \( \Delta f \). This fact is also derived differently below in Fig. 1. Equation (1) is the wavelength of the jth photon. Regardless of nondegeneracy in \( \chi^{(2)} \), all pairs are symmetrically detuned, whose corresponding phase difference is \( \pm \delta f \tau = \pm \zeta_j \), where \( \tau \) is the relative delay between paired photons for measurements.

The coincidence measurements between output ports a and b on a beam splitter BS1 are for the second-order intensity correlation \( g^{(2)}(\tau) \), where the jth output intensities are as follows (see Fig. S1 of the Supplementary Information):

\[
I_a^j(\tau, t) = I_0 \left[ 1 + \sin \left( \frac{\zeta_j}{2} \right) \right],
\]
Phase $\zeta'_{j}$ is described as:

$$I^{(1)}_{\alpha} = \sum_j I^{(2)}_{j\alpha},$$
$$I^{(1)}_{\beta} = \sum_j I^{(2)}_{j\beta}.$$ (2)

For all $\delta f_j$-dependent photon pairs, $I_{\alpha} = \sum_j I^{(2)}_{j\alpha}$ and $I_{\beta} = \sum_j I^{(2)}_{j\beta}$. Equation (3) represents four different sources of the induced phase $\zeta'_{j}$. The first, $\frac{k_0}{2} \Delta L_1$, is a center frequency-related fundamental oscillation as a function of $\Delta f$. The second, $(\delta k_j \Delta L_1)$, is the detuning-caused slow oscillation, resulting in $\Delta f^{-1}(\tau)$-dependent decoherence. The third, $(\delta \phi_j)$, is for a fixed relative phase $\pi/2$ between the signal and idler photons in each pair. The last, $(2\delta k_j r_s - \delta \omega_j t_s)$, is for $\Delta f$-independent frequency beating between the paired photons, resulting in a fixed phase. Because of the wide spectrum in Fig. 1a, this beating results in a $\Delta f^{-1}(\tau)$-dependent wide envelope. Thus, Eq. (3) becomes a function of $\Delta L_1$ (or $\tau$) only. However, all $\delta f_j$-caused phase factors in Eq. (3) cancel each other out due to the $\pm \delta f_j$ distribution of all photon pairs except for the fixed $\delta \phi_j$ at coincidence detection. Thus, the mean values of the output intensities are uniform, resulting in $\langle I_{\alpha} \rangle = \langle I_{\beta} \rangle = I_0$ because $\langle \sin(\zeta'_j) \rangle = 0$, where the signal and idler photons are interchangeable. This is the physical origin why there is no $g^{(1)}$ correlation in $g^{(2)}(\tau)$ in the first MZI. As analyzed for the second MZI below, this is also the physical origin of how $g^{(1)}$ is retrieved in $g^{(2)}(\tau)$ as previously observed. By the way, in Ref. 24, a typical HOM dip without $g^{(1)}$ fringe is observed in the first MZI, while the $\lambda$-dependent $g^{(2)}$ correlation for PBW has been observed in the second MZI. The observed PBWs in the second MZI, however, have been interpreted as the result of the HOM dip in the first MZI, which is contradictory to the present analysis.

According to the definition of intensity correlation $g^{(2)}_{\alpha\beta}(\tau) = \langle I_{\alpha}(t) I_{\beta}(t+\tau) \rangle / \langle I_{\alpha} \rangle \langle I_{\beta} \rangle$, the following equation results from Eqs. (1) and (2):

$$g^{(2)}_{\alpha\beta}(\tau, \delta f_j) = \cos^2 \left( \zeta'_j(\tau) \right).$$ (4)

To satisfy anticorrelation of $g^{(2)}_{\alpha\beta}(\tau = 0, \delta f_j) = 0$ in a SPDC-based HOM dip, $\delta \theta_j = \pm \pi/2$ must be satisfied for each photon pair. If $\tau \neq 0$, Eq. (4) gradually decays and shows a typical HOM dip curve as a function of $\delta f_j$. The observed $g^{(1)}$ correlation with $g^{(2)}$ being trivial in the first MZI is also inconsistent with the present analysis.

**Figure 2.** Numerical simulations of the intensity correlation in a typical HOM dip with $\delta \theta_j = \pm \pi/2$. (a) Photon distribution. (b) $\tau$ versus $\delta f_j$. (c) and (d) Sum $g^{(2)}(\tau)$ for all $\delta f_j$ for different coverage $\Delta f$. Dotted: $\delta \theta_j = 0$. 

$$I^{(1)}_{\beta}(r, t) = I_0 \left[ 1 - \sin \left( \zeta'_j \right) \right].$$ (2)
delay time $\tau_c = (\Delta f)^{-1}$, where decay time $\tau_c$ in $g^{(2)}(\tau) = \sum_{j} g^{(2)}_j (\tau, \delta_j)$ is preset according to the inverse of the SPDC-generated photon bandwidth $\Delta f$, as shown in Fig. 2.

Figure 2 shows numerical calculations for Eq. (4). Figure 2a shows the Gaussian distribution of SPDC-generated photon pairs with the bandwidth of $0.5 \times 10^7$ radians. According to Fig. 1a, the jth photon pair has different detuning at $\delta f_j$, whose corresponding $g^{(2)}(\tau, \delta f_j)$ is shown in Fig. 2b. In Eq. (4), the jth photon pair must contribute to different $g^{(2)}(\tau)$ only because of the detuning dependent $\zeta_j$. By definition, $g^{(2)}(\tau)$ is obtained by averaging all $g^{(2)}_j(\tau, \delta f_j)$ in Eq. (2).

In the second MZI in Fig. 1b, the $\Delta L_2$ effect can be classified for bunched photons only on BS1 if $\Delta L_1 \sim 0$. To classify the $\Delta L_2$ effect in the bunched photons, all other terms become zero except for $g^{(2)}_j(\tau) = g^{(2)}_j(\tau, \delta f_j)$ in Eq. (5) (see Fig. S3 of the Supplementary Information). The normalized coincidence detection measurement becomes $R^{\|}_{AB} = \left(1 + \cos 2\varphi_j\right) R^{(2)}_{AB}(\tau, \delta_f) = 1$ from Eq. (4) for the case of $\varphi_j = 0$. The resulting classical feature of the intensity product from a single MZI, satisfying $g^{(2)}(\tau, \delta f_j) = 0$ for photon bunching in a HOM dip violates $R^{\|}_{AB} = \left(1 + \cos 2\varphi_j\right)$ in Ref. 24, as shown in Fig. S3 of the Supplementary Information. Thus, the observations of $\cos 2\varphi_j$ modulation in Ref. 24 are not from simultaneous satisfaction of nonclassical features in both MZIs of Fig. 1b. For N ≥ 4, an inter-MZI superposition scheme can be a quantum solution as proposed in Ref. 22 and demonstrated in Ref. 24. Otherwise, an intrinsically multi-photon entangled photon pair must be involved with violation of HOM dip in the first MZI.

Figure 3 shows a coherence version of the entangled photon-pair generation comparable with Fig. 1. Because MZI works for either a single photon or coherent light equivalently, there is no difference in the photon characteristics. The photons propagating along different paths of MZI 1 are strongly coupled by the relative phase of $\varphi_j$ created from the first BS, regardless of the input photon's wavelength $\lambda$.

The matrix representations for Fig. 3a are as follows without considering $\Delta L_1$: $I_a = \frac{f_0}{2} (1 - \cos \theta)$, $I_B = \frac{f_0}{2} (1 + \cos \theta)$, $I_A = \frac{f_0}{2} [1 - \sin 2\varphi_j \sin \theta]$, and $I_B = \frac{f_0}{2} [1 + \sin 2\varphi_j \sin \theta]$, (see Fig. S4 of the Supplementary Information). Using an acousto-optic modulator (AOM) driven by an RF pulse generator with an RF frequency of $f_{rf}$, the role of $f_{rf}$-caused random phases in Fig. 1 can be satisfied by a 50% duty cycle of AOM between 0 and $f_{rf}$, as shown in Fig. 3b. In other words, the zeroth (original $f_0$) and first-order ($f_0 + f_{rf} \cdot T$) diffracted light pulses are combined, where T is the RF pulse duration. If $2\pi f_{rf} \cdot T = \pi$, the output direction is reversed. Thus, the average of each output intensity is uniform, $I_a = I_B = I_A = I_B = I_0$, satisfying randomness. Including the $\Delta L_1$ effect in $\zeta$, the revised output intensities are as follows:

$$I_a = \frac{I_0}{2} (1 - \cos \zeta')$$
Figure 3. Schematic of deterministic entangled photon-pair generations. (a) A coupled MZI structure. (b) Basis randomness for $\zeta(0, \pi)$. (c) Numerical calculations for $R_{ij} = I_iI_j \neq 4$ at $k\Delta L_1 = \frac{\pi}{2}$. (Top row) For $I_A$ and $I_B$. (bottom row) For $I_A$ and $I_B$. $I_i$ and $I_j$ are interchangeable on behalf of AOM.

$$I_\beta = \frac{I_0}{2}(1 + \cos\zeta'),$$  \hspace{1cm} (9)

$$I_A = \frac{I_0}{2}[1 - \sin\phi\sin\zeta'],$$  \hspace{1cm} (10)

$$I_B = \frac{I_0}{2}[1 + \sin\phi\sin\zeta'],$$  \hspace{1cm} (11)

where $\zeta' = \zeta + k\Delta L_1(2\pi f_f T)$.

Figure 3c shows numerical calculations for Eqs. (8)–(11) (see also Figs. S4 and S5 of the Supplementary Information). For $\zeta' = \zeta + \pi/2$, Eqs. (8)–(11) are rewritten as $I_A = \frac{I_0}{2}(1 + \sin\zeta')$, $I_\beta = \frac{I_0}{2}(1 - \sin\zeta')$, $I_A = \frac{I_0}{2}(1 + \sin\psi\cos\zeta')$, and $I_B = \frac{I_0}{2}(1 - \sin\psi\cos\zeta')$. The normalized intensity product $R_{ij}$ between $I_i$ and $I_j$ is the same as $g^{(2)}_{AB}(\zeta) = \frac{1}{2}(1 - \cos2\phi$) for MZI 1 and $g^{(2)}_{AB}(\phi) = \frac{1}{2}(1 - \sin^2\psi\sin^2\zeta')$ for MZI 2 due to the randomness by AOM. To satisfy the anticorrelation condition for $g^{(2)}_{AB}(\zeta), \zeta = \pm\pi/2$ is obtained as shown in the top panels of Fig. 3c. For the same conditions of $\zeta = \pm\pi/2$, however, there is no way to satisfy the quantum feature between $I_A$ and $I_B$, unless $\Delta L_1$ is changed. For $R_{AB} = 0, \zeta = \pm n\pi$ must be satisfied as shown in the bottom panels, where $n=0,1,2\ldots$. As analyzed in Fig. 2, this also proves the violation of the quantum feature analysis in Ref.24. In a short summary, the correct condition for the quantum feature generation in Fig. 3a for the final outputs is to break the anticorrelation condition in $\zeta$. Neither way, the PBW cannot be possible in the directly coupled MZI scheme due to this reason, where Fig. 3c is just for the diffraction limit of the Rayleigh criterion in the intensity product: $R_{AB} = (1 + \cos2\phi)/2$. As presented elsewhere, such PBW can be achieved by CBW via path superposition28. For this, an intermediate dummy MZI must be inserted between two MZIs in Fig. 3a.

Conclusion
In conclusion, the quantum features of anticorrelation and PBW were analyzed in a doubly coupled MZI system using pure coherence optics, where SPDC-generated symmetrically distributed entangled photon pairs played an essential role in both $g^{(1)}$ disappearance in the first MZI and the $g^{(1)}$ retrieval in the second MZI. Based on the $\chi^{(2)}$-generated entangled photon-pair distribution, the relative $\pi/2$ phase difference between all paired photons was derived as an essential condition for anticorrelation, i.e. a HOM dip. Moreover, the anticorrelation condition in the first MZI violated quantum feature generation conditions in the second MZI. In other words, satisfying the anticorrelation in one MZI resulted in destruction of quantum features in the other MZI. As a result, PBW could not be generated from the doubly coupled MZI system if a HOM dip condition is satisfied simultaneously. Instead, quantum superposition between MZIs via a dummy MZI can be used to create PBW21–23,28. Otherwise, a single MZI is available for PBW only with preset higher-order entangled photon pairs such as a N00N state via an action of quantum operators for a consecutive measurement process according to the particle nature of quantum mechanics29. Finally, a deterministic coherence version of entangled light pair generation was proposed.
and analyzed using pure coherence optics applicable to both single photons and coherent light without violation of quantum mechanics.

Methods
The numerical calculations in Figs. 2 and 3 were performed by MATLAB using the equations in the text. The data that support the findings of this study are available from the corresponding author upon reasonable request.

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Author contributions
B.S.H. solely wrote the manuscript with ideas, figures and calculations.

Competing interests
The author declares no competing interests.

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