Simultaneous analysis and optimal design of truss structures via displacement method

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Abstract. This study proposes an efficient technique called displacement method of analysis and applies three meta-heuristic algorithms, namely Colliding Bodies Optimization (CBO), Enhanced Colliding Bodies Optimization (ECBO), and Vibrating Particles System (VPS), to perform the simultaneous analysis and optimal design of trusses. The proposed method was applied to the minimum weight design of some planar and spatial truss structures. To investigate the accuracy and effectiveness of the presented method, the problems were designed using the same meta-heuristic algorithms through pure force and pure displacement methods as analysis tools (non-simultaneous). Then, the resulting structural weights were compared.

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1. Introduction

In engineering problems of multiphysics nature, developing methods of higher computational efficiency is an important issue. The analysis and design of structures characterized by a large number of members require large memory size and high computational time. This rather expensive computation has to be repeated in an optimal design many times (e.g., over 5,000 times) since the cross-section size of the members is not determined in the early stages of designing these structures. Thus, reducing the size of structural matrices and eliminating undue repetitions in the analysis and design of structures can ensure high computational efficiency [1]. The aforementioned objective is realized in this paper through meta-heuristics algorithms and the indirect minimization of the energy function. Further to this, the design process and minimization of the weight of a structure are combined with the analysis process.

One of the recently developed, powerful meta-heuristic techniques is the Vibrating Particles System (VPS). The VPS is a population-based optimization procedure which is inspired by the free vibration of single-degree-of-freedom systems with viscous damping [2]. In this algorithm, the solution candidates are considered as agents that gradually approach their equilibrium positions. To ensure a proper balance between exploration (global search) and exploitation (local search), equilibrium positions are obtained from the current population and historically best positions.

Meta-heuristic algorithms are shown to be powerful tools for optimizing problems with search spaces being complex, nonlinear, and non-convex. This is especially the case when near-global optimum solutions are sought after using a limited amount of computational effort. Some examples of meta-heuristic algorithms consist of Genetic Algorithms (GA) [3], Particle Swarm Optimization (PSO) [4], Ant Colony Optimization (ACO) [5], Harmony Search (HS) [6], Big Bang-Big Crunch (BB-BC) [7], Firefly Algorithm (FA)
[8], Magnetic Charged System Search (MCSS) [9], Bat Algorithm (BA) [10], Teaching Learning Based Optimization (TLBO) [11], Colliding Bodies Optimization (CBO) [12], Water Cycle, Mine Blast and Improved Mine Blast algorithms (WC-MB-IMB) [13], Search Group Algorithm (SGA) [14], the Ant Lion Optimizer (ALO) [15], the whale optimization [16], and Vibrating Particles System (VPS) [17]. Metaheuristic algorithms have many applications, some of which are given in [18].

Weight structural optimization can be achieved by minimizing the complementary strain energy for analysis [1] instead of applying the direct solution of classic equations, which not only prevents repetitive computations for the design and analysis, but also does not require finding the inverse of large matrices. Therefore, one needs to formulate necessary equations based on the minimum energy principle and use them in an efficient optimization procedure. In this paper, the metaheuristic algorithms and the displacement method are combined to perform simultaneous analysis and design by CBO, ECBO, and VPS. To this end, strain energy formulation is used and the related variables constitute design variables and analysis variables (nodal degrees of freedom of the structure).

The rest of this paper is structured as follows. In Section 2, energy formulation based on the displacement method is presented and CBO, ECBO, and VPS algorithms are applied to the analysis procedure. In Section 3, weight minimization is performed by considering the analysis procedure as a constraint in CBO, ECBO, and VPS methods. In Section 4, four structural design examples are studied. Some concluding remarks are given in Section 5.

2. Analysis by displacement approach and metaheuristic algorithms

The main purpose of this section is to minimize the strain energy using the metaheuristic algorithms, satisfying all the necessary compatibility conditions. The formulation is based on the minimum work principle provided by Kaveh and Rahami [1].

Let $p = \{p_1, p_2, \ldots, p_{\alpha N}\}$ and $v = \{v_1, v_2, \ldots, v_{\alpha N}\}$ be the joint loads and joint displacements of a structure, respectively. The force-displacement relationship for the structure can be written as follows:

$$\{p\} = [K] \{v\},$$

where $[K]$ is the symmetric $\alpha N \times \alpha N$ matrix, known as the stiffness matrix of the structure [1], [19]. The strain energy, $U$, can be expressed as follows:

$$U = \frac{1}{2} \{v\}^T [K] \{v\} - \{v\}^T \{p\}. \quad (2)$$

Now, $\{v\}$ should be calculated such that $U$ reaches its minimum point by metaheuristic algorithms.

![Figure 1. A simple planar truss.](image)

In order to minimize $U$, CBO, ECBO, and VPS algorithms are used that are based mainly on the algorithms used in [12], [20], and [2], respectively. To demonstrate the accuracy of the analysis by the present approach, one example is presented.

A simple truss with 11 bar elements is considered, as shown in Figure 1. This structure has eight degrees of kinematic indeterminacy. Thus, $U$ should be formed in terms of eight unknowns.

The exact calculation of $U_c$ and $U$ is performed by the force method and displacement method respectively, the values of $U$ and $\{v\}$ obtained using the present approach are shown in Table 1. The population size of this example in all of the three algorithms is set to 20.

3. Optimal design using displacement approach and metaheuristic algorithms

In this section, design and optimization processes are added to the analysis presented in the previous section. The objective function in the simultaneous analysis and design of an optimal structure is formulated by the following approach:

To minimize weight, Eq. (2) is modified such that its minimum value becomes zero. To this end, when the sum of complementary energy and strain energy is zero, the structure is in equilibrium and compatible state. Therefore, the sum of the complementary energy and the strain energy is used as a constraint and the analysis criteria.

In this respect, $U$ has been previously introduced. If the matrix $F_m$ is constructed, then the complementary energy can be calculated below [1, 21]:

$$U_c = \frac{1}{2} \{R\}^T [F_m] \{R\}, \quad (3)$$

where $[F_m]$ is the unassembled flexibility matrix of the structure, and $\{R\}$ is the member force vector. For equilibrium, $U$ is negative and $U + U_c$ is equal to zero. The objective function in metaheuristic algorithms $f$ is selected as $f = W \left(1 + \alpha (U + U_c)^2\right)$, where the first term belongs to the optimization and the second term
Table 1. Comparison of the magnitudes of $U$.

|        | Exact          | Exact          | CBO            | ECBO           | VPS            |
|--------|----------------|----------------|----------------|----------------|----------------|
|        | (force method) | (displacement  |                |                |                |
| $v$    | $R$            | $v$            | $R$            | $v$            | $R$            |
| v8     | 46.9856        | 6.1945         | 0              | 6.1945         | 0              | 6.1945         |
| v11    | 24.9255        | 6.2082         | 0              | 6.2082         | 0              | 6.2082         |
| -2.7988| 0              | 0              | -2.7988        | 0              | -2.7988        | 0              | -2.7988        |
| 2.9985 | -8.276         | 2.9985         | -8.276         | 2.9985         | -8.276         | 2.9985         |
| 3.0521 | 12.2083        | 3.0521         | 12.2083        | 3.0521         | 12.2083        | 3.0521         |
| -2.7587| -30.3131       | -2.7587        | -30.3131       | -2.7587        | -30.3131       | -2.7587        |
| -3.656 | 12.389         | -3.656         | 12.389         | -3.656         | 12.389         | -3.656         |
| 4.6647 | -16.9856       | 4.6647         | -16.9856       | 4.6647         | -16.9856       | 4.6647         |
| -5.5575| 24.2023        | -5.5575        | 24.2023        | -5.5575        | 24.2023        | -5.5575        |
| 4.5978 | 0              | 4.5978         | 0              | 4.5978         | 0              | 4.5978         |
| -3.7481| 24.9255        | -3.7481        | 24.9255        | -3.7481        | 24.9255        | -3.7481        |
| v12    | -8.3965        | -8.3965        | -8.3965        | -8.3965        | -8.3965        | -8.3965        |

$U_c = 339.5552 \quad U = -359.5552 \quad U = -339.5552 \quad U = -339.5552$

corresponds to the analysis, and can be considered as a large number. Obviously, $f$ will ultimately approach the weight $W$ as $(U + U_c)^2$ becomes zero.

If a structure includes other constraints, then they should be normalized and added to the above function with a penalty function. Thus, the ultimate formulation of the objective function is given below:

$$
\text{Min } F(v, A) = \left(1 + \varepsilon_1 \left(\sum_{m=1}^{nc} \max(0, g_m(A))\right)^{\varepsilon_2}\right)^2 W \left(1 + \alpha(U + U_c)^2\right),
$$

where $g_m(A)$ is the sum of the violations of the design constraints. Their values can be written in the form of the absolute value of the existing value to permissible value minus one. The constant $\varepsilon_1$ is set equal to 1, while $\varepsilon_2$ starts from 1.5 and linearly increases to 3.

For large-scale structures, since large flexibility (or stiffness) matrices do not require a solution or inverse, the proposed method is more efficient. Instead of applying direct analysis, it is simply required to consider minimizing energy function in the proposed analysis.

3.1. Non-simultaneous displacement method

To make a better comparison of the results, the non-simultaneous force method and the non-simultaneous displacement method were applied to ensure an optimal design of some truss structures. The design variables of these two methods include only cross-section (A), and their objective function is given below:

$$
\text{Min } F(A) = \left(1 + \varepsilon_1 \left(\sum_{m=1}^{nc} \max(0, g_m(A))\right)^{\varepsilon_2}\right)^2 W. \quad (5)
$$

In the following, the optimal design of four trusses is performed in four different cases:

- **Case 1: Simultaneous displacement method.** In this method, simultaneous analysis and design of trusses is performed by minimizing Eq. (4) through CBO, ECBO, and VPS algorithms. In this method, design variables and analysis variables include the cross-section of members (A) and nodal displacement ($v$), respectively.

- **Case 2: Simultaneous force method.** In the case of this method, Kaveh and Bijari applied CBO, ECBO, and VPS algorithms to perform simultaneous analysis and design of trusses [22]. In this method, design variables and analysis variables include the cross-section of the members (A) and redundant forces ($q$), respectively.

- **Case 3: Non-simultaneous displacement method.** In this method, the optimal design of trusses is achieved by minimizing Eq. (5) through CBO, ECBO, and VPS algorithms. In this method, design variables include the cross-section of the members (A).

- **Case 4: Non-simultaneous force method.** In this method, the optimal design of trusses is achieved by minimizing Eq. (5) through CBO, ECBO, and VPS algorithms. In this method, design variables include the cross-section of the members (A).
4. Examples

4.1. Example 1: A 10-bar planar truss
Optimal design of a 10-bar planar truss, shown in Figure 2, is considered. Table 2 contains the data concerning the design of this truss. This structure has 8 degrees of kinematical indeterminacy. The obtained results are shown in Table 3. Figure 3 shows the comparison of the values of weight obtained by four different methods. It can be seen that the minimum value has been obtained by the simultaneous displacement method using CBO algorithm (5061.71 lb) as compared to the values obtained by the other three methods and the ones in the literature ([23] (5095.46 lb) and [1] (5061.9 lb)). In this structure, the non-simultaneous force method has outperformed the non-simultaneous displacement method. Figure 4 shows the comparison of convergence histories for CBO, ECBO, and VPS algorithms by the simultaneous displacement method. It can be seen that the CBO algorithm has converged at fewer iterations than other algorithms and achieved better results.

4.2. Example 2: A 25-bar spatial truss
Figure 5 shows the schematic of a spatial truss and its members grouping. Table 4 provides the necessary

![Figure 2. Geometry of a 10-bar planar truss.](image)

| Table 2. Design data for the 10-bar planar truss. |
|-----------------------------------------------|
| Variables: $A_1, A_2, A_4, A_6; A_5; A_7, A_9; A_{10}; A_{11}; v_1; v_2; v_3; v_4; v_6; v_7; v_8$ |
| Material property and constraint data |
| Elastic modulus: $E = 10^7$ psi = 6.895 $\text{e7}$ MPa |
| Material density: $\rho = 0.1$ lb/in$^3 = 0.00277$ kg/cm$^3$ |
| Stress constraints |
| $|\sigma| \leq 25$ ksi (172.375 MPa); $i = 1, \ldots, 10$ |
| Nodal displacement constraint in all directions of the coordinated system |
| $|\Delta| \leq 2$ in (5.08 cm); $i = 1, \ldots, 6$ |
| List of the available profiles |
| $A_i \geq 0.1$ in$^2$ (0.6452 cm$^2$); $i = 1, \ldots, 10$ |

| Table 3. Comparison of optimal design for the 10-bar planar truss. |
|-----------------------------------------------|
| Area (in$^2$) | Simultaneous force [22] | Non-simultaneous force | Simultaneous displacement | Non-simultaneous displacement |
|-------------|--------------------------|------------------------|--------------------------|-------------------------------|
|             | CBO ECBO VPS             | CBO ECBO VPS           | CBO ECBO VPS             | CBO ECBO VPS                 |
| A1          | 30.8430 31.5033 30.3     | 30.5099 29.164 30.5118 | 30.8548 29.9053 31.3807   | 30.5236 30.1275 30.582       |
| A2          | 0.1 0.1 0.1              | 0.1 0.1 0.1026         | 0.1001 0.1 0.1015         | 0.1361 0.1 0.1207            |
| A3          | 23.6834 22.5822 23.3     | 23.0586 23.0336 23.5   | 23.1646 23.0366 22.9111   | 22.9757 23.4931 23.2501      |
| A4          | 15.2329 16.0048 15.1    | 15.2442 14.9697 15.1231 | 15.0233 14.8476 14.526    | 14.9044 16.3261 15.2605     |
| A5          | 0.1 0.1 0.1              | 0.1 0.1 0.1019         | 0.1 0.1 0.1053            | 0.1 0.1 0.1012              |
| A6          | 0.522 0.4634 0.5        | 0.3896 0.5236 0.3863   | 0.5755 0.3356 0.463       | 0.5721 0.6709 0.6636        |
| A7          | 7.4652 7.5275 7.4       | 7.5305 7.5215 7.4372   | 7.4267 7.4626 7.5761      | 7.5881 7.4276 7.4104        |
| A8          | 20.9774 20.6941 21      | 21.0923 21.9314 20.738  | 20.6790 21.1664 21.3159   | 21.4434 20.7521 20.9994     |
| A9          | 21.0875 21.1023 21.8    | 21.1461 21.9553 21.7535 | 21.847 22.1978 21.3952    | 21.4369 21.1649 20.8611     |
| A10         | 0.1 0.1 0.1              | 0.1 0.1 0.1033         | 0.1 0.1 0.1014            | 0.1 0.1 0.102               |
| Weight (lb) | 5064.1 5066.6 5063.6    | 5063.5 5066.4 5064.5   | 5061.7 5062.9 5066.9      | 5066.2 5066.7 5068.9        |
Figure 3. The comparison of the obtained values of weight for the 10-bar planar truss.

Figure 4. Convergence curves obtained for the 10-bar planar truss by simultaneous displacement method.

Figure 5. Schematic of a 25-bar spatial truss and grouping of the members.

Figure 6. The comparison of the obtained values of weight for the 25-bar spatial truss.

Figure 6. The simultaneous displacement method by all the three algorithms has achieved acceptable results similar to other methods and, yet, not as favorable as the results found in the literature ([11] (467.629 lb) and [24] (467.746 lb)). The non-simultaneous force method and the non-simultaneous displacement method have achieved almost the same results. Figure 7 illustrates the comparison of the convergence histories for CBO, ECBO, and VPS algorithms using the simultaneous displacement method. Obviously, all of the three algorithms have shown the same trend.

4.3. Example 3: A 72-bar spatial truss
The schematic of a 72-bar spatial truss is shown in Figure 8 as the third design example. The necessary data for the design and constraints are shown in Table 6. This structure has 48 degrees of kinematical indeterminacy. The elements are divided into sixteen groups using symmetry as follows:

(1) $A_1 - A_4$,  (2) $A_5 - A_{12}$,
(3) $A_{13} - A_{16}$,  (4) $A_{17} - A_{18}$,
(5) $A_{19} - A_{22}$,  (6) $A_{23} - A_{26}$,
(7) $A_{27} - A_{34}$,  (8) $A_{35} - A_{36}$,
(9) $A_{37} - A_{40}$,  (10) $A_{41} - A_{48}$,
(11) $A_{49} - A_{52}$,  (12) $A_{53} - A_{54}$,
(13) $A_{55} - A_{58}$,  (14) $A_{59} - A_{62}$,
(15) $A_{63} - A_{66}$,  (16) $A_{67} - A_{72}$.

The structure is subjected to the two load cases, as shown in Table 7. Table 8 compares the results obtained by CBO, ECBO, and VPS algorithms with those of other optimization methods. The comparison of the obtained weight values by four various methods is shown in Figure 9. It can be seen that the CBO and ECBO algorithms using the simultaneous displacement method have achieved better results (384.43 lb, 382.2287 lb) than the other three methods and not as
acceptable as the results found in the literature ([25]
(392.8483 lb)). The non-simultaneous force method
(382.663 lb, 383.3211 lb) and the non-simultaneous
displacement method (385.4571 lb, 384.1302 lb) have
achieved optimized weights close to each other by CBO
and ECBO algorithms. The corresponding convergence
curves are compared in the case of the simultaneous
displacement method, shown in Figure 10. As is clear,
the ECBO algorithm has obtained better results at
fewer iterations.

4.4. Example 4: A 120-bar dome truss
A 120-bar dome structure is considered as the fourth
design example. Geometry and member grouping
structures are shown in Figure 11. This structure
has 111 degrees of kinematical indeterminacy. The
necessary data for the design and the constraints are
shown in Table 9. The loading condition is considered
as follows:
1. Vertical load at node 1 equal to −13.49 kips (−60
kN).

| Table 4. Member grouping of the 25-bar spatial truss. |
|---------------------------------|
| Group number | Members |
|----------------|----------|
| 1 | 1-2 |
| 2 | 1-4, 2-3, 1-5, 2-6 |
| 3 | 2-5, 2-4, 1-3, 1-6 |
| 4 | 3-6, 4-5 |
| 5 | 3-4, 5-6 |
| 6 | 3-10, 6-7, 4-9, 5-8 |
| 7 | 3-8, 4-7, 6-9-5-10 |
| 8 | 3-7, 4-8, 5-9-6-10 |

**Table 5. Design and analysis variables in simultaneous displacement method**

**Variables:** $A_1; A_2; A_3; A_4; A_5; A_6; A_7; A_8; v_1; v_2; \ldots; v_{18}$

**Material property and constraint data**

**Elastic modulus:** $E = 1.0 \times 10^7$ psi = $6.895 \times 10^6$ MPa

**Material density:** $\rho = 0.1$ lb/in$^3$ = 0.00277 kg/cm$^3$

**Stress constraints**

$|\sigma_i| \leq 40$ ksi ($275.8$ MPa); $i = 1, \ldots, 25$

**Displacement constraint in the directions of X and Y in the coordinated system**

$|\Delta_i| \leq 0.35$ in ($0.8890$ cm); $i = 1, 2$

**List of the available profiles**

**Continuous sections**

$A_i \geq 0.1$ in$^2$ ($0.6452$ cm$^2$)

**Loading data**

| Node | $P_x$: kips (kN) | $P_y$: kips (kN) | $P_z$: kips (kN) |
|------|------------------|------------------|------------------|
| 1    | $(1.448)$        | $-10$ (−44.48)  | $-10$ (−44.48)  |
| 2    | 0                | $-10$ (−44.48)  | $-10$ (−44.48)  |
| 3    | 0.5(2.224)       | 0                | 0                |
| 6    | 0.6(2.6688)      | 0                | 0                |
Table 5. Comparison of the optimal designs for the 25-bar spatial truss.

| Area (in²) | Simultaneous force, Kaveh and Hijari [22] | Non-simultaneous force CBO | ECBO | VPS | Simultaneous displacement CBO | ECBO | VPS | Non-simultaneous displacement CBO | ECBO | VPS |
|------------|------------------------------------------|-----------------------------|------|-----|-------------------------------|------|-----|-----------------------------------|------|-----|
| A1         | 0.1                                      | 0.1                         | 0.1169 |     |                               |      |     |                                   |      |     |
| A2         | 0.1029                                   | 0.1                         | 0.1038 |     |                               |      |     |                                   |      |     |
| A3         | 3.5539                                   | 3.5683                      | 3.6151 |     |                               |      |     |                                   |      |     |
| A4         | 0.1056                                   | 0.1                         | 0.1012 |     |                               |      |     |                                   |      |     |
| A5         | 1.9539                                   | 1.9592                      | 1.9546 |     |                               |      |     |                                   |      |     |
| A6         | 0.7876                                   | 0.7893                      | 0.7857 |     |                               |      |     |                                   |      |     |
| A7         | 0.1499                                   | 0.1461                      | 0.1325 |     |                               |      |     |                                   |      |     |
| A8         | 3.9437                                   | 3.9354                      | 3.9202 |     |                               |      |     |                                   |      |     |
| Weight (lb)| 467.304                                   | 467.36                      | 467.382 |    |                               |      |     |                                   |      |     |

Table 6. Design data for the 72-bar spatial truss.

Design and analysis variables in simultaneous displacement method
Variables: \( A_1; A_2; \ldots; A_{16}; v_1; v_2; \ldots; v_{18} \)
Material property and constraint data
Elastic modulus: \( E = 1 \times 10^7 \) psi = 6.895×10^7 MPa
Material density: \( \rho = 0.1 \) lb/in³ = 0.00277 kg/cm³
Stress constraints
\[ [\sigma_i] \leq 25 \text{ ksi (172.37 MPa)}; i = 1, \ldots, 72 \]
Displacement constraint in the directions of X and Y in the coordinated system
\[ [\Delta_i] \leq 0.25 \text{ in (0.635 cm)}; i = 1, 2, 3, 4 \]
List of the available profiles
\( A_i \geq 0.1 \) in² (0.6452 cm²)

2. Vertical loads at nodes 2–14 equal to -6.744 kips (~30 kN).
3. Vertical loads in the rest of the nodes equal to -2.248 kips (~10 kN).

The comparison of the optimal designs of 120-bar

Figure 8. Schematic of a 72-bar spatial truss.

Figure 9. The comparison of the obtained values of weight for the 72-bar spatial truss.
Table 7. Loading conditions for the 72-bar spatial truss.

| Nodes | Load Case 1 | | Load Case 2 | | |
|-------|------------|------------|------------|------------| |
|       | Px (kips) | Py (kips) | Pz (kips) | Px (kips) | Py (kips) | Pz (kips) |
| 1     | 5         | 5         | -5        | 0         | 0         | -5        |
| 2     | 0         | 0         | 0         | 0         | 0         | -5        |
| 3     | 0         | 0         | 0         | 0         | 0         | -5        |
| 4     | 0         | 0         | 0         | 0         | 0         | -5        |

Table 8. Comparison of the optimal designs for the 72-bar spatial truss.

| Area (in²) | Simultaneous force [22] | Non-simultaneous force | Simultaneous displacement | Non-simultaneous displacement |
|------------|--------------------------|-------------------------|---------------------------|-------------------------------|
|            | CBO  | ECBO | VPS | CBO  | ECBO | VPS | CBO  | ECBO | VPS | CBO  | ECBO | VPS |
| A1         | 1.9  | 2.0364 | 1.9501 | 1.9732 | 1.8881 | 1.9931 | 1.897 | 1.8368 | 2.1025 | 1.9422 | 2.1589 | 1.2837 |
| A2         | 0.5125 | 0.51 | 0.4888 | 0.4913 | 0.4745 | 0.5234 | 0.5201 | 0.5242 | 0.5283 | 0.5084 | 0.4819 | 0.4419 |
| A3         | 0.1  | 0.1001 | 0.1  | 0.1  | 0.106 | 0.1250 | 0.1001 | 0.143 | 0.106 | 0.1019 | 0.137 |
| A4         | 0.1  | 0.1168 | 0.1038 | 0.1148 | 0.1096 | 0.1163 | 0.1163 | 0.11  | 0.11  | 0.1272 |
| A5         | 1.2155 | 1.4082 | 1.3009 | 1.3184 | 1.2655 | 1.2909 | 1.1794 | 1.19  | 1.3707 | 1.2192 | 1.1905 | 1.3388 |
| A6         | 0.5303 | 0.505 | 0.5048 | 0.5237 | 0.5242 | 0.5016 | 0.478 | 0.5039 | 0.578 | 0.5947 | 0.5402 | 0.6259 |
| A7         | 0.1  | 0.1003 | 0.1113 | 0.1022 | 0.1122 | 0.101 | 0.1065 | 0.1271 | 0.1089 | 0.1  | 0.1199 |
| A8         | 0.1054 | 0.1115 | 0.1  | 0.11  | 0.124 | 0.105 | 0.1  | 0.103 | 0.1084 | 0.1  | 0.1077 |
| A9         | 0.5168 | 0.5104 | 0.5015 | 0.556 | 0.6301 | 0.5079 | 0.6433 | 0.6047 | 0.4356 | 0.6837 | 0.5528 | 0.5163 |
| A10        | 0.5063 | 0.4598 | 0.5184 | 0.5383 | 0.5256 | 0.5289 | 0.5446 | 0.5346 | 0.448 | 0.4732 | 0.4834 | 0.6188 |
| A11        | 0.1  | 0.1112 | 0.1023 | 0.1034 | 0.1  | 0.1  | 0.108 | 0.1  | 0.103 | 0.1  | 0.1554 |
| A12        | 0.1095 | 0.1034 | 0.1  | 0.1016 | 0.1  | 0.1794 | 0.2298 | 0.1  | 0.1256 | 0.2858 |
| A13        | 0.169 | 0.1544 | 0.1552 | 0.1737 | 0.1678 | 0.1627 | 0.1634 | 0.1539 | 0.1638 | 0.1616 | 0.16  | 0.157 |
| A14        | 0.5767 | 0.5369 | 0.5545 | 0.5211 | 0.5246 | 0.5543 | 0.5691 | 0.5423 | 0.5683 | 0.4929 | 0.5032 | 0.6393 |
| A15        | 0.4301 | 0.4365 | 0.4203 | 0.379 | 0.424 | 0.3019 | 0.4166 | 0.4147 | 0.2506 | 0.4829 | 0.5401 | 0.5002 |
| A16        | 0.5661 | 0.6062 | 0.5854 | 0.6005 | 0.6814 | 0.6297 | 0.5297 | 0.5286 | 0.7493 | 0.5436 | 0.5984 | 0.453 |

Weight (lb) 381.8569 381.3952 382.6638 383.3211 384.5238 384.4313 385.2287 390.3201 385.4571 384.1502 385.2405 380.2607

Figure 10. Convergence curves obtained for the 72-bar spatial truss by simultaneous displacement method.

done truss is shown in Table 10. Figure 12 presents the comparison of the obtained weight values by four different methods. In the simultaneous displacement method, the obtained weight values by all the three algorithms are lower than those found in the literature ([24] (33241.99 lb)) and, also, the optimized weight by ECBO algorithm (31886 lb) is well consistent with the values obtained by the simultaneous force method and the non-simultaneous force method. The weight obtained by VPS algorithm using the non-simultaneous displacement method (31888 lb) is very close to that found by the non-simultaneous force method. Figure 13 shows the comparison of the convergence curves of the best results obtained by CBO, ECBO, and VPS algorithms using the simultaneous displacement method. It appears that the CBO algorithm has converged at fewer iterations; however, the ECBO algorithm has achieved better results.

5. Concluding remarks

In this paper, an efficient method was proposed to
Table 9. Design data for the 120-bar spatial truss

Variables: $A_1$; $A_2$; $A_3$; $A_4$; $A_5$; $A_6$; $A_7$; $v_1$; $v_2$; ...; $v_{111}$

Material property and constraint data

| Elastic modulus: $E$ | 30150 Ksi | 210000 MPa |
|---------------------|-----------|------------|
| Material density: $\rho$ | 0.288 lb/in$^3$ | 7971.810 kg/cm$^3$ |
| For all members: $0.775 \leq A_i \leq 20$ in$^2$, $i = 1, ..., 120$ |

Constraints

$\lambda_i = \frac{F_i}{E A_i}$, $r = \sqrt{0.4 \times \lambda_i}$, $C_v = \sqrt{\frac{2 \pi^4 E}{r^4 \rho}}$

For tensile members

$\lambda_i \leq 300$

$F_a \leq 0.6 F_y$

For compressive members

$\lambda_i \leq 200$

$F_a = \left[ \left( 1 - \frac{\lambda_i}{\sqrt{r^2 \pi^4 E \rho}} \right) \frac{F_c}{\lambda_i} \right]$ for $\lambda_i \leq C_v$

$F_a = \frac{12 \pi^4 E}{25 \lambda_i^2}$ for $\lambda_i > C_v$

$[\sigma_i] \leq 58$ ksi (400 MPa); $i = 1, ..., 120$

Displacement constraint in the directions of $X$, $Y$ and $Z$ at all unsupported nodes

$|\Delta_i| \leq 0.1969$ in (0.500126 cm)

Figure 11. Schematic of a 120-bar dome truss.

Figure 12. The comparison of the obtained values of weight for the 120-bar spatial truss.

perform simultaneous analysis, design, and optimization of structures using CBO, ECBO, and VPS algorithms to prevent the formation of the inverse for large structural matrices, especially for structures with a large number of members. These metaheuristic algorithms and the displacement method were applied simultaneously to analyze and design different kinds of large-scale structures. The results were compared with those of the non-simultaneous force method and non-simultaneous displacement method. Benchmark problems were studied in order to show the performance of the presented method. The proposed tech-
Table 10. Comparison of the optimal designs for the 120-bar spatial truss.

| Area (in²) | Simultaneous force [22] | Non-simultaneous force | Simultaneous displacement | Non-simultaneous displacement |
|------------|--------------------------|------------------------|---------------------------|-------------------------------|
|            | CBO          | ECBO       | VPS         | CBO          | ECBO       | VPS         | CBO          | ECBO       | VPS         |
| A1         | 2.2164       | 2.2484     | 2.4646      | 2.2164       | 2.2484     | 2.4646      | 2.2164       | 2.2484     | 2.4646      |
| A2         | 15.5525      | 16.2365    | 15.773      | 15.8888      | 15.8888    | 15.8754      | 14.1522      | 15.7986    | 15.5359      |
| A3         | 5.6267       | 5.3103     | 5.3939      | 5.4508       | 5.4888     | 5.4958      | 5.9065       | 5.5503     | 5.5287      |
| A4         | 2.4648       | 2.4548     | 2.467       | 2.4657       | 2.4622     | 2.4621      | 2.4688       | 2.4631     | 2.4857      |
| A5         | 9.0497       | 8.9467     | 8.946       | 8.9808       | 8.9481     | 8.9811      | 9.2361       | 8.9568     | 8.9281      |
| A6         | 3.5581       | 3.4806     | 3.721       | 3.697        | 3.6577     | 3.5199      | 3.7272       | 3.5267     | 3.7868      |
| A7         | 1.9181       | 1.9782     | 1.9709      | 1.9779       | 1.9748     | 1.9538      | 2.0818       | 1.9435     | 2.0058      |

Weight (lb) | 31884  | 31900  | 31888  | 31884  | 31881  | 31882  | 31880  | 31881  | 31882  |

Figure 13. Convergence curves obtained for the 120-bar spatial truss by simultaneous displacement method.

nique performs better optimal designs for three of the four problems investigated than the simultaneous force method, the non-simultaneous force method, and the non-simultaneous displacement method. The results demonstrate the capability and accuracy of the metaheuristic algorithms and displacement method when simultaneously utilized for the analysis, design, and optimization of constrained problems. The comparison of the optimal designs using this work and those of the other researchers is shown in Tables 3, 5, 8, and 10.

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Biographies

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