Single-photon-assisted entanglement concentration of a multi-photon system in a partially entangled W state with weak cross-Kerr nonlinearity

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We propose a nonlocal entanglement concentration protocol (ECP) for N-photon systems in a partially entangled W state, resorting to some auxiliary single photons and the parity-check measurement based on cross-Kerr nonlinearity. One party in quantum communication first performs a parity-check measurement on her photon in an N-photon system and an ancillary photon, and then she picks up the even-parity instance for obtaining the standard W state. When she obtains an odd-parity instance, the system is in a less-entanglement state and it is the resource in the next round of entanglement concentration. By iterating the entanglement concentration process several times, the present ECP has the total success probability approaching to the limit in theory. The present ECP has the advantage of a high success probability. Moreover, the present ECP requires only the N-photon system itself and some auxiliary single photons, not two copies of the systems, which decreases the difficulty of its implementation largely in experiment. It maybe have good applications in quantum communication in future.

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I. INTRODUCTION

Entanglement is a key important resource in quantum information and quantum computation [1]. The advantage of quantum computer, the powerful computation, comes from multipartite entanglement, compared with classical computer. Also, entanglement is used as the information carries in quantum communication, such as quantum key distribution (QKD) [2–6], quantum teleportation [7], quantum dense coding [8], quantum secret sharing [10–17], quantum state sharing [18–22], controlled teleportation [23, 24], and so on. In a long-distance quantum communication, entanglement is used to construct quantum repeaters. However, entanglement is fragile to channel noise. In a practical transmission or the process for storing an entangled quantum system, it inevitably suffers from channel noise and its environment. The noise will make the system decoherent, which will decrease the security of QKD protocols and the fidelity of quantum teleportation and dense coding. There are some interesting ways for dealing with the issue of decoherence in quantum communication, such as decoherence-free subspaces [26–29], faithful qubit distribution [30, 31], faithful qubit transmission [32], error-rejecting codes [33], faithful entanglement distribution [34], and so on. Most of them are very useful for overcoming a collective noise by encoding a logical qubit with several physical qubits. There is a fundamental hypothesis that the noise is a collective one. These methods are used to deal with the photon systems before they are transmitted over a noise channel.

Entanglement purification and entanglement concentration are two interesting quantum techniques with which the users can obtain some high-fidelity entangled photon systems after they are transmitted over a noisy channel or stored in a practical environment and they are in a less-entanglement state. In detail, entanglement purification is used to extract some high-fidelity entangled systems from a less-entangled ensemble in a mixed state [35–46]. Entanglement concentration is used to obtain a subset of photon systems in a maximally entangled state from a set of systems in a partially entangled pure state. Although entanglement purification is more general than entanglement concentration in the practical applications because an entangled photon system is usually in a mixed entangled state after it is transmitted over a noisy channel, entanglement concentration is more efficient for the two remote parties in quantum communication, say the sender Alice and the receiver Bob, to distill some maximally entangled systems from an ensemble in a less-entangled state because entanglement purification should consume a great deal of quantum resource for improving the fidelity of systems in a mixed entangled state, not obtain a maximally entangled state directly. Entanglement concentration is useful in some particular cases, such as the decoherence of entanglement arising from the storage process or the imperfect entanglement source.

Since Bennett et al. [47] proposed the original entanglement concentration protocol (ECP) in 1996, there have been some interesting and typical ECPs for photon systems [47–55] and atom systems [56, 57]. For example, Bose et al. [48] proposed an ECP based on entanglement swapping in 1999. Subsequently, Shi et al. [49] presented an ECP based on a collection unitary evolution on a qubit in a multi-qubit system and an ancillary qubit. In 2001, an ECP based on polarizing beam splitters (PBSs) was proposed [50, 51]. In 2008, Sheng, Deng and Zhou proposed an interesting ECP [52] with cross-Kerr nonlinearities. In 2010, they presented the first single-photon ECP [53] with cross-Kerr nonlinearities. In 2012, Sheng et al. [54] proposed a single-photon-assisted ECP for partially entangled multi-photon systems. Recently, an optimal nonlocal multipartite ECP for photon systems in a partially entangled Bell-type state is proposed [55], resorting to a parity-check measurement on one photon in the system and an ancillary single photon and the projection measurement on the ancillary photon with cross-Kerr nonlinearities.
Although there exist some interesting ECPs, most of them are used to distill some maximally entangled Bell states or Greenberger-Horne-Zeilinger (GHZ) states. There are few schemes for concentrating the non-maximally entangled pure W-class states. In essence, W state are equivalent to the GHZ states as they cannot be converted into each other under stochastic local operations and classical communication (SLOCC). Moreover, a W state is more robust than GHZ states with the loss of one or two photons. Therefore, it is interesting to discuss the entanglement concentration on the partially entangled W state. By far, there are three ECPs for photon systems in a partially entangled W state [58–60]. The first one is a linear optical scheme for entanglement concentration of two known partially entangled three-photon W states [59]. The second one is a linear-optics-based entanglement concentration of unknown partially entangled three-photon W states [59]. It is proposed by Wang, Zhang, and Yeon [59] in 2010. In 2011, Xiong and Ye [60] proposed another ECP for a partially entangled W state with cross-Kerr nonlinearity. Both these two interesting ECPs are used to deal with an unknown multi-photon W-class state. There is no ECP for a known multi-photon W-class state.

In this paper, we proposed an nonlocal ECP for N-photon systems in a known partially entangled pure W state, resorting to ancillary single photons and the parity-check measurement based on cross-Kerr nonlinearity. It does not depend on two copies of N-photon systems in a partially entangled W-class state in each round of concentration, just each system itself and some ancillary single photons, which makes it far different from other ECP for W states [59, 60]. In the present ECP, only one of the parties in quantum communication, say Alice, first operates her photon and the ancillary single photons for concentrating the entanglement of an N-photon system and then tells the others to retain or discard the system, which greatly simplifies the complication of classical communication as others require all the parties operate the entanglement process in the same way, similar to the works for a Bell-type state [54, 55]. Moreover, the present ECP has a higher total success probability approaching to the limit in theory by iterating the entanglement concentration process several times. All these advantages make our ECP more feasible than others. Maybe it have good applications in quantum communication in future.

II. ENTANGLEMENT CONCENTRATION OF PARTIALLY ENTANGLLED THREE-PHOTON W STATES

Before we discuss the principle of our ECP for a partially entangled three-photon W states, we first introduce the principle of a parity-check detector (PCD) on the polarization states of two photons with cross-Kerr nonlinearity. In fact, the principle of the PCD here is similar to those in Refs. [46, 55, 61]. In detail, the Hamiltonian of a cross-Kerr nonlinearity can be written as follows [61]:

\[ H_{ck} = \hbar \chi a_+^s a_p a_+^p \]  

\( (1) \)

where \(a_+^s\) and \(a_+^p\) are the creation operations, and \(a_i\) and \(a_p\) are the destruction operations. The subscripts \(s\) and \(p\) represent the signal light and the probe light, respectively. \(X\) is the coupling strength of the cross-Kerr nonlinearity. If a signal light in the state \(|\Psi\rangle_s = c_0|0\rangle_s + c_1|1\rangle_s\) (\(|0\rangle_s\) and \(|1\rangle_s\), denote that there are no photon and one photon respectively in this state) and a coherent probe beam in the state \(|\alpha\rangle_p\) couple with a cross-Kerr nonlinearity medium, the evolution of the whole system can be described as \([46, 55, 61]\):

\[ U_{ck}|\Psi\rangle_s|\alpha\rangle_p = e^{i\theta t}|\alpha\rangle_p + c_0|0\rangle_s|\alpha\rangle_p + c_1|1\rangle_s|\alpha e^{i\theta}\rangle_p, \]  

\( (2) \)

where \(\theta = \chi t\) is the phase shift of the probe beam, which depends on the interaction time \(t\) and the coupling strength \(\chi\). That is, the coherent beam \(P\) picks up a phase shift \(\theta\) directly proportional to the number of the photons in the signal light in the Fock state \(|\Psi\rangle_s\). Based on this feature of a cross-Kerr nonlinearity, the principle of our PCD is shown in Fig.1 similar to those in Refs. [46, 55, 61]. Here \(|X\rangle\langle X|\) represents an X quadrature measurement with which one can not distinguish the the states \(|\alpha e^{i\theta}\rangle_p\) [61, 62]. With the two cross-Kerr nonlinearities \(ck_1\) and \(ck_2\), one can distinguish the superpositions and mixtures of the polarization states \(|HH\rangle\) and \(|VV\rangle\) from \(|HV\rangle\) and \(|VH\rangle\) based on the different phase shifts. That is, the probe beam \(|\alpha\rangle_p\) will pick up a phase shift \(\theta\) if the two photons is in the state \(|HH\rangle_{bh}\) or \(|VV\rangle_{bh}\), while it picks up a phase shift 0 when the two photons is in the state \(|VH\rangle_{bh}\) or \(|HV\rangle_{bh}\). In other words, when the parity of the two photons is even, the coherent beam will pick up a phase shift \(\theta\); otherwise it will pick up a phase shift 0. By detecting the phase

![FIG. 1: The principle of a parity-check detector (PCD) on the polarizations of two photons, the same as that in Ref. [46, 55]. PBS represents a polarizing beam splitter which transmits the photon in the horizontal polarization \(|H\rangle\) and reflects the photon in the vertical polarization \(|V\rangle\). \(ck_1\) and \(ck_2\) represent two cross-Kerr nonlinearities which will lead to the phase shift \(+\theta\) and \(-\theta\) when there is a photon passing through the media, respectively. \(|X\rangle\langle X|\) represents an X quadrature measurement with which one can not distinguish the the states \(|\alpha e^{i\theta}\rangle_p\).]
The principle of the present ECP is shown in Fig. 2. In the process of concentrating a three-photon system, Alice prepares an ancillary photon $a$. It is in the polarization state $|\varphi\rangle_a$. Here

$$|\varphi\rangle_a = \frac{1}{\sqrt{\alpha^2 + \beta^2}}(\alpha|H\rangle + \beta|V\rangle)_a.$$  

Before Alice preforms a parity-check measurement on her photon $A$ and the ancillary photon $a$, the composite system composed of the four photons $CBAa$ is in the state

$$|\Phi\rangle_{CBAa} = |\varphi\rangle_{CBA} \otimes |\varphi\rangle_a$$

With the parity-check measurement on the photons $A$ and $a$, Alice can divide the state of the four-photon system $CBAa$ into two classes. In the first one, it is in a state in which each item has the same parameter, that is,

$$|\Psi_1\rangle_{CBAa} = \frac{1}{\sqrt{3}}[(|H\rangle|H\rangle|H\rangle)_{BA} + |H\rangle|H\rangle|H\rangle)_{BA}] + (|H\rangle|H\rangle|H\rangle)_{BA}].$$  

In the second one, the system is in a state with less entanglement and different parameters, that is,

$$|\Psi_1'\rangle_{CBAa} = \frac{1}{\sqrt{\alpha^2 + 2\beta^2}}[(\alpha^2|H\rangle|H\rangle|H\rangle)_{BA}] + (\beta^2(|H\rangle|H\rangle|H\rangle)_{BA}].$$

The probability that Alice obtains an even parity is

$$P_1 = \frac{3\alpha^2\beta^2}{\alpha^2 + \beta^2}.$$  

And the probability that Alice obtains an odd parity is

$$P_1' = \frac{\alpha^2 + 2\beta^2}{\alpha^2 + \beta^2}.$$  

Alice can measure the ancillary photon $a$ for obtaining the standard three-photon W state from the four-photon state $|\Psi_1\rangle_{CBAa}$ with the basis $X$ (i.e., $|\pm x\rangle_a = \frac{1}{\sqrt{2}}(|H\rangle \pm |V\rangle)$). When she obtain the state $|+x\rangle_a$, the three-photon system is in the standard W state $|W_3^+\rangle$. Here

$$|W_3^+\rangle_{CBA} = \frac{1}{\sqrt{3}}[(|H\rangle|H\rangle|H\rangle)_{BA} + |H\rangle|H\rangle|H\rangle)_{BA}].$$

When she obtain the state $|-x\rangle_a$, the three-photon system is in another standard W stat

$$|W_3^-\rangle_{CBA} = \frac{1}{\sqrt{3}}[(|H\rangle|H\rangle|H\rangle)_{BA} - |H\rangle|H\rangle|H\rangle)_{BA}].$$

Alice can transform the state $|W_3^-\rangle_a$ into the state $|W^+_3\rangle$ by performing a phase-flip operation $\sigma_z = |H\rangle\langle H| - |V\rangle\langle V|$ on her photon $A$. 

Certainly, in a practical application, it is not difficult for the parties to obtain information about the parameters $\alpha$ and $\beta$ by detecting a subset of three-photon systems, similar to the case for Bell-type states\cite{56}.
As for the less-entanglement state \(|\psi'_1\rangle_{CBA}\), Alice can measure the ancillary photon \(\alpha\) with the basis \(X\) to transform it into a three-photon state with less entanglement. That is,

\[
|\psi'_2\rangle_{CBA} = \frac{\alpha^2}{\sqrt{\alpha^2 + 2\beta^2}}(H)C(H)B|V\rangle_A \\
+ \frac{\beta^2}{\sqrt{\alpha^2 + 2\beta^2}}((H)C|V\rangle_B + |V\rangle_C(H)B)|H\rangle_A.
\] (13)

In detail, when Alice obtains the state \(|+\alpha\rangle\), the three-photon system is in the state \(|\psi'_2\rangle_{CBA}\). When Alice obtains the state \(|-\alpha\rangle\), she need only perform a phase-flip operation on her photon \(A\) and she will obtain the state \(|\psi'_2\rangle_{CBA}\).

It is not difficult to find that the state \(|\psi'_2\rangle_{CBA}\) shown in Eq.\((13)\) has the same form as the state \(|\varphi\rangle_{CBA}\) shown in Eq.\((3)\) but different parameters. We need only replace the parameters \(\alpha\) and \(\beta\) in Eq.\((3)\) with the parameters \(\alpha' = \frac{\beta'}{\sqrt{\alpha^2 + 2\beta^2}}\) and \(\beta' = \frac{\alpha'}{\sqrt{\beta^2 + 2\alpha^2}}\), respectively. That is, Alice can also concentrate the state \(|\psi'_2\rangle_{CBA}\) as the same as the state \(|\varphi\rangle_{CBA}\). The probability that Alice, Bob, and Charlie obtain the standard three-photon W state from each system in the state \(|\psi'_2\rangle_{CBA}\) is

\[
P_2 = \frac{3 \cdot \frac{\alpha^4}{\alpha^2 + 2\beta^2} \cdot \frac{\beta^4}{\alpha^2 + 2\beta^2}}{\frac{\alpha^4}{\alpha^2 + 2\beta^2} + \frac{\beta^4}{\alpha^2 + 2\beta^2}} = \frac{3 \alpha^4 \beta^4}{(\alpha^2 + \beta^2)(\alpha^2 + 2\beta^2)}. \] (14)

Certainly, the probability that Alice, Bob, and Charlie obtain the three-photon state with less entanglement from each system in the state \(|\psi'_2\rangle_{CBA}\) becomes

\[
P'_2 = \left(\frac{\alpha^4}{\alpha^2 + 2\beta^2}\right)^2 + 2 \cdot \left(\frac{\beta^4}{\alpha^2 + 2\beta^2}\right)^2 = \frac{\alpha^8 + 2\beta^8}{(\alpha^2 + \beta^2)(\alpha^2 + 2\beta^2)}. \] (15)

After Alice performs the entanglement concentration process for \(n\) times, the total probability that Alice, Bob, and Charlie obtain the standard three-photon W state \(|\psi'_n\rangle_{CBA}\) is

\[
P(n) = P_1 + P'_1P_2 + P'_1P'_2P_3 + \cdots + P'_1P'_2 \cdots P'_n = 3[\frac{\alpha^2 \beta^2}{\alpha^2 + \beta^2} + \frac{\alpha^4 \beta^4}{(\alpha^2 + \beta^2)(\alpha^2 + 2\beta^2)} + \frac{\alpha^8 \beta^8}{(\alpha^2 + \beta^2)(\alpha^2 + 2\beta^2)} + \cdots + \frac{\alpha^{2n} \beta^{2n}}{(\alpha^2 + \beta^2)(\alpha^2 + 2\beta^2) \cdots (\alpha^2 + \beta^2)}]. \] (16)

Let us assume that the parameter \(|\alpha|^2 \leq |\beta|^2\). One can see that the maximal success probability that Alice, Bob, and Charlie can distill a standard W state from the partially entangled state \(|\varphi\rangle_{CBA} = \alpha|H\rangle_C|H\rangle_B|V\rangle_A + \beta(|H\rangle_C|V\rangle_B|H\rangle_A + |V\rangle_C(H)B|H\rangle_A)\) is \(3|\alpha|^2\) and \(|\alpha|^2 \in [0, 1/3]\). Let us assume \(F = 3|\alpha|^2\). The relation between the total probability \(P(n)\) and \(F\) is shown in Fig.\(3\). Generally, when Alice repeats her entanglement concentration 5 times, the total success probability \(P(n)\) approaches the parameter \(F\), the limit in theory.

**FIG. 3:** The relation between the total success probability \(P(n)\) and the parameter \(F = 3|\alpha|^2\) when \(|\alpha|^2 \leq |\beta|^2\) for the cases \(n = 1\) (dot curve), \(2\) (dash-dot curve), \(3\) (dash-dot-dot curve), \(4\) (dash curve), and \(5\) (solid curve), respectively.

For a partially entangled W-class state with less entanglement, Alice need only iterate the process for 2 or 3 times for obtaining the total success probability approaching to the limit.

### III. ENTANGLEMENT CONCENTRATION OF PARTIALLY ENTANGLED N-PHOTON W STATES

In principle, it is not difficult to generalize our ECP for partially entangled \(N\)-photon W states. Let us assume that there is a partially entangled \(N\)-photon W-class state

\[
|\varphi\rangle_{ABC\cdots Z} = \alpha|H\rangle_Z \cdots |H\rangle_C|H\rangle_B|V\rangle_A + \beta(|H\rangle_Z \cdots |H\rangle_C|V\rangle_B|H\rangle_A + |H\rangle_Z \cdots |V\rangle_C(H)B|H\rangle_A) + \cdots + |V\rangle_Z \cdots |V\rangle_C(H)B|H\rangle_A). \] (17)

The subscript \(A, B, C, \ldots, Z\) represent the photons in W-class states shared by Alice, Bob, Charlie, ..., and Zach, respectively. Here, the parameters \(\alpha_1\) and \(\beta_1\) satisfy the following relation

\[
\alpha_1^2 + (N - 1)\beta_1^2 = 1. \] (18)

For obtain a standard \(N\)-photon W state from each system in the state \(|\varphi\rangle_{ABC\cdots Z}\), Alice prepares an ancillary photon \(\alpha_1\) in the stat \(|\varphi\rangle_{n_1} = \frac{1}{\sqrt{\alpha_1^2 + \beta_1^2}}(\alpha_1|H\rangle + \beta_1|V\rangle)_{n_1}\), similar to the case in the entanglement concentration of a three-photon system. Then the state of the composite system can be written as

\[
|\Phi\rangle_{Z\cdots CBA_{n_1}} = |\varphi\rangle_{Z\cdots CBA} \otimes |\varphi\rangle_{n_1},
\]

\[
= \frac{1}{\sqrt{\alpha_1^2 + \beta_1^2}}(\alpha_1\beta_1|H\rangle_Z \cdots |H\rangle_C|H\rangle_B|V\rangle_A|V\rangle_{n_1})
\]
which takes place with the probability

$$P_1''' = \frac{N\alpha_1^4 + (N - 1)\beta_1^4}{\alpha_1^2 + \beta_1^2}. \quad (23)$$

By measuring the ancillary photon $a_1$ in the $(N+1)$-photon system in the state $|\Psi_1''\rangle_{Z-CBAa_1}$, the $N$ parties can obtain the standard $N$-photon state

$$|W_N^+\rangle_{Z-CBA} = \frac{1}{\sqrt{N}}[(H)Z\cdots(H)C|V_B \rangle + (H)Z\cdots|V_C \rangle |H_B \rangle]$$

which takes place with the probability

$$P_1'' = \frac{N\alpha_1^2 + (N - 1)\beta_1^2}{\alpha_1^2 + \beta_1^2}. \quad (24)$$

In summary, we have proposed an ECP for nonlocal $N$-photon systems in a partially entangled pure $W$-class state, resorting to ancillary single photons and parity-check measurement based on cross-Kerr nonlinearity. Only one of the $N$ parties in quantum communication prepares ancillary photons and operates the entanglement concentration process for obtaining the standard $N$-photon $W$ state from each partially entangled pure $W$-class state. She need only communicate the classical information to other parties for retaining or discarding their photons, which greatly simplifies the complication of classical communication, similar to the works for a Bell-type state [54, 55]. Third, it has a higher success probability than others as its total success probability approaches to the limit in theory. These advantages maybe makes our ECP more feasible than others.

IV. DISCUSS AND SUMMARY

By far, there are no ECP for photon systems in a known $W$-class state, although there are two ECPs for photon system in an unknown $W$-class state [59, 60]. In fact, in a practical application of entanglement concentration, it is not difficult for the $N$ parties in quantum communication to obtain information about the $W$-class state shared by them. They need only measure a subset of samples. The present ECP is the first one for a known $W$-class state and it is more practical in the application in future. Compared with other two ECPs for $W$-class states [59, 60], the present ECPs has some advantages. First, the present ECP requires only an $N$-photon system in each round of entanglement concentration, not two copies of two $N$-photon entangled systems, which decreases the difficulty of its implementation largely. Second, only one of the $N$ parties in quantum communication perform the local unitary operation for reconstructing the standard $W$ state from the $W$-class state and she need only communicate the classical information to other parties for retaining or discarding their photons, which greatly simplifies the complication of classical communication, similar to the works for a Bell-type state [54, 55]. Third, it has a higher success probability than others as its total success probability approaches to the limit in theory. These advantages maybe makes our ECP more feasible than other ECPs.
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[1] M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information (Cambridge University, 2000).
[2] A. K. Ekert, "Quantum cryptography based on Bell’s theorem," Phys. Rev. Lett. 67, 661 - 663 (1991).
[3] C. H. Bennett, G. Brassard, and N. D. Mermin, "Quantum cryptography without Bell’s theorem," Phys. Rev. Lett. 68, 557 - 559 (1992).
[4] G. L. Long and X. S. Liu, "Theoretically efficient high-capacity quantum-key-distribution scheme," Phys. Rev. A 65, 032302 (2002).
[5] F. G. Deng and G. L. Long, "Controlled order rearrangement encryption for quantum key distribution," Phys. Rev. A 68, 042315 (2003).
[6] X. H. Li, F. G. Deng, and H. Y. Zhou, "Efficient quantum key distribution over a collective noise channel," Phys. Rev. A 78, 022321 (2008).
[7] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, "Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels," Phys. Rev. Lett. 70, 1895 - 1899 (1993).
[8] C. H. Bennett and S. J. Wiesner, "Communication via one and two-particle operators on Einstein-Podolsky-Rosen states," Phys. Rev. Lett. 69, 2881 - 2884 (1992).
[9] X. S. Liu, G. L. Long, D. M. Tong, and L. Feng, "General scheme for superfundee coding between multiparties," Phys. Rev. A 65, 022304 (2002).
[10] M. Hillery, V. Buzek, and A. Berthiaume, "Quantum secret sharing," Phys. Rev. A 59, 1829 - 1834 (1999).
[11] A. Karlsson, M. Koashi, and N. Imoto, "Quantum entanglement for secret sharing and secret splitting," Phys. Rev. A 59, 162 - 168 (1999).
[12] L. Xiao, G. L. Long, F. G. Deng, and J. W. Pan, "Efficient multiparty quantum-secret-sharing schemes," Phys. Rev. A 69, 052307 (2004).
[13] F. G. Deng, G. L. Long, and H. Y. Zhou, "Bidirectional quantum secret sharing and secret splitting with polarized single photons," Phys. Lett. A 337, 329 - 334 (2005).
[14] F. G. Deng, X. H. Li, and H. Y. Zhou, "Efficient high-capacity quantum secret sharing with two-photon entanglement," Phys. Lett. A 372, 1957 - 1962 (2008).
[15] Z. J. Zhang, Y. Li, and Z. X. Man, "Multiparty quantum secret sharing," Phys. Rev. A 71, 044301 (2005).
[16] F. L. Yan and T. Gao, "Quantum secret sharing between multiparty and multiparty without entanglement," Phys. Rev. A 72, 012304 (2005).
[17] F. G. Deng, X. H. Li, H. Y. Zhou, and Z. J. Zhang, "Improving the security of multiparty quantum secret sharing against Trojan horse attack," Phys. Rev. A 72, 044302 (2005).
[18] A. M. Lance, T. Symul, W. P. Bowen, B. C. Sanders, and P. K. Lam, "Tripartite quantum state sharing," Phys. Rev. Lett. 92, 177903 (2004).
[19] F. G. Deng, X. H. Li, C. Y. Li, P. Zhou, and H. Y. Zhou, "Multiparty quantum-state sharing of an arbitrary two-party state with Einstein-Podolsky-Rosen pairs," Phys. Rev. A 72, 044301 (2005).
[20] F. G. Deng, X. H. Li, C. Y. Li, P. Zhou, and H. Y. Zhou, "Quantum state sharing of an arbitrary two-qubit state with two-photon entanglements and Bell-state measurements," Europ. Phys. J. D 39, 459 - 464 (2006).
[21] X. H. Li, P. Zhou, C. Y. Li, H. Y. Zhou, and F. G. Deng, "Efficient symmetric multiparty quantum state sharing of an arbitrary m-qubit state," J. Phys. B 39, 1975 - 1983 (2006).
[22] Y. B. Sheng, F. G. Deng, and H. Y. Zhou, "Efficient and economic five-party quantum state sharing of an arbitrary m-qubit state," Eur. Phys. J. D 48, 279 - 284 (2008).
[23] A. Karlsson and M. Bourennane, "Quantum teleportation using three-particle entanglement," Phys. Rev. A 58, 4394 - 4400 (1998).
[24] C. P. Yang, S. I. Chu, and S. Han, "Efficient many-party controlled teleportation of multiqubit quantum information via entanglement," Phys. Rev. A 70, 022329 (2004).
[25] F. G. Deng, C. Y. Li, Y. S. Li, H. Y. Zhou, and Y. Wang, "Symmetric multiparty-controlled teleportation of an arbitrary two-particle entanglement," Phys. Rev. A 72, 022338 (2005).
[26] Z. D. Walton, A. F. Abouraddy, A. V. Sergienko, B. E. A. Saleh, and M. C. Teich, "Decoherence-free subspaces in quantum key distribution," Phys. Rev. Lett. 91, 087901 (2003).
[27] J. C. Boileau, D. Gottesman, R. Laflamme, D. Poulin, and R. W. Spekkens, "Robust polarization-based quantum key distribution over a collective-noise channel," Phys. Rev. Lett. 92, 017901 (2004).
[28] J. C. Boileau, R. Laflamme, M. Laforest, and C. R. Myers, "Robust quantum communication using a polarization-entangled photon pair," Phys. Rev. Lett. 93, 220501 (2004).
[29] X. H. Li, F. G. Deng, and H. Y. Zhou, "Efficient quantum key distribution over a collective noise channel," Phys. Rev. A 78, 022321 (2008).
[30] T. Yamamoto, J. Shimamura, S. K. Ozdemir, M. Koashi, and N. Imoto, "Faithful qubit distribution assisted by one additional qubit against collective noise," Phys. Rev. Lett. 95, 040503 (2005).
[31] X. H. Li, B. K. Zhao, Y. B. Sheng, F. G. Deng, and H. Y. Zhou, "Efficient fault tolerant quantum transmission with frequency degree of freedom," Opt. Commun. 282, 4025 - 4027 (2009).
[32] X. H. Li, F. G. Deng, and H. Y. Zhou, "Faulty qubit transmission against collective noise without ancillary qubits," Appl. Phys. Lett. 91, 144101 (2007).
[33] F. G. Deng, X. H. Li, and H. Y. Zhou, "Passively self-error-rejecting qubit transmission over a collective-noise channel," Quantum Inf. Comput. 11, 0913 - 0924 (2011).
[34] Y. B. Sheng and F. G. Deng, "Efficient quantum entanglement distribution over an arbitrary collective-noise channel," Phys. Rev. A 81, 042332 (2010).
[35] C. H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J. A. Smolin, and W. K. Wootters, "Quantum teleportation via noisy channels," Phys. Rev. Lett. 76, 722 - 725 (1996).
[36] D. Deutsch, A. Ekert, R. Jozsa, C. Macchiavello, S. Popescu, and A. Sanpera, "Quantum privacy amplification and the security of quantum cryptography over noisy channels," Phys. Rev. Lett. 83, 4221 - 4224 (1999).
[37] J. W. Pan, C. Simon, and A. Zellinger, "Entanglement purification for quantum communication," Nature (London) 410, 1067 - 1070 (2001).

[38] C. Simon and J. W. Pan, "Polarization entanglement purification using spatial entanglement," Phys. Rev. Lett. 89, 257901 (2002).

[39] Y. B. Sheng, F. G. Deng, and H. Y. Zhou, "Efficient polarization-entanglement purification based on parametric down-conversion sources with cross-Kerr nonlinearity," Phys. Rev. A 77, 042308 (2008).

[40] Y. B. Sheng and F. G. Deng, "Deterministic entanglement purification and complete nonlocal Bell-state analysis with hyper-entanglement," Phys. Rev. A 81, 032307 (2010).

[41] X. H. Li, "Deterministic polarization-entanglement purification using spatial entanglement," Phys. Rev. A 82, 044304 (2010).

[42] Y. B. Sheng and F. G. Deng, "One-step deterministic polarization-entanglement purification using spatial entanglement," Phys. Rev. A 82, 044305 (2010).

[43] F. G. Deng, "One-step error correction for multipartite polarization entanglement," Phys. Rev. A 83, 062316 (2011).

[44] C. Wang, Y. Zhang, and G. S. Jin, "Entanglement purification and concentration of electron-spin entangled states using quantum-dot spins in optical microcavities," Phys. Rev. A 84, 032307 (2011).

[45] C. Wang, Y. Zhang, and G. S. Jin, "Polarization-entanglement purification and concentration using cross-kerr nonlinearity," Quantum Inf. Comput. 11, 0988 - 1002 (2011).

[46] F. G. Deng, "Efficient multipartite entanglement purification with the entanglement link from a subspace," Phys. Rev. A 84, 052312 (2011).

[47] C. H. Bennett, H. J. Bernstein, S. Popescu, and B. Schumacher, "Concentrating partial entanglement by local operations," Phys. Rev. A 53, 2046 - 2052 (1996).

[48] S. Bose, V. Vedral, and P. L. Knight, "Purification via entanglement swapping and conserved entanglement," Phys. Rev A 60, 194 - 197 (1999).

[49] B. S. Shi, Y. K. Jiang, and G. C. Guo, "Optimal entanglement purification via entanglement swapping," Phys. Rev. A 62, 054301 (2000).

[50] T. Yamamoto, M. Koashi, and N. Imoto, "Concentration and purification scheme for two partially entangled photon pairs," Phys. Rev. A 64, 012304 (2001).

[51] Z. Zhao, J. W. Pan, and M. S. Zhan, "Practical scheme for entanglement concentration," Phys. Rev. A 64, 014301 (2001).

[52] Y. B. Sheng, F. G. Deng, and H. Y. Zhou, "Nonlocal entanglement concentration scheme for partially entangled multipartite systems with nonlinear optics," Phys. Rev. A 77, 062325 (2008).

[53] Y. B. Sheng, F. G. Deng, and H. Y. Zhou, "Single-photon entanglement concentration for long-distance quantum communication," Quantum Inform. Comput. 10, 0272 - 0281 (2010).

[54] Y. B. Sheng, L. Zhou, S. M. Zhao, and B. Y. Zheng, "Efficient single-photon-assisted entanglement concentration for partially entangled photon pairs," Phys. Rev. A 85, 012307 (2012).

[55] F. G. Deng, "Optimal nonlocal multipartite entanglement concentration based on projection measurements," Phys. Rev. A 85, 022311 (2012).

[56] M. Yang, Y. Zhao, W. Song, and Z. L. Cao, "Entanglement concentration for unknown atomic entangled states via entanglement swapping," Phys. Rev. A 71, 044302 (2005).

[57] Z. L. Cao, L. H. Zhang, and M. Yang, "Concentration for unknown atomic entangled states via cavity decay," Phys. Rev. A 73, 014303 (2006).

[58] H. F. Wang, S. Zhang, and K. H. Yeon, "Linear optical scheme for entanglement concentration of two partially entangled three-photon W states," Eur. Phys. J. D 56, 271 - 275 (2010).

[59] H. F. Wang, S. Zhang, and K. H. Yeon, "Linear-optics-based entanglement concentration of unknown partially entangled three-photon W states," J. Opt. Soc. Am. B 27, 2159 - 2164 (2010).

[60] W. Xiong and L. Ye, "Schemes for entanglement concentration of two unknown partially entangled states with cross-Kerr nonlinearity," J. Opt. Soc. Am. B 28, 2030 - 2037 (2011).

[61] K. Nemoto and W. J. Munro, "Nearly deterministic linear optical controlled-not gate," Phys. Rev. Lett. 93, 250502 (2004).

[62] S. D. Barrett, P. Kok, K. Nemoto, R. G. Beausoleil, W. J. Munro, and T. P. Spiller, "Symmetry analyzer for nondestructive Bell-state detection using weak nonlinearities," Phys. Rev. A 71, 060302 (2005).