A CROSS-CORRELATION ANALYSIS OF WMAP AND EGRET DATA IN WAVELET SPACE

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ABSTRACT

We cross-correlate the first-year Wilkinson Microwave Anisotropy Probe (WMAP) data and the diffuse gamma-ray intensity maps from the Energetic Gamma-Ray Experiment Telescope (EGRET) using spherical-wavelet approaches. Correlations at the 99.7% significance level have been detected, at scales around 15° in the WMAP foreground-cleaned W-band and Q-band maps, based on data from regions that are outside the most conservative WMAP foreground mask; no significant correlation is found with the Tegmark cleaned map. The detected correlation is most likely of Galactic origin and thus can help probe the origins of possible Galactic foreground residuals and ultimately remove them from measured microwave sky maps.

Subject headings: cosmic microwave background — cosmic rays — diffuse radiation — methods: data analysis

1 INTRODUCTION

The study of anisotropies in the cosmic microwave background (CMB) is a powerful tool in cosmology. Results from the Wilkinson Microwave Anisotropy Probe (WMAP) provide us with the angular power spectrum and its cosmological implications (Bennett et al. 2003a; Spergel et al. 2003; Page et al. 2003). However, unwanted signals due to foregrounds would contaminate any intrinsic signals, most importantly on large angular scales (Tegmark & Efstathiou 1996; de Oliveira-Costa et al. 2003).

The WMAP team chose the CMB-dominated bands (two Q-band maps at 40.7 GHz, two V-band maps at 60.8 GHz, and four W-band maps at 93.5 GHz) and combined them to obtain a map with enhanced signal-to-noise ratio. Despite the fact that the maps at the selected frequencies are dominated by the CMB, Galactic foregrounds—as well as extragalactic point sources—contribute significantly to the map. The WMAP team performed a foreground template fit (thermal dust from Finkbeiner et al. 1999; free-free from Finkbeiner 2003 and Schlegel et al. 1998; synchrotron from Haslam et al. 1982) to avoid the Galactic emissions and separated them from the underlying CMB signal according to the frequency dependence of the foregrounds (Bennett et al. 2003b). Since the foreground fluctuations depend on the multipole moment $l$ as well (Bouchet et al. 1995), $l$-dependent statistical weights were applied by Tegmark et al. (2003), who undertook an independent foreground analysis. Several works that subsequently detected WMAP non-Gaussianities found residual foregrounds or other systematic effects to be the source (Chiang et al. 2003; Chiang & Naselsky 2004; Naselsky et al. 2003, 2004; Hansen et al. 2004; Liu & Zhang 2005).

Wibig & Woffendale (2005) have presented evidence that the Tegmark cleaned map contains residual foregrounds possibly induced by cosmic rays, where the Energetic Gamma Ray Experiment Telescope (EGRET) diffuse gamma-ray intensity map (Hunter et al. 1997) was adopted as the Galactic tracer. Diffuse Galactic gamma-ray emission is supposedly produced by interactions of Galactic cosmic rays with the interstellar gas and radiation field, and it thus provides us with an indirect measurement of cosmic rays in various locations in the Galaxy.5

In this Letter, we cross-correlate the WMAP first-year data and the EGRET maps, which are based on more data and more complete point-source subtraction (Cillis & Hartman 2005), in wavelet space to probe the origins of potential residual foregrounds with characteristic scales. Both the WMAP combined foreground-cleaned maps (Bennett et al. 2003b) and the Tegmark cleaned map (Tegmark et al. 2003) have been used.

2 CROSS-CORRELATION IN WAVELET SPACE

A measure of the correspondence of two data sets on the sphere is the angular cross-correlation function $CCF(\theta)$, which represents how two measurements on the sky separated by an angle $\theta$ are correlated. Previous works have performed cross-correlations of the CMB data with nearby galaxy density tracers in a search for the integrated Sachs-Wolfe effect (Boughn & Crittenden 2002, 2004; Fosalba et al. 2003; Nolta et al. 2004; Fosalba & Gaztañaga 2004; Afshordi et al. 2004; Vielva et al. 2006). As illustrated by Vielva et al. (2006), cross-correlation studies can also be made in wavelet space.

The wavelet approach is very useful for detecting signals with a characteristic scale, such that a most optimal detection can be made by filtering the data at that scale to amplify the corresponding structures. It has been adopted in CMB-related analyses of non-Gaussianity (Hobson et al. 1999; Barreiro et al. 2000; Aghanim et al. 2003; Cayón et al. 2001, 2003; Martínez-González et al. 2002; Mukherjee & Wang 2004; Vielva et al. 2004; McEwen et al. 2005; Liu & Zhang 2005). In wavelet space, the cross-correlation covariance at a given scale $a$ is defined as

$$\text{Cov}_{\text{WF}}(a) = \frac{1}{N_a} \sum_p \psi_{\text{CMB}}(a,p) \psi_{\text{EGRET}}(a,p)$$ (1)

(Vielva et al. 2006), where $\psi_{\text{CMB}}(a,p)$ and $\psi_{\text{EGRET}}(a,p)$ are the wavelet coefficients of the WMAP and EGRET data at a position $p$ on the sky map and the sum $\sum_p$ is extended over all the pixels ($N_a$) that are not removed by the Galactic mask “$Kp0$” (Bennett et al. 2003b). Equation (1) gives the autocorrelation

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5 See Bertsch et al. (1993) and Hunter et al. (1997) for three-dimensional modeling.
covariance (ACC) when the two data sets are the same. To make the analysis more robust and less sensitive to any discrepancies between CMB data and simulations, we use the dimensionless and normalized cross-correlation coefficient $C_{W}(\theta)$ as the test statistic, which is given by $C_{W}(\theta) = \frac{\text{Cov}_{W}(\theta)}{\sigma_{W}^{+} \sigma_{W}^{-}}$, where $\sigma_{W}^{+} = \text{ACC}_{W}$ and $\sigma_{W}^{-} = \text{ACC}_{E}$ are the WMAP and EGRET autocorrelation covariances, respectively.

The wavelet coefficients are obtained by convolving the map with a certain spherical wavelet basis at a given scale:

$$\omega_{\ell}(p, a) = \int d\Omega' D(p + p') \Psi_{\ell}(\theta', a),$$

where $\Psi_{\ell}(\theta', a)$ is the spherical wavelet basis and $D(p + p')$ is the data set to be analyzed. The spherical wavelet can be obtained from its Euclidean counterpart using the inverse stereographic projection suggested by Antoine & Vanderheynst (1998). A new, self-consistent, and practical approach to wavelet filtering on the sphere has recently been developed (Wiaux et al. 2005a). The corresponding fast algorithms and the exact expression of the wavelet coefficients can be found in Wiaux et al. (2005b, 2005c), Martínez-González et al. (2002) have described the projection for the spherical Mexican hat wavelet (SMHW), as well as its properties, whereas the spherical Morlet wavelet (SMW) has been applied to non-Gaussianity detection in the WMAP data (McEwen et al. 2005; Liu & Zhang 2005). In this correlation study we have adopted both wavelets, of which SMW seems to be more sensitive than SMHW. We only present results from the SMW analysis here, even though similar correlations are also found when applying SMHW.

The preprocessing pipelines for the WMAP CMB data originally in the HEALPix scheme (Górski et al. 2005) and Monte Carlo simulations are the same as in Liu & Zhang (2005), whereas the gamma-ray intensity maps are degraded to the same resolution as the CMB data to be cross-correlated, all in an equiangular spherical grid pixelization. The preprocessed maps are shown in Figure 1. Ten thousand simulations of the WMAP data have been applied, following the cosmological model given in Table 1 of Spergel et al. (2003), to obtain the significance levels of any detected correlations. We determine these levels in a robust way by taking into account the probability distribution of the wavelet cross-correlation coefficient at each scale. Although the map-making algorithm presented by Cillis & Hartman (2005) may have produced some systematic effects in the EGRET data, these can be calibrated out with simulations.

3. RESULTS

Cross-correlation with the EGRET maps in wavelet space has been performed for both the Q-V-W combined WMAP map and the Tegmark cleaned map to detect foreground residuals (throughout, all WMAP CMB maps used are foreground-removed). We have also performed the analysis for (1) the WMAP map in each band separately, that is, Q, V, and W; (2) systematic beam and noise maps created by subtracting maps from receivers at the same frequency; and (3) a foreground map almost free of CMB, made by subtracting the two Q-band receivers and the two V-band receivers from the four receivers at W band (Vielva et al. 2004).

For every CMB-related map, we performed the analysis separately using EGRET maps in different energy ranges, that is, 30–100 MeV, 100–300 MeV, 300–1000 MeV, and above 1 GeV; only the results from the latter map are shown here, because (1) it has been stated by Cillis & Hartman (2005) that the intensity map above 1 GeV has the best statistical accuracy compared with those from lower energy ranges, at which possible correlations can be smeared out by statistical uncertainty fluctuations, and (2) the analyses at the different gamma-ray energy ranges stated above generally give similar correlations, with the significance level seeming to increase a little with the energy of the adopted gamma-ray data. Note that this variance of correlation significance does not conflict with the power-law energy spectrum, since the cross-correlation coefficient is a normalized quantity.

The cross-correlation coefficients $C_{W}(\theta)$ are illustrated in Figures 2a and 2b, where the corresponding size $\theta$ on the sky can be obtained from equation (3) in Liu & Zhang (2005); we have analyzed scales from 1° to 90°, but we are only concerned with results at scales over 4° because of the angular resolution limit of the EGRET data. We show the ACCs (normalized by the map dispersion $\sigma_{m}$ in real space) of the WMAP combined map, the Tegmark cleaned map, the foreground-component map, and the diffuse gamma-ray intensity map in Figure 2c, which indicates that the general patterns of $C_{W}(\theta)$ in Figures 2a and 2b are caused by the convolution of the gamma-ray data and the wavelets, not by systematic artifacts. Note that at all the scales of interest, the foreground map correlates more significantly with the EGRET map, consistent with the assumption that most of the foregrounds have been removed from the CMB maps. Results from analyzing the systematic beam and noise maps are not shown, since their correlation coefficients are all consistent with zero values within statistical fluctuations.

Fig. 1.—Preprocessed EGRET diffuse gamma-ray intensity map (>1 GeV; Cillis & Hartman 2005) and CMB anisotropy maps from the WMAP team (Bennett et al. 2003a) and as cleaned by Tegmark et al. (2003) to be analyzed in the cross-correlation study. Note that we have analyzed the EGRET diffuse gamma-ray intensity maps for different energies separately, that is, 30–100 MeV, 100–300 MeV, 300–1000 MeV, and above 1 GeV, whereas only the latter map is shown here. All maps are plotted in Galactic coordinates with the Galactic center (l, b) = (0, 0) in the middle and Galactic longitude l increasing to the left.
For the WMAP CMB maps, correlations at the 99.7% significance level are detected at scales from about 14° to 16° in the W-band and Q-band maps; less significant correlations (96.8%) are found at larger scales, from about 43° to 48°, in the W-band map only. In sum, the W-band and Q-band maps exhibit more significant correlations than those from the V-band map, at all the relevant scales. This frequency dependence can be easily understood according to Figure 10a of Bennett et al. (2003b), which shows evidence that the detected correlations are caused by residual foreground signals. In order to test whether the detected signal has a Galactic origin, we show in Figure 2d the cross-correlation coefficients as a function of Galactic latitude around $\theta = 15^\circ$. Note that the sum of the cross-correlation coefficients at all the latitudes in Figure 2d corresponds to $C_{W,E}(15^\circ)$ in Figure 2a. Here the oscillatory pattern is given by the spherical Morlet wavelet, and we have tested that the outline profile due to the EGRET intensity maps. Tests at other scales also present similar profiles. The results show some consistency with the cosecant law for the Galactic components and also exhibit a north-south asymmetry, with the correlation being stronger in the southern hemisphere.

Analysis of the Tegmark cleaned map does not show any significant correlation at any of the scales of concern, evidence that it is “cleaner” than the WMAP combined map.

4. CONCLUSIONS AND DISCUSSION

We have performed a cross-correlation analysis of the WMAP first-year data with the EGRET diffuse gamma-ray intensity maps in wavelet space, finding correlations in the WMAP foreground-cleaned maps based on regions that are outside the most conservative WMAP foreground mask. Analysis of the WMAP W- and Q-band maps reveals a correlation at the 99.7% significance level at scales around 15° in the sky; at scales around 45°, a less significant correlation (96.8%) is found only in the WMAP W-band map. The Tegmark cleaned map seems to be compatible with pure CMB simulations at all scales under consideration.

These cross-correlation signals are not caused by systematic beams or noise, because (1) the analysis of the systematic beam and noise map shows almost zero correlation results and (2) the correlations are detected at scales where noise and beam...
effects can be ignored (Tegmark et al. 2003). We thus conclude that these cross-correlation signals are most probably caused by foreground residuals, because (a) correlations from the Q and W bands are more significant than those from the V band, consistent with the frequency-dependent nature of the foreground; (b) the correlation coefficients from the CMB maps present similar patterns to those from the foreground map, whereas random simulations do not show correlations at the detected level; (c) the Tegmark cleaned map, which has been commonly believed to be cleaner than the WMAP combined map in terms of the foreground removal, shows no significant correlation with the EGRET map in the analysis; and (d) the detected cross-correlation coefficient as a function of Galactic latitude appears to be consistent with a cosecant law, evidence of a Galactic origin. It is possible that these foreground residuals were induced by cosmic rays, since the Galactic diffuse gamma-ray emission is supposedly produced by interactions of cosmic rays with gas and ambient photon fields and thus can be an indicator of cosmic rays in various locations in the Galaxy.

The detected foreground residuals can be located in the coefficient map at a certain scale. It would therefore be worthwhile to perform a detailed correlation study with the cosmic-ray spatial and spectral distributions. Note that the diffuse gamma-ray emission has not been well understood, in that at energies above 1 GeV the observed intensity in the inner Galaxy displays a GeV excess at a level of 60% compared with predictions (Hunter et al. 1997), a topic that has been addressed by several models (Strong et al. 2000; de Boer 2005). Further cross-correlation work must be carried out to fully understand the nature of foreground residuals and finally remove them from the CMB maps completely, in order to minimize the impact of foreground residuals on cosmological studies of the CMB.

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