Design of preview controller for a type of discrete-time interconnected systems

Hao Xie¹, Fucheng Liao¹, Usman¹ and Jiamei Deng²

Abstract
This article proposes and studies a problem of preview control for a type of discrete-time interconnected systems. First, adopting the technique of decentralized control, isolated subsystems are constructed by splitting the correlations between the systems. Utilizing the difference operator to the system equations and error vectors, error systems are built. Then, the preview controller is designed for the error system of each isolated subsystem. The controllers of error systems of isolated subsystems are aggregated as a controller of the interconnected system. Finally, by employing Lyapunov function method and the properties of non-singular M-matrix, the guarantee conditions for the existence of preview controllers for interconnected systems are given. The numerical simulation shows that the theoretical results are effective.

Keywords
Discrete-time interconnected systems, preview control, error system, Lyapunov function, non-singular M-matrix

Date received: 3 August 2019; accepted: 5 December 2019

Introduction
In many practical cases, future reference or disturbance signal of control systems is either partly or completely known, such as the flight path of aircraft, the processing path of numerically controlled machine tools, the driving path of vehicles, and so on. This future information is fully utilized to improve the quality of system, which is the preview control problem. Since Sheridan¹ put forward the concept of preview control, which was further upheld by Masayoshi Tomizuka, Tohru Katayama, and other scholars in the 1960s, preview control has attracted extensive attention in theoretical research and application, and formed a set of relatively complete theories and methods.²–⁴ Preview control theory has been widely combined with various systems in recent years, which has produced many important results. In Liao et al.,⁵ preview control theory and descriptor systems are merged to investigate preview control for linear causal descriptor systems. The theory results of preview control were extended to the cooperative consensus problem of multi-agent systems, and the sufficient conditions to guarantee the achievement of cooperative preview tracking control were given.⁶ In addition, the theory of multi-rate systems preview control and random systems preview control has made progress.⁷,⁸ At the same time, preview control has also been exploited in many engineering control problems, such as robot system, active suspension system, electromechanical servo, and aircraft.⁹–¹¹

The so-called interconnected system refers to the system with complex structure, comprehensive functions, numerous factors, and large scale. Interconnected system is also called large-scale system. The power systems, urban transportation networks, and water resources systems are the examples of actual interconnected systems, which can be seen in daily life.¹²–¹⁴ For interconnected systems, if the controller is designed by centralized control method, it will be difficult to centralize and deal with a large amount of information, which makes the control difficult to achieve. Therefore, decentralized aggregation method is adopted to design the controller.¹⁵–¹⁷ From a mathematical perspective, that is, the large-scale systems decomposition method. First, the associated terms are deleted artificially to obtain several low-dimensional systems (called isolated subsystems), and controllers are designed to meet

¹School of Mathematics and Physics, University of Science and Technology Beijing, Beijing, China
²School of Computing, Creative Technologies & Engineering, Leeds Beckett University, West Yorkshire, UK

Corresponding author:
Fucheng Liao, School of Mathematics and Physics, University of Science and Technology Beijing, 100083, China.
Email: fcliao@ustb.edu.cn

Creative Commons CC BY: This article is distributed under the terms of the Creative Commons Attribution 4.0 License (https://creativecommons.org/licenses/by/4.0/) which permits any use, reproduction and distribution of the work without further permission provided the original work is attributed as specified on the SAGE and Open Access pages (https://us.sagepub.com/en-us/nam/open-access-at-sage).
certain requirements. Then, the controller of the interconnected system is obtained through a certain method of synthesis.\textsuperscript{18–20} So far, there have been many control theories for interconnected systems. The tracking problem of interconnected systems through decentralized iterative learning control was studied in the following literature.\textsuperscript{21–23} Zhang and Feng\textsuperscript{23} reported the problem of controller design for fuzzy interconnected systems, and its stability is analyzed using piecewise Lyapunov function. Furthermore, Koeln\textsuperscript{24} discusses the decentralized control of interconnected systems with a special structure, and studies its application in large refrigeration and air conditioning systems.

Under the current circumstances that theories of preview control and interconnected systems have made great progress, it has important theoretical and practical significance to associate with the two theories. Until now, only Liao et al.\textsuperscript{25} solved a type of preview tracking control problems related to continuous-time interconnected systems. This article designs a controller with preview effect for a type of discrete-time interconnected systems, taking into account the previewable situation of both the external disturbance signal and the reference signal.

Research contents are arranged as follows: The introduction is given in section “Introduction.” Section “Preliminaries” consist of the elementary knowledge, which gives the key concepts needed in this paper. Section “Problem formulation” presents preview control problems for a type of interconnected systems and gives fundamental assumptions. The design of error control problems related to continuous-time interconnected systems are discussed in sections “Controller of error system of isolated subsystems” and “Preview controller design for interconnected systems,” respectively. Section “Numerical simulation” explains numerical simulation. Finally, a brief conclusion is given in section “Conclusion.”

Throughout this paper, $A \in \mathbb{R}^{n \times m}$ represents $A$ as the real matrix of $n \times m$; $R > 0 (R \geq 0)$ shows the matrix $R$ is a symmetric positive definite (semi-positive definite); $B \cdot C$ indicates the Hadamard product of matrices $B$ and $C$; $\rho(\cdot)$ is the spectral radius of matrix; $\Delta$ denotes the difference operator, which means $\Delta \xi(k) = \xi(k) - \xi(k - 1)$; and $\|A\|$ means the norm of matrix $A$ derived from Euclid norm of vector.

**Preliminaries**

For readability, the definition and partial properties of Hadamard product and non-singular M-matrix are given here.

**Definition 1.** Set $B = [b_{ij}]$, $C = [c_{ij}] \in \mathbb{R}^{n \times n}$. $B \cdot C$ is a matrix obtained by multiplying the corresponding element of $B$ and $C$, that is, $B \cdot C = [b_{ij}c_{ij}]$. Let us call $B \cdot C$ the Hadamard product of matrices $B$ and $C$.\textsuperscript{26}

The following properties can be obtained instantly from Definition 1 and the definition of matrix multiplication.

**Property 1.** Setting $a = [a_1, a_2, \ldots, a_n]^T \in \mathbb{R}^n$, $d = [d_1, d_2, \ldots, d_n]^T \in \mathbb{R}^n$, $z_i \in R(i = 1, 2, \ldots, n)$, there is

$$[z_1 \ z_2 \ \cdots \ z_n](a \cdot d) = a^T \text{diag}(z_1, z_2, \ldots, z_n)d$$

**Definition 2.** Let $A \in \mathbb{R}^{n \times n}$ be defined as

$$A = sI_n - B$$

where $s > 0$, each element in matrix $B$ is non-negative. If $s > \rho(B)$, then $A$ is called a non-singular M-matrix.

**Lemma 1.** If the non-diagonal elements of matrix $A$ are less than or equal to zero, then the necessary and sufficient condition for $A$ to be a non-singular M-matrix is that one of the following conditions must be true:

1. For any $\alpha \geq 0, A + \alpha I$ is non-singular and
2. There is a matrix $K = \text{diag}(k_1, k_2, \ldots, k_n) \geq 0$ that makes $KA + A^TK > 0$.

It can be proved that the non-singular M-matrix also has the following property.

**Theorem 1.** If $A$ is a non-singular M-matrix, $G = (I - A)(I + A)^{-1}$, then there is diagonal matrix $K > 0$, such that $K - G^TKG > 0$.

**Proof.** From Lemma 1, there is a diagonal matrix $K > 0$, so that $KA + A^TK > 0$. We know from the obvious equality

$$2(KA + A^TK) = A^TKA + KA + A^TK + K - (A^TKA - KA - A^TK + K) = (I + A)^TK(I + A) - (I - A)^TK(I - A) = \Phi$$

that $\Phi$ is a positive definite matrix. Since $(I + A)^{-1}$ exist, there is

$$[I + A]^T \Phi(I + A)^{-1} = K - G^TKG$$

which means $K - G^TKG$ and $\Phi$ are congruent. Because the congruent matrix has the same positivity, $K - G^TKG > 0$. Theorem 1 is proved.

**Problem formulation**

Consider discrete-time interconnected system
Clearly, system (1) is able to be equivalently expressed as

\[
\begin{align*}
\begin{bmatrix}
    x_1(k+1) \\
x_2(k+1) \\
\vdots \\
x_N(k+1)
\end{bmatrix} &= 
\begin{bmatrix}
    A_1 & A_{12} & \cdots & A_{1N} \\
    A_{21} & A_2 & \cdots & A_{2N} \\
    \vdots & \vdots & \ddots & \vdots \\
    A_{N1} & A_{N2} & \cdots & A_N
\end{bmatrix}
\begin{bmatrix}
    x_1(k) \\
x_2(k) \\
\vdots \\
x_N(k)
\end{bmatrix}
\begin{bmatrix}
    B_1 & 0 & \cdots & 0 \\
    0 & B_2 & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & \cdots & B_N
\end{bmatrix}
\begin{bmatrix}
    u_1(k) \\
    u_2(k) \\
    \vdots \\
    u_N(k)
\end{bmatrix}
\begin{bmatrix}
    E_1 & 0 & \cdots & 0 \\
    0 & E_2 & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & \cdots & E_N
\end{bmatrix}
\begin{bmatrix}
    d_1(k) \\
d_2(k) \\
\vdots \\
d_N(k)
\end{bmatrix}
\end{align*}
\]

(1)

Here, \( x_i(k) \in \mathbb{R}^n \), \( u_i(k) \in \mathbb{R}^m \), \( d_i(k) \in \mathbb{R}^q \), \( y_i(k) \in \mathbb{R}^r \), \( A_j \in \mathbb{R}^{n \times n} \), \( A_{ij} \in \mathbb{R}^{n \times m} \), \( B_i \in \mathbb{R}^{n \times q} \), \( E_i \in \mathbb{R}^{n \times n} \), \( C_j \in \mathbb{R}^{r \times n} \) are constant matrices; \( A_{ij} (i \neq j) \) is the correlation matrix.

Remark 1. Assumption 1 and Assumption 2 are fundamental assumptions for the original system. In the design of the controller, it is necessary to build the error system (10) and take the performance index function of system (11). The controller of interconnected system is obtained under the conditions where \((\Phi_i, G_i)\) is stabilizable and \((Q_i^{1/2}, \Phi_i)\) is observable. Naturally, it is necessary to give conditions that the original system satisfies, so that \((\Phi_i, G_i)\) is stabilizable and \((Q_i^{1/2}, \Phi_i)\) is observable. According to Katayama et al., Assumption 1 can guarantee that \((\Phi_i, G_i)\) is stabilizable. Assumption 2 can guarantee that \((Q_i^{1/2}, \Phi_i)\) is observable. Assumption 3 and Assumption 4 are the standard assumptions for preview control. In fact, by the characteristics of the control systems, only a period of previewable information has obvious impact on quality of the systems. The value beyond previewable steps has a little effect on the characteristics of the systems. Therefore, in the theory of preview control, the future value outside the previewable information is usually assumed to be a constant. The tracking error \(e(k)\) can be defined as follows

\[
e(k) = r(k) - y(k)
\]

(3)

The aim of this article is to adopt optimal control theory to design a previewable controller that allows the output \(y(k)\) of the system (2) to asymptotically track \(r(k)\). In other words, \(\lim_{k \to \infty} e(k) = \lim_{k \to \infty} [r(k) - y(k)] = 0\).

Controller of error system of isolated subsystems

For the sake of designing the controller, the method of decentralized control is utilized. First, we cut off the linkage between subsystems to form isolated subsystems and design controller for each isolated subsystem. Then, the controllers of the isolated subsystems are combined to get the controller of the interconnected systems. Finally, by discussing the stability of interconnected system, the constraints of associated terms are obtained.

Based on the output of system (2), we rewrite \(e(k)\) as follows

\[
e(k) = \sum_{i=1}^{N} e_i(k) = \sum_{i=1}^{N} [a_i r(k) - y_i(k)]
\]

(4)
where \( \alpha_i(i = 1, 2, \ldots, N) \) are constant and satisfy \( \sum_{i=1}^{N} \alpha_i = 1 \). According to equation (4), if for any \( e_i(k) = \alpha_i r(k) - y_i(k)(i = 1, 2, \ldots, N) \), there is \( \lim_{k \to \infty} e_i(k) = 0 \), then \( \lim_{k \to \infty} e(k) = 0 \).

**Remark 2.** We can think of \( y_i(k) \) as the output of \( i \)th subsystem. Equation (4) means that if output \( y_i(k) \) of \( i \)th subsystem tracks \( \alpha_i r(k)(i = 1, 2, \ldots, N) \), then the output \( y(k) = \sum_{i=1}^{N} y_i(k) \) of interconnected system (1) can track \( r(k) \). The parameter \( \alpha_i(i = 1, 2, \ldots, N) \) gives us the freedom of choice. For example, we can choose \( \alpha_1 = \alpha_2 = \cdots = \alpha_N = (1/N) \), which means that all \( y_i(k) \) keep track of \( (1/N)r(k) \). If \( \alpha_i = 0 \), it indicates that the output of the \( i \)th subsystem tracks the zero vector, and the task of tracking \( r(k) \) is completed by the output of other subsystems, and so on.

The equation of \( i \)th isolated subsystem is

\[
\begin{align*}
\dot{x}_i(k + 1) &= A_i x_i(k) + B_i u_i(k) + E_i d_i(k) \\
y_i(k) &= C_i x_i(k)
\end{align*}
\]

(5)

Currently, the error system is constructed for the isolated subsystem by the method of usually preview control. As a result, the tracking problem of isolated subsystem is turned into the error system regulation problem. Since the error system of interconnected system is still needed in the construction of the controller of the interconnected system, the error system (2) is constructed first to avoid the repetition calculations. Then, the correlation term is cut off to obtain the error systems of the isolated subsystems.

The \( \Delta \) is applied on both ends of the state equation of system (2) to get

\[
\Delta x_i(k + 1) = A_i \Delta x_i(k) + \sum_{j \neq i} A_{ij} \Delta x_j(k) + B_i \Delta u_i(k)
\]

\[
+ E_i \Delta d_i(k) \quad (i = 1, 2, \ldots, N)
\]

(6)

Utilizing \( \Delta \) to both sides of \( e_i(k + 1) = \alpha_i r(k + 1) - y_i(k + 1)(i = 1, 2, \ldots, N) \), we get

\[
\Delta e_i(k + 1) = \alpha_i \Delta r(k + 1) - \Delta y_i(k + 1)
\]

\[
= \alpha_i \Delta r(k + 1) - C_i \Delta x_i(k + 1)
\]

(7)

Notice that \( \Delta e_i(k + 1) = e_i(k + 1) - e_i(k) \), then substitute equation (6) into equation (7) to get

\[
e_i(k + 1) = e_i(k) + \alpha_i \Delta r(k + 1) - C_i A_i \Delta x_i(k)
\]

\[
- \sum_{j \neq i} C_{ij} A_{ij} \Delta x_j(k) - C_i B_i \Delta u_i(k)
\]

\[
- C_i E_i \Delta d_i(k) \quad (i = 1, 2, \ldots, N)
\]

(8)

Combine equations (6) and (8) to get

\[
X_i(k + 1) = \Phi_i X_i(k) + \sum_{j = 1}^{N} \Phi_{ij} Y_j(k) + G_i \Delta u_i(k)
\]

\[
+ G_i \Delta r(k + 1) + G_i \Delta d_i(k) \quad (i = 1, 2, \ldots, N)
\]

(9)

Here

\[
X_i(k) = \begin{bmatrix} e_i(k) \end{bmatrix}, \quad \Phi_i = \begin{bmatrix} I & -C_i A_i \\ 0 & A_i \end{bmatrix},
\]

\[
\Phi_{ij} = \begin{bmatrix} 0 & -C_i A_{ij} \\ 0 & A_{ij} \end{bmatrix}, \quad G_i = \begin{bmatrix} -C_i B_i \\ B_i \end{bmatrix},
\]

\[
G_{ij} = \begin{bmatrix} \alpha_i I \\ 0 \end{bmatrix}, \quad G_d = \begin{bmatrix} -C_i E_i \\ E_i \end{bmatrix}
\]

System (9) is the error system of interconnected system (2).

Noted that \( e_i(k) \) is a partial vector of \( X_i(k) \), so if there is \( \lim X_i(k) = 0 \) in system (9), there is \( \lim e_i(k) = 0(i = 1, 2, \ldots, N) \). In this way, \( y(k) \) of the interconnected system (2) can track \( r(k) \) asymptotically.

The error system of isolated subsystems is collected by cutting off the correlation item in system (9). The error system of the \( i \)th \((i = 1, 2, \ldots, N)\) isolated subsystem is

\[
X_i(k + 1) = \Phi_i X_i(k) + G_i \Delta u_i(k)
\]

\[
+ G_i \Delta r(k + 1) + G_i \Delta d_i(k)
\]

(10)

For the sake of utilizing the results of optimal control, a quadratic performance index function is defined for error system (10)

\[
J_i = \sum_{k = 1}^{\infty} \left[ X_i^T(k) Q_i X_i(k) + \Delta u_i^T(k) H_i \Delta u_i(k) \right]
\]

(11)

where

\[
Q_i = \begin{bmatrix} Q_{i e} & 0 \\ 0 & Q_{i v} \end{bmatrix} \in R^{(n + n_i) \times (n + n_i)}, \quad Q_{i e} > 0,
\]

\[
Q_{i v} \geq 0, \quad H_i \in R^{m_i \times m_i}, \quad H_i > 0.
\]

**Remark 3.** Obviously, the input \( \Delta u_i(k) \) of the system (10) is used in system (11). For original system (2), it is to quote \( \Delta u_i(k) \) (not \( u_i(k) \)) in the performance index function. This causes the controller to include integrators, which helps to eliminate static errors.\(^2\)

From the known conclusion in Katayama et al.,\(^4\) Theorem 2 can be proved directly.

**Theorem 2.** Let us assume that \((\Phi_i, G_i)\) is stabilizable, \((Q_i^{1/2}, \Phi_i)\) is observable, and Assumption 3 and Assumption 4 hold, then the controller of the system (10), which minimizes the performance index function of system (11), has the form of
closed-loop system tends to zero asymptotically.

\[
(9) \quad \text{asymptotically approaches the zero vector.}
\]

The interconnected systems can be obtained, here is constructed, where \( (\Phi_j, G_j) \) is stabilizable and \( (Q_j^{1/2}, \Phi_j) \) is observable \((i = 1, 2, \ldots , N)\). From Katayama et al., if \((\Phi_j, G_j)\) is stabilizable, \((Q_j^{1/2}, \Phi_j)\) is observable, Assumption 3 and Assumption 4 are established, then there is a unique positive definite solution matrix \( P_i \) for Riccati equation (13). Using \( P_i \) to construct \( V_l(X) = X^T P_i X_i \), it is a positive definite quadratic form of \( X_i \). Take difference to \( V_l(X) \) along the system (16) trajectory to obtain

\[
\Delta V_l(16) = X_i^T (k + 1) P_i X_i (k + 1) - X_i^T (k) P_i X_i (k)
\]

Next, a sufficient condition is given to assure system (15) asymptotically approaches the zero vector.
Notice $\eta_i = -\lambda_{\max}[\xi^T P_i \xi_i - P_i]$, further to

$$
\Delta V_{i|16} \leq - \eta_i \|X_i(k)\|^2 + 2 \|X_i^T(k)\| \|\xi_i^T\| \|P_i\| \sum_{j \neq i} \|\Phi_{ij} [X_i(k)]\| + \left[ \sum_{j \neq i} \|\Phi_{ij} [X_i(k)]\| \right]^2 \|P_i\|
$$

Continuously, use the properties of norms to obtain

$$
\Delta V_{i|16} \leq - \eta_i \|X_i(k)\|^2 + 2 \|X_i^T(k)\| \|\xi_i^T\| \|P_i\| \sum_{j \neq i} \|\Phi_{ij} [X_i(k)]\| + \left[ \sum_{j \neq i} \|\Phi_{ij} [X_i(k)]\| \right]^2 \|P_i\|
$$

Substitute $L$ into equation (18) to get

$$
G = [I - (T - S)(T + S)^{-1}] [I + (T - S)(T + S)^{-1}]^{-1}
$$

So, $K = (ST^{-1})^T K (ST^{-1}) > 0.$

Let $i = 1, 2, \ldots, N$ to get

$$
\begin{bmatrix}
\Delta V_1|16 \\
\Delta V_2|16 \\
\vdots \\
\Delta V_N|16 \\
\end{bmatrix}
\leq - \begin{bmatrix}
\|X_1(k)\| & \|X_2(k)\| & \cdots & \|X_N(k)\| \\
\|X_2(k)\| & \|X_3(k)\| & \cdots & \|X_N(k)\| \\
\vdots & \ddots & \ddots & \vdots \\
\|X_N(k)\| & \cdots & \|X_1(k)\| \\
\end{bmatrix}
\begin{bmatrix}
O(T - S) \\
O(T + S) \\
\end{bmatrix}
\begin{bmatrix}
\|X_1(k)\| & \|X_2(k)\| & \cdots & \|X_N(k)\| \\
\|X_2(k)\| & \|X_3(k)\| & \cdots & \|X_N(k)\| \\
\vdots & \ddots & \ddots & \vdots \\
\|X_N(k)\| & \cdots & \|X_1(k)\| \\
\end{bmatrix}
\begin{bmatrix}
\|X_1(k)\| & \|X_2(k)\| & \cdots & \|X_N(k)\| \\
\|X_2(k)\| & \|X_3(k)\| & \cdots & \|X_N(k)\| \\
\vdots & \ddots & \ddots & \vdots \\
\|X_N(k)\| & \cdots & \|X_1(k)\| \\
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_N \\
\end{bmatrix}
$$

Using $K$ and $O$, the Lyapunov function of system (16) is taken as positive definite quadratic

$$
V = \sum_{i=1}^{N} \frac{k_i}{v_i} V_i = \begin{bmatrix} \frac{k_1}{v_1} & \frac{k_2}{v_2} & \cdots & \frac{k_N}{v_N} \end{bmatrix} \begin{bmatrix} V_1 \\
V_2 \\
\vdots \\
V_N \end{bmatrix}
$$

Then, the difference of $V$ along system (16) trajectory is

$$
\Delta V|16 = \begin{bmatrix} \frac{k_1}{v_1} & \frac{k_2}{v_2} & \cdots & \frac{k_N}{v_N} \end{bmatrix} \begin{bmatrix} \Delta V_1|16 \\
\Delta V_2|16 \\
\vdots \\
\Delta V_N|16 \end{bmatrix}
$$

By substituting system (17) and using Property 1, we get

$$
G = (I - L)(I + L)^{-1}
$$
Because system (16) is asymptotically stable and according to Assumption 3, there is $\lim_{n \to \infty} \lambda = 0 (i = 1, 2, \ldots, N)$.

Assumption 3 and Assumption 4, when $k$ tends to infinity, $\Delta x(k)$ and $\Delta d(k)$ tend to zero vectors. In view of the relationship between $\Theta(k)$ and $\Delta r(k)$, $\Delta d(k)$, it is easy to get $\lim_{k \to \infty} \Theta(k) = 0 (i = 1, 2, \ldots, N)$. Therefore, the difference of $V$ along with system (16) trajectory is negative definite. As a result, the zero solution of system (16) is asymptotically stable.

Then, we prove that $\lim_{k \to \infty} \Theta(k) = 0 (i = 1, 2, \ldots, N)$. According to Assumption 3 and Assumption 4, when $k$ tends to infinity, $\Delta x(k)$ and $\Delta d(k)$ tend to zero vectors. In view of the relationship between $\Theta(k)$ and $\Delta r(k)$, $\Delta d(k)$, it is easy to get $\lim_{k \to \infty} \Theta(k) = 0 (i = 1, 2, \ldots, N)$.

Because system (16) is asymptotically stable and $\lim_{k \to \infty} \Theta(k) = 0 (i = 1, 2, \ldots, N)$, according to Theorem 1 of Chen, there is $X(k) = 0 (i = 1, 2, \ldots, N)$ in system (15). Hence, Theorem 3 is proved.

Below, we use the relevant parameters of interconnected system (2) to give the conditions, which ensure that $(\Phi_t, G_t)$ can be stabilized and $(\tilde{Q}_t^{1/2}, \Phi_t)$ can be observed.

According to Katayama et al.⁴ the sufficient and necessary condition for $(\Phi_t, G_t)$ to be stabilizable is that $\text{rank} \begin{bmatrix} A_t - I_{n_t} & B_t \\ C_t & 0 \end{bmatrix} = n_t + p$ and $(A_i, B_i)$ is stabilizable $(i = 1, 2, \ldots, N)$. If $A_i$ is invertible and $(C_i, A_i)$ is observable, then $(\tilde{Q}_t^{1/2}, \Phi_t)$ is observable $(i = 1, 2, \ldots, N)$.

To sum up, one of the main theorems in this paper is as follows.

**Theorem 4.** Suppose

1. Assumption 1 to Assumption 4 hold;
2. $L = (T - S)(T + S)^{-1}$ is a non-singular $M$-matrix;
3. $Q_t > 0, H_t > 0 (i = 1, 2, \ldots, N)$;
4. Let $u_t(k) = 0, r_t(k) = 0, \chi_t(k) = 0, d_t(k) = 0 (i = 1, 2, \ldots, N)$ for $k < 0$;

then the controller with preview effect, which enables the output signal of system (2) to track the reference signal asymptotically, is

$$u_t(k) = \begin{bmatrix} u_{t1}(k) & u_{t2}(k) & \cdots & u_{tn}(k) \end{bmatrix}^T$$

where
\[ u(k) = u(0) + F_c \sum_{j=1}^{k} e(j) + F_{x}(x(k) - x(0)) \]
\[ + \sum_{j=1}^{M_0} F_c(j)[r(k+j) - r(j)] \]
\[ + \sum_{j=0}^{M_0} F_d(j)[d(k+j) - d(j)] \]
\[ (i = 1, 2, \ldots, N) \]

Remark 4. (21).

When steps 1–3 of this theorem are true, all the conditions of Theorem 3 are satisfied, so the conclusion of Theorem 3 is true. The controller of system (1) can be achieved by solving \( u(k)(i = 1, 2, \ldots, N) \) from equation (14) or equation (12).

For a given \( i(i = 1, 2, \ldots, N) \)

\[ u(s) - u(s-1) = F_c e(s) + F_c(x(s) - x(s-1)) \]
\[ + \sum_{j=1}^{M_0} F_c(j)[r(s+j) - r(s-1 + j)] \]
\[ + \sum_{j=0}^{M_0} F_d(j)[d(s+j) - d(s-1 + j)] \]
\[ (s = 1, 2, \ldots, k) \]

is obtained from equation (12). In equation (23), taking \( s = 1, 2, \ldots, k \), adding the two sides and moving \( u(0) \) to the right side of equation to get equation (22), then combining \( u(k)(i = 1, 2, \ldots, N) \), the controller of the interconnected system (2) is attained, that is, equation (21).

Numerical simulation

Two examples are given to illustrate the effectiveness of the designed controller in this section.

Example 1. Consider interconnected system with two subsystems \((i.e., N = 2), n_1 = 3, n_2 = 2\) and the coefficient matrices are

\[ A_i = \begin{bmatrix} 0.5 & 2 & 1.5 \\ -1.5 & 0 & -5 \end{bmatrix}, \quad A_{i2} = \begin{bmatrix} 0.005 & 0.003 \\ -0.002 & -0.005 \end{bmatrix} \]
\[ B_i = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad E_i = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad C_i = [0.5 \ 1 \ 1] \]

\[ A_2 = \begin{bmatrix} 1 & 3 \\ 0 & -5 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} 0.001 & 0.0015 & -0.001 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad C_2 = [1 \ 1] \]

By adopting the Popov-Belevitch-Hautus (PBH) rank criterion, it is known that \((A_i, B_i)(i = 1, 2)\) is controllable and \((A_i, C_i)(i = 1, 2)\) is observable. In addition, it is easy to verify \( \text{rank} \left[ \begin{bmatrix} A_i - I & B_i \\ C_i & 0 \end{bmatrix} \right] = n_i + p(i = 1, 2) \), and \( A_i(i = 1, 2) \) is an invertible matrix.

Let the weight matrix of the performance index function of system (11) be

\[ Q_1 = \begin{bmatrix} Q_{c1} & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad Q_{c1} = 30, \quad H_1 = 2 \]
\[ Q_2 = \begin{bmatrix} Q_{c2} & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad Q_{c2} = 20, \quad H_2 = 3 \]

The solution of Riccati equation of two isolated subsystems and the feedback gain matrix of the controller are calculated using MATLAB

\[ \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} F_{x1} \\ F_{x2} \end{bmatrix} = \begin{bmatrix} 0.217624980452534 \\ 0.163512352801475 \end{bmatrix}, \quad F_{c1} = [1.387583158168247, -0.260959136238370, 4.321642830283796] \]
\[ F_{c2} = [-0.417176668167282, 3.728417778359146] \]

Besides
The eigenvalues of $L$ are 0.001984526309942 and 0.000102982681743, which means that $L$ is a non-singular M-matrix.

First, considering the non-disturbance situation, that is, $d_i(k) = 0$ ($i = 1, 2$), $r(k)$ can be chosen as

$$r(k) = \begin{cases} 
0, & k \leq 15 \\
0.05(k - 15), & 15 < k \leq 35 \\
1, & k > 35 
\end{cases}$$

The reference signal is still in the form of equation (24). At this time, the output curve of the interconnected system is depicted in Figure 2.

The tracking error of interconnected system is depicted by Figure 3. It can be clearly seen from Figures 2 and 3 that the output of interconnected system is able to track the reference signal asymptotically, even if there are disturbance signals. Moreover, the controller with preview effect can apparently decrease the tracking error and the overshoot caused by disturbance.

**Example 2.** Consider interconnected system (2), where $N = 2$, $n_1 = n_2 = 2$

$$A_1 = \begin{bmatrix} 0.1 & 1 \\ 0 & 0.2 \end{bmatrix} \quad A_{12} = \begin{bmatrix} 0.1 & 0.05 \\ 0.02 & 0.1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0.1 & 1 \\ 0 & -0.3 \end{bmatrix} \quad A_{21} = \begin{bmatrix} -1.5 & 2 \\ 0 & 0.05 & 0.1 \end{bmatrix}$$

After verification, $(A_1, B_1)(i = 1, 2)$ can be stabilized (but not controllable), $(A_i, C_i)(i = 1, 2)$ can be observed,
and the matrix $\begin{bmatrix} A_i - I & B_i \\ C_i & 0 \end{bmatrix}$ $(i = 1, 2)$ is of full row rank. Let

$$Q_{x_1} = \begin{bmatrix} 20 & 0 \\ 0 & 0 \end{bmatrix}, \quad Q_{x_1} = 30, \quad H_1 = 2$$

$$Q_{x_2} = \begin{bmatrix} 20 & 0 \\ 0 & 0 \end{bmatrix}, \quad Q_{x_2} = 30, \quad H_2 = 3$$

Similarly, the solution of Riccati equation for two isolated subsystems and the feedback gain matrix of the controller are obtained

$$P_2 = \begin{bmatrix} 36.3042685703614 & 1.3602596383232 & 12.574782360793279 \\ 1.3602596383232 & 20.438323123878305 & 4.214645954245819 \\ 12.574782360793279 & 4.214645954245819 & 42.131239677202679 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 52.575702989830198 & -2.756669712831776 \\ -2.756669712831776 & 20.496692403145200 & -4.544892821345821 \\ 24.738103635858831 & -4.544892821345821 & 44.533491801009028 \end{bmatrix}$$

$$F_{e_1} = 1.360259638323257, \quad F_{e_1} = [0.438323123878289]$$

$$F_{e_2} = 1.837779808554515, \quad F_{e_2} = [-0.331128268763466]$$

By calculation, there is

$$L = \begin{bmatrix} 0.337474647269200 & -0.237820233607235 \\ -0.129901485275353 & 0.093322541770793 \end{bmatrix}$$

$L$ is a non-singular $M$-matrix.

Reference signal and disturbance signals are selected as

Selecting $a_1 = 0.3, \quad a_2 = 0.7$, the initial value is set as $x_1(0) = [0.02]^T, \quad u_1(0) = 0, \quad x_2(0) = [0.01 0]^T, \quad u_2(0) = 0$. We carried out numerical simulations for two cases $M_r = 0, \quad M_d = 0, \quad M_d = 0$, and $M_r = 6, \quad M_d = 4, \quad M_d = 3$.

Figure 4 indicated that the $r(k)$ can be tracked by the $y(k)$ of the interconnected system without static error, and the controller with preview effect can shorten the adjustment time and restrain the disturbance signal to a certain extent.

**Conclusion**

This paper investigates the previewable controller for a type of discrete-time interconnected systems. Initially, using the basic scheme of preview control, the previewable controller is designed for the error system of each isolated subsystem. Then, the controller of the error system of the isolated subsystem is combined as the controller of error interconnected system. In resort of Lyapunov function and the properties of non-singular $M$-matrix, the stability of error interconnected system is discussed, then the criterion to ensure its stability is given. Finally, the guarantee conditions for the existence of the preview controller, then the controller for the original interconnected system are derived. The theoretical results and numerical simulation show that the designed controller is able to make the output of the system to track reference signal without static error regardless of the existence of the disturbance signal, and the tracking performance is improved with the increase of the preview steps.

**Declaration of conflicting interests**

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

**Funding**

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work was supported by the Oriented Award Foundation for Science and Technological Innovation, Inner Mongolia Autonomous Region, China (Grant no. 2012) and National Key R&D Program of China (Grant no. 2017YFF0207401).

**ORCID iD**

Fucheng Liao https://orcid.org/0000-0001-5450-1861
References

1. Sheridan TB. Three models of preview control. *IEEE Trans Hum Fact* 1966; HFE-7(2): 91–102.
2. Tomizuka M and Rosenthal DE. On the optimal digital state vector feedback controller with integral and preview actions. *J Dyn Syst Meas Control: Trans ASME* 1979; 101(2): 172–178.
3. Katayama T and Hirono T. Design of an optimal servo-mechanism with preview action and its dual problem. *Int J Control* 1987; 45(2): 407–420.
4. Katayama T, Ohki T, Inoue T, et al. Design of an optimal controller for a discrete-time system subject to previewable demand. *Int J Control* 1985; 41(3): 677–699.
5. Liao F, Cao M, Hu Z, et al. Design of an optimal preview controller for linear discrete-time causal descriptor systems. *Int J Control* 2012; 85(10): 1616–1624.
6. Liao F, Lu Y and Liu H. Cooperative optimal preview tracking control of continuous-time multi-agent systems. *Int J Control* 2016; 89(10): 2019–2028.
7. Liao F and Guo Y. Optimal preview control for discrete-time systems in multirate output sampling. *Math Probl Eng* 2016; 2016: 6924324 (10 pp.).
8. Wu J, Liao F and Tomizuka M. Optimal preview control for a linear continuous-time stochastic control system in finite-time horizon. *Int J Syst Sci* 2017; 48(1): 129–137.
9. Li P, Lam J and Cheung KC. Multi-objective control for active vehicle suspension with wheelbase preview. *J Sound Vib* 2014; 333(2): 5269–5282.
10. Yim S. Design of preview controllers for active roll stabilization. *J Mech Sci Tech* 2018; 32(4): 1805–1813.
11. Takase R, Hamada Y and Shimomura T. Aircraft gust alleviation preview control with a discrete-time LPV model. *SICE J Control Meas Syst Integr* 2018; 11(3): 190–197.
12. Zribi M, Mahmoud MS, Karkoub M, et al. $H_\infty$-controlers for linearised time-delay power systems. *IET Proc: Gener Transm Distrib* 2000; 147(6): 401–446.
13. Siljak DD. *Large-scale dynamic systems stability and structure*. New York: Dover Publications, 2007.
14. Zhou Z, Schutter BD, et al. Two-level hierarchical model-based predictive control for large-scale urban traffic networks. *IEEE Trans Control Syst Techn* 2017; 25(2): 496–508.
15. Wang M. Decomposition of equations in stability theory. *Science Record* 1960; 4(1): 1–5.
16. Lunze J. *Feedback control of large-scale systems*. London: Prentice Hall, 1992.
17. Huang R, Zhang J and Lin Z. Decentralized adaptive controller design for large-scale power systems. *Automatica* 2017; 79: 93–100.
18. Vellaboyana BR and Taylor JA. Optimal decentralized control of DC-segmented power systems. *IEEE Tran Autom Control* 2018; 63(10): 3616–3622.
19. Michel AN and Miller RK. *Qualitative analysis of large-scale dynamical systems*. New York: Academic Press, 1977.
20. Qu Q, Zhang H, Feng T, et al. Decentralized adaptive tracking control scheme for nonlinear large-scale interconnected systems via adaptive dynamic programming. *Neurocomputing* 2017; 225: 1–10.
21. Ruan X, Bien ZZ and Park KK. Iterative learning controllers for discrete-time large-scale systems to track trajectories with distinct magnitudes. *Int J Syst Sci* 2005; 36(4): 221–233.
22. Ruan X, Bien ZZ and Park KK. Decentralized iterative learning control to large-scale industrial processes for nonrepetitive trajectory tracking. *IEEE Trans Syst Man Cyber Part A Syst Hum* 2008; 38(1): 238–252.
23. Zhang H and Feng G. Stability analysis and controller design of fuzzy large-scale systems based on piecewise Lyapunov functions. *IEEE Trans Syst Man Cyber Part B Cyber* 2008; 38(3): 1390–1401.
24. Koeln J. *A decentralized control design approach to a class of large-scale systems*. MD Thesis, University of Illinois at Urbana-Champaign, Urbana, IL, 2013.
25. Liao F, Wang Y, Lu Y, et al. Optimal preview control for a class of linear continuous-time large-scale systems. *Trans Inst Mea Control* 2018; 40: 4004–4013.
26. Styan GPH. Hadamard products and multivariate statistical analysis. *Line Alg Appl* 1973; 6: 217–240.
27. Plemmons RJ. M-matrix characterizations I-nonsingular M-matrices. *Line Alg Appl* 1977; 18(12): 175–188.
28. Zhong Y. *Optimal control*. Beijing, China: Tsinghua University Press, 2015.
29. Chen W. On convergent properties for states of nonhomogeneous linear systems. *J Tianjin Univ Light Industry* 2003; 18(1): 1–3.