Quantum Gravity Resolution to the Cosmological Constant Problem

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Abstract

A finite quantum gravity theory is used to resolve the cosmological constant problem. A fundamental quantum gravity scale, $\Lambda_G \leq 10^{-3} \text{ eV}$, is introduced above which the quantum corrections to the vacuum energy density coupled to gravity are exponentially suppressed by a graviton vertex form factor, yielding an observationally acceptable value for the particle physics contribution to the cosmological constant. Classical Einstein gravity retains its causal behavior as well as the standard agreement with observational data.

1 Introduction

The cosmological constant problem is considered to be the most severe hierarchy problem in modern physics. We shall propose a quantum gravity solution to the problem, based on a nonlocal, finite quantum field theory and quantum gravity theory. We can define an effective cosmological constant

$$\lambda_{\text{eff}} = \lambda_0 + \lambda_{\text{vac}},$$

where $\lambda_0$ is the ‘bare’ cosmological constant in Einstein’s classical field equations, and $\lambda_{\text{vac}}$ is the contribution that arises from the vacuum density $\lambda_{\text{vac}} = 8\pi G \rho_{\text{vac}}$. Already at the standard model electroweak scale $\sim 10^2 \text{ GeV}$, a calculation of the vacuum density $\rho_{\text{vac}}$, based on local quantum field theory results in a discrepancy with the observational bound

$$\rho_{\text{vac}} \leq 10^{-47} (\text{GeV})^4,$$

of order $10^{55}$, resulting in a severe fine tuning problem, since the virtual quantum fluctuations giving rise to $\lambda_{\text{vac}}$ must cancel $\lambda_0$ to an unbelievable degree of accuracy. The bound on $\lambda_{\text{vac}}$ is

$$\lambda_{\text{vac}} \leq 10^{-84} \text{ GeV}^2.$$  

If we choose the quantum gravity scale $\Lambda_G \leq 10^{-3} \text{ eV}$, then our nonlocal quantum gravity theory leads to a damping of the gravitational quantum corrections to $\lambda_0$ for $q^2 \gg \Lambda_G^2$, where $q$ is the Euclidean internal loop momentum. This suppresses $\lambda_{\text{vac}}$ below the observational bound. Since the graviton tree graphs
are identical to the standard point like, local tree graphs of perturbative gravity, we retain classical, causal GR and Newtonian gravity theory, and the measured value of the gravitational constant $G$. Only the quantum gravity loop graphs are suppressed above energies $\leq 10^{-3}$ eV.

The scales $\Lambda_{SM}$ and $\Lambda_G$ are determined by the quantum non-localizable nature of the standard model (SM) particles as compared to the graviton. The SM particle radiative corrections have a nonlocal scale at $\Lambda_{SM} > 1 - 10$ TeV or a length scale $\ell_{SM} < 10^{-16}$ cm, whereas the graviton radiative corrections are localizable down to an energy scale $\Lambda_G \leq 10^{-3}$ eV or a length scale $\ell_G < 1$ cm. Thus, the fundamental energy scales in the theory are determined by the underlying physical nature of the particles and fields and do not correspond to arbitrary cut-offs, which destroy the gauge invariances of the field theory. The underlying explanation of these physical scales must be sought in a more fundamental theory.

The ‘fuzziness’ of the SM particles and the graviton, due to the nonlocal nature of the quantum field theory, gives rise to the ‘composite’ nature of the particles. An attempt to incorporate a composite graviton in a toy model field theory was made by Sundrum [23]. In this model, the ‘stringy’ graviton was coupled to a stringy halo surrounding the SM particles in the loop coupled to external gravitons.

In Section 2, we describe the local action of the theory and in Section 3, we provide a review of the basic properties of the finite quantum field theory as a perturbative scheme. In Section 4, we develop the formalism for quantum gravity, while in Section 5, we analyze the results of gluon and gravitational vacuum polarization calculations. In Section 6, we use the quantum gravity theory to resolve the cosmological constant problem and in Section 7, we end with concluding remarks.

## 2 The Action

We begin with the four-dimensional action

$$W = W_{\text{grav}} + W_{YM} + W_H + W_{\text{Dirac}} + W_M,$$

where

$$W_{\text{grav}} = -\frac{2}{\kappa^2} \int d^4x \sqrt{-g}(R + 2\lambda_0),$$

$$W_{YM} = -\frac{1}{4} \int d^4x \sqrt{-g} \text{Tr}(F^2),$$

$$W_H = -\int d^4x \sqrt{-g} \left[ \frac{1}{2} D_\mu \phi^i D^\mu \phi^i + V(\phi^2) \right],$$

$$W_{\text{Dirac}} = \frac{1}{2} \int d^4x \sqrt{-g} \bar{\psi} i \gamma^\mu \epsilon_{\mu [\partial_\mu - \omega_\mu] \psi - D(A_\mu) \psi} + h.c.$$
Here, we use the notation: \( \mu, \nu = 0, 1, 2, 3 \), \( g = \det(g_{\mu \nu}) \) and the metric signature of Minkowski spacetime is \( \eta_{\mu \nu} = \text{diag}(-1, +1, +1, +1) \). The Riemann tensor is defined such that

\[
R^\lambda_{\mu \nu \rho} = \partial_{\rho} \Gamma_{\mu \nu}^\lambda - \partial_{\nu} \Gamma_{\mu \rho}^\lambda + \Gamma_{\mu \nu}^\alpha \Gamma_{\rho \alpha}^\lambda - \Gamma_{\mu \rho}^\alpha \Gamma_{\nu \alpha}^\lambda.
\]

Moreover, h.c. denotes the Hermitian conjugate, \( \bar{\psi} = \psi^\dagger \gamma^0 \), and \( e^a_\mu \) is a vierbein, related to the metric by

\[
g_{\mu \nu} = \eta_{ab} e^a_\mu e^b_\nu, \tag{10}
\]
where \( \eta_{ab} \) is the four-dimensional Minkowski metric tensor associated with the flat tangent space with indices \( a, b, c \ldots \). Moreover, \( F^2 = F_{\mu \nu} F^{\mu \nu} \), \( R \) denotes the scalar curvature and

\[
F_{\mu \nu} = \partial_{\nu} A_{i \mu} - \partial_{\mu} A_{i \nu} - e f_{ikl} A_{k \mu} A_{l \nu}, \tag{11}
\]
where \( A_{i \mu} \) are the gauge fields of the Yang-Mills group with generators \( f_{ikl} \), \( e \) is the coupling constant and \( \kappa^2 = 32 \pi G \) with \( c = 1 \). We denote by \( D_{\mu} \) the covariant derivative operator

\[
D_{\mu} \phi^i = \partial_{\mu} \phi^i + e f_{ikl} A_{k \mu} \phi^l. \tag{12}
\]

The Higgs potential \( V(\phi^2) \) is of the form leading to spontaneous symmetry breaking

\[
V(\phi^2) = \frac{1}{4} g (\phi^i \phi^i - K^2)^2 + V_0, \tag{13}
\]
where \( V_0 \) is an adjustable constant and the coupling constant \( g > 0 \).

The spinor field is minimally coupled to the gauge potential \( A_{i \mu} \), and \( D \) is a matrix representation of the gauge group \( SO(3,1) \). The spin connection \( \omega_{\mu} \) is

\[
\omega_{\mu} = \frac{1}{2} \omega_{\mu ab} \Sigma^{ab}, \tag{14}
\]
where \( \Sigma^{ab} = \frac{1}{4} [\gamma^a, \gamma^b] \) is the spinor matrix associated with the Lorentz algebra \( SO(3,1) \). The components \( \omega_{\mu ab} \) satisfy

\[
\partial_{\mu} e^\sigma_a + \Gamma_{\mu \sigma}^\tau e^\tau_a - \omega_{\mu a}^\sigma e^\sigma_a = 0, \tag{15}
\]
where \( \Gamma_{\mu \sigma}^\tau \) is the Christoffel symbol. The field equations for the gravity-Yang-Mills-Higgs-Dirac sector are

\[
R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R - \lambda g_{\mu \nu} = - \frac{1}{4} \kappa^2 T_{\mu \nu}, \tag{16}
\]

\[
g^{\rho \mu} \nabla_{\rho} F^i_{\mu \nu} = g^{\rho \mu} \left( \partial_\rho F^i_{\mu \nu} - \Gamma_{\rho \mu}^\sigma F^i_{\sigma \nu} - \Gamma_{\rho \nu}^\sigma F^i_{\mu \sigma} + [A_\rho, F_{\mu \nu}]^i \right) = 0, \tag{17}
\]

\[
\frac{1}{\sqrt{-g}} D_\mu [\sqrt{-g} g^{\mu \nu} D_\nu \phi^i] = \left( \frac{\partial V}{\partial \phi^i} \right) \phi^i, \tag{18}
\]
\[ \gamma^a e^\mu_a [\partial_\mu - \omega_\mu - D(A_\mu)]\psi = 0. \] (19)

The energy-momentum tensor is
\[ T_{\mu\nu} = T_{\mu\nu}^{YMH} + T_{\mu\nu}^{\text{Dirac}} + T_{\mu\nu}^{M}, \] (20)

where

\[
\begin{align*}
T_{\mu\nu}^{YMH} &= F_{\mu\sigma}^i F_{\nu}^{i\sigma} + D_\mu \phi^i D_\nu \phi^i - \frac{1}{2} g_{\mu\nu} \left[\frac{1}{2} \text{Tr}(F^2) + D_\sigma \phi^i D^\sigma \phi^i + V(\phi^2) \right], \\
T_{\mu\nu}^{\text{Dirac}} &= -i \bar{\psi} \gamma_\mu [\partial_\nu - \omega_\nu - D(A_\nu)] \psi, \\
\end{align*}
\] (21) 

and \( T_{\mu\nu}^{M} \) is the energy-momentum tensor of non-field matter.

### 3 Finite Quantum Field Theory Formalism

An important development in nonlocal quantum field theory was the discovery that gauge invariance and unitarity can be restored by adding series of higher interactions. The resulting theory possesses a nonlinear, field representation dependent gauge invariance which agrees with the original local symmetry on-shell but is larger off-shell. Quantization is performed in the functional formalism using an analytic and convergent measure factor which retains invariance under the new symmetry. An explicit calculation was made of the measure factor in QED \[8\], and it was obtained to lowest order in Yang-Mills theory \[11\]. Kleppe and Woodard \[15\] obtained an ansatz based on the derived dimensionally regulated result when \( \Lambda \to \infty \), which was conjectured to lead to a general functional measure factor in nonlocal gauge theories.

In contrast to string theory, we can achieve a genuine quantum field theory, which allows vertex operators to be taken off the mass shell. The finiteness draws from the fact that factors of \( \exp[\mathcal{K}(p^2)/2\Lambda^2] \) are attached to propagators which suppress any ultraviolet divergences in Euclidean momentum space, where \( \Lambda \) is an energy scale factor. An important feature of the field theory is that only the quantum loop graphs have nonlocal properties; the classical tree graph theory retains full causal and local behavior.

We shall consider the 4-dimensional spacetime to be approximately flat Minkowski spacetime. Let us denote by \( f \) a generic local field and write the standard local Lagrangian as
\[ \mathcal{L}[f] = \mathcal{L}_F[f] + \mathcal{L}_I[f], \] (23)

where \( \mathcal{L}_F \) and \( \mathcal{L}_I \) denote the free part and the interaction part of the action, respectively, and
\[ \mathcal{L}_F[f] = \frac{1}{2} f_i \mathcal{K}_{ij} f_j. \] (24)

In a gauge theory the action would be the Becchi, Rouet, Stora, Tyutin (BRST) gauge-fixed action including ghost fields in the invariant action required to fix
The gauge\[23\]. The kinetic operator $K$ is fixed by defining a Lorentz-invariant distribution operator
\[ E \equiv \exp\left(\frac{K}{2\Lambda^2}\right) \]
and the operator:
\[ O = \frac{E^2 - 1}{K} = \int_0^1 d\tau \exp\left(\tau \frac{K}{\Lambda^2}\right). \]

The regularized interaction Lagrangian takes the form
\[ \hat{L}_I = -\sum_n (-g)^n F^n, \]
where $g$ is a coupling constant and $F$ is a vertex function form factor. The decomposition of $I$ in order $n = 2$ is such that the operator $O$ splits into two parts $F^2/K$ and $-1/K$. For Compton amplitudes the first such term cancels the contribution from the corresponding lower order channel, while the second term is just the usual local field theory result for that channel. The action is then invariant under an extended nonlocal gauge transformation. The precise results for QED were described in ref. [8].

The regularized action is found by expanding $\hat{L}_I$ in an infinite series of interaction terms. Since $F$ and $O$ are entire function of $K$ the higher interactions are also entire functions of $K$. This is important for preserving the Cutkosky rules and unitarity, for an entire function does not possess any singularities in the finite complex momentum plane.

The regulated action is gauge invariant under the transformation
\[ \delta f = ig \int d^4y d^4z T[f](x,y,z)\theta(y)f, \]
where $\theta$ is the infinitesimal gauge parameter and $T$ is a spinorial matrix for $f(x) = \psi(x)$ as well as a function of a gauge potential. An explicit construction for QED [8] was given using the Cutkosky rules as applied to the nonlocal field theory, whose propagators have poles only where $K = 0$ and whose vertices are entire functions of $K$. The regulated action satisfies these requirements which guarantees unitarity on the physical space of states. The local action is gauge fixed and then a regularization is performed on the BRST theory.

Quantization is performed using the definition
\[ \langle 0|T^*(O[f])|0\rangle_{\mathcal{E}} = \int [Df]\mu[f](\text{gauge fixing})O[f]\exp(i\hat{W}[f]). \]

where $\hat{W}$ is the regulated action. On the left-hand side we have the regulated vacuum expectation value of the $T^*$-ordered product of an arbitrary operator $O[f]$ formed from the local fields $f$. The subscript $\mathcal{E}$ signifies that a regulating Lorentz distribution has been used. Moreover, $\mu[f]$ is a gauge invariant measure factor and there is a gauge fixing factor, both of which are needed to maintain perturbative unitarity in gauge theories.
The new Feynman rules for are obtained as follows: Every leg of a diagram is connected to a local propagator,

\[ D(q^2) = \frac{i}{K(q^2) + i\varepsilon} \]  

(30)

and every vertex has a form factor \( \mathcal{F}^k(q^2) \), where \( q \) is the momentum attached to the propagator \( D(q^2) \), which has the form

\[ \mathcal{F}^k(q^2) \equiv \mathcal{E}^k(q^2) = \exp\left(\frac{K^2}{2\Lambda^2}\right), \]  

(31)

where \( k \) denotes the particle nature of the external leg in the vertex. The formalism is set up in Minkowski spacetime and loop integrals are formally defined in Euclidean space by performing a Wick rotation. This facilitates the analytic continuation; the whole formalism could from the outset be developed in Euclidean space.

Renormalization is carried out as in any other field theory. The bare parameters are calculated from the renormalized ones and \( \Lambda \), such that the limit \( \Lambda \to \infty \) is finite for all noncoincident Green’s functions, and the bare parameters are those of the local theory. The regularizing interactions are determined by the local operators.

The regulating Lorentz distribution function \( \mathcal{E} \) must be chosen to perform an explicit calculation in perturbation theory. We do not know the unique choice of \( \mathcal{E} \). However, once a choice for the function is made, then the theory and the perturbative calculations are uniquely fixed. A standard choice in early papers is \( \mathcal{E}_m \):

\[ \mathcal{E}_m = \exp\left(\frac{\partial^2 - m^2}{2\Lambda^2}\right). \]  

(32)

In the tree order, Green’s functions remain local except for external lines which are unity on-shell. It follows immediately that since on-shell tree amplitudes are unchanged by the regularization, the Lagrangian preserves all symmetries on-shell. Also all loops contain at least one regularizing vertex function and therefore are ultraviolet finite.

The on-shell tree amplitudes agree with the local, unregulated action, while the loop amplitudes disagree. This seems to contradict the Feynman tree theorem \( \mathcal{20} \), which states that loop amplitudes of local field theory can be expressed as sums of integrals of tree diagrams. If two local theories agree at the tree level, then the loop amplitudes agree as well. However, the tree theorem does not apply to nonlocal field theories. The tree theorem is proved by using the propagator relation

\[ D_F = D_R + D^+ \]  

(33)

to expand the Feynman propagator \( D_F \) into a series in the on-shell propagator \( D^+ \). This decomposes all terms with even one \( D^+ \) into trees. The term with no \( D^+ \)s is a loop formed with the retarded propagator and vanishes for local interactions. But for nonlocal interactions, this term generally survives and new
physical effects occur in loop amplitudes, which cannot be predicted from the local on-shell tree graphs.

4 Finite Perturbative Quantum Gravity

As is well-known, the problem with perturbative quantum gravity based on a point-like graviton and a local field theory formalism is that the theory is not renormalizable \[27, 28\]. Due to the Gauss-Bonnet theorem, it can be shown that the one-loop graviton calculation is renormalizable but two-loop is not \[29\]. Moreover, gravity-matter interactions are not renormalizable at any loop order.

We shall now formulate the gravitational sector in more detail. This problem has been considered previously in the context of four-dimensional GR \[7, 8, 22\]. We expand the gravity sector about flat Minkowski spacetime. In fact, our quantum gravity theory can be formulated as a perturbative theory by expanding around any fixed, classical metric background \[27\]

\[
  g_{\mu\nu} = \bar{g}_{\mu\nu} + \eta_{\mu\nu},
\]

where \(\bar{g}_{\mu\nu}\) is any smooth background metric field, e.g. a de Sitter spacetime metric. For the sake of simplicity, we shall only consider expansions about flat spacetime. Since the gravitational field is weak up to the Planck energy scale, this expansion is considered justified; even at the standard model energy scale \(E_{\text{SM}} \sim 10^2\) GeV, the curvature of spacetime is very small. However, if we wish to include the cosmological constant, then we cannot strictly speaking expand about flat spacetime. This is to be expected, because the cosmological constant produces a curved spacetime even when the energy-momentum tensor \(T_{\mu\nu} = 0\). Therefore, we should use the expansion (34) with a curved background metric. But for energy scales encountered in particle physics, the curvature is very small, so we can approximate the perturbation calculation by using the flat spacetime expansion and trust that the results are valid in general for curved spacetime backgrounds including the cosmological constant.

Let us define \(g^{\mu\nu} = \sqrt{-g}g^{\mu\nu}\). It can be shown that \(\sqrt{-\bar{g}} = \sqrt{-\bar{g}}\), where \(g = \det(g^{\mu\nu})\) and \(\partial_{\rho}g = g_{\alpha\beta}\partial_{\rho}g^{\alpha\beta}\). We can then write the local gravitational action \(W_{\text{grav}}\) in the form \[30\] :

\[
  W_{\text{grav}} = \int d^4x L_{\text{grav}} = \frac{1}{2\kappa^2} \int d^4x \left\{ \left( g^{\rho\sigma} \partial_{\rho}g_{\lambda\mu} \partial_{\sigma}g^{\lambda\mu} - \frac{1}{2} g^{\rho\sigma}g_{\lambda\mu}g_{\sigma\lambda} \right) \right. \\
  - \frac{2}{\sqrt{\bar{g}}} \bar{g}^{\rho\sigma}g_{\lambda\mu}g_{\sigma\lambda} \partial_{\rho}g^{\lambda\mu} \partial_{\rho}g^{\lambda\mu} \right. \\
  - \frac{1}{\kappa^2} \partial_{\rho}g^{\mu\nu} \partial_{\rho}g^{\kappa\lambda} \eta_{\mu\kappa} + \bar{C}^{\nu}_{\rho \mu \lambda} X^{\rho}_{\mu \lambda} C^{\lambda}, \tag{35}
\]

where we have added a gauge fixing term with the parameter \(\alpha\), \(C^{\mu}\) is the Fadeev-Popov ghost field and \(X^{\mu}_{\nu \lambda}\) is a differential operator.

We expand the local interpolating graviton field \(g^{\mu\nu}\) as

\[
  g^{\mu\nu} = \delta^{\mu\nu} + \kappa^{\mu\nu} + O(\kappa^2). \tag{36}
\]
Then,
\[ g_{\mu\nu} = \eta_{\mu\nu} - \kappa \gamma_{\mu\nu} + \kappa^2 \gamma_{\mu}^{\alpha} \gamma_{\alpha\nu} + O(\kappa^3). \]  

The gravitational Lagrangian density is expanded as
\[ \mathcal{L}_{\text{grav}} = \mathcal{L}^{(0)} + \kappa \mathcal{L}^{(1)} + \kappa^2 \mathcal{L}^{(2)} + ... . \]

We obtain
\[ \mathcal{L}^{(0)} = \frac{1}{2} \partial_\sigma \gamma_{\lambda\rho} \partial^\sigma \gamma^{\lambda\rho} - \partial_\lambda \gamma^{\mu\nu} \partial_\kappa \gamma^{\lambda}_{\nu} - \frac{1}{4} \partial_\rho \partial^\rho \gamma \]
\[ + \beta \partial_\gamma \lambda \partial_\kappa \gamma^{\lambda} + \bar{C}^{\lambda} \partial_\sigma \partial^\sigma C_\lambda, \]

\[ \mathcal{L}^{(1)} = \frac{1}{4} \left( -4 \gamma_{\lambda\rho} \partial_\mu \gamma^{\mu\nu} \partial_\nu \gamma^{\lambda\rho} + 2 \gamma_{\mu\nu} \partial_\rho \gamma^{\mu\nu} \partial_\rho \gamma \right) \]
\[ + \hat{C}^{\nu} \gamma_{\lambda\rho} \partial_\nu \partial^\rho C_\nu + \bar{C}^{\nu} \partial_\gamma \lambda \partial_\kappa \gamma^{\lambda\mu} \partial_\mu \gamma - \hat{C}^{\nu} \partial_\rho \gamma^{\mu\nu} \partial_\rho \partial_\mu \gamma \],

\[ \mathcal{L}^{(2)} = \frac{1}{4} \left( 4 \gamma_{\nu\lambda} \gamma^{\nu\lambda} \partial_\mu \gamma^{\mu\nu} \partial_\nu \gamma^{\lambda\rho} \partial_\rho \gamma^{\lambda\nu} + (2 \gamma_{\nu\lambda} \gamma_{\mu\nu} - \gamma_{\mu\nu} \gamma_{\nu\lambda}) \right) \partial_\rho \gamma^{\mu\nu} \partial_\rho \gamma^{\nu\lambda} \]
\[ - 2 \gamma_{\lambda\nu} \partial_\mu \gamma^{\nu\lambda} \partial_\mu \gamma - \gamma_{\mu\nu} \partial_\rho \gamma^{\mu\nu} \partial_\rho \gamma \gamma^{\nu\lambda} \]
\[ + \gamma \partial_\rho \gamma^{\mu\nu} \partial_\rho \gamma^{\nu\lambda} - 2 \gamma_{\nu\lambda} \gamma^{\nu\lambda} \partial_\rho \gamma^{\mu\nu} \partial_\rho \gamma^{\nu\lambda} \).

where \( \gamma = \gamma^{\alpha}_{\alpha} \).

In the limit \( \alpha \to \infty \), the Lagrangian density \( \mathcal{L}_{\text{grav}} \) is invariant under the gauge transformation
\[ \delta \gamma_{\mu\nu} = X_{\mu\nu\lambda} \xi^{\lambda}, \]

where \( X_{\mu\nu\lambda} \) is an infinitesimal vector quantity and
\[ X_{\mu\nu\lambda} = \kappa (- \partial_\mu \gamma_{\nu\lambda} + 2 \eta^{\mu\lambda} \gamma_{\nu\nu} \partial^\kappa) + (\eta_{\mu\lambda} \partial_\nu) - \eta_{\mu\nu} \partial_\lambda. \]

However, for the quantized theory it is more useful to require the BRST symmetry. We choose \( \xi^{\lambda} = C^{\lambda} \sigma \), where \( \sigma \) is a global anticommuting scalar. Then, the BRST transformation is
\[ \delta \gamma_{\mu\nu} = X_{\mu\nu\lambda} C^{\lambda} \sigma, \quad \delta \hat{C}^{\nu} = - \partial_\mu \gamma^{\mu\nu} \left( \frac{2\sigma}{\alpha} \right), \quad \delta C_\nu = \kappa C^{\mu} \partial_\mu C_\nu \sigma. \]

We now substitute the operators
\[ \gamma_{\mu\nu} \to \hat{\gamma}_{\mu\nu}, \quad C_\lambda \to \hat{C}_\lambda, \quad \hat{C}_\nu \to \hat{\hat{C}}_\nu, \]

where
\[ \hat{\gamma}_{\mu\nu} = \mathcal{E}^{-1} \gamma_{\mu\nu}, \quad \hat{C}_\lambda = \mathcal{E}^{-1} C_\lambda, \quad \hat{\hat{C}}_\nu = \mathcal{E}^{-1} \hat{C}_\nu. \]

The regularized Lagrangian density up to order \( \kappa^2 \) is invariant under the extended BRST transformations [12]:
\[ \hat{\delta}_0 \gamma_{\mu\nu} = X_{\mu\nu\lambda} C^{\lambda} \sigma = (\partial_\kappa C_\mu + \partial_\mu C_\nu - \eta_{\mu\nu} \partial_\lambda C^{\lambda}) \sigma, \]

8
\[ \delta^1 \gamma_{\mu\nu} = \kappa \mathcal{E}_0^2 X_{\mu\nu}^{(1)} C_\sigma^\lambda = \kappa \mathcal{E}_0^2 (2\gamma_{\rho\mu} \partial^\rho C_\nu - \partial^\lambda \gamma_{\mu\nu} C^\lambda - \gamma_{\mu\nu} \partial_\lambda C^\lambda) \sigma, \]  
\[ \hat{\delta}_0 \tilde{C}^\nu = 2 \partial_\mu \gamma^{\mu\nu} \sigma, \]  
\[ \hat{\delta}_1 C_\nu = \kappa \mathcal{E}_0^2 C^\mu \partial_\mu C_\nu \sigma. \]  

The order \( \kappa^2 \) transformations are

\[ \hat{\delta}_2 \gamma_{\mu\nu} = \kappa^2 \mathcal{E}_0^2 (2\partial^\rho C_\nu \tilde{D}_\rho \partial^\mu \partial^\nu \partial^\mu \partial^\nu B^{\kappa\lambda} + H^{\kappa\lambda}) \]  
\[ - C^\rho \tilde{D}_{\mu\nu\kappa\lambda} (\partial_\rho B^{\kappa\lambda} + \partial_\rho H^{\kappa\lambda}) - \partial_\rho C^\rho \tilde{D}_{\mu\nu\kappa\lambda} (B^{\kappa\lambda} + H^{\kappa\lambda}) \]  
\[ + 2\gamma_{\rho(\mu} \tilde{D}^{\text{ghost}}_{\nu)\kappa} \partial^\rho H^{\kappa\lambda} - \partial_\mu \gamma_{\mu\nu} \tilde{D}^{\text{ghost}}_{\rho\kappa} H^{\kappa\lambda} - \gamma_{\mu\nu} \tilde{D}^{\text{ghost}}_{\rho\kappa} \partial_\rho H^{\kappa\lambda} \sigma, \]  
\[ \hat{\delta}_2 C_\nu = - \kappa^2 \mathcal{E}_0^2 (\partial_\mu C_\nu \tilde{D}^{\text{ghost}}_{\rho\kappa} H^{\kappa\lambda} + C_\mu \tilde{D}^{\text{ghost}}_{\rho\kappa} \partial_\rho H^{\kappa\lambda}) \sigma. \]  

Here, we have

\[ H^{\alpha\beta} = - (\partial^{\alpha\beta} C_{\rho\sigma} \partial^\rho C^\sigma + \partial^\rho C^{(\alpha \beta)} C_{\rho\sigma} + \partial^\rho \partial^{(\alpha \beta)} C^{\sigma}) \]  
\[ H^{\rho} = \gamma_{\lambda\kappa} \partial^\lambda \partial^\rho C^\kappa + \partial^\rho \gamma_{\lambda\kappa} \partial^\lambda C^\kappa - \partial_\lambda \gamma_{\rho\sigma} \partial^\lambda C^\lambda - \partial_\lambda \gamma_{\rho\sigma} \partial^\lambda C^\lambda; \]  
\[ \tilde{H}^{\rho} = \partial^\lambda \tilde{C}^{\alpha\beta \kappa\lambda} \partial^\beta \gamma_{\rho\kappa} + \partial^\lambda \partial^\rho \tilde{C}^{\alpha\beta \kappa\lambda} + \partial^\rho \partial^{\alpha\beta \kappa\lambda}. \]  

Moreover, \( \tilde{D}_{\alpha\beta\mu\nu} \) is the “stripping” propagator for the graviton in the gauge \( \alpha = -1 \):

\[ \tilde{D}_{\alpha\beta\mu\nu}(p) = \frac{1}{2} (\eta_{\alpha\mu} \eta_{\beta\nu} + \eta_{\alpha\nu} \eta_{\beta\mu} - \eta_{\alpha\beta} \eta_{\mu\nu}) \mathcal{O}_0(p), \]  

while the ghost stripping propagator is given by

\[ \tilde{D}^{\text{ghost}}_{\mu\nu}(p) = \eta_{\mu\nu} \mathcal{O}_0(p), \]  

where

\[ \mathcal{O}_0(p) = \frac{\mathcal{E}_0^2 - 1}{p^2}. \]  

We see that the local propagator can be obtained from the nonlocal propagator minus the stripping propagator

\[ \frac{1}{p^2} = \frac{\exp(p^2/\Lambda_g^2)}{p^2} - \mathcal{O}_0(p). \]  

The stripping propagators are used to guarantee that the tree-level graviton-graviton scattering amplitudes are identical to the local, point-like tree-level amplitudes, which couple only to physical gravitons.

The graviton propagator in the fixed de Donder gauge \( \alpha = -1 \) in momentum space is given by

\[ D_{\mu\nu\rho\sigma}(p) = \frac{\eta_{\mu\rho} \eta_{\nu\sigma} + \eta_{\mu\sigma} \eta_{\nu\rho} - \eta_{\mu\nu} \eta_{\rho\sigma}}{p^2 - i\epsilon}, \]
while the graviton ghost propagator in momentum space is

\[ D_{\mu\nu}^{\text{ghost}}(p) = \frac{\eta_{\mu\nu}}{p^2 - i\epsilon}. \] (61)

As in the case of the Yang-Mills sector, the on-shell vertex functions are unaltered from their local antecedents, while virtual particles are attached to nonlocal vertex function form factors. This destroys the gauge invariance of e.g. graviton-graviton scattering and requires an iteratively defined series of “stripping” vertices to ensure the decoupling of all unphysical modes. Moreover, the local gauge transformations have to be extended to nonlinear, nonlocal gauge transformations to guarantee the over-all invariance of the regularized amplitudes. Cornish has derived the primary graviton vertices and the BRST symmetry relations for the regularized \( \hat{W}_{\text{grav}} \) \([12, 13]\), using the nonlinear, nonlocal extended gauge transformations suitable for the perturbative gravity equations.

Because we have extended the gauge symmetry to nonlinear, nonlocal transformations, we must also supplement the quantization procedure with an invariant measure

\[ \mathcal{M} = \Delta(g, \bar{C}, C) D[g_{\mu\nu}] D[C_\lambda] D[C_\sigma] \] (62)

such that \( \delta \mathcal{M} = 0 \).

As we have demonstrated, the quantum gravity perturbation theory is invariant under generalized, nonlinear field representation dependent transformations, and it is finite to all orders. At the tree graph level all unphysical polarization states are decoupled and nonlocal effects will only occur in graviton and graviton-matter loop graphs. Because the gravitational tree graphs are purely local there is a well-defined classical GR limit. The finite quantum gravity theory is well-defined in four real spacetime dimensions or in any higher D-dimensional spacetime.

We quantize by means of the path integral operation

\[ \langle 0 | T^{\ast} (O[g]) | 0 \rangle \mathcal{E} = \int [Dg] \mu[g] (\text{gauge fixing}) O[g] \exp(i\hat{W}_{\text{grav}}[g]). \] (63)

The quantization is carried out in the functional formalism by finding a measure factor \( \mu[g] \) to make \( [Dg] \) invariant under the classical symmetry. To ensure a correct gauge fixing scheme, we write \( \hat{W}_{\text{grav}}[g] \) in the BRST invariant form with ghost fields; the ghost structure arises from exponentiating the Faddeev-Popov determinant \([31]\). The algebra of extended gauge symmetries is not expected to close off-shell, so one needs to introduce higher ghost terms (beyond the normal ones) into both the action and the BRST transformation. The BRST action will be regularized directly to ensure that all the corrections to the measure factor are included.
5 Standard Model and Gravitational Vacuum Polarization

A basic feature of our regularized field theory is that the vertex function form factors $\mathcal{F}(q^2)$ in momentum space are determined by the nature of the vertex function. For a SM gauge boson, such as the $W$ or $Z$ boson connected to a standard model particle and anti-particle, the vertex function form factor in Euclidean momentum space is

$$\mathcal{F}^{\text{SM}}(q^2) = \exp\left(-\frac{q^2}{2\Lambda_{\text{SM}}^2}\right),$$  \hspace{1cm} (64)

while for a vertex with a graviton attached to a standard model particle and an anti-particle, the vertex function form factor will be

$$\mathcal{F}^{G}(q^2) = \exp\left(-\frac{q^2}{2\Lambda_{G}^2}\right).$$  \hspace{1cm} (65)

Thus, when two vertices are drawn together to make a loop graph, the energy scale dependence $\Lambda$ will be determined by the external legs attached to the loop. If we ignore the weak effects of gravity in SM calculations, then the graviton scale $\Lambda_{G}$ can be ignored, as is usually the case in SM calculations.

A calculation of the one-loop gluon vacuum polarization gives the tensor in D-dimensions \[11]\:

$$\Pi^{\mu
u}_{ik}(p) = \frac{e^2}{2^{D-n-2}} f_{ilm} f_{klm} (p^2 \eta^{\mu\nu} - p^\mu p^\nu) \Pi(p^2),$$  \hspace{1cm} (66)

where $p$ is the gluon momentum and

$$\Pi(p^2) = 2 \int_0^{1/2} dy \Gamma(2 - D/2, y p^2 / \Lambda_{\text{SM}}^2) [y(1 - y)p^2]^{D/2-2}$$

$$\times [2(D - 2)y(1 - y) - \frac{1}{2}(D - 6)].$$  \hspace{1cm} (67)

We observe that $\Pi_{ik,\mu}^{\mu}(0) = 0$ a result that is required by gauge invariance and the fact that the gluon has zero mass.

The dimensionally regulated gluon vacuum polarization result is obtained by the replacement

$$\Gamma(2 - D/2, y p^2 / \Lambda_{\text{SM}}^2) \rightarrow \Gamma(2 - D/2)$$  \hspace{1cm} (68)

and choosing $p^2 \ll \Lambda_{\text{SM}}^2$. In four-dimensions we get

$$\Pi(p^2) = 2 \int_0^{1/2} dy E_i(y p^2 / \Lambda_{\text{SM}}^2)[4y(1 - y) + 1],$$  \hspace{1cm} (69)

where we have used the relation

$$\Gamma(0, z) \equiv E_i(z) = \int_z^{\infty} dt \exp(-t)t^{-1}. \hspace{1cm} (70)$$
The lowest order contributions to the graviton self-energy will include the standard graviton loops, the ghost field loop contribution and the measure loop contribution. In the regularized perturbative gravity theory the first order vacuum polarization tensor $\Pi_{\mu\nu\rho\sigma}$ must satisfy the Slavnov-Ward identities [33]:

$$p_\mu p_\rho D^{G\mu\nu\alpha\beta}(p)\Pi_{\alpha\beta\gamma\delta}(p)D^{G\gamma\delta\rho\sigma}(p) = 0. \quad (71)$$

By symmetry and Lorentz invariance, the vacuum polarization tensor must have the form

$$\Pi_{\alpha\beta\gamma\delta}(p) = \Pi_1(p^2)\eta_{\alpha\beta}\eta_{\gamma\delta} + \Pi_2(p^2)\eta_{\alpha\gamma}\eta_{\beta\delta} + \Pi_3(p^2)\eta_{\alpha\delta}\eta_{\beta\gamma} + \Pi_4(p^2)\eta_{\alpha\beta}\eta_{\delta\gamma} + \Pi_5(p^2)\eta_{\alpha\beta}\eta_{\gamma\delta}. \quad (72)$$

The Slavnov-Ward identities impose the restrictions

$$\Pi_2 + \Pi_4 = 0, \quad 4(\Pi_1 + \Pi_2 - \Pi_3) + \Pi_5 = 0. \quad (73)$$

The basic lowest order graviton self-energy diagram is determined by [34, 35, 36, 37, 38]:

$$\Pi_{\mu\nu\rho\sigma}(p) = \frac{1}{2} \kappa^2 \int d^4q U_{\mu\nu\rho\sigma\delta\tau}(p, -q, q - p, q) F^{G\mu\nu}(q^2) D^{G\alpha\beta\delta\tau}(q) \times D^{G\gamma\delta\rho\sigma}(q - p, q - p, q - p, q), \quad (74)$$

where $U$ is the three-graviton vertex function

$$U_{\mu\nu\rho\sigma\delta\tau}(q_1, q_2, q_3) = -\frac{1}{2} \left[ q_2(\mu q_3\nu) \left( 2\eta_{(\delta\eta_{\rho\tau})\sigma} - \frac{2}{D-2} \eta_{\mu\nu} \eta_{\delta\tau} \right) + q_1(\rho q_3\sigma) \left( 2\eta_{(\delta\eta_{\rho\tau})\nu} - \frac{2}{D-2} \eta_{\mu\nu} \eta_{\delta\tau} \right) + \cdots \right], \quad (75)$$

and the ellipsis denotes similar contributions.

To this diagram, we must add the ghost particle diagram contribution $\Pi^2$ and the measure diagram contribution $\Pi^3$. The dominant finite contribution to the graviton self-energy will be of the form

$$\Pi_{\mu\nu\rho\sigma}(p) \sim \kappa^2 \Lambda_2^4 Q_{\mu\nu\rho\sigma}(p^2) \sim \frac{\Lambda_2^4}{M_{PL}^4} Q_{\mu\nu\rho\sigma}(p^2), \quad (76)$$

where $M_{PL}$ is the reduced Planck mass and $Q(p^2)$ is a finite remaining part.

For renormalizable field theories such as quantum electrodynamics and Yang-Mills theory, we will find that the loop contributions are controlled by the incomplete $\Gamma$-function. If we adopt an ‘effective’ quantum gravity theory expansion in the energy [39], then we would expect to obtain

$$\Pi_{\mu\nu\rho\sigma}(p) \sim \kappa^2 G(2 - D/2, p^2/\Lambda_2^2) Q_{\mu\nu\rho\sigma}(p^2). \quad (77)$$
where \( G \) denotes the functional dependence on the incomplete \( \Gamma \)-function. By making the replacement

\[
G(\Gamma(2 - D/2), p^2/\Lambda^2) \rightarrow G(\Gamma(2 - D/2)),
\]

we would then obtain the second order graviton loop calculations using dimensional regularization [34, 35, 36, 37, 38, 27]. The dominant behavior will now be \( \ln(\Lambda^2/q^2) \) and not \( \Lambda^4 \). However, in a nonrenormalizable theory such as quantum gravity, the dimensional regularization technique may not provide a correct result for the dominant behavior of the loop integral and we expect the result to be of order \( \Lambda^4 \). Indeed, it is well known that dimensional regularization for massless particles removes all contributions from tadpole graphs and \( \delta^4(0) \) contact terms. On the other hand, our regularized field theory takes into account all leading order contributions and provides a complete account of all counterterms. Because all the scattering amplitudes are finite, then renormalizability is no longer an issue.

The function

\[
Q^{\mu\nu\sigma\rho}(p^2) \sim p^4
\]

as \( p^2 \rightarrow 0 \). Therefore, \( \Pi^{\mu\nu\sigma\rho}(p) \) vanishes at \( p^2 = 0 \), as it should from gauge invariance and for massless gravitons. We now find that

\[
\Pi^G(p^2) \sim \frac{\Lambda_G^4}{M_{PL}^2}.
\]

Thus, the pure graviton self-energy is proportional to \( \Lambda_G^4 \). We shall choose \( \Lambda_G \ll 10^{-3} \) eV, so that the pure gravitational quantum corrections to the bare cosmological constant \( \lambda_0 \) are cut off for energies above \( \sim 10^{-3} \) eV.

In contrast to recent models of branes and strings in which the higher-dimensional compactification scale is lowered to the TeV range [24], we retain the classical tree graph GR gravitation picture and its Newtonian limit. It is perhaps a radical notion to entertain that quantum gravity becomes weaker as the energy scale increases towards the Planck scale \( \sim 10^{19} \) Gev, but there is, of course, no known experimental reason why this should not be the case in nature. However, we do not expect that our weak gravity field expansion is valid at the Planck scale when \( G\xi^2 \sim 1 \), although the damping of the quantum gravity loop graphs could still persist at the Planck scale. This question remains unresolved until a nonperturbative solution to quantum gravity is found.

### 6 Resolution to the Cosmological Constant Problem

Zeldovich [39] showed that the zero-point vacuum fluctuations must have a Lorentz invariant form

\[
T_{\mu\nu} = \lambda_{\text{vac}} g_{\mu\nu},
\]

(81)
consistent with the equation of state \( \rho_{\text{vac}} = -p_{\text{vac}} \). Thus, the vacuum within the framework of particle quantum physics has properties identical to the cosmological constant. In quantum theory, the second quantization of a classical field of mass \( m \), treated as an ensemble of oscillators each with a frequency \( \omega(k) \), leads to a zero-point energy \( E_0 = \sum \frac{1}{2} \hbar \omega(k) \). The experimental confirmation of a zero-point vacuum fluctuation was demonstrated by the Casimir effect [40].

A simple evaluation of the vacuum density obtained from a summation of the zero-point energy modes gives

\[
\rho_{\text{vac}} = \frac{1}{(2\pi)^2} \int_0^{M_c} dk k^2 (k^2 + m^2)^{1/2} \sim \frac{M_c^4}{16\pi^2},
\]

(82)

where \( M_c \) is the cutoff. Already at the level of the standard model, we get \( \rho_{\text{vac}} \sim (10^2 \text{ GeV})^4 \) which is 55 orders of magnitude larger than the bound (2). To agree with the experimental bound (2), we would have to invoke a very finely tuned cancellation of \( \lambda_{\text{vac}} \) with the ‘bare’ cosmological constant \( \lambda_0 \), which is generally conceded to be theoretically unacceptable.

We shall consider initially the basic lowest order vacuum fluctuation diagram computed from the matrix element in flat Minkowski spacetime

\[
M_{(2)}^{(0)} \sim e^2 \int d^4 p d^4 p' d^4 k \delta(k + p - p') \delta(k + p - p')
\]

\[
\times \frac{1}{k^2 + m^2} \text{Tr} \left( \frac{i\gamma^\sigma p_\sigma - m_f}{p^2 + m_f^2} \gamma^\mu \frac{i\gamma^\sigma p'_\sigma - m_f}{p'^2 + m_f^2} \gamma^\mu \right)
\]

\[
\exp \left[ -\frac{p^2 + m_f^2}{2\Lambda_{SM}^2} - \frac{p'^2 + m_f^2}{2\Lambda_{SM}^2} - \frac{k^2 + m_f^2}{2\Lambda_{SM}^2} \right],
\]

(83)

where \( e \) is a coupling constant associated with the standard model. We have considered a closed loop made of a SM fermion of mass \( m_f \), an antifermion of the same mass and an internal SM boson propagator of mass \( m \); the scale \( \Lambda_{SM} \sim 10^2 - 10^3 \text{ GeV} \). This leads to the result

\[
M_{(2)}^{(0)} \sim 16\pi^2 g_y^4 \delta^4(a) \int_0^\infty dp p^3 \int_0^\infty dp' p'^3 \left[ \frac{-P^2 + p^2 + p'^2 + 4m_f^2}{(P + a)(P - a)} \right]
\]

\[
\times \frac{1}{(p^2 + m_f^2)(p'^2 + m_f^2)} \exp \left[ -\frac{(p^2 + p'^2 + 2m_f^2)}{2\Lambda_{SM}^2} - \frac{P^2 + m_f^2}{2\Lambda_{SM}^2} \right],
\]

(84)

where \( P = p - p' \) and \( a \) is an infinitesimal constant which formally regularizes the infinite volume factor \( \delta^4(0) \). We see that \( \rho_{\text{vac}} \sim M_{(2)}^{(0)} \sim \Lambda_{SM}^4 \). To maintain gauge invariance and unitarity, we must add to this result the contributions from the ghost diagram and the measure diagram.

In flat Minkowski spacetime, the sum of all disconnected vacuum diagrams \( C = \sum_n M_n^{(0)} \) is a constant factor in the scattering S-matrix \( S' = SC \). Since the S-matrix is unitary \( |S'|^2 = 1 \), then we must conclude that \( |C|^2 = 1 \), and all the
disconnected vacuum graphs can be ignored. This result is also known to follow
from the Wick ordering of the field operators. However, due to the equivalence
principle gravity couples to all forms of energy, including the vacuum energy
density $\rho_{\text{vac}}$, so we can no longer ignore these virtual quantum fluctuations in
the presence of a non-zero gravitational field.

We can view the cosmological constant as a non-derivative coupling of the
form $\lambda_0 \sqrt{-g}$ in the Einstein-Hilbert action $\mathcal{L}$. This classical tree-graph
coupling has the effect of de-stabilizing Minkowski spacetime. Quantum corrections
to $\lambda_0$ come from loops formed from massive SM states, coupled to external gravita-
tion lines at essentially zero momentum. The massive SM states are far off-shell.
Experimental tests of the standard model involving gravitational couplings to
the SM states are very close to being on-shell. Important quantum corrections
to $\lambda_0$ are generated by a huge extrapolation to a region in which gravitons couple
to SM particles which are far off-shell.

Let us now consider the dominant contributions to the vacuum density aris-
ing from the graviton loop corrections. As explained above, we shall perform
the calculations by expanding about flat spacetime and trust that the results
still hold for an expansion about a curved metric background field, which is
strictly required for a non-zero cosmological constant. Since the scales involved
in the final answer, including the predicted smallness of the cosmological con-
stant, correspond to a very small curvature of spacetime, we expect that our
approximation is justified.

We shall adopt a simple model consisting of a massive vector meson $V_\mu$, which has the standard model mass $m_V \sim 10^2$ GeV. We have for the vector
field Lagrangian density

$$\mathcal{L}_V = -\frac{1}{4} (-g)^{-1/2} g^{\mu\nu} g^{\alpha\beta} F_{\mu\alpha} F_{\nu\beta} + m_V^2 V_\mu V^\mu, \quad (85)$$

where

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu. \quad (86)$$

We include in the Lagrangian density an additional piece $-\frac{1}{2} (\partial_\mu V^\mu)^2$, and the vector field propagator has the form

$$D_{\mu\nu}^V = \frac{\eta_{\mu\nu}}{p^2 + m_V^2 - i\epsilon}. \quad (87)$$

The graviton-V-V vertex in momentum space is given by

$$V_{\alpha\beta\lambda\sigma}(q_1, q_2) = \eta_{\lambda\sigma} q_{1\alpha} q_{2\beta} - \eta_{\sigma\beta} q_{1\alpha} q_{2\lambda} - \eta_{\lambda\beta} q_{1\alpha} q_{2\sigma} + \eta_{\alpha\beta} (\eta_{\lambda\sigma} q_{1\alpha} q_{2\lambda} - q_{1\sigma} q_{2\lambda}), \quad (88)$$

where $q_1, q_2$ denote the momenta of the two $V$s connected to the graviton with
momentum $p$. We use the notation $A_{(\alpha B_\beta)} = \frac{1}{2} (A_\alpha B_\beta + A_\beta B_\alpha)$.

The lowest order correction to the graviton vacuum loop will have the form

$$\Pi_{\mu\nu;\alpha\beta}^{GV}(p) = -\kappa^2 \int d^4 q V_{\mu\nu;\alpha\beta}(p, -q, q - p) F^G(q^2) D^V_{\lambda\delta}(-q)$$

$$-\frac{1}{2} (\partial_\mu V^\mu)^2, \quad (85)$$

$$D_{\mu\nu}^V = \frac{\eta_{\mu\nu}}{p^2 + m_V^2 - i\epsilon}. \quad (87)$$

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where $q_1, q_2$ denote the momenta of the two $V$s connected to the graviton with
momentum $p$. We use the notation $A_{(\alpha B_\beta)} = \frac{1}{2} (A_\alpha B_\beta + A_\beta B_\alpha)$.

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$$\Pi_{\mu\nu;\alpha\beta}^{GV}(p) = -\kappa^2 \int d^4 q V_{\mu\nu;\alpha\beta}(p, -q, q - p) F^G(q^2) D^V_{\lambda\delta}(-q)$$
\[ \langle -p, p - q, q \rangle \mathcal{F}^G((q - p)^2)D^V \alpha \kappa (q - p). \]  

We obtain

\[
\Pi^{GV}_{\mu \nu \rho \sigma}(p) = -\kappa^2 \int \frac{d^4q}{(q^2 + m_V^2)(q - p)^2 + m_V^2} K_{\mu \nu \rho \sigma}(p, q)
\times \exp \left[ -(q^2 + m_V^2)/2\Lambda_G^2 \right] \exp \left[ -(m^2_V + m^2_V)/2\Lambda_G^2 \right],
\]

(90)

where in D-dimensions

\[
K_{\mu \nu \rho \sigma}(p, q) = p_\alpha p_\beta p_\rho p_\sigma + q_\alpha p_\beta p_\rho p_\sigma - q_\alpha q_\beta p_\rho p_\sigma + (1 - D)q_\alpha q_\beta q_\rho p_\sigma
\]

\[ - (1 + D)p_\alpha q_\beta q_\rho q_\sigma + (D - 1)p_\alpha q_\beta q_\rho q_\sigma + Dq_\alpha q_\beta q_\rho q_\sigma. \]

(91)

As usual, we must add to the contributions from the fictitious ghost particle diagrams and the invariant measure diagram.

We observe that from power counting of the momenta in the integral (90), we obtain

\[
\Pi^{GV}_{\mu \nu \rho \sigma}(p) \sim \kappa^2 \Lambda_G^4 N_{\mu \nu \rho \sigma}(p^2)
\]

\[
\sim \frac{\Lambda_G^4}{M_{PL}^2} N_{\mu \nu \rho \sigma}(p^2),
\]

(92)

where \( N(p^2) \) is a finite remaining part of \( \Pi^{GV}(p) \). We have as \( p^2 \to 0 \):

\[
N_{\mu \nu \rho \sigma}(p^2) \sim p^4.
\]

(93)

Thus, \( \Pi^{GV}_{\mu \nu \rho \sigma}(p) \) vanishes at \( p^2 = 0 \), as it should because of gauge invariance and the massless graviton.

We now have

\[
\Pi^{GV}(p^2) \sim \frac{\Lambda_G^4}{M_{PL}^2}.
\]

(94)

If we choose \( \Lambda_G \leq 10^{-3} \) eV, then the quantum correction to the bare cosmological constant \( \lambda_0 \) is suppressed sufficiently to satisfy the bound (3), and it is protected from large unstable radiative corrections. This provides a solution to the cosmological constant problem at the energy level of the standard model and possible higher energy extensions of the standard model. The universal fixed gravitational scale \( \Lambda_G \) corresponds to the fundamental length \( \ell_G \leq 1 \) cm at which virtual gravitational radiative corrections are cut off.

The vector field vertex form factor, when coupled to SM gauge bosons, will have the form

\[
\mathcal{F}^{SM}(q^2) = \exp \left[ -(q^2 + m_V^2)/2\Lambda_{SM}^2 \right].
\]

(95)

If we choose \( \Lambda_{SM} > 1 - 10 \) TeV, then we will reproduce the SM experimental results, including the running of the SM coupling constants, and \( \mathcal{F}^{SM}(p^2) \) becomes \( \mathcal{F}^{SM}(0) = 1 \) on the mass shell \( q^2 = -m_V^2 \).
We observe that the required suppression of the vacuum diagram loop contribution to the cosmological constant, associated with the vacuum energy momentum tensor at lowest order, demands a low gravitational energy scale $\Lambda_G \lesssim 10^{-3} \text{eV}$, which controls the quantum gravity loop contributions. This is essentially because the external graviton momenta are close to the mass shell, requiring a low energy scale $\Lambda_G$. This seems at first sight a radical suggestion that quantum gravity corrections are weak at energies higher than $\lesssim 10^{-3} \text{eV}$, but this is clearly not in contradiction with any known gravitational experiment. Indeed, as has been stressed in recent work on large higher dimensions, there is no experimental knowledge of gravitational forces below 1 mm. In fact, we have no experimental knowledge at present about the strength of graviton radiative corrections. The SM experimental agreement is achieved for SM particle states close to the mass shell. However, we expect that the dominant contributions to the vacuum density arise from SM states far off the mass shell. In our perturbative quantum gravity theory, the tree graphs involving gravitons are identical to the tree graphs in local point graviton perturbation theory, retaining classical, causal GR and Newtonian gravity. In particular, we do not decrease the strength of the classical gravity force.

In order to solve the severe cosmological constant hierarchy problem, we have been led to the surprising conclusion that, in contrast to the conventional folklore, quantum gravity corrections to the classical GR theory are negligible at energies above $\lesssim 10^{-3} \text{eV}$, a result that will continue to persist if our perturbative calculations can be extrapolated to near the Planck energy scale $\sim 10^{19} \text{GeV}$. Since the cosmological constant problem already results in a severe crisis at the energies of the standard model, our quantum gravity resolution based on perturbation theory can resolve the crisis at the standard model energy scale and well beyond this energy scale.

7 Conclusions

The ultraviolet finiteness of perturbative quantum field theory in four-dimensions is achieved by applying the nonlocal field theory formalism. The nonlocal quantum loop interactions reflect the quantum, non-point-like nature of the field theory. Thus, as with string theories, the point-like nature of particles is ‘fuzzy’ for energies greater than the scale $\Lambda$. One of the features of superstrings is that they provide a mathematically consistent theory of quantum gravity, which is ultraviolet finite and unitary. Our nonlocal theory focuses on the basic mechanism behind string theory’s finite ultraviolet behavior by invoking a suppression of bad vertex behavior at high energies, without compromising perturbative unitarity and gauge invariance. It provides a mathematically consistent theory of quantum gravity at the perturbative level. If we choose $\Lambda_G \lesssim 10^{-3} \text{eV}$, then quantum radiative corrections to the classical tree graph gravity theory are perturbatively negligible to all energies greater than $\Lambda_G$, provided that the perturbative regime is valid.

The important gauge hierarchy problem, associated with the Higgs sector,
can also be resolved in our nonlocal field theory. It can be shown that an
exponential damping of the Higgs self-energy in the Euclidean $p^2$ domain occurs
for $p^2 \gg \Lambda_H^2$, and for a $\Lambda_H$ scale in the electroweak range $\sim 10^2 - 10^3 \text{ GeV}$.

A damping of the vacuum polarization loop contributions to the vacuum
energy density-gravity coupling at lowest order can resolve the cosmological
constant hierarchy problem, if the gravity loop scale $\Lambda_G \leq 10^{-3} \text{ eV}$, by sup-
pressing virtual gravitational radiative corrections above the energy scale $\Lambda_G$.
We expect that the SM energy scale $\Lambda_{\text{SM}}$ to be much larger than the electroweak
scale $\sim 10^2 - 10^3 \text{ GeV}$, and it could be as large as grand unification theory (GUT)
$\Lambda_{\text{SM}} \sim 10^{16} \text{ GeV}$, allowing for possible GUT unification schemes.

Recently, new supernovae data have strongly indicated a cosmic acceleration
of the present universe [41]. This has brought the status of the cosmological con-
stant back into prominence, since one possible explanation for this acceleration
of the expansion of the universe is that the cosmological constant is non-zero but
very small. We can, of course, accommodate a small non-zero cosmological con-
stant by choosing carefully the gravity scale $\Lambda_G$. Indeed, this new observational
data can be viewed as a means of determining the size of $\Lambda_G$.

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References

[1] S. Weinberg, Rev. Mod. Phys. 61, 1 (1989).
[2] V. Sahni and A. Starobinski, astro-ph/9904396.
[3] E. Witten, hep-ph/0002297.
[4] S. M. Carroll, hep-th/0004075.
[5] N. Straumann, astro-ph/0009386.
[6] S. E. Rugh and H. Zinkernagel, hep-th/0012253.
[7] J. W. Moffat, Phys. Rev. D 41, 1177 (1990).
[8] D. Evens, J. W. Moffat, G. Kleppe and R. P. Woodard, Phys. Rev. D 43, 49 (1991).
[9] J. W. Moffat and S. M. Robbins, Mod. Phys. Lett. A 6, 1581 (1991).
[10] G. Kleppe and R. P. Woodard, Phys. Lett. B 253, 331 (1991).
[11] G. Kleppe and R. P. Woodard, Nucl. Phys. B388, 81 (1992).
[12] N. J. Cornish, Mod. Phys. Lett. 7, 631 (1992).
[13] N. J. Cornish, Mod. Phys. Lett. 7, 1895 (1992).
[14] B. Hand, Phys. Lett. B275, 419 (1992).
[15] G. Kleppe and R. P. Woodard, Ann. of Phys. 221, 106 (1993).
[16] M. A. Clayton, L. Demopolous and J. W. Moffat, Int. J. Mod. Phys. A9, 4549 (1994).
[17] J. Paris, Nucl. Phys. B450, 357 (1995).
[18] J. Paris and W. Troost, Nucl. Phys. B482, 373 (1996).
[19] G. Saini and S. D. Joglekar, Z. Phys. C76, 343 (1997).
[20] S. D. Joglekar, hep-th/0003104, hep-th/0003077.
[21] A. Basu and S. D. Joglekar, hep-th/0004128.
[22] J. W. Moffat, hep-th/9808091. Talk given at the XI International Conference on Problems in Quantum Field Theory, Dubna, Russia, July, 1998. Proceedings published by World Scientific, Singapore, 1999; J. W. Moffat, Talk given at the IV Workshop on Quantum Chromodynamics, June, 1998, eds. H. M. Fried and B. Müller. Proceedings published by World Scientific, Singapore, 1999.
[23] R. Sundrum, JHEP 9907, 001 (1999), hep-ph/9708329 v2.
[24] E. Witten, Nucl. Phys. B471, 135 (1996); J. D. Lykken, Phys. Rev. D54, 3693 (1996); I. Antoniadis, Phys. Lett. B246, 377 (1990); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B429, 263 (1998); N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Rev. D59, 105002 (1999); K. Dienes, E. Dudas, and T. Gherghetta, Nucl. Phys. B537, 47 (1999); L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999), hep-th/9905221.
[25] C. Becchi, A. Rouet, and R. Stora, Comm. Math. Phys. 42, 127 (1975); I. V. Tyutin, Lebedev Institute preprint N39 (1975).
[26] R. P. Feynman, Acta Phys. Pol. 24, 697 (1963); Magic Without Magic, edited by J. Klauder (Freeman, New York, 1972), p. 355; Feynman Lectures on Gravitation, edited by B. Hatfield, (Addison-Wesley publishing Co. 1995.)
[27] G. ’t Hooft and M. Veltman, Ann. Inst. Henri Poincaré, 30, 69 (1974).
[28] S. Deser and P. van Nieuwenhuizen, Phys. Rev. Phys. Rev. 10, 401 (1974); Phys. Rev. 10, 411 (1974).
[29] M. Goroff and A. Sagnotti, Nucl. Phys. B266, 709 (1986).
[30] J. N. Goldberg, Phys. Rev. 111, 315 (1958).
[31] E. S. Fradkin and I. V. Tyutin, Phys. Rev. D2, 2841 (1970).
[32] T. de Donder, La Grafique Einsteinienne (Gauthier-Villars, Paris, 1921); V. A. Fock, Theory of Space, Time and Gravitation (Pergamon, New York, 1959).
[33] D. M. Capper and M. R. Medrano, Phys. Rev. 9, 1641 (1974).
[34] D. M. Capper, G. Leibbrandt, and M. R. Medrano, Phys. Rev. 8, 4320 (1973).
[35] M. R. Brown, Nucl. Phys. B56, 194 (1973).
[36] J. F. Donoghue, Phys. Rev. D50, 3874 (1994).
[37] D. M. Capper, M. J. Duff, and L. Halpern, Phys. Rev. 10, 461 (1974).
[38] M. J. Duff, Phys. Rev. 9, 1837 (1974).
[39] Ya. B. Zeldovich, Pis’ma Zh. Eksp. Teor. Fiz. 6, 883 [JETP Lett. 6, 316 (1967)].
[40] H. B. G. Casimir, Proc. K. Ned. Akad. Wet. 51, 635 (1948).
[41] S. Perlmutter et al., Nature 391, 51 (1998); Ap. J. 517, 565 (1999); A. Riess, et al., Astron. Journ. 117, 207 (1998); B. Schmidt et al., Ap. J. 507, 46 (1998); P. Garnavich et al., Ap. J. 509, 74 (1998).