Influence of Hole-Pressure Error on Pasty Propellant Rheological Test Results

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Abstract. Pasty propellants are viscoelastic fluids that exhibit a normal stress difference in shear flow. This phenomenon causes the pressure data measured by the pressure tap to be inconsistent with the expected value, which is represented by the hole-pressure error. In order to find out its influence on the rheological measurement results, this paper simulate the tube flow with the pressure tap based on PTT model. Results show that the pressure value in the circular section decreases with the increase of radial distance, and there is a weak secondary flow and lateral flow at the entrance of the pressure tap. By analyzing the pressure drop data of tube flow, the power law model is used to fit the desired pressure drop and the measured pressure drop respectively. It indicates that the hole-pressure error results in a small fitting value. However, the discrepancy between the power law model and the material has a certain positive effect. It can reduce the influence of the hole-pressure error on the forward calculation results by selecting the appropriate working range.

1. Introduction

Compared with the solid propellant engine, the pasty propellant engine is easy to achieve thrust adjustment and multiple work, and has a wide application space [1]. The existing pasty propellant engines mostly adopt the feeding mode of the squeeze flow in the tube [2, 3]. Therefore, when the pasty engine works, it is necessary to pressurize the pasty propellant in the tank to force it to flow into the combustion chamber through the tube.

![Figure 1. Schematic diagram of the pressure tap.](image)
In tube flow, it is necessary to measurement the pressure inside the tube to study the flow characteristics of the pasty propellant [4-6]. With the pressure tap, we can achieve the measurement of pressure. In the laminar flow state, for Newtonian fluid, the point pressure \( P_g \) and the desired pressure \( P \) are nearly equal; but for a viscoelastic fluid such as pasty propellant, \( P_g < P \), and the difference between the two is called hole-pressure error, that is, \( \Delta = P - P_g \). As early as 1968, lodge and his collaborators proposed the existence of the hole-pressure error. This is caused by the presence of first normal stress difference in the shear flow of a viscoelastic fluid [7-9].

![Figure 2. Normal stress difference.](image)

The power law model [5] is one of the most commonly used constitutive equations for pasty propellants. Within a certain range, this model has a good degree of conformity with this material, and is convenient to use. According to the theory of viscosity test [10], tube flow is often used for viscosity testing at higher shear rates. Due to the existence of the hole-pressure error, the measured pressure data has obvious deviation, resulting in a significant deviation of the power law model fitting results. It is feasible to analysis this deviation using a differential viscoelastic constitutive model-PTT model. This model can predict the first normal stress difference phenomenon [2]. This paper will introduce the PTT mode parameters for the tube flow simulation, and use the power law model to fit the flow rates and pressure data. The difference between the expected pressure drop and the test pressure drop is compared and the influence of the hole-pressure error is analyzed.

2. Model

2.1. Constitutive Model

This paper deals with two commonly used non-Newtonian fluid constitutive models, namely the power law model and the PTT model. Both are incompressible fluid models, and this paper only studies isothermal flow conditions.

1) The expression of the isothermal flow power law model is [10]

\[
\tau = K \dot{\gamma}^n
\]

and

\[
\eta = K \dot{\gamma}^{n-1}
\]

The power law model has a good degree of conformity to the material within a certain range, but the constant viscosity under low shear and high shear cannot be predicted.

2) The expression of the isothermal flow PTT model is [2-12]:

\[
\tau = \eta \frac{\partial \dot{\gamma}}{\partial t} + K \dot{\gamma}^n
\]
where

\[ \tau = \tau_1 + \tau_2 \]

\[ \tau_2 = 2\eta_2 D \]

\[ \eta_1 = \eta - \eta_2 \]

\[ \frac{\Delta}{\tau_1} = \frac{D\tau_1}{Dt} + \tau_1 \cdot \nabla v^T + \nabla v \cdot \tau_1 \]  

(4)

\[ \frac{v}{\tau_1} = \frac{D\tau_1}{Dt} - \tau_1 \cdot \nabla v - \nabla v^T \cdot \tau_1 \]  

(5)

\( \varepsilon \) and \( \xi \) are material parameters for controlling shear viscosity and elongational behavior, respectively; \( \lambda \) is the material relaxation time; \( D \) is the rate-of-deformation tensor; \( \tau \) is the total extra-stress tensor, and is decomposed into a viscoelastic component \( \tau_1 \) and a purely viscous component \( \tau_2 \).

The PTT model can predict the shear thinning and the first normal stress difference phenomenon, and can predict constant viscosity at low shear rates. Often used in viscoelastic fluid research.

2.2. Theory of Rheological Measurement

The tube flow is a simple shear deformation and is one of the commonly used viscosity measure flows [7]. This section describes the method for testing the rheological parameters of a non-Newtonian power law model for tube flow. The perfect tube viscosity measure flow should meet the following conditions:

1) The flow in the tube is laminar;
2) The flow of each section perpendicular to the axis in the tube is consistent, and the velocity distribution function has a unique argument radius \( r \);
3) No wall slip.

Applying the power law model to the perfect tube flow equation, we can get the following relationship:

\[ Q = \frac{n\pi}{3n+1} \left( \frac{\Delta P}{2KL} \right)^{\frac{l}{n}} R^{\frac{3n+1}{n}} \]  

(6)

\( K \) and \( n \) is the power law model material parameters, \( R \) is the radius of the tube, \( r \) is the radial distance, \( L \) is the length of the tube, \( \Delta P \) is the pressure drop, and \( Q \) is the volume flow rates.

In the actual experiment, the shear rates and shear stress at the wall position are used for the fitting of the power law model:

\[ \dot{\gamma}_w = \frac{3n+1}{n\pi R^3} Q \]  

(7)

\[ \tau_w = \frac{R\Delta P}{2L} \]  

(8)

It can be seen from the above equation that the flow condition is only related to \( n \), but it is unknown before fitting, we can do as:
\[ \dot{\gamma}_f = \frac{4Q}{\pi R^4} \]  

(9)

This formula derive from Newtonian fluid. Fit \( Q \) and \( \Delta P \) to get \( K \) and \( n \), \( n \) is the exact value, \( \bar{K} \) and \( K \) have the following relationship [11]:

\[ K = \left( \frac{4n}{3n+1} \right)^n \bar{K} \]  

(10)

If we can get these data, \( Q \), \( \Delta P \), radius \( R \) and length \( L \), the power law model material parameters can be fitted according to Equation (1), (2), (8), (9) and (10).

2.3. Geometric Model  

The main pipe and the pressure tap of the tube flow form the fluid domain as shown in Fig 3.a, and B4 is the pressure measure point. The following expressions are the values of the cross section. The following test values are the values of this section, and the position of the pressure measuring point is this section.

![Figure 3. Geometric model and grid.](image)

The grid is shown in Figure 3.b using ICEM, grid is divided into hexahedral structured type. Use O-block to improve mesh quality in wall areas and increase mesh intensity in wall and intersecting areas.

Boundary conditions B1 is inflow, B5 is outflow, B2, B3, and B4 are zero wall velocity (\( v_n = v_s = 0 \)), and B1 and B5 are completely developed flow;

SolverPolyflow, its built-in isothermal PTT material model;

Solving parameters relax = 0.06, \( xi = 0.15 \), eps = 0.0152.

The steady state solution is a non-linear problem and does not converge. Therefore, using the evolution solution, the result is still steady state. Set the inlet and outlet flows to evolution parameters in the simulation. The evolution function is \( F(S) = S \), where \( S \) is evolution variable. \( Q \) is calculated in order from small to large, and the calculation result of the previous working condition can be used as the initial value of the next working condition. We can improve the calculation efficiency by setting reasonable \( Q \) and matching the appropriate \( S \).

3. Results

3.1. Flow results  

The flow inside the tube is laminar. According to the streamline diagram in Figure 4, the Streamline protrudes toward the pressure tap, and the shear rate changes slightly along the radial direction, where the main pipe is close to the pressure tap. At the same time, a weak secondary flow is formed at the pressure tap. A similar phenomenon is confirmed in the literature of P. Townsend [8], which differing from the literature is a gap flow. Due to the geometric asymmetry, there is also a lateral flow from the center to the sides in the pressure tap, as shown in Figure 4.c.
Figure 4. Streamline diagram.

In figurea is Streamline diagram, b is section streamline diagram, c is local Streamline diagram and d is local section streamline diagram.

3.2. Pressure distribution
As shown in Figure 4 of each section pressure cloud, the pressure changes sharply near the pressure tap. The minimum pressure appears in the secondary flow area at the inlet of the pressure tap. The internal flow of the hole is weak and the pressure distribution is evenly distributed.

Figure 5. Section pressure cloud.

A pressure extreme occurs near the line connecting the pressure tap to the main pipe. As shown in Fig. 5, in the middle section, there is a minimum value near the upstream intersection point which is much smaller than the pressure of the circular section of the main pipe at that point. There is a maximum value near the downstream intersection point is much larger than the pressure of the circular section of the main pipe at that point. It is precisely because of the secondary flow that the pressure in the vicinity of the intersecting line changes drastically.
Figure 6. Pressure curve at the pressure tap.

In figure the pressure curve at the axis of the pressure tap on the middle section to the wall of the main tube, and the pressure curve at the near wall of the upper and lower pressure taps on the middle section (the direction is the same as the axis of the pressure tap); X=-0.002 ~0.002 is along the diameter of the main tube, and x>0.002 is located in the pressure gap.

The flow and pressure drop data extracted from the simulation results are shown in Table 1.

Table 1. Pressure drop data.

| Volume flow rates Q \( (\times 10^{-4} \text{m}^3/\text{s}) \) | Expect pressure drop \( \Delta P^* \) \( (\text{MPa}) \) | Test pressure drop \( \Delta P_0 \) \( (\text{MPa}) \) | Measurement error \( \Delta \) \( (\text{MPa}) \) | Relative error \( (\%) \) |
|---|---|---|---|---|
| 0.201082 | 1.517625 | 1.455215 | 0.06241 | 4.112347 |
| 0.251327 | 1.59644 | 1.52727 | 0.06917 | 4.332766 |
| 0.301611 | 1.656942 | 1.585642 | 0.0713 | 4.303107 |
| 0.502484 | 1.868965 | 1.799375 | 0.06959 | 3.72345 |
| 0.754024 | 2.124346 | 2.046096 | 0.07825 | 3.683487 |
| 1.006114 | 2.38738 | 2.29497 | 0.09241 | 3.87077 |
| 1.256763 | 2.61815 | 2.50536 | 0.11279 | 4.308004 |

Note The average pressure difference between the two ends of the tube is called the expected pressure drop, and the difference between the pressure measured point and the average pressure of the outlet is called the test pressure drop, the same below.

3.3. Data Processing
The power law model is fitted to the data to obtain shear stress and viscosity curves [11].
Figure 7. Shear stress-shear rate curve and viscosity-shear rate curve.

In figure result 1 is fitted by $\Delta P^*$ and result 2 is fitted by $\Delta P_0$.

The shear stress and viscosity bound by the test pressure drop are small due to the small pressure drop caused by the hole-pressure error.

Figure 8. Pressure drop curve and relative deviation curve.

In figure a $\Delta P^*$ is the desired pressure drop, $\Delta P_0$ is the test pressure drop; $\Delta P_1$ is the pressure drop obtained by the forward solution of the fitting result 1; and $\Delta P_2$ is the pressure drop obtained by the forward solution of the fitting result 2.

In figure b $\delta_0$ is the deviation of $\Delta P_0$ from $\Delta P^*$, that is, the test deviation; $\delta_1$ is the deviation of $\Delta P_1$ from $\Delta P^*$, that is, the Fitting deviation; $\delta_2$ is the deviation of $\Delta P_2$ from $\Delta P^*$, that is, the Comprehensive deviation.

According to the fitting result, the pressure drop is calculated in the forward direction, as shown in Figure 8. The difference between $\Delta P_0$ and $\Delta P^*$, $\Delta P_2$ and $\Delta P_1$, reflects the degree of inconsistency between the power law model and the material rheological properties.

It can be clearly seen from the deviation curve that the hole-pressure error $\Delta$ leads to the smaller test pressure drop. Using the power-law model material parameters fitted by the test pressure drop data, a small pressure drop is obtained in the forward tube flow calculation. Under the given material parameters, the Q in 0.2-1.25:

1) The degree of non-conformity between the power law model and the material is expressed as the value of the two ends of the a curve is too small, the value of the middle is too large, and there is a maximum value when Q is about 0.55, and the absolute value is the largest when Q is maximum;

2) The effect of the pressure error is that $\delta_2$ is smaller than $\delta_1$;

3) The combined effect of the two is that when Q is between 0.3 and 0.9, the absolute value of $\delta_2$ is less than the absolute value of $\delta_1$, that is, the error of $\Delta P_2$ is less than $\Delta P_1$. 

...
4. Conclusion
Due to the influence of the hole-pressure error, there are deviations in the power law model parameters obtained by the measurement method of the tube flow, as the result the shear stress and viscosity are small. The pressing hole error is consistent with the change trend of the first normal stress difference, and the larger the flow rate (ie. the larger the shear rate), the larger the pressing hole error.

Due to the existence of the hole-pressure error and the deviation between the power law model and the material, the forward pressure drop calculation result in the research range, the error at both ends is larger, and the middle portion is smaller. When the actual tube flow test fits the power law model, the test range can be appropriately widened, and the middle segment data of the fitting result can be used to reduce the forward calculation error.

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