Optical forces and torques exerted on coupled silica nanospheres: novel contributions due to multiple scattering

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Abstract
When illuminated by light, optically coupled nanoparticles suffer the action of multiple electromagnetic forces. In general, two kinds of forces are assumed: binding forces that make the particles attract/repel each other and scattering forces that push the system forwards. Tangential forces and orbital torques can also be induced to align the interacting particles with the electric field. In this work, new degrees of freedom were found for two coupled silica nanospheres under illumination with linearly-polarized plane waves. The results have a general validity for arbitrary mesoscale systems: multiple scattering of light induces unusual torques and deviation forces. These torques include spin contributions to the movement of the whole system. The results are supported by previous works and pave the way for the engineering of nanoscale devices and nanorotators. Any application based on photonics at mesoscales should take into account the new movements predicted here.

Keywords: silica nanospheres, dimers, optical forces, spin torques, Mie resonances, multiple scattering, spin–orbit coupling

(Some figures may appear in colour only in the online journal)

1. Introduction

It has been known for more than a century that light carries linear and angular momentum [1, 2] in addition to energy. When light is scattered or absorbed by a particle, the transfer of momenta can cause the particle to move and/or rotate. Thus light can be used to manipulate particles, molecules or mesoscale objects [3–6]. Direct manipulation of objects through light-induced forces has led to formidable progress, impacting research in many areas ranging from ultra-cold matter physics [7], biology [8, 9], microfluidics [10, 11], optical printing [12], to optical engineering [13–17] among other fields. For example, demonstration of levitation and trapping of micron-sized particles by radiation pressure dates back to 1970 [18]. Since the 1990s, even new states of matter have been conceived with manipulation by optical forces [19]. In particular, one important consequence of electromagnetic two-particle interaction is the optical binding first noted by Burns et al [20–22]. Two particles excited by a common field can be bound in stable or pseudo-stable positions [21–24].

Equally important to get such manipulatable forces is to make proper theoretical/predictive models. In particular, the optical forces exerted on a small, single scatterer are typically assumed as the contributions due to its dipole moment [25]. In this regime, two-force components are usually found, namely the gradient force and the scattering force [26]. Moreover, the latter component is usually associated with the radiation pressure exerted by light on the system [27]. These components can be easily distinguished when the light is a single plane wave. However, non-trivial contributions can exist for ‘structured’ light carrying spin densities [28]: the more complex the electromagnetic field around the system, the more complex to model the exerted force and torque components [29–31].
For two coupled particles, there are no analytical formulations of the force far beyond dipole–dipole coupling, even for simple illumination like plane waves [32–34]. It is commonly assumed that the system can undergo binding forces in addition to scattering forces, both kinds generated by field gradients in response to the incident field [35]. The former forces are contributions that make the particles attract/repel each other and the latter forces, or the radiation pressure, push the system forwards with respect to the direction of the incident wave. Many works have studied the optical forces exerted on two interacting particles under the influence of light fields [32, 36–39]. In particular, Haefner et al [40] have discovered the presence of spin torques in pairs of equal nanospheres when these pairs are illuminated symmetrically with plane waves having linear or circular polarization. In the present work, however, new degrees of freedom have been found for the movement of electromagnetically coupled systems that have general validity irrespective of material and shape properties. A complete set of induced forces under linearly-polarized incident waves is reported here. In addition to the usual exerted forces and torques predicted in [40], new spin contributions to the movement of the system were found. These induced, non-trivial torques are a product of the multiple interactions occurring in the system. There is no need to use complex helical fields to obtain angular momentum transfer [30, 41]. The spin motors or ‘nanorotors’ have been traditionally discussed based on optical traps created with circularly polarized light or vortex beams [42–44].

A well-known method of discrete dipole moments, namely the discrete dipole approximation (DDA), is used to perform the calculations [45–49]. To give a simple explanation of the phenomena, the results are compared with similar results obtained by simplified systems consisting of a few dipole moments. This procedure deduces how the new torques appear naturally as a result of asymmetrical internal fields induced on the system in reaction to the incident wave.

For the sake of simplicity, this study focuses on the response of two interacting silica nanospheres under three configurations of illumination variation. The Mie resonances dominate the optical response by silica spheres, which in dielectric bodies are also known as morphology-dependent resonances (MDRs) [50, 51]. In contrast to plasmonic nanoparticles, which exhibit surface resonances, dielectric nanoparticles exhibit volume resonances [52].

Similar results have been reported recently for two-dimensional (2D) systems of bound cylinders calculated with an integral formulation [52–55]. As the present work uses a different method of solving Maxwell equations and different dimensions, the results of this paper complete those previous studies with a generalization of the phenomena that is independent of the materials, dimensions, and geometries involved.

This paper also shows that in general the retardation effects cannot be neglected in a scattering problem. Even when a few multipolar terms are not taken into account, the dynamics of the system under time-harmonic fields can be seriously affected. In this way, the results play a key role in the correct design of optical nanoscale devices and nanorotors. Although an exact electromagnetic method is used here, neither thermal nor Brownian forces are considered [56]. Besides, no ‘dynamic’ forces are calculated, i.e. forces that take into account initial velocities and accelerations of the particles [57]. Then, no complete dynamics is obtained for the system. Yet, the new dynamical features presented here are essential for the functionality and efficiency of small optical devices and they should be considered for future applications involving photonic forces.

2. Remarks on force and torque calculations

The DDA is a well-known method to solve electromagnetic scattering problems. It solves these problems by dividing the scatterer’s domain into small subvolumes which respond to the electromagnetic field by means of induced dipole moments. A complete version of the DDA for non-magnetic particles can be found in [45, 49]. Let us summarize the methodology required for the force and torque calculations involving isotropic materials that respond to a time-harmonic field: DDA states a set of coupled dipole equations for the exciting fields at each subvolume

$$E_{\text{exc},n} = E_{0,n} + k_0^2 \sum_{m=n}^{N} \hat{\alpha}_{nm} \alpha_m E_{\text{exc},m},$$

where $\alpha_m$ is the polarizability of the $m$-volume element, as

$$\alpha_m = \frac{\alpha_{0,m}}{1 - ik_0^2 \alpha_{0,m}/(6\pi)}, \quad \alpha_{0,m} = 3V_m \left( \frac{\epsilon_m - 1}{\epsilon_m + 2} \right).$$

and $\alpha_{0,m}$ is the quasistatic polarizability. The formulation automatically includes the so-called radiative corrections [58–60], which are related to the imaginary part of the Green tensor, so it is fully consistent with the optical theorem. In this work, a plane-wave illumination, $E_{0,0}(\mathbf{r}) = E_0 e^{ik_0 r}$ is assumed.

From the solution in equations (1), one can obtain the dipole moments and the total internal fields as follows

$$p_n = \epsilon_0 \alpha_n E_{\text{exc},n},$$

$$E_n = \frac{1}{\epsilon_0 (\epsilon_n - 1)} p_n.$$ 

To calculate the net time-average optical force exerted on a body or a system, one must use the proper dipole moments that represent each scatterer region in the frame of the DDA. We consider here that each body $b$ is composed of $N_b$ dipole moments and then the subindex $n$ runs only over all the dipoles of the chosen body, i.e. $n = 1, 2, ..., N_b$. Note, on the other hand, that the equation (1) use $N$ or the number of dipole moments in the whole system. This number accounts for the multiple interactions between the dipole moments. In general, $N = N_b$ for several coupled bodies. In this way, the components of the net force exerted on the body $b$ in question, are represented by

$$F_{i,b} = \sum_{n=1}^{N_b} F_{n,i,b}.$$
The $i$-component of this force, $F_{n,i}$, can be obtained from the time-averaged force on a particle within the Rayleigh approximation [26]. This is

$$F_{n,i} = \frac{1}{2} \text{Re} \left\{ p_n^t \left[ \partial E^t (r, \omega) \right]_{r=r_b} \right\}.$$ (6)

The derivatives of the total field $\partial E(r, \omega)_{r=r_e}$ at the dipoles’ position $r_b$ of body $b$ can be obtained from the following equations [61]:

$$\partial E(r, \omega)_{r=r_e} = \partial E_0(r, \omega)_{r=r_e} + \frac{k_0^2}{\epsilon_0} \sum_{m=1}^{N} \left( \partial \mathbf{G}(r, r_m)_{r=r_e} \mathbf{p}_m \right).$$ (7)

The optical torques can also be calculated from the DDA method, as given in [62]:

$$\mathbf{N}_b = \sum_{n=1}^{N_b} \mathbf{r}_n \times \mathbf{F}_n + \frac{1}{2 \epsilon_0} \sum_{n=1}^{N_b} \text{Re} \left\{ \mathbf{p}_n \times \left( \mathbf{p}_n / \alpha_0 \right)^\ast \right\}.$$ (8)

The first term in equation (8) involves the so-called extrinsic torque and the second term is called the intrinsic torque. Their significance has been discussed in [30, 31, 62], among others, but the main difference between them lies in the spatial dependence on the first term. However, it is important to mention that $\mathbf{N}_b$ can represent both orbital or spin torque (do not confuse the torque $\mathbf{N}_b$, the number of dipole moments of the body, $N_b$). The character of spin or orbital of the torque in equation (8) will depend on the choice of the reference system to define the positions $\mathbf{r}_n$ of the dipole moments that compose the examined body. Then, spin torques are defined when the associated positions are taken with respect to the center of each body. Otherwise, the reference system is assumed to be located at the CM of the whole system and therefore orbital torques are set. The forces involved in the calculation of the extrinsic torque are the forces exerted on each dipole $n$ composing particle $b$.

### 3. Results

The general configuration of this study can be seen in figure 1. A silica two-sphere system is positioned with its axis parallel to the $y$-axis of a rectangular coordinate system. The gap is $d$ and the radii of the spheres are $R = 240$ nm. The incident wave approaches with a wavevector $\mathbf{k}_0$, which is determined by the angles $\theta$, $\phi$ given by the spherical coordinates. The incident wave is fully determined by the incident wavelength $\lambda = 2\pi/k_0$ and a third angle $\zeta$, which is the polarization angle (see also inset scheme in figure 1). Silica nanospheres are simulated by a refractive index $n = 1.59 + \iota10^{-7}$. Unless otherwise stated, it is assumed that $N_1 = N_2 = N/2$ in equations (5) and (8).

The optical response by two spheres is explored below in terms of the mechanical magnitudes. The results are scaled by proper factors, but they represent forces of an order of piconewtons when the intensity of the illumination reaches a few $\text{mW} / \text{um}^2$. In the same way, the obtained torques reach an order of $\text{pN} \text{nm}$ when the same order of illumination intensity is used.

#### 3.1 Spectral variation

In the study of the spectral forces (figure 2), the incident wave is assumed to have a direction given by $\mathbf{k}_0 = k_0 \mathbf{\hat{z}}$ and a polarization $E_0 = E_0 \mathbf{\hat{y}}$. A gap of $d = 500$ nm is used. The whole system suffers the action of binding forces ($\Delta$, figure 2(a)) due to the electromagnetic coupling between the spheres and the incident field. Simultaneously, the system is also pushed by radiation pressure ($F_{\mu\nu}$, figure 2(b)) along with the forward direction. The binding force is defined as
\[ \Delta = F_1 - F_2, \] the difference between the force components along the axis of the pair, while the scattering force is defined as \( F_{sc} = F_1 + F_2, \) the total force corresponding to the force exerted on the center of mass (CM) of the system. The forces are scaled to the magnitude \( 3V_n k_0 u_E, u_E = \frac{1}{2} e_0 |E|^2 \) being the electric energy density of the incident field and \( V_n \) being the subvolume of the discretization used in the corresponding calculation.

The excitations of the MDRs can be seen in the spectra in figure 2. Note how the binding force alternates its sign by means of these excitations (figure 2(a)). A repulsive (\( \Delta > 0 \)) or attractive character (\( \Delta < 0 \)) is obtained depending on which resonance is excited: the curve oscillates around the zero-value of force (blue line). The values of the radiation pressure are relatively much larger than those obtained for binding forces. In addition, some resonance shifts can be found between both kinds of spectra [52, 54]. Thus, as seen in previous works, the optical forces of the system can ‘feel’ the electromagnetic modes and even serve as near-field observables of the scatterer system.

For the configuration of illumination in figure 2, neither spin nor orbital torque was expected in the literature [40]. Logically, there are no orbital torques because we are dealing with a symmetric method of two equal particles [52]. However, when the results by DDA are introduced, optical spin torques can be found with respect to the x-axis of the system (black solid line and red line with circles in figure 3). These induced torques cancel each other out because of the symmetry and give zero torque for the whole system.

This kind of spin has been recently found in 2D systems by means of an integral formulation [52–54]. Note that the phenomenon is exclusively obtained with bodies represented by multiple dipole moments; the induced spins for bodies represented by single dipole moments are zero (dashed line in figure 3). The new dynamics can be obtained only when realistic interactions between the scatterers are simulated, i.e. the complete multiple scattering must be considered.

### 3.2. The origin of the spin torques

This subsection shows how the distortions induced in the internal fields by the multiple scattering cause the spins. For this purpose, five maps of the internal fields were calculated for an arbitrary value of wavelength, i.e. \( \lambda = 600 \text{ nm} \) (figure 4). The first three maps (figures 4(a)–(c)) correspond to different plane cuts for a single sphere of \( R = 240 \text{ nm} \) located at the center of the coordinate system in figure 1. The calculations were performed with 2553 dipole moments.

For the single sphere, the multiple scattering between all the dipoles originates an inhomogeneous internal field but this is always symmetric with respect to the incident wave field (figures 4(b)–(d)). The resultant dipoles are, of course, oriented as the electric field or polarization vector (not shown) but they generate symmetric forces which together give the typical scattering component exerted on the sphere, i.e. the
radiation pressure. Note that several overlapped MDRs can be excited at this particular value of $\lambda$.

The maps of figures 4(d)–(e) show the distributions of the internal fields calculated for the configuration in figures 2–3 at the chosen wavelength. Figures 4(d) and (e) correspond to the cut $x = 0$ of the field distribution $y, z$ inside the left (right) sphere, 1(2). These two maps were calculated with 2378 dipole moments. Note that the fields inside the spheres present almost the same distribution as the map $y - z$ shown for the single sphere (figure 4(c)). However, the distribution changes in this case due to the interaction between the spheres. Although this is a symmetric configuration with respect to the incidence, the multiple scattering between the spheres results in bent patterns for the internal fields. Consequently, these asymmetric patterns generate the net tangential forces that produce the spin torques.

Let us now define a two-particle system with a minimal amount of dipole moments in order to obtain asymmetrical inner fields. Then, the simplest configuration to obtain spins is a system represented by three dipole moments under symmetric illumination (figure 5(a)). In this case, one sphere is represented by a single dipole moment, $N_1 = 1$, and the other one is represented by two dipole moments, $N_2 = 2$, placed in symmetric positions with respect to the polarization direction and the center of the sphere. In this way, note that the spin can only be induced on the sphere composed of two dipole moments (red arrows in figure 5(a)). The spin for the sphere 1 in figure 5(a) is identically zero. The resultant inhomogeneity of the internal field inside sphere 2 makes the whole particle rotate because the induced tangential forces for each dipole’s subvolume are different (see the elemental forces $F_{21}, F_{22}$ in the scheme; internal field not shown for this configuration). In this example, the symmetry of the whole system is preserved from the point of view of the net forces. The induced force $F_1$ on sphere 1 is perfectly compensated with the induced force $F_2 = F_{21} + F_{22}$ on sphere 2 along the polarization direction, $\hat{y}$. As a result, a binding force (i.e. relative attraction/repulsion) between the particles can exist but the whole system moves solely forward due to the radiation pressure. In other words, there is no transversal force along the $y$-direction and this simple example is fully representative of the studied effects.

To explore the new degrees of freedom that can be induced in sphere pairs, other illumination configurations were studied. In the following sections, 3.3 and 3.4, the results correspond to angular variations in the illumination at a fixed value of the wavelength, namely $\lambda = 600$ nm. The results can be approximately reproduced in an experiment having any laser wavelength near this value such as a He–Ne laser, $\lambda = 632.8$ nm.

3.3. Azimuthal variation

The variations in the induced forces with the azimuthal angle can be seen in figure 6. The panels show the binding forces (figure 6(a)), the module $F_{pr}$, $|F_{pr}| = \sqrt{F_{pr}^2 + F_{pr,CM}^2}$, and the deviation $\delta = \phi(F_{pr}) - \phi$ of the scattering force (figure 6(b)), and the deviation $\delta = \phi(F_{pr}) - \phi$ of the direction of the scattering force with respect to the direction of the incident wave (figure 6(c)). Here the angle of the scattering force is defined as $\phi(F_{pr}) = \arctan(F_{y,CM}/F_{x,CM})$ and the components of the CM are defined as $F_{ji,CM} = F_{j1} + F_{j2}$ where $j = x, y$. The solid lines are calculated with $N = 2378$ and they are compared against the same results but simulating the particles by single dipole moments (dashed line, case $N = 2$). The other angles of the illumination are set at $\theta = 90^\circ$ and $\zeta = 0^\circ$.

All the curves in figure 6 are periodic with the variation of $\phi$; they differ from sinusoidal forms because of the multiple scattering at $\lambda = 600$ nm. The binding force presents a small value of attraction at $\phi = 0^\circ$, $180^\circ$ (solid line) and it changes as $\phi$ increases, oscillating from attraction to repulsion.
these angles of illumination. By contrast, the values of the incident field modulation of the MDRs excited. As a result, the whole dynamical torques exerted on the CM of the system. The exerted torques under this configuration has been studied previously in [40]. However, some of the induced torques were not predicted; this work completes the information about the transient movement of the system under this configuration. The other illumination angles were set at the values \( \theta = 180^\circ \) and \( \phi = 0^\circ \). The behavior of the induced forces is first analyzed in figure 8 for the cases of spheres made with \( N_1 = 1189 \) (solid lines with or without circles) and \( N_1 = 1 \) (dashed lines with/without circles). The black lines without circles (red lines with circles) in figure 8(a) correspond to the net tangential forces induced on the left (right) sphere, i.e. sphere 1 (2). As pointed out in [40], the tangential forces (figure 8(a)) turn out to be unusual and they already predict the presence of non-trivial orbital torques.

The resultant binding forces are plotted in figure 8(b) and the module of the radiation pressure is plotted in figure 8(c). In this configuration, the direction corresponding to the radiation pressure is always directed parallel to the versor \( \hat{z} \), but its module varies smoothly with the polarization angle. Observe that all the curves present a clear sinusoidal form due to the symmetry of the system with equal spheres, the study of the variation in the angle \( \theta \) is ignorable and its configuration can be described by the rest of the analysis carried out so far. In this subsection, the polarization angle \( \zeta \) was changed. A fixed wavelength \( \lambda = 600 \) nm and a gap of \( d = 2R = 480 \) nm were chosen for this example. In particular, this illumination configuration has been studied previously in [40]. However, some of the induced torques were not predicted; this work completes the information about the transient movement of the system under this configuration. The other illumination angles were set at the values \( \theta = 180^\circ \) and \( \phi = 0^\circ \).

The wide variation in deviation is an exclusive phenomenon induced by the multipolar structure of the scattering of the system. In case \( N = 2 \) there is also an angular deviation (dashed line in figure 6(c)) because, in general, the response of a two-dipole system is quadrupolar [63]. If the illumination is not aligned with any principal direction for this quadrupole, the scattering force becomes deviated. However, the effect of deviation is vanishing in this case; the maxima do not exceed 1.6%.

Now more degrees of freedom are explored to study the dynamics of the system. The exerted torques under this configuration are presented in figure 7. The results of both cases \( N = 2378 \) (solid lines with/without symbols) and \( N = 2 \) (dashed lines) were added. Even in case \( N = 2 \), the symmetry breaking induced by the illumination direction with respect to the system axis allows the presence of orbital torques in the structure (figure 7(a)). In agreement with the broken symmetry, in general the spin torques exerted on each particle are different (figure 7(b)). The only non-zero components correspond to torques aligned with the \( z \)-axis. In order to approach the movement for the whole system, the green lines with squares were added to the figures. They represent the resultant torques for the CM of the system, as given by \( N(CM) = N_1 + N_2 \); for both the orbital (figure 7(a)) and the spin components (figure 7(b)). In general, the predicted movement is not trivial. Both particles can follow similar spin phases in a wide range of values of \( \phi \), so they will spin in the same sense of rotation in those angular values.

Again, the spin torques result in zero for the case of two dipole moments (dashed line in figure 7(b), this curve represents \( N_1 = N_2 \) for clarity in the figure). All the torques for the system vanish when the illumination direction is given by the values \( \phi = 0^\circ, 90^\circ, 180^\circ, 270^\circ \).

Given the complex movement that the curves in figures 6 and 7 imply, a rough approach to the movement of the system could be obtained by taking only curves \( N = 2 \), as many works do throughout the literature [12, 64–66]. A discussion about the use of the dipole–dipole approximation can be found in the appendix.

### 3.4. Variation of polarization

Due to the symmetry of the system with equal spheres, the study of the variation in the angle \( \theta \) is ignorable and its configuration can be described by the rest of the analysis carried out so far. In this subsection, the polarization angle \( \zeta \) was changed. A fixed wavelength \( \lambda = 600 \) nm and a gap of \( d = 2R = 480 \) nm were chosen for this example. In particular, this illumination configuration has been studied previously in [40]. However, some of the induced torques were not predicted; this work completes the information about the transient movement of the system under this configuration. The other illumination angles were set at the values \( \theta = 180^\circ \) and \( \phi = 0^\circ \).

The behavior of the induced forces is first analyzed in figure 8 for the cases of spheres made with \( N_1 = 1189 \) (solid lines with or without circles) and \( N_1 = 1 \) (dashed lines with/without circles). The black lines without circles (red lines with circles) in figure 8(a) correspond to the net tangential forces induced on the left (right) sphere, i.e. sphere 1 (2). As pointed out in [40], the tangential forces (figure 8(a)) turn out to be unusual and they already predict the presence of non-trivial orbital torques.

The resultant binding forces are plotted in figure 8(b) and the module of the radiation pressure is plotted in figure 8(c). In this configuration, the direction corresponding to the radiation pressure is always directed parallel to the versor \( \hat{z} \), but its module varies smoothly with the polarization angle. Observe that all the curves present a clear sinusoidal form due
to the phase variation in the incident field with respect to the symmetry of the system.

The presence of the tangential forces is not a phenomenon exclusively occurring with many dipole moments (solid lines with or without symbols). The tangential forces are also induced for each sphere in case $N = 2$ (dashed lines with or without symbols). This means that there will be orbital torques also for $N = 2$ when the angle $\zeta$ is varied, such as when $\phi$ is varied. Remarkably, the module of the tangential forces is reduced for case $N = 2378$ and these forces change their signs with respect to case $N = 2$ (figure 8(a)). The variations in the binding force and the radiation pressure with the angle $\zeta$ are also reduced with the presence of several dipole moments (figures 8(b) and (c)). Such behaviors constitute another illustration of the errors one can make when trying to describe the dynamics of the system with only a few dipole moments (see the appendix for remarks about the dipole–dipole approximations).

Figure 8(b) shows that the repulsion between the spheres is very attenuated with the response of the multiple dipoles; the negative values are not as pronounced for $N = 2378$ as in case $N = 2$. In addition, the curves for the tangential forces in figure 8(a) differ in phase in $90^\circ$ with respect to the curves of the forces in figures 8(b) and (c). In other words, symmetric inductions such as the radiation pressure and the binding force prevail for symmetric electric fields induced in $\zeta = 90^\circ$, $270^\circ$. By contrast, asymmetric inductions such as the tangential forces cancel out for these values of $\zeta$.

Finally, unusual torques found for this illumination configuration are discussed. Figure 9 follows the same color and symbol code as figure 8; however, the torques $N_{x1}$, $N_{y2}$ in case $N = 2$ are equal so they are only shown with a single line type, i.e. dashed line. As cases $N = 2378$ and $N = 2$ are in opposite phases in figure 8(a), the orbital torques in figure 9(a) show a similar property. The presence of many dipole moments in the interaction between the spheres also reduces the absolute values of the orbital torques. New spin components are shown in figures 9(b)–(d). Figures 9(b)–(d) correspond to spin torques around the $x$, $y$, and $z$ axes respectively. Again, there are no spin components for two interacting dipole moments.

Note that the spin torques found appear in ‘coordinated’ form as observed in the first illumination configuration (figure 3); the black solid lines and red lines with circles are equal in value but have opposite signs (figures 9(b)–(c)). By contrast, the black solid line and the red line with circles are totally equal in figure 9(d). This coordination of the spins preserves the symmetry of the system. The system ‘feels’ an asymmetry with respect to the $x$ and $y$ axes due to this illumination variation. Yet, the components of the total spin torque are identically zero but their individual components, i.e. for each particle, are not. In addition, figure 9(b) shows that the $x$-component never vanishes for this particular wavelength, in agreement with the results in figure 3 when $\zeta = 0^\circ$. Remarkably, this component of the spin torque reaches the largest values. The $x$- and $y$- components reach larger values than the $z$-component.

Now the ‘gear’ mechanism of the light scattering already appears in 2D results, see [52–54]. In the present three-dimensional case, this mechanism appears for the torques along the coordinates $x$ and $y$. In [52–54], this kind of induction appears in only one dimension, here analogous to what we found along the $x$ direction, i.e. spin rotation with respect to the $x$-axis. Now the new component, namely the $y$-spin, makes the particles rotate with respect to the $y$-axis in coaxial counter rotation.

It is worth mentioning that the phases of the curves of the orbital torque and $z$-spin torques are opposite for $N = 2378$. 

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**Figure 8.** Scaled optical forces exerted on a silica two-sphere system of $R = 240$ nm and $d = 480$ nm as a function of the polarization angle $\zeta$. The incident wavelength is $\lambda = 600$ nm and the other angles are $\theta = 180^\circ$ and $\phi = 0^\circ$. (a) Tangential force $F_1 = F_2$; black lines without circles (red lines with circles) correspond to forces exerted on sphere 1 (2); (b) binding force; (c) module of the radiation pressure. The solid (dashed) lines with/without circles correspond to calculations with $N = 2378$ ($N = 2$). The blue lines in (a) and (b) highlight the zero of the scales.

**Figure 9.** Scaled optical torques exerted on a silica two-sphere system of $R = 240$ nm and $d = 480$ nm as a function of the polarization angle $\zeta$. The incident wavelength is $\lambda = 600$ nm, and the other angles are $\theta = 180^\circ$ and $\phi = 0^\circ$ and $\zeta = 0^\circ$. (a) Orbital torque. (b)–(d) Spin torques. The solid lines and the lines with circles correspond to calculations with $N = 2378$. The dashed lines correspond to $N = 2$. Black lines without symbols (red lines with symbols) correspond to torques $N_{x1}$ ($N_{y2}$) exerted on sphere 1 (2).
This makes sense since some spin–orbit coupling is expected. The total torque exerted on the whole system must be zero on average because the illumination has no net angular momentum. Consequently, the electromagnetic field around the system must have a net contribution to the rate of angular momentum, which is exactly compensated with the rate of angular momentum taken for the movement of the system. An indication of such an effect was shown in Appendix for 2D systems of plasmonic dimers. A similar phenomenon is expected in three-dimensional systems (not shown here).

Note that the scaling factors of the orbital torque and the spin torques differ in the constant value $k_0d$. Although it appears that the $z$-spin torque module is larger than the orbital torque module for the scaled values, it is not. To compare all the values in figure 9 one must multiply the orbital torque by the adimensional factor $k_0d$ in such a way that it takes the same units as the spin torques, i.e. $3V_uu_0$. In this particular case, the factor is $k_0d \approx 5$. Then, the maxima of the orbital torque are 1.4 times larger than the maxima of the $z$-component of the spin torque. However, the maxima of the $x$-component of the spin torque are larger than the maxima of the orbital torque by $\approx 2.9$ times in this case.

Note that the curve for the $x$-component of the spin torque differs in phase in $90^\circ$ with respect to the curve for the $y$-component. This is a phase difference of geometrical origin because these components are orthogonal to each other. However, the whole movement of the system is complex in general and, of course, the relative phases and modules of the torques differ when other wavelengths are observed.

### 4. Conclusions

This paper outlines some novel consequences of the multiple scattering between two nanospheres when illuminated with time-harmonic fields that do not carry angular momentum. In particular, the study focuses on the optical forces and torques exerted on two silica spheres and it is supported by calculations with DDA. Unusual spin torques were found for illumination with plane waves having a linear polarization. The results were explained in terms of asymmetries in the internal fields of the spheres.

The variation in the induced forces with the angles of illumination was also studied. Remarkably, new degrees of freedom were predicted for the movement of the system. It was found that both spheres, as a whole, can deviate from the direction expected by the radiation pressure. When the polarization of the illumination was changed, new spin torques were found in coordination with orbital torques.

Regarding the experimental demonstration of these torques, no device capable of measuring such magnitudes has been reported previously. These measures are considered non-trivial for two reasons: first, the rotations correspond to transient torques; second, the torques are not directly caused by the illumination but by the scattered field. Thus, a device with optical traps could be designed since different field shapes should anyway cause non-trivial torques. However, this device should be sensitive enough to the scattered field.

In addition, the problem of approaching the system by single dipole–dipole interactions was compared against the complete problem that involves many dipole moments for each particle. The results clearly show how the multipolar interactions between the particles are essential in considering the realistic dynamics of the system.

The results of this work can be helpful in the design of light-controlled micro/nanomachines [67] as well as in the context of optomechanical systems now widely explored in precision measurements, thermodynamics, and quantum sensing [68–70].

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### Appendix. Remarks about dipole–dipole approximations

Although not realistic, in general the dipole–dipole interaction results in a more intuitive interpretation of the electromagnetic coupling of the scatterers. This simple picture can explain some features of the interaction as the sign of the binding forces in figure 6(a) (dashed line) because the attraction/repulsion is seen as generated by the forces exerted between equal or opposite charges in the dipole moments. It can also explain the origin of the orbital torques in figure 7(a) for case $N=2$ as a result of the alignment of the total dipole of the system with the field. The picture of the system dynamics based on the alignment of dipole moments should be taken with caution because the induced dipole moments have complex components. Two perpendicular vectors of the dipole moments can be defined with respect to the incident field, namely the components parallel and perpendicular to it. They are both complex quantities and different from zero under illumination with an arbitrary angle $\phi$. Yet, the particular situation under illumination with right angles, i.e. $\phi = 0^\circ$, $90^\circ$, $180^\circ$, $360^\circ$, is different because the perpendicular components of the dipole moments are equal to zero. Moreover, if a small particle approximation can be applied (of course it is not the general case), almost zero perpendicular components of the dipole moments can be obtained no matter what the value $\phi$ is and the imaginary parts of the induced dipole moments are compensated (compensation of the phases). It is in this context that the two-dipole interpretation is useful. Still, if the system is observed under angles of illumination close to the right angles, the system tries to obtain one of two stable configurations [35], depending on the relative phases for each dipole: one is the position of cancelled dipole moment for the system; the other one is the complete alignment of parallel dipoles to form a net dipole moment for the system. However, the phases of the dipole moments (imaginary parts of the components) are not compensated if the particles are not relatively small enough with respect to the wavelength, and this makes the dipole–dipole
interpretation more difficult to deal with. More importantly, the spin torques are completely avoided within the frame of this formulation. This may be the reason why the spins were not predicted a long time ago [40].

It is interesting to describe the dipole–dipole dynamics in figures 8 and 9(a) (dashed lines with/without circles). Note that there are two sets of angular positions where the system gains some stability. One set corresponds to $\zeta = 0^\circ, 180^\circ$ where the system finds an equilibrium of zero tangential forces. However, it is unstable because a little angular perturbation around these positions would force the system to move off from its alignment with the incident electric field. Observe the values and signs of the tangential forces in figure 8(a) and the torques in figure 9(a). At $\zeta = 0^\circ, 180^\circ$, the two complex dipole moments become parallel to the incident electric field such that they are oriented along their connecting line (→→). Thus, we obtain attractive forces between them because the opposite charges of the dipoles attract each other. However, this alignment is not preferred as soon as the system leaves the configuration of $\zeta = 0^\circ, 180^\circ$.

The other equilibrium set is obtained for $\zeta = 90^\circ, 270^\circ$. In this case, the system finds a stable equilibrium; the tangential forces are zero but small deviations from these angle values result in a ‘rapid’ realignment with the original position. The system prefers to align its axis to the perpendicular direction with respect to the incident electric field. The two complex dipole moments become parallel with each other and with the incident electric field. Thus, we obtain repulsive forces between them because the positive and negative charges of the dipoles coincides in position and repel each other (↑↑). Also note that for this set of angles, the values of the radiation pressure and the binding force reach their extremes ($N = 2$ in figures 8(b) and (c)).

In conclusion, the complex dynamics predicted in this work cannot be simply approached by taking the interaction between a few dipole moments in the system. The complete multiple scattering must be taken into account to observe all the induced mechanical features.

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