Teleparallel Version of the Levi-Civita Vacuum Solutions and their Energy
Contents

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Abstract

In this paper, we find the teleparallel version of the Levi-Civita metric and obtain tetrad and the torsion fields. The tensor, vector and the axial-vector parts of the torsion tensor are evaluated. It is found that the vector part lies along the radial direction only while the axial-vector vanishes everywhere because the metric is diagonal. Further, we use the teleparallel version of Möller, Einstein, Landau-Lifshitz and Bergmann-Thomson prescriptions to find the energy-momentum distribution of this metric and compare the results with those already found in General Relativity. It is worth mentioning here that momentum is constant in both the theories for all the prescriptions. The energy in teleparallel theory is equal to the corresponding energy in GR only in Möller prescription for the remaining prescriptions, the energy do not agree in both theories. We also conclude that Möller’s energy-momentum distribution is independent of the coupling constant λ in the teleparallel theory.

Keywords: Teleparallel Theory, Axial-Vector, Energy.

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1 Introduction

Ever since the Einstein’s theory of General Relativity (GR) was proposed, the subject of the localization of energy has been lacking of a definite answer. It is well-known that the definition of energy is an oldest, thorny, most interesting and most controversial problem in GR [1]. Many researchers have proposed the several energy-momentum complexes to resolve this problem. As a pioneer, Einstein [2] proposed an expression for the energy-momentum density of the gravitational field. In fact, this quantity is a pseudo-tensor, i.e., it is a coordinate dependent object. Later on, Landau-Lifshitz [3], Papapetrou [4], Bergmann [5], Tolman [6], Weinberg [7] and Möller [8] proposed their own prescriptions to resolve this issue. These prescriptions, except Möller’s [8], are restricted to perform the calculations in Cartesian coordinates.

Virbhadra and his collaborators re-opened this issue and showed that several prescriptions could give the same results for a given spacetime [9-14]. He also found that, for a general non-static spherically symmetric metric of the Kerr-Schild class, the four different complexes ELLPW (Einstein, Landau-Lifshitz, Papapetrou and Weinberg) yield the same result as found by [15,16] in the context of quasi-local mass. However, some other people [17,18] found that different energy-momentum complexes might give different results for a given spacetime.

Alternate representations of a theory are usually important and reflect a valuable insight. The notion of tetrad field was first introduced by Einstein [19] to unify gravitation and electromagnetism, besides that he could not succeed. Later on, Hayashi nad Nakano [20] formulated the tetrad theory of gravitation, also called teleparallel gravity (TPG) or New General Relativity which corresponds to a gauge theory of translation group [21,22]. The basic entities of this theory are the non-trivial tetrad fields $h^a_{\mu}$ and is defined on Weitzenböck spacetime [23], which is endowed with the affine connection $\Gamma^\theta_{\mu\nu} = h^a_\nu \partial_\mu h^a_{\mu}$, called Weitzenböck connection. The curvature tensor, constructed out of this connection, vanishes identically but torsion remains non-zero.

In TPG, gravitation is attributed to torsion [22] which plays the role of force [24] while it geometrizes the underlying spacetime in the case of GR. The translational gauge potentials appear as a non-trivial part of the tetrad field and induce a teleparallel (TP) structure on spacetime which is directly related to the presence of a gravitational field. In some other theories [21-25], torsion is only relevant when spins are important [26]. This point of view
indicates that torsion might represent some additional degrees of freedom as compared to curvature and some new physics may be associated with it. TP is naturally formulated by gauging external (spacetime) translations which are closely related to the group of general coordinate transformations underlying GR. Thus the energy-momentum tensor represents the matter source in the field equations of tetradic theories of gravity like in GR.

Some authors [27,28] hoped that the problem of localization of energy might be settled in the framework of TPG and the results may coincide with those already found in the context of GR. Mikhail et al. [27] gave the TP version of Möller prescription and Vargas [29] constructed the TP version of Einstein, Landau-Lifshitz and Bergmann-Thomson prescriptions. Using the TP version of Einstein and Landau-Lifshitz prescriptions, Vargas showed that total energy of the closed FRW universe is zero which agrees with the results obtained by Rosen [30]. After this, many authors [31] explored the energy-momentum distribution of different spacetimes by using the TP version of the above mentioned prescriptions. It is found that the results are same in both the theories for some spacetimes while they disagree in some cases.

Pereira, et al. [32] obtained the TP versions of the Schwarzschild and the stationary axisymmetric Kerr solutions of GR. They proved that the axial-vector torsion plays the role of the gravitomagnetic component of the gravitational field in the case of slow rotation and weak field approximations. Recently [33], we have found the TP versions of the Friedmann models and Lewis-Papapetrou spacetimes which lead to some interesting results. We have also explored the energy-momentum distribution of the Lewis-Papapetrou spacetime by using TP version of Möller prescription [34]. It has been extended to the stationary axisymmetric solutions of Einstein-Maxwell field equations [35] and the class of static axially symmetric solutions of EFEs [36] together with their energy contents. The irreducible parts of the torsion tensor and the axial-vectors are also investigated.

This paper is devoted to find the TP version of the Levi-Civita vacuum solutions and then calculate the irreducible parts of the torsion tensor. The energy-momentum distribution of the solutions is also explored by using the TP version of Möller, Einstein, Landau-Lifshitz and Bergmann-Thomson prescriptions. The results for energy contents are compared with those found in the framework of GR [18]. The description of this paper is as follows. Section 2 contains the review of the basic concepts of the TP theory and the TP version of Möller, Einstein, Landau-Lifshitz and Bergmann-Thomson prescriptions. In section 3, we shall find the TP version of the Levi-Civita
vacuum solutions and the irreducible parts of the torsion tensor. Section 4 is devoted to the evaluation of the energy-momentum distribution for Levi-Civita metric using these prescriptions. The last section provides summary and discussion of the results obtained.

2 Teleparallel Theory and Energy-Momentum Prescriptions

The basic entity of the theory of TPG is the non-trivial tetrad \([37] h^a_\mu\) whose inverse is denoted by \(h^a_\nu\) and satisfy the following relations:

\[
h^a_\mu h^a_\nu = \delta^\nu_\mu; \quad h^a_\mu h^a_\mu = \delta^a_a.
\]  

(1)

The theory of TPG is described by the Weitzenböck connection given by

\[
\Gamma^\theta_\mu\nu = h^\theta_\alpha \partial_\nu h^\alpha_\mu.
\]  

(2)

which comes from the condition of absolute parallelism [22]. This implies that the spacetime structure underlying a translational gauge theory is naturally endowed with a TP structure [21,22]. In this paper, if not mentioned specifically, the Latin alphabet \((a, b, c, \ldots = 0, 1, 2, 3)\) will be used to denote the tangent space indices and the Greek alphabet \((\mu, \nu, \rho, \ldots = 0, 1, 2, 3)\) to denote the spacetime indices. The Riemannian metric in TPG arises as a by product [22] of the tetrad field given by

\[
g_{\mu\nu} = \eta_{ab} h^a_\mu h^b_\nu,
\]  

(3)

where \(\eta_{ab}\) is the Minkowski spacetime such that \(\eta_{ab} = diag(+1, -1, -1, -1)\).

For the Weitzenböck spacetime, the torsion is defined as \([38]\)

\[
T^\theta_\mu\nu = \Gamma^\theta_\nu\mu - \Gamma^\theta_\mu\nu
\]  

(4)

which is antisymmetric w.r.t. its last two indices. Due to the requirement of absolute parallelism, the curvature of the Weitzenböck connection vanishes identically [37]. The Weitzenböck connection and the Christoffel symbol satisfy the following relation

\[
\Gamma^{0\theta}_\mu\nu = \Gamma^\theta_\mu\nu - K^\theta_\mu\nu.
\]  

(5)
where $\Gamma^\theta_{\mu\nu}$ are the Christoffel symbols and $K^\theta_{\mu\nu}$ denotes the \textbf{contorsion tensor} given as

$$K^\theta_{\mu\nu} = \frac{1}{2}[T^\theta_{\mu\nu} + T^\theta_{\nu\mu} - T^\theta_{\mu\nu}],$$

(6)

The torsion tensor of the Weitzenböck connection can be decomposed into three irreducible parts under the group of global Lorentz transformations [22]: the tensor part

$$t_{\lambda\mu\nu} = \frac{1}{2}(T_{\lambda\mu\nu} + T_{\mu\lambda\nu}) + \frac{1}{6}(g_{\nu\lambda}V_{\mu} + g_{\nu\mu}V_{\lambda}) - \frac{1}{3}g_{\lambda\mu}V_{\nu},$$

(7)

the vector part

$$V_{\mu} = T^\nu_{\mu\nu}$$

(8)

and the axial-vector part

$$A^\mu = \frac{1}{6}\epsilon^{\mu\nu\rho\sigma}T_{\nu\rho\sigma},$$

(9)

where

$$\epsilon^{\lambda\mu\nu\rho} = \frac{1}{\sqrt{-g}}\delta_{\lambda\mu\nu\rho}.$$  

(10)

Here $\delta = \{\delta_{\lambda\mu\nu\rho}\}$ and $\delta^* = \{\delta_{\lambda\mu\nu\rho}\}$ are completely skew symmetric tensor densities of weight -1 and +1 respectively [22].

The TP version of the Einstein, Landau-Lifshitz and Bergman-Thomson energy-momentum complexes, by setting $c = 1 = G$, are respectively given by [29]

$$hE^\mu_{\nu} = \frac{1}{4\pi}\partial_\lambda(U^{\nu\mu\lambda}),$$

(11)

$$hL^{\mu\nu} = \frac{1}{4\pi}\partial_\lambda(hg^{\mu\beta}U_{\beta\nu\lambda}),$$

(12)

$$hB^{\mu\nu} = \frac{1}{4\pi}\partial_\lambda(g^{\mu\beta}U_{\beta\nu\lambda}),$$

(13)

where $U^{\nu\mu\lambda}$ is the Freud’s superpotential and is given by

$$U^{\nu\mu\lambda} = hS^{\nu\mu\lambda}.$$  

(14)

Here $S^{\nu\mu\lambda}$ is a tensor quantity, which is skew symmetric in its last two indices, and is defined as

$$S^{\nu\mu\lambda} = m_1T^{\nu\mu\lambda} + \frac{m_2}{2}(T^{\nu\mu\lambda} - T^{\lambda\nu\mu}) + \frac{m_3}{2}(g^{\nu\lambda}T_{\beta\mu\beta} - g^{\mu\lambda}T_{\beta\nu\beta}),$$

(15)
where $m_1$, $m_2$ and $m_3$ are three dimensionless coupling constants of TPG [22]. TPG equivalent of GR may be obtained by considering the following particular choice

$$m_1 = \frac{1}{4}, \quad m_2 = \frac{1}{2}, \quad m_3 = -1 \quad (16)$$

It is mentioned here that $hE_0^0$, $hL_0^0$, $hB_0^0$ are the energy density components and $hE_i^0$, $hL_i^0$, $hB_i^0$, ($i = 1, 2, 3$) are the momentum density components of the Einstein, Landau-Lifshitz and Bergman-Thomson prescriptions respectively.

The superpotential of the Möller tetrad theory is given by Mikhail et al. [27] as

$$U_{\mu}^{\nu\beta} = \frac{\sqrt{-g}}{2\kappa} P_{\chi\rho\sigma}^{\nu\beta} [V_\rho g^{\sigma\chi} g_{\mu\tau} - \lambda g_{\tau\mu} K^{\chi\rho\sigma} - g_{\tau\mu}(1 - 2\lambda) K^{\sigma\rho\chi}], \quad (17)$$

where

$$P_{\chi\rho\sigma}^{\nu\beta} = \delta_\chi^\tau g^{\nu\beta}_{\rho\sigma} + \delta_\rho^\tau g^{\nu\beta}_\chi - \delta_\sigma^\tau g^{\nu\beta}_\chi, \quad (18)$$

while $g^{\nu\beta}_{\rho\sigma}$ is the tensor quantity and is defined by

$$g^{\nu\beta}_{\rho\sigma} = \delta_\rho^{\nu} \delta_\sigma^{\beta} - \delta_\sigma^{\nu} \delta_\rho^{\beta}, \quad (19)$$

$K^{\sigma\rho\chi}$ is the contortion tensor, $g$ is the determinant of the metric tensor $g_{\mu\nu}$, $\lambda$ is free dimensionless coupling constant of TPG, $\kappa$ is the Einstein constant and $V_\mu$ is the basic vector field. We can write the Möller energy-momentum density components as

$$\Xi_\mu^\nu = U_{\mu}^{\nu\rho} ; \quad (20)$$

where comma means ordinary differentiation. Here $\Xi^0_0$ and $\Xi^0_i$, ($i = 1, 2, 3$) are the energy and momentum density components respectively.

3 Teleparallel Version of the Levi-Civita Solutions

In 1917, Levi-Civita obtained the most general static cylindrically symmetric vacuum solutions [39]. Since then, many explicit cylindrical solutions, for different fluids, have been found. These solutions are mostly local and no global analysis has been usually discussed. Recently, Bicak et al. [40] have
studied the global properties of static cylindrically symmetric space times with perfect fluid matter. The global existence of these solutions and the finiteness of radius of fluid cylinder is shown. It is also shown that when the fluid cylinder has a finite extension, it is possible to glue it smoothly with Levi-Civita solutions and obtain a global solution. The metric is given by [41]

\[ ds^2 = \rho^{4s} dt^2 - \rho^{4s(2s-1)} (d\rho^2 + dz^2) - \alpha^2 \rho^{2(1-2s)} d\phi^2, \]  

(21)

where \( \alpha \) is a parameter and \( s \) is the charge density parameter. The following interpretations are somewhat expected for:

- \( s = 0, \alpha = 1/2 \), the above metric reduces to locally flat spacetime
- \( s = 0, \alpha = 1 \), it represents the Minkowski spacetime
- \( s = 0, \alpha \neq 1 \), we have cosmic string.

It is mentioned here that \( \gamma \)-metric is one of the most interesting metrics of the family of the Weyl solutions. It is also known as Zipoy-Voorhes metric [42]. The Levi-Civita metric can be obtained from the \( \gamma \)-metric in the limiting case when the length of its Newtonian image source tends to infinity.

The tetrad field satisfying Eq.(3) is given as

\[
\begin{bmatrix}
  \rho^{2s} & 0 & 0 & 0 \\
  0 & \rho^{2s(2s-1)} & 0 & 0 \\
  0 & 0 & \alpha \rho^{(1-2s)} & 0 \\
  0 & 0 & 0 & \rho^{2s(2s-1)} \\
\end{bmatrix}
\]

(22)

with its inverse

\[
\begin{bmatrix}
  \rho^{-2s} & 0 & 0 & 0 \\
  0 & \rho^{2s(1-2s)} & 0 & 0 \\
  0 & 0 & \frac{1}{\alpha} \rho^{2s-1} & 0 \\
  0 & 0 & 0 & \rho^{2s(1-2s)} \\
\end{bmatrix}
\]

(23)

The non-vanishing components of the Weitzenböck connection can be found by using Eqs.(22) and (23) in Eq.(2) as

\[
\Gamma^0_{01} = \frac{2s}{\rho}, \quad \Gamma^1_{11} = \frac{2s(2s-1)}{\rho}, \\
\Gamma^2_{21} = \frac{1-2s}{\rho}, \quad \Gamma^3_{31} = \frac{2s(2s-1)}{\rho}.
\]

(24)

The corresponding non-vanishing components of the torsion tensor are

\[
T^0_{01} = -\frac{2s}{\rho} = -T^0_{10}, \quad T^2_{21} = \frac{2s-1}{\rho} = -T^2_{12},
\]
\[ T_{31}^3 = \frac{2s(1 - 2s)}{\rho} = -T_{13}^3. \]  

When we make use of these values in Eqs.(7)-(9), we get the following non-vanishing components of the tensor part:

\[
\begin{align*}
t_{010} &= -\frac{1}{6}(4s^2 - 8s + 1)\rho^{4s-1} = t_{100}, \quad t_{001} = -2t_{001}, \\
t_{212} &= \frac{\alpha^2}{3}(2s^2 + 2s - 1)\rho^{1-4s} = t_{122}, \quad t_{221} = -2t_{212}, \\
t_{313} &= -\frac{1}{6}(8s^2 - 4s - 1)\rho^{8s^2-4s-1} = t_{133}, \quad t_{331} = -2t_{313}
\end{align*}
\]

and the vector part

\[ V_1 = -\frac{1}{\rho}(4s^2 - 2s + 1), \]

respectively. The components of the axial-vector part all vanish, i.e.,

\[ A^i = 0, \quad i = 0, 1, 2, 3 \]

which is due to the diagonal metric similar to the Schwarzschild case [32].

## 4 Energy-Momentum Distribution of the Levi-Civita Solutions

In this section, we shall use the TP version of Möller, Einstein, Landau-Lifshitz and Bergmann-Thomson prescriptions, given by Eqs.(20), (11), (12) and (13) respectively, to find the energy-momentum distribution of the Levi-Civita solutions.

### 4.1 Möller Prescription

When we multiply Eq.(27) by \( g^{11} \), it turns out

\[ V^1 = (4s^2 - 2s + 1)\rho^{-8s^2+4s-1}. \]

The non-vanishing components of the contorsion tensor, in contravariant form, become

\[ K^{010} = 2s\rho^{-8s^2-1} = -K^{100}, \]
\[ K^{212} = \frac{1}{\alpha^2} (2s - 1) \rho^{-8s^2 + 8s - 3} = -K^{122}, \]
\[ K^{313} = 2s(1 - 2s) \rho^{-16s^2 + 8s - 1} = -K^{133}. \] (30)

Clearly, the contorsion tensor is antisymmetric w.r.t. its first two indices. By substituting Eqs. (29)-(30) in Eq. (17), the required non-vanishing components of the superpotential in Möller’s tetrad theory are

\[ U^{01}_0 = -\frac{\alpha}{\kappa} (4s^2 - 4s + 1) = -U^{10}_0. \] (31)

Using Eq. (31) in (20), the energy-momentum density components vanish, i.e.,

\[ \Xi^0_i = 0, \quad i = 0, 1, 2, 3. \] (32)

This shows that both energy and momentum become constant in Möller’s tetrad theory which implies that Möller energy-momentum distribution is independent of the coupling constant \( \lambda \).

4.2 Einstein Prescription

For Einstein, Landau-Lifshitz and Bergmann-Thomson prescriptions, it is necessary to use Cartesian coordinate system to get meaningful results. When we transform the line element (21) into Cartesian coordinates, we obtain

\[ ds^2 = \rho^{4s}dt^2 - \rho^{4s(2s-1)} \left\{ \left( \frac{x \rho^{4s^2 - 2s - 1}}{\rho} \right)^2 + \left( \frac{y \rho^{4s^2 - 2s - 1}}{\rho} \right)^2 \right\} - \alpha^2 \rho^{2(1 - 2s)} \left( \frac{xdy - ydx}{\rho^2} \right)^2. \] (33)

The tetrad field corresponding to this metric is

\[ h^a_\mu = \begin{bmatrix}
\rho^{2s} & 0 & 0 & 0 \\
0 & x \rho^{4s^2 - 2s - 1} & y \rho^{4s^2 - 2s - 1} & 0 \\
0 & \alpha y \rho^{-2s - 1} & -\alpha x \rho^{-2s - 1} & 0 \\
0 & 0 & 0 & \rho^{2s(2s-1)}
\end{bmatrix} \] (34)

with its inverse

\[ h^a_\mu = \begin{bmatrix}
\rho^{-2s} & 0 & 0 & 0 \\
0 & \frac{x}{x^2 - y^2} \rho^{-4s^2 + 2s + 1} & -\frac{y}{y^2 - x^2} \rho^{-4s^2 + 2s + 1} & 0 \\
0 & \frac{y}{\alpha(y^2 - x^2)} \rho^{1 + 2s} & -\frac{x}{\alpha(x^2 - y^2)} \rho^{1 + 2s} & 0 \\
0 & 0 & 0 & \rho^{2s(1 - 2s)}
\end{bmatrix}. \] (35)
The non-vanishing components of the Weitzenböck connection are

\[ \Gamma^0_{01} = \frac{2sx}{\rho^2}, \quad \Gamma^0_{02} = \frac{2sy}{\rho^2}, \]

\[ \Gamma^1_{11} = \frac{2sx}{\rho^4}(2sx^2 - \rho^2), \quad \Gamma^2_{22} = \frac{2sy}{\rho^4}(2sy^2 - \rho^2), \]

\[ \Gamma^1_{12} = \frac{2sy}{\rho^4}(2sx^2 - \rho^2), \quad \Gamma^2_{21} = \frac{2sx}{\rho^4}(2sy^2 - \rho^2), \]

\[ \Gamma^1_{21} = \frac{y}{\rho^4}(4s^2x^2 - \rho^2), \quad \Gamma^2_{12} = \frac{x}{\rho^4}(4s^2y^2 - \rho^2), \]

\[ \Gamma^3_{31} = \frac{2sx}{\rho^2}(2s - 1), \quad \Gamma^3_{31} = \frac{2sy}{\rho^2}(2s - 1) \] (36)

and the components of the torsion tensor, in contravariant form, are

\[ T^{001} = 2sx\rho^{-8s^2-2} = -T^{010}, \]
\[ T^{002} = 2sy\rho^{-8s^2-2} = -T^{020}, \]
\[ T^{112} = -\alpha^{-2}y(1 - 2s)\rho^{-8s^2+8s-2} = -T^{121}, \]
\[ T^{221} = -\alpha^{-2}x(1 - 2s)\rho^{-8s^2+8s-2} = -T^{212}, \]
\[ T^{331} = 2sx(1 - 2s)\rho^{-16s^2+8s-2} = -T^{313}, \]
\[ T^{332} = 2sy(1 - 2s)\rho^{-16s^2+8s-2} = -T^{323}. \] (37)

Using Eqs.(16) and (37) in Eq.(15), the required non-vanishing components of the tensor \( S^\mu_{\nu\lambda} \), in mixed form, are

\[ S^0_{01} = -x(4s^2 - 10s + 1)\rho^{-8s^2+4s-2}, \] (38)
\[ S^0_{02} = -y(4s^2 - 10s + 1)\rho^{-8s^2+4s-2}. \] (39)

When we make use of these values and \( h = \alpha\rho^{8s^2-4s} \) in Eq.(14), the non-zero components of the Freud’s superpotential turn out to be

\[ U^0_{01} = -\alpha x(4s^2 - 10s + 1)\rho^{-2}, \] (40)
\[ U^0_{02} = -\alpha y(4s^2 - 10s + 1)\rho^{-2}. \] (41)

Using Eqs.(40)-(41) in Eq.(11), the components of energy-momentum density become

\[ hE^0_\mu = 0, \quad \mu = 0, 1, 2, 3 \] (42)

which gives constant energy-momentum.
4.3 Landau-Lifshitz Prescription

When we use Eqs.(40)-(41) and the values of $h$ and $g^{\mu\nu}$ in Eq.(12), the components of energy-momentum density in this prescription become

$$hL_{00} = -\frac{2\alpha^2 s \pi}{8s^2 - 8 s - 2},$$

$$hL_{0i} = 0, \quad i = 1, 2, 3.$$  \hspace{1cm} (43)

Here momentum also becomes constant.

4.4 Bergmann-Thomson Prescription

Now we replace Eqs.(40)-(41) and the values of $g^{\mu\nu}$ in Eq.(13), so that the components of energy-momentum density in Bergmann-Thomson prescription become

$$hB_{00} = \frac{\alpha s \pi}{8s^2 - 10 s + 1} \rho^{-4s-2},$$

$$hB_{0i} = 0, \quad i = 1, 2, 3.$$  \hspace{1cm} (45)

Here again we have constant momentum.

5 Summary and Discussion

The debate of the localization of energy-momentum has been an open issue since the time of Einstein when he formulated the well-known relation between mass and energy. Misner et al. [1] concluded that energy can only be localized in spherical coordinates. But, soon after, Cooperstock and Saracciino [43] demonstrated that if the energy is localizable in spherical systems then it can be localized in any system. Bondi [44] rejected the idea of non-localization of energy in GR due to the reason that there should be some form of energy which contributes to gravitation and hence its location can, in principle, be found. Many authors believed that a tetrad theory should describe more than a pure gravitation field [45]. In fact, Möller [46] considered this possibility in his earlier attempt to modify GR.

This paper continues the investigation for the following two issues: Firstly, we find the TP version of the Levi-Civita vacuum solutions and evaluate the irreducible parts of the torsion tensor. Secondly, we evaluate the energy-momentum density components of the Levi-Civita vacuum solutions by using
the TP version of Möller, Einstein, Landau-Lifshitz and Bergmann-Thomson prescriptions. The axial-vector torsion vanishes because the metric is diagonal similar to the case of Schwarzschild metric [32]. The energy-momentum distributions for each prescription are given in the following table:

**Table: Energy-Momentum Densities of the Levi-Civita Metric in TP**

| Prescription       | Energy Density          | Mom. Density          |
|--------------------|-------------------------|-----------------------|
| Möller             | $\Xi_0^i = 0$           | $\Xi_0^0 = 0$         |
| Einstein           | $hE_0^0 = 0$            | $hE_0^i = 0$          |
| Landau-Lifshitz    | $hL^{00} = \frac{2s^4}{\pi}(4s^3 - 14s^2 + 11s - 1)\rho^{8s^2 - 8s - 2}$ | $hL_0^0 = 0$ |
| Bergmann           | $hB^{00} = \frac{s^3}{\pi}(4s^2 - 10s + 1)\rho^{-4s - 2}$ | $hB_0^0 = 0$ |

These results show that momentum is constant in each prescription which coincides with the results of GR [18]. Further, energy density becomes similar for both the theories only in the case of Möller prescription. However, energy density is different in the remaining prescriptions and do not match with the corresponding densities in GR. We also note that Möller energy-momentum distribution is independent of the coupling constant $\lambda$ in TPG. It is worth mentioning here that energy-momentum becomes constant both in GR and TPT when we choose $s = 0$, as expected for Minkowski spacetime.

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