Parabose Squeezed Operator and Its Applications

Weimin Yang and Sicong Jing
Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, P.R.China

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Abstract

By virtue of the parabose squeezed operator, propagator of a parabose parametric amplifier, explicit form of parabose squeezed number states and normalization factors of excitation states on a parabose squeezed vacuum state are calculated in this letter, which generalize the relevant results from ordinary Bose statistics to parabose case.

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1 Introduction

A fundamental unsolved question in physics is whether all particles in the nature are necessarily either bosons or fermions. Theoretical investigations of other possibilities in local, relativistic quantum field theory show that may exist more general particle statistics. Parastatistics was introduced by Green as an exotic possibility extending the Bose and Fermi statistics

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and for the long period of time the interest to it was rather academic. Nowadays it finds some applications in the physics of the quantum Hall effect \[2\] and (probably) it is relevant to high temperature superconductivity \[3\], so more and more attentions are paid to it in recent years. Even though there are no observed paraparticles in nature, the possibility exists for unobserved particles which obey the parastatistics. Some developments in interacting many-particle systems have also shown that the quasiparticles in such systems may exhibit features far more exotic than those permitted to ordinary particles, and it appeara quite possible that parastatistics may be realized in condensed matter physics. In the case of parastatistics, there is the additional motivation of the possible production of \( p > 1 \) paraparticles (\( p \) is the parastatistics order) at the high energies of new and future colliders (the Tevatron, the LHC, the NLCs, etc).

Since the paraquantization, carried out at the level of the algebra of creation and annihilation operators, involves trilinear (or double) commutation relations in place of the bilinear relations that characterize Bose and Fermi statistics, sometimes it is highly non-trivial to generalize some interesting results from ordinary Bose or Fermi statistics to parastatistics case. In spite of these difficulties, many progresses have been made in the past years \[4\].

It is also well known that \( SU(2) \) and \( SU(1,1) \) are the simplest non-Abelian Lie groups, and the relevant Lie algebras \( su(2) \) and \( su(1,1) \) have many important applications in various areas of physics. For instance, the \( su(1,1) \) coherent state, has some intrinsic relations to the squeezed state, one of the most important non-classical states in quantum optics. Using the ordinary Bose oscillator, one can get a very simple realization of the \( su(1,1) \) algebra. By virtue of the parabose oscillator, one can also do this, that is, introducing notations \( K_\pm \) and \( K_0 \) defined by

\[
K_+ = \frac{1}{2} a^{\dagger 2}, \quad K_- = \frac{1}{2} a^2, \quad K_0 = \frac{1}{4} \left( a^{\dagger} a + a a^{\dagger} \right),
\]

where \( a^{\dagger} \) and \( a \) are the parabose creation and annihilation operators respectively, which satisfy the following trilinear commutation relations

\[
[a, \{a^{\dagger}, a\}] = 2a, \quad [a, a^{\dagger 2}] = 2a^{\dagger}, \quad [a, a^2] = 0,
\]

one has the \( su(1,1) \) Lie algebra

\[
[K_0, K_\pm] = \pm K_\pm, \quad [K_+, K_-] = -2K_0.
\]
In terms of the generators $K_\pm$, the parabose squeezed operator $S(z)$ takes the form of
\[ S(z) = \exp \left( \frac{1}{2} za^2 - \frac{1}{2} z^* a^\dagger^2 \right). \] (4)

In this letter, using this parabose squeezed operator, we generalize some interesting results of the ordinary boson systems to systems made of a single type of parabosons, which include the propagator of a parametric amplifier in parabose coherent state representation, the explicit form for parabose squeezed number states and the normalization factors of excitation states on a parabose squeezed vacuum state.

2 Propagator of a parabose parametric amplifier

In this section, we take the parameter $z$ in $S(z)$ as an imaginary number $z = -ir$. Using the disentangling theorem of $su(1,1)$, we have
\[ S(-ir) = \exp \left( -i \tanh r \frac{a^\dagger^2}{2} \right) \exp \left( \ln \text{sech} r \frac{\{a^\dagger, a\}}{2} \right) \exp \left( -i \tanh r \frac{a^2}{2} \right). \] (5)

Differentiating eq.(5) with the parameter $r$ and using the following operator identities
\[ \exp \left( \lambda a^\dagger^2 \right) \{a^\dagger, a\} \exp \left( -\lambda a^\dagger^2 \right) = \{a^\dagger, a\} - 4\lambda a^\dagger^2, \] (6)
\[ \exp \left( \lambda \{a^\dagger, a\} \right) a \exp \left( -\lambda \{a^\dagger, a\} \right) = a e^{-2\lambda}, \] (7)
\[ \exp \left( \lambda a^\dagger^2 \right) a^2 \exp \left( -\lambda a^\dagger^2 \right) = a^2 - 2\lambda \{a^\dagger, a\} + 4\lambda^2 a^\dagger^2, \] (8)

one gets
\[ \frac{\partial}{\partial r} S(-ir) = -\frac{i}{2} (a^2 + a^\dagger^2) S(-ir), \] (9)
with a boundary condition $S(r = 0) = 1$. In fact, eq.(9) also may be derived from eq.(4) directly by differentiating $S(-ir)$ with $r$.

Noticing that $S(-ir)$ is a unitary operator, furthermore, when the parameter $r$ is a function of time $t$ and the initial value of $r$ is taken as $r(t = t_0) = 0$,
one may denote $S(-ir)$ as $S(t, t_0)$ and rewrite eq.(9) as

$$i \frac{\partial}{\partial t} S(t, t_0) = \frac{1}{2}(a^2 + a^{12}) S(t, t_0) \frac{\partial r}{\partial t},$$

(10)

with the condition $S(t_0, t_0) = 1$. Particularly, for $\partial r/\partial t = 2f(t)$, eq.(10) will lead to

$$i \frac{\partial}{\partial t} S(t, t_0) = f(t)(a^2 + a^{12}) S(t, t_0).$$

(11)

If one introduces a free Hamiltonian

$$H_0 = \frac{\omega}{2}\{a^\dagger, a\}$$

for the parasystem under investigation, and uses eq.(7) and its conjugate identity, one may rewrite $f(t)(a^2 + a^{12})$ as

$$H_I(t) = f(t)(a^2 + a^{12}) = e^{iH_0 t} f(t)(a^2 e^{2i\omega t} + a^{12} e^{-2i\omega t}) e^{-iH_0 t}.$$  

(12)

Thus eq.(11) becomes

$$i \frac{\partial}{\partial t} S(t, t_0) = H_I(t) S(t, t_0),$$

(13)

which means that one can treat $S(t, t_0)$ and $H_I(t)$ as a time-displacement operator and the interaction Hamiltonian in the interaction picture respectively. According to the standard picture transformation theory in quantum mechanics, a Hamiltonian resulting the squeezed effect in the Schrödinger picture is

$$H_S = \frac{\omega}{2}\{a^\dagger, a\} + f(t) \left( a^2 e^{2i\omega t} + a^{12} e^{-2i\omega t} \right).$$

(14)

Obviously here $H_S$ is a time-dependent Hamiltonian. When the $a^\dagger$ and $a$ in $H_S$ are ordinary bose creation and annihilation operators, $H_S$ describes a kind of non-linear optical phenomena, that is, the interaction of two types of light beams in a non-linear optical coupler. $f(t)$ includes a pumping factor (treated as a classical quantity) and another factor related to two-order of susceptibility of some optical medium. Especially, when $f(t) = k$, or $r = 2k(t - t_0)$, $H_S$ is exactly the Hamiltonian of a degenerate parametric amplifier. So the $H_S$ in eq.(14) is a parabose generalization of this kind of Hamiltonian. Also from the transformation theory between interaction picture and Schrodinger picture, the time-displacement operator in the Schrödinger picture is

$$S_S(t, t_0) = e^{-iH_0 t} S(t, t_0) e^{iH_0 t_0},$$

(15)
which satisfies the following equation

\[ i \frac{\partial}{\partial t} S_t S = H S S. \]  \hfill (16)

Now let us consider the propagator of the degenerate parametric amplifier in a parabose coherent state representation \[ \langle z | S_S(t, t_0) | z_0 \rangle, \] where \[ |z\rangle \] is the parabose coherent state given by

\[ |z\rangle = E(|z|^2)^{-1/2} \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{|n|!}} |n\rangle, \]  \hfill (17)

|n\rangle being the number state of the parabose Fock space

\[ |n\rangle = \frac{a^n |0\rangle}{\sqrt{|n|!}}, \quad a|n\rangle = \sqrt{|n|} |n-1\rangle, \quad a^\dagger|n\rangle = \sqrt{|n+1|} |n+1\rangle, \]  \hfill (18)

and

\[ [n] = n + \frac{p - 1}{2} (1 - (-)^n), \quad E(x) = \sum_{n=0}^{\infty} \frac{x^n}{[n]!}, \]  \hfill (19)

where \[ [n]! = [n][n-1] \cdots [1], \] \[ [0]! \equiv 1, \] and \[ |0\rangle \] is the unique vacuum state of the Fock space, which satisfies \[ a|0\rangle = 0 \] and \[ aa^\dagger|0\rangle = p|0\rangle, \] here \( p \) is the parastatistics order \( (p = 1, 2, 3, \cdots) \). Using eq.(7) one has

\[ \exp(iH_0 t_0)|z_0\rangle = e^{ip\omega t_0/2} |z_0 e^{i\omega t_0}\rangle. \]  \hfill (20)

Furthermore, according to the disentangling formula (5), it is easily to get the desired propagator

\[ \langle z | S_S(t, t_0) | z_0 \rangle = e^{-i\omega(t-t_0)/2} E(|z|^2)^{-1/2} E(|z_0|^2)^{-1/2} \times (\text{sech} \, 2k(t-t_0))^{p/2} E \left( z^* z_0 e^{-i\omega(t-t_0)} \text{sech} \, 2k(t-t_0) \right) \times \exp \left( -\frac{i}{2} \text{tanh} \, 2k(t-t_0) (z_0^2 e^{2i\omega t_0} + z^* z e^{-2i\omega t}) \right). \]  \hfill (21)

When \( p \rightarrow 1 \), this propagator will reduce to the ordinary result \[ 3. \]
3 Parabose squeezed number states

In this section we consider the resulting states from the parabose squeezed operator \( S(r) = \exp(\frac{r}{2}a^2 - \frac{r}{2}a^\dagger) \) acting on the parabose number states \(|n\rangle\)

\[
|r, n\rangle = S(r)|n\rangle, \quad (n = 0, 1, 2, 3, \cdots) \tag{22}
\]

here for the sake of simplicity, we take the squeezed parameter \( z \) as a real number \( r \). We call \(|r, n\rangle\) the parabose squeezed number states. Obviously, \(|r, n\rangle\) form a complete and orthonormal state-vector set for the system made of a single type of parabosons:

\[
\langle r, n| r, m\rangle = \langle n| m\rangle = \delta_{n,m}, \quad \sum_{n=0}^{\infty} |r, n\rangle \langle r, n| = 1. \tag{23}
\]

Using the following transformations

\[
S(r) a S(r)^{-1} = \cosh r a + \sinh r a^\dagger,
S(r) a^\dagger S(r)^{-1} = \cosh r a^\dagger + \sinh r a \tag{24}
\]

and the disentangling formula (5), we have

\[
|r, n\rangle = \left(\frac{\text{sech } r}{\sqrt{n!}}\right)^{p/2} \left(\cosh r a^\dagger + \sinh r a\right)^n e^{-\tanh r a^\dagger/2}|0\rangle. \tag{25}
\]

In terms of the deformed Hermite polynomials \( H_{n}^{(p)}(x) \) (see the appendix), the explicit form of \(|r, n\rangle\) is

\[
|r, n\rangle = \left(\frac{\text{sech } r}{\sqrt{n!}}\right)^{p/2} \left(-\frac{\tanh r}{2}\right)^{n/2} H_{n}^{(p)}\left(\frac{a^\dagger}{\sqrt{\sinh 2r}}\right) e^{-\tanh r a^\dagger/2}|0\rangle. \tag{26}
\]

We prove eq.(26) by induction. Firstly, from (25), and using

\[
e^{\tanh r a^\dagger/2} a^{n} e^{-\tanh r a^\dagger/2} = (a - \tanh r a^\dagger)^{n}, \tag{27}
\]

for \( n = 1 \) case, we have

\[
|r, 1\rangle = \left(\frac{\text{sech } r}{\sqrt{1!}}e^{-\tanh r a^\dagger/2}\right) a^\dagger e^{-\tanh r a^\dagger/2}|0\rangle
\]

\[
= \left(\frac{\text{sech } r}{\sqrt{1!}}\right)^{p/2} \left(-\frac{\tanh r}{2}\right)^{1/2} H_{1}^{(p)}(\chi)e^{-\tanh r a^\dagger/2}|0\rangle, \tag{28}
\]
where $\chi$ stands for $a^\dagger /i\sqrt{\sinh 2r}$. Then supposing

$$|r, n-1\rangle = \frac{(\text{sech } r)^{p/2}}{\sqrt{|n-1|!}} \left(-\frac{\tanh r}{2}\right)^{\frac{n-1}{2}} H_{n-1}^{(p)}(\chi) e^{-\tanh r a^\dagger/2} |0\rangle,$$  

(29)

we will have

$$|r, n\rangle = \frac{1}{\sqrt{|n|!}} (\cosh r a^\dagger + \sinh r a) |r, n-1\rangle
$$

$$= \frac{(\text{sech } r)^{p/2}}{\sqrt{|n|!}} \left(-\frac{\tanh r}{2}\right)^{\frac{n+1}{2}} \cosh r a^\dagger H_{n-1}^{(p)}(\chi) e^{-\tanh r a^\dagger/2} |0\rangle
$$

$$+ \frac{(\text{sech } r)^{p/2}}{\sqrt{|n|!}} \left(-\frac{\tanh r}{2}\right)^{\frac{n+1}{2}} \sinh r e^{-\tanh r a^\dagger/2} (a - \tanh r a^\dagger)
$$

$$\times H_{n-1}^{(p)}(\chi) |0\rangle.$$  

(30)

By use of the relations

$$[a, a^n] = a^{n-1} \left(n + \frac{p-1}{2} (1 - (-)^n) R\right),$$

$$[a^\dagger, a^n] = -a^{n-1} \left(n + \frac{p-1}{2} (1 - (-)^n) R\right),$$

(31)

where $R$ is a reflection operator (see eq.(38)), it is easily to find that

$$a H_{n-1}^{(p)}(\chi) |0\rangle = \frac{2[n-1]}{i \sqrt{\sinh 2r}} H_{n-2}^{(p)}(\chi) |0\rangle.$$  

(32)

Substituting eq.(32) into (30), we obtain

$$|r, n\rangle = \frac{(\text{sech } r)^{p/2}}{\sqrt{|n|!}} \left(-\frac{\tanh r}{2}\right)^{n/2} \left(2\chi H_{n-1}^{(p)}(\chi) - 2[n-1] H_{n-2}^{(p)}(\chi)\right)
$$

$$\times e^{-\tanh r a^\dagger/2} |0\rangle.$$  

(33)

At last, by virtue of a recursion relation for the deformed Hermite polynomials (see the appendix)

$$H_{n+1}^{(p)}(x) - 2x H_n^{(p)}(x) + 2[n] H_{n-1}^{(p)}(x) = 0,$$  

(34)

eq(26) is proved.
4 Excitations on a parabose squeezed vacuum state

In this section we consider the excitations on a parabose squeezed vacuum state which defined by
\[ \lvert n, r \rangle = a^\dagger n \lvert 0, r \rangle, \]
where \( \lvert 0, r \rangle = S(r) \lvert 0 \rangle \) is the parabose squeezed vacuum state. It is easily to see that \( \lvert 0, r \rangle \) is normalized, however, the states \( \lvert n, r \rangle \) have not been normalized. We would like to point out that the states \( \lvert n, r \rangle \) may be normalized in terms of the deformed Legendre polynomials \( P_n^{(p)}(x) \) (see the appendix) as
\[ \langle n, r \lvert n, r \rangle = [n]!(\cosh r)^n P_n^{(p)}(\cosh r). \]

We prove eq.(36) by induction again. Firstly, we have
\[ a \lvert 0, r \rangle = (\text{sech} r)^{p/2} a e^{-\tanh r a^\dagger 2/2} \lvert 0 \rangle = (\text{sech} r)^{p/2} e^{-\tanh r a^\dagger 2/2} (a - \tanh r a^\dagger) \lvert 0 \rangle = -\tanh r a^\dagger \lvert 0, r \rangle. \]

By virtue of the so-called \( R \)-deformed Heisenberg algebra \[7\], one can rewrite the trilinear commutation relations (2) as
\[ [a, a^\dagger] = 1 + (p - 1)R, \quad \{R, a\} = \{R, a^\dagger\} = 0, \quad R^2 = 1, \]
where \( R \) is the reflection operator. Using the \( R \)-deformed commutation relation (38) and noticing that \( R \lvert 0 \rangle = \lvert 0 \rangle \), we find
\[ \langle 1, r \lvert 1, r \rangle = \langle 0, r \lvert aa^\dagger \lvert 0, r \rangle = \langle 0, r \lvert (1 + a^\dagger a + (p - 1)R) \lvert 0, r \rangle \]
\[ = p + \tanh^2 r \langle 1, r \lvert 1, r \rangle = [1]! \cosh r P_1^{(p)}(\cosh r). \]

Then supposing eq.(36) is true for \( m \leq n \), that is,
\[ \langle m - 1, r \lvert m - 1, r \rangle = [m - 1]!(\cosh r)^{m-1} P_{m-1}^{(p)}(\cosh r), \]
we show that eq.(36) works. In fact, using eq.(31) we have
\[ \langle n, r \lvert n, r \rangle = \langle 0, r \lvert a^{n-1}a^\dagger a^\dagger^{(n-1)} \lvert 0, r \rangle \]
\[\begin{align*}
&= \langle 0, r | a^{n-1} (1 + a^\dagger a + (p-1)R) a^{\dagger(n-1)} | 0, r \rangle \\
&= \langle n-1, r | n-1, r \rangle + (p-1)(-)^{n-1} \langle n-1, r | n-1, r \rangle \\
&+ \langle 0, r | (a^\dagger a^{n-1} + [n-1]a^{n-2}) (a^{\dagger(n-1)}a + [n-1]a^{\dagger(n-2)}) | 0, r \rangle \\
&= \text{tanh}^2 r \langle n, r | n, r \rangle - [n-1]^2 \langle n-2, r | n-2, r \rangle \\
&+ (2[n-1] + 1 + (p-1)(-)^{n-1}) \langle n-1, r | n-1, r \rangle,
\end{align*}\]  

or

\[\begin{align*}
\langle n, r | n, r \rangle &= -\cosh^2 r |n-1|^2 \langle n-2, r | n-2, r \rangle \\
&+ \cosh^2 r \left(2[n-1] + 1 + (p-1)(-)^{n-1}\right) \langle n-1, r | n-1, r \rangle.
\end{align*}\]

Substituting (40) into (42), we get

\[\begin{align*}
\langle n, r | n, r \rangle &= -\cosh^n r |n-1|! [n-1] P_{n-2}^{(p)}(\cosh r) \\
&+ \cosh^{n+1} r |n-1|! \left(2[n-1] + 1 + (p-1)(-)^{n-1}\right) P_{n-1}^{(p)}(\cosh r).
\end{align*}\]

Noticing that \(2[n-1] + 1 + (p-1)(-)^{n-1} = [2n-1]\) and using the following recursion relation for \(P_n^{(p)}(x)\) (see the appendix),

\[\begin{align*}
[n+1]P_{n+1}^{(p)}(x) - [2n+1]xP_n^{(p)}(x) + [n]P_{n-1}^{(p)}(x) = 0,
\end{align*}\]

we finally arrive at eq.(36). Thus we see that the deformed Legendre polynomials indeed can be used to normalize the excitation states on a squeezed vacuum state for a single parabose mode.

In summary, in this letter, we generalize some interesting results from the ordinary Bose statistics to the parabose statistics which are related to the squeezed operator. In these generalizations, some deformed polynomials and the relevant recursion relations are used which will be defined and explained in the appendix.

### Appendix

**Deformations of ordinary Hermite and Legendre polynomials**

In order to explain the definitions and properties of the new kind of deformation of the ordinary Hermite polynomials and Legendre polynomials, let us
introduce a kind of deformation of the ordinary derivative operator, which was proposed for developing coordinate representation theory of parabose system [8]. The deformed derivative operator $D$ is defined by

$$Df(x) \equiv \frac{D}{Dx} f(x) = \frac{df}{dx} f(x) + \frac{p-1}{2x} (1 - R) f(x)$$

$$= df(x) + \frac{p-1}{2x} (f(x) - f(-x)), \quad (45)$$

where $df = df/dx$ and $R$ is the reflection operator which has a property $Rf(x) = f(-x)$ for any $x$ dependent function $f(x)$. Let us consider solutions of a second-order differential equation based on the deformed derivative operator $D$

$$D^2 f(x) - 2xDf(x) + \mu f(x) = 0. \quad (46)$$

When the parameter $\mu$ takes eigenvalues $\mu = 2[n], n = 0, 1, 2, 3, \cdots$, for each given parastatistics order $p$, eq.(46) has the following solutions

$$H_{n}^{(p)}(x) = [n]! \sum_{k=0}^{[n/2]} \frac{(-)^k (2x)^{n-2k}}{k!(n-2k)!}, \quad (47)$$

where $[k]'$ in the above of $\sum$ stands for the largest integer smaller than or equal to $k$. We call polynomials (47) the deformed Hermite polynomials because when $p \to 1$ (47) will reduce to the ordinary Hermite polynomials. The first few of $H_{n}^{(p)}(x)$ have the following explicit forms

$$H_0^{(p)}(x) = 1, \quad H_1^{(p)}(x) = 2x, \quad H_2^{(p)}(x) = 4x^2 - [2]!$$

$$H_3^{(p)}(x) = 8x^3 - 4[3]x, \quad \cdots \quad (48)$$

In terms of the deformed operator $D$, $H_{n}^{(p)}(x)$ also have its differential form

$$H_{n}^{(p)}(x) = (-)^n e^{x^2} D^n e^{-x^2}. \quad (49)$$

The generating function of $H_{n}^{(p)}(x)$ is

$$e^{-t^2 E(2tx)} = \sum_{n=0}^{\infty} \frac{t^n}{[n]!} H_{n}^{(p)}(x). \quad (50)$$
There are some definite relations between neighbouring $H_n^{(p)}(x)$ and their derivatives. The main recursion relations are the following two:

$$ DH_n^{(p)}(x) - 2[n]H_{n-1}^{(p)}(x) = 0, $$

$$ H_{n+1}^{(p)}(x) - 2xH_n^{(p)}(x) + 2[n]H_{n-1}^{(p)}(x) = 0. $$

Another second-order differential equation also based on the deformed operator $D$

$$(1 - x^2)D^2 f(x) - 2x Df(x) + \mu f(x) = 0$$

will lead to the notion of the deformed Legendre polynomials when $\mu = [n][n + 1], n = 0, 1, 2, 3, \cdots$,

$$ P_n^{(p)}(x) = \sum_{k=0}^{[n/2]} \frac{(-)^k [2n - 2k]! x^{n-2k}}{2^k k!(n-k)!(n-2k)!}. $$

The first few of $P_n^{(p)}(x)$ are

$$ P_0^{(p)}(x) = 1, \quad P_1^{(p)}(x) = x, \quad P_2^{(p)}(x) = \frac{1}{2}([3]x^2 - [1]), $$

$$ P_3^{(p)}(x) = \frac{1}{2}([5]x^3 - [3]x), \quad \cdots. $$

A differential expression for $P_n^{(p)}(x)$ is

$$ P_n^{(p)}(x) = \frac{1}{2^n n!} D^n(x^2 - 1)^n. $$

The main recursion relations of $P_n^{(p)}(x)$ are

$$ [n + 1]P_{n+1}^{(p)}(x) - [2n + 1]xP_n^{(p)}(x) + [n]P_{n-1}^{(p)}(x) = 0, $$

$$ DP_{n+1}^{(p)}(x) - xDP_n^{(p)}(x) - [n + 1]P_n^{(p)}(x) = 0, $$

$$ xDP_n^{(p)}(x) - DP_{n-1}^{(p)}(x) - [n]P_n^{(p)}(x) = 0, $$

$$ DP_{n+1}^{(p)}(x) - DP_{n-1}^{(p)}(x) - [2n + 1]P_n^{(p)}(x) = 0, $$

$$ (x^2 - 1)DP_n^{(p)}(x) - [n]xP_n^{(p)}(x) + [n]P_{n-1}^{(p)}(x) = 0. $$

Note added in proof. The authors would like to thank one of the referees for drawing to their attention a paper by Saxena and Mehta [9], in which the parabose squeezed vacuum state was discussed, and in the present work the excitations on the parabose squeezed vacuum state was considered and normalized in terms of a new kind of deformed Legendre polynomials.
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