The robustness of worldwide metro networks

Longfeng Zhao, Wei Li, Yueying Zhu
1 Key Laboratory of Quark and Lepton Physics (MOE) and Institute of Particle Physics, Central China Normal University, Wuhan 430079, China
E-mail: zlfccnu@mails.ccnu.edu.cn (L. Zhao), liw@mail.ccnu.edu.cn (W. Li)

Abstract. The robustness of worldwide metro networks have been revealed by using random attack and target attack strategies. We find that the metro networks are more robust towards random attack. The robustness of those metro networks have also been compared with two model networks: random networks and scale-free networks. The metro networks are more robust than scale-free networks under target attack but more fragile under random attack. Comparing to random networks, the metro networks are vulnerable under both random and target attacks.

1. Introduction
Our daily life are networked by many infrastructure systems among which the most important one is the transportation network[1]. Metro network is the most important part of modern urban transportation. The transportation cost of metro network is relatively low compare with other commuter ways such as bus and self-driving. The low commute time of metro network relies on its stability. Thus its robustness is of great importance.

Recently many operation failures of metro networks have been reported such as the 2010 bombings in Moscow’s subway network and the 2008 London subway accident. Those are malicious terrorist attacks. Thus it has been extremely important to quantify the robustness of the metro networks from both random failure and target attack.

The complex network theory has taken over most of the science ranging from physics, biology to economics from nearly two decades ago. Since then its wide application to those fields have been extremely fruitful[2, 3, 4, 5, 6]. Thus the use of network representation to characterize the metro networks is quite straightforward. In this paper, we have analyzed the robustness of 28 worldwide metro networks under random attack and target attack. The largest connected component is used as a metric of functionality. We found that the metro networks are relatively robust under random attack compare to target attack. We also simulate two different model networks: Barabási-Albert model(BA network) and Erdős-Rényi model(ER network) as reference models[7, 8]. When comparing to BA networks, the metro networks are more vulnerable under random attack and more robust under target attack. But when comparing to ER networks, the metro networks are more vulnerable under both random and target attacks.

The paper is organized as follows. In Sec. 2 we present the basic statistics of the metro networks. In Sec. 3 we give the main empirical results. The last section provides our conclusion.

2. Basic statistics of metro networks
We have analyzed 28 worldwide metro networks. The basic statistics of those metro networks have been listed below. The basic topological structures of those metro networks have been
discussed in our previous paper[3] and the data source has also been provided therein.

**Table 1.** Topological parameters of those 28 metro networks. $N$ is the number of stations for each metro network; $N_L$ is the number of lines; $m$ is the number of edges; $\langle k \rangle$ is the average degree; $\langle s \rangle$ is the average strength of the metro networks; $\rho$ is the density of the metro networks; $l$ is the average shortest-path length; $C$ is the clustering coefficient; $N_\triangle$ and $N_{\square}$ are the number of 3-node loops and the number of 4-node loops.

| Cities     | $N$   | $N_L$ | $m$    | $\langle k \rangle$ | $\langle s \rangle$ | $\rho$ | $l$   | $C$  | $N_\triangle$ | $N_{\square}$ |
|------------|-------|-------|--------|----------------------|----------------------|-------|-------|------|--------------|--------------|
| Barcelona  | 137   | 153   | 2.234  | 2.277                | 0.016                | 10.87 | 0.037 | 3    | 4            |              |
| Beijing    | 225   | 249   | 2.213  | 2.222                | 0.010                | 14.419| 0      | 0    | 3            |              |
| Berlin     | 307   | 356   | 2.319  | 3.225                | 0.008                | 12.681| 0.015 | 3    | 6            |              |
| Boston     | 120   | 127   | 2.117  | 2.217                | 0.018                | 12.858| 0.033 | 2    | 4            |              |
| Busan      | 120   | 123   | 2.050  | 2.050                | 0.017                | 14.962| 0      | 0    | 0            |              |
| Chicago    | 157   | 177   | 2.255  | 2.408                | 0.015                | 12.353| 0.037 | 4    | 16           |              |
| Delhi      | 137   | 138   | 2.015  | 2.015                | 0.015                | 16.088| 0      | 0    | 0            |              |
| Guangzhou  | 129   | 135   | 2.093  | 2.093                | 0.016                | 13.398| 0      | 0    | 2            |              |
| Hamburg    | 150   | 169   | 2.253  | 3.107                | 0.015                | 9.893 | 0.031 | 3    | 2            |              |
| Hong Kong  | 84    | 87    | 2.071  | 2.214                | 0.025                | 11.064| 0.027 | 1    | 0            |              |
| London     | 289   | 342   | 2.367  | 2.547                | 0.008                | 13.550| 0.041 | 9    | 21           |              |
| Madrid     | 275   | 312   | 2.269  | 2.291                | 0.008                | 15.727| 0.030 | 5    | 13           |              |
| Melbourne  | 216   | 220   | 2.037  | 3.361                | 0.010                | 22.227| 0.035 | 3    | 1            |              |
| Mexico     | 163   | 183   | 2.245  | 2.245                | 0.014                | 11.728| 0.011 | 1    | 2            |              |
| Milan      | 210   | 230   | 2.190  | 2.724                | 0.011                | 10.885| 0      | 0    | 2            |              |
| Montreal   | 68    | 69    | 2.029  | 2.029                | 0.030                | 10.595| 0      | 0    | 0            |              |
| Moscow     | 150   | 174   | 2.320  | 2.333                | 0.016                | 9.916 | 0.066 | 7    | 11           |              |
| New York   | 595   | 741   | 2.491  | 2.807                | 0.004                | 12.312| 0.032 | 20   | 56           |              |
| Osaka      | 97    | 112   | 2.309  | 2.371                | 0.024                | 8.335 | 0      | 0    | 4            |              |
| Paris      | 301   | 357   | 2.372  | 2.445                | 0.008                | 12.079| 0.031 | 7    | 12           |              |
| Seoul      | 497   | 567   | 2.282  | 2.511                | 0.005                | 18.922| 0.020 | 6    | 5            |              |
| Shanghai   | 251   | 275   | 2.191  | 2.271                | 0.009                | 14.986| 0.007 | 1    | 2            |              |
| Shenzhen   | 118   | 126   | 2.136  | 2.136                | 0.018                | 13.618| 0.018 | 1    | 1            |              |
| Taipei     | 101   | 13    | 2.040  | 2.040                | 0.020                | 12.312| 0      | 0    | 0            |              |
| Tokyo      | 216   | 261   | 2.417  | 2.574                | 0.011                | 10.543| 0.048 | 8    | 7            |              |
| Valencia   | 141   | 146   | 2.071  | 2.596                | 0.015                | 15.317| 0.017 | 1    | 0            |              |
| Vienna     | 90    | 96    | 2.133  | 2.133                | 0.024                | 9.929 | 0      | 0    | 1            |              |
| Washington | 86    | 88    | 2.047  | 3.023                | 0.024                | 11.065| 0      | 0    | 0            |              |

In table 1 the basic topological parameters have been given. The number of stations or the number of nodes $N$ in the metro networks range from 68 for Montreal to 595 for New York. Although the number of lines in the metro networks $N_L$ and number of links $m$ vary from city to city, but the average degree $\langle k \rangle$ of those metro networks are very close to 2. The lack of 3-node loops $N_\triangle$ and 4-node loops $N_{\square}$ make those metro networks very similar to tree graphs with very small clustering coefficient $C$, relatively large shortest path length $l$ and quite sparse network density $\rho$. Those topological quantities give us a very detailed description of the geometrical structures of the metro networks. In the following context we will try to evaluate the robustness of those metro networks from two attack strategies: random attack and target
3. Robustness of metro networks

Here we use two attack strategies to model the failure of metro networks. The first one is random attack strategy which means there are \(1 - p\) fraction of nodes will be randomly removed from the metro networks. The second one is the target attack strategy. Here we use the degree based attack strategy which means \(1 - p\) fraction nodes will be removed based on their degree ranking. Thus after the attack, there are only \(p\) fraction of nodes left in the metro networks. We use the largest connect component \(G\) as a metric of robustness of the networks. In this way, we assume that only the largest connected component can remain functional after the attack.

![Figure 1](image_url)

**Figure 1.** The largest connected components of 28 metro networks under both random attack and target attack. The red lines denote the largest connected components as a function of remaining node fraction \(p\) under random attack strategy and the blue lines represent the largest connected components under target attack strategy.

In figure 1, we have given the largest connected component \(G\) of those 28 metro networks under both random and target attacks. The red lines are the largest connected component \(G\) as a function of remaining node fraction \(p\) under random attack and the blue lines are the results for target attack. Very distinctive patterns for different attack strategies manifest the robustness of metro networks under different failure types. Obviously we see that the metro networks are more robust under random failure. The very sharp decrease of the largest connect component \(G\) under degree based attack gives a hint about the importance of those hub nodes in the metro networks. Those stations are extremely important for the maintenance of the functionality. Thus those large degree stations should be taken very strict protection.

In order to further quantify the robustness of those metro networks, we have simulated 28 Barbási-Albert networks\(^{10}\). Those BA networks are simulated with the same sizes as those 28 metro networks. Figure 2 gives the largest connected components \(G\) of metro networks and
the largest connected components $G$ of the corresponding BA networks under both random attack and target attack. In figure 2(a) we can see that average size of giant components for BA networks are larger than that of metro networks under random attack. Whilst in figure 2(b) the size of giant component for BA networks are smaller than that of metro networks. The different behavior of metro networks compare to BA networks under different attack strategies means that the design of metro networks is more robust under target attack.

Meanwhile, we also simulated 28 ER networks with the same size as those 28 metro networks. Figure 3 gives the giant component of both metro networks and ER networks under two attack strategies. We observe that the size of giant component for metro networks are consistently smaller than that of ER networks for different remaining node fraction $p$. Thus we conclude that the metro networks are less robust compare with ER networks under both random and target attacks. This maybe rooted in the spatial nature of metro networks which will hinder the formation of shortcuts during the growth of metro networks. Thus the lack of shortcuts and long range connections make the metro networks extremely vulnerable compare to random networks.
4. Conclusion
In conclusion, we have analyzed the robustness of 28 worldwide metro networks. By using two attack strategies: random attack and target attack, we can access the robustness of metro networks from the size of largest connected component. We find that the metro networks are more robust under random attack, but fragile under target attack. This gives a hint about the importance of those hub stations which deserve strict protection.

We also simulate 28 BA networks and 28 ER networks with the same sizes of corresponding metro networks. Under two different attack strategies, we find metro networks are more robust than BA networks under target attack and less robust than BA networks under random attack. Meanwhile, the metro networks have been proved to be less robust than ER networks under both random and target failures.

Those results presented here can be recognized as a guidance for future metro system design. There also exist some research directions such as robustness evaluation of metro networks from network control, the robustness of metro networks coupled with other infrastructure systems, e.g., bus networks and railway networks. It is also very interesting to access the robustness of metro networks from their community structure\cite{11, 12, 13}. Those will subject to further researches.

Acknowledgement
This work is supported in part by the Programme of Introducing Talents of Discipline to Universities under grant NO. B08033 and the program of China Scholarship Council (No. 201606770023).

References
[1] Barabási A L 2011 Nature Physics 8 14–16
[2] Newman M E J 2003 SIAM Rev 45 167
[3] Boccaletti S, Latora V, Moreno Y, Chavez M and Hwang D U 2006 *Physics Reports* **424** 175-308
[4] Zhao L, Li W and Cai X 2016 *Physics Letters A* **380** 654–666
[5] Zhao L, Wang G J, Wang M, Bao W, Li W and Stanley H E 2018 *Physica A*, https://doi.org/10.1016/j.physa.2018.05.039.
[6] Zhao L, Li W, Fenu A, Podobnik B, Wang Y and Stanley H E 2018 *Journal of Statistical Mechanics: Theory and Experiment* **2018** 23402
[7] Erdös P and Rényi A 1959 *Publicationes Mathematicae (Debrecen)* **6** 290–297
[8] Barabási A L and Albert R 1999 *Science* **286** 509–512
[9] Li W, Gu J, Liu S, Zhu Y, Deng S, Zhao L, Han J and Cai X 2014 *Preprint* 1403.7844
[10] Albert R and Barabási A L 2002 *Reviews of Modern Physics* **74** 47–97
[11] Fortunato S and Hric D 2016 *Physics Reports* **659** 1–44
[12] Han J, Li W, Su Z, Zhao L and Deng W 2016 *The European Physical Journal B* **89** 272
[13] Han J, Li W, Zhao L, Su Z, Zou Y and Deng W 2017 *PLoS ONE* **12** e0188655