Shape Design of Cable-net for Parabolic Cylindrical Deployable Antenna

Han Zhang¹²³, Bo Dong¹²³, Yiqun Zhang¹²³ and Na Li¹²³

¹Key Laboratory of Electronic Equipment Structure Design, Ministry of Education, Xi’an, 710071, China
²School of Electromechanical Engineering, Xidian University, Xi’an, 710071, China
³Collaborative Innovation Center of Information Sensing and Understanding at Xidian University, Xi’an, 710071, China

Abstract. A method of cable-net shape design based on the equilibrium matrix method is proposed for a new parabolic cylindrical deployable antenna structure with fewer modules. And the inverse iteration method is adopted to find the shape of cable-truss structure with considering the truss deformation induced by cable tension. Firstly, the ideal geometrical configuration of the locally symmetric support cable is designed for the given truss. Then, the pretension distribution of the cable is solved by the equilibrium matrix method under the circumstance of the unchanged topology of cable-net structure, position of nodes and boundary condition. In addition, the inverse iteration method is adopted to find the shape of cable-truss structure. Finally, the validity of the method is verified by simulation analysis.

1 Background

The space-borne parabolic cylindrical deployable antenna has become one of the new development directions of space-borne antennas with its merits of strong directivity, high gain, and easy automatic beam scanning [1]. At present, some developed countries such as the United States and Japan, have begun to apply parabolic cylindrical antennas to many types of spacecraft like precipitation radars and communication satellites [2].

The parabolic cylindrical deployable antenna is mainly composed of a deployable truss, a supporting cable-net structure, a reflecting surface and an adjustable device. The shape design of the mesh deployable antenna is one of the fundamental parts, which is of great significance for ensuring the shape accuracy and structural stability of reflector. On the one hand, shape accuracy of reflector surface is an important index to measure and evaluate the performance of antenna. The metal reflective mesh is attached to the convex side of front net, and its shape accuracy is limited by the geometry of cable-net structure. On the other hand, the flexible cable-net structure is a tension structure, and the tension balance of overall structure needs to be obtained by finding the shape and state to obtain the required stiffness and structural stability [3].

For the shape design of the parabolic cylindrical deployable antenna, we design the initial pretension of supporting cable-net structure at the first step. Without considering the deformation of truss structure, points connecting the cable-net structure and truss are constraint nodes, and the node position of cable-network boundary points remain unchanged during the morphological analysis of cable-net structure. Then, the independent pretension optimization design of cable-net structure is carried out by using the equilibrium matrix method. The optimization result is used as the design result of the initial pretension of cables. Finally, based on the finite element model of cable-truss structure, the idea of inverse iteration is adopted, and the initial pretension design result of cable-net is used as the initial value of inverse iteration. The shape accuracy of antenna with taking the truss deformation into consideration would meet the design requirement by updating the coordinates of front cable-net iteratively and the morphological analysis of cable-truss structure would be realized.

2 Initial geometry of the supporting cable-net structure

Generally, the initial shape design of supporting cable-net structure mainly includes two aspects, shape and state [4]. Shape refers to the accuracy of reflector. Compared with cable membrane structure of general construction, the mesh antenna requires extremely high reflective precision to ensure electromagnetic performance of antenna. Generally speaking, the RMS of reflector should be less than 1/30~1/50 of wavelength [5]. The higher the frequency of antenna, the higher the accuracy of reflective surface. State refers to the cable tension distribution of antenna. Uniform cable tension not only helps to improve the accuracy of reflector, but also avoids the phenomenon of cable slack in the harsh space thermal environment.

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2.1 Initial geometry of the front cable-net

The reflecting surface of parabolic cylindrical antenna is formed by small planes and there is inevitably a certain error between actual reflector and ideal reflector. It is called principle error of reflector which is introduced by the process of utilizing planes to approximate the ideal reflecting surface. Generally, principle error is determined by the configuration of front net and location of the cable nodes.

In order to ensure high shape accuracy of the cable-net, initial geometric configuration of front net can generally be determined by the minimum root mean square value \( (\delta_{\text{rms}}) \) method [6]. The method ensures that principle error is within a certain range by controlling the size of the grid. Given that the equation of reflective surface of parabolic cylindrical antenna is parabola, the equation could be approximated by a circle with radius of \( R \) based on the minimum root mean square value \( \delta_{\text{rms}} \). Fig. 1 shows the geometric relationship between the circle and parabola, and they coincide at the apex and the surrounding points. The following relationship is obtained

\[
R=2\sqrt{f\left(\frac{f}{2}\right)}
\]

where, \( R \) is the radius of circle, \( f \) is the focal length and \( D \) is the aperture of antenna.

Cables are straight caused by the pretension under the microgravity space environment, and the arc is approximated by a straight line. Assuming that the length of cables is approximately equal, the resulting surface error is improved by using the modified minimum RMS \( \delta_{\text{rms}} \) value which approximates Eq. 2

\[
\delta_{\text{rms}} = \frac{L}{26.2R}
\]

Furthermore, the maximum length limited formula of the cable-net structure of parabolic cylindrical antenna is equal to

\[
L_{\text{max}} < \sqrt{26.2 \times \delta_{\text{rms}} \left(2f + D^2/32f\right)}
\]

Fig. 1. Parabolic arc approximation.

2.2 Initial geometry of the rear cable-net

Due to the limitations of rocket carrying space and capacity, the vertical bars of the antenna back frame should not be too high [7]. In order to ensure that the unfolding height of parabolic cylindrical antenna meets the requirements, the rear cable-net of parabolic cylindrical antenna is modularized by the modular idea as shown in Fig. 2. Firstly, the initial geometric configuration of front cable-net is divided into two segments by the boundary point \( pB \) along the parabola direction. Secondly the local front cable-net on the left side is obtained by doing an anti-symmetric projection about the local coordinate system \( x' \) under the local coordinate system \( x_o'z'_o \). And the initial geometric configuration of the left rear cable-net module as shown in Fig. 2 is obtain by translating the projection to the distance of \( \Delta z \) in the negative direction of the axis \( z'_o \). Where, the origin \( o' \) of local coordinate system \( x'_o'z'_o \) is midpoint of line which connects the leftmost point \( pA \) of the front cable to the boundary point \( pB \). Similarly, the initial geometry of the rear cable-net module on the right side of Fig. 2 is generated. Finally, the vertical cables are established by connecting the front to rear cable-net nodes.

![Fig. 2. Modular design of the rear net of antenna.](image)

3 Initial pretension design of cable-net structure

After initial geometric configuration of cable-net is established, the whole cable-net structure attributed to the flexible tension structure should maintain the required rigidity and structural stability by finding tension-balanced shape and state. The geometry of cable-net is unchanged during the designed force-finding process, and the shape accuracy of reflective surface is easy to ensure. The equilibrium matrix method is a typical design method of finding force by shape design. Firstly, the ideal geometry of cable net structure is given. Then, the force balance equations of internal nodes are established under the condition of unchanged geometry and the pretension distribution of cable-net satisfying the condition is obtained by solving the equilibrium equation.

The equilibrium equation of the cable-net structure without external load can be expressed as

\[
B \cdot T = 0
\]

where \( B_{\text{cnet}} \) is the balance matrix, and \( T_{\text{cnet}} \) is the cable tension column vector. \( n \) is the number of internal nodes of cable-net structure, and \( m \) is the number of cable elements of cable-net structure.

The cable pretension can be expressed as a linear combination of zero-space orthogonal bases of the equilibrium matrix by the balanced matrix method [8]

\[
T = [V]_{cnet} [\beta]_{cnet}
\]

where, \([V]_{cnet}\) is the zero-space orthogonal basis matrix of the balance matrix \( B \). \([\beta]_{cnet}\) is the combined coefficient column vector. And \( s \) is the independent self-internal force modal number of cable-net structure.

The number of nodal force balance equation of cable-net structure \( 3n \) is usually smaller than the number of cable elements \( m \). There is a problem of multiple solutions for such equilibrium equations, according to the theory of linear algebra. And the initial pretension design of cable-net structure can be translated into how to find a
combination of a set of self-internal force modes so that the pretension is as uniform as possible and the cable-net structure is in equilibrium under the action of pretension. The optimization model can be described as that the design variable is the combination coefficient of internal force mode, and minimizing the maximum tension ratio of mesh cable segment is optimization target. Besides, the constraint conditions include the equilibrium equation of the cable-net structure and the upper and lower limits of cable pretension. The optimal value is solved by an optimization method which can be expressed as

\[
\mathbf{\beta} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_s \end{bmatrix} \quad (i = 1, \ldots, s)
\]

\[
\text{Find} \quad \mathbf{\beta} = \begin{bmatrix} \beta_i \end{bmatrix} 
\]

\[
\text{Minimize} \quad f = \max \left( \frac{\max(T_{\text{front}})}{\min(T_{\text{front}})} \cdot \frac{\max(T_{\text{back}})}{\min(T_{\text{back}})} \cdot \frac{\max(T_{\text{ver}})}{\min(T_{\text{ver}})} \right)
\]

\[
\text{Subject to} \quad T_{\text{min}} < \mathbf{T} < T_{\text{max}}
\]

where, \( T_{\text{front}}, T_{\text{back}}, \text{and} T_{\text{ver}} \) are the pretension vectors of front cable-net, rear cable-net and the vertical cable respectively. \( T_{\text{min}} \) and \( T_{\text{max}} \) are the upper and lower limitations of cable tension required by the design respectively. \( \mathbf{\beta} \) is a column vector composed of \( s \) combined coefficients \( \{\beta_i\} (i = 1 \ldots s) \).

4 Cable-truss combination structure design

For cable-truss structure, when antenna truss structure is deformed deeply due to the tension of cables, the position of boundary supporting points of cables would change, which will affect the distribution of tension and surface accuracy of reflector. For the highest reflector accuracy, the pretension of cable-net structure needs to be further adjusted to compensate the influence of truss deformation. Based on the finite element model of the cable-back frame combined structure, the coordinates of front cable-net nodes are updated with the inverse iterative idea to make root mean square error satisfy the design requirements of reflector accuracy after the cable-truss structure in static self-balanced. The inverse iterative method is an analytical method based on the principle of similarity. For the example in Fig. 3, the iterative process can be expressed as below

Fig. 3. The schematic diagram of the inverse iteration method.

(1) Fig. 3(a) describes the initial geometric configuration of structure. Assuming that \( k \) is the number of iterations and a set of cable tensions are given, static analysis of structure is carried out. And the deformation \( d_i \) at point \( M \) is shown in Fig. 3(b).

(2) The deformation \( \sum_{i=1}^{k} d_i \) is added to initial geometry of the structure reversely with same cable tension given, which is shown in Fig. 3(c). After static self-balancing, the deformation at point \( M \) is \( d_{k+1} \) as shown Fig. 3(d).

(3) Repeat step(1) and (2) until the deformation of structure \( d_{k+1} \) at point \( M \) with the initial given cable tension satisfies the convergence condition, which means \( d_{k+1} \leq \varepsilon \), where \( \varepsilon \) is the convergence accuracy of deformation.

5 Simulation analysis

For a parabolic cylindrical antenna with \( 30m \times 100m \) aperture, suppose that it works in L-band and its reflector accuracy requirement is \( \delta_{\text{rms}} > 5 \text{mm} \), the principle error of cable-net structure design in this paper (The error introduced by reflecting surface approaching ideal parabolic reflecting surface) is better than 1.5 \( \text{mm} \), and the offset distance is \( \rho = 0.8 \text{m} \), and the focal length of front cable-net is \( f_1 = 24 \text{m} \). According to the improved minimum root mean square value (\( \delta_{\text{rms}} \)), the main cables in the direction of paraboloid need to be divided into at least 24 segments. Then, the principle error of antenna reflector surface is about \( 1.21 \text{mm} \leq 1.5 \text{mm} \) which meets the design accuracy requirement.

The points connecting cable-net structure to antenna truss are used as constrained nodes, and the independent pretension optimized design of cable-net structure is carried out by the equilibrium matrix method. The design values are shown in Tab. 1. The finite element model of cable-truss structure is shown in Fig. 4. The material of vertical cable and cable-net are all aramid. The material properties are \( E = 20 \text{GPa}, \nu = 0.3 \), and the diameter is \( 1.4 \text{mm} \). All the bars are carbon fiber, and the material properties are \( E = 206 \text{GPa}, \nu = 0.3 \). The vertical bar and horizontal bar is the same with \( 47 \text{mm} \) inner diameter and \( 50 \text{mm} \) outer diameter. As for diagonal bar and short bar, the inner diameter are \( 57 \text{mm} \) and \( 17 \text{mm} \), and the outer diameter are \( 60 \text{mm} \) and \( 20 \text{mm} \) respectively.

Tab. 1 The design value of initial pretension of cable-mesh antenna

|                     | Maximum(N) | Minimum(N) |
|---------------------|------------|------------|
| Lateral tension     | 59.18      | 44.21      |
| Vertical tension    | 11.79      | 8.87       |
| Vertical cable tension | 3.08     | 1.0        |

Based on the finite element model of cable-truss
structure shown in Fig. 4, the inverse iterative method regarding the initial pretension design result of the cable-net as the initial value is adopted, and then reflector accuracy \( \delta'_{rms} \) of the deformed truss structure satisfies convergence condition \( \varepsilon \) under the condition of updating node coordinates of front cable-net so that the shape design of cable-truss structure would be realized. The iterative curve of shape accuracy is shown in Fig. 5, where the convergence precision is \( \varepsilon = 10^{-5} \text{m} \).

It can be seen from Fig. 5 that the initial shape accuracy RMS of antenna is 2.85 mm after the antenna truss is subjected to the pretension of cable. The inverse iterative method is used to do form-finding of cable-truss structure, and it satisfies convergence condition only after 7 iterations. Finally, the shape accuracy RMS is improved to 0.0095 mm, which shows the effectiveness of cable-net shape design method greatly.

![Fig. 5. The inverse iteration graph of cable-truss structure.](image)

### 6 Conclusion

In order to realize a cable-net shape design of a parabolic cylindrical deployable antenna based on modularization, the initial geometric configuration of partially symmetric supporting cable-net structure is established, and the initial shape design of the supporting cable-net structure is realized by the inverse iterative method in this paper.

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