Constraints on Nuclear Saturation Properties from Terrestrial Experiments and Astrophysical Observations of Neutron Stars

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Abstract

Taking into account the terrestrial experiments and the recent astrophysical observations of neutron stars and gravitational-wave signals, we impose restrictions on the equation of state (EoS) for isospin-asymmetric nuclear matter. Using the relativistic mean-field model with SU(3) flavor symmetry, we investigate the impacts of effective nucleon mass, nuclear incompressibility, and slope parameter of nuclear symmetry energy on the nuclear and neutron star properties. It is found that the astrophysical information of massive neutron stars and tidal deformabilities, as well as the nuclear experimental data, plays an important role to restrict the EoS for neutron stars. In particular, the softness of the nuclear EoS due to the existence of hyperons in the core gives stringent constraints on those physical quantities. Furthermore, it is possible to put limits on the curvature parameter of nuclear symmetry energy by means of nuclear and astrophysical calculations.

Unified Astronomy Thesaurus concepts: Neutron stars (1108); Nuclear astrophysics (1129); Nuclear physics (2077)

1. Introduction

Neutron stars are known to be the densest objects that are visible in our universe. Since the first discovery of a pulsar (Hewish & Oyoye 1965; Hewish et al. 1968), many theoretical studies regarding the understanding of neutron star characteristics have been performed because the nature of neutron stars is broadly determined by the nuclear equation of state (EoS). At present, neutron stars are believed to be cosmological laboratories for dense nuclear matter (Glendenning 2000; Lattimer & Prakash 2007). Conversely, the accurate data from astrophysical observations can enable us to select the appropriate EoSs for neutron stars; thus, it may be possible to figure out the properties of nuclear matter around and beyond the nuclear saturation density.

Thanks to advanced technological developments in science, some invaluable information due to astrophysical observations has been reported in the last decade. In particular, Shapiro delay measurements of a 2 M_{\odot} neutron star have had a great impact on the astrophysical and nuclear communities because it is very difficult to explain such massive neutron stars using the existing EoSs with exotic degrees of freedom, such as hyperons and/or kaon condensates (Glendenning & Moszkowski 1991; Schaffner-Bielich 2008). The binary millisecond pulsar J1614−2230 had a mass of 1.97 ± 0.04 M_{\odot} (Demorest et al. 2010), and it was recently updated to 1.908 ± 0.016 M_{\odot} (Arzoumanian et al. 2018). The mass of PSR J0348+0432 is estimated to be 2.01 ± 0.04 M_{\odot} by a combination of radio timing and precise spectroscopy of the white dwarf companion (Antoniadis et al. 2013). Furthermore, an extremely massive millisecond pulsar, J0740+6620, has been found, and the mass is measured to be 2.14^{+0.10}_{−0.08} M_{\odot} (Cromartie et al. 2019).

In addition to the observations of massive neutron stars, a new type of observational data has been established by the gravitational waves (GWs) from binary neutron star mergers detected by the Advanced LIGO and Advanced Virgo observatories (Abbott et al. 2018, 2019). Because the GW signals of binary neutron star inspirals can potentially yield robust information on the nuclear EoS, it is quite useful to consider the tidal deformability of a neutron star (Hinderer 2008; Hinderer et al. 2010). The dimensionless tidal deformability, \Lambda, is defined as \Lambda = \frac{3}{k^2}\left(\frac{R}{M}\right)^5, where k2 is the second Love number and M and R are, respectively, the mass and radius of a neutron star. Recently, many theoretical discussions have focused on the GW information and the tidal deformability (Fattoyev et al. 2013; Annala et al. 2018; De et al. 2018; Lim & Holt 2018; Most et al. 2018; Raithel et al. 2018; Capano et al. 2020). They have reported that the tidal deformability of a canonical 1.4 M_{\odot} neutron star, \Lambda_{1.4}, is highly sensitive to its radius, R_{1.4}, and the GW signal is of great use to give stringent constraints on the EoS for neutron star matter (Chatziioannou et al. 2018; Kim et al. 2018; Malik et al. 2018; Radice et al. 2018; Tews et al. 2018, 2019; Zhao & Lattimer 2018; Lourenço et al. 2019; Wei et al. 2019; Li & Xie 2020). Although nucleons and/or quark matter are included to construct the EoSs for neutron stars, hyperons, which may be the first candidate of exotic degrees of freedom in the core, are almost ignored in the recent studies. Hence, the explicit inclusion of hyperons as well as nucleons in the core of a neutron star is very important in studying the GW information and tidal deformability.

From the viewpoint of nuclear physics, the nuclear symmetry energy, E_{sym}, is recognized to be a significant physical quantity to explain the properties of finite nuclei and nuclear matter at low densities (Li et al. 2008; Danielewicz & Lee 2009). Although the characteristics of isospin-asymmetric nuclear matter in the density region beyond the nuclear saturation density are still under debate, much progress has...
been made in understanding the density dependence of $E_{\text{sym}}$ based on various analyses of terrestrial experiments such as heavy-ion collisions (Li et al. 2014, 2019; Zhang et al. 2018; Xie & Li 2019, 2020; Miyatsu et al. 2020). Recently, many calculations have focused on the correlation between $E_{\text{sym}}$ and $\Lambda$ to determine the EoSs for neutron-rich nuclear matter at suprahigh densities (Fattoyev et al. 2018; Krastev & Li 2019; Raithel & Ozel 2019; Zhang & Li 2019).

In the present study, we restrict the nuclear EoSs based on the recent data of terrestrial experiments and astrophysical observations using the relativistic mean-field (RMF) model with a nonlinear potential (Walecka 1974; Boguta & Bodmer 1977; Serot & Walecka 1986; Todd-Rutel & Piekarewicz 2005; Fattoyev et al. 2010). In particular, the existence of hyperons in the core of neutron stars is explicitly treated to clarify their impact on the EoS for dense nuclear matter. We take into account the semiempirical data deduced from the realistic $N-N$ interaction (Katayama & Saito 2013; Sammarruca et al. 2015; Drischler et al. 2020) and the analyses of heavy-ion collisions (Tsang et al. 2012; Russotto et al. 2016) in the intermediate-density region. Not only the tidal deformabilities (Abbott et al. 2018) but also the observed maximum masses of neutron stars (Antoniadis et al. 2013; Cromartie et al. 2019) are exploited to investigate the neutron star properties. Moreover, hyperons, as well as nucleons, are explicitly included in the core of a neutron star within SU(3) flavor symmetry (Katayama et al. 2012; Miyatsu et al. 2012, 2013a; Weissenborn et al. 2012), since the exotic degrees of freedom are known to drastically soften the EoSs for neutron star matter, and their effect is still an open question in the multimessenger era (Kumar et al. 2017; Li et al. 2018; Paschalidis et al. 2018; Zhou et al. 2018; Zhu et al. 2018; Li & Sedrakian 2019; Ribes et al. 2019; Sahoo et al. 2019; Fortin et al. 2020). At last, we present the calibrated EoSs for neutron stars and the preferable relations in nuclear saturation properties, such as effective nucleon mass, nuclear incompressibility, and slope and curvature parameters of $E_{\text{sym}}$.

This paper is organized as follows. In Section 2, a brief review of the RMF model in SU(3) flavor symmetry is presented. Numerical results compatible for nuclear and neutron star matter are presented with detailed discussions concerning the correlations among the important physical quantities in Section 3. Finally, we give a summary in Section 4.

### 2. Theoretical Framework

For describing the properties of nuclear and neutron star matter, we employ the usual Lagrangian density in RMF approximation (Walecka 1974; Serot & Walecka 1986). In addition, for the purpose of studying the impact of strangeness in the core of a neutron star, not only the $\sigma$, $\omega$, and $\rho$ mesons but also the strange mesons, namely the isoscalar, Lorentz scalar ($\sigma^*$) and vector ($\phi$) mesons, are taken into account in SU(3) flavor symmetry (Katayama et al. 2012; Miyatsu et al. 2012, 2013a; Weissenborn et al. 2012). Since the charge neutrality and $\beta$ equilibrium conditions are imposed in neutron star calculations, leptons must be introduced as well. The Lagrangian density is thus chosen to be

$$
\mathcal{L} = \sum_B \bar{\psi}_B [\gamma_\mu \partial^\mu - M_B(\sigma, \sigma^*) - g_{\omega B} \gamma_\mu \omega^\mu - g_{\rho B} \gamma_\mu \rho^\mu - i B_\mu \psi_B] + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) + \frac{1}{2} (\partial_\mu \sigma^* \partial^\mu \sigma^* - m_{\sigma^*}^2 \sigma^{*2}) \\
+ \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \omega_{\mu \nu} \omega^{\mu \nu} + \frac{1}{2} m_\rho^2 \rho_\mu \rho^\mu - \frac{1}{4} R_{\mu \nu} \cdot R^{\mu \nu} \\
- U_{NL}(\sigma, \omega^\mu, \rho^\mu) + \frac{1}{3} \sum_\ell \bar{\psi}_\ell (i \gamma_\mu \partial^\mu - m_\ell) \psi_\ell,
$$

where

$$
W_{\mu \nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \quad P_{\mu \nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu, \quad \text{and} \quad R_{\mu \nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu,
$$

is the baryon (lepton) field, $I_B$ is the isospin matrix for the baryon, and $m_\ell$ is the lepton mass. The sum $B$ runs over the octet baryons, $N$ (proton and neutron), $\Lambda$, $\Sigma^+, \Sigma^0, \Sigma^-$, and the sum $\ell$ is for the leptons, $e^-$ and $\mu^-$. The $\omega$, $\phi$, and $\rho$-B coupling constants are respectively denoted by $g_{\omega B}$, $g_{\rho B}$, and $g_{\phi B}$. The effective baryon mass, $M_B^*$, in matter is simply expressed as

$$
M_B^*(\sigma, \sigma^*) = M_B - g_{\omega B} \sigma - g_{\rho B} \sigma^* - \frac{1}{3} \Lambda_{\omega B} (\omega_\mu \omega^\mu) (\rho_\mu \rho^\mu),
$$

is introduced in Equation (1). Here the potential involves three coupling constants, $g_{\omega B}$, $g_{\rho B}$, and $\Lambda_{\omega B}$. In the present study, the hadron, meson, and lepton masses are taken as follows: $M_N = 939$ MeV, $M_P = 1116$ MeV, $M_{\Sigma} = 1193$ MeV, $M_{\Lambda} = 1318$ MeV, $m_\sigma = 500$ MeV, $m_\omega = 783$ MeV, $m_\rho = 770$ MeV, $m_{\sigma^*} = 975$ MeV, $m_\Lambda = 1020$ MeV, $m_\Sigma = 511.5$ MeV, and $m_\ell = 105.7$ MeV.

In RMF approximation, the meson fields are replaced by the constant mean-field values: $\bar{\sigma}$, $\bar{\omega}$, $\bar{\rho}$, $\bar{\phi}$, and $\bar{\rho}$ (the $\rho^0$ field). The equations of motion for the baryon and meson fields in uniform matter are thus given by

$$
[i \gamma_\mu \partial^\mu - M_B^*(\bar{\sigma}, \bar{\sigma}^*) - g_{\omega B} \gamma_\mu \bar{\omega} - g_{\rho B} \gamma_\mu \bar{\rho} - g_{\phi B} \gamma_\mu (I_B) \bar{\phi}] \times \psi_B = 0,
$$

$$
m_\sigma^2 \bar{\sigma} + g_{\omega B} \bar{\omega}^2 + g_{\rho B} \bar{\rho}^2 = \sum_B g_{\omega B} \bar{\rho}_B^2,
$$

$$
m_\sigma^2 \bar{\sigma}^* + \sum_B g_{\sigma B} \rho_B^2,
$$

$$
(m_\omega^2 + 2 \Lambda_{\omega B} \bar{\rho}^2) \bar{\omega} = \sum_B g_{\omega B} \rho_B,
$$

$$
m_\rho^2 \bar{\phi} = \sum_B g_{\rho B} \rho_B,
$$

$$
(m_\phi^2 + 2 \Lambda_{\rho B} \bar{\rho}^2) \bar{\rho} = \sum_B g_{\phi B} (I_B) \bar{\phi}_B.
$$
where the scalar density, $\rho_\sigma^0$, and the baryon density, $\rho_B$, read

$$\rho_B = \frac{1}{\pi^2} \int_0^{k_{FB}} dk \frac{k^2 \sqrt{k^2 + M_B^2(\sigma, \sigma^* \bar{\sigma})}}{k^2 + M_B^2(\sigma, \sigma^*)},$$

$$\rho_B = \frac{k_{FB}}{3\pi^2},$$

with $k_{FB}$ being the Fermi momentum for baryon $B$.

With the self-consistent calculations of the meson fields given in Equations (4)–(8), the total energy density, $\varepsilon$, and pressure, $P$, in neutron star matter are given by

$$\varepsilon = \sum_B \frac{1}{\pi^2} \int_0^{k_{FB}} dk \frac{k^2 \sqrt{k^2 + M_B^2(\sigma, \sigma^*)}}{k^2 + M_B^2(\sigma, \sigma^*)} + \sum_\ell \frac{1}{\pi^2} \int_0^{k_{FL}} dk \frac{k^2 \sqrt{k^2 + m_\ell^2}}{k^2 + m_\ell^2},$$

$$\varepsilon = \frac{1}{\pi^2} \int_0^{k_{FB}} dk \frac{k^4}{k^2 + M_B^2(\sigma, \sigma^*)} + \frac{1}{\pi^2} \int_0^{k_{FL}} dk \frac{k^4}{k^2 + m_\ell^2},$$

$$P = \frac{1}{3} \sum_B \frac{1}{\pi^2} \int_0^{k_{FB}} dk \frac{k^4}{k^2 + M_B^2(\sigma, \sigma^*)} + \frac{1}{3} \sum_\ell \frac{1}{\pi^2} \int_0^{k_{FL}} dk \frac{k^4}{k^2 + m_\ell^2} - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 - \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{2} m_\rho^2 \rho^2 + \frac{3}{2} \Lambda_{\omega/\rho} \bar{\sigma}^2 \bar{\rho}^2,$$

$$P = \frac{1}{3} \sum_B \frac{1}{\pi^2} \int_0^{k_{FB}} dk \frac{k^4}{k^2 + M_B^2(\sigma, \sigma^*)} + \frac{1}{3} \sum_\ell \frac{1}{\pi^2} \int_0^{k_{FL}} dk \frac{k^4}{k^2 + m_\ell^2} - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 - \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{2} m_\rho^2 \rho^2 + \frac{3}{2} \Lambda_{\omega/\rho} \bar{\sigma}^2 \bar{\rho}^2.$$

3. Numerical Results and Discussions

Here we present how to calibrate the nuclear EoS using the RMF model with a nonlinear potential. First, the coupling constants for the nucleon ($N$) are determined so as to reproduce the saturation properties of nuclear matter from terrestrial experiments, as well as theoretical calculations. Then, the calibrated EoSs are applied to the calculations of neutron star matter, compared with the recent data of astrophysical observations, such as the maximum mass of a neutron star, $M_{\text{max}}$, and the tidal deformability of a canonical 1.4 $M_\odot$ neutron star, $\Lambda_{\text{tide}}$. In particular, we focus on the effect of hyperons ($Y$) in the core of a neutron star. Finally, we give some constraints on the nuclear saturation properties, which are not well known, so as to satisfy the data based on both nuclear experiments and astrophysical observations.

In order to deal with the properties of nuclear matter, it is very useful to consider the expansion of isospin-asymmetric EoS with a power series in the isospin asymmetry, $\delta = (\rho_n - \rho_p)/\rho_B$, where the total baryon density is denoted as $n_B = \sum_B \rho_B$ (Chen et al. 2009). The binding energy per nucleon is generally written as $E(n_B, \delta) = E_0(n_B) + E_{\text{sym}}(n_B)\delta^2 + \mathcal{O}(\delta^4)$, where $E_0(n_B)$ is the binding energy of symmetric nuclear matter and $E_{\text{sym}}(n_B)$ is the nuclear symmetry energy,

$$E_{\text{sym}}(n_B) = \frac{1}{2} \frac{\partial^2 E(n_B, \delta)}{\partial \delta^2} \bigg|_{\delta=0}. \quad (13)$$

Besides, $E_0(n_B)$ and $E_{\text{sym}}(n_B)$ can be expanded around the nuclear saturation density, $n_0$, as

$$E_0(n_B) = E_0(n_0) + \frac{K_0}{2!} \delta^2 + \frac{J_0}{3!} \delta^3 + \mathcal{O}(\delta^4),$$

$$E_{\text{sym}}(n_B) = E_{\text{sym}}(n_0) + L\delta + \frac{K_{\text{sym}}}{2!} \delta^2$$

$$+ \frac{J_{\text{sym}}}{3!} \delta^3 + \mathcal{O}(\delta^4),$$

with $\chi = (n_B - n_0)/3n_0$ being a dimensionless variable characterizing the deviations of $n_B$ from $n_0$. The incompressibility coefficient of symmetric nuclear matter, $K_0$, and the slope and curvature parameters of nuclear symmetry energy, $L$ and $K_{\text{sym}}$, are respectively given by

$$K_0 = 9n_0^3 \frac{d^2 E_0(n_0)}{dn_B^2} \bigg|_{n_B=n_0}, \quad L = 3n_0 \frac{dE_{\text{sym}}(n_0)}{dn_B^3} \bigg|_{n_B=n_0},$$

$$K_{\text{sym}} = 9n_0^2 \frac{d^2 E_{\text{sym}}(n_0)}{dn_B^2} \bigg|_{n_B=n_0}. \quad (16)$$

In addition, the third-order incompressibility coefficients of symmetric nuclear matter and nuclear symmetry energy, $J_0$ and $J_{\text{sym}}$, are respectively defined as

$$J_0 = \frac{27n_0^3}{2} \frac{d^3 E_0(n_0)}{dn_B^3} \bigg|_{n_B=n_0}, \quad J_{\text{sym}} = \frac{27n_0^3}{2} \frac{d^3 E_{\text{sym}}(n_0)}{dn_B^3} \bigg|_{n_B=n_0}. \quad (17)$$

3.1. Terrestrial Experiments and Theoretical Calculations

The nuclear saturation properties have been extensively studied so far, and the binding energy per nucleon and nuclear symmetry energy at the saturation density, $E_0(n_0)$ and $E_{\text{sym}}(n_0)$, are determined with a highly accurate precision ( Dutra et al. 2012, 2014; Baldó & Burgio 2016). However, the effective nucleon mass, $M_0^*$, and the higher-order physical quantities, e.g., the incompressibility coefficient, $K_0$, and the slope and curvature parameters, $L$ and $K_{\text{sym}}$, still have a large ambiguity even at $n_0$ (Li & Han 2013; Zhang et al. 2018). As for the quantities, the recent standard values are hence employed as follows. We set $E_0 = -16.0$ MeV and $E_{\text{sym}} = 32.0$ MeV at $n_0 = 0.16$ fm$^{-3}$, but $M_0^*$, $K_0$, and $L$ are supposed to be varied in the range of $0.50 < M_0^*/M_0 < 0.80$, $180 < K_0$ (MeV) $< 320$, and $30 < L$ (MeV) $< 100$, respectively. Here we consider a large range of $K_0$ to extract constraints on the physical quantities without any prejudice because the standard value of $K_0$ from the experimental data is, even now, not fully understood (Blaizot 1980; Stone et al. 2014).

The determining procedure of the coupling constants in the present study follows that in Miyatsu et al. (2013a). Using SU(3) flavor symmetry in the couplings of vector mesons to nucleons, the isoscalar coupling constants, $g_{\omega N}$, $g_{\omega N}$, 6, and $g_8$, are determined so as to reproduce $E_0$, $P$, $M_0^*$, and $K_0$ at $n_0$ with the assumption of $g_{\omega N} = 0$. The values of a mixing angle, $\theta_\rho$, and a ratio of the octet to singlet couplings, $\zeta$, are respectively chosen to be $\theta_\rho = 37^\circ50$ and $\zeta = 0.1949$, which are the suggested values in the Nijmegen extended soft-core model.
(Rijken et al. 2010). Moreover, the coupling constants related to the isovector mesons, $g_{\sigma N}$ and $\Lambda_{\sigma n}$, are fixed to duplicate $E_{\text{sym}}$ and $L$ at $n_0$.

In order to study a limit on $M_N^*$ from the experiments and theoretical calculations, we consider the single-nucleon potential, $U_N^{\text{SEP}}$, based on the so-called Schrödinger-equivalent potential (SEP; Jaminon et al. 1981; Chen et al. 2007),

$$U_N^{\text{SEP}}(k, \varepsilon_k) = \sum_k(k) - \frac{E_N(k)}{M_N} \sum_k(k) + \frac{1}{2M_N}(\sum_k(n(k))^2 - [\sum_k(k)]^2),$$

where the nucleon kinetic energy, $\varepsilon_k$, reads $\varepsilon_k = E_N - M_N$, with $E_N$ being the single-particle energy. The Lorentz-covariant scalar and vector self-energies for the nucleon are respectively given by $\sum_k = -g_s\bar{\sigma} - g_s\bar{\sigma}^*$ and $\sum_k = -g_s\bar{\omega} - g_s\bar{\omega} - g_s\bar{\nu}_k\bar{\rho}$ with the meson fields in Equations (4)–(8). We show $U_N^{\text{SEP}}$ in symmetric nuclear matter at $n_0$ with some experimental data in Figure 1. The shaded band reveals the results of nucleon optical model potential extracted from analyzing the nucleon–nucleus scattering data, denoted by Li et al. (2013). We also include the results of $U_N^{\text{SEP}}$ obtained by the Dirac phenomenology for elastic proton–nucleus scattering data calculated by Hama et al. (1990). It is found that the satisfied value of $M_N^*$ is roughly estimated to be $0.65 \leq M_N^*/M_N \leq 0.75$ in the current energy region. We note that the case of $M_N^*/M_N = 0.69$ is the optimum condition for satisfying the scattering data.

In Figure 2, the density dependence of $E_{\text{sym}}$ and $E_0$ is depicted in the three cases of $M_N^*/M_N = 0.65$, 0.70, and 0.75, which are deduced from the restriction of $M_N^*$ shown in Figure 1. In Figure 2(a), we also present the experimental results obtained from heavy-ion collisions (Tsang et al. 2012; Russotto et al. 2016). It is found that $E_{\text{sym}}$ strongly depends on $L$ below $n_0$, and $L$ should be larger than 40 MeV to match the experimental data in all cases. Because of the lesser dependence of $K_0$, we only show the results in the case of $K_0 = 240$ MeV.

The $E_0$ in the intermediate-density region is presented in Figure 2(b), where $K_0$ varies from 180 to 300 MeV. Although the difference among the results with various $K_0$ is not so large below $n_0$, $E_0$ is sensitive to $K_0$ and $M_N^*$ above $n_0$. On the other hand, since $L$ has little influence on $E_0$, we only show the results in the case of $L = 70$ MeV, which is a middle value in the acceptable range of $L$ explained in Figure 2(a). Compared with the realistic calculations in the Dirac–Brueckner–Hartree–Fock (DBHF) theory (Katayama & Saito 2013) and the chiral effective field theory (χEFT; Sammarruca et al. 2015; Drischler et al. 2020), it is possible to impose some constraints on the nuclear EoSs at higher densities. We find that the allowed range of $K_0$ can be restricted to $180 \leq K_0 (\text{MeV}) \leq 230$, $210 \leq K_N (\text{MeV}) \leq 270$, and $235 \leq K_0 (\text{MeV}) \leq 305$ for $M_N^*/M_N = 0.65, 0.70,$ and 0.75, respectively.

### 3.2. Astrophysical Observations of Neutron Stars

The characteristics of a neutron star are, in general, estimated by solving the Tolman–Oppenheimer–Volkoff equation with the EoS for neutron star matter in which the charge neutrality and $\beta$ equilibrium under weak processes are imposed (Tolman 1934; Oppenheimer & Volkoff 1939). Since the radius of a neutron star is remarkably sensitive to the EoS at very low densities, we adopt the EoS for nonuniform matter below $n_0 = 0.068$ fm$^{-3}$, where nuclei are taken into account using the Thomas–Fermi calculation (Miyatsu et al. 2013b). Moreover, the coupling constants for hyperons are determined so as to fit the experimental data of hypernuclei and the Nagara event in SU(3) flavor symmetry $U_N^{\text{flav}} = -28$ MeV, $U_N^{\text{flav}} = +30$ MeV, $U_N^{\text{flav}} = -18$ MeV, and $U_N^{\text{flav}} \approx -5$ MeV, with $U_N^{\text{flav}}$ being the potential depth for $Y$ in the matter of the baryon species $j$ (Schaffner & Mishustin 1996; Takahashi et al. 2001; Yang & Shen 2008; Miyatsu et al. 2013a). As for a potential depth for $Y$, we employ $U_N^{\text{SEP}}$ given in Equation (18).

The mass–radius relations of a neutron star with and without hyperons are presented in Figure 3. As is well known, if hyperons are taken into account in the core, the maximum mass of a neutron star, $M_{\text{max}}$, is drastically reduced. Thus, in order to explain the observed masses of heavy neutron stars (Antoniadis et al. 2013; Cromartie et al. 2019), it is possible to give more severe constraints on the neutron star EoSs by considering hyperons. It is found that $M_{\text{max}}$ is very sensitive to $M_N^*$ at $n_0$, and the EoSs for $M_N^*/M_N > 0.70$ are ruled out by the observation data once hyperons are included. If we combine both restrictions of $M_{\text{max}}$ (with hyperons) and $U_N^{\text{SEP}}$, shown in Figure 1, we can estimate the more refined condition of $M_N^*$ as $0.65 \leq M_N^*/M_N \leq 0.70$ at $n_0$. Meanwhile, $L$ and $K_0$ have a small impact on $M_{\text{max}}$, and they somewhat affect the neutron star radius.

In Figure 4, the dimensionless tidal deformability of a neutron star with hyperons, $\Lambda$, is presented with the astrophysical constraint on the GW signals from a binary neutron star merger, GW170817, detected by the Advanced LIGO and Advanced Virgo observatories (Abbott et al. 2018, 2019). Hereafter, hyperons are taken into account in all calculations. With the observation data, it is possible to impose constraints on the EoSs for neutron stars using the tidal deformability of a canonical 1.4 $M_\odot$ neutron star, $\Lambda_{1.4}$. It is found that $\Lambda$ becomes small as $M_N^*$ at $n_0$ increases, but the larger $L$, which gives the larger radius of a neutron star shown in Figure 3, consequently brings the larger $\Lambda$, as shown in Figure 4(a). We also see that, in Figure 4(b), there is a strong
correlation between $K_0$ and $\Lambda$, and the smaller $K_0$ is preferred to support the astrophysical data of $\Lambda_{1.4}$. The contour lines of $M_{\text{max}}$ in the $K_0 - L$ plane are presented in Figure 5. In Figure 5(a), we show several lines of $M_{\text{max}}/M_0$ in the case of $M_N^*/M_N = 0.69$, which is the favorable condition for supporting $U_{\text{SEP}}$, as already explained in Figure 1. It is found that $M_{\text{max}}$ is sensitive to $K_0$, while $L$ has a little influence on $M_{\text{max}}$. The larger $K_0$ and smaller $L$ are required to support the heavier $M_{\text{max}}$; thus, the preferable correlation between $K_0$ and $L$ is restricted in the right-hand region above the yellow

Figure 2. (a) Nuclear symmetry energy, $E_{\text{sym}}$, and (b) binding energy per nucleon, $E_0$, as a function of $n_B$. In both top (middle) [bottom] panels, we give the results in the case of $M_N^*/M_N = 0.65$ (0.70) [0.75]. The shaded bands in panel (a) show the experimental data based on heavy-ion collisions, indicated by HIC and ASY-EOS (Tsang et al. 2012; Russotto et al. 2016). In panel (b), we also present the theoretical results based on realistic $N$-$N$ interactions using the DBHF calculation (Katayama & Saito 2013) or the $\chi$EFT (Sammarruca et al. 2015; Drischler et al. 2020).

Figure 3. Mass–radius relations of a neutron star (a) with or (b) without hyperons in the various combinations of $M_N^*/M_N$, $L$, and $K_0$. In both panels, the red (green) [blue] lines correspond to the case of $M_N^*/M_N = 0.65$ (0.70) [0.75], the solid (dashed) [dotted–dashed] lines are for $L = 40$ (60) [80] MeV, and the case of $K_0 = 240$ (280) MeV is given by the thick (thin) lines. The shaded bands also show the observation data of PSR J0348+0432 ($2.01 \pm 0.04$ $M_\odot$) and MSP J0740+6620 ($2.14_{-0.09}^{+0.10}$ $M_\odot$; Antoniadis et al. 2013; Cromartie et al. 2019).
Figure 4. Tidal deformability of a neutron star with hyperons, $\Lambda$, as a function of $M/M_\odot$. The left panel (a) shows the influence of $M_N^\Lambda/M_N$ and $L$ for $K_0 = 240$ MeV, and the right panel (b) expresses the dependence of $K_0$ for $M_N^\Lambda/M_N = 0.70$ and $L = 60$ MeV. The shaded band also shows the astrophysical constraint on the tidal deformability of a canonical 1.4 $M_\odot$ neutron star from the merger event, GW170817 ($70 < \Lambda_{1.4} < 580$; Abbott et al. 2018).

Figure 5. Schematic representation of $M_{\text{max}}$ in the $K_0$–$L$ plane. We show (a) $M_{\text{max}}/M_\odot$ for $M_N^\Lambda/M_N = 0.69$ and (b) $M_{\text{max}}/M_\odot$ for $M_{\text{max}}/M_\odot = 2.01$. For details, see the text.

Figure 6. Same as Figure 5 but for $\Lambda_{1.4}$. We show (a) $\Lambda_{1.4}$ for $M_N^\Lambda/M_N = 0.69$ and (b) $M_N^\Lambda/M_N$ for $\Lambda_{1.4} = 580$. For details, see the text.
line of $M_{\text{max}}/M_0 = 2.01$ to explain the observed mass of PSR J0348+0432 (Antoniadis et al. 2013). In addition, the effect of $M_{\text{max}}^a/M_N$ at $n_0$ on $M_{\text{max}}/M_0$ is depicted in Figure 5(b). We find that the smaller $M_0$ can easily satisfy the $2 M_0$ constraint with the smaller $K_0$, while the larger $K_0$ is needed as $M_0$ increases. We emphasize here that, owing to the softness of the EoSs for neutron stars, as shown in Figure 3.

In Figure 6, we also present the schematic representation of $\Lambda_{1.4}$ in the $K_0$-$L$ plane. Each line indicates the specific values of $\Lambda_{1.4}$ for $M_{\text{max}}^a/M_N = 0.69$ in Figure 6(a). It is found that $\Lambda_{1.4}$ is quite sensitive to $K_0$ and $L$, and, for example, the larger $K_0$ and $L$ lead to the larger $\Lambda_{1.4}$. According to the upper limit based on the GW signals, $\Lambda_{1.4} = 580$ (Abbott et al. 2018), the region below the yellow line can be permitted. Moreover, in Figure 6(b), we consider the influence of $M_{\text{max}}^a/M_N$ on $\Lambda_{1.4}$ in the $K_0$-$L$ plane, focusing on the upper boundary of $\Lambda_{1.4}$. As $M_{\text{max}}^a$ decreases, $L$ becomes sensitive to $\Lambda_{1.4}$ compared to $K_0$. We note that, contrary to the case of $M_{\text{max}}$ shown in Figure 5, the existence of hyperons in the core of a neutron star does not affect $\Lambda_{1.4}$ because hyperons do not have any influence on the radii of neutron stars, as shown in Figure 3.

3.3. Constraints on Nuclear Saturation Properties

By combining the results obtained from the calculations of nuclear and neutron star matter, we put restrictions on the nuclear properties at $n_0$. In Figure 7(a), we show the constrained parameter region in the $K_0$-$L$ plane for $M_{\text{max}}^a/M_N = 0.69$, which is the best-fit value for reproducing $U_N^{\text{SEP}}$ at $n_0$. As explained in Figure 2, the lowest limit of $L$ is presented by the data of the HIC analysis based on terrestrial experiments, and $K_0$ is theoretically constrained by the DBHF (TUS) calculations. Moreover, we see that the $M_{\text{max}}$ of PSR J0348+0432 and $\Lambda$ of GW signals, which are based on astrophysical observations, are very useful for giving constraints on the relations between $K_0$ and $L$. In addition, it is possible to restrict $K_{\text{sym}}$ using the closed, meshed region of $K_0$ and $L$ in Figure 7(b). It is found that the satisfied ranges of $K_0$ and $L$ can be respectively estimated to be $215 \leq K_0 (\text{MeV}) \leq 260$ and $40 \leq L (\text{MeV}) \leq 85$, and the corresponding value of $K_{\text{sym}}$ roughly lies in the range of $-128 \leq K_{\text{sym}} (\text{MeV}) \leq -33$, which is more severe than that of extensive surveys of over 520 theoretical predictions, $-400 \leq K_{\text{sym}} (\text{MeV}) \leq 100$ (Dutra et al. 2012, 2014; Li & Magnon 2020). The present result is almost the same as the constraints calculated by Malik et al. (2018), $-113 \leq K_{\text{sym}} (\text{MeV}) \leq -52$ and $-141 \leq K_{\text{sym}} (\text{MeV}) \leq 16$. Additionally, the restricted relations between $K_0$ and $L$ lead to the constraints on the high-density EoS parameters given in Equations (14) and (15). The corresponding value of $J_0$ is in the range of $-520 \leq J_0 (\text{MeV}) \leq -300$, which is close to the recent calculations by Tews et al. (2017); $-800 \leq J_0 (\text{MeV}) \leq 400$ and Xie & Li (2021); $-390 \leq J_0 (\text{MeV}) \leq 70$. The estimated value of $J_{\text{sym}}$ is also given to be $45 \leq J_{\text{sym}} (\text{MeV}) \leq 1535$ in the present calculations based on the RMF Lagrangian with the minimum set of a nonlinear potential.

Figure 8 shows another acceptable parameter region using the more massive neutron star condition, $M_{\text{max}}/M_0 = 2.14$, for $M_{\text{max}}^a/M_N = 0.65$. If the heavier mass of an observed neutron star, MSP J0740+6620, is taken into account, then the parameter region is further restricted, and $K_0$ and $L$ are respectively estimated to be $215 \leq K_0 (\text{MeV}) \leq 235$ and $40 \leq L (\text{MeV}) \leq 65$ in Figure 8(a). Consequently, the more stringent constraint on $K_{\text{sym}}$ is given to be $-84 \leq K_{\text{sym}} (\text{MeV}) \leq -10$ in Figure 8(b). The allowed regions of $J_0$ and $J_{\text{sym}}$ in the present study are also estimated to be $-450 \leq J_0 (\text{MeV}) \leq -320$ and $1040 \leq J_{\text{sym}} (\text{MeV}) \leq 1540$, respectively.

4. Summary

We have studied the properties of nuclear and neutron star matter using the RMF model with a nonlinear potential. In order to restrict the EoS for isospin-asymmetric nuclear matter in the extensive density region, the terrestrial experiments and recent astrophysical observations of neutron stars and GW signals have been taken into account. Moreover, we have investigated the effects of important physical quantities, namely $M_{\text{max}}^a$, $K_0$, and $L$, which are still unknown even at $n_0$. As for the analyses of nuclear properties in the density region below $n_B = 0.4$ fm$^{-3}$, we have employed the data based on some nuclear experiments and theoretical results. We have found that $M_{\text{max}}^a$ at $n_0$ is roughly estimated to be $0.65 \leq M_{\text{max}}^a/M_N \leq 0.75$ so as to reproduce the $U_N^{\text{SEP}}$ obtained.
from the nucleon–nucleus and elastic proton–nucleus scattering data (Hama et al. 1990; Li et al. 2013). It has also been found that $E_{\text{sym}}$ strongly depends on $L$ below $n_0$, and $L$ should be larger than 40 MeV to satisfy the experimental results obtained from heavy-ion collisions (Tsang et al. 2012; Russotto et al. 2016). In addition, compared with the realistic calculations based on the DBHF theory and the $\chi$EFT (Katayama & Saito 2013; Sammarruca et al. 2015; Drischler et al. 2020), $K_0$ can be restricted to $180 \lesssim K_0 (\text{MeV}) \lesssim 230$, $210 \lesssim K_0 (\text{MeV}) \lesssim 270$, and $235 \lesssim K_0 (\text{MeV}) \lesssim 305$ in the cases of $M_0^*/M_N = 0.65, 0.70$, and 0.75, respectively.

Concerning the neutron star neutron calculations, we have adopted the astrophysical constraints on $M_{\text{max}}$ and $\Lambda_{1.4}$ (Antoniadis et al. 2013; Abbott et al. 2018, 2019; Cromartie et al. 2019). We have also considered the existence of hyperons in the core to restrict the realistic EoS for neutron stars using SU(3) flavor symmetry (Katayama et al. 2012; Miyatsu et al. 2012, 2013a; Weissenborn et al. 2012). It has been found that $M_{\text{max}}$ is very sensitive to $M_0^*$ at $n_0$, and the neutron star EoSs in the cases of $M_0^*/M_N > 0.70$ are ruled out to support 2 $M_\odot$ neutron stars with hyperons. Additionally, we have found a strong correlation between $K_0$ and $\Lambda$, and the smaller $K_0$ is preferred to satisfy the astrophysical data of $\Lambda_{1.4}$.

At last, by combining both calculations of nuclear and neutron star matter, we have presented the constrained relations between $K_0$ and $L$ in Figures 7 and 8. It has been found that, in the case of $M_0^*/M_N = 0.69$ and $M_{\text{max}}/M_\odot = 2.01$, $K_0$ and $L$ can be respectively estimated to be $215 \lesssim K_0 (\text{MeV}) \lesssim 260$ and $40 \lesssim L (\text{MeV}) \lesssim 85$, and the corresponding value of $K_{\text{sym}}$ roughly lies in the range of $-128 \lesssim K_{\text{sym}} (\text{MeV}) \lesssim -33$. If we consider the higher limit of a neutron star mass, it can be possible to impose severe constraints on $K_0$, $L$, and $K_{\text{sym}}$.

In conclusion, it has been found that the astrophysical information of massive neutron stars and tidal deformabilities, as well as the terrestrial nuclear experimental data, plays an important role in restricting the EoS for neutron stars. In particular, the softness of the nuclear EoS due to the existence of hyperons in the core gives stringent constraints on the physical quantities, $K_0$, $L$, and $K_{\text{sym}}$. Since the other exotic degrees of freedom in the core of a neutron star and/or the phase transition from hadrons to quarks may influence $M_{\text{max}}$ and $\Lambda_{1.4}$ (Alford et al. 2013; Miyatsu et al. 2015; Han & Steiner 2019), we have to include their effects as well as hyperons. We leave them for future works.

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