Analysis of Centroid Variation of Three-floating Gyroscope Based on Error Sensitivity

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Abstract. The surface error of the three-floating gyroscope caused the centroid variation of the three-floating gyroscope, which caused the navigation accuracy to decrease. In this paper, the error sensitivity model of three-floating gyroscope was established by using the geometric error transfer theory, the influence of each surface error to the three-floating gyroscope's centroid was calculated, and the results were verified using modeling software. Finally, this paper evaluated the assembly capabilities of the assembly surface. This paper has guiding significance for the three-floating gyroscope to reduce production cost under the premise of ensuring navigation accuracy.

1. Introduction
The three-floating gyroscope has been widely used in industrial control, aerospace, automotive and other fields. With the improvement of navigation accuracy requirements in these industries, the accuracy performance of the three-floating gyroscope is particularly important [1]. Accuracy design is an important part of ensuring the accuracy of the gyroscope. In the traditional gyroscope design, the tolerance level of each part of the gyroscope is mainly determined by the empirical method, but due to the size of the part and its position in the assembly, each mating surface has a different degree of influence on the accuracy of the assembly. If the influence of the surface error on the assembly accuracy cannot be quantified, the precision of the machining will not be reasonably determined, resulting in excessive production cost or low system assembly accuracy. Therefore, it is particularly important to quantitatively calculate the influence of these surface errors on the assembly accuracy of the system.

The three-floating gyroscope is used as an inertial device, and the assembly accuracy is reflected in the variation of the assembly's centroid. By establishing the accuracy model of the three-floating gyroscope, the relationship between the error of mating surface and the error of the assembly's centroid can be reflected. In the analysis of mating surface error sensitivity based on error transfer, many scholars have done a lot of research. He Boxia et al. proposed the concept of part basis deviation and matching reference deviation [2]; Lu Cheng et al. analyzed the problem of assembly joint surface error modeling and tolerance optimization design between parts under various tolerance coupling conditions [3]; Wang Ge et al. analyzed the geometric error transfer model problem [4]. Based on the above research, the paper accumulates the process of geometric error transfer caused by the mating surface error of the three-floating gyroscope during the assembly process, and calculates the sensitivity of the mating surface error to the centroid of the assembly.
2. Structure description and establishment of coordinate system

2.1. Structure Description
There are many kinds of components in the three-floating gyroscope, which are mainly composed of four parts: the housing component, the float component, the torque device/sensor base component and the bellows component. The center-of-gravity offset of the three-floating gyroscope is mainly caused by the misalignment of the center of the float component and the center of the buoyancy support [5]. Since the float component has the greatest influence on the working accuracy of the three-floating gyroscope, this paper mainly studies the centroid variation of the float component. The float component consists of a frame, 2 magnetic suspension rotors (axial), 2 magnetic suspension rotors (radial), a torque rotor, a sensor rotor, a motor assembly, and a pontoon, as shown in Figure 1.

![Fig 1. Schematic diagram of the three-floating gyroscope](image1)

Fig 1. Schematic diagram of the three-floating gyroscope

![Fig 2. Three-floating gyroscope coordinate system diagram](image2)

Fig 2. Three-floating gyroscope coordinate system diagram

2.2. Coordinate system establishment
Taking the ideal centroid point of the pontoon as the origin, the axial direction of the frame is the X axis, and the axial direction of the motor is the Y axis, the Z axis direction is determined according to the Cartesian coordinate system, and the reference coordinate system is established, as shown in Fig. 2. It can be seen from the assembly relationship of the float component that all the parts except the frame are directly matched with the frame, it belongs to a single process. After calculation, the ideal position of each part's centroid is shown in Table 1.

| NO. | Part Name                                  | X/mm    | Y/mm | Z/mm  |
|-----|-------------------------------------------|---------|------|-------|
| 1   | Frame                                     | -0.1076 | 0    | -0.1382 |
| 2   | X+ magnetic suspension rotors (radial)     | 2.5     | 0    | 0     |
| 3   | X- magnetic suspension rotors (radial)     | -2.5    | 0    | 0     |
| 4   | X+ magnetic suspension rotors (axial)      | 0.5     | 0    | 0     |
| 5   | X- magnetic suspension rotors (axial)      | -0.5    | 0    | 0     |
| 6   | Torque rotor                              | 3.0403  | 0    | 0     |
| 7   | Sensor rotor                              | -4.6263 | 0    | 0     |
| 8   | Pontoon                                   | 0       | 0    | 0     |
| 9   | Motor assembly                            | 0       | 0    | 0     |

3. Modeling of the error sensitivity

3.1. Calculating of the centroid variation
The most important reason for the variation of the assembly's centroid is that the geometric error of the part mating surface causes the actual mating surface to be inconsistent with the ideal mating surface. If
the mating surface is flat, the positional variation of the actual mating surface and the ideal mating surface can be expressed by the six degrees of freedom variation; If the mating surface is a cylindrical surface, the positional variation of the actual mating surface and the ideal mating surface can be expressed by the six degrees of freedom variation of the position and direction of the cylindrical surface axis. In summary, the change between the actual mating surface and the ideal mating surface can be expressed as [4]:

\[ U = [\delta_x \ \delta_y \ \delta_z \ \delta_x \ \delta_y \ \delta_z]^T \]  

where \( \delta_x, \delta_y, \delta_z \) is the three tiny rotational components of the pose variation; \( \delta_x, \delta_y, \delta_z \) is the three tiny translational components of the pose variation.

The homogeneous transformation matrix of the ideal mating surface coordinate system to the mating assembly surface coordinate system can be expressed as

\[
T = \begin{bmatrix}
1 & -\delta_z & \delta_y & \delta_x & 0 & 0 \\
\delta_z & 1 & -\delta_x & 0 & \delta_y & 0 \\
-\delta_y & \delta_x & 1 & 0 & 0 & \delta_z \\
0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}
\]  

Set the ideal centroid coordinate to \( X \) (x, y, z), the homogeneous transformation matrix is

\[
X_1 = [x \ y \ z \ 1]^T
\]

The homogeneous transformation matrix of the actual centroid can be expressed as

\[
X_2 = T \times X_1 = \begin{bmatrix}
x - y\delta_z + z\delta_y + \delta_x \\
y - x\delta_z - z\delta_x + \delta_y \\
z - x\delta_y + y\delta_x + \delta_z \\
1
\end{bmatrix}
\]

The matrix of centroid variation is

\[
\Delta = X_2 - X_1 = \begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z \\
0
\end{bmatrix} = \begin{bmatrix}
-y\delta_z + z\delta_y + \delta_x \\
-x\delta_z - z\delta_x + \delta_y \\
-x\delta_y + y\delta_x + \delta_z \\
0
\end{bmatrix}
\]

Combined centroid drift formula

\[
\Delta c = \begin{bmatrix}
\Delta x_c \\
\Delta y_c \\
\Delta z_c
\end{bmatrix} = \begin{bmatrix}
\sum_{i=1}^{m_i} \delta x_i \\
\sum_{i=1}^{m_i} \delta y_i \\
\sum_{i=1}^{m_i} \delta z_i
\end{bmatrix}, \Delta c = \begin{bmatrix}
\sum_{i=1}^{m_i} \frac{\Delta x_i}{\delta x_i} \\
\sum_{i=1}^{m_i} \frac{\Delta y_i}{\delta y_i} \\
\sum_{i=1}^{m_i} \frac{\Delta z_i}{\delta z_i}
\end{bmatrix}
\]

the assembly centroid variation can be obtained.

3.2. Error sensitivity

The influence of each mating surface error on the assembly's centroid can be expressed by error sensitivity. Define the absolute value of the partial derivative of \( \Delta c \) for \( U \) as the error sensitivity.

\[
S_k = \left| \frac{\partial \Delta c}{\partial U_k} \right| = \begin{bmatrix}
\frac{\partial \Delta x_c}{\partial \delta x_k} & \frac{\partial \Delta y_c}{\partial \delta y_k} & \frac{\partial \Delta z_c}{\partial \delta z_k}
\end{bmatrix}
\]

where \( w = x, y, z; k \) is the number of the mounting surface.

3.3. Sensitivity uncertainty analysis

In order to verify the correctness of the error sensitivity, the uncertainty of the sensitivity is analyzed. Using the point cloud data measured by the coordinate measuring instrument to obtain the fitting
surface of each mating surface, and analyzed the centroid changes of the assembly using CREO software modeling. The specific steps are as follows:

- The part is ideally modeled and assembled to measure the centroid position O₁(X₁, Y₁, Z₁) of the assembly.
- The part mating faces are modified to fit the fitting surface and assembled to measure the centroid position O₂(X₂, Y₂, Z₂) of the assembly.
- Calculation error sensitivity uncertainty.

The error sensitivity is measured as

\[ S'_{k} = \frac{\Delta P}{d} \quad \text{or} \quad S'_{k} = \frac{\Delta P}{\alpha \times 0.017} \]  

where \( d \) is the distance from the fitting surface to ideal surface; \( \alpha \) is the angle from fitting surface to the ideal surface; \( \Delta P \) is the distance the centroid moves.

\[ \Delta P = \sqrt{(X_1 - X_2)^2 + (Y_1 - Y_2)^2 + (Z_1 - Z_2)^2} \]  

Uncertainty can be expressed as

\[ u_{rel} = \frac{|S_k - S_k'|}{S_k} \times 100\% \]  

3.4. Surface assembly ability evaluation

To reflect the influence of surface error on assembly's centroid, the assembly capability index \( C_{AP} \) can be defined as

\[ C_{AP} = \frac{\Delta P_{max}}{\Delta P} \]  

where \( \Delta P_{max} \) is the position change of the assembly's centroid caused by the tolerance of mating surface; \( \Delta P \) is the distance from the actual assembly's centroid to the ideal assembly's centroid.

4. Example

4.1. Sensitivity calculation

The error of each assembly surface is shown in Figure 3:

![Fig 3. Each mating surface error](image)

The mass of each part is shown in Table 2.

| Part number | 1    | 2    | 3    | 4    | 5    |
|-------------|------|------|------|------|------|
| Mass/g      | 68.4975 | 0.8239 | 0.8239 | 2.6496 | 2.6496 |
| Part number | 6    | 7    | 8    | 9    |
| Mass/g      | 4.9059 | 2.6403 | 48.0479 | 119.2581 |
Through the above data, combined with the formula (5), (6), (7) and Table 1, the sensitivity of each error can be obtained. After the normalization process, the sensitivity analysis histogram can be drawn, as shown in Figure 4–6.

Fig 4. Sensitivity coefficients for each error associated with $\Delta x_c$

Fig 5. Sensitivity coefficients for each error associated with $\Delta y_c$

Fig 6. Sensitivity coefficients for each error associated with $\Delta z_c$

4.2. Calculation of sensitivity uncertainty
From the above results, it is understood that the error sensitivity of the mating surface 8 and 9 is large, so calculate the sensitivity uncertainty of the mating surface 8 and 9. The data measured by the coordinate measuring instrument can get the error of the matching surface 8 and 9. The centroid variation of the float component can be obtained by CREO software modeling, as shown in Table 3.

| $d_{x8}$ | $d_{y8}$ | $d_{z8}$ | $d_{x9}$ | $d_{y9}$ | $d_{z9}$ |
|----------|----------|----------|----------|----------|----------|
| U/mm     | 0.21     | 0.13     | 0.16     | 0.09     | 0.05     | 0.03     |
| $\Delta P$/mm | 0.0403   | 0.0250   | 0.0307   | 0.0428   | 0.0238   | 0.0143   |

From the above data, the error sensitivity of each mating surface can be obtained as shown in Table 4:

| $d_{x8}$ | $d_{y8}$ | $d_{z8}$ | $d_{x9}$ | $d_{y9}$ | $d_{z9}$ |
|----------|----------|----------|----------|----------|----------|
| $u_{rel}$ | 0.02%    | 0.18%    | 0.04%    | 0.19%    | 0.05%    | 0.04%    |

The results show that the sensitivity calculation method is feasible and the calculation level is reliable (the uncertainty is less than 1%).

4.3. Evaluation of assembly capability
From the tolerance of mating surface, the maximum centroid variation of the float component can be obtained by CREO software modeling. Calculated assembly ability of each assembly surface as shown in Table 5.
The results show that the error of the mating surface of pontoon, motor assembly with frame is within the tolerance range, but the assembly capability of the mating surface of motor assembly with frame is poor, and it is necessary to focus on the translation error in the X direction.

4.4. Analysis of results
The centroid position of float component is influenced by the rotation error and the translational error, it is mainly influenced by the translational error. The translational error of the mating surface of pontoon, motor assembly with frame have a significant influence on the centroid position. Therefore, the values of these three errors should be controlled in the precision design.

5. Summary
This study proposed a method for calculating the influence of mating surface error on assembly's centroid variation based on the single-process error transfer principle. The main contribution of the paper is as follows:
- An error sensitivity analysis model based on the principle of single-process error transfer has been established.
- A method for verifying the correctness of the error sensitivity analysis model has been proposed.
- An evaluation method for the mating surface assembly capability has been proposed.

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