Chiral symmetry and classical solution of the NLSUSY model

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Abstract. The classical solution of the equation of the motion for the Nambu-Goldstone fermion of the nonlinear representation of supersymmetry and its physical significance are discussed, which gives a new insight into the chiral symmetry of the standard model for the low energy particle physics.

Supersymmetry (SUSY) [1], which constitutes the space-time symmetry, may be the most promising notion for unifying two Standard Models (tSM), i.e., the SM for the low energy particle physics and Einstein General Relativity theory (GR) for space-time. The linear representation of SUSY (LSUSY) [2] is realized on the multiplet of fields. Various models based upon \(N = 1\) LSUSY have been studied extensively and remarkable phenomenological and field theoretical results have been obtained [1]. The minimal supersymmetric standard model (MSSM), which may stabilize the mass of light Higgs particle, looks severely constrained by the LHC experiment. The unpleasant point of LSUSY model is that the mechanism and the physical meaning of the spontaneous SUSY breaking, e.g., the origin of \(D\) term, is not clear. While the nonlinear representation of SUSY (NLSUSY) [3] is given by the Nambu-Goldstone fermion corresponding to [superPoincaré/Poincaré], which is realized geometrically on specific flat space-time equipped with the robust spontaneous SUSY breaking encoded in the nature of space-time itself. The action containing higher orders of the stress-energy-momentum tensors of the fermion field is almost unique up to the higher-order derivative terms.

Despite the apparent non-renormalizability it may be worth exploring in the NLSUSY framework a new paradigm for the unified theory of nature in terms of the fermion field, which is the long standing challenge. To incorporate the space-time dynamics we assume that the ultimate shape of nature is the four dimensional (Riemann) manifold possessing the NLSUSY degrees of freedom on tangent space, i.e., tangent space is specified by the Grassmann coordinates \(\psi_\alpha\) besides the Minkowski coordinates \(x_\alpha\). NLSUSY can be easily fused with the general relativity (GR) principle, which produces the nonlinear supersymmetric general relativity theory (NLSUSYGR) [4, 5] in the form of Einstein-Hilbert (EH) action of the ordinary GR equipped with the NLSUSY cosmological term indicating the NLSUSY nature of tangent space. NLSUSYGR would break down spontaneously (Big Decay) to EH...
action for ordinary Riemann space-time (graviton) and NLSUSY action for massless Nambu-Goldstone(NG) fermion (superon) corresponding to the spontaneous space-time SUSY breaking: [super GL(4, R)/GL(4, R)] [4, 5] called superon graviton model (SGM).

Although NLSUSY is the non-renormalizable highly nonlinear theory for the fermion, it is shown by the systematic linearizations of NLSUSYGR in flat space-time that the NLSUSY cosmological term is recast (equivalent) to the familiar broken LSUSY theory, i.e., the familiar broken LSUSY theory emerges in the true vacuum of NLSUSY theory as the low energy (effective) theory, where all fields including $D$ term are composed of NG fermion degrees of freedom [5, 6].

The NLSUSYGR scenario has a potential to give new insights into the SUSY effects which can be tested in cosmology and the low energy particle physics [5, 6] In this talk we study the classical solution of the equation of motion for the fermion of NLSUSY model regarded as an asymptotic flat-space limit of NLSUSYGR, which may explain the chiral eigenstates for fermions in the SM(MSSM) for the low energy particle physics.

The $N = 1$ NLSUSY model $L_{\text{NLSUSY}}$ in flat space-time is given by Volkov and Akulov as follows [3]:

$$L_{\text{NLSUSY}} = -\frac{1}{2\kappa^2} |w| = -\frac{1}{2\kappa^2} \left[ 1 + t_a^a + \frac{1}{2}(t_a^a t_b^b - t_b^a t_a^b) + \cdots \right],$$

where

$$|w| = \det w_{ab} = \det(\delta_{ab} + t_a^b), \ t_a^b = -i\kappa^2 \bar{\psi}\gamma^a \partial_b \psi$$

and $\psi$ is a four components Majorana spinor. $L_{\text{NLSUSY}}$ is invariant under $N = 1$ NLSUSY transformation,

$$\delta_{\zeta} \psi = \frac{1}{\kappa} \zeta - i\kappa \bar{\zeta} \gamma^a \partial_a \psi,$$

where $\zeta$ is a global Majorana spinor parameter, and $\psi$ is the coordinates of the coset space $\text{Super-Poincare}/\text{Poincare}$, and subsequently recasted as the NG fermion in the vacuum of $L_{\text{NLSUSY}}$. The constant $\kappa$ with the dimension [length]$^{-1/2}$ gives the strength of the coupling of NG fermion to the vacuum, which is analogous to the pion decay constant $f_\pi$.

The variation of (1) with respect to $\bar{\psi}$ gives the following equation of motion for $\psi$:

$$\bar{\phi}\psi - i\kappa^2 \left\{ T_a^a \bar{\theta}\psi - T_b^b \gamma^b \partial_a \psi + \frac{1}{2}(\partial_a T_a^b - \partial_b T_a^a) \gamma^a \psi \right\}$$

$$-\frac{1}{2}(-i\kappa^2)^2 \epsilon_{abcd} e^j fg^i (T_a^a T^b_b \gamma^c \partial_d \psi + T_a^a \partial_b T^d_d \gamma^c \psi) = 0,$$

where $T_a^b = i\kappa^2 t_a^b = \bar{\psi}\gamma^a \partial_b \psi$. The highest order term with $O(T^4)$ in the NLSUSY action (1) is known to vanish identically for $N = 1$ SUSY [7].

It is not easy to solve (4) in general. However, considering $\psi$ is the NG fermion it is natural to argue the (free) massless solution, namely, we consider the case,

$$\bar{\phi}\psi(x) = 0,$$

which eliminates the lowest order term of (4). In NLSUSYGR scenario, the NG fermion $\psi(x)$ is besides $x^a$ the Grassmann degrees of freedom on tangent space of ultimate space-time $(x^a, \psi; \bar{x}^b)$.

1 Minkowski space-time indices are denoted by $a, b, \cdots = 0, 1, 2, 3$ and the flat metric is $\eta^{ab} = \text{diag}(+, - , - , -)$.

Gamma matrices satisfy $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$ and we define $\sigma^{ab} = \frac{i}{2}[\gamma^a, \gamma^b]$. 
This means that the exact solution of $\psi(x)$ is dictated by the nontrivial geometry of space-time, even in asymptotic flat space-time.

In fact, we have shown in supergravity [8, 9] that the nontrivial exact classical solution of the massless (free) gravitino under the Coulomb type condition $\psi_\mu = \delta_0 \psi$ exists in flat space-time of the Kerr solution of GR, not in that of Schwarzschild solution [10]. Therefore, without restricting to the specific (plane wave) case, the condition (5) is worth considering seriously from the physical point of view, which allow the non-trivial (localized) solution depending on the geometry of space-time. We will see throughout the discussion that Eq.(4) is satisfied identically at each order in $\kappa$, provided $\psi$ is a single chiral state which satisfies (5), i.e. $\vartheta \psi_L(x) = 0$ or $\vartheta \psi_R(x) = 0$. We show below some details of the computation.

First let us study the term $T^a b \gamma^b \partial_a \psi$ in the second term at $O(\kappa^2)$ of Eq.(4), for the first and the third terms at $O(\kappa^2)$ vanish due to $\vartheta \psi$ (5). By using the Fierz transformation and considering Eq.(5) and the (anti)commuting properties characteristic of the Majorana spinor in tensor products, many terms containing the product of $\gamma$ matrices with (pseudo)scalar and (axial) tensor structures vanish provided $\psi$ is chiral and also terms containing $\vartheta \psi(x)$ vanish by (5). Consequently, the following (axial)vector-type terms survive apparently in the second term:

$$T^a b \gamma^b \partial_a \psi \sim \frac{i}{2} \epsilon^{abcd} ( \partial_a \bar{\psi} \gamma_b \gamma_c \partial_d \psi - \partial_a \bar{\psi} \gamma_b \gamma_a \partial_c \psi \gamma_c \partial_d \psi ).$$

Again by the Fierz transformation and the subsequent similar arguments the terms (6) become,

$$-\frac{i}{4} \epsilon^{abcd} ( \partial_a \bar{\psi} \gamma_b \gamma_c \partial_d \psi \gamma_5 \gamma_c \gamma^c \psi - \partial_a \bar{\psi} \gamma_b \gamma_a \partial_c \psi \gamma_c \gamma^c \psi ),$$

which vanishes for the chiral eigenstate of $\psi$. Next we discuss the terms at $O(\kappa^4)$ in Eq.(4).

From the relation,

$$\epsilon_{abcd} \epsilon^{efgd} T^e c T^b f \gamma^c \partial_g \psi$$

$$= -(T^a d T^b c - T^b d T^a c) \vartheta \psi - 2(T^a c T^b c - T^c c T^a b) \gamma^b \partial_a \psi,$$

we give an example of calculations for $T^a c T^c b \gamma^b \partial_a \psi$ in the third term of Eq.(8). By using repeatedly the Fierz transformation and performing the similar arguments the following term survives,

$$T^a c T^c b \gamma^b \partial_a \psi \sim \frac{1}{4} \gamma_a \sigma_{de} \sigma_{fg} \partial^a \psi \bar{\psi} \gamma_b \sigma^{de} \partial^b \psi \bar{\psi} \gamma_c \sigma^{fg} \partial^c \psi.$$

However, it can be shown that the term (9) is expressed by only terms proportional to $\vartheta \psi$ by using Clifford algebra of $\gamma$ matrices and becomes trivial. As for the last term in Eq.(4), the similar argument shows that it vanishes identically for the single chiral state satisfying Eq.(5). This means that the NG fermion with higher order self-interactions of NLSUSY is the massless chiral fermion which satisfies either of the equation $\vartheta \psi_L(x) = 0$ or $\vartheta \psi_R(x) = 0$. This situation is interesting, for the lowest order of $T^a b$, i.e., the kinetic term does not constrain the chirality of the (left-right symmetric) solution by itself, however higher order self-interaction terms with NLSUSY structure as a whole constrain the chirality of the solution to one (left or right) chiral eigenstate, i.e. $\psi$ should be recasted as the chiral eigenstate. If NLSUSYGR is the underlying principle beyond the SM, this mechanism may explain the chiral eigenstate for all massless fermions in the SM, which is a basic assumption within the SM framework. Considering that the observed fermion in the SM is inconsistent with the NG fermion, the SM may be the composite (effective) theory of the NG fermion as in NLSUSYGR scenario.
In fact, in $N = 2$ minimal SGM scenario ($N = 1$ is unphysical [11] in this scenario), LSUSY QED (and probably the MSSM as well) emerges in the true vacuum of NLSUSY, where all particles for the LSUSY multiplet are composites of superons.

For example, the massless fermion $\lambda^I$ ($I = 1, 2$ for leptons and quarks) in Wess-Zumino gauge is composed of superon $\psi^I$ as follows [6];

$$\lambda^I = \left\{ \psi^I |w| - \frac{i}{2} \kappa^2 \partial_a (\gamma^a \psi^I \bar{\psi}^J \psi^J |w|) \right\},$$

(10)

which shows that quarks and leptons $\lambda^I$ are chiral and composites fermions, because the constituent $\psi^I$ (superon) is chiral. The above results do not depend on the representation of $\gamma$-matrices. Here it may be useful to see an explicit form of the chiral eigenstate in a specific basis of $\gamma$-matrices. For example, for a real four-component $\psi = (\psi_1, \psi_2, \psi_3, \psi_4)^T$ Majorana spinor in the Majorana basis [1] of $\gamma$-matrices, $\psi_L$ is expressed as follows,

$$\psi_L = \frac{1}{2} \begin{pmatrix} \psi_1 - i \psi_3 \\ \psi_2 - i \psi_4 \\ \psi_3 + i \psi_1 \\ \psi_4 + i \psi_2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} u_a \\ i u_a \end{pmatrix}, (a = 1, 2)$$

(11)

which is equivalent to a complex two-components Weyl spinor $u_a$.

Finally before conclusions, we mention that NLSUSYGR (SGM) scenario constrains the dimensions of space-time to four, provided we require $SO(D-1, 1) \cong SL(d, \mathbb{C})$, i.e., $\frac{D(D-1)}{2} = 2(d^2 - 1)$ for the Lorentz invariance, which holds for only $D = 4$, $d = 2$ besides the exceptional case $D = 2, d = 1$.

Now we summarize the result as follows. The classical solution of NLSUSY model predicts the chiral eigenstates for the NG fermion, where higher order self-interaction terms of the NG fermion constrain the chirality of itself. The chiral eigenstate for quarks and leptons in the SM (MSSM), i.e. the chiral symmetry, is understood naturally, if quarks and leptons are composites of the chiral fermion(superon), which achieves simultaneously the true vacuum of NLSUSY as demonstrated in the SGM scenario of NLSUSYGR [4, 5].

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