Neutrino Decay in the Doublet Majoron Model

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Abstract

A new Majoron model is presented within the framework of the seesaw mechanism. Its Higgs sector consists of only doublet representations and the lepton-number violation takes place at the same scale of the electroweak symmetry breaking. This model is different from the singlet- or triplet-Majoron model in several respects: it is free from the $\rho$-parameter constraint and it provides moderately fast neutrino decay, but the constraint from the stellar cooling of red giants is satisfied only with an imposed approximate symmetry. A $\tau$ neutrino as heavy as 10 MeV is possible in this model despite various cosmological and astrophysical constraints.
1. Introduction

We present in this paper a new Majoron model. Its Higgs sector consists only of doublet representations under the standard electroweak gauge group $G = \text{SU}(2) \times \text{U}(1)_Y$ and the lepton number symmetry $\text{U}(1)_L$ breaks down spontaneously together with the breaking of $G$. The Majoron, the Nambu-Goldstone (NG) boson associated with the $\text{U}(1)_L$ breaking, is induced as a linear combination of only doublet Higgs bosons which is of course not the case in the singlet Majoron (SM) model [1] or the triplet Majoron (TM) model [2]. In this respect, we refer to the present model as the doublet Majoron (DM) model. There is also a model in which the Majoron belongs to the doublet representation [3], but that version is now ruled out experimentally by the nonobservation of $Z^0 \rightarrow$ invisible light scalars. In the present model the light neutrino masses come about through the seesaw mechanism [4], while in the model of Ref. [3] they are induced radiatively.

One motivation of Majoron models in general is that they may induce neutrino decay through Majoron emission [4]. This decay can provide a way to evade the cosmological constraint on the neutrino mass $m_\nu$, that is, $m_\nu < 100 \text{ eV}$ or $m_\nu >$ a few GeV for a stable neutrino. In fact, this decay in both the TM and SM models is strongly suppressed and it turns out to be almost cosmologically irrelevant: in the TM model the decay only takes place through a loop diagram [3], while in the SM model it is suppressed at tree level by the fourth power of the seesaw factor $(m_\nu/M)$ ($M$ is the mass scale for the right-handed gauge-singlet neutrino) [3]. In the DM model, on the other hand, the decay occurs by the same mechanism as in the SM model, but the suppression of the seesaw factor is weaker; the power reduces to three instead of four. This means that neutrino decay in the DM model could well be cosmologically relevant.

Some time ago, the possibility of a $\tau$ neutrino of about 10 MeV in mass was proposed in a scenario of baryogenesis at the electroweak phase transition in the SM model [7]. However it appears difficult to have such a mass range which also satisfies the cosmological constraint
unless one chooses an unnaturally small \( M \). We aim to investigate the decay of such a neutrino more closely for a similar mass range using the DM model. We will also take into account the constraint of Majoron emission from the supernova 1987A \( \text{[8]} \) and that of nucleosynthesis \( \text{[9, 10]} \).

There is another motivation for looking into Majoron models. Although the seesaw mechanism is an attractive idea to account for the mass hierarchy of the neutrinos compared to the charged leptons \( \text{[4]} \), once it is combined with the observation that the anomalous baryon and lepton number \( B + L \) violating process is in thermal equilibrium at an early stage of the Universe \( \text{[11]} \), it leads to a very strong bound on the neutrino masses \( \text{[12, 13]} \). If both the \( B + L \) violation from the anomalous process and the \( L \) violation from the gauge-singlet neutrino mass term are active together at a certain time during the Universe’s evolution, any primordial excess in \( B \) or \( L \) is wiped out. To avoid this situation, a neutrino mass must be less than 1 eV as long as the anomalous process is fast enough up to a temperature of about \( 10^{12} \) GeV \( \text{[14]} \). This makes any neutrino an unsuitable candidate for a component of dark matter.

One way to evade the above constraint is to make \( U(1)_L \) an exact symmetry above the electroweak phase transition. In the DM model, the \( L \) symmetry is exact before spontaneous symmetry breaking even though the gauge-singlet neutral fermion has mass, and it breaks down at the electroweak scale, which is a natural consequence of its Higgs representation. Provided that this phase transition is first-order and that the anomalous process is suppressed after it \( \text{[15]} \), the \( L \) and \( B + L \) violating processes cannot coexist above or below the phase transition.

2. The model

We now describe the DM model. A basic observation that characterizes the DM model is that a Majorana mass for the gauge-singlet neutral fermions \( N_a \) \((a = 1, 2, 3\) denotes genera-
tions) does not immediately imply $L$ violation. Since $N_a$ belong to a different representation from the doublet leptons $L_a$, they can have a lepton number different from that of $L_a$. We assign zero lepton number to $N_a$ so that their Majorana mass terms do not violate $L$. Instead, we introduce a Higgs doublet $H_1$, different from the ordinary one $H_0$, and assign $L = -1$ to it. (The hypercharge $Y$ is the same for all the doublet Higgs bosons, $Y = 1$.) The Yukawa coupling between $L_a$ and $N_a$ is provided by $H_1$. The spontaneous breakdown of $G$, as $H_0$ and $H_1$ get vacuum expectation values, is accompanied by the violation of $U(1)_L$.

Actually, this simple extension of the minimal standard model to a Majoron model turns out not to be enough. The Majoron obtained this way has components in both $H_0$ and $H_1$. As a result, the Majoron couples to the charged leptons through the hypercharge current $j^Y_\mu$ as well as through the lepton current $j^L_\mu$. This coupling allows Majoron emission in Compton scattering and significantly contributes to the stellar cooling of red giants [3,10]. To avoid this coupling, we introduce another Higgs doublet $H_2$ and assign $L = 1$ to it. As we will see below, we can suppress the Majoron coupling to the charged leptons by a cancellation between $H_1$ and $H_2$.

The Lagrangian is the same as in the standard model except for the part including the extra Higgs bosons and gauge-singlet neutral fermions. The corresponding part is

$$\mathcal{L} = \sum_{\alpha} |D_\mu H_\alpha|^2 - V(H_\alpha)$$

$$+ \sum_a \left[ iL_a^\dagger(D_\mu \bar{\sigma}^\mu)L_a + iC_a^\dagger(D_\mu \bar{\sigma}^\mu)C_a + iN_a^\dagger(\partial_\mu \bar{\sigma}^\mu)N_a \right]$$

$$+ \sum_{a,b} \left[ H_0^\dagger \Gamma_{ab}(L_a^T i \sigma^2 C_b) + \bar{H}_1^\dagger \Gamma_{ab}(L_a^T i \sigma^2 N_b) + \frac{1}{2} M_{ab} N_a^T i \sigma^2 N_b + (\text{h.c.}) \right], \quad (1)$$

where all the lepton fields have been written in left-handed two-component notation; $L_a$ denotes the doublet leptons

$$L_a = \begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix};$$

$C_a$ the singlet charged leptons $e^c, \nu^c$, and $\tau^c$; $N_a$ the singlet neutral fermions; $\bar{H} = i \tau^2 H^*$; $D_\mu$
is the covariant derivative and \( \bar{\sigma}^\mu = (1, -\sigma) \); Note that \( H_2 \) cannot have a Yukawa coupling because of its quantum number \( Y = 1 \) and \( L = 1 \).

The Higgs potential which is invariant under \( G \) and \( \text{U}(1)_L \) consists of thirteen terms, i.e., three of the type \( (H^\dagger_\alpha H_\alpha) \), three of the type \( (H^\dagger_\alpha H_\alpha)^2 \), three of the type \( (H^\dagger_\alpha H_\alpha)(H^\dagger_\beta H_\beta) \), three of the type \( (H^\dagger_\alpha H_\beta)(H^\dagger_\beta H_\alpha) \), and \( (H^\dagger_0 H_1)(H^\dagger_0 H_2) \) plus its hermitian conjugate. For the last term, we have used the rephasing of \( H_\alpha \) to absorb a possible complex phase factor. For \( V \) has thirteen real parameters and has no explicit CP violation. For a wide range of parameters, \( V \) breaks \( G \times \text{U}(1)_L \) down to \( \text{U}(1)_{\text{em}} \) spontaneously with nontrivial real expectation values \( v_\alpha \) of \( H_\alpha \). Let us write the scalar fields as

\[
H_\alpha = \left( \frac{h_\alpha}{\Phi_\alpha} \right), \quad \Phi_\alpha = e^{i(\varphi_\alpha/\sqrt{2}v_\alpha)} \left( v_\alpha + \frac{1}{\sqrt{2}} \phi_\alpha \right). \tag{3}
\]

These thirteen parameters are phenomenologically constrained for the following reason. In the imaginary parts of the neutral sector of the Higgs bosons, there are two NG bosons associated with the violation of \( \text{U}(1)_Y \) and \( \text{U}(1)_L \). An appropriate linear combination is the one absorbed into \( Z^0 \), and the other orthogonal combination remains massless and becomes the Majoron. The specific form of the linear combination for the Majoron in terms of \( \varphi_\alpha \) is obtained by working with the currents \( j^L_\mu \) and \( j^Y_\mu \);

\[
\begin{align*}
  j^L_\mu &= i \sum\alpha L(H_\alpha) \{ H^\dagger_\alpha D^\mu H_\alpha - (D^\mu H_\alpha)^\dagger H_\alpha \} + \sum_f L(f) (f^\dagger \bar{\sigma}^\mu f),
  \end{align*}
\]

where \( L(H_\alpha) \) and \( L(f) \) denote the lepton number assignment for \( H_\alpha \) and the fermion species \( f \); \( j^Y_\mu \) is given by the same equation with \( Y(H_\alpha) \) and \( Y(f) \), the hypercharges, instead of \( L(H_\alpha) \) and \( L(f) \). We write the current-conservation equations using Eq. (3). They are

\[
\begin{align*}
\partial_\mu \partial^\mu \sum\alpha L(H_\alpha)v_\alpha \varphi_\alpha &= -\frac{g}{\sqrt{2} \cos \theta_W} \left( \sum\alpha L(H_\alpha)v_\alpha^2 \right) \partial_\mu Z^\mu + \frac{1}{\sqrt{2}} \partial_\mu j^L_\mu(f) + ... \tag{5} \\
\partial_\mu \partial^\mu \sum\alpha v_\alpha \varphi_\alpha &= -\frac{g}{\sqrt{2} \cos \theta_W} \left( \sum\alpha v_\alpha^2 \right) \partial_\mu Z^\mu + \frac{1}{\sqrt{2}} \partial_\mu j^Y_\mu(f) + ... \tag{6}
\end{align*}
\]

where \( \theta_W \) is the Weinberg angle and \( j^L_Y(f) \) denotes the fermion component in each current. These equations describe the motion of the NG bosons; the right-hand sides represent the
interactions with the other fields. The Majoron is the combination that does not have
the linear $Z_\mu$ term in its equation. We can thus simply use Eqs. (5) and (6) to extract out
the equation for the Majoron. This procedure reveals that the Majoron has a coupling of the
form $g_{\varphi ee} J^{Y^\mu}$ with the strength $g_{\varphi ee}$ proportional to

$$\sum_\alpha L(H_\alpha) v_\alpha^2.$$  \hspace{1cm} (7)

This coupling is highly constrained to be less than about $10^{-13}$ divided by the electron mass
from the astrophysical consideration already mentioned [3, 10].

The reason we employed $H_2$ is to cancel the $H_1$ contribution in (7) [17]. Note that it
vanishes for $L(H_1) = -L(H_2)$ and $v_1 = v_2$. Hence we parametrize $V$ as

$$V = \sum_\alpha \lambda_\alpha (H_\alpha^\dagger H_\alpha - v_\alpha^2)^2 + \sum_{\alpha<\beta} \eta_{\alpha\beta} (H_\alpha^\dagger H_\alpha - v_\alpha^2)(H_\beta^\dagger H_\beta - v_\beta^2)$$

$$+ \sum_{\alpha<\beta} \zeta_{\alpha\beta} \left[ (H_\alpha^\dagger H_\alpha)(H_\beta^\dagger H_\beta) - (H_\alpha^\dagger H_\beta)(H_\beta^\dagger H_\alpha) \right]$$

$$+ \xi \left[ (H_0^\dagger H_0)(H_1^\dagger H_1) + (H_0^\dagger H_0)(H_2^\dagger H_2) - (H_0^\dagger H_1)(H_0^\dagger H_2) - (H_1^\dagger H_0)(H_2^\dagger H_0) \right],$$ \hspace{1cm} (8)

and further require it to be symmetric under the exchange $H_1 \leftrightarrow H_2$. The parameters are
then constrained: $v_1 = v_2 \equiv v_L$, $\lambda_1 = \lambda_2 \equiv \lambda$, $\eta_{01} = \eta_{02} \equiv \eta$, and $\zeta_{01} = \zeta_{02} \equiv \zeta$. $V$ has
a minimum at the same expectation value for $\Phi_1$ and $\Phi_2$ and the Majoron coupling to $J^{Y^\mu}$
vanishes.

Since the couplings to fermions are different for $H_1$ and $H_2$, the above exchange symmetry
is no longer exact once fermion-loop corrections are included. If we adopt the “effective-
potential” method for evaluating the quantum corrections to $V$, they are typically of order $\gamma^4$ ($\gamma$ is the Yukawa coupling of $H_1$ to the neutral fermions). We assume these corrections
are fine-tuned so that they will not have a significant contribution to the $J^{Y^\mu}$ coupling.

We now find out explicitly the particle spectrum in the Higgs sector. For the charged
scalars $h_\alpha$, the mass matrix is given by

$$
\begin{pmatrix}
2(\zeta + \xi)v_L^2 & -(\zeta + \xi)v_0v_L & -(\zeta + \xi)v_0v_L \\
-(\zeta + \xi)v_0v_L & (\zeta + \xi)v_0^2 + \zeta_12v_L^2 & -\zeta_12v_L^2 \\
-(\zeta + \xi)v_0v_L & -\zeta_12v_L^2 & (\zeta + \xi)v_0^2 + \zeta_12v_L^2
\end{pmatrix}.
\tag{9}
$$

The mass eigenstates and their masses are readily evaluated:

$$
h_G = \cos \beta h_0 + \frac{\sin \beta}{\sqrt{2}} (h_1 + h_2); \quad m^2 = 0,
\tag{10}
$$

$$
h_L = \frac{1}{\sqrt{2}} (h_1 - h_2); \quad m^2 = (\zeta + \xi)v_0^2 + 2\zeta_12v_L^2,
\tag{11}
$$

$$
h_H = -\sin \beta h_0 + \frac{\cos \beta}{\sqrt{2}} (h_1 + h_2); \quad m^2 = (\zeta + \xi)(v_0^2 + 2v_L^2),
\tag{12}
$$

where the angle $\beta$ is defined by

$$
\tan \beta = \frac{\sqrt{2}v_L}{v_0}.
\tag{13}
$$

The zero eigenstates $h_G$ correspond to the NG bosons associated with the SU(2) breaking and they are absorbed into the $W^\pm$ gauge bosons. For the real part of the neutral sector, the mass matrix is

$$
\begin{pmatrix}
4\lambda_0v_0^2 & 2\eta v_0v_L & 2\eta v_0v_L \\
2\eta v_0v_L & 4\lambda_0v_0^2 + \xi v_0^2 & 2\eta_12v_L^2 - \xi v_0^2 \\
2\eta v_0v_L & 2\eta_12v_L^2 - \xi v_0^2 & 4\lambda_0v_L^2 + \xi v_0^2
\end{pmatrix},
\tag{14}
$$

and the mass eigenstates and masses are

$$
\phi_L = \frac{1}{\sqrt{2}} (\phi_1 - \phi_2); \quad m^2 = (4\lambda - 2\eta_12)v_L^2 + 2\xi v_0^2, 
\tag{15}
$$

$$
\phi_+ = \cos \alpha \phi_0 + \frac{\sin \alpha}{\sqrt{2}} (\phi_1 + \phi_2); \quad m^2 = 4\lambda_0v_0^2 + \tan \alpha \frac{4\eta v_0v_L}{\sqrt{2}},
\tag{16}
$$

$$
\phi_- = -\sin \alpha \phi_0 + \frac{\cos \alpha}{\sqrt{2}} (\phi_1 + \phi_2); \quad m^2 = (4\lambda + 2\eta_12)v_L^2 - \tan \alpha \frac{4\eta v_0v_L}{\sqrt{2}},
\tag{17}
$$

where $\alpha$ is defined by

$$
\cot \alpha - \tan \alpha = \frac{\sqrt{2}}{\eta} \left[ \lambda_0 \frac{v_0}{v_L} - \left( \lambda + \frac{\eta_12}{2} \right) \frac{v_L}{v_0} \right], \quad -\frac{\pi}{4} < \alpha < \frac{\pi}{4}.
\tag{18}
$$

Similarly for the imaginary part, the mass matrix is

$$
\xi \begin{pmatrix}
4v_L^2 & -2v_0v_L & -2v_0v_L \\
-2v_0v_L & v_0^2 & v_0^2 \\
-2v_0v_L & v_0^2 & v_0^2
\end{pmatrix}.
\tag{19}
$$
The mass eigenstates are
\[
\varphi_A = -\sin \beta \varphi_0 + \frac{\cos \beta}{\sqrt{2}} (\varphi_1 + \varphi_2); \quad m^2 = 2\xi (v_0^2 + 2v_L^2), \tag{20}
\]
\[
\varphi_L = \frac{1}{\sqrt{2}} (\varphi_1 - \varphi_2); \quad m^2 = 0 \tag{21}
\]
\[
\varphi_G = \cos \beta \varphi_0 + \frac{\sin \beta}{\sqrt{2}} (\varphi_1 + \varphi_2); \quad m^2 = 0. \tag{22}
\]

The combination \(\varphi_G\) is absorbed into the \(Z^0\) gauge boson and \(\varphi_L\) is the Majoron.

The motion of the Majoron, especially its interaction with the other fields, is now solely described by the \(j_L^\mu\) conservation, Eq. ⑶. We obtain the effective interaction Lagrangian \(\mathcal{L}_{\text{eff}}\) by requiring that the resulting Euler-Lagrange equation with respect to \(\varphi_L\) coincides with Eq. ⑶. Up to cubic terms, we get
\[
\mathcal{L}_{\text{eff}} = \left( \frac{\partial^\mu \varphi_L}{v_L} \right) \left[ \frac{1}{2} j_L^\mu(f) + \frac{g v_L}{\cos \theta_W} Z_\mu \phi_L - gv_L \left( W_\mu^+ h_L^\dagger + W_\mu^- h_L \right) ight. \\
+ \frac{1}{2} (\partial_\mu \varphi_L) (\cos \alpha \phi_+ + \sin \alpha \phi_+) + cos \beta (\partial_\mu \varphi_A) \phi_L \\
+ \left( \frac{i}{2} \right) \cos \beta \left\{ (\partial_\mu h_H^\dagger) h_L - h_H^\dagger (\partial_\mu h_L) + (\partial_\mu h_L^\dagger) h_H - h_L^\dagger (\partial_\mu h_H) \right\}. \tag{23}
\]

Reflecting on the NG-boson nature of \(\varphi_L\), we have written its interaction in the derivative-coupling form. The first term in the bracket includes the coupling to the neutrinos and induces neutrino decay. The second term is important since it describes the coupling to \(Z^0\).

Remember that the mass of the accompanying scalar particle, \(\phi_L\), is \((4\lambda - 2\eta_{12})v_L^2 + 2\xi v_0^2\). The values for \(v_0\) and \(v_L\) are constrained by \(\sqrt{v_0^2 + 2v_L^2} = 174\) GeV, but otherwise they are free. The expression for the \(\phi_L\) mass can naturally give a bigger value than the \(Z^0\) mass. Hence the decay \(Z^0 \rightarrow \varphi_L \phi_L\) can be forbidden kinematically or else this model would be ruled out by the present LEP experiments. Note also that in the DM model all the Higgs bosons belong to the doublet representation and \(v_L\) and \(v_0\) are free from the \(\rho\)-parameter constraint.
3. Neutrino decay

We now look into the neutrino decay

\[ \nu_\tau \to \nu_\mu \varphi_L \text{ or } \nu_e \varphi_L. \]  

(24)

They are induced by the first term in the bracket in Eq. (23). The existence of these flavor-changing processes can be seen in two steps. First, the conservation of the neutrino portion of \( j^L_\mu(f) \) is violated in the symmetry-broken phase by the appearance together of both the Dirac mass,

\[ m_{Dab} = v_L \Gamma^1_{ab}, \]  

(25)

and the Majorana mass \( M_{ab} \). This explains why the Majoron coupling to a charged lepton is suppressed even though \( j^L_\mu(f) \) has charged-lepton components: their masses are necessarily of the Dirac type, hence that part of \( j^L_\mu(f) \) is automatically conserved at tree level. Second, although \( j^L_\mu(f) \) is diagonal with respect to \( \nu_a \), the mass diagonalization procedure involves both \( \nu_a \) (\( L = 1 \)) and \( N_a \) (\( L = 0 \)), and this generates nondiagonal flavor-changing vertices. This is in contrast to the TM model, where no singlet neutrino is involved and the current is flavor-diagonal even after the diagonalization of the Majorana mass matrix for \( \nu \).

Let us see this second point in detail for the seesaw mass matrix and obtain the neutrino decay vertex explicitly. In the symmetry-broken phase the physical fields are the mass eigenstates. We write the neutrino fields in \( j^L_\mu(f) \) in terms of these. It is done by the following replacement:

\[ \begin{pmatrix} \nu \\ N \end{pmatrix} \to U \begin{pmatrix} \nu \\ N \end{pmatrix} = \begin{pmatrix} U_1 & U_2 \\ U_3 & U_4 \end{pmatrix} \begin{pmatrix} \nu \\ N \end{pmatrix}, \]  

(26)

where we have suppressed the generation index; \( U_i \) are \( 3 \times 3 \) matrices. The unitary matrix \( U \) is obtained by

\[ U^T \begin{pmatrix} 0 & m_{D} \\ m_{D}^T & M \end{pmatrix} U = \text{diag}(m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}, m_{N_1}, m_{N_2}, m_{N_3}). \]  

(27)
Although $U$ is unitary, $U_1$ is not necessarily so. Thus the light neutrino components in $j^f_\mu(f)$ after the replacement, $\nu^i U_1^\dagger U_1 \tilde{\sigma}^\mu \nu$, are not diagonal in general and allow the flavor-changing decay. For a typical seesaw mass matrix ($m_D \ll M$), $U_2$ and $U_3$ are of order $m_D/M$ and the deviation of $U_1$ from a unitary matrix is small, of order $(m_D/M)^2$. The matrices $U_i$ have been obtained order by order in $m_D/M [6]$. $U_1$ is given explicitly up to second order as

$$U_1 = \left(1 - \frac{1}{2} m_D^* \frac{1}{M^2} m_D^T \right) V,$$

where the unitary matrix $V$ is defined by

$$V^T \left(-m_D \frac{1}{M} m_D^T \right) V = \text{diag}(m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}),$$

and in solving $U_1$ we have assumed without loss of generality that $M_{ab}$ is already diagonal with positive eigenvalues. The eigenvalue $m_\nu$ has the typical seesaw size, $m_D^2/M$. The neutrino decay vertex is now given by

$$\mathcal{L} = \frac{\phi_L}{2v_L} \sum_{a,b} \partial_\mu R_{ab}(\nu_a^\dagger \tilde{\sigma}^\mu \nu_b), \quad R \equiv V^\dagger m_D^* \frac{1}{M^2} m_D^T V$$

and the flavor-changing mixing is of order $m_\nu/M$.

The decay width of $\nu_\tau$ is readily evaluated. For $m_{\nu_\tau} \gg m_{\nu_\mu}, m_{\nu_e}$, we obtain

$$\Gamma = \sum_{a=\mu,e} \frac{1}{64\pi} |R_{\nu_\tau \nu_a}|^2 \frac{m_{\nu_\tau}^3}{v_L^2} \approx \frac{\gamma^2 \sin^2 \theta}{64\pi} \left(\frac{m_{\nu_\tau}}{M} \right)^3 m_{\nu_\tau},$$

where we parametrize $|R|$ as $(m_{\nu_\tau} \sin \theta/M)$ with a mixing angle $\theta$ and $(m_{\nu_\tau}/v_L)^2$ as $(\gamma^2 m_{\nu_\tau}/M)$ with the Yukawa coupling $\gamma$ for the $\tau$ neutrino.

In the SM model this decay is also similarly described but the power of $m_{\nu_\tau}/M$ in the final result (31) is 4 instead of 3. This is because the corresponding scalar expectation value $v_L$ in the SM model is related to the mass of the gauge-singlet neutrinos.

Let us compare this width with the cosmological bound. The thermal history of neutrinos in the expansion of the Universe is well studied\[^1\][18]. Their relic density parameter $\Omega_{\nu\bar{\nu}}$ is

\[^1\] The key process to get the relic energy density of a neutrino species in Ref. [18] is $\nu\bar{\nu} \leftrightarrow f\bar{f}$. In the following, we assume that extra processes, intrinsic in a Majoron model such as $\nu\nu \leftrightarrow \phi_L \phi_L$, will not change the results drastically.
given by
\[ \Omega_{\nu\bar{\nu}} h^2 \sim \frac{m_\nu}{91.5\text{eV}} \]
for each light neutrino, \textit{i.e.} that which is relativistic at its decoupling temperature, or by
\[ \Omega_{\nu\bar{\nu}} h^2 \sim \left( \frac{m_\nu}{\text{GeV}} \right)^{-2} \]
for each heavy neutrino, \textit{i.e.} that which is nonrelativistic at its decoupling temperature.

[The notation is the same as in Ref. [18]. Since we are interested in a narrow mass range, around 10 MeV, we have neglected a logarithmic dependence on \( m_\nu \) in Eq. (33).] Thus a neutrino species that is heavier than 100 eV or lighter than a few GeV must decay. For the lifetime constraint we use here that which comes from a consideration on “structure formation” [13], which gives a much stronger bound than the condition of not overclosing the Universe by the relic density of the decay products [20]. This requires the relic density parameter of the decay product to be smaller than that of the radiation, \textit{i.e.} the photon and the light neutrinos,
\[ \Omega_{\gamma\nu} h^2 \sim 4 \times 10^{-5}. \]

The relic density of the relativistic decay product decreases faster than that of the nonrelativistic matter and is approximately given by \( \Omega_{\nu\bar{\nu}} R_D \), where \( R_D \) is a scale parameter related to the lifetime \( \Gamma^{-1} \) by
\[ \Gamma^{-1} \sim \frac{4.9 \times 10^9 R_D^2}{(\Omega_{\gamma\nu} h^2)^{1/2}} \text{ year}. \]

Thus we get
\[ \Gamma^{-1} < 3 \times 10^2 \left( \frac{m_\nu}{1\text{MeV}} \right)^{-2} \text{ sec for light neutrino,} \]
\[ \Gamma^{-1} < 4 \times 10^{-2} \left( \frac{m_\nu}{1\text{MeV}} \right)^4 \text{ sec for heavy neutrino.} \]

We demonstrate in Fig. 1 that it is indeed possible for \( \nu_\tau \) to be as heavy as 10 MeV for a choice of parameters \( \gamma^2 = 10^{-2} \) and \( \sin^2 \theta = 10^{-2} \) as an example. We depict the allowed area
in the \((m_{\nu_\tau} - M)\) plane. The lines AB and BC come from the cosmological bound for light and heavy neutrino, respectively. They are obtained by Eq. (36) or (37) with (31). We have also taken into account a constraint coming from supernova cooling by Majoron emission \[8\], which is shown by the line CD. The region allowed by the cosmological constraint mostly corresponds to that of “Majoron trapping” and it gives \[8\]

\[
\left( \frac{m_{\nu}}{1\text{MeV}} \right) \left( \frac{1\text{GeV}}{v_L} \right)^2 > 3.3 \times 10^{-3}.
\]

Note that the values for \(m_{\nu_\tau}\) and \(M\) implicitly determine the value of \(v_L\) by the relation \(\gamma v_L \sim \sqrt{M m_{\nu_\tau}}\). There is of course also the laboratory upper bound of 35 MeV on \(m_{\nu_\tau}\).

The values of \(v_L\) for the allowed region in Fig. 1 are relatively low compared with \(v_0 \sim 170\) GeV; they are typically a factor of 5 or so less. This predicts a relatively light Higgs boson, the \(\phi_-\) of Eq. (17). Since this Higgs boson is orthogonal to \(\phi_L\) which couples the Majoron \(\varphi_L\) to the \(Z^0\) boson, there is no conflict with the experimental data for the \(Z^0\) width. It can, however, be a rare decay product of \(Z^0\) in

\[
Z^0 \rightarrow \phi_- f \bar{f}.
\]

The branching fraction of this process is suppressed by

\[
(- \cos \beta \sin \alpha + \sin \beta \cos \alpha)^2 \sim 10^{-2}
\]

compared with that of the single Higgs boson of the minimal standard model and thus \(\phi_-\) can still have a mass below the latter’s experimental lower bound of about 60 GeV. Through its \(\phi_0\) component, \(\phi_-\) decays into visible channels such as charged fermion pairs, and may thus be observed in future Higgs-boson search experiments. We have drawn the line AE that corresponds to \(v_L = 10\) GeV in Fig. 1; \(\phi_-\) with this mass has roughly the same branching fraction as the standard Higgs boson of 60 GeV in the process \(39\) \[21\].

Finally we consider the effect of nucleosynthesis. The number of relativistic degrees of freedom at the temperature of about 1 MeV strongly affects the abundance of light elements.
Using the best current nuclear physics data and astronomical observations, the number in terms of corresponding light neutrino species, $N_{\nu}$, is restricted to be less than 3.3. The Majoron contribution to $N_{\nu}$ has been considered extensively in Ref. [9]. The key process for the Majoron to be in thermal contact with the neutrinos is $\nu \nu \leftrightarrow \varphi_L \varphi_L$. Its decoupling temperature $T_D$ is estimated by comparing the inverse mean free path of the process with the expansion rate of the universe,

$$T_D \sim 10^3 \sqrt{g_*(T_D)} \frac{v_L^4}{m_{\nu}^2 m_{Pl}}. \quad (41)$$

The decoupling temperature from the $\tau$ neutrino is the lowest among the three neutrinos and it is much lower than 1 MeV for the parameters that are allowed in Fig. 1. The Majoron keeps thermal equilibrium as long as the $\tau$ neutrino does. Thus its decoupling temperature cannot be high enough to suppress its contribution to $N_{\nu}$, which is given by $(8/7)(1/2) \sim 0.6$.

To keep $N_{\nu}$ within the above-mentioned bound, the $\tau$ neutrino needs to be nonrelativistic at its decoupling temperature and its energy density must be small compared to that of the radiation. Since the energy density of massive matter after decoupling decreases at a slower rate than that of the massless degrees of freedom, it may significantly contribute to the total energy density again afterwards. If this matter domination takes place at the time of nucleosynthesis, it would affect the primordial abundance of light elements. The cosmological constraint on the $\tau$ neutrino lifetime we have used does not allow its energy density or that of its decay products to exceed that of the light degrees of freedom. But it may still contribute partially to $N_{\nu}$. Based on a detailed study of the $\tau$ neutrino lifetime and mass constraints from nucleosynthesis, we thus further require its lifetime to be shorter than 1 second [22, 23]. We show the corresponding boundary by a dashed line in Fig. 1, the region to the right of which remains allowed. Under the conditions that $\tau$ neutrino is strongly nonrelativistic at the decoupling temperature and it decays fast enough before primordial nucleosynthesis, $N_{\nu}$ is given by the sum of the contributions from the Majoron and the two
lighter neutrinos, i.e. 2.6.

In summary, we have presented a new Majoron model. It allows the \( \tau \) neutrino to be in the 10-MeV mass range.

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Figure caption

Fig. 1 Region of the \((m_{\nu_e} - M)\) plane allowed as the result of various constraints for \(\gamma^2 = \sin^2 \theta = 10^{-2}\).
This figure "fig1-1.png" is available in "png" format from:

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