Numerical Analysis of Forced Convection Heat Transfer around Spherical Particles Packed in Fluid Flow

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Abstract. In order to make clear the forced convection heat transfer phenomena around spherical particles packed in fluid flow, we numerically analyzed the heat transfer and flow pattern of the air using a single sphere and then the closest packed structure arrangement of spherical particles. We used 3-dimensional thermo fluid computation code "STAR-CCM+". We calculated the forced convection heat transfer coefficient for spheres of 10 mm diameter with Reynolds number 63 - 6340. Our calculation results of the average heat transfer coefficient for a single sphere agree with the correlation equation proposed by Ranz and Marshall. Local heat transfer coefficient is high at portions where local flow impinges to the surface of spheres for packed spherical particles. Our calculation results of the average heat transfer coefficient for packed spherical particles are close to the correlation equation proposed by Wakao et al.

1. Introduction
Heat transfer between fluid and particles packed in fluid flow appear in many kinds of thermal equipment such as active magnetic refrigerators [1], heat storage tanks filled with particles, and fluidized beds. It is important to understand the forced convection heat transfer phenomena around spherical particles densely packed in fluid flow in order to better design the thermal performance of this kinds of equipment.

Many studies on convection heat transfer and flow pattern around particles in fluid flow have been made. Ranz and Marshall [2, 3] measured the evaporation heat transfer rate from a sphere and proposed a correlation equation of forced convection heat transfer coefficient for a single sphere in fluid flow. Wakao et al. [4] proposed a correlation equation of particle-to-fluid forced convection heat transfer coefficient in packed beds summarising much of the experimental data found in published literature. Inabe and Fukuda [5] proposed a correlation equation of particle-to-fluid forced convection heat transfer coefficient in a heat storage tank filled with cylindrical particles based on thier experimental data. Ozaki and Inaba [6] numerically analysed the forced convection heat transfer coefficient for sphere particles considering the boundary-wall effect using a simple model and reported that their calculation results agreed with the experimental results. Taneda [7], Fujita and Watanabe [8] reported flow pattern around a single spherical particle in fluid flow with changing Reynolds number. Tsuji et al. [9, 10] reported their experimental results of flow pattern around two spheres lined along the flow direction and also the flow pattern around many spherical particles.
In this work, we numerically analyse the forced convection heat transfer coefficient for a single sphere and then for packed spherical particles (near the closest packed structure arrangement) in air flow using 3-dimensional thermo fluid computation code.

2. Analytical method
In this work, we used 3-dimensional thermo fluid computation code "STAR-CCM+" [11] to calculate flow velocity and temperature distributions in air flow packed with heat generating spherical particles. We used the finite volume method scheme for the numerical calculations. The conditions of incompressible fluid, unsteady laminar flow, constant properties, and discretization with polyhedral mesh were used in the calculations. We calculated temperature distribution only in the fluid and did not calculate temperature distribution in the spherical particles. The reason for this is to understand the basic convection heat transfer coefficient for standard boundary conditions with constant wall temperature and also then for constant wall heat flux. So, the calculation region is in the fluid region between the spaces created by spheres. In this work we calculated using only spheres of 10 mm in diameter. The boundary condition at the surface of the spheres has no-slip wall. The fluid is air with an inlet temperature $T_a = 300$ K, density $\rho = 1.18$ kg/m$^3$, specific heat capacity $C_p = 1000$ J/(kg K), viscosity $\mu = 1.86 \times 10^{-5}$ (Pa s), and thermal conductivity $\lambda = 0.0260$ W/(m K). Thermal boundary conditions at the surface of the spherical spaces are a constant temperature $T_w = 400$ K, constant heat flux $q = 13000$ W/m$^2$, or adiabatic. The boundary condition of the outside-wall of the flow region is symmetric. Time step of unsteady calculation is 0.05 s, iteration number at each time step is 200, and steady flow velocity and temperature distributions are obtained after more than 3 s with less than 0.1% change of velocity and temperature.

3. Calculation result for a single sphere
At first, in order to check the numerical calculation accuracy, we calculated flow velocity and temperature distribution of a wide air flow region with a single heating spherical particle. Figures 1 and 2 show the calculation model and calculation mesh in a central cross section of the flow region. The flow region is 50 mm in width, 50 mm in height, and 150 mm in total length. The distance between the entrance of the flow region and the centre of the sphere is 25 mm, and the diameter of the sphere is 10 mm. The minimum size of the mesh is 0.1 mm near the sphere and the maximum size of the mesh is 5 mm in the main flow region. Total number of mesh is 28,000. We calculated with cases of inlet air flow velocity $v = 0.1$, 1, and 10 m/s, which correspond to Reynolds number Re ($= \rho v D / \mu$) = 63.4, 634, and 6340. The average heat transfer coefficient on the surface of the sphere $\alpha_m$ was calculated by equation (1) with average surface temperature of the sphere $T_w$, inlet air temperature $T_a$, total heat transfer rate of the sphere $Q$, and surface area of the sphere $A$.

$$\alpha_m = \frac{Q}{A(T_w - T_a)} \quad (1)$$

Local heat transfer coefficient on the surface of sphere $\alpha$ was obtained from the local heat flux, which was calculated using the temperature gradient of fluid near the surface, the local surface temperate and the inlet air temperature. The angle-from-flow-direction is $\theta$ and the rotation-angle with central axis to flow direction is $\phi$, and are defined as shown in Figure 3.

Figures 4 and 5 show the calculation results of flow velocity vector and temperature distribution in a central horizontal cross section of flow region with an inlet air flow velocity $v = 1$ m/s (Re = 634) and a constant surface temperature of the sphere at $T_w = 400$ K. Our calculation result of the flow velocity vector is similar to that reported by Fujita and Watanabe [8]. There is an asymmetric vortex in the wake, which is probably caused by asymmetric calculation mesh, and the vortex does not move periodically. Figure 6 shows the local heat transfer coefficient on the surface of the sphere. Figure 7
Figure 1. Calculation model of single sphere.

Figure 2. Calculation mesh in central cross section for calculation of single sphere.

Figure 3. Angles $\theta$ and $\phi$.

Figure 4. Velocity vector for single sphere ($Re=634$).

Figure 5. Temperature distribution in fluid for single sphere ($Re=634$, $T_w=400$K).

Figure 6. Local heat transfer coefficient of single sphere ($Re=634$, $T_w=400$K).

Figure 7. Relation between local heat transfer coefficient and angle from flow direction for single sphere ($Re=634$, $T_w=400$K).

Figure 8. Relation between local heat transfer coefficient and angle from flow direction for single sphere ($Re=634$, $q=13000$ W/m$^2$).
show the relation between local heat transfer coefficient $\alpha$ and angle-from-flow-direction $\theta$. The local heat transfer coefficient is large at upper stream side and small in wake at downstream side. Local heat transfer coefficient is constant along the rotation-angle $\phi$. Figure 8 shows the relation between local heat transfer coefficient $\alpha$ and angle-from-flow-direction $\theta$ for inlet air flow velocity $v = 1 \text{ m/s} \ (\text{Re} = 634)$ and constant heat flux $q = 13000 \text{ W/m}^2$. The calculation results in Figure 8 for constant heat flux condition, is almost same as that in Figure 7 for constant surface temperature condition. Average heat transfer coefficient $\alpha_{av}$ is $39 \text{ W/(m}^2 \text{ K)}$ for constant heat flux condition in Figure 8 and $46 \text{ W/(m}^2 \text{ K)}$ for constant surface temperature condition in Figure 7. The values are almost same and the value for constant surface temperature condition is a little larger than that for constant heat flux condition.

Figure 9 shows the relation between Reynolds number $Re = \frac{\rho v D}{\mu}$ and Nusselt number $Nu = \frac{\alpha m D}{\lambda}$ for inlet air flow velocity $v = 0.1-10 \text{ m/s} \ (\text{Re} = 63 - 6340)$ and constant surface temperature of the sphere at $T_w = 400 \text{ K}$. The correlation equation of forced convection heat transfer coefficient for a single sphere proposed by Ranz and Marshall [3] (shown in equation (2)) is shown by a red broken line in Figure 9.

$$Nu = 2 + 0.6 \text{Re}^{0.5} \text{Pr}^{0.333} \quad (1 < Re < 10^5, \ 0.6 < Pr < 380) \quad (2)$$

Here, $Pr$ is Prandtl number. Our calculation results agree with the correlation equation proposed by Ranz and Marshall [3] within 10 %, and the accuracy of our numerical calculation is found to be good.

4. Calculation result for packed spherical particles

We calculated flow velocity and temperature distribution in the air flow region with packed spherical particles (near the closest packed structure arrangement). We calculated using the face-centred cubic cell as it is the closest packed structure arrangement, shown in Figure 10. Standard flow direction is the flow direction-A shown with a red arrow in Figure 10, and we also calculate for the flow direction-B shown with a yellow arrow. The flow direction-A is a diagonal line on a side surface of the face-centred cubic cell and the three spheres of the side surface align in series to the flow direction. This is the flow direction which has the largest value in the minimum cross section area of all directions for the face-centred cubic cell. So, the flow direction-A is the flow direction with minimum pressure drop. In this work, the standard arrangement of spheres has 1 mm distance between spheres of 10 mm diameter, because we found that accurate calculation was difficult using extremely fine mesh for contact condition of spheres. We also calculate the case using 0.2 mm distance between spheres and estimate the contact condition of the spheres. Figures 11 and 12 show the calculation model and calculation mesh in a central cross section of the flow region for the case of the flow direction-A and 1 mm distance between spheres. Boundary condition at the outside-wall of the flow region is symmetric.
We calculated using one row of four spheres and found the calculation result for the third sphere was closest to the conditions when there are many spherical particles. The flow region is 15.6 mm in width, 11 mm in height, and 103 mm in total length. The distance between the entrance of the flow region and the centre of the first sphere is 25 mm, and the diameter of the sphere is 10 mm. Minimum size of mesh is 0.1 mm near the sphere and maximum size of mesh is 2 mm in the main flow region. Total number of mesh is 120,000. We calculated for cases of inlet air flow velocity $v = 0.1, 1, \text{ and } 10 \text{ m/s}$, which correspond to Reynolds number $Re = 63.4, 634, \text{ and } 6340$. In order to reduce the effect of the change of fluid temperature on the heat transfer coefficient by the heating of other spheres, we also calculated the case of heating only one sphere to examine the heat transfer coefficient and adiabatic condition at other spheres.

Figures 13 and 14 show calculation results of flow velocity vector and temperature distribution in a central horizontal cross section of the flow region for flow direction-A, with 1 mm distance between spheres, an inlet air flow velocity $v = 1 \text{ m/s} (Re = 634)$, and a constant surface temperature of all spheres.

**Figure 10.** Flow directions-A and B in face-centred cubic cell.

**Figure 11.** Calculation model of lined spheres (flow direction-A).

**Figure 12.** Calculation mesh in centre cross section for calculation of lined spheres (flow direction-A).

**Figure 13.** Velocity vector for lined spheres (flow direction-A, $Re=634$).

**Figure 14.** Temperature distribution in fluid for lined spheres ($Re=634, \; T_w=400K$).

**Figure 15.** Local heat transfer coefficient of third sphere in lined spheres (flow direction-A, $Re=634, \; T_w=400K$).

**Figure 16.** Relation between local heat transfer coefficient and angle from flow direction for third sphere (flow direction-A, $Re=634, \; T_w=400K$).
spheres at \( T_w = 400 \) K. We found a strong impinge flow at angle-from-flow-direction \( \theta = 45^\circ \) on the upper side of the surface of the third sphere in Figure 13. Figure 15 shows the local heat transfer coefficient on the surface of the third sphere when the surface temperature of only the third sphere is constant at \( T_w = 400 \) K and adiabatic condition at other spheres. Figure 16 shows the relation between local heat transfer coefficient \( \alpha \) on the surface of the third sphere and angle-from-flow-direction \( \theta \). The local heat transfer coefficient shown in Figure 16 is the average value along the \( \phi \) direction. We find that the local heat transfer coefficient is large at \( \theta = 45^\circ \) which is the position of the strong impinge flow in Figure 13. Symbol \( \circ \) and black line in Figure 17 show the relation between average heat transfer coefficient \( \alpha_{av} \) and number of spheres in the row (1 - 4) where a specified sphere has a constant surface \( T_w = 400 \) K and the other spheres are assigned adiabatic conditions. Symbol \( \circ \) and blue one-dot chain line in Figure 17 show the relation between the average heat transfer coefficient and number of spheres in the row where all spheres are assigned a constant surface temperature \( T_w = 400 \) K. The average heat transfer coefficient is calculated using inlet air temperature. Air temperature rises along the row when all spheres are at 400 K and so the average heat transfer coefficient after the second sphere in the row decreases. Symbol \( \triangle \) and red broken line in Figure 17 show the average heat transfer coefficient corrected for the effect of average air temperature rise along the row. The average heat transfer coefficient corrected for the effect of air temperature rise (shown with symbol \( \Delta \)) agrees with the average heat transfer coefficient for the case of one sphere at 400 K (shown with symbol \( \circ \)) except for the second sphere in the row. For the second sphere in the row, correction of the average air temperature rise along the row cannot be applied well. The average heat transfer coefficient is highest at the third sphere in the row. This is because the flow velocity in the spaces between the spheres is weak in the first and second spheres in the row, and flow velocity at the downstream side of the fourth sphere in the row is also weak. We calculated for the case of 6 spheres in a row and the average heat transfer coefficient of the fifth sphere in the row is 89 W/(m\(^2\) K) and it is similar to that of the third sphere in the row 91 W/(m\(^2\) K) in Figure 17 (the details of the calculation are not shown). So, the average heat transfer coefficient of the third row in Figure 17 is close to that of spheres when there are many spherical particles. The average heat transfer coefficient of the third sphere is 87 W/(m\(^2\) K) for case of constant heat flux condition \( q \) = 13000 W/m\(^2\). The value for constant surface temperature condition is almost same as that for constant heat flux condition.

Figure 18 shows the relation between Reynolds number \( Re \) and Nusselt number \( Nu \) for inlet air flow velocity \( v = 0.1 - 10 \) m/s (\( Re = 63 - 6340 \)) when the surface temperature of only the third sphere is constant at \( T_w = 400 \) K. The correlation equation of forced convection heat transfer coefficient for a single sphere proposed by Ranz and Marshall [3] (shown in equation (2)) is shown with a red broken line in Figure 18. The correlation equation of forced convection heat transfer coefficient for packed particles proposed by Wakao et al. [4] (shown in equation (3)) is shown with a brown one-dot chain line in Figure 18.

\[
Nu = 2 + 1.1 Re^{0.6} Pr^{0.333} \quad (10 < Re < 10^4) \tag{3}
\]

Correlation equation of forced convection heat transfer coefficient of tube banks in staggered tube rows proposed by Zukauskas [12] (shown in equation (4)) is shown with a blue two-dot chain line in Figure 18.

\[
Nu = 0.4 Re_{max}^{0.6} Pr^{0.36} (Pr/Pr_{crit})^{0.25} \quad (10^3 < Re_{max} < 2 \times 10^5, (Y/X) > 2) \tag{4}
\]

Here, \( Re_{max} \) is Reynolds number using velocity at the minimum cross section area, where \( Pr \) and \( Pr_{crit} \) are Prandtl numbers at temperature of main flow and heat transfer surface, and \( (Y/X) \) is the ratio between tube pitch perpendicular to flow direction, and that with flow direction. Nusselt number of our calculation result is about twice as big as the correlation equation for a single sphere proposed by Ranz and Marshall [3], is about 1.4 times larger than the correlation equation of tube banks proposed by Zukauskas [12] and is about 70% smaller than the correlation equation for packed particles proposed by Wakao et al. [4].
Figures 19 shows the calculation result of the ratio of average heat transfer coefficient of the third row for the case of 0.2 mm distance between spheres, and inlet air flow velocity $v = 1\, \text{m/s} \ (Re = 634)$ when the surface temperature of only the third sphere is constant at $T_w = 400\, \text{K}$. The ratio of average heat transfer coefficient means the value divided by the value of 1 mm distance between spheres. When the distance between spheres is narrow 0.2 mm, the flow between spheres becomes strong and the local heat transfer coefficient is large at the position of the strong impinge flow. From this calculation result we can guess that the average heat transfer coefficient of 0 mm distance between spheres, which means the contact condition of spheres, is about 1.6 times larger than that of 1 mm distance between spheres. We think this is the reason that our calculation result is about 70% smaller than the correlation equation for packed particles proposed by Wakao et al. [4]. So, the average heat transfer coefficient of the contact condition of spheres with the face-centred cubic cell can be estimated with the correlation equation for packed particles proposed by Wakao et al. [4] within 15%.

Next we calculated for the case of the flow direction-B in Figure 10. Figure 20 shows the calculation model for the case of the flow direction-B and 1 mm distance between spheres. Figures 21 and 22 show the calculation results of flow velocity vector and temperature distribution in central horizontal cross section of flow region for the case of inlet air flow velocity $v = 1\, \text{m/s} \ (Re = 634)$ and

**Figure 17.** Relation between average heat transfer coefficient and number of spheres in row of lined spheres ($Re=634$).

**Figure 18.** Relation between Nusselt number and Reynolds number for third sphere in lined spheres.

**Figure 19.** Relation heat transfer coefficient and distance between spheres.

**Figure 20.** Calculation model of lined spheres (flow direction-B).

**Figure 21.** Velocity vector for lined spheres (flow direction-B, $Re=634$).

**Figure 22.** Temperature distribution in fluid for lined spheres ($Re=634, T_w=400\, \text{K}$).
constant surface temperature of all spheres at $T_w = 400$ K. Figure 23 shows the local heat transfer coefficient on the surface of the third sphere when the surface temperature of only the third sphere is constant at $T_w = 400$ K and adiabatic condition at other spheres. Figure 24 shows the relation between local heat transfer coefficient $\alpha$ and the angle-from-flow-direction $\theta$. As arrangement of spheres is different from that in Figure 11, distribution of local heat transfer coefficient is different from that in Figure 16 but the local heat transfer coefficient is also large at the position of the strong impinge flow. The average heat transfer coefficient of the third row is $87 \text{ W/(m}^2\text{K)}$ is almost same as that of $91 \text{ W/(m}^2\text{K)}$ for flow direction-A in Figure 17.

5. Summary
We numerically analysed the forced convection heat transfer coefficient of a single sphere and that of packed spherical particles (near the closest packed structure arrangement) in air flow using the 3-dimensional thermo fluid computation code. The following results are obtained.

1. Calculation results of the average forced convection heat transfer coefficient for a single sphere agree with the correlation equation proposed by Ranz and Marshall [3] (shown in equation (2)) within 10 %.

2. Local heat transfer coefficient is large at the position of the strong impinge flow to the surface of spheres for packed spherical particles.

3. Calculation results of the average forced convection heat transfer coefficient for packed spherical particles (near the closest packed structure arrangement) are close to the correlation equation proposed by Wakao et al. [4] (shown in equation (3)).

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