Recent theoretical developments on QCD matter at finite temperature and density

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Theoretical challenges for QCD at finite density

For RHIC Beam Energy Scan-II program:

Equation of State for $\mu_B/T \sim 2.5-3$.

Measure the curvature of chiral crossover line.

Look for possible existence of critical end-point in the phase diagram.

Understanding the relevant degrees of freedom near the crossover.

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[http://www.bnl.gov/rhic/news]
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Lattice techniques at finite $\mu_B$-Taylor expansion

- Conventional Monte-Carlo methods suffer from sign problem at finite $\mu_B$.
- Two methods presently allow to go to thermodynamic and continuum limits.
- Taylor expansion of physical observables around $\mu = 0$ in powers of $\mu_B/T$ is one such method [Bi-Swansea collaboration, 02]

\[
\frac{P(\mu_B, T)}{T^4} = \frac{P(0, T)}{T^4} + \frac{1}{2} \left( \frac{\mu_B}{T} \right)^2 \frac{\chi_2^B(0, T)}{T^2} + \frac{1}{4} \left( \frac{\mu_B}{T} \right)^4 \frac{\chi_4^B(0)}{3!} + \ldots
\]

- The series for $\chi_2^B(\mu_B)$ should diverge at the critical point. On finite lattice $\chi_2^B$ peaks, ratios of Taylor coefficients equal, indep. of volume [Gavai& Gupta, 03]
Eq. of State of QCD at finite density

- In most central heavy-ion experiments typically:
  \[ n_S = 0, \text{ Strangeness neutrality,} \]
  \[ \frac{n_Q}{n_B} = \frac{n_P}{n_P + n_N} = 0.4. \]
  [Bi-BNL collaboration, 1208.1220, HotQCD collaboration, 1701.04325]

- For lower \( \sqrt{s} \) collisions: Need to understand baryon stopping!
- Imposes non-trivial constraints on the variation of \( \mu_S \) and \( \mu_Q \).
- Possible to vary them by only varying \( \mu_B \) through
  \[ \mu_S = s_1 \mu_B + s_3 \mu_B^3 + s_5 \mu_B^5 + \ldots \]
  \[ \mu_Q = q_1 \mu_B + q_3 \mu_B^3 + q_5 \mu_B^5 + \ldots \]
Central values of $P_4, P_6$ already deviate from Hadron Resonance gas model at $T > 145$ MeV [HotQCD collaboration, 1701.04325].

$P_6$ has characteristic structure at $T > T_c \rightarrow$ remnant of the chiral symmetry due to the light quarks. Effects of $U_A(1)$ anomaly?

Essentially non-perturbative $\rightarrow$ cannot be predicted within Hard Thermal Loop perturbation theory.
Summary for the EoS

- The EoS is well under control for $\mu_B/T \sim 2.5$ already with $\chi^B_6$.
- Continuum estimates from two different fermion discretization agree for $\mu_B/T \leq 2$.

[Bielefeld-BNL-CCNU collaboration, 1701.04325, Borsanyi et. al, 1606.07494].
Do we understand the degrees of freedom around $T_c$?

- For a strongly interacting medium a quasi-particle description may not exist.
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Initially believed that thermodynamic properties of QCD near chiral crossover can be very well described by a non-interacting gas of hadrons+resonances.

[Dashen, Ma and Bernstein, 69,71, Prakash and Venugopalan, 92]
Do we understand the degrees of freedom around $T_c$?

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  [Dashen, Ma and Bernstein, 69,71, Prakash and Venugopalan, 92]

- With very precise lattice data we now know that a naive HRG description breaks down much below $T_c$.
  
  [A. Bazavov et. al. HotQCD coll. 1404.6511]
Why should naive HRG description break down?

- All baryon channels do not have resonant interactions $\rightarrow$ interacting baryons cannot be always mimicked by resonances
- There are non-trivial in-medium modification of baryon masses

[G. Aarts et. al., 1812.07393].
Why should naive HRG description break down?

- Thermal width of the resonances may be significant

[G. Aarts et. al., 1812.07393]
Constraining different HRG models from QCD

- Higher order fluctuations of conserved charges are more sensitive to the departure from PDG-HRG → needs a more accurate measurement
- Repulsive baryon interactions?
- Lattice data for higher order baryon number fluctuations are getting precise enough to allow for a comparison between different scenarios → additional resonances from quark-models are necessary + interactions not very well constrained.

[P. Huovinen, P. Petreczky, 1811.09330]
Constraining different HRG models from QCD

- Including repulsive interaction for baryons + non-interacting mesons + resonances, new versions of HRG has been studied → significant deviation from non-interacting HRG.
  
  [See also V. Vovchenko, M. I. Gorenstein and H. Stoecker 1609.03975]

- Lattice data can constrain such models strongly!
  
  Currently none of these models are perfect to describe QCD at freezeout.
Off-diagonal fluctuations more sensitive to deviation from naive HRG

- Naive HRG already inadequate!

- Resonances dominate

- HotQCD preliminary

- Free quark gas
More from Cross-correlations

\[ \frac{(\chi_{112}^{BQS} + 5\chi_{211}^{BQS})}{6} \]

HRG: \( \Xi + \Omega \)

PDG-HRG
QM-HRG

\( N_\tau = 6 \)
8
12

HotQCD preliminary

Large number of excited states for multi-strange baryons.
Curvature of the chiral crossover line

- Since $m_{u,d} \ll \Lambda_{QCD}$ the $SU_L(2) \times SU_R(2)$ is a near exact symmetry of 2 + 1 flavor QCD.
- Though not strictly a phase transition, however all chiral observables show observable changes at a certain temperature. It thus makes sense to talk about a precise $T_c(\mu_B = 0) = 156.5 \pm 1.5$ MeV. [HotQCD collaboration, 1812.08235]
Curvature of the chiral crossover line

\[ \frac{T_c(\mu X)}{T_c(0)} = 1 - \kappa_2^X \frac{\mu_X^2}{T_c(0)^2} - \kappa_4^X \frac{\mu_X^4}{T_c(0)^4} \]

For strangeness neutral system, continuum results available!
\( \kappa_2^B = 0.012(4), \kappa_4^B \sim 0 \) with Taylor expansions and HISQ fermions.

[HotQCD collaboration, 1812.08235]
Curvature of the chiral crossover line

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\]

Consistent with imaginary chemical potential method and stout fermions

\[\kappa_2^B = 0.0135(20) \] [C. Bonati et. al., 1805.02960] and
\[\kappa_2^B = 0.0153(18) \] [BW collaboration, 2002.02821].

removes earlier tension between two methods!

[Figure courtesy BW collaboration, arxiv:2002.02821]
Curvature of freeze-out line vs chiral crossover line

- For lines $P = \text{const}$, the entropy density changes by 15% → better description of LCP for viscous medium formed in heavy-ion collisions? [HotQCD collaboration, 1701.04325].

- Different LCP’s agree within 2 MeV for $\mu_B/T \leq 2$ [HotQCD collaboration, 1812.08235].

- STAR results favor steeper curvature for the freezeout curve → consistent at large $\mu_B$ [arXiv:1412.0499].

- Agreement with the recent ALICE analysis of freezeout parameters [arXiv:1408.6403].
The Taylor series for $\chi_B^2(\mu_B)$ should diverge at the critical end-point.
Critical-end point search from Lattice

- The Taylor series for $\chi_2^B(\mu_B)$ should diverge at the critical end-point.
- For a singularity on the real $\mu_B$ line, all the $\chi_n^B > 0$. 

[Definition: $r_2^n \equiv \sqrt{2n(2n-1)} |\chi_2^B|^{2n+2}$]

• Strictly defined for $n \to \infty$. How large $n$ could be on a finite lattice?
• Signal to noise ratio deteriorates for higher order $\chi_n^B$. 

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Radius of convergence will determine the location of the critical end-point. [Gavai & Gupta, 03]
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Critical-end point search from Lattice

Higher order fluctuations like $\chi_8^B$ are positive only for $T \sim 135$ MeV.

[HotQCD collab. 2001.08530, BW collab. 1805.04445]
Critical-end point search from Lattice

- **Current bound for CEP:** $\mu_B / T > 2.5$ for $135 \leq T \leq 140$ MeV
  
  [HotQCD coll., 1701.04325, update 2018].

- Ultimately all estimates will agree in the continuum limit!

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**Graphical Content:***

- Imaginary $\mu$ data is consistent
- Systematics?

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**Plot Details:**

- 2017: lower bound for $r_4^{\chi}$ estimator
- D'Elia et al., 2016, $r_4^{\chi}$
- Datta et al., 2016
- Fodor, Katz, 2004

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**Legend:**

- $r_2^{\chi}$, $r_4^{\chi}$, $r_6^{\chi}$, HRG
- disfavored region for the location of a critical point

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**Graph Title:**

Imaginary $\mu$ data is consistent

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**Graph X-Axis:**

$T$ [MeV]

---

**Graph Y-Axis:**

$r_2^{\chi}$, $r_4^{\chi}$, $r_6^{\chi}$, HRG

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**Graph Note:**

Disfavored region for the location of a critical point

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**Graph Legend Entries:**

- $r_2^{\chi}$, $r_4^{\chi}$, $r_6^{\chi}$, HRG
- Imaginary $\mu$ data is consistent
- Systematics?
With the current precision $\kappa_4 = \kappa_6 = \kappa_8 = \ldots \sim 0$, radius of curvature estimates tell us $T_{\text{CEP}} \sim 0.86 T_c(0)$ and $\mu_B / T_{\text{CEP}} > 2.5$.

On the other hand if $\kappa_4 \sim 0.1 \kappa_2$, will contribute to the curvature for $\mu_B / T_{\text{CEP}} > 2.5 \rightarrow$ its precise determination is imp.

Steeper curvature would imply slow convergence of $r_n$ with order $n$
Critical end-point & chiral crossover line: current status

With the current precision \( \kappa_4 = \kappa_6 = \kappa_8 \ldots \sim 0 \),

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Steeper curvature would imply slow convergence of \(r_n\) with order \(n\).
Can lattice data tell us anything about the fireball?

- In equilibrium thermodynamics one measures fluctuations of globally conserved quantum numbers like baryon no., electric charge, strangeness.

- In the experiments, the hadron multiplicities are measured within a rapidity window → not a conserved number!

- Feed-down decays may push them out of the rapidity window.

- Hadron multiplicities may not have information about whether thermal equilibrium is achieved near freezeout.
Revisiting the statistical hadronization model

- $\sigma^2/M$ for net-charge and net-protons affected due to resonance decays.
- A new proposal to look for ratios $\sigma_{QK}^2/\sigma_K^2$, $\sigma_{pK}^2/\sigma_K^2$, and $\sigma_{Qp}^2/\sigma_p^2$ and $S\sigma_{p,K,Q}$. A fit to these observables consistent with thermal HRG model and same freezeout conditions. [S. Gupta et. al., 2004.04681].

Thermalization observed for $\sqrt{s} \geq 19.6$ GeV

![Diagram showing thermalization observed for $\sqrt{s} \geq 19.6$ GeV]
Thermal fits were done including only statistically independent particle yield ratios for most central collisions at mid-rapidity. [Bhattacharyya et. al, 1911.04828].

Including/excluding strangeness neutrality condition gave same $T_f$ for a large set of hadrons → simultaneous equilibration?

Thermal fits to hadron yields in central Pb-Pb collision at the LHC have been re-analyzed taking into account additional QM states. [Andronic et. al., 2011.03826]. Strong influence from resonances $N^*$, $\Delta^*$. If corrections due to pion-nucleon interactions are properly included gives $T_f \sim 156$ MeV.
Fluctuations measured at freezeout: Are these thermalized?

First to second moment:

\[
\frac{M_B}{\sigma_B^2} = \frac{\mu_B}{T} + O\left(\frac{\mu_B}{T}\right)^3
\]

\(\mu_B/T\) is an unknown parameter and model dependent. Instead represent it by \(\frac{M_B}{\sigma_B^2}\). [Karsch et. al., arxiv:1512.06987]

Clear deviation from naive HRG prediction at larger \(\mu_B\).
[HotQCD collaboration, arxiv:2001.08530].

Hence any ratio can be expressed as model independent manner

\[
R_{31}^B = \frac{S_B \sigma_B^3}{M_B} = \frac{\chi_4^B}{\chi_2^B} + \frac{1}{6} \left[ \frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2 \right] \left(\frac{M_B}{\sigma_B^2}\right)^2 + ..
\]
Extrapolated STAR data for $R_{31}^P(\mu_B = 0)$ tantalizingly close to QCD prediction for $R_{31}^B = \frac{\chi_4^B}{\chi_2^B} \rightarrow$ Accidental coincidence or hints of thermalization?

**Challenges** Need to perform more such tests with higher order cumulants. Proton number \( \neq \) Baryon number
$R_{62}^P$ is difficult to measure! $R_{51}^P$ will be less noisy.
More from Cross-correlations

- $\chi^{11}_{BS}/\chi^S_2$ shows $\sim 15\%$ deviation between 155 and 165 MeV. Analysis with ALICE [A. Andronic et. al., 16] consistent with lattice at $T_c \sim 155$ MeV. Including $\Sigma^* \rightarrow N\bar{K}$ will make the ratio lower!

- Similar observables at higher $\mu_B$ would be interesting! [A. Chatterjee et. al., STAR collaboration, 2019]
Preparing for BES-II runs: LQCD EoS important for hydrodynamic modeling of QGP. For $\mu_B/T \leq 2 \rightarrow \sqrt{s_{NN}} \geq 11$ GeV already under control with $\chi_6^B$. 

Lines of constant $\epsilon$, $p$ consistent with LQCD estimates of curvature of chiral crossover line. Higher order cumulants of baryon no. will also help in bracketing the possible CEP. Recent LQCD calculations suggest $\mu_B(CEP)/T \geq 2.5$, $T(CEP) \sim 0.86 T_c$. 

Several new studies suggest that the fireball produced in most central HIC may be thermalized at chemical freezeout for $\sqrt{s_{NN}} \geq 19.6$ GeV. For many new updates, please visit the exciting talks and posters in QCD & HI sessions.
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Outlook

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