Quantum Pumping with Ultracold Atoms on Microchips: Fermions versus Bosons

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We present a design for simulating quantum pumping of electrons in a mesoscopic circuit with ultra-cold atoms in a micro-magnetic chip trap. We calculate theoretical results for quantum pumping of both bosons and fermions, identifying differences and common features, including geometric behavior and resonance transmission. We analyze the feasibility of experiments with bosonic $^{87}$Rb and fermionic $^{40}$K atoms with an emphasis on reliable atomic current measurements.

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Pumping is any cyclical time-varying mechanism that generates sustained flow. Quantum pumping $^{1,2}$ in mesoscopic solid state circuits is a coherent quantum process for generating directed transport of charge with time dependent potentials, but no applied bias field. With its promise of precise and reversible flow control at the single electron level and extension to transport of spin $^3$ and entangled electron pairs $^4$, quantum pumping has been the subject of considerable theoretical research $^5$. Despite potential technological applications, quantum pumping experiments in solid state system have not been successful, partly due to dominant competing rectification effects associated with electrically charged carriers $^6,7,8$. Neutral ultra-cold atomic systems present a possible path around the current bottleneck by avoiding such complications. An atomic circuit using a Bose-Einstein condensate (BEC) or a degenerate Fermi gas (DFG) can test basic theoretical predictions, while also providing a reference for experiments in solid state systems.

In this letter, we present a design for an experiment to test quantum pumping theory with ultra-cold atoms in a micro-magnetic potential on a chip. Ultra-cold atoms open up the possibility of studying not only fermion quantum pumping but also boson pumping, as well as the influence of variable interactions and long range order, in a fully controlled and tunable system. We present theoretical results for both types of atoms in prototypical pumping schemes and we analyze the feasibility of a cold atom based experiment with numerical simulations.

Mesoscopic circuit with atom chips: A prototypical mesoscopic circuit consists of a device, e.g. a quantum dot, connected by nanowires to macroscopic contacts. At low temperatures, electrons and holes can have mean free paths longer than the nanowires, so they can be described as freely propagating particles in one dimension (1D). The device presents a scattering potential for the particles, so that transport is reduced to a scattering problem $^4$.

We can simulate this setup with the atom chip based scheme shown in Fig. 1. Atom chips are substrates on which currents in lithographically imprinted wires generate a micro-magnetic trapping potential for ultra-cold atoms. These chips can efficiently produce both BECs and DFGs with temperatures in the 10 nK to 1 µK range $^10,11$. Two reservoirs connected by a 1D quantum wire can be implemented on an atom chip by two 3D micro-magnetic traps connected by a quasi-1D magnetic guide, generated by co-propagating currents in two parallel wires (red in Fig. 1) on the substrate, with a constriction for the tighter 1D section. The atoms are trapped in the plane of the wires, with the substrate between them removed $^{12}$, which also allows optical access from above and below. The trapping potential is harmonic along all principal axes, including the axial one due to a current through the two ‘end cap’ wires (dashed-green in Fig. 1) below the trapping plane $^{13}$. Residual defects in the trap potential can be suppressed by applying an AC current through the principal trapping wires, while keeping the external axial magnetic field and the current in the end-cap wires constant $^{14}$. The “device”, or scattering potential, can be realized with a dipole laser focused onto the 1D section.

![FIG. 1: (a) Configuration for generating two micro-magnetic trap reservoirs connected by a 1-d channel. The red wires provide radial confinement, while the dashed green ‘end-cap’ wires, located 50 µm below, provide axial confinement. The large red arrows are probe lasers for measurements on the trapped atoms, represented by the blue structure. The vertical (purple) laser implements the pump potential; (b) a 1 µK equipotential for alkali atoms trapped by 250 mA and 10 mA in the red and green wires, respectively, along with a 1 G axial magnetic field (the transverse/axial scale is 37); (c) transverse isopotential curves along the 1-d channel from 50 to 1000 µK showing its symmetry and significant trap-depth.](image)
The generated atomic current can be determined from a measurement of the momentum distribution of the particle flow, since the average current can be written as $J = (\hbar/m) \int |\psi(k)|^2 dk$. Bragg spectroscopy is ideally suited for measurements of the momentum distribution, since it can be selectively applied to atoms in the 1-d channel and combined with fluorescence imaging for high signal-to-noise detection. A spectroscopic flag can be attached to the kicked atoms by adding the hyperfine splitting to the base detuning of the probe lasers, thus changing their hyperfine level. A large fraction of the scattered photons can be collected by a microscope lens located a few millimeters above the atom chip and imaged onto a high sensitivity camera. We calculate that roughly a hundred photons per atom can be detected with a fluorescence pulse of a few hundred microseconds.

**Theory of bosonic and fermionic pumps:** Quantum pumping has been studied exclusively for fermions in solid state systems, and primarily in the adiabatic regime where the pump period exceeds the dwell time of the carriers at the potential. With atomic experiments in mind, we extend the theory of pumping to include bosons.

As with electrons in nanowires, the dynamics of atoms in the central segment is quasi-1D with quasi-continuum description along the transport direction and quantized transverse channels $(n)$. The axial and transverse components can be factorized, $Ψ(x, y, z, t) = \sum_k \psi_n(x, z, t) \phi_n(x)$, (in cylindrical symmetry): $\int dz |\psi_n(x, z, t)|^2$ is the population fraction in the $n$-th channel. Scattering influences the evolution of the axial functions, with little effect on the transverse profiles. For weak interactions, phase fluctuations of degenerate bosons can be neglected in the 1D section, so the axial dynamics has an effective description in degenerate bosons can be neglected in the 1D section, so the axial dynamics has an effective description in degenerate bosons at wavevector $k_0$ by $J_B(k) = 2\pi \hbar k_0^2/2$. For a BEC at rest, one sets $k = 0$, after the derivative.

Essential features of quantum pumps can be understood with models involving time-varying single barrier potentials: with variable strength $V_1(x, t) = U(t)\delta(x)$ and translating uniformly $V_2(x, t) = U\delta(x-\nu t)$. For adiabatic variation, Eqs. (3) and (4) give the pumped currents:

$$J_{F}^{(1)}(x, t) = \frac{1}{2\pi} \int dx' \frac{\partial}{\partial x'} \psi^*_{k,n}(x') \psi_{k,n}(x), \quad J_{F}^{(2)} = 1 \frac{k \nu U}{k^2 + U^2}$$

$$J_{B}^{(1)}(x, t) = \frac{U(1-k^2)}{U^2 + k^2} \frac{1}{U^2 + k^2}$$

They show the role of symmetry: $V_1$ generates no net particle transport from one reservoir to the other over a period, due to antisymmetry with respect to $x$; while $V_2$ being symmetric leads to net transport. In general, both symmetric and antisymmetric parts can be present.

The fermionic current for $V_2$ is always in the direction of motion of the potential, but the bosonic current can flow opposite (Fig. 2). When the bosons have sufficient energy $k^2 > U$, the transmitted fraction dominates, and particles going against the barrier have a higher transmission probability; for fermions, the averaging over states washes out this effect. Over a period $T$, the net pumped particles, $\int J \times T$, is independent of the velocity $v$ and depends only on the parameter path traversed by the potential. It is a geometric quantity analogous to a geometric phase, a feature shared by all adiabatic quantum pumps.
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current differs dramatically from the case of the delta
analytical expressions for the pumped current can be like-

(ii) the finite width creates oscillations due to resonance
transmission. For bosons (Fig. 2(c)), the oscillations are

The pumped current at arbitrary barrier velocities for
V2 can be found using a Galilean transformation to be:

\[ J_B = \frac{2U^2(U^2 + v^2 - k^2)}{U^4 + 2U^2(v^2 + k^2) + (v^2 - k^2)^2} \]
\[ J_F = \frac{U^2}{4\pi} \left| \frac{U^2 + (k_F + v)^2}{U^2 + (k_F - v)^2} \right| \]

The adiabatic expressions in Eq. (6) are retrieved with
a Taylor expansion for \( v \ll k, k_F \). The fermionic current
vanishes for \( k_F = 0 \), as the number density vanishes; but
for a stationary BEC, \( J_B(k=0) = vU^2/(v^2 + U^2) \),
eturally due to reflection. At high barrier velocity \( v \gg k \),
\( J_B \sim J_B(k=0) \) and \( J_F \sim k_F vU^2/(\pi(v^2 + U^2)) \)
like the adiabatic limit with \( k_F \) and \( v \) interchanged.

Even for a finite translating square barrier (SB) \( V_{SB} = (U/a)\delta(x - vt)\delta(x + vt + a) \) of width \( a \) and height \( U/a \),
analytical expressions for the pumped current can be
likely calculated, too lengthy to be shown, but plots based
upon those solutions are shown in Fig. 2. The pumped current
diffs dramatically from the case of the delta barrier:
(i) the finite height allows particle energy to ex-
ced the barrier potential leading to sharp transitions at
\( \frac{1}{2}(k + v)^2 = U \), the classical cutoffs for transmission, and
(ii) the finite width creates oscillations due to resonance
transmission. For bosons (Fig. 2(c)), the oscillations are
pronounced, with the current vanishing and reversing for
some velocities; but less so for fermions (Fig. 2(d)) due
to averaging over wavevectors. For a translating barrier,
both classical and quantum features are manifest, but for
a translating well, the behavior is quantum mechanical.

The quantum nature of pumping becomes truly signif-
ificant in a turnstile pump comprised of two barriers with
heights oscillating out of phase with each other. This
model has been studied for fermions, and here we present
results for bosons contrasted with fermions. Essential features can be understood with two delta func-
tion potentials \( U_{\pm}(t)\delta(x \mp a) \) with oscillating strengths
\( U_+(t) = 1 + \cos(\omega t) \) and \( U_-(t) = 1 + \sin(\omega t) \), that trace out
a circle over a period \( T = 2\pi/\omega \). In this limiting case,
the current is entirely due to quantum interference [19].
Reversing the cycle reverses the flow.

The insets in Figure 3 show that the currents on the
left and the right of the potential are not in sync and vary
over time, but their time integrals over a full cycle are
equal. Resonant transmission effects are prominent due
to finite barrier separation, 2a: The particle transport, \( q \),
in a pump cycle displays oscillations and peaks as a function
of the barrier separation, Fig. 3(a,d), and also as a
function of the wavevector \( k \) or \( k_F \), Fig. 3(b,e). Fermions
display less pronounced resonance behavior, due to aver-
aging over momentum states. There has been recent
interests in testing resonance transport through double
barrier structures [20], quantum pumps demonstrate this
by periodic cycling of the potentials. The geometric na-
ture of adiabatic pumps is clearly seen in Fig. 3(c,f), since
particle transport per cycle is independent of \( \omega \).

Feasibility Analysis: Pump potentials can be im-
plemented with blue-detuned lasers at 532 nm focused
to 1-5 \( \mu \)m gaussian-profile barriers. The lasers need to
translate at velocities \( v \sim l_v \omega_r \approx 0.5 \) cm/s or vary in
intensity at frequencies \( \sim \omega_r \approx 2 \) kHz, easily achievable.

Bosonic pumps at non-zero \( |k| \) can be implemented
with a broad (relative to pump potential) wavepacket
split into counterpropagating momentum states by a
Bragg pulse [13]. In this scenario, the resonators can be
removed. For \(^{87}\text{Rb}\) in the \( F = 2, m_F = +2 \) state in the
set-up of Fig. 4 the transverse and axial trap frequen-
cies are \( \omega_r, 1D = 2\pi \times 5.1 \) kHz and \( \omega_{axial} = 2\pi \times 3.6 \)
Hz. For a wavepacket of 1000 \(^{87}\text{Rb}\) atoms with scattering
length of \( a_s = 99 a_0 \), a variational calculation [21] yields
the effective 1D non-linear constant \( g_{1D} = 67.3 \) and axial
Thomas-Fermi width 587 \( l_v \). After the axial trap is
turned off and the Bragg pulse applied, the split wave-
packet evolves in the presence of the pump potential.
We simulate this numerically by solving the NLSE with
a split-step operator method; results are shown in Fig. 4.
In the absence of nonlinearity (\( g_{1D} = 0 \)), the wavepacket
simulations with square barriers are consistent with an-
alytical results (Figs. 2(c) and 3(b)) obtained assuming
plane waves, validating the method. Figure 4 also shows
that Gaussian profile barriers and nonlinearity lead to
some qualitative changes, but the pumped current or
charge remains significant. The nonlinearity reduces the
signal somewhat, and for the turnstile, the broader bar-
rriers and barrier-separation lead to more closely spaced oscillations in the current as packet velocity (k) varies. Numerical simulations [22] show that the pump signal for the turnstile is more sensitive to chip trap roughness than the translating barrier scheme, but AC suppression of roughness [14] is sufficient for a robust signal.

A 1000 atom wavepacket has initial peak density $7.3 \times 10^{14}$ cm$^{-2}$ and chemical potential $\mu_{3d} = 0.26\mu k = 1.09h\omega_c$. The number of atoms can be significantly increased, considering: (i) More atoms mean stronger non-linearity and faster expansion, requiring longer traps to allow sufficient interaction times with the pump; without the reservoirs the axial length can be extended upto 1000 $\mu$m. (ii) To remain in the transverse ground state (for single channel), $\mu < 2h\omega_c$; our variational calculations gives $\mu \simeq 1.6h\omega_c$ with $N_{1D} = 2.0 \times 10^4$ atoms.

For fermion pumps with $^{40}$K the currents listed for Fig. 1 produce trap frequencies a factor of $\sqrt{mk/m_h} \simeq 1.5$ higher than with $^{87}$Rb. Energetically, the 1D section can contain $\omega_{c,1D}/\omega_{axial} \simeq 1400$ spin-polarized fermions in the lowest transverse channel due to the Pauli principle. Since the size of harmonic oscillator eigenstates scales as $\sqrt{2N}$, for the axial oscillator length $l_c = 6.9$ $\mu$m, the 250 $\mu$m 1D section will hold about 700 atoms; each reservoir contains 50 times more. The lowest channel can accommodate Fermi vectors up to $k_F = 1.4l_c^{-1}$.

**Conclusions:** Our analysis has shown that quantum pumping experiments can be done with current atom-chip technology, allowing a broad survey of a process that has eluded confirmation in solid state systems. In addition to simulating fermion pumping, ultra-cold atom based experiments open up the possibility of studying quantum pumping of bosons which we expect to show enhanced resonant tunneling and current reversal. Furthermore, the scheme can be adapted to search for conductance quantization for pumping with periodic lattices [1] by imposing a moving optical lattice on the 1-d quantum channel. In a broader context, our design is easily adapted to a variety of mesoscopic transport experiments, important in electronic systems, like conductance quantization and spin transport, yet hardly explored with ultracold atoms.

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