Effects of Heavy States on the Effective $N = 1$ Supersymmetric Action

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(November 12, 2017)

Abstract

Using the power of superspace formalism, we investigate the decoupling effects of heavy states in $N = 1$ supersymmetric field theory. We find that “mixed” couplings in the superpotential between the heavy and light fields contribute to the effective superpotential at the leading order, and also contribute to the effective Kähler potential (in the next to leading order). Mixed couplings in the Kähler potential always contribute to the effective Kähler potential at the leading order. Several examples are presented which illustrate the effects explicitly.
I. INTRODUCTION

In this paper, we examine the decoupling of heavy states in $N = 1$ supersymmetric field theories. While we are motivated by the analysis of a class of quasi-realistic string models after vacuum restabilization, the analysis has general applications. We specifically concentrate on the effects of gauge neutral fields. We also do not consider effects due to (soft) supersymmetry breaking.

In accordance with the well-known Applequist-Carazzone decoupling theorem for non-supersymmetric theories, the tree-level exchange of heavy fields leads to nonrenormalizable terms in the effective potential of the light fields. The supersymmetric generalization as studied in [4] reveals that the leading contribution is to the superpotential. However, we also find that in general, the decoupling of the heavy fields leads to nonrenormalizable modifications of the Kähler potential of the light fields of the theory (as was also pointed out in [5]). The effects of decoupling then lead to nonrenormalizable interactions which are competitive at each order with other nonrenormalizable terms present in string models (those include nonrenormalizable terms at a given order calculated directly in string theory and those generated from higher-order terms which involve the fields with the large VEV’s), leading to a tower of nonrenormalizable terms to be classified in the model.

In our analysis of decoupling effects, we utilize the power of supersymmetry by employing superspace formalism. For the sake of simplicity, we consider the case of gauge singlet fields. We discuss the conventional method of integrating out the massive modes in the superspace functional integral and use the corresponding supergraphs to illustrate the results. However, we focus on an alternative method to determine the decoupling effects, which is to solve the equation of motion for the heavy superfields, and then determine the effective supersymmetric action for the light superfields of the theory. In some cases, it is possible to obtain a complete solution to the equations of motion for the heavy fields, and thus in principle obtain the contributions to the effective action at all orders in the nonrenormalizable terms.

The paper is structured as follows. In Section II, we develop the formalism for both methods (supergraphs and minimization of the action) of examining the decoupling of the heavy fields. In Section III, we investigate the effects of different types of superpotential terms (motivated from string models) involving the heavy fields, first taking the simplest case of a superpotential with a mass term for the heavy fields and a term linear in the heavy field coupled to an arbitrary function of the light fields. We then consider additional superpotential terms, such as a term bilinear in the heavy fields coupled to an arbitrary function of the light fields, as well as a trilinear self-interaction term for the heavy fields (we

*In a class of quasi-realistic string models with an anomalous $U(1)$, the standard anomaly cancellation mechanism generates a Fayet-Iliopoulos contribution to the $D$-term of the anomalous $U(1)$ at genus-one. The FI term triggers certain scalar fields to acquire vacuum expectation values (VEV’s) of $O(M_{\text{String}} \sim 5 \times 10^{17} \text{GeV})$ along $D$- and $F$- flat directions, leading to a “restabilized” supersymmetric string vacuum. Effective mass terms are generated via the superpotential coupling of the fields in the model to the fields with string-scale VEV’s, and hence a number of states acquire string-scale masses and decouple from the theory. See e.g. [1,2] and references therein.
do not consider higher-order interactions, as they are immediately non-renormalizable and thus of higher order). We also consider the effects of non-minimal Kähler potential terms which mix the heavy and light states. Finally, in Section IV we present the summary and conclusions.

II. FORMALISM

The supersymmetric Lagrangian of gauge singlet chiral superfields is determined by two functions of the chiral superfields \( \{ \varphi_i \} \): (1) the Kähler potential \( K(\varphi_i, \varphi^\dagger_i) \), and (2) the superpotential \( W(\varphi_i) \), where \( K \) is a real function and \( W \) a holomorphic function of the chiral multiplets of the theory. The supersymmetric action is given by

\[
S = \int d^4x d^2\theta d^2\bar{\theta} K(\varphi_i, \varphi^\dagger_i) + \left\{ \int d^4x d^2\theta W(\varphi_i) + h.c. \right\},
\]

in which we use the notation and conventions of [6].

We assume that a subset of the fields \( \vec{\Phi} = \{ \Phi_l \} \) acquire heavy masses \( \mathcal{O}(M) \), and all other fields (denoted by \( \vec{\varphi} = \{ \varphi_i \} \)) are light or massless. The Kähler potential of the theory can be written as

\[
K = K_{\text{min}} + K',
\]

in which \( K_{\text{min}} \) is the minimal canonical Kähler potential for the theory:

\[
K_{\text{min}} = \vec{\varphi}^\dagger \vec{\varphi} + \vec{\Phi}^\dagger \vec{\Phi}.
\]

\( K' \) includes possible non-minimal terms which can mix the heavy and light fields; such terms are treated as interaction terms.

We parameterize the superpotential in powers of the heavy fields \( \Phi_l \):

\[
W = \sum_{\{l_m\}} \sum_{n=0}^{\infty} \frac{1}{n!} \Phi_{l_1} \ldots \Phi_{l_n} W_n^{\{l_1 \ldots l_n\}}(\varphi_i);
\]

in which the \( W_n(\varphi_i) \) are holomorphic functions of the light fields. In particular, \( W_2 \) includes mass terms for \( \{ \Phi_l \} \).

Let us rewrite the action in term of its heavy and light components:

\[
S(\vec{\varphi}, \vec{\Phi}) = S_{\text{light}}(\vec{\varphi}) + S_0(\vec{\Phi}) + S_{\text{int}}(\vec{\varphi}, \vec{\Phi}).
\]

In this expression, \( S_{\text{light}}(\vec{\varphi}) \) includes the free action and the self-interactions of the light fields, such that

\[
S_{\text{light}}(\vec{\varphi}) = \int d^4x d^2\theta d^2\bar{\theta} \vec{\varphi} \vec{\varphi}^\dagger + \left\{ \int d^4x d^2\theta W_0(\vec{\varphi}) + h.c. \right\}.
\]

\( S_0(\vec{\Phi}) \) is the free action of the heavy fields \( \vec{\Phi} = \{ \Phi_l \} \), and \( S_{\text{int}}(\vec{\varphi}, \vec{\Phi}) \) includes the interactions between light fields and \( \vec{\Phi} \), i.e.:
\[ S_0(\vec{\Phi}) = \int d^4x d^2\theta d^2\bar{\theta}^\dagger \vec{\Phi} + \left\{ \int d^4x d^2\theta d^2\bar{\theta}^T \mathbf{M} \vec{\Phi} + \text{h.c.} \right\}, \]  

(7)

\[ S_{\text{int}}(\vec{\varphi}, \vec{\Phi}) = \left\{ \int d^4x d^2\theta \sum_{\{l_m\}}^{\infty} \frac{1}{n!} \Phi_{l_1} \ldots \Phi_{l_n} W_{n}^{\{l_1 \ldots l_n\}}(\varphi_i) - \vec{\Phi}^T \mathbf{M} \vec{\Phi} + \text{h.c.} \right\} + \int d^4x d^2\theta d^2\bar{\theta} K', \]  

(8)

in which \( \mathbf{M} \) is the mass matrix for \( \vec{\Phi} \), with eigenvalues of \( \mathcal{O}(M) \). At energy scales lower than \( M \), the heavy fields decouple from the theory, leading to nonrenormalizable contributions to the effective action of the light fields which are suppressed by inverse powers of \( M \). The conventional method to obtain the effective supersymmetric action is to integrate out the heavy fields from the theory using superspace functional integral techniques, as discussed in [4]. We briefly describe this method in part (A). However, we concentrate on an alternative method in this paper, by solving the full set of equations of motion for \( \{\Phi_l\} \). We introduce this formalism in part (B).

### A. Superspace Functional Integral Formalism

In general, if a supersymmetric theory contains light or massless fields \( \{\varphi\} \) and heavy fields \( \{\Phi_l\} \), the effective action \( S_{\text{eff}}(\vec{\varphi}) \) can be derived from the full action \( S(\vec{\varphi}, \vec{\Phi}) \) by functionally integrating over the heavy fields:

\[
\exp(iS_{\text{eff}}(\vec{\varphi})) = \int [d\Phi] \exp(iS(\vec{\varphi}, \vec{\Phi})) = \exp(iS_{\text{light}}(\vec{\varphi})) \int [d\Phi] \exp[iS_0(\vec{\Phi}) + iS_{\text{int}}(\vec{\varphi}, \vec{\Phi})].
\]  

(9)

This expression is then rewritten as follows:

\[
\exp(iS_{\text{eff}}(\vec{\varphi})) = \exp(iS_{\text{light}}(\vec{\varphi})) \exp(iS_{\text{int}}(\delta, \delta)) \int [d\Phi] \exp(iS_0(\vec{\Phi}))|_{j_l=j_m=0},
\]  

(10)

where \( j = \{j_l(z)\} \) are introduced as chiral sources for \( \vec{\Phi} \), and \( \vec{\Phi} \) and \( \vec{\Phi}^\dagger \) are replaced in \( S_{\text{int}} \) by \( \{\delta j_l\} \) and \( \{\delta j_m\} \), respectively. In the above expressions, \( \int [d\Phi] \exp(iS_0(\vec{\Phi})) \) is the free generating function \( Z_0[j, \tilde{j}] \) for \( \vec{\Phi} \) in superspace:

\[
Z_0[j, \tilde{j}] = \exp \left( -\frac{1}{2} i \int d^4x d^4\theta d^4x' d^4\theta' [j(z), \tilde{j}(z')] \Delta_{\text{GRS}}(z, z') \right),
\]  

(11)

where \( z = (x, \theta, \bar{\theta}) \) are the usual superspace coordinates, \( d^4\theta = d^2\theta d^2\bar{\theta} \), and \( \Delta_{\text{GRS}}(z, z') \) is the superspace propagator given by:

\[
\Delta_{\text{GRS}}(z, z') = (\Box 1 - M^2)^{-1} \left[ \begin{array}{c} \frac{M D^2}{4\theta} \\ \frac{1}{4\theta} \end{array} \right].
\]  

(12)
Treating $S_{int}$ as a perturbation, the effective action can be achieved via the expansion:

$$\exp(iS_{eff}(\vec{\varphi})) = \exp(iS_{light}(\vec{\varphi}))\{1 + iS_{int}\left(\frac{\delta}{\delta j_l}, \frac{\delta}{\delta j^*_m}\right) + \ldots\}|_{j_l=j^*_m=0}. \quad (13)$$

The contributions to the effective action may arise from all orders in the expansion. The corrections to the superpotential take the form of the type $\Pi_i \varphi^n_i / M^{N-3}$ (with $N = \sum_i n_i$), and the corrections to the Kähler potential take the form of the type $\Pi_{l,j} \varphi^n_i \varphi^{*n}_j / M^{N-2}$ (with $N = \sum_{i,j} n_i n_j$).

An advantage of the path integral formalism is that an inspection of the supergraphs illustrates (and in some cases gives a systematic and compact answer to) the new contributions to the effective action.

**B. Solving the equation-of-motion of the heavy field in superspace**

An alternative approach is to solve the full equation of motion of the heavy fields $\{\Phi_l\}$ and substitute the solution into the full Lagrangian to obtain the effective Lagrangian $L_{eff}(\vec{\varphi}, \vec{\varphi}^*)$ of the light fields.

Assuming the form of the Kähler potential (2) and the form of the superpotential given in (4), the equation of motion for the heavy field $\Phi_l$ is

$$- \frac{D^2}{4} \Phi^*_l - \frac{D^2}{4} \left( \frac{\partial K'}{\partial \Phi_l} \right) + \frac{\partial}{\partial \Phi_l} \left( \sum_{\{l_k\}} \sum_{n=1}^{\infty} \frac{1}{n!} \Phi_{l_k} \ldots \Phi_{l_n} W_{n}^{(l_1 \ldots l_n)}(\varphi_{i}) \right) = 0. \quad (14)$$

This equation can be solved iteratively, assuming $\Phi_l = \sum_{n=0}^{\infty} \Phi_l^{(n)}$. The solution of the equation of motion yields an expression for $\vec{\Phi}$ as a function of $\vec{\varphi}$ and $\vec{\varphi}^*$ in the form of a series. This expression is then put back into the full supersymmetric action to derive the effective superpotential and Kähler potential for the light fields. As $\vec{\Phi}$ in general is a function of both $\vec{\varphi}$ and its conjugate, there are terms of the form $\varphi^n_i \varphi^{*n}_j$ from the expansion of the superpotential, which are corrections to the effective Kähler potential. Similarly, there are terms of form $\varphi^n_i$ from the expansion of $\vec{\Phi}^* \vec{\Phi}$ in the Kähler potential, which become corrections to the effective superpotential. Hence, the new terms appearing in the effective action can be carefully grouped to derive $K_{eff}$ and $W_{eff}$.

In general, it is hard to obtain the full solution to the equation of motion for $\Phi_l$. In the next sections, we investigate several examples of differing choices of $W$ and $K'$ involving the heavy fields in which the solution to the equation of motion can be written in a compact form. In one case, we find that the corrections to the effective action can be determined to all orders in the nonrenormalizable terms through this method, and that the calculation is in precise agreement with the results of the calculation of the relevant supergraphs. However, we find for most cases it is necessary to work to a certain order in the calculation of the effective superpotential and effective Kähler potential. We calculate the first few nonleading corrections to the effective action in these cases.
III. RESULTS

A. Choices of Superpotential

We now consider several examples of the superpotential terms that involve the interactions between the heavy and light fields. At present, we assume the minimal Kähler potential (3). Our examples are motivated by the types of terms that are generically present in the superpotentials of string models.

For tree-level exchange of heavy fields, the simplest case is to assume the presence of an interaction term of the type $\Phi_l^T W_1^T (\vec{\varphi}) \vec{\Phi} + W_0(\vec{\varphi})$. We then consider slightly more complicated examples, keeping in mind that many such terms are immediately nonrenormalizable. We choose to analyze the case with an interaction term between one light field and two heavy fields (i.e. a nontrivial choice of $W_2(\vec{\varphi})$ in (4)), as well as a cubic self-interaction among the heavy fields.

• Example 1.

We take

$$W = \vec{\Phi}^T M \vec{\Phi} + \vec{W}_1^T (\vec{\varphi}) \vec{\Phi} + W_0(\vec{\varphi}).$$  \hfill(15)

In this case, $S$ is

$$S = \int d^4x d^2\theta d^2\bar{\theta} \bar{\Phi} \Phi^{\dagger} + \{ \int d^4x d^2\theta (\vec{\Phi}^T M \vec{\Phi} + \bar{\Phi}^T \vec{W}_1) + h.c. \} + S_0,$$  \hfill(16)

with $S_0$ given in (7).

The equation of motion for $\vec{\Phi}$ can be written from (14):

$$-\frac{\bar{D}^2}{4} \bar{\Phi}^{\dagger} + \bar{\Phi}^T M + \bar{W}_1^T (\vec{\varphi}) = 0.$$  \hfill(17)

This set of equations can be solved iteratively, assuming

$$\vec{\Phi} = \sum_{n=0}^{\infty} \vec{\Phi}^{(n)},$$  \hfill(18)

and that $-\frac{\bar{D}^2}{4} \vec{\Phi} \ll \vec{\Phi}^T M + \bar{W}_1(\vec{\varphi})$, such that the kinetic terms for $\vec{\Phi}$ are small compared to the mass terms. The zeroth order solution is obtained by neglecting the kinetic energy terms:

$$\vec{\Phi}^{(0)} = -M^{-1} \bar{W}_1.$$  \hfill(19)

Similarly, the first order correction is given by

$$\vec{\Phi}^{(1)} = M^{-1} \frac{\bar{D}^2}{4} \vec{\Phi}^{(0)}. $$  \hfill(20)
Repeating the procedure, the solution to the equations of motion for $\Phi$ takes the form of a geometric series

$$
\Phi = \Phi^{(0)} + M^{-1} \frac{\bar{D}^2}{4} \Phi^{(0)} \ast + M^{-1} \frac{\bar{D}^2}{4} M^{-1} \frac{\bar{D}^2}{4} \Phi^{(0)}
+ M^{-1} \frac{\bar{D}^2}{4} M^{-1} \frac{\bar{D}^2}{4} M^{-1} \frac{\bar{D}^2}{4} \Phi^{(0)} \ast + \ldots ,
$$

(21)

which can be summed exactly. Using the identity $\bar{D}^2 D^2 \Phi = \Box \Phi$ (for chiral superfields, which satisfy $\bar{D} \Phi = 0$), the solution can be written in the compact form

$$
\Phi = -(M^2 - \Box)^{-1} \left[ M\tilde{W}_1 + \frac{\bar{D}^2}{4} \tilde{W}_1 \ast \right].
$$

(22)

The solution is then substituted into the action $S$ to determine the effective action for the light fields. The contributions to the effective superpotential and Kähler potential are extracted from each term, using the identity that under an $x$-integration $d^2 \theta = -\bar{D}^2$ (and similarly $d^2 \bar{\theta} = -D^2$).

Summing up all of the contributions, the effective action

$$
S_{\text{eff}} = \int d^4 x d^2 \theta d^2 \bar{\theta} K_{\text{eff}} + \left\{ \int d^4 x d^2 \theta W_{\text{eff}} + h.c. \right\}
$$

(23)

can be expressed in terms of the exact effective Kähler potential and superpotential written in a closed form:

$$
W_{\text{eff}} = W_0(\tilde{\varphi}) - \tilde{W}_1^T (M^2 - \Box) M \tilde{W}_1,
$$

(24)

$$
K_{\text{eff}} = K_{\text{light}} + \tilde{W}_1^T (M^2 - \Box) \tilde{W}_1.
$$

(25)

This result can be obtained using the functional integral formalism as well; the relevant supergraphs are shown in Figure 1. In this case, there is a $\Phi_i \Phi_j$ propagator, as well as a $\Phi_i \Phi_j^\ast$ propagator. The supergraph with the $\Phi_i \Phi_j$ propagator gives the result for the effective superpotential $W_{\text{eff}}$, while the supergraph with the $\Phi_i \Phi_j^\ast$ propagator yields the above expression for the effective Kähler potential $K_{\text{eff}}$.

The effective scalar potential can be derived from $W_{\text{eff}}$ and $K_{\text{eff}}$ via:

$$
V_{\text{eff}} = \left[ \frac{\partial^2 K_{\text{eff}}}{\partial \varphi_i \partial \varphi_j} \right]^{-1} \frac{\partial W_{\text{eff}}}{\partial \varphi_i} \left( \frac{\partial W_{\text{eff}}}{\partial \varphi_j} \right)^\dagger.
$$

(26)

Thus, the corrections to the effective scalar potential include not only the corrections to the superpotential, but also the effects of the corrections to the Kähler potential. This effect leads to additional nonrenormalizable terms in the scalar potential. The corrections from the effect of the kinetic energy of the heavy particle, which are the terms that include the factor $\Box / M^2$, are higher-order derivative interactions in the theory.

To illustrate the techniques we have developed above, we consider the following example, taking one heavy field $\Phi$ for simplicity. We assume the following form for the superpotential, consistent with (17):

$$
\begin{align*}
\text{(17)}
\end{align*}
$$

6
\[ W = \frac{M}{2} \Phi^2 + \Phi \varphi_1^2 + \varphi_1 \varphi_2^2; \]  

(27)
i.e., \( W_0 = \varphi_1 \varphi_2^2 \) and \( W_1 = \varphi_1^2 \). In this case, \( \Phi_0 = -\varphi_1^2/M \). Assuming the minimal Kähler potential for the light fields (3), we determine the effective superpotential and Kähler potential from (24) and (25):

\[ W_{\text{eff}} = \varphi_1 \varphi_2^2 - \frac{1}{1 - \frac{\varphi_1^4}{M^2}} \varphi_1^4; \quad K_{\text{eff}} = \varphi_1 \varphi_1^\dagger + \varphi_2 \varphi_2^\dagger + \frac{1}{1 - \frac{\varphi_1^2 \varphi_2^2}{M^2}}. \]  

(28)

To obtain the effective scalar potential, we use (26) and neglect all terms involving \( \Box / M^2 \).

The effective scalar potential for the theory is therefore

\[ V_{\text{eff}} = \frac{|\varphi_2^2 - \varphi_1^3/M|^2}{1 + |\varphi_1|^2/M^2} + 4|\varphi_1 \varphi_2|^2, \]  

(29)
in which the denominator of the first term demonstrates the effect of the modified Kähler potential. In particular, this effect leads to an additional term of \( \mathcal{O}(1/M^2) \) in the scalar potential \( (|\phi_1|^2 |\phi_2|^4/M^2) \).

- Example 2.

We consider the superpotential

\[ W = W_0(\varphi_i) + \Phi W_1(\varphi_i) + \Phi^2 W_2, \]  

(30)
in which we assume only one heavy field \( \Phi \) for the sake of simplicity, and we take

\[ W_2 = \frac{M}{2} (1 + 2 \tilde{W}_2 M). \]  

(31)

The action \( S \) of the heavy fields is therefore given by

\[ S = \int d^4x d^2\theta d^2\bar{\theta}(\Phi \Phi^\dagger) + \left\{ \int d^4x d^2\theta (\Phi^2 W_2 + \Phi W_1) + h.c. \right\}. \]  

(32)

The equation of motion for \( \Phi \) is

\[- \frac{\tilde{D}^2}{4} \Phi^\dagger + W_1(\varphi_i) + 2 \Phi W_2(\varphi_i) = 0.\]  

(33)

As in the previous example, this equation can be solved iteratively in the limit that the kinetic energy term is small. The zeroth order solution is

\[ \Phi^{(0)} = - \frac{W_1}{2W_2} = -(1 + \frac{2\tilde{W}_2}{M})^{-1} \frac{W_1}{M}, \]  

(34)

and the first order correction is

\[ \Phi^{(1)} = \frac{\tilde{D}^2}{8W_2} \Phi^{(0)} = (1 + \frac{2\tilde{W}_2}{M})^{-1} \frac{\tilde{D}^2}{4M} \Phi^{(0)} \dagger. \]  

(35)
Repeating the procedure to obtain the higher order terms in the expansion, the series can be formally summed to obtain

\[
\Phi = \frac{1}{1 - \frac{\bar{D}^2}{8W_2}\left(\frac{D^2}{8W_2}\right)} \left[ \Phi^{(0)} + \left(1 + \frac{2\bar{W}_2}{M}\right)^{-1}\frac{\bar{D}^2}{4M} \Phi^{(0)}\right] = \frac{1}{1 - \left(1 + \frac{2\bar{W}_2}{M}\right)^{-1}\frac{\bar{D}^2}{4M} \left(1 + \frac{2\bar{W}_2}{M}\right)^{-1}\frac{\bar{D}^2}{4M} \Phi^{(0)}\right].
\]

The form of (36) indicates that in this case, it is not tractable to extract the exact corrections to all orders in \((1/M)\) to the effective action of the light fields. Therefore, we work to the first nonleading contribution to the effective action, which is the \((1/M^2)\) correction to the effective superpotential and the \((1/M^3)\) correction to the effective Kähler potential. Substituting the solution (36) into (32) and keeping only the terms to the appropriate order, the result is

\[
W_{\text{eff}} = W_0 - \frac{W_1^2}{2M} + \frac{W_1^2\bar{W}_2}{M^2} + \mathcal{O}\left(1/M^3\right)
\]

\[
K_{\text{eff}} = K_{\text{light}} + \frac{W_1^2\bar{W}_1}{M^2} + 2\frac{W_1^2\bar{W}_1(\bar{W}_2 + \bar{W}_2^\dagger)}{M^3} + \mathcal{O}\left(1/M^4\right).
\]

The relevant supergraphs illustrate this result, and are shown in Figure 2.

• Example 3.

We consider the cubic self-interactions of one heavy field \(\Phi\), with the superpotential

\[
W = W_0(\phi_i) + \frac{M}{2} \Phi^2 + \Phi W_1(\phi_i) + \frac{\lambda}{3} \Phi^3,
\]

so that the supersymmetric action involving the heavy fields is given by

\[
S = \int d^4xd^2\theta d^2\bar{\theta}(\Phi\Phi\dagger) + \left\{ \int d^4xd^2\theta(\frac{M}{2} \Phi^2 + \Phi W_1 + \frac{\lambda}{3} \Phi^3) + h.c. \right\}.
\]

The equation of motion for \(\Phi\) is

\[
- \frac{\bar{D}^2}{4} \Phi\dagger + W_1(\phi_i) + M\Phi + \lambda\Phi^2 = 0.
\]

In this case, the zeroth order iterative solution \(\Phi^{(0)}\) is obtained from the quadratic equation

\[
\lambda\Phi^{(0)} + M\Phi^{(0)} + W_1 = 0,
\]

with the solution

\[
\Phi^{(0)} = -\frac{M}{2\lambda} \pm \sqrt{\left(\frac{M}{2\lambda}\right)^2 - \left(\frac{W_1}{\lambda}\right)}.
\]
When the solution is expanded in the limit $W_1 \ll M^2$, it is evident that the positive root is the physical solution (that corresponds to the previous result with $\lambda = 0$). The first few terms of this solution are

$$\Phi^{(0)} = -\frac{W_1}{M} - \frac{\lambda W_1^2}{M^3} - \frac{2\lambda^2 W_1^3}{M^5} + \ldots. \tag{44}$$

The iterative procedure leads to the first and second order solutions

$$\Phi^{(1)} = (1 + 2\frac{\lambda \Phi^{(0)}}{M})^{-1} \bar{D}^2 \frac{\Phi^{(0)}}{4M}, \tag{45}$$

$$\Phi^{(2)} = (1 + 2\frac{\lambda \Phi^{(0)}}{M})^{-1} \left( \frac{\bar{D}^2}{4M} \Phi^{(1)} \right) - \frac{\lambda}{M} \sum_{i=1}^{n-1} \Phi^{(i)} \Phi^{(n-i)}. \tag{46}$$

In this case, the solution can not be summed into a compact form. However, the terms in the series are determined from the recursion relation

$$\Phi^{(n)} = (1 + 2\frac{\lambda \Phi^{(0)}}{M})^{-1} \left( \frac{\bar{D}^2}{4M} \Phi^{(n-1)} \right) - \frac{\lambda}{M} \sum_{i=1}^{n-1} \Phi^{(i)} \Phi^{(n-i)}. \tag{47}$$

Therefore, to obtain the effective supersymmetric action of the light fields, it is necessary to work to a given order in the nonrenormalizable terms. The solution (44) indicates that the first nonleading correction to the superpotential from the cubic self-interaction term is of $O(1/M^3)$, and hence we work to that order (and to $O(1/M^4)$ in the effective Kähler potential). The results are presented below:

$$W_{eff} = W_0 - \frac{W_1}{2M} - \frac{\lambda W_1^3}{3M^3} + O \left( \frac{1}{M^5} \right) \tag{48}$$

$$K_{eff} = K_{light} + \frac{W_1 \Phi}{M^2} + \frac{\lambda W_1 \Phi^2}{M^4} + \frac{\lambda W_1^2 \Phi}{M^4} + O \left( \frac{1}{M^6} \right). \tag{49}$$

We display the corresponding supergraphs in Figure 3.

**B. Choices of Kähler Potential**

We now consider the effects of non-minimal Kähler potential terms involving the heavy fields.

- **Example 1.**

The simplest non-trivial example in this category is $K' = \Phi F(\varphi_i, \varphi_j^\dagger) + h.c.,$ in which $F(\varphi_i, \varphi_j^\dagger)$ is a function of either $\varphi_j^\dagger$ only or $\varphi_i \varphi_j^\dagger$ (but not a function of $\varphi_i$ only), such that $\Phi F(\varphi_i, \varphi_j^\dagger)$ is not holomorphic. These terms are non-renormalizable and hence are suppressed by $1/M^{N-2}$, where $N$ is the total power of the term.

For this example, we take

$$K = K_{light} + \Phi^\dagger \Phi + (\Phi F(\varphi_i, \varphi_j^\dagger) + h.c.), \tag{50}$$

$$W = \Phi W_1 (\varphi_i) + \frac{M}{2} \Phi^2. \tag{51}$$
The supersymmetric action involving the heavy field $\Phi$ is therefore

$$S = \int d^4x d^2\theta d^2\bar{\theta} \left\{ \Phi \Phi^\dagger + (\Phi F(\varphi_i, \varphi_j^\dagger) + h.c.) \right\} + \int d^4x d^2\theta \left\{ \left( \frac{M}{2} \Phi^2 + \Phi W_1 \right) + h.c. \right\}. \quad (52)$$

The equation of motion for $\Phi$ is

$$-\frac{\bar{D}^2}{4} \Phi^\dagger - \frac{\bar{D}^2}{4} F + M \Phi + W_1 = 0. \quad (53)$$

In this case, we can solve the equation for $\Phi$ iteratively to all orders. However, as $\Phi F + h.c.$ is already non-renormalizable, we only work with the lowest order solutions which lead to non-trivial effects in the effective action. The zeroth order solution is

$$\Phi^{(0)} = -\frac{W_1}{M} + \frac{\bar{D}^2}{4M} F; \quad (54)$$

and the higher order solutions can be calculated using the iteration equation

$$\Phi^{(n)} = \frac{\bar{D}^2}{4M} \Phi^{(n-1)} \dagger. \quad (55)$$

Keeping the lowest orders while expanding the superpotential and the Kähler potential, the corrections to the effective action are

$$S_{\text{eff}} = S_0 + S_{W_1}$$

$$+ \int d^4x d^2\theta d^2\bar{\theta} \left\{ \left( -\frac{W_1}{M} F - F \frac{\bar{D}^2}{4M^2} W_1^\dagger + \frac{1}{2} F \frac{D^2}{4M} F \right) + h.c. + \mathcal{O}(1/M^4) \right\}. \quad (56)$$

$S_{W_1}$ includes the contributions from the superpotential term $\Phi W_1$ only, which are the corrections to the effective superpotential and Kähler potential given in (24) and (25).

To investigate the type of corrections in the third term of the previous expression for $S_{\text{eff}}$, we split $F$ into its two types of terms as follows:

$$F_1 = \frac{1}{M^{N-1}} (\varphi_1 \varphi_2 \ldots \varphi_k)(\varphi_{k+1}^\dagger \ldots \varphi_N^\dagger); \quad (57)$$

and

$$F_2 = \frac{1}{M^{N-1}} \varphi_1^\dagger \varphi_2^\dagger \ldots \varphi_N^\dagger. \quad (58)$$

As $\varphi_i$ are chiral fields, effectively $F_1 \sim \varphi \varphi^\dagger$ and $F_2 \sim \varphi^\dagger$. We also recall that $W_1(\varphi_i)$ is a holomorphic function of $\varphi_i$. It is clear that for the $F_1$ terms the highest components of $-\frac{W_1}{M} F$, $-F \frac{\bar{D}^2}{4M^2} W_1^\dagger$ and $\frac{1}{2} F \frac{D^2}{4M} F$ are the $\theta \theta \bar{\theta}$ components; hence, they are corrections to $K_{\text{eff}}$.

For the $F_2$ terms, the highest components of $-\frac{W_1}{M} F$ are the $\theta \theta \bar{\theta}$ components. In addition the highest component of $-F \frac{\bar{D}^2}{4M^2} W_1^\dagger$ is also the $\theta \theta \bar{\theta}$ component in this case. To see this more clearly, note that
\[ \bar{D}(\bar{D}^2 W_1^\dagger) = 0, \]  
(59)

as \( \bar{D} \) is an anti-commuting two-component operator. Therefore, \( \bar{D}^2 W_1^\dagger \) is effectively a chiral field of type \( \varphi \), so that \(-F \frac{\bar{D}^2}{4M^2} W_1^\dagger \sim \varphi \varphi \), which has the \( \theta \theta \bar{\theta} \bar{\theta} \) component as its highest component. Similar reasoning leads to the same result for \( \frac{1}{2} F_2 \frac{\bar{D}^2}{4M} F_2 \). We conclude that the leading corrections from the non-minimal Kähler potential are corrections to the effective Kähler potential (with no direct corrections to the effective superpotential):

\[ K_{\text{eff}} = K_{\text{light}} + \frac{W_1^\dagger W_1}{M^2} - \left( \frac{W_1}{M} F + F \frac{\bar{D}^2}{4M^2} W_1^\dagger - \frac{1}{2} F \frac{\bar{D}^2}{4M} F + \text{h.c.} \right) + O(1/M^4). \]  
(60)

**Example 2.**

We consider the non-minimal Kähler potential terms that take the form \( \Phi^\dagger F(\varphi_i, \varphi_j^\dagger) \Phi \), in which \( F(\varphi_i, \varphi_j^\dagger) \) is a real function of the light fields \( \varphi_i, \varphi_j^\dagger \). This term is therefore suppressed by \( 1/M^N \), where \( N \) is the total power of the light fields in \( F \).

We take

\[ K = K_{\text{light}} + \Phi^\dagger \Phi + \Phi^\dagger F(\varphi_i, \varphi_j^\dagger) \Phi \]  
(61)
\[ W = \Phi W_1 + \frac{M}{2} \Phi^2. \]  
(62)

such that the supersymmetric action for the heavy fields is

\[ S = \int d^4x d^2\theta d^2\bar{\theta} \left\{ \Phi \Phi^\dagger + \Phi F(\varphi_i, \varphi_j^\dagger) \Phi^\dagger \right\} + \int d^4x d^2\theta \left\{ \left( \frac{M}{2} \Phi^2 + \Phi W_1 \right) + \text{h.c.} \right\}. \]  
(63)

The equation of motion for \( \Phi \) is

\[ -\frac{\bar{D}^2}{4}(\Phi^\dagger) - \frac{\bar{D}^2}{4}(F\Phi^\dagger) + M\Phi + W_1 = 0. \]  
(64)

As \( F \) is of higher order in \( 1/M \), the zeroth order solution is

\[ \Phi^{(0)} = -\frac{W_1}{M}, \]  
(65)

the higher order solutions can be calculated using the iteration equation

\[ \Phi^{(n)} = \frac{\bar{D}^2}{4M} \Phi^{(n-1)^\dagger} + \frac{\bar{D}^2}{4M} (F\Phi^{(n-1)^\dagger}). \]  
(66)

To demonstrate the non-trivial effects of \( F \), we retain the terms up to third order in \( (1/M) \) in the expansion of the superpotential and the Kähler potential. As in the previous example, the non-minimal Kähler potential term contributes to the effective Kähler potential only (and not the effective superpotential):

\[ K_{\text{eff}} = K_{\text{light}} + \frac{W_1^\dagger W_1}{M^2} + \frac{W_1^\dagger FW_1^\dagger}{M^2} + \frac{1}{M^3} \left[ W_1^\dagger \frac{\bar{D}^2}{4}(FW_1^\dagger) + (FW_1^\dagger) \frac{\bar{D}^2}{4} W_1^\dagger + (FW_1^\dagger) \frac{\bar{D}^2}{8} (FW_1^\dagger) + \text{h.c.} \right] + O(1/M^4). \]  
(67)
Note that the last term in the set of $1/M^3$ terms has two factors of $F$, and thus is potentially of higher order than the other two terms. The effective superpotential includes contributions from the term $\int d^4 x d^2 \theta \Phi W_1$ as usual, which is given in (24).

This example has relevance for the issue of decoupling in gauge theories as well. In this case, $F$ is a function of the corresponding vector supermultiplets. For example, in the case of an Abelian gauge theory in which $\Phi$ has $U(1)$ charge $q$, the gauge invariant kinetic energy term for $\Phi$ is of the type $\Phi^\dagger e^{gqV} \Phi^\dagger$, where $g$ is the gauge coupling and $V$ denotes the vector supermultiplet. In the Wess-Zumino gauge, $F(V) = e^{gqV} - 1 = qgV + q^2g^2V^2/2$, and we can apply the techniques we have developed previously. We find that the lowest order correction to the effective superpotential involving $F(V)$ is $\int d^4 d^2 \theta d^2 \bar{\theta} (1/M^2) W_1(qgV + q^2g^2V^2/2) W_1^\dagger$. In contrast to $F(\phi_i, \phi_j^\dagger)$, $F(V)$ does not have additional suppressions in powers of $(1/M)$. Hence, the lowest order correction is comparable to the lowest contribution from $\Phi W_1$ in the superpotential.

IV. CONCLUSIONS

We addressed the effects of the decoupling of heavy fields in the $N = 1$ supersymmetric action. While the study of these effects is motivated by vacuum restabilization of string vacua due to an anomalous $U(1)$, the discussion is given in a general context of $N = 1$ supersymmetric field theories (of gauge singlet fields).

We employed an iterative procedure to solve equations of motion for the heavy chiral superfields $\vec{\Phi}$, which allows for a controlled study of the corrections at each order to both the effective superpotential and Kähler potential; in some specific cases, in particular the case with the “mixed” couplings between the heavy and light chiral superfields $\vec{\phi}$ of the type $\vec{\Phi}^T \vec{W}_1(\vec{\phi})$, the full summation is possible. This method is also illustrated in a complementary way by employing the corresponding supergraphs.

For specific examples of mixed couplings appearing in the superpotential, we demonstrated that the leading correction is indeed to the superpotential $\mathcal{W}$. However, the next-order corrections are not only to the superpotential but also to the Kähler potential, and signify new effects to the effective potential of the theory. In general, the corrections to the Kähler potential are one order higher than the corrections to the effective superpotential of the theory.

In examples with the mixed couplings arising in the Kähler potential we found that these terms always contribute to the leading corrections in the effective Kähler potential (and are generically of the higher order than the leading correction due to the mixed couplings appearing in the superpotential).

In the case of supersymmetric gauge field theories, corrections due to the coupling of the heavy fields to gauge fields naturally appear in the Kähler potential. We found that these effects again correct the effective Kähler potential (in the leading order); however, further study that may explore possible corrections to the effective gauge function (specifying the effective gauge coupling) is underway.
V. ACKNOWLEDGMENTS

This work was supported in part by U.S. Department of Energy Grant No. DOE-EY-76-02-3071. We thank J. R. Espinosa for his participation in the initial stages of the collaboration, and for many helpful discussions. We also thank P. Langacker, K. Dienes, V. Kaplunovsky, and M. Porrati for useful discussions.
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FIG. 1. The supergraphs contributing to (a) the effective superpotential, and (b) the effective Kähler potential for Example 1, with $W = \Phi^T \bar{W}_1$.  

FIG. 2. The supergraphs contributing to (a) the effective superpotential, and (b) the effective Kähler potential for Example 2, with $W = \Phi W_1 + \Phi^2 W_2$.  

FIG. 3. The supergraphs contributing to (a) the effective superpotential, and (b) the effective Kähler potential for Example 3, with $W = \Phi W_1 + \Phi^3$. 