THE $e^-e^-$ MODE OF LC: OPPORTUNITIES TO DISCOVER
LOOP-LEVEL LEPTON FLAVOR (NUMBER) VIOLATION

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The $e^-e^-$ mode with lepton number $L = +2$ of the initial state is particularly
suitable for studying lepton and flavor number violating reactions (LFV). In this
brief report we give a summary of the study of two loop-level reactions which
violate lepton and flavor numbers: we first consider seesaw type models with heavy
Majorana neutrinos at the TeV scale and study the reactions $e^-e^- \rightarrow \ell^-\ell^-$, and
then similar reactions $e^-e^- \rightarrow \ell^-e^- (\ell = \mu, \tau)$ in supersymmetric models where
LFV is due to slepton mixing. More details on the calculations and numerical
tools used can be found in Refs. 1 while summary of the theoretical scenarios is
presented in the summary report of the $e^-e^-$ session 2.

1 $e^-e^- \rightarrow \ell^-\ell^- (\ell = \mu, \tau)$ With TeV Scale Majorana Neutrinos

To produce a detectable signal, heavy Majorana neutrinos (HMN), besides
having masses in the TeV range, must have interactions which are not sup-
pressed by the mixing matrices as it happens in the one family seesaw me-
chanism, where $\theta \simeq \sqrt{m_\nu/M_N}$. With three generations, more free pa-
rameters are at our disposal, and the “two miracles” of not so large masses and
non negligible mixing, are obtained imposing suitable relations among the el-
ements of the matrices $m_D$ and $M_R$: examples of these models are proposed
in Refs. 3. Experimentally one cannot put bounds on single mixing matrix
elements, but on some combinations of them, assuming that each charged lep-
ton couples only to one heavy neutrino with significant strength. Light-heavy
mixing has to be inferred from low-energy phenomenology and from global
fits performed on LEP data identifying the following effective mixing angles
$s^2_\ell = \sum_j |B_{\ell N_j}|^2 \equiv \sin^2 \theta_{\ell N_j}$ with upper bounds 4: $s^2_\ell < 0.0054$, $s^2_\mu < 0.005$,
$s^2_\tau < 0.016$, where $B$ is the mixing matrix appearing in the charged current
weak interaction lagrangian. Under these assumptions, the coupling of neu-
trissimos to gauge bosons and leptons is numerically fixed to $gB_{HN}$. Since the
width of the heavy states grows as $M_N^3$, at a certain value it will happen that
$\Gamma_N > M_N$, signaling a breakdown of perturbation theory. The perturbative
limit on $M_N$ is thereby estimated requiring $\Gamma_N < M_N/2$, which gives an upper bound of $\simeq 3$ TeV for the numerical values of the mixing used. We study the process $e^- e^- \rightarrow \ell^- \ell^- \ (\ell = \mu, \tau)$, described by box diagrams with two HMN and two $W$ gauge bosons in the loop. One expects an enhancement of the cross section at $\sqrt{s} \simeq 161$ GeV $\simeq 2M_W$, the threshold for on-shell $WW$ gauge boson production, at which the four-point functions develop an imaginary part. The enhancement due to the threshold singularity of the loop amplitude is more pronounced for values of Majorana masses close to $M_W$ and is drastically reduced increasing $M_{N_i} \approx M_{N_j}$ to $\mathcal{O}$(TeV). The main contribution comes from the graph with two Goldstone bosons (the calculation is performed in the 't Hooft-Feynman gauge) since their coupling is proportional to $M_{N_i}$. Moreover the chiral structure of the coupling selects the mass term in the numerator of the Majorana neutrino propagators. When these masses are much larger then the other quantities, the amplitude scales like $M_{N_i}^2/M_{N_j}^2, M_{N_i}^2/M_{N_j}^2 \simeq M_{N_i}M_{N_j}$, i.e. is proportional to the square of the heavy masses. As shown in Fig. 1, (where both the two previous effects are easily seen) for $M_{N_i} = M_{N_j} = 3$ TeV the signal does reach the level of $10^{-1}, 10^{-2}$ fb respectively for the ($\tau\tau$) and the ($\mu\mu$) signals at $\sqrt{s} = 500$ GeV, which for an annual integrated luminosity of 100 fb$^{-1}$ would correspond respectively to 10 and 1 event/year. At higher energies, $\mathcal{O}$ (TeV), one could get even larger event rates (30 and 3) respectively: this is largest because the upper limits on the mixing are less stringent.

Figure 1: Total unpolarized cross sections as function of $\sqrt{s}$ for some values of HMN masses.
In the limit of massless external particles the process is dominated by a well defined helicity amplitude: $e_L e_L \rightarrow \ell_L \ell_L$. With left polarized initial beams one can enhance the signal by a factor of four (for ideal 100% polarization).

2  $e^- e^- \rightarrow \ell^- e^-$ ($\ell = \mu, \tau$) In R-Conserving Supersymmetric Models

The diagonalization of slepton mass matrices induce LFV couplings in the lepton-slepton-gaugino vertices: off diagonal entries in slepton mass matrices are generated, for example, by the seesaw mechanism embedded in the MSSM with R-parity conservation and mSUGRA boundary conditions at high energy. Anyway our approach is model independent: in order to keep the discussion simple enough, the maximal mixing of only two generations is considered, and accept the flavor violating entries of slepton mass matrices as large as allowed by the experimental bounds on rare LFV decays. The essential parameter which controls the LFV signal is $\delta_{LL} = \Delta m^2 / \tilde{m}^2$. It is assumed that the two lightest neutralinos are pure bino and pure wino with masses $M_1$ and $M_2$ respectively, while charginos are pure charged winos with mass $M_2$, $M_1$ and $M_2$ being the gaugino masses in the soft breaking potential. Numerical results are obtained using the mSUGRA relation $M_1 \approx 0.5 M_2$ for gaugino masses while $\Delta m^2$ and the slepton masses are taken to be free phenomenological parameters.

The are three contributing amplitudes: $M_{E1} = \mathcal{M}(e^- e^- \rightarrow \ell^- e^-)$, $M_{E2} = \mathcal{M}(e^- e^r \rightarrow \ell^- e^r)$, $M_{E3} = \mathcal{M}(e^- e^L \rightarrow \ell^- e^L)$. $M_{E1}$ has $J_z = 0$, is flat and forward-backward symmetric because of the antisymmetrization. $M_{E2}$ and $M_{E3}$ describe P-wave scattering with $J_z = +1$ and $J_z = -1$ respectively and are peaked in the forward direction and in the backward direction. The signal cross section is dominated by the amplitude $M_{E1}$. The analysis of the corresponding total cross section as a function of $\sqrt{s}$ shows that at $\sqrt{s} = 2 \tilde{m}_L$ a changes of orders of magnitude giving a sharp peak that is smeared only by large values of $\Delta m^2$. This can be easily understood considering the threshold behavior of the cross section for slepton pair production: $\beta = \sqrt{1 - 4 m^2_L / s}$ the selectron velocity, the amplitude of the intermediate state $e^- e^- \rightarrow \bar{e}^- \bar{e}^L$ behaves like $\beta^3$. With SUSY masses not much larger than $\sim 200$ GeV the signal is of order $\mathcal{O}(10^{-2})$ fb for $\delta_{LL} > \mathcal{O}(10^{-1})$. In addition the cross section is practically angle independent and thus insensitive to angular (or tranverse momentum) cuts.

The phenomenological points of the SUSY parameter space corresponding to gaugino masses $(M_1, M_2) = (80, 160)$ GeV or (100, 200) GeV and to slepton masses $m_L = 100 - 200$ GeV and $\delta_{LL} > 10^{-1}$ (which implies $\Delta m^2 > 10^3$ GeV$^2$)
can give in the $e^-e^-$ mode a detectable LFV signal ($e^-e^- \rightarrow \ell^-e^-$) although at the level of $\mathcal{O}(1 - 25)$ events/yr with $L_0 = 100$ fb$^{-1}$. Higher sensitivity to the SUSY parameter space could be obtained with larger $L_0$. On the other hand the experimental bounds on rare lepton decays $\mu, \tau \rightarrow e\gamma$ set constraints on the LFV violating parameters $\Delta m^2$ or $\delta_{LL}$. Fig. 2 shows that the bound from $\tau \rightarrow e\gamma$ does not constrain the region of the $(\delta_{LL}, m_L)$ plane compatible with an observable LFV signal and therefore the reaction $e^-e^- \rightarrow \tau^-e^-$ could produce a detectable signal within the highlighted regions of the parameter space. The process $e^-e^- \rightarrow \mu^-e^-$ is observable only in a small section of the parameter space since the allowed region from the $\mu \rightarrow e\gamma$ decay almost does not overlap with the collider “discovery” region except for a very small fraction in the case of gaugino masses ($M_1 = 80$ GeV and $M_2 = 160$ GeV). The compatibility of values of $\delta_{LL} \approx 1$ is due to a cancellation among the diagrams that describe the $\ell \rightarrow \ell'\gamma$ decay in particular points of the parameter space.
3 Conclusions

In summary, using the maximum experimentally allowed mixings, that masses of heavy Majorana neutrinos up to $2 - 3$ TeV can be explored with the reaction $e^- e^- \to \ell^- \ell^-, (\ell = \mu, \tau)$, because the amplitude gets an enhancement at the threshold for two gauge bosons production and then shows a non-decoupling behavior with the mass of the virtual heavy states. For the similar reaction $e^- e^- \to \ell^- e^-, (\ell = \mu, \tau)$ induced by slepton mixing in supersymmetric models, in certain regions of the parameter space, the signal can reach the level of $10^{-2}$ fb around the threshold for selectrons pair production. The possibility of employing beams with high degree of left longitudinal polarization is also essential to enhance the signal. These signals have the unique characteristic of a back to back high energy lepton pair and no missing energy. Sources of Standard Model background like the reaction $e^- e^- \to \nu_e\bar{\nu}_e W^- W^-\ast$ followed by the decays $W^- W^- \to \ell^- \bar{\nu}_e \ell^- \bar{\nu}_e$, are studied in details in Ref. 1 where it is shown that with reasonable cuts on the transverse momenta of the leptons and on the missing energy, it will be drastically reduced, without affecting significantly the signal.

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