On some properties of di-hadronic states

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Abstract

The binding energies of di-hadronic states have been calculated assuming a ‘molecular’ interaction provided by the asymptotic expression of the residual confined gluon exchange potential between the component hadrons in the system. Meson–meson, meson–baryon and baryon–baryon states have been studied in detail and a mass formula has been used to calculate total mass of the ‘molecules’. The calculated data are found to match available experimental values. The calculations allow us to identify exotic states such as \( f_0(0.982) \), \( h_1(1.17) \), \( f_2(2.01) \), etc as di-hadronic molecules.

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1. Introduction

Quantum chromodynamics (QCD) allows for the existence of various multi-quark states, and the same can also be predicted by various phenomenological models [1–6]. While the properties of mesons (\( q\bar{q} \)) and baryons (\( qqq \)) have been well documented both in theory and experiment, the exotics (states that do not fit into \( q\bar{q} \) or \( qqq \) states) remain less known and understood. The exotics include the tetraquark, pentaquark and hexaquark states, and hybrids such as \( q\bar{q}g \) and \( gg \) or \( ggg \) glueball states [7]. Recent reports on the discovery of the tetraquark X(3872) [8] and the pentaquark \( \Theta^+ (1540) \) [9, 10] have revived interest in the study of di-hadronic states. Investigations into the existence of multi-quark states using the quark and bag models [11, 12] began in the early days of QCD. Non-relativistic potential models have also been used to study these systems [2, 13].

The tetraquark states have been studied extensively, especially with regard to the X(3872) particle [14–16]. QCD spectral sum rules have been used to test the nature of the heavy meson. The tetraquark has been studied in the framework of the string (flux tube) model [17], where it is assumed that di-quark–di-antiquark are connected by colour flux. The heavy chiral unitary approach has been used to investigate interactions between heavy vector mesons and light pseudoscalar mesons [18]. s-wave tetraquarks have also been studied using chromomagnetic interaction with full account for flavour symmetry breaking [19].

The spectra of tetraquark and pentaquark states have been evaluated, and a complete classification has been provided in terms of spin flavour, colour and spatial degrees of freedom for tetraquarks [20]. The same has been done for five quark systems in terms of spin flavour \( SU(6) \) representation [21]. However, little success has been achieved in understanding tetraquark and pentaquark states as di-hadronic molecules due to the non-perturbative nature of QCD at the hadronic scale.

The present paper seeks to examine the tetraquark, pentaquark and hexaquark states as di-hadronic molecules (meson–meson, meson–baryon or baryon–baryon). A molecular interaction of the van der Waals type is assumed between the constituent hadrons [22, 23]. A mass formula has been employed to calculate the total mass of the molecule. The binding energy of the molecules is calculated using two different wavefunctions obtained from the statistical model [24] and spin hyperfine interactions have also been taken into consideration. The results obtained using each of the two wavefunctions are compared with each other and with available experimental data. The approach has been clearly outlined in section 2, with the calculations being presented in section 3. The conclusion is given in section 3, which comprises a detailed analysis of the nature of results obtained and how well they compare with experimental data and the results obtained by other groups.

2. Method

The di-hadron is composed of either a meson–meson, a meson–baryon or a baryon–baryon held together by van der Waals type interaction. The formula for calculating the low-lying di-hadronic molecular mass is taken as

\[
M_{\text{Tot}} = M_1 + M_2 + E_{\text{BE}} + E_{\text{SD}},
\]

(1)
where $M_1$ and $M_2$ represent the masses of the constituent hadrons, respectively, and $E_{BE}$ represents the binding energy of the di-hadronic system and $E_{SD}$ represents the spin-dependent term, taken separately.

$E_{BE}$ can be calculated using the formula

$$E_{BE} = \langle \Psi(r_{12}) | V(r_{12}) | \Psi(r_{12}) \rangle,$$  

(2)

where $r_{12}$ is taken to be the radius parameter of the di-hadronic molecule and $V(r_{12})$ is the di-hadronic molecular potential [22, 23]. $\Psi(r_{12})$ is the wavefunction of the di-hadronic state [24]. The wavefunction substituted here is that suggested by the statistical model for hadrons, which is found to be very successful in describing the different properties of hadrons. In this model, the hadron is assumed to be composed of a virtual $q\bar{q}$ in addition to the valence partners, which determines the quantum number of its available phase space. The valence quarks are assumed to be non-interacting with each other and considered to be moving almost independently in conformity with the experimental finding of asymptotic freedom. However, the valence quarks are considered to be moving in an average smooth background potential due to their interaction with the virtual sea. Now, we imbibe the concept of number density of quarks and antiquarks for its distribution in their respective phase spaces. The quarks and antiquarks (real and virtual) are confined within a sphere whose radius is taken as the parameter corresponding to the size of the hadron. Then the effective number density of quarks/antiquarks $\rho_q^{eff}(r_{12})$ is equated to the probability density, i.e. the probability of finding the particle in the volume $|\Psi(r_{12})|^2$. Hence $\rho_q^{eff}(r_{12}) = \rho_q(r_{12}) = |\Psi(r_{12})|^2$. With the above considerations, the wavefunctions for the ground state of the hadronic molecule as obtained from the statistical model are:

$$|\Psi(r_{12})|^2 = \frac{315}{64\pi r_0^{9/2}} (r_0 - r_{12})^{3/2} \theta(r_0 - r_{12}),$$  

(3)

corresponding to linear type background potential, and

$$|\Psi(r_{12})|^2 = \frac{8}{\pi r_0^2} (r_0 - r_{12})^{3/2} \theta(r_0 - r_{12}),$$  

(4)

corresponding to harmonic type background potential, where $r_0$ is the radius of the molecule and $\theta(r_0 - r_{12})$ represents the step function. The residual interaction of the confined gluon is considered similar to van der Waals interaction and is assumed to be due to asymptotic expression $(r_{12} \to \infty)$ of the residual confined one-gluon exchange interaction with strength $k_{mol}$ [22, 23]. The potential is given by

$$V(r_{12}) = -\frac{k_{mol}}{r_{12}} e^{-C r_{12}^{1/2}},$$  

(5)

where $k_{mol}$ is the residual strength of the strong interaction molecular coupling and $C$ is the effective colour screening of the confined gluons. The radius $r_0$ of the di-hadronic molecule in equations (3) and (4) is obtained by employing an additive rule for the radii of constituent hadrons. We take $r_0 = r_1 + r_2$, where $r_1$ and $r_2$ represent the individual radii of the hadrons constituting the molecule, respectively.

Now, using equations (2), (3) and (5) we get an integral for $E_{BE}$, which yields

$$E_{BE} = \frac{315 \times 0.114286 k_{mol}}{16r_0} [2F_2[(1.5, 1), (2.75, 2.25), -\beta]]$$  

(6)

if Re $\beta > 0$, where $2F_2$ is the relevant hypergeometric function. Now, using equations (2), (4) and (5) we get

$$E_{BE} = \frac{32k_{mol}}{\pi r_0} \times \left[ -0.75 + 0.5\beta + \frac{e^{-\beta}}{\beta^{5/2}} + \frac{0.66467e^{-\beta}\text{Erfi} [\sqrt{\beta}]}{\beta^{5/2}} \right],$$  

(7)

where Erfi$[\sqrt{\beta}]$ represents an error function and $\beta = C^2 r_{12}^2/2$ and $C = 50$ MeV [25] have been substituted. As the experimental mass for X(3872) [8] is known, it is utilized for evaluation of $k_{mol}$. Swanson [16] indicated that X(3872) may be a mesonic molecule of $D^0D^0$ and $D^0\bar{D}^0$. These two particles would be bound together just like two hydrogen atoms bound together to make a gaseous hydrogen molecule. The radius considered for the $D^0$ is $4.5$ GeV$^{-1}$ [26]. The masses of the constituent hadrons ($D^0$ and $D^0$), taken to be $1864.5$ MeV [27], are substituted in equation (1) to find the corresponding binding energy. This is then used in equations (6) and (7) to calculate the value of $k_{mol}$. It is found that $k_{mol} = 0.59$, corresponding to linear type background potential, which is fixed as the strength of the residual confined gluonic interaction corresponding to the di-hadronic wavefunction in equation (3). Working in the same way, $k_{mol} = 0.65$, for the harmonic type background potential, is calculated for the wavefunction in equation (4). The radii considered are $r(\sigma) = 5.38$ GeV$^{-1}$, $r(K) = 4.77$ GeV$^{-1}$, $r(\rho) = 4.75$ GeV$^{-1}$ [28], $r(D_s) = 4.3$ GeV$^{-1}$ [26], $r(\omega) = 4.765$ GeV$^{-1}$ and $r(\phi) = 5.00$ GeV$^{-1}$ [29]. Similarly, $r(\bar{p}) = 6$ GeV$^{-1}$, $r(n) = 4.7$ GeV$^{-1}$ [30], $r(\Lambda_c) = 5.727$ GeV$^{-1}$, $r(\Sigma_c^+) = 3.386$ GeV$^{-1}$, $r(\Xi_c^0) = 2.404$ GeV$^{-1}$ [31] and $r(\Sigma) = 3.9$ GeV$^{-1}$ [32]. Radii used here have been calculated within the framework of the statistical model. To get an estimate of the radii of $\pi$, $K$, $p$ and $n$, their form factor is expanded to get a relation of the radius in terms of charge radius. By substituting the experimentally measured charge radii an estimate of hadron radius is obtained [28, 30]. The variational method has also been employed to calculate the radius parameter of the hadron. The expectation value of the Hamiltonian is minimized with respect to the radius to find the value of $r_0$ for $D_s$, $\Sigma^+$, $\Lambda_c$ and $\Xi_c^0$ [26, 31]. Experimental values of the respective meson and baryon masses have been considered for calculation [27].

$E_{SD}$, the contribution to total mass from spin hyperfine interaction, is given by the following expression [33]:

$$E_{SD} = \frac{8}{9 M_1 M_2} \bar{S}_1 \cdot \bar{S}_2 |\Psi(0)|^2,$$  

(8)

where $M_1$ and $M_2$ are the individual masses of the constituent hadrons in the di-hadronic molecule, $\alpha_s$ is the strong
interaction constant taken as 0.59 for light hadrons and 0.2 for the heavier sector [34] and $S_1$ and $S_2$ are the spins of the hadrons involved. $|\Psi(0)|^2$ is the value of the di-hadronic wavefunction at the origin. The spin contribution, on calculation, is found to be extremely small in comparison with the binding energy of the molecules. The values of the binding energy, the spin-dependent term and the total mass calculated for the di-hadronic molecules are displayed in table 1 (tetraquark states), table 2 (pentaquark states) and table 3 (hexaquark states). Available experimental data are also presented in the tables to provide a ready comparison with obtained values.

3. Conclusion

The binding energies and masses of exotic states (tetraquark, pentaquark and hexaquark) have been calculated by considering the exotic states as di-hadronic molecules.
The values are calculated using two separate wavefunctions (equations (3) and (4)) obtained using linear and harmonic potentials, respectively. The values obtained in both cases are found to be in close agreement with no serious discrepancy. This may be considered to suggest that for any wavefunction arising from a smooth background potential, the calculated values will be independent of the actual potential. The experimental states are exotics whose spin-parity does not match the expected quark–anti-quark structure for mesons or the 3-quark structure for baryons. Hence, pseudoscalar–pseudoscalar di-mesonic states and vector–vector di-mesonic states are assigned parity charge-conjugation PC ‘++’ and pseudoscalar-vector di-mesonic states have PC ‘+-’. The values of $J^{PC}$ have also been indicated in the tables and those experimental states whose $J^{PC}$ values match the predicted values have been considered for comparison. In the case of di-mesonics, a number of experimentally observed states were available, with which the calculated values have been compared, wherever possible. The calculated results are found to be in good agreement with experiment. The $\pi - K$ state is found to have a mass of approximately 0.76 GeV, which is nearly the same as that calculated by Rai et al ($\sim 0.72$ GeV) [25]. The binding energies of tetraquark states are found to lie within the range of 0.12–0.142 GeV, while the masses range from 0.397 to 2.607 GeV. The experimentally observed $X(3872)$ [8] meson has been chosen as reference for calculating the values of the interaction parameter. The obtained values have been compared with the few available experimental data and are found to be in favourable agreement. The calculated total mass for the $\Sigma - K$ state (1.792 GeV) is found to be close to the estimate given by Rai et al (1.787 GeV) [25]. The well-known $\Lambda$ particle (1.405 GeV) [35] is found to match the predicted $\pi - \Sigma$ molecule (1.433 GeV) closely. So is the case with the pentaquark $\Theta^+$ [9] whose observed mass is 1.54 GeV, whereas our estimate is 1.568 GeV. The binding energies of meson–baryon molecules are found to range from 0.106 to 0.168 GeV, whereas the total mass ranges from 1.189 to 3.387 GeV. In calculating the binding energies and total mass of hexaquark particles, experimental results were not available for comparison. Despite theory suggesting that the hexaquark state should be more stable than the pentaquark state, experimental evidence has been reported in support of the latter, but not yet for the former. Binding energies of the hexaquark states are found to range from 0.122 to 0.166 GeV, and mass varies from 2.04 to 2.502 GeV.

The approach of the current work is simple as far as dynamical variables are concerned: only the effect of the masses of hadrons are taken into account. However, this succeeds in reproducing experimental results to a fair degree. To improve on our work further, we plan to consider other dynamical effects such as the form factor and scaling properties in our future work.

Recently, we have successfully predicted the mass of the pentaquark $\Theta^+$ using a di-quark approach [36], which suggests that the same may be extended to studying other multi-quark states such as $X(3872)$.

The good agreement of the theoretically calculated results with available experimental data and other theoretical works encourages us to suggest that many of the predicted states may be experimentally detected in the near future.

References

[1] Rosner J L 1974 Phys. Rep. 11 89
[2] Barnes T, Dooley K and Isgur N 1987 Phys. Rev. Lett. B 183 210
[3] Jaffe R L 1977 Phys. Rev. D 38 195
[4] Fleck S, Grignoux C and Richard J 1989 Phys. Lett. B 220 616
[5] Zouzou S et al 1986 Z. Phys. C 30 457
[6] Lipkin H J 1986 Phys. Lett. B 172 242
[7] Cass C et al (PDG) 1998 Eur. Phys. J. C 3 1
[8] Choi S K et al 2003 Phys. Rev. Lett. 91 262001
[9] Nakano T et al (LEPS collaboration) 2003 Phys. Rev. Lett. 91 012002
[10] Alt C et al (NA 49 collaboration) 2004 Phys. Rev. Lett. 92 012003
[11] Jaffe R L 1977 Phys. Rev. D 15 281
[12] Strottman D 1979 Phys. Rev. D 20 748
[13] Weinstein J and Isgur N 1983 Phys. Rev. D 27 588
[14] Thomas C E and Close F E 2008 Phys. Rev. D 78 034007
[15] Maiani L et al 2005 Phys. Rev. D 71 014028
[16] Swanson E S 2006 Phys. Rep. 429 243
[17] Iwasaki M and Fukutome T 2005 Phys. Rev. D 72 094016
[18] Guo F K, Shen P N and Chiang H C 2007 Phys. Lett. B 647 133
[19] Buccella H et al 2007 Eur. Phys. J. C 49 743
[20] Santopinto E and Guiseppie G 2007 Phys. Rev. C 75 4
[21] Bijker R, Giannini M M and Santopinto E 2004 Eur. Phys. J. A 22 319
[22] Vinodkumar P C, Pandya J N and Khadikar S B 1992 Pramana–J. Phys. 39 47
[23] Vinodkumar P C, Pandya J N and Rai A K 2003 DAE-BRNS Symp. on Nucl. Phys. 46B p334
[24] Bhattacharya A et al 1998 Eur. Phys. J. C 2 671
[25] Bhattacharya A et al 2005 Nucl. Phys. B (Proc. Suppl.) 142
[26] Bhattacharya A et al 2002 Mod. Phys. A 17 4939
[27] Bhattacharya A et al 1985 Z. Phys. C 24 447
[28] Bhattacharya A et al 1983 Ann. Phys. (NY) 150 956
[29] Rai A K, Pandya J N and Vinodkumar P C 2006 Indian J. Phys. 80 387
[30] Chakrabarti B 1997 Mod. Phys. Lett. A 12 2133
[31] Yao W M et al 2006 J. Phys. G: Nucl. Part. Phys. 33 1
[32] Banerjee S N et al 1987 Int. J. Mod. Phys. A 2 1829
[33] Banerjee S N et al 1989 Int. J. Mod. Phys. A 4 943
[34] Banerjee S N et al 1988 Can. J. Phys. 66 749
[35] Chakrabarti B et al 2000 Phys. Scr. 61 49
[36] Povh B and Hifines J 1990 Phys. Lett. B 245 653
[37] Rai A K, Pandya J N and Vinodkumar P C 2007 Nucl. Phys. A 782 406c–9c
[38] Lucha W et al 1991 Phys. Rep. 200 168
[39] Lutz M et al 2002 Nucl. Phys. A 700 193
[40] Nacher J et al 1999 Phys. Lett. B 455 55
[41] Chakrabarti B et al 2007 Nucl. Phys. A 782 392c