Understanding of Defect in Photonic Crystal using Solitons and Spectral Theory

Vivek S. Sharma

Department of Applied Science and Humanities, MPSTME NMIMS, Shirpur, India
Email id: vivek.sharma@nmims.edu

Abstract. The concept of mechanical deformation is generally understood in terms of propagation of crystal defect. The deformability of the crystals such as uranium can be studied if its local vibrations are known. These local vibrations also give insight into the nature of the defect if its dynamics and bifurcation are known. One such local vibration is soliton or topological soliton. Solitons also form solutions to the differential equations modelling thermophoretic motion in superconductors such as graphene and traveling wave type solutions to elastic rods with deformations. Also spectral theory is widely used to study defects in structures. In this paper, soliton theory and spectral theory approaches are combined and analytical and numerical analysis is carried out for a 10*7 um dielectric material with 1.5 refractive index. The defect is introduced by changing the refractive index, changing the dimensions of one of the dielectric rod in the crystal and removing one of the dielectric rods using FDTD simulations. The results obtained are new and have not been reported in literature.

Keywords: Defect Analysis, Solitons, PDEs, Sine-Gordon Equations, Maxwell’s Equations, Resonance Photonic Crystals, Photonic Band Gap Crystals.

1. Introduction
The solitons are exceptionally useful structures that widely occur in nature. The unique characteristic of solitons is their shape preserving nature. Because of this reason, they have applications in numerous domains of science and engineering such as fiber optics, plasma flows, fluid dynamics, high-energy physics, chemical kinematics, chemical physics and geochemistry. Crystal defect modeling for non-linear periodic systems is a recent branch of engineering studied using nonlinear partial differential equations (PDEs) having solitonic solutions. These problems are of utmost importance from both theoretical and experimental point of view. Three most widely used PDEs for studying such systems are the Maxwell’s equations, the nonlinear Schrodinger equations and the non-linear Sine-Gordan equations.

The solitonic solutions to Sine-Gordan equations particularly explains diverse range of physical phenomenon. The Resonance photonic crystal (RPhC) are one of such systems where light propagation is described by Sine-Gordan Equations [1], [2]. The non-linearity has its own importance as it detunes the band gap locally as energy can propagate in the form of solitary waves [3]. Igor et al. [4] have shown for the first time that Gap Solitons exists in resonant photonic crystals. Gap solitons are stable in the deep of gap and near the band edge they feature specific oscillatory instabilities similar to those observed in fiber Bragg gratings [6], [7]. They have used Schrodinger equations [6] and Laplace equations [7] to explain the propagation of light. Photonic crystals with resonant nonlinearities have applications in manufacturing of periodic bragg semiconductor Nano-structures.

Whereas, Schrodinger’s equations are basic quantum mechanical equations used to model wave-particle nature of photons. Soliton solutions to these equations are extensively studied in literature. These equations along with Maxwell’s equations are widely used in the study of photonic band gap crystals (PBG). The Photonic Bandgap (PBG) materials are dielectric structures which are periodic in nature. They form the energy structure of photons allowing or forbidding the propagation of electromagnetic waves having very specified ranges of frequency. The working of PBGs crystals is similar to that of periodic potential for electrons in atomic crystals as discussed by Maka et al. [5]. In general, a defect is
any interruption of the translation symmetry [8]. The defects can be either point defects or extended defects including rows or planes of atoms.

This paper is arranged as follows: In section 1, the resonance photonic crystals (RPhc) and photonic band gap crystals [PBG] are briefly discussed. In section 2, the Maxwell equation modelling defect are discussed. Also the design specifications of the simulated environment using Optiwave software are discussed. In section 3, FDTD analysis of the defect is carried out and the results are presented. In section 4, Graphs and Plots of the observer point and plane with and without defect are plotted and specifications discussed. In section 5, the simulations of the photonic crystal with and without defects are compared. In section 6, conclusions and inferences are drawn and future directions using solitons are discussed.

2. THE MAXWELL’S WAVE EQUATION IN BRIEF

One of the most important theoretical analogy between photonic and solid state crystals is the band of photons. It enables one to create two widely applicable features of photonic band gap and localization of light [9]. In solid state physics one of the most fundamental equations (Maxwell’s wave equation) is changed into an eigenvalue equation having a periodic boundary condition and is solved using computations. Also the photonic Band-Gap Crystals contains periodic array of optical materials called Photonic Band Gap (PBG) fibers. Particle physics at smaller scale can be studied using higher accelerating gradients. These accelerating gradients can be created by small structures with large field strengths [3], [21]. Therefore, in a photonic crystal, light cannot enter the crystal and photons cannot be emitted by it. Implying that the doping of an optical item with impurity or defect creates the opposite situation of the strong localization of the resonant photons [9]. The term band gap means a range of energies and the direction in which the electrons are not allowed to travel [11]. As a photonic bandgap [PBG] crystal behaves like an insulator of light, flexible and dense photonic circuit such as an electronic LSI [Large Scale Integrated circuit] can be created. The light velocity in PBG crystal is changed from vacuum velocity \(c\) to zero (stopping condition) so that the light matter interaction can be arbitrarily controlled. Point defects may be defined within PBG materials to build embedded optical cavities. Such cavities which contains an emitting material may be used to inhibit spontaneous emission [10]. One of the exiting domains where photonic crystals has wide applications is optical biosensors [12]. In photonic crystals, if the refractive indices of dielectrics differ greatly, the modulation of the light waves would be sufficient, and so a photonic bandgap may form, in which optical models are absent [16]. The variation of refractive indices is the most common way to introduce defect in a dielectric photonic crystal. The other ways that can be used to introduce a point defect are 1. Removal of a dielectric rod from the structure and 2. Changing the dimensions of a dielectric rod. In this section, firstly, a brief introduction to Maxwell’s equations which forms basis of the Finite Domain Time Difference (FDTD) method applied here to analyze the defects in the photonic bandgap crystal is given. Further, the design specifications of a photonic bandgap crystal using Optiwave software are given.

The Maxwell’s Equations

The Maxwell’s equations are four equations which together form a complete description of the production and interrelation of electric and magnetic fields. If the free space propagation model is assumed, then these equations become

\[
\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} = 4\pi k \rho \\
\n\nabla \cdot \mathbf{B} = 0
\]

(1)
\[
\frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\varepsilon_0} \nabla \times \mathbf{H} \quad \text{and} \\
\frac{\partial \mathbf{H}}{\partial t} = -\frac{1}{\mu_0} \nabla \times \mathbf{E}
\]  

(2)

Where, \( \mathbf{E} = \) Electric field, \( \mathbf{B} = \) Magnetic Field, \( \mathbf{H} = \) Magnetic Field Strength, \( \mu_0 = \) Permeability of the medium and \( \varepsilon_0 = \) Permittivity of free space. Where equation (1) specifically is Gauss law of Electricity and Magnetism and equation (2) is Faraday’s Law of Induction. The constant \( k = \frac{1}{4\pi \varepsilon_0} = \) Coulomb’s Constant. With \( \nabla \cdot \mathbf{E} \) and \( \nabla \times \mathbf{E} \) and similarly \( \nabla \cdot \mathbf{B} \) and \( \nabla \times \mathbf{H} \) representing the vector operations of divergence and curl respectively.

Of the above two equations, the equation (2) is discretized using lee’s scheme [17] following leap-frog algorithm for implementation of Finite Domain Time Difference [FDTD] Scheme. The details of this scheme can be found in [17].

3. Design Specifications of the Photonic Band Gap (PBG) Crystal

The layout of the Photonic bandgap crystal is designed using waveguides. An anisotropic dielectric material with wafer dimensions of 10x7 um is Consider. The refractive index of the 2-dimensional wafer dielectric material used being 1.5 and with PBG atom have unit width and thickness. The PBG lattice atom is of 2-dimensional hexagonal type with elliptic waveguide. Two input planes are considered with the photonic source emitting a sine-modulated Gaussian pulse with wavelength of 1.55 um. Three observation points and an observation area is selected on the design. As seen in Figure 1(a), 1(b) and 1(c), the 6 equally spaced dielectric rods are used in the design. The design has three observers and an observation area A.
Simulation Specifications and detector analysis of the Photonic Band Gap (PBG) Crystal

The design environment is 2-D TE(s-polarized) simulated with mesh size of 0.05*0.05 and for 12000 time steps and the simulation images are obtained and the detector analysis of PBG crystal is carried out at the three observation points and the observation area. In detector analysis all observation points at ordinates (0.00, 0.00, 0.25), (0.00, 0.00, 8.5) and (0, 0, 9.5) are selected and spectral analysis is done using Discrete Fourier Transform (DFT). These normalized frequency Discrete Fourier Transform graphs are plotted with wavelength range from 1.5 um to 2.3 um and at 1000 sample points Fig 2 (a). The observation area integral surface is obtained for DFT with wavelength 1.9 um and center position being (1.78, 0.00, 4.11) as shown in Fig 2(b) and Fig 2(c).

Figure 1(a). Layout of Designed Photonic Bandgap Crystal

Figure 1(b). 3-Dimensional view of the designed PBG crystal

Figure 1(c). Grid View of PBG crystal with grid size 101*71.
The DFT plots show that spectral plots in form of Discrete Fourier Transform (DFT) give insight into the output of the FDTD scheme for dielectric crystal considered. This crystal is without any defect in it. In the next section, the PBG crystal is designed with point defects.

By changing the major and minor axis radius of a cell for the dielectric crystal

By removing a dielectric rod from the crystal lattice and the environment is simulated. Further detector analysis is carried out similar to crystal without defect. Graphs and Plots of the observer points and plane are plotted with defects.

4. Defect Study of the Photonic Bandgap Crystal

The DFT plots show that spectral plots in form of Discrete Fourier Transform (DFT) give insight into the output of the FDTD scheme for dielectric crystal considered. This crystal is without any defect in it. In the next section, the PBG crystal is designed with point defects.

i. By changing the major and minor axis radius of a cell for the dielectric crystal

ii. By removing a dielectric rod from the crystal lattice and the environment is simulated. Further detector analysis is carried out similar to crystal without defect. Graphs and Plots of the observer points and plane are plotted with defects.

Study of defect by changing radius of major and minor axis

The major radius and minor radius of the cell for dielectric rods with ordinates (0, 0, 2) are changed from both 0.2 to new major radius of 0.3 and new minor radius of 0.25. The design and simulations thus obtained are noted in Fig 3(a, b, c).

Fig 3(a, b, c) depict Design and Simulation of 2D Photonic Bandgap Crystal with defect introduced by changing major and minor radius in cell (0,0,2).

Further, the normalized Discrete Fourier Transform (DFT) graphs are plotted for the detector points and area A. These DFT values are plotted for all the three observers and the observer plane. The integral surface and amplitude plots are plotted for the DFT in Figure 4(a, b, c).
5. **Study of defect by removing one of the dielectric rod in the PBC lattice**

The major radius and minor radius of the cell for dielectric rods with ordinates (0, 0, 2) are not changed and they remain 0.2 but one of the dielectric rod is removed from the position (0, 0, 3). The design and simulations thus obtained are noted in Fig 5(a, b, c).
Further, the normalized Discrete Fourier Transform (DFT) graphs are plotted for the detector points and area A. These DFT values are plotted for all the three observers and the observer plane. The integral surface and amplitude plots are plotted for the DFT in Figure 6(a,b,c).

Figure 5(a). Design of defective PBG crystal by by removing dielectric rod at (0,0,3)

Figure 5(b). 3-Dim. Layout of defective PBG crystal by by removing dielectric rod at (0,0,3)

Figure 5(c). Simulation of 2D Photonic Bandgap Crystal with defect introduced by removing a dielectric rod at positon cell (0,0,3)

Figure 6(a). The DFT plot for PBC with defect at (0,0,3)
6. Remark
Spatial Kerr Soliton solutions exist for Maxwell equation discussed in equations (1) and (2) in the form of both in bright and dark version, as TM [19]. These solutions are linearly polarized, exactly integrable one-dimensional solitons. In two-dimensional solids, these solutions are obtained as azimuthally polarized and circularly symmetric dark solitons. In the current simulations of Photonic Band Gap Crystal (PBG) and its defect it is observed the integral surfaces obtained seem to have dark soliton type solutions which can be inferred by their DFTs. It also seems to be in agreement with the analytical results obtained by Ciattonni et al. [19] and Snyder et al. [20].

7. Conclusion and Future Research
The design and simulation of crystal defects in Photonic Band Gap crystal is carried out in this work. The defects considered are introduced by increasing the radius of major and minor axis of the cell of dielectric crystal and by removing a dielectric rod from the PBG crystal. The simulations are carried out using optiwave software assuming three observers and one observation plane. These simulations are new and have not been reported in literature.

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