Configuration Design and Analysis of Double Tetrahedron Superposition Symmetrical Coupling Mechanism

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Abstract. A new hexahedral single degree of freedom developable symmetrical coupling mechanism is proposed based on the spatial structure characteristics of regular tetrahedron. Firstly, in order to design the equivalent basic model of hexahedral mechanism, three kinds of substitute components are designed which based on the analysis of the spatial structure characteristics of the new hexahedron. The basic movable element is obtained by adding branched chains to change the constraints. A hexahedron coupling mechanism is designed by combining and reconstructing the elements. Secondly, based on the screw theory, the independent motion shunt labeling method is applied to calculate the degree of freedom and motion properties. Finally, the three-dimensional model of the coupled mechanism is established, and the displacement variations of the end and nodes under different driving conditions are simulated and revealed with time-displacement curves by Adams. Simulation results show that the novel mechanism is a symmetrical deployable coupled mechanism with one translation $DOF$.

1. Preface

In mechanism science, spatial multi ring coupling mechanism is a more complex hybrid mechanism. As a new type of mechanism, it mainly combines series and parallel mechanism effectively, giving full play to the advantages of parallel mechanism such as high precision, high bearing capacity, large working space and flexible control. Its structural form is more diversified, and it can realize some special and complex movements, which has gradually become an important research direction and research hotspot in the field of mechanism [1-6].

At present, the mechanism configuration synthesis of multi ring coupling mechanism is usually based on polyhedron, especially polyhedron symmetrical coupling mechanism. Because of its novel structure and flexible output motion, it is favored by researchers in the field of mechanism science. However, because of its complex structure, the synthesis and analysis methods of series and parallel mechanisms cannot be directly used. Therefore, it is of great significance to carry out the research on the synthesis theory and method of polyhedral symmetrical coupling mechanism, and promote its development and application.

Due to the compactness and high symmetry of regular polyhedron structure, it can provide the basis for the design of polyhedron coupling mechanism. In reference [7-10], several kinds of polyhedron coupling mechanisms are designed. These mechanisms change the vertices and edges of polyhedron...
and use other polygons as coupling nodes to achieve the performance of extension and contraction. With the expansion and contraction of each branch of the coupling mechanism, scholars at home and abroad have designed a variety of tetrahedral robots that can realize roll or transfer motion. For example, Li Yezhuo et al. [11] applied a scaling technique based on parallelogram element to polyhedral rolling mechanism, designed a folding double triangle cone (hexahedron) rolling coupling mechanism, and realized two rolling modes. Yang Yi [12] et al. Started with a special kind of spatial polyhedron centripetal mechanism, based on the analysis of its structure and motion characteristics, proposed a new hexahedron mechanism unit and designed a new type of extension arm mechanism. Of course, considering the multiple mesh or ring-shaped element structures that make up the coupling mechanism, researchers at home and abroad have integrated different types of spatial polyhedron coupling mechanism by combining different structural elements [13-16].

Based on the model of parallel mechanism similar to the polyhedron structure, according to the constraint relationship of the mechanism, the way of adding series branch chain or parallel nesting is adopted. Scholars have also synthesized different types of spatial polyhedron coupling mechanism [17-23]. These coupling mechanisms meet the functional requirements by changing the shape of the coupling mechanism, and most of them have been widely used in aviation and aerospace. It can be seen that this kind of mechanism has good extension and contraction performance. It can be contracted to the limit position in the transportation process, and can be extended to the specific position to realize the special function in the working time.

In this paper, a new type of hexahedron symmetrical coupling mechanism with single degree of freedom is proposed. By adding branch chain to change the constraint system, a movable basic element body is obtained, which has a multi ring symmetric coupling mechanism with moving degrees of freedom. Based on the theory of helix, the free degree of freedom and the analysis of motion property are calculated by using the independent motion shunt marking method [24-25], and the desired symmetrical coupling mechanism is designed. In CATIA, a three-dimensional solid model of the coupling mechanism is established, and different driving conditions are set up by ADAMS to carry out dynamic simulation respectively. It is verified that the new mechanism proposed is a single degree of freedom mobile symmetric coupling mechanism and has scalability.

2. The design of a double tetrahedron superposition symmetrical coupling mechanism model

As shown in Figure 1, the regular tetrahedron ABCD is a space closed figure surrounded by four equal regular triangles, all edges are equal in length, so the convergence connection of edges at each vertex is exactly the same, as shown in figure 1. The hexahedron mechanism proposed in this paper is to get a new spatial polyhedron by superposing the triangle faces of two tetrahedrons. The branch structure of the mechanism is simple and symmetrical. The whole mechanism framework is based on the spatial polyhedron structure obtained by the combination of two regular tetrahedrons. According to the characteristics of the geometric elements that make up the regular polyhedron, two regular tetrahedrons with the same structure size, namely ABCD and A'B'CD', are selected to overlap the BCD and B'CD' of the regular triangular faces contained in the two regular tetrahedrons, a new spatial polyhedron ABCDA' or A B'C'D'A' is obtained, among them, B and B 'coincide, C and C' coincide, and D and d 'coincide, so they are complementary to tetrahedral mechanism in performance, as shown in figure 2. According to the spatial structure layout of the hexahedron, the equivalent geometric basic model is established, the basic unit structure is established by adding branch chain to change the constraint system, and the corresponding coupling mechanism is established by combining the basic unit design.
As shown in figure 2, according to the Euler's theorem of polyhedron, the number of faces \( F \), the number of vertices \( V \) and the number of edges \( E \) have the following relations:

\[
V + F - E = 2
\]  

(1)

Where 
- \( V \) -- polyhedron contains all vertex points
- \( F \) -- Polyhedron contains all faces
- \( E \) -- Polyhedron includes all edges

The hexahedron is composed of five vertices, nine edges and six identical regular triangles, which satisfies the Euler's theorem of polyhedron [1].

2.1. Establish the basic framework of coupling mechanism

According to the layout of hexahedron spatial structure, the corresponding coupling basic frame is established, and the types of each component are determined by defining the vertex and edge types in the frame, so as to obtain all the branches of the hexahedron symmetric coupling mechanism, as shown below.

(1) Determine the type of replacement component

The hexahedron proposed in this paper is a closed space figure surrounded by two tetrahedrons, namely six equal regular triangles. Figure 2 is the schematic diagram of its structural form. In the figure, two vertices a and a' of the hexahedron are connected with three edges, and the two vertices are replaced by the regular triangle component as shown in figure 3a, while the vertices B, C and D are connected with four edges. The three vertices are replaced by the regular quadrilateral component as shown in figure 3b, and the nine edges are replaced by the component as shown in figure 3c. The vertex replacement component shown in figure 3a is a regular triangle component, the three rotating auxiliary axes of each regular triangle component are all located in the triangle plane, and the included angle between adjacent axes is 60°; the vertex replacement component shown in figure 3b is a regular quadrilateral component, and the four rotating auxiliary axes are all located in the plane of the component itself, and the included angle between adjacent axes is 90°, and the opposite axes are parallel to each other; the edge replacement component shown in figure 3c is a double pole component, the two rotating auxiliary axes are parallel.
Figure 3. The substitutive components

(2) Determine the dynamic platform and frame components of the coupling mechanism

Due to the symmetry of the coupling frame structure, the equivalent vertices A and A’ with the same convergence and symmetrical spatial distribution are selected as the dynamic platform and frame components of the coupling mechanism model.

(3) Determine the component of coupling node and branch chain composition

The three vertices directly connected with the equivalent vertices B, C and D are staggered and distributed symmetrically in space. Each vertex gathers three normal quadrilaterals, and there is a coupling relationship between the two adjacent vertices due to a common edge. Therefore, these three vertices are defined as coupling vertices, all of which are regarded as coupling node components. Then, each coupling branch chain connected by six coupling node components between the moving platform A and the frame A’ can be determined.

2.2. Construct equivalent coupling mechanism model

Figure 4. Frameworks of coupled structure
According to the spatial layout of the coupling structure frame, all the substitute members are connected by r-pair in order to build the coupling mechanism model. Therefore, replacing all vertices and edges of hexahedron with triangle component, quadrilateral component and secondary member shown in figure 3 respectively, and combining and connecting the above substitute components with rotation pair (R pair) according to their structural layout, a hexahedron like equivalent geometric foundation model is obtained as shown in figure 4. The components M, T1, T2, T3 and D in the figure respectively correspond to the five vertices A, B, C, D and A’ in Figure 1, and the components L1~L9 correspond to the nine edges AB, AC, Ad, BC, BD, BA’, CD, CA’ and DA’ of the hexahedron respectively. At this time, the hexahedron multi ring coupling mechanism includes 11 faces, 27 edges and 18 vertices, which meet the Euler formula of the polyhedron [1].

2.3. Kinematic analysis of basic unit ring members
In order to avoid the complete constraint of the designed multi ring coupling mechanism, it is necessary to judge the internal constraint of the basic unit ring which makes up the model, so that it can be moved.

The equivalent geometric foundation model shown in figure 4 can be seen as the sequential connection of six identical ring structures shown in figure 5. Each ring structure corresponds to an equilateral triangle in the hexahedral configuration shown in figure 2. According to the symmetry, the constraints of the six ring structures are the same. Taking the ring structure shown in figure 5 as an example, the constraint system of the ring structure of the unit is analysed. Taking component D as the frame and component M as the moving platform, the structure can be seen as the parallel connection of branch 1: D-L1-T3- and branch 2: D-L3-T2-L4-T3-.

![Figure 5. Structure of basic unit ring](image)

According to the screw theory [1], branch 1 provides four constraints for the moving platform, including two constraint force couples and two binding forces. The directions of two constraint force couples are perpendicular to the direction of the rotating auxiliary axis of rod L1, one binding force is along the length direction of rod L1, and one binding force is along the direction of the rotating auxiliary axis of rod L1. Branch 2 provides two constraints for the moving platform, including one constraint couple and one constraint. The direction of the constraint couple is perpendicular to the plane where component T3 is located, and the constraint is coincident with the line direction obtained from the intersection of all rotating auxiliary axes of bars L3 and L5. Therefore, the movement and rotation degrees of freedom of the ring structure shown in figure 5 in any direction of space are constrained and are rigid components.
2.4. Design of movable unit

By adding the branch chain method to change the constraint screw system of each branch, the spatial layout of the components can make the circular structure become the desired form of the moving mechanism. Considering the diversity of the structure of the coupling mechanism, this paper only considers the mechanism with the rotating pair (R pair), so the spiral system of the movable mechanism obtained by adding the branch chain is composed of the vector of the if trunk line. In general, the expansion mechanism has reciprocating motion. According to the geometric judgment method of helix correlation and phase inversion [1], the line vector that is opposite to the movement helix system of the branch must be perpendicular to all the even vectors and intersect with all the line vectors. Therefore, in the branch 2, add a rod group with two rods and three pairs and the axis of the rotation pair parallel to each other, as shown in figure 6. In this way, adding a branch chain can reduce one binding force provided by the branch 2 to the moving platform and make it move perpendicular to the axis direction of the rod group, so as to obtain the movable moving ring shown in Fig. 6b and design the coupling mechanism as a basic unit component.

2.5. Multi ring coupling mechanism with branch chain added

The basic unit shown in figure 7 is used to replace the other five immovable ring structures corresponding to the equivalent geometric basic model, and a kind of hexahedron coupling mechanism is obtained by combination, as shown in figure 8. At this time, the mechanism includes 11 faces ($V = 11$), 21 vertices ($F = 21$) and 30 edges ($E = 30$), which meet the Euler formula shown in formula (1).

It can be seen that this kind of hexahedral coupling mechanism is composed of six basic elements with the same structure. Each basic element is a 7-bar space single ring structure with 1 regular triangle and 2 regular quadrilateral members. The whole mechanism consists of 2 regular triangle members, 3 regular quadrilateral members and 3 pairs of bars. 21 rotation pairs are connected to form a complex multi ring mesh mechanism. The mechanism is composed of 5 joint members and 9 bar groups. The straight lines passing through the center of gravity of each joint member and perpendicular to their respective plane intersect at a point $O$, which is the geometric center of the mechanism at a certain time. Since the whole mechanism is symmetrical about this point, the configuration of the mechanism at this time is recorded as the initial configuration of the coupling mechanism.
3. Analysis of the degree of freedom of the coupling mechanism

Based on the screw theory, the number of degrees of freedom and the motion properties of the end piece of the multi ring coupling mechanism are analyzed by using the independent motion shunt marking method [24-25], so as to verify whether the proposed basic coupling mechanism is correct.

Firstly, the mechanism is decoupled into several independent motion branches according to the independent motion shunt marking method to determine the motion transmission route from the frame to the end piece; secondly, a reasonable coordinate system is established according to the geometric symmetry of the mechanism, and the constraints imposed by each independent motion branch to the end piece are analyzed based on the screw theory, and it is equivalent to an independent motion single chain relative to the frame; Finally, the equivalent parallel mechanism of the coupling mechanism is established and the motion and constraint screw of the equivalent mechanism are analysed to determine the number and nature of the degrees of freedom of the mechanism.

3.1. Independent movement shunt mark of coupling mechanism

If the components $D$ and $M$ are triangle replacement components and the branch chains are connected by rotation pairs, the rotation pairs of the replacement parts are marked as $R_{1j}$ and $R_{5j}$ ($j=1, 2, 3$); If the members $T_1$, $T_2$, and $T_3$ are quadrilateral replacement members and the branch chains are connected by rotation pairs, the rotation pairs of the replacement parts are marked as $R_{2j}$, $R_{3j}$, and $R_{4j}$ ($j=1, 2, 3, 4$).

$D$ is the fixed frame and $M$ is the moving platform. According to the independent movement shunt marking method, since the three rotating pairs connected to the moving platform $m$ are marked $R_{51}$, $R_{52}$, and $R_{53}$ respectively, the multi ring coupling mechanism is divided into three independent kinematic chains. Mark the motion transmission route of each branch in the mechanism with dotted arrows, as shown in figure 9.
Where, the kinematic connection between the frame and the rotating pair on the moving platform is recorded as $C_i$ ($i = 1, 2, 3$), that is, the branches where $R_{31}$, $R_{32}$ and $R_{33}$ are located are $C_1$, $C_2$ and $C_3$ respectively, so $C_1$, $C_2$ and $C_3$ are the branch coupling structures of the basic coupling mechanism, at this time, the three branches of the coupling mechanism can be respectively: the first branch $C_1$-$R_{31}$-, the second branch $C_2$-$R_{32}$-, and the third branch $C_3$-$R_{33}$-. The coupling structure $C_1$, $C_2$ and $C_3$ can be equivalent to three independent parallel branch chains $C_{i1}$, $C_{i2}$ and $C_{i3}$ ($i = 1, 2, 3$) respectively and connected with frame $D$. The structure composition block diagram of corresponding branches is shown in fig.10.

$$
D = \begin{bmatrix}
  C_{i1} : R_{i1} - R_3 - R_{i3} \\
  C_{i2} : R_{i2} - R_2 - R_{i2} - T_1 - R_{i1} - R_{i3} - R_{i2} \\
  C_{i3} : R_{i3} - R_3 - R_{i2} - T_3 - R_{i1} - R_{i3} - R_{i2}
\end{bmatrix}
- T_1 - R_{i3} - R_3 - R_{i3} - R_{i2} -

$$

**Figure 9.** Motion flow sign sketch of novel mechanism

**Figure 10.** Three branches of novel mechanism

As shown in fig.10, at the initial moment, the whole basic coupling mechanism is decoupled into three independent kinematic chains, and the three independent kinematic chains split are symmetrical with respect to the geometric center point $O$, so when analysing the degrees of freedom, one of them can be analysed. Taking the first branch $C_1$ as the research object, $C_1$-$R_{31}$, the constraints provided by
the branch chain to the end piece are analyzed. The mark map of branch 1 structure movement shunt is shown in fig.11.

![Diagram of branch movement shunt mark of branch 1](image)

**Figure 11.** Structure movement shunt mark of branch 1

3.2. $C_1$-$R_5$-kinematic constraint analysis of the first branch

3.2.1. Analysis of branch $C_{11}$ of independent kinematic chain. According to fig. 11, at the initial moment of the coupling mechanism, the first branch $C_1$ is equivalent to three parallel branches, which are labeled as $C_{11}$, $C_{12}$ and $C_{13}$ respectively, wherein the parallel branches $C_{12}$ and $C_{13}$ are symmetrical with respect to $C_{11}$ and the motion constraints are the same. Parallel branches $C_{11}$: D-$R_{11}$-$R_3$-$R_{31}$, $C_{12}$: D-$R_{12}$-$R_2$-$R_{21}$-$T_2$-$R_{23}$-$R_5$-$R_{33}$-$C_{13}$: D-$R_{13}$-$R_1$-$R_{41}$-$T_3$-$R_{43}$-$R_6$-$R_{32}$, where the axes of $R_{11}$, $R_3$ and $R_{31}$, $R_{12}$, $R_2$ and $R_{21}$, $R_{13}$, $R_1$ and $R_{41}$ are parallel and perpendicular to the axes of $R_{23}$, $R_5$, $R_{43}$ and $R_6$, respectively.
Figure 12. Analysis coordinate sketch of branch 1

Establish the local coordinate system $O_1X_1Y_1Z_1$ as shown in fig. 12. Take the center point of $R_{31}$ axis as the coordinate origin $O_1$, make $Y_1$ axis coincide with $R_{31}$ axis direction, $Z_1$ axis perpendicular to the plane $D$ of the rack, judge $X_1$ axis perpendicular to $X_1Y_1Z_1$ plane according to the right-hand rule, then $C_{11}$ is in $X_1O_1Z_1$ plane, $C_{12}$ and $C_{13}$ are symmetrical about $X_1O_1Z_1$.

In the coordinate system $O_1X_1Y_1Z_1$, assume that the position coordinate of $R_{21}$ axis center point is recorded as $A_1 = (a_1, c_1, 0)$. According to the structural symmetry, the position coordinate $A_2$ of $R_{41}$ axis center point is $(a_1, -c_1, 0)$. In this coordinate system, the movement spiral system of branch chain $C_{11}$ is in the local coordinate system is,

$$
\begin{align*}
&d_1, f_1 —— Three components of the dual part of spinor \mathbf{s}_{11}; \\
&d_2, f_2 —— Three components of the dual part of spinor \mathbf{s}_{11}; \\
\end{align*}

In formula (2), $d_1, f_1, d_2$ and $f_2$ are all variables related to the position of the axis of the rotating pair, whose size can be solved by position analysis, and has nothing to do with the problem of analysing the movement and constraint of the mechanism. The bottom corner of screw $\mathbf{s}_{11}, \mathbf{s}_3, \mathbf{s}_{31}$ corresponds to the bottom corner of the rotating pair one by one. By solving the inverse helix of equation (2), the constraint helix of the branch chain is obtained as follows:
\[
\begin{align*}
S'_{11} &= \begin{pmatrix} 0 & 1 & 0; & 0 & 0 & 0 \end{pmatrix} \\
S'_{33} &= \begin{pmatrix} 0 & 0 & 0; & 1 & 0 & 0 \end{pmatrix} \\
S'_{31} &= \begin{pmatrix} 0 & 0 & 0; & 0 & 0 & 1 \end{pmatrix}
\end{align*}
\] 

(3)

In equation, $S'_v$ —— anti helix of branch chain $C_i$ (i, j = 1, 2, 3).

Formula (3) shows that the constraint imposed on $T_1$ by branch chain $C_{ij}$ is 1 constraint and 2 constraint couple. Among them, the binding force $S'_{11}$ restricts the movement of $T_1$ along the axis of rotation pair $R_{31}$; the directions of constraint couple $S'_{33}$ and $S'_{31}$ are perpendicular to the axis direction of rotation pair $R_3$, which restrict the rotation of $T_1$ around axis $X_1$ and axis $Z_1$ respectively.

3.2.2. Analysis of branch $C_{12}$ of independent kinematic chain. According to the symmetry, in the coordinate system $O_1$-$X_1Y_1Z_1$, the moving spiral system of the branch chain $C_{12}$ is:

\[
\begin{align*}
S_{12} &= \begin{pmatrix} a_1 & c_1 & 0; & d_3 & e_3 & f_3 \end{pmatrix} \\
S_2 &= \begin{pmatrix} a_1 & c_1 & 0; & d_4 & e_4 & f_4 \end{pmatrix} \\
S_{21} &= \begin{pmatrix} a_1 & 0 & c_1; & 0 & 0 & f_3 \end{pmatrix} \\
S_{23} &= \begin{pmatrix} 0 & 0 & 1; & d_6 & e_6 & 0 \end{pmatrix} \\
S_5 &= \begin{pmatrix} 0 & 0 & 1; & d_7 & e_7 & 0 \end{pmatrix} \\
S_{33} &= \begin{pmatrix} 0 & 0 & 1; & d_8 & e_8 & 0 \end{pmatrix}
\end{align*}
\] 

(4)

Find the reverse helix of the above formula, the constraint helix of the branch chain is,

\[
S'_{12} = \begin{pmatrix} 0 & 0 & 0; & -c_1 & a_1 & 0 \end{pmatrix}
\] 

(5)

Equation (5) shows that there is a local degree of freedom in the branch chain $C_{12}$, and the branch chain is applied to a constraint couple of $T_2$ members of the node, which limits the rotation of $T_2$ around the axis direction of $O_1A_1$ line.

Since the branched chain $C_{12}$ and $C_{13}$ are symmetrical about the branched chain $C_{11}$, according to the symmetry of the mechanism, the spiral system of the branched chain $C_{13}$ is as follows:

\[
\begin{align*}
S_{13} &= \begin{pmatrix} a_1 & -c_1 & 0; & -d_3 & e_3 & f_3 \end{pmatrix} \\
S_4 &= \begin{pmatrix} a_1 & -c_1 & 0; & -d_4 & e_4 & f_4 \end{pmatrix} \\
S_{41} &= \begin{pmatrix} a_1 & -c_1 & 0; & 0 & 0 & f_5 \end{pmatrix} \\
S_{43} &= \begin{pmatrix} 0 & 0 & 1; & -d_6 & e_6 & 0 \end{pmatrix} \\
S_6 &= \begin{pmatrix} 0 & 0 & 1; & -d_7 & e_7 & 0 \end{pmatrix} \\
S_{32} &= \begin{pmatrix} 0 & 0 & 1; & -d_8 & e_8 & 0 \end{pmatrix}
\end{align*}
\] 

(6)

Find the reverse helix of the above formula, the constraint helix of the branch chain is
Equation (7) shows that there is a local degree of freedom in the branch chain $C_{13}$, which is applied to a constraint couple of $T_3$ members.

To find the quadratic anti helix from the combination of equations (3), (5) and (7), we obtain

$$
$$_{13}^r = \left( 0 \ 0 \ 0; \ c_1 \ a_1 \ 0 \right) \quad (7)
$$

Equation (7) shows that there is a local degree of freedom in the branch chain $C_{13}$, which is applied to a constraint couple of $T_3$ members.

To find the quadratic anti helix from the combination of equations (3), (5) and (7), we obtain

$$
$$_{11}^g = \left( 0 \ 0 \ 0; \ 1 \ 0 \ 0 \right) \quad (8)
$$

In equation $$_{11}^g$$, the second anti helix of ring structure

Equation (8) includes two moving spirals, indicating that the independent movement of node component $T_1$ relative to the frame can be equivalent to two moving movements $P_1$ and $P_2$. Therefore, the independent motion of branch $C_1$ relative to the frame can be equivalent to the generalized motion $P_1$-$P_2$-$R_{34}$-$R_{9}$-$R_{51}$.

3.2.3. Analysis of the first branch motion constraint. As shown in fig.12, in the coordinate system $O_1$-$X_1$-$Y_1$-$Z_1$, the kinematic spiral system of the first branch equivalent kinematic chain is expressed as:

\[
\begin{align*}
&$$_{p1}^s = \left( 0 \ 0 \ 0; \ 1 \ 0 \ 0 \right) \\
&$$_{p2}^s = \left( 0 \ 0 \ 0; \ 0 \ 0 \ 1 \right) \\
&$$_{34}^s = \left( 0 \ 1 \ 0; \ p_1 \ 0 \ r_1 \right) \\
&$$_{34}^s = \left( 0 \ 1 \ 0; \ p_2 \ 0 \ r_2 \right) \\
&$$_{51}^s = \left( 0 \ 1 \ 0; \ p_3 \ 0 \ r_3 \right)
\end{align*}
\] \quad (9)

In equation, $p_1, r_1$ —— the component of the dual part of spinor $$_{34}^s$;

$p_2, r_2$ —— the component of the dual part of spinor $$_9^s$;

$p_3, r_3$ —— the component of the dual part of spinor $$_{51}^s$;

To get reverse helix, the constraint spiral system of the equivalent branch chain is obtained as follows:

$$
$$_{1}^g = \left( 0 \ 1 \ 0; \ 0 \ 0 \ 0 \right) \quad (10)
$$

The above equation (10) shows that there are two local degrees of freedom in the first branch, and the constraint helix applied by the first branch to the moving platform $m$ is one constraint and two constraint couple, which restrict the movement of the moving platform $m$ along the $Y_1$ axis and around the $X_1$ axis and $Z_1$ axis respectively.

3.3. Analysis of the whole mechanism

To sum up, the three independent chains $C_1$-$R_{51}$, $C_2$-$R_{52}$ and $C_3$-$R_{53}$ of the coupling mechanism are symmetrical to each other. From equation (10), it can be determined that the constraint screw system applied to the moving platform $M$ by the other two branches contains one constraint couple and two
constraints. Each constraint couple is located in the symmetry plane of each branch, and the constraint is perpendicular to the symmetry plane of each branch, and parallel to the plane of frame $D$.

Establish the global coordinate system $O$-XYZ, so that the o-point coincides with the geometric center of the rack $D$, $XOY$ is in the plane where $D$ is located, then establish the $O$-XYZ coordinate system as shown in Figure 13, so that it is the coordinate system of the whole coupling mechanism. The top view and coordinate system setting of frame $D$ are shown in the following figure.

![Figure 13. Top view of base $D$](image)

Taking the coordinate system $O$-XYZ as the whole coordinate system, the three branches of the coupling mechanism provide the constraint screw of the moving platform as follows:

$$
\begin{align*}
S_1^{1r} &= \begin{pmatrix} 0 & 1 & 0; & 0 & 0 & 0 \end{pmatrix} \\
S_2^{2r} &= \begin{pmatrix} 0 & 0 & 0; & 1 & 0 & 0 \end{pmatrix} \\
S_3^{3r} &= \begin{pmatrix} 0 & 0 & 0; & 0 & 0 & 1 \end{pmatrix} \\
S_2^{1r} &= \begin{pmatrix} 1 & 1 & 0; & u_1 & v_1 & w_1 \end{pmatrix} \\
S_2^{2r} &= \begin{pmatrix} 0 & 1 & 0; & u_2 & v_2 & w_2 \end{pmatrix} \\
S_2^{3r} &= \begin{pmatrix} 0 & 1 & 0; & u_3 & v_3 & w_3 \end{pmatrix} \\
S_3^{1r} &= \begin{pmatrix} 0 & 1 & 0; & u_4 & v_4 & w_4 \end{pmatrix} \\
S_3^{2r} &= \begin{pmatrix} 0 & 1 & 0; & u_5 & v_5 & w_5 \end{pmatrix} \\
S_3^{3r} &= \begin{pmatrix} 0 & 1 & 0; & u_6 & v_6 & w_6 \end{pmatrix}
\end{align*}
$$

Equation (11) shows that the secondary anti spiral is:

$$
S'' = \begin{pmatrix} 0 & 0 & 0; & 0 & 0 & 1 \end{pmatrix}
$$

Formula (12) shows that the moving platform $M$ has a movement along the $Z$-axis, that is, the moving platform $M$ moves up and down along the $Z$-axis vertical direction under the initial configuration.

3.4. Analysis of DOF of output parts of equivalent parallel mechanism

The parallel mechanism shown in fig.14 is the equivalent mechanism diagram of the whole new hexahedral coupling mechanism, so the degree of freedom of the end piece of the coupling mechanism can be determined by the degree of freedom of the equivalent parallel mechanism.
The equivalent parallel mechanism consists of 15 kinematic pairs, 14 members and 3 chains, each of which provides a binding force and 2 constraint couple to the moving platform. Therefore, the moving platform is subject to 9 constraint screws. Among them, three constraints restrict the movement of the moving platform in the plane of the frame, and six constraint pairs restrict the rotation of the moving platform in any direction around the space. Therefore, there is one common constraint \((\lambda=1)\), three redundant constraints \((\nu=3)\), and six local degrees of freedom \((\zeta=6)\). According to the modified G-K formula [3], the degree of freedom of the moving platform \(M\) is calculated as follows:

\[
M = (6 - \lambda) (n - g - 1) + \sum_{i=1}^{g} f_i + \nu - \zeta = 1
\]

In formula, \(M\)——Mechanism moving platform;
\(\lambda\)——Number of common constraints;
\(n\)——Number of fixed platform components;
\(g\)——Number of motor pairs;
\(f_i\)——The degree of freedom of the \(i\)th pair;
\(\nu\)——Parallel redundant constraints;
\(\zeta\)——The local degree of freedom in the new coupling mechanism;

![Figure 14. Equivalent mechanism sketch of coupled mechanism](image-url)

No matter which form of motion of the moving platform leaves the initial position, the geometric relationship between the kinematic pairs of the mechanism remains unchanged, and the coordinate system \(O-XYZ\) can always be established, that is to say, according to the above splitting principle, the constraint screw of each branch also remains unchanged, which can still be equivalent to the parallel mechanism shown in fig.14. Therefore, the degree of freedom of the coupling mechanism is always 1, and the mechanism has full cycle.

### 4. Analysis of kinematic characteristics of coupling mechanism

According to the above analysis, the dynamic platform of the new hexahedral coupling mechanism has only one degree of freedom. In order to verify the correctness of the theoretical analysis intuitively, a three-dimensional solid model of the coupling mechanism is established based on CATIA, and then it is imported into Adams to establish a kinematic model for kinematic simulation, and then the degrees of freedom of the hexahedral coupling mechanism and the kinematic characteristics of each component are further analyzed.
4.1. Establish simulation model of coupling mechanism

Establish the three-dimensional model of the new hexahedron coupling mechanism and the motion reference coordinate system $O-XYZ$ of the mechanism, as shown in fig.15. Where the $XOY$ plane is parallel to the surface of frame $D$ and the coordinate system origin $o$ coincides with the geometric center of frame $D$, the $Y$ axis is parallel to the $R_{11}$ axis, the $X$ axis is vertical to the $R_{11}$ axis and bisects, the $Z$ axis is obtained by the right-hand spiral rule, and fig.13 shows the top view of frame $D$. The kinematic model of the mechanism established in ADAMS is shown in fig.16. Under the given driving conditions, the displacement curve of the geometric center of each component is obtained.

![Figure 15. Coordinate sketch of coupling mechanism](image1)

![Figure 16. Kinematic model of mechanism](image2)

4.2. Analysis of component motion characteristics

In order to verify the kinematic characteristics of the mechanism, drives are set on the $R_{11}$, $R_{12}$, $R_{13}$ and $R_{8}$ axes respectively. The drives are shown by the blue arrow in figure 16. The driving rotation direction is determined according to the kinematic requirements of the mechanism. In this paper, the simulation takes the upward movement of the mechanism as an example to set drive 1. The rotational speeds of the four drives are all 10.0 rad / s, and the total simulation time is 5S. The displacement curve obtained by taking the geometric center of the moving platform as the measurement point is shown in figure 17. It can be seen from the figure that in the whole simulation process, the displacement of the moving platform $M$ in the z-axis exists and rises slowly, and the displacement in the x-axis and y-axis is both 0, that is to say, the moving platform $M$ has only one z-dof, which is consistent with the theoretical analysis results above, and proves the rationality of the mechanism design.

![Figure 17. Displacement variations of moving platform along axis](image3)
In order to study the motion characteristics of each node component, the time displacement curves of the geometric center points of each node component \( T_i (i=1, 2, 3) \) in the X, Y and Z axes are extracted respectively as shown in figure 18, figure 19 and figure 20.

![Figure 18. Schematic diagram of displacement change of each node component along X axis](image1)

![Figure 19. Schematic diagram of displacement change of each node component along Y axis](image2)

![Figure 20. Schematic diagram of displacement change of each node component along Z axis](image3)

In figure 18, the displacement of the joint component \( T_i (i=1, 2, 3) \) on the X-axis tends to decrease and approach to the center, where the geometric center of the \( T_1 \) component is on the \( XOZ \) plane, and the movement along the X-axis is relatively faster, and the motion curves of the \( T_2 \) and \( T_3 \) components coincide, that is, the same direction of the same displacement speed, indicating that the \( T_2 \) and \( T_3 \) components are symmetrical about the \( XOZ \) plane, which is consistent with the theory.

In figure 19, the displacement of \( T_1 \) component in Y-axis is 0. According to the establishment characteristics of datum coordinate system above, the geometric center of node component \( T_1 \) is in \( XOZ \) plane, so the projection in Y-axis is 0. The simulation is consistent with the theory. The displacements of \( T_2 \) and \( T_3 \) on the X-axis are the same and opposite, and they are decreasing to zero gradually, which shows that they are symmetrical about the \( XOZ \) plane and close to the center at the same speed, which is consistent with the theory.

In figure 20, the displacement curve of node component \( T_i (i=1,2,3) \) in Z axis coincides, i.e. the movement displacement is the same in the same direction, indicating that the node component maintains the same height in Z axis during the whole simulation process, which is consistent with the theoretical analysis direction.

It can be seen from fig.17-20 that in the whole simulation process, the hexahedral coupling mechanism has only one degree of freedom, the node components \( T_2 \) and \( T_3 \) are symmetrical about the \( XOZ \) plane, the \( T_1 \) component is kept in the \( XOZ \) plane, the node components are all close to the center,
with centripetal characteristics, which is completely consistent with the theoretical analysis, and the spatial motion curve is shown in fig.21.

Figure 21. Schematic diagram of space motion track curve

4.3. Ductility of symmetrical coupling mechanism

In order to verify the telescopic motion characteristics of the coupling structure, the driving direction of the $R_8$ axis is modified based on the driving 1 above, and the moving platform $M$ moves downward, and the motion state at a certain time is shown in figure 22a. Set the mechanism to move downward as drive 2, and change the drive direction of $R_8$ axis at the same time. The motion state of the whole mechanism at a certain time in the motion process is shown in Figure 22b and figure 22c, respectively. It can be seen from the figure that: in the motion process, the height of node component $T_i$ ($i=1, 2, 3$) relative to frame $D$ is the same at all times and opens along the geometric center $O$ of the mechanism, and the moving platform $M$ moves up and down alternately. It is proved that the new hexahedral symmetric coupling mechanism is a flexible mechanism with one degree of freedom by synthesizing figures 21 and 22.

Figure 22. Schematic diagram of expansion and change of deployable structure

a telescopic change of posture 1  b telescopic change of posture 2  c telescopic change of posture 3

5. Conclusion

(1) According to the structural characteristics and symmetry of the double tetrahedron superposition mechanism, a new type of hexahedron space coupling centripetal mechanism is designed by adding a series of branch chains to change the branch constraint system.

(2) Based on the theory of helix, the movement property of the mechanism is analyzed by using the independent movement shunt marking method, which shows that the mechanism has one degree of
freedom. Through modeling, simulation and analysis, it is verified that the freedom of the mechanism is correct and the mechanism design is reasonable, and its centripetal reciprocating characteristics can meet the requirements of the telescopic motion of the engineering robot.

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