Correctness Analysis of IBFT

Roberto Saltini
PegaSys (ConsenSys)
{name}.{surname}@consensys.net

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Abstract
In this paper we analyse the correctness of Istanbul BFT (IBFT), which is a Byzantine-fault-tolerant (BFT) proof-of-authority (PoA) blockchain consensus protocol that ensures immediate finality. We show that the IBFT protocol does not guarantee Byzantine-fault-tolerant consistency and liveness when operating in an eventually synchronous network, and we propose modifications to the protocol to ensure both Byzantine-fault-tolerant consistency and liveness in eventually synchronous settings.

1 Introduction
In this paper we perform a correctness analysis of the IBFT (Istanbul Byzantine-fault-tolerant) consensus protocol which was developed around early 2017 by AMIS Technologies [11]. IBFT was created to provide the standard Ethereum blockchain protocol [17] with a Byzantine-fault-tolerant (BFT) proof-of-authority (PoA) consensus protocol with immediate finality, well suited for either private or consortium blockchains.

Blockchains are data structures that maintain an ordered set of transactions organised into blocks. For this reason, blockchains are also referred to as distributed transaction ledgers. A blockchain starts with an initial block called the genesis block. Blocks are added to the blockchain, linking them to a previous block, usually by referencing the hash of the previous block. The first widely adopted blockchain, Bitcoin [12], and most like it use the transactions to transfer tokens between accounts. The Ethereum blockchain augments the standard value transfer capability of each transaction with the possibility to specify instructions of a Turing-complete language to execute on a sandboxed runtime. The runtime, called the Ethereum Virtual Machine (EVM) modifies the Ethereum global state maintained by each node. This means that any user of the Ethereum blockchain has the capability to create decentralised applications, called smart contracts, that can govern the interaction between the different users of the system. One of the first use cases for Ethereum was the creation of escrow smart contracts eliminating the need for a trusted 3rd party.

Within the blockchain context, the role of consensus protocols is to define algorithms by which all nodes on the network can agree on the canonical block sequence. Most of these protocols have some level of Byzantine-fault-tolerance (BFT) [10], which means that the protocols can cope with some fraction of the nodes, called Byzantine nodes, being faulty and behaving in any arbitrary way, including colluding and delaying message delivery to honest nodes. The standard Ethereum consensus protocol is designed to operate in a permissionless setting where any node can join or leave the network at any point in time and can propose blocks to be added to the blockchain. Ethereum requires each node to spend compute effort in resolving a hard cryptographic puzzle before it can propose a block. This type of technique is called proof of work (PoW) [5, 12] and ensures blockchain immutability with a high probability. Proof of stake (PoS) [15], proof of space (PoSpace) [13] and proof of elapsed time (PoET) [4] are examples of alternative techniques for guaranteeing blockchain immutability in a permissionless context that have been proposed over the years. In contrast, the IBFT consensus protocol falls in the PoA category as only a set of special nodes, called validators, are authorised to propose new blocks. In the context of IBFT, the set of validators is dynamic as validators can vote to add or remove nodes to/from this set. This makes IBFT well suited for consortium or private blockchains where there are
limitations on which nodes can join the network and the permissioning level of each of the nodes. In a private blockchain only nodes from a single organisation can join the peer-to-peer network to view transactions. In a consortium network a group of organisations form the consortium, and nodes from any member of the consortium can join the network. In some consortium networks read access is made available to the public. In both private and consortium networks block creation is limited to a subset of nodes.

While IBFT specifies some modifications to the standard Ethereum block header structure and block validation ruleset, all of the standard Ethereum capabilities (e.g. smart contracts) are maintained.

Roughly speaking, IBFT adapts the state machine replication protocol Practical BFT (PBFT) \cite{1} to work in a blockchain setting and integrates it with the dynamic validator set voting mechanism originally designed for Clique \cite{16} that allows validators to be added to, or removed from, the set.

While IBFT is a very valuable consensus protocol that opens up the way for deploying Ethereum EVM based solutions within private and consortium blockchains, to the best of our knowledge, unlike the correctness proof for the PBFT protocol \cite{2}, to date no correctness analysis of the IBFT consensus protocol has been performed. Our contribution is to perform such a correctness analysis, showing its limitations, and finally proposing modifications to the original protocol to address shortcomings, particularly within the context of eventually synchronous networks.

The main differences between PBFT and IBFT are as follows:

- in IBFT there is no client submitting requests to the network, but rather all of the validators can in turn propose a block to the network of validators;
- IBFT allows for two types of nodes: validators that take part in the consensus protocol and standard nodes that validate blocks but do not participate in the consensus protocol;
- the set of validators in PBFT is static whereas IBFT features a dynamic validator-set where validators can be added to or removed from the set;
- IBFT specifies a simplified version of the View-Change message of PBFT and does not include the New-View message included in the PBFT protocol;
- while IBFT does not use checkpoints explicitly, each IBFT block can be considered the IBFT equivalent of a PBFT checkpoint.

The work is organised as follows. In Section 2 we present our analysis model, and define robustness for the IBFT protocol as logical conjunction of two properties: persistence and liveness. Persistence guarantees blockchain consistency and immutability amongst all honest nodes, while liveness guarantees that transactions submitted to the system will eventually be included in the blockchain (distributed transaction ledger). In Section 3 we describe the IBFT protocol. Section 4 introduces a series of definitions that will be used in the following analysis of the IBFT protocol. In Section 5 we present the persistence analysis of the IBFT protocol which shows that IBFT does not guarantee Byzantine fault tolerance when operating in an eventually synchronous network. We then describe and analyse modifications to the IBFT protocol that will ensure optimal Byzantine fault tolerance under eventual synchrony. Section 6 discusses the liveness property of the protocol showing that IBFT is not live in eventually synchronous networks, and explores two potential modifications to the IBFT protocol that will ensure liveness under these conditions.

2 Model

2.1 System Model

Asynchronous nodes. We consider a system composed of an unbounded number of asynchronous nodes, each of them maintaining a local copy of the blockchain obtained by adding blocks to it as specified by the IBFT protocol. We assume that all nodes have the same genesis block.

Network Model. The IBFT protocol relies on the Ethereum \(\mathcal{D}E\mathcal{V}p2p\) protocol for the delivering of all protocol messages. We model the gossip network as an eventually synchronous network, also called partially synchronous network, as defined in Dwork et al. \cite{6}, where there exists a point in time called global stabilisation time (GST), after which the message delay is bounded by a constant, \(\Delta\). Before GST there is not bound on the message delay.
Failure Model. We consider a Byzantine failure mode system, where Byzantine nodes can behave arbitrarily. In contrast, honest nodes never diverge from the protocol definition. We denote the maximum number of Byzantine nodes that an eventually synchronous network of $n$ nodes can be tolerant to with $f(n)$. As proved in Dwork et al. [6], the relationship between the total number of nodes, $n$, and the maximum number of Byzantine nodes can be expressed as follows:

$$f(n) \equiv \left\lfloor \frac{n - 1}{3} \right\rfloor$$  \hspace{1cm} (1)

which, by transforming the floor function into a ceiling function, can also be written as follows:

$$f(n) \equiv \left\lceil \frac{n}{3} \right\rceil - 1$$  \hspace{1cm} (2)

Cryptographic Primitives. The IBFT protocol uses the Keccak hash function variant as per Ethereum Yellow Paper [17] to produce digests of blocks. We assume that the Keccak hash function is collision-resistant.

We assume a digital signature scheme that ensures uniqueness and unforgeability. Uniqueness means that the signatures generated for two different messages are also different with high probability. The unforgeability property ensures that Byzantine nodes, even if they collude, cannot forge digital signatures produced by honest nodes.

We use $\langle m \rangle_{\sigma_v}$ to denote a message $m$ signed by validator $v$.

IBFT Robustness Property. For the purpose of defining the robustness property, we consider the IBFT protocol as implementing a distributed permissioned transaction ledger with immediate finality. A distributed transaction ledger maintains an append-only fully-ordered set of transactions and ensures its consistency amongst all honest nodes that participate in the protocol.

Compared to a public transaction ledger where any node can add transactions to the ledger, in a permissioned transaction ledger only a limited set of nodes, called validators, can add transactions to it and participate in ensuring the consistency of the ledger.

Each node maintains a local copy of the transaction ledger organised as a chain of blocks, or blockchain. Our definition of robustness for the IBFT protocol is based on the definition of robustness for public transaction ledgers provided in Garay et al. [9].

For the purpose of this definition, the position of a transaction within the transaction ledger implemented by the IBFT protocol is defined as a pair with the first component corresponding to the height of the block including the transaction and the second component corresponding to the position of the transaction within the block.

Definition 1. The IBFT protocol implements a robust distributed permissioned transaction ledger with immediate finality and $t$-Byzantine-fault-tolerance if, provided that no more than $t$ validators are Byzantine, it guarantees the following two properties:

- **Persistence.** If an honest node adds transaction $T$ in position $i$ of its local transaction ledger, then (i) $T$ is the only transaction that can ever be added in position $i$ by any other honest node, (ii) $T$ will eventually be added to the local transaction ledger of any other honest node.
- **Liveness.** Provided that a transaction is submitted to all honest validators, then the transaction will eventually be included in the distributed permissioned transaction ledger.

3 Protocol Description

The presentation of the IBFT protocol provided in this section is based on EIP 650 [11] and the actual implementation available on GitHub [14].

The IBFT blockchain maintained by each node is constituted of finalised blocks where each finalised block contains an Ethereum block and a finalisation proof used to attest that consensus was reached on the Ethereum block included in the finalised block. As common in the blockchain literature, we define the height of a finalised block $FB$ as the distance in finalised blocks between $FB$ and the genesis block that has height 0. The first finalised block after the genesis block has height 1, the next one has
height 2 and so on.

For each block height, the IBFT consensus protocol can be considered running sequential different instances of what we call the IBFT-block-finalisation-protocol. The aim of the \( h \)-th instance of the IBFT-block-finalisation-protocol is to decide on the block to be added at height \( h \), generate the related finalised block and broadcast it to all nodes. Only a subset of the entire set of IBFT nodes can participate in the \( h \)-th instance of the block finalisation protocol. We call this set of nodes the validators for height/instance \( h \) and refer to each member of this set as a validator for height/instance \( h \). We also refer to all of the nodes not included in the validator set for height/instance \( h \) as standard nodes. We often omit for height/instance \( h \) when this is clear from the context. The set of validators for each instance \( h \) of the IBFT-block-finalisation-protocol is deterministically computed as function of the chain of blocks from the genesis block until the block with height \( h - 1 \).

Once the IBFT-block-finalisation-protocol for an honest validator reaches a decision on the block to be included in the finalisation block with height \( h \), it creates a finalised block by adding the finalisation proof to the block and propagates the finalised block to all other nodes in the network, both validators and standard nodes. Finalised blocks are transmitted using the standard Ethereum block synching protocol by which each validator (i) transmits a newly created finalised block using the Devp2p gossip protocol and (ii) asks its peers for the availability of new finalised blocks either when the validator starts or when the validator receives a finalised block with an height higher than the next expected finalised block height. The purpose of the finalisation proof is to allow any node, even nodes that did not participate in the IBFT-block-finalisation-protocol, to verify that a decision on the block inclusion in the blockchain at height \( h \) was reached by a correct execution of the IBFT-block-finalisation-protocol despite the presence of a potential number of Byzantine validators. As described in Section 3.1, the finalisation proof is composed of signatures over the Ethereum block, called commit seals, that are sent by validators as part of the Commit messages exchanged during the IBFT-block-finalisation-protocol. The IBFT consensus protocol is described by the pseudocode in Algorithms 1 and 2 where:

- Each of the \textbf{upon} blocks in the pseudocode is assumed to be executed atomically when the condition specified after the \textbf{upon} keyword is satisfied;
- All functions in \textit{typewriter} font are defined in the remainder of this section, whereas all functions in \textit{italic} font are defined in the pseudocode;
- \texttt{numOfReceived}(\texttt{m from} \texttt{V, v}) corresponds to the number of messages \texttt{m} sent by validators included in set \( V \) that have been received by validator \( v \);
- \texttt{received}(\texttt{m from} \texttt{V, v}) is true if and only if validator \( v \) has received at least one message \( m \) sent by validators included in set \( V \), i.e \( \texttt{received}(\texttt{m from} \texttt{V, v}) \equiv \texttt{numOfReceived}(\texttt{m from} \texttt{V, v}) \geq 1 \);
- If \texttt{from} \( V \) is omitted (i.e \texttt{received}(\texttt{m, v})\), then \texttt{received} is true if \( m \) has been received by \( v \) regardless of who the sender is;
- The symbol \(*\) denotes any value.
- \texttt{blockHeight}() is defined as the height of the finalised block \( FB \);
- Each validator \( v \) stores its local blockchain in \texttt{chain} \( v \);
- \texttt{chain} \( v \)[\( n \)] corresponds to the finalised block with height \( n \), while \texttt{chain} \( v \)[\( n : m \)] corresponds to a sub-chain including all of the finalised blocks from height \( n \) to height \( m \);
- The blockchain height is defined as the height of the last finalised block added to the blockchain.
- \texttt{validators}() represents the set of authorised validators for instance \( h \) of the IBFT-block-finalisation-protocol. The definition of the \texttt{validators}() function is not presented here as it is outside the scope of this work and does not have relevance to the results presented here. For the same reason, we do not describe the protocol that can be used to add or remove validators to/from the validator set of each instance of the IBFT-block-finalisation-protocol;
- \texttt{n}() represents the number of validators for instance \( h \) of the IBFT-block-finalisation-protocol, i.e \( \texttt{n}(\texttt{chain} \[0 : h - 1\]) \equiv \texttt{validators}(\texttt{chain} \[0 : h - 1\]) \).
- \texttt{extractFinalisationProof}() denotes the finalisation proof included in the finalised block \( FB \);
- \texttt{extractBlock}(\texttt{FB}) denotes the block included in the finalised block \( FB \);
- The function \( \texttt{isValidBlock}(\texttt{B, B_{parent}}) \) is defined to be true if and only if block \( B \) is a valid Ethereum block with parent \( B_{parent} \). For the purpose of this work, we consider that
isValidBlock($B, B_{parent}$) only verifies the following fields of the standard Ethereum header: parentHash, stateRoot, transactionsRoot, receiptsRoot, logsBloom, number, gasLimit, gasUsed. These fields are verified as specified in [17]. The IBFT protocol implementation [14] actually verifies also the other fields but in a different way than specified in [17]. We do not describe how these fields are verified as this is out of the scope of this work and does not affect our results. By abuse of language, we say that two blockchains of length $h$ are identical if and only if the chains obtained by considering only the blocks included in the finalisation blocks (and not the finalisation proofs) are identical. Formally, $chain_v[0 : h] = chain_v[0 : h] \rightarrow \forall 0 \leq h' \leq h : extractBlock(chain_v[h']) = extractBlock(chain_v[h'])$.

Algorithm 1 defines the function isValidFinalisedBlock($FB, v$) to be true if and only if all of the following conditions are met:

- at least Quorum($n$) $\equiv f(n) \cdot 2 + 1$ of the commit seals included in the finalisation proof of $FB$ are signed by validators included in the set of validators for the $h$-th instance of the IBFT-block-finalisation-protocol, where $n \equiv n(chain_v[0 : h - 1])$;
- the block included in $FB$ is a valid Ethereum block.

We say that a finalised block $FB$ is valid for validator $v$ if and only if isValidFinalisedBlock($FB, v$) is true.

The IBFT protocol is essentially described by the upon block at line 13 of Algorithm 1 where $h_v$ corresponds to the height of the next finalised block to be added to the local blockchain of node $v$. If $v$ is a validator for the $h_v$-th instance of the IBFT-block-finalisation-protocol, then $h_v$ also corresponds to the number of the instance of the IBFT-block-finalisation-protocol that $v$ is currently running. When a node $v$ receives a valid finalised block for height $h_v$, then (i) $v$ adds the finalised block to its local blockchain, (ii) if $v$ is a validator for the $h_v$-th instance of the IBFT-block-finalisation-protocol then

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**Algorithm 1: IBFT protocol for IBFT node $v$**

1: Functions:
2:  
3:  Quorum($n$)$\equiv$
4:  
5:  $f(n) \cdot 2 + 1$
6:  
7:  isValidFinalisationProof($FB, v$)$\equiv$
8:  
9:  $\geq$ Quorum($n(chain_v[0 : h_v - 1])$) of the commit seals included in
10:  
11:  extractFinalisationProof($FB$) are signed by validators included in
12:  
13:  validators($chain_v[0 : h_v - 1]$)
14:  
15:  isValidFinalisedBlock($FB, v$)$\equiv$
16:  
17:  isValidFinalisationProof($FB, v$)$\wedge$
18:  
19:  isValidBlock(extractBlock($FB$),extractBlock($chain_v[h_v - 1]$))
20:  
21:  Initialisation:
22:  
23:  $chain_v[0]$ $\leftarrow$ genesis block
24:  
25:  $h_v$ $\leftarrow$ 1
26:  
27:  if $v$ in validators($chain_v[0 : h_v - 1]$) then start the $h_v$-th instance of the
28:  
29:  IBFT-block-finalisation-protocol
30:  
31:  Upon Blocks:
32:  
33:  upon received($\langle$FINALISED-BLOCK, $FB\rangle, v$) do
34:  
35:  if blockHeight(extractBlock($FB$)) = $h_v$ then
36:  
37:  if isValidFinalisedBlock($FB, v$) then
38:  
39:  $chain_v[h_v]$ $\leftarrow$ $FB$
40:  
41:  if $v$ in validators($chain_v[0 : h_v - 1]$) then stop the $h_v$-th instance of the
42:  
43:  IBFT-block-finalisation-protocol
44:  
45:  $h_v$ $\leftarrow$ $h_v$ + 1
46:  
47:  if $v$ in validators($chain_v[0 : h_v - 1]$) then start the $h_v$-th instance of the
48:  
49:  IBFT-block-finalisation-protocol
50:  
51:  end
52:  
53:  end
54:  
55:  end
\(v\) aborts the \(h_v\)-th instance of the IBFT-block-finalisation-protocol, (iii) \(v\) advances \(h_v\) to \(h_v + 1\) and (iv) if \(v\) is a validator for the \(h_v\)-th instance of the IBFT-block-finalisation-protocol then \(v\) starts a new instance of the IBFT-block-finalisation-protocol.
3.1 IBFT-block-finalisation-protocol

Algorithm 2: $h$-th instance of the IBFT-block-finalisation-protocol for validator $v$.

1: Macro Expansions:
2: $n_{h,v}$ expands to: $n(chain_v[0:h-1])$
3: validators$_{h,v}$ expands to: validators$(chain_v[0:h-1])$
4: proposer$_{h,v}(r_{h,v})$ expands to: proposer$(chain_v[0:h-1], r_{h,v})$

5: Initialisation:
6: lock$_{h,v}$ ← ⊥
7: StartNewRound$(0, h, v)$

8: Procedures:
9: def moveToNewRound$(r, h, v)$:
10:  $r_{h,v}$ ← $r$
11:  roundAlreadyStarted$_{h,v}$ ← false
12:  if lock$_{h,v}$ = ⊥ then
13:    acceptedPrePrepare$_{h,v}$ ← ⊥
14:  end
15: def StartNewRound$(r, h, v)$:
16:  moveToNewRound$(r, h, v)$
17:  roundAlreadyStarted$_{h,v}$ ← true
18:  commitSent$_{h,v}$ ← false
19:  finalisedBlockSent$_{h,v}$ ← false
20:  set roundTimer$_{h,v}[r_{h,v}]$ to expire after roundTimerTimeout$(r_{h,v})$
21:  if $v$ = proposer$_{h,v}(r_{h,v})$ then
22:    if lock$_{h,v}$ ≠ ⊥ then
23:      $B$ ← lock$_{h,v}$
24:  else
25:    $B$ ← createNewProposedBlock$(h, v)$
26:  end
27:  multicast $(PRE-PREPARE, h, r_{h,v}, B)$$_{σ_v}$ to validators$_{h,v}$
28: end
29: def moveToNewRoundAndSendRoundChange$(r, h, v)$:
30:  moveToNewRound$(r, h, v)$
31:  multicast $(ROUND-CHANGE, h, r_{h,v})$_{σ_v} to validators$_{h,v}$
32: Upon Blocks:
33: upon received$(PRE-PREPARE, h, r_{h,v}, B)$ from proposer$_{h,v}(r_{h,v}, v)$ do
34:  if acceptedPrePrepare$_{h,v}$ = ⊥ then
35:    if $(lock_{h,v} = ⊥$ or lock$_{h,v}$ = $B))$ and isValidBlock$(B, chain_v[h-1])$ then
36:      acceptedPrePrepare$_{h,v}$ ← $B$
37:      multicast $(PREPARE, h, r_{h,v}, KEC(B))$_{σ_v} to validators$_{h,v}$
38:    else
39:      moveToNewRoundAndSendRoundChange$(r_{h,v} + 1, h, v)$
40:    end
41: end
42: upon numOfReceived$(PREPARE, h, r_{h,v}, KEC(B))$ from validators$_{h,v}, v)$ ≥ Quorum$(n_{h,v})$ and
43:  commitSent$_{h,v}$ = false do
44:  if acceptedPrePrepare$_{h,v}$ = $B$ then
45:    lock$_{h,v}$ ← $B$
46: end
Algorithm 2: $h$-th instance of the IBFT-block-finalisation-protocol for validator $v$ (continue).

upon (received((PRE-PREPARE, $h$, $r_{h,v}$, $B$) from proposer$_{h,v}(r_{h,v}), v$) or received((PREPARE, $h$, $r_{h,v}$, KEC($B$)) from validators$_{h,v}, v$) and commitSent$_{h,v} = \text{false}$) do

if lock$_{h,v} = B$ then

multicast ($\text{COMMIT, } h, r_{h,v}, \text{KEC}(B), \text{CS}(B, v))_v$ to validators$_{h,v}

commitSent$_{h,v} \leftarrow \text{true}$

end

upon numOfReceived((COMMIT, $h$, $r_{h,v}$, KEC($B$), $\ast$) from validators$_{h,v}, v$) $\geq$ Quorum($n_{h,v}$) and finalisedBlockSent$_{h,v} = \text{false}$ do

if acceptedPrePrepare$_{h,v} = B$ then

finalisedBlockSent$_{h,v} \leftarrow \text{true}$

if (all of the (COMMIT, $h$, $r_{h,v}$, KEC($B$), $\ast$) received from validators$_{h,v}$ contain a commit seal of correct size) and wellFormedToAddFinalisationProof($B$) then

lock$_{h,v} \leftarrow B$

finalisationProof$_{h,v} \leftarrow$ all commit seals included in the (COMMIT, $h$, $r_{h,v}$, KEC($B$), $\ast$) received from validators$_{h,v}$

FB $\leftarrow$ ($B$, finalisationProof$_{h,v}$)

broadcast (FINALISED-BLOCK, FB) to all nodes

else

moveToNewRoundAndSendRoundChange($r_{h,v} + 1$, $h, v$)

lock$_{h,v} \leftarrow \bot$

end

end

upon numOfReceived((ROUND-CHANGE, $h$, $r_{rc}$) from validators$_{h,v}, v$) $\geq f(n_{h,v}) + 1$ do

if $r_{rc} > r_{h,v}$ then

moveToNewRoundAndSendRoundChange($r_{rc}, h, v$)

end

upon expiry of roundTimer$_{h,v}[r_{h,v}]$ do

moveToNewRoundAndSendRoundChange($r_{h,v} + 1$, $h, v$)

upon numOfReceived((ROUND-CHANGE, $h$, $r_{rc}$) from validators$_{h,v}, v$) $\geq$ Quorum($n_{h,v}$) do

if $r_{rc} > r_{h,v}$ or ($r_{rc} = r_{h,v}$ and roundAlreadyStarted$_{h,v} = \text{false}$) then

StartNewRound($r_{rc}, h, v$)

end

end

This sub-section describes a generic $h$ instance of the IBFT-block-finalisation-protocol for validator $v$ as specified by the pseudocode in Algorithm 2.

The IBFT-block-finalisation-protocol is organised in rounds, starting from round 0, where validators progress to the next round once they suspect that in the current round they will not be able to decide on the block to be included in the finalised block with height $h$. Both in Algorithm 2 and here, the current round for the $h$-th instance of the IBFT-block-finalisation-protocol for validator $v$ is denoted by $r_{h,v}$.

For each round, one of the validators is selected to play the role of block proposer. This selection is operated by the evaluation of proposer($\text{chain}_v[0 : h - 1], r_{h,v}$) where proposer($\cdot$) is a deterministic function of the chain of blocks from the genesis block until the block with height $h - 1$ and the current round number.

The pseudocode at lines 2 to 4 introduces the following macros:

- $n_{h,v}$: number of validators for the $h$-th instance of the IBFT-block-finalisation-protocol for validator $v$;
- validators$_{h,v}$: validators for the $h$-th instance of the IBFT-block-finalisation-protocol for vali-
idator $v$;
- $\text{proposer}_{r_{h,v}}(r_{h,v})$: proposer for round $r_{h,v}$ of the $h$-th instance of the IBFT-block-finalisation-protocol for validator $v$.

These macros are used both in the pseudocode and in this section to simplify the notation when describing the $h$-th instance of the IBFT-block-finalisation-protocol for validator $v$.

For the purpose of this work, we do not define the proposer selection function, but we state that it ensures that all of the validators for the $h$-th instance of the IBFT-block-finalisation-protocol are selected for any sequence of $n_{h,v}$ consecutive rounds. The IBFT protocol provides the logic to select the proposer for round $0$ in two different ways:

- **Sticky Proposer.** The proposer for round $0$ corresponds to the proposer of the block included in the previous finalised block;
- **Round-Robin Proposer.** The proposer for round $0$ corresponds to the proposer of the proposer selection sequence that comes after the proposer of the block included in the previous finalised block.

During the IBFT-block-finalisation-protocol, when specific conditions are met, a validator $v$ can lock on a block $B$, which corresponds to the pseudocode in Algorithm 2 setting $\text{lock}_{h,v}$ to a value different from $\bot$ (see line 14). However, regardless of whether a validator $v$ is locked or not, $v$ always unlocks (sets $\text{lock}_{h,v}$ to $\bot$) when a new IBFT block finalisation instance starts (see line 0).

As specified by the $\text{StartNewRound}$ procedure (line 15), at the beginning of the current round $r_{h,v}$, if $v$ is the selected block proposer for round $r_{h,v}$, then $v$ multicasts a Pre-prepare message $\langle \text{PRE-PREPARE}, h, r_{h,v}, B \rangle_{\sigma_{r_{h,v}}}$ to all validators (including itself) where, if $v$ is locked on a block then $B = \text{lock}_{h,v}$, otherwise $B$ can be any valid block for height $h$. The pseudocode uses $\text{createNewProposedBlock}(h, v)$ to represent the creation of a new block with height $h$ by validator $v$. Honest validators employee a fair transaction selection algorithm to decide which transactions to include in the next block. The definition of such algorithm is outside the scope of this work.

As specified by lines 33 to 36, a validator $v$ accepts a Pre-prepare message $\langle \text{PRE-PREPARE}, h_{pp}, r_{pp}, B \rangle$ if and only if all of the following conditions are met:

- $v$ is currently running the IBFT-block-finalisation-protocol instance $h_{pp}$, i.e. $h_{pp} = h$;
- $v$ is in round $r_{pp}$, i.e. $r_{pp} = r_{h,v}$;
- the message is signed by the selected proposer for round $r_{h,v}$ and instance $h$ of the IBFT-block-finalisation-protocol;
- $v$ has not already accepted a Pre-prepare message for round $r_{h,v}$ in the $h$-th instance of the IBFT-block-finalisation-protocol;
- $v$ is not locked on a block different from $B$;
- block $B$ is a valid block for height $h$.

When a validator $v$ accepts a Pre-prepare message, it multicasts a Prepare message $\langle \text{PREPARE}, h, r_{h,v}, \text{KEC}(B) \rangle_{\sigma_v}$ (see line 37) to all validators (including itself) where $\text{KEC}(B)$ corresponds to the application of the Keccak hash function to block $B$.

As specified by the upon block at line 12 upon reception of either a Pre-prepare or Prepare message, if all of the following conditions are satisfied, then validator $v$ locks on block $B$:

- $v$ has accepted a Pre-prepare message for block $B$, height $h$ and round $r_{h,v}$;
- $v$ has received at least $\text{Quorum}(n_{h,v})$ Prepare messages for height $h$, round $r_{h,v}$ and Keccak hash matching $\text{KEC}(B)$ from validators $s_{r_{h,v}}$.

As specified by the upon block at line 46 the first time that all of the conditions listed below are verified for validator $v$, $v$ multicasts a Commit message $\langle \text{COMMIT}, h, r_{h,v}, \text{KEC}(B), CS(B, v) \rangle_{\sigma_v}$ to all validators (including itself), where $CS(B, v)$, called commit seal, corresponds to the signature of $v$ over the block $B$.

- $v$ is locked on block $B$;
- either
  - $v$ has accepted a Pre-prepare message for block $B$, height $h$ and current round $(r_{h,v})$;
  - or $V$ has received a Prepare message for block hash $\text{KEC}(B)$, height $h$ and current round $(r_{h,v})$.

The pseudocode uses the state variable $\text{commitSent}_{h,v}$ to indicate that the Commit message is sent
only the first time that all of the conditions listed above are met. Indeed, \texttt{commitSent}_{h,v} is set to \texttt{true} at line 49 and reset to \texttt{false} in the \textit{StartNewRound} procedure at line 18. In the IBFT implementation [14], \(CS(B,v)\) is actually a signature over a modified version of block \(B\), but in this work we consider the simplified definition provided above as the differences between this definition and the complete one do not affect the results presented here.

The IBFT protocol also includes the following optimisation that, for brevity, we have omitted from the pseudocode description. Commit messages are treated as Prepare messages when evaluating the conditions for \textit{locking} and sending Commit messages. For example, if \(Quorum(n_{h,v}) = 4\) and \(v\) accepts a Pre-prepare message for height \(h\), round \(r_{h,v}\) and block \(B\), 2 Prepare messages for height \(h\), round \(r_{h,v}\) and block \(B\) and Keccak hash matching \(KEC(B)\), and 2 Commit messages for the same height, round \(r_{h,v}\) and block hash, then \(v\) \textit{locks} on \(B\) and sends a Commit message for height \(h\), round \(r_{h,v}\) and block hash \(KEC(B)\).

As specified by lines 51 to 58, the first time that all of the following conditions are verified for validator \(v\), (i) \(v\) \textit{locks} on block \(B\) (line 53) and (ii) broadcasts a finalised block including block \(B\) and related finalisation proof:

- \(v\) has accepted a Pre-prepare message for height \(h\), round \(r\) and block \(B\);
- \(v\) has received at least \(Quorum(n_{h,v})\) Commit messages for height \(h\), round \(r_{h,v}\) and Keccak hash matching \(KEC(B)\);
- all of the commit seals received (as part of Commit messages) are of the correct size;
- \(B\) is well formed to allow adding the finalisation proof to it.

As indicated by line 57, the finalisation proof includes all of the commit seals included in Commit messages for height \(h\), current round \((r_{h,v})\) and Keccak hash matching \(KEC(B)\). The pseudocode uses the state variable \texttt{finalisedBlockSent}_{h,v} to trigger the transmission of a finalised block only the first time that the conditions listed above are met. \texttt{finalisedBlockSent}_{h,v} is set at line 53 and reset in the \textit{StartNewRound} procedure at line 19.

In alignment with PBFT, IBFT relies on a round change sub-protocol to detect whether the selected proposer may be Byzantine and causing the protocol to never terminate. When one of the conditions listed below is satisfied for validator \(v\) while in round \(r_{h,v}\), \(v\) moves to a new round \(r'\) and multicasts a Round-Change message \((ROUND-CHANGE,h,r')_{\sigma_v}\) to all validators (including itself).

- **Round Timer Expiry** (line 68). Expiration of the round timer started by each validator at the beginning of every round (see line 20). The length of the round time is exponential to the round number. In this case \(v\) moves to round \(r' = r_{h,v} + 1\).
- **Pre-prepare message not matching locked block** (lines 58 to 59). Reception of a Pre-prepare message sent by the selected proposer for round \(r_{h,v}\) with proposer block not matching the block on which validator \(v\) is \textit{locked}. In this case \(v\) moves to round \(r' = r_{h,v} + 1\).
- **Reception of \(f(n) + 1\) Round-Change messages for future round** (line 64). Reception of \(f(n) + 1\) Round-Change messages for height \(h\) and round \(r'\) with \(r' > r_{h,v}\). In this case \(v\) moves to round \(r'\).
- **Failure in creating the finalisation proof** (lines 63 to 64). \(v\) has received at least \(Quorum(n_{h,v})\) Commit messages for height \(h\), current round and Keccak hash \(KEC(B)\) and at least one of the following conditions is verified:
  - at least one of the commit seals included in the Commit messages received by a validator for round \(r_{h,v}\) and height \(h\) is of the wrong size;
  - block \(B\) included in the accepted Pre-prepare message is not formatted correctly and does not allow adding the finalisation proof to it.

If this condition is verified, then \(v\) also \textit{unlocks} before moving to the next round (see line 61).

When validator \(v\) moves to a new round \(r'\), if and only if it is not \textit{locked} on any block, then it resets the state variable \texttt{acceptedPrePrepare}_{h,v} to \(\bot\) (see line 53). As it can be noted from the pseudocode, moving to a new round (line 6) does not imply starting a new round (line 15). As specified by the \texttt{upon} block at line 74, when a validator \(v\) receives either \(Quorum(n_{h,v})\) Round-Change messages for height \(h\) and round \(r'\) with \(r' > r_{h,v}\) or \(Quorum(n_{h,v})\) Round-Change messages for height \(h\) and round \(r'\) matching the current round (i.e. \(r' = r_{h,v}\)) but the current round has yet to be started, then \(v\) starts round \(r'\).

Starting round \(r'\) includes executing the following actions:

- \(v\) starts the round timer for round \(r'\) with length \texttt{roundTimerTimeout}(\(r'\));
if \( v \) is the proposer for round \( r' \), then \( v \) multicasts the following Pre-prepare message to all validators:

\[
\langle \text{PRE-PREP ARE}, h, r', B', \sigma_{p_r} \rangle
\]

where, if \( p_r \) is locked on block \( B \) then \( B' = B \), otherwise \( B' \) can be any valid block for height \( h \).

From here on the protocol proceeds as described above.

## 4 Definitions

In this section we provide a series of definitions that will be used in the presentation of our persistence and liveness analysis in Sections 5 and 6.

We define \( t \)-Byzantine-fault-tolerant persistence as follows:

**Definition 2** (\( t \)-Byzantine-fault-tolerant persistence). The IBFT protocol ensures \( t \)-Byzantine-fault-tolerant persistence if and only if the following statement is true: provided that no more than \( t \) validators are Byzantine, the IBFT protocol guarantees the persistence property of distributed permissioned transactions ledgers (see Definition 1).

The aim of the \( h \)-th instance of the IBFT-block-finalisation-protocol is to have all honest validators to eventually decide on the block to be included in the finalised block with height \( h \) and broadcast the finalised block to all the nodes.

In the context of the IBFT-block-finalisation-protocol we define safety as follows:

**Definition 3** (\( t \)-Byzantine-fault-tolerant safety for the IBFT-block-finalisation-protocol). The IBFT-block-finalisation-protocol ensures \( t \)-Byzantine-fault-tolerant safety if and only if it guarantees the validity of the following statement: in the presence of no more than \( t \) Byzantine validators, the protocol ensures that no two valid finalised blocks including different blocks for the same height can ever be produced.

In relation to the safety property of the IBFT-block-finalisation-protocol, we define Byzantine-fault-tolerant safety threshold as follows:

**Definition 4** (Byzantine-fault-tolerant safety threshold). Byzantine-fault-tolerant safety threshold for a protocol that guarantees \( t \)-Byzantine-fault-tolerant safety is defined as the maximum number of Byzantine nodes that the protocol can withstand while ensuring safety, i.e. \( t \).

As proved in Dwork et al. [6], for a network of \( n \) nodes, the maximum Byzantine-fault-tolerant safety threshold for any consensus protocol operating in an eventually synchronous network corresponds to \( f(n) \equiv \left\lfloor \frac{n}{3} \right\rfloor - 1 \) (as defined in Section 2). Our following definitions of optimal Byzantine-fault-tolerant safety and persistence follow directly from this known lower limit:

**Definition 5** (Optimal Byzantine-fault-tolerant safety threshold for the IBFT-block-finalisation-protocol). The IBFT-block-finalisation-protocol guarantees optimal Byzantine-fault-tolerant safety threshold provided that for any instance \( h \) its Byzantine-fault-tolerant safety threshold corresponds to \( f(n_h) \) where \( n_h \) is the number of validators for the \( h \)-th instance of IBFT-block-finalisation-protocol.

**Definition 6** (Optimal Byzantine-fault-tolerant persistence threshold for the IBFT protocol). The IBFT protocol guarantees optimal Byzantine-fault-tolerant persistence threshold provided that any instance of the IBFT-block-finalisation-protocol guarantees optimal Byzantine-fault-tolerant safety threshold.

The following two definitions are related to the liveness property of the IBFT protocol.

**Definition 7** (\( t \)-Byzantine-fault-tolerant liveness). The IBFT protocol ensures \( t \)-Byzantine-fault-tolerant liveness if and only if the following statement is true: provided that no more than \( t \) validators are Byzantine, the IBFT protocol guarantees the liveness property of distributed permissioned transactions ledgers (see Definition 4).

**Definition 8** (\( t \)-Byzantine-fault-tolerant weak-liveness of the IBFT-block-finalisation-protocol). The IBFT-block-finalisation-protocol guarantees \( t \)-Byzantine-fault-tolerant weak-liveness if and only if, provided that no more than \( t \) validators are Byzantine, it guarantees that for any \( h \) instance of the IBFT-block-finalisation-protocol at least one honest validator will eventually be able to produce a valid finalised block for height \( h \).
5 Persistence Analysis

In this section we analyse the persistence property of the IBFT protocol that in conjunction with the liveness property determines that overall robustness of the IBFT protocol as defined in Definition 1.

**Lemma 1.** If the IBFT-block-finalisation-protocol does not guarantee $t$-Byzantine-fault-tolerant safety, then the IBFT protocol does not guarantee $t$-Byzantine-fault-tolerant persistence.

**Proof.** Assume that $v$ and $v'$ are two honest nodes such that the height of their local blockchain is $h$. According to the Lemma, in presence of $t$ Byzantine validators for height $h$, the IBFT-block-finalisation-protocol can produce two different valid finalised blocks including blocks $B$ and $B'$ respectively, with $B \neq B'$. Assume that transactions $T$ and $T'$, with $T \neq T'$, are included at position $i$ of blocks $B$ and $B'$ respectively. This is possible as $B \neq B'$. If this happens before GST, then $v$ could receive the finalised block including $B$ while $v'$ could receive the finalised block including $B'$. Since both finalised block are valid, $v$ adds the finalised block including $B$ in position $h$ of its local blockchain while $v'$ adds the finalisation block including $B'$ in position $h$ of its local blockchain. This equates to $T$ being added in position $(h, i)$ of the local ledger of $v$ and $T'$ being added in the same position $(h, i)$ of the local ledger of $v'$, with $T \neq T'$.

**Lemma 2.** A transaction $T$ cannot appear in two different positions of the local blockchain of an honest node.

**Proof.** An honest node, before adding a finalised block to its local blockchain, checks the block included in the finalised block for validity (line 6 in Algorithm 1). As detailed in Section 4 block validity, as far as transactions are concerned, is verified as specified in the Ethereum Yellow Paper [17]. Equation (58) of [17] mandates that a transaction is valid only if the transaction nonce matches the expected nonce of the transaction sender. Equation (61) of [17] states that the expected nonce of the transaction sender is incremented when a transaction is executed. Thus, the same transaction $T$ cannot be included more than once in the same block and cannot be included in two different blocks of the same chain.

**Lemma 3.** If the IBFT-block-finalisation-protocol guarantees $t$-Byzantine-fault-tolerant safety, then the longest common prefix of the local blockchains of any two honest nodes is always identical.

**Proof.** The proof is by induction on the length $h$ of the longest common prefix between any two honest nodes. When $h = 0$ the Lemma is verified as we assume that any two honest nodes have the same genesis block.

For the inductive step, assume that the Lemma holds for $h'$. We show that then the Lemma also holds for $h = h' + 1$. Say that $v$ and $v'$ are two honest nodes such that the common prefix of their local blockchains is of length $h'$. The proof is now by contradiction. Let us assume that $v$ adds finalised block $FB$ including block $B$ in position $h$ of its local blockchain and $v'$ adds finalised block $FB'$ including block $B'$ in position $h$ of its local blockchain with $B \neq B'$. Since honest nodes only add valid finalised blocks to their local blockchain, then this means that both $FB$ and $FB'$ are valid finalised blocks for height $h$ including different block. This is in contradiction to the definition of $t$-Byzantine-fault-tolerant safety of the IBFT-block-finalisation-protocol (see Definition 3).

**Lemma 4.** If the IBFT-block-finalisation-protocol guarantees $t$-Byzantine-fault-tolerant safety, then the IBFT protocol guarantees condition (i) of $t$-Byzantine-fault-tolerant persistence.

**Proof.** Obvious from Lemmas 2 and 3.

**Lemma 5.** $n - 1 > f(n)$ for any $n \geq 2$

**Proof.**

\[
\begin{align*}
  f(n) &\equiv \left\lfloor \frac{n - 1}{3} \right\rfloor \\
  &\leq \frac{n - 1}{3}
\end{align*}
\]

It is easy to see the $\frac{n - 1}{3} < n - 1$ for any $n \geq 2$. 

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Lemma 6. The IBFT protocol does not guarantee condition (ii) of $t$-Byzantine-fault-tolerant persistence even if the IBFT-block-finalisation-protocol guarantees $t$-Byzantine-fault-tolerant safety and $t$-Byzantine-fault-tolerant weak-liveness.

Proof. The proof is by counterexample. Indeed, we assume that the IBFT-block-finalisation-protocol guarantees both $t$-Byzantine-fault-tolerant safety and $t$-Byzantine-fault-tolerant weak-liveness and show a sequence of events that, if occurring before GST, lead the IBFT protocol to a state where condition (ii) of the persistence property is violated. We denote the number of validators for the $h$-th instance of the IBFT-block-finalisation-protocol with $n_h$. Considering that no Byzantine-fault-tolerant safety or weak-liveness can be ensured if $t = 0$, we assume $t \geq 1$. Since $f(n_h)$ is the upper limit for $t$, this implies $f(n_h) \geq 1$, which in turns implies $n_h \geq 4$. We also assume that all validators are running the $h$-th instance of the IBFT-block-finalisation-protocol.

1. Honest validator $v$ produces a finalised block $FB$, containing block $B$, for height $h$. This will eventually happen as we assume that the IBFT-block-finalisation-protocol guarantees $t$-Byzantine-fault-tolerant weak-liveness.

2. However, only validator $v$ receives its own produced $FB$, while all other validators will never receive it. This is possible as GST has yet to be reached. As consequence of this, $v$ terminates its $h$-th instance of the IBFT-block-finalisation-protocol and starts the $h+1$-th instance, while all other nodes are still running the $h$-th instance of the IBFT-block-finalisation-protocol.

3. $t$ of the $n-1$ validators still running instance $h$ of the IBFT-block-finalisation-protocol stop communicating and never restart. This is possible as we assume that up to $t$ validators can be Byzantine and act arbitrarily. As consequence of this, $t+1$ of the $n_h$ validator do not participate any more in the $h$-th instance of the IBFT-block-finalisation-protocol as $t$ validators stopped and one, $v$, stopped the $h$-th instance of its IBFT-block-finalisation-protocol and started instance $h+1$. Since $t+1 > t$, weak-liveness of the IBFT-block-finalisation-protocol does not hold any more and therefore there is no guarantee that any new finalised block in instance $h$ will be ever produced. Also, only one validator is operating in instance $h+1$ which equates to $n_{h+1}-1$ validators behaving like if they stopped. According to Lemma 5, $n_{h+1}-1 > f(n_{h+1})$ if $n_{h+1} \geq 2$. Considering that $f(n_h)$ is the upper bound for $t$ (i.e $t \leq f(n_h)$), this implies that weak-liveness of the IBFT-block-finalisation-protocol does not hold for the instance $h+1$ either and therefore there is no guarantee that any new finalised block in instance $h+1$ will be ever produced.

In the system state resulting from the last event of the sequence of events presented above there is no guarantee that any new finalised block for height $h$ will ever be broadcast. Considering that we assume that finalised block $FB$ produced by $v$ will never be received, there is no guarantee that any of the transactions included in block $B$ (which is included in finalised block $FB$) will ever be added to the transaction ledger of any other honest node.

5.1 Modification IBFT-M1: Ensure condition (ii) of the Persistence property

In this section we describe a modification to the IBFT protocol to ensure condition (ii) of the persistence property (see Definition 1) when the IBFT-block-finalisation-protocol guarantees $t$-Byzantine-fault-tolerant safety and weak-liveness. We denote the protocol resulting from this modification as IBFT-M1.

The IBFT-M1 protocol is obtained by requiring the following addition to the protocol used by IBFT to transmit finalised blocks over the $\mathcal{P}_2p$ protocol: nodes query their peers about the availability of new finalised blocks on a regular basis.

5.1.1 Persistence Analysis of IBFT-block-finalisation-protocol-M1

Lemma 7. The IBFT-M1 protocol guarantees condition (ii) of $t$-Byzantine-fault-tolerant persistence.

Proof. If transaction $T$ is added to the local transaction ledger of an honest validator $v$, it implies that a finalised block, say $FB$, including a block, say $B$, containing $T$ is added to the local blockchain of $v$. Finalised blocks are transmitted using the block transmission protocol of IBFT-M1 by which nodes continuously ask their peers for the availability new finalised blocks. Hence, once GST is reached, the
finalised block $FB$ added by $v$ will be eventually transmitted to $v$’s peers that will, in turn, eventually transmit it to their peers as well. In this way, finalised block $FB$, which includes transaction $T$ as part of block $B$, will be eventually transmitted and added to the local blockchain of any honest node. This equates to transaction $T$ being added to the local transaction ledger of any honest node. This proves condition (ii) of the definition of $t$-Byzantine-fault-tolerant persistence.

**Theorem 1.** The IBFT-M1 protocol guarantees $t$-Byzantine-fault-tolerant persistence if and only if the IBFT-block-finalisation-protocol protocol guarantees $t$-Byzantine-fault-tolerant safety.

**Proof.** Direct consequence of Lemmas 1, 4 and 7.

### 5.1.2 Safety Analysis of the IBFT-block-finalisation-protocol

In this section we prove that the IBFT-block-finalisation-protocol as described in Algorithm 2 and Section 3.1 is not Byzantine-fault-tolerant when operating in an eventually synchronous network. According to Theorem 1 this implies that the IBFT-M1 protocol, when operating in an eventually synchronous setting, is not Byzantine-fault-tolerant either.

When analysing the generic $h$-th instance of IBFT-block-finalisation-protocol, we use the inductive assumption of Lemma 3 which states that the local blockchains of all honest nodes are identical until finalised block with height $h - 1$. Therefore, since the set of validators for the $h$-th instance of the IBFT-block-finalisation-protocol-M1 is a function of the local blockchain until the block with height $h - 1$, this set is identical amongst all honest validators. We denote the total number of validators for the $h$-th instance of the IBFT-block-finalisation-protocol-M1 with $n_h$.

**Lemma 8.** $n - 1 \geq \text{Quorum}(n)$ for any $n \geq 4$.

**Proof.** The Lemma is proved by the following chain of inequalities and equalities:

$$
\text{Quorum}(n) \equiv 2 \cdot \left\lceil \frac{n - 1}{3} \right\rceil + 1
\leq 2 \cdot \frac{n - 1}{3} + 1
= \frac{2n + 1}{3}
$$

It is easy to prove that $\frac{2n + 1}{3} \leq n - 1$ for any $n \geq 4$.

**Lemma 9.** The IBFT finalisation protocol is not Byzantine-fault-tolerant when operating in an eventually synchronous network.

**Proof.** The proof is by counterexample. Below we provide a possible sequence of events that may occur before GST which leads two honest nodes to create two valid finalised blocks for the same height $h$ that include two different blocks. We assume $f(n_h) \geq 1$ which implies $n_h \geq 4$. Let $v$ be an honest validator, $W$ be a set of size $n_h - 1$ including all validators except $v$ and $W_{\text{honest}}$ be the subset of all honest validators included in $W$.

1. Let $r$ be any round such that no honest validator has locked on any block. This is always the case for $r = 0$. The proposer for round $r$, $p_r$, multicasts a $\langle \text{PRE-PREPARE}, h, r, B \rangle_{\sigma_p}$ message to all validators (including itself).
2. All validators (both Byzantine and honest) receive the Pre-prepare message sent by the proposer and multicast the corresponding $\langle \text{PREPARE}, h, r, \text{KEC}(B) \rangle$ message to all validators (including themselves).
3. All honest validators ($W_{\text{honest}} \cup \{v\}$) receive the $n_h$ Prepare messages sent by all validators. Since $n_h \geq \text{Quorum}(n_h)$, all honest validators lock on block $B$ and multicast a $\langle \text{COMMIT}, h, r, \text{KEC}(B), \text{CS}(B, \text{sender}) \rangle_{\sigma_{\text{sender}}}$ message to all other validators.
4. Byzantine validators also send a well-formed $\langle \text{COMMIT}, h, r, \text{KEC}(B), \text{CS}(B, \text{sender}) \rangle_{\sigma_{\text{sender}}}$ message to honest validator $v$.
5. However, Byzantine validators send Commit messages containing a commit seal of the wrong size to all honest validators included in set $W_{\text{honest}}$. 
6. As result of events 3 to 5, each of the $n_h$ validators (whether Byzantine or honest) has sent a Commit message to all the honest validators, $v$ will receive only well-formed Commit messages, whereas the Commit messages sent by Byzantine validators that the validators in $W_{\text{honest}}$ will receive will include a commit seal of the wrong size. Assume, without loss of generality, that at least one of the Commit messages with the commit seal of the wrong size is included in the first $\text{Quorum}(n_h)$ Commit messages received by each of the honest validators included in set $W_{\text{honest}}$. Therefore, once each validator included in $W_{\text{honest}}$ receives $\text{Quorum}(n_h)$ Commit messages, it unlocks and sends a $\langle \text{ROUND-CHANGE}, h, r' = r + 1 \rangle$ message to all validators (including itself).

In contrast, $v$ only receives Commit messages with valid commit seals and therefore it creates a valid finalised block including block $B$ and broadcasts it to all the nodes. However, we assume that the finalised block created by $v$ will not be received by any validator before the end of the last event of this sequence of events. This is possible as GST has yet to be reached.

7. Like validators in $W_{\text{honest}}$, all Byzantine validators in $W_{\text{honest}}$ also send a $\langle \text{ROUND-CHANGE}, h, r' \rangle$ message to all validators.

8. All validators in $W_{\text{honest}}$ receive all of the Round-Change messages for round $r'$ sent by all validators in $W$, both Byzantine and honest. According to Lemma 8, since $n_h \geq 4$, the following holds $n_h - 1 \geq \text{Quorum}(n_h)$. Therefore all validators in $W_{\text{honest}}$ start round 1.

9. Assume, without loss of generality, that the proposer of round $r' = r + 1$, $p_{r'}$, is not $v$. $p_{r'}$ multicasts a $\langle \text{PRE-PREPARE}, h, r', B' \rangle_{\sigma_{p_{r'}}}$ message with $B' \neq B$ to all validators. This is possible as $p_{r'}$ is either Byzantine or honest but included in set $W_{\text{honest}}$ and has therefore unlocked at event 6.

10. All validators in $W$ receive the Pre-prepare message sent by $p_{r'}$ and therefore broadcast a $\langle \text{PREPARE}, h, r', \text{KEC}(B') \rangle$ message to all validators (including themselves).

11. All honest validators included in $W_{\text{honest}}$ receive the $n_h - 1$ Prepare messages sent by all validators included in set $W$. According to Lemma 8 since $n_h \geq 4$, the following holds $n_h - 1 \geq \text{Quorum}(n_h)$. Therefore all honest validators included in set $W_{\text{honest}}$ lock on block $B'$ and multicast a $\langle \text{COMMIT}, h, r', \text{KEC}(B'), \text{CS}(B', \text{sender}) \rangle_{\sigma_{\text{sender}}}$ message to all other validators. Byzantine validators also multicast the same Commit message. All Commit messages sent for this round include a valid commit seal.

12. All honest validators included in set $W_{\text{honest}}$ receive the $n_h - 1$ Commit messages sent by all validators included in set $W$. According to Lemma 8 since $n_h \geq 4$, the following holds $n_h - 1 \geq \text{Quorum}(n_h)$. Therefore, all honest validators in set $W_{\text{honest}}$ create a valid finalised block for block $B'$.

This concludes the proof as two valid finalised blocks including different blocks ($B$ and $B'$) have been created at events 6 and 12.

**Theorem 2.** The IBFT-M1 protocol (as well as the IBFT protocol) does not guarantee Byzantine-fault-tolerant persistence when operating in an eventually synchronous network.

**Proof.** Direct consequence of Lemma 9 and Theorem 1.
5.2 Modification IBFT-block-finalisation-protocol-M1: Ensure Byzantine-fault-tolerance safety

Algorithm 3: $h$-th instance of the IBFT-block-finalisation-protocol-M1 for validator $v$.

1: Macro Expansions:
2: $n_{h,v}$ expands to: $n \left( \text{chain}_v[0 : h - 1] \right)$
3: validators$_{h,v}$ expands to: validators$\left( \text{chain}_v[0 : h - 1] \right)$
4: proposer$_{h,v}(r_{h,v})$ expands to: proposer$\left( \text{chain}_v[0 : h - 1], r_{h,v} \right)$

5: Initialisation:
6: $\text{lock}_{h,v} \leftarrow \bot$
7: $\text{StartNewRound}(0, h, v)$

8: Procedures:
9: def $\text{moveToNewRound}(r, h, v)$:
10: $r_{h,v} \leftarrow r$
11: roundAlreadyStarted$_{h,v} \leftarrow false$
12: if $\text{lock}_{h,v} = \bot$ then
13: acceptedPrePrepare$_{h,v} \leftarrow \bot$
14: end
15: def $\text{StartNewRound}(r, h, v)$:
16: $\text{moveToNewRound}(r, h, v)$
17: roundAlreadyStarted$_{h,v} \leftarrow true$
18: commitSent$_{h,v} \leftarrow false$
19: finalisedBlockSent$_{h,v} \leftarrow false$
20: set roundTimer$_{h,v}$[r$_{h,v}$] to expire after roundTimerTimeout(r$_{h,v}$)
21: if $v = \text{proposer}_{h,v}(r_{h,v})$ then
22: if $\text{lock}_{h,v} \neq \bot$ then
23: $B \leftarrow \text{lock}_{h,v}$
24: else
25: $B \leftarrow \text{createNewProposedBlock}(h, v)$
26: end
27: multicast $\langle \text{PRE-PREPARE}, h, r_{h,v}, B \rangle_{\sigma_v}$ to validators$_{h,v}$
28: end
29: def $\text{moveToNewRoundAndSendRoundChange}(r, h, v)$:
30: $\text{moveToNewRound}(r, h, v)$
31: multicast $\langle \text{ROUND-CHANGE}, h, r_{h,v} \rangle_{\sigma_v}$ to validators$_{h,v}$
32: Upon Blocks:
33: upon received($\langle \text{PRE-PREPARE}, h, r_{h,v}, B \rangle$ from proposer$_{h,v}(r_{h,v}), v$) do
34: if acceptedPrePrepare$_{h,v} = \bot$ then
35: if (lock$_{h,v} = \bot$ or lock$_{h,v} = B$) and isValidBlock(B, chain$_v[h - 1]$) then
36: acceptedPrePrepare$_{h,v} \leftarrow B$
37: multicast $\langle \text{PREPARE}, h, r_{h,v}, \text{KEC}(B) \rangle_{\sigma_v}$ to validators$_{h,v}$
38: else
39: $\text{moveToNewRoundAndSendRoundChange}(r_{h,v} + 1, h, v)$
40: end
41: end
42: upon numOfReceived($\langle \text{PREPARE}, h, r_{h,v}, \text{KEC}(B) \rangle$ from validators$_{h,v}, v$) $\geq$ Quorum($n_{h,v}$) and
43: commitSent$_{h,v} = false$ do
44: if acceptedPrePrepare$_{h,v} = B$ then
45: $\text{lock}_{h,v} \leftarrow B$
46: end
In this section we describe the first modification to the IBFT-block-finalisation-protocol to ensure Byzantine-fault-tolerant safety. We denote the IBFT-block-finalisation-protocol produced by this modification as IBFT-M1-IBFT-block-finalisation-protocol-M1 and denote the overall IBFT-M1 protocol resulting by replacing the IBFT-block-finalisation-protocol with the IBFT-block-finalisation-protocol-M1 as IBFT-M1-IBFT-block-finalisation-protocol-M1. As shown in Section 5.2.1 while IBFT-block-finalisation-protocol-M1 enhances the IBFT-block-finalisation-protocol by providing Byzantine-fault-tolerant safety, it fails to guarantee optimal Byzantine-fault-tolerant safety.

The IBFT-block-finalisation-protocol-M1, specified by the pseudocode in Algorithm 3 is obtained by applying the following list of modifications to the IBFT-block-finalisation-protocol:

1. Only consider Commit messages that have commit seal signed by the sender of the Commit message. Compare line 51 of Algorithm 3 with line 51 of Algorithm 2. Since we assume a digital signature scheme that ensures signature uniqueness, verifying the signature implies verifying that the size of the commit seal is correct.
2. Embed the verification that the block included in the accepted Pre-prepare message allows adding a finalisation proof to it in the series of verifications performed by the isValidBlock() function and, therefore, remove the wellFormedToAddFinalisationProof(B) check from the list of verifications listed at line 54 of Algorithm 2. This modification is not required to ensure Byzantine-fault-tolerant safety but to ensure that Byzantine validators can not stall the IBFT-block-finalisation-protocol by sending Pre-prepare messages including a block that does not allow adding a finalisation proof to it.
3. Remove the Failure in creating the finalisation proof condition from the list of conditions that trigger sending Round-Change messages. This corresponds to the condition “all of the
\texttt{(COMMIT, }h, r_{h,v}, \text{KEC}(B), *) \texttt{ received from validators}_{h,v} \texttt{ contain a commit seal of correct size}” at line [44] and block lines [59] to [61] of Algorithm 2. This means that this condition is never checked and therefore no validator can unlock as consequence of this condition being true.

5.2.1 Safety Analysis of the IBFT-block-finalisation-protocol-M1

In this section we analyse the safety property of the IBFT-block-finalisation-protocol-M1 and show that the IBFT-block-finalisation-protocol-M1 guarantees Byzantine-fault-tolerant safety, but that it does not guarantee optimal Byzantine-fault-tolerant safety threshold. We denote the Byzantine-fault-tolerant safety threshold of the IBFT-block-finalisation-protocol-M1 with $f_{\text{IBFT-M1}}(n)$, where $n$ is the total number of validators participating in the protocol. We first derive the definition of $f_{\text{IBFT-M1}}(n)$ and then we show that there exist values of $n$ such that $f_{\text{IBFT-M1}}(n) < f(n)$.

Similarly to Section 5.1.2, when we analyse the generic $h$-th instance of IBFT-block-finalisation-protocol-M1, we assume that the local blockchains of all honest nodes are identical until finalised block with height $h - 1$. Therefore, since the set of validators for the $h$-th instance of the IBFT-block-finalisation-protocol-M1 is a function of the local blockchain until block with height $h - 1$, this set is identical amongst all honest validators. We denote the total number of validators for the $h$-th instance of the IBFT-block-finalisation-protocol-M1 with $n_h$.

Lemma 10. For any upon or initialisation event $e$, if an honest validator $v$ locks on block $B$ during the execution of the event $e$, then $\text{acceptedPrePrepare}_{h,v} = B$ was true when the execution of the event $e$ started.

Proof. $v$ can only lock on block $B$ at lines [44] and [53]. Both of these lines can only be executed if $\text{acceptedPrePrepare}_{h,v} = B$ is true at the beginning of the execution of each respective upon block. This proves the Lemma. \hfill \Box

Lemma 11. For any upon or initialisation event $e$, if an honest validator $v$ locks on block $B$ during the execution of the event $e$ then $\text{acceptedPrePrepare}_{h,v} = B$ is true when the execution of the event ends.

Proof. $v$ can only lock on block $B$ at lines [44] and [53]. None of the upon blocks including these lines modify the value of $\text{acceptedPrePrepare}_{h,v}$. This, together with Lemma 10 implies the Lemma. \hfill \Box

Lemma 12. For any upon or initialisation event $e$, if an honest node $v$ is locked on a block $B$ at the beginning of the execution of $e$, then either $\text{acceptedPrePrepare}_{h,v}$ is not changed during the execution of $e$ or $\text{acceptedPrePrepare}_{h,v}$ is set to $B$.

Proof. $\text{acceptedPrePrepare}_{h,v}$ is set only at lines [13] and [39]. Line [13] is executed only if $v$ is not locked on any block. Line [39] is executed only if $v$ is either not locked or locked on block $B$. \hfill \Box

Lemma 13. If an honest validator $v$ is locked on block $B$ while in round $r$ of the $h$-th instance of the IBFT-block-finalisation-protocol-M1, then $v$ will be locked on block $B$ for any round $r' \geq r$ of the same instance $h$.

Proof. Let $e_i$ denote the temporally ordered sequence of upon events occurring during the $h$-th instance of the IBFT-block-finalisation-protocol-M1 such that no event is ever executed between the time that event $e_i$ and event $e_{i+1}$ are executed. Therefore, the value of any state variable at the beginning of event $e_{i+1}$ corresponds to the value that that state variable had at the end of event $e_i$. We let $e_0$ correspond to the initialisation event of the $h$-th instance of the IBFT-block-finalisation-protocol-M1. We prove the Lemma by proving the following conditions

(a) If an honest validator $v$ locks on block $B$ during the event $e_j$ of the $h$-th instance of the IBFT-block-finalisation-protocol-M1, then $v$ will be locked on block $B$ at the end of the execution of any event $e_k$ with $k \geq j$ for the same instance $h$ of the IBFT-block-finalisation-protocol-M1;
(b) If an honest validator \(v\) locks on block \(B\) during the event \(e_j\) of the \(h\)-th instance of the IBFT-block-finalisation-protocol-M1, then \(\text{acceptedPrePrepare}_h,v = B\) is true at the end of the execution of any event \(e_k\) with \(k \geq j\) for the same instance \(h\) of the IBFT-block-finalisation-protocol-M1.

Since \(v\) is not locked on any block at the initialisation of any instance \(h\) and the round number \(r_{h,v}\) increments monotonically with the time, condition (a) implies the Lemma.

The proof of conditions (a) and (b) is by induction on \(k\). For the base case we consider \(k = j\). Lemma \([11]\) implies the proof for the base case for (b) while the fact that upon events are executed atomically is sufficient to imply the proof for the base case of (a).

For the inductive step, we assume that conditions (a) and (b) are verified for an event \(e_{k'}\) and then we show that they are also verified for event \(e_k\) with \(k = k' + 1\). The proof of both conditions is by contradiction.

For the proof of condition (a), let us consider two cases: (1) during event \(e_k\), \(v\) locks on block \(B'\) with \(B' \neq B\) and (2) during event \(e_k\), \(v\) unlocks.

Case (1): during event \(e_k\), \(v\) locks on block \(B'\) with \(B' \neq B\). According to Lemma \([10]\) this implies that \(\text{acceptedPrePrepare}_h,v = B'\) when event \(e_k\) starts executing. This, in turn, implies that \(\text{acceptedPrePrepare}_h,v = B'\) after the execution of event \(e_k\) but this is in contradiction with the inductive assumption that condition (b) holds for \(e_{k'}\).

Case (2): during event \(e_k\), \(v\) unlocks. This is not possible as \(v\) can only unlock during the initialisation.

For the proof of condition (b), let us consider two cases: (1) after the execution of event \(e_k\), \(\text{acceptedPrePrepare}_h,v = B'\) with \(B' \neq B\) and (2) after the execution of event \(e_k\), \(\text{acceptedPrePrepare}_h,v = \bot\).

Case (1): after the execution of event \(e_k\), \(\text{acceptedPrePrepare}_h,v = B'\) with \(B' \neq B\). According to Lemma \([12]\) this implies that either \(\text{acceptedPrePrepare}_h,v = B'\) or \(v\) is locked on block \(B'\) at the beginning of the execution of event \(e_k\). This is in contradiction with the inductive assumptions on conditions (a) and (b) which imply that \(v\) is locked on block \(B\) and \(\text{acceptedPrePrepare}_h,v = B\) at the beginning of the execution of event \(e_k\).

Case (2): after the execution of event \(e_k\), \(\text{acceptedPrePrepare}_h,v = \bot\). The only line where \(\text{acceptedPrePrepare}_h,v\) is set to \(\bot\) is line \([13]\). This line can be executed only if \(v\) is not locked on any block (see line \([14]\)). Therefore, if case (2) is true, then \(v\) is not locked on any block at the beginning of event \(e_k\). This is in contradiction with the inductive assumption on conditions (a) which implies that \(v\) is locked on block \(B\) at the beginning of the execution of event \(e_k\).

\[\Box\]

Lemma 14. If an honest validator \(v\) sends a Commit message for block \(B\) (i.e. \(\langle \text{COMMIT}, h, r, \text{KEC}(B), \text{CS}(B,v)_{\sigma_v} \rangle\)), then validator \(v\) is locked on \(B\).

Proof. The only line of the pseudocode of Algorithm \([8]\) where a Commit message is sent is line \([11]\). This line is executed only if condition at line \([14]\) is verified, i.e. validator \(v\) is locked on block \(B\).

Lemma 15. If the intersection of any two sets of Quorum(n) validators, in a set of \(n\) validators where there are no more than \(j_{\text{IBFT-protocol-M1}}(n)\) Byzantine validators, is guaranteed to include at least one honest validator, then the following statement is verified: no two valid finalised blocks for the same height \(h\) including different blocks can be produced by the IBFT-block-finalisation-protocol-M1.

Proof. A block for height \(h\) is considered finalised if it contains a valid finalisation proof. A finalisation proof is valid if and only if it contains at least Quorum(n) commit seals signed by different validators in \(\text{validators}_{h,v}\). The proof is by contradiction. Assume that (a) there exist two finalised blocks \(FB\) and \(FB'\) including block \(B\) and \(B'\) respectively, with \(B \neq B'\). Since the intersection of any two sets of Quorum(n) validators is guaranteed to include an honest validator and we assume unforgeability of digital signature, assumption (a) implies that there exists one honest validator \(v\) that produced both a commit seal for block \(B\) and commit seal for block \(B'\). This, in turn, implies that validator \(v\) sent a Commit message with block hash matching \(\text{KEC}(B)\) and a Commit message with block hash matching \(\text{KEC}(B')\) as commit seals are only included in Commit messages. However, according to Lemma \([11]\) honest validators only send Commit message for the block that they are locked on. Therefore, since we
assume that $\text{KEC}(\cdot)$ is a collision-resistant hash function (i.e $B \neq B' \rightarrow \text{KEC}(B) \neq \text{KEC}(B')$ w.h.p), for assumption (a) to be satisfied $v$ must have locked on both block $B$ and $B'$. This is in clear contradiction with Lemma \[\text{13}\]

\textbf{Lemma 16.} If the intersection of any two sets of Quorum$(n)$ validators, in a set of $n$ validators where there are no more than $f_{\text{IBFT-M1}}(n)$ Byzantine validators, is not guaranteed to include at least one honest validator, then the IBFT protocol finalisation protocol does not guarantee Byzantine-fault-tolerant safety.

\textbf{Proof.} The proof is by counterexample. Let $V$ and $W$ be two sets of size Quorum$(n_h)$ that do not intersect in any honest validator. Without loss of generality, we assume that the proposer $p_0$ for round 0 is in set $V$ and Byzantine. The following sequence of events leads to the honest validators in set $V$ to produce a valid finalised block for height $h$ including block $B$ and the honest validators in set $W$ to produce a valid finalised block for the same height $h$ including block $B'$, with $B \neq B'$.

1. Proposer $p_0$ sends a Pre-prepare message for block $B$ $\langle \text{PRE-PREPARE}, h, 0, B\rangle_{\sigma_{p_0}}$ to the honest validators in set $V$ and a Pre-prepare for block $B'$ $\langle \text{PRE-PREPARE}, h, 0, B'\rangle_{\sigma_{p_0}}$ to the honest validators in set $W$. $p_0$ can send Pre-prepare messages for different blocks as we assume that it is Byzantine.

2. Honest validators in set $V$ multicast a Prepare message for block $B$ $\langle \text{PREPARE}, h, 0, \text{KEC}(B)\rangle$ to all validators while honest validators in set $W$ multicast a Prepare message for block $B'$ $\langle \text{PREPARE}, h, 0, \text{KEC}(B')\rangle$ to all validators.

3. Byzantine validators in set $V$ (resp. $W$), regardless of whether they are also part of set $W$ (resp. $V$), send a Prepare message for block $B$ (resp. $B'$) to honest validators in set $V$ (resp. $W$). To be noted that since Byzantine validators can act arbitrarily, they can send both a Prepare message for block $B$ and a Prepare message for block $B'$.

4. The honest validators in set $V$ (resp. $W$) receive Quorum$(n_h)$ Prepare messages for block $B$ (resp. $B'$) in addition to the Pre-prepare message for block $B$ (resp. $B'$). Therefore the honest validators in set $V$ (resp. $W$) lock on block $B$ (resp. $B'$) and send a Commit message for block $B$ $\langle \text{COMMIT}, h, 0, \text{KEC}(B), \text{CS}(B, \text{sender})\rangle_{\sigma_{\text{sender}}}$ (resp. $B'$) where $\text{CS}(B, \text{sender})$ indicates the commit seals over $B$ signed by the sender of the Commit message.

5. Byzantine validators in set $V$ (resp. $W$), regardless of whether they are also part of set $W$ (resp. $V$), send a Commit message for block $B$ (resp. $B'$) to honest validators in set $V$ (resp. $W$). To be noted that since Byzantine validators can act arbitrarily, they can send both a Commit message for block $B$ and a Commit message for block $B'$.

6. The honest validators in set $V$ receives Quorum$(n_h)$ Commit messages for block $B$ and therefore they create a valid finalised block for height $h$ including block $B$. The honest validators in set $W$ receives Quorum$(n_h)$ Commit messages for block $B'$ and therefore they create a valid finalised block for height $h$ including block $B'$.

\textbf{Lemma 17.} The IBFT-block-finalisation-protocol guarantees safety if and only if the intersection of any two sets of Quorum$(n)$ validators, in a set of $n$ validators where there are no more than $f_{\text{IBFT-M1}}(n)$ Byzantine validators, is guaranteed to include at least one honest validator.

\textbf{Proof.} The forward direction of the Lemma is proved by Lemma \[\text{13}\] while the reverse direction is proved by Lemma \[\text{16}\]

\textbf{Lemma 18.} The intersection of any two sets of Quorum$(n)$ validators, in a set of $n$ validators, is guaranteed to include at least one validator if and only if $n \neq 2 \land n \neq 3 \land n \neq 6$.

\textbf{Proof.} Let us assume that $V$ and $W$ are any two sets of validators of size Quorum$(n)$. The minimum number of validators contained in the intersection of $V$ and $W$ is expressed by the following equation:

$$\min(||V \cap W||) = \max \left(2 \cdot \text{Quorum}(n) - n, 0\right)$$

$$= \max \left(2 \cdot (2 \cdot f(n) + 1) - n, 0\right)$$

(5)
For the Lemma to be verified, the following inequality must be verified as well:

\[
\max \left( 2 \cdot (2 \cdot f(n) + 1) - n, 0 \right) \geq 1 \to
\]

\[
2 \cdot (2 \cdot f(n) + 1) - n \geq 1 \to
\]

\[
4 \cdot f(n) - n + 1 \geq 0 \to
\]

\[
4 \cdot \left( \left\lceil \frac{n}{3} \right\rceil - 1 \right) - n + 1 \geq 0
\]

The following chain of inequalities and equalities proves that \( 4 \cdot \left( \left\lceil \frac{n}{3} \right\rceil - 1 \right) - n + 1 \geq \frac{n}{3} - 3 \).

\[
4 \cdot \left( \left\lceil \frac{n}{3} \right\rceil - 1 \right) - n + 1 \geq 4 \cdot \left( \frac{n}{3} - 1 \right) - n + 1
\]

\[
= \frac{n}{3} - 3
\]

It is easy to see that the \( \frac{n}{3} - 3 \geq 0 \) if and only if \( n \geq 9 \), so the Lemma is proved for \( n \geq 9 \).

The following table proves the Lemma for \( n < 9 \) by evaluating \( \max (2 \cdot \text{Quorum}(n) - n, 0) \geq 1 \) for each \( n < 9 \).

\[
\begin{array}{cccc}
\hline
n & f(n) & \text{Quorum}(n) & 2 \cdot \text{Quorum}(n) \\
\hline
1 & 0 & 1 & 2 \\
2 & 0 & 1 & 2 \\
3 & 0 & 1 & 2 \\
4 & 1 & 3 & 6 \\
5 & 1 & 3 & 6 \\
6 & 1 & 3 & 6 \\
7 & 2 & 5 & 10 \\
8 & 2 & 5 & 10 \\
\hline
\end{array}
\]

\[
\text{Lemma 19. The intersection of any two sets of Quorum}(n) \text{ validators, in a set of } n \text{ validators, is guaranteed to include at least one honest validator if and only if (i) } n \neq 2 \land n \neq 3 \land n \neq 6 \text{ and (ii) the number of Byzantine validators is } \leq (f(n) - ((n - 1) \mod 3)).
\]

\[\text{Proof. Each of the two conditions is proved separately below.}\]

\textbf{Condition (i)} For the intersection to include at least one honest validator, it is obviously required that the intersection contains at least one validator (either honest or Byzantine). Lemma \ref{lem:quorum} proves that this condition is verified if and only if \( n \neq 2 \land n \neq 3 \land n \neq 6 \).

\textbf{Condition (ii)} Let us assume that \( V \) and \( W \) are any two sets of validators of size \( \text{Quorum}(n) \). Let \( H \) be the set of honest validators included in the intersection of \( V \) and \( W \), and \( t \) be the number of Byzantine validators in the system. The minimum size of \( H \) is expressed by the following equation:

\[
\min(\|H\|) = \max \left( 2 \cdot (2 \cdot f(n) + 1) - n - t, 0 \right)
\]

(8)

For the set \( H \) to contain at least one honest validator, the following inequalities must hold:

\[
\max \left( 2 \cdot (2 \cdot f(n) + 1) - n - t, 0 \right) \geq 1 \to
\]

\[
2 \cdot (2 \cdot f(n) + 1) - n - t \geq 1 \to
\]

\[
t \leq \max (4 \cdot f(n) + 1 - n, 0)
\]

(9)

For the purpose of finalising this proof, we show that \( n \) can be expressed as \( 3 \cdot f(n) + (n \mod 3) + 1 \):

\[
f(n) = \left\lfloor \frac{n - 1}{3} \right\rfloor \to
\]

\[
n - 1 = 3 \cdot f(n) + ((n - 1) \mod 3) \to
\]

\[
n = 3 \cdot f(n) + ((n - 1) \mod 3) + 1
\]

(10)

(11)

By rewriting \( n \) as \( 3 \cdot f(n) + (n \mod 3) + 1 \) we get:

\[
t \leq \max (4 \cdot f(n) + 1 - 3 \cdot f(n) - ((n - 1) \mod 3) - 1, 0) \to
\]

\[
t \leq \max (f(n) - ((n - 1) \mod 3), 0)
\]

(12)
Table 1: Optimal Byzantine-fault-tolerance safety threshold \( f(n_h) \) vs the IBFT-block-finalisation-protocol-M1 Byzantine-fault-tolerance safety threshold \( f_{\text{IBFT-M1}}(n_h) \). “-” in the \( f_{\text{IBFT-M1}}(n_h) \) column for \( n_h = 6 \) indicates that no safety is provided in that case even if all of the validators are honest.

| \( n_h \) | \( f(n_h) \) | \( f_{\text{IBFT-M1}}(n_h) \) |
|----------|-------------|-----------------|
| 4        | 1           | 1               |
| 5        | 1           | 0               |
| 6        | 1           | -               |
| 7        | 2           | 2               |
| 8        | 2           | 1               |
| 9        | 2           | 0               |
| 10       | 3           | 3               |
| 11       | 3           | 2               |
| 12       | 3           | 1               |

Let us define condition (a) as \( f(n) - ((n - 1) \mod 3) \geq 0 \) for all \( n \) such that \( n \neq 2 \land n \neq 3 \land n \neq 6 \). It is easy to see that if (a) is verified, then the last inequality above proves condition (ii) of the Lemma.

Condition (a) is obviously verified when \( f(n) \geq 2 \) which is the case for any \( n \geq 7 \). It is then easy to verify the veracity of condition (a) for the missing cases, i.e. \( n = 1 \vee n = 4 \vee n = 5 \). This proves condition (a) and consequently condition (ii) of the Lemma.

Lemma 20. The IBFT-block-finalisation-protocol-M1 guarantees Byzantine-fault-tolerant safety if and only if (i) the total number \( n_h \) of validators for any instance \( h \) is \( n_h \neq 2 \land n_h \neq 3 \land n_h \neq 6 \) and (ii) the number of Byzantine validators \( \leq (f(n) - ((n_h - 1) \mod 3)) \).

Proof. The proof follows from Lemma 17 and Lemma 19.

Corollary 20.1. For any \( h \) instance of the IBFT-block-finalisation-protocol-M1, if the total number \( n_h \) of validators is \( n_h = 2 \vee n_h = 3 \vee n_h = 6 \), then the IBFT-block-finalisation-protocol-M1 does not guarantee safety even if no validators are Byzantine.

Proof. Assume that all validator of instance \( h \) of the IBFT-block-finalisation-protocol-M1 are honest. When condition \( n_h = 2 \vee n_h = 3 \vee n_h = 6 \) applies, then according to Lemma 17 there is no guarantee that the intersection of any two Quorum\( (n_h) \) set of validators contains a validator. When this is the case, i.e. there exist two sets of Quorum\( (n_h) \) validators such that their intersection is empty, then it is the easy to identify a sequence of events similar to the one described in Lemma 16 that leads two honest validators to produce two finalised blocks including two different blocks.

Corollary 20.2. \( f_{\text{IBFT-M1}}(n) \equiv f(n) - ((n - 1) \mod 3) \) for \( n \neq 2 \land n \neq 3 \land n \neq 6 \).

Proof. Obvious from the definition of \( f_{\text{IBFT-M1}}(n) \) and Lemma 20.

Corollary 20.3. The IBFT-block-finalisation-protocol-M1 does not guarantee optimal Byzantine-fault-tolerant safety threshold for any instance \( h \) such that \( n_h \geq 4 \) and \( ((n_h - 1) \mod 3) > 0 \). If \( n_h = 6 \), then no safety is guaranteed even if all of the validators are honest.

Proof. Obvious from the definition of optimal Byzantine-fault-tolerant safety threshold (Definition 5), Corollary 20.2, Corollary 20.1, and the fact that Byzantine-fault-tolerance can only be provided for \( n_h \geq 4 \).

Theorem 3. The IBFT-M1-IBFT-block-finalisation-protocol-M1 protocol does not guarantee optimal Byzantine-fault-tolerance persistence threshold.

Proof. Obvious from Definition 6 of optimal Byzantine-fault-tolerant persistence threshold and Corollary 20.3.
Table 1 shows a comparison between the optimal Byzantine-fault-tolerant safety threshold, i.e. $f(n_h)$, and the Byzantine-fault-tolerant safety threshold provided by the IBFT-block-finalisation-protocol-M1, i.e. $f_{IBFT-M1}(n_h)$.

5.3 Modification IBFT-block-finalisation-protocol-M2: Ensure optimal Byzantine-fault-tolerance safety

In this section we describe how the IBFT-block-finalisation-protocol-M1 can be modified to achieve optimal Byzantine-fault-tolerance safety. We denote the protocol obtained by applying such a modification as IBFT-block-finalisation-protocol-M2 and denote the overall IBFT-M1 protocol resulting by replacing the IBFT-block-finalisation-protocol with the IBFT-block-finalisation-protocol-M2 as $IBFT-M1-IBFT-block-finalisation-protocol-M2$.

Essentially, the IBFT-block-finalisation-protocol-M2 is obtained by replacing the Quorum($n$) function of the IBFT-block-finalisation-protocol-M1 with the Quorum$_{opt}$(n) function defined below.

Definition 9.

$$Quorum_{opt}(n) \equiv \left\lceil \frac{2n}{3} \right\rceil$$

5.3.1 Safety Analysis of the IBFT-block-finalisation-protocol-M2

In this section we analyse the safety property of the IBFT-block-finalisation-protocol-M2 and show that it provides optimal Byzantine-fault-tolerant safety.

Similarly to Sections 5.1.2 and 5.2.1, when we analyse the generic $h$-th instance of IBFT-block-finalisation-protocol-M1, we assume that the local blockchains of all honest nodes are identical until the finalised block with height $h - 1$. Therefore, since the set of validators for the $h$-th instance of the IBFT-block-finalisation-protocol-M2 is a function of the local blockchain until block with height $h - 1$, this set is identical amongst all honest validators. We denote the total number of validators for the $h$-th instance of the IBFT-block-finalisation-protocol-M1 with $n_h$.

Lemma 21. For any $n \geq 1$ with $n$ corresponding to the total number of validators, the intersection of any two sets of Quorum$_{opt}(n)$ validators is guaranteed to include an honest validator, provided that the number of Byzantine validators is $\leq f(n)$.

Proof. Let us assume that $V$ and $W$ are any two sets of validators of size Quorum$_{opt}(n)$. Let $H$ denote the set of honest validators included in the intersection of $V$ and $W$. The minimum size of $H$ is expressed by the following equation:

$$\min(\|H\|) = \max \left( 2 \cdot Quorum_{opt}(n) - n - f(n), 0 \right) \quad (13)$$

The following chain of equalities and inequalities proves that $\min(\|H\|) \geq \frac{1}{3}$:

$$\min(\|H\|) = 2 \cdot Quorum_{opt}(n) - n - f(n)$$

$$= 2 \left\lceil \frac{2n}{3} \right\rceil - \left\lceil \frac{n - 1}{3} \right\rceil - n$$

$$\geq 2 \cdot \frac{2n}{3} - \frac{n - 1}{3} - n$$

$$= \frac{1}{3}$$

Since $\min(\|H\|)$ is an integer value, the inequality $\min(\|H\|) \geq \frac{1}{3}$ implies that $\min(\|H\|) \geq 1$. □

Lemma 22. For any $n \geq 1$ with $n$ corresponding to the total number of validators, the intersection of any two sets of validators of size $< Quorum_{opt}(n)$ is not guaranteed to include at least one honest validator if the number of Byzantine validators is $f(n)$.
Proof. Let us assume that $V$ and $W$ are any two sets of validators of size $< \text{Quorum}_{opt}(n)$. Let $H$ denote the set of honest validators included in the intersection of $V$ and $W$. The minimum size of $H$ is expressed by the following equation:

$$\min(\|H\|) = \max\left(2 \cdot (\text{Quorum}_{opt}(n) - 1) - n - f(n), 0\right)$$

(15)

The following chain of equalities and inequalities proves that $\min(\|H\|) \leq \frac{1}{3}$:

$$\min(\|H\|) = 2 \cdot \left(\left\lceil \frac{2n}{3} \right\rceil - 1\right) - n - \left\lfloor \frac{n - 1}{3} \right\rfloor$$

$$= 2 \cdot \left(\frac{2n - 1}{3} \right) - n - \left(\frac{n}{3} - 1\right)$$

$$\leq 2 \cdot \frac{2n - 1}{3} - n - \left(\frac{n}{3} - 1\right)$$

$$= \frac{1}{3}$$

Since $\min(\|H\|)$ is an integer value, the inequality $\min(\|H\|) \leq \frac{1}{3}$ implies that $\min(\|H\|) \leq 0$.

Lemma 23. For any $n \geq 1$ with $n$ corresponding to the total number of validators, $\text{Quorum}_{opt}(n)$ corresponds to the minimum size required for the intersection of any two sets of validators to be guaranteed to include at least one honest validator provided that the number of Byzantine validators is $\leq f(n)$.

Proof. The proof follows from Lemma 21 and Lemma 22.

Lemma 24. The following inequality is verified for any $n \geq 0$:

$$\text{Quorum}_{opt}(n) + f(n) \leq n$$

Proof. The following chain of equalities and inequalities proves that $\text{Quorum}_{opt}(n) + f(n) \leq n + \frac{1}{3}$:

$$\text{Quorum}_{opt}(n) + f(n) = \left\lceil \frac{2n}{3} \right\rceil + \left\lfloor \frac{n - 1}{3} \right\rfloor$$

$$= \frac{2n - 1}{3} + 1 + \left\lfloor \frac{n - 1}{3} \right\rfloor$$

$$\leq \frac{2n - 1}{3} + 1 + \frac{n - 1}{3}$$

$$= n + \frac{1}{3}$$

Since $\text{Quorum}_{opt}(n) + f(n)$ is an integer value, the inequality $\text{Quorum}_{opt}(n) + f(n) \leq n + \frac{1}{3}$ implies that $\text{Quorum}_{opt}(n) + f(n) \leq n$.

Lemma 25. The liveness property of the IBFT protocol is not affected if $\text{Quorum}(n)$ is replaced by $\text{Quorum}_{opt}(n)$ provided that the number of Byzantine validators for any instance $h$ of the IBFT-block-finalisation-protocol-M2 is no more than $f(n_h)$.

Proof. Lemma 24 implies that for any round of the $h$-th instance of the IBFT-block-finalisation-protocol-M2, there exists at least $\text{Quorum}_{opt}(n_h)$ honest validators. This, in turn, implies that once GST is reached, if all $n_h$ honest validators are unlocked and the proposer for the current round is honest, then Byzantine validators cannot prevent honest validators from receiving $\text{Quorum}_{opt}(n_h)$ Prepare messages and $\text{Quorum}_{opt}(n_h)$ Commit messages for the block proposed by the block proposer.

Lemma 26. The IBFT-block-finalisation-protocol-M2 achieves optimal Byzantine-fault-tolerant safety threshold.
Proof. The proof is direct consequence of Lemmas 17, 23 and 25.

Theorem 4. The IBFT-M1-IBFT-block-finalisation-protocol-M2 protocol guarantees optimal Byzantine-fault-tolerance persistence threshold.

Proof. Obvious from Definition 39 of optimal Byzantine-fault-tolerance persistence threshold and Lemma 26.

6 Liveness Analysis

In this section we analyse the liveness property of the IBFT protocol. Specifically, we show that the IBFT protocol does not guarantee liveness when operating in an eventually synchronous network model when either a Byzantine or a fail-stop failure model are considered.

Lemma 27. The IBFT-M1 protocol guarantees $t$-Byzantine-fault-tolerant liveness only if the IBFT-block-finalisation-protocol guarantees $t$-Byzantine-fault-tolerant weak-liveness.

Proof. If the IBFT-block-finalisation-protocol does not guarantee $t$-Byzantine-fault-tolerant weak-liveness, then there could exist an instance $h$ of the IBFT-block-finalisation-protocol such that no finalised block is ever produced. This implies that no finalised block with height $\geq h$ will ever be added to the local blockchain of any honest node as honest nodes must add a finalised block with height $h$ to their local blockchain before being able to add any finalised block with height higher than $h$. Hence, any transaction sent to all honest validators that is not already included in a finalised block with height $< h$ will never be added to the local blockchain of any honest validator.

6.1 Analysis of the IBFT-block-finalisation-protocol, IBFT-block-finalisation-protocol-M1 and IBFT-block-finalisation-protocol-M2

Lemma 28. The following inequality is verified for any $n$ such that $n \geq 4 \land n \neq 6$:

$$n - Quorum(n) < Quorum(n)$$

(18)

Proof. It is easy to see that the following inequality implies the Lemma:

$$2 \cdot Quorum(n) > n, \text{ if } n \geq 4 \land n \neq 6$$

(19)

We now prove the Lemma by showing that (a) $2 \cdot Quorum(n) \geq 4 \cdot \frac{n}{3} - 2$ and (b) $4 \cdot \frac{n}{3} - 2 > n$ when $n \geq 4 \land n \neq 6$.

The following chain of equalities and inequalities proves (a).

$$2 \cdot Quorum(n) = 4 \cdot f(n) + 2$$

$$= 4 \left( \left\lceil \frac{n}{3} \right\rceil - 1 \right) + 2$$

$$= 4 \left\lceil \frac{n}{3} \right\rceil - 2$$

$$\geq 4 \cdot \frac{n}{3} - 2$$

It is easy prove that $4 \cdot \frac{n}{3} - 2 > n$ when $n > 6$. Therefore the Lemma is proved for $n > 6$.

For $n = 4$ or $n = 5$, we have $2 \cdot Quorum(n) = 6 > n$. This concludes the proof.

Lemma 29. The following inequality is verified for any $n > 0$:

$$n - Quorum_{opt}(n) < Quorum_{opt}(n)$$

(21)
Proof. It is easy to see that the following inequalities imply the Lemma:

\[ 2 \cdot \text{Quorum}_{opt}(n) > n \]

The Lemma is proved by the following sequence of inequalities and equalities:

\[ 2 \cdot \text{Quorum}_{opt}(n) = 2 \cdot \left[ \frac{2n}{3} \right] \geq 2 \cdot \frac{2n}{3} = \frac{4n}{3} > n \]

Lemma 30. For any \( h \) instance of the IBFT-block-finalisation-protocol and IBFT-block-finalisation-protocol-M1 where \( n_h \geq 4 \), the IBFT-block-finalisation-protocol and IBFT-block-finalisation-protocol-M1 do not guarantee weak-liveness in the presence of a single faulty node. For any \( h \) instance of the IBFT-block-finalisation-protocol-M2 where \( n_h \geq 2 \), the IBFT-block-finalisation-protocol-M2 does not guarantee liveness in the presence of a single faulty node. This is true even if the only type of node failure considered is fail-stop.

Proof. The proof is by counterexample. This is achieved by providing a possible sequence of events that may occur before GST which lead the protocol to be unable to create any new finalised block even once GST is reached. We consider events occurring during the \( h \)-th instance of the IBFT-block-finalisation-protocol, IBFT-block-finalisation-protocol-M1 and IBFT-block-finalisation-protocol-M2, and assume that one of the \( n_h \) validators for the \( h \)-th instance is faulty and therefore can stop communicating and never restart. Also, let us assume that all of the events outlined below happen before GST. We consider three possible cases.

Case 1: IBFT-block-finalisation-protocol or IBFT-block-finalisation-protocol-M1 with \( n_h \geq 4 \wedge n_h \neq 6 \). Let \( W \) be a subset of \( \text{Quorum}(n_h) \) validators containing the proposer for round 1, \( p_1 \), and the faulty validator. Let \( V \) be the complement of \( W \). The size of \( V \) is therefore \( n_h - \text{Quorum}(n_h) \) which, according to Lemma 28 is \( < \text{Quorum}(n_h) \) for \( n_h \geq 4 \wedge n_h \neq 6 \). The immediate consequence of the definition above is that all validators in \( V \) are honest.

1. The proposer for round 0, \( p_0 \), multicasts a Pre-prepare message for block \( B \) \( \langle \text{PRE-PREPARE}, h, 0, B \rangle_{\sigma_{p_0}} \).
2. All validators reply with a Prepare message for block \( B \) \( \langle \text{PREPARE}, h, 0, \text{KEC}(B) \rangle \).
3. All validators in \( V \) receive the Pre-prepare message for block \( B \) and all of the Prepare messages. As consequence of this, all validators in \( V \) lock on block \( B \).
4. The round-timer expires before any other validator, except for those in set \( V \), receive any of the Prepare messages. To be noted that since \( ||V|| < \text{Quorum}(n_h) \) and only validators in \( V \) may send a Commit messages for block \( B \), no honest validator will ever decide on block \( B \) as effect of the Commit messages sent in round 0.
5. All validators send a Round-Change message \( \langle \text{ROUND-CHANGE}, h, 1 \rangle \) to move to round 1.
6. All validators receive \( \text{Quorum}(n_h) \) Round-Change messages and move to round 1.
7. \( p_1 \) sends a Pre-prepare message for block \( B' \) \( \langle \text{PRE-PREPARE}, h, 1, B' \rangle_{\sigma_{p_1}} \), with \( B' \neq B \). This is possible as \( p_1 \) is not locked on any block as it is part of the set \( W \) which is disjoint from the set \( V \).
8. All validators included in set \( V \) are locked on \( B \), therefore when they receive the Pre-prepare message sent by \( p_1 \) they reply with a Round-Change \( \langle \text{ROUND-CHANGE}, h, 2 \rangle \) to move to round 2.
9. All of the validators in set \( W \) respond to the Pre-prepare for block \( B' \) that they received with a Prepare message for block \( B' \) and round 1 \( \langle \text{PREPARE}, h, 1, \text{KEC}(B') \rangle \). However, one of these validators is faulty and stops just after sending a Prepare message and before sending any Commit message. Set \( W \) now contains only \( \text{Quorum}(n_h) - 1 < \text{Quorum}(n_h) \) active validators.
Case 2: IBFT-block-finalisation-protocol or IBFT-block-finalisation-protocol-M1 with and V messages for this block, then no validator will ever receive the one presented for Case 1 with the additions of events 9 to 14.

1. The proposer for round 0, \( p_0 \), multicasts a Pre-prepare message for block \( B \) to all validators (including itself).
2. All validators reply with a Prepare message for block \( \langle \text{PRE-PREPARE}, h, 0, B \rangle_{\sigma_{p_0}} \) to the proposer for round 0.
3. All validators in \( V \) receive the Pre-prepare message for block \( B \) and all of the Prepare messages. As consequence of this, all validators in \( V \) lock on block \( B \).
4. The round-timer expires before any other validator, except for those in set \( V \), receive any of the Prepare messages. To be noted that since \( |V| < \text{Quorum}(n_h) \) and only validators in \( V \) may send a Commit messages for block \( B \), no honest validator will ever decide on block \( B \) as effect of the Commit messages sent in round 0.
5. All validators send a Round-Change message (ROUND-CHANGE, h, 1) to move to round 1.
6. All validators receive \( \text{Quorum}(n_h) \) Round-Change messages and move to round 1.
7. \( p_1 \) sends a Pre-prepare message for block \( B' \) \( \langle \text{PRE-PREPARE}, h, 1, B' \rangle_{\sigma_{p_1}} \), with \( B' \neq B \).
8. All validators included in set \( V \) are locked on \( B \), therefore when they receive the Pre-prepare message sent by \( p_1 \) they reply with a Round-Change (ROUND-CHANGE, h, 2) to move to round 2.
9. All of the validators in sets \( Z \) and \( W \) respond to the Pre-prepare for block \( B' \) that they received with a Prepare message for block \( B' \) and round 1 \( \langle \text{PREPARE}, h, 1, \text{KEC}(B') \rangle \).
10. All validators in \( Z \) receive the Pre-prepare message for block \( B' \) and all of the Prepare messages sent by validators in set \( Z \) and \( W \). Since \( |Z \cup W| \geq |W| = \text{Quorum}(n_h) \), all validators in \( Z \) lock on block \( B' \).
11. The round-timer four round 1 expires before any other validator, except for those in sets \( V \) and \( Z \), receive any of the Prepare messages. To be noted that since \( |Z| < \text{Quorum}(n_h) \) and only validators in \( Z \) may send a Commit messages for block \( B' \), no honest validator will ever produce a finalised block including block \( B' \) as effect of the Commit messages sent in round 1.
12. All validators send a Round-Change message (ROUND-CHANGE, h, 2) to move to round 2.
13. All validators receive \( \text{Quorum}(n_h) \) Round-Change messages and move to round 2.
14. \( p_2 \) sends a Pre-prepare message for block \( B'' \) \( \langle \text{PRE-PREPARE}, h, 2, B'' \rangle_{\sigma_{p_2}} \), with \( B'' \neq B' \neq B \). This is possible as \( p_2 \) is not locked on any block as it is part of the set \( W \) which is disjoint from the union of sets \( V \) and \( Z \).
15. All validators included in sets \( V \) and \( Z \) are locked on \( B \) and \( B' \) respectively, therefore when they receive the Pre-prepare message sent by \( p_2 \) they reply with a Round-Change to move to round 3. All of the validators in set \( W \) respond with a Prepare message for block \( B'' \).
and round 2. However, one of these validators is faulty and stops just after sending the Prepare message and before sending the related Commit message. Set $W$ now contains only $Quorum(n_h) - 1 < Quorum(n_h)$ active validators.

16. All honest validators in set $W$ receive all the Prepare messages for block $B''$ sent by all validators in $W$ including the Prepare message sent by the faulty validator before stopping. Since $|W| = Quorum(n_h)$, all honest validators in set $W$ lock on block $B''$.

Let $W_{active}$ be the subset of active validator in $W$. By the end of the last step of the sequence of events presented above, all validators, except the faulty one that stopped, are locked on a block. Specifically, validators in $V$ are locked on $B$, validators in $Z$ are locked on $B'$ and validators in $W_{active}$ are locked on $B''$. In the IBFT-block-finalisation-protocol honest validators can release the lock only if they receive $Quorum(n_h)$ Commit messages with height, round and block matching the Pre-prepare message, while in the IBFT-block-finalisation-protocol-M1 (and IBFT-block-finalisation-protocol-M2) honest validators never release a lock (see Lemma 13). Since (i) the union of sets $W_{active}$, $V$ and $Z$ corresponds to the entire set of all honest and active validators, (ii) all sets $W_{active}$, $V$ and $Z$ have size $< Quorum(n_h)$ and (iii) honest validators that are locked on a block only send messages for this block, then no validator will ever receive $Quorum(n_h)$ Commit messages for the same block and therefore no honest validator will either unlock or create a finalised block.

**Case 3: IBFT-block-finalisation-protocol-M2 with $n_h \geq 2$.**

This case corresponds to Case 1 with $Quorum(n_h)$ replaced by $Quorum_{opt}(n_h)$. In this case, the size of $V$ is $n_h - Quorum_{opt}(n_h)$ which, according to Lemma 29, is $< Quorum_{opt}(n_h)$ for $n_h \geq 1$. The resulting sequence of events leads the IBFT-block-finalisation-protocol-M2 to a state where it will be unable to create any new finalised block even once GST is reached.

**Remark.** For $n_h = 2 \lor n_h = 3$, the IBFT-block-finalisation-protocol and the IBFT-block-finalisation-protocol-M1 guarantee pathological weak-liveness as in this case $Quorum(n_h) = 1$ and therefore each honest validator can create valid finalised block without requiring the interaction with any other validator.

**Theorem 5.** When operating in an eventually synchronous network model, the IBFT protocol does not guarantee liveness. This is true even if the only type of node failure considered is fail-stop.

**Proof.** Direct consequence of Lemmas 27 and 30 and the fact that we are interested only in the cases where Byzantine-fault-tolerance can be guaranteed, i.e the number of validators is $\geq 4$.

### 6.2 Modification IBFT-block-finalisation-protocol-M3: Ensure liveness

In this section we explore two options of how the IBFT protocol can be modified to guarantee liveness when operating in an eventually synchronous network.

#### 6.2.1 PBFT-like solution

The first option that we present is based on the original PBFT protocol [3]. The key change to IBFT is to remove the locking mechanism and ensure safety between round changes via a round change sub-protocol similar to the one in the PBFT protocol. The key modifications can be summarised as follows:

**(S1.M-1)** Remove the locking logic.

**(S1.M-2)** Add a prepared certificate to the Round-Change message.

   In contrast to PBFT, no checkpoint is required here as the finalisation proof added to each IBFT block serves the role of checkpoint.

**(S1.M-3)** Add a New-Round message to be sent by the proposer of the new round once $Quorum_{opt}(n)$ valid Round-Change messages for the same new round are received by the proposer.

The content of the New-Round message is similar to the content of the New-View message of the PBFT protocol, but with the following differences:

**(S1.M-3.1)** Since IBFT decides on a block at a time, only one Pre-prepare message is included in the New-Round message.
(S1.M-3.2) The Pre-prepare message included in the New-Round message always contains a valid block. If the set of Round-Change messages included in the New-Round message contains at least one valid prepared certificate, then the block included in the Pre-prepare message must match the block that has a valid prepare certificate with the highest round number. Otherwise, the Pre-prepare message can contain any valid block. This is in contrast with PBFT where the Pre-prepare messages included in the New-View message may contain null requests.

Modification (S1.M-1) obviously solves the liveness issue, but if introduced in isolation, it would make the protocol unsafe in the case that a round change occurs. Modification (S1.M-2) and (S1.M-3) ensure safety without compromising liveness.

A few protocol optimisations are required to this solution to reduce the size of Round-Change and New-Round messages. One of the potential solutions for reducing the size of the New-Round messages is replacing the block included in the signed Pre-prepare messages with a digest of the block and piggyback the full block to the signed Pre-prepare, Round-Change and New-Round messages. Other solutions are available but their discussion is outside the scope of this work.

The correctness proof of this solution is quite extensive and therefore it will be presented as part of a separate body of work.

6.2.2 Tendermint-like solution

In contrast, the second option looks at implementing some of the concepts from the Tendermint variant of the PBFT protocol [1]. The availability of this type of solution stem from a private conversation with Clearmatics. The key idea here is borrowing the relocking mechanism from the Tendermint protocol [1]. The list of modifications that should be applied to IBFT for applying this solution are summarised below:

(S2.M-1) Add the locked round value to the Pre-prepare message. The locked round value corresponds to the latest round number where a validator locked on the locked block.

(S2.M-2) Allow relocking if a validator receives Quorum opt \((n)\) Prepare messages for round \(r\) and one Pre-prepare message with locked round equal to \(r\) provided that \(r\) is higher than the current round.

6.2.3 Comparison between the two solutions

The two proposed solutions differ in behaviour and performance only when either the proposer is Byzantine or the network delay is longer than the duration of the round timer for round 0. If the proposer for round 0 is honest and all messages are delivered within the duration of the round timer for round 0, then the two solutions behave identically from a practical perspective. Performance wise, the main differences between the two solutions can be summarised as follows:

- The PBFT-like solution ensures that after GST, if the proposer of the current round is honest, then every honest node decides within that round. In contrast, the Tendermint-like solution may require to go through \(n_h - 1\) round changes before an honest validator reaches a decision where \(n_h\) is the number of validator for the generic \(h\)-th instance of the IBFT-block-finalisation-protocol;
- The PBFT-like solution can be extended to achieve block finalisation in two communication phases only (Pre-prepare and Prepare) in the optimal case that no Byzantine validators are present and the network delay is longer than the duration of the round timer for round 0. This can be achieved by adapting the protocol for very fast learning presented by Dutta et al. [5] to work within the definition of the IBFT protocol;
- The Tendermint-like solution requires lower overall bandwidth than the PBFT-like solution when the protocol moves to a new round.

7 Conclusions

IBFT is a proof-of-authority consensus protocol with immediate finality applicable to Ethereum blockchains. However, to the best of our knowledge, no correctness analysis of the IBFT consensus protocol
has been carried out so far. With this work we fill this gap by analysing the robustness of the IBFT protocol as logical conjunction of two properties: persistence and liveness. Persistence is related to maintaining blockchain consistency and immutability amongst all honest nodes, while liveness ensures that a transaction sent to all honest validators will eventually be included in the blockchain. As outcome of the persistence analysis, we show that the IBFT protocol does not guarantee Byzantine-fault-tolerant persistence under eventual synchrony. As outcome of the liveness analysis, we show that the IBFT protocol does not guarantee liveness under an eventually synchronous network model even if the only failure mode considered is fail-stop. As part of this work, we also show that the IBFT protocol can be slightly modified to achieve optimal Byzantine-fault-tolerant safety. Finally, we propose two possible ways to modify the IBFT protocol to ensure liveness when operating in eventually synchronous networks. However, these two potential solutions are just sketched here and no correctness proof is provided. Much more work is required to analyse pros and cons of each solution and perform related robustness analysis.

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