Entanglement-based quantum key distribution over noisy channels

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Quantum key distribution (QKD) is one of the most important subjects in quantum information theory. There are two kinds of QKD protocols, prepare-measure protocols and entanglement-based protocols. For long-distance communications in noisy environments, entanglement-based protocols might be more reliable since they could be assisted with distillation procedures, which however, often result in a large consumption of states while not all states can be distilled. This paper aims to study the entanglement-based QKD over certain noisy channels including Pauli noises, amplitude damping noises, phase damping noises, collective noises and mixtures of them, without distillations. We modify the standard entanglement-based protocol by adding testing states and choosing checking states carefully such that it can be implemented over noisy channels as in noiseless ones and with improvements of the efficiency.

Keywords: Quantum key distribution; Testing state; Qubit; Noisy channel; Entanglement.

I. INTRODUCTION

Cryptography is one of the most important subjects in the information era. However, the security of the most-used RSA public-key system is based on the low capacity of classical computers and arithmetics on factorizing large integers. When quantum arithmetics are taken into account, it is no longer secure [1]. Nowadays, the only algorithm proven to be secure is the private-key algorithm of which the security is guaranteed by the secure distribution of one-time pads, however, can not be verified in classical information theory. But on the other hand, quantum cryptographic protocols can provide private-key distribution, whose security only relies on physical laws of quantum systems.

Suppose that two legitimated parties, Alice and Bob, want to communicate by using a shared one-time pad, a string of numbers (private-key) they agree with each other but private for others. The main problem is how to transmit such a key securely. Assume that Alice and Bob have both quantum channels and authenticated classical channels which might not be private, such that the eavesdropper, Eve, may eavesdrop classical communications and access quantum channels. The security is guaranteed in the sense that if Eve gets enough information about the key, she is detectable by the legitimated parties who can abort the key. Quantum key distribution (QKD) gives methods on how the legitimated parties can share a key securely by using quantum information theory.

There are mainly two classes of QKD protocols. The first class is prepare-measure protocols, in which Alice prepares states and sends to Bob who verifies the states by measurements. Examples can be found in [2–9]. Another class is entanglement-based protocols. The main point of such protocols is using distributed entangled states. Examples can be found in [6, 10–13].

However, the main restriction in implementing a QKD protocol, no matter for the prepare-measure type or the entanglement-based type, is that quantum channels might be noisy [11, 14]. The most researched noises might be collective noises such as collective dephasing (CD) and collective rotation (CR) [15–18]. Other researches include Pauli noises [19–22], amplitude damping (AD) and phase damping (PD) noises [23–28], as well as the related ones [25, 29–32].

In this paper, we study entanglement-based QKD schemes with Pauli noises, AD and PD noises, collective (CP and CR) noises, as well as their mixtures. We give a method against such noises with any strength by using testing states. Our schemes can be implemented in noisy channels without errors, like in noiseless ones. On the other hand, comparing with the standard entanglement-based protocol, our protocols may consume less states by choosing checking states carefully.

II. THE STANDARD ENTANGLEMENT-BASED QKD PROTOCOL

A standard entanglement-based QKD protocol includes the distribution of (maximally) entangled states from which Alice and Bob can generate a raw secret key, and the checking procedure from which they can estimate whether the security of the key is acceptable. For the qubit case, the following standard protocol can accomplish an entanglement-based QKD task for Alice and Bob over noiseless channels.

1. Alice creates 2N maximally entangled states \( |\varphi\rangle_{AB} = \frac{1}{\sqrt{2}} \sum_{j=0}^{1} |j\rangle_A |j\rangle_B \), where \( \{ |j\rangle | j = 0, 1 \} \) is an
III. ENTANGLEMENT-BASED QKD OVER NOISY CHANNELS

Concerning entanglement-based QKD over noisy channels, one might run distillation procedures, which however, often result in a large consumption of states, while states such as bound entangled ones can not be distilled \([33–41]\). In the following, we investigate how an entanglement-based QKD protocol can resist certain noises without entanglement distillations. The scheme is similar to the standard protocol except using testing states against noises and choosing checking states carefully to improve the efficiency. The general protocol is stated as follow:

**Step 1:** Alice creates key states, checking states and testing states. The key states are used to generate the key, the checking states are used to estimate security, while the testing states are used to modify the bits over noisy channels.

**Step 2:** Alice sends B partite of each state to Bob in a randomly chosen order, which guarantees the security together with checking states.[42]

**Step 3:** After Bob published his receipt, Alice publishes the order she chose, and informs Bob which are checking states and which are testing states. Alice also publishes what the checking and testing states are if needed.

**Step 4:** Bob measures the checking and testing states under certain bases while Alice and Bob measure the key states under certain bases. The outcomes(bits) of key states and checking states should be modified by outcomes of testing states.

**Step 5:** Alice and Bob calculate the error rate on checking states and generate a raw key by key states if it is acceptable.

The above steps will be repeated several times until a long enough raw secret key is obtained and then followed by error correcting and privacy amplification procedures if needed.

In subsections below, we will discuss how to choose testing and checking states for special noises.

### A. Pauli noises

Pauli noises act on each qubit sent by the channel via Pauli operators, \(I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \ X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\).

\(ZX = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\) under the computational basis, with corresponding probabilities \(p_I, p_Z, p_X\) and \(p_{ZX}\), summing to 1.

Generally, it is hard to deal with such noises. In the following, we assume that if Alice sends two qubits together via the same channel, they will encounter the same effects.

#### 1. Two Pauli channel

In two Pauli channels, \(p_I, p_Z, p_X, p_{ZX}\) might all be non-zero. The final states might be completely mixed.

**Step 1:** To share a N-bit key string, Alice creates N states \(|\varphi\rangle_{AB_1} |0\rangle_{B_2} = \frac{1}{\sqrt{2}} \left( |00\rangle + |11\rangle \right)_{AB_1} |0\rangle_{B_2}\), M states \(|x\rangle_{B_3} |x\rangle_{B_4}\), where \(x \in \{0, 1, \pm\}\) is chosen randomly for each state. Hence, partita \(B_1\) is for key, partita \(B_3\) is for checking, and partita \(B_2, B_4\) are for testing.

**Step 2:** Alice mixes such states and sends partite B (including \(B_1, B_2, B_3, B_4\)) to Bob with a random order such that the B partite of a state are always sent together and assumed to suffer the same effects.

**Step 3:** After Bob published his receipt, Alice publishes which states are checking states, which are testing states(the order) and what they are.

**Step 4:** Bob measures partite \(B_3\) and \(B_4\) via the same basis depending on Alice’s declaration, namely, via \(\{|0\rangle, |1\rangle\}\) if Alice sent \(|0\rangle_{B_3} |0\rangle_{B_4}\) or \(|1\rangle_{B_3} |1\rangle_{B_4}\), while via \(\{|+, -\rangle\}\) if Alice sent \(|+\rangle_{B_3} |+\rangle_{B_4}\) or \(|-\rangle_{B_3} |-\rangle_{B_4}\). Alice and Bob also measure partite \(A, B_1, B_2\) via \(\{|0\rangle, |1\rangle\}\).

**Step 5:** An error occurs when Bob’s outcomes on \(B_3\) and \(B_4\) do not agree and he calculates the error rate. If the error rate is acceptable, they agree a key string by the measurement outcomes on partite \(A, B_1, B_2\). In more details, if Alice’s result does not agree with Bob’s then Bob gets outcome 1 on partita \(B_2\), and otherwise, he gets 0.

To agree a N-bit key string, Alice and Bob consume \(N\) two-qubit maximally entangled states, \(N\) single qubit states, \(M\) two-qubit product states. Totally, they consume \(3N + 2M\) qubits. In the case \(M = N\), they consume \(5N\) qubits.

#### 2. One Pauli channel

In one Pauli channels, states suffer two of the four Pauli operators, one of which is I.

Let us assume that states suffer I with a probability \(p\) and Z with a probability \(1-p\). The procedures are exactly the same as above. However, some of the states
can be saved. Instead of sending M states $|x\rangle_{B_1},|x\rangle_{B_4}$ and N states $|\varphi\rangle_{AB_1},|0\rangle_{B_3}$, Alice only needs to send $\frac{M}{4}$ states $|y\rangle_{B_3}|y\rangle_{B_4}$ and $\frac{N}{4}$ states $|z\rangle_{B_3}|z\rangle_{B_4}$, where randomly chosen $y \in \{+, -, \}$, $z \in \{0, 1\}$ for each state, are for checking and testing, while $\frac{M}{4}$ states $|\varphi\rangle_{AB_1},|\varphi\rangle_{AB_1} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{AB_1}$ and $\frac{N}{4}$ states $|z\rangle_{B_3}|z\rangle_{B_4}$ are for key. This is because that outcomes of $|\varphi\rangle_{B_3}|\varphi\rangle_{B_4}$ and $|\varphi\rangle_{AB_1}$ are not affected by the noise when measuring via $\{0\}, \{1\}$. Hence, partite $B_2$ can be saved and partite $B_4$ of $|\varphi\rangle_{B_3}|\varphi\rangle_{B_4}$ can be used for checking too.

Now, to agree a N-bit key string, Alice and Bob consume $2N + 1.5M$ qubits. For the case $M = N$, they consume 3.5N qubits.

Other kinds of one Pauli channels can be analyzed exactly in the same way. If the states suffer I with a probability $p$ and X with a probability 1-p, Alice can operate $H$ on all partite of states she creates. Alice and Bob operate $H$ on all bases they choose to measure. And if the states suffer I with a probability $p$ suffering $\{+, -\}$, while via $H$, they calculate the error rate and generate a key string by the outcomes of key states if it is acceptable (Step 5).

Alice and Bob agree an average (1-p)N-bit key string by consuming N two-qubit states, N single qubit states, and $\frac{N}{2}$ two-qubit product states. Totally, they consume $3N + M$ qubits. In the case $M = N$, they consume 4N qubits.

C. Phase damping (PD)

Phase damping noises have Kraus operators $E_0 = \begin{bmatrix} 1 & 0 \\ \sqrt{1 - p} & \sqrt{p} \end{bmatrix}$, $E_1 = \begin{bmatrix} \sqrt{p} & 0 \\ 0 & 0 \end{bmatrix}$, $E_2 = \begin{bmatrix} 0 & \sqrt{p} \\ 0 & \sqrt{1 - p} \end{bmatrix}$, where $p$ is the error rate, meaning that a state sent through the channel remains unchanged with a probability 1-p and suffers an error with a probability $p$.

In this case Alice creates $\frac{N}{4}, \frac{N}{4}$ states $|\varphi\rangle_{AB_1}|\varphi\rangle_{B_3}B_4 = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{AB_1}$, $E_2$ of partite $B_2, B_3$, and $\frac{N}{4}$ states $|\varphi\rangle_{AB_1}|\psi\rangle_{B_3}B_4 = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{AB_1}$, $E_2$ of partite $B_2, B_3$. For key, they are for checking and testing while others are for key (Step 1). Alice always sends the three B partite of a state in a random order, which is published after Bob received, and they are assumed to suffer the same effect (Step 2, 3). On average, only 1-p of the states can be received without any loss, which are not affected by noises. For such states, Bob measures partite $B_2, B_3$ via basis $\{0\}, \{1\}$ or $\{+, -\}$, in coincident on the two partite for each state and randomly for different states. Alice and Bob also measure partite $A, B_1$ via $\{0\}, \{1\}$ or $\{+, -\}$, in coincident on the two partite for each state and randomly for different states (step 4).

An error occurs if Bob’s results on partite $B_2$ and $B_3$ agree when measuring via $\{0\}, \{1\}$, agree when measuring via $\{+, -\}$ for $|\varphi\rangle$ or do not agree when measuring via $\{+, -\}$ for $|\psi\rangle$. They calculate the error rate and generate a key string by the outcomes of key states if it is acceptable (Step 5).

Hence, Alice and Bob agree an average (1-p)N-bit key string by consuming $2N$ two-qubit maximally entangled states. Totally, they consume 4N qubits.

D. Collective dephasing (CD)

The collective dephasing noises act on each qubit sent though the channel equivalently via $\begin{bmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{bmatrix}$ under the computational basis, where $\phi$ is a parameter depending on the noise.

Note that $|\varphi\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{AB}$ becomes $|\varphi^\phi\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + e^{i\phi}|11\rangle)_{AB}$ being affected by the noise, which can be used for key by measuring via basis $\{|j\}, j = 0, 1\}$, but can not used for checking if Eve purifies states via direction $\{|j\}, j = 0, 1\}$. Making them be $|\varphi^\phi\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + e^{i\phi}|11\rangle)_{AB}$. To overcome this problem, Alice adds some checking states which are invariant under CD noises for checking. The protocol could be implemented as follow for a N-bit key string.
Alice creates $N/4$ maximally entangled states $|\varphi\rangle_{AB_1}|\varphi\rangle_{AB_2} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{AB_1}\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{AB_2}$ for key and $N/4$ maximally entangled states $|\psi\rangle_{B_1B_2} = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)_{B_1B_2}$, $M$ two-qubit states $|t\rangle_{B_3} |l\rangle_{B_4}$, where $t, l \in \{0, 1\}$ are chosen randomly, for checking (Step 1). Alice sends B partite in a random order to Bob (Step 2). After Bob published his receipt, Alice publishes the order and so Bob knows which states are key states and which are checking states. Alice also publishes what checking states are (Step 3). Bob measures partite $B_2, B_3$ via basis $\{|0\rangle, |1\rangle\}$ if Alice sent $|t\rangle |l\rangle$ while via $\{|+, -\rangle\}$ if Alice sent $|\psi\rangle$. They also measure the key states via $\{|0, 1\rangle\}$ on both partite $A$ and $B_1$ (Step 4). An error occurs when Bob getting outcome 1 if Alice sent $|0\rangle$, outcome 0 if Alice sent $|1\rangle$, or if the outcomes on $B_2$ and $B_3$ agree if Alice sent $|\varphi\rangle$, on checking states. Bob calculates the error rate. They generate a raw key by key states if it is acceptable (Step 5).

To agree a N-bit key string, Alice and Bob consume $N + \frac{M}{2}$ maximally entangled states and $\frac{M}{2}$ two-qubit states. Totally, they consume $2N + 1.5M$ qubits and in the case $M=N$, they consume $3.5N$ qubits.

E. Collective rotation (CR)

Collective rotation noises act on each qubit sent by the channel equivalently via

$$
\begin{bmatrix}
\cos \theta & \sin \theta \\
\sin \theta & -\cos \theta
\end{bmatrix}
$$

under the computational basis, where $\theta$ is a parameter depending on the noise and evolves upon time.

Being affected by such a noise, both $|\varphi\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and $|\psi\rangle_{B_1B_2} = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ are unchanged upon a global phase. Thus, Alice creates N key states $|\Psi\rangle_{AB_1B_2} = \frac{1}{\sqrt{2}}(|0\rangle|\varphi\rangle + |1\rangle|\psi\rangle)_{AB_1B_2}$ and M checking states $|\varphi\rangle_{B_1B_2} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{B_1B_2}$, instead (Step 1). She sends partite B to Bob with a random order (step 2) and publishes the order after Bob received (step 3). Bob measures via the same basis on both $B_3$ and $B_4$, randomly in $\{|0\rangle, |1\rangle\}$ or $\{|+, -\rangle\}$ for each checking state while measures partite $B_1$ via $\{|+, -\rangle\}$ after providing a C-Not gate on partite $B_1, B_2$ with partite $B_2$ being aborted for each key state. Alice measures partite A via $\{|0\rangle, |1\rangle\}$ (step 4). If the outcomes on $B_3$ and $B_4$ are different, Bob considers it as an error and he calculates the error rate. They generate a raw key by the outcomes on key states if the error rate is acceptable (Step 5). In more details, Alice and Bob agree 0 if the outcomes are 0 or + and 1, otherwise.

To agree a N-bit key string, Alice and Bob consume N tripartite entangled states and M two-qubit maximally entangled states. Totally, they consume $3N + 2M$ qubits and in the case $M = N$, they consume $5N$ qubits.

F. Mixture of different noises

We can deal with mixtures of different types of noises by combining the above strategies. Let us discuss two kinds of them.

1. Mixture of PD, Pauli noises and CD

Following the discussions above, Alice creates $N/4$ states $|\varphi\rangle_{AB_1}|\psi\rangle_{B_2B_3} |0\rangle_{B_4} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{AB_1}\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)_{B_2B_3} |0\rangle_{B_4}$, $N/4$ states $|\varphi\rangle_{AB_1}|\psi\rangle_{B_2B_3} |0\rangle_{B_4} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{AB_1}\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)_{B_2B_3} |0\rangle_{B_4}$, $N/4$ states $|\varphi\rangle_{AB_1}|\psi\rangle_{B_2B_3} |1\rangle_{B_4} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{AB_1}\frac{1}{\sqrt{2}}(|10\rangle - |01\rangle)_{B_2B_3} |1\rangle_{B_4}$, and $N/4$ states $|\varphi\rangle_{AB_1}|\psi\rangle_{B_2B_3} |1\rangle_{B_4} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{AB_1}\frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)_{B_2B_3} |1\rangle_{B_4}$, where partite $B_2$ and $B_3$ are for checking, partite A, $B_1$ are for key and partite $B_4$ are for testing (Step 1). Alice always sends the four B partite of a five-partita state together such that they suffer the same effect. For each state, Alice sends the four B partite in a randomly chosen order, which is published together with the state of partite $B_4$ after Bob received (Step 2, 3). On average, only 1-p of the states can be received without any loss. For such states, Bob measures partite $B_2, B_3$ via basis $\{|0\rangle, |1\rangle\}$ or $\{|+, -\rangle\}$ randomly but in coincident on partite $B_2$ and $B_3$ for each state. Alice and Bob also measure other partite via $\{|0\rangle, |1\rangle\}$ (step 4). Errors occur with the same conditions in PD case, and Bob calculates the error rate. If the error rate is acceptable, Alice and Bob generate a key string by the measurement outcomes on key states and modify them by testing states as in the Pauli noises case (Step 5).

Hence, Alice and Bob agree an average (1-p)N-bit key string by consuming 2N two-qubit maximally entangled states and N single qubit states. Totally, they consume $5N$ qubits.

2. Mixture of PD, Pauli noises and CR

Similar to above, Alice creates N states $|\Psi\rangle_{AB_1B_2}|\psi\rangle_{B_3B_4} = \frac{1}{\sqrt{2}}(|0\rangle|\varphi\rangle_{B_1B_2} + |1\rangle|\psi\rangle_{B_1B_2})|\psi\rangle_{B_3B_4}$, where partite $B_3$ and $B_4$ are for testing and checking, while partite $A$, $B_1$ and $B_2$ are for key (Step 1). Alice always sends the four B partite of a five-partita state together such that they suffer the same effect. For each state, Alice sends the four B partite together in a randomly chosen order, which is published together with the state of partite $B_3, B_4$ after Bob received (Step 2, 3). On average, only 1-p of the states can be received without any loss. For such states, Bob measures partite $B_3, B_4$ via basis $\{|0\rangle, |1\rangle\}$ or $\{|+, -\rangle\}$, in coincident on partite $B_3$ and $B_4$ for each state and randomly for different states. Errors occur with the same conditions as in PD case.
and Bob calculates the error rate. Alice and Bob also measure other partite as in CR case and generate a key string by the measurement outcomes on key states if the error rate is acceptable (Step 4, 5).

Hence, Alice and Bob agree an average (1-p)N-bit key string by consuming N three-qubit states and N two-qubit states. Totally, they consume 5N qubits.

The state $|\psi'\rangle$ is unchanged under collective noises. Such a state is called decoherence-free. For a mixture of PD, CP, CR and Pauli noises, in general, we can use two orthogonal decoherence-free states to handle. However, this is impossible in $C^2 \otimes C^2$, since $|\psi'\rangle$ is the unique state with this property. But in $C^4 \otimes C^4$, this can be done $^{43}$.

IV. SECURITY, EFFICIENCY AND NOTE

The security of the protocols comes from the security of the ordinary protocol and the order-rearrangement techniques $^{3, 12}$. Using above special checking states to estimate the error rate is equivalent to the ordinary checking procedure by randomly publishing part of the bit string.

It is worth noting that our protocols could be more efficient than the standard one. For example, the standard protocol consumes 4N qubits in noiseless channels for N secret bits while our protocol against CD noises or one Pauli noises only consumes 3.5N qubits by selecting checking states carefully.

Also note that our method seems like the decoy state method $^{44, 45}$, however, they are not the same. The decoy state method is used to handle photon-number-splitting attacks and only cares about the photon number (but not what states are) of a signal while our method is used to avoid noises by special states. Our method somehow might be viewed as an error detecting and correcting procedure.

V. CONCLUSION

In this paper, we have investigated a method for the entanglement-based quantum key distribution protocol against certain noises and presented modified protocols, which can be implemented in noisy channels without giving rise to errors, like in noiseless ones. Our schemes could consume less states comparing with the standard protocol by selecting checking states carefully. The collective noises can be easily dealt with and have been considered for prepare-measure QKD protocols without Trojan Horse attacks $^{13}$. For Pauli noises and PD, AD noises, we have used a testing state strategy to verify the noises and to modify the outcomes of measurements. The same arguments are also suitable for mixtures of noises. Our schemes might be viewed as error detecting and collecting schemes and are suitable for any strength of noises.

An entanglement-based QKD scheme might not be efficient comparing with a prepare-measure one. However, the distribution of entangled states has comprehensive employments. For example, it could be used for implementing direct communications via teleportations $^{46}$.

Our methods may also be applied to prepare-measure QKD protocols and could be extended to qudits. One may also take into account problems with an untrusted third party or imperfect maximally entangled states. Other considerable problems are calculating classical communications needed in implementing such QKD protocols and the efficiency on consumptions of both classical bits and qubits.

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