Selection of the optimal parameters of a reinforced concrete rectangular beam with single reinforcement

V S Kuznetsov, Yu A Shaposhnikova, A A Yandiev

Department of Reinforced Concrete and Stone Structures, Moscow State University of Civil Engineering, 26, Yaroslavl highway, Moscow 129337, Russia

E-mail: yuliatalyzova@yandex.ru

Abstract. The article is devoted to the problem of choosing the optimal parameters of reinforced concrete rectangular beams with single reinforcement. The analysis of the dependence of the cost of a single-span hinged-supported reinforced concrete beam on the selected parameters of its cross section under the action of a uniformly distributed load. The optimal range of cross-sectional parameters of reinforced concrete articulated beams (height, width, class of concrete and reinforcement, area and reinforcement pitch) was determined depending on the value of the uniformly distributed external load. Strength calculation of reinforced concrete beams is based on the traditional method given in the SR.63.13330.2018 "Concrete and reinforced concrete structures." The cost of reinforced concrete beams was calculated under the influence of various uniformly distributed loads. The first option, where the width of the beams was constant and the section heights varied between 250 and 1000 mm. The second option, when the heights of the beams were taken equal, and the widths of the beams were in the range from 100 to 500 mm. The recommended height and width of the beam section for the specified value span in the investigated range of loads was set.

1. Introduction

The essence of design designs is not only to ensure safe operation, but also to minimize financial costs. The process of design of reinforced concrete structures includes two main stages: assignment of assumed geometric parameters of cross-section, class of concrete and reinforcement and selection of required reinforcement. The components of reinforced concrete, steel and concrete, differ significantly in cost. For this reason, the question arises, which ratio of steel to concrete in the structure is most optimal. It is also interesting to know how the cost of the design changes with this ratio. And from a practical point of view, it is important to know which range of reinforced concrete beam configurations of the rectangular section is most cost-effective.

2. Literature review

The question of the optimal design of reinforced concrete beams is dealt with by many Russian and foreign scientists, for example, Karpenko N.I., Baykov V.N., Skladnev I.O., Alekseytsev A.V. and others.

A great contribution to the development of methods for calculating reinforced concrete structures was made by Karpenko N. I., Baikov V. N. In the work of Karpenko N. I., a general theory of deformation and destruction of concrete and reinforced concrete under various types of loading was constructed [1]. Baykov V. N. made a significant contribution to the development of the calculation of...
construction from precast reinforced concrete [2]. Tamrazyan A.G., Alekseytsev A.V. are engaged in modern methods of optimization of structural solutions for load-bearing structures [3-4]. Also, the problem of optimal design reinforced concrete construction, taking into account reliability and efficiency, was addressed by Skladnev N. N. [5]. Kuznetsov V.S. investigated the work of monolithic floors, as well as reinforced concrete beams in the zones of changing the pitch of the transverse rods [6-7].

Chakrabarty B.K. studied the relationship between beam cost and unit cost of materials and beam sizes [8]. Jensen C. and Lapko A. investigated the design of shear reinforcement in reinforced concrete beams [9]. Carlos Coello Coello, Filiberto Santos Hernández and Francisco Alonso Farrera used genetic algorithms for optimal design beams [10]. Also Garstecki A., Glema A. and Scigallo J. developed a software package for the optimal design of reinforced concrete beams and columns [11]. Demby Michal addressed the problem of the optimal and safe design of reinforcement structures in reinforced concrete elements [12]. F.H. Turner studied the fatigue of reinforced concrete in beams [13]. Nemirovsky Ju. V. considers critical characteristics of concrete failure, such as shrinkage, temperature sensitivity, and the influence of production technology [14].

Currently, reinforced concrete beams in the Russian Federation are calculated in accordance with SR 63.13330.2012 “Concrete and reinforced concrete structures” [15]. In Europe, EN 1992 Eurocode 2: Design of concrete structures is used [16].

3. Materials and methods

Object of study: single-span, articulated-reinforced concrete beam of rectangular cross section, made of concrete of class B20 ($R_b = 11500$ kN/m$^2$, $R_{bt} = 900$ kN/m$^2$), longitudinal reinforcing bars of class A500 ($R_s = 435000$ kN/m$^2$, $R_{s0} = 400000$ kN/m$^2$, $\xi_R = 0.493$, $\alpha_R = 0.372$). As transverse reinforcement, reinforcing bars of class A240 ($R_s = 210$ kN/m$^2$, $R_{s0} = 170$ kN/m$^2$), diameter 8 mm, rod area 0.503 cm$^2$ were used, two flat welded frames are located in the cross-section $n = 2$, $A_{sw} = 2 \times 0.503 = 1.06$ cm$^2$. Today's estimated span beams $l_0 = 5$m. Beam width $b$ is variable, beam height $h$ is variable. The working section height $h_0 = h - a$, where $a = 0.03$m. The full uniform load was taken taking into account the own weight $q = 10, 30, 50$ kN/m (Figure 1).

Strength calculation of reinforced concrete beams is based on the traditional method given in [1-2, 17-20].

External moment.

$$M = \frac{q l_0^2}{8}$$

(1)

Transverse force in normal section from external load

$$Q_{\text{max}} = \frac{q l_0}{2}$$

(2)

Coefficient $\alpha_m$

$$\alpha_m = \frac{M}{R_s b h_0^2}$$

(3)

Checking the need for compressed fittings

$$\alpha_m \leq \alpha_R$$

(4)

Required cross-sectional area of reinforced reinforcement

$$A_s = \frac{R_s b h_0 (1 - \sqrt{1 - 2\alpha_m})}{R_s}$$

(5)
Knowing the dimensions of the cross-section, the required area of the reinforcement section was obtained. The diameter of longitudinal rods was taken according to the classification [15, 17-18].

The condition for the strength of the inclined section

\[ Q \leq Q_b + Q_{sw} \]  

(6)

where \( Q \) – is the transverse design force in the cross section on the support; \( Q_b \) – is the shear force perceived by concrete; \( Q_{sw} \) – is the transverse force perceived by transverse reinforcement (clamps) [1]. The minimum transverse force perceived by concrete \( Q_b \) was calculated under the assumption that the projection of the inclined section \( c = 3h_0 \) [2].

\[ M_b = 1,5R_{c}b h_0^2 \]  

(7)

\[ Q_b = \frac{M_b}{c} \]  

(8)

where \( M_b \) – is the moment perceived by concrete, \( c \) – is the length of the projection of the inclined section.

In accordance with [17-18], we assign the pitch of the transverse reinforcement. Figure 2 shows the arrangement of transverse reinforcement.

Accept

\[ Q = Q_{max} \]  

(9)

Actual cut load on the cross rods at the pitch of the rods \( s_{w1} \)

\[ q_{sw} = \frac{R_{sw} A_{sw}}{s_{w1}} \]  

(10)

transverse force perceived by clamps

\[ Q_{sw} = 0,75q_{sw}c_0 \]  

(11)

The condition was then checked for compliance (6).

If this condition is not satisfied, the pitch of the transverse reinforcement \( s_{w1} \) decreases.

On the basis of the obtained data, the cost of the beam was calculated without taking into account the transportation of materials and manufacturing. Tables 1-2 show the average values of materials presented on the market of the Russian Federation in 2020 [17-18].
Table 1. The cost of materials.

| Material                                      | Cost        |
|-----------------------------------------------|-------------|
| Concrete of class B20                         | 3400 rub/m³ |
| Reinforcing bars of class A240 diameter 8 mm  | 43037.9 rub/t |
| Reinforcing bars of class A500                | 37099.6 rub/t |

The cost of concrete in the structure minus the volumes of longitudinal and transverse reinforcement

\[ C_b = (bhl_0 - mA_{sw}l_{sw} - nA_{lr}l_{lr})x_b \]  

(12)

where \( x_b \) – is the price of concrete per m³; \( m \) – the number of transverse rods in the beam, \( n \) – the number of longitudinal rods, \( A_{sw} \) – cross-sectional area of the transverse reinforcement, \( l_{sw} \) – the length of one clamp (transverse reinforcement)

\[ m = \frac{l_0}{2} \left( \frac{1}{s_{w1}} + \frac{1}{s_{w2}} \right) \]  

(13)

Cost of shear reinforcement

\[ C_{A_{240}} = ml_{sw}A_{sw} \rho x_{A_{240}} \]  

(14)

where \( x_{A_{240}} \) – is the cost of the transverse reinforcement, \( \rho \) is the density of the reinforcement, t/ m³,

The cost of longitudinal working reinforcement

\[ C_{A_{500}} = A_{lr}l_{lr} \rho x_{A_{500}} \]  

(15)

where \( x_{A_{500}} \) – is the cost of longitudinal working reinforcement.

The cost of the upper longitudinal reinforcement

\[ C'_{A_{500}} = 2A'_{lr}l_{lr} \rho x_{A_{500}} \]  

(16)

where \( A' \) – is the area of one rod of diameter \( d = ... \) of the upper longitudinal reinforcement in m².

The cost of reinforced concrete beam materials is

\[ C = C_b + C_{A_{240}} + C_{A_{500}} + C'_{A_{500}} \]  

(17)

4. Results of the research

4.1. The first stage

The first stage of the calculation was that with a different beam height equal to 1/5, 1/7, 1/10, 1/12, 1/15, 1/17, 1/20 of l, and with a constant beam width of 200 mm, the required reinforcement area in cross section was calculated, as well as the cost of the beam materials. The percent utilization of the cross section for various beam heights is estimated. The results are presented in tables 2-7 and in the graphs (Figure 3-4).

Table 2. Settlement area of fittings (cm²) at \( b \times h \) (mm).

| Load, kN/m | 200×1000 | 200×750 | 200×500 | 200×450 | 200×350 | 200×300 | 200×250 |
|------------|----------|---------|---------|---------|---------|---------|---------|
| 10         | 0.75     | 1.01    | 1.58    | 1.78    | 2.42    | 2.97    | 3.93    |
| 30         | 2.27     | 3.12    | 5.11    | 5.92    |         |         |         |
| 50         | 3.85     | 5.37    | 9.43    |         |         |         |         |

Note to table 2-4.
In black cells - the condition $\alpha_m < \alpha_R$ is not satisfied;

**Figure 3.** Scheme of beam reinforcement by transverse reinforcement.

From the graph on Figure 3 it can be seen that with reduction of section height from 1000 mm to 250 mm (4 times) the required area section of longitudinal stretched reinforcement increases more than 5 times from 0.75 cm$^2$ to 3.93 cm$^2$ at load level of 10 kN/m. At load 30 kN/m and section height less than 450 mm – compressed reinforcing bars is required. At a load of 50 kN/m, compressed reinforcing bars are required already at a height of less than 500 mm.

To assess the percentage of use of the cross-section, for the accepted parameters of the cross-section of the beam, table 3 shows the result of calculating the ratio of the boundary height of the compressed zone $\xi$ to the tabular value $\xi_R = 0.493$, depending on the class of reinforcement, and in table 4 the percentage of reinforcement of the cross-section, depending on the geometric dimensions section of the beam.

If we accept in accordance with the joint venture that the full use of the material is achieved at $\xi = \xi_R \ (\alpha_m = \alpha_R)$, then the actual bearing capacity of the beam is characterized by the parameter ($\xi$ or $\alpha$) that the beam reaches in the first place.
Table 3. The ratio $\xi / \xi_R$ (percentage of section usage), $b \times h$.

| Load, kN/m | 200×1000 | 200×750 | 200×500 | 200×450 | 200×350 | 200×300 | 200×250 |
|------------|----------|----------|----------|----------|----------|----------|----------|
| 10         | 2.95     | 5.39     | 12.89    | 16.28    | 28.98    | 42.19    | 68.51    |
| 30         | 8.99     | 16.63    | 41.72    | 54.08    |          |          |          |
| 50         | 15.22    | 28.60    | 76.99    |          |          |          |          |

Table 4. Percentage of section reinforcement $\mu$, %.

| Load, kN/m | 200×1000 | 200×750 | 200×500 | 200×450 | 200×350 | 200×300 | 200×250 |
|------------|----------|----------|----------|----------|----------|----------|----------|
| 10         | 0.04     | 0.07     | 0.16     | 0.20     | 0.35     | 0.49     | 0.79     |
| 30         | 0.11     | 0.21     | 0.51     | 0.66     |          |          |          |
| 50         | 0.19     | 0.36     | 0.94     |          |          |          |          |

The graph (Figure 4) shows the dependence of the percentage of section usage on the dimensions of the beam section within the specified loads.

It can be seen from the graph in Figure 4 that with a decrease in the section height from 1000 mm to 250 mm, the percentage of section use increased from 2.95 to 68.51 (23 times) at a load of 10 kN/m. You can also notice that under heavy loads, an increase in the percentage of use of the section with a decrease in the height of the section is faster.

With different beam heights equal to 1/5, 1/7, 1/10, 1/12, 1/15, 1/17, 1/20 of $l$, and with a constant beam width of 200 mm, the cost of the beam materials was calculated (table 5) and the total cost of the beam (table 6).

Table 5. The cost of the components of the beam, rub.

| Load, kN/m | 200×1000 | 200×500 | 200×500 | 200×450 | 200×450 | 200×350 | 200×350 | 200×300 | 200×250 |
|------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 10         | $C_b$ 3274.80 | 2452.37 | 1629.73 | 1465.41 | 1132.35 | 967.64 | 801.88 |
|            | $C_{A500}$ 109.21 | 147.07 | 230.07 | 259.20 | 352.39 | 432.48 | 572.27 |
|            | $C_{A240}$ 347.63 | 569.18 | 944.01 | 1077.74 | 1936.82 | 2214.85 | 2621.51 |
|            | $C'_{A500}$ 82.42 | 82.42 | 82.42 | 82.42 | 82.42 | 82.42 | 82.42 |
| 30         | $C_b$ 3272.26 | 2448.82 | 1623.77 | 1458.42 |          |          |          |
|            | $C_{A500}$ 330.55 | 454.32 | 744.10 | 862.05 |          |          |          |
|            | $C_{A240}$ 352.35 | 577.60 | 960.96 | 1097.43 |          |          |          |
|            | $C'_{A500}$ 82.42 | 82.42 | 82.42 | 82.42 |          |          |          |
| 50         | $C_b$ 3269.63 | 2445.07 | 1616.55 |          |          |          |          |
|            | $C_{A500}$ 560.62 | 781.96 | 1373.16 |          |          |          |          |
|            | $C_{A240}$ 355.71 | 583.69 | 974.64 |          |          |          |          |
|            | $C'_{A500}$ 82.42 | 82.42 | 82.42 |          |          |          |          |

From the data of table 6 it can be seen that for each loading level there is an optimal height of the beam section (highlighted in green), for which the cost of materials is minimal. With an increase or decrease in cross-sectional height, the cost of the beam increases, due to the more expensive component – reinforcement or too much concrete overrun. It is also seen that this extremum shifts toward the beams with a higher section height with increasing load level, which does not allow us to introduce a universal criterion for the optimality of an element that is independent of the external load.
Table 6. The total cost of the beam, rub.

| Load, kN/m | 200×1000 | 200×750 | 200×500 | 200×450 | 200×350 | 200×300 | 200×250 |
|------------|-----------|---------|---------|---------|---------|---------|---------|
| 10         | 3814.06   | 3251.04 | 2886.24 | 2884.77 | 3503.98 | 3697.39 | 4078.09 |
| 30         | 4037.58   | 3563.16 | 3411.25 | 3500.32 |         |         |         |
| 50         | 4268.38   | 3893.14 | 4046.77 |         |         |         |         |

It is worth noting that for each load level you can get two beams with the same cost, but one of them will be more reliable due to the greater percentage of reinforcement at a lower section height (see table 4). At a load of 30 kN/m or more, such beam sections are located in Table 6 next to such beam configurations that require reinforcement in the compressed section zone.

It seems interesting that the optimal height (with a minimum cost) does not always indicate the full use of the section (table 7).

Thus, it is possible to recommend the use of the most optimal section with a height of 500 mm with a width of 200 mm and other accepted input data. A beam of 200x500 mm is most optimally used when applying an “average” load level of 30 kN/m, which provides the highest percentage of cross-sectional use (41.72%) in this case.

4.2. The second stage
The second stage of the calculation was that with the adopted optimal beam height l/10 = 500 mm, obtained from the calculation at the first stage, with a variable width equal to 100, 150, 200, 250, 300, 350, 400, 450, 500 mm, the required reinforcement area in cross section and the cost of the beam materials were calculated. The percent utilization of the cross section for various beam heights is estimated. The results are presented in tables 8-13 and graphs (Figure 5-6).

Table 7. The parameters of the cross section of the optimal beams.

| Load, kN/m | The total cost of the beam, rub. | Percentage of section usage $\xi/\xi_R$ | Percentage of section reinforcement $\mu$ |
|------------|---------------------------------|----------------------------------------|----------------------------------------|
| 10         | 2884.77 (200×450 mm)            | 16.28                                  | 0.20                                   |
| 30         | 3411.25 (200×500 mm)            | 41.72                                  | 0.51                                   |
| 50         | 3893.14 (200×750 mm)            | 28.60                                  | 0.36                                   |

Table 8. The estimated area of the reinforcement (cm$^2$) at $b \times h$.

| Load, kN/m | 100×500 | 150×500 | 200×500 | 250×500 | 300×500 | 350×500 | 400×500 | 450×500 | 500×500 |
|------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 10         | 1.64    | 1.60    | 1.58    | 1.57    | 1.56    | 1.56    | 1.55    | 1.55    | 1.55    |
| 30         | 6.07    | 5.35    | 5.11    | 4.99    | 4.91    | 4.86    | 4.82    | 4.79    | 4.77    |
| 50         | 9.43    | 8.92    | 8.64    | 8.47    | 8.34    | 8.25    | 8.18    |         |         |

Note to tables 8-12.
In black cells - the condition $\alpha_m < \alpha_R$ is not satisfied;
Figure 5. Percentage of section use depending on the height of the beam, with a constant section width of 200 mm.

It can be seen from the graph in Figure 5 that with an increase in the cross-sectional width from 100 mm to 500 mm (5 times), with a constant cross-sectional height of 500 mm, the required cross-sectional area of the longitudinal tensile reinforcement practically does not change – from 1.64 cm$^2$ to 1.55 cm$^2$ (decreases by 5.8%), with a load level of 10 kN/m. When a load of 30 kN/m is applied, with an increase in the cross-sectional width from 100 mm to 500 mm, the required cross-sectional area of the longitudinal tensile reinforcement decreases from 6.07 cm$^2$ to 4.77 cm$^2$, i.e. by 27.3%. When a load of 50 kN/m is applied, with an increase in the cross-sectional width from 200 mm to 500 mm, the reinforcement area decreases from 9.43 cm$^2$ to 8.18 cm$^2$, i.e. by 15.3%, and with a cross-section width of less than 200 mm, it is required compressed fittings.

To assess the percentage of use of the cross-section, for the accepted parameters of the cross-section of the beam, Table 9 shows the results of calculating the ratio of the boundary height of the compressed zone $\xi$ to the tabular value $\xi = 0.493$, depending on the class of reinforcement, and in Table 9, the percentage of reinforcement of the cross-section, depending on the geometric dimensions section of the beam.

Figure 6. The estimated area of the reinforcement (cm$^2$).
Table 9. The ratio $\xi / \xi_R$, % (percentage of section usage).

| Load, kN/m | 100×500 | 150×500 | 200×500 | 250×500 | 300×500 | 350×500 | 400×500 | 450×500 | 500×500 |
|------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 10         | 26.71    | 17.38    | 12.89    | 10.24    | 8.50     | 7.26     | 6.34     | 5.62     | 5.05     |
| 30         | 99.03    | 58.28    | 41.72    | 32.56    | 26.71    | 22.65    | 19.67    | 17.38    | 15.57    |
| 50         | 76.99    | 58.28    | 47.04    | 39.49    | 34.05    | 29.93    | 26.71    |           |           |

Table 10. Percentage of section reinforcement $\mu$.

| Load, kN/m | 100×500 | 150×500 | 200×500 | 250×500 | 300×500 | 350×500 | 400×500 | 450×500 | 500×500 |
|------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 10         | 0.08     | 0.05     | 0.04     | 0.03     | 0.03     | 0.02     | 0.02     | 0.02     | 0.02     |
| 30         | 0.30     | 0.18     | 0.13     | 0.10     | 0.08     | 0.07     | 0.06     | 0.05     | 0.05     |
| 50         | 0.24     | 0.18     | 0.14     | 0.12     | 0.10     | 0.09     | 0.08     |           |           |

From the table 10 you can see that the percentage of reinforcement of the cross section for the entire range of the considered beams in principle does not exceed 0.3%.

The graph (Figure 7) shows the dependence of the percentage of use of the section on the dimensions of the section of the beam.

Figure 7. Percentage of section use depending on the beam height, with a constant section width of 200 mm.

The graph in Figure 6 shows that with an increase in the cross-sectional width from 100 mm to 500 mm, at a constant height of 500 mm, the percentage of use of the cross-section decreases: 5.3 times for a load of 10 kN/m, 6.4 times for a load 30 kN/m, 2.9 times for a load of 10 kN/m. With an average load level of 30 kN/m, with the smallest section width of 100 mm, almost full use of the section is observed − 99%.

With different beam widths equal to 100, 150, 200, 250, 300, 350, 400, 450, 500 mm, and at a constant height of 500 mm, the cost of the beam materials (table 11) and the total cost of the beam (table 12) were calculated.
Table 11. The cost of the components of the beam, rub.

| Load, kN/m | 100×500 | 150×500 | 200×500 | 250×500 | 300×500 | 350×500 | 400×500 | 450×500 | 500×500 |
|-----------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| \( C_b \) | 810.25  | 1220.01 | 1629.73 | 2039.44 | 2449.15 | 2858.84 | 3268.54 | 3678.23 | 4087.92 |
| \( C_{A500} \) | 238.81  | 232.99  | 230.07  | 228.62  | 227.16  | 227.16  | 225.70  | 225.70  | 225.70  |
| \( C_{A240} \) | 803.31  | 873.60  | 944.01  | 1014.49 | 1084.98 | 1155.52 | 1226.01 | 1296.56 | 1367.11 |
| \( C'_{A500} \) | 82.42   | 82.42   | 82.42   | 82.42   | 82.42   | 82.42   | 82.42   | 82.42   | 82.42   |

Table 12. The total cost of the beam, rub.

| Load, kN/m | 100×500 | 150×500 | 200×500 | 250×500 | 300×500 | 350×500 | 400×500 | 450×500 | 500×500 |
|-----------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 10        | 1934.79 | 2409.01 | 2886.24 | 3364.97 | 3843.70 | 4323.94 | 4802.67 | 5282.91 | 5763.15 |
| 30        | 2609.62 | 2982.62 | 3427.06 | 3889.39 | 4357.69 | 4830.46 | 5304.72 | 5780.48 | 6257.73 |
| 50        | 4046.77 | 4452.17 | 4891.31 | 5346.59 | 5807.73 | 6274.75 | 6744.71 |         |         |

From the table 10 you can see that, with an increase in the width of the section, the cost increases due to an increase for concrete, which makes up a significant proportion of the cost of the beam. With an increase in the width of the section, the price of the beam is less dependent on the external load. Obviously, beams with the smallest cross-sectional width (100 mm with a load of 10-30 kN/m and 200 mm with a load of 50 kN/m) are the most cost-effective.

Thus, it is possible to recommend the use of the most optimal section with a height of 500 mm and a width of 100 mm (and other accepted source data). A beam of 100x500 mm is the most optimally used when applying an “average” load level of 30 kN/m, which provides a percentage of section use of 99%.

Table 13. The parameters of the section of the optimal beams.

| Load, kN/m | The total cost of the beam, rub. | \( \xi/\xi_R \) | Percentage of section usage \( \xi/\xi_R \) | Percentage of section reinforcement \( \mu \) |
|-----------|--------------------------------|----------------|---------------------------------|----------------|
| 10        | 1934.79 (100x500 mm)            | 26.71           | 26.71                           | 0.08            |
| 30        | 2609.62 (100x500 mm)            |                | 99.03                           | 0.30            |
| 50        | 4046.77 (200x500 mm)            |                | 76.99                           | 0.24            |
5. Conclusions
1. For each level of loading, there is an optimal height of the beam section with a constant section width, for which the cost of beam materials is minimal.
2. For each load level, you can get two beams with the same cost, but one of them will be more reliable due to the greater percentage of reinforcement at a lower section height.
3. It seems interesting that the optimal height (with a minimum cost) does not always mean full use of the section.
4. Thus, it is possible to recommend the use of the most optimal section with a height of 500 mm and a width of 100 mm (and other accepted initial data), which is most optimally used when applying the "average" load level of 30 kN/m, which ensures almost one hundred percent use of the section.

References
[1] Karpenko N I 1996 General models of reinforced concrete mechanics (Stroyiszdat)
[2] Baykov V N, Sigalov A E 2009 Reinforced concrete structures. General course (Stroyiszdat)
[3] Alekseytsev A V and Tamrazyan A G 2019 Optimal design of load-bearing structures of buildings taking into account the relative risk of accidents Vestnik MGSU 7 819–830
[4] Tamrazyan A G and Alekseytsev A V 2020 Modern methods for optimizing structural solutions for the bearing systems of buildings and structures Vestnik MGSU 1 12–30
[5] Skladnev N N 1979 Optimal design of reinforced concrete structures taking into account the requirements of profitability, manufacturability, reliability, durability (Moscow) p 354
[6] Kuznetsov V S and Talyzova Yu A 2010 The effect of torques on the strength of crossbars in bending Vestnik MGSU 4
[7] Kuznetsov V S and Shaposhnikova Yu A 2014 Calculation of beam-free monolithic overlap in the ultimate equilibrium Vestnik MGSU 6
[8] Chakrabarty B K 1992 Models for optimal design of reinforced concrete beams Engineering
[9] Jensen B C and Lapko A 2009 On shear reinforcement design of structural concrete beams on the basis of theory of plasticity Materials Science
[10] Coello C C, Hernandez F S and Farrera F A 2014 Optimal design of reinforced concrete beams using genetic algorithms Journal of Intelligent Learning Systems and Applications 6(4)
[11] Garstecki A, Glema A and Scigallo J 2000 Optimal design of reinforced concrete beams and frames Proc. Engineering pp 440–458
[12] Demby M and Scigallo J 2017 Design Aspects of the Safe Structuring of Reinforcement in Reinforced Concrete Bending Beams Proc. Engineering pp 211–217
[13] Turner F H 2020 Fatigue design of concrete beams
[14] Nemirovsky Ju V 2014 Problems and Methods of Structural Design of Reinforced Concrete Constructions Materials Science
[15] SR 63.13330.2012 Concrete and reinforced concrete structures. The main provisions. Updated edition of SNiP 52-01-2003, 2015
[16] EN 1992 Eurocode 2: Design of concrete structures, 1998
[17] SR 52-103-2007 Monolithic reinforced concrete structures of buildings (Moscow) 2007
[18] A guide for the design of concrete and reinforced concrete structures made of heavy concrete without prestressing reinforcement (to SR 52-101-2003) (Moscow) 2005
[19] Paille G M 2013 Calculation of reinforced concrete structures (AFNOR)
[20] Seinturiere P, Service Limit State, IUT, Civil Engineering of Grenoble, 2006