Determination of material parameters of rubber and composites for computational modeling based on experiment data

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Abstract. The article deals with determination of material parameters of elastomer (rubber) for computational modeling based on experiment data - hardness and also determination of materials parameters of specific composite materials for next-generation tires. Mooney-Rivlin parameters of elastomers can be determined on the basis of the Shore A hardness. There are exist equations which can be used conversion of the mentioned hardness to material parameters of elastomers. The procedure is such that the Shore A hardness is converted to the elastic or shear modulus and then Mooney-Rivlin material parameters are determined from the modulus. But these equations can lead to different results for the same hardness. In this paper, these results are comparison. The results can be used as input material data of rubber seals of pump and compressor machinery for their computational modeling. Methods of determination of the modulus of elasticity of composite with elastomer matrix are necessary created based on experimental data of static tensile test of specimens of a radial tire casing. Next results can be applied for simulations of tires with interaction roads which used for modern passenger cars and trucks and the results can be applied for simulation of conveyor belts where elastomer is used.

1. Introduction

The topic and content of this paper, which deals with the determination of the Mooney-Rivlin material parameters of elastomers (rubbers) based on experimental data hardness Shore A and the determination of the modulus of elasticity of composites with elastomer based on experimental data, is also current in terms of the needs and requirements of the tire industry not only in Slovak Republic because the results from the project can be applied for the development of tire casings for modern passenger cars and trucks as well as for production of the products relating to other areas where they can be used appropriately (e.g. conveyor belts). The results can be used as input data to computational modelling of composite materials with elastomer matrix and new-generation tires.

For the next-generation tires, there are high requirements in relation to the tire casings, including increase in tire resistance as a whole to higher vehicle speeds, low tire rolling resistance and other requirements in terms of the economical operation of cars (reducing of the fuel consumption or increasing of the driving range of electric vehicles).
The computational modeling of next-generation tires by Finite Element Method allows solving of various ways of loading. It is possible deal with static tire models if only static load should be applied (a car stands, the target is to determine deformation conditions of inflated tires loaded by the vertical force) or dynamic tire models, which will represent real states of tires during car such as tire impact on the specific barrier at a certain hybrid electric vehicle speed.

It is important to experimentally determine material parameters used for the description of each individual part of tire-casings for computational modeling of tires [1]. Tire casings have different constructions depending on the type of transport means. The constructions of tire casings are different for passenger cars [2, 3], trucks [4, 5], off-highway cars and sport cars. A standard automobile radial tire casing consists of elastomer parts and parts with textile-cords (e.g. PA 6.6, PES textile fibers) and steel-cords in a tire tread as reinforcements [2]. The structure composite parts applied into passenger car radial tire casings are: textile carcass plies, a textile cap ply (called an overlap belt) and steel-cord belts [6].

These structures of a tire have a different geometry and material structure – cord material, cord angle (e.g. for a steel belt there is applied an angle of 21–27° in a radial tire casing for a passenger car), the construction of cord and number of layers (single-layer or multi-layer). Therefore, tires have specific deformation properties [6, 7].

For computational simulation of tire casing the results from experimental data are needed. Data about cross-sections, construction-reinforcing plies and moduli of elasticity are a necessary input for the creation of computational models of tire casings and belts for the strain-stress analysis of a tire under the vertical load, modal analysis etc. E.g. material parameters as the modulus of elasticity and Poisson ratio describe the textile fiber [1].

2. Literature review

For the description of elastomer parts of tire-casings there are used several material models of the viscoelastic behavior of the material – constitutive models, the Mooney-Rivlin (MR) model is the most commonly used. To determine of MR parameters, it is necessary to carry out the tensile test for elastomer samples by standard ISO 37 [8].

Next way, MR parameters can be determined on the basis of the Shore A hardness. The procedure is such that the Shore A hardness (it is expressed as $A$ in equations) is converted to the elastic modulus $E$ [MPa] or shear modulus $G$ [MPa] and then MR parameters are determined from the modulus. Following equations can be used conversion of the mentioned hardness:

- Gent equation [9]:

$$ E = \frac{0.0981 \cdot (56 + 7.62336 \cdot A)}{0.137505 \cdot (254 - 2.54 \cdot A)} $$  \hspace{1cm} (1)

- equation [10]:

$$ E = \exp(0.0235 \cdot A - 0.5403) $$  \hspace{1cm} (2)

- equation [11], but the elastic modulus is expressed in [psi]:

$$ E = 11.427 \cdot A - 0.4445 \cdot A^2 + 0.0071 \cdot A^3 $$  \hspace{1cm} (3)

- Batterman/Köhler equation [12] is based on expression dependence between shear modulus and Shore A hardness:

$$ G = 0.086 \cdot 1.045^A $$  \hspace{1cm} (4)

These equations can lead to different results of moduli for the same Shore A hardness.
Mooney-Rivlin parameters, such as $C_{10}$ and $C_{01}$ [MPa] are calculated according to:

$$ G = 2 \cdot (C_{10} + C_{01}) $$  \hspace{1cm} (5)

where $C_{el} = (\text{from } 0.2 \text{ to } 0.25) \cdot C_{10}$.

The parameter of incompressibility ($d$) [MPa$^{-1}$] can be calculated as:

$$ d = \frac{2 \cdot (1 - 2\nu)}{C_{10} (5\nu - 2) + C_{01} (11\nu - 5)} $$  \hspace{1cm} (6)

where $\nu$ [-] is Poisson's ratio, which has values close to 0.5 for incompressible elastomers and it is commonly considered to be 0.4995 for better convergence of calculations.

The moduli of elasticity of composite structures of tire-casings such as steel-cord belt [13] are needed as material input to tire computation models too.

The steel-cord [1, 2, 14] of typical tire casings has the construction of $2 + 2 \times 0.28$ mm (the cord consists of four filaments) or $2 \times 0.30$ mm (two filaments). Such as example, the steel-cord belts consist of two plies with a density (it is a number of ends per one meter of width, marked as EPM) of 961 m$^{-1}$ and the cord construction is $2 \times 0.30$ mm with cord angle $\pm 23^\circ$ for specific tire Matador 165 R13 [1, 6], which is taken as an etalon in this research. The thickness of the steel cord belt (two plies altogether) is 1.90 mm. The spacing between cords is 1.04 mm and volume of cords in one layer is approx. 28.6%.

Tests of composite structure parts of tire casings can be divided into:

- Static tensile tests (uniaxial and biaxial) under different temperatures;
- Static bend tests;
- Static compression tests;
- Static shear tests;
- Tests of cyclic loading (uniaxial and biaxial);
- Corrosion tests in a corrosion chamber (exposition time) etc.

The paper is oriented to static uniaxial tensile tests. Tests of specific long-fiber composite materials with an elastomer are not standardized [1]. Before performing the tests, the shape and geometric parameters of the test specimens must be designed.

In article [15], which deals with the mechanical properties of composite parts of tire casings, the geometrical parameters of single-layer tensile test specimens are given with the orientation of the cord filaments angle of $0, 10, 22, 40, 55, 72$ and $90^\circ$. The samples are dimension 152.4 x 25.4 mm.

Publications [16, 17] described different standards for testing of composites. The ASTM D 3039 standard [18] prescribes the width of the test specimens of 25 mm width, 2 to 4 mm thickness, the total length of 250 mm and a working length of 150 mm, while allowing for smaller widths, e.g. 10 mm.

A comparison of the shapes, geometrical parameters and test conditions for selected standards for laminate composites in the form of clear and comprehensible tables is provided in a publication [19]. Other publications [20, 21] deal with cord-elastomer composites with module determination based on laminate theory. But methods of determination of the moduli of elasticity of tire steel-cord belts and geometry parameters of specimens are not defined so it is necessary to create a suitable method for moduli calculation. Therefore, are proposed various methods for the determination of the moduli and calculated values of moduli of elasticity using different methods are published in this paper. The moduli are determined on the basis of the statical tensile and bend tests [1] and they can be used for verification analyses between tests and computational modeling of structure parts of tire-casings. Also, specific low-cyclic tensile tests of tire textile carcass [22] are also important for obtaining material parameters too.
3. Research methodology

The Shore A durometer is used for hardness measurement of elastomer. The constitutive two parameter Mooney-Rivlin (MR) model is used according to Equation (5) based on Equations (1–4). Verification of MR parameters is on the basis of computational model of interaction between indenter of Shore A durometer and elastomer samples. The program ANSYS is used for computational modeling.

The test machines Hounsfield H20K–W and Shimadzu Autograph AGX plus 5 kN are used for tests of composites. As the first method, there is used a method based on 0–8 % strain. The principle of the method lies in calculating the value of 8 % strain of the specimens. From the intersection of the dependencies of force-deformation and stress-strain from the uniaxial static tensile test, the values for applied force for given deformation and given specific strain are read and these values are inserted into equation 5 for calculating moduli of elasticity.

\[
E = \frac{\varepsilon}{\sigma} = \frac{F \cdot l}{\Delta l \cdot S} = \frac{(F_n - F_o) \cdot l}{(l_n - l_o) \cdot S}
\]

(7)

where \( E \) [MPa] is the modulus of elasticity, \( \sigma \) [MPa] = tensile stress, \( \varepsilon \) [%] = strain, \( F \) [N] = tensile force, \( l \) [mm] = initial length of the specimen between clamps of test machine, \( S \) [mm²] = initial cross-section area of specimen, \( \Delta l \) [mm] = length through which the specimen is elongated amount, \( F_n \) [N] = ending tensile force for concrete method, \( F_o \) [N] = starting tensile force for concrete method, \( l_n \) [mm] = ending elongation for concrete method, \( l_o \) [mm] = starting elongation for concrete method.

Other methods are similar, but the range of values of the strain is changed for the calculation: 4–8 %, 0–10 % and 4–10 %. The last method is based on reading values of the linear part of dependence. The strain into a tire casing during moving [1] can be between 0–10 %. Due to the most accurate determination of modulus of elasticity, these computing methods are suggested.

As a sample, the calculations of moduli of elasticity are made on 5 specimens with a different two-layer steel-cord belt of a radial tire casing. The specifications of specimens are shown in Table 1. The specimens are taken from tire productions making specific real parts of tire casings but some specimens are unspecified and one special specimen was created with bigger thickness of layers. The cord constructions are similar. The specimens were carved using a water jet.

| Specimen No. | Steel-cord belt accordant with the tire | Cord-angle (°) | Specimen geometry width x thickness (mm) |
|--------------|----------------------------------------|---------------|------------------------------------------|
| 1            | 165 R13                                | ±23           | 14.3 x 2.4                               |
| 2            | 235/65 R17                             | ±32           | 14.0 x 1.0                               |
| 3            | MATADOR unspecified                    | ±23           | 14.0 x 1.5                               |
| 4            | special specimen (not from real tire)  | ±23           | 14.3 x 4.5                               |
| 5            | BARUM unspecified                      | ±31           | 14.0 x 1.0                               |

The specific initial conditions of uniaxial static tensile tests are the speed of loading 10 mm/min and the initial length of specimen 80 mm between the clamps of the test machine. The specimens after the tensile tests are in Figure 1 (only specimen No. 4 did not fail).
4. Results

Tables 2 and 3 show the calculated moduli for Shore A hardness of 80 and for Shore A hardness of 90, MR parameters as well as parameter of incompressibility. It is important to point out that $C_{01}$ parameter is 0.2 multiple of $C_{10}$ parameter.

**Table 2.** Shore A hardness of 80, Poisson’s ratio $\nu = 0.4995$, $C_{01} = 0.2 \cdot C_{10}$.

| Equation | $E$ (MPa) | $G$ (MPa) | $K$ (MPa) | $C_{10}$ (MPa) | $C_{01}$ (MPa) | $C_{10} - C_{01}$ (MPa) | $d$ (MPa$^{-1}$) |
|----------|-----------|-----------|-----------|----------------|----------------|------------------------|-----------------|
| 1        | 9.3513    | 3.1181    | 3.117     | 1.5590         | 0.3118         | 1.2472                 | 0.00064         |
| 2        | 3.4545    | 1.1519    | 1.151     | 0.5759         | 0.1151         | 0.4607                 | 0.00173         |
| 3        | 11.7525   | 3.9188    | 3.917     | 1.9594         | 0.3918         | 1.5675                 | 0.00051         |
| 4        | 8.7252    | 2.9093    | 2.909     | 1.4546         | 0.2909         | 1.1637                 | 0.00068         |

**Table 3.** Shore A hardness of 90, Poisson’s ratio $\nu = 0.4995$, $C_{01} = 0.2 \cdot C_{10}$.

| Equation | $E$ (MPa) | $G$ (MPa) | $K$ (MPa) | $C_{10}$ (MPa) | $C_{01}$ (MPa) | $C_{10} - C_{01}$ (MPa) | $d$ (MPa$^{-1}$) |
|----------|-----------|-----------|-----------|----------------|----------------|------------------------|-----------------|
| 1        | 20.8439   | 6.9503    | 6.947     | 3.4751         | 0.6950         | 2.7801                 | 0.00028         |
| 2        | 4.3697    | 1.4570    | 1.456     | 0.7285         | 0.1457         | 0.5828                 | 0.00137         |
| 3        | 17.9531   | 5.9863    | 5.984     | 2.9931         | 0.5986         | 2.3945                 | 0.00033         |
| 4        | 13.5500   | 4.5181    | 4.518     | 2.2590         | 0.4518         | 1.8072                 | 0.00044         |

Based on the values in Tables 2 and 3, it can be stated that there is also influence of hardness change because one equation can be suitable for hardness of 80 and any other can be suitable for hardness of 90. The resulting MR parameters have to be verified because this is the only one possible way how to find out the most suitable equation for the given application. The mentioned fact is the reason for creation of FEM computational model of interaction between indenter and elastomeric material. The given model is used for simulation of hardness testing process by Shore A method.

The computational model of interaction between indenter of Shore A durometer and elastomer samples is on the Figure 2. The problem is modelled as planar stress (with type of element PLANE 182). The common construction steel is specified to be material for indenter, the elastic modulus is 200 000 MPa and Poisson's ratio is 0.3. The model of tested elastomeric sample is based on the two parameter MR model. The parameters, which can be found in Tables 2 and 3, were used in relation to the computational modeling. Non-linear contact is used between indenter and tested elastomeric sample. In the Figure 3, there is an example of the strain of tested elastomeric sample and it is based
on definition of MR parameters, which were calculated by help of equation with designation as (1) and it was for hardness of 80.

![Computational model](image1.png)

**Figure 2.** Computational model for verification of calculated MR parameters with detailed image of contact area between indenter and tested elastomeric sample [23].

![Displacement filed](image2.png)

**Figure 3.** Displacement field of the tested sample after indenter displacement by 0.5 mm [23].

Table 4 reflects the forces in the spring during experimental measurement as well as there are resulting reaction forces corresponding to indenter displacement by 0.5 and 0.25 mm in relation to the predetermined equations (equations with designation from 1 to 4). The preload or prestress of spring is 0.55 N. In the case of calculated reaction forces, the given preload or prestress has to be also taken into account.

Considering the results, it can be stated that more accurate results for hardness of 80 and 90 are obtained by help of equation, which is designated as 1 because after comparison of experimental results with the calculated results (using equation with designation as 1), the difference is 3.5 % for hardness of 80 and 6 % for hardness of 90. Based on the mentioned fact above, MR parameters for the range of hardness from 80 to 90 are recommended to be obtained from equation with designation as 1.
Table 4. Calculated reaction forces with corresponding Shore A hardness.

| Equation | Shore A hardness of 80 – indenter shift by 0.5 mm | Shore A hardness of 90 – indenter shift by 0.25 mm |
|----------|-----------------------------------------------|-----------------------------------------------|
|          | Spring force with preload (prestress) (N) | Resulting reaction force (N) | Optimized reaction force by preload (prestress) (N) | Hardness (HSA) | Spring force with preload (prestress) (N) | Resulting reaction force (N) | Optimized reaction force by preload (prestress) (N) | Hardness (HSA) |
| 1        | 6.55                                          | 5.77                                          | 6.32                                          | 77              | 7.3                                          | 6.31                                          | 6.86                                          | 84              |
| 2        | 6.55                                          | -                                             | -                                             | -               | 7.3                                          | 1.32                                          | 1.87                                          | 17              |
| 3        | 6.55                                          | 7.25                                          | 7.80                                          | 97              | 7.3                                          | -                                             | -                                             | -               |
| 4        | 6.55                                          | -                                             | -                                             | -               | 7.3                                          | 4.10                                          | 4.65                                          | 55              |

*it was not determined because there was not convergence of calculation*

Figure 4. Force-deformation dependences from tensile tests of specimens.

The tensile force-deformation dependences are in Figure 4. The stress-strain dependences for obtain of moduli of elasticity are in Figure 5.

After deduction of the values of the graph for each method are substitute into the form and calculated the modulus of elasticity by Equation (7).
Figure 5. Stress-strain dependences from tensile tests of specimens.

For comparison of the results has been made synoptic Table 5. From the table we can see differences in the methods for determination of the modulus of elasticity for each specimen 1–5. These moduli of elasticity are in longitudinal direction of tires.

From the results was the task as precisely as possible the method for determination of the modulus of elasticity, which is suitable for other specimens of steel-cord belt configurations.

Because of nonlinear character given depending, force-deformation dependences, we have identified several methods that we consider most appropriate for determination of modulus of elasticity. The comparing of the results of modulus of elasticity obtained by individual methods between the specimens, we concluded that the most appropriate method to calculate of modulus of elasticity of steel-cord belts is a method based on determination of the modulus of elasticity of 4–8\% elongation (in the Table 5 are the results of the moduli of elasticity by this method shown in bold).

5. Conclusions
Based on the obtained results, it can be concluded that we recommend to use Gent equation (with designation as 1) for calculation of MR parameters for hardness of 80 and 90.

The values of moduli of elasticity are used as inputs into computational models of tires for strain-stress analyses of the tire. The Finite Element Method using the program system ANSYS is applied to the computational simulation. The verification of the calculation is very difficult because we do not have the results of calculations of modulus of elasticity for these specific composite specimens by other methods, whether from static tensile tests, or other such non-destructive methods of determination of the modulus of elasticity using other test equipment.

In the evaluation results must also consider the mistake which could be shown in the results. Mistake can be caused by several reasons. The reason may be wrong of measuring device. For example, it can be sliding of specimens from the brackets (such as specimen No. 4). Of course, mistake can be caused by methods of determination too. Accuracy of results is sufficient, because the results of moduli of elasticity can be entered as initial values for material description of steel-cord belts and will be used for tire computational models.
The evaluation of various methods, it was concluded that the most appropriate method for determination of the moduli of elasticity is method of 4–8 % elongation.

Further research work will be focused on tests of cyclic loading of composites using uniaxial and biaxial tensile loading at different temperatures with determination of material parameters such as moduli of elasticity for computational tire simulations.

Table 5. Moduli of elasticity for specific tire composite structure specimens.

| Specimen No. | Methods start–end of strain (%) | E (MPa) |
|--------------|---------------------------------|---------|
| 1            | 0–8                             | 225.8   |
|              | 4–8                             | 218.5   |
|              | 0–10                            | 223.1   |
|              | 4–10                            | 233.1   |
|              | linear part                     | 265.4   |
|              | 0–8                             | 55.4    |
|              | 4–8                             | 53.6    |
| 2            | 0–10                            | 58.6    |
|              | 4–10                            | 59.5    |
|              | linear part                     | 200.0   |
|              | 0–8                             | 357.1   |
|              | 4–8                             | 380.9   |
| 3            | 0–10                            | 380.9   |
|              | 4–10                            | 412.6   |
|              | linear part                     | 1 023.1 |
|              | 0–8                             | 109.2   |
|              | 4–8                             | 65.5    |
| 4            | 0–10                            | 101.3   |
|              | 4–10                            | 67.0    |
|              | linear part                     | 64.0    |
|              | 0–8                             | 106.7   |
|              | 4–8                             | 151.7   |
| 5            | 0–10                            | 160.7   |
|              | 4–10                            | 154.7   |
|              | linear part                     | 295.2   |

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