CHALLENGES FOR NON-MINIMAL HIGGS SEARCHES AT FUTURE COLLIDERS

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Abstract
In models with a non-minimal Higgs sector, the lightest scalar state may be a neutral CP-even Higgs boson, whose properties are nearly identical to those of the minimal Higgs boson of the Standard Model. In such a scenario, the other Higgs scalars are significantly heavier than $m_Z$; their effects rapidly decouple from the low-energy theory. The decoupling limit of the most general CP-conserving two-Higgs doublet model is formulated. Detection of evidence for a non-minimal Higgs sector in the decoupling limit presents a formidable challenge for Higgs searches at future colliders.

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1. Introduction

Let us suppose that a candidate for the Higgs boson is discovered in a future collider experiment. What are the expectations for its properties? Will it resemble the Higgs boson of the minimal Standard Model or will it possess some distinguishing trait? If the properties of this scalar state are difficult to distinguish from the Standard Model Higgs boson, what are the requirements of future collider experiments for detecting the existence (or non-existence) of a non-minimal Higgs sector? See e.g., refs. 2 and 3 for earlier attempts to address these questions.

I shall address these questions in the context of the (CP-conserving) two-Higgs doublet model. In this model, the physical scalar states consist of a charged Higgs pair ($H^\pm$), two CP-even scalars ($h^0$ and $H^0$, with $m_{h^0} \leq m_{H^0}$) and one CP-odd scalar ($A^0$). The ultimate conclusions of this paper will survive in models with more complicated scalar sectors. The working hypothesis of this paper is that $h^0$, assumed to be the lightest scalar state, will be the first Higgs boson to be discovered. Moreover, the mass gap between $h^0$ and the heavier scalars is assumed to be sufficiently large so that the initial experiments which can detect $h^0$ will not have sufficient energy and luminosity to discover any of the heavier scalar states.

How will $h^0$ be discovered and where? Present LEP bounds imply that $m_{h^0} \gtrsim 60$ GeV. This bound will be improved by LEP-II, which will be sensitive to Higgs masses up to roughly $\sqrt{s} - m_Z - 10$ GeV. The LEP search is based on $e^+e^- \rightarrow Z \rightarrow Zh^0$ where one of the two Z’s is on-shell and the other is off-shell. At hadron colliders, an upgraded Tevatron with an integrated luminosity of 10 fb$^{-1}$ can begin to explore the intermediate-mass Higgs regime ($80 \lesssim m_{h^0} \lesssim 130$ GeV). The Higgs search at the LHC will significantly extend the Higgs search to higher masses (although the intermediate mass regime still presents some significant difficulties for the LHC detector collaborations). The dominant mechanism for Higgs production at hadron colliders is via $gg$-fusion through a top-quark loop. If $m_{h^0} > 2m_Z$, the “gold-plated” detection mode is $h^0 \rightarrow ZZ$; each Z subsequently decays leptonically, $Z \rightarrow \ell^+\ell^-$ (for $m_{h^0} \gtrsim 130$ GeV, $h^0 \rightarrow ZZ^*$, where $Z^*$ is off-shell, provides a viable signature). Other decay modes are required in the case of the intermediate mass Higgs (for recent reviews, see refs. 7 and 9). At a future $e^+e^-$ linear collider (NLC), the Higgs mass reach of LEP-II will be extended. In addition, with increasing $\sqrt{s}$, Higgs boson production via $W^+W^-$ fusion begins to be the dominant production process. Finally, Higgs production via $\gamma\gamma$-fusion through a $W^+W^-$ and a $t\bar{t}$ loop may be detectable at the $\gamma\gamma$ collider option of the NLC, depending on the particular Higgs final state decay. Note that almost all of the Higgs search techniques outlined above involve the $h^0ZZ$ (and in some cases the $h^0W^+W^-$) vertex. In a few cases, it is the $h^0t\bar{t}$ vertex (and possibly the $h^0b\bar{b}$ vertex) that plays the key role.
In section 2, I briefly review the Higgs sector of the minimal supersymmetric extension of the Standard Model (MSSM). In this context, I discuss under which circumstances one might expect $h^0$ to be the lightest scalar whose properties are nearly identical to those of the Standard Model Higgs boson. In section 3, I place the results of section 2 in a more general context. I define the “decoupling limit” of the general two-Higgs doublet model; in this limit, $h^0$ is indistinguishable from the Standard Model Higgs boson. In section 4, I discuss the phenomenological challenges of the decoupling limit for the Higgs search at future colliders. After briefly mentioning the prospects for non-minimal Higgs detection at the LHC, I consider in more detail the prospects for the discovery of the non-minimal Higgs sector at the NLC. Conclusions are presented in section 5.

2. The Higgs Sector of the MSSM—A Brief Review

The Higgs sector of the minimal supersymmetric extension of the Standard Model (MSSM) consists of two complex doublet scalar fields $H_1$ and $H_2$ of hypercharge $-1$ and $+1$, respectively.\textsuperscript{13} Because of the underlying supersymmetry, the tree-level Higgs masses and couplings are determined in terms of two free parameters: $m_{A^0}$ and $\tan \beta = v_2/v_1$ [where $v_2$ ($v_1$) is the vacuum expectation value of the Higgs field that couples to up-type (down-type) fermions]. Then, the other (tree-level) Higgs masses are given by

\[ m_{H^\pm}^2 = m_W^2 + m_{A^0}^2, \]
\[ m_{H^0, h^0}^2 = \frac{1}{2} \left( m_{A^0}^2 + m_Z^2 \pm \sqrt{(m_{A^0}^2 + m_Z^2)^2 - 4m_Z^2m_{A^0}^2 \cos^2 2\beta} \right). \] (2.1)

The mass eigenstates $H^0$ and $h^0$ are linear combinations of the original Higgs fields of definite hypercharge

\[ H^0 = (\sqrt{2} \Re H_1^0 - v_1) \cos \alpha + (\sqrt{2} \Re H_2^0 - v_2) \sin \alpha \]
\[ h^0 = -(\sqrt{2} \Re H_1^0 - v_1) \sin \alpha + (\sqrt{2} \Re H_2^0 - v_2) \cos \alpha \] (2.2)

which defines the CP-even Higgs mixing angle $\alpha$. Explicit formulae for $\alpha$ can also be derived. Here, I shall note one particularly useful relation

\[ \cos^2(\beta - \alpha) = \frac{m_{h^0}^2 (m_Z^2 - m_{h^0}^2)}{m_{A^0}^2 (m_{H^0}^2 - m_{h^0}^2)}. \] (2.3)
Consider the limit where $m_{A^0} \gg m_Z$. Then, from the above formulae,

$$
\begin{align*}
    m_{h^0}^2 &\simeq m_Z^2 \cos^2 2\beta, \\
    m_{H^0}^2 &\simeq m_{A^0}^2 + m_Z^2 \sin^2 2\beta, \\
    m_{H^\pm}^2 &= m_{A^0}^2 + m_W^2, \\
    \cos^2 (\beta - \alpha) &\simeq \frac{m_Z^4 \sin^2 4\beta}{4m_{A^0}^4}.
\end{align*}
$$

Two consequences are immediately apparent. First, $m_{A^0} \simeq m_{H^0} \simeq m_{H^\pm}$, up to corrections of $O(m_Z^2/m_{A^0})$. Second, $\cos(\beta - \alpha) = 0$ up to corrections of $O(m_Z^2/m_{A^0}^2)$. Although the radiative corrections to the Higgs masses can have a profound effect on the phenomenology, the overall size of such corrections is never larger than $O(m_Z)$, and hence the consequences of eq. (2.4) noted above remain valid.

The phenomenological implications of these results may be discerned by reviewing the coupling strengths of the Higgs bosons to Standard Model particles (gauge bosons, quarks and leptons) in the two-Higgs doublet model. The coupling of $h^0$ and $H^0$ to vector boson pairs or vector-scalar boson final states is proportional to either $\sin(\beta - \alpha)$ or $\cos(\beta - \alpha)$ as indicated below.\cite{1,13}

\begin{align*}
    \cos(\beta - \alpha) &\simeq \frac{m_Z^4 \sin^2 4\beta}{4m_{A^0}^4} \\
    \sin(\beta - \alpha) &\simeq \frac{m_Z^4 \sin^2 4\beta}{4m_{A^0}^4}.
\end{align*}

Note in particular that all vertices in the theory that contain at least one vector boson and exactly one heavy Higgs boson state ($H^0$, $A^0$ or $H^\pm$) are proportional to $\cos(\beta - \alpha)$. This can be understood as a consequence of unitarity sum rules which must be satisfied by the tree-level amplitudes of the theory.\cite{15}

In models with a non-minimal Higgs sector, the Higgs couplings to quarks and leptons are model-dependent. In the MSSM, one Higgs doublet ($H_1$) couples exclusively to down-type fermions and the second Higgs doublet ($H_2$) couples exclusively to up-type fermions. In this case, the couplings of the neutral CP-even Higgs bosons to $f \bar{f}$ relative to the corresponding Standard Model value (using 3rd

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\textit{Note:} The above text is a simplified version of the original content, focusing on the mathematical expressions and their implications. Further details and context are provided in the referenced sources.
family notation) are given by

\[ h^0 b \bar{b} : \quad -\frac{\sin \alpha}{\cos \beta} = \sin(\beta - \alpha) - \tan \beta \cos(\beta - \alpha), \]

\[ h^0 t \bar{t} : \quad \frac{\cos \alpha}{\sin \beta} = \sin(\beta - \alpha) + \cot \beta \cos(\beta - \alpha), \]

\[ H^0 b \bar{b} : \quad \frac{\cos \alpha}{\cos \beta} = \cos(\beta - \alpha) + \tan \beta \sin(\beta - \alpha), \]

\[ H^0 t \bar{t} : \quad \frac{\sin \alpha}{\sin \beta} = \cos(\beta - \alpha) - \cot \beta \sin(\beta - \alpha). \]

In contrast to the Higgs couplings to vector bosons, none of the couplings in eq. (2.5) vanish when \( \cos(\beta - \alpha) = 0 \). The significance of \( \cos(\beta - \alpha) = 0 \) is now evident: in this limit, couplings of \( h^0 \) to gauge boson pairs and fermion pairs are identical to the couplings of the Higgs boson in the minimal Standard Model. More precisely, in the limit of \( m_{A^0} \gg m_Z \), the effects of the heavy Higgs states \( (H^\pm, H^0, A^0) \) decouple, and the low-energy effective scalar sector is indistinguishable from that of the minimal Standard Model.

3. Decoupling Properties of the Two-Higgs Doublet Model\(^{16}\)

The decoupling properties of the MSSM Higgs sector are not special to supersymmetry. Rather, they are a generic feature of non-minimal Higgs sectors.\(^{17}\) In this section, I demonstrate this assertion in the case of the most general CP-conserving two-Higgs doublet model. Let \( \Phi_1 \) and \( \Phi_2 \) denote two complex \( SU(2)_L \) doublet scalar fields.\(^{\dagger}\) The most general gauge invariant scalar potential is given by

\[ V = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}] \]

\[ + \left\{ \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \right\} \quad (3.1) \]

\[ + \left\{ \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + [\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\}. \]

In principle, \( m_{12}^2, \lambda_5, \lambda_6 \) and \( \lambda_7 \) can be complex. Here, I shall ignore the possibility of CP-violating effects in the Higgs sector by choosing all coefficients in eq. (3.1)

\(^*\) Likewise, the \( h^0 h^0 h^0 \) and \( h^0 h^0 h^0 h^0 \) couplings also reduce to their Standard Model values when \( \cos(\beta - \alpha) = 0 \), while in the same limit the other Higgs self-couplings (which involve \( H^0, A^0, \) and/or \( H^\pm \)) do not vanish.

\(^\dagger\) In terms of the \( Y = \pm 1 \) fields of the previous section, \( H_1^\pm = \epsilon_y \Phi_i^\dagger \) and \( H_2 = \Phi_2 \).
to be real. The scalar fields will develop non-zero vacuum expectation values if the mass matrix $m_{ij}^2$ has at least one negative eigenvalue. Imposing CP invariance and $U(1)_{EM}$ gauge symmetry, the minimum of the potential is $\langle \Phi_i^0 \rangle \equiv v_i/\sqrt{2}$ ($i = 1, 2$), where the $v_i$ can be chosen to be real and positive. It is convenient to introduce the following notation: $v^2 \equiv v_1^2 + v_2^2 = (246 \text{ GeV})^2$ and $t_\beta \equiv \tan \beta \equiv v_2/v_1$. Eliminating $m_{11}^2$ and $m_{22}^2$ by minimizing the scalar potential, the squared masses for the CP-odd and charged Higgs states are obtained:

$$m_{A^0}^2 = \frac{m_{12}^2}{s_\beta c_\beta} - \frac{1}{2} v^2 (2 \lambda_5 + \lambda_6 t_\beta^{-1} + \lambda_7 t_\beta),$$

$$m_{H^\pm}^2 = m_{A^0}^2 + \frac{1}{2} v^2 (\lambda_5 - \lambda_4),$$

where $s_\beta \equiv \sin \beta$ and $c_\beta \equiv \cos \beta$. The two CP-even Higgs states mix according to the following squared mass matrix:

$$M^2 = m_{A^0}^2 \begin{pmatrix} s_\beta & -s_\beta c_\beta \\ -s_\beta c_\beta & c_\beta^2 \end{pmatrix} + v^2 \begin{pmatrix} M_{11}^2 & M_{12}^2 \\ M_{12}^2 & M_{22}^2 \end{pmatrix},$$

where

$$\begin{pmatrix} M_{11}^2 & M_{12}^2 \\ M_{12}^2 & M_{22}^2 \end{pmatrix} \equiv \begin{pmatrix} \lambda_1 c_\beta^2 + 2 \lambda_6 s_\beta c_\beta + \lambda_5 s_\beta^2 & (\lambda_3 + \lambda_4) s_\beta c_\beta + \lambda_6 c_\beta^2 + \lambda_7 s_\beta^2 \\ (\lambda_3 + \lambda_4) s_\beta c_\beta + \lambda_6 c_\beta^2 + \lambda_7 s_\beta^2 & \lambda_2 s_\beta^2 + 2 \lambda_7 s_\beta c_\beta + \lambda_5 c_\beta^2 \end{pmatrix}$$

(3.4)

It is convenient to define four squared mass combinations:

$$m_L^2 \equiv M_{11}^2 \cos^2 \beta + M_{22}^2 \sin^2 \beta + M_{12}^2 \sin 2\beta,$$

$$m_D^2 \equiv (M_{11}^2 M_{22}^2 - M_{12}^2)^{1/2},$$

$$m_T^2 \equiv M_{11}^2 + M_{22}^2,$$

$$m_S^2 \equiv m_{A^0}^2 + m_T^2.$$

In terms of these quantities,

$$m_{H^0,H^0}^2 = \frac{1}{2} \left[ m_S^2 \pm \sqrt{m_S^4 - 4m_{A^0}^2 m_L^2 - 4m_D^4} \right],$$

(3.6)

and

$$\cos^2(\beta - \alpha) = \frac{m_L^2 - m_{H^0}^2}{m_{A^0}^2 - m_{H^0}^2}.\quad (3.7)$$

Suppose that all the Higgs self-coupling constants $\lambda_i$ are held fixed such that $\lambda_i \lessapprox 1$, while taking $m_{A^0}^2 \gg \lambda_i v^2$. This will be called the decoupling limit of
the model. Then the $M^2_{ij} \sim \mathcal{O}(v^2)$, and it follows that:

\[ m_{h^0} \simeq m_L, \quad m_{H^0} \simeq m_{A^0} \simeq m_{H^\pm}, \quad (3.8) \]

and

\[ \cos^2(\beta - \alpha) \simeq \frac{m_L^2 (m_T^2 - m_L^2) - m_D^4}{m_{A^0}^4}. \quad (3.9) \]

Comparing these results with those of eq. (2.4), one sees that the MSSM results are simply a special case of the more general two-Higgs doublet model results just obtained. In particular, eq. (3.9) implies that $\cos(\beta - \alpha) = \mathcal{O}(m_Z^2/m_{A^0}^2)$ in the decoupling limit, which means that the $h^0$ couplings to Standard Model particles match precisely those of the Standard Model Higgs boson.

It is interesting to attempt to circumvent the decoupling limit while maintaining the hierarchy of Higgs masses, $m_{h^0} \ll m_{H^0}, m_{A^0}, m_{H^\pm}$. The latter implies [using eq. (3.6)] that

\[ 0 < m_{A^0}^2 m_L^2 + m_D^4 \ll m_S^4, \quad (3.10) \]

Eq. (3.10) is satisfied in one of two cases:

(i) $m_Z^2, m_L^2, m_D^4/m_{A^0}^2 \ll m_{A^0}, m_S^2$. That is, each term on the left-hand side of eq. (3.10) is separately smaller in magnitude than $m_S^4$, or

(ii) $m_{A^0}^2 m_L^2$ and $m_D^4$ are both of $\mathcal{O}(m_S^4)$, but due to cancelation of the leading behavior of each term, the sum satisfies eq. (3.10).

In case (i), one finds that

\[ m_{h^0}^2 \simeq \frac{m_{A^0}^2 m_L^2}{m_S^2} + \frac{m_D^4}{m_S^2} + \frac{m_{A^0}^2 m_A^4}{m_S^6} + \mathcal{O}\left(\frac{m_L^4}{m_S^4}\right), \quad (3.11) \]

and $m_{H^0}^2 \sim \mathcal{O}(m_S^2)$. In the same approximation,

\[ \cos^2(\beta - \alpha) \simeq \frac{m_L^2}{m_S^2} \left(1 - \frac{m_{A^0}^2}{m_S^2}\right) + \frac{1}{m_S^4} \left[ m_A^4 \left(\frac{2m_{A^0}^2}{m_S^2} - 3m_{A^0}^4\right) - m_D^4\right]. \quad (3.12) \]

The behavior of $\cos(\beta - \alpha)$ depends crucially on how close $m_{A^0}^2/m_S^2$ is to 1. If $m_T^2 \ll m_S^2$ [see eq. (3.5)], we recover the results of the decoupling limit [see eqs. (3.8) and (3.9)]. On the other hand, if $m_T^2 \sim \mathcal{O}(m_S^2)$, then eq. (3.12) implies that $\cos(\beta - \alpha) \sim \mathcal{O}(m_Z/m_{A^0})$. This is a particular region of the parameter space where some of the $\lambda_i$ are substantially larger than 1, and yet the $h^0$ couplings do not significantly deviate from those of the Standard Model. Nevertheless, the onset
of decoupling is slower than the \( \cos(\beta - \alpha) \sim \mathcal{O}(m_Z^2/m_{A_0}^2) \) behavior found in the decoupling regime. In order to find a parameter regime which exhibits complete non-decoupling, one must consider case (ii) above. In this case, despite the fact that \( m_{h_0} \ll m_{H_0}, m_{A_0}, m_{H^\pm} \), one nevertheless finds that \( \cos(\beta - \alpha) \sim \mathcal{O}(1) \), which implies that the couplings of \( h^0 \) deviate significantly from those of the Standard Model Higgs boson. Although it might appear that case (ii) requires an unnatural cancelation, it is easy to construct a simple model of non-decoupling. Consider a model where:

\[
\begin{align*}
    m_{h_0}^2 &= \lambda_6, \\
    m_{H_0}^2 &= \lambda_1, \\
    m_{A_0}^2 &= -\lambda_5, \\
    m_{H^\pm}^2 &= \frac{1}{2} \lambda_3 v^2,
\end{align*}
\]

and \( \cos^2(\beta - \alpha) = c_\beta^2 \). Note that in this model, the heavy Higgs states are not approximately degenerate (as required in the decoupling limit).

4. Phenomenological Challenges of the Decoupling Limit

In the decoupling limit, the couplings of \( h^0 \) to Standard Model gauge bosons and fermions approach those of the Standard Model Higgs boson. Suppose that a future experiment has already discovered and studied the properties of \( h^0 \). What are the requirements of experiments at future colliders for proving the existence or non-existence of a non-minimal Higgs sector? To be specific, let us assume in this section that \( h^0 \) has been discovered with couplings approximating those of the Standard Model Higgs boson and \( m_{A_0} \geq 250 \text{ GeV} \).

At the LHC, the rate for \( gg \rightarrow A_0, H^0 \) and \( gb \rightarrow H^- t \) provides a substantial number of produced Higgs bosons per year (assuming that the heavy Higgs masses are not too large).\(^{18}\) Unfortunately, there may not be a viable final state signature. For example, since \( \cos^2(\beta - \alpha) \ll 1 \), the branching ratio of \( H^0 \rightarrow ZZ \) is significantly suppressed, so that the gold-plated Standard Model Higgs signature is simply absent. At present, there is no known proven technique for detecting \( A_0, H^0 \) and \( H^\pm \) signals at the LHC in the decoupling regime of parameter space. An attempt to isolate a Higgs signal in \( t\bar{t} \) final states has been discussed in ref. 19. Another method consists of a search for Higgs signals in \( t\bar{t}t\bar{t}, t\bar{t}b\bar{b}, \) and \( bb\bar{b}\bar{b} \) final states.\(^{20}\) These can arise from \( gg \rightarrow QQ'(H^0, A^0 \text{ or } H^\pm) \), where \( Q \) is a heavy quark (\( b \) or \( t \)), and the Higgs boson subsequently decays into a heavy quark pair. As noted below eq. (2.5), even in the decoupling limit, the couplings of \( H^0, A^0 \) and \( H^\pm \) to heavy quark pairs are not suppressed. Whether such signals can be extracted from the substantial QCD backgrounds (very efficient \( b \)-tagging is one of the many requirements for such a search) remains to be seen.*

* For an optimistic assessment, see ref. 7. In the large \( \tan \beta \) regime, the enhanced couplings of the heavy Higgs bosons to \( bb \) could lead to an observable signal in \( gg \rightarrow bb(H^0 \text{ or } A^0) \) followed by Higgs decay to either \( bb \) or \( \tau^+\tau^- \).
Let us now turn to $e^+e^-$ colliders. Consider the process $e^+e^- \rightarrow h^0A^0$ (via $s$-channel $Z$-exchange). Since the $Zh^0A^0$ coupling is proportional to $\cos(\beta - \alpha)$, the production rate is suppressed in the decoupling regime. For example, in the MSSM, if $m_{A^0} > m_{h^0}$, then LEP-II will not possess sufficient energy and/or luminosity to directly produce the $A^0$. Of course, with sufficient energy, one can directly pair-produce the heavy Higgs states via $e^+e^- \rightarrow H^+H^-$ and $e^+e^- \rightarrow H^0A^0$ without a rate suppression. At the NLC, with $\sqrt{s} = 500$ GeV and 10 fb$^{-1}$ of data, it has been shown$^{21,22}$ that no direct signals for $A^0$, $H^0$, and $H^\pm$ can be seen if $m_{A^0} \gtrsim 230$ GeV. These results support the following general conclusion: *evidence for the non-minimal Higgs sector at the NLC requires a machine with energy $\sqrt{s} > 2m_{A^0}$, sufficient to pair-produce heavy Higgs states.* Can this conclusion be avoided? There are two possible methods: (i) produce one heavy Higgs state in association with lighter states, or (ii) make precision measurements of $h^0$ couplings to Standard Model particles in order to detect a deviation from the Standard Model expectations.

First, consider the processes that yield a singly produced heavy Higgs state. In section 2, I noted that whenever a single heavy Higgs state couples directly to a gauge boson plus other light particles, the coupling is suppressed by $\cos(\beta - \alpha)$. To avoid this suppression, one must couple the heavy Higgs state to either fermions or scalars. For example, the production rates for $e^+e^- \rightarrow Q\bar{Q}'(H^0, A^0, \text{or } H^\pm)$, where $Q = b$ or $t$, have been computed by Djouadi et al.$^{23}$ Unfortunately, the three-body phase space greatly suppresses the rate once the heavy Higgs state is more massive than the $Z$. In particular, for $m_{A^0} > \sqrt{s}/2$, these processes do not provide viable signatures for the heavy Higgs states. A similar conclusion is obtained when the heavy Higgs state couples to light scalars. Thomas and I have computed the rate for $e^+e^- \rightarrow h^0H^0Z$, assuming that the dominant contribution is due to the $s$-channel $Z$-exchange, where the virtual $Z^*$ decays via $Z^* \rightarrow Zh^0\rightarrow Zh^0H^0$. For $m_{H^0} \gg m_{h^0}, m_Z$, we obtained$^{16}$

$$
\frac{\sigma(e^+e^- \rightarrow h^0H^0Z)}{\sigma(e^+e^- \rightarrow h^0Z)} \approx \frac{g_{H^0h^0H^0}^2}{32\pi^2s^3m_{H^0}^2}\left\{ (s - m_{h^0}^2) \left[ (s - m_{H^0}^2)^2 + 12sm_{H^0}^2 \right] -6m_{H^0}^2s(s + m_{H^0}^2)\ln\left( \frac{s}{m_{H^0}^2} \right) \right\}, \tag{4.1}
$$

where the $H^0h^0h^0$ coupling in the decoupling limit [i.e., when $\cos(\beta - \alpha) = 0$] in the MSSM is given by

$$
g_{H^0h^0h^0} = -\frac{3igm_Z}{4\cos\theta_W} \sin 4\beta. \tag{4.2}
$$

The rate obtained in eq. (4.1) is too small for a viable Higgs signal.
Fig. 1. Contour lines of fractional deviation, $\Delta BR/BR(\text{SM})$, are exhibited for $h^0 \rightarrow b\bar{b}$, where $\Delta BR \equiv BR(\text{MSSM}) - BR(\text{SM})$ is the difference between the corresponding branching ratios in the MSSM and the Standard Model. A given value of $\tan \beta$ and $m_{A^0}$ (the latter expressed in GeV units above) determines the $h^0$ mass and couplings. (Leading log radiative corrections have also been included, with $m_t = 175$ GeV, and a common mass of 1 TeV for all supersymmetric particles.)

Second, consider precision measurements of $h^0$ branching ratios at the NLC. In a Monte Carlo analysis, Hildreth et al.\textsuperscript{24} evaluated the anticipated accuracy of $h^0$ branching ratio measurements at the NLC, assuming $\sqrt{s} = 500$ GeV and a data set of 50 fb$^{-1}$. For example, taking $m_{A^0} = 120$ GeV, they computed an extrapolated error of $\pm 7\%$ for the 1-$\sigma$ uncertainty in $BR(h^0 \rightarrow b\bar{b})$ and $\pm 14\%$ for $BR(h^0 \rightarrow \tau^+\tau^-)$. Other channels yielded substantially larger uncertainties. To see whether these are significant measurements, Hildreth and I compared the $h^0$ branching ratios computed in the MSSM (including one-loop leading logarithmic radiative corrections\textsuperscript{14}) as a function of $m_{A^0}$ and $\tan \beta$ to the corresponding Standard Model branching ratios.\textsuperscript{25} (See ref. 21 for a related analysis.) The fractional deviation of $BR(h^0 \rightarrow b\bar{b})$ from the corresponding Standard Model value is shown in Fig. 1. [The analogous plot for $BR(h^0 \rightarrow \tau^+\tau^-)$ is nearly identical to Fig. 1.] Note that the (negative) deviation shown in Fig. 1 is about 7% for values of $m_{A^0}$ as large
as 450 GeV and about 15% for values of $m_{A^0}$ as large as 250 GeV. These results imply that a precision measurement of $h^0 \to b\bar{b}$ has the potential for detecting the existence of a non-minimal Higgs sector even if the heavier Higgs states cannot be directly detected at the NLC. Of course, one will have to push the precision of this measurement beyond the present expectations to obtain a significant result, since a 2-σ deviation is not compelling evidence for new physics. Other Higgs decay channels are not competitive.

![Fig. 2. Contour lines of fractional deviation, $|\text{BR(MSSM)} - \text{BR(SM)}|/\text{BR(SM)}$, are exhibited for $h^0 \to \gamma\gamma$. See caption to Fig. 1.](image)

There is one novel approach which could extend the discovery limits for heavy Higgs bosons at the NLC. A high energy, high luminosity photon beam can be produced by the Compton backscatter of an intense laser beam off a beam of electrons. This provides a mechanism for turning the NLC into a high energy, high luminosity $\gamma\gamma$ collider. All neutral Higgs states couple to $\gamma\gamma$ at one-loop via loops of charged matter. Since the couplings of the heavy Higgs states to fermions and scalars are not suppressed in the decoupling limit, the $\gamma\gamma$ couplings of the heavy Higgs states are also not suppressed relative to the $h^0\gamma\gamma$ coupling. Thus, one can search for the non-minimal Higgs sector at the $\gamma\gamma$ collider by either
measuring the $h^0\gamma\gamma$ coupling with sufficient precision or by directly producing $A^0$ and/or $H^0$ in $\gamma\gamma$ fusion. In the decoupling regime, the $h^0\gamma\gamma$ coupling approaches the corresponding coupling of the Standard Model Higgs boson, as illustrated in Fig. 2. As a result, this is not a viable method for detecting deviations from the Standard Model. Thus, one must focus on $\gamma\gamma \rightarrow (A^0, H^0)$. In ref. 11, Gunion and I showed that parameter regimes exist where one could extend the heavy Higgs mass reach above $\sqrt{s}/2$. For example, at a 500 GeV $\gamma\gamma$ collider, a statistically significant $A^0$ signal in $b\bar{b}$ and $Zh^0$ final states could be seen above the background if $m_{A^0} < 2m_t$.

5. Conclusions

In the most general CP-conserving two Higgs doublet model, a decoupling limit can be defined in which the lightest Higgs state is a CP-even neutral Higgs scalar, whose properties approach those of the Standard Model Higgs boson. In the MSSM, the decoupling limit corresponds to $m_{A^0} \gg m_Z$ (independent of $\tan\beta$). Moreover, the approach to decoupling is rapid once $m_{A^0}$ is larger than $m_Z$. Thus, over a very large range of MSSM parameter space, the couplings of $h^0$ to gauge bosons, quarks and leptons are nearly identical to the couplings of the Standard Model Higgs boson.

If the $h^0$ is discovered with properties approximating those of the Standard Model Higgs boson, then the discovery of the non-minimal Higgs sector will be difficult. At the LHC, $A^0$, $H^0$ and $H^\pm$ production rates via gluon-gluon fusion are not suppressed. However, isolating signals of the heavy Higgs states above background presents a formidable challenge. At the NLC, if $\sqrt{s} > 2m_{A^0}$, then pair production of $H^+H^-$ and $H^0A^0$ is easily detected. Below pair-production threshold, detection of the non-minimal Higgs sector is problematical. For example, the cross sections for single heavy Higgs boson production (in association with light particles) are too small to be observed. However, experiments at the $\gamma\gamma$ collider may be able to extend the NLC discovery limits of the heavy Higgs states (via $\gamma\gamma$ fusion to $H^0$ or $A^0$). Precision measurements of $h^0 \rightarrow b\bar{b}$ may also provide additional evidence for a non-minimal Higgs sector. However, current experimental expectations at the NLC predict only a 2-$\sigma$ deviation from Standard Model expectations if $m_{A^0}$ is just beyond the kinematic limit for pair production of heavy scalar states.

Once the first evidence for the Higgs boson is established, it will be crucial to ascertain the underlying dynamics of the electroweak symmetry breaking sector. If the data reveals a single neutral CP-even Higgs boson, with the precise properties

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* The contributions of supersymmetric loops to the $h^0\gamma\gamma$ amplitude vanish in the limit of large supersymmetric particle masses.
expected of the minimal Higgs boson of the Standard Model, then one will have to look for alternative techniques of exploring the physics of electroweak symmetry breaking. The challenge for future collider experiments is to develop new strategies for directly probing the scalar sector in order to see beyond the minimal Higgs boson.

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