COMMENT ON THE BAIER-KATKOV QUASICLASSICAL OPERATOR APPROACH TO THE LANDAU-POMERANCHUK-MIGDAL EFFECT

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Abstract

I demonstrate that the Baier-Katkov approach to the Landau-Pomeranchuk-Migdal effect based on their quasiclassical operator method is conceptually wrong in quantum regime of interaction of the charged particles with medium constituents which takes place for electrons and positrons in real media.
Ever since the celebrated papers by Landau and Pomeranchuk [1], and Migdal [2] the influence of multiple scattering on the radiation processes in medium (the so-called Landau-Pomeranchuk-Migdal (LPM) effect) attracted much attention (see, for instance, the monograph [3] and recent review [4]). The new activity in the LPM effect in QED has been stimulated by the first accurate experimental investigation of this effect by the SLAC E-146 collaboration [5]. In Ref. [6] (see also Refs. [7, 8, 10, 11]) I have developed a new rigorous light-cone path integral (LCPI) approach to the LPM effect in QED and QCD. I have expressed the radiation rate through solution of a two-dimensional Schrödinger equation with an imaginary potential. The representation for the radiation rate derived in Ref. [6] is similar to the one obtained long ago in Refs. [11, 12] within the Baier-Katkov quasiclassical operator (QO) approach to the radiation processes in QED [13]. In Ref. [12] the radiation rate has been evaluated for a medium without an external field, and in Ref. [12] for a medium with allowance for a smooth external field. The representation of Refs. [11, 12] was recently used for analysis of the LPM effect in several papers by Baier and Katkov [14, 15, 16, 17]. Due to the coincidence of the final expressions for the radiation rate of Ref. [6] and Refs. [11, 12], Baier and Katkov in Ref. [14] conclude that the results of the LCPI approach [6] to the LPM effect coincide with that of the QO method.

In this comment I would like to point out that the analysis of the LPM effect of Refs. [11, 12] is conceptually wrong in the case of real media, and the approach developed cannot be regarded as a consistent quantum theory of the LPM effect. As far as the coincidence of the formulas for the radiation rate obtained in Refs. [11, 12] and in Ref. [6] is concerned, it is of accidental nature. It is an artifact of an interplay of two conceptual errors made by the authors of Refs. [11, 12]. As will be seen below the incorrectness of the analysis of the LPM effect given in Refs. [11, 12] is almost evident. Nonetheless, it seems, this fact is not known among the experts.

Let us discuss for definiteness the $e \rightarrow e\gamma$ transition in an infinite medium in the presence of an external field considered in Ref. [12]. Its authors assume that the emission probability can be evaluated by averaging over the classical electron trajectories and atomic positions of the expression for the radiation rate given by the QO method for a given classical trajectory. The corresponding QO formula (Eq.(2.1) of Ref. [12]) reads

$$dw = \frac{\alpha}{(2\pi)^2} \frac{d^3k}{\omega} \int dt_1 \int dt_2 R^*(t_1)R(t_2) \exp \left\{ -\frac{i\varepsilon}{\varepsilon'}[k x(t_2) - k x(t_1)] \right\}, \quad (1)$$

where $\alpha = 1/137$, $k = (\omega,k)$ is the four-momentum of the photon, $\varepsilon$ and $\varepsilon'$ are the initial and final electron energy, $x(t) = (t, r(t))$, $t$ is the time, and $r$ is the electron location on a classical trajectory, the factor $R(t)$ can be expressed through the electron spinors, its specific form is not important for us. In Ref. [12] the averaging over the trajectories and the atomic positions is performed with the aid of the distribution function, averaged over atomic positions. This leads to the following expression (Eq. (2.4) of Ref. [12]) for the emission probability per unit time

$$dW = \langle \frac{dw}{dt} \rangle = \frac{\alpha}{(2\pi)^2} \frac{d^3k}{\omega} \text{Re} \int_0^\infty d\tau \exp \left( -\frac{i\varepsilon}{\varepsilon'} \omega \tau \right)$$
\[ \times \int d^3v d^3v' d^3r d^3r' L(\theta', \theta) F_i(r, v, t) F_f(r', v', \tau; r, v) \exp \left\{ i \frac{\varepsilon}{\varepsilon'} \mathbf{k}(r'-r) \right\}. \tag{2} \]

Here, \( L(\theta', \theta) = 2R^*(t+\tau)R(t) \), the angle \( \theta \approx v_\perp \) (\( v_\perp \) is the component of the electron velocity perpendicular to the vector \( \mathbf{k}/\omega \)). The distribution function \( F_i \) satisfies the kinetic equation

\[ \frac{\partial F_i(r, v, t)}{\partial t} + v \frac{\partial F_i(r, v, t)}{\partial r} + w \frac{\partial F_i(r, v, t)}{\partial v} = n \int d^3v' \sigma(v, v') [F_i(r, v', t) - F_i(r, v, t)], \tag{3} \]

where \( n \) is the number density of the medium, \( w \) is the electron acceleration in the external field, and \( \sigma(v, v') \) is the electron scattering cross section on the medium constituent (atom). The distribution function \( F_i \) satisfies the same equation. The normalization condition for \( F_i \) and initial one for \( F_f \) read

\[ \int d^3r d^3v F_i(r, v, t) = 1, \tag{4} \]

\[ F_f(r', v', 0; r, v) = \delta(r - r')\delta(v - v'). \tag{5} \]

Using Eqs. (2)-(3) after some algebra the authors of Ref. [12] obtain the following expression for the spectral distribution of emission probability per unit time (Eq. (2.18) of Ref. [12] )

\[ \frac{dW}{d\omega} = \alpha \omega \text{Re} \int_0^\infty d\tau \exp \left( -i \frac{a\tau}{2} \right) \left[ \frac{\omega^2}{\gamma^2 \varepsilon'^2} \varphi_0(0, \tau) - i \left( 1 + \frac{\varepsilon^2}{\varepsilon'^2} \right) \nabla \varphi(0, \tau) \right], \tag{6} \]

where \( \varphi_\mu \) satisfies equation

\[ \frac{\partial \varphi_\mu(x, \tau)}{\partial \tau} - \frac{ib}{2} \Delta_x \varphi_\mu(x, \tau) + iwx \varphi_\mu(x, \tau) = n \varphi_\mu(x, \tau) \int d^2\theta [\exp(i\theta x) - 1] \sigma(\theta), \tag{7} \]

\[ \varphi_0(x, 0) = \delta(x), \quad \varphi(x, 0) = -i \nabla \delta(x), \]

\( a = \omega m_e^2/\varepsilon' \), \( b = \omega \varepsilon/\varepsilon' \), \( \gamma = \varepsilon/m_e \), \( \sigma(\theta) \) is the electron scattering cross section written through the angular variable \( \theta \). A similar derivation of the radiation rate in the absence of external field (\( w = 0 \)) has been given in Ref. [11]. By changing the two-dimensional variable \( x \) by the transverse coordinate \( \rho = x/\omega \) one can cast the result of Refs. [11, 12] in the form obtained in my paper [8] (there I have discussed the situation without external field, however, it can be trivially included). In Ref. [8] the analogue of the right hand side of Eq. (7) contains the dipole cross section \( \sigma_d(\rho) \) of scattering of \( e^+e^- \) pair on the medium constituent. In order to represent Eq. (7) in the form of Ref. [8] one should use the relation

\[ \sigma_d(\rho) = 2 \int d^2\theta [1 - \exp(i\theta \varepsilon \rho)] \sigma(\theta) \tag{8} \]

between the differential cross section \( \sigma(\theta) \) and dipole cross section \( \sigma_d(\rho) \). It is crucial that to represent Eqs. (8), (9) in the form obtained in Ref. [8] one should use the quantum electron scattering cross section \( \sigma(\theta) \).

My criticism of the derivation of the radiation rate given in Refs. [11, 12] is based on the following two facts:
1. The QO expression (1) is valid if the motion of the electron in the total potential (which equals the sum of the medium potential and external one) is quasiclassical. In the quasiclassical limit the typical transverse scale at which the quantum interference of the electron trajectories are important becomes considerably smaller than the typical scale of variation of the medium constituent potential. As a result, in such a regime in scattering on a medium constituent there exists an approximate correspondence between the scattering angle and the electron impact parameter, and the concept of the classical trajectory is justified. Eq. (1) derived neglecting the variation of the field acting on the electron for the quantum fluctuations of the electron trajectory is valid only in this quasiclassical limit. In the quantum regime the above correspondence between the scattering angle and the electron impact parameter in interaction with a medium constituent is lost. In this case one must take into account accurately the variation of the field acting on the electron in evaluating the radiation rate. Eq. (1) (obtained for a smooth field) does not make any sense in the quantum regime. It is well known that for a screened Coulomb potential the quasiclassical situation takes place at \(Z\alpha \gg 1\). For real media, when \(Z\alpha < 1\), we have essentially quantum regime. For this reason, it is evident that Eqs. (1), (2) are not justified for real media. The use of Eqs. (1) and (2) which are not valid in the quantum regime is the main conceptual error in the treatment of the LPM effect given in Refs. [11, 12].

2. The authors of Refs. [11, 12] compensate the incorrectness of Eq. (1) by another error in the procedure of averaging over the electron trajectories. According to their logic the QO expression (1) must be averaged over all possible classical trajectories (and the authors say this). It is evident that in this case the distribution functions entering (2) should satisfy the kinetic equation in which the collisional term contains the classical scattering cross section. However, the authors use in the kinetic equation (3) the quantum cross section. As was said only in this case the formula for the radiation rate of Refs. [11, 12] coincides with that of Ref. [6].

The above facts make it clear that the approach to the LPM effect of Refs. [11, 12] based on the Baier-Katkov QO method cannot be regarded as a consistent quantum theory of the LPM effect. It is only justified in the quasiclassical limit which does not take place for real media. The formal coincidence of the final expression for the radiation rate obtained in Refs. [11, 12] with the one obtained by me in the LCPI approach [6] is of accidental nature. It is a consequence of an interplay of the use by the authors of Refs. [11, 12] of the incorrect expression (2) and replacement of the classical scattering cross section by the quantum one in the kinetic equation (3).

In conclusion I would like to emphasize that my criticism concerns namely the conceptual aspects of the Baier-Katkov QO approach to the LPM effect, and does not concern the technical details of the subsequent analyses by Baier and Katkov [14, 15, 16, 17]. However, one has to bear in mind that the starting expression for the radiation rate used in Refs. [14, 15, 16, 17] has never been rigorously derived within the Baier-Katkov QO method.

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1The fact that for a purely Coulomb potential the quantum scattering cross section coincides with the classical one, of course, does not justify the use of Eqs. (1), (2). In addition, for the screened potential the quantum and classical cross sections differ.
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