Cutting and tearing thin elastic sheets: Two novel period-single cracks and the first period-doubling crack

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Two novel forms of quasi-static crack propagation are observed in the experiments of cutting a folded sheet with a blunt object and tearing thin brittle sheet under the guidance of a meter-stick. The crack propagates in saw-tooth way in both experiments, however, the shape of the two saw-tooth cracks are totally different. Additionally, we also observe a period-doubling crack in tearing experiment. We cut and tear the sheet in different directions; the experiment results suggest the anisotropy of the thin sheet play an important role in the formation of these two kinds of saw-tooth cracks. We show the formation of the period-doubling crack is closely correlated with the changing of the contact region between the sheet and the meter-stick. We show that the growth process of the cutting crack is a Logistic growth process (S-curve), while the tearing cracks propagates in the form of approximate power-law function.

The classical fracture theories initially proposed by Griffith [1] falls short of predicting the path of a crack as it propagates through a solid [2]. The study of the fracture path and the associated instabilities has been the subject of much interest. A body of unstable cracks were observed in the laboratory, such as branching cracks and cracks with rough surface [3–7], oscillatory cracks [6–15], shark-fin-like cracks [16–21], spiral cracks [22–26], crescent cracks [24], tongue-like cracks [27–30], zigzag cracks [31, 32], en passant cracks [33–35], helical cracks [36, 37], sideway cracks [38, 39], etc. The thin sheets are ubiquitous [40], almost everyone has already had the experience of tearing and cutting a sheet, such as opening an envelope, tearing a piece of paper in half, etc. A number of studies have been carried out on these common experiences. Number of crack growth ways were revealed [41]. Most of previous studies assumed the thin sheet is isotropic, although most of thin sheets are anisotropic because of their inherent micro-structure and the manufacturing process. This raises a question: are there any undiscovered ways of crack propagation in the fracture of anisotropic sheet? Additionally, the previously discovered oscillatory cracks are all period-single crack [6–21], is there a period-doubling [42, 43] crack?

Here, we study the cutting of a folded sheet with a blunt tool and the tearing of a sheet under the guidance of a meter-stick. In both experiments, we used anisotropic sheets of bi-axially oriented polypropylene (widely used in fields of product packaging) of thickness $t=40\mu m, 53\mu m$ with strength varying from 120MPa to 180MPa depending on the direction. All experiments were performed quasi-statically. Schematic diagram of these two experiments are shown in Fig. 1. After tearing and cutting, the sheets was digitized using a scanner, and the morphology of the fracture path was measured.

The first experiment [Fig. 1(a)] is setup to study the crack propagation of cutting a folded thin sheet with a tool, which mimics the process of opening an envelope with a knife. The process to cut is as follows, firstly, a thin sheet is folded in half, note that no crease is formed in the sheet during the folding process. Secondly, this folded sheet is placed on a platform. Thirdly, a plane is put on it and an external force is applied on top surface of this plane, and cut an initial notch (orange) with a knife. Lastly, a blunt object (red) is used to cut this sheet. Two shapes of tools are used: rectangle and circular. The angle between the object and the sheet is $\alpha \approx 35^\circ \pm 5^\circ$.

Examples of the crack formed by cutting with two kinds of tools are shown in Fig. 2(a). The crack path is related to the shape of the tool, which is different with the fracture path in clamped thin sheets [16–21]. The fracture path is a straight line when $\beta > \beta^*$ ($\beta$ is the angle between the $x$ direction and the direction of the least strength of sheet). The value of $\beta^*$ is associated with the size $s$ (thickness or diameter) of the tool. As shown in Fig. 2(b), $\beta^*$ increases dramatically at small size, but the rate of increase slows gradually with a further increase of the tool size, it is expected that $\beta^*$ will reach a saturation value when the size $s$ goes beyond a certain threshold. The transition boundary between straight crack and no-straight crack follows $\beta \approx a(1 - e^{-s/b})$. For thin enough tools, $\beta^* = 0$, the crack path is a straight line, as one would expect.

The amplitude of the fracture path is approximately the same as the thickness/diameter of the tool. The average wavelength $\lambda$ of the fracture path is dependent of the $\beta$, $\lambda \propto 1/\cos \beta$, as shown in Fig. 2(c).

These results suggest the formation of no-straight
There are four stages for each fracture period: I, II, III, IV [Fig. 4(a)]. Among them, the propagation direction of crack in stage I and III have a certain angle with the cutting direction. For $\beta > 0$, the crack in stage I basically propagates along the orthogonal of the cutting direction, while the crack in stage III does not. This is because $\theta$ (the angle between the propagation direction of crack and the direction of the least strength of sheets) in these two stages are different, $\theta_{111} < 90^\circ$ in stage III, and $\theta_1 > 90^\circ$ in stage I. The absolute value of slope $S$ of crack in stage III is negatively correlated with $\beta$. In stage II and IV, the crack basically propagates along the cutting direction.

Cracks have something to do with the anisotropy of the sheets. A possible explanation is as follows. For smaller $\beta$, such as $\beta \approx 0^\circ$ (the whole system is not completely symmetrical, it is difficult to ensure $\beta^* = 0$), as the tool moving to an initial symmetrical crack [Fig. 3(a), also see Supplementary Movies [44]], the crack opening displacement has to be increased in order to accommodate the tool motion. So both sides of the crack were subjected to two forces (in-plane force $F^I$ and out-of-plane force $F^O$). Due to $\sigma^*_w < \sigma^*_s$ ($\sigma^*_s$, $\sigma^*_w$ denote the strength of strong and weak direction of the sheet, respectively), the crack tends to grow along an upward inclined direction [Fig. 3(b)] rather than the cutting direction (perpendicular to the strongest direction). The out-of-plane force applied by the tool to the crack side of the sheet is decreased to zero when the crack reaches to the up edge of the tool. The crack begins to propagate along the cutting direction under the action of in-plane tensile force, and the out-of-plane force applied on the down side of the crack is increasing during this process. When the out-of-plane force increases to a critical value, the crack begins to grow along a downward inclined direction [Fig. 3(c)], and reaches to the down edge of the tool, then propagates along the cutting direction. Then the above processes are repeated until the cutting is completed. However, for larger $\beta$, such as $\beta \approx 90^\circ$, although the sheet is also subject to out-of-plane force from the tool, the crack is not affected by this force due to $\sigma^*_w < \sigma^*_s$, and always follows a straight path parallel to the cutting direction [see Supplementary Movies [44]].

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FIG. 3. (color online) Sequence of the crack propagation for $\beta \approx 0^\circ$. (a) Initial symmetrical crack. The crack grows (b) along upward inclined direction, (c) downward inclined direction and (d) upward inclined direction.
However, for larger $\beta$, the length $l$ of the crack in stage IV is significantly longer than that in state II. This can be attributed to the different crack path in stage I and III. The crack path in stage I is nearly perpendicular to the axial of the tool, i.e. $S_I > S_{III}$. Thus the out-of-plane force applied on the crack of stage I is larger than that on the crack of stage III, i.e. $F^o_I > F^o_{III}$, thus $l_{III} < l_{IV}$. As shown in Fig. 2(a), the slope of the crack in stage I is not sensitive to $\beta$, thus the length of the crack in stage II changes little with $\beta$.

The formation process of the cutting cracks can be explained by using the logistic growth model. The formation of the no-straight crack is relate to the out-plane force $F^o$, and the growth rate of the crack $dy/dx$ is positively correlated with the $F^o$, i.e. $dy/dx \sim F^o$. $F^o$ increases firstly and then decreases during the formation of crack in stage I and stage III, so does $dy/dx$. We assume $dy/dx$ follows the logistic growth model

$$\frac{dy}{dx} = ky(1 - \frac{y}{s})$$  \hspace{1cm} (1)

where $k$ is the growth parameter. The size $s$ of the tool is the limit value of the function of the crack path $y$. The solution of Eq. (1) is

$$y = \frac{s}{1 + e^{-k(x-x')}}$$  \hspace{1cm} (2)

The growth parameter $k$ should be related to $\beta$. However, because the cutting process is such a highly nonlinear process, it is difficult for us to accurately construct a formula to describe the relationship between $k$ and $\beta$. The growth parameter $k$ is confirmed by fitting Eq. (2) to experiment data. Fig. 4(a) shows that the prediction of the shape of the cutting crack, $y$ (the solid line) by logistic growth model, is in good agreement with the experiments (the green scatter). As shown in Fig. 4(b), for the stage II to the stage IV(blue), the fitting results suggest the parameter $k$ (absolute value) decrease as the increasing of $\beta$ ($\theta_{III}$ decrease with the increasing of $\beta$), as expected. However, for the stage IV to the stage II (black), $k$ first increases and then decreases with the increasing of $\beta$. The physical origin of this phenomenon still needs to be uncovered.

The second experiment [Fig. 1(b)] is setup to study the crack propagation of the tearing of a sheet under the guidance of a meter-stick. The process to tear is as follows, firstly, a thin sheet is placed on a platform. Secondly, a meter-stick ($l=160\,\text{mm}$, $w=150\,\text{mm}$) is put on it and an external force is applied on each end of this meter-stick to ensure the sheet does not slide out during the tearing process, and cut an initial notch (orange) with a knife. Then tear this sheet along a line (red dashes line), which the angle $\varphi$ between this line and the edge of the sheet is about $45^\circ$.

Fig. 5(a) presents typical scanned photographs of the tearing cracks. It can be seen that the cracks formed by tearing are completely different with the cutting cracks [Fig. 2(a)]. Depending on the angle $\beta$, the crack grows straight ($\beta > 55^\circ$) or no-straight about the meter-stick. For $\beta < 20^\circ$, the cracks in thick sheets ($53\,\mu\text{m}$) follow period-doubling (one small and one big [42, 43]) path when the tearing distance exceeds a critical value. By contrast, the period-doubling cracks are rarely formed in thin sheets ($40\,\mu\text{m}$). During last 40 years, several different types of period-single cracks have been reported, e.g. oscillating fracture in thermal quenching experiments [8], bi-axially stretched rubber [10], pure uniaxial tension thin brittle gels [10], clamped thin sheets [16–21] and thin coatings [24], etc. While the period-doubling cracks have never been reported in the literature, the cracks presented here is the first period-doubling crack.

We measured the average wavelength and amplitude of the fracture paths. As shown in Fig. 5(b), the wavelength/amplitude increases during the tearing process because of the changing of the loading direction (the crack front to the loading point). The relation between the wavelength/amplitude and $\beta$ is not monotonic function, we have no good explanation for this result, additional research is needed to confirm and explain it. However, the ratio of wavelength to amplitude increases exponentially with the increasing of $\beta$ [Fig. 5(c)], i.e. $\lambda/A \approx c + d e^{R \beta}$, the value of $c$, $d$ and $R$ are related to the thickness of the sheet ($40\,\mu\text{m}$: $c = 2.12$, $d = 0.3$ and $R = 17.15$; $53\,\mu\text{m}$: $c = 2.10$, $d = 0.347$ and $R = 20$).

Like the cutting configuration, the formation of the tearing crack is also related to the anisotropy of the material, its formation process can also be divided into four stage: I, II, III, IV [Fig. 5(a) and Fig. 7]. However, the formation process of the tearing and cutting cracks are entirely different. The tearing cracks in stage I and stage

![FIG. 4. (color online) (a) Detail of the cutting crack path ($\beta \approx 30^\circ$, $t = 40\,\mu\text{m}$). (b) The growth parameter $k$ for different $\beta$.](image-url)
IV are formed on the side of the meter-stick (x-z plane), while the cracks in in stage II and stage III are formed on the upper surface of the meter-stick (x-y plane). For smaller $\beta$, such as $\beta \approx 0^\circ$, the crack (Mode I+II) in stage II propagates along the meter-stick[Fig. 6(a), also see Supplemental Movies [44]] under two in-plane forces $F_x$ and $F_y$ (the crack tends to grow along X direction under $F_x$, and grow along Y direction under $F_y$). During this process, the tooth gradually becomes larger. As a result, the stress $\sigma$ in crack tip gradually increases, i.e. $\sigma_a \propto \int_0^L \tau_x dx \propto L$. When the stress $\sigma_x$ exceeds a certain critical value, the crack (Mode I+III) then propagates (stage III) along the vertical direction of the meter-stick [Fig. 6(b), also see Supplemental Movies [44]] due to $\sigma_a < \sigma_x^*$, mainly under in-plane force $F_x$, since the out-of-plane force $F^O$ applied by the meter-stick to the crack tooth is small because the sheet is very thin. However, for larger $\beta$, such as $\beta \approx 90^\circ$, the crack always follows a straight path parallel to the meter-stick for the same reason [see Supplemental Movies [44]].

The shape of tearing cracks [Fig. 7] are well fit by the following equation

$$y = \begin{cases} A|x - x^*|^p_i & x < x^* \\ A|x - x^*|^p_r & x > x^* \end{cases}$$

where $x^*$ is the valley of the tearing crack, which indicate that the tearing cracks grow in the form of power-law [45], rather than S-curve.

To understand why the period-doubling cracks are formed, we recorded the formation process of the tearing cracks [see Supplemental Movies [44]]. The deformation morphology of the sheet when the big tooth and the small tooth have just been formed are shown in Fig. 6(c)
and 6(d), respectively. The red line is a boundary. On its left, the sheet is attached to the meter-stick. On its right, it is the opposite. It was found that the boundary in Fig. 6(c) and Fig. 6(d) are very different. In Fig. 6(d), it is a straight line. By contrary, it is a curve in Fig. 6(d). We viewed the recorded video over and over again. We found the straight boundary does not appear in the formation process of period-single cracks. It seems that the straight boundary appears only after the formation of the big tooth. A small tooth is always formed after its appearing.

In conclusion, we have reported two new period-single cracks and a period-doubling crack in thin elastic sheet, and showed they are the cooperative result of the anisotropy of the sheet and the interaction between the sheet and the tool or meter-stick. We expect that our experimental results will motivate the development of new theoretical models to accurately predict the growth process of these cracks and to understand how the anisotropy of material affects the fracture path [46–48]. We believe that these robust cracks can be used as good test cases for theoretical models that couple anisotropy of material and fracture.

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