Monopoles in non-Abelian Dirac-Born-Infeld Theory

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We investigate monopole solutions for the Born-Infeld Higgs system. We analyze numerically these solutions and compare them with the standard ’t Hooft-Polyakov monopoles. We also discuss the existence of a critical value of \( \beta \) (the Born-Infeld “absolute field parameter”) below which no regular solution exists.

I. INTRODUCTION

Classical solutions for the Dirac-Born-Infeld (DBI) theories are useful in the understanding of brane dynamics \([1]-[3]\). In this respect, bions and soliton like solutions have recently attracted much attention \([2]-[4]\); in particular, vortex and monopole solutions have been investigated \([2]-[7]\).

Concerning monopoles, it was shown in \([6]\) that it is possible to construct a DBI action coupled to a Higgs scalar in such a way that the usual BPS monopole solution to the Yang-Mills-Higgs theory also solves the resulting (first-order) equations of motion. To this end, one has to endow the Higgs field with dynamics also described by a square-root Born-Infeld like Lagrangian and also consider the Prassad-Sommerfield \( \lambda \to 0 \) limit for the symmetry breaking potential.

Being the solution that of a BPS monopole, one does not capture any features associated with Born-Infeld dynamics and, in particular, the resulting solution is insensitive to the value of \( \beta \), the “absolute field” parameter in Born-Infeld models. In contrast, a critical value \( \beta_c \) was discovered in a previous investigation of vortex solutions in Abelian DBI theories \([7]\), such that no soliton solution exists for \( \beta \leq \beta_c \), this showing how DBI dynamics determines the nature of soliton solutions.

In this work we shall discuss monopole solutions in \( SO(3) \) DBI gauge theories coupled to a Higgs triplet which enters through the usual kinetic energy term, \( L_{\text{Higgs}} \sim \text{tr}(D_\mu \phi D^\mu \phi) \). Concerning the way in which the non-Abelian DBI scalar Lagrangian is defined, there exist different possibilities among which we consider taking (i) the usual trace over internal indices of the square root DBI Lagrangian defined through its power series expansion and (ii) the “symmetric trace” advocated by Tseytlin \([8]\) as a way to make contact with the low energy effective action derived from superstring theories.

The paper is organized as follows: we present in Section II the \( SO(3) \) DBI-Higgs action, discuss the spherically symmetric ansatz and derive the radial equations of motion both for the usual and the symmetric trace. In Section III we describe our numerical solutions and discuss their main properties. We give in Section IV analytical arguments giving support to the existence of critical values for \( \beta \) below which the monopole solution ceases to exist. Finally we present in Section V a summary of our results and the conclusions.

II. THE LAGRANGIAN AND THE MONOPOLE ANSATZ

The ’t Hooft-Polyakov monopole solution \([3]-[10]\) to the equations of motion of the Yang-Mills-Higgs Lagrangian owes its existence and main properties to the non-Abelian character of an ansatz for the gauge and scalar fields, mixing
space-time and internal indices in such a way that ensures topologic non-triviality and regularity of the resulting solution. In order to look for analogous solutions in the DBI theory, one should necessarily start from a non-Abelian version of the Born-Infeld theory and also decide how the Higgs field will be coupled to the gauge field.

The definition of the DBI theory for a non-Abelian gauge group is not unique and several alternatives have been discussed in the literature, [8], [11]-[16]. The simplest extension amounts to define the gauge field Lagrangian in the form

\[ L_{DBI} = \beta^2 Tr \left( 1 - \sqrt{1 + \frac{1}{2\beta^2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{8\beta^4} (F_{\mu\nu} \tilde{F}^{\mu\nu})^2} \right). \]  

(1)

Here \( F_{\mu\nu} \) is the field strength taking values in the Lie algebra of the gauge group (which we take for simplicity as \( SO(3) \)),

\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \epsilon [A_\mu, A_\nu] \]  

(2)

\[ A_\mu = A_\mu^a t^a, \quad t^a = \frac{\vec{A}_\mu}{\sqrt{2}}, \]  

(3)

\[ \tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta} \]  

(4)

and “tr” in eq. (1) represents the usual trace on \( SO(3) \) indices, with generators normalized so that

\[ \text{tr}(t^a t^b) = \delta^{ab} \]  

(5)

A second possibility is to define a symmetric trace operation,

\[ \text{Str}(t_1, t_2, \ldots, t_N) \equiv \frac{1}{N!} \sum_{\pi} \text{tr}(t_{\pi(1)} t_{\pi(2)} \ldots t_{\pi(N)}) \]  

(6)

with the sum extending over all permutations \( \pi \) of the product of \( N \) given \( t \)'s. Then, the DBI Lagrangian is defined as

\[ L^{\text{Str}}_{DBI} = \beta^2 \text{Str} \left( 1 - \sqrt{1 + \frac{1}{2\beta^2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{8\beta^4} (F_{\mu\nu} \tilde{F}^{\mu\nu})^2} \right). \]  

(7)

Remarkably, the r.h.s. in (7) can be written in this case in terms of a determinant,

\[ L^{\text{Str}}_{DBI} = \beta^2 \text{Str} \left( 1 - \sqrt{-\det(g_{\mu\nu} + \frac{1}{2\beta} F_{\mu\nu})} \right). \]  

(8)

thus making contact with the tree level open string effective action for branes, [8]. Of course, \( g_{\mu\nu} \) in eq. (8) is the 3 + 1 usual Minkowski space-time metric, \( g_{\mu\nu} = \text{diag}(1, -1, -1, -1) \), and not the pullback of the \( d + 1 \) dimensional Minkowski metric to the \( p + 1 \) dimensional world volume of the p-brane. It should be mentioned that odd powers of the field strength \( F \) are absent from the expansion of \( L_{\text{Str}} \) this implying that \( F \) (although possibly large) should be slow varying since \( F^3 \sim [D, D] F^2 \). In this sense using Str amounts to some kind of Abelian approximation. It should be noted that some unsolved problems related to the use of a symmetric trace have been signaled. They refer to discrepancies between the results that arise from a symmetrized non-Abelian Born Infeld theory and the expected spectrum of brane theories [2].

Apart from this alternatives related to the way the trace operation is defined, one has to decide how the Higgs field dynamics is introduced. In previous analysis, DBI monopoles were constructed by demanding that the usual Yang-Mills-Higgs BPS relations also hold in the DBI case [4]. This amounts to define a Higgs field Lagrangian in a Born-Infeld-like way (i.e., also under a square root) in such a way that the model has a supersymmetric extension [4], [14]-[16]. Being the BPS relations the same as in the Yang-Mills-Higgs case, the resulting DBI monopole solutions are identical to the well-honored Prassad-Sommerfield exact solutions and have no specific features resulting from the DBI dynamics. Instead, we shall consider here the usual \( SO(3) \) Higgs field Lagrangian and a symmetry breaking potential not necessarily in the BPS limit. We then propose the following Lagrangian for the Higgs field:
\[ L_{\text{Higgs}} = \frac{1}{2} D^\mu \phi^\ast D_\mu \phi - V[\phi] \] (9)

with the scalar triplet written in the form
\[ \phi = \phi^a t^a = \vec{\phi} \cdot \vec{t}, \] (10)

the symmetry breaking potential given by
\[ V[\phi] = \frac{\lambda}{4} (\vec{\phi} \cdot \vec{\phi})^2 - \frac{\mu^2}{2} \vec{\phi} \cdot \vec{\phi} \] (11)

and the covariant derivative defined as
\[ D_\mu \phi = \partial_\mu \phi + e \vec{A}_\mu \wedge \vec{\phi}. \] (12)

\textbf{(i) The equations of motion for } L_{\text{DBI-Higgs}}^f

When the trace operation “tr” is used, the DBI-Higgs Lagrangian reads
\[ L_{\text{DBI-Higgs}}^f = \beta^2 \text{tr} \left( 1 - \sqrt{1 + \frac{1}{2\beta^2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{8\beta^4} (F_{\mu\nu} \tilde{F}^{\mu\nu})^2} \right) + \frac{1}{2} D^\mu \vec{\phi} \cdot D_\mu \vec{\phi} - V[\phi]. \] (13)

From here on we shall consider purely magnetic configurations for which \( F_{\mu\nu} \tilde{F}^{\mu\nu} = 0 \). Then the equations of motion take the form
\[ D^\mu \left( \frac{\tilde{F}_{\mu\nu}}{\sqrt{1 + \frac{1}{4\beta^2} \tilde{F}_{\mu\nu} \cdot \tilde{F}^{\mu\nu}}} \right) = e \vec{\phi} \times D_\nu \vec{\phi}, \] (14)

\[ D^\mu D_\mu \vec{\phi} = \mu^2 \vec{\phi} - \lambda \phi^2 \vec{\phi}. \] (15)

We shall consider the usual spherically symmetric ’t Hooft-Polyakov ansatz \[ (16)-(18) \],
\[ \vec{A}_i(\vec{r}) = \frac{K(r) - 1}{e} \vec{\Omega} \wedge \partial_i \vec{\Omega}, \] (16)
\[ \vec{A}_0(\vec{r}) = 0, \] (17)
\[ \vec{\phi}(\vec{r}) = \frac{H(r)}{er} \vec{\Omega}, \] (18)
\[ \vec{\Omega} = \vec{\Omega}(\theta, \varphi) = \frac{1}{r} \vec{r}, \] (19)

with the appropriate boundary conditions for \( K \) and \( H \),
\[ \lim_{r \to \infty} K(r) = 0, \quad \lim_{r \to \infty} \frac{1}{r} H(r) = \frac{\mu e}{\sqrt{\lambda}} \] (20)

together with the conditions at the origin
\[ K(0) = 1, \quad H(0) = 0. \] (21)

Inserting ansatz \[ (16)-(18) \] into the eqs. of motion \[ (14)-(15) \] one gets
\[ r^2 K'' - r^2 \frac{R'}{R} K' = K (R H^2 + K^2 - 1) \]
\[ r^2 H'' = 2 H K^2 - \mu^2 r^2 H (1 - \frac{\lambda}{e^2 \mu^2 r^2} H^2) \] (22)
where
\[ R = \sqrt{1 + \frac{1}{\beta^2 e^2 r^4} (r^2 K'^2 + \frac{1}{2} (K^2 - 1)^2)}. \] (23)

It will be convenient to define new dimensionless variables and parameters,
\[ \rho = \frac{e\mu r}{\sqrt{\lambda}}, \]
\[ \lambda_0 = \frac{\lambda}{e^2}, \]
\[ \beta_0 = \frac{\beta \lambda}{e \mu^2}, \] (24)

so that one finally has
\[ \rho^2 K'' = K (RH^2 + K^2 - 1) + \rho^2 R' R K', \] (25)
\[ \rho^2 H'' = 2HK^2 - \lambda_0 H (\rho^2 - H^2), \] (26)
\[ R = \sqrt{1 + \frac{1}{\beta_0 \rho^4} (\rho^2 K'^2 + \frac{1}{2} (K^2 - 1)^2)}. \] (27)

With this ansatz, we can write the energy for the monopole solution in the form
\[ E = \frac{4\pi \mu}{\sqrt{\lambda} e} \int d\rho \rho \left\{ 2\beta_0^2 (R - 1) + \frac{1}{2\rho^2} \left[ (H' - \frac{H}{\rho})^2 + \frac{2}{\rho^2} H^2 K^2 \right] + \frac{\lambda_0}{4} \left( \frac{H^2}{\rho^2} - 1 \right)^2 \right\} \] (28)

This expression reduces to the ‘t Hooft-Polyakov monopole mass formula in the $\beta \to \infty$ limit, as expected.

The electromagnetic $U(1)$ field strength $F_{\mu\nu}$ is defined as usual [9] in the form
\[ F_{\mu\nu} = \frac{1}{|\phi|} \hat{\phi}_{\mu} \cdot \hat{F}_{\nu} - \frac{1}{|\phi|^3} \hat{\phi} \cdot (D_{\mu} \hat{\phi} \wedge D_{\nu} \hat{\phi}). \] (29)

Now, since we are considering DBI dynamics, we have to distinguish between the magnetic induction $\vec{B}$ and the magnetic intensity $\vec{H},$
\[ B^i = \frac{1}{2} \varepsilon^{ijk} F_{jk}, \quad H^i = \frac{1}{2} \varepsilon^{ijk} \hat{F}_{jk}. \] (30)

Using ansatz (16)-(18) one easily finds that
\[ B^i = \frac{x^i}{er^3}, \] (31)
so that the magnetic flux at infinity,
\[ \Phi = \int_{S_{\infty}} dS_i B^i = \frac{4\pi}{e}, \] (32)
corresponds to that of a unit magnetic monopole located at the origin. The magnetic flux $\Phi$ can alternatively be defined in terms of $\vec{H}$, this leading to the same answer (32).

(ii) The equations of motion for $L^{Str}_{DBI-Higgs}$

When the symmetric trace operation is used, the DBI-Higgs Lagrangian is defined as
\[ L^{Str}_{DBI-Higgs} = \beta^2 \ \text{Str} \left( 1 - \sqrt{1 + \frac{1}{2\beta^2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{8\beta^2} (F_{\mu\nu} \hat{F}^{\mu\nu})^2} \right) + \frac{1}{2} D^\mu \hat{\phi} D_\mu \hat{\phi} - V[\phi]. \] (33)
Again we will only consider purely magnetic configurations so $F_{\mu\nu}\tilde{F}^{\mu\nu} = 0$. Because of the use of the symmetric trace, deriving the equations of motion in this case becomes rather involved. Indeed, one has first to expand the square root in $L^{Str}$ in powers of $1/\beta^2$ and at each order $N$, consider the $N!$ terms which are included in $Str$. For example, up to order $1/\beta^2$ one has for the purely DBI Lagrangian

$$
L_{DBI}^{Str} = -\frac{1}{4} F_{\mu\nu} \cdot \tilde{F}^{\mu\nu} + \frac{1}{96\beta^2} \left( (F_{\mu\nu} \cdot \tilde{F}^{\mu\nu})^2 + 2(F_{\mu\nu} \cdot \tilde{F}_{\rho\sigma})(F^{\mu\nu} \cdot \tilde{F}^{\rho\sigma}) \right) + O(\frac{F_4^6}{\beta^4}).
$$

(34)

Already at this order, this DBI Lagrangian differs from the one arising when one expands $L_{DBI}^0$:

$$
L_{DBI}^0 = -\frac{1}{4} F_{\mu\nu} \cdot \tilde{F}^{\mu\nu} + \frac{1}{32\beta^2} (F_{\mu\nu} \cdot \tilde{F}^{\mu\nu})^2 + O(\frac{F_4^6}{\beta^4}).
$$

(35)

The $1/\beta^4$ term in the expansion of $L_{DBI}^{Str}$ involves 120 terms containing the sixth power of the field strength, this making the search of a solution using a numerical approach too complicated. We shall here consider the problem to the $1/\beta^2$ order given in equation (34) and analyze how the solution differs from the one obtained using the more simple “tr” operation.

The equations of motion for the gauge field resulting from (34) read

$$
D_\mu \left( \tilde{F}^{\mu\nu} - \frac{1}{12\beta^2} \left( (\tilde{F}_{\rho\sigma} \tilde{F}^{\rho\sigma}) \tilde{F}^{\mu\nu} + 2(\tilde{F}_{\rho\sigma} \cdot \tilde{F}^{\mu\nu}) \tilde{F}^{\rho\sigma} \right) \right) = e\tilde{\phi} \times D^\nu \tilde{\phi}
$$

(36)

while those associated to the Higgs field remain unchanged.

After using the spherically symmetric ansatz (14)-(18), eq. (20) becomes

$$
K''(\rho) = -\left( 3K(\rho) - 6\rho^4 \beta_5^2 K(\rho) + 6\rho^4 \beta_5^4 H(\rho)^2 K(\rho) - 17K(\rho)^3 + 6\rho^4 \beta_5^2 K(\rho)^3 + 45K(\rho)^5 - 71K(\rho)^7 + 70K(\rho)^9 - 42K(\rho)^{11} + 14K(\rho)^{13} - 2K(\rho)^{15} - 4K'(\rho) + 8\rho K(\rho)^{2} K'(\rho) - 4\rho K(\rho)^{4} K'(\rho) + 2\rho^2 K(\rho)^{3} K'(\rho)^2 - 8\rho K(\rho)^{3} - 4\rho^3 K'(\rho)^3 + 32\rho K(\rho)^{2} K'(\rho)^3 - 48\rho K(\rho)^{4} K'(\rho)^3 + 32\rho K(\rho)^{6} K'(\rho)^3 - 8\rho K(\rho)^{8} K'(\rho)^3 - 12\rho^2 K(\rho) K'(\rho)^4 + 36 \rho^2 K(\rho)^{3} K'(\rho)^4 - 36 \rho^2 K(\rho)^{5} K'(\rho)^4 + 12 \rho^2 K(\rho)^{7} K'(\rho)^4 - 8 \rho^3 K'(\rho)^7 - 2\rho^2 K(\rho) K'(\rho)^2 \right) \times \frac{1}{S},
$$

(37)

$$
S = \rho^2 \left( 1 - 6\rho^4 \beta^2 - 2 K(\rho)^2 + K(\rho)^4 + 6 K'(\rho)^2 + 6 \rho^2 K'(\rho)^2 + 24 K(\rho)^2 K'(\rho)^2 + 36 K(\rho)^4 K'(\rho)^4 - 24 K(\rho)^6 K'(\rho)^4 + 6 K(\rho)^8 K'(\rho)^4 + 28 \rho^2 K'(\rho)^6 \right)
$$

(38)

while the equation for $H(\rho)$ is still given by equation (22). Finally, the energy associated to the monopole is given by

$$
E = \frac{4\pi\mu}{\sqrt{\lambda e}} \int d\rho \left\{ \frac{1}{\rho^2} \left( \rho^2 K'^2 + \frac{1}{2} (K'^2 - 1)^2 \right) - \frac{1}{6\rho^3 \beta_5^2} \left( \rho^2 K'^2 + \frac{1}{2} (K'^2 - 1)^2 \right)^2 + \left( \rho^2 K'^4 + \frac{1}{2} (K'^2 - 1)^2 \right)^2 \right\} + O(\beta^4).
$$

(39)

III. NUMERICAL RESULTS

To obtain a detailed profile of the monopole solution, we solved numerically the differential equations (22)-(23) for the case of the trace operation “tr” and (37), (38), and (24) for the symmetric trace “Str”. We employed a relaxation method for boundary value problems [18]. Such method determines the solution by starting with an initial guess and improving it iteratively. The natural initial guess was the exact Prasad-Sommerfield solution [14] (which corresponds to $\lambda_0 = 0$ and $\beta \to \infty$).
(i) The usual trace

For $\beta \gtrsim 10$, the solutions to eqs. (25)-(27) do not differ appreciably from the ’t Hooft-Polyakov monopole solution (see for example [20] for a plot of the ’t Hooft-Polyakov solution). As $\beta$ decreases, the solution changes slowly: the monopole radius decreases and the (radial) magnetic field $\vec{H}$ concentrates at the origin. Some of the solutions profile are depicted in figures (1) and (2).

For $\beta \sim 1$ new features become apparent from our numerical analysis. In particular, we found that:

1- For $\lambda_0 = 0$ the behavior of the Higgs field at large distances depends on $\beta$,  

$$H(r) \to \phi_0 r + c(\beta/\mu^2) \quad \text{for} \quad r \to \infty$$  

(40)

(here $\phi_0$ is the Higgs field v.e.v. in the Prassad-Sommerfield limit). Of course, for $\beta \to \infty$, $c(\beta/\mu^2) \to 1$ and one has the usual asymptotic behavior of the Higgs field for the ’t Hooft-Polyakov monopole. For finite $\beta$, however, it is interesting to note that the $1/r$ falloff, which in the Yang-Mills-Higgs case is related to the massless dilaton associated to the scale invariance of the BPS regime, has now a $\beta$ dependent coefficient.

2- There is a critical value of $\beta$, which we shall denote as $\beta_c$, such that for $\beta \leq \beta_c$ there is no (numerical) solution to the equations of motion (25)-(27). Some values for $\beta_c$ are: $\beta_c = 0.41$ for $\lambda_0 = 0$, $\beta_c = 0.62$ for $\lambda_0 = 0.5$.

This peculiar characteristic of the solutions appears to be a consequence of the high nonlinearity of the equations and not a fictitious artifact of the numerical method. Moreover, the energy of the solutions seems to be singular at $\beta = \beta_c$ (see figure (3)).

(ii) The symmetric trace

For this case we solved the eqs. (23), (28) and (29) with the same numerical approach. Since the equations are valid to order $1/\beta^2$, our analysis cannot be reliable for small $\beta$. We see that for $\beta \gtrsim 4$ the solutions do not differ notably from those arising when the trace operation “tr” is considered. The profile of the solutions are indistinguishable from the solid-line curves of figures (1) and (2). In view of our approximation, we could not analyze the region where one expects the existence of $\beta_c$ in this case.

IV. ANALYSIS OF $\beta_c$

As noted in the introduction, the existence of a critical value of the absolute field below which the solution to the equations of motion of the DBI-Higgs system ceases to exist, was already noticed for vortices in the Abelian case and should be considered as a distinctive feature of soliton solutions in DBI theories.

In order to better understand the origin of $\beta_c$ let us introduce the following scaling argument. For a monopole like solution, there is a characteristic radius $R_W$ that can be associated with the monopole core; outside this core, the gauge field approaches to its asymptotic value. For Yang-Mills-Higgs monopoles (or Nielsen-Olesen vortices), this radius should be necessarily related to $\mu$, the sole parameter carrying dimensions, $R_W \sim 1/\mu$. The size $R_W$ is fixed so as to minimize the sum of the energy stored in the magnetic field outside the core and the energy due to the scalar field gradient inside the core. The resulting value $R_W$ is in this case $R_W = 1/M_W$, with $M_W$ the mass of the gauge boson, $M_W = (e/\sqrt{\lambda})\mu$. A second length playing a rôle in the monopole configuration is related to the size of the region outside of which the Higgs scalar takes practically its vacuum expectation value. We shall call the radius of this region $R_H$. For the Yang-Mills-Higgs system one has $R_H = 1/\mu$. From $R_W$ and $R_H$ we can define a dimensionless parameter $v$ measuring the relative intensity of the two coupling constants in the theory,  

$$v \equiv \frac{R_H}{R_W} = \frac{e}{\sqrt{\lambda}}$$  

(41)

(In the vortex case $1/v$ coincides with the Ginzburg-Landau parameter separating the two types of superconductivity). For $v \sim 1$ one has a well defined monopole configuration.

Now, when DBI-Higgs monopoles are considered, there is, apart from $\mu$, a second dimensionful parameter, $\beta$, $[\beta] = [\mu]^2$. Then $R_W$ and $R_H$ could in principle depend both on $\mu$ and $\beta$ and the configuration minimizing the energy will result from the matching of both parameters determining the size of the monopole. It may happen that in some
region of the \((\beta, \mu)\) domain such a matching becomes not possible. The outcome will be the non-existence of solutions in a range of values of \(\beta\) with size related to \(\beta_c\). In view of the complexity of the non-linear coupled system (25)-(27), let us analyze this possibility by using an approximate monopole configuration sharing the main features of the true solution:

\[
K_{\text{app}}(r) = \left(1 - \frac{r}{R}\right) \theta(R - r) \tag{42}
\]

\[
H_{\text{app}}(r) = r \left(1 - \frac{r_0^2}{r^2}\right)^2 \theta(r - r_0). \tag{43}
\]

Here \(R\) and \(r_0\) are parameters controlling the shape of the gauge field and scalar field configurations and they have to be determined by minimizing the energy \(E\) of the configuration. One can relate \(R\) and \(r_0\) with \(R_W\) and \(R_H\) by searching in (42)-(43) for the values of \(r\) for which the gauge and scalar field configurations differ in \(\frac{1}{\epsilon}\) from its asymptotic. This gives \(R_W \sim (1/2)R\), \(R_H \sim 3r_0\) and in this sense one can think that \(R = R(\beta, \mu)\) and \(r_0 = r_0(\beta, \mu)\). Let us finally note that using eq.\((41)\) one can write \(v\) in terms of \(r_0\) and \(R\),

\[
v \sim 6 \frac{r_0}{R} \equiv 6 x. \tag{44}
\]

We have seen that for large \(\beta\) (say \(\beta \gtrsim 5\)), the DBI-Higgs theory just gives the same answer as the Yang-Mills-Higgs model so that one should always find in this region values for \(R\) and \(r_0\) minimizing \(E\). In particular, in the \(\beta \to \infty\) case we found using our approximate configurations that one has, for \(\lambda_0 = 0.5\), \(x = 0.128\). This giving for \(v\) the result approximate result \(v_{\text{app}} \sim 6x \sim 0.8\) to be compared with the “exact” result for Yang-Mills-Higgs theory, \(v = 1/\sqrt{\lambda_0} = \sqrt{2}\).

Now, for small \(\beta\) the situation radically changes. Indeed, using (42) and (43) one finds for the energy, to second order in \(\beta\) (apart of an irrelevant additive constant):

\[
E = -R \left(-0.33 + 0.03\beta + 3.04 x + 3.33 x^2 - 8.53 x^3 + 2 x^4 - 0.13 x^6 + 0.01 x^8 + 8 x^2 \log(x)\right) + 2.1 R^3 \lambda x^3 + O(\beta^3)
\]

\[
= 2.1 R^3 \lambda x^3 - R \left(f(x) + 0.03\beta\right) + O(\beta^3). \tag{45}
\]

It is not difficult to show that for small \(\beta\) (and any \(\lambda\)) the above expression does not have a minimum for any \(R\) and \(x\). We conjecture that this phenomenon occurring for an approximate configuration also takes place for the actual monopole solution: below a critical \(\beta\) value, there is no possible matching between the monopole core and the size of the region where the Higgs scalar is different from its vacuum value, in such a way the energy is minimized.

V. SUMMARY AND CONCLUSIONS

We have discussed in this work monopole solutions for an \(SO(3)\) Dirac-Born-Infeld gauge theory coupled to a Higgs scalar. We considered two alternative Lagrangians for the theory, differing in the way the trace over group indices is taken. Concerning the Higgs field, we have chosen the usual kinetic energy term and symmetry breaking potential.

As in the case of the Yang-Mills-Higgs system, a spherically symmetric ansatz leads to a system of coupled non-linear radial equations that have to be solved numerically. We have seen that the magnetic field corresponds, as in the ’t Hooft-Polyakov case, to that of a monopole with unit charge. When the absolute field \(\beta\) parameter is large (\(\gtrsim 5\)) the profile of the monopole solution is practically the same as the corresponding to the Yang-Mills-Higgs model. As \(\beta\) decreases, the monopole radius becomes smaller and the magnetic field concentrates more and more near the origin.

A remarkable effect takes place for small \(\beta\): there exists a critical value \(\beta_\epsilon\) such that for \(\beta \leq \beta_\epsilon\) the solution ceases to exist. The actual value of \(\beta_\epsilon\) depends on the choice of the other free parameters. We presented an scaling argument that supports this result: using an approximate solution that depends only on the dimensions of the configuration we showed that for small values of \(\beta\) it is impossible to adjust the size parameters to minimize the energy.

The monopole solution we have presented has many remarkable features that make worth a thorough investigation. In particular, the analysis of dyon solutions, which implies the inclusion of the \((F \tilde{F})^2\) term in the DBI action should reveal new features related to the existence of the dimensionfull parameter \(\beta\). We hope to discuss this problem in a future work.
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FIG. 1. Plot of the functions $K(r)$ and $H(r)/r$ (in dimensionless variables) for the monopole solution with $\lambda = 0$. The solid line corresponds to the solution with $\beta = 10$ and the dashed line corresponds to the solution with $\beta = 0.5$. 
FIG. 2. Plot of the functions $K(r)$ and $H(r)/r$ (in dimensionless variables) for the monopole solution with $\lambda = 0.5$. The solid line corresponds to the solution with $\beta = 10$ and the dashed line corresponds to the solution with $\beta = 0.8$. 
FIG. 3. Energy of the monopole configuration as a function of $\beta$ for different values of $\lambda$. 