Weak target echo detection based on low rank signal in multi-static passive radar

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Abstract—This paper considers weak target echo detection in multi-static passive radar system. A new detector adopting a test statistic based on the cross correlation of the principal left singular vectors of the reference and surveillance signal-plus-noise matrices is devised. Due to the exploitation of the low-rank structure of transmitted signal, it offers better performance than traditional detector. Simulation results verify the effectiveness of the proposed detection method.

1. INTRODUCTION

Passive radar which uses signals of opportunity as transmitted signal, such as television, audio broadcast and mobile communications, can offer many benefits compared with conventional active radars [1]. A receiver having two channels, namely reference channel and surveillance channel, is often implemented and the detection is based on the cross correlation of the signal-plus-noise matrices of these two channels. But this kind of detector has some drawbacks: first, it cannot reach performance optimality; second, it always needs long time coherent integration to detect weak target echo [2]. Therefore, some new methods have been developed to deal with the detection problem in passive radar. They can be grouped into categories as follows: detection methods exploiting the structure characteristic, detection methods using classification, detection methods based on fractal theory, detection methods adopting deep neural networks, detection methods using priori knowledge of the environment and detection methods based on space-time Radon Fourier transform.

In this paper, we exploit the inherent low-rank signal structure of the transmitted signal. A more rigorous asymptotic theoretical analysis of the detector is obtained through taking advantage of random matrix theory. We adopt the centralized processing scheme, which can jointly process all available measurement data to detect the target. Because this scheme can take advantage of correlations within the received signals that are not available in decentralized scheme. So it can achieve better detection performance than other approaches [3-4]. Further, we use some priori knowledge of the received signal in reference channel to reduce the computation load of the detector [5]. In addition, the information theoretic bound of SNR is analyzed and the detector is effective on condition that SNR satisfies the requirement of this bound [6].

The paper is organized as follows. In Section II, the signal model is established which will be used in our analysis. Then the detector is presented in Section III. Simulation result is given in Section IV and conclusion is drawn in Section V.
2. SIGNAL MODEL
Suppose the transmitted signal is a weighted periodic combination of the same waveform. Let there be \(N\) pulses in a processing interval. So the transmitted signal
\[
s(t) = \sum_{i=1}^{N} c_i u(t - iT)
\]
(1)
Where \(T\) is the period, \(u(t)\) is the waveform repeated.
The received signal in reference channel is
\[
s_r(t) = \sum_{i=1}^{N} u_{ri} u(t - iT - \tau_r) + n_r(t)
\]
(2)
Where \(u_{ri}\) includes contributions from the random message modulations as well as the channel attenuations, \(\tau_r\) denotes the propagation delay from the illuminator to the passive receiver, \(n_r(t)\) denotes the Gaussian white noise.

In the presence of a target at delay \(\tau_s\), doppler \(f_d\), the received signal in surveillance channel is
\[
s_s(t) = \sum_{i=1}^{N} u_{si} u(t - iT - \tau_s) e^{j2\pi f_d t} + n_s(t)
\]
(3)
Where the attenuations \(u_{si}\) vary from pulse to pulse due to target fluctuations and the random message modulations in the transmitted symbols from pulse to pulse, \(n_s(t)\) denotes the Gaussian white noise. The received target echo in surveillance channel is much weaker and is often masked by the direct path signal. Therefore, it is important to cancel it in the surveillance channel. Apart from the direct path signal coming through the side lobe of the receiver antenna, the target echo is also affected by clutter and multipath interference. In our system, we suppose that these disturbances are removed before the detection process.

We consider that the system consists of \(p\) illuminators and \(M\) receivers, and there is no synchronization problem in this system. Therefore, the received signals of surveillance channel and reference channel are
\[
\begin{bmatrix}
  s_s(n) \\
  s_r(n)
\end{bmatrix} =
\begin{bmatrix}
  \theta H_s \\
  H_r
\end{bmatrix} u(n) +
\begin{bmatrix}
  n_s(n) \\
  n_r(n)
\end{bmatrix}
\]
(4)
Where \(H_s\) and \(H_r\) represent the \(M \times p\) channels from the transmitters to the surveillance and reference receivers respectively. \(\theta \in \{0,1\}\) determines whether or not there is a signal in the surveillance channel \([7-8]\).

3. DETECTOR
Let \(\tilde{U}_s\) be an \(M \times K\) matrix whose \(K\) columns denote the \(K\) dominant left singular vectors of the data matrix \(s_s(n)\). Similarly \(\tilde{U}_r\) contains the \(K\) dominant left singular vectors of the data matrix \(s_r(n)\). Then, the low rank SVD test statistic is
\[ T = \sum_{k=1}^{K} \sum_{j=1}^{K} u_{sk}^H u_{rj}^T \]  

(5)

Where \( u_{sk} \) and \( u_{rj} \) denote the \( k \)th dominant singular vectors. Here, we assume that the rank of the signal subspace \( K \) is unknown, so we should estimate in the detection process \([9]\).

In our detector, we use KS test to discriminate between the null and alternative. KS test statistic is expressed as

\[ KS = \max_x \left| \hat{F}_1(x) - \hat{G}_1(x) \right| \]

(6)

Where \( \hat{F}_1(x) \) and \( \hat{G}_1(x) \) are the empirical distributions corresponding to the unspecified distributions \( F(x) \) and \( G(x) \) respectively. They are defined as

\[
\begin{align*}
\hat{F}_1(x) &= \begin{cases} 
0, & x < T_{(0,1)} \\
\frac{P}{P}, & T_{(0,p)} < x < T_{(0,p+1)} \\
1, & x \geq T_{(0,p)} 
\end{cases} \\
\hat{G}_1(x) &= \begin{cases} 
0, & x < T_{(1,1)} \\
\frac{P}{P}, & T_{(1,p)} < x < T_{(1,p+1)} \\
1, & x \geq T_{(1,p)} 
\end{cases}
\end{align*}
\]

(7)

(8)

If the empirical distributions are well discriminated, the test statistic assumes unity and when the distributions are not well separated, the test statistic assumes smaller positive values.

In the detection process, if there is some prior knowledge of the received signal in reference channel, the detection can be accomplished by using fewer measurements.

Because the rank of \( s_n(n) \) is unknown, it should be estimated first. It can be calculated as \([10-12]\)

\[ r = \frac{\| \phi \|^2}{\| \phi \|^2} \]

(9)

Where \( \phi \) is the correlation matrix of the reference channel.

After getting the rank of \( s_n(n) \), a shift is made as follows

\[ \lambda \phi = \eta \hat{U}_r \hat{V}_r \]

(10)

Where \( \hat{V}_r \) and \( \hat{U}_r \) are the first \( r \) left and right singular vectors of \( \phi \) respectively, \( \eta \in \{0,1\} \).

In the theory of information theoretic bounds, there exists a SNR threshold, below which the detector cannot succeed.

Let
\[ SNR^{upper} = 2\sqrt{h(\gamma) + \gamma \log 2} \]  

And

\[ SNR^{lower} = \begin{cases} 
1\gamma \geq 0.6 \\
\sqrt{2\gamma\omega\left(\frac{1}{2\gamma\sqrt{e}}\right)e^{-4l}/81} \leq \gamma < 0.6 \\
\sqrt{g(\gamma)e^{-4l}/81} & \gamma \leq e^{-4l}/81 \end{cases} \]  

Where

\[ h(\gamma) = -\gamma \log \gamma - (1-\gamma) \log (1-\gamma) \]
\[ g(\gamma) = 4\gamma \left(-\log \gamma - 2.1\sqrt{-2\log \gamma - \frac{3}{2} \log \frac{3e}{1-\gamma}}\right) \]
\[ \gamma \in [0,1] \text{ and } \omega(\gamma) \text{ for the root } x \text{ of } xe^x = \gamma. \]

If SNR is larger than \( SNR^{upper} \), the detection is information-theoretically possible and if SNR is smaller than \( SNR^{lower} \), the detection is information-theoretically impossible.

4. SIMULATION RESULT

We use the proposed detector to deal with the transmitted signal of rank-k, and the pulse number and sample number are set to be 50 and 13 respectively. We compare the proposed detector with the traditional cross correlator and GLRT detector. Two different SNRs in reference channel, namely, 0dB and -10dB are considered.

Figure1. Reference SNR=0dB
The simulation result in figure 1 shows that SVD based detector is superior to the other detection methods. But when the SNR in surveillance channel is too low, it cannot achieve the detection purpose. This result coincides with the information theoretical bound in the discussion.

The comparison between simulation result in figure 1 and that in figure 2 illustrates that if the priori knowledge in reference signal is more accurate, the detection performance is better.

5. CONCLUSION

In this paper, an SVD-based detector is proposed to detect the weak target echo in multi-static passive radar system. It exploits the underlying low-rank structure of the transmitted waveform from noncooperative illuminators. A new method which aims to reduce the measurements is introduced and SNR threshold is analyzed to ensure the effectiveness of the detector. Compared with the GLRT detectors, the proposed one shows better performance. Future research will exploit the sparse and low rank feature at the same time and use more priori knowledge to assist the detector.

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