Optimization of irrigation water allocation by using linear programming: case study on Belitang irrigation system

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Abstract. The irrigation water requirement in the Belitang Irrigation System cannot be fulfilled during dry season due to the limited water availability. Therefore, some agricultural land in the Belitang Irrigation System receive inadequate irrigation water. To solve this problem, it is necessary to optimize the allocation of irrigation water, thus, the existing potential area could be optimized. This study aimed to obtain optimum cropping area and maximum benefits by optimizing the allocation of irrigation water based on the dependable discharge and existing area. The optimum cropping area was obtained by making several alternative cropping patterns. The linear programming was used to analyze the optimization of irrigation water allocation. The results showed that the irrigation water allocation with a dependable discharge of 80% exhibited an optimum cropping area of 58,609 ha, cropping intensity of 271.36% and maximum benefits of IDR 1,041,186,630,000.00. From the results of these studies, the cropping pattern that can be applied by considering the water availability and the existing area to obtain the optimum cropping area and maximum benefit per year is paddy-paddy/paddy/second crop.

1. Introduction
The use of irrigation in agricultural activities is one of the factors that can support the agricultural productivity. In agricultural areas that have lower rainfall than crop water requirement, the use of irrigation is a priority to fulfill water requirements [1]. The agricultural water requirement consumes a lot of water. Therefore, increasing the efficiency of agricultural water use by optimizing the allocation of water resources is expected to ensure food security and promote sustainable agricultural development [2].

South Sumatera is one of the provinces in Indonesia that contributes to national food productivity, which is ranked 5th as a national main food producer. This is supported by 620,632 ha of rice fields with an irrigation area of 117,757 ha in 2015 [3]. One of the districts in South Sumatra Province with the most extensive use of irrigation to fulfill the needs of paddy fields is Ogan Komering Ulu Timur (OKU Timur) covering an area of 43,506 Ha in 2015 [4]. The source of irrigation water comes from the Komering River which is abstracted and distributed to several irrigated areas.

Belitang Irrigation System is a part of the irrigation area in OKU Timur which has a command area of about 21,598 ha and is divided into 3 regions, namely Belitang I, II and III. The problem of irrigation water allocation that needs to be handled from the irrigation system, occurs during the rainy and dry season. During the rainy season, irrigation water is available in very large quantities but the utilization is not optimal, so that water is discharged through drainage which ultimately contributes to
flooding in several rivers. During the dry season, the available water is limited, so that some agricultural land in the Belitang Irrigation System receive inadequate irrigation water. Therefore, not all farmers can cultivate crops in the cropping season III. Thus, the water allocation for crop water requirement needs to be optimized, so that water supply can be carried out optimally and farmers can get maximum benefit.

Optimization efforts in irrigation water allocation can be planned using a decision support system. There were several studies managed the proper water allocation. Giusti and Marsilli-Libelli [5] showed a Fuzzy decision support system to improve irrigation by considering commodities and location characteristics. Rudson et al. [6] used linear programming with solver facility in Microsoft Excel Software to optimize cropping patterns in the Kosinggolan Irrigation Area. Juwono et al. [7] used linear programming approach to optimize water allocation in the Parsanga Irrigation Area to obtain optimal cropping area and maximum benefits.

From several previous studies, this study would adopt the use of linear programming for decision making related to water allocation optimization. Linear programming was used because of its ability to solve a system with quite a lot of variables and constraints, its accuracy, and its result reliability [8]. This study used several variables including water availability, water requirements, cropping area, cropping patterns, production costs and agricultural productivity. This study aimed to obtain optimal cropping area and maximum benefits by optimizing the allocation of irrigation water based on the dependable discharge and existing area.

2. Research methods

2.1. Study area

Belitang Irrigation System is located in Ogan Komering Ulu Timur Regency, South Sumatra Province, Indonesia. Administratively, it is divided into three regions including Belitang I, Belitang II, and Belitang III. This irrigation system irrigates 21,598 ha of agricultural land whose water source comes from the Komering River abstracted through Belitang irrigation canal.

![Administrative map of OKU Timur Regency](study_area.png)

**Figure 1.** Administrative map of OKU Timur Regency (Study area).

2.2. Data collection

Research data were obtained from several agencies. Maps and schemes of irrigation networks, water discharge, irrigation institutions, irrigation area, and cropping patterns were obtained from the Perjaya Dam Office and Regional Office of Belitang Irrigation System. Plant commodities and agricultural productivity were obtained from the Agricultural Agency of OKU Timur Regency. Rainfall, air temperature, humidity, sun brightness, and wind velocity were obtained from Meteorological,
Climatological, and Geophysical Agency of Palembang. Sale value and production costs were obtained from Central Bureau of Statistics of OKU Timur Regency.

2.3. Data processing

2.3.1. Water availability. The irrigation water availability was obtained based on the available discharge, namely dependable discharge of 80%. The water discharge used was the actual data in the field of operational results in 2015-2019. It could also be interpreted that the possibility of river discharge being lower than dependable discharge was 20% [9]. The dependable discharge analysis was carried out with an event probability approach using the Weibull method. The equation used was as follows:

\[ P = \frac{m}{n+1} \times 100\% \]  

(1)

Where \( P \) is the probability (%), \( m \) is the number of discharge data, and \( n \) is the number of data.

2.3.2. Irrigation water requirement. The irrigation water requirement was determined using the Van Den Goor method [7]. One of the factors that determine crop water requirement is evapotranspiration which was determined by the Penman method [9]. Rainfall and climatological data used were measured data in the Belitang Irrigation System in 2015-2019. The irrigation water requirement (NFR) was determined by the following equation:

\[ NFR = Cu + Pd + NR + P - R_{eff} \]  

(2)

Where NFR is the water need on the rice field (mm/day), \( Cu \) is the crop water requirement (mm/day), \( Pd \) is the water need for land processing (mm/day), \( NR \) is the water need for nursery (mm/day), \( P \) is the percolation (mm/day), and \( R_{eff} \) is the effective rainfall (mm/day).

2.3.3. Analysis of irrigation benefit. Utilization of irrigation water in a certain period of time would generate benefits in agricultural production [8]. This benefit was determined by multiplying the selling price of agricultural production (IDR/kg) and agricultural productivity (kg/ha), then the product of this multiplication was reduced by the total cost of production (IDR/ha).

2.4. Analysis of optimization using linear programming

2.4.1. Optimization. The optimization method used was linear programming by utilizing the solver facility in Microsoft Excel. The advantage of using this method was the ability to solve a system with a lot of variables and constraints, its ease of use and accuracy, and its result reliability [8,10].

2.4.2. Analysis of mathematical model. Linear programming had three basic elements, namely the objective function, constraint function, and decision variable. The objective function showed the mathematical function that must be maximized and reflects the objectives to be achieved. In this study, the objective was to obtain optimum cropping area and maximum benefits. The mathematical equation of objective function was as follows:

\[ \text{maxZ} = \sum_{k=1}^{p} a X_{jMtk} - k + \sum_{k=1}^{p} b X_{jMtk} - k \]  

(3)

Where maxZ is the maximum benefit per year (IDR), \( a/b \) is the net benefit of yield (IDR/ha), \( X \) is the irrigation area (ha), \( k \) is the cropping season, \( p \) is the number of cropping season, and \( j \) is the commodity.
The constraint function showed the mathematical function that becomes constraints for efforts to maximize the objective function and represents the constraints that must be faced by decision makers. In this study, there were two constraint functions, namely water availability constraints and cropping area constraints. The mathematical equation for the constraint function of water availability is as follows:

\[ \sum_{k=1}^{p} X_{jMt-k} K_{jk} \leq Q_{jk} \]  \hspace{1cm} (4)

Where \( K \) is the volume of water need (m
\[^3\]/ha), and \( Q \) is the volume of water availability (m
\[^3\]).

The mathematical equation for the constraint function of water availability is as follows:

\[ \sum_{k=1}^{p} X_{jMt-k} \leq X_m \]  \hspace{1cm} (5)

Where \( X_{jMt-k} \) is the irrigation area of commodity \((j)\) in cropping season \(k\) (ha), and \( X_m \) is the existing irrigation area in each cropping season (ha).

The decision variable showed the mathematical function to be sought and provided the best value for the objectives achieved. In this study, the decision variable was the total cropping area for various crop commodities obtained after optimization analysis. The mathematical equation of decision variable was as follows:

\[ X_{jMt-k} \geq 0 \]  \hspace{1cm} (6)

Where \( X_{jMt-k} \) is the irrigation area of commodity \((j)\) in cropping season \(k\) (ha).

2.4.3. Scenarios of cropping pattern. Optimization of irrigation water allocation based on water availability with dependable discharge of 80% in this study was carried out in 4 cropping pattern scenarios. Existing-CP is a field condition (existing) which was used as a comparison. The cropping pattern scenario can be seen in Table 1. Cropping patterns that had been carried out in the irrigated area include cropping season I (Oct 2\(^{nd}\) week-Nov-Dec-Jan-Feb 1\(^{st}\) week), cropping season II (Feb 2\(^{nd}\) week-Mar-Apr-May-Jun 1\(^{st}\) week), and cropping season III (Jun 2\(^{nd}\) week-Jul-Aug-Sep-Oct 1\(^{st}\) week).

| Scenario of Cropping Pattern | Cropping Season | Explanation |
|------------------------------|-----------------|-------------|
| Existing-CP                  | MT I            | MT II       | MT III       |
| Scenario 1-CP                | Paddy           | Paddy       | Paddy/Second crop | Comparison |
| Scenario 2-CP                | Paddy           | Paddy       | Second crop   | Optimization |
| Scenario 3-CP                | Paddy           | Paddy       | Paddy/Second crop | Optimization |

3. Results and discussion

3.1. Irrigation water availability

Irrigation water from the Belitang Irrigation System is intended to irrigate agricultural areas in 3 regions. Based on the results of dependable discharge analysis using the Weibull method with a probability of 80%, the availability of irrigation water in a half-monthly period can be seen in Figure 2. The availability of irrigation water in the cropping season I and II was greater than the cropping season III. The discharge data would be used in the optimization analysis as a constraint function of water availability.
3.2. Irrigation water requirement

Irrigation water requirement was influenced by crop water requirement, water requirement for land preparation, water requirement for nursery, percolation and effective rainfall. The value of irrigation water requirement was determined by the cropping pattern of paddy-paddy-paddy/second crop with a half-monthly period starting from the second week of October. The analysis results of irrigation water requirement and water balance can be seen in Figure 3. The irrigation water requirement in cropping season I and II could be fulfilled, while cropping season III occurs with water deficit.

3.3. Analysis of optimization

3.3.1. Benefit of irrigation. The net benefit was derived from the income less the production cost. Production costs for agricultural cultivation consist of land processing cost, seed purchase cost, cropping cost, fertilizer cost, spraying cost and harvest cost. The results of production benefits can be seen in Table 2. Based on these benefits, the objective function can be formulated as in equation 7.
\[
\text{max} \ Z = \sum_{k=1}^{3} 18,435,000 X_{\text{paddy} \ Mt-k} + \sum_{k=1}^{3} 15,280,000 X_{\text{second crop} \ Mt-k}
\]  

(7)

3.3.2. Cropping area. The total of cropping area in Belitang Irrigation System can be seen in Table 3. The planting area was based on the cropping area in each cropping season with the existing cropping pattern, namely paddy-paddy-paddy/second crop. The cropping area in Table 3 became basis for determining the constraint function of cropping area in optimization analysis of irrigation water allocation. The constraint function of cropping area can be seen in equation 8, 9, 10, and 11.

Table 3. Cropping area in the Belitang Irrigation System before optimization (Existing).

| Commodity          | Cropping Area (ha) | Total of Cropping Area (ha) | Cropping Intensity (IP) |
|--------------------|--------------------|-----------------------------|-------------------------|
|                    | MT I | MT II | MT III | Total |                      |
| Paddy              | 21,598 | 21,598 | 7,770 | 50,966 | 235.98 %            |
| Second crop        | -    | -     | 4,677 | 4,677 | 21.65 %             |
| Total              | 21,598 | 21,598 | 12,447 | 55,643 | 257.63 %            |

\[
\sum_{j=1}^{2} X_{j \ Mt-1} \leq 21,598
\]  

(8)

\[
\sum_{j=1}^{2} X_{j \ Mt-2} \leq 21,598
\]  

(9)

\[
\sum_{j=1}^{2} X_{j \ Mt-3} \leq 21,598
\]  

(10)

\[
X_{j \ Mt-3} \leq 12,447
\]  

(11)

3.3.3. Volume of available water and water requirement in cropping season. Volume of available water and water requirement was determined based on the volume of water collected in the half-monthly period in each cycle of the cropping season. Volume of available water and water requirement can be seen in Table 4. The volume of water in the table was used as the basis for determining the constraint function of water availability formulated in equations 12, 13 and 14.

Table 4. Volume of available water and water requirement in cropping season.

| Cropping Pattern | Cropping Season | Water requirement (m$^3$/ha) | Volume of Available Water (10$^6$ m$^3$) |
|------------------|-----------------|------------------------------|------------------------------------------|
| Existing-CP      | I : Paddy       | 6,353.562                   | 555.297                                  |
|                  | II : Paddy      | 8,021.664                   | 511.466                                  |
|                  | III : Paddy/Second crop | 24,729.847 | 163.037                                  |
| Scenario 1-CP    | I : Paddy       | 6,353.562                   | 555.297                                  |
|                  | II : Paddy      | 8,021.664                   | 511.466                                  |
|                  | III : Paddy     | 24,729.847                  | 163.037                                  |
| Scenario 2-CP    | I : Paddy       | 6,353.562                   | 555.297                                  |
|                  | II : Paddy      | 8,021.664                   | 511.466                                  |
|                  | III : Second crop | 0.000                    | 163.037                                  |
| Scenario 3-CP    | I : Paddy       | 6,353.562                   | 555.297                                  |
|                  | II : Paddy      | 8,021.664                   | 511.466                                  |
|                  | III : Paddy/Second crop | 24,729.847 | 163.037                                  |
\[ \sum_{j=1}^{2} X_{j}^{M_{t-1}} K_{j}^{M_{t-1}} \leq 555.297 \times 10^6 \]  \hspace{1cm} (12)

\[ \sum_{j=1}^{2} X_{j}^{M_{t-2}} K_{j}^{M_{t-2}} \leq 511.466 \times 10^6 \]  \hspace{1cm} (13)

\[ \sum_{j=1}^{2} X_{j}^{M_{t-3}} K_{j}^{M_{t-3}} \leq 163.037 \times 10^6 \]  \hspace{1cm} (14)

### 3.4. Result of optimization

The result of optimization, which was the objective function in this study, was obtained by inserting the constraint function of water availability and cropping area in the solver facility of Microsoft Excel. The recapitulation of optimum cropping area and maximum benefits with several cropping pattern scenarios can be seen in Table 5 and Table 6, respectively.

**Table 5.** Optimization results of cropping area and cropping intensity in the cropping season.

| Cropping Pattern | Cropping Season | Commodity  | Total of cropping area (ha) | Cropping intensity (%) |
|------------------|----------------|-----------|-----------------------------|------------------------|
| Existing-CP      | MT I           | Paddy     | 21,598                      | 257.63                 |
|                  | MT II          | Paddy     | 21,598                      |                        |
|                  | MT III         | Paddy     | 7,770                       |                        |
|                  |                | Second crop | 4,677                      |                        |
| Scenario 1-CP    | MT I           | Paddy     | 21,598                      | 242.46                 |
|                  | MT II          | Paddy     | 21,598                      |                        |
|                  | MT III         | Paddy     | 9,171                       |                        |
| Scenario 2-CP    | MT I           | Paddy     | 21,598                      | 257.63                 |
|                  | MT II          | Paddy     | 21,598                      |                        |
|                  | MT III         | Second crop | 12,447                     |                        |
| Scenario 3-CP    | MT I           | Paddy     | 21,598                      | 271.36                 |
|                  | MT II          | Paddy     | 21,598                      |                        |
|                  | MT III         | Second crop | 12,447                     |                        |

Based on Table 5, the highest total of cropping area and cropping intensity is in the scenario 3-CP, while the lowest is in the scenario 2-CP. With the same cropping pattern between the existing-CP and the optimized scenario 3-CP, the cropping area and cropping intensity in the scenario 3-CP increased in area of 2,966 ha and cropping intensity of 13.73%. The increase in cropping area specifically occurred in the cropping season 3 with an increase in the cropping area of second crop and a decrease in the cropping area of paddy. This occurs because the consumption of irrigation water for second crop is lower than for paddy. The increase in cropping area has an effect on the maximum benefits obtained.

**Table 6.** Recapitulation of maximum benefit based on the optimization results in cropping season

| Cropping season | Maximum benefit (IDR) |
|-----------------|------------------------|
|                 | Existing-CP | Scenario 1-CP | Scenario 2-CP | Scenario 3-CP |
| MT I            | 398,159,130,000 | 398,159,130,000 | 398,159,130,000 | 398,159,130,000 |
| MT II           | 398,159,130,000 | 398,159,130,000 | 398,159,130,000 | 398,159,130,000 |
| MT III          | 214,704,510,000 | 169,069,228,500 | 190,190,160,000 | 244,868,370,000 |
| Benefit per year | 1,011,022,770,000 | 965,387,488,500 | 986,508,420,000 | 1,041,186,630,000 |
The maximum benefit was determined based on the sum of benefits for each cropping season. Based on Table 6, the highest maximum benefit per year was obtained in the scenario 3-CP of IDR 1,041,186,630,000.00. This is consistent with the highest total cropping area in this scenario. Therefore, the cropping pattern that can be applied by considering the water availability and the existing area to obtain the optimum cropping area and maximum benefit per year is paddy-paddy-second crop.

4. Conclusions
The allocation of irrigation water with dependable discharge of 80% obtained an optimum cropping area of 58,609 ha, cropping intensity of 271.36% and maximum benefit of IDR 1,041,186,630,000.00. The optimum cropping area and maximum benefit occurs in the scenario 3-CP with cropping pattern (paddy-paddy-paddy/second crop).

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