RESEARCH ON CASCADING FAILURE MODES AND ATTACK STRATEGIES OF MULTIMODAL TRANSPORT NETWORK

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Abstract. Cascading failure overall exists in practical network, which poses a risk of causing significant losses. Studying the effect of different cascading failure modes and attack strategies of the network is conducive to more effectively controlling the network. In the present study, the uniqueness of multimodal transport network is investigated by complying with the percolation theory, and a cascading failure model is built for the multimodal transport network by considering recovery mechanisms and dynamics. Under the three failure modes, i.e., node failure, edge failure and node-edge failure, nine attack strategies are formulated, consisting of random node attacking strategy (RNAS), high-degree attacking strategy (HDAS), high-closeness attacking strategy (HCAS), random edge attacking strategy (REAS), high-importance attacking strategy (HIAS1), high-importance attacking strategy (HIAS2), random node-edge attacking strategy (RN-EAS), high degree-importance1 attacking strategy (HD-I1AS), as well as high closeness-importance2 attacking strategy (HC-I2AS). The effect of network cascading failure is measured at the scale of the affected network that varies with the failure ratio and the network connectivity varying with the step. By conducting a simulation analysis, the results of the two indicators are compared; it is suggested that under the three failure modes, the attack strategies exhibiting high node closeness as the indicator always poses more effective damage to the network. Next, a sensitivity analysis is conducted, and it is concluded that HCAS is the most effective attack strategy. Accordingly, the subsequent study on the cascading failure of multimodal transport network should start with the nodes exhibiting high closeness to optimize the network.

1. Introduction. Cascading failure is common in practical network [26, 12], which involves power grid system [21, 25], transportation system [20, 31], communication network [32], etc. If some nodes in the network fail as impacted by attacks, the scale of failure will expand continuously via the interaction of coupling nodes; as a result, the network is partially or overall collapsed. This phenomenon is termed as cascading failure [16, 4]. In the network, cascading failure may cause catastrophic
consequences [7]. Thus, the recent study on cascading failure problems in practical network has aroused wide concern [19, 33, 22, 24, 9, 6].

The study on cascading failure problems can fall to three types. First, the differences of cascading failure effect on different network structures are studied. To be specific, Motter et al. verified that intentional attacks can pose more damage to complex networks, investigated the cascading failure in networks that displayed a highly heterogeneous distribution of loads, and determined that heterogeneity triggers network failure [17]. Li et al. built a model of cascading dynamics for the asymmetric dependency network. They proved that the asymmetric dependency network exhibits stronger robustness than the symmetric dependency network [14]. Bao et al. explored the performances of small world network and scale-free network when intentionally attacked at the vertex and edge, and they proved that scale-free network is more robust to intentional edge attacks, as opposed to small world network [1]. Liu et al. studied the cascading failure of multi-layer network with mutual dependence and demonstrated that asymmetric multi-layer network with mutual dependence exhibits stronger robustness than symmetric network [15]. Buldyrev et al. analyzed the cascading failure in a two-layer network and compared it with a single network. As revealed from the results, the wider the degree distribution of a single network node, the stronger the robustness to random failure will be; the opposite is true for interdependent networks [3]. Second, the effects of different factors on cascading failure in a single network are studied. Here are the following examples. Tian et al. studied the scale-free modular network with different coupling preferences, and they found that increasing the connection between communities can improve the robustness of the scale-free modular network [23]. Jiang et al. formulated various link orientation strategies in urban traffic networks to improve the robustness of the network, and verified the effectiveness of the strategies [11]. Wang et al. compared the effect of two attack strategies on scale-free network; when the corresponding parameters are altered, the attack strategies will achieve unexpected results [27]. Chen et al. delved into cascading failure of weighted complex network and built a capacity allocation model of a load capacity optimal relationship, proving that the model is capable of improving the robustness of the network at the minimum cost [5]. Witthaut et al. conducted a study on the nonlocal effects far away from the initial fault based on the complex power supply network. As suggested from the results, high clustering and small average path length of the network can overall suppress nonlocal overloads [30]. Wang et al. developed a structural and functional robustness model for the networked critical infrastructure systems; they considered the corresponding measurement indicators and cascading process to assess the effect of different failure modes on the robustness and subsequently verified the effectiveness of the model [28]. Third, the cascading failure mitigation methods are studied. To be specific, Yuan et al. proposed the introduction of a small number of reinforced nodes in interdependent networks, which can impact and support their neighbors. As revealed from the results, the reinforced nodes are capable of effectively reducing the effect of cascading failure in the network [34]. Zhao et al. proposed two immune strategies for disease transmission in multilayer network, i.e., multiplex node-based random (targeted) immunization and layer node-based random (targeted) immunization. Moreover, they proved that the two strategies are effective in ER random network [35]. Parshani et al. studied the cascading failure in a two-layer network. As revealed from the results, reducing the
coupling strength between network can effectively regulate the cascading failure in a dependent network [18].

Most of the existing studies highlighted cascading failure without recovery mechanism, whereas this study analyzes cascading failure in multimodal transport networks. Multimodal transport network is integrated transport network, which is limited by infrastructures in reality. Cascading failure refers to a multimodal transport network that pertain to problems with restoration mechanism and time dynamics. This study employs percolation theory to quantify cascading failure processes in multimodal transport network. The percolation theory was originally proposed by Broadbent and Hammersley [2] in 1957 to analyze the diffusion of the fluid passing through the medium. Given the difference of percolation state, it can fall to discrete percolation and continuous percolation. The essential difference is manifested in the difference of the network node distribution. Based on the difference of percolation probability, it can be split to uniform percolation and non-uniform percolation. The cascading failure of multimodal transport network is significantly similar to the diffusion of fluid in the percolation theory. This is indicated by three points below. The first is propagation environment: the risk propagation of multimodal transport network is determined by the flow of load, and the percolation depends on the fluid diffusion. Both of them propagate in the network with non-uniform distribution, with strong similarity. The second refers to propagation process: both of them can only propagate to the adjacent points and edges, and no process of cross-point or cross-edge propagation is carried out. The propagation process of multimodal transport highly complies with the mixed percolation mode in the percolation theory. The third indicates propagation direction: both are undirected network, and the load and fluid are allowed to propagate in any direction within the optional range. Accordingly, it is considered that propagation theory can be used to quantify cascading failure in multimodal transport network.

In this study, the percolation theory is introduced to analyze the cascade failure of the multimodal transport network. In the failure process, the time factor is characterized by the step size of the network percolation, and the cascading methods of different failure models are defined by exploiting the characteristics of the multimodal transport network. A cascading failure model in the multimodal transport network is built. The calculation is conducted in the Sichuan-Tibet multimodal transport network, and the effects of 9 failure modes on the multimodal transport network are compared. As revealed from the results, the high-closeness attacking strategy (HCAS) is more destructive than other attack strategies, so the nodes exhibiting high closeness should be started with to improve the robustness of the network.

The rests of this study are arranged as follows. In the second section, the characteristics of the multimodal transport network are described, and the failure mode and attack mode in the network are defined. In the third section, the model of the cascading failure of the multimodal transport network is built, and the network failure process is presented. In the fourth section, the simulation results and corresponding analysis are presented. In the fifth section, the work of this study is concluded.

2. Analysis of intermodal transport network characteristics. Multimodal transport refers to the effective connection of two or more modes of transport to provide the whole process integrated organization of goods transport services. Via
the interaction of coupling nodes between different transport modes, the multimodal transport network connects multiple transport subnetwork into a multimodal transport network. The multimodal transport network consisting of railway, highway and air transport modes is selected for analysis. In the multimodal transport network of the layer transport subnetwork, the coupling mode of “multi to multi part coupling” is employed between the nodes of the two-way connected network. Based on the physical structure of the multimodal transport network, the topological structure of each layer of the subnetwork is abstracted, and the railway transport network, road transport network and air transport network are respectively expressed as undirected graphs $G_r, G_h, G_a$.

2.1. **Network structure.** In the topological structure of multimodal transport network $G = (V, E), V = \{V_r, V_h, V_a\}$, denotes the set of nodes in the network, and $E = \{E_r, E_h, E_a\}$ represents the set of connected edges in a network. $G_r = \langle V_r, E_r, A_r \rangle$ indicates the topology of the railway transportation network, where $V_r = \{R_1, R_2, \cdots, R_N\}$ represents the node set of railway stations, $E_r = \{E_{ij} | i, j \in [R_1, R_N] \& i < j\}$ indicates the set of connected edges of railway routes, $A_r$ is the adjacency matrix, expressing the connection between the railway and other transportation network. Likewise, $G_h = \langle V_h, E_h, A_h \rangle$ and $G_a = \langle V_a, E_a, A_a \rangle$ respectively represent the topological structure of road transport network and air transport network. Different transport subnetworks are interconnected, and the topology is presented in Fig. 1. In the respective transport subnetwork, the connecting edges between the nodes are two-way connected. Moreover, only the capacity of the nodes and edges is required to be considered, instead of the direction of transportation.

![Multimodal transport network topology](image)

**Figure 1.** Multimodal transport network topology

2.2. **Network failure modes.** The failure of one or a few nodes or links will cause the failure of other nodes via the coupling relationship between nodes, then generate cascading effect, and eventually lead to the collapse of a considerable number of nodes or even the entire network. This phenomenon is termed as cascading failure phenomenon [17]. In the multimodal transport network, for the limitation of the basic settings, the nodes and connecting sides in the network are interrelated, allowing the cascading failure in the multimodal transport network to be particular. Then, the particularity is defined:
Definition 1: When a node failure occurs in a subnetwork in the multimodal transport network, the failed node acts as a directional circulation node. The load can only flow out, instead of flowing in, which is irreversible. Moreover, the load on the failed node can only flow to other subnetworks.

Definition 2: When a node in an intermodal network becomes overloaded, the overload node turns out to be a directional circulation node, and the load can only flow out, instead of flowing in. This process is reversible. After returning to normal, the overloaded node resumes disconnected.

Three trigger modes are set for cascading failure in a multimodal transport network, i.e., node failure, edge failure, and node-edge failure. ① Node failure: when a node fails, the load on it can no longer enter the corresponding connecting edge, nodes are disconnected from connected edges in the same transport subnetwork, and the load on them can only flow to other subnetwork via coupled nodes. ② Edge failure: when the connecting edge fails, the load on the connecting edge returns to the outflow node. The two connected nodes are disconnected from the connecting edge. ③ Node-edge failure: nodes and edges fail simultaneously. Then, network connectivity and load flow are the superposition of node failure states and edge failure states.

The flow of load in an intermodal network acts as an indicator determining the status of nodes or edges. In the stable state of the network, when the node or edge cannot connect with the network and the load on it cannot flow, the node or edge is in the failure state. When the load on a node or edge exceeds the maximum load and the flow speed of the load in the network is less than the normal speed, the network is overloaded. Both the failure state and the overload state are abnormal, thereby affecting the normal transportation of the network.

2.3. Network attack mode. We study 3 causes of node failure: random node attacking strategy (RNAS), high-degree attacking strategy (HDAS) and high-closeness attacking strategy (HCAS). RNAS means that a certain percentage of nodes in the network are randomly selected for failure handling. HDAS is a widely used deliberate attack strategy [26, 6, 10]. By descending the degree of nodes \( k_i \) in the network, a certain proportion of nodes are taken for failure treatment. Node closeness was initially proposed by Freeman in 1979 to measure the centrality of nodes in the network location [8], and it often acts as an indicator in attack strategies. HCAS classifies the closeness of the nodes in \( C_i \) descending order, which selects a certain proportion of nodes in turn for failure treatment. The calculation method of node closeness is expressed in Eq. (1):

\[
C_i = \frac{N_G - 1}{\sum_{j=1}^{N_G-1} d(i, j)}
\]

Where \( N_G \) is the total number of nodes in the network, \( d(i, j) \) represents the distance between node \( i \) and \( j \) node in the network.

We have also studied the 3 causes of edge failure, including: random edge attacking strategy (REAS), high-importance attacking strategy (HIAS1) and high-importance attacking strategy2 (HIAS2). REAS refers to randomly selecting a certain percentage of edges from the network for failure treatment. HIAS1 is to arrange the edge importance \( I_1(i, j) \) based on node degree in descending order, and select
Table 1. Network attacking strategy

| Failure mode     | Attack strategy |
|------------------|-----------------|
| Node failure     | RNAS            |
|                  | HDAS            |
|                  | HCAS            |
| Edge failure     | REAS            |
|                  | HIAS1           |
|                  | HIAS2           |
| Node-edge failure| RN-EAS          |
|                  | HD-I1AS         |
|                  | HC-I2AS         |

a certain percentage of edges in turn for failure treatment. We define the edge importance function $I_1(i,j)$ as expressed in Eq. (2); HIAS2 needs to first calculate the importance of all edges based on closeness $I_2(i,j)$. We define the edge importance function two $I_2(i,j)$ as expressed in Eq. (3), sort the importance values in descending order, and select a certain percentage of edges in turn for failure treatment.

$$I_1(i,j) = \frac{k_ik_j}{\sum_{i \neq j} k_ik_j}$$  \hspace{1cm} (2)

$$I_2(i,j) = \frac{C_iC_j}{\sum_{i \neq j} C_iC_j}$$  \hspace{1cm} (3)

Thus, we mainly study 9 attack strategies of the network, as shown in Table 1:

3. Establishment of cascaded failure model based on cascading failure theory. When the nodes and edges in the network fail, the load needs to be redistributed. The flow process of the load in the network is similar to the percolation process of the fluid in the porous medium [29]. Accordingly, this study introduces percolation theory to study cascading failure. Percolation theory was first proposed by Broadbent and Hammersley in 1957 [2], it means that when the percolation probability of the network nodes and edges does not exceed the percolation threshold, the failed nodes will only form isolated clusters. In contrast, the failed nodes will form a huge percolation cluster running throughout the network. Failure problems in multimodal transport network can be considered continuous percolation processes [13]. The present study analyzes the cascading failure of the network by defining node capacity, percolation probability and network cluster status in the network.

3.1. Node capacity. At the initial phase of network equilibrium, the initial load of the node $V_i$ is $L_i(0)$, maximum capacity is $C_i$, node degree is represented as $k_i$. By employing the Motter-Lai model [17], the load—capacity model is optimized as:

$$C_i = (1 + \alpha)L_i(0) = (1 + \alpha)k_i^\beta$$  \hspace{1cm} (4)

Where $\alpha, \beta$ are variable parameters adopted to regulate the relationship between node degree and initial load.

3.2. Percolation probability. The distribution of multimodal transport network is uneven. When a node $V_i$ in the network should redistribute the load, the probability $p_{j/i}$ of the load $L_i$ required to be redistributed to each node $V_j$ is different; the higher the probability, the more load the node $V_j$ will receive. Subsequently,

$$p_{j/i} = \frac{L_j(0)}{\sum_{f \in \Gamma_i} L_f(0)}$$  \hspace{1cm} (5)
Where $\Gamma_i$ denotes the set of neighboring nodes of the node $V_i$. When flowing in the multimodal transport network, the load always tends to flow to the nodes exhibiting high aggregation and large capacity, i.e., to the nodes with large node degree, as an attempt to ensure the mobility of the load, which is consistent with the actual situation.

### 3.3. Percolation theory

#### Node failure

The number of nodes in the network $G$ is $N_G$. When $t = 0$, if the node with the proportion $\varphi_1$ in the network $G$ fails, the load of the corresponding node is flowed out of the node, and then the failed node with the proportion $\varphi_1$ and its connected edges are deleted from the network, some normal and connected edges turn out to be independent clusters due to disconnection from the network. Furthermore, under the comprehensive effect, the topology of the network $G$ will vary to $G_1$. Here, the number of failed nodes in the network is $N_{G_1}$, connectivity is $C_{G_1}$, and then:

$$N_{G_1} = N_G \cdot (\varphi_1 + \varphi_2) \quad (6)$$

$$C_{G_1} = 1 - \frac{\sum_{i \in N_G} k_i}{\sum_{i \in N_G} k_i} \quad (7)$$

Where $\varphi_2$ denotes the proportion of independent clusters in the network affected by failure nodes. Then, the change of the network topology from $G$ to $G_1$ is the first stage of the network, and the failed node causes the change of the network structure to vary.

After the load $L_i(0)$ flows out from the failed node $V_i$, it can only flow to the coupling nodes in other transportation subnetwork. After coupling node $V_j$ receives additional load $L_j(0)$, a certain proportion of $\omega_t$ loads will be overloaded due to the limitation of node capacity $C_i$, causing node of proportion $\varphi_3$ to be overloaded. Then, the node will complete the transportation of the overcapacity load in time $\Delta t$ to restore the node to normal, and no longer receive the load inflow within time $\Delta t$. Moreover, the nodes connected to the nodes $V_j$ will cause nodes of proportion $\varphi_4$ in the network to be overloaded due to the accumulation of the load in time $\Delta t$. The network topology varies from $G_1$ to $G_2$, the number of overloaded nodes in the network is $N_{G_2}$, and Network connectivity is $C_{G_2}$. Subsequently,

$$N_{G_2} = N_G \cdot (\varphi_3 + \varphi_4) \quad (8)$$

$$C_{G_2} = C_{G_1} - \frac{\sum_{i \in N_{G_2}} k_i}{2 \sum_{i \in N_G} k_i} - \frac{2 \sum_{i \in N_G} k_i + \sum_{i \in N_{G_2}} k_i}{2 \sum_{i \in N_G} k_i} \quad (9)$$

Network varies from state to state $G_1$ is $G_2$ the second stage, and it is the first percolation of the network. After the first percolation, the network will conduct two or more times of percolation in time $t > \Delta t$, the overload node in the network should flow out the overload to restore normal, similar to the second stage process, cycle in turn till there is no overload node in the network. The simplified change of the network state in such process is illustrated in Fig. 2.
Figure 2. State change of node failure network

Here, “●” is set as normal nodes; “○” represents a failed node; “⊘” is overload node; “—” indicates the connection in the transport subnet; “■ ■ ■ ■” represents the connection between transport subnetwork.

At time $t = 0$, the network is in normal state; at $t \sim (0, \Delta t)$, node $R4$ in the railway network, node $H3$ in the highway network, and node $A2$ in the air traffic network fail, and the edges connected to them are disconnected. Then, the node $A4$ turns out to be isolated clusters since all the connected nodes have failed. Moreover, nodes $R3$ and $A3$ are overloaded as impacted by to the percolation process. At $t > \Delta t$, there will be two or more percolation processes in the network until the overload node in the network returns to normal. Then, the network scale $S_1$ affected by the failure node in the network is written as:

$$S_1 = \frac{N_{G1} + N_{G2} + \cdots}{N_G}$$  \hspace{1cm} (10)

**Edge failure**

The number of nodes in the network is expressed as $N_G$, and the number of connected sides is $M_G$, take the load flowing from a node with a small number to a node with a large number as an example. At $t = 0$, edges with proportion $\lambda_1$ in the network $G$ fail, the load on the connected edge returns to the outflow node, the connected edge is disconnected from the corresponding node, the network topology varies from $G$ to $G_3$, the node status in the network remains unchanged, and the connectivity is $C_{G_3}$:

$$C_{G_3} = 1 - \lambda_1$$  \hspace{1cm} (11)

At $t \sim (0, \Delta t)$, as impacted by the change of network topology, the node with the proportion $\varphi_5$ in the network is overloaded, and the node will complete the transportation of overload at $\Delta t$ to make the node return to normal. At $\Delta t$, the node no longer receives the inflow of the load, while the nodes connected to it will cause a certain proportion of $\varphi_6$ the nodes to be overloaded under the accumulation of the load. Here, the network varies from $G_3$ to $G_4$, the number of overloaded nodes in the network is $N_{G4}$, and the connectivity $C_{G_4}$ is written as:

$$N_{G4} = (\varphi_5 + \varphi_6) N_G$$  \hspace{1cm} (12)

$$C_{G_4} = 1 - \lambda_1 - \frac{\sum_{i \in N_{G4}} k_i}{2 \sum_{i \in N_G} k_i}$$  \hspace{1cm} (13)
At $t > \Delta t$, the network will have two or more percolation processes until the overload nodes in the network return to normal. The network state changes as illustrated in Fig. 3.

\[ S_2 = \frac{N_{G_3} + N_{G_4} + \cdots}{N_G} \]  

\[ C_{G_5} = 1 - \lambda_1 - \frac{\sum_{i \in N_{G_5}} k_i}{\sum_{i \in N_G} k_i} + \lambda_2 \]  

When $t \sim (0, \Delta t)$, as impacted by percolation, the network generates an overload node with the proportion of $\varphi_8$. Moreover, the node does not receive the inflow of load during the recovery process, which will lead to an overload node with the proportion of $\varphi_9$. Then, the network structure varies from $G_5$ to $G_6$, the overload node proportion in the network is $N_{G_6}$, and the connectivity $C_{G_6}$ is:

\[ N_{G_6} = N_G (\varphi_8 + \varphi_9) \]
When \( t > \Delta t \), the network will follow two or more percolation processes till the overload node in the network returns to normal. The network state changes as illustrated in Fig. 4.

\[
C_{G_0} = 1 - \lambda_1 - \frac{2 \sum_{i \in N_{G_5}} k_i + \sum_{i \in N_{G_6}} k_i}{2 \sum_{i \in N_G} k_i} + \lambda_2
\]  

(18)

Figure 4. State change of node-edge failure network

Here, “•” is set as normal nodes; “○” is a failed node; “⊘” represents overload node; “—” indicates the connection in the transport subnet; “■” represents the connection between transport subnetwork.

When \( t = 0 \), nodes \( R_3, H_2 \) and edges \( E_{R2R3}, E_{A1A3} \) fail, and the corresponding nodes are disconnected from the connection edge; when \( t \sim (0, \Delta t) \), nodes \( R_2, H_3, A_1 \) become overloaded, and the overloaded nodes percolate to resume normal operation; when \( t > \Delta t \), have two or more percolation processes till the overload node in the network returns to normal. Subsequently, the network scale affected by the failure nodes and edges in the network is \( S_3 \):

\[
S_3 = \frac{N_{G_5} + N_{G_6} + \cdots}{N_G}
\]

(19)

4. Case study and simulation analysis. In the present section, we focus on the effect of multiple attack modes on multimodal transport network. According to the query of this study on the website of the Ministry of transport of People’s Republic of China, when the goods are transported in the multimodal transport network, only the node selection of the main road is considered. In the practical environment, the number of nodes in the main road of multimodal transport network is not too large. Sichuan Tibet multimodal transport network is adopted for analysis, total nodal point \( N_G = 71 \), where the railway transport subnetwork consists of nodes \( N_{G_r} = 23, V_r = \{ R1, R2, \ldots, R23 \} \) road transport subnetwork contains nodes \( N_{G_h} = 43, V_h = \{ H1, H2, \ldots, H43 \} \) air transport subnetwork contains nodes \( N_{G_a} = 5, V_a = \{ A1, A2, \ldots, A5 \} \). The corresponding parameter values are achieved as \( \beta = 1.5, \alpha = 1.1 \), according to the data originating from China Railway Survey and Design Institute. Taking the average value of 50 independent experiments on the network, the effect of different attack strategies on the multimodal transport network can be obtained. To determine the effect of network cascading failure, two judgment indicators are set, i.e., affected network size with failure ratio
and network connectivity with step size. Fig. 6 illustrates the effect of network size and connectivity under a range of attack strategies.

![Diagram of network size and connectivity under different failure modes](image)

**Figure 5.** Impact of different attack strategies on the network

Fig. 5 (a) and Fig. 6 (b) indicate that as the failure node proportion increases, the entire network will eventually be affected by the failure node. At the identical
failure ratio, HCAS causes the poorest network connectivity. As suggested from the comparison of the effects of the three strategies, HCAS exerts the most significant cascading failure effect and performs the most effective attack. According to Fig. 5 (c) and 5 (d), unlike the node failure mode, the edge failure mode in the network will not ultimately alter network connectivity. The network connectivity only fluctuates during load percolation. For the edge failure mode, HIAS2 exerts the most noticeable impact on the network, and the attack is the most effective. In contrast, RN-EAS causes the network to exhibit the slowest crash speed and the narrowest impact range, while HC-I2AS is the opposite, as shown in Fig. 5 (e) and 5 (f). Attacks are more effective. As revealed from the results, under different failure modes, the attack strategy exhibiting high node proximity is constantly the most effective. To more specifically determine the effect of different attack strategies on the network, we choose HCAS, HIAS2, HC-I2AS for sensitivity analysis. As shown in Fig. 6. According to Fig. 6 (a) and 6 (b), with the increase of failure ratio, the entire network will be ultimately affected, whereas different attack strategies have different effects on the final network connectivity. HCAS causes the worst connectivity, while HIAS2 is the opposite. Overall, HCAS is considered the most effective of all attack strategies. It is therefore concluded that high-closeness attacking strategy is the most effective among 9 attack strategies.

![Figure 6. Sensitivity analysis](image)

5. Conclusion. In brief, the cascading failure of the multimodal transport network is analyzed by complying with percolation theory, and the characteristic of the multimodal transport network is highlighted. Moreover, a multimodal transport network is built in accordance with the network recovery mechanism and time dynamics Cascading failure model by exploiting network size and network connectivity affected by the failure as indicators. 9 attack strategies are compared under 3 failure modes (node failure, edge failure and node-edge failure): random node attacking strategy (RNAS), high-degree attacking strategy (HDAS), high-closeness attacking strategy (HCAS), random edge attacking strategy (REAS), high-importance attacking strategy (HIAS1), high-importance attacking strategy (HIAS2), random node-edge attacking strategy (RN-EAS), high degree-importance1 attacking strategy (HD-I1AS), as well as high closeness-importance2 attacking strategy (HC-I2AS).
By conducting a simulation analysis under different failure modes, the attack strategy exhibiting the high closeness of the node as an indicator is suggested to maintain an more extensive and faster impact than other strategies. Furthermore, the sensitivity of three attack strategies, i.e., HCAS, HIAS2 and HC-I2AS, is analyzed under the high closeness nodes. As revealed from simulation result, HCAS is considered the most effective of the 9 attack strategies.

According to the results, the risk management of multimodal transport network is found to be able to start from the nodes exhibiting high degree of closeness (e.g., facilitating the configuration of safety equipment of nodes, improving the failure recovery ability of nodes, and optimizing the network structure by introducing novel nodes and edges to make the network more balanced). This study has not been completed.

In subsequent study, further insights into the network robustness optimization strategy will be gained based on the results of this study, and a complete research system to solve cascading failure in multimodal transport network is expected to be developed.

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