ONE-POINT PROBABILITY DISTRIBUTION FUNCTIONS OF SUPersonic TURBULENT FLOWS IN SELF-GRAVITATING MEDIA

RALF S. KLESSEN
Sterrewacht Leiden, Postbus 9513, 2300-RA Leiden, Netherlands, and Max-Planck-Institut für Astronomie, Heidelberg, Germany
Received 1999 January 20; accepted 2000 January 21

ABSTRACT

Turbulence is essential for understanding the structure and dynamics of molecular clouds and star-forming regions. There is a need for adequate tools to describe and characterize the properties of turbulent flows. One-point probability distribution functions (PDFs) of dynamical variables have been suggested as appropriate statistical measures and applied to several observed molecular clouds. However, the interpretation of these data requires comparison with numerical simulations. To address this issue, smoothed particle hydrodynamics (SPH) simulations of driven and decaying, supersonic, turbulent flows with and without self-gravity are presented. In addition, random Gaussian velocity fields are analyzed to estimate the influence of variance effects. To characterize the flow properties, the PDFs of the density, of the line-of-sight velocity centroids, and of the line centroid increments are studied. This is supplemented by a discussion of the dispersion and the kurtosis of the increment PDFs, as well as the spatial distribution of velocity increments for small spatial lags. From the comparison between different models of interstellar turbulence, it follows that the inclusion of self-gravity leads to better agreement with the observed PDFs in molecular clouds. The increment PDFs for small spatial lags become exponential for all considered velocities. However, all the processes considered here lead to non-Gaussian signatures, differences are only gradual, and the analyzed PDFs are in addition projection dependent. It appears therefore very difficult to distinguish between different physical processes on the basis of PDFs only, which limits their applicability for adequately characterizing interstellar turbulence.

Subject headings: ISM: clouds — ISM: kinematics and dynamics — MHD — turbulence

1. INTRODUCTION

Turbulence is an important ingredient for understanding the properties and characteristics of molecular clouds and star-forming regions. Turbulent gas motions are highly supersonic, as indicated by the superthermal line widths ubiquitously observed throughout molecular clouds (Williams, Blitz, & McKee 2000). These motions carry enough energy to halt global collapse and act as a stabilizing agent for the entire cloud. However, it can be shown that interstellar turbulence decays quite rapidly on timescales of the order of the free-fall time of the system (Mac Low et al. 1998; Stone, Ostriker, & Gammie 1998; Padoan & Nordlund 1999). To explain the observed long lifetimes, turbulence in molecular clouds must be constantly driven (Gammie & Ostriker 1996; Mac Low et al. 1999). The interplay between self-gravity on the one hand (leading to local collapse and star formation) and turbulent gas motion on the other hand (trying to prevent this process) plays a key role in determining the structure of molecular clouds. Altogether, an understanding of the characteristics of compressible, supersonic, and constantly replenished turbulence in self-gravitating media is an important ingredient for an adequate description of molecular clouds dynamics. Vice versa, from analyzing the spatial and dynamical structure of molecular clouds, we can gain insight into the phenomenon of turbulence (for an overview over interstellar turbulence, see Franco & Carraminana 1999).

Unfortunately, a complete and comprehensive theory of turbulence does not exist. Because of the enormous complexity of the problem, progress has been slow since Kolmogorov’s pioneering work in 1941, where he derived simple scaling laws for incompressible, stationary, and homogeneous turbulence by postulating a self-similar energy cascade downward from the driving scale to the dissipation range. Most effort has since been put in finding an adequate closure procedure, i.e., in finding a way to express the highest-order correlation in the hierarchy of equations governing turbulent motion (for an excellent overview, see Lesieur 1997; also Boratav, Eden, & Erzan 1997). However, a satisfying description of turbulence has yet to be found.

Correlation and distribution functions of dynamical variables are frequently deployed for characterizing the kinematical properties of turbulent molecular clouds. Besides using two-point statistics (e.g., Scalo 1984; Kleiner & Dickman 1987; Kitamura et al. 1993; Miesch & Bally 1994; LaRosa, Shore, & Magnani 1999), many studies have concentrated on one-point statistics, namely, on analyzing the probability distribution function (PDF) of the (column) density and of dynamical observables, e.g., of the centroid velocities of molecular lines and their increments. The density PDF has been used to characterize numerical simulations of the interstellar medium by Vázquez-Semadeni (1994), Padoan, Nordlund, & Jones (1997), Passot & Vázquez-Semadeni (1998), and Scalo et al. (1998). Velocity PDFs for several star-forming molecular clouds have been determined by Miesch & Scalo (1995) and Miesch, Scalo, & Bally (1999). Lis et al. (1996, 1998) analyzed snapshots of a numerical simulation of mildly supersonic, decaying turbulence (without self-gravity) by Porter, Pouquet, & Woodward (1994) and applied the method to observations of the ρ Ophiuchus cloud. Altogether, the observed PDFs exhibit strong non-Gaussian features; they are often nearly exponential with possible evidence for power-law tails in the outer parts. This disagrees with the nearly Gaussian behavior typically found in experimental measurements and numerical models of incompressible turbulence. The observed centroid velocity increment PDFs are more strongly peaked and show stronger deviations from Gauss-
ianity than numerical models of incompressible turbulence predict. Furthermore, the spatial distribution of the largest centroid velocity differences (determining the tail of the distribution) appears “spotty” across the face of the clouds; there is no convincing evidence for filamentary structure. Miesch et al. (1999) conclude that turbulence in molecular clouds involves physical processes that are not adequately described by incompressible turbulence or mildly super- sonic decay simulations (see also Mac Low & Ossenkopf 2000).

It is the principal goal of this paper to extend previous determinations of PDFs from numerical models into a regime more applicable for interstellar turbulence by (1) calculating fully supersonic flows, (2) including self-gravity, and (3) incorporating a (simple analytic) description of turbul- ent energy input. For comparison with molecular cloud observations, I discuss the dynamical properties of decaying and stationary (i.e., driven), supersonic, isotropic turbulence in self-gravitating isothermal gaseous media. The PDFs for the density, for the line centroid velocity, and for their increments are derived as function of time and the evolu- tionary state of the turbulent model.

The structure of this paper is as follows. Section 2 intro- duces and defines the statistical tools applied in the current study. It is followed in § 3 by a description of the numerical scheme used to compute the time evolution of the turbulent flows. Section 4 shows that already simple variance effects in random Gaussian fields are able to introduce strong non-Gaussian distortions to the PDFs, which makes a clear-cut interpretation difficult. Section 5 contains the analysis of decaying, initially highly supersonic turbulence without self-gravity. This effect is then added to the simulations pre- sented in § 6. The model most relevant for molecular cloud dynamics is discussed in § 7. It includes a simple driving term to replenish the turbulent cascade. Finally, in § 8 all results are summarized.

2. PDFS AND THEIR INTERPRETATION

2.1. Turbulence and PDFs

The Kolmogorov (1941) approach to incompressible turbulence is a purely phenomenological one and assumes the existence of a stationary turbulent cascade. Energy is injected into the system at large scales and cascades down in a self-similar way. At the smallest scales it gets converted into heat by molecular viscosity. The flow at large scales is essentially inviscid; hence for small wavenumbers the equation of motion is dominated by the advection term. If the stationary state of fully developed turbulence results from random external forcing, then one naively expects the velocity distribution in the fluid to be Gaussian on timescales larger than the correlation time of the forcing, irrespective of the statistics of the forcing term. This follows from the central limit theorem. However, the situation is more complex (e.g., Frisch 1995; Lesieur 1997). One of the most striking (and least understood) features of turbulence is its intermittent spatial and temporal behavior. The structures that arise in a turbulent flow manifest themselves as high peaks at random places and at random times. This is reflected in the PDFs of dynamical variables or passively advect- ed scalars. They are sensitive measures of deviations from Gaussian statistics. Rare strong fluctuations are responsible for extended tails, whereas the much larger regions of low intensity contribute to the peak of the PDF near zero (for an analytical approach, see, e.g., Forster, Nelson, & Stephens 1977; Falkovich & Lebedev 1997; Chertkov, Kolokolov, & Vergassola 1997; Balkovsky et al. 1997; Balkovsky & Falkovich 1998). For incompressible turbu- lence the theory predicts velocity PDFs that are mainly Gaussian with only minor enhancement at the far ends of the tails. The distribution of velocity differences (between locations in the system separated by a given shift vector $\Delta r$) is expected to deviate considerably from being normal and is likely to resemble an exponential. This finding is sup- ported by a variety of experimental and numerical determinations (e.g., Kida & Murakami 1989; Vincent & Meneguzzi 1991; Jayesh & Warhaft 1991; She 1991; She, Jackson, & Orszag 1991; Cao, Chen, & She 1996; Vainshtein 1997; Lamballais, Lesieur, & Métais 1997; Machiels & Deville 1998). Compressible turbulence has remained too complex for a satisfying mathematical analysis.

2.2. PDFs of Observable Quantities

It is not clear how to relate the analytical work on incompressible turbulence to molecular clouds. In addition to the fact that interstellar turbulence is highly supersonic and self-gravitating, there are also observational limitations. Unlike the analytical approach or numerical simulations, molecular cloud observations allow access only to dimen- sionally reduced information. Velocity measurements are possible only along the line of sight, and the spatial structure of a cloud is seen only in projection onto the plane of the sky, i.e., as variations of the column density. Although some methods can yield information about the three-di- mensional spatial structure of the cloud (see Stutzki & Güsten 1990; Williams, De Geus, & Blitz 1994), the result is always model dependent and equivocal (see also Ballesteros-Paredes, Vázquez-Semadeni, & Scalo 1999).

A common way of obtaining knowledge about the velocity structure of molecular clouds is to study individual line profiles at a large number of various positions across the cloud. In the optically thin case, line shapes are in fact histograms of the radial velocities of gas sampled along the telescope beam. Falgarone & Phillips (1990) and Falgarone et al. (1994) showed that line profiles constructed from high- sensitivity CO maps exhibit non-Gaussian wings and attributed this to turbulent intermittency (see also Falgarone et al. 1998 on results from the IRAM Key Project). Dubinski, Narayan, & Phillips (1995) demonstrated that non-Gaussian line profiles can be produced from any Gaussian random velocity field if variance effects become important (which is always the case for very steep or truncat- ed power spectra). They concluded that non-Gaussian line profiles do not provide clear evidence for intermittency.

Another method of inferring properties of the velocity distribution in molecular clouds is to analyze the PDF of line centroid velocities obtained from a large number of individual measurements scanning the entire projected surface area of a cloud (Miesch & Scalo 1995; Lis et al. 1998; Miesch et al. 1999). Each line profile (i.e., the PDF along the line of sight) is collapsed into one single number, the centroid velocity, and then sampled perpendicular to the line of sight. Hence, the two functions differ in the direction of the sampling and in the quantity that is considered. A related statistical measure is the PDF of centroid velocity increments, which samples the velocity differences between the centroids for line measurements that are offset by a given separation. The observational advantage of using cen-
troid and increment PDFs is that the line measurements can typically be taken with lower sensitivity as only the centroid has to be determined instead of the detailed line shape. These measures are also less dependent on large-scale systematic motions of the cloud, and they are less effected by line broadening due to the possible presence of warm dilute gas. However, to allow for a meaningful analysis of the PDFs especially in the tails, the number of measurements needs to be very large and should not be less than about 1000. In order to sample the entire volume of interstellar clouds, the molecular lines used to obtain the PDFs are optically thin. I follow this approach in the present investigation and use a mass-weighted velocity sampling along the line of sight to determine the line centroid. This zero-opacity approximation does not require any explicit treatment of the radiation transfer process.

The observed PDFs are obtained from averaged quantities (from column densities or line centroids). To relate these observational measures to quantities relevant for turbulence theory, i.e., to the full three-dimensional PDF, numerical simulations are necessary, as only they allow unlimited access to all variables in phase space. A first attempt to do this was presented by Lis et al. (1996, 1998), who analyzed a simulation of mildly supersonic decaying hydrodynamic turbulence by Porter et al. (1994). Since their model neither included self-gravity nor considered flows at high Mach number or mechanisms to replenish turbulence, the applicability to the interstellar medium remained limited. This fact prompts the current investigation which extends the previous ones by calculating highly supersonic flows, and by including self-gravity and a turbulent driving scheme. The current study does not consider magnetic fields. Their influence on the PDFs needs to be addressed separately. However, the overall importance of magnetic fields and MHD waves on the dynamical structure of molecular clouds may not be large. The energy associated with the observed fields is of the order of the (turbulent) kinetic energy content of molecular clouds (Crutcher 1999). Magnetic fields cannot prevent the decay of turbulence (e.g., Mac Low et al. 1998), which implies the presence of external driving mechanisms. These energy sources replenish the turbulent cascade and may excite MHD waves explaining the inferred equipartition between turbulent and magnetic energies.

2.3. Statistical Definitions

The one-point probability distribution function \( f(x) \) of a variable \( x \) is defined such that \( f(x) \, dx \) measures the probability for the variable to be found in the interval \([x, x + dx]\). The density PDF (\( \rho \)-PDF) discussed in this paper is obtained from the local density associated with each smoothed particle hydrodynamics (SPH) particle. It is basically the normalized histogram summed over all particles in the simulation; i.e., a mass-weighted sampling procedure is applied. The line-of-sight velocity centroid PDF (\( v \)-PDF) is more complicated to compute. The face of the simulated cube is divided into \( 64^2 \) equal-sized cells. For each cell, the line profile is computed by sampling the normal (line-of-sight) velocity component of all gas particles that are projected into that cell. The line centroid is determined as the abscissa value of the peak of the distribution. This procedure corresponds to the formation of optically thin lines in molecular clouds, where all molecules within a certain column through the clouds contribute equally to the shape and intensity of the line. To reduce the sampling uncertainties, this procedure is repeated with the location of the cells shifted by one-half a cell size in each direction. Altogether about 20,000 lines contribute to the PDF. This is procedure is repeated for lines of sight along all three system axes to identify projection effects. The line centroid increment PDF (\( \Delta v \)-PDF) is obtained in a similar fashion. However, the sampled quantity is now the velocity difference between line centroids obtained at two distinct locations separated across the face of the cloud by a fixed shift vector \( \Delta \). The \( \Delta v \)-PDF for a spatial lag \( \Delta r \) is obtained as azimuthal average, i.e., as superposition of all individual PDFs with shift vectors of length \( \Delta r \).

Also, statistical moments of the distribution can be used to quantify the spread and shape of PDFs. For the current analysis I use the first four moments. Mean value \( \mu \) and standard deviation \( \sigma \) (the 1. and 2. moments) quantify the location and the width of the PDF and are given in units of the measured quantity. The third and fourth moments, skewness \( \theta \) and kurtosis \( \kappa \), are dimensionless quantities characterizing the shape of the distribution. The skewness \( \theta \) describes the degree of asymmetry of a distribution around its mean. The kurtosis \( \kappa \) measures the relative peakedness or flatness of the distribution. I use a definition by which \( \kappa = 3 \) corresponds to a normal distribution. Smaller values indicate existence of a flat peak compared to a Gaussian, larger values point toward a stronger peak or equivalently toward the existence of prominent tails in the distribution. A pure exponential results in \( \kappa = 6 \). Gaussian random fields are statistically fully determined by their mean value and the two-point correlation function, i.e., by their first two moments, \( \mu \) and \( \sigma \). All higher moments can be derived from those. The two-point correlation function is equivalent to the power spectrum in Fourier space (e.g., Bronstein & Semendjajew 1979).

Besides using moments, there are other possibilities for characterizing a distribution. Van der Marel & Franx (1993) and Dubinski et al. (1995) applied Gauss-Hermite expansion series to quantify non-normal contributions in line profiles. A more general approach has been suggested by Vio et al. (1994), who discuss alternatives to the histogram representation of PDFs. However, as astrophysical data sets typically are histograms of various types and as histograms are the most commonly used method to describe PDFs, this approach is also adopted here.

3. THE NUMERICAL MODEL

3.1. SPH in Combination with GRAPE

SPH (smoothed particle hydrodynamics) is a particle-based scheme to solve the equations of hydrodynamics. The fluid is represented by an ensemble of particles, each carrying mass, momentum, and hydrodynamic properties. The time evolution of the fluid is represented by the time evolution of the particles, governed by the equations of motion which are supplemented by a prescription to modify the hydrodynamic properties. At any location these properties are obtained by averaging over an appropriate set of neighboring particles. Excellent overviews over the method provide the reviews by Benz (1990) and Monaghan (1992). For the current study I use SPH because it is intrinsically Lagrangian and because it is able to resolve very high density contrasts. Another reason for choosing SPH is the possibility to use it in combination with the special-purpose
The code is based on a version originally developed by Benz (1990) and is used with a standard description of a von Neumann-type artificial viscosity (Monaghan & Gingold 1983) with the parameters $\alpha_1 = 1$ and $\beta_1 = 2$ for the linear and quadratic terms. The system is subject to periodic boundary conditions (Klessen 1997) and is integrated in time using a second-order Runge-Kutta-Fehlberg scheme, allowing individual time steps for each particle. Furthermore, the smoothing volume over which hydrodynamic quantities are averaged in the code is freely adjustable in space and time such that the number of neighbors for each particle remains approximately 50. When including self-gravity, regions with masses exceeding the Jeans limit become unstable and collapse. Once a highly condensed core has formed in the center of a collapsing gas clump, that core is substituted by a “sink” particle (Bate, Bonnell, & Price 1995) that inherits the combined masses, linear momenta, and “spin” angular momenta of the particles it replaces. It also has the ability to accrete further SPH particles from its infalling gaseous envelope.

For simulations of turbulent flows, one also has to take into account that an explicit viscosity term is introduced in the SPH method. This fact demands attention when studying dissipative processes, especially in the subsonic regime. The current study focuses on the properties of highly supersonic turbulent flows. In this regime, direct comparison between SPH and grid-based methods has proven the close correspondence of both methods (Mac Low et al. 1998; Klessen, Heitsch, & Mac Low 2000). If one bears the above caveats in mind, the SPH method calculates the time evolution of gaseous systems very reliably and accurately and offers large spatial and dynamical flexibility.

### 3.2. Models

The numerical models discussed here describe isothermal gas. The hydrodynamic equations are extended to include self-gravity (in §§ 6 and 7) and to incorporate a random turbulent driving mechanism (in § 7). All physical constants are set to unity. The same applies to mass and length scales; i.e., the total mass is $M = 1$ and the simulated volume is the cube $[ -1, 1 ]^3$. The mean density is thus $\rho = \frac{1}{3}$. The initial configuration of all dynamical systems discussed in this paper is a homogeneous gas distribution with a Gaussian velocity field. Without turbulence, the time evolution depends on one parameter, the ratio between internal and gravitational energy, $\epsilon \equiv \epsilon_{\text{int}} / \epsilon_{\text{pot}}$. This quantity can be interpreted as dimensionless temperature and determines the number of thermal Jeans masses contained in the system. Molecular clouds are characterized by line widths that largely exceed the thermal broadening. The evolution away from the homogeneous initial state is thus strongly influenced by the adopted initial velocity distribution and depends on whether turbulence is decaying or driven. Large turbulent kinetic energy can considerable slow down or even prevent the collapse of thermally Jeans unstable gas. The situation is very complex and depends on the shape and strength of the turbulent velocity spectrum (Klessen et al. 2000; for an analytical approach, see also see Bonazzola et al. 1992; Vázquez-Semadeni & Gazol 1995).

To generate and maintain turbulent flows, Gaussian velocity fields are introduced. The spatial variations of each component of the velocity vector $\mathbf{v}$ are described as superpositions of plane waves with wavenumbers $k = (k_x, k_y, k_z)$, where the phase of each wave is random and sampled from a uniform distribution in the interval $[0, 2\pi]$. Also, the amplitude is random but selected from a Gaussian distribution centered on zero and with a width determined by the power spectrum $P(k) = A_k k^\nu$. Gaussian fields are isotropic and depend only on the absolute value of the wave vector $k = |k|$. Only waves in the range $1 \leq k \leq k_{\text{max}}$ are considered. For large cutoff wavenumbers $k_{\text{max}}$, the Gaussian statistics is very well sampled. If only very few modes are used to generate the field, variance effects become strong and individual realizations of the field can deviate significantly from the ensemble average (see § 4). The field is then transformed back into real space and the resulting velocities are assigned to individual SPH particles using the “cloud-in-cell” scheme (Hockney & Eastwood 1988). For the initial field, all velocities are multiplied by the appropriate factor to reach the desired rms Mach number of the flow. In case of driven turbulence, this velocity field is also used to “kick” the SPH particles at every time step such that a constant level of kinetic energy is maintained (see Mac Low 1999).

### 4. PDFs from Gaussian Velocity Fluctuations

Variance effects in poorly sampled Gaussian velocity fields can lead to considerable non-normal contributions to the $v$- and $\Delta v$-PDFs. If a random process is the result of a sequence of independent events (or variables), then in the limit of large numbers, its distribution function will be a Gaussian around some mean value. However, only the properties of a large ensemble of Gaussian fields are determined in a statistical sense. Individual realizations may exhibit considerable deviations from the mean. The effect is strongest when only few (spatial) modes contribute to the field or, almost equivalently, when the power spectrum falls off very steeply. In this case, most kinetic energy is in large-scale motions.

This is visualized in Figure 1, which shows $v$-PDFs for homogeneous gas (sampled by $64^3$ SPH particles placed on a regular grid) with Gaussian velocity fields with power spectra $P(k) = \text{const.}$ that are truncated at different wavenumbers $k_{\text{max}}$ ranging from (Fig. 1a) $k_{\text{max}} = 2$ to (Fig. 1d) $k_{\text{max}} = 32$. Each realization is scaled such that the rms velocity dispersion is $\sigma_v = 0.5$. The figure displays the PDFs for the $x$-, $y$-, and $z$-components of the velocity. The PDFs of the strongly truncated spectrum (Fig. 1a) do not at all resemble normal distributions. The Gaussian statistics of the field is very badly sampled with only very few modes. Note that the PDFs of the same field may vary considerably for different velocity components, i.e., for different projections. With the inclusion of a larger number of Fourier modes, this situation improves, and in Figure 1d the PDFs of all projections sample the expected Gaussian distribution very well.

A similar conclusion can be derived for the $\Delta v$-PDF. This measure is even more sensitive to deviations from Gaussian statistics. Figure 2 plots the $\Delta v$-PDFs for the same sequence of velocity fields. For brevity, only the line-of-sight component parallel to the $x$-axis is considered. Furthermore, from the sequence of possible $\Delta v$-PDFs (defined by the spatial lag $\Delta r$), only three are shown, at small ($\Delta r = 1/32$; top curve), medium ($\Delta r = 10/32$; middle...
Fig. 1.—PDFs of line centroids for a homogeneous gaseous medium with Gaussian velocity field. The power spectrum is $P(k) = \text{const.}$ with wavenumbers in the intervals (a) $1 \leq k \leq 2$, (b) $1 \leq k \leq 4$, (c) $1 \leq k \leq 8$, and (d) $1 \leq k \leq 32$. All other modes are suppressed. Each figure plots PDFs of the $x$, $y$, and $z$-component of the velocity offset by $\Delta v = 0$. The length of the error bars is determined by the square root of the numbers of entries per velocity bin. The Gaussian fit from the first two moments is shown with dotted lines.

curve), and large spatial lags ($\Delta r = 30/32$; bottom curve). Sampling the Gaussian field with only two modes (Fig. 2a) is again insufficient to yield increment PDFs of normal shape. The velocity field is very smooth, and the line centroid velocity difference between neighboring cells is very small. Hence, for $\Delta r = 1/32$ the PDF is dominated by a distinct central peak at $\Delta v = 0$. The tails of the distribution are quite irregularly shaped. The situation becomes “better” when sampling increasing distances, as regions of the fluid separated by larger $\Delta r$ are less strongly correlated in velocity. For $\Delta r = 10/32$ and $\Delta r = 30/32$ the PDFs follow the Gaussian distribution more closely although irregularities in the shapes are still present. In Figures 2b and 2c the $\Delta v$-PDFs for medium to large lags are very well fit by Gaussians. Deviations occur only at small $\Delta r$, and the PDFs are exponential (and the distribution for $k_{\text{max}} = 4$ is still a bit cuspy). Finally, Figure 2d shows the three $\Delta v$-PDFs for the case in which all available spatial modes contribute to the velocity field ($1 \leq k \leq 32$). The PDFs follow a Gaussian for all spatial lags.

This behavior is also seen in the variation of the moments of the distribution as function of the spatial lag $\Delta r$. Applied to the above sequence of Gaussian velocity fields, Figure 3 displays the dispersion $\sigma$ and the kurtosis $\kappa$ of the distribution. The corresponding models are indicated at the right-hand side of each plot. The width of the distribution, as indicated by the dispersion $\sigma$ (Fig. 3a), typically grows with increasing $\Delta r$, reflecting the relative peakedness of the distribution at small lags. For example, the distribution (Fig. 3a) yields a slope of 0.3 in the range $-0.6 \leq \log_{10} \Delta r \leq -0.4$ and (Fig. 3b) leads to a value of 0.2 in relatively large interval $-1.5 \leq \log_{10} \Delta r \leq -0.5$. The effect disappears for the better sampled fields. Typical values for that slope in observed molecular clouds are $-0.3$ to $-0.5$ (Miesch et al. 1999). A direct measure of the peakedness of the distribution is its fourth moment, the kurtosis $\kappa$ (Fig. 3b). At small lags $\Delta r$, clearly the PDFs of model (a) are more strongly peaked than exponential ($\kappa = 6$). Comparing the entire sequence reveals again the tendency of the PDFs to become Gaussian at decreasing $\Delta r$ with

Note that Miesch et al. (1999) are plotting the function $\sigma^2$ versus the spatial lag $\Delta r$. For a comparison with the present study, their numbers have to be divided by a factor of two. Furthermore, they use a relatively narrow range of $\Delta r$-values to compute the slope of the function; larger intervals would on average tend to decrease these values (see their Fig. 14). In addition, Miesch et al. (1999) applied spatial filtering to remove large-scale velocity gradients in the clouds. These would lead to steeper slopes. The fact that in the present study the functions $\sigma$ and $\kappa$ level out for large spatial lags $\Delta r$ is a consequence of the periodic boundary conditions that do not allow for large-scale gradients.
increasing number of modes considered in the construction of the velocity field.

Taking it all together, it is advisable to consider conclusions about interstellar turbulence derived from solely analyzing one-point probability distribution functions from molecular clouds with caution. Similar to what has been shown by Dubinski et al. (1995) for molecular line profiles, deviations from the regular Gaussian shape found in $v$- and $\Delta v$-PDFs need not be the signpost of turbulent intermittency. Gaussian velocity fields that are dominated by

---

**Fig. 2.**—PDFs of line centroid increments for the same systems as in Fig. 1: (a) $1 \leq k \leq 2$, (b) $1 \leq k \leq 4$, (c) $1 \leq k \leq 8$, and (d) $1 \leq k \leq 32$. Each plot shows the distribution of centroid velocity differences between locations separated by the distance $\Delta r$: $\Delta r = 1/32$ (top curve), $\Delta r = 10/32$ (middle curve), and $\Delta r = 30/32$ (bottom curve). Only the velocity component for the line of sight parallel to the $x$-axis is considered. Again, the dotted lines represent the best fit Gaussian, except for the upper curve in (b) and (c), where the best exponential fit is shown.

---

**Fig. 3.**—(a) Second, dispersion $\sigma$, and (b) fourth moment, kurtosis $\kappa$, of the distribution of velocity increments displayed in Fig. 2 as functions of spatial lag $\Delta r$. The letters on the right-hand sight indicate correspondence to the previous figure. Each plot is offset by $\Delta \log_{10} \sigma = 0.5$ and $\Delta \log_{10} \sigma = 0.5$, and in (b) the horizontal dotted line indicates the value for a Gaussian $\kappa = 3$ ($\log_{10} \kappa = 0.48$).
5. ANALYSIS OF DECAYING SUPersonic TURBULENCE WITHOUT SELF-GRAVITY

In this section the PDFs of freely decaying initially highly supersonic turbulence without self-gravity are discussed. They are calculated from an SPH simulation with 350,000 particles (model G of Mac Low et al. 1998). Initially the system is homogeneous with a Gaussian velocity distribution with \( P(k) = \text{const.} \) in the interval \( 1 \leq k \leq 8 \). The rms Mach number of the flow is \( M = 5 \).

After the onset of the hydrodynamic evolution, the flow quickly becomes fully turbulent, resulting in rapid dissipation of kinetic energy. The energy decay is found to follow a power law \( t^{-\eta} \) with exponent \( \eta = 1.1 \pm 0.004 \). The overall evolution can be subdivided into several phases. The first phase is very short and is defined by the transition of the initially Gaussian velocity field into fully developed supersonic turbulence. It is determined by the formation of the first shocks, which begin to interact with each other and build up a complex network of intersecting shock fronts. Energy gets transferred from large to small scales and the turbulent cascade builds up. The second phase is given by the subsequent self-similar evolution of the network of shocks. Even though individual features are transient, the overall properties of this network change only slowly. In this phase of highly supersonic turbulence, the loss of kinetic energy is dominated by dissipation in shocked regions. In the transonic regime, i.e., the transition from highly supersonic to fully subsonic flow, energy dissipation in vortices generated by shock interactions becomes more and more important. Only the strongest shocks remain in this phase. Surprisingly, the energy decay law does not change during this transition. It continues to follow a power law with exponent \( \eta \approx 1 \). In the subsonic phase the flow closely resembles incompressible turbulence. Its properties are similar to those reported from numerous experiments and simulations (e.g., Porter et al. 1994; Lesieur 1997; Boratav et al. 1997). The simulation is stopped at \( t = 20.0 \), when the flow has decayed to an rms Mach number of \( M = 0.3 \). Since the energy-loss rate follows a power law, the duration of each successive phase grows.

This sequence of evolutionary stages is seen in the PDFs of the system. One noticeable effect is the decreasing width of the distribution functions as time progresses. As the kinetic energy decays the available range of velocities shrinks. This leads not only to “smaller” \( v \)- and \( \Delta v \)-PDFs but also to a smaller \( \rho \)-PDF, since compressible motions lose influence and the system becomes more homogeneous. This is indicated in Figure 4, which displays (Fig. 4a) the \( \rho \)-PDF and (Fig. 4b) the \( v \)-PDF at the following stages of the dynamical evolution (from top to bottom): shortly after the start, at \( t = 0.2 \) when the first shocks occur; then at \( t = 0.6 \), when the network of interacting shocks is established and supersonic turbulence is fully developed; during the transonic transition at \( t = 3.5 \); and finally at \( t = 20.0 \), when the flow has progressed into the subsonic regime. The rms Mach numbers at these stages are \( M = 5.0, 2.5, 1.0, \) and \( 0.3 \), respectively. The density PDF always closely follows a log-normal distribution; i.e., it is Gaussian in the logarithm of the density. Also the distribution of line centroids at the four different evolutionary stages of the system is best described by a Gaussian with only minor deviations at the far ends of the velocity spectrum.

For the same points in time, Figure 5 shows the \( \Delta v \)-PDFs for the \( x \)-component of the velocity. The displayed spatial lags are selected by analogy to Figure 2. Note the different velocity scaling in each plot, reflecting the decay of turbulent energy as the system evolves in time. Throughout the entire sequence, spatial lags larger than about 10% of the system size always lead to \( \Delta v \)-PDFs very close to Gaussian shape (middle and bottom curves). Considerable deviations occur only at small spatial lags (top curves). For those, the increment PDFs exhibit exponential wings during all stages of the evolution. When scaling the PDFs to the same width, the distribution in the subsonic regime (Fig. 5d) appears to be more strongly peaked than during the supersonic or
transonic phase (Figs. 5a–5c). There, the central parts of the PDFs are still reasonably well described by the Gaussian obtained from the first two moments, whereas in (Fig. 5d) the peak is considerably narrower, or vice versa, the tails of the distribution are more pronounced.

These results can be compared with the findings by Lis et al. (1998). They report increment PDFs for three snapshots of a high-resolution hydrodynamic simulation of decaying mildly super-sonic turbulence performed by Porter et al. (1994). They analyze the system at three different times corresponding to rms Mach numbers of $M \approx 0.96, 0.88$, and 0.52. Their first two data sets thus trace the transition from supersonic to subsonic flow and are comparable to the phase of the current model shown in Figure 5c; their last data set corresponds to that in Figure 5d. In the transonic regime both studies agree: Lis et al. (1998) report enhanced tails in the increment PDFs for the smallest spatial lags that they considered and near Gaussian distributions for larger lags (however, the largest separation they study is about 6% of the linear extent of the system). In the subsonic regime, Lis et al. (1998) find near Gaussian PDFs for very small spatial lags ($<1\%$) but find extended wings in the PDFs for lags of 3% and 6% of the system size. They associate this with the “disappearance” of large-scale structure. Indeed, their Figure 7 exhibits a high degree of fluctuations on small scales, which they argue become averaged away when considering small spatial lags in the $\Delta v$-PDF. In comparison of the PDF with spatial lags of 3% (top curves in Fig. 5; cf. the PDFs labeled $\Delta = 15$ in Lis et al. 1998), both studies come to the same result. At these scales the $\Delta v$-PDFs tend to exhibit more pronounced wings in the subsonic regime than in the supersonic regime. The SPH calculations reported here do not allow for a meaningful construction of $\delta v$-PDFs for $\Delta r < 3\%$. The Gaussian behavior of PDFs for very small spatial lags reported by Lis et al. (1998) therefore cannot be examined. However, neither of the purely hydrodynamic simulations lead to PDFs that are in good agreement with the observations. Observed PDFs typically are much more centrally peaked at small spatial separation (see, e.g., Fig. 4 in Lis et al. 1998; Miesch et al. 1999).

Figure 6 shows the spatial distribution of centroid velocity differences between cells separated by a vector lag of $\Delta r = (1/32, 1/32)$ (i.e., between neighboring cells along the diagonal). Data are obtained at the same times as above. Each figure displays the array of the absolute values of the velocity increments $\Delta v_x$ in linear scaling as indicated at the top. Note the decreasing velocity range reflecting the decay of turbulent energy. The distribution of $\Delta v_x$ appears random; there is no clear indication for coherent structures. This corresponds to most observations. Miesch et al. (1999) find for their sample of molecular clouds that high-amplitude velocity differences for very small spatial lags
6. ANALYSIS OF DECAYING TURBULENCE WITH SELF-GRAVITY

In this section, I discuss the properties of decaying, initially supersonic turbulence in a self-gravitating medium. Figure 7 displays an SPH simulation with 200,000 particles at six different times of its dynamical evolution. Since the model is subject to periodic boundary conditions, every figure has to be considered infinitely replicated in each direction. By analogy to the previous model, the system is initially homogeneous and its velocity field is generated

typically are well distributed, resulting in a spotty appearance. Note, however, that using azimuthal averaging Lis et al. (1998) report finding filamentary structures for the ρ Ophiuchus cloud. Altogether, filamentary structure is difficult to define, and a thorough mathematical analysis is seldom performed (for an astrophysical approach, see Adams & Wiseman 1994; for a discussion of the filamentary vortex structure in incompressible turbulence, consult Frisch 1995 or Lesieur 1997). The visual inspection of maps is often misleading and influenced by the parameters used to display the image. Larger velocity bins for instance tend to produce a more “filamentary” structure than very fine sampling of the velocity structure. Further uncertainty may be introduced by the fact that molecular clouds are seen only in one projection, as the signatures of the dynamical state of the system can strongly depend on the viewing angle.
Fig. 7.—Three-dimensional gas distribution in the SPH simulation of initially supersonic, decaying turbulence at six different stages of the dynamical evolution. Only every fourth of the 200,000 SPH particles is displayed. (a) Homogeneous initial density field. Further snapshots of the system are taken at (b) $t = 0.5$, (c) $t = 1.0$, (d) $t = 1.5$, (e) $t = 2.0$, and (f) $t = 2.5$, where time is measured in units of the free-fall timescale. The system evolves into a network of interacting shocks creating a filamentary density structure. As the turbulent flow decays, local collapse becomes possible. Dense cores (replaced by “sink” particles) are indicated by dark dots. In (e) the mass accumulated in collapsed cores is 40% of the total gas mass; in (f) this value is 61%.

with $P(k) = \text{const.}$ using modes with wavenumbers $1 \leq k \leq 8$. From the choice $\alpha = 0.01$ it follows that the system contains 120 thermal Jeans masses. The initial rms velocity dispersion is $\sigma_v = 0.5$, and with the sound speed $c_s = 0.082$ the rms Mach number follows as $M = 6$. These values imply that the initial turbulent velocity field contains sufficient energy to globally stabilize the system against gravitational collapse. Scaled to physical units using a
density $n(H_2) = 10^5$\ cm$^{-3}$, which is typical for massively star-forming regions (e.g., Williams et al. 2000), the system corresponds to a volume of [0.32 pc]$^3$ and contains a gas mass of $200 M_\odot$. As the simulation starts, the system quickly becomes fully turbulent and loses kinetic energy. As in the case without self-gravity a network of intersecting shocks develops leading to density fluctuations on all scales. If the mass of a fluctuation exceeds the (local) Jeans limit, it begins to contract because of self-gravity. During the early evolution, there is enough kinetic energy to prevent this collapse process on all scales (Fig. 7a: $t = 0.5$) and the properties of the system are similar to those of pure hydrodynamic turbulence. However, as time progresses and turbulent energy decays, the effective Jeans mass decreases. Local collapse of shock generated density fluctuations sets in despite the fact that the system is still globally stabilized by turbulence (see also Klessen et al. 2000). The central high-density cores of collapsing clumps are indicated by black dots. The cores form mainly at the intersection of filaments, where the density is highest and local collapse is most likely to set in. When turbulence is decayed sufficiently large-scale collapse also becomes possible. Gas clumps follow the global flow pattern toward a common center of gravity, where they may merge or subfragment. Gradually a cluster of dense cores is built up. In the isothermal model this process continues until all available gas is accreted onto the “protostellar” cluster (for more details, see Klessen & Burkert 2000).

The PDFs of (Fig. 8a) the density and of (Fig. 8b) the $x$-component of the line centroid velocities for the above six model snapshots are displayed in Figure 8. The corresponding time is indicated by the letters at the right-hand side of each panel. During the dynamical evolution of the system, the density distribution develops a high-density tail. This is the imprint of local collapse. The densities of compact cores are indicated by filled circles (at $t = 2.0$ and $t = 2.5$). Virtually all particles in the high-density tails at earlier times (at $t = 1.0$ and more so at $t = 1.5$) are accreted onto these cores. The bulk of matter roughly follows a log-normal density distribution, as indicated by the dotted parabola. The $v$-PDFs are nearly Gaussian as long as the dynamical state of the system is dominated by turbulence. Also, the width of the PDF remains roughly constant during this phase. This implies that the decay of turbulent kinetic energy is in balance with the gain of kinetic energy due to gravitational (“quasi-static”) contraction on large scales. The timescale for this process is determined by the energy dissipation in shocks and turbulent eddies. However, once localized collapse is able to set in, accelerations on small scales increase dramatically and the evolution “speeds up.” For times $t > 2.0$ the centroid PDFs become wider and exhibit significant deviations from the original Gaussian shape. The properties of the PDFs are similar to those observed in star-forming regions (Miesch & Scalo 1995; Lis et al. 1998; Miesch et al. 1999). This is expected since gravitational collapse is a necessary ingredient for forming stars.

Gravity creates nonisotropic density and velocity structure structures. When analyzing $v_\perp$- and $\Delta v$-PDFs, their appearance and properties will strongly depend on the viewing angle. This is a serious point of caution when interpreting observational data, as molecular clouds are seen in only one projection. As illustration, Figure 9 plots the centroid PDF at the time $t = 2.0$ for the line-of-sight projection along all three axes of the system. Whereas the PDFs for the $x$- and $y$-components of the velocity centroid are highly structured (top and middle curves—the latter one is even double peaked), the distribution of the $z$-component (bottom curve) is smooth and much smaller in width, comparable to the “average” PDF at earlier stages of the evolution. As the variations between different viewing angles or equivalently

![Fig. 8.—PDFs of (a) the density and (b) the $x$-component of line centroids for the simulation of initially supersonic, decaying turbulence in self-gravitating gas. The time sequence is the same as in the previous figure as denoted by the corresponding letter to the right of each PDF. In the left panel, the initial density is indicated by the vertical line at $\rho = \frac{1}{3}$. The density contributions from collapsed cores forming in the late stages of the evolution are indicated by filled circles. The core density corresponds to a mean value computed from the core mass divided by its accretion volume. In both figures, each PDF is offset by $\Delta \log_{10} N = 2.0$, with the base $\log_{10} N = 0.0$ indicated by horizontal dashed lines. The best-fit Gaussian curves are shown as dotted lines.](image-url)
Fig. 9.—Centroid velocity PDFs for the simulation of initially supersonic, decaying turbulence in self-gravitating gas at $t = 2.0$ for the line of sight being along the $x$-axis (top curve, which is identical to the fifth PDF in Fig. 8b), along the $y$-axis (middle curve), and along the $z$-axis of the system (bottom curve). Each distribution is offset by $\log_{10} N = 2.0$, with the horizontal lines indicating the base $\log_{10} N = 0.0$. The PDFs of various projections and velocity components can differ considerably.

Gravity affects the $\Delta v$-PDF. Figure 10 displays the increment PDFs at small, intermediate, and large spatial lags, analogous to Figures 2 and 5. The time ranges from (Fig. 10a) $t = 1.0$ to (Fig. 10d) $t = 2.5$, corresponding to Figures 7c–7f. The PDFs for $t = 0.0$ and $t = 0.5$ are not shown since at these stages supersonic turbulence dominates the dynamics of the system and the PDFs are comparable to those without gravity (Fig. 5). This still holds for $t = 1.0$. The increment PDFs for medium to large spatial lags appear Gaussian; however, the PDF for the smallest lag follows a perfect exponential all the way inward to $\Delta v = 0$. Unlike in the case without gravity, the peak of the distribution is not “round”; i.e., it is not Gaussian in the innermost parts (when scaled to the same width). It is a sign of self-gravitating systems that the increment PDF at smallest lags is very strongly peaked and remains exponential over the entire range of measured velocity increments. This behavior is also seen Figures 10b–10d. At these later stages of the evolution in addition non-Gaussian behavior is also found at medium lags. This results from the existence of large-scale different velocity components can be very large, statements about the three-dimensional velocity structure from only observing one projection can be misleading.

Gravity effects the $\Delta v$-PDF. Figure 10 displays the increment PDFs at small, intermediate, and large spatial lags, analogous to Figures 2 and 5. The time ranges from (Fig. 10a) $t = 1.0$ to (Fig. 10d) $t = 2.5$, corresponding to Figures 7c–7f. The PDFs for $t = 0.0$ and $t = 0.5$ are not shown since at these stages supersonic turbulence dominates the dynamic of the system and the PDFs are comparable to those without gravity (Fig. 5). This still holds for $t = 1.0$. The increment PDFs for medium to large spatial lags appear Gaussian; however, the PDF for the smallest lag follows a perfect exponential all the way inward to $\Delta v = 0$. Unlike in the case without gravity, the peak of the distribution is not “round”; i.e., it is not Gaussian in the innermost parts (when scaled to the same width). It is a sign of self-gravitating systems that the increment PDF at smallest lags is very strongly peaked and remains exponential over the entire range of measured velocity increments. This behavior is also seen Figures 10b–10d. At these later stages of the evolution in addition non-Gaussian behavior is also found at medium lags. This results from the existence of large-scale
filaments and streaming motions. The same behavior is found for the increment PDFs from observed molecular clouds (for ρ Ophiuchus, see Lis et al. 1998; for Orion, Mon R2, L1228, L1551, and HH 83 see Miesch et al. 1999). In each case, the distribution for the smallest lag (one pixel size) is very strongly peaked at \( \Delta r = 0 \), in some cases even more than exponential. The deviations from the Gaussian shape remain for larger lags but are not so pronounced. The inclusion of self-gravity into models of interstellar turbulence leads to good agreement with the observed increment PDFs. However, this result may not be unique because in molecular clouds additional processes that could also lead to strong deviations from Gaussianity are likely to be present.

The time evolution of the statistical moments of the \( \Delta \nu \)-PDFs for various spatial lags is presented in Figure 11. It plots (Fig. 11a) the dispersion \( \sigma \) and (Fig. 11b) the kurtosis \( \kappa \). The letters on the right-hand side indicate the corresponding time in Figure 7. At \( t = 0.0 \) the width \( \sigma \) of the PDF is approximately constant for all \( \Delta r \) and the kurtosis \( \kappa \) is close to normal value of 3. Both indicate that Gaussian statistics very well describes the initial velocity field. As turbulent energy decays, gravitational collapse sets in. Because of the gravitational acceleration, the amplitudes of centroid velocity differences between separate regions in the cloud grow larger, the width \( \sigma \) of the \( \Delta \nu \)-PDFs increases. This becomes more important when sampling velocity differences on larger spatial scales; hence \( \sigma \) also increases with \( \Delta r \). The slope is \( d \log_{10} \sigma / d \log_{10} \Delta r \approx 0.2 \). For \( \log_{10} \Delta r > -0.4 \) it levels out, which is a result of the adopted periodic boundary conditions. They do not allow for large-scale velocity gradients. The increasing peakedness of the \( \Delta \nu \)-PDF is reflected in the large values of the kurtosis \( \kappa \) at the later stages of the evolution. For small spatial lags the PDFs are more centrally concentrated than exponential (i.e., \( \kappa > 6 \)), and even at large spatial separations they are still more strongly peaked than Gaussian (\( \kappa > 3 \)). The slope at \( t = 2.5 \) is \( d \log_{10} \kappa / d \log_{10} \Delta r \approx -0.4 \) which is indeed comparable to what is found in observed star-forming regions (Miesch et al. 1999).

For the above simulation of self-gravitating, decaying, supersonic turbulence, Figure 12 plots the two-dimensional distribution of centroid increments for a vector lag \( \Delta r = (1/32, 1/32) \). The velocity profiles are sampled along the x-axis of the system. The magnitude of the velocity increment \( \Delta \nu \) is indicated at the top of each plot. The spatial distribution of velocity increments during the initial phases appears random. Later on, gravity gains influence over the flow and creates a network of intersecting filaments where gas streams onto and flows along toward local potential minima. At that stage, the velocity increments with the highest amplitudes tend to trace the large-scale filamentary structure. This is the sign of the anisotropic nature of gravitational collapse motions.

7. ANALYSIS OF DRIVEN TURBULENCE WITH SELF-GRAVITY

Figure 13 displays the gas distribution at different evolutionary stages of a simulation of driven, supersonic, self-gravitating turbulence. The number of SPH particles is 205,379. Again, the system is initially homogeneous in space and has a random Gaussian velocity field with a flat power spectrum in the wavenumber interval \( 3 \leq k \leq 4 \). It contains 64 thermal Jeans masses and turbulence is continuously driven as described in § 3.2. The initial evolution into equilibrium between the energy input by the driving force and the decay of turbulent kinetic energy is computed without self-gravity, then it is turned on. This phase is displayed in Figure 13a. In this state the turbulent Jeans mass (on scales larger than the maximum driving wave length) exceeds the total mass in the system by a factor of 2; the cloud is therefore stabilized by turbulence against gravitational collapse on global scales. However, local collapse (on scales at or below the driving scale) is still possible and does occur. As in the previous case without driving, the dynamical evolution of the system leads to the formation of a cluster of dense collapsed cores. This is shown in Figures 13b–13d, which display the system when 20%, 40%, and 60% of the gas mass has accumulated in dense collapsed cores (at times...
Fig. 12.—Two-dimensional distribution (in the $yz$ plane) of centroid increments for velocity profiles along the $x$-axis of the system between locations separated by a vector lag $\Delta r = (1/32, 1/32)$ for the simulation of self-gravitating, decaying, supersonic turbulence. As in Fig. 10, the data are displayed for times (a) $t = 1.0$, (b) $t = 1.5$, (c) $t = 2.0$, and (d) $t = 2.5$. The magnitude of the velocity increment $\Delta v_x$ is indicated at the top of each plot.

$t = 1.8, 3.2, \text{and } 4.8$, respectively). However, in the presence of the driving source, the timescales for accretion are longer and the cluster is less dense.

The PDFs of (Fig. 14a) the density and (Fig. 14b) the $x$-component of the line centroid velocities corresponding to the above four snapshots are displayed in Figure 14. As in the previous model, the bulk of gas particles that are not accreted onto cores build up an approximately log-normal $\rho$-PDF (dotted lines). Also the $v$-PDF remains close to the Gaussian value. This is different from the case of purely decaying self-gravitating turbulence, where at some stage global collapse motions set in and lead to very wide and distorted centroid PDFs. This is not possible in the simulation of driven turbulence, as it is stabilized on the largest scales by turbulence. Collapse occurs only locally, which leaves the width of the PDFs relatively unaffected and only mildly alters their shape.

Also the $\Delta v$-PDFs show no obvious sign of evolution. For the $x$-component of the velocity, these functions are displayed in Figure 15, again for three different spatial lags. The chosen times correspond (Fig. 15a) to the equilibrium state at $t = 0.0$ and (Fig. 15b) to $t = 4.8$, which is the final state of the simulation. The PDFs only marginally grow in width. At every evolutionary stage, the PDF for the smallest spatial lag is exponential, whereas the PDFs for medium and large shift vectors closely follow the Gaussian curve.
Fig. 13.—Three-dimensional gas distribution in the simulation of constantly driven turbulence in self-gravitating gas. Once the turbulent kinetic energy reaches the equilibrium level, gravity is turned on. This stage is displayed in (a). The next three snapshots of the system are taken at times (b) $t = 1.8$, when 20% of the gas mass is in dense collapsed cores (*filled circles*; cf. Fig. 7), at (c) $t = 3.2$, when the mass in cores is 40% of the total mass, and at (d) $t = 4.8$, when the cluster of cores contains 60% of the system mass. Time is given in units of the free-fall time, but unlike in the previous cases it is counted from the point gravity is turned on.

Fig. 14.—PDFs of (a) the density and (b) the $x$-component of line centroids for the simulation of driven turbulence in self-gravitating gas. The time sequence is the same as in the previous figure as indicated by the letters to the right. Each PDF is offset by $\Delta \log_{10} N = 2.0$, with the base $\log_{10} N = 0.0$ indicated by horizontal dashed lines. The best-fit Gaussian curves are shown as dotted lines. The density contributions in (a) coming from collapsed cores are indicated by *filled circles*. 
Fig. 15.—PDFs of the $x$-component of the centroid velocity increments for three spatial lags: $d \Delta r = 1/32$ (*top curve*), $d \Delta r = 10/32$ (*middle curve*), and $d \Delta r = 30/32$ (*bottom curve*). The functions are computed for the simulation of driven, self-gravitating, supersonic turbulence at (a) $t = 0.0$ and (b) $t = 4.8$. As in the previous models the increment PDFs for small spatial lags are approximately exponential; however, the PDFs for larger separations remain close to Gaussian throughout the evolution.

defined by the first two moments of the distribution (*dotted lines*). The functions are similar to the ones in the previous model before the large-scale collapse motions set in (Figs. 10a and 10b). Only overall contraction will affect $\Delta r$-PDF at medium to large lags. This behavior also follows from comparing the statistical moments. Figure 16 plots (Fig. 16a) the dispersion $\sigma$ and (Fig. 16b) the kurtosis $\kappa$ as function of the spatial lag $\Delta r$. Figures 11a and 16a are very similar, as soon as turbulence is established the width $\sigma$ of the PDF increases with $\Delta r$ with a slope of $\frac{d \log_{10} \sigma}{d \log_{10} \Delta r} \lesssim 0.2$ for small to medium lags and levels out for larger ones. However, when comparing the peakedness of the PDF as indicated by $\kappa$ (Figs. 11b and 16b), the model of decaying self-gravitating turbulence yields much higher values since the PDFs are more strongly peaked because of the presence of large-scale collapse motions.

Figure 17 finally shows the spatial distribution of the $x$-component of the line centroid increments for a vector lag $d \Delta r = (1/32, 1/31)$. Since the increment maps at different evolutionary times are statistically indistinguishable, only times (Fig. 17a) $t = 0.0$ and (Fig. 17b) $t = 4.8$ are displayed in the figure. As in the case of supersonic, purely hydrodynamic turbulence, the spatial distribution of velocity increments appears random and uncorrelated.

The adopted driving mechanism prevents global collapse. The bulk properties of the system therefore resemble hydrodynamic, non–self-gravitating turbulence. However, local collapse motions do exist and are responsible for noticeable distortions away from the Gaussian statistics. As the non-local driving scheme adopted here introduces a bias toward Gaussian velocity fields, these distortions are not very large. There is a need to introduce other, more realistic driving agents into this analysis. These could lead to much stronger non-Gaussian signatures in the PDFs.

8. SUMMARY

SPH simulations of driven and decaying, supersonic, turbulent flows with and without self-gravity have been analyzed in this study. It extends previous investigations of mildly supersonic, decaying, non–self-gravitating turbulence (Lis et al. 1996, 1998) into a regime more relevant for molecular clouds, by (1) considering highly supersonic flows and by including (2) self-gravity and (3) a driving source for turbulence.

![Fig. 16](image-url)
The flow properties are characterized by using the probability distribution functions of the density, of the line-of-sight velocity centroids, and of their increments. Furthermore the dispersion and the kurtosis of the increment PDFs are discussed, as well as the spatial distribution of the velocity increments for the smallest spatial lags.

1. To assess the influence of variance effects, simple Gaussian velocity fluctuations are studied. The insufficient sampling of random Gaussian ensembles leads to distorted PDFs similar to the observed ones. For line profiles this has been shown by Dubinski et al. (1995).

2. Decaying, initially highly supersonic turbulence without self-gravity leads to PDFs that also exhibit deviations from Gaussianity. For the trans- and subsonic regime, this has been reported by Lis et al. (1996, 1998). However, neglecting gravity and thus not allowing for the occurrence of collapse motions, these distortions are not very pronounced and cannot account well for the observational data (Lis et al. 1998; Miesch et al. 1999).

3. When including gravity into the models of decaying initially supersonic turbulence, the PDFs get into better agreement with the observations. During the early dynamical evolution of the system, turbulence carries enough kinetic energy to prevent collapse on all scales. In this phase the properties of the system are similar to those of non-gravitating hydrodynamic supersonic turbulence. However, as turbulent energy decays, gravitational collapse sets in. At first it is localized and on small scales, but as the turbulent support continues to diminish collapse motions include increasingly larger spatial scales. The evolution leads to the formation of an embedded cluster of dense protostellar cores (see also Klessen & Burkert 2000). As the collapse scale grows, the $p_\rho$, $v_\rho$, and $\Delta v$-PDFs get increasingly distorted. In particular, the $\Delta v$-PDFs for small spatial lags are strongly peaked and exponential over the entire range of measured velocities. This is very similar to what is observed in molecular clouds (for $\rho$ Ophiuchus, see Lis et al. 1998; for Orion, Mon R2, L1228, L1551, and HH 83, see Miesch et al. 1999).

4. The most realistic model for interstellar turbulence considered here includes a simple (nonlocal) driving scheme. It is used to stabilize the system against collapse on large scales. Again non-Gaussian PDFs are observed. Despite global stability, local collapse is possible and the system again evolves toward the formation of an embedded cluster of accreting protostellar cores. As the adopted driving scheme introduces a bias toward maintaining a Gaussian velocity distribution, the properties of the PDFs fall in between those of pure hydrodynamic supersonic turbulence and those observed in systems where self-gravity dominates after sufficient turbulent decay. This situation may change when considering more realistic driving schemes.

5. A point of caution: the use of $v$- and $\Delta v$-PDFs to unambiguously characterize interstellar turbulence and to identify possible physical driving mechanisms may be limited. All models considered in the current analysis lead to non-Gaussian signatures in the PDFs; differences are only gradual. In molecular clouds the number of physical processes that are expected to give rise to deviations from Gaussian statistics is large. Simple statistical sampling effects (§ 4) and turbulent intermittency caused by vortex motion (Lis et al. 1996, 1998), as well as the effect self-gravity (§ 6) and of shock interaction in highly supersonic flows (Mac Low & Ossenkopf 2000), all will lead to non-Gaussian signatures in the observed PDFs. Also stellar feedback processes, galactic shear and the presence of magnetic fields will influence the interstellar medium and create distortions in the velocity field. This needs to be studied in further detail. In addition, the full three-dimensional spatial and kinematical information is not accessible in molecular clouds, as measured quantities are always projections along the line of sight. As the structure of molecular clouds is extremely complex, the properties of the PDFs may vary
considerably with the viewing angle. Attempts to disentangle the different physical processes influencing interstellar turbulence therefore should not rely on analyzing velocity PDFs alone: they require additional statistical information.

I thank A. Burkert, F. Heitsch, and M.-M. Mac Low for many fruitful and stimulating discussions, and the editor S. Shore for his comments on the paper and his help with an extremely slow and nonresponsive (anonymous) referee. This study made extensive use of ADS.

REFERENCES
Adams, F. C., & Wiseman, J. J. 1994, ApJ, 435, 693
Balkovksy, E., & Falkovich, G. G. 1998, Phys. Rev. E, 57, 1231
Balkovksy, E., Falkovich, G., Kolokolov, I., & Lebedev, V. 1997, Phys. Rev. Lett., 78, 1452
Ballesteros-Paredes, J., Vázquez-Semadeni, E., & Scalo, J. M. 1999, ApJ, 515, 286
Bate, M. R., Bonnell, I. A., & Price, N. M. 1995, MNRAS, 277, 362
Benz, W. R., in The Numerical Modeling of Nonlinear Stellar Pulsations, ed. J. R. Buchler (Dordrecht: Kluwer), 269
Bonazzola, S., Perault, M., Puget, J. L., Heyvaerts, J., Falgarone, E., & Panis, I. 1992, J. Fluid Mech., 245, 1
Boratav, O., Eden, A., & Erzan, A. 1997, Turbulence Modeling and Vortex Dynamics (Heidelberg: Springer)
Bronstein, I. N., & Semendjajew, K. A. 1979, Taschenbuch der Mathematik (Leipzig: Teubner)
Cao, N., Chen, S., & She, Z.-S. 1996, Phys. Rev. Lett., 76, 3711
Crutcher, R. M. 1999, ApJ, 520, 706
Crutcher, R. M., Narayan, R., & Phillips, T. G. 1995, ApJ, 448, 226
Dubinski, E., Mazikino, J., & Vázquez-Semadeni, E. 1997, A&A, 303, 204
Falgarone, E., Lis, D. C., Phillips, T. G., Pety, J., & Falgarone, E. 1996, ApJ, 463, 623
Falgarone, E., Heitsch, F., & Mac Low, M.-M. 2000, ApJ, 535, 887
Gammie, C. F., & Ostriker, E. E. C. 1996, ApJ, 466, 814
Hockney, R. W., & Eastwood, J. W. 1988, Computer Simulation Using Particles, (Bristol: IOP)
Jayesh, & Warhaft, Z. 1991, Phys. Rev. Lett., 67, 3503
Kida, S., & Murakami, Y. 1989, Fluid Dyn. Res., 4, 347
Kitamura, Y., Sunada, K., Hayashi, M., & Hasegawa, T. 1993, ApJ, 413, 221
Kleiner, S. C., & Dickman, R. L. 1987, ApJ, 312, 837
Klessen, R. S. 1997, MNRAS, 292, 11
Klessen, R. S., & Burkert, A. 2000, ApJS, in press (astro-ph/9904090)
Klessen, R. S., Heitsch, F., & Mac Low, M.-M. 2000, ApJ, 535, 887