Production of electroweak bosons in $e^+e^-$ annihilation at high energies

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Production of electroweak bosons in $e^+e^-$ annihilation into quarks and into leptons at energies much greater than 100 Gev is considered. We account for double-logarithmic contributions to all orders in electroweak couplings. It is assumed that the bosons are emitted in the multi-Regge kinematics. The explicit expressions for the scattering amplitudes of the process are obtained. It is shown that the cross sections of the photon and $Z$ production have the identical energy dependence and asymptotically their ratio depends only on the Weinberg angle whereas the energy dependence of the cross section of the $W$ production is suppressed by factor $s^{-0.4}$ compared to them.

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I. INTRODUCTION

Annihilation of $e^+e^-$ in the double-logarithmic approximation (DLA) was considered first in Ref. [1]. In this work it was shown that when the total energy of the annihilation is high enough, the most sizable radiative QED corrections to $e^+e^-$ annihilation into $\mu^+\mu^-$ are double-logarithmic (DL). These corrections were calculated in Ref. [1] to all orders in $\alpha$. The DL contributions to this process appear when the final $\mu^+\mu^-$-pair is produced in the Regge kinematics, i.e. when the muons move (in cmf) closely to the initial $e^+e^-$-beam direction. According to the terminology introduced in Ref. [1], the process where $\mu^+$ moves in the $e^+\ (e^-)$-direction is called forward (backward) annihilation. Generalisation of these results to QCD (the forward and backward annihilation of quarks into quarks of other flavours) and to the EW theory (the backward annihilation of the left handed leptons into the right handed leptons) was obtained in Ref. [2] and Ref. [3] respectively. The forward and backward annihilation of $e^+e^-$ into quarks, all chiralities accounted for, was considered recently in Ref. [4]. One of the features obtained in Refs [1]-[4] is that the forward scattering amplitudes in DLA are greater than the backward ones in QED, in QCD and in EW theory.

Besides these $2 \rightarrow 2$, i.e. elastic processes, it is interesting also to study the $2 \rightarrow 2+n$ -exclusive processes accounting for emission of $n$ bosons accompanying the elastic $2 \rightarrow 2$ annihilation. The point is that besides the conventional, (soft) bremsstrahlung there can be emitted harder bosons. Emission of such bosons can be also studied in DLA to all orders in the couplings, providing the hard bosons are emitted in cones with opening angles $\ll 1$ around the initial $e^+e^-$ beams, i.e. in the multi-Regge kinematics. In this case, the most important part of the inelastic scattering amplitudes accounting for emission of $n$ bosons consists of the kinematic factor $\sim (1/k_1^\perp) \ldots (1/k_n^\perp)$ multiplied by some function $M$ which is called the multi-Regge amplitude of the process. The energy dependence of $M$ is controlled by $n+1$ electroweak Reggeons propagating in the crossing channel. Description of the multi-Regge photon production in the backward $e^+e^- \rightarrow \mu^+\mu^-$ -annihilation was considered in Ref. [5] and Ref. [6]. The multi-Regge amplitudes for gluon production in the backward annihilation of quark-antiquark pairs were considered in Ref. [7].

In the present paper we calculate the scattering amplitudes for electroweak boson production in $e^+e^-$ annihilation into quarks and leptons assuming that the bosons are emitted in the multi-Regge kinematics. We use the approach of Refs. [3,4] and account for electroweak double-logarithmic contributions to all orders in the electroweak couplings. The paper is organised as follows: in Sect. II we consider emission of one EW boson in $e^+e^-$ -annihilation into a quark-antiquark pair. We compose the infrared evolution equations (IREE) for the amplitudes of these processes. The IREE are solved in Sect. III. A generalisation of these results to the case of emission of $n$ bosons is given in Sect. IV. Emission of the EW bosons in $e^+e^-$ annihilation into leptons is considered in Sect. V. Results of numerical calculations are presented and discussed in Sect. VI. Finally, Sect. VII is for conclusive remarks.

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II. EMISSION OF ONE ELECTROWEAK BOSON IN THE MULTI-REGGE KINEMATICS

Let us start by considering the process $e^+(p_2)e^-(p_1) \rightarrow q(p'_1)\bar{q}(p'_2)$ accompanied by emission of one electroweak boson with momentum $k$. Energies of the bosons are assumed to be $\gg M_Z$. There are two kinematics for this process that yield DL radiative corrections. First of them is the kinematics where $p'_1 \sim p_1$, $p'_2 \sim p_2$. Obviously,

$$s = (p_1 + p_2)^2 \gg t_{1,2}, \quad t_1 = q_1^2 = (p'_1 - p_1)^2, \quad t_2 = q_2^2 = (p'_2 - p_2)^2$$

in this region. Eq. (1) means that the final particles are in cones with opening angles $\theta \ll 1$ around the $e^+e^-$ beams.

The second kinematics is the one where $p'_1 \sim p_2$, $p'_2 \sim p_1$ and therefore

$$s = (p_1 + p_2)^2 \gg u_{1,2}, \quad u_1 = q_1^2 = (p'_2 - p_1)^2, \quad u_2 = q_2^2 = (p'_1 - p_2)^2. \quad (2)$$

Eq. (2) means that the final particles are also in cones with the cm opening angles $\pi - \theta \ll 1$ around the $e^+e^-$ beams. Through this paper we call kinematics (1) the $t$ -kinematics and the kinematics (2) - the $u$ -kinematics. Both of them are of the Regge type and studying them is similar in many respects.

Instead of directly calculating inelastic amplitudes $A^{(\gamma,Z,W)}$ describing emission of any of $\gamma, Z, W$, it is possible to calculate first the amplitudes $A^{(0)}$ and $A^{(r)}$ $(r = 1, 2, 3)$ describing emission of the isoscalar and the isovector bosons respectively. When expressions for such amplitudes are obtained, the standard relations between the fields $\gamma, Z, W$ and the fields corresponding to the unbroken $SU(2) \otimes U(1)$ can be used in order to express $A^{(\gamma,Z,W)}$ in terms of $A_0, A_r$. This way of calculating $A^{(\gamma,Z,W)}$ is technically simpler than the direct one because when the radiative corrections are taken into account in DLA, contributions proportional to masses in propagators of all virtual EW bosons are neglected and therefore both the isoscalar and the isovector fields act as independent ones. It makes more convenient operating with virtual isoscalar and isovector bosons than with $\gamma, Z, W$ -bosons.

It is also convenient to discuss a more general process where lepton $l'(p_1)$ (instead of $e^-$) and its antiparticle $\bar{l}'(p_2)$ (instead of $e^+$) annihilate into the quark-antiquark pair $q^i(p'_1)\bar{q}^j(p'_2)$ and a boson. The emitted boson can be either the isoscalar boson $A_c$, with $c = 0$ or an isovector one $A_r$, with $c = 1, 2, 3$. We consider first the most difficult case when both $l^i$ and $q^j$ are left-handed particles, transitions to the other chiralities are easy to do. The scattering amplitude of this process is $q_j\bar{q}'^j(Mc)^{ij}_{ij'}l'_{ij'}$ where the matrix amplitude $(Mc)^{ij}_{ij'}$ is the object to calculate. In order to simplify operating with the isospin matrix structure of $(Mc)^{ij}_{ij'}$, it is convenient to regard the process in the crossing channel, i.e. in the $t$ -channel for kinematics (1) and in the $u$ -channel for kinematics (2). When the process $l^il'_{ij'}\rightarrow q^j\bar{q}'^jA_c$ is considered in the $t$ -channel, its amplitude can be expressed through the same matrix $(Mc)^{ij}_{ij'}$, however with initial (final) $t$-state being $q_jl^i(A_cq^j\bar{l}'_{ij'})$:

$$M = \frac{2}{k_\perp^2} A_c q^j\bar{l}'_{ij'} (Mc)^{ij}_{ij'} q_j l^i. \quad (3)$$

We have extracted the kinematic factor $2/k_\perp^2$ in order to simplify the matching condition (3) we will use. The initial cross-channel state $q_jl^i$ in Eq. (4) can be expanded into the sum of the isoscalar and of the isovector irreducible $SU(2)$ representations:

$$q_jl^i = \left[ \frac{1}{2} \delta^i_0 \delta^j_0 + 2 (tm)_j^i (tm)_a^b \right] q_bl^a. \quad (4)$$

The same is true for the final $q'^j\bar{l}'_{ij'}$ -pair. Therefore, $(Mc)^{ij}_{ij'}$ can be represented as the sum

$$(Mc)^{ij}_{ij'} = \sum_{k=0}^{4} (P^c_k)^{ij}_{ij'} M_k. \quad (5)$$

of the invariant amplitudes $M_k$ $(k = 0, 1, 2, 3, 4)$, each multiplied the projection operator $(P^c_k)^{ij}_{ij'}$ corresponding to an irreducible $SU(2)$ -representation. $k = 0, 1$ correspond to emission of the isoscalar field and $k = 2, 3, 4$ correspond to the isovector fields emission. Then, the projection operator $(P^c_0)^{ij}_{ij'}$ describes the case (see Fig. 1) when both the initial $t$ -channel fermion state and the final one are $SU(2)$ singlets.

Obviously, in this case the emitted boson can be isoscalar only, i.e. $c = 0$. Therefore

$$\left( P^c_0 \right)^{ij}_{ij'} = \frac{1}{2} \delta^i_0 \delta^j_0 \delta^i_1. \quad (6)$$
The multi-Regge invariant amplitudes \( M \) to the isoscalar Reggeons; whereas the zigzag lines stand for the isovector ones. The dashed lines denote isoscalar vector bosons and the waved line correspond to the isovector boson.

The projection operators \( P_k \), with \( k = 1, 2 \) describe the cases when both the initial and the final \( t \)-channel states are the isovector \( SU(2) \) states. However, \( P_1 \) corresponds to the case when the emitted boson is isoscalar,

\[
(P_1^{t^c})_{ij} = 2 \delta_0 (t_m)_{ij}^2 (t_m)^i_j,
\]

whereas \( P_2 \) describes emission of isovector fields:

\[
(P_2^{t^c})_{ij} = (t_b)^i_j (T^c)_{ba} (t_a)^i_j.
\]

\( T^c (c = 1,2,3) \) in Eq. (8) stands for \( SU(2) \) generators in the adjoint (vector) representation.

Projector \( P_3 \) correspond to the case when the initial fermion state is the \( SU(2) \) singlet whereas the final one is the \( SU(2) \) vector. Projector \( P_4 \) describes the opposite situation. The emitted boson is isovector in both these cases. Therefore,

\[
(P_3^{t^c})_{ij} = (t_c)^i_j \delta_j^i, \quad (P_4^{t^c})_{ij} = \delta_i^j (t_c)_j^i.
\]

All operators in Eqs. (3, 4) are orthogonal:

\[
(P_A^{t^c})_{ij} (P_B^{t^c})_{ij}^* \sim \delta^{AB}.
\]

Below (see Eq. (12)) we will show that the invariant amplitudes \( M_3, M_4 \) do not have DL contributions. It leaves us with amplitudes \( M_{0,1,2} \) to calculate. These invariant amplitudes account for radiative corrections to all powers in the EW couplings in the DLA. The arguments of \( M_k \) are

\[
s_1 = (p_1^2 + k^2) \approx 2p_1 k, \quad t_1 = q_1^2 = (p_1 - p_1')^2,
\]

\[
s_2 = (p_2^2 + k^2) \approx 2p_2 k, \quad t_2 = q_2^2 = (p_2 - p_2')^2,
\]

so that

\[
s_1 s_2 = s k_\perp^2.
\]

The kinematics is the \( t \)-channel multi-Regge kinematics when

\[
s_{1,2} \gg t_{1,2} \geq M_Z^2.
\]

Similarly, in order to simplify studying the isotopic structure of \((M^c)_{ij}^{t^c}\) of Eq. (3) in kinematics (2), it is convenient to consider it in the \( u \)-channel where it can be expressed through \( u \)-channel invariant amplitudes \( M_k^u \):

\[
(M^c)_{ij}^{t^c} = \sum_{k=0}^2 (P_k^{t^c})_{ij} M_k^u.
\]

Operators \((P_k^{t^c})_{ij}^{t^c}\) describe irreducible \( SU(2) \) representations, which for this channel are either symmetrical or antisymmetrical two-quark states:

\[
P_0^{c^c} = \frac{1}{2} \delta_0 \left[ \delta_i^j \delta_j^i - \delta_i^j \delta_j^i \right], \quad P_1^{c^c} = \frac{1}{2} \delta_0 \left[ \delta_i^j \delta_j^i + \delta_i^j \delta_j^i \right],
\]

\[
P_2^{c^c} = \frac{1}{2} \left[ \delta_i^j (t^c)_{ij}^i + (t^c)_{ij}^i \delta_j^i + \delta_i^j (t^c)_{ij}^i + (t^c)_{ij}^i \delta_j^i \right].
\]
Operators $P_0^\prime$, $P_1^\prime$ describe emission of the isoscalar field whereas $P_2^\prime$ describes the isovector field emission. Let us note that operator

$$P_3^\prime c = \frac{1}{2} \left[ \delta^\prime_{ij} (t^\prime)^\prime_{ij} + (t^\prime)^\prime_{ij} \delta^\prime_{ij} - \delta^\prime_{ij} (t^\prime)^\prime_{ij} - (t^\prime)^\prime_{ij} \delta^\prime_{ij} \right]$$

(16)

must be accounted for\(^1\) (cf Ref. [8]). However, the invariant amplitude $M_3^\prime$ related to $P^\prime$ does not yield DL contributions in the case of $SU(2)$ though it does in the case of $SU(N)$ with $N > 2$. It leaves us with three invariant amplitudes $M_0^\prime_{1,2}$ (just like it was in the case of the $t$-kinematics). They depend on $s_{1,2}$ and on $u_{1,2}$. In the multi-Regge kinematics (4)

$$s_{1,2} \gg u_{1,2} \geq M_Z^2 .$$

(17)

In order to specify the multi-Regge $t$ ($u$)-kinematics completely, we assume that

$$t_1 \gg t_2 , \quad (u_1 \gg u_2).$$

(18)

The opposite case can be considered similarly. The kinematics where $t_1 \sim t_2$ ($u_1 \sim u_2$) means emission of soft electroweak bosons. This kinematics will be considered below separately.

From the point of view of the Regge theory, accounting for radiative corrections in kinematics (15) can be expressed through exchange of Reggeons propagating in the cross channels. Therefore operators $P_0$ ($P_0^\prime$) of Eq. (1) (Eq. (15)) imply that amplitude $M_0$ ($M_0^\prime$) is controlled by two isoscalar Reggeons whereas the projection operators $P_{1,2}$ of Eqs. (3) (operators $P_{1,2}$ of Eqs. (15)) imply that the energy dependence of amplitudes $M_{1,2}(M_{1,2}^\prime)$ is controlled by two isovector Reggeons. In contrast to it, one of the Reggeons in amplitudes $M_{3,4}$ is isoscalar and the other is isovector.

Besides $s_{1,2}$ and $t_1$ ($u_2$), invariant amplitudes $M_{0,1,2}$ ($M_{0,1,2}^\prime$) depend also on the infrared (IR) cut-off $\mu$ introduced in order to avoid IR singularities from integrating over virtual particle momenta. We use the IR cut-off $\mu$ in the transverse space. However, definition of $\mu$ for radiative amplitudes differs from the definition for elastic amplitudes. In this paper we introduce $\mu$ the same way as it was done in [8]. Let us denote $k_{\perp}^{ab}$ to be the component of a virtual particle momenta $k_{\perp}$ transverse to the plane formed by momenta $a$ and $b$, with $a \neq b$. Then, the IR cut-off $\mu$ obeys

$$\mu < k_{\perp}^{ab}$$

(19)

for all $l = 1, \ldots$ when $a, b = p_1, p_2, p_1^\prime, p_2^\prime, k$. In the present paper we assume that $\mu \approx M_Z$.

In order to calculate $M$, we generalise to the EW theory the technique applied earlier to investigation of the similar inelastic processes in QED [13,14] and in QCD [16]. The essence of the method is factorizing DL contributions from the virtual particles with minimal $k_{\perp}^{ab}$ and differentiating with respect to $\ln \mu^2$. At $t_1, t_2 \gg \mu^2$, such particles can only be virtual EW bosons. Factorizing their contributions leads to the IREE for amplitude $(M^\prime)^{t,j}_{ij}$. This equation is depicted in Fig. 2.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig2.png}
\caption{IREE for $M_Z$. Letters inside the blobs stand for infrared cutoffs.}
\end{figure}

\(^1\) The gluon fields In Ref. [7] were described in terms of matrices $(t^c A^c)^b$.
Applying to it the projector operators of Eqs. (8 - 10) leads to the following IREE for $M_r$ (see Refs. [3, 4, 5] for technical details):

$$\frac{\partial M_r}{\partial \rho_1} + \frac{\partial M_r}{\partial \rho_2} + \frac{\partial M_r}{\partial \rho_3} + \frac{\partial M_r}{\partial y_2} = -\frac{1}{8\pi^2} \left[ b_r \ln(s/\mu^2) + h_r(y_1 + y_2) + m_k y_1 \right] M_r .$$ (20)

We have used in Eq. (20) the fact that, according to our assumption Eq. (18), $k^2_+ \approx t_1(u_1)$ and introduced the logarithmic variables

$$\rho_{1,2} = \ln(s_{1,2}/\mu^2) , \quad y_{1,2} = \ln(t_{1,2}/\mu^2) .$$ (21)

The numerical factors $b_r, h_r$ and $m_r$ in Eq. (20) are:

$$b_0 = \frac{g^2(Y - Y')^2}{4} , \quad b_1 = b_2 = 2g^2 + \frac{g^2(Y - Y')^2}{4} ,$$

$$h_0 = \frac{3g^2}{4} + \frac{g^2YY'}{4} , \quad h_1 = h_2 = -\frac{g^2}{4} - \frac{g^2YY'}{4} ,$$

$$m_0 = m_1 = 0 \quad m_2 = g^2 .$$ (22)

The IREE for the invariant amplitudes $M'_{0,1,2}$ can be obtained similarly. It has the same structure as Eq. (20), though everywhere $t_1,2$ should be replaced by $u_{1,2}$. It means in particular that $y_{1,2}$ should be replaced by $y'_{1,2} = \ln(u_{1,2})/\mu^2$. This replacement and replacement of operators $P_{0,1,2}$ by $P'_{0,1,2}$ results in replacement of the factors $b_r, h_r$ by $b'_{r}, h'_{r}$ :

$$b'_0 = \frac{g^2(Y + Y')^2}{4} , \quad b'_1 = b'_2 = 2g^2 + \frac{g^2(Y + Y')^2}{4} ,$$

$$h'_0 = \frac{3g^2}{4} - \frac{g^2YY'}{4} , \quad h'_1 = h'_2 = -\frac{g^2}{4} - \frac{g^2YY'}{4} .$$ (23)

Factors $m'_r$ in the IREE for $M'_{r}$ coincide with factors $m_r$ in Eq. (22). Therefore after replacements $y_{1,2} \to y'_{1,2}$, and $b_{0,1,2} \to b'_{0,1,2}, h_{0,1,2} \to h'_{0,1,2}$, Eq. (20) for amplitudes $M_r$ in kinematics [13] holds for amplitudes $M'_{r}$ describing $e^+e^- \to q\bar{q}$ -annihilation in kinematics [17].

### III. SOLVING THE EVOLUTION EQUATIONS FOR $M_r$

In order to solve Eq. (20), it is convenient to operate with the Mellin amplitude $F_r$ related to $M_r$ through the Mellin transform:

$$M_r = \int_{-\infty}^{1}\frac{d\omega_1}{2\pi i} \frac{d\omega_2}{2\pi i} e^{\omega_1 \rho_1+\omega_2 \rho_2} F_r(\omega_1, \omega_2, y_1, y_2) .$$ (24)

In the $\omega$ -representation, multiplying by $\rho_i$ corresponds to $-\partial/\partial \omega_i$. Using this and Eqs. (12, 24), we can rewrite Eq. (20) as

$$\omega_1 F_r + \omega_2 F_r + \frac{\partial F_r}{\partial y_1} + \frac{\partial F_r}{\partial y_2} = b'_r \left( \frac{\partial F_r}{\partial \omega_1} + \frac{\partial F_r}{\partial \omega_2} \right) + \left( \frac{1}{8\pi^2} \right) \left[ (b_r - h_r - m_r)y_1 - h_r y_2 \right] F_r .$$ (25)

For further simplification, it is convenient to introduce variables $x_{1,2}$ and $z_{1,2}:

$$x_{1,2} = \omega_{1,2}/\lambda_r , \quad z_{1,2} = -\lambda_r y_{1,2}$$ (26)

where $\lambda_r = \sqrt{b_r/8\pi^2} .

In terms of $x_i, z_i$, the differential operator in the left hand side of Eq. (24) acquires symmetrical and simple form. Thus, we arrive at

$$\frac{\partial F_r}{\partial x_1} + \frac{\partial F_r}{\partial x_2} + \frac{\partial F_r}{\partial z_1} + \frac{\partial F_r}{\partial z_2} = \left[ (x_1 + x_2) + (1 + \beta_r)z_1 + \gamma_r z_2 \right] F_r$$ (27)

where $\beta_r = -(h_r + m_r)/b_r, \quad \gamma_r = -h_r/b_r.$
The general solution to Eq. (27) can be written as

\[ F_r = \Phi_r(x_1 - z_2, x_2 - z_2, x_1 - z_1) \exp \left[ \frac{x_1^2 + x_2^2}{2} + (1 + \beta_r) \frac{z_1^2}{4} + \gamma_r \frac{z_2^2}{2} \right] \]  

(28)

where unknown function \( \Phi_r \) has to be specified. It can be done in particular through matching

\[ F_r(x_1, x_2, z_1, z_2) \big|_{z_2=0} = \tilde{F}_r(x_1, x_2, z_1) \]  

(29)

where \( \tilde{F}_r \) is related through the Mellin transform (24) to amplitude \( \tilde{M}_r \) of the same process in the kinematics Eqs. (13,18) through with \( q_0^2 \sim \mu^2 \). The IREE (30) for \( \tilde{F}_r \) differs from the IREE of Eq. (27) for \( F_r \) in the following two respects. First, there is no \( z_2 \) dependence in Eq. (27). Second, in contrast to Eq. (27), the IREE for \( \tilde{F}_r \) contains an additional term (that we denote \( dQ_r(x)dx \)) in the rhs:

\[ \frac{\partial \tilde{F}_r}{\partial x_1} + \frac{\partial \tilde{F}_r}{\partial x_2} + \frac{\partial \tilde{F}_r}{\partial z_1} = [(x_1 + x_2) + (1 + \beta_r)z_1 + \frac{dQ_r(x_2)}{dx_2}] \tilde{F}_r \]  

(30)

This new term corresponds to the situation when the particles with the minimal transverse momenta are the \( t_2 \)-channel virtual quark pair (see Fig. 3).

![FIG. 3: The soft fermion contribution to the IREE for \( \tilde{M}_Z \)](image)

This contribution is \( \mu \)-dependent only when \( t_2 \approx \mu^2 \). The intermediate two-particle state in Fig. 3 factorizes amplitude \( \tilde{M}_r \) into a convolution of the same amplitude and the elastic amplitude \( E_r \). The explicit expressions for the elastic electroweak amplitude \( E_r \) were obtained in Ref. [4]. The particular case where the produced particles were a right handed lepton and its antiparticle was studied in Ref. [4]. For all cases, the Mellin amplitude \( f_r \) is related to \( E_r \) through the Mellin transform

\[ E_r = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} e^{i\rho} f_r(\omega) , \]  

(31)

with \( \rho = \ln(s/\mu^2) \), can be expressed through the Parabolic cylinder functions \( D_{p_r}(x) \) with different values of \( p_r \):

\[ f_r(x) = \frac{1}{p_r} \frac{d[\ln(e^{x^2/4}D_{p_r}(x))]}{dx} \equiv \frac{1}{p_r} \frac{D_{p_r-1}(x)}{D_{p_r}(x)} \equiv \frac{1}{p_r} \frac{dQ_r(x)}{dx} . \]  

(32)

The term \( dQ_r(x_2)/dx_2 \) in the rhs of Eq. (30) corresponds to the contribution of the right blob in Fig. 3 to the IREE (30).

The general solution to Eq. (30) is

\[ \tilde{F}_r = \hat{\Phi}_r(x_1 - z_1, x_2 - z_1) \frac{1}{D_{p_r}(x_2) \exp(x_2^2/4)} \exp \left[ \frac{x_1^2 + x_2^2}{2} + (1 + \beta_r) \frac{z_2^2}{2} \right] \]  

(33)

where there is, again, an unknown function \( \hat{\Phi}_r \).

In order to specify \( \hat{\Phi}_r \), we use the factorisation of bremsstrahlung bosons with small \( \xi \perp \) which takes place (see Refs. [8],[9],[10]) both in Abelian and in non-Abelian field theories. In the context of the problem under consideration it states that when \( z_1 = 0 \), the radiative amplitude \( \tilde{M}_r \) and the elastic amplitude \( E_r \) are related:

\[ \tilde{M}_r|_{z_1=0} = E_r G_r . \]  

(34)

\( E_r \) in Eq. (34) are the invariant amplitudes of the elastic annihilation process (see [4]); \( G_r = g(Y \pm Y')/2 \) for \( r = 0 \) (for invariant amplitude \( M_0(M_0') \) the sign is "+" ("-")); \( G_r = g \) for \( r = 1, 2, 3 \).
Eq. (34) means that when $z_1 = 0$, the two Reggeons in every amplitude $M_r$ converge into one Reggeon that controls $E_r$ energy dependence. However, such convergence is possible in the DLA only when both Reggeons are either isoscalar or isovector. This rules amplitudes $M_{3,4}$ out of consideration. Obviously, this property of the multi-Regge amplitudes holds for the more complicated cases when the number of involved Reggeons is more than two. This property was first obtained in Ref. [8] and was called “Reggeon diagonality”.

The matching (34) can be rewritten in terms of Mellin amplitudes $\phi_r(x_1, x_2) \equiv \tilde{F}_r(\omega_1, \omega_2, z_1)|_{z_1=0}$ and $f_r(\omega)$:

$$
\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{i\omega_1p_1 + i\omega_2p_2} \phi_r(\omega_1, \omega_2) = G_r \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{i\omega(p_1 + p_2)} f_r(\omega) .
$$

(35)

We have used in Eq. (33) that according to Eq. (24) $\rho = p_1 + p_2$ when $z_1 = 0$. For the amplitudes with positive signatures that we discuss in the present paper, the transform inverse to Eq. (24) can be written as

$$
M_r(p_1, 1_1, y_1) = \int_0^\infty dp_1 dp_2 e^{-i\omega_1p_1 - i\omega_2p_2} F_r(\omega_1, \omega_2, y_1) .
$$

(36)

Applying this transform to Eq. (35) at $z_1 = 0$ leads to

$$
\phi_r(\omega_1, \omega_2) = G_r \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int_0^\infty dp_1 dp_2 e^{i(\omega - \omega_1)p_1 + i(\omega - \omega_2)p_2} f_r(\omega) =
G_r \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{(\omega - \omega_1)(\omega - \omega_2)} f_r(\omega) .
$$

(37)

Choosing the integration contours in Eq. (37) so that $\Re \omega < \Re \omega_1, 2$ allows us to integrate over $\omega$ by closing the contour to the right, which does not involve dealing with singularities of $f_r$. When this integration is done, we arrive at

$$
\phi_r = \frac{f_r(\omega_1) - f_r(\omega_2)}{\omega_2 - \omega_1} .
$$

(38)

After that it is easy to obtain the following expression for $M_r$:

$$
M_r = G_r \int_{-\infty}^{\infty} \frac{d\omega_1}{2\pi} \frac{d\omega_2}{2\pi} \frac{s_1}{q_1^2} \omega_1 \left( \frac{s_2}{q_2^2} \right) \omega_2 \frac{f_r(x_1) - f_r(x_2)}{\omega_2 - \omega_1} .
$$

$$
D_{p_r}(x_2 - z_1) \exp \left[ \frac{(1 - 2\beta_r)}{4} \frac{z_2^2}{z_1^2} - \frac{(1 - 2\gamma_r)}{4} \frac{z_2^2}{z_1^2} \right] .
$$

(39)

If we choose $\Re x_1 < \Re x_2$, Eq. (39) takes simpler form:

$$
M_r = G_r R_r .
$$

(40)

where

$$
R_r = \int_{-\infty}^{\infty} \frac{d\omega_1}{2\pi} \frac{d\omega_2}{2\pi} \frac{s_1}{q_1^2} \omega_1 \left( \frac{s_2}{q_2^2} \right) \omega_2 \frac{1}{\omega_2 - \omega_1} \frac{D_{p_r}(x_1 - z_1)}{D_{p_r}(x_2 - z_2)} \exp \left[ \frac{(1 - 2\beta_r)}{4} \frac{z_2^2}{z_1^2} - \frac{(1 - 2\gamma_r)}{4} \frac{z_2^2}{z_1^2} \right] .
$$

(41)

The amplitudes $M^{(q)} = M^{(q)}(0^\pm)$ of the electroweak boson production are easily expressed through $R_r$:

$$
M^{(q)} = \cos \theta_W M^{(0)} + \sin \theta_W M^{(3)} = g \cos \theta_W (R_0 + R_1) + g \sin \theta_W R_2 ,
$$

$$
M^{(z)} = -\sin \theta_W M^{(0)} + \cos \theta_W M^{(3)} = -g \sin \theta_W (R_0 + R_1) + g \cos \theta_W R_2 ,
$$

$$
M^{(W^\pm)} = (1/\sqrt{2})[M^{(1)} \pm iM^{(2)}] = (g/\sqrt{2}) R_2
$$

(42)

when the boson are produced in kinematics (33). For kinematics (47), the boson production amplitudes are expressed through $R'_{r}$:

$$
M^{(q)} = g \cos \theta_W (R'_0 + R'_1) + g \sin \theta_W R'_2 ,
$$

$$
M^{(z)} = -g \sin \theta_W (R'_0 + R'_1) + g \cos \theta_W R'_2 ,
$$

$$
M^{(W^\pm)} = (g/\sqrt{2}) R'_2 .
$$

(43)
The exponent in Eq. (41) is the Sudakov form factor for this process. It accumulates the soft DL contributions, with virtualities $\leq z_i^2$. The harder DL contributions are accounted through $D_{pr}$ -functions. It is convenient to perform integration over $\omega_{1,2}$ by taking residues. Such residues are actually zeros $\bar{x}_k(r)$ $(k = 1, \ldots)$ of involved $D_{pr}$ -functions, so

$$R_r \sim \sum_{k=1}^{\infty} \left( \frac{s_1}{q_1} \right)^{\lambda_r \bar{x}_k(r)} \left( \frac{s_2}{\sqrt{q_1 q_2}} \right)^{\lambda_r \bar{x}_k(r)} \quad (44)$$

Position of $\bar{x}_k(r)$ depend on values of $p_r$ in such a way that the greater $p_r$, the greater are $\Re \bar{x}_k(r)$. In particular, the real part of the rightmost zero $\equiv \bar{x}(r)$ is positive when $p_r > 1$. In other words, $R_r$ increase with the total energy when $p_r > 1$. Ref. \[4\] states that

$$p'_0 = \frac{3 - YY' \tan^2 \theta_W}{(Y + Y')^2 \tan^2 \theta_W}, \quad p'_1 = p'_2 = -\frac{1 + YY' \tan^2 \theta_W}{8 + (Y + Y')^2 \tan^2 \theta_W} \quad (45)$$

and

$$p_0 = \frac{3 + YY' \tan^2 \theta_W}{(Y - Y')^2 \tan^2 \theta_W}, \quad p_1 = p_2 = -\frac{1 - YY' \tan^2 \theta_W}{8 + (Y - Y')^2 \tan^2 \theta_W} \quad (46)$$

Therefore, only $M_0$ and $M'_0$ grow with increase of the annihilation energy whereas the amplitudes $M_{1,2}$ and $M'_{1,2}$ are falling.

Let us discuss the asymptotics of $R_r$ first. The asymptotics of the energy dependence of each $R_r$ is controlled by two identical isoscalar (isovector) leading Reggeons. Intercepts $\Delta_j$ $(j = S, V, S', V')$ of these Reggeons are related to the position of the rightmost zero $\bar{x}(j)$ of the $D_{pr}$ -functions so that

$$\Delta_S = \lambda_0 \bar{x}(p_0), \quad \Delta_S' = \lambda'_0 \bar{x}(p'_0), \quad \Delta_V = \lambda_1 \bar{x}(p_1), \quad \Delta_V' = \lambda'_1 \bar{x}(p'_1) \quad (47)$$

We remind that $\lambda_j = \sqrt{b_j/8\pi^2}$. Therefore we arrive at the following asymptotics:

$$M_0 \sim g's^{\Delta_S}, \quad M_1 \sim g's^{\Delta_V}, \quad M_2 \sim gs^{\Delta_V'}, \quad M'_0 \sim g's^{\Delta_S'}, \quad M'_1 \sim g's^{\Delta_V'}, \quad M'_2 \sim gs^{\Delta_V'} \quad (48)$$

As the intercepts of the isoscalar Reggeons are greater than the ones of the isovector Reggeons, the asymptotics of the exclusive cross sections $\sigma^{(\gamma)}$ and $\sigma^{(Z)}$ of the photon and $Z$ -production is given by contributions of the isoscalar Reggeons with intercepts $\Delta_S$ and $\Delta_S'$. Therefore, the only difference between these cross sections is the different couplings of these fields to the isoscalar Reggeons. So, we conclude (see Eqs. \(42\),\(43\)) that asymptotically

$$\frac{\sigma^{(Z)}}{\sigma^{(\gamma)}} \approx \tan^2 \theta_W \quad (49)$$

Accounting for contributions of other zeros, $\bar{x}(r)$ changes the values of $\sigma^{(\gamma)}$ and $\sigma^{(Z)}$ but does not change Eq. (49). In contrast to Eq. (43), the asymptotics of the ratio $\sigma^{(\gamma)}/\sigma^{(W)}$ depends on $s$. The point is that the exclusive cross section $\sigma^{(W)}$ of $W$ -production involves the isovector Reggeons with smaller intercepts. So, asymptotically this cross section obeys the following relation:

$$\frac{\sigma^{(\gamma)}}{\sigma^{(W)}} \sim s^{2(\Delta_S - \Delta_V)} = s^{-0.36} \quad (50)$$

However, contributions of other zeros of $D_{pr}$ -functions change this asymptotic relation. Results of numerical calculation of $\sigma^{(W^+)}$, $\sigma^{(Z)}$ and $\sigma^{(\gamma)}$, and accounting for non-leading DLA contributions are discussed in Sect. \[6\]

IV. EMISSION OF $n$ VECTOR BOSONS IN THE MULTI-REGGE KINEMATICS

The arguments of the previous Sects. can be extended in the straightforward way to the case when the $e^+e^-$ -annihilation into quarks is accompanied by emission of $n$ isoscalar or isovector bosons with momenta $k_1, \ldots, k_n$ in the multi-Regge kinematics. It is not difficult to generalise expressions of Eqs. \(48\),\(49\) for projection operators to the case of the $n$ boson emission and obtain new projector operators. First of all, let us note that all non-diagonal
projectors should be ruled out of consideration by the same reason as it was done in Sect. 1. Therefore, the invariant amplitudes of emission of \( n \) bosons involve \( n + 1 \) identical intermediate Reggeons. The isotopic quantum numbers of the Reggeons depend on the initial fermion state and on the isospin of the emitted bosons. If the initial fermion state is isoscalar (or antisymmetric), the same is true for all intermediate Reggeons and therefore only isoscalar bosons can be emitted in these cases. The projector operators for this case are again \( P_{0,1}^{(a)} \) with trivial adding factors \( \delta_{v,0} \) for every isoscalar boson. If the initial fermion state is isovector (or symmetrical), the emitted bosons can be both isoscalar or isovector gauge fields. Accounting for emission of the isoscalar bosons does not require any changes of the projectors. When \( r \ (r \leq n) \) isovector bosons \( c_1, c_2, \ldots, c_r \) are emitted in the \( t \)-kinematics, the operators \( P_2 \) of Eq. (3) should be replaced by

\[
(P_2^{(c_1, c_2, \ldots, c_n)})_{ij}^{ij'} = (t_b)^{ij'}(T^{c_1})_{o_1, a_1} \ldots (T^{c_2})_{a_2, a_1} (T^{c_1})_{a_2, a_1} (t_{a_1})_{ij}^{ij'} .
\] (51)

A similar generalisation of operator \( P_2^{(c)} \) is also easy to obtain. The new invariant amplitudes \( M_j(M_j^{(c)}) \) corresponding to these operators depend on \( n + 1 \) variables \( s_i \):

\[
s_1 = 2p_1k_1 , \quad s_2 = 2k_1k_2 , \ldots , \quad s_{n+1} = 2k_np_2
\] (52)

and on \( t_i(u_i) \) in the case of the \( t \) (\( u \)) - kinematics:

\[
t_i = q_i^2 , \quad u_i = q_i^2
\] (53)

where

\[
q_1 = p_1' - p_1 , \quad q_2 = q_1 - k_1 , \ldots , \quad q_{n+1} = q_n - k_n = p_2 - p_2' , \\
q_1' = p_2' - p_1 , \quad q_2' = q_1 - k_1 , \ldots , \quad q_{n+1}' = q_n - k_n = p_2 - p_1' .
\] (54)

The kinematics is the multi-Regge \( t \)-kinematics if

\[
s_i \gg t_j \geq M_Z^2
\] (55)

and it is the multi-Regge \( u \)-kinematics if

\[
s_i \gg u_j \geq M_Z^2 ,
\] (56)

with \( i, j = 1, \ldots, n + 1 \). In order to define these kinematics completely, one should fix relations between different \( t_i \) (different \( u_i \)). In this paper we consider the simplest case of the monotonic ordering. We assume that

\[
t_1 \gg t_2 \gg \ldots \gg t_{n+1}
\] (57)

for the case of the multi-Regge \( t \)-kinematics and the similar monotonic ordering

\[
u_{1} \gg u_{2} \gg \ldots \gg u_{n+1}
\] (58)

for the case of the multi-Regge \( u \)-kinematics.

Eqs. (52, 53) read that in the both kinematics

\[
s_1 \ldots s_{n+1} = sk_1^2 \ldots k_n^2
\] (59)

It is also convenient to introduce variables \( \rho_i = \ln(s_i/\mu^2) \) and \( y_i \) where \( y_i = \ln(t_i/\mu^2) \) for the forward kinematics \( (y_i' = \ln(u_i/\mu^2) \) for the case of the backward one). In these terms, The IREE for \( M_j^{(n)} \) looks quite similar to Eq. (20):

\[
\frac{\partial M_j^{(n)}}{\partial \rho_1} + \ldots + \frac{\partial M_j^{(n)}}{\partial \rho_{n+1}} + \frac{\partial M_j^{(n)}}{\partial y_1} + \ldots + \frac{\partial M_j^{(n)}}{\partial y_{n+1}} = - \frac{1}{8\pi^2} \left[ b_j \ln(s/\mu^2) - h_j(y_1 + y_{n+1}) + \sum l m_l y_l \right] M_j^{(n)} ,
\] (60)

---

2 Scattering amplitudes for other multi-Regge kinematics can be calculated similarly (see Ref. [7]). It is worth to mention that amplitudes for kinematics (5) and (6) yield main contributions to the inclusive cross section when integration over the EW boson momenta is performed.
where \( b_j, h_j \) are given by Eqs. (22,23); \( m_l = q^2 \) if the boson \( l \) with momentum \( k_l \) is isovector, otherwise \( m = 0 \). Let us consider for simplicity the case of emission of isoscalar bosons. Introducing the Mellin amplitude \( F_n \) through the transform

\[
M_n = \int_{-\infty}^{\infty} \frac{d \omega_1 \cdots d \omega_1+1}{2\pi i} e^{i \omega_1 \rho_1 + \cdots + i \omega_1+1 \rho_1+1} F_n(\omega_1, \ldots, \omega_n+1, y_1, \ldots, y_n+1) \tag{61}
\]

and using notations \( x_i, z_i \) defined as

\[
x_i = \omega_i / \lambda , \quad z_i = -\lambda y_i , \tag{62}
\]

we transform Eq. (61) into the following one:

\[
\frac{\partial F_n}{\partial x_1} + \cdots + \frac{\partial F_n}{\partial x_n+1} + \frac{\partial F_n}{\partial z_1} + \cdots + \frac{\partial F_n}{\partial z_n+1} = \left[ (x_1 + \cdots + x_n+1) + z_1 + \cdots + z_n + h(z_1 + z_n+1) \right] F_n , \tag{63}
\]

with the general solution

\[
F_n = \Phi_n(x_1 - z_n+1, x_2 - z_n+1, \ldots, x_n+1 - z_n+1; z_1 - z_n+1, \ldots, z_n - z_n+1) \cdot \exp \left[ S_n+1(x) + S_n(z) + \frac{h(z_1^2 + z_n+1^2)}{2} \right] \tag{64}
\]

where we have denoted \( S_r(a) = \sum_i a_i^2 / 2 \). An unknown function \( \Phi_n \) can be specified through the matching

\[
F_n|_{z_n+1=0} = \tilde{F}_n \tag{65}
\]

where the Mellin amplitude \( \tilde{F}_n \) describes the same process in the multi-Regge kinematics (22,24,57) though with \( q_{n+1}^2 = \mu^2 \). The IREE for \( \tilde{F}_n \) is

\[
\frac{\partial \tilde{F}_n}{\partial x_1} + \cdots + \frac{\partial \tilde{F}_n}{\partial x_n+1} + \frac{\partial \tilde{F}_n}{\partial z_1} + \cdots + \frac{\partial \tilde{F}_n}{\partial z_n+1} = \left[ (x_1 + \cdots + x_n+1) + z_1 + \cdots + z_n+1 + h_1 z_1 + \frac{dQ_r(x_{n+1})}{dx_{n+1}} \right] \tilde{F}_1 \tag{66}
\]

where \( Q \) is defined by Eq. (22). The general solution to Eq. (64) can be obtained quite similar to the one of Eq. (63):

\[
\tilde{F}_n = \tilde{\Phi}_n(x_1 - z_n+1, x_2 - z_n+1, \ldots, x_n+1 - z_n+1; z_1 - z_n, \ldots, z_n-1 - z_n) \cdot \exp [S_n+1(x) + S_n(z) + \frac{z_1^2}{2}] . \tag{67}
\]

It also contains an unknown function \( \tilde{\Phi} \). In order to specify it we use factorisation of the photons with small \( k_\perp \):

\[
\tilde{M}_n|_{z_n=0} = \tilde{M}_{n-1}(s_1, \ldots, s_n; q_1^2, \ldots, q_{n-1}^2, \mu^2) . \tag{68}
\]

Rewriting this equation in terms of the Mellin amplitudes and performing the transform inverse to Eq. (61), we express \( \tilde{F}_n \) through amplitude \( \tilde{F}_{n-1} \):

\[
\tilde{F}_n|_{z_n=0} = \frac{1}{(\omega_n - \omega_{n+1})} \left[ \tilde{F}_{n-1}(x_1, \ldots, x_n+1; z_1, \ldots, z_n) - \tilde{F}_{n-1}(x_1, \ldots, x_n+1; z_1, \ldots, z_n-1) \right] . \tag{69}
\]

Combining this equation with Eqs. (61,67) leads to the following recurrent formula for \( F_n \):

\[
F_n(x_1, \ldots, x_n+1; z_1, \ldots, z_n+1) = \frac{1}{\omega_n - \omega_{n+1}} \tilde{F}_{n-1}(x_1 - z_n, \ldots, x_n - z_n; z_1 - z_n, \ldots, z_n-1 - z_n) \cdot \frac{D_p(x_{n+1} - z_n)}{D_p(x_{n+1} - z_{n+1})} \exp \left[ S_{n+1}(x) - S_{n+1}(x - z_n) + S_n(z) - S_{n-1}(z - z_n) + \frac{h}{2} (z_1^2 + z_{n+1}^2 - (z_1 - z_n)^2) \right] . \tag{70}
\]
Using this formula leads to the following expression for the amplitude $M_j^{(n)}$ of emission of $n$ isoscalar bosons in the ordered kinematics (77,78):

$$M_j^{(n)} = \left(\frac{g'(Y + Y')}{2}\right)^n \int_{-1}^{1} \frac{dw_1}{2\pi} \cdots \frac{dw_n+1}{2\pi} \left(\frac{s_1}{q_1}\right)^{\omega_1} \cdots \left(\frac{s_{n+1}}{\sqrt{n_2 q_{n+1}}^{2}}\right)^{\omega_{n+1}} \frac{D_{p_r-1}(x_1 - z_1)}{D_{p_r}(x_1 - z_1)} \cdot$$

$$\frac{D_{p_r}(x_2 - z_1)}{D_{p_r}(x_2 - z_2)} \cdots \frac{D_{p_r}(x_{n+1} - z_n)}{D_{p_r}(x_{n+1} - z_{n+1})} \exp \left[\frac{(b_r - 2h_r)}{4h_r} (z_1^2 + z_2^2)\right].$$

(71)

where $Y = Y'$ corresponds to the kinematics (77,78) respectively. Eq. (71) implies that the contours of integrations obey $\Re x_1 < \ldots < \Re x_{n+1}$. After that one can perform integration in Eq. (71) by taking residues in the $D_{p_r}$ zeros.

When $k$ of the bosons are replaced by the isovector ones, $(g'(Y + Y')/2)^n$ in Eq. (71) should be replaced by $g(k g'(Y + Y')/2)^{n-k}$; the factor $(m/2h_r)z_i^2$ for each of the emitted isovector bosons should be added to the last exponent. Using the standard relation between gauge fields $A_\mu$ and $\gamma, Z, W$, one can easily rewrite the gauge boson production amplitudes of Eq. (71) in terms of amplitudes for the electroweak bosons production. Asymptotics of the scattering amplitudes of the photon and $Z$ -production are governed by the isoscalar Reggeons whereas $W$ -production involves the isovector Reggeons. Eq. (71) can be used for obtaining different relation between cross sections of different radiative processes in the multi-Regge kinematics. For example,

$$\frac{\sigma^{(nZ)}}{\sigma^{(n\gamma)}} \approx \tan^{2n} \theta_W,$$

whereas the energy dependence of the ratio $\sigma^{(n\gamma)}/\sigma^{(nW)}$ is less trivial. Asymptotically,

$$\frac{\sigma^{(n\gamma)}}{\sigma^{(nW)}} \sim s^{-0.36}.$$ 

(73)

Results of accounting for the non-leading Reggeon contributions for $\sigma^{(n\gamma)}/\sigma^{(nW)}$ can be obtained from Fig. 6 because $\sigma^{(n\gamma)}/\sigma^{(nW)} = (\sqrt{s}/g)^{n-1}\sigma^{(\gamma)}/\sigma^{(W)}$. In obtaining Eqs. (72,73) from Eq. (71) we have used that according to Eq. (59), $(s_1)^{A_1} \ldots (s_{n+1})^{A_{n+1}} \sim s^{\Delta}$. 

V. EMISSION OF EW BOSONS IN $e^+e^-$-ANNIHILATION INTO LEPTONS

Inelastic annihilation of $e^+e^-$, with the $e^-$ being left handed, into another left handed lepton $l'$ (i.e. into $\mu$ or $\tau$) and its antiparticle $\bar{l}'$ can be considered quite similarly to the annihilation into quarks discussed above. In particular, explicit expressions for the new invariant amplitudes, $L_j^{(n)}$ of this process$^3$ can be obtained from Eqs. (29,40,71) by putting $Y' = Y = -1$ in Eqs. (29,40,71) for the factors $h_r, h, b_r, b$. However having done it, we obtain that $b_0 = 0$ (see Eq. (29)). It means that the IR evolution amplitudes for the scattering amplitude $L_0^{(n)}$ of the inelastic annihilation $e^+e^-\rightarrow l'l' + n\gamma + (n-1)Z$ in the kinematics (1) do not contain contributions proportional to $\ln(s/\mu^2)$ in the r.h.s and therefore the Mellin amplitudes $f_0^{(n)}$ (related to $L_0$ through the Mellin transform (29), Eq. (74)) do not have the partial derivatives with respect to $\omega_j$ (cf Eq. (71)). In order to obtain expressions for the new scattering amplitudes $L_0^{(n)}$, let us consider first the simple case of emission of one isoscalar boson accompanying the forward $e^+e^-\rightarrow l'l'.\bar{l}'$ -annihilation, assuming that both $e^-$ and $l'$ are left particles. It is obvious that for this case, the IREE of Eq. (29) for scattering amplitude $M_0$ has to be replaced by the simpler one,

$$\frac{\partial L_0^{(1)}}{\partial p_1} + \frac{\partial L_0^{(1)}}{\partial p_2} + \frac{\partial L_0^{(1)}}{\partial y_1} + \frac{\partial L_0^{(1)}}{\partial y_2} = -\frac{1}{8\pi^2} \tilde{h}_0(y_1 + y_2) L_0^{(1)}$$

(74)

where we have denoted $\tilde{h}_0 = (3g^2 + g'^2)/4$.

In terms of the Mellin amplitude $f_0^{(1)}$, Eq. (74) takes the following form:

$$(\omega_1 + \omega_2) f_0^{(1)} + \frac{\partial f_0^{(1)}}{\partial y_1} + \frac{\partial f_0^{(1)}}{\partial y_2} = \tilde{h}_0(y_1 + y_2) f_0^{(1)}.$$ 

(75)

$^3$ the kinematic factor $2/k_\perp$ is also extracted from $A_0^{(n)}$, like it was done for amplitudes $M_0^{(n)}$. 

The solution to Eq. (73) respecting the matching condition (24) is (cf Eq. (33)):

\[ L_0^{(1)} = \frac{g^\prime Y}{2} \int_{-\infty}^{\infty} \frac{d\omega_1}{2\pi} \frac{d\omega_2}{2\pi} \left[ \frac{s_1}{q_1^2} \right]^{\omega_1} \left[ \frac{s_2}{q_2^2} \right]^{\omega_2} \left( \frac{\tilde{f}_0(x_1) - \tilde{f}_0(x_2)}{(\omega_2 - \omega_1)} \right) e^{(1/8\pi^2)\tilde{f}_0(\omega_1)(y_1-y_2) - (\bar{h}_0/2)(y_1^2 + y_2^2)} \]

(76)

where the Mellin amplitude \( \tilde{f}_0 \) for the elastic \( e^+e^- \rightarrow \mu^+\mu^- \) annihilation is (see 2, 3, 4)

\[ \tilde{f}_0 = 4\pi^2 \left[ \omega - \sqrt{\omega^2 - (3g^2 + g^\prime)^2/8\pi^2} \right]. \]

(77)

The last exponent in Eq. (76) is the Sudakov form factor accumulating the DL contribution of the soft virtual EW bosons only. The other terms in the integrand account for harder contributions. The leading singularity (intercept), \( \omega_0 \) of the integrand of Eq. (76) is given by the position of the branch point of the rhs of Eq. (77). Therefore we obtain

\[ \omega_0 = \sqrt{\frac{\alpha}{2\pi} \left( \frac{3}{\sin^2 \theta_W} + \frac{1}{\cos^2 \theta_W} \right)} = 0.13, \]

(78)

so asymptotically

\[ L_0 \sim s^{0.13}. \]

(79)

The invariant amplitudes \( L_0^{(n)} \) for production of \( n \) isoscalar bosons in the kinematics (57) when \( e^+e^- \) annihilate into another lepton pair can be obtained similarly. The IREE for the amplitudes \( L_0^{(n)} \) is

\[ \left[ \frac{\partial L_0^{(n)}}{\partial \rho_1} + ... + \frac{\partial L_0^{(n)}}{\partial \rho_{n+1}} \right] + \left[ \frac{\partial L_0^{(n)}}{\partial y_1} + ... + \frac{\partial L_0^{(n)}}{\partial y_{n+1}} \right] = -\frac{1}{8\pi^2} \tilde{f}_0(y_1 + ... + y_{n+1})L_0^{(n)} \]

(80)

and its solution is

\[ L_0^{(n)} = \left( \frac{g^\prime Y}{2} \right)^n \int_{-\infty}^{\infty} \frac{d\omega_1}{2\pi} ... \frac{d\omega_{n+1}}{2\pi} \left[ \frac{s_1}{q_1^2} \right]^{\omega_1} \left[ \frac{s_2}{q_2^2} \right]^{\omega_2} ... \left[ \frac{s_{n+1}}{q_{n+1}^2} \right]^{\omega_{n+1}} \left( \frac{\tilde{f}_0(\omega_1)}{\omega_{n+1} - \omega_n} ... (\omega_2 - \omega_1) \right) e^{(1/8\pi^2)\tilde{f}_0(\omega_1)(y_1-y_2)+...+\tilde{f}_0(\omega_{n+1})(y_n-y_{n+1})} e^{-(\bar{h}_0/2)(y_1^2 + y_2^2) + ... + y_{n+1}^2)} \]

(81)

when \( \Re(\omega_i) < \Re(\omega_{i+1}), i = 1, ..., n \). Their asymptotic s-dependence is also given by Eq. (73). The results of numerical calculations for the cross section of \( \gamma, Z \) and \( W \) production in \( e^+e^- \rightarrow l^+l^- \) are presented in Fig. 6.

**VI. NUMERICAL RESULTS**

In order to estimate at what energy scale one might hope to observe the predicted asymptotical behaviour of cross sections of exclusive \( W^\pm \) and \( Z, \gamma \) production we have first to account for all non-leading DL amplitudes for left and right chiralities of initial \( e^+e^- \) and final \( q\bar{q} \) or \( ll \) pairs. There are many such amplitudes, but all of them can be easily calculated as described in previous sections. The results for Regge intercepts for the forward (\( t \)-channel) and backward (\( u \)-channel) kinematics are collected in Table III for the final \( q\bar{q} \) and in Table IV for the final \( ll \).

Evidently, in far asymptotics the leading contribution for \( W^\pm \) production comes from \( F_1 \) (isotriplet) of the backward \( e_L^+e_L^- \rightarrow q\bar{q}, q\bar{l} \) whereas the leading contribution to \( (Z, \gamma) \) production comes from \( F_0 \) (isosinglet) of the forward \( e_L^+e_L^- \rightarrow q\bar{l}, q\bar{l} \). However differences between the non-leading and leading intercepts are small, and one can expect the role of the first to be essential at real energies scales. Moreover, the effects of non-leading intercepts of the same amplitude can be also large enough at real energies. Therefore it seems reasonable to numerically compute the energy dependent amplitudes, \( M_r \), by taking the inverse transform of the IREE solutions \( F_r(\omega) \), and to calculate with them the inelastic cross sections of boson production in central region (in cmf) with \( k_T^2 \sim \mu^2 = M^2_Z \). It seems also suitable to sum over the final \( q\bar{q} \) or \( ll \) isotopic states, fixing only the emitted boson isotopic state.

Easy but cumbersome calculations lead to the following results. \( W^\pm \) production in the forward and backward kinematics is described by the same formula (though with different amplitudes involved):

\[ \sigma(W^\pm) = g^2 \left[ |M_{LLT}|^2 + |M_{RL}|^2 \right], \]

(82)
TABLE I: Rightmost zeros \( x_0 \) of parabolic cylinder functions \( D_\nu(x) \) determining the values of the leading singularities \( \omega_0 \) of different Mellin transform amplitudes \( F_\nu(\omega) \) for \( e^+e^- \rightarrow q\bar{q} \) annihilation in forward and backward kinematics.

| \( F_\nu(\omega) \) | \( p \) | \( x_0 \) | \( \lambda \) | \( \omega_0 \) | \( p \) | \( x_0 \) | \( \lambda \) | \( \omega_0 \) |
|-----------------|------|--------|--------|--------|------|--------|--------|--------|
| \( F_{LLS} \)   | 5.796 | -3.23  | 0.026  | 0.083  | 24.68 | 8.65   | 0.013  | 0.111  |
| \( F_{LLT} \)   | -0.129 | -2.52±1.62 | 0.106 | -0.267±0.171 | -0.805 | -1.98±2.62 | 0.039 | -0.076±0.101 |
| \( F_{LRu} \)   | -0.083 | -2.65±1.48  | 0.077  | -0.205±0.115  | 0.124  | -1.85   | 0.063  | -0.117  |
| \( F_{Lrd} \)   | 0.062  | -2.25    | 0.063  | -0.142  | -0.05  | -2.81±1.35 | 0.071 | -0.199±0.095 |
| \( F_{RL} \)    | -0.042 | -2.87±1.31 | 0.077  | -0.222±0.102 | 0.05   | -2.365  | 0.071  | -0.167  |
| \( F_{RRu} \)   | -0.24  | -2.33±1.86 | 0.064  | -0.15±0.12   | 6.0    | 3.32    | 0.013  | 0.0428  |
| \( F_{RRd} \)   | 0.75   | -0.34    | 0.026  | -0.009  | -0.188 | -2.40±1.76 | 0.051 | -0.124±0.090 |

TABLE II: Rightmost zeros \( x_0 \) of parabolic cylinder functions \( D_\nu(x) \) determining the values of the leading singularities \( \omega_0 \) of different Mellin transform amplitudes \( F_\nu(\omega) \) for \( e^+e^- \rightarrow l\bar{l} \) annihilation in forward and backward kinematics. Notations for isodoublet components of \( l \) are taken as for muon doublet.

| \( F_\nu(\omega) \) | \( p \) | \( x_0 \) | \( \lambda \) | \( \omega_0 \) | \( p \) | \( x_0 \) | \( \lambda \) | \( \omega_0 \) |
|-----------------|------|--------|--------|--------|------|--------|--------|--------|
| \( F_{LLS} \)   | \( \infty \) | ---    | ---    | 0.132  | 2.41  | 1.32   | 0.039  | 0.051  |
| \( F_{LLT} \)   | -0.090 | -2.63±1.50 | 0.103  | -0.270±0.154 | -0.602 | -2.06±2.40 | 0.053 | -0.109±0.127 |
| \( F_{LRu} \)   | ---    | ---    | ---    | ---    | ---   | ---    | ---    | ---    |
| \( F_{Lrd} \)   | 0.172  | -1.64   | 0.066  | -0.108  | -0.102 | -2.59±1.54 | 0.086 | -0.221±0.132 |
| \( F_{RL} \)    | 0.172  | -1.64   | 0.066  | -0.108  | -0.102 | -2.59±1.54 | 0.086 | -0.221±0.132 |
| \( F_{RRu} \)   | ---    | ---    | ---    | ---    | ---   | ---    | ---    | ---    |
| \( F_{RRd} \)   | \( \infty \) | ---    | ---    | 0.077  | -0.25  | -2.32±1.88 | 0.077 | -0.179±0.145 |

where \( \sigma_0 \) is the common Born cross section of the elastic process (see \([1]\)), \( M_{LLT} \) denotes \( M_2 \) amplitude of \( e^+_L e^-_L \rightarrow q_L \bar{q}_L \) and \( M_{RL} \) denotes the amplitude of \( e^+_R e^-_R \rightarrow q_L \bar{q}_L \) (and similar for annihilation to leptons). Let us remind that \( W^\pm \) are produced first as isovector boson \( A_1 \), \( A_2 \) states and then transform to observable boson states. In contrast, \((Z, \gamma)\) are being produced first as isoscalar \( B \) or isovector \( A_3 \) fields, and then transform to the observable states, \( Z \) mainly comes from \( A_3 \) and \( \gamma \) - from \( B \). Easy but cumbersome calculation lead to the following cross sections for production of \( B \) and \( A_3 \) bosons:

\[
\sigma(Z, \gamma) = \sigma(A_3) + \sigma(B),
\]

\[
\frac{\sigma(A_3)}{\sigma_0} = g^2 \left[ |M_{LLT}|^2 + \frac{1}{4} \left( |M_{LRu}|^2 + |M_{Lrd}|^2 \right) + \frac{1}{2} |M_{RL}|^2 \right],
\]

where again the amplitudes \( M \) involved are either forward or backward amplitudes: \( M_{LLT} = M_2 \) of \( e^+_Le^-_L \rightarrow q_L \bar{q}_L \), \( M_{LRu} \) stands for \( e^+_Le^-_L \rightarrow \nu_R\bar{\nu}_R \), \( M_{Lrd} \) stands for \( e^+_Le^-_L \rightarrow \bar{d}_R d_R \) and \( M_{RL} \) for \( e^+_R e^-_R \rightarrow q_L \bar{q}_L \), and

\[
\frac{\sigma(B)}{\sigma_0} = g^2 \left[ \frac{(Y_{e\gamma} + Y_{q\gamma})^2}{4} \left( \frac{1}{4} |M_{LLS}|^2 + \frac{5}{4} |M_{LLT}|^2 + \frac{1}{2} |M_{LRS} M_{LLT}|^2 \right) + \frac{(Y_{e\gamma} + Y_{\nu\gamma})^2}{4} |M_{LRA}|^2 \right] + \frac{(Y_{e\gamma} + Y_{d\gamma})^2}{4} |M_{RLd}|^2 + \frac{(Y_{e\gamma} + Y_{\nu\gamma})^2}{4} (2|M_{RL}|^2) + \frac{(Y_{e\gamma} + Y_{d\gamma})^2}{4} |M_{RRd}|^2 \left( \frac{1}{2} |M_{RL}|^2 \right),
\]

where "\( + \)" denotes that "\( - \)" sign corresponds to forward amplitudes and "\( + \)" sign to backward amplitudes denoted above.

The same formulae can be used for \( e^+e^- \rightarrow l\bar{l} \) annihilation channel: one has to substitute the appropriate amplitudes \( M_l \) and to replace electro-weak charge \( Y_q \) with the appropriate \( Y_l \).

As the Regge kinematics is dominating in the cross sections, we sum the contributions of forward and backward kinematics in what follows. The results of numerical calculations presented in Figs. \([4, 5]\) show that at energies \( \sqrt{s} < 10^6 \) GeV \( W^\pm \) and \((Z, \gamma)\) are mainly produced in \( e^+e^- \rightarrow l\bar{l} \) annihilation. And only at \( \sqrt{s} > 10^6 \div 10^7 \) GeV their yields from \( e^+e^- \rightarrow q\bar{q} \) annihilation become greater (see Fig. \([5]\)).
FIG. 4: Dependence of exclusive $W^\pm$ and $(Z, \gamma)$ production on the total energy of $e^+e^-$ annihilation. The cross sections are divided by the differential elastic Born cross section $\sigma_0$ to make differences in energy dependencies more clear.

FIG. 5: Total energy dependence of $W^\pm$ and $(Z, \gamma)$ production in different channels of $e^+e^-$ annihilation: $e^+e^- \rightarrow \ell \bar{\ell}$ – solid curves and $e^+e^- \rightarrow q\bar{q}$ – dashed curves.

The explicit asymptotical dominance of exclusive channel $q\bar{q} + (Z, \gamma)$ over the channel $\ell\bar{\ell} + (Z, \gamma)$ stems from the fact that despite the leading $F_{LLS}$ amplitude in the Table I has the greater intercept $\omega_0 \approx 0.132$ than $\omega_0 \approx 0.111$ of the leading $F_{LLS}$ in the Table II its contribution is multiplied by the zero factor in Eq. (85).

The numerical calculation for the ratio of $W^\pm$ to $(Z, \gamma)$ production summed over the both annihilation channels $e^+e^- \rightarrow \ell\bar{\ell}$ and $e^+e^- \rightarrow q\bar{q}$ is shown in Fig. 6. Thus the DLA predicts rather slow energy dependence of the ratio till $\sqrt{s} \sim 10^4$GeV and then its relatively rapid decrease.

The energy dependence of the ratio $Z$ to $\gamma$,

$$\frac{\sigma(Z)}{\sigma(\gamma)} = \frac{\sigma(A_3) + \sigma(B) \tan^2 \theta_W}{\sigma(A_3) \tan^2 \theta_W + \sigma(B)}, \quad (86)$$

is shown in Fig. 7. In far asymptotics radiation of isoscalar field $B$ dominates over radiation of isovector field $A$ and the ratio tends to the fixed value $\tan^2 \theta_W \approx 0.28$. 
FIG. 6: Total energy dependence of $W^\pm$ to $(Z, \gamma)$ rate in $e^+e^-$ annihilation.

![Graph showing the ratio $\sigma_{W^\pm} / \sigma_{(Z, \gamma)}$ vs. $\sqrt{s}/\text{GeV}$](image)

FIG. 7: Total energy dependence of $Z$ to $\gamma$ rate in $e^+e^-$ annihilation. The dashed line shows the asymptotical value of the ratio: $\tan^2 \theta_W \approx 0.28$.

![Graph showing the ratio $\sigma_Z / \sigma_\gamma$ vs. $\sqrt{s}/\text{GeV}$](image)

Worthwhile to note that apart from the results of [1] for pure QED, the figures of the Table II show that backward $e_L^+e_L^\to l_Ll_L^\to l_L$ amplitude (i.e. when antilepton follows the direction of initial electron) for isoscalar channel in EW theory has the positive intercept though small enough if compared to forward annihilation amplitude.

Let us emphasise that the demonstrated in Fig. 5 excess of $Z$ production over $\gamma$ production at $\sqrt{s} < 10^3 \div 10^5$ GeV as well as excess of $(Z, \gamma)$ emission over $W^\pm$ emission in the same energy range shown in Fig. 4 and the dominance of $ll$ channel over $qq$ channel shown in Fig. 3 may all happen to be just artifacts of the DLA. To get more reliable predictions for the cross sections one has at least to account for single logarithmic corrections as well. The presented figures show that account of the non-leading DLA effects can make observation of the theoretically correct predictions (49) and (50) hardly possible even in far future.
VII. CONCLUSION

In the present paper we have obtained explicit expressions for the scattering amplitudes for the $e^+e^-$ annihilation into quarks and into leptons at the annihilation energies $\sqrt{s} \gg 100$ Gev accompanied by emission of $n$ electroweak bosons in the multi-Regge kinematics, i.e. in the kinematics where the final particles are in cones with opening angles $\ll 1$ around the initial $e^+e^-$ beams. We accounted for the double-logarithmic contributions to this process to all orders in the EW couplings. We have shown that it is convenient to calculate amplitudes of this process in terms of the isoscalar and of the isovector amplitudes. The isoscalar amplitudes describe production of the isoscalar gauge fields. They are controlled by $n+1$ isoscalar Reggeons propagating in the crossing channel. The leading intercepts of these Reggeons are positive ($\Delta_S' = 0.11$ and $\Delta_S = 0.08$) and therefore such scattering amplitudes grow when $s$ increases. The isovector amplitudes bring sub-leading contributions to the production of the isoscalar bosons and in the same time give the leading contributions to production of the isovector gauge fields. They are governed by $n+1$ isovector Reggeons with negative intercepts $\Delta_V' = -0.08$ and $\Delta_V = -0.27$. It means that the amplitudes for isovector production decreases when $s$ grows. These results lead in particular to the fact that production of each $Z$ boson is always accompanied by production of a hard photon with the same energy $\gg 100$ Gev. In DLA, such hard photons are never produced without $Z$ bosons. The cross sections of production of these photons and the $Z$ bosons have identical energy dependence, however they are different numerically due to difference in the couplings. They are related by Eq. (72) at asymptotically high energies ($\geq 10^7$ Gev). The $s$ -dependence of the ratio $\sigma^{nZ}/\sigma^{n\gamma}$ for lower energies is given in Fig. 5. The energy dependence of cross section for the $W$ production is weaker than the one for the photons and the $Z$ bosons by factor $s^{-0.36}$ at asymptotically high energies. The $s$ -dependence of these cross sections is shown in Figs. 4-6. Through this paper we consider only the monotonically ordered multi-Regge kinematics (57) and (58). Accounting for the other kinematics can be done in a similar way. Though it is likely to bring corrections to explicit formulae for the invariant amplitudes $M_{n}(r)$, it cannot change the asymptotic relation of Eq. (72) and the fact (see Eq. (73)) that $\sigma^{(n\gamma)}/\sigma^{(nW)}$ decreases with $s$.

VIII. ACKNOWLEDGEMENT

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