Abstract

We develop quantization aspects of our Liouville approach to non-critical strings, proposing a path-integral formulation of a second quantization of string theory, that incorporates naturally the couplings of string sources to background fields. Such couplings are characteristic of macroscopic string solutions and/or D-brane theories. Resummation over world-sheet genera in the presence of stringy (σ-model) soliton backgrounds, and recoil effects associated with logarithmic operators on the world sheet, play a crucial rôle in inducing such sources as well-defined renormalization-group counterterms. Using our Liouville renormalization group approach, we derive the appropriate second-order equation of motion for the D brane. We discuss within this approach the appearance of open strings, whose ends carry non-trivial Chan-Paton-like quantum numbers related to the $W_\infty$ charges of two-dimensional string black holes.
1 Introduction

A key issue in string theory is how to go beyond perturbation theory in a fixed space-time background, describable as one of many apparently consistent classical string vacua, each of which is characterized by a unique conformal field theory on the world sheet. One would like to understand how to interpolate between such backgrounds, and how to treat the non-perturbative transitions between them. A related issue is how to treat fluctuations in the topology of the string world sheet, which appear in the integration over higher genera.

Resolution of these two issues necessarily involves a departure from criticality. This is because, on the one hand, transitions between different classical vacua, i.e., critical string theories, must traverse some broader framework in which these are embedded. Moreover, as we discuss in more detail later, quantum fluctuations in higher genera may in general be viewed as sources that perturb the world-sheet field theory away from criticality. The minimal broader framework that can be considered is that of general renormalizable two-dimensional theories on the world sheet. Renormalization may be implemented using the Liouville field on the world sheet as a local renormalization scale [1]. A suitable version of the Zamolodchikov metric [2] can be introduced as a positive metric on the relevant space of unitary two-dimensional field theories, and their renormalization group flow is monotonic with respect to a suitable version of the Zamolodchikov $C$ function [2].

We have shown [1] how the Liouville field may be identified with the time variable [1, 3, 4]. Starting from a generalized light-cone formulation of critical string theory, in which time does not appear as a target coordinate, we have shown that renormalization counterterms associated with transitions to supercritical string theories [2] naturally yield a negative metric for the Liouville field. An explicit calculation within this framework reproduces the metric of the two-dimensional (spherically-symmetric four-dimensional) string black hole [3]. The monotonicity of the Zamolodchikov $C$ function and the semigroup nature of the renormalization group translate into a monotonic increase in the entropy of the effective target-space theory [1, 5].

The Zamolodchikov renormalization-group flow through the space of world-sheet theories is itself subject to quantum fluctuations, induced in particular by the sum over higher genera mentioned above. We have demonstrated that this flow satisfies the Helmholtz conditions for consistent first quantization [7]. This result suggests that it may be possible to define suitably a functional integral over configurations of the world sheet. The purpose of this paper is to embark on this programme and to derive as a first consequence the action of a $D$ brane coupled as a source to the string effective action [8] (in standard notation):

$$S = \int d^Dy \sqrt{g} [R + 4(\partial \Phi)^2 - \frac{1}{12} e^{-2\Phi} H_{MNP}^2 + \ldots] +$$
\[
\frac{1}{4\pi\alpha'} \int_{\Sigma} d^{2}\sigma [\partial_{\alpha}X^{M}\partial^{\alpha}X^{N} e^{\frac{1}{2\alpha'} - \frac{1}{2}} g_{MN} + \epsilon_{\alpha\beta} \partial_{\alpha}X^{M} \partial_{\beta}X^{N} B_{MN} + \Phi(X) R^{(2)}] \tag{1}
\]

in which the latter appears as a renormalization counterterm.

The outline of this paper is as follows. In section 2 we review the appearance of quantum fluctuations in couplings of the world-sheet field theory, as a consequence of higher-genus wormhole-like configurations of the world sheet, emphasizing the role of logarithmic operators \[9, 10\]. We start section 3 by recalling the consistency of non-critical Liouville string theory with canonical quantization of the space of string theories \[7\], and then introduce a prescription for the path integral over string configurations. A key element in our approach is the treatment of the Liouville field as a local renormalization scale \[1\]. The renormalization programme is developed in section 4, and we demonstrate explicitly how the action (1) emerges from renormalization counterterms. We also derive the appropriate second-order equation of motion for the \(D\) brane. Section 5 develops an important application of this formalism to the problem of black holes and information loss in string theory, inspired by previous work in a two-dimensional pilot example. We demonstrate \[11\] the appearance of open strings attached to the event horizon viewed as a \(D\) brane \[12\], whose ends carry Chan-Paton-like quantum numbers related to the \(W_{\infty}\) charges of the two-dimensional string black hole. In our interpretation, it is the “leakage” of these charges that leads to information loss at the horizon \[1\], as perceived by an external observer. Conclusions are presented in section 6.

2 Formulation of string theory in resummed high-genus world sheets

In this section we introduce the formulation of the sum over world-sheet configurations of a string theory. We recall that, in the normal first-quantized treatment of a string model at fixed genus, the background fields are determined classically, and may be regarded as coupling constants. However, the sum over world-sheet topologies entails quantization of these couplings, as we shall now argue. Features of this quantization can be seen in the non-perturbative matrix model formulation of two-dimensional strings, in which the sum over world-sheet topologies is handled via a discrete triangulation of the world sheet. Similar conclusions have been reached in the ‘wormhole calculus’ approach to the sum over topologically non-trivial space times in four-dimensional quantum gravity \[13\]. We shall bring out the analogies during the course of our development.

We start our development by reviewing relevant results from \(c = 1\) Liouville theory \[14\], which can be viewed as the asymptotic spatial and temporal limit of the two-dimensional black hole \[6\]. We consider the extra logarithmic divergences that arise, as we now discuss, from degenerate handles in the summation over higher-genus Riemann surfaces. These may in general be formulated as long, thin tubes.
connecting two Riemann surfaces $\Sigma_{1,2}$. At the one-loop (torus) order, the relevant configuration is a long, thin handle attached to a sphere $\Sigma$. This may be regarded as a world-sheet wormhole, in close analogy with conventional wormholes in four-dimensional quantum gravity [13].

String propagation on such a world sheet may be described formally by adding bilocal world-sheet operators $B_{ii}$ on the world-sheet sphere [15]:

$$B_{ii} = \int d^2z V_i(z) \int d^2w V_i(w) \frac{1}{L_0 + \mathcal{T}_0 - 2}$$  \hspace{1cm} (2)

where the last factor represents the string propagator, $\Delta_S$, on the handle, with the symbols $L_0, \mathcal{T}_0$ denoting Virasoro generators as usual. Inserting a complete set $\mathcal{E}_\alpha$ of intermediate string states, we can rewrite (2) as an integral over the parameter $q \equiv e^{2\pi i \tau}$, where $\tau$ is the complex modular parameter characterizing the world-sheet tube. The string propagator over the world-sheet tube then reads

$$\Delta_S = \sum_\alpha \int dq d\tau \frac{1}{q^{1 - h_\alpha} \tau^{1 - \bar{h}_\alpha}} \{ \mathcal{E}_\alpha(z_1) \otimes (\text{ghosts}) \otimes \mathcal{E}_\alpha(z_2) \}_{\Sigma_1 \oplus \Sigma_2}$$  \hspace{1cm} (3)

where $h_\alpha, \bar{h}_\alpha$ are the conformal dimensions of the states $\mathcal{E}_\alpha$. The sum in (3) is over all states propagating along the long, thin tube connecting $\Sigma_1$ and $\Sigma_2$, which are both equal to the sphere in the degenerating torus handle case of interest. As indicated in (3), the sum over states must include the ghosts, whose central charge cancels that of the world-sheet matter theory in any critical string model.

States with $h_\alpha = \bar{h}_\alpha = 0$ may cause extra logarithmic divergences in (3) which are not included in the familiar $\beta$-function analysis on $\Sigma$ [16]. This is because such states make contributions to the integral of the form $\int dq d\tau / q\tau$ in the limit $q \to 0$, which represents a long, thin tube. We assume that such states are discrete in the space of states, i.e., they are separated from other states by a gap. In this case, there are factorizable logarithmic divergences in (3) which depend on the background surfaces $\Sigma_{1,2}$, e.g., the sphere in the case of the degenerating torus.

The bilocal term (2) can be cast in the form of a local contribution to the world-sheet action, if one employs the trick, familiar from the wormhole calculus, of rewriting it as a Gaussian integral [13, 17]:

$$e^{B_{ii}} \propto \int d\alpha e^{-\alpha_i^2 + \alpha_i \int V_i}$$  \hspace{1cm} (4)

where the $\alpha_i$ are to be viewed as quantum coupling constants/fields of the world-sheet $\sigma$-model. Once it becomes apparent (4) that the couplings $\alpha_i$ must be treated as quantum variables, it is natural to replace the factor $e^{-(\alpha_i)^2}$ by a more general Gaussian distribution of width $\Gamma$ [13, 17]:

$$\mathcal{P} = \frac{1}{\sqrt{2\pi\Gamma}} e^{-\frac{1}{2\Gamma}(\alpha_i)^2}$$  \hspace{1cm} (5)
The extra logarithmic divergences associated with degenerate handles that we mentioned above: \( \propto \ln \epsilon \) where \( q \sim \epsilon \sim 0 \), have the effect of causing the width parameter \( \Gamma \) also to depend logarithmically on the cutoff scale \( \epsilon \). Upon identification of the cutoff scale with the Liouville field, and identifying the latter with the target time variable, we infer that the distributions of the couplings \( \alpha_i \) become time-dependent [18, 17].

We have assumed in the above treatment that the Virasoro operator \( L_0 \) may be diagonalized simply in the basis of string states, with eigenvalues their conformal dimensions \( h_\alpha \):

\[
L_0 |E_\alpha > = h_\alpha |E_\alpha >
\]

However, this simple diagonalization may fail in the presence of a non-trivial solitonic background [10], as has been shown in a range of conformal models, including flat-space \( c = 1 \) Liouville theory [19], two-dimensional black holes [1, 10], and certain models of disorder in condensed-matter physics [20]. In these cases, there are logarithmic operators \( C \) associated with degeneracies in the spectrum of conformal dimensions \( h_\alpha \). In particular, other operators may have dimension zero, and hence be degenerate with the identity operator. Examples in the flat-space \( c = 1 \) Liouville theory are the two conjugate dressings of the identity operator: \( e^{\sqrt{2} \phi} \), \( \phi e^{\sqrt{2} \phi} \), where \( \phi \) is the Liouville field [19].

In the presence of logarithmic operators, the OPE features anomalous logarithmic factors, and there is a non-trivial Jordan cell in which the Virasoro operator is not diagonal. These logarithmic operators therefore mix in string propagators, and may yield additional logarithmic factors in the presence of a degenerate handle, if the logarithmic operators carry non-trivial weight in the sum over intermediate states [10]. This is not the case in the flat-space \( c = 1 \) Liouville string example mentioned above, because the dressings of the identity operator carry zero weight in the sum over intermediate states. In such a case, the bilocal operator (2) yields distributions for the quantum couplings \( \alpha_i \) which have constant widths \( \Gamma \), as in standard wormhole examples in four-dimensional quantum gravity with fluctuating space-time topology. However, the logarithmic operators in generic soliton backgrounds do have non-trivial weights [10], and hence contribute to the scale- and hence time-dependence of the distributions of the couplings which we mentioned earlier as a possibility, via divergences of double-logarithmic type: \( \int dq dq' \frac{1}{|q|} \ln |q| \ldots \), in the region \( q \sim 0 \). The double logarithm arises from the form of the string propagator over the world-sheet tube used in (2), which, in the presence of generic logarithmic operators \( C \) and \( D \), takes the form

\[
\int dq dq' q^{h_C - 1} q'^{-1} < CD | \left( \begin{array}{c} 1 \\ 0 \\ \frac{1}{|q|} \\ 1 \end{array} \right) | CD > < CD | \left( \begin{array}{c} 1 \\ 0 \\ \frac{1}{|q'|} \\ 1 \end{array} \right) | CD >
\]

As was shown in ref. [10], this mixing between \( C \) and \( D \) states along degenerate handles leads formally to divergent string propagators in the amplitudes, whose
integrals have leading divergences of the form
\[ \int \frac{dq dq}{q^2} \left[ \ln q \int d^2 z_1 D(z_1) \int d^2 z_2 C(z_2) + c.c. \right] \]
\[ \sim (\ln^2 \epsilon) \int d^2 z_1 D(z_1) \int d^2 z_2 C(z_2) + c.c. \]  

(8)
giving a leading singularity \( \propto \ln \ln \epsilon \). Besides this term, there will be terms \( \propto \ln \epsilon \), corresponding to
\[ \int d^2 z_1 D(z_1) \int d^2 z_2 D(z_2) \] and
\[ \int d^2 z_1 C(z_1) \int d^2 z_2 C(z_2) \] terms.

To see the origin of such non-trivial logarithmic operators in a generic string-soliton case, consider the scattering of a light string mode off a soliton, including recoil effects. The centre of mass of the soliton must be treated as a collective coordinate [16], whose integration ensures momentum conservation: the centre of mass is shifted during a scattering process, as a result of the recoil. This effect can be described in the world-sheet \( \sigma \)-model path-integral formalism by making a constant shift in the spatial coordinates \( X^\mu \): 
\[ X^\mu(z, \bar{z}) \rightarrow X^\mu(z, \bar{z}) + q^\mu; \quad q^\mu = \text{const} \]  

(9)
As discussed in refs. [16, 10], such a deformation can be expressed as a total world-sheet derivative 
\[ \mathcal{O}_I = \mathcal{N} (\text{ghosts}) \otimes \partial_\alpha \{ g^I(X) \frac{\delta}{\delta(\partial_\alpha X_I)} V_j(\partial_\beta X; J) \} \equiv \mathcal{N} (\text{ghosts}) \otimes \partial_\alpha J^{\alpha}_I \]  

(10)
where \( \mathcal{N} \) is a normalization factor, \( J \) are Kac-Moody currents, and \( J^{\alpha}_I \) is a two-dimensional Noether current. We consider as an example here the deformations associated with translations (9), which are labelled by a latin index \( I \) running over the (target-space) soliton-background zero modes:
\[ J^{\alpha}_I \equiv \frac{\delta S}{\delta(\partial_\alpha X_I)} \]  

(11)
where \( S \) is a world sheet action. The ghost insertions in (10), which are of the form \( \tau_c \), must be included in order to treat correctly the ghost zero modes, whose presence shifts the anomalous dimension of the operator from (1,1) to (0,0).

It was shown in ref. [10] that the (normalized) Zamolodchikov metric [2] corresponding to \( \mathcal{O} \), which is defined by [3]:
\[ G_{\mathcal{O}\mathcal{O}} = 4\pi \delta^{IJ} = |z|^4 < \mathcal{O}^I(z, \bar{z}), \mathcal{O}^J(0) > = \mathcal{N}^2 |z|^4 < (\overline{\partial J}^I(z, \bar{z}) + c.c.)(\partial J^J(0,0) + c.c.) > \]  

(12)
is well-defined and finite if one postulates that in the soliton background of ref. [10] the currents \( J^I \) are not the usual \((1,0)\) currents, but the logarithmic currents obeying the following OPE [10, 4, 20]
\[ < J^I(z) J^J(0) > = -\kappa G^{IJ} |z|^2 \]  

(13)
with $\kappa$ a normalization constant. We have used a non-trivial target-space background metric $G_{IJ}$ above, so as to incorporate general covariance, in a straightforward generalization of the flat space result of ref. [10]. It is now also straightforward to verify that (13) leads to a finite factor $\mathcal{N}^2$ and a well-defined Zamolodchikov metric (12)

$$\mathcal{N}^2 = 2\kappa N^2 = 4\pi;$$

We contrast this with the conventional case, assumed in ref. [16], in which the currents $J^I$ satisfy a standard non-logarithmic OPE

$$< J^I(z) J^J(0) > \sim \frac{\kappa G_{IJ}}{z^2}$$

The resulting ‘normalized’ metric (12) would have a logarithmically-divergent coefficient, $\mathcal{N} = \log \epsilon$, and the un-normalized Zamolodchikov metric tensor would vanish logarithmically [16]. The importance of the finite metric (12) for the central result of this work, namely the derivation of the action (1), will become apparent in section 4.

We draw the reader’s attention to the analogy between the above discussion and the treatment of conifold singularities in Calabi-Yau compactifications [21]. There, the metric in moduli space has an apparent logarithmic singularity if massless quantum black holes are not taken correctly into account. Here also, the inclusion of new massless states associated with logarithmic operators restores the regularity of the metric in theory space.

In the particular example developed above, the extra logarithmic divergence (13), and hence the corresponding logarithmic operator and extra divergence in the distribution function (5), result from the correct imposition of momentum conservation via soliton recoil as in (9). This is just one example of a transition between two different conformal field theories, characterized in this case by different locations of the string soliton. Analogous logarithmic operators also appear in the quantum treatment of the two-dimensional string black hole, e.g., in the calculation of instanton effects [22, 1] - which reflect transitions between black holes described by conformal field theories with different central charges and hence represent black-hole decays, and in the treatment of world-sheet monopole-antimonopole configurations - which reflect the creation and annihilation of such a string black hole [1]. These are also examples of transitions between different conformal field theories.

Analogous transitions, logarithmic operators and the associated zero modes are clearly generic, and their treatment requires going beyond the conventional first-quantized approach to string theory. As we have argued elsewhere [7], and as we
review at the start of the next section, they necessitate a quantum treatment of the background couplings. The generic soliton situation outlined above corresponds to a saddle point in the eventual string field theory path integral, and later in the next section we make a proposal towards the formulation of such a path integral based on a second-quantized version of Liouville string theory. This accommodates string solitons in a natural way, as well as the eventual emergence of $D$ branes.

3 Path-Integral Formulation of Quantum String Theory Space

We have argued above that the summation over world-sheet topologies necessarily induces the second quantization of string theory [1]. The correct formulation of this problem in generic higher-dimensional string theory is far from complete. In two-dimensional string theory, the summation over genera can be performed exactly in the so-called matrix model approach [23], which amounts to a discretization of the world sheet. There is no rigorous proof that the continuum limit of such a model yields the appropriate sum over genera of a world-sheet continuum $\sigma$-model in an arbitrary background. However, the summation over genera can be performed in a flat space-time background, by using the inverted harmonic oscillator potential of the $c = 1$ matrix model. From the point of view of Liouville strings, the interesting case is that of fluctuating black-hole space-time backgrounds. The incorporation of such backgrounds in the matrix-model picture is still not very well understood, though there are attempts [24] to incorporate fixed black-hole backgrounds in the matrix-model framework, by utilizing deformations of the inverted harmonic oscillator potential. However, it is not yet known how to incorporate a fluctuating (or, a fortiori, an evaporating) black hole at the desired level of rigour. Nevertheless, for the purposes of our discussion below, we shall assume that the summation over genera in black-hole string theory can be defined satisfactorily, and we shall present a qualitative discussion of its properties, based on a geometric description of theory space.

As we have argued elsewhere [1], the natural framework in which to discuss the space of string theories is that of renormalizable two-dimensional $\sigma$ models on the world sheet, which is endowed with a natural metric structure [2]. Motion through this theory space is determined at the classical level by the renormalization $\beta$ functions of the $\sigma$-model couplings, \{\(g^i\)\}, which are given by the gradients of a suitable form of the Zamolodchikov $C$ function [2]:

\[
\beta^i = G^{ij} \partial_j C[g] \quad : \quad G_{ij} = 2|z|^4 < V_i(z, \bar{z}), V_j(0) > 
\]

where the $V_i$ are the appropriate vertex operators on the world sheet, describing the emission of target-space states by the fields/couplings $g^i$. This classical motion along Renormalization Group trajectories may equivalently be expressed in terms
of an action principle, with the integrated Zamolodchikov $C$ function serving as the action \[ \Gamma[g] = \int dt C(g) = \int dt (p_i \dot{g}^i - H) \] (17)

The effective target-space Hamiltonian $H$ is defined by a Legendre transformation of (17), and provides the following representation of the classical equation of motion:

\[ \partial_t \rho = -\{ \rho, H \} - G_{ij} \beta^j \partial_p \rho \] (18)

where the $\{,\}$ are classical Poisson brackets, $\rho(g^i, p_k)$ denotes the matter density matrix, the coordinates $g^i$ in string theory space are identified with the $\sigma$-model couplings, and the vertices $V_i$ are identified with the canonical conjugate momenta $p_i$, upon identification of the Liouville scale field with time [1, 5].

We have argued in the previous Section that the summation over higher genera necessitates quantization of the background fields/couplings $g_i$. Canonical quantization with the commutation relations:

\[ [g^i, g^j] = [p_i, p_j] = 0 \quad [g^i, p_j] = i\hbar \delta^i_j \] (19)

is possible only if the dynamical system obeys the Helmholtz conditions [25]. If this is the case, total time derivatives of functionals of $g^i$ are then defined by commutators with the Hamiltonian operator $H$

\[ \frac{df(g)}{dt} = -i[f(g), H] \] (20)

and equation (18) becomes [1, 3]:

\[ \partial_t \rho = i[\rho, H] + iG_{ij} \beta^j [g^i, \rho] \] (21)

where here and henceforth we set $\hbar = 1$.

We have shown elsewhere [7] that non-critical strings satisfy the Helmholtz conditions [25], and here we only recall some key features of the proof. Formally, given a Newtonian-type equation of motion for a generic mechanical system with coordinates $g^i$, a sufficient condition for canonical quantization is the existence of a non-singular symmetric matrix $w_{ij}$ and scale factor $Q$ with the property that [25]

\[ w_{ij} (\ddot{g}^j - Q \dot{g}^j) = \frac{d}{d\phi} \left( \frac{\partial L}{\partial \dot{g}^i} \right) - \frac{\partial L}{\partial g^i} \] (22)

where the dots denote derivatives with respect to the Liouville mode $\phi$, for some function $L$ to be identified as the off-shell Lagrange function of the system. If the appropriate conditions for the existence of $w_{ij}$ and $Q$ are met, the solution for $w_{ij}$ is [25]

\[ w_{ij} = \frac{\partial^2 L}{\partial \dot{g}^i \partial \dot{g}^j} \] (23)
We have shown that, in the case of ‘off-shell’ strings, discussed here, the lagrangian $L(t)$ is just the Zamolodchikov $C$ function:

$$L(t) = -\beta^i G_{ij} \beta^j$$

(24)

and $w_{ij}$ may be identified (up to a sign) with the Zamolodchikov metric:

$$w_{ij} = -G_{ij}$$

(25)

In this case, the Helmholtz conditions are satisfied as a result of non-trivial features of the Zamolodchikov framework, in particular the facts that $G_{ij}$ is only a functional of the $g^i$ and not the $\beta^i$, and that it obeys the rescaling property

$$\frac{d}{dt} G_{ij} = Q G_{ij}$$

(26)

under the action of the on-shell Liouville time derivative $d/dt$ for some suitable scale factor $Q$, which coincides in this case with the ordinary renormalization group operator for the couplings $g^i$. Due to the Liouville-renormalization-group invariance of $Q$, (24) implies a linearly-expanding [3] scale factor for the metric in string theory space,

$$G_{ij}[t, g(t)] = e^{Qt} \hat{G}_{ij}[t, g(t)]$$

(27)

where the notation $\hat{A}$ is used for any Liouville-renormalization-group invariant function $A$. This is exactly the form of the Zamolodchikov metric in Liouville strings, as discussed in ref. [7].

The above results suggest that a path-integral formalism for the string theory space parametrized by the $g^i$ should be available, and we now make a proposal for its construction. The starting point for our proposal is the formalism of Osborn and Shore [26, 27], in which the background fields $g^i$ are allowed to depend explicitly on the world-sheet coordinates, which we have applied previously to the case of a fixed-genus Riemann surface [1]. Here we generalize the approach of [26, 27] to the summation over world-sheet topologies: motivated by the matrix model results, and replacing the background fields $g^i$ by operators $\hat{g}^i$ in target space, we show later how the form of the Zamolodchikov metric can be induced dynamically when we perform a path integral over the variables $g^i$. This approach assumes a Lagrangian formalism in coupling-constant space, obtained by integration over the conjugate momenta $p_i$ in the target-space string effective action $\Gamma[g]$ (17). Our proposal is to write the path integral over string theories in the form

$$\int \mathcal{D}g^i e^{-\Gamma[g, A]} = \int \mathcal{D}g^i \int DX D\phi \delta(A - \int_{\Sigma} e^{\phi} e^{-(S^* + \int_{\Sigma} (g_i V_i(X) + \partial_\alpha g^i G_{ij} \partial_\alpha g^j + \Phi(g) R^2 + \ldots))}}$$

(28)

where $S^*$ is a fixed point world-sheet action, the $X$ are the target spatial coordinates, and $\phi$ is the Liouville field, interpreted as the target time coordinate. We have factored out the integral over $A$, the global area of the world sheet, which serves...
as a renormalization-scale evolution parameter, as described above. The index \( i \) includes a summation over the target-space zero modes \([1]\), which play a crucial role in the quantization of the \( g^i \), as we discussed after \((14)\) in the previous section. These should not be confused with the dependences on the space-time coordinates \( X \) and \( \phi \), which we treat separately. The world-sheet integral is over the lowest-genus Riemann surface \( \Sigma \), since the summation over higher genera is represented by the quantization of the couplings \( g^i \), treated here as path-integration variables. Classical string vacua, which are described by conformal backgrounds corresponding to fixed points of the renormalization group flow, appear as saddle points in the path integral \((28)\). This is a \( \sigma \)-model vision of what a string field theory might be, which is not necessarily complete, but captures the features of interest to us for the derivation of \((1)\).

The interpretation of \((28)\) is of a ‘string’ moving in an abstract space of possible theories, whose coordinates are the background fields \( g^i \). This is to be distinguished on the one hand from the usual motion of a string in target space-time \((X, \phi)\), and, on the other hand, from the usual interpretation of the renormalization group as the motion of a point-like particle in the space of possible field-theoretical couplings. Formally, \((28)\) resembles a \( \sigma \) model in theory space, and we now demonstrate that the quantization induced by the summation over higher genera endows this space with its metric \( G_{ij} \), which appears in the integral

\[
\mathcal{Z} = \int Dg^i e^{-\int_\Sigma \left\{ \partial_\alpha g^i \partial_\alpha g^i + \Phi(g) R^{(2)} + C[g] \right\}}
\]  

(29)

The unconventional \( G_{ij} \)-dependent terms in \((29)\) appear as renormalization counter-terms. They would be absent on a flat world sheet, since they vanish as \( \partial_\alpha g^i \to 0 \), for \( \alpha = z, \bar{z} \). Their presence is essential, though, for the consistency of the quantization in curved space \([26]\).

To see how the terms in \((29)\) arise, we first recall \([27, 26]\) that renormalizability implies the following local renormalization group equation:

\[
\mathcal{D}\mathcal{L} \equiv (\epsilon - \hat{\beta}, \partial_\xi g^i - \hat{\beta}_\lambda, \partial_\lambda)\mathcal{L} = 0 \quad \lambda \equiv (\Phi, \Lambda)
\]  

(30)

where \( \mathcal{L} \) is a first-quantized world-sheet \( \sigma \)-model lagrangian, and the dot includes integration over world-sheet coordinates. The \( \hat{\beta} \) functions refer to the structures appearing in \((29)\), with \( \Phi \) denoting dilaton counterterms, and \( \Lambda \) representing the extra counterterm involving \( G_{ij} \). These arise from the explicit world-sheet coordinate dependence of the renormalized couplings \( g^i \) on the curved world sheet:

\[
\hat{\beta}_g^i = \epsilon g^i + \beta^i(g) \quad ; \quad \hat{\beta}_\Phi = \epsilon \Phi + \beta^\Phi \quad ; \quad \hat{\beta}_\Lambda = \epsilon \Lambda + \beta^\Lambda
\]

(31)

where \( \chi_{ij} \) in \((31)\) is defined via

\[
(\epsilon - \hat{\beta}^k \partial_k) G_{ij} - (\partial_i \hat{\beta}^k) G_{kj} - (\partial_j \hat{\beta}^k) G_{ik} = \chi_{ij}
\]  

(32)
It can be shown [26] that $\chi_{ij}$ is related to the coincidence limit of the Zamolodchikov metric [2], defined through the two-point functions of vertex operators $V_i$.

In order to discuss the antisymmetric-tensor term in (1), we also need to address the appearance of torsion terms in coupling constant space. Such terms can be non-zero on the boundaries between ‘patches’ in theory space. The formal inclusion of such terms presents no essential difficulties [26]. It should be noted that in the off-shell corollary of the Zamolodchikov $C$-theorem [2], which relates off-shell variations of the string effective action in target-space, $\Gamma$, to the $\sigma$-model $\beta$-functions, such coupling-constant-space torsion terms appear explicitly [26]:

$$\frac{\delta}{\delta g^i} \Gamma[g] = \chi_{ij} \beta^j + (\partial_i W_j - \partial_j W_i) \beta^j$$

where the $W_i$ are well-defined functions of renormalized couplings, related to total derivatives on the world sheet in the expression for the trace $\Theta$ of the $\sigma$-model stress tensor in terms of the (renormalized) couplings/fields $g^i$ [26]:

$$\Theta = \beta^i [V_i] + \partial_{\alpha} Z^\alpha \ ; \quad Z^\alpha = W_i \partial^\alpha g^i$$

The bracket in (34) denotes a normal product with respect to the renormalization group flow variable that we identify with time. Due to their total-derivative form, the counterterms $W_i$ play a non-trivial rôle in the case of world sheets with boundaries, as appear when we incorporate open strings in the spectrum of physical theories. These and related issues will be discussed in the next section.

Having identified the most relevant structures in the path integral (28), we now discuss consistency conditions that it must satisfy. Integrating (28) over $X$ and the Liouville conformal factor $\phi$, we arrived at the path integral (29). This makes sense as a quantum field theory on the world-sheet when one imposes the absence of the conformal anomalies, which correspond to the vanishing of the $\beta$ functions associated with the quantities $G_{ij}$, $\Phi(g)$ and $C[g]$, considered as couplings of a $\sigma$ model formulated over a target manifold parametrized by coordinates $g_i$. Thus, their $\beta$ functions should not be confused with the ordinary renormalization-group coefficients pertaining to the target-space fields $g^i$. Moreover, conformal invariance of any $\sigma$ model on a target manifold implies the vanishing, not of the usual renormalization-group $\beta$ functions, but of the Weyl anomaly coefficients, which are invariant under target-space diffeomorphisms [27]. In our case, it is known that a renormalization-scheme change corresponds to diffeomorphisms in the coordinates $g^i$, in the sense of local field redefinitions [28].

The conformal invariance of the last term in (29) is ensured by taking into account our use of the global world-sheet area $A$ as an ordinary renormalization-group scale, which implies that it coincides with the usual renormalization-group invariance of the Zamolodchikov function $C[g] \sim \Gamma[g]$. Conformal invariance of the middle ‘dilaton’
term in (29) restricts the form of $\Phi$ in a manner which does not concern us here. Finally, the conformal-invariance conditions for the first term in the action in (29), namely the Zamolodchikov metric background, read

$$\frac{d}{dt}G_{ij} = \nabla_i(W_j)$$

We now recall (32), which we write in the form

$$\chi_{ij} = G_{ij}^{(1)} + L_\xi G_{ij} = \frac{1}{2}(\partial_t + \hat{\beta}^k \partial_k)G_{ij} + \nabla_i(\hat{\beta}^k G_{jk}) - \hat{\beta}^k \Gamma_{kij} \equiv D G_{ij} + \nabla_i(V_j)$$

where $(ij)$ denotes symmetrization of indices, $L_\xi$ is the Lie derivative with respect to a coordinate transformation $\xi$, and $\Gamma_{kij}$ is the Christoffel symbol for the ‘metric’ $G_{ij}$. Thus, up to diffeomorphism and connection terms, which can be removed by appropriate scheme choice, the right-hand side of (32) corresponds to the usual renormalization group operator acting on $G_{ij}$. The scheme for which the connection $\Gamma_{ijk}$ vanishes corresponds to a ‘normal coordinate choice’, which will be adopted in the definition of our path integral (28).

We now explore the consequences of (35), and check its consistency with standard $\sigma$-model lore. To this end, we first observe from the middle expression in (36) that a change in renormalization scheme relates $\chi_{ij}$ to the residue of the simple pole in $\epsilon$ of $G_{ij}$. From arguments in the string literature, we also know [29] that there exists a scheme which relates $\chi_{ij}$ to the coincidence limit of the original Zamolodchikov tensor [2]. From our conformal invariance condition (35), then, it follows that

$$\chi_{ij} = \nabla_i(W_j - V_j)$$

As we shall argue now, it will be necessary that

$$W_i - V_i = \nabla_i C[g]$$

where $C[g]$ is identified with the Zamolodchikov function, up to an irrelevant constant. To see this, we use the definitions

$$[O_i(x)] = \frac{\delta S}{\delta g^i(x)}$$

for the renormalized vertex operators $[O_i]$, where $S$ is the $\sigma$-model action on a world-sheet with coordinates $x$. From this, we can easily derive the following expression:

$$< [O_i](x)[O_j](0) > = \frac{\delta^2 \Gamma[g]}{\delta g^i(x) \delta g^j(0)} - \frac{\delta}{\delta g^i(x)} [O_i](0)$$

\footnote{Connection terms are related to contact terms of moduli operators, and the reader might worry that such schemes cannot be easily adopted in string theory, where moduli fields play an important rôle. However, in our case there will be no such problem. Our normal coordinate choice implies vanishing connection with respect to the metric $G$, which is different from the Zamolodchikov metric $G_{ij}$ that is directly related to the moduli fields.}
The first term takes a Kähler form, whilst the second is non-zero only in the coincidence limit $x \to 0$, and in fact is proportional to a world-sheet $\delta$ function, since $\delta g^i(x)/\delta g^j(y) = \delta^i_j \delta^{(2)}(x - y)$. Such local terms do not contribute to the Zamolodchikov metric [29], either because one can always defines the latter at large scales $|x| \to \infty$, or else because such local terms are subtracted to ensure the renormalization-group invariance of the pertinent two-point function. Details have been given in ref. [29]. As we have argued there, one can always find a class of renormalization schemes for which the generating functional of connected string amplitudes $\Gamma[g]$ can be transformed to the Zamolodchikov function $C[g]$. This can be achieved by redefinitions that involve only the metric $G_{ij}$ and not the connection in coupling-constant space. Thus, such schemes are always operational in the ‘normal coordinate’ system $\frac{i}{2}$ in string theory space. Therefore, (37) is consistent with (10) if and only if the choice (38) is satisfied.

This completes our discussion of the conformal invariance of (29). In turn, this justifies the path-integral ansatz (28): the latter can indeed be regarded as a generalized $\sigma$ model with quantized couplings, as suggested previously [4].

As a corollary to the above proof, we remark that if we complexify the coupling constant space, as is appropriate for the $N = 2$ superconformal theories of interest to string theory with non-trivial duality symmetries [30], then (38) implies a Kähler form for the metric (37). As we have seen above, this Kähler form of the Zamolodchikov metric appears consistent with its standard definition in terms of the divergences of two-point functions which are not removed by conventional renormalization. This will be important for section 5, when we shall present arguments in favour of a fundamental rôle played by two-dimensional target-space strings in the path integral (28).

We close this section with a few important comments. We stress once again that the tensor $G_{ij}$ is not, in general, the same as the Zamolodchikov metric: $G_{ij} \propto \chi_{ij}$. Rather, the renormalization-group derivative of $G_{ij}$ coincides with $\chi_{ij}$ in certain schemes. In our quantum version of the theory space, it is $G_{ij}$ that appears as the ‘metric’ of the $\sigma$-model on the manifold $g^i(x)$. Thus, the effect of the quantum corrections due to higher genera is that the ‘string’ should be regarded as a particle described by a $\sigma$ model with target space metric $G_{ij}$ and not $G_{ij}$. One can argue, though, that there exists a class of renormalization schemes - compatible with canonical quantization of the coupling/fields $g^i$ - in which the Zamolodchikov metric has the structure

$$\chi_{ij} = G_{ij} = Q^2 G_{ij} + \ldots$$

(41)

where $Q^2 = \frac{1}{3}(C[g] - 25)$, with $C[g]$ the Zamolodchikov $C$-function, which in the renormalization scheme chosen above is equivalent to the effective action of the low-

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\(^2\)The subtleties with the moduli fields in certain target-space supersymmetric string theories must be dealt with in the way described above.
energy field theory. The ... denote terms that either refer to moduli fields (exactly marginal deformations) of the string, or to antisymmetric tensor backgrounds, which drop out of the symmetrical counterterm involving $G_{ij}$ in (28) (c.f. (12) below). The moduli-dependent terms are important to ensure passage from one background to another in a smooth way, but their details can be ignored for our present purposes. The relation (11) follows from a combination of the off-shell corollary (13) of the $C$-theorem [26], which allows an identification of $\chi_{ij}$ with $G_{ij}$, the relation (33), and the conditions (26,27) for canonical quantization. We argue in the next section that one finds in this class of schemes consistent string soliton solutions, which need sources for their support [8]. The latter solutions play an important rôle in understanding the structure of string theory, especially from the point of view of recent developments in the context of duality symmetries [21].

4 D branes and solitons in string field theory

We now show how the above path-integral framework (28) leads to an action of the form (1), in which a target-space string effective lagrangian is combined with a first-quantized world-sheet action, the latter describing the coupling of the string background to a test string source [8]. We start by considering the local counterterm

$$\Delta S = \int_{\Sigma} d^2 \sigma \partial_\alpha g^i G_{ij} \partial^\alpha g^j$$

(42)

in the exponent of (28). As discussed in the previous section, this arises from the consistent renormalization of the $\sigma$-model action on a curved world sheet, in the context of the summation over higher genera. This approach [26] relied on couplings $g^i$ becoming local functions of the world-sheet variables $z, \bar{z}$. This dependence was allowed to be arbitrary in ref. [26], whereas it was taken to be through the Liouville mode [1, 27] in the first-quantized approach to the Liouville string. However, when one proceeds to second quantization of the Liouville string, the $g^i$ also depend on $z, \bar{z}$ via the target-space $\sigma$-model fields $X^M(z, \bar{z})$. This dependence is in addition to any target-space zero-mode dependence included among the indices $i$ of the $\sigma$-model couplings $g^i$.

We use this implicit dependence on the world-sheet coordinates via the target-space coordinates $X^M(z, \bar{z})$ to rewrite the counterterm (12) in the path integral

3In fact, the dimensionality of the $X^M(z, \bar{z})$ may be different from that of the original target space we started with, and in this way one can dynamically generate target-space dimensions starting from low-dimensional target-space strings, e.g. the two-dimensional black hole [6]. More discussion on this point will be presented in the next section, in connection with a conjectural fundamental rôle of two-dimensional target-space strings in the path integral [28].
We use now the fact that the metric $G_{ij}$ is related through (41) to $\chi_{ij}$, which is connected in a given scheme $[26, 29]$ to the coincidence limit of the Zamolodchikov metric $G_{ij}$, which is in turn related to two-point functions of vertex operators in the generalized world-sheet $\sigma$ model. Applying the $\sigma$-model equations of motion, we can rewrite (43) in the form

$$\Delta S = \int d^2\sigma \partial_\alpha X^M \partial^\alpha X^N \frac{1}{Q^2} \frac{1}{Q^2} \frac{1}{Q^2} \frac{1}{Q^2} < \partial^\alpha J_\alpha^M (z, \bar{z}) \partial^\beta J_\beta^N (w, \bar{w}) > (44)$$

The currents $J^M$ appearing in (44) are the same Noether currents as in (11), which have the unconventional OPE (13) in a soliton background, which we relate in turn to the finite Zamolodchikov metric (14). By virtue of (13), then, the expression (44) may in turn be re-expressed in terms of the target-space metric $G_{MN}$

$$\Delta S \propto \int d^2\sigma \partial_\alpha X^M \partial^\alpha X^N G_{MN} (X) (45)$$

The absence of explicit logarithmic divergences in (45) is consistent with the conformal-invariance conditions (35) of the string-theory-space path integral (28). Notice that this construction works only for string backgrounds with logarithmic currents. For conventional currents $J^M$ satisfying the non-logarithmic OPE (15), the unrenormalized Zamolodchikov metric $G_{ij}$ vanishes logarithmically [16], and hence there is no associated contribution to the path integral (28).

This discussion can be extended to antisymmetric-tensor backgrounds, which are also of interest for string solitons and membranes [31]. In their presence, there are additional structures which lead to an antisymmetric tensor coupling in (45), and the relevant renormalization-group counterterms in (28) take the form

$$\int d^2\sigma \epsilon^{\alpha\beta} \partial_\alpha g^i \partial_i W_j - \partial_j W_i) \partial_\beta g^j (46)$$

The counterterm (46) is dictated by the $\sigma$-model renormalization group relations (13), (34), which imply that the metric in theory space of the stringy $\sigma$-model contains a torsion (antisymmetric) part. Replacing $\partial_\alpha g^i$ by $\partial_\alpha X^M \frac{\delta g^i}{\delta X^M}$, with the $X^M$ depending on the world-sheet coordinates, one obtains an antisymmetric-tensor source term at the point $X^M$ in the effective target-space action

$$\int d^2\sigma \epsilon^{\alpha\beta} \partial_\alpha X^M \partial_\beta X^N B_{MN} (X)$$

$$B_{MN} = \partial_N \Lambda_M (X) - \partial_M \Lambda_N (X) \equiv \frac{\delta W_i}{\delta X^M} \frac{\delta g^i}{\delta X^N} \equiv \frac{\delta W_i}{\delta X^M} \frac{\delta g^i}{\delta X^N} (47)$$

For regular string background fields $g^i$, such terms do not lead to non-trivial source terms, as the corresponding antisymmetric tensor fields can be gauged away. However, for points in $g^i$ space that have singularities at $X^M$ (e.g., solitonic string
backgrounds such as black holes), \(\Lambda(X)\) may be singular at \(X\), thereby leading to a non-trivial field strength for the antisymmetric field \(B_{MN}\) defined as in (47). The presence of logarithmic operators in such backgrounds guarantees the smooth character of the antisymmetric-tensor coupling at the source point \(X\), in a way similar to the graviton coupling (15).

Finally, we also observe that terms in the path integral (28) that couple to the world-sheet curvature \(R^{(2)}\) through ‘dilatons’ \(\Phi[g]\) in theory space \(\{g^I\}\) lead to a dilaton \(\Phi(X)\) contribution to the target space effective action, \(C[g]\), and hence to the string-source \(\sigma\)-model action in order \(\alpha'\).

Combining the above results, we find that, in a low-energy effective-action treatment of \(C[g]\), the complete exponent of (28) in a soliton background can be written as

\[
\hat{S} = C[g] + \text{string source terms} = \\
\frac{1}{g_s^2} \int d^D X \sqrt{G} e^{-2\Phi} [R + 4(\nabla \Phi)^2 - \frac{1}{12} H_{MNR}^2 + O(\alpha')] + \\
\frac{1}{4\pi \alpha'} \int d^2 \sigma [\partial_{\alpha} X^M \partial^\alpha X^N G_{MN}(X) + \epsilon^{\alpha\beta} \partial_{\alpha} X^M \partial_{\beta} X^N B_{MN} + \Phi(X) R^{(2)}] 
\] (48)

where the metric \(G_{MN}\) is the \(\sigma\)-model metric, \(g_s\) is the string coupling constant

\[
g_s \equiv e^{<\Phi>} 
\] (49)

and \(\chi\) is the Euler characteristic of the world-sheet manifold. For world-sheets with the topology of the sphere, \(\chi = 2\), whilst for those with the topology of a disk, as appropriate for a theory containing open strings in its spectrum, \(\chi = 1\). Note that the appearance of the string coupling constant in front of the effective-action part of (18) is a result of the non-trivial \(Q^2\) normalization factor appearing in (11), which enters the relevant expression for the counteterm (12). Eq. (18) describes the action for a macroscopic string (soliton) solution coupled to a string source \(\Phi\). In order to pass to a renormalization scheme where the Einstein term in the target-space effective action assumes the canonical form (1), one has to rescale the metric \(G_{MN} \rightarrow e^{-\frac{2\Phi}{g_s^2}} G_{MN} \equiv g_{MN}\). In terms of \(g_{MN}\), the action assumes the form (1). It is a highly non-trivial fact that solutions to the above coupled system of world-sheet and target-space objects are found. From our point of view such solutions correspond to stationary points of the path integral in string ‘theory space’ (28).

The explicit appearance of the string coupling constant in (18) motivates the definition of a new scale parameter

\[
\alpha'_s \equiv \alpha' / g_s^\chi 
\] (50)

Using a system of units based on the new fundamental scale \(\alpha'_s\) (50), the natural cut-off scale that appears in the low-energy target-space effective action in (1) becomes
\( \alpha'_s g_s \). In the case of \( D \) branes \([32]\), which are associated with open strings, \( \chi = 1 \), and the new scale is \( \alpha'_s g_s \).

For a four-dimensional string, the scale \( \alpha'_s g_s \) appearing in the rescaled target-space action should be identified with the Planck scale of quantum gravity. Such a scale, involving the string coupling constant, has been conjectured to exist in quantum solitonic string theory and in \( D \)-brane theory \([33]\), and has also appeared in the two-dimensional stringy black-hole example of ref. \([1]\), where it has been related to the entropy of a decaying stringy black hole \([34]\). We remark that the flat-space scattering of soliton strings and \( D \) branes, both systems characterized by a no-net-force condition, cannot probe scales shorter than \( \alpha'_s \) \([35]\). Thus, it is still not known how one can probe scales of order \( \alpha'_s g_s \) using weakly-coupled strings with \( g_s < 1 \). However, it may well be that such scales are probed by superscattering \( S \)-matrix elements of soliton strings and \( D \) branes in highly-curved (black-hole) target space times. In this respect we note that the field redefinition of the metric field which allows a passage from (48) to (1) has been made in an essentially off-shell effective action. Usually, such local redefinitions leave the on-shell \( S \)-matrix elements invariant, and therefore do not affect the low-energy physics. However, in the presence of a string source or macroscopic string, and in general in string backgrounds such as black holes, etc., where an \( S \)-matrix cannot always be defined, local redefinitions may well affect the physics, as a result of the off-shell character of the target space action in \([1, 36]\).

At this point we address the key question how open string states appear in our formulation of quantized string field theory \([28]\), which would allow \( D \)-brane structures \([32]\) to emerge. To see how the approach leading to \([28]\) naturally incorporates world-sheet boundaries, which are associated with open string states, we consider world-sheet dependences of the couplings \( g^i \) such that \( \partial_\alpha g^i_\alpha = 0 \) in some regions of the two-dimensional surface appearing in the integrand of \([28]\). In such cases, the imposition of appropriate conditions on the boundaries separating these regions from the rest of the Riemann surface leads to interesting saddle-point solutions of \([28]\) such as \( D \) branes \([32]\). The reason is that in the presence of a hole in the world sheet there is a boundary contribution coming from the counterterm \([12]\)

\[
\int_{\partial \Sigma} < g^i V_i \partial^\alpha J_{\alpha, M} > \partial_n X^M \tag{51}
\]

where \( \partial_n \) denotes derivatives normal to the world-sheet boundary \( \partial \Sigma \). On the other hand, from the torsion counterterms \([46]\) one gets

\[
\int_{\partial \Sigma} < g^i \partial_i W_{ij} \frac{\delta g^j}{\delta X^M} > \partial_\tau X^M \tag{52}
\]

where \([\ldots]\) denotes antisymmetrization of the respective indices, and \( \partial_\tau \) denotes derivatives tangential to the boundary \( \partial \Sigma \). By appropriate choice of the functions \( X^M \) on the boundary, one may induce in this way the \( U(1) \) gauge fields that appear
in $D$-brane theories $^{32}$. For instance, in Type II superstring theories, if one chooses Neumann boundary conditions for some coordinates of the target manifold, $X^i$, while the rest, $X^I$, obey fixed Dirichlet $^{32}$ boundary conditions, the term (51) leads to transverse excitations of the $D$ brane, which are represented by insertions of the operator

$$\int_{\partial \Sigma} A_I \partial_n X^I$$

(53)

On the other hand, the term (52) leads to longitudinal excitations, corresponding to an internal gauge field associated with the gauge invariance of the antisymmetric tensor field: $B_{MN} \rightarrow B_{MN} + \partial_{[M} A_{N]}$:

$$\int_{\partial \Sigma} A_i \partial_\tau X^i$$

(54)

The gauge field operators (53,54) induced in this way may then be absorbed in redefinitions (scheme choices) of target-space $U(1)$ gauge fields, which appear naturally in any theory involving open strings$^{4}$.

The construction above makes evident the similarity of $D$ branes to string solitons, and $D$ branes have indeed been viewed in the literature $^{32, 38}$ as sources for closed string fields obeying the standard vanishing $\beta$-function conditions of critical string theory outside the $D$ brane. A fixed boundary is analogous to a fixed string source, as studied above, as far as the breaking of certain target-space symmetries is concerned. By an appropriate choice of boundary conditions $^{32}$, the whole boundary conformal field theory is mapped onto a single point in target space $\{X\}$. However, the integration over boundaries (i.e. over $\{X\}$) in (28), dictated by the renormalization-group approach $^{26}$, should restore these symmetries. Logarithmic operators should also play a rôle in this case, by analogy with the restoration of translational or diffeomorphism invariance through soliton-recoil/back-reaction effects in the case of macroscopic strings $^{14, 15}$ or stringy black-holes $^{11, 39}$. Indeed, while this paper was close to completion, an interesting work $^{40}$ appeared in which logarithmic operators describing $D$-brane recoil have been explicitly constructed in the spirit of ref. $^{10}$. For instance, in the simplified example of a 0 brane (point particle) discussed in ref. $^{40}$, the logarithmic operators describing recoil are given by the boundary contribution

$$V_{\text{recoil}} = v_I \int d^2 \sigma \partial_\alpha (X^0 \Theta(X^0) \partial^\alpha X^I)$$

(55)

where $v_I$ is the (center-of-mass) velocity of the 0 brane, the position of its centre of mass as a function of time $X^0$ being $v_I X^0$, and $\Theta(X^0)$ is the Heaviside step function. The time $X^0$ obeys the standard Neumann boundary conditions on the boundary of the world sheet, whilst the $X^I$ obey fixed Dirichlet boundary conditions, corresponding to the location of the 0 brane, regarded as a collective coordinate.

$^{4}$It is to be understood that one uses in such cases the formal technology of $\sigma$ models with boundaries $^{37}$, appropriately extended in the framework of $^{28}$. 
According to the analysis of ref. [40], the operator (55), viewed as a deformation of the 0 brane on the disc, cancels logarithmic divergences of amplitudes for matter excitations in the 0-brane background on the annulus, in complete analogy to the closed-string soliton case, discussed in ref. [10] and in section 2. This interpretation of (55) as solitonic recoil passes the consistency check of yielding a soliton mass $\propto 1/g_s$, where $g_s$ is the string loop coupling constant. Moreover, it is consistent with the above unified approach to strings and D branes from the non-critical-string path integral (28) point of view. The deformation (55) has the general form of the contribution (53) to the D-brane boundary state [38], and may be thought of as corresponding to a Lorentz boost with time $X^0$. In our approach in which the Liouville field is interpreted as time [1, 3, 4], the step function appearing in (55) is a reflection of the monotonic flow that results from the semigroup nature of the renormalization group [1, 5].

Further formal evidence for the consistency of such an identification is provided by the exact flat-space renormalization-group equation for the wave functional of the D brane, postulated [11] on the basis of Polchinski’s extension of the Wilsonian approach [12]. Let $\Psi[X^I]$ be the wave functional of the D brane, which is related by $Z = \int dX^I \Psi[X^I]$ to the partition function $Z$ of the first-quantized version of the D brane, formulated on world sheets with boundaries and resummed over genera. The wave functional is, thus, given by [11]

$$\Psi[X^I] \sim e^{S_D} ; \quad S_D \equiv \sum_{h=1}^{\infty} g^h s_s S_{(h)}$$

where $h$ denotes the number of holes (the sum over handles has been suppressed in the above notation), and $S_D$ sums up all one-particle irreducible connected world-sheet amplitudes $S_{(h)}$ whose boundaries are mapped onto the world-volume of the D brane.

The formal exact renormalization-group equation for a D brane in flat space reads [11]

$$\epsilon \partial_\epsilon \Psi[X^I] = \frac{1}{2} \int_{\partial \Sigma} \int_{\partial \Sigma} \epsilon \partial_\epsilon G_{IJ}[X^I, X^J] \frac{\delta^2}{\partial X^I \partial X^J} \Psi[X^I]$$

where $\epsilon$ is the world-sheet renormalization scale, related, for example, to the degeneracy of handles as discussed in section 2, and $G_{IJ}[X^I, X^J]$ denotes the two-point function of the $\sigma$-model fields $X^I$, obeying Dirichlet boundary conditions. We expect, on the basis of our analysis in section 2, that such two-point functions contain $\log \epsilon$ divergencies due to the zero-mode contributions

$$\operatorname{Lim}_{\epsilon \to 0} G_{IJ}[X^I, X^J] \sim -|\overline{X}'|^2 \delta^{IJ} \log \epsilon$$

where the $\overline{X}'$ denote c-number zero modes of the transverse coordinates, which are independent of the world-sheet coordinates of the D brane. For instance, in the
case of the 0 brane discussed in ref. [40], it is straightforward to verify (58) by computing the two-point function of the operator (55) describing translation of the transverse coordinates of the 0 brane. Concentrating on the $X^0$ part, sufficient for demonstrating (58), one obtains for the zero-mode contributions

$$<v_I X^0 \Theta(X^0) v_J X^0 \Theta(X^0)>_{\text{zero mode}} \sim |v_I|^2 |\delta_{IJ}| \Theta(X^0)(z = i \frac{1}{2} \epsilon)^2 = |v_I X^0 \Theta(X^0)|^2 \delta_{IJ} \left( \int \frac{dr}{r^2} e^{-r^2} \right)^2 \sim -|v_I X^0 \Theta(X^0)|^2 \delta_{IJ} \log \epsilon$$

(59)

where we have used an integral representation for the $\Theta$ function, and taken into account the fact that, near the boundary of the world sheet $z = i \frac{1}{2} \epsilon$, the v.e.v of the recoil operator (55) is divergent [40] as a result of the zero modes, and needs to be regulated by a proper-time cut-off $\epsilon$.

Equipped with (58), equation (57) can be verified by a direct computation of the infinities of the annulus amplitudes due to the massless zero modes (logarithmic operators) [10] associated with the translation of the transverse coordinates of the $D$ brane and related to recoil [10, 10]. Such divergences are target-space infrared in nature, as we mentioned in section 2 and in the above discussion, and are associated with bilocal operator insertions on the disk amplitudes of the translation operator (53):

$$V_T \equiv \int_{\partial \Sigma} A_I(X^0, \ldots X^p) \partial_n X^I = A_I \frac{\partial}{\partial X^I}$$

(60)

for a $D$ brane with $10 - p$ collective coordinates, i.e., with $p$ coordinates $X^0, \ldots, X^p$ obeying Neuman boundary conditions. To leading order in the string coupling constant $g_s$, only the disk and annulus amplitudes contribute. From (56), (60) it is, then, immediate to write down the cut-off dependence of the $D$-brane wave function boosted with velocity $V$ [41]

$$\Psi_V[X^I] = e^{-\log \frac{1}{2} g_s \nabla_X^2} \Psi[X^I]$$

(61)

which is consistent with the identification of $-\log \epsilon$ with a Euclideanized target time $T$. Equation (61) is an approximate expression which yields a diffusion-like equation of motion for the wave functional of the $D$ brane [41]

$$\frac{d}{dT} \Psi_V = -\frac{1}{2M_D} \nabla_X^2 \Psi_V$$

(62)

where $M_D \propto 1/g_s$ is the $D$-brane mass.
However, the above analysis is restricted to a $D$ brane moving in flat target space, and is modified in an essential way when curved backgrounds are taken into account, as we now discuss. We emphasize first that, in the presence of curved target-space backgrounds, not all boundary conditions are compatible with conformal invariance. For instance, in the presence of a linear-dilaton background [3] fixed Dirichlet boundary conditions are not conformal invariant [32]. To restore conformal invariance, one needs to incorporate certain extra boundary operators [43]. From our non-critical string approach, this phenomenon is compatible with the quantum string nature of the $D$ branes. The modification of the boundary conditions to restore conformal symmetry is the analogue of Liouville dressing in our non-critical string approach [1].

A possible connection between such boundary modifications and logarithmic operators in the boundary, that induce interactions with the bulk, might be anticipated in view of the above discussion. This is in close analogy to the case of chiral edge currents of the Hall systems in condensed matter physics [44]. There, the presence of a boundary condition on the edge current induces extra local interactions of $\delta$-function type in the potential of the quantum-mechanical Schrödinger equation. Such interactions exhibit non-trivial renormalization-group scaling properties [44]. A rigorous formulation of such issues in the context of (28) is under study at present.

Nevertheless, from the discussion above, we can infer some useful properties of the time evolution of $D$ branes with fixed Dirichlet boundary conditions in such non-conformal curved backgrounds. We take the point of view that, instead of modifying the boundary conditions so as to ensure conformal invariance [43], one should rather view this type of problem with fixed non-conformal boundary conditions on the collective coordinates of the string soliton as an effective non-critical string problem requiring dressing by the Liouville field, interpreted as a local renormalization-group scale to be identified with target time [1, 5]. In this way, we derive the correct equation of motion of the $D$ brane wave function in the spirit of our picture (28), which incorporates $D$ branes as saddle-point solutions of ordinary string theories in soliton backgrounds. Indeed, a generic $D$-brane world-sheet action can always be viewed as an open-string action in appropriate gauge-field backgrounds, with all the coordinates of the string obeying standard Neumann boundary conditions [32, 35]. In this interpretation, the two terms in (53,54) can be interpreted as standard open-string gauge background fields, upon the replacement of the normal derivative $\partial_n X^I$ by a tangential derivative $\partial_\tau X^I$, with the $X^I$ obeying standard Neumann boundary conditions. The formal reason for this equivalence is that the following relation is valid on the world-sheet boundary:

$$<\partial_\tau X^I(\tau)\partial_\tau X^J(\tau')>_{\text{Dirichlet}}=-<\partial_n X^I(\tau)\partial_n X^J(\tau')>_{\text{Neumann}}=\frac{2\alpha'\delta^{IJ}}{(\tau-\tau')^2}$$

(63)

Using the abovementioned equivalence, the target-space effective action of the $D$ brane on a flat target space is then given by the Born-Infeld Lagrangian [15]

$$\mathcal{L}_{\text{BI}} \propto \sqrt{-\det(\eta_{\mu\nu}+2\pi\alpha'F_{\mu\nu})}, \quad \mu, \nu = 1, \ldots 10$$

(64)
where $\eta_{\mu\nu}$ is a Minkowski target-space flat metric, and $F_{\mu\nu}$ is the field strength of the background (Abelian) gauge field appearing in (53,54). Going back to the Dirichlet picture, the above string effective action yields the standard Nambu-Goto world-volume action of a $p$-brane [32, 33], expressed in terms of Dirichlet coordinates:

$$L_{p-\text{brane}} \propto \sqrt{-\det(\eta_{\alpha\beta} + \partial_\alpha X^M \partial_\beta X^M)} \quad ; \quad \alpha, \beta = 1, \ldots, p \quad M = p+1, \ldots, 10$$

(65)

where $\eta_{\alpha\beta}$ is the flat world-volume metric.

In the case of curved backgrounds for the world volume, conformal invariance may be lost, as discussed above [43, 32] for the fixed Dirichlet case, in the sense that non-conformal deformations of the $\sigma$ model appear. In the string picture, the effective Born-Infeld Lagrangian depends also on the gravitational target-space metric field, and in general on non-conformal background fields $g^i$. As in our two-dimensional black-hole picture [1], criticality is restored by Liouville dressing. The deviation from criticality in a curved background is parametrized by by $Q^2 \propto C[g] - 25$, where the Zamolodchikov function exceeds $C[g] > 25$, so that the theory is critical and the Liouville field $\phi$ has negative time-like metric. As discussed elsewhere [1, 46], this approach leads to a second-order renormalization-group equation for the Liouville-dressed couplings $\lambda^i(\phi)$ [1]:

$$\ddot{\lambda}^i(\phi) + Q \dot{\lambda}^i(\phi) = -\beta^i(\lambda)$$

(66)

where the dots denote differentiation with respect to $\phi$, and the $\beta^i$ are the flat-world-sheet $\beta$-functions. Notice that the friction term proportional to $Q$ in (66) can be absorbed into a ‘mass-shift’ for the mode $\lambda^i$ [3] but, contrary to the case of ref. [3], the mass is real here, despite the Minkowskian signature of the field $\phi$ when interpreted as target time [1]:

$$[\partial^2_\phi - \frac{1}{4}Q^2]e^{-\frac{1}{2}Q^2} \lambda^i = \beta^i(e^{-\frac{1}{2}Q^2} \lambda^i)$$

(67)

Coming now to our main objective, namely the derivation of an equation of motion for the $D$-brane wave functional $\Psi[X^I]$, in our interpretation of time as a Liouville renormalization-group scale, we observe that, because of the abovementioned equivalence (54,55) of the $D$ brane to an open string in background fields, one can represent $\Psi$ as

$$\Psi[X^I] = e^{-C[\lambda]}$$

(68)

where the Zamolodchikov function $C[\lambda]$ is the generating functional of connected Green functions, which depends on the various non-conformal backgrounds. In the dual $D$-brane picture it corresponds to $S_D$ in (54). The source of the violation of conformal invariance is the soliton background, which is characterized by the Dirichlet boundary conditions of its collective coordinates or, equivalently in a closed string framework, by the presence of a conformally-non-invariant macroscopic string source [1], as in the discussion of this paper. Let us then denote by $Q^2$ the effective
vacuum energy of the soliton, which will include its mass $M_D$. It is known in the non-critical string picture that the effective central charge obeys a second-order equation near a fixed point:

$$
\ddot{C}[g, \phi] + Q[g, \phi] \dot{C}[g, \phi] = -\beta^i \frac{\partial}{\partial \lambda^i} C[\lambda] + O[\dot{\lambda}^2 \frac{\partial^2 C[\lambda]}{\partial \lambda^i \partial \lambda^j} \dot{\lambda}] \leq 0 \text{ for } C \geq 25 \quad (69)
$$

where we have made use of the conventional flat-world-sheet $C$-theorem [2, 1, 46]: $-\beta^i \frac{\partial}{\partial \lambda^i} C[\lambda] \leq 0$. From (68), (69), it is then straightforward to generalize the renormalization-group equation (57) to the Liouville case, and derive an evolution equation for the wave functional of the $D$ brane:

$$
\left[\partial^2_{X^0} - \frac{1}{4} Q^2 - \nabla^2_x - \ldots\right] e^{-\frac{1}{2} Q^2 X^0} \Psi = 0 \quad Q^2 \equiv M_D \propto \frac{1}{g_s} \quad (70)
$$

where $X^0 \equiv \phi/Q$ is a covariant renormalization-group scale, which behaves as a Minkowskian evolution parameter, and may thus be identified with the target time [1, 3], and the dots denote possible additional terms including interactions among the $D$ branes, which are not relevant for our purposes here. The above ‘derivation’ of (70) exploits the equivalence of $D$ branes to some closed solitonic string backgrounds, which follows from our path-integral ansatz (28). In this respect, equation (70) is a direct consequence of the fact that the $D$ branes are nothing other than saddle-point solutions of (28). In standard $D$-brane/string language, this equivalence arises via certain duality symmetries [32]. It would be interesting, as a further consistency check of our approach, to derive (70) in an independent way, extending the derivation of (62) [41] directly to the case of curved world-brane volumes.

We close this section with some comments on previous attempts to derive the action (48) from standard $\sigma$-model theory. It has been argued, correctly, in ref. [47] that the action (48) can be considered the result of world-sheet resummation in $\sigma$-models. The non-conformal invariance of the $\sigma$-model involved is traced back to the loop corrections to the $\beta$ functions. In particular a heuristic derivation of this action has been given by attaching a world-sheet wormhole operator (2) to a $\sigma$ model on a Riemann surface of lower genus. This is similar in spirit to our analysis above. The crucial difference in our approach, however, is the proper incorporation of recoil effects on the soliton background via logarithmic operators on the world sheet [1, 33, 10]. In our opinion, this provides a more complete treatment of the problem. As we have seen above, in the version (28) of string-theory-space quantization which we advocate in this work, derived from a computation of such recoil/back-reaction effects in a $\sigma$-model language, the crucial source/background coupling terms for string solitons originate from a particular renormalization-group counterterm (12). This term is also responsible for a dynamical determination of the metric in theory.

\footnote{In the presence of other background deformations, there will be other background-dependent terms.}
space via the conditions (35). Such terms have not been considered in the previous literature on the subject [47]. Although our derivation above was formal, it has the merit of providing an explicit demonstration that membrane theories are intimately connected with a quantum target-space string theory. Moreover, our quantization scheme (28), which is consistent with the non-critical nature of the (quantum) strings involved and our identification of time in string theory with the Liouville mode [1], allows for a proper transition between the quantum and classical worlds in target space-time, via decoherence effects associated with the solitonic backgrounds [18].

5 Application to Black Holes and Information Loss

Basing our arguments on the two-dimensional black-hole prototype, we suggested some time ago that the Hawking-Bekenstein entropy of a black hole could be understood as the number of string states [34], in agreement with the previous suggestion of ref. [48] based on string duality. This suggestion has been repeated recently in the context of $D$-brane studies, where the black-hole entropy may be interpreted as the number of open string states that terminate on the horizon [12]. We have gone further, and argued that, to the extent that these black-hole string states are unobserved, even unobservable, they necessarily lead to microscopic information loss [1]. The amount of this information loss was related, in our interpretation, to the loss of internal $W_\infty$ quantum numbers carried by the two-dimensional stringy black hole [1, 19, 50]. In this section, we use the development of the previous sections to discuss further aspects of the $D$-brane approach to black hole information problem, indicating how features of our previous two-dimensional analysis may be carried over into the $D$-brane approach.

We start by recalling that open strings also appear naturally in the two-dimensional black-hole model, when one takes correctly into account its asymptotic space-time geometry [11]. The reason is that, like all maximally-extended Schwarzschild geometries in General Relativity, the original two-dimensional black-hole model contains two asymptotically-flat domains. These may be reduced to a single asymptotic domain by applying the familiar orbifold technique of modding out by a $Z_2$ factor [11]. This construction leads to open unoriented string states attached to the orbifold singularity at the origin. In this two-dimensional model, such states are necessarily discrete delocalized states, and therefore cannot be detected in local scattering experiments. This exemplifies the loss of information to which we have drawn attention previously [1]. An interesting duality exists [11] that maps the model to a theory of open strings in a black hole background.

The above discussion refers to the space-time singularity at the core of the black hole. As has been discussed elsewhere, this singularity may be represented as a topologically non-trivial world-sheet monopole configuration [51, 11]. In this picture, it has been argued [11] that the horizon of the black hole may also be represented
as a world-sheet (anti)monopole. The picture outlined in the previous paragraph therefore implies that the horizon of the two-dimensional black hole should also admit open-string states [6] in accord with the intuitive picture of ref. [54], according to which a closed string state falling into the horizon of the black hole may lie partly inside and partly outside the horizon.

There are similarities, but also important differences, between our previous two-dimensional (2D) picture and this emerging higher-dimensional (HD) one, which deserve attention. In the 2D picture, the open-string states are discrete [11] and non-propagating, and carry the gauged $W_\infty$ ‘quantum hair’ of the black hole [1]. In the HD picture, one starts from a string living in some higher-dimensional space time with some dimensions compactified. This leads to modes [12] winding around a compact dimension, characterized by discrete momenta. In the dual description which is appropriate for a $D$-brane representation of the horizon of the black hole [12], such modes are associated with open-string states whose ends are confined to the $D$ brane, and also have discretized momenta. Such states may be either left- or right-movers. These two-dimensional substructures play an important rôle [12] in the excitation of the various black-hole states that correspond to a given set of macroscopic classical hair (mass, charge and angular momentum) [53]. Such excitations are obtained by considering the excitation of oppositely-moving pairs of such open strings, with their end points attached to the black-hole horizon [12]. This corresponds to the intuitive ‘splitting’ picture developed in ref. [54] for a closed string state falling inside the horizon of a string black hole. The counting of such states matches in a certain perturbative limit [12] the classical Hawking-Bekenstein law. The existence of discrete-momentum, essentially two-dimensional, structures on the horizon of the black hole, which are responsible for the degeneracy of the black-hole states, is very similar to the situation in the 2D picture of [6, 1, 11].

However, an important feature of the 2D picture, which in our opinion does not yet seem to have been take properly into account in the emerging HD picture, is the peculiar nature of the quantum hair carried by the discrete open-string states of the two-dimensional black hole. This is essential to any discussion of the maintenance or otherwise of quantum coherence in black-hole decay [1, 34]. Estimates of entropy production along the lines in ref. [12], although very important, do not address the nature of the precise mechanism of the information transfer from propagating to non-propagating (open-string) states of the ($D$-brane) black hole. On the other hand, two-dimensional strings are known to possess $W_\infty$ symmetries with a coherence-preserving property that maintains unitarity in the temporal evolution at the fundamental string level [1]. This admits a more-or-less conventional conformal-invariant world sheet description [1, 55], provided that one takes into account the discrete modes of the two-dimensional string.

---

[6] The explicit duality symmetry [12] that maps the singularity of the two-dimensional stringy black hole [6] onto the horizon offers further support for this picture.
However, the treatment of quantum-gravitational fluctuations necessitates further analysis when one considers the time evolution of a propagating closed-string mode in the presence of a stringy black hole [1]. Since a low-energy observer is restricted to making local measurements, which are insensitive to the non-local discrete modes which carry information via their $W_\infty$ quantum numbers, the time evolution in target space does not take the familiar unitary form. The analogies between the 2D and HD pictures outlined above, in particular the fact that two-dimensional stringy black holes [3] also admit [11] open string structures after orbifolding, suggests that analogous modifications of unitary evolution may also occur in the HD case. This motivates the study of open string states - especially their $W_\infty$ quantum numbers - that we carry out below.

Before embarking on this analysis, we first discuss how the non-critical two-dimensional black-hole string model may be embedded in a higher-dimensional theory within our approach. One may introduce observable matter fields as perturbations on the black-hole $\sigma$ model. One may also consider tensoring the black hole with an initially-decoupled flat-space-time non-critical string of the type introduced in [3]:

$$S_{nc} = \int d^2z [\delta_{MN} \partial X^M \partial X^N + QX^M R^{(2)}]$$ (71)

where $Q^2 = \frac{1}{3}(D-25)$ in the case of a bosonic string, which we consider for simplicity. Once the integration over the backgrounds $g^i$ in (28) is considered, the two theories couple to each other, and one encounters the typical Liouville-dressing problem of a non-critical string with a single Liouville coordinate. The latter represents Minkowski target time if the central charge of the theory is higher than 25 [3], as is the case of the two-dimensional black hole string perturbed by world-sheet instanton deformations [1]. These may be used to represent the integration over the global string modes of the string black hole which is necessarily made by any low-energy observer. The role played by two-dimensional strings is fundamental in this approach, since they characterize the global, non-propagating, string modes which are also present in higher-dimensional theories. Their presence creates an essentially ‘unobservable’ environment in quantum gravity theories [1], leading to the loss of microscopic loss of quantum coherence engendered by the loss of the $W_\infty$ charge information which we now discuss.

It is essential for this purpose to understand in more detail the nature of the hair carried by a black hole, extending our previous analysis of the $W$ hair that appears in the 2D case to the emerging HD treatment based on $D$ branes. This extension must take account of the special nature of the $W$ hair, which is associated with a gauged phase-space $W_\infty$ symmetry [26, 1]. The corresponding $W$ currents are non-local [57], and hence the $W$ charges resemble ‘quantum’ hair à la Aharonov-Bohm, rather than conventional classical gauge hair. Indeed, we have shown [1, 55, 50] how the $W$ hair is carried by discrete non-propagating modes in the 2D model, and how
it may in principle be measured by (an infinite number of) Aharonov-Bohm phase measurements.

To see how this picture may be reflected in the HD approach, we consider the physics of the black-hole horizon, viewed as a membrane with open strings attached to it. The effective action appropriate for an observer at infinity is given by (1), as derived from the string path integral (28) in Section 3, for the back-ground-field configurations appropriate for the description of a stringy black hole. In our approach, the string world sheet has two space-like dimensions, with target time appearing as the Liouville mode. There is therefore a one-to-one map between the lowest-genus world-sheet configuration and the black-hole horizon. The world-sheet of a string source can loop the horizon of the black hole [58], and this is a quantum effect from an effective target-space point of view. An open string appears as a handle attached to the horizon at defects in the world sheet, each of which we may surround by a small circle, whose radius we call $L$. The emergence of such defects from the path integral (28) appears consistent with the above picture. The physics of this boundary may be described by a $c = 1$ matrix model on a circle of radius $L$. This is motivated by the fact that in our approach we view the world-sheet of the string as representing a (smooth Euclidean) target space of another string theory [59]. This allows the representation of the world-sheet as a $c = 1$ matrix model, which is a theory of free fermions on a Euclidean two-dimensional space time [23]. One can introduce a ‘time’ for such configurations by identifying it with the Liouville mode of the full black-hole string theory that has Minkowskian signature. In this sense, the free fermions that live on the closed lines representing the defects constitute a ‘string theory’ living on a $(1 + 1)$-dimensional target space-time. Then, the whole picture, consisting of a world sheet and a Liouville mode, represents a membrane whose boundaries are 2d Liouville theories. Such constructions are known explicitly to exist [60], and are based on gravitational Chern-Simons theories on three-dimensional manifolds with boundaries. In this work we base our discussion on their mathematical consistency.

It has recently been argued [61] that such a $c = 1$ matrix model is equivalent non-perturbatively to a two-dimensional pure Yang-Mills theory of $U(N \to \infty)$ (or, under suitable restrictions, $SU(N \to \infty)$) type on a torus, with the large-$N$ limit appearing as $L \to 0$. The world-sheet boundary circles are closed circles in target space, with target time the second dimension. If $g$ is the gauge coupling constant, the corresponding $c = 1$ string coupling constant is $4/g^2LN$. We are indeed interested in the limit where the radius $L$ of the defect in the world sheet vanishes, which we combine with the large-$N$ limit so that the product $NL$ remains finite. The above discussion then tells us that a weakly-coupled string theory formulated on the boundary of the world-sheet defect is equivalent to strongly-coupled gauge field theory formulated at the ends of the open string.

The ‘tachyon’ of the Das-Jevicki Hamiltonian [62], which appears in the continuum limit of the $c = 1$ matrix model as a momentum mode of the $c = 1$ string living on
the degenerating $L \to 0$ world-sheet boundary, appears in the $SU(N \to \infty)$ gauge field theory as a discrete winding mode. This model may be described in terms of free fermions $\psi$ located at positions $\theta_i : i = 1, 2, \ldots, N \to \infty$ living on the $L \to 0$ circle, with Hamiltonian

$$H = -\left(\frac{g^2 L}{2}\right) \sum_{i=1}^{N} \frac{\partial^2}{\partial \theta_i^2}; \quad 0 \leq \theta_i < 2\pi \quad (72)$$

Fermionization is a consequence of the appearance of the Vandermonde determinant in the wave function: $\Delta \equiv \prod_{i<j} \sin \frac{1}{2}(\theta_i - \theta_j)$. One can, therefore, construct a second quantized effective field theory of $(72)$ which is that of the $c = 1$ matrix model in the free-fermion $\psi$ representation $[23]$. Bosonization yields the Das-Jevicki Hamiltonian $[62]$ on a circle.

In terms of the fermion fields $\psi$, the generators of the $W_{1+\infty}$ symmetry are found by first decomposing the fermion field on the circle into two oppositely-moving Weyl fermions $\chi^{(\pm)}$ which describe the relevant degrees of freedom in the vicinity of the Fermi surface of the model, as appropriate for an effective field-theory limit,

$$\psi(\theta, t) = \frac{1}{\sqrt{L}} [e^{ip_F(\theta-p_F t)}\chi^{(+)\prime}(\theta-p_F t)+e^{-ip_F(\theta+p_F t)}\chi^{(-)\prime}(\theta+p_F t)]; \quad p_F = \frac{\pi(N - 1)}{L} \quad (73)$$

with $p_F$ the Fermi momentum. For each chirality, the relevant generators of the associated $W_{1+\infty}$ symmetry are then given by $[23, 63]$ (we omit the chirality index $\pm$ for simplicity)

$$V^m_n = \int_0^{2\pi} d\theta \chi^\dagger(\theta) : e^{-i\theta} (i\partial_\theta)^m : \chi(\theta) \quad (74)$$

where $: \ :$ denotes normal ordering, which satisfy the commutation relations of a quantum $W_{1+\infty}$ algebra with central extensions $[64]$

$$[V^i_n, V^m_m] = (jn - im) V^{i+j-1}_{m+n} + q(i, j, m, n) V^{i+j-3}_{m+n} + \ldots + \delta_{m+n, 0} c(n, i, j) \quad (75)$$

Above, the functions $q$ represent quantum corrections $[14]$, and $c$ denotes the central extensions.

The $SU(N \to \infty)$ Yang-Mills gauge theory, which the above free-fermion problem $[72]$ is equivalent to $[61]$, is the counterpart in configuration space of the $W_\infty$ symmetry $[23]$ of the two-dimensional strings, which may be viewed as a gauge symmetry in the two-dimensional phase space $[56]$. The appearance of the $SU(N)$ symmetry allows us to introduce more-or-less conventional Chan-Paton factors at the ends of the open string, where it couples to the horizon membrane. This procedure may be viewed, in some sense, as the $D$-brane extension of the fermionic representation on the world-sheets boundary of the conventional Chan-Paton factors of open strings $[65]$. These $SU(N \to \infty)$ gauge charges are the counterpart of the $W_\infty$ charges $[66]$ which we identified in the $2D$ model as playing a key role in the black-hole information problem $[1]$. 
In the 2D case, we have argued that these $W_\infty$ charges label the different black-hole states, and that the inability of a low-energy observer performing local experiments to measure these $W_\infty$ charges is responsible for the loss of information in a black-hole environment. We have, moreover, related [1] the loss of information directly to the “leakage” of $W_\infty$ charge to unobservable modes in our effective non-critical string treatment of low-energy dynamics. The above analysis demonstrates explicitly how this picture may be translated into the emerging HD picture. It has been pointed out [12] that the entropy of the black hole may be related to the multiplicity of open-string states living on the horizon surface, and we have demonstrated above that these carry the HD analogues of the $W_\infty$ charges. We believe that a full treatment of the low-energy dynamics outside the black-hole horizon will confirm the loss of information through “leaks” of the black-hole quantum numbers via these open string states.

We conclude this section with a brief comment on the treatments of the black-hole singularity within the 2D and HD approaches. It is thought that the black-hole singularity has a topological nature, and there is indeed an explicit topological description of the singularity in the 2D case in terms of a model with twisted $N = 2$ supersymmetry [6, 67]. This model can again be mapped into a $c = 1$ string theory on a circle, which may also be described in terms of fermions, interacting this time via potential terms. There is again a duality symmetry that maps this model into a $U(N)$ gauge theory with fermionic matter in some representation $R$ which is related to the string-model interactions. Consider, for example, the simplified case of a $U(N)$ gauge theory with a heavy colour source, represented by a time-like Wilson line, whose action is [68]:

$$\frac{1}{4} \int d^2 x F_{\mu \nu}^2 + \int dt \overline{\Psi}(i \partial_t - g A_0(x = 0)^a T_R^a + M) \Psi$$

(76)

where $M$ is the mass of the heavy colour source, and the $T_R^a$ are the generators of the gauge group $U(N)$ in the representation $R$. This problem can be mapped into a $c = 1$ string theory which may be written in terms of interacting fermions $\psi_i$, located at positions $\theta_i, i = 1, 2, \ldots N$ on a circle, with Calogero-Sutherland type interactions [68]:

$$H = -\frac{g^2 L}{2} \sum_i \frac{\partial^2}{\partial \theta_i^2} + \frac{g^2 L}{8} \sum_{i\neq j} \frac{2C_{2m}/(N - 1) + 2L_i^a L_j^a}{\sin^2 \frac{1}{2}(\theta_i - \theta_j)}$$

(77)

where the $L_i^a$ are generators of the $U(N)$ group, and $C_{2m}$ is the quadratic Casimir for the $m$-fold symmetric representation of $SU(N)$. The limit $L \to 0$, appropriate for our two-dimensional string description, can be taken in [7], at the expense of the appearance of a strongly-coupled ($g^2 \to \infty$) gauge field theory living on a space point, which is the limit of a circle of vanishing length. This simplified example manifests the complexity arising when a specific type of heavy matter is incorporated in the two-dimensional Yang-Mills theory. The point we wish to emphasize here is...
that, in the effective field theory description of the Calogero-Sutherland model, one can construct explicitly a $W_{1,\infty}$ spectrum-generating symmetry algebra, in an analogous way to the free-fermion case \[63\]. This suggests that such a symmetry also plays a key rôle at the black-hole singularity. However, the situation there is much more complicated, and the pertinent analysis still incomplete. Technical complications arise from the twisted space-time supersymmetry \[67 \] which gives rise to an enhanced topological super-$W_{\infty}$ \[68 \] at the the black-hole singularity, which possesses a bosonic $W_{\infty} \times W_{\infty}$ subsymmetry. We expect that the emerging HD picture will also contain such topological features and an enhanced symmetry at the singularity, but the investigation of these points lies beyond the scope of this paper.

6 Summary and Conclusions

In the approach followed in this paper, critical string theory emerges as an “effective” description, analogous to the chiral lagrangian description of QCD in the conventional low-temperature vacuum containing quark and gluon condensates. The spectrum of this bosonized version of QCD contains solitons, that can be identified as baryons à la Skyrme. As one approaches from below the critical temperature for the hadron/quark-gluon transition in QCD, the quark condensate vanishes, as does the baryon mass. Above the critical temperature, the appropriate description of QCD is in terms of light quarks. As has been suggested previously, black holes in string theory are analogous to baryons \[49, 50 \], and the vanishing of their masses is also believed to herald the appearance of a new vacuum. A complete treatment of QCD must go beyond the effective lagrangian treatment. Likewise, a complete treatment of string theory must include black holes and the non-perturbative transitions they induce, and its natural description may well not be in terms of “effective” strings.

We have discussed in this paper an approach to the formulation of string field theory which is based on our non-critical Liouville string description of quantum fluctuations in the space-time background. As we have discussed in section 2, these arise inevitably when one considers higher-genus configurations of the string world sheet. Particularly important in this respect are configurations with degenerate handles, which induce quantum fluctuations in the couplings of the world-sheet field theory ($\sigma$ model) characterizing the “effective” string theory sitting on a typical classical string vacuum. In the generic case, extra logarithmic operators appear, which signal transitions between different conformal field-theory backgrounds. Examples of this problem include the collective coordinates of string solitons, among which are included string black holes. The correct treatment of these logarithmic operators is essential for obtaining the appropriate finite Zamolodchikov metric in theory space.
We have proceeded in section 3 to discuss the consistent canonical quantization of string theory space. As we have shown previously [7], the Liouville renormalization group approach to this problem obeys the Helmholtz conditions which guarantee the existence of such a canonical quantization scheme. We have proposed a path-integral formulation (28) of this string field theory, and demonstrated that it passes certain essential consistency checks.

We went on to show in section 4 how this path-integral approach may be used to treat extended-object backgrounds (solitons) in string theory, demonstrating the emergence of the various terms in the effective action (11) as world-sheet renormalization-group counterterms. In the recent string literature, the collective coordinates characterizing such solitonic backgrounds have been implemented in a simple and elegant way by the imposition of suitable Dirichlet boundary conditions on open-string world sheets. Duality connects such D-branes with closed-string solitons. In our path-integral approach (28), open strings and D-branes emerge after an appropriate choice of the local renormalization group scheme on the world sheet. The precise formulation of string dualities within our path-integral approach remains to be elucidated, but will presumably impose interesting restrictions on the form of the measure in the string path integral (28). Using our Liouville renormalization group approach, we have derived the appropriate second-order equation of motion for the D brane.

In section 5 we have applied this formalism to the discussion of string black holes and the problem of information loss. We have drawn the reader’s attention to analogies between the emerging higher-dimensional (HD) analysis of these questions and our previous two-dimensional (2D) approach. In particular, we have argued that the open strings appearing on the black-hole horizon, treated as a D brane in the HD approach, carry Chan-Paton-like quantum numbers corresponding to the $W_\infty$ charges first identified in the 2D model. It is the leakage of information via these quantum numbers which is responsible, within our interpretation, for information loss from black holes.

Although we believe we have identified some key ingredients in the eventual non-perturbative formulation of string field theory, and shown how they may be applied to problems of physical interest, we are amply aware that many important formal issues remain to be elucidated. Among these, we mention in particular the correct treatment of the logarithmic operators associated with the back-reaction of matter on string black-hole space times, which may be regarded as a string soliton recoil problem. Also, as we have mentioned above, the important restrictions imposed by duality on the measure of the string path integral (28) remain to be explored. Moreover, we have not considered in this paper the possible rôle of supersymmetry, in particular in the stabilization of the soliton backgrounds.
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Note Added

Since the appearance of our paper as hep-th/9605046, two new papers have appeared which discuss the hair of $D$ branes and its possible measurement [69, 70]. The paper closer to ours in its philosophy is [70], which discusses the quantum hair of $D$ branes and the possible measurement of associated generalized Aharonov-Bohm phases in the scattering of highly-excited string states. These ideas were proposed previously in Refs. [49, 50], on the basis of explicit studies [71] in the 1 + 1-dimensional black-hole model of [1]. We linked these ideas to the appearance of leg poles in string scattering amplitudes, as also noted in [70]. This author also mentions that BRST transformations become non-trivial in the presence of $D$ branes: we made the same remark in connection with world-sheet instantons related to transitions among 1 + 1-dimensional black holes in [1]. In Sections 4 and 5 of this paper, we have developed the relation between our earlier work and subsequent $D$-brane studies, and shown how open string states carry analogous $W$ quantum numbers.

The author of [70] states a belief that his interpretation of his results differs from ours [49, 50]. As may be concluded from the previous paragraph, we agree with the above-mentioned aspects of his work, which is along the lines of [49, 50]. We have gone on elsewhere [1] to develop the point of view that an experimentalist capable only of low-energy experiments using lowest-level string states would not be able to disentangle all the string black-hole states that could be distinguished by measurements using highly-excited string states [50, 70]. The author of ref. [70] suggests that one may be able to reconstruct all the information derivable from the scattering of excited string states by considering external states with an arbitrarily large number of lowest-level particles. It remains to be seen whether all the black-hole information can be recovered this way in a feasible experimental programme.

In the mean time, we observe that the results of [70] confirm our proposals in [49, 50], and are in line with the results of Section 5 of this paper.

References

[1] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B293 (1992), 37;
Lectures presented at the Erice Summer School, 31st Course: From Supersymmetry to the Origin of Space-Time, Ettore Majorana Centre, Erice, July 4-12
1993, published in Proc. Subnuclear Series Vol. 31, p.1 (World Scientific, Singapore 1994);
Mod. Phys. Lett. A10 (1995), 425; and hep-th/9305117.

[2] A.B. Zamolodchikov, JETP Lett. 43 (1986), 730; Sov. J. Nucl. Phys. 46 (1987), 1090.

[3] I. Antoniadis, C. Bachas, J. Ellis and D.V. Nanopoulos, Phys. Lett. B211 (1988), 393; Nucl. Phys. B328 (1989), 117; Phys. Lett. B257 (1991), 278.

[4] J. Polchinski, Nucl. Phys. 324 (1989), 123;
D.V. Nanopoulos, in Proc. International School of Astroparticle Physics, HARC (Houston) (World Scientific, Singapore, 1991), p. 183.

[5] I. Kogan, preprint UBCTP-91-13 (1991); Proc. Particles and Fields 91, p. 837, Vancouver 18-21 April 1991 (eds. D. Axen, D. Bryman and M. Comyn, World Sci. 1992); see also Phys. Lett. B265 (1991), 269.

[6] E. Witten, Phys. Rev. D44 (1991), 314.

[7] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Mod. Phys. Lett. A10 (1995), 1685.

[8] A. Dabholkar and J. Harvey, Phys. Rev. Lett. 63 (1989), 478;
A. Dabholkar, G. Gibbons, J. Harvey and F. Ruiz Ruiz, Nucl. Phys. B340 (1990), 33;
for recent developments see A.A. Tseytlin, Phys. Lett. B363 (1995), 223.

[9] V. Gurarie, Nucl. Phys. B410 (1993), 535;
M.A. Flohr, preprint CSIC-IMAFF-42, hep-th/9509166;
H.G. Kausch, preprint DAMTP-95-52, hep-th/9510149.

[10] I. Kogan and N.E. Mavromatos, preprint OUTP-95-50P, hep-th/9512210, Phys. Lett. B in press.

[11] P. Horava, Phys. Lett. B289 (1992), 293.

[12] C. Callan and J. Maldacena, preprint PUPT-1591, hep-th/9602043;
S. Das, preprint KEK-TH/472,TIFR-TH/96-11, hep-th/9602172.

[13] S. Coleman, Nucl. Phys. B310 (1988), 643;
T. Banks, I. Klebanov and L. Susskind, SLAC-PUB-4705 (1988).

[14] F. David, Mod. Phys. Lett. A3 (1988), 1651;
J. Distler and H. Kawai, Nucl. Phys. B321 (1989), 509.
[15] J. Polchinski, Nucl. Phys. B307 (1988), 61; *ibid.* B357 (1995), 241.

[16] W. Fischler, S. Paban and M. Rozali, Phys. Lett. B352 (1995), 298.

[17] C. Schmidhuber, Nucl. Phys. B435 (1995), 156.

[18] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, *Some Physical Aspects of Liouville String Dynamics*, contributions by J.E. and N.E.M. in *Phenomenology of Unification from Present to Future*, Roma 23-26 March 1994, p. 187 (World. Sci. 1994), hep-th/9405190.

[19] A. Bilal and I. Kogan, preprint PUPT-1482, hep-th/9407151 (unpublished); Nucl. Phys. B449 (1995), 569.

[20] J.S. Caux, I.I. Kogan and A. Tsvelik, preprint OUTP-95-62, hep-th/9511130.

[21] For a concise recent review see: A. Strominger, preprint UCSBTH-95-29, hep-th/9510207; and references therein.

[22] A.V. Yung, Int. J. Mod. Phys. A9 (1994), 591; *ibid.* A10 (1995), 1553.

[23] E. Brézin and V.A. Kazakov, Phys. Lett. B236 (1990), 144;
M. Douglas and A. Shenker, Nucl. Phys. B335 (1990), 635;
D. Gross and A.A. Migdal, Phys. Rev. Lett. 64 (1990), 127;
For a review see, e.g., I. Klebanov, in *String Theory and Quantum Gravity*, Proc. Trieste Spring School 1991, ed. by J. Harvey et al. (World Scientific, Singapore, 1991), and references therein.

[24] A. Jevicki and T. Yoneya, Nucl. Phys. B411 (1994), 64.

[25] S.A. Hojman and L.C. Shepley, J. Math. Phys. 32 (1991), 142;
F. Pardo, J. Math. Phys. 30 (1989), 2054.

[26] H. Osborn, Nucl. Phys. B294 (1987), 595; *ibid.* B308 (1988), 629; Phys. Lett. B222 (1989), 97.

[27] G. Shore, Nucl. Phys. B286 (1987), 349.

[28] For a review see A.A. Tseytlin, Int. J. Mod. Phys. A4 (1989), 4249.

[29] N.E. Mavromatos and J.L. Miramontes, Phys. Lett. B212 (1988), 33;
N.E. Mavromatos, Phys. Rev. D39 (1989), 1659;
N.E. Mavromatos and J.L. Miramontes, Phys. Lett. B226 (1991), 291.

[30] V. Periwal and A. Strominger, Phys. Lett. B235 (1990), 261;
S. Cecotti and C. Vafa, Nucl. Phys. B367 (1991), 359;
H. Ooguri and C. Vafa, Nucl. Phys. B361 (1991), 469; *ibid.* B367 (1991), 83.
[31] M. Duff, R. Khuri and X.J. Lu, Phys. Rep. 259 (1995), 213.

[32] J. Polchinski, Phys. Rev. D50 (1994), 6041; NSF-ITP-95-122 preprint, hep-th/9510017;
    J. Polchinski, S. Chaudhuri and C. Johnson, NSF-ITP-96-003 preprint, hep-th/9602052, and references therein.

[33] S. Shenker, preprint Rutgers RU-95-53, hep-th/9509132.

[34] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B278 (1992), 246.

[35] C. Bachas, preprint NSF-ITP-95-144, CPTH-S388-1195; hep-th/9511043.

[36] A. A. Tseytlin, Phys. Lett. B363 (1995), 223.

[37] H. Osborn, Nucl. Phys. B363 (1991), 486;
    D. McAvity and H. Osborn, Nucl. Phys. B406 (1993), 655; Nucl. Phys. B455 (1995), 522.

[38] C. Callan and I. Klebanov, preprint PUPT-1578, hep-th/9511173;
    C. Schmidhuber, preprint PUPT-1585, hep-th/9601003.

[39] In a different context see: N. Mavromatos and D.V. Nanopoulos, preprint ACT-19/95, CTP-TAMU-55/95, OUTP-95-52P, quant-ph/9512021.

[40] V. Periwal and O. Tafjord, preprint PUPT-1607, hep-th/9603156.

[41] S.J. Rey, preprint SNUTP 96-030, hep-th/9604037.

[42] J. Polchinski, Nucl. Phys. B231 (1984), 413;
    J. Hughes, J. Liu and J. Polchinski, Nucl. Phys. B316 (1989), 15.

[43] M. Li, preprint BROWN-HET-1027, hep-th/9512042.

[44] V. John, G. Jungman and S. Vaidya, Nucl. Phys. B455 (1995), 505;
    G. Amelino-Camelia and D. Bak, Phys. Lett. B343 (1995), 231.

[45] E.S. Fradkin and A.A. Tseytlin, Phys. Lett. B163 (1985), 123; 
    A. Abouelsaood, C. Callan, C.R. Nappi and S.A. Yost, Nucl. Phys. B280 (1987) 599;
    C. Callan, C. Lovelace, C.R. Nappi and S.A. Yost, Nucl. Phys. B308 (1988) 221.

[46] C. Schmidhuber and A.A. Tseytlin, Nucl. Phys. B426 (1994), 187;
    H. Dorn, preprint HU-Berlin-IEP-94-21, hep-th/9410084.

[47] A.A. Tseytlin, Phys. Lett. B251 (1990), 530.
[48] S. Kalara and D.V. Nanopoulos, Phys. Lett. B267 (1991), 343.

[49] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B284 (1992), 27.

[50] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B284 (1992), 43.

[51] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B289 (1992), 25; ibid. B296 (1992), 40.

[52] A. Giveon, Mod. Phys. Lett. A6 (1991), 2843.

[53] F. Larsen and F. Wilczek, Princeton preprint, hep-th/9604134.

[54] L. Susskind, Rutgers preprint, hep-th/9309143.

[55] L. Susskind and J. Uglum, Phys. Rev. D50 (1994), 2700.

[56] S. Chaudhuri and J. Lykken, Nucl. Phys B396 (1993), 270.

[57] S. Das, A. Dhar, G. Mandal and S. Wadia, Int. J. Mod. Phys. A7 (1992), 5165.

[58] In a rather different context, the idea of wrapping the world-sheet around the horizon of a spherically-symmetric four-dimensional target-space black hole appeared first in: I. Kogan, Mod. Phys. Lett. A6 (1991), 3297;

Subsequently a similar discussion appeared in S. Giddings, J. Harvey, J. Polchinski, S. Shenker and A. Strominger, Phys. Rev. D50 (1994), 6422.

[59] M. Green, Nucl. Phys. B293 (1987), 593.

[60] I. Kogan, Phys. Lett. B256 (1991), 369; Nucl. Phys. B375 (1992), 362;

S. Carlip, Nucl. Phys. B362 (1991), 111;

M.C. Ashworth, Mod. Phys. Lett. A10 (1995), 2749.

[61] J. Minahan and A. Polychronakos, Phys. Lett. B312 (1993), 155.

[62] S. Das and A. Jevicki, Mod. Phys. Lett. A5 (1990), 1639.

[63] R. Caracciolo, A. Lerda, and G. Zemba, Phys. Lett. B352 (1995), 304.

[64] I. Bakas, Phys. Lett. B228 (1989), 57;

C. Pope, X. Shen and L. Romans, Nucl. Phys. B339 (1990).

[65] N. Marcus and A. Sagnotti, Phys. Lett. B188 (1987), 58.

[66] E. Floratos, J. Iliopoulos and G. Tiktopoulos, Phys. Lett. B17 (1989), 285.

[67] T. Eguchi, Mod. Phys. Lett. A7 (1992), 85.
[68] J. Minahan and A. Polychronakos, Phys. Lett. B326 (1994), 288.

[69] A. Strominger, *Statistical Hair on Black Holes*, Rutgers Univ. preprint RU-96-47, hep-th/9606016.

[70] T. Banks, *Quantum Hair on D-branes and Black-Hole Information in String Theory*, Rutgers Univ. preprint RU-96-49, hept-th/9606026.

[71] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B267 (1991), 465; *ibid.* Phys. Lett. B272 (1991), 261.