Applying Mamdani’s method to categorize mathematical literacy of public middle school students in Kupang

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Abstract. The study objective was to describe the application of Mamdani’s fuzzy inference method to the categorization of public middle school students’ mathematical literacy in Kupang, East Nusa Tenggara. Having been done not only because of the low achievement of Indonesian 15-year-old students in PISA, but the research has also become the answer to the lack of surveys regarding students’ mathematical literacy in Kupang. Consequently, this study provides valid and actual data that the government should refer to. As many as 377 respondents of ninth-grade students from public schools were selected using a two-stage cluster randomized sampling so that they could best represent six districts in Kupang. Their ability was evaluated by fifteen math literacy problems adopted from PISA 2012 and the products of some development research; as a result, the reliability could be assured. At the fuzzification stage, scores obtained from three process domains of math literacy: formulating, employing, and interpreting, became the crisp input since those domains were the input variables. After being analyzed, the scores then produced crisp output indicating student mathematical literacy–as the output variable–at the defuzzification stage. Overall, public middle school students’ mathematical literacy in Kupang was classified as a low group.

1. Introduction
Educational evaluations, which include measuring and assessing student achievement and learning outcomes, can identify the quality of education in a country [1]. National Assessment (AN) is an educational evaluation used by the Indonesian Government to evaluate learning outcomes in every education level, including Minimum Competency Assessment (AKM). The AKM covers literacy and numeracy or a mathematical literacy test and would be conducted for the first time in 2021. Besides, internationally, Programme for International Student Assessment (PISA) is one of the educational evaluations organized by OECD (Organization for Economic Co-operation). The main aim is to measure 15-year-old students' essential skills in participating countries. One of the abilities assessed in PISA is mathematical literacy.

Mathematical literacy is one of the fundamental abilities that 15-year-old students should have to prepare them to deal with real-world challenges when it comes to formulate, employ, and interpret mathematics in various contexts [2, 3]. However, the performance of Indonesian students in PISA still acquired the bottom rank [4-6]. Indonesian students' mathematical literacy scores gained in PISA 2009, 2012, and 2015 were respectively 384, 375, and 386, statistically significantly below the average score of OECD countries [4-6]. Besides, the latest PISA survey in 2018 revealed that 15-year-old Indonesian students’ mean score in mathematics was 379, nowhere near as high as the OECD
average, 489 [7]. Moreover, there is still a lack of research to map and analyze student math literacy in East Nusa Tenggara, especially in Kupang.

Mathematical literacy can be identified as a fuzziness phenomenon because it cannot be determined immediately just by looking at the score. One alternative way to implement this is by using a Fuzzy Inference System (FIS) based on the fuzzy logic concept, which is understandable, flexible, able to tolerate data inaccuracy, and that the expert experience can be applied directly in its modeling system [8-11]. The fuzzy inference method used was Mamdani's method. The strengths of this method are that it is intuitive, widely accepted, and is very suitable for human input [9]. By using the fuzzy set principle that utilizes the role of membership degree, determining the category of students' mathematical literacy can avoid weaknesses in the crisp set [12].

This study differs from the other previous research in the method, location, and sample. Retrospective research examined student math literacy in Bantul regency and Kupang city using statistical methods [13, 14]. The other research attempted to utilize Mamdani’s fuzzy inference method to determine the most outstanding teachers [15] and the success rate of lecturers in teaching [16]. Therefore, this study aimed at applying Mamdani’s fuzzy inference method to categorize the mathematical literacy of public junior high school students in Kupang.

2. Method
This research was classified as a survey with a descriptive quantitative approach. It was conducted on 21 March until 16 April 2019. The respondents were 15-year-old students (ninth grade students) according to the recommendation of the OECD [17], and two-stage cluster randomized sampling was applied to select 377 students of 2018/2019 academic year representing six public middle schools each of which are located in six distinct districts in Kupang. Since the population was 6,029 ninth grade pupils, the appropriate sample size was estimated at least 364 respondents [18]. They were then asked to do fifteen mathematical literacy problems, twelve of which were adopted from PISA 2012 [2] and three of which were the products of development research [19-21]. They completed the questionnaire under a sufficient time limit of 120 minutes (estimated 8 minutes per items) with the researcher as the invigilator, compared to the PISA survey where the students have to do 85 problems within 210 minutes (estimated 2.5 minutes per items) [22].

The data analysis utilized was a mixture of Mamdani’s fuzzy inference method and statistical method. Students’ mathematical literacy was categorized referring to the normative reference of deviation standard, adopted from Ebel and Frisbie [14] as shown in table 1.

Table 1. The normative reference of deviation standard.

| Score Range                  | Criteria        |
|-----------------------------|-----------------|
| $M_i + 1.5Sd_i < X < M_i + 3Sd_i$ | Very High      |
| $M_i + 0.5Sd_i < X < M_i + 1.5Sd_i$ | High           |
| $M_i - 0.5Sd_i < X < M_i + 0.5Sd_i$ | Medium         |
| $M_i - 1.5Sd_i < X < M_i - 0.5Sd_i$ | Low            |
| $M_i - 3Sd_i < X < M_i - 1.5Sd_i$ | Very Low       |

Annotation:

$M_i$ = the ideal mean = $\frac{1}{2}$ (the ideal maximum + the ideal minimum)

$Sd_i$ = the ideal deviation standard = $\frac{1}{6}$ (the ideal maximum – the ideal minimum)

$X$ = the total score achieved by students
There were three stages involved in generating the FIS: fuzzification, rules design, and defuzzification. In the first stage, fuzzification is the process of converting crisp input into fuzzy input based on the role of the membership function of linguistic variables. The first step was to determine the membership function of each linguistic value. The selected input variables were the three process domains of the mathematical literacy assessed on the test instrument, namely formulating, employing, and interpreting. The student test scores in each of these domains would be crisp input from each variable. Since the output variable was mathematical literacy, the crisp output was in the form of a test score for the mathematical literacy of each student. The fuzzy sets of each variable used the trapezoidal membership function approximation, as illustrated in Table 2.

Table 2. Fuzzy sets of input and output variables.

| Linguistic Variable | Linguistic Value | Universal Set | Numeric Value |
|---------------------|-----------------|---------------|---------------|
| Formulate (x)       | Very High       | [12.5,18]     |               |
|                     | High            | [9.5,14]      |               |
|                     | Medium          | [0,18]        | [6.5,11]      |
|                     | Low             | [3.5,8]       |               |
|                     | Very Low        | [0,5]         |               |
| Employ (y)          | Very High       | [20,28]       |               |
|                     | High            | [15.33,21.67] |               |
|                     | Medium          | [0,28]        | [10.67,17]    |
|                     | Low             | [6,12.34]     |               |
|                     | Very Low        | [0,7.67]      |               |
| Interpret (z)       | Very High       | [19.25,27]    |               |
|                     | High            | [14.75,20.75] |               |
|                     | Medium          | [0,27]        | [10.25,16.25] |
|                     | Low             | [5.75,11.75]  |               |
|                     | Very Low        | [0,7.25]      |               |
| Output (u)          | Very High       | [54.75,73]    |               |
|                     | High            | [42.58,54.75] |               |
|                     | Medium          | [0,73]        | [30.42,42.58] |
|                     | Low             | [18.25,30.42] |               |
|                     | Very Low        | [0,18.25]     |               |

This research evaluated students' mathematical literacy based on three process domains: formulating, employing, and interpreting. As a result, of fifteen problems on the questionnaire, four items assess how respondents can formulate situations mathematically, with the maximum ideal score of 18. Seven other questions evaluate how far students can employ mathematical concepts and procedures, contributing as many as 28 of the total score. The remaining four items examined students' ability to interpret mathematical outcomes, with a maximum score of 27. In total, a possible highest grade students' can obtain is 73. The linguistic value in Table 2 was categorized using the trapezoidal membership function. Meanwhile, the numerical value interval for each variable was obtained using the normative reference of deviation standard in Table 1.

The following step is that inference rules were determined using the IF-THEN rule based on the input and output variables. The rules play a fundamental role in supporting the Fuzzy Inference System (FIS). These rules are responsible for evaluating the crisp input and transforming it into a fuzzy input value using the Min (Minimum) implication function. The Min implication function uses the AND operator on the antecedent of the rules so that it determines the intersection of two or more
fuzzy sets in the antecedent and consequent of the rules. The antecedent contains input variables, while the consequent of the rules contains output variables. Based on the number of fuzzy sets formed on each input and output variable, the number of rules to be determined was 125 rules. The following are some selected rules as shown in table 3.

| Rules Number | Rules                                                                 |
|--------------|----------------------------------------------------------------------|
| [R1]         | IF (Formulate is very high) AND (Employ is very high) AND (Interpret is very high) THEN (Mathematical Literacy is very high) |
| [R18]        | IF (Formulate is very high) AND (Employ is low) AND (Interpret is medium) THEN (Mathematical Literacy is high)              |
| [R60]        | IF (Formulate is medium) AND (Employ is high) AND (Interpret is very low) THEN (Mathematical Literacy is medium)             |
| [R98]        | IF (Formulate is low) AND (Employ is very low) AND (Interpret is medium) THEN (Mathematical Literacy is low)                |
| [R125]       | IF (Formulate is very low) AND (Employ is very low) AND (Interpret is very low) THEN (Mathematical Literacy is very low)    |

Finally, the defuzzification stage aims to take a crisp value from the fuzzy output to obtain exactly one crisp output value. The defuzzification method used was the centroid method (composite moment), by taking the center point of the fuzzy set field to produce a crisp output. In general, the formula of the centroid method is as in equation (1) [8].

$$x = \frac{\int x \mu(x) dx}{\int \mu(x) dx}$$  \hspace{1cm} (1)

After being produced, all the crisp output, which was the mathematical literacy test scores of students, were then described using descriptive statistics to identify the characteristic of the data. Besides, the percentage of student achieved scores were also calculated using the formula in equation (2) [14].

$$P = \frac{\sum B}{\sum T} \times 100\%$$  \hspace{1cm} (2)

P is the percentage of student achieved scores; B is the score obtained by students; T is the ideal maximum score.

To generalize the result to all populations, an estimation of the confidence interval for $\mu$ was then examined with a significance level of $(1 - \alpha)$% as shown in equation (3) [23].

$$\bar{x} - t(\frac{\alpha}{2}, v) \frac{s}{\sqrt{n}} < \mu < \bar{x} + t(\frac{\alpha}{2}, v) \frac{s}{\sqrt{n}}$$  \hspace{1cm} (3)

$t(\frac{\alpha}{2}, v)$ is the value of $t$ distribution with a freedom degree of $v = n - 1$; $\bar{x}$ and $s$ are respectively the mean and deviation standard of crisp output.

However, before doing the estimation, the normality of the crisp output was evaluated using skewness and kurtosis tests to avoid the sensitivity and unreliability of Kolmogorov-Smirnov and
Shapiro-Wilk tests due to the large sample size. The data are from a normally distributed population if and only if either the absolute value of skewness is more than two or the absolute value of kurtosis is more than seven [24].

3. Result and discussion

The generating process of FIS was assisted by a fuzzy logic designer on MatLab, consisting of three input variables (formulate, employ, and interpret) and one output variable (student mathematical literacy). The diagram of input and output variables as to how it looks on the fuzzy logic designer is illustrated in figure 1.

![Input and output variables on Fuzzy Logic Designer, MatLab.](image)

The membership functions of both the input and output variables were respectively shown in figure 2, figure 3, figure 4, and figure 5.

![The Membership Function of Formulate.](image)

![The Membership Function of Employ.](image)

![The Membership Function of Interpret.](image)

![The Membership Function of Math Literacy.](image)

As many as 125 rules had been defined to evaluate the crisp value of each input variable. After the crisp input had been evaluated, they were then composed by the Max (Maximum) method aiming to find the union of fuzzy sets formed by all rules so that the fuzzy output value could be obtained. At this point, the fuzzy output continued to the defuzzification stage, where a number located at the center
of the fuzzy set field would be taken as the crisp output. After that, all the crisp output will be used to
categorize students' mathematical literacy.

The following is the crisp output value, indicating the mathematical literacy test scores of some
selected public junior high school students in Kupang, as shown in table 4.

**Table 4.** The crisp output of several students.

| Students | Input Variable(s) | Output Variable |
|----------|-------------------|-----------------|
|          | Crisp Input       | Crisp Output    |
|          | Formulate (0-18)  |                 |
|          | Employ (0-28)     |                 |
|          | Interpret (0-27)  |                 |
|          | Mathematical Literacy (0-73) |     |
| NE       | 10                | 26              | 17   | 63.9 |
| JLRG     | 11                | 20.88           | 14.75| 58.2 |
| GYAP     | 7                 | 20.13           | 17.5 | 51   |
| AA       | 9.88              | 10.75           | 21.75| 50.1 |
| YDTR     | 5.25              | 8.5             | 18.5 | 36.5 |
| IM       | 10                | 12              | 6.88 | 33.5 |
| MGBT     | 2                 | 10.75           | 6.25 | 25.3 |
| SNH      | 2.25              | 7.38            | 5.13 | 20.3 |
| MAPD     | 4.13              | 7               | 5.81 | 16.4 |
| NT       | 3.56              | 3.56            | 0.19 | 9.59 |

Meanwhile, table 5 below provides information about the descriptive statistics obtained from the
crisp output of students' test scores data analysis.

**Table 5.** Descriptive statistics of students' test scores (crisp output).

| Description       | Students' Mathematical Literacy |
|-------------------|---------------------------------|
| Mean              | 21.99                           |
| Median            | 20.30                           |
| Deviation Standard| 13.46                           |
| Ideal Maximum     | 73                              |
| Maximum           | 63.9                            |
| Ideal Minimum     | 0                               |
| Minimum           | 9.13                            |
| Range             | 54.77                           |
| Sample Size       | 377                             |

It is evident from table 5 that the average mathematical literacy test scores were low, indicated by
the mean and median of the data. There was a significant difference between both the mean and
median and the ideal maximum score. Thus, it is undeniable that most of the respondents achieved a
relatively lower score, far less than the ideal maximum. Moreover, from the deviation standard of the
data, it is apparent that the average number of public middle school students in Kupang obtained
scores comparatively close to the mean and median, meaning few students could solve the math
literacy problems correctly. Furthermore, although the maximum score achieved was 63.9, far higher
than the minimum score, 9.13, the number of students who got a higher score (close to the maximum)
was approximately 10% compared to the total students who obtained a lower score (near the minimum
score). Hence, the big gap, indicated by the range of the minimum and the maximum score achieved
by respondents, 54.77, between respondents who obtained a lower score and those who gained a
higher score was insufficient to push the mean to a point near the maximum score. All in all, the
statistics revealed that most of the public junior high school students' mathematical literacy still
occupied the low category. Meanwhile, figure 6 below displays the data about the distribution of students based on their category level of mathematical literacy, the output of Mamdani’s fuzzy inference method.

Figure 6. The distribution of respondents based on their level of mathematical literacy.

Figure 6 shows that the majority of respondents were in a very-low group at just under half of them, and next came the low-ability group at just under a third. This was followed by the medium category at nearly a tenth, just ahead of the high category at 7%, which was twice as many as respondents in the very-high class at a mere 3%. The bar graph also highlights the statistics that the lower the category level of students' ability, the more the number of students, and vice versa. The figure rocketed from the medium to the very-low category, which means more than half of public middle school students in Kupang had low mathematical literacy. Besides, figure 7 reveals the calculation of students' achieved score percentage and the distribution of achieved scores between respondents' mathematical literacy category level acquired from Mamdani’s Fuzzy Inference System (FIS) output. The bar graph in figure 7 illustrates that the very-high group successfully gained 83% of the ideal maximum, while the very-low students could only obtain nearly a fifth of the total score, which was far less than the other groups. The high-level respondents could gain around two-thirds of the maximum score, followed by the medium group who could afford just under half of the ideal score, while the low-level students could only make it to nearly a third. At a glance, the percentage of scores successfully achieved by students was decreasing from the very-high to the very-low class. The lower the category level of students' math literacy, the less the scores they achieved, and vice versa. The amount of fall, which was pretty significant, happened between the low and the very-low group, as much as 18%, far more than the decrease witnessed in between other classes. Therefore, it is evident that the higher the level of students' ability, the easier for them to solve all math literacy problems correctly, and vice versa.

Meanwhile, table 6 and table 7 respectively provide information about the skewness and kurtosis of the data and the estimation of the confidence interval.

| Description | Statistics | Std. Error |
|-------------|------------|------------|
| Skewness    | 1.148      | 0.126      |
| Kurtosis    | 0.753      | 0.251      |

Table 6. Skewness and kurtosis of respondents' test scores.

| Description            | Statistics          |
|------------------------|---------------------|
| Mathematical Literacy  | 95% Confidence      |
|                        | Lower Bound         | 20.63           |
| Mathematical Literacy  | Interval for Mean   | 23.35           |

Table 7. The confidence interval for $\mu$ of test scores.
The normality test revealed that the data of public middle school students' mathematical literacy test scores were from a normally distributed population because, according to Table 6, the absolute value of skewness was less than two and the absolute value of kurtosis was less than seven [24]. The confidence interval for $\mu$ lied between 20.63 and 23.35 (Table 7). Therefore, according to the numerical value of output variable in Table 2, the mathematical literacy of public junior high school students in Kupang was generally in a low category. These results were similar to some retrospective research [13, 14]. Furthermore, the result was also in line with the latest PISA assessment conducted in 2018, where the mathematical literacy of the majority of Indonesian 15-year-old students belonged to the low group [7]. The low achievers shared 76% of Indonesian participants, far more than the OECD average, which only 24% of the students categorized as low achievers [7].

4. Conclusion

The application of Mamdani’s fuzzy inference method in categorizing student math literacy involved several stages: determining the input and output variables, generating the membership functions, designing the rules, and processing the crisp input into crisp output later used to determine student ability. In general, the mathematical literacy of public middle school students in Kupang was classified as a low category. Although having contributed a lot to providing valid prior data about public middle school students' mathematical literacy in Kupang city, this research has several weaknesses. Firstly, the test was conducted in the morning, so this study only examined students’ performance under the specific time. Secondly, as this research utilized toolbox Matlab, a fuzzy logic designer, to generate the fuzzy inference system, the control system cannot automatically produce students’ ability category level based on the crisp output released by the FIS. Therefore, for further research, it is highly recommended to improve the system, to conduct the same study whichever methods are used, either statistical or fuzzy control system, or both of them, to map students' math literacy in other regions in Indonesia.

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