Radar and Lightcurve Observations and a Physical Model of Potentially Hazardous Asteroid 1981 Midas

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Abstract

We report observations of the Apollo-class potentially hazardous asteroid 1981 Midas, which passed 0.090 au from Earth (35 lunar distances) on 2018 March 21. During this close approach, Midas was observed by radar both from the Arecibo Observatory on March 21 through 25 (five nights) and from NASA’s Goldstone Deep Space Communications Complex on March 19 and 21. Optical lightcurves were obtained by other observers during four apparitions (1987, 1992, 2004, and 2018), which showed a rotation period of 5.22 hr. By combining the lightcurves and radar data, we have constructed a shape model for Midas. This model shows that Midas has two lobes separated by a neck, which, at its thinnest point, is about 60% of the width of the largest lobe. We also confirm the lightcurve-derived rotation period and show that Midas has a pole direction within 6° of ecliptic longitude and latitude (λ, β) = (39°, −60°) and dimensions of (3.41 ± 9%) × (1.90 ± 11%) × (1.27 ± 29%) km. Analysis of gravitational slopes on Midas indicates that nearly all of the surface has a slope less than the typical angle of repose for granular materials, so it does not require cohesion to maintain its shape. In addition, we measured a circular polarization ratio of 0.83 ± 0.04 at Arecibo’s 13 cm wavelength, which is the highest seen to date for any near-Earth asteroid with visible and near-infrared spectral type V.

Unified Astronomy Thesaurus concepts: Near-Earth objects (1092); Asteroids (72); Radar astronomy (1329); Radar telescopes (1330); Light curves (918)

Supporting material: figure sets

1. Introduction

Asteroid 1981 Midas was discovered in 1973 by Charles Kowal at Palomar Observatory (IAUC 2532). It is an Apollo-class asteroid that crosses the orbits of Venus, Earth, and Mars and has been classified as a “Potentially Hazardous Asteroid” (PHA) by the Minor Planet Center. Midas has visible and near-infrared spectral type V (Binzel et al. 2001, 2004, 2019), which suggests a basaltic composition (Bus & Binzel 2002 and references therein). Previous observations have found a semimajor axis of 1.776 au, eccentricity of 0.650, inclination of 39°, and orbital period of 2.37 yr.

Existing lightcurves reported in Mottola et al. (1995), Wysinski et al. (1997), Kuinonen et al. (2007), and Franco et al. (2018) revealed a rotation period of 5.22 hr and a large amplitude that suggests an elongated body. The amplitude of about 1.0 mag, seen on several nights, implies an elongation (ratio of maximum to minimum cross sections) of a/b ≈ 2.5 (Harris et al. 2014).

Using mid-infrared data from AKARI, Usui et al. (2011) found an albedo of 0.293 ± 0.025; they combined that with an absolute magnitude of H = 15.60 ± 0.20 (assuming G = 0.23 ± 0.11).

Observations of Midas from Goldstone in 1987 September yielded Doppler astrometry and a circular polarization ratio measurement of 0.65 ± 0.13 at 8495 MHz (Ostro et al. 1991; Benner et al. 2008).

No shape models or pole directions of Midas have previously been reported. Recently acquired high-resolution radar images from the 2018 March close approach of Midas, supplemented by lightcurves taken with a variety of viewing geometries, make Midas a promising target for shape modeling. Here we describe radar observations, lightcurve observations, and the resulting three-dimensional shape model for Midas.

2. Observations

2.1. Radar Observations

Radar observations enable spatially resolved images of near-Earth asteroids (NEAs) to be taken from ground-based telescopes. A powerful series of radio waves is transmitted toward the asteroid. Some of those radio waves reflect off the asteroid, and the echoes are received on Earth after the round-trip time (RTT). Decoding the recorded echoes in time (delay) and Doppler frequency yields two-dimensional images of the target.

During the 2018 March close approach of Midas (0.0896 au or 34.9 lunar distances), radar data were acquired over two nights by...
the Goldstone Solar System Radar 70 m DSS-14 dish and on five nights by the Arecibo 305 m telescope. All radar observations of Midas were monostatic, in which the telescope transmitted toward Midas for approximately the RTT, then switched to receiving just as the first echoes were about to return to Earth. After the last echoes were received, the telescope switched back to transmission. Each cycle of transmission and reception is called a “run” or a “scan.” The RTT sets the duration of data recording for each scan. That duration, in turn, determines the finest possible frequency resolution, which is the reciprocal of the recording duration—longer durations allow for finer frequency resolution. However, it is often preferable to process the data with coarser frequency resolution, so that there are multiple independent samples (looks) and a higher signal-to-noise ratio (S/N) in each frequency channel. The data acquisition and reduction techniques used are consistent with those previously described by other publications (e.g., Magri et al. 2007).

In 2017 September, Arecibo Observatory was damaged by Hurricane Maria. The strong winds distorted the dish, reducing the S-band gain (effective area) by about 30%. Arecibo did not have a full set of working generators during the Midas observations in 2018 March, so the transmitted power was only about 370 kW. These observations are summarized in Table 1 and sky positions are shown in Figure A1. Prior to the Arecibo observations, Midas had been expected to have an S/N of roughly 100 per run, a rough estimate that accounted for its previously known rotation period and its approximately known size (based on $H$). The observed S/N of Midas was close to this. For discussion of Midas’s radar-scattering properties and S/N, see Sections 4.3 and 5.

Over the seven nights of radar observations, continuous-wave echo power (CW) spectra were obtained. Arecibo and Goldstone each transmitted circularly polarized signals, and then the reflected opposite-sense circular (OC) and same-sense circular (SC) polarization echoes were received and recorded. The radar transmissions are at a single frequency, but the echoes get smeared out into a finite range of frequencies (bandwidth) due to the target’s rotation. Some areas on its surface are rotating toward the observer (relative to its center of mass), and other areas are rotating away from the observer. The relation between the target’s observed Doppler bandwidth ($B$) in a CW spectrum and the asteroid’s properties is

$$B = \frac{4\pi D}{\lambda P} \cos \phi \quad (1)$$

where $D$ is the diameter (breadth) of the asteroid, $\lambda$ is the radar wavelength, $P$ is the rotation period of the asteroid, and $\phi$ is the sub-observer latitude. Visual inspection of Arecibo observations from the first night showed a bandwidth of about 17 Hz. Given the previously known rotation period of 5.22 hr, this bandwidth implies a breadth of at least 3.2 km. Later Arecibo CW observations showed bandwidths varying from about 13 to 17 Hz, but the bandwidth only varied by up to 1.5 Hz on any single night, implying that the breadth of Midas only changed by about 10% during the Arecibo CW observations on any night. All CW spectra are provided in Figure Set A2.

Delay-Doppler observations were obtained during all five nights of Arecibo observations. For our delay-Doppler observations, the phase of the transmitted signal was pseudo-randomly modulated in order to resolve the reflected signal in both Doppler frequency and delay. Decoding the detected echoes in round-trip travel time (delay) and Doppler-shifted frequency allows the generation of two-dimensional delay-Doppler images. These images (see Figure 1) were taken with baud lengths of 0.2 $\mu$s and 0.5 $\mu$s, which correspond to

| Observing Date (UT) | Receive Time (HH:MM:SS) | Telescope | Type | Baud ($\mu$s) | spb | Freq Resolution (Hz) | Runs | Power (kW) | R.A. (°) | Decl. (°) | $\Delta$ (au) |
|---------------------|--------------------------|-----------|------|--------------|-----|----------------------|------|------------|----------|-----------|---------------|
| 2018 Mar 19         | 02:46:44–04:43:50        | G         | CW   | ...         | ... | 10                   | 36   | 96          | 127       | 46        | 0.099       |
| 2018 Mar 21         | 01:06:39–02:45:09        | G         | CW   | ...         | ... | 5                    | 33   | 96          | 108       | 37        | 0.090       |
| 2018 Mar 22         | 03:52:42–04:48:48        | G         | CW   | ...         | ... | 5                    | 19   | 95          | 126       | 46        | 0.089       |
| 2018 Mar 23         | 05:32:07–06:51:36        | G         | CW   | ...         | ... | 5                    | 27   | 98          | 106       | 36        | 0.090       |
| 2018 Mar 24         | 22:17:55–22:25:17        | A         | CW   | ...         | ... | 0.37                 | 3    | 370         | 101       | 31        | 0.090       |
| 2018 Mar 25         | 22:32:01–22:59:18        | A         | DD   | 0.5        | 1   | 0.95                 | 9    | 376         | 100       | 31        | 0.090       |
| 2018 Mar 22         | 22:32:01–22:59:18        | A         | DD   | 0.2        | 4   | 0.30                 | 20   | 359         | 100       | 31        | 0.090       |
| 2018 Mar 23         | 21:22:09–21:41:44        | A         | CW   | ...         | ... | 0.48                 | 7    | 339         | 93        | 25        | 0.091       |
| 2018 Mar 24         | 21:48:01–21:49:24        | A         | CW   | ...         | ... | 0.48                 | 1    | 347         | 93        | 25        | 0.091       |
| 2018 Mar 25         | 22:00:26–22:29:08        | A         | DD   | 0.2        | 4   | 0.30                 | 10   | 341         | 93        | 25        | 0.091       |
| 2018 Mar 24         | 22:37:26–23:54:39        | A         | DD   | 0.5        | 1   | 0.95                 | 26   | 358         | 93        | 24        | 0.091       |
| 2018 Mar 25         | 20:53:21–21:18:51        | A         | CW   | ...         | ... | 0.46                 | 8    | 377         | 87        | 18        | 0.095       |
| 2018 Mar 24         | 21:26:06–22:06:18        | A         | DD   | 0.5        | 1   | 0.48                 | 13   | 395         | 86        | 18        | 0.095       |
| 2018 Mar 25         | 22:26:03–23:06:51        | A         | DD   | 0.5        | 1   | 0.48                 | 13   | 391         | 86        | 18        | 0.095       |
| 2018 Mar 24         | 23:18:57–23:29:54        | A         | DD   | 0.5        | 1   | 0.48                 | 4    | 389         | 86        | 17        | 0.095       |
| 2018 Mar 25         | 20:28:52–20:53:59        | A         | CW   | ...         | ... | 0.43                 | 8    | 388         | 81        | 12        | 0.100       |
| 2018 Mar 25         | 20:59:34–22:55:58        | A         | DD   | 0.5        | 1   | 0.95                 | 33   | 395         | 81        | 11        | 0.101       |
| 2018 Mar 25         | 20:19:56–20:48:05        | A         | CW   | ...         | ... | 0.40                 | 8    | 390         | 77        | 6         | 0.108       |
| 2018 Mar 25         | 20:57:03–22:20:18        | A         | DD   | 0.5        | 1   | 0.95                 | 20   | 394         | 76        | 6         | 0.108       |

Note. This table lists the observing date in UT, the receive start and end times, the telescope that acquired the data (G for Goldstone, A for Arecibo), the type of radar data (CW for continuous wave or DD for delay-Doppler), the baud length used (if delay-Doppler), the number of samples per baud (spb, for delay-Doppler), the frequency resolution, the number of runs conducted, the power of the transmitted signal, the asteroid’s right ascension (R.A.) and declination (decl.), and the geocentric distance of the asteroid ($\Delta$).
Figure 1. Delay-Doppler images of Midas from Arecibo in 2018. All of these images show single scans. Upper left: midreceive time March 21, 22:37:37 UTC; frequency resolution 0.954 Hz, delay resolution 0.50 μs. Lower left: March 22, 22:01:08 UTC; 0.298 Hz × 0.05 μs (from 0.20 μs baud with four samples per baud). Upper middle: March 23, 21:59:14 UTC; 0.477 Hz × 0.50 μs. Lower middle: March 23, 22:37:38 UTC; 0.477 Hz × 0.50 μs. Upper right: March 23, 23:06:08 UTC; 0.477 Hz × 0.50 μs. Lower right: March 24, 22:08:58 UTC; 0.954 Hz × 0.50 μs. In these delay-Doppler images, and for all other delay-Doppler images shown in this work, Doppler frequency is plotted on the horizontal axis, increasing from left to right. Delay (range) is plotted on the vertical axis, increasing from top to bottom. The original data had different delay resolutions, but all images have been stretched length corresponding to 6.0 km. This means that the original delay-Doppler pixels can appear rectangular here.

2.2. Lightcurve Observations

We have obtained optical lightcurve data from 20 nights between the years of 1987 and 2018. A summary of these observations is given in Table 2, and sky positions can be found in Figure A1. The large amplitudes of some lightcurves suggest that Midas is a highly elongated object (all lightcurves are shown in Appendix Figure Set A4). In some cases, the lightcurve data points’ nominal uncertainties (error bars) appeared to be significantly smaller than the typical scatter between neighboring data points, so we had to manually increase their uncertainties. The large quantity of lightcurves available enabled us to better determine the orientation of Midas’s rotational axis (hereafter called the pole direction) and was especially crucial in determining a preference between prograde and retrograde rotation. Given the north–south ambiguity present in radar images (Ostro et al. 2002), lightcurves were necessary for breaking this ambiguity and determining a preference between models.

At the start of this work, we were not aware of the four nights of lightcurve observations from 2018 March 5 to 11 (listed in Table 2) taken by M. Husárik at the Skalnaté Pleso Observatory (Slovakia), so they were not originally used in the shape model. Once we obtained them, later on in the modeling process, they were incorporated and acted as an independent check on the progress of our shape model. These had the longest duration of any of our lightcurve observations, covered different viewing geometries from the other observations, and included a range of phase angles for which we previously did not have lightcurve coverage (see Table 2). These lightcurves provided more evidence that led us to prefer a southern pole direction (retrograde rotation) over a northern pole direction (prograde rotation). For further discussion, see Section 4.1.

3. Shape Modeling

3.1. SHAPE Software

To develop this model of Midas, we used the SHAPE software created by Hudson (1994) and later enhanced significantly by Magri et al. (2007). SHAPE combines radar data and optical lightcurves to find a best-fit model by iterating through a set of parameters. These parameters include optical and radar scattering properties, spin-state parameters, lengths of each of the three axes, and the positions of the vertices (which make up the triangular mesh that represents the model). For any
fit, each of these parameters can be held constant at a user-specified value or allowed to vary. SHAPE iterates through the parameters that are allowed to vary, varying each parameter slightly to minimize the value of the model’s objective function, which is the weighted sum of squared residuals (chi-square) plus penalty terms. SHAPE calculates the chi-square by computing a model frame and comparing that to the corresponding data for each frame of delay-Doppler, light-curve, and CW observations. We apply penalty functions to mathematically discourage physically unrealistic models, such as models with very sharp spikes.

Shape inversion from radar data is not a trivial problem to solve, even with software that is optimized for this purpose. SHAPE often gets stuck in local minima of this many-dimensional parameter space, so human judgment is necessary to guide SHAPE to the best-fit model. Getting SHAPE out of these local minima can require adjusting penalty weights or manually adjusting the position of vertices using Blender, open-source 3D computer graphics software that was first used in asteroid shape modeling by Crowell et al. (2017). We used this software to access the individual vertices of the 3D model and shift their positions by hand. It is important to note that Blender does not use the actual data. We used it as a tool to shift individual vertices when SHAPE failed to reproduce specific features, in order to provide better initial models for SHAPE.

3.2. Modeling Process

In asteroid shape modeling, one typically starts with a simple shape like an ellipsoid or ovoid, then gradually moves to more complex shapes. For Midas, because of the asymmetric shape seen in radar images, we started with an ovoid model to get rough dimensions of the asteroid. Then, because radar images indicated that Midas is a bilobed object, we applied those base dimensions to a preexisting asteroid model of another bilobed asteroid, 8567 (1996 HW1) (Magri et al. 2011). We chose to use another bilobed asteroid as a starting model because with software like SHAPE that makes small, iterative adjustments, it is much easier and quicker to shift a preexisting “neck” between the lobes of the asteroid than it is to create a neck from a simple model made up of an ovoid, or even multiple ellipsoids. This progression from the 1996 HW1 model to the Midas model is shown in Figure 2.

When moving from the 1996 HW1 model to a more accurate Midas model, we used a spreadsheet that included the coordinates of all of the vertices to adjust the location of the neck. 1996 HW1 has a strongly bifurcated shape, so compared to the radar images, the neck was consistently located farther from the center than we see in the Midas data. In Figure 3(a), we see the vertices of the 1996 HW1 model, and then in Figure 3(b), we show the model after applying linear shifts to the x-coordinates of all vertices. Using a spreadsheet enabled us to apply three different linear shifts to the vertices in three regions of the model (left lobe, neck, and right lobe) while ensuring that the model remained smooth at the regions’ boundaries. In other words, no vertices in the neck region were to the left of vertices in the left lobe, or to the right of vertices in the right lobe, which would have caused the model to fold in on itself.

From the beginning, we assumed that Midas is in a simple rotation state, rotating about its shortest principal axis (that with the greatest moment of inertia) in approximately the rotation period seen in the lightcurves. Previous data showed no indication of non-principal-axis rotation, and principal-axis rotation provided good fits to our data, so we did not have to revisit this assumption.
When constructing the shape model of Midas, we alternated between refining the pole direction and adjusting the positions of the vertices. SHAPE will not easily determine the best pole direction because changing the pole direction requires compensating with changes in other parameters’ values as well. Because SHAPE changes the parameters sequentially (one at a time), it cannot make simultaneous changes to multiple parameters, so changing only the pole longitude or latitude almost always results in a worse chi-square value. To find an accurate pole direction, we began with a coarse search over the entire celestial sphere, then searched a broad spacing of fixed candidate pole directions and gradually performed finer searches around our best-fit pole directions. After each pole search, we would take one or two of the best models and allow SHAPE to vary the positions of each vertex, in order to optimize the shape of the model at the different fixed pole directions. We then checked the chi-square values of these candidate pole directions and visually inspected the fits. A plot of example grid search results is shown in Figure A5. Pole searches used all data (radar and lightcurves), but given the greater variety of viewing geometries in our lightcurve observations, the lightcurve data proved to be the most useful in determining the pole direction.

SHAPE would often get stuck in a local chi-square minimum, so it was necessary to manually make larger changes to the model. Blender became an invaluable tool to do this, as it allows for adjustment of the positions of individual vertices as well as the addition of vertices in places that need finer resolution. For Midas, we had some difficulties fitting the vertices in the neck area, so we increased the number of vertices there, in order to give SHAPE the ability to make finer adjustments to the neck geometry. We also used Blender to make large enough vertex shifts for SHAPE (as seen in Figure 4) to be able to get away from the local chi-square minimum that it was stuck in. Then, by putting the model edited in Blender back into SHAPE and running a model where we let all of the vertices vary, SHAPE would generally produce a model much closer to the data than before the edits were made in Blender. Of course, this is highly dependent on the ability to make reasonable adjustments manually, using visual assessment.

To model the lightcurves, we used two different types of optical scattering laws, Kaasalainen (weighted average of Lambert and Lommel–Seeliger; Kaasalainen et al. 2001) and Hapke (2012). Both gave acceptable fits to the lightcurve data, but the Kaasalainen law was slightly better, so we used that for the final shape models. The best model had a mixing parameter of 0.000001, which is almost pure Lommel–Seeliger. (For SHAPE’s implementation of the Kaasalainen law, the mixing parameter can vary from zero to one, with 0.0 being pure Lommel–Seeliger and 1.0 being pure Lambert.) All lightcurves were treated as relative photometry, so we cannot provide any independent information on the absolute magnitude or visible albedo of Midas.

The north–south ambiguity present in radar imaging creates a challenge for shape modeling. In many cases, both senses of rotation may yield equally good fits to the radar data, so it is only possible to determine two preferential models, one with a pole direction north of the ecliptic plane (which, for Midas, would be prograde rotation) and one with a southern pole direction (retrograde rotation). We began constructing our model with this in mind, running separate fits on both a prograde model and a retrograde model. By doing this, we remove the bias toward either orientation that would arise from running a southern pole-direction search on a vertex model that had been created with a fixed northern pole direction (which would create a bias against the southern pole direction) or vice versa.

The prograde model started out as a viable model, but after folding in additional lightcurve observations, it was clear that prograde rotation did not fit all of the data. This was largely due to having lightcurves that were taken when Midas was at a variety of viewing geometries (see Table 2). By using these lightcurve data, we were able to see that the models with prograde rotation not only have higher chi-square values but also are visibly worse fits. These prograde models could fit either the delay-Doppler images or the lightcurves, but not both. In the example shown in Figure 5, the model is a good fit for the lightcurve data and a poor fit for the delay-Doppler images, while the example in Figure 6 is a bad fit for the lightcurve data and a better fit for the delay-Doppler images.

4. Results

4.1. Pole Direction

Finding the orientation of the rotational axis (pole direction) in terms of ecliptic longitude and latitude was one of the more human-power-intensive parts of the shape modeling process because SHAPE does not easily determine the best pole direction. By using the grid search method discussed in Section 3.2, we found a final pole direction of (39°, –60°). The obliquity of Midas is 152°.

In order to determine the uncertainty of this pole direction, we ran a grid search of models with fixed pole directions near the best-fit pole. For these models, we allowed SHAPE to change the length along each axis, the rotation phase, and the photometric properties. A plot of the pole directions of these models and the resulting reduced chi-square values is given in Figure A6. Based on visual inspection of numerous models, we determined the 1σ uncertainty in the pole direction to be 6°. We
could not avoid having to make some subjective decisions about which models could be considered acceptable, which also was done in Magri et al. (2007, 2011), and Nolan et al. (2013).

A grid search over a range of densely spaced rotation periods yielded a comb of possible solutions, as was found for 101955 Bennu (Nolan et al. 2013), with spacing that corresponds to (very nearly) an integer or half-integer number of rotations between the various apparitions. There is no unique solution for the rotation period of Midas, but lightcurves from one or two additional apparitions may be sufficient to break this degeneracy. For the sidereal rotation period of Midas, we adopt a best-fit value of 5.2216 hr, with an uncertainty of 0.0017 hr, which is in agreement with rotation periods derived from existing lightcurves (Mottola et al. 1995; Wisniewski et al. 1997; Muinonen et al. 2007; Franco et al. 2018). This is one of the shortest known rotation periods of any published NEA contact binary; the only faster rotators are Castalia (4.0 hr, Hudson & Ostro 1994) and 2001 KZ66 (4.99 hr, Zegmott et al. 2021). Rotational acceleration (YORP) is not necessary to fit the entire optical data set, given our uncertainty in the spin period.

4.2. Shape Model

We present the shape model of the potentially hazardous asteroid 1981 Midas from lightcurve and radar observations. Our best shape model of Midas is shown in Figure 7, and additional views of the model with comparisons to the data are shown in the Appendix. This model is made up of 532 vertices that define 1060 triangular facets, with a mean edge length of 0.17 km. Its properties are given in Table 3.

The shape appears rather flat and angular, but features like the knob on the small lobe seem persistent across many models. (The largest knob on the smaller lobe is about 500 m across, so it probably is too large to be a boulder.) This flatness is uncommon among modeled contact binary NEAs but closely resembles that of Kuiper Belt object 486958 Arrokoth (Spencer et al. 2020). Though Arrokoth and Midas share similarities in their shapes, they may have formed by different mechanisms because they are different sizes and are located in very different regions of the solar system.

Increasing the weights of SHAPE’s penalties forces the software to make changes to the model (e.g., smoothing) that yield worse fits to some of the data. However, we found that greatly increasing the weights of SHAPE’s nonsmooth and concavity penalties yields similar models—the depth of the concavity is about the same, and the small lobe has the same bump. Somewhat surprisingly, the lightcurves seem more sensitive to small changes in the shape than the radar data. As we increased the penalty weights, we saw that relative chi-square values increased more quickly for the lightcurves than for the radar data. Increasing the weights for the lightcurve data sets, thus forcing SHAPE to fit those better at the expense of worse fits to the radar data, led to models with some questionable angular features.

To find the 1σ uncertainties of Midas’s dimensions, we ran a set of models with varying lengths along each axis. We found

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10. [https://echo.jpl.nasa.gov/lance/binary.neas.html](https://echo.jpl.nasa.gov/lance/binary.neas.html)
relative uncertainties of 9% for the length along the \( x \) direction (longest principal axis), 11% for \( y \), and 29% for \( z \) (shortest axis). Given the pole direction, the sub-radar latitude on Midas during the Arecibo imaging observations ranged from \(-7^\circ\) to \(-40^\circ\), so Midas’s equatorial dimensions are better constrained than its polar dimension, and details of the shape at high northern latitudes are poorly constrained. Some of the best constraints on the \( z \) length come from lightcurves in early 2018 March, in which the sub-observer latitudes were highest (greater than \(+50^\circ\)). Models with very different \( z \) lengths do not reproduce the amplitudes of those lightcurves. Marshall et al. (2017) similarly found that lightcurves gave better constraints than radar data for the polar dimension of NEA 162421 (2000 ET70), with the lightcurves favoring a shape model that was shorter along its \( z \)-axis than the radar-only model of Naidu et al. (2013). With its long axis measuring 3.4 km, Midas is among the largest NEA contact binaries reported to date. Of the 40 NEAs that were clearly resolved in delay-Doppler images by Virkki et al. (2022), 11 (28%) appear to be contact binaries.

4.3. Radar Scattering Properties

Radar scattering from Midas’s surface is represented with a cosine law (Mitchell et al. 1996). Table 4 lists the radar cross sections in the two circular polarizations, radar albedos, and circular polarization ratios that were measured in the CW spectra from Arecibo.

There are three sources of error for the cross-section measurements (\( \sigma_{OC} \) and \( \sigma_{SC} \)): thermal noise from the receiver’s finite system temperature, self noise from the statistical properties of observed radar signals (Ulaby et al. 1982), and systematic errors in the radar systems’ calibration due to imperfections in telescope pointing and transmitter power.

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Figure 5. Example views of the model with a northern pole direction that best fit the lightcurve data. From left to right, the delay-Doppler frames (c), (d) are as follows: delay-Doppler data, model, plane-of-sky view of the model. The colored arrows are as in Figure 2. The model is a good fit for the lightcurve data (a), (b) but is a poor fit for the delay-Doppler images (c), (d). This is compelling evidence to confirm the preference of a southern pole direction (retrograde rotation) over this northern pole direction (prograde rotation).
measurements. When averaging over multiple observations, the errors from thermal noise and self noise add in quadrature, so their relative contributions decrease. However, the relative uncertainty from calibration remains constant, no matter how many observations are taken. Therefore, the calibration uncertainty is the dominant source of error. Based on past experience (Brozović et al. 2011; Nolan et al. 2013; Virkki et al. 2022), we have assigned relative uncertainties of 25% for cross sections measured from Arecibo data, and these propagate into the uncertainty of the radar albedo.

When taking a ratio of cross sections, those calibration errors mostly cancel, but not perfectly, because the system temperatures may differ slightly between the two polarizations. When calculating the circular polarization ratios, \( \mu_C = \frac{\sigma_{\text{SC}}}{\sigma_{\text{OC}}} \), we therefore add a relative error of 5% in quadrature with the uncertainties from thermal noise and self noise. However, radar albedo \( \delta = \frac{\sigma}{\lambda \pi} \) has an even larger relative uncertainty than radar cross section because the 30% uncertainty of each projected area also contributes to the uncertainty of the albedo.

Midas is an elongated object, so its projected area can vary considerably as it rotates. However, during the CW observations on all five nights from Arecibo, we happened to see Midas broadside—its long axis was always within 30° of being

**Figure 6.** Example views of the model with a northern pole direction that best fit the radar data. The delay-Doppler frames are organized as in Figure 5. The model is a poor fit for the lightcurve data (a), (b), but is a better fit for the delay-Doppler images (c), (d). This is further evidence to confirm the preference for retrograde rotation rather than prograde rotation.
perpendicular to our line of sight, but we saw both of its sides. This explains why we only saw the bandwidth varying by about 10% during the Arecibo CW observations on any single night.

The derived radar albedo values hint that Midas has a higher albedo near its south pole than near its equator. However, because these differences are comparable to the uncertainty in each night’s radar albedo, we conclude that there is no convincing evidence for inhomogeneities in the radar-scattering properties of Midas’s surface.

Ostro et al. (1991) measured a circular polarization ratio (CPR) of $\mu_C = 0.65 \pm 0.13$ at 3.5 cm. We measured a CPR of $\mu_C = 0.83 \pm 0.04$ at Arecibo’s 13 cm wavelength. According to Benner et al. (2008), such a high polarization ratio would imply a surface that is rough at decimeter scales (possibly with coherent backscattering). However, the interpretation of a simple relation between CPR and roughness is disputed, as the role that dielectric properties play in CPR has not been properly demonstrated. Lauretta et al. (2019) found that the surface of 101955 Bennu is very rough, despite Nolan et al. (2013) having observed a low CPR, $0.18 \pm 0.03$ at 13 cm.

This CPR is the highest seen to date for any NEA with visible and near-infrared spectral type V (Binzel et al. 2001, 2004, 2019); it even overlaps the distribution of polarization ratios seen for some E-class NEAs, though the visible and near-infrared spectroscopy of Midas is inconsistent with the E class. Benner et al. (2008), looking at data for six V-type NEAs, found that they have a mean polarization ratio of 0.603, with a standard deviation of 0.088. Previously, the V-type NEA with the greatest known polarization ratio at 2380 MHz was 3908 Nyx (Benner et al. 2002), with a ratio of $0.78 \pm 0.02$. The nominal ratio for Midas falls slightly above

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**Figure 7.** The principal-axis views of the best model of Midas. Facets with yellow shading were seen at incidence angles greater than 60° (or not seen at all) in all delay-Doppler images and therefore are not well constrained.

**Table 3**

Properties of 1981 Midas

| Parameter                          | Value  | Uncertainty | Rel. Unc. |
|------------------------------------|--------|-------------|-----------|
| Dimensions along principal axes (km)$^a$ | $x$    | 3.41        | 0.29      | 9%        |
|                                    | $y$    | 1.90        | 0.21      | 11%       |
|                                    | $z$    | 1.27        | 0.37      | 29%       |
| Surface area (km$^2$)$^a$          | $A$    | 13.03       | 3.85      | 30%       |
| Volume (km$^3$)$^a$                | $V$    | 2.88        | 1.28      | 44%       |
| Volumetric mean diameter (km)$^a$  | $D$    | 1.77        | 0.26      | 15%       |
| DEEVE extents (km)                 | $2a$   | 3.59        | 0.31      | 9%        |
|                                    | $2b$   | 1.52        | 0.17      | 11%       |
|                                    | $2c$   | 1.01        | 0.29      | 29%       |
| Moment of inertia ratios$^a$       | $I_{x}/I_{z}$ | 0.219 | 0.042       | 19%       |
|                                    | $I_{y}/I_{z}$ | 0.915 | 0.025       | 3%        |
| Sidereal rotation period (hr)      | $P$    | 5.2216      | 0.0017    | 0.03%     |
| Pole direction (ecliptic)          | ($\lambda$, $\beta$) | (39°, $-60^\circ$) | 6°        |

Notes. Parameters and 1σ uncertainties for the shape model of 1981 Midas.

$^a$ Our analysis showed that the uncertainties of the lengths along the three axes are all positively correlated with each other. The uncertainties in the model’s surface area, volume, and other derived quantities are therefore greater than what they would be if those uncertainties were all uncorrelated. The volumetric mean diameter is the diameter of a sphere that would have the same volume as this shape model. The dynamically equivalent equal volume ellipsoid (DEEVE) is the uniform-density ellipsoid that has the same volume and moment of inertia ratios as the shape model. The moment of inertia ratios were derived with the assumption that the model’s density is homogeneous.
Table 4  
Radar Scattering Properties of Midas

| Observing Date (UT) | Sub-radar lon, lat | $Λ_{proj}$ (km$^2$) | # Runs | $σ_{OC}$ (km$^2$) | $δ_{OC}$ | $σ_{SC}$ (km$^2$) | $δ_{SC}$ | $μ_c$ |
|---------------------|--------------------|----------------------|--------|------------------|--------|------------------|--------|------|
| 2018 Mar 21         | 259, −7            | 3.0                  | 2      | 0.86             | 0.29   | 0.74             | 0.25   | 0.85 |
| 2018 Mar 22         | 104, −16           | 3.0                  | 8      | 0.85             | 0.28   | 0.68             | 0.23   | 0.81 |
| 2018 Mar 23         | 281, −25           | 3.1                  | 7      | 0.83             | 0.26   | 0.68             | 0.22   | 0.83 |
| 2018 Mar 24         | 101, −32           | 3.3                  | 8      | 1.37             | 0.41   | 1.15             | 0.35   | 0.84 |
| 2018 Mar 25         | 258, −39           | 3.4                  | 8      | 1.14             | 0.33   | 0.91             | 0.27   | 0.80 |

Overall $0.31 \pm 0.12$ $0.26 \pm 0.10$ $0.83 \pm 0.04$

Note. Radar scattering properties of Midas at 12.6 cm, from Arecibo $S$-band observations. A sub-radar longitude of zero is within $2^\circ$ of the $+x$ direction shown in Figure 7. Sub-radar coordinates and projected area values ($Λ_{proj}$) are derived from the best-fit shape model. # runs is the number of CW scans from which cross sections could be measured; there were a few scans for which telescope-pointing errors were too large to allow for reliable cross-section measurements. $σ_{OC}$ and $σ_{SC}$ are the measured cross sections for the opposite-sense (OC) and same-sense (SC) circular polarizations. $δ_{OC} = \frac{σ_{OC}}{Λ_{proj}}$ and $δ_{SC} = \frac{σ_{SC}}{Λ_{proj}}$ are the radar albedos for the two polarizations. $μ_c$ is the circular polarization ratio.

that range, but the circular polarization ratios for Midas and Nyx are consistent within their uncertainties.

4.4. Astrometric Measurements

The shape model tells us Midas’s dimensions, so we know where its center of mass is relative to the leading edges in the various delay-Doppler images. We used the final shape model to obtain astrometric measurements of Midas that are more accurate than what one can obtain from visual inspection of the images. Our final delay measurements are given in Table 5.

For the Arecibo images, the accuracy of the measured ranges is mainly limited by uncertainty in the dimensions of the shape model, rather than uncertainty in where the center of mass is relative to the observed leading or trailing edges. We note that Midas’s center of mass is at a substantially greater delay (by several baud lengths) than what we had estimated from visual inspection when entering preliminary delay measurements based on images in which Midas was seen broadside. In other words, Midas’s center of mass is several pixels past its (rather faint) apparent trailing edge in many images.

We are only reporting delay measurements because the precision of Doppler measurements would be limited by the frequency resolution of the processed data. The radar data have frequency resolution that is coarser (greater) than the a priori Doppler uncertainties; therefore, we cannot significantly refine Midas’s orbit with any Doppler measurements.

For the next Midas apparition, in 2032, the new shape-based radar astrometry included in JPL orbit solution s227 reduces the plane-of-sky uncertainty by 23.1% compared to an optical-only solution, even with Midas having been tracked optically over 20 orbit periods. With s227, the 1σ uncertainty in the delay of Midas around the time of its closest approach in 2032 will be about 240 μs, so delay measurements in 2032 will further improve our knowledge of Midas’s orbit.

Using the 2018 radar astrometry, the interval over which its Earth encounters can be calculated deterministically is unchanged, from the year 1237 to 2930, not including any currently undetected Yarkovsky acceleration. However, the timing of the encounters at the extreme dates is adjusted by 2–16 minutes, and their 3σ uncertainties are reduced by up to 20 minutes in the s227 radar solution, relative to an optical-only solution.

Table 5

| Date       | Time (UT) | Measurement (s) | Uncertainty (μs) |
|------------|-----------|-----------------|-----------------|
| 2018 Mar 21| 22:59:00  | 89.35836122     | 1.11            |
| 2018 Mar 22| 23:24:00  | 90.86319154     | 1.53            |
| 2018 Mar 23| 22:02:00  | 94.51675378     | 1.35            |
| 2018 Mar 23| 22:26:00  | 94.81132308     | 1.19            |
| 2018 Mar 24| 21:24:00  | 100.30994632    | 1.29            |
| 2018 Mar 25| 21:34:00  | 108.10872141    | 1.65            |

Note. 1981 Midas astrometric measurements derived from the final shape model determination of the center of mass. The measurement reference point for Arecibo is the center of curvature of the dish. The uncertainties come from two terms added in quadrature: the baud length of the images (0.5 μs for all images used here) and the uncertainty in Midas’s delay depth (which depends on its orientation at the time of each observation).

4.5. Gravitational Environment

Based on approximating Midas as two spheres with a volume (mass) ratio of about 2:1, its density must be greater than about 1500 kg m$^{-3}$ to avoid rotational fission (Scheeres 2007). HED meteorites are the closest analog to a V-type asteroid like Midas. These meteorites have a grain density of about 3400 kg m$^{-3}$ (Consolmagno et al. 2008), which would be the greatest plausible density for Midas, if it has negligible porosity. From comparison with measured densities of other type-V asteroids (Carr 2012), Midas likely has a density that is closer to the lower limit.

Using the polyhedron method of Werner & Scheeres (1996) and Naidu et al. (2020), we generated maps of the accelerations and slopes on the surface of the Midas shape model (Figure 8). We assumed a uniform density of 2000 kg m$^{-3}$. The acceleration on the surface is the sum of the gravitational acceleration from the asteroid’s mass and the centrifugal acceleration from its rotation. Figure 8(a) shows the magnitude of the total acceleration, which ranges from about 0.15 mm s$^{-2}$ at the end of the small lobe to 0.45 mm s$^{-2}$ at high latitudes on the large lobe. Centrifugal acceleration is highest at the equator and acts in a direction opposite to the gravitational acceleration, hence the total acceleration is lowest at the equator. The magnitude of the acceleration generally increases as latitude increases; thus, centrifugal acceleration is significant. The acceleration decreases as one moves farther from the center of mass (at a given latitude).
Figure 8 (b) shows each facet’s gravitational slope, which is defined as the angle between the inward-pointing surface normal and the local acceleration vector. The greatest slopes, approximately 46°, occur at low latitudes on some of the small lobe’s bumps. These slopes are just slightly greater than the typical angle of repose for granular materials (approximately 40°). However, because nearly all of the surface slopes are less than that angle of repose, and only a
few facets have slightly greater slopes, Midas probably does not need any cohesion to maintain its shape; it could be a rubble pile. This is quite different from 65803 Didymos, which has many facets with slopes greater than the angle of repose (Naidu et al. 2020).

5. Future Work

The next opportunity for radar observations of Midas will be in 2032. On September 14, it will pass 0.0864 au (33.6 lunar distances) from Earth. On that date, it will be at declination +14°. The sub-observer latitude will be about −34°, similar to what was seen on 2018 March 24. With a system equivalent to Arecibo’s pre-Maria performance, the S/N of Midas on 2032 September 14 would be about 600 per scan and 4000 per day (“day” meaning a full track, which at Arecibo was a maximum of about 2.5 hr), enough for images with much finer resolution than those from 2018. Monostatic observations with Goldstone DSS-14 would yield an S/N of about 30 per scan and 400 for a six-hour track, which would cover more than one full rotation. For bistatic observations with DSS-14 transmitting and the GBT receiving, the S/N would be about 50 per scan (86 seconds) and 800 for a six-hour track. (For comparison, the S/N for the Arecibo observations on 2018 March 21 was about 100 per scan and 700 per day.) The sub-observer latitudes will be negative around the time of the closest approach in 2032. Starting on September 21, when Midas will be 0.135 au from Earth and receding, the sub-observer latitudes will be positive. Observations from September 21 onward will provide information on the shape at high northern latitudes that were not seen during the radar observations in 2018.

There are no preexisting shape models of Midas to compare with this model, but because Midas is a large PHA, acquiring radar data should be prioritized during its 2032 close approach. Creating another model from these future observations would allow for a check on the accuracy of this model as well as the development of a more detailed model with better radar coverage. Radar-ranging observations in 2032 could also result in the detection of the Yarkovsky effect for Midas, which would constrain the mass and density of the object. Additionally, because Midas has a bimodal mass distribution, it cannot be well approximated by simple shapes, such as spheroids and ellipsoids. For orbital dynamics studies, the shape of the object is important for a good representation of the gravity field in order to better study the dynamical environment around it, consequently assisting in small-bodies space mission planning (Werner & Scheeres 1996; Fahnestock & Scheeres 2007; Agrusa et al. 2020).

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Appendix

The sky positions of Midas at the time of each observation are summarized in Figure A1. We present the radar observations in Figures A2 and A3, and lightcurve observations in Figure A4. Figure A5 shows an example of an early grid search to determine the pole direction of the model, and Figure A6 shows the pole-direction chi-square values within 1σ of the best-fit pole-direction.
Figure A1. Sky positions (R.A. and decl.) of Midas during observations. Yellow-green triangles indicate Arecibo radar observations, cyan triangles indicate Goldstone radar observations, and purple circles indicate lightcurve observations.
Goldstone opposite-sense circular (OC) CW spectra. Data are plotted in red with model spectra overlaid in blue and the plane-of-sky view of Midas to the right. In the plane-of-sky view, the magenta arrow shows the rotational axis, and the red and green shafts show the long and intermediate principal axes, respectively (the short axis is aligned with the rotational axis). All of the Goldstone CW spectra were summed over many scans and decimated in frequency so that each Goldstone sum that was used for shape modeling had thousands of looks (independent samples). Therefore, self-noise is negligible for all of the Goldstone spectra, and the uncertainty of each data point is nearly equal to the thermal standard deviation. For the Arecibo CW spectra, self-noise was not negligible. These spectra were decimated in frequency so that each has 30 or 40 looks. Thus, for data points with a bright signal, self-noise dominates, and the relative uncertainty is approximately $1/\sqrt{N_{\text{looks}}}$, or about 15%–20%.

(The complete figure set (6 images) is available.)
Figure A3. Arecibo delay-Doppler images, 2018 March 21. Each column shows from left to right, the observed delay-Doppler image, corresponding delay-Doppler model image, and plane-of-sky view of the model. In the plane-of-sky view, the magenta arrow shows the rotational axis, and the red and green shafts show the long and intermediate principal axes, respectively (the short axis is aligned with the rotational axis).

(The complete figure set (5 images) is available.)
Figure A4. 1987 September lightcurve observations from Table Mountain Observatory and Steward Observatory. Observations are plotted in red, with model lightcurves overlaid in blue. In the plane-of-sky view, the magenta arrow shows the rotational axis, and the red and green shafts show the long and intermediate principal axes, respectively (the short axis is aligned with the rotational axis).

(The complete figure set (5 images) is available.)

Figure A5. An early grid search (left, southern/retrograde; right, northern/prograde) to determine the pole direction of the model using lightcurve data. Ecliptic longitude and latitude are plotted on the $x$- and $y$-axes, respectively. Each point is color-coded with the reduced chi-square value of the model compared to lightcurve data only, with purple indicating a lower reduced chi-square value and yellow indicating a higher reduced chi-square value. On the left, squares are good models, diamonds are marginal models, and circles indicate models that are outside the $1\sigma$ threshold. On the right, the square indicates the best of the prograde models, diamonds are some of the next-best models, and circles are worse models. Looking at these figures, it becomes clear that the models having a northern pole direction (the “best” such model having a reduced chi-square value of about 17) are much worse fits for the lightcurve data.
Figure A6. Plot of pole-direction chi-square values. The ecliptic longitude is plotted on the horizontal axis, and the ecliptic latitude on the vertical axis; it is colored by the reduced chi-square value of the model compared to the entire data set (delay-Doppler images, CW spectra, and lightcurves), with purple indicating a lower reduced chi-square value and yellow indicating a higher reduced chi-square value. Squares are good models, diamonds are marginal models, and circles indicate models that are outside the 1σ threshold.

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