Effect of resonance states of the weakly bound $^6$Li on total fusion in reactions with targets $^{28}$Si, $^{96}$Zr and $^{209}$Bi

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Abstract. Continuum Discretized Coupled-Channel calculations of total fusion cross sections for reactions induced by the weakly bound nucleus $^6$Li with targets $^{28}$Si, $^{96}$Zr and $^{209}$Bi at energies around the Coulomb barrier are presented. In the cluster structure frame of $^6$Li$\rightarrow\alpha+d$, short-range absorption potentials are considered for the interactions between the $\alpha$- and $d$-fragments with the targets. In the model, the effect of resonance ($l = 2, J^* = 3^+, 2^+, 1^+$) and non-resonance breakup states of $^6$Li on fusion is studied by using two approaches: (1) by omitting the resonance states from the full discretized CDCC breakup space and 2) by considering only the resonance discretized sub-space. Among other things, it is found that resonance breakup states produce strong repulsive polarization potentials that lead to fusion suppression. The particular effect on fusion of a single resonance is calculated. It is found that resonances $2^+$ and $1^+$ have a more significant role on incomplete fusion, while the $3^+$ on complete fusion.

1. Introduction
One of the most interesting and important topics of research of reactions with weakly bound nuclei is to understand the effects of breakup on elastic scattering and fusion processes. It is already well accepted that couplings to continuum breakup states of weakly bound projectiles have a very strong effect on these nuclear mechanisms. As a matter of fact, consideration of breakup is essential to fit the experimental data [1, 2, 3, 4, 5]. The characteristic low binding energy associated to weakly bound projectiles can affect reaction processes mainly in two ways. Namely, the static effect due to the large diffusivity matter density and the high breakup probability. The stretched density lowers the Coulomb barrier and hence enhances fusion. On the other hand, the high breakup probability produces strong repulsive couplings that raise the barrier and then suppresses fusion. Also, reactions with weakly bound projectiles present characteristic nuclear processes like complete (CF) and incomplete (ICF) fusion. Complete fusion can proceed via a direct (DCF) or sequential (SCF) processes. DCF is a mechanism similar to fusion between strongly bound nuclei, that is, fusion that occurs without a previous excitation of breakup channels. SCF takes place when all projectile fragments fuse to the target after a previous breakup. ICF occurs when only some fragments are absorbed by the target, while the others fly away to the continuum. Another important process is the elastic breakup (EB), that is, when none of the fragments is captured by the target.
From an experimental viewpoint, DCF and SCF are very difficult to distinguish, since the evaporation products of these processes are very similar. On the other hand, only until recently, it has been possible to perform separate measurements of complete and incomplete fusion. These measurements have been restricted to reactions with light weakly bound projectiles on medium and heavy mass targets. In this case, the compound nucleus decays by emission of uncharged particles, mainly neutrons and α-particles [6, 7, 8], thus separate measurements of CF and ICF cross sections can be achieved [9, 10]. In contrast, for light targets, thermal evaporation occurs by an important contribution of charged particles and measurements become more difficult. Therefore, most of fusion measurements for reactions with weakly bound projectiles, correspond to Total Fusion (TF), which is the sum of complete and incomplete fusion. From an experimental viewpoint, DCF and SCF are very difficult to distinguish, since the evaporation products of these processes are very similar. On the other hand, only until recently, it has been possible to perform separate measurements of complete and incomplete fusion. These measurements have been restricted to reactions with light weakly bound projectiles on medium and heavy mass targets. In this case, the compound nucleus decays by emission of uncharged particles, mainly neutrons and α-particles [6, 7, 8], thus separate measurements of CF and ICF cross sections can be achieved [9, 10]. In contrast, for light targets, thermal evaporation occurs by an important contribution of charged particles and measurements become more difficult. Therefore, most of fusion measurements for reactions with weakly bound projectiles, correspond to Total Fusion (TF), which is the sum of complete and incomplete fusion.

From a theoretical standpoint, for a complete description of reactions with weakly bound nuclei all of the aforementioned reaction mechanisms should be included in a single calculation. The CDCC (Continuum Discretized Coupled-Channel) formalism [11, 12, 13] is the most widely applied theoretical tool to study breakup states in the continuum. Likewise, the effects of continuum couplings on processes like elastic scattering and fusion can be studied. In fact, the CDCC model has been successfully applied in many nuclear systems [14, 15, 16, 17, 18, 19], since it provides a good description for the observed NCBU, elastic and TF cross sections [14, 15, 20, 21, 22]. Notwithstanding, most CDCC calculations have a shortcoming [21, 23], as they cannot calculate the ICF and CF cross sections separately. That is, a simultaneous description of complete fusion and incomplete fusion is very difficult to achieve. However, recently some efforts have been made in which the simultaneous CDCC calculation of CF and ICF has been carried out. For instance, reactions between the weakly bound projectiles $^6, ^7\text{Li}$ with targets $^{209}\text{Bi}$ and $^{198}\text{Pt}$ [24] and $^6\text{Li}$ with $^{144}\text{Sm}$ and $^{154}\text{Sm}$ [25] have been studied. In these calculations, separate incomplete fusion of the fragments $a-d$ of $^6\text{Li}$ or $a-t$ of $^7\text{L}$ are determined. The technique used in these works, consists of introducing two short-range Woods-Saxon potentials to account for absorption between the fragments, say $a$ and $b$, and the target, while CF is calculated with a different potential in the incident channel. Subsequently, two types of calculations are performed, first, one of the fragment-target potentials, say $W_a$, is turned-off so as to calculate fusion capture of the other fragment $b$ and the incident channel. In a second calculation, this process is repeated for the other fragment, that is, $W_b$ is turned-off, so fusion of fragment $a$ and the incident channel is determined. The contribution to incomplete fusion corresponding to any fragment is then determined by subtracting these results from total fusion. However, there is a serious drawback in this treatment as pointed out in Ref. [26], that is, the competition and correlation between both fragment-target capture processes have been neglected. That is, the capture of a given fragment affects the absorption of the other. Besides, it is assumed in Ref. [24] that breakup of $^6\text{Li}$ happens mainly in the $a-d$ channel, however, recent experimental data show that other important breakup triggered by neutron transfer and deuteron pick processes are significantly important [27]. The effect of neglecting the competition and correlation between the fragment capture absorption processes has been discussed in Ref. [26] for the $^6\text{Li}+^{209}\text{Bi}$ system. More realistic approaches have been proposed to separately
calculate CF and ICF, for instance, the classical dynamical reaction model described in Refs. [28, 29, 30, 31]. This model unambiguously calculates individual contributions to incomplete fusion from the fragments, besides the contributions to CF from sequential and direct processes are explicitly determined. Among other things, it is shown in this model that sequential complete fusion has a more important role than previously assumed. However, this model being a classical model, can not treat fusion for energies below the Coulomb barrier, where tunneling effects are essential.

The CDCC model has been used to calculate total fusion cross sections of several weakly bound systems. For instance, it has been applied for reactions between the projectiles $^6$Li with targets $^{59}$Co and $^{209}$Bi [16], or the nucleus $^{11}$Be on the target $^{208}$Pb [15]. One of the most important aspects studied in Refs. [16, 15], is to clarify whether breakup produces enhancement or suppression effects on total fusion, as well as its energy dependence. It may be mentioned that for the projectile $^{11}$Be, total fusion is calculated in Ref. [15] by the absorption of the its center of mass. That is, since breakup occurs mainly by the process $^{11}$Be $\rightarrow ^{10}$Be + n, thus the capture of the center of mass of $^{11}$Be implies the absorption of $^{10}$Be, which carries most of the mass and charge of the projectile. However, this is not the case for nuclei such as $^6$Li, for which the main breakup channels are $^6$Li $\rightarrow \alpha + d$ and $^7$Li $\rightarrow \alpha + t$. In both cases the fragment masses are not so different, thus fusion can not be calculated by the absorption of their center of mass. For this reason, two short-range absorption potentials are used to account for capture of $\alpha$ and $d$ for $^6$Li or $\alpha$ and $t$ for $^7$Li as done in Ref. [16]. In this way, complete fusion (both direct and sequential) is accounted for when both fragments are captured, that is, both are inside the region of absorption. Similarly, incomplete fusion is taken into account when only one fragment is inside its respective short-range potential while the other is outside and flies to the continuum. Clearly, this procedure can not distinguish whether CF occurs by a direct or sequential process.

Hence, in the present work, we present a systematic CDCC calculations of total fusion for the weakly bound projectile $^6$Li with targets $^{28}$Si, $^{96}$Zr and $^{209}$Bi at energies around the Coulomb barrier. Following Ref. [16], two short-range imaginary volume Woods-Saxon potentials will be used to account for fusion absorption of the fragments $\alpha$ and $d$ by the target. Besides determining the effect of breakup on fusion, the effects of resonance and nonresonance scattering excited states $^6$Li are studied by extracting the corresponding sub-space from the full discretized space. An interpretation of these effects will be given in terms of the polarization potentials that result from the corresponding couplings. An analysis of effects on fusion from resonance and nonresonance states as function of the target mass is also presented. In fact, so as to isolate the effects of the resonances, couplings to excited states of the target will be ignored.

It is important to mention that this work is a sequence of our previous CDCC calculations of elastic scattering angular distributions and the effect of resonance and nonresonance breakup continuum states of $^6$Li on targets $^{28}$Si, $^{58}$Ni [32] and $^{144}$Sm [33]. In these calculations systematic Woods-Saxon volume and surface potentials were used for the interactions between the $\alpha$-particle and deuteron with the target. The effect of resonance and nonresonance couplings was determined by following two different approaches. First, by omitting couplings to/from resonance states from the full discretized CDCC breakup space, secondly by considering only couplings to/from the resonance sub-space. For the calculations with resonance couplings (nonresonance space excluded), the elastic scattering angular distributions were very much similar to that with the full discretized energy space. Therefore, consideration of these states is essential. On the other hand, when resonance states are excluded (nonresonance couplings included) a larger deviation is present, particularly for $^{58}$Ni and $^{144}$Sm at energies below the barrier and for backward dispersion angles. The effects of resonance and nonresonance sub-spaces were explained in terms of the polarization potentials that appear from the corresponding couplings. It was determined that the polarization potentials produced by resonance couplings are strongly repulsive, therefore suppress fusion. Those from the nonresonance sub-space are
weakly repulsive and thus have a smaller effect. A comparison regarding the effects of resonance and nonresonance couplings on elastic scattering and those presented in this work for total fusion will be also discussed.

The recent work of Ref. [26] for the $^{6}$Li+$^{209}$Bi system establishes the significant importance of $2^+$ and $1^+$ resonance states of $^6$Li on ICF and NCBU processes. This is explained in terms of the shorter half-life of these states with respect to the $3^+$ resonance. For the $3^+$-resonance having a longer half-life, delayed breakup takes place. Thus, breakup of the $3^+$-state can occur far from the target, at the outgoing part of the trajectory, thus a small effect on ICF is expected. If breakup happens near the target, then it has a more important effect on CF than on ICF. On the other hand, for the $2^+$ and $1^+$ states prompt breakup takes place thus a larger effect on ICF is expected. In a final calculation, we study the effect on fusion of a given resonance by omitting couplings to only this state, then a discussion of the results is presented.

As a last comment, it is necessary to emphasize that our calculations are restricted to the assumption that the main breakup mechanism of $^6$Li occurs into the $\alpha$ and $d$ channel. Recent measurements [27] have established that significant breakup of $^6$Li is triggered by neutron transfer leading to sequential $p-\alpha$ breakup, or via $d$-pickup by $^6$Li leading to sequential breakup of $^8B \rightarrow \alpha + \alpha$. However, these measurements also show that the main breakup dissociation of $^6$Li occurs through the $\alpha - d$ channel. Treatment of more complicated four-body CDCC calculations of sequential breakup triggered by particle transfer is of current theoretical interest but not attempted in this work.

In section II, a very brief description of the CDCC model is presented as well as the numerical calculations. In section III, a summary and conclusions are presented.

2. Brief CDCC description and calculations

A detailed description of the CDCC formalism is given in Refs.[11, 12, 13]. Also, a formal description of how the CDCC is used to study the effects of resonance and nonresonance states is presented in Refs. [32, 33]. In this section, only the basic equations that are required to perform our calculations are presented. The two-body cluster model structure of $^6$Li ($\alpha-d$) with ground state energy $E_{g.s.} = -1.47$ MeV is assumed. The model space for constructing discrete breakup states is that described in Ref. [16]. The total wave function of the three-body system ($\alpha-d$-target) is given by,

$$
\Psi(R, r) = \sum_{\beta} F_{\beta}(R) \psi_{\beta}^{P}(r) \Phi_{0}^{T},
$$

where $\Phi_{0}^{T}$ is the ground state of the target. $F_{\beta}(R)$ represents the projectile-target relative radial motion, where the projectile is in the discrete breakup state $\beta$. Here, $R$ is the relative distance between the target and the center of mass of the cluster $^6$Li. The $\beta$-channel can be the elastic incident channel or any discrete excited state of the projectile. $\psi_{\beta}^{P}(r)$ describes the radial relative motion of the ($\alpha,d$)-system, where $r$ is the relative distance between the fragments. The coupled-channel equations are obtained by projecting the equation of motion $\hat{H}\Psi = E\Psi$ onto the discrete radial wave function $\psi_{\beta}^{P}(r)$ of the projectile.

$$
[\hat{T}_{R,K} + U_{\beta,\beta'}^{(J)}(R) - (E - \epsilon_{0} - \epsilon_{\beta})]F_{\beta}^{(J)}(R) = - \sum_{\beta' \neq \beta} U_{\beta,\beta'}^{(J)}(R) F_{\beta'}^{(J)}(R).
$$

Where, $\epsilon_{0}$ is the ground state energy of the target (excited states are not considered) and $\epsilon_{\beta}$ the ground and discrete breakup states of the projectile. $U_{\beta,\beta'}^{(J)}$ are the diagonal ($\beta = \beta'$) and
non-diagonal ($\beta \neq \beta'$) coupling matrix elements given by,

$$U^{(j)}_{\beta \beta'}(R) = < u_\beta | \hat{V}_{dT}(r_{dT}) + \hat{V}_{dT}(r_{dT}) | u_{\beta'} >,$$

where $u_\beta(r)$ are the normalized square-integrable wave functions known as bin states [11, 32, 33], which are functions of the relative distance $r$ between the fragments and that satisfy $r_{dT} = R + \frac{6}{5}r$. $\hat{V}_{dT}$ and $\hat{V}_{dT}$ are the $\alpha - T$ and $d - T$ real interactions that include Coulomb and nuclear parts and depend on the distance between the fragments to the target. To account for fusion, two short-range imaginary potentials $W_{\alpha T}$ and $W_{dT}$ will be used for the $\alpha - T$ and $d - T$ interactions, i.e.,

$$W(r_i) = W_0 f^{-1}(r_i), \quad f(r_i) = 1 + exp[(r_i - R_0)/a_0],$$

where $r_i = r_{dT}$ and $r_{dT}$ are distances between the fragments to the target, $a_0$ is the diffuseness and $R_0 = r_0(A_1^{1/3} + A_2^{1/3})$, $A_T$, $A_\alpha$ and $A_d$ being the target, $\alpha$-particle and deuteron mass numbers respectively. These potentials are equivalent to the use of an incoming boundary condition inside the Coulomb barrier. Total fusion is calculated by,

$$\sigma_{TF} = \frac{2}{h v} \sum_{J, \beta} \frac{1}{2J + 1} < F^{(j)}_{\beta}(R)|W^{(j)}_{\beta \beta'}(R)|F^{(j)}_{\beta}(R) >,$$

where $v$ is the projectile-target collision velocity in the center of mass and $W^{(j)}_{\beta \beta'}(R) = < u_\beta | \hat{W}_{dT}(r_{dT}) + \hat{W}_{dT}(r_{dT}) | u_{\beta'} >$.

The procedure to construct bin discretized states of $^6$Li is essentially the same as that used in the CDCC calculations of elastic scattering of Refs. [32, 33]. That is, the ground (binding energy -1.47 MeV), resonance and nonresonance bin scattering states of $^6$Li $\rightarrow d + \alpha$ are generated from the Hamiltonian $\hat{h}_{ud}(r, k)$, using the $\alpha - d$ nuclear and spin-orbit interactions of Ref. [16]. Careful attention is paid to construct resonance states $l = 2, j^z = 3^+, 2^+, 1^+$ with centroid energies and widths close to the experimental values and to avoid double-counting. The discretization procedure to obtain convergent calculations is carried out as follows: bin states are constructed from an initial energy $\varepsilon_0 = 0$ MeV (above the threshold energy $E_{\text{thres}} = -1.47$ MeV) up to a maximum energy $\varepsilon_{\text{max}} = 6.8$ MeV for energies below and around the barrier and $\varepsilon_{\text{max}} = 8$ MeV for higher energies. The relative angular momentum $l$ between the fragments $\alpha$ and $d$ varies from 0 to 3, higher values do not have an effect on the convergent calculations. Partial waves of the relative angular momentum between the center of mass of projectile and of the target are taken up to $L = 60$, while potential multipoles of nuclear and Coulomb interactions are considered up to $K = 4$. Following Ref. [16], continuum (nonresonant and resonant) breakup states are discretized in equally spaced momentum bins with respect to the relative wave number $k$ between the fragments. Nonresonance bin states ($l = 0, J^z = 1$), ($l = 1, J^- = 0, 1, 2$), ($l = 3, J^- = 2, 3, 4$) and resonance states $l = 2, J^z = 3^+, 2^+, 1^+$ are discretized with the same densities as reported in Ref. [16].

The calculations of total fusion cross sections are performed with the use of the FRESCO code described in Ref. [34]. It is important to point out that a similar systematic analysis was reported in Refs. [32, 33], for elastic scattering angular distributions of $^6$Li with targets $^{28}$Si, $^{58}$Ni and $^{144}$Sm. However in these works, global nuclear and imaginary absorption potentials [35, 36, 37] were used for the $d - T$ and $\alpha - T$ interactions. In the present calculations, use is made of the same real nuclear interactions, however to account for fusion absorption, short-range volume Woods-Saxon potentials of Eq.(4), are used for the fragment-target interactions. In the calculations here presented, the parameters of these potentials will be set to, reduced radius
CDCC total fusion cross section calculations (solid-lines) induced by $^6$Li with targets $^{28}$Si, $^{96}$Zr and $^{209}$Bi. In the calculations, the complete discretized breakup space of the projectile is used.

\[ r_{dT} = r_{aT} = 1.0 \text{ fm}, \text{ diffuseness } a_0 = 0.1 \text{ fm and strength } W_0 = 50 \text{ MeV for all targets. Notice that, with these parameters the potentials are inside the Coulomb barrier for all targets.} \]

**Figure 1.** CDCC total fusion cross section calculations (solid-lines) induced by $^6$Li with targets $^{28}$Si, $^{96}$Zr and $^{209}$Bi. In the calculations, the complete discretized breakup space of the projectile is used.

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Figures 1a-1c show the results of total fusion for the $^6$Li projectile with targets $^{28}$Si, $^{96}$Zr and $^{209}$Bi respectively. The solid-lines correspond to the calculations with couplings with the full discretized breakup space. The data shown for $^{28}$Si are those of Ref. [38] (solid dots) and [39] (empty dots), for $^{96}$Zr of Ref. [40] and for $^{209}$Bi of Ref. [41]. Regarding the sensitivity of the TF calculations respect to the potential parameters of the short-range potentials, we noticed that smaller values of the reduced radius parameter do not have an appreciable effect as long as the potential is well inside the Coulomb barrier. As for the strength $W_0$ and diffuseness $a_0$, there is not appreciable effect if the range of the potential is larger than the mean free path of the fragment inside the barrier [16].

It is observed in Fig. 1 that the calculations are above the data for $^{28}$Si nucleus, while for the target $^{96}$Zr, the data [40] include lower limits of incomplete fusion measurements, therefore a larger total fusion data is expected. For the nucleus $^{209}$Bi the calculated TF agrees reasonably well with the data. It is important to mention that, we have used systematic nuclear potentials
for the interactions of the fragments $\alpha$ [36, 37] and deuteron [35] with the targets. These global potentials depend on the incident fragment energy and are the result of a large systematic study of elastic scattering data in a large range of target masses. So, for that reason some disagreement with the data may appear. Also, as mentioned above, we assume that breakup of $^6$Li occurs into the $\alpha$ and deuteron channel. Although, this is the main dissociation channel, other important mechanisms [27], like $d \rightarrow p + n$, with $n$-transfer to the target, are very important. In order to have a complete CDCC description, these breakup possibilities should be included in conjunction with the one treated here. However, treatment of deuteron breakup, with a subsequent neutron transfer, constitutes a four-body calculation which computationally is very difficult to achieve.

Respect to the effect on total fusion of resonance ($l = 2$, $j^\pi = 3^+, 2^+, 1^+$) and nonresonance discretized breakup states of $^6$Li, use is made of the same procedure as in Refs. [32, 33], where a study of the effect of these resonances on elastic scattering was presented. Two different types of calculations are performed; a) fusion is calculated when resonance states are excluded from the full discretized energy space and, b) only couplings between the resonance sub-space are considered. The results are shown in Fig. 2a-2c for $^{28}$Si, $^{96}$Zr and $^{209}$Bi respectively. The solid-lines represent the calculations of Figs. 1a-1c, with couplings with the full discretized energy space. The dashed-lines correspond to the case when only couplings between resonance states are included, while the dashed-dotted lines when resonance states are omitted from the full discretized space. The calculations of fusion through the elastic-channel, i.e., fusion without breakup of the projectile, correspond to the dotted-lines. These are obtained when only the term $\beta' = \beta = 0$ is considered in Eq. (3) and quantify the effect on fusion from the breakup of $^6$Li. In all cases, the elastic-channel calculations are above those with couplings between the complete breakup space. This means that, when breakup states are considered, fusion is suppressed. Thus, a net repulsive polarization potential emerges from couplings to breakup discrete states, which in turn increases the nominal Coulomb barrier and hence suppresses fusion. Now, when couplings between resonance states are considered (dashed-lines), the calculations are closer but still above those with the full space. Respect to the elastic-channel calculation (dotted-lines), resonance couplings produce repulsive polarizations for all targets and energies but still some repulsive contribution should come from nonresonance states. In fact, for the heavier target $^{209}$Bi, the resonance calculations are very similar to those with the full discretized space. Hence, couplings to these states become increasingly important as the target mass increases.

It is also observed in Figs. 2a-2c, that for $^{28}$Si, the calculation with couplings between only nonresonance states (dashed-dotted line), shows suppression respect to the elastic-channel (dotted-lines). That is, a net repulsive polarization potential appears from these couplings at energies above the barrier. For $^{96}$Zr at the lowest energies some enhancement is observed. That is, the polarization potential turns attractive, which means that the nuclear component of this potential dominates over the repulsive. For the target $^{209}$Bi, this effect is more evident. At high energies, the nonresonance calculation is very similar to the elastic-channel. However, as the energy decreases towards the barrier, an increasing enhancement appears respect to the elastic-channel calculation. As a matter of fact, the increasing attractive character of nonresonance couplings is compensated by a stronger repulsive polarization from resonance states as observed in Figs. 2b and 2c.

In order to quantify the effect of resonance and nonresonance breakup states of $^6$Li on fusion, we define the ratio $\Gamma$ by,

$$\Gamma_i = 1 - \frac{\sigma_{TF}}{\sigma_{Fi}},$$

being $\sigma_{Fi}$, fusion through the elastic channel $i = 1$; 2 and nonresonance $i = 3$ states. $\sigma_{TF}$ is total fusion with the full discretized breakup space. Figs. 3a-3c show the results of $\Gamma_i$ in terms of $E_{c.m.}/V_B$. The effect of the breakup of the projectile $\Gamma_1$, is shown by the dotted-lines. The dashed-lines correspond to resonance states $\Gamma_2$, while the dashed-dotted lines from
Figure 2. Total fusion calculations of $^6$Li with targets $^{28}$Si, $^{96}$Zr and $^{209}$Bi for the elastic channel (dotted-lines), resonance(dashed-lines) and nonresonance (dashed-dotted-lines) couplings. The solid-lines correspond to the calculations with the full breakup space of the projectile nonresonance states $\Gamma_3$. In these figures, positive values of $\Gamma_i$ mean fusion enhancement respect to the calculation with the full breakup space, while negative values indicate suppression. For all nuclear systems, $\Gamma_1$, $\Gamma_2$ and $\Gamma_3$, increase as the energy decreases towards the barrier $V_B$ to then decrease for lower energies. Also, $\Gamma_2$ has lower values than $\Gamma_1$ and $\Gamma_3$, that is fusion through resonance states is closer to the full calculation. In fact, for the heavier targets $\Gamma_2$ maintains values close to zero and even becomes negative at sub-barrier energies. For $^{96}$Zr, $\Gamma_3$ remains close to $\Gamma_1$, this shows that couplings to these states have a small effect on fusion. However for the heavier target $^{209}$Bi, $\Gamma_3$ becomes even larger than $\Gamma_1$ for energies below the barrier. That is, couplings to nonresonance states strongly enhances fusion. This effect is correlated to the increasing suppression effect produced by resonance states in this energy region.

In order to understand the particular effect on fusion of a given resonance of the projectile, we show in Fig. 4 three types of calculations, namely, fusion is calculated when the states of a single resonance are extracted from the resonance sub-space. The short-dashed-line corresponds
Figure 3. Effects on fusion $\Gamma_i$ as defined in Eq. (6), for elastic (dotted-lines), resonance (dashed-lines) and nonresonance (dashed-dotted-lines) sub-spaces.

to the case when the $3^+$-state is extracted from the resonance sub-space. That is, the $2^+$ or $1^+$ are included. The dashed-dotted- and dotted-lines present the results when the $2^+$ and $1^+$ resonances are respectively excluded from the resonance sub-space. The solid-line represents total fusion with the full breakup space (resonance and nonresonance) while, the long-dashed-line with the resonance sub-space. For the $^{28}$Si-target, we observe that for energies above the barrier, the effect of the $3^+$-resonance state is stronger than for the $2^+$ and $1^+$ resonances. That is, the absence of couplings to the $3^+$-state (short-dashed-line) produces a larger enhancement respect to the calculation with the full resonance sub-space (long-dashed-line) and respect to the other two calculations (dashed-dotted and dotted-lines) in which it is included. It is known that for reactions with weakly bound projectiles, total fusion is dominated by complete fusion at high energies, while at energies around and below the barrier, incomplete fusion becomes increasingly important. Contrary to the elastic scattering process [32], the fusion mechanism strongly depends on the incident excited states of the projectile. For example, for nonresonance states prompt breakup takes place, thus these states have a more significant role on NCBU and on ICF processes, particularly at energies around and below the barrier. As for resonance states,
they have very particular effects on fusion due to the mean-half lives. That is, since the typical collision time is of the order of $10^{-21}$ s, resonance states $2^+$ and $1^+$ with half-lives $3.7 \cdot 10^{-22}$ s and $4.2 \cdot 10^{-22}$ s, can have a more important effect on incomplete fusion than on complete fusion. On the other hand, the $3^+$ resonance, with a longer half-life $262.1 \cdot 10^{-22}$ s, dissociates later and can contribute more to complete fusion.

This is the reason why, as seen in Fig. 4a for the $^{28}$Si-target, when the $3^+$ state is considered the results are closer to the TF calculation (solid-line) at energies above the barrier. For the $^{96}$Zr-target as seen in Fig. 4b, the extraction of any single state from the resonance sub-space has similar effects at energies above the barrier, however below the barrier the extraction of the $3^+$-resonance has a more significant effect on fusion. That is, fusion through the $2^+$- and $1^+$-resonances (short-dash-line) approaches the TF values (solid-line) where fusion is dominated by incomplete fusion. For the $^{209}$Bi-target (Fig. 4c) at energies above the barrier, the inclusion of the $3^+$-state gives results closer to TF (solid-line), but at energies well below the barrier, as seen in Fig. 4d, the calculations with the $2^+$- and $1^+$-resonances becomes closer to TF. This findings agrees with those of Ref. [26] for the $^{6}$Li+ $^{209}$Bi system.
3. Summary

CDCC calculations of total fusion cross section have been studied for the weakly bound nucleus $^6$Li with targets $^{28}$Si, $^{96}$Zr and $^{209}$Bi at energies above and below the nominal Coulomb barriers. In the cluster structure scheme of $^6$Li, imaginary short-range potentials have been used to account for absorption of the fragments $\alpha$ and $d$ by the target. When both or one of the fragments are/is inside the range of the absorption potential are/is captured and contribute to fusion. So, complete and incomplete fusion are accounted for in the calculations.

The effects of resonance/nonresonance breakup state couplings of the projectile on fusion have been determined by restricting the calculations to separate resonance and nonresonance breakup sub-spaces. It has been found that resonance states produce a strong effect by suppressing fusion, in fact, this effect is stronger for the heavier targets. At energies above the barrier where, fusion is dominated by the complete fusion process, couplings to the $3^+$-resonance have a significant importance. However, at very low energies where total fusion is dominated by incomplete fusion, the $2^+$ and $1^+$ states have a more important role than the $3^+$-state.

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