Fast Measurement Method of Gear Parameters Based on Convexity Defect

Chen Guo-hua, Liu Xiao-long, Zhang Ai-jun and Wang Jun-yi
Beijing University of Chemical Technology

E-mail: chengh@mail.buct.edu.cn

Abstract: Gear parameters measurement based on machine vision is an important part in the field of intelligent manufacturing. Existing methods, which are all based on gear center, may cause accumulative error and have the weakness of large computation and low efficiency due to the repeated use of all contour points. In view of these problems, a new measurement method of gear parameters based on convexity defect is presented. Firstly, a gear contour is extracted from the gear image. Secondly, convex hull and defects of the gear contour are computed using image processing approach, then the number of the gear teeth and key points on the dedendum circle and addendum circle are determined, respectively. Finally, each radius of the dedendum circle and addendum circle is obtained separately by means of circle fitting with the corresponding key points. In this method, the key points are calculated by using the properties of convex defects, avoiding the calculation of the gear center and greatly increasing the computing speed by using a limited number of key points, precisely, two times the number of gear teeth. Compared with the traditional methods, the experimental results show that the speed of this method is increased by 1.5 times and it can meet the real-time requirement.

Keywords: Gear, Parameters Measurement, Convexity Defect

1. Introduction

Gear is one of the most important parts in mechanical transmission, whose quality and accuracy affect the performance and service life of a machine. The gear’s fast measurement, therefore, is necessary in real gear inspection. However, it is very difficult to perform precise measurements fleetly due to complicated shapes and varied features of a gear (Yin et al 2013). Gear parameters are usually artificially measured in conventional measurement methods, which is inefficient and prone to subjective misreading. With the rapid development of computer vision technology, visual measurement has become a research hotspot in recent years for the characteristics of real time, non-touching and high automation (Ma and Zhang 1998, Lin et al 2015, Klancnik et al 2015, Simunovic et al 2016, Wang and Mottershead 2016, Ge et al 2017).

According to the requirements of the gear’s on-line detection and fast manufacturing, researchers have applied computer vision to gear parameters measurement. Zhang et al (2004) studied a non-contact measuring method for spur gears based on digital image processing technology, in which the gear geometric center is obtained by center-of-gravity method and the teeth number is calculated by drawing an auxiliary circle. As the method of Li and Liu (2009), the edge contour of a gear is firstly extracted using edge detection, then the center of the gear is computed by means of random Hough transform. Chen and Li (2010) use a similar method to Li and Liu's method. Wang (2011) proposed a method based on mathematical morphology for gear center localization and gear teeth number.
counting, in which the gear center is obtained through fitting a circle with the least squares method, and edge detection is uniquely not needed.

In almost all the existing methods, the gear center is the basis to calculate other geometric parameters. After obtaining the gear contour, various methods such as the least squares method and the random Hough transform were applied to gain the gear center. But these methods all have their own shortcomings more or less. The least squares method itself has calculation error which will lead to positioning error of the center, pointed by Zhang et al (2004). In the process of extracting the circle and calculating its center by the random Hough transform, the choice of data may cause positioning error, as Li and Liu (2009) said in their paper. The positioning error of the center can subsequently result in the error of the other parameters of the gear, which will affect the accuracy of the measurement.

Moreover, in the existing methods, all the points on the contour will be utilized almost in each stage of calculations. Hence the amount of calculation is relatively large. By analyzing the gear shape, we found that convex hull and convexity defects may become the key to solve the problem, which is often used for gesture recognition (Mykyta et al 2014).

In this paper, a fast measurement method to calculate the gear parameters by using a finite number of key points on the contour was proposed, which greatly reduces the computational complexity and significantly improves the detection efficiency. Particularly, the gear center does not need to be positioned because this method does not require it to be the basis for calculating other geometric parameters. Based on the extracted key points, which is shown in figure 1 as the blue and pink circles, it is easy to find dedendum circle and addendum circle, the two bigger yellow circles below, by fitting the key points. And then the radii are not difficult to calculate.

![Figure 1. Key points of gear contour and fitted circles based them](image)

**2. Acquisition of convex hull and defect of gear contour**

2.1. Analyses of spur gear shape

From the appearance point of view, the teeth of spur gear are periodic with the nature of uniform distribution. Gear tooth and tooth space appear alternately on the outer edge of the gear and both are
evenly distributed around the center of the gear. Obviously, the number of gear teeth equals to the number of teeth space. To calculate the number of gear teeth, radius of the dedendum circle and addendum circle, all needed is a point on the dedendum circle and addendum circle respectively and the gear center, ideally. The point on the dedendum circle, which is called P1 in this paper, is the lowest point in the tooth space and the point on the addendum circle, which is called P2, is the highest point of on the gear tooth. Points like P1 and P2 are called key points.

\[
\begin{align*}
\sum P1 &= \sum p2 = z \\
\sum \text{Keypoint} &= \sum p1 + \sum p2 = 2z
\end{align*}
\] (1)

Based on the above considerations, the problem of how to get the gear parameters converts to how to get the key points of the gear.

This paper presents a method, which extracts the key points from the convex hull and convexity defect of the gear contour and after that one can calculate the gear parameters directly and simply. Compared with existing methods, this method does not need to determine the gear center, use contour extraction to instead the edge detection and is not necessary to calculate all the pixels on the contour, reduce the amount of calculation greatly, significantly improve the detection efficiency.

2.2. Extraction of the gear contour

As shown below in figure 2, image preprocessing such as grayscale transformation and threshold segmentation should be performed firstly before extracting the gear contour. In this paper, the contour extraction refers to acquiring a set of points with coordinate information, which constitutes the contour. Subsequent calculations are based on these points.

![Figure 2](image)

**Figure 2.** The flow chart of measurement process

A border following method (Suzuki and Be 1985) instead of edge detection like Canny’s method that is commonly used at present is applied to get the gear contour. The advantage of using the border following method is that it can easily handle gear images with partially highlight areas (as illustrated in figure 3), while using the edge detection will obtain a lot of redundant information that must be removed by additional treatments.
Figure 3. (a) Original gear image (b) Binary image.

Figure 4(a) depicts the contour that was achieved from figure 3(b) by the border following method, and figure 4(b) shows the edges acquired using the Canny’s method. Obviously, the latter contains much redundancy and needs additional treatments.

Figure 4. (a) Gear contour (b) Gear edge.

Another difference between the two methods is the form of outputs. The border following method can directly output the desired points composing the contour, while edge detection methods will give an image of gear edges, and the edge images should be further processed in order to obtain the points.

Figure 5 shows a common example in which the image does not contain superfluous information like highlight area as in figure 3. Figure5 (b) displays the binary image preprocessed from figure 5(a), and figure 5(c) depicts the gear contour extracted by border following method.
2.3. Convex hull acquisition

In order to extract the key points on the top circle from the point set, this paper first obtains the convex hull of the point set. The convex hull of a finite point set S is the set of all convex combinations of its points. In a convex combination, each point $x_i$ in S is assigned a weight or coefficient $\alpha_i$ in such a way that the coefficients are all non-negative and sum to one, and these weights are used to compute a weighted average of the points. For each choice of coefficients, the resulting convex combination is a point in the convex hull, and the whole convex hull can be formed by choosing coefficients in all possible ways. Expressing this as a single formula, the convex hull is the set:

$$Conv(S) = \left\{ \sum_{i=1}^{[S]} \alpha_i x_i \mid (\forall i : \alpha_i \geq 0 \land \sum_{i=1}^{[S]} \alpha_i = 1) \right\}$$

For instance, when S is a bounded subset of the plane, the convex hull may be visualized as the shape enclosed by a rubber band stretched around S. Convex hull of a point set is shown in figure 6.

![Convex hull of a points set](image)

**Figure 6.** Convex hull of a points set

In this paper, the convex hull of the gear contour is the smallest convex polygon that can contain the contour of the gear, which is acquired by the GCH algorithm (Sklansky 1982). And the vertexes of the polygon are distributed on the top of each tooth. For the gear contour in figure 5(c), it’s convex hull is shown in figure 7.
2.4. Acquisition of the convexity defects

Any deviation of the object from this hull can be considered as convexity defect. That means an area that do not belong to the object but located inside of its convex hull. Expressing convexity defects as a formula.

\[ \text{ConvexityDefects} = \text{Conv} (\text{Contour}) - \text{Contour} \]  

(3)

In the field of machine vision, convexity defects are often used for gesture recognition and limb motion recognition. For convenience, convexity defects can be represented by a quaternion which contains these values: [start, end, farthest, depth].

\[ \text{Defects} = \sum \Delta (\text{start, end, farthest}) \]  

(4)

Start and end denote the start and end points of the defect area on the contour, corresponding to point A and point B in figure 8, respectively. Farthest represents the point furthest from the convex hull in the defect area, corresponding to point C in figure 8. Depth represents the shortest distance between the farthest point and the convex hull, which is shown in figure 8 by the line with arrow. For the sake of convenience, this paper uses the triangle consisting of point A, B, C to describe a defect.

In this paper, a defect between the contours of the gear and the convex hull is shown below in figure 9. The starting point A and the end point B of the defect are located on the top of the adjacent teeth. In other words, point A and B are located on the addendum circle. Obviously, the farthest point C is
Figure 9. Triangle ABC presents a defect between the contours and the convex hull.

3. Fast calculation of gear parameters
The use of contour extraction of the new method proposed in this paper, instead of the edge extraction process in traditional methods, omits the need to extract point sets from edge images. After obtaining the convex hull and the convexity defect of the gear contour, extract the limited number of key points of the gear profile. The process greatly reduce the amount of calculation, one can quickly calculate the gear parameters.

3.1. The number of the gear teeth
According to the nature of the gear, the number of gear teeth is equal to the number of defects, also equal to one-third of the number of key points.

\[ \text{Num(teeth)} = \text{Num(defects)} \]
(5)

3.2. Radius of the dedendum circle and addendum circle
After obtaining the defects between the contour and the convex hull, the starting point and the end point of each defect triangle are on the crest circle, and the farthest point is on the root circle. The minimum circumscribed circles is fitted according to the starting point (or end point) point set and the farthest point set, respectively.

The radius of the minimum circumscribed circle of the starting point (or end point) set is equal to the tooth top circle radius, and the radius of the minimum circumscribed circle of the farthest point set is equal to the radius of the tooth root circle.

The dedendum circle and addendum circle obtained by fitting the key points of the gear contour in figure 5(c) is shown in figure 10.

The radius is determined by the coordinates in the image, measured in pixels. Once the camera is calibrated, the number of pixels can be converted to the actual unit of measure, thus the radius can be expressed in actual dimensions.

Figure 10. The dedendum circle and addendum circle
4. Experiment
The experiment consists of two parts, hardware and software. Hardware includes desktop computer with Windows operating system, two megapixel DaHeng Mercury GigE series industrial camera (MER-200-14GM). As for software part, the algorithm is implemented in C++ based on Visual Studio and data processing is done by Python. This paper compares the new methods proposed in this paper and the classical methods proposed in paper [9].

The gear diameter measured by the vernier caliper is treated as the standard parameter, which is compared with the results obtained by the new method and the classical method, respectively. The results are shown in table 1.

| Table 1. Results obtained by the new method and the classical method |
|---------------------------------------------------------------|
| Standard value | New method | Classical method |
| Radius of addendum circle (mm) | 17.86 | 17.8879 | 17.8879 |
| Radius of dedendum circle (mm) | 15.78 | 15.8405 | 15.3017 |
| Gear teeth number | 34 | 34 | 34 |

Both methods are performed 50 cycles, then record the run time. Finally draw a line chart, shown below in figure 11.

As can be seen from table 1, for the top circle, the measurement errors obtained by the two methods both are 1.56 ‰, and for the root circle, the measurement error obtained by our method is 3.82% and 3.13% is achieved by the classical one.

As for the time consumed for 50 running cycles, the new method is 1.97 times less than the classical one, meaning nearly twice improvement of the performance.

![Figure 11. Time contrast of performing 50 times.](image)

5. Conclusion
This paper presents a method for rapid measurement of gear parameters based on contour convex hull and convexity defects. Firstly, a series of pre-processing, including gray-scale transformation, threshold segmentation and contour extraction, is performed on the gear image. And then obtains the finite key points on the gear profile by finding the convexity defects of the gear contour. After that, gear parameters like gear teeth number, radius of the dedendum circle and addendum circle can be calculated quickly and easily. In the process of image preprocessing, the new method uses the contour extraction instead of edge detection to get the point set of gear contour. So the operation of extracting
a set of points from the edge image is omitted. Parameter calculation based on key points makes it unnecessary for the new method to have all the points of the gear profile involved in the calculation, which greatly reducing the amount of calculation. The comparative experiments show that the new method speeds up nearly two times in the case of maintaining the same accuracy, compared with the classic method.

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