Simplified Chua’s attractor via bridging a diode pair

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Published in The Journal of Engineering; Received on 2nd February 2015; Accepted on 11th February 2015

Abstract: In this paper, a simplified Chua’s circuit is realised by bridging a diode pair between a passive LC (inductance and capacitance in parallel connection - LC) oscillator and an active RC (resistance and capacitance in parallel connection - RC) filter. The dynamical behaviours of the circuit are investigated by numerical simulations and verified by experimental measurements. It is found that the simplified Chua’s circuit generates Chua’s attractors similarly and demonstrates complex non-linear phenomena including coexisting bifurcation modes and coexisting attractors in particular.

1 Introduction

In the realm of chaotic circuits, Chua’s circuit [1] and its many variants [2, 3] have attracted much attention because of the generations of chaotic signals. The original fundamentals to achieve chaotic behaviours in those circuits are non-linear elements, for example, the well-known Chua’s diodes. Minimising both the number of physical components and the number of idealised elements are two effective ways for measuring the simpleness of a system [4]. Therefore a natural extension is to simplify the physical implement of Chua’s circuit, on which Chua’s attractors can be maintained to generate. Recently, Srisuchinwong and Munmuangsaen [5] demonstrated a four current tunable chaotic oscillators in set of two diode reversible pairs and Kandiban et al. [6] proposed a simple three-dimensional autonomous circuit with a diode pair, in which the physical circuits are achieved for simplified Chua’s circuit. In this paper, a simplified Chua’s circuit is proposed, which is realised by bridging a diode pair between a passive LC oscillator and an active RC filter. The proposed circuit has three determined equilibrium points and its dynamics mainly relies on its circuit parameters. Owing to special properties of these equilibria, complex phenomena of the coexisting attractors are discovered. The simplified Chua’s attractor and their coexisting attractors are revealed by numerical simulations based on the circuit model and verified by experimental measurements based on the physical circuit.

2 Simplified Chua’s circuit

By bridging a diode pair between a passive LC oscillator and an active RC filter, a simplified Chua’s circuit is easily constructed as depicted in Fig. 1. The simplified Chua’s circuit is physically realisable and contains only a diode pair to realise its non-linearity. The constitutive relations of the voltage $v_k$ across and the current $i_k$ pass through diode $D_k (k=1,2)$ are described as [7]

$$i_k = I_k(e^{\rho v_k} - 1)$$

(1)

where $\rho = 1/(n V_T)$, $I_k, n$ and $V_T$ denote the reverse saturation current, emission coefficient and thermal voltage of the diode, respectively. Letting that $i$ and $v$ represent the voltage across and the total current flowing through the diode pair, the voltage-current relation of the diode pair is mathematically modelled by

$$i = 2I_k \sinh (\rho v) = 2I_k \sinh [\rho(v_2 - v_1)]$$

(2)

where $v = v_2 - v_1$.

The proposed circuit in Fig. 1 has three dynamic elements of the capacitor $C_1$, the capacitor $C_2$ and the inductor $L$, corresponding to three-state variables of $v_1, v_2$ and $i_2$.

Applying the Kirchhoff’s circuit laws and the constitutive relations of the circuit elements, a set of state equation is obtained as

$$\frac{dv_1}{dt} = - \frac{1}{C_1} i_1 + \frac{1}{RC_1} v_1$$

$$\frac{dv_2}{dt} = - \frac{1}{C_2} i_2 - \frac{1}{C_2} i$$

$$\frac{di}{dt} = \frac{1}{L} v$$

(3)

To investigate the dynamics of the simplified Chua’s circuit, we solve (3) numerically. Moreover, an analogue electronic circuit is made by commercially available elements. The typical parameters of the circuits are $R = 1.55 \, k\Omega$, $L = 45 \, mH$, $C_1 = 80 \, nF$, $C_2 = 150 \, nF$ and AD711KN is used with a bipolar ±15 V supply. Two 1N4148 diodes are utilised and the parameters are $I_s = 6.89 \, nA$, $n = 1.83$ and $V_T = 26 \, mV$.

Several chaotic and periodic orbits in the $v_1$-$v_2$-plane of the circuit are obtained numerically and experimentally, as shown in Fig. 2. As further evidence of complex dynamical behaviours, the right column of Fig. 2 shows several phase portraits for different values of $R$ experimentally, from which the chaotic attractor and the coexisting attractors are observed correspondingly. In

Fig. 1 Schematic of simplified Chua’s circuit via bridging a diode pair

J Eng, 2015, Vol. 2015, Iss. 4, pp. 125–127
doi:10.1049/joe.2015.0018
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By setting the right side of (3) equal to zero, the equilibrium points for the typical parameters are obtained as $S_0 = (0, 0, 0)$ and $S_c = (±0.5117, 0, ±0.0003)$ numerically. The three roots at each
equilibrium points are calculated as

\[ S_0 : \lambda_1 = 8060.89, \quad \lambda_2 = -0.97 \pm j12171.61 \]
\[ S_\pm : \lambda_{1,2} = 1241.17 \pm j9482.75, \quad \lambda_3 = -127129.67 \]  

(4)

There are two complex conjugate roots with positive real part and one negative root at \( S_0 \), implying that the pair of non-zero equilibrium points are two unstable saddle-foci, therefore, a double-scroll chaotic attractor can be emerged at the neighbourhoods of two non-zero equilibrium points.

For the typical parameters, the resistance \( R \) is regarded as a bifurcation parameter and varies from 1.4 to 2.6 k\( \Omega \). The bifurcation diagram of the state variable \( v_1 \) and the corresponding first two Lyapunov exponents are depicted in Figs. 3a and b, respectively.

The denotations and the initial values are the same as those in Fig. 2. When \( R \) increases gradually, the dynamics of the circuit begins with chaotic behaviours and finally settles down periodic behaviours via a reverse period-doubling bifurcation route. Correspondingly, the maximum Lyapunov exponent has a transition from positive to zero. There are several periodic windows in which the maximum Lyapunov exponent equals to zero. Moreover, it is noted that for the non-linear circuit, their attractors are all emerged from the unstable equilibria, that is, an attractor with a basin of attraction is associated with an unstable equilibrium point. Additionally, when the coexisting phenomena occur in the parameter regions of \( 1.82 \text{ k}\Omega \leq R \leq 2.6 \text{ k}\Omega \), two independent basins of attractions are formed, resulting in the generations of coexisting attractors for different initial values.

4 Conclusion

A simplified Chua’s circuit is presented. The proposed circuit is modelled mathematically by three-order differential equations. The research results indicate that the circuit has an unstable saddle point and a pair of unstable saddle-foci in certain parameter regions. The dynamical characteristics with the variations of the parameter \( R \) are investigated. Complex non-linear phenomena of the coexisting bifurcation modes and the coexisting attractors are demonstrated in particular. To ensure the occurrence of complex non-linear phenomena, an analogue electronic circuit is performed, on which the phase portraits are captured to validate the theoretical analyses.

5 Acknowledgments

This work is supported by the grants from the National Natural Science Foundations of China (grant no. 51277017), the Natural Science Foundations of Jiangsu Province (grant no. BK20120583) and the Natural Science Foundations for colleges and universities in Jiangsu Province (grant no. 14KJB430004).

6 References

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