Evidence of a Phonon Hall Effect in the Kitaev Spin Liquid Candidate $\alpha$-RuCl$_3$

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The material $\alpha$-RuCl$_3$ has been the subject of intense scrutiny as a potential Kitaev quantum spin liquid, predicted to display Majorana fermions as low energy excitations. In practice, $\alpha$-RuCl$_3$ undergoes a transition to a state with antiferromagnetic order below a temperature $T_N \approx 7$ K, but this order can be suppressed by applying an external in-plane magnetic field of $H_\parallel = 7$ T. Whether a quantum spin liquid phase exists just above that field is still an open question, but the reported observation of a quantized thermal Hall conductivity at $H_\parallel > 7$ T by Kasahara and co-workers [Kasahara et al., Nature 559, 227 (2018)] has been interpreted as evidence of itinerant Majorana fermions in the Kitaev quantum spin liquid state. In this study, we re-examine the origin of the thermal Hall conductivity $\kappa_{xy}$ in $\alpha$-RuCl$_3$. Our measurements of $\kappa_{xy}(T)$ on several different crystals yield a temperature dependence very similar to that of the phonon-dominated longitudinal thermal conductivity $\kappa_{xx}(T)$, for which the natural explanation is that $\kappa_{xy}$ is also mostly carried by phonons. Upon cooling, $\kappa_{xx}$ peaks at $T \approx 20$ K, then drops until $T_N$, whereupon it suddenly increases again. The abrupt increase below $T_N$ is attributed to a sudden reduction in the scattering of phonons by low-energy spin fluctuations as these become partially gapped when the system orders. The fact that $\kappa_{xy}$ also increases suddenly below $T_N$ is strong evidence that the thermal Hall effect in $\alpha$-RuCl$_3$ is also carried predominantly by phonons. This implies that any quantized signal from Majorana edge modes would have to come on top of a sizable – and sample-dependent – phonon background.

I. INTRODUCTION

The quasi-2D Mott insulator $\alpha$-RuCl$_3$ has attracted much interest since it was proposed as a promising material for realizing a Kitaev spin liquid state, thanks to its honeycomb lattice with anisotropic bond-directional spin interactions [1, 2]. The Kitaev model predicts the existence of mobile Majorana fermions as quasiparticles that would manifest as topologically protected heat carriers on the edges of the sample. In principle, such Majorana edge modes could be detected by measuring the thermal Hall effect [3–5], and their signature would be a half-integer quantized thermal Hall conductivity per plane $\kappa_{2D}$ = $\kappa_{xy}d$ at low temperature ($d$ is the honeycomb interlayer distance). However, because $\alpha$-RuCl$_3$ orders antiferromagnetically at low temperature, below $T_N = 7$ K, in order to access the putative spin liquid phase one must apply an in-plane magnetic field in excess of a critical field $H_\parallel = 7$ T to suppress the magnetic order. The possibility thus arises that a spin liquid phase could emerge immediately above that critical field. An inelastic neutron scattering study finds a magnetically disordered state at $T \rightarrow 0$, sandwiched between the ordered phase below $H_\parallel = 7$ T and a field-polarized phase above $H_\parallel \approx 9$ T, which is reminiscent of a quantum spin liquid [6]. A few studies of heat transport have reported the observation of a quantized $\kappa_{xy}$ in some samples of $\alpha$-RuCl$_3$, in a narrow range of in-plane fields ($\hbar < H_\parallel < 9$ T) and temperatures ($3 < T < 6$ K) [7–9]. However, since then, a number of studies have reported $\kappa_{xy}$ signals in various insulators and attributed them to phonons [10–14]. The $\kappa_{xy}$ response can have either sign, and the magnitude of $\kappa_{xy}$ is easily as large as that found in $\alpha$-RuCl$_3$, often larger. Typically, in those studies that attribute the thermal Hall effect to phonons, one finds that the magnitude of $\kappa_{xy}$ scales roughly with the magnitude of the phonon thermal conductivity $\kappa_{xx}$, with a ratio $|\kappa_{xy}/\kappa_{xx}|$ of order $10^{-3}$. These studies raise the possibility that phonons might also generate a thermal Hall effect in $\alpha$-RuCl$_3$.

In this paper, we explore that possibility. We measured the thermal conductivity $\kappa_{xx}$ and thermal Hall conductivity $\kappa_{xy}$ in several samples of $\alpha$-RuCl$_3$, coming from two different sources. We find a substantial variation in the magnitude of $\kappa_{xx}$ and $\kappa_{xy}$ from sample to sample, but the same qualitative behaviour. Upon cooling from 80 K, $\kappa_{xx}$ and $\kappa_{xy}$ exhibit similar behaviours, namely a peak in $\kappa_{xx} / T$ and $\kappa_{xy} / T$ at roughly the same temperature $T \approx 20$ K, followed by a dip towards $T_N$. If the magnetic field is applied along the c direction, both $\kappa_{xx}/T$ and $\kappa_{xy}/T$ exhibit a rapid rise below $T_N$. If a magnetic field is applied such that its in-plane component is $H_\parallel = 7$ T, thereby suppressing the magnetic order, both $\kappa_{xx}/T$ and $\kappa_{xy}/T$ continue their monotonic decrease as $T \rightarrow 0$. The fact that $\kappa_{xy}(T)$ mimics the phonon-dominated $\kappa_{xx}(T)$ is strong evidence that the thermal Hall signal is also dominated by phonons. Moreover, we find that $\kappa_{xy}/\kappa_{xx} \approx$
10^{-3} at low temperature, a magnitude typical of the phonon Hall effect in other insulators. Finally, the fact that there is a sizeable $\kappa_{xx}$ signal below $T_N$, in the antiferromagnetic phase, shows that it does not simply arise from the excitations of a pure spin liquid phase. All this calls into question any interpretation of prior data exclusively in terms of Majorana fermions.

II. METHODS

Samples. — We measured a variety of samples coming from two different groups: Oak Ridge National Laboratory (ORNL) and the University of Toronto (UofT). Single crystals from ORNL were grown using the vapor transport method on $\alpha$-RuCl$_3$ powder coming from Furuya Metal Co., Ltd; the growth technique is detailed elsewhere [15]. Single crystals from UofT were grown using the same method but using powder coming from Sigma-Aldrich. The powder used was composed of 45%-55% ruthenium and was sealed in a quartz tube under vacuum. The latter was placed inside a two-zone tube furnace using a temperature gradient of 70°C (warmest side was 850°C). The powder was annealed over two days, followed by a 4°C/hour cooldown while maintaining the temperature gradient (see Ref. [16] for more details). Here we report data on four samples from ORNL (labelled O1, O2, O3 and O4) and one sample from UofT (labelled T1). The samples were cut into thin rectangular platelets of typical dimensions 1 × 1 mm, with thicknesses ranging from 20 to 140 µm. The contacts on the samples were made by glueing thin silver leads with silver paste.

Measurement technique. — Measurements were performed by a steady-state method using a standard four-terminal technique, with the thermal current applied along the length $l$ of the sample within the honeycomb layers (perpendicular to the Ru-Ru bonds; $J |a$). The thermal conductivity $\kappa_{xx}$ was measured by employing a standard one-heater–two-thermometers method, using a 5 kΩ resistor and two in situ calibrated bare chip CX-1050 Cernox sensors. A constant heat current $Q$ was injected at one end of the sample, while at the other end the sample is well heat sunk to a copper block referenced at a temperature $T_0$. The heat current was generated by sending an electrical current through a 5 kΩ strain gauge whose resistance is marginally dependent of temperature and magnetic field. A longitudinal thermal gradient $\Delta T_x = T^+ - T^-$ is measured at two points along the length of the sample, separated by a distance $l$. The longitudinal thermal conductivity is given by $\kappa_{xx} = Q/\Delta T_x |a|$, where $\alpha = ut/l$ is the geometric factor of the sample (width $w$, thickness $t$). Under a magnetic field applied perpendicular to the basal plane, a transverse thermal gradient $\Delta T_y$ resulted (the field can also be tilted at an angle, such that it will have a component in the plane ($H_\parallel$), as well as normal to the plane ($H_\perp$)). This transverse gradient was measured using a differential type-E thermocouple. The thermal Hall conductivity is then given by $\kappa_{xy} = -\kappa_{yy} (\Delta T_x/\Delta T_y) (l/w)$ after having antisymmetrized the thermal Hall gradient via $\Delta T_y (H) = [\Delta T_y (T, H) - \Delta T_y (T, -H)]/2$. The error bars on the absolute values of thermal coefficients come mostly from the uncertainty in estimating the dimensions ($l$, $w$ and $l$) of the samples, approximately ±20%. The applied current was chosen such that $\Delta T_y/H$ is 5-10%; the resulting $\kappa_{xx}$ was independent of $\Delta T$, indicating that there was no heat loss. Any contamination of $\Delta T_y (H)$ coming from the copper heat sink was ruled out by a previous study that compared heat sinks made of Cu vs LiF (see Supplementary Information in Ref. [14]). Moreover, $\Delta T_y$ data obtained with thermocouples were found to be in good agreement with $\Delta T_y$ data obtained with Cernox sensors applied to the same sample.

III. RESULTS

In Fig. 1(a), we show the thermal conductivity of $\alpha$-RuCl$_3$ measured in five crystals, plotted as $\kappa_{xx}/T$ vs $T$. We see that there is a considerable variation in the magnitude of $\kappa_{xx}$ amongst samples, by a factor 3 or so, with the largest $\kappa_{xx}(T)$ value seen in sample T1. Prior data by Leahy et al. [17] and Henrich et al. [18] fall within the range of magnitudes of our own samples. Kasahara et al. [19] find a $\kappa_{xx}(T)$ that is 2-3 (7-8) times larger than our data on sample T1 (O2). This variation in magnitude is attributed to different levels of disorder, perhaps structural (associated with domains that form upon cooling through the structural transition at 130 K). Despite this quantitative variation, the qualitative behavior of $\kappa_{xx}(T)$ is the same in all samples. There is a peak in $\kappa_{xx}/T$ vs $T$ at $T \approx 20$ K, below which $\kappa_{xx}/T$ drops as $T \rightarrow T_N$. As argued by Henrich et al. [18], the dominant carriers of heat in $\alpha$-RuCl$_3$ are phonons, and these become increasingly scattered by low-energy antiferromagnetic spin fluctuations upon cooling below 80 K. It is this scattering that causes $\kappa_{xx}/T$ to drop as $T \rightarrow T_N$ (from above). Application of a magnetic field in the plane gaps the low-energy spin fluctuation spectrum, causing $\kappa_{xx}$ at $T = 10$ K ($> T_N$) to increase rapidly for $H > 10$ T [18].

As we cool below $T_N$, at $T < 1$ K ($< T_N$), we see that $\kappa_{xx}/T$ (in zero field) shoots up immediately in all samples (Fig. 1a and Refs. [17–19]), presumably because the magnetic order also causes a gapping of the low-energy spin fluctuation spectrum. This V-shaped dependence of $\kappa_{xx}/T$ vs $T$ at $H = 0$ T is mimicked by a similar V-shaped dependence of $\kappa_{xx}$ vs $H$ at $T < 1$ K, with the minimum at the critical field of 7 T [20]. In summary, the thermal conductivity of $\alpha$-RuCl$_3$ at low temperature ($T < 50$ K) can be understood essentially in terms of phonons scattered by spin fluctuations.

In Fig. 1(b), we show the thermal Hall conductivity of $\alpha$-RuCl$_3$ measured in the same 5 samples, plotted as $\kappa_{xy}/T$ vs $T$ for a field $H = 15$ T applied normal to the plane ($H |c$). There is considerable variation in the magnitude of $\kappa_{xy}$ across samples, even larger than that seen in $\kappa_{xx}$. Prior data by Henrich et al. [21] (with $H = 16$ T) yield a $\kappa_{xy}(T)$ curve very similar, quantitatively and qualitatively, to the curve for our sample O2 (for which $H = 15$ T), Kasahara et al. [19] find a $\kappa_{xy}(T)$ curve that is 3–4 times larger than that. Despite the quantitative variation, the qualitative behavior of $\kappa_{xy}(T)$ is
FIG. 1. (a) Thermal conductivity of $\alpha$-RuCl$_3$ in zero magnetic field, plotted as $\kappa_{xx}/T$ vs $T$, for our five different samples: T1 (black), O1 (green), O2 (red), O3 (blue) and O4 (gold). (b) Thermal Hall conductivity of the same five samples, measured in a magnetic field of $H = 15$ T applied normal to the honeycomb layers ($ab$-plane), plotted as $\kappa_{xy}/T$ vs $T$. The $\kappa_{xy}$ data for sample T1 (black) have been multiplied by a factor 0.4. Lines are a guide to the eye.

same in all samples from all groups – at least for $T > T_N$. The thermal Hall signal is positive, and there is a peak in $\kappa_{xy}/T$ vs $T$ at $T \approx 20$ K, below which $\kappa_{xy}/T$ drops as $T \rightarrow T_N$. In other words, $\kappa_{xy}(T)$ mimics the phonon-dominated $\kappa_{xx}(T)$.

Below $T_N$, we observe an increase in $\kappa_{xy}(T)$ upon cooling (Fig. 1(b)), in all our samples. Although $\kappa_{xy}(T)$ dips down to a minimum at $T_N$, in one case getting close to zero (sample O4), it always remains positive. Data by Henrich et al. [21] on one sample show a $\kappa_{xy}(T)$ curve that dips to zero at $T_N$, becoming perhaps very slightly negative; however, no data were reported for $T < 7$ K. The same is true for sample T1, whose $\kappa_{xy}$ amplitude is 3-4 times larger (at $T \approx 20$ K; Fig. 1). For our five samples, the ratio at $T = 20$ K and $H = 15$ T ranges from 0.03 % to 0.10 %. This is consistent with prior data by Henrich et al. [21] and Kasahara et al. [19], where $\kappa_{xy}/\kappa_{xx} \approx 0.05 \%$. As we will discuss below, a ratio of this magnitude is typical for a phonon thermal Hall effect. In Fig. 3, we show the effect of applying a component of the field parallel to the honeycomb layers, so that the antiferromagnetic order is suppressed. The red curves are for $H_\perp = 7$ T and no in-plane field ($H_\parallel = 0$ T). Both curves – $\kappa_{xx}(T)$ (panel a) and $\kappa_{xy}(T)$ (panel b) – are very similar to those in Fig. 2, where $H_\perp = 15$ T (and $H_\parallel = 0$ T). The only difference is quantitative: the magnitude of $\kappa_{xy}$ is down roughly by a factor 7 T / 15 T (the reduction in $H_\perp$). Adding an additional 7 T field component in the plane...
FIG. 3. (a) Thermal conductivity $\kappa_{xx}$ of sample O2, plotted as $\kappa_{xx}/T$ vs $T$, for an applied magnetic whose component normal to the honeycomb layers is $H_\perp = 7$ T and whose component parallel to the layers is either $H_1 = 0$ T (red) or $H_1 = 7$ T (blue). (b) Same as in (a), for $\kappa_{xy}$. The horizontal dashed line marks the quantized value ($\kappa_{\text{HQ}}$) expected for Majorana edge modes, divided by 10. The arrows mark $T_N$. Lines are a guide to the eye.

(blue curves) has only a small effect above 7 K, but a dramatic one below 7 K: now $\kappa_{xx}(T)$ and $\kappa_{xy}(T)$ no longer suddenly increase below 7 K ($= T_N$) but continue to decrease smoothly through 7 K, in the absence of ordering. This makes sense for phonons, which continue to be scattered by low-lying spin fluctuations that remain ungapped down to the lowest temperature. So again, the striking similarity between $\kappa_{xy}(T)$ and the phonon-dominated $\kappa_{xx}(T)$ argues for a thermal Hall signal carried by the phonons.

IV. DISCUSSION

Traditionally, the thermal Hall effect from phonons has been considered very small [22]. First observed in 2005, its magnitude in Tb$_2$Ga$_2$O$_4$ was detected to be $\kappa_{xy} \approx 0.02$ mW/K m at $H = 3$ T and $T = 5$ K [23]. This is two orders of magnitude smaller than the signal first reported in $\alpha$-RuCl$_3$, in 2017: $\kappa_{xy} \approx 4$ mW/K m at $H = 6$ T and $T = 20$ K [19]. So it was perhaps natural to rule out phonons in 2017, although that year a large thermal Hall effect was reported in the antiferromagnetic insulator Fe$_2$Mo$_3$O$_8$ and attributed to phonons (and their coupling to spins), with $\kappa_{xy} \approx 30$ mW/K m at $H = 14$ T and $T = 35$ K [24], one order of magnitude larger than in $\alpha$-RuCl$_3$. Since then, it has become abundantly clear that phonons can carry a sizable thermal Hall conductivity in various insulators, whether magnetic, as in the antiferromagnetic cuprate Mott insulator La$_2$CuO$_4$ [10, 13], or non-magnetic, as in the quantum paraelectric SrTiO$_3$ [12]. The magnitude of $\kappa_{xy}$ (at $H = 15$ T and $T = 20$ K) can vary from $|\kappa_{xy}| \approx 1$ mW/K m in Tb$_2$Ti$_2$O$_7$ [11, 22] to $|\kappa_{xy}| \approx 200$ mW/K m in Nd$_2$CuO$_4$ [14] and become as large as $|\kappa_{xy}| \approx 1000$ mW/K m in Cu$_3$TeO$_6$ [25]. Although the mechanisms by which phonons acquire a handedness (become chiral) in a magnetic field are still unclear in these materials, the cumulative evidence that phonons are the carriers of the thermal Hall effect in these insulators is strong: although $|\kappa_{xy}|$ varies by three orders of magnitude, the ratio of $\kappa_{xy}$ to the phonon-dominated $\kappa_{xx}$ is roughly the same, namely $|\kappa_{xy}/\kappa_{xx}| \approx 2 - 5 \times 10^{-3}$ in all cases (see Table 1 in [25]). In other words, what really varies from material to material is the ability of phonons to conduct heat, i.e. the magnitude of $\kappa_{xx}$.

Being now aware of these more recent studies, it seems very likely that phonons must also generate a sizable $\kappa_{xy}$ signal in the antiferromagnetic insulator $\alpha$-RuCl$_3$, especially given the measured ratio $|\kappa_{xy}/\kappa_{xx}| \approx 1 \times 10^{-3}$ (at $H \approx 15$ T and $T = 20$ K; Fig. 2(b)). The immediate implication of having a sizable phonon contribution to $\kappa_{xy}$ in $\alpha$-RuCl$_3$ is that the total measured value of $\kappa_{xy}$ in any sample would include a sizable phonon background, to which any contribution from Majorana fermions would add. In this context, the reported observation of a plateau in $\kappa_{xy}/T$, over a small range of in-plane fields ($6 < H_{\parallel} < 9$ T) and temperatures ($3 < T < 6$ K) [7], with a measured total value of $\kappa_{xy}/T$ having precisely the half-quantized value of $\pi k_B^2/12 \hbar$ per plane expected theoretically for Majorana edge modes [3], can only be meaningful if the phonon background is precisely zero. As we have argued, this condition is unlikely to be satisfied in $\alpha$-RuCl$_3$, at least when there is a component of the magnetic field normal to the plane (and to the heat current), which was the case in the study of Ref. 7. To investigate the same configuration as in Ref. 7, we turn to our data in a tilted field, displayed in Fig. 3 (blue curves). If the entire signal is attributed to Majorana fermions, it is difficult to understand why our value of $\kappa_{xy}/T$ at $H_{\parallel} = 7$ T and $T = 6$ K is only $\kappa_{xy}/T \approx 0.04$ mW/K$^2$ m (Fig. 3b), while the value reported in Ref. 7 is $\kappa_{xy}/T \approx 0.8$ mW/K$^2$ m, a factor 20 larger. On the other hand, if the signal is attributed to phonons, then the likely reason for the much smaller $\kappa_{xy}$ in our sample is simply that its $\kappa_{xx}$ is much smaller, with $\kappa_{xx} \approx 0.5$ W/K m at $T = 10$ K and $H_1 = 7$ T (Fig. 3a), while the value reported in Ref. 7 is $\kappa_{xx} \approx 5$ W/K m – a factor of 10 larger. In other words, a phonon scenario solves the puzzle of why a thermal Hall conductivity that exceeds the half-quantized value, $\kappa_{xy}/T = \kappa_{\text{HQ}} \approx 0.8$ mW/K$^2$ m has only been observed in samples that have the largest values of $\kappa_{xx}$ [7–9, 26]: a larger value of $\kappa_{xy}$ is what we expect from phonons that conduct better (and thus produce a larger $\kappa_{xx}$), as nicely demonstrated by the highly conductive samples of Cu$_3$TeO$_6$ [25].
V. SUMMARY

We have measured the thermal conductivity $\kappa_{xx}$ and the thermal Hall conductivity $\kappa_{xy}$ of $\alpha$-RuCl$_3$, for a heat current within the honeycomb layers (perpendicular to the Ru-Ru bond; $J||\alpha$) in five different single crystals, from two separate sources. Although the magnitude of $\kappa_{xx}$ and $\kappa_{xy}$ vary significantly from sample to sample, presumably because of varying degrees of structural disorder, we find the same qualitative behavior in all samples. Upon cooling, both $\kappa_{xx}(T)$ and $\kappa_{xy}(T)$ are found to have the same temperature dependence, namely a broad peak located at the same temperature $T \simeq 7$ K, when the field is applied normal to the layers, or both continue their rapid rise in tandem upon entering the antiferromagnetic phase at $T_N = 7$ K, when the field has an in-plane component sufficient to remove the antiferromagnetic order. The fact that $\kappa_{xy}(T)$ mimics the phonon-dominated $\kappa_{xx}(T)$ so well leads us to conclude that the thermal Hall effect in $\alpha$-RuCl$_3$ is carried predominantly by phonons. This interpretation is supported by the fact that the magnitude of $\kappa_{xy}$ in various samples of $\alpha$-RuCl$_3$ roughly scales with the magnitude of the phonon-dominated $\kappa_{xx}$. Moreover, the ratio $\kappa_{xy}/\kappa_{xx}$ has a magnitude comparable to that found in several other insulators where phonons have been shown or argued to cause the Hall effect, namely $|\kappa_{xy}/\kappa_{xx}| \simeq 1 \times 10^{-3}$ (see Table 1 in [25]).

If phonons contribute significantly to the $\kappa_{xy}$ signal of $\alpha$-RuCl$_3$ samples, the experimentally measured $\kappa_{xy}$ cannot be attributed directly and exclusively to theoretically predicted Majorana fermions, as has been done in some reports.

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[1] G. Jackeli and G. Khaliullin, Mott Insulators in the Strong Spin-Orbit Coupling Limit: From Heisenberg to a Quantum Compass and Kitaev Models, Physical Review Letters 102, 017205 (2009).
[2] K. W. Plumb, J. P. Clancy, L. J. Sandilands, V. V. Shankar, Y. F. Hu, K. S. Burch, H.-Y. Kee, and Y.-J. Kim, $\alpha$-RuCl$_3$: A spin-orbit assisted Mott insulator on a honeycomb lattice, Physical Review B 90, 041112 (2014).
[3] J. Nasu, J. Yoshitake, and Y. Motome, Thermal Transport in the Kitaev Model, Physical Review Letters 119, 127204 (2017).
[4] M. Ye, G. B. Halász, L. Savary, and L. Balents, Quantization of the Thermal Hall Conductivity at Small Hall Angles, Physical Review Letters 121, 147201 (2018).
[5] Y. Vinkler-Aviv and A. Rosch, Approximately Quantized Thermal Hall Effect of Chiral Liquids Coupled to Phonons, Physical Review X 8, 031032 (2018).
[6] C. Balz, P. Lampen-Kelley, A. Banerjee, J. Yan, Z. Lu, X. Hu, S. M. Yadav, Y. Takano, Y. Liu, D. A. Tennant, M. D. Lumsden, D. Mandrus, and S. E. Nagler, Finite field regime for a quantum spin liquid in $\alpha$-RuCl$_3$, Physical Review B 100, 060405 (2019).
[7] Y. Kasahara, T. Ohnishi, Y. Mizukami, O. Tanaka, S. Ma, K. Sugii, N. Kurita, H. Tanaka, J. Nasu, Y. Motome, T. Shibauchi, and Y. Matsuda, Majorana quantization and half-integer thermal quantum Hall effect in a Kitaev spin liquid, Nature 559, 227 (2018).
[8] T. Yokoi, S. Ma, Y. Kasahara, S. Kasahara, T. Shibauchi, N. Kurita, H. Tanaka, J. Nasu, Y. Motome, C. Hickey, S. Trebst, and Y. Matsuda, Half-integer quantized anomalous thermal Hall effect in the Kitaev material candidate $\alpha$-RuCl$_3$, Science 373, 568 (2021).
[9] J. A. N. Bruin, R. R. Claus, Y. Matsumoto, N. Kurita, H. Tanaka, and H. Takagi, Robustness of the thermal Hall effect close to half-quantization in a field-induced spin liquid state, arXiv:2104.12184 (2021).
[10] G. Grissonnanche, A. Legros, S. Badoux, E. Lefrançois, V. Zatko, M. Lizaire, F. Laliberté, A. Gourgout, J.-S. Zhou, S. Pyon, T. Takayama, H. Takagi, S. Ono, N. Doiron-Leyraud, and L. Taillefer, Giant thermal Hall conductivity in the pseudogap phase of cuprate superconductors, Nature 571, 376 (2019).
[11] Y. Hirokane, Y. Nii, Y. Tomioka, and Y. Onose, Phononic thermal Hall effect in diluted terbiurn oxides, Physical Review B 99, 134419 (2019).
[12] X. Li, B. Fauqué, Z. Zhu, and K. Behnia, Phonon Thermal Hall Effect in Strontium Titanate, Physical Review Letters 124, 105901 (2020).
[13] G. Grissonnanche, S. Thériault, A. Gourgout, M.-E. Boulanger, E. Lefrançois, A. Ataei, F. Laliberté, M. Dion, J.-S. Zhou, S. Pyon, T. Takayama, H. Takagi, N. Doiron-Leyraud, and L. Taillefer, Chiral phonons in the pseudogap phase of cuprates, Nature Physics 16, 1108–1111 (2020).
[14] M.-E. Boulanger, G. Grissonnanche, S. Badoux, A. Allaire, E. Lefrançois, A. Legros, A. Gourgout, M. Dion, C. H. Wang, X. H. Chen, R. Liang, W. N. Hardy, D. A. Bonn, and L. Taillefer, Thermal Hall conductivity in the cuprate Mott insulators Na$_2$CuO$_2$ and Sr$_2$CuO$_2$Cl$_2$, Nature Communications 11, 5325 (2020).
[15] A. F. May, J. Yan, and M. A. McGuire, A practical guide for crystal growth of van der Waals layered materials, Journal of Applied Physics 128, 051101 (2020).

[16] J. A. Sears, Y. Zhao, Z. Xu, J. W. Lynn, and Y.-J. Kim, Phase diagram of $\alpha$-RuCl$_3$ in an in-plane magnetic field, Physical Review B 95, 180411 (2017).

[17] I. A. Leahy, C. A. Pocs, P. E. Siegfried, D. Graf, S.-H. Do, K.-Y. Choi, B. Normand, and M. Lee, Anomalous Thermal Conductivity and Magnetic Torque Response in the Honeycomb Magnet $\alpha$-RuCl$_3$, Physical Review Letters 118, 187203 (2017).

[18] R. Hentrich, A. U. Wolter, X. Zotos, W. Brenig, D. Nowak, A. Isaeva, T. Doert, A. Banerjee, P. Lampen-Kelley, D. G. Mandrus, S. E. Nagler, J. Sears, Y.-J. Kim, B. Büchner, and C. Hess, Unusual Phonon Heat Transport in $\alpha$-RuCl$_3$: Strong Spin-Phonon Scattering and Field-Induced Spin Gap, Physical Review Letters 120, 117204 (2018).

[19] Y. Kasahara, K. Sugii, T. Ohnishi, M. Shimozawa, M. Yamashita, N. Kurita, H. Tanaka, J. Nasu, Y. Motome, T. Shibauchi, and Y. Matsuda, Unusual Thermal Hall Effect in a Kitaev Spin Liquid Candidate $\alpha$-RuCl$_3$, Physical Review Letters 120, 217205 (2018).

[20] Y. Yu, Y. Xu, K. Ran, J. Ni, Y. Huang, J. Wang, J. Wen, and S. Li, Ultralow-Temperature Thermal Conductivity of the Kitaev Honeycomb Magnet $\alpha$-RuCl$_3$ across the Field-Induced Phase Transition, Physical Review Letters 120, 067202 (2018).

[21] R. Hentrich, M. Roslova, A. Isaeva, T. Doert, W. Brenig, B. Büchner, and C. Hess, Large thermal Hall effect in $\alpha$-RuCl$_3$: Evidence for heat transport by Kitaev-Heisenberg para-magnons, Physical Review B 99, 085136 (2019).

[22] M. Hirschberger, R. Chisnell, Y. S. Lee, and N. Ong, Thermal Hall Effect of Spin Excitations in a Kagome Magnet, Physical Review Letters 115, 106603 (2015).

[23] C. Strohm, G. L. J. A. Rikken, and P. Wyder, Phenomenological Evidence for the Phonon Hall Effect, Physical Review Letters 95, 155901 (2005).

[24] T. Ideue, T. Kurumaji, S. Ishiwata, and Y. Tokura, Giant thermal Hall effect in multiferroics, Nature Materials 16, 797 (2017).

[25] L. Chen, M.-E. Boulanger, Z.-C. Wang, F. Tafti, and L. Taillefer, Large Phonon Thermal Hall Conductivity in a Simple Antiferromagnetic Insulator, arXiv:2110.13277 (2021).

[26] M. Yamashita, J. Gouchi, Y. Uwatoko, N. Kurita, and H. Tanaka, Sample dependence of half-integer quantized thermal Hall effect in the Kitaev spin-liquid candidate $\alpha$-RuCl$_3$, Physical Review B 102, 220404 (2020).