Intermittency and roughening in the failure of brittle heterogeneous materials

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Abstract
Stress enhancement in the vicinity of brittle cracks makes the macro-scale failure properties extremely sensitive to the micro-scale material disorder. Therefore, (i) fracturing systems often display a jerky dynamics, so-called crackling noise, with seemingly random sudden energy release spanning over a broad range of scales, reminiscent of earthquakes; (ii) fracture surfaces exhibit roughness at scales much larger than that of material microstructure. Here, I provide a critical review of experiments and simulations performed in this context, highlighting the existence of universal scaling features, independent of both the material and the loading conditions, reminiscent of critical phenomena. I finally discuss recent stochastic descriptions of crack growth in brittle disordered media that seem to capture qualitatively—and sometimes quantitatively—these scaling features.

(Some figures in this article are in colour only in the electronic version)

1. Introduction
Understanding how materials break is of fundamental and technological interest. As such, it has attracted much attention from engineers and physicists. Since the pioneer work of Griffith [1], a coherent theoretical framework, the linear elastic fracture mechanics (LEFM) was developed. It states that, in an elastic material, crack initiation occurs when the mechanical energy released by crack advance is sufficient to balance that needed to create new surfaces [1]. Then, once the crack starts to grow, inertial effects should be included in the energy balance (see, e.g. [2]). This continuum theory has proven to be extremely powerful to describe ideal homogeneous media, but fails to capture some features observed in heterogeneous ones.

While the mechanical energy released during crack growth is well determined by continuum theory, the dissipation processes occur within a tiny zone at the crack tip. This separation in length scales, intrinsic to fracture mechanics, makes the failure properties observed at the continuum length scale extremely sensitive to the microstructural disorder of the material. Consequences include important fluctuations in the strength exhibited by different samples of the same material [3], size effects in the strength [4, 5], crackling dynamics with seemingly random violent events of energy release prior and during the failure [6] and rough crack paths [7], among others.

I will focus in this review on two of these aspects, namely, the crackling dynamics evidenced in brittle failure and the morphology of fracture surfaces. Experimental observations performed in this context report the existence of scale-invariant laws [6, 8–10]. This has attracted much attention from the statistical physics community over the last 25 years [11–13]. In this respect, lattice models such as random fuse models (RFMs) [14] were shown to capture qualitatively these scaling features in minimal models, keeping only the two key ingredients responsible for the complexity in material breakdown: the material local disorder and the long-range elastic load redistribution after a local failure event. More recently, it was proposed to extend LEFM descriptions to heterogeneous brittle materials by taking into account the microstructural disorder through a stochastic term [15, 16]. As a result, the onset of crack propagation is analogous to a dynamic phase transition between a stable phase, where the crack remains pinned by material heterogeneities, and a moving phase, where the mechanical energy available at the crack tip is sufficient to make the front propagate. As such, it exhibits universal—and predictable—scaling laws that reproduce fairly well the intermittency statistics observed in crack dynamics [17] and the morphological scaling features exhibited by fracture surfaces [18].
This review is organized as follows. First, I provide a brief introduction to standard LEFM theory in section 2 and discuss some of its predictions in terms of fracture dynamics and crack roughness. Then, I review in section 3 the experiments and field observations that have allowed to characterize the intermittent dynamics observed in many fracturing systems, from earthquakes to laboratory fracture experiments. Section 4 is devoted to a critical overview of the morphological scaling features evidenced experimentally in various materials. RFM models and their predictions in terms of crackling dynamics and crack roughness are discussed in section 5. Section 6 discusses the stochastic descriptions of crack growth and highlights their predictions in terms of scale-free distributions, scaling laws and scale-invariant morphology for fracture in perfectly brittle disordered materials. Finally, section 7 outlines the current challenges and possible perspectives.

2. Continuum theory of crack growth

2.1. Stress concentration

Crack propagation is the basic mechanism leading to the failure of brittle materials. The crack motion in a brittle homogeneous material is classically analysed using methods from elastodynamics. Let us first consider the case of a straight running crack embedded in a linear elastic material under tension. As first noted by Orowan [19] and Irwin [20], the stress field $\sigma_{ij}$ is singular in the vicinity of the crack tip and can be written as (see, e.g. [2])

$$\sigma_{ij}(r, \theta) = \frac{K_1}{\sqrt{2\pi r}} F_{ij}(\theta, v) + T G_{ij}(\theta, v) + O(\sqrt{r}),$$

(1)

where $v$ is the crack speed, $(r, \theta)$ are the polar coordinates in the frame $(\tilde{e}_x, \tilde{e}_y)$ centred at the crack tip (see figure 1(a)) and $F_{ij}(\theta, v)$ and $G_{ij}(\theta, v)$ are some non-dimensional universal functions indicating the angular variation of the stress field, and its variation with $v$. The two prefactors $K_1$ and $T$, called stress intensity factor and T-stress, respectively, only depend on the applied loading and the specimen geometry. They determine the intensity of the singular and non-singular parts of the local field.

This can be generalized to any loading conditions. It is convenient to decompose them into the sum of three modes (figure 1(b)):

- Mode I (tensile mode) corresponds to normal separation of the crack under tensile stresses.
- Mode II (shearing mode) corresponds to a shear parallel to the direction of crack propagation.
- Mode III (tearing mode) corresponds to a shear parallel to the crack front.

In the vicinity of the crack tip, the stress field is then written as a sum of three terms the shape of which is given by the right hand term of equation (1) with prefactors $[K_1, T_1]$, $[K_{II}, T_{II}]$ and $[K_{III}, T_{III}]$ associated with the tensile, shearing and tearing modes, respectively. As we shall see in section 2.3, a crack propagating in an isotropic solid chooses its orientation so that it makes shear vanish at its tip. As a consequence, in most of the crack problems discussed thereafter, we will consider solids loaded dominantly in tension where mode II and III perturbations will be subordinated to that in mode I. Two exceptions will be discussed later in this review, namely, earthquake problems and rocks broken under compression. In both cases shear and/or tear fracture modes are dominant.

2.2. Crack propagation: Griffith theory

Because of the stress singularity, the mechanical energy released as fracture occurs is entirely dissipated within a small zone at the crack tip, referred to as the fracture process zone (FPZ). In nominally brittle materials, the absence of an outer plastic zone can be assumed. According to Griffith’s theory [1], the onset of fracture is then reached when the amount of elastic energy released by the solid as the crack propagates by a unit length is equal to the fracture energy $\Gamma$ defined as the energy dissipated in the FPZ to create two new fracture surfaces of unit area. The form taken by the stress field at the crack tip (equation (1)) allows relating the mechanical energy release $G$ at the onset of crack propagation ($v = 0$) to $K_1$:

$$G = \frac{K_1^2}{E},$$

(2)

where $E$ is the Young modulus of the material. The Griffith criterion for the fracture onset then reads as

$$G \geq \Gamma$$

or equivalently $K_1 \geq K_c$ with $K_c = \sqrt{EG\Gamma}$,

(3)

where the so-called toughness $K_c$ is, like $\Gamma$, a material property to be determined experimentally. Then, once the crack starts to propagate, its motion is governed by the balance between the mechanical energy flux into the FPZ and the dissipation rate $\Gamma v$. From the form taken by the stress field at the crack tip (equation (1)), the energy flux can be related to $K_1$ and $v$, which yields (see, e.g. [2])

$$A(v) \frac{K_1^2}{E} = \Gamma$$

with $A(v) \simeq \left(1 - \frac{v}{c_R}\right)$,

(4)

where $c_R$ refers to the Rayleigh wave speed in the material. The dynamic fracture regime reached when $v$ is on the order of $c_R$ is beyond the scope of this review. The interested reader
can refer to the reviews of Ravi-Chandar [21] and Fineberg and Marder [22] for a detailed presentation of this regime. In the following, I will focus on the slow crack growth regime where \( v \ll c_R \). Then, the equation of motion can be rewritten as

\[
\frac{1}{\mu} v \simeq K_1 - K_c
\]

or equivalently

\[
\frac{1}{\mu'} v \simeq G - \Gamma,
\]

where the effective mobilities \( \mu \) and \( \mu' \) are given by \( \mu = 2c_R/K_c \) and \( \mu' = c_R/\Gamma \), respectively.

### 2.3. Crack path: principle of local symmetry (PLS)

Finally, to complete the continuum theory of crack growth in ideal homogeneous brittle materials, one has to add a path criterion. This is provided by the PLS of Goldstein and Salganik [23] which states that a moving crack progresses along a direction so as to remain in pure tension. In 2D systems, the crack is loaded by a combination of modes I and II only, and the PLS implies that the direction of crack propagation is chosen so that \( K_{II} = 0 \). In 3D systems, the crack load can also contain a mode III component. Then, to cancel \( K_{III} \) and to propagate in pure mode I, the crack front would have to twist abruptly around the direction of propagation, which would yield unphysical discontinuities in the crack path. In this situation, the front is commonly observed to split into many pieces and to form ‘lances’ [24] the origin of which remains highly debated (see, e.g. [25, 26] for recent works on this topic).

### 2.4. Continuum mechanics predictions

There are numerous consequences of the LEM summarized above. Here I will focus on two of them: (i) the equation of motion (equation (5)) predicts regular and continuous dynamics for crack propagation; (ii) the PLS predicts smooth surfaces at continuum scales, i.e. at scales over which the mechanical properties of the considered brittle material are homogeneous. As one will see in the next two sections, there are numerous experimental observations that have been accumulated over the past three decades that contradict both of these predictions.

### 3. Crackling dynamics in fracture

#### 3.1. Earthquake statistics

Contrary to that predicted by LEM, crack growth in heterogeneous brittle materials often displays a jerky dynamics, with seemingly random discrete jumps of a variety of sizes. The most obvious signature of this crackling dynamics can be found in the seismic activity of faults: Earth responds to the slow shear strains imposed by the continental drift through a series of violent impulses, earthquakes, spanning over many orders of magnitude, from barely noticeable to catastrophic ones (figure 2(a)). The distribution of radiated seismic energy presents the particularity to form a power law (figure 2(b)) with no characteristic scale:

\[
P(E) \propto E^{-\beta},
\]

where the exponent \( \beta \simeq 1.7 \) was proven to be very robust over different regions [27, 28].

Earthquake sizes are most commonly quantified by their magnitude \( M \). The first (and most popular) magnitude scale is the one introduced by Richter [29] that relates \( M \) to the logarithm of the maximum amplitude measured on a Wood–Anderson torsion seismometer at a given distance from the earthquake epicentre. In this respect, \( M \) is relatively easy to measure and it provides information that is directly useful for various engineering applications. Magnitude and radiated seismic energy of earthquakes are usually related via the Gutenberg–Richter empirical relation [30]:

\[
\log_{10} E = 1.5M + 11.8,
\]

where \( E \) is expressed in joules. While this relation is considered to be reasonably accurate for earthquakes of small and intermediate sizes, its validity is questioned for great earthquakes [31]. Using this energy–magnitude relation in equation (6) leads to the Gutenberg–Richter frequency-magnitude statistics:

\[
p(M) \propto \exp(-bM),
\]

where \( b \) and \( \beta \) are related through \( \beta = b/1.5 + 1 \).

The \( b \) measurement has been one of the most frequently discussed topics in seismology studies (see, e.g. [27] for a recent review). Observed \( b \) values of regional seismicity typically fall in the range 0.7–1.3 and usually take a value around 1 [27]. Its universality (or not) remains an open question [27], made particularly tough since \( b \) measurement can be affected by various factors as e.g. the type of magnitude scale chosen, the completeness and the magnitude range of the collected data, to name a few.

The radiated seismic energy is also found to scale with the rupture area \( S \) of earthquakes as [32, 33]

\[
E \propto S^{1.5}.
\]

Using this relation in equation (6) yields the following distribution for rupture area:

\[
p(S) \propto S^{-\tau},
\]

with \( \tau = 1.5\beta - 0.5 \simeq 2 \).

The time occurrence of earthquakes exhibits also scale-free features expressed by Omori’s law [8], which states that a main earthquake is followed by a sequence of aftershocks the frequency of which decays with time as \( t^{-\alpha} \), with \( \alpha \simeq 1 \). More recently, driven by the availability of complete seismic catalogues (see, e.g. [34]) gathering the occurrence time, the magnitude and the 3D location of earthquakes, many studies [35–41] have been performed to characterize the spatio-temporal organization of earthquake events. In particular, Bak et al [35] have shown that the distribution of waiting times \( P(\Delta t) \) between earthquakes of energy larger than \( E \) within a zone of size \( L \) obeys the following generalized expression of Omori’s law (figure 2(c)):

\[
P_{E,L} (\Delta t) = \Delta t^{-\alpha} f(\Delta t/\Delta t_0) \quad \text{with} \quad \Delta t_0 \propto 1/(E^{-\beta} L^d),
\]
where $\beta$ is the exponent defined in equation (6), $\alpha \simeq 1$ is the Omori exponent, $d_f \simeq 1.2$ is a fractal dimension characterizing the spatial distribution of epicentres and $f(x)$ is a function constant for $x \ll 1$, and decaying rapidly with $x$ for $x \gg 1$.

3.2. AEs in tensile failure

Signature of crackling dynamics is also evidenced in the AE that goes with mode I failure of many brittle materials: the distributions of energy and silent time between two successive events have been computed in many fracture experiments [45–51] and found to follow power laws similar to what observed in earthquakes (see equations (6) and (11)). It should be emphasized, however, that the relation between AE energy and released elastic energy remains largely unknown (see, e.g., [52] for recent work in this context). The exponents $\beta_{AE}$ and $\alpha$ associated with AE energy and silent time were reported to depend on the considered material: they were found to be e.g. $\{\beta_{AE} \simeq 1.5, \alpha \simeq 1.9\}$ in wood [46, 47], $\{\beta_{AE} \simeq 2, \alpha \simeq 2.7\}$ in fibreglass [46, 47], $\{\beta_{AE} \simeq 1.3–1.8, \alpha \simeq 1–1.5\}$ in polymeric foams [50, 51], $\{\beta_{AE} \simeq 1, \alpha \simeq 1.25\}$ in sheets of paper [48].

As a synthesis, the values for $\beta_{AE}$ and $\alpha$ measured in mode I fracture experiments range typically between 1 and 2. Santucci et al [53, 54] have also observed directly the slow subcritical

$^{1}$ In polymeric foams, both $\beta_{AE}$ and $\alpha$ were also found to depend on the loading conditions (load-imposed or displacement-imposed) and on the temperature [50, 51].
intermittent crack growth in 2D sheets of paper, up to their final breakdown. In these experiments, the crack tip is found to progress through discrete jumps the size of which is power law distributed with an exponent close to 1.5 up to a stress-dependent cutoff that diverges as a power law at $K_c$. The waiting time $\Delta t$ between two jumps is found to exhibit power-law distribution with Omori’s exponent $\alpha$ close to 0.7 for $\Delta t$ smaller than the mean value, and $\alpha = 2$ for $\Delta t$ larger than the mean value.

It is worth noting that most of the AE fracture experiments reported in the literature are non-stationary: usually, one starts with an intact specimen and loads it up to its catastrophic failure. In these tests, the recorded AE reflects more the micro-fracturing processes preceding the initiation of the macroscopic crack than the growth of this latter. More recently, Koivisto et al [49] and Salminen et al [55] investigated the AE statistics in a steady regime of crack propagation in experiments of paper peeling. The distribution of AE energy follows a power law with an exponent $\beta_{AE} \simeq 1.8 \pm 0.2$ significantly higher than that observed in standard tensile tests starting from an initially intact paper sheet [48]. Nevertheless, the silent time between successive events was found to display statistics and correlations significantly more complex than those of Omori’law.

### 3.3. Intermittent crack growth along planar heterogeneous interfaces

AE methods are very powerful to localize micro-fracturing events in time. On the other hand, to relate quantitatively the AE energy to the local crack dynamics or the mechanical energy released through these events remains a rather difficult task (see, e.g. [52] for recent work in this context). This has motivated Måløy, Santucci and co-workers to investigate experimentally the dynamics of crack propagation in a simpler configuration, namely, the one of a planar crack propagating along the weak heterogeneous interface between two sealed transparent Plexiglas plates [56, 57]. Using a fast camera, they directly observe crack propagation (figure 3(a), from [57]).

To characterize the local dynamics, they computed the time spent by the crack front at each location $(z,x)$ of the recorded region. A typical grey-scale image of this so-called waiting-time map, $w(z,x)$, is shown in figure 3(b) (from [57]). The intermittency reflects in the numerous and various regions of grey levels. The local speed $v(z,x)$ of the crack front as it passes through a particular location $(z,x)$ is then proportional to $1/w(z,x)$. Quakes are then defined [57] as connected zones where $v(z,x) \geq c(v)$ where $(v)$ denotes averaging over all locations and $c$ is a constant ranging from 2 to 20 (figure 3(c), from [57]). Recent analyses performed by Grob et al [58] reveal many similarities between the statistics of these experimental quakes and that of real earthquakes: first, the distribution of quake area, $S_a$, follows a power law, characterized by an exponent $\tau_a \simeq 1.7$ [57], which was suggested [58] to be analogous to the energy distribution characterized by an exponent $\beta \simeq 1.7$ observed in real earthquakes (equation (6), figure 4); second, the distribution of time occurrence and spatial distance between the epicentres of two successive events were found to be the same in both experimental quakes and real earthquakes (equations (11) and (12), respectively). All these distributions were found to be independent of the clipping parameter $c$ used to define the quakes in the waiting-time maps [57, 58].

Let us draw some main conclusions from the experimental and fields’ observations reviewed here:

(i) Failure of brittle heterogeneous materials displays crackling dynamics, with sudden random events of energy release and/or jumps in the crack growth. The event statistics is characterized by various power-law distributions in e.g. energy, size or silent time between successive events. This crackling dynamics is observed in the micro-fracturing processes occurring prior to the initiation of a macroscopic crack as well as during the slow propagation of this latter.

(ii) The exponents associated with the distribution of energy and silent times between the micro-fracturing events preceding the initiation of a macroscopic crack are generally observed to depend on the material, loading conditions, environment parameters, etc

(iii) The intermittency observed during slow crack propagation in an heterogeneous brittle medium loaded under tension shows strong similarities to earthquake dynamics of faults. The exponents associated with the different power law distributions seem to take universal values.

Finally, it is worth mentioning that crackling dynamics has also been observed, at the nanoscale, in cleavage experiments performed on pure crystals such as mica [59] and sapphire [60, 61]. In the second case, both the distributions of released energy and silent time between two successive nanofracture
In both plots, the straight line corresponds to a power-law for a wide range of clipping parameter \((r, h)\) are statistically invariant through the affine transformation from \((57)\).

et al the waiting-time maps computed in the model experiment of Måløy et al (see figure 3). The notation \(S = S_0\) was the one used in \((57)\). The various symbols correspond to various values of mean front velocity \((v)\) and pixel resolution \(a\). Inset: distribution of quake area for a wide range of clipping parameter \(c\). The axes are logarithmic. In both plots, the straight line corresponds to a power-law distribution with \(tau_c \approx 1.7\). (Courtesy of Santucci and Måløy, from \((57)\)).

events have been computed. They are found to obey power law similar to that observed in AE during the fracture of brittle heterogeneous materials or in earthquakes.

4. Morphology of cracks

We turn now to crack roughness. The morphology of fracture surfaces has been widely investigated over the last century—fractography is now routinely used to determine the causes of failure in engineering structures (see, e.g. \([7]\) for a recent review). In section 2.3, we saw that LEFM states that crack propagation in brittle materials remains in pure tension as long as \(v\) remains small. This would lead to smooth fracture surfaces. Experimentally, the failure of disordered media leads to rough surfaces at length scales much larger than that of the microstructure (figure 5).

4.1. Self-affine scaling features of crack profiles

Starting from the pioneering work of Mandelbrot et al \([9]\), many experiments have revealed that fracture surfaces exhibit self-affine morphological scaling features. In other words, crack profiles such as the one shown in figure 6(a) are statistically invariant through the affine transformation \((r, h) \rightarrow (\lambda r, \lambda^\xi h)\) where \(r\) and \(h(r)\) refer to the in-plane and out-of-plane coordinates, respectively. The exponent \(\xi\) is called the Hurst or roughness exponent. Experimentally, this affine invariance can be checked by computing the distribution \(P_{\Delta r}(\Delta h)\) of height increments between two points separated by a distance \(\Delta r\) (figure 6(h)): for self-affine crack profiles, \(P_{\Delta r}(\Delta h)\) takes the following form (figure 6(c))

\[
P_{\Delta r}(\Delta h) = \Delta r^{-\xi} f(\Delta h/\Delta r^\xi).
\]

The shape of the generic distribution \(f\) is often found to be very close to a Gaussian (see, e.g. \([65, 66]\) and figure 6(c)). Then, \(\xi\) allows one to characterize entirely the statistics of crack roughness. In this respect, its determination was an important issue and led to many studies over the last 25 years. Besides the full computation of the distribution \(P_{\Delta r}(\Delta h)\) that presents the drawback to necessitate a large amount of data, several other methods were developed to measure \(\xi\). They will not be reviewed here. The interested reader may refer to the work of Schmittbuhl et al \([67]\) for more information on these methods and their limitations. In the following, we will use one of the most common ones, which consists of computing the height–height correlation function:

\[
\Delta h(\Delta r) = \left< (h(r + \Delta r) - h(r))^2 \right>^{1/2},
\]

where \(< >\) denotes average over all positions \(r\) along a given profile. For a self-affine profile, one gets

\[
\Delta h(\Delta r) \propto \Delta r^\xi.
\]

It is important to emphasize that this method—as well as many others—allows one to measure the roughness exponent provided that the profile is actually self-affine, but does not ensure this self-affinity by itself.

Possible universality in the morphological scaling features of cracks was first mentioned by Temionia and Meakin \([68]\); they made use of a minimal 2D molecular model of material failure to address the question and observed that the fractal dimension of the final crack was independent of the elastic constants over a wide range of values. Bouchaud et al \([69]\) were the first to suggest a universal value for the roughness exponent \(\xi\) on the basis of experimental observations: in their seminal series of fractography experiments, the authors observed that \(\xi \approx 0.8\) in four specimens of aluminium alloy on which different heat treatments had conferred different microstructures and toughness. This universality seemed to be corroborated by Maloy et al \([70]\) who observed similar \(\xi\) values in six different brittle materials. Thereafter, many experimental works were focused on the measurement of \(\xi\)—the review written by Bouchaud \([10]\) synthesizes most of them up to 1997. Reported \(\xi\) values are between 0.5 and 0.9 in metallic alloys \([9, 69, 71, 72]\), around 0.9 in graphite \([70]\), around 0.75 in porcelain \([70]\), around 0.85–0.9 in Bakelite \([70]\), between 0.5 and 0.8 in oxide glasses \([73, 74]\), around 0.8–0.9 in wood \([75]\) and in ice \([76]\), around 0.45–0.5 in Sandstone \([66, 77, 78]\), around 0.45–0.5 in Basalt \([78]\), around 0.7–0.8 in mortar \([79]\) and around 0.4–0.5 in glassy ceramics made of sintered glass beads with various porosities \([80, 81]\). As a result, these fractography experiments performed over a period spreading from 1984 to 2006 led to mitigated opinions on the relevance or not of this universality \([71, 82]\). As we will see in the two next sections, some recent observations allow one to reconcile this apparent spreading in the measurements of \(\xi\) with the scenario of universal morphological scaling features.

All the measurements reported in the preceding paragraph concern the roughness of fracture surfaces resulting from the failure of 3D specimens. Crack morphology was also explored in 2D geometries. Crack lines in paper broken in tension
were also found to be self-affine with a roughness exponent $\zeta \simeq 0.7$ [65, 83, 84]. Note that a small but clear difference was found in $\zeta$ between the slow (subcritical) and the fast growth regimes [85]. Crack fronts in the interfacial fracture experiments presented in section 3.3 were also reported to exhibit self-affine scaling features. The associated roughness exponent was found to be $\zeta_H \simeq 0.6$ [86, 87]. Let us end this section by mentioning that this picture of simple self-affine cracks was recently questioned. First, multi-scaling, characterized by non-constant scaling exponents $\zeta_q$ between the higher order height–height correlation function $\Delta h_q(\Delta r) = \langle (h(r + \Delta r) - h(r))^q \rangle^{1/q}$ and $\Delta r$, was invoked in both 2D geometries [88] and 3D geometries [89]. This multi-scaling seems, however, to disappear at
large scales [65, 84, 90]. Second, fracture surfaces exhibit anomalous scaling [75, 91]: the introduction of an additional global roughness exponent, \( \xi_{\text{glob}} \), is necessary to describe the scaling between the global crack width and the specimen size. Experiments reveal that \( \xi_{\text{glob}} \) depends on the material and on the fracture geometry [75, 91].

4.2. Family–Vicsek scaling of fracture surfaces

To uncover the primary cause of the apparent spreading in the measured roughness exponents, scaling anisotropy was looked for [62–64] in the fracture surfaces of various materials (quasicrystals, glasses, metallic alloys, wood and mortar) broken under various conditions (cleavage, stress corrosion, tension) and various velocities, ranging from \( 10^{-12} \) to \( 10^2 \) m s\(^{-1} \). At this point, it is important to set the notations (figure 14): in all the following, the axis \( \hat{e}_x \), \( \hat{e}_y \) and \( \hat{e}_z \) will refer to the direction of crack propagation, the direction of tensile loading and the mean direction of the crack front, respectively. Special attention was paid to ensure that fracture surfaces were observed in a steady-state regime, far enough from crack initiation so that roughness becomes statistically invariant of the position \( x \) along the direction of crack growth. By computing the height–height correlation function \( \Delta h(\Delta r) \) for profiles making various angles \( \theta \) with \( \hat{e}_x \), the existence of anisotropy in the scaling features (figure 7) was evidenced: \( \xi(\theta) \) varies from \( \xi(\theta = 0) = \xi_\parallel \approx 0.6 \) (profiles taken parallel to the direction of crack growth) to \( \xi(\theta = 90^\circ) = \xi_\perp \approx 0.8 \) (profiles taken perpendicular to the direction of crack growth, parallel to the mean direction of the crack front).

As a consequence, a full characterization of the scaling statistical properties of fracture surfaces calls for the computation of the 2D height–height correlation function:

\[
\Delta h(\Delta z, \Delta x) = \langle (h(z + \Delta z, x + \Delta x) - h(z, x))^2 \rangle_{\Delta z, \Delta x}
\]

This was shown [62–64] to take a peculiar form, referred to as Family–Vicsek scaling [92] (figure 8):

\[
\Delta h(\Delta z, \Delta x) \propto \Delta x^{\xi} f(\Delta z/\Delta x^{\xi/\xi_\parallel}),
\]

where \( f(u) \propto \begin{cases} 1 & \text{if } u \ll c, \\ u^{-\xi_\parallel} & \text{if } u \gg c. \end{cases} \) (17)

Such Family–Vicsek scalings are classically observed in interface growth processes. In this respect, the exponent \( \xi_\parallel \) and the ratio \( \xi_\perp/\xi_\parallel \) can be mapped to a growth exponent and a dynamic exponent, respectively. It is worth mentioning that such an anisotropic Family–Vicsek scaling of fracture surfaces was first suggested by the phenomenological Langevin description of crack propagation proposed by Bouchaud et al [93] detailed thereafter in section 6.3.

The two exponents \( \xi_\parallel = 0.75 \pm 0.05 \) and \( \xi_\perp = 0.6 \pm 0.05 \) were found to be universal, independent of the considered material, the failure mode and the crack growth velocity over the whole range from ultra-slow stress corrosion fracture (picometre per second) to rapid failure (a few hundred metres per second). On the other hand, the range of length scales over which these Family–Vicsek scaling features are observed is limited and material dependent. This will be discussed in section 4.3.

By rescaling the distances \( \Delta x \) and \( \Delta z \) by the topothesies \( \ell_x \) and \( \ell_z \), defined as the scales at which the out-of-plane increment becomes equal to the in-plane one, i.e. \( \Delta h(\Delta z = 0, \Delta x = \ell_x) = \ell_z \) and \( \Delta h(\Delta z = \ell_z, \Delta x = 0) = \ell_x \), one can rewrite the Family–Vicsek scaling [62]:

\[
\Delta h(\Delta z, \Delta x) = \ell_x (\Delta x/\ell_x)^{\xi} g \left( u = \frac{\ell_x}{\ell_z} \frac{\Delta z}{\Delta x^{\xi/\xi_\parallel}} \right),
\]

where \( g(u) = \begin{cases} 1 & \text{if } u \ll 1, \\ u^{-\xi_\parallel} & \text{if } u \gg 1. \end{cases} \) (18)

The exponent of \( f \) is then found to be universal [62], independent of the considered material, of the failure mode and of the crack growth velocity (figure 9).

It is important to insist that this universal Family–Vicsek scaling is observed in fracture surfaces far enough from crack initiation, in a regime where roughness becomes statistically invariant with respect to translations along the direction of crack growth \( \hat{e}_x \). The transient roughening regime starting from an initial straight notch, was extensively studied by Morel et al [75, 94] and Lopez et al [91]. It was found to display a scaling significantly more complex than that expected in the Family–Vicsek scenario. It involves the global roughness exponent \( \xi_{\text{glob}} \) defined at the end of section 4.1 as well as a

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**Figure 7.** (a) 1D height–height correlation function measured parallel and perpendicular to the crack propagation direction. The axes are logarithmic. (b) Variation of roughness exponent \( \xi \) as a function of the angle \( \theta \) between the considered profile and the direction of crack propagation. Data were measured in quasi-crystals (from [62]).
new independent dynamic exponent. These two exponents are found to depend on both the material and specimen geometry.

4.3. On the relevant length scales

Several fractography observations reported in the literature are found not to be compatible with the Family–Viseck scaling (equation (17)) and its associated exponents \( \zeta_\perp \approx 0.75, \zeta_\parallel \approx 0.6 \). In particular, experiments in sandstone \([66, 77, 78]\) and in granular packings made of sintered glass beads \([18, 80, 81]\) have shown self-affine scaling properties characterized by roughness exponents significantly smaller, around 0.4–0.5. In the case of granular packings made of sintered glass beads, the 2D height–height correlation function was found to exhibit Family–Viseck scaling as for the materials investigated in the previous section, but with \( \{ \zeta_\perp \approx 0.4, \zeta_\parallel \approx 0.5 \} \) \([18]\) (figure 10). This suggests the existence of a second universality class for post-mortem fracture surfaces.

To uncover the origin of these two distinct series of observed exponents, the range of length scales over which these two regimes are observed was examined \([18]\). In all experiments compatible with \( \{ \zeta_\perp \approx 0.8, \zeta_\parallel \approx 0.6 \} \), fracture surfaces were observed at small length scales, below a given cutoff length \( \xi \) of e.g. a few nanometres in quasi-crystals \([62]\), a few tens of nanometres in glass \([18, 73, 95]\), a few hundred micrometres in metallic alloys \([72, 73]\), a few millimetres in wood \([63, 75]\), a few tens of millimetres in mortar \([79, 94]\), etc. On the other hand, observations compatible with \( \{ \zeta_\perp \approx 0.4, \zeta_\parallel \approx 0.5 \} \) were observed at large scales, well above the microstructure scale, up to a cutoff length set by the specimen size \([66]\).

It was then proposed, in \([18]\), that the upper cutoff length \( \xi \) that limits the \( \{ \zeta_\perp \approx 0.8, \zeta_\parallel \approx 0.6 \} \) Family–Viseck scaling regime is set by the size of FPZ. This conjecture was proven.
to be true in oxide glasses [18], quasi-crystals [62] and mortar [94]. Work in progress [96] seems to indicate that it is also true in metallic alloys: CT specimen was broken at various temperatures, ranging from 20 to 148 K—this allows one to tune the FPZ size, from 20 \( \mu \)m to 1 mm. And in all these tests, the cutoff length \( \xi \) of the self-affine \( \xi \approx 0.8 \) regime is found to be roughly proportional to the process zone size.

It is worth mentioning that no anomalous scaling was reported in Sandstone fractography experiments [66]. This suggests that the anomalous scaling commonly associated with the \( \xi \approx 0.8 \) self-affine regime [75,91] is absent in the large scale \( \xi \approx 0.4 \) self-affine regime. It is then appealing to associate the observation of this anomalous scaling to damage spreading at a scale smaller than the FPZ.

Let us also note that a third isotropic scaling regime arising at small scale, characterized by a roughness exponent \( \xi \approx 0.5 \) was observed in ductile materials such as Ti3Al-based super-\( \alpha_2 \) intermetallic alloy broken under fatigue within the 1–10 \( \mu \)m range [10,73,97], and in Zr-based metallic glass within the range 100 nm–1 mm [98]. It should be emphasized that in the second case, the distribution of height increments does not display the collapse given by equation (13) expected for truly self-affine profiles. A similar \( \xi \approx 0.5 \) isotropic regime was also claimed [10,73] to be observed on the AFM fracture surfaces of soda-lime broken under stress corrosion, within 1 nm–10 nm range. However, this observation has been recently called into question since the observations remain confined within (in-plane) AFM resolutions. Moreover, it was not reproduced in nanosaved fracture surfaces of various silicate glasses (silica, soda-lime, borosilicate and aluminosilicate) broken under stress corrosion with crack velocities as small as the picometre per second [18,63]. As a result, this third small-scale regime characterized by \( \xi \approx 0.5 \) is suspected to be inherent to ductile fracture. As such, it will not be discussed in the following.

Let us conclude this section by drawing the outlines of the fractography experiments reviewed here:

(i) Failure of brittle materials leads to rough fracture surfaces, with self-affine morphological features.
(ii) Far enough from crack initiation, the morphology of fracture surfaces exhibits anisotropic Family–Viseck scaling, characterized by two distinct roughness exponents \( \xi_{\parallel} \) and \( \xi_{\perp} \), measured parallel and perpendicular to the direction of crack propagation, respectively. By analogy with interface growth problems, the exponent \( \xi_{\parallel} \) and the ratio \( \xi_{\perp}/\xi_{\parallel} \) can be mapped to a growth and a dynamic exponent, respectively.
(iii) Two distinct sets of universal exponents are observed: \( \xi_{\parallel} \approx 0.8, \xi_{\parallel} = 0.6 \) at small scales and \( \xi_{\parallel} \approx 0.4, \xi_{\parallel} = 0.5 \) at large scale.
(iv) The cutoff \( \xi \) of the small-scale regime is set by the FPZ size.

5. Numerical observations in lattice models

The two preceding sections allowed to illustrate that the failure of brittle heterogeneous materials leads to (i) intermittent crackling dynamics and (ii) rough fracture surfaces. In both cases, one observes scaling invariance in terms of distribution and morphology. Some of these scaling features appear universal—the associated scaling exponents are the same in a wide range of materials and loading conditions. This is reminiscent of critical phenomena where the balance between local (thermal or quenched) disorder and simple interactions between microscopic objects may select such complex scale-invariant organization of the system (see, e.g. [99] for an introduction to critical phenomena).

In this context, it was proposed, initially by de Arcangelis et al [100], to exploit the analogy between scalar mode III elasticity and electricity and sketch an heterogeneous brittle material as a network of fuses of unit ohmic resistance and randomly distributed breakdowns (figure 11). In these so-called RFMs, current and voltage are analogous to force and displacement, respectively. The goal is not to reproduce exactly the failure of real brittle materials but to see to what extent one can reproduce the scaling features presented in sections 3 and 4 in such minimalist materials, keeping only the two main ingredients characterizing material failure, namely, the microstructure randomness and the long-range coupling that accompanies the load redistribution after each element breakdown.
5.1. Intermittency

A typical ‘fracture’ experiment performed on 2D RFM is represented in figure 12. The ‘loading’ is increased by increasing the voltage $U$ of the nodes belonging to the upper electrode while keeping the bottom ones at zero. Voltages at each node and currents crossing each element are determined such that Kirchhoff law and Ohm’s law are satisfied everywhere. At some point, a fuse breaks. As a result, the current increases abruptly in the remaining elements. In turn, this may trigger a cascade, avalanche, of elements breakdowns that can lead either to a stable situation or to the failure of the whole structure. Typical evolution of the size of these avalanches as a function of the imposed voltage $U$ is plotted in figure 13. Avalanches are analogous to the AE events observed prior to the catastrophic failure of real brittle materials, as discussed in section 3.2.

The distribution $P(S)$ of avalanche sizes, i.e. the number of fuses participating in a breakdown cascade, has been investigated numerically, first by Hansen and Hemmer [101] and later by other teams [102–105]. It was found to follow a power law $P(S) \propto S^{-\tau}$ up to a cutoff $S_0$ that scales with the lattice size $L$ as $S_0 \propto L^D$ where $D$ refers to the avalanche fractal dimension. Initially, 2D simulations yielded a universal value $\tau \simeq 2.5$ [101, 102]. However, recent large scale simulations suggest that $\tau$ depends slightly on the lattice type in 2D systems: $\tau = 2.75$ and $\tau = 3.05$ in diamond and triangular lattice, respectively [104]. On the other hand, $D$ was found to be universal: $D \simeq 1.18$. In 3D simulations, both exponents were found to be universal [105]: $\tau = 2.5$ and $D = 1.5$. 

5.2. Morphology of fracture surfaces

The morphology of final cracks has also been investigated numerically [103, 104, 106–111]. As in the experiments, it was found to exhibit self-affine scaling features. In 2D RFM such as the one shown in figure 12, large scale simulations yield a universal roughness exponent $\zeta = 0.71$ independent of the lattice topology, the presence or not of a pre-existing initial notch and of the disorder in breaking thresholds [104, 111]. In 3D systems, $\zeta \simeq 0.42$ [103, 108, 110].

It is interesting to note that the value of $\zeta$ measured in 2D RFM is pretty close to the $\zeta \simeq 0.7$ observed experimentally in quasi-two-dimensional materials [65, 83, 84]. In 3D RFM, the value $\zeta \simeq 0.42$ is significantly smaller than the $\zeta \simeq 0.8$ observed experimentally in a wide range of materials (see sections 4.1 and 4.2). On the other hand, it is pretty close to the value $\zeta \simeq 0.4 - 0.5$ observed at scale larger than the FPZ size in sandstone [66, 77], glassy ceramics made of sintered glass beads [80, 81] and mortar [94]. It should be emphasized that the morphological scaling analyses reported in [103, 108, 110] were performed by averaging together the results of different simulations starting from configurations without initial notch, and therefore no prescribed direction of crack propagation. As a consequence, the scaling anisotropy reported in experiments and described in section 4.2 cannot be captured.

Let us finally mention that large scale numerical simulations revealed the existence of anomalous scaling in both 2D and 3D RFM [104, 111]. The origin of this anomalous scaling remains unclear. It seems to be intrinsic to the scalar dimensionality of RFM since it disappears in random beam models [112].
6. Stochastic theory of crack growth

Experimental observations gathered in sections 3 and 4 revealed that slow crack growth in brittle disordered materials (i) exhibits an intermittent dynamics with scale-free distributions in space, time and energy and (ii) leads to rough fracture surfaces with scale-invariant morphological features. As was exposed in section 5, these two aspects can be qualitatively reproduced in simple numerical models such as RFM. However, these models rely on important simplifications which make quantitative comparisons with experiments difficult.

We now turn to another approach—pioneered by Gao and Rice [15] and later extended by Bower and Ortiz [113], Schmittbuhl et al [16], Larralde and Ball [114], Ramanathan et al [115] and Lazarus [116], which consists of using LEFM to describe crack growth in an elastic isotropic material in the presence of deterministic or random obstacles. As we will see below, to first order, the equation of motion and the equation of path are decoupled and can be solved independently.

6.1. Equation of motion

Let us consider the situation depicted in figure 14 of a crack front that propagates within a 3D elastic solid. Provided that the motion is slow enough, the local velocity of a point $M(z, x = f(z, t), y = h(z, x = f(z, t)))$ is proportional to $G(M) - \Gamma(M)$ (equation (5)). Obstacles and inhomogeneities in the material structure are then captured by introducing a fluctuating component into the fracture energy: $\Gamma(M) = \Gamma^0 (1 + \eta(M))$ where $\eta(M)$ denotes the fluctuating part of fracture energy. This induces distortions of the front, which in turn generates perturbation in $G(M)$. One can then use the three-dimensional weight-function theory derived by Rice [117] to relate $G(M)$ to $\{f, h\}$. To first order, $G(M)$ depends on the in-plane distortions $f(z, t)$ only and the resulting equation of motion reads [16, 115] as

$$\frac{1}{\mu} \frac{\partial f}{\partial t} = F + \frac{G^0}{2\pi} \int f(z) - f(z') \frac{d z'}{(z - z')^2} + \Gamma^0 \eta(z, f(z, t)).$$

(19)

where $F = G^0 - \Gamma^0$ and $G^0$ denotes the reference mechanical energy release which would result from the same loading with a straight front at the same mean position. $G^0$ depends on both the specimen geometry and the loading conditions. It is worth mentioning that this equation of motion was recently shown by Dalmas et al [118] to reproduce quantitatively experiments which consists of making a planar crack propagate along a weak interface with simple mono-dimensional regular patterns in a thin film stack (single band or periodic bands drawn parallel to the direction of crack propagation).

In the case of heterogeneous materials, $\eta(M)$ is chosen to be random, with short range spatial correlation, zero mean and constant variance. Let us first consider the situation of constant remote stress loading conditions, i.e. constant $F$. This assumption will be relaxed later in this section. As first noticed by Schmittbuhl et al [16], the front motion described by equation (19) exhibits a so-called depinning transition controlled by the ‘force’ $F = G^0 - \Gamma^0$ and its position with respect to a critical value $F_c$ (see figure 15(a)).

- When $F < F_c$ the crack front is pinned by disorder and does not propagate.
- When $F \gg F_c$ the crack front moves with a mean velocity $\bar{v} = (\partial f/\partial t)_{z,t}$ proportional to $F$: $\bar{v}/\mu = G^0 - \Gamma^0$. One recovers the equation of motion expected by LEFM (equation (5)).
- When $F = F_c$ a critical state is reached.

The consequences of this criticality will be extensively commented on later in this section. But it is now time to
and the loading conditions (see, e.g. [17] for their value in the case of the interfacial experiment described in section 3.3). The crack motion can then be decomposed as follows (figure 15):

\[ F(t, \langle f \rangle) = c t - k \langle f \rangle(t), \]

where \( c \) and \( k \) are constants depending on the precise geometry and the loading conditions (see, e.g. [17] for their value in the case of the interfacial experiment described in section 3.3). The crack motion can then be decomposed as follows (figure 15(b)): when \( F \leq F_c \), the front remains pinned and the effective force increases with time. As soon as \( F \geq F_c \), the front propagates, \( \langle f \rangle(t) \) increases, and, as a consequence, \( F \) is reduced. This retro-action process keeps the system attracted to the critical state, as in models of self-organized criticality [119]. For high values of \( c \), the front moves smoothly with mean velocity \( \langle v \rangle = c/k \). For low values of \( c \), stick–slip motion is observed and the crack propagates through distinct avalanches between two successive pinned configurations. In the limit \( c \to 0 \) and \( k \to 0 \), criticality is reached and one expects to observe universal scaling features:

- independent of the microscopic and macroscopic details of the system, i.e. the considered material and the loading conditions;
- identical to those observed in other systems belonging to the same universality class, as e.g. interface motion in disordered magnets [120, 121] and wetting of rough substrates [122, 123].

These scaling features can be predicted using theoretical tools issued from statistical physics like functional renormalization methods (FRG) or numerical simulations. In particular, for \( F \) slightly above \( F_c \), the front \( f(z) \) is self-affine and exhibits Family–Viseck dynamic scaling form up to a correlation length \( \xi \):

\[ \langle (f(z + \Delta z, t + \Delta t) - f(z, t))^2 \rangle^{1/2} \propto \Delta t^{\zeta_H/\kappa} g(\Delta z/\Delta t^{\zeta_H/\kappa}), \]

where

\[ g(u) \propto \begin{cases} 1 & \text{if } u \ll c \\ u^{\zeta_H/\kappa} & \text{if } u \gg c \end{cases} \]

(21)

\( \zeta_H \) and \( \kappa \) refer to the roughness exponent and the dynamic exponent, respectively. It is worth recalling that \( \zeta_H \) is associated with the in-plane roughness of the crack front and is therefore different from \( \xi \) defined in section 4 to characterize the out-of-plane roughness. Above \( \xi \), the front is no longer self-affine and exhibits logarithmic correlation.

The exponents \( \zeta_H \) and \( \kappa \) were estimated using FRG methods. They were found to be between \( \zeta_H = 1/3, \kappa = 7/9 \) [123] (first order) and \( \zeta_H = 0.47, \kappa = 0.66 \) [124] (second order). More recently, they were precisely evaluated by Rosso and Krauth [125] and Duenmmer and Krauth [126] using numerical simulations:

\[ \zeta_H = 0.385 \pm 0.005, \quad \kappa = 0.770 \pm 0.005. \]

(22)

The correlation \( \xi \) is set by the distance between the force \( F \) and its critical value \( F_c \). In situations where \( F \) is kept constant, \( \xi \) diverges at \( F_c \) as

\[ \xi \propto (F - F_c)^{-\nu} \quad \text{with} \quad \nu = \frac{1}{1 - \zeta_H} = 1.625 \pm 0.010, \]

(23)

where the relation between \( \nu \) and \( \zeta_H \) comes from a specific symmetry of equation (19), referred to as the statistical tilt symmetry [127]. In situations where \( F \) varies with time and crack length according to equation (20), \( \xi \) is set by the ‘strength’ of the feedback term. To be more precise,
where the cutoff scales as $T_0 \propto \xi^\kappa$, which implies

$$T_0 \propto k^{-1/\Delta} \quad \text{with} \quad \Delta = 1/\kappa v_0 = 1.300 \pm 0.010. \quad (29)$$

Presently, these scaling predictions on the global dynamics of the crack front were not confronted directly to experiments. Work in progress [128] seems to indicate that the size and duration of the mean front jumps observed in the interfacial experiment presented in section 3.3 are distributed according to equations (25) and (28), respectively. On the other hand, the local dynamics of the stochastic description presented here was directly confronted to the interfacial experiment presented in section 3.3 [18]. Equation (19) with $F$ given by equation (20) was solved for various values of parameters and the results were analysed using the procedure described in section 3.3: the map of waiting times (figure 17(a)), i.e. the time spent by the crack front at each point $(z, x)$ of the recorded region was computed, as in the experiments presented in section 4.3. Then, the avalanches were defined by thresholding these maps of waiting time, also as in section 4.3 (figure 17(b)). The statistics of the obtained avalanches was finally computed and found to be the same as that observed in the interfacial experiment, with, in particular, a power-law distribution for avalanches area characterized by an exponent $\tau_w \simeq 1.7$ (figure 17(c)). Presently, the relation between $\tau_w$ and the standard exponents defined in equations (22)–(29) remains unknown.

It is finally worth discussing the roughness exponent $\zeta_H \simeq 0.385$ expected for the in-plane crack front (equation (22)). This value is significantly smaller than the value $\zeta_H \simeq 0.6$ reported in interfacial experiments [86, 87]. However, recent experimental results [129] shed light on the apparent disagreement between the two and report at large scales a roughness exponent $\zeta_H \simeq 0.35$ compatible with theoretical predictions of equation (22).

$\xi$ is defined [121] as the length for which the feedback term $-k(\xi')/\xi$ in equation (20) balances the elastic term $(G_0^0/2\pi) \int (f(z) - f(z'))/(z - z')^2 \, dz'$ in equation (19). This leads to $k\xi/L \approx G_0^0 f'/\xi$ where $L$ refers to the specimen width. Hence, equation (23) should be replaced by [121]

$$\xi \propto k^{-\nu_0} \quad \text{with} \quad \nu_0 = 1/2. \quad (24)$$

The avalanche size $S$ is then shown [123] to obey the power law distribution:

$$P(S) \propto S^{-\tau} f(S/S_0)$$

with $\tau = 2 - 1 + 2\nu \zeta_H/\nu(1 + \zeta_H) = 1.280 \pm 0.010, \quad (25)$

where the cutoff scales as $S_0 \propto \xi^{1+\nu_0}$, i.e. after having used equation (24):

$$S_0 \propto k^{-1/\nu} \quad \text{with} \quad \nu = 2/1 + \zeta_H = 1.445 \pm 0.005. \quad (26)$$

The avalanche duration $T$ scales with the avalanche size $\ell$ as $T \propto \ell^\kappa$. Since the avalanche size $S$ scales as $\xi^{1+\nu_0}$, one gets

$$T \propto S^\alpha \quad \text{with} \quad \alpha = \frac{\kappa}{1 + \zeta_H} = 0.555 \pm 0.005. \quad (27)$$

And, after having changed the variables accordingly in equation (25), one shows that $T$ is distributed as

$$P(T) \propto T^{-\alpha} f(T/T_0)$$

with $\alpha = 1 + (\nu - 1)/\kappa v = 1.500 \pm 0.010, \quad (28)$

where the cutoff scales as $T_0 \propto \xi^\kappa$, which implies

$$T_0 \propto k^{-1/\Delta} \quad \text{with} \quad \Delta = 1/\kappa v_0 = 1.300 \pm 0.010. \quad (29)$$

Figure 16. (a) Sketch of the front propagation between two pinned configurations. The avalanche area $A$ scales with the size $\ell$ of the depinned zone as $A \propto \ell^{1+\nu_0}$. (b) Typical evolution of the mean crack length. The crack progresses through jumps of size $S$ proportional to $A$. (c) Typical evolution of the energy released $E$. The energy released during an avalanche is also proportional to $S$. Note the similarity between this curve and the one presented in figure 2(a).
6.2. Path equation

We now turn to out-of-plane crack roughness. The universal morphological scaling features observed both in experiments and in lattice simulations reported in sections 3 and 4.2, respectively, have led to intense theoretical development over the last 20 years. We will focus here on the stochastic descriptions of crack path in 3D elastic disordered materials, in the spirit of what was done in the preceding section about motion equation, returning in section 6.3 to a brief discussion of alternative interpretations of the morphological scaling features.

Let us again consider the situation depicted in figure 14 where a crack front propagates within a 3D isotropic elastic solid remotely loaded in mode I. Provided that the motion is slow enough, PLS imposes that the path chosen by the crack front at each time \( t \) (section 2.3): \( K_II(M(t)) = 0 \). The tough part of the problem is to relate \( K_II(M(t)) \) to the crack roughness \( f(z, t) \) and \( h(z, x) \) (figure 14). This has been made possible thanks to the work of Larraalde and Ball [114] later refined by Movchan et al [130] which shows that, to first order, \( K_II(M(t)) \) only depends on \( h \).

It can then be written as the sum of two contributions: a first contribution arises from the coupling of the singular mode I component \( K^0_0 \) of the stress field of the unperturbed crack with the position along the crack edge and a second contribution comes from the coupling between the slope of the crack surface and the non-singular \( T^0 \) normal stress in the direction of crack propagation. Both contributions can be computed analytically (see [130] for the complete form), but the second contribution was shown to be negligible compared with the first one when one considers roughness length scales small with respect to the system size [114]. To the remaining contribution, a stochastic 3D term is added to capture the heterogeneous nature of the material, and a constant term \( K_II^0 \) is added to take into account the inherent misalignment in any loading system. Introducing all these terms in the PLS \( K_II(M(t)) = 0 \) leads to the following path equation [18]:

\[
\frac{\partial h}{\partial t} = \frac{K_II^0}{K_II^0} + \frac{1}{\pi} \frac{2 - 3v}{2(2 - v)} \int_{-\infty}^{\infty} h(z) - h(z') \frac{1}{(z - z')^2} dz' + \eta(z, x, h(z, x)). \tag{30}
\]

At this point, it is worth recalling that this path equation was derived within elastostatic and, as such, cannot work for dynamic fracture when the crack speed exceeds a few per cent of the Rayleigh wave speed.

The form taken by this equation is very similar to that of equation (19) which describes the in-plane motion of the crack front. Two main differences are, however, worth discussing:

(i) The absence of explicit time \( t \) in equation (30). This comes from the fact that to first order, \( K_II \) only depends on the out-of-plane roughness \( h(z, x) \). This yields a decoupling between this path equation and equation (19) describing the crack dynamics. It implies that the scaling properties of fracture surfaces will not depend on the crack velocity \( v \) or crack loading \( G^0 \), contrary to those of the in-plane projection of the crack front.

(ii) The 3D nature of the stochastic term \( \eta(z, x, h(z, x)) \). This second point makes equation (30) extremely difficult—if not impossible—to solve. Two limit cases can, however, be considered.

The first limit consists [114, 115] of considering very smooth fracture surfaces so that the 3D stochastic term \( \eta(z, x, y = h(z, x)) \) reduces to an effective ‘thermal’ term \( \eta(z, x) \). Fracture surfaces are then predicted to exhibit logarithmic scaling. This prediction seems in apparent disagreement with the experimental observations reported in section 4. However, recent observations performed by Dalmas et al [95] on the nanoscale phase separated glasses reveals logarithmic roughness at large length scales.

The second approximation consists [18] of writing \( \eta(z, x, y = h(z, x)) \) as the sum of two 2D terms: \( \eta(z, x, y = h(z, x)) = \eta(z, x) + \eta(z, y = h(z, x)). \) The morphology of the fracture surface \( h(z, x) \) is then given by the motion of the elastic string \( h(z) \) that ‘creeps’—the \( x \) coordinate playing the role of time—within a random potential \( \eta(z, h(z, x)) \) due to the ‘thermal’ fluctuations \( \eta(z, x) \). The fracture surface \( h(z, x) \) then obeys Family–Viseck anisotropic self-affine scaling (equation (17)) with \( |z_1| \approx 0.385, \langle z_1 \rangle \approx 0.5 \) in perfect agreement with observations reported at large scales, as e.g. in sandstone [66, 77, 78], in artificial rocks [88] and granular packing made of sintered glass beads [18, 80, 81] (see also section 4.3).
Let us finally mention that a similar stochastic approach has recently been developed by Katsav et al. [131] in simpler 2D elastic materials. In this case, the path equation can be solved analytically and leads to apparent self-affine scaling features at small scales, with $\xi = 0.5$, and departure from self-affinity at large scales. However, these predictions seem contradictory to experimental observations [65, 83, 84] which report a unique $\xi \simeq 0.7$ self-affine scaling regime in quasi-two-dimensional materials.

6.3. Alternative interpretations for self-affine crack roughness

The stochastic theory of crack growth in elastic brittle disordered materials described in section 6.2 seems to reproduce quite well the morphological scaling properties of cracks for length scale above the FPZ size. On the other hand, it cannot describe observations performed below the FPZ size. In particular, in essence, it cannot reproduce the small-scale anisotropic Family–Viseck scaling characterized by a roughness exponent $\xi \simeq 0.8$ reported in a wide range of materials and discussed in section 4. In this respect, we will now present some alternative approaches that were developed to account for the morphological scaling features observed in fractography experiments as well as in RFM simulations.

Hansen et al. [106] have studied the morphology of crack lines (2D case) or crack surfaces (3D) obtained after the breakdown of RFM with perfectly plastic fuses, i.e. fuses that act as unit Ohmic resistors up to a threshold voltage and that carry a constant current above. They have shown that the problem can be mapped to that of a directed polymer (2D case) or a minimum energy surface (3D case) in random potential. As a result, crack path can be mapped to a KPZ equation [132] and crack lines/surfaces are expected to display self-affine morphological isotropic scaling features with a universal roughness exponent equal to $\xi = 2/3$ in 2D and $\xi \simeq 0.42$ in 3D. They conjectured that these results, demonstrated rigorously on elastic–perfect-plastic model, can also be relevant for brittle fracture. Indeed, these predictions are in close (but not perfect) agreement with the $\xi \simeq 0.71$ (2D) and $\xi \simeq 0.42$ (3D) obtained from numerical simulations of brittle RFM and presented in section 4.2. However, the anomalous scaling is not recovered. Hansen et al.’s predictions are also very close to the $\xi \simeq 0.7$ observed experimentally in 2D sheet of paper [65, 83, 84] and the $\xi \simeq 0.4–0.5$ observed in sandstone [66, 77, 78] or glassy ceramics made of sintered glass beads [18, 80, 81].

Bouchaud et al. [93] proposed to model the fracture surface as the trace left by a line moving through a 3D disordered landscape—the dynamics of which is described through a phenomenological nonlinear Langevin equation derived initially by Ertas and Kardar [133] to describe the motion of vortex lines in superconductors. The resulting fracture surfaces are then predicted to display anisotropic Family–Viseck scaling with $\xi_\perp \simeq 0.35$, $\xi_\parallel \simeq 0.5$. This first work was then extended by Daguer et al. [73] who showed that, depending on the crack velocity $v$, two distinct self-affine regimes can be expected: a large length-scale regime with $\xi_\perp \simeq 0.75$, $\xi_\parallel \simeq 0.5$, and a small length-scale regime with $\xi_\perp \simeq 0.5$, $\xi_\parallel \simeq 0.4$. The crossover $\xi$ between the two is expected [10, 73] to diverge with $v$ as $\xi \propto v^{-1/\xi_\perp^{\xi_\perp-1}}$. The exponents values obtained at large scales/large velocities are in excellent agreement with those observed in many brittle materials at scales smaller than the FPZ size (see section 4.3). However, the prediction of a distinct self-affine regime with smaller exponents at small scales/small velocities is incompatible with experiments.

More recently, Hansen, Schmittbuhl and Batrouni [134, 135] suggested that the universal scaling properties of fracture surfaces are due to the fracture propagation being a damage coalescence process described by a stress-weighted percolation phenomenon in a self-generated quadratic damage gradient.

Then, assuming that the damage profile is proportional to the system size $L$, the rms roughness $w$ of the crack line/surface is expected to scale as

$$w \propto L^\xi$$

with $\xi = \frac{2v_{perc}}{1 - 2v_{perc}}$, \hspace{1cm} (31)

where $v_{perc}$ is the exponent describing the divergence of the correlation length $\xi_{perc} \propto (p - p_c)^{-v_{perc}}$ as the density of broken bonds $p$ is reaching the critical value $p_c$ in standard percolation problems (see, e.g. [136] for an introduction to percolation theory).

In percolation theory, the exponent $v_{perc}$ depends only on the system dimension and dimensionality. In standard scalar percolation, $v_{perc} = 4/3$ in 2D and $v_{perc} = 0.88$ in 3D. This leads to $\xi = 8/11$ in 2D and $\xi \simeq 0.64$ in 3D. The first value is compatible with the $\xi \simeq 0.7$ measured in 2D brittle RFM (see section 4.2 or [104, 111]), while the second one is significantly higher than the $\xi \simeq 0.42$ observed in 3D brittle RFM (see section 4.2 or [103, 108, 110]). In tensorial elastic percolation, $v_{perc} = 2$ in 3D, which leads to $\xi = 4/5$. The value of the roughness exponent is in excellent agreement with $\xi \simeq 0.8$ observed in many brittle materials observed at scales smaller than the FPZ size (see section 3), but the Family–Viseck anisotropic scaling is not captured.

This interpretation yields recent controversies. First, the analysis of large scale RFM simulations [137] suggests that, contrary to what was assumed, (i) damage profile is not quadratic and (ii) the width of the damage profile is not linear with the system size. Second, it was mentioned [138] that the exponent $\xi$ in equation (31) cannot be interpreted as a ‘standard’ roughness exponent since, strictly speaking, the front is not self-affine. Indeed, in such a model, the fracture perimeter displays substantial overhangs the size of which is comparable to their width. To be more precise, self-similarity (i.e. isotropic scaling) is expected for length scales smaller than $w$, and a trivially flat front beyond. In response, Hansen, Schmittbuhl and Batrouni [139] pointed out that experimental fracture fronts are also found to display significant overhangs while remaining self-affine at large scales. They argue that the non-isotropic scaling given by equation (31) is sufficient to define self-affine surfaces.
Table 1. Summary of the exponents characterizing the various scale-free distributions and morphological scaling features reported in fields observations, laboratory experiments, numerical simulations and stochastic models derived from LEFM.

| Energy a | Size b | Silent time c | Roughness d |
|----------|--------|---------------|-------------|
| **Earthquakes [27, 28]** | $\beta \simeq 1.7$ | $\tau \simeq 2$ | $\alpha \simeq 1$ | — |
| Lab experiments, 3D | Non-universal $\beta_{AE}$ | — | Non-universal $\alpha$ | Small scale: $\xi_\perp \simeq 0.75$, $\xi_\parallel \simeq 0.6$ [62–64] |
| Lab experiments, interfacial | — | $\tau_W \simeq 1.7$ [58] | $\alpha \simeq 1$ [58] | Small scale: $\xi_H \simeq 0.6$ [86, 87] |
| Lab experiments, 2D | $\beta_{AE} \simeq 1$ [48] | $\tau \simeq 1.5$ [48] | Non-universal $\alpha$ | $\xi \simeq 0.7$ [65, 83, 84] |
| RFM simulations, 3D | — | $\tau = 2.5$ [105] | — | $\xi \simeq 0.42$ [103, 108, 110] |
| RFM simulations, 2D | — | — | Non-universal $\alpha$ | $\xi \simeq 0.71$ [104, 111] |
| Stochastic LEFM, 3D | $\beta \simeq 1.280$ | $\tau \simeq 1.280$, $\tau_W \simeq 1.7$ [17] | — | $\xi_\perp \simeq 0.385$, $\xi_\parallel \simeq 0.5$ [18] or logarithmic [114, 115] |
| Stochastic LEFM, interfacial | $\beta \simeq 1.280$ | $\tau \simeq 1.280$, $\tau_W \simeq 1.7$ [17] | — | $\xi_H \simeq 0.385$ at small $v$ [16, 115] |
| Stochastic LEFM, 2D | No power law | No power law | No power law | Small scale: $\xi \simeq 0.5$ (Large scale: not self-affine [131]) |

\(a\) The exponent $\beta$ characterizes the distribution of released energy during fracture events. The exponent $\beta_{AE}$ characterizes the energy distribution of AE triggered by these fracture events. The relation between the two remains unknown.

\(b\) The exponent $\tau$ characterizes the distribution of rupture area associated with fracture events. The exponent $\tau_W$ characterizes the distribution of quake events defined from the waiting-time map (see section 3.3 for details). The relation between the two remains unknown.

\(c\) The exponent $\alpha$ characterizes the distribution of silent time between two successive fracture events.

\(d\) The exponent $\xi$ refers to the roughness exponent measured on self-affine crack profiles (2D) or isotropic self-affine fracture surfaces (3D). When fracture surfaces exhibit anisotropy in scaling, $\xi_\parallel$ and $\xi_\perp$ refer to the roughness exponents measured parallel and perpendicular to the direction of crack propagation, respectively. The exponent $\xi_H$ refers to the roughness exponent measured on crack fronts in interfacial geometries.

7. Concluding discussion

Many experiments and field observations have revealed that brittle failure in materials exhibit scale-invariant features (see table 1 for a summary). In particular: (i) fracturing systems display jerky crackling dynamics with random impulsive energy release, as suggested from the AE accompanying the failure of various materials and, at a much larger scale, the seismic activity associated with earthquakes. The energy distribution of these discrete events forms a power law that spans over many orders of magnitude, with no characteristic scale and (ii) roughness of cracks exhibits self-affine morphological features, characterized by roughness exponents. These observations are common to many brittle materials and can be reproduced qualitatively in the electrical breakdown of the random fuse network. On the other hand, they cannot be captured by standard LEFM continuum theory.

Some of these scale-free distributions and scale-invariant morphological features are universal. Others are not. The experiments and lattice simulations presented in sections 3, 4 and 5 allows one to distinguish two cases:

(A) Damage spreading processes within a brittle material preceding the initiation of a macroscopic crack. The associated micro-fracturing events release energy impulses which are power law distributed. The associated exponents are non-universal, but depend on the considered materials, loading conditions environment parameters (see section 3.2). This transient damage spreading is also suggested [64, 94] to be responsible for the anomalous non-universal scaling exhibited in the initial transient roughening regime of fracture surfaces following crack initiation [75, 91, 94].

(B) Macroscopic crack growth within a brittle material. When this propagation is slow enough, it exhibits an intermittent crackling dynamics, with sudden jumps and energy release events the distributions of which form power law with apparent universal exponents. Crack growth leads to rough fracture surfaces that display Family–Viseck universal scaling far enough from crack initiation. Quite surprisingly, seismicity associated with earthquakes seems to belong to the second case and exhibits quantitatively the same statistical scaling features as that observed in experiments of interfacial crack growth along weak disordered interfaces [58].

The (non-universal) scaling features associated with the micro-fracturing events observed experimentally in case (A) are reproduced qualitatively in lattice models such as the RFM presented in section 4. Nevertheless, what sets precisely the distribution and the value of scaling exponents in experiments remains largely unknown. On the other hand, the recent stochastic extensions of LEFM theory presented in section 6 seems efficient to capture quantitatively the observations performed in case (B). In these stochastic LEFM descriptions, the onset of crack growth is found to be analogous...
to a critical transition between a stable phase where the crack remains pinned by the material heterogeneities and a moving phase where the mechanical energy available at the crack tip is sufficient to make the front propagate \[16, 115\]. While growing, the crack decreases its mechanical energy and gets pinned again. Provided that the growth is slow enough, this retro-action process keeps the system close to the critical point during the whole propagation, as for self-organized-critical systems \[140\]. As seen in section 6.1, the resulting dynamics exhibits spatio-temporal intermittency characterized by universal statistical features that can be predicted theoretically using FRG methods and are compatible with experimental observations.

The analogy between the onset of crack propagation and a critical dynamic transition has numbers of other consequences. Roux \textit{et al} \[141\] and Charles \textit{et al} \[142\] made use of the universality manifested around the depinning onset to determine the distribution of effective macroscopic toughness in heterogeneous materials. Their main results are (i) universal scaling between toughness variance and specimen size and (ii) the existence of a universal toughness distribution for large enough specimens. These predictions were shown \[143\] to reproduce fairly well the statistics of crack arrest lengths observed in indentation experiments performed in various brittle materials. More recently, Ponson \textit{et al} \[144, 145\] investigate the role of a finite temperature in this stochastic LEFM description of crack growth and propose a creep law to relate crack velocity and stress intensity factor in the subcritical failure regime. This was recently shown to describe rather well experiments of paper peeling \[49\] and subcritical crack growth in sandstone \[145\].

It should be emphasized that the stochastic descriptions presented in section 6 were developed within elastostatic approximation. As such, they cannot account for the dynamic stress transfers through acoustic waves occurring as a dynamically growing crack is interacting with the material disorder \[146–150\]. The recent availability of analytical solutions for weakly distorted cracks within full 3D elastodynamics framework \[151, 152\] suggests promising developments in this context in the near future (see \[153, 154\] for recent theoretical work in this context).

As presented in section 6.2 this stochastic LEFM description seems to capture fairly well the self-affine scaling features exhibited by fracture surfaces. In particular, the apparent disagreements between its predictions and experimental observations presented in Bouchaud’s 1997 review \[10\] are now uncovered \[18, 95\]: the ‘classical’ self-affine scaling regime characterized by a roughness exponent $\zeta \approx 0.8$ widely reported in the literature is in fact observed below the FPZ size, where in essence stochastic LEFM descriptions cease to be relevant.

While the problem of crack propagation in disordered brittle materials has been widely addressed theoretically, the role of disorder on the formation of seed cracks remains far less studied (see \[155, 156\] for recent theoretical work in this topic). The fact that fracture surfaces observed at small scale exhibit a universal small scale $\zeta \approx 0.8$ Family–Viseck scaling in very different materials with various damage processes as, e.g., plastic deformation, crack blunting, ductile cavity growth or microcracking remains largely unexplained. This suggests that it may be possible to find a generic unified statistical description of damage spreading within disordered brittle materials within the FPZ, independent of the precise nature of the considered material. In this context, damage descriptions in terms of universal stress-weighted percolation processes as proposed by Schmittbuhl and Hansen \[135\] appear promising. Their development, their implications in terms of damage dynamics and their careful confrontation to experiments and/or simulations represent interesting challenges for future investigations.

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