Walking wheel

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Abstract. The dynamics of a 2D walking wheel motion down an inclined plane is analytically investigated in nonlinear formulation. It is the simplest model of a bipedal walking. The possible cases of the motion of the walking wheel are investigated at various values of the inclination of the support surface and the initial angular velocity of the wheel. It is shown that various modes of motion of the walking wheel are possible. The most interesting of which is the existence of a stable periodic solution (self-oscillations).

1. Introduction

The problem of design and motion control automatic bipedal robots capable of moving like a human has attracted the attention of scientists and engineers since the time of Leonardo da Vinci. There are various approaches to solving the problem of motion control of bipedal vehicles. Some of them are given in [1–17]. If the biped has large controlled feet, its movement can be organized within the framework of static stability. However, in order to organize the movement of a bipedal vehicle capable of moving within the framework of dynamic stability (including an vehicle without controlled feet) the control system must to solve a very complex problem of controlling a motion of an unstable mechanical system (system with a deficit of controls). In this case, it is impossible to provide an arbitrary programmed motion in all degrees of freedom of the vehicle. A periodic programmed motion is usually constructed and algorithms for stabilization of this motion are developed. When a vehicle is in motion, capable of changing the parameters of its movement, the motion control system must to integrate the differential equations of motion of the biped when moving along simple terrain (a flat horizontal surface or a surface with small irregularities). At the same time, a human during his movement does not integrate differential equations. When a human is moving along a simple relatively flat terrain, he does not even think about the process of organizing walking. Everything is done at the subconscious level. It can be assumed that the motion control of the bipedal vehicle can be implemented much easier. May be a walking is an ongoing process of falling, in which, to prevent falling, a human put another leg on the support surface to prevent falling. At the same time, he can easily change the parameters of his movement. May be the motion control system is constructed quite simply due to the existence of stable periodic movements in a part of the variables (self-oscillations) and then to control the movement of a part of the variables it is possible to use open loop control system (without feedback). Motion control can be constructed in the same way as in the passive dynamic walkers [10, 11, 14] and jumping vehicles with elastic elements in their legs [19–22]. The parameters of these stable periodic solutions (self-oscillations) can change due to the operation of independent motion control loops for other degrees of freedom of the vehicle.
This work is the first step in trying to take a deal with this hypothesis. We begin with the study of an object substantially simpler than a bipedal vehicle – a walking wheel or legged wheel, rimless wheel (Figures 1-2), which was proposed by A.M.Formalsky [1]. Regardless of him, this model was considered by T.McGeer [14] a little later. He used these results to create an vehicle that demonstrates passive bipedal walking without using actuators in legs joints. The walking wheel is the simplest model of the motion of a 2D biped vehicle. If during the movement of the bipedal vehicle (human) he, in order not to fall forward, substitutes a new leg, then during the motion of a walking wheel the next leg comes in contact with the supporting surface as a result of its rotation (rolling). Such a model of a waking vehicle is of interest because of its simplicity. The 2D motion of a solid body is investigated, while the walking vehicle (human) is a system of several bodies having actuators in the hinges connecting these bodies. The walking wheel also suggested using D.J.Todd [18] in the construction of wheeled off-road robots. The “walking" wheel, which we will call the wheel, resembles a ship’s steering wheel, gear wheel, a cart-wheel without a rim, a hard wheel with large lugs or a wheel with a very coarse tread as they move along a hard surface.

When setting foot of "walking" wheel on the supporting surface take place an impact. It is assumed that in this case an absolutely inelastic impact occurs and the foot does not slide on the supporting surface. Upon impact, a loss of energy occurs and, as a result, during motion on a horizontal surface, the velocity of motion slows down and the wheel stops [1]. During a motion down an inclined plane with a sufficiently large angle of inclination to the horizon, the wheel enters a stable periodic mode of motion (self-oscillation). In this case, the flow of energy into the system is provided by the work of gravity. This process was approximately investigated by T.McGeer [14] in the linearized motion model assuming that the angle between the legs of the walking wheel and the angle of inclination of the surface to the horizon are small. In this paper, this problem is solved analytically in a nonlinear formulation.

The investigation of the walking wheel motion by the methods of computer simulation and hardware prototyping was carried out in [24-28]. In [28], an interactive mathematical model of the a walking wheel motion with animation of its movement, construction of phase trajectories and the possibility of setting the parameters of the wheel and the angle of inclination of the slope along which it moves is available on the Internet.

Figure 1. Walking wheel as a bipedal walking model

Figure 2. Structural scheme of the walking wheel on an inclined plane
2. Statement of the problem

Consider a solid body (Figures 2–3) – a disk with \( n \) identical rods attached to its edge, where \( n \geq 3 \). The outer ends of these rods form a regular polygon. Let's call this body a "walking" wheel. The segment connecting the center of mass of the body \( C \) with the outer end of the rod will be called the leg (virtual leg). The outer end of the rod will be called the foot. Suppose that the lengths of all legs are the same and equal \( l \). The angle between adjacent legs is \( \alpha \) where \( \alpha = \pi/n \). The mass is equal \( m \), \( J_C = mp^2 \) – the moment of inertia relative to the center of mass, \( \rho \) – the radius of inertia relative to the center of mass. The position of the wheel is determined by the angle of rotation \( \varphi \), measured from the vertical to the supporting leg. For the positive direction of the angle reading, we take the direction in the clockwise direction.

Consider the 2D motion of the "walking" wheel down an inclined plane with an angle of inclination to the horizon \( \beta \). Suppose that the supporting surface is absolutely rough and the feet cannot slide along the supporting surface. The velocity of the center of mass is denoted by \( \vec{V} \), and the angular velocity of the wheel through \( \omega \). The movement of the wheel consists in alternating the phases of rotation around the foot of the supporting leg and the impacts when changing the supporting legs.

3. Impact during a changing of support legs

Before impact (Figure 3), the wheel supports on a surface at a point \( S_{i-1} \) that has zero velocity. As a result of the rotation of a solid body around this point, an collision with supporting surface occurs at a new point of contact \( S_i \). In this case, the impact is assumed to be absolutely inelastic and after the impact, a new point of contact remains on the supporting surface.

Due to the fact that the angle of rotation of the wheel is counted from the vertical to the supporting leg, its value changes (Figure 3) upon impact (setting a new foot on the supporting surface)

\[
\varphi^- = \beta + \alpha \quad , \quad \varphi^+ = \beta - \alpha
\]

Hereinafter, the plus index corresponds to the value after the impact, and the minus index corresponds to the value before the impact.
Impact pulses of the support surface reaction occur both at the point of impact \( S_i \) and at the point of support \( S_{i-1} \). As a result of the impact, the initial point of support can remain on the supporting surface, or leave it, due to the fact that these constrain is unilateral.

Denote \( \vec{F}_i^*=(\vec{x}_{Ci}^*, \vec{y}_{Ci}^*) \), \( \vec{F}_{i-1}^*=(\vec{x}_{Ci}^*, \vec{y}_{Ci}^*) \), \( \omega_i^- \), \( \omega_i^+ \) the speed of the center of mass \( C \) and the angular velocity of the wheel, respectively, before the impact and after the impact. For the positive direction of the angular velocity is taken the clockwise direction.

At the moment of collision, impact impulses arise at the points of support of the legs (impact reactions of the supporting surface); we denote their projections on the coordinate axes \( X_{i-1}, Y_{i-1} \) in the foot \( S_{i-1} \) and \( X_i, Y_i \) in the foot \( S_i \) (Figure 3).

There are unilateral constrains on the coordinates of the feet of the supporting legs (the feet of the supporting legs can leave the supporting surface and move upwards). Consequently, the vertical components of impact reactions at the points of support of the legs are not negative.

\[
Y_{i-1} \geq 0, \quad Y_i \geq 0 . \tag{1}
\]

If the foot is lift off from the supporting surface as a result of the impact, then after the impact the foot velocity is directed upwards

\[
\dot{y}_{S_{i+1}}^+ = \omega_i^- 2l \sin \alpha > 0 . \tag{2}
\]

In addition, we accept the hypothesis that the reactions at the point of support of this leg are equal zero [1]

\[
X_{i-1}=0, \quad Y_{i-1}=0 . \tag{3}
\]

This assumption is equivalent to the absence of impulse controls in the degrees of leg mobility, which can additionally make a “push” in the foot that is lift off from the supporting surface.

Before the impact, the body rotated around the stationary foot \( S_{i-1} \) and at the moment of collision of the foot \( S_i \) with the supporting surface (before the impact) had an angular velocity

\[
\omega_i^- > 0 , \tag{4}
\]

then

\[
\dot{x}_{Ci}^- = \omega_i^- l \cos \alpha , \quad \dot{y}_{Ci}^- = -\omega_i^- l \sin \alpha . \tag{5}
\]

By virtue of the theorems on the center of mass motion and the change in the kinetic momentum of the system relative to the center of mass upon impact, we have

\[
m(\dot{x}_{Ci}^- - \omega_i^- l \cos \alpha ) = X_{i-1} + X_i ,
\]

\[
m(\dot{y}_{Ci}^- + \omega_i^- l \sin \alpha ) = Y_{i-1} + Y_i ,
\]

\[
mp^2 (\omega_i^+ - \omega_i^-) = (Y_{i-1} - Y_i) l \sin \alpha - (X_{i-1} + X_i) l \cos \alpha . \tag{6}
\]

Two different types of an impact are possible.

1. As a result of the impact, both feet remain on the supporting surface. There are impact reactions in the points of support of both legs. After the collision the wheel stops

\[
\dot{x}_{Ci}^* = 0 , \quad \dot{y}_{Ci}^* = 0 , \quad \omega_i^+ = 0 .
\]

Substituting these relations into (6), we have
\[-m\omega_i^+ l \cos \alpha = X_{i+1} + X_i,\]
\[m\omega_i^+ l \sin \alpha = Y_{i+1} + Y_i,\]
\[-m\rho^2 \omega_i^- = (Y_{i+1} - Y_i) l \sin \alpha - (X_{i+1} + X_i) l \cos \alpha.\]

Then
\[Y_{i+1} = -\frac{\rho^2 + l^2 \cos 2\alpha}{2l \sin \alpha} m\omega_i^+, \quad Y_i = \frac{\rho^2 + l^2}{2l \sin \alpha} m\omega_i^- .\] (7)

The condition that the vertical components of the impact reactions (1) are not negative due to (4) and (7) is equivalent to the condition
\[\cos 2\alpha \leq -\left(\frac{\rho}{l}\right)^2 < 0 .\] (8)

2. After the impact, the wheel begins to rotate around the supporting foot \(S_i\), with an angular velocity.
\[\omega_i^+ > 0 , \quad x_{C_i}^+ = \omega_i^+ l \cos \alpha , \quad y_{C_i}^+ = \omega_i^+ l \sin \alpha .\]

Substituting these relations and (3) into (6), we have
\[m(\omega_i^+ - \omega_i^-) l \cos \alpha = X_i ,\]
\[m(\omega_i^+ + \omega_i^-) l \sin \alpha = Y_i ,\]
\[m\rho^2 (\omega_i^+ - \omega_i^-) = -Y_i l \sin \alpha - X_i l \cos \alpha .\]

Then
\[\omega_i^+ = k \omega_i^- ,\] (9)
\[Y_i = 2\frac{\rho^2 + l^2 \cos^2 \alpha}{\rho^2 + l^2} m\omega_i^- l \sin \alpha ,\]

where
\[k = \frac{\rho^2 + l^2 \cos 2\alpha}{\rho^2 + l^2} .\] (10)

Rotation around the foot \(S_i\) occurs as a result of an impact if and only if, when \(\omega_i^+ > 0, \ Y_i > 0\). By virtue of (4), (9) and (10), these conditions are met if \(k > 0\) or
\[\cos 2\alpha > -\left(\frac{\rho}{l}\right)^2 .\] (11)

From (8), (11) it follows that the impact model is correct (deterministic). For any values of the parameters, only one of the two possible types of impact occurs. Note that the nature of the movement of the wheel after the impact depends only on the design parameters and does not depend on the angular velocity of the wheel before the impact.
Angle $\alpha = \pi/n$, where $n$ is the number of legs. If $n \geq 5$ the angle between neighboring legs $2\alpha$ is acute then the condition (11) is always satisfied. If $n = 4 \cos 2\alpha = 0$ and condition (11) is satisfied only if $\rho \neq 0$. Note that $\rho = 0$ corresponds to the case, when the mass of the wheel is concentrated in its center of mass. If $n = 3 \cos 2\alpha = -1/2$ and condition (11) is satisfied only if $\rho > \sqrt{2}l$.

In the following, we will consider only the case when condition (11) is fulfilled and, as a result of an impact a foot $S_{i-1}$ lift off from the supporting surface and the rotation around the foot $S_i$ begins. Note that in this case

$$0 < k < 1 .$$

Then by virtue of (9) $0 < \omega_i^- < \omega_i^+$, i.e. upon impact, a loss of energy occurs. With an increase in the number of legs, the angle $\alpha = \pi/n$ decreases and the coefficient $k$ increases. In the limit if $n \to \infty$ the coefficient $k \to 1$. The limiting transition to an infinite number of legs can be interpreted as a transition to a conventional wheel.

4. Rotation around the supporting leg.

After impact, the wheel rotates around the foot $S_i$ (Figure 2). In accordance with the theorem on the change of the kinetic momentum relative to the point $S_i$

$$(\rho^2 + l^2)\dot{\varphi} = gl \sin \varphi ,$$

where $\varphi$ – the angle of the wheel rotation. Denote the wheel angular velocity $\omega = \dot{\varphi}$. At the beginning of this phase of motion $\varphi_0 = \varphi_i^- = \beta - \alpha$, $\omega_0 = \omega_i^+$. Equation (13) is the equation of motion of an inverted pendulum.

Equation (13) has an energy integral

$$\frac{\omega^2}{2} + \frac{gl}{\rho^2 + l^2} \cos \varphi = \frac{(\omega_i^-)^2}{2} + \frac{gl}{\rho^2 + l^2} \cos(\beta - \alpha) .$$

When $\beta < \alpha$, if the wheel reaches a critical position, when the center of mass is above the point of support $S_i$ (i.e. $\varphi = 0$) with a non-zero angular velocity, it will pass through this position and collides with the supporting surface by the next foot $S_{i+1}$. For this, due to the energy integral (14), it is necessary and sufficient that

$$\omega_i^+ > \omega_{sp} ,$$

where

$$\omega_{sp} = \left( \frac{2gl}{\rho^2 + l^2} (1 - \cos(\beta - \alpha)) \right)^{1/2} .$$

If $\beta = \alpha$ then $\omega_{sp} = 0$.

If $\beta > \alpha$ the position of the wheel supported on the legs $S_{i-1}$ and $S_i$ statically unstable and under the action of gravity at any value $\omega_i^+ \geq 0$ the wheel will rotate around the foot $S_i$ and will continue to move down the slope and collides by the next foot $S_{i+1}$. At the same time, even from the state of rest, the wheel will start to move down the slope.
5. The condition of the movement without loss of walking wheel feet contact with the supporting surface.
When the wheel moves, the constraint at the point of support of the leg is unilateral. If the angular velocity of the wheel is high then a separation of the supporting leg foot with the surface may occur. In this case, the wheel goes from “walking” to “running”. This occurs when the normal reaction of the supporting surface necessary for the wheel rotation \( Y_i < 0 \). In this paper, we consider that this is unacceptable, i.e. we are restricted to “walking” mode of the wheel motion. For this, it is necessary and sufficient that at each step \( Y_i \geq 0 \).

In accordance with the theorem on the center of mass motion

\[
\Phi(\omega^i, \beta, \alpha) > 0, \tag{16}
\]

where

\[
\Phi(\omega^i, \beta, \alpha) = 
\min_{\varphi=\beta-\alpha, \beta+\alpha}[g \cos \beta - \frac{gl^2}{\rho^2 + l^2} [2 \cos(\varphi - \beta) \cos(\beta - \alpha) - \cos \varphi] - l \sin(\varphi - \beta) \sin \varphi - l \cos(\varphi - \beta)(\omega^i)^2]
\]

6. Self-oscillations. Poincare map.
If \( \omega^i < \omega_{sp} \), then the wheel does not reach a critical position and, under the action of gravity, begins to rotate in the opposite direction and collides with the supporting surface with the previous foot \( S_{i-1} \).

Upon this impact due to the energy integral \( \omega^{i+1} = -\omega^i \), then after the impact

\[
\omega^{i+1} = f_i(\omega^i) = -k\omega^i. \tag{17}
\]

This relation is the Poincare map for changing the angular velocity in the \( i \)-th step. It is linear, and has one fixed point \( \omega_0 = 0 \). Which corresponds to a stable equilibrium position with support on two legs \( S_{i-1} \) and \( S_i \), by virtue of the Koenig’s theorem [29], because

\[
\left| \frac{df}{d\omega^i} \right| = k < 1.
\]

The value of \( k \) is determined by the relation (10) and \( 0 < k < 1 \).

The Poincare map (17) is shown in Fig. 4. In this case, the angular velocity decreases in the geometrical progression. It can be shown that the duration of each step decreases in the geometrical progression too. There is an endless series of impacts, but the total duration of this series of impacts is finite.

If \( \omega^i = \omega_{sp} \), then the wheel will endlessly reach a critical position.

If \( \beta \geq \alpha \) or \( \beta < \alpha \) and \( \omega^i > \omega_{sp} \), then the wheel will roll over the supporting leg. From the energy integral (14), we determine the angular velocity of the wheel when the foot \( S_{i+1} \) collides with the supporting surface. Take in account that in this position \( \theta = \theta^i = \beta + \alpha \), we obtain
where $\omega^i_{i+1} > 0$ is the angular velocity of the wheel at the end of the $i$-th step or, which is the same at the beginning of the $i$-th step.

From relation (9) for a foot strike and relation (18), we obtain the Poincare map for changing the angular velocity of the wheel in the $i$-th step, which consists of the stage of rotation around the foot and the subsequent foot strike.

From relation (9) for a foot $S_{i+1}$ collision with supporting surface and relation (18), we obtain the Poincare map for changing the angular velocity of the wheel in the $i$-th step, which consists of the stage of rotation around the foot $S_i$ and the subsequent impact of the foot $S_{i+1}$

$$\omega^*_{i+1} = k \left( \omega^i_{i+1} \frac{4gE}{\rho^2 + l^2} \sin \alpha \sin \beta \right)^{1/2}.$$  

This map has a fixed point

$$\omega_* = \left( \frac{k^2}{1-k^2} \frac{2gE}{\rho^2 + l^2} \sin \alpha \sin \beta \right)^{1/2},$$

In accordance with the Koenig’s theorem [29], this fixed point corresponds to a stable limit cycle, since

$$\left| \frac{df}{d\omega} \right| = k^2 < 1 \quad \text{when} \quad \omega = \omega_*.$$  

![Figure 4. Poincare map in the case when $\beta \geq \alpha$ or $\beta < \alpha$ and $\omega^i_{i+1} < \omega_{xp}$](image1)

![Figure 5. Poincare map in the case when $\beta < \alpha$, $\omega^i_{i+1} > \omega_{xp}$ and $\omega_* > \omega_{xp}$](image2)

However, the wheel will actually have this stable periodic solution if $\beta \geq \alpha$ or if $\beta < \alpha$, $\omega^i_{i+1} > \omega_{xp}$ and $\omega_* > \omega_{xp}$. This map Poincare map (19) is shown in Fig. 5.
Note that this stable periodic mode of motion occurs if at each step the condition of the movement without loss of walking wheel feet contact with the supporting surface is fulfilled. For this, it is necessary and sufficient that this condition is satisfied at the limit cycle and at the first step, i.e.

$$\Phi(\omega_i, \beta, \alpha) > 0, \quad \Phi(\omega^*, \beta, \alpha) > 0.$$ 

When $\beta < \alpha, \omega^* > \omega_{sp}$ and $\omega_i < \omega_{sp}$, there is no periodic solution corresponding to a fixed point $\omega_i$. It is explained by the fact that the Poincare map (19) is valid only as long as the wheel turns over through the supporting leg. Within a few steps, the angular speed of the wheel will decrease until it becomes smaller than $\omega_{sp}$. After that, the wheel mark time in accordance with the Poincare map (17). The Poincare map for this case is shown in Fig. 6.

![Figure 6. Poincare map in the case when $\beta < \alpha$, $\omega_i > \omega_{sp}$ and $\omega_i < \omega_{sp}$.](image)

![Figure 7. Poincare map in the case when $\beta < \alpha$, $\omega_i > \omega_{sp}$ and $\omega_i = \omega_{sp}$.](image)
When $\beta < \alpha$, $\omega^*_i > \omega_{sp}$ and $\omega_* = \omega_{sp}$, the wheel will tend to a periodic solution corresponding to a fixed point $\omega_*$. But this periodic solution is not stable, because when $\omega^*_i < \omega_{sp}$ the wheel moves in accordance with the Poincare map (17). The Poincare map for this case is shown in Fig. 7.

7. Conclusion
The dynamics of the walking wheel downward along an inclined plane is analytically investigated. The walking wheel is the simplest model of passive two-legged walking. During its movement, energy is supplied to the system due to the work of gravity. It is shown that the point map of the change in the angular velocity of the wheel per step (Poincare map) in the vast majority of cases has one fixed point. This fixed point corresponds to either a stable periodic solution (self-oscillation), which is the rolling of the wheel down an inclined plane, or the movement of the wheel ends with its stopping as a result of an endless series of impacts when the wheel swings on two legs. In the degenerate case, the Poincare map has two fixed points. One of them corresponds to an unstable limit cycle corresponding to wheel rolling, and the second corresponds to a stop of the wheel. In this case, the limit cycle is stable outside, and unstable inside itself.

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