The Cosmological Constant Problem and Inflation in the String Landscape

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ABSTRACT: An earlier paper points out that a quantum treatment of the string landscape is necessary. It suggests that the wavefunction of the universe is mobile in the landscape until the universe reaches a meta-stable site with its cosmological constant \( \Lambda_0 \) smaller than the critical value \( \Lambda_c \), where \( \Lambda_c \) is estimated to be exponentially small compared to the Planck scale. Since this site has an exponentially long lifetime, it may well be today’s universe. We investigate specific scenarios based on this quantum diffusion property of the cosmic landscape and find a plausible scenario for the early universe. In the last fast tunneling to the \( \Lambda_0 (\leq \Lambda_c) \) site in this scenario, all energies are stored in the nucleation bubble walls, which are released to radiation only after bubble collisions and thermalization. So the \( \Lambda_0 \) site is chosen even if \( \Lambda_0 \) plus radiation is larger than \( \Lambda_c \), as long as the radiation does not destabilize the \( \Lambda_0 \) vacuum. A consequence is that inflation must happen before this last fast tunneling, so the inflationary scenario that emerges naturally is extended brane inflation, where the brane motion includes a combination of rolling, fast tunnelings, slow-roll, hopping and percolation in the landscape. We point out that, in the brane world, radiation during nucleosynthesis are mostly on the standard model branes (brane radiation, as opposed to radiation in the bulk). This distinction may lead to interesting dynamics. We consider this paper as a road map for future investigations.

KEYWORDS: the string landscape, the brane world, the cosmological constant problem, inflation.

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1. Introduction

The simplest and most compelling explanation of the discovery of dark energy \([1]\) is that we are living in a vacuum state with a tiny positive cosmological constant (i.e., vacuum energy density) \(\Lambda_0\). Phenomenologically, it is not unexpected, as pointed out by Weinberg \([2]\). However, from the conceptual point of view, this is one of the deepest puzzle in fundamental physics, since a naive cosmological constant would have a value dictated by the Newton’s constant, or about \(10^{120}\) bigger than the observed value.

In Einstein theory, the cosmological constant is simply an input parameter. In string theory, there are 10-dimensional spacetime. To agree with observations, 6 of the spatial dimensions must be compactified into a very small size. Recent analysis of flux compactifications shows that string theory has exponentially (probably infinitely) many meta-stable vacuum solutions \([3, 4]\), with a wide range of the cosmological constant which is dynamically generated. This is referred to as the cosmic string landscape, which includes ones with very small positive cosmological constants. This is encouraging, since, if string theory is
correct, our universe with a very small cosmological constant must be one of its solutions. However, this does not solve the puzzle, since the question why we end up at such a low positive vacuum energy state remains unanswered.

Recently, Ref.\cite{5} speculates that the vastness of the cosmic landscape may have another property, namely, that the universe is mobile in the landscape. The vastness of the landscape is a consequence of the large number (∼d) of moduli. If one is allowed to treat the string landscape as a complicated multi-dimensional effective potential in which sits the wavefunction of the universe, one can borrow from the knowledge accumulated in condensed matter physics and make more precise statements supporting the above speculation \cite{6}:

1. due to the high dimension d of the moduli space and non-zero vacuum energies, mobility of the wavefunction of the universe high up (still much below the Planck scale) in the landscape is a consequence of its resulting coherence and fast tunnelings;
2. the wavefunction of the universe is mobile in the landscape until it reaches a vacuum site with its cosmological constant Λ0 smaller than a critical value Λc;
3. Once it enters this Λ0 site, it loses its coherence (due to decoherence). Its decay lifetime from this meta-stable (with Λ0 < Λc) site would be exponentially long; this sharp transition (from mobile (conducting) to trapped (insulating or localized)) is due to a second order phase transition at the mobility edge Λc, and the critical Λc is crudely estimated to be exponentially small compared to the string or the Planck scale.
4. Assuming that tunneling to negative Λ sites are ignored due to the resulting big crunches \cite{7}, the universe reaches the Λ0 ≥ 0 site via fast tunneling from a site higher up (Λ ≳ Λc) in the landscape.

In this scenario, even if the universe starts at a large Λ site in the landscape, its mobility allows it to move freely down the landscape until it reaches a site with an exponentially small Λ (0 ≤ Λ < Λc) where it will stay for an exponentially long time before it tunnels to another site, probably a supersymmetric site with a zero vacuum energy. Presumably, we are living at this meta-stable (exponentially long lived) site with an exponentially small Λ0. In this picture, the quantum diffusion properties are crucial, i.e., the cosmic landscape is fully quantum, and cannot be treated semi-classically. One may view the universe in the landscape as an excited state making transitions to the lower excited states.

Intuitively, this qualitative property of the landscape may be understood by considering a quantum mechanical potential. The vastness of the landscape translates to a complicated potential in a high d dimension. In general, a low-dimensional attractive potential with bound states may become too weak to trap a particle in higher dimensions, even if the depth and size of the potential are kept fixed. (The attractive δ-function potential is a well-known example.) So some classical local minima in the landscape are strong enough to bind the wavefunction (meta-stable vacua) while some are too weak to bind (unstable vacua). If the binding is weak, the wavefunction has a long tail, so the meta-stable vacuum can easily tunnel to a neighboring vacuum. Together with the resonance tunneling effect \cite{5, 6} in high dimensions, the wavefunction is much harder to be trapped in the landscape.

Another important enhancement of tunneling is the effect of the vacuum energy via gravity. For the same barrier, quantum tunneling can be much faster (the Euclidean instan-
ton action in the WKB approximation can be exponentially smaller) when the wavefunction is high up in the landscape compared to low regions. This double exponential enhancement of the tunneling rate can be seen in both the Coleman-De Luccia tunneling \[7\] and in the Hawking-Moss tunneling \[9\]. It is a property of gravity and so is absent in usual quantum mechanical system.

For the above reasons, it is easy to envision why the wavefunction is mobile in the landscape. As we go to smaller \( \Lambda \) sites, the distance from such a low \( \Lambda \) meta-stable site to a neighboring site with a lower \( \Lambda \) increases, so the barrier grows and the tunneling becomes suppressed. The presence of the conducting-insulating phase transition tells us that fast tunneling is shut off once we reach a meta-stable site with \( \Lambda_0 \) below the critical value.

Now, what does the above properties tell us about the early universe? More specifically, how does inflation fit in? In this paper, we suggest a plausible scenario. Consider the last step of fast tunneling, that is, from a site with \( \Lambda_+ > \Lambda_c \) to a \( \Lambda_0 \) (\(< \Lambda_c \)) site. According to Coleman-De Luccia \[7\], at least in the thin wall approximation, all energies released from the tunneling process are stored in the nucleation bubble walls, so that the inside of the bubbles are in the pure \( \Lambda_0 \) vacuum. The energies stored at the bubble walls are only released later to radiation when bubbles collide. Since this last tunneling is still fast, many bubbles are formed and the bubbles quickly collide. In the language of Guth and Weinberg \[10, 11\], the system percolates (no region with \( \Lambda_+ \) remains) and thermalizes.

In this scenario, the \( \Lambda_0 \) site is chosen even if \( \Lambda_0 \) plus radiation is larger than \( \Lambda_c \) (actually it is expected to be close to \( \Lambda_c \)). After bubble collisions, the universe is heated up to a high temperature so that symmetry (chiral and/or electroweak) restorations may take place. As long as the resulting radiation does not destabilize the \( \Lambda_0 \) vacuum, the universe will eventually cool back down to the pure \( \Lambda_0 \) vacuum. In this picture, this will be the second time our universe is entering the pure \( \Lambda_0 \) vacuum, if it does not decay in the meantime (e.g., to a 10-dimensional Minkowski supersymmetric vacuum).

In this scenario, inflation takes place before the last fast tunneling in the landscape. Typically we expect inflation to take place while the \( D3 \)-branes are rolling, scattering and tunneling in the landscape, in addition to its motion inside the bulk. This leads us to consider an extended brane inflationary scenario that is a combination of rolling (maybe with some slow-roll), fast tunnelings, hopping and percolation. A simplified version of such a scenario has been briefly considered already \[12\]. The scatterings and tunneling events are frequent enough so that the course-grain behavior of the wavefunction may mimic that of a slow-roll inflaton \[13\]. However, bubble nucleation and collisions in fast tunneling can produce gravitational waves and non-Gaussianities, while the bouncing around can produce entropic perturbations and non-Gaussianities; so we argue that this tunneling/percolating inflationary scenario ending with a fast tunneling predicts potentially observable non-Gaussianity and gravitational waves.

How reasonable is the above assumption that the resulting radiation from bubble collisions does not destabilize the \( \Lambda_0 \) vacuum, as the radiation plus \( \Lambda_0 \) is mostly likely bigger than \( \Lambda_c \), in which case, we expect it to fast tunnel again. Also, there is another puzzle (as recently emphasized in Ref.\[14, 15\]) that we have to address: Today’s cosmological constant was dynamically irrelevant in the early universe. Big bang nucleosynthesis requires
that the universe was once hot, with a temperature of at least 1 MeV, sitting at the present site in the landscape. At that stage, $\Lambda_0 \sim 10^{-11} \text{eV}^4$ is dominated by the radiation/matter density $\sim 1 \text{MeV}^4 = 10^{24} \text{eV}^4$. Since both the vacuum energy and the radiation/matter density contribute to the stress tensor that couples to gravity, how come the gravitational dynamics pick a site with such a small $\Lambda$ and not one with a larger $\Lambda$ (say, one closer to 1 MeV$^4$)? Does the critical $\Lambda_c$ really refer to the vacuum energy only, or more generally it includes radiation/matter as well? In the former case, how does it pick out such a small $\Lambda_0$ when the vacuum energy is overwhelmed by the radiation/matter? In the latter case, there is at least a 35 orders of magnitude $\Lambda$ problem remaining.

We do not have ready answers to these questions. However, in the brane world scenario in Type IIB string theory, there is a distinction between the radiation before nucleosynthesis and the vacuum energy. Let us approximate the wavefunction of the universe by that of a collection of branes. In the brane world, standard model particles are open string modes localized to the branes, say a stack of D3-branes. The radiation present during (or just before) the big bang nucleosynthesis is that of the standard model particles, so they are localized on the branes. This (4-dimensional) brane radiation is distinct from the (10-dimensional) radiation in the bulk, which is consisted of close string modes only. In the symmetric electroweak phase, the electroweak vacuum energy density $\Lambda_{EW}$ is again localized on the branes, which simply raises the effective brane tension. This (4-dimensional) vacuum energy density is very different from the (10-dimensional) vacuum energy density in the string landscape, even though both contributes to the vacuum energy density we have been studying so far. At distance scales large compared to the compactification scale, the $\Lambda$ in the string landscape emerges from the interplay between the open and the close string dynamics.

A typical radiation in the bulk probably enhances tunneling. However, a localized radiation or vacuum energy density (like $\Lambda_{EW}$) on a brane behaves like an addition to the brane tension; it is analogous to a particle with an increased mass, which typically suppresses, not enhances its tunneling. So it is reasonable to assume that this radiation/vacuum energy density on the branes plays a different role in the tunneling and do not destabilize the $\Lambda_0$ vacuum. In this scenario, bubble collisions after the last fast tunneling should lead to an efficient heating of the branes. The transfer of energy to the brane modes may be somewhat similar to that in the (p)reheating in brane inflation. If there is a selection mechanism that leads to an exponentially small $\Lambda_0$, the interplay between closed and open string dynamics in the brane world must play a crucial role.

We review the application of the scaling theory to the landscape in Sec. 2., explaining the presence of a second order phase transition between the mobility (conducting) and the trapped (insulating) phases, and the existence of a critical $\Lambda_c$, which is exponentially small compared to the string/Planck scale. In Sec. 3, we review the Coleman-De Luccia tunneling in the thin-wall limit. Because of its importance, we discuss why there is no radiation inside the nucleation bubbles. We also show that tunneling is exponentially enhanced when it happens high up in the landscape. This is a huge effect for the landscape. In Sec. 4, we consider the last fast tunneling, that is, tunneling from a $\Lambda_+ (> \Lambda_c)$ false vacuum to a vacuum with its local minimum at $\Lambda_0 < \Lambda_c$. We argue that the tunneling is fast enough
so that thermalization of the universe (via bubble collisions) takes place. We explain how part of the cosmological constant problem may be solved in this scenario. In Sec. 5, we discuss how the picture is changed in the brane world. The distinction between brane radiation/vacuum energy and bulk radiation/vacuum energy most likely would lead to a more complicated and richer scenario. The resulting inflationary scenario is discussed in Sec. 6. In this scenario, inflation takes place in the landscape, suggesting an extended brane inflationary scenario. In Sec. 7, implications of the last fast tunneling that is fast enough for percolation but not thermalization is discussed. The resulting inflationary scenarios reduces to that of slow-roll in an open universe. Its possible observational consequences are mentioned. We also explain why this scenario is not preferred. A summary, further remarks and some open questions are collected in Sec. 8. Appendix A gives a quantum mechanical example illustrating why a typical classically stable local vacuum in the landscape may not be able to trap the wavefunction. This provides an intuitive understanding of the mobility in the landscape. Appendix B contains a toy model illustrating the role of brane radiation/vacuum energy density.

A clarification may be helpful here, since the terms “phase transition” and “percolation” appear in 2 places in this paper, with different meanings. Unless explicitly stated otherwise, they carry the following meanings. In Sec. 2, the phase transition refers to the second order phase transition between mobility (conducting) phase and the insulating (trapped) phase in the landscape. Here percolation refers to the long distance connectivity in the landscape via classical scatterings. Phase transition in Sec. 3 refers to a first order phase transition from one vacuum (meta-stable site) to another in the landscape associated with CDL tunneling. Percolation in Sec. 4 refers to the completion of the tunneling process in an expanding universe, that is, no region in the universe remains in the old vacuum, thus avoiding eternal inflation in the old vacuum.

2. The scaling theory applied to the string landscape

One way to understand qualitatively in string theory why our universe has a small cosmological constant today is the mobility of the universe in the string landscape. One may get an intuitive understanding of this property of the landscape by considering some simple quantum mechanical systems. It is a well known fact in quantum mechanics that an attractive 1-dimensional δ-function potential has a bound state, but an attractive 3-dimensional δ-function potential does not. More generally, a low-dimensional attractive potential with bound states becomes too weak for binding as we go to higher dimensions, when the strength of the potential is kept fixed. (As an illustration, we review a known quantum mechanical example in Appendix A.) In the landscape, with large d, many classically stable local minima may turn out to be too weak to trap the wavefunction. Such vacua are quantum mechanically unstable. (It is important to point out that this property can be explicitly checked by considering a typical local minimum in the string landscape.) The classically stable local minima that are strong enough to bind are meta-stable. For a meta-stable vacuum that binds weakly, the wavefunction spreads far so that tunneling out of it to lower Λ sites is fast. Together with resonance tunneling effects, such meta-
stable states would have relatively short lifetimes, so the universe tunnels rapidly down to lower \( \Lambda(>0) \) sites. As we go to smaller \( \Lambda \) sites, the distance from a meta-stable site to a neighboring site with a lower \( \Lambda \) increases, so tunneling becomes slower. In this scenario, tunneling from a meta-stable site in the cosmic landscape is fast if the site has a relatively large \( \Lambda \). This fast tunneling process can happen repeatedly, until the wavefunction of the universe reaches a site with \( \Lambda \) smaller than the critical value \( \Lambda_c > 0 \). Its lifetime at this low \( \Lambda_0(\leq \Lambda_c) \) site is exponentially long, and this low \( \Lambda_0 \) meta-stable site may describe today’s universe.

For this scenario to work, the universe should never have entered into eternal inflation. Otherwise, the universe would arrive at a meta-stable site with some intermediate \( \Lambda \), and have enough time to expand away the radiation/matter present and enter into eternal inflation. If this happens, some (most) parts of our universe would still be in an eternally inflationary phase today. This would lead to the difficult question why we are not inside an eternally inflating (and presumably exponentially large) bubble. So, in realizing the above scenario, the shut off of mobility and fast tunneling must be sharp; that is, there should be a phase transition between fast tunneling and exponentially long tunneling, at an exponentially small (compared to the Planck or string scale) critical \( \Lambda_c \). Effects due to cosmological evolution will be discussed later.

Let \( \Psi(a,...,\varphi_j,\phi_i,...) \) be the wavefunction of the universe in the landscape, where \( a \) is the cosmic scale factor, \( \varphi_j \) are the values of the closed string moduli and \( \phi_i \) are the positions of the \( D \)-branes (and fluxes) in the compactified bulk (which can decompactify). In general, the \( \phi_i \) and the \( \varphi_j \) are all coupled, so movements (and creations/annihilations) of the branes/fluxes will shift the moduli. Suppose, in some situation, this wavefunction can be approximately described by the positions of the \( D3 \)-branes only. If the moduli are fixed, the wavefunction of a \( D3 \)-brane further reduces to \( \Psi(a,\phi_i) \), where \( \phi_i \) are the 6 fields measuring the position of the brane in the 6-dimensional compactified manifold. Suppose we are interested in a \( D3 \)-brane moving in the landscape, so \( \Psi \) is a function of \( d \) number of fields. In the realistic situation, the stringy cosmic landscape in this simplified picture is still very complicated. In Ref.[6], the landscape is treated as a \( d \)-dimensional random potential in the Schrödinger approach, where the number of moduli and brane/flux positions, namely \( d \), is large. Using the scaling theory developed in condensed matter physics [16], the transition from fast tunneling to exponentially slow tunneling is shown to be a second order phase transition, which happens at an exponentially small \( \Lambda_c \). We note that \( d \) around any meta-stable site may be measured by the number of light scalar modes at that vacuum state. It is clear that \( d \) varies from region to region in the string landscape. As a consequence, the critical \( \Lambda_c \) also depends on the neighborhood of the site we are interested in. To simplify the discussion, our analysis is carried out in the neighborhood where 6 spatial dimensions are compactified and 3 spatial dimensions stay large.

Here we briefly review some key elements of the analysis. The physics is dictated by the (dimensionless) conductance \( g \). At microscopic scale, the tunneling probability to a neighboring site at a distance \( s \) is \( T(s) \simeq |g(s)|^2 \), where

\[
g(s) \simeq |\psi(s)| \simeq e^{-s/\xi}
\]  

(2.1)
where $\psi(s)$ is the enveloping amplitude and $\xi$ is the size of the site (dictated by, say, the inverse mass of a light modulus). Knowing this property at a specific small distance scale $s$,

![Figure 1: The $\beta$-function $\beta_g (g)$ in the Shapino model [19] as a function of $\ln g$. The $\beta$-function vanishes at the finite critical conductance $g_c (d)$ for $d > 2$. Note that we are interested in large $d$.](image)

what is $g(L)$ at a larger distance $L$? In this scaling analysis, there are 2 possibilities: fast tunneling translates to a conducting (or mobile) behavior while slow tunneling translates to an insulating (or trapped) behavior. At distance $L$ as $L \to \infty$, $g \sim L^{d-2}$ implies a conducting behavior while $g \sim e^{-L/\xi}$ implies an insulating behavior. Based on dimensional and phenomenological arguments, the $\beta$-function

$$\beta_d (g_d (L)) = \frac{d \ln g_d (L)}{d \ln L}$$

depends only on the dimensionless conductance $g_d (L)$. So $\beta \to d - 2$ ($\beta \to \ln g$) if the medium is conducting (insulating). Smooth continuity of the $\beta$-function is assumed. Ref. [14] further assumes a monotonous $\beta$-function, which is backed up by experiments. The resulting $\beta$-function for different $d$ and $g$ is shown in Fig. 1. For $d < 2$, the $\beta$-function is always negative, so the RG flow is towards small $g$ and the medium is always insulating. For $d > 2$, $\beta$-function always crosses zero. If the microscopic $g$ has a positive value of $\beta$-function, the RG flow is towards large $g$ so the medium is conducting. It tends to be more conducting as $d$ increases.

That the localization property is very sensitive to the spatial dimension $d$ rather than the strength of the randomness of the potential may be somewhat surprising. One way
to appreciate the importance of dimensionality is to consider the probability of a random walker, at time $t_1 > 0$ onward, that it will return to its original position ($r = 0$ at $t = 0$) \[17\]. The probability density of the random walker is given by $P(r, t) = (4\pi Dt)^{-d/2} \exp(-r^2/4\pi Dt)$ (where $D$ is the diffusion constant), so the desired probability is given by

$$\lim_{T \to \infty} \int_{t_1}^{T} P(0, t) dt = \lim_{T \to \infty} \int_{t_1}^{T} (4\pi Dt)^{-d/2} dt$$

For $d = 1, 2$, the integral diverges, implying that it will always return to the origin, while for $d \geq 3$, the integral goes like $t_1^{1-d/2}$, so it vanishes as $t_1 \to \infty$.

That is, low enough in a $d \leq 2$ landscape, the wavefunction is always trapped, and eternal inflation is unavoidable \[18\]. High enough in the landscape (to be explained in the next section) and/or for $d \geq 3$, the medium can be conducting; that is, the wavefunction can be mobile in the landscape. For $d > 2$, the $\beta$-function vanishes at some critical value $g_c(d)$. This critical value is exponentially small for large $d$,

$$g_c(d) \sim e^{-(d-1)}.$$

Since the slope of the $\beta$-function is positive at $g_c(d)$, it is an unstable fixed point. $g(s) > g_c(d)$ leads to conducting while $g(s) < g_c(d)$ leads to insulation. This is a second order phase transition. Comparing this to Eq.(2.1), we see that mobility requires

$$d \geq s/\xi + 1$$

(2.4)

The smallness of $g_c(d)$ for large $d$ implies that even if tunneling to a neighboring site is typically exponentially small, the wavefunction may still be mobile due to the large $d$ (that is, many possible channels to tunnel to). In the conducting phase, the wavefunction maintains coherence over large distances. Our scenario is quite different from the usual assumption in the literatures on the string landscape and eternal inflation. Here we see that the eternal inflation does not happen as long as the tunneling rate is larger than $e^{-2(d-1)}$.

To be concrete, a simple $\beta$-function formula is given in Ref.\[19\],

$$\beta_g = (d - 1) - (g + 1) \ln(1 + 1/g).$$

(2.5)

where the $d = 1$ case was first given in Ref.\[20\], based on analytic and phenomenological considerations. (Other models have very similar behaviors.) We are interested in large $d$, in the range of a few dozen to over a thousand.

In general, we expect the brane motion in the landscape to include rolling, scatterings and tunneling. Classical scattering in the landscape raises the issue of percolation, that is, whether such scattering will shut down the long distance mobility. Including this effect, the conductance $g$ also depends on a percolation probability $p$ ($0 \leq p \leq 1$). Extending the above analytic formula to include percolation \[19\] :

$$\beta_g(g, p) = (d - 1) \left( 1 + \frac{1-p}{p} \ln(1-p) \right) - (g + 1) \ln(1 + 1/g),$$

(2.6)
Figure 2: The $\beta$-function $\beta_g(g, p)$ with inclusion of percolation for $d = 6$ and the $\beta$-function $\beta_p(p)$ for the running of $p$, for different $p$ and $d$. The black line corresponds to $\beta_p(g, p) = 0$.

where the running of $p$ is given by

$$\beta_p(p) = \frac{\partial p}{\partial \ln L} = p \ln p - (d - 1)(1 - p) \ln(1 - p).$$

(2.7)

The $\beta$-functions $\beta_g(g, p)$ and $\beta_p(p)$ are shown in Fig. 2. For large $d$, we see that $\beta_p(p) > 0$ for almost all values of $p$, so the flow is towards $p = 1$, that is, the system percolates. In this case, $\beta_g(g, p)$ approaches $\beta_g(g)$ in Eq.(2.5).

One can also get an order of magnitude estimate of the mobility edge, i.e., the critical $\Lambda_c$ that divides the mobility (conducting) region from the trapped (insulating) region; that
is fast tunneling for sites with $\Lambda > \Lambda_c$ and exponentially slow tunneling for sites with $\Lambda < \Lambda_c$. It is easy to convince oneself that $\Lambda_c$ should be exponentially small compared to the Planck scale $[8]$. Since tunneling from a false vacuum to another one which has a larger vacuum energy is highly suppressed, we consider only downward tunneling, that is, tunneling of a false vacuum to another one which has a smaller (or comparable) vacuum energy. Let $s(\Lambda)$ be the typical separation between a $\Lambda$ site and one of its neighboring sites which has a vacuum energy equal to or smaller than $\Lambda$ (so it can tunnel to). Let us consider the scaling behavior of $s(\Lambda)$. Assume that the distribution of the meta-stable sites with $\Lambda \geq 0$ in the string landscape is random and goes like $\sim \Lambda^{q-1}$ ($q > 0$) so that the fraction of sites with a positive cosmological constant smaller than (or equal to) $\Lambda$ is given by

$$f(\Lambda) = (\Lambda/\Lambda_s)^q$$   \hfill (2.8)

where $\Lambda_s$ is that of the the string scale (or the Planck scale), and the dimension of landscape at the region of interest is $d$. The case of $q = 1$ corresponds to a flat $\Lambda$ distribution. Let $N_T$ be the total number of sites with $\Lambda \leq \Lambda_s$ inside a region of the landscape of size $L$. The number of the sites within a box of size $L$ with vacuum energies below a given $\Lambda$ is

$$N(\Lambda) = f(\Lambda)N_T = f(\Lambda)\left(\frac{L}{s(\Lambda_s)}\right)^d = \left(\frac{L}{s(\Lambda)}\right)^d$$   \hfill (2.9)

So we have

$$s(\Lambda) = s(\Lambda_s)\left(\frac{\Lambda}{\Lambda_s}\right)^{-q/d}$$   \hfill (2.10)

As expected, the typical distance $s(\Lambda)$ from a $\Lambda$ site to one of its neighboring sites that it can tunnel to increases as $\Lambda$ decreases. At the string (or Planck) scale, quantum effects dominate, so tunnelings are not suppressed (say, of order 1), that is, $s(M_s) \sim \xi \sim 1/M_s$. Assuming the variation of $\xi$ is small compared to $s(\Lambda)$, we have from Eqs.(2.4) and (2.10),

$$\Lambda_c \sim d^{-d/q}M_s^d$$   \hfill (2.11)

For a reasonable choice, one may consider $d \sim 50$ and $q \sim 1$. Another choice may be $d \sim 1000$ and $q \sim 30$. Clearly, the value of $\Lambda_c$ is very sensitive to the details. A careful analysis beyond these very crude estimates is important when the structure of the landscape is better understood.

To summarize, we argue that a quantum treatment of the string landscape is necessary especially at part of the landscape with electroweak to GUT scale vacuum energies. By quantum treatment we mean the description of our position in the landscape in terms of a wave function as if it is a solution of a Schrodinger equation in some complicated random (versus periodic) effective potential. We apply methods of condensed matter physics to such a wave function. We do not assume the evolution of the wave function is coherent. It is a consequence that emerges from the analysis. Such a feature (coherent and mobile versus decoherent and localized) follows from the scaling theory, which shows that this feature depends more on dimensionality than on the randomness of the potential. As the universe moves down the landscape, it starts to loose its coherence. The scaling theory points out
that it does not lose its coherence slowly, but rather abruptly (a phase transition). After
the phase transition, one can describe the evolution semi-classically. The estimate is that
the phase transition happens at a very low cosmological constant $\Lambda_c$ (the so-called mobility
edge).

3. Coleman-De Luccia tunneling

There are exponentially many classically stable vacua in string landscape. Those with
positive vacuum energies are either unstable or meta-stable. The latter ones will decay to
some other vacua with lower cosmological constants. An important issue is to calculate
the tunneling rate for a meta-stable vacuum. However the structure of string landscape is
very complicated. Here we like to focus on the last fast tunneling, from a meta-stable site
with $\Lambda_+ > \Lambda_c$ to a meta-stable site with $\Lambda_- > 0$. We review the argument that there is no
radiation inside the bubble, even if the universe starts at the $\Lambda_+$ vacuum with radiation.
Since $\Lambda_+$ is expected to be much smaller than the string scale, and we are interested in
the region of the landscape where 6 spatial dimensions are compactified, we may treat this
tunneling process in the approximation of a 4-dimensional effective field theory. We expect
tunneling to take place along the direction, say $\phi$ (either a brane mode or a bulk mode),
where the tunneling rate is fastest. Instead of $d$ scalar fields, we may now further simplify
the problem by considering the single scalar field $\phi$. This leads us to consider the vacuum
decay for the potential $U(\phi)$ in Fig. 3. Since the decay of the meta-stable vacuum state

![Figure 3: The potential $U(\phi)$. Tunneling from $U_+$ at $\phi_+$ to $U_-$ at $\phi_-$ is fast if $U_+ > \Lambda_c$.](image)

at $\phi_-$ is exponentially long, we may treat it as the true vacuum in this approximation.
So in this simple potential, there are the false $\phi_+$ vacuum and the $\phi_-$ vacuum at $\phi = \phi_+$
and $\phi = \phi_-$ respectively. Vacuum decay proceeds through the quantum materialization of
a bubble of true vacuum which is separated by a bubble wall from the surrounding false vacuum. In \cite{Coleman:1980} Coleman and De Luccia (CDL) calculated the tunneling rate with inclusion of gravitation in the thin wall limit. Thin wall approximation means that the bubble size is much larger than the thickness of the bubble wall. In \cite{DeLuccia:1986} the authors propose a wave-function approach to describe the false vacuum decay. This method can be used to treat the case with a thick wall. For a low, broad barrier, the Hawking-Moss approach comes in handy \cite{Hawking:1983}.

Although tunneling is treated in the semi-classical approximation in the CDL analysis, it may be applied in more general situations. For example, it is still applicable in resonance tunneling \cite{Resonance}, where the tunneling probability approaches unity. Here, when the CDL instanton action approaches unity or less, the WKB estimate becomes unreliable. However, for our purpose here, we simply interpret a very small CDL instanton action to mean that the resulting tunneling probability is not exponentially suppressed. We shall simply assume that the resulting tunneling rate should be of the order of the mass scales involved and so is fast, while its actual value is not crucial for our description of what is going on.

The CDL instanton is a solution with the topology of a four sphere $S^4$ in Euclidean space. For scalar fields, the instanton configuration with $O(4)$ symmetry has the smallest Euclidean action \cite{Diakonov:1987, DeLuccia:1986}. This should be also the case with inclusion of gravitation. The geometry after bubble nucleation is described by the analytic continuation of the CDL instanton to Lorentzian signature. The metric with $O(4)$ symmetry in Euclidean space is given by

\[
 ds^2 = d\chi^2 + r^2(\chi)d\Omega_3^2,
\]

where $d\Omega_3^2$ is the element of distance on $S^3$ and $r$ is the radius of this $S^3$. The radial coordinate $\chi$ is continued to $\chi = it$ and the metric in Lorentzian frame is

\[
 -ds^2 = -dt^2 + r^2(it)d\Omega_{H^3}^2,
\]

where the metric is multiplied by an overall minus sign and $d\Omega_{H^3}$ is the element of length for a unit hyperboloid with timelike normal. The metric within the bubble describes a spatially open Friedmann-Robertson-Walker Universe.

The tunneling rate depends on the the size of the bubble. Larger is the critical bubble size, smaller is the tunneling rate. Another key point is that quantum tunneling obeys the law of conservation of energy. In order to compensate the positive energy stored in the bubble wall, the size of the bubble for the case with radiations inside should be larger than the size of empty bubble at the time of materialization. So an empty bubble (that is, without radiation inside) is favored by the quantum tunneling. Since this fact is important for our scenario, let us elaborate on this point.

In \cite{Coleman:1980} the tunneling rate for the potential in Fig. 3 is computed in the thin wall limit. In the semiclassical limit, the tunneling rate per unit volume takes the form

\[
 \Gamma = Ae^{-B}
\]
When we include gravitation the effective 4-dimensional action for a single scalar field is given by

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - U(\phi) - \frac{R}{2\kappa} \right], \quad (3.4)$$

where $\kappa = 8\pi G$. The tunneling rate is determined by the minimum value of the Euclidean action. The Euclidean action is defined as minus the formal analytic continuation of the action in Lorentzian frame to imaginary time. Such a tunneling process is described by the CDL instanton which exhibits an $O(4)$ symmetry in Euclidean space. The metric of the instanton in Euclidean space is given in Eq.(3.1) and the Euclidean action is

$$S_E = 2\pi^2 \int d\chi \left[ r^3 \left( \frac{\phi'^2}{2} + U \right) - \frac{3r}{\kappa} (r'^2 + 1) \right] \quad (3.5)$$

where the prime denotes $d/d\chi$. The scalar field equation of motion is

$$\phi'' + \frac{3r'}{r} \phi' = \frac{dU}{d\phi} \quad (3.6)$$

The Einstein equation yields one non-trivial equation,

$$r'^2 = 1 + \frac{1}{3} \kappa r^2 (\frac{\phi'^2}{2} - U). \quad (3.7)$$

and the equation of motion for $r(\chi)$ follows from Eqs.(3.6,3.7). Using Eq.(3.7) to simplify the Euclidean action, one obtains

$$S_E = 4\pi^2 \int d\chi \left[ r^3 U - \frac{3r}{\kappa} \right] \quad (3.8)$$

The coefficient $B$ in the tunneling rate (3.3) is

$$B = S_E(\phi) - S_E(\phi_+). \quad (3.9)$$

We divide the integration for $B$ into three parts. Outside the bubble, $\phi = \phi_+$ and thus

$$B_{\text{out}} = 0. \quad (3.10)$$

In the wall, we have

$$B_{\text{wall}} = 2\pi^2 r^3 \tau, \quad (3.11)$$

where $r$ is the bubble size and $\tau$ is the tension of the wall which is decided by the barrier between the false and true vacua. Here we take the thin wall approximation and $B_{\text{wall}}$ is the energy stored in the thin wall. Inside the bubble, $\phi$ is a constant and Eq.(3.7) becomes

$$d\chi = dr \left( 1 - \kappa r^2 U/3 \right)^{-1/2}. \quad (3.12)$$

Hence

$$S_{E,\text{in}}(\phi) = -\frac{12\pi^2}{\kappa} \int_0^r \tilde{r} d\tilde{r} \left( 1 - \kappa U(\phi) \tilde{r}^2 / 3 \right)^{1/2}. \quad (3.13)$$
Summing the 3 parts of $B$, we obtain
\[
B = 2\pi^2 r^3 \tau + \frac{12\pi^2}{\kappa^2} \left[ \frac{1}{U_-} \left( (1 - \kappa r^2 U_-/3)^{3/2} - 1 \right) - \frac{1}{U_+} \left( (1 - \kappa r^2 U_+/3)^{3/2} - 1 \right) \right].
\] (3.14)

The decay coefficient $B$ is stationary at $r = R$ which satisfies
\[
\frac{1}{R^2} = \frac{\epsilon^2}{9\tau^2} + \frac{\kappa(U_+ + U_-)}{6} + \frac{\kappa^2 \tau^2}{16},
\] (3.15)

where $\epsilon = U_+ - U_-$. According to Eq.(3.15), we can easily check that the bubble radius at the moment of materialization is not larger than the event horizon of the de Sitter space in false vacuum $R_+ = (\kappa U_+ / 3)^{-1/2}$. This is reasonable; otherwise the bubble cannot be generated causally. Keeping $\tau$ and $U_+$ fixed, we find
\[
\frac{dB}{d\epsilon} \bigg|_{r=R} = -\frac{\partial B}{\partial U_-} \frac{\partial U_-}{\partial \epsilon} = \frac{12\pi^2}{\kappa^2 U_-^2} \left[ \left( 1 - \kappa R^2 U_- / 3 \right)^{1/2} \left( 1 + \kappa R^2 / 6 U_- \right) - 1 \right] \leq 0.
\] (3.16)

So a larger difference of the vacuum energies corresponds to a larger tunneling probability, as expected. Keeping $\tau$ and $\epsilon$ fixed, we have
\[
\frac{dB}{dU_+} \bigg|_{r=R} = \frac{\partial B}{\partial U_+} + \frac{\partial B}{\partial U_-} \frac{\partial U_-}{\partial U_+} = \frac{\partial B}{\partial U_+} + \frac{\partial B}{\partial U_-} \left( \frac{1}{U_+^2} \left( 1 - \kappa R^2 U_+/3 \right)^{1/2} \left( 1 + \kappa R^2 U_+/6 \right) - 1 \right) - (U_+ \rightarrow U_-) \leq 0,
\] (3.17)

which implies that the tunneling probability increases as the vacuum energy in the false vacuum increases, even when the energy difference $\epsilon$ and the bubble tension are fixed. So the lifetime of the false vacuum with large vacuum energy can be very short, even much shorter than the Hubble time. If so, there is no eternal inflation at high energy scale.

For the special case with $r \ll R_+$, the bubble size is much smaller than the curvature radius of the background and gravity does not play a big role. In this limit $B$ becomes
\[
B = 2\pi^2 r^3 \tau - \frac{\pi^2}{2} r^4 \epsilon.
\] (3.18)

The parameter $B$ is stationary at
\[
r = R = \frac{3\tau}{\epsilon},
\] (3.19)

which is consistent with Eq.(3.15). This result is nothing but the energy conservation. Since the curvature of the background is small, the energy conservation reads
\[
\frac{4\pi}{3} R^3 U_+ = 4\pi R^2 \tau + \frac{4\pi}{3} R^3 U_-,
\] (3.20)

and thus we recover Eq.(3.19). Requiring $R \ll R_\pm$ yields
\[
\frac{\epsilon^2}{\tau^2} \ll \frac{1}{\kappa \Lambda}.
\] (3.21)
and the tunneling rate per unit volume is given by
\[ \Gamma \sim \exp \left( -\frac{27\pi^2}{2} \frac{4}{\epsilon^3} \right). \quad (3.22) \]

It is also interesting for us to ask how to modify the tunneling rate if there are radiations in the false vacuum and/or in the true vacuum. In the case with the neglected background curvature, the energy conservation implies that the bubble size should be
\[ R \simeq \frac{3\tau}{\epsilon + g_+ T_+^4 - g_- T_-^4}, \quad (3.23) \]
where \( g_\pm \) measures the effective light degrees of freedom and \( g_\pm T_\pm^4 \) is the energy density of radiations in the false/true vacuum. The parameter \( B \) is now modified to be
\[ B = 2\pi^2 r^3 \tau - \frac{\pi^2}{2} r^4 (\epsilon + g_+ T_+^4 - g_- T_-^4). \quad (3.24) \]

The bubble size at the stationary point is just that given in Eq.(3.23), as expected. Here we need to stress that the barrier becomes smaller and the tension of bubble \( \tau \) is smaller than that without \( T_\pm \) radiation. Even allowing a mild dependence of \( \tau \) on \( T_\pm \), we see that the tunneling rate \((3.3)\) becomes exponentially more likely when there is no radiation in true vacuum, i.e., \( T_- = 0 \), that is,
\[ \Gamma \sim \exp \left( -\frac{27\pi^2}{2} \frac{4}{(\epsilon + g_+ T_+^4)^3} \right). \quad (3.25) \]

A larger \( T_\pm \) results in a faster tunneling rate, as expected.

Away from the thin wall approximation, the analysis becomes more complicated. In general, \( T_- \) depends on the details of the model. If \( \phi \) is a close string modulus, its coupling to standard model particles is of gravitational strength and so may be very much suppressed. In this case, \( T_- \) should remain negligibly small.

### 3.1 Enhancement of tunneling by the vacuum energy

We see in Eq.(3.17) that, with fixed energy difference \( \epsilon \) and the bubble tension \( \tau \), the tunneling rate is faster if the wavefunction is higher up in the landscape. Here we like to point out that this is a huge effect. Let us go back to Eq.(3.14) for \( B \).

It is worth discussing another limit of \( r \simeq R_+ \) which corresponds to \( U_+ \simeq U_- = U \gg U_s = \frac{2\kappa^2}{3\kappa \tau^2} + \frac{3\pi^2}{8} \). It happens at the high energy scale in the landscape. In this case the bubble radius is given by
\[ R \simeq \sqrt{\frac{3}{\kappa U}}. \quad (3.26) \]

Now \( B \) is dominated by the first term in Eq.(3.14), namely
\[ B \simeq 6\sqrt{3\pi^2 \tau (\kappa U)^{-3/2}}. \quad (3.27) \]

In the unit of \( M_p = 1 \), or equivalently \( \kappa = 1 \), the tension of the bubble satisfies \( \tau \ll 1 \). In Planck region \((U \sim 1)\), \( B \ll 1 \) and \( \Gamma \sim 1 \). At low energy scale, the background curvature
Figure 4: Huge variation of the CDL tunneling rate $\Gamma \sim e^{-B}$ from $U_+ (= U_- + \epsilon)$ to $U_-$ via a fixed barrier between them. This point is illustrated by a specific example here. Keeping fixed the domain wall tension $\tau \sim 10^{-12}$ and the energy difference $\epsilon \sim 10^{-22}$ (both in Planck units), $B \sim -\log \Gamma$ is given as a function of the potential $U_-$. In this case, we see that $B$ varies 30 orders of magnitude. Tunneling is exponentially enhanced when the wavefunction is higher up (i.e., larger $U_-$) in the landscape.

radius is quite large and the bubble size is relatively small, and then the tunneling rate is insensitive to the vacuum energy of the false vacuum.

As an illustration, let us consider a concrete example in which the height of barrier is $\Delta U \sim 10^{-20}$ in the unit of $M_p = 1$ and the bubble wall tension is $\tau \sim \Delta \phi \sqrt{\Delta U} \sim 10^{-12}$ (to be specific, we take $\Delta \phi \sim 10^{-2} M_p$). We also consider $\epsilon \sim 10^{-22} \ll \Delta U$. Keeping $\epsilon$ and $\tau$ fixed, for different $U_-$ we have

$$\Gamma(U_- = 10^{-1}) \sim e^{-3 \cdot 10^{-9}} \sim 1$$

while

$$\Gamma(U_- = 10^{-20}) \sim e^{-3 \cdot 10^{19}} \ll 1$$

In this case $U_s \sim 2\epsilon^2/3\tau^2 \sim 10^{-20}$ and therefore $B$ is roughly a constant for $U < U_s$. See Fig. 4 in detail, where we see that $B$ changes by 30 orders of magnitude.

If the barrier is broad and not high, one may use the Hawking-Moss formula [9], where the same effect is easier to see. Consider Hawking-Moss tunneling, which is really quantum fluctuation up a potential barrier. Suppose the initial wavefunction is at $A$ and it has to go over the barrier with the top at $D$, then classically roll down to $C$, a local minimum on the other side of the barrier $D$, where $U_D > U_A > U_C$. In units of $M_p = 1$,

$$\Gamma(A \to C) \simeq \exp \left( \frac{3}{8} \left( \frac{1}{U_D} - \frac{1}{U_A} \right) \right)$$
Fixing the barrier height (potential difference) to be $U_D - U_A = \delta$, we have

$$B = -\log \Gamma(A \rightarrow C) = \frac{3}{8} \frac{\delta}{U_D U_A} = \frac{3}{8} \frac{\delta}{U_A(U_A + \delta)}$$

(3.28)

For fixed potential barrier $\delta$, we see that the Hawking-Moss tunneling rate is substantially enhanced when it happens higher up in the landscape (as $U_A$ increases). This scenario is quite realistic, since the barrier has to do with local properties while the overall vacuum energy can receive contributions from, for example, the presence of a $D3$-brane somewhere else in the bulk.

To get an idea, suppose $\delta = 10^{-20}$ and $U_A > 10^{-9}$. Eq. (3.28) then yields $B \simeq -\log \Gamma \sim 0$, so there is no exponential suppression in the tunneling rate. At a lower vacuum energy, say $U_A < 10^{-12}$, we have $B > 10^3$, which implies a very suppressed tunneling rate.

Note that the tunneling should happen along the direction with minimum barrier. There, $\delta$ or $\Delta U$ can easily be orders of magnitude smaller than the typical (average or median) value of the barrier height. This is especially true for large $d$.

Overall, we see that tunneling is doubly exponentially enhanced when it happens higher up in the landscape. This phenomenon is absent in condense matter systems. High enough in the landscape, fast tunneling will allow the wavefunction to be mobile even if $d \ell e 2$. We believe that this effect tends to sharpen the phase transition.

4. Last fast tunneling in the string landscape

Instead of the decay rate per unit volume $\Gamma$, it is convenient to introduce the dimensionless parameter $\gamma$ given by

$$\gamma = \frac{\Gamma}{H^4}$$

(4.1)

where $H$ is the Hubble parameter in false vacuum, so $\gamma$ is a measure of the nucleation rate relative to the expansion rate of the universe. Let the probability of an arbitrary point remaining in a false de Sitter vacuum at time $t$ be $P(t)$. Guth and Weinberg [11] show that there is a critical $\gamma_P$, so that for $\gamma > \gamma_P$ the system percolates, that is, $P(t) \rightarrow 0$. This is fast tunneling, and numerous nucleation bubbles are formed. In this case, no region of spacetime remains in the false vacuum. The bubbles grow and then collide. Energies in the bubble walls are released to heat up the universe. Then $P(t)$ behaves as

$$P(t) \sim \exp\left(-\frac{4\pi}{3} \gamma H t\right),$$

(4.2)

Thus the lifetime of the field in the false vacuum is estimated as

$$t_F \simeq \frac{3}{4\pi H \gamma}.$$  

(4.3)

However, thermalization (a homogeneous and isotropic universe in radiation) is not assured. For $\gamma \gtrsim \gamma_P$, a typical cluster of bubbles is dominated by a large bubble surrounded by tiny bubbles, and the collisions of tiny bubbles with the large bubble may not heat up the inside of the large bubble. So thermalization requires a faster tunneling than just
percolation. Let us introduce another critical value $\gamma_T > \gamma_P$, so that thermalization is complete only if $\gamma > \gamma_T$. Precise values of $\gamma_P$ and $\gamma_T$ are not known; fortunately, we do not need them for a qualitative discussion. For a crude feeling [11, 24], we have in mind $\gamma_P \lesssim 1/4$ and $\gamma_T \simeq 9/4\pi$. If $\gamma \gg 1$, the lifetime of the false vacuum is much shorter than the Hubble time. For $\Gamma \sim M_4^4 e^{-B}$, this is not difficult to arrange, as we shall see.

Now let us consider the 3 possibilities for the last fast tunneling:
1. $\gamma > \gamma_T$ (thermalization);
2. $\gamma_T > \gamma > \gamma_P$ (percolation but no thermalization) and
3. $\gamma < \gamma_P$ (no percolation - eternal inflation).

In case (3), some region of the spacetime remains in the false vacuum. Inflation in these regions of the false vacuum leads to eternal inflation. With fast tunneling, we consider this case to be unlikely for $\Lambda \sim \Lambda_c$. Recalling Eq. (2.11) and the critical local tunneling rate $\Gamma_c \sim M_4^4 e^{-2(d-1)}$ (4.4)

we find that, at the mobility edge, with $H_c^2 \sim \Lambda_c/M_P^2$,

$$\gamma_c = \frac{\Gamma_c}{H_c^4} \sim \left(\frac{M_P}{M_s}\right)^4 \exp\left(\frac{2d}{q} \ln d - 2(d-1)\right)$$

(4.5)

For any reasonable values of $d$ and $q$ (say $d \sim 50$ and $q \sim 1$), we see that $\gamma_c \gg 1$. For the last fast tunneling, we expect $\Lambda_+ \sim \Lambda_c$, so $\gamma \sim \gamma_c \gg \gamma_T$, so we are in the thermalization case. If the last fast tunneling has any chance to be case (2), $\gamma_c \lesssim 1$. That requires $q > \ln d$. In case (2), the percolation but no thermalization case, the resulting scenario is not favorable for a very small $\Lambda_0$. That is, slow-roll inflation is necessary after the last fast tunneling in the landscape, and a fine-tuning problem remains for $\Lambda_0$. Fortunately, we consider case (1) to be most likely, case (2) to be unlikely and case (3) to be least likely. So let us discuss case (1) here and postpone the discussion of case (2) to Sec. 7.

Let us first give a summary of the scenario we have in mind for case (1), namely $\gamma > \gamma_T$, before discussing some of the details. In this scenario, the last tunneling in the landscape is fast enough so that the universe is thermalized after the tunneling. We expect this to be the case with fast tunneling from the $U_+$ vacuum to the $U_- = \Lambda_0$ vacuum, since we are still in the conducting phase. All energies released from the tunneling process are stored in the nucleation bubble walls, so that the insides of the bubbles are in the pure $\Lambda_0$ vacuum. The energy at the bubble walls are only released later to radiation when the bubbles collide. There are enough bubbles around for them to quickly collide with each other. Such collisions release the energies stored in the bubble walls and heat up the universe to a high enough temperature to start the hot big bang era that eventually leads to nucleosynthesis. This is what we mean by thermalization, in which all the bubbles come together to form a single thermalized region, restoring homogeneity and isotropy. If the temperature is high enough, symmetry (chiral and/or electroweak) restorations may take place. As long as the resulting radiation does not destabilize the $\Lambda_0$ vacuum, which is a reasonable requirement, the universe will eventually cool back down to the pure $\Lambda_0$ vacuum (we are not quite there yet as of today). In this picture, we note that this will
be the second time the universe is in the pure $\Lambda_0$ vacuum. It is entirely possible that our universe will decay (e.g., tunnel to a 10-dimensional Minkowski supersymmetric universe) before it enters into eternal inflation in this $\Lambda_0$ vacuum.

There are a number of issues that we like to briefly discuss.

- Does the tunneling from $U_+$ prefer to go directly to the minimum $\Lambda_0$ at $\phi_0$, or maybe to $U_- > \Lambda_0$ at $\phi_-$ slightly away from $\phi_0$? Following the tunneling rate (3.25), for a fixed bubble wall tension $\tau$, we see that tunneling is fastest for largest $\epsilon$, so the universe will seek out the lowest $\Lambda$ minimum in the region it fast tunnels to. To check this in the thin wall approximation, let $U_b$ be the height of the potential along the tunneling path between $\phi_+$ and $\phi_-$, so $\Delta \phi = |\phi_+ - \phi_-|$ and the height of the barrier $\Delta U = U_b - U_+$. In general we expect

$$\Delta U \gg \epsilon \gg \Delta \epsilon$$

(4.6)

Here bubble wall tension $\tau = c\Delta \phi \sqrt{2\Delta U}$, where $c \lesssim 1$ parameterizes the shape of the potential. Does tunneling prefer to go directly to $\Lambda_0$, or somewhere $\phi_-$ that is $\Delta \epsilon = U_- - \Lambda_0$ higher up the potential and then roll/drop down to $\Lambda_0$? Let us consider the tunneling rate (3.25).

First consider the square well case, where $\phi_- = \phi_0$ and $c = 1$. It is clear that, with fixed $\tau$, a larger $\epsilon$ (that is, $\Lambda_0$) is preferred. However, $\tau$ does vary a little. The fractional change in $\tau$ between the 2 situations ($\epsilon$ versus $\epsilon - \Delta \epsilon$) is $\Delta \epsilon / \Delta U$ while the fractional change in $\epsilon$ is $\Delta \epsilon / \epsilon$. With (4.6), the change in the tension is negligible compared to the change in $\epsilon$ and the tunneling chooses to hit the minimum $\Lambda_0$ directly. For the $\lambda \phi^4$ model, $c = 1/3$.

In general, the above argument goes through for any generic potential in the thin wall approximation.

- Besides CDL tunneling via nucleation bubbles, one should also consider Hawking-Moss tunneling, especially when the thin wall approximation is not valid. A proper interpretation of the HM tunneling is given in Ref. [25, 26]. The field $\phi$ experiences quantum fluctuations with amplitude $\delta \phi \sim H/2\pi$. These quantum fluctuations lead to local changes in $\phi$, so that $\phi$ can stochastically climb up the potential from the minimum to a neighboring top or saddle point. Once $\phi$ reaches the top (or the saddle point) of the potential, it can simply roll down (classically) to a nearby local minimum. This process is more like a (quantum) hopping or jumping than tunneling. In general, this quantum hopping mechanism is more likely for a low, broad barrier in the potential. The rolling down can lead to some sort of (p)reheating. Note that this hopping is most efficient if all energy is dispensed, so there is no radiation at the early stages of the classical rolling down.

In some situation, the wavefunction of the universe can be approximately described by the positions of the $D3$-branes only. Then this is an extension of brane inflation where the position of a brane includes not only its position inside a particular compactified bulk, but also its position in the landscape, moving from one compactified bulk to the next. As pointed out recently, the Dirac-Born-Infeld (DBI) action of a $D3$-brane may enhance the tunneling rate by a significant amount [27]; that is, with the same potential barrier, tunneling with a DBI kinetic term can be exponentially faster than that with a canonical kinetic term.
5. Tunneling in the brane world

Now we like to discuss the puzzle that is mentioned in the introduction and emerges again in the above scenario. We do not have answers to them. However, our goal here is point out that the usual formulation of the puzzles do not hold in the brane world. Here we can ask, in the brane world scenario, what properties of the string landscape in the brane world scenario will yield such a small $\Lambda_0$?

5.1 The puzzle

As mentioned in the introduction, the radiation during big bang nucleosynthesis leads to a puzzle \[4, \[5\]: Eq.(3.25) suggests that tunneling from $\Lambda_+$ may prefer to go to the site with the smallest $\Lambda_-$. Suppose $\Lambda_+ > 1 \text{ MeV}^4$ and $\Lambda_0 \sim 10^{-11} \text{ eV}^4$. Now consider another possible tunneling path from $\Lambda_+$ to a $U = \Lambda_1$ site where $\Lambda_+ \gg \Lambda_1 \gg \Lambda_0$ (say, $\Lambda \sim 1 \text{ eV}^4$). Since $\Lambda_0$ plus radiation roughly equals $\Lambda_+$ and so is $\Lambda_1$ plus radiation, with the radiation component dominating in both cases, one sees no reason why tunneling to $\Lambda_0$ is preferred over $\Lambda_1$ or any of the many other nearby vacua with orders of magnitude larger vacuum energy densities. That is, why our universe does not have a larger $\Lambda$? Of course, this question becomes more acute if $\Lambda_+ > 100 \text{ GeV}^4$. This puzzle emerges because both radiation and vacuum energy contributes in the same way to the stress tensor that gravity couples to, and the tiny $\Lambda_0$ is completely masked by the radiation.

One may argue that radiation/matter and vacuum energy do behave differently in Einstein’s theory of gravity. This may allow a possible (yet unknown) dynamics to distinguish between them. However, one can see the fallacy of this argument if one goes to a higher temperature. Suppose the hot big bang universe was once at a temperature $T > 100 \text{ GeV}$, so the electroweak interaction was in the symmetric phase, with $\Lambda_{EW} \sim (100 \text{ GeV})^4$. (This is believed to be necessary for the generation of matter-antimatter asymmetry of our universe via baryogenesis/leptogenesis.) As the temperature drops, spontaneous symmetry breaking takes place and the vacuum energy also drops to a new lower value. Dimensional arguments suggest the new value to be smaller but within a few orders of magnitude of the $\Lambda_{EW}$. For the new value to be today’s $\Lambda_0$, we must fine-tune $\Lambda_{EW}$ to a precision within $10^{-11} \text{ eV}^4/(100 \text{ GeV})^4 \sim 10^{-55}$. If we consider the chiral symmetry breaking in QCD (with vacuum energy density $\Lambda_\chi \sim (100 \text{ MeV})^4$ before chiral symmetry breaking), we may lower the minimum hot big bang temperature by a few orders of magnitude, but the fine-tuning required on the $\Lambda$ before chiral symmetry breaking so that the final $\Lambda_0$ comes out right is still more than 43 orders of magnitude.

Related to the above puzzle is the following question: how reasonable is it to assume that the radiation does not destabilize our vacuum? If we start from a site with $U_+ > \Lambda_c$, its decay to $U_- < \Lambda_c$ will be accompanied by a radiation energy density such that radiation plus $U_-$ is close to $U_+$ and so is bigger than $\Lambda_c$, implying that the universe should fast tunnel out of this site.

5.2 The brane world scenario

A possible way to evade this puzzle requires different gravitational dynamics for the radia-
tion and the vacuum energy. This is what happens in the brane world. Let us first review some relevant properties of the brane world scenario. In the brane world, standard model particles are open string modes with their ends ending on branes. Let us assume these branes are a stack of D3-branes (the discussion can easily be generalized to Dp-branes wrapping a (p-3)-cycle) in the bulk. At distance scales larger than the compactification scale, the effective 4-dimensional vacuum energy density is given by the vacuum energy density of the brane $\Lambda_{D3}$, which is simply the brane tension $T_3$, and the 10-dimensional vacuum energy density $\Lambda_{10}$ integrated over the 6-dimensional compactification volume $V_6$.

In a toy model with no warped geometry, we have

$$\Lambda = \Lambda_{10}V_6 + \Lambda_{D3} = \Lambda_B + T_3 \quad (5.1)$$

So this brane vacuum energy density $\Lambda_{D3}$ is very different from the 10-dimensional vacuum energy density $\Lambda_{10}$ in the string landscape, even though both contribute to the 4-dimensional effective vacuum energy density in the effective theory (at distance scales larger than the compactification scale) we have been studying so far. In the symmetric electroweak phase, the 4-dimensional $\Lambda_{EW}$ is again localized on the stack of branes. It simply raises the effective brane tension,

$$\Lambda_{D3} = T_3 + \Lambda_{EW} \quad (5.2)$$

In the case where $M$ D3-branes are bound together, then $\Lambda_{D3} \approx MT_3$. This distinction also applies to the radiation. In general, any radiation present can also be divided into bulk radiation density $\rho_B$ and brane radiation density $\rho_{D3}$. The radiation present during (or just before) the big bang nucleosynthesis is that of the standard model particles, so they are localized on the branes and contributes to $\rho_{D3}$ only. This brane radiation is distinct from the radiation in the bulk, $\rho_B$, which is composed of close string modes.

Naively, radiation leads to a finite temperature effect that may enhance the tunneling. However, if we are dealing with the tunneling of a D3-brane, then an additional brane vacuum energy density (like $\Lambda_{EW}$ or $\Lambda_\chi$) behaves like an addition to the brane tension; it is analogous to a particle with an increase in its mass, which typically suppresses, not enhances, its tunneling. We may see this effect in the following toy model (see Appendix B for a brief argument). The brane mode is given by $\phi = \sqrt{T_3 \tau}$. Increasing its tension is equivalent to a redefinition of $\phi \rightarrow \hat{\phi} \gg \phi$, which tends to suppress its tunneling (Appendix B). It is reasonable to assume that brane radiation has a similar effect. So it is reasonable to assume that this radiation/vacuum energy density on the branes do not destabilize the $\Lambda_0$ vacuum.

Recall that $\varphi_j$ are moduli and $\phi_i$ are brane modes in the landscape potential $U(\varphi_j, \phi_i)$ without brane radiation, it becomes $\hat{U}(\varphi_j, \hat{\phi}_i)$ when brane radiation is included. Note that bulk radiation will have a different effect on the potential, akin to the finite temperature ($T$) effect in field theory: $U(\varphi_j, \phi_i) \rightarrow U(\varphi_j, \phi_i, T)$. Based on the above distinction, it is plausible to absorb the brane radiation effect entirely into the redefinition of $\phi_j \rightarrow \hat{\phi}_j$ and study $\hat{U}(\varphi_j, \hat{\phi}_i)$ only; in this scenario, we have to deal with $\Lambda$ and bulk radiation only. So, in Eq.(3.25), the radiation term $g_+ T_+^4$ comes from the bulk radiation only and $\epsilon$ measures
the difference of the total vacuum energy densities. In this scenario, bubble collisions after each tunneling should lead to an efficient heating of the branes. The transfer of energy from the closed string modes to the brane modes should be somewhat similar to that in the (p)reheating in brane inflation. As an illustration, suppose fast tunneling goes from $U_+ \sim 100 \text{ MeV}^4$ to $U_1 \sim 10 \text{ keV}^4$ to $U_2 \sim 10 \text{ eV}^4$ to $U_3 \sim 10^{-5} \text{ eV}^4$ to $U_0 \sim 10^{-11} \text{ eV}^4$, while the brane is being heated to a temperature of a few MeV.

In a realistic flux compactification scenario, warped geometry appears. The brane modes and the moduli are non-trivially coupled and the correlation can be subtle. Clearly a better understanding of the potential is necessary to see if and how this interplay of the brane tension/position and bulk vacuum energy density works out in detail, whether the scenario is natural or not. It is important that the chiral and/or electroweak phase transitions are either second order or are completed fast enough for thermalization if they are first order.

5.3 Other possibilities

In Eq. (3.24), we see that the bulk radiation in the false vacuum enhances the tunneling rate. In fact, if the total energy density (vacuum energy plus bulk radiation) is bigger than $\Lambda_c$, we expect generic fast tunneling. After tunneling from $U_+$ site to the $\Lambda_0$ site and thermalization, one may argue that the bulk radiation plus $\Lambda_0$ (which is negligible compared to radiation) is larger than $\Lambda_c$ and will lead to fast tunneling again, that is, the $\Lambda_0$ vacuum is destabilized by the radiation. Fast tunneling above $\Lambda_c$ is a generic feature of the landscape. However, since the complicated potential looks random, there can be rare isolated sites that have exceptionally long lifetimes. Presumably, these trapped sites are very rare. At $\Lambda \gtrsim \Lambda_c$, such trapped sites may be rare but not as rare as when $\Lambda$ is larger. Suppose the bulk radiation in the $U_-$ site (with total energy density above $\Lambda_c$) leads to a fast tunneling out of this site, it will simply go to another low $\Lambda$ site. If the resulting bulk radiation again leads this site to tunnel out fast, the process can repeat any number of times, until (1) the bulk energy is mostly transferred to the branes, and/or (2) the universe reaches one of those rare trapped sites even when the bulk total energy density is above $\Lambda_c$.

In the latter case, efficient energy transfer to the branes is still necessary. As the universe cools, the universe will settle down at this $\Lambda_0$ site.

Suppose the potential barrier is lower between smaller $\Lambda$ sites; that is, $\Delta U$ is smaller between low $\Lambda$ sites than that between high $\Lambda$ sites. This is not an unreasonable property. In this situation, the universe may tunnel for $m$ number of steps: from $U_+$ to $U_1$ to $U_2$ to ...to $U_n$ to ...$U_{m-1}$ to $U_-$, where $U_{n-1} > \Lambda_c > U_n$. The last steps are still fast tunneling because the bulk radiations are large enough. Now the tunneling paths are dictated not by the difference in $\epsilon$ but by the smaller $\tau$s. As energy is being drained by the expansion of the universe and by the heating up of the brane modes, the small vacuum energy $\Lambda_0$ begins to play a larger role in enhancing the tunneling. This possibility may be checked by a detailed analysis of the landscape around a typical low $\Lambda$ site.

In the each step of this scenario, many bubbles are nucleated since the tunneling rate is large. These bubbles grow and collide with each other. Because of the spherical shape of the bubbles, a single expanding bubble does not generate gravitational waves. But
the many bubble collisions break the spherical symmetry and the gravitational waves are produced. With some luck, they may be detectable [29].

The above fast tunneling scenario provides a dynamical argument why today’s cosmological constant $\Lambda_0$ can be so small. This realization requires that the last tunneling in the landscape is from a $\Lambda > \Lambda_c$ site to a $0 \leq \Lambda_0 < \Lambda_c$ site, provided that enough of the energy released are efficiently transferred to the radiation in the branes. Also, the phase transition between mobile and trapped phases in the landscape may become more intricate in the brane world.

6. Inflation in the landscape

The above scenario puts tight constraints on the inflationary scenario in the early universe. Here we like to discuss some of the consequences of this scenario, what type of constraints appears, how the scenario is linked to inflation, and what are the possible detectable signatures.

In short, in the case where the universe is thermalized immediately after the last fast tunneling in the landscape, inflation should happen before this last fast tunneling, while the universe is still roaming in the landscape. In terms of the motion of a $D3$-brane, one may view this as an extension of brane inflation. That is, the brane is moving not just in a particular fixed 6-dimensional compactified manifold, but it is also moving from one manifold to another. Its motion in the landscape involves repeated fast tunneling, quantum hopping, scattering and ordinary rolling, may be even some slow-roll.

The potential for the inflation in this scenario is illustrated in Fig. 5. In this figure, we (over)simplify the picture of tunneling/percolation path of our universe to be one-dimensional. This picture is similar in looks to that proposed in Ref. [30], where a saltatory relaxation (i.e., relaxation by jumps) of the cosmological constant takes place as the universe tunnels repeatedly down the potential. Here we give an explicit dynamical realization of this behavior that happens in the landscape due to its vastness. Despite the presence of classically local minima, the wavefunction may simply roll past them since these minima are either too weak to trap the wavefunction, or strong enough to trap the wavefunction, but the trapping is so weak that the wavefunction can easily tunnel out. (For a $D3$-brane moving down such a random potential, its mobility would be further enhanced by its kinetic energy as well as bulk radiation, since these tend to further suppress binding. The DBI action may significantly enhance the tunneling rate [27]. In addition, as shown in Eq. (3.17), the tunneling is faster when the brane is higher up in the landscape. On the other hand, brane radiation, cosmological expansion of the universe and other damping mechanisms would tend to counteract, providing an interesting non-trivial dynamics.) The other key new input is the phase transition from the mobility phase to the trapped phase at an exponentially small $\Lambda_c$ due to the multi-dimensional and the complicated (random-looking versus periodic) nature of the landscape, which is necessary in the realization of the scenario.

That inflation takes place when a $D3$-brane is roaming in the landscape, not just moving inside a specific compactified bulk, should not be a big surprise. A similar issue
Figure 5: Cartoon for the tunneling path of the universe in the string landscape. The universe rolls, percolates and tunnels down the landscape. Tunneling is fast at sites with $\Lambda > \Lambda_c$, and exponentially slow at sites with $\Lambda < \Lambda_c$, where $\Lambda_c$ is exponentially small compared to the Planck scale. It is important to note that the actual path is multi-dimensional and the scale of this picture is not accurate.

in the KKLT scenario [4] has been studied in Ref. [31]. There, the height of the barrier $U(\text{barrier})$ separating a meta-stable site (with a tiny cosmological constant similar to our vacuum) at volume modulus $\rho_0$ from the 10 dimensional Minkowski vacuum (at modulus $\rho \to \infty$) is given by $\kappa U(\text{barrier})/3 \sim m_{3/2}^2$, where $m_{3/2}$ is the gravitino mass. To achieve inflation, one introduces an inflaton potential and the resulting Hubble scale can easily be bigger than $m_{3/2}$. This would very likely destabilize the volume modulus. Viewed from this bottom up perspective, one should not expect that, during inflation, the $D3$-brane is already trapped in today’s vacuum. In general, the $D3$-brane is not trapped in any particular 6-dimensional manifold during inflation. This is especially true when the $D3$-brane is high up in the cosmic landscape, where the vacuum energy is large so inflation is rapid and tunneling is fast. So we expect inflation to take place while the $D3$-brane is tunneling, hopping, (slow) rolling and bouncing around (classical scattering) in the landscape, finding its way down the landscape to a rest place at $\Lambda < \Lambda_c$.

An analysis of the properties of this inflationary scenario is most important but beyond the scope of this paper. Instead we shall try to get some ideas on what to expect. Let us consider the scenario of extended $D3$-brane inflation in the landscape. In the case of repeated scatterings and fast tunneling events within a single e-fold of inflation, the coarse-
grain behavior of the inflaton may mimic that of the slow-roll scenario. Let us now consider what differences and possible new features can emerge.

Since slow-roll (or DBI motion) is well studied, we shall briefly discuss the tunneling and scattering parts of this inflationary scenario. In addition to fast tunnelings, we expect the D3-brane to be scattered in the landscape. For a complicated landscape where the vacuum sites are distributed somewhat randomly, a D3-brane may easily fall into a very low \( \Lambda \) site. If it tunnels into it, the bulk radiation will enable it to quickly tunnel out. If it rolls into it, its kinetic energy will allow it to roll out. So classically, the D3-brane is not easily trapped. Quantum mechanically, it is a well known fact that any attractive 1-dimensional potential has at least one bound state, but this is no longer true in higher dimensions. A higher dimensional attractive potential that is either too narrow or too shallow has no bound state. In the landscape, with large \( d \), one expects that the wavefunction cannot be trapped by many classically stable local minima, especially those with large \( \Lambda \)s. (Since this also provides an intuitive way to see why a D3-brane is mobile in the landscape, we review a known example in Appendix A as an illustration.) So inflation most likely does not end once the D3-brane hits a low \( \Lambda > 0 \) meta-stable site.

Let us first consider the scatterings of the inflaton as it percolates in the landscape. This mimics the multi-field model of inflation. In this case, the classical inflaton trajectory can in general be decomposed into the longitudinal field, which parameterizes the motion along the trajectory, and transverse fields, which describes the directions perpendicular to the trajectory. Fluctuation along the trajectory yields adiabatic (curvature) perturbations and fluctuations orthogonal to the trajectory yields entropic (or isocurvature) perturbations. For this reason, the longitudinal field is sometimes called the adiabatic field and the transverse fields are called entropic fields. In general, correlations of these 2 types of perturbations would yield distinctive signatures that may be detected. Here the D3-brane bounces around in the random potential, causing sharp turns in the trajectory. Such sharp turns in the trajectory can convert entropic/isocurvature perturbations into adiabatic/curvature perturbations, even on superhorizon scales. In particular, this can give rise to interesting features in the primordial power spectrum and non-Gaussianity \[32\]. It will be exciting if non-Gaussianity can be detected \[33\].

Now let us consider fast tunnelings. Restricting to only repeated tunnelings, this extended brane inflationary scenario reduces to the chain inflation model \[12\]. In general, we expect a multi-step tunneling process. Each tunneling process is essentially the “old” inflationary scenario originally proposed by Guth \[10\], except that with fast tunneling, the graceful exit problem is automatically solved. We expect numerous steps of fast tunneling to take place as the universe moves down the landscape. Although a single step of fast tunneling (together with percolation and maybe thermalization) does not allow enough e-folds of inflation \[11\], it does contributes some inflation (say an e-fold or a big enough fraction) so multiple fast tunneling together may be sufficient. So enough inflation takes place as the universe (or D3-brane) percolates down the landscape, via fast tunneling, scattering and/or rolling.
In the conducting phase, the tunneling rate is
\[ \Gamma = M^4 e^{-2s/\xi}, \]  
where \( \xi \) measures the size of the site and \( s \) is the typical distance between sites in \([6]\). According to Eq. (4.3), the lifetime of a metastable vacuum is roughly given by
\[ t_F = \frac{H^3}{M^4_s} e^{2s/\xi}. \]  
To simplify the discussion, let us assume, as a toy model, that inflation is dominated by repeated fast tunneling events. Following \([34]\), we can easily estimate the amplitude of density perturbation
\[ \delta_H \sim Ht_F \sim \frac{H^4}{M^4_s} e^{2s/\xi}. \]  
COBE normalization is \( \delta_H \sim 10^{-5} \) and thus \( t_F \sim 10^{-5} H^{-1} \) which implies that there are roughly \( 10^5 \) steps during one e-fold. Since \( e^{2s/\xi} > 1 \), the Hubble parameter \( H \) during inflation must be less than the fundamental scale \( M_s \). Assume the change of energy density in each step is roughly a constant and \( \delta_H \) does not vary much during the whole inflationary epoch. Requiring the total number of e-folds to be at least 60 yields \( 60 \cdot 10^5 \cdot \epsilon < V \), or \( \epsilon < 1.7 \times 10^{-7} V \). The amplitude of the tensor perturbation is only related to the Hubble scale
\[ \delta_T \sim \frac{H}{M_p}. \]  
The tensor-scalar ratio is given by
\[ r = \frac{\delta_T^2}{\delta_H^2} \approx \frac{1}{M^2_p t_F^2}. \]  
A small tensor-scalar ratio \( r < 0.30 \) from WMAP+SDSS \([33]\) implies that the lifetime of the meta-stable vacuum should be longer than the Planck time. Even though the lifetime is much shorter than the Hubble time, the motion of the scalar field does not really roll down along a smooth potential, but a bumpy potential. Usually the small bumps do not affect the amplitude of density perturbations, but it may infer a large distinctive non-Gaussianity \([36]\). In our case the tunneling/percolation time is much shorter than the Hubble time, which means that the “period” of the bump is very short and the non-Gaussianity feature from bumps may be too oscillatory to be picked up observationally. In the case with a combination of tunneling, percolation and slow-roll, the non-Gaussianity from the features may be detectable.

Recall that \( s(\Lambda) \) is the typical separation between a \( \Lambda \) site and one of its neighboring sites which has a vacuum energy equal to or smaller than \( \Lambda \) (so it can tunnel to). The scaling behavior of \( s(\Lambda) \) is given in Eq. (2.10). The time scale of the tunneling is estimated in Ref.\([6]\).
\[ t_F \sim \left( \frac{s(\Lambda)}{L} \right)^{d-1} \frac{1}{\epsilon \Lambda^{-3/4}}, \]  

\( – 26 – \)
where \( \epsilon \) is the change of the vacuum energy density in a tunneling event. On the other hand, the COBE normalization implies
\[
t_F \simeq 10^{-5} H^{-1} = 10^{-5} \frac{M_p}{\Lambda^{1/2}} \tag{6.7}
\]
Combining the above two equations with Eqs.\((2.8,2.9)\), we find a scaling behavior for \( \epsilon \)
\[
\epsilon(\Lambda) = 10^5 N_T^{-\frac{5}{2q}} \left( \frac{s(\Lambda)}{L} \right)^{(1-\frac{5}{2q})d-1} \frac{\Lambda^{\frac{5}{2}}}{M_p}. \tag{6.8}
\]
If the tunneling rate is dominated by CDL instanton, \( \tau^4 \epsilon^3 \sim \frac{s(\Lambda)}{\xi} \) and thus
\[
\tau(\Lambda) \sim s(\Lambda)^{\frac{1}{2}(1-\frac{5}{2q})d-\frac{1}{2}}, \tag{6.9}
\]
where we assume \( \xi \) is insensitive to \( \Lambda \). The barrier between two sites and the tension of bubble increase as the cosmological constant decreases. So the index of \( s(\Lambda) \) in Eq. \((6.9)\) should be positive, namely
\[
\frac{1}{q} < \frac{4}{5} \left( 1 - \frac{2}{3d} \right). \tag{6.10}
\]
On the other hand, \( \epsilon(\Lambda) \) should decrease as \( \Lambda \) decreases, or equivalently \( s(\Lambda) \) increases. So the index of \( \epsilon(\Lambda) \) in Eq.\((6.8)\) should be negative, i.e.
\[
\frac{1}{q} > \frac{4}{5} (1 - \frac{1}{d}). \tag{6.11}
\]
Combining Eq.\((6.10)\) and Eq.\((6.11)\), the parameter \( q \) can be parameterized as
\[
\frac{1}{q} = \frac{4}{5} \left( 1 - \frac{2 + \eta}{3 + \eta d} \right), \tag{6.12}
\]
with \( \eta > 0 \). For \( d \gg 1, q \simeq 5/4 \) and the scaling behaviors are summarized as follows
\[
\begin{align*}
\Lambda_c & \sim d^{-\frac{5}{2d}}, \\
s(\Lambda) & \sim \Lambda^{-\frac{5}{2d}}, \\
\epsilon(\Lambda) & \sim s(\Lambda)^{-\frac{5}{8q}}, \\
\tau(\Lambda) & \sim s(\Lambda)^{\frac{1}{8(3+\eta)}},
\end{align*} \tag{6.13}
\]
Here \( \eta \) is a free parameter. The distribution of the cosmological constant is fixed for this simple toy model in which inflation is dominated by repeated fast tunnelings.

### 7. The open universe scenario

In this percolation but no thermalization case (i.e., case (2) where \( \gamma_T > \gamma > \gamma_P \)), it is likely that we end up inside a single large nucleation bubble after the last fast tunneling in the landscape. In this scenario, bubble collisions are still expected. If these collisions happen
\footnote{The convention in \texttt{3} is \( \Lambda = \Lambda^4 \) and then \( \Delta \Lambda \simeq \epsilon \Lambda^{-3/4} \).}
between the big bubble that we live in and the small bubbles at the edge of our bubble, the radiation released from such bubble wall collisions and annihilations may not have time to thermalize. This introduces the homogeneity and isotropy problem. As pointed out originally by CDL, the bubble inside is described by a FRW open geometry. At the moment of the bubble formation, the negative curvature contribution typically dominates over the vacuum energy density of the universe. This re-introduces the flatness problem.

A simple way to solve these problems is for the inside of the bubble to inflate, say via slow-roll. This is not unreasonable, since tunneling from $U_+$ may not reach $U_-$ directly: it may reach a point slightly higher than $U_-$ and then slow-roll towards $U_-$; even if it reaches $U_-$ directly, $U_-$ may be the local minimum of the potential only along the particular $\phi$ direction. It may not be a true local minimum, in the sense that it can roll down another direction orthogonal to $\phi$ to reach its true local minimum at $U = \Lambda_0$. During such rolling, and if the potential is flat enough, slow-roll inflation can solve the flatness, homogeneity and isotropy problems. This is the “open inflationary universe” scenario that has been studied extensively [37]. In this scenario, (p)reheating happens after or about the same time when $\phi$ reaches the $\Lambda_0$ site, that is, radiation/matter fields appear only after the $\Lambda_0$ site has been identified by the inflaton. In this scenario, as bubble collisions happen between the big bubble that we live in and the small bubbles at the edge of our bubble, the radiation released from such bubble wall collisions and annihilations may be observable [38].

Note that there is no constraint on $U_-(\phi)$: it can be bigger or smaller than $\Lambda_c$. As long as $U_0(\phi) < \Lambda_c$, $U_+(\phi) \rightarrow U_-(\phi)$ is the last fast tunneling, and we are stuck in $U_0(\phi)$ for a cosmologically long time.

In the conventional inflationary scenario for an open universe [37], the spatial curvature is inflated away and a spatial flat universe is predicted. However many authors suggested that our universe begins in a false vacuum corresponding to an old inflationary epoch during which any pre-existing inhomogeneities are redshifted away. The smoothness and horizon problems are solved during this period. Then the universe tunnels to its “true” vacuum via the nucleation of a single bubble inside which a second period of inflation takes place. After bubble materialization, the surfaces on which the inflaton field $\phi$ is constant are surfaces of constant negative spatial curvature. If the period of inflation is not too long, a detectable negative curvature is expected and the correction to the primordial quantum perturbations has also been investigated [37]. In this scenario, some part of the universe inevitably remains in the false vacuum, so eternal inflation is unavoidable. Our picture is slightly different, but in a crucial way. Because of fast tunneling, percolation is complete so there is no eternal inflation.

Unfortunately, the fine-tuning problem remains in this scenario. The $\phi_-$ site with vacuum energy density $U_-$ is chosen as the end point of the fastest tunneling path from $U_+$, irrespective of the value $\Lambda_0$. The minimum of the potential that the wavefunction would end up after slow-rolling from $U_-$ should have a value smaller than $U_-$, but not necessary many orders of magnitude smaller. Presumably $U_- \gg 1$ MeV$^4$, so there is no dynamical reason why $\Lambda_0 \approx 10^{-11}$ eV$^4$. In this sense, this slow-roll inflation after the last fast tunneling scenario is not very attractive.
If inside of each bubble has a large negative curvature, what is the curvature of the universe after bubble nucleation and thermalization? To simplify the problem, let the false vacuum to be flat. Is there a flatness problem in this case? This question is relevant not only here, but in any first order phase transition in the early universe after inflation, for example, in baryogenesis (leptogenesis) in a first order electroweak phase transition.

We believe there is no flatness problem in the $\gamma > \gamma_T$ case where the universe thermalizes. Lacking a proof, we like to give an intuitive (and convincing, we believe) argument here. In this scenario, inflation happens before the last fast tunneling. So it is reasonable to assume that the false vacuum is flat. At first sight, it is surprising that an infinite open universe can fit inside an expanding bubble of finite size. This is a result of different foliations (time slices) used. We know from the FRW equations that if a universe is precisely homogeneous and isotropic for all times, and if it is initially flat, then it will stay flat forever. For this fast tunneling case here, numerous microscopic size bubbles are produced and collide before any has grown much, as shown by Eq. (4.3) where the size of a typical bubble just before collision is clearly smaller than the Hubble size, i.e., $Ht_F \ll 1$. Although homogeneity and isotropy are broken at length scales of order $t_F$, homogeneity, flatness and isotropy at macroscopic scales ($\geq H^{-1}$) are maintained throughout the phase transition and the nucleation process. So the universe remains flat after tunneling and thermalization. Even if homogeneity and isotropy are destroyed by the randomness of bubble nucleation and collisions, but are then restored by thermalization, one expects that the temporary loss of homogeneity and isotropy does not change the original flatness property. So it is reasonable to assume that there is no new flatness problem in this scenario.

Now we also see the reasonableness of the input assumption that the false vacuum is flat. This false vacuum is arrived at via an earlier fast tunneling from another vacuum with a higher $\Lambda$. Suppose we start with a curved universe high up in the landscape. Inflation takes place during repeated tunneling, percolation and slow-roll. With enough inflation, any curvature would be inflated away. By the time the universe ends up in the $U_+$ vacuum just before the last fast tunneling, it would be flat in any practical sense.

8. Summary and remarks

In Einstein theory, the cosmological constant problem is a fine-tuning problem, namely, why $\Lambda_0 \sim 10^{-122} M_{\text{Planck}}^4$? In the brane world in string theory, the string scale is expected to be some orders of magnitude below the Planck scale. So, in terms of the fundamental string scale, the fine-tuning problem is somewhat ameliorated, though still substantial.

In the string landscape, which has exponentially many classically stable vacua, including ones with exponentially small $\Lambda$, this problem becomes a selection problem; the question is whether there exists a dynamical selection mechanism that argues for the natural emergence of such a small $\Lambda$ vacuum. In this and an earlier paper, we argue that such a mechanism exists. This requires that the universe is mobile in the string landscape, which is natural if the classically stable vacua are too weak to trap the wavefunction, or

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2We thank Alan Guth for sharing his view on this issue.
if they are barely strong enough to trap the wavefunction, so tunneling out of it can be fast. For a mobile $D3$-brane (representing the universe) in the landscape, its kinetic energy and the bulk radiation would further prevent trapping, so its mobility should be expected. Also we require that our particular universe is entered via a fast tunneling from another false vacuum. Together they impose strong constraints on the inflationary scenario, that it is likely to take place in the string landscape where the wavefunction of the universe is fully mobile. If the conducting-insulating phase transition happens at $\Lambda_c \sim d^{-d}M_s^4$, where $d$ is the number of effective dimension of the string landscape, the landscape allows mobility above this critical vacuum energy, so inflation most likely takes place in the mobile phase, before the last fast tunneling described above. The resulting inflationary picture bears some resemblance to the original “old” inflationary scenario, except that rolling and scattering are likely and tunneling is now fast and probably happens repeatedly, so there is no graceful exit problem.

In slow-roll inflation, the slow motion of the inflaton is due to a relatively flat potential. Here, the motion includes rolling, fast tunneling, quantum hopping and bouncing around in the landscape. In general, all these take time, providing a natural mechanism to slow down the motion of the inflaton. As these scattering/tunneling happens repeated within one e-fold, the coarse-grain behavior of the inflaton may look remarkably like that in the slow-roll scenario. In this sense, inflation in the landscape may be quite natural, allowing many e-folds of inflation. However, scatterings/tunnelings in the landscape may lead to interesting signatures in the CMB. Even if the power spectrum turns out to be almost scale-invariant, as dictated by the data, measurable bi-spectrum and tri-spectrum may reveal the true nature of the inflaton.

In the above scenario, eternal inflation is naturally avoided. One can hope that, as our knowledge of the string landscape improves, we can see better why the selection of our particular vacuum is reasonable, just as a planet like earth is not unexpected in any solar system. We are hopeful that mobility in the landscape provides a road map to a better understanding of the cosmological constant problem.

Towards the end of the inflationary epoch in the landscape, the branes of the standard model particles are (p)reheated as the universe undergoes its last fast tunneling in the landscape. On the other hand, the brane radiation does not participate directly in the tunneling process as the bulk radiation or the vacuum energy density. This offers the plausibility why today’s $\Lambda$ can be so small.

There are many open questions remain in the above scenario. Some of them are:

- The structure and property of the landscape: (1) locally, to check if a typical locally stable vacuum in the landscape is too weak to trap the wavefunction; (2) also, the structure of our vacuum site (or some other typical low $\Lambda$ sites) and its neighborhood should be mapped; (3) globally, does the landscape have the structure to allow for many e-folds of inflation?
- A key assumption of the whole scenario is that tunneling from a $\Lambda > 0$ site to a $\Lambda < 0$ site may be ignored, since the resulting negative $\Lambda$ site will end up in a big crunch. The meaning and implication of this big crunch remain poorly understood, so the justification of ignoring such tunnelings remains open. Here this crunch property has to be analyzed in the brane world scenario instead of in 4 dimensional spacetime.
• The different roles played by the brane energy density and the bulk energy density need further investigations. Also, what is the finite temperature effect of the bulk radiation on the tunneling paths and rates?
• The signatures of the extended brane inflationary scenario. One may use the almost scale-invariant power spectrum in the CMB as a constraint on the structure of the string landscape. It is unclear if the non-Gaussianity and/or other features are large enough and distinct enough to be detected.

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A. Difficulty trapping the wavefunction in high dimensions

Here we like to recall the well-known feature of quantum mechanics, that a potential that traps a particle in low spatial dimensions may be too weak to trap a particle in higher dimensions. Consider a quantum mechanical particle with mass $m$ in a $d$-dimensional localized attractive spherical potential $V(r)$ as a function of the radius $r$. To see if the particle can be trapped or not, let us consider the ground state $S$-wave only: $\psi(r) = r^{-(d-1)/2} \eta(r)$, where $\eta(r)$ satisfies

$$\eta(r)'' + \left(2m[E - V(r)] - \frac{(d-1)(d-3)}{4r^2}\right) \eta(r) = 0 \quad (A.1)$$

where the prime denotes derivative with respect to $r$. Suppose the attractive potential is a $\delta$-function: $V(r) = -g^{d}(r)$, for any strength $g$. It is well known that there is a bound state for $d = 1$ (with binding energy $E = -g^2 m/2$) but no bound state for $d = 3$.

Let us consider in some detail another well-known example, namely, the spherical square well potential $V(r)$,

$$V(r) = \begin{cases} 
-V_0 & r < R \\
0 & r > R 
\end{cases} \quad (A.2)$$

where the constant $V_0 > 0$. Introducing $k_0^2 = 2mV_0$, we see that $k_0$ measures the depth and $R$ measures the size of the attractive potential, so the dimensionless number $k_0R$ measures the overall strength of the potential. In a theory with a single scale, this number is expected to be of order unity. Here $\psi(r)$ (or $\eta(r)$) describes a bound state only if its
energy $E < 0$. For the bound state, $\psi(r)$ must drop exponentially for $r > R$, $\psi(r) \sim e^{-ar}$ where $a^2 = 2m|E|$. Let $l = (d - 3)/2$. the equation can be easily solved (consider integer $l \geq 0$):

$$\begin{align*}
\eta(r) &= k r A j_l(kr) & r < R \\
\eta(r) &= i a r B h_l(iar) & r > R
\end{align*}$$

(A.3)

where $A$ and $B$ are normalization constants, $k^2 = 2m(V_0 - |E|)$, and $j_l(z)$ and $h_l(z)$ are spherical Bessel functions (first and third kind):

$$\begin{align*}
j_0(z) &= \sin z/z, & h_0(z) &= -ie^{iz}/z \\
j_1(z) &= \sin z/z^2 - \cos z/z, & h_1(z) &= (-i/z^2 - 1/z)e^{iz}
\end{align*}$$

(A.4)

The continuity condition at $r = R$ leads to

$$iaR \left( \frac{h_{l+1}(iaR)}{h_l(iaR)} \right) = kR \left( \frac{j_{l+1}(kR)}{j_l(kR)} \right)$$

(A.5)

Let $k_0R = x_0$, $k/k_0 = \xi$ and $aR = x_0\sqrt{1 - \xi^2}$, we have $0 \leq \xi \leq 1$ and the continuity condition can be rewritten as

$$\tan(x_0\xi) = F_1(x_0, \xi)$$

(A.6)

where

$$F_0(x_0, \xi) = -\xi/\sqrt{1 - \xi^2}$$

$$F_1(x_0, \xi) = \frac{x_0\xi}{1 + \frac{\xi^2}{1 - \xi^2}(1 + x_0\sqrt{1 - \xi^2})}$$

(A.7)

and $F_l(x_0, \xi)$ gets complicated for large $l$. One can solve Eq.(A.6) numerically. For fixed $x_0 = k_0R$, a solution to Eq.(A.6) yields the bound state with energy $E = -k_0^2(1 - \xi^2)/(2m)$.

In the $d = 1$ case, there is always at least one bound state. In the $d = 3$ case, there is no bound state if $x_0 < \pi/2$. For large (odd) $d$, one finds that there is no bound state solution if $x_0 = k_0R$ is less than a critical value $P_c$,

$$k_0R < P_c \quad P_c \simeq 0.58d + 0.5$$

(A.8)

We see that a larger and deeper potential is needed to trap the particle as $d$ increases. For a potential barely strong enough to trap the particle, i.e., $k_0R \gtrsim P_c$, we see that $\xi \lesssim 1$ and $aR$ is very small. So its wavefunction $\psi(r) \sim e^{-ar}$ spreads to $r \gg R$ and its tunneling to a neighboring site can be fast. (Instead of $k_0R$, one may consider $k_0\bar{R}$, where $\bar{R}^d = V(d)$, where $V(d)$ is the volume of the potential. In this case, the bound becomes, for large $d$,

$$P_c \simeq P_c \sqrt{2e/\pi/d} \simeq 2.4\sqrt{d}.$$.

If the particle has kinetic energy, binding will be further suppressed. However, even without binding, the particle will be scattered by the attractive potential. For a theory with a single scale, like string theory, the dimensionless quantity $k_0R$ should be of order unity. In the presence of warped geometry, where a hierarchy of scales appears, one may expect that other possibilities are easy to find. However, since $k_0R$ is dimensionless, it is
likely that it stays unwarped, even if \( R \) and \( k_0 \) are warped. Of course, this feature should be checked in explicit examples.

This quantum mechanical example provides an intuitive understanding why mobility is likely in the vast landscape: many of the classically stable local minima may not be able to trap the wavefunction. For other sites, the binding may be so weak that the bound state wavefunction has a long tail and so its tunneling out of it to a nearby site with a lower \( \Lambda \) can be fast. Comparing \( \psi(r) \sim e^{-ar} \) to Eq. (2.1), we see that \( \xi \sim 1/a \). The tunneling probability goes like \( e^{-2as} \), where \( s \) is the separation between the 2 local minima. For a classical local minimum that is too weak to bind, namely an unstable vacuum, \( a \) becomes pure imaginary: \( \text{Re}(a) = 0 \). This is the limiting case of fast tunneling: that is, the lifetime of this vacuum is zero.

**B. Effect on tunneling due to a change in mass (or brane tension)**

In the quantum mechanical example in Appendix A, we see that binding is stronger for a more massive particle. Also, the exponential damping of a bound state wavefunction is stronger for a larger mass. It follows that tunneling is suppressed as the mass increases. Here we consider tunneling for a scalar field with similar results.

Following \[22\], let us start with the effective Langrangian density

\[
L = \frac{1}{2} \frac{\partial \phi}{\partial \mu} \frac{\partial \phi}{\partial \mu} - U(\phi)
\]

where

\[
U(\phi) = \frac{\lambda}{8} \left( \phi^2 - \frac{\mu^2}{\lambda} \right)^2 + \frac{e}{2a} (\phi - a) + \Lambda_p
\]

and \( a^2 = \mu^2/\lambda \), so the false vacuum is at \( \phi = a \) and the “true” vacuum is at \( \phi = -a \). Here \( \Lambda_p \gtrsim \epsilon \) so that tunneling is to a positive vacuum energy site. The bubble wall tension is given by

\[
\tau = \frac{\mu^3}{3\lambda}
\]

Suppose we are studying a D3-brane tunneling somewhere inside the bulk. Then \( \phi = \sqrt{T_3} \), where \( r \) is the position of the brane in some coordinate. If there is a radiation component on the brane and if some symmetry is restored, then an additional vacuum energy density will effectively increase \( T_3 \) to \( \Lambda_{D3} > T_3 \). The treatment of a vacuum energy term on the brane is easier so let us consider only such a term here. To maintain the canonical kinetic term for the scalar field, we rescale \( \phi \) to \( \hat{\phi} \),

\[
\hat{\phi} = \frac{\sqrt{\Lambda_{D3}}}{\sqrt{T_3}} \phi = \phi/b
\]

where \( b \leq 1 \). With the same potential (B.2) but expressed in terms of \( \hat{\phi} \), one obtains

\[
\hat{U}(\hat{\phi}) = \frac{\hat{\lambda}}{8} \left( \hat{\phi}^2 - \frac{\hat{\mu}^2}{\hat{\lambda}} \right)^2 + \frac{\epsilon}{2\hat{a}} (\hat{\phi} - \hat{a}) + \Lambda_p
\]

where \( \hat{\lambda} = b^4 \lambda, \hat{\mu} = b\mu \) and \( \hat{a} = a/b \), while \( \epsilon \) and \( \Lambda_p \) do not change. With (B.3), \( \tau \rightarrow \mathring{\tau} = \tau/b \), This can be interpreted that the bubble wall thickens due to the increase in its
tension. Following Eq. (3.25), we see that an increase in brane tension leads to an increase in bubble wall tension \( \tau \) which in turn decreases the tunneling probability.

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