The chiral condensate in neutron matter

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We calculate the chiral condensate in neutron matter at zero temperature based on nuclear forces derived within chiral effective field theory. Two-, three- and four-nucleon interactions are included consistently to next-to-next-to-leading order (N^3LO) of the chiral expansion. We find that the interaction contributions lead to a modest increase of the condensate, thus impeding the restoration of chiral symmetry in dense matter and making a chiral phase transition in neutron-rich matter unlikely for densities that are not significantly higher than nuclear saturation density.

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The understanding of the phase diagram of matter is a current frontier in nuclear physics. At high temperatures and vanishing net baryon density, the properties of strongly interacting matter have been studied in first-principle lattice QCD calculations. It is found that at a temperature of 154 ± 9 MeV matter exhibits a chiral and deconfinement crossover transition from the low-temperature hadronic phase, where chiral symmetry is spontaneously broken, to the chirally symmetric high-temperature phase, the quark-gluon plasma [1]. An order parameter for characterizing this transition is the chiral condensate [2–4].

Owing to the fermion sign problem, matter at non-zero baryon densities cannot be directly studied in lattice QCD. Therefore, there are no first-principle QCD results for the phase diagram in particular at low temperatures and high densities. These conditions are probed in neutron stars, which can reach several times nuclear saturation density in their interiors [5–8]. However, there has been much speculation on possible exotic phases that may appear in the center of neutron stars, such as Lee-Wick abnormal matter, pion- as well as kaon-condensed matter, hyperon matter and quark matter [5]. It has also been conjectured, see, e.g., Fig. 1 in Ref. [9], that in neutron matter, the QCD phase transition may begin already below saturation density.

Recent observations of neutron stars with 2 M⊙ masses [10] provide general constraints on the equation of state (EOS) of cold strongly interacting matter, and put into question whether exotic phases that tend to soften the EOS are realized in neutron stars. At densities n ≲ n0, where n0 = 0.16 fm⁻³ denotes nuclear saturation density, the properties of nuclear systems have been studied systematically based on nuclear forces derived within chiral effective field theory (EFT) [12] [13] and using renormalization group methods [14] [15]. In this paper, we use chiral EFT interactions to study the chiral condensate as a function of density in neutron matter, based on perturbative calculations around the first-order Hartree-Fock energy [16] [17].

The chiral condensate can be obtained from the energy using the Hellman-Feynman theorem [3] [19] [20],

\[ \langle \bar{q}q \rangle_n - \langle \bar{q}q \rangle_0 = n \frac{\partial}{\partial m_q} \left[ \frac{E_{\text{free}}(m_q, k_F)}{N} + \frac{E_{\text{int}}(m_q, k_F)}{N} \right], \]

where \( \langle \bar{q}q \rangle_n \) and \( \langle \bar{q}q \rangle_0 \) are the chiral condensates at finite baryon density \( n = k_F^3/(3\pi^2) \) (with Fermi momentum \( k_F \)) and in vacuum, respectively. Moreover, \( E_{\text{free}}/N = m_N + 3k_F^2/(10n_N) \) is the energy per particle of a system consisting of \( N \) noninteracting degenerate neutrons in the nonrelativistic limit, \( E_{\text{int}} \) is the corresponding interaction energy, \( m_q \) denotes the average of the u and d quark masses, \( \bar{q}q = \bar{u}u + \bar{d}d \), and \( m_N \) is the neutron mass.

The contribution from the nucleon mass to the chiral condensate is proportional to the pion-nucleon sigma term \( \sigma_{\pi N} \), which accounts for the scalar quark density in the nucleon [3] [21]:

\[ \sigma_{\pi N} = \langle N | m_q \bar{q}q | N \rangle = m_q \frac{\partial m_N}{\partial m_q} \]

Here \( |N\rangle \) represents the state of a nucleon at rest. The value of the pion-nucleon sigma term has been determined within different frameworks [21] [25]. As a baseline we use the value \( \sigma_{\pi N} \approx 50 \text{ MeV} \) [20]. The chiral condensate in neutron matter relative to the vacuum is then given by [3]

\[ \frac{\langle \bar{q}q \rangle_n}{\langle \bar{q}q \rangle_0} = 1 - \frac{n \sigma_{\pi N}}{f_\pi^2 m_N^2} \left( 1 - \frac{3k_F^2}{10n_N} + \ldots \right) \frac{n \frac{\partial}{\partial m_q} E_{\text{int}}(m_q, k_F)}{f_\pi^2 \frac{\partial m_q}{m_N}} \]

where we have used the Gell-Mann–Oakes–Renner relation \( m_q \langle \bar{q}q \rangle_0 = -f_\pi^2 m_N^2 \). In our calculation we use for the pion mass the charge average \( m_\pi = 138 \text{ MeV} \) and for the pion decay constant \( f_\pi = 92.4 \text{ MeV} \).
The leading $c_1$ contribution to the chiral condensate in Eq. (3), which is due to the mass term in $E_{\text{free}}/N$, is linear in density and is shown in Fig. 1 by the dashed line. By extrapolating this linear density dependence, one finds restoration of chiral symmetry at a density around $(2.5-3)n_0$ \cite{[19]}. For the density range shown in Fig. 1 where chiral EFT interactions can be applied with confidence, the kinetic energy contribution to $E_{\text{free}}/N$ gives only a 4% correction relative to the leading term. Relativistic corrections, indicated by the dots in Eq. (3), are negligible at these densities. The next term $9k_F^2/5\pi m_N^3$ is a 0.3% correction. While the first correction to the chiral condensate in Eq. (3) is a consequence of the finite nucleon density, the long-range contributions from $E_{\text{int}}$ can be attributed to the modification of the scalar pion density $\Delta n_\pi = n \partial(E_{\text{int}}/N)/\partial m_\pi$ due to the interactions between nucleons (cf. Ref. \cite{[26]}).

The energy per particle of neutron matter has recently been calculated based on chiral EFT interactions to N$^3$LO, including two-nucleon (NN), three-nucleon (3N), and four-nucleon (4N) forces \cite{[17],[18]}. The pion-mass dependence of nuclear forces arises from two sources: First, due to the explicit $m_\pi$ dependences in the long-range pion-exchange interactions, and second, implicitly, due to the quark-mass dependence of the pion-nucleon coupling $g_A$, the pion decay constant $f_\pi$, as well as the leading NN contact interactions $C_S$ and $C_T$, and higher-order pion-nucleon and short-range NN and 3N contact interactions.

We calculate the explicit $m_\pi$ dependence of nuclear forces by varying the value of the pion mass in the pion-exchange NN, 3N, and 4N interactions. At the NN level, we use the N$^3$LO potentials of Epelbaum, Glöckle, and Mei$\ss$ner (EGM) \cite{[27],[28]} with cutoffs 450/500 and 450/700 MeV (and their N$^2$LO versions to study the order-by-order convergence). With these NN interactions neutron matter is perturbative at the densities considered here \cite{[17],[18]}. This result was recently validated using first Quantum Monte Carlo calculations with chiral EFT interactions \cite{[29]}. We vary $m_\pi$ by 0.5% in the corresponding potential routines. The derivative of the interaction energy with respect to $m_\pi$ in Eq. (3) is then computed numerically for different densities.

We estimate the impact of the quark-mass dependence of $g_A$ and $g_\pi$ using the results of Refs. \cite{[30],[31]}. In a perturbative calculation, the interaction energy per particle $E_{\text{int}}/N$ is a polynomial in $g_A$ and in $1/f_\pi$. Consider a term in this polynomial, in which $g_A$ enters with the power $\alpha$, $[E_{\text{int}}/N]_\alpha$. For the corresponding contribution to the chiral condensate, due to the pion-mass dependence of $g_A$, we thus have

$$-\frac{n}{f_\pi^2} \frac{\partial}{\partial m_\pi^2} \left[ \frac{E_{\text{int}}}{N} \right]_\alpha = -\frac{n}{f_\pi^2} \frac{\partial g_A}{\partial m_\pi^2} \left[ \frac{E_{\text{int}}}{N} \right]_\alpha$$

$$\approx -(4.4 - 5.7) \times 10^{-4} \text{MeV}^{-1} \left[ \frac{E_{\text{int}}}{N} \right]_\alpha.$$  \hspace{1cm} (4)

Here, we have used the range for $\frac{\partial g_A}{\partial m_\pi}$ from Ref. \cite{[31]}. Similarly, an interaction term, in which $1/f_\pi$ enters with the power $\beta$, $[E_{\text{int}}/N]_\beta$, leads to a contribution to the chiral condensate

$$-\frac{n}{f_\pi^2} \frac{\partial}{\partial m_\pi^2} \left[ \frac{E_{\text{int}}}{N} \right]_\beta = -\frac{n}{f_\pi^2} \frac{\partial f_\pi}{\partial m_\pi^2} \frac{\partial}{\partial f_\pi} \left[ \frac{E_{\text{int}}}{N} \right]_\beta$$

$$\approx (2.6 - 5.0) \times 10^{-4} \text{MeV}^{-1} \left[ \frac{E_{\text{int}}}{N} \right]_\beta,$$  \hspace{1cm} (5)

using $\frac{\partial f_\pi}{\partial m_\pi^2}$ from Ref. \cite{[31]}. The uncertainty is larger in this case, because of the $c_3$ and $c_4$ uncertainties, which are taken as in the N$^3$LO calculations of Refs. \cite{[17],[18].

For the leading-order one-pion-exchange NN interaction, which is proportional to $g_A^2/f_\pi^2$ and contribute $\sim 10$ MeV per particle at $n_0$, the terms (4) and (5) give a contribution to the chiral condensate ranging from $-0.006$ to $+0.001$. The leading N$^2$LO two-pion-exchange 3N forces also provide $\sim 10$ MeV per particle at $n_0$. These terms are proportional to $g_A^2/f_\pi^2$ and the corresponding contribution to the chiral condensate lies in the range $-0.001$ to $+0.011$. Combined, these corrections amount to at most a 25% increase of the uncertainty band in...
Fig. 1. We expect the contributions from the short-range interactions to start at a similar level. However, the extrapolation of their $m_\pi$ dependence from lattice QCD results at heavier pion masses to the physical point is uncertain. This will be improved in the future once lattice QCD results for NN and 3N interactions for physical pion masses will become available. Because the estimated effects beyond the explicit $m_\pi$ dependence are small compared to the band in Fig. 1, we do not include these contributions in the present paper.

In Fig. 1 we show our results for the chiral condensate in neutron matter at N³LO, based on the two N³LO NN potentials and including 3N and 4N interactions to N³LO. The calculations include all interactions at the Hartree-Fock level plus N³LO NN and N²LO 3N interactions to second order and including particle-particle/hole-hole third-order contributions, using a free or a Hartree-Fock spectrum, as in Refs. [17, 18]. The bands include the uncertainties of the many-body calculation, of the $c_3$ couplings of 3N forces, and those resulting from the 3N/4N cutoff variation (see Refs. [17, 18] for details). The width of the bands are dominated by the uncertainties of the $c_3$ coupling and by the sensitivity of the many-body calculation on the single-particle spectrum used.

We find that the density dependence of the chiral condensate is dominated by the leading $\sigma_{\pi N}$ term and therefore the chiral condensate decreases almost linearly with increasing density. The interaction contributions lead to a positive correction, thus impeding the restoration of chiral symmetry with increasing density. Consequently, for moderate densities, below say $n = 0.3\,\text{fm}^{-3}$, which is below the linear extrapolation $n = (2.5 - 3)n_0$, a chiral phase transition seems unlikely in neutron matter. However, we note that, based on calculations of the type presented here, where only the broken symmetry phase...
is considered, we cannot exclude the possibility of a first-order transition, where the order parameter changes discontinuously. If the transition occurs below \( n = 0.3 \text{ fm}^{-3} \), this would have to be a very strong first-order transition.

The results for the chiral condensate in neutron matter based on the two EGM 450/500 MeV and 450/700 MeV \( N^3\text{LO} \) potentials are in very good agreement within the uncertainty bands in Fig. 1. At nuclear saturation density, \( \langle \bar{q}q \rangle_n / \langle \bar{q}q \rangle_0 \) lies in the range \( 67.3 - 69.8\% \) and \( 67.8 - 69.5\% \), respectively. In comparison, the uncertainty of the leading \( \sigma_{\pi N} \) term is much larger. Using \( \Delta \sigma_{\pi N} = 8 \text{ MeV} \) [21], we find an uncertainty on the order of \( 10\% \) at \( n = 0.2 \text{ fm}^{-3} \).

The individual interaction contributions to the chiral condensate are shown in Fig. 2. In the upper row, the Hartree-Fock \( N^2\text{LO} \) NN, \( N^2\text{LO} \) 3N, and \( N^3\text{LO} \) 3N and 4N results are given. In the lower row, second-order and particle-particle/hole-hole third-order contributions beyond Hartree-Fock are shown, grouped into the different contributions where \( N^2\text{LO} \) 3N forces are included as density-dependent two-body interactions. This follows the notation of Ref. [18]. The most important contributions are the NN and 3N Hartree-Fock terms as well as higher-order correlation effects due to NN interactions. The latter are sensitive to the single-particle spectrum used. This is because the Hartree-Fock single-particle energies depend on the pion mass, so that the derivative with respect to \( m^2_{\pi} \) yields additional contributions. For the Hartree-Fock spectrum, the NN correlation contributions are then only about half as large as for the free spectrum. For the NN-3N and 3N-3N correlation contributions, we find a similar sensitivity, but they are relatively small.

As shown in Fig. 2 at nuclear saturation density the NN Hartree-Fock contribution of the EGM 450/500 MeV potential is smaller than that of the EGM 450/700 MeV potential by a factor two. However, for the higher-order correlations the situation is reversed, so that the sum of the Hartree-Fock and higher-order NN contributions of the two NN potentials are in very good agreement, as shown in the left panel of Fig. 3. The total interaction contribution, including 3N and 4N forces, is shown in the right panel of Fig. 3 and yields a \( 6 \pm 2\% \) enhancement of the chiral condensate at saturation density. We again find a very good agreement within the uncertainty bands of the two NN potentials.

The order-by-order convergence of the chiral EFT calculation for the chiral condensate is shown in Fig. 4 from \( N^2\text{LO} \) to \( N^3\text{LO} \). Going from \( N^2\text{LO} \) to \( N^3\text{LO} \), the enhancement of the condensate is weakly reduced and the width of the uncertainty band is reduced by roughly a

![Graph](image-url)
factor of two. A similar reduction of the bands was found in the $^3\text{LO}$ calculation of the equation of state \[17\].

We note that the increase of the chiral condensate due to interactions corresponds to a decrease of the scalar pion density in the interacting system. While the iterated NN one-pion-exchange interaction yields an enhancement of the scalar pion density \[26\], the interference between the one-pion-exchange and shorter-range NN parts induces the opposite effect. Thus, the sign of the interaction contribution to the chiral condensate is governed by a competition between these two contributions.

Our results agree with those of Kaiser and Weise \[32\], who calculated the interaction corrections to the chiral condensate in chiral perturbation theory with explicit $\Delta$'s including one- and two-pion exchange contributions up to three-loop order in the energy density. This leads to a $\approx 5\%$ contribution at $n_0$. Extrapolating their results to higher densities, chiral symmetry restoration is found at $\approx 3n_0$. The chiral condensate in neutron matter was also calculated to NLO by Lacour \[33\] et al. using in-medium chiral perturbation theory \[34\]. They found only small interaction corrections, which reduce the chiral condensate at this level. Lacour \[35\] et al. also calculated the chiral condensate for the $u$ and $d$ quarks separately and showed that the condensate for the $u$ quarks is larger than for the $d$ quarks by $\approx 7\%$ at $n = 0.2 \text{ fm}^{-3}$. This difference is however smaller than the uncertainty from the $\sigma_{\pi N}$ term.

Finally, Kaiser \[36\] et al. also computed the interaction corrections to the chiral condensate in symmetric nuclear matter in the same scheme. Both in neutron and symmetric matter, they find a strong enhancement of the condensate owing to correlation diagrams involving the excitation of the $\Delta$. In our work, the corresponding contribution is included mainly through 3N interactions. On a qualitative level, our results agree with those of Refs. \[4\] \[19\]. However, the various interaction contributions and the magnitude of the enhancement seem rather different. This may be due to differences in the calculational schemes, but also due to differences in the system considered (neutron matter versus symmetric nuclear matter). In the future, we will study symmetric matter, so that a direct comparison can be made.

In summary, we find that nuclear interactions impede the restoration of chiral symmetry in neutron matter at zero temperature. The net effect of interactions remains below 10\% for $n \lesssim 0.2 \text{ fm}^{-3}$, but grows with increasing density. The dominant source of uncertainty is the $\sigma_{\pi N}$ term. We conclude that for moderate densities, say $n \lesssim 0.3 \text{ fm}^{-3}$, a chiral phase transition in neutron-rich matter therefore seems unlikely, although we cannot exclude a strong first-order transition. For the densities considered here, we find a good convergence of the chiral condensate from $^2\text{LO}$ to $^3\text{LO}$ in chiral EFT. Clearly it would be very interesting to calculate the chiral condensate also for higher densities. While a systematic calculation in chiral EFT is difficult at densities much higher than $n = 0.2 \text{ fm}^{-3}$, astrophysical observations shed light on matter at high densities (see Refs. \[6\] \[18\] \[20\] for the equation of state).

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