MASSLESS DECOUPLED DOUBLERS: CHIRAL YUKAWA MODELS AND CHIRAL GAUGE THEORIES*

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We present a new method for regularizing chiral theories on the lattice. The arbitrariness in the regularization is used in order to decouple massless replica fermions. A continuum limit with only one fermion is obtained in perturbation theory and a Golterman–Petcher like symmetry related to the decoupling of the replicas in the non–perturbative regime is identified. In the case of Chiral Gauge Theories gauge invariance is broken at the level of the regularization, so our approach shares many of the characteristics of the Rome approach.

1. INTRODUCTION

In order to make some progress on a rigorous definition of quantum field theories and study their non–perturbative effects a crucial step is to use a lattice as regulator. In the case of a Chiral Gauge Theory, any attempt in this direction [1,2] is based on a solution of the well known difficulties (doubling problem) one finds for theories with fermion fields on the lattice. Several ways of dealing with this problem, which have been proposed recently [3], have been reviewed in references [1] and [4], so that our interest here is concentrated mainly on our proposal for the decoupling of the doublers.

1.1. Philosophy of our approach

We will start by saying the philosophy of our approach.

As replica fermions look inevitable, let us try to live with decoupled but massless, (i.e. harmless), replica fermions. This has to be contrasted with the usual ways of decoupling the replica by giving them a mass of the order of the cut–off. Our first tentative were the works in reference

* TALK PRESENTED AT THE TOPICAL WORKSHOP “NON PERTURBATIVE ASPECTS OF CHIRAL GAUGE THEORIES”, ACCADEMIA NAZIONALE DEI LINCEI, ROMA, 9-11 MARCH,1992.
[5] and some results in two spacetime dimensions can be found in reference [6].

To implement this idea, we use the arbitrariness in the regularization in order to couple, by hand, in the action, only one of the $2^d$ fermions to the scalars, in the case of Chiral Yukawa Models, and to the gauge fields and the scalars, in the case of CGT. In the case of CGT this “decoupling by hand” can be done at the expense of working with a non gauge invariant regularization, so our approach shares many of the characteristics of the Rome approach [7].

Given that dimensional regularization, the paradigm of a gauge invariant preserving regularization, does not work very well in the case of CGT [8], it seems natural to investigate the chances offered by a gauge non–invariant regularization on the lattice.

1.2. Properties

In both, CGT and CYM, we are able to describe fermions with arbitrary chiral content (a chiral field, $\Psi_L$ for instance, does not need the presence of a complementary one, $\Psi_R$). Also our procedure involves a minimal field content (no mirror fermions, for instance).

For the case of CYM the regularization method keeps all symmetries of the continuum: hermiticity, invariance under discrete rotations and translations and (global) chiral symmetry (see section 3 for some comments about the symmetries of the quantum theory).

Besides, no extra tuning is needed in order to give finite mass to the physical fermions (and to decouple the replicas). Also the global chiral symmetry we have (in the case of a CGT we lose only the corresponding local symmetry) avoids the occurrence of a mass counterterm for the fermion field: the masses of the fermions are protected.

1.3. State of the art

a) We have proved perturbatively that in the continuum limit only one fermion is coupled to the scalars, in the case of a CYM, and to the scalars and the gauge fields, in the case of a CGT (in this case at one loop level).

b) We have (partially) found the phase diagram of a CYM.

c) We have a nice Golterman–Petcher like symmetry [9] related to the decoupling of the replicas in the non perturbative regime of both CGT and CYM.

2. THE LATTICE ACTION FOR A CGT AND THE DECOUPLING SYMMETRY

Concerning a CGT we have done little but proposing our regularization. The action has five terms (for details of notation, see J. Smit at the Capri Conference [1]),

$$I(\Psi, \Phi, U) = I_U + I_B(\Phi, U) + I_F(\Psi) + I_{INT}(\Psi, U) + I_Y(\Phi, \Psi),$$

where,

$$I_U = \frac{1}{g^2 a^{4-d}} \sum_{x, \mu, \nu} Tr[U_{Lx\mu\nu} + U_{Rx\mu\nu}] + (L \leftrightarrow R),$$

is the usual plaquette term for the gauge fields made out of link variables $U_{x\mu}$,

$$I_B(\Phi, U) = - \sum_x \frac{1}{2} Tr[\Phi_x^+ \Phi_x + \lambda (\Phi_x^+ \Phi_x - 1)^2] + \frac{k}{2} \sum_{x, \mu} Tr[\Phi_{x+\mu}^+ U_{Lx\mu}^+ \Phi_x U_{Rx\mu}] + h.c.,$$
is the lattice action for a complex scalar field. $I_F(\Psi)$, the free action, is,

$$I_F(\Psi) = I_N(\Psi, U = 1),$$

$I_N$ being the naive action,

$$I_N(\Psi, U) = - a^{d-1} \frac{1}{2} \sum_{x, \mu} [\Psi_{Lx} \gamma_\mu U_{Lx\mu} \Psi_{Lx+\hat{\mu}} - \Psi_{Lx+\hat{\mu}} \gamma_\mu U_{Lx\mu}^+ \Psi_{Lx}] + (L \leftrightarrow R). \quad (4)$$

The interaction term,

$$I_{INT}(\Psi, U) = I_N(\Psi^{(1)}, U) - I_N(\Psi^{(1)}, 1), \quad (5)$$

contains the coupling of the fermions to the gauge fields. (Note that if we put $U = 1$, then $I_{INT}(\Psi, U) = 0$).

Finally, the Yukawa term is

$$I_Y(\Phi, \Psi) = - y \sum_x (\bar{\Psi}^{(1)}_{Lx} \Phi_x \Psi^{(1)}_{Rx} + \bar{\Psi}^{(1)}_{Rx} \Phi_x \Psi^{(1)}_{Lx}). \quad (6)$$

The way to implement the decoupling of the replica fermions is based on the use, in the interaction terms, of the component $\Psi^{(1)}$. In momentum space $\Psi^{(1)}$ is given by

$$\Psi^{(1)}(\theta) = F(\theta)\Psi(\theta), \quad F(\theta) = \prod_\mu f(\theta_\mu),$$

$$f(\theta) = \cos\left(\frac{\theta}{2}\right), \quad \theta \in (-\pi, \pi]. \quad (7)$$

Note that $f(0) = 1$, $f(\pi) = 0$. As we see, $\Psi^{(1)}$ is the original $\Psi$ field but modulated by a form factor which is responsible of the decoupling of the replica fermions at the tree level. This is achieved by imposing that the form factor vanishes for the momenta corresponding to an elementary hypercube in the positive directions,

$$\Psi^{(1)}_x = \int_\theta \exp\{i\theta(x + \frac{1}{2} \sum_\mu \hat{\mu})\} F(\theta) \Psi(\theta)$$

$$= \frac{1}{2^d} [\Psi_x + \sum_{n=1}^d \sum_{\mu_1 < \cdots < \mu_n} \Psi_{x+\hat{\mu}_1+\cdots+\hat{\mu}_n}]$$

$$= \frac{1}{2^d} [\Psi_x + \Psi_{x+\hat{1}} + \Psi_{x+\hat{2}} + \Psi_{x+\hat{3}} + \Psi_{x+\hat{4}} + \Psi_{x+\hat{1}\hat{2}} + \Psi_{x+\hat{1}\hat{3}} + \cdots]. \quad (8)$$

All terms in (1) which include the fermion field break the gauge invariance,

$$\Psi_x \rightarrow (\Omega_{Lx} P_L + \Omega_{Rx} P_R) \Psi_x, \quad (9)$$

$$\bar{\Psi}_x \rightarrow \bar{\Psi}_x (\Omega_{Lx}^+ P_R^+ + \Omega_{Rx}^+ P_L), \quad (10)$$

$$U_{Lx\mu} \rightarrow \Omega_{Lx} U_{Lx\mu} \Omega_{Lx+\hat{\mu}}^+, \quad (11)$$

$$U_{Rx\mu} \rightarrow \Omega_{Rx} U_{Rx\mu} \Omega_{Rx+\hat{\mu}}^+, \quad (12)$$

$$\Phi_x \rightarrow \Omega_{Lx} \Phi_x \Omega_{Rx}^+. \quad (13)$$

$I_{INT}(\Psi, U)$ breaks gauge invariance because in $\bar{\Psi}^{(1)}_{Lx} \gamma_\mu U_{Lx\mu} \Psi^{(1)}_{Lx+\hat{\mu}}$ the same gauge variable, $U_{Lx\mu}$, couples $\Psi$’s in different points. The same argument ($U \leftrightarrow \bar{\Psi}$) applies to $I_Y(\Phi, \Psi)$.

As emphasized by the Rome group [7], the model has to be defined in the presence of the gauge fixing and Fadeev–Popov terms, so that one must keep such terms also at the non-perturbative level. At the same time one has to include all the counterterms which correspond to gauge non invariant terms of dimension less or equal to four required in order to recover a gauge invariant theory in the continuum limit.

The consistency of the model has been established at one loop level when the fermion field content is anomaly free [7].

A non-perturbative tuning associated to the quadratically divergent mass term for each gauge field is present due to the non–invariance of the regularization (see L.Maiani, in these proceed-
A difference of our method versus that of Rome is that our fermion sector has a global chiral symmetry which prevents the occurrence of linearly divergent mass counterterms, of the type $\bar{\Psi}\Psi$, for the fermion fields. This reduces the number of non-perturbative tunings needed.

At the same level at which the consistency has been proved (i.e. at one loop), we have enough $F(\theta)$ factors to assure the decoupling of the replica fermions. As we will see in the next section, the best proof of this is based on the Reisz power counting theorem [10] concerning Feynman integrals.

In that section we will state this theorem and it will be used to prove the decoupling of the replica fermions in CYM, a case in which the problems related to the gauge non invariance of our regularization (which prevent us to go beyond one loop) are not present.

Now, we will present an argument of plausibility to argue that the decoupling works also in the non-perturbative regime. This argument is based on a symmetry of our action which is a consequence of the use of the component $\Psi^{(1)}$ in the interaction term. Actually, the action is invariant under the $2^d-1$ transformations of the fermion field,

$$\Psi_x \rightarrow \Psi_x + \epsilon_x^{(i)}; \quad \epsilon_x^{(i)} = e^{i\theta^{(i)}_{\mu}} \epsilon_x \quad (14)$$

$$\bar{\Psi}_x \rightarrow \bar{\Psi}_x + \bar{\epsilon}_x^{(i)}; \quad \bar{\epsilon}_x^{(i)} = e^{i\bar{\theta}^{(i)}_{\mu}} \bar{\epsilon}_x$$

where $\theta^{(i)}_{\mu} = 0, \pi$ and at least one component is different from zero (momenta corresponding to the replica fermions).

An illustration, in two dimensions, of these $2^d-1$ transformations is in figure 1, where the value of the translation in the fermionic field in each point of the lattice is shown for $\theta = (0, \pi)$ and $\theta = (\pi, \pi)$.

The invariance of the interaction terms $I_Y$ and $I_{INT}(\Psi, U)$ follows from the invariance of the $\Psi^{(1)}$ component, entering on these terms, under transformations (14). This invariance results from the fact that the sum of the $\epsilon$'s in an elementary hypercube is zero. By inspection it is straightforward to check that the naive term, $I_F(\Psi)$ in (1), is also invariant under these transformations. In fact it is also invariant under the transformation $\epsilon_{x+\bar{\mu}} = \epsilon_x$.

It is normal to have $2^d-1$ transformations of symmetry which are translations in the fermionic variable if one expects to have $2^d-1$ free massless fermions [9].
As we have said, we expect that, due to this symmetry, the decoupling will work also in the non–perturbative regime.

In fact, in the case of a CYM, it is easy to derive the Ward identities associated to these symmetries [11]. The result, for finite $a$, is:

1) The position of the replica fermions in momentum space is not changed by quantum corrections.

2) The one-particle irreducible fermion Green functions do not see the doublers, in the sense that any 1PI amputated Green function vanishes if at least one external fermion line has $\theta = \theta^{(i)}$.

Consequently, the identification of this symmetry can be used to argue that also in a non–perturbative regime the replica fermions decouple, IF a sensible continuum limit exists with only the terms of the action of the CYM (which are those exhibiting the symmetry). Without this IF the proof of the decoupling in the non–perturbative regime of a CYM would be complete. The IF converts the proof in only “an argument of plausibility”.

Because this transformation of symmetry acts only on the fermion field the former conclusion about the decoupling is extensible to a CGT but now the IF is a stronger assumption. In fact, in the case of a CGT the existence of a sensible continuum limit with only terms of dimension less or equal to four is an open problem, even perturbatively (as only at one–loop level has consistency been proved [7]). To progress on the question of the existence of this limit is one of the points of this Workshop.

This “decoupling symmetry” is in correspondence with the Golterman–Petcher symmetry [9] in the case of the Wilson–Yukawa formulation of the Standard Model which allows one to prove the decoupling of the right–handed neutrino in the continuum limit.

3. CHIRAL YUKAWA MODELS ON THE LATTICE AND DECOUPLING

The simplest fermion–scalar sector of a CGT has the following global $U(1) \times U(1)$ invariant action,

$$I(\Psi, \Phi) = I_B(\Phi) + I_F(\Psi) + I_Y(\Phi, \Psi),$$

where,

$$I_B(\Phi) = - \sum_x \Phi_x^+ \Phi_x$$

$$+ \frac{k}{2} \sum_{x,\mu} (\Phi_x^{+\mu} \Phi_x + \Phi_x^+ \Phi_{x+\mu})$$

$$- \lambda \sum_x (\Phi_x^+ \Phi_x - 1)^2,$$

$$I_F(\Psi) = - a^{d-1} \sum_{x,\mu} (\bar{\Psi}_x \gamma_\mu \Psi_{x+\mu}$$

$$- \bar{\Psi}_{x+\mu} \gamma_\mu \Psi_x),$$

$$I_Y(\Phi, \Psi) = - y \sum_x (\bar{\Psi}_{Lx}^{(1)} \Phi_x \Psi_{Rx}^{(1)$$

$$+ \bar{\Psi}_{Rx}^{(1)} \Phi_x^+ \Psi_{Lx}^{(1)}).$$

Now, we are going to prove, in a weak coupling perturbative analysis, using power counting arguments, that the doublers are decoupled. We would like to emphasize that this will be a rigorous proof (in contrast with the previous, non–perturbative, argument of plausibility).

The proof is based on the Reisz power counting theorem which, essentially, says #2:

#2 The basic idea of the theorem is to bound from below the denominators of the propagators of a lattice Feynman integral. This, as we will see, is not possible for the denominator of the naive propagator (because of the doubling problem).
Theorem

Let $I_F$ be a lattice Feynman integral,

$$I_F = \int d^4k^1 \ldots d^4k^L \frac{V(k, q; a)}{C(k; a)},$$

(19)

where $k^1, \ldots, k^L$ are the loop momenta, $q$'s are the external momenta, $V$ includes all vertex factors and the numerators of the propagators and

$$C(k; a) = \prod_i C_i(k^i; a).$$

$C_i(k^i; a)$ are the denominators of the propagators. $C_i$ is required to have the following properties (these assumptions, less strong than those originally assumed by Reisz, were established by Lüscher [12]):

1. There is a smooth function $G_i$ ($2\pi$-periodic in the momentum $ak^i$) such that

$$C_i(k^i; a) = a^{-2}G_i(ak^i).$$

(20)

2. The continuum limit of $C_i$ exists and is given by

$$\lim_{a \to 0} C_i(k^i; a) = (k^i)^2.$$  

(21)

3. There are two positive constants, $a_0$ and $A$, such that

$$|C_i(k^i; a)| \geq A(\hat{k})^2,$$  

(22)

for all $a \leq a_0$ and all $k^i$, where

$$\hat{k}^i_\mu = \frac{2}{a} \sin \left(\frac{ak^i_\mu}{2}\right).$$

Note that the free naive propagator verifies (20) and (21) but not (22).

It is not worthwhile to specify the requirements $V(k, q; a)$ must satisfy because they are not very restrictive and are satisfied in all (local) lattice models we know of.

Suppose, in addition, that the degree of divergence of $I_F$ [12] is less than zero.

Then, the Reisz power counting theorem, states that the continuum limit of $I_F$ exists and is given by

$$\lim_{a \to 0} I_F = \int d^4k^1 \ldots d^4k^L \lim_{a \to 0} \frac{V(k, q; a)}{\prod_i \lim_{a \to 0} C_i(k^i; a)},$$

(23)

where the integral of the r.h.s. is absolutely convergent.

The important modification incorporated by Reisz to the BPHZL [13] approach consists in replacing Taylor operators by Taylor polynomials in “lattice momenta” $(\sin(qa)/a)$.

Then he shows that the subtractions can be written as counterterm contributions to the lattice action [14]. The method works for fields carrying spin and internal symmetries like colour, etc.

The conclusion is that when the vertices and the propagators of a model in the lattice verify the conditions of the Reisz theorem, then the continuum limit of any Green function calculated perturbatively with the model in the lattice coincides, after renormalization, with the Green function of the model in the continuum (because of the interchange of limits in (23) ).

Next we will see that our lattice vertices and propagators satisfy, in an effective sense, the conditions of the Reisz theorem.

Before doing that, it is enlightening to perform an explicit one-loop computation of the fermion propagator (see appendix of ref. [11]). The result is that the one-loop self-energy vanishes when the external momentum coincide with one of the replicas. Consequently the position of replicas in momentum space are not affected by the quantum corrections. This is an important point if we want that our form factor, $F(\theta)$, will work beyond the tree level. In fact we can see the proof of the decoupling, based on the Reisz theorem, as a powerful way of proving this to all orders...
in perturbation theory.

The reason why the theorem does not work in the case of the naive fermions is that the denominator, $D_F(k, a) = (1/a^2) \sum_\mu \sin^2(ak_\mu)$, of the naive propagator is, for $k_\mu$ near $\pi/a$, of order one (doubling problem) while $A\hat{k}^2$ can be arbitrarily large for $a \to 0$ independently of the value of $A$.

On the contrary, for our regularization this theorem works.

The reason is that, for any Feynman diagram, an internal propagator is always accompanied by two $F(\theta)$ factors coming from the vertices. Then, in the power counting theorem, the naive denominator is replaced by an effective denominator, $\hat{D} = D_F(k, a)/F^2(ak)$, which now can be bounded by $A\hat{k}^2$ for all $k$ \#3. Note that each vertex differs from the naive vertex in two $F(\theta)$ factors, one for each fermion field entering the vertex.

When applied to our case, the result we have obtained (i.e. the continuum limit of any Green function coincides, after renormalization, with the Green function of the model in the continuum limit with only one fermion) means that the replica fermions do not give any contribution to Green functions with finite external momenta.

Also, one can check performing an explicit one loop computation of the fermion propagator, $S(p)$, that the behaviour of $S(p)$ around a point $\bar{p}$ ($\bar{p}$ denotes the position of the replica poles) is the corresponding one to the lattice naive free fermion propagator. In fact, defining $k = p - \bar{p}$, for $k = O(a^0)$ one finds

$$S_{1\text{-loop}}(p) = -i \sum_\mu \gamma_\mu k_\mu \cos(\bar{p}_\mu a) \sum_\mu k_\mu^2 + O(a),$$

which is the result one would expect if the denominator is replaced by an effective denominator, $\hat{D} = D_F(k, a)/F^2(ak)$, which now can be bounded by $A\hat{k}^2$ for all $k$ \#3. Note that each vertex differs from the naive vertex in two $F(\theta)$ factors, one for each fermion field entering the vertex.

The zeros of $F^2(ak)$ kill the zeros in $\sin(ak_\mu)$ for $k_\mu = \pi/a$ and then both terms in (22) are of the same order in $1/a$. Then it is enough to take $A$ sufficiently small to satisfy the bound.

#4 It is easy to check that the generalization of (7) which picks out each replica is,

$$\Psi^{(i)}(\theta) = \prod_\mu \cos(\frac{\theta_\mu + \theta^{(i)}_\mu}{2})\Psi(\theta), \quad i = 1, \ldots, 2^d.$$

For $\theta^{(i)}_\mu = 0$ we obtain (7). Also $\Psi_x = \sum_i \Psi^{(i)}_x$. 

One last comment about the symmetries of the quantum theory defined by our action (15). As our regularization preserves the global $U(1) \times U(1)$ symmetry, this symmetry is realized by the quantum theory. Of course in this realization the massless decoupled replica fermions play an essential rôle. Note that, under the $U(1) \times U(1)$ transformation, “the replica fermions” \#4 transform in the same way as the “interacting $\Phi^{(1)}$ fermion”. Currents $\bar{\Psi}\gamma_\mu \Psi$ and $\bar{\Psi}\gamma_\mu \gamma_5 \Psi$ are conserved as a consequence of cancellation of contributions of the interacting and decoupled replica fermions. But the physical relevant currents, made up from the $\Psi^{(1)}$ component, should not be simultaneously conserved.

Let us finish up with a comment about other regularizations. One can ask himself how the assumptions of the Reisz power counting theorem are satisfied in other regularization approaches. Of course the Wilson propagator of the Rome regularization satisfies those assumptions. The situation is different for the model based on the Wilson–Yukawa term (Smit–Swift model [1,3] for instance). In fact, in the paramagnetic phase with weak Yukawa couplings (PMW) of a CYM, $(\Phi) = 0$, the propagator is the naive one and the vertex, $V$, does not have the adequate behaviour to cause any definition of an effective propagator to satisfy (22). Therefore, in this phase the decoupling is not assured. Actually we know that one finds $2^d$ massless fermions in the PMW phase. In the ferromagnetic phase with weak Yukawa couplings (FMW) one recov-
ers, because $v =: \langle \Phi \rangle \neq 0$, essentially the Wilson propagator BUT, as we approach the phase transition line, $v$ approaches zero, then the Reisz theorem is not satisfied and fermion doubling occurs [1,15,16].

Of course, the nondecoupling effects of the doublers at one loop level found by Dugan and Randall in other regularizations [17], are not present in our case as our decoupling method is not based on massive doubler fermions.

4. THE PHASE DIAGRAM FOR THE $U(1) \times U(1)$ CHIRAL YUKAWA MODEL

For the moment, we have found, in a mean field computation, the phase structure of the model defined by (15) for the case in which $\Phi \in U(1)$ and for small $y$. We hope to complete this phase diagram, soon [18]. For a review of this topic see reference [19]. For a recent comparison of the phase structure of different models see reference [20].

Before showing our results, we must say something about the symmetries of the phase diagram, because they are not exactly the same as in other regularizations and have relevant consequences.

We have some of the usual symmetries. For instance, for $y = 0$ the action is invariant under $k \rightarrow -k, \quad \Phi_x \rightarrow \epsilon_x \Phi_x, \quad \epsilon_x = (-1)^{x_1+x_2+x_3+x_4}$. For $y \neq 0$ it is invariant under

$$\Phi_x \rightarrow -\Phi_x \quad y \rightarrow -y,$$

so we take $y \geq 0$ with no loss of generality.

But we have lost the symmetry $k \rightarrow -k \quad y \rightarrow -iy$, usual in other models [16,19].

In fact, in the implementation of this symmetry, try the term $I_B(\Phi)$ of the action,

$$I_B(\Phi) = \frac{k}{2} \sum_{x,\mu} (\Phi^+_x \Phi_{x+\bar{\mu}} + \Phi^+_{x+\bar{\mu}} \Phi_x),$$

forces one to balance the change in $k$ with the change $\Phi_x \rightarrow \epsilon_x \Phi_x$ (do not forget this), i.e., this implies a change in $I_Y$ which should be balanced, too. Obviously this change can not be balanced with only a change in $y$, we need some change in $\Psi^{(1)}$ also. This change in $\Psi^{(1)}$ must be induced by a change in $\Psi$ which should not produce, in turn, any effect on $I_F(\Psi)$. In other regularizations this is achieved by

$$\Psi_x \rightarrow e^{i\epsilon_x \alpha} \Psi_x, \quad \bar{\Psi}_x \rightarrow e^{i\epsilon_x \alpha} \bar{\Psi}_x, \quad (25)$$

because, then,

$$\bar{\Psi}_x \Psi_{x+\bar{\mu}} \rightarrow e^{i\alpha(\epsilon_x + \epsilon_x + \mu)} \bar{\Psi}_x \Psi_{x+\bar{\mu}} = \bar{\Psi}_x \Psi_{x+\bar{\mu}}, \forall \alpha.$$

Usually, in other regularizations, there is a value of $\alpha(\pi/4)$ and a change in $y$ ($\rightarrow -iy$) which keeps $I_Y$ unchanged. But, in our case, in

$$\bar{\Psi}^{(1)}_x \Phi^{(1)}_x = \frac{1}{28} \left( \bar{\Psi}_x + \bar{\Psi}_{x+1} + \bar{\Psi}_{x+2} + \ldots \right) \Phi_x$$

$$\left( \Psi_x + \Psi_{x+1} + \Psi_{x+2} + \ldots \right),$$

there are terms, such as $\bar{\Psi}_x \Phi_x \Psi_{x+1}$, which stay unaffected by (25), but which change when $\Phi_x \rightarrow \epsilon_x \Phi_x$. Therefore, in our case, we do not have the symmetry $k \rightarrow -k, \quad y \rightarrow -iy$.

The consequence of this fact is that “the phase transition line PM–AFM is not determined by the phase transition line FM–PM”, as usually happens [16]. In our model, a simple mean field computation yields that both lines meet in a point. We will return to this point later on. Then, it is also possible that the mean field approach yields a ferrimagnetic phase to the right of this point, making it a quadruple point [18] (an interesting possibility already found in numerical simulations [16,19]).
Also concerning the limiting case $y \to \infty$, we have a peculiarity similar to that in the Chiral Yukawa model with hypercubic coupling [21]. In fact, a rescaling of the fermion field $\Psi \to (1/\sqrt{y})\Psi$, suppresses, as always, the fermion kinetic term but, in our case, the term $I_Y$ also couples fermions in different points (because of the splitting in $\Psi^{(1)}$) so that the fermion can now propagate and it does not decouples in the limit $y \to \infty$. So, the phase diagram is not, in this limit, that of a pure $\Phi^4$ model.

The next step is to make a mean field analysis of the phase diagram.

The more significant characteristic of our mean field computation are:

1) We have taken the saddle point approach to the mean field calculation [22].

2) To handle the four fermion interaction which occurs when developing, for small $y$, the term $e^{-I_Y}$, we must, before doing the integration over the fermi fields, decouple the composite field $\bar{\Psi}(x)\Psi(x)$ by using the identity [23]

$$\exp\left\{\frac{1}{2}(\bar{\Psi}_x\Psi_x)^2V^{-1}\right\} = [\text{det}(V_x)]^{\frac{1}{2}} \times \int \frac{d\lambda(x)}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\lambda(x)^2V_x + \lambda(x)\bar{\Psi}_x\Psi_x\right\}.$$ 

The phase diagram for small $y$ is shown in figure 2.

The FM–PM phase transition is second order and the transition line is,

$$2k_c d = 1 - \bar{y}^2 2^4 \int \frac{d^4 \theta}{(2\pi)^d} \frac{F^4(\theta)}{\sum_\lambda \sin^2 \theta_\lambda}.$$ 

For $d = 4$,

$$\int \frac{d^4 \theta}{(2\pi)^4} \frac{F^4(\theta)}{\sum_\lambda \sin^2 \theta_\lambda} = 1.61 \times 10^{-2}. \quad (26)$$

The PM–AFM phase transition is also second order and the transition line is,

$$2k_c d = -1 - \bar{y}^2 2^4 \int \frac{d^4 \theta}{(2\pi)^d} \frac{F^2(\theta)F^2(\theta + \pi)}{\sum_\lambda \sin^2 \theta_\lambda}.$$ 

Fig. 2. Phase diagram for the $U(1) \times U(1)$ chiral Yukawa model for small $y$ and $\Phi \in U(1)$.

For $d = 4$,

$$\int \frac{d^4 \theta}{(2\pi)^4} \frac{F^2(\theta)F^2(\theta + \pi)}{\sum_\lambda \sin^2 \theta_\lambda} = 8.4 \times 10^{-5}, \quad (27)$$

so that $k_c$ changes very little with $y$.

The reason why these two lines meet in a point is that in (26) $F^4(\theta)$ kills the most important contribution of the replica while in (27) $F^2(\theta)F^2(\theta + \pi)$ kills the replica AND the “normal” fermions, which implies a very small value for the integral.

In the naive case,

$$F^4(\theta) = F^2(\theta)F^2(\theta + \pi) = 1,$$

we recover the $k \to -k \quad y \to -iy$ symmetry and both lines are parallel.

Note that, in the regularized theory, the value of $\bar{y}$ in which the two lines meet decreases as the width of the form factor, $F(\theta)$, decreases. Of course, in a hypothetical scaling limit the phys-
ical quantities should be the same for any form factor which kills the replicas.

Now, we are studying the intermediate and the large $y$ values considering, in our mean field analysis, the general situation of a ferrimagnetic phase. To solve the saddle point equations in this general situation is rather involved [18].

5. OUTLOOK

Almost everything is still to be done:

i) Estimation of the upper bounds on fermion masses (and comparison with the symmetry breaking scale). This question may be relevant to the real world. From LEP data and comparison with radiative corrections in the SM one concludes that if $m_t < 250$ GeV (to make sure that the one loop calculations are dominant) then $100 \text{GeV} < m_t < 150 \text{ GeV}$. However those perturbative analysis are not valid for very heavy top. In fact, for $y_t^2/(4\pi^2) > 1$ the contribution to $\Delta \rho$ ($\rho = 1 + \Delta \rho$; $\rho$ measures the relative strength of charged-current and neutral-current effective couplings) beyond the perturbative regime is not known. Then, the present precision weak interaction measurements could be compatible with very big values of $m_t$.

At this point a comment on the decoupling theorem of Appelquist and Carazzone [24] is pertinent.

This theorem does not hold in the PERTURBATIVE regime of gauge theories with spontaneous symmetry breaking (SSB). This happens because of the presence of large Yukawa couplings which grow with heavy fermion masses. In a way, this is nice because it allows us to see the effect of heavy fermions at low energies. But, on the other hand, it is surprising that a fermion of, for instance, infinite mass may have significance in real life. Perhaps the decoupling theorem of Appelquist and Carazzone also holds in the presence of SSB but, in order to notice it, one must go beyond the perturbative regime.

Well, we hope to compute the contributions to $\Delta \rho$ which are responsible for the perturbative bounds on $m_t$ beyond the perturbative regime, to see if these contributions grow up or go down with $y_t$.

ii) Of course, one must study the phase diagram for other CYM, as $SU(2) \times SU(2)$, for instance.

iii) The bounds on the Higgs mass should also be reexamined in the presence of heavy fermions.

iv) Also the connection of CYM and four fermion interaction models should be investigated.

v) At the level of a CGT all we know is that the one loop perturbative analysis from the Rome approach can be directly applied to our case: everything else remains to be done. Nevertheless we hope that the global chiral symmetry present with this action will make things a little easier.

Acknowledgements

We wish to thank the organizers for giving us the opportunity to participate in this very stimulating workshop. We thank to D. Espriu and B. Grinstein for very interesting discussions. This work was partially supported by the CICYT (proyecto AEN 90-0030).

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