1. Girders of modern truss bridge
Modern bridge truss girders are mainly “Warren” trusses – “W” bracing layout (Fig. 1). This arrangement is economic in terms of self-weight and assembly (Ryżyński et al. 1985). Large spacing of flange nodes, reaching 15 m (Siekierski 2010a), is possible due to deck construction. Cross beams transfer loads to truss flange nodes and to truss flange members, between nodes. Truss girder flange, loaded in this way, carries substantial bending moments. Simultaneously, large axial forces are caused by truss action. Such flange is often referred to as “rigid flange”. The idea is also utilized in trussed decks of modern cable-stayed bridges (Nan et al. 2014; Zhang et al. 2011).

Rigid flanges have usually I cross-section (Alkhafaji et al. 1998), shown in Fig. 2, or box cross-section (Ahlgrimm, Lohrer 2005; Gao 2012; Reintjes, Gebert 2006; Reintjes 2009). Bracing members are connected to gusset plates that are welded to top plates of the I-section or those are inserts in box side walls.

Rigid flange described above may be considered as a continuous beam stiffened (and strengthened) by truss. It is also referred to as a braced beam (bracing in the plane of bending) (Megson 2005). Usually the system is analysed as a beam combined with auxiliary hinged-node truss as its bottom flange (Goremikins, Serduiks 2010a), top flange (Goremikins, Serduiks 2010b) or a member of inverted cable-stayed system (Gesualdo et al. 2014). However, in

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**Fig. 1.** Scheme of Warren truss with rigid bottom flange (cross beams are shown)

**Fig. 2.** Truss rigid flange of “I” cross-section during bridge refurbishment (RC slab removed); cross beam spacing is four times smaller than flange node spacing
the case of truss rigid flange, all nodes are rigid and bracing members carry bending moments.

Since rigid flange carries substantial bending moments, its preliminary design cannot be accomplished as in the case of classic truss flange that is loaded only at nodes (Caglayan et al. 2012). Thus, the method of analysis of isolated rigid flange is suggested. This approach may be applicable also for verification of results of other analysis methods.

The main advantage of the method is that modelling of the whole truss girder (Miyachi et al. 2012) is not necessary. Instead, a continuous beam with appropriate boundary conditions is analysed. This approach enables better understanding of what factors influence stress level in rigid flange members and clear assessment of the extent of this influence.

2. Analysis of rigid flange

2.1. Analysis overview

As mentioned above there are two internal forces that are crucial for assessment of rigid flange load carrying capacity, i.e. axial forces and bending moments. They may be computed separately, in two stages:

– stage I: analysis of axial forces distribution,
– stage II: analysis of bending moments distribution.

The analysis involves equivalent model of truss girder as simply supported beam (as auxiliary model) and model of isolated rigid flange as continuous beam (main model).

2.2. Stage I of analysis

Stage I is aimed at computation of axial forces in rigid flange. The following assumptions are made:

– truss girder is a plane hinged truss,
– loads are applied to truss nodes.

Preparation for this stage consists of replacement of actual loads along rigid flange (forces at cross beam joints) with equivalent loads applied at flange nodes. Then any analytical method of truss solving can be applied. Axial forces may be computed using method of joints or method of sections (Megson 2005). Method of sections (Ritter’s method) is advised since it is more efficient.

Axial forces should be computed with regard to actual height of truss girder – distance between top and bottom flange axes. Results of stage I are the axial forces in rigid flange.

It is possible that there are eccentricities of actual flange axis with respect to theoretical axis (at rigid flange to cross bracing connections). In such case, additionally, axial forces are to be computed with regard to theoretical rigid flange axis. This must be done for the sake of stage II analysis.

2.3. Stage II of analysis

Stage II concerns computation of bending moments in rigid flange. The following are regarded:

– actual values and location of forces transferred from cross beams – according to actual cross beam layout,
– actual eccentricities of rigid flange axis at nodes – theoretical truss nodes may be situated away from truss flange neutral axis, to reduce size of gusset plates (Siekerski 2010b).

Continuous beam model of rigid flange is assumed. The beam intermediate supports are elastic in terms of rotation and vertical displacement. Location of supports complies with location of flange nodes. The supports are described by coefficient of vertical elasticity ($k_u$) and coefficient of rotational elasticity ($k_\phi$).

The computational model for the stage II is suggested in Fig. 3. Symbol description:

– 0, 1, 2, … – "m" – subsequent nodes of rigid flange („0” – node over bearing);
– $D_1$, $D_2$, $D_p$ – members of rigid flange;
– $M_{0p}$, $M_1$, $M_2$, … – concentrated bending moments caused by forces transferred from cross bracing members (if eccentricities exist); the moments are computed as multiplication of axial forces at the level of theoretical axis of rigid flange and eccentricities of actual axis of rigid flange;
– $k_{\phi 0}$, $k_{\phi 1}$, $k_{\phi 2}$, … – coefficients of rotational elasticity of supports 0, 1, 2, …;
– $k_{u0}$, $k_{u1}$, $k_{u2}$, … – coefficients of vertical elasticity of supports 0, 1, 2, ….

2.3.1. Coefficient of rotational support elasticity

It is assumed that rigid flange rotation at its nodes is restrained by flexural stiffness of cross bracing members connected to considered node. The boundary conditions at opposite ends of cross bracing members depend on the other flange flexural stiffness in comparison to stiffness of cross bracing members. In the case of relatively small stiffness of the other flange (common case), hinged connection may be assumed – Fig. 4. Otherwise, fixed connection
should be considered. In the case of hinged connection coefficient of rotational elasticity of support equals:

\[ k_{qm} m = \frac{M_m}{\varphi_m} = \sum_{i=1}^{n} \frac{3EJ_{Bi}}{L_{Bi}}, \quad (1) \]

where \( k_{qm} m \) – coefficient of rotational elasticity support under node “m” (bending moment generated at elastic support “m” by its unit rotation), kNm; \( n \) – number of cross bracing members connected to node “m” (usually \( n = 2 \); \( n = 1 \) over bearing); \( L_{Bi} \) – theoretical length of cross bracing member “i”, connected at node “m”; \( m \); \( E \) – elastic modulus for cross bracing members, kPa; \( J_{Bi} \) – moment of inertia in bending for cross bracing member “i”, connected at node “m”, m4.

In the case of fixed connection of cross bracing members to the other flange, coefficient of support elasticity due to rotation equals:

\[ k_{qm} = \frac{M_m}{\varphi_m} = \sum_{i=1}^{n} \frac{6EJ_{Bi}}{L_{Bi}}. \quad (2) \]

### 2.3.2. Coefficient of vertical support elasticity

Firstly, moments of inertia of rigid flange (\( J_{flange} \)) and truss girder (\( J_{TR} \)) are compared. Assume that rigid flange has bisymmetrical cross-section and negligible web area, shown in Fig. 5.

Geometrical characteristics of flange cross-section are:

\[ A_{flange} = 2tb, \quad (3a) \]
\[ J_{flange} \approx 2tb\left(\frac{h}{2}\right)^2 = 0.5tbh^2. \quad (3b) \]

Geometrical characteristics of truss girder cross-section are calculated assuming:

– actual distance between flange axes (\( H \));
– equal area of flanges;
– 30% reduction of moment of inertia due to cross bracing elasticity (multiplier 0.7).

Characteristics of truss girder cross-section are:

\[ A_{truss} = 2(2tb), \quad (4a) \]
\[ J_{truss} = 0.7 \cdot 2tb\left(\frac{H}{2}\right)^2 = 0.7tbH^2, \quad (4b) \]

where \( H \) – actual height of truss girder, m.

Height of rigid flange (\( h \)) is not related to truss girder height (\( H \)). However, inspection of existing bridge spans (Siekerski 2010a) shows that ratio \( H/h \) is at least 5. So:

\[ \frac{J_{flange}}{J_{truss}} \leq \frac{0.5tbh^2}{0.7tb(5h)^2} = \frac{1}{35}. \quad (5) \]

It can be seen that \( J_{flange} < 0.03J_{truss} \).

Vertical displacements of rigid flange nodes depend on flexural rigidity of truss girder. To provide that rigidity in the model of isolated rigid flange as a continuous beam, elastic supports must have large coefficients of elasticity in comparison to flexural rigidity of the flange. On the other hand, rigid flange, as a member of the girder, deflects always over whole span length, no matter if the loading is distributed or concentrated. Assumed model of isolated rigid flange as a continuous beam on elastic supports of large coefficient of elasticity cannot replicate such behaviour.

However, it is possible to keep the independence of flange deflection and flange loading in the model of isolated rigid flange as a continuous beam on elastic supports. To achieve this, coefficients \( k_{um} \) must depend on loading intensity and distribution. In this situation, it is convenient to replace concept of elastic vertical supports (under nodes of rigid flange) by concept of imposed vertical displacements of the nodes. The displacements depend on loading and truss girder flexural stiffness. However, to find the displacements, truss girder modelling is not necessary.

To find vertical displacement of rigid flange nodes under given loading a model of truss girder as a beam of equivalent moment of inertia is analyzed. Flexural stiffness of the truss girder is:

\[ J = J_{TF} + A_{TF}a_{TF}^2 + J_{BF} + A_{BF}a_{BF}^2, \quad (6) \]

where \( J \) – moment of inertia of truss girder, m4; \( J_{TF}, J_{BF} \) – moment of inertia of top (\( TF \)) and bottom (\( BF \)) flanges respectively, m4; \( A_{TF}, A_{BF} \) – cross-sectional area of top (\( TF \)) and bottom (\( BF \)) flanges respectively, m2; \( a_{TF}, a_{BF} \) – distances between centre of gravity of truss girder cross-section and centre of gravity of top (\( TF \)) and bottom (\( BF \)) flange cross-section respectively, m.

Bracing members of trusses provide smaller shear stiffness in comparison to plate girders. Hence, effective moment of inertia of truss girder is smaller than calculated above. The difference depends on truss static scheme – type of supports and loading. In the case of bridge truss girders, a simply supported and uniformly loaded beam may be assumed as representative scheme. On the basis of (Pałkowski 2001), the moment of inertia of a truss girder respecting its actual shear stiffness is:

\[ J_{TR} = \frac{J}{1 + \frac{48}{5} \frac{EJ}{S_{V}L_{truss}^2}}, \quad (7) \]

where \( J_{TR} \) – truss girder moment of inertia respecting actual shear stiffness, m4; \( E \) – elastic modulus for steel, kPa;
where $\alpha$ – an angle between bracing members and horizontal plane, (rad); $A_B$ – area of bracing member, m². In the case of various bracing members, the average area may be assumed.

Displacements of rigid flange nodes under given loading (distributed or/and concentrated) may be computed as superposition of effects of several concentrated loads on simply supported beam of $J_{TR}$ moment of inertia.

Vertical displacement $u_m$ of flange node “$m$” equals (Fig. 6):

\[
\begin{align*}
\text{if } x_m & \leq x_P: \\
& \frac{P (L_t - x_P) x_m}{6E J_{TR} L_t} \left( L_t^2 - (L_t - x_P)^2 - x_m^2 \right), \\
\text{if } x_m & > x_P: \\
& \frac{P x_P (L_t - x_m)}{6E J_{TR} L_t} \left( L_t^2 - x_P^2 - (L_t - x_m)^2 \right),
\end{align*}
\]

where $x_m$ and $x_P$ are ordinates of node “$m$” and concentrated load $P$ along rigid flange respectively, m. They are explained in Fig. 6.

Thus all date necessary to create the model of isolated rigid flange as continuous beam are found (supports at flange nodes).

3. Working example

3.1. Test loading

Four truss girders of twin railway truss bridges were tested. The scheme of truss girder is shown in Fig. 7. Member symbols and cross beam numbering are given.

Geometrical characteristics of truss girders are put together in Table 1. For member symbols see Fig. 7. The explanation of indexes: 1) half of element near D11, 2) half of element near D13, 3) half of element near D13, 4) half of element near D21, 5) half of element near D14, 6) half of element near D22.

The same locomotive set was used for both spans (Fig. 8). In the case of both spans, railway track is located symmetrically between truss girders. It may be concluded that four truss girders were test loaded in the same way.

During testing the following were recorded:

- vertical displacement of flange nodes “1”, “2”, “3” (Fig. 7), under loading scheme as in Fig. 8;
- strains at the top of truss bottom flanges at cross-section located 3.5 m away from the node “1” (Fig. 7), towards midspan, during locomotive set crossing the span at very low speed ($\leq 5$ km/h) – quasi-static loading.

3.2. Rigid flange modelling

To find the efficiency of described rigid flange modelling, two computational models were created. Computer programme Robot was used for this purpose. The analyzed models were:

- model of truss girder (model “T”) with rigid flange as its part – Fig. 9,
3.3. Analysis results

Table 5 shows recorded and computed displacements. Recorded values are mean values for population of four truss girders. Positive sign marks displacement downwards.

It can be seen that accuracy of displacement assessment is similar for both models. Recorded values are smaller than computed ones because bridge deck, disregarded in both models, enhances span flexural stiffness in reality.

Symbols D1, D2, D3, D4 in Figs 9–10 mark rigid flange members.

Both models regard eccentricities in connections of cross bracings members to bottom flange and disregard existence of bridge deck. Both models were used to assess internal forces and deflection under test loading (Fig. 8).

Characteristics of structural elements were assumed according to design specifications. The characteristics are put together in Table 1. Symbols are explained in Fig. 7.

Since both models do not concern bridge deck, the forces shown in Fig. 8 have to be replaced by equivalent loading applied at cross beam to truss girder connections. Results are given in Table 2.

Model “T” was analyzed. Internal forces in rigid flange as well as vertical displacements of rigid flange nodes were computed.

For the sake of stage I of analysis of isolated rigid flange, loads in Fig. 8 were redistributed to rigid flange nodes. Results are given in Table 3.

Then axial forces in flange members were computed using section method. Due to eccentricity of rigid flange actual axis in respect to theoretical axis, the axial forces were calculated at actual and theoretical levels of axis of rigid flange members. Taking into account the values of axial forces at theoretical levels of axis of rigid flange members (eccentricity 0.85 m), concentrated bending moments were calculated at actual and theoretical levels of axis of rigid flange (stage I of analysis). They are shown in Table 4.

Then coefficients of rotational elasticity of supports due to cross bracing flexural rigidity were computed (kint in Fig. 10). Hinged connection to the other flange was assumed (Eq (1)). Equivalent model of truss girder as simply supported beam was used to compute flange nodal displacements (Eq (9)). Loads as in Table 3 were used for this purpose. Computed displacements were then imposed in the model of isolated rigid flange (u_{m} in Fig. 10) – the analysis was carried out in compliance with the principles stated above. Data for computational model of stage II are given in Table 4. Sign convention of bending moment refers to Fig. 10.

Table 4. Data for computation of bending moments in rigid flange (stage II of analysis)

| Node number (Fig. 10) | Nodal load, kN | Coefficient of rotational elasticity of support k_{f} kN/m/rad | Imposed vertical displacement u_{m}, m |
|-----------------------|----------------|---------------------------------------------------------------|-------------------------------------|
| “0”                   |                |                                                              |                                     |
| “1”                   |                |                                                              |                                     |
| “2”                   |                |                                                              |                                     |
| “3”                   |                |                                                              |                                     |
| “4”                   |                |                                                              |                                     |

Table 5. Comparison of displacements of rigid flange nodes

| Source | “1” | “1” | “2” | “2” | “3” | “3” |
|--------|-----|-----|-----|-----|-----|-----|
| rec.   | 8.16| 1.00| 11.78| 1.00| 8.07| 1.00|
| “T”    | 10.26| 1.26| 14.56| 1.24| 10.12| 1.25|
| “F”    | 10.75| 1.32| 15.16| 1.29| 10.75| 1.33|
| “F”/“T”| 1.05| –   | 1.05| –   | 1.06| –   |

Note: rec. – recorded; “T”, “F” – model symbols.
length. In both models locations of extreme mid-span bending moment were the same.

It can be seen that differences in results provided by two analysed models are limited to 9%. Model "F" seems to be as adequate as model "T" in terms of preliminary rigid flange analysis.

Average strains recorded at the top of truss bottom flanges during locomotive set crossing the span at a very low speed (≤5 km/h) represent normal stress of about 8.9 MPa. The stress values based on both models analysis of axial forces and then computation of bending moments.

Comparison of internal forces in rigid flange based on model "T" and "F" analyses

| Location | Axial force, kN | Bending moment, kNm |
|----------|----------------|---------------------|
|          | "T" | "F" | "T" | "F" | "T" | "F" | "T" | "F" |
| 0"       | 443 | 418 | 0.94 | 511 | 526 | 1.03 |
| 0–1"     | 988 | 987 | 1.00 | 428 | 425 | 0.99 |
| 1"       | 973 | 986 | 1.01 | 146 | 159 | 1.09 |
| 2"       | 429 | 417 | 0.97 | 338 | 356 | 1.05 |
| 3–4"     | –   | –   | –   | –382| –413| 1.08 |

4. Conclusions

1. A rigid flange of a truss girder may be analysed separately taking into account appropriate boundary conditions.

2. The analysis may be carried out in two stages: computation of axial forces and then computation of bending moments.

3. Computational model of a rigid flange may be assumed as continuous beam. Rotational elasticity of supports and imposed vertical support displacements are additional boundary conditions.

4. The model of an isolated rigid flange (model "F") and the model of a truss girder as plane frame (model "T") offer similar accuracy of assessment of internal forces and vertical displacements distribution in rigid flange. The values if internal forces, obtained from the "F" model analysis, are 0.94÷1.09 of respective results obtained from the "T" model analysis. The values of vertical displacements of rigid flange nodes, obtained from "F" model analysis, are 1.05 of respective results obtained from the "T" model analysis.

5. Presented approach to rigid flange modelling may be applied in preliminary design, in investigation on factors influencing rigid flange behaviour and for verification purposes.

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