On the nature of mass-energy constituents of the universe

V.E. Kuzmichev, V.V. Kuzmichev

Bogolyubov Institute for Theoretical Physics, National Academy of Sciences of Ukraine, Kiev, 03143 Ukraine

Abstract

On the basis of our quantum cosmological approach we show that there can be two previously unknown types of collective states in the universe. One of them relates to a gravitational field, another is connected with a matter (scalar) field which fills the universe on all stages of its evolution. The increase in number of the quanta of the collective excitations of the gravitational field manifests itself as an expansion of the universe. The collective excitations of the scalar field above its true vacuum reveal themselves mainly in the form of dark matter and energy. Under the action of the gravitational forces they decay and produce the non-baryonic dark matter, optically bright and dark baryons. We have calculated the corresponding energy densities which prove to be in good agreement with the data from the recent observations.

1. Introduction

The recent astrophysical observations provide the strong empirical evidence that the total energy density of the universe equals to $\Omega_0 = 1$ to within 10% (in units of critical density) [1, 2]. At the same time the mean matter density consistent with big-bang nucleosynthesis is estimated by the value $\Omega_M \approx 0.3$ [2, 3, 4]. It is assumed that the density $\Omega_M$ is contributed by the optically bright baryons ($\Omega_* \approx 0.003$ [3, 5, 6]) and dark matter which consists of the dark baryons ($\Omega_{DB} \approx 0.04$) and matter of uncertain origin and composition (known as cold dark matter, $\Omega_{CDM} \approx 0.3$). The contribution from neutrinos $\Omega_\nu$ depends on the neutrino mass and its lower limit is comparable with $\Omega_*$ [3, 4]. The lack of density $\Omega_X \approx 0.7$, where $\Omega_X = \Omega_0 - \Omega_M$, is ascribed to so-called dark energy [3, 4, 7, 8]. Its nature is unknown and expected properties are unusual. This dark energy is unobservable (in no way could it be detected in galaxies), spatially homogeneous and, as it is expected, it has large, negative pressure. The last property should guarantee an agreement with the present-day accelerated expansion of the universe observed in the type Ia supernova Hubble diagram [9, 10].

Thus modern cosmology poses the principle question about the nature of the components of our universe and their percentage in the total energy density. In this paper we make an attempt to explain the matter content of our universe on the basis of the conjuncture that the collective excitations of previously unknown type exist in it. At the heart of this approach we put the quantum cosmology formulated in [11, 12, 13, 14, 15]. It is shown that there are two types of collective states in the quantum universe. One of them relates to a gravitational field, another is connected with a matter (scalar) field which fills the universe on all stages of its evolution. The quantization of the fields which describe the geometrical and matter properties of the universe is made. We demonstrate that in the early epoch the non-zero energy of the vacuum in the form of the primordial
scalar field is a source of the transitions of the quantum universe from one state to another. During these transitions the number of the quanta of the collective excitations of the gravitational field increases and this growth manifests itself as an expansion of the universe. The collective excitations of the scalar field above its true vacuum reveal themselves mainly in the form of dark (nonluminous) matter and energy. Under the action of the gravitational forces they decay and produce the non-baryonic dark matter, optically bright and dark baryons and leptons. For the state of the universe with large number of the matter field quanta the total energy density, density of (both optically bright and dark) baryons, and density of non-baryonic dark matter are calculated. The theoretical values prove to be in good agreement with the data from the recent observations in our universe.

Below we use dimensionless variables. The modified Planck values are chosen as units of length \( l = \sqrt{2G/(3\pi)} = 0.74 \times 10^{-33} \text{ cm} \), mass/energy \( m_{Pl} = \sqrt{3\pi/(2G)} = 2.65 \times 10^{19} \text{ GeV} \), and time \( t_{Pl} = \sqrt{2G/(3\pi)} = 2.48 \times 10^{-44} \text{ s} \), where \( \hbar = c = 1 \). The unit of density is \( \rho_{Pl} = 3/(8\pi G l^2) = 9/16 \tilde{\rho}_{Pl} \), where \( \tilde{\rho}_{Pl} \) is the standard Planck density. The scalar field \( \phi \) is measured in units \( \phi_{Pl} = \sqrt{3/(8\pi G l)} = l\sqrt{\rho_{Pl}} \), while the potential \( V(\phi) \) in \( \rho_{Pl} \).

2. Quantum cosmology

2.1 Equation of motion

Let us consider a homogeneous, isotropic and closed universe filled with primordial matter in the form of the uniform scalar field \( \phi \). The potential \( V(\phi) \) of this field will be regarded as a positive definite function of \( \phi \). The quantum analog of such universe is described by the equation \[ 2iT\Psi = \left[ \partial_a^2 - \frac{2}{a^2} \partial_\phi^2 - U \right] \Psi , \] where its wavefunction \( \Psi(a,\phi,T) \) depends on the scale factor \( a \) (\( 0 \leq a < \infty \)), the scalar field \( \phi \) (\( -\infty < \phi < \infty \)), and the time variable \( T \) (\( -\infty < T < \infty \)). The last is related to the synchronous proper time \( t \) by the differential equation \( dt = a dT \). The functional \[ U = a^2 - a^4 V(\phi) \] plays the role of an effective potential of the interaction between the gravitational and matter fields. It should be emphasized that (1) is not an ordinary Schrödinger equation with a potential (2). It is a constraint equation for the wavefunction (see e.g. [16][17]). The famous Wheeler-DeWitt equation for the minisuperspace model (the wavefunction depends on the finite number of variables) is the special case (\( \partial_T \Psi = 0 \)) of the more general equation (1). The discussion of the possible solutions of the Wheeler-DeWitt equation, its interpretation, and the corresponding references can be found in [18][19]. In the region \( a^2V > 1 \) the solutions of the Wheeler-DeWitt equation and the solutions of (1) as well can be associated with the exponential expansion of the universe [11][15][20].

A general solution of (1) can be represented by a superposition \[ \Psi(a,\phi,T) = \int_{-\infty}^{\infty} dE \, e^{\frac{i}{\hbar}FET} \, C(E) \, \psi_E(a,\phi) \]
of the states $\psi_E$ which satisfy a stationary equation

\[
\left( -\partial_a^2 + \frac{2}{a^2} \partial_\phi^2 + U - E \right) \psi_E = 0.
\] (4)

Here $E$ is the eigenvalue. It has a physical dimension of action, while in (4) it is written in units $\hbar/2$. The function $C(E)$ characterizes the distribution in $E$ of the states of the universe at the instant $T = 0$.

Let us compare (1) and (4) with the corresponding Schrödinger equations. It is well known that when a Hamiltonian does not depend explicitly on the time variable the quantum system is described by the states with the time-dependence in the form $\exp(-iEt)$. The time variable $t$ and the classical energy $\mathcal{E}$ can be related with each other through the formal transformation $\mathcal{E} \rightarrow i\partial_t$ \cite{21, 22}. In the case of quantum cosmology there is an analogous formal transformation between the classical parameter $E$, which enters into the energy-momentum tensor of radiation

\[
\tilde{T}_0^0 = \frac{E}{a^4}, \quad \tilde{T}_1^1 = \tilde{T}_2^2 = \tilde{T}_3^3 = -\frac{E}{3a^4},
\]

and the time variable $T$, $E \rightarrow -2i\partial_T$ (minus originates from the specific character of the gravitational problem, while the additional factor 2 results from the choice of units). This analogy demonstrates that the role, played in ordinary quantum mechanics by the time variable $t$, is assumed by the variable $T$. Thus in the framework of the minisuperspace model the equation (1) solves the problem of correct definition of the time coordinate in quantum cosmology. The role of time in quantum gravity and the difficulties with the introduction of time variable suitable for the description of the dynamics in quantum cosmology are elucidated in \cite{17, 18}.

The effective potential $U$ has the form of barrier of the finite width and height in the variable $a$. It is convenient to represent the solution of (4) in the form

\[
\psi_E(a, \phi) = \int_{-\infty}^{\infty} d\epsilon \varphi_\epsilon(a, \phi) f_\epsilon(\phi; E),
\] (6)

where the function $\varphi_\epsilon$ satisfies the equation

\[
\left( -\partial_a^2 + U - \epsilon \right) \varphi_\epsilon = 0.
\] (7)

The eigenvalue $\epsilon$ and eigenfunction $\varphi_\epsilon$ depend parametrically on $\phi$. The functions $\varphi_\epsilon$ describe the states of the continuous spectrum. Using the explicit form of the solution of (7) in the region $a^2V \gg 1$ \cite{11, 15}, one can obtain the normalization condition for the functions $\varphi_\epsilon$

\[
\int_0^\infty da \varphi_\epsilon^*(a, \phi) \varphi_{\epsilon'}(a, \phi) = \delta(\epsilon - \epsilon').
\] (8)

Then the function $f_\epsilon$ will satisfy the equation

\[
\partial_\phi^2 f_\epsilon + \int_{-\infty}^{\infty} d\epsilon' K_{\epsilon\epsilon'} f_{\epsilon'} = 0,
\] (9)
where the kernel is

\[
K_{\epsilon'}(\phi; E) = \int_0^\infty da \varphi_e^* \partial_\phi^2 \varphi e + 2 \int_0^\infty da \varphi_e^* \partial_\phi \varphi_e \partial_\phi \\
+ \frac{1}{2} (\epsilon' - E) \int_0^\infty da \varphi_e^* a^2 \varphi e.
\]

This equation can be reduced to a solvable form if one defines concretely the problem. In order to find the function \(\psi_E\), the equation (10) must be supplemented with the boundary condition.

### 2.2 Scenario of the origin of quantum universe

From (4) and (7) it follows that these equations coincide at \(a^2 \to \infty\). It means that in the asymptotic region \(a^2 \gg 1\) the wavefunction \(\psi_E\) can be taken in the form of the function \(\varphi\) (or superposition of such functions) which in turn can be chosen as the sum of the wave \(\varphi_{\epsilon^-(+)}\) incident upon the barrier \(U\) and the outgoing wave \(\varphi_{\epsilon^+(+)}\). The amplitude of the scattered wave has the poles in the complex plane of \(\epsilon\) at \(\epsilon = \epsilon_n + i \Gamma_n\), where \(\epsilon_n\) and \(\Gamma_n\) are the positions and widths of the quasistationary levels with the numbers \(n = 0, 1, 2 \ldots\)

Just as in classical cosmology which uses a model of the slow-roll scalar field \([19, 23]\), in quantum cosmology based on (4) it makes sense to consider a scalar field \(\phi\) which slowly evolves into its true vacuum state, \(V(\phi_{\text{vac}}) = 0\), from some initial state \(\phi = \phi_{\text{start}}\), where \(V(\phi_{\text{start}}) \sim \rho_{\text{Pl}}\). The latter condition makes it possible to consider the evolution of the universe in time in the classical sense. This allows to describe the process of formation of quantum universe in time (see below).

For the slow-roll potential \(V\), when \(|d \ln V/ d\phi|^2 \ll 1\), the function \(\varphi_\epsilon\) describes the universe in the adiabatic approximation. In this case one can separate the slow motion (with respect to the variable \(\phi\)) from the rapid one (with respect to the variable \(a\)) in the universe. This means that the universe expands (or contracts) more rapidly than the state of matter has time to change.

The calculations demonstrate \([11, 12, 15]\) that the first \((n = 0)\) quasistationary state appears when \(V\) decreases to \(V_{in} = 0.08 = 0.045 \rho_{\text{Pl}}\). It has the parameters: \(\epsilon_0 = 2.62 = 1.31h\), \(\Gamma_0 = 0.31 = 1.25 \times 10^{43} \text{s}^{-1}\). The lifetime of the universe in this state, \(\tau = 0.8 \times 10^{-43} \text{s}\), is three times greater than the Planck time \(t_{\text{Pl}}\). The instant of the origination of the first quasistationary state can be taken as a reference point of time against the scale of \(T\).

A further decrease of \(V\) leads to an increase of the number of quasistationary states of the universe. The levels which have emerged earlier shift towards the oscillator values \(\epsilon_0^0 = 4n + 3\) which they would have in the limit \(V \to 0\), while their widths

\[
\Gamma_n \approx \exp \left\{ -2 \int_{a_1}^{a_2} da \sqrt{U - \epsilon_n} \right\} \quad \text{at} \quad \Gamma_n \ll \epsilon_n
\]

decrease exponentially. Here \(a_1\) and \(a_2\) \((a_1 < a_2)\) are the turning points specified by the condition \(U = \epsilon_n\). As a result the lifetime of the universe in such states is many orders greater than the Planck time and at \(V \sim 10^{-122}\) it reaches the values \(\tau \sim 10^{61} \sim 10^{17} \text{s}\) comparable with the age of our universe.

Thus the whole scenario can be represented in the following way. In the quantum cosmological system of the most general type which is described by a superposition of the
waves $\varphi^{(\pm)}$ incident upon the barrier and scattered by the barrier \[11\] \[12\] \[15\], it occurs
the formation of the fundamentally new state as a result of a slow decreasing of the potential $V$ (vacuum energy density \[18\] \[19\]). When the potential $V$ reaches the value $V_{\text{in}}$ the wave $\varphi_\epsilon$ with $\epsilon \approx \epsilon_0$ penetrates into the prebarrier region \[15\] and it results in the transition of the cosmological system to a quasistationary state. In this state the system is characterized by the expectation values of the scale factor $\langle a \rangle_{n=0} = 1.25 = 0.93 \times 10^{-33}$ cm and total energy density $\langle \rho \rangle_{n=0} = 1.16 = 0.65 \tilde{\rho}_{\text{Pl}} = 3.35 \times 10^{93}$ g/cm$^3$ \[12\] \[15\]. These parameters are of Planck scale. Such a new formation has a lifetime which exceeds the Planck time and one can consider it as a quantum cosmological system with well-defined physical properties. We call it the quantum universe. Such universe can evolve by means of change of its quantum state. In every quantum state it can be characterized by the energy density, observed dimensions, lifetime (age), proper dimensions of non-homogeneities of the matter density, amplitude of fluctuation of radiation temperature, power spectrum of density perturbations, angular structure of the radiation anisotropies, deceleration parameter, total entropy, and others \[15\].

3. Collective states

3.1 Collective excitations of the gravitational field

Within the lifetime the state of the universe can be considered with a high accuracy as a stationary state which takes the place of a quasistationary one when its width becomes zero \[24\]. Taking into account that such states emerge at $V \ll 1$, while in the prebarrier region ($a < a_1$) $a^2 V < 1$ always, the solutions of \[17\] can be written in the form of expansion in powers of $V$ on the interval $\Delta \phi = |\phi_{\text{vac}} - \phi_{\text{start}}|$. We have

$$\varphi_n = \langle n \rangle - \frac{V}{4} \left[ \frac{1}{8} \sqrt{N(N-1)(N-2)(N-3)} \langle n-2 \rangle ight. 
+ \sqrt{N(N-1)} \left( N - \frac{1}{2} \right) \langle n-1 \rangle 
- \sqrt{(N+1)(N+2)} \left( N + \frac{3}{2} \right) \langle n+1 \rangle 
- \frac{1}{8} \sqrt{(N+1)(N+2)(N+3)(N+4)} \langle n+2 \rangle 
- O(V^2) \right]$$

(12)

for the wavefunction and

$$\epsilon_n = \epsilon_0 - \frac{3}{4} V [2N(N+1) + 1] - O(V^2)$$

(13)

for the position of the level, where $N = 2n + 1$. Here $\langle n \rangle$ is the eigenfunction, $\epsilon_0 = 2N + 1$ is the eigenvalue of the equation for an isotropic oscillator with zero orbital angular momentum

$$(-\partial_o^2 + a^2 - \epsilon_n^0) \langle n \rangle = 0.$$ 

(14)

The wavefunction $\langle a | n \rangle$ describes the geometrical properties of the universe as a whole. Since the gravitational field is considered as a variation of space-time metric \[25\], then this wavefunction also characterizes the quantum properties of the gravitational field.
which is considered as an aggregate of the elementary excitations of quantum oscillator \[|\text{vac}\rangle\]. It should be emphasized that in this approach the role of the dynamic variables is taken by the variables \((a, \phi)\) of the minisuperspace and after the quantization such elementary excitations cannot be identified with gravitons. Introducing the operators

\[
A^\dagger = \frac{1}{\sqrt{2}}(a - \partial_a), \quad A = \frac{1}{\sqrt{2}}(a + \partial_a),
\]

the state \(|n\rangle\) can be represented in the form

\[
|n\rangle = \frac{1}{\sqrt{N!}}(A^\dagger)^n|\text{vac}\rangle, \quad A|\text{vac}\rangle = 0,
\]

\[
|\text{vac}\rangle = \left(\frac{4}{\pi}\right)^{1/4}\exp\left\{-\frac{a^2}{2}\right\}.
\]

Since \(A^\dagger\) and \(A\) satisfy the ordinary canonical commutation relations, \([A, A^\dagger] = 1, [A, A] = [A^\dagger, A^\dagger] = 0\), then one can interpret them as the operators for the creation and annihilation of quanta of the collective excitations of the gravitational field (in the sense indicated above). We shall call it g-quantum. The integer \(N\) gives the number of g-quanta in the n-th state of the gravitational field.

Passing in (14) to the ordinary physical units it is easy to see that the states (16) can be interpreted as those which emerge as a result of motion of some imaginary particle with the mass \(m_{Pl}\) and zero orbital angular momentum in the field with the potential energy \(U(R) = k_{Pl}R^2/2\), where \(R = la\) is a “radius” of the curved universe, while \(k_{Pl} = (m_{Pl}c^2)^3(\hbar c)^{-2}\) can be called a “stiffness coefficient of space (or gravitational field)” \(, k_{Pl} = 4.79 \times 10^{85} \text{GeV/cm}^2 = 0.76 \times 10^{83} \text{g/s}^2\). This motion causes the equidistant spectrum of energy \(E_n = \hbar \omega_{Pl} (N + \frac{1}{2}) = 2 \hbar \omega_{Pl} (n + \frac{1}{2})\), where \(\hbar \omega_{Pl} = m_{Pl}c^2\) is the energy of g-quantum, \(\omega_{Pl} = a_{Pl}^{-1}\) is the oscillation frequency.

The interaction of the gravitational field with a vacuum which has non-zero energy density \((V(\phi) \neq 0)\) in the range \(\Delta \phi\) results in the fact that the wavefunction \([12]\) is a superposition of the states \(|n\rangle, |n + 1\rangle\) and \(|n - 1\rangle\). From \([12]\) and \([13]\) it follows that the quantum universe can be characterized by a quantum number \(n\).

The physical state \(|n\rangle\) (16) is chosen so that \(|0\rangle\) describes an initial state of the gravitational field. From the point of view of the occupation number representation there is only one g-quantum in such state. When the gravitational field transits from the state \(|n\rangle\) into the neighbouring one \(|n + 1\rangle\) two g-quanta are created, while the corresponding energy increases by \(2\hbar \omega_{Pl}\). In the inverse transition two g-quanta are absorbed and the energy decreases. The transitions with the creation/absorption of one g-quantum are forbidden. The vacuum state \(|\text{vac}\rangle\) in (16) describes the universe without any g-quantum.

The g-quanta are bosons. Since the probability of the creation of a boson per unit time grows with the increase in number of bosons in a given state (see e.g. \([21]\)), then an analogous effect must reveal itself in the quantum universe during the creation of g-quantum. As the average value is \(\langle a \rangle \sim \sqrt{(N + 1)/2}\), then the growth in number \(N\) of g-quanta (or number of levels \(n\)) means the increase in expectation value of the scale factor. In other words the expansion of the universe reflects the fact of the creation of g-quantum (under the transition to a higher level). The increase in probability of the creation of these quanta per unit time leads to the accelerated expansion of the universe. This phenomenon becomes appreciable only when \(n\) reaches the large values. The observations of type Ia...
supernovae provide the evidence that today our universe is expanding with acceleration \cite{4,9,10}. The above-mentioned qualitative explanation of this phenomenon is confirmed by the concrete quantitative calculations of the deceleration parameter \cite{15}. On this point the theory is in good agreement with the measurements of type Ia supernovae.

3.2 Collective excitations of the matter field

When the potential $V$ decreases to the value $V \ll 0.1$ the number of available states of the universe increases up to $n \gg 1$. By the moment when the scalar field will roll in the location where $V(\phi_{\text{vac}}) = 0$ the universe can be found in the state with $n \gg 1$. This can occur because the emergence of new quantum levels and the (exponential) decrease in width of old ones result in the appearance of competition between the tunneling through the barrier $U$ and allowed transitions between the states, $n \rightarrow n$, $n \pm 1$, $n \pm 2$. A comparison between these processes demonstrates \cite{11,12,15} that the transition $n \rightarrow n + 1$ with creation of g-quanta is more probable than any other allowed transitions or decay. The vacuum energy of the early universe originally stored by the field $\phi$ with a potential $V(\phi_{\text{start}})$ is a source of transitions.

According to accepted model the scalar field $\phi$ descends to the state with zero energy density, $V(\phi_{\text{vac}}) = 0$. Then due to the quantum fluctuations the field $\phi$ begins to oscillate with a small amplitude about equilibrium vacuum value $\phi_{\text{vac}}$. We shall describe these new states of the quantum universe assuming that by the moment when the field $\phi$ reaches the value $\phi_{\text{vac}}$ the universe transits to the state with $n \gg 1$. This means that the “radius” of the universe has become $\langle a \rangle > 10 l$. The wavefunction of the universe is $\psi_{E} = \sum_{n} \psi_{n}$, where the function $\psi_{n}$ describes the universe in the state with a given $n$ and takes the following form up to the terms $\sim O(V^{2})$

$$\psi_{n}(a, \phi) = \langle a | n \rangle f_{n}(\phi; E) \quad \text{at} \quad n \gg 1,$$

where the function $f_{n}$ satisfies the equation

$$\left[ \partial^{2}_{x} + z - V(x) \right] f_{n} = 0. \quad (18)$$

Here $x = \sqrt{m/2} (2N)^{3/4} (\phi - \phi_{\text{vac}})$ characterizes the deviation of the field $\phi$, $z = (\sqrt{2N/m}) (1 - E/(2N))$, $V(x) = (2N)^{3/2} V(\phi)/m$, $m$ is a dimensionless parameter which it is convenient to choose as $m^{2} = \left[ \partial^{2}_{\phi} V(\phi) \right]_{\phi_{\text{vac}}}$. The formulae (17) and (18) follow from (6), (9), and (12) if one takes into account resonance behaviour of the wave $\varphi_{\epsilon}$ at $\epsilon = \epsilon_{n}$ in the prebarrier region \cite{13,15}.

Since $\langle a \rangle = \sqrt{N/2}$, where averaging was performed over the states (17), then $V(x)$ is a potential energy of the scalar field contained in the universe with the volume $\sim \langle a \rangle^{3}$. Expanding $V(x)$ in powers of $x$ we obtain

$$V(x) = x^{2} + \alpha x^{3} + \beta x^{4} + \ldots, \quad (19)$$

where the parameters are

$$\alpha = \frac{\sqrt{2}}{3} \frac{\lambda}{m^{5/2}} \frac{1}{(2N)^{3/4}}, \quad \beta = \frac{1}{6} \frac{\nu}{m^{3}} \frac{1}{(2N)^{3/2}}, \quad (20)$$

and $\lambda = \left[ \partial^{2}_{\phi} V(\phi) \right]_{\phi_{\text{vac}}}$, $\nu = \left[ \partial^{4}_{\phi} V(\phi) \right]_{\phi_{\text{vac}}}$. Since $N \gg 1$, then the coefficients are $|\alpha| \ll 1$ and $|\beta| \ll 1$ even at $m^{2} \sim \lambda \sim \nu$. Therefore (18) can be solved using the perturbation
theory for stationary problems with a discrete spectrum. We take for the state of the unperturbed problem the state of the harmonic oscillator with the equation of motion
\[
\left[ \partial_x^2 + z^0 - x_0^2 \right] f_n^0 = 0.
\] (21)
In the occupation number representation one can write
\[
f_{n s}^0 = \frac{1}{\sqrt{s!}} (B_n^\dagger)^s f_{n_0}^0, \quad B_n f_{n_0}^0 = 0,
\]
\[
f_{n_0}^0(x) = \left( \frac{1}{\pi} \right)^{1/4} \exp \left\{ -\frac{x^2}{2} \right\}.
\] (22)
with \( z^0 = 2s + 1 \), where \( s = 0, 1, 2 \ldots \), and
\[
B_n^\dagger = \frac{1}{\sqrt{2}} (x - \partial_x), \quad B_n = \frac{1}{\sqrt{2}} (x + \partial_x).
\] (23)
Here \( B_n^\dagger (B_n) \) can be interpreted as the creation (annihilation) operator which increases (decreases) the number of quanta of the collective excitations of the scalar field in the universe in the \( n \)-th state. We shall call them the matter quanta. The variable \( s \) is the number of matter quanta in the state \( n \gg 1 \). It can be considered as an additional quantum number.

Using (18), (19) and (22) we obtain
\[
z = 2s + 1 + \Delta z, \quad \Delta z = \frac{3}{2} \beta \left( s^2 + s + \frac{1}{2} \right) - \frac{15}{8} \alpha^2 \left( s^2 + s + \frac{11}{30} \right) - \frac{\beta^2}{16} \left( 34s^3 + 51s^2 + 59s + 21 \right)
\] (25)
takes into account a self-action of matter quanta. In order to determine the physical meaning of the quantities which enter (18) with the potential (19) we rewrite it in the ordinary physical units,
\[
\left( -\frac{\hbar^2}{2\mu} \partial_r^2 + \frac{1}{2} \mu \omega^2 r^2 + \mathcal{E}_1 \left( \frac{r}{l} \right)^3 + \mathcal{E}_2 \left( \frac{r}{l} \right)^4 + \ldots - \mathcal{E} \right) f_n(r) = 0,
\] (26)
where we denote \( \mu = m_{Pl} m^{-1} \), \( r = lx \), \( \omega = m m_{Pl} l \), \( \mathcal{E}_1 = m m_{Pl} c^2 \alpha / 2 \), \( \mathcal{E}_2 = m m_{Pl} c^2 \beta / 2 \), \( \mathcal{E} = m m_{Pl} c^2 z / 2 \). Here \( l = \sqrt{\hbar / (\mu \omega)} \) is the Planck length. According to (20) an imaginary particle with a mass \( \mu \) performs the anharmonic oscillations and generates the energy spectrum
\[
\mathcal{E} = \hbar \omega \left( s + \frac{1}{2} + \frac{\Delta z}{2} \right),
\] (27)
where \( \hbar \omega = m m_{Pl} c^2 \) is the energy of matter quantum, and \( m \) can be interpreted as its mass (in units \( m_{Pl} \)). The quantity
\[
M = m \left( s + \frac{1}{2} \right) + \Delta M,
\] (28)
where $\Delta M = m \Delta z/2$, is a mass of the universe with $s$ matter quanta. Since we are interested in large values of $s$, then for the estimation of $\Delta M$ let us assign the values $s \sim 10^{80}$ and $n \sim 10^{122}$. These parameters describe our universe, where $s$ is equal to the equivalent number of baryons in it and $\langle a \rangle \sim 10^{28}$ cm is a size of its observed part\textsuperscript{[13, 14]}. In this case $\Delta M \sim O((\nu/m^2)10^{-24}, (\lambda/m^2)^210^{-24})$. Hence when the number of the matter quanta becomes very large their self-action can be a fortiori neglected.

4. The matter content of the quantum universe

4.1 Einstein-Friedmann equation in terms of the average values

In the early epoch (on the interval $\Delta \phi$) there is no matter in the ordinary sense in the universe, since the field $\phi$ is only a form of existence of the vacuum, while the vacuum energy density is decreasing with time. In spite of the fact that $V(\phi_{\text{vac}}) = 0$, the average value $\langle V \rangle \neq 0$. It contributes to the vacuum energy density in the epoch when the matter quanta are created and determines the cosmological constant $\Lambda = 3\langle V \rangle$. The universe is filling with matter in the form of aggregate of quanta of the collective excitations of the primordial scalar field.

Let us consider the possible matter content of the universe from the point of view of quantum cosmology. For this purpose we change from the quantum equation (4) to the corresponding Einstein equation for homogeneous, isotropic, and closed universe filled with the uniform matter and radiation. Averaging over the states $\psi_E$ we obtain the Einstein-Friedmann equation in terms of the average values

$$\left( \frac{\partial_t \langle a \rangle}{\langle a \rangle} \right)^2 = \langle \rho \rangle - \frac{1}{\langle a \rangle^2},$$

(29)

where we have neglected the dispersion, $\langle a^2 \rangle \sim \langle a \rangle^2$, and $\langle a^6 \rangle \sim \langle a \rangle^6$. The average value

$$\langle \rho \rangle = \langle V \rangle + \frac{2}{\langle a \rangle^6} \langle -\partial_\phi^2 \rangle + \frac{E}{\langle a \rangle^4}$$

(30)

is a total energy density in some fixed instant of time with the Hubble constant $H_0 = \partial_t \langle a \rangle / \langle a \rangle$. The first term is the energy density of the vacuum with the equation of state $p_v = -\rho_v = -\langle V \rangle$, the second term is the matter energy density, while the last describes the contribution of the radiation.

Using the wavefunction (17) and passing in (16) and (22) to the limit of large quantum numbers, for the energy densities of the vacuum $\Omega_v$ and matter $\Omega_{qm}$ we obtain

$$\Omega_v = \frac{M}{12\langle a \rangle^3 H_0^2}, \quad \Omega_{qm} = \frac{16 M}{\langle a \rangle^3 H_0^2},$$

(31)

where $M = m \left( s + \frac{1}{2} \right)$ is the mass of the universe with “radius” $\langle a \rangle = \sqrt{N/2}$. From the definition of $z$ in (13) it follows that $\langle a \rangle = M + E/(4\langle a \rangle)$. For the matter-dominant era, $M \gg E/(4\langle a \rangle)$, from (29) and (31) we find

$$\Omega = 1.066, \quad \Omega_v = 0.006, \quad \Omega_{qm} = 1.060,$$

(32)

where $\Omega = \langle \rho \rangle H_0^{-2}$. From (32) it follows that the quantum universe in all states with $n \gg 1$ and $s \gg 1$ looks like spatially flat. A main contribution to the energy density
of the universe is made by the collective excitations of the scalar field above its true vacuum. The total density $\Omega$ can be compared with the present-day density of our universe, $\Omega_0 = 1 \pm 0.12$ \[1\] or $\Omega_0 = 1.025 \pm 0.075$ \[2\].

The matter quanta are bosons. They have non-zero mass/energy, while their state according to (22) depends on the state of the gravitational field. This means that the matter quanta are subject to the action of gravity. Due to this fact they can decay into the real particles (e.g. baryons and leptons) that have to be present in the universe in small amounts (because of the weakness of the gravitational interaction). The main contribution to the energy density will still be made by the matter quanta.

### 4.2 The matter quantum decay

In order to make a numerical estimate we consider the matter quantum ($\phi$) decay scheme

$$\phi \rightarrow \phi' \nu n \rightarrow \phi' \nu p e^- \bar{\nu},$$

where $\phi'$ is the quantum of the residual excitation, which reveals itself in the universe in the form of the non-baryonic dark matter. Neutrino $\nu$ takes away the spin. Assuming that the quanta $\phi$, just as the neutrons $n$, decay independently, we obtain the law of production of protons in the form

$$s_p = s \left[ 1 + \frac{1}{\Gamma_n - \Gamma_\phi} \left( \Gamma_\phi e^{-\Gamma_\phi \Delta t} - \Gamma_n e^{-\Gamma_n \Delta t} \right) \right],$$

where $s$ is the number of quanta $\phi$ at some arbitrarily chosen initial instant of time $t'$, $\Gamma_n$ and $\Gamma_\phi$ are the decay rates of neutron and quantum $\phi$ respectively, $\Delta t = t - t'$ is time interval during which $s_p$ protons were produced. Since we are interested in matter density in the universe today, then for numerical estimations we choose $\Delta t$ equal to the age of the universe, $\Delta t = 14$ Gyr \[3\]. For experimentally measured decay rate of neutron, $\Gamma_n = 1.13 \times 10^{-3}$ s$^{-1}$ \[3\] we have: $\Gamma_n \Delta t = 5 \times 10^{14} \gg 1$. We shall suppose that the decay of quantum $\phi$ is caused by the action of the gravitational forces. This means the following condition must be fulfilled

$$\Gamma_\phi \ll \Gamma_n.$$ \hspace{1cm} (35)

Taking into account these two remarks one can simplify (34)

$$s_p = \bar{s} = s \left[ 1 - e^{-\Gamma_\phi \Delta t} \right],$$ \hspace{1cm} (36)

where $\bar{s}$ is an average number of the quanta $\phi$ which decay during the time interval $\Delta t$. The law of proton production (36) implies that all primordial neutrons have to decay up to now. The quanta $\phi'$ in (33) are assumed to be the stable particles (with lifetime greater than $\Delta t$), and their number is equal to $\bar{s}$. The decay rate $\Gamma_\phi$ is unknown and it must be determined theoretically on the basis of vertex modelling of complex decay (33) or extracted from the astrophysical data. Since, generally speaking, the decay rate of the quanta $\phi$ may change with time, then by $\Gamma_\phi$ it should be implied the mean probability of decay on the time interval $\Delta t$,

$$\Gamma_\phi = \frac{1}{\Delta t} \int_{t'}^{t} dt \zeta(t) \gamma(t),$$ \hspace{1cm} (37)
where $\gamma(t)$ is the decay rate at the instant of time $t$ when the energy density in the universe equals to $\rho(t)$, and weight function $\zeta(t)$ takes into account influence of other accompanying processes on the decay rate (e.g. space-time curvature).

The density of (optically bright and dark) baryons is

$$\Omega_B = \frac{16 \bar{s} m_p}{\langle a \rangle^3 H_0^2},$$

(38)

where $m_p = 0.938$ GeV is a proton mass. Taking into account (31) and (36) we find

$$\Omega_B = \Omega_{qm} \sqrt{\frac{3\pi}{2g}} \frac{m_p}{m_{Pl}} \left(1 - e^{-\Gamma_\phi \Delta t}\right),$$

(39)

where $g = G m^2$ is the gravitational coupling constant for the quantum $\phi$ with mass $m$. Since we suppose that $\Gamma_\phi \sim g$, the density $\Omega_B$ is the function on the coupling constant $g$. This function vanishes at $g = 0$ and tends to zero as $g^{-1/2}$ at $g \to \infty$. It has one maximum. Let us fix the coupling constant $g$ by maximum value of $\Omega_B(g)$. Then we obtain

$$\Gamma_\phi \Delta t = 1.256.$$  

(40)

For $\Delta t = 14$ Gyr it gives

$$\Gamma_\phi = 2.840 \times 10^{-18} \text{s}^{-1}.$$  

(41)

This rate satisfies inequality (35). In addition

$$\Gamma_\phi > H_0,$$

(42)

where $H_0 = 71 \text{ km s}^{-1} \text{ Mpc}^{-1}$ is the present-day value of the Hubble expansion rate [3]. This condition means that on average at least one interaction has occurred over the lifetime of our universe.

4.3 The model of $\Gamma_\phi$

Since the final product of decay of the quantum $\phi$ through the intermediate creation of neutron is proton, in a first approximation one can write a simple expression for the mean probability of the decay $\Gamma_\phi$

$$\Gamma_\phi = \alpha g \Delta m,$$

(43)

where $\alpha$ is a fine-structure constant, $\Delta m = 1.293$ MeV is a difference in masses of neutron and proton. Using the numerical values of the parameters from (41) and (43) we find

$$g = 19.8 \times 10^{-38}.$$  

(44)

Then the density of baryons is

$$\Omega_B = 0.131,$$

(45)

and the mass of matter quantum $\phi$ corresponding to such coupling constant is equal to $m = 5.433$ GeV. The density of the non-baryonic dark matter is

$$\Omega_{CDM} = \Omega_B \frac{m_\phi}{m_p},$$

(46)
where, according to (33), $m_{\phi'} = m - m_n - m'_{\nu} - Q$ is a mass of the quantum $\phi'$, $m'_{\nu} \equiv m_{\nu}^2/(2p_{\nu})$, $m_{\nu}$ and $p_{\nu}$ are the neutrino mass and momentum, $Q$ is the energy of the relative motion of all particles. Since the contribution from $\Omega_{CDM}$ to the matter density $\Omega_M$ of our universe is not at least smaller than $\Omega_B$ \cite{2,3,4}, then the mass $m_{\phi'}$ can possess the values within the limits

$$0.938 \text{ GeV} \leq m_{\phi'} \leq 4.493 \text{ GeV} \quad (47)$$

depending on the value of $Q$. Such particles are non-relativistic and non-baryonic dark matter is classified as cold. Hence we find

$$0.131 \leq \Omega_{CDM} \leq 0.627 \quad (48)$$

The total matter density $\Omega_M = \Omega_B + \Omega_{CDM}$ can possess any values within the limits

$$0.262 \leq \Omega_M \leq 0.758 \quad (49)$$

We neglect the contribution from neutrino to $\Omega_M$ assuming that $m'_{\nu} \ll m_n$. A spread in theoretical values of the densities $\Omega_{CDM}$ and $\Omega_M$ is caused by the fact that the energy $Q$ is an undefined parameter of the theory. In principle it can be fixed from the astrophysical data for $\Omega_M$. With the regard for this remark one can compare (49) with the matter density of our universe, $\Omega_M^{\text{obs}} = 0.3 \pm 0.1$ \cite{3}, where a spread in values is related with inaccuracy of measurements.

According to (32) and (49) the residual density $\Omega_X = \Omega_{qm} - \Omega_M$ can possess the values within the limits

$$0.302 \leq \Omega_X \leq 0.798 \quad (50)$$

The observations in our universe give the restriction $\Omega_X^{\text{obs}} = 0.7 \pm 0.1$ \cite{3}. These values agree with (50).

In our approach according to (31), (38), and (46) the residual density $\Omega_X$ has only dynamical nature and it can be attributed to the optically dark (nonluminous) energy.

The present-day density of optically bright and dark baryons in our universe consistent with the big-bang nucleosynthesis is estimated as $\Omega_B^{\text{obs}} = 0.039 \pm 0.004$ \cite{3}. This value does not contradict with (45), since the latter determines the maximum possible baryon matter density. The observed value of the cold dark matter density $\Omega_{CDM}^{\text{obs}} \sim 0.3$ falls within the limits of its theoretical counterpart (48). According to (38) the density of the optically bright baryons in our approach can be estimated as

$$\Omega_* = \frac{1}{16} \Omega_B \quad (51)$$

This relation agrees with the data of astronomical observations which indicate that the baryons in stars account for about 10% of all baryons \cite{3,4}.

Then according to (45) the contribution from the optically bright baryons must not exceed

$$\Omega_* \sim 0.008 \quad (52)$$

In this sense this value is in agreement with the observations of the bright stars, gaseous content of galaxies, groups and clusters, $\Omega_*^{\text{obs}} = 3_{-2}^{+1} \times 10^{-3}$ \cite{3,5,6}. 
In our approach the densities $\Omega_M$ (19) and $\Omega_X$ (50) are of the same order. Hence the known problem of present-day coincidence between dark matter and dark energy components in our universe [27] is solved automatically.

Thus the model of matter quantum decay rate (43) gives the values of the energy density components in the universe close (on the order of magnitude) to observed. Let us note that the possible mass (47) of particle of the non-baryonic dark matter lies in the range of baryon resonances and mesons. Any known particle here will not do for the role of quantum $\phi'$.

It is interesting that the range of masses (47) is close to one of the possible limits on the mass of the light gluino, $m_{\tilde{g}} \lesssim 5$ GeV [3]. However, since the gluino is the colour octet Majorana fermion partner of the gluon, it also will not fit for the role of non-baryonic dark matter particle $\phi'$.

One can solve an inverse problem. Namely, using the observed value $\Omega_{B}^{\text{obs}}$, it is possible to restore the coupling constant $g$ and then find the masses $m$, $m_{\phi'}$ and densities $\Omega_B$, $\Omega_*$, $\Omega_M$ and $\Omega_X$.

### 4.4 The inverse problem

Let us set $\Omega_* = 0.0025$. Fixing $g$ by the maximum of the function $\Omega_B(g)$ as above, we find

$$g = 212 \times 10^{-38}, \quad m = 17.78 \text{ GeV}.$$  

(53)

The corresponding values of mass of the quantum $\phi'$ lie in the range

$$0.938 \text{ GeV} \leq m_{\phi'} \leq 16.84 \text{ GeV},$$

with mean value $m_{\phi'} = 8.889$ GeV, for possible energies of the decay of matter quantum in the domain $0 \leq Q \leq 15.90 \text{ GeV}$. According to (51) the baryon density is equal to $\Omega_B = 0.04$. This value practically coincides with $\Omega_B^{\text{obs}}$. Repeating the calculations such as above, we obtain the following limits on the components of energy density for the parameters (53)

$$0.04 \leq \Omega_{CDM} \leq 0.718,$$

with mean value $\overline{\Omega}_{CDM} = 0.379$,

$$0.08 \leq \Omega_M \leq 0.758,$$

with mean value $\overline{\Omega}_M = 0.419$,

$$0.302 \leq \Omega_X \leq 0.98,$$

with mean value $\overline{\Omega}_X = 0.641$, and $\Omega_{DB} = 0.0375$ for dark baryons, $\Omega_{DB} = \Omega_B - \Omega_*$. All these arithmetic mean values are in good agreement with corresponding experimental data given above.

The lightest supersymmetric particles can be considered as the candidates for the quantum $\phi'$ of non-baryonic cold dark matter. Some accelerator experiments give the bound $m_{\tilde{\chi}_1^0} > 10.9$ GeV for the mass of the lightest stable neutralino $\tilde{\chi}_1^0$ [28] which falls into the range of $m_{\phi'}$ (54).
5. Concluding remarks

The mechanism of origin of mass-energy constituents of the universe by means of the collective excitations of the primordial scalar field, which we propose, can explain the fact that our universe appears to be populated exclusively with matter than antimatter.

For the mass $m_{\phi'} \sim 9$ GeV, the energy $Q \sim 8$ GeV is released in the decay (33). Such kinetic energy corresponds to the temperature $\sim 10^{14}$ K. This temperature can be in the primary plasma under thermal equilibrium in the early universe at $\Delta t \sim 10^{-8}$ s. In early, very dense universe among the particles of primary hot plasma in addition to primordial protons, electrons and $\nu \bar{\nu}$-pairs there must be other particle-antiparticle pairs with masses $< m_p$, secondary neutrons and photons. In this case the number of light particles will exceed the number of baryons (cp. [20]). Such very hot and dense universe will expand and cool down in accordance with standard big-bang model (see e.g. [31, 20]). The questions concerning initial heating of the early universe and transition to the radiation dominated phase in our approach exceed the aims of this paper and require a separate investigation.

Another interesting question which we shall mention here is connected with the cosmological constant in our universe. Using the obtained value of the energy density $\Omega_v$ (32) for the critical density of the universe $\rho_c = 1.879 \times 10^{-29} h^2$ g cm$^{-3}$ [3] we find the energy density of small quantum fluctuations of the scalar field about equilibrium vacuum state $\phi_{\text{vac}}$

$$\rho_v = 1.125 \times 10^{-31} h^2 \text{ g cm}^{-3}. \quad (58)$$

Such density forms the cosmological constant in the universe

$$\Lambda = 2.097 \times 10^{-58} h^2 \text{ cm}^{-2}. \quad (59)$$

If one suppose that the cosmological constant in our universe arises due to the residual (dark) energy $\Omega_X$ [4] then for $\Omega_X = 0.7$ its value will be

$$\Lambda_X = 2.454 \times 10^{-56} h^2 \text{ cm}^{-2}. \quad (60)$$

In this case one can neglect the contribution from $\rho_v$ to the total vacuum energy density.

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