Iterative map-making methods for Cosmic Microwave Background data analysis

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Abstract. The map-making process of Cosmic Microwave Background data involves linear inversion problems which cannot be performed by a brute force approach for the large timelines of most modern experiments. We present optimal iterative map-making methods, both COBE and Wiener, and apply these methods on simulated data. They produce very well restored maps, by removing nearly completely the correlated noise that appears as intense stripes on the simply pixel-averaged maps.

INTRODUCTION

The Cosmic Microwave Background (CMB hereafter) is being extensively studied nowadays, thanks to the improvement of instrument and detector performances. Since the COBE experiment (Smoot et al. 1992), which performed the first detection of CMB anisotropies, the size of time-ordered information (TOI hereafter) has been widely increased. The data processing and analysis is thus still a challenge for large timeline data, already in processing or still to come. The Cosmic Microwave Background data analysis is usually performed in three steps, the first being the map-making process, the second the \( \langle \ell \rangle \) estimation from the maps and the noise covariance matrices, the third the cosmological parameters estimation from the \( \langle \ell \rangle \) power spectrum. In this article, we focus on the map-making step, which, in the context of large timelines, cannot be performed by a brute-force approach, which would imply the manipulation and inversion of tera-element large matrices. Non optimal map-making methods have been developed, such as destriping using the scan intercepts (see Delabrouille 1998 and Dupac & Giard 2001). We aim in this paper to apply the optimal methods to large timelines, developing algorithms to avoid computation trouble. The map-making methods for CMB are known to be optimal when following desirable properties. These are linear map-making methods, well known as the COBE method (Janssen & Gulkis 1992) and the Wiener filter (Wiener 1949). The matrix expressions of these optimal methods can be found, e.g., in Tegmark (1997).

Applying these optimal methods to large timelines is not straightforward, because of computer limitations. The several tens or hundreds of mega-elements in a bolometer timeline have to be processed by vector-only methods, that we aim to present in this paper. We mean by "vector-only methods" computations in which one never needs to compute or even to store any large matrix.

OBSERVATION STRATEGY

We present here a large-coverage observation strategy, simulated from simple and usual technical requirements. We simulate balloon-borne CMB experiments, required to scan the sky making constant elevation circles at a rather constant rotation speed (as do Archeops: Benoît et al. 2001, and TopHat: http://topweb.gsfc.nasa.gov). This allows to observe a large area on the sky, thanks to the rotation of the Earth (and, eventually, to the moving of the balloon on the Earth). We have simulated a balloon-borne experiment timeline with a constant scanning elevation of 35 degrees above the horizon, a sampling frequency of 100 Hz and a rotation speed of the gondola of 2 rpm. The flight is 24 hours long, launched from a polar place (Kiruna for instance). The winter time in this region provides polar nights allowing to make 24 hours flight avoiding contamination from the sun. The coverage for this polar flight is 35 % of the sky.
SIMULATION PROCESS

We have made our simulated skies from three components: the CMB fluctuations, the dipole, and the Galaxy, at 2 mm wavelength. The simulated Universe we have simulated is Λ dominated, with ΩΛ = 0.7, ΩCDM = 0.25, Ωbar = 0.05, H0 = 50 and a scalar spectral index of the fluctuations equal to 1. The C_l simulated spectrum (with no cosmic variance) is made thanks to the CMBFAST software (Seljak & Zaldarriaga 1996). The sky gaussian random field (i.e. a realization of a Universe with the cosmological parameters we chose) is then simulated with the SYNFAST tool of the HEALPix package (http://www.eso.org/science/healpix). The dipole is added thanks to its COBE/DMR (i.e. a realization of a Universe with the cosmological parameters we chose) is then simulated with the SYNFAST code. The Galaxy at 2 mm wavelength is extrapolated from the composite 100 μm IRAS-COBE/DIRBE all sky dataset (Schlegel et al. 1998). The noise we introduce in our simulated timelines can be characterized by its statistical power spectrum which follows a 1/f law: \( P(f) \propto (1+f_c/f)^n \), where \( f_c \) is the level of white noise (i.e. the only noise at high frequency), \( f_c \) the cut frequency and \( n \) the power index. The noise we introduced is characterized by \( n = 1, f_c = 0.1 \) Hz and \( l_{inf} = 100 \) μK CMB rms. This white noise rms is approximatively the level expected for Planck (Tauber 2000) bolometers with a 100 Hz sampling rate. Of course this quite low value is the level of the non-correlated noise in the timelines, but the total amount of noise introduced is much larger.

MAP-MAKING METHODS APPLIED ON LARGE TIMELINES

We use the HEALPix pixel scheme (http://www.eso.org/science/healpix) to make our maps. Reprojecting timelines on maps is not only a domain to domain transform, but the simplest way to estimate the true map of the sky, by averaging the samples of a same pixel on the sky. The noise is therefore reduced by a factor square root of the number of samples in the pixel (the weight). We will not investigate the beam deconvolution in this article, and therefore consider only 1-and-0 point-spread matrices.

The optimal map-making methods use the noise (N) and sky (S) covariance matrices: these are impossible to invert or even to store for such large timelines. However, if the noise is stationary in the time domain, as it is the case for 1/f noise and white noise (usual bolometer noises), the noise correlation matrix in the time domain is circulant, i.e. multiplying a vector by this matrix is a convolution, which is filtering in the Fourier domain. The sky covariance matrix is stationary in the map domain, as far as the Cosmic Microwave Background is a gaussian random field (for timelines without the Galaxy).

In this case the sky covariance matrix in the map domain is circulant.

The COBE equation:

\[
\tilde{x} = [A'N^{-1}A]^{-1}A'N^{-1}y,
\]

cannot be directly applied with vector-only algorithms, because of the matrix inversions needed. (\( A \) is the point-spread matrix, \( N \) the noise covariance matrix in the time domain, \( y \) the data timeline and \( \tilde{x} \) the optimal reconstructed sky map.) Thus the trick is to solve rather:

\[
[A'N^{-1}A] x = A'N^{-1} y
\]

This form prevents from the heavy inversion, but needs an iterative scheme. The general iterative scheme for this equation is:

\[
\alpha \hat{x}_{n+1} = \alpha \hat{x}_n + A'N^{-1} y - [A'N^{-1}A] \hat{x}_n
\]

where \( \alpha \) is any linear operator on a vector, that is, any square matrix. We have tested this algorithm on simulations and real data from the Archeops experiment (Benoît et al. 2001), with \( \alpha \) being a scalar. By testing the method with different \( \alpha \), we find that the identity is the best iterator.

Another scheme can be developed, by making the noise map converge instead of the sky map, as mentioned by Prunet (2001). This can be better, as the signal can be more tricky than the instrumental noise for the stability of the iterative scheme: hot galactic points for example may induce stripes on the maps. The noise-iterating scheme works with the following trick: we change the variable \( \tilde{x} \) to \( \hat{x} = [A'A]^{-1}A' y - \tilde{x} \). It is straightforward to show that this is the noise map plus the reconstruction error. It leads to:

\[
\alpha \hat{x}_{n+1} = \alpha \hat{x}_n + A'N^{-1} z - [A'N^{-1}A] \hat{x}_n
\]

where \( z = A[A'A]^{-1}A' y - \tilde{x} \). If this algorithm converges, then the converging limit is exactly the optimal solution of the map-making problem. The case of \( \alpha = 1 \) is actually the simplest iterator one can imagine, but works well on the simulations and real data that we have processed.

The iterative scheme that we have developed for the Wiener method is close to the COBE one:

\[
\alpha \hat{x}_{n+1} = \alpha \hat{x}_n + u \cdot [S^{-1} + A'N^{-1}A] \hat{x}_n
\]

where \( u = A'N^{-1}[A[A'A]^{-1}A' - y] + S^{-1}[A'A]^{-1}A' y \). As we have shown for the COBE iterative method, here the converging limit is the exact solution of the Wiener map-making equation. This iterative scheme needs to handle both the N matrix, noise covariance matrix in the timeline, that we process as a filter in the Fourier domain like we do for
the COBE iterative method, and the S matrix, sky covariance matrix in the map domain. Handling this as a matrix is not possible for a small scale pixelization that we need for CMB experiments of today, thus we have to process it as a filter in Fourier space, like we do for filtering timelines. The HEALPix RING scheme is stationary with respect to the sphere, because it pixelizes it making a ring around the sphere from the north pole to the south pole, with equal pixel surfaces. So filtering a HEALPix vector (i.e. a map) in Fourier space is optimal, to the condition that there must not be large holes in the map, that would harm the stationarity of the sky in the HEALPix scheme.

RESULTS AND CONCLUSION

We have applied these methods to the polar flight simulated data: the method reaches the residual noise at about 50 iterations, and this residual is about $21.6 \mu K_{CMB}$ rms. We can check that we have reached the convergence by observing the evolution of the global residual noise rms, but also the evolution for some individual pixels, the map aspect and the $C_l$ power spectrum. We have to compare this result to the white noise amount in the map: the rms level of white noise in the polar flight map is $21.06 \mu K_{CMB}$ rms, which is very close to the residual noise amount. This shows how good the reconstruction is, as it is clear that the correlated noise is significantly removed from the map. The reconstructed map exhibits no visible difference with the true map. The noise spectrum is nearly the one of a white noise above about $l=50$, but exhibits some weak residual correlation at lower scales. Even if the residual noise amount is very low, this deviation from the flat spectrum could have to be taken into account for very precise measurements of the low $l$.

This kind of vector-only methods seems to us unavoidable to make optimal maps from CMB experiments of today, or still to come. The reduction of the information in CMB data is a heavy work, from gigabytes of rawdata to essentially 12 cosmological numbers with their error bars. Since the computer facilities are limited and unsufficient for brute force approaches (and it will be still the case for Planck data reduction), it is an interesting challenge to process each step of this reduction work without losing information. Using stationarity properties of a signal in a given domain (sphere, map, timeline...) to transform a matrix inversion problem into a vector-only solution, could be probably also developed for other CMB reduction steps, such as the component separation.

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