New Grand Unified Models with Intersecting D6-branes, Neutrino Masses, and Flipped SU(5)

Mirjam Cvetič\textsuperscript{a} and Paul Langacker\textsuperscript{a,b}

\textsuperscript{a} Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, PA 19104-6396, USA
\textsuperscript{b} School of Natural Sciences, Institute for Advanced Study, Einstein Drive, Princeton, NJ 08540, USA

Abstract

We construct new supersymmetric SU(5) Grand Unified Models based on $\mathbb{Z}_4 \times \mathbb{Z}_2$ orientifolds with intersecting D6-branes. Unlike constructions based on $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifolds, the orbifold images of the three-cycles wrapped by D6-branes correspond to new configurations and thus allow for models in which, in addition to the chiral sector in $10$ and $\bar{5}$ representations of SU(5), only, there can be new sectors with $(15 + \bar{15})$ and $(10 + \bar{10})$ vector-pairs. We construct an example of such a globally consistent, supersymmetric model with four-families, two Standard Model Higgs pair-candidates and the gauge symmetry $U(5) \times U(1) \times Sp(4)$. In a $N = 2$ sector, there are $5 \times (15 + \bar{15})$ and $1 \times (10 + \bar{10})$ vector pairs, while another $N = 1$ sector contains one vector-pair of 15-plets. The $N = 2$ vector pairs can obtain a large mass dynamically by parallel D6-brane splitting in a particular two-torus. The 15-vector-pairs provide, after symmetry breaking to the Standard Model (via parallel D-brane splitting), triplet pair candidates which can in principle play a role in generating Majorana-type masses for left-handed neutrinos, though the necessary Yukawa couplings are absent in the specific construction. This model can also be interpreted as a flipped SU(5) $\times U(1)_X$ Grand Unified Model where the 10-vector-pairs can play the role of Higgs fields, though again there are phenomenological difficulties for the specific construction.
1 Introduction

The explanation of the origin of small neutrino masses in string constructions is a notoriously difficult problem. In particular, most of the intersecting D brane constructions of the semi-realistic Standard Model string vacua allow for Dirac neutrino masses; however, it is typically difficult to ensure small Dirac neutrino masses while on the other hand providing for an acceptable mass hierarchy in the quark and charged lepton sector \[1, 2, 3, 4, 5\]. No examples of intersecting D brane constructions leading to Majorana masses have been given. Within heterotic string theory, it is possible in principle to realize the usual minimal seesaw model\(^1\). In practice, however, it is difficult to simultaneously generate a large Majorana mass for the singlet neutrino and a Dirac mass coupling for the doublet and singlet neutrinos, while preserving supersymmetry at large scales and respecting the necessary consistency conditions for the string construction \[8\]-\[14\], with the few examples being non-minimal (i.e., involving a higher power of the heavy mass in the denominator \[9\], \[10\]) or not GUT-like \[11\], \[12\], \[13\], and often invoking additional dynamical assumptions. A systematic survey of a class of \(Z_3\) orbifold constructions did not find a single example of a minimal seesaw \[14\], and a study of \(Z_6\) constructions did not find any examples to the order considered if \(R\)-parity is imposed \[15\]. Similar problems may occur for theories with additional low energy symmetries \[16\].

Within the framework of particle physics model building, one intriguing possibility of generating small Majorana masses is via vector pairs in \((\mathbf{3} + \mathbf{3})\) representations of \(SU(2)_L\) with unit hypercharge \[17\], \[18\], \[19\]. If the \(\mathbf{3}\) couples to a pair of lepton doublets and the \(\mathbf{\bar{3}}\) to a pair of up-type Higgs doublets (or the \(\mathbf{3}\) to a pair of down-type Higgs), then lepton number is violated. If there is also a large supersymmetric mass \(M_T\) for the \(\mathbf{3} + \mathbf{\bar{3}}\)

\(^1\)For reviews of neutrino mass mechanisms, see, for example, \[6\], \[7\].
pair, then the neutral component of the 3 will acquire a tiny expectation
value of order $|\langle H_0^u \rangle|^2/M_T$, leading to the so-called type II seesaw mecha-
nism. (If there is no $\mathbf{3} H_u \mathbf{H}_u$ coupling, the $\mathbf{3} H_d \mathbf{H}_d$ coupling generates a
mass of order $M_T^{-2}$ [16].) The possibility of realizing such a triplet seesaw
mechanism within heterotic string constructions was considered in [20]. An
$SU(2)_L$ triplet with unit hypercharge could be obtained by a diagonal em-
bedding of $SU(2)_L$ into $SU(2) \times SU(2)$ (i.e., a higher affine Kac-Moody level
construction). It was shown that such a construction would most naturally
lead to an off-diagonal mass matrix, and therefore to distinct phenomenolog-
ical features (e.g., an inverted hierarchy with observable neutrino-less double
beta decay and a mixing $U_{e3}$ close to the present experimental limit), very
different from triplet models motivated from bottom-up or non-string moti-
vations. Explicit constructions were given with many, but not all, of the
necessary ingredients.

Higgs triplet pairs with unit hypercharge can arise as a decomposition
of $(\mathbf{15} + \overline{\mathbf{15}})$ pairs of the $SU(5)$ Grand Unified Theory (GUT) [21]. (For
reviews see [22, 23].) The purpose of this paper is to realize this mecha-
nism within explicit, globally consistent supersymmetric string constructions.
The concrete realization is based on intersecting D6-brane constructions on
toroidal orbifolds. (For a review see [24] and references therein.) This frame-
work provides a natural mechanism to realize supersymmetric $SU(5)$ GUT
constructions [25, 26]. In these constructions the $\mathbf{10}$-plets (and $\mathbf{15}$-plets)

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2For the original work on non-supersymmetric chiral intersecting D-branes, see [27, 28, 29]. For chiral supersymmetric ones, see [31, 25] and also [32]. For supersym-
metric chiral constructions within Type II rational conformal field theories, see [33, 34] and references therein. For the study of the landscape of intersecting D-brane constructions, see [25, 36].

For flipped $SU(5)$ GUT constructions, see [37, 38]. For recent GUT constructions with
intersecting D6-branes, see also [39] and references therein. For related studies of features
of GUTs in the Type II context, see [40, 41]. For proton decay calculations within
arise from the intersections of the $U(5)$ D6-brane configuration and its orientifold image. The appearance of 15-plets turns out to be ubiquitous in such constructions [26]. The major drawback of these constructions is the absence of the up-quark Yukawa couplings to the Standard Model (SM) Higgs; they are zero in perturbative Type IIA theory [26, 41], due the conservation of the “anomalous” $U(1)$ part of the $U(5)$ GUT symmetry.

The known supersymmetric GUT constructions with intersecting D6-branes are based on $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifolds, where the orbifold images of three-cycles, which are “inherited” from the toroidal ones, are the same as the original three-cycles. Therefore, D6-brane configurations that wrap such three-cycles result in massless open-string sectors that effectively arise from a single set of D6-brane configurations, inherited from the toroidal ones. The massless spectrum in each such sector is either associated with the $N=1$ supersymmetric chiral sector or purely non-chiral $N=2$ supersymmetric ones. As a consequence, intersecting D6-brane constructions on $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifolds cannot account for the appearance of $N=2$ vector pairs of 15-plets (and/or $N=2$ vector pairs of 10-plets), without the introduction of the chiral “excess” of the same number of $\overline{15}$ as there are chiral 10-plets. Namely, the 10-plets should be chiral to be identified with the fermion families. However, since they arise from the same sector as 15’s, the latter are also necessarily chiral. This feature also applies to the flipped $SU(5)$ constructions [37, 38], which require in addition to chiral matter in 10 and $\bar{5}$ representations of $SU(5)$, also additional GUT Higgs multiplets in $(10 + \overline{10})$ representations. However, within $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold constructions the appearance of the GUT Higgs pairs necessarily requires a net number of chiral $\overline{15}$’s, which is the same as the number of chiral 10-plets, and thus the $SU(5)$ anomaly cancellation requires additional 5’s.

In this paper we therefore turn to constructions of supersymmetric $SU(5)$ intersecting D6-brane constructions (and their strong coupling limits), see [42, 43].
GUT's that are based on orientifolds whose orbifold action produces new D6-brane configurations, and thus in addition to the original brane configurations, with say, a chiral sector, one now has new sectors, associated with the orbifold images, that can provide, say, non-chiral sectors. In order to demonstrate the existence of such constructions, we shall focus on a specific orientifold, which we choose for simplicity to be the $\mathbb{Z}_4 \times \mathbb{Z}_2$ orientifold. In addition, we choose only the three-cycles inherited from the toroidal constructions, i.e., for simplicity, we do not include fractional D-brane configurations, associated with the three-cycles wrapping orbifold singularities. A class of such orientifold constructions was discussed in detail in [44], with a goal to obtain three-family Standard Models. Here, our aim is to employ such an orientifold to construct supersymmetric GUT models with the features described above. In particular, we shall describe in detail an explicit, globally consistent supersymmetric construction with four-families and the gauge group structure of $U(5) \times U(1) \times Sp(4)$. Note that this explicit construction is meant to demonstrate specific features GUT spectrum, that in particular the non-chiral sector allows for matter candidates that may have interesting implications for neutrino masses. [Within the framework of flipped $SU(5)$ GUT constructions we shall see that the construction of the type presented in this paper can also be interpreted as a flipped $SU(5)$ GUT with no net $15$'s, while the GUT Higgs candidates of flipped $SU(5)$, i.e., $(10 + \overline{10})$-pairs, arise from the $N = 2$ sector of the construction.] Of course, one can also pursue constructions on other orbifolds, such as, e.g., [45 46], and involve there more general cycles with fractional D6-branes, e.g., [45 46 47], which is a topic of further research.

The paper is organized as follows. In Section 2 we discuss features of the $\mathbb{Z}_4 \times \mathbb{Z}_2$ orientifold, such as orbifold and orientifold actions and the corresponding O6-planes. In Section 3 we discuss in detail the spectrum, global consistency conditions and supersymmetry conditions for the open string
sector of D6-branes wrapping the three-cycles of the $\mathbb{Z}_N \times \mathbb{Z}_M$ orientifold, emphasizing the geometric aspects of the spectrum and consistency conditions for three-cycles inherited from the six-torus. This section also serves as a set-up for intersecting D6-brane constructions on three-cycles inherited from the six-torus for more general orientifolds than the one discussed in this paper. In Section 4 we discuss general features of the spectrum and couplings of the GUT models in the intersecting D6-brane constructions. In Section 5 we provide explicit expressions for the global consistency, K-theory constraints and supersymmetry conditions, as well as intersection numbers of the massless matter supermultiplets for open string sectors of a $\mathbb{Z}_4 \times \mathbb{Z}_2$ orientifold. In Section 6 we construct an explicit example of a supersymmetric, globally consistent four-family GUT model and discuss in detail the open string sector massless spectrum as well as features of Yukawa couplings. In Subsection 6.1 we also address the interpretation of the spectrum in the context of flipped $SU(5) \times U(1)_X$ and show that the choice of the $U(1)_X$ gauge symmetry is non-anomalous with the massless gauge boson. In Section 7 we summarize the results at the spectrum level and point toward future constructions that may overcome phenomenological difficulties at the level of Yukawa couplings.

2 $\mathbb{Z}_4 \times \mathbb{Z}_2$ Orientifold

The construction is based on the $\mathbb{T}^6/(\mathbb{Z}_4 \times \mathbb{Z}_2)$ orientifold. We consider $\mathbb{T}^6$ to be a six-torus factorized as $\mathbb{T}^6 = \mathbb{T}^2 \times \mathbb{T}^2 \times \mathbb{T}^2$ whose complex coordinates are $z_i$, $i = 1, 2, 3$ for the $i$-th two-torus, respectively. The $\theta$ and $\omega$ generators for the orbifold group $\mathbb{Z}_4 \times \mathbb{Z}_2$ are associated with twist vectors $(1/4, -1/4, 0)$ and $(0, 1/2, -1/2)$, respectively; they act on the complex coordinates of $\mathbb{T}^6$ as

$$\theta : (z_1, z_2, z_3) \rightarrow (iz_1, -iz_2, z_3),$$
Table 1: Wrapping numbers of the four O6-planes, fixed under the $Z_2 \times Z_2$ action of $\theta^2$ and $\omega$ generators. $b$ is equal to 0 and $\frac{1}{2}$ for rectangular and tilted third two-torus, respectively.

| Orientifold Action | O6-Plane | $(n^1,m^1) \times (n^2,m^2) \times (n^3,m^3)$ |
|--------------------|----------|-------------------------------------------|
| $\Omega R$         | 1        | $(1,0) \times (1,1) \times (4b,-2b)$      |
| $\Omega R\omega$   | 2        | $(1,0) \times (1,-1) \times (0,1)$        |
| $\Omega R\theta^2\omega$ | 3     | $(0,-1) \times (1,1) \times (0,1)$       |
| $\Omega R\theta^2$ | 4        | $(0,-1) \times (-1,1) \times (4b,-2b)$    |

\[
\omega : (z_1, z_2, z_3) \rightarrow (z_1, -z_2, -z_3).
\] (1)

The orientifold projection is implemented by gauging the symmetry $\Omega R$, where $\Omega$ is world-sheet parity, and $R$ acts as

\[
R : (z_1, z_2, z_3) \rightarrow (\overline{z}_1, \overline{z}_2, \overline{z}_3)
\] (2)

We briefly review the basics of the constructions first for the model [44, 48] where the torus configurations and the corresponding O6 planes as well as their images are depicted in Figures 1 and 2, respectively. [One can also consider a somewhat different construction with the second two-torus modified to be of the same type as the first two-torus and vice versa. In addition, constructions on other type of toroidal orbifolds, such as $Z_4$-orientifold [45], $Z_6$-orientifold [46] and $Z_2 \times Z_2$-orientifold with torsion [47], and inclusion of more general cycles at orbifold singularities [45, 46, 47], resulting in fractional D-brane configurations branes are of interest. These aspects of constructions will be discussed elsewhere.]
Figure 1: The locations of O6-planes, fixed under the orientifold actions \( \Omega R \), \( \Omega R\omega \), \( \Omega R\omega^2 \), and \( \Omega R\theta^2 \) (denoted by bold solid lines) for a factorized six-torus with the third two-torus tilted.
Figure 2: The locations of O6-planes fixed under the orientifold actions $\Omega R\theta$, $\Omega R\omega\theta$, $\Omega R\omega\theta^3$, and $\Omega R\theta^3$ (denoted by bold solid lines) for the factorized six-torus with the third two-torus tilted.
When a specific brane configuration is invariant under these orbifold actions, the corresponding Chan-Paton factors are subject to their projections, as discussed in the following subsections. [The fact that D6-branes are invariant under orbifold projections does not imply that their intersection points will be. The final spectrum, however, turns out to be rather insensitive to this subtlety in the case of the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold construction. See [25] for further discussions.]

There are four kinds of orientifold 6-planes (O6-planes) due to the action of $\Omega R$, $\Omega R\omega$, $\Omega R\theta^2\omega$, and $\Omega R\theta^3$, respectively. Their configurations are tabulated in Table 1 and presented geometrically in Figure 1, respectively. The corresponding images under the orbifold actions $\theta$ are given in Table 2 and Figure 2.
Table 3: Chiral spectrum for intersecting D6-branes wrapping three cycles \( \pi_a \) [24]. We choose a convention that the negative intersecting numbers below correspond to the left-handed chiral superfields in the representations displayed in the first column.

| Representation | Multiplicity |
|----------------|--------------|
| \( \pi_a \)   | \( \frac{1}{2} (\pi_a \circ \pi_{a'} + \pi_a \circ \pi_{O6}) \) |
| \( \pi_{a'} \) | \( \frac{1}{2} (\pi_a \circ \pi_{a'} - \pi_a \circ \pi_{O6}) \) |
| \( (\pi_a, \pi_b) \) | \( \pi_a \circ \pi_b \) |
| \( (\pi_a, \pi_{b'}) \) | \( \pi_a \circ \pi_{b'} \) |

3 Tools for \( \mathbb{Z}_N \times \mathbb{Z}_M \) orientifolds

3.1 Massless Open String Spectrum

For the orientifold models with intersecting D6-branes wrapping three-cycles, inherited from the six-torus, the chiral spectrum, arising from open string sectors, can be determined geometrically from the intersection numbers of the three-cycles the D6-branes are wrapped around. For \( N_a \) D6-branes that wrap three-cycles, not invariant under the anti-holomorphic involution, the gauge symmetry is \( U(N_a) \). For this case the general rule for determining the massless left-handed chiral spectrum is presented in Table 3 (for details see, e.g., [24]). Open strings stretched between a D-brane and its \( \sigma \) image are the only ones left invariant under the combined operation \( \Omega \circ \sigma (-1)^{F_L} \). Here \( F_L \) refers to the left-moving world-sheet fermion number. Therefore, they transform in the antisymmetric or symmetric representation of the gauge group, indicating more general representations in an orientifold background. These representations play an important role in the construction of \( SU(5) \)
Grand Unified Models (GUT’s).

To apply Table 3 to concrete models, one has to compute the intersection numbers of three-cycles. We focus only on the three-cycles $\pi_a$ that are “inherited” from the three-cycles of the six-torus. In the case of toroidal orbifolds, such as $T^6/(\mathbb{Z}_N \times \mathbb{Z}_M)$, the application and geometric interpretation of the Table 3 for such cycles can be made explicit.

The spectrum of Table 3 implies the computation for the intersection numbers on the orbifold, and not on the ambient torus (see also [24]). For three-cycles $\pi_a$ on the orbifold space, which are inherited from the torus, the three-cycles $\pi^T_a$ on the torus are arranged in orbits of length $N$ and $M$, under the $\mathbb{Z}_N \times \mathbb{Z}_M$ orbifold group, i.e.,

$$
\pi_a = \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} \theta^i \omega^j \pi_{T}^a ,
$$

where $\theta^i$ and $\omega^j$ denote the generators of $\mathbb{Z}_N$ and $\mathbb{Z}_M$, respectively. The definition of the orientifold image cycle $\pi_{a'}$ is analogous, with $\pi_{T}^a$ replaced by the orientifold image on the torus, denoted by $\pi_{T}^a$. The three-cycles $\pi_{O6}$ of the O6 planes, fixed under the orientifold action, take the following analogous form:

$$
\pi_{O6} = \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} \theta^i \omega^j \pi_{O6}^T .
$$

Such orbits can then be considered as a three-cycle of the orbifold, where the intersection number is given by

$$
\pi_a \circ \pi_b = \frac{1}{NM} \left( \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} \theta^i \omega^j \pi_{T}^a \pi_{T}^b \right) \circ \left( \sum_{i'=0}^{N-1} \sum_{j'=0}^{M-1} \theta^{i'} \omega^{j'} \pi_{T}^a \pi_{T}^b \right) .
$$

[In addition to these untwisted three-cycles, certain twisted sectors of the orbifold action can give rise to so-called twisted three-cycles, associated with the fractional D-branes at orbifold singularities, but we shall not include these cycles in our consideration.] Table 3 only gives the chiral spectrum of
an intersecting D6-brane model. To compute the non-chiral spectrum one has to employ the enhanced supersymmetry associated with a specific $T^2$, as will be discussed in a concrete case for the $\mathbb{Z}_4 \times \mathbb{Z}_2$ orientifold in Section 5.

For a factorizable product of three-one cycles on the six-torus, $\pi_a^T$ can be explicitly written in terms of wrapping numbers $(n^a_i, m^a_i)$ along the fundamental cycles $[a^i]$ and $[b^i]$ on each $T^2$. Note also, that for the specific orbifold $\mathbb{Z}_4 \times \mathbb{Z}_2$, the generators $\theta^2$ and $\omega$ are those of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ subgroup; these group elements transform each D-brane configuration into itself, while $\theta$ and $\theta^3$ produce a new image of the original D6 brane configuration, and thus belong geometrically to a different open-string sector. We shall explicitly employ this geometric feature of the construction and obtain distinguished features of the spectrum in different open string sectors.

3.2 Homological Tadpole Cancellation and K-Theory Constraints

The equation of motion for the Ramond-Ramond (R-R) field strength $G_8 = dC_7$ takes the form:

$$\frac{1}{\kappa^2} d \star G_8 = \mu_6 \sum_a N_a \delta(\pi_a) + \mu_6 \sum_a N_a \delta(\pi_a') - 4\mu_6 \delta(\pi_{O6})$$

where $\delta(\pi_a)$ denotes the Poincaré dual three-form of $\pi_a$ cycles, $\pi_a'$ its orientifold image, $\kappa$ is the 10-dimensional Planck constant and $\mu_6$ is the D6-brane tension.

Since the left hand side of eq. (6) is exact, the R-R tadpole cancellation condition reduces to a simple condition on the homology classes (see, [24] and references therein.):

$$\sum_a N_a (\pi_a + \pi_a') - 4\pi_{O6} = 0$$

The above condition implies that the overall three-cycle wrapped by D-branes and orientifold planes is trivial in homology. Again, for toroidal-type
compactifications with D6-branes wrapping factorizable three-cycles inherited from the six-torus, the explicit expression \( [5] \), which are specified by wrapping numbers \( (n^i, m^i) \) along the fundamental cycles \( [a^i] \) and \( [b^i] \), take a simpler form written down in the following sections.

K-Theory constraints can be formulated \([49]\) in terms of probe D6-branes that wrap three-cycles of \( O_6^i \) planes, and whose gauge symmetry is \( Sp(2k_i) \). The K-theory constraints imply that the massless spectrum associated with the intersection of such probe D6-branes with the D6-brane configurations of the model has an even number of fundamental representations \( 2k_i \) of \( Sp(2k_i) \), and thus the construction is free from discrete global anomalies \([50]\). This condition can again be expressed in terms of intersection numbers of cycles associated with \( \theta^i \omega^j \pi^T_{O_6} \) planes with the cycles \( \pi_a \) of the D6-brane configurations and can be written schematically in the form:

\[
(\pi_a + \pi'_a) \circ (\theta^i \omega^j \pi^T_{O_6}) \in 2\mathbb{Z}.
\]

### 3.3 Supersymmetry

The supersymmetry condition for a three-cycle \( \pi_a \) requires that it is a special Lagrangian. Namely, the restriction of the Kähler form \( J \) of the Calabi-Yau space on the cycle vanishes, i.e., \( J|_{\pi_a} = 0 \) and the three-cycle is volume minimizing, i.e., the imaginary part of the three-form \( \Omega_3 \) vanishes when restricted to the cycle, \( \Im(e^{i\varphi_a} \Omega_3)|_{\pi_a} = 0 \). The parameter \( \varphi_a \) determines which \( N = 1 \) supersymmetry is preserved by the branes. This supersymmetry condition also ensures that the Neveu-Schwarz-Neveu-Schwarz (NS-NS) tadpoles are cancelled as well.

For factorizable three-cycles of toroidal compactifications these conditions become geometric conditions:

\[
\phi_1^a + \phi_2^a + \phi_3^a = 0 \mod 2\pi,
\]

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where $\phi^a_i$ is the angle with respect to the one-cycle on the i-th two-torus of the orientifold plane O6$^1$ of Table 1. This condition can be rewritten in terms of $\tan \phi^a_i$’s as:

$$
\sum_{i=1}^{3} \tan \phi^a_i = \prod_{i=1}^{3} \tan \phi^a_i , \quad \cos(\sum_{i=1}^{3} \phi^a_i) > 0.
$$

(10)

Again, these condition can be expressed in terms of the three-cycle wrapping numbers and toroidal complex structure moduli $U^{(i)} \equiv \frac{R^{(i)}}{R^{(1)}}$.

### 4 $SU(5)$ Grand Unified Model Constructions

In the following we shall summarize the features of the spectrum and couplings of the spectrum of the Grand Unified Models (GUT) for intersecting D6-branes on $\mathbb{Z}_N \times \mathbb{Z}_M$ orientifolds.

The intersecting D6-branes on orientifolds allow for the construction of GUT models, based on the Georgi-Glashow $SU(5)$ gauge group. [Such supersymmetric Type IIA GUT constructions have a strong coupling limit, which is represented as a lift on a circle to M-theory compactified on singular seven dimensional manifolds with $G_2$ holonomy (see [52], and references therein).]

The key feature in these constructions is the appearance of antisymmetric (and/or symmetric) representations, i.e., $10$ ($15$) of $SU(5)$, which appear at the intersection of a D6-brane with its orientifold image (see Table 3). Therefore $10$-plets, along with the bi-fundamental representations ($\overline{5}$, $N_b$) at the intersections of $U(5)$ branes with $U(N_b)$ (or $Sp(N_b)$) branes, constitute chiral fermion families of the Georgi-Glashow $SU(5)$ GUT model. [Note that the gauge boson for the $U(1)$ factor of $U(5)$ is massive, and the anomalies associated with this $U(1)$ are cancelled via the generalized Green-Schwarz mechanism, ensured by the cancellation of the homological R-R tadpole conditions (7) and the Chern-Simons terms in the expansion of the Wess-Zumino]
D6-brane action. For details, as applied to the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold, see the Appendix of [25].

For toroidal orbifolds with D6-branes wrapping factorizable three-cycles, inherited from the torus, there are three chiral superfields in the adjoint representations on the world-volume of the D6-branes. In general, there are additional chiral superfields in the adjoint representation, associated with the intersections of the D6-brane configuration $a$ and its non-equivalent orbifold images of the type $\theta^i \omega^j a$. The first set of fields are moduli associated with the D6-brane splitting (or continuous Wilson lines), and are a consequence of the fact that such cycles are not rigid. The second set of fields are moduli associated with the brane recombination and can also lead to further breaking of the $U(N_a)$ gauge symmetry. In the effective theory, this geometric picture corresponds respectively to turning on vacuum expectation values (VEVs) of D-brane splitting and D-brane recombination moduli fields, and it can spontaneously break $SU(5)$ down to the Standard Model (SM) gauge group. Since in the case of parallel brane splitting all the SM gauge group factors arise from D6-branes that wrap parallel, homologically identical, cycles, this framework automatically ensures that there is a gauge coupling unification.

[Within this framework one can in principle address the long standing problem of doublet-triplet splitting, i.e., ensuring that after the breaking of $SU(5)$ the doublet of $5_H$, the Higgs multiplet responsible for the electroweak symmetry breaking, remains light while the triplet becomes heavy. In the strong coupling limit, i.e., within M-theory compactified on $G_2$ holonomy spaces [51], this mechanism was addressed via discrete Wilson lines with different quantum numbers for the doublet and the triplet fields. However, in the present context the Wilson lines, associated with the D-brane splitting mechanism, are continuous, due to the non-rigidity of the three-cycles. This problem can be remedied by introducing rigid three-cycles associated with orbifold singularities, see [47]; however, no explicit example of a chiral GUT]
model with discrete Wilson lines was found there.]

The most important drawback of these constructions is the absence of Yukawa couplings of the up-quark sector. Namely, the SM Higgs candidates are in fundamental ($\mathbf{5}_H$ and $\mathbf{5}_H$) representations of $U(5)$, and thus only Yukawa couplings of the type: $\mathbf{5} \, \mathbf{10} \, \mathbf{5}_H$ are present, while the couplings of the type $\mathbf{10} \, \mathbf{10} \, \mathbf{5}_H$ are absent due to the $U(1)$ charge conservation. In the strongly coupled limit of Type IIA theory, which corresponds to M-theory compactified on singular $G_2$ holonomy space, the absence of perturbative Yukawa couplings to the up-quark families may be remedied by non-perturbative effects, though see [41].

The explicit supersymmetric GUT model was first constructed on the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold model [25], and a systematic construction of three-family models was given in [26]. There are on the order of twenty models with three families; however, they necessarily include in addition to three copies of $\mathbf{10}$-plets also three copies of $\mathbf{15}$-plets, i.e., the chiral supermultiplets in the symmetric representations of $SU(5)$. These additional $\mathbf{15}$-plets transform under $SU(3)_C \times SU(2)_Y \times U(1)_Y$ as:

$$\mathbf{15} \rightarrow (6, 1)(-\frac{2}{3}) + (1, 3)(+1) + (3, 2)(+1) + (1, \mathbf{6}).$$ (11)

These multiplets therefore provide candidate exotic SM fields, in particular, $\mathbf{3}$’s of $SU(2)_L$. Since the $\mathbf{15}$-plets can couple to $\mathbf{5}$ and $\overline{\mathbf{5}}_H$, after symmetry breaking down to the Standard Model, triplets $\mathbf{3}$ of $SU(2)_L$ could couple to the Standard Model Higgs fields and/or leptons and could in principle provide appropriate Majorana-type couplings for neutrino masses. However, as described in the Introduction, for such a mechanism to be effective one requires the $\mathbf{15}$ to occur in a vector pair of $(\mathbf{15} + \overline{\mathbf{15}})$ which could become

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3In the flipped $SU(5)$ context [37, 38] the absence of such couplings corresponds to those in the down-quark sector and thus their absence may be a somewhat less severe problem.

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very massive, as well as the needed couplings. [For further supersymmetric constructions of such models, see [37]. For examples without chiral 15-plets, which are obtained after the inclusion of Type IIA fluxes, leading to AdS$_4$ vacua, see [38].]

Note, however, that for $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold models the orbifold images of the three-cycles wrapped by D6-branes are the same as the original brane configuration, and thus the spectrum associated with the appearance of 10 and 15 arises from a single chiral sector; therefore, in this case there is no possibility of generating from this single sector chiral 10-plets and vector-pairs of 15 and $\mathbf{15}^\ast$-plets, associated with the genuine $\mathcal{N} = 2$ supersymmetric sector. Only in the case where such a mass spectrum can be realized, could the $\mathcal{N} = 2$ vector pairs of 15-plets dynamically obtain masses by the D-brane-splitting mechanism. On a specific two-torus where the one-cycles of the brane configuration $a$, its orientifold image $a'$ and the specific orientifold planes O6$^i$ are parallel, the D-brane splitting mechanism would be responsible for generating the mass for these pairs.

As we have emphasized earlier, for other orbifolds, such as $\mathbb{Z}_N \times \mathbb{Z}_M$ the massless matter appears not only from the intersection of D6-branes of the original configuration $a$ and its orientifold image $a'$, but also in the sector associated with the orbifold images $\theta^i \omega^j a$. In particular, for the $\mathbb{Z}_4 \times \mathbb{Z}_2$ orbifold for each D6-brane configuration $a$ there is the sector associated with its $\mathbb{Z}_4$ orbifold image $\theta a$, and thus some sectors may result in chiral and others in non-chiral representations. It is this feature of more general orbifolds that we shall explore for the construction of the GUT Models which result in the desirable vector-pair representations of the 15-plets in one sector and the chiral matter in the 10 representation in another sector.
5 Model construction

We shall now apply the general tools described in Section 3 for model construction to the case of the \( \mathbb{Z}_4 \times \mathbb{Z}_2 \) orientifold of Section 2. For this model the form of the toroidal configuration and the O6 planes are represented in Figures 1-2. The explicit assignment of the O6\(^i\) and \( \theta \)O6\(^i\) planes in terms of the wrapping numbers \((n_i, m_i)\) for the basis one-cycles \([a_i]\) and \([b_i]\) is given in Tables 1-2.

5.1 Global Consistency Constraints

From the general expressions for the global R-R consistency conditions (7) we obtain the following conditions on the D-brane wrapping numbers (see also [44]):

\[
\sum_{a=1}^{K} N_a A n_3^a = 32 \tag{12}
\]

\[
\sum_{a=1}^{K} N_a B (m_3^a + b n_3^a) = -\frac{32}{4^b},
\]

where

\[
A \equiv n_1^a n_2^a - m_1^a m_2^a + n_1^a m_2^a + m_1^a n_2^a
\]

\[
B \equiv -n_1^a n_2^a + m_1^a m_2^a + n_1^a m_2^a + m_1^a n_2^a,
\]

and \( b = 0, \frac{1}{2} \) for the third two-torus untilted and tilted, respectively. \( A \) and \( B \) are completely symmetric with respect to the interchange of \((n_1^a, m_1^a)\) and \((n_2^a, m_2^a)\) wrapping numbers.

In the calculation of homological R-R tadpole cancellation we have taken into account for each \( a \) configuration with the wrapping numbers:

\[
a-configuration : \quad (n_i^a, m_i^a), \quad i = 1, 2, 3, \tag{14}
\]
its \(\theta a\) image with the wrapping numbers:

\[
\theta a\text{ – image} : \left[\left(-m_1^a, n_1^a\right), \left(m_2^a, -n_2^a\right), \left(n_3^a, m_3^a\right)\right],
\]

(15)

and the orientifold images, specified by the wrapping numbers:

\[
a'\text{ – orientifold image} : \left(n_1^a, -m_1^a\right), \left(m_2^a, n_2^a\right), \left(n_3^a, -m_3^a - 2b n_3^a\right),
\]

(16)

\[
\theta a'\text{ – orientifold image} : \left(-m_1^a, n_1^a\right), \left(-n_2^a, m_2^a\right), \left(n_3^a, -m_3^a - 2b n_3^a\right),
\]

(17)

As for the action of the \(\mathbb{Z}_2 \times \mathbb{Z}_2\) subgroup elements, \(\theta^2\), \(\omega\) and \(\theta^2 \omega\), which render the D6-brane configuration elements invariant, we appropriately normalized the intersection numbers à la (5).

Allowing for \(4k_i\) branes wrapping the three-cycles of \(O6^i\) planes (Table 1) and their \(\theta\) images \(\theta O6^i\) (Table 2) with the resulting gauge symmetry \(Sp(2k_i)\), the homological R-R tadpole cancellation condition (12) can be written in the following way:

\[
\begin{aligned}
\sum_{a=1}^{K'} N_a A n_3^a &= 32 - 8 \sum_{i=1,4} k_i \\
\sum_{a=1}^{K'} N_a B (m_3^a + b n_3^a) &= -\frac{32}{4^6} + \frac{8}{4^6} \sum_{i=2,3} k_i ,
\end{aligned}
\]

(18)

where now on the left-hand side the sum is only over the wrapping numbers of the D6-brane configurations that are not parallel with the \(O6^i\) plane, and are associated with the \(U\left(\frac{N_a}{2}\right)\) gauge symmetry.

The K-theory constraints (8) take the form:

\[
\begin{aligned}
\sum_{a=1}^{K} N_a A (m_3^a + b n_3^a) &\in \frac{4\mathbb{Z}}{4^6} \\
\sum_{a=1}^{K} N_a B n_3^a &\in 4\mathbb{Z} .
\end{aligned}
\]

(19)

As discussed in Section 3 these conditions ensure that the probe branes wrapping cycles of \(O6^i\) branes (and their \(\theta\) images), which are associated with the
appearance of the Sp(2k_i) gauge symmetry, induce a massless spectrum at
the intersections with the U(N_a/2) D6-branes that have an even number of
chiral superfields in the fundamental 2k_i - representation of Sp(2k_i), and it
is thus free of discrete global gauge anomalies [50].

5.2 Supersymmetry Constraints

For the supersymmetry conditions (10), the expressions for tan φ_i^a of the
angles φ_i^a with respect to the O6^1 plane take the form [44]:

\[
\tan \phi_1^a = \frac{m_1^a}{n_1^a}, \quad \tan \phi_2^a = \frac{(m_2^a - n_2^a)}{(n_2^a + m_2^a)}, \quad \tan \phi_3^a = -\frac{(m_3^a + b n_3^a)U^{(3)}}{n_3^a},
\]

(20)

where \( U^{(3)} \equiv \frac{\kappa_3^2}{\kappa_1^2} \) is the complex structure modulus for the third two-torus.

The supersymmetry conditions (10) in turn take the form

\[
\frac{m_1^a}{n_1^a} + \frac{(m_2^a - n_2^a)}{(n_2^a + m_2^a)} + \frac{(m_3^a + b n_3^a)U^{(3)}}{n_3^a} = \frac{m_1^a (m_2^a - n_2^a) (m_3^a + b n_3^a)U^{(3)}}{n_1^a (n_2^a + m_2^a) n_3^a},
\]

\[-An_3^a + B(m_3^a + b n_3^a)U^{(3)} \leq 0. \]

(21)

We can solve the above conditions for \( U^{(3)} \):

\[
U^{(3)} = -\frac{n_3^3 B}{(m_3^a + b n_3^a)A} > 0, \quad \frac{n_3^3}{A} (A^2 + B^2) \geq 0. \]

(22)

These conditions provide strong constraints on the allowed wrapping numbers
\((n_i^a, m_i^a)\) of different D6-brane configurations, since (22) should be satisfied
for each such configuration.

5.3 Spectrum

The gauge symmetry of \( N_a \)-D6 branes, with the general wrapping numbers
\((n_i^a, m_i^a)\) is \( U(N_a/2) \). For \( N_a \) D6-branes positioned at any of the O6 planes
in Table [1] (and depicted in Figure [1]), the additional orientifold projection
introduces an $Sp(N_a/2)$ gauge group in the open string sector on the D-brane, i.e., in this case a multiple of 4 branes is needed. The spectrum on the $U(N_a)$ D6-brane consists of 3-fields in the adjoint representation, corresponding to the D-brane splitting moduli as well as 

$$I'_{a\theta a} = \left[ (n_1^a)^2 + (m_1^a)^2 \right] \left[ (n_2^a)^2 + (m_2^a)^2 \right]$$ \hspace{1cm} (23)

D-brane recombination moduli, also in the $N = 2$ sector of the adjoint representation. In the $Sp(2k_i)$ sector, the multiplicity of the D-brane splitting and recombination moduli is 3 and 2, respectively, and they are in the symmetric representation of $Sp(2k_i)$ symmetry group \[44\].

In the following we shall focus on the symmetric and anti-symmetric representations of the D-brane configurations, associated with the GUT $SU(5)$ symmetry group. As discussed in Section 3, we shall split the calculation into sectors associated with a-unitary brane configuration and its $Z_4$ orbifold image $\theta a$. Employing the expression for the intersection numbers (Table 3 and eqs. (5)) we obtain the following expressions for the intersection numbers:

$$I_{a a'} = 4n_1^a m_1^a \left[ (n_2^a)^2 - (m_2^a)^2 \right] n_3^a (m_3^a + b n_3^a) ,$$

$$I_{(\theta a) a'} = 4 \left[ (n_1^a)^2 - (m_1^a)^2 \right] n_2^a m_2^a n_3^a (m_3^a + b n_3^a) .$$ \hspace{1cm} (24)

Note that $I_{a(\theta a')} = I_{(\theta a') a'}$.

The intersection of the $a$-D6-brane configuration with the orientifold planes can be split into the one with intersections with $O6_{tot} \equiv \sum_{i=1}^{4} O6^i$ planes (see Table 4 for the wrapping numbers of $O6^i$) and the second one, for intersections with $\theta O6_{tot} \equiv \sum_{i=1}^{4} \theta O6^i$ (see Table 5 for the wrapping numbers of $\theta O6^i$). Due to the symmetry of the configurations it turns out that these intersection numbers are the same for both sectors, and they take the form:

$$I_{a O6_{tot}} = I_{a \theta O6_{tot}} = 4 \left[ A (m_3^a + b n_3^a) + B b n_3^a \right] .$$ \hspace{1cm} (25)

Due to the symmetry of the construction the intersection number of the $\theta a$ configuration with the $O6_{tot}$ and $\theta O6_{tot}$ sector has the same intersection
number as above, i.e.,

$$I_{(\theta a)O6\text{tot}} = I_{aO6\text{tot}}.$$  \hfill (26)

The multiplicity of the symmetric and anti-symmetric representations in $a$ and $\theta a$ sectors is determined by the following expressions:

$$I_{a}^{\text{symm,antisymm}} = \frac{1}{2} \left( I_{a a'} \pm \frac{1}{2} I_{aO6\text{tot}} \right),$$

$$I_{\theta a}^{\text{symm,antisymm}} = \frac{1}{2} \left( I_{(\theta a) a'} \pm \frac{1}{2} I_{(\theta a)O6\text{tot}} \right).$$  \hfill (27)

In the calculation of the intersection numbers (27) we have accounted for the multiplicity of the equivalent configurations associated with the $\mathbb{Z}_2 \times \mathbb{Z}_2$-part of the orbifold action, i.e., those associated with $\mathbb{Z}_2 \times \mathbb{Z}_2$ group elements: $\theta^2$, $\omega$ and $\theta^2 \omega$.

An important observation is in order: since there are two separate sectors, associated with the appearance of symmetric and anti-symmetric representations, there is now a possibility that in one sector the representations are for example $15$-plets and in another sector, $\overline{15}$-plets (and analogously for the anti-symmetric representations). However, note that such $15$-plets and $\overline{15}$ arise from the $N = 1$ sector and are thus chiral in nature.

We require there are only chiral $10$ representations, and no net chiral $15$-plets. In addition, we shall require that there is a genuine $N = 2$ sector associated with the vector pairs of $15$ (and $10$). The necessary conditions to ensure these constraints are: say, $I_{(\theta a) a'} = 0$ and $I_{a a'} = I_{aO6\text{tot}} \neq 0$. Namely, the first condition is a necessary condition to have a genuine $N = 2$ sector, and the second condition then automatically ensures that there are no net chiral $15$-plets (see eq. (27))\footnote{Note that in the case of $\mathbb{Z}_2 \times \overline{\mathbb{Z}_2}$ orientifold one has only the sector associated with $a$ and $a'$ configuration. Therefore the necessary condition to have vector pairs requires $I_{aa'} = 0$ which automatically implies the same number of chiral $10$'s and $\overline{15}$'s (or $\overline{10}$'s and $15$'s).}. To address quantitatively the appearance of vector pairs in the $N = 2$ sector, we have to focus on sectors that involve the
two-torus where both the one-cycle for $a$ and $\theta a'$ configurations are parallel as well as that of specific $\theta O6^i$-planes. In this subsector there are consequently no intersections associated on the specific two-torus, and the spectrum is that of $N = 2$ vector pairs, which can be determined in terms of the intersection numbers on the remaining four-torus as:

$$I_{a;N=2}^{\text{symm,antisymm}} = I'_{a \theta a'} \pm \sum_i I'_{a \theta O6^i},$$

where $i$ refers to the intersection numbers in the remaining four-torus and the summation is only over intersections with $\theta O6^i$-planes which are parallel with the one-cycle of $a$ configuration in the specific two-torus.

The conditions for the number of bi-fundamental representations associated with $a$ and $b$ branes take the following expression (see also [44]):

$$I_{a,b} + I_{a,\theta b} = (-n^a_3 m^b_3 + m^a_3 n^b_3)$$
$$\times[(n^a_1 m^b_1 - m^a_1 n^b_1)(-n^a_2 m^b_2 + m^a_2 n^b_2) + (n^a_1 n^b_1 + m^a_1 m^b_1)(n^a_2 n^b_2 + m^a_2 m^b_2)],$$

$$I_{a,\nu} + I_{a,\theta \nu} = (+2b n^a_3 n^b_3 + n^a_3 m^b_3 + m^a_3 n^b_3)$$
$$\times[(n^a_1 m^b_1 + m^a_1 n^b_1)(n^a_2 n^b_2 - m^a_2 m^b_2) + (n^a_1 n^b_1 - m^a_1 m^b_1)(n^a_2 m^b_2 + m^a_2 n^b_2)].$$

We would like reiterate that we have chosen a convention that the left-handed chiral superfields in representations, according to Table 3, correspond to the negative values of the above intersection numbers. (For the positive values of these intersection numbers the left-handed chiral field representations correspond to the charge-conjugated ones.)

At this point we are equipped with all the tools to construct a specific model, with relevant interesting implications for neutrino masses.

6 Explicit four-family GUT Model

A specific, globally consistent supersymmetric model with the wrapping numbers of D6-branes and their intersection numbers (expressions given in the
previous Section 5) is depicted in Table 4. The explicit chiral and non-chiral spectrum is presented in Table 5. For this model the homological (18) and K-theory (19) tadpole constraints (with respective contributions $-\frac{3}{2} \times (10 + 2)$ and $1 \times (10 + 2)$) are satisfied, and the supersymmetry conditions (21) are satisfied for the value of the complex structure modulus $U^{(3)} = \frac{2}{3}$.

Table 4: D6-brane configurations and intersection numbers for globally consistent four family Grand Unified model.

| stack | $N$ | $(n^1, m^1) \times (n^2, m^2) \times (n^3, m^3)$ | $n$ | $n'$ | $b$ | $b'$ | 1 |
|-------|-----|-----------------------------------------------|-----|-----|-----|-----|---|
| a     | 10  | $(1, 2) \times (1, 0) \times (1, -1)$         |     |     | $(5+1)\times$pairs | 4 | +1×pair | 2 | -2 | 1 |
| b     | 2   | $(1, 0) \times (0, 1) \times (1, -2)$         |     |     | -2  | 0   |     | 1 |
|       | 1   | $(1, 0) \times (1, -1) \times (0, 1)$         |     |     |     |     |     | 1 |

The gauge symmetry of the model is $U(5) \times U(1) \times Sp(4)_2$, where $Sp(4)_2$ is associated with the $\Omega R\omega$ action, i.e., O6$^2$-plane in Table 1. One can satisfy the homological and K-theory tadpoles also by replacing $Sp(4)_2$ with $Sp(2)_2 \times Sp(2)_3$, where these gauge group factors arise from the D6-branes on O6$^2$ and O6$^3$ planes, respectively.

In addition to the four-family chiral spectrum $4 \times (10 + \overline{5})$, the model possesses two pairs of the Standard model Higgs candidates $2 \times (5 + \overline{5})$. The $N = 2$ non-chiral sector consists of 5 vector pairs of $(15 + 1\overline{5})$ and 1 vector pair of $(10 + 1\overline{10})$. These vector pairs can obtain a mass due to the parallel D-brane splitting in the second two-torus. There is an additional vector pair of $(15 + 1\overline{15})$; however, its origin is chiral, i.e., it arises from the $N = 1$ sector, and thus it cannot obtain a mass from parallel D-brane splitting.

In general, one would expect that there are non-zero Yukawa couplings of both 10’s as well as 15’s to bi-linears of $\overline{5}$’s, which play a role of down-sector Standard Model Higgs fields and/or lepton doublets. In principle there could
Table 5: The chiral spectrum and non-chiral spectrum, as obtained from the information for the configuration and the intersection numbers listed in Table 4.

| Sector | $U(5) \times U(1) \times Sp(4)_2$ | Fields |
|--------|---------------------------------|--------|
| $aa$   | $(3 + 5) \times (25, 0, 1)$     | D-brane-splitting + recombination moduli |
| $bb$   | $(3 + 1) \times (1, 0, 1)$      | D-brane-splitting + recombination moduli |
| $cc$   | $(3 + 2) \times (1, 0, 6)$      | D-brane-splitting + recombination moduli |
| $aa'$  | $3 \times (10, 0, 1) + 1 \times (15, 0, 1)$ | chiral 10-fermion families + chiral-15 |
| $\theta aa'$ | $1 \times (10, 0, 1) + 1 \times (15, 0, 1)$ | chiral 10-fermion family + chiral-15 |
| $bb'$  | $1 \times (1, -2, 1)$           | non-chiral pairs of 10 + 15 |
| $\theta bb'$ | $1 \times (1, -2, 1)$           | “hidden sector” chiral matter |
| $ab$   | $2 \times (5, 1, 1)$            | “hidden sector” chiral matter |
| $ab'$  | $2 \times (5, 1, 1)$            | chiral 5-fermion families |
| $ac$   | $1 \times (5, 0, 4)$            | chiral SM up-Higgs |
| $bc$   | $1 \times (1, -1, 4)$           | “hidden sector” chiral matter |

Also be Yukawa couplings of $\overline{10}$’s as well as $\overline{15}$’s to bi-linears of 5’s, which are the up-sector Standard Model Higgs candidates. (For the full conformal field theory calculation of such couplings see [53], and for a detailed analysis of the classical part of the Yukawa couplings, see [54].) However, for the specific construction the only surviving Yukawa couplings are those of bi-linears of $(5, 0, 4)$’s to $(10, 0, 1)$’s, two components of $(5, 0, 4)$’s playing a role of the Standard Model (SM) down-Higgs, and two components corresponding to two fermion families.\(^5\) Unfortunately, due to the gauge invariance constraints, in this specific construction the Yukawa couplings to 15’s, $\overline{15}$’s (as well as $\overline{10}$’s) are absent. Had the couplings of 15 and $\overline{15}$ vector pairs to 5 and

\(^5\)There are also Yukawa couplings of $(5, 1, 1) \times (5, 0, 4) \times (1, -1, 4)$.
5 multiplets been present, they would have played an important role for generating Majorana type neutrino masses. Note, however, that in principle in other related constructions there does not seem to be any obstruction for such couplings to exist.

6.1 Flipped SU(5) GUT Interpretation

We would also like to point out that the above construction can be interpreted as a flipped SU(5) construction. A linear combination of $U(1)_5$ of $U(5)$, $U(1) \equiv U(1)_1$ and the Abelian subgroup $U(1)_4$ of $Sp(4)$ (which can be obtained after the D-brane splitting mechanism\footnote{For such a D6-brane splitting analysis as well as a complementary field theoretical Higgs mechanism, see \cite{55}.}) provide an adequate $U(1)_X$ of the flipped SU(5) construction:

$$Q_X = \frac{1}{4}[Q_5 - 5(Q_1 \pm Q_4)].$$

(For a brief summary of features of the flipped SU(5) see, e.g., \cite{37}.) For the specific $Q_X$ charges of the model and the spectrum interpretation, see Table 6.

This specific combination of $Q_X$ turns out to be non-anomalous, i.e. the gauge boson for $U(1)_X$ is massless. This result could be suspected from the absence of field theoretical triangular anomalies associated with the $U(1)_X$ gauge field, and therefore the generalized Green-Schwarz contributions to these anomaly cancellations are absent\footnote{For a detailed analysis of the cancellation of gauge and gravitational anomalies for the $\mathbb{Z}_2 \times \mathbb{Z}_2$, see the Appendix of \cite{25}. A generalization to other orientifolds is straightforward, but it involves a careful bookkeeping of the orbifold images of D6-brane configurations.}. In the following we show that the Chern-Simons term, which plays a role in the generalized Green-Schwarz mechanism and is responsible for the mass of the $U(1)_X$ gauge boson, is indeed absent.
The specific Chern-Simons term, responsible for the mass of the $U(1)_a$ gauge field for the D6-brane configuration $a$, arises in the expansion of the D6-brane Wess-Zumino action. (For details and an application to $\mathbb{Z}_2 \times \mathbb{Z}_2$-orientifold see the Appendix of [25].) It has the following form:

$$2N_a (p_I^a + p_{I \theta}^a) \int_{\mathbb{R}^{1,3}} B_I \wedge F_a.$$  \hfill (31)

In the above expression, the factor of two accounts for the same contribution from the orientifold images, $F_a$ is the $U(1)_a$ gauge field strength, $B_I$'s are the two-form fields (dual to the axion fields $\Phi_I$ of toroidal moduli), and $(p_I^a, p_{I \theta}^a)$ are respective wrapping numbers of the D6-brane configuration $a$ and its $\theta a$ image along the the $I$-th three-cycle of the lattice $\Lambda^I$:

$$\Lambda^I = \{-[b_1^I] \times [b_2^I] \times [b_3^I], [b_1^I] \times [a_2^I] \times [a_3^I], [a_1^I] \times [b_2^I] \times [a_3^I], [a_1^I] \times [a_2^I] \times [b_3^I]\}$$  \hfill (32)

where $([a_i^I], [b_i^I])$ are the one-cycles, parallel with and perpendicular to the

| Sector | Flipped $U(5) \times U(1) \times Sp(4)_2$ | $U(1)_X$ | Fields |
|--------|----------------------------------------|-----------|--------|
| $aa'$  | $3 \times (10, 0, 1) + 1 \times (15, 0, 1)$ | $\frac{1}{2}$ | chiral $10$-fermion families + $15$ |
| $\theta aa'$ | $1 \times (10, 0, 1) + 1 \times (15, 0, 1)$ | $\frac{1}{2} + (-\frac{1}{2})$ | chiral $10$-fermion family + $15$ |
| $bb'$  | $1 \times (1, -2, 1)$ | $\frac{5}{2}$ | chiral charged lepton |
| $\theta bb'$ | $1 \times (1, -2, 1)$ | $\frac{5}{2}$ | chiral charged lepton |
| $ab$   | $2 \times (\overline{5}, 1, 1)$ | $-\frac{3}{2}$ | chiral $5$-fermion families |
| $ab'$  | $2 \times (\overline{5}, 1, 1)$ | $-1$ | chiral SM up-Higgs |
| $ac$   | $1 \times (\overline{5}, 0, 4)$ | $2 \times \left(-\frac{3}{2} + 1\right)$ | chiral $5$-fermion families + SM down-Higgs |
| $bc$   | $1 \times (1, -1, 4)$ | $2 \times \left(\frac{3}{2} + 0\right)$ | chiral charged leptons + exotics |

Table 6: The charge assignments and the matter spectrum interpretation for a flipped $SU(5) \times U(1)_X$ GUT. $U(1)_X$ is shown to be non-anomalous.
O6$^1$-plane, respectively. Note, $\Lambda^I$ is dual to the lattice $\Sigma_I$:

$$\Sigma_I = \{ [a_1^o] \times [a_2^o] \times [a_3^o], [a_1^o] \times [b_2^o] \times [b_3^o], [b_1^o] \times [a_2^o] \times [b_3^o], [b_1^o] \times [b_2^o] \times [a_3^o] \}, \quad (33)$$

with the property that $\Lambda^I \circ \Sigma_J = \delta^I_J$.

Table 7: Wrapping numbers $p_I^a$ and $p_I^{b,a}$ along the three-cycles of the dual lattice $\Lambda^I$. Again, $b = 0, \frac{1}{2}$ for the untitled and tilted third two-torus, respectively.

| $I$ | $p_I^a$ | $p_I^{b,a}$ |
|-----|---------|-------------|
| 1   | $m_1^a(n_2^a - m_2^a)(m_3^a + bn_3^a)$ | $n_1^a(n_2^a + m_2^a)(m_3^a + bn_3^a)$ |
| 2   | $m_1^a(n_2^a + m_2^a)n_3^a$ | $-n_1^a(n_2^a - m_2^a)n_3^a$ |
| 3   | $-n_1^a(n_2^a - m_2^a)n_3^a$ | $m_1^a(n_2^a + m_2^a)n_3^a$ |
| 4   | $n_1^a(n_2^a + m_2^a)(m_3^a + bn_3^a)$ | $m_1^a(n_2^a - m_2^a)(m_3^a + bn_3^a)$ |

For a configuration with wrapping numbers $(n_i^a, m_i^a)$ (with respect to the basis one cycles $([a_i], [b_i])$ of the original six-torus), the values of $(p_I^a, p_I^{b,a})$ are listed in Table 7. The contribution of the Chern-Simons terms (31) to the $U(1)_X$ field strength involves the following linear combination of the coefficients $p_I^a$:

$$5(p_I^a + p_I^{b,a}) - 5(p_I^b + p_I^{b,b}), \quad (34)$$

where $a$ and $b$ refer to the respective configurations for $U(5)$ ($N_a = 10$) and $U(1)$ ($N_b = 2$), and whose wrapping numbers are given in Table 4. Note also, that it is the linear combination of $\frac{1}{4}(Q_5 - 5Q_1)$ charges that contributes to $U(1)_X$, and that $U(1)_4$, since it arises from the non-Abelian gauge symmetry $Sp(4)$, is automatically non-anomalous. It is now straightforward to show that these linear combinations are indeed zero for all four I’s.

While $U(1)_X$ is a suitable anomaly free candidate for the flipped $SU(5) \times U(1)_X$, the model suffers from a number of phenomenological problems.

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There is also the absence of Yukawa couplings of the GUT Higgs candidates $\mathbf{10}$ to the SM Higgs candidates $\mathbf{5}$’s, and thus the model does not address a part of doublet-triplet splitting problem, and, of course, the model also suffers from the absence of the down-sector Yukawa couplings (just as the standard $SU(5)$ GUT’s in this framework do not have perturbative top-sector Yukawa couplings). Nevertheless, the constructions of that type provide a net number of chiral $\mathbf{10}$-plets (and no net number of chiral $\mathbf{15}$-plets) as well as potential flipped $SU(5)$ GUT Higgs candidates, as $(\mathbf{10} + \mathbf{10})$ vector pairs.

In the conclusion of this Section we would like to emphasize that although at the coupling level we are faced with specific obstacles, the explicit construction presented in this paper provides us with a geometric approach to identify a desirable massless spectrum of GUT constructions. We would also like to emphasize that the geometric interpretation of the origin of the spectrum in our case allows for the clear identification of genuine $N = 2$ vector pairs of $\mathbf{15}$-plets as well as those that arise from the $N = 1$ sector. Therefore, we are able to determine which pairs can obtain mass after D-brane splitting and which are protected due to their chiral origin. At the coupling level we also have explicit techniques to calculate Yukawa couplings, although zero values of such couplings are typically determined already at the level of gauge invariance.

7 Conclusions

In this paper we have provided detailed technical tools for the construction of $SU(5)$ grand-unified models (GUT’s) with intersecting D6-branes on orbifolds different from $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifolds. Specifically, we chose the $T^6/(\mathbb{Z}_4 \times \mathbb{Z}_2)$ orientifold and the three-cycles wrapped by D6-branes that are inherited from the original six-torus $T^6$. In particular, we highlighted the new features of the spectrum, that allows in addition to the chiral sector of
10’s and 5’s also the appearance of the vector pairs of 15’s and 10’s. In the genuine $N = 2$ sector such vector pairs can obtain a mass due to a parallel D6-brane splitting in a specific two-torus. We have constructed such a globally consistent, supersymmetric model with $U(5) \times U(1) \times Sp(4)$ gauge symmetry, four-families of $(10 + \bar{5})$’s and two pairs of the Standard Model Higgs candidates. In the case that 15’s (and 10) could couple to 5 (and 5) bilinears, such Yukawa couplings would provide, after symmetry breaking (via parallel D-brane splitting) down to the Standard Model (SM), the relevant couplings to generate small Majorana-type masses for left-handed neutrinos. Unfortunately, such couplings are not present in the concrete construction, although we do not see any obstruction in principle to having such couplings in related constructions.

We have also pointed out that this construction can have an interpretation of the flipped $SU(5) \times U(1)_X$ GUT model, where we have shown that the $U(1)_X$ gauge boson remains massless. For this interpretation the $(10 + \bar{10})$-vector pairs can play the role of the GUT Higgs candidates, while there is only a net number of chiral $(10 + \bar{5})$’s, as family candidates, i.e. there are no net chiral 15’s. The concrete model has two additional singlets which could play a role of right-handed neutrinos. There are some phenomenological problems at the Yukawa coupling level. Nevertheless we expect that related constructions may well produce more realistic flipped $SU(5)$ GUT models with interesting phenomenological implications.

With an aim to construct related models that pass the tests not only at the spectrum but also at the coupling level, we plan to turn to constructions of models on other orientifolds as well as three-cycles associated with the orbifold singularities.
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