Strong CP violation in nuclear physics

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Electric dipole moments of nuclei, diamagnetic atoms, and certain molecules are induced by CP-violating nuclear forces. Naive dimensional analysis predicts these forces to be dominated by long-range one-pion-exchange processes, with short-range forces entering only at next-to-next-to-leading order in the chiral expansion. Based on renormalization arguments we argue that a consistent picture of CP-violating nuclear forces requires a leading-order short-distance operator contributing to $^3S_0$-$^3P_0$ transitions, due to the attractive and singular nature of the strong tensor force in the $^3P_0$ channel. The short-distance operator leads to $\mathcal{O}(1)$ corrections to static and oscillating, relevant for axion searches, electric dipole moments. We discuss strategies how the finite part of the associated low-energy constant can be determined in the case of CP violation from the QCD $\theta$ term by the connection to charge-symmetry violation in nuclear systems.

Introduction. Electric dipole moments (EDMs) of nuclei, atoms, and molecules are excellent probes of new sources of CP violation [1]. CP violation in the quark and lepton mixing matrices of the standard model (SM) leads to immeasurably small values for EDMs [2, 3], implying that any nonzero measurement is either due to the so-far undiscovered QCD $\theta$ term or from beyond-the-SM (SM) sources of CP violation. Current experimental EDM limits [4–6] set strong constraints on BSM models with additional CP-violating phases such as supersymmetry, leptoquarks, multi-Higgs, or left-right symmetric models, and various scenarios of electroweak baryogenesis [7]. In the framework of the SM effective field theory (SMEFT), EDM limits constrain a large set of CP-odd dimension-six operators at the multi-TeV scale, well above limits from high-energy collider experiments [8].

The interpretation of EDM experiments requires care. It is a non-trivial task to connect EDMs of complex objects such as nuclei or molecules to the underlying CP-violating source at the quark level. Recent years have seen significant theoretical improvements towards model-independent first-principle calculations of EDMs from a combination of lattice QCD [9–11], chiral EFT ($\chi$EFT) [12–14], and nuclear calculations [15–19]. The chain of logic is roughly as follows: the SMEFT framework allows for the derivation of a general set of dimension-four (the QCD $\theta$ term) and -six CP-violating operators involving light quarks, gluons, and photons. $\chi$EFT, the low-energy EFT of QCD, is used to construct the corresponding CP-violating interactions among the relevant low-energy degree of freedoms: pions, nucleons, and photons. Each interaction in the chiral Lagrangian comes with a low-energy constant (LEC) that encodes the nonperturbative QCD dynamics that is ideally calculated from lattice QCD (LQCD). EDMs can be then be calculated in terms of the LECs in the CP-odd chiral Lagrangian.

The $\chi$EFT framework provides an expansion of hadronic and nuclear amplitudes in terms of $p/\Lambda_\chi$ where $p \sim k_F \sim m_\pi \sim \mathcal{O}(100\text{MeV})$ and $\Lambda_\chi \sim 4\pi F_\pi \sim \mathcal{O}(1\text{GeV})$ [20, 21], where $F_\pi \simeq 92.4\text{ MeV}$ is the pion decay constant. The electric dipole form factors of nucleons were calculated up to next-to-next-to-leading order (N$^2$LO) in the chiral expansion [22–25]. EDMs of nuclei require the derivation of both CP-conserving and CP-violating forces and currents. The CP-odd nucleon (NN) potential was calculated up to N$^3$LO in Refs. [19, 26] and used to calculate EDMs of light nuclei and diamagnetic atoms [15–19].

The derivation of the CP-odd NN potential of Refs. [19, 26] is based on Weinberg’s power-counting scheme [27]. In this scheme, the CP-odd potential arises from one-pion-exchange (OPE) diagrams, whose LECs can in principle be fixed from processes involving just nucleons and pions (only in principle as $\pi N$ scattering experiments are not sufficiently accurate). Chiral symmetry does not forbid purely nuclear short-distance interactions with LECs that can only be fixed in nuclear systems. Indeed, in the CP-conserving potential the leading-order (LO) potential consists of OPE diagrams and two non-derivative contact interactions in $^1S_0$ and $^3S_1$ waves. In the CP-violating case, NN interactions require at least one space-time derivative and Weinberg’s power-counting scheme predicts short-distance operators to enter at N$^2$LO in the chiral expansion. This is welcome news, as it implies that nuclear EDMs can be calculated in terms of only a few LECs and ratios of EDMs can be used to pinpoint the underlying CP-violating source [28].

Weinberg’s power counting scheme is based on naive dimensional analysis (NDA) of the NN LECs [29] which is not always reliable for nuclear physics. NDA does not in all cases lead to order-by-order renormalized nuclear amplitudes [30, 31], as required in a consistent EFT. This is most clear in partial waves where OPE is attractive and non-perturbative, such as the $^3P_0$ channel, where
phase shifts show oscillatory limit-cycle-like cut-off dependence [32] that cannot be renormalized at LO in Weinberg’s scheme. The same problem affects external currents inserted in NN scattering states in perturbation theory [33, 34]. In this work, we investigate long-distance CP-violating OPE potentials and demonstrate that renormalization requires a LO short-distance operator for $^1S_0-^3P_0$ transitions. This has direct consequences for the interpretation of EDM experiments in terms of the QCD $\theta$ term or higher-dimensional operators, and axion searches via oscillating nuclear EDM experiments [35, 36].

Setup of the calculation. We first consider the case of strong CP violation from the QCD $\theta$ term. The relevant Lagrangian is given by [37, 38]
\[
\mathcal{L} = \bar{q}i\gamma \cdot \partial q - \bar{q} (\mathcal{M} - i\gamma_5 m, \partial) q ,
\]
where $q = (u, d)^T$ denotes the quark field, $\mathcal{D}_\mu$ is the color and electromagnetic covariant derivative, $\mathcal{M} = \text{diag}(m_u, m_d)$ the quark mass matrix, $m_\epsilon = m_u m_d/(m_u + m_d)$, and the QCD angle $\theta$. The relevant chiral Lagrangian can be constructed with well-known methods [39], and the leading CP-even and CP-odd pion-nucleon interactions are given by
\[
\mathcal{L}_{\pi N} = -\frac{g_A}{2F_\pi} \nabla \vec{\pi} \cdot \vec{N} \vec{\pi} \sigma N + \bar{g}_0 \vec{N} \vec{\pi} \cdot \tau N + \cdots,
\]
in terms of the non-relativistic nucleon doublet $N = (p, n)^T$ and the pion triplet $\vec{\pi}, g_A \approx 1.27$ is the nucleon axial coupling, and $\bar{g}_0 = \mathcal{O}(m_\epsilon/F_\pi)$ a CP-odd LEC. The dots denote interactions involving more pions. The QCD $\theta$ term is related by a chiral rotation to the isospin-breaking component of the quark masses [22], giving a precise determination of $\bar{g}_0$ [40]
\[
\bar{g}_0 = \delta m_N^\text{str} (1 - \epsilon^2)/4F_\pi \epsilon = - (14.7 \pm 2.3) \cdot 10^{-3} \bar{\theta},
\]
where $\delta m_N^\text{str}$ is the quark-mass induced part of the proton-neutron mass splitting that has been calculated with LQCD [41] and $\epsilon = (m_u - m_d)/(m_u + m_d)$. The value of $\bar{g}_0$ agrees with a LQCD extraction [11].

From the interactions in Eq. (2) we calculate the OPE NN potentials
\[
V_{\text{str}, \pi} = -\frac{1}{(2\pi)^3} \left( \frac{g_A}{2F_\pi} \right)^2 \bar{\tau}_1 \cdot \tau_2 \left( \sigma_1 \cdot q \right) \left( \sigma_2 \cdot q \right) q^2 + m_\pi^2,
\]
\[
V_{\text{90}} = -\frac{1}{(2\pi)^3} \frac{g_A \bar{g}_0}{2F_\pi} \bar{\tau}_1 \cdot \tau_2 i \left( \sigma_1 - \sigma_2 \right) \cdot q q^2 + m_\pi^2,
\]
where $q = p - p'$ is the momentum transfer between in- and outgoing nucleon pairs with relative momenta $p$ and $p'$ respectively ($|p| = p$ and $|p'| = p'$), and $m_\epsilon$ denotes the pion mass. In addition, we consider CP-even NN interactions in the $^1S_0$, $^3S_1$, and $^3P_0$ waves
\[
V_{\text{str}, \text{sd}} = \frac{1}{(2\pi)^3} \left( C_s P_s + C_t P_t + \frac{1}{4} p p' C_P P_p \right),
\]
where $P_{s,t,p}$ project respectively on the $^1S_0$, $^3S_1$, and $^3P_0$ waves. In Weinberg’s power counting the $S$-wave contact terms appear at LO while the $P$-wave counter term enters at $N^2$LO. To obtain the strong NN scattering wave functions we solve a Lippmann-Schwinger (LS) equation
\[
T_{\text{str}} = V_{\text{str}} + V_{\text{str}} G_0 T_{\text{str}}, \quad G_0 = (E - p^2/m_N + i\epsilon)^{-1},
\]
with $V_{\text{str}} = (V_{\text{str}, \pi} + V_{\text{str}, \text{sd}}) f_A(p, p')$, where $f_A(p, p')$ is a regulator function
\[
f_A(p, p') = e^{-(p/p')^4} e^{-((p'/p)^4},
\]
in terms of a momentum space cut-off $\Lambda$. The LS equation is solved numerically for a wide range of $\Lambda$ to ensure that observables are cut-off independent.

We briefly discuss results for waves with total angular momentum $j = 0, 1$ and give explicit results in the Appendix. Solving the LS equation for just the strong OPE potential leads to $^1S_0$ and $^3S_1$, $^3D_1$ phase shifts and mixing angles that are cut-off dependent. In the $^3P_1$ waves, the strong OPE potential lead to cut-off independent phase shifts that at low energies agree well with experimental data. In the $^3P_0$ channel, however, the phase shifts arising from OPE are strongly cut-off dependent and undergo a dramatic limit-cycle like behavior, see Fig. 4. In Weinberg’s power counting, the regulator dependence of the $S$-wave phase shifts can be absorbed into the LO counter terms $C_s$ and $C_t$ but there is no counter term for the $^3P_0$ channel. Following Ref. [32], we promote $C_P$ to LO and fit $C_{s,t,p}$ to the phase shifts at a center-of-mass energy $E_{CM} = 5$ MeV. The resulting phase shifts are $\Lambda$ independent for a wide range of energies demonstrating that the strong wave functions are properly renormalized. The LECs $C_{s,t,p}$, of course, show significant $\Lambda$ dependence, but this is of no concern as they are not observable. All results are in agreement with Refs. [32, 42].

Having obtained renormalized scattering states, we insert the $CP$-odd potential $V_{\text{90}}$ which causes $^1S_0^3P_0$ and $^3S_1^3P_1$ transitions. We can treat $V_{\text{90}}$ to very good accuracy in perturbation theory and write
\[
T_{\text{90}} = V_{\text{90}} + V_{\text{90}} G_0 T_{\text{str}} + T_{\text{str}, \text{sd}} G_0 V_{\text{90}} + T_{\text{str}, \text{sd}} G_0 V_{\text{90}} G_0 T_{\text{str}}.
\]

The on-shell scattering matrix $T = T_{\text{str}} + T_{\text{90}}$ is related to the $S$ matrix
\[
S(E_{CM}) = 1 - i\pi m_N^3/2 E_{CM}^{1/2} T(p = p' = \sqrt{E_{CM} m_N}),
\]
where $m_N$ is the nucleon mass. In the $j = 0$ channel we parametrize the $S$ matrix by
\[
S_{j=0} = \begin{pmatrix}
    e^{2i\delta_{1S0}} & e^{i\delta_{3S0} + i\delta_{3P0}} \\
    -e^{i\delta_{3S0}} e^{i\delta_{3S0} + i\delta_{3P0}} & e^{2i\delta_{3P0}}
\end{pmatrix},
\]
where $P_{s,t,p}$ project respectively on the $^1S_0$, $^3S_1$, and $^3P_0$ waves. In Weinberg’s power counting the $S$-wave contact terms appear at LO while the $P$-wave counter term enters at $N^2$LO. To obtain the strong NN scattering wave functions we solve a Lippmann-Schwinger (LS) equation
where \( \epsilon_{SP}^0 \sim \bar{\theta} \) denotes the small \( ^1S_0-^3P_0 \) mixing angle. The \( j = 1 \) channel is more complicated due to strong \( ^3S_1-^3D_1 \) mixing, and for simplicity we expand in the small \( S-D \) mixing angle \( \epsilon \). Up to \( \mathcal{O}(\epsilon^3) \) corrections we write

\[
S_{j=1} = \left( \begin{array}{cc}
\frac{e^{2i\delta_{S1}+\delta_{1P1}}}{2} \cos 2\epsilon & ie^{i[\delta_{S1}+\delta_{1P1}]} \sin 2\epsilon \ x_{SP} \\
\frac{ie^{i[\delta_{S1}+\delta_{1P1}]} \sin 2\epsilon}{2} & e^{2i\delta_{1P1}} \cos 2\epsilon \ x_{DP} 
\end{array} \right)
\]

\[
x_{SP} = [\epsilon_{SP} + i\epsilon_0 \epsilon_{DP}] e^{i[\delta_{S1}+\delta_{1P1}]},
\]

\[
x_{DP} = [\epsilon_{DP} + i\epsilon_0 \epsilon_{SP}] e^{i[\delta_{1P1}+\delta_{1P1}]},
\]

in terms of two \( CP \)-odd mixing angles \( \epsilon_{SP}^0 \) and \( \epsilon_{DP}^0 \). \( S \) is antisymmetric in the \( S-P \) and \( P-D \) elements due to time-reversal violation. The \( CP \)-odd mixing angles \( \epsilon_{SP}^0 \) and \( \epsilon_{DP}^0 \) are in principle observable in, for example, spin rotation of polarized ultracold neutrons on a polarized hydrogen target [43], but it is unlikely that these experiments can reach a sensitivity that is competitive with EDM experiments, although neutron transmission experiments using heavy target nuclei might be up to the task [44, 45]. Nuclear EDMs can be written as linear combinations of the mixing angles in addition to contributions from \( CP \)-odd electromagnetic currents such as constituent nucleon EDMs.

The \( CP \)-odd mixing angles are observable and should be independent of the value of \( \Lambda \). We find that this is the case for \( \epsilon_{SP}^0 \) and \( \epsilon_{DP}^0 \) which quickly converge as shown in the top panel of Fig. 1. However, \( \epsilon_{SP}^0 \) shows an oscillatory behavior and even changes sign as function of \( \Lambda \).

There is no sign of convergence whatsoever. We have checked that no regulator dependence appears for any \( j = 2 \) transition after renormalizing the strong \( j = 2 \) scattering states. The difference between the behavior of the \( ^1S_0-^3P_0 \) and \( ^3\{S, D\}_{-1} \cdot ^1P_1 \) arises from the absence of a strong counter term in the \( ^1P_1 \) channel. The observed regulator dependence arises from divergences in diagrams contributing to \( T_{\bar{g}_0} \) with topology of the left diagram in Fig. 2, where \( V_{\bar{g}_0} \) is dressed on both sides by a strong short-distance interaction (an infinite number of related LO diagrams are generated by adding additional strong interactions on either side). At LO this only occurs for \( ^1S_0-^3P_0 \) transitions. In \( \chi \)EFT calculations using Weinberg’s power counting, \( P \)-wave counter terms appear at \( N^2 \)LO, but are iterated to all orders in the solution of the LS equation [46]. Divergent diagrams with the topology of Fig. 2 reappear and the \( CP \)-odd transitions become regulator dependent. In practice, this might be hard to see numerically as regulators are only varied in a tiny window around \( \Lambda = 450 \) MeV [17, 19].

The need for a counter term. The observation that \( \epsilon_{SP}^0 \) is cut-off dependent implies that \( CP \)-odd nuclear observables that depend on \( ^1S_0-^3P_0 \) mixing cannot be directly calculated from \( \bar{g}_0 \), and thus \( \bar{\theta} \) via Eq. (3). An observable that shows regulator dependence in an EFT calculation indicates there must be an associated counter term that encapsulates missing short-distance physics and absorbs the divergence. In the present context, such counter terms are provided by short-range \( CP \)-odd interaction, see the right diagram of Fig. 2, of the form [13, 14]

\[
\mathcal{L}_{NN} = \tilde{C}_0 \left[ \tilde{N} \bar{\sigma} N \cdot \nabla (\tilde{N} \bar{\sigma} N) + \frac{1}{3} \tilde{N} \bar{\sigma} \tau \bar{\sigma} N \cdot \nabla (\tilde{N} \bar{\sigma} \tau \bar{\sigma} N) \right],
\]

which projects on \( ^1S_0-^3P_0 \). \( \tilde{C}_0 \) is a LEC that depends on \( \Lambda \) in such a way to make \( \epsilon_{SP}^0 \) \( \Lambda \)-independent. NDA suggests \( \tilde{C}_0 = \mathcal{O}(m_u \bar{\theta} / (F_\pi^2 \Lambda^2)) \) and a \( N^2 \)LO contribution, but renormalization enhances \( \tilde{C}_0 \) by \( (4\pi)^2 \) making it LO instead.

We now show that promoting \( \tilde{C}_0 \) to LO indeed renormalizes the \( ^1S_0-^3P_0 \) transition. We fit \( \tilde{C}_0 \) at a specific kinematical point to a fictitious measurement of \( \epsilon_{SP}^0 \), picking \( \epsilon_{SP,fit}^0 = 0.01 \bar{g}_0 \) at \( E_{CM} = 5 \) MeV for concreteness. The regulator dependence of \( \tilde{C}_0 \) is shown in the bottom panel of Fig. 1 and shows a limit-cycle-like behaviour driven by \( CP \). The resulting \( \epsilon_{SP}^0 \) is regulator-in-
dependent for a wide range of energies as depicted by the dashed lines in the top panel of Fig. 1. While this method accounts for the regulator-dependent part of the short-distance contributions and renormalizes the CP-odd amplitude, it cannot account for possible finite contributions from \( \bar{C}_0 \). That is, the results in Fig. 1 can shift up or down (they remain flat) if we were to pick different values for \( \epsilon_{SP,\pi}^0 \). The best way to obtain the total short-distance contribution is by fitting to a measurement of \( \delta m_N^{str} \). This is at present not possible, and even if there was data it would not be satisfactory. We would like to use such data to extract a value of \( \theta \).

**Fixing the value of the short-distance LEC.** We discuss two potential methods to obtain a value for \( \bar{C}_0 \) in the absence of data. The first one is to perform a LQCD calculation of \( NN \to NN \) scattering in the presence of a nonzero \( \bar{\theta} \) background. There have been significant recent developments in calculations of the nucleon EDM arising from the QCD \( \bar{\theta} \) term by applications of the gradient flow [11, 47], and the same techniques could be used to study four-point functions in a \( \bar{\theta} \) vacuum. A major challenge will be to control the signal-to-noise. Already for CP-conserving \( NN \to NN \) scattering, signal-to-noise considerations demand pion masses well above the physical point [48]. Going to smaller pion masses is even more daunting in case of CP violation from the \( \bar{\theta} \) term, as the signal scales as \( \sim \bar{\theta} m_\pi^2 \). If such a LQCD calculation is possible, we can obtain \( \bar{C}_0 \) from a matching calculation of \( \chi \) EFT to the lattice data after taking the appropriate continuum and infinite-volume limits.

On a shorter time-scale a more promising approach is to apply chiral-symmetry relations between the \( \bar{\theta} \) term and the quark masses similar to the relation between \( \bar{g}_0 \) and \( \delta m_N^{str} \) in Eq. (3). Using SU(2)\(_L\) \( \times \) SU(2)\(_R\) \( \chi \) EFT, the operators in Eq. (17) arise from the structures

\[
\mathcal{L}_{NN} = - \frac{i \bar{C}_0}{8} \text{Tr}[\chi_-] \left[ N \bar{\sigma}N \cdot \nabla(\bar{N}N) + \frac{1}{3} \bar{N} \tau \bar{\sigma}N \cdot \nabla(\bar{N}\tau N) \right] ,
\]

where \( \chi_- = u^\dagger \chi u - u \chi^\dagger u = \exp(i \bar{\tau} \cdot \hat{\pi} / (2F_\pi)) \), \( \chi = 2B(M + im_\star \bar{\theta}) \), and \( B = -\langle \bar{q}q \rangle / F_\pi^2 \) related to the chiral condensate. Expanding out the trace gives \( \bar{C}_0 = (Bm_\theta \bar{\theta})C_0 \) and a relation to the CP-conserving but isospin-breaking \( NN\pi \) operators [26]

\[
\mathcal{L}_{NN,\pi} = \frac{C_0 B(m_d - m_u)}{2F_\pi} \left[ \bar{N} \sigma \cdot \nabla(\bar{N}N) + \frac{1}{3} \bar{N} \tau \bar{\sigma}N \cdot \nabla(\bar{N}\tau N) \right] .
\]

These operators contribute to charge-symmetry-breaking (CSB) in \( NN \to NN\pi \) processes [49–52]. One of the LO contributions to this CSB process arises from the \( N\pi\pi \) vertex related to \( \delta m_N^{str} \) by chiral symmetry

\[
\mathcal{L}_{CSB} = - \delta m_N^{str} \frac{\bar{N} \tau \bar{\sigma}N}{4F_\pi^2} \cdot \nabla(\bar{N}\tau N) .
\]

The contact operator in Eq. (14) contributes at N\(^2\)LO in Weinberg’s counting (in agreement with Ref. [51] that regulate counter terms to N\(^4\)LO in an expansion in \( \sqrt{p/\Lambda}_\chi \)). At the pion threshold, where final-state \( \pi\pi \) interactions can be neglected, the transition operator for the process \( \chi N^0 \to \pi^0 \pi^0 \) due to Eq. (15) is of the same form as \( V_{\bar{g}_0} \). As such, the regulator dependence seen in Fig. 1 appears and \( C_0 \) must be promoted to LO for renormalization. Unfortunately the simplest process where CSB data is available, \( pn \to d\pi^0 \), is not sensitive to \( C_0 \) due to the isosinglet nature of the deuteron. This motivates an investigation of \( dd \to \alpha \pi^0 \) using renormalized \( \chi \) EFT to fit \( \bar{C}_0 \) to CSB data [53], which would provide a determination of \( \bar{C}_0 = (Bm_\theta \bar{\theta})C_0 \).

**Consequences for other sources of CP or P violation.** At the dimension-six level in the SMEFT there appear other CP-odd sources involving light quarks. The most relevant operators for the present discussion are quark chromo-EDMs and chiral-breaking four-quark operators, which are induced in a wide range of BSM models [28, 54]. In addition to the isoscalar \( \bar{g}_0 \) term in Eq. (2), the LO CP-odd chiral Lagrangian contains an isovector term

\[
\mathcal{L}_{\pi N} = \bar{g}_1 \bar{N} \pi N .
\]

A potential isotensor term is subleading for all dimension-six operators [13]. In combination with the strong \( g_A \) vertex, an OPE involving \( \bar{g}_1 \) causes \( 1S_0-3P_0 \) and \( 3S_1-3P_1 \) transitions. Strong \( 3P_1 \) interactions arise solely from long-distance OPE such that the divergent diagrams in Fig. 2 do not appear and we expect no regulator dependence for \( 3S_1-3P_1 \) transitions. This is confirmed by explicit calculations. The \( j = 0 \) transition, up to an isospin factor, shows the same regulator dependence as the \( \bar{g}_0 \) case and thus a LO isospin-breaking counter term is needed. The associated operator takes the form

\[
\mathcal{L}_{NN} = \bar{C}_1 \left[ \bar{N} \tau \bar{\sigma}N \cdot \nabla(\bar{N}N) + \bar{N} \sigma \pi \cdot \nabla(\bar{N}\tau N) \right] ,
\]

which projects unto \( 1S_0-3P_0 \), but only for the neutron-neutron and proton-proton case. The simplest EDM that depends on \( \bar{g}_1 \) is the deuteron EDM [55], which is targeted in storage-ring experiments [56]. Due to the isosinglet nature of the deuteron, its EDM only depends on...
$^3S_1-^3P_1$ transitions which do not require a counter term for renormalization. There is no such selection rule for more complex EDMs such as $^3$He, $^{199}$Hg, or $^{229}$Ra [16–19, 57, 58], and $C_1$ must be included at LO.

Finally, the finiteness of $^3S_1-^3P_1$ transitions is relevant for the field of hadronic parity ($P$) violation [59]. The LO $P$-odd, but CP-even, chiral Lagrangian induced by $P$-odd four-quark operators contains a single $\pi N$ term [60], usually parametrized as $(\vec{h}_\pi/\sqrt{2})\bar{N}(\vec{r}\times\vec{r})^3\vec{N}$ that in combination with $g_\pi$ leads to $^3S_1-^3P_1$ transitions [61, 62]. We have checked explicitly that no regulator dependence appears and no counter terms are needed. The value of $h_\pi$ has been recently determined from $P$-violating asymmetries in $\bar{n}p \rightarrow d\gamma$ [63], can thus be directly applied in calculations of other $P$-odd observables.

**Conclusion.** We have argued the need for a leading-order short-range CP-violating counter term in $^1S_0-^3P_0$ transitions that affects calculations of EDMs and CP violation in neutron-nucleus scattering at the $O(1)$ level. This directly affects the interpretation of experimental limits, and hopefully future signals, in terms of the QCD $\theta$ term and other $CP$-odd sources, and the interpretation of axion dark matter searches via oscillating EDMs. For CP violation from the $\theta$ term, we have proposed strategies to obtain the value of the associated low-energy constant, $C_0$, from existing data on charge-symmetry-breaking in pion production in few-body systems. We hope our results stimulate determinations of $C_1$ using lattice QCD and analyses of CSB data, and calculations of the impact of the short-range operator on observables of experimental interest such as (oscillating) EDMs and time-reversal-odd scattering observables.

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The LS equation is given by
\[
\int d\sigma^\alpha(p', p, \Lambda) = V_{\text{str}}^\alpha(p', p) + \sum_{\alpha'} \int dp'' p''^2 V_{\text{str}}^{\alpha'}(p', p'') G_0(p''^2) T_{\text{str}}^{\alpha'}(p'', p, \Lambda),
\]
which we solve numerically after introducing the regulator function \( f_\Lambda(p, p') \) in Eq. (7).

The phase shifts and mixing angles calculated using just the OPE potential are cut-off dependent in the \( 1S_0 \) and \( ^3S_1 - ^3D_1 \) channels, see the top panel of Fig. 3. This is resolved by including the short-distance counter terms \( C_s \) and \( C_t \) acting in the \( 1S_0 \) and \( ^3S_1 \) waves. We fit the LECs...
Using just the strong OPE potential leads to cut-off independent phase shifts in the $^1P_1$ and $^3P_1$ channels, see the top panel of Fig. 4. In the $^3P_0$ wave, however, the strong tensor force is attractive leading to phase shifts that are very sensitive to short-distance physics and the phase shifts show a limit-cycle behaviour as a function of $\Lambda$. Unlike for the $^1S_0$ and $^3S_1$ channels there does not appear a counter term that can absorb this regulator dependence in Weinberg’s power counting. We therefore promote the $^3P_0$ counter term with LEC $C_P$ in Eq. (5) to LO and fit $C_P$ to the $^3P_0$ phase-shift at $E_{CM} = 5$ MeV. With this modified power counting the phase-shifts becomes cut-off independent, see top panel of Fig. 4. The regulator dependence of $C_P$ is given in the bottom panel of Fig. 4.