Field Test of Vibration Wave Attenuation Coefficient Model in Casing Medium

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ABSTRACT-The casing string is a very good down-hole signal transmission channel due to its rigidity which means it can reduce attenuation in vibration wave propagation. Based on elastic mechanics theory, we obtain the wave equations of vibration propagating in the free casing, analyze the transmitting law of the vibration wave in the well fluid-casing-cement system and found that the attenuation of the vibration wave is mainly caused by the energy absorbed including the well fluid, the casing and the cement ring. Based on the attenuation factors which affect the wave propagation, we get the vibration wave attenuation coefficient model and calculate the relative attenuation coefficients in different casing strings. By detecting and analyzing the drilling plug signal in the second spud, the field test results prove the feasibility of long-distance transmission and the low attenuation of vibration wave in the casing string, and get the attenuation coefficient of casing size 9	extsuperscript{7}/8, which is similar to the simulation result.

1. INTRODUCTION

The casing string is a very good down-hole signal transmission channel due to its rigidity which means it can reduce attenuation in vibration wave propagation. In recent years, we have relied on the casing vibration signal for casing-damage location detection [1], adjacent well anti-collision monitoring when drill bit approaching [2, 3], perforating monitoring [4, 5] and so on.

The vibration wave propagation in the casing is a very complicated problem, and its attenuation can be divided into the following two situations. The first situation is when the vibration wave propagates in a completely free casing string. The energy attenuation is mainly related to the length of the casing string, the wall thickness of the casing, and the liquid content inside and outside the casing. Besides, casing coupling makes the casing string a periodic structure, causing a band-stop effect. The second situation is when a wave propagates in a casing with the surrounding medium, the attenuation of wave
energy is not only affected by the casing itself, but also affected by the medium surrounding, including the cement bonding quality, the elastic modulus and thickness of the cement ring, and the nature of the formation. To get accurate attenuation characteristics of casing vibration wave, in this paper, we analyze the transmitting law of the vibration wave in the well fluid-casing-cement whole system and get the attenuation coefficient model. Compare the results of field tests to ensure the accuracy of the model calculation results.

2. VIBRATION WAVE SPREAD IN CASING

2.1. Attenuation coefficient model

The coordinate system is shown in Figure 1, when the wall thickness of casing is small compared with other geometric dimensions (nominal radius, length) and the diameter-thickness ratio is greater than 10, the shell model is used for research. For an infinitely long cylindrical shell, assuming that the casing wall material is isotropic and the casing wall motion is linear elastic, using the Flugge cylindrical shell model [6], the motion equations of the cylindrical shell structure are:

\[ \begin{align*}
    \left( 1 + \frac{\mu}{2} \frac{d^2}{d\theta^2} - \frac{1}{2} \frac{d}{dx} \left( 1 - \mu \right) \frac{d}{dx} \right) U &= \left[ \frac{1}{2} \mu \frac{d^2}{d\theta^2} - \frac{1}{2} \frac{d}{dx} \left( 1 + \frac{\mu}{2} \right) \right] V \\
    \left( 1 + \frac{\mu}{2} \frac{d^2}{d\theta^2} - \frac{1}{2} \frac{d}{dx} \left( 1 - \mu \right) \frac{d}{dx} \right) W &= 0 \\
    \left( 1 + \frac{\mu}{2} \frac{d^2}{d\theta^2} - \frac{1}{2} \frac{d}{dx} \left( 1 + \frac{\mu}{2} \right) \right) W &= \left[ \frac{3}{2} \mu \frac{d^2}{d\theta^2} + \frac{3}{2} \frac{d}{dx} \left( 1 - \mu \right) \frac{d}{dx} \right] V \\
    \left( 1 + \frac{\mu}{2} \frac{d^2}{d\theta^2} - \frac{1}{2} \frac{d}{dx} \left( 1 + \frac{\mu}{2} \right) \right) W &= 0 \\
    \left( 1 + \frac{\mu}{2} \frac{d^2}{d\theta^2} - \frac{1}{2} \frac{d}{dx} \left( 1 - \mu \right) \frac{d}{dx} \right) W &= \left[ \frac{2}{2} \mu \frac{d^2}{d\theta^2} + \frac{2}{2} \frac{d}{dx} \left( 1 + \frac{\mu}{2} \right) \right] V \\
    \left( 1 + \frac{\mu}{2} \frac{d^2}{d\theta^2} - \frac{1}{2} \frac{d}{dx} \left( 1 - \mu \right) \frac{d}{dx} \right) W &= 0
\end{align*} \]

Where \( U \) represents axial displacement, \( V \) represents circumferential displacement, \( W \) represents radial displacement, \( a \) represents shell radius, \( \mu \) represents Poisson coefficient, \( E \) represents Young's modulus of shell material, \( \rho \) represents density of shell material, \( h \) represents shell thickness, \( \beta^2 \) represents Shell thickness factor, \( \beta^2 = h^2/2 \), \( \nabla^2 \) represents Laplacian, \( \nabla^2 = a^2 \partial / \partial a^2 + \partial / \partial a^3 \). The solution of the equations is:

\[ \begin{align*}
    U &= U_0 \cos(n\theta) \exp(-ik_0 x + i\omega t + i\pi/2) \\
    V &= V_0 \sin(n\theta) \exp(-ik_0 x + i\omega t) \\
    W &= W_0 \cos(n\theta) \exp(-ik_0 x + i\omega t)
\end{align*} \]
Where $k_w$ represents axial wave number, $\omega$ represents wave frequency (rad/s), $n$ represents number of circumferential modes, $U_w$ represents axial displacement amplitude, $V_w$ represents circumferential displacement amplitude, $W_w$ represents radial displacement amplitude.

When the vibration wave propagate in the well fluid-casing-cement whole system, shown in Figure 2, the attenuation of the vibration wave is mainly caused by the energy absorbed including the well fluid, the casing and the cement ring.

If the energy reduction caused by the expansion of the sphere is ignored, we can only consider the loss caused by the absorption of energy by the medium, the vibration wave propagation equation (2) can be abbreviated as:

$$A_h = A_r e^{-\alpha h}$$ (3)

Where $A_r$ represents initial amplitude of casing vibration wave, $h$ represents distance along the casing, $m$, $\alpha$ represents attenuation coefficient of the fluid-casing-cement system, 1/m.

The specific expression is as follows:

$$\alpha = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$$ (4)

Where $\alpha_1$ represents the energy attenuation coefficient caused by the vibration wave radiated into the liquid in the wellbore, 1/m, $\alpha_2$ represents energy attenuation coefficient of vibration wave propagating in casing, 1/m, $\alpha_3$ represents the energy attenuation coefficient caused by vibration wave radiation into the cement ring, 1/m, $\alpha_4$ represents energy attenuation coefficient caused by vibration wave at casing coupling, 1/m.

Based on the impact dynamics and wave spread theory[7], the energy attenuation coefficient of vibration wave propagating in casing is:

$$\alpha_2 = 2 \alpha_1 \omega \rho_1 \rho_2 \sqrt{\frac{v_s}{E_s}} \frac{R^3}{\pi E_s}$$ (5)

The energy attenuation coefficient caused by vibration wave radiation into the cement ring is:
According to (5) and (6), find out the vibration wave attenuation coefficient model as (7).

\[
\alpha = \alpha_1 \left( \frac{\omega^2 \rho_1 \rho_1 V_p^2 R}{E_i a} - \frac{\alpha_1}{C_3} \right) + \alpha_3
\]

(7)

Where \( \omega \) represents angular frequency of vibration wave, Hz, \( \rho_1 \) represents the density of the liquid in the wellbore, \( \text{kg/m}^3 \), \( \rho_2 \) represents density of casing, \( \text{kg/m}^3 \), \( R \) represents the inner radius of the casing, m, \( V_p \) represents propagation speed of vibration wave in casing, m/s, \( E_i \) represents elastic modulus of casing, MPa, \( a \) represents casing wall thickness, m, \( C_3 \) represents the propagation velocity of vibration waves in the formation, m/s.

2.2. Simulation calculation

Use common casings of different sizes, as shown in Table 1.

### Table 1  Size parameters of different casing

| Inner radius (mm) | In wall thickness (mm) | Inner radius (mm) | In wall thickness (mm) |
|-------------------|------------------------|-------------------|------------------------|
| 4 1/2             | 7.37                   | 99.56             | 9 3/8                  |
| 5 1/2             | 7.72                   | 124.26            | 10 7/8                 |
| 7 3/8             | 9.52                   | 174.635           | 13 3/8                 |

We assume that: the elastic modulus of casing \( E_i = 2.085 \times 10^9 \text{Pa} \), the density of casing \( \rho_2 \) is 7800 m/s; the density of the liquid in the wellbore \( \rho_1 \) is 1000 \( \text{kg/m}^3 \), the propagation speed of vibration wave in casing \( V_p \) is 5000 m/s, the propagation velocity of vibration waves in the formation \( C_3 \) is 2500 m/s. Assuming that the wellbore is water-based drilling fluid with a water content of more than 80%, the energy attenuation coefficient of vibration wave in water can be obtained from experimental data, \( \alpha_1 = 2.5 \times 10^{-1}/\text{m} \). Assuming that the connection at the casing collar is tight and rigid, there is no energy attenuation caused by the casing coupling, \( \alpha_3 = 0 \). According to the vibration wave attenuation coefficient model (7), calculate the attenuation coefficient of the vibration wave of different frequencies (take the frequency range of 50~1000Hz) as shown in Figure 3.

![Figure 3 The attenuation coefficient of vibration wave in different frequency (50~1000Hz)](image)
coefficient does not change much at different frequencies. Within 0–400Hz, the attenuation coefficient of vibration wave propagation in different sizes of casing changes slowly, and the order of magnitude is one thousandth, which shows that the casing can be used as a vibration wave transmission.

3. FIELD TEST DATA ANALYSIS
The field test (Figure 4) was performed on Well X (casing size: 9⅝, top of cement: to ground, float collar position: 1390m) in a certain oil field of East China. The accelerometers are installed at the surface of the technical casing using the drilling-plug Monitoring system to collect the signal. The sampling frequency is 4000Hz, and the data collection interface shown in Figure 5 (selected part while drilling plug).

In Figure 4, the cellar is filled with drilling fluid, isolating the upper part of the casing from the ground, which can effectively absorb the interference vibration of the ground noise. It can be seen from Figure 5 that the acceleration sensor at the surface of the casing head can accurately and clearly collect the vibration signal propagating along the casing generated by the drill bit when drilling a cement plug.

In order to obtain the casing propagation attenuation coefficient, playback the 50-300Hz filtered and reconstructed vibration signal of the casing collected of the well (part of the time period is from 1:18:22 to 1:36:06), as shown in Figure 6.
The drill bit has a certain speed of penetration during the drilling process, that is, it takes a certain amount of time for the drill bit to drill 1m. Suppose the depth of a certain point is \( h \), and the average value of the acceleration amplitude of the vibration wave collected when the drill bit drills 0.5m above and below the depth of the well is taken as the acceleration amplitude at the depth \( h \). The relationship between the corresponding well depth and amplitude is shown in Table 2.

![Figure 6 Playback time domain signal while drilling plug](image)

**Figure 6 Playback time domain signal while drilling plug**

Because under the same wellbore structure conditions, low drilling speed and low weight-on-bit drilling are used during the plugging process, the weight-on-bit drilling speed changes very little. It can be approximated that the bit is in stable working condition and the vibration wave generated by the rock breaking action. The energy is the same, that is, the \( A_r \) is the same. For the same well, we believe that its attenuation coefficient is independent of the well depth, that is, \( \alpha \) is constant. According to the attenuation formula (3) of the vibration wave in the casing, the transformation model has the following form:

\[
\ln A_h = -\alpha h + \ln A_r
\]  

Take \( \ln A_h \) as Y-axis and \( h \) as X-axis, the model can be changed into a straight line, and the slope of the straight line is the attenuation coefficient of the casing. Perform data fitting on the data obtained from the test well drill plug, as shown in Figure 7.

**Table 2 Amplitude of vibration signal when drilling the plug**

| Well depth/m | the acceleration amplitude (mm/s²) | Well depth/m | he acceleration amplitude (mm/s²) |
|--------------|-----------------------------------|--------------|-----------------------------------|
| 1390         | 5.9554                            | 1395         | 5.9186                            |
| 1391         | 5.9532                            | 1396         | 5.9172                            |
| 1392         | 5.9465                            | 1397         | 5.9134                            |
| 1393         | 5.9443                            | 1398         | 5.9116                            |
| 1394         | 5.9298                            | 1399         | 5.9013                            |
In Figure 7, we can get the attenuation coefficient of vibration wave propagation in the casing of this well \( \alpha = 0.0011, \) m\(^{-1}\). The attenuation coefficient of vibration wave propagating through the casing obtained by the actual test is not much different from the vibration wave attenuation coefficient tentatively calculated in Figure 2, which is basically in the same order of magnitude. This shows that the theoretically calculated data has a certain degree of credibility and it proves that the casing as a transmission medium of vibration signals has low attenuation characteristics.

4. CONCLUSIONS
The main factors affecting the propagation and attenuation of the vibration wave in the casing are theoretically analyzed, and the following conclusions are obtained through the vibration signal acquisition experiment of the drilling plug:

1. The vibration wave propagation in the casing decays exponentially, which is mainly related to the size, length, wall thickness of the casing, the liquid inside and outside the casing, the coupling and the medium around the casing, etc.

2. The propagation attenuation coefficients of vibration wave signals of different frequencies in the casing are different, and the casing has low attenuation characteristics as the transmission medium of the vibration signal in the low frequency band.

3. It is feasible to install a sensor at the surface of the casing to detect the down-hole vibration signal propagating along the casing.

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