AUTOMATED RESUMMATION OF
JET OBSERVABLES IN QCD

GIULIA ZANDERIGHI
IPPP and University of Durham
E-mail: giulia.zanderighi@dur.ac.uk

We present a master formula, with applicability conditions, which allows
us to automate the resummation of infrared and collinear logarithms
appearing in distributions of jet observables in QCD at next-to-leading
logarithmic accuracy.

1 Jet observables: fixed order and resummation

Jet observables, event shapes and jet rates, revealed themselves as one of the
richest laboratories to explore QCD. Being infrared and collinear safe (IRC),
they can be predicted with perturbative (PT) techniques, but their high sensi-
tivity to low energy emissions allows us to investigate the fairly unknown non-
perturbative regime. Most discriminatory studies make use of distribu-
tions. In integrated distributions \( \Sigma(v) \) one requires that the value of the observable
\( V(\{\vec{p}\}, k_1, \ldots, k_n) \) – a function of all secondary final state momenta \( k_i \) and of
the Born momenta after recoil from the emissions \( \{\vec{p}\} \) – be less than a fixed
value \( v \)

\[
\Sigma(v) = \int dV \frac{1}{\sigma} \frac{d\sigma}{dV} \theta(v - V(\{\vec{p}\}, k_1, \ldots, k_n)).
\]  

The inclusive phase space region, where \( v = \mathcal{O}(1) \), is dominated by events with
hard jets, which move the value of the observable far away from its Born value,
\( v = 0 \). These events can be described with fixed order PT expansions, however
they are quite rare, every additional jet being suppressed by an additional
factor of \( \alpha_s \). More common events are characterised by a large number of soft-
collinear emissions which modify only slightly the value of the observable from
its Born value, so that \( v \ll 1 \). Here fixed order predictions fail since every
power of \( \alpha_s \) is accompanied by up to two large logarithms \( L = \ln(1/v) \) of the
value of the observable. A reorganization of the PT expansion is then needed
in order to resum all leading (LL, exp\{\alpha_s L^{n+1}\}) and next-to-leading (NLL, exp\{\alpha_s L^n\}) logarithmic terms.

In the past few years the analytical resummation for a variety of observables have been presented in $e^+e^-$-collision\cite{1, 2} and DIS\cite{3}. The matching of resummed predictions with fixed order results allowed tests of QCD, measures of the coupling constant and studies of non-perturbative corrections\cite{4}. However, the need for a separate analytical calculation for every observable has limited the experimental use of resummed predictions. When dealing with multi-jet observables, analytical calculations become quite unfeasible, involving many integral transforms in order to write the distribution in a factorized form\cite{5}. Also in some cases it turns out not to be possible to resum the observable analytically\cite{6}.

We present then here a general approach to resummation based on a preliminary automated analysis of the observable, to establish its relevant properties with respect to soft-collinear emissions; in a subsequent step this information is used as an input of a general master formula.

2 Automated resummation

2.1 Analysis and applicability conditions

We start by considering a Born event consisting of \(n\) hard partons (‘legs’), \(n_i\), of which are incoming, with momenta \(p_1 \ldots p_n\). We resum \((n+1)\)-jet observables in the \(n\)-jet limit. The (positive defined) observable should then

1. vanish smoothly as a single extra \((n+1)\) parton of momentum \(k\) is made soft and collinear to a leg \(\ell\), with the functional dependence

\[
V(\{\vec{p}\}, k) = d_\ell \left( \frac{k_t}{Q} \right)^{a_\ell} e^{-b_\ell \eta} g_\ell(\phi).
\]  

\(2\)

Here \(Q\) is a hard scale of the process and the secondary emission \(k\) is defined in terms of its transverse momentum \(k_t\) and rapidity \(\eta\) with respect to leg \(\ell\), and where relevant, by an azimuthal angle \(\phi\) relative to a Born event plane. By requiring the functional form (2) (in practise, almost always valid), the problem of analyzing the observable reduces in part to identifying, for each leg \(\ell\), the coefficients \(a_\ell, b_\ell, d_\ell\) and the function \(g_\ell(\phi)\). IRC safety demands \(a_\ell > \max\{0, -b_\ell\}\).
2. be recursively IRC safe, i.e. given an ensemble of arbitrarily soft and collinear emissions, the addition of a relatively much softer or more collinear emission should not significantly alter the value of the observable, condition required for exponentiation of the leading logarithms. Observables like jet-rates in the Jade algorithm are then excluded, and many other examples of non-exponentiating IRC safe observables exist[7].

3. be continuously global[3] — this means that for a single soft emission, the observable’s parametric dependence on the emission’s transverse momentum (with respect to the nearest leg) should be independent of the emission direction. In practice this is perhaps the most restrictive of the conditions. It implies $a_1 = a_2 = \ldots = a_n \equiv a$.

To enable our computer program CAESAR (Computer Automated Expert Semi-Analytical Resummation) to establish these properties with the desired degree of reliability and precision, we have found it useful to make use of multiple-precision arithmetic[8].

### 2.2 Master formula

Given the above conditions, one can derive the following NLL master resummation formula for the distribution $\Sigma(v)[7]$:

$$
\ln \Sigma(v) = -\sum_{\ell=1}^{n} C_\ell \left[ r_\ell(L) + r'_\ell(L) \left( \ln \bar{d}_\ell - b_\ell \ln \frac{2E_\ell}{Q} \right) \right] + B_\ell T \left( \frac{L}{a + b_\ell} \right)
$$

$$
+ \sum_{\ell=1}^{n_\infty} \ln \frac{f_\ell(x_\ell, v \alpha_s(\mu_\ell^2))}{f_\ell(x_\ell, \mu_\ell^2)} + \ln S \left( T(L/a) \right) + \ln \mathcal{F}(C_1 r'_1, \ldots, C_n r'_n),
$$

(3)

where $C_\ell$ is the color factor associated with Born leg $\ell$, and $E_\ell$ is its energy, $B_\ell$ is $-3/4$ for quarks and $-(11C_A - 4T(r)) / (12C_A)$ for gluons, $\ln \bar{d}_\ell = \ln d_\ell + \int_{0}^{2\pi} \frac{d\phi}{2\pi} \ln g_\ell(\phi)$, and for incoming legs, $f_\ell$ are the parton densities. We note that (3) is independent of the frame in which one determines the $d_\ell$ and (to NLL accuracy) of the choice of hard scale $Q$.

The functions $r_\ell(L)$, which contain all the LL (and some NLL) terms, are

$$
r_\ell(L) = \int_{v^{2\pi}Q^2} \frac{dk_\ell^2 \alpha_s(k_\ell)}{k_\ell^2 \pi} \ln \left( \frac{k_\ell}{v^{1/2}Q} \right) + \int_{v^{2\pi}Q^2} \frac{dk_\ell^2 \alpha_s(k_\ell)}{k_\ell^2 \pi} \ln \left( \frac{k_\ell}{Q} \right),
$$

(4)
here $\alpha_s$, in the Bremsstrahlung scheme[2], runs at two-loop order. $r'_\ell = \partial_L r_\ell$ and $T(L)$ are relevant only at NLL level

$$T(L) = \int_{e^{-2LQ^2}}^{Q^2} \frac{dk_t^2 \alpha_s(k_t)}{k_t^2}. \tag{5}$$

The process dependence associated with large-angle soft radiation is described by $S(T(L/a))$, whose form depends on the number $n$ of legs. For $n = 3$

$$\ln S(t) = -t \left[ C_A \ln \frac{Q_g q g'}{Q_{qg} Q} + 2C_F \ln \frac{Q_{q'g'}}{Q} \right], \tag{6}$$

where $Q_{ab}^2 = 2p_a.p_b$ and $q, q'$ and $g$ denote the (anti)-quarks and gluon. The simpler case $n = 2$ can be read from (6) setting $C_A = 0$. The $n = 2, 3$ formulae apply then to $e^+e^-$, DIS and Drell-Yan production, while a process such as $gg \to \text{Higgs + g}$ involves simply different color factors. The (more involved) $n = 4$ case needed to describe hadronic dijet production can be found in[7].

We examine now the factor $\mathcal{F}$. Without it, eq. (3) corresponds essentially to the probability of vetoing all emissions $k$ with $V(\{\tilde{p}\}, k) > v$. However at NLL subtle effects enter: a simple veto on single emissions turns out to be insufficient, since events might have $V(\{\tilde{p}\}, k_1, \ldots, k_m) > v$ though all emissions separately had $V(\{\tilde{p}\}, k_i) < v$, or vice versa. This multiple emission effect, encoded in $\mathcal{F}$, is connected to how all secondary emission coherently determine the value of the observable and can be computed in a general way in[6].

While this talk concentrates on the method, new results obtained with it were presented at this conference in[9] (and other results can be found in[7]).

### 3 Final remarks

This project started with the development of an algorithm to numerically compute the non-trivial NLL terms associated with multiple emissions. It was initially applied by hand to three new observables [6]. Progress since then includes the derivation of a general master formula, together with a precise, automatically verifiable list of conditions on the observable for the resummation to be valid at NLL and the full numerical implementation of this. Though no human intervention is needed, results have the quality of analytic NLL predictions, so that any hadronisation model can be applied, studies of renormalization and
factorization scale dependence can be carried out and a matching with fixed order is feasible. The most important results obtained up to now are the first resummations in hadronic dijet production, for a variety of observables at a time. We now aim at automating the matching with fixed order results[10], this will open up the possibility to carry out a vast amount of phenomenological studies.

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