Blockmodels: A R-package for estimating in Latent Block Model and Stochastic Block Model, with various probability functions, with or without covariates.

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Abstract

Analysis of the topology of a graph, regular or bipartite one, can be done by clustering for regular ones or co-clustering for bipartite ones.

The Stochastic Block Model and the Latent Block Model are two models, which are very similar for respectively regular and bipartite graphs, based on probabilistic models.

Initially developed for binary graphs, these models have been extended to valued networks with optional covariates on the edges.

This paper present a implementation of a Variational EM algorithm for Stochastic Block Model and Latent Block Model for some common probability functions, Bernoulli, Gaussian and Poisson, without or with covariates, with some standard flavors, like multivariate extensions. This implementation allow automatic group number exploration and selection via the ICL criterion, and allow analyze networks with thousands of nodes in a reasonable amount of time.

1 Introduction

Complex networks are being more and more studied in different domains such as social sciences and biology. Statistical methodology have been developed for analysing complex data such as networks or bipartite networks in a way that could reveal underlying data patterns through some form of classification.

The models used in this paper are the Stochastic Block Model, introduced by [Nowicki and Snijders 2001] and Latent Block Model introduced by [Govaert and Nadif 2003], which are the same model for regular and bipartite networks.

This paper introduce a Gnu R package using Variational-EM algorithm as introduced by [Mariadassou et al. 2010] for Stochastic Block Model. The same method is applied for Latent Block Model.

For group number selection, the ICL [Biernacki et al. 2000] is used, and automatic exploration is done to found the optimal group number selection. This package use automatic reinitialization with other group number result to found coherent results.

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The implementation is done in C++ for CPU intensive step, using the linear algebra library armadillo [Sanderson, 2010]. The interfacing with GNU R is done by the package RcppArmadillo [Eddelbuettel and Sanderson, 2014], which itself is using the package Rcpp [Eddelbuettel et al., 2011] to provide a easy way to interface C++ code in packages in GNU R.

This package is the successor of previous implementation done by the same author in C++ [Leger, 2014]. This implementation is more efficient for matrix computation, have more available models, and more flexibility. This new implementation also have an R interface, and work both for Stochastic Block Model as well for Latent Block Model.

All non time consuming operations are done in GNU R, and the user interface is integrally usable in R. This package use parallelism to run in the same time on different initialization via the parallel package in R.

2 Model framework
Stochastic Block Model and Latent Block Model models are described with the following notation.

2.1 Stochastic Block Model
2.1.1 Notations
Considering the following notation: $[1,n]$ the set of nodes with a graph on $n$ nodes. Let $X_{ij} \in G$ the weight of the edge $(i,j) \in [1,n]^2$. For example, for a binary graph, $G = \{0,1\}$, and $X$ is named the adjacency matrix. Or, for a univariate weighed graph $G = \mathbb{R}$, and $X$ is named the weighted adjacency matrix. $Y_{ij}$ the covariates vector associated to the edge $(i,j) \in [1,n]^2$, if there is covariates.

We consider $Q$ classes on nodes, and let $Z$ the membership matrix defined as $Z_{iq} = 1$ if and only if node $i$ is a member of class $q$, for $i \in [1,n]$ and $q \in [1,Q]$. Let $Z_i$ the $i$-th row of the matrix $Z$.

As above, and in all this article, for Stochastic Block Model, $i$ and $j$ are denoting nodes indices (in $[1,n]$), $q$ and $l$ are denoting class indices (in $[1,Q]$).

2.1.2 Model
Latent layer : The class memberships of nodes are driven by independent identically distributed multinomial distribution.

$$Z_i \sim_{i.i.d.} M(1, \alpha),$$

with $i \in [1,n]$

Where $\alpha \in \mathbb{R}_+$ is a parameter as $\sum_{q=1}^{Q} = 1$. 

2
Observed layer for Stochastic Block Model: The model is defined by giving the distribution of each edge \((i,j)\) conditionally to the membership of node \(i\) in the \(q\)-th class and node \(j\) in the \(l\)-th class.

\[ X_{ij} | Z_{iq} Z_{jl} = 1 \overset{\text{ind}}{\sim} F_{ql}^{Y_{ij}} \]

with \((i,j) \in \mathbb{I},n\)^2 and \(i \neq j\).

The choice of \(F\) can lead to a large range of models, depending or not on covariates effect.

Observed layer for symmetric Stochastic Block Model: For symmetric Stochastic Block Model, edges \((i,j)\) and \((j,i)\) are considered to be the same, and only observed one time. The model is the same, but only observed of \(i<j\):

\[ X_{ij} | Z_{iq} Z_{jl} = 1 \overset{\text{ind}}{\sim} F_{ql}^{Y_{ij}} \]

with \((i,j) \in \mathbb{I},n\)^2 and \(i < j\).

2.2 Latent Block Model

2.2.1 Notations

Considering two types of nodes. The set of the two type are \([1,n^{(1)}]\) and \([1,n^{(2)}]\), with \(n^{(1)}\) nodes and \(n^{(2)}\) nodes of each set. The edges only consist of edges between different type nodes (i.e. for edges \((i,j) \in [1,n^{(1)}] \times [1,n^{(2)}]\)). Let \(X_{ij} \in \mathbb{G}\) be the weight of the edges \((i,j) \in [1,n^{(1)}] \times [1,n^{(2)}]\). For example, for a binary graph, \(\mathbb{G} = \{0,1\}\), and \(X\) is named the adjacency matrix. Or for a univariate weighed graph \(\mathbb{G} = \mathbb{R}\), and \(X\) is named the weighted adjacency matrix. \(Y_{ij}\) the covariates vector associated to the edge \((i,j) \in [1,n^{(1)}] \times [1,n^{(2)}]\), if there is covariates.

We consider \(Q^{(1)}\) classes on nodes for type 1 nodes, and \(Q^{(2)}\) classes of nodes for type 2 nodes, and let for type 1, \(Z^{(1)}\) the membership matrix defined as \(Z_{iq}^{(1)} = 1\) if and only if node \(i\) is a member of class \(q\), for \(i \in [1,n^{(1)}]\) and \(q \in [1,Q^{(1)}]\), and \(Z^{(2)}\) the membership matrix defined as \(Z_{jl}^{(2)} = 1\) if and only if node \(j\) is a member of class \(l\), for \(j \in [1,n^{(2)}]\) and \(l \in [1,Q^{(2)}]\). Let \(Z_{i}^{(1)}\) the \(i\)-th row of the matrix \(Z^{(1)}\), and \(Z_{j}^{(2)}\) the \(j\)-th row of the matrix \(Z^{(2)}\).

As above, and in all this article, for Latent Block Model, \(i\) is denoting node index of the first type (in \([1,n^{(1)}]\)), \(j\) is denoting nodes index of the second type (in \([1,n^{(2)}]\)), \(q\) is denoting class index of the first class (in \([1,Q^{(1)}]\)) and \(l\) is denoting class index of second type (in \([1,Q^{(2)}]\)).

2.2.2 Model

Latent layer: For each type, the node membership are driven by independent identically distributed multinomial distribution:
\[
\begin{align*}
Z^{(1)}_i & \overset{\text{i.i.d.}}{\sim} M(1, \alpha^{(1)}), \\
Z^{(2)}_j & \overset{\text{i.i.d.}}{\sim} M(1, \alpha^{(2)}),
\end{align*}
\]
with \(i \in [1, n^{(1)}], j \in [1, n^{(2)}]\).

**Observed layer** The model is defined by giving the distribution of each edge \((i, j) \in [1, n^{(1)}] \times [1, n^{(2)}]\) conditionally to the membership of node \(i\) in the \(q\)-th class of type 1 \((q \in [1, Q^{(1)}])\) and node \(j\) in the \(l\)-th class of type 2 \((l \in [1, Q^{(2)}])\).

\[X_{ij} | Z^{(1)}_{iq} Z^{(2)}_{jl} = 1 \overset{\text{iid}}{\sim} F_{Y_{ij}}\]
with \((i, j) \in [1, n^{(1)}] \times [1, n^{(2)}]\).

The choice of \(F\) can lead to a large range of models, depending or not on covariates effect.

### 3 Estimation procedure

The used estimation procedure is from Mariadassou et al. [2010], with a variational expectation maximization. As Mariadassou et al. [2010] the ICL criterion is used for group number selection.

#### 3.1 Variational-EM algorithm

As done by Mariadassou et al. [2010], the following criterion is used from a variational approximation of the likelihood, for Stochastic Block Model:

\[J = \sum_{i,q} \tau_{iq} \log(\alpha_q) + \sum_{i,j;i \neq j} \sum_{q,l} \tau_{iq} \tau_{jl} \log f_{Y_{ij}}(X_{ij})\]

Where \(\tau_i\) is the variational parameter of the multinomial distribution which approximate \((Z_i|X)\).

For Latent Block Model, the following criterion is used:

\[J = \sum_{i,q} \tau^{(1)}_{iq} \log(\alpha^{(1)}_q) + \sum_{j,l} \tau^{(2)}_{jl} \log(\alpha^{(2)}_l) + \sum_{i,j} \sum_{q,l} \tau^{(1)}_{ij} \tau^{(2)}_{q,l} \log f_{Y_{ij}}(X_{ij})\]

The EM with variational approximation is translated is two steps, which are repeated until convergence:

1. **Pseudo-E step**: Maximisation with respect to variational parameters, \(\tau\) for Stochastic Block Model, and \((\tau^{(1)}, \tau^{(2)})\) for Latent Block Model.

2. **M-step**: Maximisation with respect to original parameters, \(\alpha\) and model function parameters for Stochastic Block Model and \((\alpha^{(1)}, \alpha^{(2)})\) and model function parameters for Latent Block Model.
The maximisation with respect to variational parameters is done by iterating a fixed point equation. The maximization with respect to original parameters is explicit for $\alpha$ or $(\alpha^{(1)}, \alpha^{(2)})$. For model function, the maximization can be done with explicit formula or by a numerical maximization algorithm.

3.2 ICL criterion

For group number selection, the ICL from Biernacki et al. [2000] is used.

3.3 Initialization and reinitialization

As many algorithm based on the EM algorithm, this method have a huge dependency of the initialization quality. Two type of initialization are used by this package.

Absolute Eigenvalues Spectral Clustering : This variant of Spectral Clustering seems to give very good first approximation of a classification for Stochastic Block Model case. Furthermore, of Stochastic Block Model, with Bernoulli distribution without covariate, the Absolute Eigenvalues Spectral Clustering is consistent, see Rohe et al. [2011]. For small graphs (less than 1000 nodes), the obtained clustering seems not to be a very good results to be used as is, but it is a good start point for this package.

To take care of covariates, where there are ones, the Absolute Eigenvalues Spectral clustering is run on the residual graphs of the regression (which is in fact the residual graphs for the one-group model).

For Stochastic Block Model, this residual graphs is directly used as a input of the Absolute Eigenvalues Spectral Clustering. For Latent Block Model, the residual graph is projected on each node type, and a Absolute Eigenvalues Spectral Clustering is done for each node type.

Reinitialization : The obtained results by the method for a group number is used to provide new initialization for previous groups number (by merging groups) and next groups number (by splitting groups).

Due to the high number of reinitialization proposed, in some case, the criterion is evaluated on each provided reinitializations, and only best ones are used (the number of used iterations each step is depending of a constant and the group number, the constant can be changed by user).

The process of reinitialization is done while reinitialization improve the criterion.

3.4 Group number exploration

To explore the group number, the model is run for a beginning set of group number (which can be changed by the user). After that, for the selected number of group (the ICL maximum), the exploration is done to a maximal number of groups which is a constant (by default 1.5, user modifiable) times the selected number of groups.
It is important to explore after the maximum, oversplitted groups (after the ICL maximum) can provide good reinitialization by merging, and change the maximum location.

4 Architecture

In this section the architecture of the package is described. The package use C++ for CPU intensive operations and R for other operations. The general architecture is describe in the following section and each type of code is describe below.

4.1 General architecture

The package is usable inside GNU R, therefore the interface of the package is in R, user provide data to the package in R, and the time-consuming operation is written in C++.

Model definition: The user define a object with the model, Stochastic Block Model or Latent Block Model and model function, and the data, adjacency of the network and covariates if there are ones.

The returned object have methods which provide estimation and access to the results.

4.2 R code

All the R code use RefClass (S4) class. These type of class, are the equivalent in R of class in most other programming language where the methods of the class can modify the object itself.

All estimation for different initialization of a number of group are run in parallel, via the parallel GNU R package on platforms which support parallelism (Linux, Solaris, *BSD, MacOS). On Windows, estimations for different initialization are run sequentially because the parallel package does not support this OS.

4.2.1 Memberships

The memberships, Stochastic Block Model or Latent Block Model, have specific functions, for estimation and for results access. They inherit from a virtual class membership which if the one considered by other functions.

4.2.2 Model functions

All model functions inherit from a virtual class which is used by other functions. Some model functions inherit between themselves when a model function is the extension of another. This code contains non-time consuming model specific functions, as normalization, or displaying functions.

A example file is given in the source code which indication how to write a model specific class.
4.3 C++ code

The EM is implemented in C++, via templated function. Generic template functions are written, and the EM function is evaluated at compilation for each model function and each membership type (Latent Block Model or Stochastic Block Model).

Templated function are defined, with generic code. Each model need template specialization to specify the model functions. The specialization can be done in higher lever when vectorized function exists (for the fixed point equation, for explicit maximum in the M step if there is one, or for the gradient calculation) or can be done in the lower level with only providing the model functions and derivatives.

An example file is given in the source code which indicating the function to specialize to describe a new model function.

5 Implemented model functions

Common model functions are implemented, they are described below. Some model have vectorized specialization to provide fast code when this is possible.

5.1 Bernoulli family

5.1.1 Bernoulli

This is the common Stochastic Block Model or Latent Block Model. Links are valued in $G = \{0, 1\}$. The model is defined as below:

$$p_{ql}^Y_{ij} = \mathcal{B}(\pi_{ql})$$

Parameters:

- $\pi_{ql} \in [0, 1]$, $(q, l) \in [1, Q]^2$ for Stochastic Block Model or $(q, l) \in [1, Q^{(1)}] \times [1, Q^{(2)}]$ for Latent Block Model. This is the group effect.

The implementation is vectorized for the E-step, and have an explicit maximum computed with vectorized formula in the M-step.

The model is accessible by \texttt{EM_bernoulli}

5.1.2 Bernoulli multiplex

This model is a multivariate non-independent Bernoulli distribution. Links are valued in $G = \{0, 1\}^p$.

The model is defined as follow for Stochastic Block Model:

$$\forall x \in \{0, 1\}^p \quad P(X_{ij} = x|Z_{iq}Z_{jl} = 1) = \pi_{ql}[x]$$

and for Latent Block Model:

$$\forall x \in \{0, 1\}^p \quad P\left(X_{ij} = x|Z_{iq}^{(1)}Z_{jl}^{(2)} = 1\right) = \pi_{ql}[x]$$
Parameters:

- \( \pi_{ql}[x] \in [0, 1] \) \((q,l) \in [1,Q]^2\) for Stochastic Block Model or \( (q,l) \in [1,Q^{(1)}] \times [1,Q^{(2)}] \) for Latent Block Model, \( x \in \{0,1\}^p \), under the constraint \( \forall q,l; \sum_x \pi_{ql}[x] = 1 \). This is the group effect.

The implementation is vectorized for the E-step, and have a explicit maximum computed with vectorized formula in the M-step.

The model is accessible by `BM_bernoulli_multiplex`.

5.1.3 Bernoulli with covariates

This model provide a logistic regression with a group effect which is the intercept and a covariates effect.

Links are valued in \( G = \{0,1\} \). The model is defined as below:

\[
F_{ql}^{Y_{ij}} = \mathcal{B} \left( \logit^{-1}(m_{ql} + \beta^T Y_{ij}) \right)
\]

where \( \logit(p) = \log \left( \frac{p}{1-p} \right) \).

Parameters:

- \( m_{ql} \in \mathbb{R}, (q,l) \in [1,Q]^2 \) for Stochastic Block Model or \( (q,l) \in [1,Q^{(1)}] \times [1,Q^{(2)}] \) for Latent Block Model. This is the group effect.
- \( \beta \), the covariates effect.

Two implementation of this model is describe below.

**Standard implementation** : Due to the non-separability between the group effect and the covariates effect, even in polynomial form, the specialization must be done in the lower level. This implementation is very slow.

The E-step is not vectorized, the maximum is numerically computed in the M-step without vectorized gradient calculation.

This implementation is accessible by `BM_bernoulli_covariates`.

**Fast implementation with approximation** : Alternatively to the previous implementation, which is exact, a fast implementation is provided using a approximation.

Let \( g \) defined as:

\[
g : x \mapsto \frac{1}{2} x + \log \left( 1 - \frac{1}{1 + \exp(-x)} \right)
\]

With this function, the log likelihood for a link can be expressed as:

\[
\log f_{ql}^{Y_{ij}}(X_{ij}) = \left( X_{ij} - \frac{1}{2} \right) (m_{ql} + \beta^T Y_{ij}) + g \left( m_{ql} + \beta^T Y_{ij} \right)
\]

Terms using \( \log f_{ql}^{Y_{ij}}(X_{ij}) \) and variational parameters are summed over \( i,j \) and \( q,l \). As for above, due to the form of \( g \), this sum can not be separated.
To separate the sum and vectorize the computation, \( g \) is substituted by a polynomial which approximate the function. The polynomial involve powers of the term \( m_{ql} + \beta^T Y_{ij} \). Terms are separated by power of \( m_{ql} \) and \( \beta^T Y_{ij} \) using the Binomial theorem.

By changing the summing order, and considering the sum over terms as lower priority, we can separate and vectorize the computation for each term, involving only powers of \( m_{ql} \) and \( \beta^T Y_{ij} \).

The function \( g \) is even, so the polynomial approximation involves only even power terms. The polynomial is chosen of degree 14, in order to approximate the best the function \( g \) on \([-15, 15]\). The function \( g \) is concave, though the polynomial approximating the function \( g \) does not need to be concave to be a good approximation. Still an upper bound constraint is added on the second derivative. This last constraint provides a good numerical stability with a very small approximation loss.

In simulation, for all \( i, j, q, l \), if \( m_{ql} + \beta^T Y_{ij} \in [-15, 15] \), the approximation method gives the same results as the exact method.

Due to asymptotic branch of the polynomial which go very quickly to \(-\infty\), in general case, this fast method have the same behavior of the logistic regression under the constraint:

\[
\forall i, j, q, l \left| m_{ql} + \beta^T Y_{ij} \right| \leq 15
\]

The E-step is vectorized, the maximum is numerically computed in the M-step with a vectorized gradient calculation.

This implementation is accessible by `BM_bernoulli_covariates_fast`.

### 5.2 Gaussian family

#### 5.2.1 Gaussian

This is the Stochastic Block Model or Latent Block Model with normally distributed values on links. Links are valued in \( G = R \).

The model is defined as below:

\[
\mathcal{F}_{ql}^{Y_{ij}} = \mathcal{N}(\mu_{ql}, \sigma^2)
\]

Parameters:

- \( \mu_{ql} \in \mathbb{R}, (q, l) \in [1, Q]^2 \) for Stochastic Block Model or \( (q, l) \in [1, Q^{(1)}] \times [1, Q^{(2)}] \) for Latent Block Model. This is the group effect.

- \( \sigma^2 \), the parameter of the variance.

The implementation is vectorized for the E-step, and have a explicit maximum computed with vectorized formula in the M-step.

The model is accessible by `BM_gaussian`
5.2.2 Gaussian multivariate

This is the Stochastic Block Model or Latent Block Model with multivariate normally distributed values on links. Links are valued in $\mathbf{G} = \mathbb{R}^p$.

The model is defined as below:

$$F_{ql}^{Y_{ij}} = \mathcal{N}(\mu_{ql}, \Sigma)$$

Parameters:

- $\mu_{ql} \in \mathbb{R}^p$, $(q, l) \in \mathbb{[1,Q]}^2$ for Stochastic Block Model or $(q, l) \in \mathbb{[1,Q^{(1)}]} \times \mathbb{[1,Q^{(2)}]}$ for Latent Block Model. This vector is the group effect.
- $\Sigma$, the covariance matrix.

Three flavors of this model are provided depending on the shape of the covariance matrix.

All the flavors have vectorized E-step and an explicit maximum with vectorized computation in the M-step.

**Independent homoscedastic case** : This case considers the components are independent and have same variance. We consider $\Sigma = \sigma^2 I_p$ where $I_p$ is the identity matrix of size $p$.

This model is accessible by `BM_gaussian_multivariate_independent_homoscedastic`.

**Independent case** :

This case considers the components are independent and have same variance. We consider $\Sigma$ is a diagonal matrix.

This model is accessible by `BM_gaussian_multivariate_independent`.

**General case** We only assume that $\Sigma$ is semi-definite positive matrix, which is contained in the likelihood.

This model is accessible by `BM_gaussian_multivariate`.

5.2.3 Gaussian with covariates

This model is a standard linear regression on the covariates for Stochastic Block Model and Latent Block Model. Links are valued on $\mathbf{G} = \mathbb{R}$. The model is defined as below:

$$F_{ql}^{Y_{ij}} = \mathcal{N}(\mu_{ql} + \beta^T Y_{ij}, \sigma^2)$$

Parameters:

- $\mu_{ql} \in \mathbb{R}$, $(q, l) \in \mathbb{[1,Q]}^2$ for Stochastic Block Model or $(q, l) \in \mathbb{[1,Q^{(1)}]} \times \mathbb{[1,Q^{(2)}]}$ for Latent Block Model. This is the group effect.
- $\beta$, the covariates effect,
• $\sigma^2$ the parameter of variance.

This model have vectorized E-step, a numerically maximum with vectorized computation in the M-step.

This model is accessible by BM\_gaussian\_covariates

5.3 Poisson family

5.3.1 Poisson

This is the Stochastic Block Model or Latent Block Model with Poisson distributed values on links. Links are valued in $G = \mathbb{N}$.

The model is defined as below:

$$ F_{\lambda_{ql}}^{Y_{ij}} = \mathcal{P}(\lambda_{ql}) $$

Parameters:

• $\lambda_{ql} \in \mathbb{R}$, $(q, l) \in [1, Q]^2$ for Stochastic Block Model or $(q, l) \in [1, Q^{(1)}] \times [1, Q^{(2)}]$ for Latent Block Model. This is the group effect.

The implementation is vectorized for the E-step, and have a explicit maximum computed with vectorized formula in the M-step.

The model is accessible by BM\_poisson

5.3.2 Poisson with covariates

This model is a Poisson regression on the covariates for Stochastic Block Model and Latent Block Model. Links are valued on $G = \mathbb{N}$. The model is defined as below:

$$ F_{\lambda_{ql}}^{Y_{ij}} = \mathcal{P}(\lambda_{ql} \exp(\beta^T Y_{ij})) $$

Parameters:

• $\lambda_{ql} \in \mathbb{R}$, $(q, l) \in [1, Q]^2$ for Stochastic Block Model or $(q, l) \in [1, Q^{(1)}] \times [1, Q^{(2)}]$ for Latent Block Model. This is the group effect.

• $\beta$, the covariates effect,

This model have vectorized E-step, a numerically maximum with vectorized computation in the M-step.

This model is accessible by BM\_poisson\_covariates
6 Execution time on a example

6.1 Methodology

All this tests are done with Stochastic Block Model.

For each model, a network is simulated accordingly, in the way documented in manuals.

Four conditions are simulated:

- 5 groups and 100 nodes
- 5 groups and 200 nodes
- 10 groups and 100 nodes
- 10 groups and 200 nodes.

In order for the result to be comparable, the automatic group number exploration is disabled, and the exploration is force to explore all groups number between 1 and twice the number of simulated groups.

Each estimation is repeated 5 times, the median is the result retained and reported in the table below. The reported time is the CPU time which cumulates the execution time of all parallel process. The real execution time is less than the CPU time, due to parallelism. All computation are run on the same machine, with a Intel Xeon X5675 CPU.

6.2 Results

The results are:

|          | 100 nodes | 200 nodes |
|----------|-----------|-----------|
|          | 5 groups  | 10 groups | 5 groups | 10 groups |
| Bernoulli| Standard  | 10 s      | 16 s     | 3 m 30 s |
|          | Multiplex | 9 s       | 43 s     | 3 m 57 s |
|          | Covariates (exact) | 135 h 55 m 03 s | 761 h 53 m 40 s |
|          | Covariates (fast) | 3 h 10 m 58 s | 38 m 00 s |
|          |           | 11 h 38 m 35 s | 16 h 01 m 32 s |
| Gaussian | Standard  | 8 s       | 19 s     | 2 m 51 s |
|          | Multivariate (indep. homosc.) | 11 s | 28 s | 2 m 49 s |
|          | Multivariate (indep.) | 10 s | 37 s | 2 m 52 s |
|          | Multivariate | 5 s | 10 s | 1 m 03 s |
|          | Covariates | 51 s | 2 m 59 s | 1 h 40 m 25 s |
| Poisson  | Standard  | 10 s      | 27 s     | 4 m 23 s |
|          | Covariates | 1 m 17 s | 4 m 06 s | 3 h 49 m 14 s |

The results are highly dependent on the network and the signal to noise ratio. Therefore, no comparison should be done between families, which use different models to simulate networks.

Users should keep in mind that above times have been computed on simulated graphs generated from the true model. Thus these timings are provided only as a guide. Times for applications on real graph may differ.
7 Application on the Debian keyring signing network

7.1 The Debian project
7.2 The data
7.3 Estimation procedure
7.4 Results

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