1. Introduction

It is well known that a powder bed is sheared in one of the following three ways: 1) Powder bed reaches the steady flow state after yielding with expansion. 2) Powder bed reaches the steady flow state at the start of yielding without volume change. 3) Powder bed reaches the steady flow state after yielding with contraction.

To experimentally examine the above yielding phenomena, the accurate measurement of expansion and contraction of the powder bed is essential. Although cell-type direct shear testers are widely used for the shear test, it is difficult to uniformly load the shear stress on these testers and the accurate measurement of volume change in the powder bed is impossible because of the effect of the cell wall especially for fine powders with high void fraction. Therefore, the results measured by a single tester for the whole yield locus over the range of expansion and contraction on yielding of the powder bed have been scarcely obtained.

In this experiment, a direct shear tester with parallel plates is used. This tester has been made by modifying the testers originally designed by Hiestand or Budny. It is suitable for the accurate measurement of expansion and contraction in the shear process because the powder bed is not restricted by the cell wall. The data obtained by the tester are discussed on Roscoe condition diagram. Moreover, the bed consolidated by pre-shearing on the basis of Jenike's method and the bed consolidated uniaxially without pre-shearing are tested to examine the effect of the pre-shearing on the powder yield locus.

2. Experimental apparatus and procedures

The outline of the direct shear tester with parallel plates is shown in Fig. 1. In the present shear test, calcium carbonate (mass median diameter \(D_p = 3.5 \mu m\)) and silica powder \((D_p = 5.5 \mu m)\) were used as test materials after drying at 120°C for 48 hours and removing the agglomerate with a 32 mesh sieve. These materials were consolidated in two ways: the uniaxial consolidation for the examination of the shear process with expansion or contraction of powder beds and the shear consolidation for comparison with the shear test due to Jenike's method.

For the uniaxial consolidation test, a sample powder bed of 1 mm thickness was packed on a fixed plate ①, and then gently twisted by a movable plate ② mounted on the bed. After pre-consolidation under the normal stress \(a_p\) for five minutes, the bed was sheared under the normal stress \(a\) by weight placed on the movable plate. The shear test was conducted under the tensile force by motor ③ under the condition that the normal stress was less than the stress due to the movable plate or was negative (including the tensile strength). It became possible by using this tester to conduct not only the shear test under slight stress but also the tensile strength test which produced the same failure plane as the shear plane.

For the shear consolidation, on the other hand, the same procedure as Jenike's method was adopted. The powder bed under the normal stress \(a_v\) on the movable plate was sheared until the steady state where the stress and the volume were constant was achieved. Then this powder bed was sheared under the normal stress \(a\) less than \(a_v\). Furthermore, Jenike's tester was also used for comparison with these results.

In both tests, the displacement in the direction of the normal stress was measured within an error of about 0.5 \(\mu m\) by a noncontacting...
displacement meter 6 plugged into the fixed plate.

3. Shear process and yield locus

Figure 2 shows Roscoe condition diagram which represents yielding characteristics of the powder bed. Consider the shear process of the powder bed with a void fraction $\varepsilon = \varepsilon_b$. At the normal stress $a = a_b$, the shear stress $\tau$ increases from the point C to the point E until the powder bed becomes the steady flow state at the point E. When $a = a_d (a_d < a_b)$, $\tau$ increases from the point H to the point P at which point the yielding starts with expansion, and then $\tau$ decreases gradually along PpZ with increasing the void fraction from $\varepsilon_b$ to $\varepsilon_d$ until the powder bed becomes the steady flow state at the point Z. When $a = a_c (a_c > a_b)$, $\tau$ increases from the point F to the point Q at which point
the yielding starts with contraction, and then \( \tau \) increases gradually along \( Q_qK \) with decreasing the void fraction from \( \varepsilon_b \) to \( \varepsilon_e \) until the powder bed becomes the steady flow state at the point \( K \). As a result, the yield locus (Y.L.) which gives the correlation of \( \tau \) and \( \sigma \) at the yield point of the powder bed is displayed by a line \( A'R'SQ' \) which is projected as a line \( A'P'EQ \) on \( \sigma - \tau \) plane. This line represents the yield locus for the powder bed with the void fraction \( \varepsilon_b \) characterized by pre-consolidation stress because \( \tau \) and \( \sigma \) are obtained at the start point of yielding.

The yield locus obtained from \( \tau \) and \( \sigma \) at the steady flow state becomes a line \( OZ'SK' \) which is projected as a line \( E_1ZEK_e \) on \( \sigma - \tau \) plane. This yield locus is the critical state line (C.S.L.) proposed by Schwedes[1]. The void fraction \( \varepsilon \) decreases as \( \sigma \) increases; that is, the specified values of \( \tau \) and \( \sigma \) exist for a certain value of \( \varepsilon \). If the yield locus is obtained from \( \tau \) and \( \sigma \) at some shear displacement instead of at the start point of yielding, this locus becomes a line \( a'p'Sq' \) which is projected as a line \( apEq \) on \( \sigma - \tau \) plane. In this line, the void fractions at the points \( p, E, q \) are expressed as \( \varepsilon_p, \varepsilon_e, \varepsilon_f \), respectively. It is also shown in Fig. 2 that the yield locus approaches a C.S.L. with increasing the yield criterion which is defined by the volume change in the powder bed, since the void fraction increases with progress of shearing. Therefore, the shear stress at the start point of the yielding should be accurately measured to obtain the yield locus of the powder bed prepared under a given consolidation stress.

4. Results and discussion

4.1 Measurement of yield locus by the shear tester with parallel plates[7,8]

Figure 3 shows the relation between the displacement of the powder bed in the direction of the normal stress, \( \Delta n \) (the sign is positive when expansion takes place), and the shear stress, \( \tau_i \) in the shear process of calcium carbonate pre-consolidated at 4.9 kPa. For the low normal stress in shearing \( (\sigma_0 \sim \sigma_2) \), the powder bed expanded and the shear stress decreased clearly with progress of yielding. In this case, the shear stress at the start of yielding could easily be obtained with high reproducibility. For the high normal stress in shearing \( (\sigma_4 \sim \sigma_10) \), the shear stress at the start of yielding was difficult to determine because the powder bed contracted gradually with increasing the shear stress. Therefore, the criterion of yielding of the powder bed based on the displacement in the direction of the normal stress \( \Delta n \) was adopted in this work because the definition of the yield of the powder bed should be clarified to obtain the accurate yield locus.

Figure 4 shows the effect of the yield criterion \( \Delta n \) on the powder yield locus. It is found from this figure that the yield locus approached to the critical state line with increasing the yield criterion \( \Delta n \) from 2 \( \mu \)m to 20 \( \mu \)m and this tendency would agree with the description in the previous section. The suitable value of the yield criterion \( \Delta n \) obtained in this experiment might be considered 2 \( \mu \)m that was about a half of the diameter of the sample powder because the consolidation yield locus showed reasonable shape. This implies that the value of \( \Delta n \) would be dependent on a particle size of the sample powder. Hence, the accurate measurement of the value of \( \Delta n \) up to the same
size as that of a particle is required to obtain a satisfactory yield locus.

Figure 5 shows the yield locus and the critical state line of calcium carbonate which was pre-consolidated at \( a_p = 4.9 \) kPa. As a consequence of using the yielding criterion based on the displacement in the direction of the normal stress \( \Delta n \), the consolidation yield locus could be obtained experimentally. Therefore, the whole range of the powder yield locus including the tensile strength and the cohesion could be expressed as a single smooth curve, although there had been almost no powder yield locus given by a single tester over the whole range.

Recently, Williams et al.\(^9\) reported the reduced yield locus based on the yield locus obtained from the shear test and the tensile strength test. In their method, the normal stress corresponding to the end point of the yield locus in the range accompanied by expansion of the powder bed must be determined; however, the decision of this point is very troublesome. In this paper, the pre-consolidation stress \( a_p \) is used instead of the normal stress at the end point; the reduced yield locus is obtained by assuming that the similarity can be held independently of \( a_p \) for any yield locus. Figures 6 and 7 show that the relation between the reduced yield loci given by the dimensionless stress \( \tau / (\sigma_p - \sigma_T) \) instead of \( \tau \) and \( (\sigma - \sigma_T) / (\sigma_p - \sigma_T) \) instead of \( \sigma \). Although silica powder shown in Figure 6 or calcium carbonate shown in Figure 7 had its individual shear characteristics, each reduced yield loci of these two materials could be expressed as a nearly single curve, independently of \( a_p \). Therefore, if the reduced yield locus of the powder bed pre-consolidated at an arbitrary normal stress \( a_p \) is known, a yield locus pre-consolidated at a normal stress other than \( a_p \) can be estimated by only a measured value of the tensile strength \( \sigma_T \).

4.2 Effect of the consolidation method of the powder bed \(^6\)

In order to find how the yield locus was affected by the consolidation method of the powder bed, the shear test was performed concerning the two kinds of powder beds prepared by different methods: the powder bed pre-sheared on the basis of Jenike’s method (shear-consolidated bed) and the powder bed consolidated uniaxially (uniaxially consolidated bed). The effect of the consolidation methods on the yield locus was examined for the both powder beds.

Experimental results of the shear-consolidated bed obtained by Jenike’s tester were compared with those by the parallel plate-type tester. Figure 8 shows the critical state line and the yield loci after pre-sheared at the normal stress of 4.9 kPa for silica powder. The yield
loci obtained by both the testers agreed well and C.S.L. agreed perfectly. This is probably because the shear plane would reach the steady flow as stated previously and a certain particle arrangement resulted from the normal stress would be formed during pre-shearing.

Figure 9 shows the experimental results of the uniaxially consolidated bed compared with those of the shear-consolidated bed (σ_p = 4.9 kPa). The yield locus of the bed uniaxially consolidated at the normal stress σ_p = 4.9 kPa was lower than that of the bed shear-consolidated at the shear-consolidation stress σ_v = 4.9 kPa. The uniaxially consolidated bed was consolidated only with the normal stress σ_p, while the shear-consolidated bed was consolidated with both the normal stress and the shear stress. Since the maximum principal stress obtained was 8.35 kPa as indicated in Fig. 8, the yield locus of the uniaxially consolidated bed pre-consolidated at σ_p = 8.35 kPa was compared with that of the shear-consolidated bed pre-sheared at σ_v = 4.9 kPa; the former was found to be still lower than the latter. So, further pre-consolidation stress for the uniaxially consolidated bed was applied by the method of trial and error and it was found that a yield locus of the uniaxially consolidated bed pre-consolidated at σ_v = 24.5 kPa was nearly equal to that of the shear-consolidated bed pre-sheared at σ_v = 4.9 kPa. That stress σ_v was five times as large as the pre-sheared stress σ_v and was three times as large as the maximum principal stress of the shear-consolidated bed. Thus, it became clear that the normal stress required for pre-consolidation of the uniaxially consolidated bed would be much larger than the pre-shear stress to obtain the same yield locus for the uniaxially consolidated bed and the shear-consolidated bed.

Average void fractions of the consolidated bed by Jenike’s tester and of the uniaxially consolidated bed by a parallel plate-type shear tester are shown in Table 1. The void fractions of the shear-consolidated bed were much smaller than that of the uniaxially consolidated bed at the same consolidation stress, and they were close to that of the uniaxially consolidated bed pre-consolidated at σ_v.

5. Conclusion

From the results of shear test using a parallel

Table 1 Effect of consolidation method on void fraction

|                | pre-shearing (by Jenike's tester) | uniaxially consolidation (by parallel plates) |
|----------------|----------------------------------|-----------------------------------------------|
|                | σ_p [kPa] | ε [-] | σ_v [kPa] | ε [-] | σ_v [kPa] | ε [-] |
| silica         | 4.9      | 0.587 | 4.9      | 0.639 | 24.5     | 0.601 |
| CaCO_3         | 4.9      | 0.725 | 4.9      | 0.784 | 32.5     | 0.735 |
plate-type shear tester, the relation between the shear characteristics and the yield locus was examined. By using a Jenike’s shear tester as well as this tester, the effect of the consolidation method on the yield locus was also examined. The following results could be obtained by examining the experimental data.

1) The yield locus in the low normal stress range where the powder bed expanded with yielding could be determined reproducibly by a parallel plate-type shear tester. On the other hand, the yield locus in the high normal stress range where the powder bed contracted with yielding varied with the yield criterion. Therefore, it was considered to be essential to establish the clear yield criterion for determining an accurate yield locus. The yield criterion was defined by the displacement in the direction of the normal stress, and the suitable value of the displacement obtained in this experiment was about a half of the median diameter of the sample powders.

2) With increasing the yield criterion, the yield locus approached the critical state line. The determination of the shear stress at the start point of yielding was found to be important in order to obtain exactly the yield locus of the powder bed prepared at the fixed void fraction.

3) The reduced yield locus in terms of the two dimensionless stresses, that is, \( \frac{\tau}{(\sigma_p - \sigma_r)} \) instead of \( \tau \) and \( \frac{\sigma - \sigma_r}{(\sigma_p - \sigma_r)} \) instead of \( \sigma \), could be expressed as a single line for the powder beds under various pre-consolidation stresses. If the reduced yield locus of the powder bed pre-consolidated at an arbitrary normal stress \( \sigma_p \) is known, a yield locus of the powder bed pre-consolidated at a normal stress other than \( \sigma_p \) may be easily estimated by only a measured value of the tensile strength \( \sigma_r \).

4) The yield locus of the powder bed pre-sheared on the basis of Jenike’s method was higher than that of the powder bed pre-consolidated uniaxially at the same normal stress. This implies that the uniaxially consolidated powder bed would require larger pre-consolidation stress than the maximum principal stress of the pre-sheared powder bed to obtain the same extent of yield locus.

Nomenclature

- \( \Delta n \): displacement in the direction of the normal stress [\( \mu m \)]
- \( e \): void fraction of powder bed [-]
- \( \sigma \): normal stress in shear process [kPa]
- \( \sigma_p \): pre-consolidation stress of uniaxially consolidated bed [kPa]
- \( \sigma_r \): normal stress at the steady shear state of shear-consolidated bed [kPa]
- \( \sigma_r \): tensile strength of powder bed [kPa]
- \( \tau \): shear stress in shear process [kPa]

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