Dual Supersymmetry Algebras from Partial Supersymmetry Breaking

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Abstract

The partial breaking of supersymmetry in flat space can be accomplished using any one of three dual representations for the massive $N=1$ spin-3/2 multiplet. Each of the representations can be “unHiggsed”, which gives rise to a set of dual $N=2$ supergravities and supersymmetry algebras.
1 Introduction

The transition from string theory to the standard model can be characterized in terms of a hierarchy of supersymmetries: from $N = 8$ to $N = 0$. These supersymmetries must be spontaneously broken, either all at once, to $N = 0$, or partially, first to $N = 1$ (or higher) and then to $N = 0$.

For phenomenological applications of weak-scale supersymmetry, one would like to construct the effective field theory that describes the breaking of $N = 8$ to $N = 1$. In this paper we will focus on a simpler case, that of $N = 2$ broken to $N = 1$. We will construct a set of effective supergravity theories that contain an unbroken, linearly realized $N = 1$ supersymmetry, as well as a spontaneously broken, nonlinearly realized, $N = 2$.

Heuristically, it might seem impossible to partially break $N = 2$ to $N = 1$. The argument runs as follows. Start with the $N = 2$ supersymmetry algebra

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma^m_{\alpha\dot{\alpha}} P_m,$$
$$\{S_\alpha, \bar{S}_{\dot{\alpha}}\} = 2\sigma^m_{\alpha\dot{\alpha}} P_m,$$  (1)

where $Q_\alpha$ and its conjugate $\bar{Q}_{\dot{\alpha}}$ denote the first, unbroken supersymmetry, and $S_\alpha, \bar{S}_{\dot{\alpha}}$ the second. Suppose that one supersymmetry is not broken, so

$$Q |0\rangle = \bar{Q} |0\rangle = 0.$$  (2)

Because of the supersymmetry algebra, this implies that the Hamiltonian also annihilates the vacuum,

$$H |0\rangle = 0.$$  (3)

Then, according to the supersymmetry algebra,

$$(\bar{S}S + SS) |0\rangle = 0.$$  (4)

For a positive definite Hilbert space, this leads one to conclude that

$$S |0\rangle = \bar{S} |0\rangle = 0.$$  (5)

This argument can be evaded by two loopholes. The first is that in a spontaneously broken theory, one can only consider the algebra of the currents, since the charges of the spontaneously broken symmetries do not exist rigorously. The second exploits the fact that in covariantly-quantized supergravity theories, the gravitino $\psi_{ma}$ is a gauge field with negative-norm components, so the Hilbert space does not have positive norm.

There are by now many examples of partial supersymmetry breaking which take advantage the first loophole. The first was given by Hughes, Liu and Polchinski [1], who showed that supersymmetry is partially broken on the world volume of an $N = 1$ supersymmetric 3-brane propagating in six-dimensional superspace. Later, Bagger and Galperin [2, 3] used the techniques of Coleman, Wess, Zumino [4], and Volkov [5] to construct an effective field theory of partial supersymmetry breaking, with the broken supersymmetry realized nonlinearly. They found that the Goldstone fermion could belong to an $N = 1$ chiral or an $N = 1$ vector multiplet. Antoniadis, Partouche and Taylor discovered another realization in which the Goldstone fermion is contained in an $N = 2$ vector multiplet [6].
Each of these examples relies on the fact that in partially broken supersymmetry, the current algebra can be modified as follows,

\[
\begin{align*}
\{ \bar{Q}_{\dot{a}}, J^1_{\alpha m} \} &= 2 \sigma^{\alpha}_{\dot{a} \dot{\alpha}} T_{mn} \\
\{ \bar{S}_{\dot{a}}, J^2_{\alpha m} \} &= 2 \sigma^{\alpha}_{\dot{a} \dot{\alpha}} (v^4 \eta_{mn} + T_{mn}) ,
\end{align*}
\]  

where the \( J^i_{\alpha m} \) (\( i = 1, 2 \)) are the supercurrents and \( T_{mn} \) is the stress-energy tensor. The shift in the second stress-energy tensor in (6) prevents the current algebra from being integrated into a charge algebra, and circumvents the no-go theorem.

In gravity, however, a shift in the stress-energy tensor corresponds to a shift in the vacuum energy. This suggests that the mechanism of partial breaking might be different in supergravity theories. Indeed, theories with partial breaking were constructed by Cecotti, Girardello and Porrati, and by Zinov’ev [7], starting from linearly realized \( N = 2 \) supergravity. (A geometrical interpretation was given in [8].) These authors considered scenarios with vector- and hypermultiplets and found that the gravitational couplings exploited the second loophole. It is natural to ask whether their results apply more generally in supergravity theories.

In this paper we will address this question using a model-independent approach with a minimal field content motivated by the superHiggs-effect. We will see that partial breaking in flat space can be accomplished using three dual representations for the \( N = 1 \) massive spin-3/2 multiplet. When coupled to gravity, the dual representations give rise to new \( N = 2 \) supergravities with new \( N = 2 \) supersymmetry algebras.

In each case, our technique will be as follows: We will start with the Lagrangian and supersymmetry transformations for the massive \( N = 1 \) spin-3/2 multiplet. We shall then “unHiggs” the representation by adding appropriate Goldstone fields and coupling it to gravity.

2 The SuperHiggs Effect in Partially Broken Supersymmetry

2.1 Dual Versions of Massive \( N = 1 \) Spin-3/2 Multiplets

The starting point for our investigation is the massive \( N = 1 \) spin-3/2 multiplet. This multiplet contains six bosonic and six fermionic degrees of freedom, arranged in states of the following spins,

\[
\begin{pmatrix}
\frac{3}{2} \\
1 \\
\frac{1}{2}
\end{pmatrix}.
\]  

(7)

The traditional representation of this multiplet contains the following fields [9]: one spin-3/2 fermion, one spin-1/2 fermion, and two spin-one vectors, each of mass \( m \). The dual representations have the same fermions, but one or two antisymmetric tensors in place of one or two of the vectors. As we shall see, each representation gives rise to a distinct \( N = 2 \) supersymmetry algebra.
The traditional representation is described by the following Lagrangian [9],

\[
\mathcal{L} = \epsilon^{mn \rho \sigma} \bar{\psi}_m \sigma_n \partial_\rho \psi_\sigma - i \bar{\psi} \sigma^m \partial_m \zeta - \frac{1}{4} A_{mn} \tilde{A}^{mn} \\
- \frac{1}{2} m^2 A_m \tilde{A}^m + \frac{1}{2} m \zeta \zeta + \frac{1}{2} m \tilde{\zeta} \tilde{\zeta} \\
- m \psi_m \sigma^{mn} \tilde{\psi}_n - m \bar{\psi}_m \sigma^{mn} \tilde{\psi}_n .
\]

Here \( \psi_m \) is a spin-3/2 Rarita-Schwinger field, \( \zeta \) a spin-1/2 fermion, and \( A_m = A_m + i B_m \) a complex spin-one vector. This Lagrangian is invariant under the following \( N = 1 \) supersymmetry transformations,

\[
\delta_\eta A_m = 2 \psi_m \eta - i \frac{2}{\sqrt{3}} \bar{\zeta} \sigma_m \eta - \frac{2}{\sqrt{3} m} \partial_m (\zeta \eta) \\
\delta_\eta \zeta = \frac{1}{\sqrt{3}} \bar{A}_{mn} \sigma^{mn} \eta - i \frac{m}{\sqrt{3}} \sigma^{mn} \eta A_m \\
\delta_\eta \psi_m = \frac{1}{3 m} \partial_m (A_n \sigma^{rs} \eta + 2 m \sigma^n \eta A_n) - i \frac{1}{2} (H_{+mn} \sigma^n + \frac{1}{3} H_{-mn} \sigma^n) \eta \\
- \frac{2}{3} m (\sigma^n A_n \eta + \bar{A}_m \eta) ,
\]

where \( H_{\pm mn} = A_{mn} \pm \frac{i}{2} \epsilon_m r s A^{rs} \) and \( A_{mn} = \partial_m A_n - \partial_n A_m \).

A dual Lagrangian and its supersymmetry transformations can be found by using a Poincaré duality which relates a massive vector field to a massive antisymmetric tensor field of rank two. This duality can be used to relate the vector \( B_m \) to an antisymmetric tensor \( B_{mn} \) by \( B_{mn} = 1/m \epsilon_m r s \partial^r B^s \) or \( B_m = v_m /m \) [10].

This dual representation is special in the sense that it can also be written in \( N = 1 \) superspace formulation\(^1\) [14]. It has the following component Lagrangian,

\[
\mathcal{L} = \epsilon^{pqr s} \tilde{\psi}_p \sigma_q \partial_r \psi_s - i \bar{\psi} \sigma^p \partial_p \zeta - \frac{1}{4} A_{mn} A^{mn} + \frac{1}{2} v^m v_m \\
- \frac{1}{2} m^2 A_m A^m - \frac{1}{4} m^2 B_{mn} B^{mn} + \frac{1}{2} m \zeta \zeta + \frac{1}{2} m \tilde{\zeta} \tilde{\zeta} \\
- m \psi_m \sigma^{mn} \tilde{\psi}_n - m \bar{\psi}_m \sigma^{mn} \tilde{\psi}_n ,
\]

where \( A_{mn} \) is the field strength associated with the real vector field \( A_m \), and \( v_m = \frac{1}{2} \epsilon_m r s \partial^r B^s \) is the field strength for the antisymmetric tensor \( B_{mn} \). This Lagrangian is invariant under the following \( N = 1 \) supersymmetry transformations:\(^2\)

\[
\delta_\eta A_m = (\psi_m \eta + \bar{\psi}_m \bar{\eta}) + \frac{i}{\sqrt{3}} (\bar{\eta} \sigma_m \zeta - \bar{\zeta} \sigma_m \eta) - \frac{1}{\sqrt{3} m} \partial_m (\zeta \eta + \tilde{\zeta} \bar{\eta}) \\
\delta_\eta B_{mn} = \frac{2}{\sqrt{3}} (\eta \sigma_m \zeta + \frac{i}{2m} \partial_m \bar{\zeta} \sigma_m \eta) + \eta \sigma_m \bar{\psi}_n + \frac{1}{m} \eta \psi_{mn} + h.c.
\]

\(^1\) The massive version of the de Wit/van Holten formulation (see [11] and references therein) leads to a reducible supersymmetry representation.

\(^2\) Here, the square brackets denote antisymmetrization, without a factor of 1/2.
\[\delta_\eta \zeta = \frac{1}{\sqrt{3}} A_{mn} \sigma^{mn} \eta - \frac{im}{\sqrt{3}} \sigma^m \bar{\eta} A_m - \frac{1}{\sqrt{3}} m \sigma_{mn} \eta B^{mn} - \frac{1}{\sqrt{3}} v_m \sigma^m \bar{\eta} \]

\[\delta_\eta \psi_m = \frac{1}{3m} \partial_m (A_{rs} \sigma^{rs} \eta + 2im \sigma^n \bar{\eta} A_n) - \frac{i}{2} (H^A_{+mn} \sigma^n + \frac{1}{3} H^A_{-mn} \sigma^n) \bar{\eta} \]

\[= \frac{2}{3} m (\sigma_m A_n \eta + A_m \eta) + \frac{1}{3m} \partial_m (2v_n \sigma^n \bar{\eta} - m \sigma_{rs} \eta B_{rs}) \]

\[= \frac{2i}{3} (v_m + \sigma_{mn} v^n \eta) - \frac{im}{3} (B_{mn} \sigma^n \bar{\eta} + i \epsilon_{mnrs} B^{nr} \sigma^s \bar{\eta}) \],

(11)

A third representation can be found by dualizing the remaining vector, \(A_n\). Its derivation is straightforward, so we will not write its Lagrangian and transformations here.

Each of the three dual Lagrangians describe the dynamics of free massive spin-3/2 and 1/2 fermions, together with their supersymmetric partners, massive spin-one vector and tensor fields. They can be regarded as “unitary gauge” representations of theories with additional symmetries: a fermionic gauge symmetry for the massive spin-3/2 fermion, as well as additional gauge symmetries associated with the massive gauge fields.

### 2.2 UnHiggsing Massive \(N = 1\) Spin-3/2 Multiplets

To study partial breaking, these Lagrangians must be unHiggsed by including appropriate gauge and Goldstone fields. In each case we need to add a Goldstone fermion and Goldstone bosons and then gauge the full \(N = 2\) supersymmetry. In this way we can construct theories with \(N = 2\) supersymmetry nonlinearly realized, and \(N = 1\) represented linearly on the fields. The resulting effective field theories describe the physics of partial supersymmetry breaking, well below the scale \(v\) where the second supersymmetry is broken.

In what follows we will focus on the first two cases presented above; the example with two antisymmetric tensors can be worked out in a similar fashion. In each case we introduce Goldstone fields by a Stückelberg redefinition. We unHiggs the complex massive vector \(A_m\) by replacing

\[A_m \rightarrow A_m - \frac{\sqrt{2}}{m} \partial_m \phi ; \]

(12)

for the dual representation, we take

\[A_m \rightarrow A_m - \frac{1}{m} \partial_m \phi \]

\[B_{mn} \rightarrow B_{mn} - \frac{1}{m} \partial_m [B_n] . \]

The introduction of the Goldstino \(\nu\) requires an additional shift

\[\psi_m \rightarrow \psi_m - \frac{1}{\sqrt{6} m} (2 \partial_m \nu + im \sigma_m \bar{\nu}) \]

(13)

to obtain a proper kinetic term for \(\nu\).

In Figure 1(a) the physical fields of the traditional representation for the massive spin-3/2 multiplet are arranged in terms of massless \(N = 1\) multiplets. The lowest superspins
Figure 1: The unHiggsed versions of the (a) traditional and (b) dual representations of the $N=1$ massive spin-$3/2$ multiplet.

form an $N=1$ chiral and an $N=1$ vector multiplet. These fields may be thought of as $N=1$ "matter." The remaining fields are the gauge fields of $N=2$ supergravity. In unitary gauge, the two vectors eat the two scalars, while the Rarita-Schwinger field eats one linear combination of the spin-1/2 fermions. This leaves the massive $N=1$ multiplet coupled to $N=1$ supergravity. As we shall see, Figure 1 only illustrates the field content; it does not describe the $N=1$ multiplet structure of the unHiggsed theory.

The resulting Lagrangian is as follows,

$$e^{-1}L = -\frac{1}{2\kappa^2}R + \epsilon^{mnr}s_{\tau}^m\sigma^m_{\tau}D_r\psi^j_s - i\chi^m\sigma^m D_m\lambda - i\lambda^m\sigma^m D_m\phi + D^m\phi D^m\phi$$

$$- \frac{1}{4} A_{mn} - \left[ \frac{1}{\sqrt{2}} m \psi_1^m + \frac{1}{\sqrt{2}} \kappa \sqrt{2} \bar{\psi}_1^m \lambda \right]$$

$$+ m \psi_2^m \sigma^m \psi_2^m + \frac{\kappa}{\sqrt{2}} \psi_1^m \sigma^m \psi_1^m + \frac{\kappa}{\sqrt{2}} \lambda \sigma^m \bar{\psi}_1^m$$

$$+ \frac{\kappa}{\sqrt{2}} \psi_2^m \sigma^m \psi_2^m + \frac{\kappa}{\sqrt{2}} \epsilon^{mns} \psi_2^m \sigma^m \psi_2^m + h.c.) ,$$

where $\kappa$ denotes Newton’s constant, $m = \kappa v^2$, and $D_m$ is the covariant derivative. The supercovariant derivatives take the form

$$\hat{D}_m \phi = \partial_m \phi - \frac{\kappa}{\sqrt{2}} \psi_1^m \chi - \frac{1}{\sqrt{2}} \kappa v^2 A_m$$

$$\hat{A}_{mn} = A_{mn} + \kappa \psi_1^{[m} \psi_1^{n]} - \frac{\kappa}{\sqrt{2}} \bar{\lambda} \sigma_1^{[n} \psi_1^{m]} .$$

This Lagrangian is invariant under two independent abelian gauge symmetries, as well as the following supersymmetry transformations,
\[ \delta e^a_m = i\kappa (\eta^i \sigma^a \bar{\psi}_{mi} + \bar{\eta}^i \sigma^a \psi_{mi}) \]
\[ \delta \psi^i_m = \frac{2}{\kappa} D_m \eta^i \]
\[ + \left( -\frac{i}{2} \hat{H}_{+mn} \sigma^m \eta^1 + \sqrt{2} D_m \phi \eta^1 - \kappa \psi^1_m (\bar{\chi} \eta^1) + i v^2 \sigma_m \bar{\eta}^2 \right) \delta^i_\eta \]
\[ \delta A_m = 2 \epsilon_{ij} \bar{\psi}^i_m \eta^j + \sqrt{2} \lambda \sigma_m \eta^1 \]
\[ \delta \lambda = \frac{i}{\sqrt{2}} \bar{A}_{mn} \sigma^{mn} \eta^1 - i \sqrt{2} v^2 \eta^2 \]
\[ \delta \chi = i \sqrt{2} m \sigma_1 \bar{A}_{m} \eta^1 + 2 v^2 \eta^2 \]
\[ \delta \phi = \sqrt{2} \chi \eta^1 , \quad (16) \]

for \( i = 1, 2 \). This result holds to leading order, that is, up to and including terms in the transformations that are linear in the fields. Note that this representation is irreducible in the sense that there are no subsets of fields that transform only into themselves under the supersymmetry transformations.

Let us now consider the dual case with one massive tensor. The degree of freedom counting is shown in Figure 1(b). This time, however, the “matter” fields include an \( N = 1 \) vector multiplet together with an \( N = 1 \) linear multiplet. In unitary gauge, one vector eats one scalar, while the antisymmetric tensor eats the other vector. These are the minimal set of fields that arise when coupling the alternative spin-3/2 multiplet to \( N = 2 \) supergravity.

The Lagrangian and supersymmetry transformations for this system can be worked out following the same procedures described above. They can also be derived by dualizing first the scalar \( \phi_B \) and then the vector \( B_m \) using the method described in [13]. As \( \kappa \to 0 \), the dualities relating a massless antisymmetric tensor \( B_{mn} \) to a massless scalar \( \phi \) and a massless vector \( A_m \) to another vector \( B_m \) reduce to the simple expressions \( v_m = -\partial_m \phi \) and \( F^B_{mn} = 1/2 \epsilon_{mnr} F^{A_{rs}} \).

The Lagrangian is as follows,

\[ e^{-1} \mathcal{L} = \]
\[ -\frac{1}{2\kappa^2} \mathcal{R} + \epsilon^{pqrs} \bar{\psi}_m \bar{\sigma}_q D_r \psi_s - i \bar{\chi} \bar{\sigma}^m D_m \chi - i \bar{\lambda} \bar{\sigma}^m D_m \lambda - \frac{1}{2} \bar{D}^m \phi D_m \phi \]
\[ - \frac{1}{4} F_{amn}^A F^{A_{mn}} - \frac{1}{4} F_{amn}^B F^{B_{mn}} + \frac{1}{2} v^m v_m - \left( \frac{1}{\sqrt{2}} m \psi_2^m \bar{\sigma}^m \bar{\chi} + m \psi_2^m \sigma^m \bar{\chi} \right) \]
\[ + \sqrt{2} m i \chi + \frac{1}{2} m \chi + m \psi_2^m \sigma^{mn} \psi_2^n + \frac{\kappa}{2\sqrt{2}} \epsilon_{ij} \bar{\psi}^i_m \bar{\psi}^j_n \]
\[ + \frac{\kappa}{2} \sigma^m \bar{\sigma}^n \psi_1^m D_n \phi + \frac{\kappa}{2} \bar{\lambda} \sigma_m \psi_1^n F_{+}^{B_{mn}} + \frac{\kappa}{2} \epsilon^{pqrs} \bar{\psi}_p \bar{\sigma} q^1 \psi^1_r D_s \phi \]
\[ - i \frac{\kappa}{2} \chi \sigma^{m} \bar{\sigma}^n \psi_1^m v_n - i \frac{\kappa}{2} \epsilon^{pqrs} \bar{\psi}_p \bar{\sigma} q^1 \psi^1_r v_s + h.c. \]  \( (17) \)

\( ^{3}\) The transformations (19) do not appear to be dual to (18), because the vectors \( A_m \) and \( B_m \) in (19) have been rotated to simplify the transformations.
where, as before, $m = \kappa v^2$, and

\[
\mathcal{D}_m \phi = \partial_m \phi - \frac{m}{\sqrt{2}}(A_m + B_m)
\]

\[
\mathcal{F}^A_{mn} = \partial_m A_n + \frac{m}{\sqrt{2}}B_{mn}
\]

\[
\mathcal{F}^B_{mn} = \partial_m B_n - \frac{m}{\sqrt{2}}B_{mn}.
\]

(18)

This Lagrangian is invariant under an ordinary abelian gauge symmetry, an antisymmetric tensor gauge symmetry, as well as the following two supersymmetries,

\[
\delta_\eta \epsilon^a_m = i \kappa (\eta^a \sigma^a \psi_{m i} + \bar{\eta}^a \bar{\sigma}^a \psi_{m i})
\]

\[
\delta_\eta \psi^1_m = \frac{2}{\kappa} \mathcal{D}_m \eta^1
\]

\[
\delta_\eta A_m = \sqrt{2} \epsilon_{ij} (\psi^j_m \eta^i + \bar{\psi}^j_m \bar{\eta}^i)
\]

\[
\delta_\eta B_m = \eta^1 \sigma_m \lambda + \bar{\lambda} \sigma_m \eta^1
\]

\[
\delta_\eta B_{mn} = 2 \eta^1 \sigma_{mn} \chi + i \eta^1 \sigma_{[m} \bar{\psi}^{n]}_{\bar{1}} + i \eta^2 \sigma_{[m} \bar{\psi}^{n]}_{\bar{1}} + \text{h.c.}
\]

\[
\delta_\eta \lambda = i \mathcal{\hat{F}}^B_{mn} \sigma^m \eta^1 - i \sqrt{2} v^2 \eta^2
\]

\[
\delta_\eta \chi = i \sigma^m \bar{\eta}^1 \mathcal{\hat{D}}_m \phi - \bar{v}_m \sigma^m \eta^1 + 2 v^2 \eta^2
\]

\[
\delta_\eta \psi^2_m = \frac{2}{\kappa} \mathcal{D}_m \eta^2 + iv^2 \sigma^m \eta^2 - \frac{i}{\sqrt{2}} \mathcal{\hat{F}}^A_{mn} \sigma^n \eta^1
\]

\[
\delta_\eta \phi = \chi \bar{\eta}^1 + \bar{\chi} \eta^1
\]

(19)

up to linear order in the fields. The supercovariant derivatives are given by

\[
\mathcal{\hat{D}}_m \phi = \mathcal{D}_m \phi - \frac{\kappa}{2} (\psi^1_m \chi + \bar{\psi}^1_m \bar{\chi})
\]

\[
\mathcal{\hat{F}}^A_{mn} = \mathcal{F}^A_{mn} + \frac{\kappa}{\sqrt{2}} (\psi^2_m \psi^1_n + \bar{\psi}^2_m \bar{\psi}^1_n)
\]

\[
\mathcal{\hat{F}}^B_{mn} = \mathcal{F}^B_{mn} - \frac{\kappa}{2} (\bar{\lambda} \sigma_{[m} \psi^1_{n]} + \bar{\psi}^1_m \sigma_{[n} \lambda)
\]

\[
\bar{v}_m = v_m + \left( i \kappa \psi^1_n \sigma^m_n \chi - \frac{i \kappa}{2} \epsilon_{n r s} \psi^1_n \sigma_r \bar{\psi}^2_s + \text{h.c.} \right).
\]

(20)

These fields form an irreducible representation of the $N = 2$ algebra.

Each of the two Lagrangians has a full $N = 2$ supersymmetry (up to the appropriate order). The first supersymmetry is realized linearly. The second is realized nonlinearly: it is spontaneously broken. In each case, the transformations imply that

\[
\zeta = \frac{1}{\sqrt{3}} (\chi - i \sqrt{2} \lambda)
\]

(21)

does not shift, while

\[
\nu = \frac{1}{\sqrt{3}} (\sqrt{2} \chi + i \lambda)
\]

(22)

does. Therefore $\nu$ is the Goldstone fermion for $N = 2$ supersymmetry, spontaneously broken to $N = 1$.
3 Dual Algebras from Partial Supersymmetry Breaking

Now that we have explicit realizations of partial supersymmetry breaking, we can see how they avoid the no-go argument presented in the introduction. We first compute the second supercurrent. In each case it turns out to be

\[ J_{\alpha m}^2 = v^2 \left( \sqrt{6} i \sigma_{\alpha \dot{\alpha}} \bar{\nu}^{\dot{\alpha}} + 4 \sigma_{\alpha \beta m n} \psi^{2n} \right) , \]

plus higher-order terms. The commutator of the second supercharge with the second supercurrent is then

\[ \{ \bar{S}_\alpha, J_{\alpha m}^2 \} = 0 + \text{terms at least linear in the fields} . \] (24)

From this we see that the stress-energy tensors in the current algebra \([8]\) do not differ by a constant shift. The supergravity couplings must exploit the second loophole to the no-go theorem.

To check this assertion, note that the operators \(J_{\alpha m}^i\) and \(T_{mn}\) contain contributions from all of the fields, including the second gravitino. When covariantly-quantized, the second gravitino gives rise to states of negative norm. Indeed, we find

\[ (\bar{S} S + S \bar{S}) |0\rangle = 0 , \] (25)

even though

\[ S |0\rangle \neq 0 \quad \bar{S} |0\rangle \neq 0 . \] (26)

To elucidate the role of the bosonic symmetries associated with partial supersymmetry breaking, let us now compute the closure of the first and second supersymmetry transformations to zeroth order in the fields. In this way we can identify the Goldstone fields associated with any spontaneously broken bosonic symmetries.

For the traditional representation, (Figure 1(a)), we find

\[
\begin{align*}
[\delta_{\eta_1}, \delta_{\eta_2}] \phi &= 2\sqrt{2} v^2 \eta_1 \eta_2 \\
[\delta_{\eta_1}, \delta_{\eta_2}] A_m &= 4 \kappa \partial_m (\eta_1 \eta_2) \\
[\delta_{\eta_1}, \delta_{\eta_2}] B_m &= 2 \sqrt{2} i v^2 \eta_1 \sigma_m \eta_2 , \end{align*}
\] (27)

This shows that the complex scalar \(\phi\) is indeed the Goldstone boson for a gauged central charge. Moreover, in unitary gauge, where

\[ \phi = \nu = 0 , \] (28)

this Lagrangian reduces to the usual representation for a massive \(N = 1\) spin-3/2 multiplet \([3]\).

For the dual representation (Figure 1(b)), we have

\[
\begin{align*}
[\delta_{\eta_1}, \delta_{\eta_2}] \phi &= 2 \sqrt{2} v^2 (\eta_1 \eta_2 + \bar{\eta}_1 \bar{\eta}_2) \\
[\delta_{\eta_2}, \delta_{\eta_1}] A_m &= 2 \sqrt{2} i \kappa \partial_m (\eta_1^2 \bar{\eta}_2 + \bar{\eta}_1 \bar{\eta}_2) - \sqrt{2} i v^2 (\eta_1^2 \sigma_m \bar{\eta}_1 - \eta_1^1 \sigma_m \bar{\eta}_1) \\
[\delta_{\eta_2}, \delta_{\eta_1}] B_m &= \sqrt{2} i v^2 (\eta_1^1 \sigma_m \bar{\eta}_1 - \eta_1^1 \sigma_m \bar{\eta}_1) \\
[\delta_{\eta_2}, \delta_{\eta_1}] B_{mn} &= 2 \sqrt{2} i \kappa D_{m[n} (\eta_1^2 \sigma_{n]} \bar{\eta}_1 - \eta_1^1 \sigma_{n]} \bar{\eta}_1) .
\end{align*}
\] (29)
The real vector \(- (A_m - B_m)/\sqrt{2}\) is the Goldstone boson for a gauged vectorial central extension of the \(N = 2\) algebra. In addition, the real scalar \(\phi\) is the Goldstone boson associated with a single real gauged central charge. In unitary gauge, with

\[
- \frac{1}{\sqrt{2}} (A_m - B_m) = \phi = \nu = 0 ,
\]

this Lagrangian reduces to the dual representation for the massive \(N = 1\) spin-3/2 multiplet [12].

Finally, for the case with two tensors \(A_{mn} = A_{mn} + iB_{mn}\) and two Goldstone vectors \(A_m = A_m + iB_m\), the algebra is

\[
[\delta_{\eta^2}, \delta_{\eta^1}] A_m = \frac{4}{\kappa} D_m (\bar{\eta}^1 \eta^2) - 4i v^2 \eta^2 \sigma_m \bar{\eta}^1 ,
\]

\[
[\delta_{\eta^2}, \delta_{\eta^1}] A_{mn} = - \frac{4i}{\kappa} D_{[m} (\eta^2 \sigma_n \bar{\eta}^1) ,
\]

This case requires two vectorial central extensions of the supersymmetry algebra.

4 Discussion and Conclusion

In this paper we have examined the partial breaking of supersymmetry in flat space. We have seen that partial breaking can be accomplished using either of three representations of the massive \(N = 1\) spin-3/2 multiplet. We unHiggsed the representations, and found a new \(N = 2\) supergravity and a new \(N = 2\) supersymmetry algebra.

Each of these theories gives rise to different \(N = 1\) multiplet structures in the limit \(\kappa \to 0\). For the traditional representation, we find a massless chiral multiplet, \((\chi, \phi)\), together with a pair of “twisted” massless \(N = 1\) multiplets, \((\psi^2_m, A_m, \lambda)\). The twisted multiplets transform irreducibly into each other under the first, unbroken supersymmetry. They can be untwisted with the help of a second unbroken supersymmetry which appears in this limit.\(^4\) The second supersymmetry transformations are obtained from (16) (in the \(\kappa \to 0\) limit) by \(A_m \to \bar{A}_m, \lambda \to -\lambda\). We see that the twisted multiplet is actually a massless \(N = 2\) multiplet.

In the case of the dual representation, the \(N = 1\) transformations (13) reduce, in the \(\kappa \to 0\) limit, to those of a massless vector multiplet, \((B_m, \lambda)\), a linear multiplet, \((\chi, B_{mn}, \phi)\), and a massless spin-3/2 multiplet, \((\psi^2_m, A_m)\).\(^5\) This multiplet structure can also be obtained by an explicit superfield unHiggsing of the massive spinor superfield \(\Psi_\alpha\) in the Ogievetsky/Sokatchev formulation [12],

\[
\mathcal{L} = - \frac{1}{2} (\Psi \bar{\Psi}) \pi^L \left( \begin{array}{c} \Psi \\ \bar{\Psi} \end{array} \right) + \frac{1}{2} m (\Psi \bar{\Psi} + \bar{\Psi} \Psi) ,
\]

\(^4\)We are indebted to W. Siegel for pointing this out.

\(^5\)The transformations that mix the gravitino and the antisymmetric tensor are physically irrelevant because the transformations of the corresponding field strengths vanish on-shell.
where $\pi^\perp = \sqrt{\Pi_1}$ and $\Pi_1$ is the superspin-1 projector for a spinor superfield $[14]$. The St"uckelberg redefinition $\Psi_\alpha \rightarrow \Psi_\alpha - iD_\alpha V + L_\alpha + 2iW_\alpha/m + D_\alpha L/4m$ leads to

$$\mathcal{L} \rightarrow \mathcal{L}^\perp + 2W^\alpha D_\alpha V - \frac{1}{8}L^2 + \frac{1}{2}m((\Psi_\alpha - iD_\alpha V + L_\alpha)^2 + h.c.) ,$$  \hspace{1cm} (32)

where $V$ is a real vector and $L_\alpha$ a chiral spinor superfield; $W_\alpha$ and $L$ are the corresponding field strengths. The correct multiplet structure is obtained in the limit $m \rightarrow 0$. Note, however, that the auxiliary superspin-0 is lost so we expect $1/m$ singularities in the supersymmetry transformations.

The multiplet structure of the dual theory with two antisymmetric tensors consists of the $N = 2$ representation, $(\psi_m^2, A_m + iB_m = A_m, \lambda)$, as well as a linear multiplet with two antisymmetric tensors, $(\chi, A_{mn} + iB_{mn} = A_{mn})$. The argument that prevents the coupling of this multiplet to supergravity (see $[15]$ and references therein) does not apply here since the “non-closure” terms in the supersymmetry algebra are cancelled by terms from the variation of $\psi_m^2$.

The Lagrangian for the traditional representation is a truncation of the supergravity coupling found by Cecotti, Girardello, and Porrati, and by Zinov’ev $[7]$. Their results were based on linear $N = 2$ supersymmetry; they involved at least one $N = 2$ vector-multiplet and one hypermultiplet. The Lagrangians for the dual cases are new. They contain new realizations of $N = 2$ supergravity.

In each case, the couplings presented here are minimal and model-independent. They describe the superHiggs effect in the low-energy effective theories that arise from partial supersymmetry breaking.

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