Drop analysis of Active Magnetic Bearing with rolling-sliding integrated auxiliary bearing

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Abstract. The Active Magnetic Bearing (AMB) technology is introduced in the High Temperature Reactor-Pebble-bed Modules (HTR-PM) demonstration nuclear power plant, which is being constructed in Shandong province, China. The auxiliary bearing is one of the most important components guaranteeing the reliability of the AMB. It also has an important impact on the reliability of the whole reactor system. Compared with the traditional auxiliary bearing, a novel one proposed by the authors has a smaller impact force and the rotor center orbit is much more concentrated during the rotor drop. This paper establishes an analytical model of drop of rolling-sliding integrated auxiliary bearing to analyze the above phenomena. Based on the Hertz contact theory, the complex structure inside the rolling bearing is simplified through a spring damping model. The overall impact model of the rolling-sliding integrated auxiliary bearing is established. Then, according to the structural characteristics of the rolling-sliding integrated auxiliary bearing, the tangential force inside the rolling-sliding integrated auxiliary bearing can be obtained by applying the angular momentum theorem. Finally, a four-degree-of-freedom horizontal rotor drop model is established to analyze and calculate the center orbit and motion state of the rotor. The analytical model is helpful in the selection and design of auxiliary bearing for AMB. In further research this contact model can be used to calculate the center orbit and contact forces in the application of the rolling-sliding integrated auxiliary bearing.

1. Introduction
The Active Magnetic Bearing (AMB) is a typical type of mechanical and electronic product. It has a lot of advantages, such as active suppression of vibration, ultra-high speed, lubrication-free, no mechanical friction, high energy density, low operation and maintenance costs and so on. AMB can solve the problem of traditional mechanical dynamics, and it gradually becomes a focus of research on the application of the high-speed rotating machinery [1]. In the High Temperature Reactor-Pebble-bed Modules (HTR-PM) demonstration nuclear power plant, which is being constructed in Shandong province, China, the AMB is used to suspend the rotor of the main helium blower [2, 3]. The rotor will lose support and fall on the auxiliary bearings in case of emergency such as power failure, overload, or
outage for maintenance. The auxiliary bearings are also called backup bearings or protection bearings. When the rotor falls off, the auxiliary bearing accelerates rapidly and bears the collision and friction, which can avoid the damage caused by the rotor falling off. Therefore, auxiliary bearing is one of the most important components guaranteeing the reliability of the AMB. It also directly affects the reliability of the whole reactor system.

Generally speaking, there are two basic types of auxiliary bearings, rolling bearings and sliding bearings. Compared with sliding bearings, the rolling auxiliary bearings can effectively reduce the friction inside the bearing when the rotor falls down. However, the contact between the rolling element and the inner and outer rings is point contact, which has large local stress. And the rolling auxiliary bearing is more susceptible to damage. Prashad et al. [4] proposed the Double Decker Rolling Bearing (DDR) formed by connecting two high-precision rolling bearings to each other via an adapter ring. As an auxiliary bearing, it will effectively reduce the dynamic impact of the rotor drop. But due to its complicated structure and difficult design, its application is limited. The sliding auxiliary bearing has a simple structure and large starting friction resistance. It is mostly used in low-speed and heavy-load rotor system. OKBM institute of Russia [5~7] applied sliding auxiliary bearing with damping systems in the helium turbine modular high temperature gas cooled reactor to provide temporary support for the rotor. Moreover, they applied the electromagnetic bearing to the 600MW GT-MHR system. In addition, some scholars improved the structure of auxiliary bearings according to different engineering application requirements, and proposed Planetary Gear Auxiliary Bearings, Zero Clearance Auxiliary Bearings (ZCAB) [8], External Clearance Auxiliary Bearings [9] and hybrid auxiliary bearings which combined different types of auxiliary bearings or mechanical parts. The auxiliary bearing can effectively avoid the catastrophic accident of the electromagnetic bearing suspension rotor system. [10]

G. J. Yang et al. [11] presented a new-type auxiliary bearing called rolling-sliding integrated auxiliary bearing. It is a design of novel arrangement with outer clearance structure. For this novel arrangement, the rolling bearing is fixed on the rotor, and there is interference fit or small clearance fit between the inner ring of the rolling bearing and the rotor. The speed of the outer ring will be equal to or smaller than the speed of the rotor. When the magnetic bearing system fails, the rolling bearing will fall on the bearing housing with the rotor. J. Q. Chen et al. [12, 13] studied the performance of this new auxiliary bearing by simulation with software LS-Dyna. X. N. Liu et al. [14] conducted a rotor drop experiment on a horizontal high-speed motor with an AMB system, and compared the rotor center orbit and the maximum impact force of the rolling-sliding integrated auxiliary bearing and the traditional auxiliary bearing.

The structure of the rolling-sliding integrated auxiliary bearing is shown in Fig. 1. In this paper, a drop dynamics model for the rolling-sliding integrated auxiliary bearing is proposed. The dynamic characteristics and internal friction between the rolling-sliding integrated auxiliary bearing and the rotor are studied in the falling process of the horizontal rotor, so as to analyze the dynamics of the dropping rotor.

![Figure 1. The schematic of the structure of the novel auxiliary bearing.](image-url)
2. Hertz Contact

According to the Hertz contact theory, the Hertz contact force \( F \) is expressed as:

\[
F = \begin{cases} 
K\delta^e + C\delta, & \delta > 0 \\
0, & \delta \leq 0
\end{cases}
\]

(1)

where \( \delta \) represents the relative normal deformation of the contacting bodies, \( K \) and \( C \) are respectively the generalized stiffness parameter and generalized damping parameter, \( e \) is the contact coefficient and its value is determined by the contact type: for point contact, \( e=3/2 \), for line contact, \( e=10/9 \) and for surface contact, \( e=1 \) [15]. The generalized parameter \( K \) is dependent on contact type and material properties.

There are two main types of contact between horizontal rotor and the rolling-sliding integrated auxiliary bearing. The radial contact between the rolling bearing and the bearing housing is line contact, while between the internal rolling element of the rolling bearing and the inner and outer rings is point contact. The bearing characteristics of the rolling-sliding integrated auxiliary bearing are determined by the above two contact types.

According to the relative normal deformation of the contacting bodies \( \delta \), the line contact stiffness parameter can be calculated by the empirical formula proposed by Palmgren [16]:

\[
K_1 = 0.7l^{8/9} \left[ \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right]^{-1}
\]

(2)

where \( l \) is the line contact length, \( E_1, E_2 \) are respectively the elastic modulus of two contact surfaces, \( \nu_1, \nu_2 \) are the Poisson's ratios of two contact surfaces.

If no load is applied, the contact between the rolling element and the inner and outer rings is a point contact. When the load is applied, the contact is a surface contact, and the contact surface is elliptical. It is considered that point contact occurs between the rolling element and the inner and outer rings. Point contact stiffness parameter \( K_p \) can be expressed as [17]:

\[
K_p = \frac{1}{2} \sqrt{\frac{2\pi^3\mu^3}{3 \left[ \frac{(1 - \nu_1^2)}{E_1} + \frac{(1 - \nu_2^2)}{E_2} \right]^2 \Gamma^3 \Sigma \rho}}
\]

(3)

where \( \mu \) is the Hertz contact coefficient, \( \Gamma \) is the complete elliptic integral of the first kind and \( \Sigma \rho \) is the sum of radius of curvature of two contact bodies on two orthogonal planes.

3. Internal Contact Stiffness Parameter of Rolling Bearings

The rolling bearing is an important component of the rolling-sliding integrated auxiliary bearing. When the horizontal rotor falls down, the internal forces of the rolling bearing directly affect the rotor drop. This section will analyze the contact force inside the rolling bearing. According to the structural characteristics of the rolling-sliding integrated auxiliary bearing, the rotor and inner ring are considered as a whole. The load is applied to the bearing and transmitted to the bearing housing through the rolling element and outer ring. It is considered that the rotor is rigid and elastic deformation occurs in the contact process between the rotor and the auxiliary bearing. Considering material damping, the internal contact of rolling bearing can be simplified through a spring damping model with mass block.

Hertz contact will occur inside the rolling-sliding integrated auxiliary bearing when the rotor falls. Although radial load can reduce the contact angle of angular contact ball bearing, it has little effect on its size. Considering the material elasticity and ignoring the influence of the radial load and position of the rolling element on the size of the contact angle, the contact angle of auxiliary bearing is constant during the loading process. The internal contact stiffness parameter of angular contact ball bearing is shown in Fig. 2.
In Fig. 2, $K_{IJ}$, $K_{OI}$ are respectively the contact stiffness parameter between the $j$-th rolling element and the inner and outer rings. According to equation (3), the contact stiffness parameter between all rolling elements and the inner and outer rings has nothing to do with the position of the rolling element. $K_{rL}$ is the contact stiffness parameter between the outer ring and the bearing housing, and $\alpha$ is the bearing contact angle. When the horizontal rotor falls down, the axial force of the rolling-sliding integrated auxiliary bearing is small, which has little effect on the motion state of the rotor [14]. Therefore, the axial contact stiffness parameter is not analyzed here. The spring structure in contact with the inner and outer rings of the rolling element is regarded as a series spring, and all rolling elements are regarded as parallel springs. According to the structure of the bearing, the total stiffness of the $j$-th rolling element (series connection of the inner and outer rings) can be expressed as:

$$K_{Bj} = \frac{1}{\left(\frac{K_{IJ}}{3} + \frac{K_{OI}}{3}\right)^{3/2}}$$

(4)

The radial displacement of the $j$-th rolling element is $\delta_{rBj}$, the radial force of the $j$-th rolling element can be expressed as:

$$K_{rBj} = K_{Bj} \left(\frac{\delta_{rBj}}{\cos \alpha}\right)^{3/2} \cos \alpha = K_{Bj} \cdot \delta_{rBj}^{3/2} \cdot (\cos \alpha)^{-1/2}$$

(5)

Based on equations (4–6), the radial component of the total stiffness parameter of the $j$-th rolling element can be expressed as:

$$K_{rBj} = \frac{1}{\left(\frac{K_{IJ}}{3} + \frac{K_{OI}}{3}\right)^{3/2} (\cos \alpha)^{-1/2}}$$

(6)

If the external radial load is $F_r$, the radial deformation of the rolling element in the direction of the maximum radial load is $\delta_{rB}$, and the included angle between the adjacent rolling element and the center of the rolling bearing is $\beta$. According to the formula of spring series and parallel connection, and considering the force balance:

$$F_r = K_{rBj} \delta_{rB}^{3/2} + 2K_{rBj} (\delta_{rB} \cdot \cos \beta)^{3/2} \cos \beta + 2K_{rBj} (\delta_{rB} \cdot \cos 2\beta)^{3/2} \cos 2\beta + \cdots$$

(7)

If $N_B$ is the number of rolling elements, the equation (7) can be simplified as [16]:

$$F_r = K_{rBj} \frac{N_B}{4.36} \delta_{rB}^{3/2}$$

(8)
According to the combination of contact stiffness parameter of all stressed rolling elements, the radial contact stiffness parameter of the rolling element in the direction of the maximum radial load is obtained by

\[ K_{RB} = \frac{K_{RB}N_B}{4.36} \]  

(9)

The rolling element is simplified as the spring damping model, and the normal contact force is replaced by the linear spring damping force. When the rotor collides with the rolling-sliding integrated auxiliary bearing, the rolling element’s comprehensive radial embedding depth \( \delta_{RB} \) is positive, i.e., \( \delta_{RB} > 0 \). According to the Hertz contact theory, the comprehensive radial contact force between the rolling element and the inner and outer rings can be obtained as follows:

\[ F_{RB} = K_{RB}\delta_{RB}^{3/2} + C_{RB}\delta_{RB} \]  

(10)

Thus, we can obtain the relationship between the overall deformation amount of the bearing (or elastic approaching amount) and the load. The contact stiffness parameter of the bearing can be obtained after some manipulation. The comprehensive radial contact force between the rolling element and the inner and outer rings can also be calculated.

4. The Contact Model of Rolling-Sliding Integrated Auxiliary Bearing

When the electromagnetic bearing system fails, the rotor immediately loses the electromagnetic control and falls on the auxiliary bearing. In the falling process, the various degrees of freedom of the rotor have extremely strong coupling relationships. The rotor drop process is highly nonlinear. The study of the contact force exerted by the dropping rotor on the auxiliary bearing is based on the accurate contact model. In this section, with the help of the Hertz contact theory and considering the damping characteristics in the contact process, a nonlinear contact force model of dropping rotor on the auxiliary bearing is established.

According to the Hertz contact theory, the internal contact force of the rolling-sliding integrated auxiliary bearing is analyzed. When the rotor falls, line contact occurs between the outer ring and bearing housing. The radial depth \( \delta_{RL} \) of insertion between outer ring and bearing housing satisfies \( \delta_{RL} > 0 \). The radial contact force between the outer ring and the bearing housing can be obtained as follows:

\[ F_{RL} = K_{RL}\delta_{RL}^{10/9} + C_{RL}\delta_{RL} \]  

(11)

where \( K_{RL}, C_{RL} \) are respectively the line contact stiffness parameter and damping parameter between the outer ring and bearing housing. Therefore, the overall impact model of the rolling-sliding integrated auxiliary bearing is shown in Fig. 3.

![Figure 3. The contact model of rolling-sliding integrated auxiliary bearing.](image)
According to the model structure:

\[ F_r = F_{rB} = F_{rL} \quad (12) \]
\[ \delta_{rB} + \delta_{rL} = \delta_r \quad (13) \]

here \( F_r \) is the comprehensive contact force between the rotor and the bearing housing and \( \delta_r \) is the comprehensive radial embedding depth of the auxiliary bearing. With \( \delta_r \), the coupling formula (10–13) can be used to calculate the magnitude of \( F_r, \delta_r \) and \( \delta_r \). The expressions of contact stiffness parameters are given above. Due to the complexity of bearing damping calculation, damping is generally not involved in the analysis of bearing in the existing literature. When damping is required, it is usually measured experimentally. The internal elasticity and damping of the rolling-sliding integrated auxiliary bearing greatly buffer the force on the bearing housing when the rotor falls down.

5. Tangential Contact Force Between the Horizontal Rotor and the Rolling-Sliding Integrated Auxiliary Bearing

When the electromagnetic bearing system fails, the rotor falls on the rolling-sliding integrated auxiliary bearing. The interaction inside the bearing is shown in Fig. 4.

![Figure 4. The drop model of the rotor](image_url)

In the falling process of the horizontal rotor, the rotation process of the left and right rolling-sliding integrated auxiliary bearings is driven by the radial friction of the bearing housing and the rotor. When the rolling element rotates, there is not only a rotation around its own central axis, but also a revolution around the center of the bearing. In the case of the rotation of the rolling element, the rolling element rotates around a central axis. The angular velocity is the absolute angular velocity relative to the ground, such as relative to a fixed horizontal line. It is stipulated that the vertical paper facing outwards is the positive direction. Based on the angular momentum theorem, the rotational motion of the rotor and the left and right rolling-sliding integrated auxiliary bearings can be obtained.

\[ \dot{y}l = -(F_{tb1} + F_{tb2})R_r \quad (14) \]
\[ \dot{\theta}_b1I_b = -(F_{tb1} + F_{tb1})R_b \quad (15) \]
\[ \dot{\theta}_b2I_b = -(F_{tb2} + F_{tb2})R_b \quad (16) \]
\[ \dot{\theta}_o1I_o = (F_{tb1} - F_{tl1})R_o \quad (17) \]
\[ \dot{\theta}_o2I_o = (F_{tb2} - F_{tl2})R_o \quad (18) \]
where \( J, I_b, I_O \) are respectively the moment of inertia of rotor (including inner ring), rolling element and outer ring, \( R_r, R_b, R_O \) are the radius of the rotor (including inner ring), rolling element and the outer ring, \( \dot{\gamma} \) is the speed of rotor, \( \dot{\theta}_{b1}, \dot{\theta}_{b2} \) are the rolling element speed of the left and right auxiliary bearing, \( \dot{\theta}_O \) is the speed of outer ring, \( F_{tb1}, F_{tb2} \) are respectively tangential forces between the left and right auxiliary bearing rolling element and the rotor, \( F_{tb1}^*, F_{tb2}^* \) the tangential forces between the left and right auxiliary bearing rolling element and the outer ring and \( F_{tl1}, F_{tl2} \) the tangential forces between the left and right auxiliary bearing outer ring and the bearing housing.

\[
\begin{align*}
-\dot{I}_b \dot{\theta}_{b1} \cos \alpha + m_b R_m \beta_1 &= F_{tb1} R_r - F_{tb1}^* R_O \\
-\dot{I}_b \dot{\theta}_{b2} \cos \alpha + m_b R_m \beta_2 &= F_{tb2} R_r - F_{tb2}^* R_O
\end{align*}
\]

(19) (20)

where \( R_m \) is the revolution radius of the rolling element and \( \beta_1, \beta_2 \) are respectively the revolution speed of the rolling element of the left and right rolling-sliding integrated auxiliary bearings. Moreover, the contact angle of the rolling element is ignored here. The spin axis of the rolling element is considered to be perpendicular to the surface of the paper.

According to the characteristics of the structure of the rolling-sliding integrated auxiliary bearing, the moment of inertia of the rolling element and the outer ring is considered to be very small compared to the rotor. In order to simplify the calculation model, it is assumed that sliding friction always occurs between the outer ring and the bearing housing and rolling friction always occurs between the rolling element and the inner and outer rings during the falling of the horizontal rotor. As for the structure, the rolling bearing can be regarded as a planet wheel. When standing on the rolling element (planet wheel) for observation, the rotor, the rolling element and the outer ring all rotate around a central axis. The angular velocities of the three should be subtracted from the velocity of the rotating coordinate system, that is, the velocity of the rolling element.

\[
\begin{align*}
R_r (\dot{\gamma} - \dot{\beta}_1) &= R_b (\dot{\theta}_{b1} - \dot{\beta}_1) = R_O (\dot{\theta}_{o1} - \dot{\beta}_1) \\
R_r (\dot{\gamma} - \dot{\beta}_2) &= R_b (\dot{\theta}_{b2} - \dot{\beta}_2) = R_O (\dot{\theta}_{o2} - \dot{\beta}_2)
\end{align*}
\]

(21) (22)
The following formula can be obtained by using the pure rolling hypothesis:

\begin{align}
F_{\text{tL1}} &= \mu_d F_{\text{rL1}} \\
F_{\text{tL2}} &= \mu_d F_{\text{rL2}}
\end{align}

(23)

(24)

where \(\mu_d\) is the coefficient of dynamic friction between the outer ring and the bearing housing. \(F_{\text{rL1}}\), \(F_{\text{rL2}}\) are respectively the radial forces between the left and right auxiliary bearing outer ring and bearing housing, and the calculation formula has been given above. Considering the revolution and rotation of the rolling element, \(F_{\text{tb1}}, F_{\text{tb2}}, F_{\text{tb1}}^*, F_{\text{tb2}}^*\) can be obtained according to the formula (14~24). The results can be obtained explicitly.

In fact, the friction between the rolling-sliding integrated auxiliary bearing and the bearing housing is more complex in the falling process of the rotor. Rolling friction and sliding friction occur staggered in the whole falling process. Because the moment of inertia of the outer ring is small, the outer ring is rapidly slowed down by the bearing housing in the rotor fall behind. After bouncing, the outer ring is subjected to the force transmitted by the rolling element, and the rotating speed has a certain degree of improvement. Ignoring acceleration and deceleration time, rotor spinning and other state, it can be considered that only sliding friction occurs between the outer ring and the bearing housing, and only rolling friction occurs between the rolling element and the inner and outer rings.

6. 4 DOF Horizontal Rotor Drop Model

In order to ensure the stability of the suspension rotor in normal operation, the rotor is controlled in six degrees of freedom. The axial rotation of the rotor is controlled by the motor. The four degrees of freedom of the rotor – in horizontal and vertical direction, are controlled by two radial magnetic bearings respectively. The axial degree of freedom of the rotor is controlled by an axial electromagnetic bearing. It is assumed that the rotor is rigid and affected by the gravity of the rotor itself and the auxiliary bearings during the falling process of the rotor. A four-degree-of-freedom horizontal rotor falling model is established as follows.

\[\text{Figure 6. Horizontal rotor diagram.}\]

In Fig. 6, \(a\) and \(b\) are respectively represent the positions of the left and right auxiliary bearing sections. The Cartesian coordinate system is established at the center of the rotor with the bearing housing centroid as the origin.

\[\text{Figure 7. Radial friction between rotor and auxiliary bearing.}\]
When the electromagnetic bearing fails, the rotor immediately loses its control in the x, y directions and the support of four degrees of freedom. Radial contact occurs between dropping rotor and auxiliary bearing.

According to the contact model of the rolling-sliding integrated auxiliary bearing established above, the motion state of the rotor is analyzed. In Fig. 7, O is the bearing housing centroid, C is the rotor centroid, S is the rotor barycenter, \( \rho \) is the distance between O and C, \( e \) is the eccentricity, \( \alpha \) is the rotor vortex speed, \( \gamma \) is the rotor rotation speed and \( \varphi \) is the initial phase. According to the geometric relationship, the displacements of the rotor barycenter S are as follows:

\[
\begin{align*}
    x_s &= \rho \cos \alpha + e \cos(\gamma + \varphi) \\
    y_s &= \rho \sin \alpha + e \sin(\gamma + \varphi)
\end{align*}
\]  

(25)

\[
\begin{align*}
    \dot{x}_s &= \dot{\rho} \cos \alpha - \ddot{\rho} \sin \alpha - e \dot{\gamma} \sin(\gamma + \varphi) \\
    \dot{y}_s &= \dot{\rho} \sin \alpha + \ddot{\rho} \cos \alpha + e \dot{\gamma} \cos(\gamma + \varphi)
\end{align*}
\]

(26)

Denote the displacements of the left and right auxiliary bearing centroid in the x direction and the y direction as respectively, \( x_1, x_2, y_1, y_2 \), then the displacements of the rotor centroid C are as follows:

\[
\begin{align*}
    x &= \frac{bx_1 + ax_2}{a + b} \\
    y &= \frac{by_1 + ay_2}{a + b}
\end{align*}
\]

(27)

The rotation angles of the rotor around the X-axis and Y-axis are \( \Theta_x \) and \( \Theta_y \) respectively:

\[
\begin{align*}
    \Theta_x &= -\frac{y_1 - y_2}{a + b} \\
    \Theta_y &= -\frac{x_1 - x_2}{a + b}
\end{align*}
\]

(28)

The displacement of the left and right auxiliary bearing centroid in the x and y directions can be expressed:

\[
\begin{align*}
    x_1 &= \rho \cos \alpha - a \Theta_y \\
    y_1 &= \rho \sin \alpha - a \Theta_x \\
    x_2 &= \rho \cos \alpha + b \Theta_y \\
    y_2 &= \rho \sin \alpha + b \Theta_x
\end{align*}
\]

(29)

Under the global coordinate system, the acting forces of the left and right auxiliary bearings on the rotor \( F_{x1}, F_{y1}, F_{x2}, F_{y2} \) are:

\[
F_{x1} = -F_{r1} \frac{x_1}{\sqrt{x_1^2 + y_1^2}} + F_{tb1} \frac{y_1}{\sqrt{x_1^2 + y_1^2}}
\]

(30)

\[
F_{y1} = -F_{r1} \frac{y_1}{\sqrt{x_1^2 + y_1^2}} - F_{tb1} \frac{x_1}{\sqrt{x_1^2 + y_1^2}}
\]

(31)

\[
F_{x2} = -F_{r2} \frac{x_2}{\sqrt{x_2^2 + y_2^2}} + F_{tb2} \frac{y_2}{\sqrt{x_2^2 + y_2^2}}
\]

(32)

\[
F_{y2} = -F_{r2} \frac{y_2}{\sqrt{x_2^2 + y_2^2}} - F_{tb2} \frac{x_2}{\sqrt{x_2^2 + y_2^2}}
\]

(33)

where \( F_{r1}, F_{r2} \) are the radial contact force between the left and right auxiliary bearing and the rotor, \( F_{tb1}, F_{tb2} \) are the tangential force between the rolling element of the left and right auxiliary bearing
and the rotor. The calculation formulas have been given above. After the horizontal rotor falls, components of the force exerted by the rolling-sliding integrated auxiliary bearing on the rotor are:

\[
F_x = F_{x1} + F_{x2} \quad (34)
\]

\[
F_y = F_{y1} + F_{y2} \quad (35)
\]

In order to analyze the mechanical properties of the falling rotor, the dynamic model of the horizontal rotor is established according to the Lagrange equations as follows:

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_1} \right) - \frac{\partial T}{\partial q_1} + \frac{\partial V}{\partial \dot{q}_1} = Q_1 \quad (36)
\]

\[
T = \frac{1}{2} m (\dot{x}_s^2 + \dot{y}_s^2) + \frac{1}{2} I_p \dot{\theta}_x^2 + \frac{1}{2} I_p \dot{\theta}_y^2 + \frac{1}{2} J \dot{\gamma}^2 \quad (37)
\]

\[
V = mg \rho \sin \alpha + mge \sin (\gamma + \varphi) \quad (38)
\]

where \( q_1 \) is the generalized coordinate, \( Q_1 \) is the generalized force, \( T \) is the kinetic energy of the rotor, \( V \) is the gravitational potential energy of the rotor, \( m \) is the quality of rotor, \( I_p \) is the equatorial moment of inertia of the rotor, and \( J \) is the polar moment of inertia of the rotor.

According to the above analysis, the rotor state during the rotor falling process can be obtained as follows:

\[
\ddot{a} = \frac{1}{\rho} \left[ -\frac{F_t}{m \rho} - e^2 \cos(\alpha - \gamma - \varphi) - g \cos \alpha - e^2 \sin(\alpha - \gamma - \varphi) - 2 \rho \ddot{a} \right] \quad (39)
\]

\[
\ddot{\rho} = -\frac{F_n}{m} - e^2 \sin(\alpha - \gamma - \varphi) + e^2 \cos(\alpha - \gamma - \varphi) + \dot{a}^2 \rho - g \sin \alpha \quad (40)
\]

\[
I_p \dot{\theta}_x = a F_{y1} - b F_{y2} - f \dot{\gamma} \dot{\theta}_y \quad (41)
\]

\[
I_p \dot{\theta}_y = a F_{x1} - b F_{x2} - f \dot{\gamma} \dot{\theta}_x \quad (42)
\]

where \( F_n \) and \( F_t \) respectively represents the normal contact force and tangential friction between the rotor and the auxiliary bearing:

\[
F_n = - \left( F_x \cos \alpha + F_y \sin \alpha \right) \quad (43)
\]

\[
F_t = F_x \sin \alpha - F_y \cos \alpha \quad (44)
\]

According to the relative displacement of the rotor and the auxiliary bearing center, the collision state can be evaluated. The forward insertion depth indicates radial contact between the rotor and the auxiliary bearing. Otherwise, no contact occurs. The contact force can be obtained according to the relation between the Hertz contact force and the deformation, which has been described in detail above.

7. Results and Outlook

Based on the Hertz contact theory, the complex structure inside the rolling bearing is simplified to the spring damping model, and obtain the comprehensive radial contact force inside the rolling bearing. The overall impact model of the rolling-sliding integrated auxiliary bearing is established. Then, according to the structural characteristics of the rolling-sliding integrated auxiliary bearing, the tangential force inside the rolling-sliding integrated auxiliary bearing can be obtained by applying the angular momentum theorem. Finally, a 4-degree-of-freedom horizontal rotor drop model is established.
to analyze and calculate the center orbit and motion state of the rotor. Thus, the drop analysis of the horizontal dropping rotor of the rolling-sliding integrated auxiliary bearing is carried out.

At present, the experimental verification of the impact model has been completed. The research in near future will focus on the solution of the model and the comparison of calculation and experimental results. Furthermore, a quantitative comparison will be made between the rolling-sliding integrated auxiliary bearing and the traditional rolling bearing to verify the model predictability.

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