Fluctuation spectrum and size scaling of river flow and level
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Abstract
We describe the statistical properties of two large river systems: the Danube and the Mississippi. The properties of the two rivers are compared qualitatively to the general properties of a critical steady state system. Specifically, we test the recent suggestion by Bramwell, Fennell, Holdsworth and Portelli [Europhys. Lett. 57, 310 (2002)] that a universal probability density function (PDF) exists for the fluctuations in river level, namely the Bramwell-Holdsworth-Pinton (BHP) PDF. The statistical properties investigated in this paper are: the PDF of the river flow and river level; moment scaling with basin area; moment to moment scaling or relative scaling; and power spectral properties of the data. We find that the moments of the flow scale approximately with basin area and that the seasonally adjusted flows exhibit relative moment scaling.

Compared to the Mississippi, the Danube shows large deviations from spatial scaling and the power spectra show considerable dependence on system size. This might be due to water use and regulations as well as inhomogeneities in the basin area. We also find that the PDF of level data in some, but not all, cases can be qualitatively approximated by the BHP PDF. We discuss why this coincidence appears to be accidental.

Keywords: River systems; universal fluctuations; finite size scaling; multiscaling.

1 Introduction
The spatial structure of river basins and the statistics and dynamics of the flow of rivers have been analysed by a number of authors using methods from statistical mechanics. Rinaldo found that river basins are spatial fractals [Rodriguez-Iturbe and Rinaldo, 1997]. The flow of rivers were found to exhibit scaling [Thomas and Benson, 1970] and multiscaling [Gupta and Waymire, 1990] with basin area for basins ranging from 10 to $10^3$ km$^2$. Temporal correlations, when analysed by means of the power spectrum, are also considered to be multiscaling or multifractal with respect to time [Gupta and Waymire, 1990, Pandey et al., 1998, Tessier et al., 1996]. Perhaps the most remarkable link to systems in statistical mechanics is an observation by Bramwell, Fennell and Portelli [Bramwell et al., 2002]. They found that the probability density function (PDF) of the fluctuations of river level at Nagymaros, which is on the Danube, is in good agreement with a PDF found for the magnetic fluctuations in the classical XY-model close to criticality [Bramwell et al., 2000b]. We will refer to this PDF as the Bramwell-Holdsworth-Pinton (BHP) PDF. The data from the Nagymaros was published in [Jánosi and Gallas, 1999] and the fit proposed by Bramwell et al. was checked visually in [Bramwell et al., 2002].

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The BHP PDF has been described and characterised by Bramwell et. al. [Bramwell et al., 2000b]. It has also been found in one experimental system and in computer simulations of SOC-models [Bramwell et al., 1998, Bramwell et al., 2000a, Dahlstedt and Jensen, 2001, Pinton et al., 1999, Sinha-Ray et al., 2001]. These investigations indicated that the BHP PDF describes fluctuations in many very different systems. It was suggested that the BHP PDF might be universal and ubiquitous in nature arising in systems which lack a characteristic length scale and exhibit finite size scaling similar to critical equilibrium systems.

Inspired by these observed links to statistical mechanics and scale invariance, we want to address the following questions: Do the statistics of river flow and level height typically exhibit scaling and, if so, what type of scaling? Is the observation of the BHP fluctuation spectrum at Nagymaros representative of fluctuations of river level in general and, if so, can the observed BHP fluctuation spectrum be related to scale free aspects of the river system? It is not clear a priori which quantity one should look for when searching for the BHP fluctuation spectrum [Bramwell et al., 2000b, Bramwell et al., 1998, Bramwell et al., 2000a, Dahlstedt and Jensen, 2001, Pinton et al., 1999, Sinha-Ray et al., 2001]. We will therefore analyse level as well as flow data.

We approach these issues by analysing data collected from several stations, corresponding to different basin areas, along the rivers Danube and Mississippi, see table [1]. We also consider data from one very small river, the Wye at Cefn Bryn with a basin area of 10.6 km$^2$ [NWA, 2002] and one very large river, the Rio Negro at Manaus with a basin area of $3 \times 10^6$ km$^2$ [Richey et al., 1989]. This allows us to carry out a finite size scaling analysis similar to that used in statistical mechanics to study the scale free behaviour in systems with a diverging correlation length [Cardy, 1990].

It is useful to explain what we mean by finite size scaling. In equilibrium critical systems one expects that the standard deviation $\sigma$ and the mean of the order parameter $\langle m \rangle$ scale with system size (close to criticality) such that a rescaled PDF becomes independent of system size. This is expressed by the hyperscaling relation $\langle m \rangle \propto \sigma \propto L^\alpha$ and the finite size scaling (FSS) hypothesis [Cardy, 1990, Aji and Goldenfeld, 2001].

The FSS hypothesis can generally be written as

$$P(m) = L^{-\beta} f(m L^{-\alpha}, L/\xi)$$

which holds if the system is in a critical state [Bramwell et al., 2000b, Aji and Goldenfeld, 2001]. If the spatial correlation length $\xi$ is infinitely larger than $L$, i.e. $L/\xi \rightarrow 0$, then we can make the approximation

$$P(m) = L^{-\beta} f(m L^{-\alpha}).$$

The FSS hypothesis leads to a scaling of the moments which we can write as

$$\langle m^n \rangle \propto L^{n\theta}$$

where $\theta = \alpha$. This will be referred to as simple scaling. The simple scaling of the moments can be extended to multiscaling by letting the moments of the order parameter $m$ scale as

$$\langle m^n \rangle \propto L^{n\theta(n)}.$$
The FSS hypothesis does not hold in the following three cases: the correlation length does not diverge and corrections due to the finite length of $\xi$ in relation to the system size have to be accounted for; the moments are multiscaling and the hypothesis needs to be changed; more than one scaling field exists in the system, such that

$$\langle m \rangle \propto L^{a_1} L^{a_2} \ldots L^{a_N}.$$  \hfill (5)

In the last case, the FSS hypothesis can hold approximately if there is one dominant scaling field and all the other scaling fields are irrelevant [Forgacs et al., 1991].

### Table 1: River sites used

| Mississippi | Danube      |
|-------------|-------------|
| **Name**    | **Name**    | **Basin area** |
| Royalton    | Dillingen   | 30 040 km$^2$  |
| St. Paul    | Ingolstadt  | 95 300 km$^2$  |
| Clinton     | Nagymaros   | 221 700 km$^2$ |
| Keokuk      | Orsova      | 308 200 km$^2$ |
| Memphis     | Ceatal Izmail| 2 416 000 km$^2$ |

The paper is structured in the following way. We begin by considering river flow. In Secs. 2, 3 and 4 we discuss the PDF, the scaling and multiscaling analysis and the power spectrum analysis of river flow. In Sec. 5 we present the PDF of the fluctuations in the river level and Sec. 6 contains a discussion and our conclusions.

To avoid misunderstandings we mention that we use the following notation. The river flow is denoted by $q$ and the height of the river level by $h$. The seasonally adjusted flow and height (defined in the appendix) is denoted by $s_q$ and $s_h$ respectively.

## 2 River Flow

In this section we present the comparison of the PDFs of seasonally adjusted flow data with varying basin area from the sites listed in Table 1. The signal we consider is equal to the deviation from the average, over all years, at a specific day, say 5th of February, normalised by the standard deviation of that particular day. We defer details to the appendix. This seasonally adjusted signal has by construction a zero mean and a standard deviation equal to one. Thus no further scaling is needed in order to plot the PDF of the seasonally adjusted data in the form used previously to check for Gaussian and BHP like behaviour, see e.g. [Bramwell et al., 1998, Bramwell et al., 2000a].

Fig. 1 shows the PDF of the seasonally adjusted flow for the river Mississippi. We plot the seasonally adjusted flow along the x-axis and the logarithm of the PDF along the y-axis. A normal distribution will appear as a parabola in this plot. We note that the tails of the distributions vary slightly but there is an approximate data collapse. Hence adjusting for seasonal variation cancels the dependence on system size.

In Fig. 2 we show a similar plot for the Danube. In this plot, we can see a definite change of the PDF shape from the small basin area site, at Dillingen, to the large basin area site at Ceatal Izmail. The shift in PDF is towards a Gaussian and it suggests that the spatial correlation length of the system is shorter than the system size at the largest site leading
to a sum of uncorrelated contributions to the flow. From the PDF analysis we expect the
hyperscaling relationship and simple scaling to hold for the river Mississippi but not for the
Danube. We also note that the PDF of the fluctuations in the flow deviates strongly from
the BHP PDF, indicated by the dashed line in both figures. As we shall see in Sec. 5, the
situation is more subtle for river level data.

Let us now consider the mean and the standard deviation of the “raw” data for which no
adjustment for seasonal variation has been carried out. Fig. 3 compares the scaling of the
mean with the basin area, $L$, and the scaling of the standard deviation with $L$. The mean
and standard deviation scale approximately as $\langle q \rangle \propto L^{1.08}$ and $\sigma_q \propto L^{1.05}$ for the Mississippi
and for the river Danube approximately as $\langle q \rangle \propto L^{0.87}$ and $\sigma_q \propto L^{0.79}$. The exponents for
$\langle q \rangle$ and $\sigma_q$ are close, i.e. the hyperscaling relationship is approximately fulfilled for both
rivers, though with greatest accuracy for the Mississippi data.

The analysis shows that the data collapse of the Mississippi flow data is in fairly good
agreement with the FSS hypothesis of equation 1 and simple scaling of the first two mo-
ments. The Danube has a larger deviation from the hyperscaling relationship, which we
also expected from the analysis of the PDFs. This deviation might be due to a spatial
decorrelation of the process, as we mentioned earlier. There may be several reasons for the
decorrelation, such as inhomogeneities in the basin area and human regulation of the river.

3 Scaling of higher moments and multiscaling

We now present data on the scaling of the higher moments of the flow for the Mississippi and
the Danube. We have shown that both rivers approximately fulfil the hyperscaling relation-
ship. However, both rivers show deviations from simple scaling that look like multiscaling.
The scaling exponents can be expressed as a function of moment-order $n$. This function is
shown in Fig. 4

We have also looked at the scaling of skewness $\langle (m - \langle m \rangle)^3 \rangle / \sigma^3$ and kurtosis $\langle (m - \langle m \rangle)^4 \rangle / \sigma^4 - 3$ of the data. Given the analysis above and assuming the data is multisca-
ling the skewness and kurtosis should not be constants and they should scale with basin
area. For both the river Mississippi and the Danube, skewness and the kurtosis as functions
of basin area do not fit a power-law, as shown in Fig. 5. Despite this, there is a simple
power-law relationship between the skewness and kurtosis. This is given by $kurt \propto skew^{2.75}$
for the Mississippi and $kurt \propto skew^{1.78}$ for the Danube. Thus both rivers have system-
atice corrections to simple scaling that certainly look like multiscaling but, since the higher
rescaled moments do not scale with basin area, neither of the rivers multiscale according to
the standard definitions.

The deviations from multiscaling can also be seen in the seasonally adjusted data. The
moments of the seasonally adjusted river Mississippi flow as a function of basin area do not
fit a power-law as seen in Fig. 6. Even so, the higher moments scale as power laws of the
$3^{rd}$ moment with different exponents. We refer to this type of scaling as relative scaling of
moments. The exponents are shown in Fig. 7 as functions of moment-order $n$.

We define the relative scaling as

$$\langle m^n \rangle / \sigma^n \propto (\langle m^{n'} \rangle / \sigma^{n'})^{(n/n')}K(n,n') n \neq n' > 2.$$  \hspace{1cm} (6)

Assuming that the moments follow simple scaling it is easy to see that $K(n,n') = 0$, and if
the moments are multiscaling, relative scaling should be satisfied with the exponent $K(n,n')$
related to $\theta(n)$. So for the Mississippi and the Danube we found that multiscaling is not fulfilled and hence the observed relative scaling must have a different origin.

Relative scaling is also possible if we consider a weak secondary scaling field, such that the moments scale as

$$\langle m^n \rangle = L_1^{\alpha n} L_2^{\beta(n)}. \tag{7}$$

If $\theta(2) \approx 0$ and $\langle m \rangle = 0$, then $\sigma \propto L_1^{\alpha}$ and $\langle m^n \rangle / \sigma^n \propto L_2^{\beta(n)}$. In this way relative scaling is possible even if $\langle m^n \rangle / \sigma^n$ does not scale with $L_1$. For a related situation in equilibrium systems see [Forgacs et al., 1991] [Fisher and Gelfand, 1988].

The assumption of more than one scaling field is supported by a previous study of river basin characteristics. Regression analysis of different basin characteristics has shown that river flow is affected by many other features, in addition to basin area, such as channel slope, precipitation and others [Thomas and Benson, 1970].

### 4 Power Spectra of River Flow

We now want to examine the signs of scale free behaviour from a temporal point of view and study the long time correlations by means of a power spectral analysis for the rivers Danube, Mississippi, Rio Negro and the Wye. Generally the power spectra of rivers have two different frequency regimes of power-law-like nature [Tessier et al., 1996]. The two regimes are separated by a cross-over frequency $f_c$. Most river flow data is also considered to be multiscaling/multifractal with respect to time [Gupta and Waymire, 1990] [Pandey et al., 1998] [Tessier et al., 1996]. In these studies data from different sites and rivers are considered to be realisations of the the same process and the power spectra are averaged over all sites and there are generally no references to how the cross-over frequency changes with basin area. Here we explicitly study the dependence of the power spectrum on the basin area and we find that $f_c$ changes with basin area for the Danube but not for the Mississippi.

The Mississippi sites all have approximately the same spectrum which consists of two power-law-like regimes separated by a cross-over frequency $f_c$ corresponding to approximately 20-40 days. In the low frequency domain the power-law has an exponent close to $-1$ corresponding to logarithmic decaying correlations in the long time limit (see e.g. [Jensen, 1998]) and for high frequencies an exponent close to $-3$, as can be seen in Fig. 8. This indicates that the flow of the Mississippi is determined by the intrinsic, system size independent, dynamics which produces time correlations that are longer than the longest observation time. Thus, within the basin area size range investigated here we see critical-system-like behaviour: above we found that the PDF shows data collapse and that the moments scale; in addition we observe that the power spectra are similar for all sites in the system and exhibit $1/f$ behaviour down to the lowest recorded frequencies.

For the Danube the crossover $f_c$ shifts with basin area with frequencies corresponding to approximately $1/f_c = 60$ days for Cetăţuia Izmail and Orsova, 20 days for Nagymaros and 10 days for Ingolstadt and Dilligen. In the low frequency region the spectrum does to a degree behave like $1/f$, see Fig. 9. However, we suggest that the trend is that as the basin area is increased, the spectrum moves towards a Lorentzian-like form, i.e $(1 + (f/f_c)^2)^{-1}$. This would be indicative of exponentially decaying time correlations with a characteristic decay time $t_c = 1/f_c$ and no essential time correlations beyond $t_c$. The behaviour described here evince that the dynamics of the Danube is non critical and that its correlation time is determined by the considered system size, or basin area.
We have also considered the flow of the Rio Negro at Manaus, with a very large basin area \(3 \times 10^6 \text{ km}^2\), and the flow of the Wye at Cefn Brwyn, with a very small basin area 10.6 \(\text{km}^2\). The power spectra are shown in Fig. 10.

For the Rio Negro the spectrum seems to have a smooth transition between the low and the high frequency regime. The spectrum resembles, as in the case of the Danube at large basin areas, a Lorentzian form. Clearly it is not possible to identify accurately the characteristic frequency, but a value of about \(1/f_c = 600\) days seems reasonable. Thus the Rio Negro does not show critical power-law time correlation, though its characteristic time is very long. If we assume that long time corresponds to large spatial scales, our analysis suggests that the Rio Negro is so large that its size exceeds the correlation length of the river flow. The river therefore behaves as uncorrelated at the largest spatial and temporal scales.

The dynamics of the river Wye appears to be able to support \(1/f\) correlations down to a characteristic frequency \(f_c\) corresponding to about 60 days. For frequencies lower than \(f_c\) the spectrum becomes white, thus the river system is so small that it cannot support correlations on time scales longer than these 60 days.

The PDFs of the flow of these two rivers are shown in Fig. 11. There is a remarkable difference since the Wye has a nearly exponential PDF and the Rio Negro has a Gaussian PDF. The Gaussian is consistent with our finding above that the Rio Negro is so big that its dynamics are decorrelated. The exponential tail found for the small river Wye is probably related to spatial correlations extending up to the system size. This together with the \(1/f\) behaviour down to a certain frequency might reflect that the dynamics of the river Wye is critical, but since the system is rather small, the scale free range is limited.

## 5 River-Level

In this section, we present the results of the analysis of river level. Fig. 12 shows four PDFs of river level. The data is from the stations Clinton on the Mississippi, and Dillingen, Ingolstadt and Nagyramos on the Danube. The seasonally adjusted PDFs are similar at the sites of Ingolstadt, Nagyramos and Clinton and significantly different for the Dillingen site. The data that collapses seems to do so on or close to the BHP PDF but it is also close to a first order extreme value, Fisher-Tippet-Gumbel (FTG), PDF [Gumbel, 1958]. It is difficult to determine visually from the histogram how good the fit is.

To get a better estimate of the fit, table 2 shows the seasonally adjusted higher moments of the data compared to the moments of the BHP and a FTG PDF [Bramwell et al., 2000b]. This shows that quantitatively the fit is poor.

| Name/Moment | 3   | 4   | 5   | 6   |
|-------------|-----|-----|-----|-----|
| FTG         | 1.14| 5.40| 18.57| 91.39|
| BHP         | 0.8907| 4.415| ±   | ±   |
| Clinton     | 0.77| 3.05| 5.67| 17.37|
| Nagyramos   | 0.54| 3.66| 6.18| 27.31|
| Ingolstadt  | 0.76| 3.64| 7.84| 28.44|

Table 2: Seasonally adjusted moments \(\langle s_h^n \rangle\) for three of the sites, the 1st order extreme value PDF and the BHP PDF
We also investigated whether the moments of a river level fulfill the hyperscaling relationship and scale with basin area. The relationship between the mean and the standard deviation is shown in Fig. 13. For the Danube we can make an ad hoc linear fit, which does not suggest hyperscaling since the slope is negative. From a scaling analysis with basin area (not shown here) we conclude that river level is not scaling with basin area. Thus the hyperscaling and scaling is not the cause of the apparent data collapse of the data from the Ingolstadt and Nagymaros sites. Although some of the data collapses close to the BHP, we can only conclude that river level does not in general have a single universal PDF upon rescaling. Furthermore, in the case where river level fluctuations are reasonably well described by the BHP form, the lack of scaling prevents us from concluding that the agreement with the BHP PDF arises as a consequence of underlying scale free behaviour.

6 Discussion and Conclusion

We have shown that PDFs of the fluctuations of the river flow of the river Mississippi for different basin areas can all be approximately collapsed on to one functional form by size scaling. However, this scaling function is not of the BHP form. In addition to the data collapse we have shown that the flow follows approximately simple scaling, with deviations. In the case of the river Mississippi, the deviations resemble multiscaling though quantitative analysis reveals that the behaviour is inconsistent with multiscaling. We suggested a simple form with a weak second scaling field, independent of basin area which might explain the observed relative scaling of the seasonally adjusted moments. In the case of the river Danube, the deviations from simple scaling have features in common with multiscaling. However, in this case the deviations were found to be related to spatial decorrelation between the sites along the river consistent with the lack of size scaling collapse of the PDFs. Our conclusion is that the river Mississippi behaves like one correlated whole whereas for the Danube, correlations are destroyed as the basin area is increased.

We summarise that some degree of scale invariance can be found for rivers of intermediate size. Hence, our analysis confirms and extends previous observations of scaling in smaller river systems [Thomas and Benson, 1970, Gupta and Waymire, 1990].

We set out to determine whether measurements of river level fluctuations consistently have a universal PDF close to that of the BHP PDF. For two out of the four river sites we investigated, the average daily level PDFs approximately collapse onto one PDF. This PDF resembles to the BHP PDF but quantitatively the fit is poor. The two other sites have a somewhat different shape of the PDF. We also found that there is no size scaling with basin area for river level. Hence even when the BHP form is found, it cannot be related to an underlying scale free behaviour of the Danube. It would have been very interesting to be able to analyse the level data for the Mississippi. Unfortunately we could only obtain flow data for the Mississippi. Nevertheless, our analysis suggests that fluctuations in river level do not in general follow the BHP functional form. The observations in Bramwell et al., 2002 appear to be accidental.

In addition to the analysis reported here we have investigated financial data, weather data, tree ring data and other data and could not find the BHP PDF. River level data appeared to be an exception rather than the rule. Thus, in general we are compelled to conclude that the BHP PDF is rather difficult to find in nature.
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8 Appendix

The flow of a river is generally highly dependent upon the seasons. Therefore all the data has been seasonally adjusted with the seasonal mean and seasonal standard deviation denoted $\langle x \rangle_d$ and $\sigma_d$. Let $x(d, y)$ denote the measured value of either the flow $q$ or the level $h$ measured at day number $d$ in year number $y = 1, \ldots, N_y$. The adjusted data is defined as

$$s_x = \frac{x(d, y) - \langle x \rangle_d}{\sigma_d}.$$  

The seasonal mean and the seasonal standard deviation are defined as

$$\langle x \rangle_d = \frac{1}{N_y} \sum_{y=1}^{N_y} x(d, y)$$

and

$$\sigma_d = \sqrt{\frac{1}{N_y} \sum_{y=1}^{N_y} (x(d, y) - \langle x \rangle_d)^2}.$$  

The adjusted data have $\sigma_s = 1$ and $\langle s \rangle = 0$ which can easily be checked.

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Figure 1: The PDF of the flow for the five sites along the river Mississippi on a semi-log plot. For reference, a normal PDF (solid line), a BHP PDF (dashed line) and a power-law tail (dotted line) have been plotted.
Figure 2: The PDF of the flow for the five sites along the Danube on a semi-log plot. For reference, a normal (solid line) and a BHP (dashed line) PDF have been plotted.

Figure 3: The scaling of mean and standard deviation $\sigma$ with basin area on a log-log plot. River Mississippi mean [○] and $\sigma$ [∗]. River Danube mean [□] and $\sigma$ [△]. The dashed line is the fit $\langle q \rangle \propto L^{0.86779}$, the dashed-dotted line is the fit $\sigma_q \propto L^{0.78994}$, the solid line is the fit $\langle q \rangle \propto L^{1.0763}$ and the dotted line is the fit $\sigma_q \propto L^{1.0477}$. The data from the Danube has been shifted with a factor of 10 for clarity.
Figure 4: Multiscaling function $n\Theta(n)$. The solid line is the simple scaling behaviour: the Danube [•], $\Theta(n) = 0.89278 - 0.021216n$ and the river Mississippi [○], $\Theta(n) = 1.1198 - 0.046448n + 0.0026748n^2$. The error bars shown are the errors from the regression analysis.

Figure 5: Scaling of skewness and kurtosis to basin area on a log-log plot. River Mississippi skewness[○] and kurtosis[•]. River Danube skewness[□] and kurtosis[△]. The data from the Danube has been shifted with a factor of 10 for clarity.
Figure 6: Higher seasonally adjusted moments for the river Mississippi plotted against river basin area on a log-log plot. 3rd moment [o] to 8th moment [*].

Figure 7: The scaling of higher seasonally adjusted moments for the river Mississippi [o] and the river Danube [*].
Figure 8: Power spectra for the 5 sites along the Mississippi, for comparison two power-laws with exponents -1 and -3 are shown. The unit of frequency is inverse days. The inset shows the averaged power spectra for all sites with approximate crossover of 30 days.

Figure 9: Power spectra for the 5 sites along the Danube, for comparison two power-laws with exponents -1 and -3 are shown. The unit of frequency is inverse days.
Figure 10: Power spectra for the rivers Rio Negro at Manaus and Wye at Cefn Brwyn. The straight line indicates $1/f$ behaviour. The unit of frequency is inverse days.

Figure 11: Seasonally adjusted PDF for the rivers Rio Negro at Manaus and Wye at Cefn Brwyn on a semi-log plot, compared with an exponential and a Gaussian PDF.
Figure 12: The PDF of river level for the sites Ingolstadt, Dillingen, Nagymaros and Clinton. As reference a normal (solid line), a BHP (dashed line) and a FTG (dotted line) PDF have been plotted.

Figure 13: The scaling of river level mean to standard deviation for the Danube [*] and Mississippi [o].