Relations between elementary particle masses

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Relations between elementary particle masses are given using only known physical constants, without any arbitrary number.

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The large and increasing number of hadronic particles, suggests to find new classifications, in addition to those already existing based on their quantum numbers (isospin, spin, charge conjugation and parities). A possible approach is to look to eventual fractal properties of these particles. We recall that Nottale [1] noted that the "lepton e, µ, τ mass ratios followed a power-law sequence, namely, $m_\mu \approx 3 \times 4.1^3 \times m_e = 105.656$ MeV, and $m_\tau \approx 3 \times 4.1^4 \times m_e = 1776.1$ MeV".

In the same mind, we give relations between elementary particle masses: quarks, bosons, and (an other relation) between leptons. All experimental masses, except those specifically indicated, are taken from Review of Particle Physics (PDG) [2].

A. Quark masses

We present here power law relations between quark masses. Six quarks are known and classified into two families. The charge of the first family is 2/3, and the quarks are: "u", "c", and "t". The charge of the second family is -1/3, and the quarks are: "d", "s", and "b". These masses are:

$\bar{m}_u = 1.5 - 3.3$ MeV, we take $m_u = 2.28$ MeV
$m_c = 1.27 \pm 0.07$ GeV
$m_t = 171.2 \pm 2.1$ GeV [8]

$m_d = 3.5 - 6.0$ MeV, we take $m_d = 5.1$ MeV
$m_s = 104 \pm 3$ MeV
$m_b = 4.20 \pm 0.12$ GeV

The ratio between $m_u$ and $m_d$, r=0.45, corresponds to the value used in theoretical calculations.

In each family, we attribute successively, the numbers 1, 2, and 3 to the three quarks.

In order to get the relations between all quark mass ratios, the remarkable ratio $r = 1/134.8$ between the charm quark and the top quark masses, suggests to use the fine structure constant $\alpha = e^2/4\pi\epsilon_0\hbar c = 1/137.036$ (see [4]).

$$m_{n+1}^{(1)} = 2^{2(2-n)} m_n^{(1)}/\alpha,$$

$$\ln \left( m_{n+1}^{(1)}/m_n^{(1)} \right) = \ln \left( 2^{2(2-n)} \right) - \ln \alpha. \tag{2}$$

The application of the formula gives:
$m_c = 1.24977$ GeV, and $m_t = m_c/\alpha = 171.295$ GeV.

The relation between quark masses of the second family is:

$$m_{n+1}^{(2)} = (m_\pi/m_p) m_{n+1}^{(1)}$$

$$\ln \left( m_{n+1}^{(2)}/m_{n}^{(2)} \right) = \ln \left( (m_\pi/m_p)m \right) - \ln \alpha \tag{4}$$

where $m_\pi/m_p = 0.14875236$ is the ratio of the pion mass to the proton mass. The application of the formula gives: $m_s = 103.961$ MeV, and $m_b = 4240.0$ MeV. The masses obtained by these formulas are shown in table I.

The masses of eventual higher mass quarks, are successively for both families: $M^{(1)} = 5865$ GeV, and $M^{(2)} = 257$ GeV. The present experimental limit is $m_\tau \geq 256$ GeV [5] and $m_\nu \geq 128$ GeV in case of (SM4) (a possible fourth generation particles [8]).

The previous relations (1) and (3) allow to get the following relation between the masses of the first $m_{n+1}^{(1)}$ and the second $m_{n+1}^{(2)}$ quark families:

$$m_{n+1}^{(2)}/m_{n+1}^{(1)} = (m_{n+1}^{(1)}/m_n^{(1)}) 2^{2(n-2)} (m_\pi/m_p)n \tag{5}$$
n = 0 gives M = 0, corresponding to the photon and the experimental Z mass equals 3 $10^{-3}$ GeV.

n = 5 gives M = 90.917 MeV; the relative shift from the experimental masses (in GeV).

These bosons are:
- the photon with a mass $m_\gamma \leq 1 \cdot 10^{-18}$ ev,
- the gluon without mass,
- the W, $m_W = 80.398 \pm 0.025$ GeV, and
- the Z, $m_Z = 91.1876 \pm 0.0021$ GeV.

The following relation allows to get the experimental masses, with a shift from the experimental values, however much larger than the experimental precisions.

$$M = m_p \alpha^{-1} (n/10)^{1/2}$$  \quad (6)

n = 0 gives M = 0, corresponding to the photon and the gluon mass,

n = 4 gives M = 81.319 GeV; the relative shift from the experimental W mass equals 1.15 $10^{-2}$,

n = 5 gives M = 90.917 MeV; the relative shift from the experimental Z mass equals 3 $10^{-3}$ \[7\]. The masses obtained by this formula are shown in table II. It is difficult to predict a rule for the mass incrementation. Indeed we use "n" = 0, or 4, or 5. If we choose "n" = 9 for the next mass, which can be the mass of the Higgs boson, we get $M_H \approx 122$ GeV. If we choose simply M = $m_p \alpha^{-1}$, we get $M_H \approx 128.6$ GeV. It is also possible to propose two Higgs boson masses, using the incrementation "n" = 0, 4, 5, 9, and 10, and the masses of both bosons will be: $M_H \approx 122$ GeV et $M_H \approx 128.6$ GeV.

The Higgs boson mass limits was recently suggested to lie between 110 and 200 GeV \[10\] and a recent analysis concluded that the Higgs boson mass should range between 115 and 148 GeV \[8\].

We are aware of the incrementation jump, leaving out "n" = 1, (40.716 GeV), "n" = 2 (57.58 GeV), "n" = 3 (70.52 GeV), "n" = 6 (99.73 GeV), "n" = 7 (107.72 GeV), and "n" = 8 (115.16 GeV). Do some of these masses correspond to still unobserved particles, in case of (SM4); and if it is the case, why do have they never been observed? \[12\].

### B. Gauge boson masses

The relations between all quark masses in one hand and between all gauge boson masses in the other hand, were given previously. Therefore, we show only the relation between one gauge boson and the quark masses, more precisely between the Z boson mass and the "u" and "c" quark masses:

$$m_Z = m_p/(4\sqrt{2})(m_c/m_u)$$  \quad (7)

We point out that by inverting the previous relations, we get the proton mass by using the gauge boson mass(es), just as the ratio of quark masses by means of the proton and the Z boson masses, and the pion mass by means of the proton and the ratio between two quark masses. All these masses are related between them by the fine structure constant $\alpha$.

### D. Lepton masses

A power-law-type sequence between $e$, $\mu$, and $\tau$ leptons was given by Nottale \[11\] as already noted.

We propose here another relation, which gives less exact leptonic masses than Nottale’s relation, but which also calculates masses for two neutrinos, and is obtained without introduction of “external” numbers. The six leptons: $\nu_e$, $\nu_\mu$, $\nu_\tau$, $e$, $\mu$, and $\tau$, are associated with index "k" = 1, 2, 3, 4, 5, and 6. Writing "n" = "k - 3", we have:

$$m^{(k)} = m_\pi \alpha^{(2-n)} p(n(n+1)/2 + (-1)^n)/h(n)$$  \quad (8)

where

$$h(n) = |n^2 - ((n + 1)/4)((-1)^n - 1)|$$  \quad (9)

where r = 1.001378 is the ratio of the neutron mass to the proton mass. It corresponds to a correction term, very close to 1. The h(n) term vanishes for "n" = 0", preventing the determination of the $\nu_\tau$ mass through such a mass relation.

The masses obtained by this formula are shown in table III.
TABLE III: Comparison of lepton calculated and experimental masses.

| lepton | calculated mass | experimental mass | relative difference |
|--------|-----------------|------------------|---------------------|
| $\nu_e$ | 0.3 eV | $0.05 \leq m_{\nu_e} \leq 0.23$ eV [11] | |
| $\nu_{\mu}$ | 54 eV | $\leq 0.19$ MeV | |
| $\nu_{\tau}$ | | $\leq 18.2$ MeV | |
| $m_e$ | 0.511004 MeV | 0.5109989 MeV | $8 \times 10^{-6}$ |
| $m_{\mu}$ | 105.62 MeV | 105.65837 MeV | $3.6 \times 10^{-4}$ |
| $m_{\tau}$ | 1768.97 MeV | 1776.84 MeV | $4.4 \times 10^{-3}$ |

The calculated mass of an eventual heavier lepton is: $m \approx 505.5$ GeV, very close to the value $m \approx 502$ GeV found by Nottale [1]. The experimental lower mass limit of this heavier lepton is very unprecise, of the order of $m \geq 100$ GeV. Several calculations propose an eventual fermion additional generation (see for example [12]). Since the lepton masses depend only on the pion mass, and to a small amount, on the ratio of neutron over proton masses, a relation between lepton masses and quark (boson) masses is meaningless. However, these masses are related to the other masses, discussed previously, through the pion mass. The relations are rather simple.

In conclusion, we are able to rely nearly all elementary particle masses with each other, by using proton, pion, and neutron masses, and fine structure constant. These relations should be helpful for a better knowledge of particle properties. They allow to predict the masses of still unobserved elementary particles, specially possible masses for the Higgs bosons at $m_H = 122$ and 128.6 GeV.

[1] L. Nottale, The Theory of Scale Relativity: Non-Differentiable Geometry and Fractal Space-Time. In : Computing Anticipatory Systems. CASYS03 - Sixth international Conference (Liège 2003). D.M. Dubois, Ed., American Institute of Physics Conference proceedings, 718, p.68 (2004).

[2] C. Amsler et al., Phys. Lett. B667, 1 (2008).

[3] T. Aaltonen et al., Phys. Rev. D 81, 052011 (2010). A recent mass measurement yields a somewhat larger value for the top quark mass: $174.8 \pm 2.4$ (stat + JES) $\pm 1.3$ (syst) GeV.

[4] With the new top quark mass $174.8$ GeV, the ratio between charm and top quark masses $r = 1/137.6$ is still nearer to the a value.

[5] D. Acosta et al., Phys. Rev. Lett. 90, 131801 (2003).

[6] B. Holdom, W.S. Hou, T. Hurth, M. Mangano, S. Sultansoy, and G. Ünel, [arXiv:0904.4695v2 [hep-ph] (2009)].

[7] It is possible to reduce these shifts, by using the following two factors:

\[ v = \left( \frac{m_{\nu_e}}{m_{\pi^+}} \right)^{1/2} = 0.9834. \]

A shift of 429 MeV remains therefore, between the experimental and the calculated Z masses, which must be compared to the experimental imprecision: 2.1 MeV.

[8] J. Erler, [arXiv:1002.1320] [hep-ph] (2010).

[9] More precise Z mass, but especially with a regular incrementation, can be obtained at the expense of a more complicate relation. We attribute the indexes "k" = 1, 2, 3, and 4 to photon, gluon, W, and Z bosons. Their masses are given by the relation:

\[ M_k = m_\nu \alpha^{-1} (f(k)/10)^{1/2} \]

where $f(k) = 2(k-1)(k-2)g(k)$, and $g(k) = 2 + (\pi/2)^{k+1}$. The masses obtained with this relation are:

- $k = 1$, $f_k = 0$, $M_1 = 0$,
- $k = 2$, $f_2 = 0$, $M_2 = 0$,
- $k = 3$, $f_3 = 4$, $M_3 = 81.319$ GeV (same value as before),
- $k = 4$, $f_4 = 5.03$, $M_4 = 91.1898$ GeV (exact value).

From this relation the possible Higgs mass ("k" = 5) is $M_H = 199.2$ GeV and the mass of an eventual additional heavier boson is $M = 607.5$ GeV.

[10] T. Aaltonen et al, Phys. Rev. Lett. 104, 061803 (2010).

[11] F. R. Joaquim, Phys. Rev. D68, 033019 (2003).

[12] E. Jens and L. Paul, arXiv.org/abs/1003.3211.