Time-dependent model of particle acceleration in the vicinity of approaching magnetohydrodynamic flows

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Abstract. Supernova explosions and powerful stellar winds with strong shocks can convert a sizeable part of the kinetic energy release into energetic particles. The starforming regions are argued as a favorable site of energetic particle acceleration and could be efficient sources of nonthermal emission. We present the analytic solution for the time-dependent diffusion-convection equation for the case of two approaching shocks. We also present here time-dependent model of particle acceleration in the vicinity of two closely approaching fast magnetohydrodynamic (MHD) shocks. Such MHD flows are expected to occur in rich young stellar cluster where a supernova is exploding in the vicinity of a strong stellar wind of a nearby massive star. We find that the spectrum of the high energy particles accelerated at the stage of two closely approaching shocks can be harder than that formed at a forward shock of an isolated supernova remnant. The presented method can be applied to model particle acceleration in a variety of systems with colliding MHD flows.

1. Introduction

Diffusive shock acceleration (DSA) mechanism thought to be operating at the fast shocks of young supernova remnants (SNRs) is likely responsible for the galactic cosmic ray (CR) acceleration up to the knee-region energies (see e.g. [1, 2]) and may even exceed $10^{17}$ eV as it was advocated by [3] for the case of nuclei accelerated in an isolated young Type IIIb SNRs. The basic features of the DSA process have been revealed in the pioneering papers of [4], [5], [6] and [7] where the high efficiency of the acceleration mechanism was shown. Effects of nonlinear backreaction of the accelerated particles on the structure of the supersonic shock flow were discussed later by [8], [9], [10], [11], [12], [13] and [14]. It has been found that the pressure of accelerated particles can modify the bulk plasma flow in the shock upstream and that may result in a substantial increase of the flow compression and flattening in the particle spectra at the maximal energies.

The maximum energy of accelerated particles strongly differs for different types of supernovae (see e.g.[3]), it depends on the circumstellar medium around the supernova progenitor star. Moreover, core-collapsed supernovae produced by massive stars often occur in OB-star associations where the intense radiation of hot massive stars, powerful stellar winds and supernova shocks strongly modify the interstellar environment, producing large hot cavities of a few tens of parsec size, called superbubbles.
In the present paper we model a class of a few parsec size particle accelerators associated with collision of a young supernova shock with a fast stellar wind of a massive star. Some basic features of such systems were noticed in the previous paper [15]. The modeled stage starts a few hundred years before the supernova shock collides with the wind termination shock. At this stage the maximal energy particles accelerated via DSA at the SNR shock reach the fast wind termination shock and are scattered back by magnetic fluctuations carried by the fast stellar wind. Therefore, the high energy particles that have mean free path \( \Lambda(p) \) larger than the distance between the two shocks \( L_{12} \) start to be accelerated by the converging fast flows. This is the most favorable circumstance for the efficient Fermi acceleration. While the structure of the MHD flow in the vicinity of a supernova shell colliding with the stellar wind termination shock is rather complex, it is possible to consider a simplified model. When the magnetic field fluctuations are strong enough to provide the so-called Bohm diffusion regime with \( \Lambda(p) = \xi r_g(p) \), where \( p \) is the particle momentum, \( r_g(p) \) is the particle gyroradius in the mean magnetic field and \( \xi \gtrsim 1 \), the high energy particles bouncing between the converging flows are likely to have a spectrum harder than that produced at an individual shock and may contain a sizeable fraction of the total kinetic energy of the converging MHD flows.

2. Analytic time-dependent solution

Consider a model describing a population of high energy CR particles with \( \Lambda(p) \gtrsim L_{12} \) in a vicinity of two approaching shocks with \( R_{sh} = R_{sw} \gg L_{12} \), so planar geometry can be applied. In Figure 1 we illustrate a simplified scheme of the local flow with two approaching shocks. At \( x < 0 \) there is the upstream region of the shock 1 and at \( x > 0 \) there is the upstream region of the shock 2.

To derive the distribution function at the shock in the case of the two colliding shock fronts we employ a time-dependent diffusion-convection equation with the CR particle injection rate:

\[
\frac{\partial f_i(t, x, p)}{\partial t} = \frac{\partial}{\partial x} D_i(x, p) \left( \frac{\partial f_i(t, x, p)}{\partial x} - \frac{p}{3} \frac{\partial u_i}{\partial x} \frac{\partial f_i(t, x, p)}{\partial p} \right) + Q_0 \delta(x) \delta(p-p_0) H(t-t_0)
\]  

(1)
where index $i = 1$ corresponds to the $x < 0$ region and $i = 2$ for $x > 0$ (see Fig.1), $Q_0$ - is the injection rate of the particle, $\delta(x)$ - is Dirac’s delta-function, $H(\tau)$ - is the Heaviside step function, $D_i(x, p)$ - is the diffusion coefficient in the upstream area 1 and 2 (for simplicity we will assume $D_i$ to be independent of $x$).

Velocity profile has been chosen as

$$u_i = \begin{cases} u_1, & \text{if } x < 0; \\ -u_2, & \text{if } x > 0; \end{cases}$$

The distribution function of the particles $f(t, x, p) = 0$ at $t = 0$. Boundary conditions at the point $x = 0$ can be obtained by integration of the Eq.1 by $x$:

$$f_1 = f_2,$$

$$D_1 \frac{\partial f_1}{\partial x} + \frac{u_1}{3} p \frac{\partial f_1}{\partial p} = D_2 \frac{\partial f_2}{\partial x} - \frac{u_2}{3} p \frac{\partial f_2}{\partial p} + Q_0 \delta(p - p_0) H(t - t_0).$$

On taking the Laplace transform with respect to time:

$$g_f(s, x, p) = \int_0^\infty \exp(-st) f(t, x, p) dt,$$

equation (1) and boundary conditions (2),(3) become:

$$s g_{f_1} + u_i \frac{\partial g_{f_1}}{\partial x} = D_i \frac{\partial^2 g_{f_1}(s, x, p)}{\partial x^2},$$

$$D_1 \frac{\partial g_{f_1}}{\partial x} + \frac{u_1}{3} p \frac{\partial g_{f_1}}{\partial p} = D_2 \frac{\partial g_{f_2}}{\partial x} - \frac{u_2}{3} p \frac{\partial g_{f_2}}{\partial p} + \frac{Q_0}{s} \delta(p - p_0).$$

In order to satisfy boundary conditions $g_{f_1}(s, x, p) \to 0$ as $x \to \pm \infty$ we look for the solution $g_{f_1} \propto \exp(\beta_i x)$, where

$$\beta_i = \frac{u_i}{2D_i} \left(1 + \sqrt{1 + \frac{4D_i}{u_i^2}}\right).$$

Thus we have:

$$D_1 \beta_1 g_{f_0} - D_1 \beta_2 g_{f_0} + \frac{1}{3} p \frac{\partial g_{f_0}}{\partial p} = \frac{1}{s} Q_0 \delta(p - p_0).$$

where $g_{f_0} = g_f(s, 0, p)$ - Laplace transform at $x = 0$.

In general case inverse transform of the equation (8) cannot be evaluated. But for physically important values $t \gg D_i/u_i^2$ one can easily perform inverse Laplace transform by taking into account that for this case $\sqrt{1 + \frac{4D_i}{u_i^2}} \approx (1 + 2D_i u_i^2)$.

Therefore, solution of the equation (1) reads:

$$f_i(x, p, t) = \frac{3Q_0}{u_1 + u_2} \left(\frac{p}{p_0}\right)^{-3} H(p - p_0) H(t - \tau_a) \exp \left(-\frac{u_i |x|}{D_i(p)}\right) \propto \frac{1}{p^3},$$

and the particle spectrum:

$$\frac{\partial N}{\partial p} \propto p^2 f \propto \frac{1}{p},$$

where

$$\tau_a(p^*) = \int_{p_0}^{p^*} \frac{3}{(u_1 + u_2)} \left(\frac{D_1}{u_1} + \frac{D_2}{u_2}\right) \frac{dp}{p}.$$
is the CR acceleration time up to the momentum $p^*$. 

Presented acceleration mechanism can provide efficient creation of a nonthermal particle population with a very hard energy spectrum, containing a substantial part of the kinetic energy released by the supernova. The high energy nonthermal emission of the sources is characterized by a very hard spectral energy distribution peaked at the maximal photon energies and have the apparent properties of the dark accelerators.

3. Time-dependent model simulations

Via time-dependent DSA simulations we calculated the energy spectra of cosmic ray protons and electrons at a SNR shock approaching a strong wind of a nearby early type star. For relativistic electrons/positrons we account for the energy losses due to synchrotron and inverse Compton (IC) radiation. We solve one-dimensional diffusion-advection equations (see [16, 17, 18]) for the pitch-angle-averaged phase space distribution function of protons, $f_p(x,p,t)$, and electrons, $f_e(x,p,t)$, given by

$$\frac{\partial g_p}{\partial t} + u_i \frac{\partial g_p}{\partial x} = \frac{1}{3} \frac{\partial u_i}{\partial x} \left( \frac{\partial g_p}{\partial y} - 4g_p \right) + \frac{\partial}{\partial x} \left( D(p) \frac{\partial g_p}{\partial x} \right)$$

where $g_p = p^4 f_p$, $g_e = p^4 f_e$, $y = \ln(p)$. The dimensionless particle momentum is expressed in the units of $m_pc$. The cooling term $b(p) = -dp/dt$ describes the electron synchrotron and IC losses. These equations were solved with integration-interpolation algorithm and the standard Crank-Nicolson scheme. The model allows to calculate $f_{e,p}(x,p,t)$ at any position between the shocks and in the post-shock flows.

In Fig. 2 we present the results of calculations of the proton distribution function $f_p(x,p,t)$ that were obtained by time-dependent colliding shocks flow model and proton spectrum $dN_p(p)/dp$ at the moving left shock for the corresponding times comparing to the spectrum predicted by analytic test-particle solution (Eq. 9).

The left shock is propagating in the positive direction of the $x$-axis and the right shock is moving in the opposite direction. In Fig. 2 the CR proton distribution function $f(x,p,t)$ in the phase space $(p,x)$ is shown at different moments that correspond to the distances between the shocks of 0.6 pc, 0.5 pc, 0.3 pc, 0.1 pc. Spectral evolution of the proton distribution function between the colliding shocks is clearly seen. The CRs are initially concentrated around the shocks, and proton spectrum is close to the spectrum of the identical SNR shock $dN_p(p)/dp \propto 1/p^2$. But as the inter-shock distance reduces the proton spectrum gets harder and CRs concentrate in between the shocks. It can be seen from the Fig. 2 that when the distance between the shocks becomes 0.1 pc proton spectrum almost coincides with the analytic solution (Eq. 9).

Assuming that the amplified magnetic field near the shock is $\sim 100 \mu G$ as it was inferred from young SNR observations recently reviewed by [19], the distance between the shock fronts $L_{1/2} \sim 1$ pc, we used $D_{is} = 100 \times D_{sn}$ and $D_{sw} \approx D_{sn}$ for the Bohm-type diffusion, where $D_{is}, D_{sn}, D_{sw}$ - are interstellar, SNR-upstream and SW-upstream diffusion coefficients respectively. These parameters are convenient for the efficient particle acceleration in that system (Bykov A., Gladilin P., Osipov S., 2012, to be published).

4. Conclusion

The model describing the time-dependent particle acceleration in the vicinity of two colliding shocks is presented. This model predicts an unusual hard spectrum of CR protons and electrons confined in the flow and allows to accelerate particles up to the $10^{14}$ eV. We concentrated here on
Figure 2. On the left: proton distribution function $f(x, p) p^3$ as a function of the CR momentum and the observer position $x$, presented for four different distances between the shocks - from top to bottom: 0.6 pc, 0.5 pc, 0.3 pc, 0.1 pc. On the right: proton spectrum $dN_p/dp$ at the moving left shock for a corresponding times; dashed line - spectrum predicted by the test-particle solution $dN_p/dp \propto 1/p$ (eq. 9,10), solid line - simulated spectrum.

The test-particle time-dependent solutions without taking into account CR pressure modification of the shocks fronts. The non-linear effects of the described system will be discussed in the following paper (Bykov A., Gladilin P., Osipov S., 2012, to be published).

The stage of the closely approaching SNR and stellar wind flows that is favorable for particle acceleration typically lasts for 200-1,000 years depending on the stellar wind velocity and the termination shock radius. Therefore, such sources can likely contribute to the galactic CRs population.
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References
[1] Hillas A M 2005 Journal of Physics G Nuclear Physics 31 95–131
[2] Aharonian F, Bykov A, Parizot E, Ptuskin V and Watson A 2012 Space Sci. Rev. 166 97–132 (Preprint 1105.0131)
[3] Ptuskin V, Zirakashvili V and Seo E S 2010 ApJ 718 31–36 (Preprint 1006.0034)
[4] Axford W I, Leer E and Skadron G 1977 International Cosmic Ray Conference vol 11 pp 132–137
[5] Krymskii G F 1977 Akademiia Nauk SSSR Doklady 234 1306–1308
[6] Bell A R 1978 MNRAS 182 147–156
[7] Blandford R D and Ostriker J P 1978 ApJ 221 29–32
[8] Blandford R and Eichler D 1987 Phys. Reports 154 1–75
[9] Berezhko E G and Krymskij G F 1988 Uspekhi Fizicheskikh Nauk 154 49–91
[10] Bell A R 1987 MNRAS 225 615–626
[11] Malkov M A and O’C Drury L 2001 Reports on Progress in Physics 64 429–481
[12] Blasi P 2004 Astroparticle Physics 21 45–57 (Preprint arXiv:astro-ph/0310507)
[13] Amato E and Blasi P 2005 MNRAS 364 76–80 (Preprint arXiv:astro-ph/0509673)
[14] Vladimirov A E, Bykov A M and Ellison D C 2008 ApJ 688 1084–1101 (Preprint 0807.1321)
[15] Bykov A M, Gladilin P E and Osipov S M 2011 MemSAIt 82 800 (Preprint 1111.2587)
[16] Skilling J 1975 MNRAS 172 557–566
[17] Toptygin I N 1985 Cosmic rays in interplanetary magnetic fields (Moscow: Izdatel’stvo Nauka)
[18] Kang H 2011 Journal of Korean Astronomical Society 44 49–58 (Preprint 1102.3109)
[19] Vink J 2012 Astron. Astroph. Reviews 20 49 (Preprint 1112.0576)