Single Pulse Variability in Gamma-ray Pulsars

M. Kerr

1 Space Science Division, Naval Research Laboratory, Washington, DC 20375–5352, USA

ABSTRACT

The Fermi Large Area Telescope receives \( \ll 1 \) photon per rotation from any \( \gamma \)-ray pulsar. However, out of the billions of monitored rotations of the bright pulsars Vela (PSR J0835–4510) and Geminga (PSR J0633+1746), a few thousand have \( \geq 2 \) pulsed photons. These rare pairs encode information about the variability of pulse amplitude and shape. We have cataloged such pairs and find the observed number to be in good agreement with simple Poisson statistics, limiting any amplitude variations to \(< 19\% \) (Vela) and \(< 22\% \) (Geminga) at \( 2\sigma \) confidence. Using an array of basis functions to model pulse shape variability, the observed pulse phase distribution of the pairs limits the scale of pulse shape variations of Vela to \(< 13\% \) while for Geminga we find a hint of \( \sim 20\% \) single-pulse shape variability most associated with the pulse peaks. If variations last longer than a single rotation, more pairs can be collected, and we have calculated upper limits on amplitude and shape variations for assumed coherence times up to 100 rotations, finding limits of \( \sim 1\% \) (amplitude) and \( \sim 3\% \) (shape) for both pulsars. Because a large volume of the pulsar magnetosphere contributes to \( \gamma \)-ray pulse production, we conclude that the magnetospheres of these two energetic pulsars are stable over one rotation and very stable on longer time scales. All other \( \gamma \)-ray pulsars are too faint for similar analyses. These results provide useful constraints on rapidly-improving simulations of pulsar magnetospheres, which have revealed a variety of large-scale instabilities in the thin equatorial current sheets where the bulk of GeV \( \gamma \)-ray emission is thought to originate.

1. INTRODUCTION

Our understanding of pulsar magnetospheres is improving rapidly: sophisticated simulations are providing the first nearly self-consistent realizations of magnetospheres, and the Fermi Large Area Telescope (LAT) continues its groundbreaking discovery and characterization of the large population of \( \gamma \)-ray pulsars needed to test these models.

These new particle-in-cell (PIC) simulations rest on half a century of theoretical work. Soon after Hewish et al. (1968) discovered pulsars, Goldreich & Julian (1969) proposed a “force-free” (FF) model of the pulsar magnetosphere in which a co-rotating charge-separated plasma with density \( \rho_{GJ} \sim \Omega \cdot B \) screens the parallel electric field \( (E_\parallel) \), a picture which largely holds up today. However, FF magnetospheres by definition cannot accelerate particles, and the necessity of strong acceleration with concomitant \( e^+ / e^- \) pair formation in active pulsars was quickly recognized (Sturrock 1971). Moreover, solving the coupled currents and electrodynamics even with the FF constraint proved intractable, and the best available analytic model was that of a magnetic dipole rotating in vacuo (Deutsch 1955), also clearly far from reality.

These difficulties led to long threads of research which compartmentalized the problem. To characterize particle acceleration, theorists supposed that in an otherwise FF magnetosphere, small regions might lack the \( \rho = \rho_{GJ} \) plasma needed to screen the electric field. Such vacuum “gaps” could accelerate particles to high energy and form the pairs needed to achieve the assumed force-free conditions elsewhere. The ideas included a gap operating above the magnetic polar cap (Ruderman & Sutherland 1975), a “slot gap” arising around the polar cap rim (Arons 1983), and an “outer gap” operating above the null charge surface \( (\rho_{GJ} = 0) \) near the light cylinder (Cheng et al. 1986).

\( \gamma \) rays are ideal for testing these models, as they directly trace particle acceleration. Observations by COS-B and EGRET (e.g. Thompson et al. 1999) provided both pulse profiles and spectra for a half-dozen young pulsars. Even in small numbers, it was clear that the light curves differed substantially from radio pulses, arriving later in the neutron star rotation (by a phase lag commonly noted \( \delta \)) and demonstrating a now-canonical morphology of two cuspy peaks separated by \( \Delta \sim 0.2–0.5 \) rotations. Comparison to data was achieved using a “geometric” method which propagated \( \gamma \)-ray emissitivities to an observer using the Deutsch
field. Approximately offsetting effects from time delay and field sweepback produced caustic-like features (Romani & Yadigaroglu 1995) more reminiscent of the data than the predictions of low-altitude models (Daugherty & Harding 1996), lending some credence to an outer magnetospheric origin. However, the small pulsar numbers and incomplete magnetic field information could not support strong conclusions.

Thirty years after the proposal of the idea, Contopoulos et al. (1999) and Timokhin (2006) numerically solved the FF magnetohydrodynamic equations for an aligned ($\Omega \parallel B$) magnetosphere while later work (Spitkovsky 2006; Kalapotharakos & Contopoulos 2009) addressed the nonaligned cases. These breakthrough solutions finally provided an exact description of both fields and currents, revealing a key feature: current flows out of the polar cap into a broad volume of the magnetosphere, but much of it returns in a very thin layer in the equatorial plane, splitting at the “separatrix” near the light cylinder and returning to the rim of the polar cap along the boundary of the closed and open magnetic field zones. This equatorial current sheet (ECS) is a hallmark of FF magnetospheres, though less of the return current flows through the ECS for large magnetic inclinations. Its extremely high areal current density makes it a natural candidate for magnetic reconnection and accelerating fields, but assessing these processes cannot be done with FF MHD.

This development set the stage for an observational revolution: soon after its launch, the Fermi Large Area Telescope (LAT, Atwood et al. 2009) detected populations of radio-loud, radio-quiet (Abdo et al. 2009a), and millisecond pulsars (Abdo et al. 2009b), establishing all classes as efficient $\gamma$-ray emitters capable of converting $\sim$1–30% of their available spin-down power (Abdo et al. 2013) into GeV photons. Early analyses of the LAT data strongly supported the outer magnetosphere as the most likely site of substantial $\gamma$-ray production. First, the spectral signatures of bright pulsars lacked evidence for attenuation in the strong magnetic fields above polar caps (Abdo et al. 2009c). Second, the fraction of pulsars with radio-trailing, cuspy double-peaked light curves was too large for low-altitude emission modeling. Instead, these authors invoked a “separatrix layer model” operating along the ECS, extending even further into the outer magnetosphere than the classic outer gap model. Relaxing the FF assumptions by introducing resistivity into the magnetosphere—thus emulating conversion of the fields and currents into high-energy particles—yielded even better agreement. Kalapotharakos et al. (2014) used a prescription which was FF inside the light cylinder and resistive outside, with the resulting tweaks to the magnetic field and emission geometry producing the first excellent agreement with the observed $\delta$-$\Delta$ properties of the LAT pulsar sample (Abdo et al. 2013). This and other work cemented the paradigm that $\gamma$-ray pulsars have nearly force-free magnetospheres and produce most of their $\gamma$ rays in the ECS near and beyond the light cylinder.

Recent PIC simulations have confirmed this picture with, for the first time, nearly self-consistent modeling. In PIC simulations, macroparticles representing collections of many charged particles are evolved iteratively with the electromagnetic fields tabulated at the cell boundaries. The particles produce the (non-background) fields and are in turn accelerated by them. The dynamic range of the simulations is still many orders of magnitude below a true magnetosphere, so e.g. maximum particle Lorentz factors reach $\sim$1000. Thus the key process of pair production must still be put in manually. Timokhin & Arons (2013) demonstrated that a current density $J \equiv c \rho$ of $J/J_{\text{GJ}} < 0$ or $J/J_{\text{GJ}} > 1$ is indicative of intense particle acceleration and pair production, while for $0 < J/J_{\text{GJ}} < 1$, current can be sustained by space-charge limited flow with no pair production. This provides a common prescription for PIC simulations: by requiring that $J/J_{\text{GJ}} > 1$, authors can assume pair production occurs locally and replace the primary particles with secondaries and $\gamma$ rays, thus providing both the copious plasma needed to screen the electric field and tracking the emission of $\gamma$ rays for comparison to observation.

With such prescribed pair injection, PIC simulations have been successful in reproducing the general structure of FF magnetospheres but offer more insight into the dynamics, in particular the response of the magnetosphere to the location and extent of pair production. E.g., Philippov et al. (2015a) found that the incorporation of General Relativistic effects near the neutron star surface suppresses $\rho_{\text{GJ}}$ locally, facilitating the pair production needed to explain the presence of radio pulsations. However, Chen et al. (2020, and see also Kalapotharakos et al. (2018) for similar studies), simulating magnetospheres over a range of pair production rates, highlight the importance of pairs within the ECS to achieving a near-FF magnetosphere due to the difficulty in screening $E_\parallel$ there. If pairs produced elsewhere (Cerutti et al. 2015; Brambilla et al. 2018) are unable to enter the ECS, or the sheet itself is no longer able to sup-

---

1 Simulations are often scaled to $\rho_{\text{GJ}}$, so this condition also appears in the literature as a magnetization exceeding unity.
port pair production, the magnetosphere may transition to a static, charge-separated electrosphere. In the intermediate regime, the magnetosphere fluctuates between these states, with γ-ray production and global currents fluctuating in step.

As PIC simulations have been refined, they have become more predictive for γ-ray data. Kalapotharakos et al. (2018) and Philippov & Spitkovsky (2018) built γ-ray light curves by tracking emitted photons and found their pattern on the sky in reasonable agreement with data. Philippov & Spitkovsky (2018) postulate that γ rays are primarily produced by synchrotron emission from reconnection in the ECS (Uzdensky & Spitkovsky 2014), near the LC in the case of aligned rotators and from beyond the LC for oblique rotators, which are also somewhat less efficient emitters. On the other hand, Kalapotharakos et al. (2017) and Kalapotharakos et al. (2019) have noted that γ-ray properties lie on a “fundamental plane” according to their γ-ray luminosity, spectral cutoff energy, magnetic field, and spindown luminosity $\dot{E}$. By connecting the cutoff energy to the maximum particle Lorentz factors, they argue that the scaling may be indicative of curvature radiation. The upcoming third Fermi pulsar catalog will provide data for many more pulsars and provide a touchstone to determine which of these mechanisms is operant. Thus, after more than 50 years of work, theory and data are poised to deliver a nearly-complete picture of the pulsar magnetosphere and how it emits γ rays.

Yet while the overall static picture is becoming clear, the PIC simulations have opened the field of magnetosphere dynamics and their observability. Notably, kink instabilities in the ECS are a common feature in PIC simulation results—especially near the Y-point/separatrix where the return current converges (Chen & Beloborodov 2014)—with the local tangent to the ECS varying by $\sim 0.1$ rad or more, which could affect the beaming of emitted γ rays. If the γ rays are emitted via magnetic reconnection, Uzdensky & Spitkovsky (2014) argue that the maximum size of coherent regions should be $<100 \times$ the width of the current sheet, only about 1 m! Thus many reconnecting “islands” contribute to the current sheet and this could suppress such coherent effects, whereas dynamic instabilities could affect curvature radiation emission more significantly. On the other hand, reconnecting regions could merge into larger ones, driving γ-ray variability (Philippov et al. 2019; Ceylan Andaç et al. 2022). Further, Kirk & Giacinti (2017) invoke a bulk Lorentz factor of $\sim 10^4$ in the wind of the Crab pulsar, in which case reconnection cannot proceed due to time dilation. In any case, for weaker pulsars, transitions between FF magnetospheres and “dead” electrospheres are common (Kalapotharakos et al. 2018; Philippov & Spitkovsky 2018) and transitions can happen on timescales of one neutron star rotation (Chen et al. 2020) and longer.

γ-ray pulses, tracing both of these processes, would provide an invaluable probe for dynamics of the equatorial current sheet and the global magnetosphere. Accordingly, this work presents the first systematic search for and characterization of single- and few-pulse γ-ray pulse variability. Despite the recent theoretical motivation, such studies have heretofore been lacking primarily due to observational constraints: even the sensitive LAT records on average only $\sim 0.003$ photons from Vela, the brightest γ-ray, during each 89 ms rotation, so measuring pulse-to-pulse profile or amplitude is impossible.

However, the LAT has observed billions of rotations of Vela, and $\sim 10^{-6}$ of the rotations have 2 or more recorded photons. These multi-photon events are a far cry from a pulse, yet they still carry information about the variability of the pulsed GeV emission on short (neutron star rotation) time scales. For instance, any deviations from a constant flux would inevitably increase the number of observed pairs over the simple Poisson prediction. Likewise, pulse shape variations can be probed by examining the joint distribution of pulse phase for photon pairs. If certain portions of the pulse profile are brighter during some rotations than in others, this will manifest as a correlation in phase. Most generally, we expect simultaneous amplitude and shape variations: e.g., in a two-peaked pulse profile, a reduction in the intensity of one of the peaks could entail a reduction in total intensity, or it could be accompanied by an offsetting increase in the brightness of the other peak.

Below, we carry out a search for single- and multipulse variations in the two brightest γ-ray pulsars, Vela and Geminga. §2 presents our data preparation and selection of “photon pairs”. In §3, we search for amplitude variations independently and (finding none), we use the resulting limits as guidelines for a pulse shape variation search (§4) in which the total flux doesn’t vary by more than the established limits. We extend the analysis to longer timescales (up to 100 rotations) in §5, improving the constraints. We discuss other γ-ray pulsars, which are too faint to be of use, in §6, and we conclude in §7, placing our results in context and discussing the scant prospects for additional γ-ray variability measurements.

2. DATA PREPARATION AND SELECTION

We obtain “Pass 8” (Atwood et al. 2013) event reconstruction version P8R3 (Bruel et al. 2018) from the Fermi Science Support Center and process it using the Fermi Science Tools v2.0.0. We select about 12 years of data, between 2008 Aug 04 (MJD 54698) and an end date which depends on the range of validity of the pulsar timing solution (Table 1). We further

2 The software developed for this analysis is archived at: https://doi.org/10.5281/zenodo.6636586.

3 https://fermi.gsfc.nasa.gov/ssc/data/access/
restrict attention to events with reconstructed energy $0.1 \text{ GeV} < E < 10 \text{ GeV}$, measured zenith angle $< 100^\circ$, and a reconstructed incidence direction placing the photon within $3^\circ$ of the pulsar position. To enhance the sensitivity of the analyses (Bickel et al. 2008; Kerr 2011, 2019), using the 4FGL DR2 sky model (Abdollahi et al. 2020; Ballet et al. 2020) and the P8R3_SOURCE.V3 instrument response function, we assign each photon a weight using gtsrcprob and restrict attention to events with $w > 0.05$. This selection retains most of the signal while eliminating background photons that increase computational costs.

We obtain timing solutions for each pulsar using the maximum likelihood approach presented in Ajello (2022). Using PINT (Luo et al. 2021), we computed the absolute pulse phase for each event, and we define pairs as those photons with identical integer phase. This definition is somewhat arbitrary, but we have centered the pulse profile (Figure 1) within the phase window such that pairs associated with the same pulse are selected. (In §5, we adopt a simpler method of selecting photons whose total phase difference is simply less than some threshold, and we find no substantive difference.) Triples and higher-order coincidences follow the same definition.

Pursuing this method for Vela, we identified 6388 pairs, 12 triples, and no higher order coincidences. Because some photons come from the background, this total comprises three cases: two background photons, a single pulsed photon with a background photon, and two bona fide pulsed photons. The breakdown can be estimated from the photon weights, where $w_1$ and $w_2$ are the probability weights of the two photons in the observed pair; $\sum_i (1 - w_{1i})(1 - w_{2i}) = 188.9$ background pairs; $\sum_i (1 - w_{1i})w_{2i} + w_{1i}(1 - w_{2i}) = 1835.4$ with a single pulsed photon; and $\sum_i w_{1i}w_{2i} = 4363.6$ bona fide pairs.

Similar arguments apply to the triples and we expect $\sum_i w_{1i}w_{2i}w_{3i} = 5.4$ bona fide triples. These triples are too rare to be significantly constraining, so in the analysis below we focus on pairs. Table 2 summarizes these results for Vela and Geminga, including predictions accounting for the instrument exposure (see below).

### Table 2. Observed total events and probability-weighted pairs and triples.

| Type   | Obs. | Chance | bona fide | Pred. |
|--------|------|--------|-----------|-------|
| Vela   | –    | –      | 2,720,089.2 | –     |
| . . events | 3,295,429 | –      | 4,436.3 | 4,295.4 |
| . . pairs  | 6,388  | 2,024.4 | 4,363.6 | 4,295.4 |
| . . triples | 12    | 6.6    | 5.4      | 4.7   |

### Table 1. Some basic properties of the target sources and the data set. All data sets begin MJD 54698.

| Name             | Nickname | MJD End | Years |
|------------------|----------|---------|-------|
| J0835–4510       | Vela     | 59170   | 12.3  |
| J0633+1746       | Geminga  | 59249   | 12.5  |
| J0534+2200       | Crab     | 59000   | 11.9  |
| J1709–4429       | –        | 59312   | 12.6  |
| J1057–5226       | –        | 59249   | 12.5  |

Figure 1. Two rotations of the Vela pulsar, showing a weighted histogram (500 bins per rotation, blue) of the full dataset and an analytic model for $f(\phi)$ overlaid (orange).
intervals and assume it is the same for each pulsar rotation. As a further simplification, we ignore the spin evolution of the pulsar and simply use the mean spin rates of $\bar{\nu} = 11.187519$ Hz for Vela and $\bar{\nu} = 4.217539$ Hz for Geminga. With this prescription, the expected number of source photons per rotation is given by

$$\lambda_{30} = \frac{1}{\bar{\nu}} \int N(E) \epsilon(E, \cos(\theta_{30})) \, dE, \quad (1)$$

with $N(E)$ the photon spectral density (ph cm$^{-2}$ s$^{-1}$) for the pulsar and $\epsilon$ the effective area (cm$^2$) for the given energy $E$ at an incidence angle $\theta_{30}$ taken to be constant over the 30 s interval. We evaluate $\lambda_{30}$ using the methods developed by Kerr (2019), particularly the package *godot*\(^4\). In brief, this evaluation uses the tabulated spacecraft pointing history (“FT2 file”), with 30 s resolution, to determine $\cos(\theta_{30})$ and hence the appropriate value of the tabulated effective area. Gaps in data taking are accounted for with the tabulated “Good Time Intervals” (GTI). The method also assumes a universal energy weighting $\alpha E^{-2}$, which roughly describes the overall distribution of LAT photon energies. To further tune this approach for pulsars, which have spectral cutoffs at a few GeV, we perform the exposure calculation using 8 logarithmic bins spanning 100 MeV to 10 GeV. In each band, we then estimate the constant of proportionality to enforce $\sum \lambda_{30} w_i = \sum u_i / 30s$. This accounts for any “local” differences in the distribution of photons from $\alpha E^{-2}$ as well as for the reduction in effective exposure at low energies because the point spread function (PSF) is broader than the 3° region of interest. We further note that we tabulate events which convert in the “front” (evtype 1) and “back” (evtype 2) of the tracker separately. We exclude any intervals with $\cos(\theta_{30}) < 0.4$, when the source is near or outside the edge of the LAT field-of-view. We further discard the lowest 0.5% of intervals, which eliminates a tail of low-exposure instances. The resulting rates (per rotation) for the two pulsars are shown in Figure 2. It is interesting to note that despite similar median photon rates (0.0032 and 0.0025 per rotation), Vela has three times as many photon pairs. As Figure 2 shows, Vela accumulates much more “high rate” exposure than Geminga, and this leads to more pair production. This intuitive explanation is borne out by the computations below and illustrates the importance of correcting for the instantaneous instrument exposure.

With the computed rates, we can predict the total number of pairs expected to be observed by using the Poisson distribution to evaluate the probability mass function of observing 0, 1, 2, or 3 photons during each rotation and summing these probabilities over all observed pulsar rotations. Using a simple model in which the pulsed flux is constant and only the instrumental exposure varies, for Vela this process yields $4,295.4 \pm 65.5$ pairs (uncertainty assumes Poisson variance), in good agreement (1.0$\sigma$) with the observed, probability-weighted value of 4,363.6. Although triples are too rare to be constraining, the predicted number is 4.7, again in good agreement with the observation of 5.4. For Geminga, the prediction (1,417.0 ± 37.6) is higher than the observed value (1,378.1) and agrees at 1.0$\sigma$. All results are tabulated in Table 2.

---

\(^4\) https://www.github.com/kerrm/godot
Figure 4. The joint distribution of phase for photon pairs, weighted by $w_1 \times w_2$. The left panel shows the observed distribution sorted such that $\phi_1 < \phi_2$. The center panel shows the expected null distribution $f(\phi_1, \phi_2) = f(\phi_1)f(\phi_2)$. The right panel gives the distribution of the sorted phase differences for the observed pairs, again weighted by $w_1 \times w_2$, along with the expectation from the null distribution, obtained by randomizing phases from the entire sample. Error bars are computed from the Poisson variance, which neglects the small correlations between bins.

Figure 5. As Figure 4 for Geminga.

To gauge how large intrinsic flux variations could be without entering serious disagreement with the pair count, we consider two models of amplitude variations. First, we use a log normal distribution, with parameters $\mu$ and $\sigma$ for the underlying normal distribution. If we fix $\mu = -\sigma^2/2$, the distribution has unit mean, which preserves a constant photon rate but increases fluctuations between pulses. We then scale each value of $\lambda_{30}$ with a random draw from the distribution. (It is not necessary to draw a random variable for every pulsar rotation because the distribution is already sufficiently sampled by the millions of values of $\lambda_{30}$.) Second, we consider a uniform distribution with total width $2W$, which allows the relative values of $\lambda_{30}$ to vary by up to $1 + W$, and we restrict $W \leq 1$ to avoid negative photon rates. These distributions differ strongly: in particular, the log normal distribution has heavy tails, allowing for very bright pulses, while the brightest pulses are capped in the uniform case.

The results of the study for Vela are shown in Figure 3. Increasing the scale parameters $\sigma$ and $W$ increases the number of predicted pairs until the predictions are no longer compatible with the observed value. If we adopt a $2\sigma$ discrepancy for an upper limit, then scale parameters larger than $\sigma = 0.21$ and $W = 0.38$ are excluded. To put the models on equal footing we can compare their standard deviation ($\sigma_w = W/\sqrt{3}$ for the uniform case) and find $\sigma \approx \sigma_w$, and so in a relatively model independent fashion, we can exclude fluctuations whose typical strength exceeds 21%. A similar analysis for Geminga gives limits of $\sigma = 0.16$ and $W = 0.28$, limiting fluctuations to about 16%.
These results depend on the observed values, and so Geminga gives superior results despite having fewer pairs because the null pair prediction exceeds the observed value. We can instead simply consider the capability of the data to constrain variations without regard for the particular realization of it. With $\sim 4300$ predicted (null hypothesis) pairs, the largest fractional increase in pair rate is $2 \times 4300^{-1} = 0.3\%$, and for Geminga it is $2 \times 1400^{-1} = 0.3\%$. These values correspond to $\sigma$ and $W$ parameters of 0.17 and 0.30 for Vela, and for Geminga 0.23 and 0.40, or relative variations of 17% and and 23% respectively.

4. PULSE SHAPE VARIATIONS

We now consider limits on the variation in pulse shape. Let the time-averaged pulse profile be $f(\phi)$. If normalized such that $\int_0^1 f(\phi) = 1$, this also is the probability density function (pdf) to observe a photon at the given phase. The pdf for the photon pairs is $f(\phi_1, \phi_2)$, normalized in the same way, and in the null hypothesis $\phi_1$ and $\phi_2$ are independent variables $f(\phi_1, \phi_2) = f(\phi_1) f(\phi_2)$. We can analyze any pairwise pulse shape variations using this distribution.

First, we characterize the distribution empirically. Figure 4 shows the observed joint probability distribution $f(\phi_1, \phi_2)$ for Vela as estimated with a weighted histogram, and the null expectation, namely $f(\phi_1) f(\phi_2)$. The data and this null model are visually in good agreement, though the small counts preclude assessment of goodness of fit. We have also examined the distribution of $\phi_2 - \phi_1$, which also encodes information about correlations and is more sensitive to fluctuations with a constant scale $\delta \phi$. E.g., an enhancement of probability to observe a photon pair from either peak should produce an excess near the origin and a deficit elsewhere. This distribution, likewise, agrees well (within the uncertainty estimates) with the null hypothesis expectation, which we estimated by drawing random pairs from the much larger parent sample.

We now seek to quantify the largest shape variations the data can accommodate. In doing so, we are guided by the results from §3 and the empirical analysis: the time-averaged distribution of photon pairs is very similar to $f(\phi)$, and if pulse shape variations are accompanied by intensity variations, they cannot be much larger than 20%. We also seek a physically motivated model: although the $\gamma$-ray emission contributing to a single spin phase can be accumulated from a large portion of the magnetosphere through caustic-like effects, the portions of the magnetosphere contributing to, e.g., P1, P2, and the bridge/P3 in Vela are distinct. Thus, considering changes localized to pulse profile features will target large-scale but distinct regions of the magnetosphere.

We thus adopt a family of $N_b$ basis functions, $b_i(\phi)$, which describe the variation in the pulse profile from rotation to rotation. Over many rotations, the individual pulse profiles must yield the long-term light curve, so by construction $\sum_i b_i(\phi) / N_b = f(\phi)$. For both Vela and Geminga, with well-defined two-peaked light curves and bridge emission, we choose $N_b = 6$ corresponding to an increment and decrement of each of the three main features. The basis functions are defined via a strength parameter $\alpha$, with each basis function reducing to the null hypothesis: $b_i(\alpha, \phi) \rightarrow f(\phi)$ as $\alpha \rightarrow 0$. Figure 6 (7) illustrates the basis functions for Vela (Geminga) with $\alpha = 0.15$ (15%) variations. Even this modest total variation results in changes of the relative peak amplitudes by about 2. And indeed, $\alpha$ cannot exceed $\sim 0.2$ as this results in the removal of gross features of the light curve and a negative pdf. This is not a significant restriction.

Figure 6. An example set of six basis functions for a model of single-pulse variations for Vela. They correspond to an addition (or loss) of 15% of the total flux from each of the three major components, P1, P2, and P3/the bridge. The time-averaged pulse profile is shown in black for reference. By construction, the mean of the six basis functions is equal to this pulse profile.

Figure 7. As Figure 6 for Geminga, but note different phase range.
since $\alpha = 0.2$ already captures very large shape variations, and it ensures that the models tested in this analysis are consistent with the amplitude variation limits established above.

We proceed via maximum likelihood. On a given rotation, the photons are emitted according to one of the basis functions. Considering just a single pair of photons, the probability density function is then

$$P(X; \alpha, \phi) = \sum_i \pi_i [w_i(1-w_2) b_i(\alpha, \phi_1) + w_2(1-w_1) b_i(\alpha, \phi_2) + w_1 w_2 b_i(\alpha, \phi_1) b_i(\alpha, \phi_2)],$$

where the $\pi_i$ describe the probability of the $i$th basis function as being the correct one for the rotation in question. For convergence to the time-averaged profile, $\pi_i > 1/N_b$. We have no a priori knowledge of $\pi_i$ for any given pulse, so we let it assume the time-averaged value $\pi = 1/6$, allowing direct evaluation of the likelihood as a function of $\alpha$. The resulting log likelihood surface is shown in Figure 8. The likelihood peak for Vela is near $\alpha = 0.08$, but the significance is consistent with $\alpha = 0$. Consequently, we assume a uniform prior on $\alpha$ and estimate a 95% confidence upper limit of $\alpha < 0.12$.

Next, we consider the empirical distribution function for Geminga (Figure 5). As with Vela, it appears to largely follow the null hypothesis. The projected distribution in $\phi_2 - \phi_1$ does show a small excess near the origin, indicating a possible small correlation for photons to arrive nearby in phase. Using the same basis-based technique (see Figure 7 for the basis functions for Geminga), we find the maximum likelihood value occurring around $\alpha = 0.18$, with $\delta \log L = 3.67$, near the edge of the validity of our model. Because this model satisfies the conditions of Wilks’ Theorem, we can quickly estimate the significance of the alternative hypothesis as 99.3%. Thus there is modest evidence for pulse shape variability for Geminga, with an amplitude consistent with the largest allowed intensity variations. By skipping marginalization, we can determine the likelihood-maximizing basis function for each pair and thus obtain the distribution of preferred bases. We find bases which modify P1 and P2 are most prevalent, with P3 “hi” being least preferred. Thus, if the pulse shape variation is real, it is concentrated in the peaks, consistent with the empirical analysis above. However, we caution that this result strongly depends on our assumption of a particular basis.

5. MULTI-ROTATION CORRELATIONS

We have also considered correlations on time scales greater than one neutron star rotation, which could be expected e.g. from magnetospheric state switching on longer time scales. We carried out a similar procedure, identifying photons with the requisite phase separation and using their weights to calculate the probability that they both originate from the pulsar. Here, we apply a simpler criterion, selecting as “pairs” those photons whose phase separation lies below a given threshold, say $\delta \phi < \Phi$. A threshold of $\Phi = 0.5$ approximately reproduces the single-rotation selection above, while $\Phi = 1$ produces about twice as many pairs, because it has contributions from two pulses.

As before, we examine amplitude fluctuations by studying the total number of pairs up to an assumed coherence of $\Phi = 100$ rotations, with results shown in Figure 9 for the two bright pulsars. Counting pairs based on exposure is more difficult in this case, because...


Figure 10. Limits on the strength of the variability parameter $\alpha$ (see main text) when including pairs collected over the indicated number of rotations. The solid lines give the results for the data, while dashed lines give the average of 10 Monte Carlo realizations in which any correlations in the real data have been broken. The scaling in both cases is in good agreement with the total number of pairs to the one-fourth power.

Each rotation of the pulsar is no longer statistically independent. Thus, to calibrate the mean expectation, we use a simple Monte Carlo approach in which we permute the integer phase of each photon. This preserves the overall exposure pattern but breaks any short-term correlations. The resulting predictions, along with the corresponding Poisson uncertainties, are drawn in the figure. Because each of the observed pair counts lies within this band, we conclude there is no evidence for amplitude variations. By assuming longer coherence scale, the number of pairs increases substantially, and the limits on amplitude fluctuations tighten. The scaling simply depends on the total number of pairs, thus on $\Phi$, and we arrive at the limits on relative amplitude fluctuation for an assumed coherence time $\Phi$ in neutron star rotations: $<21\% (\Phi/0.5)^{-1/2}$ for Vela and $<16\% (\Phi/0.5)^{-1/2}$ for Geminga. For $\Phi = 10$ this is 4.7% and 3.6%, and for $\Phi = 100$, 1.5% and 1.1%.

Likewise, we can feed this larger population of pairs into the pulse shape analysis of §4. Here, because of the marginalization over the unknown basis for each pulse pair, the scaling with the number of pulses is less straightforward, and we have carried out the analysis directly on the data for the increasingly larger populations of pairs. The results, as a function of assumed coherence length $\Phi$, are shown in Figure 10. N.B. that the modest evidence for pulse shape variability in Geminga detected in §4 drops as soon as additional rotations are added and the limit quickly asymptotes to the same dependence as for Vela. For $\Phi = 10$, the 95% limit on the shape variation parameter $\alpha$ is about 5% for both pulsars, and for $\Phi = 100$ about 3%. The limits on the parameter seem to scale as the total observed pairs to the one-fourth power, rather than the one-half power as expected for Poisson uncertainties.

6. OTHER BRIGHT PULSARS

The Crab pulsar (J0534+2200) is an interesting target due to its known giant pulse phenomenon and the recent discovery of enhanced X-ray emission associated with such giant pulses (Enoto et al. 2021). However, its fast spin frequency yields a typical weighted photon rate per rotation of only $1.5 \times 10^{-4}$, and consequently we have observed only 39.5 bona fide pairs, in reasonably good agreement with the null hypothesis prediction of 35.6. These are simply too few pairs to probe single pulse variations beyond this simple sanity check.

Another “EGRET” pulsar PSR J1709—4429 is bright but embedded deep in the Galactic plane, giving a relatively high background rate, and consequently only 10% of the observed pairs are likely to be pulsed. It has a median weighted photon rate of $4.7 \times 10^{-4}$ and produces 89.9 bona fide pairs compared to a prediction of 84.8, in excellent agreement. Finally, PSR J1057—5226 has a mean weighted photon rate of $2.2 \times 10^{-4}$, and its exposure pattern leads to mean predicted and observed pair rates of 10.0 and 9.2, respectively. The next few brightest LAT pulsars all also have faster spin periods and consequently even fewer pairs.

7. DISCUSSION

Thus, for at least two pulsars, we can restrict variations in amplitude and pulse shape to $\leq 20\%$. (Though we caution that with our definition, shape variations of 20% still encapsulate substantial changes to the pulse profile.) These constraints allow us to rule out large, coherent variations of the full $\gamma$-ray emitting volume.

Consider the case of a nearly aligned rotator, with $\gamma$-rays emitted within $\sim R_{LC}$ (light cylinder radius) of the separatrix. Kink instabilities within the ECS observed in PIC simulations can deform this region substantially, producing deviations of 0.1 rad or more, although the amplitudes are likely to be smaller in magnetospheres with realistic magnetic fields (Cerutti et al. 2016). Beaming of the $\gamma$-ray emission would then result in either removal of the flux from the line-of-sight or a

| Type                | Obs. | Chance | bona fide | Pred. |
|---------------------|------|--------|-----------|-------|
| J0534+2200 events   | 695,267 | —    | 398,271.7 | —    |
| J0534+2200 pairs    | 125   | 85.5  | 39.5      | 35.6  |
| J1709-4429 events   | 1,177,343 | —    | 359,382.0 | —    |
| J1709-4429 pairs    | 918   | 828.1 | 89.9      | 84.8  |
| J1057-5226 events   | 283,529 | —    | 90,115.8  | —    |
| J1057-5226 pairs    | 113   | 103.8 | 9.2       | 10.0  |

Table 3. As Table 2, for fainter pulsars.
shift to a different spin phase, either of which could be detected with this method. Thus, either the amplitude of such instabilities must be smaller, or else the \( \gamma \)-ray emission must arise on different spatial scales.

As discussed in §1, one possibility is that magnetic reconnection in the ECS could fragment into many small islands which damp large-scale instabilities. Another possibility is that the \( \gamma \)-ray emission instead arises over many \( R_{\text{LC}} \), i.e. within the striped wind (Pétrí 2012), such that material corresponding to many neutron star rotations contributes to an apparent pulse, and the observational effects of instability average away. Intriguingly, An et al. (2020) detected orbital modulation of pulsed emission from a millisecond pulsar, and one possible way to arrange this is via inverse-Compton scattering of light from the companion by \( \gamma \)-ray emitting particles in the striped wind. However, this process may be inefficient and instead such modulation may rely critically on the presence of the companion star (Clark et al. 2021).

We can also rule out wholesale state changes between FF and electrosphere configurations. Such state changes seem to increase as a pulsar ages, and are potentially related to the pair multiplicity: Chen et al. (2020) note that the duty cycle of these oscillations is related to the time it takes for the global current to deplete the charge cloud formed when the pulsar is in the active (ECS present) state. Thus, oscillation could operate on a range of timescales, from one period to many periods. Using the results of §5, we can limit the time spent in such “dead” states to less than a few percent. Moreover, the variability models for both shape and amplitude did not include the extreme case of nulling, so the constraints are even stronger. Comparison to behavior found by, e.g. Chen & Beloborodov (2014) in which the magnetosphere “breathes” at the spin period, varying the position of the Y-point, must await more quantitative predictions from simulation.

The lack of observable state changes is perhaps not surprising for Vela and Geminga, energetic pulsars whose bright \( \gamma \)-ray luminosity requires the formation of many \( e^+/e^- \) pairs. However, such changes have been observed in other pulsars over a wide range of time scales. In the most relevant case, substantial \( \gamma \)-ray pulse profile and flux changes, with correlated spin down changes, have been observed in the energetic, radio-quiet, Geminga analogue PSR J2021+4026 (Allahfort et al. 2013) with an apparent variability timescale of about 3 years. Kerr et al. (2016) found evidence for quasi-periodic modulation of the spindown rate with 1–3 year timescale in a sample of energetic, radio-loud pulsars. In the same sample of high spindown-power radio pulsars—thus likely \( \gamma \)-ray emitters—Brook et al. (2016) found additional evidence of aperiodic, year-scale correlations between radio profile variability and spindown rate. Such timescales are difficult to account for with any magnetospheric source of pulsar timing noise, and are more reminiscent of older, less energetic radio pulsars, which demonstrate long term nulling with decreased spindown rate (e.g. Kramer et al. 2006) but also less dramatic correlated spindown-rate and pulse profile changes on shorter timescales (Lyne et al. 2010).

An unusual magnetic configuration may be required for such strong observable changes. J2021+4026 may be nearly aligned and viewed near the spin equator, accounting both for its lack of radio emission and very broad (much of its emission is unpulsed) \( \gamma \)-ray profile. Further, Philippov et al. (2015b) note that more oblique pulsars have a suppressed kink instability. And interestingly, on even shorter (hour) timescales, Hermsen et al. (2013) directly observed changes in the open and closed field zones via correlated radio and X-ray emission for PSR B0943+10, which is also thought to be nearly aligned. However, the discovery of a nearly-orthogonal rotator with similar radio/X-ray correlated mode changes (Hermsen et al. 2018) precludes any simple criteria relating metastable equilibria and magnetic field alignment.

From the standpoint of simulations, much additional work is needed to make such observational features accessible. A typical PIC simulation lasts only about a few neutron star rotations, so measuring instability properties robustly—e.g. fluctuation spectra of the ECS deformation—will require a substantially larger investment of computational resources. Ceyhan Andaç et al. (2022) recently took a step in this direction, using two-dimensional PIC simulations to analyze variability in pulse flux over tens of rotations. The resulting distribution of “subpulse” fluxes may be in tension with the results reported here.

Extending this variability analysis to more \( \gamma \)-ray pulsars could reveal more rare cases like J2021+4026 and thus better connect with simulations and multiwavelength observations, but this is unlikely. Analyses on such short timescales were not envisaged at all prior to the launch of Fermi, and are only possible thanks to improvements in the event reconstruction (Atwood et al. 2013) and the long dataset. Together these yield enough pulsed pairs to provide meaningful constraints for the two brightest pulsars. And indeed additional LAT data (another \( \sim \)10 years) will improve these constraints by a few tens of per cent, which may be enough to determine if the modest evidence for Geminga pulse shape variations grows into a real detection. However, this improvement is not good enough to increase the sample of useful pulsars beyond two.

A 10× sensitivity improvement—the canonical threshold for a new experiment—would make measurements for dozens more young pulsars accessible. (Single-pulse studies of millisecond pulsars would require about 1000× improved sensitivity!) But such an observatory would require a much larger payload than the LAT, which is already fairly efficient relative to its geometric cross section, and with more complete \( \gamma \) conversion requiring
either thicker tungsten foils (degrading the PSF) or a much deeper tracker (decreasing the field-of-view). This larger instrument could be made about 30% lighter per unit area by sacrificing >10 GeV performance with a thinner calorimeter that still fully contains 1 GeV showers. For a fixed geometric area, an improved PSF would help substantially for pulsars embedded in the Galactic plane. E.g., while for Vela and Geminga, the number of pairs is "signal dominated"—more come from the pulsar than from the background—this is not at all true for PSR J1709−4429, where only about 10% of the pairs are bona fide. In this case, improving the angular resolution by 2\times (reducing the background by \sim 4\times) is about as effective as boosting the effective area by a factor of 3. Thus, a “super LAT” four times larger than LAT and with an improved PSF could meet this 10\times threshold. Such a configuration would also maintain the squat configuration preferred for a large field of view because the additional depth needed to achieve the better PSF would be offset by the larger area. Unfortunately no such instrument is under development.

The Fermi LAT Collaboration acknowledges generous ongoing support from a number of agencies and institutes that have supported both the development and the operation of the LAT as well as scientific data analysis. These include the National Aeronautics and Space Administration and the Department of Energy in the United States, the Commissariat à l’Énergie Atomique and the Centre National de la Recherche Scientifique / Institut National de Physique Nucléaire et de Physique des Particules in France, the Agenzia Spaziale Italiana and the Istituto Nazionale di Fisica Nucleare in Italy, the Ministry of Education, Culture, Sports, Science and Technology (MEXT), High Energy Accelerator Research Organization (KEK) and Japan Aerospace Exploration Agency (JAXA) in Japan, and the K. A. Wallenberg Foundation, the Swedish Research Council and the Swedish National Space Board in Sweden.

Additional support for science analysis during the operations phase is gratefully acknowledged from the Istituto Nazionale di Astrofisica in Italy and the Centre National d’Études Spatiales in France. This work performed in part under DOE Contract DE-AC02-76SF00515.

Work at NRL is supported by NASA. The author is grateful to David Smith, Jean Ballet, and Zorawar Wadiasingh for helpful suggestions, to Alice Harding for a careful review of the manuscript, and to the anonymous journal referee, whose suggestions improved this work.

Facilities: Fermi

REFERENCES
Abdo, A. A., Ackermann, M., Ajello, M., et al. 2009a, Science, 325, 840, doi: 10.1126/science.1175558
—. 2009b, Science, 325, 848, doi: 10.1126/science.1176113
Abdo, A. A., Ackermann, M., Atwood, W. B., et al. 2009c, ApJ, 696, 1084, doi: 10.1088/0004-637X/696/2/1084
Abdo, A. A., Ajello, M., Allafort, A., et al. 2013, ApJS, 208, 17, doi: 10.1088/0067-0049/208/2/17
Abdollahi, S., Acero, F., Ackermann, M., et al. 2020, ApJS, 247, 33, doi: 10.3847/1538-4365/ab6bcb
Ajello, Marco, e. 2022, TBD
Allafort, A., Baldini, L., Ballet, J., et al. 2013, ApJL, 777, L2, doi: 10.1088/2041-8205/777/1/L2
An, H., Romani, R. W., Kerr, M., & Fermi-LAT Collaboration. 2020, ApJ, 897, 52, doi: 10.3847/1538-4357/ab93ba
Arons, J. 1983, ApJ, 266, 215, doi: 10.1086/160771
Atwood, W., Albert, A., Baldini, L., et al. 2013, arXiv e-prints, arXiv:1303.3514.
Ballet, J., Burnett, T. H., Digel, S. W., & Lott, B. 2020, arXiv e-prints, arXiv:2005.11208.
https://arxiv.org/abs/2005.11208
Bickel, P., Kleijn, B., & Rice, J. 2008, ApJ, 685, 384, doi: 10.1086/590399
Brambilla, G., Kalapotharakos, C., Timokhin, A. N., Harding, A. K., & Kazanas, D. 2018, ApJ, 858, 81, doi: 10.3847/1538-4357/aab3e1
Brook, P. R., Karastergiou, A., Johnston, S., et al. 2016, MNRAS, 456, 1374, doi: 10.1093/mnras/stv2715
Bruel, P., Burnett, T. H., Digel, S. W., et al. 2018, arXiv e-prints, arXiv:1810.11394.
https://arxiv.org/abs/1810.11394
Cerutti, B., Philippov, A., Parfrey, K., & Spitkovsky, A. 2015, MNRAS, 448, 606, doi: 10.1093/mnras/stv042
Cerutti, B., Philippov, A. A., & Spitkovsky, A. 2016, MNRAS, 457, 2401, doi: 10.1093/mnras/stw124
Ceyhun Andaç, I., Cerutti, B., Dubus, G., & Ekşi, K. Y. 2022, arXiv e-prints, arXiv:2205.01942.
https://arxiv.org/abs/2205.01942
Chen, A. Y., & Beloborodov, A. M. 2014, ApJL, 795, L22, doi: 10.1088/2041-8205/795/1/L22
Chen, A. Y., Cruz, F., & Spitkovsky, A. 2020, ApJ, 889, 69, doi: 10.3847/1538-4357/ab5c20
Cheng, K. S., Ho, C., & Ruderman, M. 1986, ApJ, 300, 500, doi: 10.1086/163829
Clark, C. J., Nieder, L., Voisin, G., et al. 2021, MNRAS, 502, 915, doi: 10.1093/mnras/staa3484
Contopoulos, I., Kazanas, D., & Fendt, C. 1999, ApJ, 511, 351, doi: 10.1086/306652
Daugherty, J. K., & Harding, A. K. 1996, ApJ, 458, 278, doi: 10.1086/176811
Deutsch, A. J. 1955, Annales d’Astrophysique, 18, 1
Enoto, T., Terasawa, T., Kisaka, S., et al. 2021, Science, 372, 187, doi: 10.1126/science.abb4659
Goldreich, P., & Julian, W. H. 1969, ApJ, 157, 869, doi: 10.1086/150119
Hermsen, W., Hessels, J. W. T., Kuiper, L., et al. 2013, Science, 339, 436, doi: 10.1126/science.1230960
Hermsen, W., Kuiper, L., Basu, R., et al. 2018, MNRAS, 480, 3655, doi: 10.1093/mnras/sty2075
Hewish, A., Bell, S. J., Pilkington, J. D. H., Scott, P. F., & Collins, R. A. 1968, Nature, 217, 709, doi: 10.1038/217709a0
Kalapotharakos, C., Brambilla, G., Timokhin, A., Harding, A. K., & Kazanas, D. 2018, ApJ, 857, 44, doi: 10.3847/1538-4357/aab550
Kalapotharakos, C., & Contopoulos, I. 2009, A&A, 496, 495, doi: 10.1051/0004-6361:200810281
Kalapotharakos, C., Harding, A. K., & Kazanas, D. 2014, ApJ, 793, 97, doi: 10.1088/0004-637X/793/2/97
Kalapotharakos, C., Harding, A. K., Kazanas, D., & Brambilla, G. 2017, ApJ, 842, 80, doi: 10.3847/1538-4357/aa713a
Kalapotharakos, C., Harding, A. K., Kazanas, D., & Wadiasingh, Z. 2019, ApJL, 883, L4, doi: 10.3847/2041-8213/ab3e0a
Kerr, M. 2011, ApJ, 732, 38, doi: 10.1088/0004-637X/732/1/38
—. 2019, ApJ, 885, 92, doi: 10.3847/1538-4357/ab459f
Kerr, M., Hobbs, G., Johnston, S., & Shannon, R. M. 2016, MNRAS, 455, 1845, doi: 10.1093/mnras/stv2457
Kirk, J. G., & Giacinti, G. 2017, PhRvL, 119, 211101, doi: 10.1103/PhysRevLett.119.211101
Kramer, M., Lyne, A. G., O’Brien, J. T., Jordan, C. A., & Lorimer, D. R. 2006, Science, 312, 549, doi: 10.1126/science.1124060
Luo, J., Ransom, S., Demorest, P., et al. 2021, ApJ, 911, 45, doi: 10.3847/1538-4357/abe62f
Lyne, A., Hobbs, G., Kramer, M., Stairs, I., & Stappers, B. 2010, Science, 329, 408, doi: 10.1126/science.1186683
Pétri, J. 2012, MNRAS, 424, 2023, doi: 10.1111/j.1365-2966.2012.21350.x
Philippov, A., Uzdensky, D. A., Spitkovsky, A., & Cerutti, B. 2019, ApJL, 876, L6, doi: 10.3847/2041-8213/ab1590
Philippov, A. A., Cerutti, B., Tchekhovskoy, A., & Spitkovsky, A. 2015a, ApJL, 815, L19, doi: 10.1088/2041-8205/815/2/L19
Philippov, A. A., & Spitkovsky, A. 2018, ApJ, 855, 94, doi: 10.3847/1538-4357/aaabbc
Philippov, A. A., Spitkovsky, A., & Cerutti, B. 2015b, ApJL, 801, L19, doi: 10.1088/2041-8205/801/1/L19
Pierbattista, M., Harding, A. K., Grenier, I. A., et al. 2015, A&A, 575, A3, doi: 10.1051/0004-6361/201423815
Romani, R. W., & Yadigaroglu, I.-A. 1995, ApJ, 438, 314, doi: 10.1086/175076
Ruderman, M. A., & Sutherland, P. G. 1975, ApJ, 196, 51, doi: 10.1086/153393
Spitkovsky, A. 2006, ApJL, 648, L51, doi: 10.1086/507518
Sturrock, P. A. 1971, ApJ, 164, 529, doi: 10.1086/150865
Thompson, D. J., Bailes, M., Bertsch, D. L., et al. 1999, ApJ, 516, 297, doi: 10.1086/307083
Timokhin, A. N. 2006, MNRAS, 368, 1055, doi: 10.1111/j.1365-2966.2006.10192.x
Timokhin, A. N., & Arons, J. 2013, MNRAS, 429, 20, doi: 10.1093/mnras/sts298
Uzdensky, D. A., & Spitkovsky, A. 2014, ApJ, 780, 3, doi: 10.1088/0004-637X/780/1/3
Watters, K. P., Romani, R. W., Weltevrede, P., & Johnston, S. 2009, ApJ, 695, 1289, doi: 10.1088/0004-637X/695/2/1289