Transport in Luttinger Liquids

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Abstract

We give a brief introduction to Luttinger liquids and to the phenomena of electronic transport or conductance in quantum wires. We explain why the subject of transport in Luttinger liquids is relevant and fascinating and review some important results on tunneling through barriers in a one-dimensional quantum wire and the phenomena of persistent currents in mesoscopic rings. We give a brief description of our own work on transport through doubly-crossed Luttinger liquids and transport in the Schulz-Shastry exactly solvable Luttinger-like model.

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I am very happy to be here at the symposium held to felicitate Prof. Rajaraman, who has taught me many things in quantum field theory as well as condensed matter physics. Since Prof. Rajaraman has had contributions in various diverse fields of physics, such as nuclear physics, particle physics, formal field theory and condensed matter physics, the audience here is also varied and has representatives from all fields. Hence, I will start my talk by first giving a brief introduction to the words such as Luttinger liquid and transport in the title of my talk. I will explain why the field is both very important at the current time and at the same time theoretically fascinating. I will then give a quick review of some of the important results, before I go on to describe some work that we have done and are doing in this field.

I. Introduction

What is a Luttinger liquid?

Let us first remind ourselves of a Fermi liquid. Usual condensed matter systems deal with a collection of electrons, which are fermions. If the fermions are non-interacting, we have a Fermi gas, with single particle eigenstates, which can be filled up to the Fermi level. Excitations over the ground state are quasiparticles (above the Fermi surface) and quasiholes (below the Fermi surface), which have the same quantum numbers as that of the original electrons or holes. The idea behind Fermi liquid theory, is that interactions can change the ground state, modify the excitations and their energies and so on, but essentially, one continues to have single-particle fermion like excitations even after inclusion of the interactions. These excitations (called Landau quasiparticles) can have their masses, couplings, etc renormalised, but basically each state is in one-to-one correspondence with the non-interacting states. Such a system is called a Fermi liquid system.

The Luttinger liquid\cite{1} on the other hand, is the ground state of an interacting system which no longer has quasiparticles similar to that of the non-interacting case. Instead, it has collective excitations, which bear no resemblance to the original fermions - they are bosonic. Also, for a fermion, its charge and spin move together. In a Luttinger liquid, the charge and spin degrees of freedom move independently. At a more technical level, instead of

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a pole in the single particle propagator, even when interactions are included, as one would expect for a Fermi liquid, here one finds anomalous non-integer exponents. These anomalous exponents in various correlation functions or response functions is the hallmark of Luttinger liquid behaviour.

In three dimensions, most electronic phenomena can be understood within the framework of Fermi liquid theory. Two dimensions is still a doubtful case, where for some phenomena, it is not clear whether Fermi liquid theory is really applicable. For instance, many people believe that high $T_c$ superconductivity needs non-Fermi liquid behaviour. But in one dimension, it is well-known that Fermi liquid theory breaks down and hence the relevance of Luttinger liquid theory has been understood for quite a while.

**Transport**

Transport is an important concept in condensed matter physics. Here, by transport, we mean electronic transport or conductance. We apply a voltage across a wire and measure the current through it. This gives us the conductance. The aim is to compute the conductance as a function of the voltage, temperature, presence of impurities or disorder and so on. Normally, when currents are measured in wires, one does not worry about quantum effects, because wires are still macroscopic objects. But here, we shall be talking about one-dimensional ‘mesoscopic’ wires, so quantum effects will be important. In fact, whenever the physical dimensions of the conductor becomes small, (it need not be really one-dimensional), the usual Ohmic picture of conductance where the conductance is given by

$$G = \sigma \frac{W}{L} = \sigma \frac{\text{width of conductor}}{\text{length of conductor}}$$

where $\sigma$ is a material dependent quantity, breaks down. A whole new field called ‘mesoscopic physics’ has now been created to deal with electronic transport in such systems. The term ‘mesoscopic’ in between microscopic and macroscopic is used for systems, where the sizes of the devices are such that it is comparable with a) the de Broglie wavelength (or kinetic energy) of the electron, b) the mean free path of the electron and c) the phase relaxation length (the length over which the particle loses memory of its phase) of the electron. For a macroscopic object, the size is much larger than any of these lengths. These lengths actually vary greatly depending on the material and
also on the temperature. Typically, at low temperatures, they vary between a nanometer for metals to a micrometer for quantum Hall systems.

For mesoscopic wires, in general, quantum effects need to be taken into account. The conductances are computed using the usual quantum mechanical formulation of transmission and reflection through impurities. This formulation is called the Landauer-Buttiker formulation and works for Fermi liquids. However, when we really go to one dimensional wires, interactions change the picture dramatically, since the quasi-particles are no longer fermion-like. Hence the Landauer-Buttiker formalism cannot be applied and one needs to compute conductances in Luttinger wires taking interactions into account right from the beginning. We shall review the theoretical results of transport in Luttinger wires after giving a brief motivation as to why the study of transport through Luttinger wires is interesting.

**Motivation**

The main motivation in this field is that recently, advances in nanotechnology, and the discovery of new one-dimensional materials such as carbon nanotubes have enabled the fabrication of extremely narrow wires. Experiments of fundamental theoretical importance have been performed on these wires such as those that look for coherent scattering and measure the phase of the transport of the electron through barriers in these wires. One can also try to look for Luttinger liquid behaviour in these wires by measuring their transport properties. However, it is hard to observe Luttinger liquid behaviour because any residual disorder or any deviation from one dimensionality, obscures the power laws which are characteristic of Luttinger liquids.

Some of the experiments looking for Luttinger liquid behaviour include

- Experiments on semi-conductor wires.
  These are quantum wires because they are of mesoscopic sizes and are at low temperatures. But it not yet clear whether Luttinger liquid behaviour has been seen in these wires or whether the experiments can be explained by Fermi liquid theory.

- Tunneling into edge states in the Fractional Quantum Hall effect.
  Here, Luttinger liquid behaviour has actually been seen. For Fermi liquid behaviour (which is seen for the $\nu = 1$ state), $I \propto V$. But for
Luttinger liquids, the exponent changes. It was theoretically predicted to be $I/V \sim V^2$ and $G \sim T^2$ for $\nu = 1/3$ state, and experimentally found to be $I/V \sim V^{1.7\pm0.06}$ and $G \sim T^{1.75\pm0.08}$, where $G$ is the conductance as a function of the temperature.

- Experiments on single carbon nano-tubes.

Transport along a single carbon nano-tube has been experimentally measured and the results have been similar to those for semi-conductor wires.

There have also been predictions that armchair nanotubes form a Luttinger liquid and the appropriate power laws for various conductances have been calculated. But the main point that I wish to emphasize is that at the moment, several experiments are being done on 1-D systems. Since, in 1-D systems, interactions exist and change the physics drastically, it becomes important to take it into account. Theory can hence lead to predictions which can be immediately tested. At a deeper level, the field involves a fascinating interplay of concepts from strong correlations, impurities and disorder as well as mesoscopic systems.

**Review of Important Results**

I will now review a couple of important results in the field which are required to explain my work.

**Persistent currents**

In the presence of an external magnetic field, it is possible to have persistent currents in small metal rings. This is a very simple quantum mechanical phenomena, which can be understood on the basis of the Aharanov-Bohm effect. The idea is that we have a small metal ring of circumference $L$ and thread a magnetic flux through it. Hence, the wave-function of an electron that goes along the ring, picks up a phase $\psi(x + L) = e^{2i\pi\phi/\phi_0}\psi(x)$ after it completes a circuit. Since the sign of the phase is different for left-movers and right-movers, this breaks the degeneracy between them on the ring. Thus, for a given chemical potential, there are more rightmovers than left (or vice-versa), which leads to a current. One can compute the current by adding up the contribution of all the levels below the Fermi level and plot it against the flux to get the sawtooth picture depicted in Fig.(1). The periodicity in
\[ \phi/\phi_0 \text{ is clearly because the flux } \phi_0 \text{ where } \phi_0 = hc/e \text{ is the unit of flux is indistinguishable from no flux.} \]

\[ \Phi/\Phi_0 \]

\[ I/I_0 \]

Fig 1. Current versus flux through the ring. The periodicity for \[ \phi = n\phi_0 \] is clearly visible.

**Kane-Fisher results**

Kane and Fisher in a pathbreaking paper in 1992, showed how interactions in a one-dimensional system, changed transmission through barriers dramatically. We shall describe the idea behind their work in some detail here.

The simplest Luttinger liquid model can be described by two bosons, one for the charge and one for the spin degree of freedom. The action (in one space, one time dimension) is given by

\[ S = \int dx d\tau \left( \frac{v_\rho g_\rho}{2} [ (\nabla \phi_\rho)^2 + \frac{1}{v_\rho^2} (\partial_\tau \phi_\rho)^2 ] + \frac{v_\sigma g_\sigma}{2} [ (\nabla \phi_\sigma)^2 + \frac{1}{v_\sigma^2} (\partial_\tau \phi_\sigma)^2 ] \right) \]  

(2)

where the field \( \phi_\rho \) and \( \phi_\sigma \) denote the charge and spin degrees of freedom respectively and the velocities of the two fields can be different, since spin and charge degrees of freedom decouple in a Luttinger liquid. \( g_\rho \) and \( g_\sigma \) are parameters that are a measure of the strengths of interaction of the original fermions. Equivalently, the \( g \)-parameters can be identified with the radii of compactification \( R \) of the free bosons of a \( c = 1 \) conformal field theory, with the free fermion point being identified with a particular value of \( R \). The point to note is that these parameters do not denote interactions in the bosonic
model. \( g_\rho = 2 \) (\( g_\rho = 1 \) for spinless fermions) denotes the free fermion point for the charge degree of freedom. Generically, unless there is a magnetic field, \( g_\sigma = 2 \) in order to respect the \( SU(2) \) symmetry.

So now, we have a one-dimensional wire made up of Luttinger bosons. What happens when we put a voltage across the wire and measure the current? Initially, Kane and Fisher claimed that the conductance depended on the Luttinger parameter and was given by \( I/V = g_\rho e^2/h \), but now, it is generally accepted\[8\] that for a pure Luttinger wire, \( I/V = e^2/h \), just as it is for a non-interacting Fermi liquid wire. The reason for this is that if the wire has no impurity, then its only resistance comes from the contacts at the leads, - the one-dimensional wire is connected to three-dimensional Fermi liquid leads at the two ends of the wire. Since the resistance comes only from the contacts, it does not matter whether the fermions in the wire are interacting or not. This explains why the conductance is the same as that for non-interacting fermions.

Now, we may ask: ‘What happens if we introduce an impurity?’ (The impurity may be a barrier, a constriction or a localised impurity.) The impurity is modelled by a potential \( V(x) \) at or around the origin, so that the Hamiltonian (for spinless fermions) is modified by

\[
\delta H = \int dx V(x) \psi(x) \psi^\dagger(x) \quad (3)
\]

in the weak barrier limit. In the strong barrier limit, it is more appropriate to think of two independent wires to the left and right of the origin - i.e., a wire which is cut at the origin - and then allow a small hopping, given by

\[
\delta H = -t[\psi_+^\dagger(x = 0) \psi_-(x = 0) + h.c.]. \quad (4)
\]

In either case, we can bosonise these terms and use perturbation theory in \( V \) or \( t \) to obtain the renormalisation group equations given by

\[
\frac{dV}{dl} = (1 - g)V \quad \text{or} \quad \frac{dt}{dl} = (1 - \frac{1}{g})t. \quad (5)
\]

Hence, for \( g < 1 \), the barrier term is relevant and grows (and from the other limit, the hopping term is irrelevant and becomes weak). So for \( g < 1 \), either from the weak coupling or the strong coupling side, the result is that the impurity ‘cuts’ the wire and there is no transmission. For \( g > 1 \), on the
other hand, $V$ decreases and $t$ increases. In other words, even if we start with a barrier, it vanishes under RG and transmission becomes perfect — i.e., the wire is ‘healed’.

$g < 1$ corresponds to repulsive interactions in the original fermionic model, whereas $g > 1$ corresponds attractive interactions. $g = 1$ is the free fermion limit for spinless fermions, where both the perturbing operators, either from the weak barrier or strong barrier side, are marginal. Here, as we know from usual one-dimensional quantum mechanics, one can have both transmission and reflection. Similar results can also be found for electrons with spin.

The next thing is to study transmission through two barriers. For $g < 1$, one may expect that since even one barrier cuts the wire, there is no chance of transmission. However, surprisingly, it is still possible to have resonant transmission. The idea is that one can have quasi-bound states between the barriers, which will correspond to the energies for resonant transmission. For weak barriers, this happens at the energies at which backscattering from both sides can be tuned to be zero. For strong barriers, this happens when the energy on the island between the two barriers is degenerate for two states, which is again arranged by tuning the chemical potential on the island, so that the energy to add another electron is zero. Hence, one finds resonances as a function of the gate voltage.

This is somewhat reminiscent of what is called Coulomb blockade physics for non-interacting electrons. Even for non-interacting electrons, the mesoscopic length scale of the island, implies that it has a small capacitance - it can only hold so much charge. To add another electron to the island costs Coulomb charging energy $e^2/C$, where $C$ is the capacitance of the island. (Note that this is different from the interaction represented by the Luttinger parameter $g < 1$, which is only a measure of the short-range part of the repulsive interaction between electrons.) If the charging energy can be neutralised by changing the chemical potential on the island by a gate voltage, then there is no energy required to add another electron and one can get resonant transmission. This can only happen for specific values of the gate voltage. At other values of the voltage, there is no transmission because the Coulomb energy blocks the passage of the electron. Hence, one gets peaks in the conductance at particular values of the gate voltage or equivalently, one gets plateaux and jumps in the graph of the current versus the gate voltage, which is called the Coulomb staircase. This is depicted in Fig. (2).
Fig 2. Current $I$ versus voltage $V$ and conductance $G$ versus $V$ at zero temperature for a Luttinger liquid wire with two barriers. Note the sharpness of the jumps, which is the feature that sets it apart from the analogous Coulomb blockade for non-interacting electrons.

However, the physics which causes the resonances in the interacting model is analogous but not identical to the Coulomb blockade physics. Unlike for the non-interacting case, where there is a finite width to the resonances, for the Luttinger liquid model, the resonances become infinitely sharp at zero temperature.

Our work

Here, we will report briefly on two pieces of work, which uses some of the results that we have reviewed above. One of them is on transmission through a particular geometry of Luttinger liquid wires, which is interesting - that of doubly crossed Luttinger wires\cite{9}. The second, which is still incomplete, is on some exact results in a toy model of Luttinger liquid\cite{10}.

Double-crossed Luttinger wires

One motivation for studying crossed Luttinger wires was that in the standard two Luttinger chain problem, with couplings all along the wire between the two chains, the coupling was relevant and led to a flow away from the Luttinger liquid fixed point of a single chain. So our aim was to try and include couplings between chains at several points and see whether Luttinger liquid behaviour is destroyed. But interestingly, point-like couplings even for one and two points lead to unusual transport features. Even for just a single crossing of two Luttinger liquids, it was found\cite{11} that the current in one
wire was extremely sensitive to the voltage drop across the other wire. Here, we study two Luttinger liquid wires coupled at two points and connected to external constant voltage sources. The aim was to see whether one can tune resonant transmission in both wires by applying gate voltages. Our conclusion was that in the limit when the external biases tended to zero, a single gate voltage was sufficient to tune for resonances in both wires.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3.png}
\caption{Two Luttinger liquids coupled together at two points ($x = -d$ and $x = +d$) and connected to external reservoirs held at constant voltages $U_1^1, U_1^2$ on the left and $U_2^1, U_2^2$ on the right.}
\end{figure}

Now, let us see how one gets this result in a little more detail. We start with spinless fermions, (spin is an added complication, which can be incorporated at a later stage), bosonise them and describe the Luttinger liquid as

$$H = \frac{\hbar v_F}{2g} \sum_{A=1}^2 \int dx [g(\partial_x \phi_A)^2 + g^{-1}(\partial_x \theta_A)^2].$$

(6)

The external voltage biases are incorporated as boundary conditions on the boson fields at $-L$ and $+L$, where $L$ is the length of both the wires. At the two coupling points, we allow density-density couplings and single particle tunnelings. In fact, for repulsive interactions, single particle tunnelings just renormalise the density-density couplings, so we only need to introduce the interaction term given by

$$V_{\text{den}} = \lambda_1 \rho_1^1(-d)\rho_2^2(-d) + \lambda_2 \rho_1^1(+d)\rho_2^2(+d).$$

(7)
With standard normalisation, these operators have scaling dimension $2g$. Since for a bulk operator, relevance or irrelevance depends on whether $g$ is less than or greater than one, we see that for $1/2 < g < 1$, these couplings are irrelevant and there is perfect transmission in both the wires -

$$I^A = \frac{e^2(U_1^A - U_2^A)}{h} \equiv \frac{e^2 U^A}{h}. \quad (8)$$

But for $1/2 < g < 1$, the operators are relevant. However, in this case, we find that the model can be mapped to decoupled wires as shown in Fig.(4).

\[ \begin{align*}
+ & \text{ Wire} \\
& U_1^+ U_1^- \\
& U_2^+ U_2^- \\
\end{align*} \]

\[ \begin{align*}
- & \text{ Wire} \\
& U_1^- U_1^+ \\
& U_2^- U_2^+ \\
\end{align*} \]

- **Fig 4.** Two decoupled wires with barriers at $(x = -d$ and $x = +d)$. The values of the external voltages have changed.

The external biases have changed so that the potential drops are now $U_1^+ + U_2^+$ in the $+$ wire and $U_1^- - U_2^-$ in the $-$ wire. The coupling constant $g \rightarrow \tilde{g} = 2g$ and hence the dimensions of the barrier operators or the RG equations for the barriers have changed, but now the problem is easy to analyse because it maps exactly into two copies of the Kane-Fisher problem. Hence, we can directly take over their results. Their resonance condition for the two wires (with our changed parameters) is given by

$$4eg^2 \Delta \phi_G^\pm = \pi h v_F / 2d \quad (9)$$

where $\Delta \phi_G^\pm$ is the spacing of the gate voltages. In the Kane and Fisher analysis, they did not worry about the charging of the barriers. This was
later included by other workers in the field. If we include those effects as well, we get

\[ 4e g^2 \Delta \phi_G^\pm = \left[ \frac{2d}{\pi \hbar v_F} + \frac{2C^\pm (2g)}{e^2} \right]^{-1} \]  

where \( C^\pm \) is the capacitance of the barriers. The inclusion of the barrier capacitance decreases the spacing of the gate voltages where we get resonances. One can understand this as follows. The charging of the barriers increases the island’s capacity to hold charge. Since the spacing of the gate voltage is inversely proportional to the capacitance of the island, this decreases the spacing and we get more resonances within a given range of the gate voltage. For strong barriers, the capacitances have no dependence on the external biases, and \( C^+ = C^- \). So since the lengths of the two wires between the barriers are also the same, \( (d^+ = d^-) \), the + and - wires satisfy the same condition for resonance and \( \Delta \phi_G^+ = \phi = \Delta \phi_G^- \).

In terms of the original wires 1 and 2, resonance implies the condition that the current in each of the wires before the crossing is equal to that after the crossing - \( i.e., I_1 = I_1' \) and \( I_2 = I_2' \). We can consider two cases. Let us first consider the case when one of the wires, say wire 2 is unbiased and \( I_2 = I_2' = 0 \). We want to look for when there is resonant tunneling through wire 1. We see that when the resonance condition is satisfied in both the + and - wires, we get \( I^+ = I^- \), which in turn gives us \( I_1 = I_1' \), since there is no current in wire 2. We can also consider the case when one of the decoupled wires is unbiased, for instance, the - wire is unbiased. In that case, we have the same current flowing through both the wires - \( i.e., I_1 = I_1' = I_2 = I_2' \) when \( \Delta \phi_G = \Delta \phi_G^1 + \Delta \phi_G^2 = 2\phi \) is a constant. In this case, resonant tunneling takes place through both wires. Note that even in case 1 where tunneling only occurs in one wire, the situation is still different from that of two originally decoupled wires, because the resonance condition has changed from that of a single wire.

In general, without resonant transmission, \( I_1 \neq I_1' \) and \( I_2 \neq I_2' \). So one needs four current probes to measure the current characteristics, in terms of a four by four matrix.

**Transport in an exactly solvable toy model of Luttinger liquid**

Recently, a toy model of a Luttinger liquid was proposed by Schulz and Shastry\[13\]. It is a model with two species of fermions with a pseudospin
index $\sigma = \pm$, and a Hamiltonian given by

$$H = \sum_{i\sigma}(p_{\sigma i} + \sigma A_{\sigma}(x_{-\sigma i}))^2$$

(11)

where $A_{\sigma}$ is a gauge potential which for a positive pseudospin particle depends on all the negative pseudospin particles and vice-versa - i.e.,

$$A_{\sigma}(x) = \sum_j V(x - x_{-\sigma j}).$$

(12)

The mainpoint is that since the potential depends on the total number of particles of the opposite pseudospin, every time a $+$ particle is added, the energy levels of all the $-$ particles change and vice versa. This model is easy to solve because one can make a gauge transformation to remove the gauge field so that

$$H \rightarrow \sum_{\sigma i} p_{\sigma i}^2$$

at the expense of changing the boundary conditions on the wave-functions. So if we take the particles to be on a ring, instead of quantising

$$k_i = \frac{2\pi n_i}{L},$$

the changed boundary conditions lead to the changed quantisation condition given by

$$k_{\pm i} = \frac{2\pi}{L}(n_{\pm i} \pm \frac{N_{\pm} \delta}{2\pi})$$

(13)

where $N_{\mp}$ is the total number of particles of the $\mp$ in the wire. Clearly the Hilbert space of states for $N_{\mp} \delta/2\pi = \text{fractional}$ is different from that of a non-interacting model with $N_{\mp} \delta/2\pi = \text{integer}$. Hence, even though, it ‘looks’ like a non-interacting theory, as far as the Hamiltonian is concerned, the changed boundary conditions incorporate the interactions of the model that existed before gauge transformation. In the original paper, they computed the correlation functions in this model and showed that they could get fractional exponents, which is the hallmark of Luttinger liquids. With the motivation of studying coupled chains of Luttinger liquids, we tried to generalise this model. However, the natural generalisation led to a model which was more like a multi-band single chain model, which we analysed[14] and obtained correlation functions.

Currently, we are studying transport in this model[10]. Since interactions in this model can only be introduced through a change in boundary conditions and consequently quantisation conditions, we study the model on a ring.
Hence, the driving force is a flux through the ring rather than external voltage sources as for an open wire. Like for free fermions, we expect to get persistent currents. On explicitly introducing barriers (potentials), we expect to see results similar to those in the Kane-Fisher model. Repulsive interactions will cut the wire and attractive interactions will heal the wire. However, here, since mesoscopic length scales are involved, the cutting and healing may not be perfect.

In our opinion, the importance of this model lies in the fact that even in the original fermion language, the model is almost free, with interactions only being introduced through quantisation conditions. Hence, it should be possible to get results for the model and consequently for a Luttinger liquid, without going through bosonisation. We are hence, trying to see whether we can reproduce the Kane-Fisher results on the ring, without going through bosonisation in this model.

**Conclusions**

In conclusion, I would like to emphasize that the field of transport in Luttinger liquids is a highly relevant field at the moment, because a lot of experiments are likely to be performed in the near future on wires operating in the single channel limit, on carbon nanotubes, etc. Hence, the Landauer-Buttiker formalism for mesoscopic wires needs to be redone for these strongly interacting electrons or for the Luttinger bosons.

There are other interesting phenomena in this general area, which we have not touched upon in this talk. For instance, inclusion of spin will lead to the formation of Kondo resonances. Inclusion of AC voltages can lead to novel phenomena. New materials are constantly being made, which could have new physics. Examples are the amchair carbon nanotubes, and the chiral nanotubes.

Hence, both at the theoretical level and at the experimental level, we expect the field to expand considerably in the near future.

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