Investigation of \(P_{cs}(4459)^0\) pentaquark via its strong decay to \(\Lambda J/\Psi\)

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Recently the observation of a new pentaquark state, the hidden-charmed strange \(P_{cs}(4459)^0\), was reported by the LHCb Collaboration. The spin-parity quantum numbers of this state were not determined as a result of insufficient statistics. To shed light on its quantum numbers, we investigate its decay, \(P_{cs}(4459)^0 \rightarrow J/\Psi\Lambda\), the mode that this state has been observed, within the QCD sum rule framework. We obtain the width of this decay assigning the spin-parity quantum numbers of \(P_{cs}(4459)^0\) state as \(J^P = \frac{3}{2}^+\) and its substructure as diquark-diquark-antiquark. To this end, we first calculate the strong coupling constants defining the considered decay and then use them in the width calculations. The obtained width is consistent with the experimental observation, confirming the quantum numbers \(J^P = \frac{3}{2}^+\) and compact pentaquark nature for \(P_{cs}(4459)^0\) state.

I. INTRODUCTION

In the past two decades, starting with the observation of many exotic hadrons candidates for tetraquarks [2] and pentaquarks [3–5]. The first observation of pentaquark states was announced in 2015 by the LHCb collaboration [3] and two pentaquark states in \(J/\psi p\) invariant mass spectrum of the \(\Lambda_b^0 \rightarrow J/\psi pK^-\) decays were reported with the following resonance parameters [3]: \(m_{P_c(4380)}^+ = 4380 \pm 8 \pm 29\ MeV\), \(\Gamma_{P_c(4380)}^+ = 205 \pm 18 \pm 86\ MeV\) and \(m_{P_c(4450)}^+ = 4449.8 \pm 1.7 \pm 2.5\ MeV\), \(\Gamma_{P_c(4450)}^+ = 39 \pm 5 \pm 19\ MeV\). The LHCb Collaboration supported this observation later, in 2016, with a full amplitude analysis for \(\Lambda_b^0 \rightarrow J/\psi p\pi^-\) decays [4]. In 2019, a new pentaquark resonance, \(P_c(4312)^\mp\), was reported by the LHCb Collaboration with the following mass and width [5]: \(m_{P_c(4312)}^\mp = 4311.9 \pm 0.7 \pm 6.8\ MeV\) and \(\Gamma_{P_c(4312)}^\mp = 9.8 \pm 2.7 \pm 5.5\ MeV\). Together with the \(P_c(4312)^\mp\) state, the LHCb also announced the split of the peak corresponding to \(P_c(4450)^\pm\) into two peaks which have the following masses and widths: \(m_{P_c(4440)}^\pm = 4440.3 \pm 1.3 \pm 4.1\ MeV\), \(\Gamma_{P_c(4440)}^\pm = 20.6 \pm 4.9 \pm 5.7\ MeV\) and \(m_{P_c(4457)}^\pm = 4457.3 \pm 0.6 \pm 1.1\ MeV\), \(\Gamma_{P_c(4457)}^\pm = 6.4 \pm 2.0 \pm 5.7\ MeV\). These observations and the advances in experimental facilities and techniques indicate the possibility to observe more exotic states in the future.

On the other hand, there is still uncertainties in the sub-structures and quantum numbers of these observed pentaquark states. In that matter, there are different proposals and theoretical works about these resonances in the literature analyzing their parameters and giving consistent predictions with their observed properties. It is obvious that deeper investigations are required not only to differentiate these proposals but also to help better identify the nature of these states. Understanding the inner structures and properties of these exotic states may also support their future investigations. Besides, they may provide improvements in understanding the dynamics of the quantum chromodynamics (QCD) in its nonperturbative domain. With their non-conventional quark substructures that are different from the conventional baryons composed of three quarks/antiquarks or mesons composed of a quark and an antiquark, they provide an attractive ground for the understanding of the nonperturbative nature of strong interaction. Although the investigations of such exotic states extend before their observations, with their observations the pentaquarks have become a hot topic in all these respects. With these motivations and the excitement brought by their observations, their various properties were investigated widely with different approaches to shed light on their nonspecific sub-structures and quantum numbers. Based on their close masses to meson-baryon threshold, they were assigned as meson baryon molecular states in Refs. [6–14]. They were interpreted with diquark-diquark-antiquark [15–28] and diquark-triquark [27, 29] models. To investigate their properties in Ref. [30] a variant of the D4-D8 model and

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in Ref. [31] the topological soliton model were used. They were also explained as kinematical effects [32–36]. Besides the observed ones, the possible other candidate pentaquark states were also considered in the literature with different quark contents [37–51].

Recently, in a talk, implications of LHCb measurements and future prospects, the evidence for a pentaquark including a strange quark in its quark content was first announced by the LHCb Collaboration [52] and later it was reported in the Ref. [53]. The $P_{cs}(4459)^0$ was observed in $\Xi_c^0 \to J/\psi K^- \Lambda$ decays with the following measured mass and width [53]:

$$M = 4458.8 \pm 2.9^{+4.7}_{-1.1} \text{ MeV}, \quad \Gamma = 17.3 \pm 6.5^{+8.0}_{-5.7} \text{ MeV},$$  \hspace{1cm} (1)

with statistical significance exceeding 3$\sigma$ and there is no determination for its spin parity quantum numbers, yet. With a mass just below $\Sigma_c D^*$ threshold, the $P_{cs}(4459)^0$ was interpreted as $D^* \Sigma_c$ hadronic molecular state in Ref. [54]. The analyses were conducted using QCD sum rule method and the results supported its possibility to be $D^* \Sigma_c$ molecular state with either $J^P = \frac{1}{2}^-$ or $J^P = \frac{3}{2}^-$ giving mass values consistent with the experimentally reported one [54].

Molecular explanation for the $P_{cs}(4459)^0$ was also discussed in Ref. [55] using effective field formalism and the masses were predicted considering the $D^* \Sigma_c$ molecular pictures with $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$ as 4469 MeV and 4453 – 4463 MeV, respectively. With these results the spin of the $P_{cs}(4459)^0$ state was suggested to possibly be $J^P = \frac{3}{2}^-$. In Ref. [56] molecular interpretation was taken into account using one-boson-exchange model and $P_{cs}(4459)^0$ was interpreted as a coupled $\Xi_c D^*/\Xi_c^* D/\Xi_c^* D^*/\Xi_c^* D^*$ bound state that has $I(J^P) = 0(3^-)$. In Ref. [57] the mass analysis was made via QCD sum rule approach for a pentaquark state containing strange quark with an interpolating current in the scalar-diquark-scalar-diquark-antiquark form. Based on the mass value obtained for the state as $M = 4.47 \pm 0.11$ MeV, which was consistent with the experimentally observed one, $P_{cs}(4459)^0$ was assigned to have the quantum numbers $J^P = \frac{3}{2}^-$. As is seen, the quantum numbers for the $P_{cs}(4459)^0$ state were not determined by the experiment, and from different studies there are different assumptions for its quantum numbers and sub-structure, indicating the necessity for further investigations of the properties of this state. Inspired by this, we investigate the $P_{cs}(4459)^0$ state through its strong decay via QCD sum rule method [58–60]. This method has a wide range of applications in the literature, which resulted in successful predictions consistent with the experimental observations. To provide the width, we first calculate the strong coupling constants defining the decay $P_{cs}(4459)^0 \to J/\psi \Lambda$ using three-point QCD sum rule approach with an interpolating current in the scalar-diquark-scalar-diquark-antiquark form of $J^P = \frac{3}{2}^-$. Then, the obtained results for the strong coupling constants are used to determine the corresponding width value. We compare the obtained result with the experimental observation to shed light on the quantum numbers and quark sub-structure of the considered state.

The organization of the paper is as follows: In next section we give the details of the QCD sum rule calculations for the strong coupling constants defining the $P_{cs}(4459)^0 \to J/\psi \Lambda$ decay. The numerical analyses of the obtained sum rules as well as the width of the considered decay are also presented in Sec. II. Last section is devoted to a summary and comparison of the obtained result for the width to that of the experiment.

II. THE STRONG DECAY $P_{cs}(4459)^0 \to J/\psi \Lambda$

In this section the details of the calculations for the strong coupling constants and the width of the strong decay $P_{cs}(4459)^0 \to J/\psi \Lambda$ and their numerical analyses are given. The correlation function required for the calculations has the following form:

$$\Pi_{\mu}(p, q) = i^2 \int d^4xe^{-ipx} \int d^4ye^{ip'y} \langle 0 | T \{ \eta^\Lambda(y) \eta^{J/\psi}_{\mu}(0) \eta^{P_{cs}}(x) \} | 0 \rangle,$$  \hspace{1cm} (2)

where the $\eta^{P_{cs}}$, $\eta^\Lambda$ and $\eta^{J/\psi}$ are the interpolating currents of the considered states which have the same quantum numbers with these states and $T$ is used to represent the time ordering operator. The interpolating currents are given as:

$$\eta^{P_{cs}} = \epsilon^{ia} \epsilon^{ijk} \epsilon^{lmn} u_j^T C \gamma_5 d_k^c s_m^l C \gamma_5 c_n C \gamma_5^T,$$

$$\eta^\Lambda = \frac{1}{\sqrt{6}} \epsilon^{lmn} \sum_{i=1}^{2} \left[ 2(u_i^T C A_1 d_m) A_2^\ast s_n + (u_i^T C A_1's_m) A_2^\ast d_n + (d_n^T C A_1's_m) A_2^\ast u_1 \right],$$

$$\eta^{J/\psi}_{\mu} = \bar{c}_i \gamma_{\mu} c_i,$$  \hspace{1cm} (3)
where the sub-indices, $a, i, j, k, l, m, n$ represent the color indices $u, d, s, c$ are the quark fields, $C$ is charge conjugation operator; and $A^1_I = I, A^2_I = A^1_I = \gamma_5$ and $A^2_2 = \beta$ is an arbitrary parameter to be determined from the analyses. The above correlation function is calculated in two representations which are called hadronic and QCD representations. The QCD sum rules for the physical quantities are obtained from the matches of the coefficients of the same Lorentz structures attained on both sides.

In the hadronic representation of the correlation function, the interpolating currents are treated as operators creating or annihilating the hadronic states. To proceed in the calculation of this side, complete sets of related hadronic states that have the same quantum numbers with the given interpolating currents are inserted inside the correlator. After taking four integrals the results turn into

$$
\Pi^\text{Had}_\mu(p, q) = \frac{(0|\eta^\Lambda|\Lambda(p', s'))(0)_{\mu}\bar{\psi}|\psi(q)\rangle\langle J/\psi(q)|\Lambda(p', s')|P_{cs}(p, s)\rangle\langle P_{cs}(p, s)|\eta^{P_{cs}}|0\rangle}{(m^2_{\Lambda} - p^2)(m^2_{J/\psi} - q^2)(m^2_{P_{cs}} - p^2)} + \cdots. \tag{4}
$$

where $\cdots$ is used to represent the contributions of higher states and continuum, the $p, p'$ and $q$ are the momenta of the $P_{cs}$ and $\Lambda$ and $J/\psi$ states, respectively. The matrix elements in this result are defined in terms of the masses and current coupling constants, and they have the following forms:

$$
\begin{align*}
(0|\eta^{P_{cs}}|P_{cs}(p, s)) &= \lambda_{P_{cs}}u_{P_{cs}}(p, s), \\
(0|\eta^{\Lambda}|\Lambda(p', s')) &= \lambda_{\Lambda}u_{\Lambda}(p', s'), \\
(0)_{\mu}\bar{\psi}|\psi(q)\rangle &= f_{J/\psi}m_{J/\psi}\varepsilon_\mu,
\end{align*}
$$

where $\varepsilon_\mu$ is the polarization vector and $f_{J/\psi}$ is the decay constant of the $J/\psi$ state, $\lambda_{P_{cs}}, \lambda_{\Lambda}$ are the current coupling constants of the $P_{cs}$ and $\Lambda$ states, $u_{P_{cs}}$ and $u_{\Lambda}$ are the corresponding spinors, respectively. $|P_{cs}(p, s)\rangle$ is used to represent one-particle pentaquark state with negative parity. The matrix element $\langle J/\psi(q)|\Lambda(p', s')|P_{cs}(p, s)\rangle$ is given in terms of the coupling constants, $g_1$ and $g_2$ as

$$
\langle J/\psi(q)|\Lambda(p', s')|P_{cs}(p, s)\rangle = \epsilon^{\nu}u_{\Lambda}(p', s')\left[ g_1\gamma_\nu - \frac{i\sigma_{\nu\alpha}}{m_\Lambda + m_{P_{cs}}}q^\alpha g_2 \right] \gamma_5 u_{P_{cs}}(p, s). \tag{6}
$$

In the next step, the matrix elements given in Eqs. (5) and (6) are placed in the Eq. (4) and following summations over spins of spinors and polarization vectors are applied

$$
\begin{align*}
\sum_s u_{P_{cs}}(p, s)\bar{u}_{P_{cs}}(p, s) &= (\not{p} + m_{P_{cs}}), \\
\sum_{s'} u_{\Lambda}(p', s')\bar{u}_{\Lambda}(p', s') &= (\not{p'} + m_{\Lambda}), \\
\varepsilon_\alpha\varepsilon^*_\beta &= -g_{\alpha\beta} + \frac{q_\alpha q^\beta}{m^2_{J/\psi}}, \tag{7}
\end{align*}
$$

And finally, after the Borel transformation, which is applied to suppress the contributions coming from higher states and continuum, the result of physical side is obtained as

$$
\begin{align*}
\tilde{\Pi}^\text{Had}_\mu(p, q) &= e^{-\frac{m^2_{P_{cs}}}{M^2}}e^{-\frac{m^2_{\Lambda}}{M^2}}\frac{f_{J/\psi}\lambda\lambda\lambda\lambda m_{\Lambda}}{m_{J/\psi}(m_\Lambda + m_{P_{cs}})\left( m_{J/\psi}^2 + Q^2 \right)} \left[ -g_1(m_\Lambda + m_{P_{cs}})^2 + g_2m^2_{J/\psi} \right] \not{p}\not{p}\gamma_5 \\
&+ e^{-\frac{m^2_{P_{cs}}}{M^2}}e^{-\frac{m^2_{\Lambda}}{M^2}}\frac{f_{J/\psi}\lambda\lambda\lambda\lambda m_{J/\psi}m_{\Lambda}}{m_\Lambda + m_{P_{cs}}} \left[ m^2_{J/\psi} + Q^2 \right] \left[ g_1(m_\Lambda + m_{P_{cs}}) + g_2(m_\Lambda - m_{P_{cs}}) \right] \not{p}\not{p}\gamma_5 \\
&+ \text{other structures} + \cdots. \tag{8}
\end{align*}
$$

where $M^2$ and $M'^2$ are the Borel parameters to be determined from the analyses imposing some necessary criteria and $Q^2 = -q^2$. The result contains more Lorentz structures than the ones given explicitly in Eq. (8). However, in the last equation, we present only the ones that are used directly in the analyses, and the others and the contribution of the excited states and continuum are represented as other structures $+ \cdots$.

The second representation of the correlation function is obtained using the interpolating currents explicitly in the correlation function. To this end, the possible contractions between the quark fields are attained using Wick’s theorem.
that renders the result to the one given in terms of heavy and light quark propagators as:

$$
\Pi^{OPE}_\mu(p, p', q) = i^2 \int d^4x e^{-i p x} \int d^4y e^{i p' y} \epsilon^{k l m n} \epsilon^{i j l m} \epsilon^{i' j' k' l'} 1 \sqrt{6} \sum_{i=1}^{2} \left\{ -2Tr[\gamma_5 C S^{k l}_{U}] (y - x) CA_i S^{k l}_{U} (y - x) \right\} \\
\times A_2 S^{m n}_{v} (y - x) C \gamma_\mu C S^{m n}_{c} (y - x) C + A_2 S^{m n}_{s} (y - x) \gamma_5 C S^{m n}_{c} (y - x) C + A_2 S^{m n}_{s} (y - x) \gamma_5 C S^{m n}_{c} (y - x) C \\
\times \gamma_5 C S^{m n}_{c} (y - x) C \gamma_\mu C S^{m n}_{c} (y - x) C \\
\times S^{m n}_{c} (y - x) C \gamma_\mu C S^{m n}_{c} (y - x) C, \\
\tag{9}
$$

where $S^{ab}_q(x) = S^{ab}_{u,d,s}(x)$ and $S^{ab}_c(x)$ are the light and heavy quark propagators with the following explicit expressions:

$$
S^{ab}_q(x) = i \frac{x}{2 \pi^2 x^2} \delta_{ab} - \frac{m_q}{4 \pi^2 x^2} \delta_{ab} - \frac{\langle h \rangle}{12} \left( 1 - i \frac{m_q}{4} \frac{x}{f} \right) \delta_{ab} - \frac{x^2}{192} m_q^2 \langle \langle h \rangle \rangle \left( 1 - i \frac{m_q}{6} \frac{x}{f} \right) \delta_{ab} \\
- \frac{ig_s G^{\sigma_\eta}_{ab}}{32 \pi^2 x^2} \left\{ \frac{f \sigma_{\theta_1} + \sigma_{\theta_2}}{\langle h \rangle} - \frac{x^2 G^{\sigma_\eta}_{ab}}{774} \right\} \delta_{ab} + \cdots, \\
\tag{10}
$$

and

$$
S^{ab}_c(x) = \frac{i}{(2 \pi)^4} \int d^4 k e^{-i k x} \left\{ \frac{\delta_{ab}}{k - m_c} + \frac{g_s G^{\sigma_\eta}_{ab} \sigma_{\alpha_\beta}(k + m_c) + (k + m_c) \sigma_{\alpha_\beta}}{4} \frac{1}{(k^2 - m_c^2)^2} \right\} \\
+ \frac{\pi^2}{3} \left( \frac{\alpha_s G}{\pi} \right) \delta_{ab} m_c \frac{k^2 + m_c k}{(k^2 - m_c^2)^2} + \cdots. \\
\tag{11}
$$

The same Lorentz structures obtained in the hadronic side are also present in this side, and the ones used in our analyses are $\not\! p_\mu \gamma_5$ and $\not\! p_\mu \gamma_5$, whose contributions are represented in the below equation explicitly, and the contributions of the others are represented with the last term stated as other structures.

$$
\Pi^{OPE}_\mu(p, q) = \Pi_1 \not\! p_\mu \gamma_5 + \Pi_2 \not\! p_\mu \gamma_5 + \text{other structures}. \\
\tag{12}
$$

To obtain the coefficients, $\Pi_i$, of these Lorentz structures, we use the propagators explicitly in Eq. (9) and transform the results into momentum space. After computation of the four integrals the spectral densities, $\rho_i$ are obtained from the imaginary part of the results, $\rho_i(s, s', q^2) = \frac{1}{\pi} Im[\Pi_i]$. These spectral densities are used in the following dispersion relation:

$$
\Pi_i = \int ds \int ds' \rho_i^{pert}(s, s', q^2) + \rho_i^{non-pert}(s, s', q^2), \\
\tag{13}
$$

giving us the final results of the OPE representation of the correlation function. In the last equation $i = 1, 2, \ldots, 12$ and $\rho_i^{pert}(s, s', q^2)$ and $\rho_i^{non-pert}(s, s', q^2)$ represent the perturbative and non-perturbative parts of the spectral densities, respectively. The results of the spectral densities that are used in the analyses ($i = 1, 2$) are:

$$
\rho_1^{pert} = \int_0^1 ds \int_0^{1-x} dy \left\{ \frac{1}{1024 \sqrt{6} \pi^3 \chi^3} (1 + 5\beta) m_s x y (Q^2 x y + s' \chi x' + m_c^2 \chi^2)^2 \Theta[L(s, s', Q^2, x, y)], \right\} \\
- \frac{\langle Q^2 \rangle}{3664 \sqrt{6} \pi^4 \chi^4 \beta} (1 + 5\beta) m_c x^2 y^2 + \frac{\langle Q^2 \rangle}{9216 \sqrt{6} \pi^4 \chi^4} (1 + 5\beta) m_s x y [9 x^2 + 9 (y - 1)^2 + x (19 y - 18)] \\
+ \frac{1}{256 \sqrt{6} \pi^4 \chi^4} [m_s^2 ((1 + 2) \langle \tilde{d} \rangle + \langle \tilde{s} s \rangle (1 + 5 \beta) + (1 - 1) \langle \tilde{u} u \rangle) x y] \Theta[L(s, s', q^2, x, y)], \\
\tag{14}
$$

$$
\rho_2^{pert} = \int_0^1 ds \int_0^{1-x} dy \left\{ - \frac{1}{128 \sqrt{6} \pi^3 \chi^3 \alpha} [((1 - 1) \langle \tilde{d} \rangle + \langle \tilde{ss} \rangle (1 + 5 \beta) + (1 - 1) \langle \tilde{uu} \rangle) x y (Q^2 x y + s' \chi x' + m_c^2 \chi^2)] \\
- \frac{\langle Q^2 \rangle}{3664 \sqrt{6} \pi^4 \chi^4 \beta} (1 + 5\beta) m_c x^2 y^2 + \frac{\langle Q^2 \rangle}{9216 \sqrt{6} \pi^4 \chi^4} (1 + 5\beta) m_s x y [9 x^2 + 9 (y - 1)^2 + x (19 y - 18)] \\
+ \frac{1}{256 \sqrt{6} \pi^4 \chi^4} [m_s^2 ((1 + 2) \langle \tilde{d} \rangle + \langle \tilde{s} s \rangle (1 + 5 \beta) + (1 - 1) \langle \tilde{u} u \rangle) x y] \Theta[L(s, s', q^2, x, y)], \\
\tag{14}
$$
and

\[ \rho_{2}^{\text{pert}} = \int_{0}^{1} dx \int_{0}^{1-x} dy \frac{1}{2048 \sqrt{6 \pi^6} \lambda^4} \left[ - (1 + 5 \beta) m_e m_s (Q^2 x y + m^2 \chi'^2 + s'(x^2 + (y - 1)y + x(2y - 1))^2) \right] \times \Theta[L(s, s', Q^2, x, y)] \]

\[ \rho_{2}^{\text{non-pert}} = \int_{0}^{1} dx \int_{0}^{1-x} dy \left\{ \frac{1}{512 \sqrt{6 \pi^4} \lambda^4} \left[ m_s(\langle s \bar{s} \rangle(1 + 5 \beta) - 2(\beta - 1)\langle \bar{u}u \rangle)\chi(2Q^2 x y + s' \chi') - 2m_e(\langle s \bar{s} \rangle(1 + 5 \beta) + (\beta - 1)\langle \bar{u}u \rangle)\chi(2Q^2 x y + s' \chi') + m^2 \chi'^2 \right] \right\} + \frac{(\alpha_s G_F)}{147456 \sqrt{6 \pi^4} \lambda^4} \left[ (1 + 5 \beta) m_e m_s x \chi'(4x^2 - 3xy + 4y^2) - 8m_s \chi(2x^4 - x^3y - xy^3 + 2y^4) \right] - \frac{(\alpha_s G_F)}{36864 \sqrt{6 \pi^4} \lambda^4} \left[ (1 + 5 \beta) - (Q^2 + s - s') x^2 y \chi + 2m_s m_s \chi'^2 (9x^2 + 9(y - 1)^2 + x(17y - 18)) \right] - \frac{m^2}{1536 \sqrt{6 \pi^4} \lambda^4} \left[ 2m_s(\langle s \bar{s} \rangle(1 + 5 \beta) - 3(\beta - 1)\langle \bar{u}u \rangle)xy + 3m_e(\langle s \bar{s} \rangle(1 + 5 \beta) + (\beta - 1)\langle \bar{u}u \rangle)\chi^2 \right] + 3(\beta - 1)\langle \bar{d}d \rangle \left[ - 2m_s x y + m_s \chi'^2 \right] + \frac{1}{5184 \sqrt{6 \pi^4} \lambda^4} (1 + 5 \beta) g^2 \left( \langle \bar{d}d \rangle^2 + \langle s \bar{s} \rangle^2 + \langle \bar{u}u \rangle^2 \right) xy + \frac{1}{48 \sqrt{6 \pi^2} \lambda^4} \left[ (\beta - 1)\langle s \bar{s} \rangle\langle \bar{u}u \rangle + \langle \bar{d}d \rangle \left( (\beta - 1)\langle s \bar{s} \rangle + \langle \bar{u}u \rangle + 5(\beta - 1)\langle \bar{u}u \rangle \right) \right] \Theta[L(s, s', Q^2, x, y)] \]

where

\[ \chi = (x + y - 1), \]
\[ \chi' = (x + y), \]
\[ L(s, s', Q^2, x, y) = -Q^2 x y - s'(x + y - 1)(x + y) - m_s^2(x + y)^2, \]

with \( \Theta[\ldots] \) being the unit-step function.

Completing the calculations of both representations, we match the results considering the coefficients of the same Lorentz structures. This step gives two results both containing \( g_1 \) and \( g_2 \). Solving these two coupled expressions together, we obtain the sum rules giving the considered coupling constants, \( g_1 \) and \( g_2 \), analytically as

\[ g_1 = e^{\frac{\beta_0}{2\pi}} e^{\frac{\alpha_s}{2\pi}} \frac{m_{J/\psi}(m^2_{J/\psi} + Q^2)}{f_{J/\psi} \lambda_\Lambda \lambda_{P_s} m_s (m^2_\Lambda + m^2_{J/\psi} - m^2_{P_s})} \]
\[ g_2 = e^{\frac{\beta_0}{2\pi}} e^{\frac{\alpha_s}{2\pi}} \frac{(m_{J/\psi} + m_{\Lambda})(m^2_{J/\psi} + Q^2)}{f_{J/\psi} \lambda_\Lambda \lambda_{P_s} m_s (m^2_\Lambda + m^2_{J/\psi} - m^2_{P_s})}, \]

where \( \Pi_i \) is the Borel transformed form of the \( \Pi_i \) function. As is seen from the results for computations of these coupling constants, we are in need of some input parameters. These parameters are collected in Table I. In the calculations, the masses of light \( u \) and \( d \) quarks are taken as zero. Besides the given parameters in Table I, there are five additional parameters which are the threshold parameters \( s_0 \) and \( s'_0 \), Borel parameters, \( M^2 \) and \( M'^2 \) and the mixing parameter \( \beta \) which is coming from the interpolating current of the \( \Lambda \) baryon. These parameters are determined from the analyses of the results imposing the standard criteria of the method such as weak dependence of the results on the auxiliary parameters, pole dominance and OPE convergence. Considering these conditions, the threshold parameters are fixed as follows:

\[ 21.0 \text{ GeV}^2 \leq s_0 \leq 23.0 \text{ GeV}^2, \]
\[ 1.7 \text{ GeV}^2 \leq s'_0 \leq 2.3 \text{ GeV}^2. \]

The upper limits of Borel parameters are determined by imposing the pole dominance for the selected working regions of continuum thresholds. For the calculations of their lower limits, the convergence of the series of OPE is considered: the dominance of perturbative part over the nonperturbative ones and "the higher the dimension of the nonperturbative operator, the lower its contribution". With these conditions, the Borel parameters are fixed as

\[ 5.0 \text{ GeV}^2 \leq M^2 \leq 7.0 \text{ GeV}^2, \]
\[ 1.4 \text{ GeV}^2 \leq M'^2 \leq 2.6 \text{ GeV}^2. \]
Values

| Parameters | Values            |
|------------|-------------------|
| $m_s$      | $93^{+11}_{-5}$ MeV [2] |
| $m_c$      | $(1.27 \pm 0.02)$ GeV [2] |
| $m_{P_{c\bar{s}}}$ | $(4.47 \pm 0.11)$ GeV [57] |
| $m_{J/\psi}$ | $(3096.900 \pm 0.006)$ MeV [2] |
| $m_{\Lambda}$ | $(1115.683 \pm 0.006)$ MeV [2] |
| $\lambda_{P_{c\bar{s}}}$ | $(1.86 \pm 0.31) \times 10^{-3}$ GeV$^2$ [57] |
| $\lambda_{\Lambda}$ | $(0.013 \pm 0.02)$ GeV$^3$ [61] |
| $f_{J/\psi}$ | $(481 \pm 36)$ MeV [62] |
| $\langle q\bar{q} \rangle$ | $(-0.24 \pm 0.01)^3$ GeV$^3$ [63] |
| $\langle s\bar{s} \rangle$ | $0.8\langle q\bar{q} \rangle$ [63] |
| $m_0^2$ | $(0.8 \pm 0.1)$ GeV$^2$ [63] |
| $(g_5\sigma Gq)^2$ | $m_0^2\langle q\bar{q} \rangle$ |
| $(g_5^2 G^2)$ | $4\pi^2(0.012 \pm 0.004)$ GeV$^4$ [64] |

TABLE I. Some input parameters entering the calculations.

| Coupling Constant | $g_0$ | $c_1$ | $c_2$ |
|-------------------|-------|-------|-------|
| $g_1$             | 4.22  | 1.54  | 1.16  |
| $g_2$             | 10.54 | 1.54  | 1.16  |

TABLE II. : Parameters of the fit functions for coupling constants, $g_1$ and $g_2$.

As the final parameter, we determine the working intervals of $\beta$ from the analyses by considering a parametric plot of the results as functions of $\cos \theta$ where $\beta = \tan \theta$. We select the regions that show least variations with respect to the changes in $\cos \theta$, which read

$$-1 \leq \cos \theta \leq -0.5 \quad \text{and} \quad 0.5 \leq \cos \theta \leq 1.$$  \hspace{1cm} (20)

Our analyses show that the physical quantities show weak dependence on the auxiliary parameters in the above windows for $s_0$ and $s_0'$, $M^2$, $M'^2$ and $\cos \theta$.

Using the given input parameters in Table I and the determined windows for auxiliary parameters, we calculate the strong coupling constants for the considered decay channel. The following fit functions represent the $Q^2$-behavior of the strong coupling form factors:

$$g_i(Q^2) = g_0 e^{c_1 \frac{Q^2}{m_{P_{c\bar{s}}}} + c_2 (\frac{Q^2}{m_{P_{c\bar{s}}}})^2},$$  \hspace{1cm} (21)

with $g_0$, $c_1$ and $c_2$ being the fit parameters that take the values given in Table II. We, then, use the fit functions to determine the coupling constants at $Q^2 = -m_{J/\psi}^2$ as

$$g_1 = 2.64 \pm 0.31 \quad \text{and} \quad g_2 = 5.25 \pm 0.63,$$  \hspace{1cm} (22)

where the errors are due to the uncertainties present in the input parameters entering the calculation and in the determinations of the auxiliary parameters, as well.

Having determined the strong coupling constants, the next task is to compute the corresponding width for $P_{c\bar{s}} \rightarrow J/\psi \Lambda$ decay channel in terms of the strong coupling constants and other related parameters. The standard calculations lead to the width formula as

$$\Gamma = \frac{f(m_{P_{c\bar{s}}}, m_{J/\psi}, m_{\Lambda})}{16\pi m_{P_{c\bar{s}}}} \left[ -\frac{2(m_{J/\psi}^2 - (m_{\Lambda} + m_{P_{c\bar{s}}})^2)}{m_{J/\psi}(m_{\Lambda} + m_{P_{c\bar{s}}})^2} \left( g_1^2 m_{J/\psi}^2 (m_{J/\psi}^2 + 2(m_{\Lambda} - m_{P_{c\bar{s}}})^2) ight) 
+ 6g_1 g_2 m_{J/\psi}^2 (m_{\Lambda} - m_{P_{c\bar{s}}})(m_{\Lambda} + m_{P_{c\bar{s}}}) + g_1^2 (2m_{J/\psi}^2 + (m_{\Lambda} - m_{P_{c\bar{s}}})^2)(m_{\Lambda} + m_{P_{c\bar{s}}})^2 \right],$$  \hspace{1cm} (23)

where

$$f(x, y, z) = \frac{1}{2x} \sqrt{x^4 + y^4 + z^4 - 2xy - 2xz - 2yz}.$$  \hspace{1cm} (24)
Using the values of the strong coupling constants, we compute the width for the considered channel to be

$$\Gamma(P_{cs} \rightarrow J/\psi \Lambda) = (15.87 \pm 3.11) \text{ MeV}. \quad (25)$$

### III. SUMMARY AND CONCLUSION

The recently observed pentaquark state, the hidden-charmed strange $P_{cs}(4459)^0$, added a new member to the pentaquark family. Its experimentally observed mass and width were reported as $M = 4458.8 \pm 2.9^{+4.7}_{-1.7}$ MeV and $\Gamma = 17.3 \pm 6.5^{+8.0}_{-5.5}$ MeV, respectively [53]. However, its quantum numbers, $J^P$, could not be determined as a result of insufficient statistics in the experiment [53]. Using the QCD sum rule method, $P_{cs}(4459)^0$ state was studied in the molecular form assigning its quantum numbers as $J^P = \frac{1}{2}^-$ or $\frac{3}{2}^-$ [54] and in the diquark-diquark-antiquark form with quantum numbers $\frac{3}{2}^-$ [57] and its mass was obtained in these studies to shed light on its nature. Both of these interpretations resulted in mass predictions consistent with the experimental data creating a need for further investigations of this state.

In this study, we investigated the strong $P_{cs} \rightarrow J/\psi \Lambda$ decay and obtained the strong coupling constants representing the amplitude of this decay using the QCD sum rule method. To this end, we adopted an interpolating current in the diquark-diquark-antiquark form for the substructure of this particle. In the analysis, we considered the quantum numbers of $P_{cs}(4459)^0$ state as $J^P = \frac{1}{2}^-$. The obtained strong coupling constants were used in the determination of the corresponding width, which is obtained as $\Gamma(P_{cs} \rightarrow J/\psi \Lambda) = (15.87 \pm 3.11) \text{ MeV}$. Compared to the experimental value, the obtained width is in good consistency with experimental data, which favors the quantum numbers $J^P = \frac{1}{2}^-$ and compact pentaquark nature of diquark-diquark-antiquark form for $P_{cs}(4459)^0$ state.

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