On the Modeling and Simulation of Anti-Windup Proportional-Integral Controller

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Abstract—This paper investigates the chattering and deadlock behaviors of the proportional-integral (PI) controller with an anti-windup (AW) limiter recommended by the IEEE Standard 421.5-2016. Depending on the simulation method, the controller may enter a chattering or deadlock state in some combinations of parameters and inputs. Chattering and deadlock are analyzed in the context of three numerical integration approaches: explicit partitioned method (EPM), execution-list based method (ELM), and implicit trapezoidal method (ITM). This paper derives the chattering stop condition for EPM and ELP, and analyzes the impacts of step size and convergence tolerance for simultaneous integration method. The deduced chattering stop conditions and deadlock behavior is verified with numerical simulations.

Index Terms—Proportional integral (PI) controller, anti-windup limiter, power system simulation, discontinuity.

I. PROBLEM STATEMENT

DISCONTINUITIES in power system simulations are intricate problems that require careful handling. Anti-windup (AW) limiter is one type of discontinuous component for modeling the saturation of devices. The IEEE Standard 421.5-2016 [1] recommends a proportional-integral (PI) controller block [2], [3] with an AW limiter [4]–[6] as shown in Fig. 1 and the recommended implementation is as follows:

\[
\begin{align*}
\dot{x} &= z_i K_p u , \\
0 &= (K_p u + x) - y , \\
0 &= (z_i y + z_u w_{\min} + z_l w_{\max}) - w .
\end{align*}
\]

(1)

The uniqueness of this model is that the AW limiter on the integrator is conditional, depending on the hard limiter status. The differential-algebraic equation (DAE) formulation introduces one differential variable \(x\) and two algebraic variables, \(y\) and \(w\), as follows:

\[
\begin{align*}
\dot{x} &= z_i K_p u , \\
0 &= (K_p u + x) - y , \\
0 &= (z_i y + z_u w_{\min} + z_l w_{\max}) - w .
\end{align*}
\]

(2)

It is common to use boolean variables to implement the AW effect and use a piecewise equation to set the hard limits [7].

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the chattering behavior using the EPM and ELM. Section IV discusses the deadlock using ITM and the impacts of step size and convergence tolerance. Section V concludes the finding.

II. NUMERICAL INTEGRATION WORKFLOWS

As seen in Fig. 1, the integrator state variable is dependent on the output \(y\), an algebraic variable. The equation evaluation workflow of integration methods will directly affect the chattering and deadlock behaviors of the controller. Three most commonly used methods for power system transient dynamic simulation are discussed in the following.

1) Explicit Partitioned Method (EPM): algebraic equations that approximate fast electromagnetic transients are first solved, followed by the integration of differential equations using an explicit formula [10]:

\[
0 = g(x_i,y_{t+h},u_{t+h}),
0 = x_{t+h} = f(x_i,y_{t+h},u_{t+h}),
\]

where \(x\), \(y\), and \(u\) are the states, algebraic variables, and discrete states, respectively. To simulate from \(t\) to \(t+h\), the two equation sets in (3) are solved sequentially. Although iterations can be applied until \(y_{t+h}\) and \(x_{t+h}\) stop changing, iterative implicit methods offer better numerical stability when iterative approaches are needed. The non-iterative EPM is the most commonly used method in commercial simulation tools.

2) Implicit Methods: the differential equations are solved along with algebraic equations:

\[
0 = g(x_i,y_{t+h},u_{t+h}),
0 = x_{t+h} - f(x_i,y_{t+h},u_{t+h}).
\]

There are a variety of implicit methods utilized in power systems, being the implicit trapezoidal method (ITM) the most popular one [11]. The ITM applied to (3) leads to:

\[
0 = g(x_i,y_{t+h},u_{t+h}),
0 = x_{t+h} - x_i - 0.5h(f(x_i,y_{t+h},u_{t+h}) - f_i),
\]

where \(x_i\) and \(f_i\) are the known vectors of state variables and differential equations, respectively, evaluated at step \(t\). Due to robustness and generality, ITM is also widely adopted in both open-source [12], [13] and commercial tools.

3) Execution List-based Method (ELM): algebraic and differential equations are solved sequentially for each block. Blocks are solved sequentially as defined in the execution list [14]. Equations are not grouped like in a power system simulation tool. Rather, the evaluation sequence is based on the data flow specific to the model. For example, to integrate from \(t\) to \(t+h\) for the PI controller under discussion, ELM evaluates \(\dot{x}_{t+h}\), \(x_{t+h}\) (based on \(z_{i,t}\), \(y_{t+h}\) with the corresponding \(z_{i,t+h}\), and \(u_{t+h}\) in sequence, based on [3]. ELM has the advantage of fully representing the control logic and signal flow in digital controllers.

It is worth mentioning that ELM and the non-iterative form of EPM introduce a “delay” equal to the time step \(h\) between \(x\) and \(y\). Implicit methods are iterative for nonlinear systems and can guarantee solutions to satisfy all equations for each step, but they may show convergence issues for discontinuous right-hand side equations, i.e. when the equations model anti-windup limiters. Due to the workflow and stop criteria, chattering can only happen when solved with non-iterative methods, while deadlock can only occur with iterative ones.

Following sections study the conditions in which chattering or deadlock can be stopped or avoided. All following analyses start from a generic time \(t-h\) and consider a decreasing input \((u < 0\) in the proximity of \(t-h\)). We also assume the output \(y_{t-h}\) is at the upper limit that initially locks the integrator.

III. PI CONTROLLER CHATTERING

A. Chattering with Explicit Partitioned Method

The most relevant characteristic of EPM to the PI controller is that the change in output \(y\) can unlock the integrator for the current time step. However, the change in state \(x\) is only reflected at the next time step. Based on the assumptions in Section II initial conditions in EPM are given by:

\[
y_{t-h} = K_p u_{t-h} + x_{t-2h} = w_{\text{max}},
\Rightarrow \ z_{i,t-h} = 0 & x_{t-h} = x_{t-2h}.
\]

At time \(t\), as \(u\) decreases, \(y_{t}\) drops below the upper limit, unlocking the integrator for time \(t\), as given by:

\[
y_{t} = K_p u_{t} + x_{t-h} = y_{t-h} + K_p \Delta u_{t} < w_{\text{max}},
\Rightarrow \ z_{i,t} = 1 & x_{t} = x_{t-1} + \Delta x_{t},
\]

where the unlocked limiter will be reflected in \(x_{t}\), as well as \(y_{t+h}\) due to the “delay” in EPM.

For the subsequent time steps, if the output \(y_{t+kh}\) \((k = 1, ..., N)\) do not return to \(w_{\text{max}}\), the controller is considered to have stopped chattering. For the immediate next step \(t+h\), the condition is given by [3]:

\[
y_{t+h} = y_{t-h} + K_p \Delta u_{t} + \Delta u_{t+h} + \Delta x_{t} < w_{\text{max}},
\Rightarrow \ K_p (\Delta u_{t} + \Delta u_{t+h}) + \Delta x_{t} < 0.
\]

Since \(\Delta x_{t}\) is integrated from \(\dot{x}_{t}\), which only depends on \(u_{t}\), \(\Delta x_{t}\) will evaluate to \(h K_i u_{t}\) regardless of the integration formula. The generalized condition for the subsequent step \(t+kh\), where \(k\) is the number of steps ahead, is given by

\[
K_p \sum_{i=0}^{k} \Delta u_{t+i+h} + h K_i (k u_{t} + \sum_{i=0}^{k-1} k \Delta u_{t+i+h}) < 0.
\]

Equation [9] must hold for a sufficient number of steps to avoid chattering. Note that it depends on the initial condition and trajectory of \(u_{t}\) and is case-specific. When such condition is satisfied, the integrator will not become locked again after being unlocked.

B. Chattering with Execution List-based Method

ELM is different from EPM in the equation evaluation sequence. Since the integrator outputs to the summation block, state \(x_{t}\) is integrated based on \(z_{i,t-h}\), which is from the
previous time step, before calculating \( y_t \). The initial conditions for ELM at \( t - h \) are given by
\[
y_{t-h} = K_p u_{t-h} + x_{t-h} = w_{\text{max}}, \quad \text{and} \quad z_{i,t-h} = 0.
\] (10)

At time \( t \), the output \( y_t \) is given by
\[
x_t = x_{t-h} \quad \Rightarrow \quad y_t = y_{t-h} + K_p \Delta u_t < w_{\text{max}}, \quad \text{and} \quad z_{i,t} = 1.
\] (11)

Similarly, \( y_{t+h} \) needs to remain smaller than \( w_{\text{max}} \).
\[
x_{t+h} = x_t + \Delta x_{t+h}, \quad \text{and} \quad y_{t+h} = y_{t-h} + K_p(\Delta u_t + \Delta u_{t+h}) + \Delta x_{t+h} < w_{\text{max}}.
\] (12)

The comparison between (12) and (9) shows that state \( x \) in ELM is one step ahead in terms of the differential variable. Similar to (9), (13) needs to hold for sufficient steps starting from \( k = 1 \) to exit chattering.
\[
K_p \sum_{i=0}^{k} \Delta u_{t+ih} + hK_i(ku_t + \sum_{i=1}^{k} k\Delta u_{t+ih}) < 0.
\] (13)

IV. PI CONTROLLER DEADLOCK

The PI controller may enter a deadlock state when simulated with ITM, which implements an inner iteration loop. Fig. 2 shows the inner iteration loop for one integration step. Hard limiter status is based on the input and updated before equation evaluation. Anti-windup limiter equations are dependent on both variables and equations and are thus updated after equation evaluation.

Suppose at time \( t-h \), the input and output satisfy conditions
\[
\begin{align*}
\text{Input:} & \quad y > 0, \quad \dot{u} < 0, \\
\text{Output:} & \quad \begin{cases}
\quad w = w_{\text{max}} \leq y, \\
\quad z_u = 1, \quad z_i = z_l = 0
\end{cases}
\end{align*}
\] (14)

At time \( t \), the integrator is disabled for the first iteration, and deadlock will happen if iterations meet the conditions:

- If the input decreases to a value that renders \( y < w_{\text{max}} \), iterations will continue due to the increment for \( y \). As a result, the AW will be enabled for the next iteration.
- Next, if the integrator output is so large that \( y \geq w_{\text{max}} \), iterations will continue due to the increment for \( y \) and \( w \). As a result, the AW will be disabled again.

A. Impact of Integration Step Size

When non-convergence happens, an ITM solver can decrease the step size based on predefined criteria \([11]\). This variable step size approach is commonly used because software can take advantage of the numerical stability of ITM by using a larger size for most cases and shrinking it when necessary. The technique, however, does not reduce the occurrence of deadlock. Consider a generic iteration \( i \) of simulation time \( t \), \( y_t^{(i)} \) satisfies
\[
y_t^{(i)} = K_p u_t + x_t^{(i)},
\] where
\[
x_t^{(i)} = \begin{cases}
\quad x_{t-h}, & \text{if} \quad z_{i,t} = 0, \\
\quad x_{t-h} + \Delta x_t^{(i)}, & \text{if} \quad z_{i,t} = 1,
\end{cases}
\] (15)

where \( \Delta x_t^{(i)} \) is the increment for iteration \( i \). To avoid deadlock, if the integrator is unlocked at iteration \( i \), \( y_t^{(i+1)} \) should stay below \( w_{\text{max}} \) to remain the integrator unlocked. Therefore,
\[
y_t^{(i+1)} = K_p u_t + x_t^{(i+1)},
\] (16)
\[
y_{t-h} + (K_p \Delta u_t + \Delta x_t^{(i+1)}) < w_{\text{max}}, \quad \Rightarrow \quad K_p \Delta u_t + \Delta x_t^{(i+1)} < 0,
\]
applying ITM on (16) and observing that $\dot{x}_{t-h} = 0$, and $u_t > 0$, the step size needs to satisfy (17) to avoid deadlock.

$$h < -(2K_p/K_i)(\Delta u_t/u_t).$$  \hfill (17)

If we assume $u$ is the output of a low-pass filter and is thus differentiable, applying ITM on (16) yields

$$h(\dot{u}_t + \dot{u}_{t-h}) < -2(K_p/K_i)(\dot{u}_t + \dot{u}_{t-h}) - 2u_{t-h}. \hfill (18)$$

Note that $u$ decreases in the proximity of $t - h$, hence $(\dot{u}_t + \dot{u}_{t-h}) < 0$. Dividing (18) by $\dot{u}_t + \dot{u}_{t-h}$ yields

$$h > -2(K_p/K_i) - 2u_{t-h}/(\dot{u}_t + \dot{u}_{t-h}). \hfill (19)$$

(19) shows that the step size needs to be greater than a minimum value to remain unlocked. The variable step techniques, however, are designed to decrease the step size when non-convergence happens. This explains why in the case of deadlock, the simulation program cannot improve the convergence by reducing the step size. On the other hand, the integration step size has to be adequately small for systems with fast dynamics. As will be shown in Section V, the minimum step size to avoid deadlock given in (19) can be too large to achieve. Therefore, in some combinations of parameters and inputs, deadlock could be inevitable because the step size condition cannot be achieved.

### B. Impact of Convergence Tolerance

Convergence tolerance also affects the deadlock in terms of the iteration exit condition. As shown in Figure 2, the inner Newton iteration will be deemed converged if the maximum variable increment is smaller than the tolerance $\epsilon$. It explains why even if the step size in (17) is not achievable, ITM can still converge.

Consider a deadlock scenario at time $t$ that after iteration $i - 1$, the integrator is unlocked. The maximum increment, if we omit the subscript $t$, is given by

$$\max\left\{\left|\Delta x^{(i)}\right|, \left|\Delta y^{(i)}\right|, \left|\Delta w^{(i)}\right|\right\}, \hfill (20)$$

where $\left|\Delta y^{(i)}\right| = \left|\Delta x^{(i)}\right|$, and

$$\Delta w^{(i)} = \left|y^{(i)} - w_{\max}\right| \leq \left|(w_{\max} + \Delta x^{(i)}) - w_{\max}\right| = \left|\Delta x^{(i)}\right|. \hfill (21)$$

Applying ITM and observing $\dot{x}_{t-h} = 0$, $\Delta x_t^{(i)}$ is given by

$$\Delta x_t^{(i)} = 0.5h\dot{x}_t^{(i)}, \hfill (22)$$

where $\dot{x}_t^{(i)}$ ($i \in [0, N_{\text{iter}}]$) toggles between 0 and $K_iu_t$. The deadlock can exit only when the increment for iteration $i$ is smaller than $\epsilon$, given by

$$\left|\Delta x_t^{(i)}\right| = 0.5hK_iu_t < \epsilon, \hfill (23)$$

which indicates that, for a fixed step size, a temporarily lift in tolerance can be implemented to stop the deadlock. On the other hand, given a fixed tolerance, (23) allows to calculate the maximum allowed (i.e., adequately small) step size to stop deadlock with a variable step approach.

### V. Simulation Verification

Numerical simulations in Simulink and ANDES [13] are performed to verify the analyses. Simulations are performed on a standalone PI controller as recommended by the IEEE standard. The PI controllers parameters are from [9] with $K_p = 1$, $K_i = 20$, and

$$\dot{u} = \begin{cases} -1 & 2 < t < 6, \\ 1 & \text{otherwise}. \end{cases} \hfill (24)$$

Substitute parameters into (9) and (13) under step size $h = 10^{-3}$ and enumerate $k$ from 1 to 10, we can obtain the input value starting from which the controller can remain unlocked.

Calculations show that for EPM and ELM, $u$ need to drop below 0.0595 and 0.0605, respectively, to prevent the integrator from switching back to the locked state. Numerical integration in Simulink using ODE1 verifies the calculation, as shown in Figure 4. It can be observed that the last relocking happens when the input drops to 0.0605. When the input further decreases, the controller gets unlocked and stays unlocked thereafter.

For ITM, the initial step size $h$ and the convergence tolerance $\epsilon$ are both set to $10^{-3}$. A heuristic algorithm for adjusting the step size is employed. The algorithm is based on the following rules:

$$h_{t+h} = \begin{cases} h_t + 10^{-6} & \text{if } N_t \leq 3, \\ h_t - 10^{-6} & \text{if } N_t \geq 15, \\ h_t & \text{otherwise}. \end{cases} \hfill (25)$$

where $N_t$ is the number of iterations taken to converge for a generic time $t$. The theoretical minimum step size to avoid deadlock from (19) is $h_{\min} = 0.1915$. Compared with the cycle time of a 50/60 Hz power system, this step size is apparently too large for systems under disturbance.

Simulation results using ANDES [13] are shown in Fig. 5. When the deadlock begins at $t = 3.709$ s, the input value $u_t = 0.2915$.

Next, Using (23) and a step size cap of $10^{-3}$, the step size needs to satisfy $h < 0.3431$ ms to exit deadlock. Figure 6 shows the step size change and verifies the calculation.
non-convergence of the inner Newton iteration loop is explained. The impacts of step size and convergence tolerance on the ITM deadlock are also discussed.

The most interesting conclusion from the analysis is that, for some combinations of parameters and inputs, deadlock is inevitable since the step size requirement to avoid deadlock cannot be achieved. However, after shrinking the step size using the variable step approach, ITM will exit the deadlock once the convergence tolerance is satisfied.

VI. CONCLUSIONS

This paper investigates the chattering and deadlock issue of the PI controller recommended by the IEEE Standard 421.5-2016 under three most commonly used numerical integration methods. For the non-iterative EPM and ELM, the chattering issue is discussed with the chattering stop conditions deduced, respectively. For the iterative ITM, the deadlock caused by

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