Topological tensor current of \( \tilde{p} \)-branes in the \( \phi \)-mapping theory

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Abstract

We present a new topological tensor current of \( \tilde{p} \)-branes by making use of the \( \phi \)-mapping theory. It is shown that the current is identically conserved and behave as \( \delta(\tilde{\phi}) \), and every isolated zero of the vector field \( \tilde{\phi}(x) \) corresponds to a ‘magnetic’ \( \tilde{p} \)-brane. Using this topological current, the generalized Nambu action for multi \( \tilde{p} \)-branes is given, and the field strength \( F \) corresponding to this topological tensor current is obtained. It is also shown that the ‘magnetic’ charges carried by \( \tilde{p} \)-branes are topologically quantized and labeled by Hopf index and Brouwer degree, the winding number of the \( \phi \)-mapping.

I. INTRODUCTION

Extended objects with \( p \) spatial dimensional, known as ‘branes’, play an essential role in revealing the non-perturbative structure of the superstring theories and \( M \)-theories \[1-4\]. Antisymmetric tensor gauge fields have been widely studied in the theories of \( p \)-branes \[5-7\]. In the context of the effective \( D = 10 \) or \( D = 11 \) supergravity theory a \( p \)-brane is a \( p \)-dimensional extended source for a \((p + 2)\)-form gauge field strength \( F \). It is well-known that the \((p + 2)\)-form strength \( F \) satisfies the field equation

\[
\nabla_\mu F^{\mu\mu_1\cdots\mu_{p+1}} = j^{\mu_1\cdots\mu_{p+1}}
\]

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where $j^{\mu_1 \cdots \mu_{p+1}}$ is a $(p + 1)$-form tensor current and corresponding to electric source, and the dual field strength $*F$ satisfies

$$\nabla_\mu * F^{\mu_1 \cdots \mu_{p+1}} = j^{\mu_1 \cdots \mu_{p+1}}$$

in which $j^{\mu_1 \cdots \mu_{p+1}}$ is a $(p + 1)$-form tensor current and corresponding to magnetic source [8] [9] [10].

The $\phi$-mapping theory proposed by Prof. Duan [11,12] is important in studying the topological invariant and topological structure of physics systems and has been used to study topological current of magnetic monopole [11], topological string theory [12], topological structure of Gauss-Bonnet-Chern theorem [13], topological structure of the SU(2) Chern density [14] and topological structure of the London equation in superconductor [15]. We must pointed out that the $\phi$-mapping theory is also a powerful tools to investigate the topological defects theory [16–18], and here the vector field $\vec{\phi}$ is looked upon as the order parameters of the defects.

In this paper, we present a new topological tensor current of ‘magnetic’ $\tilde{p}$-branes by making use of the $\phi$-mapping theory. One shows that the each isolated zero of the $d$-dimensional vector field $\vec{\phi}(x)$ corresponds to a $\tilde{p}$-brane ($\tilde{p} = D - d - 1$), and this current is proved to be the general current density of multi $\tilde{p}$-branes. Using this current, the generalized Nambu action for multi $\tilde{p}$-branes is obtained. This topological tensor current will give rise to the inner structure of the field strength $F$ including the contribution of the ‘magnetic’ $\tilde{p}$-branes. Finally, we show that the charges carried by multi $\tilde{p}$-branes are topologically quantized and labeled by the Hopf index and Brouwer degree, the winding number of the $\phi$-mapping.

II. THE TOPOLOGICAL TENSOR CURRENT OF $\tilde{P}$-BRANES

Let $X$ be a $D$-dimensional smooth manifold with metric tensor $g_{\mu\nu}$ and local coordinates $x^\mu (\mu, \nu = 0, \cdots, D - 1)$ with $x^0 = t$ as time, and let $\mathbb{R}^d$ be an Euclidean space of dimension
We consider a smooth map \( \phi : X \rightarrow \mathbb{R}^d \), which gives a \( d \)-dimensional smooth vector field on \( X \)

\[
\phi^a = \phi^a(x), \quad a = 1, 2, \cdots, d.
\]  

(1)

The direction unit field of \( \vec{\phi}(x) \) can be expressed as

\[
n^a = \frac{\phi^a}{||\phi||}, \quad ||\phi|| = \sqrt{\phi^a \phi^a}.
\]  

(2)

In the \( \phi \)-mapping theory, to extend the theory of magnetic monopoles [11] and the topological string theory [12], we present a new topological tensor current, with the unit ‘magnetic’ charge \( g_m \), defined as

\[
\tilde{j}^{\mu_1 \cdots \mu_{D-d}} = \frac{g_m}{A(S^{d-1})(d-1)!} \left( \frac{1}{\sqrt{g}} \right)^{\mu_1 \cdots \mu_{D-d} \mu_{D-d+1} \cdots \mu_{D}} \epsilon_{a_1 a_2 \cdots a_d} \partial_{\mu(D-d+1)} n^{a_1} \partial_{\mu(D-d-2)} n^{a_2} \cdots \partial_{\mu_D} n^{a_d}
\]  

(3)

where \( g \) is the determinant of the metric tensor \( g_{\mu\nu} \). Obviously, this ‘magnetic’ tensor current is identically conserved,

\[
\nabla_{\mu_{i}} \tilde{j}^{\mu_1 \cdots \mu_{D-d}} = 0, \quad i = 1, \cdots, D - d.
\]  

(4)

From (2) we have

\[
\partial_{\mu} n^a = \frac{1}{||\phi||} \partial_{\mu} \phi^a + \phi^a \partial_{\mu} \left( \frac{1}{||\phi||} \right)
\]  

(5)

\[
\frac{\partial}{\partial \phi^a} \left( \frac{1}{||\phi||} \right) = -\frac{\phi^a}{||\phi||^3}
\]  

(6)

Using the above expressions, the general tensor current can be rewritten as

\[
\tilde{j}^{\mu_1 \cdots \mu_{D-d}} = \frac{g_m}{A(S^{d-1})(d-1)(d-2)!} \left( \frac{1}{\sqrt{g}} \right)^{\mu_1 \cdots \mu_{D-d} \mu_{D-d+1} \cdots \mu_{D}} \epsilon_{a_1 a_2 \cdots a_d} \partial_{\mu(D-d+1)} \phi^a \partial_{\mu(D-d-2)} \phi^{a_2} \cdots \partial_{\mu_D} \phi^{a_d} \frac{\partial}{\partial \phi^a} \frac{\partial}{\partial \phi^{a_1}} \left( \frac{1}{||\phi||^{d-2}} \right).
\]  

(7)

If we define a generalized Jacobians tensor as
\[ \varepsilon^{a_1 \cdots a_d} J^{\mu_1 \cdots \mu_{D-d}} = \varepsilon^{\mu_1 \cdots \mu_{D-d} \mu_{D-d+1} \mu_{D-d+2} \cdots \mu_D} \partial_{\mu(D-d+1)} \phi^{a_1} \partial_{\mu(D-d+2)} \phi^{a_2} \cdots \partial_{\mu_D} \phi^{a_d} \] (8)

and make use of the generalized Laplacian Green function relation in \( \phi \)-space

\[ \frac{\partial}{\partial \phi^a} \frac{\partial}{\partial \phi^a} \left( \frac{1}{||\phi||^{d-2}} \right) = \frac{4\pi^d}{\Gamma(d/2 - 1)} \delta(\vec{\phi}), \] (9)

we obtain a \( \delta \)-function like tensor current \[12\]

\[ \tilde{j}^{\mu_1 \cdots \mu_{D-d}} = g_m \delta(\vec{\phi}) J^{\mu_1 \cdots \mu_{D-d}} (\vec{\phi}) \left( \frac{1}{\sqrt{g}} \right). \] (10)

We find that \( \tilde{j}^{\mu_1 \cdots \mu_{D-d}} \neq 0 \) only when \( \phi = 0 \). So, it is essential to discuss the solutions of the equations

\[ \phi^a(x) = 0, \quad a = 1, \cdots, d \] (11)

Suppose that the vector field \( \vec{\phi}(x) \) possesses \( l \) isolated zeroes, according to the deduction of Ref. \[12\] and the implicit function theorem \[19\] \[20\], when the zeroes are regular points of \( \phi \)-mapping, i.e. the rank of the Jacobian matrix \( [\partial_{\mu} \phi^a] \) is \( d \), the solution of \( \vec{\phi}(x) = 0 \) can be parameterized by

\[ x^\mu = z_i^\mu (u^1, u^2, \cdots, u^{D-d}), \quad i = 1, \cdots, l, \] (12)

where the subscript \( i \) represents the \( i \)-th solution and the parameters \( u = u(u^1, \cdots, u^{D-d}) \) span a \( (D - d) \)-dimensional submanifold of \( X \), denoted by \( N_i \), which corresponds to a \( \tilde{p} \)-brane \( (\tilde{p} = D - d - 1) \) with spatial \( \tilde{p} \)-dimension and \( N_i \) is its worldvolume. One see that the tensor current \( \tilde{j}^{\mu_1 \cdots \mu_{D-d}} \) is not vanished only on the worldvolume manifolds \( N_i \) \( (i = 1, \cdots, l) \), each of which corresponds to a \( \tilde{p} \)-brane. Therefore, every isolated zero of \( \vec{\phi}(x) \) on \( X \) corresponds to a magnetic \( \tilde{p} \)-branes. These ‘magnetic’ \( \tilde{p} \)-branes had been formally discussed and not studied based on the topology theory \[22\] \[23\]. Here, we must pointed out that the \( \tilde{p} \)-branes, sometimes, may be considered as topological defects \[10\] \[24\], in this case for our theory the vector field \( \phi^a(x) \) \( (a = 1, \cdots, d) \) may be looked upon as the generalized order parameters \[18\] for \( \tilde{p} \)-branes.
In the following, we will discuss the inner structure of the topological tensor current \( \tilde{j}^{\mu_1 \cdots \mu_{D-d}} \). It can be proved that there exists a \( d \)-dimensional submanifold \( M \) in \( X \) with the parametric equation

\[
x^\mu = x^\mu(v^1, \cdots, v^d), \quad \mu = 1, \cdots, D,
\]

which is transversal to every \( N_i \) at the point \( p_i \) with

\[
g_{\mu\nu} \frac{\partial x^\mu}{\partial u^I} \frac{\partial x^\nu}{\partial u^A} \Big|_{p_i} = 0, \quad I = 1, \cdots, D - d, \quad A = 1, \cdots, d.
\]

This is to say that the equations \( \vec{\phi}(x) = 0 \) have the isolated zero points on \( M \).

As we have pointed in Ref. [13,14], the unit vector field defined in (2) gives a Gauss map \( n : \partial M_i \to S^{d-1} \), and the generalized Winding Number can be given by this Gauss map

\[
W_i = \frac{1}{A(S^{d-1})(d-1)!} \int_{\partial M_i} n^* (\epsilon_{a_1 \cdots a_d} n^{a_1} dn^{a_2} \wedge \cdots \wedge dn^{a_d})
\]

\[
= \frac{1}{A(S^{d-1})(d-1)!} \int_{\partial M_i} \epsilon_{a_1 \cdots a_d} n^{a_1} \partial_{A_2} n^{a_2} \cdots \partial_{A_d} n^{a_d} dv^{A_2} \wedge \cdots \wedge dv^{A_d}
\]

\[
= \frac{1}{A(S^{d-1})(d-1)!} \int_{M_i} \epsilon^{A_1 \cdots A_d} \epsilon_{a_1 \cdots a_d} \partial_{A_1} n^{a_1} \partial_{A_2} n^{a_2} \cdots \partial_{A_d} n^{a_d} d^dv.
\]

(15)

where \( \partial M_i \) is the boundary of the neighborhood \( M_i \) of \( p_i \) on \( M \) with \( p_i \notin \partial M_i, M_i \cap M_j = \emptyset \). Then, by duplicating the derivation of (3) from (10), we obtain

\[
W_i = \int_{M_i} \delta(\vec{\phi}(v)) J(\vec{\phi}/v) d^dv.
\]

(16)

where \( J(\vec{\phi}/v) \) is the usual Jacobian determinant of \( \vec{\phi} \) with respect to \( v \)

\[
\epsilon^{a_1 \cdots a_d} J(\vec{\phi}/v) = \epsilon^{A_1 \cdots A_d} \partial_{A_1} n^{a_1} \partial_{A_2} n^{a_2} \cdots \partial_{A_d} n^{a_d}.
\]

(17)

According to the \( \delta \)-function theory [25] and the \( \phi \)-mapping theory, we know that \( \delta(\vec{\phi}(v)) \) can be expanded as

\[
\delta(\vec{\phi}(v)) = \sum_{i=1}^{l} \beta_i \eta_i \delta^d(\vec{v} - \vec{v}(p_i))
\]

(18)
on $M$, where the positive integer $\beta_i = |W_i|$ is called the Hopf index of the map $v \rightarrow \bar{\phi}(v)$ and $\eta_i = sgn(J(\bar{\phi}(v)))|_{p_i} = \pm 1$ is the Brouwer degree \[13,15\]. One can find the relation between the Hopf index $\beta_i$, the Brouwer degree $\eta_i$, and the winding number $W_i$

$$W_i = \beta_i \eta_i,$$  

(19)

One see that the Eq. (18) is only the expansion of $\delta(\bar{\phi}(x))$ on $M$. In order to investigate the expansion of $\delta(\bar{\phi}(x))$ on the whole manifold $X$, we must expand the $d$-dimensional $\delta$-function of the singular point in terms of the $\delta$-function on the singular submanifold $N_i$ which had been given in Ref. \[25\]

$$\delta(N_i) = \int_{N_i} \delta^D(x - z_i(u)) \sqrt{g_u} d^{(D-d)} u, \quad i = 1, \cdots, l$$  

(20)

in which

$$g_u = \det(g_{\mu\nu} \frac{\partial x^\mu}{\partial u^I} \frac{\partial x^\nu}{\partial u^J}), \quad I, J = 1, \cdots, (D - d).$$

Then, from Eqs. (18), and by considering the property of the $\delta$-function, one will obtain

$$\delta(\bar{\phi}(x)) = \sum_{i=1}^l \beta_i \eta_i \int_{N_i} \delta^D(x - z_i(u)) \sqrt{g_u} d^{(D-d)} u.$$  

(21)

Therefore, the general topological current of the $\tilde{p}$-branes can be expressed directly as

$$\tilde{j}^{\mu_1 \cdots \mu_{D-d}} = \frac{1}{\sqrt{g}} J^{\mu_1 \cdots \mu_{D-d}} \frac{\bar{\phi}}{x} \sum_{i=1}^l \beta_i \eta_i \int_{N_i} \delta^D(x - z_i(u)) \sqrt{g_u} d^{(D-d)} u,$$  

(22)

which is a new topological current theory of $\tilde{p}$-branes based on the $\bar{\phi}$-mapping theory.

If we define a Lagrangian as

$$L = \sqrt{\frac{1}{(D-d)!}} g_{\mu_1 \nu_1} \cdots g_{\mu_{(D-d)} \nu_{(D-d)}} \tilde{j}^{\mu_1 \cdots \mu_{D-d}} \tilde{j}^{\nu_1 \cdots \nu_{D-d}},$$  

(23)

which is just the generalization of Nielsen’s Lagrangian \[24\], from the above deductions, we can prove that

$$L = \left(\frac{1}{\sqrt{g}}\right) \delta(\bar{\phi}(x)).$$  

(24)
Then, the action takes the form

\[ S = \int_X L \sqrt{g} d^D x = \int_X \delta(\vec{\phi}(x)) d^D x. \]  

(25)

By substituting the formula (21) into (25), we obtain an important result

\[ S = \int_X \sum_{i=1}^l \beta_i \eta_i \int_{N_i} \delta^D(x - z_i(u)) \sqrt{g_u} d^{(D-d)} u d^D x \]

\[ = \sum_{i=1}^l \beta_i \eta_i \int_{N_i} \sqrt{g_u} d^{(D-d)} u, \]

(26)

i.e.

\[ S = \sum_{i=1}^l \eta_i S_i, \]

(27)

where \( S_i = \beta_i \int_{N_i} \sqrt{g_u} d^{(D-d)} u \). This is just the generalized Nambu action for multi \( \tilde{p} \)-branes\( (\tilde{p} = D - d - 1) \), which is the straightforward generalization of Nambu action for the string world-sheet action [27]. Here this action for multi \( \tilde{p} \)-branes is obtained directly by \( \phi \)-mapping theory, and it is easy to see that this action is just Nambu action for multi-strings when \( D - d = 2 \) [12].

\section*{III. THE GAUGE FIELD CORRESPONDING TO THE TOPOLOGICAL CURRENT}

In this section, we will study the antisymmetric tensor gauge field corresponding to the topological tensor current presented in above section. We know that \( p \)-branes naturally acts as the ‘electric’ source of a rank \( p + 2 \) field strength

\[ F = dA, \]

(28)

where \( A \) is a \( (p+1) \)-form as the tensor gauge potential and satisfies the gauge transformation

\[ A \rightarrow A + d\Lambda_p. \]

From Eq. (28), one have the Bianchi identity

\[ \]
\[ dF \equiv 0. \]  

(29)

And the ‘electric’ current density associated with the source can be expressed as

\[ j^{\mu_1 \cdots \mu_{\bar{p}+1}} = \nabla_{\mu} F^{\mu \mu_1 \cdots \mu_{\bar{p}+1}}. \]  

(30)

Just as the usual Maxwell’s equation, we know that Eqs. (28), (29) and (30) imply the presence of an ‘electric’ charge, i.e. \( p \)-branes, but no ‘magnetic’ source \[10].

Now, let us discuss the case when there exists the ‘magnetic’ source. For this case, one must introduce another \( (p+2) \)-form \( G \) for the magnetic source, and the field strength \( F \) must be modified to

\[ F = dA + G, \]  

(31)

which is the generalized field strength including the contribution of the ‘magnetic’ source, i.e. ‘magnetic’ branes: \( \tilde{p} \)-branes with \( \tilde{p} = D - p - 4 \).

To obtain the explicit expression for \( G \), let us consider the current density corresponds to magnetic source which is given by

\[ \tilde{j}^{\mu_1 \cdots \mu_{D-1}} = \nabla_{\mu} F^{\mu \mu_1 \cdots \mu_{D-1}}. \]  

(32)

Using (31) and (32), we obtain

\[ \tilde{j}^{\mu_1 \cdots \mu_{\tilde{p}+1}} = \frac{1}{\sqrt{g}} \partial_{\mu} (\sqrt{g} \perp^\mu_{\mu_1 \cdots \mu_{\tilde{p}+1}} G_{\mu_{\tilde{p}+2} \cdots \mu_{D-1}}), \]  

(33)

It has been pointed out in the above section that the current density of the ‘magnetic’ branes is a topological current given by Eq. (4), which can be rewritten as

\[ \tilde{j}^{\mu_1 \cdots \mu_{(D-1)}} = \frac{g_m}{A(S^{d-1})(d-1)!} \left( \frac{1}{\sqrt{g}} \right) \partial_{\mu_1} (\perp_{\mu_2 \cdots \mu_{D-1}} n^{a_1} \partial_{\mu_2} n^{a_2} \cdots \partial_{\mu_{d-1}} n^{a_d}), \]  

(34)

where \( (D - d) = \tilde{p} + 1 \), i.e. \( \tilde{p} = D - d - 1 \). Comparing the Eq. (33) to (34), we can obtain

\[ G_{\mu_1 \cdots \mu_{d-1}} = \frac{(-1)^{(D-d)} g_m}{A(S^{d-1})(d-1)!} \perp_{a_1 \cdots a_d} n^{a_1} \partial_{\mu_1} n^{a_2} \cdots \partial_{\mu_{d-1}} n^{a_d}, \]  

(35)
and
\[
G = \frac{(-1)^{(D-d)}g_m}{A(S^{d-1})(d-1)!} \epsilon_{a_1a_2\cdots a_d} n^{a_1}dn^{a_2}\wedge \cdots \wedge dn^{a_d}. \tag{36}
\]

Of equal interest is the ‘magnetic’ charge carried by the multi $\tilde{p}$-branes, which is given by
\[
Q^M = \int_{\Sigma} \tilde{j}^{\mu_1\cdots\mu_{\tilde{p}+1}} \sqrt{g}d\sigma_{\mu_1\cdots\mu_{\tilde{p}+1}} \tag{37}
\]
where $\Sigma$ is a $d$-dimensional ($d = p + 3$) hypersurface in $X$, while $d\sigma_{\mu_1\cdots\mu_{\tilde{p}+1}}$ is the covariant surface element of $\Sigma$ \[28\]. From (32) and (37), it is easy to prove that
\[
Q^M = \int_{\partial \Sigma} F,
\]
where $\partial \Sigma$ is the boundary of $\Sigma$ and a $(p+2)$-dimension hypersurface. Substituting (22) into (37), we have
\[
Q^M = g_m \int_{\Sigma} J^{\mu_1\cdots\mu_{\tilde{p}+1}}(x) \sum_{i=1}^{l} \beta_i \eta_i \int_{N_i} \delta^D(x - z_i(u)) \sqrt{g_u d^{(D-d)}} ud\sigma_{\mu_1\cdots\mu_{\tilde{p}+1}}, \tag{38}
\]
from (8), and the relation
\[
\frac{1}{(\tilde{p} + 1)!} \epsilon^{\mu_1\cdots\mu_{\tilde{p}+1}\nu_1\cdots\nu_d} d\sigma_{\mu_1\cdots\mu_{\tilde{p}+1}} = dx^{\nu_1} \wedge \cdots \wedge dx^{\nu_d},
\]
the expression (38) can be rewritten as
\[
Q^M = \hat{g}_0 \int_{\phi(\Sigma)} \sum_{i=1}^{l} \beta_i \eta_i \int_{N_i} \frac{1}{\sqrt{g}} \delta^D(x - z_i(u)) \sqrt{g_u d^{(D-d)}} ud^{(d)}\phi. \tag{39}
\]
Since on the singular submanifold $N_i$ we have
\[
\phi^a(x)|_{N_i} = \phi^a(z^1_i(u), \cdots, z^D_i(u)) \equiv 0, \tag{40}
\]
which leads to
\[
\partial\phi^a \frac{\partial x^\mu}{\partial u^l}|_{N_i} = 0. \tag{41}
\]
Using this expression, one can prove
\[ J_{\mu_1\cdots\mu_{D-d}}^{\mu} \bigg|_{\bar{\phi}=0} = \frac{\sqrt{g}}{\sqrt{g_\mu}} \varepsilon^{I_1\cdots I_{(D-d)}} \frac{\partial x^{\mu}}{\partial u^{I_1}} \cdots \frac{\partial x^{\mu}}{\partial u^{I_{(D-d)}}}. \] (42)

Then we obtain a useful formula

\[ d^{(d)} \sqrt{g_\mu} d^{(D-d)} u = \sqrt{g} d^D x. \] (43)

By making use of the above formula and (39), we finally get

\[ Q^M = g_m \sum_{i=1}^l \beta_i \eta_i \int_X \delta^D (x - z_i(u)) d^D x = g_m \sum_{i=1}^l \beta_i \eta_i. \] (44)

The above expression shows that the \( i \)-th brane carries the ‘magnetic’ charge \( Q^M_i = g_m \beta_i \eta_i = g_m W_i \), which is topologically quantized and characterized by Hopf index \( \beta_i \) and Brouwer degree \( \eta_i \), the winding number \( W_i \) of the \( \phi \)-mapping.

**IV. CONCLUSION**

In this paper the \( \phi \)-mapping theory is introduced to study the \( \tilde{p} \)-branes theory, which is development of our former theories of magnetic monopoles and topological strings. We present a new topological tensor current of magnetic multi \( \tilde{p} \)-branes and discuss the inner structure of this current in detail. It is shown that every isolated zero of the vector field \( \bar{\phi} \) (i.e. order parameters) is just corresponding to a magnetic brane, \( \tilde{p} \)-brane \( \tilde{p} = D - d - 1 \).

The generalized Nambu action for multi \( \tilde{p} \)-branes can be obtained directly in terms of this topological current. The topological structure of the charges carried by \( \tilde{p} \)-branes shows that the magnetic charges is topologically quantized and labeled by the Hopf index and Brouwer degree, the winding number of the \( \phi \)-mapping. The theory formulated in this paper is a new concept for topological \( \tilde{p} \)-branes based on the \( \phi \)-mapping theory.

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