Research Article

Robust Feedback Control for Nonminimum Phase, Delayed, or Unstable Systems with Multiple Inputs

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Received 11 October 2019; Revised 13 January 2020; Accepted 10 February 2020; Published 4 April 2020

1. Introduction

The collaboration of multiple inputs at a time to regulate a single measurable output (MISO control) is often considered in process control to increase control authority, performance, or flexibility. How to use the additional degrees of freedom has led to different designations, structures, and design methods in the scientific literature. On the basis of the inputs differing significantly in their dynamic effect on the output and the relative cost of manipulating each one, valve position control [1] and habituating control [2] pursue the participation of each input along different frequency bands. Thus, a chained intervention of gradually slower loops takes place whenever the output is deviated from its set point. This also makes the slowest inputs reset the fastest inputs (input resetting control [3]) to convenient set points (midranging control [4]), which prevents the saturation of actuators and may report economical benefits in different ways. For example, since the cheapest input manages the stationary [2, 5], or since the midrange set points condition a more profitable equilibria of the whole set of manipulated inputs [6, 7]. A serial arrangement of feedback controllers [2, 8] offers a more transparent design methodology in accordance with the previous objectives. Alternatively, a parallel arrangement (Figure 1) is more general and flexible [2]. In fact, it provides the ability of several inputs collaborating along the same frequency band [9], and eventually, the control bandwidth distribution among the inputs as desired [10]. It is also common to separate a master centralised controller from the parallel structure of branch elements [11] (slaves), as in load-sharing applications [12]. This paves the way to much more complex distribution structures [13] and specific algorithms among which the model predictive control and optimization techniques are frequent [2, 4, 9]. In general, it can be regarded as a hierarchical structure where the high-level linear controller faces the single output regulation by yielding a single virtual control input. Then, a low-level control allocation algorithm coordinates the different manipulated inputs such that they together produce the desired virtual control effort, if possible [14]. Nonlinear approaches to control allocation are common in flight control, marine applications, automotive, and other industries where mechatronics prevails [15, 16]. Optimization-based allocation methods are referred in [17]. The relevance of different
multi-input control approaches is supported by recent applications in several fields such as process control [7, 18, 19], medical systems [20–22], robotics [23–25], unmanned aerial vehicles [26, 27], automotive industry [28], and electrical power systems [29].

Nonminimum phase (NMP) behaviours (or delays) are frequent in MISO process control. The design methods in [1, 2, 30] pay special attention to them inside serial arrangements of controllers. Those strategies make the inputs collaborate over different bands of frequency, such that the NMP input handles the low-frequency band and does not limit the achievable performance. However, as long as the inputs cannot collaborate along the same frequencies, a control effort reduction is not possible by using redundant control inputs. On the other hand, authors in [9, 11] present methods for some kind of parallel structure, which allows a potential collaboration of inputs over the same frequencies. Alvarez-Ramirez et al. [9] illustrate how less-expensive control effort can be obtained with the addition of redundant control inputs, but such a cost saving is obtained at the expense of a sluggish closed-loop response if any additional control input is of the NMP nature. Thus, the drawback of the method is that the inputs are forced to collaborate along the same frequency band. Schroek et al. [11] propose a master-slaves structure and develop a PQ method for control design. Firstly, the slaves are designed bearing in mind the following issues: (a) they condition the zeros of the equivalent slaves-plants avoiding that they are located in the right half-plane (RHP) and may limit the achievable closed-loop performance; (b) they determine the relative magnitude contribution to the output (a separated contribution of the inputs along the frequency band is pursued); (c) the relative phase contribution may be critical if they are out of phase when their magnitude contribution is nearly the same (destructive interference). Secondly, the master achieves the desired performance and stability. The PQ method has been proved in a hard disk drive application [11] and in a turbocharged engine control [31]. Some drawbacks of the method are the inputs eventually collaborate in different frequencies, the slaves are designed open loop with no reference to the closed-loop specifications, which can negatively interfere with the master task, and the control structure and the design method are complex when the number of inputs grows. Finally, any specific method in the scientific literature has dealt with the potential collaboration of unstable systems to the best of the author’s knowledge.

The frequency domain appears to be the most meaningful approach to determine the best input participation, under the assumption that the system can be approximated by linear models. And, the parallel structure in Figure 1 is being used to freely distribute the frequency band. A proper scaling of all $u_i$ input ranges allows fairer comparisons as it considers saturation constraints. Then, the frequency response $(y_i(u_i), i.e., p_i(j\omega)$, reveals the relative “capacity” of each input; $p_i$ plant models can include uncertainty, for example, corresponding to unmodelled dynamics or small-signal linear approximations of nonlinearities. The capacity of each $p_i(j\omega)$ plant is in reference to the closed-loop bandwidth (required performance). Inside the framework of quantitative feedback theory (QFT), the bounds express, at discrete frequencies, the set of closed-loop specifications in terms of the nominal open-loop transfer function, which initially matches the nominal plant [32, 33]. In this way, special bounds [10] depict in a transparent and quantitative way the capacity of each plant of the MISO system to achieve the specifications. A comparison of those bounds helped to distribute the control bandwidth among loops in order to get the set of controllers of minimum gain at each frequency [10]. Several inputs could either collaborate at a certain frequency to reduce the individual control effort or be inhibited to avoid useless fatigue and saturation risk of actuators. However, only magnitude contributions of plants (inputs) were taken into account, which only works well for minimum-phase (MP) plants. This paper develops a method that takes into account the relative phase of the branches under the potential presence of RHP dynamics (zeros or poles) or delays. If branches were out of phase, they would fight each other and the total output magnitude would drop considerably.

The paper is organised as follows: Section 2 formulates the feedback control problem to be solved inside a QFT framework. Then, it puts forward the challenges to decide the best control bandwidth distribution when output/input relations are not in the minimum phase; in particular, the unstable case is analysed by an example. Finally, the section summarises the methodology to find the set controllers of smallest gain at each frequency. Section 3 applies it for the dual-input control of a continuous stirred-tank reactor (CSTR), where a convenient distribution of feedback improves the system controllability despite the nonminimum phase behaviour of one manipulated input. Conclusions are in Section 4.

2. MISO Control to Reduce the Feedback Demand

2.1. Statement of the Robust Regulation Problem. Figure 1 depicts an architecture of $n$ parallel feedback controllers $c_{i=1,...,n}(s)$. Each plant $p_i(s)$ models the dynamic response from the manipulated input $u_i$ to the single controlled

![Figure 1: Parallel control structure for MISO systems.](image-url)
output \( y \). The plant \( p_d(s) \) determines how a nonmeasurable disturbance \( d \) deviates the output \( y \) from its set point \( r \). Each actuator contribution in the steady state can be conveniently selected by set points \( r_{n_i} \) for \( n-1 \) manipulated inputs at most. At least a plant (i.e., \( p_1 \)) will work in the low-frequency band, and \( r_{n_{j+1}} \) will compensate any output deviation in the steady state. External input \( v \) represents the high-frequency noise associated with the single sensor that measures the output \( y \).

Uncertainty of plant models is being explicitly considered in the collaborative control design. Thus, let us take a \( 1 \times (n + 1) \) vector of plant transfer functions \( P = [p_{i=1,...,n}, p_d] \), which can be any element of

\[
\mathcal{P} = \{P(s, q_i) : q_i \in \mathcal{Q}\},
\]

where \( q_i \) is a vector with the \( m \) uncertain parameters of plant models and \( \mathcal{Q} \) is the set in \( \mathbb{R}^m \) that is defined by all its possible values.

Then, if a certain collaborative model is desired for \( d \) disturbance rejection at the \( y \) output, QFT defines the robust specification in the frequency domain \( s = j\omega, \omega = [0, \infty) \) as

\[
\left| T_d(j\omega) = \frac{y(j\omega)}{d(j\omega)} \right| = \left| \frac{p_d(j\omega)}{1 + l_i(j\omega)} \right| \leq |W_d(j\omega)|; \quad \forall P(j\omega), \forall \omega,
\]

where \( l_i = \sum_{j=1}^{n} l_j \) denotes the total open-loop transfer function that is contributed by branches \( l_i = p_ic_i \). Additionally, occasionally changing in \( r_{n_i} \) may be of interest, and robust specifications on \( (y_r/r_d) \) responses can be also taken into account. From a mathematical point of view, these can be treated identically to (2). Thus, they are here being omitted for simplicity.

Robust stability in \( s = j\omega, \omega = [0, \infty) \) is being defined by the set

\[
\left| T_i(j\omega) = \left[ \frac{l_i(j\omega)(1 + \sum_{j=1}^{n} l_j(j\omega))}{1 + l_i(j\omega)(1 + \sum_{j=1}^{n} l_j(j\omega))} \right] \right| \leq W_s, \quad i = 1, \ldots, n; \quad \forall P(j\omega), \forall \omega,
\]

where the upper tolerance \( W_s \) means the minimum distance from \( (l_i(j\omega)(1 + \sum_{j=1}^{n} l_j(j\omega)))/(1 + l_i(j\omega)(1 + \sum_{j=1}^{n} l_j(j\omega))) \) to the critical point \(-1\). It is usually of interest to assure a certain phase margin \( \text{PM}_i \) by taking

\[
W_s = \frac{1}{2 \sin(\text{PM}_i/2)},
\]

or, if preferred, to choose a certain gain margin \( \text{GM}_i \) by taking

\[
W_s = \frac{1}{\text{GM}_i - 1}, \quad \text{GM}_i > 1.
\]

Robust stability (3) is straightforward related to individual loops, while robust performance (2) is a collaborative task among loops.

The \( c_i \) controller design is being executed by loop-shaping a nominal open-loop \( l_{c_i}(j\omega) = p_{a_i}(j\omega)c_i(\omega) \) on a mod-arg chart, \( l_{c_i}(j\omega) \) has to meet the QFT bounds \( \beta_{l_i}(\omega) \), which are computed at discrete frequencies \( \omega \in \Omega \). These \( \beta_{l_i} \) bounds translate the closed-loop robust specifications (2)–(3) in terms of \( l_{c_i} \), and they can be computed by using the command \texttt{genbnds} of Terasoft QFT Toolbox [34]. It handles specifications in the general form

\[
\frac{A + BG}{C + DG} \leq |W|,
\]

where \( G \) is the \( c_i \)-controller to be designed and functions \( A, B, C, D, \) and \( W \) can be easily identified by comparison with (2)–(3).

The aim is obtaining the feedback control solution \( c_{i=1,...,n} \) that achieves (2)–(3) using as small as possible control gain at each frequency. The challenge is how to distribute the collaborative task over the frequency band among loops despite the fact that plants \( p_{i=1,...,n} \) contain RHP poles, RHP zeros, or delays.

2.2. Challenges in the Collaboration of Plants with RHP Dynamics. Let us suppose a system where two inputs can intervene in the output regulation. The uncertainty-free plant models are

\[
p_1(s) = \frac{1}{s + 1}, \quad p_2(s) = \frac{1}{s - 1}, \quad p_d(s) = 1.
\]

The disturbance rejection model

\[
W_d(s) = \frac{1.75s}{(s + 12)}
\]

defines the desired \( y/d \) behavior (2). For a minimum phase margin of 40° at both loops (3), the tolerance choice (4) is

\[
W_{s_{i=1,2}} = 1.462.
\]

QFT bounds will represent those closed-loop specifications in terms of open-loop transfer functions at the discrete set of \( \omega \)-frequencies:

\[
\Omega = [0.1, 0.3, 1, 2, 3, 6, 10, 20, 50] \text{ [rad/s]}
\]

In order to compare the capacity of each plant in the regulation task, let us firstly solve the problem assuming that only one of the two plants (inputs) participates at a time. For the case of \( p_1 \) alone participation, the bounds \( \beta_i \) in Figure 2(a) show the feedback demand in terms of \( l_i = l_{c_i} \). The initial location of this open-loop function is \( p_1 \) since \( c_1 = 1 \). A possible solution \( l_i = p_i c_i \) that fulfills the bounds is given by

\[
c_1 = \frac{7.3(s + 1)}{s ((s/25)^2 + (2 \times 0.6/25)s + 1)}
\]

On the other hand, let us only use \( p_2 \) in the regulation. As long as \( p_2 \) is not a minimum phase plant, the QFT method allows two ways of conducting the loop-shaping. The former is depicted in Figure 2(b) where bounds \( \beta_{c_2} \) express the
feedback gain in terms of $l_1 = l_2 = c_2 p_2$, which is not minimum phase and initially $(c_2 = 1)$ matches $p_2$. Another procedure \cite{35, 36} is shown in Figure 2(c). In this case, bounds $\beta_l$ express the feedback demand in terms of $l_2$, which initially matches $p_2'$. This plant corresponds to a $p_2$ plant whose RHP pole has been mirrored to the LHP (in this example, $l_2$ is equal to $p_1$). Then, $l_2'$ and $p_2'$ are minimum phase, and the phase shift due to $p_2$ RHP pole has been translated to the bounds $\beta_l'$. Let us note as the relative position between plant frequency responses ($p_2$ or $p_2'$) and their respective bounds ($\beta_l$ or $\beta_l'$) is the same. As expected, both procedures can yield the same solution, for example, $c_2 = \frac{7.3(s/0.8 + 1)}{s((s/25)^2 + (2 \times 0.7/25)s + 1)}$ \hspace{1cm} (12)

Placing the open-loop function exactly onto the bounds at each design frequency means using the minimum controller gain to achieve the specifications, which favours a smaller control gain beyond the gain cross-over frequency $\omega_{gc}$ \cite{32, 37}, where feedback becomes useless or even harmful (e.g., high-frequency unmodelled dynamics or sensor noise amplification that can saturate the control input). Then, the minimum quantity of feedback to achieve the specification at each design frequency is the distance between the plant frequency response and the corresponding bound. To quantify it, Rico-Azagra et al. \cite{10} proposed the use of bounds $\beta_l(\omega)$, which were calculated scaling classical bounds $\beta_l(\omega)$ by the nominal plant magnitude $|p_2(\omega)|$, i.e., $\beta_l(\omega) = (|p_2(\omega)|/|p_2(\omega)|)$. Performing this on bounds of Figure 2 yields the bounds of Figure 3.

At any frequency, several locations onto the bound are possible for the loop-shaping: one for each phase in the horizontal axis. Thus, the design phase becomes important. Let us analyse the lowest frequency $\omega = 0.1$ in Figure 2. Either the control solution through $p_1$ (11) or through $p_2$ (12) contains an integrator to achieve zero steady-state error $r - y$ when a $d$-step happens. The integrator is usually the first element added during the loop-shaping, which produces a phase lag of $-90^\circ$. Secondly, the control gain is raised to fulfil the low-frequency bound. After performing these operations in Figures 2(a) and 2(b), the same control element ($7.3/s$) makes $l_1(\omega_0)$ and $l_1(\omega_0)$ to fulfil the lowest frequency bounds, $\beta_l(0.1)$ and $\beta_l(0.1)$, respectively. Obviously, as long as both plants (7) have the same magnitude at low frequencies, both demand the same control gain. However, the design phase to measure and compare the demanded gain is different: about $-90^\circ$ for $l_1$ in Figure 2(a) and about $-180^\circ$ for $l_2$ in Figure 2(b). The exact quantities can be measured on Figures 3(a) and 3(b), respectively. Applying the same reasoning for other frequencies may be a challenging task since it is not so obvious which design phases have to be compared between $l_1$ and $l_2$. However, those phases for comparisons become the same on the pictures for $l_1$ and $l_2$, see Figures 2(a) and 2(c). In summary, the bound height (feedback demand) can be easily compared between bounds in Figures 3(a) and 3(c) looking at the same phase. Then, bearing only this criterion in mind, both plants $p_1$ and $p_2$ could be indistinctly used to solve the regulation problem since both require the same control gain at any frequency.

However, the fact that two plants require the same feedback when working by their own does not necessarily imply that a collaboration of both would have that quantity of feedback inside an MISO structure. This is only true for minimum-phase plants: for example, the use of two twin plants $p_1$ would yield two controllers ($c_1/2$). On the contrary, $p_1$ and $p_2$ (or $l_1$ and $l_2$) are completely in counter phase at $\omega = 0$, and are only in phase at $\omega = \infty$, as shown in Figures 2(a) and 2(b). Then, they can collaborate over the high frequencies but not over the low frequencies to reduce the individual feedback gain since the function $l_1$ is the vectorial sum of $l_1 + l_2$. Any attempt of designing minimum-phase controllers $c_1$ and $c_2$ to make $p_1$ and $p_2$ collaborate at low

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**Figure 2:** Loop-shaping bounds: (a) $\beta_{l1}$, (b) $\beta_{l2}$, and (c) $\beta_{l2}$; frequencies in different colours.
frequencies would yield controllers of higher gains than $c_1$ (11) and $c_2$ (12). As a general rule of thumb, to build a vector $l_1 (j\omega)$ with vectors $l_1 (j\omega)$ and $l_2 (j\omega)$ that have a smaller magnitude than $l_1$, the phase difference between $l_1 (j\omega)$ and $l_2 (j\omega)$ has to be less than $\pm 90^\circ$. Looking back at Figure 2, the information about the $p_i$ phase that can be read in Figure 2(b) has been translated to the bounds that appear shifted in Figure 2(c). Therefore, the phase shift between true plants $p_1$ and $p_2$ can be read in the phase shift between bounds $\beta_{c_1}$ and $\beta_{c_2}$ in Figure 3; the vertical segment is taken as a reference that marks the critical phase ($-180^\circ$) on the bounds.

Once detected at which frequencies the two plants are not out of phase ($\omega \geq 1$), the amount of feedback that each one would demand becomes of interest. This focuses on the height difference (magnitude in dB) between bounds $\beta_{c_1}$ and $\beta_{c_2}$ in Figure 3 at the design phase $\Theta_d$ that is selected for each frequency $\omega \geq 1$. Regardless of the bound height difference, a collaboration would always reduce the gain of controllers (let us remind the loops are not out of phase). However, these reductions may not justify the major complexity of performing the design. A magnitude difference of $20\log_2$ between bounds is a practical guideline to attempt a collaboration, and it is funded on the ideal collaboration of two equal plants, which would reduce the controller gain two times.

Applying previous guidelines, Table 1 summarises the frequency band distribution between the two loops of the example (7). The next step is the loop-shaping of $l_1$ and $l_2$ that achieves it, for which viewing the real phase of $l_1$ and $l_2$ becomes of great importance. Thus, bounds of the kind of $\beta_{c_1}$ in Figure 2(c) are not advisable, but bounds $\beta_{c_2}$ in Figure 2(b) are preferred. The standard procedure to compute $\beta_{c_2}$ bounds and to perform iteratively the loop-shaping of each $l_i$ is fully described in [10]. The main steps for the present example are illustrated in Appendix A.

The final elements that integrate the MISO control are

\[
c_1 = \frac{63.47s(s/1.3 + 1)}{(s/0.2 + 1)(s/0.5 + 1)(s/32)^2 + (2 \times 0.6/32)s + 1} \quad \text{(13)}
\]

\[
c_2 = \frac{7.01(s/2.7 + 1)(s/1.7 + 1)(s/0.23 + 1)}{s((s/0.8)^2 + (2 \times 0.8/0.8)s + 1)((s/32)^2 + (2 \times 0.6/32)s + 1)} \quad \text{(14)}
\]

\[
c_e = \frac{4(s/0.8 + 1)(s + 1)}{s^2((s/35)^2 + (2 \times 0.7/35)s + 1)} \quad \text{(15)}
\]

to fulfill the specifications. It has forced both plants to collaborate over the whole frequency band, which has implied a huge gain at frequencies where both plants are out of phase ($\omega < 1$). Over the frequencies where only one branch works, the MISO control is obviously equal to any SISO control ($\omega > 1$).
Table 1: Frequency band distribution between control inputs of system (7).

| ω  | 0.1 | 0.3 | 1   | 2   | 3   | 6   | 10  | 20  | 50  |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \(P_1\) | ×   | ×   | ×   | ×   | ×   | ×   | ×   | ×   | ×   |
| \(P_2\) | ×   | ×   | ×   | ×   | ×   | ×   | ×   | ×   | ×   |

2.3. Proposed Methodology

2.3.1. Stage 1: Feedback Sharing at Each Frequency of Design. This first stage distributes the control bandwidth among the plants that could contribute to the robust regulation problem (2)–(3) in order to reduce as much as possible the amount of feedback (gain) that is demanded by the controllers at each frequency. These decisions are made by the comparison of control bounds \(\beta_i^c(ω)\), \(i = 1, \ldots, n\).

\(\beta_i^c(ω)\) quantifies the necessary feedback \(|\varepsilon_i(jω)|\) at each discrete frequency \(ω\) in the event that a single plant \(p_i(jω)\) worked to achieve the robust control requirements. In order to calculate the said bounds, nominal MP plants \(p_{0}\) must be employed. For those plants that were not minimum phase, \(p_{0}\) is obtained from \(p_{0}\) by mirroring all RHP dynamics into the LHP and neglecting any possible delay (it fulfils \(|p_{0}(jω)| = |p_{0}(jω)|\)). Nominal plants can be any \(P(jω)\) of the uncertainty set (1). Then, common QFT bounds on the nominal open-loop function \(l_0^c(jω)\) are computed, which yields \(\beta_i^c(ω)\). These are computed in the event that only \(p_{0}(jω)\) assumes the regulation task. Eventually, \(\beta_i^c(ω) = (\beta_i^c(ω)|p_{0}(jω)|)\). Bounds \(\beta_i^c(ω)\) shift with respect to 180° an amount equal to the phase difference between \(p_{0}(jω)\) and \(p_{0}(jω)\).

The \(\beta_i^c(ω)\) bound comparison among loops \(i = 1, \ldots, n\) focuses at the desired design phase \(θ_d \in (−360°, 0°]\), which is particularly chosen for each frequency and is unique for all loops (\(θ_d\) is the phase in which the nominal open-loop \(l_0^c\) will be modified through the addition of gain, pole, or zero elements during the loop-shaping at Stage 2). The criteria for the control bandwidth distribution are

(a) **Criterion 1:** \(m\) plants \((m ≥ 2)\) can collaborate at \(ω\), if the phase differences between the \(m\) bounds \(\beta_i^c(ω)\) are less than ±90° and if the heights of the said bounds do not differ from each other more than ±20 log \(m\) dB at the design phase \(θ_d\). Additionally, there is no bound \(j \neq k\) of lesser height. Consequently, the remaining plants \(j \neq k\) should be inhibited, reducing their control gain as much as possible at \(ω\) during the design stage.

(b) A single plant is in charge of the regulation at \(ω\) if criterion 1 is not met.

(c) The highest \(m\) is sought \((m ≤ n)\) since the gain saving on each one of the \(m\) branch controllers is proportional to \(m\), i.e., to the number of plants that collaborate.

The amount 20 log \(m\) is a practical guideline. It is funded on the ideal collaboration of \(m\) equal plants, which would reduce the controller gain \(m\) times. The phase shifting between bounds is vital when plants contain RHP dynamics and delays. This argument condition determines if the loops are not out of phase, and therefore, \(|l_0^c(jω)| = |\sum l_0^c(jω)|\) can be built with \(|l_0^c(jω)| ≤ |l_0^c(jω)|\).

2.3.2. Stage 2: Design of Controllers. The designs of the \(i = 1, \ldots, n\) loops are carried out sequentially such that \(l_0^c(jω)\) fulfills the specifications (bounds \(\beta_i^c(ω)\)) according to the feedback sharing that was established in Stage 1. The general procedure was provided in [10], and it does not depend on the plants being MP or not. Bounds \(\beta_i^c\) are computed and referenced to the true plant \(p_{0}\) being MP or not.

3. Application: MISO Control of a Continuous Stirred-Tank Reactor

In this section, the benefits of the proposed methodology are shown on a recurrent benchmark of the chemical and materials industry, the continuous stirred-tank reactor (CSTR) of unquestionable importance in the transformation of raw materials into valuable chemicals or products [6, 38, 39].

3.1. The Process and the Control Design. The control setup of the MISO plant is depicted in Figure 5. A coolant flow (usually water) through a jacket \(u_2 = F_j\) removes the necessary energy to prevent the exothermic and irreversible reaction runaway and to regulate the reactor temperature \(y = T_r\). Due to the limited heat-removal capacity of the coolant flow, the manipulation of the reactant flow \(u_1 = F\) can contribute to the temperature control. The linear dynamic models linked to these manipulated inputs are \(p_1\) and \(p_2\). The work [6] presented a MISO control strategy inside a series control architecture following the VPC of [40]. In this paper, the parallel architecture of Figure 1 is used. The jacket coolant flow (the fastest actuation) midranges around \(r_{u_1} = F\), while large reaction temperature excursions are compensated with the feed flow rate (the slowest actuation). Thus, beyond the dual-input collaboration in disturbance
compensation, the MISO strategy can achieve the highest possible production rate: $F$ is maximised when $F_j$ set point is set to maximum. Three disturbance inputs are being considered: the feed concentration $d_1 = C_{a_{in}}$, the feed temperature $d_2 = T_{in}$, and the inlet coolant temperature $d_3 = T_{j_{in}}$, whose contributions to the output $T_r$ deviation are being represented by linear plants $p_d$, $p_{d_1}$, and $p_{d_2}$, respectively. Appendix B details the procedure on how to obtain all linear plant models from nonlinear differential equations of dynamic mass and energy balances \[6\]. Small-signal linearisation is executed about the system operating point

$$ \mathbf{q}_j = [T_r, F_e, F_{j_r}, C_{a_{in}}, T_{in}, T_{j_{in}}] $$

which relates the equilibrium (subscript “e”) of main process variables. A discrete set of 192 possible operating points is being considered (see Appendix B), which yields uncertainty about linear plant models. Thus, there is a plant matrix $\mathcal{P}(s)$ that collects 192 vectors $\mathbf{P}(s; \mathbf{q}_j) = [p_1, p_2, p_d, p_{d_1}, p_{d_2}]$. Figure 6 shades the envelopes of magnitude frequency responses of all plant cases and highlights the nominal plants.

which correspond to the nominal values of equilibrium (16): $T_{re} = 350 \text{ K}$, $F_o = 4.377 \times 10^{-3} \text{ (m}^3/\text{s})$, $F_{j_o} = 11.3 \times 10^{-3} \text{ (m}^3/\text{s})$, $C_{a_{no}} = 8.01 \text{ (kmol/m}^3)$, and $T_{in} = T_{j_{in}} = 294 \text{ K}$.

Regarding Figure 6, the gain of $p_1$ is higher than that of $p_2$ over the whole frequency band; this reveals the feed flow $F$ is the most powerful regulation variable. Its counterpart is the feed temperature $T_{in}$ which gets a significant impact on dynamic controllability. When the feed is colder than the reactor ($T_{in} < T_r$), the immediate effect of increasing the feed flow rate is a temporary decrease in the reactor temperature, i.e., $p_1$ behaves as a nonminimum phase (NMP) plant. In particular, the RHP-plane uncertain zero of $p_1$ at $\omega \in [0.0014, 0.0029]$ makes the $p_1$ participation unsuitable in control tasks over high frequencies. Then, a convenient frequency distribution of MISO dynamic controllability is of importance and can be quantitatively accomplished through the robust frequency domain method that this paper develops. Let us also mention that $p_2$ plant has a negative gain since raising the jacket flow rate $F_j$ makes the reactor temperature $T_r$ to drop.

Three performance specifications for robust disturbance rejection are defined following (2), being $p_d$ equal to $p_{d_1}$, $p_{d_2}$, and $p_{d_3}$, respectively. The required performance is that in the case of a maximum disturbance happens, the reactor temperature deviation has to be less than 1 K; and this maximum deviation has to be reduced to 0.25 K no later than 2.5 h after and fully extinguished in the steady state (settling time less than 4.0 h to stay within a ±5% band). To ensure these conditions, the performance upper model is

$$ W_d(j\omega) = \frac{6796j\omega}{(j\omega/0.0004 + 1)^2} $$

Besides, two robust stability specifications (3) adopt as upper tolerance
to guarantee minimum phase margins of 40° on each of the two loops (4). The discrete set of \( \omega \)-frequencies to compute QFT bounds is

\[
\Omega = [1, 2, 3, 5, 8, 10, 50] \times 10^{-4} \text{ [rad/s]}.
\] (24)

### 3.1.1. Stage 1: Feedback Sharing at Each Frequency of Design.

Bounds \( \beta' \) are calculated. Figure 7 depicts them at the most illustrative frequencies, which yields the following conclusions according to the criteria in Section 2.3:

At low frequencies \( \omega \leq 3 \times 10^{-4} \), bounds \( \beta'_{1o} \) and \( \beta'_{2o} \) are found phase-shifted less than ±90°. However, \( \beta'_{1} \) and \( \beta'_{2} \), which make the participation of \( p_2 \) useless in the regulation. Thus, the branch of \( p_1 \) should assume only the \( y \) regulation, and the branch of \( p_2 \) should be switched off at \( \omega \leq 3 \times 10^{-4} \).

At medium frequencies \( 3 \times 10^{-4} < \omega \leq 8 \times 10^{-4} \), bounds \( \beta'_{1o} \) and \( \beta'_{2o} \) are found phase-shifted less than ±90° and present a similar height around \( \Theta_d = -90° \). Thus, plants \( p_1 \) and \( p_2 \) can collaborate over this interval of frequencies.

At frequencies \( \omega > 8 \times 10^{-4} \), the phase shifting between \( \beta'_{1o} \) and \( \beta'_{2o} \) increases beyond ±90° such that the collaboration between plants is no longer possible. Thus, \( p_2 \) should assume the control tasks while \( p_1 \) should be inhibited by making \( l_1(j \omega) \approx 0 \), which avoids the limitations introduced by the RHP zero of \( p_1 \).

### 3.1.2. Stage 2: Loop-Shaping of Controllers.

To fulfil the objectives set in Stage 1, the loop-shaping of the two nominal open-loop functions is carried out following a sequential methodology as detailed in [10]. Figure 8 shows the final arrangement of bounds \( \beta_{1,2} \) at design frequencies (24) and the nominal open-loop transfer functions \( l_{4,1,2} \) that satisfy them. Let us note that \( l_{4o} \) contains an RHP zero. The resulting controllers are

\[
c_1 = \frac{3.2 \times 10^{-5} (s/0.0003 + 1)(s/0.0084 + 1)}{s (s/0.0017 + 1)^2 ((s/0.015)^2 + (2 \times 0.5/0.015)s + 1)}.
\] (25)

\[
c_2 = \frac{-0.37 (s/0.0003 + 1)(s/0.0018 + 1)}{(s/0.001 + 1)((s/0.0024)^2 + (2 \times 0.8/0.0024)s + 1)}.
\] (26)

### 3.2. Results: Analysis and Comparatives.

Figure 9 proves the fulfillment of control specifications for the whole set of 192 plant cases: Figure 9(a) shows what concerns the robust rejection at the output of the three disturbances and the performance specification (2) and (22), and Figure 9(b) shows what concerns the robust stability of the two loops and the stability specification 3 (23). Figure 10 illustrates the frequency band allocation between both loops: \( l_1(j \omega) \) matches \( l_1(j \omega) \) over low frequencies, \( l_2(j \omega) \) matches \( l_2(j \omega) \) over high frequencies, and \( l_3(j \omega) \) over midfrequencies.

Closed-loop time responses of main system variables are shown in Figure 11. These results have been obtained using the proposed control system in the nonlinear model of CSTR (at this point it is recommended to see Appendix B for a full understanding of the physical units and experiments). The simulation shows the system behavior for step-type disturbances in the feed temperature (\( \Delta T_{in} = 5 \text{ K at } t = 2 \text{ h} \)), in the coolant temperature (\( \Delta T_{in} = -5 \text{ K at } t = 8 \text{ h} \)), and in the feed concentration (\( \Delta C_{in} = 5% \text{ at } t = 14 \text{ h} \)). Measured output \( T_r \) incorporates sensor noise, which is generated by a band-limited white noise source \( v(t) \) (noise power: 0.02; sample time = 100). Several set points on the coolant flow rate \( r_C_r \) between 65% and 85% of \( F_r \) maximum capacity have been tested. In all cases, the reactor temperature \( T_r \) deviates less than the maximum permitted (1 K) and recovers the set point (350 K) in less than 4.0 h after any
disturbance happens. The steady-state feed flow $F$ increases when the coolant set point $r_{F_j}$ increases. This proves the smart VPC strategy [6] of indirectly maximizing the production rate by using a dual-input control. The collaboration of inputs (plants) in disturbance rejection is as follows: when a disturbance happens, the coolant flow rate $F_j$ quickly reacts to compensate the reactor temperature deviation. Then, as the feed flow $F$ takes control of the situation, $F_j$ is progressively reset. Eventually, $F_j$ returns to $r_{F_j}$ after approximately 30 min. $F$ cannot return to a particular set point $r_F$ while any disturbance persists. However, $r_{F_j}$ can be wisely moved to bring $F$ towards a particular range: a larger $r_{F_j}$ involves a larger $F$, which increases the production rate; on the other hand, a smaller $r_{F_j}$ pursues saving the amount of $F_j$. Both $F$ and $F_j$ move between their margins of operation [0% and 100%], despite the fact perturbations are of maximum amplitude. The evolution of other internal system variables, like $T_j$ and $C_{aj}$, is also included in the simulations.

Following, there are argued benefits of MISO control versus SISO control. In this last case, the same robust control
specifications are being satisfied with solely a loop intervention. Let us denote $F$-SISO control when $u_1 = F$ is in charge of the regulation tasks, and thus $u_2 = F_j$ remains at a constant value. In that case, the control design process yields

\begin{equation}
    c_{1_{\text{SISO}}} (s) = \frac{3.1354 \times 10^{-5} \left( s/0.0012 + 1 \right) \left( s/0.0004 + 1 \right) \left( s/0.00045 + 1 \right) }{s \left( s/0.003 + 1 \right) \left( s/0.006 + 1 \right)}.
\end{equation}

(27)

On the other hand, let us denote $F_j$-SISO control when $u_2 = F_j$ is in charge of the regulation task and $u_1 = F$ remains at a constant value. Thus, the single controller results in

\begin{equation}
    c_{2_{\text{SISO}}} (s) = \frac{-1.2402 \times 10^{-4} \left( s/\left(3.488 \times 10^{-4}\right) \right)^2 + \left( 2 \times 0.76/\left(3.488 \times 10^{-4}\right) \right)s + 1 }{s \left( s/0.001 + 1 \right) \left( s/0.004 + 1 \right)}.
\end{equation}

(28)
Figures 12 and 13 depict frequency and time responses, respectively; their first and second columns of plots correspond to $F$-SISO and $F_j$-SISO control, respectively. The beam of frequency responses in Figures 12(c)–12(f) corresponds to the 192 cases of plant uncertainty. The time experiments of Figure 13 are run in a nonlinear simulator of CSTR and under the same conditions than the MISO control. In particular, Figure 11(e) details the changes of disturbance inputs, and five cases are being considered: for $F$-SISO, the coolant flow $F_j(t)$ takes constant values $65\%$, $70\%$, $75\%$, $80\%$, and $85\%$; and for $F_j$-SISO, the feed flow $F(t)$ adopts constant values $55\%$, $57.5\%$, $60\%$, $62.5\%$, and $65\%$. Correlation between time and frequency responses agrees with the superiority of MISO over SISO control as follows.

The required performance and stability cannot be achieved by $p_1$ on its own due to the limited bandwidth imposed by the RHP zero; $F$-SISO (27) yields a reasonable trade-off solution between disturbance rejection and stability commitments. Figure 12(e) shows how $T_{d_1}$ and $T_{d_3}$ slightly violate $W_d$ over midfrequencies, despite the fact this worsening is hardly noticeable in Figure 13(a) in comparison with Figure 11(a). Additionally, Figure 12(e) shows violations of robust stability $W_{a_1}$, which means a reduction of the phase margin from $40^\circ$ to $35^\circ$. Nevertheless, the major drawback of $F$-SISO control can be noticed from the
comparison of control gains in Figure 12(a). $p_1$-plant, which has an RHP zero at $\omega \in [0.0014, 0.0029]$, has been forced to work beyond $\omega = 0.0008$ at the expense of a huge increase of the controller gain $|c_{1_{\text{SISO}}}(j\omega)|$ over high frequencies. It produces an inadmissible sensor noise amplification at the control input $F(t)$, see Figure 13(c), which did not happen in MISO control—see Figures 11(c) and 11(d). In that case, a minimum phase $p_2$-plant worked over the high-frequency band, despite apparently being less powerful (less gain) than a nonminimum phase $p_1$-plant, see Figure 6(a). Therefore, $|c_2(j\omega)| \ll |c_{1_{\text{SISO}}}(j\omega)|$ over high frequencies reports important benefits. Additionally, $|c_1(j\omega)| \approx |c_{1_{\text{SISO}}}(j\omega)|$ at low
Figure 13: SISO control of CSTR. Closed-loop time responses: (a), (c), and (e) $T_1$-SISO; (b), (d), and (f) $F_j$-SISO.

Figure 14: Continued.
frequencies, which means that the same $F(t)$ is needed to manage the steady states at Figure 13(c) and at Figure 11(c) if the noise components are dismissed from Figure 13(c).

On the contrary, Figures 12(d) and 12(f) check how the minimum phase $p_2$-plant can achieve the specifications on its own, which means the temperature recovery $T_r(t)$ meets...
the time requirements in Figure 13(b) as it happened in Figure 11(a). The counterpart is that the plant has to do the plant work at $\omega \leq 0.0003$, despite the fact that $p_1$ is less powerful (gain) than $p_1$ (see Figure 6(a)). It results in higher control gains $|c_{2silo}(j\omega)| > |c_1(j\omega)|$ over low frequencies and specifically at $\omega = 0$. In this way, $\delta F$, in each steady state, see Figure 13(f), is larger than $\delta F$ in each steady state, see Figure 11(c). As an example, after the step change of the feed temperature ($\delta T_{wp} = 5 \text{K}$) at $t = 2 \text{h}$, the SISO control (case $F = 65\%$) requires a steady-state variation $\delta F_j$ of $14\%$, which approaches the available maximum of $F_j$; meanwhile, the MISO control (case $r_{FP} = 85\%$) only required a steady-state variation $\delta F$ of $5\%$. Additionally, $|c_2(j\omega)| = |c_{2silo}(j\omega)|$ at high frequencies, which means the same sensor noise amplification at $F_j$ in Figure 13(f) and in Figure 11(d).

In summary, the superiority of MISO versus the SISO control of CSTR relies on the following facts. A better performance for reactor temperature control can be achieved without high sensor noise amplification at the manipulated inputs, which prevents actuator saturation and fatigue. One input $F$ is in charge of the steady state, which resets the fastest input $F_j$ to prescribed values $r_{FP}$. This is important from two points of view. As the fastest input returns to the set point after a disturbance rejection, it preserves the initial manoeuvring range to quickly react to new disturbances. Additionally, $r_{FP}$ gives places to infinite combinations of manipulated inputs to achieve the same reactor temperature. Some convenient ones are increasing $r_{FP}$ to maximize the production rate $F$ or decreasing $r_{FP}$ to reduce the coolant cost $F_j$.

4. Conclusions

This work has presented a control solution where multiple inputs to a system can be used to govern its single measurable output. The method aimed at parallel controllers with the smallest possible gain at each frequency to achieve prescribed robust stability and robust performance in disturbance rejection. At a frequency, the gain of a branch controller was reduced when the magnitude of the associated plant contributed significantly to increasing the total magnitude of the vector sum of the branches (each branch included a controller and a plant). However, this was not easily to foresee due to counter-phase effects of RHP dynamics (or delays), due to uncertainties in the plant models, or due to the intervention of a great number of plants (inputs). The quantitative feedback theory (QFT) framework succeeds in the challenge. Then, some QFT control bounds were proposed to quantify the capacity of each plant to achieve the performance at each frequency. The lower the height the control bounds represented, the lower the control magnitude required by the associated plant. Besides, RHP dynamics and delays resulted in a phase shifting of these control bounds. Thus, comparing not only their relative magnitude but also their relative phase, the criteria of plant collaboration or inhibition at certain frequency to yield the control solution of the smallest gain were established. At the end of this first stage, the frequency band was allocated amongst the plants. At a second stage, the loop-shaping of the controllers carried out the predefined regulation sharing. Classical QFT bounds were computed to guide each loop-shaping. To illustrate the method, the MISO control of an unstable system was conducted.

As a challenging example, the temperature of a continuous stirred-tank reactor was regulated through the coolant and feed flows pulling together along frequency bands that slightly overlapped. The feed flow capacity dominated the steady state, but from a dynamic point of view, it involved a temporary decrease in the reactor temperature. This nonminimum phase behaviour made useful the contribution of the coolant flow in the high-frequency band to improve the performance. Several feed flow temperatures, cooling flow temperatures, and reactant concentrations were considered. The provided control design method achieved a quantitative frequency distribution of MISO dynamic controllability in order to meet the prescribed robust performance and stability. Besides, a conveniently chosen set point for the coolant flow (midrange variable) governed indirectly the feed flow in the steady state, which conditioned the production rate.

Appendix

A. Design of Controllers for a Dual-Input System with an Unstable Plant

As part of the example in Section 2.2, the sequential loop-shaping of $c_1$ and $c_2$ is performed with Table 1 aims. Figure 14 illustrates the procedure in detailed steps (marked with superscripts). Steps 0, I, and II belong to the same first iteration, which explains the design of $c_1$ and $c_2$. Further iterations (step III) are required to optimize both designs looking for the strictly minimum amount of feedback (gain) at each frequency, which usually trades off with a reasonable order of the controllers.

0 Initial bound computation is drawn with $c^{0}_{1} = 0$ for the loop 1 (Figure 14(a)) and with $c^{0}_{2} = 0$ for the loop 2 (Figure 14(b)).

(I) Loop-shaping of $l_2$ (see Figure 14(d)): to achieve $l_2 \approx l_2$ at $\omega < 1$, $l_2$ is located onto bounds $\beta_1(0.1)$ and $\beta_2(0.3)$. At $\omega \geq 1$, $l_2$ will collaborate with $l_1$, so $l_2$ can slightly violate the bounds. After step I, the controllers are

$$c^{I}_{1} = c^{0}_{1} = 0,$$

$$c^{I}_{2} = \frac{7.5(s/1.8 + 1)}{s((s/32)^2 + (2 \times 0.6/32)s + 1)} \quad (A.1)$$

As a consequence, new bounds $\beta_{HI}$ result in Figure 14(c).

(II) Loop-shaping of $l_1$ (see Figure 14(c)): $l_1$ should fulfil all the bounds if no more iterations were desired. However, this would make hard the design, and require a high-order controller. Further iterations are recommended. A simpler controller is designed despite the fact it temporarily violates the bounds that are hard to satisfy ($\omega = 1$). It yields...
three nonlinear first-order differential equations describe the dynamic balance of component A inside the reactor (irreversible exothermic liquid-phase reaction A → B)

\[
\frac{dC_A(t)}{dt} = \frac{F(t)}{V_r} \left( C_{in}(t) - C_A(t) \right) - k_a e^{-\left( E/R \right)RT_r(t)} C_A(t).
\]  

(B.1)

(ii) The dynamic balance of energy inside the reactor

\[
\frac{dT_r(t)}{dt} = \frac{F(t)}{V_r} \left( T_{in}(t) - T_r(t) \right) - \frac{\lambda}{\rho c_p} k_a e^{-\left( E/R \right)RT_r(t)} C_A(t) + \frac{Q}{V_r} - \frac{U A_j}{V_j \rho c_p} \left( T_r(t) - T_j(t) \right).
\]  

(B.2)

(iii) The dynamic balance of energy inside the jacket

\[
\frac{dT_j(t)}{dt} = \frac{U A_j}{V_j \rho c_p} \left( T_r(t) - T_j(t) \right) - \frac{F_j(t)}{V_j} \left( T_j(t) - T_{in}(t) \right),
\]  

(B.3)

whose parameters are defined in Table 2.

Equilibrium values of main process variables (denoted with subscript “e”)

\[
q_e = \left[ T_{re}, F_e, F_{je}, C_{ane}, T_{ine}, T_{jne} \right],
\]  

(B.4)

are related by the steady state of the nonlinear differential equations (B.1)–(B.3). Thus, \( q_e \) (B.4) denotes an operating point of the system. A particular equilibrium point is adopted as the nominal and denoted with subscript “o” in Table 2.

The nonlinear equations (B.1)–(B.3) are being linearised. The procedure is based on the expansion of the nonlinear function into a Taylor series about the operating point and the retention of only the linear terms.

Then, after taking the Laplace transform, and conveniently substituting and rearranging the equations, a small variation of reactor temperature about equilibrium \( \delta T_r(t) = T_r(t) - T_{re} \) can be expressed as an \( s \)-Laplace function \( \delta T_r(s) \). It depends on two manipulated inputs, the reactant flow \( \delta F(s) \) and the jacket cooling flow \( \delta F_j(s) \); and on three disturbance inputs, the feed concentration \( \delta C_{ane}(s) \), the inlet temperature \( \delta T_{in}(s) \), and the inlet coolant temperature \( \delta T_{jne}(s) \). All these inputs are \( s \)-Laplace transforms of \( \delta F(t) = F(t) - F_e, \delta F_j(t) = F_j(t) - F_{je}, \delta C_{ane}(t) = C_{ane}(t) - C_{ane,e}, \delta T_{in}(t) = T_{in}(t) - T_{ine}, \) and \( \delta T_{jne}(t) = T_{jne}(t) - T_{jne,e} \), respectively. Eventually, it yields five 3rd-order output-input transfer functions:
\[
m_{j=1,\ldots,5}(s) = \frac{b_{j_5}s^2 + b_{j_4}s + b_{j_3}}{s^3 + a_1s^2 + a_1s + a_0}, \tag{B.5}
\]

where \(m_1 = (\Delta T_1/\Delta F), m_2 = (\Delta T_2/\Delta F), m_3 = (\Delta T_3/\Delta C_{a_0}), m_4 = (\Delta T_4/\Delta T_0),\) and \(m_5 = (\Delta T_5/\Delta T_0);\) coefficients \(b_{j_k}\) and \(a_{k_0}\) \((j = 1, \ldots, 5; k = 0, \ldots, 2)\) of (B.5) depend on process parameters (Table 2) and equilibrium values in (B.4).

Several operating points (B.4) are being considered as follows: a single equilibrium value is adopted for the controlled variable \(T_{\text{in}} = T_{\text{ja}}.\) The three nonmanipulated inputs are taken as independent variables, whose equilibrium values are estimated as

\[
C_{a_0\text{n}} = [0.9, 1, 1.1] \times C_{a_0}, \tag{B.6}
\]

which represents a variation range of \(\pm10\%\) around nominal \(C_{a_0}\), and

\[
T_{\text{in}} = T_{\text{ja}} = [284, 289, 294, 299], \tag{B.7}
\]

which represents a variation range of \(-10/5\) K around nominal \(T_{\text{in}}, T_{\text{ja}}.\) Finally, the steady state of jacket flow rate is allowed to reach values in-between 65% and 85% of full capacity:

\[
F_{j_0} = (0.65, 0.7, 0.75, 0.85) \times F_{\text{jmax}}, \tag{B.8}
\]

A grid combination of discrete values in (B.6), (B.7), and (B.8) yields 192 cases. Then, the dependent variable \(F_{j_0}\) for each case is computed from the steady state of the nonlinear differential equations (B.1)–(B.3). Finally, a set of 192 operating points \(q_{j_0}\) (B.4) is obtained. Substituting them in the coefficients of (B.5) yields a plant matrix that collects 192 vectors of size 1 \times 5 each one.

Multivariable systems must be conveniently scaled for control design tasks [41]. Here, the presence of a single output makes only necessary to scale the range of the five inputs to the normalized range \([-1, +1].\) The scaling gain for \(T_{\text{in}} = 5, T_{\text{ja}} = 5,\) and \(C_{a_0} = 0.1C_{a_0}\) according to the smallest distance from the nominal equilibrium to the edge value in the variation range. The scaling gain for \(F_{j_0}\) is 0.2\(F_{\text{jmax}}\) according to the smallest distance from any equilibrium value (B.8) to the maximum flow rate capacity of installation \(F_{\text{jmax}}.\) In a similar way, a scaling gain of 0.29\(F_{\text{jmax}}\) for \(F\) is computed. These scaling gains are applied to the plants (B.5). Eventually, it is obtained a plant matrix \(\mathcal{P}\) that collects 192 vectors \(\mathcal{P}(s; q_{j_0}) = [P_1,p_2,p_d, p_d, p_d, p_d],\) whose magnitude frequency response is depicted in Figure 6. Let us cite the most relevant information: (i) all \(p_1\)-plant cases contain a nonminimum phase zero, which locates over \(\omega \in [0.0014, 0.0029]\) rad/s; (ii) all \(p_2\)-plant cases have inverse gain; (iii) the maximum resonant pick of the set of plant cases takes place at \(\omega = 0.000271\) rad/s and reaches the values 32.2 dB for \(p_1,\) 23.5 dB for \(p_2,\) 12.1 dB for \(p_d,\) 16.8 dB for \(p_d,\) and 17.1 dB for \(p_d.\)

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Disclosure

The authors are currently at Edificio Departamental - C/San José de Calasanz, 31, 26004 Logroño, La Rioja.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The authors are grateful for the assistance provided by the University of La Rioja.

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