Entanglement harvesting of accelerated detectors versus static ones in a thermal bath

Zhihong Liu¹, Jialin Zhang¹,² * and Hongwei Yu¹,² †

¹ Department of Physics and Synergetic Innovation Center for Quantum Effects and Applications, Hunan Normal University,
36 Lushan Rd., Changsha, Hunan 410081, China
² Institute of Interdisciplinary Studies, Hunan Normal University,
36 Lushan Rd., Changsha, Hunan 410081, China
(Dated: February 27, 2023)

Abstract

We make a detailed comparison between entanglement harvesting for uniformly accelerated detectors in vacuum and static ones in a thermal bath at the Unruh temperature and find that, for a small energy gap relative to the Heisenberg energy of the detectors, static detectors in the thermal bath can harvest more entanglement and possess a comparatively larger harvesting-achievable range than the uniformly accelerated ones; however, as the energy gap grows sufficiently large, the uniformly accelerated detectors are instead likely to harvest more entanglement and possess a relatively larger harvesting-achievable range than inertial ones in the thermal bath. In comparison with static detectors in vacuum, there exist phenomena of acceleration-assisted entanglement harvesting but never that of thermal-noise-assisted one. A notably interesting feature is that, although both the amount of entanglement harvested and the harvesting-achievable interdetector separation for static detectors in a thermal bath are always a decreasing function of temperature, they are not always so for uniformly accelerated detectors as acceleration (Unruh temperature) varies, suggesting the existence of the anti-Unruh effect in the entanglement harvesting phenomena of the accelerated detectors.

* Corresponding author. jialinzhang@hunnu.edu.cn
† Corresponding author. hwyu@hunnu.edu.cn
I. INTRODUCTION

The vacuum state of any free quantum field is entangled in the sense that it can maximally violate Bell’s inequalities \([1]\). It is known that the Minkowski vacuum entanglement can be extracted by two particle detectors via local interactions with vacuum quantum fields for a finite time, even if the detectors are spacelike separated \([2,3]\). This phenomenon has been dubbed entanglement harvesting \([5,6]\). Up to now, the entanglement harvesting phenomenon has been extensively studied by using the Unruh-DeWitt (UDW) detector model in various circumstances, and it is found that entanglement harvesting is quite sensitive to spacetime dimensionality \([6]\), topology \([7]\) and curvature \([8,16]\), the presence of boundaries \([17,19]\), and the characters of detectors such as their intricate motion \([5,19,22]\) and energy gap \([23,24]\).

For the influence of intricate motion of detectors on entanglement harvesting, let us note that the range finding of entanglement for detectors (the harvesting-achievable range of interdetector separation) in different scenarios of uniform acceleration has been discussed in Refs. \([5]\) by using the saddle-point approximation. It was argued there that entanglement harvesting can be enhanced only in a special acceleration scenario, i.e., the antiparallel acceleration. More recently, a more comprehensive investigation in the entanglement harvesting phenomenon which includes not only the harvesting-achievable separation range but also the amount of entanglement harvested has been performed with both analytical analyses and numerical calculations in Refs. \([19,22]\). It is demonstrated that acceleration increases the amount of harvested entanglement and enlarges the harvesting-achievable range in all the scenarios (parallel, antiparallel and mutually perpendicular acceleration) once the energy gap is sufficiently large relative to the interaction duration \([22]\), and there is no evidence of the phenomena of entanglement resonance argued previously using the saddle-point approximation \([5]\), which seems to suggest that the conclusions based upon such an approximation may not be reliable.

It is worth noting that, associated with acceleration, there is a striking effect in quantum field theory, known as the Unruh effect, which predicts that a uniformly accelerated observer in the Minkowski vacuum will perceive a thermal bath of particles at a temperature proportional to its proper acceleration (the Unruh temperature) \([25]\). The Unruh effect establishes an amazing equivalence between accelerated motion and a thermal bath in terms of the response rate of an Unruh-Dewitt particle detector. Then, an interesting question naturally
arises as to whether the entanglement harvesting phenomena that involve two uniformly accelerated detectors would also show such an equivalence of acceleration and thermal bath.

In the present paper, we will try to answer the question by making a detailed comparison between entanglement harvesting for uniformly accelerated detectors in the Minkowski vacuum and static ones in a thermal bath in Minkowski spacetime, focusing upon the amount of entanglement harvested and the harvesting-achievable range of interdetector separation. Let us note that the harvesting-achievable separation range of two static detectors in a Minkowski thermal bath has been studied in Refs. [5, 8] by using certain method of approximation, such as the Taylor expansion or the saddle-point approximation, and it is implied that the static detectors in the Minkowski thermal bath at the Unruh temperature always have a comparatively larger harvesting-achievable range than the (parallel) accelerated ones [5]. This conclusion, as we will demonstrate later, is, however, not universal but rather crucially dependent on the energy gap of the detectors.

In contrast, we will approach the problem in the present paper with a more reliable strategy. Specifically, our investigation will be carried out with numerical integration rather than the saddle-point approximation adopted in Ref. [5] or the Taylor expansion in Refs. [8]. Moreover, the amount of entanglement harvested will also be studied and cross-compared.

We will demonstrate that the entanglement harvesting of accelerated detectors in terms of both the harvested entanglement amount and the harvesting-achievable separation range displays features fundamentally different from those of static detectors in a thermal bath at the Unruh temperature. The energy gap of the detectors plays a critical role in determining which scenario for the detectors is more conducive to entanglement harvesting. Our results suggest that equivalence between acceleration and thermal bath as seen by a single detector via the detector’s response is lost when the entanglement harvesting of two detectors are considered. Let us note here that the loss of the equivalence between temperature and acceleration in the entanglement dynamics for two detectors has been discussed within the framework of open quantum systems in Refs. [26, 28].

It is now worthwhile to point out that there is a circular version of the Unruh effect for detectors undergoing circular motion with constant centripetal acceleration instead of linear motion with uniform acceleration, which predicts that the detectors will also perceive radiation, albeit with a non-Planckian spectrum [29, 30], and it has been argued that if one takes a circularly orbiting electron in a constant external magnetic field as the Unruh-DeWitt de-
tector then the Unruh effect physically coincides with the Sokolov-Ternov effect [31, 32]. Let us also note that the entanglement harvesting phenomenon for circularly moving detectors has recently been studied in Ref. [20], where it was found that centripetal acceleration has significant impacts on entanglement harvesting and the amount of harvested entanglement rapidly degrades with increasing acceleration or interdetector separation.

The rest of the paper is organized as follows. We briefly review, in Sec. II, the basics of the UDW detector model and the protocol of entanglement harvesting. In Sec. III, we, respectively, calculate and compare the entanglement harvested by uniformly accelerated detectors in the Minkoswki vacuum and static ones in a thermal bath at the Unruh temperature. Approximate analytical results will be presented in some particular cases along with detailed numerical calculations for the two scenarios in general. We end with a conclusion in Sec. IV. For convenience, we adopt the natural units $\hbar = c = k_B = 1$ throughout this paper.

II. BASIC FORMALISM

We consider two identical two-level Unruh-DeWitt detectors $A$ and $B$ interacting locally with a massless scalar field $\phi[x_D(\tau)]$. The spacetime trajectory of the detector, $x_D(\tau)$ ($D \in \{A, B\}$), is parametrized by its proper time $\tau$. Then, the interaction Hamiltonian of the detector and the field is given by

$$H_D(\tau) = \lambda \chi(\tau) \left[ e^{i\Omega\tau} \sigma^+ + e^{-i\Omega\tau} \sigma^- \right] \phi[x_D(\tau)], \quad D \in \{A, B\},$$

(1)

where $\lambda$ is the coupling strength, $\chi(\tau) = \exp[-\tau^2/(2\sigma^2)]$ is the Gaussian switching function with parameter $\sigma$ controlling the duration of the interaction, and $\Omega$ is the detectors’ energy gap between the ground state $|0_D\rangle$ and the excited state $|1_D\rangle$ ($D \in \{A, B\}$). Here, $\sigma^+ = |1_D\rangle\langle 0_D|$ and $\sigma^- = |0_D\rangle\langle 1_D|$ denote SU(2) ladder operators.

Suppose that initially the two detectors are prepared in their ground state. Then, to leading order in the coupling strength, the density matrix for the final state of the detectors can be obtained by tracing out the field degrees of freedom in the
basis\{0_A | 0_B \rangle, |0_A | 1_B \rangle, |1_A | 0_B \rangle, |1_A | 1_B \rangle\}  \[6, 14\],

\[\rho_{AB} = \begin{pmatrix} 1 - P_A - P_B & 0 & 0 & X \\ 0 & P_B & C & 0 \\ 0 & C^* & P_A & 0 \\ X^* & 0 & 0 & 0 \end{pmatrix} + \mathcal{O}(\lambda^4) , \tag{2}\]

where the transition probability \( P_D \) reads

\[P_D := \lambda^2 \int d\tau d\tau' \chi(\tau)\chi(\tau')e^{-i\Omega(\tau-\tau')}W(x_D(t), x_D(t')) \quad D \in \{A, B\} , \tag{3}\]

and the quantities \( C \) and \( X \), which characterize correlations, are given by

\[C := \lambda^2 \int d\tau d\tau' \chi(\tau)\chi(\tau')e^{-i\Omega(\tau-\tau')}W(x_A(t), x_B(t')) , \tag{4}\]

\[X := -\lambda^2 \int d\tau d\tau' \chi(\tau)\chi(\tau')e^{-i\Omega(\tau+\tau')}\left[\theta(t' - t)W(x_A(t), x_B(t')) + \theta(t - t')W(x_B(t'), x_A(t))\right] , \tag{5}\]

where \( W(x, x') \) is the Wightman function of the quantum fields [e.g., for the quantum field in the Minkowski vacuum state, \( W(x, x') := \langle 0_M | \phi(x)\phi(x') | 0_M \rangle \)] and \( \theta(t) \) represents the Heaviside theta function. Note that the detector’s coordinate time is a function of its proper time in the above equations, i.e., \( t = t(\tau) \). The amount of harvested entanglement can be quantified by concurrence [33], which, for the density matrix [2], is given by [7, 13]

\[C(\rho_{AB}) = 2 \max \left[ 0, |X| - \sqrt{P_A P_B} \right] + \mathcal{O}(\lambda^4) . \tag{6}\]

Obviously, the amount of entanglement acquired by the detectors is determined by the competition between the correlation term \( X \) and the geometric mean of the transition probabilities, which in general depend on the motion status and the energy gap of the detectors.

### III. ENTAILMENT HARVESTING FOR DETECTORS IN UNIFORM ACCELERATION AND IN A THERMAL BATH

In this section, we are going to analyze and compare entanglement harvesting for detectors in uniform acceleration and in a thermal bath. For this purpose, we first need to calculate \( P_D \) and \( X \) in the expression of concurrence [6].
A. Scenario of uniform acceleration

We assume the two identical detectors are accelerating along the x direction with acceleration $a$; then, the corresponding trajectories can be written as

$$x_A := \{ t = a^{-1} \sinh(a \tau_A), x = a^{-1} \cosh(a \tau_A), y = z = 0 \},$$

$$x_B := \{ t = a^{-1} \sinh(a \tau_B), x = a^{-1} \cosh(a \tau_B) + L, y = z = 0 \},$$

(7)

where $L$ represents the constant interdetector separation as measured in the laboratory reference frame, i.e., that seen by an inertial observer.

The Wightman function for vacuum massless scalar fields in four-dimensional Minkowski spacetime reads [34]

$$W (x, x') = -\frac{1}{4\pi^2} \left( t - t' - i\epsilon \right)^2 - |x - x'|^2.$$  

(8)

Substituting Eq. (7) and Eq. (8) into Eq. (3), we obtain [19, 20]

$$P_D = \frac{\lambda^2 T_U \sigma}{2\sqrt{\pi}} \int_0^\infty ds \frac{\cos(s \gamma) e^{-s^2 \alpha} \left( \sinh^2 \tilde{s} - \tilde{s}^2 \right)}{\tilde{s}^2 \sinh^2 \tilde{s}} + \frac{\lambda^2}{4\pi} \left[ e^{-\Omega^2 \sigma^2} - \sqrt{\pi} \Omega \sigma \text{Erfc}(\Omega \sigma) \right],$$  

(9)

where $T_U := a/(2\pi)$ is the Unruh temperature, $\gamma := \Omega/(\pi T_U)$, $\alpha := 1/(2\pi T_U \sigma)^2$, and Erfc(z) denotes the complementary error function. Similarly, the correlation term $X$, denoted here by $X_{acc}$ for acceleration, is given by [22]

$$X_{acc} = -\frac{\lambda^2 T_U^2}{8} \int_0^\infty d\tilde{y} \int_{-\infty}^\infty d\tilde{x} e^{-\tilde{x}^2 + \tilde{y}^2 - i\tilde{x} \Omega} F(\tilde{x}, \tilde{y}),$$  

(10)

where

$$F(\tilde{x}, \tilde{y}) := \left\{ \left[ \pi T_U L - e^{-\tilde{x} \pi T_U} \sinh(\tilde{y} \pi T_U) \right] \left[ \pi T_U L + e^{-\tilde{x} \pi T_U} \sinh(\tilde{y} \pi T_U) \right] - i\epsilon \right\}^{-1}$$

$$+ \left\{ \left[ \pi T_U L + e^{-\tilde{x} \pi T_U} \sinh(\tilde{y} \pi T_U) \right] \left[ \pi T_U L - e^{-\tilde{x} \pi T_U} \sinh(\tilde{y} \pi T_U) \right] - i\epsilon \right\}^{-1}. \quad (11)$$

Obviously, an evaluation of Eqs. (9) and (10) calls for numerical integration.

B. Scenario of thermal bath

The Wightman function for the fields in a thermal state at the Unruh temperature $T_U$ is given by [34]

$$W (x, x') = -\frac{1}{4\pi^2} \sum_{m=-\infty}^\infty \frac{1}{(t - t' - im/T_U - i\epsilon)^2 - |x - x'|^2}. \quad (12)$$
For an inertial detector in a thermal bath, one can verify that the corresponding transition probability is exactly the same as that of the detector in uniform acceleration, which is nothing but what the well-known Unruh effect means\(^{[34]}\). Adapted to the two detectors at rest with an interdetector separation \(L\), the Wightman function Eq. (12) becomes

\[
W (x, x') = \frac{T_U}{8\pi L} \left\{ \coth \left[ \pi T_U (L - t + t' + i\epsilon) \right] + \coth \left[ \pi T_U (L + t - t' - i\epsilon) \right] \right\}. \tag{13}
\]

Inserting Eq. (13) into Eq. (5), the correlation term \(X\), denoted here by \(X_{th}\) for a thermal bath at temperature \(T_U\), can be written as

\[
X_{th} = -\frac{\lambda^2 e^{-\Omega^2 \sigma^2} T_U \sigma}{4\sqrt{\pi} L} \int_0^\infty ds e^{-s^2/4\sigma^2} \left\{ \coth \left[ \pi T_U (L + s) \right] + \frac{\cosh \left[ \pi T_U (L - s) \right]}{\sinh \left[ \pi T_U (L - s) \right] - i\epsilon} \right\}. \tag{14}
\]

Equations (10) and (14) show that \(|X_{acc}|\) is different from \(|X_{th}|\), and this implies that the entanglement harvesting in two scenarios will be not equivalent in general. For a qualitative understanding of the entanglement harvesting, we first analytically estimate \(X_{acc}\) and \(X_{th}\) in some special cases.

### C. Analytical approximation

For a small acceleration or Unruh temperature with respect to the Heisenberg energy (i.e., \(a \ll 1/\sigma\) or \(T_U \ll 1/\sigma\)), the correlation terms \(X_{acc}\) and \(X_{th}\) can be respectively approximated as

\[
X_{acc} \approx -\frac{i\lambda^2 \sigma e^{-(L^2+4\Omega^2 \sigma^4)/(4\sigma^2)\text{Erfc}[iL/(2\sigma)]}}{4L\sqrt{\pi}} - \frac{i\lambda^2 \pi^{3/2} T_U^2 e^{-(L^2+4\Omega^2 \sigma^4)/(4\sigma^2)}}{24\sigma} \\
\times \left\{ 3(4L^2 \sigma^2 + 4\sigma^4 - L^4) \Omega^2 - 9L^2 - 6\sigma^2 \right\} \sigma^2 + 2L^4 \right\}, \tag{15}
\]

\[
X_{th} \approx -\frac{i\lambda^2 \sigma e^{-(L^2+4\Omega^2 \sigma^4)/(4\sigma^2)\text{Erfc}[iL/(2\sigma)]}}{4L\sqrt{\pi}} - \frac{\lambda^2 \pi T_U^2 \sigma^2}{6} e^{-\Omega^2 \sigma^2}. \tag{16}
\]

If a small interdetector separation (\(L/\sigma \ll 1\)) is further assumed, Eqs. (15) and (16) can then be simplified as

\[
|X_{acc}| \approx \frac{\lambda^2 e^{-\Omega^2 \sigma^2} \sigma}{4\sqrt{\pi} L} - \frac{\lambda^2 e^{-\Omega^2 \sigma^2} L}{16\pi^{3/2} \sigma} (\pi - 2) + \frac{\lambda^2 e^{-\Omega^2 \sigma^2}}{4L} \left( 2\Omega^2 \sigma^2 - 1 \right) \pi^{3/2} T_U^2 \sigma^3, \tag{17}
\]

\[
|X_{th}| \approx \frac{\lambda^2 e^{-\Omega^2 \sigma^2} \sigma}{4\sqrt{\pi} L} - \frac{\lambda^2 e^{-\Omega^2 \sigma^2} L}{16\pi^{3/2} \sigma} (\pi - 2) + \frac{\lambda^2 e^{-\Omega^2 \sigma^2} \pi^{1/2} \sigma L T_U^2}{6}. \tag{18}
\]
It is easy to find that $|X_{\text{acc}}| - |X_{\text{th}}| \propto (2\Omega^2 \sigma^2 - 1)$, suggesting that $|X_{\text{th}}|$ is larger or smaller than $|X_{\text{acc}}|$ depending on whether the energy gap $\Omega$ is smaller or larger than $1/(\sqrt{2} \sigma)$, i.e., $\Omega \sigma < 1/\sqrt{2}$ or $\Omega \sigma > 1/\sqrt{2}$ . Let us recall the transition probability of the detector \([9]\), which can now be approximated as

$$P_D \approx \frac{\lambda^2}{4\pi} \left[ e^{-\Omega^2 \sigma^2} - \sqrt{\pi} \Omega \sigma \text{Erfc}(\Omega \sigma) \right] + \frac{\lambda^2 \pi}{6} (T_U \sigma)^2 e^{-\Omega^2 \sigma^2} . \quad (19)$$

Then, according to Eq. \([6]\), we can see that the detectors both in uniform acceleration and in a thermal bath could harvest entanglement when the acceleration (the Unruh temperature) and the interdetector separation are small. Furthermore, static detectors in a thermal bath can harvest more (less) entanglement than accelerated detectors if $\Omega \sigma < 1/\sqrt{2}$ ($\Omega \sigma > 1/\sqrt{2}$).

On the other hand, if the acceleration (the Unruh temperature) and the interdetector separation are large enough (\(L/\sigma \gg a \sigma > 1\) and $a \gg \Omega$, to be precise), we have

$$|X_{\text{acc}}| \approx \frac{\lambda^2 e^{-\Omega^2 \sigma^2} \sigma^2}{2\pi L^2} + \frac{\lambda^2 \sigma^2}{2\pi^3 T_U^2 L^4} e^{(8\pi^2 T_U^2 - \Omega^2) \sigma^2} \cos(4\pi T_U \Omega \sigma^2) , \quad (20)$$

$$|X_{\text{th}}| \approx \frac{\lambda^2 T_U \sigma^2}{2L} e^{-\Omega^2 \sigma^2} . \quad (21)$$

It is clear that $|X_{\text{acc}}|$ and $|X_{\text{th}}|$ in general are decreasing functions of interdetector separation, and $|X_{\text{acc}}| \propto L^{-2}$, $|X_{\text{th}}| \propto L^{-1}$. Therefore, as the interdetector separation $L$ grows, $|X_{\text{acc}}|$ degrades more rapidly than $|X_{\text{th}}|$. As a result, the scenario of thermal bath possesses a comparatively large harvesting-achievable range of the interdetector separation as opposed to the scenario of uniform acceleration.

D. Numerical results

In general, the concurrence \([6]\) can be obtained by numerical integration. As shown in Figs. 1 and 2, the concurrence is plotted as a function of interdetector separation and acceleration (the Unruh temperature), respectively. Obviously, the concurrence is a monotonically decreasing function of interdetector separation, which is in accordance with the conclusions in the previous literature \([7, 19, 22]\). The static detectors in a thermal bath always harvest less entanglement than those in the Minkowski vacuum. In other words, no thermal noise-assisted entanglement harvesting occurs. In contrast, there clearly exists acceleration-assisted entanglement harvesting as long as the energy gap is large enough [see,
for example, Fig. [1(i)] or Fig. [2(i)]. Comparing the case of thermal bath with that of uniform acceleration, we find that the detectors at rest in the Minkowski thermal bath could harvest more entanglement if the energy gap $\Omega$ is much less than the detectors’ Heisenberg energy $1/\sigma$ ($\Omega \sigma < 1/\sqrt{2}$, to be more precise). However, as the energy gap grows large enough ($\Omega \sigma \gg 1$), the accelerated detectors may instead harvest comparatively more entanglement [e.g., see Fig. [1(g)] or Fig. [2(g)]. This is in accordance with our analytical analysis. An interesting hallmark worthy of being noted here is that the amount of entanglement harvested is always a monotonically decrease function of temperature for static detectors in a thermal bath, but it is not always so for uniformly accelerated detectors. In fact, for a sufficiently large energy gap, the harvested entanglement may first increase then decrease as acceleration grows [see, for example, Fig. [2(e)]. This kind of nonmonotonicity of the harvested entanglement as a function of acceleration can be regarded the anti-Unruh effect [35–38] in terms of the amount of entanglement harvested.

We now turn our attention to the role of interdetector separation in determining when the detectors in uniform acceleration could harvest more entanglement than static ones in a thermal bath. For this purpose, we introduce $L_{\text{crit}}$ to stand for the critical interdetector separation, below which (i.e., $L < L_{\text{crit}}$) the accelerated detectors could harvest more entanglement than the inertial ones in a thermal bath. As shown in Fig. 3, for not too large energy gap ($\Omega \sigma < 1$), $L_{\text{crit}}$ is a monotonically decreasing function of acceleration (or the Unruh temperature), while as the energy gap grows to large enough, although $L_{\text{crit}}$ on the whole still behaves as a decreasing function of acceleration, it undergoes some oscillation over the regime of small $a\sigma$. Let us note that when the interdetector separation $L$ is much smaller than the characteristic length $1/a$ (i.e., $L \ll 1/a$) the interdetector interaction between two accelerated detectors behaves almost like that of two inertial detectors in the Minkowski vacuum [39]. Therefore, we could use $\sim 1/a$ as a characteristic separation to signal that the entanglement harvesting of two accelerated detectors would behave more or less like that of two inertial detectors. As long as the energy gap is not too small (i.e., $\Omega \sigma > 1/\sqrt{2}$), Fig. 3 shows that when the interdetector separation lies in the inertial regime ($L \ll 1/a$), the detectors in the acceleration scenario could harvest more entanglement due to the fact that there does not exist thermal noise-assisted entanglement harvesting but rather thermal noise-caused degradation. Meanwhile, the curve of $L_{\text{crit}}$ seems to have a chance to be above the curve of $L = 1/a$ if the energy gap is large enough. This implies that a pair of acceler-
FIG. 1: The concurrence is plotted as a function of the interdetector separation with $\Omega \sigma = \{0.50, 1.20, 2.00\}$ in top-to-bottom order and $a \sigma = 2\pi T_U \sigma = \{0.50, 1.00, 3.00\}$ in the left-to-right order. Note that the dashed line corresponds to the scenario of detectors at rest in the Minkowski vacuum. For convenience, all the physical quantities are rescaled to be unitless in units of $\sigma$. 

(a) $\Omega \sigma = 0.50, \ a \sigma = 2\pi T_U \sigma = 0.50$
(b) $\Omega \sigma = 0.50, \ a \sigma = 2\pi T_U \sigma = 1.00$
(c) $\Omega \sigma = 0.50, \ a \sigma = 2\pi T_U \sigma = 3.00$

(d) $\Omega \sigma = 1.20, \ a \sigma = 2\pi T_U \sigma = 0.50$
(e) $\Omega \sigma = 1.20, \ a \sigma = 2\pi T_U \sigma = 1.00$
(f) $\Omega \sigma = 1.20, \ a \sigma = 2\pi T_U \sigma = 3.00$

(g) $\Omega \sigma = 2.00, \ a \sigma = 2\pi T_U \sigma = 0.50$
(h) $\Omega \sigma = 2.00, \ a \sigma = 2\pi T_U \sigma = 1.00$
(i) $\Omega \sigma = 2.00, \ a \sigma = 2\pi T_U \sigma = 3.00$
FIG. 2: The plots of the concurrence versus the acceleration (or the Unruh temperature) with $\Omega \sigma = \{0.50, 1.20, 2.00\}$ in the top-to-bottom order and $L/\sigma = \{0.50, 1.00, 2.00\}$ in the left-to-right order. The dashed lines in all plots indicate the case of detectors at rest ($a = 0$) in Minkowski vacuum.
ated detectors separated by a noninertial distance could instead harvest comparatively more entanglement due to the effect of acceleration-assisted entanglement harvesting.

![Plot](image)

**FIG. 3:** The plots of $L_{\text{crit}}/\sigma$ versus the acceleration with $\Omega\sigma = \{0.80, 1.00, 1.20, 1.50, 2.00\}$. The dashed black curve represents the function $L = 1/a$ that indicates the size of the inertial region. Note that a non-negative $L_{\text{crit}}$, according to the aforementioned discussion, should require $\Omega\sigma > 1/\sqrt{2} \sim 0.707$.

Now, we are in a position to analyze how two situations differ in the harvesting-achievable range of interdetector separation. We introduce a parameter, $L_{\text{max}}$, to characterize the maximum harvesting-achievable interdetector separation, beyond which entanglement harvesting cannot occur. As shown in Fig. 4, for a small energy gap ($\Omega\sigma < 1$), $L_{\text{max}}$ for both the scenario of detectors in uniform acceleration in vacuum and that of static ones in a thermal bath is a monotonically decreasing function of acceleration or the Unruh temperature, and the thermal bath scenario may possess a comparatively large harvesting-achievable range.
FIG. 4: The maximum separation, $L_{\text{max}}$, between two detectors when entanglement harvesting almost does not occur is plotted as a function of the acceleration (or the Unruh temperature). Here, we have set $\Omega\sigma = \{0.50, 1.00, 3.00\}$. The dashed horizon line shows the case of $a = 0$, i.e., the scenario of inertial detectors in Minkowski vacuum.

Notably, it was argued in Ref. [5] that the degradation of entanglement harvesting for two uniformly accelerated detectors is different from that of two inertial detectors in a thermal bath. More specifically, according to Eqs. (3.1) and (3.3) and Fig. 2 in Ref. [5], which are based on the saddle-point approximation, one can infer that the static detectors in the Minkowski thermal bath at the Unruh temperature have a comparatively larger harvesting-achievable range than the (parallel) accelerated ones. However, our numerical plots in Fig. 4 show that this conclusion is not universal but contingent upon a small energy gap. In fact, for a large enough energy gap ($\Omega\sigma \gg 1$), the acceleration scenario may instead have a comparatively large $L_{\text{max}}$ [see Fig. 4(c)].

It is rather interesting to note that for $\Omega\sigma \gg 1$ the $L_{\text{max}}$ curve in the acceleration case is not an exactly decreasing function of acceleration but exhibits some oscillation. This reveals that the detectors in uniform acceleration can possess a comparatively larger entanglement harvesting-achievable interdetector separation than those in a thermal bath when the energy gap is sufficiently large although the static detectors enjoy a comparatively larger harvesting-achievable separation when the energy gap is small as was found previously. More remarkably, acceleration can even enlarge the harvesting-achievable range of interdetector separation as compared to the inertial vacuum case. In contrast, the noise of a thermal bath
can never enlarge the maximum harvesting-achievable interdetector separation. It is worth pointing out that the oscillation is an indication of the anti-Unruh effect again in terms of entanglement harvesting for accelerated detectors since it means nonmonotonicity of the harvesting-achievable separation as acceleration varies.

Noteworthily, although only the entanglement harvesting of accelerated detectors in the parallel acceleration scenario is compared with that of inertial detectors in a thermal bath, our conclusions remain qualitatively unchanged if other acceleration scenarios, e.g., those of antiparallel and mutually perpendicular acceleration, are instead adopted. This is because, for a sufficiently large energy gap of the detectors, acceleration increases, in all the scenarios, the amount of harvested entanglement and enlarges the harvesting-achievable range \[22\], in contrast to a previous claim \[5\]. Furthermore, the harvesting-achievable range is not a monotonic function of acceleration in all the scenarios as one can see from Fig. 5, in which we plot \(L_{\text{max}}\) versus \(a\sigma\) for parallel, antiparallel and mutually perpendicular acceleration scenarios.

IV. CONCLUSION

Within the framework of the entanglement harvesting protocol, we have made a detailed comparison between entanglement harvesting in the sense of both the amount of entanglement harvested and the harvesting-achievable separation range for uniformly accelerated detectors in vacuum and static ones in a thermal bath at the Unruh temperature and find that the entanglement harvesting for two uniformly accelerated detectors is markedly different from that for two static detectors in a thermal bath, suggesting that equivalence between acceleration and a thermal bath in terms of the response of a single detector is lost for the entanglement harvesting for two detectors.

Regarding the amount of entanglement harvested by detectors, we find that static detectors in a thermal bath always harvest less entanglement than those in vacuum. In other words, no thermal noise-assisted entanglement harvesting ever occurs. In contrast, there exists acceleration-assisted entanglement harvesting as long as the energy gap is large enough. A cross-comparison shows that static detectors in a thermal bath can harvest more entanglement than uniformly accelerated ones if the energy gap of the detectors is much smaller than the detectors’ Heisenberg energy, while as the energy gap grows large enough, the accelerated
FIG. 5: The maximum harvesting-achievable interdetector separation, $L_{\text{max}}$, is plotted as a function of acceleration for parallel, antiparallel and mutually perpendicular acceleration scenarios. Here, we have set $\Omega \sigma = 2.00$, and the dashed curve denotes the case of detectors at rest.

Detectors may instead harvest comparatively more entanglement. There is a critical value of interdetector separation, below which accelerated detectors can acquire comparatively more entanglement. It is interesting to note that the critical interdetector separation is in general a decreasing function of acceleration or the Unruh temperature, and however it has a chance to become larger than the effective inertial regime ($\sim 1/a$) when the energy gap is large enough, signaling the phenomenon of acceleration-assisted entanglement harvesting.

With respect to the harvesting-achievable separation range, we find that, for a small energy gap ($\Omega \sigma < 1$) the thermal bath scenario possesses a comparatively larger harvesting-achievable range than the uniform acceleration scenario, which is in accordance with that obtained in Ref. [5] based on the saddle-point approximation. However, for a large enough energy gap ($\Omega \sigma \gg 1$), the accelerated detectors have, in contrast, a larger harvesting-achievable...
separation range. Moreover, acceleration can even enlarge the harvesting-achievable range of interdetector separation in comparison with the inertial vacuum case for a large energy gap. However, thermal noise can never enlarge the harvesting-achievable range but only monotonically shorten the harvesting-achievable interdetector separation as temperature increases.

A notably interesting feature is that, although both the amount of entanglement harvested and the harvesting-achievable interdetector separation for static detectors in a thermal bath are always a decreasing function of temperature, they are not always so for uniformly accelerated detectors as acceleration (Unruh temperature) varies. In fact, the amount of entanglement harvested may first increase and then decrease in the regime of small acceleration for sufficiently large energy gap, while the harvesting-achievable interdetector separation exhibits some oscillation in the regime of small acceleration for sufficiently large energy gap.

Finally, let us stress that the nonmonotonicity of the two physical quantities characterizing the entanglement harvesting phenomenon in the acceleration case (i.e., the amount of entanglement harvested and harvesting-achievable interdetector separation) as a function of acceleration, which physically indicates that the existence of the anti-Unruh effect in terms of entanglement harvesting, and acceleration-assisted enhancement of the amount of entanglement harvested and enlargement of the harvesting-achievable interdetector separation are two remarkable properties which distinguish the entanglement harvesting of the accelerated detectors from that of inertial detectors in a thermal bath, since the amount of entanglement harvested and the harvesting-achievable interdetector separation are all monotonic functions of temperature and are never enhanced or enlarged by thermal noise.

Acknowledgments

We are grateful to Jiawei Hu for a number of helpful discussions. This work was supported in part by the NSFC under Grants No. 12175062 and No. 12075084 and the Research Foundation of Education Bureau of Hunan Province, China, under Grant No. 20B371.

[1] S. J. Summers and R. Werner, J. Math. Phys. (N.Y.) 28, 2448 (1987).
[2] A. Valentini, Phys. Lett. A 153, 321 (1991).
[3] B. Reznik, Found. Phys. 33, 167 (2003).
[4] B. L. Hu, S.-Y. Lin, and J. Louko, Classical Quantum Gravity 29, 224005 (2012).
[5] G. Salton, R. B. Mann, and N. C. Menicucci, New J. Phys 17, 035001 (2015).
[6] A. Pozas-Kerstjens and E. Martin-Martinez, Phys. Rev. D 92, 064042 (2015).
[7] E. Martin-Martinez, A. R. H. Smith, and D. R. Terno, Phys. Rev. D 93, 044001 (2016).
[8] G. L. Ver Steeg and N. C. Menicucci, Phys. Rev. D 79, 044027 (2009).
[9] Y. Nambu, Entropy 15, 1847 (2013).
[10] S. Kukita and Y. Nambu, Entropy 19, 449 (2017).
[11] K. K. Ng, R. B. Mann, and E. Martin-Martinez, Phys. Rev. D 97, 125011 (2018).
[12] K. K. Ng, R. B. Mann, and E. Martin-Martinez, Phys. Rev. D 98, 125005 (2018).
[13] L. J. Henderson, R. A. Hennigar, R. B. Mann, A. R. H. Smith, and J. Zhang, Classical Quantum Gravity 35, 21LT02 (2018).
[14] L. J. Henderson, R. A. Hennigar, R. B. Mann, A. R. H. Smith, and J. Zhang, J. High Energy Phys. 05 (2019) 178.
[15] M. P. G. Robbins, L. J. Henderson, and R. B. Mann, arXiv:2010.14517 [hep-th].
[16] K. Gallock-Yoshimura, E. Tjoa, and Robert. B. Mann, Phys. Rev. D 104, 025001 (2021).
[17] W. Cong, E. Tjoa, and R. B. Mann, J. High Energy Phys. 06 (2019) 021.
[18] W. Cong, C. Qian, M. R. Good, and R. B. Mann, J. High Energy Phys. 10 (2020) 067.
[19] Z. Liu, J. Zhang, and H. Yu, J. High Energy Phys. 08 (2021) 020.
[20] J. Zhang and H. Yu, Phys. Rev. D 102, 065013 (2020).
[21] C. Suryaatmadja, W. Cong, and R. B. Mann, arXiv:2205.14739 [quant-ph].
[22] Z. Liu, J. Zhang, R. B. Mann, and H. Yu, Phys. Rev. D 105, 085012 (2022).
[23] H. Hu, J. Zhang, and H. Yu, J. High Energy Phys. 05 (2022) 112.
[24] H. Maeso-Garcia, T. Rick Perche, and E. Martin-Martinez, Phys. Rev. D 106, 045014 (2022).
[25] W. G. Unruh, Phys. Rev. D 14, 870 (1976).
[26] J. Zhang and H. Yu, Phys. Rev. D 75, 104014 (2007).
[27] J. Hu and H. Yu, Phys.Rev. A 91, 012327 (2015).
[28] A. P. C. M Lima, G. Alencar, and R. R. Landim, Phys. Rev. D 101, 125008 (2020).
[29] J. S. Bell and J. M. Leinaas, Nucl. Phys. B 212, 131 (1983).
[30] S. Biermann, S. Erne, C. Gooding, J. Louko, J. Schmiedmayer, W. G. Unruh, and S. Wein-furtner, Phys. Rev. D 102, 085006 (2020).
[31] E. T. Akhmedov and D. Singleton, Int. J. Mod. Phys. A 22, 4797 (2007).

[32] E. T. Akhmedov and D. Singleton, JETP Lett. 86, 615 (2008).

[33] W. K. Wootters, Phys. Rev. Lett. 80, 2245 (1997).

[34] N. D. Birrell and P. C. W. Davies, Quantum Fields in Curved Space (Cambridge University Press, Cambridge, United Kingdom, 1984).

[35] W. G. Brenna, R. B. Mann, and E. Martin-Martinez, Phys. Lett. B 757, 307 (2016).

[36] L. J. Garay, E. Martin-Martinez, and J. de Ramon, Phys. Rev. D 94, 104048 (2016).

[37] P.-H. Liu and F.-L. Lin, J. High Energy Phys. 07 (2016) 084.

[38] Y. Zhou, J. Hu, and H. Yu, J. High Energy Phys. 09 (2021) 088.

[39] S. Cheng, W. Zhou, and H. Yu, Phys. Lett. B 834, 137440 (2022).