ZERO-POINT RADIATION, INERTIA AND GRAVITATION

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In this paper it is shown that the forces which resist the acceleration of the mass of the electron, $m_e$, arising from the Compton effect, the Klein-Nishina-Kann formula for its differential cross section and the transversal Doppler effect when the electron moves in a straight line coincide, with $\epsilon < 1.16 \times 10^{-4}$, with the force required to propel me with the same acceleration, if the radius of the electron is equal to its classical radius and if the forces which rise from the interaction of the electron and zero-point radiation are equal to those deriving from the electrostatic repulsion of the charge of the electron against itself (Poincare’s tensions). The equations worked in this paper show that there is no difference between inertial mass and gravitational mass and may be used to determine the value of the gravitational constant.

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0.1 INTRODUCTION

Sparnaay’s experiments in 1958 [4] showed the existence of zero-point radiation which Nernst had considered as a possibility in 1916. This radiation is inherent to space, and for that to be the case its spectral density function must be inversely proportional to the cubes of its wavelengths. In other words, the number of photons of wavelength $\lambda$ which strike a given area during a given period must be inversely proportional to $\lambda^3$. In 1969, Timothy H. Boyer [4] deduced that the spectral density function of zero-point radiation is:

$$f_\lambda(\lambda) = \frac{1}{2\pi^2} \frac{1}{(\lambda_\alpha)^3}$$

where $\lambda_\alpha$ is the number which measures the wavelength $\lambda$.

To this function there corresponds the energy function

$$E_\lambda(\lambda) = \frac{1}{2\pi^2} \frac{h c}{\lambda (\lambda_\alpha)^3}$$

For $\lambda = 0$, $E_\lambda(\lambda) = \infty$. There must therefore exist a threshold for $\lambda$ which will henceforth be written as $q_\lambda$.

In order to simplify the following arguments, it is convenient to use the $(e, m_e, c)$ system of measurements in which the basic magnitudes are the quantum of electrical charge, the mass of the electron and the speed of light. In this system the units of length and time are, respectively, $l_e = e^2/m_e c^2$ and $t_e = e^2/m_e c^3$. Moreover, the following results and formulae obtained in titles [2] and [3] of the references are basic.

- The flow of the zero-point photons of wavelength $nq_\lambda$ through an area $(q_\lambda)^2$ is one photon in a lapse of time $n^3 q_\tau$ where $q_\tau = q_\lambda/c$.

Therefore $(k_\lambda/n)^3$ photons of wavelength $nq_\lambda$ will flow through an area $(l_e)^2$ during any lapse of time $t_e$, where $k_\lambda = l_e/q_\lambda = t_e/ q_\tau$. See the final result of [2].

- The minimum wavelength in zero-point radiation is $xq_\lambda$. See [3], p. 7.

- The energy flow of zero-point radiation per $(q_\lambda)^2$, expressed in the $(e, m_e, c)$ system is:

$$W_x = \frac{2\pi (k_\lambda)^2 m_e c^2}{3\alpha x^3} \text{per } (q_\lambda)^2, \text{ See [3] p. 7.}$$

(1)

- The energy transferred by a photon of wavelength $nq_\lambda$ to an electron is $E_{t_n} = E_n \pi [A_m]$.

$$E_n = \frac{hc}{nq_\lambda} = \frac{2\pi k_\lambda}{\alpha n} m_e c^2;$$

$$[A_m] = \lim_{m \to \infty} \left[ \frac{7}{12} - \frac{11}{10} A^2 + \cdots + (-1)^{m-1} \{m\} A^m \right]$$

$$A = \frac{2\pi k_\lambda}{\alpha n};$$

$$\{m\} = \frac{1}{m+1} + \frac{2}{m+2} - \frac{3}{m+3} - 1 - \frac{m(m-1)}{6};$$
hence

\[ E_{T_n} = \frac{2\pi^2}{\alpha} \frac{k_{\lambda}}{n} [A_m] m_e c^2. \] (2)

see [3], p. 8.

- The zero-point radiation transfers to the electron the energy flow

\[ W_{T_n} = \frac{2\pi^2}{\alpha} \frac{(k_{\lambda})^2}{x^3} [B]_m \frac{m_e c^2}{t_e} \text{ per } (q_{\lambda})^2 \] (3)

where \( B = \frac{2\pi k_{\lambda}}{\alpha} x \),

\[ [B]_m = \frac{7}{48} B + \cdots + (-1)^m \frac{\{m\}}{m+3} B^m \]

see [3], p. 9.

- \( x^3 = \frac{4\pi^3 (k_{\lambda})^4 (r_x)^4 [B]_m^{}/3\alpha} \), where \((r_x)\) is the radius of the electron expressed in \( l_e \).

For \( r_x = r_e = l_e \) we have

\[ x^3 = \frac{4\pi^3 (k_{\lambda})^4 [B]_m^{}/3\alpha} \] (4)

see [3], p. 10.

- \( x^3 = \frac{2\pi^2 (k_{\lambda})^2 (r_x)^2 [B]_m^{}/3\alpha G_e} \).

For \( r_x = r_e = l_e \) we have

\[ x^3 = \frac{2\pi^2 (k_{\lambda})^2 [B]_m^{}/3\alpha G_e} \] (5)

see [3], p. 10.

From (4) and (5) we obtain

\[ k_{\lambda} = \left( \frac{1}{2\pi G_e} \right)^{1/2} \] (6)

From here \( q_{\lambda} = \left(2\pi \alpha\right)^{1/2} L_P \), where \( L_P \) is the Planck’s length, and

\[ k_{\lambda} = 8.143375 \times 10^{20} \]

\[ x = 5.257601 \times 10^{27} \]

\[ z = \frac{k_{\lambda}}{x} = 1.548877 \times 10^{-7} \]

The value of \( k_{\lambda} \) means that the minimum wavelength of a photon is

\[ l_e/8.143375 \times 10^{20} = 1.227992 \times 10^{-21}l_e \]

### 0.2 ON ZERO-POINT RADIATION WITHIN A REFERENCE SYSTEM \( S' \) MOVING IN A STRAIGHT LINE AND WITH UNIFORM ACCELERATION, AND ON ITS INTERACTION WITH AN ELECTRON AT REST IN \( S' \)

Let us consider a reference system \( S' \) which coincides with the inertial system \( S_0 \), until at \( t = t_1 \) it begins to move along the axis \( \Omega X' \) at a uniform acceleration “\( a \)”, and which after the lapse \( t_1 \) coincides with the inertial system \( S_1 \) which is moving at a velocity \( v_1 = at_1 \) relative to \( S_0 \). It is obvious that the zero-point radiation at \( S_1 \) has the same spectral composition as at \( S_0 \), but this does not happen with \( S' \), which continues to move at a uniform acceleration “\( a \)”.

After a minimal lapse of time \( q_r \), the zero-point radiation photons of wavelength \( \lambda \) in \( S_1 \), which at \( t = t_1 \) arrive at \( P \) along a line which makes an angle \( \theta \) with \( \Omega'X' \) (see Fig. 1) are perceived in \( S' \) as if proceeding from a source of light \( F' \), situated on a line which passes through \( P \) and makes an angle \( \theta \) with \( \Omega'X' \), and moving at velocity \( \Delta v = a q_{r} \) relative to \( O' \), where \( a_q \) is the acceleration “\( a \)” expressed through a system in which the unit of length is \( q_{\lambda} \) and the unit of time is \( q_r \). Therefore:

\[ \Delta v = a_q \frac{q_{\lambda}}{(q_r)^2} q_r = a_q c, \]

which can also be written:

\[ \Delta v = a_q \frac{k_{\lambda} q_{\lambda}}{(k_{\lambda} q_r)^2} k_{\lambda} q_r = a \frac{l_e}{(t_e)^2} l_e, \]

which reveals that the increase in velocity per \( q_r \) given by an acceleration of \( a_q \frac{q_{\lambda}}{(q_r)^2} \) is the same as the increase of velocity per \( t_e \) given by an acceleration of \( a_q \frac{l_e}{(t_e)^2} \); this allows us to write:

\[ \Delta v = at_e = a \frac{l_e}{(t_e)^2} t_e = ac. \]

We should point out here the enormous size of the unit of acceleration in the \((e, m_e, c)\) system, compared with the unit of the same magnitude in the \((c.g.s.)\) system. The relation between them is given by:

\[ \frac{l_e}{(t_e)^2} = \frac{c}{t_e} = \frac{2.997925 \times 10^{10} \text{ cm/s}}{9.399639 \times 10^{-24} \text{ s}} = 3.189404 \times 10^{33} \text{ cm/(s)^2}. \]
We have already established that the flow of zero-point radiation photons of wavelength $n q \lambda$ through a frame of area $(l_e)^2$ is one of $(k \lambda / n)^3$ photons for each lapse $t_e$. Viewed from the system of reference $S'$, moving at velocity $a t_e$ relative to the inertial system $S_1$, and along the axis $O \hat{X}$, this number of photons appears to pass through a frame of area:

$$(l'_e)^2 = \frac{(l_e)^2}{(1 - a^2)}$$

for every lapse $t'_e = \frac{t_e}{(1 - a^2)^{1/2}}$; this gives a photon flow of $\left( \frac{k \lambda}{n} \right)^3 (1 - a^2)^{3/2}$ through a frame of area $(l'_e)^2$, per $t_e$; however since the number $a$ must surely be inferior to $10^{-23}$, this flow does not differ significantly from $\left( \frac{k \lambda}{n} \right)^3$.

The fact that the intensity of the flow $N'_{n \theta}$ of the photons of wavelength $\lambda$ which arrive at $O'$ following a trajectory which makes an angle $\theta$ with $O'X'$, through the frame of area $(l'_e)^2$ during every lapse $t'_e$, does not differ significantly from the intensity of the analogous flow $N_{n \theta}$, enables us to establish the value of $N'_{n \theta}$ without difficulty. From Fig. 2 we can see that if $N_n$ is the number of photons of wavelength $n q \lambda$ which arrive from all directions of space at a frame of area $(l_e)^2$ within a lapse $t_e$, and if that number corresponds to the area $4 \pi N_n^2$ of a spherical surface of radius $N_n$, the number, $N'_{n \theta}$, of those which arrive at the said frame within the same lapse of time and which make an angle $\theta$ with $O'X'$, corresponds to the area produced by the arc $\hat{A} \hat{B} = N_n a \theta$ when rotated around $O \hat{X}$, whence:

$$N'_{n \theta} = \frac{1}{2} \left( \frac{k \lambda}{n} \right)^3 \sin \theta d \theta$$

(8)

for every $(l_e)^2$ during every $t_e$.

Figure 3 shows the equatorial section of an electron at rest at $O'$, in system $S'$. Because of the Doppler lateral relativistic effect, the photons of wavelength $\lambda$ emitted by $F_\varphi$, moving at velocity $-a \Delta v = -a t_e$ relative to $O'$ along a straight line parallel to $O'X'$, and which on arrival at points $P_1$ and $P_2$ make an angle $\theta$ with $O'X'$ are perceived at those points as having a wavelength:

$$\lambda'_\theta = \lambda \frac{1 - a \cos \theta}{(1 - a^2)^{1/2}}$$

One result of this is that the acceleration required to move in one second from rest to the speed of light; which is the (c.g.s) system is $2.997925 \times 10^{10} \text{ cm/(s)}^2$, is $9.399639 \times 10^{-24} l_0 / (t_e)^2$ when expressed in the (e, m, c) system. This shows that the values of “$a$” are minuscule for the accelerations which can be produced in physical reality, excepting those which could be produced in the process of the Big-Bang. It is therefore reasonable to conclude that “$a$” is an extremely small number, certainly less than $10^{-23}$.
According to (8), the number of photons of wavelength \( nq \lambda \) which arrive at all the points \( P \) of intersection between the surface of the electron and the cone having its apex at \( Q \) and making an angle \( 2\theta \) at that apex, is

\[
\frac{1}{2} \left( \frac{k \lambda}{n} \right)^3 \sin \theta d\theta.
\]

Therefore, the energies transferred during each lapse \( t_e \) to the electron by these photons, and by those which arrive from the opposite direction, \( \pi + \theta \), are respectively given by:

\[
W'_{T_{n\theta}^e} = \frac{\pi^2}{\alpha} \left( \frac{k \lambda}{n} \right)^4 \frac{1}{1 - a \cos \theta} \left[ A'_{n\theta} \right] \sin \theta d\theta \frac{m_e c^2}{t_e} \\
W'_{T_{n(\pi+\theta)}^e} = \frac{\pi^2}{\alpha} \left( \frac{k \lambda}{n} \right)^4 \frac{1}{1 + a \cos \theta} \left[ A'_{n(\pi+\theta)} \right] \sin \theta d\theta \frac{m_e c^2}{t_e}
\]

per \( (l_e)^2 \), where:

\[
A'_{n\theta} = \frac{2\pi k \lambda}{\alpha} \frac{(1 - a^2)^{1/2}}{n \ 1 - a \cos \theta} ;
\]

\[
A'_{n(\pi+\theta)} = \frac{2\pi k \lambda}{\alpha} \frac{(1 - a^2)^{1/2}}{n \ 1 + a \cos \theta} ;
\]

and where the developments of \( [A'_{n\theta}] \) and \( [A'_{n(\pi+\theta)}] \) are analogous to the development of \( [A_n] \) for \( A = \frac{2\pi k \lambda}{\alpha} \frac{1}{n} \), which appears in (2).

In (9) the number to be considered in each of the formulas is the number arriving from a half-space, so that \( \pi^2/\alpha \) appears there instead of \( 2\pi^2/\alpha \) as in (2).

The total \( W'_{T_{n\theta}^e} + W'_{T_{n(\pi+\theta)}^e} \) per \( (l_e)^2 \) is:

\[
\frac{m_e c^2 \pi^2}{t_e} \alpha \sin \theta d\theta \left\{ \frac{7}{12} \left( \frac{2\pi}{\alpha} \right)^5 \left( \frac{k \lambda}{n} \right)^5 \frac{(1 - a^2)^{2/2}}{[1 - (a \cos \theta)^2]^{1/2}} \right\} \\
+ \cdots + (-1)^{m-1} \left\{ m \right\} \left( \frac{2\pi}{\alpha} \right)^{m+4} \left( \frac{k \lambda}{n} \right)^{m+4} \frac{(1 - a^2)^{m+1}}{[1 - (a \cos \theta)^2]^{1/2}} \frac{1}{2a \cos \theta} + \cdots
\]

\times \frac{1 - a^2}{[1 - (a \cos \theta)^2]^{m+1}} \left[ 2(m + 1)a \cos \theta + 2 \left( \frac{m + 1}{3} \right) a^3 \cos^3 \theta + \cdots \right].
Since \( a < 10^{-23}, (1-a^2)^{m+1} \) and \( (1-(a \cos \theta)^2)^{m+1} \) do not differ significantly from 1, while the terms
\[
2 \left( \frac{m + 1}{3} \right) a^3 \cos^3 \theta, \quad 2 \left( \frac{m + 1}{5} \right) a^5 \cos^5 \theta,
\]
ecc., are insignificant with respect to \( 2(m + 1)a \cos \theta \), and can be ignored so that we can write:
\[
W'_{T_\theta} + W'_{T_n} = \frac{m_e c^2 \pi^2}{\alpha \theta} \sin \theta \cos \theta d\theta
\]
\[
\times \left[ \frac{7}{12} \left( \frac{2\pi}{\alpha \lambda} \right) \left( \frac{k\lambda}{n} \right)^5 \right] 4 + \cdots
\]
\[
+(-1)^{m+1}\{m\} \left( \frac{2\pi}{\alpha \lambda} \right)^{m+4} \left( \frac{k\lambda}{n} \right)^{m+4} 2(m+1)
\]
\[
(10)
\]
per \((l_e)^2\).

The energy flow which is transferred to the electron by all the zero-point radiation photons which arrive at \( P_1 \) and \( P_2 \) with trajectories which, respectively, make angles \( \theta \) and \( \pi + \theta \) with \( OX' \) can be obtained by adding up the values of \( n \) between \( n = x \) and \( n \to \infty \), where \( xq\lambda \) is the wavelength of the zero-point radiation photon which possesses most energy. Since
\[
\sum_{n=x}^{n=m} = \frac{1}{m-1}.
\]
\[
\frac{1}{x^{m-1}}, \text{ where } \varepsilon < \frac{1}{2x}, \text{ and } x \text{ is a very large number, we can accept the value }
\]
\[
\frac{1}{m-1} \cdot \frac{1}{x^{m-1}}
\]
do not differ significantly from \( \sum_{n=x}^{n=m} \) and by introducing it in (10) we obtain
\[
W'_{T_\theta} + W'_{T_n} \approx \frac{m_e c^2 \pi^2}{\alpha \theta} \sin \theta \cos \theta \cos \theta d\theta
\]
\[
\times \left[ \frac{14}{48} \left( \frac{k\lambda}{x} \right)^3 \right] \frac{1}{x^3} \cdot \frac{1}{x^3} \cdot \frac{1}{x^3}
\]
\[
(11)
\]
per \((l_e)^2\), where
\[
\frac{1}{48} \left( B_x \right) - \frac{33}{50} \left( B_x \right)^2 + \cdots
\]
\[
+(-1)^{m+1}\{m\} \frac{m+1}{m+3} \left( B_x \right)^{m+1}
\]
and \( B_x = \frac{2\pi k\lambda}{\alpha x} \).

Hence we obtain the force \( F'_{\theta} = \frac{W'_{T_\theta}}{c} \), whose projection along \( OX' \)
\[
W'_{T_\theta} \cos \theta = \frac{2\pi^2 k\lambda}{\alpha x} \left( \frac{k\lambda}{x} \right)^3 \left( B_x \right) \frac{m_e c^2}{\theta d\theta}
\]
\[
(12)
\]
per \((l_e)^2\).

Figure 4 shows that for an area \((q\lambda)^2\) situated on the surface of the electron at \( P \) on \( OX' \), the sum of the projections \( F'_{\theta} = \cos \theta \) is:
\[
F'_{\theta} = \frac{2\pi^2 k\lambda}{\alpha x^3} \left[ B_x \right] \frac{m_e c}{\theta d\theta}
\]
per \((q\lambda)^2\), whence
\[
F'_{\theta} = \frac{2\pi^2 k\lambda}{\alpha x^3} \left[ B_x \right] \frac{m_e c}{\theta d\theta}
\]
per \((q\lambda)^2\) (13)
where \((k\lambda)^2\) appears instead of \( k\lambda \left( \frac{k\lambda}{x} \right)^3 \) because (12) expresses forces per \((l_e)^2\); while we are now talking of forces per \((q\lambda)^2\), i.e. the former divided by \((k\lambda)^2\).

Figure 4: Fig. 1

The sum \( F_P \) of the projections along \( OX' \) of the forces which come from zero-point radiation and the Doppler lateral relativistic effect is
the same at every point inside the circle whose center is at $O'$ and whose radius is $O'P_P$, perpendicular to $O'X'$. Therefore, the force which comes from the presence of zero-point radiation and the Doppler relativist effect is

$$F = \frac{2\pi^3 (k_\lambda)^4 (r_x)^2}{3\alpha x^3} + B_x + \frac{a m_e c}{\lambda e}$$

(14)

where $r_x$ is an integer. For $r_x = 1$

$$F = \frac{2\pi^3 (k_\lambda)^4}{3\alpha x^3} + B_x + \frac{a m_e c}{\lambda e}$$

(15)

By introducing in this last formula the values obtained in [3] for $k_\lambda$ and $x$, which are reproduced in the Introduction we obtain $F_0 = 0.999884a m_e c/\lambda e$, instead of $F = a m_e c/\lambda e$: the difference is less than $1.16 \times 10^{-4}$.

For $a = G_e/d^2$ (15) gives:

$$F_G = \frac{2\pi^3 (k_\lambda)^4}{3\alpha x^3} + B_x + \frac{G_e m_e c}{d^2 \lambda e}$$

By equalising with $F_d = \frac{2\pi^3 k_\lambda^3 |B| m_e c}{3\alpha x^3 d^2 \lambda e}$ (see [3], formula (18)) we obtain:

$$G_e = \frac{|B| m}{\pi (k_\lambda)^2 + B_x}$$

(16)

instead of $G_e = \frac{1}{2\pi (k_\lambda)^2}$ as in (6). The quotient:

$$\frac{G_e}{|B| m} = \frac{1}{\pi}$$

is equal to 2 for $B = 0$

$$B = \frac{2\pi}{\alpha} z$$

must be $< 1$ which means that

$$z < \alpha/2\pi = 1.16 \times 10^{-3}$$

For $z = 1.1 \times 10^{-3}$, very near to its upper limits, we have $\frac{G_e}{|B| m} = 1.358$

By starting with $G_{cgs} = 6.6742 \times 10^{-8}$; $G_e = 2.400575 \times 10^{-43}$ we obtained $F_I = 0.999884635a m_e c/\lambda e$. The maximum value found for $F_I$, $F_I = 0.999899a m_e c/\lambda e$, there is for $G_e = 2.399999 \times 10^{-43}$ which it corresponds to $G_{cgs} = 6.672599 \times 10^{-8}$, which is very near to the estimation in CODATA 1986: $G_{cgs} = 6.67259 \times 10^{-8}$.

0.3 CONCLUSIONS

1. The results obtained through the set of equations (4), (5), (6) and (15) show that inertia may be considered as the opposition to any change in the motion of electrons immersed in zero-point radiation because of the lateral relativistic Doppler effect (Therefore, there remains a difference equal to $1.1438 \times 10^{-6}$% to be explained).

2. The equations worked in this paper show that there is no difference between inertial and gravitational mass.

3. With abstraction of the aforesaid difference, equation (15) constitutes a new validation of Einstein’s Theory of Special Relativity.

4. There are no gravitons. The photons of zero-point radiation are the messengers of gravity.

5. Quantum theory plays an important role to explain gravity. The extremely large and the extremely small are closely linked.

6. The equations (4) and (5) come from title [3] of the references and form a system of two equations with two variables, $x$ and $k_\lambda$, which might be the result of mere imagination. With equation (15) we have a system of three equations with the same variables to explain three different phenomena, apparently independent. This system would have no solution if the equations were the result of mere imagination. Therefore they must be truly linked with the physical reality.

7. Formula (16) may be useful, possibly, to obtain better values for $G_e$ and, therefore, for $G$.

8. Formula (15) expressed the resistance of zero-point radiation to a change in the state of motion of the mass $m_e$ with acceleration “a”. This resistance can be identified by calling it $F_{a0}$. Therefore we have

$$F_{a0} = \frac{2\pi^3 (k_\lambda)^4}{3\alpha x^3} + B_x + \frac{a m_e c}{\lambda e}$$

The mass $m_e$ is the mass which has been considered in the equations in article [4], which leads to equations (4), (5) and (6) in this paper. We have not considered any difference between gravitational mass and inertial mass.

Equation (19) in paper [4] expresses the force produced by the “shadow effect” between 2 electrons immersed in zero-point radiation, which is later identified with the gravitational attraction between them, which leads
to equations (5) and (6) of this paper. If we call this force $F_{0G}$, we can write:

$$F_{0G} = \frac{2\pi^2 (k\lambda)^2 [B]_m m_e c}{\alpha x^3 \frac{d^2}{d^2 t_c}};$$

whence, by using the remaining equations, we arrive at:

$$\frac{F_{0a}}{F_{0G}} = \frac{[B_x]}{2[B]_m} = \left(\frac{4}{7}B - \frac{1}{2}B^2 + \cdots + (-1)^{m+1}m(m+3)B^m\right) - \left(\frac{1}{2}B + \frac{1}{2}B^2 + \cdots + (-1)^m m(m+3)B^m\right)$$

We get the same result by considering the electrostatic repulsion between two electrons, instead of gravitation

$$F_e = \frac{e^2}{(dl_e)^2} = \frac{1}{d^2 m_e} \cdot \frac{l_e}{(t_e)^2}$$

where $d, l_e$ is the distance between their centres.

$$\frac{F_{0a}}{F_e} = \frac{2\pi^3 (k\lambda)^4 [B]_m m_e l_e}{3\alpha x^3 \frac{d^2}{d^2 t_e} [B_x]}$$

By introducing the equation (4), $x^3 = 4\pi^3 (k\lambda)^4 [B]_m / 3\alpha$, we obtain

$$\frac{F_{0a}}{F_e} = \frac{[B_x]}{2[B]_m}$$

There is no question of inertial, gravitational or electrostatic mass, but of forces against the resistance of zero-point radiation to the change of motion of masses. If we measure any of them considering the acceleration which produces in a known mass and the equation $F_a = ma$, we obtain the value 1.0001006$F_R$, being $F_R$ the force really necessary to produce the acceleration “a” in the mass $m$. 
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