Experiment proposed for the observation of UCN interaction with matter moving with giant acceleration

A.I. Frank, D.V. Kustov, G.V. Kulin, S.V. Goryunov, D.V. Roshchupkin, D.V. Irzhak

1 Frank Laboratory of Neutron Physics, Joint Institute for Nuclear Research, Joliot-Curie str. 6, Dubna, Russia, 141980
2 Institute of Microelectronics Technology and High-Purity Materials Russian Academy of Sciences, Academician Ossipyan str. 6, Chernogolovka, Russia, 142432

E-mail: frank@nf.jinr.ru

Abstract

It was assumed that in neutron optics the concept of the effective potential has a limited region of validity, which is unlikely in the case of giant acceleration of matter. In the current work an experimental approach to test this hypothesis has been proposed.

1. Introduction

It is known that the nature of the refractive index is associated with the interference of the incident and scattered waves. The scattering of slow neutrons is isotropic and the scattering amplitude in most cases is constant (taken to be \(-b\)). In this case, the phase of the scattered wave differs from the phase of the incident wave by the value \(\delta = -kb\), where \(k\) is the wave number in vacuum and \(b\) is the scattering length. The phase shift \(\delta\) is responsible for refraction. The refractive index \(n\) satisfies the Foldy formula [1, 2]

\[
 n^2 = 1 - \frac{4\pi\rho b}{k^2},
\]

where \(\rho\) is the atomic density of nuclei. The interaction of neutrons with matter can be described by the effective potential

\[
 U_{\text{eff}} = \frac{2\pi\hbar^2}{m} \rho b,
\]

where \(m\) is the neutron mass.

Representations (1) and (2) are completely equivalent to each other. For this reason, dispersion law (1) is often called ‘potential’. The validity of equations (1) and (2) for a uniformly moving medium is indubitable, because the transition to the coordinate system, where matter is at rest for significantly non-relativistic problems characteristic of neutron optics, results only in a change in the phase of wave function [3].

It was predicted in [4, 5] that the energy of neutrons passing through a layer of matter moving at acceleration should be different from the initial energy. The theory was based on the assumption of the
validity of dispersion law (1) in an accelerated medium. The change in energy was really detected in experiments with ultracold neutrons [6, 7], where the acceleration of the sample reached 75 m/s$^2$ and the energy transferred to neutrons was fractions of neV and was in satisfactory agreement with the calculation. Consequently, the assumption of the validity of the potential dispersion law under the indicated condition is justified.

However, the general conclusion on the validity of the dispersion law in an accelerated medium would probably be erroneous. Indeed, the theory of dispersion, which is, in essence, the theory of multiple scattering of waves, involves a very substantial assumption of sphericity of interfering scattered waves. At the same time, in a non-inertial coordinate system associated with accelerated matter, the concept of spherical waves is invalid, which can affect the condition of their interference [8].

The acceleration, at which a significant deviation from dispersion law (1) is expected, can be estimated as follows [9]:

$$\frac{ma d^2}{4E} \ll b, \quad a \ll \frac{4Eb}{md^2} = W_c,$$

(3)

where $a$ is the acceleration of matter and $d$ is the interatomic spacing. According to eq. (3), the critical acceleration $W_c$ is proportional to the energy of neutrons.

Using $E = 100$ neV, $b = 5 \cdot 10^{-13}$ cm and $d = 5 \cdot 10^{-8}$ cm in equation (3), we will obtain $W_c = 4 \cdot 10^7$ cm/s$^2$ for ultracold neutrons. This acceleration is achievable in laboratory experiments. Moreover, an experiment on the observation of neutron interaction with an object moving at such acceleration was performed many years ago [10]. However, the results of experiment [10] do not contradict estimate (3) above, as it was carried out not with ultracold neutrons (UCN), but with cold neutrons and in accordance with (3) the value $W_c$ was at least two orders of magnitude greater than the sample acceleration. A similar experiment with UCN would answer the important question of the validity of the dispersion law of neutron waves in a medium moving at giant acceleration.

In this paper we briefly discuss a possible approach to testing the validity of the main equation of neutron optics in the case of giant acceleration of matter.

2. Interaction of neutrons with oscillating potential

Unfortunately, the dispersion law of the neutron wave in accelerating matter is unknown. Nevertheless, certain information about it can be obtained, if we perform an experiment on the observation of neutron interaction with a potential structure oscillating in space and compare the results with the theoretical predictions made in the assumption of the validity of eq.(2). This experiment should be performed both for a relatively low $a \ll W_c$, and for the largest possible acceleration of matter $a \gg W_c$.

Due to the relatively low intensity of UCNs, it is difficult to conduct the experiment in the reflection geometry, as it was done in [10], because such an experiment requires not only monochromatization, but also relatively good beam collimation, which leads to a considerable loss in intensity. Therefore, following ref. [11], we turned to the problem of the neutron tunneling through the oscillating barrier.

The time evolution of the wave function is defined by Schrödinger equation

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = H \psi(x,t).$$

(4)

The initial wave function is represented by a wave packet defined by

$$\psi(x,p) = \frac{1}{(2\pi\delta^2)^{1/4}} \exp\left(\frac{-(x-x_0)^2}{4\delta^2} - ipx\right).$$

(5)

Where $x_0$ is the initial position of its maximum, $p$ is its momentum, and $\delta$ is its space dispersion. The potential is confined to a finite spatial region and is given by

$$V(x,t) = U(x-A \cos(\omega t + \phi)),$$

(6)

where $U(x)$ is, in the general case, any potential structure moving as a whole

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2 Eq. (3) differs from eq. (6) of ref. [9] by a factor 2 because of mistake in [9].
Here \( U(x) = \begin{cases} V_0(\xi) & 0 \leq x \leq L \\ 0 & \text{otherwise.} \end{cases} \) \( (7) \).

The maximum of the incoming wave packet corresponded to the energy of 135 neV, which was exactly equal to the maximum of NIF transmission, and its dispersion was selected in a way to provide the travelling time through the observation point being much greater than the oscillation period. It corresponded to the degree of monochromatization \( \Delta p/p \approx 10^{-3} \div 10^{-4} \).

The calculations were done for two regimes of NIF oscillation. The frequencies and amplitudes of oscillation were, respectively, 100 KHz and 280nm for the low-frequency regime, and 5MHz and 5nm for the high-frequency regime. The maximal acceleration of NIF \( a_{\text{max}} = A \omega^2 \) was \( a_{\text{max}} = 1.1 \times 10^7 \text{cm/s}^2 \) and \( a_{\text{max}} = 5 \times 10^8 \text{cm/s}^2 \) for both regimes, which was less and greater than the critical acceleration \( W_c \).

In both cases, the transmitted state was characterized by the line spectrum with the splitting \( \Delta E = h\Omega \), \( \Omega = 2\pi f \), where \( f \) is the frequency of oscillation. But, unfortunately, it is very difficult to perform a spectroscopic experiment for the measurement of such spectra. In the low-frequency regime the value of splitting \( \Delta E \) is of the order of 0.3 neV, which is less than the resolution of the existing spectroscopic methods. In the high-frequency regime the splitting of the spectrum is 20 neV, which looks
reasonable, but for realistic values of the oscillation amplitudes the intensities of the first-order line turn out to be very small.

However, it was found that instead of measuring the spectrum it is possible to measure the time dependence of the transmitted flux. In Figure 2 the time oscillation of the flux density of the transmitted neutrons is shown for the case of the low-frequency regime. This value is proportional to the count rate of the detector placed behind NIF.

![Figure 2](image)

**Figure 2.** Time oscillation of the flux density at the point behind the potential in the low-frequency regime (see text)

This dependence of the intensity can be easily understood. The smooth flux growing and decreasing during the time of the order of $10^{-4}$s demonstrates the travelling of the wave packet (5) trough the point of observation. Oscillations of the flux are conditioned by the space oscillation of the NIF. The periodical variation of NIF velocity $V_{\text{NIF}}(t) = V_0 \sin(\omega t + \varphi)$ leads to the Doppler shift of the neutron energy in the reference system of NIF. In the case under study, $V_0 \approx 18 \text{ cm/s}$, which should be compared with the neutron velocity in the laboratory system $v \approx 5 \text{ m/s}$. The periodical change in the neutron energy $\Delta E/E = 4V_0/v$ in the reference system of NIF is greater than the width of the NIF transmission function and it transmits neutrons only during a certain part of the period, when its velocity is relatively small. As it can be seen in Figure 2, during the oscillation period of $10^{-5}$s, as expected, two maxima of intensity appear.

In this case the above-mentioned classical interpretation is correct, as the period of NIF oscillation of $10^{-5}$ s is two orders of magnitude greater than the time of neutron transmission through NIF, or, equivalently, the life time of neutron quasi-bound state, which is of the order of $10^{-7}$ s. [16].

In the high-frequency regime (see Figure 3) oscillations are also seen clearly, but their amplitude is much less than in the first case, while the maximal NIF velocity $V_0$ is of the same order of magnitude as in the low-frequency regime. The point is that the oscillation period of $2 \times 10^{-7}$ s is now comparable with the transmission time and the classical interpretation becomes inapplicable.
3. Possible experimental implementation

The possible experimental implementation is illustrated in Figures 4, 5. In the low-frequency regime the wafer with the deposed films forming the potential structure of NIF is fixed at the ring-shaped Piezo driver. The advantage of this approach is that neutrons do not pass through the driver and the latter may have the thickness necessary for providing the required vibration frequency. Besides, in this approach it is relatively easy to vary the sample - NIF. The UCN detector must be placed next to the wafer. It may be a silicon detector with a thin (0.2 mkm) layer of $^{10}$B.

An alternative approach is shown in Figure 5. In this case the films forming the potential structure of NIF are deposed on the quartz plate, which serves as a resonant Piezo driver. The thickness of the plate must be equal or multiple of a half of ultrasonic wavelength. For the operation at a frequency of 5MHz it must be of the order of 0.5 mm. Such a plate is transmittive enough for UCN. The vibration amplitude may be varied smoothly in certain limits without exceeding the value of the order of 5nm.
5. Conclusion

In the current paper it was assumed that in neutron optics the concept of the effective potential has a limited region of validity, which is unlikely in the case of giant acceleration of matter. An experimental approach to test this hypothesis has been proposed.

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