Reduce Transmission Delay for Cache-Aided Two-Layer Network

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Abstract—In this paper, we consider a two-layer caching-aided network, where a single server consisting a library of $N$ files connects with multiple relays through a shared noiseless link, and each relay connects with multiple users through a shared noiseless link. We design a caching scheme that exploits the spared transmission time resource by allowing the concurrent transmission between the two layers. It is shown that the caching scheme is order optimal and can further reduce the transmission delay compared to the previously known caching scheme. Also, we show that for the two-relay case, if each relay’s caching size $M_1$ equals to $0.382N$, our scheme achieves the same delay as $M_1 = N$, which implies that increasing $M_1$ will not always reduce the transmission delay.

Index Terms—Caching, relay network, delay

I. INTRODUCTION

Cache is used as an important technique to release the traffic load on the Internet during a network peak time. A representative approach is taking use of memories in end users or other terminals to store some contents in advance. If user caches parts of the information which is demanded during the network congestion time, therefore, only the contents not cached in local memories need to be delivered, resulting in a reduction on communication load. The whole procedure in the caching system is divided into two phases: the placement phase, where each user prefetches some contents to store in its local cache, and the delivery phase, where each user informs the contents they need to the server and the server delivers the contents needed to the users according to the information cached in users.

The coded caching scheme proposed by Ali and Niesen in [1] aroused great concern. In [1] and [2] the authors proposed two typical coded caching schemes, namely the centralized caching scheme and the decentralized caching scheme, which main differ in the way of storing contents in the placement phase. They both focus on a caching system with a server connected to some end users. A two-layer network is considered in [3], in which a server is connected to users with the help of some relay nodes, and each relay connects to a set of users with a fixed number. For this network, the authors proposed a hierarchical coded caching (HCC) scheme which achieves the optimal communication rates within a constant multiplicative and additive gap. Different two-layer caching-aided networks have been studied in [4], [5]. Besides, there are many related works which make big progress using the coded caching scheme in some other directions, such as caching problems with nonuniform user demands [6], device-to-device caching [7], caching system with multi-server [8], interference management [9], [10], and exploiting coded caching and computing to establish the optimal trade-off between communication and computation load [11], [12].

In this paper, we consider the two-layer caching-aided network proposed in [3]. In this setup, a single server consisting a library of $N$ files connects with multiple relays through a shared noiseless link, and each relay connects with multiple users through a shared noiseless link. Each relay and user are equipped with a cache memory of $M_1$ and $M_2$ files, respectively. We design a caching scheme that in the placement phase, the relays and users independently apply the the decentralized placement as in [2], and in the delivery phase, the server and the relays concurrently send XOR symbols which exploits the spared transmission time resource. It is shown that the caching scheme is order optimal and can further reduce the transmission delay compared to the HCC scheme. Also, we show that for the two-relay case, if the relay’s caching size $M_1 = 0.382N$, our scheme achieves the same delay as $M_1 = N$, which implies that increasing $M_1$ will not always reduce the transmission delay.

The rest of the paper is organized as follows. Section II introduces the system model considered in this paper. Section III reviews the related work and describes the motivation on our work. Section IV presents our main results and the proof is given in V. Section VI concludes the paper.

II. PROBLEM DEFINITION

Consider a two-layer delivery network in Fig. 1 which includes a single server, $K_1$ relays and $K_2$ users. The server has a library of $N$ independent files $W_1, \ldots, W_N$. Each $W_n$, $n = 1, \ldots, N$, is is uniformly distributed over

$$[2^F] \triangleq \{1, \ldots, 2^F\},$$

for some positive integer $F$. Every relay node has a cache memory of size $M_1 F$ bits, $M_1 \in [0, N]$, and is connected to the server through a noiseless shared link. Meanwhile, each relay connects with $K_2$ users, each equipped with a cache memory of size $M_2 F$ bits, for $M_2 \in [0, N]$, through a noiseless shared link. Let the $j$-th user attached to relay $i$ be $u^j_i$, for $i \in [K_1]$ and $j \in [K_2]$, and define

$$U_i \triangleq \{u^1_i, \ldots, u^{K_2}_i\}, \ U \triangleq U_1 \cup \cdots \cup U_{K_1} \quad (1)$$

$$\sum_{i=1}^{K_1} |U_i| \leq M_1 F,$$
and produces symbol $X_i \triangleq f_{i,d}(Z_i, X)$, for some encoding functions

$$f_d : [2^F]^N \rightarrow [[2^{FR_1}]],$$

$$f_{i,d} : [[2^{FM_1}]] \times [[2^{FR_1}]] \rightarrow [[2^{FR_2}]]$$

where $R_1$ and $R_2$ denote the rate transmitted in the first layer and second layer, respectively.

User $u_i^j$ perfectly observes the symbol sent by relay $i$, and decodes its desired message as

$$\hat{W}_{d_i^j} = \psi_{j,i}(X_i, Z_j^i),$$

for some decoding function

$$\psi_{j,i} : [[2^{FR_2}]] \times [[2^{FM_2}]] \rightarrow [2^F].$$

We define the worst-case probability of error as

$$P_e \triangleq \max_{d \in [2^F]^N} \max_{i \in [K_1]} \max_{j \in [K_2]} \Pr \left( \hat{W}_{d_i^j} \neq W_d^j \right).$$

A caching scheme $(M_1, M_2, R_1, R_2)$ consists of caching functions (4), encoding functions (7) and decoding functions (8). We say that the rate region $(M_1, M_2, R_1, R_2)$ is achievable if for every $\epsilon > 0$ and every large enough file size $F$, there exists a caching scheme such that $P_e$ is less than $\epsilon$.

Suppose that there exists a caching scheme $(M_1, M_2, R_1, R_2)$ that can successfully deliver the required files to all users in $T_e$ transmission slots, the coding delay for request $d$ is defined as the transmission slots normalized to the size of file, i.e.,

$$T(d) \triangleq \frac{T_e}{F}.$$  

The optimal coding delay is defined as:

$$T^* \triangleq \min_{d \in [2^F]^N} \max_{d \in [2^F]^N} T(d).$$

The goal is to design a caching scheme such that the coding delay at the time of content delivery is minimized.

Notice that there may exist concurrent transmissions between the server and relays due to the orthogonal links between the two layers. Thus, if in a caching scheme $(M_1, M_2, R_1, R_2)$ the server and all relays concurrently send symbols throughout all transmission slots, then the corresponding coding delay is

$$T(d) \triangleq \max \{ R_1, R_2 \}.$$  

On the other hand, if there exists a relay which starts transmission until the server finishes its transmission, then we have

$$T(d) \triangleq R_1 + R_2.$$  

III. PRELIMINARY AND MOTIVATION

In this section, we recall the former related work and present several interesting observations that inspire our work.
A. Preliminary

In [2], a single-layer network is considered, where a server connects with $K$ users through a shared noiseless link. Each user is provided with a cache memory of size $M$ files. The authors show that the decentralized caching scheme achieves the coding delay $r(M/N, K)$, where $r(\cdot, \cdot)$ is given by

$$ r\left(\frac{M}{N}, K\right) \triangleq \left[ K \left(1 - \frac{M}{N}\right) \frac{N}{KM} \left(1 - \left(1 - \frac{M}{N}\right) K\right) \right]^+ $$

(14)

where $[x]^+ \triangleq \max\{x, 0\}$.

In [3], the authors considered the two-layer network as described in Section II. They propose three hierarchical caching schemes, namely HCC-A, HCC-B and HCC-C, based on the single-layer decentralized caching scheme. We recall these three schemes for future reference and comparison as follows.

- Scheme HCC-A: The main idea is that each relay is considered as a “tycoon user” that wishes to cover all the files requested by its attached users, the server firstly delivers the requested files to all relays, and after the relays decoding all their required files, they concurrently send the requested files to their attached users.

  More specifically, in the placement phase, each relay $i$ and user $u_i$ independently cache a subset of $M_i F$ and $M_i K$ bits of file $W_i$, respectively, chosen uniformly at random, for $n \in [N]$. The delivery phase consists of two subphases. In the first subphase, the server considers each relay $i$ as a tycoon user requesting files $(W_{d_i1}, \ldots, W_{d_iK})$ and broadcasts XOR files to all relays using the single-layer decentralized caching scheme. It is easy to obtain that to ensure all relays perfectly know $(W_{d_i1}, \ldots, W_{d_iK})$, the rate satisfies

$$ R_{HCC-A,1} \triangleq K_2 \cdot r(M_1/N, K_1). $$

(15a)

In the second subphase, after the relays successfully decode all the requested files of their attached users through the first subphase, they concurrently deliver the requested files to their attached users by the single-layer decentralized caching scheme, which leads to an achievable rate

$$ R_{HCC-A,2} \triangleq r(M_2/N, K_2). $$

(15b)

- Scheme HCC-B: In scheme HCC-B the caching memories of the relays are completely ignored, and the relays forward relevant parts of the server transmissions to the corresponding users. In the placement phase, $K_1 K_2$ users independently cache a subset of $M_i F$ bits of file $W_i$, chosen uniformly at random, for $n \in [N]$. In the delivery phase, the server ignores the relays’ caches and uses the decentralized caching scheme to deliver the requested files of $K_1 K_2$ users. Thus, we have the rate of the first layer:

$$ R_{HCC-B,1} \triangleq r(M_2/N, K_1 K_2). $$

(16a)

and the rate of the second layer:

$$ R_{HCC-B,2} \triangleq r(M_2/N, K_2). $$

(16b)

- Scheme HCC-C: Informally, HCC-C is a mixture scheme HCC-A and scheme HCC-B. The system is divided into two subsystems with two fixed parameters $\alpha, \beta \in [0, 1]$. The first subsystem includes the entire cache memory of each relay, a $\alpha$ fraction of each file in the library and a $\beta$ fraction of cache memory for each user, and the second subsystem includes the remaining $1 - \alpha$ fraction of each file in the server and a $1 - \beta$ fraction of each user’s cache memory. Obviously, scheme HCC-A and HCC-B can be implemented in the first subsystem and second subsystem, respectively. Thus, we have the rate of the first layer:

$$ R_{HCC-C,1} \triangleq \alpha K_2 r\left(\frac{M_1}{\alpha N}, K_1\right) + (1 - \alpha) \cdot r\left(\frac{(1 - \beta) M_2}{(1 - \alpha) N}, K_1 K_2\right), $$

(17a)

and the rate of the second layer:

$$ R_{HCC-C,2} \triangleq \beta M_2 r\left(\frac{1}{\alpha N}, K_1\right) + (1 - \beta) \cdot r\left(\frac{(1 - \beta) M_2}{(1 - \alpha) N}, K_2\right). $$

(17b)

The approximately optimal $\alpha$ and $\beta$ is given as

$$ \alpha(\alpha, \beta) = \begin{cases} \left(\frac{M_1}{N}, \frac{M_1}{N}\right), M_1 + M_2 K_2 \geq N, 0 \leq M_1 \leq N/4, \\ \left(\frac{M_1}{N}, \frac{M_1 + M_2 K_2}{N}\right), M_1 + M_2 K_2 < N, \\ \left(\frac{M_1}{N}, 1\right), M_1 + M_2 K_2 \geq N, N/4 < M_1 \leq N. \end{cases} $$

(17c)

Scheme HCC-C reduces to scheme HCC-A when $\alpha = \beta = 1$, and to scheme HCC-B when $\alpha = \beta = 0$. Since the transmission in two layers proceeds in a sequential progress, the coding delay of scheme HCC-C is

$$ T_{HCC-C} = R_{HCC-C,1} + R_{HCC-C,2}. $$

(18)

In [3], it shows that $R_{HCC-C,1}$ and $R_{HCC-C,2}$ achieve the optimal rates to within a constant multiplicative and additive gap.

B. Motivation

Note that in the first subphase of scheme HCC-A, the server takes the relays as tycoon users and sends files only with the help of relays’ caches, ignoring the users’ caches. This means that the server may send some redundant contents that have already been stored in users’ caches. Furthermore, the second subphase can only start after the first subphase, i.e., the relays can only deliver files after they successfully decoded all requested files of their attached users through the first subphase. These will waste the transmission resources and cause unnecessary coding delay. Consider the following examples.
that both HCC-A and HCC-B require the coding delay larger than \(r(M_2/N, K_1 K_2)\). In fact, if we use a pipeline forward scheme where the relay connects the two layers as one pipeline (the information flow moves smoothly through the pipeline), then the two-layer network is equivalent to the single-layer network in which a server connects with \(K_1 K_2\) users through a shared noiseless link. From (2), we achieve the coding delay \(r(M_2/N, K_1 K_2)\). Note that the improvement is in essence from the fact that the relays and server can concurrently deliver data. 

We conclude this section by listing the following insights:

- Concurrent transmission between the two layers can reduce the coding delay.
- For some cases, such as Example 2 having partial size \((M_1 < N)\) of the library at the relays may achieve the same coding delay as the case \(M_1 = N\). In other words, increasing the size of the relay's cache memory may not always reduce the coding delay.

IV. MAIN RESULTS

We now present a upper bound on the coding delay of the network depicted in Fig. 1.

**Theorem 1.** For \(\alpha, \beta \in [0, 1]\), define \(M_2' \triangleq \beta M_2\), \(N' \triangleq \alpha N\) and

\[
R_{s_1} \triangleq \alpha \left[ r \left( \frac{M_1}{N}, K_1 \right) - K_1 \left( 1 - \frac{M_1}{N} \right)^{K_1} \right] r \left( \frac{M_2'}{N'}, K_2 \right),
\]

\[
R_{s_2} \triangleq \alpha \left( 1 - \frac{M_1}{N} \right)^{K_1} r \left( \frac{M_2'}{N'}, K_1 K_2 \right),
\]

\[
R_{s_3} \triangleq \alpha \frac{M_1}{N'} r \left( \frac{M_2'}{N'}, K_2 \right),
\]

\[
R_e \triangleq \alpha \left( 1 - \frac{M_1}{N} \right)^{K_1} \frac{1 - M_2'}{N} r \left( \frac{M_2'}{N'}, (K_1 - 1) K_2 \right).
\]

For all \(M_1, M_2 \in [0, N]\) and \(\alpha, \beta \in [0, 1]\),

\[
T^* \leq T_{\text{propose}} = R' + R''
\]

where

\[
R' \triangleq R_{s_1} + R_{s_2} + \max \{ R_{s_3} - R_e, 0 \},
\]

\[
R'' \triangleq (1 - \alpha) r \left( \frac{1 - \beta M_2}{(1 - \beta) N}, K_1 K_2 \right).
\]

**Proof.** See proof in Section V.

**Corollary 1.** Consider the two-relay case \((K_1 = 2)\). we have,

\[
T^* \leq \left\{ \begin{array}{ll}
    r \left( \frac{M_2}{N}, K_2 \right), & \text{if } M_1 \frac{1}{N} \geq \left( 1 - \frac{M_1}{N} \right)^2 \left( 1 - \frac{M_2}{N} \right)^2, \\
    \frac{M_1}{N} \left( 1 - \frac{M_1}{N} \right) r \left( \frac{M_2}{N}, K_2 \right) + \left( 1 - \frac{M_1}{N} \right)^2 r \left( \frac{M_2}{N}, 2 K_2 \right), & \text{otherwise}.
\end{array} \right.
\]

**Proof.** Setting \(\alpha = \beta = 1\) in (19), we have \(R'' = 0\) and \(T^* \leq R'\). Given \(K_1 = 2\), if \(R_{s_3} \geq R_e\), i.e.,

\[
M_1 \frac{1}{N} \geq \left( 1 - \frac{M_1}{N} \right)^2 \left( 1 - \frac{M_2}{N} \right)^2,
\]

TABLE I

| Server | Relay 1 | Relay 2 | Rate |
|--------|---------|---------|------|
| A2 ⊕ C1 | 2/1 | 2/1 | 1 |
| B2 ⊕ D1 | 1/2 | 1/2 | 1 |

TABLE II

| Server | Relay 1 | Relay 2 | Rate |
|--------|---------|---------|------|
| A2 ⊕ C1 | A1 | C1 | 2/1 |
| B2 ⊕ D1 | B2 | D1 | 1/2 |
| A2 | C2 | 1/2 |
| B2 | D2 | 1/2 |
then \( R' = R_{s1} + R_{s2} + R_{s3} - R_e = r \left( \frac{M_2}{N}, K_2 \right) \). If \( R_{s3} < R_e \), then \( R' = \frac{M_1}{N} \left( 1 - \frac{M_1}{N} \right) r \left( \frac{M_2}{N}, K_2 + 1 - \frac{M_1}{N} \right) + \frac{M_1}{2K_2} \).

From Corollary 1 it is surprising to find that when condition (21) holds, we achieve the coding delay \( r(M_2/N, K_2) \), which is the optimal coding delay with \( M_1 = N \) under the decentralized placement. Note that \( (1 - M_2/N)^2 \leq 1 \), if

\[
M_1 \geq \left( 1 - \frac{M_1}{N} \right)^2 \implies M_1 \geq 0.382N,
\]

then (21) is satisfied. It implies that 0.382N is a threshold for \( M_1 \) to achieve the coding delay as \( M_1 = N \). In other words, when the relays’ cache memory \( M_1 \) equals to 0.382N, increasing \( M_1 \) will no longer reduce the coding delay. This in turn explains the observation in Example 2 showing that \( M_1 = N/2 \) achieves the same coding delay as \( M_1 = N \). We emphasize this result as the following corollary.

**Corollary 2.** For the two-relay two-layer network as depicted in Fig. 1 when using decentralized caching placement and \( M_1 \) equals to the threshold 0.382N, we can achieve the optimal coding delay same as \( M_1 = N \).

The following theorem presents the lower bound on the coding delay.

**Theorem 2.** For all \( M_1, M_2 \in [0, N] \), \( s_1 \in [K_1] \) and \( s, s_2 \in [K_2] \).

\[
T^* \geq \max \left\{ s_1 s_2 - \frac{s_1 M_1 + s_1 + s_2 M_2}{N/(s_1 s_2)}, s - \frac{s M_2}{N/s} \right\}.
\] (22)

**Proof.** The first term on right hand of (22) follows from the similar cut-set bound given in [3, Appendix A]. The second term is obtained by the cut-set bound assuming \( M_1 = N \). In this case, the relays access the full library and the two-layer network is equivalent to the single-layer network where a server connects with \( K_2 \) users each caching \( M_2 \) files, and thus from [1, Theorem 8], we obtain the second term on right hand of (22).

Comparing the upper bound in Theorem 1 with the HCC-C upper bound (18) and the lower bound (22), we have

**Theorem 3.** For all \( M_1, M_2 \in [0, N] \).

\[
T_{\text{HCC-C}} - T_{\text{propose}} \geq \begin{cases} 
(1 - \frac{M_1}{N}) r \left( \frac{M_2}{N}, K_2 \right), & \text{regime I,} \\
M_2 K_2 \left( \frac{M_1 + M_2 K_2}{N K_2}, K_2 \right), & \text{regime II,} \\
M_1 + M_2 K_2 \left( 1 - \frac{M_1}{N} \right) r \left( \frac{3 M_2}{4(N - M_1)}, K_2 \right), & \text{regime III,}
\end{cases}
\] (23a)

\[
T_{\text{propose}} \leq c \cdot T^* 
\] (23b)

where regime 1, 2 and 3 represent \( M_1 + M_2 K_2 \geq N, 0 \leq M_1 \leq \frac{N}{3}, M_1 + M_2 K_2 < N \) and \( M_1 + M_2 K_2 \geq N, \frac{N}{3} < M_1 < \frac{N}{2} \), increasing \( M_1 \) does not reduce the coding delay, which indicates that we could even lower the threshold given in Corollary 2 for some cases.

**V. PROOF OF THEOREM 1**

We first propose a caching scheme and carefully design the algorithms to allocate the delivery of subfiles cached in server or relays. The decentralized placement procedure is applied to cache memory in relays and users independently, which results in that subfiles may be stored only in users, only in relays or both in users and relays. For each user \( u_i \in U \), we divide the subfiles required to be delivered into three parts: Subfiles I are those cached by other relays except relay 1, and
will be sent using a block decode-forward strategy; Subfiles II are those not cached by any relay, and will be sent by the pipeline forward strategy introduced in Example[3]. Subfiles III are those cached by relay \( i \), and will be sent by the single-layer decentralized caching scheme. When the server sends Subfiles II, there are some parts which are redundant for relay \( i \). We let relay \( i \) send parts of Subfiles III if the server’s signal is not useful. Another caching scheme is to simply use the pipeline forward strategy to send all the requested files. Combine these two caching schemes in a similar way as HCC-C scheme, we obtain the coding delay in Theorem[1]

A. Concurrent Caching Scheme

The scheme is divided into the placement phase and the delivery phase. In the placement phase, firstly, each relay \( i \) uses caching function \( \phi_i \) to map the \( N \) files into its \( M_1 F \)-bit cache randomly and independently. Similarly, each user \( u_j \) uses caching function \( \phi_j \) to map the files into its \( M_2 F \)-bit cache randomly and independently, which is showed in Algorithm 1. Therefore, each file \( W_n \) is divided into multiple subfiles, i.e., for \( n = 1, \ldots, N \)

\[
W_n = \{ W_{n,S}^Q, \text{for all } Q \subseteq [K_1], S \subseteq U \}
\]

The subfiles stored in user \( u_j^i \) is

\[
\{ W_{1,S}^Q, \ldots, W_{N,S}^Q, \text{for all } Q \subseteq [K_1], S \subseteq U \text{ with } u_j^i \in S \}
\]

The subfiles stored in relay \( i \) is

\[
\{ W_{1,S}^Q, \ldots, W_{N,S}^Q, \text{for all } Q \subseteq [K_1], S \subseteq U \text{ with } i \in Q \}
\]

Algorithm 1 Placement Phase

1: for \( n \in [N] \) do
2: \hspace{1em} for \( i \in [K_1] \) do
3: \hspace{2em} relay \( i \) independently caches a subset of \( M_1 F \) bits of file \( W_n \), chosen uniformly at random
4: \hspace{2em} for \( j \in [K_2] \) do
5: \hspace{3em} User \( u_j^i \) independently caches a subset of \( M_2 F \) bits of file \( W_n \), chosen uniformly at random
6: \hspace{2em} end for
7: \hspace{1em} end for
8: end for

Note that the request vector \( d \) is not informed during this phase and all caching functions select contents to cache completely arbitrarily. When the file size \( F \) is large, by the law of large numbers, the subfile size with high probability can be written as

\[
|W_{n,S}^Q| \approx \left( \frac{M_1}{N} \right)^{|Q|} \left( 1 - \frac{M_1}{N} \right)^{K_1 - |Q|} \cdot \left( \frac{M_2}{N} \right)^{|S|} \left( 1 - \frac{M_2}{N} \right)^{K_2 - |S|} F.
\] (24)

It’s easy to verified that under the placement given in Algorithm 1, each relay and user fill to \( M_1 F \) and \( M_2 F \) bits to its caches, respectively.

In the delivery phase, each user \( u_j^i \) requests a file \( W_{d_j,i} \), and the request vector \( d \) is promoted to the server and relays. Our objective is to get the upper bound on coding delay in the worst request case, so we assume that each of the users makes unique request in the following discussion. The subfiles of \( W_{d_j,i} \) requested by user \( u_j^i \) can be characterized into three types:

- Subfiles I: the subfiles cached by other relays except relay \( i \), i.e., \( \forall S \subseteq U, Q \subseteq [K_1] \) with \( i \notin Q \).
- Subfiles II: the subfiles not cached by any relay, i.e., \( \forall S \subseteq U \).
- Subfiles III: the subfiles cached by relay \( i \), i.e., \( \forall S \subseteq U, Q \subseteq [K_1] \) with \( i \in Q \).

Next we illustrate the transmission of the three types of subfiles. For ease of notation, the corresponding transmission is labelled as Transmission I, II, III, respectively.

- Transmission I: Since there is no communication or cooperation among the relays, Subfiles I can only be delivered from the server to users. The server sends the following symbol to the relays

\[
\oplus_{i \in R} \left( \oplus_{u_j^i \in S} W_{d_j,i,S \setminus \{u_j^i\} \cup T}^R \right),
\] (25a)

for each \( j \in [K_2] \) and

\[
R \subseteq [K_1]: |R| = r, r = K_1, K_1 - 1, \ldots, 2,
\]

\[
S_j \subseteq U: u_j^i \in S_j \text{ and } |S_j| = s, s \in [K_2],
\]

\[
T \subseteq U \setminus U_i: |T| = t, t \in \{K_1 K_2 - K_2, \ldots, 0\}.
\] (25b)

For a given tuple of parameters \((j, R, S_i, T)\), after relay \( i \in R \) observing the symbol in (25), it decodes the message

\[
\oplus_{u_j^i \in S_i} W_{d_j,i,S_i \setminus \{u_j^i\} \cup T}^R
\] (26)

and forwards it to its attached users. After observing the message (26), each user \( u_j^i \in S_i \) decodes the following requested subfile based on the cached content

\[
W_{d_j,i,S_i \setminus \{u_j^i\} \cup T}^R.
\]

Notice that the relays can simultaneously receive and transmit signals, that means when each relay \( i \in R \) decodes and forwards the message (26), the server can keep sending the symbol for a different tuple of parameters \((j, R, S_i, T)\). This procedure is similar to the block-Markov coding scheme in [13], where in every current block the transmitter sends a new source message, and the relay decodes and forwards the signal received from the previous block.
According to the delivery strategy described above, Subfiles I can be perfectly known at the corresponding users. The delay of Transmission I, denoted by $R_{T1}$, is thus

$$R_{T1} = \left[ r\left(\frac{M_1}{N}, K_1\right) - K_1 (1 - \frac{M_1}{N}) r\left(\frac{M_2}{N}, K_2\right) \right]. \quad (27)$$

- Transmission II: Subfiles II also need to be sent originally to the server. The server sends symbol

$$\oplus_{u_i \in S} W^0_{d_j, S \setminus \{u_i\}}, \quad (28)$$

for each subset $S \subseteq U : |S| = s, s \in [K_1 K_2]$. Thus the delay of sending all Subfiles II, denoted by $R_{T2}$, is

$$R_{T2} = \left(1 - \frac{M_1}{N}\right) K_1 r\left(\frac{M_2}{N}, (K_1 - 1)K_2\right). \quad (29)$$

For a given subset $S$ with $S \cap U_i = \emptyset$, the symbol $W^0_{d_j, S \setminus \{u_i\}}$ contains the requested subfiles for relay $i$’s attached users. We call this kind of symbol the useful symbol of Subfiles II for relay $i$. Relay $i$ uses pipeline forward scheme as described in Example 3 to send the message

$$\oplus_{u_i \in S} W^0_{d_j, S \setminus \{u_i\}}, \quad (28)$$

Each user $u_i' \in S$ decodes its requested subfile $W^0_{d_j, S \setminus \{u_i\}}$ based on the cached content.

For a given set subset $S$ with $S \cap U_i = \emptyset$, the symbol $W^0_{d_j, S \setminus \{u_i\}}$ does not contain any subfile requested by relay $i$’s attached users. We call this kind of symbol the redundant symbol of Subfiles II for relay $i$. The rate of this part, denoted by $R_e$, can be computed as

$$R_e = \left(1 - \frac{M_1}{N}\right) K_1 r\left(\frac{M_2}{N}, (K_1 - 1)K_2\right).$$

Here, we exploit this spared time resource by letting each relay $i$ send some parts of Subfiles III. This is possible since Subfiles III have already been stored in the relay’s cache memory during the placement phase. More specifically, when the server sends the symbol $W^0_{d_j, S \setminus \{u_i\}}$ with $S \cap U_i = \emptyset$, each relay $i$ sends

$$\oplus_{u_i' \in S_i} W^{R'}_{d_j, S_i \setminus \{u_i'\} \cup T'}, \quad (30a)$$

for some set

$$R' \subseteq [K_1] : i \in R', \quad (30b)$$

$$S' \subseteq \cup U : |S'| = s, s \in [K_2], \quad (30c)$$

$$T' \subseteq \cup U : |T'| = t, t = K_1 K_2 - K_2, \ldots, 0, \quad (30d)$$

such that the rate of delivering these symbols equals to $\min\{R_e, R_{s3}\}$, where $R_{s3}$ denotes the rate required to send Subfiles III. Due to the different sizes of symbols in (28) and (30), we may not be able to find $(R', S', T')$ such that the the delay of sending symbols (30) exactly equals to $R_e$. One can obviate this problem by splitting

Subfiles III into the smaller pico-files and sending these pico-files in the same way as (30).

Each User $u_i'$ decodes Subfiles II $W^0_{d_j, S \setminus \{u_i\}}$ and part of Subfiles III $W^{R'}_{d_j, S \setminus \{u_i'\} \cup T'}$ based on its cached content.

- Transmission III: Review the three subfiles summarized above, when Transmission I and II finished, only the remaining parts of the Subfiles III need to be transmitted from the relays to their attached users. The server does not send any symbol in Transmission III. All the relays concurrently send signals to their attached users, i.e., relay $i$ concurrently sends the symbol same as (30) except that $(R', S', T')$ are replaced by $(R'', S''', T''')$, respectively, and $R' \cup R'' = [K_1], S' \cup S''' = U_i$ and $T' \cup T'' = U \cup U_i$. The rate for sending Subfiles III, denoted by $R_{s3}$, is

$$R_{s3} = \left(1 - \frac{M_1}{N}\right) K_1 r\left(\frac{M_2}{N}, K_2\right). \quad (31)$$

Thus the delay of Transmission III, denoted by $R_{T3}$, is

$$R_{T3} = \max\{R_{s3} - R_e, 0\}. \quad (32)$$

Combines (27), (29) and (32), we obtain the delay of the whole transmission, denoted by $R_{propose}$, as

$$R_{propose} = R_{T1} + R_{T2} + R_{T3}. \quad (33)$$

The delivery phase is summarized in Algorithm 2. Table III shows the order of transmitting Subfiles I, II and III. In order to illustrate the caching and delivery scheme described above, we consider an example as below.

**Example 4.** Consider the two-layer network with $N = 4$ files $(A, B, C$ and $D), K_1 = K_2 = 2,$ and $M_1, M_2 \in \{0, N\}$, as depicted in in Fig. 4.

In the placement phase, using Algorithm 1, relay 1 and relay 2 independently store a random $M_1 F/4$-bit subset of each file, and four users independently store a random $M_2 F/4$-bit subset of each file. Let $A^0_S$ denote the subset of file $A$ that are stored in the cache memories of users in $S$ and relays in $Q$, where $S \subset \{1, 2, 3, 4\}, Q \subset \{1, 2\}$. For example, $A^1_{2,3}$ is the subset of $A$ cached by user 2, 3 and relay 1.

$$A = \begin{cases} A^0_0, A^0_1, A^0_2, A^0_3, \ldots, A^0_{1,2,3,4} \\ A^1_0, A^1_1, A^1_2, A^1_3, \ldots, A^1_{1,2,3,4} \\ A^2_0, A^2_1, A^2_2, A^2_3, \ldots, A^2_{1,2,3,4} \\ A^3_0, A^3_1, A^3_2, A^3_3, \ldots, A^3_{1,2,3,4} \end{cases}. \quad (34)$$
Algorithm 2 Delivery Phase

1:  Procedure 1 Delivery of Subfiles I
2:  for \( r = K_1, K_1 - 1, \ldots, 2 \) do
3:      for \( \mathcal{R} \subseteq [K_1] : |\mathcal{R}| = r \) do
4:          for \( s \in [K_2] \) do
5:              for \( j \in [K_2] \) do
6:                  for \( i \in \mathcal{R} \) do
7:                      for \( S_i \subseteq \mathcal{U}_i : |S_i| = s \) and \( u^i_j \in S_i \) do
8:                          for \( t = K_1 K_2 - K_2, \ldots, 1, 0 \) do
9:                              for \( T \subseteq \mathcal{U}_i \setminus \mathcal{U}_i : |T| = t \) do
10:                                 \( \oplus_{i \in \mathcal{R}} \left( \oplus_{u^j_i \in S_i} W_{d_j^{-1}, S_i \setminus \{u^j_i\} \cup T} \right) \)
11:                              end for
12:                          end for
13:                      end for
14:                  end for
15:              end for
16:          end for
17:      end for
18:  end for
19:  End Procedure
20:  
21:  Procedure 2 Delivery of Subfiles II
22:  for \( s \in [K_1 K_2] \) do
23:      for \( S \subseteq \mathcal{U}_i : |S| = s \) do
24:          \( \oplus_{u^j_i \in S} W_{d_j^{-1}, S_i \setminus \{u^j_i\}} \)
25:      end for
26:  end for
27:  End Procedure
28:  
29:  Procedure 3 Delivery of Subfiles III
30:  for \( \mathcal{R} \subseteq [K_1] : i \in \mathcal{R} \) do
31:      for \( S_i \subseteq \mathcal{U}_i : |S_i| = s \) do
32:          for \( t = K_1 K_2 - K_2, \ldots, 0 \) do
33:              for \( T \subseteq \mathcal{U}_i \setminus \mathcal{U}_i : |T| = t \) do
34:                  \( \oplus_{u^j_i \in S_i} W_{d_j^{-1}, S_i \setminus \{u^j_i\} \cup T} \)
35:              end for
36:          end for
37:      end for
38:  end for
39:  End Procedure
40:  
41:  Procedure 4 Redundant Symbols in Subfiles II For Relay \( i \)
42:  for \( s \in [K_1 K_2 - K_2] \) do
43:      for \( S \subseteq \mathcal{U}_i : |S| = s \) and \( S \cap \mathcal{U}_i = \emptyset \) do
44:          \( \oplus_{u^j_i \in S} W_{d_j^{-1}, S_i \setminus \{u^j_i\}} \)
45:      end for
46:  end for
47:  End Procedure

\[
\left| W_{d_j^{-1}, S} \right| \approx \left( \frac{M_1}{4} \right)^{|S|} \cdot \left( 1 - \frac{M_1}{4} \right)^{2-|S|} \cdot \left( \frac{M_2}{4} \right)^{|S|} \cdot \left( 1 - \frac{M_2}{4} \right)^{4-|S|} F
\]

In the delivery phase, we apply Algorithm 2 to send Subfiles I, II and III. More specifically, the transmission for Subfiles I is

\[
A_0^Q \oplus C_0^Q, B_0^Q \oplus D_0^Q, A_2^Q \oplus B_2^Q \oplus C_1^Q \oplus D_1^Q,
\]

\[
A_3^Q \oplus C_1^Q, A_4^Q \oplus C_2^Q, B_2^Q \oplus D_1^Q, B_3^Q \oplus D_2^Q,
\]

\[
A_2^Q \oplus C_1^Q, A_4^Q \oplus C_2^Q, B_2^Q \oplus D_1^Q, B_3^Q \oplus D_2^Q,
\]

\[
A_2^Q \oplus C_1^Q, A_4^Q \oplus C_2^Q, B_2^Q \oplus D_1^Q, B_3^Q \oplus D_2^Q,
\]

\[
A_2^Q \oplus C_1^Q, A_4^Q \oplus C_2^Q, B_2^Q \oplus D_1^Q, B_3^Q \oplus D_2^Q,
\]

\[
A_3^Q \oplus C_1^Q, A_4^Q \oplus C_2^Q, B_2^Q \oplus D_1^Q, B_3^Q \oplus D_2^Q,
\]

\[
A_3^Q \oplus C_1^Q, A_4^Q \oplus C_2^Q, B_2^Q \oplus D_1^Q, B_3^Q \oplus D_2^Q,
\]

\[
A_3^Q \oplus C_1^Q, A_4^Q \oplus C_2^Q, B_2^Q \oplus D_1^Q, B_3^Q \oplus D_2^Q,
\]

For a large enough file size \( F \), this requires a communication rate

\[
R_{S_1} = \frac{M_1}{4} \cdot \left( 1 - \frac{M_1}{4} \right)^r \left( \frac{M_2}{4} \right)^{r} F
\]

The transmission for Subfiles II is

\[
A_0^Q \oplus B_0^Q \oplus C_0^Q, D_0^Q,
\]

\[
A_2^Q \oplus B_1^Q, A_3^Q \oplus C_1^Q, A_4^Q \oplus D_1^Q, C_2^Q \oplus B_3^Q, D_2^Q \oplus B_4^Q, C_4^Q \oplus D_3^Q,
\]

\[
A_2^Q \oplus B_1^Q, A_3^Q \oplus C_1^Q, A_4^Q \oplus D_1^Q, C_2^Q \oplus B_3^Q, D_2^Q \oplus B_4^Q, C_4^Q \oplus D_3^Q,
\]

\[
A_2^Q \oplus B_1^Q, A_3^Q \oplus C_1^Q, A_4^Q \oplus D_1^Q, C_2^Q \oplus B_3^Q, D_2^Q \oplus B_4^Q, C_4^Q \oplus D_3^Q,
\]

\[
A_2^Q \oplus B_1^Q, A_3^Q \oplus C_1^Q, A_4^Q \oplus D_1^Q, C_2^Q \oplus B_3^Q, D_2^Q \oplus B_4^Q, C_4^Q \oplus D_3^Q,
\]

\[
A_2^Q \oplus B_1^Q, A_3^Q \oplus C_1^Q, A_4^Q \oplus D_1^Q, C_2^Q \oplus B_3^Q, D_2^Q \oplus B_4^Q, C_4^Q \oplus D_3^Q,
\]

The communication rate is

\[
R_{S_2} = \left( 1 - \frac{M_1}{4} \right)^{2r} \left( \frac{M_2}{4} \right)^{4r} F
\]

The transmission for Subfiles III contains two parts:

- From relay 1 to user 1 and 2

\[
A_2^Q \oplus B_1^Q, A_3^Q \oplus B_2^Q, A_4^Q \oplus B_3^Q, A_5^Q \oplus B_4^Q,
\]

\[
A_2^Q \oplus B_1^Q, A_3^Q \oplus B_2^Q, A_4^Q \oplus B_3^Q, A_5^Q \oplus B_4^Q,
\]

\[
A_2^Q \oplus B_1^Q, A_3^Q \oplus B_2^Q, A_4^Q \oplus B_3^Q, A_5^Q \oplus B_4^Q,
\]

\[
A_2^Q \oplus B_1^Q, A_3^Q \oplus B_2^Q, A_4^Q \oplus B_3^Q, A_5^Q \oplus B_4^Q,
\]

\[
A_2^Q \oplus B_1^Q, A_3^Q \oplus B_2^Q, A_4^Q \oplus B_3^Q, A_5^Q \oplus B_4^Q,
\]

\[
A_2^Q \oplus B_1^Q, A_3^Q \oplus B_2^Q, A_4^Q \oplus B_3^Q, A_5^Q \oplus B_4^Q,
\]

\[
A_2^Q \oplus B_1^Q, A_3^Q \oplus B_2^Q, A_4^Q \oplus B_3^Q, A_5^Q \oplus B_4^Q,
\]

\[
A_2^Q \oplus B_1^Q, A_3^Q \oplus B_2^Q, A_4^Q \oplus B_3^Q, A_5^Q \oplus B_4^Q,
\]

\[
A_2^Q \oplus B_1^Q, A_3^Q \oplus B_2^Q, A_4^Q \oplus B_3^Q, A_5^Q \oplus B_4^Q,
\]

\[
A_2^Q \oplus B_1^Q, A_3^Q \oplus B_2^Q, A_4^Q \oplus B_3^Q, A_5^Q \oplus B_4^Q,
\]

\[
A_2^Q \oplus B_1^Q, A_3^Q \oplus B_2^Q, A_4^Q \oplus B_3^Q, A_5^Q \oplus B_4^Q,
\]
• From relay 2 to user 3 and 4

\[
\begin{align*}
C_{0}^{Q_{1}} & = D_{0}^{Q_{1}}, C_{4}^{Q_{1}} + D_{4}^{Q_{1}} & & \quad C_{1}^{Q_{1}}, D_{1}^{Q_{1}}, D_{2}^{Q_{1}}, D_{3}^{Q_{1}}, \\
C_{0}^{Q_{2}} & = D_{0}^{Q_{2}}, C_{4}^{Q_{2}} + D_{4}^{Q_{2}} & & \quad C_{1}^{Q_{2}}, D_{1}^{Q_{2}}, D_{2}^{Q_{2}}, D_{3}^{Q_{2}}, D_{4}^{Q_{2}}
\end{align*}
\]

Here \( Q_{1} \) denotes the subsets of the relay \( \{1, 2\} \) which includes relay 1. And in this case, \( Q_{1} = \{1, 2\} \). Similarly, \( Q_{2} \) denotes the subsets of the relay \( \{1, 2\} \) which includes relay 2, and here \( Q_{2} = \{2, 1, 2\} \). Relay 1 and relay 2 can transmit these subfiles simultaneously and respectively, so the normalized rate is

\[
R_{s_{3}} = \frac{M_{1}}{4} \cdot r\left(\frac{M_{2}}{N}, 2\right).
\]

The redundant symbols of Subfiles II for the relays are as below:

- Relay 1 does not need

\[
C_{0}^{Q_{1}}, D_{0}^{Q_{1}}, C_{4}^{Q_{1}} + D_{4}^{Q_{1}}.
\]

- Relay 2 does not need

\[
A_{0}^{Q_{2}}, B_{0}^{Q_{2}}, A_{4}^{Q_{2}} + B_{4}^{Q_{2}}.
\]

The rate of sending redundant symbols for each relay is

\[
R_{e} = \left(1 - \frac{M_{1}}{4}\right)^{2} \left(1 - \frac{M_{2}}{4}\right) \cdot r\left(\frac{M_{2}}{4}, 2\right). \tag{36}
\]

Thus, the coding delay of this scheme is

\[
T = R_{s_{1}} + R_{s_{2}} + \max\{R_{s_{3}} - R_{e}, 0\}, \tag{37}
\]

which is identical to the delay given in Corollary 1.

**B. Pipeline Forward Scheme**

We can completely ignore the relays’ caches and use a pipeline forward scheme as introduced in Example 3. More specifically, in the placement phase, we only use the parts about the \( K_{1}K_{2} \) users in Algorithm 1. In other words, each cache memory in a user stores a \( M_{2}F/N \)-bit subset of each file, independently and randomly. In the delivery phase, the server broadcasts XOR files to \( K_{1}K_{2} \) users using the decentralized coding scheme \( 2 \). Each relay connects the two layers as one pipeline, and thus the data sent from the server moves like a information flow smoothly through the pipeline. This makes the two-layer network equivalent to the single-layer network where a server connects with \( K_{1}K_{2} \) users through a shared noiseless link. From \( 3 \), this pipeline forward caching scheme achieves the coding delay \( r(M_{2}/N, K_{1}K_{2}) \).

**C. General Caching Scheme**

Apply the similar method described in \( 3 \), Sec. V-C \) to combine two schemes described above. Divide the system model into two subsystems with two parameters \( \alpha, \beta \in [0, 1] \). The first subsystem includes the entire cache memory in each relay and a \( \beta \) fraction of each user’s cache memory, and the second subsystem holds the remaining \( 1 - \beta \) fraction of each user’s cache memory. Additionally, each file is split into two parts of size \( \alpha F \) and size \( (1 - \alpha)F \) bits, as showed in Fig. 5. After that, the proposed caching scheme in Section IV-A is applied to the first subsystem to recover the \( \alpha F \) bits of each file, and the forward caching scheme in Section V-B \) is applied to the second subsystem to recover the \( (1 - \alpha)F \) bits of each file.

Consider the first subsystem. The equivalent file size, the user’s cache memory and and the relay’s memory are \( \alpha F, M_{1}F/\alpha \), and \( \beta M_{2}F/\alpha \), respectively. By \( 33 \), we obtain the coding delay of the first subsystem as

\[
R' = R_{s_{1}} + R_{s_{2}} + \max\{R_{s_{3}} - R_{e}, 0\} \tag{38}
\]

where \( R_{s_{1}}, R_{s_{2}}, R_{s_{3}} \) and \( R_{e} \) are defined in Theorem 1.

Similarly, consider the involved parameters in the second subsystem, the equivalent file size and user cache memory are \( (1 - \alpha)F \) and \( (1 - \alpha)M_{2}F/\alpha \), respectively, thus the coding delay of the second subsystem by using pipe-line forward scheme is

\[
R'' = (1 - \alpha) \cdot r\left(\frac{(1 - \beta)M_{2}}{(1 - \alpha)N}, K_{1}K_{2}\right) \tag{39}
\]

Merge the two subsystems into a complete system, we obtain the coding delay

\[
T_{\text{proposed}} = R' + R'' \tag{40}
\]

**VI. Conclusions**

In this paper, we proposed a coded caching scheme for the two-layer cache-aided network, where a server accesses a library of files and wishes to communicate with users with the help of caches and relays. We design a caching scheme that exploits the spared time resource by allowing the concurrent transmission between the two layers. It is shown that the caching scheme is order optimal and can further reduce the transmission delay compared to the previously known caching scheme. Also, we show that for the two-relay case, if each relay’s caching size equals to 38.2% of full library’s
size, our scheme achieves the same transmission delay as if all relays had accessed the full library. This implies that increasing the relay’s caching memory will not always reduce the transmission delay.

APPENDIX A
PROOF OF THEOREM

We first prove (23a) and then show that $T_{\text{propose}}/T^* \leq c$.

A. Proof of (23a)

1) Regime I: $M_1 + M_2 K_2 \geq N$ and $0 \leq M_1 \leq N/4$. In this regime, scheme HCC-C chooses $(\alpha, \beta) = (M_1/N, M_1/N)$. Although the optimal choice of $T_{\text{propose}}$ could be different, we apply the same choice, and have

$$T_{\text{HCC-C}} = r \left( \frac{M_2}{N}, K_2 \right) + \left( 1 - \frac{M_1}{N} \right) r \left( \frac{M_2}{N}, K_1 K_2 \right),$$

$T_{\text{propose}} \leq \frac{M_1}{N} r \left( \frac{M_2}{N}, K_2 \right) + \left( 1 - \frac{M_1}{N} \right) r \left( \frac{M_2}{N}, K_1 K_2 \right)$, and

$$T_{\text{HCC-C}} - T_{\text{propose}} \geq \left( 1 - \frac{M_1}{N} \right) r \left( \frac{M_2}{N}, K_2 \right).$$  \quad (41)

2) Regime II: $M_1 + M_2 K_2 < N$. In this regime, scheme HCC-C chooses $(\alpha, \beta) = (M_1/(M_1 + M_2 K_2), 0)$. Although the optimal choice of $T_{\text{propose}}$ could be different, we apply the same choice, and have

$$T_{\text{HCC-C}} = \frac{M_1 K_2}{M_1 + M_2 K_2} r \left( \frac{M_1 + M_2 K_2}{N}, K_1 \right)$$

$$+ \frac{M_2 K_2}{M_1 + M_2 K_2} r \left( \frac{M_1 + M_2 K_2}{N K_2}, K_1 K_2 \right)$$

$$+ \frac{M_1 + M_2 K_2}{M_1 + M_2 K_2} r \left( \frac{M_1 + M_2 K_2}{N K_2}, K_2 \right)$$

$$+ \frac{M_1 + M_2 K_2}{M_1 + M_2 K_2} r \left( \frac{M_1 + M_2 K_2}{N}, K_1 \right)$$

$$\leq \frac{M_1 K_2}{M_1 + M_2 K_2} r \left( \frac{M_1 + M_2 K_2}{N}, K_1 \right) + \frac{M_1 K_2}{N}$$

$$+ \frac{M_2 K_2}{M_1 + M_2 K_2} r \left( \frac{M_1 + M_2 K_2}{N K_2}, K_1 K_2 \right)$$

$$- (K_1 - 1) \frac{M_1 K_2}{M_1 + M_2 K_2} \left( 1 - \frac{M_1 + M_2 K_2}{N} \right)^{K_1},$$

and

$$T_{\text{HCC-C}} - T_{\text{propose}} \geq \frac{M_2 K_2}{M_1 + M_2 K_2} r \left( \frac{M_1 + M_2 K_2}{N K_2}, K_2 \right)$$

$$+ \frac{(K_1 - 1) M_1 K_2}{M_1 + M_2 K_2} \left( 1 - \frac{M_1 + M_2 K_2}{N} \right)^{K_1}$$

$$+ \frac{M_2 K_2}{M_1 + M_2 K_2} r \left( \frac{M_1 + M_2 K_2}{N K_2}, K_2 \right) \quad (a)$$

where (a) follows from $M_1 + M_2 K_2 < N$.

3) Regime III: $M_1 + M_2 K_2 \geq N$ and $N/4 < M_1 \leq N$. In this regime, scheme HCC-C chooses $(\alpha, \beta) = (M_1/N, 1/4)$. Although the optimal choice of $T_{\text{propose}}$ could be different, we apply the same choice, and have

$$T_{\text{HCC-C}} = \left( 1 - \frac{M_1}{N} \right) r \left( \frac{3 M_2}{4(N - M_1)}, K_1 K_2 \right)$$

$$+ \left( 1 - \frac{M_1}{N} \right) r \left( \frac{3 M_2}{4(N - M_1)}, K_2 \right)$$

$$+ \frac{M_1}{N} r \left( \frac{M_2}{4 M_1}, K_2 \right),$$

$$T_{\text{propose}} \leq \left( 1 - \frac{M_1}{N} \right) r \left( \frac{3 M_2}{4(N - M_1)}, K_1 K_2 \right)$$

$$+ \frac{M_1}{N} r \left( \frac{M_2}{4 M_1}, K_2 \right),$$

and

$$T_{\text{HCC-C}} - T_{\text{propose}} \geq \left( 1 - \frac{M_1}{N} \right) r \left( \frac{3 M_2}{4(N - M_1)}, K_2 \right).$$  \quad (43)

B. Proof of $T_{\text{propose}}/T^* \leq c$

From (23a), we have

$$T_{\text{propose}} \leq T_{\text{HCC-C}} = T_{\text{HCC-C-1}} + T_{\text{HCC-C-2}}.$$  \quad (44)

By (22), we rewrite the lower bound, for all $M_1, M_2 \in [0, N]$, $s_1 \in \{1, \ldots, K_1\}$ and $s_2 \in \{1, \ldots, K_2\}$,

$$T^* \geq \max \{ T_1^*, T_2^* \}$$  \quad (45)

where

$$T_1^* \triangleq s_1 \frac{M_1 + s_1 + s_2 M_2}{N/(s_1 s_2)},$$  \quad (46)

$$T_2^* \triangleq s - \frac{s M_2}{N/s}.\quad (47)$$

In [3] Sec. V] the authors showed that

$$R_{\text{HCC-C-1}} \leq c_1 T_1^*,$$  \quad (48)

thus if we prove $R_{\text{HCC-C-2}} \leq c_2 T_2^*$, then there must exist a $c$ such that $T_{\text{propose}} \leq cT^*$, where $c_1$, $c_2$ and $c$ are finite positive constants independent of all the problem parameters.

From (3) we know

$$R_{\text{HCC-C-2}} \leq 4 r(M_2/N, K_2).$$  \quad (49)

In [2], the authors show that the decentralized caching scheme achieves the coding delay $r(M_2/N, K_2)$ and for some $s \in \{1, \ldots, K_2\}$,

$$r(M_2/N, K_2) \leq 12 T_2^*.$$  \quad (50)

From (49) and (50), we have

$$R_{\text{HCC-C-2}} \leq 64 \cdot T_2^* = c_2 T_2^*.$$  \quad (51)

Combine (44), (48) and (51), we have $T_{\text{propose}}/T^* \leq c$. 
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