Dynamics of Nonintegrable Phases in Softly Broken Supersymmetric Gauge Theory with Massless Adjoint Matter

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Abstract

We study $SU(N)$ supersymmetric Yang-Mills theory with massless adjoint matter defined on $M^3 \otimes S^1$. The $SU(N)$ gauge symmetry is broken maximally to $U(1)^{N-1}$, independent of the number of flavor and the boundary conditions of the fields associated with the Scherk-Schwarz mechanism of supersymmetry breaking. The mass of the Higgs scalar is generated through quantum corrections in the extra dimensions. The quantum correction can become manifest by a finite Higgs boson mass at low energies even in the limit of small extra dimensions thanks to the supersymmetry breaking parameter of the Scherk-Schwarz mechanism.

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1 Introduction

The dynamics of nonintegrable phases is one of the most important phenomena when one studies (supersymmetric) gauge theory in a space-time where one of the space coordinates is compactified on a topological manifold \([1]\). The dynamics are caused essentially by quantum effects in extra dimensions, reflecting the topology of the extra dimensions.

Component gauge fields for compactified directions can develop vacuum expectation values. The vacuum expectation values, which correspond to the constant background fields, are also related with the eigenvalues of the Wilson line integrals for the compactified direction. Therefore, they are dynamical variables and cannot be gauged away. By studying the effective potential for the phases, in perturbation theory, for example, one can investigate how gauge symmetry is broken dynamically. This shows that the quantum effects in the extra dimensions are remarkable and should be taken into account when we study the theory.

The dynamics of the nonintegrable phases have been studied extensively\([2, 3, 4]\) since a pioneering work \([1]\). In nonsupersymmetric gauge theories, one can, in principle, determine how the gauge symmetry is broken dynamically. It has been known that the gauge symmetry breaking patterns depend on the number, the boundary conditions of the fields and the representation under the gauge group of matter \([5, 6, 7]\). On the other hand, in supersymmetric gauge theories, the effective potential for the phases vanishes as long as supersymmetry is not broken. This is because contributions coming from bosons and fermions in a supermultiplet to the constant background field cancel each other. One natural way to break supersymmetry is to resort to the Scherk-Schwarz mechanism \([8, 9]\). Supersymmetry is broken by the boundary conditions of the fields for compactified directions in the Scherk-Schwarz mechanism. By using symmetry degrees of freedom of the theory, one can twist the boundary conditions of the field in such a way that they are different between bosons and fermions in a supermultiplet. The boundary condition associated with the \(U(1)_R\) symmetry is a candidate and breaks supersymmetry softly \([8, 9, 10]\). And it has been also pointed out that the supersymmetry breaking terms resulting from the mechanism have attractive features such as flavor blindness and only two parameters \([10, 11]\). Once the supersymmetry is broken, we obtain nonvanishing effective potential for the nonintegrable phases in perturbation theory and can discuss how the gauge symmetry is broken dynamically.

In a previous paper \([11]\), the author studied the dynamics of the nonintegrable phases in the softly broken supersymmetric gauge theories with matter defined on \(M^3 \otimes S^1\), where \(M^3, S^1\) are three-dimensional Minkowski space-time and a circle, respectively. In that paper the gauge group was assumed to be \(SU(2)\). This paper is a generalization of the previous work. We shall consider the \(SU(N)\) gauge group and study the dynamics of
the nonintegrable phases in a model of $SU(N)$ supersymmetric Yang-Mills theory with $N_F$ numbers of massless adjoint matter. We resort to the Scherk-Schwarz mechanism to break the supersymmetry in the model.

In nonsupersymmetric gauge theory with massless adjoint matter, the gauge symmetry breaking patterns are rich, depending on the values of the boundary conditions of the matter field [5]. On the contrary, the $SU(N)$ gauge symmetry is broken dynamically to its maximal commutative subgroup, i.e., $U(1)^{N-1}$ in our model. We have an unique gauge symmetry breaking pattern. It is remarkable that this does not depend on the boundary conditions of the fields associated with the Scherk-Schwarz mechanism of supersymmetry breaking.

The component gauge field for the compactified direction acquires mass through the quantum correction in the extra dimensions and becomes a Higgs scalar in the adjoint representation under the $SU(N)$ gauge group at low energies [1]. We obtain the mass of the Higgs scalar in our model. The mass explicitly depends on the gauge coupling, supersymmetry breaking parameter, and the number of flavor and suffers from a correction of the compactification scale.

The mass spectrum of the model at low energies is also obtained, and we discuss low-energy effective theory. If we take the limit of small extra dimensions, all the effects of the extra dimensions whose mass scales are given by the compactification scale are decoupled from low-energy physics. It implies that the effect of the extra dimensions never appears at low energies. The nontrivial limit of small extra dimensions, however, is possible thanks to the supersymmetry breaking parameter of the Scherk-Schwarz mechanism. The effect of the extra dimensions can become manifest by the Higgs scalar as having finite mass at low energies.

In the next section we shall start with reviewing briefly the effective potential for the nonintegrable phases. In order to make discussions clearer and carry out analytic calculations, we take our space-time to be $M^3 \otimes S^1$ as in the previous work. In section 3 we shall discuss the gauge symmetry breaking through the dynamics of the nonintegrable phases. We obtain the mass spectrum in three dimensions and discuss the low-energy effective theory in the limit of small extra dimensions in section 4. Conclusions are given in section 5.

2 Effective Potential for Nonintegrable Phases

Let us first consider, $SU(N)$ supersymmetric Yang-Mills theory defined on $M^3 \otimes S^1$, where $M^3$ is three-dimensional Minkowski space-time and $S^1$ is a circle. The coordinates of $M^3$ and $S^1$ are denoted by $x^\mu$ and $y$, respectively. $x^{\hat{\mu}}$ stands for $x^{\hat{\mu}} = (x^\mu, y)$ and $L$ is the length of the circumference of $S^1$. 
The on-shell Lagrangian is given by

$$L = \text{tr} \left[ -\frac{1}{2} F_{\hat{\mu}\hat{\nu}} F^{\hat{\mu}\hat{\nu}} - i \lambda \sigma^\mu D_\mu \bar{\lambda} + i D_\mu \lambda \sigma^\mu \bar{\lambda} \right].$$

(1)

Here $\lambda$ is the gaugino, the superpartner of the gauge boson $A_\hat{\mu}$. The covariant derivative and field strength are defined by

$$F_{\hat{\mu}\hat{\nu}} = \partial_\hat{\mu} A_\hat{\nu} - \partial_\hat{\nu} A_\hat{\mu} - ig[A_\hat{\mu}, A_\hat{\nu}], \quad D_\mu \lambda = \partial_\mu \lambda - ig[A_\mu, \lambda],$$

(2)

respectively. Here $g$ is the gauge coupling constant, and we normalize generators of $SU(N)$ as $\text{tr}(T^a T^b) = \delta_{ab}/2$, where $a, b = 1, \cdots, N^2 - 1$.

It has been pointed out that the component gauge field for the $S^1$ direction, denoted by $A_3 \equiv \Phi$, can develop vacuum expectation values, reflecting the topology of $S^1$ \[1\]. One can parametrize the vacuum expectation value as

$$\langle \Phi \rangle = \frac{1}{gL} \text{diag}(\theta_1, \theta_2, \cdots, \theta_N) \quad \text{with} \quad \sum_{i=1}^{N} \theta_i = 0.$$  

(3)

The constant background (3) is also equivalent to introducing the nontrivial Wilson line integral for the $S^1$ direction

$$W_c \equiv \mathcal{P} e^{-ig \int_{S^1} dy \langle A_3 \rangle} = \begin{pmatrix} e^{-i\theta_1} & e^{-i\theta_2} & \cdots & e^{-i\theta_N} \end{pmatrix}, \quad (\theta_i \text{ (mod } 2\pi)).$$

(4)

The gauge symmetry is broken in the Cartan subgroup of $SU(N)$ by nontrivial values of $\theta_i$. The residual gauge symmetry is generated by generators commuting with $W_c$, i.e. $[W_c, T^a] = 0$. The phase $\theta_i$ is called the nonintegrable phase \[1\].

If we expand the fields around the vacuum expectation value and integrate out the fluctuating fields up to the quadratic terms, we obtain the effective potential for the nonintegrable phases in a one-loop approximation. As we have noted in the introduction, we need to break supersymmetry to obtain nonvanishing effective potential at least in perturbation theory. We shall resort to the Scherk-Schwarz mechanism \[8, 9\]. According to the mechanism, supersymmetry is broken by the nontrivial boundary conditions of the gaugino field $\lambda$ for the $S^1$ direction. By using the $U(1)_R$-symmetry degrees of freedom in the theory, one can impose the boundary conditions on the field, which are defined by \[8, 9, 10\]

$$\lambda(x^\mu, y + L) = e^{i\beta} \lambda(x^\mu, y).$$

(5)

The gauge field $A_\hat{\mu}$ satisfies the periodic boundary conditions. Thus, $A_\hat{\mu}$ and $\lambda$ obey the different boundary conditions, so that supersymmetry is broken by the mechanism.
The effective potential for the nonintegrable phases is calculated, in the Feynman gauge, as

\[ V_{\text{SYM}}(\theta) = \frac{-2}{\pi^2 L^4} \sum_{n=1}^{\infty} \sum_{i,j=1}^{N} \frac{1}{n^4} \left( \cos[n(\theta_i - \theta_j)] - \cos[n(\theta_i - \theta_j - \beta)] \right). \]  

(6)

The first and second terms in the potential come from the gauge boson and gaugino, respectively. As we can see, if we take \( \beta = 0 \), the supersymmetry is restored to yield vanishing effective potential. We can also recast the potential as

\[ V_{\text{SYM}}(\theta) = \frac{-2}{\pi^2 L^4} \sum_{n=1}^{\infty} \frac{1}{n^4} \left( 1 - \cos(n\beta) \right) \left( N + 2 \sum_{1 \leq i < j \leq N} \cos[n(\theta_i - \theta_j)] \right). \]  

(7)

The leading term \( N \) comes from the diagonal part with respect to \( i \) and \( j \). It is important to note that the nontrivial phase \( \beta \), which has a role in breaking supersymmetry, does not affect the location of absolute minima of the potential though the potential energy at the minimum depends on the phase.

Let us introduce \( N_F \) numbers of massless adjoint matter into the theory. The chiral superfield for the matter belongs to the adjoint representation under \( SU(N) \), and the on-shell degrees of freedom in the chiral superfield are a complex scalar (squark) \( \phi \) and a two-component Weyl spinor (quark) \( q \). We ignore the flavor index. We impose the boundary conditions associated with the \( U(1)_R \) symmetry on the squark field\(^1\).

\[ \phi(x^\mu, y + L) = e^{i\beta} \phi(x^\mu, y). \]  

(8)

The phase \( \beta \) is common to all flavors as noted in Refs.\(^{10,11}\).

By expanding the fields around the background (3), we obtain the effective potential for the nonintegrable phases coming from the massless adjoint matter

\[ V_{\text{ADJ}}(\theta) = \frac{2N_F}{\pi^2 L^4} \sum_{n=1}^{\infty} \sum_{i,j=1}^{N} \frac{1}{n^4} \left( \cos[n(\theta_i - \theta_j)] - \cos[n(\theta_i - \theta_j - \beta)] \right). \]  

(9)

Here \( 2N_F \) accounts for the on-shell degrees of freedom for the massless adjoint matter. The first and second terms come from the quark and squark, respectively. Hence, the effective potential for the supersymmetric Yang-Mills theory with \( N_F \) numbers of massless adjoint matter is given by

\[ V(\theta) \equiv V_{\text{SYM}}(\theta) + V_{\text{ADJ}}(\theta) = \frac{(2N_F - 2)}{\pi^2 L^4} \sum_{n=1}^{\infty} \sum_{i,j=1}^{N} \frac{1}{n^4} \left( \cos[n(\theta_i - \theta_j)] - \cos[n(\theta_i - \theta_j - \beta)] \right) \]  

\[ = \frac{(2N_F - 2)}{\pi^2 L^4} \sum_{n=1}^{\infty} \frac{1}{n^4} (1 - \cos(n\beta)) \left( N + 2 \sum_{1 \leq i < j \leq N} \cos[n(\theta_i - \theta_j)] \right) \]  

(10)

\(^1\)Strictly speaking, one has to consider massive adjoint matter in order to have the boundary conditions associated with the \( U(1)_R \) symmetry. The discussion here corresponds to the massless limit.
Let us note, again, that the nontrivial phase $\beta$ does not affect the location of the absolute minima of the potential.

We immediately see that if $N_F = 1$, the potential vanishes for any values of $\beta$. This is because the supersymmetric Yang-Mills theory with one massless adjoint matter is nothing but $\mathcal{N} = 2$ supersymmetric gauge theory in four dimensions, and $\mathcal{N} = 1$ supersymmetry still remains even after imposing the boundary conditions (3) and (8) associated with the $U(1)_R$ symmetry. This, however, implies partial supersymmetry breaking through the boundary conditions associated with the $U(1)_R$ symmetry. In order to have nonvanishing effective potential for this case, one needs to impose the boundary conditions associated with the $SU(2)_R$ [or $U(1)_J$ in $\mathcal{N} = 1$ language] symmetry in addition to the $U(1)_R$.

### 3 Gauge Symmetry Breaking

Let us discuss the gauge symmetry breaking through the dynamics of the nonintegrable phases. We first study the effective potential (4) for the case of the supersymmetric Yang-Mills theory. The potential is minimized when

$$\theta_i - \theta_j = 0. \quad (11)$$

Since $\sum_{i=1}^{N} \theta_i = 0$, we have

$$\theta_i = \frac{2\pi m}{N} \quad (m = 0, 1, \cdots, N-1). \quad (12)$$

It gives $e^{i\theta_i} = e^{2\pi im/N}$, so that $W_c$ is an element of the center of $SU(N)$ and commutes with all the generators of $SU(N)$. Therefore, the gauge symmetry is not broken in this theory. This is the same result as in the case for the Yang-Mills theory [1]. The potential energy at the minimum (11) is calculated as

$$V_{SYM} = -\frac{N(N+1)}{\pi^2 L^4} \times \frac{\beta^2(\beta - 2\pi)^2}{48}, \quad (13)$$

where we have used the formula

$$\sum_{n=1}^{\infty} \frac{1}{n^4} \cos(nt) = -\frac{1}{48} t^2(t - 2\pi)^2 + \frac{\pi^4}{90} \quad (0 \leq t \leq 2\pi). \quad (14)$$

The potential energy at the minimum is negative [2]. This reflects the fact that the supersymmetry breaking of the Scherk-Schwarz mechanism is not a spontaneous breaking of supersymmetry, but an explicit breaking in our model.

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[2] The contribution from matter to the potential energy can make the potential energy positive as we will see later. There may be a possibility to have zero energy by adding an appropriate amount of matter.
Let us next study the gauge symmetry breaking in $SU(N)$ supersymmetric Yang-Mills theory with $N_F$ numbers of massless adjoint matter. The effective potential is given by Eq. (10). It may be convenient to recast it as

$$V(\theta) = \frac{(2N_F - 2)}{\pi^2 L^4} \sum_{n=1}^{\infty} \frac{1}{n^4} \left(N \left(1 - \cos(n\beta)\right)\right)$$

+ \sum_{1 \leq i < j \leq N-1} 2 \cos[n(\theta_i - \theta_j)] - \cos[n(\theta_i - \theta - \beta)] - \cos[n(\theta_i - \theta_j + \beta)]

+ 2 \cos[n(2\theta_1 + \theta_2 + \cdots + \theta_{N-1})]

- \cos[n(2\theta_1 + \theta_2 + \cdots + 2\theta_{N-1} - \beta)] - \cos[n(2\theta_1 + \theta_2 + \cdots + 2\theta_{N-1} + \beta)]

+ \cdots

+ 2 \cos[n(\theta_1 + \theta_2 + \cdots + 2\theta_{N-1})]

- \cos[n(\theta_1 + \theta_2 + \cdots + 2\theta_{N-1} - \beta)] - \cos[n(\theta_1 + \theta_2 + \cdots + 2\theta_{N-1} + \beta)]$$

where we have used $\theta_N = -\sum_{i=1}^{N-1} \theta_i$. It is important to note that the potential is invariant under

$$\theta_i \leftrightarrow \theta_j \quad (i \neq j), \quad \beta \leftrightarrow -|\beta|.$$  \hspace{1cm} (16)

Classifications depending on the sign of $\theta_i - \theta_j - \beta$ are necessary when we apply the formula (14) to the effective potential. Thanks to the symmetries (10), however, the region given by $\theta_i - \theta_j \leq \beta$ is equivalent to that given by $\theta_i - \theta_j \geq |\beta|$. It is enough for us take only the region of $\theta_i - \theta_j \geq \beta$ into account. It also follows that the region $0 < \beta \leq \pi$ is enough to study the potential.

After straightforward calculations, we arrive at the expression given by

$$V(\theta) = \frac{(2N_F - 2)}{\pi^2 L^4} \beta^2 \left[\frac{N}{48} (\beta - 2\pi)^2 + \frac{N(N-1)}{48} (\beta^2 + 4\pi^2)\right]$$

+ $\frac{N}{4} \left(\sum_{i=1}^{N-1} \theta_i^2 + \sum_{1 \leq i < j \leq N-1} \theta_i \theta_j\right) - \frac{2\pi}{2} \sum_{i=1}^{N-1} (N - i)\theta_i$.$$ \hspace{1cm} (17)

The absolute minima of the potential is given by solving $\partial V(\theta)/\partial \theta_k = 0 (k = 1, 2, \cdots, N-1)$, which is read as

$$\frac{N}{2} \left(\sum_{i=1}^{N-1} 2\theta_i \delta_{ik} + \sum_{1 \leq i < j \leq N-1} (\theta_j \delta_{ik} + \theta_i \delta_{jk})\right) = \pi \sum_{i=1}^{N-1} (N - i)\delta_{ik}, \quad k = 1, 2, \cdots, N-1.$$ \hspace{1cm} (18)

This can also be written in the form

$$\frac{N}{2} \begin{pmatrix} 2 & 1 & \cdots & 1 \\ 1 & 2 & \cdots & \vdots \\ \vdots & \ddots & \vdots & \vdots \\ 1 & \cdots & \ddots & 2 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{N-1} \end{pmatrix} = \pi \begin{pmatrix} N-1 \\ N-2 \\ \vdots \\ 2 \\ 1 \end{pmatrix}.$$ \hspace{1cm} (19)

The effective potential is also invariant under $\beta \rightarrow \beta + 2\pi im, m \in \mathbb{Z}$. This is traced back to $\lambda \rightarrow e^{2\pi mi} \lambda$. Note that physical region of $\beta$ is given by $-\pi \leq \beta \leq \pi$, except $\beta = 0$. 

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All the (off-)diagonal elements of the matrix in Eq. \((19)\) are \(2(1)\). The inverse of the matrix is given by

\[
\frac{1}{N} \begin{pmatrix}
N - 1 & -1 & \cdots & -1 \\
-1 & N - 1 & \vdots \\
\vdots & & & \ddots \\
\vdots & & \ddots & \vdots \\
-1 & \cdots & \cdots & N - 1
\end{pmatrix},
\]

where all the (off-)diagonal elements of the matrix are \(N - 1(-1)\). Therefore, the solution to Eq. \((18)\) is

\[
\begin{pmatrix}
\theta_1 \\
\theta_2 \\
\theta_3 \\
\vdots \\
\theta_{N-2} \\
\theta_N
\end{pmatrix} = \frac{\pi}{N} \begin{pmatrix}
N - 1 \\
N - 3 \\
N - 5 \\
\vdots \\
-(N - 5) \\
-(N - 3)
\end{pmatrix}
\]

or \(\theta_i = \frac{\pi}{N} (N - (2i - 1)) \quad (i = 1, \cdots, N - 1)\). \((21)\)

Let us note that \(\theta_N = -\sum_{i=1}^{N-1} \theta_i = -\pi(N - 1)/N\). We have found that the absolute minimum of the potential is located at

\[
\langle \Phi \rangle = \frac{\pi}{gL} \text{diag} \left( \frac{N - 1}{N}, \frac{N - 3}{N}, \cdots, 0, -\frac{(N - 3)}{N}, -\frac{(N - 1)}{N} \right) \pmod{2\pi},
\]

where \(\theta_{i=(N+1)/2} = 0\).

As an example, let us present results for the cases of \(SU(2)\), \(SU(3)\), and \(SU(5)\):

\[
\langle \Phi \rangle = \frac{\pi}{gL} \text{diag} \left( \frac{1}{2}, \frac{1}{2} \right) + \text{permutations} \quad \text{for} \quad SU(2),
\]

\[
\langle \Phi \rangle = \frac{\pi}{gL} \text{diag} \left( \frac{2}{3}, 0, -\frac{2}{3} \right) + \text{permutations} \quad \text{for} \quad SU(3),
\]

\[
\langle \Phi \rangle = \frac{\pi}{gL} \text{diag} \left( \frac{4}{5}, \frac{2}{5}, 0, -\frac{2}{5}, -\frac{4}{5} \right) + \text{permutations} \quad \text{for} \quad SU(5).
\]

Since \(\theta_i\) is a module of \(2\pi\), the configuration for \(SU(5)\) is equivalent to

\[
\langle \Phi \rangle = \frac{\pi}{gL} \text{diag} \left( \frac{4}{5}, \frac{2}{5}, 0, \frac{8}{5}, \frac{6}{5} \right) + \text{permutations}.
\]

The configuration \((22)\) breaks the \(SU(N)\) gauge symmetry maximally to \(U(1)^{N-1}\). It should be noted that this does not depend on the number of flavor \(N_F\) and the supersymmetry breaking parameter \(\beta\), which is the boundary condition of the fields \(\lambda, \phi\) for the \(S^1\) direction. We have arrived at the conclusion that in our model the \(SU(N)\) gauge symmetry is broken dynamically to its maximal commutative subgroup, \(i.e., U(1)^{N-1}\), independent of \(\beta\) and \(N_F\). This is very different from the nonsupersymmetric gauge theories, in which the symmetry breaking patterns depend on the boundary conditions of the matter fields \([3, 4]\).
We also depict potential energies for the possible gauge symmetry breaking patterns of $SU(3), SU(5)$ in Figs. 1 and 2. We compare the potential energy for each pattern given by

\[
\begin{align*}
SU(3) & \rightarrow \begin{cases} 
    A : & U(1) \times U(1) \cdots \frac{\pi}{3}(2, 0, -2) + \text{permutations}, \\
    B : & SU(2) \times U(1) \cdots \frac{\pi}{3}(1, 1, -2) + \text{permutations}, \\
    C : & SU(3) \cdots \frac{\pi}{3}(0, 0, 0), 
\end{cases} \\
SU(5) & \rightarrow \begin{cases} 
    A : & U(1)^4 \cdots \frac{2\pi}{5}(4, 2, 0, 8, 6) + \text{permutations}, \\
    B : & SU(2)^2 \times U(1)^2 \cdots \frac{2\pi}{5}(0, 3, 3, 7, 7) + \text{permutations}, \\
    C : & SU(3) \times SU(2) \times U(1) \cdots \frac{2\pi}{5}(0, 0, 0, 1, 1) + \text{permutations}, \\
    D : & SU(4) \times U(1) \cdots \frac{2\pi}{5}(1, 1, 1, 1, 6) + \text{permutations}, \\
    E : & SU(5) \cdots \frac{2\pi}{5}(0, 0, 0, 0, 0). 
\end{cases}
\end{align*}
\]

It is clear from the figures that the lowest energy of the potential is always given by the case of the maximal breaking of the original gauge symmetry and that the boundary condition $\beta$ never affects the symmetry breaking patterns.

The potential energy at the minimum is calculated as

\[
V(\theta) = (2N_F - 2)\frac{4\pi^2 \beta^2}{L^4} \left[ \frac{N}{12}(\beta - 1)^2 + \frac{N(N - 1)}{12}(\beta^2 + 1) - \frac{1}{3}(N - 1)(N - 2) \right], 
\]

where we have rescaled $\beta$ as $\beta = 2\pi \bar{\beta}$. It is obvious to see that the energy is positive for $SU(2)$. If the gauge group becomes larger than $SU(2)$, the sign of the energy depends on the values of $N$ and $\beta$. Let us define

\[
D(N) \equiv N(\bar{\beta} - 1)^2 + N(N - 1)(\bar{\beta}^2 + 1) - 4(N - 1)(N - 2). 
\]

Then, we find that

\[
\begin{align*}
V(\theta) & \geq 0 \quad \text{for} \quad 2 \leq N \leq N_+ \equiv \frac{6 - \bar{\beta} + \sqrt{9\bar{\beta}^2 - 12\bar{\beta} + 12}}{3 - \bar{\beta}^2} < 4, \\
V(\theta) & < 0 \quad \text{for} \quad 4 \leq N.
\end{align*}
\]

The zero energy can be realized only by $N = 3$ with $\bar{\beta} = \frac{1}{3}$. The potential energy is positive definite for the other values of $\beta$ when $N = 3$.

4 Mass Spectrum and Low-energy Effective Theory

\footnote{The vacuum energy may be estimated by taking account of the vacuum expectation values of the squark $\langle \phi \rangle \equiv a \in \mathbb{C}$ in addition to $\langle \Phi \rangle$. Originally, this corresponds to the flat direction satisfying the $D$-term condition. The flat direction may be lifted in our case due to $\beta$. The model we are considering has essentially two kinds of order parameters $\langle \Phi \rangle$ and $\langle \phi \rangle$, though the leading contribution to the potential may be dominated by $\langle \Phi \rangle$. This is inferred from the fact that the effective potential (11) does not depend on the gauge coupling at the one-loop level. We are focusing here on the dynamics of the nonintegrable phases.}
4.1 Higgs Scalar

Let us study the mass of the Higgs scalar $\Phi \equiv A_3$, which is originally the component gauge field for the $S^1$ direction. The mass term for the Higgs scalar, which is zero at the tree level, is generated through the quantum corrections in the extra dimensions [1]. After the compactification is carried out by integrating the coordinate of $S^1$, the lowest Kaluza-Klein mode of $\Phi$ behaves as the adjoint Higgs scalar, which transforms as the adjoint representation under the gauge group.

Let us now evaluate the mass term for the Higgs scalar. It is given by estimating the second derivative of the effective potential (17) at the absolute minimum (21). Alternatively, after taking the second derivative of Eq. (15) with respect to $\theta_i$, one can use

$$\sum_{n=1}^{\infty} \frac{1}{n^2} \left( 2 \cos(nx) - \cos[n(x-\beta)] - \cos[n(x+\beta)] \right) = -\frac{\beta^2}{2},$$

(30)

which is independent of $x$. The two approaches give the same result, as they should.

Then, we obtain

$$\text{mass term} \equiv \frac{1}{2} \theta_i (M^\Phi)_{ij} \theta_j \quad (i, j = 1, \cdots, N - 1),$$

(31)

where

$$(M^\Phi)_{ij} \equiv \frac{\partial^2 V(\theta)}{\partial \theta_i \partial \theta_j} = \frac{(2NF - 2) \beta^2}{\pi^2 L^4} \times 2N \times \begin{pmatrix} 2 & 1 & \cdots & \cdots & 1 \\ 1 & 2 & \cdots & \cdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 1 & \cdots & \cdots & \cdots & 2 \end{pmatrix}. \quad (32)$$

The indices $i, j$ run from 1 to $N - 1$. The matrix in Eq. (32) is the same as the one in Eq. (19).

In order to study the mass of the Higgs scalar more clearly, let us change the variable $\theta_i$ to another variable. To this end, let us define

$$\langle \Phi \rangle \equiv \frac{1}{gL} \sum_{m=1}^{N-1} v_m H_m,$$

(33)

where $H_m (m = 1, 2, \cdots, N - 1)$ is the diagonal generator of the Cartan subalgebra of $SU(N)$ and is a $N \times N$ matrix. $v_m$ is a real parameter. We choose the form

$$(H_m)_{ij} = \frac{1}{\sqrt{2m(m+1)}} \left( \sum_{k=1}^{m} \delta_{i,k} \delta_{j,k} - m\delta_{i,m+1} \delta_{j,m+1} \right).$$

(34)
Then, the $\theta_i$’s in Eq. (3) are related with $v_m$ by

$$\begin{align*}
\theta_1 &= \frac{v_1}{2} + \frac{v_2}{2\sqrt{3}} + \cdots + \frac{v_m}{\sqrt{2m(m+1)}} + \cdots + \frac{v_{N-1}}{\sqrt{2N(N-1)}}, \\
\theta_2 &= -\frac{v_1}{2} + \frac{v_2}{2\sqrt{3}} + \cdots + \frac{v_{N-1}}{\sqrt{2N(N-1)}}, \\
\theta_3 &= -\frac{v_2}{\sqrt{3}} + \frac{v_3}{2\sqrt{6}} + \cdots + \frac{v_{N-1}}{\sqrt{2N(N-1)}}, \\
&\vdots \\
\theta_{m+1} &= \frac{-mv_m}{\sqrt{2m(m+1)}} + \cdots + \frac{v_{N-1}}{\sqrt{2N(N-1)}}, \\
&\vdots \\
\theta_{N-1} &= \frac{-(N-2)v_{N-2}}{\sqrt{2(N-1)(N-2)}} + \frac{v_{N-1}}{\sqrt{2N(N-1)}}. 
\end{align*}$$

(35)

$\theta_N = -\sum_{i=1}^{N-1} \theta_i$ is given by $-(N-1)v_{N-1}/\sqrt{2N(N-1)}$. It follows from Eq. (33) that $\langle \Phi^m \rangle = v_m/gL$. We obtain an equation which relates $\theta_i$ with $\langle \Phi^m \rangle$ as follows:

$$\begin{pmatrix}
\theta_1 \\
\theta_2 \\
\vdots \\
\theta_{N-1}
\end{pmatrix} = gL \times B \begin{pmatrix}
\langle \Phi^1 \rangle \\
\langle \Phi^2 \rangle \\
\vdots \\
\langle \Phi^{N-1} \rangle
\end{pmatrix},$$

(36)

where $B$ is a $(N-1) \times (N-1)$ matrix given by

$$B = \begin{pmatrix}
\frac{1}{2} & \frac{1}{2\sqrt{3}} & \cdots & \cdots & \frac{1}{\sqrt{2m(m+1)}} & \cdots & \frac{1}{\sqrt{2N(N-1)}} \\
-\frac{1}{2} & \frac{1}{2\sqrt{3}} & \cdots & \cdots & \frac{1}{\sqrt{2m(m+1)}} & \cdots & \frac{1}{\sqrt{2N(N-1)}} \\
0 & -\frac{1}{\sqrt{3}} & \frac{1}{2\sqrt{6}} & \cdots & \cdots & \cdots & \frac{1}{\sqrt{2N(N-1)}} \\
\vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \frac{1}{\sqrt{2N(N-1)}} \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \frac{-(N-2)}{\sqrt{2N(N-1)}}
\end{pmatrix}.$$

(37)

The row vector in the matrix $B$ is just the weight vector $\nu^i (i = 1 \sim N-1)$ in the fundamental representation of $SU(N)$. Thus, we obtain \footnote{One can diagonalize the matrix $(M^\Phi)_{ij}$ by a real symmetric matrix $U$. The eigenvalues are $1[(N-2)$-degeneracy] and $N$. If we rescale $\theta_i$ as $\theta_i \rightarrow \theta_i/\sqrt{2}(i = 1, \cdots, N-2)$ and $\theta_{N-1} \rightarrow \theta_{N-1}/\sqrt{2N}$, then the matrix $U$ becomes $B$ in the text. Accordingly, all the eigenvalues are scaled to be $N/2$. This also implies that the inverse of $B$ is given by taking the transposition of $B$ and multiplying the first $N-2$ numbers of the row vectors by 2 and the last $(N-1)$th row vector by $2N$.}

$$\text{mass term} = \frac{1}{2} (gL)^2 \theta_i (M^\Phi)_{ij} \theta_j = \frac{1}{2} \Phi^i \left[ B^T M^\Phi B \right]_{ij} \Phi^j,$$

(38)
where
\[ B^T M \Phi B = \left(\frac{N_F - 1}{\pi^2} \right) \frac{g^2 \beta^2 N}{L^2} \frac{1}{2} (N-1) \times (N-1). \tag{39} \]

We have found that the masses for the Higgs scalars \( \Phi^1, \Phi^2, \ldots, \Phi^{N-1} \) are all degenerate and are given by
\[ m^2_\Phi = \left(\frac{N_F - 1}{\pi^2} \right) \frac{g^2 \beta^2 N}{L^2} \frac{1}{2}. \tag{40} \]

The mass term respects the residual gauge symmetry \( U(1)^{N-1} \) and explicitly depends on the gauge coupling \( g \), the supersymmetry breaking parameter \( \beta \), and the number of flavor \( N_F \). The Higgs boson mass \( m^2_\Phi \) suffers from a correction of \( O(1/L^2) \). If we would consider massive adjoint matter instead of a massless one, the mass of the Higgs scalar would have a “Boltzman factor” such as \( e^{-mL} \) for large \( mL \), where \( m \) is the mass of the adjoint matter \([12]\).

If we take \( \beta = 0 \), the supersymmetry is restored, and all the components of the adjoint Higgs scalar become massless. This reflects the fact that the effective potential is flat for all directions with respect to \( \Phi^a (a = 1, \ldots, N^2 - 1) \). Since the origin of the mass of the Higgs scalar is the quantum corrections in the extra dimensions, even though the gauge symmetry is not broken dynamically, nonvanishing mass terms respecting the unbroken gauge symmetry may appear.

### 4.2 Mass Spectrum in Three Dimensions

Since the \( SU(N) \) gauge symmetry is broken to \( U(1)^{N-1} \) in our model, the gauge boson \( (A_\mu) \), gaugino \( (\lambda) \), squark \( (\phi) \), and quark \( (q) \) become massive. In order to obtain the mass terms at the tree level, it may be convenient to use the parametrization \([33]\) and to expand the fields as

\[ F = F^m H_m + F^{(\alpha)} E_\alpha, \quad F \equiv A_\mu, \lambda, \phi, q. \tag{41} \]

\( E_\alpha \) is the raising and lowering operator corresponding to the roots \( \alpha \) of the Lie algebra.

Using the commutation relations
\[ [H_m, H_n] = 0, \quad [H_m, E_{\pm\alpha}] = \pm \alpha_m E_{\pm\alpha} \quad (E_{-\alpha} = E^\dagger_\alpha), \quad \text{tr}(E_\alpha E^\dagger_\beta) = \frac{1}{2} \delta_{\alpha \beta}, \tag{42} \]

we find that
\[ -g^2 \text{tr}([\langle \Phi \rangle, A_\mu])^2 = \frac{1}{2L^2} \sum_\alpha (v_m \alpha_m)^2 |A^{(\alpha)}|^2, \]
\[ 2g^2 \text{tr}([\langle \Phi \rangle, \phi^\dagger][\langle \Phi \rangle, \phi]) = -\frac{1}{L^2} \sum_\alpha (v_m \alpha_m)^2 |\phi^{(\alpha)}|^2, \]
\[ g \text{tr}(-\lambda \sigma^3 [\langle \Phi \rangle, \bar{\lambda}] + [\langle \Phi \rangle, \lambda] \sigma^3 \bar{\lambda}) = \frac{1}{L} \sum_\alpha (v_m \alpha_m) \lambda^{(\alpha)} \sigma^3 \bar{\lambda}^{(\alpha)}, \]
\[ g \text{tr}(-\lambda \sigma^3 [\langle \Phi \rangle, q] + [\langle \Phi \rangle, q] \sigma^3 \bar{q}) = \frac{1}{L} \sum_\alpha (v_m \alpha_m) q^{(\alpha)} \sigma^3 \bar{q}^{(\alpha)}, \tag{43} \]
where \( v_m \) is obtained by Eq. (22) through Eq. (33).

The field \( F^{(a)} \) behaves as “charged” field under the residual gauge symmetry \( U(1)^{N-1} \). If we decompose the Weyl spinor in four dimensions into the one in three dimensions, we have

\[
\psi^{(a)} \sigma^3 \bar{\psi}^{(a)} \rightarrow \frac{1}{2} (\bar{\psi}_1^{(a)} \psi_1^{(a)} + \bar{\psi}_2^{(a)} \psi_2^{(a)}), \quad (\psi^{(a)} = \lambda^{(a)}, q^{(a)}).
\]  

(44)

Let us proceed to the mass spectrum in three dimensions by integrating the coordinate \( S^1 \). The Kaluza-Klein mass is generated through the kinetic term for the compactified direction. In \( S^1 \) compactification, there appears no Kaluza-Klein mass for the Higgs scalar \( \Phi \) because there is no coupling of \( \text{tr}(F_{ab})^2 \), where \( a, b \) stand for the compactified coordinates. By straightforward calculations, we obtain the mass terms for gauge boson, gaugino, squark, quark, and the Higgs scalar. These are summarized as follows:

\[
A_{\hat{\mu}} = \begin{cases} 
A_{(n)\mu}^n & \cdots & \left( \frac{2\pi n}{L} \right)^2, \\
A_{(a)\mu}^n & \cdots & \left( \frac{2\pi n}{L} + \frac{(v_m \alpha_m)}{L} \right)^2, 
\end{cases}
\]

(45)

\[
\Phi(= A_3) = \begin{cases} 
\Phi_{(n)}^m & \cdots & \left( \frac{(N_F-1)g^2\beta^2N}{2\pi^2L^2} \right), \\
\Phi_{(a)}^n & \cdots & \text{massless}, 
\end{cases}
\]

(46)

\[
\lambda = \begin{cases} 
\lambda_{(n)i}^m & \cdots & \left( \frac{2\pi}{L} (n + \frac{\beta}{2\pi}) \right), \\
\lambda_{(a)i}^n & \cdots & \left( \frac{2\pi}{L} (n + \frac{\beta}{2\pi}) - \frac{(v_m \alpha_m)}{L} \right), 
\end{cases}
\]

(47)

\[
\phi = \begin{cases} 
\phi_{(n)}^m & \cdots & \left( \frac{2\pi}{L} (n + \frac{\beta}{2\pi}) \right)^2, \\
\phi_{(a)}^n & \cdots & \left( \frac{2\pi}{L} (n + \frac{\beta}{2\pi}) + \frac{(v_m \alpha_m)}{L} \right)^2, 
\end{cases}
\]

(48)

\[
q = \begin{cases} 
q_{(n)i}^m & \cdots & \frac{2\pi}{L}, \\
q_{(a)i}^n & \cdots & \frac{2\pi}{L} - \frac{(v_m \alpha_m)}{L}. 
\end{cases}
\]

(49)

The Kaluza-Klein mode is denoted by \((n)\). The index \( i(= 1, 2) \) stands for the three-dimensional Majorana spinor.

We observe that all the mass terms are proportional to the compactification scale \( 1/L \) and do not depend on the gauge coupling constant \( g \) except for the mass of the Higgs scalar \( \Phi \). The independence of the gauge coupling in the form of \((v_m \alpha_m)/L\) shows that the mass generated through the dynamical gauge symmetry breaking is the leading effect besides the Kaluza-Klein mode even though the dynamics itself is caused by the quantum corrections in the extra dimensions. The dependence of the Higgs mass on the gauge coupling means that the mass is generated by the quantum effects. Both components \( F^m \) and \( F^{(a)} \) acquire the Kaluza-Klein mass. The zero modes for \( \lambda_{(n)i}^m \) and \( \phi_{(n)}^m \) are removed by the Scherk-Schwarz mechanism. Let us note that \( \lambda_{(n=0)}^{(a)} \) and \( \phi_{(n=0)}^{(a)} \) can be massless for special values of \( \beta = \pm (v_m \alpha_m)/2\pi \), for example, when the gauge symmetry breaking occurs. This is one of special features resulting from the existence of the supersymmetry breaking parameter \( \beta \).
4.3 Low-energy Effective Theory

In this section we discuss low-energy effective theories in three dimensions. We are, in particular, interested in the low-energy effective theory, in which relevant energy scale is much smaller than the compactification scale $1/L$. If we take $L$ to be small, it corresponds to small extra dimensions.

The particle whose mass scale is proportional to $1/L$ becomes superheavy if we take the small extra dimensions, and it is decoupled from low-energy physics. Only massless particles survive at low energies, so that the low-energy effective theory consists of them. This means that all the effects of the quantum corrections in the extra dimensions disappear in low energies as long as we take the limit of the small extra dimensions. Nevertheless, the nontrivial limit of the small extra dimensions is possible thanks to the supersymmetry breaking parameter $\beta$ of the Scherk-Schwarz mechanism.

Let us first consider the naive limit of small extra dimensions. As we have mentioned above, only the massless mode survives at low energies. The massless particles are $A_{(n=0)\mu}^m$, $q_{(n=0)i}^m$, and $\Phi_{(n=0)}^{(\alpha)}$, which consist of the low-energy effective theory. This is summarized in (i) of Table I. If we do not take the quantum corrections in the extra dimensions into account and consider the small extra dimensions, the particle contents in the low-energy effective theory are given by the trivial compactification. This is also listed in (ii) of Table I. Let us note that the superparticles, gaugino ($\lambda^m$) and squark ($\phi^m$), do not have massless modes due to the supersymmetry breaking parameter $\beta$, so that they do not appear in the low-energy effective theories.

Let us consider the limit by which some particle masses remain finite even in the limit of small extra dimensions. Suppose that $\bar{\beta}$ and $L$ are the same order, and we take $\bar{\beta}, L \to 0$, keeping the ratio of $\bar{\beta}$ and $L$ finite:

$$\frac{\bar{\beta}}{L} = \text{fixed as } \bar{\beta}, \ L \to 0, \quad (50)$$

where $\beta \equiv 2\pi \bar{\beta}$. In this limit, as seen from Eq. (40), the mass of the the Higgs scalar $m_{\Phi}^2$ is finite, and the Higgs scalar survives at low energies. The limit also makes the masses for $\lambda_{(n=0)i}^m$ and $\phi_{(n=0)}^{(\alpha)}$ finite, so that they, superparticles, also take part in the low-energy physics. The limit gives us a technique for generating mass in low-energy effective theory through compactification. Let us note that as long as $(v_m \alpha_m)/L$ is nonzero, the masses for $\lambda_{(n=0)i}^{(\alpha)}$ and $\phi_{(n=0)}^{(\alpha)}$ become superheavy even in this nontrivial limit. The particle contents in the low-energy effective theory are summarized in (iii) of Table I.

Table I
| Limit | Particle contents | Symmetry |
|-------|-------------------|----------|
| (i)   | $A^m_{(n=0)\mu}$, $\Phi^{(\alpha)}_{(n=0)}$, $q^m_{(n=0)i}$ | $U(1)^{N-1}$ |
| (ii)  | $A^m_{(n=0)\mu}$, $A^{(\alpha)}_{(n=0)}$, $\Phi^m_{(n=0)}$, $\Phi^{(\alpha)}_{(n=0)}$, $q^m_{(n=0)i}$, $q^{(\alpha)}_{(n=0)i}$ | $SU(N)$ |
| (iii) | $A^m_{(n=0)\mu}$, $\Phi^m_{(n=0)}$, $\Phi^{(\alpha)}_{(n=0)}$, $q^m_{(n=0)i}$, $\lambda^m_{(n=0)i}$, $\phi^m_{(n=0)i}$ | $U(1)^{N-1}$ |

Table I: Particle contents in the low-energy effective theories of $SU(N)$ supersymmetric Yang-Mills theory with massless adjoint matter. (i) stands for the naive limit of $L \to 0$ for the theory taking the quantum corrections in the extra dimensions into account. (ii) is the trivial compactification and (iii) represents the nontrivial limit defined by Eq. (50).

There is no supersymmetry in the effective theories because of the nontrivial phase $\beta$, and the gauge symmetry is broken to $U(1)^{N-1}$ through the dynamics of the nonintegrable phases except in the case of (ii) in Table I. We observe that the superparticles $\lambda^m_{(n=0)i}$, $\phi^m_{(n=0)i}$ come into play in the low-energy physics in the case of the limit (50). The quantum corrections in the extra dimensions become manifest in the low-energy physics by the mass of the adjoint Higgs scalar $\Phi^m_{(n=0)}$. It should be noted that this is possible due to the existence of the unique parameter $\beta$ associated with the Scherk-Schwarz mechanism of supersymmetry breaking.

The above discussion on low-energy effective theory is based on classical considerations of the mass terms (except for the Higgs scalar $\Phi$) in the previous subsection. In general, if we take into account quantum corrections for the mass terms, they suffer from the correction of order $O(1/L)$, like the Higgs scalar. This may modify the massless modes at the tree level and make them superheavy. They are decoupled from low-energy physics. Therefore, the effective theory may be different from the one obtained from the classical consideration. If $L$ is large, the quantum corrections are suppressed, so that the classical consideration may be a good approximation for the effective theory. The particle can still remain massless even after taking into account the quantum corrections if they are the Nambu-Goldstone boson, for example, associated with the breakdown of symmetry. And the limit defined in Eq. (50) is an example of obtaining massive particles at low energies.

5 Conclusions

We have studied the dynamics of the nonintegrable phases in $SU(N)$ supersymmetric Yang-Mills theory with $N_F$ numbers of massless adjoint matter. We have resorted to the Scherk-Schwarz mechanism, by which supersymmetry is broken softly, in order to obtain the nonvanishing effective potential for the phases in perturbation theory.
We have found that the $SU(N)$ gauge symmetry is broken dynamically to its maximal commutative subgroup, i.e., $U(1)^{N-1}$. This result does not depend on the values of the supersymmetry breaking parameter $\beta$, which is the boundary condition of the fields $\lambda, \phi$ for the $S^1$ direction. This is remarkable if we compare our model with nonsupersymmetric gauge theories, in which rich symmetry breaking patterns are possible, depending on the values of the boundary conditions of the fields.

We have obtained the mass of the Higgs scalar $\Phi$, which is originally the component gauge field for the compactified direction. The mass is generated through the quantum corrections in the extra dimensions. The mass term respects the residual gauge symmetry $U(1)^{N-1}$ and explicitly depends on the gauge coupling $g$, the supersymmetry breaking parameter $\beta$, and the number of flavor $N_F$. The Higgs boson mass suffers from a correction of $O(1/L^2)$.

We have also obtained the mass spectrum in three dimensions and discussed the low-energy effective theory in the limit of the small extra dimensions. All the effects of the extra dimensions are decoupled from low energies in the naive limit of $L \to 0$ since the relevant mass scale is given by the compactification scale $1/L$ as shown in Eqs. (45)-(49). We have considered the nontrivial limit defined by Eq. (50). The nontrivial limit is possible thanks to the supersymmetry breaking parameter $\beta$ of the Scherk-Schwarz mechanism. The mass of the Higgs scalar becomes finite in the limit, and the Higgs scalar survives at low energies. This implies that the quantum corrections in the extra dimensions become manifest in the low-energy physics even in the limit of the small extra dimensions. The limit also makes the masses of the gaugino $\lambda$ and squark $\phi$ finite, so that the superparticles come into play at low energies.

Concerning the limit defined by Eq. (50), let us comment on the gauge coupling constant in the low-energy effective theory. If we start with a space-time $M^{D-m(=4)} \otimes T^m(m$-torus) and assume that the size of the extra dimensions is equal to $L^m$, then the dimensionless gauge coupling constant in $M^{D-m}$ is given by $\tilde{g} = g/L^{m/2}$. The trilinear and quartic coupling constants arising from the covariant derivative have the form $g(2\pi n/L + \beta/L)/L^{m/2} = \tilde{g}(2\pi n/L + \beta/L)$ and $g^2/L^m = \tilde{g}^2$, respectively. And the mass of the Higgs scalar is scaled on the dimensional ground as $m^2_\Phi \sim g^2L^2\beta^2/L^D = \tilde{g}^2(\beta/L)^2$, where $D = 4 + m$. As long as $\tilde{g}$ is finite and we take the limit (50), the coupling constants and the mass of the Higgs scalar are finite at low energies.

It may be interesting to introduce the matter fields belonging to the fundamental representation under the $SU(N)$ gauge group in addition to the adjoint matter. Then, unlike the present case, we may expect rich patterns of gauge symmetry breaking and realistic low-energy effective theories. These are under investigation.
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FIG. 1. Potential energies for the gauge symmetry breaking patterns of $SU(3)$, Eq. (25). The horizontal axes stand for the supersymmetry breaking parameter $\beta$ of the Scherk-Schwarz mechanism. In calculating the potential energy numerically, we have ignored the factor as well as the terms which do not depend on $\theta_i$ in the effective potential (15).
FIG. 2. Potential energies for the gauge symmetry breaking patterns of $SU(5)$, Eq. (26). The horizontal axes stand for the supersymmetry breaking parameter $\beta$ of the Scherk-Schwarz mechanism. In calculating the potential energy numerically, we have ignored the factor as well as the terms which do not depend on $\theta_i$ in the effective potential [13].