Measurement-based local quantum filters and their ability to transform quantum entanglement

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We introduce local filters as a means to detect the entanglement of bound entangled states which do not yield to detection by witnesses based on positive (P) maps which are not completely positive (CP). We demonstrate how such non-detectable bound entangled states can be locally filtered into detectable bound entangled states. Specifically, we show that a bound entangled state in the orthogonal complement of the unextendible product bases (UPB), can be locally filtered into another bound entangled state that is detectable by the Choi map. We reinterpret these filters as local measurements on locally extended Hilbert spaces. We give explicit constructions of a measurement-based implementation of these filters for 2 ⊗ 2 and 3 ⊗ 3 systems. This provides us with a physical mechanism to implement such local filters.

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I. INTRODUCTION

Ever since its introduction by Schrödinger [1, 2] in the context of the EPR paradox [3], quantum entanglement has played a central role in quantum theory. While entanglement is responsible for the non-classical correlations leading to the violation of Bell’s inequalities [4, 5], it also plays a key role in quantum computing where it is connected with the exponential advantage of quantum algorithms over their classical counterparts [6]. Studies of entanglement have led to a well developed mathematical theory of entanglement where positive maps (P) which are not completely positive (CP) [7–14] and unextendible product bases (UPB) [15–17] play an important role [18–21]. These mathematical advances have led to the discovery of bound entangled states [22]: states from which one cannot distill EPR pairs although they are still provably non-separable.

Quantum states (pure or mixed) are represented by positive definite Hermitian operators ρ ∈ B(H) with unit trace. For the special case when the rank of ρ is one, it represents a pure state. For a bipartite composite system where states are defined on B(HA⊗HB), a state ρ is said to be separable if it can be written as a convex sum:

ρ = ∑ j p j ρ j A ⊗ ρ j B , p j > 0 , ∑ j p j = 1 ; (1)

where ρ j A and ρ j B are states in B(H A) and B(H B) respectively. A state is entangled, if it cannot be written in the above form. The fundamental problem of determining whether a given state ρ is separable or entangled remains open for general states of systems which are beyond 2 ⊗ 2 and 2 ⊗ 3.

Allowed quantum evolutions are those P maps which are CP. P maps which are not CP are not physically allowed quantum evolutions, because entangled states can lose their positivity when such a map is applied to one part of the system. Therefore such maps act as entanglement witnesses. The partial transpose operation which is a particular entanglement witness, plays an important role in the identification of entangled states [23]. States which reveal their entanglement by acquiring one or more negative eigenvalues under partial transposition are called NPT (not-positive under partial transpose) while the rest are called PPT (positive under partial transpose). NPT states are entangled while PPT states can be either entangled or separable. In 2 ⊗ 2 and 2 ⊗ 3 dimensional systems, it has been shown that a state is separable if and only if it is PPT [23, 24]. Therefore, in dimensions 3 ⊗ 3 and larger, there are entangled states which are PPT [25]. These states in general require other entanglement witnesses to implicate their entanglement and cannot be distilled to give EPR pairs. Such states (with non-distillable entanglement) are called PPT or bound entangled states.

Choi map and its generalizations have been used to detect the entanglement of certain classes of bound entangled states [26]. However, even for the 3 ⊗ 3 system, the detection of bound entangled states is far from complete. Local operations, including measurement and filtering, cannot alter the status of the state from PPT to NPT and therefore can be used to convert one NPT state to another NPT state or one PPT state to another PPT state. Gisin et. al. showed that, starting with a mixed entangled state of a 2 ⊗ 2 system that does not violate Bell inequalities, one can set up a local filtration scheme based on measurements such that the filtered states violate Bell inequalities [27]. In this case only NPT states were involved as there are no PPT entangled states for the 2 ⊗ 2 case. These results have been extended, used and experimentally validated by a number of researchers [28–32]. Our work is a generalization

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and extension of these results into the domain of PPT states of $3 \otimes 3$ systems. In this work, we introduce local filters which convert PPT entangled states not detectable by the Choi map, to states which are detectable by the Choi map. In particular we are able to show that the PPT states obtained from the UPB can be converted into states detectable by the Choi map. Furthermore, we provide an explicit scheme for implementation of our filtration protocol via local projective measurements involving local ancillas. Although the notation of entanglement is well defined for infinite dimensional spaces, we restrict ourselves to finite dimensional Hilbert spaces in this paper.

The material in this paper is arranged as follows: In Section II we describe our local filtration scheme. Two examples are taken up in the Section II B where such schemes are used to manipulate entangled states. Section III describes a measurement-based scheme to realize the local filters while Section III A describes the general scheme. Section III B describes the implementation of filters used by Gisin and Section III C describes the implementation of filters on three-level systems. Section IV offers some concluding remarks.

II. LOCAL FILTRATION AND ENTANGLEMENT DETECTION

A. Local filters

Local filters are local non-unitary operators represented by $L \otimes M$ where $L$ and $M$ are invertible operators acting in the state spaces of their respective systems. Given a bipartite quantum state $\rho \in B(\mathcal{H}_A \otimes \mathcal{H}_B)$, the filter acts on the state giving a new state

$$\rho^f = (L \otimes M)\rho(L \otimes M)^\dagger \tag{2}$$

which is a positive Hermitian operator belonging to the same space and its trace can be brought to one by dividing by an appropriate positive number. For every invertible set of operators $L$ and $M$ we thus have a filter.

Proposition II.1. Let $L$ and $M$ be two full rank operators. Then the map $\rho \mapsto (L \otimes M)\rho(L \otimes M)^\dagger$ does not change the Schmidt number of the state.

Proof. Terhal and Horodecki [33] defined the Schmidt rank of a general density matrix. A bipartite state $\rho$ has Schmidt rank $k$ if

1. For any ensemble decomposition of $\rho$ as $\{p_j \geq 0, |\psi_j\rangle\}$ where $\rho = \sum_j p_j |\psi_j\rangle \langle \psi_j|$ at least one of the vectors $|\psi_j\rangle$ has Schmidt rank $k$.

2. There exists a decomposition of $\rho$ where all vectors $\{|\psi_j\rangle\}$ in the decomposition have a Schmidt rank at most $k$.

Therefore, we need only to show that the Schmidt number of pure bipartite states $|\psi\rangle = \sum_j \lambda_j |e_j\rangle \otimes |f_j\rangle$ is invariant under the operation $|\psi\rangle \mapsto (L \otimes M)|\psi\rangle$.

The Schmidt rank (SR) of $|\psi\rangle$ is the matrix rank of $\sum_j \lambda_j |f_j\rangle \langle e_j|$.

Thus

$$\text{SR}((L \otimes M)|\psi\rangle) = \text{Rank} \sum_j \lambda_j M |f_j\rangle \langle e_j| L^\dagger. \tag{3}$$

Let $L = U_1 D_1 V_1$ and $M = U_2 D_2 V_2$ be the singular value decompositions of $L$ and $M$ respectively, where $U_1, V_1, U_2, V_2$ are unitary operators. Then

$$\text{SR}((L \otimes M)|\psi\rangle) = \text{Rank} \sum_j \lambda_j U_2 D_2 V_2 |f_j\rangle \langle e_j| D_1^T U_1^\dagger = \text{Rank} \sum_j \lambda_j D_2 |f_j\rangle \langle e_j| D_1^T \tag{4}$$

where $|e_j\rangle = V_1 |e_j\rangle$ and $|f_j\rangle = V_2 |f_j\rangle$ are mutually orthogonal bases of the first and second systems respectively. Since $L$ and $M$ are of full rank, the diagonal matrices $D_1$ and $D_2$ are also of full rank, and the above assertion holds, i.e. SR$|\psi\rangle$ is invariant under these operations. \hfill \Box

The above proposition further shows that, entanglement is not created or destroyed by the above operations.

Let us choose a standard basis $\{|j\rangle\}$, $j = 1 \cdots n$ in an $n$-dimensional state space $\mathcal{H}$. Any density operator $\rho \in B(\mathcal{H})$ can then be written as $\rho = \sum_{i,j} \rho_{ij} |i\rangle \langle j|$. Transpose operation is defined through its action on $\rho$

$$\rho \xrightarrow{T} \rho^T = \sum_{i,j} \rho_{ij}^\dagger |i\rangle \langle j| \tag{5}$$

A bipartite state $\rho$ is defined to be PPT if and only if $(1 \otimes T)\rho \geq 0$ where $T$ is the transpose operation defined on $\mathcal{H}_B$ as described in Equation (5).

Proposition II.2. The PPT or NPT character of a state is invariant under an invertible local filtration operation.

Proof. Now consider $\{|j\rangle\}$, $j = 1 \cdots n_1$ to be the standard basis in $\mathcal{H}_A$. We can write $\rho = \sum_{i,j} |i\rangle \langle j| \otimes \rho_{ij}$, where $\rho_{ij} \in B(\mathcal{H}_B)$. We then have

$$\rho^f = (L \otimes M)\rho(L \otimes M)^\dagger = \sum_{i,j} L^\dagger |i\rangle \langle j| L^\dagger \otimes M \rho_{ij} M^\dagger \tag{6}$$

After the application of the transpose operation on the
The Choi map.

of UPB can be locally filtered into states detectable by the PPT entangled states in the orthogonal complement is detectable by the Choi map. Similarly we show that the Choi maps have been used to unearth new PPT entangled states from the old ones.

These maps provide us with important entanglement witnesses, the filtered state reveals its entanglement by the filtering operation. Therefore for a given PPT entangled state, the filtered state is another PPT entangled state. If the original state is entangled, the nature of its entanglement was not detectable via the Choi map, however it is not always detected by the Choi maps given in equations (8) and (9). For instance, set $t = \frac{1}{20}$, then for $0.6044 < x < 0.6554$, the state is not detectable by the Choi map. Consider a local filter

$$L_3 \otimes M_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{5}{8} & 0 \\ 0 & 0 & \frac{5}{8} \end{pmatrix} \otimes I_3$$

(11)

The filtered state after the application of this filter is obtained as

$$\rho^f(x, t) = (L_3 \otimes M_3) \rho(x, t)(L_3 \otimes M_3)^\dagger$$

(12)

We now apply the Choi map given in equation (8) on the first system via $\phi \otimes I_3$ to the filtered as well as non-filtered density operator to obtain

$$\rho(x, t)^{\otimes I_3} = \rho_{\text{Choi}}(x, t)$$

(13)

$$\rho^f(x, t)^{\otimes I_3} = \rho_{\text{Choi}}^f(x, t)$$

(14)

For the operator $\rho_{\text{Choi}}^f(x, t)$ for $t = \frac{1}{20}$ and for $0.6044 < x < 0.6554$, the minimum eigenvalue turns out to be negative, indicating that the state is entangled. On the other hand, the minimum eigenvalue of the operator $\rho_{\text{Choi}}(x, t)$ which is obtained by the application of Choi map without filtering, is positive. This shows that the local filter defined in equation (12) has converted the state $\rho(x, t)$ whose entanglement was not detectable via the Choi map into a state $\rho^f(x, t)$ whose entanglement is detectable via the Choi map.

An important example of bound entangled states is provided by the well known UPB construction known as ‘TILES’ [15] for a $3 \otimes 3$ system

$$|\psi_0\rangle = \frac{1}{\sqrt{2}} |0\rangle (|0\rangle - |1\rangle), \quad |\psi_2\rangle = \frac{1}{\sqrt{2}} |2\rangle (|1\rangle - |2\rangle),$$

$$|\psi_1\rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) |2\rangle, \quad |\psi_3\rangle = \frac{1}{\sqrt{2}} (|1\rangle - |2\rangle) |0\rangle,$$

$$|\psi_4\rangle = \frac{1}{3} (|0\rangle + |1\rangle + |2\rangle) (|0\rangle + |1\rangle + |2\rangle)$$

(15)
The mixed state
\[ \rho_{\text{upb}}^{\text{f}} = \frac{1}{4} \left( I_3 \otimes \sum_{i=0}^{d} |\psi_i\rangle \langle \psi_i| \right). \] (16)

provides an example of a PPT entangled state \cite{15}. Choi maps, applied directly, can not detect the entanglement of such states. Consider the local filter

\[ L_3' \otimes M_3' = I_3 \otimes \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}, \] (17)

Applying this filter gives a new filtered state given by
\[ \rho_{\text{upb}}^{\text{f}} = (L_3' \otimes M_3') \rho_{\text{upb}}^{\text{f}} (L_3' \otimes M_3')^\dagger \] (18)
We now apply the second Choi map given in equation (9) on the second system via \( I_3 \otimes \psi \) to the filtered as well as non-filtered density operator to obtain
\[ \rho_{\text{upb}}^{\text{f}} \xrightarrow{I_3 \otimes \psi} \rho_{\text{Choi}} \] (19)
\[ \rho_{\text{upb}}^{\text{f}} \xrightarrow{I_3 \otimes \psi} \rho_{\text{f}}^{\text{upb}} \] (20)
The operator \( \rho_{\text{Choi}}^{\text{f}} \) has a negative eigen value which reveals the entanglement of \( \rho_{\text{upb}}^{\text{f}} \) while \( \rho_{\text{Choi}}^{\text{upb}} \) does not have a negative eigen value. This shows that the entanglement of the state \( \rho_{\text{upb}}^{\text{upb}} \) is not directly revealed by the Choi map, however, it can be filtered into a state that is detected by the Choi map. This is directly related to the construction given in terms of the automorphisms in \cite{20} and is much simpler than the construction given in \cite{18}.

### III. IMPLEMENTATION OF LOCAL FILTERS

#### A. General scheme

We now turn to the question of the physical interpretation and implementation of the local quantum filtration process introduced in the previous section. A filter is defined through its action given in equation (2) and comprises of invertible operators \( L \) and \( M \) where \( L \) acts locally on \( \mathcal{H}_A \) (the Hilbert space of Alice) and \( M \) acts locally on on \( \mathcal{H}_B \) (the Hilbert space of Bob). We choose the standard bases in \( \mathcal{H}_A \) and \( \mathcal{H}_B \). Each of these operators has a singular valued decomposition given by
\[ L = U_1 D_1 V_1 \quad \text{and} \quad M = U_2 D_2 V_2. \] Here \( U_1, U_2, V_1, V_2 \) are unitary operators and \( D_1, D_2 \) are diagonal with real positive definite diagonal entries. The unitary operators correspond to Hamiltonian evolutions and can hence be physically realized in a straightforward way. We therefore focus here on the implementation of diagonal matrices \( D_1 \) and \( D_2 \).

Consider the implementation of \( D_1 \) on \( \mathcal{H}_A \). The diagonal matrix \( D_1 = \text{Diag}[d_1, d_2, \cdots, d_n] \) has diagonal entries \( d_j \) such that \( 0 < d_j < 1, \quad (j = 1 \cdots n) \). We now show that such a \( D_1 \) can be implemented by first extending the Hilbert space by adding one qubit as ancilla and then measuring an appropriate projection operator \( P \). To achieve this, we first consider a set of \( n \) orthogonal but un-normalized vectors of the form \( |u_j\rangle = \sqrt{\delta_j} |j\rangle \) in the \( n \) dimensional system Hilbert space. We extend each of these vectors into a \( 2n \) dimensional Hilbert space to form a new set of vectors \( \{|\xi_j\rangle\} \) given by \( |\xi_j\rangle = \sqrt{\delta_j} |j\rangle + \sqrt{1 - \delta_j} |j + n\rangle \). In addition to being mutually orthogonal, these vectors are also normalized. Thus we have constructed an orthonormal set of \( n \) vectors in a \( 2n \) dimensional Hilbert space formed from the \( n \)-dimensional system and a two-dimensional ancilla. Corresponding to each of these vectors, we can construct a projection operator \( P_j \) given by
\[ P_j = |\xi_j\rangle \langle \xi_j| \] (21)
where the \( n \times n \) matrices are given by:
\begin{align*}
\eta_j &= d_j |j\rangle \langle j| , \\
\eta_j' &= (1 - d_j) |j\rangle \langle j| , \\
\delta_j &= \sqrt{d_j (1 - d_j)} |j\rangle \langle j| \quad \text{(22)}
\end{align*}
The projection operator obtained by adding these mutually orthogonal projectors is given by
\[ P = \sum_{j=1}^{n} P_j = \left( \frac{D_1}{\Delta} \right)_{2n \times 2n} \] (23)
where \( D_1 = \eta_1 + \cdots + \eta_n \), is the original operator that we wanted to implement, \( D_1' = \eta_1' + \cdots + \eta_n' \) is a complementary operator obtained from \( D_1 \) and \( \Delta = \delta_1 + \cdots + \delta_n \) represents the cross terms.

Now consider the system to be in an arbitrary state \( \rho_A \) and the one-qubit ancilla to be in the state \( |0\rangle \otimes |0\rangle \). Consider a measurement of \( P \) on this composite system. If the outcome of the measurement is positive, we retain the state. The state after such a selection is given by the action of the projection operator \( P \) on the composite state:
\[ P |0\rangle \otimes \rho_A \] (24)
We further carry out a projective measurement of the projector operator \( |0\rangle \otimes |0\rangle \) on the ancilla qubit and retain the state only if the outcome is positive. We now discard the ancilla which is anyway decoupled from the system. This completes the implementation of the map \( D_1 \) on the system density operator \( \rho_A \). Thus to realize the operator \( D_1 \) in the space of the system, we need to carry out two projective measurements: first measure the projector \( P \)
Implement \( V \) and the unitaries \( U \) with several copies of the shared state \( \rho \). Evolution of the non-unitary part represented by \( D \) ancilla space and retain the state if the outcome is positive and then make another measurement of the projector \( |0\rangle \langle 0| \). As discussed in Section III A, by adding a one-qubit ancilla these two-dimensional vectors can be extended into four-dimensional orthonormal vectors.

\[
\rho' = (L \otimes M) \rho (L \otimes M)^\dagger
\]

This protocol obviously requires classical communication between Alice and Bob because they need to know the outcome of the measurements that the other performs. The situation is schematically depicted in Figure 1.

**B. Filtration of two-qubit systems**

An interesting example of quantum filtration was introduced by Gisin [27] for an entangled mixed state of two spin-\( \frac{1}{2} \) particles not violating the Bell-CHSH inequality. In this scheme by using a polarized beam splitter one can convert such an input state to an output state which remains entangled but, this time, its entanglement can be detected by a Bell inequality violation. We recast this filtration scheme and connect it with our results.

Let us suppose that Alice and Bob share a \( 2 \otimes 2 \) system \( \rho \in B(\mathcal{H}_A \otimes \mathcal{H}_B) \) between themselves. Interpreting the Gisin filter in our formalism reveals that in that case \( \rho \mapsto (L_2 \otimes M_2)\rho(L_2 \otimes M_2)^\dagger \), where the operators \( L_2 \) and \( M_2 \) are given by

\[
L_2 = \begin{pmatrix}
\kappa & 0 \\
0 & 1
\end{pmatrix} \quad \text{and} \quad M_2 = \begin{pmatrix}
1 & 0 \\
0 & \kappa
\end{pmatrix}
\]

(25)

In the notation of reference [27], \( \kappa = \sqrt{\frac{\beta}{\alpha}} \), \( \alpha \) and \( \beta \) are two real numbers such that \( \alpha > \beta > 0 \) and \( \alpha^2 + \beta^2 = 1 \) so that \( 0 < \kappa < 1 \). Let us consider the implementation of the non-unitary operator \( L_2 \). Since the operator \( L_2 \) is acting locally, we need to look for the map \( \rho_A \mapsto L_2 \rho_A L_2 \).

Since \( L_2 \) is a diagonal matrix (\( L_1 \) = \( L_2 \)), we do not need to undertake a singular value decomposition. We directly introduce two mutually orthogonal but un-normalized vectors

\[
|u_1\rangle = \begin{pmatrix}
\sqrt{\kappa} \\
0
\end{pmatrix}, \quad |u_2\rangle = \begin{pmatrix}
0 \\
1
\end{pmatrix}.
\]

(26)

As discussed in Section III A, by adding a one-qubit ancilla these two-dimensional vectors can be extended into four-dimensional orthonormal vectors

\[
|\xi_1\rangle = \begin{pmatrix}
\sqrt{\kappa} \\
0 \\
\sqrt{1-\kappa} \\
0
\end{pmatrix}, \quad |\xi_2\rangle = \begin{pmatrix}
0 \\
1 \\
0 \\
0
\end{pmatrix}
\]

(27)

Constructing the corresponding mutually orthogonal...
When the outcomes of measurement of all four projectors are positive, Bob does a similar exercise to implement another projective measurement in the ancilla space and retain it if the outcome of measurement of an arbitrary state \(\rho\) on a single three-level system is an arbitrary state \(\rho\) and the ancilla qubit is in the state \(\rho_u = |0\rangle \langle 0|\).

Next introducing a qubit as an ancilla and extending the above vectors in the composite Hilbert space of 6 dimensions, we obtain

\[
|\xi_1\rangle = \begin{pmatrix} \sqrt{d_1} \\ 0 \\ 0 \end{pmatrix}, |\xi_2\rangle = \begin{pmatrix} 0 \\ \sqrt{d_2} \\ 0 \end{pmatrix}, |\xi_3\rangle = \begin{pmatrix} 0 \\ 0 \\ \sqrt{d_3} \end{pmatrix}
\]

which are orthonormal.

The projector operators corresponding to these vectors are given by \(P_j = |\xi_j\rangle \langle \xi_j|\) and can be written explicitly as

\[
P_1 = \begin{pmatrix} d_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 - d_1 \end{pmatrix}
\]

\[
P_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & 1 - d_2 \end{pmatrix}
\]

\[
P_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 - d_3 \end{pmatrix}
\]

This is the result of measurement of \(P\) on the composite system for the case when the outcome is positive. To extract the top left block of the above matrix, we perform another projective measurement in the ancilla space and retain it if the outcome of measurement of \(|0\rangle \langle 0|\) is positive. Bob does a similar exercise to implement \(M_2\) in his laboratory and both Alice and Bob retain the state only when the outcomes of measurement of all four projectors are positive. This completes the protocol.

C. The case of 3 \(\otimes\) 3 systems

In Section II B we demonstrated the role of local filters in strengthening the entanglement detection capabilities of entanglement witnesses for 3 \(\otimes\) 3 systems. Here we delineate the implementation of such a filtration process for these systems. We begin by discussing the filtration on a single three-level system \(\hat{\rho}_A \in \mathcal{B}(\mathcal{H}_A)\). We consider the following transformation \(\hat{\rho}_A \mapsto L_3 \hat{\rho}_A L_3^\dagger\) where \(L = U_1 D_1 V_1\) is the singular value decomposition of \(L\) and

\[
D_1 = \begin{pmatrix} d_1 & d_2 \\ d_2^* & d_3 \end{pmatrix}
\]

with \(0 < d_1, d_2, d_3 \leq 1\).

We now introduce three mutually orthogonal and unnormalized vectors, in a three-dimensional Hilbert space

\[
|u_1\rangle = \begin{pmatrix} \sqrt{d_1} \\ 0 \\ 0 \end{pmatrix}, |u_2\rangle = \begin{pmatrix} 0 \\ \sqrt{d_2} \\ 0 \end{pmatrix}, |u_3\rangle = \begin{pmatrix} 0 \\ 0 \\ \sqrt{d_3} \end{pmatrix}
\]

The projector \(P\) corresponding to the operator \(L_3\) is thus given by

\[
P = P_1 + P_2 + P_3 = \begin{pmatrix} L_3 & \Delta_3 \\ \Delta_3^\dagger & L_3 \end{pmatrix}
\]

For any operator \(\hat{\rho}_A \in \mathcal{B}(\mathcal{H}_A)\) we use a one-qubit ancillary system in the state \(|0\rangle \langle 0|\) and measure the projector \(P\). This leads us to an equation which is the same as the Equation (30) with \(L_2\) and \(\Delta_2\) replaced by \(L_3\) and \(\Delta_3\). Similarly we can do the analysis from Bob’s point of view and arrive at the projector \(Q\) corresponding to \(D_2\) (\(M_3 = U_2 D_2 V_2\)). Using this projector and a one qubit ancilla he sets up his measurements. Then they both follow the protocol steps 1-3 given in the last part of Section III A to complete the filtration process and obtain the new joint density operator.
IV. CONCLUSIONS

In this work, we discussed the role of local filters in transforming one PPT entangled state to another PPT entangled state. It may turn out that the entanglement of the new state is detectable by a P map which is not CP while the entanglement of the original state is not detectable by the map. It is in this sense that local filters can enhance the power of an entanglement witness in detecting entanglement. We give two concrete examples where this actually occurs. In the first example, a new class of bound entangled states becomes detectable by the Choi map and in the second example PPT entangled states in the orthogonal complement of UPB become detectable by the Choi map.

We then undertook the analysis of these filtration schemes as explicit local projective measurements coupled with local unitaries. It turns out that we need to add a one-qubit ancilla for both the parties involved in order to implement the non-unitary part of these filters as local measurements. We have constructed explicit projection operators corresponding to the filters that we have used. A point worth emphasizing is that these local filters do not change the NPT or PPT status of a state. Gisin exploited this fact to convert NPT states of two qubits which do not violate Bell’s inequalities into the ones which violate Bell’s inequalities. We have used these filters to convert one PPT entangled state into another PPT entangled state such that the PPT entanglement is detectable by a given entanglement witness.

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