Classical to quantum crossover in high-current noise of one-dimensional ballistic wires

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Abstract

Microscopic current fluctuations are inseparable from conductance. We give an integral account of both quantized conductance and nonequilibrium thermal noise in one-dimensional ballistic wires. Our high-current noise theory opens a very different window on such systems. Central to the role of nonequilibrium ballistic noise is its direct and robust dependence on the statistics of carriers. For, with increasing density, they undergo a marked crossover from classical to strongly degenerate behavior. This is singularly evident where the two-probe conductance shows quantized steps: namely, at the discrete subband-energy thresholds. There the excess thermal noise of field-excited ballistic electrons displays sharp and large peaks, invariably larger than shot noise. Most significant is the nonequilibrium peaks’ high sensitivity to inelastic relaxation within the open system. Through that sensitivity, high-current noise provides unique clues to the origin of quantized contact resistance and its evolution towards normal diffusive conduction.

I. INTRODUCTION

In this work we address a major, yet largely neglected, issue of mesoscopic transport: the nature of high-field thermal fluctuations and of what they reveal about transport dynamics. Noise and conductance in electronic transport are intertwined. Knowledge of one illuminates the other. Nowhere is their symbiosis clearer, or more critical to physical understanding, than in mesoscopic conduction.

Carrier fluctuations, measured as noise, change dramatically in the physical setting of ballistic transport \cite{footnote2}. Noise yields insights into the innermost aspects of carrier motion that cannot be gained from conductance alone. Noise at high currents is the most revealing. Little is known about high-field mesoscopic noise, despite the ease with which even modest driving potentials can push a mesoscopic system beyond linear response \cite{footnote3}. It is acknowledged that this unfamiliar but important region must be opened up and mapped \cite{footnote4}. 


We preface our study with a brief recollection of ballistic transport. So-called mesoscopic conductors have sizes comparable to the scale for electron scattering. Such systems no longer exhibit bulk Ohmic behavior. This was strikingly demonstrated in the recent experiments of de Picciotto et al. The resistance of a ballistic, almost one-dimensional (1D) quantum wire was probed, noninvasively, at two inner contacts well separated from the large diffusive leads acting as current source and drain. Across the inner probes and at finite current, the wire’s voltage drop, and hence the intrinsic resistance, were deduced to be essentially zero.

Reference also reported two-probe current-voltage data that included the source and drain in series with the 1D channel. In their well characterized 1D wire they found good agreement with the standard Landauer-Büttiker (LB) prediction of quantized jumps in the two-terminal conductance, as a function of carrier density.

At this point we come upon the core issue in ballistic transport. How can the finite conductance (albeit with sharp steps) be reconciled with the property of totally resistance-free transport within the central ballistic segment?

The LB theory accounts for this coexistence by simple kinematics. A 1D wire admits a sparse set of discrete subband states for electron transmission, while its attached leads have a much richer quasicontinuous density of such states. As well, they are collisionally coupled to a rich phonon spectrum. Bottlenecks arise as carriers try to funnel into and fan out of the small set of 1D states. That mismatch destroys perfect conduction and manifests as the “contact resistance” of the leads.

According to the LB account, the contact resistance is quantized because its bottleneck is regulated solely by the occupancy of the quantized ballistic states. Each time the electrons’ Fermi level accesses an additional higher 1D subband, the ideal two-probe conductance is shown to jump by $e^2/\pi\hbar$.

In the Landauer-Büttiker picture, therefore, the two-probe conductance appears to be free of any influence from extraneous parameters such as the asymptotic density of states and scattering rates for the macroscopic leads; those can enter only implicitly in the transport. One must suppose that the physics of relaxation in the carrier reservoirs remains concealed – in a nonspecific fashion – within each lead’s equilibrium state as it fixes the boundary conditions for the sample.

II. A PHYSICAL DILEMMA

We make an important observation. If a 1D wire is imperfectly transmissive (finite resistance), LB predicts departures from the ideal staircase for two-probe conductance. On the other hand, if one knows for sure that the 1D wire transmits perfectly (zero resistance) LB is bound to say, unequivocally, that the two-probe conductance must scale strictly universally. The published two-probe results of Ref. record a somewhat imperfect “universal” conductance, yet the four-probe results record perfect transmission. We will return to this apparent dilemma.

Here noise enters as a wholly different kind of probe. Beside the linear LB theory for conductance, there is a corresponding theory for low-field mesoscopic fluctuations (in the strictly linear regime, these should always be be tightly circumscribed by the fluctuation-dissipation theorem). The most authoritative survey of LB noise theory to date is Blanter...
and Büttiker’s [1].

Inherently nonlinear mesoscopic noise is out of reach to the LB approach, which can treat only voltages much below the Fermi energy [1]. We now pose our key question: What can those little-known fluctuations reveal about the wire-lead interaction and the contact resistance?

We predict the surprising existence of large peaks in the thermal, or hot-electron, noise of a ballistic system. These noise structures are much larger than shot noise which also peaks at the stepwise two-probe conductance transitions. For high-field thermal noise, more can be said. Its maxima carry unique quantitative information tracking the interplay of inelastic and elastic scattering in the leads. Nonlinear fluctuations, governed by inelasticity, are simply inaccessible to elastic-only, low-field descriptions. For our part, we build upon existing nonequilibrium kinetics [2,6].

In the following Section we describe our kinetic model for the conductance of a 1D ballistic wire fully open to an incoherent dissipative environment. In Sec. 4 we extend the kinetic description to the nonequilibrium current noise in the system, and demonstrate the rich behavior of its spectral density. Section 5 has our numerical results, confirming that hot ballistic electrons, in their transition from a classical to a degenerate regime, yield dramatic thermal-fluctuation peaks. Finally, in Sec. 6 we sum up; our predictions for nonequilibrium ballistic noise have direct implications for corroborating the recent seminal four-probe data [3].

III. KINETIC DESCRIPTION

Figure 1 depicts our model system. An ideal 1D wire of length $L$ is connected to two large reservoirs which are always at equilibrium; the identical arrangement of Landauer-Büttiker, as for Ref. [3]. Elastic (nondissipative) and inelastic (dissipative) collisions occur within the leads. A kinetic description will cover both wire and leads. Inelastic and elastic mean free paths (MFPs) cannot be shorter than $L$.

We stress that it is utterly essential to incorporate, within the microscopic equations of motion, at least the leading-order effects of dissipative inelastic collisions in the leads. It is the kinetics of dissipative relaxation alone that secures the energetic stability of transport over the system [5], by establishing steady state.

There is a second crucial consequence of inelastic relaxation in the leads: the total loss of phase memory for carriers crossing the ballistic region of the system. From the ballistic carriers’ perspective, they no longer bear the detailed signature of their collision history, which thus becomes Markovian.

The only kinetic parameters that survive phase breaking are (a) the effective scale of the scattering mean free paths (matching the ballistic length), and (b) the size of the driving field that impels carriers across the collision-free segment. It is precisely through phase breaking that the ballistic system becomes insensitive to the particulars of geometry and material structure of its asymptotic leads. On that basis, one can make a semi-classical analysis of the ballistic kinetics.

Our Fermi-liquid perspective is very closely related to that of Kamenev and Kohn [7]. We generalize their coherent, closed-circuit, linear-response approach to deal with a phase-
broken, open system driven far from equilibrium. We guarantee the open system’s all-
important gauge invariance by inserting an explicit flux $I/e$ of electrons at the source, and its matched sink $-I/e$ at the drain (equivalently, a hole flux of strength $I/e$).

Formally, the direct inclusion of boundary flux sources is strictly mandated by global – as distinct from local – charge and current conservation in open conductors, with nontrivial electromotive forces (EMFs) acting from outside [9]. Physically, the flux-source regions at the lead–wire boundaries fulfill a unique, and absolutely necessary, dynamical role. They are the active sites for the nonequilibrium scattering processes that directly determine the relaxation in the leads.

First we recover the full Landauer-Büttiker conductance by our kinetic analysis. Consider any single, occupied 1D subband. From the viewpoint of their time-stationary distribution, incoherence in the leads means that carriers inside the finite wire will perceive a homogeneous physical environment, not only within the wire proper (internal homogeneity is also assumed in the LB description) but at a far longer range. They will behave as though the physical wire were embedded, seamlessly, in a very long effective 1D host. This effective host conductor, “standing in” for the leads, is itself nonideal on a scale to match the ballistic length $L$.

The boundaries and field sources are as if at infinity, even though the actual current injector and extractor can be adjacent to the wire. Explicit carrier relaxation in the boundary neighborhoods, in concert with the EMF (Landauer’s resistivity dipole), asymptotically “prepares” the nonequilibrium and uniform steady state of the carriers crossing the inner ballistic region. The size $V$ of the EMF quantifies the injected carriers’ self-consistent adjustment to relaxation inside the embedding host (the leads). The collision-mediated relation between $V$ and $I$ follows [9].

Let the steady-state electron distribution in the wire be $f_k$ for states $k$ within our 1D subband (spin label implied). The collision-time form for the kinetic equation, for EMF field $-E$ in the source-to-drain direction, is [10,2]

$$
e E \frac{\partial f_k}{\partial k} = -\frac{1}{\tau_{\text{in}}(\varepsilon_k)} f_k - \frac{\langle \tau_{\text{in}}^{-1} f \rangle}{\langle \tau_{\text{in}}^{-1} f^{\text{eq}} \rangle} f_{\text{eq}} - \frac{1}{\tau_{\text{el}}(\varepsilon_k)} \frac{f_k - f_{-k}}{2}.
$$

Here $\tau_{\text{in}}(\varepsilon_k)$ and $\tau_{\text{el}}(\varepsilon_k)$ are the inelastic and elastic scattering times, in general energy-dependent. The leading, inelastic collision term has a restoring contribution proportional, in the general case, to the expectation

$$\langle \tau_{\text{in}}^{-1} f(t) \rangle \equiv \int_{-\infty}^{\infty} \frac{dk}{2\pi} \tau_{\text{in}}^{-1}(\varepsilon_k) f_k(t).$$

The inelastic collision term respects continuity and (local) gauge invariance [11,14] dynamically and, as in Eq. (1), statically. As usual, the underlying equilibrium distribution is $f_k^{\text{eq}} = 1/[1 + \exp((\varepsilon_k + \varepsilon_i - \mu)/k_B T)]$ where $\varepsilon_i$ is the band-threshold energy. Only one chemical potential, $\mu$, enters the problem [2,4,13]. Finally, the elastic collision term restores symmetry.

A microscopic formulation such as Eq. (1) does not need to segregate left- and right-moving carriers for exclusive treatment [3]. Nor, as Kamenev and Kohn have rigorously shown [7], does such a formalism need the special creation of chemical potentials exclusive to different movers [4].
Let us now specialize, just as in Landauer-Büttiker, to energy-independent scattering rates. Stanton [10] has solved Eq. (1) analytically with a particular linear transformation:

$$\hat{f}_k \equiv \frac{1}{2} \left( 1 + \sqrt{\frac{\tau_{in}}{\tau}} \right) f_k + \frac{1}{2} \left( 1 - \sqrt{\frac{\tau_{in}}{\tau}} \right) f_{-k}$$

where $\tau^{-1} = \tau_{in}^{-1} + \tau_{el}^{-1}$ is recognizable as the Matthiessen relaxation rate. The composite function $\hat{f}$ satisfies a simpler form of Eq. (1) in which $\sqrt{\tau_{in}}$ appears in place of $\tau_{in}$ and there is no elastic collision term. Its exact solution is [10]

$$\hat{f}_k = \lambda \int_{-\infty}^{k} dk' e^{-\lambda(k-k')} f_{k'}^{eq},$$

where $\lambda = \hbar/(eE\sqrt{\tau_{in}})$. The physical solution is easily recovered, as is the expectation value of the current:

$$I = \langle ev_k f_k \rangle = \frac{e\hbar}{m^*} \langle k f_k \rangle = \frac{e\hbar}{m^*} \sqrt{\frac{\tau}{\tau_{in}}} \langle k \hat{f}_k \rangle$$

for a parabolic subband with effective mass $m^*$. Integration at subband density $n$ yields the time-honored result

$$I = \sqrt{\frac{\tau}{\tau_{in}}} \frac{ne\hbar}{m^*} = \frac{ne^2 \tau}{m^*} E.$$  

Near equilibrium the ballistic hypothesis applies. The dominant mean free paths in the effective 1D host conductor, namely the Fermi ones $v_F\tau_{in}$ and $v_F\tau_{el}$, will each span $L$, the ideal collision-free length of the uniform sample. Let us therefore equate each MFP to $L$; then $\tau = L/2v_F$. The uniform-state solution Eq. (5) gives, on writing the subband density as $n = 2m^*v_F/\pi\hbar$,

$$I = \frac{2m^*v_F}{\pi\hbar} \frac{e^2 EL}{2m^*v_F} = \frac{e^2}{\pi\hbar} V$$

in which $V = EL$ is the external EMF. Equation (6) exhibits precisely the Landauer-Büttiker conductance.

This result complements the Kamenev-Kohn approach, which derives Eq. (6) by applying orthodox Fermi-liquid principles (via microscopic Kubo and quantum-transmission theories) in the context of a closed, nondissipative, phase-coherent mesoscopic circuit [7]. Equally, the present derivation builds upon standard Fermi-liquid microscopics and applies it (via kinetic theory) to the open, dissipative, phase-breaking context of mesoscopic circuits in the laboratory [2,6].

### IV. NONEQUILIBRIUM BALLISTIC FLUCTUATIONS

Our open-system kinetic theory for ballistic transport clearly leads to the Landauer-Büttiker quantized conductance. Now we obtain the nonequilibrium hot-electron noise,
which the linear-response LB picture cannot attain. For this we need the retarded, space-
time dependent Green function \( R_{kk'}(x, x'; t - t') = \theta(t - t') \delta f_k(x, t)/\delta f_{k'}(x', t') \) for the
dynamical form of Eq. (4). The standard, manifestly gauge-invariant kinetic equation for the
dynamic response \( R \) is \[1,2,12,13\]
\[
\left[ \frac{\partial}{\partial t} + v_k \frac{\partial}{\partial x} + \frac{eE}{\hbar} \frac{\partial}{\partial k} \right] R_{kk'}(x, x'; t - t') = 2\pi \delta(k - k') \delta(x - x') \delta(t - t')
\]
\[
- \frac{1}{\tau_{\text{in}}} \left( R_{kk'} - \langle R_{kk'} \rangle \right) - \frac{1}{2\tau_{\text{el}}} (R_{kk'} - R_{-k,k'}). \tag{7}
\]

The first step in solving \( R_{kk'}(x, x'; t - t') \) is to Fourier transform it to \( R_{kk'}(q, q'; \omega) \) in the
momentum-frequency domain \((q, q'; \omega)\). Next, a frequency-dependent continuation of Stan-
ton's transformation for \( \hat{f} \), Eq. (2), leads one to a composite Green function \( \hat{R}_{kk'}(q, q'; \omega) \). That object, a well formed linear combination of matrix elements of the physical response \( R_{kk'}(q, q'; \omega) \), has an equation of motion more readily solved than Eq. (7). The las-
t major step is to isolate the purely transient component of \( \hat{R} \). This is embodied in the correlated
propagator \[2,12\]
\[
\hat{C}_{kk'}(q, q'; \omega) \equiv \hat{R}_{kk'}(q, q'; \omega) - j_0(qL/2)(\hat{R}_{kk'}(0, q'; \omega)/2\pi) \frac{n_{\hat{f}_k}}{n}, \tag{8}
\]
whose second right-hand term represents the adiabatic (low-frequency dominant) contribu-
tion to \( \hat{R} \), now subtracted \[1\].

The current-current spectral density, taken over all carriers in the sample, can now be regai-
ded. It is a linear superposition of convolutions of the correlation \((\pm v_k)(\pm v_{k'})\hat{C}_{\pm k, \pm k'}\),
evaluated for all sign choices, with the steady-state mean-square fluctuation distribution
\[
\Delta \hat{f}_{k'} = \lambda \int_{-k'}^{k'} dk'' e^{-\lambda(k' - k'')} k_B T \frac{\partial f_{eq_k}}{\partial \mu} \tag{9}
\]
With the same linear transformation used in Eq. (2), this uniquely maps the static electron-
hole pair correlation at equilibrium (the last derivative factor in the integrand of Eq. (9)) to
its fully nonequilibrium counterpart \[2\].

In the dynamics of a spontaneous thermal excursion of the steady state, Eq. (9) fixes
the mean initial strength of the background electron-hole fluctuation \( \Delta \hat{f} \), perturbing the
nonequilibrium system at any given instant. From Eq. (8), \( \hat{C} \) then provides its subsequent
relaxation.

The flux autocorrelation function has the raw form \( \langle \langle \nu \hat{C}_{k'} \Delta \hat{f} \rangle \rangle'_t \). After some tedious
but straightforward linear algebra, the original quantities \( C_{kk'}(q, q'; \omega) \) and \( \Delta f_{k'} \) are re-
constructed and, with them, we finally obtain the complete and explicit physical form

\[1\] The spherical Bessel function \( j_0(qL/2) \) is the transform of the step function \( \theta(L/2 - |x|) \), delimi-
ting the finite ballistic wire.
This defines the thermal-noise spectral density in the ballistic wire, specializing now to the ith subband:

\[ S_i(q, q'; \omega) \equiv 4 \Re \left\{ \langle \langle -e v'/L \rangle \rangle C_i(q, q'; \omega)(-e v'/L) \Delta f_i' \rangle \rangle' \right\}. \quad (10) \]

The long-time average of the hot-electron fluctuations, integrated over the wire for a parabolic subband at density \( n_i = 2m^* v_{Fi}/\pi \hbar \), comes from the long-wavelength static form of Eq. (10):

\[ S_{xs}^i(V) = S_i(0, 0; 0) - 4G_i k_B T \]
\[ = 4G_i k_B T \frac{\partial \ln n_i}{\partial \mu} \frac{e^2 V^2}{2m^* L^2} \left( \tau_{in;i}^2 + 2\tau_{in;i} \tau_{el;i} - \tau_{el;i}^2 \right); \quad (11) \]

note that the (dissipative) Johnson-Nyquist term \( 4G_i k_B T \) is removed. The subband conductance is \( G_i = (2v_{Fi} \tau_{el;i}/L) G_0 \) with \( G_0 = e^2/\pi \hbar \). We do not necessarily assume, as for Eq. (6), collision times \( \tau_{el;i} \) and \( \tau_{in;i} \) equal to the transit time \( L/v_{Fi} \) and predicated on a potentially singular density dependence of the relaxation rates.

Equation (11) is clearly sensitive to the ratio \( \tau_{in;i}/\tau_{el;i} \). It probes the dynamics imposed by the leads upon each individual subband. It is our foremost outcome.

There are three points more.

(ı) The excess thermal noise is nonlinear in the EMF. Although, like shot noise, it is nondissipative \([2,14]\), Eq. (11) does not describe shot noise \([1]\). We emphasize that \( S_{xs}^i \) is an entirely separate and experimentally distinguishable effect.

(ıı) For strong inelastic scattering \( S_{xs}^i/G_i \) vanishes. At weak inelasticity it diverges; as \( \tau_{in;i} \to \infty \), carriers have no way to shed excess energy and the hot-electron distribution broadens without limit. Fluctuations are all. Thus, transport models based solely on elastic relaxation have, at best, a marginal thermodynamic stability.

(ııı) At strong degeneracy \( S_{xs}^i \) scales as \( k_B T \partial \ln n_i/\partial \mu = k_B T/2(\mu - \epsilon_i) \). This is a feature generic to the dense nonequilibrium electron gas \([2,13]\). For classical electrons, the same factor goes to unity, and classical excess noise is then independent of \( T \).

V. NUMERICAL RESULTS

In the light of (ııı) above, the nonequilibrium behavior of Eq. (11) at a subband threshold is extremely interesting. An increase in electron density within each level \( i \) takes its ballistic carrier population out of the classical domain \( (\mu \ll \epsilon_i) \) and makes it cross over to the highly degenerate regime \( (\mu \gg \epsilon_i) \). Thus the kinematic \( T \)-scaling of \( S_{xs}^i \) starts to attenuate it strongly while, at the same time, the factor \( G_i \) in Eq. (11) grows from almost nothing at low subband occupancy to values near \( G_0 \) at high density. This pattern repeats itself as each level is filled in succession.

Evidently, classical statistics naturally dominates whenever \( \mu - \epsilon_i \leq k_B T \). If one insists on suppressing it by imposing a strict but inappropriate degeneracy there, the development of \( S_{xs}^i \) will echo that for the Landauer-Büttiker conductance model: \( G_i \to \theta(\mu - \epsilon_i) G_0 \). Then \( S_{xs}^i \) behaves pathologically, and quite unphysically. Forgoing this unnatural choice we shall
adopt a proper finite-temperature representation, with no artificially enforced degeneracy at the subband crossings.

The Ansatz in Eq. (6), whereby \( \tau_i \sim 1/v_{F_i} \) as \( v_{F_i} \to 0 \), must be corrected for its spurious divergence when the chemical potential falls below the subband edge. The MFP for elastic impurity scattering stays close to \( L \) and is insensitive to hot-electron effects, while the inelastic MFP shortens as phonon emission sets in. We write the elastic time as \( \tau_{\text{el};i} = L/u_i(\mu) \) for the well-behaved and characteristic velocity average \( u_i(\mu) \equiv 2\langle |v|f_{\text{eq}} \rangle_i/n_i \).

Near and below the subband crossing \( u_i \) is the classical, thermal velocity; above, \( u_i \to v_{F_i} \).

We will map the behavior of \( S^{\text{x}s} \) as a functional of the more labile inelastic time, which we parametrize as \( \tau_{\text{in};i} = \zeta_i \tau_{\text{el};i} \).

Our results for a two-band model are in Fig. 2. We choose inelastic-to-elastic ratios \( \zeta_1 = 1 \) and \( \zeta_2 = 1, 0.8, \) and \( 0.6 \). Our total excess noise is \( S^{\text{x}s} = S_1^{\text{x}s} + S_2^{\text{x}s} \), total conductance \( G = G_1 + G_2 \). Subband edges are \( \varepsilon_1 = 5k_B T \) and \( \varepsilon_2 = 17k_B T \). We keep \( V \) constant at \( 9k_B T/e \) in each of three plots of \( S^{\text{x}s}(V) \) as a function of chemical potential.

In \( S^{\text{x}s} \), with increasing \( \mu \), we see dramatic peaks where \( G \) has characteristic steps. The peaks reveal an orderly metamorphosis of 1D hot-electron fluctuations, from their classical (\( T \)-independent) regime at the steps to their degeneracy-suppressed (\( T \)-scaled) state at the plateaux. This striking, chameleon-like transformation bears no relation to shot noise (1).

Such an effect can be readily probed in the laboratory, by appropriately biasing a gate voltage that couples to the electron density in the ballistic channel (3,15).

We also see how the thermal-noise maxima change significantly more with \( \zeta_i \) than \( G \) itself does; \( S^{\text{x}s}(V) \) is truly a fine marker for nonuniversal effects. Experimentally, this could well include the observation of higher-order phenomena (lead geometry, inter-subband transitions (16), etc.) not covered at our present level of treatment.

It is instructive to continue the study of \( S^{\text{x}s}(V) \) down to low EMFs, comparing it with the well known Landauer-Büttiker “noise crossover” formula, which combines all excess thermal noise into a single entity with shot noise (1):

\[
S^{\text{LB}}(V) = \sum_i 4G_i k_B T \left( 1 - \frac{G_i}{G_0} \right) \left[ \frac{eV}{2k_B T} \coth \left( \frac{eV}{2k_B T} \right) - 1 \right].
\]

The quantitative contrast between \( S^{\text{x}s} \) and the total LB noise is made clear in Fig. 3 for the identical two-band situation of Fig. 2, but now at \( V = 0.9k_B T/e \). Even in this low-field regime we find that \( S^{\text{x}s} \), computed strictly within an orthodox and fully gauge-invariant kinetic approach, still dominates the result of the widely held crossover model (3).

2 The corresponding LB transmission factors (1) are given by \( T_i = G_i/G_0 \). In the degenerate limit this means that \( T_i \to 2\zeta_i/(1 + \zeta_i) \) so that, for our chosen values \( \zeta_i \), one obtains the respective LB factors \( T_1 = 1 \) and \( T_2 = 1, 0.89, \) and \( 0.75 \).

3 The Landauer-Büttiker noise formula is not inherently gauge-invariant. Therefore its underlying dynamical fluctuations are not guaranteed to conserve charge. See Ref. (1), remarks following their Eq. (51).
VI. CONCLUSION: A NEW BALLISTIC-NOISE EXPERIMENT

We end by recalling the apparent impasse raised by the de Picciotto et al. data [3], vis à vis the LB theory’s categorical and uncompromising prediction of perfect, universal conductance steps [1,4]. Resolution depends crucially on the physics of the noninvasive probes. Measurements of the nonequilibrium noise would provide a unique opportunity to reveal how, precisely, that physics acts.

Suppose the noninvasive contacts do access the local potentials. Had they detected any voltage at all at probe separation $l$ inside the wire, it would have been the resistivity-dipole potential $E_l$. The probes saw nothing. This implies total neutralization of Landauer’s resistivity dipole within the wire body; $E = 0$. Its canceling counter-fields must be from dipoles sitting hard by the boundaries. That makes them highly localized.

The notion of strong additional localized counter-charges, over and above the natural Hartree displacement [7] that sets up the resistivity dipole in the first place, is energetically unlikely. Landauer’s self-consistent dipole is the fundamental element of mesoscopic transport [4,9]. Further substructures, at screening scales shorter than natural, would invite a logical reductio ad absurdum.

Now suppose instead that the probes couple capacitively to the 1D wire. At most, they will record variations of carrier density between their locations. But the wire is uniform, so there is no variation. Capacitive probes will report no difference. Thus there is no reason for the resistivity dipole to cancel out.

The conclusion of Ref. [3], that their noninvasive probe arrangement gives genuine and direct access to internal voltages, may now be checked by quite different means. The nonequilibrium thermal ballistic noise (totally separate from any considerations of the shot noise [1]) can provide independent experimental verification.

In brief: if the driving field $E$ throughout the ballistic structure is indeed zero, there will be little excess noise of the kind we predict. If, on the other hand, $E \neq 0$ then thermal noise must appear – and in copious amounts – at the subband crossings.

Here, the outcome is plain in terms of nonequilibrium noise. The experimental import of our work is much wider, however. It covers not just noise in a heterostructure-based device [3,15], but also fluctuations at the intense fields sustainable in metallic carbon nanotubes [17]. There, the quantum-confinement energies are huge by comparison with normal semiconductor-based 1D channels.

Carbon nanotubes provide a far more challenging testing-ground for transport and fluctuation theories in the extremely high-field domain. It is an area that is certainly wide open – and ripe – for exciting explorations.
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FIGURES

FIG. 1. An ideal, uniform ballistic wire. Its diffusive leads (S, D) are at equilibrium. A paired source and sink of current $I$ at the boundaries drive the transport. Local charge clouds (shaded), induced by the influx and efflux of $I$, set up the dipole potential $E(I)L$ between D and S.

FIG. 2. Left scale: excess thermal noise $S^{xs}$ of a ballistic wire at voltage $V = 9k_B T/e$, as a function of chemical potential. Right scale: two-probe conductance $G$. At the subband crossing points of $G$, the excess noise peaks. Noise is high at the crossing points, where subband electrons are classical, low at the plateaux where subband degeneracy is strong. $S^{xs}$, far more than $G$, is sensitive to the scattering-time ratios $\zeta_i = \tau_{ini}/\tau_{eli}$. Dashed line: ideal LB shot-noise prediction (see Eq. (12)) corresponding to our full line ($\zeta_1 = \zeta_2 = 1$). The predicted shot noise is much smaller.

FIG. 3. Full line: low-field excess thermal noise $S^{xs}$ of our two-band ballistic wire, at $V = 0.9k_B T/e$. The scattering-time ratios are ideal: $\zeta_1 = \zeta_2 = 1$. Dashed line: the corresponding shot-noise prediction $S^{LB}$ given by the Landauer-Büttiker crossover formula [1], Eq. (12). Our standard kinetic-theoretical prediction for the excess noise dominates the crossover even in the weak-field regime.
FIG. 1
FIG. 2

\[ \frac{S_{xs}}{4G_0 k_B T} \]

\[ \mu / k_B T \]

\[ \zeta_1 = 1 \quad \zeta_2 = 1 \]

\[ \frac{C}{C_0} \]

FIG. 3

\[ \frac{S_{xs}}{4G_0 k_B T} \]

\[ \mu / k_B T \]