Comparison of Curved Roofs of Various Configurations

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Abstract. Curved roofs of various configurations have been compared considering the snow load distribution pattern. The dependence of the roof slope, which is the determining factor affecting the snow accumulation, on the relative roof height has been determined. The pattern of snow accumulation on curved roofs with circumferential, square parabolic, sinusoidal, and elliptical shapes, most widespread in construction, have been considered. The snow deposition patterns on curved roofs of various shapes have been analyzed. Recommendations are given for choosing the most effective roof configurations, considering the snow load distribution.

1. Introduction
Analysis of incidents that have occurred in recent years shows the damage to various structures, associated with exceeding the permissible snow load [1-2]. Quite often, arched structures experience damage caused by unforeseen snow accumulations [3]. Very many factors affecting the pattern of snow deposition on the surfaces of various structures, the multivariance of events, significant variability of parameters characterizing the physical and mechanical properties of snow, and the specifics of construction site natural and climatic conditions determine the probabilistic nature of snow accumulation [4-6]. In this regard, when designing buildings and structures, all the available factors that may affect the snow accumulation on roofs should be considered to the maximum possible extent. One of the factors affecting the snow load on the structure is the roof configuration. To identify the snow load formation specifics, the patterns of snow distribution on curved surfaces have been studied [7-8]. For a more detailed study of the impact of various roof forms on snow retainability, consider the pattern of snow distribution on arched structures of various shapes.

2. Theoretical
In the regulatory document in force, governing the procedure for accumulating loads on curvilinear structures SP 20.13330.2011, Loads and Impacts, the snow load is determined depending on the snow shape coefficient $\mu$. The coefficient value depends on the slope of a horizontal line $\alpha$ and is determined by the formula $\mu = \cos 1.5\alpha$. In most cases, the snow load distribution on curvilinear roofs, determined by this formula (Fig. 1), is in good agreement with the real snow accumulation pattern for a significant part of Russia.
In this case, the snow accumulation pattern will be largely determined by the slope angle $\alpha$, which depends on the roof configuration. To clarify the regularities of the snow load distribution over the roofs, perform a comparative analysis of arched structures of various configurations for snow accumulation on the roof. As a criterion, we take the slope of a horizontal line $\alpha$. The slope angle is affected by both the roof configuration and the building height $f$ to span $l$ ratio. Consider the dependence of the slope angle $\alpha$ on the factors affecting the snow accumulation pattern, exemplified by the most common circumferential, square parabolic, sinusoidal, and elliptical roof shapes.

a) Circumferential shape:

The equation for the axis of a circumferential arch has the form

$$y = \sqrt{R^2 - \left(\frac{l}{2} - x\right)^2} - R + f$$

(1)

Since the first derivative, as is known, can be interpreted as the tangent of the slope angle, we will have

$$tg \alpha = \frac{dy}{dx} = \frac{l-2x}{\sqrt{4R^2-(l-2x)^2}}$$

(2)

From which, we get

$$l - 2x = \frac{2R \cdot tan \alpha}{\sqrt{1+tan^2\alpha}} = 2R \sin \alpha.$$  

(3)

Considering that the curvature radius is determined by the following equation

$$R = \frac{f}{2} + \frac{l^2}{8f}$$

(4)

and representing the coordinates in relative terms for the convenience of subsequent analysis, we obtain the following dependence for the slope angle on the geometric parameters of the arch axis

$$\alpha = \arcsin \frac{1-2x}{l}.$$

(5)

Since the circumferential arch rise cannot exceed the curvature radius in the absence of vertical walls, the maximum ratio of the arch rise to the symmetrical arch span will be less than $f/l=0.5$. 

Figure 1. Geometric Diagram of a Curved Roof.
The curve (Fig. 2) plotted according to the formula (5) obtained indicates the snow accumulation pattern depending on the roof slope, which is largely predetermined by the arch rise-to-span ratio. For shallow arches including those with a rise-to-span ratio \( f/l < 1/8 \ldots 1/10 \), the slope angle of the span along the entire length is less than 60°, and snow will accumulate over the entire surface. For steep arches at \( f/l > 1/4 \ldots 1/3 \), snow will egress from the sections adjacent to the supports, and the snow load will decrease.

b) Square parabolic shape:
For the square parabolic arch axis, the equation has the form

\[
y = \frac{4fx(l-x)}{l^2}.
\]  

The first derivative equal to the slope tangent will be

\[
tg \alpha = \frac{dy}{dx} = \frac{4f(l-2x)}{l^2}.
\]  

From which, we get in relative terms

\[
\alpha = arctg \left[ \frac{4f(l-2x)}{l(l-x)} \right].
\]  

The slope angle is also affected by the relative arch rise \( f/l \) (Fig. 3).
c) Sinusoidal shape:
For the sinusoidal arch axis, the equation has the form

\[ y = f \left( \sin \frac{\pi x}{l} \right) \]  \hspace{1cm} (9)

After differentiating this function, we obtain the following final equation for the roof slope

\[ \alpha = \arctan \left( \frac{\pi f}{l} \cos \frac{\pi x}{l} \right). \]  \hspace{1cm} (10)

For this roof configuration, snow will egress at a relative arch rise exceeding \( f/l = 0.5 \), when the roof slope is more than 60° (Fig. 4).

d) Elliptical shape:
For the elliptical arch axis, the equation has the form

\[ y = \frac{4x^2}{l^2} + \frac{y^2}{a^2} = 1. \]  \hspace{1cm} (11)

Figure 3. Dependence of the Slope Angle for a Square Parabolic Arch.

Figure 4. Dependence of the Slope Angle for a Sinusoidal Arch.
Having performed similar transformations, we obtain the following final equation for the roof slope angle in relative terms

\[
\alpha = \arctg \left( \frac{x f}{l} \frac{4}{\sqrt{1 - \frac{4x^2}{l^2}}} \right),
\] (12)

From elliptically curved roofs, snow can only egress from small areas near supports, even at a low building height (Fig. 5).

![Figure 5. Dependence of the Slope Angle for an Elliptical Arch.](image)

3. Analysis
Comparative analysis of the slope angles for the curved roofs of various configurations has shown that the snow load is significantly affected by the relative roof rise \( f/l \) (formulas 5, 8, 10, 12). The maximum snow load is generated in slightly sloping curved roofs with a small relative rise not exceeding \( f/l=0.5 \). For arched roofs of a steep configuration, snow egresses, as a rule, mainly from the support areas. The largest snow egress areas, as plots (Figs. 3-4) show, have square parabolic and sinusoidal roofs. However, the highest intensity of snow egress from near-support areas is observed for circumferential and elliptical roofs (Figs. 2, 5) characterized by the maximum slope in these zones. Herewith, the maximum snow accumulation occurs on elliptical and circumferential roofs characterizing by the longest slight-slope sections.

4. Conclusions
The snow load distribution over curved roofs is largely determined by the roof configuration and the relative rise of the structures.

The maximum snow accumulation occurs on elliptical and circumferential curved roofs.

For curved structures with slightly sloping roofs, sinusoidal arches are more adapted to the perception of snow load.

The minimum snow load is observed for steep curved roofs with a relative structure rise \( f/l>0.5 \).

5. References
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