Exotic see-saw mechanism for neutrinos and leptogenesis in a Pati-Salam model

Andrea Addazi
Dipartimento di Fisica, Università di L’Aquila, 67010 Coppito, AQ
LNGS, Laboratori Nazionali del Gran Sasso, 67010 Assergi AQ, Italy

Massimo Bianchi
Dipartimento di Fisica, Università di Roma Tor Vergata, I.N.F.N. Sezione di Roma Tor Vergata, Via della Ricerca Scientifica, 1 00133 Roma, Italy

Giulia Ricciardi
Dipartimento di Fisica E. Pancini, Università di Napoli Federico II, I.N.F.N. Sezione di Napoli, Complesso Universitario di Monte Sant’Angelo, Via Cintia, 80126 Napoli, Italy

Abstract

We discuss non-perturbative corrections to the neutrino sector, in the context of a D-brane Pati-Salam-like model, that can be obtained as a simple alternative to SO(10) GUT’s in theories with open and unoriented strings. In such D-brane models, exotic stringy instantons can correct the right-handed neutrino mass matrix in a calculable way, thus affecting mass hierarchies and modifying the see-saw mechanism to what we name exotic see-saw. For a wide range of parameters, a compact spectrum of right-handed neutrino masses can occur that gives rise to a predictive scenario for low energy observables. This model also provides a viable mechanism for Baryon Asymmetry in the Universe (BAU) through leptogenesis. Finally, a Majorana mass for the neutron is naturally predicted in the model, leading to potentially testable neutron-antineutron oscillations. Combined measurements in neutrino and neutron-antineutron sectors could provide precious informations on physics at the quantum gravity scale.

1 Introduction

In [1], Majorana proposed the existence of extra mass terms of the form $m \psi \psi^\dagger$, in which $\psi$ is a neutral fermion, such as a neutrino or a neutron. Majorana’s proposal has never seemed to be so up-to-date and intriguing as today. In fact, from several measures of atmospheric, solar, accelerator and reactor neutrinos, neutrino oscillations...
have been fully confirmed. These observations represent evidence that neutrinos are
massive. Majorana’s proposal goes even beyond the mass issues: a Majorana mass
term for neutrinos or for the neutron leads to violation of Lepton (L) and Baryon
(B) numbers as $\Delta L = 2$ and $\Delta B = 2$, respectively. The Standard Model (SM) does
not offer an adequate explanation of the observed Matter-Antimatter asymmetry in
our Universe, i. e. the SM does not generate the necessary Lepton and/or Baryon
number asymmetries in the primordial Universe. The possibility of a Majorana mass
term for neutrino or neutron can disclose new paths towards the origin of the observed
asymmetry and its possible dynamical generation, through a viable mechanism for
baryogenesis.

See-saw Type I mechanism is considered one of the most elegant ways to explain
the observed smallness of neutrino masses [2, 4, 5, 6]. In see-saw Type I, right-
handed (RH) neutrinos with masses much higher than the electroweak (EW) scale are
required. Remarkably, this mechanism offers a simple and natural solution for lepto-
genesis, a model of baryogenesis where the lightest RH neutrino can decay into lighter
particles [7]. In the primordial universe, near the EW phase transition, leptons, quarks
and Higgs also interact via $B + L$ violating non-perturbative interactions, generated
by sphalerons, leading to an effective conversion of part of the initial lepton number
asymmetry into a baryonic one [15]. Moreover, the complex Yukawa couplings of the
RH neutrinos can provide new sources of CP violation. All Sakharov’s conditions to
dynamically generate baryon asymmetry [8] are satisfied: 1) out of thermal equilibrium
condition; 2) CP violations; 3) baryon number violation. The sphaleron-mediated ef-
fective interactions were calculated for the first time by t’Hooft [14]. These effects are
strongly suppressed in our present cosmological epoch but, in the primordial thermal
bath, they are expected to be unsuppressed, leading to non-negligible corrections to
the chemical potentials.

The see-saw mechanism can be naturally embedded in a Pati-Salam (PS) model
$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ or $SU(4)_c \times Sp(2)_L \times Sp(2)_R$ [16]. As suggested
in [6] Majorana masses for neutrinos can be elegantly connected to a spontaneous
symmetry breaking of parity and to leptogenesis. In fact the RH masses are related
to Left-Right scale and $U(1)_{B-L} \subset SU(4)_c$ spontaneous symmetry breaking scale. On
the other hand, a RH neutrino mass scale of order $M_R \sim 10^{9±13}$ GeV is necessary for
consistent leptogenesis [27].
As a natural step beyond a PS-model, $SO(10)$ GUT could unify the SM with $U(1)_{B-L}$ via an intermediate $SU(4)_c \times SU(2)_L \times SU(2)_R$ PS-like gauge group. However, let us recall that the $SO(10)$ GUT scenario presents some challenging theoretical problems, that are generally solved at the cost of some complications of the initial GUT model. Problems such as proton destabilization and the imperfect unification of coupling constants are generally alleviated in SUSY $SO(10)$ GUT. With or without SUSY, the most serious hierarchy problem for $SO(10)$ and other GUTs is the doublet-triplet splitting. The standard Higgs doublet is contained in $10_H$ (or $5_H + 5^*_{H}$ in $SU(5)$), leading to dangerous scale-mixing diagrams between standard doublets and heavier Higgs triplets inside $10_H$. In other words, a stabilization of the ordinary doublet at much smaller scales than $M_{GUT} \simeq 10^{15-16}$ GeV is highly unnatural, i.e. it reintroduces another Higgs hierarchy problem even if one assumes 1 TeV SUSY breaking scale.

In $SO(10)$, the quark-lepton symmetry makes the reconciliation of leptogenesis and see-saw mechanism more problematic. In fact, assuming the spontaneous symmetry breaking scale of $SU(4)_c \times SU(2)_L \times SU(2)_R \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$ around $\Lambda_R \simeq 10^{11}$ GeV, the lightest RH eigenstate $N_1$, which is generally the main responsible for generating a lepton asymmetry, acquires a mass $M_{R_1} \ll 10^9$ GeV. Unfortunately, this value is well below the Davidson-Ibarra (DI) bound $M_{DI} \gtrsim 10^9$ GeV, guaranteeing a sufficient production of lepton asymmetry from RH neutrino decays. There are basically three ways out of this difficulty. One possibility is to consider leptogenesis where crucial contributions arise via the decays of heavier RH neutrinos, with masses above the DI limit. Alternatively, one can assume a highly compact spectrum. Finally, in a situation in which one pair of RH neutrinos is highly degenerate, the DI bound can be avoided through a resonant enhancement of CP asymmetries. Let us observe that the latter two scenarios are not easily incorporated in $SO(10)$.

Lastly, it is undoubtable that $SO(10)$ cannot provide a way to unify gravity with the other interactions. Indeed, $SO(10)$ scenarios are not the only possible completion of PS-like models. In IIA and IIB superstring theory, a natural way to construct a PS-
like model can be achieved through a system of intersecting D-branes stacks wrapping some sub-manifold (‘cycles’) in a Calabi-Yau (CY) compactifications with open strings ending on them. In this class of models, a different kind of \textit{geometric unification} can be achieved, including gravity – even if string theory were incomplete, even if quantum gravity were only understood partially. Recently, a simple D-branes PS-like model was suggested in [47]. In [47], we have noticed that a Higgs sector composed of $\Delta(10^*,1,1)$, $\Delta^c(10^*,1,1)$, $\phi_{LL}(1,3,1)$, $\phi_{RR}(1,1,3)$ and $h_{LR}(1,2,2)$, the latter containing SM Higgses, can reproduce the right pattern of fermion masses. However, the above Higgses cannot break $SU(4) \times SU(2)_R$ down to $SU(3) \times U(1)_Y$ in the desired way. This spontaneous symmetry breaking can be obtained through Higgs superfields $\bar{H}(4,1,2)$ and $H(4,1,2)$. In $SO(10)$, they are usually contained in $16_H, \bar{16}_H$. $\bar{H}$ has the same representation $F_R$ of the standard fermions and their super-partners, while $H$ is in the conjugate one. They can be decomposed in components as

$$\bar{H}(4,1,2) = (u^c_R, d^c_R, e^c_R, \nu^c_R) \quad (1)$$

$$H(4,1,2) = (\bar{u}^c_R, \bar{d}^c_R, \bar{e}^c_R, \bar{\nu}^c_R) \quad (2)$$

The vacuum expectation values (VEVs) along the “sneutrino” components

$$\langle \bar{H} \rangle = \langle \nu^c_R \rangle, \quad \langle H \rangle = \langle \bar{\nu}^c_R \rangle \quad (3)$$

break $SU(4) \times SU(2)_R$ to $SU(3) \times U(1)_Y$. VEVs (3) have to be higher than $\langle \Delta^c \rangle, \langle \phi_{RR} \rangle$ in order to guarantee the right symmetry breaking pattern. In this model a Majorana mass for the neutron and extra terms in the RH neutrino mass matrix are generated by Euclidean D2-branes (or E2-branes), wrapping a different 3-cycle with respect to the ordinary D6-branes. Such E2’s are called exotic instantons. They are a different kind of instantons not present in gauge theories. The effect of E2s are calculable and controllable in models like our one. Unlike ‘gauge’ instantons, ‘exotic’ instantons do not admit an ADHM construction. See [49, 50] for useful reviews of these

\footnote{As in GUTs, also in these models we can find some difficult theoretical problems: i) the identification of the precise CY singularity for the D-brane construction, ii) the quantitative stabilization of geometric moduli for the particular realistic particle physics model considered. These problems are expected to be solved by including fluxes and the effects of stringy instantons. For the moment, awaiting for a more precise quantitative UV completion (global embedding) of our model, we can neglect these problematics. Our attitude is to consider effective string-inspired models, locally free from anomalies and tadpoles and interesting for phenomenology of particle physics and cosmology. On the other hand, attempts to solve the problems mentioned above are the main topics of an intense investigation. For example, see [44, 45, 46] for recent discussions.}

\footnote{For this reason, a TeV-ish Left-Right symmetry breaking is not favored by our precise model. Comments on phenomenological aspects made in [37] can be valid in quivers inspired by the present one but with extra nodes.}
The main new peculiar feature of exotic instantons is that they can violate vector-like symmetries like baryon and lepton numbers! B/L-violations by exotic E2-instantons are not necessarily suppressed: suppression factors depend on the particular size of the 3-cycles wrapped in the CY compactification by exotic E2-instantons. A dynamical violation of a symmetry is something “smarter” than an explicit one: all possible dangerous operators are not generated by exotic instantons, only few interesting operators can be generated. For instance, an effective operator \((u^c d^c)^2/\Lambda_{n\bar{n}}^5\) is generated in our model, without proton destabilization: a residual discrete symmetry is preserved by exotic instantons, avoiding \(\Delta B = 1\) processes but allowing \(n-\bar{n}\) \((\Delta B = 2)\) transitions \[43\]. In particular, such transitions are mediated by three color scalar sextets present in our model. E2-instantons generate an effective superpotential term \(W_{E2} = \Delta(6)_{u^c u^c} \Delta(6)_{d^c d^c} \langle S(1) \rangle / M_E\), where \(\Delta(6) = (6,1)_{+2/3}\) and \(S = (1,1)_{-2}\) are contained in \((10,1,1)\) of \(SU(4)_c \times Sp(2)_L \times Sp(2)_R\). When \(S\) takes an expectation value, spontaneously breaking \(U(1)_{B-L}\), an effective trilinear interaction for \(\Delta(6)_{s}\) is generated at low energies of order \(M_E \sim M_S\), where \(M_S\) is the string scale. \(n-\bar{n}\) transition can be obtained from \(W_{E2}\) and renormalizable operators, present in our model and coded in a quiver, \(\Delta(6)_{u^c u^c} u^c u^c\) and \(\Delta(6)_{d^c d^c} d^c d^c\), with \(\Lambda_{n\bar{n}}^5 \approx M_E M_{\Delta(6)_{u^c u^c}}^2 M_{\Delta(6)_{d^c d^c}}^2 M_{\text{SUSY}}/v_{B-L}\) where \(M_{\text{SUSY}}\) is the SUSY breaking scale, \(v_{B-L}\) the \(U(1)_{B-L}\) breaking VEV. Its scale can be as low as \(\Lambda \approx 1000\) TeV, corresponding to \(n-\bar{n}\) transitions in vacuum (no magnetic-fields, outside nuclei) with \(\tau_{n\bar{n}} \approx 100\) yr, \(\approx 10^{33}\) yr. The next generation of experiments promises to test exactly this scale, enhancing the current best limits for \(\tau_{n\bar{n}}\) \[52\] by two orders of magnitude \[54, 55\]. In string theory, \(M_S\) needs not be necessarily close to the Planck scale, it can easily stay at a lower scale. Similarly the SUSY breaking scale is not necessarily at the TeV scale - since we are only interested in SUSY as a symmetry for superstring theory, we will consider it to be around the String scale \[49\]. Direct limits on color sextet scalars can be obtained from FCNCs as discussed in \[59, 60\], usually stronger than LHC ones \[61, 62\]. In the present paper, we discuss quantitative predictions of our PS-like model for low energy observables in neutrino physics. An alternative mechanism for Baryon Asymmetry of the Universe (BAU) can be envisaged. As proposed in \[43, 56, 57, 58\], a Post-Sphaleron Baryogenesis mediated by color scalar sextets could be a viable alternative to a Leptogenesis-Sphaleron mechanism. An intriguing possibility is to test this scenario in Neutron-Antineutron physics. Color scalar sextets are naturally embedded not only in \(SO(10)\), but also in our model with intersecting D-branes, as extensively discussed in \[47\].

\[\text{References:}\]

9See \[51\] for a recent paper on D-brane instantons in chiral quiver theories.

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11For other D-branes model generating a Majorana mass for the neutron and other intriguing signatures for phenomenology, in Ultra Cold Neutron Physics, Ultra High Energy Cosmic Rays, FCNCs and LHC, see \[63, 64, 65, 66, 67, 68, 69\].
physics, as done in the literature for \(SO(10)\) GUT’s. We show that our model can be remarkably predictive for neutrino physics, exposing a quark-lepton symmetry and a compact spectrum of RH neutrinos with masses above the DI bound for leptogenesis. The compactness of the mass spectrum of RH neutrinos is related to the geometrical proprieties of the relevant mixed disk amplitudes. Our model provides a theoretical framework where a compact RH spectrum emerges naturally. In our phenomenological analysis, we will take into account a non vanishing value of the lepton mixing angle \(\theta_{13}\), as measured in \([70, 71, 72]\), assuming the best fit value given in \([72]\). We will see how the compactness of the RH neutrino mass spectrum leads to consistent solution with a non-zero Dirac phase \(\delta \neq 0\), in the Pontecorvo-Maki-Nagakawa-Sakata (PMNS) mixing matrix. The solutions obtained then fix the other unknown low energy parameters: the PMNS CP violating phases \(\delta, \alpha, \beta\) (modulo signs) and the left-handed (LH) neutrino mass scale \(M_1\). We also predict the RH neutrino masses. The numerical approach follows the path drawn in the context of \(SO(10)\) GUT, where a compact RH spectrum represented a somewhat arbitrary assumption \([35, 36]\). The plan of the paper is as follows. In Sect. 2 we review and amend a Pati-Salam-like model with gauge \(U(4) \times Sp(2)_L \times Sp(2)_R\) based on unoriented D-branes proposed in \([47]\). In Sect. 4 we calculate relevant parameters for leptogenesis in a case where the right order of magnitude and sign of the BAU is recovered, a non trivial result in view of the high level of predictability of the present model.

2 Pati-Salam-like D-brane models

The effective theory, in the low energy limit, is described by a Pati-Salam gauge group \(U(4) \times Sp(2)_L \times Sp(2)_R\). \(U(4)\) is generated by a stacks of 4 D6-branes and their images \(U'(4)\) under \(\Omega\). \(Sp(2)_{L,R}\) are supported on two stacks of two D-branes each lying on top of the \(\Omega\)-plane. We also consider three Euclidean \(D2\)-branes (or \(E2\)-branes) on top of the \(\Omega\)-plane, corresponding to three Exotic \(O(1)\) Instantons. Let us call

\footnote{Let us recall that \(\Omega\)-planes are introduced for quantum consistency and tadpole cancellations. See references \([73, 74, 75, 76, 77, 78, 79, 80, 81, 77, 78, 79]\) for a complete discussion of these aspects.}

\footnote{Let us note that, generically, in D-brane models, one cannot construct directly \(SU(N)\) gauge groups. For this reason we cannot obtain directly a PS model, but an extended one, with \(U(4)\) rather than \(SU(4)\) and \(Sp(2)_{L,R}\) rather than \(SU(2)_{L,R}\). In fact, \(N\) parallel branes stacked together (with open strings ending on them) will produce, at lower energy limit, \(U(N), SO(N), Sp(2N)\) gauge theories. In particular, \(U(N)\) is obtained if the D-brane stack does not lie on the \(\Omega\)-plane. On the other hand, if the D-brane stack lies on the \(\Omega\)-plane, one obtains \(SO(N)\) or \(Sp(2N)\) (for \(\Omega^+\) respectively). \(\Omega\)-planes seem necessary in order to produce realistic gauge groups, in which chiral matter can be embedded \([83, 84]\).}
Figure 1: On the left, the unoriented quiver for a Pati-Salam-like model $U(4) \times Sp(2)_L \times Sp(2)_R$ is shown. Circles, labeled by $4, 2_L, 2_R$, correspond to the $U(4), Sp(2)_L, Sp(2)_R$ gauge groups, respectively. The $U(4)$ stack is identified with its mirror image through an $\Omega^+$-plane. $Sp(2)_{L,R}$ correspond to stacks of two D6-branes lying on the $\Omega^+$-plane. The triangles are $E_2$-branes lying on the $\Omega^+$ plane, corresponding to $O(1)$ instantons. $E_2^\prime, E_2^\prime\prime$-instantons generate a quartic superpotential for $\Delta(10, 1, 1)$ and $\Delta^c(10, 1, 1)$, leading to an effective Majorana mass for the neutron. On the right, the effective unoriented quiver theory after Higgsing via $H, \bar{H}$ is shown. From the quiver on the left to the one on the right, extra undesired modulini appear, that are assumed to be lifted by a combination of higgsing and fluxes. The $E_2$-instanton generates a PMNS mass matrix for neutrinos. The PS-like quiver generates the (MS)SM-like quiver on the right side after splitting the $Sp(2)_R$ D-branes from the $\Omega^+$-plane.

these $E_2, E_2^\prime, E_2^\prime\prime$. Quarks and leptons in Left and Right fundamental representations $F_{L,R} \equiv (4, 2_L, 4^*, 2_R)$, are reproduced as open strings stretching from the $U(4)$-stack to the Left or Right $Sp(2)_{L,R}$-stacks (respectively). Analogously, but at variant w.r.t. the original model [47], Higgs $\tilde{H} = (4^*, 2_R)$ and its conjugate $H = (4, 2_R)$ are introduced as extra intersections of the $U(4)$-stack with $Sp(2)_R$. Extra color states $\Delta = (10, 1, 1)$, and their conjugates, are obtained as open strings stretching from the $U(4)$-stack to its $\Omega$ image $U(4)'$-stack. $\phi_{LL} = (1, 3, 1)$ and $\phi_{RR} = (3, 1, 1)$ correspond to strings with both end-points attached to the $Sp(2)_{L,R}$ (respectively). Higgs fields $h_{LR} = (2, 2, 1)$ are massless strings stretching from $Sp(2)_L$ to $Sp(2)_R$. The quiver on the left of Fig. 1 automatically encodes the following super-potential terms [47]:

$$W_{Yuk} = Y^{(0)} h_{LR} F_L F_R + \frac{Y^{(1)}}{M_{F_1}} F_L \phi_{LL} F_L \Delta + \frac{Y^{(2)}}{M_{F_2}} F_R \phi_{RR} F_R \Delta^c$$  (4)
by two $E$-side of Fig.1). In fact, fermionic modulini

\[ \text{string scale, as well as at lower scales} \]

\( i.e. \) they depend on the particular completion of our model, \( i.e. \) the standard model) is

\( \text{final electroweak symmetry breaking.} \)

Decuplets decompose as \( \Delta^c \) (\( \text{Stu stands for St" uckelberg, see below} \)) and \( \Delta^c \)

\[ \text{CY}_3 \text{moduli, associated to 3-cycles of the} \]

\( 15 \) or in [85, 86, 87, 89] in different contexts

fermionic modulini, we exactly recover the interactions \( (8) \) and \( (6) \), as shown in [47]

\[ \text{mass terms} \]

\( \text{in} \) \( \text{Majorana masses for neutrinos are completely generated by exotic instantons.} \)

\[ Y^{(-)} \text{are} 3 \times 3 \text{Yukawa matrices; the mass scales} \]

\( M_\text{F} \text{are considered as free parameters:} \)

they depend on the particular completion of our model, \( i.e. \) they could be near \( M_S \), the string scale, as well as at lower scales.\(^{14}\) The super-potential terms \( (6) \) can be generated by two \( E2 \)-brane instantons shown in Fig.1: \( O(1)', O(1)'' \) intersect twice the \( U(4) \) stack and \( O(1) \) intersects twice the \( U(4) \)-stack and once the \( Sp(2)R \)-stack (2\(_R \) on the left side of Fig.1). In fact, fermionic modulini \( \tau_i, \tau'_i, \omega'_{\alpha} \) appear as massless excitations of open strings ending on \( U(4)-O(1) \), \( U(4)-O(1)' \), \( Sp(2)R-O(1)' \) respectively; \( i = 1, 4 \) and \( \alpha = 1, 2 \) are indices of \( U(4) \) and \( Sp(2)R \) respectively. Integrating over the fermionic modulini, we exactly recover the interactions \( (8) \) and \( (6) \), as shown in [47]

or in [85] [86] [87] [89] in different contexts.\(^{15}\) The dynamical scales generated in \( (6) \) are \( \mathcal{M}_0'^{'} = Y^{(1)} \text{M}_{S e^c + S e'^c} \) and \( \mathcal{M}_0'' = Y^{(1)} \text{M}_{S e^c + S e'^c} \), where \( S_{E2', E2''} \) depend on geometric moduli, associated to 3-cycles of the \( CY_3 \), around which \( E2', E2'' \) are wrapped.

The spontaneous breaking pattern down to the (MS)SM (minimal supersymmetric standard model) is

\[ U(4) \times Sp(2)_L \times Sp(2)_R \longrightarrow SU(4) \times Sp(2)_L \times Sp(2)_R \]

\[ \longrightarrow \quad SU(3) \times Sp(2)_L \times U(1)_Y \]

\( \text{(Stu stands for St" uckelberg, see below) and} \) \( h_{LR} \) contain the standard Higgses for the final electroweak symmetry breaking. Decuplets decompose as \( \Delta^c = \Delta^c_6 + T^c + S^c \), with \( \Delta_6 = 6Y_{+2/3} \), \( T = 3Y_{-2/3} \), \( S = 1Y_{-2} \), and the singlet \( S \) takes a VEV.

\(^{14}\) The mass terms \( m_\Delta \) and \( m_{LR} \) can be generated by R-R or NS-NS 3-forms fluxes in the bulk, in a T-dual Type IIB description, \( i.e. m_\Delta \sim \Gamma^{ijk}(\tau H_{ijk} + iF_{ijk}) \), \( m_{LR} \sim \Gamma^{ijk}(\tau H^{LR}_{ijk} + iF^{LR}_{ijk}) \), with \( H_3 \) RR-RR and \( F_3 \) NS-NS 3-forms. In general, \( H_3, F_3 \) are not flavour diagonal since fluxes through different cycles, wrapped by different D-branes, could be different. For recent discussions of mass deformed quivers and dimers see [90].

\(^{15}\) In [85] [86] [87] [89] Majorana masses for neutrinos are completely generated by exotic instantons.
Let us note that the extra $U(1)_4 \subset U(4)_c$ is anomalous in gauge theory. In string theory a generalization of the Green-Schwarz mechanism can cure these anomalies. Generalized Chern-Simons (GCS) terms are generally required in this mechanism. The new vector boson $Z'$ associated to $U(1)_4$ gets a mass via a Stückelberg mechanism.

The final effective (MS)SM embedding quiver that we will consider is obtained from the previous SUSY PS-like quiver through a splitting of nodes $4 \rightarrow 3+1$ and $2_{R} \rightarrow 1+1'$. In this new quiver, $E2$ intersects $U(1)$ and $\hat{U}(1)'$ as shown on the right of Fig. 1, where $\hat{U}'(1)$ is the $\Omega$-image of $U'(1)$. In the Higgsing from SUSY PS-like quiver to SUSY SM-like, extra undesired modulini are obtained. In particular, colored modulini at $E2-U(3)$ intersections. We assume that these modulini are lifted out by Higgsings and fluxes. This technical aspect deserves future investigation beyond the purposes of this paper. As a consequence, an extra mass matrix term is non-perturbatively generated

$$W_{E2} = \frac{1}{2} M_{ab} N^a_R N^b_R$$

where $N^a_R$ are RH neutrinos ($a = 1, 2, 3$ label neutrino species), contained, as singlet, inside $F_R$. The generated mass matrix is $M_{ab} = Y_{ab}^{(0)'} M_S e^{-S_{E2}}$, where $Y_{ab}^{(0)'}$ is the Yukawa matrix parameterizing masses and mixings among RH neutrinos, depending of course on the particular $E2$ intersections with ordinary D6-branes stacks. Let us note that the superpotential (8) can be generated only after spontaneous symmetry breaking of $U(4)_c$ down to $U(3)_c$, and $Sp(2)_R$ down to $U'(1)$. This will impose bounds on the parameters that we will discuss in Section 2.

Now, let us discuss electroweak symmetry breaking in our present model: as mentioned before, this is due to the VEVs $\langle h_{LR} \rangle$ of the complex Higgs bi-doublets $h_{LR}$ yielding the tree-level mass relations for leptons and quarks

$$m_d = m_e \quad \text{and} \quad m_u = m_D$$

where $m_D$ are Dirac masses of neutrinos. From (9), tight hierarchy constraints on RH neutrino masses are predicted: as a result the neutrino’s hierarchy is related to the up-
quarks. It is interesting to observe that the hierarchy obtained at the perturbative level (with closed-string fluxes generating the $M_2$ scale) is corrected by exotic instantons, parametrized by $M_{ab}$. Left-Right symmetry breaking pattern implies

$$m_D = m_u \quad \text{and} \quad V_L = V_{CKM}$$

(10)

with $V_{CKM}$ the Cabibbo-Kobayashi-Maskawa matrix. We obtain the mass matrix

$$M = \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix}$$

(11)

In our case, RH neutrino masses are

$$M_R = M^P_R + M^{E2'}_R$$

where

$$M^P_R = \langle \phi_{RR} \rangle \langle S^c \rangle / M_2$$

and

$$M^{E2}_R = M'_{ab}$$

as shown in [47].

From the usual see-saw formula one obtains the light neutrino mass matrix $m_\nu$

$$m_\nu \simeq -m_D \left( M^P_R + M^{E2}_R \right)^{-1} m_D$$

(12)

A natural situation for our quiver is that $E2'$ induce non-perturbative mass terms for RH neutrinos of the same order, i.e. $M^{E2}_{R\{1,2,3\}} \simeq M^{E2}_{R\{2,3\}} \simeq M^{E2}_{R\{3\}}$ where 1,2,3 are generation indices. As a consequence, $M^{E2}_{R\{1,2,3\}} \simeq 10^{9+13}$ GeV and we obtain a highly degenerate RH mass spectrum in a good range for leptogenesis, non-perturbative mass corrections are higher than or at least of the same order as the perturbative ones. Naturally, such a situation does not imply a highly degenerate LH mass spectrum, since a large quark-lepton hierarchy remains encoded in $m_D$. The see-saw formula can be inverted as

$$M_R = M^P_R + M^{E2}_R \simeq -m_D m_\nu^{-1} m_D$$

(13)

since in our model $m_D = m_D^T$. From (13) one can get information on the RH neutrino mass matrix $M_R$ by using data on LH neutrino mass matrix $m_\nu$, and assuming a quark-lepton symmetry. In general, a quark-lepton symmetry complicates BAU mechanisms because it imposes a strong hierarchy in the neutrino sector: under the assumption that
$v_1 v_2 / M_2 \simeq 10^{11-13} \text{GeV}$ with $v_1 = \langle \phi_{RR} \rangle$ and $v_2 = \langle \Delta \rangle$, the lightest RH eigenstate $N_1$ takes a mass much smaller than the Davidson-Ibarra bound $M_{N_1} \ll 10^9 \text{GeV}$, i.e. $N_1$ decays cannot guarantee a sufficient production of lepton asymmetry. Fortunately, non perturbative $E2$ contributions can generate a compact RH neutrino spectrum above the DI bound, i.e. the mass eigenvalues of RH neutrino mass matrix are highly degenerate and higher than $10^9 \text{GeV}$. We would like to stress that, unlike $SO(10)$ GUTs, our model provides a natural mechanism to obtain a compact RH neutrino hierarchy.

Let us also observe that, after the splitting in Fig. 1, we obtain an effective cubic interaction term $(\langle S \rangle / M_0) \epsilon_{ijk} S^{SU(3)} \epsilon_{i'j'k'} S^{SU(3)} \Delta_6 \Delta_6 \Delta_6$ which violates Baryon number as $\Delta B = 2$ and generates a Majorana mass for neutrons $[47]$, as mentioned in the introduction. On the other hand, exotic instantons can preserve discrete sub-symmetries $Z_2^{(B,L=1)}$, avoiding proton destabilization, but allowing $\Delta L, \Delta B = \pm 2$ processes. However, $\Delta B = 2$ violating operators can also destabilize the proton if one consider all $\Delta L = 1$ mixing terms among $F_{LR}$ and $H, \bar{H}$ in $[4]$. Higher order operators of this kind are generated by fluxes, so that one can naturally assume that they are suppressed by a mass scale larger than $M_S$.

So, potentially dangerous terms are

$$W_{\gamma(5), \Delta L=1} = Y^{(5)} h_{LR} F_{L} \langle \bar{H} \rangle \quad \text{and} \quad W_{\mu, \Delta L=1} = \mu'' F_R H$$

These terms are easily understood: $\bar{H}$ is like a fourth generation of $F_R$. So that, calling $F_R^{f=1,4} = (F_{R}^{f=1,3}, \bar{H})$, they generically mix through

$$W_{\mu_f} = \mu_f H \bar{H} + \mu'' H F_R$$

Such mass terms can be diagonalized so that the mixing term $HF_R$ can be rotated away in the mass eigenstate basis. Similarly, $W_{\gamma(5)}$ can be incorporated in the standard Yukawa term as

$$W_{\gamma_f} = \gamma_{f'=1,3,f=1,4} h_{LR} F_{L}^{f'} F_{R}^{f} = h_{LR} \left[ Y^{(0)}_{f'=1,3,f=1,3} F_{L}^{f'} F_{R}^{f} + Y^{(5)}_{f'=1,3} F_{L}^{f'} \bar{H} \right]$$

In order to avoid proton destabilization, we can impose the following condition on matrices $\mu_f$ and $\gamma_f$

$$\mu_f \gamma_f = 0 \quad (14)$$

Relation $[14]$ automatically guarantees matrices of the form

$$\mu_f = (\mu', 0, 0, 0)^T$$
\[ Y_f = (0, Y_f^{(0)}, Y_f^{(0)}, Y_f^{(0)}) \]

in the basis \( F_R^f = (F_R^{f=1,2,3}, \bar{H}) \).

A natural geometric explanation of Eq. (14) could come from global intersecting D-brane models, consistently completing our local one in the Calabi-Yau singularity. The quiver in Fig. 1 apparently seems to democratically consider different flavors, like \( F_R^f \). However, the presence of internal bulk R-R or NS-NS fluxes can discriminate different intersections of two stacks of D6-branes i.e. different flavors from one another. Alternatively, one can consider that the local quiver theory in Fig. 1 could come from a (or more) Gepner model(s). In Gepner models, the Calabi-Yau space has a more complicated geometry than for example a toroidal orbifold, inducing accidental discrete symmetries in the low energy limits. For example, the intersections of two stacks on a torus are geometrically equivalent, while in a complicated topological deformation of a torus “flavor democracy” is broken. This affects the vertex operators of an open string massless fermion \( V_F = V_S \Sigma \int f \), where \( V_S = u^{\alpha}(k) S_\alpha e^{-\phi/2} e^{ikX} \) accounts for the space-time part, while \( \Sigma \int f \) is an internal spin field depending on the flavor. Similarly for massless scalars \( V_B = \Psi_{\int f}^\alpha e^{-\phi} e^{ikX} \), with \( \Psi_{\int f}^\alpha \) being a chiral primary operator. A Yukawa coupling, like \( h_{LR} F_L F_R \), will give rise to a flavor matrix \( Y_{f_1 f_2 f_3} \) proportional to \( \langle \Psi_{\int f_1}^\alpha \Sigma_{\int f_2} \Sigma_{\int f_3} \rangle \). As a consequence, the suppression of \( \mathcal{W}_{Y_{1},\Delta L=1} \) can be geometrically understood as emerging from different inequivalent intersections among the same stacks of branes.

### 2.1 Free parameters

In this section we will comment on the relevant parameters in our model and clarify our assumptions.

#### 2.1.1 Supersymmetry and string scale

First, let us clarify the role of supersymmetry in our considerations. Clearly, if the SUSY breaking scale is assumed to be \( M_{SUSY} \approx 1 \text{ TeV} \), this will introduce several extra parameters relevant for leptogenesis. A TeV-scale SUSY will complicate one-loop (n-loops) contributions, introducing extra CP-violating phases in RH-neutrino decays. Here, we will assume that supersymmetry has nothing to do with the hierarchy problem of the Higgs mass, i.e. SUSY has the role to stabilize instanton calculations and to...
eliminate tachyonic states from the present string model. While the second aspect is crucial for the consistency of our model, saving us from "fighting" with instabilities, and imposing a bound on the SUSY-scale as \( M_{\text{SUSY}} \simeq M_S \), the first aspect is "less fundamental", since it only has the role of simplifying istanton calculations. This requires \( M_{\text{SUSY}} \simeq M_S e^{-S_{E2}} \gtrsim 10^9 \text{GeV} \). As a result, supersymmetric particles do not give any relevant contributions to RH neutrino decays\(^1\).

2.1.2 Relevant effective Lagrangian and free parameters

After the spontaneous breaking of SUSY, \( U(4) \) symmetry and Left-Right symmetry, the effective Lagrangian in the neutrino sector reads

\[
\mathcal{L}_{\text{eff}}^\nu = Y^{(0)} h_u l \nu_R + \frac{Y^{(2)}}{M_2} \nu_R \langle \varphi_{RR} \rangle \nu_R \langle \delta^c \rangle + Y^{(0)'} M_S e^{-S_{E2}} \nu_R \nu_R
\]

where \( h_u \) is the scalar component of the superfield \( H_u \) contained in the bi-doublet superfield \( h_{LR} \), \( \nu_R \) are the RH neutrinos, the fermionic component of the the RH neutrino supermultiplets, \( \varphi_{RR}, \delta^c \) are the scalar components of the supermultiplets \( \phi_{RR}, \Delta^c \).

Therefore, the number of relevant free parameters in the neutrino sector is

\[
N_{\text{f.p.}} = n_{Y0} + n_{Y2} + n_{Y0'} + n_{V_{EV1}} + n_{V_{EV2}} + n_{\text{Flux}} + n_{E2} = 22
\]

(\( f.p. \) stands for free-parameters) where \( n_{Y0,2} = 6 \) are the number of free parameters in the Yukawa matrices \( Y^{(0)}, Y^{(2)}, Y^{(0)'} \) respectively; \( n_{V_{EV1,2}} \) account for the number of ratios between extra VEVs \( v_{1,2} \) with respect to \( v_{EW} \), i.e. \( z_1 = v_1/v_{EW} \) and \( z_2 = v_2/v_{EW} \); \( n_{\text{Flux}} = 1 \) is the number of non-perturbative scales generated by fluxes entering in the neutrino sector, i.e \( M_{F2} \) (or \( z_3 = M_{F2}/v_{EW} \)); \( n_{E2} \) parameterizes the size of the 3-cycle wrapped by \( E2 \)-brane.

Under reasonable assumptions, the number of free parameters can be significantly reduced. In the following analysis, we will suppose a dominance of non-perturbative effects: \( M_{E2}^R \gg M_P^R \) (all matrix parameters). In this case, \( n_{V_{EV1,2},\text{Flux},Y2} \) are irrelevant, as they are related to tiny extra corrections. In this case, the mass matrix of RH neutrinos is practically completely generated by the \( E2 \)-instanton! AB: The hierarchy \( M_{E2}^R \gg M_P^R \) can be understood as follows. The \( E2 \)-instanton generates a mass matrix for neutrinos with an absolute value \( M_S e^{-\Pi_3/g_s} \), where \( \Pi_3 \) is the volume\(^1\).

\(^{18}\)One could speculate that dark matter is a hidden parallel system of intersecting D-branes. Implications in direct detection of such a scenario was studied in [144].
of 3-cycles wrapped by the $E2$-instanton on $CY_3$. Volumes of 3-cycles (in string units) can be as small as $\Pi_3 \simeq 1$, or as large as $\Pi_4 >> 1$. In other words, the hierarchy among RH neutrino masses and the string scale can be considered as a free parameter. On the other hand, the $Y^{(2)}$-term is suppressed by the scale of the non-perturbative flux, that can easily be near the string-scale so as to justify the assumed hierarchy $M_R^{E2} \gg M_R^p$.

As a consequence, the number of relevant parameters will simply be

$$N_{f.p.} \simeq n_{Y0} + n_{Y0'} + n_{E2} = 6 + 6 + 1 = 13$$

Let us note that such a situation requires $v_1 v_2 / M_{F2} \ll 10^9$ GeV. But $v_{1,2} < v_R$ with $v_R \gtrsim 10^9$ GeV: exotic instanton effects are related to a Stückelberg mechanism for $U(1)_{B-L}$, otherwise they will violate the B-L gauge symmetry. On the other hand, $v_R \gtrsim 10^9$ GeV since exotic instantons have to distinguish RH neutrinos from $E^c$ at this very scale! As a consequence, $M_{F2} \gg 10^9$ GeV satisfies these bounds. This situation seems natural: $M_{F2}$ are related to closed-string fluxes, i.e. another kind of quantum gravity effects.

3 Phenomenology in neutrino physics

In this section we derive our predictions for yet-unknown low energy neutrino parameters, the mass of the lowest neutrino state and the phases of the PMNS (Pontecorvo-Maki-Nakagawa-Sakata) matrix.

3.1 Conditions for a compact RH neutrino spectrum

As mentioned in Sect. 2 the Dirac neutrino mass matrix $m_D$ is symmetric, thus it can be diagonalized by a single unitary matrix $V_L$.

$$m_D = V_L^\dagger m_D^{diag} V_L^*$$

where $m_D^{diag} = diag(m_{D1}, m_{D2}, m_{D3})$ with real and non-negative eigenvalues $m_{(D1,D2,D3)}$. The seesaw condition expressed in Eq. (13) yields

$$M_R = -V_L^\dagger m_D^{diag} A m_D^{diag} V_L^*$$

where we have defined a matrix $A$, symmetric by construction, as

$$A = V_L^* m_{\nu}^{-1} V_L^\dagger$$
In terms of the matrix elements of $A$ and $V_L$, the RH mass matrix elements become

\[
M_{R11} = -A_{11}V_{L11}^2m_{D1}^2 - A_{22}V_{L21}^2m_{D2}^2 - A_{33}V_{L31}^2m_{D3}^2 + 2A_{12}V_{L11}V_{L21}^*m_{D1}m_{D2} - 2A_{13}V_{L11}V_{L31}^*m_{D1}m_{D3} - 2A_{23}V_{L11}V_{L21}^*m_{D2}m_{D3}
\]

\[
M_{R12} = -A_{11}V_{L11}^2V_{L12}^2m_{D1}^2 - A_{22}V_{L21}^2V_{L22}^2m_{D2}^2 - A_{33}V_{L31}^2V_{L32}^2m_{D3}^2 + A_{12}(V_{L11}^2V_{L21} + V_{L11}^*V_{L22})m_{D1}m_{D2} - A_{13}(V_{L11}^2V_{L31} + V_{L11}^*V_{L32})m_{D1}m_{D3} + A_{23}(V_{L21}^2V_{L31} + V_{L21}^*V_{L32})m_{D2}m_{D3}
\]

\[
M_{R13} = -A_{11}V_{L11}^2V_{L13}^2m_{D1}^2 - A_{22}V_{L21}^2V_{L23}^2m_{D2}^2 - A_{33}V_{L31}^2V_{L33}^2m_{D3}^2 + A_{12}(V_{L11}^2V_{L21} + V_{L11}^*V_{L23})m_{D1}m_{D2} - A_{13}(V_{L11}^2V_{L31} + V_{L11}^*V_{L33})m_{D1}m_{D3} + A_{23}(V_{L21}^2V_{L31} + V_{L21}^*V_{L33})m_{D2}m_{D3}
\]

\[
M_{R22} = -A_{11}V_{L12}^2m_{D1}^2 - A_{22}V_{L22}^2m_{D2}^2 - A_{33}V_{L32}^2m_{D3}^2 + 2A_{12}V_{L12}V_{L22}^*m_{D1}m_{D2} - 2A_{13}V_{L12}V_{L32}^*m_{D1}m_{D3} - 2A_{23}V_{L22}V_{L32}^*m_{D2}m_{D3}
\]

\[
M_{R23} = -A_{11}V_{L12}^2V_{L13}^2m_{D1}^2 - A_{22}V_{L22}^2V_{L23}^2m_{D2}^2 - A_{33}V_{L32}^2V_{L33}^2m_{D3}^2 + A_{12}(V_{L12}^2V_{L22} + V_{L12}^*V_{L23})m_{D1}m_{D2} - A_{13}(V_{L12}^2V_{L32} + V_{L12}^*V_{L33})m_{D1}m_{D3} + A_{23}(V_{L22}^2V_{L32} + V_{L22}^*V_{L33})m_{D2}m_{D3}
\]

\[
M_{R33} = -A_{11}V_{L13}^2m_{D1}^2 - A_{22}V_{L23}^2m_{D2}^2 - A_{33}V_{L33}^2m_{D3}^2 + 2A_{12}V_{L13}^2V_{L23}^2m_{D1}m_{D2} + 2A_{13}V_{L13}^2V_{L33}^2m_{D1}m_{D3} - 2A_{23}V_{L23}^2V_{L33}^2m_{D2}m_{D3}
\]

(21)

Since the matrix $M_R$ is also symmetric by construction, one has $M_{Rij} = M_{Rji}$ for any $i, j = 1, 2, 3$. Motivated by quark-lepton symmetry, we assume, as for quarks, a large hierarchy in the eigenvalues of the Dirac mass matrix for leptons, that is

\[
m_{D1} \ll m_{D2} \ll m_{D3}
\]

(22)

The hierarchy assumption in (22) implies that the elements of $A$ are at most mildly hierarchical, and the same holds for the RH neutrino spectrum. Therefore only specific constraints on the $A$ matrix can enforce the conditions that ensure that the RH neutrino spectrum is compact. We can immediately see that a generically compact RH spectrum would result by suppressing the entries proportional to $A_{23}$ and $A_{33}$. In that case, all
matrix elements become of the same order of magnitude, that is \( m_{D1} m_{D3} \sim m_{D2}^2 \). In first approximation, we can set

\[
A_{23} = A_{33} = 0. \tag{23}
\]

Let us stress that while the approximation (23) has the virtue of simplifying the analysis, a generic compact RH neutrino spectrum can be obtained by fixing the \( A_{23} \) and \( A_{33} \) values to any sufficiently small number.

The precise form of the \( V_L \) matrix is not crucial to ensure the compactness of the RH spectrum, provided it does not have unnaturally large matrix elements. Guided by the symmetries of the model, discussed in Sect. 2, we assume that in the diagonal basis for the down-quarks and charged leptons mass matrices, the unitary rotation \( V_L \) that diagonalizes the symmetric matrix \( m_D \) coincides with the Cabibbo-Kobayashi-Maskawa (CKM) matrix that diagonalizes \( m_u \). In other terms, we set, according to Eq. (10)

\[
V_L = V_{CKM} \tag{24}
\]

where \( V_{CKM} \) is the CKM matrix encoding quark mixing.

### 3.2 Low Energy Observables

The PMNS matrix is the lepton counterpart of the CKM mixing matrix in the quark sector. If neutrinos are Majorana particles, there are two more physical phases with respect to the CKM matrix. By adopting the standard parametrization in terms of three Euler mixing angles \( \theta_{12}, \theta_{23}, \theta_{13} \), a Dirac phase \( \delta \), and two Majorana phases \( \alpha \) and \( \beta \), the PMNS mixing matrix can be written as:

\[
U_{PMNS} = U'_{PMNS}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) \times \text{diag} (1, e^{i\alpha}, e^{i\beta}) \tag{25}
\]

where

\[
U'_{PMNS} = \begin{pmatrix}
    c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\
    -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & -s_{23} c_{13} \\
    s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13}
\end{pmatrix} \tag{26}
\]

Here \( c_{ij} = \cos \theta_{ij} \) and \( s_{ij} = \sin \theta_{ij} \), with \( i \) and \( j \) labeling families that are coupled through that angle \( (i,j = 1, 2, 3) \). In the basis in which the charged lepton mass matrix is diagonal, \( U_{PMNS} \) diagonalizes the effective neutrino mass matrix

\[
m_\nu = U_{PMNS}^\dagger m_{\nu}^{diag} U_{PMNS} \tag{27}
\]
where

\[ m_\nu^{\text{diag}} = \text{diag}(m_1, m_2, m_3) \]  

(28)

Since the matrix \( V_L \) is also unitary, we choose the same parameterization as for the PMNS matrix, Eq. (26), distinguishing the \( V_L \) parameters with a prime superscript: \( s'_{12}, s'_{23}, s'_{13}, \delta' \). Their values are the same as the ones in the CKM matrix because of the assumption \( V_L = V_{CKM} \), discussed in Sect. 3.1.

In Sect. 2.1.2, we have operated a counting of the fundamental free parameters of the model, and found 13 real parameters in the case of dominance of non-perturbative effects. Under the assumption of symmetry expressed by Eq. (10), the values of these 13 real parameters are constrained by observables in the up-type quark and neutrino sectors. They are: the three quark masses \( m_u, m_c, m_t \), the two neutrino mass-squared differences \( \Delta m^2_{21}, \Delta m^2_{32} \), the three CKM mixing angles \( \theta'_{12}, \theta'_{23}, \theta'_{13} \) and the three PMNS mixing angles \( \theta_{12}, \theta_{23}, \theta_{13} \), amounting to 11 real observables. Imposing on the complex elements of the matrix \( A \) the two additional conditions in (23), \( A_{23} = A_{33} = 0 \), implies that the set of real fundamental parameters must satisfy two additional requirements, that is \( \text{Re}(A_{23}) = \text{Re}(A_{33}) = 0 \). Thus the parameter space of the model remains completely determined, allowing to obtain a quantitative prediction for the absolute neutrino mass scale \( m_1 \).

The matrix \( A \) can be expressed in terms of the observables \( V_L, U_{PMNS} \) and \( m_\nu^{\text{diag}} \) as

\[
A = (V_L U_{PMNS}^*)^* \frac{1}{m_\nu^{\text{diag}}} (V_L U_{PMNS}^*)^\dagger.
\]  

(29)

This equality connects \( A \) to the observables listed before, and the conditions \( A_{23} = A_{33} = 0 \) determine two relations among them, that we generically indicate with

\[
f([\theta'_{ij}, \delta', \theta_{12}, \theta_{23}, \theta_{13}; \Delta m^2_{21}]; \delta, m_1, \alpha, \beta) = 0 \quad (30)
\]

\[
g([\theta'_{ij}, \delta', \theta_{12}, \theta_{23}, \theta_{13}; \Delta m^2_{31}]; \delta, m_1, \alpha, \beta) = 0 \quad (31)
\]

where \( f \) and \( g \) are known functions. We have eliminated \( m_2 \) and \( m_3 \) by using their relations with their mass-squared differences, \( m_2^2 = m_1^2 + \Delta m^2_{21} \) and \( m_3^2 = m_1^2 + \Delta m^2_{31} \). By projecting \( f \) and \( g \) onto their absolute values, we obtain two relations between real quantities connecting the mass \( m_1 \) and the PMNS phase \( \delta \). Extracting imaginary parts from equations (30) and (31) gives nontrivial relations between the observable \( \delta' \) and the PMNS phases, and allows to determine \( \alpha \) and \( \beta \) in terms of \( m_1, \delta \), and the known mixing angles and mass squared differences.
Table 1: Input parameters. We use the up-quark masses renormalized to the scale $\Lambda = 10^9$ GeV given in Table IV in Ref. [139]. Neutrino’s mass squared differences are taken from the global fit in Ref. [141] and renormalized to the scale $\Lambda$ with a multiplicative factor $r^2$ with $r = 1.25$ according to the prescription in Ref. [140]. The CKM mixing angles $\theta'_{ij}$ and CKM phase $\delta'$ are derived from the values of the Wolfenstein parameters given by the PDG [142]. The PMNS mixing angles are taken from the global fit in Ref. [141]. Renormalization effects for the CKM and PMNS parameters have been neglected.

In Eqs. (30) and (31) the input parameters are listed in square brackets. Their approximate averages, which for our purpose represent an adequate level of approximation, are reported in Table 1. Neutrinos mass squared differences are taken from the global fit in Ref. [141] and renormalized to the scale $\Lambda = 10^9$ GeV ($\sim M_R$), with a multiplicative factor $r^2$ ($r = 1.25$, according to the prescription in Ref. [140]). The up-quark masses, renormalized to the scale $\Lambda$, are taken from Table IV in Ref. [139]. The CKM mixing angles $\theta'_{ij}$ and CKM phase $\delta'$ are derived from the values of the Wolfenstein parameters given by the PDG [142]. The PMNS mixing angles are taken from the global fit in Table 1 of Ref. [141], under the assumption of normal hierarchy of the neutrino masses. Renormalization effects for the CKM and PMNS parameters have been neglected. It is worth noting that the $|V_{ub}|$ puzzle keeps affecting the uncertainty of the small $\theta'_{13}$ value\textsuperscript{19}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Quark sector & & Neutrino sector & \\
\hline
$m_u(\Lambda)$ & 0.00067 GeV & $\Delta m^2_{21}(\Lambda)$ & $11.71 \times 10^{-5}$ eV$^2$ \\
m_\ell(\Lambda) & 0.327 GeV & $\Delta m^2_{31}(\Lambda)$ & $3.84 \times 10^{-3}$ eV$^2$ \\
m_t(\Lambda) & 99.1 GeV & & \\
\hline
$\theta'_{12}$ & 13.03° & $\theta_{12}$ & 33.5° \\
$\theta'_{23}$ & 2.37° & $\theta_{23}$ & 42.3° \\
$\theta'_{13}$ & 0.24° & $\theta_{13}$ & 8.5° \\
$\delta'$ & 1.19 rad & & \\
\hline
\end{tabular}
\end{table}

\textsuperscript{19}For reviews on the $V_{ub}$ uncertainties see e. g. [145, 147, 148, 149, 150, 151].
Figure 2: Plots of $m_1$ in meV as a function of $\delta$, when $(\theta_{12}, \theta_{23}) = (-33.5^\circ, -42.3^\circ)$. The points of intersections represent possible solutions for $(m_1, \delta)$.

solutions $(m_1, \delta)$ correspond to the intersections between the two lines. Exploiting the constraints on the imaginary parameters given by the same eqs. (30)-(31) results in predictions for $\alpha$ and $\beta$ as well. Summarizing, the yet-unknown neutrino parameters $m_1$, $\delta$, $\alpha$ and $\beta$ are given, in our approach, by the following two possibilities

$$m_1 \simeq 2.5 \times 10^{-3} \text{ eV} \quad \delta \simeq \pm 0.6 \quad \alpha \simeq \mp 1.4 \quad \beta \simeq \mp 0.9 \quad (32)$$

which correspond to the upper or lower sign of the three phases. Current experimental data have recently started to put constraints on the Dirac CP-violating phase and we can compare with a recent result of global 3$\nu$ oscillation analysis which give a 1$\sigma$ range $\delta/\pi \in [1.12, 1.77]$ for normal hierarchy [163]. However, at 3$\sigma$, all values $[0, 2]$ are still allowed.

4 Leptogenesis

Most of the interest in the values of the masses of RH neutrinos lies in their double role in the see-saw mechanism and in leptogenesis. Without loss of generality, it is convenient to work in the basis where the RH neutrino mass matrix $M_R$ is diagonal. Since $M_R$ is symmetric, it can be brought to diagonal form $M_R^{\text{diag}} = \text{diag}(M_1, M_2, M_3)$ with real and positive entries by means of a unitary matrix $W$:

$$M_R^{\text{diag}} = W^\dagger M_R W^*.$$  (33)
We indicate the Dirac mass matrix in this basis as
\[ \hat{m}_D = m_D W^*. \] (34)

In this section we discuss the same case study of Sect. 3.2 by setting \( (\theta_{12}, \theta_{23}) = (-|\theta_{12}|, -|\theta_{23}|) \). By arranging the ordering of RH neutrino masses according to \( M_1 < M_2 < M_3 \), our predictions for the RH masses are
\[ M_1 \simeq 3.5 \times 10^9 \text{ GeV} \quad M_2 \simeq M_3 \simeq 8.7 \times 10^9 \text{ GeV} \] (35)
The numerical differences between the absolute values of each pair of solutions for \( \delta \) are negligible. There is no large hierarchy between the masses, and the RH spectrum is compact, with values in the correct range for leptogenesis. Let us observe that the degeneracy of the eigenstates \( M_2 \simeq M_3 \) is lifted when the condition \( (23) \) is only approximately satisfied.

The CP asymmetry in the decay of the RH neutrino \( N_i \) \( (i = 1, 2, 3) \) to a lepton \( \ell_\alpha \) \( (\alpha = e, \mu, \tau) \) is given by [152, 153, 154]
\[ \epsilon_{i\alpha} = \frac{1}{8\pi v^2} \sum_{k \neq i} \frac{\text{Im} \left[ (\hat{m}_D^\dagger)_{i\alpha} (\hat{m}_D)_{ak} (\hat{m}_D^\dagger \hat{m}_D)_{ik} \right]}{f_{LV} \left( \frac{M_k^2}{M_i^2} \right)} \]
\[ + \frac{1}{8\pi v^2} \sum_{k \neq i} \frac{\text{Im} \left[ (\hat{m}_D^\dagger)_{i\alpha} (\hat{m}_D)_{ak} (\hat{m}_D^\dagger \hat{m}_D)_{ki} \right]}{f_{LC} \left( \frac{M_k^2}{M_i^2} \right)}, \] (36)
where \( v = 174 \text{ GeV} \) is the EW VEV. The loop functions are
\[ f_{LV}(x) = \sqrt{x} \left[ \frac{1 - x}{(1 - x)^2 + \left( \frac{\Gamma_i}{M_i} - x \frac{\Gamma_k}{M_k} \right)^2} + 1 - (1 + x) \log \frac{1 + x}{x} \right], \]
\[ f_{LC}(x) = \frac{1 - x}{(1 - x)^2 + \left( \frac{\Gamma_i}{M_i} - x \frac{\Gamma_k}{M_k} \right)^2}, \] (37)
where
\[ \Gamma_i \equiv \frac{M_i}{8\pi v^2} (\hat{m}_D^\dagger \hat{m}_D)_{ii} \] (38)
is the total \( N_i \) width. The first term in eq. (36) comes from lepton-number-violating wave and vertex diagrams, while the second term is from the lepton-number-conserving (but lepton-flavour-violating) wave diagram. The rescaled decay width
\[ \tilde{m}_i \equiv \frac{8\pi v^2}{M_i^2} \Gamma_i = \frac{(\hat{m}_D^\dagger \hat{m}_D)_{ii}}{M_i}, \] (39)
Washout projectors

| $P_{1e}$ | $P_{1\mu}$ | $P_{1\tau}$ |
|-----------|-------------|-------------|
| $0.02$   | $0.42$     | $0.56$      |
| $P_{2e}$ | $P_{2\mu}$ | $P_{2\tau}$ |
| $7.40 \times 10^{-5}$ | $1.62 \times 10^{-3}$ | $0.99$ |
| $P_{3e}$ | $P_{3\mu}$ | $P_{3\tau}$ |
| $7.42 \times 10^{-5}$ | $0.42$     | $0.99$      |

Washout parameters

| $\tilde{m}_1$ | $\tilde{m}_2$ | $\tilde{m}_3$ |
|--------------|----------------|----------------|
| $7.6 \times 10^{-2}$ eV | $565$ eV | $565$ eV |

Table 2: Leptogenesis washout projectors and parameters

which is also known as the effective washout parameter, parameterizes conveniently the departure from thermal equilibrium of $N_i$-related processes (the larger $\tilde{m}_i$, the closer to thermal equilibrium the decays and inverse decays of $N_i$ occur, thus suppressing the final lepton asymmetry).

The washout projector, $P_{i\alpha}$, projects the decay rate over the $\alpha$ flavour, that is, it corresponds to the branching ratio for $N_i$ decaying to $\ell_{\alpha}$, and can be written as

$$P_{i\alpha} = \frac{\begin{pmatrix} \hat{m}_D^\dagger \end{pmatrix}_{ia} (\hat{m}_D)_{\alpha i}}{(\hat{m}_D^\dagger \hat{m}_D)_{ii}}. \quad (40)$$

Finally, the combination $P_{i\alpha} \tilde{m}_i$ projects the washout parameter over a particular flavour direction, and determines how strongly the lepton asymmetry of flavour $\alpha$ is washed out.

Our results for the washout projectors and parameters are collected in Table 2, given the values found in Eq. (32) (differences for $\delta > 0$ or $\delta < 0$ are negligible). Our results for the CP asymmetries are collected in Table 3, for positive and negative values of $\delta$, respectively.

In order to calculate the baryon asymmetry, we need to solve a set of Boltzmann equations (BE) derived as in Ref. [36]. We report here such derivation for convenience’s sake. By including for simplicity only decays and inverse decays, the BE for the RH neutrino densities $Y_{N_i}$ and for $Y_{\Delta_\alpha}$, that is the asymmetry density of the charge $B/3-L_{\alpha}$ normalized to the entropy density $s$, take the form:

$$sHz \frac{dY_{N_i}}{dz} = -\gamma_{N_i} \left( \frac{Y_{N_i}}{Y_{N_i}^{eq}} - 1 \right),$$

$$sHz \frac{dY_{\Delta_\alpha}}{dz} = - \sum_i \left[ \epsilon_{i\alpha} \gamma_{N_i} \left( \frac{Y_{N_i}}{Y_{N_i}^{eq}} - 1 \right) - \frac{\gamma_{N_{i\alpha}}}{2} \left( \frac{Y_{\Delta_\alpha}}{Y_{\Delta_\alpha}^{eq}} + \frac{Y_{\Delta_\alpha}}{Y_{H}^{eq}} \right) \right], \quad (41)$$
Table 3: CP asymmetries. The first and second values in parenthesis refer to positive and negative values of $\delta$, respectively, as given by Eq. (32).

where $Y_{eq}^{N} = \frac{45}{4\pi^2 g_*} z^2 \mathcal{K}_2(z)$ is the equilibrium density for the RH neutrinos with $g_* = 106.75$ and $\mathcal{K}_2$ the second order modified Bessel function of the second kind, $2Y_{\ell}^{eq} = Y_{\ell}^{eq} = \frac{15}{4\pi^2 g_*}$ are respectively the equilibrium densities for lepton doublets and for the Higgs, and the integration variable is $z = M/T$ with $T$ the temperature of the thermal bath. Here $Y_{\Delta_{\alpha}} \equiv Y_{\Delta B}/3 - Y_{\Delta_{L_{\alpha}}}$ where $Y_{\Delta_{L_{\alpha}}}$ is the total lepton density asymmetry in the $\alpha$ flavour which also includes the asymmetries in the RH lepton singlets. Since RH neutrinos only interact with lepton doublets, the right hand side of the second equation of eqs. (41) involves only the LH lepton doublets density asymmetry in a given flavour $\alpha$, $Y_{\Delta_{L_{\alpha}}} = A_{\alpha\beta} Y_{\Delta_{\beta}}$ with $A_{\alpha\beta}$ the flavour mixing matrix \[156\] given in Eq. (42). In equation (41) it is also used $Y_{\Delta H} = C_{\beta} Y_{\Delta_{\beta}}$ the Higgs density asymmetry with $C_{\beta} \[157\]$ given in (42) and $\gamma_{N_i} = P_i \gamma_{N_i}$ (no sum over $i$). The $A$ flavour mixing matrix and the $C$ vectors in the relevant temperature regime are given by \[158\]

$$
A = \frac{1}{2148} \begin{pmatrix}
-906 & 120 & 120 \\
75 & -688 & 28 \\
75 & 28 & -688
\end{pmatrix},
$$

$$
C = -\frac{1}{358} (37, 52, 52). 
$$

(42)

We have solved numerically the BE in eq. (41) and found the baryon asymmetry generated through leptogenesis according to the relation \[159\]

$$
Y_{\Delta B} = \frac{28}{79} \sum \alpha Y_{\Delta_{\alpha}}. 
$$

(43)

Our average result is

$$
Y_{\Delta B} \simeq 2.19 \times 10^{-10}
$$

(44)

which correspond to the input parameters in eq. (32) with positive $\delta$. By comparing with experimental data, we find it sufficiently close to the experimental value to be phenomenologically acceptable. Indeed, recent combined Planck and WMAP CMB
measurements \cite{161,162} yield, at 95\% c.l.

\[ Y_{\Delta B}^{P/WMAP} = (8.58 \pm 0.22) \times 10^{-11}. \]  

(45)

Let us underline that it is not a trivial result to recover the sign and the order of magnitude of the experimental data, given the high degree of predictability of our model.

Comparison with data allows us to discard the second possibility granted by (32), corresponding to \( \delta < 0 \), which results in a negative value \( Y_{\Delta B} \simeq -0.23 \times 10^{-11} \). Let us observe that a small difference of input parameters can have a non negligible impact on the values of leptogenesis asymmetries, in contrast to what happens for the values of masses \( m_1 \) and \( M_i \).

5 Phenomenology in neutron-antineutron physics

The mass matrix \( M_{RH}^{NP} \) has to have eigenvalues smaller than the LR symmetry breaking scale \( v_R \):

\[ M_{RH,1,2,3}^{E2} < v_{1,2} < v_R \]

On the other hand, we have assumed that

\[ M_{RH,1,2,3}^{E2} \gg \frac{v_1 v_2}{M_{F2}} \]

So, the scale \( M_{F2} \) has to be \( M_{F2} \gg 10^9 \text{GeV} \). This case is compatible with the natural situation \( M_{F2} \simeq M_S \).

On the other hand, the string scale has necessary to be higher than the RH neutrino mass, i.e \( M_S \gg 10^9 \text{GeV} \). These bounds have important implications for other signatures in phenomenology.

Neutron-antineutron transitions generated by new physics at a scale \( 300 \div 1000 \text{TeV} \) can be tested in the next generation of experiments. In particular the AB-model predicts this signature, even if the precise scale is unknown. The strength of neutron-antineutron transitions is

\[ G_{n-\bar{n}} \simeq \frac{g_3^2}{16\pi M_{\Delta c\bar{u}u}^2} \frac{f_{11}^2 v_2}{M_{SUSY} M_0} \]  

(46)

where \( f_{11} = \tilde{f}_{11} v_1 / M_2 \) with \( \tilde{f}_{11} \) Yukawa couplings \( \tilde{f}_{11} v_1 Q^c Q^c \Delta^c / M_{F2} \), including \( f_{11} \Delta_{\Delta c\bar{u}u} \Delta_{\Delta c\bar{d}d} \) and \( f_{11} \Delta_{\Delta c\bar{d}d} \Delta_{\Delta c\bar{d}d} \) are the sextets contained in \( \Delta^c \). This can be rewritten

\footnote{As a consequence, our model is not compatible with a TeV-ish LR symmetric model}
as the following bound on the sextets

\[
\frac{1}{f_{11}}M_{\Delta_{e\nu e}}^2M_{\Delta_{e\nu e}}^2 > \frac{(300 \, \text{TeV})^3 v_2}{M_{SUSY} M_{S_e - S_{E2}^\prime}}
\]

A conservative assumption on the sextets, in order to avoid FCNCs bounds, is \(M_{\Delta_{e\nu e}} \simeq M_{\Delta_{e\nu e}} > 100 \, \text{TeV}\) (with \(f_{11} \simeq 1\)). Calling \(x = v_2/M_{SUSY}\), FCNCs bounds will constrain \(M_{S_e - S_{E2}^\prime}, x\) as

\[
x^{-1} M_{S_e + S_{E2}^\prime} > 100 \, \text{TeV}
\]

at system with \(M_{SUSY} > 10^9 \, \text{GeV}\), \(v_{1,2} < v_R\) and \(M_{SUSY} \leq M_S\). These bounds correspond to several different regions of the parameters space, compatible with neutrino physics. As a consequence, our model provides a viable way to generate a Majorana mass for the neutron testable in the next generation of experiments\(^{21}\). On the other hand, the generation of such a \(B - L\) violating operator can be dangerous in combination with \(B + L\) violating sphalerons: they can wash-out an initial lepton number asymmetry generated by RH neutrino decays. Of course, they can regenerate the correct amount of baryon asymmetry through a post-sphaleron mechanism, as discussed in \[6, 18\]. On the other hand, from a string theory prospective, it is reasonable to consider the case in which the strength of the effective operators coupling six quarks increases as a dynamical field from the early Universe to the present epoch. Moduli stabilization is one of the most challenging problem in string theory, because it necessary involves non-perturbative effects such as fluxes and stringy instantons. In string theory, coupling constants, such as \(\alpha_{em}\) and so on, are functions of dynamical moduli \(f(\phi_i)\), that in turn have to be somehow stabilized. However, in principle, moduli can undergo a slow cosmological evolution rather than being exactly constant in time. As a result, a slowly growing coupling can be naturally envisaged in string inspired models. A natural ansatz can be a solitonic solution in time connecting to constant asymptotes. The naturalness of such a proposal is also supported by the fact that usually the dependence of coupling constants on moduli is of exponential type. In our case, we can suggest a solitonic solution growing from \(G_{n\bar{n}}(t \ll t_{e.w}) \ll G_{n\bar{n}}(t_{e.w} \ll \bar{t} \ll t_{BBN})\) to \(\bar{G}_{n\bar{n}},\) where \(\bar{G}_{n\bar{n}}\) is bounded by direct laboratory limits. Under this general assumption, we also avoid cosmological limits from BBN (Big Bang Nucleosynthesis). Let us remark that the moduli dependence of \(G_{n\bar{n}}\) could enter from the non-perturbative mixing of 10-plets \(\Delta, i.e\) in instantonic geometric moduli. Of course, such a proposal deserves future investigations in global stringy models, beyond the purposes of this paper.

\(^{21}\)Neutron-Antineutron transitions could be also an intriguing test for new interactions, as discussed in \[145\].
6 Conclusions and remarks

In this paper, we have considered an alternative see-saw mechanism produced by exotic instantons rather than by spontaneous symmetry breaking. We have named this mechanism “exotic see-saw” mechanism, since exotic instantons generate the main contribution to the mass matrix of RH neutrinos. We have embedded such a mechanism in an (un)oriented string model with intersecting D-branes and E-branes, giving rise to a Pati-Salam like model in the low energy limit, plus extra non-perturbative couplings. The specific unoriented quiver theory that we have considered was largely inspired by the one suggested in [47]. The present model has a predictive power in low energy observables, not common to other see-saw models.

Our model makes precise predictions for low energy physics, from the acquisition of 11 inputs from neutrino physics. Seven degrees of freedom parameterize the geometry of the mixed disk amplitudes, i.e. of $E2$-instanton intersecting $D6$-branes’ stacks. We have reconstructed the seven geometric parameters associated to the exotic instanton and we have predictions to compare with the next generation of experiments. This will allow to indirectly test if the $E2$-instanton considered really dominates the mass terms in the neutrino sector. We have considered a class of mixed disk amplitudes producing a RH neutrino mass matrix with quasi degenerate spectrum of eigenvalues. The compactness of the RH neutrino spectrum is geometrically understood in terms of mixed disk amplitudes and it is a favorable feature for predictability. As shown, this mechanism can also realize a successful baryogenesis through RH neutrino decays.

In our model, a $\theta_{13} \neq 0$ is compatible with leptogenesis and other neutrino physics bounds. Our model is also suggesting other possible signatures in neutron-antineutron transitions [47]. On the other hand, our model is assuming a supersymmetry breaking scale $M_{SUSY} \gg 1$ TeV as well as a Left-Right symmetry scale $M_{LR} \gg 1$ TeV. A possible discover of Supersymmetry of Left-Right symmetry at LHC or future high energy colliders would rule out our model. In conclusion, our model provides a unifying picture of particles and interactions that will be indirectly tested from different low energy channels in neutrino physics, flavor changing neutral currents, neutron-antineutron transitions and LHC.

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References

[1] E. Majorana, Nuovo Cimento 14 (1937) 171.
[2] P. Minkowski, Physics Letters B 67 (1977) 421.
[3] P. Ramond, in Sanibel Conference, CALT-68-700 (1979), hep-ph/9809459
[4] M. Gell-Mann, P. Ramond and R. Slansky, Conf. Proc. C 790927 (1979) 315 arXiv:1306.4669 [hep-th].
[5] T. Yanagida, Prog. Theor. Phys. 64 (1980) 1103.
[6] R. N. Mohapatra, G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912.
[7] M. Fukugita, T. Yanagida, Phys. Lett.B 174 (1986) 45.
[8] A. D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. 5 (1967) 32 [JETP Lett. 5, 24 (1967)]; Sov. Phys. Usp. 34, 392 (1991) [Usp. Fiz. Nauk 161, 61 (1991)]
[9] G. Chen, M. J. Pérez and P. Ramond, Phys. Rev. D 92 (2015) 076006 arXiv:1412.6107 [hep-ph].
[10] J. Kile, M. J. Pérez, P. Ramond and J. Zhang, Phys. Rev. D 90 (2014) 013004 arXiv:1403.6136 [hep-ph].
[11] Y. Mambrini, N. Nagata, K. A. Olive, J. Quevillon and J. Zheng, Phys. Rev. D 91 (2015) 095010 arXiv:1502.06929 [hep-ph].
[12] J. Hisano, Y. Muramatsu, Y. Omura and M. Yamanaka, Phys. Lett. B 744 (2015) 395 arXiv:1503.06156 [hep-ph].
[13] F. Wang, W. Wang and J. M. Yang, JHEP 1503 (2015) 050 [arXiv:1501.02906 [hep-ph]].

[14] G.'t Hooft, Phys. Rev. D 14 (1976) 3432.

[15] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B 155 (1985) 36.

[16] J. C. Pati, A. Salam, Phys. Rev. D10 (1974) 275.

[17] S. Dimopoulos and F. Wilczek, Proceedings Erice Summer School, ed. A. Zichichi (1981) ss4.

[18] K.S. Babu and R.N. Mohapatra, Phys. Rev. Lett. 74 (1995) 2418.

[19] Z. Berezhiani, G. Dvali, Bull. Lebedev Phys. Inst, (1989).

[20] R. Barbieri et al., Nucl. Phys. B391, 487 (1993).

[21] L. Randall and C. Csaki, In *Palaiseau 1995, SUSY 95* 99-109 [hep-ph/9508208].

[22] P. Candelas et al., Nucl. Phys. B258, 46 (1985).

[23] L.J. Dixon et al., Nucl. Phys. B261, 678 (1985).

[24] L.J. Dixon et al., Nucl. Phys. B274, 285 (1986).

[25] L. E. Ibanez et al., Phys. Lett. B187, 25 (1987).

[26] L. E. Ibanez et al., Phys. Lett. B191, 282 (1987).

[27] S. Davidson and A. Ibarra, Phys. Lett. B535 (2002) 25.

[28] G. Engelhard, Y. Grossman, E. Nardi and Y. Nir, Phys. Rev. Lett. 99, (2007) 081802.

[29] P. Di Bari, Nucl. Phys. B727 (2005) 318.

[30] O. Vives, Phys. Rev. D73 (2006) 073006.

[31] A. Abada, P. Hosteins, F. X. Josse-Michaux and S. Lavignac, Nucl. Phys. B809 (2009) 183.

[32] P. Di Bari and A. Riotto, Phys. Lett. B671 (2009) 462.
[33] P. Di Bari and A. Riotto, JCAP 1104 (2011) 037.

[34] S. Blanchet, D. Marfatia and A. Mustafayev, JHEP 1011, 038 (2010).

[35] F. Buccella, D. Falcone and L. Oliver, Phys. Rev. D83, 093013 (2011).

[36] F. Buccella, D. Falcone, C. S. Fong, E. Nardi and G. Ricciardi, Phys. Rev. D 86 (2012) 035012 [arXiv:1203.0829 [hep-ph]].

[37] A. Pilaftsis and T. E. J. Underwood, Nucl. Phys. B692 (2004) 303.

[38] A. Pilaftsis and T. E. J. Underwood, Phys. Rev. D72 (2005) 113001.

[39] A. Pilaftsis, Phys. Rev. Lett. 95, (2005) 081602.

[40] L. Marzola, arXiv:1410.5565 [hep-ph].

[41] P. Di Bari, L. Marzola and M. Re Fiorentin, Nucl. Phys. B 893 (2015) 122 arXiv:1411.5478 [hep-ph].

[42] C. S. Fong, D. Meloni, A. Meroni and E. Nardi, JHEP 1501 (2015) 111 arXiv:1412.4776 [hep-ph].

[43] R. N. Mohapatra, R. E. Marshak, Phys. Rev. Lett. 44, 1316–1319 (1980).

[44] F. Quevedo, Mod. Phys. Lett. A 30, 1530004 (2015) arXiv:1404.5151 [hep-th].

[45] S. Krippendorf, M. J. Dolan, A. Maharana and F. Quevedo, JHEP 1006 (2010) 092 arXiv:1002.1790 [hep-th].

[46] J. P. Conlon, A. Maharana and F. Quevedo, JHEP 0905 (2009) 109 arXiv:0810.5660 [hep-th].

[47] A. Addazi and M. Bianchi, arXiv:1502.08041 [hep-ph].

[48] P. Anastasopoulos, G. K. Leontaris and N. D. Vlachos, JHEP 1005 (2010) 011 arXiv:1002.2937 [hep-th].

[49] M. Bianchi and M. Samsonyan, Int. J. Mod. Phys. A 24 (2009) 5737 arXiv:0909.2173 [hep-th].

[50] M. Bianchi and G. Inverso, Fortsch. Phys. 60 (2012) 822 arXiv:1202.6508 [hep-th].
[51] S. Franco, A. Retolaza and A. Uranga, arXiv:1507.05330 [hep-th].

[52] Particle Data Group, Phys. Rev. D 86, 010001 (2012).

[53] M. Baldo-Ceolin et al., Z. Phys. C 63, 409 (1994).

[54] K.S. Babu, et al., arXiv:1311.5285
K. Babu, et al. arXiv:1310.8593.

[55] A.S. Kronfeld, R.S. Tschirhat, U. Al. Binni, W. Altmannshofer, C.Ankenbrandt, K.Babu, S. Banerjee and M.Bass et al. Project X: Physics Opportunities, arXiv:1306.5009 [hep-ex]

[56] K. S. Babu, R. N. Mohapatra and S. Nasri, Phys. Rev. Lett. 98, 161301 (2007) arXiv:hep-ph/0612357;

[57] K. S. Babu, R. N. Mohapatra and S. Nasri, Phys. Rev. Lett. 97, 131301 (2006) arXiv:hep-ph/0606144;

[58] K. S. Babu, P. S. Bhupal Dev and R. N. Mohapatra, Phys. Rev. D 79, 015017 (2009) arXiv:0811.3411 [hep-ph].

[59] K. S. Babu, P. S. Bhupal Dev, E. C. F. S. Fortes and R. N. Mohapatra, Phys. Rev. D 87, 115019 (2013) arXiv:1303.6918 [hep-ph].

[60] E. C. F. S. Fortes, K. S. Babu, R. N. Mohapatra arXiv:1311.4101

[61] R. N. Mohapatra, N. Okada and H. -B. Yu, Phys. Rev. D 77, 011701 (2008) arXiv:0709.1486 [hep-ph].

[62] Y. C. Zhan, Z. L. Liu, S. A. Li, C. S. Li and H. T. Li, Eur. Phys. J. C (2014) 74:2716.

[63] A. Addazi and M. Bianchi, JHEP 1412 (2014) 089 arXiv:1407.2897 [hep-ph].

[64] A. Addazi, JHEP 1504 (2015) 153 arXiv:1501.04660 [hep-ph].

[65] A. Addazi and M. Bianchi, JHEP 1507 (2015) 144 arXiv:1502.01531 [hep-ph].

[66] A. Addazi, arXiv:1504.06799 [hep-ph].

[67] A. Addazi, arXiv:1505.00625 [hep-ph].
[68] A. Addazi, arXiv:1505.02080 [hep-ph].

[69] A. Addazi, arXiv:1506.06351 [hep-ph].

[70] T. Schwetz, M. Tortola and J. W. F. Valle, New J. Phys. 13 (2011) 109401.

[71] G. L. Fogli, E. Lisi, A. Marrone, A. Palazzo and A. M. Rotunno, Phys. Rev. D84 (2011) 053007.

[72] M. C. Gonzalez-Garcia, M. Maltoni and J. Salvado, JHEP 1004 (2010) 056.

[73] A. Sagnotti, IN *CARGESE 1987, PROCEEDINGS, NONPERTURBATIVE QUANTUM FIELD THEORY* 521-528 AND ROME II UNIV. - ROM2F-87-025 (87,REC.MAR.88) 12p [hep-th/0208020]. M. Bianchi and A. Sagnotti, Phys. Lett. B 231 (1989) 389. doi:10.1016/0370-2693(89)90681-3 M. Bianchi and A. Sagnotti, ROM2F-88-040, C88-06-06.6. M. Bianchi and A. Sagnotti, Phys. Lett. B 211 (1988) 407. doi:10.1016/0370-2693(88)91884-9

[74] A. Sagnotti, Phys. Lett. B294 (1992) 196 [hep-th/9210127].

[75] C. Angelantonj and A. Sagnotti, Phys. Rept. 1 [(Erratum-ibid.) 339] arXiv:hep-th/0204089.

[76] G. Pradisi and A. Sagnotti, Phys. Lett. B 216 (1989) 59.

[77] M. Bianchi and A. Sagnotti, Phys. Lett. B 247 (1990) 517.

[78] M. Bianchi and A. Sagnotti, Nucl. Phys. B 361 (1991) 519.

[79] M. Bianchi, G. Pradisi and A. Sagnotti, Nucl. Phys. B 376 (1992) 365.

[80] A. M. Uranga, Nucl. Phys. B 598, 225 (2001) [hep-th/0011048].

[81] G. Aldazabal, S. Franco, L. E. Ibanez, R. Rabadan and A. M. Uranga, J. Math. Phys. 42, 3103 (2001) [hep-th/0011073].

[82] M. Bianchi and J. F. Morales, JHEP 0003 (2000) 030 [hep-th/0002149].

[83] C. Angelantonj, M. Bianchi, G. Pradisi and Y. S. Stanev, Phys. Lett. B 385 (1996) 96 [hep-th/9606169].

[84] C. Angelantonj, M. Bianchi, G. Pradisi, A. Sagnotti and Y. S. Stanev, Phys. Lett. B 387 (1996) 743 [hep-th/9607229].
[85] R. Blumenhagen, M. Cvetic and T. Weigand, Nucl. Phys. B 771 (2007) 113 [hep-th/0609191].

[86] L.E. Ibanez, A.M. Uranga, JHEP 0703 (2007) 052.

[87] L.E. Ibanez, A.N. Schellekens, A.M. Uranga, JHEP 0706 (2007) 011.

[88] S. Antusch, L.E. Ibanez and T. Macr, JHEP 0709 (2007) 087.

[89] R. Blumenhagen, M. Cvetic, D. Lust, R. Richter and T. Weigand, Phys. Rev. Lett. 100 (2008) 061602 [arXiv:0707.1871 [hep-th]].

[90] M. Bianchi, S. Cremonesi, A. Hanany, J. F. Morales, D. R. Pacifici and R. K. Seong, JHEP 1410 (2014) 27 [arXiv:1408.1957 [hep-th]].

[91] P. Anastasopoulos, M. Bianchi, E. Dudas and E. Kiritsis, JHEP 0611 (2006) 057 [hep-th/0605225].

[92] J. De Rydt, J. Rosseel, T.T. Schmidt, A. Van Proeyen and M. Zagermann, Class. Quant. Grav. 24 (2007) 5201 [arXiv:0705.4216] [INSPIRE].

[93] D. Feldman, Z. Liu and P. Nath, Phys. Rev. D 75 (2007) 115001 [hep-ph/0702123 [INSPIRE].

[94] D. Feldman, Z. Liu and P. Nath, AIP Conf. Proc. 939 (2007) 50 [arXiv:0705.2924 [INSPIRE].

[95] B. Körs and P. Nath, Phys. Lett. B 586 (2004) 366 [hep-ph/0402047] [INSPIRE].

[96] B. Körs and P. Nath, JHEP 12 (2004) 005 [hep-ph/0406167] [INSPIRE].

[97] B. Körs and P. Nath, [hep-ph/0411406] [INSPIRE].

[98] B. Körs and P. Nath, JHEP 07 (2005) 069 [hep-ph/0503208] [INSPIRE].

[99] P. Anastasopoulos, F. Fucito, A. Lionetto, G. Pradisi, A. Racioppi and Y.S. Stanev, Phys. Rev. D 78 (2008) 085014 [arXiv:0804.1156] [INSPIRE].

[100] C. Corianó, N. Irges and E. Kiritsis, Nucl. Phys. B 746 (2006) 77 [hep-ph/0510332] [INSPIRE].

[101] M. Bianchi and E. Kiritsis, Nucl. Phys. B 782 (2007) 26 [hep-th/0702015].
[102] B. de Wit, P.G. Lauwers and A. Van Proeyen, Nucl. Phys. B 255 (1985) 569 [INSPIRE].

[103] N. Arkani-Hamed, H. Georgi and M. D. Schwartz, Annals Phys. 305 (2003) 96.

[104] V. A. Rubakov, arXiv:hep-th/0407104.

[105] S. L. Dubovsky, JHEP 0410, 076 (2004).

[106] A. Addazi and S. Capozziello, Int. J. Theor. Phys. 54 (2015) 6, 1818 arXiv:1407.4840 [gr-qc]].

[107] A. Addazi, arXiv:1508.04054 [gr-qc].

[108] M. Bianchi and A. V. Santini, JHEP 0612 (2006) 010 [hep-th/0607224]

[109] A. Addazi and G. Esposito, Int. J. Mod. Phys. A 30 (2015) 15, 1550103 arXiv:1502.01471 [hep-th]].

[110] A. Addazi, arXiv:1505.07357 [hep-th].

[111] L. Ibanez, A. Schellekens, and A. Uranga, Nucl.Phys. B865 (2012) 509?540, arXiv:1205.5364 [hep-th].

[112] L. E. Ibanez and G. G. Ross, Nucl.Phys. B368 (1992) 3?37.

[113] M. Berasaluce-Gonzalez, L. E. Ibanez, P. Soler, and A. M. Uranga, JHEP 1112 (2011) 113, arXiv:1106.4169 [hep-th].

[114] P. Anastasopoulos, M. Cvetic, R. Richter, and P. K. Vaudrevange, JHEP 1303 (2013) 011, arXiv:1211.1017 [hep-th].

[115] G. Honecker and W. Staessens, JHEP 1310 (2013) 146, arXiv:1303.4415 [hep-th].

[116] H. Abe, K.-S. Choi, T. Kobayashi, and H. Ohki, Nucl.Phys. B820 (2009) 317?333, arXiv:0904.2631 [hep-ph].

[117] M. Berasaluce-Gonzalez, P. Camara, F. Marchesano, D. Regalado, and A. Uranga, JHEP 1209 (2012) 059, arXiv:1206.2383 [hep-th].

[118] F. Marchesano, D. Regalado, and L. Vazquez-Mercado, JHEP 1309 (2013) 028, arXiv:1306.1284 [hep-th].
[119] Y. Hamada, T. Kobayashi, and S. Uemura, arXiv:1402.2052 [hep-th].

[120] T. Kobayashi, H. P. Nilles, F. Ploger, S. Raby and M. Ratz, Nucl. Phys. B 768 (2007) 135 hep-ph/0611020.

[121] T. Dijkstra, L. Huiszoon, and A. Schellekens, Phys.Lett. B609 (2005) 408?417, arXiv:hep-th/0403196 [hep-th].

[122] T. Dijkstra, L. Huiszoon, and A. Schellekens, Nucl.Phys. B710 (2005) 3?57, arXiv:hep-th/0411129 [hep-th].

[123] P. Anastasopoulos, T. Dijkstra, E. Kiritsis, and A. Schellekens, Nucl.Phys. B759 (2006) 83?146, arXiv:hep-th/0605226 [hep-th].

[124] R. Blumenhagen, JHEP 0311 (2003) 055, arXiv:hep-th/0310244 [hep-th].

[125] R. Blumenhagen and T. Weigand, arXiv:hep-th/0408147 [hep-th].

[126] R. Blumenhagen and T. Weigand, Phys.Lett. B591 (2004) 161?169, arXiv:hep-th/0403299 [hep-th].

[127] R. Blumenhagen and T. Weigand, JHEP 0402 (2004) 041, arXiv:hep-th/0401148 [hep-th].

[128] G. Aldazabal, E. C. Andres, M. Leston, and C. A. Nunez, JHEP 0309 (2003) 067, arXiv:hep-th/0307183 [hep-th].

[129] S. Govindarajan and J. Majumder, JHEP 0402 (2004) 026, arXiv:hep-th/0306257 [hep-th].

[130] G. Aldazabal, E. Andres, and J. Juknevich, JHEP 0405 (2004) 054, arXiv:hep-th/0403262 [hep-th].

[131] I. Brunner, K. Hori, K. Hosomichi, and J. Walcher, JHEP 0702 (2007) 001, arXiv:hep-th/0401137 [hep-th].

[132] E. Kiritsis, B. Schellekens, and M. Tsulaia, JHEP 0810 (2008) 106, arXiv:0809.0083 [hep-th].

[133] E. Kiritsis, M. Lennek, and B. Schellekens, Nucl.Phys. B829 (2010) 298?324, arXiv:0909.0271 [hep-th].
[134] P. Anastasopoulos, G. Leontaris, and N. Vlachos, JHEP 1005 (2010) 011, arXiv:1002.2937 [hep-th]

[135] P. Anastasopoulos, R. Richter and A. N. Schellekens, JHEP 1506 (2015) 189, arXiv:1502.02686 [hep-th].

[136] T. Takagi, Japanese J. Math. 1 (1924) 83.

[137] B. Pontecorvo, Sov. Phys. JETP 6 (1957) 429 [Zh. Eksp. Teor. Fiz. 33 (1957) 549].

[138] B. Pontecorvo, Sov. Phys. JETP 26 (1968) 984 [Zh. Eksp. Teor. Fiz. 53 (1967) 1717].

[139] Z. z. Xing, H. Zhang and S. Zhou, Phys. Rev. D 77, 113016 (2008), arXiv:0712.1419 [hep-ph].

[140] G. F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, Nucl. Phys. B 685, 89 (2004) hep-ph/0310123.

[141] M. C. Gonzalez-Garcia, M. Maltoni and T. Schwetz, JHEP 1411, 052 (2014), arXiv:1409.5439 [hep-ph].

[142] K. A. Olive et al. [Particle Data Group Collaboration], Chin. Phys. C 38, 090001 (2014).

[143] P. Di Bari, L. Marzola and M. Re Fiorentin, Nucl. Phys. B 893, 122 (2015), arXiv:1411.5478 [hep-ph].

[144] A. Addazi, Z. Berezhiani, R. Bernabei, P. Belli, F. Cappella, R. Cerulli and A. Incicchitti, arXiv:1507.04317 [hep-ex].

[145] A. Addazi, Nuovo Cim. C 038 (2015) 01, 21.

[146] G. Ricciardi et al., Eur. Phys. J. Plus 130, 209 (2015) arXiv:1507.05029 [hep-ph].

[147] G. Ricciardi, AIP Conf. Proc. 1701, 050014 (2016) arXiv:1412.4288 [hep-ph].

[148] G. Ricciardi, Mod. Phys. Lett. A 29, 1430019 (2014) arXiv:1403.7750 [hep-ph].

[149] G. Ricciardi, Mod. Phys. Lett. A 28, 1330016 (2013) arXiv:1305.2844 [hep-ph].
[150] G. Ricciardi, PoS Beauty 2013, 040 (2013) [arXiv:1306.1039 [hep-ph]].

[151] G. Ricciardi, Mod. Phys. Lett. A 27, 1230037 (2012) [arXiv:1209.1407 [hep-ph]].

[152] L. Covi, E. Roulet and F. Vissani, Phys. Lett. B 384, 169 (1996) [hep-ph/9605319].

[153] W. Buchmuller and M. Plumacher, Phys. Lett. B 431, 354 (1998) [hep-ph/9710460].

[154] A. Anisimov, A. Broncano and M. Plumacher, Nucl. Phys. B 737, 176 (2006) [hep-ph/0511248].

[155] S. Davidson, E. Nardi and Y. Nir, Phys. Rept. 466 (2008) 105.

[156] R. Barbieri et al., Nucl. Phys. B 575 (2000) 61; T. Endoh, T. Morozumi and Z. h. Xiong, Prog. Theor. Phys. 111 (2004) 123.

[157] E. Nardi, Y. Nir, J. Racker and E. Roulet, JHEP 0601 (2006) 068.

[158] E. Nardi, Y. Nir, E. Roulet and J. Racker, JHEP 0601 (2006) 164.

[159] J. A. Harvey and M. S. Turner, Phys. Rev. D 42 (1990) 3344.

[160] C. S. Fong, M. C. Gonzalez-Garcia and E. Nardi, Int. J. Mod. Phys. A 26 3491 (2011).

[161] P. A. R. Ade et al. [Planck Collaboration], Astron. Astrophys. 571 (2014) A16 [arXiv:1303.5076 [astro-ph.CO]].

[162] C. L. Bennett et al. [WMAP Collaboration], Astrophys. J. Suppl. 208 (2013) 20 [arXiv:1212.5225 [astro-ph.CO]].

[163] F. Capozzi, G. L. Fogli, E. Lisi, A. Marrone, D. Montanino and A. Palazzo, Phys. Rev. D 89 (2014) 093018 [arXiv:1312.2878 [hep-ph]].