Quantitative Study of Thermal Disturbances Due to Nonuniformly Perfused Tumors in Peripheral Regions of Women’s Breast

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ABSTRACT

BACKGROUND: Mathematical modeling of biothermal processes is widely used to enhance the quantitative understanding of thermoregulation system of human body organs. This quantitative knowledge of thermal information of various human body organs can be used for developing clinical applications. In the past, investigators have studied thermal distribution in hemisphere-shaped human breast in the presence of sphere-shaped tumor. The shape and size of the breast as well as tumor may also affect thermal distribution which can have serious implications in thermography. In this article, a model of thermal disturbances in peripheral regions of ellipsoid-shaped human breast involving ellipse-shaped nonuniformly perfused tumor has been developed for a 2-dimensional steady-state case. The modeling study will provide biomedical scientists vital insights of thermal changes occurring due to the shape and size of breast and tumor which can influence the development of protocols of thermography for diagnosis of tumors in women’s breast.

METHOD: We have incorporated the significant parameters such as blood flow, metabolic activity, and thermal conductivity in the thermal model for normal and malignant tissues. The controlled metabolic activity has been incorporated for normal tissues, and uncontrolled metabolic activity has been incorporated for tumor regions. The peripheral regions of breast are divided into 3 major layers, namely, epidermis, dermis, and subdermal tissues. An ellipse-shaped nonuniformly perfused tumor is assumed to be present in dermal layers. The nonuniformly perfused tumor is divided into 2 natural components, namely, the necrotic core and tumor periphery. The outer surface of the breast is assumed to be exposed to the environment, and the heat loss takes place by conduction, convection, radiation, and evaporation. The finite element approach is used to obtain the solution. The numerical results have been used to study the effect of shape and size of tumor on temperature distribution in matured breast of different shapes.

RESULTS: By selecting appropriate model parameters, we have shown the spatial thermal variation in matured breast of different shapes which could be replicated by the proposed model. We have also shown the thermal disturbances caused by different shapes and sizes of tumors by selecting appropriate values of parameters. In addition, the thermal information from our model provides us the basis for prediction of shape and size of tumors in terms of change of the slope of temperature profiles at the junction of tumor and normal tissues and tumor periphery and tumor core.

CONCLUSIONS: The proposed model was successfully used to study the impact of different sizes and shapes of nonuniformly perfused tumor on thermograms in peripheral regions of ellipse-shaped woman’s breast. The proposed model is more realistic in terms of shape and size of tumors and woman’s breast in comparison with earlier models reported in the literature. The finite element discretization of breast into large number of triangular ring elements effectively models the heterogeneity of region. The changes in slope of the thermal curves at the junctions of various peripheral and tumor layers are due to the nonhomogeneous nature of the region. The location of major thermal disturbances in the tissues indicates the presence of tumor. The change in the slope of the thermal curves gives us idea about the position, type, and size of the tumors in the peripheral tissues. This thermal information can be exploited for detection of tumors by thermographic techniques.

KEYWORDS: Woman’s breast, shape function, thermal disturbances and finite element discretization

Background

Temperature is a very special variable which is used as a tissue response in clinical situation since ancient times in traditional medicine. Thermography is a noncontact and noninvasive temperature screening method used widely in the medical arena as it is economic and does not inflict any pain on the patient. It can detect the variation of temperature on the human skin surface. One of the notable examples is of detection of cancer by thermography. Thermography is nondestructive in nature and is one of the main motivations behind the preferable use of thermography to detect breast tumor rather than existing known imaging techniques such as mammography. Mammography is the most popular technique for detection of tumors. The second popular technique is ultrasound. Thermography lags behind mammography and ultrasound. However, none of these techniques are completely accurate. It is reported in a study that the combination of mammography
and thermography boosts the sensitivity and specificity, which is useful for detecting tumors in early stage. Thermography has not matured to detect disturbances and temperature due to small tumor or early-stage tumor. These facts motivate us for further research in this area for development of thermography.

The study of temperature distribution problems of human body is of clinical interest. The human body is equipped with a thermoregulation system which maintains the body core temperature at a uniform temperature of 37°C by maintaining the balance between heat generation within the body cells and heat loss to the environment. The various heat transfer processes, such as thermal conduction and convection due to blood perfusion, take place in peripheral regions of human body organs to transport heat from body core to the surface. The heat is lost from the surface to the environment by conduction, convection, radiation, and evaporation. The heat is generated in the body cells due to metabolic activities.

The peripheral region, namely, the skin and subdermal tissues, is a heterogeneous medium, and it plays an important role in thermoregulation. This peripheral region consists of 3 layers, namely, epidermis, dermis, and subdermal tissues. The dermis is made up of matted masses of connective tissues and elastic fibers, blood vessels, lymphatics, and nerves. The population density of blood vessels in the dermis is very thin near the interface of epidermis, i.e., there are no blood vessels in epidermis, but it increases gradually and becomes almost uniform in the subdermal part. At low atmospheric temperature, blood flows at body core temperature from trunk into arteries near the core of the breast which cools down by flowing into capillaries in the peripheral regions of breast. Furthermore, this cold blood returns from capillaries to veins at central part of the breast. Thus, portions of core of breast near the trunk are maintained at 37°C, and the portions of breast core which are at some distance from trunk will have lower temperature. Thus, at lower atmospheric temperature, the core temperature of breast is found to vary with respect to position.

Any abnormality, such as the presence of a tumor, can disturb the temperature distribution in this region. The malignant tumors are characterized by uncontrolled growth and uncontrolled metabolic activity, whereas normal tissues are characterized by self-controlled growth and self-controlled metabolic activity. Initially, the tumor is of 1 mm in diameter and it derives its nutrients from nearby blood vessels for its growth. As a tumor grows, it incorporates the nearby blood vessels into its mass, and due to the increase in requirement of nutrients to cater to its needs of uncontrolled growth, neovascularization and hypervascularization take place in the tumor region. In this way, the number of vessels in the tumor grows due to the presence of tumor angiogenesis factor. This results in higher rate of blood flow and metabolic activity in a malignant tumor. It has been found that the blood flow and metabolic activity in tumor vary 1 to 7 times than that in normal tissues. Also, the thermal conductivity of blood is higher than that of normal tissues, and as the tumor is rich in blood content, the thermal conductivity of tumor is found to be higher than that in normal tissues. The necrotic core of the tumor consists of dead cells.

Several investigations have been made by various research workers to study 1-dimensional heat flow in flat-shaped human organs. Also, some attempts have been made to study heat flow in dermal regions of flat and cylindrical human organs for 2-dimensional cases. Attempts have also been made to study temperature distribution in skin and subcutaneous tissues involving abnormalities such as tumor in flat-shaped human organs for 1-dimensional and 2-dimensional cases. The seminumerical and finite element models are reported in the literature for the study of temperature distribution in cylindrical-shaped human organs, such as human limbs, for 2-dimensional and 3-dimensional cases with and without tumors.

But very little attention has been paid to the study of heat flow in sphere-shaped human organs. The thermoregulation in human head has been investigated under cold environmental conditions. The thermal modeling of women's breast under normal environmental conditions has been performed by researchers to study relationships among various physical parameters. Theoretical investigations have also been conducted to study the effect of sphere-shaped tumors in deep tissues of sphere-shaped women's breast on the surface temperature of the breast.

Recently, a 2-dimensional finite element model to study temperature distribution in peripheral regions of extended spherical human organs is reported in the literature. Also, the human breast was considered to be of semispherical shape in the above studies. However, the breast is not exactly of spherical shape. The breast is closer to ellipsoidal shape. Furthermore, tumors are also found to be closer to ellipsoidal shape. From the literature survey, it is observed that no attempt is reported for the study of thermal effect of ellipse-shaped tumors in the peripheral part of ellipsoidal-shaped human breast. In view of the above, an attempt has been made to develop a finite element model to study temperature variation in ellipse-shaped human breast involving ellipse-shaped nonuniformly perfused tumor in peripheral layers of breast. The model has been developed for a 2-dimensional steady-state case involving fixed and variable boundary conditions due to different environmental conditions. The numerical results are obtained to study temperature distribution in skin and subcutaneous tissues of ellipse-shaped human organs with and without tumors at low, moderate, and high environmental temperatures.

Mathematical Models

The partial differential equation for heat flow in the skin and subcutaneous tissues (SST) region is expressed as:

\[
\left( \rho_c \frac{\partial T}{\partial t} \right) = \nabla \cdot (K \nabla T) + m_s c_s \left(T_a - T\right) + \bar{S}
\]  

(1)
where the effect of blood flow and metabolic heat generation is given by the terms \( m_k \rho c_p(T_k - T) \) and \( S \), respectively. Here, \( K \) is the thermal conductivity of tissue, \( m_k \) is the blood mass flow rate, \( c_p \) is the specific heat of blood, \( T_k \) is the blood temperature, \( T \) is the tissue temperature at position \( r \) measured perpendicularly from the skin surface, \( \epsilon \) is the specific heat of tissues at time \( t \), and \( \rho \) is the tissue density. We consider \( T_d = T_k \) as the arterial blood temperature, as the blood flows in arteries from core at body core temperature.

Equation (1) for temperature distribution in living tissues for a 2-dimensional steady-state case in elliptical coordinates is given as follows:

\[
\frac{1}{d^2} \left( \sinh^2 \mu + \sin^2 \nu \right) \left[ K \frac{\partial}{\partial \mu} \left( \frac{\partial T}{\partial \mu} \right) + K \frac{\partial}{\partial \nu} \left( \frac{\partial T}{\partial \nu} \right) \right] + m_k \rho c_p(T_d - T) + S = 0
\]

Here, \( d \) is the eccentricity of elliptical layer, and \( \mu \) and \( \nu \) are the radial and angular coordinates of the ellipsoid-shaped human breast. To incorporate the metabolic heat generation in normal and malignant tissues, the term \( S \) is expressed as follows:

\[
\tilde{S} = S + W
\]

Here, \( S \) represents controlled metabolic heat generation in normal tissues and \( W \) represents uncontrolled rates of metabolic heat generation in malignant tissues. The terms \( S \) and \( W \) will depend on position. In the regions of normal tissues, \( W = 0 \), whereas in malignant tissues, \( W \) dominates over \( S \) and therefore \( \tilde{S} = S \).

The outer surface of the region is exposed to the environment, and heat loss at this surface takes place mainly due to conduction, convection, radiation, and evaporation.\(^{14}\) Hence, the boundary condition imposed at the outer surface is given as follows:

\[
-k \frac{\partial T}{\partial r} = h(T - T_a) + LE \text{ at } \mu = \mu_4, \nu \in (0, \pi)
\]

where \( h \) is the heat transfer coefficient, \( T_a \) is the atmospheric temperature, and \( L \) and \( E \) are latent heat and rate of sweat evaporation, respectively.

The 2 types of boundary conditions are imposed at the inner boundary as given below.

**Type 1 boundary condition**

At medium and higher atmospheric temperatures, the inner boundary is maintained at uniform core temperature \( T_k \). Thus, inner boundary is assumed to be at constant temperature \( T_k \). Hence, the condition at inner boundary is given as follows:

\[
T(\mu, \nu) = T_k \text{ at } \mu = \mu_0
\]

**Type 2 boundary condition**

At low atmospheric temperatures, the shell temperature of human breast is variable along angular direction \( \nu \). This is because the warm blood flows in arteries at 37°C from core of the trunk to the shell of human breast, and the same blood reaching extreme parts of the breast cools down and returns from extremities of the breast through veins at lower temperature than the body core temperature. Hence, the following boundary condition is imposed:

\[
T(\mu, \nu) = F(\nu) \text{ at } \mu = \mu_0
\]

\[
F(\nu) = a_1 + a_2 \nu + a_3 \nu^2
\]

Here, \( a_1, a_2, \) and \( a_3 \) are constants.

The values of \( a_1, a_2, \) and \( a_3 \) are found using the following conditions:

\[
T(\mu_0, \nu) = \alpha \text{ at } \nu = 0
\]

\[
T(\mu_0, \nu) = \beta \text{ at } \nu = \pi / 2
\]

\[
T(\mu_0, \nu) = \gamma \text{ at } \nu = \pi
\]

Here, the values of \( \alpha, \beta, \) and \( \gamma \) are constants, which can be assigned the values based on the temperature at selected points of the shell of human breast.

The dermal region of the breast is divided into 13 concentric elliptic layers with different eccentricities \( d_1, d_2, d_3, \ldots, d_{13} \).

**Solution**

Equation (1) along with the boundary condition equations (3), (4), and (5) in the variational form is written as given below:

\[
\mathcal{I}(\nu) = \frac{1}{2} \int_{\nu, \mu} \left[ \mathcal{K}(\nu) \left( \frac{\partial T}{\partial \mu} \right)^2 + \left( \frac{\partial T}{\partial \nu} \right)^2 \right] d\mu d\nu + \int_{\nu} \left[ m_k \rho c_p(T_d - T)^2 - 2S(\tau) \right] \right] d\mu d\nu
\]

\[
+ 2 \int_{\nu} A_j(\nu) \left[ T(T_d - T)^2 + 2LET(\nu) \right] dv \text{ for } \epsilon = 1(1)416
\]

\[
A_j(\nu) = d(\nu)^2 \left( \sinh^2 \mu + \sin^2 \nu \right)
\]

where \( \mu \) and \( \mu_j \) are boundaries of \( \epsilon \)th elements \( K(\nu), M(\nu), S(\nu), T(\nu) \) and \( \mathcal{T}(\nu) \) denotes the values of \( K, M, S, T, \) and \( T' \), respectively, in \( \epsilon \)th layer.\(^4\) \( \lambda(\nu) = 1 \) for elements along the surface and \( \lambda(\nu) = 0 \) for all elements which are not along the outer surface.

A woman’s breast is assumed to be of semilircular shape. Also, the physiological structure is assumed to be symmetric along angular directions involving nonuniformly perfused
tumor as shown in Figure 1. The SST region of the breast is divided into 13 layers, namely, epidermis as 1 layer, dermis as 11 layers, and subcutaneous tissues as 1 layer. The dermis is divided into larger number of layers to incorporate the nonhomogeneity of the subregions.

Here, ellipse-shaped nonuniformly perfused tumor is assumed to be situated in the dermis. The region is discretized into 416 triangular ring elements as shown in Figure 2. These large numbers of elements have been taken to incorporate the minute details of physiology, such as structure and the heterogeneity of the region. Dividing the region into large number of elements gives us flexibility in assigning the independent values to the physical and physiological parameters in each subregion. The blood flow and metabolic activity in tumor are found to vary 0 to 7 times than that in normal tissues (Table 1).

Accordingly, the different values have been assigned to blood mass flow and metabolic heat generation in nonuniformly perfused tumors. Because the triangular ring elements used here are very small in size, the linear shape function can be assumed for the variation of temperature in each element.

The following linear shape function for variation of temperature within each element has been taken:

$$T^{(e)} = c_1^{(e)} + c_2^{(e)} \mu + c_3^{(e)} \nu$$

where $c_1^{(e)}$, $c_2^{(e)}$, and $c_3^{(e)}$ are constants for the $e$th element.

Expression (7) is rewritten as follows:

$$T^{(e)} = p^T e^{(e)}$$

where

$$p^T = [1 \mu \nu]$$

$$e^{(e)} = \begin{bmatrix} c_1^{(e)} & c_2^{(e)} & c_3^{(e)} \end{bmatrix}$$

Using nodal conditions, we have the following:

$$T_p^{(e)} = T_{i,j,k}$$

We get the following expression using equations (9), (10), and (11):

$$\overline{T}^{(i)} = p^T e^{(e)}$$

where

$$\overline{T}^{(i)} = [T_i \ T_j \ T_k]$$
Where \(T_i, T_j, T_k\) are the temperatures at \(i\)th, \(j\)th, and \(k\)th nodes, respectively. \(\mu_i, \mu_j, \mu_k\) and \(\nu_i, \nu_j, \nu_k\) are radial and angular distances at \(i\)th, \(j\)th, and \(k\)th nodes, respectively.

From equation (12), we get the following equation:

\[c_{R}T_{le}()() = \frac{1}{2} \int \int A_i^{(l)}M^{(l)}(T_{A}^{(l)})^2 + (T_{A}^{(l)})^2 d \mu d \nu\]  

(19)

\[I_i^{(l)} = \int \int (M^{(l)} T_{A}^{(l)} + S_l^{(l)} + W^{(l)}) T_{A}^{(l)} d \mu d \nu\]  

(20)

\[I_k^{(l)} = \frac{\lambda}{2} \int \int \left( \delta (T^{(l)} - T_{A})^2 + 2LET^{(l)} \right) d \mu d \nu\]  

(21)

Because the solution domain has been divided into appropriate number of elements, the variations in each layer can be incorporated by assigning different and independent values to the parameters \(K, M, S, W\) in each element. The element sizes are smaller in dermis and tumor. This is because the variations in the physical and physiological parameters are more in these subregions. However, the properties are almost uniform in subdermal tissues and epidermis; hence, elements of bigger size have been used to discretize these subregions. On the basis of subdivision, the following independent values have been assigned to the parameters \(K, M, S, W\) in each element:

**Subdermal tissues:** \((\epsilon = 1)\) (26) 391 and 2 (26) 392

The population density of blood vessels is highest and uniform in subdermal tissues among the 3 layers. Therefore, \(K, M, S, W\) are the highest in this layer and taken as constants as given below:
\[ K^{(t)} = K_1, M^{(t)} = M_1, S^{(t)} = S_1, W^{(t)} = 0 \]

**Dermis (normal tissues):** \((e=23\,(26)\,390+i; i=3\,(1)\,22 \text{ and } e=24\,(26)\,413, e=24\,(26)\,414)\)

The population density of blood vessels in the dermis is very thin near the interface of epidermis, i.e., there are no blood vessels in epidermis, but it increases gradually and becomes almost uniform in the subdermal part.\(^3\) Therefore, \(K, M,\) and \(S\) are taken to be average of that in epidermis and dermis:

\[ K^{(t)} = K_2, M^{(t)} = M_2, S^{(t)} = S_2, W^{(t)} = 0 \]

**Epidermis:** \((e=25\,(26)\,415 \text{ and } e=26\,(26)\,416)\)

The blood flow is 0 and the thermal conductivity and metabolic activity are lowest in epidermis:

\[ K^{(t)} = K_3, M^{(t)} = M_3, S^{(t)} = S_3, W^{(t)} = 0 \]

**Tumor periphery:** \((e=140, 141, 142, 143, 144, 145, 164, 165, 172, 173, 189, 190, 199, 200, 225, 226, 242, 243, 250, 251, 270, 271, 271, 273, 274, 275)\)

Tumor periphery is rich in blood, and therefore, the thermal conductivity blood flow and metabolic activity are greater than those in normal tissues. The following constant values are assigned in tumor periphery:

\[ K^{(t)} = K_4, M^{(t)} = \tau^{(t)}M_1, S^{(t)} = 0, W^{(t)} = \eta^{(t)}S_1 \]

**Tumor core:** \((e=166-171, 191-198, 217-224, \text{ and } 244-249)\)

Tumor core is devoid of blood vessels and contains dead cells in the form of necrotic core. Therefore, \(K, M,\) and \(S\) in tumor core will be less than that in normal tissues. Thus, we take

\[ K^{(t)} = K_5, M^{(t)} = \tau^{(t)}M_1, S^{(t)} = 0, W^{(t)} = \eta^{(t)}S_1 \]

Here, \(\tau\) and \(\eta\) represent that the blood flow and metabolic activity in tumor are \(\tau\) and \(\eta\) times that in normal tissues.

Integral 1 is extremized with respect to each nodal temperature \(T_i\) as shown below:

\[ \frac{dI}{dT} = 0 \quad (22) \]

\[ \frac{dI}{dT} = \left[ \frac{\partial I}{\partial T_1} \frac{\partial I}{\partial T_2} \frac{\partial I}{\partial T_3} \ldots \ldots \frac{\partial I}{\partial T_n} \right]^{T} \quad (23) \]

\[ T = \left[ \begin{array}{c} \bar{T}_1 \\ \bar{T}_2 \\ \bar{T}_3 \\ \ldots \\ \bar{T}_n \end{array} \right] \]

(24)

Here, \(T_n\) denotes the \(n\)th nodal point temperature and \(n\) is the number of nodal points. Here, the problem region contains \(N=416\) elements and \(n=238\) nodes. Thus, we have the following equation:

\[ \frac{dI}{dT} = \sum_{i=1}^{N} D^{(i)} \left[ \frac{d^{(i)}}{d\bar{T}^{(i)}} + \frac{d^{(i)}}{d\bar{T}^{(i)}} + \frac{d^{(i)}}{d\bar{T}^{(i)}} + \frac{d^{(i)}}{d\bar{T}^{(i)}} \right] B^{(i)} \quad (25) \]

\[ \frac{dI}{dT} = \sum_{i=1}^{N} D^{(i)} \left[ \frac{d^{(i)}}{d\bar{T}^{(i)}} + \frac{d^{(i)}}{d\bar{T}^{(i)}} + \frac{d^{(i)}}{d\bar{T}^{(i)}} \right] B^{(i)} \quad (26) \]

where

\[ B^{(i)} = \begin{bmatrix} 0 & 0 & 0 \\ \cdots & \cdots & \cdots \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ \cdots & \cdots & \cdots \end{bmatrix} \quad (28) \]

This leads to a system of linear algebraic equations given below:

\[ [X]_{n \times n} \left[ \bar{T}_{n+1} \right] = [Y]_{n \times 1} \quad (29) \]

Here, \([X]_{n \times n}\) is the system matrix of order \(n \times n\), and \([Y]_{n \times 1}\) is the system vector of order \(n \times 1\).

The Gauss elimination method has been used to obtain the solution of system.\(^8\) A computer program in MATLAB 7.11 is developed to find numerical solution to the entire problem. The time taken for simulation is nearly 2 minutes on Core i3 CPU M 330 @ 2.13 GHz processing speed and 3 GB memory.

**Results and Discussion**

The numerical results are obtained using the values of physical and physiological constants given below:\(^{10}\):

\[ K_1 = 0.060 \text{ cal/cm min}^\circ C, K_2 = 0.045 \text{ cal/cm min}^\circ C, K_3 = 0.030 \text{ cal/cm min}^\circ C \]
The results are mesh insensitive at this number. We have used 416 elements as the level. A confidence level of 100 implies that a saturation point only. For better clarity, we take only the ratio of 4 decimal points.

Values of parameters.\textsuperscript{8, 10}

Table 2.

| Atmospheric Temperature $T_a$, °C | $M_i = (M_g C_p)_{\text{max}}$ | $S_i = S_i$ | $E = E_i$ |
|----------------------------------|-------------------------------|-------------|-----------|
| 15                               | 0.003                         | 0.0357      | 0.0       |
| 23                               | 0.018                         | 0.018       | 0.0, 0.24 \times 10^{-3}, 0.48 \times 10^{-3} |
| 33                               | 0.315                         | 0.018       | 0.0, 0.24 \times 10^{-3}, 0.48 \times 10^{-3}, 0.72 \times 10^{-3} |

$M_a = M_i / 2, M_g = 0, S_p = S_i / 2, S_o = 0$.

\[
K = 0.0845 \text{ cal/cm min}^\circ \text{C},\ T_a = 37^\circ \text{C},
\]

\[
h = 0.009 \text{ cal/cm}^2 \text{ min}^\circ \text{C}, L = 579 \text{ cal/g}
\]

The values of $M$, $S$, and $E$ used in this study are given in Table 2.

The simulation was performed for $N=416$ elements initially. Then, again the simulation was performed by taking $N=832$ elements. We obtain a temperature of 36.2666 at node numbers 117 and 121 in tumor periphery for model with $N=832$ elements. The error is [(36.2676 − 36.2666)/100]×100 which works out to be $47 \times 10^{-6}$%.

The expression for nodal information is given below:

Radial coordinates: for $i = k + 14j$:

\[j = 0(1)13\text{ and } k = 1(1)14\]

The constant $\mu_i (i = 0(1)14)$ can be assigned any value depending on particular sample of tissues layers under study. $\mu_i$ is assigned the following values for 3 different sizes of tumors. The results are computed for a particular sample of SST region of breast as given below:

For size 1, $\mu_1 = 8$ cm; $\mu_2 = 8.5$ cm; $\mu_3 = 8.54$ cm; $\mu_4 = 8.58$ cm; $\mu_5 = 8.60$ cm; $\mu_6 = 8.64$ cm; $\mu_7 = 8.68$ cm; $\mu_8 = 8.72$ cm; $\mu_9 = 8.76$ cm; $\mu_{10} = 8.78$ cm; $\mu_{11} = 8.82$ cm; $\mu_{12} = 8.86$ cm; $\mu_{13} = 8.9$ cm; $\mu_{14} = 9.1$ cm; area of tumor = 92 mm$^2$; circumference of tumor = 84.46 mm

For size 2, $\mu_1 = 8$ cm; $\mu_2 = 8.5$ cm; $\mu_3 = 8.54$ cm; $\mu_4 = 8.58$ cm; $\mu_5 = 8.60$ cm; $\mu_6 = 8.62$ cm; $\mu_7 = 8.64$ cm; $\mu_8 = 8.66$ cm; $\mu_9 = 8.68$ cm; $\mu_{10} = 8.70$ cm; $\mu_{11} = 8.82$ cm; $\mu_{12} = 8.86$ cm; $\mu_{13} = 8.9$ cm; $\mu_{14} = 9.1$ cm; area of tumor = 73 mm$^2$; circumference of tumor = 83.19 mm

For size 3, $\mu_1 = 8$ cm; $\mu_2 = 8.5$ cm; $\mu_3 = 8.54$ cm; $\mu_4 = 8.58$ cm; $\mu_5 = 8.60$ cm; $\mu_6 = 8.63$ cm; $\mu_7 = 8.66$ cm; $\mu_8 = 8.69$ cm; $\mu_9 = 8.72$ cm; $\mu_{10} = 8.74$ cm; $\mu_{11} = 8.82$ cm; $\mu_{12} = 8.86$ cm; $\mu_{13} = 8.9$ cm; $\mu_{14} = 9.1$ cm; area of tumor = 55 mm$^2$; circumference of tumor = 81.93 mm

The tumor is taken at the following 3 locations:

1. **Location 1.** Between $\nu = 75^\circ$ and $\nu = 105^\circ$
2. **Location 2.** Between $\nu = 15^\circ$ and $\nu = 45^\circ$
3. **Location 3.** Between $\nu = 150^\circ$ and $\nu = 180^\circ$

Eccentricity is given by $d_i = dk - 1$ for $i = k + 14 j$: $j = 0(1)13$ and $k = 1(1)14$

For any ellipse, we can define a number of related eccentricities depending on the sample of human organ under study. The following sets of eccentricities have been taken as a particular case:

\[
\begin{align*}
d_1 & = 0.0030 \text{ cm}, d_2 = 0.0028 \text{ cm}, d_3 = 0.0025 \text{ cm}, \\
d_4 & = 0.0023 \text{ cm}, d_5 = 0.0021 \text{ cm}, d_6 = 0.0020 \text{ cm}, \\
d_7 & = 0.0018 \text{ cm}, d_8 = 0.0017 \text{ cm}, d_9 = 0.0015 \text{ cm}, \\
d_{10} & = 0.0014 \text{ cm}, d_{11} = 0.0012 \text{ cm}, d_{12} = 0.0010 \text{ cm}, \text{ and } d_{13} = 0.0008 \text{ cm}.
\end{align*}
\]

For a particular case, the values of $\alpha$, $\beta$, and $\gamma$ are taken as follows:

\[
\alpha = 37^\circ \text{C}, \beta = 36^\circ \text{C}, \gamma = 37^\circ \text{C}
\]

The angular positions in degrees and radians are given in Table 3.

Graphs have been plotted among $T$, $\mu$, and $v$ for different values of atmospheric temperature, $E$ and $\eta$, for 2 types of boundary conditions. It is well known that the metabolic activity of tumor is found to vary 1 to 7 times than that in normal tissues. The blood mass flow rate and metabolic heat generation are very low in the tumor core and they go on decreasing with an increase in the size of tumor. Therefore, as a special case, we assume that the rate of blood flow in tumor core is one-tenth of normal tissues. Here, for particular samples of tumor, we take $\eta = 1.0$, $\eta = 3.0$, and $\eta = 5.0$, which implies that tumor periphery has metabolic activity which is 1, 3, and 5 times that in normal tissues, respectively.

**Discussion of Results for Type 1 Boundary Condition**

Figures 3 to 5 show temperature variations along $\mu$ for normal case $T_a = 23^\circ \text{C}$ and $E = 0.24 \times 10^{-3} \text{ g/cm}^2 \text{ min}$, and
Figures 6 to 8 show temperature variations along $\mu$ and $\nu$ for normal case $T_a = 33^\circ\text{C}$ and $E = 0.48 \times 10^{-3}$ $\text{g/cm}^2 \text{min}$. We observe that the temperature falls along radial direction from body core to outer surface of the extended elliptical human organs. This is because the heat flows mainly along radial direction and is lost to the environment from the outer surface.

Table 3. Angular positions in degrees and radians.

| ANGULAR POSITION | ANGULAR POSITION | ANGULAR POSITION | ANGULAR POSITION |
|------------------|------------------|------------------|------------------|
| DEGREES          | RADIANS          | DEGREES          | RADIANS          | DEGREES          | RADIANS          | DEGREES          | RADIANS          |
| 0.0              | 0.0              | 45.0             | 0.785            | 90.0             | 1.57             | 135              | 2.355            |
| 5.0              | 0.08722          | 50.0             | 0.8722           | 95.0             | 1.657            | 140              | 2.442            |
| 10.0             | 0.17444          | 55.0             | 0.9594           | 100              | 1.744            | 145              | 2.5294           |
| 15.0             | 0.261            | 60.0             | 1.0466           | 105              | 1.8316           | 150              | 2.616            |
| 20.0             | 0.348            | 65.0             | 1.1338           | 110              | 1.9188           | 155              | 2.7038           |
| 25.0             | 0.4361           | 70.0             | 1.221            | 115              | 2.0061           | 160              | 2.7911           |
| 30.0             | 0.5233           | 75.0             | 1.30833          | 120              | 2.0933           | 165              | 2.8783           |
| 35.0             | 0.6105           | 80.0             | 1.3955           | 125              | 2.1805           | 170              | 2.9655           |
| 40.0             | 0.6977           | 85.0             | 1.4827           | 130              | 2.267            | 175              | 3.053            |
|                  |                  |                  |                  |                  |                  |                  | 180              | 3.14             |

Figure 3. Temperature distribution along $\mu$ and $\nu$ in breast without tumor for $T_a = 23^\circ\text{C}$, $E = 0.48 \times 10^{-3}$, and type 1 boundary condition.

Figure 4. Temperature distribution along $\mu$ and $\nu$ in breast with nonuniformly perfused tumor for $T_a = 23^\circ\text{C}$, $E = 0.48 \times 10^{-3}$, $\eta = 3.0$, and type 1 boundary condition, size 1, and location 1.

Figure 5. Temperature distribution along $\mu$ and $\nu$ in breast with nonuniformly perfused tumor for $T_a = 23^\circ\text{C}$, $E = 0.48 \times 10^{-3}$, $\eta = 5.0$, and type 1 boundary condition, size 1, and location 1.
of the extended elliptical human organs. Also, we observe the change in slope of the curves at the junctions of peripheral layers of the breast. This is due to different biophysical properties of each layer.

Figure 4 shows temperature variation along \( \mu \) and \( \nu \) in peripheral layers of breast involving nonuniformly perfused tumor for \( T_a = 23°C, E = 0.48 \times 10^{-3} \text{ g/cm}^2 \text{ min} \), and \( \eta = 3.0 \). Figure 5 shows temperature variation along \( \mu \) and \( \nu \) in peripheral layers of breast involving nonuniformly perfused tumor for \( T_a = 23°C, E = 0.48 \times 10^{-3} \text{ g/cm}^2 \text{ min} \), and \( \eta = 5.0 \). In Figures 4 and 5, we observe the change in the slope of the curve at the junctions of normal tissues and tumor periphery, tumor periphery and tumor core, tumor core and tumor periphery, and tumor periphery and normal tissues and also at the junctions of different layers of normal tissues. In Figures 4 and 5, we observe the major thermal disturbances in tumor region. Figure 9 shows the temperature difference between normal case (Figure 3) and peripheral regions involving tumor at \( \eta = 3.0 \) (Figure 4). Figure 10 shows the temperature difference between normal case (Figure 3) and peripheral regions involving tumor at \( \eta = 5.0 \) (Figure 5). In Figure 6, we observe that the temperature difference between normal case and peripheral regions involving tumor is 0 at \( \mu = 8.0 \) and \( \nu = 0° \) to \( \pi \), i.e., at the core of the breast, and 0 between \( \nu = 0° \) to \( 60° \) and \( \nu = 120° \) to \( 180° \); this temperature difference increases from \( \nu = 60° \) to \( 80° \) and then decreases from \( \nu = 80° \) to \( 100° \) and again it increases from \( \nu = 100° \) to \( 120° \) and further falls to 0 after \( \nu = 120° \). The temperature difference along radial direction from \( \nu = 60° \) to \( 120° \) is observed from \( \mu = 8.1 \) cm, and it increases from \( \mu = 8.5 \) to 8.72 cm where it achieves its peak; then this difference in temperature decreases from \( \mu = 8.7 \) to 8.74 cm and again increases from \( \mu = 8.74 \) to 8.78 cm and further it decreases from \( \mu = 8.78 \) to 9.1 cm. Similar results are observed in Figure 10. The maximum temperature difference observed in Figure 9 is 0.598°C, and in Figure 10, it is 0.671°C. Thus, the temperature difference in Figure 10 is more compared with that in Figure 9. This is because of higher metabolic rates in tumor in case of Figure 10.

Similar results of temperature differences are seen in Figures 11 and 12 for \( T_a = 33°C \) and \( E = 0.72 \times 10^{-3} \text{ g/cm}^2 \text{ min} \).
peak temperature differences in Figures 11 and 12 are 0.495°C and 0.589°C, respectively. From Figures 9 to 12, we observe major variations in temperature differences in tumor region. The peaks in these figures are in tumor periphery, and the steep ridges between the peaks are the regions of tumor core.

Figures 13 and 14 show temperature distribution in human breast due to the presence of tumor at locations 2 and 3, respectively, for type 1 boundary condition, size 1 of tumor, $T_a = 23°C$, $E = 0.48 \times 10^{-3}$, $\eta = 5.0$. Figures 15 and 16 show differences in temperature distribution in human breast due to the presence and absence of tumors at locations 2 and 3, respectively, for type 1 boundary condition, size 1 of tumor, $T_a = 23°C$, $E = 0.48 \times 10^{-3}$, $\eta = 5.0$. Comparing Figures 5, 15, and 16, we see the shift in locations of thermal disturbances. This is due to the presence of tumor in different locations of human breast, as shown in the figures above.

**Discussion of Results for Type 2 Boundary Condition**

Figure 17 shows temperature variation in human breast for $T_a = 15°C$, $E = 0$, and normal case.
Figures 18 and 19 show temperature variation in breast with tumor for $T_a = 15^\circ$C, type 2 boundary condition, and for $\eta = 3.0$ and $\eta = 5.0$, respectively. We observe the effect of variable boundary condition at $\mu = 0$ and $\nu = 0^\circ$ to $\pi$ with temperature $37^\circ$C at $\nu = \theta$ and $\nu = \pi$ and $36^\circ$C at $\nu = \pi/2$. We observe that the temperature falls from core to the outer skin surface between $37^\circ$C and $30^\circ$C in Figure 17, i.e., in normal case, $37^\circ$C and $30.5^\circ$C in Figure 18 and $37^\circ$C and $31^\circ$C in Figure 19. In Figures 18 and 19, we observe the change in the slope of curves at the junctions of normal tissues and tumor periphery, tumor periphery and tumor core, tumor core and tumor periphery, and tumor periphery and normal tissues. Figures 20 and 21 show temperature differences between breast with and without uniformly perfused tumors. The peak values of temperature difference in Figures 20 and 21 are, respectively, 0.65°C and 0.767°C. The peaks and ridges give us idea about the location of different layers of nonuniformly perfused tumor in the breast. The peak values of temperature difference give us idea about the accuracy with which the equipment should be able to measure the temperatures to detect the tumors.

**Conclusions**

A 2-dimensional finite element model is proposed and used to study the effect of ellipse-shaped nonuniformly perfused tumor
on temperature distribution in peripheral regions of ellipse-shaped woman’s breast. The proposed model is one of the more realistic models in comparison with earlier models reported in the literature. The shape and structure of the breast in this study are close to the real woman’s breast. The existing models are able to provide impact of spherical-shaped tumor on spherical-shaped and particular type of structure of woman’s breast. The proposed model is able to provide the impact of a shape and size of tumor on thermal patterns in different realistic shapes, sizes, and structures of woman’s breast during different stages of development. The proposed model is also capable of providing information about thermal effect of nonuniformly perfused ellipse-shaped tumor in different stages of development of woman’s breast. The discretization of the breast into large number of triangular ring elements takes care of the heterogeneity of the region. On the basis of the result, it is concluded that the changes in the slope of the curve at the junctions of various peripheral and tumor layers are caused due to the nonhomogeneous nature of the region. Also, the tumor acts as a heat source, thereby causing thermal disturbances in the tumor and peripheral regions of the breast. The major thermal disturbances in the specific location indicate the presence of a tumor. The variations in the temperature at the junction of tumor and peripheral layers give us idea of the type, size, and location of the tumor. This information can be exploited in thermography for detection of tumors. From the results, it can be concluded that the thermal impact of larger tumor is more compared with smaller tumor. Hence, predicted precision of position of tumor will certainly change with the tumor size. The position precision is proportional to the size of tumor. This means that for smaller tumors, the thermographic equipment with higher sensitivity and specificity and precision is required. The finite element method used here has proved to be quite versatile because of nonuniformly perfused tumors in peripheral layers of women’s breast. In future, it is intended to use the present model as a basis to develop 3-dimensional model to perform these investigations in more detail with more realistic and different kinds of conditions in different dimensions. Such mathematical models can be developed to generate thermal information which can be useful to biomedical scientists and engineers for designing protocol for diagnosis of tumor by thermography and treatment of tumors by hyperthermia in woman’s breast. Hyperthermia is a technique of killing the tumor by heating. The thermal information gives us the idea about how much heat dose should be given so that malignant tissues are destroyed and normal cells are saved.

The model provides interesting information about the detection of position, size, and type of tumor in the breast. The thermograms which have been generated by our model can also be obtained by thermographic equipment in practice. The present model gives idea about which point of thermograms can be used as markers of position, boundaries, size, and type of tumors. The results of the models clearly show that points of change to slope of the curves indicate changes in property of tissues which can be used as thermal markers in thermography for detection of the boundaries in various layers of tumors and normal tissues. The thermographic technology has gained scientific acceptance and has been approved for screening purpose by Food and Drug Administration and is being used for safe early working detection system. The thermographic equipment available has the capacity to measure temperatures ranging from 10°C to 55°C with accuracy of 0.1°C. Therefore, thermography can be applicable in this case. In all, it is a new research progress direction in the field of thermal biology and its applications in medical sciences.

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Author Contributions
NA: The idea, logic of model. AM: model formulation, computational and interpretation of results are provided with writing work.

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