Optimal Resource Allocation for Uplink OFDMA in 802.11ax Networks

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Abstract

In this paper, we study the scheduling and resource allocation problem for uplink OFDMA in IEEE 802.11ax WLANs. The OFDMA resource allocation problem for 802.11ax is inherently difficult to solve because consecutive subcarriers are grouped into resource units, and every user is assigned to one resource unit at most. However, since power in the uplink is allocated to each user individually, the instantaneous utility maximisation problem can be formulated as an assignment problem, and hence, solved in polynomial time by the Hungarian algorithm. In order to provide long-term performance guarantees, we focus on maximizing the utility of average user rates subject to both peak and average power constraints, and solve the problem using Lyapunov optimization. To the best of our knowledge, this is the first work that provides a comprehensive utility optimization framework for uplink OFDMA in 802.11ax networks.

Keywords: IEEE 802.11ax, OFDMA, Resource Allocation, Lyapunov Optimization

I. INTRODUCTION

The ever-growing demand for fast and ubiquitous wireless connectivity poses the challenge of delivering high data rates while efficiently managing the scarce radio resources. Orthogonal frequency division multiplexing (OFDM) has become a mainstream transmission method for broadband wireless systems. This is because it can convert a frequency selective channel into multiple flat fading subchannels and eliminate intersymbol interference, enabling in this way high achievable rates. Additionally, thanks to the independent fading of multiple users, efficient spectrum utilization can be attained by exploiting the so-called multiuser diversity. More specifically, OFDM subchannels (i.e., subcarriers) can be dynamically allocated among multiple users according to their instantaneous channel conditions. The multiuser version of OFDM, dubbed orthogonal frequency division multiple access (OFDMA), has been therefore recognised as a key technology for next-generation wireless systems.

Towards this direction, the upcoming 802.11ax amendment for high efficiency wireless local area networks (WLANs) will support OFDMA in both downlink and uplink (UL) directions. The current 802.11ac standard employs multiuser MIMO to increase the downlink throughput while stations still contend for the UL channel using carrier-sense multiple access with collision avoidance (CSMA/CA). As such, the most novel feature of 802.11ax is rather the UL OFDMA transmissions triggered by the access point (AP). In particular, AP will be able to solicit the stations to transmit through a special
control frame [2]. However, the efficiency of OFDMA transmissions mainly depends on how the AP schedules the stations and allocates the available resources. Therefore, intelligent scheduling and resource allocation is crucial for attaining the best possible system performance.

Most of the prior works on resource allocation for OFDMA assume that multiple (possibly non-consecutive) subcarriers can be assigned to a single user. Under this assumption, it has been shown that greedy subcarrier allocation, i.e., each subcarrier is allocated to the user with the best channel gain, in conjunction with waterfilling maximises the sum rate of downlink OFDMA systems [3]. Similar greedy-based resource allocation schemes have been derived for the weighted-sum rate maximisation problem as well as for the stochastic utility maximisation problem subject to minimum average rate constraints [4], [5], [6], [7]. However, none of these schemes can be applied to 802.11ax due to its peculiar subcarrier allocation model. More particularly, in 802.11ax consecutive OFDMA subcarriers are grouped into resource units (RUs) and each user is assigned to one RU at most.

There are few recent works on the scheduling and resource allocation problem for UL OFDMA in 802.11ax. In [8], the authors proposed a framework based on Lyapunov optimisation to dynamically adjust the OFDMA transmission duration so that padding overhead is minimised. To do so, they assumed simple round-robin user scheduling and fixed RU allocation. The problem of joint user scheduling and RU allocation was firstly studied in [9], [10]. Specifically, D. Bankov et al. proposed a set of multiuser schedulers using the idea that the instantaneous utility maximisation problem can be formulated as an assignment problem. However, they did not incorporate in their analysis the power constraints that users might have to satisfy neither the frequency selectivity of the wireless channel, which can significantly affect the scheduling and resource allocation decisions. In addition, they considered only maximising instantaneous performance metrics. Thus, the temporal dimension is not being exploited when the resource allocation is performed.

In this paper, we address the problem of maximising the utility of average user rates subject to both peak and average power constraints. This is accomplished as following:

(i) We employ Lyapunov optimisation [13] to convert the original problem into an instantaneous maximisation problem (IMP).

(ii) We show that the IMP can be formulated as an assignment problem, and hence, solved in polynomial time using the Hungarian algorithm. [11].

We finally derive the resource allocation policy that attains a solution arbitrarily close to the optimal one.
II. SYSTEM MODEL

Consider an UL OFDMA scenario where $K$ users seek to transmit packets to the AP over $N$ RUs. Let $\mathcal{K} = \{1, \ldots, K\}$ denote the set of users and $\mathcal{N} = \{1, \ldots, N\}$ the set of RUs. Each RU $n$ consists of $N_{\text{sc}}(n)$ data subcarriers and every subcarrier undergoes independent flat fading. Specifically, the channel gain of user $k$ over subcarrier $i$ of RU $n$ is denoted by $g_{k,i}^{(n)}$. Let $g_k = \left( g_{k,i}^{(n)} \right)_{i,n}$ be the vector containing all channel gains regarding user $k$. The channel state vector is then defined as $g = (g_1, \ldots, g_K)$.

A. Scheduling Periods and Resource Allocation

We assume that time is divided into scheduling periods denoted by $t \in \{0, 1, 2, \ldots\}$. In every scheduling period $t$, the AP triggers an UL OFDMA transmission of duration $T$. If there is an ongoing transmission at the beginning of period $t$, the UL OFDMA transmission is deferred until the channel is sensed idle, as shown in Fig. 1. The channel process $\{g(t)\}_{t=0}^{\infty}$ is assumed to be independent and identically distributed (IID). Let $p_k(t) = (p_{k,1}(t), \ldots, p_{k,N}(t))$ denote the power allocation vector of user $k$ for period $t$. If $p_{k,n}(t) > 0$ then RU $n$ is assigned to user $k$ for the UL OFDMA transmission of period $t$ and $p_{k,n}(t) = 0$, otherwise. The overall power allocation vector is given by $p(t) = (p_1(t), \ldots, p_K(t))$.

Every scheduling period $t$, the AP observes the random event $g(t)$ and chooses a power allocation vector $p(t)$ within an option set $\mathcal{P}$. This action yields a transmission rate vector $r(t) = (r_1(t), \ldots, r_K(t))$ and a power consumption vector $p_c(t) = (p_1(t), \ldots, p_K(t))$. Specifically, $r_k(t) = f_k(p(t), g(t))$ is the rate of user $k$ in period $t$ and is given by a general function $f_k(\cdot, \cdot)$ modelling the rate allocation scheme. Likewise, $p_k(t) = \sum_{n=1}^{N} p_{k,n}(t)$ is the total power allocated to user $k$ in period $t$. 

Fig. 1: Schematic representation of UL OFDMA transmissions considered in the system model. If there is an on-going transmission at the beginning of a scheduling period, then the UL OFDMA transmission is deferred until the channel is sensed idle.
The time average expected values of these two vectors are defined as

\[
\bar{r} = \lim_{t \to \infty} \sup_{\tau} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}[r(\tau)]
\]

\[
\bar{p}_c = \lim_{t \to \infty} \sup_{\tau} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}[p_c(\tau)].
\]

Power allocation decisions \( p(t) \) have to satisfy the following constraints:

C1: \( \sum_{k=1}^{K} 1\{p_{k,n}(t) > 0\} \leq 1 \quad \forall n \)

C2: \( \sum_{n=1}^{N} 1\{p_{k,n}(t) > 0\} \leq 1 \quad \forall k \)

C3: \( 0 \leq p_{k,n}(t) \leq p_{k,\text{max}} \quad \forall k, n \)

where \( 1\{\cdot\} \) denotes the indicator function. Constraint C1 ensures that each RU \( n \) is assigned to one user at most. Similarly, C2 means that each user \( k \) is assigned to one RU at most. Finally, constraints C2-C3 imply that the instantaneous transmit power of each user \( k \) cannot exceed a predescribed limit \( p_{k,\text{max}} \). The set \( \mathcal{P} \) of feasible power allocation vectors is then defined as

\[
\mathcal{P} = \{ p \in \mathbb{R}^{K \times N} \mid p \text{ satisfies C1-C3} \}
\]

III. PROBLEM STATEMENT

We seek to find a resource allocation policy that selects \( p(t) \) at each scheduling period \( t \in \{0, 1, 2, \ldots\} \) such that

\[
\max_{p(t)} U(\bar{r}) \quad \text{s.t.} \quad \bar{p}_k \leq p_{k,\text{av}}, \quad \forall k \in \mathcal{K}
\]

\[
p(t) \in \mathcal{P}
\]

where \( U(\cdot) \) is a concave, continuous and entrywise non-decreasing utility function whereas \( p_{k,\text{av}} \) corresponds to an average power consumption constraint for user \( k \). The utility function in our formulation is to balance the total throughput and fairness among users. For example, it has been shown that the so-called \( a \)-fairness can be attained by the maximizer of a class of concave utility functions [12]

\[
U_a(x) = \begin{cases} 
\frac{x^{1-a}}{1-a}, & a > 1, a \neq 0 \\
\log(x), & a = 0
\end{cases}
\]

Additionally, \( \max\text{-min} \) fairness can be attained by setting \( U(\bar{r}) = \min\{\bar{r}_1, \ldots, \bar{r}_K\} \). Since \( \mathcal{P} \) is a non-convex set, problem (1) is non-convex. In the sequel, we derive an \( O(\epsilon) \)-optimal solution using Lyapunov optimisation.
**Definition 1.** Let $U^{\text{opt}}$ be maximum utility of problem (I). A control policy is said to produce an $O(\epsilon)$-optimal solution if

$$U(\bar{r}) \geq U^{\text{opt}} - O(\epsilon)$$

and all the constraints $\bar{p}_k \leq p_k^{av}$ are satisfied.

**IV. SOLUTION VIA LYAPUNOV OPTIMISATION**

**A. Virtual Queues**

In this subsection, we explain in detail how virtual queues can be used to fulfil average constraints (14). Consider for each constraint $\bar{p}_k \leq p_k^{av}$ of (I) a virtual queue the evolves over $t$ as

$$Q_k(t + 1) = [Q_k(t) - p_k^{av} + p_k(t)]^+$$

where $p_k^{av}$ is the constant virtual service rate and $p_k(t)$ is the virtual arrival process.

**Lemma 1.** If all queues $Q_k$ are mean rate stable, i.e., $\lim_{t \to \infty} \frac{\mathbb{E}[Q_k(t)]}{t} = 0$, then constraints $\bar{p}_k \leq p_k^{av}$ are fulfilled.

**Proof.** For each $\tau \in \{0, 1, \ldots \}$, it holds

$$Q_k(\tau + 1) \geq Q_k(\tau) + p_k(\tau) - p_k^{av}$$

This is because $\max(x, 0) \geq x$. Thus,

$$Q_k(\tau + 1) - Q_k(\tau) \geq p_k(\tau) - p_k^{av}$$

Summing over $\tau \in \{0, 1, \ldots, t - 1 \}$ for some integer $t > 0$ gives (through telescoping sum)

$$Q_k(t) - Q_k(0) \geq \sum_{\tau=0}^{t-1} p_k(\tau) - t p_k^{av}.$$ 

Diving by $t$ and using the fact that $Q_k(0) = 0, \forall k$, gives

$$\frac{Q_k(t)}{t} \geq \frac{1}{t} \sum_{\tau=0}^{t-1} p_k(\tau) - p_k^{av}.$$ 

Taking $\lim_{t \to \infty} \mathbb{E}[:]$ and rearranging the terms, yields

$$\bar{p}_k(t) \leq p_k^{av} + \lim_{t \to \infty} \frac{\mathbb{E}[Q_k(t)]}{t}$$

Therefore if $Q_k$ is mean rate stable, then we have

$$\bar{p}_k(t) \leq p_k^{av}$$

\qed
B. The Transformed Problem

In 802.11ax, there is a finite set of available data rates, say $\mathcal{R} = \{r_{\min}, \ldots, r_{\max}\}$. Therefore, the variables $\{r_k\}$ are bounded. Let $\gamma(t) = (\gamma_1(t), \ldots, \gamma_K(t))$ be a vector of auxiliary variables chosen within the set $\Gamma = \mathcal{R}^K$ and $\overline{U(\gamma)} = \lim_{t \to \infty} \frac{1}{t} \sum_{t=0}^{t-1} \mathbb{E}[U(\gamma(t))]$. We consider the following transformed problem

$$\max \quad \overline{U(\gamma)}$$

s.t. $\bar{\gamma}_k \leq \bar{r}_k, \quad \forall k \in \{1, \ldots, K\}$

$\bar{p}_k \leq \bar{p}_k^{av}, \quad \forall k \in \{1, \ldots, K\}$

$\gamma(t) \in \Gamma$

$p(t) \in \mathcal{P}$

The transformed problem involves only time averages rather than nonlinear functions of time averages. Hence it can be solved with the drift-plus-penalty (DPP) algorithm. The connection between (1) and (2) is established as follows: consider a control policy $\pi$ that solves problem (2). Then maximum utility value $\overline{U(\gamma_\pi)}$ is achieved and all constraints are met, e.g., $\bar{r}_\pi \geq \bar{\gamma}_\pi$. Because $\overline{U(\cdot)}$ is concave, it holds

$$\bar{r}_\pi \geq \bar{\gamma}_\pi \Rightarrow \overline{U(\bar{r}_\pi)} \geq \overline{U(\bar{\gamma}_\pi)} \geq \overline{U(\gamma_\pi)}$$

(3)

where the last inequality is Jensen’s inequality for concave functions. What remains to show is that policy $\pi$ achieves an $O(\epsilon)$-optimal solution for the original problem (1), i.e.,

$$\overline{U(\gamma_\pi)} \geq U^{opt} - O(\epsilon)$$

If so, we have

$$U(\bar{r}_\pi) \geq U^{opt} - O(\epsilon)$$

C. The Drift-Plus-Penalty Algorithm

Consider virtual queues $Q_k$ and $Z_k$ for the constraints $\bar{\gamma}_k \leq \bar{r}_k$ and $\bar{p}_k \leq \bar{p}_k^{av}$, respectively:

$$Q_k(t + 1) = \left[Q_k(t) - p_k^{av} + p_k(t)\right]^+$$

$$Z_k(t + 1) = \left[Z_k(t) - r_k(t) + \gamma_k(t)\right]^+$$

DPP Algorithm

Set $Z(0) = Q(0) = 0$. For every scheduling period $t \in \{0, 1, 2, \ldots\}$ do:

(i) Observe $Z(t), Q(t)$ and channel state $g(t)$
(ii) Choose $\gamma(t) \in \Gamma$ such that
\[
\max \ VU(\gamma(t)) - \sum_{k=1}^{K} Z_k(t)\gamma_k(t) 
\] (4)

(iii) Choose $p(t) \in \mathcal{P}$ such that
\[
\max \ \sum_{k=1}^{K} (Z_k(t)R_k(t) - Q_k(t)p_k(t)) 
\] (5)

(iv) Update the virtual queues $Z(t)$ and $Q(t)$.

**Theorem 1.** Suppose $\{g(t)\}_{t=0}^{\infty}$ is IID over the scheduling periods. Then for a given constant $V > 0$, the drift-plus-penalty algorithm on the transformed problem achieves an $O(1/V)$-optimal solution to problem (1).

**Proof.** See Appendix. \qed

V. INSTANTANEOUS MAXIMISATION SUBPROBLEMS AND THE ALGORITHM

Let $U(\gamma) = \sum_{k=1}^{K} U_k(\gamma_k)$. For the auxiliary variable $\gamma$, we have to solve
\[
\max \ \sum_{k=1}^{K} (VU_k(\gamma_k) - Z_k\gamma_k) 
\]
\[
\text{s.t. } \gamma \in \Gamma 
\]

It is easy to see that $\gamma^*_k = \arg \max_{\gamma_k \in \mathbb{R}} (VU_k(\gamma_k) - Z_k\gamma_k)$. For the power allocation vector $p$, we introduce the binary variables $\{s_{k,n}\}$ to indicate if user $k$ is assigned to RU $n$. Furthermore, let $R(p_k, k, n)$ be the rate of user $k$ when transmits over RU $n$ with power $p_k$. Then we can rewrite the maximisation problem regarding the power allocation as
\[
\max \ \sum_{n=1}^{N} \sum_{k=1}^{K} s_{k,n} (Z_kR(p_k, k, n) - Q_kp_k) 
\]
\[
\text{s.t. } s_{k,n} \in \{0, 1\} \ \forall k, n \ \ 0 \leq p_k \leq p_k^{\max} \ \forall k \ \ 
\]
\[
\sum_{k=1}^{K} s_{k,n} \leq 1 \ \forall n \ \ 
\]
\[
\sum_{n=1}^{N} s_{k,n} \leq 1 \ \forall k \ 
\]

Since each user has its own power budget $p_k^{\max}$, problem (6) is divided into the following subproblems:
1) (optimal power allocation) For each user $k$ and RU $n$ do:
\[
\phi_{k,n}^* = \max \ Z_k R(p_k, k, n) - Q_k \rho_k \\
\text{s.t.} \quad 0 \leq p_k \leq p_{k,\text{max}}
\] (7)

2) (optimal user/RU assignment)
\[
\max \sum_{n=1}^{N} \sum_{k=1}^{K} s_{k,n} \phi_{k,n}^* \\
\text{s.t.} \quad s_{k,n} \in \{0, 1\}, \quad \forall k \in \mathcal{K}, \forall n \in \mathcal{N} \\
\sum_{k=1}^{K} s_{k,n} \leq 1, \quad \forall n \in \mathcal{N} \\
\sum_{n=1}^{N} s_{k,n} \leq 1, \quad \forall k \in \mathcal{K}
\] (8)

Problem (8) is a classic assignment problem. Hence it can be solved in $O(M^3)$ by the Hungarian algorithm, where $M = \max(K, N)$.

**Algorithm** DPP for 802.11ax
1: Set $\gamma_k^*(t) = \arg\max_{\gamma_k(t) \in \mathcal{R}} (VU_k(\gamma_k(t)) - Z_k(t)\gamma_k(t)), \forall k \in \mathcal{K}$.
2: Solve problem (7) to determine the optimal power allocation.
3: Solve problem (8) to determine the optimal RU assignment.

The computational complexity of the algorithm is $O(K + KNL + M^3)$, where $L$ is the number of available modulation and coding schemes.
VI. SIMULATION RESULTS

Fig. 2: $U(\bar{r}) = \sum_{k=1}^{K} \log(1 + \bar{r}_k), K = N = 8, p_k^{\text{max}} = 20 \text{ dBm} \ \forall k \in \mathcal{K}$. 
APPENDIX

PROOF OF THEOREM 1

Let $\Theta(t) = (Q(t), Z(t))$ and consider the quadratic Lyapunov function

$$L(\Theta(t)) = \frac{1}{2} \sum_{k=1}^{K} Q_k(t) + \frac{1}{2} \sum_{k=1}^{K} Z_k(t)$$

The conditional Lyapunov drift is defined as

$$\Delta(\Theta(t)) = \mathbb{E}[L(\Theta(t+1))] - L(\Theta(t))$$

It is easy to prove the following bound on the drift-plus-penalty expression:

$$\Delta(\Theta(t)) - \mathbb{E}[U(\gamma(t)) | \Theta(t)] \leq B - \mathbb{E}[U(\gamma(t)) | \Theta(t)] + \sum_{k=1}^{K} Q_k(t) \mathbb{E}[p_k(t) - p^a_k | \Theta(t)]$$

$$+ \sum_{k=1}^{K} Z_k(t) \mathbb{E}[\gamma_k(t) - r_k(t) | \Theta(t)]$$

where $B = \frac{1}{2} \sum_{k=1}^{K} (p^a_k)^2 + (p^m_k)^2 + 2r^2_{\max}$. Denote the drift-plus-penalty achieved by DPP as $\Delta(\Theta(t)) - \mathbb{E}[U(\gamma_\pi(t)) | \Theta(t)]$. Then we have

$$\Delta(\Theta(t)) - \mathbb{E}[U(\gamma_\pi(t)) | \Theta(t)] \leq B - \mathbb{E}[U(\gamma_\pi(t))] + \sum_{k=1}^{K} Q_k(t) \mathbb{E}[p_{k,\omega}(t) - p_{k,\pi}^a]$$

$$+ \sum_{k=1}^{K} Z_k(t) \mathbb{E}[\gamma_{k,\omega}(t) - r_{k,\omega}(t)]$$

(9)

where $\omega$ refers to any other (possibly randomized) policy. In [13], it was proven that there exists a randomized policy $\omega$ that satisfies for any $t \in \{0, 1, 2, \ldots\}$:

$$-U(\gamma_\omega(t)) \leq -U^{opt}$$

$$\mathbb{E}[p_{k,\omega}(t) - p_{k,\pi}^a] \leq 0, \quad \forall k \in \{1, \ldots, K\}$$

$$\mathbb{E}[\gamma_{k,\omega}(t) - r_{k,\omega}(t)] \leq 0, \quad \forall k \in \{1, \ldots, K\}$$

Plugging these inequalities into (9) gives

$$\Delta(\Theta(t)) - \mathbb{E}[U(\gamma_\pi(t)) | \Theta(t)] \leq B - U^{opt}$$

By using iterated expectations and telescoping sums, we take

$$\mathbb{E}[L(\Theta(t))] - \mathbb{E}[L(\Theta(0))] - V \sum_{\tau=0}^{t-1} \mathbb{E}[U(\gamma_\pi(\tau))] \leq tB - ViU^{opt}$$

(10)

(i) Constraint Satisfaction: Using the boundness assumption

$$U_{\min} \leq \mathbb{E}[U(\gamma_\pi(\tau))] \leq U_{\max}, \quad \forall \tau \in \{0, 1, \ldots\}$$
Eq. (10) gives
\[ \frac{1}{2} \sum_{k=1}^{K} \mathbb{E}[Q_k^2(t)] \leq \mathbb{E}[L(\Theta(0))] + Bt + Vt(U_{\text{max}} - U_{\text{opt}}) \]
and therefore
\[ \mathbb{E}[Q_k^2(t)] \leq 2\mathbb{E}[L(\Theta(0))] + 2Bt + 2Vt(U_{\text{max}} - U_{\text{opt}}), \quad k \in K. \]
Because the variance of \( Q_k(t) \) cannot be negative, i.e., \( \mathbb{E}[Q_k^2(t)] \geq \mathbb{E}^2[Q_k(t)] \), it holds
\[ \mathbb{E}[Q_k(t)] \leq \sqrt{2\mathbb{E}[L(\Theta(0))] + 2Bt + 2Vt(U_{\text{max}} - U_{\text{opt}})} \]
Dividing by \( t \), taking the limit \( t \to \infty \) and using that \( \mathbb{E}[L(\Theta(0))] < \infty \), we finally get
\[ \lim_{t \to \infty} \frac{\mathbb{E}[Q(t)]}{t} \leq 0 \]
hence all queues are mean rate stable.

(ii) **Optimal Value:** Rearranging the terms, Eq. (10) we get
\[ VtU_{\text{opt}} - \mathbb{E}[L(\Theta(0))] - tB \leq VtU_{\text{opt}} + \mathbb{E}[L(\Theta(t))] - \mathbb{E}[L(\Theta(0))] - tB \leq V \sum_{\tau=0}^{t-1} \mathbb{E}[U(\gamma_\pi(\tau))] \]
and therefore
\[ V \sum_{\tau=0}^{t-1} \mathbb{E}[U(\gamma_\pi(\tau))] \geq VtU_{\text{opt}} - \mathbb{E}[L(\Theta(0))] - tB \]
Dividing by \( tV \) and taking the limit \( t \to \infty \), yields
\[ \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}[U(\gamma_\pi(\tau))] \geq U_{\text{opt}} - \frac{B}{V} \]
or equivalently
\[ \overline{U(\gamma_\pi)} \geq U_{\text{opt}} - B\epsilon \]
where \( \epsilon = 1/V \). Since all virtual queues are mean rate stable, (3) holds and
\[ U(\mathbf{r}_\pi) \geq U_{\text{opt}} - O(\epsilon) \]
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