Dynamical relaxation and the orbits of low–mass extrasolar planets

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Accepted. Received; in original form

ABSTRACT

We consider the evolution of a system containing a population of massive planets formed rapidly through a fragmentation process occurring on a scale on the order of 100 au and a lower mass planet that assembles in a disc on a much longer timescale. During the formation phase, the inner planet is kept on a circular orbit due to tidal interaction with the disc, while the outer planets undergo dynamical relaxation. Interaction with the massive planets left in the system after the inner planet forms may increase the eccentricity of the inner orbit to high values, producing systems similar to those observed.

Key words: giant planet formation – extrasolar planets – dynamical relaxation – orbital elements

1 INTRODUCTION

The extrasolar planets discovered so far have masses, semi-major axes and eccentricities in the range 0.16–11 Jupiter masses ($M_J$), 0.038–4.47 au and 0–0.93, respectively. In general, high eccentricities are very difficult to explain in the context of a model where an isolated planet forms in a disc, as the disc–protoplanet interaction leads to strong eccentricity damping (Nelson et al. 2000).

Papaloizou & Terquem (2001, hereafter PT01) investigated a scenario in which a population of planetary mass objects formed rapidly through a fragmentation process occurring in a thick disc or protostellar envelope on a scale of 100 au. Such a system then underwent dynamical relaxation on a timescale of hundreds of orbits which resulted in ejection of most of the objects. It was found that the characteristics of massive eccentric extrasolar planets and the massive ‘hot Jupiter’ observed so far might be accounted for by such a model. However, planets with masses smaller than a few $M_J$ are probably too small to have formed through a gravitational instability or fragmentation process (Masunaga & Inutsuka 1999, Boss 2000) and are thus more likely to have grown through the core accretion mechanism in a protoplanetary disc (Mizuno 1980).

Here, we investigate a scenario in which we have a planet accumulated in a disc (PAID) on a timescale of $10^6$ years, being the observed lifetime of protostellar discs (Haisch, Lada & Lada 2001), together with a population of massive outer planets already formed through fragmentation processes on a much shorter timescale. During its formation phase, the PAID is kept in a circular orbit by tidal interaction with the disc while the system of outer planets undergoes dynamical relaxation. After disc dispersal occurring at $t = 10^6$ years, the eccentricity of the PAID can be pumped up to high values by interaction with the remaining bound massive planets.

In order to focus on the interaction mechanisms, we analyse the motion of a planet under a distant perturber in a highly eccentric orbit in § 2. In § 3 we present numerical simulations of the evolution of a system containing outer massive planets and an inner PAID interacting with each other. Finally, in § 4 we discuss our results.

2 SECULAR PERTURBATION THEORY AND LONG TERM CYCLES

We consider the evolution of the orbit of a PAID around a central star due to gravitational interaction with a massive long period perturber on a highly eccentric orbit that may have resulted from a prior relaxation process. We illustrate three possible effects. The first is the appearance of long period cycles in which the inner planet may attain high eccentricity. The second, occurring for stronger perturbations, is a sequence of dynamical interactions leading to
the collision of the inner planet with the central star. The third, occurring for weakly bound perturbers, is an interaction leading to the gradual ejection of the outer planet to large radii, where additional perturbations from external objects could cause it to become unbound, leaving the inner planet on a highly eccentric orbit.

To discuss these three scenarios, we denote the osculating semi-major axes of the inner planet and the perturber by \( a \) and \( a_p \), respectively. The corresponding eccentricities are \( e \) and \( e_p \), and the masses are \( m \) and \( m_p \), respectively. The central mass is \( M_\star \).

When the pericentre distance of the perturber, \( d_p = a_p(1 - e_p) \), significantly exceeds \( a \), the large difference in the orbital periods reduces the significance of short term variations such that the interaction can be discussed using secular perturbation theory, in which one considers evolution of the time averaged orbits.

Consider first the case when the orbits are coplanar. Then analytic treatment is possible in the limit \( m \ll m_p \). We denote \((r, \phi)\) and \((r_p, \phi_p)\) the cylindrical coordinates of the inner and outer mass, respectively, in a frame centered on the star. The perturbing potential energy or Hamiltonian of the inner planet, including the indirect term (which accounts for the acceleration of the origin of the coordinate system), is:

\[
H = -\frac{Gmm_p}{\sqrt{r^2 + r_p^2 - 2rr_p\mu}} + \frac{Gmm_p\mu r_p}{r_p^2},
\]

(1)

where \( \mu = \cos(\phi - \phi_p) \). For \( r/r_p \ll 1 \), we expand \( H \) in spherical harmonics retaining terms up to third order in \( r/r_p \):

\[
H = -\frac{Gmm_p}{r_p} \left[ \left( \frac{r}{r_p} \right)^2 P_2(\mu) + \left( \frac{r}{r_p} \right)^3 P_3(\mu) \right],
\]

(2)

where \( P_n \) is the Legendre polynomial of order \( n \). We have omitted the term \(-Gmm_p/r_p\) as it does not depend on the coordinates of the inner planet \((H\) is defined within the addition of a constant\)). On performing time averages over both orbits (see, e.g., Roy 1978), one obtains correct to second order in \( e \) but with no restriction on \( e_p \):

\[
H = -\frac{Gmm_p a_p^2}{(1 - e_p^2)^{3/2}} \left[ \frac{3e^2}{4} - \frac{15e e_p a \cos \varpi}{16a_p (1 - e_p^2)} \right].
\]

(3)

Here \( \varpi \) is the angle between the apsidal lines of the two orbits. For \( m \ll m_p \), we can consider that the orbit of the outer planet is fixed, i.e. that \( a_p \) and \( e_p \) are constant. The secular evolution is characterized by a constant \( a \) and oscillations of \( e \) (see, e.g., Murray & Dermott 1999). Since \( dc/dt \propto \partial H/\partial e \) (Lagrange's equation), \( e \) is maximum for \( \varpi = 0 \). During the evolution \( H(e, \varpi) \) is constant. Since \( e \) passes through zero (the inner orbit has no initial eccentricity), the maximum value of \( e \), \( e_{\text{max}} \), can then be calculated by writing \( H(0, \varpi) = H(e_{\text{max}}, 0) \). This gives \( e_{\text{max}} = 5ae_p/[2a_p (1 - e_p^2)] \).

For illustrative example we take \( e_p = 0.9 \) and \( a/a_p = 0.01 \), so that \( d_p/a = 10 \). Then \( e_{\text{max}} = 0.118 \) which is significant and independent of \( m_p \).

Below we consider the case \( a = 0.5 \) au, \( a_p = 50 \) au, \( m = 0.3 \) M\(_J\) and \( m_p = 7 \) M\(_J\). It turns out that for such small \( a \), relativistic effects have to be taken into account. This can be done following the procedure given in Lin et al. (2000), according to which the central potential is modified such that:

\[
-\frac{G M_\star}{r} \rightarrow -\frac{G M_\star}{r} \left( 1 + \frac{3GM_\star}{r c^2} \right).
\]

(4)

This amounts to adding the term \(-3(GM_\star)^2m(1 + e^2/2)/(ac^2)\) on the right hand side of equation (3). From this one readily finds that \( e_{\text{max}} \) is reduced by a factor \( 1 + 4(GM_\star)/(ac^2))(M_\star/m_p)(a_p/a)^3(1 - e_p^2)^{3/2} \). For the above parameters and \( M_\star = 1 \) M\(_\odot\), this factor is 1.93, giving \( e_{\text{max}} \approx 0.06 \).

We have calculated the orbital evolution numerically using the Bullirsh–Stoer method (e.g., Press et al. 1993). The outer planet was started at pericentre and the inner one on a circular orbit at random phase. Of course, in contrast to the above analysis, both orbits were allowed to vary. Figure 1 shows the evolution of the orbital eccentricities as a function of time. The results are in good agreement with the analysis, exhibiting eccentricity cycles with amplitude \( e_{\text{max}} \approx 0.06 \) as expected. This is much lower than typical values observed for extrasolar planets. However, much higher eccentricities can be obtained through the Kozai mechanism if the orbits are mutually inclined (e.g. Lin et al. 2000 and references therein). Analysis for an outer circular orbit suggest high eccentricities may be obtained for mutual inclinations exceeding \( \arccos \sqrt{3/5} \approx 40^\circ \). We have also followed the orbital evolution of the system when the orbits are initially mutually inclined at an angle of \( 60^\circ \) and the results are also shown in figure 1. In this case, large amplitude cycles with \( e \) attaining values up to 0.7 are obtained. This is typical for runs of this kind. Cases with \( a/a_p = 0.01 \) and \( e_p = 0.9 \) lead to long term secular variations that can provide high eccentricities of the inner orbit.

When \( a/a_p \) is larger, the ensuing interaction with the inner planet is stronger and can lead to very high eccentricities. In such cases, there can be a close approach or collision with the central star. An example of a run of this kind is illustrated in figure 2. In this case the initial mutual orbital inclination, the initial values of \( a \) and \( e_p \), and planet masses were as above, but the initial value of \( a_p \) was reduced by a factor of four. The outer planet was started at apocentre and the inner one on a circular orbit at random phase. This case led to very strong interactions at pericentre which resulted in the eccentricity of the inner planet approaching unity and...
3 N PLANETS SIMULATIONS

We consider a system of \( N \) outer planets each having a mass \( m_p = 7 \pm 8 \) M\(_J\), one inner planet with a mass \( m \), and a primary star with a mass \( M_\star = 1 \) M\(_\odot\), moving under their gravitational attraction. We take \( m = 0.3 \) M\(_J\) (Saturn mass) as, among the observed systems, this is the most extreme case of a planet forming in a disc. We suppose the outer massive planets have formed rapidly through fragmentation on a scale of 100 au, while the PAID forms in the disc surrounding the star on a much longer timescale.

3.1 Initial conditions

At \( t = 0 \), we place \( 5 \leq N \leq 20 \) massive planets taken at random from a mass distribution corresponding to a uniform density spherical shell with outer radius of 100 au and inner radius of 10 au. The planets, the orbits of which are not coplanar, are given the local circular velocity in the azimuthal direction. As shown by PT01, adopting different initial conditions, such as a mass distribution in the form of a flattened disc, leads to the same qualitative evolution as long as three dimensional relaxation occurs, so our results do not depend on these initial conditions. The system is then allowed to evolve under the influence of the gravitational attraction of the central star and the different planets (the potential of the star has been modified according to eq. (4)). We also include the tidal interaction between the star (which radius is taken to be 1 R\(_\odot\)) and a closely approaching planet (see PT01 for details). The approximation used is valid only when the planet approaches the star on a highly eccentric orbit, which is the case when tidal interactions first occur in the systems we consider here. The system undergoes dynamical relaxation as described by PT01.

We assume a PAID forms on a timescale of 10\(^6\) y. During the formation process, any eccentricity that would be excited by the outer massive planets is damped by the tidal interaction with the disc. We therefore built up the planet progressively and constrained its orbit to remain circular during this phase, with an orbital radius \( a \) in the range 0.3–10 au. Depending on \( a \), the planet may have either formed at this location or migrated there as a result of tidal interactions with the disc.

3.2 Results

We have run about 30 cases with the initial conditions described above. As found in PT01, most of the dynamical relaxation of the system of outer planets is over after a few hundred orbits, i.e. a few 10\(^7\) y. This relaxation results in most of the planets being ejected, while a few (usually between 0 and 3) of them end up on highly, sometimes close, eccentric and inclined orbits around the star. These objects may still interact with each other after most of the relaxation has occurred. As a result, or directly because of the main relaxation, they may end up close enough to the star to perturb significantly the PAID after it forms.

When there are still a few outer massive planets left at \( t = 10^6 \) y, if they make frequent incursions into the inner parts of the system, the eccentricity of the PAID is pumped up to high values and the object eventually hits the star (e.g. the calculation illustrated in figure 3).
In some cases, the PAID may approach the star closely enough for tidal effects to become important while a collision is avoided. This is illustrated in figure 4, which shows the evolution of the semi-major axes and the pericentre distances of the planets. Here \( m_p = 8 \, M_J \) and \( a = 0.2 \) au. The interaction with the outer planets left in the system at \( t = 10^6 \) y increases the eccentricity of the PAID up to high values. It approaches the star closely enough for tidal circularization to start to occur. This causes the semi-major axis to decrease while maintaining the pericentre distance almost constant equal to 0.02 au. At this stage the inclinations between the orbits of the two planets oscillate between 0 and 30\(^\circ\). When the few outer planets left from the relaxation process make at most a few incursions in the inner system, the eccentricity of the PAID may be increased up to high values by the secular interaction with one of the massive objects with appropriate orbital parameters, but without the process resulting in a collision with the star. The eccentricity then varies in a cycle as described in § 2. This also happens when there is only one outer planet left at \( t = 10^6 \) y; provided it is close enough to perturb the PAID. This case is illustrated in figure 5. Here \( m_p = 7 \, M_J \), \( N = 29 \) and \( a = 0.77 \) au. Among the massive planets, we have represented the only one which was not ejected after about \( 3 \times 10^5 \) y. We note that the pericentre distance of the outer planet is about 9 au. These results are then in agreement with those of § 2. which showed similar behaviour of the eccentricity for such a ratio of outer pericentre distance to inner semi-major axis when the initial inclination was 60\(^\circ\) (see figure 5). Here the inclination between the orbits of the two planets oscillates between 0 and 60\(^\circ\).

A complete survey of all the runs we have performed in-
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4 SUMMARY AND DISCUSSION

The analysis and results presented above show that the eccentricity of an inner PAID may be increased up to high values by secular interaction with a more distant massive planet on an eccentric orbit. The mechanism is efficient when the ratio of outer pericentre distance to inner semi-major axis is about 10 and the orbits of the planets are significantly inclined.

Such a situation arises when the dynamical relaxation of a system of outer massive planets takes place on a timescale shorter than the timescale for assembling the PAID in a disc. After disc dispersal, as was shown by PT01, the PAID would be expected to mostly interact with one more massive object orbiting the star with semi-major axis on the order of tens of au. If the relaxation is not over by the time the PAID forms, close approaches with the massive outer planets tend to result in the object colliding with the star (see figure 6).

Since giant planet formation occurs on a timescale not longer than the disc lifetime of \( \sim 10^6 \) y (Haisch et al. 2001), assuming formation of massive planets occurs on a scale of 100 au, an eccentric PAID at less than 1 au from the central star is more likely to be produced when the number of outer planets in the system is \( N \sim 15 \), although \( N = 5 \) may also produce such a system.

To illustrate the operation of the processes described here, we consider observed planets with masses in the Saturn mass range. Two of them (HD 16141 and HD 6434) have orbital eccentricities of 0.28 and 0.3, respectively. These modest eccentricity systems could be similar to that represented in figure 6. Two others (HD 83443 and HD 108147) have orbital eccentricities of 0.42 and 0.58 respectively. These larger eccentricity systems could be similar to that shown in figure 6. In these cases the eccentricities are driven by secular interactions with a more closely approaching outer planet. Note however that the probability of getting a very high eccentricity on a close orbit is reduced because the chance of hitting the central star increases as the semi-major axis decreases. Nonetheless, in some cases, the interactions (secular or not) with the outer planets increase the eccentricity of the PAID up to very high values without the object colliding with the star. The orbit can then be tidally circularized by the central star, which leads to a 'hot PAID'. Six 'hot Saturns' have been observed so far, with \( 0.24 \, M_J < m_p < 0.52 \, M_J \) and \( 0.038 \, a < 0.066 \, a_u \). Although such a process is expected to be rare within the context of this model, observational bias against observing low mass planets at larger distances makes it difficult to assess to what extent an alternative mechanism such as direct migration is required.

If the scenario we have studied in this Letter has operated, there should be a massive planet with a on the order of tens of au and a high eccentricity associated with the eccentric PAID on a close orbit. In the case represented in figure 6, for instance, the amplitude of the stellar radial velocity induced by the outer planet is about 60 m s\(^{-1}\) (that of the inner planet is between 10 and 20 m s\(^{-1}\)). Over a short period of time (\( \sim 1 \) y), only the trajectory of the inner planet would be resolved, with the mean velocity of the stellar radial velocity drift ing by an amount which depends on whether the outer companion is near apocentre or not, and which varies between a few m s\(^{-1}\) and tens of m s\(^{-1}\). Twelve systems have been selected by Fischer et al. (2001) as candidates for which such residual velocity drifts are potentially observable, and their results indicate that the kind of systems we have been describing here are on the verge of detectability. Among the systems selected by Fischer et al. (2001) with a residual drift of 50 m s\(^{-1}\) over a two year period, HD 38529 has a planet
with $m \sin i = 0.76 \, M_\odot$, $a = 0.13 \, \text{au}$ and $e = 0.27$. This is similar to the system illustrated in figure [fig:system]

**ACKNOWLEDGMENT**

J.C.B.P. acknowledges visitor support from the CNRS through a *Poste Rouge* and the IAP for hospitality.

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