Towards an accurate bioimpedance identification

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Abstract. This paper describes the local polynomial method (LPM) for estimating the time-invariant bioimpedance frequency response function (FRF) considering both the output-error (OE) and the errors-in-variables (EIV) identification framework and compare it with the traditional cross- and autocorrelation spectral analysis techniques. The bioimpedance FRF is measured with the multisine electrical impedance spectroscopy (EIS) technique. To show the overwhelming accuracy of the LPM approach, both the LPM and the classical cross- and autocorrelation spectral analysis technique are evaluated through the same experimental data coming from a nonsteady-state measurement of time-varying in vivo myocardial tissue. The estimated error sources at the measurement frequencies due to noise, \( \sigma_n^Z \), and the stochastic nonlinear distortions, \( \sigma_{NL}^Z \), have been converted to \( \Omega \) and plotted over the bioimpedance spectrum for each framework. Ultimately, the impedance spectra have been fitted to a Cole impedance model using both an unweighted and a weighted complex nonlinear least square (CNLS) algorithm. A table is provided with the relative standard errors on the estimated parameters to reveal the importance of which system identification frameworks should be used.

1. Introduction
Most of the spectral analysis techniques used for estimating the bioimpedance frequency response function (FRF) from broadband Electrical Impedance Spectroscopy (EIS) measurements are still limited today. One of the main limitations of many approaches relies on the fact that no information is gathered about the measurement noise and the non-linear distortions. In fact, it is widely common to just simply calculate the impedance frequency response as the division of the voltage and current Fourier coefficients. In other cases, the identification/measuring approach considered does not exploit the periodic nature of the excitations and leakage errors are introduced on the impedance spectrum when using the discrete Fourier transform (DFT) [1]. In the last step, data are usually fitted to a model, e.g. the Cole impedance model [2], which is an empirical complex nonlinear function model in the angular frequency \( \omega = 2\pi f \)

\[
Z(\omega, \theta) = R_{\infty} + \frac{R_0 - R_{\infty}}{1 + \left(j\frac{\omega}{\omega_c}\right)^\alpha}, \quad \text{with} \quad \theta = [R_0, R_{\infty}, \omega_c, \alpha]^T. \tag{1}
\]

From the optimal curve-fit parameters \( \theta^{\text{opt}} \), the Jacobian (\( J \in \mathbb{C}^{F \times n_d} \)) is calculated as

\[
J_{ij} = \frac{\partial Z(\omega_i, \theta_j)}{\partial \theta_j} \bigg|_{\theta_j = \theta_j^{\text{opt}}}, \tag{2}
\]
where $n_0$ and $F$ stand, respectively, for the number of parameters and measured frequencies. An estimate of the covariance matrix of the parameters follows then easily form (2), viz.

$$\text{Cov}(\theta^{\text{opt}}) \approx (2 \Re \{J^H W J\})^{-1}, \tag{3}$$

where $W$ is a diagonal matrix with $W_{ii} = \text{Var}(Z(\omega_i))^{-1}$ as diagonal elements, i.e. the identity matrix for the unweighted CNLS approach. The asymptotic standard error (SE) for the optimal parameter vector $\theta^{\text{opt}}$ equals then

$$\text{Std}(\theta^{\text{opt}}) = \sqrt{\text{diag} \left( \text{Cov}(\theta^{\text{opt}}) \right)}, \tag{4}$$

where $\text{diag}(X)$ extracts the diagonal elements of $X$. The SE is a powerful measure of how unexplained variability in the data propagates to the variability in the solution, and is a measure for the stochastic error in the model parameters (not the fitted spectrum) when modeling impedance data.

This paper focuses on the question which time-invariant spectral analysis techniques yield an accurate identification of electrical bioimpedance? To objectively assess the quality of the identification process, we use as quality tools the standard errors (4) in the Cole model parameters. We consider two periods (2 ms) of current $i(t)$ and voltage $v(t)$ time-domain signals originating from the in situ measurement of in vivo time-varying myocardium. The excitation signal is a periodic multisine signal ($F = 26$ frequencies, 1 ms, 1 kHz → 1 MHz). The error that is present in the measurements is mainly due to noise, non-linear distortions, inherent feedback conditions and the transients in the response. We process these data including the errors, first, by using the classical spectral analysis approach based on cross- and autocorrelation and, second, with the local polynomial method (LPM). In both cases, both an output-error (OE) and Error-In-Variables (EIV) identification framework are considered.

2. The Local Polynomial Method (LPM)

Briefly stated, the general LPM solution solves an over-determined set of equations in a least square (LS) sense for each spectral component of interest. This LS problem can be calculated at the excited frequencies using the Moore-Penrose algorithm or the Singular Value Decomposition.

2.1. Output-Error Local Polynomial Method (OE-LPM)

The main idea of the LPM theory is based on considering an impedance system where the measured DFT voltage spectrum $V(k)$ equals the DFT current spectrum $I(k)$ multiplied with the bioimpedance FRF $Z(k)$, plus a transient term $T(k)$ and additive noise $N(k)$, viz.

$$V(k) = Z(k) I(k) + T(k) + N(k).$$

Then, using the smoothness property of $Z(k)$ and $T(k)$, the bioimpedance FRF $Z(k)$ and the transient term $T(k)$ can locally be parametrized with a $p^{th}$ order polynomial around the excited frequency bin $k$ considering $n$ neighboring spectral points around $k$, this is the small frequency band $[k-n, k-n+1, \ldots, k+n]$ (with $2n+1$ being the width of the LPM window). The estimated bioimpedance spectrum $\hat{Z}(k)_{\text{OE-LPM}}$ is then obtained by solving the set of equations given by $V_k = K_I(k) \theta_t(k)_{\text{OE-LPM}}$, with $V_k = (V(k-n) \cdots V(k) \cdots V(k+n))^T$ and $\theta_t(k)_{\text{OE-LPM}} = (\hat{Z}(k), z_1(k), \ldots, z_p(k), T(k), t_1(k), \ldots, t_p(k))_{\text{OE-LPM}}$. $K_I(k)$ is a complex matrix that depends on the Fourier coefficients of the measured current spectrum $I(k)$. Note that $\theta_t(k)$ contains the bioimpedance FRF $Z(k)$, the polynomial bioimpedance parameters $z_i(k)$’s and the transient parameters $T(k), t_i(k)$’s. The solution for this OE problem is found as the first element in $\theta_t(k)_{\text{OE-LPM}}$, namely $\hat{Z}(k)_{\text{OE-LPM}} = \left(\hat{\theta}(k)_{\text{OE-LPM}}\right)_1 \left(\hat{K}_I(k)_{\text{opt}} \hat{K}_I(k)_{\text{opt}}^{-1}\hat{K}_I(k)_{\text{opt}}\hat{V}_k\right)_1$. Figure 1 (top) illustrates the block diagram for computing the bioimpedance FRF with the OE-LPM. The OE-LPM results are summarized in figure 2 (C).
2.2. Errors-In-Variables Local Polynomial Method (EIV-LPM)

Contrary to the LPM-OE approach where the current signal \(i(t)\) is assumed to be known and used to calculate the \(K_I\) matrix, the errors-in-variables (EIV) framework computes the LS solution w.r.t. the noisy voltage and current signals. The bioimpedance FRF is then determined as \(\hat{Z}(k)_{EIV-LPM} = \hat{Z}_{RV}(k) / \hat{Z}_{RI}(k)\), where \(\hat{Z}_{RI}(k)\) is the bioimpedance spectrum from the reference spectrum \(R(k)\) to the current spectrum \(I(k)\), namely \(\hat{Z}_{RI}(k) = \left((K_R(k)^H K_R(k))^{-1} K_R(k)^H I_k\right)_1\), and \(\hat{Z}_{RV}(k)\) from the reference spectrum \(R(k)\) to the voltage spectrum \(V(k)\), namely \(\hat{Z}_{RV}(k) = \left((K_R(k)^H K_R(k))^{-1} K_R(k)^H V_k\right)_1\) with \(X_k = [X(k-n) \cdots X(k) \cdots X(k+n)]^T\), \(X = V, I\). Note that now the regression matrix \(K_R(k)\) does not depend on the current nor the voltage measurements. Figure 1 (bottom) shows a schematic implementing this framework. The EIV-LPM results are shown in figure 2 (D).

3. Experimental myocardial bioimpedance identification and conclusions

The EIV results shown in figure 2 (B) originates from the estimation of the FRF from the reference multisine excitation to the voltage and current channels without taking into account the transient error in the data. The reader is referred to [3] for the technical explanation on the cross and autocorrelation OE-EIV frameworks (see figure 2 (A)-(B)) without transient suppression (see [4] for more details about the OE-LPM and the EIV-LPM estimators). To make a fair comparison, the measurement frequencies were chosen to be the same in number for the cases (A)-(D) in figure 2 (\(F = 22\) fitted frequencies). As it may be observed, the largest difference between the unweighted and the weighted total noise-variance (including non-linear distortions) CNLS is in the case of the classical OE spectral analysis. The fact of using the reference (i.e. allowing to work in feedback conditions), the EIV framework significantly improves the quality in both the bioimpedance and the parameter estimates. Finally, the LPM estimator shown in figure 2 have a superior accuracy in both the bioimpedance spectrum and parameter estimates due to the (very) good transient suppression.

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Figure 2. Accuracy of the bioimpedance and Cole parameters using either the cross- and autocorrelation spectral analysis technique or the Local Polynomial Method (LPM): (A) output-error (OE), (B) errors-in-variables (EIV), (C) OE-LPM and (D) EIV-LPM frameworks.

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