LARGE DEVIATION PRINCIPLE FOR EMPIRICAL SINR MEASURE OF CRITICAL TELECOMMUNICATION NETWORKS

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Abstract. For a powered Poisson process, we define Signal-to-Interference-plus-Noise Ratio (SINR) and the SINR network as a Telecommunication Network. We define the Empirical Measures (empirical powered measure, empirical link measure and empirical sinr measure) of a class of Telecommunication Networks. For this class of Telecommunication Network we prove a joint large deviation principle for the empirical measures of the Telecommunication Networks. All our rate functions are expressed in terms of relative entropies.

1. Introduction and Background

1.1 Introduction

Since the inception of the 19th century the world had experienced renaissance in the information theory of wireless communication channel, wireless networks. Further, multimedia technologies have seen significant growth in the last two decades. The use of handheld devices and obtaining services offered by the Internet has now become essential in our daily lives. Therefore, the availability of wireless networks and network quality of service (QoS) offer have become vital for mobile users. See, Hassan, Tan and Yap [13].

Currently, telecommunication is simply an electrical medium of connecting over a distance (location and battery power). Telecommunication was discovered as an electrical waves and it is suggested that they could travel a speed close to the speed of light. See, Paudel and Bhattarai [19]. The fundamental requirement of routing through any telecommunication network whether it is via voice call or data package; is that each end point on the network has a unique address which enables wireless communication. Cellular systems are now nearly universally deployed and are under ever-increasing pressure to increase the volume of data they can deliver to consumers. Refer to Andrews, Baccelli and Ganti [2] or the references therein.

Now, the advent of multimedia interactive services and the surge in the number of interconnected devices has led to investigation of new approaches able to enhance wireless capacity in 5G networks. See, example Aravanis, Lam, Muoz, Pascual-Iserte and Di Renzo [3]. Marvi, Aijaz and Khurram [17] posited that 8.3 billion hand-held devices and 3.3 billion machine-to-machine (M2M) devices will be connected by 2021. The number of connected devices would clearly exceed the expected global population of 7.8 billion by that time. The monthly global mobile data traffic is expected to reach 49

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exabytes and the annual traffic will exceed half a zettabyte by 2021.

In informatory theory and telecommunication engineering, wireless consist of nodes which connect over a wireless channel, see Gupta and Kumar [11]. Signal-to-Interference-Plus-Noise Ratio (SINR) is a tool used as rate of information transfer in wireless communication system such as networks. According to Jeske and Sampath [14], the SIR is an important metric of wireless communication link quality. SINR estimates have several important applications. These include optimizing the transmit power level for a target quality of service, assisting with handoff decisions and dynamically adapting the data rate for wireless Internet applications. Communication performance can be improved significantly by adaptive transmissions based on the quality of received signals, i.e., the signal-to-interference-plus-noise ratio (SINR) Choi, Joo, Zhang and Shroff [5].

In cellular networks, SINR is a quantity that indicates if a given frequency resource is suitable to properly maintain a communication link. This is the rationale behind the usage of SINR in network to monitor the occurrence of radio link and handover failures, see Bastidas-Puga, Galaviz and Andrade [4]. An accurate SINR estimation provides for both a more efficient system and a higher user-perceived quality of service. Thus, the SINR is popularly used in wireless connection as a way to measure the quality of wireless connection within the space.

Analogous to the SNR used often in wired communications systems, the SINR is defined as the power of a certain signal of interest divided by the sum of the interference power (from all the other interfering signals) and the power of some background noise. If the power of noise term is zero, then the SINR reduces to the signal-to-interference ratio (SIR). Conversely, zero interference reduces the SINR to the signal-to-noise ratio (SNR), which is used less often when developing mathematical models of wireless networks such as cellular networks.

Agrawal and Kshetrimavum [1] derived the IPI, ISI and average output SINR expressions for the binary phase shift keying (BSPK) modulated raised-cosine pulse in the beamforming-based mm-Wave MIMO system. At each receiving antenna, they use the coherent Rake receiver to capture the signal energy, carried by multipath components in the complete IEEE 802.15.3c channel model. Additionally, their paper presents the impacts of pulse duration and half power beam width (HPBWs) of the transmitting antennas on the average output SINR.

Choi et al., (2013) focused on developing link scheduling schemes that can achieve optimal performance under the SINR model. The underlying argument was to treat an adaptive wireless link as multiple parallel virtual links with different signal quality, building on which they develop throughput-optimal scheduling schemes using a two-stage queueing structure in conjunction with recently developed carrier-sensing techniques. Furthermore, they introduce a novel three-way handshake to ensure, in a distributed manner, that all transmitting links satisfy their SINR requirements.

Keeler, Baszczyszyn and Karray [16] worked on an explicit integral interaction for SINR distribution experienced by a typical user in the downlink channel from the k-th strongest base stations of a cellular network modelled by Poisson point process on the plane. The outcome of their work shows that whole domain of SINR was valid whenever $\text{SINR} < 1$, where one observes multiple coverage.
Aravanis et al. [3] employs MGF to provide closed form expressions for the downlink ergodic capacity for the interference limited case, and validated the accuracy of these expressions by the use of extensive Monte Carlo simulations.

Weiss [21] used large deviations techniques to analyze models of communication networks. It was assumed the points form a sequence of independent and identical distributed random variables progressing to some powerov processes in discrete or continuous time. Guiliano and Macci [10] studied on the sequences of independent and identical distributed random variables and, under suitable conditions on the (common) distribution function, they proved large deviation principles for sequences of maxima, minima and pairs formed by maxima and minima. They assumed that the independent and identical distributed random variables can be either unbounded or bounded; in the first case maxima and minima have to be suitably normalized.

Duffy, Macci, and Torrisi [9] under suitable assumptions about the large deviation behavior of the state selection and sojourn processes, proved that the empirical laws of the phase process satisfy a sample path large deviation principle. From this large deviation principle, the large deviations behavior of a class of modulated additive processes was deduced and an alternate proof of results for modulated Lévy processes were obtained. With a practical application of the results, they calculated the large deviation rate function for a process that arises as the International Telecommunications Union’s standardized stochastic model of two-way conversational speech. Other researcher have also studied large deviation behaviour of the interference in a wireless communication model. See, Ganesh and Torrisi [12].

In this article, we prove a joint large deviation principle for the empirical powered measure, empirical link measure and the empirical sinr measure of critical Telecommunication networks. In this sequel, we prove a joint large deviation principle for the empirical powered measure and empirical link measure of the Critical Telecommunication Network. See, Sekyi-Yeboah, Asiedu and Doku-Amponsah [20] for similar results for the dense Telecommunication Networks. The main techniques used in this article are the Gartner-Ellis Theorem, see Dembo and Zeitouni [6, Theorem 2.3.6] and the method of Mixtures as deployed in the Ph Thesi, Doku-Amponsah [7].

The remaining part of this article is organized in the following manner. Section 2 contain the statement of the main results; the joint LDP for the empirical powered measure, empirical link measure and the empirical sinr measure, the joint LDP for the empirical powered measure and the empirical link measure, and the conditional LDP for the empirical sinr measure given the empirical powered measure and the empirical link measure. In Section 3 we give the proofs of Theorem 2.2(i) and Theorem 2.3. Section 4 gives the proofs of Theorem 2.2(ii) and Theorem 2.1.

1.2 Background

For a fix dimension $d \in \mathbb{N}$ and a measurable set $D \subset \mathbb{R}^d$ with respect to the Borel-Sigma algebra $\mathcal{B}(\mathbb{R}^d)$. Let $\lambda \eta : D \to [0, 1]$, be a rate measure, $q$ a transition kernel from $D$ to $(0, \infty)$ and $\phi(r) = r^{-\ell}$, here $\ell \in (0, \infty)$, be a path loss function and $\gamma^{(\lambda), \ell, \delta}(r, A) : (0, \infty) \to (0, \infty)$, be technical constants. We define the Sinr Graph as follows:

- Pick $Y = (Y_i)_{i \in J}$ a Poisson Point process (PPP) with rate measure $\lambda \eta : D \to [0, 1]$.
- For $Y$, we assign each $Y_i$ a power $\rho(Y_i) = \rho_i$ independently according to the transition function $q(\cdot, Y_i)$.


For any two powered points \((Y_i, \rho_i), (Y_j, \rho_j)\) we connect a link iff 
\[
\sinr(Y_i, Y_j, Y) \geq \tau^{(\lambda)}(\rho_j) \text{ and } \text{SI}NR(Y_j, Y_i, Y) \geq \tau^{(\lambda)}(\rho_i),
\]
where 
\[
\text{SI}NR(Y_j, Y_i, Y) = \frac{\rho_i \phi(||Y_i - Y_j||)}{N_0 + \gamma^{(\lambda)}(\rho_j) \sum_{i \in I - \{j\}} \rho_i \phi(||Y_i - Y_j||)}
\]

We shall consider \(Y^\lambda(\eta, Q, \phi) = \{(Y_i, \rho_i), j \in I, E\}\) under the joint law of the powered Poisson Point process and the network. We will interpret \(Y^\lambda\) as an Sinr network and \((Y_i, \rho_i) := Y_i^\lambda\) as the power of device \(i\). We recall Proposition 1.1 from [20] as follows:

**Proposition 1.1 (SAD2020).** The link probability of the Sinr network, \(p_\lambda\), is given by 
\[
\lim_{\lambda \to \infty} \lambda^{-2} a_\lambda p_\lambda((x, \rho_x), (y, \rho_y)) = e^{-\lambda h^D_1((x, \rho_x), (y, \rho_y))},
\]

\[
h^D_1((x, \rho_x), (y, \rho_y)) = \int_D \left[ \frac{\tau^{(\lambda)}(\rho_x, \gamma^{(\lambda)}(\rho_x))}{\tau^{(\lambda)}(\rho_x, \gamma^{(\lambda)}(\rho_x)) + (||y||/||x-y||)^t} + \frac{\tau^{(\lambda)}(\rho_y, \gamma^{(\lambda)}(\rho_y))}{\tau^{(\lambda)}(\rho_y, \gamma^{(\lambda)}(\rho_y)) + (||y||/||y-x||)^t} \right] \eta(dz).
\]

We assume there a sequence of real numbers \(a_\lambda\) and a function \(h^*_+ : D \times \mathbb{R}_+ \to (0, \infty)\) such that \(\lambda^{-2} a_\lambda \to \infty\) and 
\[
\lim_{\lambda \to \infty} \lambda^{-2} a_\lambda p_\lambda((x, \rho_x), (y, \rho_y)) = h^*_+((x, \rho_x), (y, \rho_y)).
\]

We shall call \(Y^\lambda\) CriticalSinr if \(\lambda a_\lambda \to 1\), SubcriticalSinr if \(\lambda a_\lambda \to 0\) and if \(\lambda a_\lambda \to \infty\) we call it SupercriticalSinr. In this article We shall look at criticalSinr networks. i.e. 
\[
\lim_{\lambda \to \infty} \lambda a_\lambda \to 1.
\]

We define the set \(S(D)\) by 
\[
S(D) = \cup_{y \in D} \left\{ y : |y \cap B| < \infty \right\}, \text{for any bounded } B \subset D \right\}.
\]

Write \(\mathcal{Y} = S(D \times \mathbb{R}_+)\) and denote by \(\mathcal{M}(\mathcal{Y})\), the space of positive measures on the space \(\mathcal{Y}\) equipped with \(\tau-\) topology. Henceforth, we shall call \(\mathcal{Y}\) a locally finite subset of the set \(\mathcal{Y}\).

**Empirical measures of the Sinr Networks:** For any Sinr graph \(Y^\lambda\) we define a probability measure, the *empirical power measure*, \(M_1^\lambda \in \mathcal{M}(\mathcal{Y})\), by 
\[
M_1^\lambda((x, \rho_x)) := \frac{1}{\lambda} \sum_{i \in I} \delta_{Y^\lambda_i((x, \rho_x))}
\]
and a symmetric finite measure, the *empirical pair measure* \(M_2^\lambda \in \mathcal{M}(\mathcal{Y} \times \mathcal{Y})\), by 
\[
M_2^\lambda((x, \rho_x), (y, \rho_y)) := \frac{1}{\lambda} \sum_{(i,j) \in E} \left[ \delta_{(Y^\lambda_i, Y^\lambda_j)} + \delta_{(Y^\lambda_j, Y^\lambda_i)} \right]((x, \rho_x), (y, \rho_y)).
\]

Note that the total mass \(\|M_1^\lambda\|\) of the empirical power measure is \(1\) and total mass of the empirical link measure is \(2|E|/\lambda^2\).
Theorem 1.2 (SAD2020). Suppose $Y^\lambda$ is an SINR network with rate measure $\lambda \eta : D \to [0,1]$ and a
power probability function $q$ from $D$ to $(0,\infty)$ and path loss function $\phi(r) = r^{-\ell}$, for $\ell > 0$. Then, as
$\lambda \to \infty$, $M^\lambda_1$ satisfies an LDP in the space $\mathcal{M}(\mathcal{Y})$ with good rate function
\[ I_1(\sigma) = \begin{cases} H(\sigma | \eta \otimes q), & \text{if } \|\sigma\| = 1 \\ \infty & \text{otherwise}. \end{cases} \]

2. Statement of Main Results

We, define the empirical SINR measure $\mathcal{L}^{1,2}_{(x,\rho_x)}$ by
\[ \mathcal{L}^{1,2}_{(x,\rho_x)}(a) := \frac{1}{\|N^\lambda(x,\rho_x)\|} \sum_{i \in N^\lambda(x,\rho_x)} \delta_{\text{SINR}(Y^\lambda_i,(x,\rho_x),M^\lambda_1)}(a), \]
where
\[ N^\lambda(x) = \left\{ k \in I : (k,j) \in E, Y^\lambda_j = x \right\} \]
We observe that we have
\[ \|N^\lambda(x,\rho_x)\| = \lambda \int_Y M^\lambda_2((x,\rho_x),(dy,d\rho_y)) \]
and
\[ \mathcal{L}^{1,2}_{(x,\rho_x)}(a) = \frac{1}{\|N^\lambda(x,\rho_x)\|} \int_Y \Phi^\lambda_{a}(x,\rho_x), (y,\rho_y), M^\lambda_1) M^\lambda_2((dy,d\rho_y),(dx,d\rho_x)), \]
where
\[ \Phi^\lambda_{a}(x,\rho_x), (y,\rho_y), \sigma) = \mathbb{I}\left\{ \tau(\rho_y) \leq \text{SINR}(Y^\lambda_i,(x,\rho_x), \sigma) \leq a(\rho_x) \right\} \]
We write
\[ \mathcal{L}^{1,2}_{(x,\rho_x)} := \left( \mathcal{L}^{1,2}_{(x,\rho_x)}(x,\rho_x) \in \mathcal{Y} \right). \]

Theorem [2.1] is a Joint Large deviation principle for the empirical measures of the SINR network models. We recall from Subsection [1.2] the definition of $h^D_\lambda$ as
\[ h^D_\lambda((x,\rho_x),(y,\rho_y)) = \int_D \left[ \frac{\tau(\rho_x)\gamma(\rho_y)(y-x)}{\tau(\rho_x)\gamma(\rho_y)+\|y-x\|^2} + \frac{\tau(\rho_y)\gamma(\rho_y)(y-x)}{\tau(\rho_x)\gamma(\rho_y)+\|y-x\|^2} \right] \eta(dz) \]
and write
\[ h_\rho \sigma \otimes \sigma((x,\rho_x),(y,\rho_y)) := h_\rho((x,\rho_x),(y,\rho_y))\sigma((x,\rho_x))\sigma((y,\rho_y)). \]

We define $\langle f, \eta \rangle$ by
\[ \left\langle f, \eta \right\rangle_{(x,\rho_x)}(a) = \frac{1}{\eta_2((x,\rho_x))} \int_Y f_a((x,\rho_x), dz, \sigma) \eta(dz, (x, \rho)). \]

Observe that, for a finite measure $\omega$
\[ \left\langle \Phi^\lambda_{a}, \omega \right\rangle_{(x,\rho_x)}(a) = \frac{1}{\omega_2((x,\rho_x))} \int_Y (y,\rho_y), (x,\rho_x), \sigma) \omega((dy,d\rho_y),(x,\rho_x)), \]

(2.1)
is a probability measure. We write $\lim_{\lambda \to \infty} \Phi^\lambda_{a} = \Phi_a$ and note that
\[ \lim_{\lambda \to \infty} \left\langle \Phi^\lambda_{a}, \omega \right\rangle_{(x,\rho_x)}(a) = \left\langle \Phi_a, \omega \right\rangle_{(x,\rho_x)}(a), \text{ for all } a \in [\tau, \infty), \]
We write

This implies the cost \( J \) may be divided into three separate costs:

- \( \text{(i)} \) The first term is the cost of having the empirical powered measure \( \sigma \) is known. This cost is non-negative and it is zero iff \( \sigma = \eta \otimes q \).
- \( \text{(ii)} \) The second term is the cost of obtaining the empirical link measure \( \omega \) given the empirical powered measure \( \sigma \). This cost is also non-negative and it is zero iff \( \omega = h_\ast \sigma \otimes \sigma \).
- \( \text{(iii)} \) The last term is the cost obtaining the empirical link measure \( \nu \). This cost is also non-negative and it is zero iff \( \nu = \langle \Phi_\sigma, \omega \rangle \).

This implies the cost \( J_\ast (\sigma, \omega, \nu) = 0 \) iff \( \sigma = \eta \otimes q \), \( \omega = h_\ast (\eta \otimes q) \otimes (\eta \otimes q) \), and \( \nu = \langle \Phi_\rho \otimes q, h_\ast (\eta \otimes q) \otimes (\eta \otimes q) \rangle \).

We write \( B_x(s) := \{ y : \| y - x \| < s \} \), \( B^t_{(x, \rho)} := B_x \left( \left[ \frac{c_\rho}{\ell} \right] \right)^1/\ell \left( \int_D \| z \|^{-\ell} \eta(dz) \right)^{-1/\ell} \) and note that the typical behaviour of the empirical Sinr measure is as

\[
\nu_{(x, \rho)}(a) = \langle \Phi_\eta \otimes q, h_\ast (\eta \otimes q) \otimes (\eta \otimes q) \rangle_{[x, \rho]}(a)
= \int_Y \Phi_\eta ((x, \rho), (y, \rho)) \eta \boxtimes \eta \left( \int_D h_\ast ((x, \rho), [y, \rho]) \eta(dy)q(dp) \right)
= \int_{\mathbb{R}_+} \int_{\mathbb{R}_+} \mathbb{I}_{B^t_{(x, \rho)}} \mathbb{I}_{B^t_{(x, \rho)}}(x) e^{-c_\rho} h_\ast ((x, \rho), [y, \rho]) \eta(dy)dp
\]

where \( B \setminus A = B \cap A^c \) and \( \mathbb{I}_\Gamma(x) \) denote the indicator function on the set \( \Gamma \).

**Theorem 2.1.** Let \( Y^\lambda \) is a critical powered Sinr network with rate measure \( \lambda \eta : D \to [0, 1] \) and a power probability function \( q \) from \( D \) to \((0, \infty)\) and path loss function \( \phi(r) = r^{-\ell} \), for \( \ell > 0 \). Suppose \( q \) is an exponential distribution with parameter \( c \).

\[
 J_\ast (\sigma, \omega, \nu) = H(\sigma || \eta \otimes q) + \frac{1}{2} H(\nu || h_\ast \sigma \otimes \sigma) + \frac{1}{2} \int_Y H(\nu_{(x, \rho)} || \langle \Phi_\sigma, \omega \rangle_{[x, \rho]}) \omega_2((dx, d\rho_x)) \tag{2.2}
\]

**Remark 1 Interpretation of the Rate Function:** The rate function can be regarded as the cost of having a powered Sinr network corresponding to the empirical measures triplet \( (\sigma, \omega, \nu) \). This cost may be divided into three separate costs:

- \( \text{(i)} \) The first term is the cost of having the empirical powered measure \( \sigma \) is known. This cost is non-negative and it is zero iff \( \sigma = \eta \otimes q \).
- \( \text{(ii)} \) The second term is the cost of obtaining the empirical link measure \( \omega \) given the empirical powered measure \( \sigma \). This cost is also non-negative and it is zero iff \( \omega = h_\ast \sigma \otimes \sigma \).
- \( \text{(iii)} \) The last term is the cost obtaining the empirical link measure \( \nu \). This cost is also non-negative and it is zero iff \( \nu = \langle \Phi_\sigma, \omega \rangle \).

This implies the cost \( J_\ast (\sigma, \omega, \nu) = 0 \) iff \( \sigma = \eta \otimes q \), \( \omega = h_\ast (\eta \otimes q) \otimes (\eta \otimes q) \), and \( \nu = \langle \Phi_\rho \otimes q, h_\ast (\eta \otimes q) \otimes (\eta \otimes q) \rangle \).

We write \( B_x(s) := \{ y : \| y - x \| < s \} \), \( B^t_{(x, \rho)} := B_x \left( \left[ \frac{c_\rho}{\ell} \right] \right)^1/\ell \left( \int_D \| z \|^{-\ell} \eta(dz) \right)^{-1/\ell} \) and note that the typical behaviour of the empirical Sinr measure is as

\[
\nu_{(x, \rho)}(a) = \langle \Phi_\eta \otimes q, h_\ast (\eta \otimes q) \otimes (\eta \otimes q) \rangle_{[x, \rho]}(a)
= \int_Y \Phi_\eta ((x, \rho), (y, \rho)) \eta \boxtimes \eta \left( \int_D h_\ast ((x, \rho), [y, \rho]) \eta(dy)q(dp) \right)
= \int_{\mathbb{R}_+} \int_{\mathbb{R}_+} \mathbb{I}_{B^t_{(x, \rho)}} \mathbb{I}_{B^t_{(x, \rho)}}(x) e^{-c_\rho} h_\ast ((x, \rho), [y, \rho]) \eta(dy)dp
\]

where \( B \setminus A = B \cap A^c \) and \( \mathbb{I}_\Gamma(x) \) denote the indicator function on the set \( \Gamma \).

**Theorem 2.2.** Let \( Y^\lambda \) is a critical powered Sinr network with rate measure \( \lambda \eta : D \to [0, 1] \) and a power probability function \( q \) from \( D \) to \((0, \infty)\) and path loss function \( \phi(r) = r^{-\ell} \), for \( \ell > 0 \). Suppose \( q \) is an exponential distribution with parameter \( c \).
(i) Then, as \( \lambda \to \infty \), conditional on the event \( M_1^\lambda = \sigma \), \( M_2^\lambda \) satisfies a large deviation principle in the space \( \mathcal{M}(\mathcal{Y} \times \mathcal{Y}) \) with speed \( \lambda \) and good rate function

\[
I_\sigma(\omega) = \frac{1}{2} \mathcal{H}(\omega\|h_\sigma \otimes \sigma)
\]  

(2.4)

(ii) Then as \( \lambda \to \infty \), the pair \( (M_1^\lambda, M_2^\lambda) \) satisfies a large deviation principle in the space \( \mathcal{M}(\mathcal{Y} \times \mathcal{Y}) \) with speed \( \lambda \), and good rate function

\[
I(\sigma, \omega) = H\left( \sigma \mid (\eta \otimes q) \right) + \frac{1}{2} \mathcal{H}(\omega\|h_\sigma \otimes \sigma),
\]  

(2.5)

where

\[
h_\sigma \otimes \sigma((x, \rho_x), (y, \rho_y)) = h_\sigma((x, \rho_x), (y, \rho_y))\sigma((x, \rho_x))\sigma((y, \rho_y)).
\]

**Theorem 2.3.** Let \( Y^\lambda \) is a critical powered Sinr network with rate measure \( \lambda \eta : D \to [0, 1] \) and a powered probability function \( q \) from \( D \) to \((0, \infty)\) and path loss function \( \phi(r) = r^{-\ell} \), for \( \ell > 0 \). Suppose \( Y \) is an Sinr network conditional on the event \( \{(M_1^\lambda, M_2^\lambda) = (\sigma, \omega)\} \). Then, as \( \lambda \to \infty \), the empirical sinr measure \( \mathcal{L}^{1,2} \) satisfies an LDP in the space \( \mathcal{M}((\tau, \infty)) \) with speed \( \lambda \) and good rate function

\[
\bar{J}(\nu) = \frac{1}{2} \int_\mathcal{Y} H\left( \nu_{(x, \rho_x)} \right) \left( \Phi_\sigma, \omega \right)_{(x, \rho_x)} \omega_2(dx, d\rho_x),
\]

where \( \omega_2 \) denote second marginal of the finite measure \( \omega \).

3. **Proof of Theorem [2.2] and Theorem [2.1]**

3.1 **Proof of Theorem [2.2](i) by Gartner-Ellis Theorem**

Suppose \( B_1, ..., B_n \) is a decomposition of the space \( D \times \mathbb{R}_+ \). Observe that, for every \( (x, y) \in B_i \times B_j \), \( i, j = 1, 2, 3, ..., n \), \( \lambda M_2^\lambda(x, y) \) given \( \lambda M_1^\lambda(x) = \lambda \sigma(x) \) is binomial with parameters \( \lambda^2 \sigma(x) \sigma(y)/2 \) and \( p_\lambda(x, y) \). Let \( q \) be the exponential distribution with parameter \( c \). We recall the function \( R_\lambda^D \) from the previous sections as follows:

\[
h_\lambda^D((x, \rho_x), (y, \rho_y)) = \int_D \left[ \frac{\tau(x) \gamma(x) \rho_x}{\tau(x) \gamma(x) \rho_x + (\|z\|^2/\|x-y\|^2)} \right] \eta(dz).
\]

**Lemma 3.1** is key component in the application of the Gartner-Ellis Theorem, see example,

**Lemma 3.1.** Let \( Y^\lambda \) be an Sinr network with rate measure \( \lambda \eta : D \to [0, 1] \) and a powered probability function \( q \) from \( D \) to \((0, \infty)\) and path loss function \( \phi(r) = r^{-\ell} \), for \( \ell > 0 \), conditional on the event \( M_1^\lambda = \sigma \). Let \( g : \mathcal{Y} \times \mathcal{Y} \to \mathbb{R} \) be bounded function. Then,

\[
\lim_{\lambda \to \infty} \frac{1}{\lambda} \log \mathbb{E}\left\{ e^{\lambda g(M_1^\lambda)} \bigg| M_1^\lambda = \sigma \right\} = \frac{1}{2} \lim_{n \to \infty} \sum_{j=1}^n \sum_{i=1}^n \left( 1 - e^{\theta}, h_\sigma \otimes \sigma \right)_{B_i \times B_j}
\]

\[
= \frac{1}{2} \left( 1 - e^{\theta}, h_\sigma \otimes \sigma \right)_{\mathcal{Y} \times \mathcal{Y}}.
\]

**Proof.** Now we observe that

\[
\mathbb{E}\left\{ e^{\lambda g(x,y) M_2^\lambda(dx,dy)/2} \bigg| M_1^\lambda = \sigma \right\} = \mathbb{E}\left\{ \prod_{x \in \mathcal{Y}} \prod_{y \in \mathcal{Y}} e^{\lambda g(x,y) M_2^\lambda(dx,dy)/2} \right\}
\]
By the dominated convergence theorem
\[
\frac{1}{\lambda} \log E\left\{ e^{\lambda \langle g, M_1^\lambda \rangle / 2} \bigg| M_1^\lambda = \sigma \right\} = \frac{1}{\lambda} \sum_{j=1}^{n} \sum_{i=1}^{n} \int_{B_i} \int_{B_j} \log \left[ 1 - (1 - e^{g(x, y)}) p_{\lambda}(x, y) + o(\lambda) \right] \lambda^2 \sigma \otimes \sigma(dx, dy) / 2
\]
\[
\frac{1}{\lambda} \log E\left\{ e^{\lambda \langle g, M_2^\lambda \rangle / 2} \bigg| M_1^\lambda = \sigma \right\} = \lim_{\lambda \to \infty} \sum_{j=1}^{n} \sum_{i=1}^{n} \int_{B_i} \int_{B_j} \log \left[ 1 - (1 - e^{g(x, y)}) p_{\lambda}(x, y) + o(\lambda) \right] \lambda^2 \sigma \otimes \sigma(dx, dy) / 2
\]
\[
\lim_{\lambda \to \infty} \frac{1}{\lambda} \log E\left\{ e^{\lambda \langle g, M_1^\lambda \rangle / 2} \bigg| M_1^\lambda = \sigma \right\} = \frac{1}{2} \sum_{j=1}^{n} \sum_{i=1}^{n} \int_{B_i} \int_{B_j} (1 - e^{g(x, y)}) h_*(x, y) \sigma \otimes \sigma(dx, dy)
\]
\[
\lim_{n \to \infty, \lambda \to \infty} \frac{1}{\lambda} \log E\left\{ e^{\lambda \langle g, M_2^\lambda \rangle / 2} \bigg| M_1^\lambda = \sigma \right\} = \frac{1}{2} \sum_{j=1}^{n} \sum_{i=1}^{n} \left\langle 1 - e^{g}, h_* \sigma \otimes \sigma \right\rangle_{B_i \times B_j}
\]
\[
= \frac{1}{2} \left\langle 1 - e^{g}, h_* \sigma \otimes \sigma \right\rangle_{Y \times Y}
\]
Hence, by Gartner-Ellis theorem, conditional on the event \( \{ M_1^\lambda = \sigma \} \), \( M_2^\lambda \) obey a large deviation principle with speed \( \lambda \) and rate function
\[
I_\sigma(\omega) = \frac{1}{2} \sup_g \left\{ \left\langle g, \omega \right\rangle_{Y \times Y} + \left\langle 1 - e^{g}, h_* \sigma \otimes \sigma \right\rangle_{Y \times Y} \right\}
\]
which when solved, see example [7], would clearly reduces to the good rate function given by
\[
I_\sigma(\omega) = \frac{1}{2} H(\omega||h_* \sigma \otimes \sigma). \tag{3.1}
\]

### 3.2 Proof of Theorem 2.3

By Gartner-Ellis Theorem

The first step in proof of Theorem 2.3 is a large deviation principle for the sequence of measures \( \left\{ \mathcal{L}_{(x, d), (x, p)}^{1,2}, (x, p) \in Y \right\} \) conditional on the set
\[
\left\{ (M_1^\lambda, M_2^\lambda) = (\sigma, \omega) \right\}.
\]
Lemma 3.2. Let \( Y^\lambda \) be a critical powered Sinr networks with rate measure \( \lambda \eta : D \to [0, 1] \) and a powered probability function \( q \) from \( D \) to \((0, \infty)\) and path loss function \( \phi(r) = r^{-\ell} \), for \( \ell > 0 \). Then, for every \((x, \rho_x)\), we have

\[
\lim_{\lambda \to \infty} \mathbb{P}\left\{ \tau^{(\lambda)}(\rho_x) \leq \text{Sinr}([Y_i, \rho_i], (x, \rho_x), M_1^\lambda) \leq a(\rho_x), (x, \rho_x) \in \mathcal{Y} \right\} = \left\langle \Phi_\sigma, \omega \right\rangle(a)
\]

(3.2)

Proof. We compute the probability

\[
\mathbb{P}\left\{ \tau^{(\lambda)}(\rho_x) \leq \text{Sinr}([Y_i, \rho_i], (x, \rho_x), M_1^\lambda) \leq a(\rho_x) \right\} = \left\langle \Phi_\sigma, \omega \right\rangle_{(x, \rho_x)}(a)
\]

Taking limits as \( \lambda \to \infty \) on both sides we have 3.2 which ends the proof of Lemma 3.2.

Lemma 3.3. Let \( Y^\lambda \) is a critical powered Sinr graph with rate measure \( \lambda \eta : D \to [0, 1] \) and a powered probability function \( q \) from \( D \) to \((0, \infty)\) and path loss function \( \phi(r) = r^{-\ell} \), for \( \ell > 0 \). Suppose \( q \) is an exponential distribution with parameter \( e \). Then, for every \((x, \rho_x)\), we have

\[
\lim_{\lambda \to \infty} \frac{1}{\lambda} \log \mathbb{E}\left\{ e^{\int_{\mathcal{Y}} N_\lambda((dx, \rho_x)) \left( \left[M_1^\lambda, M_2^\lambda\right] = (\sigma, \omega) \right)} \right\} = \frac{1}{2} \log \left\langle e^g, \left\langle \Phi_\sigma, \omega \right\rangle \right\rangle_{[\tau, \infty)} \mathbb{E}\left\{ e^{\int_{\mathcal{Y}} N_\lambda((dx, \rho_x))/2} \left( \left[M_1^\lambda, M_2^\lambda\right] = (\sigma, \omega) \right) \right\}
\]

(3.3)

Proof. We observe that \( \left( \mathcal{L}^{1,2}_{(x, \rho_x)}, N((x, \rho_x)), (x, \rho_x) \in \mathcal{Y} \right) \) are independent distributed as

\[
\left\langle \Phi_\sigma, \omega \right\rangle_{(x, \rho_x)}, (x, \rho_x) \in \mathcal{Y}.
\]

\[
\mathbb{E}\left\{ e^{\int_{\mathcal{Y}} g, \mathcal{L}^{1,2}_{(x, \rho_x)}} N_\lambda((dx, \rho_x))/2 \left( \left[M_1^\lambda, M_2^\lambda\right] = (\sigma, \omega) \right) \right\} = \prod_{(dx, \rho_x) \in \mathcal{Y}} \mathbb{E}\left\{ \Phi_\sigma, \omega \right\}_{[x, \rho_x]} \prod_{i \in \mathcal{Y}(dx, dx)/2} e^{g(SINR(Y_i^\lambda,(x, \rho_x),\sigma))}
\]

\[
= \prod_{(dx, \rho_x) \in \mathcal{Y}} \left( \mathbb{E}\left\{ \Phi_\sigma, \omega \right\}_{[x, \rho_x]} e^{g(SINR(Y_i^\lambda,(x, \rho_x),\sigma))} \right)^{N_\lambda((dx, \rho_x))/2}
\]

\[
= \prod_{(dx, \rho_x) \in \mathcal{Y}} \int_{\tau}^{\infty} e^{g(a)} \left\langle \Phi_\sigma, \omega \right\rangle_{(x, \rho_x)} (da) N_\lambda((dx, \rho_x))/2
\]

(3.4)

Now taking limit of normalized logarithm of 3.3 and observing that \( N_\lambda((dx, \rho_x))/\lambda \to \omega_2((dx, \rho_x)), \left\langle \Phi_\sigma, \omega \right\rangle_{(x, \rho_x)} \to \left\langle \Phi_\sigma, \omega \right\rangle_{(x, \rho_x)} \) as \( \lambda \to \infty \) we have 3.3 which ends the proof of Lemma 3.3. □

Now, by the Gartner-Ellis Theorem, Conditional on the event \( \{ (M_1^\lambda, M_2^\lambda) = (\sigma, \omega) \} \) the probability measure \( \mathcal{L}^{1,2} \) obeys an LDP with speed \( \lambda \) and rate function

\[
\tilde{J}(\nu) = \frac{1}{2} \sup_{g} \left\{ \left\langle g, \nu \right\rangle_{[\tau, \infty)}, \omega_2 \right\} - \left\langle \log \left\langle e^g, \left\langle \Phi_\sigma, \omega \right\rangle \right\rangle_{[\tau, \infty)}, \omega_2 \right\} \right\}
\]
Using the variational formulation of relative entropy we have that
\[ \tilde{J}(\nu) = \frac{1}{2} \int \mathbb{H}(\nu_{(x,\rho_x)} \| \Phi_{(x,\rho_x)}) \omega_2((dx, d\rho_x)), \]
which proves Theorem 2.1.

4. Proof of Theorem 1.2(ii) and Theorem 2.1 by Method of Mixtures

For any \( \lambda \in (0, \infty) \) we define
\[ \mathcal{M}_\lambda(\mathcal{Y}) := \left\{ \sigma \in \mathcal{M}(\mathcal{Y}) : \lambda \sigma(x) \in \mathbb{N} \text{ for all } x \in \mathcal{Y} \right\}, \]
\[ \tilde{\mathcal{M}}_\lambda(\mathcal{Y} \times \mathcal{Y}) := \left\{ \omega \in \tilde{\mathcal{M}}_\rho(\mathcal{Y} \times \mathcal{Y}) : \lambda \omega(x, y) \in \mathbb{N}, \text{ for all } x, y \in \mathcal{Y} \right\}. \]
We denote by \( \Theta_\lambda := \mathcal{M}_\lambda(\mathcal{Y}) \) and \( \Theta := \mathcal{M}(\mathcal{Y}) \). With
\[ P_{\sigma_\lambda}(\eta_\lambda) := \mathbb{P}\{M_{\lambda}^2 = \eta_\lambda \mid M_{\lambda}^1 = \sigma_\lambda\}, \]
\[ P^{(\lambda)}(\sigma_\lambda) := \mathbb{P}\{M_{\lambda}^1 = \sigma_\lambda\} \]
\[ P^{(\lambda)}(\sigma_\lambda, \omega_\lambda)(\nu_{(x,\rho_x)}) := \mathbb{P}\{\mathcal{L}_{(x,\rho_x)}^{1,2} = \nu_{(x,\rho_x)}\}(M_{\lambda}^1, M_{\lambda}^2) = (\sigma_\lambda, \omega_\lambda) \]
the joint distribution of \( M_{\lambda}^1 \) and \( M_{\lambda}^2 \) is the mixture of \( P_{\sigma_\lambda}^{(\lambda)} \) with \( P^{(\lambda)}(\sigma_\lambda) \), and the joint distribution of \( \mathcal{L}_{1,2}, M_{\lambda}^1 \) and \( M_{\lambda}^2 \) is a mixture of \( \tilde{\mathcal{P}}^{\lambda} \) with \( P_{(\sigma_\lambda, \omega_\lambda)}^{(\lambda)} \) as follows:
\[ d\tilde{\mathcal{P}}^{\lambda}(\sigma_\lambda, \eta_\lambda) := dP_{(\sigma_\lambda)}^{(\lambda)}(\eta_\lambda) dP^{(\lambda)}(\sigma_\lambda). \]
(Biggin, Theorem 5(b), 2004) gives criteria for the validity of large deviation principles for the mixtures and for the goodness of the rate function if individual large deviation principles are known. The following three lemmas ensure validity of these conditions.
Observe that the family of measures \( (P^{(\lambda)} : \lambda \in (0, \infty)) \) is exponentially tight on \( \Theta_\lambda \).

Lemma 4.1.  
(i) The family of measures \( (\tilde{\mathcal{P}}^{\lambda} : \lambda \in (0, \infty)) \) is exponentially tight on \( \Theta_\lambda \times \tilde{\mathcal{M}}_\rho(\mathcal{Y} \times \mathcal{Y}) \).
(ii) The family measures \( (P_\lambda : \lambda \in (0, \infty)) \) is exponentially tight on \( \Theta \times \tilde{\mathcal{M}}_\rho(\mathcal{Y} \times \mathcal{Y}) \times \mathcal{M}([\tau, \infty)) \).

Define the function \( I : \Theta \times \mathcal{M}_\rho(\mathcal{Y} \times \mathcal{Y}) \rightarrow [0, \infty], \) by
\[ I(\sigma, \omega) = \mathbb{H}(\sigma \| q_{\eta} \bigwedge \Phi_{\omega}) + \mathbb{H}(\omega \| h_{\sigma} \otimes \sigma) \]
and recall from Theorem 2.3 that
\[ \tilde{J}(\nu) = \frac{1}{2} \int \mathbb{H}(\nu_{(x,\rho_x)} \| \Phi_{(x,\rho_x)}) \omega_2((dx, d\rho_x)). \]

Lemma 4.2.  
(i) \( I \) is lower semi-continuous.
(ii) \( \tilde{J} \) is lower semi-continuous.

By (Biggin, Theorem 5(b), 2004) the two previous lemmas and the large deviation principles we have established Theorem 2.2 and Theorem 2.3 ensure that under \( (\tilde{\mathcal{P}}^{\lambda}) \) and \( P_\lambda \) the random variables \( (\sigma_\lambda, \eta_\lambda) \) and \( (\nu, \sigma_\lambda, \eta_\lambda) \) satisfy a large deviation principle on \( \mathcal{M}(\mathcal{Y}) \times \tilde{\mathcal{M}}(\mathcal{Y} \times \mathcal{Y}) \) and \( \Theta \times \tilde{\mathcal{M}}(\mathcal{Y} \times \mathcal{Y}) \times \mathcal{M}([\tau, \infty)) \) with good rate function \( I \) and \( \tilde{J} \) respectively, which ends the proof of Theorem 2.2.
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