Non-local dynamics of Bell states in separate cavities

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Abstract

We present non-local dynamics of Bell states in separate cavities. It is demonstrated that (i) the entanglement damping speed will saturate when the cavity leakage rate $\gamma \geq 0.4$; (ii) the synchronism relationship between the fidelity and the concurrence depends on the initial state; (iii) if the initial state is $1/\sqrt{2}(|01\rangle + |10\rangle)$, the dynamics of entropy is opposite to that of fidelity.

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I. INTRODUCTION

In contrast with the extensively investigated static entanglement [1, 2, 3, 4, 5, 6], dynamic entanglement under the influence of variant environments is one of the most important and largely unexplored problems in the field of quantum teleportation, quantum computation and quantum communication [7, 8, 9, 10]. It is not only involved with the foundation of quantum mechanics, but also a fundamental issue in creating, quantifying, controlling, distributing and manipulating the entangled quantum bits, which are composed of spin-1/2 atoms in different problems [1, 11, 12, 13]. An entangled system is in such a state that cannot be factorized [14] in its Hilbert space. And the most familiar and widely used examples are Bell states. The two particles or atoms of spin-1/2 are correlated no matter how long distance is between them. Generally, due to different kinds of quantum reservoir, the entanglement degree between them vanishes asymptotically. However, if the reservoir consists of, e.g., only one or two electromagnetic field modes, then the entanglement may decrease abruptly and non-smoothly to zero in a finite time [9, 15, 16], which is a new nonlocal decoherence called entanglement sudden death (ESD). Therefore, demonstration of the dynamics of Bell states [17] would have profound implications for understanding of the physics in the realization of qubits in experiments.

So far in quantum optics experiments, Bell states can be generated with trapped ions [18] and in cavity quantum electrodynamics (CQED) [19, 20], which has attracted much attention. Based on cavity QED systems, schemes (to see Refs. 21, 22, 23, 24, 25, and references therein) have been proposed to implement quantum communications or engineer entanglement between atoms in distant optical cavities. In most of them, two separated cavities are connected via some channels, for instance, an optical fiber [25]. And in a recent paper, Yin and Li [26] investigated a system consisting of two single-mode cavities connected by an optical fiber and multiple two-level atoms trapped in the cavities. They show that ideal entangling can be deterministically realized between the distant cavities. Besides, utilizing a system of two-atoms and two photon modes, Masood and Miller [27] used the Jaynes and Cummings model [28], which is considered to be one of the most appropriate models for exploiting the dynamics of entanglement [29, 30, 31, 32], in the rotating wave approximation to study entanglement of more than one atom with vacuum.
The photon modes in their model are uncoupled, however, the leakage of cavities and the effect of temperature (Yet recently, in other models the two-qubit entanglement dynamics for a finite-temperature environment has been discussed in \([33, 34]\)) are actually not considered thoroughly.

In this paper, we consider a quantum model with two identical two-level atoms or pseudo-spins of \(1/2\) (as an open subsystem with qubits labelled \(s_1\) and \(s_2\)) and two single-mode cavities (labelled 1 and 2 correspondingly). The atom \(s_j\) (\(j = 1\) or \(2\)) is embedded in and coupled only with the cavity mode \(j\), which could be regarded as its bath or environment. The two cavities are so far departed that there is no direct interaction between them as well as the two atoms. Initially, the two qubits are prepared as a most-entangled states (Bell states). The focus of interest is their degrading quantum evolution, which are measured by the concurrence\([35, 36]\), the fidelity\([37]\) and the entropy exchange\([38, 39]\). The calculations and physical arguments will be carried out in two conditions: (i) there is leakage of photons for the cavities, which are in the vacuum states from the beginning; (ii) the cavities are so perfect that the loss of photons from them could be neglected and the two single modes are initialled in a thermal equilibrium state with the same temperature. The rest of this paper is organized as following. In Sec. \(\text{II}\) we begin with the model Hamiltonian and its analysis derivation; and then we introduce the numerical calculation procedure about the evolution of the reduced matrix for the subsystem. Detailed results and discussions can be found in Sec. \(\text{III}\). We will make a conclusion in Sec. \(\text{IV}\).

\section{Model and Method}

The master equation for a two-level atom in a single-mode cavity\([40]\), as one of the two partitions in our model, can be taken as

\[
 i \frac{d\rho_j}{dt} = [H_j, \rho_j] + i\gamma_j \left( a_j \rho_j a_j^\dagger - \frac{1}{2} a_j^\dagger a_j \rho_j - \frac{1}{2} \rho_j a_j^\dagger a_j \right). \tag{1}
\]

For density matrix \(\rho_j\), \(j\) refers to \(s_1\) or \(s_2\); for the mode operator \(a_j\) or \(a_j^\dagger\), \(j\) (1 or 2) represents the photon mode coupling with the corresponding atom. \(\gamma_j\) is the leakage rate of photons from the cavity \(j\). \(H_j\) describes the Hamiltonian for a subsystem of one atom and one cavity.
(j = 1, 2):

\[ H_j = \frac{\omega_j}{2} \sigma_j^z + (1 + \epsilon_j) \omega_j a_j^\dagger a_j + g_j \omega_j (a_j^\dagger + a_j) \sigma_j^x. \]  

where \( \omega_j \) is the energy level difference of atom \( s_j \) in cavity \( j \). \( \epsilon_j \) is the detuning parameter measuring the deviation of the photon \( j \) energy from \( \omega_j \). \( g_j \) is introduced as another dimensionless parameter which suggests the coupling strength between qubit \( s_j \) and mode \( j \). The \( x \) and \( z \) components of \( \sigma \) are the well-known Pauli operator. The two qubits are embedded in remote cavities without direct interaction. Therefore the whole Hamiltonian for this two-atom-two-cavity problem is

\[ H = H_1 + H_2. \]  

The whole state of the total system is assumed to be separable before \( t = 0 \), i.e.

\[
\rho(0) = \rho_S(0) \otimes \rho_b(0), \tag{4}
\]

\[
\rho_S(0) = |\psi(0)\rangle\langle \psi(0)|, \tag{5}
\]

\[
\rho_b(0) = \rho_{b1}(0) \otimes \rho_{b2}(0). \tag{6}
\]

The initial state \( |\psi(0)\rangle \) for the two qubits is one of the Bell states. And the two cavities are in their (i) vacuum states \( \rho_{b j}(0) = |0_j\rangle\langle 0_j| \) (in this case, we will consider \( \gamma_j \neq 0 \)) or (ii) thermal equilibrium states \( \rho_{b j}(0) = e^{-H_B/k_B T}/Z \) (in this one, we set \( \gamma \) to be zero to distinguish the effect of temperature from that of \( \gamma \)), where \( H_B \) is the pure bath part of the whole Hamiltonian and \( Z = \text{Tr} \left( e^{-H_B/k_B T} \right) \) is the partition function and the Boltzmann constant \( k_B \) will be set to 1 for the sake of simplicity.

For the former case, Eq. 1 will be exploited to calculate \( \rho(t) \). For the latter one, Eq. 1 is reduced to

\[
\rho(t) = \exp(-iHt)\rho(0)\exp(iHt). \tag{7}
\]

To determine the dynamics of the density matrix for the whole system, two factors need to be considered. The first one is the expression of the thermal bath state. In numerical calculations \[41\], we have to expand \( \rho_{b j}(0) \) \((j = 1, 2)\) to a summation of its eigenvectors with corresponding weights determined by its eigenvalues:

\[
\rho_{b j}(0) = \sum_m |\phi_{mj}\rangle \omega_{mj} \langle \phi_{mj}|, \quad \omega_{mj} = \frac{e^{-E_{mj}/T}}{Z_j}. \tag{8}
\]
Then for the two single-modes, we have

\[
\rho_{b1}(0) \otimes \rho_{b2}(0) = \sum_{mn} |\phi_{m1}\rangle |\phi_{n2}\rangle \omega_{mn} \langle \phi_{n2}| \langle \phi_{m1}|,
\]

where the subscripts \(m\) and \(n\) refer to mode 1 and 2 respectively. The second important factor is the evaluation of the evolution operator \(U(t) = \exp(iHt)\). A polynomial expansion scheme proposed by us in Ref. [42, 43, 44] is applied into the computation,

\[
U(t) = \left(\frac{1}{1 + it}\right)^{\alpha+1} \sum_{k=0}^{\infty} \left(\frac{it}{1 + it}\right)^k L^\alpha_k(H),
\]

\(L^\alpha_k(H)\) is one type of Laguerre polynomials as a function of \(H\), where \(\alpha (-1 < \alpha < \infty)\) distinguishes different types of the Laguerre polynomials and \(k\) is the order of it. The scheme is of an efficient numerical algorithm motivated by Ref. [45, 46], which is pretty well suited to many quantum problems, open or closed. Additionally, it could give results in a much shorter time compared with the traditional methods under the same numerical accuracy requirement, such as the well-known 4-order Runge-Kutta algorithm. After the density matrix \(\rho(t)\) for the whole system is obtained, the reduced density matrix \(\rho_S(t)\) for the two atoms can be derived by tracing out the degrees of freedom of the two single-mode cavities.

III. SIMULATION RESULTS AND DISCUSSIONS

We discuss three important physical quantities which indicate the time evolution of the subsystem. (i) The concurrence. It is a very good measurement for the intra-entanglement between two qubits and monotone to the quantum entropy of the subsystem when the subsystem is in a pure state. It is defined as:

\[
C = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\},
\]

where \(\lambda_i\) are the square roots of the eigenvalues of the product matrix \(\rho_S(\sigma^y \otimes \sigma^y)\rho_S^*(\sigma^y \otimes \sigma^y)\) in decreasing order. (ii) The fidelity. It is defined as

\[
F(t) = \text{Tr}_S[\rho_{\text{ideal}}(t)\rho_S(t)].
\]

where \(\rho_{\text{ideal}}(t)\) represents the pure state evolution of the subsystem only under \(H_S\), without interaction with the environment. In this study, \(H_S = \frac{\omega_1}{2} \sigma_1^z + \frac{\omega_2}{2} \sigma_2^z\). The fidelity is a
measurement for decoherence and depends on $\rho_{\text{ideal}}$. It achieves its maximum value 1 only if $\rho_S(t)$ equals to $\rho_{\text{ideal}}(t)$. (iii) The entropy exchange $E_n$ is defined as $E_n = -\text{Tr}(\rho_S \log_2 \rho_S)$. It is the von Neumann entropy of the joint state of the subsystem as composed of the two qubits in our model. It measures the amount of the quantum information exchange between the subsystem and the environment. For the subsystem consisted by two two-level atoms (its Hilbert space is $4 \times 4$), the entropy maximum is $\log_2(4) = 2.0$. When it reaches its maximum value, it means all the quantum information is cast out of the subsystem or the quantum subsystem degenerates to a classical state.

A. Dynamics at different $\gamma$

![Graph](a)C(t) ![Graph](b)F(t)

FIG. 1: Time evolution for (a) Concurrence, (b) Fidelity with the subsystem starting from $1/\sqrt{2}(|00\rangle + |11\rangle)$ at different values of anisotropic parameter: $\gamma = 0$ (solid curve), $\gamma = 0.2$ (dot dashed curve), $\gamma = 0.4$ (dashed curve), $\gamma = 0.8$ (dotted curve). The two cavities are initialized as $|0\rangle_1|0\rangle_2$.

In order to discuss the effect of $\gamma$ (We suppose the two cavities have the same loss degree: $\gamma_1 = \gamma_2 = \gamma$) and $T$, all the other parameters are fixed for the sake of simplicity and without loss of generality:

\[
\begin{align*}
\omega_1 &= \omega_2 = \omega = 0.4, \\
\epsilon_1 &= \epsilon_2 = \epsilon = -0.5, \\
g_1 &= g_2 = g = 0.2.
\end{align*}
\]
FIG. 2: Time evolution for (a) Concurrence, (b) Fidelity with the subsystem starting from $1/\sqrt{2}(|01\rangle + |10\rangle)$ at different values of anisotropic parameter: $\gamma = 0$ (solid curve), $\gamma = 0.2$ (dot dashed curve), $\gamma = 0.4$ (dashed curve), $\gamma = 0.8$ (dotted curve). The two cavities are initialed as $|0\rangle_1|0\rangle_2$.

FIG. 3: Time evolution for entropy of the subsystem from (a) $1/\sqrt{2}(|01\rangle + |10\rangle)$, (b) $1/\sqrt{2}(|00\rangle + |11\rangle)$ at different values of anisotropic parameter: $\gamma = 0$ (solid curve), $\gamma = 0.2$ (dot dashed curve), $\gamma = 0.4$ (dashed curve), $\gamma = 0.8$ (dotted curve). The two cavities are initialed as $|0\rangle_1|0\rangle_2$.

And we choose $1/\sqrt{2}(|00\rangle + |11\rangle)$ and $1/\sqrt{2}(|01\rangle + |10\rangle)$ as two different initial states for the subsystem.
We first discuss the effect of the photon loss rate $\gamma$. It is evident that with a larger $\gamma$, the entanglement degree of the subsystem will decrease in a faster speed, which could be verified by Fig. 1(a) and Fig. 2(a). The tendency of the two cases is similar. If $\gamma = 0$, the concurrence will oscillate periodically with time and will not be dissipated; but the peak value of it will never reach 1.0. On the whole, the curves of the concurrence are not perfectly harmonic, which is a little different with the results gotten in previous works. It is due to the stochastic and irrelevant microscopical processes (the spins drop from the excited state by emitting a photon or jump to the excited state by absorbing a phonon) inside the two different cavities. And the dynamics of the concurrence stems from such numerous processes, so the evolution is approximately harmonic but not perfect. When $\gamma > 0$, the concurrence drops abruptly to zero in a short time. It coincides with the description about entanglement sudden death (ESD) in Ref. [15, 47] that “after the concurrence goes abruptly to zero, it arises more or less from nowhere”. This is an example of ESD. The photons leaking out of the cavities greatly reduced the nonlocal connection between the two qubits. When $\gamma$ is bigger than 0.4, the speed of ESD is saturated and we almost cannot distinguish the curve of $\gamma = 0.4$ from that of $\gamma = 0.8$. The state $1/\sqrt{2}(|01\rangle + |10\rangle)$ seems more robust than $1/\sqrt{2}(|00\rangle + |11\rangle)$. It is verified that, for example, in the condition of $\gamma = 0.2$, the concurrence of the former state (to see the dot dashed line in Fig. 2(a)) decreases to zero at $\omega t = 17.472$ for the first time, while the latter one does at $\omega t = 15.168$.

The fidelity dynamics of the two Bell states, however, is much different from each other. In Fig. 1(b) when there is no cavity leakage, the fidelity is not synchronous with the concurrence. Concretely, along that curve, only when $\omega t = 37.376$, most component of the subsystem state is recovered to its initial one with high fidelity $F(t) = 0.944346$. While the other three peaks inside the interval of $\omega t \in [5.0, 35.0]$ along the solid line in Fig. 1(a) are just fake phenomena: although the entanglement degree between the two subsystem atoms is high, but the state of the subsystem is different from the initial one. The tendency of $\gamma = 0.2$ and $\gamma = 0.4$ is in a similar manner: the curves decrease smoothly and move towards $F = 0.3 \sim 0.4$. While the curve of $\gamma = 0.8$ has an oscillating dynamics with gradual shrinking amplitude. Finally, its value approaches to $F = 0.35$. In Fig. 2(b), however, when $\gamma = 0$, the fidelity is synchronous with the concurrence. Thus the fidelity of the state $|01\rangle + |10\rangle$ is more robust than that of $|00\rangle + |11\rangle$ under
the same environment. These contrasts between the two initial state can be noticed in other works. It implies the physics essence of them is different, although both of them are of the most entangled states. Yet the other three cases with leakage \( \gamma > 0 \) have almost the same kind of dynamics for the two states. The descend speed of the fidelity decreases with time and the value of fidelity approaches to about \( F = 0.2 \) at \( \omega t = 40.0 \).

The quantum evolution of entropy exchange \( E_n \) is depicted in Fig. 3. Initially, the entropy of the two entangled qubits equals to 0, which means that all the quantum information is kept in the entanglement between the two qubits. We find that when \( \gamma = 0 \), the evolutions of the two Bell states are a little different but the oscillation periods of them are almost the same. Although there is no leakage, but the bath, the two single-mode cavities, will absorb some of the quantum information inside the subsystem, which means the entropy never goes back to zero as initialled. To compare the solid line in Fig. 3(b) (Fig. 3(a)) with that in Fig. 1(a) (Fig. 2(a)), we notice that the tendency of the entropy is opposite to that of the concurrence. The dynamics difference from the two initial states can almost be removed by introducing non-zero \( \gamma \) as it is shown in the comparison of Fig. 3(a) and Fig. 3(b). It is evident that with a larger \( \gamma \), more quantum information of the subsystem is transferred into the bath.

**B. Dynamics under different \( T \)**

In this subsection, we turn to the effect of temperature of the cavity modes. It is found that when the temperature is comparatively low, \( T = 0.25\omega \), the entanglement degree oscillates but will not corrupt to a sudden death (to see the dot dashed line in Fig. 4(a) and Fig. 5(a)). When it goes up to a moderate temperature \( T = 0.5\omega \), the ESD happens. The first moment at which the concurrence decreases to zero and does not revive immediately is dependent on the initial state. For \( 1/\sqrt{2}(\langle 00 \rangle + |11\rangle) \), it takes place at \( \omega t = 4.192 \), which is much earlier than that of \( 1/\sqrt{2}(\langle 01 \rangle + |10\rangle) \), \( \omega t = 11.104 \). This difference coincides with the comparison in subsection IIIA which means the entanglement damping speed of the state \( |01\rangle + |10\rangle \) is slower than that of state \( |00\rangle + |11\rangle \). It is hinted that the bath influence on the former state is comparatively inapparent. Yet both of them can still revive to a certain extent \( 0.2 \sim 0.3 \) after some time. When the temperature increases to \( T \geq 0.75\omega \),
FIG. 4: Time evolution for (a) Concurrence, (b) Fidelity with the subsystem starting from $1/\sqrt{2}(|00\rangle + |11\rangle)$. The two cavities are initialed in thermal states at different temperature: $T = 0$ (solid curve), $T = 0.25\omega$ (dot dashed curve), $T = 0.5\omega$ (dashed curve), $T = 1.0\omega$ (dotted curve).

FIG. 5: Time evolution for (a) Concurrence, (b) Fidelity with the subsystem starting from $1/\sqrt{2}(|01\rangle + |10\rangle)$. The two cavities are initialed in thermal states at different temperature: $T = 0$ (solid curve), $T = 0.25\omega$ (dot dashed curve), $T = 0.5\omega$ (dashed curve), $T = 1.0\omega$ (dotted curve).

it can not revive in the future after the first sudden death happens for both Bell states. Till the temperature is as high as $T = 1.0\omega$, the concurrence of both cases falls with a very quick speed to zero. Obviously, the temperature will destroy the initial most-entangled states even if it is much lower than the energy bias $\omega$ of the two-level atoms. After ESD happened, the quantum oscillation from the local thermal bath may help to entangle the
two qubits, but this positive effect is neglectable when the temperature is high enough. Then the entanglement between the Bell states is damped forever.

Under variant temperature, the different dynamics of fidelity, which rely on the initial state, are shown in Fig. 4(b) and Fig. 5(b). In a short interval after $\omega t = 0$, higher temperature means faster damp speed for both initial states. Yet in a long time scale, their actions are totally different. For $T \leq 0.5\omega$, the four curves evolves pseudo-periodically, but the period of $1/\sqrt{2}(|00\rangle + |11\rangle)$ is much larger than that of $1/\sqrt{2}(|01\rangle + |10\rangle)$. In Fig. 4(b) the dotted curve of $T = 1.0\omega$ fluctuates with time, whose amplitude damps from the beginning time and gets some revival when $\omega t > 25.0$. In Fig. 5(b) the oscillation evolution at temperature $T = 1.0\omega$ is in the same manner as those at $T = 0.25\omega$ and $T = 0.5\omega$. And their amplitudes and peak values of the fidelity decrease with increasing temperature. Obviously, an environment at higher temperature destroys the fidelity of the subsystem even stronger.

In Fig. 6, we give the dynamics of the entropy exchange under different temperatures. Although there are some differences between the two sub-figures even when $T$ is as high as
0.5ω, but the tendency of the two cases are almost the same. With higher $T$, the entropy increases with faster speed and behaves an oscillation evolution with smaller amplitude. It is important to find that the entropy exchange in Fig. 6(a) exhibits an opposite behavior in comparison with that of the fidelity in Fig. 5(b). The periods of the two evolutions are the same and when the fidelity experiences a peak value, (For instance, for the dot dashed curves in the two figures, when $T = 0.25ω$, the four peaks appear at $ωt = 8.160$, $ωt = 18.656$, $ωt = 27.776$ and $ωt = 37.696$ in the given interval) the entropy is at the corresponding valley point and vice versa. In Ref. [48], the authors found the entropy exchange exhibit the behavior opposite to that of the concurrence. That coincides with our results. In our model, when the temperature is not too high and the initial Bell state is $1/\sqrt{2}(|01⟩ + |10⟩)$, the concurrence and fidelity evolve synchronously, then the dynamics of entropy is also opposite to that of the concurrence. However, when the temperature is high enough, the ESD will make this synchronization relationship invisible. So the opposite relationship between the entropy and concurrence is lost. Therefore we have to conclude that the more quantum information of the subsystem transfers to its bath, the more fidelity of that decreases during the time interval if $|ψ(0)⟩ = 1/\sqrt{2}(|01⟩ + |10⟩)$.

IV. CONCLUSION

In conclusion, we investigate the dynamics of two distinct uncoupled qubits embedded respectively in two single-mode cavities with leakage, which constitute the environment in our model. The subsystem consisting of the two qubits is initially prepared as one of the Bell states and the cavities as vacuum states or thermal equilibrium states. Under these two situations, the concurrence, the fidelity and the entropy exchange are used to portrait the subsystem dynamics from different views. Polynomial expansion method is applied into the numerical calculation. It is found that (i) for the leaky cavities, when the loss rate $γ ≥ 0.4$, the speed of entanglement sudden death achieves its maximal value; (ii) The evolution of different Bell states can easily be distinguished by their fidelity dynamics; (iii) the behavior of entropy exchange is opposite to that of the fidelity and concurrence in certain conditions.
Acknowledgments

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[1] M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information (Cambridge University Press, Cambridge, 2000).
[2] C. H. Bennett, H. J. Bernstein, S. Popescu, and B. Schumacher, Phys. Rev. A 53, 2046 (1996).
[3] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, arXiv:quant-ph/0702225 (2007).
[4] R. F. Werner, Phys. Rev. A 40, 4277 (1989).
[5] D. M. Greenberger, M. Horne, and A. Zeilinger, Bells Theorem, Quantum Theory, and Conceptions of the Universe (Kluwer Academic Publishers, Dordrecht, 1989).
[6] W. Dür, G. Vidal, and J. I. Cirac, Phys. Rev. A 62, 062314 (2000).
[7] T. Yu and J. H. Eberly, Phys. Rev. B 68, 165322 (2003).
[8] L. Diosi, in *Irreversible Quantum Dynamics*, edited by F. Benatti and R. Floreanini (Springer, New York, 2003), pp. 157-163.
[9] T. Yu and J. H. Eberly, Phys. Rev. Lett. 93, 140404 (2004).
[10] A. R. R. Carvalho, F. Mintert, and A. Buchleltner, Rev. Lett. 93, 230501 (2004).
[11] D. Loss, and D. P. DiVincenzo, Phys. Rev. A 57, 120 (1998).
[12] B. E. Kane, Nature (London) 393, 133 (1998).
[13] R. Tanaš and Z. Ficek, J. Opt. B 6, 90 (2004).
[14] A. Shimony, Ann. N.Y. Acad.Sci, 755 (1995).
[15] T. Yu and J. H. Eberly, Opt. Commun. 264, 393 (2006).
[16] Z. Ficek and R. Tanaš, Phys. Rev. A 74, 024304 (2006).
[17] L. M. Liang, J. Yuan, and C. Z. Li, J. Phys. B 39, 4539 (2006).
[18] C. A. Sackett, et. al, Nature, 403, 515 (2000).
[19] S. B. Zheng and G. C. Guo, J. Mod. Optics. 44, 963 (1997).
[20] A. Rauschenbeutel, et. al, Science, 288, 2024 (2000).
[21] J. I. Cirac, P. Zoller, H. J. Kimble, and H. Mabuchi, Phys. Rev. Lett. 78, 3221 (1997).
[22] T. Pellizzari, Phys. Rev. Lett. 79, 5242 (1997).
[23] L. M. Duan and H. J. Kimble, Phys. Rev. Lett. 90, 253601 (2003).
[24] S. Mancini and S. Bose, Phys. Rev. A 70, 022307 (2005).
[25] A. Serafini, S. Mancini, and S. Bose, Phys. Rev. Lett, 96, 010503 (2006).
[26] Z. Q. Yin and F. L. Li, Phys. Rev. A 75, 012324 (2007).
[27] S. S. Masood and A. Miller, arXiv:0705.0681. (2007).
[28] E. T. Jaynes and F. W. Cummings, Proc. IEEE 51, 89 (1963).
[29] S. Bose, I. Fuentes-Guridi, P. L. Knight, and V. Vedral, Phys. Rev. Lett. 87, 050401 (2001).
[30] L. Zhou, H. S. Song, Y. X. Luo, and C. Li, Phys. Lett. A 284, 156 (2001).
[31] R. W. Rendell and A. K. Rajagopal, Phys. Rev. A 67, 062110 (2003).
[32] I. Sainz and G. Björk, arXiv: 0706.3813 (2007).
[33] X. F. Zhou, Y. S. Zhang and G. C. Guo, Phys. Lett. A 363, 263 (2007)
[34] M. Ikram, F. L. Li, and M. S. Zubairy, Phys. Rev. A 75, 062336 (2007)
[35] S. Hill and W. K. Wootters, Phys. Rev. Lett. 78, 5022 (1997).
[36] W. K. Wootters, Phys. Rev. Lett. 80, 2245 (1998).
[37] L. Fedichkin and V. Privman, arXiv: cond-mat/0610756 (unpublished).
[38] B. Schumacher, Phys. Rev. A 54, 2614 (1996).
[39] B. Schumacher, Phys. Rev. A 54, 2629 (1996).
[40] M. Scala, B. Militello, A. Messina, J. Pillo, and S. Maniscalco, Phys. Rev. A 75, 013811 (2007).
[41] L. Tessieri and J. Wilkie, J. Phys. A 36, 12305 (2003).
[42] J. Jing and H. R. Ma, Phys. Rev. E 75, 016701 (2006).
[43] J. Jing and H. R. Ma, Chin. Phys. 16(06), 1489 (2007).
[44] J. Jing and Z. G. Lü, Phys. Rev. B 75, 174425 (2007).
[45] V. V. Dobrovitski and H. A. De Raedt, Phys. Rev. E 67, 056702 (2003).
[46] X. G. Hu, Phys. Rev. E 59, 2471 (1999).
[47] T. Yu and J. H. Eberly, Phys. Rev. Lett. 97, 140403 (2006).
[48] Y. Xiang and S. J. Xiong, arXiv:0705.1813v2, accepted by Phys. Rev. A.