NON LINEAR REALIZATIONS OF $SU(2) \times U(1)$ IN THE MSSM:
MODEL INDEPENDENT ANALYSIS AND $g-2$ OF $W$ BOSONS

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ABSTRACT

We perform a model-independent analysis of the spontaneously broken phase of an $SU(2) \times U(1)$ supersymmetric gauge theory, by using a non-linear parametrization of the Goldstone sector of the theory. The non-linear variables correspond to an $SL(2,C)$ superfield matrix in terms of which a non-linear Lagrangian can be constructed, and the pattern of supersymmetry breaking investigated. The supersymmetric order parameter is the V.E.V. of the neutral pseudo-Goldstone boson. Some applications of this technique are considered, in relation to the minimal supersymmetric standard model, and to determine the $g-2$ of the $W$-bosons in the limit of large top mass.
1 Preliminaries

One of the main candidate theories for an extension of the Standard Model incorporating new physics is its supersymmetric version. It is based on supersymmetric Yang-Mills theory \([1]\) with gauge group \(SU(3) \times SU(2) \times U(1)\) spontaneously broken to \(SU(3) \times U(1)_{em}\) plus chiral and vector multiplets of \(N=1\) supersymmetry\([2]\).

If supersymmetry is broken by soft terms \([3]\), then the model retains the good ultraviolet properties of supersymmetric theories \([4]\), and gives a realistic candidate for the solution of the hierarchy problem \([5]\), in agreement with present experimental constraints, and predicting a wealth of new physics in the TeV range.

Soft breaking terms may be thought of as relics of spontaneously broken supergravity \([3]\), which, in turns, may arise as the low-energy effective action of superstring theory \([7]\).

In the present letter we would like to give a model independent analysis of the spontaneously broken \(SU(2) \times U(1)\) gauge theory, in order to show that many of the predictions encoded in the Minimal Supersymmetric Standard Model (MSSM) \([8]\) are actually universal, and to show how different supersymmetric extensions of the Standard Model, with or without broken SUSY, can be classified in this contest.

As a partial application of our analysis, we will obtain the MSSM, and derive non-minimal versions of it, with softly broken supersymmetry, as well as model with unbroken SUSY. In the last case we will compute the \(g - 2\) of \(W\) vector bosons, verifying, in this particular case, a general sum rule of magnetic dipole moments of arbitrary electrically charged supersymmetric particles \([9]\).

2 Non-Linear Realizations of \(SU(2) \times U(1)\)

We want to describe the spontaneously broken phase of an arbitrary \(SU(2) \times U(1)\) gauge theory plus matter, with a residual unbroken gauge symmetry \(U(1)_{em} = T_3 + Y/2\).

The model independent part of this theory is described by a non-linear Kählerian sigma model \([10]\), constructed with the coset representatives of \(SL(2, C) \times GL(1, C)/GL(1, C)_{em}\). The coordinates of this manifold are the three Goldstone bosons of \(SU(2) \times U(1)/U(1)_{em}\) and their SUSY partners, which are pseudo-Goldstone particles. The first three are the longitudinal degrees of freedom of the \(W\) and \(Z\) particles, the others complete massive vector multiplets, in case of unbroken SUSY. An element of this coset is described by the two-by-two holomorphic matrix

\[
U(\xi) = e^{i\xi/2}, \quad \text{det} U = 1.
\]

It transforms as follows under \(SL(2, C) \times GL(1, C)\)

\[
U \rightarrow e^{+i\Lambda} U e^{-1/2i\Sigma\sigma_3}, \quad \Lambda \equiv \frac{1}{2} \Lambda_i \sigma_i.
\]
The minimal sigma model Lagrangian is a Kählerian sigma model with Kähler potential

\[ K(\xi, \bar{\xi}) = \text{Tr} \left( U^\dagger U \right) = \text{Tr} \left( U U^\dagger \right). \]  

(3)

This Lagrangian is invariant under \( SU(2) \times U(1) \) global transformations, which correspond to \( \Lambda, \Sigma \) real. It is also invariant under arbitrary local \( \Lambda, \Sigma \) transformations, provided we introduce gauge fields \( W = gW_\sigma \bar{\sigma}/2, \ B = gY_3/2 \) transforming as

\[ e^W \to e^{+i\Lambda^\dagger} e^W e^{-i\Lambda}, \]  

(4)

\[ e^B \to e^{i/2\Sigma^3} e^B e^{-i/2\Sigma^3}. \]  

(5)

We may also describe non-minimal couplings corresponding to the most general \( SU(2) \times U(1) \)-invariant Kähler potential, constructed in terms of \( U \) and \( U^\dagger \). This Kähler potential is only function of two \( SU(2) \times U(1) \)-invariant variables \( K(A, C) \), with \( A = \text{Tr} U^\dagger U, \ C = \text{Tr} U U^\dagger \sigma_3 \). The minimal Lagrangian, discussed in this paper, corresponds to \( K(A, C) = A \).

Let us define a new composite superfield

\[ \hat{U} = e^W U e^B, \quad \bar{D}_\alpha U = 0. \]  

(6)

The modified (gauged) Lagrangian is

\[ \mu^{-2}L(U, W, B) = \left[ \text{Tr} \left( U^\dagger \hat{U} \right) \right]_D = \left[ \text{Tr} \left( U^\dagger e^W U e^B \right) \right]_D = \left[ \text{Tr} \left( \hat{U} U^\dagger \right) \right]. \]  

(7)

where \( \mu \) gives the scale of \( SU(2) \times U(1) \) breaking\(^4\). It is convenient to define the two (matrix) superfields

\[ V_R = U^\dagger \hat{U}, \]  

(8)

transforming as \( V_R \to e^{i/2\Sigma^3} V_R e^{-i/2\Sigma^3} \), and

\[ V_L = \hat{U} U^\dagger, \]  

(9)

transforming as \( V_L \to e^{+i\Lambda^\dagger} V_L e^{-i\Lambda^\dagger} \). By means of these fields one can define the \( SU(2) \times U(1) \) gauge invariant superfields

\[ Z^0 = \text{Tr} (V_R \sigma_3), \]  

(10)

\[ W^\pm = \text{Tr} (V_R \sigma^\pm), \]  

(11)

whose \( \theta \sigma \bar{\theta} \) components are the \( Z^0, W^\pm \) particles up to sigma model corrections. The first is \( SU(2) \times U(1) \) invariant and the second transforms, under this group, as

\[ W^\pm \to e^{\pm i\Sigma^3} W^\pm. \]  

(12)

The complex conjugate superfields \( W^{\pm*} \) transform as

\[ \bar{W}^{\pm} \to e^{\pm i\Sigma^3} \bar{W}^{\pm}, \quad \bar{W}^{\pm} = W^{\mp*} e^{\mp g' Y}. \]  

(13)

\(^4\)Note that \( \xi = \frac{\phi_\mu}{\mu} \), in terms of dimensions 1 scalar fields \( \phi_\mu \).
The \( SU(2) \times U(1) \) gauge invariant expression for the \( g - 2 \) of the \( W \) supermultiplet is
\[
\left[ W^+ \tilde{D}_\alpha W^- V^\alpha \right]_D, \tag{14}
\]
where \( V^\alpha = \bar{D} \bar{D} D^\alpha V \) is the e.m. superfield-strength.

The invariant mass terms are simply given by
\[
\left[ W^+ W^- \right]_D, \quad \left[ (Z_0^0)^2 \right]_D. \tag{15}
\]
Instead, the invariant quartic couplings are given by
\[
\begin{align*}
&\left[ W^+ W^- W^\mu W^- \right]_D, \quad \left[ (Z_0^0)^2 Z_0^0 Z_0^\mu Z_0^\mu \right]_D, \\
&\left[ W^+ W^\pm W^\mu W^\pm \right]_D, \quad \left[ W^- W^- W^\pm W^\pm \right]_D.
\end{align*} \tag{16}
\]
where
\[
Z_0^\mu = \sigma_\mu^{\alpha\dot{\alpha}} [\bar{D}_\alpha, D_{\dot{\alpha}}] Z_0^\alpha, \quad W^\pm = \sigma_\mu^{\alpha\dot{\alpha}} [\bar{D}_\alpha, e^{\pm g' Y} D_{\dot{\alpha}}] W^\pm. \tag{17}
\]

The Kähler potential of the sigma model Lagrangian reads
\[
K(\xi, \bar{\xi}) = \text{Tr} \left( e^{-i/2 \xi} \sigma e^{i/2 \bar{\xi}} \sigma \right) = 2 \cos \frac{1}{2} \sqrt{\xi_i^2} \cos \frac{1}{2} \sqrt{\bar{\xi}_i^2} \sin \frac{1}{2} \sqrt{\xi_i^2} \sin \frac{1}{2} \sqrt{\bar{\xi}_i^2}. \tag{18}
\]

It is easy to see that this Lagrangian reduces to the non-linear Standard Model Lagrangian \[11\] if we set to zero the pseudo-Goldstone bosons.

It is now straightforward to discuss the features of this Lagrangian, even in the presence of soft supersymmetry breaking \[3\]. The most general softly broken Lagrangian\[4\] which preserves \( SU(2) \times U(1) \) is given by
\[
\mathcal{L}_{YM} + \mu^4 \text{Tr} \left[ U^\dagger \tilde{U} \right]_D - \mu^2 \frac{a}{4} \text{Tr} \left( U^\dagger U \right)_{\text{first component}} - \mu^2 \frac{b}{4} \text{Tr} \left( U^\dagger U \sigma_3 \right)_{\text{first component}}. \tag{19}
\]

The D terms produce a quartic term in the potential of the form
\[
\mu^4 g^2 \text{Tr} (U^\dagger U)^2 + \mu^4 g'^2 \text{Tr} \left( U^\dagger U \sigma_3 \right)^2. \tag{20}
\]
Notice that the \( b \) parameter can be viewed as a Fayet-Iliopoulos term \[12\], while the \( a \) parameter is a true soft breaking term. The \( a \) parameter could become a supersymmetric term in two ways. Either as a Fayet-Iliopoulos term with respect to a new \( U(1) \) gauge group \[13\], under which the \( U \) field has a definite nonzero charge, or as a relic of the supergravity coupling in the limit \( M_{\text{Planck}} \rightarrow \infty, \ m_{3/2} = \text{constant} \[3\]. Only the second possibility seems to give rise to a phenomenologically acceptable version of MSSM.

\[5\]By soft breaking terms we mean, in this context, terms which are bilinear in the \( U \) matrix.
Because of gauge invariance we may set \( \text{Re} \xi_i = 0 \), thus, we have a Lagrangian depending only on the following parameters: the complex variable \( \xi_+ = \text{Im} \xi_1 + i \text{Im} \xi_2 \), and the real variable \( \xi = \text{Im} \xi_3 \).

This Lagrangian has an absolute “minimum” at \( \xi_\pm = 0 \), and \( \xi \) given by the universal relation

\[
2\mu^2(g^2 + g'^2) = -\frac{a}{\cosh \xi} + \frac{b}{\sinh \xi}. \tag{21}
\]

We may define \( e^\xi = \tan \beta \) \((0 \leq \beta \leq \pi/2)\), then we have

\[
\begin{align*}
\sin 2\beta &= \frac{2\tan \beta}{1 + (\tan \beta)^2} = \frac{1}{\cosh \xi}, \\
\tan 2\beta &= -\frac{1}{\sinh \xi}. \tag{22}
\end{align*}
\]

Therefore, in the more conventional variable \( \beta \), eq. (21) reads

\[
2\mu^2(g^2 + g'^2) = -a \sin 2\beta - b \tan 2\beta, \quad 0 \leq 2\beta \leq \pi. \tag{23}
\]

The case \( a = b = 0 \) corresponds to unbroken supersymmetry \[3\], the \( D_3 \) and \( D_Y \) terms, proportional to \( \sinh \xi \) then vanish at \( \xi = 0 \). The case \( a = 0, b \neq 0 \) corresponds to spontaneously broken supersymmetry \[12\]. The case \( a \neq 0, b \neq 0 \) corresponds to softly broken supersymmetry.

We notice that in this analysis there is only a charged Higgs \( H^\pm \), and a neutral Higgs \( H^0 \), which are the superpartners of the \( W^\pm \) and \( Z^0 \) particles. This is the minimal set of states which must exist in any supersymmetric model, independently of whether \( SU(2) \times U(1) \) is a weakly coupled theory or a strongly coupled one (with dynamical breaking of gauge symmetry).

3 Connection with the MSSM

The MSSM is obtained by introducing a \( GL(2, C) \) matrix \( \Phi \), instead of a \( SL(2, C) \) matrix \( U \) as follows

\[
\Phi = SU = S e^{i/2 \xi} \sigma, \quad \det \Phi = S^2, \tag{24}
\]

where \( S \) is a chiral superfield. The \( \Phi \) transformation is identical to the \( U \) transformation, under \( SU(2) \times U(1) \), but now \( \Phi \) admits linear realization in terms of two doublet chiral superfields \( H_1 \), \( H_2 \), which transform linearly under \( SU(2) \times U(1) \) as follows

\[
\Phi = \left( \begin{array}{cc}
H_{(1)1} & H_{(2)1} \\
H_{(1)2} & H_{(2)2}
\end{array} \right), \quad H_{(1)} \rightarrow e^{i\lambda} e^{-i/2\Sigma} H_{(1)}, \quad H_{(2)} \rightarrow e^{i\lambda} e^{i/2\Sigma} H_{(2)}. \tag{25}
\]

It is immediate to see that \( S^2 = H_{(1)} \cdot H_{(2)} \), and that

\[
\begin{align*}
\frac{H_{(1)1} + H_{(2)2}}{2} &= S \cos \frac{\xi_+}{\sqrt{\xi_1 \xi_2}} \xi_2, \quad H_{(2)1} = i S \frac{\xi_+}{\sqrt{\xi_1 \xi_2}} \sin \frac{1}{2} \sqrt{\xi_1 \xi_2}, \\
\frac{H_{(1)1} - H_{(2)2}}{2} &= i S \frac{\xi_3}{\sqrt{\xi_1 \xi_2}} \sin \frac{1}{2} \sqrt{\xi_1 \xi_2}, \quad H_{(1)2} = i S \frac{\xi_3}{\sqrt{\xi_1 \xi_2}} \sin \frac{1}{2} \sqrt{\xi_1 \xi_2}. \tag{26}
\end{align*}
\]

\[6\]Note that this parametrization excludes the points for which \( \det \Phi = 0 \).
Therefore, the non-linear model is obtained from the linear one by integrating out the $S$ fields.

Note that if we set the pseudo-Goldstone to zero, $\text{Im} \xi = 0$, and we define $\text{Re} \xi \equiv \sigma$, $H_{(2)2} = \overline{H}_{(1)1}$, $H_{(2)1} = \overline{H}_{(1)2}$ and we recover the usual parametrization of the SM Higgs sector, where $\Phi = \sigma e^{i/2 \text{Re} \xi} \sigma^\dagger$, $\det \Phi = \Phi \Phi^\dagger = \sigma^2$.

The Higgs-sector Lagrangian is now given by the same expression as in eq. (7) with $U \to \Phi$ with an additional potential for the $S$ field

$$V(S) = g_2^2 8 \left[ (\text{Tr} \Phi^\dagger \sigma_i \Phi)^2 \right]_D + g_2^2 8 \left[ (\text{Tr} \Phi^\dagger \Phi \sigma_3)^2 \right]_D + \frac{a}{4} \text{Tr} \Phi^\dagger \Phi + \frac{b}{4} \text{Tr} \Phi^\dagger \Phi \sigma_3 + V(S).$$

(27)

The most general potential $V(S)$ contains a supersymmetric part $V(S)_{\text{SUSY}}$ plus a SUSY breaking part $V(S)_{\text{breaking}}$. By setting $V(S)_{\text{SUSY}} = 0$, $V(S)_{\text{breaking}} = -(m^2/2)S^2 |_{\text{first component}}$, we recover the MSSM. By choosing instead $V(S)_{\text{SUSY}} \neq 0$, we may obtain a non-minimal version of the SSM, with or without supersymmetry breaking, which corresponds to having integrated out an additional $SU(2) \times U(1)$-singlet chiral multiplet [13].

Let us discuss in some detail the MSSM and a supersymmetric model with $V(S) = \left[ \frac{\lambda}{3} S^3 - \tau S \right]_F$. (28)

In both these models we get a second equation which stabilizes the $S$ degree of freedom. In the second model with unbroken supersymmetry, the minimum is reached when the $D$ and $F$ terms vanish. This gives

$$\xi = 0, \quad S^2 = \frac{\tau}{\lambda}.$$  

(29)

If one defines the parameters $v_1$, $v_2$, related to $\xi$ and $S$ by the general formula $e^{\xi} = v_2/v_1$, $S^2 = v_1 v_2$, eq. (29) implies

$$v_1 = v_2, \quad v_1 \cdot v_2 = \frac{\tau}{\lambda}.$$  

(30)

In the MSSM, when the Higgs sector is parametrized by the three constants $a$, $b$, $m^2$, we obtain, for the $S$ field, the minimum condition

$$\cosh \xi = \frac{a}{m^2},$$

(31)

in the range of parameters $a^2 > m^4 > a^2 - b^2$. Eq. (31) turns out to be the standard condition relating $\sin 2\beta$, $m_1$, $m_2$, and $m_3$ [13], if one takes into account that

$$a = 2(m_1^2 + m_2^2), \quad b = 2(m_1^2 - m_2^2), \quad m^2 = 4m_3^2.$$  

(32)

In the case of generic non-minimal extensions of the SSM, and when the additional degrees of freedom are integrated out, we end up with a generic $V(S)$ in eq. (27). In this case eq. (31) still applies but with the replacements given by

$$\mu^2 \to SS, \quad a \to a + \left| \frac{1}{S} \frac{\partial W}{\partial S} \right|^2.$$  

(33)
where
\[
\frac{1}{4} (\text{Tr} \Phi^i \Phi^j) \left| \frac{1}{S} \frac{\partial W}{\partial S} \right|^2 = [V(S)_{\text{SUSY}}]_F.
\] (34)

4  \( g - 2 \) of \( W \)-Particles and Supersymmetric Sum Rules

Non-linear realizations of \( SU(2) \times U(1) \) give a powerful tool to compute some physically meaningful quantities, like the magnetic moments of elementary particles.

Recently, it has been shown that unbroken supersymmetry relates, in a model-independent way, the magnetic transitions between states of different spin within a given charged massive multiplet of arbitrary spin [9].

Any given massive multiplet with \( J_{\text{max}} = J + 1/2 \) contains four charged particles with the same charge and spin \( J + 1/2, J, J, \) and \( J - 1/2 \). Let us define the gyromagnetic ratio \( g_J \) of a given particle as
\[
\bar{\mu} = \frac{e}{2M} g_J \bar{J}.
\] (35)

Then, in a given multiplet with \( J_{\text{max}} > 1 \) we have the following sum rules
\[
g_{J+1/2} - 2 = 2Jh_J, \quad g_J - 2 = (2J + 1)h_J, \quad g_{J-1/2} - 2 = (2J + 2)h_J,
\] (36)

where \( h_J \) is the magnetic transition between spin \( J + 1/2 \) and \( J - 1/2 \). There is no magnetic transition between the two spin-\( J \) states. The sum rules read differently for \( J = 0 \) and \( J = 1/2 \) multiplets, which contain scalar particles.

For \( J = 0 \) (chiral) multiplets, the sum rule becomes \( g_{1/2} = 1/2 \), as shown a long time ago in ref. [17]. For \( J = 1/2 \) (the vector multiplet) the sum rules are instead
\[
g_{1/2} - 2 = 2h, \quad g_1 - 2 = h.
\] (37)

Here, \( h \) is the magnetic transition between the spin-0 and spin-1 states.

In a renormalizable theory of spin-1/2 and spin-1 particles, the tree-level value of the gyromagnetic moment is 2 [13–19]. Arguments have been given to show that this is a general phenomenon, which should occur in any tree-level unitary theory [18, 20]. However quantum effects can spoil this property. In a supersymmetric theory, if SUSY is unbroken, \( g - 2 = 0 \) for any chiral multiplets, as implied by our sum rules. This applies, in particular, to quarks and leptons in the MSSM. However, for spin-1 multiplets, \( h \) could be nonzero due to loop effects. Indeed, this was shown to happen in an explicit calculation, in the limit of vanishing quark and lepton masses [21].

\[\text{We see that for } J > 0 \text{ supermultiplets } g_J = 2 \text{ corresponds to vanishing transition magnetic moments } h_J.\]

For \( J = 1/2 \) (i.e. for vector multiplets) this interaction would be power-counting non renormalizable, and this fact explains why \( h_{1/2} = 0 \) at tree level in the SSM.
Eq. (37) implies that supersymmetry relates the following couplings \[ \eqref{eq:37} \]
\[
\frac{g}{M_W} (\psi^+ \sigma_{\mu\nu} \lambda^- - \psi^- \sigma_{\mu\nu} \lambda^+) F_{\mu\nu}, \quad g W^+ \nu^- F_{\mu\nu}, \]
\[
\frac{g}{M_W} \epsilon^{\mu\nu\rho\sigma} (\partial_{\mu} H^+ W^-_{\nu} - \partial_{\mu} H^- W^+_{\nu}) F_{\rho\sigma}.
\]

These Lagrangian couplings are responsible for the magnetic-moment relations \( \eqref{eq:37} \) in the case of spontaneously broken \( SU(2) \times U(1) \). To prove this statement we do not need to have a renormalizable theory, indeed, the one-loop result will satisfy eq. \( \eqref{eq:37} \) regardless of the existence of the S field, which is needed in order to have a linear realization of \( SU(2) \times U(1) \). Moreover, the same relations are insensitive to the value of the Yukawa couplings. Indeed, we may take a physically meaningful limit, wherein all quark and lepton masses vanish, with the exception of the top mass, which is sent to infinity \[23\]. In this case, the effective theory gives rise to eq. \( \eqref{eq:37} \) by a combination of one-loop graphs and the tree-level point-interactions produced by integrating out the top supermultiplet. Of course, eq. \( \eqref{eq:37} \) cannot be explained through the gauge anomaly cancellation that takes place in the renormalizable theory, as in ref. \[21\]. However, supersymmetry holds because the gauge anomalies, in the limit of infinite top mass, are cancelled by local Wess-Zumino terms, which are present in the effective action \[24\]. The quark and lepton contributions due to loops of massless quarks are added to the tree-level terms, and produce an effective interaction as in eq. \( \eqref{eq:14} \). The role of supersymmetric Wess-Zumino terms in this analysis will be discussed elsewhere.

By the same means one can also get additional informations about the MSSM. One is about the possible existence of a transition magnetic moment between the neutralinos belonging to the the S supermultiplet and the neutralinos of the \( Z^0 \) multiplet
\[
\left[ S(\bar{D}\bar{D}D_\alpha Z^0) V^\alpha \right] F.
\]
A transition magnetic moment between the two neutralinos of the \( Z^0 \) multiplet is not allowed when supersymmetry is unbroken.

## 5 Lepton-Number Violating Interactions

In the MSSM, if lepton-number conservation is not imposed, it is possible to write \( SU(2) \times U(1) \) gauge-invariant but lepton-number violating interactions which couple the leptons to the Higgs sector \[25\]. These interactions may give rise to mass mixing between lepton multiplets and Higgs multiplets, as well as transition magnetic moments between the charged leptons and the charginos. Moreover, one can get a transition magnetic moment between the neutrinos and a neutralino in the \( Z^0 \) multiplet. However, supersymmetry forbids a magnetic moment between the neutrinos and the neutralinos of the S multiplet.

We give now some explicit formulas for these new interactions. Consider the lepton doublet, transforming as
\[
E^- \rightarrow e^{+i\Lambda} e^{-i/2\Sigma} E^-,
\]

\( \Lambda \) and \( \Sigma \) are the SU(2) and U(1) charges, respectively.
Under $SU(2) \times U(1)$ the quantity $E^{-}\sigma_2$ transforms as
\[ E^{-}\sigma_2 \rightarrow E^{-}\sigma_2 e^{-i/2\Sigma} e^{-iA}. \] (41)

Then we can construct the quantity $E^{-}\sigma_2 U$, transforming as
\[ E^{-}\sigma_2 U \rightarrow E^{-}\sigma_2 U e^{-i/2\Sigma(i+\sigma_3)}. \] (42)

Therefore the up and down components of this quantity have definite $U_{e.m.}(1)$ charge
\[
(E^{-}\sigma_2 U)_{up} \rightarrow (E^{-}\sigma_2 U)_{up} e^{i\Sigma}, \\
(E^{-}\sigma_2 U)_{down} \rightarrow (E^{-}\sigma_2 U)_{down}.
\] (43) (44)

The lepton-violating interaction term is therefore
\[ \left[(E^{-}\sigma_2 U)_{down}\right]_{F}. \] (45)

In the linear theory, this corresponds to the following MSSM interaction
\[ E^{-}\sigma_2 H_2. \] (46)

If we take the positively charged lepton singlet $E^{+}$, transforming as
\[ E^{+} \rightarrow E^{+} e^{+i\Sigma} \] (47)
under $SU(2) \times U(1)$, we may form the doublet
\[ \psi = \begin{pmatrix} (E^{-}\sigma_2 U)_{up} \\ E^{+} \end{pmatrix}, \] (48)

which transforms as $\psi \rightarrow e^{-i\Sigma\sigma_3}\psi$. The lepton-chargino transition magnetic moment is
\[ \left[\psi(\bar{D}\bar{D}e^{+2B} D_\alpha W)V^\alpha\right]_{F}, \] (49)
whereas the neutrino-neutralino transition magnetic moment is
\[ \left[(E^{-}\sigma_2 U)_{down}(\bar{D}D D_\alpha \tilde{Z}^0)V^\alpha\right]_{F}. \] (50)

Eqs. (49,50) are manifestly $SU(2) \times U(1)$ invariant. Notice that all possible extensions of the interactions of the quark and lepton sector are given only by gauge-invariant combinations of the $U$ matrix, and therefore are insensitive to the detailed choice of a given linear realization of the gauge group. A detailed analysis of different models, with explicit calculations of masses and couplings will be given elsewhere.

Finally, it should be pointed out that the non-linear approach is particularly suitable to couple the system to supergravity and to disentangle the effects related to the spontaneous breaking of gauge symmetry and local supersymmetry.

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