Vanishing condition for the heat flux and slow evolution of a spherically fluid distribution

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Abstract. Recently, it has been found conditions for the heat flux where its introduction into the energy-momentum tensor, given its no mechanical nature, has no problems [1]. This has been achieved by checking the validity of the second law of thermodynamics in a fluid that is boosted by a Lorentz transformation of a non-commoving frame [2]. It is known that the condition, that turns out to be the null energy condition, involves the presence of a set of reference frames where Landau-Lifshitz frame is one of them. Moreover the entropy production remains positive, implying that there are no problems or issues that violate the second law of thermodynamics. In the present contribution we review the above condition, and apply it for a self-gravitating relativistic fluid in a spherically symmetric distribution in pure local coordinates. We find that our condition influences Schwarzschild fields, relating the condition of slow evolution with thermal quantities of the fluid.

1. Introduction

The heat flow and its constitutive equations in relativistic systems have been the subject of several discussions. In Eckart’s formalism [3] the constitutive equation of heat flow contains an acceleration term that seems to be the major source of general instabilities in the fluctuations which implies that water at room temperature becomes unstable [4], also it implies the nonexistence of the Rayleigh-Brillouin spectrum [5,6], and also the disappearance of the gravitational Jeans instability responsible for structure formation [7]. There have been proposed different constitutive equations that attempt to solve this problem [4,6,8]. Among them, one perspective comes from the relativistic kinetic theory [9], where a pressure gradient appears naturally instead of the acceleration term, and also turns the theory stable [6,8].

In the mid 80’s the relativistic extension of the Meixner Prigogine formalism was made [10]. In this theory the heat flux is not included in the energy-momentum tensor [11], it appears as an independent 4-vector and as a consequence such instabilities does not appear. It was argued that the single reason to include the heat flux in the energy momentum tensor is to fulfil the energy conservation [1]. It was claimed that because the heat is a non-mechanical form of energy, and has properties of momentum and inertia; it couldn’t be included into energy-momentum tensor.

Given such objections, the question whether to introduce the heat flux in the energy momentum tensor is posted. For its transformation properties [12], and also from the kinetic theoretical point of view the heat flux is part of the energy tensor [9]. In previous work [2], we address this issue from a
rather simple view performing Lorentz transformations. We founded that in the commoving frame the fluid has non zero heat flux while in the moving one it does not. It can be thought that the frame with no heat flux is just the Landau-Lifshitz frame [13], the so-called frame that moves with the heat. However, in [2] we obtain the condition which heat flux needs to satisfy, namely, just imposing to be smaller than a fixed amount, which is not negligible.

In this contribution we study the vanishing condition for the heat flux for an spherically symmetric self-gravitating fluid distribution. We also consider the impact of this condition on Schwarzschild field. It is noteworthy that the restriction reduces to well known conditions for the slow evolution regime.

In next sections we review the vanishing heat flux condition for the flat case and for the spherically symmetric case, sections 2 and 3 respectively. In the last one we summarize our conclusions.

2. Vanishing heat flux in a moving frame

From the point of view of kinetic theory, the energy-momentum tensor is identified as the second statistical moment of the distribution function and the heat flow \( q \) is part of the energy-momentum tensor [9].

Let us consider a dissipative fluid with a heat flux in \( x \) direction in the commoving frame, and suppose it is a component of the energy-momentum tensor. If we calculate a Lorentz transformation in \( x \) direction, we notice that the component \( q_x \), corresponding to the heat flux in that frame, can be split into two parts; a convective and non-convective flux [2]

\[
q_x = -\beta \gamma^2 (\epsilon + p) + q_x \gamma^2 (1 + \beta^2),
\]

where \( \beta = \frac{v}{c} \), \( \gamma \) Lorentz factor, \( \epsilon \) energy density, \( p \) pressure and \( q \) heat flux.

If we assume the flow is zero in the mobile system, the fluid in the commoving frame should satisfy the following condition [2]

\[
q_x < \frac{1}{2} (\epsilon + p).
\]

To make contact with the Landau-Lifshitz scheme form the Eckart’s frame, is possible to define a Lorentz frame only if \((q_x/nh)\)\(^2 = 0\), this condition is contained in Eq. (2) when \((q_x/nh)\)\(^2 < 1/4\), and where \( h = \epsilon + p/\gamma \) [9]. Notice that these imply that there is more than one reference frame where the heat flux vanishes [2]. The condition (2) is simple and can be obtained from another restriction, namely, the null energy condition. It states that the energy density of any matter distribution measured by any null observer must be non-negative [14]. Taking the heat flux in \( x \) direction and choose a simple null vector, we obtain the same condition given in (2). The difference is that we find (2) using transformations between frames [2]. In order to verify the validity of the second law on a fluid under condition (2), the entropy production is calculated in the Eckart frame [3], and it turns to be always positive and satisfies the second law of thermodynamics [2].

3. Heat flux in a spherically symmetric fluid distribution

Let us consider an spherically symmetric distribution of a fluid, which we assume to be locally anisotropic, undergoing heat flow dissipation limited by a spherical surface [15]. The line element is given in spherically symmetric by
\[ ds^2 = e^\nu dt^2 - e^\lambda dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \]

where \( \nu(t, r) \) and \( \lambda(t, r) \) are central functions. This metric need to satisfy Einstein field equations [10]

\[ G_{\mu}^{\nu} = -8\pi T_{\mu}^{\nu}. \]

In the case of heat flow we are interested in the \( T_{01} \) component given by [15]

\[ -8\pi T_{01} = -\frac{\lambda}{r}. \]

where dots denotes partial differentiation with respect to \( t \). Following Bondi let us introduce purely local Minkowski coordinates as follows

\[ dt = e^{\nu/2} dt, \quad dx = e^{\lambda/2} dr, \quad dy = r d\theta, \quad dz = rsin\theta d\phi. \]

Relative to this coordinates the energy momentum tensor component is scaled as \( e^{-(\nu+\lambda)/2} \). We suppose that, when viewed by an observer moving relative to these coordinates with velocity \( v_r \) in the radial direction, the anisotropic fluid is described by a tensor with energy density \( ne \), radial pressure \( p_r \), tangential pressure \( p_\theta \) and radial heat flux \( q_r \), which has the same form of the tensor we have considered so far. Thus, by a Lorentz transformation the \( (01) \)-component is

\[ T_{01} = -\frac{(ne+p_\theta)\beta e^{(\nu+\lambda)/2}}{1-\beta^2} + q_r e^{(\nu+\lambda)/2} \frac{(1+\beta^2)}{(1-\beta^2)}. \]

Let us notice the similarity between eqs. (7) and (1). Indeed, if we impose that convective and non-convective flows are canceled \( T_{01} = 0 \), we obtain the same condition \( q_r < \frac{1}{2}(ne + p) \), because the Swcharszchild fields are simplified. By substituting previous equation in (5) we obtain an extra condition that the field \( \lambda(t, r) \) needs to fulfill,

\[ \dot{\lambda} < 4\pi r(ne + p). \]

Unexpectedly, because of the positivity of the energy density, we can relate above condition with the so-called slowly evolving approximation [16,17]. A slowly evolving system is always close to hydrostatic equilibrium. This approximation demands, among others conditions, that \( \dot{\lambda}^2 \approx 0 \), in order to start with a system very close to the hydrostatic equilibrium. Thus, above expression becomes \( \dot{\lambda}^2 < 16\pi^2 r^2 (ne + p)^2 \), and then, to find a Lorentz frame where vanishing heat flux, it must also satisfy the slow evolving condition [16,17].

4. Conclusions

We study the heat flow behaviour contained in the energy-momentum tensor under Lorentz transformations finding there is a total heat flow that contains a convective and non-convective part. When the heat flux is in the same direction as the Lorentz transformation was showed that condition (2) is satisfied, \( q_x < \frac{1}{2}(ne + p) \). This condition can also be found through the null energy condition and was shown that do not violate the second law of thermodynamics [2]. With condition (2) is possible to find the reference systems where the heat flux vanishes, besides the so-called Landau-Lifshitz frame. For a spherically symmetric distributions of self-gravitating fluid, undergoing heat flow dissipation we find that relation (2) is satisfied by the radial component of the heat flux. When we introduce such conditions into the Schwarzschild field it turns out that \( \dot{\lambda} < 4\pi r(ne + p) \). This
condition can be related with the slow evolving condition that is the basis of the description of the life of most stars. This is because most of the relevant processes that occur in stars interiors happen on much longer time scale than the hydrostatic one [16,17]. This work is in the same direction as [7], in the sense of seeking conditions where relativistic heat flow is relevant and to what extent.

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