Structural Aspects of the Proton in the Chiral Quark Model

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ABSTRACT

We calculate with chiral symmetry the parton contents of the proton based on a two-component wave function. The calculation results give significant sea-quark contents and, especially, the intrinsic gluon polarization produced at a more fundamental level.

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How the spin of the proton is distributed among its constituents, quarks and gluons, is an important problem in hadron physics. An enormous experimental effort has been undertaken to provide precise information on this problem. In particular, measurements of polarization correlations in high momentum transfer reactions provide highly sensitive tests of the underlying structure and dynamics of hadrons.

For the past seven years, there have been several theoretical investigations on the substructure of the nucleon inspired by the EMC measurement [1] of the polarized proton structure function $g_1^p$. The inspiration in spin physics mainly originated from the EMC results, which imply that within the naïve parton model the sum of spins carried by all quarks and antiquarks almost vanishes, in contradiction to the expectation of the model [2] with sum of spins being $\frac{1}{2}$. In this letter, we shall show that the discrepancy can be resolved in the chiral quark model [3] with a two-component proton wave function [4] by giving an account of the flavor and spin contents of the proton.

Since most processes involve both low and high energy aspects, one should separate the low and high energy pieces in a multiplicative way via the idea of factorization. While in the high energy regime the Altarelli-Parisi equations [5] govern the $Q^2$-evolution ($Q$: the momentum transfer) of the structure functions, non-perturbative techniques must still be developed to attempt any kind of description of the elementary features on the flavor and spin contents of the nucleon. An important step based on the effective chiral quark theory is provided by Manohar and Georgi [3]. The successes of the chiral symmetric description of low energy hadron physics indicate that the chiral symmetry scale $\Lambda_{\chi SB} \sim 1 GeV$ is higher than the QCD confinement scale, and thus the important degrees of freedom at momentum scales relevant to hadron structure should be quarks, gluons and Nambu-Goldstone bosons. In the chiral quark model, the dominant process for the flavor content corrections is the dissociation of a quark into a quark and a Nambu-Goldstone boson, Fig.1; internal gluon effects are negligibly small [3, 6-9].

Fig.1: Fluctuation of a quark in chiral field theory.

The essential step of our investigation is to consider the two-component
proton \[4\]
\[| p \rangle = \cos \theta \, | 3q \rangle + \sin \theta \, (3q)G \], \tag{1}
where in the second component a 3q color-octet wave function with spin and isospin $\frac{1}{2}$ is coupled with a spin-1 color-octet gluon $G$ to make a color-single state with total angular momentum $J = \frac{1}{2}$. The angle $\theta$ specifies the amount of mixing. In this physical picture, the gluon density can be regarded as an intrinsic quantity.

We now calculate the proton’s flavor contents with a broken-U(3) symmetry \[9\]. The U(3) symmetry is broken at the subleading contributions in the $1/N_c$ expansion (non-planar diagrams) \[10\], where $N_c$ is the number of colors. The broken-U(3) can be implemented by taking the differently-renormalized octet and singlet Yukawa couplings, $g_0/g_8 \equiv \zeta \neq 1$ \[9\]. The interaction vertex is
\[\mathcal{L}_I = g_8 \bar{q} \bar{\phi} q + \sqrt{2} g_8 q \eta' q', \tag{2}\]
where
\[\bar{\phi} = \sum_{i=1}^{8} \lambda_i \phi_i = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \frac{\pi^+}{\sqrt{2}} & K^+ \\ \frac{\pi^-}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \end{pmatrix},\]
$q = (u, d, s)$ and $\lambda$s are the Gell-Mann matrices. The ninth singlet $\eta'$ and $\eta$ are unphysical. In this model we can express the flavor densities in terms of three parameters $\theta$, $\zeta$ and $a$, where $a$ is the probability of a $\pi^+$ emission. We suppose that the Goldstone boson fluctuation is sufficiently small to be treated as a perturbation. The probabilities of no Goldstone emission are $\cos^2 \theta [1 - \frac{\zeta^2}{3}(\zeta^2 + 8)]$ and $-\frac{1}{3} \sin^2 \theta [1 - \frac{\zeta^2}{3}(\zeta^2 + 8)]$ corresponding to the first and second component in (1), respectively. The multiplicative factor $-\frac{1}{3}$ appearing in the second probability comes from the fact that the total quark spin in the hybrid $3qG$ state has a probability of $\frac{1}{3}$ of being parallel to the total spin. After one interaction, the proton’s antiquark contents read
\[\bar{u} = (\cos^2 \theta - \frac{1}{3} \sin^2 \theta) \left[ \frac{a}{3} (\zeta^2 + 2\zeta + 6) \right], \tag{3}\]
\[\bar{d} = (\cos^2 \theta - \frac{1}{3} \sin^2 \theta) \left[ \frac{a}{3} (\zeta^2 + 8) \right], \tag{4}\]
\[\bar{s} = (\cos^2 \theta - \frac{1}{3} \sin^2 \theta) \left[ \frac{a}{3} (\zeta^2 - 2\zeta + 10) \right]. \tag{5}\]
and the quark contents are $u = 2 + \pi$, $d = 1 + \bar{d}$ and $s = \bar{s}$.

We now turn to the proton’s spin contents. A quark can change its helicity by emitting a spin zero meson, Fig. 2.

\begin{align*}
\cos^2 \theta \left\{ [1 - \frac{a}{3}(\zeta^2 + 8)] \left( \frac{5}{3} u_{\uparrow} + \frac{1}{3} u_{\downarrow} + \frac{1}{3} d_{\uparrow} + \frac{2}{3} d_{\downarrow} \right) + \frac{1}{3} & \left| \Psi(u_{\uparrow}) \right|^2 + \frac{1}{3} \left| \Psi(u_{\downarrow}) \right|^2 + \frac{2}{3} \left| \Psi(d_{\uparrow}) \right|^2 \right\} + \sin^2 \theta \left\{ [1 - \frac{a}{3}(\zeta^2 + 8)] \left( \frac{4}{3} u_{\uparrow} + \frac{2}{3} u_{\downarrow} + \frac{2}{3} d_{\uparrow} + \frac{1}{3} d_{\downarrow} \right) + \frac{2}{3} \left| \Psi(u_{\uparrow}) \right|^2 + \frac{2}{3} \left| \Psi(u_{\downarrow}) \right|^2 + \frac{1}{3} \left| \Psi(d_{\downarrow}) \right|^2 \right\},
\end{align*}

where

\begin{align*}
\left| \Psi(u_{\uparrow}) \right|^2 &= \left( \frac{2}{3} a + \frac{1}{3} \zeta^2 \right) u_{\downarrow} + a d_{\downarrow} + a s_{\downarrow}, \\
\left| \Psi(d_{\uparrow}) \right|^2 &= \left( \frac{2}{3} a + \frac{1}{3} \zeta^2 \right) d_{\downarrow} + a u_{\downarrow} + a s_{\downarrow},
\end{align*}

with $\left| \Psi(u_{\downarrow}) \right|^2$ and $\left| \Psi(d_{\downarrow}) \right|^2$ having the same but opposite helicity expressions. Thus the quark contributions to the proton spin $\Delta q = q_\uparrow - q_\downarrow + \overline{q}_\uparrow - \overline{q}_\downarrow$ are:

\begin{align*}
\Delta u &= \cos^2 \theta \left[ \frac{4}{3} - \frac{1}{9}(8\zeta^2 + 37)a \right] + \sin^2 \theta \left[ -\frac{2}{9} + \frac{1}{27}(4\zeta^2 + 23)a \right], \\
\Delta d &= \cos^2 \theta \left[ -\frac{1}{3} + \frac{2}{9}(\zeta^2 - 1)a \right] - \sin^2 \theta \left[ -\frac{1}{3} - \frac{2}{9}(\zeta^2 - 1)a \right],
\end{align*}

Fig. 2: A valence up quark changes its helicity by emitting a spin zero meson.

After one interaction, the contributions of various spin states can be read off from the proton’s flavor composition.
\[\Delta s = -(\cos^2 \theta - \frac{1}{3} \sin^2 \theta) a, \quad (8)\]

which lead to the total quark spin contribution

\[\Delta \Sigma = \Delta u + \Delta d + \Delta s\]
\[= (\cos^2 \theta - \frac{1}{3} \sin^2 \theta)[1 - \frac{2}{3}(\zeta^2 + 8)a]\]
\[= \cos^2 \theta - \frac{1}{3} \sin^2 \theta - 2\bar{d}, \quad (9)\]

and the intrinsic gluon polarization

\[\Delta G = \frac{2}{3} \sin^2 \theta + \bar{d}, \quad (10)\]

where \(\Delta G = G^\uparrow - G^\downarrow\). The general spin decomposition of the proton is \(\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + L_z\), with the orbital angular momentum \(L_z\) being taken to vanish in the ground state. In terms of the three parameters \(\theta, \zeta, \) and \(a\), we obtain the general expressions (3)-(10) for flavor and spin densities.

Our model calculations can reproduce the results of Refs. [4], [8] and [9] by adjusting the parameters. When \(\sin^2 \theta = 0\), we can obtain the results of the SU(3) octet (\(\zeta = 0\)), the results of the U(3) symmetry theory (\(\zeta = 1\)) [8] and the expressions in Ref. [9] (\(\zeta \neq 1\)); the non-vanishing gluon polarization (10) corresponds to the residual proton spin excluding the total quark spin contribution in Refs. [8] and [9]. Taking \(a = 0\) (no Goldstone boson emission) gives the expressions of Lipkin [4].

It is also instructive to look back at the SU(3) octet and the U(3) symmetry theory with \(\sin^2 \theta \neq 0\). From (3) and (4), one has \(\bar{u}/\bar{d} = 0.75\) for the SU(3) octet and \(\bar{u}/\bar{d} = 1\) for the U(3) symmetry case, while NA51 asymmetry measurement [11] gives

\[\bar{u}/\bar{d} = 0.51 \pm 0.04(stat) \pm 0.05(sys). \quad (11)\]

Clearly, the two-component proton wave function gives the same values \(\bar{u}/\bar{d} = 0.75\) and \(\bar{u}/\bar{d} = 1\), which are the same as those obtained from the original SU(6) proton wave function, do not agree with the NA51 experimental observation. The main hint of the \(\bar{u} - \bar{d}\) asymmetry comes from the
violation of the Gottfried sum rule [12], which is provided by the NMC measurement [13] of \( S_G = 0.235 \pm 0.026(Q^2 = 4\text{GeV}^2) \). Under the assumptions of isospin symmetry for the nucleon and for the sea quark distributions in the proton, the sum rule reads

\[
S_G = \int_0^1 \frac{dx}{x} [F_2^{np}(x) - F_2^{nn}(x)] \\
= \frac{1}{3} \int_0^1 dx \left[ u(x) + \bar{u}(x) - d(x) - \bar{d}(x) \right] \\
= \frac{1}{3}.
\]

It implies a large SU(2) flavor symmetry breaking in the quark sea or a suppression for the production of \( u\bar{u} \) pairs relative to \( d\bar{d} \) pairs in the proton,

\[
\bar{d} - \bar{u} = \frac{2}{3}(\cos^2 \theta - \frac{1}{3} \sin^2 \theta)(1 - \zeta)a \\
= 0.1475 \pm 0.039. \tag{12}
\]

As a rough estimate, by combining (11) with (12) and taking the current value \( \Delta \Sigma = 0.27 \) [14] for (9), we get \( \sin^2 \theta = 0.1 \). This value is different from that of Ref. [4], where \( \sin^2 \theta = \frac{15}{64} \) is fitted from the Bjorken sum rule [15]. The difference arises from a modification of the parton contents by including the Goldstone boson fluctuation.

On the other hand, the experimental uncertainties from the statistical and systematic errors give a large freedom of choices on these parameters. We now illustrate our model calculations with the following simple choice of parameters:

\[
\sin^2 \theta = 0.05, \quad a = 0.09, \quad \zeta = -1, \tag{13}
\]

where we see that the contribution from the second component of the proton wave function is suppressed. The parton contents (3)-(10) yield the values:

\[
\Delta u = 0.83, \quad \Delta d = -0.32, \\
\Delta s = -0.08, \quad \Delta G = 0.29, \\
\overline{u}/\overline{d} = 0.556, \quad \overline{d} - \overline{u} = 0.112, \tag{14}
\]

and the fractions of quarks flavor \( f_a \equiv (q_a + \overline{q}_a)/\Sigma(q + \overline{q}) \) read

\[
f_u = 0.51, \quad f_d = 0.33, \quad f_s = 0.16. \tag{15}
\]
Our results give significant strange-quark content. With the phenomenological value $\sigma_{\pi N} \approx 45\text{MeV} [16]$ extracted from the isospin symmetric part of the $\pi N$ scattering amplitude, Cheng and Li [9] deduced $(f_s)_{\sigma_{\pi N}} = 0.18$. Within these model dependent analyses, the matrix element of the strange scalar operator in the nucleon $\langle N \mid \bar{\Psi}s \mid N \rangle$ is not negligible. In addition, the magnetic moments in quark magnetons are given by

$$\mu_p = 0.687, \quad \mu_n = -0.463,$$

with a ratio

$$\frac{\mu_p}{\mu_n} = -1.484,$$

in the flavor-SU(3) limit of $m_{u,d} = m_s$. This is closer to the experimental value $-1.46$ than the original SU(6) prediction $\mu_p/\mu_n = -\frac{3}{2}$.

We have shown that a simple extension in the chiral quark model enjoys many interesting features. For example, the intrinsic gluon polarization can be drawn at a more fundamental level. In general, the sea-quark or anomalous gluonic interpretation for the violation of the Ellis-Jaffe sum rule depends on the factorization scheme defined for the quark spin density and the cross section for the photon-gluon scattering. In perturbative QCD, the polarized proton structure function is expressed as

$$g_1^p(x, Q^2) = \frac{1}{2} \sum_i e_i^2 \left\{ \int \frac{dy}{y} \Delta q_i(y, Q^2) \times \left[ \delta(1 - \frac{x}{y}) + \frac{\alpha_s(Q^2)}{2\pi} \Delta f_q(\frac{x}{y}) \right] ight. \
\left. - \frac{\alpha_s(Q^2)}{2\pi} \int \frac{dy}{y} \Delta \sigma^{\text{hard}}(\frac{x}{y}) \Delta G(y, Q^2) \right\},$$

where

$$\Delta f_q(z) = f_q(z) - \frac{4}{3}(1 + z),$$

$$\int_0^1 f_q(z) dz = 0, \quad \int_0^1 \Delta f_q(z) = -2,$$

and the hard kernels [17]

$$\Delta \sigma^{\text{hard}}(x) = (1 - 2x)(\ln \frac{Q^2}{\mu_{\text{fact}}^2} + \ln \frac{1 - x}{x} - 1) - 2(1 - x),$$

$$\Delta \tilde{\sigma}^{\text{hard}}(x) = (1 - 2x)(\ln \frac{Q^2}{\mu_{\text{fact}}^2} + \ln \frac{1 - x}{x} - 1),$$

(18)
with

\[ \int_0^1 \Delta \sigma^{hard}(x) = 0, \quad \int_0^1 \Delta \tilde{\sigma}^{hard}(x) = 1, \]

where \( \Delta \sigma^{hard}(x) \) and \( \Delta \tilde{\sigma}^{hard}(x) \) correspond to gauge-invariant factorization scheme and chiral-invariant (gauge-variant) scheme [18], respectively. In perturbative QCD, the gluonic contribution depends on the hard kernels (18); a different choice of kernels will yield a different set of parton distributions. We assume \( \Delta u_s = \Delta d_s = \Delta s \) and \( \Delta q_v = \Delta q'_v \) [19], where \( \Delta q \) and \( \Delta q' \) correspond to gauge-invariant and chiral-invariant parton spin densities, respectively. Then from (17) and (18) with their first moments, one gets

\[ \Delta s = \Delta s' - \frac{\alpha_s}{2\pi} \Delta G. \]  

(19)

We now take the threshold momentum transfer \( Q = 1 GeV \) (\( \alpha_s = 0.434, \Lambda_{QCD} = 200 MeV \)) to relate (19) with the model calculation results. It is straightforward to see that there is a good agreement with relation (19) with \( \Delta s = \Delta s_{exp} = -0.10, \Delta s' = -0.08 \) and \( \Delta G = 0.29 \) taken from (14). We will get poor agreement if the initial momentum transfer is below the threshold value, since perturbative QCD is not applicable in region below the threshold momentum transfer. Above the threshold scale, the chiral symmetry breaking occurs and the model dependent results are not applicable.

With regard to the proton spin crisis, suggestions have been made in terms of a large negative polarization of the sea quarks inside the proton [6, 20, 21] or a large positive polarization for the gluons [22]. A suitable combination of both should be more realistic. Our model calculations give a sensible combination on the first moments of the parton distributions. Further analyses would require the \( Q^2 \)- and \( x \)-dependent parton distributions in the perturbative region.

In using the Altarelli-Parisi equations [5], one has to know the initial behaviors of \( Q^2 \) and \( x \) dependences of the gauge-invariant parton distributions. In particular, the polarized parton distributions are based on the parameterizations with theoretical prejudice. To date, the only two constraints on the distributions are their first moments and the requirements of positivity of the spin-parallel and spin-antiparallel distributions [23].

Thus it is worthwhile to embark on a more detailed theoretical investigation, especially, the polarized parton distribution functions. The presented
nucleon substructure from the chiral quark model may serve as a basis for further studies.

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