Neutrino-nucleus scattering reexamined: Quasielastic scattering and pion production entanglement and implications for neutrino energy reconstruction

T. Leitner and U. Mosel
Institut für Theoretische Physik, Universität Giessen, Germany
(Dated: 23 April 2010)

We apply the GiBUU model to questions relevant for current and future long-baseline neutrino experiments, we address in particular the relevance of charged-current reactions for neutrino-disappearance experiments. A correct identification of charged-current quasielastic (CCQE) events — which is the signal channel in oscillation experiments — is relevant for the neutrino energy reconstruction and thus for the oscillation result. We show that about 20% of the quasielastic cross section is misidentified in present-day experiments and has to be corrected for by means of event generators. Furthermore, we show that a significant part of $1\pi^+$ (> 40%) events is misidentified as CCQE events mainly caused by pion absorption in the nucleus. We also discuss the dependence of both of these numbers on experimental detection thresholds. We further investigate the influence of final-state interactions on the neutrino energy reconstruction.

I. INTRODUCTION

Good knowledge of the neutrino energy is required for a precise determination of oscillation parameters in $\nu_\mu$ disappearance measurements. However, the neutrino beam is far from being mono-energetic in present experiments. Thus, $\nu_\mu$ disappearance experiments search for a distortion in the neutrino flux in the detector positioned far away from the source. Comparing both, un-oscillated and oscillated flux, one gains information about the oscillation probability and with that about mixing angles and mass squared differences.

The neutrino energy is not measurable directly but has to be reconstructed from the reaction products. Present oscillation experiments use the charged-current quasielastic (CCQE) reaction as signal event and reconstruct the energy with quasifree two-body kinematics from the outgoing muon assuming the target nucleon is at rest. Two immediate questions arise from this procedure: (1) How good is the identification of CCQE events? (2) How exact is the assumption of quasifree two-body kinematics for nucleons bound in a nucleus where many in-medium modifications are present?

CCQE is defined as $\nu n \rightarrow e^- p$ (i.e., on the nucleon). In the nucleus, CCQE is masked by final-state interactions (FSI). Thus, the correct identification of CCQE is immediately related to the question of how FSI influence the event selection. The main background to CCQE is CC1$\pi^+$ production. If the pion is absorbed in the nucleus and/or not seen in the detector, these events can be misidentified as CCQE events. Consequently, a proper understanding of both CCQE and CC1$\pi^+$ on nuclei is essential for the reconstruction of the neutrino energy.

This article addresses the questions outlined above in the framework of the GiBUU transport model. After giving the necessary model details, we start with a general introduction of event classification in typical neutrino detectors and discuss how possible detection thresholds influence the measured spectra. We further discuss both CCQE and CC1$\pi^+$ cross sections and their entanglement. Finally, we investigate how nuclear effects influence the reconstruction of the neutrino energy.

II. NEUTRINO SCATTERING IN THE GIBUU TRANSPORT MODEL

The presence of in-medium modifications and, in particular, final-state interactions inside the target nucleus requires the use of state-of-the-art theoretical methods for the extraction of elementary processes from experiments with nuclear targets (see also the review by Alvarez-Ruso [2]). For this aim we use the GiBUU transport model [3], where the neutrino first interacts with a bound nucleon. It has recently been questioned how good this impulse approximation (IA) actually is [4, 5]. Our experience with photonuclear processes [6] indicates that this approximation gives reasonably reliable results for momentum transfers $> 0.2$ GeV. The use of the IA requires a good description of both the elementary vertex and the in-medium-modifications. The final state of this initial reaction undergoes complex hadronic final-state interactions. In the following, we give a brief overview of our model and refer the reader in particular to Ref. [7] and to our website [8] for more details.

In the energy region of $E_\nu \sim 0.5 – 2$ GeV, the elementary neutrino-nucleon cross section contains contributions from quasielastic scattering (QE: $\ell N \rightarrow \ell' N'$), resonance excitation (R: $\ell N \rightarrow \ell' R$) and direct (i.e., nonresonant) single-pion production (BG: $\ell N \rightarrow \ell' \pi N'$) treated in our description as background. QE scattering is the most important process at these energies, followed by single-pion production through the excitation and subsequent decay of the $\Delta$ resonance $P_{33}(1232)$. However, we include in addition 12 $N^*$ and $\Delta$ resonances with in-
variant masses less than 2 GeV. The vector parts of the single contributions are obtained from recent analyses of electron-scattering cross sections. The axial couplings are obtained from PCAC (partial conservation of the axial current), and, wherever possible, we use neutrino-nucleon scattering data as input.

The elementary neutrino-nucleon cross section is modified in the nuclear medium. Bound nucleons are treated within a local Thomas-Fermi approximation which naturally includes Pauli blocking. The nucleons are bound in a mean-field potential depending on density and momentum which we account for by evaluating the above cross sections with full in-medium kinematics. We further consider the collisional broadening of the final-state particles within the low-density approximation \( \Gamma_{coll} = \rho v \) obtained in a consistent way from the GiBUU cross sections. Details of our model for the elementary vertex and the corresponding medium modifications can be found in Ref. [3].

In the next step, the particles propagate through the nucleus undergoing final-state interactions (FSI) which are simulated with the coupled-channel semiclassical GiBUU transport model (more information and code download on our website [3]). The GiBUU model is based on well-founded theoretical ingredients and has been tested in various very different nuclear reactions; in particular, against electron- and photon-scattering data [2,7].

The space-time evolution of a many-particle system in a mean-field potential is described by the BUU equation. For particles of species \( i \), it is given by

\[
(\partial_t + \nabla_p H \cdot \nabla_r - \nabla_r H \cdot \nabla_p) f_i(r, p, t) = I_{coll}[f_i, f_N, f_\pi, f_\Delta, ...],
\]

where the phase-space density \( f_i(r, p, t) \) depends on time \( t \), coordinates \( r \) and the four-momentum \( p \). \( H \) is the relativistic Hamiltonian of a particle of mass \( M \) in a scalar potential \( U \) given by \( H = \left( [M + U(r, p)]^2 + p^2 \right)^{1/2} \).

The scalar potential \( U \) usually depends both on four momentum and on the nuclear density. The BUU equations are coupled through the collision term \( I_{coll} \) which accounts for changes (gain and loss) in the phase-space density due to elastic and inelastic collisions between particles and also to particles decaying into other hadrons. In particular, we include two-body reactions like e.g. \( \pi N \rightarrow \pi N, N N \rightarrow N N, R N \rightarrow N N, R N \rightarrow R' N \), and three-body processes like \( \pi N N \rightarrow N N \) and \( \Delta N N \rightarrow N N N \). By this coupled-channel treatment we can describe side-feeding processes into different channels. This complex set of coupled differential-integral equations is then solved numerically with the GiBUU code.

All particles (also resonances) are propagated in mean-field potentials according to the BUU equations. Those states acquire medium-modified spectral functions (nucleons and resonances) and are propagated off shell. The medium modification of the spectral function is based both on collisional broadening and on the mean-field potentials, both of which depend on particle kinematics as well as on nuclear density.

Altogether, FSI lead to absorption, charge exchange and redistribution of energy and momentum, as well as to the production of new particles. We have shown in earlier works, that their impact on neutrino-induced pion production is dramatic [8,9]. Thus, a qualitatively and quantitatively correct treatment of these effects is of great importance, especially for the energy reconstruction as we will demonstrate in the following.

### III. EVENT SELECTION

Event selection in current neutrino experiments is a highly complicated subject. Rather than presenting a quantitative discussion for each particular setup we give a qualitative picture of how nuclear effects themselves modify the measured spectra in charged-current (CC) scattering assuming certain detection methods. Two generic detectors with the following properties are used toward this aim. We note that the event identifications used here are the ones used in the actual experiments.

**Cherenkov detector.** In a Cherenkov detector (e.g., MiniBooNE and K2K-K1kt), CCQE events are identified by a single ring from the outgoing lepton. Muons can be tagged by their decay electron. If pions are produced, they lead to additional rings either from the \( \gamma \) decay of the \( \pi^0 \) or from the decay muon of the charged pions.

For the Cherenkov detector, we identify the two relevant processes in the following way:

CCQE: \( 1\mu^- \pi^+ 0\pi^- 0\pi^0 \) \( xp \ xn \),

CC1\( \pi^+ \): \( 1\mu^- 1\pi^+ 0\pi^- 0\pi^0 \) \( xp \ xn \),

where \( xp \) and \( xn \) indicate, that any number of protons or neutrons are allowed.

The lower momentum thresholds depend on the index of refraction \( n \) via

\[
\beta_{thres} = \frac{1}{n} \leftrightarrow |p|_{thres} = \frac{m}{\sqrt{n^2 - 1}},
\]

where \( m \) is the particle mass. From this, one easily obtains the kinetic energy thresholds. Typical values for water (\( n = 1.33 \)) are \( T_{thres} \approx 55 \) MeV for muons, 75 MeV for charged pions, 0 MeV for neutral pions (identified via their \( \gamma \) decay), and 485 MeV for protons. Lower thresholds (\( \approx 10 \) MeV for muons and charged pions, \( \approx 65 \) MeV for protons) are reached with the MiniBooNE detector which is filled with mineral oil with \( n = 1.47 \) and, in addition, produces scintillation light.

**Tracking detector.** In a tracking detector (e.g., Sci-BooNE and K2K SciFi), all charged particles leave tracks which can be used to identify the particles...
and determine their properties. Thus, highly advanced event selection procedures are applied. To keep it simple, we identify

CCQE: \( \mu^+ 0\pi^+ 0\pi^- 0\pi^0 1p \) \( x n \),

CC1\( \pi^+ \): \( \mu^- 1\pi^+ 0\pi^- 0\pi^0 \) \( xp \) \( x n \).

The thresholds depend strongly on the experimental setup, e.g., the SciFi detector requires both muon (pion) kinetic energy to be above \( \approx 500 \) MeV (100 MeV) and the proton kinetic energy above 175 MeV [1].

In the following, we assume perfect particle identification above threshold in both cases and neglect any other experimental restrictions.

IV. TOPOLOGIES

A. CCQE identification

The CCQE reaction, \( \nu_\ell n \rightarrow \ell^- p \), being the dominant cross section at low energies, is commonly used to reconstruct the neutrino energy. In other words, CCQE is the signal event in the present oscillation experiments.

The experimental challenge is to identify true CCQE events in the detector, namely, muons originating from an initial QE process. To be more precise, true CCQE corresponds to the inclusive CCQE cross section including all medium effects or, in other words, the CCQE cross section before FSI. The difficulty comes from the fact that the true CCQE events are masked by FSI in a detector built from nuclei. The FSI lead to misidentified events, e.g., an initial \( \Delta \) whose decay pion is absorbed or which undergoes “pion-less decay” contributes to knock-out nucleons and can thus be counted as a CCQE event — we call this type of background event a “fake CCQE” event. We denote every event which looks like a CCQE event as “CCQE-like”.

As outlined above, in Cherenkov detectors CCQE-like events are all those where no pion is detected, whereas in tracking detectors, CCQE-like events are those where a single proton track is visible and at the same time no pions are detected. The two methods are compared in Fig. 1. The “true CCQE” events are denoted with solid lines, the CCQE-like events by dashed lines. The Cherenkov detector is able to detect almost all true CCQE events (top panel; solid vs. dash-dotted lines approximately agree) but sees also a considerable amount of “fake CCQE” (or “non-CCQE”) events (top panel; the dashed line is roughly 20% higher than the solid line). They are caused mainly by initial \( \Delta \) excitation as described in the previous paragraph (absorption of decay pion or “pion-less decay”); their contribution to the cross section is given by the dotted lines. These additional (fake) events have to be removed from the measured event rates by means of event generators, if one is interested only in the true QE events. It is obvious that this removal is better the more realistic the generator is in handling the in-medium \( \pi N \Delta \) dynamics.

On the contrary, less CCQE-like than true CCQE events are detected using the method applied in tracking detectors, which triggers both on pions and protons (lower panel, difference between dashed and solid line). The FSI of the initial proton lead to secondary protons or, via charge exchange to neutrons which are then not detected as CCQE-like any more (single proton track). We find that at tracking detectors the amount of fake events in the CCQE-like sample is less than at Cherenkov detectors (dashed and dash-dotted lines almost agree with each other in the lower panel but not in the top panel). We conclude that, even if the additional cut on the proton helps to restrict the background, an error of about 20% remains since the measured CCQE cross section underestimates the true one by that amount. Note that experimental detection thresholds are not yet taken into account. Thus, about 20% of the total cross section has to be reconstructed by using event generators. In this

FIG. 1: (Color online) Total QE cross section on \( ^{12}\text{C} \) compared to different methods on how to identify CCQE-like events in experiments (dashed lines). The top panel shows the method commonly applied in Cherenkov detectors; the lower panel shows the tracking-detector method as described in the text. The contributions to the CCQE-like events are also classified [CCQE-like from initial QE (dash-dotted) and from initial \( \Delta \) (dotted lines)]. Experimental detection thresholds are not taken into account.
FIG. 2: (Color online) Ratio of the CCQE-like to the true CCQE cross section as a function of the lower proton (pion) kinetic energy detection threshold for CC \( \nu_\mu \) on \(^{12}\)C at \( E_\nu = 1\) GeV. The solid lines are obtained using the tracking detector identification, whereas the dashed lines are for Cherenkov detectors.

To investigate further the relationship between the CCQE-like and true CCQE cross section, we show their ratio as a function of the lower proton and pion-kinetic energy detection thresholds in Fig. 2 (see Sec. III for the thresholds applied in present experiments). As the proton is not at all relevant for the CCQE identification in Cherenkov detectors, the ratio is independent of the proton kinetic energy threshold (dashed line in top panel). This is very different in tracking detectors which rely on the detected proton — here the efficiency is reduced to \( \approx 10\% \) at a proton kinetic energy threshold of 0.5 GeV (solid line in top panel). Even at \( T^p_{\text{thres}} = 0 \), the efficiency does not exceed 80\% because of charge-exchange processes that lead to the emission of undetected neutrons and because of secondary proton knock-out that leads to multiple-proton tracks. These effects cause the difference between the solid and the dashed lines in the top panel of Fig. 2. Focusing on the lower panel of Fig. 2 we find that the CCQE-like cross section increases for both detector types as \( T^p_{\text{thres}} \) increases. In this case, even more events with pions in the final state appear as CCQE-like because then these pions are below threshold and thus not detected.

The CCQE-like cross section is split into QE and non-QE sources (like \( \Delta \) excitation) in Fig. 3. Different sources are indicated: initial \( \Delta \) excitation and initial single-pion background reaction (higher resonances are negligible here and thus not shown). The top panel of Fig. 3 shows again the ability of a Cherenkov-like detector to identify over 98\% of the initial CCQE events (dashed line); the missing strength is mainly lost into pion channels, that is, the nucleons rescatter and produce pions such that the event is no longer classified as CCQE-like. This fraction almost vanishes (the dashed line gets even closer to one in the lower panel) when the pion kinetic energy threshold increases because then the CCQE-induced pions are no longer detected and the event counts again as CCQE-like.

Let us now turn to the non-QE CCQE-like cross section displayed in Fig. 4. Different sources are indicated: initial \( \Delta \) excitation and initial single-pion background reaction (higher resonances are negligible here and thus not shown). The top panel shows again the dependence on...
the proton-kinetic energy threshold, which is not relevant in the Cherenkov case, where the non-QE CCQE-like contribution adds up constantly to about 18%. However, this threshold is important for the tracking detector for the following reason: The non-QE processes lead not only to single-proton knockout but also to multi-nucleon knockout through pion absorption processes and rescattering. If the proton threshold is zero, these processes are not counted because there is more than one proton present. Increasing the threshold also increases the probability that only one proton is above threshold, in which case the event is CCQE-like. Above a certain kinetic energy on (≈ 0.1 GeV), more and more protons are below threshold and the ratio decreases again. The dependence on the pion kinetic energy threshold is displayed in the lower panel. Here the ratio increases because, with increasing threshold for the outgoing pion, more and more non-QE events are misidentified as CCQE.

We note that a realistic muon-kinetic energy threshold of roughly the same magnitude has no visible influence on the CCQE-like to true CCQE cross-section ratio since the muon kinetic energy is larger in most cases.

B. CC1π+ identification

The CC1π+ reaction is the second largest cross section at the energies of interest in this work, and the major background to the CCQE signal channel as we have seen in the previous section.

As in the case of CCQE, the detected CC1π+ events can also be masked by FSI. However, as we will now show, the misidentification is minor and independent of the detector type — both of our generic detectors identify CC1π+ in the same way. Problematic, however, is the low efficiency caused by strong pion-absorption effects. The top panel of Fig. 5 shows that already without any threshold cuts only 60% of the pions leave the nucleus.
V. NEUTRINO ENERGY RECONSTRUCTION

A. CCQE

In long-baseline (LBL) experiments, CCQE events are commonly used to determine the $\nu_\mu$ kinematics. The neutrino energy has been reconstructed from QE events and can be detected.\footnote{We have normalized the true CC1$\pi^+$ to the “no FSI” curve at $T_{thres} = 0$. Note also that we use the data of the Argonne bubble chamber experiment (ANL) as reference for our elementary pion production cross section.} Increasing the pion kinetic energy threshold decreases clearly the CC1$\pi^+$ event rate in the detector. In the lower panel, we plot the different contributions separately and find that the $\Delta$ excitation dominates. Concluding, Fig. 5 shows that the experiment sees only less than 60\% of all pions, with that number decreasing rapidly with increasing pion kinetic energy threshold. Therefore, a large part of the total pion yield has to be reconstructed. Any data on pion production thus contain a major model dependence. This makes it mandatory to use state-of-the-art and well-tested descriptions of the $\pi N \Delta$ dynamics in nuclei.

The measured muon energy, $E_\mu$, and scattering angle, $\theta_\mu$. The K2K experiment uses the same expression but with $E_B = 0$\footnote{See Fig. 11 the dash-dotted and dotted lines there have been obtained with $E_B = 0$. We note that in an event simulation the binding energy parameter could be adjusted such that the maximum of the reconstructed distribution is at the true energy. This is, however, not possible in the actual experiments where the true energy is not known.}. Eq. (3) is based on the assumption of quasifree kinematics on a nucleon at rest.

In Fig. 6 we plot the distribution of the reconstructed neutrino energy obtained using Eq. (3) for $E_\mu^\text{real} = 0.5, 0.7, 1.0$ and $1.5$ GeV. The reconstructed energy denoted by the dashed lines includes only true CCQE events, while the solid lines are obtained by reconstructing the energy with CCQE-like events under Cherenkov assumptions.

\[
E_\nu^\text{rec} = \frac{2(M_N - E_B)E_\mu - (E_B^2 - 2M_N E_B + m_\mu^2)}{2 \left[(M_N - E_B) - E_\mu + |K| \cos \theta_\mu\right]}, \tag{3}
\]

with a binding energy correction of $E_B = 34$ MeV and the measured muon energy, $E_\mu$, and scattering angle, $\theta_\mu$. The peak has a width of around 100 MeV full

\[\text{FIG. 6: (Color online) Distribution of the reconstructed neutrino energy according to Eq. (3) for} \]

\[E_\mu^\text{real} = 0.5, 0.7, 1.0 \text{ and} \]

\[E_\mu^\text{real} = 1.5 \text{ GeV.} \]

at the MiniBooNE experiment using $E_B = 34$ MeV for four fixed $E_\mu^\text{real} (0.5, 0.7, 1.0$ and $1.5$ GeV). The dashed lines show the true CCQE events only, the solid lines all CCQE-like events (using the Cherenkov definition, but without any threshold cuts). Both curves show a prominent peak around the real energy which is slightly shifted to higher $E_\nu^\text{rec}$. This shift is caused by the difference between our potential and the specific choice of $E_B$.\footnote{See Fig. 11 the dash-dotted and dotted lines there have been obtained with $E_B = 0$. We note that in an event simulation the binding energy parameter could be adjusted such that the maximum of the reconstructed distribution is at the true energy. This is, however, not possible in the actual experiments where the true energy is not known.}
width at half maximum (FWHM). This broadening is entirely caused by the Fermi motion of the nucleons — Eq. (8) assumes nucleons are rest.

While the distribution of the reconstructed energy for the true CCQE events is symmetric around the peak, this is not the case for the CCQE-like distribution. The reconstruction procedure now includes also non-CCQE events. However, Eq. (3) is entirely based on the muon kinematics and, in the case of $\Delta$-induced non-CCQE events, more transferred energy is needed than for true CCQE, so the muon energy is smaller. This lower muon energy leads then to the second smaller bump at lower reconstructed energies. Thus, the asymmetry is caused by the non-CCQE events that are identified as CCQE-like.

The asymmetry is very sensitive to detection thresholds, in particular to the kinetic energy threshold for charged pions (see Sec. III for the thresholds applied in present experiments). We have seen in the previous section that increasing this threshold also increases the CCQE-like cross section (via the non-CCQE events). Thus, a higher threshold leads to a more pronounced second bump, as seen in Fig. 7.

The reconstructed energy under tracking detector assumptions is plotted in Fig. 8. We have seen in the previous section that the tracking detector allows the extraction of a much cleaner CCQE-like sample than the Cherenkov detector — almost no fake, i.e., non-CCQE events spoil the CCQE-like sample. Consequently, the reconstructed distribution is again symmetric, but at the cost of a lower detection rate.

The previous findings, without any threshold cuts, are summarized in Table I and in Fig. 9. The former lists the expected values for the reconstructed energy and the standard deviation, while the latter shows the probability distribution of the relative discrepancy $(E_{\nu}^{\text{rec}} - E_{\nu}^{\text{true}})/E_{\nu}^{\text{true}}$ for 4 different real energies. We note that similar investigations by Blondel et al. 14 and Butkevich 15 result in smaller discrepancies. Both works consider only CCQE in the initial state, and do not include, e.g., $\Delta$ excitation.

So far, we have discussed the uncertainties in the energy reconstruction assuming a fixed, sharp neutrino energy. In reality, the energy distribution of the neutrinos is broad and thus the question arises how these flux distributions are affected by the reconstruction procedure. Therefore, we show in Fig. 10 the reconstructed energy distribution for the MiniBooNE flux (top panel) and the K2K flux (lower panel). Compared to the true CCQE, we find an enhancement at low reconstructed energies caused by the non-CCQE induced CCQE-like events in a Cherenkov-like detector (dashed vs. solid lines, corresponding to the low-energy bump in Fig. 6). In a tracking detector, the event rates are reduced (dashed vs. dash-dotted lines).

### Table I: Expected value, $E = \int_0^\infty dE' \frac{E'_{\nu}^{\text{true}}}{E_{\nu}^{\text{true}} - E'_{\nu}} \frac{d\sigma}{dE'_{\nu}}$, and standard deviation, $S = \left( \int_0^\infty dE' \frac{E'_{\nu}^{\text{true}}}{E_{\nu}^{\text{true}} - E'_{\nu}} \frac{d\sigma}{dE'_{\nu}} \right)^{1/2}$, for the distributions shown in Fig. 3 and Fig. 8.

| $E^{\text{real}}$ [GeV] | $E$ [GeV] | $S$ [GeV] |
|------------------------|----------|-----------|
| true CCQE              | 0.5      | 0.55      | 0.09 (17%) |
|                        | 0.7      | 0.74      | 0.12 (16%) |
|                        | 1.0      | 1.03      | 0.15 (15%) |
|                        | 1.5      | 1.52      | 0.16 (11%) |
| CCQE-like              | 0.5      | 0.53      | 0.11 (20%) |
| (Cherenkov)            | 0.7      | 0.70      | 0.16 (23%) |
|                        | 1.0      | 0.96      | 0.22 (23%) |
|                        | 1.5      | 1.41      | 0.27 (19%) |
| CCQE-like              | 0.5      | 0.54      | 0.10 (18%) |
| (tracking)             | 0.7      | 0.73      | 0.13 (18%) |
|                        | 1.0      | 1.02      | 0.17 (16%) |
|                        | 1.5      | 1.50      | 0.19 (13%) |

Note that the standard deviations do not reflect the low-energy tails caused by the misidentification of events.

### B. CC1π^+

The MiniBooNE Collaboration reconstructs the neutrino energy not only using the CCQE sample, but also
FIG. 8: (Color online) Same as Fig. 6 but under tracking detector assumptions for the CCQE-like events (solid lines).

FIG. 9: (Color online) Normalized distribution of the reconstructed energy vs. the relative discrepancy using Eq. (3) for $E_{\nu}^{\text{real}} = 0.5, 0.7, 1.0, \text{ and } 1.5 \text{ GeV}$. The dashed lines use true CCQE, the solid ones CCQE-like in a Cherenkov detector and the dash-dotted ones CCQE-like events in a tracking detector for the reconstruction.
FIG. 10: (Color online) Reconstructed energy distribution for the MiniBooNE flux (top panel) and K2K flux (lower panel) under different detector assumptions. Eq. (3) is used for the reconstruction, but with $E_B = 0$ in the K2K case.

FIG. 11: (Color online) Distribution of the reconstructed neutrino energy according to Eq. (4) for $E_{\nu}^{\text{true}} = 1 \text{ GeV}$. Shown is the reconstruction based on the CCQE-like sample (before and after FSI and Cherenkov assumptions) and based on the CC1$\pi^+$ sample (before and after FSI).

TABLE II: Expected value, $E = \int_{0}^{\infty} dE_{\nu} E_{\nu}^{\text{true}} E_{\nu}^{\text{true}}$, and standard deviation, $S = \left(\int_{0}^{\infty} dE_{\nu} (E_{\nu}^{\text{true}} - E) E_{\nu}^{\text{true}} \right)^{1/2}$, for the distributions shown in Fig. 11 for $E_{\nu}^{\text{true}} = 1 \text{ GeV}$.

| $E$ [GeV] | $S$ [GeV] |
|-----------|-----------|
| from CC1$\pi^+$, before FSI | 0.94 | 0.16 (17%) |
| from CC1$\pi^+$, after FSI | 0.95 | 0.19 (20%) |
| CCQE-like, before FSI | 0.97 | 0.13 (14%) |
| CCQE-like, after FSI | 0.90 | 0.21 (23%) |

Using the CC1$\pi^+$ sample. Based on the observed muon kinematics, treating the interaction as a two-body collision and assuming that the target nucleon is at rest inside the nucleus, one finds [10]

$$E_{\nu} = \frac{1}{2} \frac{2M_{N}E_{\mu} + M_{f}^{2} - M_{N}^{2} - m_{\mu}^{2}}{M_{N} - E_{\mu} + \cos \theta_{\mu} \sqrt{E_{\mu}^{2} - m_{\mu}^{2}}}$$

(4)

where $M_{N}$ is the mass of the nucleon, $m_{\mu}$ is the mass of the muon, $\theta_{\mu}$ its scattering angle, and $E_{\mu}$ its energy. $M_{f}$ is the Breit-Wigner mass of the $P_{33}(1232)$. This formula thus assumes that all pions are produced through the excitation of the $\Delta$ resonance which is taken to be a state of fixed mass, or, in other words, its spectral function is taken to be a $\delta$-function. Binding effects are neglected here. For $M_{f} = M_{N}$, this formula agrees with Eq. (3) for $E_{B} = 0$.

Fig. 11 shows the reconstructed energy distribution according to Eq. (4) for the CCQE-like sample and the CC1$\pi^+$ sample (before and after FSI). The shape of the dash-dotted and the dotted curves have been discussed before: Fermi motion broadens the peak and the fake CCQE events cause the bump at lower reconstructed energies. Also, the reconstructed energy from the pion sample is affected by Fermi motion (dashed and solid lines). A further broadening comes from the actual shape of the $\Delta$ resonance which is taken to be of $\delta$ function-like shape in Eq. (4). Overall, the reconstructed energy is centered around the true energy for both samples, although with a slight tendency to lower reconstructed energies. Table II lists the expected values for the reconstructed energy and the standard deviation. Note that the expected value is closer to the real energy when using the pion sample and that the standard deviation is smaller than in the CCQE-like case (calculated here also with $E_{B} = 0$).

VI. $Q^{2}$ RECONSTRUCTION

If one assumes a dipole ansatz for the axial form factor, $F_{A}$, the axial mass, $M_{A}$, is the only free parameter in the QE nucleon hadronic current (see, e.g., Ref. [3] for details; here we use $M_{A} = 1 \text{ GeV}$). $M_{A}$ affects both the absolute value of the cross section and the shape of the $Q^{2}$ distribution. Thus, there are two ways of extracting $M_{A}$ experimentally (we assume that the vector form factors are known): (1) $Q^{2}$-shape-only fit which has the advan-
tage that it does not require absolute flux normalization, (2) fit to the total cross section. On nuclei, the extraction of $M_A$ is much more complicated. Nuclear effects change the shape of the $Q^2$ distribution and, consequently, the extracted $M_A$ depends on the model used to relate measured rates on nuclei to nucleonic form factors. Furthermore, we saw in the previous section that FSI influence the CCQE identification. Misidentified events are likely to follow a different $Q^2$ distribution and also affect the total cross section as discussed in connection with Fig. 4.

Like the neutrino energy, $Q^2$ is not an observable — it has to be reconstructed from the measured muon properties. Using Eq. (3), we obtain the reconstructed $Q^2$ via

$$Q^2 = -m_\mu^2 + 2E_\mu(E_\mu - |k'| \cos \theta_\mu)$$  \hspace{1cm} (5)

The neutrino energy itself is reconstructed according to Eq. (3), thus Eq. (5) is also based on the assumption of quasi-free kinematics. Fig. 12 shows the CCQE-like $Q^2$ distribution (solid line) separated into CCQE-induced CCQE-like (dashed line) and fake CCQE (dash-dotted line) together with the reconstructed cross section. If the background subtraction is perfect (i.e., when the true CCQE sample is isolated and only this sample is used to reconstruct $Q^2$), then the reconstructed spectrum almost reproduces the true spectrum (dashed and double-dashed line almost coincide). If background events, namely non-QE induced events, are also taken into account for the reconstruction (“total reconstructed”) then, for the extreme case that no background at all is subtracted, we find an increase at lower $Q^2$, but then it falls off faster (dotted vs. solid line). The difference is caused by the different muon kinematics of the “fake” events. To conclude, we find that the reconstruction with the simplified formulas above turns out to be almost perfect when only true CCQE events are taken into account but not if the whole CCQE-like sample is used to reconstruct $Q^2$. The fake events affect both the height and the slope of the $Q^2$ distributions and, thus, the extracted $M_A$ values.

VII. IMPLICATIONS FOR OSCILLATION PARAMETER MEASUREMENTS IN $\nu_\mu$ DISAPPEARANCE EXPERIMENTS

We close this article with a brief discussion on why the exact knowledge of the neutrino energy is of major importance. The oscillation probability for the transition $\nu_\alpha \rightarrow \nu_\beta$ is given by (within a simplified two-flavor model)

$$P_{\text{osc}}(\nu_\alpha \rightarrow \nu_\beta) = \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 L}{4E_\nu} \right),$$  \hspace{1cm} (6)

where $\theta$ is the neutrino mixing angle, $\Delta m^2 = m_3^2 - m_1^2$ is the squared mass difference and $L$ the distance between source and detector. Consequently,

$$P_{\text{no-osc}} = 1 - P_{\text{osc}}.$$  \hspace{1cm} (7)

The oscillation probability depends directly on the neutrino energy $E_\nu$, so measuring the oscillation parameters $\theta$ and $\Delta m^2$ requires the knowledge of both $L$ and $E_\nu$. In LBL experiments, $L$ and $E_\nu$ are typically chosen such that the detector is placed in the oscillation maximum or minimum. Commonly, disappearance experiments measure the neutrino flux at the far detector and compare it to the one measured at the near detector.\(^5\) From the difference between both spectra one can determine the oscillation parameters (compare, e.g., the oscillation analysis performed at the K2K experiment [17]).

A schematic example is given in Fig. 13 for $\theta \approx 45^\circ$ and $\Delta m^2 = 2.5 \times 10^{-3}$ eV$^2$ (i.e., the parameters measured in $\nu_\mu$ disappearance [18]). It shows the K2K flux with $L = 250$ km. The un-oscillated spectrum is given by the dashed line, the survival probability by the dash-dotted line, and its convolution by the solid line. An exact reconstruction of the neutrino energy is thus necessary to resolve the oscillated flux, in particular the characteristic oscillation dip.

For further illustration, we show in Fig. 14 the convolution of the oscillation probability with the reconstructed K2K energy flux for the three detection scenarios introduced before: reconstruction using true CCQE, CCQE-like (Cherenkov), or CCQE-like (tracking detector) events. Clearly visible is the difference between the CCQE-like Cherenkov-based reconstruction and the two other methods at low neutrino energies around the oscillation minimum. Note that, in addition, also the recon-

\(^5\) This is in contrast to neutrino-appearance experiments which measure directly the appearance of a different neutrino flavor in the beam.
K2K experiment uses a tracking detector to measure the un-oscillated flux and a Cherenkov detector for the oscillated one. Then one has to extract the oscillation parameters from that difference — e.g., by comparing the dash-dotted line of Fig. 10 to the solid line of Fig. 14. A good understanding of the energy reconstruction is thus necessary to extract meaningful oscillation results.

VIII. SUMMARY

We have applied the GiBUU model to questions relevant to oscillation experiments. This model provides a theory-based and consistent treatment of in-medium modifications and final-state interactions. In particular, it describes electron- and photon-induced reactions successfully which serve as a benchmark for the neutrino-induced processes discussed here.

The present work addresses the relevance of CC reactions for neutrino-disappearance experiments. We have argued that a correct identification of CCQE events is relevant for the neutrino energy reconstruction and, thus, for the oscillation result. A significant part of CC1π+ events is detected as CCQE-like, which is mainly caused by the pion absorption in the nucleus. We have found that present-day experiments miss the total QE cross section by about 20% and the total pion yield by about 40%. These errors have to be corrected for by means of event generators so that the final experimental cross sections contain a significant model dependence. Furthermore, we have investigated the influence of these in-medium effects on the neutrino energy reconstruction and on the CCQE cross section, which is the signal channel in oscillation experiments stressing the effect of final-state interactions. We conclude that any model that aims to describe the experimental measurements must — because of the close entanglement of CCQE and CC1π+ on nuclei — describe both equally precise, and, in particular, the directly observable rates for nucleon knockout and 1π+ production.

Acknowledgments

We acknowledge useful discussions with Luis Alvarez-Ruso and the GiBUU team. This work has been supported by the Deutsche Forschungsgemeinschaft.

[1] HARP, http://harp.web.cern.ch/harp/
[2] L. Alvarez-Ruso, AIP Conf. Proc. 1222, 42 (2010).
[3] GiBUU, http://gibuu.physik.uni-giessen.de/GiBUU
[4] A. M. Ankowski, O. Benhar and N. Farina, arXiv:1001.0481
[5] M. Martini, M. Ericson, G. Chanfray and J. Marteau, Phys. Rev. C 80, 065501 (2009).
[6] B. Krusche et al., Eur. Phys. J. A 22, 277 (2004).
[7] T. Leitner, O. Buss, L. Alvarez-Ruso and U. Mosel, Phys. Rev. C 79, 034601 (2009).
[8] T. Leitner, L. Alvarez-Ruso and U. Mosel, Phys. Rev. C 73, 065502 (2006).
[9] T. Leitner, O. Buss, U. Mosel and L. Alvarez-Ruso, Phys. Rev. C 79, 038501 (2009).
[10] MiniBooNE, A. A. Aguilar-Arevalo et al., Nucl. Instrum. Meth. A 599, 28 (2009).
[11] K2K, R. Gran et al., Phys. Rev. D 74, 052002 (2006).
[12] MiniBooNE, A. A. Aguilar-Arevalo et al., Phys. Rev.
Lett. 100, 032301 (2008).
[13] K2K, M. H. Ahn et al., Phys. Rev. Lett. 90, 041801 (2003).
[14] A. Blondel, M. Campanelli and M. Fechner, Nucl. Instrum. Meth. A 535, 665 (2004).
[15] A. V. Butkevich, Phys. Rev. C 78, 015501 (2008).

[16] MiniBooNE, A. A. Aguilar-Arevalo et al., Phys. Rev. Lett. 103, 081801 (2009).
[17] K2K, M. H. Ahn et al., Phys. Rev. D 74, 072003 (2006).
[18] Particle Data Group, C. Amsler et al., Phys. Lett. B 667, 1 (2008).