Partial $K$–way negativities and three tangle for three qubit states

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We obtain, analytically, the global negativity, partial $K$–way negativities ($K = 2, 3$), Wootter’s tangle and three tangle for the generic three qubit canonical state. It is found that the product of global negativity and partial three way negativity is equal to three tangle, while the partial two way negativity is related to tangle of qubit pairs. We also calculate similar quantities for the state canonical to a single parameter ($0 < q < 1$) pure state which is a linear combination of a GHZ state and a W state. In this case for $q = 0.62685$, the state has zero three tangle and zero three-way negativity, having only W-like entanglement. The difference between the product of global and partial three way negativity and three tangle for a given state is a quantitative measure of two qubit coherences transformed by unitary transformations on canonical state into three qubit coherences.

The global negativity and partial $K$–way negativities, obtained by selective partial transpositions on multi-qubit state operator, satisfy inequalities which for three qubits are equivalent to CKW (Coffman-Kundu-Wootter) inequality.

Quantifying multipartite entanglement is an important problem in quantum information theory. A multipartite quantum system can have more than one type of qualitatively distinct quantum correlations since a given subsystem may be entangled to the rest in many different ways. Consequently, a single quantity can not characterize multipartite entanglement. The first step towards understanding the amount and nature of entanglement available to a given party holding part of the composite system, is to quantify the quantum correlations present in the composite system state.

Peres- Horodecki positive partial transpose is a widely used separability criterion for a bipartite state. Negativity of the partial transpose of an entangled state has been shown to be an entanglement monotone. In a recent paper, we defined $K$–way negativities to characterize $K$–partite quantum correlations in a multipartite state. The specific constraints applied during the construction of $K$–way partial transpose of a state $\hat{\rho} = |\Psi\rangle \langle \Psi|$ relate the partial $K$–way negativity to $K$–partite correlations present in the state in a very natural way. The global partial transpose can be written as a sum of $K$–way partial transposes. The partial $K$–way negativity, defined as the contribution of $K$–way partial transpose to global negativity, quantifies the $K$–partite coherences of the state. A pure state $\Psi$ can be transformed, through local unitary operations and classical communication (LOCC), to a state $\Psi_c$ such that both the states perform the same tasks in quantum information processing, however, the probabilities of success may be different. If $\Psi_c$ is a superposition of minimum number of local basis states and depends on coefficients that are non-local invariants, it is called a canonical state. The states $\Psi$ and $\Psi_c$ have the same entanglement content but different quantum coherences. It was conjectured that the global negativities and partial $K$–way negativities calculated for the $N$–partite canonical state are entanglement measures. In this letter, we show that for the case of three qubit states, the two-way and three-way partial negativities calculated for the generic canonical states are indeed entanglement monotones. Three tangle, introduced by coffman et al., is a widely used measure of GHZ like entanglement of a three qubit state. The three tangle has been shown to be an entanglement monotone. We show that the product of global negativity and partial three-way negativity for the generic three qubit canonical state is equal to three tangle, while the product of global negativity and partial two-way negativity for a given pair of qubits equals the tangle. To further elucidate the relation between the three tangle and partial $K$–way negativities of the canonical state, we analyze the global and partial $K$–way negativities ($K = 2, 3$) of a single parameter pure state, obtained by taking a linear combination of a GHZ state and a W state.

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I. PARTIAL $K$–WAY NEGATIVITIES AND THREE TANGLE FOR GENERIC THREE QUBIT CANONICAL STATE

The Hilbert space associated with a quantum system composed of three sub-systems is spanned by basis vectors of the form $|i_1i_2i_3⟩$, where $i_m = 0$ to $(d_m - 1)$, $d_m$ being the dimension of Hilbert space associated with $m^{th}$ sub-system. We refer to the first, second and third sub-system as $A$, $B$, and $C$. The state operator for a general tripartite state is

$$\hat{\rho} = \sum_{i_1i_2i_3,j_1j_2j_3} ⟨i_1i_2i_3| \hat{\rho} |j_1j_2j_3⟩ ⟨i_1i_2i_3⟩ |j_1j_2j_3|.$$ 

(1)

The global partial transposes $\hat{\rho}_G^{TA}$, $\hat{\rho}_G^{TB}$, and $\hat{\rho}_G^{TC}$, are obtained from the matrix $\rho$ by imposing the conditions

$$⟨i_1i_2i_3| \hat{\rho}_G^{TA} |j_1j_2j_3⟩ = ⟨j_1i_2i_3| \hat{\rho} |i_1j_2j_3⟩,$$

(2)

$$⟨i_1i_2i_3| \hat{\rho}_G^{TB} |j_1j_2j_3⟩ = ⟨i_1j_2i_3| \hat{\rho} |j_1i_2j_3⟩,$$

(3)

$$⟨i_1i_2i_3| \hat{\rho}_G^{TC} |j_1j_2j_3⟩ = ⟨i_1i_2j_3| \hat{\rho} |j_1j_2i_3⟩.$$ 

(4)

Global Negativity, defined as

$$N^p_G = \frac{1}{d_p - 1} \left( \|\rho_G^T\|_1 - 1 \right),$$

(5)

measures the entanglement of subsystem $p$ with its complement in a bipartite split of the composite system. Here $\|\rho\|_1$ is the trace norm of $\rho$. Global negativity vanishes on PPT-states and is equal to the entropy of entanglement on maximally entangled states.

The $K$–way partial transpose ($K = 2, 3$) of tri-partite state $\hat{\rho}$ with respect to subsystem $A$ is obtained by applying the following constraints:

$$⟨i_1i_2i_3| \hat{\rho}_K^{TA} |j_1j_2j_3⟩ = ⟨j_1i_2i_3| \hat{\rho} |i_1j_2j_3⟩,$$

if $\sum_{m=1}^{3} (1 - \delta_{i_m,j_m}) = K,$ 

(6)

$$⟨i_1i_2i_3| \hat{\rho}_K^{TA} |j_1j_2j_3⟩ = ⟨i_1i_2j_3| \hat{\rho} |j_1j_2i_3⟩$$

if $\sum_{m=1}^{3} (1 - \delta_{i_m,j_m}) \neq K,$

where $\delta_{i_m,j_m} = 1$ for $i_m = j_m$, and $\delta_{i_m,j_m} = 0$ for $i_m \neq j_m$. The partially transposed operators $\hat{\rho}_K^{TB}$, and $\hat{\rho}_K^{TC}$ are defined in analogous fashion. The $K$–way negativity calculated from $K$–way partial transpose of matrix $\rho$ with respect to subsystem $p$, is defined as

$$N^p_K = \frac{1}{d_p - 1} \left( \|\rho_K^T\|_1 - 1 \right).$$

(7)

A measure of $2$–way coherences involving a given pair of subsystem can be obtained from a $2$–way partial transpose constructed by restricting the transposed matrix elements of $\hat{\rho}$ to those for which the state of the third subsystem does not change. For example, $\rho_2^{TA-AB}$ is obtained from $\hat{\rho}$ by applying the condition

$$⟨i_1i_2i_3| \rho_2^{TA-AB} |j_1j_2i_3⟩ = ⟨j_1i_2i_3| \hat{\rho} |i_1j_2i_3⟩,$$

if $\sum_{m=1}^{3} (1 - \delta_{i_m,j_m}) = 2,$

(8)

$$⟨i_1i_2i_3| \rho_2^{TA-AB} |j_1j_2j_3⟩ = ⟨i_1i_2j_3| \hat{\rho} |j_1j_2i_3⟩$$

for all other matrix elements.

(9)

Similarly, matrix elements of $\rho_2^{TA-AC}$ are related to matrix elements of the state operator by

$$⟨i_1i_2i_3| \rho_2^{TA-AC} |j_1j_2j_3⟩ = ⟨j_1i_2i_3| \hat{\rho} |i_1j_2j_3⟩,$$

if $\sum_{m=1}^{3} (1 - \delta_{i_m,j_m}) = 2,$

(10)

$$⟨i_1i_2i_3| \rho_2^{TA-AC} |j_1j_2j_3⟩ = ⟨i_1i_2j_3| \hat{\rho} |j_1j_2i_3⟩$$

for all other matrix elements.
The partially transposed matrix $\rho^T_{A}$ leading to $N^2_{A-AB} = \left( \|\rho^T_{2_{A-AB}}\|_1 - 1 \right)$ and $N^2_{A-AC} = \left( \|\rho^T_{2_{A-AC}}\|_1 - 1 \right)$, measure the 2-way coherences involving the pairs of subsystems $AB$ and $AC$, respectively.

Any three qubit pure state may be transformed to canonical form with respect to qubit $A$, that reads as

$$\Psi = a |000\rangle + be^{i\varphi} |100\rangle + g |\Phi_1\rangle,$$

where $g = \sqrt{c^2 + d^2 + f^2}$ and $\Phi_1 = \frac{c}{g} |110\rangle + \frac{d}{g} |101\rangle + \frac{f}{g} |111\rangle$. The partial transpose of the state operator $\hat{\rho} = |\Psi\rangle \langle \Psi|$ for subsystem $A$ in the vector space spanned by basis vectors $|000\rangle$, $|100\rangle$, $|\Phi_1\rangle$ and $|\Phi_1^T\rangle = \frac{c}{g} |010\rangle + \frac{d}{g} |001\rangle + \frac{f}{g} |011\rangle$, is

$$\rho^T_{G} = \left[ \begin{array}{cccc} a^2 & abc^{i\varphi} & 0 & 0 \\ abc^{-i\varphi} & b^2 & be^{-i\varphi}g & ag \\ 0 & be^{i\varphi}g & g^2 & 0 \\ 0 & ag & 0 & 0 \end{array} \right].$$

The partially transposed matrix $\rho^T_{G}$ is a negative matrix with a negative eigenvalue $\lambda^- = -ag$ and the corresponding eigenvector

$$|\Psi_G^-\rangle = \frac{(a+g)}{\sqrt{ag+2}} |100\rangle - \frac{be^{i\varphi}}{\sqrt{4ag+2}} |000\rangle - \frac{be^{-i\varphi}}{\sqrt{4ag+2}} |\Phi_1\rangle - \frac{(a+g)}{\sqrt{4ag+2}} |\Phi^T_1\rangle.$$  

It is easily verified that

$$\rho^T_{G} = \rho^T_{3_{A}} + \rho^T_{2_{A}} - \hat{\rho}. \quad (14)$$

Using the relation given by Eq. (14) in

$$N^2_G = -2 \langle \Psi_G^- | \rho^T_{G} | \Psi_G^- \rangle = 2ag,$$

we obtain

$$N^4_G = E^A_{3} + E^A_{2} - E^A_{0}, \quad (15)$$

where the partial $K$-way negativities $E^A_K$ for $K = 2$ and $3$ are defined as

$$E^A_{K} = -2 \langle \Psi_G^- | \rho^T_{K} | \Psi_G^- \rangle, \quad \text{while} \quad E^A_{0} = -2 \langle \Psi_G^- | \hat{\rho} | \Psi_G^- \rangle.$$  

We may further write

$$\rho^T_{2} = \rho^T_{2_{A-AB}} + \rho^T_{2_{A-AC}} - \hat{\rho}, \quad (18)$$

and define

$$E^A_{2_{AB}} = -2 \langle \Psi_G^- | \rho^T_{2_{A-AB}} | \Psi_G^- \rangle, \quad E^A_{2_{AC}} = -2 \langle \Psi_G^- | \rho^T_{2_{A-AC}} | \Psi_G^- \rangle,$$

giving $E^A_{2} = E^A_{2_{AB}} + E^A_{2_{AC}}$. For the state of Eq. (13), we obtain

$$E^A_{3} = \frac{4a^2c^2}{2ag}, \quad E^A_{2} = \frac{4a^2(c^2 + d^2)}{2ag}, \quad (20)$$

leading to

$$\left( N^4_G \right)^2 = E^A_{3}N^A_G + E^A_{2}N^A_G.$$

The negative part of $\rho^T_{G}$ is found to be orthogonal to $\hat{\rho}$ giving $E^A_{0} = 0$. We also obtain

$$E^A_{2_{AB}} = \frac{4a^2c^2}{N^A_G}, \quad \text{and} \quad E^A_{2_{AC}} = \frac{4a^2d^2}{N^A_G}. \quad (22)$$
The tangle, defined to be a measure of entanglement of qubit $A$ with subsystem $BC$ \[11\], for the pure state of Eq. (13) is given by

$$
\tau_{A(BC)} = 4 \det (\rho^A) = 4 \det \left[ \begin{array}{cc}
a^2 & be^{i\varphi} \\
abe^{-i\varphi} & |b|^2 + |a|^2 \end{array} \right] = 4a^2g^2.
$$

If tangles $\tau_{AB}$ and $\tau_{AC}$ are measures of entanglement of $A$ with $B$ and $C$ respectively, the three tangle \[12\] defined as

$$
\tau_3 = \tau_{A(BC)} - \tau_{AB} - \tau_{AC},
$$

is known to measure the tripartite entanglement of the pure state. The value of $\tau_{AB}$ for the mixed states $\rho^{AB} = tr_C(\rho^{ABC})$ is obtained by constructing the spin flipped density matrix

$$
\tilde{\rho}^{AB} = (\sigma_y \otimes \sigma_y)(\rho^{AB})^*(\sigma_y \otimes \sigma_y),
$$

where asterisk denotes complex conjugation and $\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$. With the square roots of the eigenvalues of product matrix $\rho^{AB}\tilde{\rho}^{AB}$ given by $\lambda_1, \lambda_2, \lambda_3, \text{and} \lambda_4$, the tangle $\tau_{AB}$ is defined \[11\] as $\tau_{AB} = [\max \{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\}]^2$. For the generic state of Eq. (13) we obtain $\tau_{AB} = 4a^2c^2$. The tangle $\tau_{AC}$ calculated for $\rho^{AC} = tr_B(\rho^{ABC})$ has the value $4a^2d^2$. As such, the three tangle, for the generic three qubit canonical state of Eq. (11), is found to be $\tau_3 = 4a^2f^2$ giving

$$
E_3^A N_G^A = \tau_3, \quad E_2^A N_G^A = \tau_{AB} + \tau_{AC}, \quad (N_G^A)^2 = \tau_{A(BC)}.
$$

We also identify the products $E_2^{A-AB} N_G^A$ and $E_2^{A-AC} N_G^A$ with tangles $\tau_{AB}$ and $\tau_{AC}$, respectively. The product $E_2^{A-AB} N_G^A$ measures the $K$-way $($$K = 2$ and $3$$)$ entanglement of the canonical state. For a state obtained from the canonical state by local unitary transformations on qubits $A$, $B$, and $C$, the partial $3$-way negativity may be different from $E_3^A$, $p = 1$ to $3$, but the global negativity remains invariant. As such, for a given three qubit state, the difference $\Delta = E_3^A N_G^A - \tau_3$ is a measure of the amount of two-qubit coherences transformed to three qubit coherences by applying local unitary transformations on the corresponding canonical state.

To illustrate, how local operations on qubits transform three qubit coherences into two qubit coherences and vice versa, we consider a single parameter GHZ-like state

$$
\Psi_a = a |000\rangle + \sqrt{1 - a^2} |111\rangle, \quad 0 < a < 1,
$$

with parties $A$, $B$ and $C$ holding one qubit each. The global negativity $N_G^A = 2a\sqrt{1 - a^2}$ is equal to $E_3^A$, while $E_2^A = 0$. We define a unitary rotation of qubit $C$ by the transformation matrix

$$
U_C = \begin{bmatrix} \cos \left(\frac{\alpha}{2}\right) & \sin \left(\frac{\alpha}{2}\right) \\ -\sin \left(\frac{\alpha}{2}\right) & \cos \left(\frac{\alpha}{2}\right) \end{bmatrix}.
$$

For the state $U_C |\Psi_a\rangle$ we obtain

$$
E_3^A (\alpha) = \frac{a\sqrt{1 - a^2}}{2} (3 + \cos (2\alpha)), \\
E_2^A (\alpha) = \frac{a\sqrt{1 - a^2}}{2} (1 - \cos (2\alpha)),
$$

showing that the maximum amount of three way correlations in the state $\Psi_a$ that may be transformed to two-qubit coherences is $N_G^A E_2^A (\pi/2) = (N_G^A)^2 / 2$. We notice that a state measurement, in logical basis, on qubit $C$ completely destroys the entanglement of state $\Psi_a$, while in the case of $U_C |\Psi_a\rangle$ destroys entanglement due only to coherences measured by $E_3^A (\alpha)$. Consequently, the reduced state $tr_C (|\Psi_a\rangle \langle\Psi_a|)$ is a separable state, while $tr_C \left( U_C^\dagger |\Psi_a\rangle \langle\Psi_a| (U_C^\dagger) \right)$ is an entangled two-qubit mixed state.

For three qubit states in canonical form, GHZ-like states have $N_G^A = E_3^A$, while W-like states have $N_G^A = E_2^A$. The N-qubit states, may, likewise be classified according to the amount and type of $K$-way coherences ($2 \leq K \leq N$).
present in the canonical form. It was shown in ref. [6] that for an N partite system, the global negativity of partially transposed state operator $\hat{\rho}$ with respect to a subsystem $p$ can be written in terms of partial $K$-way negativities as

$$N_p^G = \sum_{K=2}^{N} E_K^p - E_0^p,$$

(28)

where

$$E_K^p = -\frac{2}{d_p - 1} \sum_{i} \langle \Psi_{i}^{G-} | \hat{\rho}_{K}^{T_p} | \Psi_{i}^{G-} \rangle, \quad 2 \leq K \leq N,$$

(29)

$$E_0^p = -\frac{2(N-2)}{d_p - 1} \sum_{K=2}^{N} \sum_{i} \langle \Psi_{i}^{G-} | \hat{\rho} | \Psi_{i}^{G-} \rangle,$$

(30)

and $\Psi_{i}^{G-}$ the eigenvector of $\hat{\rho}_{G}^{T_p}$ corresponding to negative eigenvalue $\lambda_i$. For the case where $E_0^p = 0$, we can affirm that

$$N_p^G \geq E_K^p, \quad 2 \leq K \leq N.$$

(31)

It is important to note that if $N_p^G > 0$ for $p$ referring to a part in all possible bipartite splits of the composite quantum system, then $E_N^p = 0$ for a given $p$ does not mean that N-partite entanglement is zero. The N-partite entanglement in this case could be caused by coherences of the order less than N. For the special case of three qubit states, we obtain two inequalities

$$N_G^p \geq E_2^p, \quad N_G^p \geq E_3^p,$$

(32)

the former being equivalent to the CKW inequality [8]

$$\tau_{A(BC)} \geq \tau_{AB} + \tau_{AC},$$

(33)

on which the definition of three tangle is based.

II. A LINEAR COMBINATION OF THREE QUBIT GHZ STATE AND W STATE

An interesting pure state for which tripartite entanglement has been studied by Lohmayer et al. [12] is a superposition of three qubit GHZ state and W state. In our paper [13], we calculated $N_G^A, E_3^A,$ and $E_2^A$ for single parameter pure states

$$\Psi^{(\pm)} = \sqrt{q} \Psi_{GHZ} \pm \sqrt{(1-q)} \Psi_{W}, \quad \hat{\rho}_2^{\pm} = \left| \Psi_2^{(\pm)} \right\rangle \left\langle \Psi_2^{(\pm)} \right|, \quad 0 \leq q \leq 1$$

(34)

and compared with three tangle for states $\Psi^{(\pm)}$, calculated from

$$\tau_3 \left( \Psi^{(\pm)} \right) = q^2 \pm \frac{8 \sqrt{6(q(1-q))^3}}{9}.$$

(35)

Here $\Psi_{GHZ} = (|000\rangle + |111\rangle) / \sqrt{2}$, is GHZ state for which $\tau_3 (\Psi_{GHZ}) = 1$ and

$$\Psi_{W} = \frac{|100\rangle + |010\rangle + |001\rangle}{\sqrt{3}},$$

is W state with $\tau_3 (\Psi_{W}) = 0$. It was found that both for the state $\Psi^{(+)}$ and $\Psi^{(-)}$ the partial 3-way negativity $E_3^A > 0$ for parameter values $0 < q < 1$. To put the record straight, the states $\Psi^{(\pm)}$ are not in canonical form. Therefore the difference $\Delta = E_3^A N_G^A - \tau_3$ for a given state, observed in [13] measures the distance of the state from the canonical state in terms of excess or deficit of three qubit coherences in comparison with that for the canonical state. The states
ψ(±) can be transformed to Schmidt like form for qubit A, by following the procedure given by Acin et al. [10]. A unitary transformation

$$U^A = \begin{bmatrix} \alpha & -\beta \\ \beta & \alpha \end{bmatrix}, \quad |\alpha|^2 + |\beta|^2 = 1$$

on qubit A, followed by Unitary transformations

$$U^C = \frac{1}{\sqrt{a^2 \beta^2 + b^2 s^2}} \begin{bmatrix} \beta a & \alpha b \\ -\alpha b & -\beta a \end{bmatrix}, \quad a = \sqrt{\frac{q}{2}}, \quad b = \pm \sqrt{\frac{1-q}{3}},$$

$$U^B = \frac{1}{\sqrt{(a^2 \beta^2 + b^2 \alpha^2)}} \begin{bmatrix} -b \alpha & a \beta \\ -a \beta & b \alpha \end{bmatrix},$$

on qubits C and B, subject to the constraint $\alpha \beta a^2 - \beta^2 ab + \alpha^2 b^2 = 0$, yield the state

$$\psi_c(q) = a_{000}|000\rangle + a_{100}|100\rangle + a_{110}|110\rangle + a_{101}|101\rangle + a_{111}|111\rangle.$$

The two possible solutions are

$$\alpha = \beta x^2 \frac{1}{2} \left(1 \pm \sqrt{1 - \frac{4}{x^3}}\right), \quad x = \frac{a}{b} = \mp \sqrt{\frac{3q}{2(1-q)}},$$

except for the case $x^3 = 4$ ($q = 0.62685$ for state $\psi^(-)$), in which case only solution is $\alpha = \beta x^2 / 2$, giving $\alpha = 0.78327$ and $\beta = 0.62169$. For $x^3 \neq 4$, the partial 3-way negativity of the state (using Eq. (20)),

$$E^A_3 = \frac{2^{a_{000}a_{111}}}{\sqrt{1 - (a_{000})^2 - (a_{111})^2}},$$

is non zero for $a_{000} \neq 0$, and $a_{111} \neq 0$.

The canonical state for $x^3 = 4$ reads as

$$\psi_c(q = 0.62685) = -0.56731 |010\rangle - 0.56731 |100\rangle + 0.18578 |110\rangle + 0.56731 |111\rangle,$$

giving $N^A_3 = E^A_3 = 0.9103, \quad E^A_3 = 0$. This result is consistent with the fact that three tangle $\tau_3(\psi^{-}) = 0$ for $q = 0.62685$. The W-like tripartite entanglement of the state $\psi_c(q = 0.62685)$ arises solely from two-body coherences measured by nonzero $E^A_2$ of the canonical state. The GHZ like tripartite entanglement of a three qubit state is due to three-body coherences (nonzero $E^A_3$ of canonical state). Two-body correlations generate W-like tripartite entanglement and pairwise entanglement of qubits. A mixed state, $\rho_{\text{mixed}}$, can be decomposed as a convex combination of projectors onto pure states as

$$\rho_{\text{mixed}} = \sum_i P_i \hat{\rho}_i, \quad \hat{\rho}_i = |\Psi_i\rangle \langle \Psi_i|,$$

where $P_i$ is the probability of finding the state $\hat{\rho}_i$. For the mixed states, we define the convex roof extended Global and K-way negativities through

$$N^p(\rho_{\text{mixed}}) = \min \sum_i P_i N^p(\rho_i).$$

Here minimum of $\sum_i P_i N^p(\rho_i)$ taken over all possible decompositions of $\rho_{\text{mixed}}$ is defined as $N^p(\rho_{\text{mixed}})$. The negativities $(N^A_G(\rho_{\text{mixed}})^A)^2$ and $(N^A_G(\rho_{\text{mixed}})^C)^2$, where $\rho_{\text{mixed}}^{AB} = tr_C(\rho)$ and $\rho_{\text{mixed}}^{AC} = tr_B(\rho)$, measures the pairwise entanglement in three qubit state $\rho$. For the state of Eq. (40), we get

$$(N^A_G(\rho_{\text{mixed}})^A)^2 = (N^A_G(\rho_{\text{mixed}})^C)^2 = 0.4143$$

that is

$$(N^A_G)^2 = (N^A_G(\rho_{\text{mixed}})^A)^2 + (N^A_G(\rho_{\text{mixed}})^C)^2 = 0.8286.$$
This analysis leads to a simple explanation for why a GHZ state cannot be converted to a W-state. It is not possible to transform three-body correlations into purely two-body correlations by local operations and classical communication.

To summarize, we have shown that for generic three qubit canonical state the product of global negativity and partial three way negativity is equal to three tangle, which is an entanglement monotone, quantifying GHZ-like three qubit quantum correlations. The product of global negativity and partial two way negativity for a given pair of qubits in the canonical state is seen to be equal to tangle for the pair. It proves that the $K$-way entropies associated with the three qubit canonical state are indeed entanglement measures. The importance of this result lies in the case with which the partial $K$-way negativities can be calculated for multipartite canonical states and the physical meaning associated to partial $K$-way negativities as measures of $K$-partite coherences. The global negativity and partial $K$-way negativities, obtained by selective partial transpositions on multi-qubit state operator, satisfy inequalities which for three qubits are equivalent to CKW (Coffman-Kundu-Wootters) inequality. The difference between the values of product of global and partial three way negativity for a given state and three tangle for the state is a quantitative measure of two qubit correlations transformed by unitary transformations on canonical state into three qubit correlations. We have also calculated the partial $K$-way negativities and three tangle for the state canonical to a single parameter ($0 < q < 1$) pure state which is a linear combination of a GHZ state and a W state. In this case for $q = 0.62685$, the state has zero three tangle as well as zero three-way negativity, having only W-like entanglement. For mixed states the relevant entanglement measures are convex roof extended global and $K$-way negativities. The GHZ-like and W-like state are found to differ in the amount and type of quantum correlations present in canonical form. The N-qubit states may also be classified according to the amount and type of $K$-way coherences ($2 \leq K \leq N$) present in the canonical form. We expect the use of partial $K$-way negativities to enhance our understanding of quantum correlations and entanglement in multipartite systems.

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