Electron scattering on circular symmetric magnetic profiles in a two-dimensional electron gas

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The quasi-bound and scattered states in a 2DEG subjected to a circular symmetric steplike magnetic profile with zero average magnetic field are studied. We calculate the effect of a random distribution of such identical profiles on the transport properties of a 2DEG. We show that a nonzero Hall resistance can be obtained, although \( \langle B_z \rangle = 0 \), and that in some cases it can even change sign as function of the Fermi energy or the magnetic field strength. The Hall and magnetoresistance show pronounced resonances apart from the Landau states of the inner core, corresponding to the so-called quasi-bound snake orbit states.

I. INTRODUCTION

The response of a two-dimensional electron gas (2DEG) to a spatially inhomogeneous magnetic field has been the subject of considerable interest in recent years.\[1\] At very low temperatures and in very pure samples, these inhomogeneities in the magnetic field can act as scattering centers for the 2DEG, perturbing the ballistic electron motion and hence altering the transport properties of the 2DEG.

These inhomogeneous magnetic fields can be realized by growing a type-II superconducting film on top of a heterojunction, containing a 2DEG, if a background magnetic field is applied, vortices will penetrate the 2DEG, where they form scattering centers. If the applied magnetic field is low, the vortices will be distributed randomly, due to the inhomogeneities in the superconducting film. Brey et al.\[3\] and Nielsen et al.\[4\] studied scattering on these vortices if distributed randomly, and if distributed on a periodic array.

In an alternative approach superconducting particles are deposited above a 2DEG. Due to the Meissner effect, magnetic flux will be expelled from the particles, which again results in a low magnetic field region underneath each of the superconducting particles. This was realized by Smith et al.\[5\] who grew lead grains on top of a heterojunction.

A logic next step would be to deposit ferromagnetic clusters as inhomogenous magnetic field creators in the 2DEG. This was realized by Ye et al.\[6\] who grew Dy-micromagnets on top of a GaAs/AlGaAs heterostructure and recently Dubonos et al.\[7\] studied scattering of electrons on the stray field of a single Dy-magnet. This problem is essentially different from the earlier problems, because now, the average magnetic field strength is zero, \( \langle B_z \rangle = 0 \).

Preliminary results on this system were already presented in Refs. 8 and 9, where scattering on the stray fields of infinitesimally flat magnetic disks with perpendicular magnetization was studied. In this paper we extend and generalize these earlier results and study cylindrical symmetric steplike profiles, with average magnetic field zero. This simplification enables us to classify the arising phenomena and understand their underlying physics. We will show that such a system can give rise to a nonzero Hall resistance, even though \( \langle B_z \rangle = 0 \). Moreover, such a system can host quasi-bound states, similar to the ones studied by Kim et al.\[7\] who investigated theoretically the electron states of a circular symmetric magnetic field profile, consisting of two regions with different magnetic field strengths inside and outside a radius \( R \). In contrast to the bound states found by Kim et al., here the electron states are quasi-bound, because we consider magnetic profiles which are finite in extent.

We will model the magnetic field by

\[
\begin{align*}
B(r < R_a) &= B_a \\
B(R_a < r < R_b) &= B_b \\
B(r > R_b) &= 0,
\end{align*}
\]

with \( B_b = -B_a/\left[(R_b/R_a)^2 - 1\right] \) such that the condition \( \langle B_z \rangle = 0 \) is satisfied. This magnetic field profile models the one of a perpendicularly magnetized ferromagnetic disk as felt by a 2DEG underneath the disk. As an example we plot in Fig. 1 the magnetic field profile (solid curve) resulting from a ferromagnet with radius \( R_a \) and thickness \( d/R_a = 1 \), grown a distance \( h/R_a = 0.1 \) above a 2DEG, as shown in the inset. The dotted curve represents the magnetic field according to our model, where we have chosen \( R_b/R_a \approx 2.8 \) to account for the shape of the profile. The cut-off at \( R_b \) is made in order to simplify the calculations. We believe that in doing so, the physics is not altered.

The parameters \( (R_a, R_b, B_a \) and \( B_b \) depend on the specific properties of the ferromagnetic material (as extent, thickness, magnetization, distance to the 2DEG). The resulting profile can also be affected by including superconducting strips, which expel the flux lines due to the Meissner effect and consequently can rearrange/guide the magnetic field lines.
The paper is organized as follows. In Sec. II we consider scattering on a single magnetic profile. First, we solve the problem classically in order to get a reference frame which describes the large energy limit. Then we concentrate on the quantum mechanical behaviour, and study the differences which arise, e.g., the existence of quasi-bound states. In Sec. III, we calculate the response of the 2DEG to a random, homogeneous distribution of these (identical) profiles over the sample. The approach is along the lines presented by Nielsen and Hedegård in Ref. 4, which was based on the Boltzmann transport equation. Again, we consider the scattering both classically and quantum mechanically, and discuss the arising differences. In Sec. IV we summarize our conclusions and discuss briefly the possibility of reproducing our results experimentally.

II. SCATTERING ON A SINGLE MAGNETIC PROFILE

A. Classical scattering

Classically, the scattering on a magnetic field profile is determined by the solution of Newton’s equation of motion, where the force is given by the Lorentz expression \( \mathbf{F} = e \mathbf{v} \times \mathbf{B} \) for a particle with charge \( e \). Outside the profile, no magnetic field is present, and consequently the path is just a straight line. Inside the profile, the electron describes an arc of a circle, of which the radius (and the direction in which it is drawn, i.e., clockwise or counterclockwise) depends on its position in the profile, since the profile considered here consists of two regions with different magnetic field strength (and sign). The respective cyclotron radii are given by \( l_c = \frac{v}{\omega_c} \), where \( v \) is the velocity of the electron and \( \omega_c = eB/mc \) is the cyclotron frequency in the local magnetic field, which is \( B = B_0 \) in the inner core and \( B = B_b \) in the outer region.

The geometry of the scattering process is determined by the following dimensionless parameters: (a) \( R_b/R_a \), i.e., the ratio of the radii of the inner and the outer circle of the magnetic field profile, and (b) \( l_a/R_a = (m/e)(v/B_0) \), which is the ratio of the cyclotron radius in the inner core, to the radius of this center part. It is clear from geometrical considerations, that it is impossible for a particle which was initially outside the magnetic profile to become trapped inside the magnetic profile.

We calculated the differential cross section \( d\sigma/d\phi \) numerically, from the different classical trajectories. In Fig. 2 we show examples of the classical trajectories (on the right) and their resulting cross sections (bold curves, on the left) for the \( R_b/R_a = 1.5 \) configuration for different \( l_a/R_a \).

In the limit of \( l_a/R_a \to 0 \), the cyclotron radius is very small compared to \( R_b \), and therefore the electron scatters on the magnetic profile as if it were bouncing off a hard wall. As a consequence the differential cross section would be symmetric in \( \phi \). From Fig. 2 we see that if \( l_a/R_a \) increases, the differential cross section changes drastically, and loses its symmetry due to the time reversal breaking magnetic field. We notice that \( d\sigma/d\phi \) is sensitive to \( l_a/R_a \): for increasing \( l_a/R_a \) it is more centered around 0, but its structure also changes significantly, which can be understood by inspection of the different classical electron trajectories shown on the right.

Suppose we consider the classical trajectories from left to right. In the case \( l_a/R_a = 0.5 \), the electrons are deflected to the left (\( \phi < 0 \)), giving rise to the orbits indicated by (1), and contributing to the differential cross section indicated by (1). Shifting the initial position of the electron further to the right, at a certain point the electron does not only feel the outer region, but is able to penetrate into the inner core, and consequently the electron is abruptly swept to the other side. The latter causes the abrupt decrease of \( d\sigma/d\phi \) at \( \phi/\pi = -0.75 \).

These trajectories contribute to part (2) in the differential cross section, and have mainly \( \phi > 0 \).

The same reasoning can be used to understand the differential cross section for \( l_a/R_a = 1 \), only now, \( d\sigma/d\phi \) is more centered around zero due to the higher velocity of the electron (with respect to the magnetic field), i.e., the electron gets less deflected.

For larger velocities, e.g. for \( l_a/R_a = 2 \) as in Fig. 2(c), the previous picture has to be extended with a new type of trajectories: those on the right (3) which again only probe the outer region, since the magnetic field in the outer region is not strong enough to deflect the electron into the inner part. These trajectories give rise to an additional peak (3) in the differential cross section.

B. Quantum mechanical scattering

1. The Schrödinger equation

We have to solve the following Schrödinger equation

\[
(H - E)\Psi(r, \phi) = 0,
\]

where \( E = \hbar^2 k^2 / 2m \) is the energy of the scattering wave. Because of cylindrical symmetry, we work with polar coordinates \( r = (r, \phi) \). We can make this equation dimensionless by rescaling the problem in the following way: length \( R_0 = R_a \), energy \( E_0 = \hbar^2 / (mR_0^2) \), time \( t_0 = mR_0^2 / \hbar \) and magnetic field \( B_0 = \hbar/\epsilon R_0^2 \). We can write the scattering wavefunction as consisting of components, separated into an angular and a radial part

\[
\Psi(r, \phi) = \sum_{m=-\infty}^{\infty} R_{km}(r) \Phi_{m}(\phi),
\]

for which the angular part equals

\[
\Phi_{m}(\phi) = \frac{1}{\sqrt{2\pi}} e^{im\phi},
\]
since the problem is cylindrical symmetric. The Schrödinger equation is then reduced to only one dimension

\[
\left[-\frac{1}{2r}\frac{d}{dr}r \frac{d}{dr} + V_m(r) - E\right] R_{km}(r) = 0, \tag{5}
\]

with

\[
V_m(r) = \frac{1}{2} \left[ A_0(r) + \frac{m^2}{r^2} \right], \tag{6}
\]

the effective potential and \( A_0(r) = (1/r) \int_0^r dr' r' B(r') \) the angular component of the vector potential. Notice that the angular quantum number satisfies \(-\infty < m < \infty\), in contrast to when the scatterers are non-magnetic, in which we have \(0 \leq m < \infty\), due to symmetry, i.e. \(m\) and \(-m\) result in the same scattered wave.

We know that the scattering process is fully determined, if we know \(\delta_m\) for every \(m\). In order to calculate these phase shifts we have to solve Eq. (5) for every \(m\) in the presence of our magnetic profile, and compare the scattered wave with the unperturbed one. The solution for \(r < R_a\) is

\[
R_{km}(r) = r^{m} e^{-\frac{1}{2}B_a r^2} [c_{m,1} M(\alpha, \beta, \gamma)] \tag{7}
\]

and for \(R_a < r < R_b\)

\[
R_{km}(r) = r^{m} e^{-\frac{1}{2}B_a r^2} \times [c_{m,2} M(\alpha', \beta, \gamma') + c_{m,3} U(\alpha', \beta, \gamma')], \tag{8}
\]

where \(M(a,b,c)\) and \(U(a,b,c)\) are confluent hypergeometric functions\([2]\) \(c_{m,i}\) are normalization constants, \(\alpha = [m|+1_m-k^2/(2B_a)]/2\), \(\beta = |m|+1\) and \(\gamma = Br^2\). \(\alpha'\) and \(\gamma'\) are the same as \(\alpha\) and \(\beta\) but with \(B_a \to B_b\). The constants \(c_{m,2}\) and \(c_{m,3}\) have to be determined such that the wavefunction is continuous at \(r = R_a\). An alternative approach is to calculate the wavefunction numerically up to \(r = R_b\).

For \(r > R_b\), Eq. (6) reduces to

\[
\left[-\frac{1}{2r}\frac{d}{dr}r \frac{d}{dr} + \frac{m^2}{2r^2} - \frac{k^2}{2}\right] R_{km}(r) = 0, \tag{9}
\]

which is the differential equation for the Bessel function of the first kind

\[
R_{km}(r) = a_m J_m(kr) + b_m Y_m(kr). \tag{10}
\]

Therefore, the resulting phase shifts can be calculated at this point, \(r = R_b\), and are derived from the condition that the logarithmic derivative of the radial wavefunction must be continuous at this boundary

\[
\frac{1}{R_{km}} \left. \frac{dR_{km}^<}{dr} \right|_{r=R_b} = \frac{1}{R_{km}} \left. \frac{dR_{km}^>}{dr} \right|_{r=R_b} \equiv \xi_{km}, \tag{11}
\]

which results in

\[
\frac{1}{R_{km}} \left. \frac{dR_{km}^<}{dr} \right|_{r=R_b} = \frac{1}{R_{km}} \left. \frac{dR_{km}^>}{dr} \right|_{r=R_b} \equiv \xi_{km}, \tag{12}
\]

where we have introduced the abbreviations \(s_m(x) = (x/2)(Z_{m-1}(x) - Z_{m+1}(x))\) with \((z, Z) = (j, J)\) or \((y, Y)\).

It is now easy to solve for \(\delta_m\)

\[
\tan \delta_m = \frac{j_m - \xi_{km} J_m(kR_b)}{y_m - \xi_{km} Y_m(kR_b)}. \tag{13}
\]

2. Resonances

In contrast to the classical problem the ratio \(l_a/R_a\) no longer determines the scattering problem completely. We need to know the exact energy and the magnetic field strength, and therefore \(l_a/R_a\) should be extended with the exact \(E\) or \(B_a\). We have chosen to fix the magnetic field strength \(B_a/B_0 = 20\), and to plot all curves as function of \(k/B_a(R_b/R_a)\) as was done in the classical case. The larger the \(k\)-value \((\sim l_a/R_a\) for fixed \(B_a\), the more classical the system is, and the more the average of it converges to our previously obtained classical result (see thin solid curves in the right part of Fig. [3]). But for lower \(k\)-values, i.e., when the wavelength of the scattering wave is comparable to the dimensions of the scatterer \(1/k \approx R_b\), quantum mechanics becomes important and results in features which cannot be understood classically, as e.g. the existence of resonances.

In order to determine at which energies these resonances occur, one should inspect the phase shift \(\delta_m\) as function of the energy or the corresponding \(k\)-vector. When a jump of \(\pi\) occurs, there is a resonance for that particular \(k\)-value for given \(m\). The lifetime of that quasi-bound state depends on the energy-interval over which this jump occurs, or on the peak width of the partial cross section \(\sigma_m\) as function of the energy.

In the following, we show the results for the case \(B_a/B_0 = 20\) and \(R_b/R_a = 1.5\). In Fig. 3 we plot the effective potential for four different \(m\)-values for this case. In Fig. 3 we plot the phase shift as function of \(k/B_a\) for \(-10 \leq m \leq 10\).

We notice that for \(m \geq -2\) well defined quasi-bound states are formed at the Landau levels of the inner core of the magnetic field profile, i.e. at \(k = \sqrt{B_a(2n+1)} = 4.471, 7.745, 10, \ldots\), or in the units of Fig. 3 \(k/B_a = 0.224, 0.387, 0.5, \ldots\). Landau states in the outer region \((B_b/B_0 = 16)\) are not possible since the total extent of the lowest Landau state \((= 2l_b = 2R_a/\sqrt{20} = 0.5R_a)\) does not fit into the outer region \((R_b - R_a = 0.5R_a)\).

Nevertheless, for \(m < -4\) there are also resonances which have an energy lower than the first Landau level of the inner core. They correspond to quasi-bound snake orbit states, which travel around the profile, propagating from the \(B_a\)-region into the \(B_b\)-region and vice versa. As an example we plot the effective potential and the corresponding radial wavefunction \(R_m\) for \(m = -8\) in
Fig. 3. Because the two wells, corresponding with the magnetic confinement in the different magnetic fields of the inner and outer regions, are joined together, they form one well which is broader—and consequently has a energy lower—than each of the separate wells. A similar effect we encountered in a previous paper for the case of electron traveling along a 1D magnetic interface. The electrons propagate classically as schematically depicted in the inset of Fig. 3.

For higher energies we also notice resonances for negative $m$, e.g. for $m = -21$ as shown in Fig. 3. They too correspond to snake orbits, but because they have a larger energy, they have to move closer to the interface, in order not to escape the magnetic field profile, since their cyclotron radius is larger. These type of states become extinct when the cyclotron radius in the outer part exceeds the radius of the outer part $R_b$, i.e. for $k l_a \approx R_b / R_a = 1.5$. We have checked this and these resonances indeed disappear.

### 3. The differential cross section

In two dimensions, the differential cross section is given by

$$\frac{d\sigma}{d\phi} = \frac{2}{\pi k} \left| \sum_{m=-\infty}^{\infty} e^{i m \phi} e^{-i \delta_m} \sin \delta_m \right|^2.$$  \hspace{1cm} (14)

We plot this together with its classical counterpart in Figs. 3 (a-c). We notice that many oscillations are present, due to interference effects. The number of these oscillations depends on the energy: the larger the energy the more oscillations. In the high energy limit, the quantum mechanical result will ultimately, on the average, converge to the classical one, except for the peak at $\phi = 0$. Its occurrence is a purely quantum mechanical effect, and is due to the fact that the electron which would classically pass by—and hence does not interact with—the scatterer, quantum mechanically has a finite overlap with the scatterer, and consequently contributes—although very little—to the cross section. Because the interaction is very little, it gets only scattered over a very small angle, and thus adds to the $\phi = 0$ peak.

### 4. The total cross section

In Fig. 3 we plot the total cross section $\sigma = \int d\phi d\sigma / d\phi$ as function of $k / R_a$. Classically, it is equal to the total diameter of the magnetic inhomogeneity $\sigma = 2R_b$ (dashed line in Fig. 3).

From Fig. 3 we notice that the quantum mechanical cross section (solid curve) is larger than the classical result. For large energies the total cross section is twice as large, for small energies the cross section is four times as large, as is the case for scattering on a spherical hard wall. We also notice the resonances mentioned before, present as small peaks, which can be attributed to a particular $m$-value. As an example, we indicated the $m = -8$ and $m = -21$ resonances, corresponding to the quasi-bound states of Fig. 3.

### III. SCATTERING ON MULTIPLE PROFILES

With the knowledge of the classical and quantum mechanical differential cross sections, it is now possible to calculate the Hall and magnetoresistance in a 2DEG subjected to a randomly distributed array of such identical profiles. We will make the assumption that the dimensions of the magnetic disks are small compared to the distance between the disks, so we do not include interference effects between different scattering events. Moreover, we neglect impurity scattering.

We solve the (classical) Boltzmann transport equation, linearized in the electric field, and follow the derivation as described in the paper of Nielsen and Hedegård. Finally, we arrive at

$$\rho_{xx} = \frac{1}{(2\pi)^2} \frac{n_0}{n_e} \frac{\hbar}{e^2} \int_0^{\pi} d\phi (1 - \cos \phi) w(k, \phi),$$  \hspace{1cm} (15a)

$$\rho_{xy} = \frac{1}{(2\pi)^2} \frac{n_0}{n_e} \frac{\hbar}{e^2} \int_0^{\pi} d\phi \sin \phi w(k, \phi),$$  \hspace{1cm} (15b)

where $n_e$ is the electron concentration, $n_0$ is the concentration of magnetic scatterers and $w(k, \phi)$ is the probability for an electron with wavevector $k$ to be scattered over an angle $\phi$. In relation to the differential cross section, we can write $w(k, \phi) = \langle kh/m \rangle (d\sigma / d\phi)$, since $\sigma v \Delta t$ is the probability for an electron with velocity $v$ to interact with a scatterer with cross section $\sigma$ in a time interval $\Delta t$.

### A. Classical result

#### 1. The magnetoresistance

In Fig. 7(a), we plot the magnetoresistance as function of $l_a / R_a$ for various $R_b / R_a$, in units of $\rho_0 = (n_0 / n_e) (\hbar / e^2)$. This is obtained by inserting the earlier calculated classical $d\sigma / d\phi$ into $w(k, \phi)$ of Eq. (15a). The magnetoresistance is zero when $l_a / R_a = 0$, because for zero energy, electrons do not move ($v = 0$) and consequently do not experience any scattering. For small values the magnetoresistance increases linearly up to a certain value after which it decreases for increasing $l_a / R_a$. This decrease is due to the fact that for higher energy, the electrons are less deflected because of a larger cyclotron radius in the magnetic inhomogeneity. We notice that for increasing $R_b / R_a$ the magnetoresistance has an overall increase, which can be explained by considering scattering on the inner and outer profile and how they
influence each other. For larger \( R_b/R_a \), the cyclotron radius of the outer part increases quadratically in \( R_b/R_a \) and therefore electrons will be less deflected for increasing \( R_b/R_a \). However, this is compensated by the fact that the cross section also increases linearly with \( R_b/R_a \), and consequently scattering on the outer part has more or less the same impact for different \( R_b/R_a \). Therefore, the increase in the resistance is due to the fact that for larger \( R_b/R_a \), the scattering on the inner and the outer region can be considered as two separate processes, which only interfere very little. This is not the case for smaller \( R_b/R_a \), where electrons interacting with the outer part are more likely to interact also with the inner part, which would deflect the electron in the opposite way, and thus diminishes the scattering effect produced by the outer part. In short we can say, that electrons which interact with the magnetic profile, on average “feel” a nonzero magnetic field which increases with increasing \( R_b/R_a \), and this results in an increase of the magneto- and Hall resistance.

The fact that for higher \( R_b/R_a \) the scattering problem can be seen as two separate scattering processes on different vortices \( (R_a,B_a) \) and \( (R_b,B_b) \), is also reflected in the dip in the magnetoresistance which arises for higher \( R_b/R_a \), e.g. \( R_b/R_a = 10 \). Actually, the magnetoresistance has two peaks of which one, the lower energy peak, corresponds to scattering on the outer region, while the second peak corresponds to scattering on the center region. For higher \( R_b/R_a \), the first peak will shift towards smaller \( l_a/R_a \), while the other peak remains in the same position, and consequently the two scattering processes will become more distinct.

2. The Hall resistance

The corresponding Hall resistance is plotted in Fig. 7(b). We notice that both the qualitative and quantitative behaviour are more sensitive to the ratio \( R_b/R_a \) than was the case for the magnetoresistance. There are two striking features: (a) again there is an overall increase of the Hall resistance with increasing \( R_b/R_a \), and (b) for small \( R_b/R_a \), i.e. \( R_b/R_a < 1.73 \), the Hall resistance can change sign as function of \( l_a/R_a \), i.e., as function of the Fermi-energy or the magnetic field strength.

The fact that the Hall resistance can change sign when \( R_b/R_a < 1.73 \) is a consequence of the interplay between scattering in the inner and the outer region of the magnetic field profile. For infinitesimally small \( l_a/R_a \), the outer part will act as a hard wall and consequently there is no Hall resistance. For increasing \( l_a/R_a \), the electrons will penetrate deeper in the outer region, but not yet in the inner core as long as \( l_a/R_a \) is small enough, i.e., \( 2b < (R_0 - R_a) \) [see the trajectory given by the bold curve in the right figure of Figs. 2(a)]. The Hall resistance is that due to a vortex with magnetic strength \( B_0 \). For increasing \( k/B_a \), the electrons will be able to penetrate into the inner region, where they get deflected to the other side and the Hall resistance consequently changes sign [see the bold curve in Figs. 2(b,c)].

The reason that this effect does not occur for higher \( R_b/R_a \) values, is due to the fact that in these cases, there is only little interplay between scattering in the inner and the outer region, as mentioned earlier. It is much more unlikely that an electron which initially only felt the outer region, for higher \( l_a/R_a \) will interact with the (relatively small) core, and be swept to the other side. Therefore the Hall resistance for large \( R_b/R_a \) always has the same sign, as generated by the outer region.

B. Quantum mechanical result

After insertion of Eq. (14) into Eqs. (15a) and (15b), we can rewrite the magnetoresistance and the Hall resistance as function of the phase shifts

\[
\rho_{xx} = \frac{n_0}{n_e} \frac{\hbar}{e^2} \sum_{m=-\infty}^{\infty} \frac{1}{\delta_m - \delta_{m+1}}, \tag{16a}
\]

\[
\rho_{xy} = \frac{n_0}{n_e} \frac{\hbar}{e^2} \sum_{m=-\infty}^{\infty} \sin[2(\delta_m - \delta_{m+1})]. \tag{16b}
\]

In Fig. 8, we plot the Hall and magnetoresistance as function of \( k/B_a \) for \( B_a/B_0 = 20 \) and \( R_b/R_a = 1.5 \). The solid curve is the quantum mechanical result, the dashed curve is our previously obtained classical result. We observe many resonances, which diminish for increasing \( k/B_a \). Except for that, on the average there is rather good agreement between both curves, which is due to the choice of \( B_a/B_0 \) being large, and consequently in this figure \( k \) and consequently the energy is large.

There are two types of resonances: (1) those which occur at the energy of the Landau levels of the inner core of the magnetic field profile (thin dotted curves), and (2) those corresponding to quasi-bound snake orbit states.

In case of the first type, the Hall resistance decreases abruptly (i.e. it has a sawtooth behaviour), while the magnetoresistance increases. This can be viewed in the right inset in Fig. 8. The reason for this is that at the energy of the Landau levels (indicated by the vertical dotted lines), electrons are (quasi-) bound into cyclotron orbits, and hence cannot (a) contribute to the conduction, and consequently the magnetoresistance increases, and (b) cannot pile up and generate a voltage difference on the left and right side, and consequently the Hall resistance decreases.

For larger energies, the cyclotron orbit in the center will increase and exceed the inner core classically at \( k/B_a = 1 \). Nevertheless, quantum mechanically the electron will “feel” the presence of the outer magnetic field, and this will change the resonant energies. The resistance jumps are also less explicit for higher Landau levels, which is due to the reduced number of states which fit in the core as a result of the large cyclotron radius.
Apart from resonant states at the Landau levels, the magnetoresistance exhibits very sharp peaks, which correspond to the snake orbit states mentioned earlier. This second type of resonances can have a lower energy than the first Landau level of both the inner and outer part, as explained before. Since for all the snake orbit resonances only one quasi-bound state is involved (in contrast to the Landau states, where various quasi-bound states exist for various m-values, see Figs. 2 and 3), these peaks are very sharp (see e.g. the left inset of Fig. 8) and are superimposed on a more continuous background.

The influence of the quasi-bound snake orbit states is also visible in the Hall resistance, but not as pronounced as in the magnetoresistance. These quasi-bound states produce small changes in the slope of the Hall resistance \( \rho_{xy} \), as is clear from the dashed curve in the left inset of Fig. 8.

**IV. CONCLUSIONS**

In this paper we studied scattering on circular symmetric magnetic field profiles with zero mean magnetic field in the 2DEG. We considered scattering on a single profile, both classically and quantum mechanically, and found different types of quasi-bound states: the Landau-states in the inner core, and different quasi-bound snake orbit states. Next, we investigated the diagonal and Hall resistivity in the presence of a randomly distributed array of these magnetic profiles, using the kinetic Boltzmann equation, for different magnetic field configurations. We obtain a nonzero Hall resistance although \( (B_z) = 0 \) and showed that the Hall resistance can change sign as function of the Fermi-energy or the magnetic field strength. We found that the electron resonances in the individual magnetic field profiles are reflected in the Hall and magnetoresistance.

There are two competing effects which make experimental measurement of these resonances difficult: in order to detect these quasi-bound states, it is necessary that the energies (bound or resonant) are not too close to each other. To obtain this, one has to make a very small magnet (since \( E_0 = \hbar^2/mR_a^2 \)), but then one encounters the problem that in order to bind the electrons in such a small area one needs a very strong inhomogeneous magnetic field (\( B_0 = \hbar e/R_a^2 \)), and currently no magnetic materials are available which can realize these strong fields.

An example of such a system is the one by Dubonos et al.\(^5\), they managed to deposit a single Dy-magnet with radius \( R \approx 0.1 \mu m \) on top of a heterojunction, containing a 2DEG. For this system, our units are given by \( E_0 = 7.63 \times 10^{-3} \text{ meV} \) and \( B_0 = 0.066 \text{ T} \). The Fermi energy in their system was about \( E_F = 17.86 \text{ meV} \), which in our units is \( E_F = 2341E_0 \) or \( k_F \approx 70 \). According to their paper, the stray field could locally generate magnetic fields of \( B_a \approx 1 T \approx 15B_0 \), which corresponds to \( k_F/B_a \approx 4.7 \), for which we are in the classical regime and the scattering process can be calculated classically.

In order to measure individual snake orbits, it is therefore necessary to include another confinement, p.e. electrical confinement, which discretizes the energies and makes measurement possible, as was also used in the paper by Nogaret et al.\(^6\).

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Footnotes:

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FIG. 1. The magnetic field profile (solid curve) and the theoretical model (dotted curve) for a magnet of radius \( R \) and thickness \( d/R = 1 \), deposited a distance \( h/R = 0.01 \) above a 2DEG, as shown in the inset. The lower inset shows a simplified top view of the magnetic field profile in the plane of the 2DEG.
FIG. 2. This figure corresponds with the situation \( B_a/B_0 = 20 \) and \( R_b/R_a = 1.5 \). On the left: The differential cross section for different \( k/B_a = l_a/R_a \), i.e., (a) \( l_a/R_a = 0.5 \), (b) \( l_a/R_a = 1 \), and (c) for \( l_a/R_a = 2 \). The bold curve corresponds to the classical result, the thin curve to the quantum mechanical result. On the right: Some classical trajectories interacting with the magnetic profile, resulting in the differential cross sections on the left. The trajectories giving rise to the different parts of the different structures, are grouped schematically.

FIG. 3. The effective potential \( V_{\text{eff}}(r) \) as function of \( r/R_0 \) for various \( m \)-values, for the case \( B_a/B_0 = 20 \) and \( R_a/R_b = 1.5 \). The horizontal lines correspond to the resonant energies.

FIG. 4. The phase shift \( \delta_m \) as function of \( k/B_a \) for different \( m \)-values, for the situation with \( B_a/B_0 = 20 \) and \( R_a/R_b = 1.5 \). Phase jumps of \( \pi \) correspond to resonant states.

FIG. 5. The effective potential \( V_{\text{eff}}(r) \) as function of \( r/R_0 \) for \( m = -8 \) and \( m = -21 \) when \( B_a/B_0 = 20 \) and \( R_a/R_b = 1.5 \), together with the radial wavefunctions (dotted curves) at the resonant energies. These quasi-bound states correspond to different types of snake orbits, propagating parallel to the magnetic edge, as depicted schematically in the inset of the figure.

FIG. 6. The cross section \( \sigma \) as function of \( k/B_a \) in the classical limit (dashed curve) and if calculated quantum mechanically (solid curve) for \( B_a/B_0 = 20 \) and \( R_a/R_b = 1.5 \). The marked resonant states correspond to the ones shown in Fig. 3 and Fig. 4. Vertical dotted lines are the Landau energies of the inner core of the magnetic field profile.

FIG. 7. The magnetoresistance (a) and the Hall resistance (b) in the classical limit as function of \( l_a/R_a \) for different \( R_b/R_a \)-configurations.

FIG. 8. The magneto- and Hall resistance as function of \( k/B_a \) in the classical limit (dashed curves) and if calculated quantum mechanically (solid curves) for \( B_a/B_0 = 20 \) and \( R_a/R_b = 1.5 \). The marked resonant states correspond to the ones shown in Fig. 3 and Fig. 4, and the vertical dotted lines are the Landau energies of the inner core of the magnetic field profile. The insets show enlargements of a quasi-bound snake orbit resonance (left) and a Landau level resonance (right) where the solid curve corresponds to \( \rho_{xx} \) and the dashed curve is \( \rho_{xy} \).
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