Constraints on a Massive Dirac Neutrino Model

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Abstract

We examine constraints on a simple neutrino model in which there are three massless and three massive Dirac neutrinos and in which the left handed neutrinos are linear combinations of doublet and singlet neutrinos. We examine constraints from direct decays into heavy neutrinos, indirect effects on electroweak parameters, and flavor changing processes. We combine these constraints to examine the allowed mass range for the heavy neutrinos of each of the three generations.

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1 Introduction

Many models of neutrinos have been proposed to accommodate light or massless neutrinos. In a model with no right handed neutrinos, it is clear that neutrinos are massless. However, if there exist additional states which can play the role of Dirac partners to the left handed states, it is perplexing why neutrinos should be massless, or at least much lighter than their charged counterparts. Of course, neutrinos can be given small masses by coupling them to the standard Higgs doublet because of an extremely small Yukawa coupling, but it is more compelling to have an explanation for their small mass. A common explanation is the so called “see–saw” mechanism, in which the neutrinos remain light because the additional right handed states have a large Majorana mass. In such a model, neutrino masses are naturally small, since they are suppressed by the ratio of Dirac to Majorana masses, which is generally taken to be small.

In this paper, we consider another viable alternative (see for ex. [2],[3],[4]). In addition to the three “right handed” neutrinos, there are three additional singlet particles. A lepton symmetry is imposed so that the only allowed mass terms are Dirac masses coupling the right handed neutrino to the standard left handed neutrino and to the additional singlet states. The consequence is that there are three heavy Dirac neutrinos, with mass determined primarily by the large mass term connecting the singlet and right handed neutrinos and three exactly massless neutrinos, the states orthogonal to the massive ones. Such a model has been considered before in several contexts; most recently it has been considered in the context of an Extended Technicolor Model with
a GIM mechanism [4]. In this type of model, the additional neutrino states
could be quite light, on the order of a GeV.

However, there are many constraints on such neutrinos. They are con-
strained from direct searches for particles which have them in their final
state, by universality constraints, and by flavor changing constraints. Cos-
mological arguments are often used to constrain neutrino masses, but the
neutrinos of this model are unstable so they do not apply. In this paper, we
put these constraints together, making reasonable assumptions on the form
of the mass matrix and mixing angles, to determine the allowed parameter
regime. Many of these constraints apply quite generally to any model in
which the left handed neutrinos mix with singlet states. Similar bounds were
considered in ref. [18]. This paper updates the bounds, integrates them with
those from LEP, and incorporates flavor changing bounds. We find with rea-
sonable assumptions described below, the lightest neutrino can be as light
as 2 GeV, although the third generation neutrino should be much heavier,
greater than 80 GeV.

The organization is as follows. We first present the model and we describe
the approximations which we use to reduce the parameter space. We then
consider constraints from meson and Z decays. Following this, we discuss the
constraints from the fact that $G_F$ will not have the same relation to standard
model parameters when the muon cannot decay to the heavy neutrino state.
We then look at flavor changing processes, which are in general permitted
when no flavor symmetries are assumed. However, we assume mixing angles
similar to those of the standard KM matrix, so there are approximate U(1)
symmetries present. We then put together the constraints and consider three
models which describe the ratio of masses of the heavy neutrinos to determine the allowed parameter regime. Finally, we conclude.

2 The Model and Simplifying Assumptions

Many models have incorporated the neutrino scenario we discuss here. For example, it has been incorporated into GUT models [2],[3]. More recently, it has been shown how to incorporate such a model in an Extended Technicolor scenario [4]. We only consider the phenomenon of the lepton sector here, so we neglect the origin of the model and focus on the neutrinos.

The standard model is extended by introducing three new left-handed neutrinos $S_L$ and three right-handed neutrinos $\nu_R$. Both left handed neutrinos are coupled to the right handed neutrinos through Dirac matrices. All other possible mass entries are forbidden by a lepton number symmetry. Thus,

$$-\mathcal{L}_{\text{mass}} = (\nu_R \ 0) \begin{pmatrix} D & S \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ S_L \end{pmatrix} + \text{h.c.}$$

This coupling results in three massive Dirac neutrinos and three massless eigenstates. The mass matrices $D$ and $S$ have different mass scales. The scale for $D$ is constrained by SU(2) symmetry breaking whereas the scale for $S$ is not, so it is reasonable to expect the masses in $S$ to be larger.

The mass of the heavy neutrinos is essentially determined by $S$. The electron, muon and tau neutrinos are a superposition of massless and massive eigenstates. The mixing to the massive neutrinos will, however, be small; it will be of the order of $M_D/M_S$, where $M_D$ and $M_S$ are typical masses in $D$. To see more precisely how this mixing occurs, we need to find the
three massless eigenstates $\nu^0$ as well as the three with mass $\nu^H$. The mass matrix can be diagonalised by multiplying on the left and the right by unitary matrices:

$$(V_0 \ 0) \begin{pmatrix} D & S \\ 0 & 0 \end{pmatrix} U = \begin{pmatrix} 0 & 0 \\ 0 & M \end{pmatrix}$$

with $$(\nu_L \ S_L) = U (\nu^0 \ \nu^H).$$

The unitary matrix $V$ diagonalises $DD^\dagger + SS^\dagger$ to give $M^2$. The unitary matrix $U$ is given by

$$U = \begin{pmatrix} U_D^\dagger \Lambda_D V'^\dagger \\ U_S^\dagger \Lambda_S V'^\dagger \end{pmatrix},$$

where the matrices $V'$, $U_D$ and $U_S$ are unitary matrices. They diagonalize $M^{-1}VD$ and $M^{-1}VS$ (where $M^{-1}$ is the inverse of the diagonal mass matrix M) to give the diagonal matrices $\Lambda_D$ and $\Lambda_S$:

$$V'^\dagger (M^{-1}VD)U_D^\dagger = \Lambda_D$$

$$V'^\dagger (M^{-1}VS)U_S^\dagger = \Lambda_S.$$

The fact that the same $V'$ appears on the left for both these diagonalizations is a consequence of the fact that the two matrix products $M^{-1}VD^\dagger V^\dagger M^{-1}$ and $M^{-1}VS^\dagger V^\dagger M^{-1}$ commute with each other, which follows in turn from the fact that their sum is the unit matrix. The most important part of $U$ is the top right $3 \times 3$ block $U_D^\dagger \Lambda_D V'^\dagger$ which links the electron, muon and tau neutrinos to the massive neutrinos $\nu^H$.

To extract bounds on the mass scales of $S$ and $D$ we need to make some simplifications to reduce the number of parameters. We will make the simplification that the matrices $D$ and $S$ are diagonalized by the same unitary matrices. In this case $V' = I_{3 \times 3}$. If we then redefine the fields $S_L$ by a
unitary transformation, absorbing the unitary matrix $U_S$, we can rewrite the matrix $U$ as:

$$
U = \begin{pmatrix}
U_D^\dagger \Lambda_S & U_D^\dagger \Lambda_D \\
\Lambda_D & \Lambda_S
\end{pmatrix},
$$

with $\Lambda_D^2 + \Lambda_S^2 = I_{3\times3}$.

In this model the mass scale of the Dirac mass $S$ is assumed to be much higher than the scale of the Dirac mass $D$. If this difference is sufficiently large, we can make the further simplification that $\Lambda_S = I_{3\times3}$. From here on the subscript $D$ on $\Lambda_D$ will be dropped.

At this point, we still have a large number of parameters. We simplify by assuming the matrix $U_D$ is similar in structure to the KM matrix for quarks. We notice that if there were no singlet left-handed neutrinos $S_L$ the matrix $U_D$ would be the lepton equivalent of the KM matrix in the quark sector. We take the individual elements to be of the same magnitude as those in the KM matrix for quarks.

We will use these approximations from now on. They leave six free parameters: $M_i$, the masses of the heavy neutrinos and $M_{Di}$, the masses induced by the mass matrix $D$ which are defined as $M_{Di} = \Lambda_i \times M_i$. In the following sections we will use experimental results to put limits on these masses.

### 3 Direct Searches for Heavy Neutrinos

Many searches for massive neutrinos have already been conducted. Massive neutrinos have been sought in the decays of $\pi^+$ [5] [6] [7], $K^+$ [8] [9] and charmed mesons [10] [11] [12], as well as in the neutral current production
of neutrino anti-neutrino pairs from $e^+ e^-$ [13] [14] [15] collisions, and, more recently the decay of the $Z$ [16] [17].

3.1 Meson Decays

If it is kinematically allowed, any process involving the production of neutrinos will be a source of heavy neutrinos. The creation process, however, will be suppressed since the weak eigenstate neutrinos contain only a small mixing of the heavy neutrinos. Leptonic decays of mesons are thus one place to look for heavy neutrinos.

At the lower end of the mass scale heavy neutrino creation in the decay of $\pi^+$ mesons has been investigated in references [5] [6] [7], and those of $K^+$ mesons in references [8] [9]. These experiments attempted to measure the mass of any heavy neutrino as it was created. This was achieved by stopping the $\pi^+$ and $K^+$ mesons and observing the energy of positrons emitted in their decay. The method did not rely on any assumptions about how the heavy neutrinos decayed. For massive neutrinos with masses less than 300 Mev these experiments placed strict limits on the mixings, $|U_{ei}|^2$ and $|U_{\mu i}|^2$ of a heavy neutrino $\nu_i^H$ into the electron and muon neutrinos. For a range of masses the matrix elements $|U_{ei}|^2$ and $|U_{\mu i}|^2$ were constrained to be less than $10^{-5}$. Since we assume the matrix $U_D$ is almost diagonal, we can get direct bounds on $\Lambda_1$ and $\Lambda_2$ of: $\Lambda_1 < 3 \times 10^{-3}$ for the mass range 35 to 360 MeV, and $\Lambda_2 < 3 \times 10^{-3}$ for the mass range 80 to 325 MeV (see Figs(1) and (2)).

Further limits on $|U_{ei}|^2$ and $|U_{\mu i}|^2$ come from the decays of charmed $D$ mesons, [10] [11] [12], (see Figs(1) and (2)). Although similar searches can in principle be performed with decays of the $B$, they have not yet been done.
In total the limits from meson decays, with a few gaps, restrict $\Lambda_1 < 3 \times 10^{-3}$ for the mass range $35 \, \text{MeV}$ to $2 \, \text{GeV}$, and $\Lambda_2 < 3 \times 10^{-3}$ for the mass range $80 \, \text{MeV}$ to $2 \, \text{GeV}$. For much of these ranges the bounds are much stricter than this.

### 3.2 $e^+ e^-$ Collisions at Low CM Energy

Heavy neutrino anti-neutrino pairs would be created by weak interaction currents in $e^+ e^-$ annihilations, but since the the center of mass energy of these collisions is less than the $W$ and $Z$ mass the cross section for the creation of these neutrinos is extremely small. Experiments [13] [14] [15] were aimed at detecting the decay of a heavy fourth generation neutrino and they thus made the assumption that the heavy neutrino had the same coupling to the $Z$ and $W$ as the other neutrinos. This is not the case for the model studied in this paper where each heavy neutrino introduces a mixing angle factor of $|U_{li}|^2$ into the weak interaction couplings. Reinterpreting the data of these experiments including the extra mixing angles results in constraints that are negligible in comparison to the other bounds studied in this paper.

### 3.3 $Z$ decays

Massive neutrinos, lighter than $M_Z$, would also be created in $Z$ decays, and the experiments [16], [17] have already conducted searches for heavy isosinglet neutrinos; the type discussed in this paper. The most abundant supply of heavy neutrinos would come from the decay of a $Z$ into one heavy neutrino and one massless neutrino. For a $Z$ decaying into a heavy neutrino $\nu_i^H$ (lighter than the $Z$) and any of the three massless anti-neutrinos $\nu_j^0$ the creation is
suppressed by:

\[ R_i = \sum_{j=1}^{3} \left| \sum_{l=\epsilon,\mu,\tau} U_{jl}^\dagger U_{l(i+3)} \right|^2 = \Lambda_i^2, \]

where \( R_{\nu_i} \) has the following meaning: if \( N \) is the number of neutrinos (from one family of the standard model) created in the experiment then the number of heavy neutrinos, \( \nu_i^H \), created is \( R_i N \).

Experiments aim to detect the neutrino by its decay. The decay of the neutrino would be quite distinctive. In general, it will decay to a high energy lepton and a virtual \( W \) or \( Z \), which would then decay into leptons, or hadrons if the neutrino is massive enough. The total decay rate can be written in terms of the rate for muon decay as follows[18]:

\[
\Gamma(\nu_i^H \to \text{leptons/hadrons}) = \sum_l |U_{l(i+3)}|^2 \left( \frac{M_i}{M_\mu} \right)^5 \Phi_l(M_i) \Gamma(\mu \to e\nu\bar{\nu})
\]

where \( \Phi_l(M_i) \) is a factor that weights the decay rate for a single channel by the effective number of channels into which there is sufficient energy to decay and takes into account the different Feynman diagrams.

There are two reasons why decays like this might not have been seen in experiments:

**Very few heavy neutrinos are produced.** If we assume that nearly all the neutrinos decay inside the detector, then the fraction, \( R_d \), that decay inside the detector is given by:

\[ R_d = R_i = \Lambda_i^2. \]

If \( R_i \) is sufficiently small, no neutrinos would be detected.
The neutrinos have a long lifetime. If the neutrinos are light, they could have a very long lifetime and thus decay almost entirely outside of the detector.

We can then calculate the fraction, $R_d$ of $Z$’s that would decay inside the detector:

$$R_d = R_i \left(1 - \exp\left(\frac{S_d}{c} \gamma^{-1} \Gamma(\nu^H \rightarrow \text{leptons/hadrons})\right)\right),$$

where $\gamma$ is the time dilation factor due to the relativistic motion of the neutrino, and for $M_Z >> M_i$ is given by $\gamma = M_Z/2M_i$, $S_d$ is the size of the detector and $c$ is the speed of light.

The experiment of reference [16] involved a search through $4 \times 10^5$ hadronic $Z$ decays and placed limits of $\Lambda_i < 0.014$ over the range 5 to 50 GeV. Above 50 GeV the phase space for heavy neutrino production becomes smaller and the limits placed on the $\Lambda_i$ become less strict. Below 5 GeV the limits are reduced due to the long lifetime of the neutrinos. In Figs (1), (2) and (3), are marked out the forbidden regions in the $M_i, \frac{1}{\Lambda_i}$ plane for the three heavy neutrinos.

4 Changes In Weak Interaction Decays and Parameters

Aside from direct searches for the heavy neutrino, the existence of the heavy neutrino will affect precision measurements of various electroweak processes. This can be the case because $G_F$ will no longer have the standard model relation to $\sin \theta_W$, since the muon decay rate will be different if the muon
cannot decay into the heavy neutrino. This would change the relation between precisely measured electroweak parameters, for example the $W$ mass or $\sin^2 \theta_W$ as measured in the forward–backward asymmetry. Furthermore, it would lead to an apparently nonunitary KM matrix.

Further constraints come from pion decay branching ratios if the heavy electron or muon neutrinos are heavier than the pion. Similarly, universality could be violated and would be seen in tau decay. Finally, the $Z$ width can be affected, both indirectly though a change in the extracted $\sin^2 \theta$, and directly if the neutrinos are heavier than the $Z$.

## 4.1 Muon decays and the Fermi coupling constant $G_F$

The Fermi coupling constant $G_F$ is the effective coupling constant for four fermi interactions and is measured extremely accurately from muon decays. If the mass of the massive neutrinos is greater than that of the muon, the decay width for the muon would be decreased, since it would not be able to decay into the heavy neutrinos; this in turn would lead to a change in the predicted value of $G_F$.

Specifically:

$$ (G_F)^2_{\text{new}} = \sum_{i,j=1}^{3} |U_{ei} U_{j \mu}^\dagger|^2 (G_F)^2_{\text{old}}, $$

which leads to

$$ \delta (G_F)^2_{\text{muon decays}} \simeq -(\Lambda_1^2 + \Lambda_2^2) $$

where $\delta$ means the fractional change. Of course $G_F$ is a measured number. What we mean here is the change in the coefficient of the four fermion operator which yields muon decay.
4.2 Semileptonic Decays and the KM Matrix

The estimates of the semileptonic processes would also be affected but to a lesser extent. The same value of $G_F$ is also used for the effective coupling constant for semileptonic decays, where elements of the KM matrix are determined. One would expect these elements to be part of a unitary matrix.

The important point to consider is that the effective coupling constants for the leptonic and semileptonic four fermion interactions would no longer be the same, and if it was assumed that they were, the predicted matrix elements for the KM matrix would no longer be those of a unitary matrix. We can check the unitarity of the KM matrix by looking at the matrix elements $(KM)_{ud}$, $(KM)_{us}$ and $(KM)_{ub}$; the sum of their square magnitudes must add up to one. The effect of having heavy neutrinos would be to make this sum slightly bigger than one. The most important shift will come from the change in nuclear beta decays used to determine the $(KM)_{ud}$ element. Consequently, what must be compared are the changes in the value of $G_F$ and in the rates for nuclear beta decays.

Specifically, as above,

$$ (G_F)_{new}^2 = \sum_{i,j=1}^3 |U_{ei}U_{j\mu}|^2 (G_F)_{old}^2, $$

which leads to

$$ \delta(G_F)_{muon\,decays}^2 \simeq - (\Lambda_1^2 + \Lambda_2^2) $$

and similarly

$$ \delta(beta\,decay) \simeq - \Lambda_1^2 $$

where $\delta$ means the fractional change. The fractional change in the width of
the muon minus the fractional change in nuclear beta decays must be less than the experimental uncertainty in the sum of the matrix elements. From reference [19]:

\[ |(KM)_{ud}|^2 + |(KM)_{us}|^2 + |(KM)_{ub}|^2 = 1 (+8.6 \times 10^{-4}, -4.7 \times 10^{-3}). \]

This leads to \( \Lambda_2 < 6 \times 10^{-2} \) \((2 \sigma)\) if the neutrinos are heavier than the muon.

### 4.3 \( M_W \) and \( \sin \theta_W \)

Changes in \( G_F \) would also affect the prediction of other weak interaction parameters. The ratio of the mass of the \( W \) and \( Z \) for example depends upon \( G_F \). Specifically [19]:

\[
\frac{M_W^2}{M_Z^2} = \frac{1}{2} \left( 1 + \frac{4\pi \alpha(1 + \delta v)}{\sqrt{2}M_Z^2G_F} \right)^{\frac{1}{2}},
\]

where \( \delta v \) is a radiative correction parameter much less than one. Using the above the change in the predicted value of \( M_W/M_Z \) due to the change in \( G_F \) is:

\[
\delta (M_W/M_Z) = 0.088 \times \delta (G_F).
\]

Current experimental bounds [20] place \( \delta (M_W/M_Z) < 7.7 \times 10^{-3} \) \((2 \sigma)\) which gives a bound for \( G_F \) of:

\[
\delta (G_F) < 8.8 \times 10^{-2} \ (2 \sigma)
\]

In fact this bound is too strong due to the uncertainty in the top quark mass. However, since it is less strict than the bound coming from the KM matrix, it will not be incorporated.
The Weinberg angle $\sin^2 \theta_W$ also depends on $G_F$ (the on shell definition is: $\sin^2 \theta_W = (1 - M_W^2/M_Z^2)$) and this can be compared with the forward backward asymmetry of the process $e^+ e^- \rightarrow f \bar{f}$, which depends on $\theta_W$. However, this too is weaker than the constraint from the unitarity of the KM matrix.

4.4 Pion Decay Branching Ratios

The ratio of the two decay channels for a $\pi^\pm; \pi \rightarrow e \nu_e$ and $\pi \rightarrow \mu \nu_\mu$, provides another bound [21]. In this model:

$$\frac{\Gamma(\pi \rightarrow e \nu_e)}{\Gamma(\pi \rightarrow \mu \nu_\mu)} = 1.233 \times 10^{-4} (1 - \sum_{i=1}^{3}(|U_{ei}|^2 - |U_{\mu i}|^2)),$$

where the factor $1.233 \times 10^{-4}$ is the theoretical value of the ratio in the standard model[22]. The mixing angle factors apply for neutrinos too heavy to be produced. Experimentally the ratio is known to be: $(1.218 \pm 0.014) \times 10^{-4}$ [23]. Using the fact that $U_D$ is almost diagonal and that $\Lambda_2 < 6 \times 10^{-2}$ leads to:

$$0.035 > (\Lambda_1^2 - \Lambda_2^2) \implies \Lambda_1 < 0.18 \quad (2\sigma)$$

4.5 Tau decays

If the neutrinos are all heavier than the tau then the decay width of the tau would also be affected. As for the case of the muon decay it can be shown that the partial width $\Gamma(\tau \rightarrow e\nu_e\nu_e)$ would be reduced by $\sim (\Lambda_1^2 + \Lambda_2^2)$ and the partial width $\Gamma(\tau \rightarrow \mu\nu_\mu\nu_\mu)$ would be reduced by $\sim (\Lambda_2^2 + \Lambda_3^2)$. Consequently, the partial width for decay into leptons would be reduced by:

$$\sim (\frac{1}{2} \Lambda_1^2 + \frac{1}{2} \Lambda_2^2 + \Lambda_3^2).$$
This fractional change minus the fractional change in the width for muon decay must be less than the experimental uncertainty of the partial width for the tau. This gives a further bound on the $\Lambda_i$:

$$|\frac{1}{2}\Lambda_1^2 + \frac{1}{2}\Lambda_2^2 - \Lambda_3^2| \leq 0.015 \ (1\sigma)$$

where the uncertainty in the partial width of the tau is 1.5% [19]. To obtain a bound for $\Lambda_3^2$ we use the following formula for calculating the error at the $1\sigma$ level of a sum of terms each with their own errors:

$$\Lambda_3^2 = \left(\left(\frac{1}{2}\Lambda_1^2\right)_{1\sigma} + \left(\frac{1}{2}\Lambda_2^2\right)_{1\sigma} + 0.015^2\right)^{\frac{1}{2}},$$

where, at the $1\sigma$ level, we use the bounds from the previous section $\Lambda_1^2 < 0.017$ and $\Lambda_2^2 < 0.0018$. This leads to a bound on $\Lambda_3$ at the $2\sigma$ level of:

$$\Lambda_3 < 0.18 \ (2\sigma),$$

### 4.6 The Width of the Z

For neutrinos heavier than the Z the width will be reduced, since the decay into the heavy neutrinos will no longer be kinematically allowed. Experimentally, the partial width, $\Gamma_{\nu\bar{\nu}}$ of the Z is known to an accuracy of 1.8% [24]. In this model

$$\Gamma_{\nu\bar{\nu}} \propto \frac{1}{3} \sum_i |1 - \Lambda_i^2| = 1 - \frac{1}{3} \sum_i \Lambda_i^2,$$

where the sum over $i$ is only over neutrinos heavier than the Z. This gives the bound:

$$\sum_i \Lambda_i^2 < 0.108 \ (2\sigma).$$
There is also an effect if muon decay is changed. However it is not numerically as important.

In Figs (1), (2) and (3) are plots of the bounds placed on the $\Lambda_i$ by all the processes considered in sections four and five.

5 Lepton Flavor Changing Processes

Flavor changing processes were also examined in this model. These processes are exactly analogs to flavor changing processes in the quark sector. Three processes with strong experimental bounds were considered [19]:

(i) $\mu \rightarrow e\gamma$; experimentally: $\frac{\Gamma(\mu \rightarrow e\gamma)}{\Gamma(\mu \rightarrow e\nu\bar{\nu})} < 5 \times 10^{-11}$,

(ii) $\mu \rightarrow e^+e^-$; experimentally: $\frac{\Gamma(\mu \rightarrow e^+e^-)}{\Gamma(\mu \rightarrow e\nu\bar{\nu})} < 10^{-12}$, and

(iii) $\mu T_i \rightarrow e T_i$; experimentally: $\frac{\Gamma(\mu T_i \rightarrow e T_i)}{\Gamma(\mu \rightarrow e\nu\bar{\nu})} < 5 \times 10^{-12}$.

These processes can only occur via loop diagrams involving the exchange of virtual neutrinos. The couplings of the neutrinos to the muon and the electron involve the unitary matrix $U$; specifically, the neutrino-$W$-muon vertex includes a factor $U_{i\mu}^\dagger$ for coupling to the $i$th neutrino, and the factor $U_{ei}$ is included with the neutrino-$W$-electron vertex. The amplitudes are obtained by summing over all intermediate states $i$. All terms proportional to the sum $\sum_{i=1}^6 U_{ei} U_{i\mu}^\dagger$ are automatically cancelled since $U$ is unitary. This is the GIM mechanism. It is an analog of the strong suppression of neutral current flavor changing processes in the quark sector. Notice this is independent of
the approximations we made.

In all cases there is very strong GIM suppression. Flavor changing processes can only proceed via an intermediate heavy neutrino state. But the coupling to the charged neutrinos is then suppressed. Therefore these constraints will only dominate in the region where there are no other strong constraints, namely for neutrinos more massive than the $Z$.

For the purpose of calculations, the masses of the electron, muon and tau and the mass matrix $D$ are generated in the standard way by coupling to the Higgs, so that the loops involved charged Higgs. t’Hooft gauge is used throughout, simplifying the form of the propagators and setting the masses of the $W$ and the charged Higgs to be the same. We will now consider, in detail, the three flavor changing processes.

5.1 $\mu \to e\gamma$

A calculation for the case where the neutrino masses are much less than $M_W$ is elaborated in Cheng and Lee [25]. Below we will give an outline of the calculation for the general case, where the neutrino masses are not assumed to be less than $M_W$. To simplify the calculations the electron is taken to be massless. The general form of the amplitude is constrained by gauge invariance, thus the gauge invariant form of the amplitude with a massless electron is given by

$$< e, \gamma | (S - 1) | \mu > = A \bar{u}_e ((1 - \gamma_5) i k^\nu \epsilon^\lambda \sigma_{\lambda \nu} i \partial) u_\mu,$$

where $k$ is the photon four momentum, $\epsilon$ is the polarization of the photon and $A$ is a constant to be determined. The partial derivative term is included to
ensure that, in the hypothetical case where the muon mass goes to zero, only
the lefthanded component of the muon coupled to the $W$ is involved in the
decay. The amplitude can then be rewritten using the Gordon decompostion
as:

$$< e, \gamma | (S - 1) | \mu > = A m_\mu \bar{u}_e [(1 + \gamma_5)(2\epsilon.p - m_\mu\not{\epsilon}] u_\mu ,$$

where $m_\mu$ is the muon mass and $p$ is the four momentum of the incoming
muon. To simplify the calculation, only the terms proportional to $\epsilon.p$ need
to be calculated. In principle there are 8 possible diagrams contributing (see
Fig.(8a)). Diagrams 4 to 8 contain only the $\not{\epsilon}$ term and thus can be ignored.
They will cancel with similar terms coming from the first four diagrams.

In evaluating diagrams 1 to 4 we can define a factor $I^i_j$ for each of the
diagrams $i = 1$ to 4, and for each of the neutrinos $j = 1$ to 6. Summing over
the six neutrinos $j$ the contribution of diagram $i$ to the constant $A$ is

$$-iK \sum_{j=1}^{6} U_{ej} U_{j\mu}^\dagger I^i_j /(32\pi^2 M^2_W)$$

where $K = -e^3/(2 \sin^2 \theta_W)$. We can also define the sum

$$I^{\mu\epsilon} = \sum_{i=1}^{4} \sum_{j=1}^{6} U_{ej} U_{j\mu}^\dagger I^i_j$$

so that the total contribution to the constant $A$ from all the diagrams is

$$-iKI^{\mu\epsilon} /(32\pi^2 M^2_W).$$

Performing the calculations gives the following results for the $I^i_j$ :

$$I^1_j = -[a_j(a_j^2 - 3a_j + \frac{31}{12}) + a_j^3(a_j - \frac{3}{2})\delta^2_j \log \delta_j],$$

$$I^2_j = -[\frac{1}{2}a_j^3 - \frac{1}{4}a_j^2 - \frac{7}{12}a_j + \frac{1}{3} + \frac{1}{2}a_j^3(a_j + 1)\delta^2_j \log \delta_j].$$
where the variable $\delta_j$ is given by $\delta_j = 0$ for $j = 1, 2, 3$ and by $\delta_j = (M_{\nu(j-3)}/M_W)^2$ for $j = 4, 5, 6$, and $a_j$ is defined as $a_j = 1/(1 - \delta_j)$. Using the expression for $U$, the sum $I_{\mu e}$ is then given by:

$$I_{\mu e} = \sum_{j=4}^{6} U_{D(ej)} U_{D_{(j-3)\mu}} A^2_{(j-3)}$$

$$\times \left[ -\frac{3}{2} a^3_j + \frac{15}{4} a^2_j - \frac{11}{4} a_j + \frac{1}{2} - \frac{3}{2} a^4_j \delta_j \log \delta_j \right].$$

This can be approximated for the two cases where the neutrino masses are all either much less than or much greater than the mass of the $W$. Thus,

$$I_{\mu e} \simeq -\sum_{j=1}^{3} U_{D(ej)} U_{D_{(j+3)\mu}} A^2_{(j+3)} \delta_{(j+3)} \text{ if all } M_j < M_W$$

$$\simeq \sum_{j=1}^{3} U_{D(ej)} U_{D_{j\mu}} A^2_j (\frac{1}{2} + \frac{1}{\delta_{(j+3)}} \frac{11}{4} + \frac{3}{2} \log \delta_{(j+3)}) \text{ if all } M_j > M_W.$$
This process can occur via extensions of the diagrams in $\mu \rightarrow e\gamma$ where the $\gamma$ is virtual and splits into an electron positron pair, or it can take place via box diagrams (see Fig (8b)). The contribution from box diagram(1) dominates the other box diagrams. This is due to the fact that the other diagrams involve exchange of virtual charged Higgs whose coupling to the electron and muon is supressed by a factor of the order of the Dirac mass $D/M_W$. The contribution from the extensions of the diagrams in $\mu \rightarrow e\gamma$ is of the same order as the first box diagram, but for an order of magnitude estimate we approximated the whole amplitude from the first box diagram only, (diagram(1) Fig(8b)).

The amplitude for the box diagram is calculated using the approximations that the neutrino masses are much greater than the muon mass, and that the electron mass is zero. Using these approximations the amplitude for the process is:

$$A = -\frac{ie^4}{32\pi^2 \sin^4 \theta_W M_W^2} \bar{u}_{q_1} P^- \gamma_\eta u_p \bar{u}_{q_2} P^- \gamma_\eta v_{q_3} J_{\mu e},$$

where the dimensionless factor $J_{\mu e}$ is:

$$J_{\mu e} = \sum_{j=1}^{6} U_{e j} U_{j \mu}^\dagger \Lambda_{(j-3)}^2 (a_j - 1 + a_j^2 \delta_j \log \delta_j)$$

with the variable $\delta_j$ given by $\delta_j = 0$ for $j = 1, 2, 3$ and by $\delta_j = (M_{\nu_{j-3}}/M_W)^2$ for $j = 4, 5, 6$, and $a_j$ defined as $a_j = 1/(1 - \delta_j)$. As for the previous flavor changing process, we can make approximations for the cases where all the neutrino masses are less than the mass of the $W$ and where they are all much
greater than the mass of the $W$. We obtain:

$$J_{\mu e} \simeq \sum_{j=1}^{3} U_{D_{j\mu}} U_{D_{j\mu}}^\dagger A_j^2 \delta(j+3)(1 + \log \delta(j+3)) \quad \text{if all } M_i < M_W$$

$$\simeq \sum_{j=1}^{3} U_{D_{j\mu}} U_{D_{j\mu}}^\dagger A_j^2 - \left[1 + \frac{1}{\delta(j+3)}(1 - \log \delta(j+3))\right] \quad \text{if all } M_i > M_W.$$

Averaging $|A|^2$ over the initial spin states and summing over the final spins and momentums leads to the decay rate:

$$\Gamma_{\mu \rightarrow e^+e^-} = \frac{G_F^2 m_\mu^5 \alpha^2}{768 \pi^5 \sin^4 \theta_W} |J_{\mu e}|^2,$$

where $\alpha$ is the fine structure constant. As before this can be compared to the decay rate $\Gamma_{\mu \rightarrow e\nu\bar{\nu}} = \frac{G_F^2 m_\mu^5}{192 \pi^3}$ to obtain the ratio which can then be compared to experiment to get an upper bound on $J_{\mu e}$:

$$\frac{\Gamma(\mu \rightarrow e^+e^-)}{\Gamma(\mu \rightarrow e\nu\bar{\nu})} = \frac{\alpha^2}{4\pi^2 \sin^4 \theta_W} |J_{\mu e}|^2 < 2 \times 10^{-12} \quad (2\sigma),$$

$$\implies |J_{\mu e}| < 2.8 \times 10^{-4} \quad (2\sigma).$$

5.3 $\mu Ti \rightarrow e Ti$

O.Shanker [26] has performed some careful calculations for $\mu e$ conversion for different nuclei. These calculations involve using an effective Hamiltonian for the muon-electron-q-q vertex where the q’s represent either two up quarks or two down quarks. This effective Hamiltonian is obtained from box diagrams very similar to the ones in the previous section except that the outgoing electrons, labeled with momenta $q_2$ and $q_3$, are replaced by an incoming and outgoing quark in the Titanium nucleus, see Fig(8c). The calculation for the
amplitude from the previous section can be carried over with very few changes to give the amplitude for the process involving the up quark; \( \mu u \rightarrow e u \):

\[
A_{\mu u \rightarrow e u} = -i \frac{G_F \alpha_{\mu e}}{\sqrt{24\pi \sin^2 \theta_W}} e_{q_1} P^- \gamma^\lambda \mu P^- \gamma_\lambda u_{q_2},
\]

and the same amplitude for the down quark. These calculations assume that the quark mass is less than the neutrino mass. We can then write an effective Hamiltonian for this interaction:

\[
H_{\text{eff}} = \frac{g}{\sqrt{2}} \bar{e} P^- \gamma^\lambda \mu (\bar{u} P^- \gamma_\lambda u + \bar{d} P^- \gamma_\lambda d),
\]

where \( g = \alpha_{\mu e} / (4\pi \sin^2 \theta_W) \). Using the calculations of O. Shanker [26], we can then obtain the ratio between the decay rate for \( \mu Ti \rightarrow e Ti \) to the rate for muon capture by the nucleus which can be compared to experiment to obtain another bound on \( J_{\mu e} \):

\[
\frac{\Gamma(\mu Ti \rightarrow e Ti)}{\Gamma(\mu^- Ti \text{ capture})} = 265.64 \frac{\alpha^2}{16\pi^2 \sin^4 \theta_W} |J_{\mu e}|^2 \leq 10^{-11} \quad (2\sigma)
\]

\[
\implies |J_{\mu e}| < 7.6 \times 10^{-5} \quad (2\sigma).
\]

The diagrams for the two processes, \( \mu Ti \rightarrow e Ti \) and muon capture, are essentially the same as the diagrams for \( \mu \rightarrow e e^+ e^- \) and \( \mu \rightarrow e \nu \bar{\nu} \) but the ratio of the decay rates of the first two processes is much greater than for the second two. Thus although the bounds placed by experiment on \( \mu Ti \rightarrow e Ti \) are not as strong as those for \( \mu \rightarrow e e^+ e^- \), it is the process \( \mu Ti \rightarrow e Ti \) which places the strongest bounds on the size of \( J_{\mu e} \).

This difference can be explained by coherence effects. The dominant process for \( \mu Ti \rightarrow e Ti \) leaves the Ti nucleus in it’s ground state [26], which is a coherent process involving summing the amplitude over all the nucleons.
Muon capture, on the other hand, is an incoherent process involving summing the square of the amplitude over all the protons.

6 Discussion and Conclusions

One of the desired results of this model was that it would provide a scenario in which weak interaction symmetry breaking could give the neutrinos a mass matrix on a scale similar to that of the electron, muon and tau, while still maintaining massless neutrinos. To investigate this all the plots discussed in this section are marked with a dashed line along which the masses $M_{D_i}$ induced by the mass matrix $D$ (from weak interaction symmetry breaking) are the same as the electron, muon and tau. For all the plots the excluded regions lie to the left of the curves.

The bounds from sections three and four are plotted separately for each of the $\Lambda_i$ in Figs (1), (2) and (3). To fulfill the scenario in which $M_{D_1} = M_e$, $M_{D_2} = M_\mu$ and $M_{D_3} = M_\tau$ we see that the mass of the third neutrino must be greater than $M_W$, the second must be heavier than 10 $GeV$ and the first heavier than 2 $GeV$.

To examine if there are further restrictions from the flavor changing processes of section five, the $\Lambda_i$ have to be plotted on the same graph since the factors $I_{\mu e}$ and $J_{\mu e}$ are functions of all three $\Lambda_i$. In fact due to the very small mixing of the third massive neutrino $\nu_i^H$ into the electron and tau neutrinos (the mixing matrix $U_D$ is chosen in this analysis to be like the KM matrix) $I_{\mu e}$ and $J_{\mu e}$ are virtually independant of $\Lambda_3$.

Figs (4), (5), and (6) plot out the constraints from the flavor changing
processes for three different scenarios. They all assumed that the masses $M_{D_i}$ generated by the mass matrix $D$ were in the same ratio as the masses of the electron muon and tau ie. $M_{D_1} : M_{D_2} : M_3 = M_e : M_\mu : M_\tau$. The dotted line, as before, marks out the line along which the masses $M_{D_i}$ are actually the same as the electron, muon and tau masses. The flavor changing processes are plotted alongside all the other constraints from sections three and four. It is immediately clear that flavor changing processes do not rule out any of the line along which $M_{D_1} = M_e$, $M_{D_2} = M_\mu$ and $M_{D_3} = M_\tau$. The bounds from $Z$ decays and, for the lightest neutrino, meson decays are much more important.

6.1 Three scenarios

In the scenarios that follow four different ratios of the neutrino masses $M_i$ are considered. The bounds given at the end of the discussion of each scenario assume that $M_{D_1} = M_e$, $M_{D_2} = M_\mu$ and $M_3 = M_\tau$.

**Scenario 1** Fig(4) $M_1 = M_2 = M_3$.

In Fig(4) are plots of the allowed regions taking into account all the experimental constraints from sections three, four and five. Areas to the left of the curves are ruled out. The plot is of the mass $M_2$ of the second heavy neutrino against $\frac{1}{\Lambda}$, the ratio between the two mass scales generated by $S$ and $D$. The most important constraint comes from the limits set by $Z$ decays on the third neutrino. If they are to lie on the dashed line all three neutrino masses are constrained to be greater than $M_W$.  

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Scenario 2 Fig(5) $M_1 : M_2 : M_3 = 1 : 15 : 60$.

Again, areas to the left are ruled out by experiment and the plot is of the mass $M_2$ of the heaviest neutrino against $\frac{1}{\Lambda_2}$, the ratio between the masses $M_2$ and $M_{D_2}$. In this case the most important constraints are those set by D decays on the mass of the first neutrino and those set by Z decays on the mass of the third neutrino. If the masses are to lie on the dashed line $M_2$ must be greater than 30 GeV. Dividing this by 15 and multiplying by 4 gives the bounds for the first and third neutrinos respectively. The bounds for the three neutrinos are thus: $M_1 > 2 GeV$ which is equivalent to $M_2 > 30 GeV$ and $M_3 > 120 GeV$.

Scenario 3 Fig(6) $M_1 : M_2 : M_3 = M_e : M_\mu : M_\tau$

In this scenario it is the constraints set by D decays on the mass of the first neutrino that are most important and the corresponding bounds for the three masses are (for masses lying on the dashed line): $M_1 > 2 GeV$, $M_2 > 400 GeV$ and $M_3 > 3500 GeV$.

6.2 Conclusions

In this paper we have examined the experimental consequences of a model of massive neutrinos and have excuded a large region of the parameter space. Specifically we have found that, in the scenario where the mass contributions, $M_{D_i}$, from weak interaction symmetry breaking are the same as those for the electron, muon and tau, the neutrino masses are approximately constrained as follows:

$$M_1 > 2 GeV, \quad M_2 > 10 GeV, \quad \text{and} \quad M_3 > 80 GeV.$$
This means that either the Dirac mass connecting standard left handed neutrinos to right handed neutrinos has entries less than their charged counterparts, or one would not expect all the neutrinos to be light. It is clearly nonetheless of interest to improve the bounds. Clearly improved statistics at LEP will give stronger constraints. Furthermore, the bound on $M_1$ can be improved by looking for heavy neutrinos in $B$ decays.

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**References**

[1] H. Harari & Y. Nir, Nucl. Phys. B 292 251-297 (1987).
[2] C.N. Leung, J.L. Rosner, Phys. Rev. D 28 2205 (1983).
[3] D. Wyler, L. Wolfenstein, Nucl. Phys. B 218 205 (1983).
[4] L. Randall, MIT-CTP-2112, Bull. Bd.: hep-ph@xxx.lanl.gov-9211268 (1992).
[5] N. De Leener-Rosier et al., Phys. Rev. D 43 3611 (1991).
[6] G. Azuelos et al., Phys. Rev. Lett. 56 2241 (1986).
[7] D.A. Bryman et al., Phys. Rev. Lett. 50 1546 (1983).
[8] T. Yamazaki et al., Proceedings of the Eleventh International conference on Neutrinos and Astrophysics at Dortmund (World scientific, Singapore, 1984) p183 (1983).
[9] J. Heintz et al., Nucl. Phys. B 149 365 (1979).
[10] J. Dorenbosch et al., Phys. Rev. Lett. B 166 473 (1986).
[11] F.J. Gilman et al., Phys. Rev. D 32 324 (1985).
[12] F. Bergsma et al., Phys. Lett. B 128 361 (1983).
[13] N.M. Shaw et al., Phys. Rev. Lett. 63 1342 (1989).
[14] H.-J. Behrend et al., Z. Phys. C 41 7 (1988).
[15] C. Wendt et al., Phys. Rev. Lett. 58 1810 (1987).
[16] O. Adriani et al., Phys. Lett. B 295 371 (1992).
[17] M.Z. Akrawy et al., Phys. Lett B 247 448(1990).
[18] M. Gronau, C.N. Leung, J.L. Rosner., Phys. Rev. D 29 2539 (1984)
[19] Review of Particle Properties, Phys. Rev. D (1992).
[20] M.E.Peskin, T. Takeuchi, Phys. Rev. D 46 381 (1992).
[21] M. Gronau, C.M. Leung, J.L. Rosner, Phys. Rev. D 29 2539 (1984).
[22] T. Kinoshita Phys. Rev. Lett. 2 477 (1959).
[23] D.A. Bryman et al., Phys. Rev. Lett. 50 7 (1983).
[24] P. Langaker, Lectures given at (TASI ’92):Black Holes and Strings to Particles, Boulder, CO, Bulletin Bd.: hep-ph@xxx.lanl.gov-9303304 (1993).
[25] T. Cheng & L. Li, Gauge Theory Of Elementary Particle Physics, Clarendon Press. Oxford (199).
[26] O. Shanker, Phys. Rev. D, 20 1608 (1979). [27] A. Blondel et al., Proceedings of the ECFA Workshop on LEP 200 Vol 1 p120 Cern 87 - 08 (1987).
Figures

Figures 1, 2 and 3 show bounds placed on the heavy neutrino masses by considering all the constraints in chapters three and four. The plots are of the neutrino masses $M_{D_i}$ against the ratio $1/\Lambda_{D_i} = M_i/M_{D_i}$. Any region to the left of a solid line is forbidden by experiment. The dashed line is the line along which the masses $M_{D_i}$ are equal to the electron, muon and tau masses.

Fig 1 Regions excluded (to the left of curves) in the $M_1, \frac{1}{\Lambda_1}$ plane from: (a) ref[5] Massive neutrinos in pion decays; (b) ref[9] and (c) ref[8] Massive neutrinos in Kaon decays; (d) ref[10] Massive neutrinos in D meson decays; (e) ref[21] / section (4.2) Banching ratio $\Gamma(\pi \to e\nu)/\Gamma(\pi \to \mu\nu)$; (f) ref[16] section (3.4) Z decays.

Fig 2 Regions excluded (to the left of curves) in the $M_2, \frac{1}{\Lambda_2}$ plane from: (a) ref[9] Massive neutrinos in Kaon decays; (b) ref[10] and (c) ref[10] Massive neutrinos in D meson decays; (d) Section (4.1.1) Changes in $G_F$, (e) ref[16] section (3.4) Z decays.

Fig 3 Regions excluded (to the left of curves) in the $M_3, \frac{1}{\Lambda_3}$ plane from: (a) section (4.1.2) Tau decays; (b) ref[16] section (3.4) Z decays.

Figures 4, 5, 6 and 7 Regions excluded (to the left of curves) in the $M_2, \frac{1}{\Lambda_2}$ plane from the experimental constraints from chapters three four and five. Each plot has the same ratio between the masses $M_{D_1}, M_{D_2}$.
and \( M_{D_3} \) induced by \( D \), This is chosen to be \( M_{D_1} : M_{D_2} : M_{D_3} = M_e : M_\mu : M_\tau \). The dashed line corresponds to the line along which \( M_{D_1} = M_e, M_{D_2} = M_\mu \) and \( M_{D_3} = M_\tau \). The different plots correspond to three different ratios of \( M_1 : M_2 : M_3 \) (the final neutrino masses).

**Fig 4** Case (1): \( M_1 = M_2 = M_3 \).
Regions excluded (to the left of curves) in the \( M_2, \frac{1}{A_2} \) plane from: (a) All the restrictions in the \( M_1, \frac{1}{A_1} \) plane studied in chapters three and four; (b) All the restrictions in the \( M_2, \frac{1}{A_2} \) plane studied in chapters three and four; (c) All the restrictions in the \( M_3, \frac{1}{A_3} \) plane studied in chapters three and four; (d) Bounds from \( \mu \to e\gamma \); (e) Bounds from \( \mu T_i \to e T_i \).

**Fig 5** Case (2): \( M_1 : M_2 = 1 : 15 : 60 \).
Regions excluded (to the left of curves) in the \( M_2, \frac{1}{A_2} \) plane from: (a) All the restrictions in the \( M_1, \frac{1}{A_1} \) plane studied in chapters three and four; (b) All the restrictions in the \( M_2, \frac{1}{A_2} \) plane studied in chapters three and four; (c) All the restrictions in the \( M_3, \frac{1}{A_3} \) plane studied in chapters three and four; (d) Bounds from \( \mu \to e\gamma \); (e) Bounds from \( \mu T_i \to e T_i \).

**Fig 6** Case (3): \( M_1 : M_2 : M_3 = M_e : M_\mu : M_\tau \).
Regions excluded (to the left of curves) in the \( M_2, \frac{1}{A_2} \) plane from: (a) All the restrictions in the \( M_1, \frac{1}{A_1} \) plane studied in chapters three and four; (b) All the restrictions in the \( M_2, \frac{1}{A_2} \) plane studied in chapters three and four; (c) All the restrictions in the \( M_3, \frac{1}{A_3} \) plane studied in chapters three and four; (d) Bounds from \( \mu \to e\gamma \); (e) Bounds from \( \mu T_i \to e T_i \).
Feynman diagrams for the flavour changing processes of chapter five.