Spin and Isospin Effects in the $NN \rightarrow NK\Lambda$ Reaction Near Threshold

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Abstract. The spin and isospin structure of the amplitudes and observables for $K^+\Lambda$ production in nucleon-nucleon collisions in the near-threshold region is analysed. It is shown that, with reasonable values for the relative strengths of the $\pi$ and $\rho$ terms in a meson-exchange model, one expects production on the neutron to be significantly stronger than that on the proton. Negative values of the spin-transfer coefficient $D_{NN}$ are also predicted due to $\pi\rho$ interference.

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1 Introduction

The study of meson production near threshold in nucleon-nucleon scattering has been a growth area over the last decade and most of the modelling of experimental data has been in terms of some form of meson-exchange model [1, 2]. The energy dependence of the total cross section is generally dominated by phase-space, folded with a strong nucleon-nucleon final-state interaction (fσi) [3]. The information that the data give on the basic driving term is therefore very limited and has to be supplemented by results from angular distributions and Dalitz plots etc. A particularly valuable constraint on theoretical models is the relative strength of the production in neutron-proton and proton-proton collisions. For $\eta$ production, it is found that $R(\eta) = \sigma(pp \rightarrow \eta pn)/\sigma(pp \rightarrow \eta pp) = 6.5 \pm 1.0$ [4]. Neglecting the differences between the $np$ and $pp$ initial and final-state interactions, the exchange of just a single pion or $\rho$ meson would lead to a factor of 5 [5], which is close to the experimental value. This is reduced by the fσi, but a quantitative agreement with the observables can be obtained with $\rho$-meson exchange being more important than that of the $\pi$ [6], though alternative scenarios are in the literature [7, 8].

The COSY11 [9,10] and COSY-TOF [11,12] collaborations have made measurements in proton-proton collisions of both $K^+\Lambda$ and $K\Sigma$ production near their respective production thresholds. The excitation functions look broadly similar to those for reactions such as $pp \rightarrow ppp\eta$, though the effects of the final state interaction are somewhat less because the hyperon-nucleon scattering lengths are much smaller than that in $pp$. Though proton-neutron data are much more sparse, there are strong indications from reactions on deuterium that $R(K^+)$ is also significantly over unity [13]. It is the purpose of this note to explore whether a large value of $R(K^+)$ could be understood within a meson exchange model.

The $pp \rightarrow pK^+\Lambda$ cross section near threshold has been estimated by several groups [14–18], but there has been no general consensus as to whether the reaction is driven mainly by the exchange of strange or non-strange mesons. In part this is due to the tremendous uncertainty in the $pK\Lambda$ coupling constant, as well as in the off-shell behaviour of the $K^+p$ scattering amplitude, which is not resonance dominated. However, the recent results from COSY-TOF [12] clearly indicate that the Dalitz plots for $pp \rightarrow pK^+\Lambda$ are dominated by the excitation of nucleon isobars, though modified by the $\Delta p$ fσi. At their lowest beam momentum (2.85 GeV/c) only the $N^*(1650)$ was seen but the $N^*(1710)$ becomes steadily more important as the momentum is raised. This suggests that strange meson exchange, which cannot excite such isobars, plays only a minor role in the process. Nevertheless, it is impossible as yet to estimate reliably the overall rate for the reaction, principally because of the uncertainty in the final $\Delta p$ wave function, especially at short distances. Different modern potentials that reproduce the limited scattering data give values of the singlet scattering length that vary from $-0.71$ fm to $-2.51$ fm [19]. We therefore limit ourselves to the evaluation of cross section ratios, which depend far less on the distortion in the final, or initial state [8].

Only three amplitudes are necessary to describe $NN \rightarrow KAN$ near threshold, and their spin and isospin structure are identified in sect. 2, where cross sections and spin observables are written in terms of them. Since there are more than five possible observables, there must be relations between some of them, and one of these is il-
Illustrated here. The contributions to the spin-isospin amplitudes are studied in sect. 3 within a meson-exchange model. Though strange and non-strange exchanges are considered, detailed evaluation is confined to the case where only the $\pi$ and $\rho$ are important. As discussed in sect. 4, the energy dependence of the total cross section is determined by the low energy $\Delta p$ scattering parameters but the normalisation depends also upon the $\Delta p$ interaction at short distances, which is largely unknown. The variation of $R(K^{+})$ with the $\pi/\rho$ strength is shown in sect. 5. With the value scaled from that used in the $\eta$ case, significantly more $K^{+}A$ production is to be expected in $pn$ than in $pp$ collisions. Furthermore, the spin-transfer parameter might be large and negative through $\pi\rho$ interference, though neither $\pi$ nor $\rho$ alone lead to a negative value. Our conclusions are reported in sect. 6.

### 2 Amplitudes and observables

The most general structure of the isoscalar and singlet $NN \rightarrow NK\Lambda$ amplitudes near threshold is

$$M_1 = \left[ W_{1,s} \eta_i \rho \cdot \epsilon_i + i W_{1,t} \rho \cdot (\epsilon_i \times \eta_i) \right] \chi_{i}^\dagger \cdot \chi_{i},$$

$$M_0 = W_{0,i} \rho \cdot \eta_i \phi^\dagger \phi_i,$$

(2.1)

where $\rho$ is the incident cm beam momentum. The initial (final) baryons couple to spin-1 or spin-0, represented by $\epsilon_i$ ($\eta_i$) and $\eta_i$ ($\eta_f$) respectively. Similarly, the $\chi_i$ ($\chi_f$) and $\phi_i$ ($\phi_f$) describe the isospin-1 and isospin-0 combinations of the initial $NN$ (final $KN$) states. It is important to note that, due to the Pauli principle, $W_{1,t} = 0$ for the analogous $pp \rightarrow p\eta\eta$ reaction and that this vanishing leads to quite different spin and isospin effects for $K$ and $\eta$ production.

After a little spin algebra, it is seen that the unpolarised intensities are given by

$$I(pp \rightarrow pK^+) = \frac{1}{2} \sum_{\text{spins}} |f |M_1| i \rangle|^2,$$

$$I(pn \rightarrow nK^+) = |f |M_0| i \rangle|^2,$$

(2.2)

where there is no interference between the two isospin amplitudes due to the spin averaging.

One may expect that, close to threshold, the amplitudes $W_{1,s,t}$ should vary little, except for the different $\Lambda N$ final-state interactions in the spin-singlet (s) and -triplet (t) systems. If we neglect these $fsi$, the corresponding total cross section becomes

$$\sigma(pp \rightarrow pK^+) = \frac{1}{64\pi^2 p^2 s} \left( m_p m_{K} (m_{K} + m_{\Lambda}) \right)^{1/2},$$

(2.3)

and similarly for the $pn$ reaction. Here the $m_i$ are the masses in the final state, $p$ is the incident proton cm momentum, $\sqrt{s}$ the total cm energy, and $Q = \sqrt{s} - \sum m_i$, the excess energy.

In the near-threshold region, both the proton analysing power and the $A$ polarisation should vanish and it is only tensor combinations that are predicted to be non-zero. Of these, the most “easily” accessible experimentally are the transverse spin-correlation ($C_{NN}$) and spin-transfer parameters ($D_{NN}$), which are given by

$$I(pp \rightarrow pK^+) C_{NN}(pp \rightarrow pK^+) = \frac{1}{2} |W_{1,s}|^2,$$

$$I(pn \rightarrow nK^+) C_{NN}(pn \rightarrow nK^+) = \frac{1}{2} |W_{0,t}|^2 - |W_{0,t}|^2,$$

(2.4)

$$I(pp \rightarrow pK^+) D_{NN}(pp \rightarrow pK^+) = -\frac{1}{2} \text{Re}(W_{1,s} W_{1,t}^*)$$

(2.5)

It follows from these relations that

$$4I(pn \rightarrow nK^+) \left[ 1 + C_{NN}(pn \rightarrow nK^+) \right] = I(pp \rightarrow pK^+) \left[ 1 + C_{NN}(pp \rightarrow pK^+) \right],$$

(2.6)

so that, in the near-threshold region, the additional measurement of the spin correlation in $np$ collisions would afford no further information. Alternatively, measuring just the two spin correlations would be sufficient to fix $R(K^+)$. 

### 3 One-boson-exchange models

Both strange and non-strange meson exchanges can contribute to $K^{+}A$ production in nucleon-nucleon collisions and the two sets of diagrams are illustrated on the left and right hand sides of fig. 1 before the inclusion of effects arising from the distortion of the initial and final waves.

Near threshold, the only significant variation is expected to arise from the spin-singlet and -triplet $fsi$ enhancements. The relevant propagators, coupling constants, masses etc. are evaluated at threshold and so merely contribute to the overall strength [6].

Employing the same technique and notation that we used for $\eta$ production, we find that

$$W_{1,s} = 2B_{\rho} + 2B_{\omega} - D_{\pi} - D_{\eta} + D_K^1,$$

$$W_{1,t} = D_{\pi} + D_{\eta} + D_K^1,$$

$$W_{0,t} = 6B_{\rho} - 2B_{\omega} + 3D_{\pi} - D_{\eta} + D_K^0,$$

(3.1)

where $D_{\pi,\eta}$ is the amplitude for the exchange of a pseudoscalar meson and $B_{\rho,\omega}$ the dominant vector-exchange term. These amplitudes have the structure of an $NNx$ coupling constant, the propagator for the meson $x$, followed by the final $xN \rightarrow K^{+}Y$ transition, which is dominated by the $S_{11}(1650)$ near threshold. The kaon exchange terms are similar, except that there is then an $N\Lambda K$ coupling constant and two isospin possibilities in $KN$ elastic scattering, leading to the two terms denoted here by $D_K^1$. However, it has been pointed out by Laget [14] that the isoscalar $K^{+}N$ scattering is dominantly $p$-wave and so would contribute relatively little here.

Though, for completeness, many terms have been included in eq. (3.1), we will concentrate our analysis on just those for $\pi$ and $\rho$ exchange. The $\eta$ and $K$ terms might
be reduced in importance by the weak coupling constants and, for \( \eta \) production, \( \omega \)-exchange was reduced rather by the final transition amplitude.

Using vector dominance to estimate the \( \rho N \to \eta N \) amplitudes, we predicted for \( \eta \) production that \( D_\pi \approx 0.7 B_\rho \) [6], which led to a reasonable agreement with the large experimental value of \( R(\eta) \) [4]. To estimate the corresponding value for \( K^+ \) production, this \( \pi/\rho \) factor should be scaled by the ratio of the amplitudes for the production of \( K^+ \) with pion and photon beams.

\[
D_\pi \approx 0.7 \left( \frac{|f(\pi^0 p \to K^0 \Lambda)|^2 |f(\gamma p \to \eta p)|^2}{|f(\pi^- p \to \eta n)|^2 |f(\gamma p \to K^+ \Lambda)|^2} \right)^{1/2} B_\rho ,
\]

where we have assumed that the same resonances are responsible for the production with pions and photons so that, in the absence of other interactions, the contributions are relatively real.

Taking the experimental data from refs. [20–23], we find that

\[
D_\pi \approx 0.7 \sqrt{\frac{(58 \pm 10)(4.6 \pm 0.2)}{(810 \pm 100)(0.19 \pm 0.04)}} B_\rho = (0.9 \pm 0.2) B_\rho .
\]

4 The \( \Delta N \) final-state interaction

To determine the overall normalisation of the \( pp \to pK^+\Lambda \) cross section, one would need reliable information on the \( Ap \) scattering wave functions, which is still sadly lacking [19]. However, the shape of the energy dependence is, to a large extent, fixed by just the \( Ap \) scattering lengths and effective ranges in the combination that gives the positions \( (\varepsilon) \) of the virtual bound states. The effect of the \( fsi \) on the shape of the cross section can be included by multiplying the threshold value of \( I \) in eq. (2.4) by the factor [15]

\[
Z = \frac{4}{\left(1 + \sqrt{1 + Q/\varepsilon} \right)^2} .
\]

A useful survey of theoretical and experimental information on the low energy \( Ap \) parameters is provided in ref. [24]. An early experiment [25] suggested that the values for the triplet and singlet energies were quite close \( (\varepsilon_t = 5.6 \text{ MeV, } \varepsilon_s = 5.1 \text{ MeV}) \) and it has been shown [10] that the statistical average of these two \( (\varepsilon = 5.5 \pm 0.6 \text{ MeV}) \) gives a good representation of the \( pK^+\Lambda \) total cross section data. Given the current theoretical uncertainty, for simplicity of presentation, we choose \( \varepsilon_s = \varepsilon_t \).

5 Results

By taking \( \varepsilon_s = \varepsilon_t \), it follows that \( R(K^+) \) should not depend upon the excitation energy \( Q \) and in Fig. 2 the prediction for this has been drawn as a function of \( x = D_\pi/B_\rho \), where it has been assumed that the \( \pi \)- and \( \rho \)-exchange amplitudes have the same phase.

As \( x \to \infty, R(K^+) \to 1 \) [26], which is very different to the factor of five expected for \( \eta \) production under similar assumptions. This difference arises, in part, because we do not include the contribution of the \( K^0 \) in the definition of \( R \), but also because the \( W_{1,t} \) term is forbidden by the Pauli principle for the \( pp \to ppp \) reaction. On the other hand, pure \( \rho \) exchange leads to \( R(K^+) \approx 2.5 \) and for a wide range of \( x \) the ratio is well over unity. It is interesting to note that the figure of 0.9, resulting from the crude scaling model of eq. (3.3), corresponds almost to the peak value of 7 shown in Fig. 2.

From eqs. (2.5) and (3.1), it is seen that, with this value of \( x \), one expects \( C_{NN}(pp \to pK^+\Lambda) = 0.43 \), to be contrasted with the 0.5 and 1.0 expected for pure \( \pi \) and \( \rho \) exchange respectively. The variation of both \( C_{NN} \) and \( D_{NN} \) with \( x \) is shown in fig. 3, where it is seen that \( D_{NN} \) has an even more interesting behaviour, with a minimum of about \(-0.7 \) for \( x = 0.9 \). This is to be contrasted with the +2/3 and 0 expected for pure \( \pi \) and \( \rho \) exchange respectively.

6 Conclusions

We have studied the charge and spin dependence of the \( pp \to pK^+\Lambda \) total cross section near threshold in the region where the final particles are in relative S-states. The
overall cross section strength is hard to estimate with any confidence, due principally to the poor knowledge of the $\Lambda N$ potential. Scaling the cross section from that for $\eta$ production using the Bargmann potential, as in ref. [15], avoids some of the problems associated with initial state interactions [8] and gives a good energy dependence. However, it leads to an estimate that is too low by a factor of up to five. This probably indicates that the short-range part of the $\Lambda p$ interaction is less repulsive than that for $pp$ [19].

Since there are only three amplitudes describing $K^+\Lambda$ production on the proton and neutron near threshold, it is clear that there should be some model-independent relations between the charge and spin dependence of the observables. To go further than this, we have worked in a simplified meson-exchange model, where the amplitudes have been assumed to be in phase. Keeping only the $\pi$ and $\rho$ terms, and scaling their relative strength from that found for $\eta$ production, we find that production of $K^+\Lambda$ on the neutron could indeed be much stronger than on the proton. However, the prediction does depend upon cancellations and is far less robust than that in the $\eta$ case.

The spin-correlation and transfer parameters are also expected to depend sensitively upon $x$, the relative $\pi/\rho$ strength. Of especial interest is $D_{NN}$ which, though $+2/3$ for $\pi$ exchange and 0 for $\rho$ exchange, is predicted to be strongly negative for our preferred value of $x$, though this does depend upon our phase ansatz. The negative value found for $D_{NN}$ in $pp \rightarrow pK^+\Lambda$ in different kinematic conditions away from threshold by the DISTO group [27] was taken as evidence for the dominance of kaon over pion exchange [14], but it is important to stress that the possibility of $\rho$ exchange was not considered by these authors.

We have only looked in detail at the consequences of $\pi$ and $\rho$ exchanges. Whether other terms are significant or not might be determined from the spin observables of eq. (2.5). In particular, the measurement of $C_{NN}$ in $pp$ collisions as well as of the $pn$ and $pp$ cross section would allow one to separate the magnitudes of the $W$ amplitudes. Since this would fix both the $\pi$ and $\rho$ amplitudes and their interference, it would then predict unambiguously the spin-transfer coefficient $D_{NN}$ and this would allow a test of the $\pi/\rho$ model.

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