$R^{\nu}_{K^{(*)}}$ and non-standard neutrino interactions

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Abstract: We discuss the modes $B \to K^{(*)}\nu\bar{\nu}$ in the context of non-standard neutrino interactions that add incoherently to the SM rates. We consider two scenarios: an additional light neutrino; and neutrino lepton flavour violation. We find that an additional light neutrino that interacts with SM fields via a non-universal $Z'$ can increase $R^{\nu}_{K^{(*)}}$ by up to a factor of two without conflicting with $B_s - \bar{B}_s$ mixing. This model then predicts rates for $B_s \to \tau^+\tau^-$ up to six times larger than the SM. In the context of neutrino lepton flavour violation mediated by leptoquarks we find that the current experimental upper bounds on $R^{\nu}_{K^{(*)}}$ are already more constraining than direct bounds from $B_s \to \tau\ell$ and $B \to K^{(*)}\tau\ell$ modes for $\ell = e, \mu$.

ArXiv ePrint:
1 Introduction

Rare $B$ decays play an important role in understanding the dynamics of the standard model (SM) as well as being a fertile ground for the search for new physics. The $B \to K^{(*)}\nu\bar{\nu}$ decays are amongst the cleanest modes to search for new physics due to their well controlled theoretical uncertainty. Experiments at Belle and Babar have already published upper limits on these modes at 2-3 times the SM rate and further improvement is expected from Belle-II, which can reach a sensitivity on the branching ratios of about 10% with 50 ab$^{-1}$ [1]. Interesting constraints for certain BSM physics can be obtained already with current bounds.

We consider two types of models that add incoherently to the SM rates. First we entertain the possibility of a fourth light neutrino that couples to SM fields through a non-universal $Z'$. We find that existing constraints on the model allow enhancements of the $B \to K^{(*)}\nu\bar{\nu}$ rates by up to factors of two and that these are correlated with the $B_s \to \tau^+\tau^-$ mode which could reach a rate up to six times larger than in the SM.

We then consider possible contributions from neutrino flavour violating final states in the context of scalar and vector leptoquarks. These contributions are correlated to the charged lepton factors of two and that these are correlated with the $B_s \to \tau^+\tau^-$ modes have the same rates so we choose to present the two modes with the strongest experimental uncertainty. Experiments at Belle and Babar have already published upper limits on these modes amongst the cleanest modes to search for new physics due to their well controlled theoretical uncertainties. The $B$ rare decays play an important role in understanding the dynamics of the standard model (SM) as well as being a fertile ground for the search for new physics. The $B \to K^{(*)}\nu\bar{\nu}$ decays are amongst the cleanest modes to search for new physics due to their well controlled theoretical uncertainty. Experiments at Belle and Babar have already published upper limits on these modes at 2-3 times the SM rate and further improvement is expected from Belle-II, which can reach a sensitivity on the branching ratios of about 10% with 50 ab$^{-1}$ [1].

Typical SM predictions obtained with flavio are

$$\mathcal{B}(B^+ \to K^+\nu\bar{\nu})_{SM} = (4.4 \pm 0.7) \times 10^{-6}, \quad \mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})_{SM} = (9.5 \pm 1.0) \times 10^{-6}. \quad (3)$$

We list the best current experimental constraints on these modes in Table 1. Belle-II is expected to improve these limits, and has produced a preliminary result $\mathcal{B}(B^+ \to K^+\nu\bar{\nu}) \leq 4.1 \times 10^{-5}$ at the 90% confidence level [9]. They have averaged this result with the previous ones to arrive at $\mathcal{B}(B^+ \to K^+\nu\bar{\nu}) = (1.1 \pm 0.4) \times 10^{-5}$ [9]. For the $K^*$ channel, Belle has also combined the charged and neutral modes to obtain the limit $\mathcal{B}(B \to K^*\nu\bar{\nu}) \leq 2.7 \times 10^{-5}$ [8] but here we will use the limit in Table 1. These results are usually presented as ratios, for which we obtain

$$R_K^\nu = \frac{\mathcal{B}(B^+ \to K^+\nu\bar{\nu})}{\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})_{SM}} = 2.5 \pm 1.0, \quad R_K^{*\nu} = \frac{\mathcal{B}(B \to K^{*0}\nu\bar{\nu})}{\mathcal{B}(B \to K^{*0}\nu\bar{\nu})_{SM}} \leq 1.9. \quad (4)$$

1These numbers agree within errors with published numbers as in [1, 5]. Neglecting isospin breaking, the neutral and charged modes have the same rates so we choose to present the two modes with the strongest experimental limits.
Table 1: Current experimental upper bounds on the modes considered.

| Mode | 90% c.l upper limit | Reference |
|------|---------------------|-----------|
| $B(B^+ \to K^+\nu\bar{\nu})$ | $1.6 \times 10^{-5}$ | Belle [8] |
| $B(B^+ \to K^{*+}\nu\bar{\nu})$ | $4.0 \times 10^{-5}$ | Belle [6] |
| $B(B^0 \to K^0\nu\bar{\nu})$ | $2.6 \times 10^{-5}$ | Belle [7] |
| $B(B^0 \to K^{*0}\nu\bar{\nu})$ | $1.8 \times 10^{-5}$ | Belle [8] |

The second number is simply the ratio of the limit in Table 1 and the central value in Eq. (3) and somewhat lower than what is used in [10].

2 Effective Hamiltonian at the $b$ scale

We can parameterise any new physics relevant for these decays through an effective Hamiltonian at the $b$ mass scale. The effective theory originates in extensions of the SM containing new particles at or above the electroweak scale that have been integrated out. In general, this results in additional contributions to $C_L$ in Eq. (2) as well as in new operators. Because our discussion is tied to two types of models, we only need to consider the following

$$\mathcal{H}_{NP} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \sum_{ij} \left( C_{Lij}^i \mathcal{O}_{Lij}^i + C_{Rij}^i \mathcal{O}_{Rij}^i \right) + C_{Lij}^r \mathcal{O}_{Lij}^r + C_{Rij}^r \mathcal{O}_{Rij}^r$$

where the operators are

$$O_{Lij}^i = (\bar{s}_L \gamma_\mu b_L)(\bar{\nu}_i \gamma^\mu (1 - \gamma_5) \nu_j)$$
$$O_{Lij}^r = (\bar{s}_R \gamma_\mu b_R)(\bar{\nu}_i \gamma^\mu (1 - \gamma_5) \nu_j)$$
$$O_{Lij}^y = (\bar{s}_L \gamma_\mu b_L)(\bar{\nu}_i \gamma^\mu \ell_j)$$
$$O_{Lij}^{10} = (\bar{s}_L \gamma_\mu b_L)(\bar{\nu}_i \gamma^\mu \gamma_5 \ell_j)$$
$$O_{Rij}^{10} = (\bar{s}_R \gamma_\mu b_R)(\bar{\nu}_i \gamma^\mu \gamma_5 \ell_j)$$

The Wilson coefficients are defined so that they only contain NP contributions and the SM is counted separately through Eq. (2). The list in Eq. (6) includes $O_{Lij}^i$, $O_{Rij}^i$, which contribute to $B^{(*)} \to K^{(*)}\nu\bar{\nu}$. It excludes operators with scalar and tensor neutrino bi-terms that have been considered in [10] because they do not appear in the models we discuss. The operators with charged leptons appear in the models we discuss with coefficients that are related to $C_{L,R}^{ij}$. The flavour diagonal operators $O_{9,10}^{ij}$ affect $b \to s\mu\bar{\nu}$ decays including the $B \to K^{(*)}\mu\bar{\nu}$ anomalies and have been studied extensively in that context.

We can classify the contributions to $B \to K^{(*)}\nu\bar{\nu}$ from these operators into two types: those that interfere with the SM, $O_{L,R}^{ij}$; and those that do not, $O_{L,R}^{ij}$ and $O_{L,R}^{ij}$. In $B \to K\nu\bar{\nu}$ only the vector current enters the hadronic matrix element so that the contributions to the rate from $O_{L,R}^{ij}$ and $O_{L,R}^{ij}$ are the same. Similarly for those from $O_{L}^{ij}$ and $O_{R}^{ij}$. At the same time, the different neutrino chirality eliminates interference between the primed and un-primed-operators for massless neutrinos. The only operators that interfere with the SM are thus the diagonal ones (in neutrino flavour) $O_{L}^{ii}$ and $O_{R}^{ii}$. The rates can be evaluated numerically using flavio [11], and the central
value (uncertainty will be shown in the figures) is given approximately by

\[ \mathcal{B}(B^+ \to K^+ \nu \bar{\nu}) \times 10^6 \approx 4.39 - 0.457 \text{Re} \sum_i \left( C_{L}^{ij} + C_{R}^{ij} \right) \\
+ 0.0357 \sum_{ij} \left( \left| C_{L}^{ij} + C_{R}^{ij} \right|^2 + \left| C_{L}'^{ij} + C_{R}'^{ij} \right|^2 \right) \]

(7)

In \( B \to K^+ \nu \bar{\nu} \) both the vector and axial-vector currents enter the hadronic matrix element resulting in different contributions for \( O_L^{ij} \) and \( O_R^{ij} \) as well as for \( O_L'^{ij} \) and \( O_R'^{ij} \). Numerically, the rate is approximately given by

\[ \mathcal{B}(B^0 \to K^{*0} \nu \bar{\nu}) \times 10^6 \approx 9.53 + \text{Re} \sum_i \left( -0.993 C_{L}^{ii} + 0.661 C_{R}^{ii} \right) \\
+ \sum_{ij} \left( 0.0775 \left( C_{L}^{ij} \right)^2 + C_{R}^{ij} \left( C_{L}'^{ij} \right)^2 + C_{L}'^{ij} \left( C_{R}'^{ij} \right)^2 \right) - 0.103 \left( C_{L}^{ij} C_{R}^{ij} + C_{L}'^{ij} C_{R}'^{ij} \right) \]

(8)

The parametric uncertainty in these predictions, as estimated by \texttt{flavio} is illustrated in Fig. 1.\footnote{The uncertainty in these predictions is around 15\% and is mostly due to the form factors for the \( B \to K \) and \( B \to K^* \) hadronic transitions, which are responsible for about 10\%, while the value of \( V_{cb} \) contributes an additional 5\%.}

We show \( \mathcal{B}(B^+ \to K^+ \nu \bar{\nu}) \) as a function of \( C_{L}^{33} \) (the figure is identical for \( C_{L,R}^{L,R} \)) and as a function of \( C_{L}^{23} \) (the figure is identical for \( C_{L,R}^{L,R} \)). The red band marks the experimental combination \( \mathcal{B}(B^+ \to K^+ \nu \bar{\nu}) = (1.1 \pm 0.4) \times 10^{-5} \)\footnote{The uncertainty in these predictions is around 15\% and is mostly due to the form factors for the \( B \to K \) and \( B \to K^* \) hadronic transitions, which are responsible for about 10\%, while the value of \( V_{cb} \) contributes an additional 5\%.} and the green band the SM in Eq. (3) at 1\( \sigma \). For \( \mathcal{B}(B^0 \to K^{*0} \nu \bar{\nu}) \) we show the dependence on \( C_{L}^{33} \) (same for all \( C_{L}^{L,R} \)), \( C_{R}^{33} \) (same for all \( C_{R}^{L,R} \)) and \( C_{L}^{23} \) (same for all \( C_{L}^{L,R} \) and \( C_{R}^{L,R} \)). In this case the red line shows the 90\% c.l. experimental upper bound from Table 1 and the green band the SM at 1\( \sigma \).

Figure 1: Top row: \( \mathcal{B}(B^+ \to K^+ \nu \bar{\nu}) \) as a function of \( C_{L}^{33} \) (left panel) and \( C_{L}^{23} \) (right panel). Bottom row: \( \mathcal{B}(B^0 \to K^{*0} \nu \bar{\nu}) \) as a function of \( C_{L}^{33} \) (left panel), \( C_{R}^{33} \) (centre panel) and \( C_{L}^{23} \) (right panel).

For example, new physics contributions allowed by the value of \( \mathcal{B}(B^+ \to K^+ \nu \bar{\nu}) \) in Eq. (7) at the 1\( \sigma \) level and taking only one non-zero parameter at a time are shown in Table 2 and can also be read off Fig. 1.
3 An additional light neutrino

The first type of new physics we consider that can increase the SM value of \( B \to K^{(*)}\nu\bar{\nu} \) consists of new light neutrinos. In fact, modes with neutrino pairs in the final state count the number of light neutrinos in the SM due to lepton universality. The existence of new light neutrinos is severely constrained by measurements of the invisible \( Z \) width and by cosmological considerations. Assuming lepton universality, the former implies that \( N_\nu = 2.9840 \pm 0.0082 \) [12]. Cosmological constraints depend on other parameters and, for example, \( \Delta N_{\text{eff}} < 0.77 \) for a Hubble constant \( H_0 = 71.3^{+1.9}_{-2.2} \) km/s/Mpc [13].

It is possible to avoid these limits with a light sterile neutrino that interacts with the SM through a \( Z' \). The contribution of this neutrino to the \( Z \) width neutrino count is proportional to the square of the \( Z - Z' \) mixing parameter and can thus be negligibly small. In addition, if the \( Z' \) is non-universal and couples predominantly to the third generation SM fermions, the new neutrino reaches thermal equilibrium with SM particles at a temperature near the \( \tau \)-lepton mass. However, at the time of big-bang nucleosynthesis the temperature is about 1 MeV and this difference results in a suppression of the contribution of this neutrino to \( \Delta N_{\text{eff}} \) to a safe level, \( \Delta N_{\text{eff}} < 0.1 \) [14].

We have previously constructed a detailed example of a model with these properties [15, 16] so we do not repeat the details here. The \( Z' \) is responsible for two new operators that contribute to \( B \to K^{(*)}\nu\bar{\nu} \) and to \( B_s \to \tau^+\tau^- \) [17]:

\[
\mathcal{H}_T = \frac{G_F}{\sqrt{2}} \frac{M_Z^2}{M_{Z'}} \cot^2 \theta_R V_{Rbs}^{\dagger} V_{Rbb}^d \bar{s}_R \gamma_\mu b_R (\bar{\nu}_{R3}\gamma^\mu \nu_{R3} - \bar{\tau}_R \gamma^\mu \tau_R) \\
\mathcal{H}_L = -\frac{G_F}{\sqrt{2}} \frac{\alpha}{\pi} \frac{M_Z^2}{M_{Z'}} \cot^2 \theta_R V_{Lbs}^{\dagger} V_{Lbb}^d \bar{\tau}_L \gamma_\mu b_L (\bar{\nu}_{R3}\gamma^\mu \nu_{R3} - \bar{\tau}_R \gamma^\mu \tau_R)
\]

In the notation of Eq. (5), the Wilson coefficients that result in this model are thus:

\[
C_L^{\tau\tau} = -C_9^{\tau\tau} = -C_{10}^{\tau\tau} = 2 \left( \frac{m_Z^2}{m_{Z'}^2} \right) \cot^2 \theta_R I(\lambda_t, \lambda_H) \\
C_R^{\tau\tau} = -C_9^{\tau\tau} = -C_{10}^{\tau\tau} = 4 \left( \frac{V_{Rbs}^{\dagger} V_{Rbb}^d}{V_{Lbs}^{\dagger} V_{Lbb}^d} \right) \left( \frac{m_Z^2}{m_{Z'}^2} \right) \cot^2 \theta_R \frac{\pi s_W}{\alpha}.
\]

The first operator in Eq. (9) originates in a flavour changing tree-level exchange of the \( Z' \). The parameters that appear in this result are: the \( Z' \) mass; a ratio parameterising the strength of the new interaction relative to the weak interaction, \( \cot \theta_R \); and two elements of the matrix that rotates the down-type quarks between the weak and the mass bases. The second operator in Eq. (9) arises from a new penguin diagram and depends on details of the scalar sector through the Inami-Lim function \( I(\lambda_t, \lambda_H) \) [18]. The existing constraints on these parameters can be summarised as:

- A combination of perturbative unitarity [15] and LHC non-production of \( Z' \) from \( bb \) annihi-
loration [19] restrict the overall strength of the new interaction to

$$\left(\frac{m_Z^2}{m_{Z'}^2}\right) \cot^2 \theta_R \lesssim 0.15.$$  \hfill (11)

- $B_s$ mixing constrains $|V^d_{Rbs} V^d_{Rbb}|$ and $I(\lambda_t, \lambda_H)$ [17]. Both the SM calculation and the experimental situation regarding $\Delta M_{B_s}$ have changed significantly so we repeat that analysis here.

In terms of the parameters of interest, the effective Hamiltonian below the $Z'$ scale is:

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} \left(\frac{m_Z^2}{m_{Z'}^2}\right) \cot^2 \theta_R \left(\frac{\alpha}{2\pi s_W} V_{tb}^* V_{ts}\right)^2 I(\lambda_t, \lambda_H)^2 \mathcal{O}_{LL}$$

$$+ s_W^2 (V^d_{Rbs} V^d_{Rbb})^2 \mathcal{O}_{RR} + 2s_W \left(\frac{\alpha}{2\pi s_W} V_{tb}^* V_{ts}\right) I(\lambda_t, \lambda_H)(V^d_{Rbs} V^d_{Rbb}) \mathcal{O}_{LR}$$  \hfill (12)

with the usual $\Delta B = 2$ operators:

$$\mathcal{O}_{LL,RR} = (\bar{s}_L, R \gamma_\mu b_{L,R}) (\bar{s}_L, R \gamma_\mu b_{L,R}) \quad \mathcal{O}_{LR} = (\bar{s}_L \gamma_\mu b_L)(\bar{s}_R \gamma_\mu b_R).$$  \hfill (13)

QCD renormalisation group running modifies the Wilson coefficients and introduces one more operator at the $b$ scale, $\mathcal{O}_{SLR} = (\bar{s}_L b_{L,R})(\bar{s}_L b_{R})$. Making use of flavio once more, we find

$$\frac{\Delta M_{B_s}}{(\Delta M_{B_s})_{SM}} \approx 1 + 1.5 \times 10^{-4} I(\lambda_t, \lambda_H)^2 - 8.2 I(\lambda_t, \lambda_H)|V^d_{Rbs} V^d_{Rbb}| + 3282. |V^d_{Rbs} V^d_{Rbb}|^2.$$  \hfill (14)

The current experimental average $\Delta M_{B_s} = (17.741 \pm 0.020)$ ps$^{-1}$ [20] combined with a recent SM prediction $(\Delta M_{B_s})_{SM} = (18.4^{+0.7}_{-1.2})$ ps$^{-1}$ [21] results in $\Delta M_{B_s}/(\Delta M_{B_s})_{SM} = 0.96^{+0.4}_{-0.6}$ and we compare this ratio to the prediction of Eq. (14) in Fig. 2 for $I(\lambda_t, \lambda_H) = 0$ and $I(\lambda_t, \lambda_H) = 3$. These two values were chosen because a scan over the parameters in the model [18] suggests $-2 \lesssim I(\lambda_t, \lambda_H) \lesssim 3$ as a range for the Inami-Lim function. The allowed range for $C_{L,R}^{\pi\tau}$, assuming that $V^d_{Rbs} V^d_{Rbb}$ is real, is then showed in the right panel of Fig. 2. The key point is that the tree level $Z'$ exchange tends to increase $\Delta M_{B_s}$ over its SM value and this is severely constrained by current data. It is the new penguin contribution to $\mathcal{O}_{LR}$ and $\mathcal{O}_{SLR}$ that allows $\Delta M_{B_s}$ to drift below its SM value. As can be seen from Eq. (14), allowing $V_{Rbs}^d V_{Rbb}^d$ to have a phase can augment the allowed parameter range but a complete phenomenological study of this general case is beyond the scope of the present work.

The allowed region in $C_{L,R}^{\pi\tau}$ shown in Fig. 2 results in an increase of the $B \to K^{(*)}\nu\bar{\nu}$ rates over their SM value and this is shown in the left panel of Fig. 3. The figure indicates a near perfect correlation between the two neutrino modes and the largest values, near two, are obtained for the upper-right corner of the region in Fig. 2. Interestingly, the same model with the constraint of Eq. (11) can no longer enhance $K \to \pi\nu\bar{\nu}$ modes by more than a few percent.\footnote{In the notation of [22] it predicts $\bar{X} \lesssim 0.1$.} The main difference between these two cases is the strong constraint on $V^d_{Rbd}$ (from $B_d$ mixing) that enters $K \to \pi\nu\bar{\nu}$ in place of $V^d_{Rbs}$, which can be close to one.

The model predicts through Eq. (10) a correlation between $B \to K^{(*)}\nu\bar{\nu}$ modes and $B_s \to \tau^+\tau^-$. The latter currently only has a weak experimental limit from LHCb $B(B_s \to \tau^+\tau^-) \leq
The allowed parameter region seen in Fig. 2, combined with $C_{10}^{TT} = -Y(x_t)/s_W^2 \sim -4.3$ [3], implies that the model allows

$$\frac{B(B_s \to \tau^+\tau^-)}{B(B_s \to \tau^+\tau^-)_{SM}} \lesssim 6.$$  \hspace{1cm} (16)$$

This can be read off Fig. 3 which illustrates the correlation with $R_K^\nu$. On the other hand, the model also allows for $(C_{10}'^{TT} - C_{10}^{TT}) \sim -4$ where the $B(B_s \to \tau^+\tau^-)$ is significantly suppressed with respect to its SM value.

The corresponding Wilson coefficients affecting the modes $b \to s\mu^+\mu^-$ exhibiting anomalies, $C_{9,10,9',10'}^{\mu\mu}$ are suppressed with respect to $C_{9,10,9',10'}^{TT}$ by factors $|V_{Rbb}^d|_{SM}$ which can be very small [24].

For this reason, this model yields predictions for $b \to s\mu^+\mu^-$ processes that are very similar to the SM. Correlations between these dimuon modes and $b \to s\nu\bar{\nu}$ modes have also been explored in other models [25, 26].
4 Models with leptoquarks

In this section we consider models that can increase $R_{K^{(*)}}^\nu$ by producing final states with neutrino pairs of different lepton flavour, but with only the three SM neutrinos. The starting point is then scalar $S$ and vector $V$ leptoquarks with couplings to SM fermions which include a left-handed neutrino $\nu_L$ of any flavour. They are [27, 28],

\[
\begin{align*}
L_S &= \lambda_{LS} \bar{q}_R^i \tau_2 \ell L \tilde{S}_0 \hat{S}_0^0 + \lambda_{L\tilde{S}_1/2} \bar{q}_R^i \ell L \tilde{S}_1^{1/2} + \lambda_{LS_1} \bar{q}_R^i \tau_2 \bar{\tau} \cdot \hat{S}_1^0 \ell L + \text{ h. c.}, \\
L_V &= \lambda_{LV_1/2} \bar{q}_R^i \gamma_\mu \ell L V_{1/2}^{1\mu} + \lambda_{LV_2} \bar{q}_R^i \ell L \gamma_\mu \bar{\tau} \cdot \hat{V}_1^{1\mu} \ell L + \text{ h. c.},
\end{align*}
\]

where the leptoquark fields and their transformation properties under the SM group are given by

\[
\begin{align*}
S_0^{1/3} &= (3, 1, 1/3), \\
\tilde{S}_0^{1/2} &= (\tilde{S}_0^{1/2}, \tilde{S}_0^{2/3}) : (3, 2, 1/6), \\
\bar{\tau} \cdot \hat{S}_1^{1/3} &= \left( S_1^{1/3}, \sqrt{2} S_1^{1/3} \right) : (3, 3, 1/3), \\
V_1^{1/2} &= (V_1^{1/2}, V_1^{4/3}) : (3, 2, 5/6), \\
\bar{\tau} \cdot \hat{V}_1^{1/3} &= \left( V_1^{1/3}, \sqrt{2} V_1^{1/3} \right) : (3, 3, 2/3).
\end{align*}
\]

Exchange of these particles at tree-level, assuming leptoquark multiplets that are degenerate in mass, generates the following effective Lagrangian

\[
\begin{align*}
L_{\text{eff}} &= \frac{\lambda_{ij}^{V} \lambda_{kl}^{L}}{2m_0^2} \left( \bar{d}_{i\ell k} \gamma_\mu d_{i\ell l} \bar{\nu}_i \gamma_\mu \nu_{L_j} + \bar{u}_{i\ell k} \gamma_\mu u_{i\ell l} \bar{e}_i \gamma_\mu e_{L_j} - \bar{u}_{L_k} \gamma_\mu d_{i\ell l} \bar{e}_i \gamma_\mu \nu_{L_j} - \bar{d}_{L_k} \gamma_\mu u_{i\ell l} \bar{\nu}_i \gamma_\mu e_{L_j} \right) \\
&+ \frac{\lambda_{ij}^{V} \lambda_{kl}^{L}}{2m_1^2} \left( \bar{d}_{i\ell k} \gamma_\mu d_{i\ell l} \bar{\nu}_i \gamma_\mu \nu_{L_j} + 2\bar{d}_{L_k} \gamma_\mu d_{i\ell l} \bar{e}_i \gamma_\mu e_{L_j} + 2\bar{u}_{L_k} \gamma_\mu u_{i\ell l} \bar{\nu}_i \gamma_\mu \nu_{L_j} + 2\bar{u}_{i\ell k} \gamma_\mu u_{i\ell l} \bar{e}_i \gamma_\mu e_{L_j} \right) \\
&- \frac{\lambda_{ij}^{V} \lambda_{kl}^{L}}{m_1^2} \left( \bar{d}_{i\ell k} \gamma_\mu d_{i\ell l} \bar{\nu}_i \gamma_\mu \nu_{L_j} + \bar{d}_{i\ell k} \gamma_\mu d_{i\ell l} \bar{e}_i \gamma_\mu e_{L_j} + \bar{u}_{i\ell k} \gamma_\mu u_{i\ell l} \bar{\nu}_i \gamma_\mu \nu_{L_j} + \bar{u}_{i\ell k} \gamma_\mu u_{i\ell l} \bar{e}_i \gamma_\mu e_{L_j} \right) \\
&+ \frac{\lambda_{ij}^{V} \lambda_{kl}^{L}}{m_{S_0}^2} \left( \bar{d}_{i\ell k} \gamma_\mu d_{i\ell l} \bar{\nu}_i \gamma_\mu \nu_{L_j} + \bar{d}_{i\ell k} \gamma_\mu d_{i\ell l} \bar{e}_i \gamma_\mu e_{L_j} + \bar{u}_{i\ell k} \gamma_\mu u_{i\ell l} \bar{\nu}_i \gamma_\mu \nu_{L_j} + \bar{u}_{i\ell k} \gamma_\mu u_{i\ell l} \bar{e}_i \gamma_\mu e_{L_j} \right). 
\end{align*}
\]

If the fermions in Eq. (19) are in their weak eigenstate basis, rotation to the mass eigenstate basis will introduce mixing angles. Here we will work with $\lambda_{ij}^{V}$ defined in a basis in which the down-type fermions are already mass eigenstates [29]. The $\nu$ and up-type quarks need to be further rotated by $u_{Lk} = (V_{KM}^*)_{km} u_{Lm}$, and $\nu_{Lk} = (V_{PMNS})_{km} \nu_{Lm}$ respectively. However, since the neutrino flavour is not measured, working in either their weak or mass basis yields the same results. Collecting the
Wilson coefficients for Eq. (6) gives,
\[
C_{ij}^{ij} = \frac{\pi}{\sqrt{2}\alpha G_F V_{tb} V_{ts}^*} \left( \frac{\lambda_{LS_{ij}}^2}{2m_{S_{ij}}^2} + \frac{\lambda_{LS_{ij}}^2}{2m_{S_{ij}}^2} - 2 \frac{\lambda_{LV_{ij}}^2\lambda_{LV_{ij}}^2}{m_{V_1}^2} \right),
\]
\[
C_{ij}^{ij} = C_{ij}^{ij} = -C_{ij}^{ij} = \frac{\pi}{\sqrt{2}\alpha G_F V_{tb} V_{ts}^*} \left( \frac{\lambda_{LS_{ij}}^2}{2m_{S_{ij}}^2} \lambda_{LS_{ij}}^2 + \frac{\lambda_{LS_{ij}}^2}{2m_{S_{ij}}^2} + \frac{\lambda_{LV_{ij}}^2\lambda_{LV_{ij}}^2}{m_{V_1}^2} \right),
\]

All of these leptoquarks contribute to \( R_{K^{(*)}}^{\nu} \) but their contributions are correlated with different modes [30–33]. We begin with the lepton flavour number violating case which adds incoherently to the minimal set of Wilson coefficients consistent with the leptoquark origin of Eq. (20) implies more than one non-zero Wilson coefficient at a time, either \( C_{ij}^{ij} = -C_{ij}^{ij} \neq 0, C_{ij}^{ij} = -C_{ij}^{ij} \neq 0, C_{ij}^{ij} = -C_{ij}^{ij} \neq 0 \) as per Eq. (20).

The best current experimental bounds on these modes as given in [20] are listed in Table 3 along with the constraints they impose on the Wilson coefficients taken one non-zero at a time. The

| Mode | 90% c.l. | one \( |C_{ij}^{\nu\bar{\nu}}| \neq 0 \) at a time | \( C_{ij}^{\nu\bar{\nu}} = -C_{ij}^{\nu\bar{\nu}} \) or \( C_{ij}^{\nu\bar{\nu}} = -C_{ij}^{\nu\bar{\nu}} \) |
|------|---------|---------------------------------|---------------------------------|
| \( B(B_s \to e^+\mu^-) \) | 5.4 \times 10^{-9} | 7.4 | 7.4 |
| \( B(B_s \to \mu^+\tau^-) \) | 4.2 \times 10^{-5} | 44 | 44 |
| \( B(B^+ \to K^+ e^-\mu^+) \) | 6.4 \times 10^{-9} | 0.6 | 0.4 |
| \( B(B^+ \to K^+ e^-\tau^+) \) | 1.5 \times 10^{-5} | 36 | 25 |
| \( B(B^+ \to K^+ \mu^-\tau^+) \) | 2.8 \times 10^{-5} | 49 | 35 |
| \( B(B^0 \to K^0 e^-\mu^+) \) | 9.9 \times 10^{-7} | 7.4 | 5.2 |
| \( B(B^0 \to K^0 e^-\tau^+) \) | 1.2 \times 10^{-7} | 2.6 | 1.8 |

Table 3: Current experimental upper bounds on lepton flavour changing modes and the limits they imply for the coefficients \( C_{ij}^{\nu\bar{\nu}} \) of Eq. (21) taken one non-zero at a time for the corresponding lepton flavour indices. The last column shows the upper bound on \( |C_{ij}^{\nu\bar{\nu}}| \) assuming that \( C_{ij}^{\nu\bar{\nu}} = -C_{ij}^{\nu\bar{\nu}} \neq 0, C_{ij}^{\nu\bar{\nu}} = -C_{ij}^{\nu\bar{\nu}} \neq 0, C_{ij}^{\nu\bar{\nu}} = -C_{ij}^{\nu\bar{\nu}} \neq 0 \) as per Eq. (20).
leptoquark couplings, in particular allowing $C_{ij}^{\ell\ell'}$ to differ from $C_{ij}^{\ell\ell}$ the tightest bounds that follow in this case are shown in the last column of Table 3.

To explore the connection with $R_{K}^{\nu}$ it is useful to consider Eq. (20) for each leptoquark multiplet separately. We see that $S_{0}$ only produces $C_{ij}^{L}$ and is therefore not correlated with CLFV modes. $S_{1}$ and $V_{i}$ generate $C_{ij}^{L}$ and $C_{ij}^{0,10}$ whereas $S_{1/2}$ and $V_{1/2}$ induce $C_{ij}^{R}$ and $C_{ij}^{9',10'}$ resulting in all cases in $R_{K}^{\nu} = R_{K}^{\nu}$. To study the numerical implications of these predictions we consider one lepton flavour pair at a time and present the results in Table 4. These numbers indicate that the CLFV are currently less restrictive on these leptoquark couplings than $R_{K}^{\nu}$, Eq. (4), except for the $e\mu$ modes.

| LQ | upper bound on $C_{L,R}^{ij}$ | $R_{K}^{\nu} = R_{K}^{\nu}$ |
|----|---------------------|---------------------|
| $S_{0}$ | $|C_{L}^{\mu\mu}| \lesssim 0.4$ | $|C_{L}^{\mu\tau}| \lesssim 26$ | $|C_{L}^{\mu\tau}| \lesssim 35$ | $R_{K}^{\mu} = R_{K}^{\mu}$ |
| $S_{1/2}$ | $|C_{R}^{\mu}\mu| \lesssim 0.4$ | $|C_{R}^{\mu\tau}| \lesssim 26$ | $|C_{R}^{\mu\tau}| \lesssim 35$ | $R_{K}^{\mu} = R_{K}^{\mu}$ |
| $S_{1}$ | $|C_{L}^{\mu\mu}| \lesssim 0.2$ | $|C_{L}^{\mu\tau}| \lesssim 13$ | $|C_{L}^{\mu\tau}| \lesssim 18$ | $R_{K}^{\mu} = R_{K}^{\mu}$ |
| $V_{1/2}$ | $|C_{R}^{\mu\mu}| \lesssim 0.4$ | $|C_{R}^{\mu\tau}| \lesssim 26$ | $|C_{R}^{\mu\tau}| \lesssim 35$ | $R_{K}^{\mu} = R_{K}^{\mu}$ |
| $V_{1}$ | $|C_{L}^{\mu\mu}| \lesssim 0.8$ | $|C_{L}^{\mu\tau}| \lesssim 52$ | $|C_{L}^{\mu\tau}| \lesssim 70$ | $R_{K}^{\mu} = R_{K}^{\mu}$ |

Table 4: Implications of limits on charged lepton flavour violation for $R_{K}^{\nu}$ for different leptoquarks shown separately for each lepton flavour combination.

4.1 The B anomalies

Leptoquark models have been studied extensively in the context of the B anomalies and we comment on that here. The neutral B anomalies are observed in $b \rightarrow s\mu\bar{\mu}$ modes, and Eq. (19) shows that, with the exception of $S_{0}$, these leptoquarks correlate $\bar{s}b\mu\bar{\mu}$ with $\bar{s}b\mu\bar{\mu}$ operators. In particular

$$S_{1} \implies C_{9}^{\mu\mu} = -C_{10}^{\mu\mu} = 2C_{L}^{\mu\mu}$$

$$V_{1} \implies C_{9}^{\mu\mu} = -C_{10}^{\mu\mu} = \frac{1}{2}C_{L}^{\mu\mu}$$

$$S_{1/2} \implies C_{9}^{\mu\mu} = -C_{10}^{\mu\mu} = 2C_{R}^{\mu\mu}$$

$$V_{1/2} \implies C_{9}^{\mu\mu} = -C_{10}^{\mu\mu} = 2C_{R}^{\mu\mu}$$

(22)

Extensive fits to data from modes induced by $b \rightarrow s\mu\bar{\mu}$ indicate that $C_{9}^{\mu\mu} = -C_{10}^{\mu\mu} = -0.46$ [34] is a possible solution whereas $C_{9}^{\mu\mu} = -C_{10}^{\mu\mu}$ is not [36]. This implies that

$$R_{K}^{\nu} = \begin{cases} 
1.02 & \text{for } S_{1} \\
1.1 & \text{for } V_{1} 
\end{cases}$$

(23)

and $R_{K}^{\nu} = 1$ for $S_{1/2}$, $V_{1/2}$.

In a similar manner $S_{0}$ correlates $\bar{s}b\bar{\tau}\nu$ with $\bar{c}b\bar{\tau}\nu$ and therefore relates $R_{K}^{\mu}$ to the so-called charged B-anomalies, $R(D^{(*)})$. We use the latest experimental and theoretical averages from [37].

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4The precise number varies depending on the fit, a more recent one gives -0.41 instead [35].
(adding errors in quadrature and using their arithmetic average of theoretical results) in terms of the ratios
\[ r_D = \frac{R(D)}{R(D)_{SM}} = 1.14 \pm 0.10, \quad r_{D^*} = \frac{R(D)}{R(D^*)_{SM}} = 1.14 \pm 0.06 \] (24)

Depending on the neutrino lepton flavour, the operator will interfere or not with the SM and both cases were considered in [38, 39]. The results are
\[ r_{D(*)} = |\Delta^{3,2}|^2 + |\Delta^{3,2}_j|^2 + |1 + \Delta^{3,3}|^2 \]
\[ \Delta^{3,j} = -\frac{\sqrt{2}}{4G_F V_{cb}} \sum_i V_{ci} \frac{\lambda_{LS0}^3 \lambda_{LS0}^{3j}}{2m_{S0}^2} \] (25)
The correlations simplify at leading order in CKM angles, where the term with \( V_{cs} \) dominates resulting in,
\[ C_{L}^{3j} = -\frac{2\pi}{\alpha} \Delta_j^{3,2} = \frac{\pi}{\sqrt{2\alpha G_F V_{cb}}} \frac{\lambda_{LS0}^{3j} \lambda_{LS0}^3}{2m_{S0}^2} \] (26)
Taking one non-zero parameter at a time for this case, and assuming that the leptoquark contribution results in the central value of Eq. (24) requires \( C_{L}^{33} \sim 54 \) or \( C_{L}^{31}, C_{L}^{32} \sim 300 \), both much larger than allowed by \( R_{\nu K}(\ast) \) as quantified in Table 2. Equivalently, the most favourable scenario from Table 2, \( C_{L}^{33} \sim 25 \) would result in
\[ r_{D(*)} \sim 1.06 \] (27)
More complex leptoquark scenarios have been invoked in the study of the B anomalies where it is possible to avoid a conflict with \( R_{\nu K}(\ast) \) [40, 41].

5 Summary

We have studied the modes \( B \to K^{(*)}\nu \bar{\nu} \) in the context of non-standard neutrino interactions. We first considered a model with an additional light neutrino that couples to a non-universal \( Z' \) and found that it can result in \( R_{\nu K}(\ast) \) close to two. The same model can also enhance \( B_s \to \tau^+\tau^- \) by up to a factor six over the SM within the parameter range allowed by \( B_s \) mixing and non-production of the \( Z' \) at LHC.

Next we considered augmenting \( R_{K^{(*)}}^{\nu} \) through neutrino lepton flavour violating modes. We parameterised this possibility through scalar and vector leptoquark exchange. This type of model correlates \( R_{K^{(*)}}^{\nu} \) with CLFV modes \( B_s \to \ell\ell' \) and \( B \to K^{(*)}\ell\ell' \) and we found that the former is currently more restrictive than \( e\tau \) and \( \mu\tau \) CLFV modes.

Finally we briefly commented on the correlation with the B anomalies. In this case we saw that global fits to \( b \to s\mu\mu \) modes constrain \( C_{L}^{\mu\mu} \) for \( S_1 \) and \( V_1 \) leptoquarks so that, by itself, it cannot add more than 10% to \( R_{K^{(*)}}^{\nu} \). Similarly, current measurements of \( R_{K^{(*)}}^{\nu} \) constrain the parameters of \( S_0 \) couplings so that \( R(D^{(*)}) \) can be at most 1.06.

Acknowledgments

This work was supported in part by the Australian Government through the Australian Research Council. XGH was supported in part by Key Laboratory for Particle Physics, Astrophysics and
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