Field-induced 3- and 2-dimensional freezing in a quantum spin liquid.

Y. Chen,† Z. Honda,‡ A. Zheludev,‡ C. Broholm,†,‡, K. Katsumata,‡ and S. M. Shapiro‡.

† Department of Physics and Astronomy, Johns Hopkins University, Baltimore, MD 21218, USA.
‡ RIKEN (The Institute of Physical and Chemical Research), Wako, Saitama 351-0198, Japan.
‡ Physics Department, Brookhaven National Laboratory, Upton, NY 11973-5000, USA.

Field-induced commensurate transverse magnetic ordering is observed in the Haldane-gap compound Ni(C$_5$D$_4$N$_2$)$_2$N$_3$(PF$_6$) by means of neutron diffraction. Depending on the direction of applied field, the high-field phase is shown to be either a 3-dimensional ordered Néel state or a short-range ordered state with dominant 2-dimensional spin correlations. The structure of the high-field phase is determined, and properties of the observed quantum phase transition are discussed.

The one-dimensional (1D) integer-spin Heisenberg antiferromagnet (AF) demonstrates unique quantum-mechanical behavior that is inconsistent with the conventional semi-classical model of magnetism. Due to zero-point quantum spin fluctuations, that destroy long-range order (LRO) even at $T = 0$, the ground state is a spin-singlet with short-range (exponentially decaying) spatial spin correlations, and is often referred to as a “quantum spin liquid”. The main feature of the magnetic excitations is the so-called Haldane energy gap $\Delta$. Quite remarkable is the behavior of 1D and quasi-1D integer-spin AFs in applied magnetic fields. The effect of the field is to suppress zero-point fluctuations, and restore a gapless spectrum. The result is a quantum phase transition at a certain critical field $H_c$, to a Néel-like state with long-range order that may be characterized as a “spin solid”. Thus, in an unusual twist, a uniform $\Delta$ induces staggered magnetization in quantum-disordered spin chains.

The magnetization process of a $S = 1$ 1D AF is now rather well understood theoretically $^3$ $^4$. For the isotropic case the problem was shown to be equivalent to Bose condensation in one dimension $^8$. Many theoretical results were confirmed in experimental studies of real quasi-1D compounds. For a long time however, the actual phase transition remained inaccessible for experimental investigation. In some materials (such as Y$_2$BaNiO$_5$, for example $^3$ $^4$), the value of $H_c$ is prohibitively high. In other compounds (e.g., NENP $^3$), the transition does not occur due to certain structural features, and is instead replaced by a broad cross-over phenomenon $^1$ $^4$. A true phase transition at $H_c$ has been experimentally observed only recently, in the quasi-1D $S = 1$ AF materials NDMAP $^4$ $^9$ and NDMAZ $^7$. Specific heat, magnetization and ESR studies have provided a comprehensive picture of the $H - T$ phase diagram $^4$ $^9$, a refined version of which is shown in Fig. 1. Nevertheless, to date, the nature of the high-field phase remained undetermined, and no direct evidence of staggered LRO has been obtained. In particular, preliminary neutron scattering experiments of Ref. $^8$ failed to detect any magnetic Bragg peaks above $H_c$. In the present work we report a high-field magnetic neutron diffraction study of NDMAP. We find that the high-field phase is characterized by a commensurate ordering of spin components perpendicular to the field direction. Surprisingly, the nature of the high-field phase is extremely sensitive to direction of applied field: depending on the experimental geometry, for $H > H_c$ the system either develops true 3D LRO, or static quasi-2-dimensional short range order.

![Field-temperature phase diagram of NDMAP.](https://example.com/ndmap_graph.png)

**FIG. 1.** Field-temperature phase diagram of NDMAP. A quantum-disordered (QD) phase with a gap is seen in low fields. The high-field phase is characterized by static long-range order (3D-LRO) or quasi-2D short-range order (2D-SRO). Open symbols from specific heat measurements, as in Ref. $^4$. Solid symbols from neutron diffraction.

Magnetic properties of NDMAP are due to $S = 1$ Ni$^{2+}$ spins that are assembled into AF chains running along the $c$ axis of the orthorhombic $Pnma$ crystal structure ($a = 18.05$ Å, $b = 8.71$ Å, $c = 6.14$ Å). Haldane gap excitations and magnetic interactions in this system were previously investigated by means of zero-field inelastic neutron scattering $^3$. In-chain AF interactions are dominant and correspond to a Heisenberg exchange constant
$J = 2.8$ meV. The triplet of Haldane excitations is split by easy-(a, b) plane anisotropy, and the gap energies are $\Delta_\perp = 1.9$ meV and $\Delta_\parallel \approx 0.47$ meV, for excitations polarized along and perpendicular to the anisotropy axis, respectively. The coupling between chains along the $b$ axis is rather weak, $J_y = 2 \cdot 10^{-3}$ meV, and barely detectable along $a$, $|J_x| < 5 \cdot 10^{-4}$ meV.

The present neutron diffraction experiments were performed at the SPINS cold-neutron 3-axis spectrometer at the National Institute of Standards and Technology Center for Neutron Research. A 150 mg sample was mounted with either the (0, $k$, $l$), ($h$, $h$, $l$) or ($h$, $k$, $k$) reciprocal-space planes in the scattering plane of the spectrometer. Most measurements were performed using ($^{58}$Niguide) $- 80' - 80' - 240'$ or ($^{58}$Niguide) $- 40' - 40' - 240'$ collimations, with a Be-filter positioned in front of the sample to eliminate higher-order beam contamination, and $E_f = 5$ meV or $E_f = 3$ meV fixed final energy neutrons. Sample environment in all cases was a pumped He-3 cryogenic insert and a 9 T or 7 T superconducting magnet, with the field applied vertically, i.e., perpendicular to the scattering plane.

![Graphs showing magnetic Bragg scattering in NDMAP](image)

**FIG. 2.** Typical elastic scans (open symbols), showing magnetic Bragg scattering in NDMAP at $T = 0.25$ K, $H = 9$ T $[0, -1, 1]$ (a,b) and $T = 0.375$ K, $H = 7$ T $[1, 0, 0]$ (c-e). The solid lines and symbols represent experimental wave vector resolution. Dashed lines as described in the text.

True 3D commensurate AF LRO was observed in a crystal mounted in the ($h$, $k$, $k$) zone, with the magnetic field applied along the $[0, -1, 1]$ direction. At $T = 0.25$ K the transition occurs at $H_c = 4.8$ T, and manifests itself in the appearance of new Bragg reflections, at ($h$, $2n + 1$, $m + 1$) ($h$-even integer; $n$-integer) reciprocal-space positions. No additional peaks were found at odd-$h$ reciprocal-space points. Figures 2(a) and 2(b) show typical scans through the (0, 0.5, 0.5) peak measured at $H = 9$ T applied field (symbols). The peaks are resolution-limited along both the (1, 0, 0) and (0, 1, 1) directions. Wave vector resolution (Fig. 2 solid lines) was either calculated or directly measured in our experiment. The multiple peaks seen in the rocking scan in Fig. 2(a) result from sample mosaic. Geometrical limitations imposed by the use of the bulky 9 T magnet prevented us from measuring the width of the peak in the vertical $[0, 1, -1]$ direction. However, the substantial values of the $b$ and $c$ axis coupling constants, as well as the resolution-limited peak widths along $a$, where magnetic interactions are weakest, suggest that the observed reflections indeed have zero intrinsic width in all three directions, and represent true 3D long-range order. This is also consistent with the measured absolute values of magnetic intensities, as discussed below.

To determine the spin arrangement in the high-field ordered phase, intensities of (0,0,5,0.5) (2,0.5,0.5) (4,0.5,0.5) (2,1.5,1.5) and (4,1.5,1.5) magnetic Bragg peaks in the ($h$, $k$, $k$) plane were measured in rocking scans at $T = 0.25$ and $H = 9$ T. Magnetic intensities were brought to the absolute scale by comparing them to nuclear intensities measured in the same configuration at room temperature (the exact low-temperature structure of NDMAP has not been determined to date). The data were analyzed using a simple collinear model. As the energy scale of in-chain AF interactions is much larger than that defined by the applied magnetic field ($J \gg g\mu_B H$), the ordered moment is expected to be perpendicular to the field direction. The ($a$, $b$)-easy plane anisotropy ensures that it should also be perpendicular to the chain-axis. For a field applied along $[0, -1, 1]$ the spins are thus along (1, 0, 0), as confirmed by recent ESR experiments [13]. This allows only one magnetic structure that would be consistent with the observed selection rule for magnetic Bragg peaks. Varying only the sublattice magnetization $m$, we obtained a good fit to the data ($\chi^2 = 1.6$) with $m = 1.13(5) \mu_B$. This value is to be compared to $m = 2 \mu_B$ for a fully saturated $S = 1$ antiferromagnet.

The measured field dependence of the (0,0.5,0.5) magnetic Bragg peak intensity at $T = 0.25$ K is shown in Fig. 3(a) (solid circles), and is consistent with our expectations for an easy-plane Haldane system at $T \to 0$. The data were analyzed assuming a power-law field dependencies of the Bragg intensity and ordered moment: $I(H) \propto (H - H_c)^\beta$. A fit of this equation to the data (Fig. 3(a), solid line) yields $H_c = 4.81(1)$ T and $\beta = 0.207(5)$. Since the transition at $H_c$ occurs through a softening of the lowest-energy Haldane gap excitation $\varepsilon$, $H_c$ is determined by the Haldane gap energies. It is straightforward to show that for a magnetic field applied at angle $\alpha$ to the anisotropy axis $c$ is given by:
\( \mu_B H_c = \Delta_z \Delta_l / \sqrt{g_z^2 \Delta_z^2 \cos^2 \alpha + g_l^2 \Delta_l \sin^2 \alpha} \). In this formula, \( g_z = 2.1 \) and \( g_l = 2.17 \) are components of the Ni\(^{2+} \) gyromagnetic tensor along and perpendicular to the anisotropy axis, respectively \[13\]. For \( H||[0, -1, 1] \), \( \alpha = 54.8^\circ \) and this equation gives \( H_c = 5.4 \) T, in reasonable agreement with our experimental result. In the case of broken axial symmetry, for a single chain in a magnetic field at \( T = 0 \), the transition is expected to fall in the 2D Ising universality class, with magnetic field taking the role of effective temperature \[3\]. The order parameter critical exponent for this model is \( \beta = 0.125 \). For NDMAP however, inter-chain interactions along the \( b \) axis can be considered substantial, and 3D Ising behavior, with \( \beta \approx 0.31 \), may be expected. The measured critical exponent falls in between these two values and is characteristic of a dimensional crossover regime.

The temperature dependence of the \( (0.5, 0.5, 0.5) \) magnetic Bragg intensity measured at \( H = 9 \) T is shown in Fig. 3(b) (solid circles). An analysis of the data assuming a power law (solid line) yields \( T_c = 1.7(1) \) K. The critical exponent was found to be indistinguishable from the mean field value \( \beta = 0.5 \), indicating that at \( T \approx 1.5 \) K the true critical region is too narrow to be investigated in the present experiment.

A striking result of this work is that for a magnetic field applied perpendicular to the chain-axis, the high-field phase is no longer a 3D-ordered state, but has predominantly 2-dimensional spin correlations. For the crystal mounted in the \( (0, k, l) \) zone, at \( T = 0.38 \) K and \( H||[1, 0, 0] \), magnetic scattering was detected at \( (0, 2m+1, 2n+1) \) \((n, m\text{-integer})\) reciprocal-space points above \( H_c = 6 \) T. \( k \)- and \( l \)-scans through the \((0, -0.5, 1.5)\) reflections contain well-defined peaks, as shown in Figs. 2(d) and 2(e) (symbols). Scans perpendicular to the \((b, c)\) plane however, reveal that the scattering is concentrated in Bragg rods along the \((1, 0, 0)\) direction, rather than Bragg peaks, as in the \( H||[0, -1, 1] \) case. Figure 2(c) shows an \( h \)-scan through the \((0, -0.5, 1.5)\) position, that could only be performed in a limited range due to geometrical constraints. The measured intensity is independent of \( h \). Note that the measured vertical resolution (Fig. 2(c), solid line), is sufficient to observe a well-defined Bragg peak. The rod nature of the magnetic scattering implies that static spin correlations along the \( a \) axis are absent in the system, and that magnetic ordering occurs only within individual \((b, c)\) planes. This is consistent with the \( a \)-axis being the direction of weakest magnetic interactions. Similar behavior was observed for a magnetic field applied along the \([1, -1, 0]\) direction, where magnetic scattering was observed above \( H_c = 5.8(2) \) T, at \( T = 0.25 \) K, and also found to be concentrated in Bragg rods parallel to the \( a \) axis. In this geometry, the rods cross the \((h, h, l)\) scattering plane at \((2n+1, 2m+1, 2l+1)\) \((n, m\text{-integer})\) positions. The measured field- and temperature dependencies of magnetic diffraction intensity at \( q = (0.5, 0.5, 0.5) \) are shown in Fig. 3(open symbols). A well-defined transition is observed in agreement with specific heat measurements (Fig. 4, diamonds).

The apparent Bragg rod intensity for \( H||[1, 0, 0] \) and \( H||[1, -1, 0] \) is substantially smaller than the intensity of magnetic Bragg peaks seen for \( H||[0, 1, -1] \) (Fig. 3). This is due to that fact that in the 2D-ordered phase magnetic intensity is spread out through an entire Bril- loun zone along the \( a \) axis, rather than being concentrated at \( h = 0 \). However, in the 2D phase, within each \((b, c)\) plane, the actual ordered moment is similar to that in the 3D ordered state. The 2D ordered moment for \( H||[1, 0, 0] \) and \( H||[1, -1, 0] \) was deduced from an analysis of measured Bragg rod intensities, paying close attention to resolution effects associated with vertical angular acceptance of the spectrometer. A series of rocking scans across the rods were collected for \( H = 7 \) T\([1, 0, 0], T = 0.38 \) K and \( H = 9 \) T\([1, -1, 0], T = 0.25 \) K, respectively. The data were normalized by the measured nuclear Bragg peak intensities. The 2-dimensional spin structure was assumed to be collinear, with spins perpendicular to both the chain axis and \( H \) in each case. Nearest-neighbor spins were assumed to be aligned antiparallel along both the \( b \) and \( c \) axes. This simple model was found to agree very well with the available data. The ordered moment was determined to be \( m = 0.66(4) \) \( \mu_B \) and \( m = 1.2(1) \) \( \mu_B \) for \( H = 7 \) T\([1, 0, 0], T = 0.38 \) K and \( H = 9 \) T\([1, -1, 0], T = 0.25 \) K, respectively. The latter value agrees very well with the 3D ordered moment measured at \( H||[0, -1, 1] \) at the same value of field and temperature.

A careful analysis of the scans across the Bragg rods for \( H||[1, 0, 0] \) reveals a finite correlation length even
within the quasi-2D ordered \((b,c)\) planes. Along the direction of strongest coupling, i.e., along the chains, the magnetic peaks are resolution-limited. This can be seen from Fig. 2(e), where the solid line represents experimental \(l\)-resolution. However, along the \(b\) axis, where magnetic interactions are weaker, the scattering rods are visibly broader than experimental resolution (Fig. 2(d), solid line). Fitting the \(k\)-scan to a Lorentzian profile convoluted with the resolution function (Fig. 2(d), dashed line) allows us to extract the intrinsic width of the rod, that corresponds to a real-space correlation length \(\xi_b = 100(20) \text{ Å} \approx 10b\). The high-field phase in this case is thus characterized as a short-range ordered state with almost-perfect correlations along the chains and short-range correlations in the transverse direction.

This type of “spin freezing” in NDMAP is very similar to that recently found in the \(S = 1/2\) quasi-1D AF SrCuO\(_2\). For a 2-dimensional Heisenberg system, the ground state is disordered and gapless for half-integer and odd-integer spin due to “instanton” topological defects. If magnetic coupling along the third direction is sufficiently weak, as it is in NDMAP, it will not be able to restore LRO in the system due to pinning of such defects within each plane. The similar behavior of two very different systems, NDMAP and SrCuO\(_2\), suggests that anisotropic spin freezing may be a rather general feature of quasi-low dimensional antiferromagnets.

What makes the case of NDMAP special is that both true 3D LRO and spin freezing can occur in the same sample, depending on the direction of applied field. This may result from the anisotropic nature of magnetic interactions along the \(a\) axis, that, due to the very long lattice spacing, are expected to be largely dipolar. The strength and even sign of such coupling depends on the orientation of the ordered moment, which, in turn, is defined by the direction of applied field. Changing the field orientation for NDMAP is thus a way to tune those magnetic interactions ultimately responsible for 3D LRO or spin-freezing behavior.

For the basic physics of Haldane spin chains, a very important experimental result is the observation of a commensurate ordering of transverse (relative to \(H\)) spin components above \(H_c\). Such staggered LRO in a quasi-1D material is a direct consequence of a divergence in the transverse spin correlation function for an isolated chain in an external field at \(q = \pi\). Theory predicts that above \(H_c\) a divergence also exists in the longitudinal correlator, but at a field-dependent incommensurate wave vector \(\xi_\parallel\). No incommensurate magnetic peaks were found in our experiments on NDMAP. Of course, a divergent susceptibility in a single chain does not guarantee the appearance of LRO at the same wave vector in a 3D material. It is also conceivable that incommensurate elastic scattering in NDMAP appears at a different wave vector transfer perpendicular to the chain axis, due to anisotropy of inter-chain interactions, and thus escapes detection in the present study. Further work will be required to fully resolve this issue.

In summary, our results provide direct experimental evidence of field-induced commensurate AF order in a Haldane-gap system, and are in good agreement with theoretical expectations. Specifics of inter-chain interactions in NDMAP result in a rich phase diagram with the high-field phase being either a true 3D ordered state or a highly anisotropic short-range ordered state.

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