Abstract

In two-step breaking of a class of grand unified theories including $SO(10)$, we prove a theorem showing that the scale ($M_I$) where the Pati–Salam gauge symmetry with parity breaks down to the standard gauge group, has vanishing corrections due to all sources emerging from higher scales ($\mu > M_I$) such as the one-loop and all higher-loop effects, the GUT-threshold, gravitational smearing, and string threshold effects. Implications of such a scale for the unification of gauge couplings with small Majorana neutrino masses are discussed. In string inspired $SO(10)$, we show that $M_I \simeq 5 \times 10^{12}$ GeV, needed for neutrino masses, with the GUT scale $M_U \simeq M_{\text{str}}$ can be realized provided certain particle states in the predicted spectrum are light.
I. INTRODUCTION

Grand unified theories based upon SUSY $SU(5), SO(10)$, nonSUSY $SO(10)$ with intermediate symmetries, and those inspired by superstrings have been the subject of considerable interest over recent years. In order to solve the strong CP problem through Peccei–Quinn mechanism and achieve small neutrino masses [1] necessary to understand the solar neutrino flux [2] and/or the dark matter of the universe, an intermediate scale seems to be essential [3]. Such a scale might correspond to the spontaneous breaking of gauged B–L contained in intermediate gauge symmetries like $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C(\equiv G_{2213})$ and $SU(2)_L \times SU(2)_R \times SU(4)_C(\equiv G_{224})$ with [3-6] or without [7] parity, or even others like $SU(2)_L \times U(1)_{I_3R} \times SU(4)_C$ and $SU(2)_L \times U(1)_{I_3R} \times U(1)_{B-L} \times SU(3)_C$. But it is well known that the predictions of a grand unified theory are more [8] or less [6,9] uncertain predominantly due to threshold [10] and gravitational smearing effects [11,12] originating from higher dimensional operators. The uncertainty in the intermediate scale prediction naturally leads to theoretical uncertainties in the neutrino mass predictions through seesaw mechanism. Therefore, an intermediate scale, stable against theoretical uncertainties, would be most welcome from the point of more accurate predictions on neutrino masses.

Another problem in SUSY GUTs having supergrand desert is the requirement of $\alpha_s(M_Z) \geq 0.12$ to achieve unification at $M_U \simeq 2 \times 10^{16}$ GeV. Even though the problem is alleviated by unknown GUT threshold and gravitational corrections [13], realization of a natural grand unification scale $M_U \simeq M_{str} \simeq 5.6 \times g_{str} \times 10^{17}$ GeV requires the presence of some lighter string states which could be the extra gauge bosons or Higgs scalars of a unifying symmetry, exotic vector–like quarks and leptons with nonconventional hypercharge assignments [14-16], or a $SU(3)_C$–octet and weak $SU(2)$–triplet in the adjoint representation of the standard gauge group [17]. But, in the absence of an intermediate symmetry, the neutrino mass predictions may fall short of the solar flux requirements by 2-3 orders. Assuming boundary conditions at the string scale to be different from a GUT–boundary condition, attempts have been made to bring down the values of intermediate scales relevant for larger
neutrino masses [18].

The presence of a $G_{224P}$ intermediate gauge symmetry, having only two couplings for $\mu > M_I$, would always guarantee gauge unification, and a demonstration of $M_I \simeq 10^{12} - 10^{14}$ GeV with $M_U \simeq M_{str}$ in SUSY inspired $SO(10)$, would solve at least two of the major problems: the string scale unification with $\alpha_s(M_Z) \simeq 0.11$ and neutrino masses needed for solar neutrino flux.

It has been shown recently that in all GUTs where $G_{224P}$ breaks spontaneously at the highest intermediate scale, the $\sin^2 \theta_W(M_Z)$ prediction is unaffected by GUT–threshold and multiloop (two–loop and higher) radiative corrections emerging from higher mass scales [6]. As a single intermediate symmetry is more desirable from minimality consideration, we confine to the single $G_{224P}$ symmetry in two–step breakings of all possible GUTs including $SO(10)$ and prove a theorem showing that all higher–scale corrections to the intermediate scale ($M_I$) prediction vanish. In SUSY $SO(10)$ inspired by superstrings [19], we find that $M_I \simeq 10^{12} - 10^{14}$ GeV is possible with $M_U \simeq M_{str}$ provided certain states in the predicted spectrum are light.

II. THEOREM ON VANISHING CORRECTIONS ON THE INTERMEDIATE SCALE

We now state the following theorem and provide its proof,

**Theorem:** In all two–step breakings of grand unified theories, the mass scale ($M_I$) corresponding to the spontaneous breaking of the intermediate gauge symmetry $SU(2)_L \times SU(2)_R \times SU(4)_C \times P(g_{2L} = g_{2R})$, has vanishing contributions due to every correction term emerging from higher scales ($\mu > M_I$).

To prove the theorem we consider the two–loop breaking pattern in SUSY or nonSUSY GUTs,

$$\text{GUT} \xrightarrow{M_U} G_{224P} \xrightarrow{M_I} G_{213} \xrightarrow{M_2} U(1)_{em} \times SU(3)_C$$
which may or may not originate from superstrings. Following the standard notations, we use the following renormalization group equations (RGEs) for the gauge couplings $\alpha_i(\mu) = g_i^2(\mu)/4\pi$,

For $M_Z \leq \mu \leq M_I$

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_i(M_I)} + \frac{a_i}{2\pi} \ln \frac{M_I}{M_Z} + \theta_i - \Delta_i,$$

$$i = Y, 2L, 3C \quad (2.1)$$

For $M_I \leq \mu \leq M_U$

$$\frac{1}{\alpha_i(M_I)} = \frac{1}{\alpha_i(M_U)} + \frac{a_i'}{2\pi} \ln \frac{M_U}{M_I} + \theta_i' - \Delta_i',$$

$$i = 2L, 2R, 4C \quad (2.2)$$

where $\Delta_i$ includes threshold effects at $\mu = M_Z(\Delta_i^Z)$ due to the top–quark and Yukawa couplings and superpartners in SUSY theories. It also includes threshold effects ($\Delta_i^I$) due to heavy particles near the intermediate scale,

$$\Delta_i = \Delta_i^Z + \Delta_i^I, \quad i = Y, 2L, 3C \quad (2.3)$$

The second (third) term in the r.h.s. of (2.1)–(2.2) is the usual one–loop (multiloop) contribution.

The GUT threshold ($\Delta_i^U$), gravitational corrections ($\Delta_i^{NRO}$), or the string threshold effects ($\Delta_i^{str}$) when the model is based upon string inspired $SO(10)$ [20], are contained in $\Delta_i'$,

$$\Delta_i' = \Delta_i^U + \Delta_i^{NRO} + \Delta_i^{str}, \quad i = 2L, 2R, 4C \quad (2.4)$$

In nonSUSY and SUSY GUTs, the $\Delta_i^{NRO}$ may emerge from higher dimensional operators scaled by the Planck mass [11] leading to a nonrenormalizable Lagrangian

$$\mathcal{L}_{NRO} =$$
where \( M_{pl} \) = Planck mass, and \( \phi \) = Higgs field which is responsible for breaking the GUT symmetry to \( G_{224P} \). For example, in \( SO(10) \), \( \phi = 54 \). These operators lead to the modifications of the GUT-scale boundary conditions on gauge couplings,

\[
\alpha_{2L}(M_U)(1 + \varepsilon_{2L}) = \alpha_{2R}(M_U)(1 + \varepsilon_{2R}) = \alpha_{4C}(M_U)(1 + \varepsilon_{4C}) = \alpha_G \tag{2.6}
\]

which imply

\[
\Delta_i^{NRO} = -\frac{\varepsilon_i}{\alpha_G}, \quad i = 2L, 2R, 4C \tag{2.7}
\]

where \( \alpha_G \) = GUT coupling and \( \varepsilon_i \) are known functions of the parameters \( \eta^{(i)} \), the vacuum expectation value of \( \phi, M_U \), and \( M_{pl} \).

Using suitable combinations of gauge couplings and eqs.(2.1)–(2.2), we obtain the following analytic formulas,

\[
\ln \frac{M_U}{M_Z} = \frac{(L_S B_I - L_\theta A_I)}{D} + \frac{(J_\theta B_I - K_\theta A_I)}{D} + \frac{(K_\Delta A_I - J_\Delta B_I)}{D} \tag{2.8}
\]

\[
\ln \frac{M_I}{M_Z} = \frac{(L_\theta A_U - L_S B_U)}{D} + \frac{(K_\theta A_U - J_\theta B_U)}{D} + \frac{(J_\Delta B_U - K_\Delta A_U)}{D} \tag{2.9}
\]

\[
D = A_U B_I - A_I B_U
\]

\[
L_S = \frac{16\pi}{3\alpha(M_Z)} \left[ \frac{\alpha(M_Z)}{\alpha_S(M_Z)} - \frac{3}{8} \right]
\]
\[ L_\theta = \frac{16\pi}{3\alpha(M_Z)} \left[ \sin^2 \theta_W(M_Z) - \frac{3}{8} \right] \]  

(2.10)

\[ A_U = 2a'_4 C - a'_2 L - a'_2 R \]

\[ B_U = \frac{5}{3} a'_{2L} - a'_{2R} - \frac{2}{3} a'_4 C \]

\[ A_I = \frac{8}{3} a_{3C} - a_{2L} - \frac{5}{3} a_Y - A_U \]

\[ B_I = \frac{5}{3} (a_{2L} - a_Y) - B_U \]

\[ J_\theta = 2\pi \left[ \theta_{2L} + \frac{5}{3} \theta_Y - \frac{8}{3} \theta_{3C} + \theta'_{2L} + \theta'_{2R} - 2\theta'_{4C} \right] \]

\[ K_\theta = 2\pi \left[ \frac{5}{3} (\theta_Y - \theta_{2L}) + \theta'_{2R} + \frac{2}{3} \theta'_{4C} - \frac{5}{3} \theta'_{2L} \right] \]

\[ J_\Delta = 2\pi \left[ \triangle_{2L} + \frac{5}{3} \triangle_Y - \frac{8}{3} \triangle_{3C} + \triangle'_{2L} + \triangle'_{2R} - 2\triangle'_{4C} \right] \]

\[ K_\Delta = 2\pi \left[ \frac{5}{3} (\triangle_Y - \triangle_{2L}) + \triangle'_{2L} + \frac{2}{3} \triangle'_{4C} - \frac{5}{3} \triangle'_{2L} \right] \]  

(2.11)

The first, second, and the third terms in the r.h.s. of (2.8)–(2.9) represent the one–loop, the multiloop, and the threshold effects, respectively. Each of these contain contributions originating from lower scales \( \mu = M_Z - M_I \), and higher scales \( \mu = M_I - M_U \). We now examine the contributions to \( \ln \frac{M_I}{M_Z} \) term by term. In the presence of the \( G_{224P} \) gauge symmetry for \( \mu \geq M_I \), \( \alpha_{2L}(\mu) = \alpha_{2R}(\mu) \). Then eq.(2.2) gives

\[ a'_{2L} = a'_{2R} \]

\[ \theta'_{2L} = \theta'_{2R} \]

\[ \triangle'_{2L} = \triangle'_{2R} \]  

(2.12)
where the $G_{224P}$ symmetry implies

$$
\Delta_{2L}^U = \Delta_{2R}^U
$$

$$
\Delta_{2L}^{NRO} = \Delta_{2R}^{NRO}, \quad \Delta_{2L}^{str} = \Delta_{2R}^{str}
$$

(2.13)

The restoration of left–right discrete symmetry in the presence of $SU(4)_C$ in $G_{224P}$ plays a crucial role in giving rise to vanishing contribution due to every type of higher scale corrections.

A. One–loop contributions

Using (2.12) we find that $B_U$ and $A_U$ are proportional to each other,

$$
B_U = \frac{2}{3} (a_{2L}' - a_{4C}') = -\frac{1}{3} A_U
$$

(2.14)

$$
D = \frac{5}{3} (a_{2L} - a_Y) A_U - \left( \frac{8}{3} a_{3C} - a_{2L} - \frac{5}{3} a_Y \right) B_U
$$

$$
= \frac{4A_U}{9} (3a_{2L} + 2a_{3C} - 5a_Y)
$$

(2.15)

Then $B_U$ or $A_U$ cancel out from the denominator and the numerator of the one-loop term in (2.9) leading to

$$
\left( \ln \frac{M_I}{M_Z} \right)_{one\,loop} = \frac{12\pi}{\alpha d} \left( \sin^2 \theta_W - \frac{1}{2} + \frac{\alpha}{3 \alpha_s} \right),
$$

$$
d = 3a_{2L} + 2a_{3C} - 5a_Y
$$

(2.16)

The fact that $a_i' (i = 2L, 2R, 4C)$ are absent from (2.16) demonstrates that the scale $M_I$ is independent of the one–loop contribution to the gauge couplings emerging from higher scales, $\mu = M_I - M_U$. But these coefficients do not cancel out from $\ln \frac{M_I}{M_Z}$, which assumes the form,
\[ \ln \frac{M_U}{M_Z} = \frac{12\pi}{\alpha d} \left( \sin^2 \theta_W - \frac{1}{2} + \frac{\alpha}{3\alpha_S} \right) + X \] (2.17)

\[ X = \frac{6\pi}{\alpha d} \left[ a_{3C} \left( 1 - \frac{8}{3}\sin^2 \theta_W \right) + a_{2L} \left( \frac{5}{3} \frac{\alpha}{\alpha_S} - 1 + \sin^2 \theta_W \right) + \frac{5}{3} a_Y \left( \sin^2 \theta_W - \frac{\alpha}{\alpha_S} \right) \right] / (a_{4C} - a'_2L) \] (2.18)

The first term in the r.h.s. of (2.17) is the one–loop contribution in (2.16).

We also note that for any standard weak doublet \((H)\)

\[ a_{3C}^{(H)} = 0, \ 3a_{2L}^{(H)} = 5a_Y^{(H)} \]

which keeps the one–loop term in (2.16) unchanged. Thus, the scale \(M_I\) is predominantly unaffected by the presence of any number of light doublets with masses \(< M_I\), degenerate or nondegenerate.

**B. Two–loop and higher–loop effects:**

Using the second term in the r.h.s. of (2.9), (2.14) and (2.15), the coefficients \(a'_i\) and terms containing \(\theta'_i\) cancel out, leading to

\[ \left( \ln \frac{M_I}{M_Z} \right)_{\text{multiloop}} = \frac{K_\theta A_U - J_\theta B_U}{D} \]

\[ = \frac{2\pi}{d} (5\theta_Y - 3\theta_{2L} - 2\theta_{3C}) \] (2.19)

showing that all multiloop contributions to the gauge couplings originating from \(\mu = M_I - M_U\) are absent in \(\ln \frac{M_I}{M_Z}\). But these multiloop effects do not cancel out from the unification mass,

\[ \left( \ln \frac{M_U}{M_Z} \right)_{\text{multiloop}} = \left( \ln \frac{M_I}{M_Z} \right)_{\text{multiloop}} + X_\theta \] (2.20)

where the first term in the r.h.s. of (2.20) is the same as in (2.19),

\[ X_\theta = \frac{9\pi}{4d(a_{4C} - a'_{2L})} \times \left\{ \frac{5}{3} \left( \theta_{2L} + \frac{5}{3} \theta_Y - \frac{8}{3} \theta_{3C} \right) + \frac{10}{3} (\theta'_{2L} - \theta'_{4C}) \right\} (a_{2L} - a_Y) \]

\[ - \left( \frac{8}{3} a_{3C} - a_{2L} - \frac{5}{3} a_Y \right) \left\{ \frac{5}{3} (\theta_Y - \theta_{2L}) + \frac{2}{3} (\theta'_{4C} - \theta'_{2L}) \right\} \] (2.21)
C. Threshold effects

Including threshold effects at $\mu = M_Z, M_I$ and $M_U$, we separate $J_\Delta$ and $K_\Delta$ into three different parts

\[ J_\Delta = J_{\Delta}^U + J_{\Delta}^I + J_{\Delta}^Z \]

\[ K_\Delta = K_{\Delta}^U + K_{\Delta}^I + K_{\Delta}^Z \]

where

\[ J_{\Delta}^U = 2\pi \left( \frac{\Delta_{2L}^U + \Delta_{2R}^U - 2\Delta_{4C}^U}{2} \right) \]

\[ J_{\Delta}^i = 2\pi \left( \frac{\Delta_{2L}^i + \frac{5}{3} \Delta_{3Y}^i - \frac{8}{3} \Delta_{3C}^i}{3} \right), \quad i = I, Z, \]

\[ K_{\Delta}^U = 2\pi \left( \frac{\Delta_{2R}^U + \frac{2}{3} \Delta_{4C}^U - \frac{5}{3} \Delta_{2L}^U}{2} \right) \]

\[ K_{\Delta}^i = \frac{10\pi}{3} \left( \Delta_{3Y}^i - \Delta_{2L}^i \right), \quad i = I, Z \] (2.22)

Using the parity restoration constraint gives

\[ K_{\Delta}^U = \frac{4\pi}{3} \left( \Delta_{4C}^U - \Delta_{2L}^U \right) = -\frac{1}{3} J_{\Delta}^U \]

and

\[ J_{\Delta}^U B_U - K_{\Delta}^U A_U = 0 \] (2.23)

Using (2.23) in the third term in (2.9) gives

\[ -\frac{9}{4d} \left( K_{\Delta}^I + \frac{J_{\Delta}^I}{3} + K_{\Delta}^Z + \frac{J_{\Delta}^Z}{3} \right) \] (2.24)

Thus, it is clear that the would be dominant source of uncertainty due to GUT–threshold effects has vanished from $\ln \frac{M_I}{M_Z}$ which contains contributions from only lower thresholds at
\( \mu = M_Z \) and \( \mu = M_I \). But the GUT–threshold contributions do not cancel out from \( \ln \frac{M_U}{M_Z} \) which has the form

\[
\left( \ln \frac{M_U}{M_Z} \right)_{\text{threshold}} = \left( \ln \frac{M_I}{M_Z} \right)_{\text{threshold}} + X_\Delta
\]

(2.25)

where

\[
X_\Delta = 2\pi \frac{\left( \Delta_{\text{4C}}^U - \Delta_{\text{2L}}^U \right)}{\left( a_{\text{4C}}' - a_{\text{2L}}' \right)} + \frac{9}{4d} \left[ \left( K_{\Delta}^I + K_{\Delta}^Z \right) \left( \frac{8}{3} a_{\text{3C}} - a_{\text{2L}} - \frac{5}{3} a_Y \right) \right. \\
- \left. \frac{5}{3} \left( J_{\Delta}^I + J_{\Delta}^Z \right) \left( a_{\text{2L}} - a_Y \right) \right] / \left( a_{\text{4C}}' - a_{\text{2L}}' \right)
\]

(2.26)

D. Gravitational smearing and string threshold effects

In the presence of left–right discrete symmetry in \( G_{224P} \), \( \Delta_{\text{2L}}^{\text{NRO}} = \Delta_{\text{2L}}^{\text{str}} \) and \( \Delta_{\text{2R}}^{\text{str}} = \Delta_{\text{2R}}^{\text{str}} \). The analysis of Sec.(C) holds true in these cases also leading to

\[
J_{\Delta}^{\text{NRO}} B_U - K_{\Delta}^{\text{NRO}} A_U = 0
\]

\[
J_{\Delta}^{\text{str}} B_U Y - K_{\Delta}^{\text{str}} A_U = 0
\]

\[
\left( \ln \frac{M_I}{M_Z} \right)_p = 0, \ p = \text{NRO, string}
\]

\[
\left( \ln \frac{M_U}{M_Z} \right)_{\text{NRO}} = \frac{2\pi}{\alpha_G} \frac{\left( \varepsilon_{2L} - \varepsilon_{4C} \right)}{\left( a_{4C}' - a_{2L}' \right)}
\]

\[
\left( \ln \frac{M_U}{M_Z} \right)_{\text{str}} = \frac{2\pi}{\alpha_G} \frac{\left( \Delta_{\text{4C}}^{\text{str}} - \Delta_{\text{2L}}^{\text{str}} \right)}{\left( a_{4C}' - a_{2L}' \right)}
\]

(2.27)

Thus, the theorem is proved demonstrating explicitly that \( \ln \frac{M_I}{M_Z} \) does not have any modification due to corrections to the gauge coupling constants at higher scales for \( \mu > M_I \). When
the Higgs scalars, fermions or gauge bosons of the full $G_{224}$ representations are taken into
account, their contributions to $\ln \frac{M_I}{M_Z}$ vanish exactly. The origin behind all cancellations is
the $G_{224}$ symmetry and the relation between the gauge couplings,

$$\frac{1}{\alpha_Y(\mu)} = \frac{3}{5} \frac{1}{\alpha_{2L}(\mu)} + \frac{2}{5} \frac{1}{\alpha_{4C}(\mu)}, \; \mu \geq M_I$$

Since no specific particle content has been used in proving the vanishing corrections, the
theorem holds true without or with SUSY and also in superstring based models.

Another stability criterion on $M_I$ with respect to contributions from lower scale correc-
tions is that, up to one–loop level, it remains unchanged by the presence of any number of
light weak doublets having masses from $M_Z$ to $M_I$.

The other byproduct of this analysis is on the stability of $M_U$ with respect to $16_H + \overline{16}_H$
pairs. In all correction terms for $\ln(M_U/M_Z)$, the higher scale one–loop coefficients appear
in the combination $a'_4C - a'_{2L}$. We note that for any $16_H$ (or $\overline{16}_H$)

$$(a'_4C)_{16_H} = (a'_{2L})_{16_H}$$

which keeps the value of $a'_4C - a'_{2L}$ unaltered. Thus, the value of $M_U$ is almost unaffected
by the presence of any number of pairs of $16_H \oplus \overline{16}_H$ between $\mu = M_I - M_{GUT}$. This has
relevance for SUSY $SO(10)$ and string inspired models.

### III. Predictions in Nonsusy SO(10)

The stability of $M_I$ in nonSUSY $SO(10)$, under the variation of $\eta^{(1)}$ in (2.5) was demon-
strated in Ref.[21] by accurate numerical estimation. According to the present theorem
$\ln \frac{M_I}{M_Z}$ is not only independent of the 5–dimensional operator and $\eta^{(1)}$, but also of other
higher dimensional operators in (2.5) and parameters arising from the GUT scale. Simi-
larly, the vanishing GUT threshold correction to $M_I$, obtained in the accurate numerical
evaluation of Ref.[22], is a part of the present theorem. Imposing the parity restoration
criteria for $\mu \geq M_I$ [23], the minimal nonSUSY $SO(10)$ with 54, 126 and 10 representations,
$\sin^2 \theta_W = 0.2316 \pm 0.0003$, $\alpha_s(M_Z) = 0.118 \pm 0.0007$, and $\alpha^{-1}(M_Z) = 127.9 \pm 0.1$ predicts [21–23],

$$M_I = 10^{13.6\pm0.16^{+0.5}_{-0.4}} \text{ GeV},$$

$$M_U = 10^{15.02\pm0.25\pm0.48\pm0.11(0.25)} \text{ GeV}$$

Where the first (second) uncertainties are due to those in the input parameters (threshold effects). In the case of $M_I$, the threshold uncertainties are due to those at $M_Z$ and $M_I$ thresholds only. The third uncertainty due to 5–dimensional operator in (2.5), which is absent in $M_I$, has been calculated for $\eta^{(1)} = \pm 5(\pm 10)$. In spite of addition of a number of extra $\underline{126}$ and $\underline{10}$ dimensional Higgs fields to build a model for degenerate and seesaw contributions to the neutrino masses in $SO(10)$ introducing $SU(2)_H$ horizontal symmetry, the scale $M_I$, according to the present theorem, is identical to that in the minimal model with the same predictions on the nondegenerate neutrino masses [24]. The proton lifetime predictions in the minimal model including NRO contribution is

$$\tau_{p \rightarrow e^+\pi^0} = 1.44 \times 10^{32.1\pm0.7\pm1.0\pm1.9\pm0.45(1.0)} \text{ yrs},$$

which might be testified by the next generation of experiments.

**IV. INTERMEDIATE SCALE IN SUSY $SO(10)$**

In the conventional SUSY $SO(10)$ employing the Higgs supermultiplets $\underline{54}$, $16_H \oplus \overline{10}_H$ and $\underline{10}$, in the usual fashion, it is impossible to achieve $M_I$ substantially lower than $M_U$. When $126_H \oplus \overline{126}_H$ are used instead of $16_H \oplus \overline{10}_H$, no intermediate gauge group containing $SU(4)_C$ has been found to be possible in Ref.[25]. But the possibilities of other intermediate gauge symmetries in string inspired SUSY $SO(10)$ including $G_{2213}$ ($g_{2L} \neq g_{2R}$) have been demonstrated [25,26] by using extra light $G_{2213}$–submultiplets not needed for spontaneous symmetry breaking, but predicted to be existing in the spectrum [19].
In the present analysis, in addition to the usual 54 with all components at the GUT scale, the pair $16_H + 10_H$ with desired components at $G_{224P}$ breaking scale, and the bidoublet $\phi(2,2,1) \subset 10$ near $M_Z$ while $(2,2,6)$ is at $M_U$, we examine the effects of other components in $45$, or in $16_H + 10_H$ not absorbed by intermediate scale gauge bosons, being lighter and having masses between $1\text{TeV} - M_I$.

The adjoint representation $45$ contains the left–handed triplet $\sigma_L(3,1,1)$, the right–handed triplet $\sigma_R(1,3,1)$ and also $\sigma(C)(1,1,1)$ under $G_{224P}$. Under the standard gauge group, $\sigma_R$ and $\sigma(C)$ decompose as

$$
\sigma_R(1,3,1) = \sigma_R^+(1,1,1) + \sigma_R^-(1,1,1) + \sigma_R^0(1,1,1)
$$

$$
\sigma(C)^{(1,1,15)} = \sigma_3(C)\left(1,\frac{2}{3},3\right) + \sigma_8(C)\left(1,-\frac{2}{3},3\right) +
$$

$$
\sigma_8(C)^{(0)}(1,0,8) + \sigma_8(C)^{(1,0,1)}
$$

The representation $16_H$ contains the $G_{224P}$ submultiplets $\chi^{(L)}(2,1,4)$ and $\chi^{(R)}(1,2,\overline{4})$ and the latter decomposes under SM gauge group as

$$
\chi^{(R)}(1,2,\overline{4}) = \chi_1^{(R)}(1,1,-1) + \chi_8^{(R)}(1,1,0) +
$$

$$
\chi_3^{(R)}\left(1,-\frac{2}{3},3\right) + \chi_3^{(R)}\left(1,-\frac{1}{3},3\right)
$$

To make the model simpler, we assume some of these lighter components from $45$ or the pair $16_H + 10_H$ to be either at $M_C \simeq 1\text{ TeV}$ while others are at $M_I$. In that case all the equations for $\ln \frac{M_I}{M_Z}$ and $\ln \frac{M_U}{M_Z}$ derived in Sec.II hold with the replacements:

$$
\ln \frac{M_I}{M_Z} \to \ln \frac{M_I}{M_C}, \quad \ln \frac{M_U}{M_Z} \to \ln \frac{M_U}{M_C}, \quad \theta_i \to \theta_i^C
$$

$$
a_i \to a_i^C (i = Y, 2L, 3C) \quad \text{and} \quad d \to d_C
$$

in (2.15)–(2.16). In addition, there are contributions to the mass scales due to evolutions from $M_Z - M_C$. We present them here only upto one–loop. The two–loop, threshold, and gravitational corrections will be estimated elsewhere [27].
\[
\left( \ln \frac{M_I}{M_C} \right)_{\text{one-loop}} = \frac{12\pi}{\alpha d_C} \left( \sin^2 \theta_W - \frac{1}{2} + \frac{\alpha}{3\alpha_s} \right) - R \ln \frac{M_C}{M_Z}
\]

\[
\left( \ln \frac{M_U}{M_C} \right)_{\text{one-loop}} = \left( \ln \frac{M_I}{M_C} \right)_{\text{one-loop}} + X_C + Y
\]

where

\[
Y = \frac{5}{8d_C(a'_{4C} - a'_{2L})}
\]

\[
\left[ \left( a_{2L}^C - a_Y^C \right) \left( 3a_{2L} + 5a_Y - 8a_{3C} \right) - \left( a_{2L} - a_Y \right) \left( 3a_{2L}^C + 5a_Y^C - 8a_{3C}^C \right) \right] \times \ln \frac{M_C}{M_Z}
\]

\[
d_C = d(a_i \rightarrow a_i^c) = 3a_{2L}^c + 2a_{3C}^c - 5a_Y^c
\]

\[
X_C = X(a_i \rightarrow a_i^c)
\]

\[
R = \frac{d}{d_c}
\]

We find that when the components under the standard gauge group given in Table I are at
\[M_C \simeq 1 \text{ TeV},\] the intermediate mass scale \[M_I = 5 \times 10^{12} - 2 \times 10^{14} \text{ GeV}\] can be achieved with \[M_U = M_{str} \simeq 6 \times 10^{17} \text{ GeV}.\] It has been emphasized that the \(SU(3)_C\)–octet and \(SU(2)_L\)–weak triplet being in the standard model adjoint representation and continuous moduli of strings, have a natural justification to keep them light [17]. In our case \(\sigma^\pm, \sigma_3, \sigma_3^c\) and \(\sigma_{(c)}\) belong to the adjoint representations (1,3,1) and (1,1,15) of \(G_{224}\) which in turn are contained in the adjoint representation \(45 \subset SO(10)\). One set of our solutions in Table I corresponds to the first three of them being as light as \[M_C \simeq 1 \text{ TeV}\] while the fourth component, the \(SU(3)_C\)–octet component in \(\sigma^c(1,1,1)\) is at \[M_I\]. We have also found a completely different type of solution where the \(SU(2)_R\)–triplet components and \(\chi_3 \oplus \chi_{\overline{3}} \subset 16_H + 16_{\overline{H}},\) but not absorbed by \(SU(4)_C\) gauge bosons, are near 1 TeV. In that case all the components in
σ(1, 1, 15) are at $M_I$. The neutrinos acquire small Majorana masses by seesaw mechanism using $SO(10)$ singlets as explained in Ref.[26]. None of the lighter scalar degrees of freedom near 1 TeV are needed to acquire vacuum expectation values as the spontaneous symmetry breakings of gauge symmetries like $SO(10)$, $G_{224P}$, and $G_{213}$ occur following the standard procedure through the vacuum expectation values of well known scalar components which are neutral under the residual gauge groups.

The left–handed neutrinos acquire small Majorana neutrino masses via seesaw mechanism where the right–handed neutrino mass $M_N$, rather then $M_I$, occurs in the seesaw formula, in both SUSY [26] and nonSUSY theories. But since $M_N$ is of the same order as $M_I$ with $M_N \leq M_I$ in a large class of models, the right–handed Majorana mass is also made correspondingly uncertain whenever $M_I$ is affected by larger uncertainties, especially due to the GUT–threshold effects with nondegenerate components of scalar representations [8] and gravitational effects due to higher dimensional operators [12,21]. This occurs in models where parity is broken at the GUT scale, but $G_{224}$ or $G_{2213}$ with $g_{2L} \neq g_{2R}$ [8], or even $SU(2)_L \times U(1)_R \times SU(4)_C$ ($\equiv G_{214}$) [29], breaks at the intermediate scale. With $G_{2213P}$ at the intermediate scale, these corrections do not vanish, although they are reduced. But in the $SO(10)$ and other GUTs, or string inspired models with $G_{224P}$ (but not $G_{2213P}$) surviving down to the intermediate scale, all major sources of uncertainties emerging from higher–scale corrections are absent in $M_I$ and, therefore, correspondingly in $M_N$, even though the latter is still undetermined within one order of magnitude below $M_I$. It is to be emphasized that in such models, the order–of–magnitude estimation of right–handed Majorana neutrino masses are much more accurate as compared to other models with intermediate scales. Consequently, the left–handed–Majorana–neutrino–mass prediction is more precise in these models. Further it is not true that imposition of the left–right symmetry at the intermediate scale always leads to vanishing higher–scale corrections. The vanishing correction occurs only in the presence of the left–right symmetric $G_{224P}$–gauge symmetry for $\mu > M_I$. Mohapatra [30] has proved a theorem on vanishing corrections due to GUT–threshold effects originating from degenerate components of $SO(10)$–Higgs representations in the presence of
other type of gauge symmetry. The present theorem emphasizes vanishing corrections due
to all sources emerging from $\mu > M_I$ in the presence of $G_{224P}$ only.

V. SUMMARY AND CONCLUSIONS

We have shown that all higher scale corrections on the intermediate–scale prediction
($M_I$), corresponding to the $G_{224P}$ gauge symmetry breaking, vanish exactly. Such correc-
tions are due to one–loop, two–loop and higher loop effects, GUT–threshold and gravita-
tional smearing effects originating from higher–dimensional operators. In string inspired
SUSY GUTs, the string–loop threshold effects have also vanishing contributions on $M_I$. In
nonSUSY $SO(10)$ models, the intermediate scale has been predicted earlier and we empha-
size that $M_I \simeq 10^{13.6}$ GeV is quite stable leading to more precise neutrino mass predictions.
The predicted proton lifetime can be testified by future experiments. The $G_{224P}$ symmetry
having only two gauge couplings guarantees unification, but the problem in SUSY $SO(10)$ is
the realization of $M_I \ll M_U$. We find solutions to this problem with $M_I \simeq 5 \times 10^{12} - 2 \times 10^{14}$
GeV and $M_U \simeq M_{str} \simeq 6 \times 10^{17}$ GeV for small $\alpha_S(M_Z)$ provided certain states in the adjoint
representation $45$ and/or $16_H + 16_H$ have masses near 1 TeV. The light states in $16_H + 16_H$
may emerge naturally from the modes not absorbed by heavy $SU(2)_R \times SU(4)_C$ gauge
bosons. String–scale unification might be possible in case of another intermediate symme-
try, such as $G_{2213}$, with parity broken at the GUT scale, when the submultiplet $\sigma^c(1, 1, 0, 8)$
is at the intermediate scale [28]; but only in the present case of $G_{224P}$ intermediate symme-
try, the scale $M_I$ has all higher–scale corrections vanishing and neutrino mass predictions in
SUSY $SO(10)$ are expected to be more precise.

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### TABLE I. Predictions for mass scales in string inspired $SO(10)$ model

| SM submultiplets | SM submultiplets | $G_{224F}$ submultiplets | $a_i^c$ | $a_i'$ | $M_I$ | $M_U$ |
|------------------|------------------|--------------------------|--------|-------|-------|-------|
| $M_Z - M_I$      | $M_C - M_I$      | $M_I - M_U$              |        |       |       |       |
| $\phi_u, \phi_d$ | $\sigma_{\pm}^R$, $\sigma_3$, $\sigma_3^\prime$ | $\sigma_L$, $\sigma_R$, $\sigma^c$, $\chi_L$, $\chi_R$, $\bar{\chi}_L$, $\bar{\chi}_R$, $\phi$ | $(47/5)$ | $(7)$ | $10^{12.5}$ | $10^{17.6}$ |
|                  | or $\sigma_{\pm}^R$, $\chi_3$, $\chi_3^\prime$ | $\bar{\chi}_R$, $\phi$ | $(-2)$ |       |       |       |
| $\phi_u, \phi_d$ | $\chi_1$, $\chi_3$, $\chi_3^\prime$, $\bar{\chi}_L$, $\bar{\chi}_R$, $\phi$ | $\chi_L$, $\chi_R$, $\bar{\chi}_L$, $\bar{\chi}_R$, $\phi$ | $(42/5)$ | $(5)$ | $10^{14.3}$ | $10^{17.8}$ |
|                  | $\chi_3$        | $\bar{\chi}_R$, $\phi$ | $(-3/2)$ |       |       |       |