Design of encoding and decoding devices in infocommunication systems with orthogonal coding

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Abstract. The orthogonal coding, which is developed by the author, allows providing the needed communication quality with a smaller power cost. The signal-to-noise ratio’s energy gain is ensured by an effective use of transmitted signals’ power with the unchanged code rate, without an increase of complexity and cost of transmitting and receiving devices. The orthogonal coding is the analogue of convolutional coding over the rational numbers’ field. Properties of orthogonality of proposed codes allow to strengthen the significance of useful signal with simultaneous weakening of the influence of AWGN. Method of the encoding and decoding devices’ design and the example are considered in details. The practical realization of the orthogonal coding has a low level of complexity: each step of the decoding process is technically reduced to a several dot products’ calculation and, consequently, to a contrasting to the zero. For this reason, the offered way of coding and design of encoding and decoding devices can be implemented in various infocommunication systems.

1. Introduction
In modern ways of noise immunity’s increase a channel is fixed as a rule. Therefore, transitional probabilities of output signals (with the fixed probabilities of input signals) don’t change [1]. As a result, the number of phases in PSK modulation in such systems doesn’t change also. For this reason there is no channel parameters dependence on the applied way of channel coding.

The basis of the developed orthogonal coding is the following feature of signals’ processing in receivers in telecommunication systems. The transmitted signals are chosen according to the decision of developers, and processing of the transmitted signals and noise is made with the use of special approach [2]. By application of orthogonal codes in this case, increase of transmitted signals and decrease of noise are provided. This characteristic exists not only in the channels with additive noise, but also in the channels with fading [3-6].

2. Synthesis of encoding and decoding matrices
It is required to implement the synthesis of matrices with equal number of lines and columns for a formation of orthogonal codes. The product of these matrices is the unitary matrix multiplied by the monomial characterizing the correcting ability of a code.

This problem is solved with the condition that elements of the matrices are polynomials of the degree 1. As a result, the class of the matrices allowing to solve practical problems of signal protection was synthesized.
Orthogonal coding as an analogue of convolutional coding is defined by couples of square matrices, the elements of which are polynomials in the variable of delay $D$ with the integer coefficients. Words of code are given by the multiplication of an information polynomial in $D$ by the encoding matrix. The decoding is performed by the multiplication by the decoding matrix.

It is required that these matrices shall satisfy the following condition

$$G(D) \cdot H(D) = \rho \cdot D^i \cdot I,$$  \hspace{1cm} (1)

where $I$ denotes the identity matrix. The multiplier $\rho \cdot D^i$ shows that the amplitude of the input signal increased $\rho$ times, and the delay of reception of symbols is $i$ time units.

Considering joint use of orthogonal coding and the differential phase-shift keying (DPSK) we will analyze only the application of the matrix $H(D)$ with polynomials in variable $D$ of degree 1.

The algorithm of synthesis of matrices satisfying the condition (1) is proposed in [7].

The algorithm of synthesis of matrices of the size $(n \times n)$ starts with synthesis of the matrix $H(D)$. First $z = 2k$, $z \leq n$, elements of the main diagonal get values $1 + D$, and the other elements of the main diagonal are 1. Even number $z$ is named the depth of the matrix [7]. Elements of odd rows and columns outside the main diagonal will be $1 - D$; elements of even rows and columns outside the main diagonal will be $1 + D$.

3. Design of encoding and decoding devices

When reducing the depth of a decoding matrix $H(D)$, the number of the corrected errors should increase, but the maximum element in an encoding matrix $G(D)$, that is, the range of symbols received at the output of the encoder, should also increase [8].

As an example, let’s consider the decoding matrix of order 4 and depth 2 [9]

$$H(D) = \begin{pmatrix}
1 + D & 1 - D & 1 - D & 1 - D \\
1 - D & 1 + D & 1 + D & 1 + D \\
1 - D & 1 + D & 1 & 1 - D \\
1 - D & 1 + D & 1 - D & 1
\end{pmatrix},$$

and the corresponding encoding matrix

$$G(D) = \begin{pmatrix}
3 + 3D & -3 + 3D & 0 & 0 \\
-3 + 3D & -5 + 3D & 4 & 4 \\
0 & 4 & 4 & -8 \\
0 & 4 & -8 & 4
\end{pmatrix}.$$  \hspace{1cm} (3)

when the matrix $G(D)$ is multiplied by the matrix $H(D)$, we get

$$G(D) \cdot H(D) = \begin{pmatrix}
12D & 0 & 0 & 0 \\
0 & 12D & 0 & 0 \\
0 & 0 & 12D & 0 \\
0 & 0 & 0 & 12D
\end{pmatrix}.$$  \hspace{1cm} (4)

In this example, all code symbols at the output of the decoding scheme should be divided by 12. This feature can be implemented to correct up to 5 errors. In this system, when the coding vector changes by no more than 5 symbols, we can get the correct code vector, whose elements will be multiples of 12 and to which the code vector with errors will be closer in terms of the Euclidean distance. With 6 errors, it is not possible to unambiguously get the correct code vector. If there are more than 6 errors, this way of errors’ correcting will provide an incorrect code vector.

An encoding scheme is designed based on an encoding matrix $G(D)$ [7]. Figure 1 shows the encoding scheme for the considered example.
Figure 1. The encoding scheme for the orthogonal code based on the encoding matrix of order 4 and depth 2.

An encoding matrix $G(D)$ generates a convolutional code. In the example, using the encoding matrix $G(D)$, we can get following generator matrix of the convolutional code

$$
G = \begin{bmatrix}
0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 & 0 \\
3 & -3 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-3 & -5 & 4 & 4 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 4 & 4 & -8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 4 & -8 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 3 & -3 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -3 & -5 & 4 & 4 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 4 & 4 & -8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 4 & -8 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

Further, using the decoding matrix $H(D)$, we can get the decoding matrix of the convolutional code. The 1st row of this matrix corresponds to the set of coefficients of the polynomials in the 1st column of the matrix $H(D)$, the 2nd row is equal to the set of coefficients of the polynomials in the 2nd column of the matrix $H(D)$, and so on.

The decoding matrix of the convolutional code has the following form

$$
H = \begin{bmatrix}
0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 & 0 & 1 & 2 & 3 & 0 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & -1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & -1 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & -1 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$
On the main diagonal of the product of matrices $G(D)$ and $H(D)$, there is $12D$, that is, the exponent of the delay variable $D$ is 1. This corresponds to the delay of one cycle, and for matrices $G(D)$ and $H(D)$ of order 4 to the delay of 4 symbols.

$$G \cdot H = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 12 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 12 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 12 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots 
\end{bmatrix}$$

(7)

Figure 2 shows the decoding scheme.

Figure 2. The decoding scheme for the orthogonal code based on the encoding matrix of order 4 and depth 2.

When the first row of the coding matrix $G(D)$ is fed to the input of the decoding scheme, we get at the first output in one cycle 12, the other three outputs will be zero. Similarly, when submitting the second row after one clock cycle, we get the second output 12, the other three outputs will be zero, and so on. Thus, the decoding scheme performs the functions of error correction [10].

4. Use of orthogonal coding in the AWGN channel

On the basis of the received relations the detailed analysis of efficiency of joint application of orthogonal coding and DPSK was made.

Information symbols were chosen from $\{+1, -1\}$ [5]. For orthogonal codes with small order of the generator matrix estimations of bit error rate were received analytically and by simulation modeling [10].
If only binary symbols from the set \{+1, −1\} are fed to the input of an encoder, for example, when using the decoding matrix of order 4 and depth 2 and the corresponding encoding matrix, signals from the set \{-22, −21, ..., 21, 22\} will appear at the output of the encoder.

For the orthogonal codes with bigger order of the generator matrix estimations of bit error rate were received only as a result of simulation modeling. Estimations of quantities of coding gain in the AWGN channel (e.g. from 3.0 dB to 4.5 dB at bit error rate \(10^{-6}\)) are shown in Figure 3 [11].

![Figure 3. Bit error rates in the AWGN channel for BDPSK and for schemes with the orthogonal coding on the basis of matrices of orders 4, 8, 16 and 32.](image)

For example, by use of orthogonal coding OC-32 bit error rate \(10^{-4}\) is assured by the signal-to-noise ratio \(E_b/N_0 = 6.14\) dB, which is 3.1 dB less, than in case of BDPSK without coding. By use of orthogonal coding OC-32 bit error rate \(10^{-6}\) is assured by the signal-to-noise ratio \(E_b/N_0 = 6.71\) dB, which is 4.5 dB less, than in case of BDPSK without coding [12].

5. Conclusions
The offered method of receiving and transmitting devices’ design can be implemented in various infocommunication systems.

Thus, the proposed way of orthogonal coding can be considered as a variety of reception in a whole of signals of M-ary DPSK with an optimum choice of a modulation code [13]. Optimization is reached by averaging of error rate on all digits of the M-ary code.

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