Anomalous valley magnetic moment of graphene

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Abstract – Carrier interactions on graphene are studied. The study shows that besides the well-known Coulomb repulsion between carriers, there also exist four-fermion interactions associated with the U-process, one of which attracts carriers in different valleys. We then calculate the contributions to the valley magnetic moment from vertex corrections and from four-fermion corrections explicitly. The relative contributions are −18% and 3%, respectively. Lastly we point out that we can mimic the heavy-quarkonium system by carrier interactions in graphene.

Introduction. – Graphene [1], newly discovered two-dimensional crystals, has attracted the attention of theorists and experimentalists more and more [2–5]. In graphene, there is a typical valley degeneracy, corresponding to the presence of two different valleys in the band structure. However, as stated in ref. [6], such degeneracy makes it difficult to observe the intrinsic physics of a single valley in experiments [7,8]. How to distinguish carriers in the two valleys is therefore always a topic attracting literatures [6,9–11].

Reference [9] pointed out that in close analogy with the spin degree, there is an intrinsic magnetic moment associated with the valley index, which was called valley magnetic moment (VMM). At the tree level the valley magnetic moment is about 30 times that of the usual spin magnetic moment, therefore, “valleytronics” provides a new and much more standard pathway to potential applications in a broad class of semiconductors as compared with the novel valley device in graphene nanoribbons [10]. However, since in graphene the effective coupling $e^2/\epsilon\hbar v \sim 1$, a question to be posed is to what extent the calculation in ref. [9] is valid.

To answer this question, we first study carrier interactions. The study shows that, in the tight-binding approximation, besides the well-known Coulomb repulsion between electrons, there are also four-fermion interactions associated with the U-process. The four-fermion interactions are type dependent and more significant, one of them attracts carriers in different valleys. Armed with the understanding of the interactions, we point out that there are two corrections to VMM at the one-loop level. One is the vertex correction and the other is the four-fermion correction. The vertex correction is similar to the anomalous magnetic moment of a particle in quantum electrodynamics (QED) except that carrier interactions on graphene are not “Lorentz covariant”. Therefore, such correction always appears even in a one-valley system. Meanwhile, the valley degree is similar to the flavor degree in particle physics or high-energy physics. To compute the anomalous magnetic moment of a particle due to the flavor degree, one should also consider the weak interaction, an interaction between flavor degrees. Our Yakawa-like four-fermion interactions are similar to the lower-energy approximation of the weak interactions. In this way, the correction to VMM due to the valley degree appears at the one-loop level. In contrast, such correction cannot occur in QED.

Our study shows that the total correction is about −15%. Furthermore, since the corrections are independent of the divergence of the loop calculations, VMM can be used to check the validity of the perturbational calculation.

Carrier interactions. – Here we study carrier interactions. The study shows that besides the well-known Coulomb repulsion, there are also four-fermion interactions between carriers at different valleys, which are not only short-range but also contacting ones.

For simplicity, we set $\hbar \equiv 1$ and $X(r-r_A')$ the normalized orbital $p_z$ wave function of the electron bound to atom (a)E-mail: liudq@mail.xjtu.edu.cn
there is no valley-valley transition during interactions. For
interactions between carriers, we first mark the coordi-

Fig. 1: Graphene hexagonal lattice constructed as a superposi-

A', i.e. it satisfies \( \int d\mathbf{r} X(r - r_A') X(r - r_A'') = \delta_{A'A''} \) [12].

The A-electron wave function \( \psi_A(k) \) in position

space is

\[ \psi_A(k) = \sqrt{\frac{\omega}{2\pi}} \sum_A e^{i\mathbf{k} \cdot \mathbf{r}_A} X(r - r_A), \]

where \( \omega = \sqrt{3}a^2/2 \) is the area of the hexagonal

\( \mathbf{r}_A \) is the position of atom \( A \) in the lattice. For the B-electron the case is similar. We then have

\[ \langle \psi_{A'}(k') | \psi_A(k) \rangle = \delta_{\mathbf{A}_0 \mathbf{A}_0'} \delta(k - k'), \]

where \( \mathbf{A}_0, \mathbf{A}_0' = A \) or \( B \).

To study carrier interactions, we consider

\[ V(k) = \frac{(2\pi)^4}{N\omega} \psi_{A_2}(k_2 + \mathbf{k}) \psi_{A_1}(k_2) \tilde{V} \psi_{A_1}(k_1 - \mathbf{k}) \psi_{A_1}(k_1), \]  

(1)

where \( A_1, A_1', A_2, A_2' \) are equal to \( A \) or \( B \). If we ignore

interchange interactions, the main contribution to

\( X'(r' - r_{A_1'})X'(r' - r_{A_2'})V X'(r - r_{A_1}) X(r - r_{A_2}) \) should be

at the vicinity of \( r_{A_1'} = r_{A_1}, r_{A_2'} = r_{A_2} \), \( r' \approx r_{A_1} \) and \( r \approx r_{A_1} \).

we get

\[ V(k) = \delta_{A_1 A_1'} \delta_{A_2 A_2'} \sum_{A_1 A_2} e^{i\mathbf{k} \cdot (r_{A_1} - r_{A_2})} \frac{\omega e^2}{N|\mathbf{r}_{A_1} - \mathbf{r}_{A_2}|}, \]

(2)

where we have fixed \( A_1 \) in the last step. We shall ignore

the two delta functions thereinafter.

Without loss of generality, we set \( A_1 = A \). To compute

interactions between carriers, we first mark the coordinates

of \( A \) and \( B \) with two integers \( n_1 \) and \( n_2 \). From fig. 1, the
coordinate of one atom \( A \) is set as \((0,0)\). Then, for

infinite large graphene, the coordinates of atom \( A \) are

depicted as \((n_1/2, \sqrt{3}(n_1 - 2n_2)/2)a \) and the coordinates

of atom \( B \) as \((n_1/2, \sqrt{3}(n_1 - 2n_2 + \frac{2}{3})/2)a \), respectively,

where \( a \) is the lattice constant and \( n_1, n_2 \) are arbitrary

integers.

We put our focus on the interactions between electrons

around \( \pm K = \pm (4\pi/3a, 0) \). We first study the case where

there is no valley-valley transition during interactions. For

this case, we suppose \(|ka| \ll 1 \). In eq. (2) the function in

the summation is a slow-moving function, therefore, the

summation can be replaced by an integral,

\[ V_{e}(k) \approx e^2 \int d\mathbf{r} \frac{e^{i\mathbf{k} \cdot \mathbf{r}}}{|\mathbf{r}|^2} = \frac{2\pi e^2}{|\mathbf{k}|}, \]  

(3a)

where we have inserted the effective permittivity \( \varepsilon \) in

the last equation to include screening effect. We thus obtain

the well-known Coulomb interaction. The type of \( A_2 \) does

not influence the results, that is, the Coulomb repulsion

works both for carriers in the same valley and for carriers in

the different valleys.

Besides the well-known Coulomb interactions, there

are other interactions which is related to valley-valley

transition. Such interactions correspond to a U-process

and therefore \( k \approx (4\pi/3a, 0) \). To deal with such case, we

substitute \( k + (4\pi/3a, 0) \) for \( k \) in eq. (2).

We first consider A-B interactions, that is, \( A_2 = B \) in

eq. (2). We get then

\[ V_d(k) \approx e^2 \sum_{n_1 \neq 0} a \frac{e^{-i\mathbf{k} \cdot \mathbf{r}_1}}{n_1} e^{i\mathbf{k} \cdot \mathbf{r}_1} \frac{\pi}{2} K_0 \left( \frac{\varepsilon_{k}|\mathbf{r}_1|}{2} \right), \]

\[ + e^2 a \sum_{n_2} \frac{e^{i\mathbf{k} \cdot \mathbf{r}_2} e^{i\mathbf{k} \cdot \mathbf{r}_1}}{|n_2 - 1/3|} \approx 0.1 e^2 a. \]  

(3b)

Compared to the long-wavelength result in eq. (3a), the

valley-valley interaction suffers a coefficient suppression

due to the large momentum transfer. However, since such

valley-valley interactions are short-range, it is not needed to

consider screening effect. We therefore does not insert the
effective permittivity \( \varepsilon \) in the above equation.

Whereas when \( A_2 = A \), one should subtract the

contribution from self interactions, which corresponds to

\((n_1, n_2) = (0,0)\) in eq. (2). The result is then

\[ V_s(k) \approx -1.55e^2 a. \]  

(3c)

Here, the large negative coefficient \(-1.55\) is due to the

subtraction.

Since Coulomb interaction is long-range, it does not

depend on the distributing detail of the adjacent electrons.

Thus, as shown in eq. (3a), such interaction is type-

independent. However, the four-fermion form of the U-

process implies that such interactions are short-range

and they therefore depend on the distributing detail.

Therefore, as shown by (3b) and (3c), such interactions

are type-dependent. References [13,14] also proposed four-

fermion interactions from different aspects. In ref. [13]

the authors add a near-neighbor interaction term and

then, when they carry out momentum integral in the first

Brillouin zone, they adhere the short-range interaction

with the usual Coulomb interaction at \( |k| = \frac{1}{2} \sqrt{3}a \), where

\( k \) is the transfer momentum. In contrast, in our approach,

there is no artificial adhering and the interactions due to

the valley transition are shown explicitly. Furthermore,
the results obtained by our approach are suitable to take the
quantum field theory calculations.
We emphasize that, besides the vertex correction, $V_s$ also contributes to VMM. Furthermore, since $V_s < 0$, it takes attracting force between electrons in different valleys. The interaction may play crucial role in superconduction phenomena [15]. Therefore, $V_s$ deserves further research.

The formal development of the Lagrangian. – We first define two two-component spinors $\varphi$ and $\chi$ as follows:

$$\varphi = \begin{pmatrix} aK(p) \\ bK(p) \end{pmatrix} \quad \text{and} \quad \chi = \begin{pmatrix} -ib - K(p) \\ ia - K(p) \end{pmatrix},$$

where $\pm K$ are two valleys. To describe the graphene dynamics in field theory language, we read the Lagrangian,

$$\mathcal{L}_0 = \varphi(i\gamma^0 \partial_t + iv \gamma \cdot \nabla - m)\varphi - e\varphi(\gamma^0 A^0 - \beta \gamma \cdot A)\varphi + \bar{\chi}(i\gamma^0 \partial_t + iv \gamma \cdot \nabla + m)\chi - e\chi(\gamma^0 A^0 - \beta \gamma \cdot A)\chi - \frac{\lambda_1}{2}\bar{\varphi}\gamma^1 \chi\gamma_1 \varphi - \frac{\lambda_2}{2}\bar{\varphi}\gamma^2 \chi\gamma_2 \varphi,$$

where $\beta = v/c, v$ is the Fermi velocity of carriers, $c$ is the effective light velocity in graphene, $\lambda_1 = -(V_s - V_d)/2$ and $\lambda_2 = -(V_s + V_d)/2$. We also set three gamma matrices as

$$\gamma^0 = \sigma_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

$$\gamma^1 = g^{11} \gamma_1 = -\gamma_1 \gamma_0 \sigma_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

and

$$\gamma^2 = g^{22} \gamma_2 = -\gamma_2 \gamma_0 \sigma_2 = \begin{pmatrix} i \\ 0 \end{pmatrix},$$

where $\sigma_i$’s are three Pauli matrices and the metric matrix $g^{\mu\nu} = \text{diag}(1, 1, -1)$. Since four-fermion interactions in eqs. (3b) and (3c) are contact interactions ones, it is necessary to introduce corresponding gauge field. In the above equation the energy gap $m$ can be used to improve the use of graphene in making transistors and is therefore the one of the hot spots of literatures. In ref. [16], the authors investigate the energy gap of graphene on a substrate BN, which is generated by the breaking of the A-B sublattice symmetry. However, such energy gap has not been observed up to now. In ref. [17] the authors report that single layer graphene on SiC has a gap of 0.26 eV, but the result is under debate [18,19].

Utilizing the definitions of the $\gamma$ matrix and boost generators corresponding to the “Lorentz” transformation, we find positive solution and negative solution for the $\varphi$-field (or negative solution and positive solution for the $\chi$-field) as $u(p) = \left(\sqrt{p^0 + m}, \frac{v\gamma \cdot p + c_0}{\sqrt{p^0 + m}}\right)^T$ and $v(p) = \left(\frac{v\gamma \cdot p - c_0}{\sqrt{p^0 + m}}, \sqrt{p^0 + m}\right)^T$, respectively. They meet $\bar{\mu}(p) = -v(p^0)m$, $\bar{v}(p')\bar{p} = -\bar{v}(p')m$, where $\bar{p} = \gamma_\mu \bar{p}^\mu$ and $\bar{p} = (p^0, v)$. However, the Lagrangian in eq. (4) is a bare one and it needs renormalization to match the observable quantities [20]. Having set $\varphi = Z_{\varphi}^{1/2} \varphi_r$, $\chi = Z_{\chi}^{1/2} \chi_r$ and $A = Z_A^{1/2} A_r$, where $\varphi_r$, $\chi_r$ and $A_r$ are renormalized quantities, we split each term of the Lagrangian into two pieces as follows:

$$\mathcal{L} = \varphi_r(i\gamma^0 \partial_t + iv \gamma \cdot \nabla - m_r)\varphi_r - e_r \varphi_r(\gamma^0 A^0_r - \beta_r \gamma \cdot A_r)\varphi_r + \bar{\chi}_r(i\gamma^0 \partial_t + iv \gamma \cdot \nabla + m_r)\chi_r - e_r \chi_r(\gamma^0 A^0_r - \beta_r \gamma \cdot A_r)\chi_r - \frac{\lambda_1}{2}\bar{\varphi}_r \gamma^1 \chi_r \gamma_1 \varphi_r - \frac{\lambda_2}{2}\bar{\varphi}_r \gamma^2 \chi_r \gamma_2 \varphi_r + \varphi_r(i\delta^0 \gamma \partial_t + i\delta_0v \gamma \cdot \nabla - \delta_m)\varphi_r + \bar{\chi}_r(i\delta^0 \gamma \partial_t + i\delta_0v \gamma \cdot \nabla + \delta_m)\chi_r - e_r \varphi_r(\delta^0 \gamma A^0_r - \beta_r \delta_c \gamma \cdot A_r)\varphi_r - e_r \chi_r(\delta^0 \gamma A^0_r - \beta_r \delta_c \gamma \cdot A_r)\chi_r - \frac{\lambda_1}{2}\bar{\varphi}_r \gamma^1 \chi_r \gamma_1 \varphi_r - \frac{\lambda_2}{2}\bar{\varphi}_r \gamma^2 \chi_r \gamma_2 \varphi_r,$$

where counterterm coefficients $\delta_2 = Z_2 - 1$, $\delta_m = Z_m - m_r$, $\delta_0 = Z_0v/v_r - 1$, $\delta_1 = Z_1Z_{\varphi}^{1/2}e/e_r - 1 \equiv Z_1 - 1$, $\delta_c = Z_1\beta/\beta_r - 1$, $\delta_1 = Z_1^{1/2}\lambda_1 - \lambda_r$ and $\delta_2 = Z_2^{1/2}\lambda_2 - \lambda_{2r}$ are determined by renormalized conditions.

Since all the quantities in the following are renormalized ones, all the subscripts $r$ will be dropped out.

Calculation of VMM. – To compute the VMM we first show Feynman rules in fig. 2(a)–(e). The contribution to VMM up to $e^2$ is depicted by fig. 2(g), two diagrams in fig. 2(h), which is denoted by $\delta T^\mu_{\nu}$, $l = 1, 2$, fig. 2(i), which is denoted by $\Gamma^\mu$, and the counterterm, fig. 2(f).
The scattering amplitude of carrier under external gauge field is
\[
i\mathcal{M} = -ie' \left\{ \bar{u}(p') (1 + \delta_1) \gamma^\mu u(p) + \Gamma^\mu + \sum_{i=1}^2 \delta \Gamma_i^\mu(q) \right\},
\]
where the counterterm \(\delta_1\) plays a similar role as \(Z_{int} - 1\) and \(Z_{ext} - 1\) in ref. [21], and \(A_i^\mu(q)\) is the Fourier transformation of the external field, \(p'\) and \(p\) are outgoing and incoming momenta of the carrier, respectively. Since we are working in the lower-energy limit, we ignore the renormalization of the Fermi velocity and charge. We get
\[
\delta \Gamma_i^\mu = -i\lambda_i \int \frac{d^3k}{(2\pi)^3} \bar{u}(p') \gamma^\mu (\vec{k} - m) \gamma^\nu (\vec{k}' - m) \gamma_\nu u(p),
\]
\[
\Gamma^\mu = \frac{2\pi ie^2}{\varepsilon^4} \int \frac{d^3k}{(2\pi)^3} \bar{u}(p') (\vec{k}')^\mu + m) \gamma^\nu (\vec{k} + m) \gamma_\nu u(p),
\]
respectively, where \(q = p' - p\), \(k' = k + q\) and \(k = q - q\). In the above equation, we do not sum over the repeated index \(l\) and the terms proportional to \(\beta^2 \sim 10^{-4}\) are neglected.

Since the external field is time-independent, \(q^0 = p^0 - p^0 = 0\) in eq. (7). If the electromagnetic field varies slowly over a large region, the Fourier components of the electromagnetic field will be concentrated about \(q = 0\). We can thus take the nonrelativistic limit, \(q \to 0\), in \(i\mathcal{M}\), which means \(|p|, |p'|, |q| \ll m/\varepsilon\). Therefore, we have the relations \(-\bar{q}^2 = v^2 q^2 > 0\) and \(\bar{p} \bar{p}' = (p^0)^2 - v^2 p \cdot p' \approx m^2\).

To study the response to the external magnetic field, we set the time component of \(A_\mu\) as zero, i.e. \(A_\mu(q) = (0, \mathbf{A}(q))\). We therefore only need to calculate the spatial part in \(i\mathcal{M}\).

Since our theory violates the “Lorentz covariance”, we should treat the result carefully. Furthermore, all the integrals in eq. (7) are divergent and therefore the result seems ambiguous. However, we have the good news that the ambiguity has no effect on VMM. After a lengthy calculation, such as Wick rotation [22] and the expansion of the result to order \(|p|, |p'|, |q|\), we write \(\Gamma^\mu\) and \(\delta \Gamma_i^\mu\) as
\[
\bar{u}(C_1 \gamma^\mu + C_2 \bar{e}^{\sigma 2/3} q^3/3) \gamma_\nu u(p),
\]
in the nonrelativistic limit, where \(e^{12} = -e^{11} = e^{11} = e^{22} = 0\, C_1, C_2\) and \(C_2\) depend on \(\Gamma^\mu, \delta \Gamma_i^\mu, \delta \Gamma_2\). For all the cases, \(C_1\) is divergent while \(C_2\) is finite. Together with the tree diagram and counterterm, \((1 + \delta_1 + C_1 (\Gamma^\mu) + C_1 (\delta \Gamma_1^\mu) + C_1 (\delta \Gamma_2)) \gamma^\nu u\) should be fixed to match renormalization conditions. Comparing with the Born approximation for scattering from a potential of carrier nearly \(p, q \to 0\), we find that \(1 + \delta_1 + C_1 (\Gamma^\mu) + C_1 (\delta \Gamma_1^\mu) + C_1 (\delta \Gamma_2)\) is just the electric charge of the carrier, in units of \(e\). Due to this, we set the renormalization condition as \(1 + \delta_1 + C_1 (\Gamma^\mu) + C_1 (\delta \Gamma_1^\mu) + C_1 (\delta \Gamma_2) = 1\) at \(p, q \to 0\). This renormalization condition corresponds with the fact that the carrier at lower energy \((p = 0)\) possesses unit (renormalized) charge \(e\) when scattered under the external potential which varies very slowly.

For the finite term \(C_2\), we have
\[
C_2(\Gamma^\mu) = \frac{e^2}{4\varepsilon m}, \quad C_2(\delta \Gamma_i^\mu) = -\frac{\lambda_i}{4\pi \varepsilon}.
\]

Ignoring term proportional to \(p + p'\), which is the contribution of the operator \(p \cdot A + A \cdot p\) in the standard kinetic-energy term of nonrelativistic quantum mechanics, we rewrite the \(\gamma^\nu u\) term as
\[
\bar{u}(p') \gamma^\nu u(p) \rightarrow -\frac{iwe^3 q^i}{m} \bar{u} \sigma^3 u.
\]
We obtain, then,
\[
i\mathcal{M} = -i2m\xi \sigma_3 \frac{e\varepsilon^\beta}{m} \left( 1 - \frac{e^2}{4\pi \varepsilon^2} + \frac{m}{4\pi \varepsilon^2} (\lambda_1 + \lambda_2) \right) B^3,
\]
where \(B^3 = -i(q^3 A_3^0 - q^2 A_3^1)\) is the magnetic field perpendicular to graphene, \(\xi = (1, 0)^T\) is the two-component spinor and \(\xi^\dagger \sigma_3 \xi = 1/2 \equiv s_3\) indicates that the electron pseudo-spin is \(1/2\).

We interpret \(i\mathcal{M}\) as the Born approximation to the scattering of the electron from a potential. The potential is just that of a magnetic moment interaction, \(V(x) = -\mu_e^3(K) B^3\), where
\[
\mu_e^3(K) = \frac{e\varepsilon^\beta}{m} \left( 1 - \frac{e^2}{4\pi \varepsilon^2} + \frac{m}{4\pi \varepsilon^2} (\lambda_1 + \lambda_2) \right) s_3
\]
is the carrier VMM parallel to \(B^3\) at \(K\) valley. For the hole, we get the same value with a necessary minus sign. Similarly, for carrier at \(-K\) valley, VMM is also the same with a minus sign. Such phenomenon is known as the broken inversion symmetry in ref. [9].

By recovering \(\hbar\), the leading term of VMM is \(e\hbar v^2 / m\), which is also obtained by ref. [9]. However, besides the leading term, there are also other contributions to VMM. The relative contribution to VMM are
\[
-\frac{e^2}{4\varepsilon^2} \frac{mV_d}{4\pi \hbar^2 v^2} = \alpha \left( -\frac{1}{4} + \frac{1.55\varepsilon}{4\pi \hbar^2 v^2} \right),
\]
where \(\alpha = e^2 / (\varepsilon v) = 0.73\) when \(e = 3\). Substituting \(a = 2.46\, \text{Å}, v \approx 10^{-8} \text{cm/s}\) into eq. (12), we find the relative modifications to VMM due to vertex correction and four-fermion interactions are about \(-18\%\) and \(3\%\), respectively, if we choose \(m = 0.26\, \text{eV}\).

It looks strange that it is not \(V_d\) but \(V_d\) which contributes to VMM. Such behavior stems from the definition of the \(\chi\)-field. From the definition of the \(\phi\)-field and the \(\chi\)-field \(V_d\) only relates to the interaction between carriers in different valleys with the same pseudo-spin so that it does not contribute to VMM. On the contrary, \(V_e\) relates interaction between carriers in different valleys with different pseudo-spins. Therefore, only \(V_e\) contributes to VMM.

Discussions. – In this paper we have discussed the carrier interactions. The study reveals that besides the well-known Coulomb repulsion between carriers, there are four-fermion interactions between carriers in different valleys. Since the interactions are short-range and
contacting ones, they depend on the atom collocation detail. Therefore, the four-fermion interactions are type dependent. Our study shows that one of the four-fermion interactions attracts carriers in different valleys, which we believe to be helpful in understanding the unusual superconduction effect in graphene.

We also compute VMM from the tree level diagram, the vertex correction and the four-fermion interactions, respectively. The contribution from the tree level diagram agrees with the result obtained in ref. [9]. The other two contributions counteract each other and therefore the total contribution to VMM is about $-15\%$ if we choose $m = 0.26$ eV and $\varepsilon = 3$.

The very high accurate measurement of spin magnetic moment is very important, both from the theoretical viewpoint and from the practical one. Similarly, our result on VMM is also significant to valleytronics in graphene, especially to the future apparatus design based on valleytronics. Our study also points out that, in close analogy to the Zeeman split, the contribution to VMM induced by $V_N$ is inherent, since it is independent of the energy gap $m$. In other words, to measure the magnetic moment induced by $V_N$, we may choose the substrate freely, although a different substrate may generate different energy gap and different effective permittivity.

From eq. (12), $\alpha$ plays the same role as the fine-structure constant in QED, $\alpha_e = \frac{\alpha}{4\pi} \approx 1/137$. However, because $\alpha$ is about 100 times larger than $\alpha_e$, it is hard to state that we mimic QED by carrier interaction in graphene. Meanwhile, when we deal with problems dominated by quantum chromodynamics (QCD), especially in heavy quarkonium, such as $cc$ system and $bb$ system, we always take $\alpha_s = \frac{g_s^2}{4\pi}$, where $g_s$ is the QCD coupling, as the estimate of the effectiveness of perturbational expansion. (In many cases when we deal with such problem we take an approach very similar to QED, up to an unimportant color factor.) At an energy scale of 740 MeV, $\alpha_s(740\text{ MeV}) \approx 0.73 \approx \alpha$ [22]. Noticing that the energy scale is close to the soft scale of $cc$ and $bb$ systems [23], the dynamics of which is described by nonrelativistic QCD, we conclude that we can mimic the heavy-quarkonium system by carrier interactions in graphene. Therefore, the study on the heavy-quarkonium system can also be carried out in graphene.

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