On the Reduced SU(N) Gauge Theory in the Weyl-Wigner-Moyal Formalism

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ABSTRACT

Weyl-Wigner-Moyal formalism is used to describe the large-$N$ limit of reduced SU(N) quenching gauge theory. Moyal deformation of Schild-Eguchi action is obtained.
Many years ago G. ‘t Hooft has shown that drastic simplification occurs in the
gauge structure of SU\((N_C)\) Quantum Chromodynamics (QCD), if one takes the
number of colors \(N_C\) to be infinite\(^1\). Large-\(N\) limit, for instance, contains genuine
non-perturbative information of field theory at the classical and quantum levels.
Chiral symmetry breaking, mass gap and color confinement are important non-
perturbative features which persist in this limit. Large-\(N\) techniques have been
also applied to describe mesons and baryons in a complete picture\(^2\) and to matrix
model approach to 2D quantum gravity and 2D string theory\(^\dagger\) when one looks
for their non-perturbative formulation. More recently large-\(N\) technique has been
used to formulate the M(atrix) Theory\(^4\) of zero D branes at the infinite momentum
frame. This theory is by now a strong candidate to realize the called \(M\) Theory\(^5\).
The power of these results enable one to study some large-\(N\) limits of various
physical systems. For instance, some two-dimensional integrable systems seem to
be greatly affected in this limit, turning out ‘more integrable’. Example of this
‘induced integrability’ occurs in the large-\(N\) limit of SU\((N)\) Nahm equations\(^6\).
Actually the origin of this drastic simplification in the field equations is not well
understood\(^7\). The results presented in this paper might will shed some light about
this mysterious limit.

The usual transition from SU\((N)\) gauge theory to the SU\((\infty)\) ones involves the
change of the Lie algebra su\((N)\) by the area-preserving diffeomorphisms algebra,
\(\text{sdiff}(\Sigma)\), on a two-dimensional manifold \(\Sigma\). The last one is an infinite dimensional
Lie algebra. As we make only local considerations we assume the space \(\Sigma\) to be a
two-dimensional simply connected and \textit{compact} symplectic manifold with local real
coordinates \(\{\tau, \sigma\}\). This space has a natural local symplectic structure given by the
local area form \(\omega = d\sigma \wedge d\tau\). \(\text{sdiff}(\Sigma)\) is precisely the Lie algebra associated with
the infinite dimensional Lie group, SDiff\((\Sigma)\), which is the group of diffeomorphisms
on \(\Sigma\) preserving the symplectic structure \(\omega\), \textit{i.e.} for all \(g \in \text{SDiff}(\Sigma)\), \(g^*(\omega) = \omega\).

Globally the symplectic form is defined by \(\omega : T\Sigma \rightarrow T^*\Sigma\) and inverse \(\omega^{-1}\):
$T^*\Sigma \rightarrow T\Sigma$. Here $T\Sigma$ and $T^*\Sigma$ are the respective tangent and cotangent bundles to $\Sigma$. While the hamiltonian (or area preserving) vector fields are $U_{H_a} = \omega^{-1}(dH_a)$ satisfying the $s\text{diff}(\Sigma)$ algebra

$$[U_{H_a}, U_{H_b}] = U_{\{H_a, H_b\}_P}, \quad \text{for all } (a \neq b), \quad (1)$$

where $\{\cdot, \cdot\}_P$ stands for the Poisson bracket with respect to $\omega$. Locally it can be written as

$$\{H_a, H_b\}_P = \omega^{-1}(dH_a, dH_b) = \omega^{ij}\partial_i H_a \partial_j H_b, \quad (2)$$

where $\partial_i \equiv \frac{\partial}{\partial \sigma_i}$, $(i = 0, 1)$, $\sigma^0 = \tau$, $\sigma^1 = \sigma$ and $H_i = H_i(\vec{\sigma}) = H_i(\sigma^0, \sigma^1)$.

The generators of $s\text{diff}(\Sigma)$ are the hamiltonian vector fields $U_{H_a}$ associated to the hamiltonian functions $H_a$ given by

$$U_{H_a} = \frac{\partial H_a}{\partial \sigma^0} \frac{\partial}{\partial \sigma^0} - \frac{\partial H_a}{\partial \sigma^1} \frac{\partial}{\partial \sigma^0}. \quad (3)$$

On the other hand, in the Ref. 8, the Lie algebra $\text{su}(N)$ is defined in a basis which appears to be very useful for our further considerations and we going briefly review.

The elements of this basis are denoted by $L_{\vec{m}}$, $L_{\vec{n}}$, etc., $\vec{m} = (m_1, m_2)$, $\vec{n} = (n_1, n_2)$, etc., and $\vec{m}$, $\vec{n}$, $\vec{m} \in I_N \subset \mathbb{Z} \times \mathbb{Z} - \{(0,0) \mod N\vec{q}\}$ where $\vec{q}$ is any element of $\mathbb{Z} \times \mathbb{Z}$. The basic vectors $L_{\vec{m}}$, $\vec{m} \in I_N$, are the $N \times N$ matrices satisfying the following commutation relations

$$[L_{\vec{m}}, L_{\vec{n}}] = \frac{N}{\pi} \text{Sin}(\frac{\pi}{N} \vec{m} \times \vec{n}) L_{\vec{m} + \vec{n}}, \quad \text{mod } N\vec{q}, \quad (4)$$

where $\vec{m} \times \vec{n} := m_1 n_2 - m_2 n_1$. 

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Now we let $N$ tend to infinity. In this case $I_\infty \equiv I = \mathbb{Z} \times \mathbb{Z} - \{(0, 0)\}$ and the commutation relations (4) read

$$[L_{\vec{m}}, L_{\vec{n}}] = (\vec{m} \times \vec{n}) L_{\vec{m}+\vec{n}}. \quad (5)$$

Consider the complete set of periodic hamiltonian functions $\{e_{\vec{m}}\}_{\vec{m} \in I}$, $e_{\vec{m}} = e_{\vec{m}}(\vec{\sigma}) := \exp[\im\sigma_1^0 + m_2\sigma_1^1]$. One quickly finds that

$$\{e_{\vec{m}}, e_{\vec{n}}\}_P = (\vec{m} \times \vec{n}) e_{\vec{m}+\vec{n}}. \quad (6)$$

Thus the mapping $F : L_{\vec{m}} \mapsto e_{\vec{m}}$, $\vec{m} \in I$, defines the isomorphism

$$\text{su}(\infty) \cong \text{Poisson algebra on } \Sigma(= T^2) \cong \text{sdiff}(T^2), \quad (7)$$

where $T^2$ is the 2-torus.

The algebra (4) is defined for the 2-torus $T^2$ but it can be extended to a Riemann surface of genus $g \geq 1 \Sigma_g$ as has been shown by I. Bars$^9$. His argument is as follows: Consider the group $\text{SU}(N)$ with $N = N_1 + \ldots + N_g$ and define the set of $N \times N$ matrices, $L_{\vec{m}_1}^{(N_1)} \oplus 1_{N_2} \oplus \ldots \oplus 1_{N_g}$, $1_{N_1} \oplus L_{\vec{m}_2}^{(N_2)} \oplus \ldots \oplus 1_{N_g}$, $1_{N_1} \oplus 1_{N_2} \oplus \ldots \oplus L_{\vec{m}_g}^{(N_g)}$ where $L_{\vec{m}_k}^{(N_k)}$ and $1_{N_k}$ are $N_k \times N_k$ matrices (for $k = 1, \ldots, g$), being the later the unit matrix.

The generalization of (4) to a Riemann surface of genus $g$ is$^9$

$$[L_{\vec{m}_1} \ldots \vec{m}_g, L_{\vec{n}_1} \ldots \vec{n}_g] = C N \sin \left( \pi \sum_{i=1}^{g} \frac{\vec{m}_i \times \vec{n}_i}{N_i} \right) L_{\vec{m}_1 + \vec{n}_1 + \ldots + \vec{m}_g + \vec{n}_g}, \quad \text{mod } (N_1 \vec{q}_1, \ldots, N_g \vec{q}_g), \quad (8)$$

where $(\vec{q}_1, \ldots, \vec{q}_g) \in \mathbb{Z}^g \times \mathbb{Z}^g$. Here the generators $L_{\vec{m}_1} \ldots \vec{m}_g$ are defined by
\[
L_{\vec{m}_1...\vec{m}_g} = (L^{(N_1)}_{\vec{m}_1} \oplus 1_{N_2} \oplus \ldots \oplus 1_{N_g}) \oplus (1_{N_1} \oplus L^{(N_2)}_{\vec{m}_2} \oplus \ldots \oplus 1_{N_g}) \oplus \ldots \oplus (1_{N_1} \oplus 1_{N_2} \oplus \ldots \oplus L^{(N_g)}_{\vec{m}_g}).
\]

(9)

It is shown in Ref. 9 that as one take the large-$N$ limit of (8) it yields

\[
[L_{\vec{m}_1...\vec{m}_g}, L_{\vec{n}_1...\vec{n}_g}] = \sum_{i=1}^g (\vec{m}_i \times \vec{n}_i) L_{\vec{m}_1+\vec{n}_1...\vec{m}_g+\vec{n}_g},
\]

(10)

which generalizes (5). The set of hamiltonian functions $e_{\vec{m}_1...\vec{m}_g}$ associated with $L_{\vec{m}_1...\vec{m}_g}$ are defined by

\[
e_{\vec{m}_1...\vec{m}_g} = \exp \left( i \sum_{i=1}^g \vec{m}_i \cdot \vec{\sigma}_i \right)
\]

(11)

and satisfy the Poisson algebra\(^9\)

\[
\{e_{\vec{m}_1...\vec{m}_g}, e_{\vec{n}_1...\vec{n}_g}\} = \sum_{i=1}^g (\vec{m}_i \times \vec{n}_i) e_{\vec{m}_1+\vec{n}_1...\vec{m}_g+\vec{n}_g}.
\]

(12)

On the other hand, Bars using a series of basic results in reduced and quenched large-$N$ gauge theories was able to derive the string theory action in a particular gauge\(^10\). In this derivation he used the above mentioned area-preserving diffeomorphisms formalism on $T^2\(^\ast\)$. A very similar (but different) approach was previously considered by Fairlie, Fletcher and Zachos in the context of large-$N$ limit of Yang-Mills theory in Ref. 8, and reviewed by Zachos in Ref. 12. There it was derived Nambu’s action from the large-$N$ approach to Yang-Mills gauge theory\(^13\). In this derivation the quadratic

\(^\ast\) Large-$N$-limit was first studied by Hoppe in the context of membrane physics\(^11\). In that case $\Sigma = S^2$, the two-sphere.
Schild-Eguchi action for strings\textsuperscript{14} arose by the first time from a gauge theory. In what follows we will take Bars approach\textsuperscript{9,10} with quenched prescriptions by Gross-Kitazawa\textsuperscript{15}.

The reduced SU\((N)\) gauge theory action is\textsuperscript{9,10}

\[
S_{\text{red}} = -\frac{1}{4} \left( \frac{2\pi}{\Lambda} \right)^d \frac{N}{g_d^2(\Lambda)} Tr(F_{\mu\nu}F^{\mu\nu}),
\]

where \(d\) is the dimension of space-time space \(M_d\), \(g_d(\Lambda)\) is the Yang-Mills coupling constant in \(d\) dimensions evaluated at certain cut-off \(\Lambda\) and \(Tr\) is an invariant bilinear form on the Lie algebra \(su(N)\), \(F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) + ig_d[A_\mu(x), A_\nu(x)]\) with \(x \in M_d, \mu, \nu = 0, 1, \ldots, d - 1\) and \(A_\mu(x)\) is the usual Yang-Mills potential on \(M_d\).

\(\hat{\text{F}}\)From now on we going follow Gross-Kitazawa paper\textsuperscript{15}. Thus the general prescription to obtain the reduced scheme from Yang-Mills gauge theory is

\[
F_{\mu\nu} = [iD_\mu, iD_\nu] \Rightarrow (F_{\mu\nu})^i_j \equiv [a_\mu, a_\nu]^i_j,
\]

where \((\cdot)^i_j\) denotes an \(N \times N\) matrix, \(D_\mu\) is the covariant derivative with respect to the Yang-Mills potential \(A_\mu\), \(iD_\mu\) must be replaced by

\[
a_\mu = P_\mu + A_\mu,
\]

where \(P_\mu\) is the quenched momentum which is a diagonal matrix and \(A_\mu\) is an \(N \times N\) matrix gauge field at \(x^\mu = 0\).

As quenched theory is a SU\((N)\) gauge theory, matrices \(A_\mu\) must satisfy a quenched gauge transformation
\[ A_\mu \rightarrow S A_\mu S^\dagger + S[P_\mu, S^\dagger], \quad (16a) \]
\[ a_\mu \rightarrow S[a_\mu] S^\dagger, \quad (16b) \]

where \( S \) is an unitary matrix. These transformations and the usual quenched relations

\[ A_\mu(x) \equiv \exp(iP \cdot x) A_\mu \exp(-iP \cdot x), \quad (17a) \]
\[ S(x) \equiv \exp(iP \cdot x) S \exp(-iP \cdot x), \quad (17b) \]

where \( P \cdot x \equiv P_\mu x^\mu \), lead to the usual gauge transformation

\[ A_\mu(x) \rightarrow S(x) A_\mu(x) S^\dagger(x) + iS(x) \partial_\mu S^\dagger(x) \quad (17c) \]

where \( S : M_d \rightarrow \text{SU}(N) \).

The quanched euclidean Feynman integral is

\[ Z_N = \int \prod_\mu \mathcal{D}a_\mu f(a_\mu) \exp\left(-S_{\text{red}}(a)\right). \quad (18) \]

Condition (15) implies a further gauge invariant constraint on function \( f(a_\mu) \) to be the quenched constraint between the eigenvalues of \( a_\mu \) and those of \( P_\mu \) given by

\[ a_\mu = V_\mu P_\mu V_\mu^\dagger, \quad (19) \]

where \( V_\mu \) diagonalizes \( a_\mu \) for each \( \mu \). At the quantum level this quenched prescription besides the usual gauge fixing terms involves an extra factor
\[ f(a) = \int \prod_{\mu} DV_\mu \delta(a_\mu - V_\mu P_\mu V_\mu^\dagger) \] (20)

in the measure of the Feynman integral (18).

Now we will use the previous discussion on area preserving diffeomorphisms to apply it to reduced large-$N$ gauge theory (for details see Refs. 8-13). Any solution of the SU($N$) reduced gauge theory equations coming from the reduced action (13) can be written in the form

\[ a_\mu = \sum_{\vec{m} \in I_N} a^{\vec{m}}(N)L_{\vec{m}} \] (21)

where the set \{\(L_{\vec{m}}\)\} satisfy algebra (4) for \(T^2\) or (8) for \(\Sigma_g\) in its corresponding basis.

The object \(F_{\mu\nu} = [a_\mu, a_\nu]\) can be written from (4) and (21) as

\[ F_{\mu\nu} = \sum_{\vec{m} \in I_N} \sum_{\vec{n} \in I_N} a^{\vec{m}}(N)a^{\vec{n}}(N) \frac{N}{\pi} \sin \left( \frac{\pi}{N} \vec{m} \times \vec{n} \right) L_{\vec{m}+\vec{n}}, \mod N\vec{q}. \] (22)

In the large-$N$ limit the above relation can be written equivalently in terms of the hamiltonian function \(F^\infty_{\mu\nu}(\vec{\sigma})\) in the basis \{\(e_{\vec{m}}(\vec{\sigma})\)\}

\[ F^\infty_{\mu\nu}(\vec{\sigma}) = \{A_\mu(\vec{\sigma}), A_\nu(\vec{\sigma})\}_P = \sum_{\vec{m},\vec{n}} a^{\vec{m}}(\vec{\sigma}) a^{\vec{n}}(\vec{\sigma}) \left( \vec{m} \times \vec{n} \right) e_{\vec{m}+\vec{n}}(\vec{\sigma}), \] (23)

where \(A_\mu(\vec{\sigma}) = A_\mu(\sigma^0, \sigma^1)\), \(e_{\vec{m}}(\vec{\sigma})\) are the generator functions of \(\text{sdiff}(T^2)\), while \{\(\sigma^0, \sigma^1\)\} are the coordinates on the 2-torus \(T^2\). It is clear that the limit

\[ a^{\vec{m}}_\mu := \lim_{N \to \infty} a^{\vec{m}}_\mu(N) \] (24)

exists for every \(\vec{m} \in I\).
Let \( S_{\text{red}}^{\infty} \) be the action which is an \( N \to \infty \) limit of \( S_{\text{red}} \). This limit can be obtained formally by the substitutions\(^8\)–\(^{13}\)

\[
\frac{(2\pi)^4}{N^3} Tr(\cdots) \to - \int \frac{d^2 \sigma}{\Sigma} (\cdots),
\]

\[(25a)\]

\[a_\mu \to A_\mu,\]

\[(25b)\]

\[[a_\mu, a_\nu] \to \{A_\mu, A_\nu\}_P.\]

\[(25c)\]

where \( d^2 \sigma \equiv d\sigma^0 d\sigma^1 \).

The large-\( N \) limit of the reduced action \( S_{\text{red}} \) is given by\(^9\),\(^{10}\)

\[S_{\text{red}}^{\infty} = 4 \left( \frac{2\pi/\Lambda}{g_\Lambda^2} \right)^{d-4} \left( \frac{N}{2\Lambda} \right)^4 \int \frac{d^2 \sigma}{\Sigma} F_{\mu\nu}^{\infty}(\sigma) F^{\infty\mu\nu}(\sigma).\]

\[(26)\]

Using (2), Eq. (23) reads

\[F_{\mu\nu}^{\infty}(\sigma) = \{A_\mu(\sigma), A_\nu(\sigma)\}_P = \omega^{ij} \partial_i A_{\mu}(\sigma) \partial_j A_{\nu}(\sigma).\]

\[(27)\]

Bars has shown that action (26) turns out to be

\[S_{\text{red}}^{\infty} \sim \int \frac{d^2 \sigma}{\Sigma} \text{det}(\partial_i A \cdot \partial_j A)\]

\[(28)\]

where “\( \cdot \)” stands for the inner product \( \partial_i A \cdot \partial_j A \equiv \partial_i A^\mu \partial_j A_\mu \).

The above action is a particular case of Polyakov’s action with a flat \( d \)-dimensional target space \( M_d \).
\[ S_{Pol} = \int_{\Sigma} d^2 \sigma \sqrt{-h} h^{ij} \partial_i A_\mu(\bar{\sigma}) \partial_j A^\mu(\bar{\sigma}), \]  

(29)

where the world-sheet metric \( h_{ij} = \partial_i A_\mu \partial_j A^\mu \) is subject to the gauge condition \( \text{det}(h) = -1 \).

Action (28) is also known as the Schild-Eguchi action\(^8,^9,^{10},^{12},^{14}\) and it is shown to be equivalent to Nambu’s action.

Just as Bars shown, the corresponding path integral is

\[ Z_\infty = \sum_g \int Dm DA f(A) \exp(S_{\text{red}}(A)) \]  

(30)

which is the bosonic string amplitude in the particular gauge \( \text{det}(h) = -1 \), \( m \) is the moduli which must be compatible with the large-\( N \) limit of \( \text{SU}(N) \) and \( f(A) \) is the large-\( N \) limit of (20) in the basis \( \{ e_{\vec{m}}(\bar{\sigma}) \} \).

Now we attempt to derive Bars results using Weyl-Wigner-Moyal formalism in quantum mechanics (see e.g. Ref. 16 and references therein). This technique has been used to obtain some new solutions of Park-Husain heavenly equation in self-dual gravity as well as to address some relations between self-dual gravity and two-dimensional field theories\(^17,^{18}\). Here we will apply this technique to rederive the string action.

\textit{Weyl correspondence} \( \mathcal{W} \) establishes a one to one correspondence between some class of linear operators \( \mathcal{B} \) acting on Hilbert space \( \mathcal{H} = L^2(\mathbb{R}) \) and the space of real smooth functions \( C^\infty(\Sigma, \mathbb{R}) \) on the phase space manifold \( \Sigma \). This correspondence is given by

\[ \mathcal{W}^{-1} : \mathcal{B} \rightarrow C^\infty(\Sigma, \mathbb{R}), \]  

(31)
\[ \mathcal{O}(\vec{\sigma}; \hbar) \equiv W^{-1}(\hat{\mathcal{O}}) := \int_{-\infty}^{\infty} <\sigma - \frac{\xi}{2}|\hat{\mathcal{O}}|\sigma + \frac{\xi}{2} > \exp\left(\frac{i}{\hbar}\xi\tau\right)d\xi, \] (32)

for all \( \hat{\mathcal{O}} \in \mathcal{B} \). Of course \( \mathcal{O}(\vec{\sigma}; \hbar) \in C^\infty(\Sigma, \mathbb{R}) \). The ‘inverse’ Weyl correspondence

\[ \mathcal{W} : C^\infty(\Sigma, \mathbb{R}) \rightarrow \mathcal{B}, \] (33)

is given by

\[ \hat{\mathcal{O}} = \mathcal{W}(\mathcal{O}(\vec{\sigma}; \hbar)) := \frac{1}{(2\pi)^2} \int_{\Omega \subset \mathbb{R}^2} \hat{\mathcal{O}}(p, q)\exp[i(p\hat{\sigma} + q\hat{\tau})]dpdq \] (34)

and

\[ \mathcal{O} = \check{\mathcal{O}}(p, q) = \int_{\Sigma} \mathcal{O}(\vec{\sigma}; \hbar)\exp[-i(p\sigma^0 + q\sigma^1)]d^2\sigma, \] (35)

where \( \check{\mathcal{O}}(p, q) \) is the Fourier transform of \( \mathcal{O}(\vec{\sigma}; \hbar) \). Here the operators \( \hat{\sigma}, \hat{\tau} \) satisfy the Heisenberg algebra and \( \{p, q\} \) are the coordinates of the Fourier dual space.

The Moyal-⋆-product on \( C^\infty(\Sigma; \mathbb{R}) \) is defined by

\[ \mathcal{O}_i \star \mathcal{O}_j := \mathcal{O}_i \exp(\frac{i\hbar}{2}\mathcal{P})\mathcal{O}_j, \quad (i \neq j) \] (36)

where \( \mathcal{P} := \frac{\partial}{\partial\sigma^1} - \frac{\partial}{\partial\sigma^0} \), \( \mathcal{O}_i = \mathcal{O}_i(\vec{\sigma}; \hbar) \). The Moyal product is an associative and non-commutative product.

With the above Moyal product definition and Eqs. (32), (34) and (35) it is very easy to check that
\[ W^{-1}(\hat{O}_i \odot \hat{O}_j) = O_i \star O_j \]  

(37)

where “\( \odot \)” stands for operator product in \( B \). Using the above results one can get the following relation

\[ W^{-1}(\frac{1}{i\hbar}[\hat{O}_i, \hat{O}_j]) = \frac{1}{i\hbar}(O_i \star O_j - O_j \star O_i) \equiv \{O_i, O_j\}_M, \]

(38)

where \([,]\) is the usual commutator and \( W^{-1}(\hat{O}_i) \equiv O_i \).

Eqs. (36) and (38) lead to

\[ \{O_i, O_j\}_M = \frac{2}{\hbar} O_i \sin(\frac{\hbar}{2} \mathcal{P}) O_j, \]

\[ = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \left( \frac{\hbar}{2} \mathcal{P} \right)^{2k} (O_i \leftrightarrow O_j). \]

(39)

Then one obtain that \( W^{-1} \) is a Lie algebra isomorphism

\[ W^{-1} : (\mathcal{B}, [,]) \to (\mathcal{M}, \{\cdot, \cdot\}_M), \]

(40)

being \( W \) its genuine inverse map. \( (\mathcal{M}, \{\cdot, \cdot\}_M) \) is called the Moyal algebra which we abbreviate as \( \mathcal{M} \).

Moyal algebra \( \mathcal{M} \) is the unique “quantum deformation” of Poisson algebras \( \text{sdiff}(\Sigma) \). We represent the Moyal algebra \( \mathcal{M} \) by \( \text{sdiff}_\hbar(\Sigma) \), where \( \hbar \) is the deformation parameter. In the limit \( \hbar \to 0 \) one recovers Poisson algebra, \( \lim_{\hbar \to 0} \text{sdiff}_\hbar(\Sigma) = \text{sdiff}(\Sigma) \). That is, Weyl-Wigner-Moyal formalism also provide the correspondence with the area-preserving formalism.
\[
\lim_{h \to 0} \mathcal{O}_i \ast \mathcal{O}_j = \mathcal{O}_i \mathcal{O}_j \quad \text{and} \quad \lim_{h \to 0} \{ \mathcal{O}_i, \mathcal{O}_j \}_M = \{ \mathcal{O}_i, \mathcal{O}_j \}_P.
\]  

(41)

In the case when \( \tau \) and \( \sigma \) be the local coordinates of a 2-torus, i.e. \( \Sigma = T^2 \). A basis of \( \text{sdiff}_h(\Sigma) \) is given by

\[
\mu_{\vec{m}} = \mu_{\vec{m}}(\vec{\sigma}) \equiv \exp[i(m_1 \sigma^0 + m_2 \sigma^1)].
\]  

(42)

Introducing the last equation into (39) we get

\[
\{ \mu_{\vec{m}}(\vec{\sigma}), \mu_{\vec{n}}(\vec{\sigma}) \}_M = 2 \bar{h} \sin \left( \frac{\bar{h}}{2} \vec{m} \times \vec{n} \right) \mu_{\vec{m}} \mu_{\vec{n}}(\vec{\sigma}).
\]  

(43)

Now we have a Fourier series instead of (34)

\[
\mathcal{W}(\mathcal{O}(\vec{\sigma}; \bar{h})) = \hat{\mathcal{O}} = \frac{1}{(2\pi)^2} \sum_{\vec{m}} \hat{\mathcal{O}}_{\vec{m}} \mu_{\vec{m}}(\vec{\sigma})
\]  

where \( \hat{\mathcal{O}}_{\vec{m}} \) is given by

\[
\hat{\mathcal{O}}_{\vec{m}} = \int_{T^2} \mathcal{O}(\vec{\sigma}; \bar{h}) \exp \left[ -i(m_1 \sigma^0 + m_2 \sigma^1) \right] d^2 \sigma.
\]  

(45)

From Eqs. (43) and (44) it is immediate to get

\[
\{ \mathcal{O}_i(\vec{\sigma}; \bar{h}), \mathcal{O}_j(\vec{\sigma}; \bar{h}) \}_M = \frac{1}{(2\pi)^4} \sum_{\vec{m}, \vec{n}} 2 \bar{h} \sin \left( \frac{\bar{h}}{2} \vec{m} \times \vec{n} \right) \hat{\mathcal{O}}_{\vec{m}} \hat{\mathcal{O}}_{\vec{n}} \mu_{\vec{m}} \mu_{\vec{n}}(\vec{\sigma}).
\]  

(46)

Using the results of Ref. 9 one can immediate generalize the algebra (43) defined on \( T^2 \) to a Riemann surface of genus \( g \), \( \Sigma_g \), as follows:
First of all consider a set of hamiltonian functions (11) \( O_{i_1...i_g} \) with the generalized Moyal product

\[
O_{i_1...i_g} \star O_{j_1...j_g} := O_{i_1...i_g} \exp \left( \frac{i\hbar}{2} \sum_{l=1}^{g} \hat{P}_l \right) O_{j_1...j_g}, \quad (i_k \neq j_k)
\]

(47)

where \( \hat{P}_l := \frac{\partial}{\partial \sigma_i^l} \frac{\partial}{\partial \sigma_i^l} - \frac{\partial}{\partial \sigma_i^l} \frac{\partial}{\partial \sigma_i^l} \) and where \( \{ \sigma_i^0, \sigma_i^1 \} (l = 1, \ldots, g) \) are the coordinates of the \( l \)-th 2-torus. These coordinates satisfy \( \{ \sigma_i^0, \sigma_i^1 \}_P = \delta_{l'}. \) With the above definitions we can find that the Moyal product for the Riemann surface is

\[
e_{\bar{m}_1...\bar{m}_g} \star e_{\bar{n}_1...\bar{n}_g} = \exp \left( i\frac{\hbar}{2} \sum_{i=1}^{g} \bar{m}_i \times \bar{n}_i \right) e_{\bar{m}_1+\bar{n}_1...\bar{m}_g+\bar{n}_g}.
\]

(48)

Thus the generalization of (43) is

\[
\{ e_{\bar{m}_1...\bar{m}_g}(\bar{\sigma}), e_{\bar{n}_1...\bar{n}_g}(\bar{\sigma}) \}_M = \frac{2}{\hbar} \sin \left( \frac{\hbar}{2} \sum_{i=1}^{g} \bar{m}_i \times \bar{n}_i \right) e_{\bar{m}_1+\bar{n}_1...\bar{m}_g+\bar{n}_g}(\bar{\sigma}).
\]

(49)

Take the deformation parameter \( \hbar \) to be

\[
\hbar = \frac{2\pi}{\sum_{i=1}^{g} N_i} = \frac{2\pi}{N}
\]

(50)

and comparing algebras (8) and (49) one can establish an isomorphism between both algebras. Thus the formalism used in this paper can be easily extended from a 2-torus to a general Riemann surface. From now on we will restrict ourselves to work on \( T^2 \) with the immediate generalization to \( \Sigma_g \) when it be required.

Now we begin from the operator \( \hat{SU}(N) \)-vauled reduced gauge theory. Here \( \hat{SU}(N) \) is the Lie group of linear unitary operators acting on the Hilbert space \( \mathcal{H} = L^2(\mathbb{R}) \). Let \( \mathcal{B} = \hat{su}(N) \) be the corresponding Lie algebra of the anti-self-dual operators in \( L^2(\mathbb{R}) \). The reduced action now takes the operator form
\[ S_{\text{red}}^{(q)} := -\frac{1}{4} \left( \frac{2\pi}{\Lambda} \right)^d \frac{N}{g_d^2(\Lambda)} \text{Tr} \left[ \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right], \]  

(51)

where \( \hat{F}_{\mu\nu} \in \mathfrak{su}(N) \), ‘Tr’ is the sum over diagonal elements with respect to an orthonormal basis \( \{|\psi_j>\}_{j \in \mathbb{N}} \) in \( L^2(\mathbb{R}) \)

\[ <\psi_j|\psi_k> = \delta_{jk}, \quad \sum_j |\psi_j> <\psi_j| = \hat{I}. \]  

(52)

The general prescription (14, 15) can be written now as

\[ \hat{F}_{\mu\nu} = \left( [i\hat{D}_\mu, i\hat{D}_\nu] \right) \Rightarrow (\hat{F}_{\mu\nu})^i_j \equiv [\hat{A}_\mu, \hat{A}_\nu]^i_j, \]  

(53)

where \( \hat{A}_\mu = \hat{P}_\mu + \hat{A}_\mu \), \( \hat{P}_\mu, \hat{A}_\mu \in \mathfrak{su}(N). \)

Action (51) can be written in the basis \( \{|\psi_i>\}_{i \in \mathbb{N}} \) as

\[ S_{\text{red}}^{(q)} = -\frac{1}{4} \left( \frac{2\pi}{\Lambda} \right)^d \frac{N}{g_d^2(\Lambda)} \hbar^2 \sum_j <\psi_j| \left\{ \left( \frac{1}{\hbar} \right) [\hat{A}_\mu, \hat{A}_\nu] \left( \frac{1}{\hbar} \right) [\hat{A}_\mu, \hat{A}_\nu] \right\} |\psi_j>. \]  

(54)

We can rewrite Eq. (54) in the following form

\[ S_{\text{red}}^{(q)} = \frac{1}{4} \left( \frac{2\pi}{\Lambda} \right)^d \frac{N}{g_d^2(\Lambda)} \hbar^2 \sum_j <\psi_j| \left\{ \left( \frac{1}{\hbar} \right) [\hat{A}_\mu, \hat{A}_\nu] \left( \frac{1}{\hbar} \right) [\hat{A}_\mu, \hat{A}_\nu] \right\} |\psi_j>. \]  

(55)

Now we arrive at the point where the Weyl-Wigner-Moyal formalism can be applied. By the Weyl correspondence \( \mathcal{W}^{-1} \) one gets the real function on the flat surface \( \Sigma \) defined as follows (see Eq. (32))
\[ A_\mu(\vec{\sigma}; \bar{h}) := W^{-1}(A) = \int_{-\infty}^{+\infty} <\sigma - \frac{\xi}{2}|\hat{A}_\mu|\sigma + \frac{\xi}{2} > \exp\left(\frac{i\tau\xi}{\bar{h}}\right) d\xi. \] \hspace{1cm} (56)

The action (55) transforms after a computation

\[ S_{red}^{(q)} = \frac{1}{4} \left(\frac{2\pi}{\Lambda}\right)^{d-1} N \frac{\bar{h}}{\Lambda g_0^2(\Lambda)} \int d^2\sigma \ F_{\mu\nu}^{(M)}(\vec{\sigma}; \bar{h}) \ast F^{(M)\mu\nu}(\vec{\sigma}; \bar{h}), \] \hspace{1cm} (57)

where

\[ F_{\mu\nu}^{(M)}(\vec{\sigma}; \bar{h}) = \{ A_\mu(\vec{\sigma}; \bar{h}), A_\nu(\vec{\sigma}; \bar{h}) \}_M. \] \hspace{1cm} (58)

Now we focus in the quenching prescription within the WWM-formalism. First we observe from the operational relation \( \hat{A}_\mu = \hat{P}_\mu + \hat{A}_\mu \) and the definition (56) that

\[ A_\mu(\vec{\sigma}; \bar{h}) = P_\mu(\vec{\sigma}; \bar{h}) + A_\mu(\vec{\sigma}; \bar{h}). \] \hspace{1cm} (59)

Operator valued quenched theory is still a \( \tilde{SU}(N) \) gauge theory and so operator-valued quenched gauge transformations (16a, b) and (17a, b) still hold. Using WWM-formalism we found that (16a) and (16b) are now

\[ A_\mu(\vec{\sigma}; \bar{h}) \to S(\vec{\sigma}; \bar{h}) \ast A_\mu(\vec{\sigma}; \bar{h}) \ast S^{-1}(\vec{\sigma}; \bar{h}) + S(\vec{\sigma}; \bar{h}) \ast \{ P_\mu(\vec{\sigma}; \bar{h}), S^{-1}(\vec{\sigma}; \bar{h}) \}_M, \] \hspace{1cm} (60a)

\[ a_\mu(\vec{\sigma}; \bar{h}) \to S(\vec{\sigma}; \bar{h}) \ast a_\mu(\vec{\sigma}; \bar{h}) \ast S^{-1}(\vec{\sigma}; \bar{h}), \] \hspace{1cm} (60b)

where now \( S(\vec{\sigma}; \bar{h}) \in W^{-1}(\tilde{SU}(N)) \equiv SU(N)_* \). The later is an infinite dimensional Lie Group which is defined as\(^{18}\)

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SU(N)⋆ := \{ S = S(\bar{\sigma}; \hbar) \in C^\infty(\Sigma)/S^{-\bar{1}}(\bar{\sigma}; \hbar) \ast S(\bar{\sigma}; \hbar) = 1, \\
S(\bar{\sigma}; \hbar) \ast S^{-\bar{1}}(\bar{\sigma}; \hbar) = 1; \quad \bar{S}(\bar{\sigma}; \hbar) = S^{-\bar{1}}(\bar{\sigma}; \hbar) \}. \quad (61)

where ‘bar’ stands for complex conjugation. Relations (17a,b) are written in WWM-formalism as

\[
A_\mu(x, \vec{\sigma}; \bar{\hbar}) \equiv \exp^\ast\left( \frac{i}{\hbar} P_\mu(\vec{\sigma}; \hbar)x^\mu \right) \ast A_\mu(\bar{\sigma}; \hbar) \ast \exp^\ast\left( -\frac{i}{\hbar} P_\mu(\vec{\sigma}; \hbar)x^\mu \right), \quad (62a)
\]

\[
S(x, \vec{\sigma}; \hbar) \equiv \exp^\ast\left( \frac{i}{\hbar} P_\mu(\vec{\sigma}; \hbar)x^\mu \right) \ast S(\bar{\sigma}; \hbar) \ast \exp^\ast\left( -\frac{i}{\hbar} P_\mu(\vec{\sigma}; \hbar)x^\mu \right), \quad (62b)
\]

where \( \exp^\ast\left( \frac{i}{\hbar} P_\mu(\vec{\sigma}; \hbar)x^\mu \right) \) is defined as \(^{20, 18}\)

\[
\exp^\ast\left( \frac{i}{\hbar} P_\mu x^\mu \right) := \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{i\hbar}{\hbar} \right)^n P \ast \ldots \ast P, \quad (n - \text{times}), \quad (63)
\]

where \( P(\vec{\sigma}; \hbar) = \sum_{n=0}^{\infty} \hbar^n P_n(\vec{\sigma}) \)^{21}.

Thus substituting Eqs. (62a,b) into (60a,b) and using (38) one can prove that the gauge transformation still holds

\[
A_\mu(x, \vec{\sigma}; \hbar) \rightarrow S(x, \vec{\sigma}; \hbar) \ast A_\mu(x, \vec{\sigma}; \hbar) \ast S^{-\bar{1}}(x, \vec{\sigma}; \hbar) + S(x, \vec{\sigma}; \hbar) \ast \partial_\mu S^{-\bar{1}}(x, \vec{\sigma}; \hbar), \quad (64)
\]

where \( S(x, \vec{\sigma}; \hbar) \) can be seen also as a smooth real function \( S : M_d \times \Sigma \rightarrow \mathbb{R} \).
At the quantum level the quenched constraint over the eigenvalues of $a_\mu$ and $P_\mu$ (19) translates to

$$a_\mu(\bar{\sigma}; \hbar) = V_\mu(\bar{\sigma}; \hbar) \ast P_\mu(\bar{\sigma}; \hbar) \ast V_\mu^{-1}(\bar{\sigma}; \hbar).$$  

(65)

Thus the quenched prescription on the eigenvalues is translated to the functional relation (65). This functional prescription can be implemented in the Feynman integral as

$$f(a_\mu(\bar{\sigma}; \hbar)) = \int \prod_\mu D V_\mu(\bar{\sigma}; \hbar) \delta \left( a_\mu(\bar{\sigma}; \hbar) - V_\mu(\bar{\sigma}; \hbar) \ast P_\mu(\bar{\sigma}; \hbar) \ast V_\mu^{-1}(\bar{\sigma}; \hbar) \right),$$  

(66)

where one have to integrate over the infinite dimensional subspace $\mathcal{L} \subset C^\infty(\Sigma)$ such the the $V$'s satisfy conditions (61). It is an easy matter to see, with the help of Eqs. (41), that one takes the limit $\hbar \to 0$ our quenched functional prescription corresponds exactly with large-$N$ quenched prescription of Bars\textsuperscript{9,10}. For instance Eq. (66) corresponds with $f(A)$ function which appears in Eq. (30).

Now we going to study the reduced quenched action (57). Expressing the Moyal bracket $\{A_\mu, A_\nu\}_M$ as a deformed Poisson bracket in the spirit of Strachan\textsuperscript{22}

$$\{A_\mu, A_\nu\}_M = \omega^{ij} \partial_i A_\mu(\bar{\sigma}; \hbar) \ast \partial_j A_\nu(\bar{\sigma}; \hbar),$$  

(67)

we define a $\ast$-deformed “world-sheet metric”

$$\hbar_{ij} \equiv \partial_i A(\bar{\sigma}; \hbar) \ast \partial_j A(\bar{\sigma}; \hbar).$$  

(68)

Now assume that this metric will transform as
\[ h_{ij}^{\star} = \omega_{lm} h_{lm}^{\star} \omega^{ij}. \]

(69)

The quenched quantum action (57) now reads proportional

\[ S_{\text{red}}^{(q)} \sim \int_{\Sigma} d^2 \sigma \ h^{\star} h_{ij} \ \partial_i A^\mu(\bar{\sigma}; h) \times \partial_j A_\mu(\bar{\sigma}; h). \]

(70)

This is the Moyal deformation of Schild-Eguchi action.

It can be shown that the above action can be derived from the “Moyal deformation of Polyakov’s action” (or quantum Polyakov)

\[ S_{\text{Pol}}^{(q)} \sim \int_{\Sigma} d^2 \sigma \sqrt{-h} h_{ij}^{\star} \partial_i A^\mu(\bar{\sigma}; h) \times \partial_j A_\mu(\bar{\sigma}; h), \]

(71)

where \( h \equiv \det(h_{ij}^{\star}) \) and \( \det: GL(2, \mathbb{R}) \to C^\infty(\Sigma, \mathbb{R}). \)

Following Strachan\(^{21}\) we assume that \( A_\mu(\bar{\sigma}; h) \) is an analytic function in \( h \) i.e.

\[ A_\mu(\bar{\sigma}; h) = \sum_{k=0}^{\infty} h^k A_\mu^{(k)}(\bar{\sigma}). \]

(72)

Taking the \( h \to 0 \) limit one can see that our quenched quantum action (70) can be reduced to the action (29) for the zero component \( A^{(0)}_\mu(\bar{\sigma}) \) of (72), i.e.

\[ \lim_{h \to 0} S_{\text{red}}^{(q)} = \int_{\Sigma} h^{ij} \partial_i A^{(0)}_\mu(\bar{\sigma}) \partial_j A^{(0)}_\mu(\bar{\sigma}) = S_{\text{red}}^{\infty}. \]

(73)

Finally we find that Feynman integral (30) is still valid in the context of WWM-formalism but besides to the usual moduli and gauge constraints there is an additional constraint on the space of \( A \)’s. It is that one must perform the functional
integration on those $\mathcal{A}$’s which admit Moyal deformation. Of course the limit $\hbar \to 0$ reproduces exactly Bars result (30).

At the present paper we have constructed another way to obtain the large-$N$ limit of reduced gauge action. We begin from the $\mathfrak{su}(N)$ reduced gauge theory and apply Weyl-Wigner-Moyal formalism and then the $\hbar \to 0$ limit. We find the Moyal deformation of the quenched gauge theory and in particular the Moyal deformation of Schild-Eguchi action, which in the $\hbar \to 0$ gives the Schild-Eguchi action. This result is valid for any underlying Riemann surface $\Sigma_g$ with ($g \geq 1$) for the phase space. Based in the recent result by Bars$^9$ we generalize the Moyal algebra for its appropriate use. Also we shown that the usual quenched prescription on the momenta$^{15}$ is translate to a functional ones (65,66). On the other hand we think that our formulation is more appropriate for the discussion of the large-$N$ limit and quantum mechanics analogies of the quenched gauge theory. This is because in Moyal algebra the limiting process ($\hbar \to 0$) is well defined unlike the limit $N \to \infty$ in matrix models. Moreover from a theoretical point of view it is more elegant to deal with a deformation of Poisson-Lie algebra than with the matrix algebras which depend on discrete parameter $N$. Following the arguments by Fairlie$^{23}$ concerning the the suitability of application of Moyal brackets to $M$ Theory, we believe that our approach can be applied$^{24}$ straightforward to the reduced matrix model$^{25}$ formulated recently in the context of the M(atrix)-Theory. It would be interesting to find the relation of our results and those found by Bars in Ref. (26) concerning SU($N$) gauge theory on discrete Riemann surfaces. Although our method needs an explicit Lie algebra homomorphism $\Psi : \mathfrak{su}(N) \to \hat{\mathfrak{su}}(N)$, which is not an easy matter, however explicit examples are given in the search of some solutions of self-dual Einstein equations for the Lie algebras $\mathfrak{su}(2)$ and $\mathfrak{sl}(2)^{6,17,18}$. This means that our method at least works for the case $N = 2$. The consideration for higher values of $N$ will be consider in a forthcoming paper.

Our results can be summarized in the next commuting diagram:
Finally the problem if the above results can be reproduced and generalized by using the geometrical framework of deformation quantization geometry used in Refs. 22 and 27 is still open.

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