Magnetic dipole–vortex interaction in a bilayer superconductor/soft-magnet heterostructure

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Recent progress in microfabrication technology has made hybrid structures composed of superconducting and ferromagnetic materials a very popular object of study. In particular, there were conducted many experimental and theoretical studies of the structures composed of superconductors (SC’s) and ferromagnetic dots (FD’s) (see, for example, Refs. [1, 2, 3] and references therein). Due to the large intrinsic magnetic moment of dots they generate diverse vortex configurations inside a SC layer [4, 5, 6, 7, 8, 9, 10, 11]. Also the dots act as effective trapping centres for the vortices causing in this way an enhancement of vortex pinning and increase of the critical magnetic field and of the critical current of a superconductor whereas the condition of the first penetration of a vortex in the superconducting constituent is found for two cases of the layer ordering, namely when the dipole is located near the superconducting or, respectively, the magnetic constituent.

We study the penetration of the nonuniform magnetic field, created by a magnetic dipole with out-of-plane magnetization, into a film heterostructure composed of a type-II superconductor layer and a soft-magnet layer. In the framework of the London approach, the energy of the magnetic dipole-vortex interaction is derived and the critical value of the dipole moment for the first appearance of a vortex in the superconducting constituent is found for two cases of the layer ordering, namely Ref. [2], we assume that thin layers of insulating oxide separate the magnetic and superconducting constituents of the heterostructure to avoid the proximity effect and exchange of electrons between them. We consider two cases of the layer ordering with respect to the MD position, namely when the SM layer is situated over the SC one (MD-SM/SC configuration, Fig. 1a) and the opposite arrangement (MD/SC SM configuration, Fig. 1b). Using the London approach, we study how parameters of the magnet layer – the thickness and the relative magnetic permeability – influence the interaction between the dipole and a single vortex in the SC layer and find the conditions for the first vortex appearance.

On the contrary, heterostructures of superconductors and soft magnets (SM’s) are studied for a short time. Soft magnets, such as Permalloy, pure iron, crooper, etc., have, as a rule, sufficiently large values of the relative permeability, very narrow hysteresis loop and possess negligible remanent magnetization. Nevertheless, they may significantly improve superconductor performance by effective shielding from the external magnetic field as well as from the transport current self-field [12, 13, 14, 15, 16, 17, 18, 19, 20].

In the present paper we consider a situation which combines both above structure types. Recently, it was shown theoretically that the magnet sheath can strongly enhance the Bean-Livingston barrier against the first entry of nonuniform magnetic field into a type-II superconductor whereas the condition of the first penetration of the uniform magnetic field is not influenced by such a sheath [21, 22]. The similar effect could be also expected in planar SC-FD heterostructures because of a strong inhomogeneity of the FD magnetic field. To clear up this question, we study the penetration of a nonuniform magnetic field, created by a small magnetic dot, into a model film structure of a finite thickness composed of a type-II SC layer and of a SM layer. Likewise some previous papers [3, 6, 12], we represent the dot as a point magnetic dipole (MD) positioned above the bilayer (Fig. 1). As was usually the case in the experiment (see, for example,
Here $\Phi_0$ is the flux quantum, $L$ is the vorticity which defines the number of flux quanta trapped by the vortex, and two-dimensional vector $\rho_v$ is the vortex position in the film plane. It is conveniently to represent the magnetic field above the heterostructure as $\mathbf{H}^{(0)} + \mathbf{h}_+$, where $\mathbf{H}^{(0)}$ is the direct contribution from the dipole and $\mathbf{h}_+$ is the field induced above the film by the supercurrent subject to the influence of the SM layer. The field $\mathbf{H}^{(0)}$ has the customary form

$$\mathbf{H}^{(0)}(\mathbf{r}) = \frac{1}{4\pi} \frac{3(\mathbf{m} \cdot \mathbf{r}) - \mathbf{m}((\mathbf{r} \cdot \mathbf{r}) \mathbf{r})}{|\mathbf{r}|^5},$$

where $\mathbf{r} \equiv \{x, y, z-a\}$ is a position vector with respect to the dipole. The magnetic field $\mathbf{h}_+$, the magnetic field below the film $\mathbf{h}_-$, the magnetic field inside the magnet layer $\mathbf{h}_M$ and the magnetic induction $\mathbf{b}$ in the whole space satisfy the Maxwell equations

$$\nabla \times \mathbf{h} = 0, \quad \nabla \cdot \mathbf{b} = 0. \quad (4)$$

We imply the existence of an insulating, nonmagnetic layer of thickness much less than $\lambda$, $d_M$, $d_S$ and $a$ between the superconductor and the magnet layer (for example, such a layer was experimentally observed in MgB$_2$/Fe wires [20]). According to this assumption, the boundary conditions for the normal (n) and tangential (t) field components, applied on the SC/SM interface, read

$$b_{S,n} = \mu_0 \mu h_{M,n}; \quad b_{S,t} = \mu_0 h_{M,t}. \quad (5)$$

Also, we apply the conventional boundary conditions [24] on the outer surfaces of the heterostructure, namely a continuity of the magnetic induction on the SC/vacuum interface and a continuity of normal components of the induction as well as tangential components of the magnetic field on the SM/vacuum interface.

To find the condition for the first vortex appearance, we consider the excess Gibbs energy of the heterostructure due to presence of the single vortex in the SC layer,

$$G = F_v - \int dV_M (h_v \cdot \mathbf{M}). \quad (6)$$

Here $F_v$ is the self-energy of a vortex, the second term describes the energy of interaction between the vortex and the magnetic dipole [24], where the integration is performed over the dipole volume $V_M$, $h_v$ is the magnetic field of the vortex in the dipole position, and $\mathbf{M} = \{0, 0, -\mu_0 m\}$ $\delta(x) \delta(y) \delta(z-a)$ is the dipole magnetization. The vortex self-energy takes the form [22]

$$F_v = \frac{1}{2\mu_0} \int dV_S (b_v \cdot Q) + \frac{1}{2} \int dS_S [\psi_{out}(Q \cdot n)]_{S_S}, \quad (7)$$

where the integration is performed over the volume $V_S$ and the surface $S_S$ of the SC layer, respectively, $n$ denotes the outer normal to the surface of the SC region, $\psi_{out}$ is the scalar potential of the vortex magnetic field outside the SC layer. After minimization of the Gibbs energy $G$ with respect to the vortex position $\rho_v$, the condition $G = 0$ determines the critical value of the dipole moment $m_{c1}$ for the first vortex appearance in the SC layer.

The free self-energy of the single vortex located in the SC layer is described in both considered configurations by the expression

$$F_v = \frac{(L\Phi_0)^2}{4\pi\mu_0\lambda} \left[\frac{d_S}{\lambda} \ln \left(\frac{\lambda}{\xi}\right) + \int_0^\infty dq \frac{\Delta_1}{k^3}\Delta_1\right], \quad (8)$$

with

$$\Delta_1 = 2\mu \left[k + g \tanh \left(\frac{k d_S}{2\lambda}\right)\right] + \tanh \left(\frac{q d_M}{\lambda}\right) + \left[(\mu^2 + 1) k + 2g \tanh \left(\frac{k d_S}{2\lambda}\right)\right], \quad (9)$$

FIG. 1: Scheme of the first vortex appearance in the bilayer heterostructure composed of the SC layer and the SM layer exposed by the field of the point magnetic dipole positioned over the SC layer (a) and over the SM layer (b).
FIG. 2: The dependence of the vortex free energy $F_v$ on the thickness $d_M$ and on the relative permeability $\mu$ of the SM layer.

\[
\Delta = \mu \left[ k + q \tanh \left( \frac{kd_s}{2\lambda} \right) \right] \\
\times \left[ k + q \coth \left( \frac{kd_s}{2\lambda} \right) \right] + \tanh \left( \frac{qd_M}{\lambda} \right) \\
\times \left( \mu^2 + 1 \right) kq \coth \left( \frac{kd_s}{\lambda} \right) + \mu^2 k^2 + q^2 \right].
\]  
(10)

Here $k = (1 + q^2)^{1/2}$ and $\xi$ is the superconductor coherence length. The calculated dependence of $F_v$ on $d_M$ and $\mu$ normalized on the value of this energy $F_v^{SC}$ for the unshielded SC film is shown in Fig. 2 for the case of $d_S = \lambda$ and $\lambda/\xi = 28$. One can see that due to the presence of SM layer the self-energy of the vortex slightly decreases. This change reaches the maximum value for the case of $d_S \approx (0.1 \div 1) \lambda$ and becomes smaller when $d_S \ll \lambda$ or $d_S \gg \lambda$.

Contrary to the free energy of the vortex, the influence of SM layer on the interaction between the magnetic dipole and the vortex, situated in the point with the position vector $\rho_v$, can be significant and is different for two considered geometries of the system. For the MD-SM/SC configuration the energy of the dipole-vortex interaction $F_{dv}^{(II)}$ takes the form

\[
F_{dv}^{(II)} = -\int dV_{MD} (h_v \cdot M)
\]

where

\[
F_{dv}^{(I)} = -\frac{L\Phi_0 m\mu}{2\pi\lambda^2} \int_0^\infty dq \frac{q\Delta_2}{k} J_0 \left( \frac{qd_v}{\lambda} \right) \exp \left[ -\frac{q(a-d_M)}{\lambda} \right],
\]  
(11)

and $J_0$ denotes the Bessel function of the 0th order. A crude approximation of this expression has shown that in the limit $\mu \gg 1$ the energy $F_{dv}^{(I)}$ decreases as $\mu^{-1}$. For the MD-SC/SM configuration the interaction energy reads

\[
F_{dv}^{(II)} = -\frac{L\Phi_0 m\mu}{2\pi\lambda^2} \int_0^\infty dq \frac{q\Delta_3}{\lambda} J_0 \left( \frac{qd_v}{\lambda} \right) \exp \left[ -\frac{q(a-d_S)}{\lambda} \right],
\]  
(13)

with

\[
\Delta_3 = \mu \left[ k + q \tanh \left( \frac{kd_S}{2\lambda} \right) \right] \\
+ \tanh \left( \frac{qd_M}{\lambda} \right) \left[ \mu^2 k^2 + q^2 \right],
\]

and in the limit of $\mu \gg 1$ has the finite value independent of $\mu$. Such a behavior of the dipole-vortex interaction energy causes the different conditions for the first vortex appearance in the SC layer.

Minimizing the Gibbs energy with respect to the vortex position we obtain that in both geometries the equilibrium position of the vortex is $\rho_v = 0$, i.e. the vortex is situated exactly under the dipole as it is shown in Fig. [I]. So, the condition of the first vortex appearance in the superconductor is determined by the equation

\[
\Delta + F_{dv} (\rho_v = 0) = 0.
\]

The calculated dependences of the magnetic moment $m_{c1} (d_M, \mu)$, normalized on the value of this moment $m_{c1}^{SC}$ for the unshielded SC film, are shown in Fig. [II] for the heterostructure with $d_S = \lambda$, $a = 5\lambda$ and $\lambda/\xi = 28$. One can see that in the case of MD-SM/SC configuration the moment $m_{c1}$ monotonically increases with increase of both $\mu$ and $d_M$ (Fig. [II,a]). With increase of $\mu$ at fixed thickness $d_M$ the $m_{c1} (\mu)$ dependence approaches to the linear one corresponding to the above mentioned limiting case $\mu \gg 1$. Notice, that such a behavior is qualitatively similar to the same dependences of the field of first magnetic flux entry into the magnetically shielded SC filament where the local magnetic field near the place of flux entry is also nonuniform. Therefore, in the MD-SM/SC configuration the SM layer can significantly prevent the penetration of vortices into the superconductor preserving the latter in the Meissner state. The $m_{c1} (d_M, \mu)$ dependence for the case of MD-SC/SM configuration (Fig. [II,b]) is quite different. The moment $m_{c1}$ monotonically decreases with increase of $\mu$ and/or $d_M$ and reaches the finite minimum value. So, in this case the presence of SM constituent decreases the magnetic moment $m_{c1}$ and promotes to the earlier appearance of the vortex in the SC layer.

The obtained difference of the dipole-vortex interaction in the considered cases can be explained in the following way. As it follows from Eq. (11), the interaction energy $F_{dv}$ is determined by the value of normal component of the magnetic field created by the vortex in the dipole position. We found that in the MD-SM/SC configuration this field is significantly reduced ($\mu$ times in the $\mu \gg 1$ limit) whereas in the MD-SC/SM configuration the field...
of the vortex in the region over the heterostructure is changed only slightly. Such a difference causes the corresponding striking change in the interaction.

Notice, that all dependences shown in Figs. 2 and 3 do not depend explicitly on the value of vorticity $L$ and, therefore, they will demonstrate the same behavior with respect to $d_M$ and $\mu$ when a vortex (or an antivortex) with arbitrary vorticity is located in the SC layer.

Though in the geometry of Fig. 3 b the SM layer promotes the earlier penetration of vortices into the superconductor, another effect occurs in this case allowing to improve the superconducting properties of the heterostructure. The dependence of the interaction energy $F_{dv}^{(II)}$, measured in units of $F_0 = L\Phi_0 m / 2\pi \lambda^2$, on the vortex position $\rho_v$ is shown in Fig. 4 for the different parameters of the heterostructure. It is clearly seen that the depth of the potential well for the vortex in the field of the dipole increases when the SM layer is present. Therefore, in the case of MD-SC/SM configuration the presence of the SM layer may enhance the single vortex pinning by the dipole.

In conclusion, we have studied in the framework of the London approach how the nonuniform magnetic field, created by the point magnetic dipole with a moment $m$, penetrates into the film heterostructure composed of a type-II SC layer and of a SM layer. We have derived the critical value of the dipole moment for the first appearance of a vortex and demonstrate that the presence of SM layer can effectively improve the superconducting properties of the heterostructure. On the one hand, the SM layer shields the SC constituent from the dipole field keeping the latter in the Meissner state when the dipole is situated above the SM constituent. On the other hand, when the dipole is positioned over the SC constituent the SM layer enhances the pinning of the vortex by the dipole. Finally, though we have restricted ourselves to the simplest model when the point dipole creates only one straight vortex in the superconductor, we suppose that, in more realistic cases, such as a creation of vortex-antivortex structures in the superconductor or heterostructures composed of superconductors and dots of finite sizes, the similar influence of the soft-magnet layer could also take place.

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[1] I. F. Lyuksyutov and V. L. Pokrovsky, Adv. Phys. 54, 67 (2005).
[2] M. Lange, M. J. Van Bael, and V. V. Moshchalkov, J. Low Temp. Phys. 139, 195 (2005).
[3] M. V. Milošević and F. M. Peeters, J. Low Temp. Phys. 139, 257 (2005).
[4] I. K. Marmorkos, A. Matulis, and F. M. Peeters, Phys. Rev. B 53, 2677 (1996).
[5] J. I. Martín, M. Vélez, A. Hoffmann, I. K. Schuller, and J. L. Vicent, Phys. Rev. Lett. 83, 1022 (1999).
[6] M. V. Milošević, S. V. Yampolskii, and F. M. Peeters, Phys. Rev. B 66, 174519 (2002).
[7] M. V. Milošević and F. M. Peeters, Phys. Rev. B 68, 024509 (2003).
[8] M. V. Milošević and F. M. Peeters, Phys. Rev. B 68, 094510 (2003); 69, 104522 (2004).
[9] M. V. Milošević and F. M. Peeters, Phys. Rev. Lett. 93, 267006 (2004).
[10] D. S. Golubović, M. V. Milošević, F. M. Peeters, and V. V. Moshchalkov, Phys. Rev. B 71, 180502(R) (2005).
[11] S. Erdin, Phys. Rev. B 72, 014522 (2005).
[12] S. L. Cheng and H. A. Fertig, Phys. Rev. B 60, 13107 (1999).
[13] L. N. Bulaevskii, E. M. Chudnovsky, and M. P. Maley, Appl. Phys. Lett. 76, 2594 (2000).
[14] M. Lange, M. J. Van Bael, Y. Bruynseraede, and V. V. Moshchalkov, Phys. Rev. Lett. 90, 197006 (2003).
[15] Yu. A. Genenko, A. Usoskin, and H. C. Freyhardt, Phys. Rev. Lett. 83, 3045 (1999).
[16] Yu. A. Genenko, A. Snezhko, and H. C. Freyhardt, Phys. Rev. B 62, 3453 (2000).
[17] M. Majoros, B. A. Glowacki, and A. M. Campbell, Physica C 334, 129 (2000); 338, 251 (2000).
[18] Yu. A. Genenko, H. Rauh, and A. Snezhko, Supercond. Sci. Technol. 14, 699 (2001).
[19] Yu. A. Genenko, H. Rauh, and A. Snezhko, Physica C 372-376, 1389 (2002).
[20] A. V. Pan and S. X. Dou, J. Appl. Phys. 96, 1146 (2004).
[21] Yu. A. Genenko, H. Rauh, and S. V. Yampolskii, J. Phys.: Condens. Matter 17, L93 (2005).
[22] S. V. Yampolskii and Yu. A. Genenko, Phys. Rev. B 71, 134519 (2005).
[23] P. G. de Gennes, Superconductivity of Metals and Alloys (New York, Addison-Wesley, 1994).
[24] L. D. Landau and E. M. Lifshitz, Electrodynamics of Continuous Media (Pergamon, Oxford, 1989).