Dynamics of Cosmic Necklaces

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Abstract

We perform numerical simulations of cosmic necklaces (systems of monopoles connected to two strings each) and investigate the conditions under which monopoles annihilate. When the total monopole energy is large compared to the string energy, we find that the string motion is no longer periodic, and thus the strings will be chopped up by self intersection. When the total monopole energy is much smaller than the string energy, the string motion is periodic, but that of the monopoles is not, and thus the monopoles travel along the string and annihilate with each other.

1 Introduction

Topological defects are a natural consequence of phase transitions in the early universe \cite{1} and are as such predicted by a number of Grand Unified Theories. They have received much attention in the past two decades, having been considered promising candidates for a number of interesting cosmological phenomena including gravity waves, baryon asymmetry, density perturbations, non-gaussianity in the cosmic microwave background and ultra-high energy cosmic rays. For a review see \cite{2}.

Topological defects can occur when some high energy particle physics symmetry group is broken to a smaller symmetry group. They are formed by the Kibble mechanism, in which regions of space that are uncorrelated (at a minimum, those that are outside the causal horizon) must independently choose vacuum states. The type of defect formed depends on the topology of the manifold of equivalent vacuum states.

Cosmic necklaces, in particular, are hybrid topological defects that can be produced in the sequence of phase transitions

\[ G \xrightarrow{\eta_m} H \times U(1) \xrightarrow{\eta_s} H \times Z_2 , \]
where $G$ is a semi-simple group. In the first phase transition, at an energy $\eta_m$, monopoles of mass $m \sim \eta_m/e$ are produced. The second transition, occurring at energy $\eta_s$, traps the magnetic flux into two strings of mass per unit length $\mu \sim \eta_s^2$ connecting monopoles to anti-monopoles. The resulting system is a network of long strings and loops with the monopoles playing the role of beads.

Previous work by Berezinsky and Vilenkin [3] was done with the idea of explaining the origin of the highest energy cosmic rays. The rather complicated underlying dynamics were, for good reason, largely ignored. They found that if one started with a low enough density of monopoles, such that one could approximate the evolution of the system using simple string evolution, and more importantly, if one could disregard the effects of monopole-antimonopole annihilation, the density of monopoles on the string would naturally increase to the point where the approximation of simple string evolution would break down. The monopole density could also be increased with a sufficiently long damping era following the formation of the network. Furthermore, they noted that a large enough density of monopoles would make loop motion non-periodic and therefore loop fragmentation very efficient. Thus, in their scenario, the network consists of a long string with monopoles in the high monopole density regime. The string intercommutes to form loops that rapidly fragment and yield cosmic rays by the annihilation of the monopoles. The estimated cosmic ray energies and fluxes produced by this model seem to fit current observations. They did leave, however, the detailed analysis of the evolution of these systems to numerical simulations and, in particular, the verification that monopole-antimonopole annihilation can initially be disregarded.

In this work we make a first attempt at understanding the dynamics of necklaces by numerically evolving single loops.

In the next section we review general string motion for completeness and derive useful results used in the rest of our paper. In the third section we look at monopole motion on cosmic necklaces when the energy of the monopoles is comparable to the energy in the string. In the fourth section we examine dynamics and monopole annihilations in the limit where the monopole energy is low compared to the string energy. We summarise and conclude in the fifth section.

2 Review of String Motion

When the typical length scale of a cosmic string is much larger than its thickness, $\delta_s \sim \eta_s^{-1}$, and when there are no long-range interactions between different string segments, as is the case for gauge strings, the string can be accurately modeled by a one dimensional object. Such an object sweeps out a two dimensional surface referred to as the string world-sheet. This surface can be described by a function of two parameters, a timelike parameter $t$ which can be identified with the time coordinate, and a spacelike parameter $\sigma$, 

$$x^\mu = x^\mu(t, \sigma).$$
The infinitesimal line element in Minkowski space-time with metric \( \eta_{\mu\nu} = \text{diag}(1, -1, -1, -1) \) is

\[
ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu = \eta_{\mu\nu} x^\mu_a x^\nu_b d\xi^a d\xi^b,
\]
where \( a = 0, 1 \) labels the internal parameters, \( t = \xi^0, \sigma = \xi^1 \) and \( x^\mu_a = \partial x^\mu / \partial \xi^a \).

One can then write the induced metric on the world-sheet of the string as

\[
\gamma_{ab} = \eta_{\mu\nu} x^\mu_a x^\nu_b.
\]

(1)

For an infinitely thin string we can use the Nambu-Goto action. It is proportional to the invariant area swept by the string,

\[
S_s = -\mu \int dA = -\mu \int d^2 \xi \sqrt{-\gamma},
\]

(2)

where \( \gamma = \det(\gamma_{ab}) \) and \( \mu \) is the mass per unit length of the string. In geometrical terms this is the action for a 1+1 dimensional space-time with a cosmological constant \( \mu \). Its variation with respect to \( x^\mu \) gives the equation

\[
\delta S_s = -\mu \int \partial_a(\sqrt{-\gamma} \gamma^{ab} x^\mu_b \delta x^\mu_a) d^2 \xi + \mu \int \partial_a(\sqrt{-\gamma} \gamma^{ab} x^\mu_b) \delta x^\mu_a d^2 \xi = 0.
\]

(3)

The first term in this equation is an integral of a total divergence that can be turned into boundary terms at the ends of the string. For ordinary strings these terms vanish because of the absence of boundaries. In the case of cosmic necklaces however, monopoles live on the boundaries of strings and so these turn out to be the most interesting terms. The second term gives the usual equations of motion for the Nambu-Goto string. If we work, as usual, in the conformal gauge,

\[
x'(\sigma, t) \cdot \dot{x}(\sigma, t) = 0
\]

(4)

\[
x'^2(\sigma, t) + \dot{x}^2(\sigma, t) = 1,
\]

(5)

and choose \( t = x^0 \), the equation of motion is

\[
x''(\sigma, t) = \ddot{x}(\sigma, t).
\]

(6)

Primes and dots denote partial derivatives with respect to \( \sigma \) and \( t \) respectively. This equation can be readily solved using

\[
x(\sigma, t) = \frac{1}{2} [a(\sigma - t) + b(\sigma + t)],
\]

(7)

then

\[
x'(\sigma, t) = \frac{1}{2} [a'(\sigma - t) + b'(\sigma + t)]
\]

(8)

\[
\dot{x}(\sigma, t) = \frac{1}{2} [-a'(\sigma - t) + b'(\sigma + t)]
\]

(9)
Figure 1: An anti-monopole attached to two monopoles. The string sources ($\sigma_{1-}$) are at the anti-monopoles and the ends ($\sigma_{1+}$) at monopoles.

and the constraints coming from (3,5) become

$$a^2(\sigma - t) = b^2(\sigma + t) = 1. \quad (10)$$

The functions $a(\sigma - t)$ and $b(\sigma + t)$, often referred to as right- and left-movers or halves of the string, are constant along the lines $\sigma = t$ and $\sigma = -t$ respectively on the string world-sheet.

3 Monopole Motion: The Massive Case

3.1 Equations of Motion

Assuming we can treat the monopoles as point particles living on the string and there are no unconfined magnetic charges, the motion for a monopole attached to two strings can be described by the action

$$S = -m \int ds - \mu \sum_{i=1}^{2} \int_{\sigma_{1-}(t)}^{\sigma_{1+}(t)} dA_i.$$

The first term is the standard action for a relativistic particle of mass $m$, the monopole, $\mu$ is the mass per unit length of the string and $A_i$ are the areas swept by each of the strings attached to the monopole. The integral over the string world-sheet is bounded by the parameters $\sigma_{1-}(t)$ and $\sigma_{1+}(t)$, the monopole parametric positions on the $i$th string. In our convention the string sources are at anti-monopoles and the ends at monopoles (see Fig. [1]). Here the number of strings attached is just two but our action can clearly be generalized to any number of strings by adding new string terms in the sum over $i$. From (3) one
can see the variation of the string part of the action yields the usual Nambu-Goto term and a total divergence. This divergence can be turned into two integrals along the world-lines of the monopoles bounding the string by the 1+1 dimensional analog of Gauss’ theorem 

\[
\int \partial_a (\sqrt{-\gamma^{ab} x^\mu_b \delta x^\mu_a}) d^2 \xi = - \int \lambda_a^- \gamma^{ab} x^\mu_b \delta x^\mu_a ds^- - \int \lambda_a^+ \gamma^{ab} x^\mu_b \delta x^\mu_a ds^+ ,
\]

where the superscripts + and − refer to monopoles and anti-monopoles respectively. The unit vector \( \lambda_a \) is orthogonal to the world-line of the monopole and points into the string world-sheet. In external coordinates it can be written

\[
\lambda_\mu(t) = \lambda_a \gamma_{ab} x^\mu_b (t),
\]

where \( \gamma_m(t) = (1 - \dot{x}_m^2(t))^{-1/2} \) is the Lorentz factor of the monopoles, and the upper sign is for anti-monopoles and the lower for monopoles. These boundary terms can be absorbed into the monopole action to give the equations of motion for the monopoles

\[
m \dot{x}_m(t) = - \mu \sum_{i=1}^2 \lambda_i^\mu(t),
\]

where the \( i = 1, 2 \) labels the strings to which the monopole is attached. The 0-component of this equation,

\[
m \dot{\gamma}_m(t) = \mu \sum_{i=1}^2 \dot{\sigma}_i(t),
\]

where \( \dot{\sigma}_i(t) \) is the rate of change of the position of the monopole on the \( i \)th string, is the equation for energy conservation. The monopoles can create string and lose kinetic energy in the process, or annihilate string and gain kinetic energy.

The spatial part in turn can be put in the form

\[
\dot{\mathbf{x}}_m(t) = \frac{\mu}{m} \gamma_m^{-3}(t) \sum_{i=1}^2 \mathbf{x}'_i(\sigma_i(t), t) \left| \mathbf{x}'_i(\sigma_i(t), t) \right|^2 = \frac{\mu}{m} \gamma_m^{-3}(t) \sum_{i=1}^2 \gamma_i(t) \hat{n}
\]

where \( \gamma_i(t) \) is the gamma factor of the string at the position of the monopole, \( \mathbf{x}'_i(\sigma_i(t), t) \) is the tangent vector of the string at the monopole, and \( \hat{n} \) is a unit vector tangent to the string pointing inwards. A similar result was obtained in [4] for the case of strings bounded by monopoles.

The rate of change of monopole position along the string is related to the energy balance of the system. A more useful expression than (14) can be obtained taking the time derivative of the constraint that the monopole must lie on the string, \( \mathbf{x}_m(t) = \mathbf{x}_i(\sigma(t), t) \),

\[
\dot{\mathbf{x}}_m(t) = \frac{d \mathbf{x}_i(\sigma_i(t), t)}{dt} = \dot{\sigma}_i(t) \mathbf{x}'_i(\sigma_i(t), t) + \dot{\mathbf{x}}_i(\sigma(t), t).
\]

Using the constraints (4) and (5), (14) can be re-written as

5
\[ \dot{\sigma}_i(t) = \frac{\dot{x}_m(t) \cdot x'_i(\sigma_i(t), t)}{|x'_i(\sigma_i(t), t)|^2}, \]  

which expresses the rate of change of the monopole parametric position along each of the two strings individually.

The equations of motion for the monopoles, (13) and (13), and for the string, (4), (5) and (6), could be solved to give the motion of our system. Except for very special initial conditions, however, the task of finding analytic solutions to this system of equations is hopeless, so we resort to numerical simulation.

### 3.2 The Numerical Algorithm

Although we cannot solve the entire system analytically, we do have an explicit solution for the interior of the string, (7). We can use this solution to give the string motion, taking as input the motion of the monopoles at the ends of the string, which we will simulate. The effect of the string will be to take excitations from the monopole (or anti-monopole) at one end and transport them to affect the anti-monopole (or monopole) at the other. We proceed as follows.

We consider, for simplicity, only a monopole attached to an anti-monopole by a string, and ignore the effects of the strings attached on the other side of either defect. Since the monopole is constrained to live on the boundary \( \sigma_i(t) \) of the string, its position in terms of the right- and left-movers can be written

\[ \mathbf{x}_m(t) = \mathbf{x}_i(\sigma_i(t), t) = \frac{1}{2} [\mathbf{a}_i(\sigma_i(t) - t) + \mathbf{b}_i(\sigma_i(t) + t)]. \]

Its velocity is then given by

\[ \dot{\mathbf{x}}_m(t) = \frac{d\mathbf{x}_i(\sigma_i(t), t)}{dt} = \dot{\sigma}_i(t) \dot{\mathbf{x}}'_i(\sigma_i(t), t) + \dot{\mathbf{x}}_i(t, t). \]

which can be expressed in terms of the right- and left-movers,

\[ \dot{\mathbf{x}}_m(t) = \frac{1}{2} \{ [\dot{\sigma}_i(t) - 1] \mathbf{a}'_i(\sigma_i(t) - t) + [\dot{\sigma}_i(t) + 1] \mathbf{b}'_i(\sigma_i(t) + t) \}. \]

For the anti-monopole, we can interpret this expression as giving the value of \( \mathbf{a}'_i \), the excitations that travel away from the defect, in terms of \( \dot{\mathbf{x}}_m \), the monopole velocity, and \( \mathbf{b}'_i \), the excitations traveling toward the defect. Similarly for the monopole, (13) gives \( \mathbf{b}'_i \) in terms of \( \dot{\mathbf{x}}_m \) and \( \mathbf{a}'_i \). The outgoing excitations \( \mathbf{a}'_i(\sigma_i(t) - t) \) and \( \mathbf{b}'_i(\sigma_i(t) + t) \) then propagate along the string segment into the future, and eventually reach the other end (see Fig. 2).

The equations for the system, after some algebra, can be cast in the form

\[ \dot{\sigma}_i(t) = \frac{\dot{x}_m^2(t) + \dot{\mathbf{x}}_m(t) \cdot \dot{\mathbf{a}}'_i(\sigma_i(t) - t)}{\dot{\mathbf{x}}_m(t) \cdot \dot{\mathbf{a}}'_i(\sigma_i(t) - t) + 1}, \]

\[ \mathbf{b}'_i = \frac{2\dot{x}_m - [\dot{\sigma}_i(t) - 1] \mathbf{a}'_i(\sigma_i(t) - t)}{\dot{\sigma}_i(t) + 1}. \]
Figure 2: The excitations on the string travel from monopole to anti-monopole and vice-versa, getting transformed with each bounce.

for the monopole, and

\[ \dot{\sigma}_i(t) = \frac{\ddot{x}_m(t) - \dot{x}_m(t) \cdot b'_i(\sigma_i(t) + t)}{\dot{x}_m(t) \cdot b'_i(\sigma_i(t) + t) - 1} \quad (21) \]

\[ a'_i = \frac{2\ddot{x}_m - [\dot{\sigma}_i(t) + 1]b'_i(\sigma_i(t) + t)}{\dot{\sigma}_i(t) - 1} \quad (22) \]

for the anti-monopole. These equations, along with (13) and

\[ x'_i(\sigma_i(t), t) = \frac{1}{2}[a'_i(\sigma_i(t) - t) + b'_i(\sigma_i(t) + t)] \]

can be used to solve for the motion of any system of this type.

In a typical time-step of our code at, say, an anti-monopole, we have its velocity, \( \dot{x}_m(t) \), and the values of \( \sigma_1(t) \) and \( \sigma_2(t) \), its parametric positions on the strings. We also have a sequence of values of \( b'_i \) emitted at past times from the other end of the string (see Fig. 2). By interpolation, we can find \( b'_i(\sigma_i(t) + t) \) and use it to compute \( \dot{\sigma}_i(t) \) and \( a'_i \) using (21) and (22). If we do this for both strings we can compute the acceleration according to (13). We can then use the acceleration and the \( \dot{\sigma}_i(t) \) we just calculated to compute the velocity and the \( \sigma_i \) at the next time-step (with an appropriate finite differencing scheme). This is what we started with and therefore all we need to continue onto the next timestep. It is, of course, necessary to store enough of the \( a'_i \) and \( b'_i \) to ensure we can perform the appropriate interpolations at every time-step. We use energy conservation, (12), as an independent check on the accuracy of our code (for example see Fig. 3).

The reward for our efforts is that in a numerical computation we can ignore the presence of the strings entirely and simply store the outward going string...
Figure 3: The position of the string can be readily reconstructed at any time $t_0$ by starting one end of the string, finding a value $\sigma_s$ on the string, interpolating for the values of $a'$ and $b'$ using $\sigma_s - t_0$ and $\sigma_s + t_0$ respectively and then integrating to find $a$ and $b$.
values of the dimensionless parameter $\mu L/Nm \sim 1$. As was anticipated, one of the most important effects is the non-periodicity of the solutions which has rather notable consequences, making the evolution of these systems very different from that of ordinary cosmic strings. In particular, the non-periodic nature of the solutions allows the system to explore the phase space. Since there are many more ways in which the string can be wiggly than straight it ends up being mostly wiggly; this is a vastly different situation from that of regular cosmic strings where the loops can get trapped in smooth low entropy configurations and never explore the phase space because of the periodicity of their motion.

Here we only show the results for one of the loop configurations we have evolved. However, the result is typical of all our other simulations and the difference between the usual cosmic string evolution and cosmic necklace evolution is very well illustrated.

Figs. 4, 5 and 6 show the evolution of a necklace using as initial conditions a Kibble-Turok loop with values for the loop parameters of $\kappa = 0.7$ and $\phi = \pi/7$, twelve monopoles and a value for the dimensionless parameter $\mu L/Nm = 5$. One can see from Fig. 5 that the monopole and string energies go through a series of maxima and minima. Bearing in mind that the motion is not truly periodic we shall nevertheless abuse the language and refer to these as oscillations.

In the early stages of evolution, when the strings are still quite straight, the monopoles can easily gain kinetic energy from the strings. However, after a
Figure 5: The energy in units of the monopole mass for the necklace loop shown in Fig. 4, as a function of time. The bottom line is the monopole energy, the second line is the string energy and the top line is the total energy. The system has been evolved for a time $t = 10000 = P\delta$, where $P$ is the number of timesteps in the code. During this time it executes approximately 300 ‘oscillations’ where what we mean by ‘oscillations’ is number of local maxima of the monopole (or string) energy. The energy is very well conserved: At the end of our simulation $\Delta E/E \sim 10^{-4}$. 
Figure 6: Plot of the same loop as in Fig. 4 at $t = 10000$, after 300 'oscillations'. Notice the size of the loop is considerably smaller: A substantial amount of the original string length has formed wiggles.

few oscillations, thirty or so, the strings become so wiggly that the monopoles can no longer gain significant amounts of kinetic energy from them. While the details of the mechanism for wiggle formation on the string elude us it is clear that they arise from the non-linear back-reaction of the monopoles on the string [20, 22]. After the wiggles are formed the energy of the monopoles is mostly just their rest mass. We have observed the behaviour described here in all loops in our simulations with values of the dimensionless parameter $\mu L/Nm \sim 1$.

Although we have not included self-intersections in our code, it seems clear that had we included them the loop would have fragmented well before reaching the wiggly state shown in Fig. 6. While the exact timescale is not obvious the general evolution is unambiguous. Because of the non-periodicity of necklace motion, necklace loops fragment into smaller daughter loops; some containing monopoles and others not. The ones not containing monopoles evolve like ordinary Nambu-Goto string loops: They self-intersect until they find themselves in stable trajectories and then decay by gravitational radiation. The ones that do contain monopoles self-intersect again until only monopole-antimonopole pairs are left on each of the loops. These pairs then radiate small loops and gravitational radiation until they decay and the monopole and anti-monopole annihilate, along the lines discussed in [4, 7].
4 Monopole Motion: The Massless Limit

The starting regime for the necklace network proposed by Berezinsky and Vilenkin \[3\] is the limit where the string energy is much larger than the monopole energy. Unless there is a sufficiently long damping era following network formation we expect the dynamics to be quite different from what we have described in the previous section.

Because of the equivalence of loops with equal values of the dimensionless parameter $\mu L/Nm$ the large string energy limit can be thought of as the limit where the monopole mass $m$ is negligible compared with the mass per unit length of the string $\mu$. Unfortunately, we can no longer use the equations derived in the previous section because they do not have a well defined $m \to 0$ limit.

It turns out to be convenient to think of the monopoles as test particles constrained to live on a 1+1 dimensional dynamical space-time, the string. In this case the monopoles do not back-react on the string and therefore also do not produce small-scale structure so we expect them to get accelerated by the curvature of the string and eventually collide and annihilate.

The motion of the string, being unaffected by the presence of the test particles, is given by some solution of the Nambu-Goto equations of motion. The monopole motion can in turn be derived by minimising the 1+1 action

$$S_m = -m \int ds = -m \int \sqrt{\gamma_{ab} \xi^a d\xi^b}$$

which is the world-line of the monopole on the string world-sheet. One can re-write this action by explicitly evaluating $\gamma_{ab}$ in the conformal gauge, yielding

$$S_m = -m \int \sqrt{(1 - \dot{x}_m^2) dt^2 - \dot{x}_m^2 d\sigma^2} = -m \int dt \sqrt{1 - \dot{x}_m^2}$$

where

$$\dot{x}_m(t) = \frac{d x_s(\sigma(t), t)}{dt} = \dot{\sigma}(t) x'_s + \dot{x}_s$$

is the monopole velocity. It should be noted that this action could have also been derived using the constraint $x_m(t) = x_s(\sigma(t), t)$ and taking the action to be the world-line of the monopole in 3+1 Minkowski space instead of the 1+1 world-sheet of the string. We feel, however, that the derivation outlined above makes the degrees of freedom of the monopole motion more manifest. The only degrees of freedom of the monopole are its parametric position $\sigma(t)$ and velocity $\dot{\sigma}(t)$ along the string.

This action yields the equation for the motion of the monopole along the string which, after some algebra, can be put in the form

$$\ddot{\sigma}(t) = (\dot{\sigma}(t) - 1)^2 \frac{x'_s(\sigma, t) \cdot \{\dot{x}_s(\sigma, t) + \dot{\sigma}(t) x'_s(\sigma, t)\}}{|x'_s(\sigma, t)|^2}$$

(23)
Figure 7: Plot of the parametric position $\sigma(t)$ for twelve monopoles on a Kibble-Turok loop with a length $L = 60$ and values for the parameters the same as in Fig. 4. The monopoles’ initial parametric position is not quite the same as in the previous figure because we want to avoid degeneracies arising from the symmetry of loop motion. The string is periodic in its length and the monopoles that cross the bottom boundary end up at the top. As can be seen from the figure the monopoles move on the string fairly randomly and all of them annihilate rather quickly, with the last pair annihilating right before the end of the second oscillation period of the loop.
This equation can be readily solved by numerically integrating the velocity \( \dot{\sigma}(t) \) and position \( \sigma(t) \) along the string of monopoles and anti-monopoles for a number of different loop trajectories.

We have again used a wide variety of initial conditions and found that, as we expected, in all cases the monopoles annihilate with the anti-monopoles within a few oscillation periods. The typical behaviour is well illustrated in Fig. 6 which shows the evolution of the parametric position of twelve monopoles on a loop identical to the one in Fig. 4. All twelve monopoles annihilate before the end the second oscillation period. Although the exact timescale does depend on the particular shape of the loop, in no case have we found it to be longer than a few oscillation periods of the loop. The reason for this is that the motion of the string is unaffected by the presence of the monopoles and is therefore periodic, while the motion of the monopoles given by (23), is not in general periodic. Being constrained to live on a string, it is inevitable that monopoles eventually collide and annihilate.

5 Concluding Remarks

We have examined the dynamics of cosmic string loops with monopoles in two limits, when the energy in the string is comparable to the monopole energy and when the string energy is much larger than the monopole energy.

The behaviour in both cases seems to point to the annihilation of the monopoles in the system. When the monopole energy is very small compared with the string energy, the monopoles on the string collide and annihilate in a few oscillation periods. When the monopole energy is large, self intersections chop the string into small loops which subsequently decay by gravitational radiation. If the monopole and string scales are not too different, this process inevitably leads to rapid monopole annihilation. On the other hand if the monopoles are very heavy and the strings very light, loops may be long-lived, as in 5. Except for this last possibility, though, it would appear that necklace loops are rather short lived.

However, one could envision an intermediate regime in which the back reaction of the monopoles on the string is not large enough to produce such a rapid process of decay by self intersections but not so small as to decay directly by annihilations. Such a regime could significantly increase the life-time of loops. Unfortunately, at present, we are not able to faithfully simulate such a regime.

Furthermore, in the case when the monopole energy is small compared to the string energy, the possibility exists that monopoles may miss each other upon colliding and avoid annihilation. The likelihood of this would depend on the cross section for monopole-antimonopole annihilation and the thickness of the string. If the probability of crossing is large then having crossed each other they would feel a force towards each other (twice the tension on the string) and undergo a bouncing process. Such a process could in principle extend the lifetime of the monopoles on the string until the strings have radiated enough energy to gravitational waves that the monopole energy becomes comparable to...
the string energy. At this point the system would decay by self-intersections, which could in principle be happening now. Our results therefore indicate that if the string energy is dominant when the system is formed and there is not sufficient damping to overturn this regime, the scenario for cosmic ray production proposed in [3] is viable only if the cross section for monopole-antimonopole annihilation is sufficiently small.

The purpose of this work was to study the dynamics of cosmic necklaces to use this knowledge in the construction of cosmologically interesting models. We have found that we can understand the behaviour of these systems in two different regimes. A more definite statement regarding the viability of these systems as a source of ultra-high energy cosmic rays and other cosmological consequences of interest depends crucially on the cosmological evolution of the dimensionless parameter $\mu L/Nm$. Work in this direction is in progress.

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