Kelvin waves on helical vortex tube in swirling flow

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Abstract. Kelvin waves formed on precessing helical vortex tube after vortex reconnection were experimentally visualized. The experiments were performed in a swirling flow of viscous fluid in optically transparent conical diffuser. Vortex visualization was facilitated by cavitation in the vortex core and carried out using high-speed video cameras. Kelvin waves of zero, first and second modes \( m = 0, 1, 2 \) were formed in packets as a result of reconnection due to vortex tube self-intersection. Kelvin waves were observed to form both on the helical vortex tube and the detached vortex ring and to propagate along the axis of the vortex tubes.

1. Introduction

The stable wave modes supported by a straight vortex tube of uniform vorticity and circular cross-section which propagate along the axis of the vortex tube were mentioned more than 130 years ago by Lord Kelvin. Azimuthal wavenumber \( m \) describes the azimuthal structure of the wave. The perturbation of a vortex tube by a single wave mode takes the form of \( m \) helices.

Kelvin waves are of immense interest in quantum vortex dynamics [1-5]. A fundamental question in quantum turbulence is the nature of dissipation in the zero-temperature limit where the effects of friction vanish. A cascade of Kelvin waves generated on vortex lines as a result of occasional close approach and by individual vortex lines reconnection events transfers energy to higher wave numbers where it is radiated as sound. These effects highlight the importance of Kelvin waves and reconnections in the transfer of energy within a turbulent vortex tangle. Arendt, Fritts and Andreassen (1997) showed that Kelvin modes form a basis for the perturbations localized in the vortex core. This means that any perturbation in the core of a Rankine vortex can be expressed by a combination of Kelvin wave modes [6]. There are just a few experimental works dedicated to Kelvin waves dynamics in viscous fluid. The latest of such works which includes the observation of large-amplitude ‘kink’ waves on the vortex cores produced by an oscillating grid in a rotating fluid was published by Maxworthy and Hopfinger more than 30 years ago [7]. It can be stated (to our knowledge), that Kelvin waves on helical vortex tube were observed for the first time.

2. Experimental setup

Research was carried out at the Institute of Thermophysics in Novosibirsk on a closed hydrodynamic loop. The test rig, experimental equipment and mean flow velocity measurements are described in detail in previous publications [8, 9]. This test rig generates a swirling flow (\( S = 0.47 \)) in a conical diffuser with throat diameter of 100 mm and allows visualizing a precessing helical vortex and the process of vortex reconnection at Reynolds number \( Re \sim 10^5 \). The diffuser is made of acrylic glass, which allows obtaining the optical access to the flow. The connected vacuum pump creates rarefaction in the loop, thus allowing visualizing vortex structures by the vapor phase formed as a result of cavitation in the vortex core.

The structure of the base flow in the conical diffuser is shown in figure 1. The main flow is directed along the walls from top to bottom and swirled in clockwise direction (view from above). A reverse flow region is formed in the axial zone. Hollow vortex tube folds in the left-handed (winding of the...
helix is opposite to the flow swirling direction) helix rotating in the same direction with main swirling flow. For current experimental conditions the frequency of the helical vortex precession is 58 Hz, which corresponds to a period of full revolution of the helix equal to 17 ms. Every 2-3 periods of precession due to unstable interaction of nearby tube coils via the velocity field that they induce on each other helical tube deforms, and helix coils approach each other, leading to self-intersection of the tube. Vortex tubes with opposite vorticity vectors are oriented parallel to each other (see figure 2 $t = 0$). Over 0.7 ms, the tubes are bent and flattened in reconnection zone, and a reconnection process takes place within 0.8 ms. Bended bridges are formed and begin to move away from each other due to induced motion.

Figure 1. Structure of the base flow in the conical diffuser: 1 – helical vortex tube, 2 – fluid rotation, 3 – trajectories of fluid near the vortex, 4 – diffusor axis, 5 – axial reverse flow, 6 – flow direction.
3. **Kelvin wave observation**

After the reconnection, strong perturbations of vortex tubes remain: sharp bending and deformation of bridges, leading to formation of Kelvin waves packets, containing various modes travelling on the tube. Despite the fact that the vortex tube in our experiments is helical, our observations coincide with Arendt’s et al. finding that any initial perturbation of such a tube evolves as a sum of Kelvin waves, so that any localized disturbance propagates away from its original location as a wave packet [2]. As a result of reconnection depending on the ratio of various magnitudes of bending, compression and flattening perturbations of the vortex tube, one of the modes turns out to be more pronounced than others, but with time another mode may become dominant due to the difference in dispersion rates. At a sufficiently short time between successive reconnection events (comparable to the lifetime of the Kelvin wave), several Kelvin wave packets are generated and simultaneously propagate on the vortex tube, see figure 3(b).

The Kelvin wave of zero mode \( m = 0 \) is visualized on the helical tube as a steep axisymmetric decrease of the diameter of vortex tube cavity, propagating along the tube upward, i.e. in the direction opposite to the tube vorticity vector according to vortex lines’ twist inducing an axial flow, see figure 3. In its turn the tube moves downstream with velocity of the same order and in result \( m = 0 \) wave can be observed for several periods of helical tube precession without dispersion. Usually such waves are separated from the helical tube by another reconnection event or are transferred with helix motion out of test section downstream. This mode is induced by axisymmetric deformation of the vortex tube, associated with a sharp local change in the diameter and vorticity of vortex tube core in the reconnected section of the tube. It should be noted that such waves are not clearly observed on the separated vortex ring.

![Figure 3. Visualization of Kelvin waves: (a) - single zero mode Kelvin wave on helical vortex tube, (b) – zero and first mode simultaneously, exposure time - 200 μs.](image)

Prior to reconnection, the connecting parts of the helical tube approach each other at an almost perpendicular orientation, leading to a sharp kink of both reconnected vortex tubes after reconnection (see figure 2) which in turn induces \( m = 1 \) Kelvin waves in the form of rotating helical deformation of the vortex tube centerline without changes in tube diameter. As one can see on figure 4(a) after reconnection the first mode waves are observed to propagate both on the helical tube and on the separated ring. On the helical tube left-handed (helix winding direction is opposite to vortex tube rotation direction) wave travels up along the tube in the direction opposite to the tube vorticity vector, as shown in figure 4(b). On the separated vortex ring \( m = -1 \) right-handed wave (helix winding direction coincides with vortex tube rotation direction) travels along the tube in the direction opposite
to vorticity vector. Both an axial propagation and an azimuthal rotation of waves are induced by self-advectio

Both an axial propagation and an azimuthal rotation of waves are induced by self-advective flow of a curved vortex tube. An important consequence of the self-induced motion of the waves $m = 1$ and $m = 2$ is that the direction of propagation is determined by the vorticity direction in the tube and the direction of the helical rotation of the wave packet. Such Kelvin waves may be also formed outside the reconnection process when two tube sections approach close enough to interact but not close enough to reconnect. The most frequently observed modes combination in our experiment is $m = 1$ and $m = 0$ shown in figure 4(c).

![Figure 4](image.png)

Figure 4. Visualization of Kelvin waves: (a) $m=1$ waves generated simultaneously on vortex ring and helical vortex, (b) single $m=1$ wave, (c) wave packet containing $m=0$ and $m=1$ modes.

Exposure times: (a), (b) – 50 µs, (c) – 200 µs.

One can clearly observe in figure 5 that $m = 1$ Kelvin waves’ packets disperse while propagating - their amplitude decreases, its period increases and the vortex tube restores its unperturbed state.

![Figure 5](image.png)

Figure 5. Visualization of $m = 1$ wave packet dispersion, exposure time - 200 µs.

Non-axisymmetric deformation of the tube - its flattening without displacement from the tube’s centerline leads to the appearance of $m = 2$ Kelvin waves. With sufficient initial deformation, such a wave splits the vortex tube into two intertwined left-handed vortex tubes of smaller diameter (see figure 6). Since vorticity is divided in half it is hard to visualize an intertwined vortex pair with cavitation because of a lower pressure drop of the vortex core. It may be noted that vorticity is not
evenly divided in half, and one vortex of the pair may be more pronounced than another. The curvature of these intertwined tubes (taking into account the influences on each other) self-induces both the tangential and axial components of motion - rotation of the double-helix pattern around the tube and axial movement up along the vortex tube as well as in the $m = 1$ mode. This mode is the least common in our experiments and was not observed on the separated vortex ring.

**Figure 6.** $m = 2$ twist wave on the helical vortex tube.

4. **Conclusion**
This work for the first presents time an experimental visualization of varying modes of Kelvin waves formed on the helical vortex tube as a result of the vortex reconnection. The $m = 0$ and $m = 2$ modes propagate along the helical vortex tube in direction opposite to the tube vorticity vector. The $m = 1$ mode forms the left-handed helix, propagating along the helical vortex tube in the direction opposite to the tube vorticity vector, and the right-handed helix, propagating on the vortex ring in the direction coinciding with the tube vorticity vector. It should be noted that modes $m = 0$ and $m = 2$ lead to a much lower pressure drop in the vortex core and in the vortex core cavity size, and the single $m = 1$ mode does not affect the size of the vortex core cavity noticeably. Further work should be devoted to a quantitative research of Kelvin waves’ properties.

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