Investigation of the $X(4020)$ peak in the $D^*\bar{D}^*$ and $\pi h_c$ channels

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(Dated: July 26, 2021)

Abstract

In this paper, the nature of $X(4020)$ observed in the vicinity of the $D^*\bar{D}^*$ threshold of the $\pi h_c$ and $D^*\bar{D}^*$ distributions is studied. Using a model containing one bare-particle state with the $\pi h_c$ and $D^*\bar{D}^*$ one-loop self energies, we make a fit to the $\pi h_c$ and $D^*\bar{D}^*$ distributions of the $e^+e^-\rightarrow \pi^+\pi^-h_c$ and $e^+e^-\rightarrow \pi^-D^{*+}\bar{D}^{*0}$ reactions reported by the BESIII Collaboration. It is found that both structures seen in the $\pi h_c$ and $D^*\bar{D}^*$ channels are described well with one resonant state which can be associated with $X(4020)$ while the obtained $X(4020)$ mass and width are found to be larger than those in the latest PDG. We evaluate the scattering length and effective range of the $S$-wave $D^*\bar{D}^*$ pair with spin $J = 1$ related to the $X(4020)$ properties which can be tested with model calculations or future lattice QCD study. Our analysis suggests that a large part of $X(4020)$ may come from a bare-particle state originating from some short-range dynamics other than the $D^*\bar{D}^*$ channel.

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I. INTRODUCTION

After the discovery of $X(3872)$ [1], many resonancelike structures in the charmonium-mass region have been observed as listed in the latest Review of Particle Physics (RPP) [2]. Among them, $Z_c(3900)$ has been paid special attention since it is found in the charged channels $\pi^\pm J/\psi$ [3] and $(D\bar{D}^*/\bar{D}D^*)^\pm$ [4], in which an additional $u\bar{d}/d\bar{u}$ component other than $c\bar{c}$ is required. Due to the closeness of the peak position to the $D\bar{D}^*$ threshold, possible interpretations as a manifestation of the $D\bar{D}^*$ threshold cusp [5] or the triangle singularity [6–8], are proposed, and the study of the $\pi J/\psi$ and $D\bar{D}^*/\bar{D}D^*$ line shapes including such kinematical effects and the dynamics of $D\bar{D}^*/\bar{D}D^*$ is done in Refs. [6, 9–11] (see also the review articles, e.g., Refs. [12–14] and references therein for more details).

An analogous charged state near the $D^*\bar{D}^*$-threshold energy is found in the $\pi h_c$ [15, 16] and the $D^*\bar{D}^*$ channels [17, 18] by the BESIII Collaboration, and many studies are devoted to clarify the $X(4020)$ properties [5, 12, 19–60]. As well as $Z_c(3900)$, $X(4020)$ found in the charged channels is a candidate for a unconventional hadron consisting of four valence quarks. In the latest RPP [2], the difference of the $X(4020)$ full width in the $\pi h_c$ and $D^*\bar{D}^*$ channels is somehow large; $7.9\pm3.7$ MeV in the $\pi^\pm h_c$ channel, $24.8\pm9.5$ MeV in the $(D^*\bar{D}^*)^\pm$ channel, and $13\pm5$ MeV for the average (the errors are added in quadrature). As pointed out in Refs. [5, 12, 21, 57], the effect of the $D^*\bar{D}^*$ threshold should be taken into account since a peaking structure can be produced by the channel opening without resonance poles. Some analyses done in Refs. [5, 12, 23, 44, 57] suggest that the enhancement in the $D^*\bar{D}^*$ or $\pi h_c$ distributions can be explained without introducing a near-threshold pole. The calculations in Refs. [25, 32, 49] suggest the existence of the virtual-state pole near the $D^*\bar{D}^*$ threshold which can enhance the enviable cusp structure at the $D^*\bar{D}^*$ threshold.\(^1\) On the other hand, the analysis with the higher-order contribution of the meson interaction suggests that the peak structure originates from a resonant state [60].

In this paper, we investigate the $\pi^\pm h_c$ and $(D^*\bar{D}^*)^\pm$ distributions in which the $X(4020)$ is seen in order to verify if the structures seen in these two channels can be described with a single resonant state by analysing the $\pi h_c$ and $D^*\bar{D}^*$ data reported by the BESIII Collaboration. The model used in the analysis will be described below, and the pole position

\(^1\) The mass of the $Z_b(10610)$ seen in some decay modes [61, 62] as a $B\bar{B}^*/\bar{B}B^*$ molecule is used as an input in Ref. [25]. Some studies on the $\pi \Upsilon(nS)$ ($n = 1, 2, 3$), $\pi h_b(mP)$ ($m = 1, 2$), and $B^{(*)}\bar{B}^{(*)}$ line shapes in the $\Upsilon$ decay can be found in Refs. [60, 63–67].
II. $e^+e^- \rightarrow \pi\pi h_c$ AND $e^+e^- \rightarrow \pi D^*\bar{D}$ AMPLITUDES

In this section, we explain the $e^+e^- \rightarrow \pi\pi h_c$ and $e^+e^- \rightarrow \pi D^*\bar{D}$ transition amplitudes which is used in our study. In this work, we assume the production of $\pi\pi h_c$ and $\pi D^*\bar{D}$ is dominated by a $Y$ resonance (see Fig. 1 for the diagram). Here, the $Y$ state is assumed to be $Y(4220)$ which is observed in the $e^+e^- \rightarrow \pi\pi h_c$ and $\pi^+\pi^- h_c$ processes [68, 69]. While there is no evidence of the $Y(4220)$ state in the $\pi D^*\bar{D}$ channel, we expect this $Y$ state plays a central role in the production process since the $e^+e^- \rightarrow \pi X(4020) \rightarrow \pi D^*\bar{D}$ Born cross section with the neutral $D^*\bar{D}$ channel at $\sqrt{s} = 4.26$ GeV is smaller than that at $\sqrt{s} = 4.23$ GeV [18], and the ratio of the cross section at $\sqrt{s} = 4.26$ GeV to that at $\sqrt{s} = 4.23$ GeV are similar in these processes. The diagrams for the $Y$ decay into the $\pi M_1 M_2$ ($M_1 M_2 = \pi h_c, D^*\bar{D}$) are shown in Fig. 2. Other than the direct production of the $\pi M_1 M_2$ pair from $Y$ [Fig. 2(a)], the production of the $X$ pole from $Y$ [Fig. 2(b)] and the contribution of the meson-meson rescattering [Fig. 2(c)] are taken into account as well. In Fig. 2(c), the meson-meson scattering amplitude is dominated by the $X$-pole contribution in the present work. The possible coupling of $\pi\psi(2S)$, whose threshold is about 3.83 GeV, is not included in this study since the interaction and the transition of the channel with a
charmonium would be small due to the absence of the light-hadron exchange.

In the production process, one can consider the \( D_1 \bar{D}^{(*)} D^* \) triangle diagram shown in Fig. 3. The triangle diagram in Fig. 3 is not considered in this study. About the \( D_1 \bar{D}^*/\bar{D}_1 D^* \bar{D}^* \) loop, the initial \( D_1 \bar{D}^* \) threshold is not so close to the \( Y(4220) \) energy. In such a case the diagram in Fig. 3 would be reduced into the one in Fig. 2(c) and included in the \( Y \to \pi D^* \bar{D}^* \) transition amplitude effectively. The contribution of the \( D_1 \bar{D}^*/\bar{D}_1 D^* \bar{D}^* \) triangle loop is expected to be small due to the absence of the \( X(4020) \) signal in the \( D \bar{D}^*/D^* \bar{D} \) distribution [4], which indicates the weak coupling of \( X(4020) \) to the \( D \bar{D}^*/D^* \bar{D} \) channel. Furthermore, with a formula for the range of the triangle singularity (see, e.g., Eq. (59) of Ref. [70]), the singularity can appear in \( m_{D^* \bar{D}^* \pi h_c} \in [3.876, 3.89] \) GeV, which is out of the region we consider in this work, \( m_{D^* \bar{D}^* \pi h_c} > 3.95 \) GeV. Then, we use the contact \( Y \to \pi D^* \bar{D}^*, Y \to \pi \pi h_c, \) and \( \pi X(4020) \) amplitudes and include the subsequent transition to the \( \pi h_c \) or \( D^* \bar{D}^* \) channel. The \( \pi^+ \pi^- \) correlation in the \( \pi^+ \pi^- h_c \) final state is not included since there is no significant structure in the Dalitz plot [15].

A. \( e^+e^- \to \gamma \to Y \)

In this work, we assume that the production of \( \pi h_c \) and \( D^* \bar{D}^* \) is dominated by a \( Y \) resonance as diagrammatically depicted in Fig. 1, with the \( Y \) state being \( Y(4220) \) seen in the \( e^+e^- \to \pi \pi h_c \) and \( \pi D \bar{D}^* \) reactions [68, 69], and the mass and width of this \( Y \) state are taken from Ref. [68] for the \( e^+e^- \to \pi^+ \pi^- h_c \) reaction. Here we note that the parameters in Ref. [69] for the \( e^+e^- \to \pi D \bar{D}^* \) process give almost the same results. The \( e^+e^- \to Y \) transition amplitude with an intermediate photon is given by

\[
-i\mathcal{M}_{e^+e^- \to Y} = -i e \bar{v} \gamma^{\mu} u \frac{\bar{i}(-g)_{\mu\nu}(ieg_0)(\epsilon_Y^*)_\nu}{s + i\epsilon} = -i g_{\mu\nu} \frac{e^2 g_0}{s + i\epsilon} \bar{v} \gamma^{\mu} u (\epsilon_Y^*)_\nu.
\]

\[
\equiv -i g_{\mu\nu} \mathcal{M}_{e^+e^- \to Y}^\mu(\epsilon_Y^*)_\nu, \tag{1}
\]
with $e$ being the electric charge unit, $g_0$ being the coupling constant of $\gamma$ to $Y$, and $s = (p_{e^+} + p_{e^-})^2$.

**B. $Y \to \pi M_1 M_2$ transition amplitude**

We provide the details of the $Y \to \pi M_1 M_2$ ($M_1 M_2 = \pi h_c, D^* \bar{D}^*$) transition amplitude used in the analysis of the $D^* \bar{D}^*$ and $\pi h_c$ distributions in this section.

With the $P$-wave $\pi h_c$ and $S$-wave $D^* \bar{D}^*$ pairs, the $Y \to \pi \pi h_c$ and $Y \to \pi D^* \bar{D}^*$ amplitudes which appear in Fig. 2(a) are given by [11]

$$-iM_{Y\to\pi\pi h_c} = i\epsilon(p_\pi, p_Y)\epsilon^{\mu\nu\rho\sigma}(p_Y)_{\mu}(\epsilon_X)_{\nu}(p_{\pi'})_{\rho}(\epsilon_{h_c})_{\sigma},$$  

$$-iM_{Y\to\pi D^*\bar{D}^*} = -d\epsilon^{\mu\nu\rho\sigma}(p_\pi)_{\mu}(\epsilon_Y)_{\nu}(\epsilon_{D^*'})_{\rho}(\epsilon_{\bar{D}^*'})_{\sigma},$$

with the totally antisymmetric Levi-Civita tensor $\epsilon^{\mu\nu\rho\sigma}$. The $Y \to \pi X$ amplitude which is needed for the diagram in Fig. 2(b) is given by [11]

$$-iM_{Y\to\pi X} = i\epsilon(p_\pi, p_Y)\epsilon_Y \cdot \epsilon_X,$$

and the decay amplitudes for $X \to \pi h_c$ and $X \to D^* \bar{D}^*$ appearing in Fig. 2(b) are given by [11, 58]

$$-iM_{X\to\pi h_c} = \frac{ig_{X,\pi h_c}}{\sqrt{2p_X^2}}\epsilon^{\mu\nu\rho\sigma}(p_X)_{\mu}(\epsilon_X)_{\nu}(p_{\pi'})_{\rho}(-\epsilon_{h_c})_{\sigma} \equiv g_{X,\pi h_c}\tilde{M}_{X,\pi h_c},$$  

$$-iM_{X\to D^*\bar{D}^*} = \frac{-g_{X,D^*\bar{D}^*}}{\sqrt{2p_X^2}}\epsilon^{\mu\nu\rho\sigma}(p_X)_{\mu}(\epsilon_{D^*})_{\nu}(\epsilon_{\bar{D}^*})_{\sigma} \equiv g_{X,D^*\bar{D}^*}\tilde{M}_{X,D^*\bar{D}^*}. $$

With these amplitudes, the $M_1'M_2' \to M_1 M_2$ amplitude with an intermediate $X$, $T_{M_1 M_2,M_1'M_2'}$, is written as follows (the channels 1 and 2 denote the $\pi h_c$ and $D^* \bar{D}^*$ channels below);

$$T_{11} = t_{11}T^{(1)\rho\sigma,\rho'\sigma'}(p_\pi)_{\rho}(\epsilon_h)_{\sigma}(p_{\pi'})_{\rho'}(-\epsilon_{h_c})_{\sigma'},$$  

$$T_{12} = t_{12}T^{(1)\rho\sigma,\rho'\sigma'}(p_\pi)_{\rho}(\epsilon_h)_{\sigma}(\epsilon_{D^*'})_{\rho}(-\epsilon_{\bar{D}^*'})_{\sigma'},$$  

$$T_{21} = t_{21}T^{(1)\rho\sigma,\rho'\sigma'}(\epsilon_{D^*'})_{\rho}(\epsilon_{\bar{D}^*'})_{\sigma}(p_\pi)_{\rho'}(\epsilon_{h_c})_{\sigma'},$$  

$$T_{22} = t_{22}T^{(1)\rho\sigma,\rho'\sigma'}(\epsilon_{D^*'})_{\rho}(\epsilon_{\bar{D}^*'})_{\sigma}(\epsilon_{D^*})_{\rho}(-\epsilon_{\bar{D}^*})_{\sigma'},$$

with

$$t_{ij} = \frac{g_{X_1}g_{X_j}}{p_X^2 - m_R^2 - \Sigma_{\pi h_c} - \Sigma_{D^*D^*}} \equiv \frac{g_{X_1}g_{X_j}}{D_X},$$  

$$\Sigma_i = g_{X_i}^2G_i^{(A)},$$  

$$T^{(1)\rho\sigma,\rho'\sigma'} = -\frac{1}{2p_X^2}\epsilon^{\mu\nu\rho\sigma}(p_X)_{\mu}\epsilon^{'\mu'\nu'\rho'\sigma'}(p_X)_{\mu'}g_{\nu\nu'},$$

5
where $p_X^\mu$ is the four momentum of the $M_1 M_2$ pair which is the decay product of $X$. See the Appendix A for the expression of the two-body loop function $G_{M_1 M_2}^{(A)}$. The amplitude for the channel with the electric charge $\pm 1$ is denoted by $t_{ij}^{( \pm)}$ below. The two-body loop functions $G_{M_1 M_2}^{(A)}$ are regularized by introducing a three-momentum cutoff $q_{\text{max}}$. Since the change of $q_{\text{max}}$ does not give noticeable modification to our results, only the results with $q_{\text{max}} = 1 \text{ GeV}$ are shown below. The effective range given by Eq. (6) is expected to be negative since the quadratic term of the $D^* \bar{D}^*$ momentum arises from the kinetic-energy part of the total energy $W = \sqrt{p_X^2} = m_{D^*} + m_{\bar{D}^*} + k^2/(2\mu_{D^* \bar{D}^*}) + \mathcal{O}(k^4)$ and the imaginary part of the $D^* \bar{D}^*$ self energy $\Sigma_{D^* \bar{D}^*}$ is negative. In the effective range expansion, the pole of the amplitude can be located in the resonant-state, virtual-state, and bound-state region with a negative effective range (see, e.g., Fig. 3 of Ref. [71]). Then, we expect these cases can be described with the amplitude Eq. (5).

With the transition amplitudes given above, the $Y \rightarrow \pi M_1 M_2$ decay amplitudes ($M_1 M_2 = \pi h_c, D^* \bar{D}^*$) in Fig. 2 are written as follows; for the $Y \rightarrow \pi^+ \pi^- h_c$ process,

$$-iM_{Y \rightarrow \pi^+ \pi^- h_c}^{(1)} = ic(p_Y)^\alpha \epsilon^{\mu \nu \rho \sigma} (p_X)_\mu (\epsilon_Y)_\nu (\epsilon_{h_c})_\rho [ (p_{\pi^-})_\alpha (p_{\pi^+})_\beta + (p_{\pi^+})_\alpha (p_{\pi^-})_\beta ],$$

$$-iM_{Y \rightarrow \pi^+ \pi^- h_c}^{(2)} = -ifg_{X h_c} q_{\pi} (p_{\pi^+} - p_Y) \epsilon^{\mu \nu \rho \sigma} (p_{X+})_\mu (\epsilon_Y)_\nu (p_{\pi^+})_\rho (\epsilon_{h_c})_\sigma,$n

$$-iM_{Y \rightarrow \pi^+ \pi^- h_c}^{(3)} = icG_{h_c}^{(A)} t_{11}^{(-)} (p_{\pi^-} - p_Y) \epsilon^{\mu \nu \rho \sigma} (p_Y)_\mu (\epsilon_Y)_\nu (p_{\pi^+})_\rho (\epsilon_{h_c})_\sigma,$n

$$-iM_{Y \rightarrow \pi^+ \pi^- h_c}^{(4)} = ic\tilde{G}_{h_c}^{(A)} t_{11}^{(-)} (p_{\pi^+} + p_Y) \epsilon^{\mu \nu \rho \sigma} (p_Y)_\mu (\epsilon_Y)_\nu (p_{\pi^-})_\rho (\epsilon_{h_c})_\sigma,$n

$$-iM_{Y \rightarrow \pi^+ \pi^- h_c}^{(5)} = icD_1^{(A)} t_{21}^{(+)} \epsilon^{\mu \nu \rho \sigma} (p_{\pi^-} - p_{\pi^+})_\mu (\epsilon_Y)_\nu (p_{\pi^+})_\rho (\epsilon_{h_c})_\sigma,$n

and

$$M_{Y \rightarrow \pi^+ \pi^- h_c} = \sum_i M_{Y \rightarrow \pi^+ \pi^- h_c}^{(i)} \equiv M^{\mu \alpha}_{Y \rightarrow \pi^+ \pi^- h_c} (\epsilon_Y)_\mu (\epsilon_{h_c})_\alpha.$$
For the $Y \rightarrow \pi^- D^+ \bar{D}^{*0}$ decay,

$$-iM_{Y \rightarrow \pi^- D^+ \bar{D}^{*0}}^{(1)} = -d_{\mu \nu \rho \sigma} (p_{\pi^-}) \mu (\epsilon_Y) \nu (\epsilon_D^+) \rho (\epsilon_{\bar{D}}^*) \sigma,$$

$$-iM_{Y \rightarrow \pi^- D^+ D^{*0}}^{(2)} = -\frac{g_{X, D^+ D^{*0}}}{\sqrt{2} p_{\pi-} X} (p_{\pi^-} \cdot p_Y) e^{\mu \nu \rho \sigma} (p_X) \mu (\epsilon_Y) \nu (\epsilon_D^+) \rho (\epsilon_{\bar{D}}^*) \sigma,$$

$$-iM_{Y \rightarrow \pi^- D^+ \bar{D}^{*0}}^{(3)} = -c (p_{\pi^-} \cdot p_Y) G^{(A)}_{\pi+ h_c} t_{12}^{(+)} e^{\mu \nu \rho \sigma} (p_Y) \mu (\epsilon_Y) \nu (\epsilon_D^+) \rho (\epsilon_{\bar{D}}^*) \sigma,$$

$$-iM_{Y \rightarrow \pi^- D^+ D^{*0}}^{(4)} = -c G^{(A)}_{\pi+ h_c} t_{12}^{(+)} e^{\mu \nu \rho \sigma} (p_Y) \mu (\epsilon_Y) \nu (\epsilon_D^+) \rho (\epsilon_{\bar{D}}^*) \sigma,$$

$$-iM_{Y \rightarrow \pi^- D^+ \bar{D}^{*0}}^{(5)} = -d G^{(A)}_{\pi+ h_c} t_{22}^{(+)} e^{\mu \nu \rho \sigma} (p_{\pi^-}) \mu (\epsilon_Y) \nu (\epsilon_D^+) \rho (\epsilon_{\bar{D}}^*) \sigma,$$

and

$$M_{Y \rightarrow D^+ D^*} = \sum_i M_{Y \rightarrow \pi^- D^+ \bar{D}^{*0}}^{(i)} \equiv M_{Y \rightarrow \pi^- D^{*+} \bar{D}^{*0}} (\epsilon_Y) \mu (\epsilon_{D}^+) \alpha (\epsilon_{\bar{D}}^*) \beta.$$  

Combining the $Y \rightarrow \pi M_1 M_2$ ($M_1 M_2 = \pi h_c, D^* \bar{D}^*$) decay amplitudes given above with the $e^+ e^- \rightarrow Y$ amplitude in Eq. (1), the $e^+ e^- \rightarrow \pi D^* \bar{D}^*$ and $e^+ e^- \rightarrow \pi \pi h_c$ transition amplitudes are given by

$$-iM_{e^+ e^- \rightarrow \pi h_c} = -iM_{e^+ e^- \rightarrow Y} (-g + \frac{Q Q}{m_Y^2} \mu \epsilon_{h_c} (\epsilon_{h_c}^* \alpha),$$

$$-iM_{e^+ e^- \rightarrow D^+ D^*} = -iM_{e^+ e^- \rightarrow Y} (-g + \frac{Q Q}{m_Y^2} \mu \epsilon_{D^+} (\epsilon_{D^+}^* \alpha \beta),$$

where $D_Y = s - m_Y^2 + i m_Y \Gamma_Y$ and $s = Q^2 = (p_{e^-} + p_{e^+})^2$.

With the amplitudes in Eqs. (7) and (8), the differential cross section of the $e^+ e^- \rightarrow \pi M_1 M_2$ reaction as a function of the invariant mass of the $M_1 M_2$ pair is given by

$$\frac{d\sigma_{e^+ e^- \rightarrow \pi M_1 M_2}}{dM_{M_1 M_2}} = \frac{q_1 q_2}{(4 \pi)^3 s_{\pi}} \sqrt{\Omega_{\pi \pi h_c M_1 M_2}} \int_{\Omega_{\pi \pi M_1 M_2}} |M_{e^+ e^- \rightarrow \pi M_1 M_2}|^2,$$

with $q_1 = q_{\text{cm}}(\sqrt{s}, m_{\pi}, M_{M_1 M_2})$, $q_1 = q_{\text{cm}}(M_{M_1 M_2}, m_1, m_2)$, and $q_2 = q_{\text{cm}}(\sqrt{s}, m_{e^+}, m_{e^-})$.

With the differential cross section Eq. (9), we fit to the $e^+ e^- \rightarrow \pi^+ \pi^- h_c$ and $e^+ e^- \rightarrow \pi^- D^{*+} \bar{D}^{*0}$ data [15, 17] to extract the quantities related to the $X(4020)$ properties and the $D^* \bar{D}^*$ interaction, such as, the pole position of the amplitude and the S-wave $J = 1 D^* \bar{D}^*$ scattering length and effective range. Before we move to the next section, we put some details of the fitting. We use the data in the range where $m_{D^{*+} \bar{D}^{*0}, \pi \pi h_c}$ larger than 3.95 GeV.
TABLE I. The best-fit parameters. The reduced $\chi^2$, $\chi^2 = 50.2/(34 + 32 - 6) = 0.84$.

| $g_{0c}$ | $g_{0d}$ | $g_{0f}$ | $m_R$ | $g_{X,\pi h_c}$ | $g_{X,D^*\bar{D}^*}$ |
|---------|---------|---------|-------|----------------|----------------------|
| (GeV$^{-2}$) | (GeV) | (GeV) | (GeV) | (GeV) | (GeV) |
| $0.13^{+0.03}_{-0.03}$ | $-1.3^{+1.5}_{-1.3}$ | $0.11^{+0.02}_{-0.02}$ | $4.10^{+0.018}_{-0.013}$ | $-5.4^{+0.8}_{-0.9}$ | $-8.7^{+0.7}_{-0.9}$ |

in order to omit the $Z_c(3900)$ signal. The $\pi^\pm h_c$ data is sum of the events with the initial $e^+e^-$ energy $\sqrt{s} = 4.23$ GeV and 4.26 GeV, and the energy $\sqrt{s}$ is fixed at 4.26 GeV in the $D^*\bar{D}^*$ case. The number of events and the cross section in Eq. (9) are related with

$$F_{e^+e^-\to\pi M_1M_2} = C_{M_1M_2} \frac{d\sigma_{e^+e^-\to\pi M_1M_2}}{dM_{M_1M_2}(\sqrt{s},M_{M_1M_2})},$$

where $C_{\pi h_c}(\sqrt{s} = 4.23$ GeV) = $2 \cdot 646/(50.2$ pb), $C_{\pi h_c}(\sqrt{s} = 4.26$ GeV) = $2 \cdot 416/(41.0$ pb), and $C_{D^*\bar{D}^*}(\sqrt{s} = 4.26$ GeV) = $560.1/(137$ pb) [15, 17]. While the statistical uncertainties of these factors are not taken into account in the fitting, about 20-percent change of $C_{M_1M_2}$ gives just a difference of several percents at most to our results. The combinatorial background in the $D^*\bar{D}^*$ data and the side-band event in the $\pi h_c$ data are subtracted as background before fitting. We make a fit to the $D^*\bar{D}^*$ and $\pi h_c$ data without including the detector resolution, which is 1.8 MeV and 2.0 MeV for the $\pi h_c$ and $D^*\bar{D}^*$ distributions, respectively, since the effect is found to be small. The fit is done with the MINUIT program [72].

III. RESULTS

In this section, we provide the results of the fitting of the $e^+e^-\to\pi^+\pi^-h_c$ and $e^+e^-\to\pi^-D^{*+}\bar{D}^{*-0}$ data [15, 17] with the transition amplitude given in Sec. II. Other than the $\pi h_c$ and $D^*\bar{D}^*$ distributions, the pole position of $X(4020)$ and the scattering length and effective range of the $S$-wave $D^*\bar{D}^*$ pair with $J = 1$ are also evaluated.

The obtained parameters are given in Table I with the statistical uncertainties, and the plots of the $\pi h_c$ and $D^*\bar{D}^*$ distributions compared with the data in Refs. [15, 17] with the subtraction of the combinatorial background and the side-band events are shown in Fig. 4. As one can see in Fig. 4, both $\pi h_c$ and $D^*\bar{D}^*$ data are fitted well and the peak structure in the $\pi h_c$ distribution around 4.02 GeV is deformed by the opening of the $D^*\bar{D}^*$ threshold. The $\pi h_c$ distribution in the whole $\pi h_c$ phase space and the Dalitz plot in the $(M_{\pi^+\pi^-},M_{\pi^+h_c})$ plane around the $D^{*+}\bar{D}^{*-0}$-threshold energy are given in Fig. 5. In the Dalitz plot, the initial
FIG. 4. The $M_{D^{++}D^{*0}}$ (left) and $M_{\pi h_c}$ (right) distributions of the $e^+e^-$ reactions with the parameters in Table II. The points with error bar are the experimental data in Refs. [15, 17] for the $D^{++}D^{*0}$ and $\pi h_c$ processes, respectively, with subtracting the combinatorial and side-band events as background. The red-solid line is the best-fit one with the parameters in Table I, and the red band shows the uncertainty given by $\Delta \chi^2 < 1$.

FIG. 5. Left: The $\pi h_c$ invariant mass distribution in the whole $\pi h_c$ phase space. The solid, dotted, and dashed lines are $F_{e^+e^-\rightarrow\pi^+\pi^-h_c}$ with $\sqrt{s} = 4.23 + 4.26, 4.23$, and $4.26$ GeV, respectively. Right: The Dalitz plot of the $e^+e^-\rightarrow\pi^+\pi^-h_c$ reaction in the $(M_{\pi^+\pi^-}^2, M_{\pi h_c}^2)$ plane at $\sqrt{s} = 4.23$ GeV. $e^+e^-$ energy is fixed at $\sqrt{s} = 4.23$ GeV. In the $\pi h_c$ distribution given in the left panel of Fig. 5, the reflection band of $X(4020)$ is seen around $M_{\pi h_c} = 3.7$ to 3.8 GeV. A similar peak structure can be seen in the experimental data, Fig. 3 of Ref. [17], while it should be noted that the distribution of Ref. [17] is the sum of the events with various $\sqrt{s}$. In the Dalitz plot given in the right panel of Fig. 4, the lower $M_{\pi^+\pi^-}^2$ region in the horizontal band of the $X(4020)$ signal around $M_{\pi h_c}^2 = 16.2 = (4.025)^2$ GeV$^2$ is larger than the higher-$M_{\pi^+\pi^-}^2$ region, which is also seen in the data in Ref. [15]. Note that the Dalitz plot in Ref. [17]
is also the sum of the events with all the initial $e^+e^-$ energies $\sqrt{s}$ and the statistics would not be enough. The $D\bar{D}^*/\bar{D}D^*$ and $Z_c(3900)$ contributions are not considered in this study since we are concerned about the $X(4020)$ structures near the $D^*\bar{D}^*$ threshold, and the analysis of the whole $\pi h_c$ phase space including the $D\bar{D}^*/\bar{D}D^*$ and $Z_c(3900)$ contributions will provide us further information on the unconventional $Z_c(3900)$ and $X(4020)$ hadrons.

The pole position, $z_X = m_X - i\Gamma_X/2$, with the statistical uncertainties given by $\Delta \chi^2 < 1$ is

$$z_X = 4.031^{+0.003}_{-0.002} + i(-1.9)^{+0.4}_{-0.6} \times 10^{-2} \text{ GeV} : (i_{RS1}, i_{RS2}) = (1, 2),$$

$$z_X = 4.029^{+0.002}_{-0.002} + i(-2.1)^{+0.4}_{-0.7} \times 10^{-2} \text{ GeV} : (i_{RS1}, i_{RS2}) = (2, 2),$$

(10)

where $i_{RS1,RS2} = 1, 2$ specify the Riemann sheet of the $\pi h_c$ and $D^*\bar{D}^*$ channels, respectively. The mass, the real part of the pole position $z_X$, is slightly larger, and the width, twice the imaginary part of $z_X$, is a few times larger than the latest PDG value [2]. The location of the pole with $(i_{RS1}, i_{RS2}) = (1, 2)$ and $(2, 2)$ are similar to each other. This would imply the subdominant role of the $\pi h_c$ channel for the resonance properties. With the $X(4020)$ pole position $z_X$ in Eq. (10), it may be possible to attribute the signal seen in the $\pi \psi(2S)$ [73, 74] to this $X(4020)$ state while it is rather listed in $X(4055)^\pm$ in the latest PDG (see also Refs. [21, 75, 76] for the study of the $\pi \psi(2S)$ distribution). However, we have to note that the $X(4055)^\pm$ is observed by the Belle Collaboration [77] and the mass is about 20 MeV larger than our result in Eq. (10). Then, we need further study including the $\pi \psi(2S)$ channel to reach a definite conclusion.

With the residue of the pole, we can give a partial width of $X(4020) \rightarrow \pi h_c$ and $D^*\bar{D}^*$; with the best-fit parameters,

$$\Gamma_{X,\pi h_c} = \frac{p_\pi}{8\pi m_X^2} |(2z_X)R_{X,\pi h_c}| \left| \tilde{M}_{X,\pi h_c} \right|^2 = 3 \text{ MeV},$$

$$\Gamma_{X,D^*\bar{D}^*} = \frac{p_{D^*}}{8\pi m_X^2} |(2z_X)R_{X,D^*\bar{D}^*}| \left| \tilde{M}_{X,D^*\bar{D}^*} \right|^2 = 38 \text{ MeV},$$

where $R_{X,i}$ is the residue of the $X(4020)$ pole in the amplitude $t_{ii}$ and $\tilde{M}_{X,M_1M_2}$ is defined in Eqs. (3) and (4). The sum of these partial widths, $\Gamma_{X,\pi h_c} + \Gamma_{X,D^*\bar{D}^*}$, gives a close value to the full width of $X(4020)$ given by twice the imaginary part of the pole position $z_X$ in Eq. (10). While the full width is not the same, the ratio $\Gamma_{X,\pi h_c}/\Gamma_{X,D^*\bar{D}^*}$ is consistent with that given in Ref. [55].
With the obtained $D^*\bar{D}^*$ amplitude, we can evaluate the scattering length and effective range which characterize the low-energy behavior of the scattering amplitude (see Eq. (6)). The obtained scattering length $a_{D^*\bar{D}^*}$ and effective range $(r_{\text{eff}})_{D^*\bar{D}^*}$ for the $S$-wave $D^*\bar{D}^*$ pair with the spin $J = 1$, isospin $I = 1$, and negative $C$ parity are

$$a_{D^*\bar{D}^*} = 0.87^{+0.10}_{-0.09} + i0.04^{+0.01}_{-0.01} \text{ fm},$$

$$(r_{\text{eff}})_{D^*\bar{D}^*} = (-1.84)^{+0.37}_{-0.39} \text{ fm},$$

where the statistical uncertainties are given together. One can see that the imaginary part of both scattering length and effective range are small compared with the real part reflecting the weak coupling to the $\pi h_c$ channel. In addition, the sign of the real part of $(r_{\text{eff}})_{D^*\bar{D}^*}$ is negative as anticipated. These values are similar to those given in Ref. [55] while we can see some difference in the real part of the effective range.

In Table I, the parameter $m_R$, about 4.1 GeV, corresponds to the mass of a bare-particle state which emerges when the couplings to the $\pi h_c$ and $D^*\bar{D}^*$ channels are turned off.\(^3\) Such a bare-particle state may be related to the state generated from some short-range physics, such as the quark or diquark dynamics discussed, e.g., in Refs. [48–50], or other hadronic channels. Here, the value of $m_R$ is not so far away from the pole position Eq. (10) involving the $\pi h_c$- and $D^*\bar{D}^*$-channel effects, the difference is less than 100 MeV. Then we expect that the contribution that arises from some dynamics other than the $D^*\bar{D}^*$ channel, such as some short-range dynamics of quark degree of freedom or other hadronic channel, is essential for the generation of the $X(4020)$ state. This is consistent with the conclusion drawn in Ref. [55] from a view point of the compositeness; the necessity of some dynamics other than the $D^*\bar{D}^*$ channel is pointed out. The compositeness can be a measure of the $D^*\bar{D}^*$ component in $X(4020)$, and this quantity is related to the threshold parameters or the pole position of the amplitude [78–81].

IV. SOME CONSIDERATIONS

In the fitting made in Sec. III, we have assumed the nonresonant part of the $\pi h_c$ distribution can also be described with the amplitude in Eq. (2) that can interfere with the

\(^3\) Note that the value of $m_R$ in Table I is the value with $q_{\text{max}} = 1$ GeV ($q_{\text{max}}$ is a cutoff parameter of $G_{M_1M_2}$ in Eq. (5)) and $m_R$ depends on $q_{\text{max}}$ even though the fit quality and the resultant pole position and threshold parameters are almost the same. The change of $q_{\text{max}}$ from 1 GeV to 0.8 or 1.2 GeV leads to a difference of a few tens of MeV in $m_R$. 11
TABLE II. The best-fit parameters. The reduced $\chi^2$, $\bar{\chi}^2 = 52.3/(34 + 32 - 6) = 0.87$.

| $g_0c$  | $g_0d$  | $g_0f$  | $m_R$  | $g_{X,\pi h_c}$ | $g_{X,D^*\bar{D}^*}$ |
|--------|--------|--------|--------|-----------------|----------------------|
| (GeV$^{-2}$) | (GeV) | (GeV) | (GeV) | (−) | (GeV) |
| $0.7^{+3.5}_{-3.8} \times 10^{-2}$ | $(−3.8)^{+1.2}_{-1.0}$ | $0.14^{+0.01}_{-0.01}$ | $4.082^{+0.011}_{-0.009}$ | $(−3.8)^{+0.5}_{-0.6}$ | $(−7.9)^{+0.6}_{-0.7}$ |

FIG. 6. The $M_{D^*\bar{D}^*}$ (left) and the $M_{\pi h_c}$ (right) distributions of $e^+e^- \rightarrow \pi\pi h_c, \pi D^*\bar{D}^*$ reactions with the parameters in Table II. The points with the error bars are obtained by subtracting the background contribution from the data [15]. The meaning of the line and band are the same as Fig. 4.

resonance term. Here, for comparison, we make a fit to the data with a further subtraction of the smooth nonresonant background from the $\pi h_c$ data [15]. The obtained parameters are provided in Table II, and the $\pi h_c$ and $D^*\bar{D}^*$ distributions and the Dalitz plot of the $e^+e^- \rightarrow \pi^+\pi^- h_c$ process in the $(M_{\pi^+\pi^-}^2, M_{\pi^+h_c}^2)$ plane are shown in Figs. 6 and 7. The pole position is

\[ z_X = 4.026^{+0.002}_{-0.002} + i(-1.3)^{+0.2}_{-0.3} \times 10^{-2} \text{ GeV} : \ (i_{RS1}, i_{RS2}) = (1, 2), \]

\[ z_X = 4.025^{+0.002}_{-0.002} + i(-1.4)^{+0.3}_{-0.3} \times 10^{-2} \text{ GeV} : \ (i_{RS1}, i_{RS2}) = (2, 2), \]  

(11)

and the scattering length and effective range are

\[ a_{D^*\bar{D}^*} = 1.06^{+0.15}_{-0.13} + i0.04^{+0.01}_{-0.01} \text{ fm}, \]

\[ (r_{\text{eff}})_{D^*\bar{D}^*} = -2.30^{+0.38}_{-0.42} \text{ fm}. \]

Both the mass and width of $X(4020)$, $z_X$, in Eq. (11) is smaller than that in Eq. (10), but these are still larger than the values in the RPP [2]. The Dalitz plot in Fig. 7 is also
FIG. 7. The Dalitz plot of the $e^+e^- \rightarrow \pi^+\pi^-h_c$ reaction in the $(M_{\pi^+\pi^-}^2, M_{\pi^+h_c}^2)$ plane.

TABLE III. The best-fit parameters. The reduced $\chi^2$, $\tilde{\chi}^2 = 25.9/(34 - 4) = 0.86.$

| Parameter | Value |
|-----------|-------|
| $g_0d$    | $(\text{GeV})$ |
| $g_0f$    | $(\text{GeV})$ |
| $m_R$     | $(\text{GeV})$ |
| $g_{X,D^*\bar{D}^*}$ | $(\text{GeV})$ |

| Parameter | Value |
|-----------|-------|
| $g_0d$    | $-1.1^{+2.3}_{-1.6}$ |
| $g_0f$    | $0.11^{+0.02}_{-0.04}$ |
| $m_R$     | $4.102^{+0.026}_{-0.015}$ |
| $g_{X,D^*\bar{D}^*}$ | $(-9.0)^{+0.9}_{-1.2}$ |

qualitatively the same as Fig. 5 obtained by fitting to the data without the subtraction of the smooth nonresonant background. The partial width of $X(4020)$ to the $\pi h_c$ and $D^*\bar{D}^*$ channels is

$$\Gamma_{X,\pi h_c} = 1 \text{ MeV}, \Gamma_{X,D^*\bar{D}^*} = 25 \text{ MeV}.$$ 

The ratio $\Gamma_{X,\pi h_c}/\Gamma_{X,D^*\bar{D}^*}$ is smaller than that in Sec. III.

In fact, the pole position and threshold parameters obtained in Sec. III are mainly fixed by the $D^*\bar{D}^*$ channel, and the $\pi h_c$ channel gives relatively small correction. Here we make a fit without the $\pi h_c$ data for comparison. Here, we simply fix the parameters $c$ and $g_{X,\pi h_c}$ related to the $\pi h_c$ channel to be zero and use the rest of the parameters to fit to the $D^*\bar{D}^*$ data. The resultant parameters and the $D^*\bar{D}^*$ distribution are shown in Table III and Fig. 8.

The pole position of $X(4020)$ is

$$z_X = 4.029^{+0.004}_{-0.004} + i(-2.2)^+_{-1.1} \times 10^{-2} \text{ GeV} : i_{RS2} = 2,$$

and the obtained threshold parameters are

$$a_{D^*\bar{D}^*} = 0.88^{+0.16}_{-0.13} \text{ fm},$$
$$a_{\text{eff}} = (-1.73)^{+0.45}_{-0.42} \text{ fm}.$$
FIG. 8. The $M_{D^+\bar{D}^*}$ distributions of the $e^+e^-$ reactions with the parameters in Table III. The meaning of the lines are the same as Fig. 4.

These quantities are quite similar to those in Sec. III, and this is plausible because the transition of the $\pi h_c$ and $D^*\bar{D}^*$ channels is expected to be suppressed by the mass of the charmed hadron and the $\pi h_c$ channel does not have a large impact on the $X(4020)$ properties appearing in the vicinity of the $D^*\bar{D}^*$ threshold.

V. SUMMARY

In this paper, we have investigated the $X(4020)^\pm$ properties observed in the $\pi^\pm h_c$ and $(D^*\bar{D}^*)^\pm$ channel and the $S$-wave $D^*\bar{D}^*$ interaction with the spin and parity $J^{PC} = 1^{+-}$ and the isospin $I = 1$. The model is constructed by introducing one bare-particle state and the effect of the coupling to the $P$-wave $\pi h_c$ and $S$-wave $D^*\bar{D}^*$ channels is incorporated with the one-loop self energies. The parameters in the model, the mass of the bare state, the coupling of the bare state to the $\pi h_c$ and $D^*\bar{D}^*$ channels, and those related to the production part which is assumed to be dominated by the $Y(4220)$ state, are fixed by fitting to the data of the two reactions, $e^+e^- \rightarrow \pi\pi h_c$ and $e^+e^- \rightarrow \pi D^*\bar{D}^*$, reported in Refs. [15, 17] by the BESIII Collaboration.

As a result of the fitting, it is found that the $X(4020)$ structures in both $\pi h_c$ and $D^*\bar{D}^*$ distributions near the $D^*\bar{D}^*$ threshold can be described well with this model. The pole position of $X(4020)$ and the $S$-wave $D^*\bar{D}^*$ scattering length are evaluated with the obtained parameters. The obtained $X(4020)$ width, twice the imaginary part of the pole position, is a few times larger than that reported in the latest RPP [2] while the mass, the real part of the
pole position, is just slightly larger than the value in Ref. [2]. With the obtained mass and width, it may be possible to associate this $X(4020)$ to the peak seen in the $\pi\psi(2S)$ channel observed by the BESIII [73, 74] Collaboration which is categorized as $X^\pm(4055)$ in the RPP [2]. In the Dalitz plot of the $e^+e^-\rightarrow\pi\pi h_c$ process, the lower side of the $X(4020)$ band in the $\pi^+\pi^-$ invariant mass is found to be larger than that in the higher energies. A similar behavior is also seen in the experiment [15] although the statistics of the data would be still not enough. Then, the comparison with the higher-statistics data will be helpful to test our model, and the future analysis of the whole $\pi h_c$ phase space including the $D\bar{D}^*/\bar{D}D^*$ and $Z_c(3900)$ contributions enables us to determine the pole position better and provide some clue to understand the pair of the unconventional $Z_c(3900)$ and $X(4020)$ states. The values of the scattering length and effective range of the $S$-wave $D^*\bar{D}^*$ system with spin $J = 1$ are close to those obtained in Ref. [55], and these quantities can be confronted with some model calculations or forthcoming lattice QCD analysis. Concerning the origin of $X(4020)$, the closeness of the mass of the bare-particle state to the pole mass evaluated with including the effects of the $\pi h_c$ and $D^*\bar{D}^*$ channels would imply that some contribution of the short-range physics, such as a compact tetra-quark state or a diquark-antidiquark state, or hadronic channels other than the $D^*\bar{D}^*$ channel plays a substantial role in the $X(4020)$ generation. It is worth noting that the conclusion of Ref. [55] from a view point of the compositeness is consistent with ours.

It is found that the obtained $X(4020)$ properties, the pole position and the $D^*\bar{D}^*$ scattering length and effective range, are mostly determined by the $D^*\bar{D}^*$ data by comparing the results with a different assumption on the $\pi h_c$ data; in one case nonresonant part of the $\pi h_c$ data is subtracted and in the other case the $\pi h_c$ data is omitted from the fitting. This conclusion is plausible since the transition amplitudes of the $\pi h_c$ channels is expected to be small due to the suppression by the exchanged charmed-meson mass indicating that the $\pi h_c$ channel is not so relevant to the nature of the $X(4020)$ state and the $D^*\bar{D}^*$ interaction.

In this work, the dominance of the $Y(4220)$ state in the $\pi\pi h_c$ and $\pi D^*\bar{D}^*$ production with a $S$-wave $D^*\bar{D}^*$ pair is assumed. As pointed out, e.g., in Ref. [23], the $D^*\bar{D}^*$ pair can be $D$ wave and the study of the dependence of the $e^+e^-\rightarrow\pi D^*\bar{D}^*$ cross section on the initial $e^+e^-$ energy should provide us valuable information to clarify the nature of $X(4020)$. 

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ACKNOWLEDGMENTS

The author thanks Prof. Atsushi Hosaka for the discussions and comments. The computer system at RCNP in Osaka University is used for some part of the numerical calculations.

Appendix A: Two-body loop function

The two-body loop functions \((G_{D\cdot D^*})^{\mu\nu\mu'\nu'}\) and \((G_{\pi_{hc}})^{\mu\nu\mu'\nu'}\) are defined by

\[
(G_{\pi_{hc}})^{\mu\nu\mu'\nu'} = i \int \frac{d^4 l}{(2\pi)^4} \frac{\eta_{\mu\mu'}\left(-g^{\mu\nu'} + \frac{(p_X - l)^{\mu'}(p_X - l)^{\nu'}}{m_{hc}^2} \right)}{[l^2 - m_\pi^2 + i\epsilon][(p_X - l)^2 - m_{hc}^2 + i\epsilon]},
\]

\[
(G_{D\cdot D^*})^{\mu\nu\mu'\nu'} = i \int \frac{d^4 l}{(2\pi)^4} \frac{\eta_{\mu\mu'}\left(-g^{\mu\nu'} + \frac{\mu\nu'}{m_{D^*}^2}\right)}{[l^2 - m_{D^*}^2 + i\epsilon][(p_X - l)^2 - m_{D^*}^2 + i\epsilon]}.
\]

Due to the structure of the \(Y \to \pi M_1 M_2\) and \(X \to M_1 M_2\) amplitudes, the term \(g^{\mu\nu'} g^{\mu'\nu'} G_{M_1 M_2}^{(A)}\) remains. The expression of \(G_{M_1 M_2}^{(A)}\) is given by

\[
G_{M_1 M_2}^{(A)} = \frac{1}{3} \left[ m_{\pi_1}^2 G_{\pi_{hc}} - \frac{p_X^2 + m_{\pi_1}^2 - m_{hc}^2}{2} G_{\pi_{hc}}^{(A)} + \frac{J_{hc}}{2} \right],
\]

\[
G_{D\cdot D^*}^{(A)} = G_{D\cdot D^*} + \frac{1}{3} \left( \frac{1}{m_{D^*}^2} + \frac{1}{m_{D^*}^2} \right) \left[ m_{D^*}^2 G_{D\cdot D^*} - \frac{p_X^2 + m_{D^*}^2 - m_{D^*}^2}{2} G_{D\cdot D^*}^{(A)} + \frac{J_{D^*}}{2} \right],
\]

with

\[
G_{M_1 M_2} = i \int \frac{1}{[l^2 - m_{M_1}^2 + i\epsilon][(p_X - l)^2 - m_{M_2}^2 + i\epsilon]},
\]

\[
J_M = i \int \frac{1}{l^2 - m_M^2 + i\epsilon},
\]

\[
G_{M_1 M_2}^{(A)} = \frac{p_X^2 + m_{M_1}^2 - m_{M_2}^2}{2p_X^2} G_{M_1 M_2} - \frac{J_{M_1}}{2p_X^2} + \frac{J_{M_2}}{2p_X^2}.
\]

These \(G_{M_1 M_2}\) and \(J_M\) are regularized with the three-momentum cutoff. The loop function in the second Riemann sheet is given by replacing \(G_{M_1 M_2}\) with \(G_{M_1 M_2} + 2iq_{M_1 M_2}/(8\pi\sqrt{p_X^2})\)

\[
[q_{M_1 M_2} = q_{cm}(\sqrt{p_X^2}, m_{M_1}, m_{M_2})].
\]

[1] S. K. Choi et al. (Belle), “Observation of a narrow charmonium - like state in exclusive \(B^\pm \to K^\pm \pi^+\pi^- J/\psi\) decays,” Phys. Rev. Lett. 91, 262001 (2003), arXiv:hep-ex/0309032 [hep-ex].
[2] P. A. Zyla et al. (Particle Data Group), “Review of Particle Physics,” PTEP 2020, 083C01 (2020).

[3] M. Ablikim et al. (BESIII), “Observation of a Charged Charmoniumlike Structure in $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ at $\sqrt{s} = 4.26$ GeV,” Phys. Rev. Lett. 110, 252001 (2013), arXiv:1303.5949 [hep-ex].

[4] M. Ablikim et al. (BESIII), “Observation of a charged $(D\bar{D}^*)^\pm$ mass peak in $e^+e^- \rightarrow \pi D\bar{D}^*$ at $\sqrt{s} = 4.26$ GeV,” Phys. Rev. Lett. 112, 022001 (2014), arXiv:1310.1163 [hep-ex].

[5] E. S. Swanson, “$Z_b$ and $Z_c$ Exotic States as Coupled Channel Cusps,” Phys. Rev. D 91, 034009 (2015), arXiv:1409.3291 [hep-ph].

[6] Qian Wang, Christoph Hanhart, and Qiang Zhao, “Decoding the riddle of $Y(4260)$ and $Z_c(3900),”$ Phys. Rev. Lett. 111, 132003 (2013), arXiv:1303.6355 [hep-ph].

[7] Adam P. Szczepaniak, “Triangle Singularities and XYZ Quarkonium Peaks,” Phys. Lett. B 747, 410–416 (2015), arXiv:1501.01691 [hep-ph].

[8] Xiao-Hai Liu, Makoto Oka, and Qiang Zhao, “Searching for observable effects induced by anomalous triangle singularities,” Phys. Lett. B 753, 297–302 (2016), arXiv:1507.01674 [hep-ph].

[9] Miguel Albaladejo, Feng-Kun Guo, Carlos Hidalgo-Duque, and Juan Nieves, “$Z_c(3900)$: What has been really seen?” Phys. Lett. B 755, 337–342 (2016), arXiv:1512.03638 [hep-ph].

[10] A. Pilloni, C. Fernandez-Ramirez, A. Jackura, V. Mathieu, M. Mikhasenko, J. Nys, and A. P. Szczepaniak (JPAC), “Amplitude analysis and the nature of the $Z_c(3900),”$ Phys. Lett. B 772, 200–209 (2017), arXiv:1612.06490 [hep-ph].

[11] Qin-Rong Gong, Zhi-Hui Guo, Ce Meng, Guang-Yi Tang, Yu-Fei Wang, and Han-Qing Zheng, “$Z_c(3900)$ as a $D\bar{D}^*$ molecule from the pole counting rule,” Phys. Rev. D 94, 114019 (2016), arXiv:1604.08836 [hep-ph].

[12] E. S. Swanson, “Cusps and Exotic Charmonia,” Int. J. Mod. Phys. E 25, 1642010 (2016), arXiv:1504.07952 [hep-ph].

[13] Hua-Xing Chen, Wei Chen, Xiang Liu, and Shi-Lin Zhu, “The hidden-charm pentaquark and tetraquark states,” Phys. Rept. 639, 1–121 (2016), arXiv:1601.02092 [hep-ph].

[14] Feng-Kun Guo, Christoph Hanhart, Ulf-G. Meißner, Qian Wang, Qiang Zhao, and Bing-Song Zou, “Hadronic molecules,” Rev. Mod. Phys. 90, 015004 (2018), arXiv:1705.00141 [hep-ph].

[15] M. Ablikim et al. (BESIII), “Observation of a Charged Charmoniumlike Structure $Z_c(4020)$ and Search for the $Z_c(3900)$ in $e^+e^- \rightarrow \pi^+\pi^- h_c,”$ Phys. Rev. Lett. 111, 242001 (2013), arXiv:1302.5137 [hep-ph].
M. Ablikim et al. (BESIII), “Observation of $e^+e^- \rightarrow \pi^0\pi^0 h_c$ and a Neutral Charmoniumlike Structure $Z_c(4020)^0$,” Phys. Rev. Lett. 113, 212002 (2014), arXiv:1409.6577 [hep-ex].

M. Ablikim et al. (BESIII), “Observation of a charged charmoniumlike structure in $e^+e^- \rightarrow (D^*\bar{D}^*)^{\pm}\pi^{\mp}$ at $\sqrt{s} = 4.26\text{GeV}$,” Phys. Rev. Lett. 112, 132001 (2014), arXiv:1308.2760 [hep-ex].

M. Ablikim et al. (BESIII), “Observation of a neutral charmoniumlike state $Z_c(4025)^0$ in $e^+e^- \rightarrow (D^*\bar{D}^*)^{0}\pi^0$,” Phys. Rev. Lett. 115, 182002 (2015), arXiv:1507.02404 [hep-ex].

Yan-Rui Liu and Zong-Ye Zhang, “The Bound state problem of S-wave heavy quark meson-antimeson systems,” Phys. Rev. C 80, 015208 (2009), arXiv:0810.1598 [hep-ph].

R. Molina and E. Oset, “The $Y(3940)$, $Z(3930)$ and the $X(4160)$ as dynamically generated resonances from the vector-vector interaction,” Phys. Rev. D80, 114013 (2009), arXiv:1310.1119 [hep-ph].

Dian-Yong Chen and Xiang Liu, “Predicted charged charmonium-like structures in the hidden-charm dipion decay of higher charmonia,” Phys. Rev. D84, 034032 (2011), arXiv:1106.5290 [hep-ph].

M. Pavon Valderrama, “Power Counting and Perturbative One Pion Exchange in Heavy Meson Molecules,” Phys. Rev. D 85, 114037 (2012), arXiv:1204.2400 [hep-ph].

A. Martinez Torres, K.P. Khemchandani, F.S. Navarra, M. Nielsen, and E. Oset, “Reanalysis of the $e^+e^- \rightarrow (D^*\bar{D}^*)^{\pm}\pi^{\mp}$ reaction and the claim for the $Z_c(4025)$ resonance,” Phys. Rev. D 89, 014025 (2014), arXiv:1310.1119 [hep-ph].

K.P. Khemchandani, A. Martinez Torres, M. Nielsen, and F.S. Navarra, “Relating $D^*\bar{D}^*$ currents with $J^P = 0^+, 1^+$ and $2^+$ to $Z_c$ states,” Phys. Rev. D 89, 014029 (2014), arXiv:1310.0862 [hep-ph].

Feng-Kun Guo, Carlos Hidalgo-Duque, Juan Nieves, and Manuel Pavon Valderrama, “Consequences of Heavy Quark Symmetries for Hadronic Molecules,” Phys. Rev. D88, 054007 (2013), arXiv:1303.6608 [hep-ph].

Jun He, Xiang Liu, Zhi-Feng Sun, and Shi-Lin Zhu, “$Z_c(4025)$ as the hadronic molecule with hidden charm,” Eur. Phys. J. C73, 2635 (2013), arXiv:1308.2999 [hep-ph].

Cong-Feng Qiao and Liang Tang, “Interpretation of $Z_c(4025)$ as the hidden charm tetraquark states via QCD Sum Rules,” Eur. Phys. J. C74, 2810 (2014), arXiv:1308.3439 [hep-ph].
[28] Wei Chen, T. G. Steele, Meng-Lin Du, and Shi-Lin Zhu, “$D^*\bar{D}^*$ molecule interpretation of $Z_c(4025)$,” Eur. Phys. J. C74, 2773 (2014), arXiv:1308.5060 [hep-ph].

[29] Xiao Wang, Yuan Sun, Dian-Yong Chen, Xiang Liu, and Takayuki Matsuki, “Simulating the charged charmoniumlike structure $Z_c(4025)$,” Eur. Phys. J. C74, 2761 (2014), arXiv:1308.3158 [hep-ph].

[30] Zhi-Gang Wang, “Reanalysis of the $Z_c(4020)$, $Z_c(4025)$, $Z(4050)$ and $Z(4250)$ as tetraquark states with QCD sum rules,” Commun. Theor. Phys. 63, 466–480 (2015), arXiv:1312.1537 [hep-ph].

[31] Gang Li, “Hidden-charmonium decays of $Z_c(3900)$ and $Z_c(4025)$ in intermediate meson loops model,” Eur. Phys. J. C73, 2621 (2013), arXiv:1304.4458 [hep-ph].

[32] Francesca Aceti, Melahat Bayar, Jorgivan Morais Dias, and Eulogio Oset, “Prediction of a $Z_c(4000)$ $D^*\bar{D}^*$ state and relationship to the claimed $Z_c(4025)$,” Eur. Phys. J. A50, 103 (2014), arXiv:1403.0810 [hep-ph].

[33] Chengrong Deng, Jialun Ping, and Fan Wang, “Interpreting $Z_c(3900)$ and $Z_c(4025)/Z_c(4020)$ as charged tetraquark states,” Phys. Rev. D90, 054009 (2014), arXiv:1402.0777 [hep-ph].

[34] Zhi-Gang Wang, “Reanalysis of the $Y(3940)$, $Y(4140)$, $Z_c(4020)$, $Z_c(4025)$ and $Z_b(10650)$ as molecular states with QCD sum rules,” Eur. Phys. J. C74, 2963 (2014), arXiv:1403.0810 [hep-ph].

[35] Xiao-Hai Liu, Li Ma, Li-Ping Sun, Xiang Liu, and Shi-Lin Zhu, “Resolving the puzzling decay patterns of charged $Z_c$ and $Z_b$ states,” Phys. Rev. D90, 074020 (2014), arXiv:1407.3684 [hep-ph].

[36] A. Esposito, A. L. Guerrieri, and A. Pilloni, “Probing the nature of $Z_c^{(')}$ states via the $\eta_{c}\rho$ decay,” Phys. Lett. B746, 194–201 (2015), arXiv:1409.3551 [hep-ph].

[37] Smruti Patel, Manan Shah, and P C Vinodkumar, “Mass spectra of four-quark states in the hidden charm sector,” Eur. Phys. J. A 50, 131 (2014), arXiv:1402.3974 [hep-ph].

[38] Sasa Prelovsek, C. B. Lang, Luka Leskovec, and Daniel Mohler, “Study of the $Z_c^+$ channel using lattice QCD,” Phys. Rev. D91, 014504 (2015), arXiv:1405.7623 [hep-lat].

[39] Lu Zhao, Wei-Zhen Deng, and Shi-Lin Zhu, “Hidden-Charm Tetraquarks and Charged $Z_c$ States,” Phys. Rev. D 90, 094031 (2014), arXiv:1408.3924 [hep-ph].

[40] Gang Li, Xiao Hai Liu, and Zhu Zhou, “More hidden heavy quarkonium molecules and their discovery decay modes,” Phys. Rev. D90, 054006 (2014), arXiv:1409.0754 [hep-ph].
[41] Ying Chen et al. (CLQCD), “Low-energy Scattering of \((D^*\bar{D}^*)^\pm\) System and the Resonance-like Structure \(Z_c(4025)\),” Phys. Rev. D\textbf{92}, 054507 (2015), arXiv:1503.02371 [hep-lat].

[42] Lu Zhao, Li Ma, and Shi-Lin Zhu, “The recoil correction and spin-orbit force for the possible \(B^*\bar{B}^*\) and \(D^*\bar{D}^*\) states,” Nucl. Phys. A\textbf{942}, 18–38 (2015), arXiv:1504.04117 [hep-ph].

[43] Wei Chen, T. G. Steele, Hua-Xing Chen, and Shi-Lin Zhu, “Mass spectra of \(Z_c\) and \(Z_b\) exotic states as hadron molecules,” Phys. Rev. D\textbf{92}, 054002 (2015), arXiv:1505.05619 [hep-ph].

[44] Zhi-Yong Zhou and Zhiguang Xiao, “Distinguishing near-threshold pole effects from cusp effects,” Phys. Rev. D\textbf{92}, 094024 (2015), arXiv:1505.05761 [hep-ph].

[45] Hong-Wei Ke and Xue-Qian Li, “Study on decays of \(Z_c(4020)\) and \(Z_c(3900)\) into \(h_c + \pi\),” Eur. Phys. J. C\textbf{76}, 334 (2016), arXiv:1601.03575 [hep-ph].

[46] Ruilin Zhu, “Hidden charm octet tetraquarks from a diquark-antidiquark model,” Phys. Rev. D\textbf{94}, 054009 (2016), arXiv:1607.02799 [hep-ph].

[47] Ming-Zhu Liu, Duo-Jie Jia, and Dian-Yong Chen, “Possible hadronic molecular states composed of \(S\)-wave heavy-light mesons,” Chin. Phys. C\textbf{41}, 053105 (2017), arXiv:1702.04440 [hep-ph].

[48] Muhammad Naeem Anwar, Jacopo Ferretti, and Elena Santopinto, “Spectroscopy of the hidden-charm \([qc][\bar{q}\bar{c}]\) and \([sc][\bar{s}\bar{c}]\) tetraquarks in the relativized diquark model,” Phys. Rev. D\textbf{98}, 094015 (2018), arXiv:1805.06276 [hep-ph].

[49] Pablo G. Ortega, Jorge Segovia, David R. Entem, and Francisco Fernández, “The \(Z_c\) structures in a coupled-channels model,” Eur. Phys. J. C\textbf{79}, 78 (2019), arXiv:1808.00914 [hep-ph].

[50] Jesse F. Giron, Richard F. Lebed, and Curtis T. Peterson, “The Dynamical Diquark Model: Fine Structure and Isospin,” JHEP \textbf{01}, 124 (2020), arXiv:1907.08546 [hep-ph].

[51] Zhi-Gang Wang, “Axialvector tetraquark candidates for the \(Z_c(3900),\) \(Z_c(4020),\) \(Z_c(4430),\) \(Z_c(4600),\)” Chin. Phys. C\textbf{44}, 063105 (2020), arXiv:1901.10741 [hep-ph].

[52] Li-Ye Xiao, Guang-Juan Wang, and Shi-Lin Zhu, “Hidden-charm strong decays of the \(Z_c\) states,” Phys. Rev. D\textbf{101}, 054001 (2020), arXiv:1912.12781 [hep-ph].

[53] Jun-Zhang Wang, Dian-Yong Chen, Xiang Liu, and Takayuki Matsuki, “Universal non-resonant explanation to charmoniumlike structures \(Z_c(3885)\) and \(Z_c(4025)\),” Eur. Phys. J. C\textbf{80}, 1040 (2020), arXiv:2007.02263 [hep-ph].

[54] Zuo-Ming Ding, Han-Yu Jiang, and Jun He, “Molecular states from \(D^{(*)}\bar{D}^{(*)}/B^{(*)}\bar{B}^{(*)}\) and \(D^{(*)}D^{(*)}/\bar{B}^{(*)}\bar{B}^{(*)}\) interactions,” Eur. Phys. J. C\textbf{80}, 1179 (2020), arXiv:2011.04980 [hep-ph].
[55] Zhi-Hui Guo and J. A. Oller, “Unified description of the hidden-charm tetraquark states $Z_{cs}(3985), Z_c(3900),$ and $X(4020),”$ Phys. Rev. D 103, 054021 (2021), arXiv:2012.11904 [hep-ph].

[56] Guang-Juan Wang, Xiao-Hai Liu, Li Ma, Xiang Liu, Xiao-Lin Chen, Wei-Zhen Deng, and Shi-Lin Zhu, “The strong decay patterns of $Z_c$ and $Z_b$ states in the relativized quark model,” Eur. Phys. J. C 79, 567 (2019), arXiv:1811.10339 [hep-ph].

[57] Xiao-Hai Liu, “Influence of threshold effects induced by charmed meson rescattering,” Phys. Rev. D90, 074004 (2014), arXiv:1403.2818 [hep-ph].

[58] M.B. Voloshin, “Radiative and pionic transitions $Z_c(4020)^0 \rightarrow X(3872)\gamma$ and $Z_c(4020)^\pm \rightarrow X(3872)\pi^\pm,”$ Phys. Rev. D 99, 054028 (2019), arXiv:1902.01281 [hep-ph].

[59] Xiang-Kun Dong, Feng-Kun Guo, and Bing-Song Zou, “Why there are many threshold structures in hadron spectrum with heavy quarks,” (2020), arXiv:2011.14517 [hep-ph].

[60] Bo Wang, Lu Meng, and Shi-Lin Zhu, “Deciphering the charged heavy quarkoniumlike states in chiral effective field theory,” Phys. Rev. D 102, 114019 (2020), arXiv:2009.01980 [hep-ph].

[61] A. Bondar et al. (Belle), “Observation of two charged bottomonium-like resonances in Y(5S) decays,” Phys. Rev. Lett. 108, 122001 (2012), arXiv:1110.2251 [hep-ex].

[62] A. Garmash et al. (Belle), “Observation of Zb(10610) and Zb(10650) Decaying to B Mesons,” Phys. Rev. Lett. 116, 212001 (2016), arXiv:1512.07419 [hep-ex].

[63] C. Hanhart, Yu. S. Kalashnikova, P. Matuschek, R. V. Mizuk, A. V. Nefediev, and Q. Wang, “Practical Parametrization for Line Shapes of Near-Threshold States,” Phys. Rev. Lett. 115, 202001 (2015), arXiv:1507.00382 [hep-ph].

[64] Yun-Hua Chen, Johanna T. Daub, Feng-Kun Guo, Bastian Kubis, Ulf-G. Meißner, and Bing-Song Zou, “Effect of $Z_b$ states on $\Upsilon(3S) \rightarrow \Upsilon(1S)\pi\pi$ decays,” Phys. Rev. D 93, 034030 (2016), arXiv:1512.03583 [hep-ph].

[65] Q. Wang, V. Baru, A. A. Filin, C. Hanhart, A. V. Nefediev, and J. L Wynen, “Line shapes of the $Z_b(10610)$ and $Z_b(10650)$ in the elastic and inelastic channels revisited,” Phys. Rev. D 98, 074023 (2018), arXiv:1805.07453 [hep-ph].

[66] V. Baru, E. Epelbaum, A. A. Filin, C. Hanhart, R. V. Mizuk, A. V. Nefediev, and S. Ropertz, “Insights into $Z_b(10610)$ and $Z_b(10650)$ from dipion transitions from $\Upsilon(10860)$,” Phys. Rev. D 103, 034016 (2021), arXiv:2012.05034 [hep-ph].
[67] Pablo G. Ortega, Jorge Segovia, and Francisco Fernandez, “The $Z_b$ structures in a constituent quark model coupled-channels calculation,” (2021), arXiv:2107.02544 [hep-ph].

[68] Medina Ablikim et al. (BESIII), “Evidence of Two Resonant Structures in $e^+e^- \rightarrow \pi^+\pi^- h_c$,” Phys. Rev. Lett. 118, 092002 (2017), arXiv:1610.07044 [hep-ex].

[69] Medina Ablikim et al. (BESIII), “Evidence of a resonant structure in the $e^+e^- \rightarrow \pi^+D^0D^{*-}$ cross section between 4.05 and 4.60 GeV,” Phys. Rev. Lett. 122, 102002 (2019), arXiv:1808.02847 [hep-ex].

[70] Feng-Kun Guo, Xiao-Hai Liu, and Shuntaro Sakai, “Threshold cusps and triangle singularities in hadronic reactions,” Prog. Part. Nucl. Phys. 112, 103757 (2020), arXiv:1912.07030 [hep-ph].

[71] Yoichi Ikeda, Tetsuo Hyodo, Daisuke Jido, Hiroyuki Kamano, Toru Sato, and Koichi Yazaki, “Structure of $\Lambda(1405)$ and threshold behavior of $\pi\Sigma$ scattering,” Prog. Theor. Phys. 125, 1205–1224 (2011), arXiv:1101.5190 [nucl-th].

[72] F. James and M. Roos, “Minuit: A System for Function Minimization and Analysis of the Parameter Errors and Correlations,” Comput. Phys. Commun. 10, 343–367 (1975).

[73] M. Ablikim et al. (BESIII), “Measurement of $e^+e^- \rightarrow \pi^+\pi^-\psi(3686)$ from 4.008 to 4.600 GeV and observation of a charged structure in the $\pi^\pm\psi(3686)$ mass spectrum,” Phys. Rev. D 96, 032004 (2017), [Erratum: Phys.Rev.D 99, 019903 (2019)], arXiv:1703.08787 [hep-ex].

[74] Medina Ablikim et al. (BESIII), “Measurement of $e^+e^- \rightarrow \pi^0\pi^0\psi(3686)$ at $\sqrt{s}$ from 4.009 to 4.600 GeV and observation of a neutral charmoniumlike structure,” Phys. Rev. D97, 052001 (2018), arXiv:1710.10740 [hep-ex].

[75] Qi Huang, Dian-Yong Chen, Xiang Liu, and Takayuki Matsuki, “Charged charmoniumlike structures in the $e^+e^- \rightarrow \psi(3686)\pi^+\pi^-$ process based on the ISPE mechanism,” Eur. Phys. J. C79, 613 (2019), arXiv:1905.05650 [hep-ph].

[76] Daniel A.S. Molnar, Igor Danilkin, and Marc Vanderhaeghen, “The role of charged exotic states in $e^+e^- \rightarrow \psi(2S)\pi^+\pi^-$,” Phys. Lett. B 797, 134851 (2019), arXiv:1903.08458 [hep-ph].

[77] X. L. Wang et al. (Belle), “Measurement of $e^+e^- \rightarrow \pi^+\pi^-\psi(2S)$ via Initial State Radiation at Belle,” Phys. Rev. D 91, 112007 (2015), arXiv:1410.7641 [hep-ex].

[78] Steven Weinberg, “Evidence That the Deuteron Is Not an Elementary Particle,” Phys. Rev. 137, B672–B678 (1965).

[79] V. Baru, J. Haidenbauer, C. Hanhart, Yu. Kalashnikova, and Alexander Evgenyevich Kudryavtsev, “Evidence that the $a_0(980)$ and $f_0(980)$ are not elementary particles,” Phys.
Lett. B 586, 53–61 (2004), arXiv:hep-ph/0308129.

[80] Zhi-Hui Guo and J. A. Oller, “Probabilistic interpretation of compositeness relation for resonances,” Phys. Rev. D 93, 096001 (2016), arXiv:1508.06400 [hep-ph].

[81] Yuki Kamiya and Tetsuo Hyodo, “Generalized weak-binding relations of compositeness in effective field theory,” PTEP 2017, 023D02 (2017), arXiv:1607.01899 [hep-ph].