Realistic Many-Body Quantum Systems vs. Full Random Matrices: Static and Dynamical Properties

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Non-equilibrium many-body quantum systems

• 1D many-body systems far from equilibrium
  • Experimentally accessible
  • Technological applications
  • Not well understood

• Goal: Understand and characterize:
  • Dynamics at different time scales
    • Dependence on chaoticity
  • Conditions for thermalization

• Use random matrices
Full Random Matrices

• Matrix filled with random numbers
  • First used by Wigner to model heavy nuclei
  • Interactions treated statistically
  • Details overlooked
  • Constrain to satisfy symmetries: real and symmetric (GOE)

• Not realistic
  • All particles interact at the same time
  • Faraway interactions as strong as nearby interactions

• Advantages
  • Analytical results
  • References and bounds for realistic systems
# Realistic Hamiltonians

| Integrable XXZ model | $H = H_{XXZ} + \epsilon_1 J S^z_1$ |
|----------------------|-----------------------------------|
| Chaotic defect model | $H = H_{XXZ} + \epsilon_1 J S^z_1 + dJS^z_{[L/2]}$ |
| Chaotic NNN model    | $H = H_{XXZ} + \lambda H_{NNN} + \epsilon_1 J S^z_1$ |

| Parameter | Value |
|-----------|-------|
| $J$       | 1     |
| $\epsilon_1$ | .1   |
| $\Delta$  | .48   |
| $d$       | .9    |
| $\lambda$ | 1     |

$H_{XXZ} = J \sum_{n=1}^{L-1} (S^n_x S_{n+1}^x + S^n_y S_{n+1}^y + \Delta S^n_z S_{n+1}^z)$

$H_{NNN} = J \sum_{n=1}^{L-2} (S^n_x S_{n+2}^x + S^n_y S_{n+2}^y + \Delta S^n_z S_{n+2}^z)$

$dJS^z_{[L/2]}$: defect in the middle

$S^z_1$: small defect on first site to break symmetries
Static Properties: Level Spacing Distribution

- Full Random matrix: Wigner Dyson
  \[ P(s) = \frac{\pi s}{2} \exp \left( -\frac{\pi s^2}{4} \right) \]

- Black curves: numerical data
- Red curve: Wigner Dyson
- Blue curve: Poisson
Static Properties: density of states

- Full Random matrix:

$$\rho^{DOS}(E) = \frac{2}{\pi \varepsilon} \sqrt{1 - \left(\frac{E}{\varepsilon}\right)^2}$$

$$-\varepsilon \leq E \leq \varepsilon$$
Static Properties: Entropies

- Shannon: $S_{sh}^{\alpha} = -\sum_k |C_k^{\alpha}|^2 \ln |C_k^{\alpha}|^2$
- Entanglement: $\rho_A = Tr_B(\rho) ; \quad S_{vN} = -Tr(\rho_A \ln \rho_A)$

- Black: Numerical Data
- Red: $\ln(0.48D)$

- Black: Normalized Entanglement Entropy
- Red: Normalized Shannon Entropy
Dynamic Properties: Survival Probability

\[ W_{\text{ini}}(t) \equiv |\langle \Psi(0) | e^{-iHt} | \Psi(0) \rangle|^2 = \left| \int P_{\text{ini,ini}}(E) e^{-iEt} dE \right|^2 \]

- LDOS: \[ P_{\text{ini,ini}}(E) = \sum_{\alpha} |C_{\text{ini}}^\alpha|^2 \delta(E - E_{\alpha}) \]

- Initial State: Néel state: \[ |\downarrow\uparrow\downarrow\uparrow\uparrow\uparrow \cdots \rangle \]
Dynamic Properties: Survival Probability

• Full Random matrix
  • \( W_{\text{int}}(t) = \frac{[J_1(2\sigma_{\text{init}})]^2}{\sigma_{\text{init}}^2 t^2} \)
  • Bessel decay
  • Envelope \( \sim t^{-3} \)

• Realistic Hamiltonians:
  • Gaussian decay
    • Even integrable system
  • Black: Numerical data
  • Red: Gaussian
Dynamic Properties: Entropies

- \( S_{sh}(t) = -\sum_{k=1}^{D} W_k(t) \ln W_k(t) \)

- \( W_k(t) = |(\phi_k | e^{-iHt} | \Psi(0))|^2 \)

- \( S_{sh}(t) = -W_{ini}(t) \ln W_{ini}(t) - \sum_{k \neq ini}^{D} W_k(t) \ln W_k(t) \)

\[ \approx -W_{ini}(t) \ln W_{ini}(t) - [1 - W_{ini}(t)] \ln \left[ \frac{1 - W_{ini}(t)}{N_{pc}} \right] \]

Survival Probability

\( N_{pc} = \langle \exp[S_{sh}(t)] \rangle \)
Dynamic Properties: Entropies (cont.)

• Full Random Matrix:

Analytical and numerical data agree perfectly for random matrices
• Linear increase for both integrable and chaotic models
• Both entropies seem to show similar behavior
Summary and Conclusion

• Random matrices
  • Analytical results
  • References and bounds for realistic systems

• Decay of initial state:
  • Random matrices: Bessel decay (fastest)
  • Realistic Systems: Gaussian decay
    • Faster than exponential!
    • Both integrable and chaotic systems!

• Shannon Entropy
  • Linear increase: exponential spreading

• Shannon and Entanglement Entropies
  • Show very similar behaviors
  • Is there any information that can only come out of Entanglement Entropy?

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Back up slides
Level Number Variance

\[ \Sigma^2(l) = \frac{2}{\pi^2} \left[ \ln(2\pi l) + \gamma_e + 1 - \frac{\pi^2}{8} \right] \]

- Black points: data
- Red Curve: logarithmic
- Blue curve: linear