Open Geometry Prover Community Project *

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Mathematical proof is undoubtedly the cornerstone of mathematics. The emergence, in the last years, of computing and reasoning tools, in particular automated geometry theorem provers, has enriched our experience with mathematics immensely. To avoid disparate efforts, the Open Geometry Prover Community Project aims at the integration of the different efforts for the development of geometry automated theorem provers, under a common “umbrella”. In this article the necessary steps to such integration are specified and the current implementation of some of those steps is described.

1 Introduction

Mathematical proof is undoubtedly the cornerstone of mathematics. All mathematics practitioner know its centrality and the difficulty in mastering it [8]. The emergence, in the last years, of computing and reasoning tools, in particular automated geometry theorem provers, has enriched our experience with mathematics immensely. Building such tools and exploring their applicability require a coherent, well-organized community of researchers working in a collaborative way, to avoid disparate efforts, as recalled by T. Han et al. [7]. Reuse of previous knowledge is vital for human beings in all kinds of learning activities, and so much more in mathematics. The reuse of practical implementations of an abstract idea is usually much harder than the reuse of the abstract idea itself. The same algorithm may be implemented several times using different programming languages and data formats due to engineering mismatches.

The Open Geometry Prover Community Project (OGPCP) aims at the integration of the different efforts for the development of geometry automated theorem provers, under a common “umbrella”. As such, a contribution to the larger goal of establishing a network of researchers working in the area of formal reasoning, knowledge-based intelligent software and geometric knowledge management, to explore efficient methodologies for the creation and reuse of electronic tools in geometry.

To bring up such a framework a series of tools and protocols must be implemented/established. The Open Geometry Prover Community Project framework, goals are:

- to provide a common open access repository for the development of Geometry Automated Theorem Provers (GATP);
- to provide an API to the different GATP in such a way that they can be easily used by users, stand-alone or integrated in other tools;
- to develop portfolio strategies to allow choosing the best GATP for any given geometric conjecture;
- to interface with repositories of geometric knowledge [27] (e.g. TGTP [24], TPTP [30]);
- to develop a GATP System Competition to be able to rate GATPs [2, 25].

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1Thousand of Geometric problems for geometric Theorem Provers, http://hilbert.mat.uc.pt/TGTP/

2Thousands of Problems for Theorem Provers, http://www.tptp.org/
Overview of the paper. The paper is organised as follows: first, in §2 the current status of the framework implementation is described. In §3 a short description of the GATP currently incorporated in the OGPCP is given. Finally, in §4 conclusions are drawn and future work is discussed.

2 OGPCP Implementation Status

The OGPCP framework is a never-ending project in the sense that new GATP can be proposed and incorporated in the project at any given moment. Nevertheless many of the steps necessary for its current use and for an easy integration of new future projects are already done.

![OGPCP Framework Diagram](image)

Figure 1: OGPCP Framework

2.1 OGPCP Source Repository

The OGPCP is hosted at GitHub\footnote{https://github.com/opengeometryprover/OpenGeometryProver}. The code is made available under the GNU General Public License\footnote{https://www.gnu.org/licenses/gpl.html} version 3 or later, and the documentation under the GNU Free Documentation License\footnote{https://www.gnu.org/licenses/fdl.html} version 1 or later.

OGPCP is available only as source-code and its installation in Unix systems is a straightforward process, provided that GNU Make, Apache Ant and OpenJDK are installed. After downloading the code from the GitHub repository, in the command line just type

```
$ make
$ make install
```

2.2 OGPCP Application Programming Interface

OGPCP API is a combination of several command-line tools, e.g., native (i.e. done by the OGPCP team and sharing a common base code) and external provers, filters, post-processors, prepared to seamlessly work together or with independent tools.

Native OGPCP provers must:
• use TPTP’s first-order format (FOF) syntax as their default conjecture format;
• accept the same command-line arguments;
• provide the same output;

All this is explained in Open Geometry Prover Community Project Programmer Manual, available at
the OGPCP’s GitHub repository.

External provers will be developed by other teams, with different base codes, even, eventually, using
different programming languages (no enforcement is done on those matters). External provers conjecture’s
format may not adhere to TPTP’s FOF syntax (see Section 4). In such cases, for OGPCP to take
advantage of those provers, and vice-versa, filters to/from the FOF syntax must be written, as well as
post-processors to interpret the output of those provers.

Example of usage and already implemented features:

Using conjectures in situ, contained in local files

$ ogp ceva.gcl 
   use of GCL prover, native language, area method

$ ogp ceva.gcl -w 
   use of GCL prover, native language, Wu’s method

$ ogp ceva.coqam 
   use of CoqAM, native language, area method

$ ogp ceva.fof 
   use of inbuilt portfolio mechanism

$ ogp -t 30 ceva.fof 
   use of inbuilt portfolio mechanism with a time limit (30 seconds)

Using conjectures in a remote repository

$ ogp --tgtp=GEO0001 gclc 
   connection to the TGTP repository (see § 2.5).

The command line OGPCP meta-syntax is the following:

ogp [<option>] [<conjecture> [<prover> [<prover-options>]]]

The available options are:

-h, --help
   prints a help message and exits — to be used alone;

-p, --provers
   lists the available (to OGPCP) GATP and exits — to be used alone;

-V, --version
   prints OGPCP version and exits — to be used alone;

-t <time>, --timeout=<time>
   redefines the default time limit, in seconds, when proving a conjecture.

The conjecture is provided to the prover in a local file or, when using the remote repository TGTP, by its
unique id in the repository, using the syntax

   --tgtp=<conjecture_id>.

When attempting to prove a conjecture, the choice of the prover, if none is indicated, proceeds according
to the following rules:

1. if the conjecture is provided in a local file, then
   (a) if file name extension is fof, then use OGPCP portfolio prover;
   (b) otherwise, use the default prover associated with that extension

2. otherwise, use OGPCP portfolio prover.

When the prover is given, a check is made to certify if the conjecture’s format is one accepted by the
prover. If that is not the case, whenever possible filters are used to convert to a format accepted by the
prover. If all this fails, an error occurs and the process ended.
2.3 OGPOR Filters & Post-processors

A set of filters are already ready to be used.

| Filter Name                  | Description                           |
|------------------------------|---------------------------------------|
| filterGCLtoFOF               | GCL language to FOF                   |
| filterGEOGEBRAtoFOF         | GeoGebra to FOF                       |
| filterJGEXtoFOF             | JGEX to FOF                           |

For the moment all these filters, filter*toFOF, assume the inclusion of the axioms of the deductive database full-angle method [6], given that these are already converted to FOF syntax. That is, a plain conversion is made and an include instruction is added at the begin with the above mentioned axiom set.

Post-processors are to be used in conjunction with independent provers. They are used to obtain information about the proof’s result, e.g., if the proof was successful or not, time, file with the proof steps, if any, etc., as the output of an independent prover must not adhere to that of a native OGPOR prover.

As of this writing, there is only one post-processor — for the Vampire ATP, to get the time of a proof.

2.4 OGPOR Portfolio

Portfolio problem solving is an approach in which for an individual instance of a specific problem, one particular, hopefully the most appropriate, solving technique is automatically selected among several available ones and used. Weidenbach [32] makes the distinction between syntactic and semantic approaches. With a Simple-Syntactic portfolio solver the selection of the core solvers is done by purely syntactic problem properties and there is no exchange of results between different core solvers. In a Sophisticated-Semantic portfolio solver the selection of the core solvers is done by semantic or structural problem properties and the solvers exchange results [32].

Already some work in the area of geometric automated theorem proving has been done, namely in the prover mechanism implemented in GeoGebra [16, 17, 22]. It is expected that this research can be incorporated into the OGPOR.

2.5 OGPOR Interface with Repositories

A server/client architecture to connect OGPOR and TGTP is already available. On the side of the TGTP repository a query-server is already implemented, always listening to client requests.

The code for the clients is open-source and available as part of the OGPOR project. The clients are build in such a way that a SQL query can be send to the TGTP database, receiving in return the code of the desired conjecture. The exchange of information between the server and the client is done using the JSON format. The implementation of new clients to other GATP it is easy and opens the use of the information contained in TGTP from any GATP. This server/client architecture is currently being used to establish a connection between the e-learning environment Web Geometry Laboratory and the TGTP repository [27, 28].

For example, using the OGPOR API we could write: `ogp --tgtp=GEO00001 gc1c`. This call will trigger the tgtptoogpor client, sending a query about the `gc1c` code for problem GEO00001 in TGTP.
the problem do not exist an error code will be returned, if the \texttt{gecl} for such a problem do not exist, the \texttt{FOF} code for that problem will be returned. After receiving an error free answer, the \texttt{ogp} command will pursue as usual.

2.6 Geometry Automated Theorem Provers Systems Competition

To be able to compare the different methods and implementations, a competition will have the virtue of pushing towards the standardization of the input language, the standardization of test sets, the direct comparability and the easier exchange of ideas and algorithmic techniques. The results of such a competition will also constitute a showcase, where potential users will look for the best GATP for their goals \cite{2,25}.

A first trial-run of the Geometry Automated Theorem Provers Systems Competition, GASC 0.2, was already run, at ThEdu’19, the 8th International Workshop on Theorem proving components for Educational software, August 2019, Natal, Brazil \cite{25,26} and a second trial-run is being prepared.

Not being directly related to the \textit{OGPCP} the GASC will be used to test the different GATP in the project, pushing towards the development of new and better implementations.

3 External Geometry Automated Theorem Provers

A set of external GATP are already part of the OGPCP. These are autonomous open source projects that recognise the \textit{OGPCP} and from which filters to/from the native syntax and FOF are already implemented, or will be implemented in a near future.

Those GATP must be downloaded and installed in a separate way, simple instructions on how to do it will be part of the \textit{OGPCP} documentation.

GeoGebra Automated Reasoning Tools. The standard version of \textit{GeoGebra}\footnote{https://www.geogebra.org/download} includes several Automated Reasoning Tools (ART):

- for conjecturing a geometric property (e.g. such three points visually “seem” to be aligned), the \texttt{Relation} command;
- for rigorously denying or confirming a given conjecture (e.g. providing an affirmative answer to the conjecture after internally verifying, using Computer Algebra tools, that some determinant involving the coordinates of the three selected points is zero), the \texttt{Prove} and \texttt{ProveDetails} commands;
- for presenting some complementary hypotheses for the truth of a given (actually false) statement (e.g. remarking that the truth of the proposed statement needs some further steps in the geometric construction describing the statement), the \texttt{LocusEquation} command.

See \cite{3} for a detailed explanation about the project and \cite{15} for a tutorial-like paper about the different commands, as well as \cite{19} for a quite updated version.

Moreover there are two other reasoning toolsets, already implemented but in (yet) non-standard versions of \textit{GeoGebra} \cite{4,20}. The first one contains the \texttt{Discover} tool and command, and the \texttt{Compare} command, can be used in the \textit{GeoGebra Discovery} fork\footnote{https://github.com/kovzol/geogebra-discovery} available in two different options: one, operating over \textit{GeoGebra Classic 5}, for \textit{MS-Windows, Mac} and \textit{Linux} systems; and, the other, working over \textit{GeoGebra Classic 6}, that requires starting it in a browser, for tablets and smartphones.\footnote{http://autgeo.online/geogebra-discovery/}
The Discover command automatically finds all theorems (of a certain kind: parallelism, congruence, perpendicularity, etc.) holding over a given element of a construction (e.g., involving a point), by considering some combinatorial heuristics to formulate different Relation tests involving always the selected element, plus some other one, and presenting as output the collection of obtained properties. The Compare command is used to find a general relationship between two quantities (for example, by comparing the sum of the lengths of the catheti \( a \) and \( b \) and the length of the hypotenuse \( c \) in a right triangle—clearly, here the relationship is an inequality, namely, \( c < a + b < \sqrt{2}c \)). This low-level command is usually called from an improved version of the Relation command [11].

The second currently on-going improvement deals with the development of an AG=Automated Geometer a “geometer” that does not require human intervention, except that of launching the computation process over a figure. It is a web-based module that allows GeoGebra to automatically produce different conjectures over the given geometric construction, and to internally confirm or deny them using tools similar to those in GeoGebra Discovery, but here not limited to exploring relations involving a single, specific element.

The algorithms behind all these tools deal with the algebraic translation of the geometric statements and the symbolic manipulation—via the embedded computer algebra system GIAC[12,13]—of the corresponding complex algebraic geometry varieties. See [14,18,21,29], for a detailed description of the involved theoretical approach.

It must be remarked that the chosen method is quite effective and is able to deal, in milliseconds and over a variety of popular electronic devices (laptops, smartphones, etc.), with very complicated statements but, on the other hand, it does not provide any human-understandable arguments for the declared truth/falsity of the involved statements.

Finally, we summarize how GeoGebra’s features can be directly exploited by OGPCP. GeoGebra offers two application programming interfaces for external programs:

- A JavaScript Application Programming Interface (API), available for web applications. The suggested method is to set up the construction via JavaScript calls and then execute the command ProveDetails to obtain the result.

- The desktop application can be directly called with an input GeoGebra file. The file structure is given in XML. By creating the XML data as input, and calling GeoGebra via command line, the debug information can be directly processed to get the result.

GCLC Automated Reasoning Tools. Within the mathematical software GCLC, there is an implementation of the area method [12] by Janičić and Quaresma, and implementations of (simple) Wu’s method and Gröbner based method, by Predović and Janičić [11]. Apart a graphical user interface all the GATP can be used in stand-alone mode, begin usable in the overall Open Geometry Prover Community Project interface.

CoqAM. The formalization within the Coq proof assistant of the area method, a decision procedure for affine plane geometry [12,23,15] It can be used in stand-alone mode (Coq must be installed), being possible its use within overall Open Geometry Prover Community Project interface.

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[11] http://autgeo.online/ag/ https://github.com/kovzol/ag
[12] Giac/Xcas, https://www-fourier.ujf-grenoble.fr/~parisse/giac.html
[13] https://dpt-info.u-strasbg.fr/~narboux/area_method.html

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**JGEX.** Java Geometry Expert, *JGEX*, is a software which combines dynamic geometry software (DGS), automated geometry theorem prover (GATP) and an approach for visually dynamic presentation of proofs. As a dynamic geometry software, *JGEX* can be used to build dynamic visual models to assist teaching and learning of various mathematical concepts.

Apart the use of the GATP systems inside the overall graphical DGS interface, they can be used stand-alone, being possible its use within the overall *Open Geometry Prover Community Project* interface.

**Generic ATP** Apart from these ATP, specific to geometry (GATP), the generic ATP can also be used. It is a question of including an axiomatic theory specific to geometry, e.g. those in the Geo domain in *TPTP* (Hilbert geometry; Tarski geometry, Rules of construction (von Plato), deductive database method, among others). Not being in the core of the project its use is, nevertheless, being taken in consideration and the *Open Geometry Prover Community Project* command line tool will process an input related to such tools.

### 4 Conclusions and Future Work

The problems related to the integration between different geometry provers can be much more harder than the presented above. Different algorithms/provers do not assume all the same mathematical setting. Different axiomatizations exist, e.g. Tarski’s, Hilbert’s, von Plato’s; Area method. Different kinds of geometry, e.g. euclidean 2D or 3D, non-euclidean. Different types of approaches, geometric, e.g. area method, algebraic, e.g. Wu’s method. More than a, maybe unrealistic, full integration, the *OGPCP* should aim to: give a simple, documented, open source, API to allow the use of GATP by experts and non-experts and to constitute itself as a forum, a space of discussion, about the deductive tools for geometry.

Apart many improvements in the existing framework, e.g. improve the API, linking with external provers, filters and post-processing, new native provers are planned: a new implementation of the full-angle method [5], the deductive database method [6] (using the axioms of the full-angle method), and a novel approach, the deductive graphs method, based on the deductive database method but using deductive graphs. Some initial work has already been done in those methods [1,9,10].

As said at the beginning the *OGPCP* is meant to be a never ending project in the sense that new improvements in the area of automated deduction will be made and incorporated in it. New methods, new implementations, improvements in the existing approaches, etc. To enlarge the usefulness and conquer new “audiences” (e.g. teachers in primary and secondary tools) the GATP need to be more modular, being able to be incorporated into “friendly” tools, that can cover the “difficult nature” of many GATP. The *OGPCP* should help on the goal of “bring the automated deduction to all geometers”.

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