Spontaneous Leptogenesis

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Abstract

I propose a new mechanism for baryogenesis, in which the out-of-equilibrium condition is not necessary. When the electroweak symmetry is broken spontaneously, left-handed neutrinos may get Majorana masses containing $C\!P$-violating phases. This induces a lepton asymmetry spontaneously, which is then converted to a baryon asymmetry in the presence of the sphaleron field.
The baryon asymmetry of the universe has been discussed extensively in the literature [1, 2]. All the mechanisms require to fulfill the three conditions proposed by Sakharov [1]. It requires baryon number violating interaction, which also violate $C$ and $CP$ symmetry. This interaction should be slower than the expansion rate of the universe, *i.e.*, satisfy the out-of-equilibrium condition.

In the present article I propose a new mechanism for baryogenesis, where the out-of-equilibrium condition is not required. Lepton number and $CP$ violation comes from the Majorana mass matrix of the left-handed neutrinos. As time evolves these massive Majorana neutrinos remains in equilibrium and hence satisfy Boltzmann statistics. But due to the rephasing invariant $CP$–violating creation phases in the Majorana mass matrix the population of the fields carrying lepton number 1 becomes different from the population of the fields carrying a lepton number -1. A lepton asymmetric universe is thus spontaneously created when the electroweak symmetry is spontaneously broken [1]. This lepton asymmetry is then converted to a baryon asymmetry in the presence of the sphalerons through anomalous baryon number violating interaction.

I work in the context of the standard model. For the neutrino mass I donot assume any particular mechanism. In the see-saw models [4], there exists right handed neutrinos whose Majorana masses breaks the lepton number. Whereas in the Zee-type models [5], a $SU(2)_L$ singlet higgs scalar breaks the lepton number. We donot subscribe to any of these specific models for this mechanism to work. We only assume that lepton number is broken before the electroweak symmetry breaking. As a result when the higgs doublet of the standard model acquires a vacuum expectation value ($vev$), the left-handed neutrinos get Majorana masses (since there is no symmetry to prevent it)

$^{1}$We studied the possibility of a lepton asymmetric universe for the heavy Majorana neutrinos recently and here a similar technique will be adopted [3, 4].
through see-saw mechanism or through radiative corrections. Once the neutrinos acquire masses the $CP$ symmetry can also be violated spontaneously. Similar to the $CP$–violating phase in the CKM matrix, there can be new $CP$–violating phase in the leptonic sector.

The Majorana mass term of the left-handed neutrinos ($\nu_i$, where $i = 1,2,3$ corresponds to three generations), which can be written as,

$$\mathcal{L} = \sum_{ij} \bar{m}_{ij}(\bar{\nu}_i)c\bar{\nu}_j$$  

(1)

can have its origin in the see-saw mechanism or the Zee-type models. But for the present purpose we start with this effective Majorana mass term in the lagrangian. This mass matrix can, in general, be complex and can have $CP$–violating complex phase. But the fields $\bar{\nu}_i$ can now be rotated to a basis $\nu_i = N_{ij}\bar{\nu}_j$, where the new mass matrix $\mathcal{M} = \text{diag} \ (m_1, m_2, m_3)$ is real and diagonal,

$$\mathcal{L} = \sum_i m_i(\nu_i)c\nu_i.$$  

(2)

However, this will introduce $CP$–violating phase in the charge current and higgs interactions of the neutrinos with the charged leptons. So although the tree level Majorana mass matrix is diagonal and real in this basis, there will be radiative corrections as given in figure 1, which will then induce off-diagonal terms (in general complex) in the mass matrix.

For the sake of simplicity and to make the physics of the problem clear I consider only two generations. Since we want to discuss the question of $CP$–violation, we distinguish the neutrino fields $|\nu>$ and the anti-neutrino fields $|\nu^c>$ to start with and then show that there is only one physical Majorana field at the end. From the charge current interactions of the fields $|\nu>$ and $|\nu^c>$ with the charged leptons

$$\mathcal{L} = g\bar{\nu}\gamma_\mu l^-W_{\mu}^+ + g\bar{\nu}^c\gamma_\mu l^+W_{\mu}^-$$  

(3)

it is possible to assign lepton numbers 1 to $|\nu>$ and -1 to $|\nu^c>$ respectively.
Figure 1: One loop graph contributing to the Majorana masses of the left-handed neutrinos.

To get the physical Majorana eigenstates of the problem we start with the effective Hamiltonian of this model given in terms of the tree level real masses $m_1$ and $m_2$ and the one loop corrections arising from the self-energy type diagrams of figure 1. The effective Hamiltonian in the basis $(|\nu^e_1 > |\nu^e_2 > |\nu_1 > |\nu_2 >)$ now reads

$$
\mathcal{H} = \begin{pmatrix}
0 & 0 & m_1 + \tilde{m}_1 & m \\
0 & 0 & m & m_2 + \tilde{m}_2 \\
m_1 + \tilde{m}_1 & \tilde{m} & 0 & 0 \\
\tilde{m} & m_2 + \tilde{m}_2 & 0 & 0
\end{pmatrix}
$$  \hspace{1cm} (4)

where,

$$
m = g^2 \left[ m_1 \sum_\alpha V_{\alpha i}^* V_{\alpha j} + m_2 \sum_\alpha V_{\alpha i} V_{\alpha j}^* \right] \left( g_{\alpha ij}^d - \frac{i}{2} g_{\alpha ij}^a \right)  \hspace{1cm} (5)$$

$$
\tilde{m} = g^2 \left[ m_1 \sum_\alpha V_{\alpha i}^* V_{\alpha j} + m_2 \sum_\alpha V_{\alpha i} V_{\alpha j}^* \right] \left( g_{\alpha ij}^d - \frac{i}{2} g_{\alpha ij}^a \right)  \hspace{1cm} (6)$$

$$
\hat{m}_i = g^2 \left[ 2m_i \sum_\alpha V_{\alpha i} V_{\alpha i}^* \right] \left( g_{\alpha ii}^d - \frac{i}{2} g_{\alpha ii}^a \right) \hspace{1cm} (7)
$$

2 There will be similar diagrams with the higgs scalars instead of the $W^\pm$, but those diagrams will be suppressed by the Yukawa couplings of the leptons.
where \( g \) is the SU\((2)_L \) gauge coupling constant, \( V_{\alpha i} \) is the mixing matrix in the charge current interactions in the basis where both the neutrino and charge lepton mass matrices are diagonal. The index \( \alpha \) corresponds to the charged lepton exchanged in the diagram. \( V_{\alpha i} \) is similar to the CKM matrix in the quark sector, except that now due to the Majorana nature of the neutrinos there is one \( CP \)-violating phase even in the two generation case. The dispersive part of the loop integral \( g_{\alpha ij}^d \) can be absorbed in the wave function and the mass renormalization. The absorptive part of the loop integral \( g_{\alpha ij}^a \) is nonvanishing as long as the momentum of the external neutrino fields are large and satisfy the condition, \( p m_W > m_l^2 \). We are interested in the range of temperatures 250 GeV (when a higgs acquire a vev) to about 50 GeV (when the sphaleron transitions become ineffective). In this limit the absorptive part of the integral is given by,

\[
g_{\alpha ij}^a = \frac{1}{16\pi}.
\]

The Hamiltonian of equation(4) can be solved exactly [4], but the expressions become somewhat involved. On the other hand without loss of generality it is possible to demonstrate the basic idea of the problem assuming a mass hierarchy \( m_1 > m_2 >> m_i, \tilde{m}, \hat{m}_i \). In this limit one can use perturbation theory to get the physical eigenstates,

\[
|\nu_1^{\text{phys}}\rangle = \frac{1}{\sqrt{N}}(|\nu_1 > + \alpha_2 |\nu_2 > + |\nu_1^c > + \alpha_1 |\nu_2^c >)
\]

\[
|\nu_1^{\text{phys}'}\rangle = \frac{1}{\sqrt{N}}(|\nu_1 > + \alpha_2 |\nu_2 > - |\nu_1^c > - \alpha_1 |\nu_2^c >)
\]

\[
|\nu_2^{\text{phys}}\rangle = \frac{1}{\sqrt{N}}(|\nu_2 > - \alpha_1 |\nu_1 > + |\nu_2^c > - \alpha_2 |\nu_1^c >)
\]

\[
|\nu_2^{\text{phys}'}\rangle = \frac{1}{\sqrt{N}}(|\nu_2 > - \alpha_1 |\nu_1 > - |\nu_2^c > + \alpha_2 |\nu_1^c >)
\]

with, \( \alpha_1 = \frac{m_1 m + m_2 \tilde{m}}{m_1^2 - m_2^2} \), \( \alpha_2 = \frac{m_1 \hat{m} + m_2^2 m}{m_1^2 - m_2^2} \) and \( N = 2 + |\alpha_1|^2 + |\alpha_2|^2 \).
As mentioned earlier we now have two physical Majorana neutrino states $|\nu^\text{phys}_1\rangle$ and $|\nu^\text{phys}_2\rangle$. The states $|\nu^\text{phys}_1\rangle'$ and $|\nu^\text{phys}_2\rangle'$ are related by $\gamma_5$ transformations to the states $|\nu^\text{phys}_1\rangle$ and $|\nu^\text{phys}_2\rangle$ respectively and are not independent.

Before the electroweak phase transition, the states $|\nu_i\rangle$ with lepton number 1 and $|\nu_c\rangle$ with a lepton number -1 were the physical states. They were evolving with time and their number densities were given by the equilibrium distribution $n_{\nu_1} = n_{\nu_2} \sim n_{\gamma}$ and hence there was no lepton asymmetry. As soon as the electroweak symmetry is spontaneously broken, the neutrinos acquire Majorana masses and $|\nu^\text{phys}_1\rangle$ and $|\nu^\text{phys}_2\rangle$ becomes the physical states. The number density of $|\nu^\text{phys}_1\rangle$ and $|\nu^\text{phys}_2\rangle$ now satisfy the equilibrium distribution $n_{\nu^\text{phys}_1} = n_{\nu^\text{phys}_2} \sim n_{\gamma}$. However, the lepton number of the universe is now given by,

$$\frac{n_L}{n_{\gamma}} = \frac{\sum_i \left( |\sum_m <\nu^\text{phys}_i|\nu^\text{phys}_m\rangle|^2 - |\sum_m <\nu^\text{phys}_i|\nu^c_m\rangle|^2 \right)}{\sum_i \left( |\sum_m <\nu^\text{phys}_i|\nu^\text{phys}_m\rangle|^2 + |\sum_m <\nu^\text{phys}_i|\nu^c_m\rangle|^2 \right)}$$

$$= \frac{g^2}{8\pi} \frac{m_1 - m_2}{m_1 + m_2} \text{Im}\left[\sum_\alpha V^*_{\alpha 1} V_{\alpha 2}\right]. \quad (9)$$

In this expression $CP$–violation comes from the $CP$–phase in the mixing matrix $V$. Since we have already made all possible phase rotations to work in the basis of real and diagonal tree level masses for the neutrinos, the only freedom left is to phase rotate the charged leptons. But then any phase rotation of the charged lepton fields $l_\alpha$ keeps this Imaginary quantity invariant. Unlike the decay scenario considered so far in the literature, we now have a new rephasing invariant combination of the mixing matrix $(m_1 - m_2)\text{Im}[\sum_\alpha V^*_{\alpha 1} V_{\alpha 2}]$, which gives rise to the $CP$–violation. This is a product of two matrix elements of $V$ and not four. In the quark sector there are no analog of this combination due to the absence of any Majorana particle. Hence this $CP$–violation is also distinct from the decay scenario.
When the doublet higgs scalar field acquires a vacuum expectation value at a temperature of around 250 GeV, a non-zero $n_L$ as given by the equation (9) is spontaneously generated from an initial value of $n_L = 0$. This lepton asymmetry is same as the $(B - L)$ asymmetry if there is no primordial baryon asymmetry of the universe. During the electroweak phase transition this lepton asymmetry will then be converted to a baryon asymmetry due to the anomalous baryon number violation in the presence of the sphalerons,

$$\frac{n_B}{n_\gamma} \sim \frac{1}{3} \frac{n_{(B-L)}}{n_\gamma} \sim \frac{1}{3} \frac{n_B}{n_\gamma}.$$\n
In equation (9) there is enough freedom for us to make the generated baryon asymmetry to be of the order of $O(10^{-8})$. This is particularly so because of our lack of knowledge of the amount of $CP$–violation in the leptonic sector.

In demonstrating how this mechanism works we have considered a two generation model and made some simplifying assumptions. In the three generation case, this mechanism works exactly in the similar way, but then there will be three $CP$–violating phases in the mixing matrix all of which can contribute to the generation of the lepton asymmetry. It is also possible to relax the assumption on the hierarchy of the masses, and solve the problem exactly \[4\], which will only increase our freedom to adjust the value of the generated baryon asymmetry. The source of $CP$–violation and the redundancy of the out-of-equilibrium condition is not altered.

To summarize, a conceptually new model of baryogenesis has been proposed. When the $SU(2)_L$ higgs doublet acquires a vev, a $CP$–violation in the Majorana mass matrix of the left-handed neutrinos will make the universe lepton asymmetric spontaneously, which then generates a baryon asymmetry in the presence of the sphaleron fields. Out-of-equilibrium condition is redundant in this scenario.
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