Development of a fluid structure interaction tool based on an actuator line model

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Abstract. In the present paper the methodology and the procedures for the implicit coupling of an Actuator Line (AL) aerodynamic code with a beam like structural code for the analysis of wind turbine rotors are detailed. Results from benchmark aeroelastic simulations of canonical inflow conditions, comparing the newly developed AL model against a standard Blade Element Momentum (BEM) model are presented in the paper. The two models provide very similar results in simple, uniform inflow, axisymmetric flow cases. The advantages of this newly developed tool emerge when more complex inflow conditions are addressed. In the present paper, besides axial flow conditions, operation under high yaw misalignment is also considered. BEM model accounts for the effect of the wake skewness through the application of an a posteriori engineering correction. Therefore, in this particular non symmetric flow case, deviations between AL and BEM are expected to be higher, especially as yaw misalignment angles increase. In the paper the above differences are assessed and interpreted.

1. Introduction

The perpetual need for wind energy cost reduction dictates continuous increase in the size and flexibility of wind turbine rotors. In connection to the above need for highly flexible designs, rotor analysis is today directed towards high fidelity numerical tools, applying elaborated, physically motivated models for the accurate prediction of the blades’ response. Among others, these tools should be able to account for aeroelastic phenomena which play a decisive role in determining dynamic behaviour and loads. With regard to structural analysis, state of the art aeroelastic design tools usually employ nonlinear beam models [1]. Concerning rotor aerodynamics, several options of varying fidelity are available today. Simplified engineering tools, such as BEM or linearized dynamic wake models [2], are widely employed in design and analysis tools. A more elaborate option is panel codes employing free wake modelling [3]. The latter can handle complex aerodynamic phenomena (yaw misalignment, high shear of the atmospheric boundary layer (ABL)) with greater accuracy, however,
they are essentially inviscid, which means that load driving conditions related to viscous effects (such as stall induced effects), can only be considered through engineering correction models. On the other end, the highest fidelity option is fully resolved, grid based CFD solvers, which however penalize substantially the computational cost. An intermediate option between the inviscid panel codes and the fully resolved CFD models are hybrid methods [4] or CFD solvers employing actuator disk/line aerodynamic models [5]. In the present work, the AL method is adopted in the aerodynamic modelling of the rotor. Similar to standard BEM models, AL models rely on measured or computed airfoil polars. However, their main advantages are that a) three dimensional flow and wake induced effects are inherently accounted for and b) modelling of inflow turbulence can be considered as part of the numerical CFD procedure. The latter is of particular importance for the better understanding of the way that turbulence structures interact with the rotor disk and evolve in the wake flow.

The paper presents a newly developed aeroelastic tool based on the AL approach and discusses implementation issues concerning the implicit (strong) coupling of the AL aerodynamic module with a structural dynamics module. The novelty of the proposed approach consists mainly in the application of the AL method in an aerelastic context. In the existing literature AL method has been mainly used in pure aerodynamic analyses of wind turbine wakes. Aeroelastic developments using AL or similar actuator based techniques have been published only by few researchers (see eg. [6] [7]). The advantage of the approach lies in the fact that it inherently accounts for 3D and turbulent wake induced effects, contrary to the standard BEM method in which such phenomena can only be considered through the application of engineering corrections models. Moreover, the coupling between the AL aerodynamic module and the structural dynamics module was achieved in a strong (implicit) way. Hence, within every time step internal iterations between the two modules are performed, until converged solution is achieved for both. In most of the existing developments loose coupling approximations are applied.

The aerodynamic module is MaPFlow [4], a standard 2nd order finite volume CFD solver that is coupled with the multi-body structural code hGAST [1]. Both MaPFlow and hGAST are in-house tools developed at NTUA. In the present work, consistency of the implementation of the aeroelastic coupling is checked through comparisons of load and deflection results predicted by the AL model against those of the standard BEM based version of hGAST code, in axial, uniform flow conditions. Furthermore, more complex inflow conditions of yawed flows are addressed and differences between the two aerodynamic models are assessed.

Simulations are performed for the conceptual DTU 10MW reference wind turbine rotor [8], at the wind speeds of 8 and 11 m/s (partial load operation) in uniform flow conditions. Yaw misalignment angles in the range -30° to +30° are considered. Aeroelastic simulations are performed considering constant rotational speed and pitch angle of the blades (open loop operation). Deformation and load results are presented as a) azimuthal variations b) radial distributions along the blade span.

2. Methodology

In the AL context, the effect of the rotor on the flow is accounted for by applying aerodynamic loads as source terms in the Unsteady Reynolds Averaged Navier Stokes equations (URANS). The above mentioned source terms are applied to the cells swept by the rotating rotor blades. There are multiple benefits from adopting such an approach; a) the computational mesh does not need to follow the motion of the rotor, b) interaction among wind turbine rotors in a wind farm configuration can be easily analyzed without needing to employ overset or chimera grids and c) the aeroelastic coupling is easy to handle, since there is no need for the CFD mesh to follow the deformation of the blades. The high accuracy of the AL model in computing aerodynamic loads suggests that an aeroelastic simulation tool of this type is a good compromise between accuracy and computational cost. On the other hand, the main shortcoming of this approach is that it relies on tabulated polars and cannot resolve aerodynamic phenomena which depend on length scales relevant to the size of the blade chord (e.g. flow separation, blades’ vortex shedding).
2.1. The aerodynamic AL module

The AL analysis is implemented within the in-house finite volume solver MaPFlow, which solves the Unsteady Reynolds-Averaged Navier Stokes (URANS) equations. It is a compressible, cell centered CFD solver, which employs both structured and unstructured grids. The convective fluxes are discretized using the approximate Riemann solver of Roe [9] with Venkatakrishnan limiter [10], while the viscous fluxes are discretized using a central 2nd order scheme. Turbulence closure includes several options; the one equation turbulence model of Spalart (SA) [11], as well as the two equation turbulence model of Menter (k-ω SST) [12]. MaPFlow can handle both steady and unsteady flows, the latter being an important requirement for aerodynamic tools employed in aeroelastic analysis. Time integration is achieved in an implicit manner permitting large CFL numbers. The unsteady calculations use a 2nd order time accurate scheme combined with the dual time-stepping technique [13] to facilitate convergence when complex unsteady flows with moving or deforming geometries are considered. Additionally, flows in the incompressible region are realized using Low Mach Preconditioning [14].

In the above described CFD framework, the AL method has been implemented following the standard approach [5] where blade loads are modelled as source terms on the mesh cells swept by the blades during their rotation. The source terms are calculated on the basis of blade element analysis along the blade span and in conjunction with tabulated 2D polars. In order to avoid singularities, they are numerically smeared across a few cells using a 3D isotropic Gaussian distribution [15].

The freestream velocity vector is directly sampled on the emission points of the actuator lines [16]. The emission points are also the centers of the blades’ bound vorticity, where the blade local flow effects (upwash and downwash created by the bound vortex) are negligible. Hence, a consistent estimation of the freestream velocity is performed. The velocity at the emission points is estimated through a distance and volume weighted interpolation to the computed velocities of the neighbouring cells using Radial Basis Functions (RBF) [17].

Even though recommended by [18], tip correction models are not used in the present work, since the three-dimensional flow field containing tip and root vortices can be fully resolved, provided that an adequately fine grid resolution is used in the vicinity of the actuator lines. With a small enough characteristic cell length ($Δx$) of up to $\frac{c}{2}$ ($\bar{c}$ is the mean chord of the blade), the standard blade force distribution suggested by [15] was found to be sufficient to resolve the tip and root vortices. For this reason and for sake of simplicity, it was preferred over more sophisticated projection techniques (e.g. [19] [20] [21] [22]).

Both Cartesian [23] and cylindrical [5] grids can be treated by the model. However, in the present analysis only Cartesian grids are considered, as they are more convenient in future development, for instance in cases where the influence of Atmospheric Boundary Layer and/or wind farm effects are to be studied. In particular, a thin equidistant region around the actuator lines is chosen to be kept structured and fine in terms of grid density ($Δx = \frac{c}{2}$), so that maximum accuracy in velocity sampling and vorticity resolution is achieved. The structured region has to sufficiently extend both upstream and downstream of the rotor disk in order to account for blade deflections. Furthermore, a region with an unstructured but still fine grid is considered around the rotor, so that the effect of near wake on the blade loads is properly accounted for. This region is chosen to reach up to one rotor diameter (1D) upstream, 2D downstream and 1D radially from the rotor center, in order to also account for wake expansion. The outer region of the grid is unstructured as well; it has a growth rate of 1.35 and extends up to 5D upstream, 12D downstream and 10D radially. Even finer grid resolutions were tested for the equidistant region, but their contribution in accuracy improvement was too small to justify the increase in computational cost. The computational cost of aeroelastic simulations is significantly increased compared to rigid aerodynamic ones, since internal iterations have to be performed within every time step until nonlinear structural dynamic equations converge.

As a rule of thumb, in most AL implementations the time step is chosen so that the tip of the actuator line sweeps no more than one cell per time step ($Δt ≤ \frac{Δx}{V_{tip}}$). In aeroelastic runs, where blade
The deflection velocity is superimposed to the rotational velocity component, a more rigorous application of the above rule of thumb is required. Hence, a time step that is half the previous rule \( \Delta t \leq 0.5 \frac{\Delta x}{V_{tip}} \) and corresponds to 540 steps per rotation is chosen. A total simulation time that lasts no less than 20 rotations is necessary, in order for the initial aerodynamic and dynamic transient effects to vanish.

As far as actuator line spacing \( \Delta r \) is concerned, accuracy of the results is maintained when it is chosen to be at most double the grid spacing \( \Delta r \leq 2\Delta x \), so that the spherical regions around the application location of body forces sufficiently overlap with each other to produce a continuous distribution of force along the blade. A uniform spacing along the blade span is utilized in the present work.

2.2. The structural dynamic module

For the elasto-dynamic modelling of the wind turbine rotor, the in-house aeroelastic tool hGAST is used. In hGAST, a multibody modelling of the rotor dynamics is applied, while flexible components are approximated as Timoshenko beams using a 1D FEM discretization. Each blade can be divided into a number of interconnected elements (sub-bodies), which are linked to each other through proper compatibility dynamic and kinematic conditions. This approach allows capturing the geometrical nonlinear effects due to large deflections and rotations using linear beam theory at the element level, but considering nonlinear effects at the sub-body level. The aeroelastic equations are solved iteratively using the 2nd order implicit Newmark method [24].

2.3. The aeroelastic coupling

Aeroelastic coupling consists of the interaction between the aerodynamic and elasto-dynamic modules within every time step of the numerical process. The distribution of the aerodynamic loads calculated by the AL model along the reference line representing the blade is fed to the structural dynamic module. In turn, the deformed coordinates of the reference line and the deflection velocities of the blade, calculated by the structural analysis, are communicated to the aerodynamic module. This procedure is repeated within every time step until convergence of both the aerodynamic and elastic solution is attained.

In the context of both the AL aerodynamic analysis and the beam structural analysis the blades are represented as straight, or in general curved lines. The reference line of the aerodynamic analysis is considered to be the quarter chord \( c/4 \) line of the blade. The structural reference line is considered to be the blade pitch axis, with respect to which structural properties of the blade are defined. The two lines are offset with respect to each other. Due to the above offset an extra twisting moment should be communicated when transferring loads from the aerodynamic line to the elastic line. The above offset must be also taken into account when transferring deflections and velocities from the elastic line to the aerodynamic line, as rotations around the structural axis induce translations at the \( c/4 \) points.

In general, the 1D discretization of the aerodynamic and structural reference lines is different. In the aerodynamic model, the reference line is divided into a number of strips. Every strip consists of two edge nodes. The centre point of every strip (midpoint between the two edge nodes) is the control point of the aerodynamic analysis. This is where the distributed aerodynamic quantities are calculated and the blade element equations are satisfied. So, the aerodynamic strip is the elementary building block of the aerodynamic analysis. In order to fully designate the blade geometry, a pair of leading and trailing edge points can be defined per strip, which is connected by the local blade chord line. With regard to the structural model and in the context of 1D FEM analysis, the elastic line is discretized into a number of FEM nodes and elements. The above definitions for the aerodynamic strips, the structural elements and the corresponding grids are shown in Figure 1. The correspondence of the aerodynamic and structural grids is defined on the basis of a material co-ordinate \( s_0 \) (see Figure 1). If the blade is straight, \( s_0 \) coincides with the radial \( y \) co-ordinate of the blade local system. If the blade is curved, \( s_0 \) is the arc length along the curved reference line (see Figure 2). The need for introducing the material co-ordinate \( s_0 \) stems from the fact that when the blade is deformed the structural nodes are displaced...
in all directions and, depending on the deformation field, the length of the deformed blade changes (increases in case of pure tension, decreases in case of bending deflection).

It is noted that appropriate interpolation procedure shall be defined in order to deal with the different grids and in particular with the transfer of loads, deflections and deflection velocities from one grid to the other. The transfer of loads from the aerodynamic grid to the structural grid is schematically illustrated in Figure 3. It is noted that aerodynamic loads are considered to be uniformly distributed over every aerodynamic strip. Aerodynamic loads are integrated over all strips lying within one FEM of the structural grid. Partial integration is performed over the elements that are not entirely lying within the same FEM (see Figure 3). The overall load is renormalized based on the length of the FEM and gets redistributed as piecewise constant load over it. The loads communicated to every FEM of the structural grid are a distribution of a normal and tangential force and the quarter chord moment plus the additional twisting moment contributed by the aerodynamic forces due to the $c/4$ offset of the aerodynamic axis with respect to the structural axis. The interpolation method presented in Figure 3 satisfies compatibility of the total thrust force; however, it does not ensure full compatibility of flapwise moments. Nevertheless, the accuracy of the approach is reasonably good for the standard grid resolutions considered both in the aerodynamic and structural analysis.

Deflections and deflection velocities are calculated on the aerodynamic control points based on the degrees of freedom (dofs) of the neighbouring elastic grid nodes and on the FEM interpolation functions (cubic for bending and linear for extension and torsion). As in the case of the loads, the offset between the two axes will generate additional displacement and linear velocity components induced by the rotational dofs.

$$u(s_0) = \begin{pmatrix} u(s_0), v(s_0), w(s_0) \end{pmatrix}^T$$

Figure 1: Definition of aerodynamic and structural grids. Unreformed state.

Figure 2: Definition of aerodynamic and structural grids. Deformed state.

Figure 3: Interpolation of aerodynamic loads to the structural grid.
3. Results

In the present section, simulation results of the newly developed AL aeroelastic model are compared against results of hGAST code employing an enhanced BEM implementation. The reference BEM code accounts for dynamic inflow effect, through the solution of a first order filter equation based on cylindrical wake consideration. It also accounts for yaw misalignment effect through the introduction of an induced velocity term due to wake skewness. Simulations are performed for the DTU10 MW rotor at the wind speeds of 8 and 11m/s (partial load operation) at axial and yawed, uniform inflow conditions. Aeroelastic simulations are performed considering constant rotational speed and pitch angle of the blades (open loop operation) as described in Table 1. Tower shadow effect is omitted in all simulations. Deformation and load results are presented as a) time histories-azimuthal variations b) radial distributions along the blade span. In the results presented below 0° azimuth corresponds to the blade being in the vertical position pointing upwards.

| Wind Speed (m/s) | Rotational Speed (rpm) | Pitch angle(°) |
|------------------|------------------------|----------------|
| 8                | 6.423                  | 0              |
| 11               | 8.837                  | 0              |

3.1. Axial uniform inflow

Figure 4 depicts the edgewise moment (Medge) at the root of the first blade for axial, uniform inflow at wind speeds of 8 and 11m/s. Almost perfect agreement between AL and BEM models is obtained. The same holds for the edgewise deflections (Uedge) shown in Figure 5. The edgewise loading and the corresponding deflections are dominated by gravitational forces. Therefore, they are not significantly affected by the choice of the aerodynamic model. The above remark can be also extended in yawed inflow conditions.

In Figure 6, the flapwise moment (Mflap) at the root of the first blade is illustrated for the same wind speeds. Good agreement is achieved in the mean value predictions of the two models. Nevertheless, an approximately 30° phase difference is noted. The 1P variation of the flapwise bending moment is caused by the 5° tilt angle of the rotor. Both models predict very small amplitude of the above variation which is not greater than 5% of the mean load. It is noted that wake skewness effect due to tilt angle is omitted in BEM modelling and this explains the above reported phase difference between the two simulation sets. Qualitatively similar with the Mflap results are obtained for the flapwise deflection (Uflap), shown in Figure 7.

Figure 4: Axial uniform inflow at 8 and 11m/s. Edgewise moment at root of the 1st blade.  
Figure 5: Axial uniform inflow at 8 and 11m/s. Edgewise deflection at tip of the 1st blade.
Good agreement is also noted in the twisting moment (Mtw), as shown in Figure 8. Moreover, as shown in Figure 9, the level difference in the mean tip torsion angle predicted by the two models is
about 0.2° at the wind speed of 8 m/s and 0.6° at the wind speed of 11 m/s. The phase difference observed in Mflap and Uflap signals is not present in Mtw and torsion signals.

It is worth highlighting that in the AL model the blades “perceive” near wake induced velocities in a different manner than in the classical BEM theory. The reason is the detailed description of the 3D flow field by the CFD method. As seen in Figure 10, the tip correction model used in BEM [25] seems to function properly, reducing the tip aerodynamic forces. Nevertheless, as shown in Figure 11, axial induction factor increases towards the tip as a result of the application of Prandtl’s tip correction function directly on the thrust coefficient Ct. On the other hand, the induced velocity predicted by the 3D actuator line model tends to zero towards the tip along with blade normal force. The conclusion drawn is that the flow fields computed by the two methods substantially differ, especially in the vicinity of the rotor tip. It is noted that average force and induction distributions of Figure 10 and Figure 11 are obtained through averaging over the 3 blades and over the last two revolutions of the simulation.

3.2. Yawed uniform inflow
In Figure 12 and Figure 13, the flapwise bending moment at the root of the blade is shown for the wind speeds of 8 and 11 m/s and for the yaw angles of ±15° and ±30°. A mean value difference between the two models arises and gets higher as the yaw angle increases, whereas the mean wind speed seems to have a smaller effect. On the other hand, the amplitude of the variation seems to be similar for both methods. The mean value difference ranges from 4% to 10%. This is justified in Figure 16 and Figure 17, which show the azimuthal variation of the axial induction coefficient. It is seen that the AL model predicts lower mean value of the induction coefficient. This difference grows with increasing yaw angle. It is also noted that the variation amplitude of the axial induction factor predicted by the AL model is higher at all wind speeds and yaw angles. In BEM model, azimuthal variation of the induction is introduced through the application of skewness correction model. However, the main and most notable difference between the two models is the about 30° to 45° phase difference. It is noted that at positive yaw angles and for both wind speeds, BEM simulations predict maximum flapwise moment close to the azimuth position of 180°. The above result is driven by the high local relative velocities seen by the blade sections when the blade passes through its lowest position. These are contributed by the in-plane component of the incoming yawed wind which in this case is added to the linear velocity due to the blade rotation. The effect of the 1P variation of induction (caused by wake skewness) on Mflap is minor, due to the low range predicted by BEM (11% to 17% of the mean value). On the other hand, in AL simulations maximum load is shifted by about 30° to 45° towards higher azimuth angles. This is due to wake induced effects which, as already mentioned, in the AL calculation have a much more pronounced variation (24% to 47% of the mean value). The much lower induced velocities of the AL simulation at 270° push the maximum load towards higher azimuth angles. At negative yaw angles, similar results are obtained but shifted by 180° (as expected) with respect to those for positive angles.

In Figure 14 and Figure 15, the torsion angle at the tip of the blade is shown for the wind speeds of 8 and 11 m/s and for the yaw angles of ±15° and ±30°. AL model predicts slightly higher mean torsion angle, in line with the axial flow simulation results, and close 1P variation amplitudes. The phase differences of the 1P variations predicted by the two models are perfectly in line with the corresponding differences in the flapwise bending moment results and they are therefore driven by wake induced effects.

A visualization of the wake produced by the aeroelastic AL model, as well as the differences between the deformed and rigid blade geometries can be seen in Figure 18 and Figure 19 respectively. Wake structures are visualized by plotting Q-Criterion iso-surfaces [26].
Figure 12: ±15° yawed uniform inflow at 8 and 11 m/s. Flapwise moment at root of the 1st blade.

Figure 13: ±30° yawed uniform inflow at 8 and 11 m/s. Flapwise moment at root of the 1st blade.

Figure 14: ±15° yawed uniform inflow at 8 and 11 m/s. Torsion angle at tip of the 1st blade.

Figure 15: ±30° yawed uniform inflow at 8 and 11 m/s. Torsion angle at tip of the 1st blade.

Figure 16: ±15° yawed uniform inflow at 8 and 11 m/s. Axial induction at 75% of the 1st blade.

Figure 17: ±30° yawed uniform inflow at 8 and 11 m/s. Axial induction at 75% of the 1st blade.
Figure 18: -30° yawed uniform inflow at 11m/s. Wake visualization.

Figure 19: -30° yawed uniform inflow at 11m/s. Deformed and undeformed blade geometries.

4. Conclusions
The paper discusses implementation methods and issues related with the coupling of an AL aerodynamic model with a multibody structural dynamics model. Benchmark comparisons of the newly developed model, against a standard BEM model coupled with the same structural dynamics code are presented.

As concerns the comparison of the results of the newly developed aeroelastic AL model against BEM aerodynamic modelling, the following remarks can be made:

- In axial flow conditions, good agreement of both the in-plane (edgewise) and out-of-plane (flapwise) loads and deflections is achieved. As concerns twisting moment and torsion angle, a small difference in the mean value is observed. The 30° phase difference between the flapwise bending moment signals predicted by the two models is due to the fact that wake skewness induced velocities due to tilt angle are omitted in the BEM model employed in hGAST.

- In yawed flow conditions, the same good agreement of the in-plane (edgewise) loads is obtained (not shown in the paper). As concerns out-of-plane (flapwise) loads, the AL model predicts slightly greater mean value, close amplitudes of the 1P variations and a consistent 30° to 45° phase shift towards higher azimuth angles which is due to different induced velocities predicted by the two models. Twist angles exhibit similar differences with those noted in flapwise loads.

Regarding implementation issues, no particular problems that require special treatment have been encountered in the implementation of the aeroelastic coupling with the AL method. No relaxation of either the external forces or of the deflection velocities is required. Overall, the numerical procedure followed in the analysis resembles the one applied in the BEM based simulation and it exhibits similar characteristics in terms of robustness and convergence rates.

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