Vortex Formation by Interference of Multiple Trapped Bose-Einstein Condensates

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We report observations of vortex formation as a result of merging together multiple $^8$Rb Bose-Einstein condensates (BECs) in a confining potential. In this experiment, a trapping potential is partitioned into three sections by a barrier, enabling the simultaneous formation of three independent, uncorrelated condensates. The three condensates then merge together into one BEC, either by removal of the barrier, or during the final stages of evaporative cooling if the barrier energy is low enough; both processes can naturally produce vortices within the trapped BEC. We interpret the vortex formation mechanism as originating in interference between the initially independent condensates, with indeterminate relative phases between the three initial condensates and the condensate merging rate playing critical roles in the probability of observing vortices in the final, single BEC.

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In a superfluid, long-range quantum phase coherence regulates the dynamics of quantized vortices and provides routes to vortex formation that are inaccessible with classical fluids. For example, in dilute-gas Bose-Einstein condensates (BECs), quantized vortices can be created using quantum phase manipulation. Vortices in BECs have also been created using methods more analogous to those in classical fluid dynamics, namely through rotating traps, turbulence, and dynamical instabilities. Yet in contrast with classical fluid dynamics, to our knowledge vortex generation via the mixing of initially isolated superfluids remains an unexplored research area. Due to the availability and relative ease of microscopic manipulation and detection techniques, BECs are well-suited to address open questions regarding superfluid mixing and associated vortex generation, along with the possible accompanying roles of phase-coherence and matter-wave interference.

In this Letter, we describe experiments demonstrating that the mixing or merging together of multiple condensates in a trap can indeed lead to the formation of potentially long-lived quantized vortices and provides routes to vortex formation that are inaccessible with classical fluids. For example, in dilute-gas Bose-Einstein condensates (BECs), quantized vortices can be created using quantum phase manipulation. Vortices in BECs have also been created using methods more analogous to those in classical fluid dynamics, namely through rotating traps, turbulence, and dynamical instabilities. Yet in contrast with classical fluid dynamics, to our knowledge vortex generation via the mixing of initially isolated superfluids remains an unexplored research area. Due to the availability and relative ease of microscopic manipulation and detection techniques, BECs are well-suited to address open questions regarding superfluid mixing and associated vortex generation, along with the possible accompanying roles of phase-coherence and matter-wave interference.

In this Letter, we describe experiments demonstrating that the mixing or merging together of multiple condensates in a trap can indeed lead to the formation of potentially long-lived quantized vortices in the resulting BEC. We ascribe the vortex generation mechanism to matter-wave interference between the initially spatially isolated but otherwise identical BECs, and show that vortex formation may be induced even for slow mixing time scales. While it is now well-known that matter-wave interference may occur between BECs, our experiment demonstrates a physical link between interference and vortex generation, providing a new paradigm for vortex formation in superfluids. We emphasize that no stirring or phase engineering steps are involved in our work, nor are any other means for controllably nucleating vortices in the trapped atomic gas; the vortex formation process itself is stochastic and uncontrollable, and depends on relative condensate phases that are indeterminate prior to condensate mixing. The vortex formation mechanism identified here may be particularly relevant when defects or roughness are present in a trapping potential, or when multiple condensates are otherwise joined together. Our experiment may also illuminate aspects of vortex formation at site defects in other superfluids, for which microscopic studies may be exceedingly difficult and questions regarding vortex formation mechanisms are unresolved.

To illustrate the basic concept underlying our experiment, we first consider our atom trap, which is formed by the addition of a time-averaged orbiting potential (TOP) and a central repulsive barrier created with blue-detuned laser light that is shaped to segment the harmonic oscillator potential well into three local potential minima. Figure 1(a) shows an example of potential energy contours in a horizontal slice through the center of our trap. We will assume throughout the ensuing descriptions that the energy of the central barrier is low enough that it has negligible effect on the thermal atom cloud; such is the case in our experiment. However, the central barrier does provide enough potential energy for an independent condensate to begin forming in each of the three local potential minima. Overlap and interference between the heretofore independent condensates would then be established as the condensates merge together into one. We have examined both scenarios.

Depending on the relative phases of the three interacting condensates and the rate at which the condensates merge together (via either process), the final merged BEC may have nonzero net angular momentum about the vertical trap axis, as we now describe. We first recall that the relative phase between two independent superfluids is indeterminate until an interference measurement is made. However, when interference occurs, a directional mass current will be established between the superfluids. A relative phase can then be determined, but it will vary...
randomly upon repeated realizations of the experiment [15, 16]. In our experiment, when the initial condensates merge together, fluid flow in the intervening overlap region is established; a straightforward model of the mass current for two overlapping but otherwise uncorrelated states shows that the direction of fluid flow depends on the sine of the relative phase between the states [17].

When our three condensates are gradually merged together while remaining trapped, fluid flow that is simultaneously either clockwise or counter-clockwise across all three barrier arms may occur with finite probability. For ease of this discussion, and keeping in mind that only relative phases carry physical meaning, we imagine that the condensates formed in the three local minima of Fig. 1(a) can be labeled with phases \( \phi_j \), where the indices \( j = 1, 2, \) and 3 identify the condensates in a clockwise order, respectively. Upon merging of the three condensates, if it turns out that for example, \( \phi_2 - \phi_1 = 0.7\pi \) and \( \phi_3 - \phi_2 = 0.8\pi \) (thus necessarily \( \phi_1 - \phi_3 = 0.5\pi \) since each \( \phi_j \) must be single valued), then some finite amount of clockwise-directed fluid flow will be established between each pair, hence also for the entire fluid. More generally, if the three merging condensates have relative phases \( \phi_2 - \phi_1, \phi_3 - \phi_2, \) and \( \phi_1 - \phi_3 \) that are each simultaneously between 0 and \( \pi \), or each between \( \pi \) and \( 2\pi \), the resulting BEC will have nonzero angular momentum, which will be manifest as a vortex within the BEC. By examining the full range of phase difference possibilities, the total probability \( P_v \) for a net fluid flow to be established in either azimuthal direction is determined to be \( P_v = 0.25 \), given statistically random phase differences for each experimental run. \( P_v \) is thus the probability for a vortex to form as the three condensates merge together.

Absent from the above description is an analysis of the phase gradients of the three condensates during the merging process, which will lead to transient interference fringes in overlapping condensates. These fringes may decay to numerous vortices and antivortices, and possibly vortex rings, similar to instabilities of dark solitons in BECs [11, 12, 18, 19]. For rapidly merged condensates, we may thus expect the observation of multiple vortex cores in an BEC image, or to find a value of \( P_v \) greater than 0.25. Yet as the condensates are merged together more and more slowly, the dependence of \( P_v \) on phase gradients becomes negligible and \( P_v \) should approach a limiting value of 0.25.

Our experiment is designed to study the presence of vortices in an \( |F=-1, m_F=-1\rangle \) \(^{87}\)Rb BEC subsequent to the merging of three condensates such created in a three-segment potential well. To create just a single BEC in a trap without a central barrier, we first cool a thermal gas to just above the BEC critical temperature in an axially symmetric TOP trap with radial (horizontal) and axial (vertical) trapping frequencies of 40 Hz and 110 Hz, respectively. We then ramp the TOP trap magnetic fields such that the final trap oscillation frequencies are 7.4 Hz (radially) and 14.1 Hz (axially). A final 10-second stage of radio-frequency (RF) evaporative cooling produces condensates of \(~4\times10^6\) atoms, with condensate fractions near 65% and thermal cloud temperatures of \(~22\) nK. The chemical potential of such a BEC is \( k_B \times 8\) nK, where \( k_B \) is Boltzmann’s constant.

To create instead three isolated condensates in a segmented trap, we modify the above procedure by ramping on an optical barrier immediately before the final 10-s stage of evaporative cooling in the weak TOP trap. The barrier itself is formed by illuminating a binary mask, illustrated in Fig. 1(b), with a focused blue-detuned Gaussian laser beam of wavelength 660 nm. After passing through the mask, the beam enters our vacuum chamber along the vertical trap axis. The mask is imaged onto the center of the atom cloud with a single lens. Due to diffraction, the beam has an intensity profile as shown in Fig. 1(c), with a maximum intensity and thus barrier energy aligned with the center of the TOP trap. The barrier’s potential energy decreases to zero over \(~35\) \( \mu \)m along three arms separated by azimuthal angles of approximately 120°. With 170 \( \mu \)W in the beam, which corresponds to a maximum barrier height of \( k_B \times 26\) nK for our beam, three condensates are created and do not merge together during their growth; a set of three BECs created under these conditions is shown in Fig. 1(d). With instead 45 \( \mu \)W in the beam, corresponding to a maximum barrier energy of \( k_B \times 7\) nK, three independent condensates again initially form, but as the condensates grow in atom number, their repulsive interatomic interactions eventually provide enough energy for the three condensates to flow over the barrier arms. The three condensates thus naturally merge together into one BEC during evaporative cooling, as shown in Fig. 1(e).

In our first study, three spatially isolated condensates

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FIG. 1: (a) An illustration of the potential energy profile of our three-well trap in a horizontal slice through the trap center. This contour plot represents the addition of our harmonic trap with the measured intensity profile of our optical barrier beam, scaled to a potential energy. (b) A computer-generated profile of the binary mask used to create our optical barrier, where white represents the transmitting area. (c) An image of our optical barrier. (d), (e) Phase-contrast images of trapped condensates representing integrated optical depths along the vertical trap axis. Each shows an area of 85 \( \mu \)m per side, as do (a) and (c). In (d), three independent condensates are generated in the presence of a strong barrier beam with 170 \( \mu \)W. (e) With 45 \( \mu \)W in the barrier beam, the initial condensates have merged together during evaporative cooling. Note that even this weak barrier displaces atoms from the center of the BEC, so the observable density dip in this image does not signify the presence of a vortex core.
were created in the presence of a strong barrier of maximum potential energy $k_B \times 26$ nK, and were then merged together by ramping down the strength of the barrier to zero linearly over a time $\tau$. Any vortex cores formed by this process in the resulting BEC have a size below our optical resolution limit, and are too small to be directly observed in the trapped BEC. We thus suddenly removed the trapping potential and observed the atom cloud using absorption imaging along the vertical axis after 56 ms of ballistic expansion. This entire process was repeated between 5 and 11 times for each of 6 different values of the barrier ramp-down time $\tau$ between 50 ms and 5 s.

In a significant fraction of our experimental runs, we observed one or more vortex cores in our condensates, a clear indication that condensate merging can indeed induce vortex formation. Moreover, the observed spatial density distributions vary from shot to shot, as would be expected with indeterminate phase differences between the initial condensates. However, many images are absent of any vortices. Example images of expanded BECs in Fig. 2(a)-(d) show the presence of vortex cores after various barrier ramp-down times. An analysis of vortex observation statistics is given in Fig. 2(e) for the different values of $\tau$ examined. We define a vortex observation fraction $F_v$ as the fraction of images, for each value of $\tau$, that show at least one vortex core. The error bars reflect the ambiguity in our ability to determine whether or not an image shows at least one vortex. For example, core-like features at the edge of the BEC image, or core-like features obscured by imaging noise, may lead to uncertainty in our counting statistics and determination of $F_v$. As the plot shows, $F_v$ reaches a maximum near 0.6 for the smaller $\tau$ values, and drops to $\sim 0.25$ for long ramp-down times. We expect that with larger sample sizes, $F_v$ should approximate $P_v$ for each $\tau$. Thus our results are consistent with our conceptual analysis, where $P_v > 0.25$ for fast merging times when interference fringes may occur, and $P_v = 0.25$ for long merging times.

For $\tau \leq 1$ s, multiple vortices were often observed in our images, as images of Fig. 2 show, indicating that phase gradients are likely to play an important role in vortex formation if condensates are quickly merged together. Furthermore, observations of multiple vortex cores may indicate the presence of vortices and antivortices. Although we are unable to determine the direction of fluid circulation around our observed vortex cores, we performed a test in which the barrier was ramped off in 200 ms, thus forming multiple vortex cores with a high probability. We then inserted additional time to hold the final BEC in the unperturbed harmonic trap before our expansion imaging step. After such a sequence, the probability of observing multiple vortices dropped dramatically: for no extra hold time, we observed an average of 2.1 vortex cores per image, whereas this number dropped to 0.7 for an extra 100 ms hold time, suggestive of either vortex-antivortex combination or other dynamical processes by which vortices leave the BEC. However, images showing single vortices were observed even after 5 s of extra hold time in our trap following the 200 ms barrier ramp, indicating a relatively long vortex lifetime in our trap.

In a second investigation, we differed from the above experiment by using a weaker barrier with a maximum energy of $k_B \times 7$ nK such that the three condensates naturally merged together into one BEC during the evaporative cooling process. We emphasize that this merging process is due solely to the increasing chemical potentials exceeding the potential energy of the barrier arms between the condensates; the barrier strength remained constant throughout the growth and merging of the condensates when vortices may form. After evaporative cooling, we ramped off the optical barrier over 100 ms and released the atoms from the trap to observe the BEC after ballistic expansion. Under these conditions, our vortex observation fraction was $F_v = 0.56 \pm 0.06$ in a set of 16 images, with example images shown in Fig. 3(a) and (b).

By adding an extra 500 ms of hold time after the final stage of BEC formation but before the start of the 100 ms barrier ramp-down and ballistic expansion, the vortex observation fraction decreased to $F_v = 0.28 \pm 0.14$. Again, this drop in probability may be due to vortex-antivortex combination during extra hold time in the weakly perturbed harmonic trap. We thus conclude that with a maximum barrier energy of $k_B \times 7$ nK, vortices are formed during the BEC creation process rather than during the ramp down of the weak barrier, consistent with our phase-contrast images of trapped BECs that show a ring-like rather than segmented final density distribution.

Barrier strengths between the two limits so far described also lead to vortex formation, either during BEC.
growth or during barrier ramp down. With a barrier strength in this range, up to at least four clearly defined vortex cores have been observed in single images, as the examples of Fig. 3(c)–(f) show. Density defects other than clear vortex cores have also appeared in our images, as in the upper left of Fig. 3(g), where a “gash”-like feature may be a possible indicator of vortex-antivortex combination; similar features have been seen in related numerical simulations [18]. Often, however, no vortices are observed; an example with no vortex cores appearing is Fig. 3(h). For comparison, an expansion image taken after creating a condensate in a trap without a barrier is shown in Fig. 3(i).

Perhaps surprisingly, single vortex cores have also appeared in ~10% of our expansion images taken in the absence of any central barrier. In other words, for our basic single BEC creation procedure as outlined above, and without a segmenting barrier ever turned on, vortices occasionally form spontaneously and are observable in expansion images. An example image is shown in Fig. 3(j). These observations may be related to predictions of spontaneous vortex formation due to cooling a gas through the BEC transition [21]. We are currently investigating these intriguing observations further.

We finally note that to generate vortices by the mechanism described in this paper, it is important for two reasons that condensates merge and interfere while trapped, as opposed to during expansion. First, in a trapped BEC, the nonlinear dynamics due to interatomic interactions would play a key role in the structural decay of interference fringes, which may be responsible for generation of multiple vortices and antivortices seen with fast merging times. In an expanding gas, the interactions become negligible as the gas density decreases. Second, by keeping condensates trapped during their mixing, we are able to study slow merging, where we believe that relative overall phases are primarily responsible for vortex generation. We conjecture that with slow merging, it may be possible to directly imprint relative phases onto three or more trapped, separated, and phase-correlated condensates to controllably engineer vortex states.

In summary, we have generated vortices by merging together isolated and initially uncorrelated condensates into one final BEC. We have shown that our vortex observation statistics are consistent with a simple conceptual theory regarding indeterminate phase differences between the initial condensates; however, quantitative theoretical examination is needed for further analysis of our results. We have also demonstrated that condensates created in the presence of weak trapping potential defects or perturbations, such as our weak optical barrier, may naturally acquire vorticity and nonzero orbital angular momentum. This result challenges the common notion that a BEC necessarily forms with no angular momentum in the lowest energy state of a trapping potential; rather, the shape of a static confining potential may be sufficient to induce vortex formation during BEC growth, a concept that may be of relevance to other superfluid systems as well.

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