Students’ errors in solving the connection cluster problem: a case study on space and shape content

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Abstract. The ability of Indonesia student is still low in solving space and shape problem. It shows that the student gets the difficulties in solving the connection cluster of Programme for International Student Assessment (PISA) particularly in space and shape content. In this study, we carried out a small-case study on analysing student error in solving space and shape problem in connection cluster problem using Newman’s Error Procedure (NEP). We conducted this study for 53 students from two schools. There were three steps in carrying out this study; selecting PISA problem, answering the problem, and interview. Three problems were selected and translated to Indonesia. The students answered the problems for 15 minutes. Then the interview was conducted to clarify the student answer related to the error. It was done only for 4 students who got the difficulties and error in solving the problem. In this research, we found that most of student error was occurred in comprehension and transformation process. Therefore, only a few of students did error for process skill and encode the problem. For future study, we suggest to continue the research about the student conceptual understanding in mathematics.

1. Introduction
Program International Student Assessment (PISA) first held in 2000 and continued for every three years. It consists of four contents that are change and relationship, space and shape, quantity, uncertainty and data [1]. It assesses student literacy of 15 years old including mathematical literacy. Unfortunately, Indonesia joins PISA since 2000 but never reach the average score. Even most Indonesia students are only able to solve until the connection cluster in mathematics. For further investigation, the ability of Indonesia students is still low in solving space and shape problems [2].

Mathematical literacy in the area of space and shape involves a range of activities such as understanding perspectives, creating and reading maps, transforming shapes with and without technology, interpreting views of three-dimensional scenes from various perspective, and constructing representations of shapes. In formal education, space and shape are taught as geometry and measurements. It shows that geometry is an essential subject learned in mathematics and able to support other subjects such as numbers and arithmetic.

The tasks in this cluster are non-routine and ask for transformation between the context and mathematical word [3]. There are three clusters of PISA problems; reproduction, connection, and reflection [4]. Reproduction, connection, and reflection cluster consist of levels 1 and 2, levels 3 and 4, levels 5 and 6, of taxonomy bloom respectively. The student gets difficulties in solving the connection cluster of PISA particularly in space and shape content.
The difficulty is the cause of errors. There are four hurdle levels in answering mathematics problems; reading, comprehension, transformation, process skills, and encode errors [5]. Previous researches show that most of the student difficulties in solving PISA occurred in reading, transformation, and process skill [2], [6]. Based on the preceding problem, we carried out a small-case study on analyzing student error in solving space and shape problems in connection clusters using Newman’s Error Procedure (NEP).

2. Methods
The research was conducted to analyses the student difficulties in solving the PISA problem within the connection cluster. There were 3 steps in carrying out this research which were selecting the PISA problem, answering the problem, and interview. First, the problems were selected form PISA tested in 2003. Specifically, the researcher selected three problems that are categorized in space and shape context and included in connection cluster of PISA. The problems are shown in Table 1. Then, they were translated by the researcher and validated by one of the lecturers in the mathematics education department. For the second step, the problem s were tested to 53 students for 15 minutes. They were in 8 grade and most of them were 15 years old. After checking and classifying the student answer sheet, finally the interview was carried out for digging up the information about the student difficulties. The researcher chooses four students who got the difficulties and errors in solving the problem related to the student answer sheet. After having the data, the researcher analyzed them using Newman’s Error Procedure (NEP) which are reading, comprehension, transformation, process skills, and encoding errors. Reading level demands the student’s ability to read the problem, in comprehension level the student should be able to recognize the meaning of the problem. Furthermore, in transformation level the student can opt for a suitable mathematical operation or procedure. Finally, student can represent the answer correctly in the encoding level.
Table 1. Space and shape content problem

| No. | Problem |
|-----|---------|
| 1   | To complete one set of bookshelves a carpenter needs the following components: 4 long wooden panels, 6 short wooden panels, 12 small clips, 2 large clips and 14 screws. The carpenter has in stock 26 long wooden panels, 33 short wooden panels, 200 small clips, 20 large clips and 510 screws. How many sets of bookshelves can the carpenter make? Answer:....... |
| 2   | Susan likes to build blocks from small cubes like the one shown in the following diagram  

Susan has a lot of small cubes like this one. She uses glue to join cubes together to make other blocks. First, Susan glues eight of the cubes together to make the block shown in Diagram A. 

Then Susan makes the solid blocks shown in Diagram B and Diagram C below. 

Now Susan wants to make a block that looks like a solid block that is 6 small cubes long, 5 small cubes wide and 4 small cubes high. She wants to use the smallest number of cubes possible, by leaving the largest possible hollow space inside the block. What is the minimum number of cubes Susan will need to make this block? Answer: .................cubes. |
| 3   | For a rock concert a rectangular field of size 100 m by 50 m was reserved for the audience. The concert was completely sold out and the field was full with all the fans standing. Which one of the following is likely to be the best estimate of the total number of people attending the concert?  

A. 2.000  
B. 5.000  
C. 20.000  
D. 50.000  
E. 100.000 |

3. Result and Discussion  
In this study, we analyse student difficulties using Newman’s Error Procedure. From the student answer sheet, we find that some students able to answer the problem correctly and some others face difficulties. Table 2. shows the summary of student difficulties analyses using Newman’s Error Procedure.
Table 2. The summary of student error analyses

| Problem | Correct Answer | Not Answer | Reading | Comprehension | Transformation | Process Skill | Encode |
|---------|----------------|------------|---------|---------------|---------------|---------------|--------|
|         | N | % | N | % | N | % | N | % | N | % | N | % | N | % | N | % | N | % | N | % | N | % |
| Problem 1 | 30 | 56 | 12 | 23 | 8 | 15 | 31 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Problem 2 | 17 | 32 | 35 | 66 | 6 | 11 | 37 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Problem 3 | 2 | 4 | 4 | 7 | 39 | 74 | 28 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 2. shows that all students tried to answer and understood the problem given although only a third of them able to answer the problem correctly. Based on the interview, most of the students answer that the problem given has the average difficulties. The student said that they can identify the information given and asked. In this research, we found that student error only occurred in comprehension, transformation, process skill, and encode. These findings are in line with the result of research by [7], [8].

The comprehension error is the biggest error that occurred in this study. Even in the second problem, the student faced the comprehension error of more than half a percent. It is in line with the previous research that stated that comprehension and transformation error are the most dominant error in solving the context-based mathematics task [3]. The comprehension error can be marked by missing some information given, misunderstanding the keyword that is used in problem-solving, and unable to distinguish the information given [3]. Figure 1 shows the example of the student answer sheet within comprehension error.

![There is 6 sides, and 9 small cubes for each. 6 x 9 = 54 cubes](image)

Figure 1. Student answer sheet with comprehension error for Problem 2.

Figure 1. shows the student answer sheet for problem 2. The Student missed some crucial information given in problem 2. He missed the information given that state about the hollow space inside the box. From the interview, we found that he understood what was asked in the problem but cannot explain the information given well. The student counted the sum of small cubes needed for making the solid block in diagram c manually. He found that every side of the block has nine small cubes, then multiplied by six as the sum of the layer of the cube.

In the third problem, most of the student errors occurred at the transformation level. The student tended to use a formula without considering whether needs it or not. Some of the student answered the problem only refer to the real situation without taking the perspective of mathematics. Then, some of them used mathematical procedure/concepts which are not relevant to the task called the transformation process as the horizontal mathematization; the activity of transferring a realistic problem to a symbolic mathematical problem. Meanwhile, the horizontal mathematization is the activities that include identifying the specific mathematics in a general context, schematizing, formulating and visualizing a problem in different ways, and discovering relations. This finding is in line with research by [9] that stated that the student has a lack of ability in horizontal mathematization. Figure 2 shows the example of student answer with transformation error.
Figure 2. Student answer sheet with transformation error for Problem 3

Figure 2 shows that the student answer the problem given in problem 3 using the circumference formula of rectangular. We do not need this formula in solving this problem. For solving this problem, the student had to find the area of the rectangular. Then, the student predicted how many people will stand on a meter square area, so that the student can predict all people attend the concert by dividing the whole area by the sum of people who stand in a meter square area. The wrong formula chosen in this problem caused the wrong answer gotten.

The fewest error found in this study is the process skill error. It is only seen in the first and third problems. This finding was in line with the research conducted by [3] but contradicted to the research conducted by [10]. Process skill error was detected by student error in calculation. Figure 3. shows the student answer sheet within the process skill error.

Figure 3. Student answer sheet with process skill error for Problem 2

Figure 3. Shows that the student wrote the process in solving the problem correctly, but faced the mistake in the last answer. The student got a mistake in multiplying the multiplication of nine by three. Until the answer of multiplication subtracted by one is wrong too. In the interview, the student admitted the mistake that done because he was wrong in memorizing the multiplication. To solve the problem using multiplication, the student always memorizes the multiplication pattern without having a conceptual understanding. Most of the errors in solving mathematics problems happened because the student have a of conceptual understanding. This finding is in line with the previous research that states that most of the transformation errors are made due to incorrect math concepts [3], [11]. Conceptual understanding is related to mathematical understanding that consists of content and context in mathematics [12]. Meanwhile [13] divided conceptual understanding to be instrumental understanding and relational understanding. There were some researches that found that student face difficulties in relational understanding [13]–[17].

The encoding error in this study include minority error. Process skill and encode error less dominant that comprehension and transformation error [3]. In this type of error, the student used the correct formula or procedure but they did not finish it. Furthermore, the student was unable to correctly interpret and validate mathematical solutions in terms of the real word problem.
Figure 4. Student answer sheet with encode error for Problem 1

Figure 4. shows that the student was able to understand, transform, and finish the prose in solving the problem. Unfortunately, the student cannot conclude the answer in the last step. The student decided and chose the wrong answer from the calculation that done. The student was not able to transform the mathematics solution to be a real-world solution. This is related to the student literacy mathematical ability. Mathematical literacy is about usability or mathematical functions that have been learned by the students in the school to everyday life in order to compete in a globalized world [18]. Furthermore, [19], [20] study showed that the mathematical literacy of secondary students is still low.

4. Conclusion
From this study, we found that most of the student errors occurred in comprehension and transformation. In comprehension error, the student missed some crucial information. On the other hand, the student transformation error was shown by the use of incorrect formula in solving the problem. Both of these errors related to the student conceptual understanding, especially relational understanding. Then, less than five percent of students did process skill and encode error. Specifically, the student was wrong in the calculation in process skill and cannot conclude the answer in encode error. The limitation of this study is we only used the PISA question from 2003, for the next study the researcher should use the question from varied years. Then, we suggest continuing the research about the student conceptual understanding in mathematics.

5. References
[1] Adams R and Wu M, 2000 PISA 2000 Technical Report. Paris: Organisation for Economic Co-operation and Development (OECD) p. 322.
[2] Sumule U Amin S M and Fuad Y, 2018 Error Analysis of Indonesian Junior High School Student in Solving Space and Shape Content PISA Problem Using Newman Procedure J. Phys. Conf. Ser. 947, 1.
[3] Wijaya A van den Heuvel-Panhuizen M Doorman M and Robitzsch A, 2014 Difficulties in solving context-based PISA mathematics tasks: An analysis of students’ errors Math. Enthus. 11, 3 p. 555–584.
[4] Stacey K, 2011 The PISA view of mathematical literacy in Indonesia J. Math. Educ. 2, 2 p. 95–126.
[5] Jha S K, 2012 Mathematics performance of primary school students in Assam (India): An analysis using Newman Procedure Int. J. Comput. Appl. Eng. Sci. II, I p. 17–21.
[6] Sari Y M and Valentino E, 2017 An Analysis of Students Error In Solving PISA 2012 And Its Scaffolding JRAMathEdu (Journal Res. Adv. Math. Educ. 1, 2 p. 90–98.

[7] Hadi S Retnawati H Munadi S Apino E and Wulandari N F, 2018 The difficulties of high school students in solving higher-order thinking skills problems Probl. Educ. 21st Century 76, 4 p. 520–532.

[8] Santoso D A Farid A and Ulum B, 2017 Error Analysis of Students Working about Word Problem of Linear Program with NEA Procedure J. Phys. Conf. Ser. 855, 1.

[9] Jupri A and Drijvers P, 2016 Student difficulties in mathematizing word problems in Algebra Eurasia J. Math. Sci. Technol. Educ. 12, 9 p. 2481–2502.

[10] Zakaria E -- I and Maat S M, 2010 Analysis of Students’ Error in Learning of Quadratic Equations Int. Educ. Stud. 3, 3 p. 105–110.

[11] Riastuti N Mardiyana M and Pramudya I, 2017 Students’ Errors in Geometry Viewed from Spatial Intelligence J. Phys. Conf. Ser. 895, 1.

[12] Kilpatrick J, 2001 The strands of mathematical proficiency.

[13] Skemp R R, 1976 Relational Understanding and Instrumental Understanding (Reprint) Math. Teach. middle Sch. 12, 2 p. 88–95.

[14] Stephens M, 2006 Describing and exploring the power of relational thinking Equivalence in Research literature Identities, Cult. Learn. Spaces July 2006 p. 479–486.

[15] Star J R Seifert C and Hall E, 2005 Re-Conceptualizing Procedural Knowledge: Flexibility and Innovation in Equation Solving J. Res. Math. Educ. 36, 5 p. 127–155.

[16] Boaler J, 1998 Open and closed mathematics: Student experiences and understandings J. Res. Math. Educ. 29, 1 p. 41–62.

[17] Donevska-Todorova A, 2016 Procedural and Conceptual Understanding in Undergraduate Linear Algebra Proc. INDRUM2016 June p. 276–285.

[18] Fery M F Wahyudin and Tatang H, 2017 Improving primary students mathematical literacy through problem based learning and direct instruction Educ. Res. Rev. 12, 4 p. 212–219.

[19] Hayati T R and Kamid K, 2019 Analysis of Mathematical Literacy Processes in High School Students Int. J. Trends Math. Educ. Res. 2, 3 p. 116.

[20] Sari R H N and Wijaya A, 2017 Mathematical literacy of senior high school students in Yogyakarta J. Ris. Pendidik. Mat. 4, 1 p. 100.

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