Dust–Acoustic Envelope Solitons in an Electron-Depleted Plasma

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Abstract—A theoretical investigation of the modulational instability (MI) of dust–acoustic waves (DAWs) by deriving a nonlinear Schrödinger equation in an electron-depleted opposite polarity dusty plasma system containing non-extensive positive ions is presented. The conditions for MI of DAWs and formation of envelope solitons are investigated. The sub-extensivity and super-extensivity of positive ions are seen to change the stable and unstable parametric regimes of DAWs. The addition of dust grains causes changes the width of both bright and dark envelope solitons. The findings of this study can help understanding the nonlinear features of DAWs in Martian atmosphere, cometary tails, solar system, laboratory experiments, etc.

Keywords: dusty plasma, dust–acoustic waves, NLSE, envelope solitons, electron depleted plasma

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1. INTRODUCTION

The ubiquitous existence of the massive dust grains in space environments (viz., supernova explosion [1], Earth polar mesosphere [2], cometary tails [3], Martian atmosphere [3], interstellar clouds [4], circumstellar clouds [4], Solar system [4], and Earth’s lower ionosphere [5], etc.) as well as laser–matter interaction [6] has created a great interest among the plasma physicists [7–11], and is considered to develop new and modified low frequency electrostatic dust–acoustic (DA) waves (DAWs) [12] and dust-ion-acoustic waves (DIAWs) [13] and associated instabilities of these new eigen–modes.

The velocity distribution of energetic miniature particles (viz., ions and electrons) in space plasma is often considered to follow the Maxwellian velocity distribution by assuming that these particles remain in thermally equilibrium in the plasma system. But the concept of Tsallis statistics [14], first recognised by Renyi [15], has superseded the Maxwellian velocity distribution and rigorously applicable in explaining the dynamics of the plasma medium which is not thermally equilibrium, and have also provided the proof of existence of non-thermal particles which motion can not be described by Maxwellian distribution function [16–21] in both space [22] and laboratory experiments [23]. Tsallis [14] introduced a parameter \( q \), also known as non-extensive parameter, in his generalized statistics in order to measure the degree of non-extensivity of the particular plasma system containing particles moving very fast in comparison with their thermal velocities. In [16] a four component dusty plasma medium (DPM) comprising opposite polarity dust grains (OPDGs) and non-extensive electrons and ions was considered, and the nonlinear propagation of DA multi-soliton in the plasma medium was investigated, and was found that the values of \( q \) has control over the generation of compressive and rarefactive DA multi-soliton. In [17] it was reported that the amplitude of the DA shock waves increases with the increase of ion non-extensivity while decreases with the increase of electron non-extensivity. In [20] authors studied the effects of electron non-extensivity on the potential structure of solitary waves in DPM, and observed that the height of the pulse increases while the width of the pulse decreases with increasing of \( q \).

Modulational Instability (MI) is a well-known nonlinear mechanism in which the nonlinear self-interaction of the carrier waves in the system causes to modulate the amplitude of the waves. Then, the evolution of the system allows to localize the energy of the wave and is also governed by the nonlinear Schrödinger equation (NLSE). A number of authors [24–32] have investigated the MI of the carrier waves in various plasma system with the help of NLSE in presence of massive dust grains (DGs). In [25] authors investigated the effects of dust concentration on the MI window of DAWs, and found that an increment in the negative dust concentration leads to shrink the stable area of DAWs. It was reported [26] that the addition of non-thermal ions possibly stabilize the DA envelope soli-
ton. In [27] a stability analysis in a warm DPM was performed, and was observed that the MI occurs for small values of wave number and only bright envelope solitons will propagate through the medium.

We assume here that electrons are significantly depleted during the charging of the negatively charged dust grains. This makes \( n_{i0}, Z_{-}n_{-0}, Z_{+}n_{+0} \gg n_{e0} \) valid. However, the minimum value of \( n_{e0}/n_{i0} > m_{e}/m_{i} \) for \( T_{e} = T_{i} \), where \( m_{e} (T_{e}) \) is the electron mass (temperature). The process of electron depletion has been identified by laboratory experiments and space observations, and successively has been considered by many researchers [33–37] to study the effects of the electron depletion in modifying the electrostatic potential structure in DPM. In [1] the non-planar DA shock waves (DASHWs) was examined in an electron depleted DPM (EDDPM) in presence of non-extensive ions and it was found that the DA solitons may exhibit rarefaction and compression for \( q > 0 \) and \( q < 0 \), respectively. DA solitary waves (DASWs) in a magnetized EDDPM were analyzed in [6]. In [34] a theoretical analysis on the DASHW structure was reported regarding an electron depleted plasma system. In [35] authors studied the DA double-layers and DASWs in EDDPM having two temperature super-thermal ions. In [36] the existence and dissipative nature of DASWs was theoretically analyzed in an unmagnetized EDDPM by considering dust-neutral collision. Also, DA shock excitations was investigated [7], and it was found that the potential of the DA shock wave depends on the various physical parameters of the EDDPM. In our article, we will observe the effects of electron depletion on the MI of DAWs and associated DA envelope excitations in an unmagnetized EDDPM consisting of OPDGs and \( q \)-distributed positive ions.

The article is arranged as follows: The governing equations describing our plasma model are presented in Section 2. A standard NLSE has been derived in Section 3. MI is given in Section 4. The envelope solitons are presented in Section 5. A brief conclusion is finally provided in Section 6.

2. GOVERNING EQUATIONS

We consider a three-component EDDPM consisting of inertial negatively charged massive dust grains (DGs), inertial positively charged DGs, and inertialess \( q \)-distributed positive ions. At equilibrium, the charge neutrality condition for our considered plasma model can be written as \( Z_{i}n_{i0} + Z_{-}n_{-0} = Z_{+}n_{+0} \), where \( Z_{i}, Z_{-}, \) and \( Z_{+} \) are the charge state of positive ions, positive and negative DGs, respectively, and \( n_{i0}, n_{-0}, \) and \( n_{+0} \) are the number densities of positive ions, positive and negative DGs, respectively. Now, the normalized form of the basic equations can be written as

\[
\frac{\partial n_{+}}{\partial t} + \frac{\partial}{\partial x} (n_{+}u_{+}) = 0, \tag{1}
\]
\[
\frac{\partial u_{+}}{\partial t} + u_{+} \frac{\partial u_{+}}{\partial x} = -\frac{\partial \phi}{\partial x}, \tag{2}
\]
\[
\frac{\partial n_{-}}{\partial t} + \frac{\partial}{\partial x} (n_{-}u_{-}) = 0, \tag{3}
\]
\[
\frac{\partial u_{-}}{\partial t} + u_{-} \frac{\partial u_{-}}{\partial x} = \nu_{i} \frac{\partial \phi}{\partial x}, \tag{4}
\]
\[
\frac{\partial^{2} \phi}{\partial x^{2}} = v_{2}n_{-} - (v_{2} - 1)n_{+} - n_{-}, \tag{5}
\]

where \( n_{+}, n_{-} , \) and \( n_{-} \) are normalized to their equilibrium values \( n_{i0}, n_{-0}, \) and \( n_{+0} \), respectively; \( u_{+} \) and \( u_{-} \) are the positive and negative dust fluid speed, respectively, normalized to the positive DA speed \( C_{+} = (Z_{+}k_{B}T_{i}/m_{+})^{1/2} \) (with \( T_{i} \) being the positive ion temperature, \( m_{+} \) being positive dust mass, and \( k_{B} \) being the Boltzmann constant); \( \phi \) represents the electrostatic wave potential normalized by \( k_{B}T_{i}/e \) (with \( e \) being the magnitude of the charge of an single electron); \( t \) and \( x \) are the time and space variables normalized by positive dust frequency \( \omega_{p+} = (4\pi e^{2}Z_{+}^{2}n_{i0}/m_{+})^{1/2} \) and positive dust Debye length \( \lambda_{D+} = (k_{B}T_{i}/4\pi e^{2}Z_{+}n_{i0})^{1/2} \), respectively. Other parameters can be defined as \( \nu_{i} = Z_{i}m_{i}/Z_{i}m_{-} \) and \( v_{2} = Z_{+}/Z_{-}/Z_{i0} \). It is important to note here that the mass of the negative DG is greater than the mass of the positive DG (i.e., \( m_{+} > m_{-} \)), the number density of the negative DG is greater than the number density of the positive DG (i.e., \( n_{-} > n_{+0} \)), and finally, the number of charges residing on the negative dust particle is greater than the positive dust particle (i.e., \( Z_{+} > Z_{-} \)).

Now, the expression for the number density of the non-extensive positive ions can be written as [16]

\[
n_{+} = [1 - (q - 1)\phi]^{(q+1)/2(q-1)}. \tag{6}
\]

The parameter \( q \), given in the Eq. (6), is a degree of measurement of non-extensivity of plasma particles. When \( q > 1 \), then the plasma system can be considered sub-extensive while the plasma system can be considered super-extensive system when \( -1 < q < 1 \), and \( q = 1 \) represents a Maxwellian system. By substituting Eq. (6) into Eq. (5), and expanding up to third order in \( \phi \), we get

\[
\frac{\partial^{2} \phi}{\partial x^{2}} + n_{+} + v_{2} = 1 + v_{2}n_{-} + J_{1}\phi + J_{2}\phi^{2} + J_{3}\phi^{3} + \cdots, \tag{7}
\]

where

\[
J_{1} = [(v_{2} - 1)(q + 1)]/2, \\
J_{2} = [(1 - v_{2})(q + 1)(3 - q)]/8, \\
J_{3} = [(1 - v_{2})(q + 1)(3 - q)(3q - 5)]/48.
\]
We may note that the term on the right hand side is the contribution of \(q\)-distributed positive ions.

3. DERIVATION OF THE NLSE

We will derive a NLSE by employing the reductive perturbation method to study the MI of DAWs. So, we first introduce the stretched co-ordinates as

\[
\xi = \epsilon (x - v_g t),
\]

\[
\tau = \epsilon^2 t,
\]

where \(v_g\) is the group speed and \(\epsilon\) is a small parameter. Then, we can write the dependent variables [24] as

\[
n_\pm = 1 + \sum_{m=1}^{\infty} \sum_{l=-\infty}^{\infty} n_{\pm}^{(m)}(\xi, \tau) \exp[i(kx - \omega t)],
\]

\[
u_\pm = \sum_{m=1}^{\infty} \sum_{l=-\infty}^{\infty} \nu_{\pm}^{(m)}(\xi, \tau) \exp[i(kx - \omega t)],
\]

\[
\phi = \sum_{m=1}^{\infty} \sum_{l=-\infty}^{\infty} \phi^{(m)}(\xi, \tau) \exp[i(kx - \omega t)],
\]

where \(k(\omega)\) is real variable which represents the carrier wave number (frequency) of the solitary wave. The derivative operators can be then written as

\[
\frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial t} - \epsilon v_g \frac{\partial}{\partial \xi} + \epsilon^2 \frac{\partial}{\partial \tau},
\]

\[
\frac{\partial}{\partial x} \rightarrow \frac{\partial}{\partial x} + \epsilon \frac{\partial}{\partial \xi},
\]

Now, by substituting Eqs. (8)–(16) into Eqs. (1)–(4), and Eq. (7), and collecting only the terms containing \(\epsilon\) for the first order \((m = 1\) with \(l = 1\)), we get the following equations

\[
\omega n_+^{(1)} = u_+^{(1)},
\]

\[
k \phi_+^{(1)} = \omega u_+^{(1)},
\]

\[
\omega n_-^{(1)} = u_-^{(1)},
\]

\[
k v_g \phi_-^{(1)} = -\omega u_-^{(1)},
\]

\[
n_+^{(1)} = k^2 \phi_+^{(1)} + J_4 \phi_+^{(1)} + v_g n_-^{(1)},
\]

\[
u_+^{(1)} = k^2 \nu_+^{(1)} + J_5 \nu_+^{(1)} + v_g \nu_-^{(1)},
\]

these equations then reduce to

\[
n_+^{(1)} = \frac{k^2}{\omega} \phi_+^{(1)},
\]

\[
u_+^{(1)} = \frac{k}{\omega} \nu_+^{(1)},
\]

The dispersion relation finally yields as

\[
\omega^2 = k^2 (1 + v_g v_2) \frac{v_2}{k^2 + J_1}.
\]

The second-order equations \((m = 2\) with \(l = 1\)) are given as

\[
n_+^{(2)} = \frac{k^2}{\omega^2} \phi_+^{(2)} + \frac{2ik(kv_g - \omega) \frac{\partial \phi_+^{(1)}}{\partial \xi}}{\omega},
\]

\[
u_+^{(2)} = \frac{k}{\omega} \phi_+^{(2)} + \frac{i(kv_g - \omega) \frac{\partial \phi_+^{(1)}}{\partial \xi}}{\omega},
\]

\[
n_-^{(2)} = -\frac{v_g k^2}{\omega^2} \phi_-^{(2)} - \frac{2iv_g k(kv_g - \omega) \frac{\partial \phi_-^{(1)}}{\partial \xi}}{\omega},
\]

\[
u_-^{(2)} = -\frac{v_g k}{\omega} \phi_-^{(2)} - \frac{i(v_g k - \omega) \frac{\partial \phi_-^{(1)}}{\partial \xi}}{\omega},
\]

with the help of compatibility condition, the group velocity of DAWs can be written as

\[
v_g = \frac{\partial \omega}{\partial k} = \frac{\omega(1 + v_1 v_2 - \omega^2)}{k(1 + v_1 v_2)}.
\]

The coefficients of \(\epsilon\) for \(m = 2\) and \(l = 2\) provide the second-order harmonic amplitudes which are found to be proportional to \(\phi_+^{(1)}\)

\[
n_+^{(2)} = J_4 \phi_+^{(1)},
\]

\[
u_+^{(2)} = J_5 \phi_+^{(1)},
\]

\[
n_-^{(2)} = J_6 \phi_-^{(1)},
\]

\[
u_-^{(2)} = J_7 \phi_-^{(1)},
\]

where

\[
J_4 = \frac{3k^4 + 2J_6 \omega^2 k^2}{20 \omega^4},
\]

\[
J_5 = \frac{k^3 + 2J_6 \omega^2}{20 \omega^3},
\]

\[
J_6 = \frac{3v_g^2 k^4 - 2J_4 \omega^2 k^2}{20 \omega^4},
\]

\[
J_7 = \frac{v_g^2 k^3 - 2J_4 \omega^2}{20 \omega^3}.
\]
\[ J_8 = \frac{2J_4\omega^4 + 3v_3V_1^2k^4 - 3k^4}{2\omega^2(k^2 + v_1V_2k^2 - 4k^2\omega^2 - J_1\omega^2)}. \]

Now, we consider the expression for \( m = 3 \) with \( l = 0 \) and \( m = 2 \) with \( l = 0 \) which leads to the zeroth harmonic modes. Thus, we obtain

\begin{align*}
    n_{(2)}^{(2)} &= J_9|\phi_0^{(0)}|^2, \\
    u_{(2)}^{(2)} &= J_{10}|\phi_1^{(0)}|^2, \\
    n_{(2)}^{(2)} &= J_{11}|\phi_1^{(0)}|^2, \\
    u_{(2)}^{(2)} &= J_{12}|\phi_1^{(0)}|^2, \\
    \Phi_0^{(2)} &= J_{13}|\phi_1^{(0)}|^2,
\end{align*}

where

\[ J_9 = \frac{\omega k^2 + J_{13}\omega^2 + 2v_3k^3}{\nu_1^2\omega^3}, \]
\[ J_{10} = \frac{k^2 + J_{13}\omega^2}{\nu_1^2\omega^3}, \]
\[ J_{11} = \frac{\omega v_1^2k^2 + 2v_3^2(\nu_1^2 - J_{13}\nu_1\omega^3)}{\nu_1^2\omega^3}, \]
\[ J_{12} = \frac{v_1^2k^2 - J_{13}\nu_1\omega^2}{\nu_1^2\omega^3}, \]
\[ J_{13} = \frac{2v_2(kv_3\omega_0 - k^3 + v_3^2k^3) - \omega k^2(1 - v_2v_1^2)}{\omega^3 + v_1v_2\omega^3 - J_{13}^3\omega_0^2}. \]

Finally, the third harmonic modes \( (m = 3) \) and \( (l = 1) \), with the help of Eqs. (22)–(41), give a set of equations which can be reduced to the following NLSE:

\[ \frac{i\partial \Phi}{\partial \tau} + P\frac{\partial^2 \Phi}{\partial \xi^2} + Q|\Phi|^2\Phi = 0, \quad (42) \]

where \( \Phi = \phi_0^{(0)} \) for simplicity. In Eq. (42), \( P \) is the dispersion coefficient and can be written as

\[ P = \frac{3v_2(kv_3\nu_1 - \nu_1^2)}{2k\omega}, \]

and \( Q \) is the nonlinear coefficient and can be written as

\[ Q = \frac{2J_3\omega^3 + 2J_3\omega^3(J_8 + J_{13}) - F}{2k^2(1 + v_1V_2)}, \]

where

\[ F = \omega k^2(J_4 + J_5) + v_1v_2\omega k^2(J_6 + J_{11}) + 2k^3(J_5 + J_{10}) + 2v_1v_2k^3(J_7 + J_{12}). \]

It may be essential to note that the values of \( P \) and \( Q \) depend on the physical parameters of the system and any change in these parameters will directly affect the nonlinearity and dispersion properties of the EDDPM.

4. MODULATIONAL INSTABILITY

The stable and unstable parametric regimes of DAWs are organised by the sign of \( P \) and \( Q \) of Eq. (42) [24, 28, 38]. When \( P \) and \( Q \) have the same sign (i.e., \( P/Q > 0 \)), the evolution of DAWs amplitude is modulationally unstable in the presence of external perturbations. On the other hand, when \( P \) and \( Q \) have opposite signs (i.e., \( P/Q < 0 \)), the DAWs are modulationally stable in the presence of external perturbations. The plot of \( P/Q \) against \( k \) yields stable and unstable parametric regimes of the DAWs. The point, at which the transition of \( P/Q \) curve intersects with the \( k \)-axis, is known as the threshold or critical wave number \( k = k_c \) [24, 28, 38].

We have depicted the variation of \( P \) with \( k \) for different values of \( \nu_1 \) in Fig. 1 and this depiction indicates that (a) the \( P \) is always negative for any values of \( k \) in the EDDPM; (b) the absolute value of \( P \) increases (decreases) with an increase in the value of positive (negative) dust mass for their constant charge state (via \( \nu_1 \)).

Figure 2 indicates how the MI of the DAWs is directly organized by the nonlinear coefficient \( Q \) of Eq. (40), and it is obvious from this figure that (a) \( Q \) can be positive/negative according to the value of \( k \) and other plasma parameters; (b) \( Q \) has positive (negative) value corresponding to the small (large) values of \( k \); (c) as \( Q \) has a combination of both positive and negative values instead of only negative value like \( P \) then the MI picture of the DAWs can be directly organized by flipping the sign of \( Q \) as well as \( P/Q \).
The criteria of the formation of dark envelope solitons associated with stable region (i.e., $\nu_1 > 0$) of DAWs as well as bright envelope solitons associated with unstable region (i.e., $\nu_1 < 0$) of DAWs can be shown in Fig. 3 in which the variation of $P/Q$ for different values of $\nu_1$ is depicted. It can be seen from this figure that (a) an increase in the value of negative (positive) dust mass enhances (reduces) the modulationally stable parametric regime for a fixed value of $\nu_1$ and $q$; (b) the modulationally stable parametric regime decreases (increases) with increasing value of $\nu_1$ ($\nu_1 < 0$) for a fixed value of positive and negative dust mass.

The effects of both sub-extensivity and super-extensivity of ions on the stable and unstable parametric regimes of DAWs can be observed from Figs. 4 and 5, respectively, and it is obvious from these figures that (a) for sub-extensive limit (i.e., $q > 1$), the $k_c$ as well as the stable parametric regime (i.e., $P/Q < 0$) of DAWs increases with an increase in the value of $q$; (b) the electrostatic dark envelope solitons associated with DAWs can generate for small values of $k$ (i.e., $k < k_c$) while the electrostatic bright envelope solitons associated with DAWs can generate for large values of $k$ (i.e., $k > k_c$); (c) on the other hand, for super-extensive limit (i.e., $-1 < q < 1$), the $k_c$ as well as the stable parametric regime (i.e., $P/Q < 0$) of DAWs increases with an increase in the value of $q$.

5. ENVELOPE SOLITONS

The bright (when $P/Q > 0$) and dark (when $P/Q < 0$) envelope solitonic solutions, respectively, can be written as [24]

$$\Phi(\xi, \tau) = \left[ \psi_0 \text{sech}^2 \left( \frac{\xi - U \tau}{\sqrt{2} W} \right) \right]^{\frac{3}{2}} \times \exp \left[ \frac{i}{2P} \left( U \xi + \left( \Omega_0 - \frac{U^2}{2} \right) \tau \right) \right],$$

(43)
where \( \psi_0 \) is the amplitude of localized pulse for both bright and dark envelope soliton, \( U \) is the propagation speed of the localized pulse, \( W \) is the soliton width, and \( \Omega_0 \) is the oscillating frequency at \( U = 0 \). The soliton width \( W \) and the maximum amplitude \( \psi_0 \) are related by \( W = \sqrt{2 \left| \rho P/\rho q \right|/\psi_0} \). We have depicted bright and dark envelope solitons profile in Fig. 6, and also numerically analyzed the effect of the physical parameter \( \nu_2 \) on the profile of bright and dark envelope solitons in Figs. 7 and 8, respectively. It is clear from these figures that the increase in the value of \( \nu_2 \) only causes to change the width of both bright and dark envelope solitons associated with DAWs but the magnitude of the amplitude of both bright and dark envelope solitons associated with DAWs does not change.

\[
\Phi(\xi, \tau) = \left[ \psi_0 \tanh^{2} \left( \frac{\xi - U \tau}{W} \right) \right]^{\frac{3}{2}} \exp \left[ \frac{i}{2P} \left( U \xi - \frac{U^2}{2} - 2PQ \psi_0 \right) \tau \right],
\]

\[ (44) \]

6. CONCLUSIONS

In our article, a NLSE was employed to study the MI and associated nonlinear features of DAWs in an unmagnetized EDDPM comprised of OPDGs and \( q \)-distributed positive ions. The physical parameters of the plasma system rigorously control the balance between the nonlinearity and dispersion of the medium hence setting the criterion for MI of DAWs as well as the formation of both bright and dark envelope solitons. The sub-extensive (super-extensive) ions help to decrease (increase) the stable region of DAWs. An addition of charge in the positive (negative) DGs increases the stability (unstability) region. The width of the solitons is changed by the variation of the plasma parameters while the amplitude of the solitons is unchanged. Finally, we hope that our investigation is useful in understanding the nonlinear behavior of DAWs and the propagation of envelope solitons in various space plasma environments, viz., cometary tails [2], Martian atmosphere [3], interstellar clouds [4], laser–matter interaction [6], etc.
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