Hunting for extra dimensions in the shadow of Sagittarius A*

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(Dated: May 17, 2022)

Recently, Vagnozzi and Visinelli’s work [Phys. Rev. D 100, 024020 (2020)] reveals that M87*’s shadow establishes an upper limit of $l \lesssim 170$ AU, which represents one astronomical unit. The Event Horizon Telescope, on the other hand, just captured the first image of the shadow of Sagittarius A* (SgrA*), the Galactic center source associated with a supermassive black hole. In this paper, we are motivated to comprehensively explore a new upper limit of $l$ with the shadow of SgrA*, and the findings suggest that $l \lesssim 0.097$ AU. Our results improve accuracy by three orders of magnitude. This is also one of the first quantifiable limitations on exotic physics derived from the remarkable first image of the shadow of SgrA*.

I. INTRODUCTION

The possible existence of extra dimensions was first proposed by Kaluza [1] and Klein [2] in the 1920s in order to unify gravity and electromagnetism (please see Ref. [3] for a fairly comprehensive review of the Kaluza-Klein (KK) supergravity theory). Although the KK theory brought extra dimensions to the attention of theoretical physicists, it gradually fell out of view because the extra dimensions in the KK theory could not be detected. But just like opening a Pandora’s box, it sparked curiosity about extra dimensions and thus the development of another theory based on extra dimensions, i.e., the braneworld theory. The braneworld theory has not only enriched people’s understanding of extra dimensions but also attracted much attention because it can well explain some difficult problems in theoretical physics, such as the hierarchy problem, the cosmological constant problem, and so on. One of the most representative models of braneworld theory is the Randall-Sundrum (RS) model [4, 5]. The RS scenario requires a five-dimensional AdS space (AdS$_5$) braneworld model with an unlimited size and a negative bulk cosmological constant. The matter fields and gauge fields of the electromagnetic, weak, and strong forces are constrained to a three-dimensional brane, however gravity is free to propagate across the AdS$_5$ spacetime. The tension on the brane is selected in such a way that General Relativity is restored in the low-energy regime [6, 7]. Ref. [8] provides a review of braneworld gravity models.

In general, the presence of braneworld extra dimensions may be observed using a range of approaches, including collider searches, precision gravity tests on small scales, and astronomical measurements [9–18]. These discoveries, in particular, allow us to restrict the AdS$_5$ radius of curvature $l$, a new parameter in the spacetime metric. The best limits on $l$ now derive from the lack of detectable macroscopic forces at laboratory distances, resulting in the restriction $l \lesssim 1$ mm [19]. Especially, in Ref. [20], the authors have shown how the image of the dark shadow of M87*, recently provided by the Event Horizon Telescope (EHT) collaboration [21–28], allows us to explore the exotic physics of extra dimensions.

They [20] demonstrated that M87*’s shadow establishes an upper limit of $l \lesssim 170$ AU within 1 AU, which represents one astronomical unit. The upper limit obtained in Ref. [20] is still not competitive with the $\mathcal{O}(\text{mm})$ constraint obtained in Ref. [19], but represents a substantial improvement over the limit obtained from GW170817 in Ref. [29].

Very recently, the EHT collaboration [30–39] just captured the first image of the shadow of Sagittarius A* (SgrA*), the Galactic center source associated with a supermassive black hole. Thus, one naturally has a question: Can the shadow of SgrA* provide a new and tougher upper bound on $l$ than the shadow of M87*? We are carrying out the present work to answer the aforesaid query. The shadow of SgrA* should be very circular in the simplest case when SgrA* is a Kerr black hole and gravity acts as expected. This expectation is satisfied in the EHT collaboration’s findings, which show that deviations from circularity in the image of the shadow of SgrA* are on the order of 10% at most [35]. One can set an initial limit of order four on relative deviations of the quadrupole moment from the standard value: $|\Delta Q/Q_{\text{Kerr}}| \equiv \varepsilon \lesssim 4$ [21, 40].

The purpose of this short paper is to explore the physics of braneworld extra dimensions in light of the EHT collaboration’s detection of the highly circular shadow of SgrA*. The remaining part of this paper is organized as follows. In Sec. II, we first go through how to compute the quadrupole moment of Kerr black holes in the RS scenario, which includes a fifth dimension with a curvature radius of $l$. Then, we estimate the relative deviation of the quadrupole moment compared to standard expectations as a function of $l$, $\varepsilon(l)$. In Sec. III Within the RS scenario, we employ the EHT collaboration’s constraint $\varepsilon(l) \lesssim 4$ to create a new and stricter upper limit on the AdS$_5$ curvature radius $l$. Finally, we present our conclusions in Sec. IV. Throughout this paper, we utilize the natural unit $c = G = 1$.

1 So far, the EHT collaboration’s series of papers [30–39] on shadow of SgrA* has not given a precise upper limit for $\varepsilon$. However, according to Refs. [21, 35], SgrA* and M87* are very similar and both can be accurately described by the Kerr metric, so we temporarily use the upper limit value of M87* in Ref. [21]: $\varepsilon \lesssim 4$. 

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II. A BRIEF REVIEW OF ROTATING BLACK HOLES IN FOUR AND FIVE DIMENSIONS

In the beginning of this section, we first revisit the computation of the quadrupole moment of Kerr black holes within the RS scenario, wherein a fifth dimension with curvature radius $l$ is present. Let us start with the four-dimensional rotating Kerr black hole, whose metric is [41]

$$ds^2 = -\left(1 - \frac{2Mr}{\Sigma}\right)dt^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 - \frac{4Mra\sin^2 \theta}{\Sigma}dt d\phi + \left(r^2 + a^2 + \frac{2Mra^2}{\Sigma}\sin^2 \theta\right)\sin^2 \theta d\phi^2,$$  

where

$$\Delta = r^2 + a^2 - 2Mr, \quad \Sigma = r^2 + a^2 \cos^2 \theta,$$  

$a$ is the rotation parameter, $M$ and $J = Ma$ are the mass and the angular momentum of the black hole, respectively. Let us define the function $\gamma(r, z)$ further by defining it as the real and positive solution to the equation $\gamma^2 - (r^2 - a^2)\gamma^2 - a^2 z^2 = 0$. Then, the Newtonian gravitational potential $\Phi(r, \theta)$ created by a source of mass $M$ is therefore described in the weak field limit by [42]

$$\Phi(r, \theta) = -\frac{Mr^3}{\gamma^2 + a^2 z^2}.$$  

The Newtonian gravitational potential, as indicated in Ref. [42], can then be extended in multipoles to provide (up to order $(a/r)^2$):

$$\Phi(r, \theta) = -\frac{M}{r} \left[1 + \left(\frac{a}{r}\right)^2 \mathcal{P}_2(\cos \theta) + \cdots\right],$$  

where $\mathcal{P}_2(\cos \theta) = (3 \cos^2 \theta - 1)/2$ is the second Legendre polynomial. The potential for $a = 0$ is spherically symmetric, as predicted, and the adjustment to spherical symmetry is provided by the second term on the right-hand side of Eq. (4). Especially, the Kerr black hole quadrupole, which is provided by $Q_{\text{Kerr}} = Ma^2$, was read off from the correction.

In the existence of a fifth dimension, we must now modify Eq. (4). In the RS model, the process of matter collapsing into a black hole has been examined in a number of significant studies [43-52]. It is worth mentioning that Ref. [44] investigated the example of a black string dwelling in the AdS5 bulk whose junction with the 3-brane appears as a black hole to an observer living on the brane. However, as described in Ref. [50], a “projected” black hole still contains information about the fifth dimension. In particular, gravitational field effects in the bulk essentially supply the Black hole with a tidal charge $\beta$, carrying information about the additional dimension and, in particular, the AdS5 curvature radius $l$, when projected onto the brane. In other words, because of the tidal effect of the bulk, the black hole dwelling on the brane bears an effective charge (which might be positive or negative).

These previous efforts concentrated on braneworld black holes in nonrotating spacetimes. The continuous spectrum of Kaluza-Klein modes in RS models, as established in Refs. [4, 5, 53], leads to a correction to the gravitational potential created by a point source, which at low energies goes as $\Delta \Phi \approx 2Ml^2/(3r^3)$. How about rotating spacetimes, such as braneworld Kerr black holes? In AdS$_5$, the line element of the Kerr black hole [54] is provided by (please see Refs. [55-64] for important research on spinning black holes in braneworld models)

$$ds^2 = -\frac{\hat{\Delta}}{\Sigma} (dt - a \sin^2 \theta d\phi)^2 + \Sigma \left(\frac{dr^2}{\Delta} + d\phi^2\right) + \frac{\sin^2 \theta}{\Sigma} \left[adt - (r^2 + a^2) d\phi\right]^2,$$  

where

$$\hat{\Delta} = r^2 + a^2 - 2Mr + \beta, \quad \Sigma = r^2 + a^2 \cos^2 \theta,$$  

in which $\beta$ is just the tidal charge of the black hole, it includes information from the bulk projected onto the brane. The metric in Eq. (5) may appear familiar: it is, in fact, completely equivalent to the four-dimensional rotating charged Kerr-Newman metric, where the charge $Q$ has been substituted by the tidal charge $\beta$ in the expression of function $\hat{\Delta}$ in Eq. (6). Indeed, it is clear that for $\beta = 0$, one recovers the Kerr metric (1). This finding is identical to that established in Ref. [50], which discovered that a Schwarzschild braneworld black hole is characterized by an Reissner-Nordström metric, with the black hole charge substituted by the tidal charge.

Even when considering spinning spacetimes, the prior estimate of $\Delta \Phi \approx 2Ml^2/(3r^3)$ is valid to order $(a/r)^2$ (i.e., the order essential for determining the quadrupole). To put it another way, the quadrupole does not get adjustments based on the spin of the black hole. This can be demonstrated by following the method of Ref. [56], which shows that the leading order correction scales as $l^2 a^2/r^4$ and so has no effect on the quadrupole computation. To sum up, up to order $(a/r)^2$, the Newtonian gravitational potential of the Kerr black hole in the RS model is [20]:

$$\Phi(r, \theta) = -\frac{M}{r} \left[1 + \left(\frac{a}{r}\right)^2 \mathcal{P}_2(\cos \theta) + \frac{2l^2}{3r^4} + \cdots\right].$$  

In all the prior equations, $\theta$ is the angle between the line-of-sight and the spin axis. As is shown in Ref. [65], for the cases of SgrA*, there are three different options for jet inclination with respect to the line-of-sight: the thermal+\kappa disk model at an inclination of $\theta_{\text{jet}} \simeq 60^\circ$ and the thermal jet model at an inclination of $\theta_{\text{jet}} \simeq 5^\circ$, and the thermal+\kappa jet model and thermal-only jet model, both at an inclination of $\theta_{\text{jet}} \simeq 90^\circ$, respectively. However, according to the results of the EHT collaboration [30], their model comparisons disfavor scenarios where the black hole is viewed at high inclination ($\theta_{\text{jet}} > 50^\circ$), therefore we dropped the two choices of $\theta_{\text{jet}} \simeq 60^\circ$ and $\theta_{\text{jet}} \simeq 90^\circ$, and kept the option of $\theta_{\text{jet}} \simeq 5^\circ$. It is worth mentioning that the option of $\theta_{\text{jet}} \simeq 5^\circ$ also coincides with the angle between the line-of-sight and the spin axis theta under the well-motivated assumption that the jet is propelled by the spin of the black hole and is approximately aligned with the spin axis (through the Blandford-Znajek mechanism [66] or variants thereof). As a result, we
will now set $\theta = \theta_{\text{jet}} \simeq 5^\circ$. Take note that at such a small observation angle, $\mathcal{P}_2(\cos \theta) \approx 1$, an approximation that we will use in the following for simplicity.

III. RESULTS

We can read off the quadrupole moment $Q_{\text{Kerr,RS}} \approx Ma^2 + 2M^2/3$ and related deviation from the Kerr value $\Delta Q \equiv Q_{\text{Kerr,RS}} - Q_{\text{Kerr}}$ using the modified Newtonian potential in the presence of extra dimensions computed in Eq. (7) and our approximation for $\mathcal{P}_2(\cos \theta) \approx 1$, with $Q_{\text{Kerr}} = Ma^2$. The deviation $\Delta Q$, and consequently the relative deviation $\varepsilon$, are given by:

$$\Delta Q \approx \frac{2Ml^2}{3},$$

$$\varepsilon = \frac{\Delta Q}{Q_{\text{Kerr}}} \approx \frac{2l^2}{3a^2}.$$

When we combine our estimate for $\Delta Q$ with the upper limit on $\varepsilon$ reported by the EHT collaboration [21] indicates:

$$\varepsilon \approx \frac{2l^2}{3a^2} \lesssim 4 \Rightarrow \frac{2l^2}{3} \lesssim 4a^2. \quad (10)$$

As we known. The angular momentum of the Kerr black hole is constrained by the Kerr limit $a \leq M$, which means that $4a^2 \leq R_s^2$, where $R_s = 2M$ is the Schwarzschild radius of the astrophysical black hole. With this additional inequality in hand, we can deduce that Eq. (10) implies:

$$l \lesssim \sqrt{\frac{3}{2}} R_s. \quad (11)$$

In Ref. [33], the mass of SgrA* was estimated to be $M_{\text{SgrA*}} = 4.0^{+1.1}_{-0.6} \times 10^6 M_\odot$, in units of the mass of the Sun $M_\odot \approx 2 \times 10^{30}$ kg. Using this outcome, We arrive to $R_s \approx 1.187 \times 10^{10} \text{m}$, which gives us the following new upper limit on AdS$_5$ curvature radius:

$$l \lesssim 1.457 \times 10^{10} \text{m} \approx 0.097 \text{AU}. \quad (12)$$

with 1 AU being one astronomical unit.

The upper limit obtained in Eq. (12) is still not competitive with the $\theta'(\text{mm})$ constraint obtained in Ref. [19], but it is significantly better than the limit obtained from the shadow of M87* [20], and improves accuracy by three orders of magnitude. More importantly, it is an independent limit and one of the first limits on exotic physics obtained by imaging the shadow of SgrA* (please see Ref. [67] for other work exploring the basic physics and properties of black holes in the imaging shadow of SgrA*).

IV. CONCLUSIONS

In this short article, we have extended Vagnozzi and Visinelli’s previous work [20] to the well-known SgrA* case and illustrated how the image of the shadow of SgrA*, very recently released by the Event Horizon Telescope collaboration, allows us to investigate the physics of extra dimensions. We have placed a new upper limit on the AdS$_5$ curvature radius $l$, finding $l \lesssim 0.097$ AU and improving accuracy by three orders of magnitude, based on the RS braneworld scenario, in which our Universe is made up of a 3-brane within an AdS$_5$ universe. Although this upper limit is also far from competing with existing extra-dimensional limitations derived from precision gravity measurements on $\theta'(\text{mm})$ scale. We believe this limitation is valuable despite its weakness since it is derived in a completely separate manner from earlier limits. It also acts as a proof-of-concept for the idea of restricting fundamental physics starting with the first image of the shadow of SgrA*.

In terms of tests of General Relativity and alternative theories of gravity, the future years will undoubtedly be quite intriguing. The Event Horizon Telescope’s success in imaging the shadows of M87* and SgrA*, as well as LIGO-Virgo-KAGRA’s effectiveness in detecting an ever-increasing number of GW events, offer us with more tests of gravity’s behavior on astronomical and cosmological scales. It will be fascinating to see how we can use existing data and build on the significant efforts made to deploy the first ever image of a black hole shadow, to investigate the physics describing the structure of our spacetime and to explore the possibility of extra dimensions beyond those that our senses can detect. We believe that future EHT and next-generation EHT results will allow us to image more central black holes in galaxies, allowing us to improve fundamental physics tests by imaging the shadows of astrophysical black holes.

ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation of China under Grants No. 11675130, and by the Doctoral Research Initiation Project of China West Normal University under Grant No. 21E028.

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2 This is also consistent with the premise [21] that the inclination angle is small (we set $\theta = \theta_{\text{jet}} \simeq 5^\circ$).
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