Non-Markovian dynamics of a microcavity coupled to a waveguide in photonic crystals

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In this paper, the non-Markovian dynamics of a microcavity coupled to a waveguide in photonic crystals is studied based on Fano-type tight binding model. Using the exact master equation, we solve analytically and numerically the temporal evolution of the cavity coherent state and the associated physical observables. A critical transition is revealed when the coupling increase between the cavity and the waveguide. In particular, the cavity field becomes dissipationless when the coupling strength goes beyond a critical value, as a manifestation of strong non-Markovian memory effect. The result also indicates that the cavity can maintain in a coherent state with arbitrary small number of photons when it strongly couples to the waveguide at very low temperature. These properties can be measured experimentally through the photon current flowing over the waveguide in photonic crystals.

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I. INTRODUCTION

Optical microcavities confine light in the micro- and submicro-scale volumes by resonant recirculation with very high quality factors [1, 2]. Micro- and submicro-scale volume ensures that resonant frequencies are more sparsely distributed in the size-dependent resonant frequency spectrum. Prototypical microcavities include the Fabry-Perot microcavities, the silica-based microdisk, microsphere, and microtoroid whispering gallery cavities, and the photonic crystal cavities. Devices based on these microcavities are already received tremendous attentions for a wide range of applications, including strong coupling cavity QED [3–5], low threshold lasers [6], biochemical detectors [7, 8], as well as optical traps [9]. In this paper, we shall study the non-Markovian dynamics of a microcavity coupled to a waveguide in photonic crystals.

A microcavity in photonic crystals is a point defect created in photonic crystals as a resonator. Its frequency can easily be tuned to any value within the band gap by changing the size or the shape of the defect and therefore can be used to enhance the efficiency of lasers. While, a waveguide in photonic crystals consists of a linear defects in which light propagates due to the coupling of the adjacent defects. By changing the modes of the resonators and the coupling configuration, the transmission properties of the waveguide can be manipulated. The most promising application of waveguide is to control the group velocity, thus potential to application in storing and buffering light by coupling to a microcavity [10–12]. While, the coupling between the microcavity and the waveguide is controllable [13], which can induce non-Markovian dissipation and decoherence phenomena [14].

The non-Markovian dynamics is an important factor in the practical applications of quantum information and quantum computation in terms of photons. Therefore, we shall use the exact master equation we developed recently to investigate the non-Markovian dynamics of the microcavity field coupled to a waveguide in different coupling regime, to explore possible new applications of microcavities in quantum optics.

The paper is organized as follow. In Sec. II, we introduce the exact master equation we developed recently for the reduced density operator of a cavity coupled to the waveguide as a reservoir. In this section, the reduced density operator as well as the temporal evolution of the cavity mode amplitude and the photon number are derived analytically. The photon current in waveguide is also calculated directly from the time-dependence of the photon number in the cavity. In Sec. III, exact non-Markovian dynamics of the cavity field is demonstrated numerically through the temporal evolution of the cavity mode amplitude as well as the photon number inside cavity. By varying the coupling between the cavity and the waveguide in different coupled configuration, the significant non-Markovian memory effect is revealed. Finally, summary and discussion are given in Sec. IV.

II. THE MICROCA VITY DYNAMICS COUPLED TO A WAVEGUIDE

A. Fano-type tight-binding model for a microcavity coupled to a waveguide

We consider a microcavity with a single mode coupled to a waveguide in photonic crystals, see a schematic plot in Fig. 1. The microcavity is a point defect created in photonic crystals as a resonator. While the waveguide consists of a linear defects in which light propagates due

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to the coupling of the adjacent defects. Therefore, the Hamiltonian of the system can be expressed as a tight binding model:

\[ H = \omega_c a^\dagger a + \sum_n \omega_0 a^\dagger_n a_n - \sum_n \xi_0 (a^\dagger_n a_{n+1} + \text{H.c.}) + \xi (a^\dagger a_1 + \text{H.c.}) . \] (1)

Here we have set \( h = 1 \). The first term in Eq. (1) is the Hamiltonian of the microcavity in which \( a^\dagger, a \) are the creation and annihilation operators of the single mode cavity field, with frequency \( \omega_c \), which can easily be tuned to any value within the band gap by changing the size or the shape of the defect. The second and third terms are the Hamiltonian of the waveguide where \( a^\dagger_n, a_n \) are the photonic creation and annihilation operators of the resonator at site \( n \) of the waveguide with an identical frequency \( \omega_0 \), and \( \xi_0 \) is the hopping rate between adjacent resonator modes. Both \( \omega_0 \) and \( \xi_0 \) are experimentally tunable. The last term is the coupling between the microcavity and the waveguide with the coupling constant \( \xi \). While, the coupling between the cavity and the waveguide is also controllable by changing the geometrical parameters of the defect cavity and the distance between the cavity and the waveguide [13].

The about system of a microcavity coupled to a waveguide in photonic crystals can also be implemented with different type of micro-resonators, such as Fabry-Perot microcavities and micro-ring resonators, with different coupling and confinement mechanism [15]. However, the dispersion relation of different kinds of microcavities and micro-waveguides are very similar, it is only characterized by the free spectral range, the quality factor of the resonators and the coupling between the resonators [16]. Therefore Eq. (1) describes indeed a large class of a microcavity coupled to a micro-waveguide. Furthermore, the Hamiltonian [11] can be re-expressed as a Fano-type model of a localized resonance interacting with continuums [17, 18]:

\[ H = \omega_c a^\dagger a + \sum_k \omega_k a^\dagger_k a_k + \sum_k [V_k a^\dagger_k a_k + \text{H.c.}] , \] (2)

where \( 0 \leq k \leq \pi \), \( \omega_k \) and \( V_k \) are given by:

\[ \omega_k = \omega_0 - 2 \xi_0 \cos(k) , \quad V_k = \sqrt{2/\pi} \xi \sin(k) , \] (3)

and \( a^\dagger_k, a_k \) are the creation and annihilation operators of the corresponding Bloch modes of the waveguide, which is defined as follow:

\[ a_k = \sqrt{\frac{2}{\pi}} \sum_{n=1}^{\infty} \sin(nk) a_n . \] (4)

After transform the Hamiltonian into this form, we can use the exact master equation we developed recently to examine the non-Markovian dynamics of the microcavity coupled to the waveguide in photonic crystal quantum optics.

### B. Exact master equation

The master equation for the cavity field is given in terms of the reduced density operator which is defined from the density operator of the total system by tracing over entirely the environmental degrees of freedom: \( \rho(t) \equiv \text{tr}_{R}(\rho_{\text{tot}}(t)) \), where the total density operator is governed by the quantum Liouville equation

\[ \dot{\rho}_\text{tot}(t) = e^{-iH(t-t_0)} \rho_\text{tot}(t_0) e^{iH(t-t_0)} . \] (5)

By integrating over all the environmental degrees of freedom, based on the Feynman-Vernon influence functional approach [20] in the framework of coherent state path-integral representation [21], we obtain the exact master equation for the reduced density operator [22, 24]:

\[ \dot{\rho}(t) = -i \omega'_c(t) \left[ a^\dagger a, \rho(t) \right] + \kappa(t) \left\{ 2 a \rho(t) a^\dagger - a^\dagger a \rho(t) - \rho(t) a^\dagger a \right\} + \tilde{\kappa}(t) \left\{ a^\dagger \rho(t) a + a \rho(t) a^\dagger - a^\dagger a \rho(t) - \rho(t) a a^\dagger \right\} , \] (6)

where the time-dependent coefficient \( \omega'_c(t) \) is the renormalized frequency of the cavity, while \( \kappa(t) \) and \( \tilde{\kappa}(t) \) describe the dissipation and noise to the cavity field due to the coupling with the reservoir. These coefficients are non-perturbatively determined by the following relations:

\[ \omega'_c(t) = -\text{Im}[\dot{\mu}(t) u^{-1}(t)] , \] (6a)

\[ \kappa(t) = -\text{Re}[\dot{\mu}(t) u^{-1}(t)] , \] (6b)

\[ \tilde{\kappa}(t) = \dot{v}(t) - 2 v(t) \text{Re}[\dot{\mu}(t) u^{-1}(t)] , \] (6c)

and \( u(t) \) and \( v(t) \) satisfy the integrodifferential equations of motion:

\[ \dot{u}(\tau) + i \omega_c u(\tau) + \int_{t_0}^{\tau} d\tau' g(\tau - \tau') u(\tau') = 0 , \] (7a)

\[ v(t) = \int_{t_0}^{t} d\tau_1 \int_{t_0}^{\tau_1} d\tau_2 \, \overline{\pi}(\tau_1) \overline{g}(\tau_1 - \tau_2) \pi(\tau_2) , \] (7b)

subjected to the initial condition \( u(t_0) = 1 \) while \( \overline{\pi}(\tau) \equiv u(t + t_0 - \tau) \).

Note that the integral kernels in the above equations are the time correlation functions of the waveguide: \( g(\tau - \tau') \) and \( \overline{g}(\tau - \tau') \). These two time-correlation
functions characterize all the non-Markovian memory structures between the cavity and the waveguide. By defining the spectral density of the waveguide: \( J(\omega) = 2\pi \sum_k |V_k|^2 \delta(\omega - \omega_k) \), the time-correlation functions are explicitly given by

\[
\begin{align*}
g(\tau - \tau') &= \int_0^\infty \frac{d\omega}{2\pi} J(\omega)e^{-i\omega(\tau - \tau')}, \\
\bar{g}(\tau - \tau') &= \int_0^\infty \frac{d\omega}{2\pi} J(\omega)\pi(\omega, T)e^{-i\omega(\tau - \tau')},
\end{align*}
\]

where \( \bar{n}(\omega, T) = \frac{1}{e^{\omega/k_B T} - 1} \) is the average number distribution of the waveguide thermal excitation at the initial time \( t_0 \). With the spectrum of the photonic crystal \( \xi \), the spectral density becomes \( J(\omega) = \frac{\xi}{\xi_0} \sqrt{4\xi_0^2 - (\omega - \omega_0)^2} \), and \( g(\omega) \) is the density of state:

\[
g(\omega) = \frac{\frac{d\xi}{\xi_0}}{\frac{\sqrt{4\xi_0^2 - (\omega - \omega_0)^2}}{\sqrt{4\xi_0^2 - (\omega - \omega_0)^2}}}, \\
V(\omega) = \frac{1}{\sqrt{2\pi} \xi_0} \sqrt{4\xi_0^2 - (\omega - \omega_0)^2}.
\]

Then the spectral density can be explicitly written as

\[
J(\omega) = \left( \frac{\xi}{\xi_0} \right)^2 \sqrt{4\xi_0^2 - (\omega - \omega_0)^2},
\]

with \( \omega_0 - 2\xi_0 < \omega < \omega_0 + 2\xi_0 \). In practical, \( \xi_0 \ll \omega_0 \), namely the waveguide has a very narrow band.

The master equation \( 4 \) is exact, far beyond the BM approximation widely used for conventional optical cavities. The back-reaction effect between the system and environment is fully taken into account by the timedependent coefficients, \( \omega(t) \), \( \kappa(t) \) and \( \bar{\kappa}(t) \), in the master equation \( 5 \) through the integrodifferential equations \( 7 \). Thus, the non-Markovian memory structure is non-perturbatively built into the integral kernels in these equations. The expression of the integrodifferential equation \( 7 \) shows that \( u(t) \) is just the propagating function of the cavity field (the retarded Green function in nonequilibrium Green function theory \( 23 \)), and \( v(t) \) is the corresponding correlation (Green) function, as we will see next. Therefore, the exact master equation \( 5 \) depicts the full nonequilibrium dynamics of the cavity system as well as the waveguide.

C. **Exact solutions of the microcavity dynamics**

The main physical observables for the microcavity are the temporal evolution of the cavity mode amplitude and the photon number inside the cavity. The cavity mode amplitude is defined by \( \langle a(t) \rangle = \mathrm{tr}[a\rho(t)] \). From the exact master equation \( 5 \), it is easy to find that \( \langle a(t) \rangle \) obeys the equation of motion

\[
\dot{\langle a(t) \rangle} = -[i\omega_0(t) + \kappa(t)]\langle a(t) \rangle = \frac{\dot{u}(t)}{u(t)}\langle a(t) \rangle.
\]

which has the exact solution:

\[
\langle a(t) \rangle = u(t)\langle a(t_0) \rangle.
\]

In other words, the temporal evolution of the cavity mode amplitude is totally determined by \( u(t) \), which indicates that \( u(t) \) is the propagating function characterizing the cavity field evolution, as we have mentioned.

Another important physical observable is the total photon number inside the cavity, which is defined by \( n(t) = \mathrm{tr}[a^\dagger a\rho(t)] \). From the exact master equation, it is also easy to find that

\[
\dot{n}(t) = -2\kappa(t)n(t) + \bar{\kappa}(t).
\]

On the other hand, Eq. \( 6b \) can be rewritten as

\[
\ddot{v}(t) = -2\kappa(t)v(t) + \bar{\kappa}(t),
\]

with \(-2\kappa(t) = [\dot{u}/u(t) + \mathrm{H.c.}]\). Combining these equations together, we obtain the exact solution of \( n(t) \) in terms of \( u(t) \) and \( v(t) \):

\[
n(t) = u(t)n(t_0)u^*(t) + v(t).
\]

In fact, the above solution is a result of the correlated Green function in nonequilibrium Green function theory \( 26 \). It contains two terms, the first term represents the temporal evolution (usually a dissipation process) of the cavity field, due to the coupling to the waveguide. The second term is a noise effect induced by thermal fluctuation of the waveguide. Therefore, Eq. \( 15 \) combines the dissipation and fluctuation dynamics together to characterize the entire cavity dynamics. The dissipation and fluctuation dynamics obeys the dissipation-fluctuation theorem, as shown from the waveguide’s time-correlation functions \( 8 \). Since \( v(t) \) is also determined by \( u(t) \), as one can see from Eq. \( 7 \), both the cavity mode amplitude and the photon number inside the cavity are completely obtained by solving the propagating function \( u(t) \).

Furthermore, to see the coherence of the cavity field, we should solve explicitly the reduced density operator. This can be done easily through the coherent state representation \( 22 \). Consider the cavity initially in a coherent state,

\[
\rho(t_0) = e^{-|\alpha_0|^2} |\alpha_0\rangle \langle \alpha_0 |,
\]

it is not difficult to find \( 24 \) that the reduced density operator at arbitrary later time \( t \) becomes

\[
\rho(t) = \exp \left\{ \frac{|\alpha(t)|^2}{1 + v(t)} \sum_{n=0}^{\infty} \frac{|v(t)|^n}{[1 + v(t)]^{n+1}} \right\} \times \left| \frac{\alpha(t)}{1 + v(t)} \right|^n \langle n, \frac{\alpha(t)}{1 + v(t)} | n \rangle,
\]

where \( | \frac{\alpha(t)}{1 + v(t)} |, \langle n, \frac{\alpha(t)}{1 + v(t)} | n \rangle \) is a generalized coherent state, and \( \alpha(t) = u(t)\alpha_0 \). It is interest to see that Eq. \( 17 \) is indeed a mixed state of generalized coherent
states $|\alpha(t)\rangle_{\tilde{\mathcal{H}}+\mathcal{H}}$, in which the photon number is given by
\[ n(t) = |u(t)\alpha_0|^2 + v(t), \]
as we expected.

Usually, $u(t)$ decays to zero due to the dissipation induced by the coupling to the waveguide. The corresponding reduced density operator asymptotically becomes a thermally state with the asymptotic photon number $n(t) = v(t \to \infty) \sim \bar{n}(\omega_c, T)$. This solution shows precisely how the cavity field loses its coherence (i.e., decoherence) due to the coupling to the waveguide. This decoherence arises from the decay of the cavity field amplitude $\alpha(t) = u(t)\alpha_0$ as well as the thermal-fluctuation-induced noise effect manifested through the correlation function $v(t)$, as shown in Eq. (17). The later describes a process of randomly losing or gaining thermal energy from the reservoir (here is the waveguide), upon the initial temperature of the waveguide.

However, when the coupling between the cavity and the waveguide is strong enough, $u(t)$ may not decay to zero, as we shall show explicitly in the numerical calculation in the next section. Then the reduced density operator remains as a mixed coherent state. On the other hand, at zero-temperature limit $T = 0$, we have $\bar{n}(\omega, T) = 0$ so that $\bar{g}(\tau - \tau') = 0$. As a result, we obtain $v(t) = 0$. The reduced density operator at zero temperature limit is given by
\[ \rho(t)_{T = 0} = e^{-|\alpha(t)|^2} |\alpha(t)\rangle \langle \alpha(t)|. \]

In other words, the cavity can remain in a coherent state in the zero temperature limit. These two features $|u(t)|$ may not decay to zero in the strong coupling regime and $v(t) = 0$ at $T = 0$ indicate that enhancing the coupling between the cavity and the waveguide and meantime lowering the initial temperature of the waveguide can significantly reduce the cavity’s decoherence effect in photonic crystals.

### III. NUMERICAL ANALYSIS OF THE EXACT NON-MARKOVIAN DYNAMICS

In this section, we will demonstrate the exact non-Markovian dynamics of a microcavity coupled to a waveguide in photonic crystals. In our calculation, based on the experiment in Ref. [19], we take the frequency of the waveguide resonators to be $\omega_0 = 12.15$ GHz $= 50.25$ µeV (in the unit $\hbar = 1$), and the coupling between the adjacent resonators to be $\xi_0 = 1.24$ µeV. The initial temperature of the waveguide is set at $T = 5K$ so that $k_B T = 430.75$ µeV $\approx 8.57\omega_0$. The frequency of the single mode cavity $\omega_c$ and the coupling $\xi$ between the cavity and the waveguide are tunable parameters by changing the geometry of the cavity and the distance between the cavity and the waveguide [27, 28]. With these experimental input parameters, we numerically calculate the exact cavity dynamics with different coupling strength for three different cavity frequency configurations: i), the cavity coupled to the waveguide at the waveguide band centre ($\omega_c = \omega_0$), ii), the cavity coupled to the waveguide near the upper band edge ($\omega_0 < \omega_c < \omega_0 + 2\xi_0$), iii), the cavity coupled apart from the band of the waveguide ($\omega_c > \omega_0$). Detailed numerical results are plotted in Figs. 2–5.
In Fig. 2 we show the exact solution of the scaled cavity field amplitude, i.e. $|\langle a(t) \rangle/\langle a(t_0) \rangle| = |u(t)|$ (see Eq. (12)), in different coupled configuration from the weak coupling to strong coupling regime. For $\omega$ lies outside the band of the waveguide, in both the weak and strong coupling regimes, $|u(t)|$ remains unchanged beside a short time very small oscillation at the beginning, see Fig. 2(a). This result indicates that when its frequency lies outside the band of the waveguide, the cavity effectively decouples from the waveguide. However, when $\omega$ lies inside the band of the waveguide, the time evolution behavior of the field amplitude is totally different in different coupling regime, see Fig. 2(b)-(c). In weak coupling regime (in terms of a dimensionless coupling rate $\eta \equiv \frac{\alpha}{\omega} < 0.7$), $|u(t)|$ decays to zero monotonically, as a typical Markov process. However, increasing the coupling such that $\eta > 1.0$, after a short time decay at the beginning, the field amplitude begins to revives, and more than that, it keeps oscillating below an nonzero value. This behavior shows that the cavity field no longer decays monotonically in strong coupling regime, as a significant non-Markovian memory effect. This effect becomes the strongest when the cavity frequency matches the band centre of the waveguide, i.e. $\omega_c = \omega_0$.

In Fig. 3 we show a 3D plot of $|u(t)|$ varying in terms of the coupling rate $\eta$ and the time $t$, where the critical transition from the Markov to non-Markovian dynamics is manifested with the critical coupling $\eta_c \approx 0.7 \sim 1.0$. To understand the underlying mechanism of this critical transition, we also plot in Fig. 3 the decay coefficient in the master equation (5), $\kappa(t) = -\text{Re}[\dot{u}(t)/u(t)]$, for different coupling configurations. The decay coefficient $\kappa(t)$ dominates the dissipation behavior of the cavity field, roughly given by the damping factor $\sim e^{-\int_0^t \kappa(t')dt'}$. As one can see, in weak coupling regime, after a short time increase, $\kappa(t)$ approaches to a stationary positive value, see Fig. 3(c). This leads to a monotonic decay for the cavity field, i.e. a dissipation process. However, in strong coupling regime, the behavior of $\kappa(t)$ is totally different, it keeps oscillation in all the time between an equal positive and negative bound value without approaching to zero, see Fig. 3(b). This oscillation process means that the cavity dissipates energy to the waveguide and then fully regains it back from the waveguide repeatedly. The overall effect of this reviving process is that no energy dissipates into the waveguide. In other words, the cavity dynamics becomes dissipationless in the strong coupling regime. Thus, the critical transition from weak to strong coupling regime reveals the transition from dissipation into dissipationless processes for the cavity dynamics, as a manifestation of the non-Markovian memory effect.

To see further the noise effect induced by thermal fluctuation in the above non-Markovian process, we examine the temporal evolution of the correlation function $v(t)$ given by (4). Physically, Eq. (15) shows that if the cavity is initially empty, then $v(t)$ is the average photon number inside the cavity, induced by the thermal fluctuation of the waveguide. In Fig. 4(b), we plot $v(t)$ with a few different coupling strength. As we see in the weak coupling regime ($\eta < 0.7$), the exact $v(t)$ increases monotonically and approaches to $\bar{n}(\omega_c, T)$ gradually. However, in strong coupling regime ($\eta > 1.0$), the behavior of $v(t)$ is qualitatively different from that in weak coupling case. It increases much faster within a very short time in the beginning, then keeps oscillation in a long time, in response to the corresponding dissipationless oscillation of the cavity amplitude $u(t)$.

To demonstrate explicitly the temperature dependence of this thermal fluctuation effect, we plot $v(t)$ in Fig. 4(a) with a very low temperature, $T = 5$ mK. The value of $v(t)$ is reduced dramatically ($< 10^{-8}$ as shown in Fig. 4(a)). This clearly shows that $v(t)$ characterizes the noise effect of the thermal fluctuation from the waveguide. Lowering the initial temperature of the waveguide can efficiently suppress the thermal noise effect. Based on the analytical solution of the reduced density matrix in the last section, if the cavity is initially in a coherent state, and if the initial temperature of the waveguide is low enough such that $v(t) \rightarrow 0$, the cavity state is given by Eq. (19) where $\alpha(t) = u(t)\alpha(0)$. As a result, we can maintain well the cavity’s coherence by enhancing coupling to the waveguide such that the dissipation can also be suppressed.

To show the total non-Markovian memory effect distributing in the dissipation and the noise processes, we examine the temporal evolution of the photon number inside the cavity in Fig. 5(a)-(b). The negative time derivation of the photon number inside the cavity corresponds to the photon current flowed into the waveguide, which is also shown in Fig. 5(c)-(d). The photon current is an important quantity to characterize the transmission of the waveguide. Eq. (16) shows that the total photon number in the cavity consists of two sources: the evolution of the initial photons in the cavity, and the thermal noise induced photons. From Fig. 5(a)-(b) one see that in a relative high temperature (above a few K), the thermal fluctuation, i.e. the contribution from $v(t)$, dominates the photon number in the cavity, where $v(t \rightarrow \infty) \sim \bar{n}(\omega_c, T)$ which is about a few tens ($\sim 50$ for $T = 5$ K, as shown in Fig. 5(b)) when $\omega_c$ is in the
microwave region. However, in a very low temperature, 
$v(t)$ approaches to zero ($v(t) < 10^{-8}$ at $T = 5$ mK, as shown in Fig. (a)). Then $n(t)$ is fully dominated by the evolution of the initial photon number in the cavity, i.e. the first term $|u(t)|^2n(t_0)$ in Eq. (15).

The difference of the time evolution of the photon number for the weak and strong couplings is mainly shown in the long time behavior. In weak coupling regime ($\eta < 0.7$), $n(t)$ approaches gradually and monotonically to $\bar{n}(\omega_c, T)$. However, in the strong coupling regime ($\eta > 1.0$), $n(t)$ quickly reaches to $\bar{n}(\omega_c, T)$ and then oscillates around $\bar{n}(\omega_c, T)$ due to the dissipationless oscillation of $u(t)$. In fact, the dissipationless oscillation of $u(t)$ in strong coupling regime indicates that the cavity field can produce subsequent pulses with a small number of photons in the low temperature region. From Fig. (a), we can see that the cavity can generate indeed single photon pulses at $T = 5$ mK when the coupling rate $\eta = \xi/\xi_0 = 2$, namely the coupling between the cavity and the waveguide is twice of the coupling between the adjacent resonators in the waveguide, which is experimentally easy to realize. Fig. (c)-(d) also plot the photon current in the waveguide, which shows the corresponding oscillation associated with the amplitude oscillation of the cavity field. Physically this result indicates that the photon tunnels between cavity and the waveguide repeatedly without loss of the coherence in the strong coupling regime. Experimentally, one can directly measure the photon current flowing over the waveguide to demonstrate these properties. These properties may provide new applications for the microcavity in photonic crystals.

From the above analysis, we find that when $\omega$ lies outside the band of the waveguide, the cavity dynamics effectively decouples from the waveguide. However, when $\omega$ locate inside the band of the waveguide, the non-Markovian memory effect can qualitatively change the dissipation as well as the noise dynamics of the cavity field. In particular, the coupling between the microcavity and the waveguide can manipulate well the dissipation behavior of the cavity dynamics. Meanwhile, in the very low temperature limit, one can also control efficiently the cavity coherence as well as the photon number. Otherwise, the thermal fluctuation can induce non-negligible noise effect.

IV. CONCLUSION

In this paper, the exact non-Markovian dynamics of a microcavity coupled to a waveguide structure is studied. By solving the exact master equation analytically, general solution of the density operator as well as the cavity mode amplitude and the photon numbers inside the cavity are obtained. We also examine the temporal evolution of the cavity mode amplitude and the photon number numerically in different coupling configurations. We show that different cavity frequency and coupling between the cavity and the waveguide would lead to totally different cavity dynamics. For the frequency lies outside the band of the waveguide, the cavity becomes isolated in both weak and strong coupling. When the cavity frequency lies inside the band of the waveguide, the non-Markovian memory effect qualitatively changes the amplitude damping behavior of the cavity field as well as the thermal noise dynamics from the weak to strong coupling regime. In particular, when the coupling strength goes beyond a critical value, the cavity field becomes dissipationless as a signature of strong non-Markovian memory effect. The result also indicates that the cavity can maintain in a coherent state with a very small photon number up to a single photon, when it strongly couples to the waveguide at very low temperature. The perfect transmission between the cavity and the waveguide in photonic crystals is also more feasible in the strong coupling regime at very low temperature. These properties can be measured experimentally through the photon current flowing over the waveguide in photonic crystals. We also hope that these properties would provide further insights for the applications of the microcavity in quantum optics.

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