LUMINOSITY FUNCTIONS OF TYPE Ia SUPERNOVAE AND THEIR HOST GALAXIES FROM THE SLOAN DIGITAL SKY SURVEY

NAOKI YASUDA 1,2 AND MASATAKA FUKUGITA 1,2,3
1 Institute for Cosmic Ray Research, University of Tokyo, Kashiwa 277-8582, Japan
2 Institute for the Physics and Mathematics of the Universe, University of Tokyo, Kashiwa 277-8568, Japan
3 Institute for Advanced Study, Princeton, NJ 08540, USA

Received 2009 May 19; accepted 2009 October 15; published 2009 November 19

ABSTRACT

The sample of 137 low-redshift type Ia supernovae (SNe Ia) with 0.05 \( \leq z \leq 0.3 \) obtained from the Sloan Digital Sky Survey (SDSS)-II supernova survey for the southern equatorial stripe of 300 deg\(^2\) is used to derive the luminosity functions (LFs) of SNe Ia and their host galaxies in the \( g, r, i \) passbands. We show that the LF of SNe Ia host galaxies matches well with that of galaxies in the general field, suggesting that the occurrence of SNe Ia does not favor a particular type of galaxy but is predominantly proportional to the luminosity of galaxies. The evidence is weak that the SNe rate varies with the color of host galaxies. The only evidence that points to possible correlation between the SN rate and star formation activity is that the SN rate in late-type galaxies is higher than that in early-type galaxies by 31\% \pm 35\%. In our low-redshift sample, the component of type Ia SN rate that is proportional to star formation activity is not evident in the integrated SN rate, while our observation is compatible with the current two-component models. The sample contains eight SNe Ia whose host galaxies were not identified, but it is shown that their occurrence is consistent with them occurring in low-luminous galaxies beyond the survey. The LF of SNe Ia is approximately Gaussian with the full width at half-maximum being a factor of \( \sigma = 0.24 \text{ mag} \) or 1.67 in luminosity. The Gaussian distribution becomes tighter if the ratio of extinction to reddening, \( R_V \), is lower than the characteristic value for the Milky Way and if luminosity is corrected for the light-curve shape. The average color excess is \( \approx 0.07 \text{ mag} \), which is significantly smaller than reddening expected for field galaxies. This color excess does not vary with the distance of the SNe from the center of the host galaxy to 15 kpc. This suggests that the major part of the color excess appears to be either intrinsic or reddening that arises in the immediate environment of SNe, rather than interstellar reddening in host galaxies, and most of SNe Ia take place in a relatively dust-free environment.

Key words: supernovae: general

1. INTRODUCTION

Recent studies have revealed that our understanding of the mechanism of type Ia supernovae (SNe Ia) is poorer than we had thought. It is suspected that there are two different progenitor types responsible for SNe Ia: explosions from old systems as in the long-accepted scenario and explosions in young stellar systems (Dallaporta 1973; Tammann 1982; Mannucci et al. 2005; Sullivan et al. 2006). The evidence, however, is not conclusive yet, and some observations at high redshift do not fit this picture; for example, there are indications for a drop in the SN rate at \( z > 1.5 \), where star formation rate (SFR) is still rising (Poznanski et al. 2007; Dahlen et al. 2008), which does not support the presence of a large prompt component.

The present paper studies the luminosity functions (LFs) of SNe Ia and their host galaxies and correlations in properties of SN Ia and of host galaxies. Miller & Branch (1990) and Richardson et al. (2002) studied the LF of various types of SNe based on the Asiago Supernova Catalog (Barbon et al. 1989). In particular, Richardson et al. (2002) showed that the LF of SNe Ia is consistent with the Gaussian distribution with \( (M_R) = -19.46 + 5 \log(H_0/60) \) and \( \sigma = 0.56 \) using 111 spectroscopically normal SNe Ia without correcting for the light-curve shape parameter. With the correction for the decline rate \( \sigma \) can be reduced to 0.11 (Phillips et al. 1999). We are not aware of a study of the LFs of SNe Ia host galaxies. We may hope that a comparison with LFs of field galaxies may provide a hint as to what type of galaxies would preferentially host SNe Ia.

We have accumulated a sample of SNe Ia acquired in the second phase of the Sloan Digital Sky Survey (SDSS; York et al. 2000). Supernovae were searched during September to November of 2005–2007 by repeated imaging for the sky area of 300 deg\(^2\) of the southern equatorial region, \(-60^\circ < R.A. < +60^\circ, -1^\circ 25 < \text{decl.} < +1^\circ 25\), every two days (Frieman et al. 2008; Sako et al. 2008; Dilday et al. 2008). Approximately 500 SNe Ia at 0.05 \( < z < 0.4 \) were identified using well-defined selection criteria and their light curves were measured (Sako et al. 2008; Holtzman et al. 2008). The advantage of the use of the SDSS is, apart from its accurate five color photometry (Fukugita et al. 1996; Smith et al. 2002), that most of the galaxies that host SNe Ia have already been photometrically observed with homogeneous preset criteria, so that one can study the properties of those galaxies and correlations between the SNe and their host galaxies. The disadvantage is that the SDSS is somewhat too shallow for this purpose due to a limitation arising from the time-delay-and-integrate mode imaging with a 2.5 m aperture telescope (Gunn et al. 1998, 2006).

The SN candidates are spectroscopically followed up with other telescopes as much as the time allows them to determine their types and redshifts (Zheng et al. 2008). After the SN light faded away, spectroscopic observations are carried out for its host galaxy to determine the redshift of the SN Ia candidate. In this paper, we use the data from the first year for which the spectroscopic observations of host galaxies are more advanced. We limit the sample effectively to \( z \lesssim 0.3 \) for the sample completeness. Five color photometry had been made for host galaxies. A 0.1 \( L^* \) galaxy at \( z \approx 0.3 \) would give \( g ∼ 22.7, r ∼ 21.9, i ∼ 21.4, z ∼ 21.1 \), so that we can sustain reasonable accuracy in SDSS photometry for the four passbands (Hogg et al. 2001; Ivezić et al. 2004). For the \( u \) passband, however, a
If we apply our light-curve criteria to all spectroscopically confirmed SNe Ia, the failure rate to confirm SNe Ia is only 10% of the light-curve selected sample tells us that spectroscopic failure rate to confirm SNe Ia is less than 2%. We also confirmed that the statistical quantities we calculated by SALT2 is less than 3 for probable and host-z SNe. For spectroscopically confirmed SNe Ia, the $\chi^2$ criteria were not applied. Here $T_{\text{rest}}$ is the rest-frame time from the epoch of peak brightness in the rest-frame $B$ passband. Two hundred and twenty-two SNe Ia among 314 passed these light-curve criteria. The known peculiar SNe Ia, SN2005gj (Aldering et al. 2006; Prieto et al. 2007) and SN2005hk (Phillips et al. 2007) are rejected here. We limit the sky area to $-50.0 < \alpha < +55.0$ and $-1.25 < \delta < +1.25$ and the time coverage to 53626 (2005 September 13) < $T_{\text{max}}$ (MJD) < 53691 (2005 November 11) to avoid the edge effects. This leaves 207 SNe Ia, the distribution of which is shown in Figure 1 as the dotted histogram. We take this as the basic SN Ia sample. These selections, up to item (5), are readily implemented in the analysis procedure. The effective area of the survey is 262.5 deg².

To make the sample incompleteness well defined for distant faint SNe and the sample suitable to an application of the $1/V_{\text{max}}$ method to compute the LF (Schmidt 1968), we make the sample magnitude limited. Figure 2 shows apparent maximum brightness in the $r$ passband as a function of redshift, which suggests that we may set the limiting magnitude conservatively to be $r_{\text{lim}} = 21.5$ mag. The figure shows that this ensures reasonable completeness of SNe Ia at $z = 0.20-0.25$. After applying this limiting magnitude, 137 SNe Ia are left with us for a magnitude-limited sample, with the mean redshift ($z$) = 0.207, which we take as our final sample for our analysis. Among magnitude-limited samples, 72 are spectroscopically confirmed, 5 are spectroscopically probable, and 60 are host-z SNe. Summary of the sample selection is shown in Table 1.

The solid histogram in Figure 1 represents the redshift distribution of our samples. The dotted histogram represents the basic sample comprising 207 SNe Ia. The solid histogram is the magnitude-limited sample; our final sample, comprising 137 SNe Ia with $r_{\text{max}} < 21.5$ mag is used to derive the LF. The shaded region shows the expected distribution when SNe Ia are distributed as $\propto (1+z)^{-2}$ with the selection criteria we adopted taken into account. Shading stands for 20% uncertainty in the normalization. The dashed curve is the expected distribution when SNe Ia rate shows no evolution in $z$.

### Figure 1. Redshift distribution of our samples. The dotted histogram represents the basic sample comprising 207 SNe Ia. The solid histogram is the magnitude-limited sample; our final sample, comprising 137 SNe Ia with $r_{\text{max}} < 21.5$ mag is used to derive the LF. The shaded region shows the expected distribution when SNe Ia are distributed as $\propto (1+z)^{-2}$ with the selection criteria we adopted taken into account. Shading stands for 20% uncertainty in the normalization. The dashed curve is the expected distribution when SNe Ia rate shows no evolution in $z$.
Figure 2. Apparent maximum brightness of 207 SNe Ia in the $r$ band as a function of redshift. The horizontal dotted line denotes the magnitude limit of $r_{\text{lim}} = 21.5$ mag. Brightness is not corrected for extinction or color excess.

Table 1
Sample Selection Summary

| Criteria                        | Number |
|--------------------------------|--------|
| Total                          | 314    |
| Spectroscopic/confirmed        | 130    |
| Spectroscopic/probable         | 16     |
| Light-curve selection          | 168    |
| Secure light curve             | 222    |
| Good sky area and date         | 207    |
| $r_{\text{lim}} < 21.5$        | 137    |
| No host galaxies               | 8      |

SuperNova ANAlysis software (SNANA;\textsuperscript{5} Kessler et al. 2009b) assuming the SN Ia rate, $r_{\text{SNe Ia}} \propto (1+z)^{1.5\pm0.6}$, which is inferred from the observed SDSS SNe Ia rate at low redshift (Dilday et al. 2008). The conditions of the SDSS observations, the software search efficiency, and the light-curve criteria are taken into account, and the normalization is determined from the number of SNe below $z = 0.15$, for which redshift range SDSS SN observation is complete. The detail of simulation is described in the Appendix. The shaded area stands for $\pm 20\%$ uncertainties of the normalization, corresponding to the Poisson error of the number of SNe Ia below $z = 0.15$. The SN frequency expected for no evolution of SN rate is shown by the dashed curve, which lies within the error indicated by the shaded region. This figure shows that our sample is likely complete to $z < 0.20$.

The sample incompleteness at higher redshift is caused mainly by spectroscopic targeting and is given by fitting the ratio of observed number of SNe in our basic SN Ia sample to the simulated number as seen in Figure 3:

$$
\epsilon(z) = \begin{cases} 
1.0, & z \leq z_c \\
1.0 - (z - z_c)/\Delta z, & z_c < z < z_c + \Delta z \\
0.0, & z \geq z_c + \Delta z 
\end{cases}
$$

(1)

where the best-fit values are $z_c = 0.162 \pm 0.029$ and $\Delta z = 0.279 \pm 0.041$.

\textsuperscript{5} http://sdssdp47.fnal.gov/sdsssn/SNANA-PUBLIC/

Figure 3. Solid line shows the ratio of the number of SNe Ia in our basic sample to the simulation as a function of redshift. The dotted line shows the completeness function Equation (1) used in this paper.

Table 2
Hostless SNe Ia

| SDSS ID | IAU Name | Redshift | Luminosity Limit ($L_\star$) | Distance of $10''$ (kpc) |
|---------|----------|----------|-----------------------------|--------------------------|
| 2943    | 2005go   | 0.2659   | 0.055                       | 40.7                     |
| 5994    | 2003ht   | 0.1885   | 0.025                       | 31.2                     |
| 6780    | 2005iz   | 0.2046   | 0.029                       | 33.1                     |
| 6924    | 2005jm   | 0.3286   | 0.089                       | 47.2                     |
| 6933    | 2005jc   | 0.2137   | 0.033                       | 34.5                     |
| 7475    | 2005jw   | 0.3188   | 0.085                       | 46.7                     |
| 7335    | 2005kn   | 0.1975   | 0.028                       | 32.6                     |
| 3565    | 2005g    | 0.2885   | 0.067                       | 43.4                     |
| 8030\textsuperscript{a} | 2005jv  | 0.4226   | 0.161                       | 55.5                     |

Note. \textsuperscript{a} Outside the magnitude-limited sample.

For each SN Ia, the nearest primary object that resides within 10 arcsec from the SN is identified as its host galaxy in Catalog Archive Server of SDSS Data Release 6 (Adelman-McCarthy et al. 2008; see also Stoughton et al. 2002), with the identification visually confirmed. When misidentification is suspected, it arises mostly from deblended galaxies. Figure 4 shows the apparent $r$-band Petrosian magnitude distribution of host galaxies for 207 SNe Ia before the magnitude cutoff with the dotted histogram. The solid histogram shows the host galaxies for our magnitude-limit sample. There are 9 SNe whose host galaxies are not identified in the basic sample of 207 SNe Ia in the 137 magnitude-limit samples) within $\sim 10$ arcsec from a SN. These hostless SNe Ia are listed in Table 2, which gives the upper limit on absolute brightness of host galaxies searched in our survey to be $0.025-0.09 \ L_\star$ for the apparent magnitude limit of $r = 22.2$ mag. The physical distance corresponding to $10''$ is also given in the table. Petrosian magnitudes of $u$, $g$, $r$, $i$, and $z$ passband are used to compute rest-frame absolute brightness of host galaxies in five passbands using kcorrect v4_1_4 (Blanton & Roweis 2007). Redshifts are fixed to the spectroscopic values.
where $\omega$ is the solid angle $105 \times 2.5 = 262.5\text{sr}$, and the index $i$ refers to SNe Ia. Since $V_{\text{max}}^i$ is regarded as the volume surveyed to find $i$th SNe, the LF is obtained by summing the inverse of $V_{\text{max}}^i$ within specified magnitude bins ($\Delta M$) assuming that the LF is not evolving over the respective redshift range. Taking into account the effect of visibility time and spectroscopic incompleteness, the LF is calculated as

$$\phi(M)\Delta M (\text{Mpc}^{-3}) = \sum_{i \in [M - \Delta M/2]} \frac{\tau}{V_{\text{max}}^i} \times T_{\text{vis}}^i \times \epsilon(z),$$

where $T_{\text{vis}}^i$ is the time of visibility in the rest frame at which each SN Ia is observed ($\tau$ in the numerator is taken to be one year if $T_{\text{vis}}$ is measured in units of yr, so that the LF is represented in units of number per Mpc$^3$). If the SN would be observed just at one epoch, as was done in a number of observations to derive the SN rate, the visibility time will be a time span over which each SN can be detected above the detection limit: in this case, a fainter SN would have a shorter visibility time. In our case, however, observations have been made for the same field of sky continuously with the magnitude-limit set for peak brightness. The visibility time will then be a time span of the survey observation. From the criteria on the date of maximum brightness, the visibility time in the observed frame is 65 days, and hence $T_{\text{vis}} = 65/(1 + z')$ days in the rest frame. For SNe Ia, $M^*$ is the absolute peak $B$-passband magnitude of SNe Ia, whose apparent rest-frame magnitude is estimated from SALT2. For host galaxies it is the absolute magnitude in the rest frame estimated using $k_{\text{correct}}$. The factor $\epsilon(z)$ is the completeness correction, Equation (1). We describe in the Appendix simulations we made to show that sample incompleteness and our corrections do not induce particular systematic errors to our analysis, and our procedures allow us to recover the true LF, SN rate, and related quantities.

4. LUMINOSITY FUNCTION OF SNe Ia

To estimate the intrinsic brightness of SNe Ia, we must correct for dust extinction within host galaxies. In fact, SNe Ia show the variation in color that could be attributed to dust extinction within host galaxies and/or interpreted as an intrinsic color variation. The color information is obtained from the color excess parameter $c$ of SALT2, which is defined by $c = (B - V)_{\text{max}} - \langle(B - V)_{\text{max}}\rangle$ at $B$-passband maximum brightness, where the second term is color of the SN Ia spectral templet. Figure 5 shows the distribution of this color excess $c = E(B - V)$. The distribution is asymmetric with respect to $E(B - V) = 0$, similar to that of the SuperNova Legacy Survey (SNLS) sample (Astier et al. 2006). If we assume the color distribution as an exponential distribution $\propto \exp(-E(B - V) / \Delta)$ smeared by intrinsic Gaussian color distribution with the dispersion of $\sigma$, the observed distribution can be fitted with $\Delta = 0.048$ and $\sigma = 0.074$ as in Figure 5. Including the data highly deviated from the fitted distribution, the mean and dispersion are $(c) = 0.176 \pm 0.280$. These values are compared with those of nearby SN Ia given by Jha et al. (2007), $\Delta = 0.138$, or $\langle E(B - V) \rangle = 0.128 \pm 0.173$ in the Gaussian fit which, however, are evaluated using late-time (35 days after the maximum) $B - V$ color distribution.

If the data points which are largely deviated by $\Delta E(B - V) \geq 0.5$, would be removed, the mean and dispersion will become $(c) = 0.061 \pm 0.107$. We note that the removal of objects with $c > 0.5$ affects $\Delta$ and $\sigma$ only by small amounts. One may
convolved with Gaussian (see the text). dimmed by extinction, unless deep enough to reach brightness significantly fainter than are the magnitude-limited sample. The curve is the function \( p(E(B-V)) \propto \exp(-E(B-V)/\Delta) \) for \( E(B-V) > 0 \) and \( p(E(B-V)) = 0 \) for \( E(B-V) < 0 \) convolved with Gaussian (see the text).

Figure 5. Distribution of color excess \( E(B-V) = c \) calculated from the magnitude-limited sample. The curve is the function \( p(E(B-V)) \propto \exp(-E(B-V)/\Delta) \) for \( E(B-V) > 0 \) and \( p(E(B-V)) = 0 \) for \( E(B-V) < 0 \) convolved with Gaussian (see the text).

Figure 6. Distribution of \( V \)-band extinction \( A_V \) for SNe Ia, estimated assuming that their color excess is due to dust reddening and \( R_V = 3.1 \) (solid histogram). The dotted histogram is \( A_V \) of field galaxies inferred from the Balmer emission line ratio \( H\alpha/H\beta \) (Nakamura et al. 2004) for comparison.

Our simulation, however, shows that the value of \( RV \) cannot be the reason for \( \Delta \), which leads to the narrower Gaussian width with \( \beta = RV + 1 = 4.1 \). We note, however, that this ratio of total to selective extinction is not well justified. There are some indications that \( RV \) for SNe Ia is lower than the Galactic value; Altavilla et al. (2004) used the bluest SNe Ia to estimate intrinsic color of SN Ia, and Reindl et al. (2005) used SNe Ia in early-type galaxies and far outlying SNe Ia in spiral galaxies. Both the authors obtained \( RV = 2.5 \). Nobili & Goobar (2008) also found a low value of \( RV = 1.75 \) simultaneously deriving templets of SNe color evolution in time and extinction. Astier et al. (2006) resulted in \( RV = 0.57 \), and Kessler et al. (2009a) gave \( RV = 2.18 \). All \( RV \) thus derived are smaller than the canonical value for the Milky Way.

Figure 7 shows the LF of SNe Ia in the \( B \) passband, which is fitted with the Gaussian distribution with the mean and dispersion given in the upper left corner of each panel. Panel (a) assumes that the color variation \( c \) arises from extinction with \( \beta = 4.1 \), giving the mean \( M_B^0 = -19.42 \) (in the Johnson zero point) and the dispersion 0.24 mag. If we would adopt a smaller value for \( RV \), the LF of SNe Ia becomes closer to a more regular Gaussian distribution with a smaller dispersion, as shown in Figure 7(b). The minimization of dispersion with respect to \( \beta \) results in \( \beta = 2.93 \), which leads to the narrower Gaussian width of 0.16 mag with \( M_B^0 = -19.32 \), which is 0.10 mag fainter than with \( \beta = 4.1 \). Thus the smaller \( RV \) gives more homogeneous SNe Ia luminosity.

This small value of \( RV \) obtained by minimizing the width of Gaussian fitted to SNe Ia LF is an alternative manifestation of the low value of \( \beta \) obtained by Astier et al. (2006) by minimizing the residual scatter in the Hubble diagram along with cosmological parameters. Kowalski et al. (2008) and Kessler et al. (2009a) indicated that minimizing the scatter in the Hubble diagram tends to give \( RV \) biased toward a value lower than the true value. Our simulation, however, shows that the value of \( \beta \) may be biased to the lower value but no more than by \( \sim 0.1 \), so that this cannot be the reason for \( RV \) being significantly smaller than 3.1. We do not conclude here that \( RV \) is actually smaller but take
\( \beta = 4.1 \) as our fiducial choice, keeping in mind the uncertainty from \( R_V \) in the analysis in what follows.

It has been argued that the maximum luminosity correlates with the light-curve shape, or more specifically the decline rate (\( \Delta m_{15} \), stretch, or SALT2’s \( x_1 \)), and the inclusion of the correlation with it makes the behavior of the LF tighter (Pskovskii 1984; Phillips 1993; Hamuy et al. 1996). We show in panels (c) and (d) of Figure 7 the LF where brightness is corrected by \( \alpha x_1 \), with \( \alpha \) chosen to minimize the width of the Gaussian. In panel (c), \( \beta \) is fixed to our fiducial value of 4.1 and the minimization gives \( \alpha = 0.052 \). In panel (d), both \( \alpha \) and \( \beta \) are chosen to minimize the width, which results in \( \beta = 2.52 \) and \( \alpha = 0.123 \). The correlation of \( x_1 \) with brightness makes the Gaussian distribution narrower, especially for the case in which both \( \alpha \) and \( \beta \) are optimized. The narrowest Gaussian is obtained with \( \alpha = 0.123 \) which corresponds to \( \alpha' = 0.76 \) where the correction for the light-curve shape to brightness is expressed as \( \alpha' \Delta m_{15} \). This value is consistent with \( 0.78 \pm 0.18 \) obtained by Hamuy et al. (1996).

Figure 8 shows the correlation between brightness of SNe with the shape parameter and the color excess. This correlation is the reason why the width of the LF decreases upon inclusions of the light-curve shape parameter and the color excess parameter (with a small \( R_V \)). The correlation in the upper panel is represented by \( M_B = -19.34 + 2.93 c \) as shown in Figure 7(b), and that in the lower panel by \( M_B - 4.1 c = -19.43 - 0.052 x_1 \) as in Figure 7(c).

5. HOST GALAXIES

The LFs of SNe Ia host galaxies, as calculated by the \( 1/V_{\text{max}} \) method, are shown in Figures 9–11 with solid histograms for the \( r \), \( g \), and \( i \) passbands. Hostless SNe are indicated by the shaded histogram at the rightmost bin. We draw with solid curves the LF of general field galaxies obtained by Blanton et al. (2001), multiplied with the luminosity. The curves show good match of the LF of SN host galaxies with that of the field galaxies, which means that the LF of galaxies derived from SNe Ia faithfully represents that of galaxies in the field. We do not see any particular deviations between the two for three color passbands \( g \), \( r \), and \( i \) we studied, meaning that the occurrence of SNe Ia is primarily proportional to the luminosity of galaxies. Matching the two LF’s gives the SNe Ia rate in the conventional SN unit (SNeu), the SN rate per \( 10^{10} \) solar luminosity per century \( r_L \). Taking \( M_r(\odot) = 4.62 \), we obtain

\[
r_L = 0.227 \pm 0.027 \text{SNeu(r),}
\]

where we use the luminosity normalization of Blanton et al. (2003) shifted to \( z = 0.2 \) using their evolution prescription with the \( Q \) parameter that represents the evolution of luminosity per redshift interval.\(^8\) This matching was done using data points at

\(^8\) The normalization of the LFs of Blanton et al. (2003) differs significantly from Blanton et al. (2001) apart from the use of different passbands defined at \( z = 0.1 \); the luminosity density of the latter is 20%–40% larger than the...
Figure 8. Correlations between the brightness of SNe and the color excess parameter \( c \) (the top panel) and the \( x_1 \) parameter (the bottom panel).

Figure 9. Luminosity functions of SNe Ia host galaxies in the \( r \) passband. The lower panel shows the number of contributing galaxies in each bin. The solid curve is the luminosity-weighted LFs of field galaxies (Blanton et al. 2001) normalized to fit the histogram. The scale factor that gives the SN rate in SNe is given at the top right of the figure. Dotted lines show the range corresponding to the fitting error. Contribution from hostless SNe is indicated in the rightmost bin with shades.

Figure 10. Same as Figure 9 but in the \( g \) passband.

Figure 12 shows that the SN rate per galaxy is proportional to luminosity of host galaxies, as one expects from the comparisons of the LFs. The slope of the solid line is fixed to the rate at 0.227 SNe\((r)\) of Figure 9. The data point at the bottom left indicated with the open circle is for the “hostless SNe” where the horizontal error bars indicate only the upper limit on host galaxy’s luminosity. We conclude that our hostless SNe Ia are consistent with them being occurred in low-luminosity galaxies beyond our detection limit but at the rate proportional to luminosity of host galaxies.

magnitude bins where the number of contributing galaxies is greater than 10, and the error is Poisson from the SN number used in the \( 1/V_{\text{max}} \) analysis. The rate given above can be treated as a mean over the redshift range of our sample, the mean redshift \( z = 0.20 \). We may convert this rate expressed in SNu to the volumetric rate by multiplying the luminosity density. With reference to the luminosity density of Blanton et al. (2003), we find the volumetric rate of SNe Ia,

\[
r_V = (3.63 \pm 0.43) \times 10^{-5} \text{ Mpc}^{-3} \text{ yr}^{-1}. \tag{5}\]

For \( g \) and \( i \) passbands, the matching of the two LF’s yields

\[
r_L = 0.278 \pm 0.036 \text{ SNe} (g) \quad \text{and} \quad r_L = 0.226 \pm 0.034 \text{ SNe} (i)
\]

taking \( M_g(\odot) = 5.07 \) and \( M_i(\odot) = 4.52 \) mag, respectively.

These SN rates yield the volumetric rate \( r_L = 0.278 \pm 0.036 \text{ SNu} (g) \) and \( r_L = 0.226 \pm 0.034 \text{ SNu} (i) \) taking \( M_g(\odot) = 5.07 \) and \( M_i(\odot) = 4.52 \) mag, respectively.

These SN rates yield the volumetric rate

\[
r_V = (3.07 \pm 0.41 \pm 0.34) \times 10^{-5} \text{ Mpc}^{-3} \text{ yr}^{-1} \quad \text{obtained directly by summing up the LF of SNe Ia in Figure 7(a).}
\]

Here the second error represents that arising from the completeness function given in Equation (1), which depends on the SN rate normalization. If we take the \( B \)-band luminosity density \( L_B = 1.44 \times 10^8 L_\odot/\text{Mpc}^3 \) at \( z = 0.2 \), obtained by interpolating across five colors, we obtain the SN rate per \( B \)-band luminosity:

\[
r_L = 0.257 \pm 0.028 \pm 0.024 \text{ SNu} (B). \tag{6}\]

This is compared with earlier measurements at low redshift compiled in Table 3.

former. The difference in the shape of the LF, however, is modest and we adopt Schechter function parameters of Blanton et al. (2001) for the LF in the \( g, r, i \) passbands at \( z = 0 \) by shifting only the normalization. For the \( g, r, i \) passbands, we interpolated across the five colors at \( z = 0 \). The luminosity densities at \( z = 0.2 \) for the \( g, r, i \) passbands adopted are

\[
\begin{align*}
\mathcal{L}_g &= 1.39 \times 10^8 L_\odot/\text{Mpc}^3, \\
\mathcal{L}_r &= 1.60 \times 10^8 L_\odot/\text{Mpc}^3, \\
\mathcal{L}_i &= 1.87 \times 10^8 L_\odot/\text{Mpc}^3.
\end{align*}
\]

\[
\text{The luminosity densities at } z = 0.2 \text{ for the } g, r, i \text{ passbands adopted are}
\]

\[
\mathcal{L}_g = 1.39 \times 10^8 L_\odot/\text{Mpc}^3, \quad \mathcal{L}_r = 1.60 \times 10^8 L_\odot/\text{Mpc}^3, \quad \mathcal{L}_i = 1.87 \times 10^8 L_\odot/\text{Mpc}^3.
\]
Figure 11. Same as Figure 9 but in the $i$ passband.

Table 3

| Reference                  | Redshift | Rate (SNu)     |
|----------------------------|----------|----------------|
| Cappellaro et al. (1999)   | 0        | $0.18 \pm 0.05$|
| Dilday et al. (2008)$^a$   | 0.09     | $0.246^{+0.076}_{-0.060}$|
| Madgwick et al. (2003)     | 0.098    | $0.196 \pm 0.098$|
| Blanc et al. (2004)        | 0.13     | $0.125^{+0.044+0.028}_{-0.034-0.028}$|
| Hardin et al. (2000)       | 0.14     | $0.22^{+0.17}_{-0.10-0.03}$|
| This work                  | 0.20     | $0.257 \pm 0.028 \pm 0.024$|
| Botticella et al. (2008)   | 0.3      | $0.22^{+0.10}_{-0.08-0.14}$|

Note. $^a$ $B$-band luminosity density of $j_B = 1.19 \times 10^8 L_\odot$ Mpc$^{-3}$ was used to convert from the value per comoving volume unit.

In Figure 13 we present the radial distance distribution of SNe Ia measured from the center of host galaxies measured in the $r$ passband in physical distance units, where we do not correct for the projection effect caused by inclination of host galaxies. The relative distance between SN and the center of host galaxy is measured within the accuracy of 0.1 arcsec, which corresponds to 0.4 kpc at $z = 0.3$ (Pier et al. 2003). The ordinate denotes numbers of the SN Ia per unit volume calculated by the formula similar to Equation (3). This radial distribution is well represented by the de Vaucouleurs profile with the half light radius of $r_e = 5.7$ kpc as drawn by the thin solid curve in Figure 13. We also draw the exponential profile (dotted curve) with the half light radius of $r_e = 3.6$ kpc, which falls off faster than the distribution of SNe Ia at an impact parameter larger than 10 kpc. The $\chi^2$ of the best-fit models, $\chi^2 = 8.2/11$ for the de Vaucouleurs profile and $\chi^2 = 11.7/11$ for the exponential profile, where fitting was done within 13 kpc using 13 data points, differ only a little but the difference is more apparent in the tail. When one uses the addition of the de Vaucouleurs and the exponential profiles, the best-fit model shows a bulge-to-disk luminosity ratio of 0.70, $r_e$(deV) = 9.1 kpc and $r_e$(exp) = 2.4 kpc with $\chi^2 = 5.0/9$. This fitted profile is drawn by a thin dashed curve.

Our result does not agree with that of Bartunov et al. (2007) who claimed that SNe Ia in spiral galaxies are in a lower rate in the central part compared to SNe Ia in elliptical galaxies based on their SN catalog obtained by a compilation of SNe in the literature. Their earlier paper (Bartunov et al. 1992) claims that the radial dependence of surface density of SNe Ia can be expressed by the exponential profile. Our result does not agree with their conclusion, either.

The thick solid curve in Figure 13 shows the empirical mean $r$-passband light profile of field galaxies, which is constructed from the aperture flux of galaxies at $z = 0.025–0.030$ in Catalog Archive Server of SDSS Data Release 6 using values of profMean. The innermost bin may be somewhat affected
by finite size seeing. The normalization is set to be the same as Figure 9 while taking into account the luminosity evolution of galaxies from $z = 0.025$–0.30 to $z = 0.2$ using the $Q$ parameter of Blanton et al. (2003) while fixing the shape of the light profile. The light profile of galaxies is consistent with the exponential disc plus de Vaucouleurs spheroid model with the bulge-to-disk luminosity ratio of 0.56. We note that the radial distribution of SNe Ia, at least for the bulk of SNe, is consistent with the light distribution ($\chi^2 = 11.2/13$), except for that at a large distance beyond 10 kpc where we see some excess occurrence of SNe. It is interesting to see that some SNe Ia occur at a large distance where the galaxy contributes little light: at $> 10$ kpc we expect from the global rate, $1.31 \times 10^{-5}$ Mpc$^{-3}$ yr$^{-1}$ SNe Ia with the light distribution of the de Vaucouleurs profile and $0.15 \times 10^{-5}$ Mpc$^{-3}$ yr$^{-1}$ SNe Ia with the exponential profile, which are compared with observed $0.55 \times 10^{-5}$ Mpc$^{-3}$ yr$^{-1}$ SNe Ia. The radial distribution of SNe Ia also supports the proposition that the occurrence of SNe Ia is primarily proportional to the luminosity of host galaxies.

We study the dependence of the SN rate on color of host galaxies to examine whether the occurrence of SNe Ia would correlate with the star formation activity. In Figure 14, the histogram represents the SN host galaxies and the solid curve shows the luminosity-weighted color function of field galaxies which is calculated using the bivariate function $\phi(M_g, g - r)$ (Blanton et al. 2001), as

$$\Phi_L(g - r)dg(g - r) = \int_{-24.25}^{-14.75} dM_g \phi(M_g, g - r) \times 10^{0.4(M_g-M_r)} dg(g - r),$$

(7)

where $\phi(M_g, g - r)$ is evaluated for $-24.25 < M_g < -14.75$ and $0.12 < g - r < 0.88$ on a $20 \times 20$ grid. The abscissa is rest-frame color after the $K$-correction. The normalization is taken to be the same as that in Figure 9. Morphological types of galaxies are indicated with the corresponding colors according to Fukugita et al. (2007) with the error bar representing the dispersion of colors in the morphologically classified sample. This figure shows that the color distribution of SNe Ia host galaxies traces well that of field galaxies, and we do not see any particular excess of SNe Ia host galaxies for bluer, late-type or irregular galaxies, beyond $1\sigma$ level. In this comparison, we do not take into account the evolution of galaxy color to $z = 0.2$.

In particular, we may be interested in the difference of the SN rate between the elliptical galaxies and other galaxies. If we select galaxies with color $0.73 < g - r < 0.81$ corresponding to elliptical galaxies (with contaminations from SO and some Sa galaxies) we obtain the SN rate $0.194 \pm 0.062$ SNeu($r$) which is compared with $0.255 \pm 0.035$ for galaxies with $g - r < 0.73$ (Sa galaxies or later). We observe a $31 \pm 35\%$ enhancement in the SN rate relative to galaxy luminosity in late-type galaxies compared to that in elliptical galaxies, though the effect is only at $1\sigma$. We do not see a particular enhancement of SN rate per luminosity, however, among late-type galaxies, where the SFR increases toward later type morphologies (Nakamura et al. 2004).

$u - g$ color is more sensitive to the star formation activity than $g - r$, but $u$-band brightness is too faint to us for most host galaxies: SN host galaxies brighter than 21 mag are less than 40% for our magnitude-limit SN sample, and we are not able to draw a meaningful conclusion from the $u - g$ color distribution. The SN host galaxies are also too faint for SDSS spectroscopy (Strauss et al. 2002), and we cannot estimate the SFR directly, unless we resort to population synthesis colors, which are not well constrained without $u$ color. Therefore, we do not find clear evidence that points to the correlation of the SN rate with the star formation activity.

Mannucci et al. (2005) and Sullivan et al. (2006) gave models of SN Ia rates with the two components of progenitors: one explosions of old binary stars with a large delay time from the formation and the other explosions with a short delay time after the system formed. Mannucci et al. (2005) gave SNe Ia rate that depends on the morphological type of host galaxies in units of SNeu($K$). We may use their rate to estimate the number of SNe Ia for each morphological type using morphological-type-dependent luminosity density of field galaxies, as given by Nakamura et al. (2003) with the aid of the conversion from the $r$ to the $K$ passband (Nagamine et al. 2006). This model gives $r_{V} = 3.0 \times 10^{-5}$ Mpc$^{-3}$ yr$^{-1}$, where 31% are the prompt component, which is consistent with the difference in SN Ia rate relative to galaxy luminosity between elliptical galaxies and spiral galaxies. The total rate also agrees with our rate in Equation (5). The detailed numbers are presented in Table 4.

Sullivan et al. (2006) modeled SN Ia rate as a function of stellar masses and mean SFRs of host galaxies. We use the morphology-dependent luminosity density of Nakamura et al. (2003) and the morphologically dependent SFR calculated from the SFR of the bulge and disk components given in Nagamine et al. (2006). The predicted SN Ia rate is $r_{V} = 2.6 \times 10^{-3}$ Mpc$^{-3}$ yr$^{-1}$, of which 13% are prompt: see also Table 4. The predicted numbers of SNe Ia are plotted in Figure 14 as dotted (Mannucci et al.’s model) and dashed (Sullivan et al.’s model) curves taking the color distribution of each morphological type as a Gaussian with mean and dispersion
| Morph. Type | $g - r$ | $j_r$ | $L_K/L_r$ | Mannucci et al. Model | Sullivan et al. Model |
|-------------|--------|-------|----------|-----------------------|-----------------------|
|             |        |       |          | $N_A$ (delayed) | $N_B$ (prompt) | $N_A + N_B$ |
|             |        |       |          | $(10^{-5} \text{ Mpc}^{-3} \text{ yr}^{-1})$ | $(10^{-5} \text{ Mpc}^{-3} \text{ yr}^{-1})$ | $(10^{-5} \text{ Mpc}^{-3} \text{ yr}^{-1})$ |
| E,S0        | 0.75   | 0.62  | 3.32     | 0.720                | 0.000                | 0.720 |
| S0/a-Sb     | 0.64   | 1.00  | 2.77     | 0.970                | 0.304                | 1.274 |
| Sbc-Sd      | 0.51   | 0.37  | 2.52     | 0.326                | 0.495                | 0.821 |
| Im          | 0.36   | 0.02  | 2.15     | 0.015                | 0.127                | 0.142 |
| Total       | 2.01   | 2.031 | 0.926    | 2.957                | 2.273                | 0.335 |
|             |        |       |          | 2.01                 | 2.031                | 0.926 |
The currently available two-component models are marginal to represent our data, but are consistent with the observed number of SNe Ia, as shown by the histogram in Figure 14 allowing for large uncertainties. The prompt component is rather minor and not very evident in the integrated rates for low $z$ SNe, as in our analysis. The model prediction for Im galaxies is low, which is due to the low luminosity density of Im galaxies in the SDSS morphologically classified sample. In Nakamura et al. (2003), Im galaxies contribute only $\sim 1\%$ of the total luminosity density, which may be due to the selection effect that disfavors Im galaxies. This contrasts to the indication from the color function of Blanton et al. (2001), which suggests that galaxies bluer than $(g - r) < 0.42$ contribute by $\sim 15\%$ of total luminosity. If the luminosity density of Im galaxies is this large, however, the number of SNe Ia in Im galaxies would be 10 times more and that would largely overshoot our observed numbers of SNe. To identify the prompt component and test the two-component hypothesis, we need a sample where the prompt component is dominant, or the sample with which both SFR and SN rate can more accurately be measured.

Our sample allows us to study if the properties of SNe Ia would depend on the property of host galaxies. It has occasionally been claimed that bright SNe Ia appear more often in late-type galaxies, while SNe Ia in early-type galaxies are subluminous (Filippenko 1989; della Valle & Panagia 1992; Reindl et al. 2005; Sullivan et al. 2006). This has been counted as one of the reasons that SN Ia calibration of nearby galaxy’s distance led to a smaller value of the Hubble constant in the past. Figure 15(a) plots the maximum brightness of SNe Ia in our sample as a function of $g - r$ color of host galaxies: mean brightness of SNe Ia does not change across colors of galaxies from E to Im. In particular, the plot does not indicate any evidence that SNe Ia in bluer galaxies are systematically brighter or those in red galaxies are fainter. Both mean and dispersion of luminosity are nearly constant with respect to galaxy color as seen in the figure. Figure 15(b) shows the plot of the $x_1$ parameter, which represents the decline rate of SNe Ia brightness, against $g - r$ color of host galaxies. The upper value of $x_1$ does not change with respect to color from E to Im galaxies, but the lower value shows the trend that it decreases from $-0.5$ (or $\Delta m_{15} \simeq 1.2$) for Im galaxies to $-3$ ($\Delta m_{15} \simeq 1.7$) for E and S0 galaxies. It is noted that large negative values of $x_1$ are seen only in early-type galaxies. This causes some dependence of $x_1$ on color of host galaxies as seen in the figure. The slope, however, is too small to cause any effect on brightness of SNe Ia through the correction of 0.12$x_1$.

Similarly, Figure 16 shows maximum brightness of SNe Ia on luminosity of host galaxies, showing that the former does not depend on luminosity of the host. Note that we expect that average metallicity of the host galaxy changes by 0.59 dex in this luminosity range (Tremonti et al. 2004). The $x_1$ parameter changes slightly as luminosity of host galaxies changes.

We also examine the dependence of the $c$ parameter and the $x_1$ parameter on the distance from the center of galaxies as shown in Figure 17. The color excess $c = E(B-V)$ stays nearly at $\approx 0.08$ with a dispersion of 0.07 and does not show a systematic change from 1 kpc to 15 kpc. In particular, we observe that the color excess does not decrease as we go farther away from the center of galaxies, where stars, and hence dust, are expected to decrease. The trend we see here does not change if we limit the SNe to $z < 0.2$ with which SNe Ia that would receive reasonably large extinction are included in the sample. This finding suggests to interpret that the observed reddening is mostly associated with individual SNe, either extinction from SN itself and/or circum supernova dust or intrinsic color variation of the SN, rather than interstellar dust in host galaxies. This would also give us an upper limit on the model as to the amount of dust extinction led to a smaller value of the Hubble constant in the past.
In the upper panel, the mean and dispersion in respective bins are also plotted. 

upon luminosity or color of host galaxies.

for galaxies. Luminosity of SNe Ia does not appear to depend identified. It is shown, however, that they are consistent with our sample contains eight SNe Ia, whose host galaxies were not are not able to differentiate SN rates among late-type galaxies. 

modest (10%–30% of the total rate), as in the current models. We 

the occurrence of SNe Ia follows star formation activity, except 

possible enhancement in the SN Ia rate is noted in late-

the occurrence of SNe Ia may likely be ascribed to intrinsic color variation or immediate neighborhood of SNe rather than extinction in host galaxies. The total column density of dust should not give extinction more than 0.2 mag in the V passband.

We thank the review group of the SDSS-II Supernova Survey Project (JoshFrieman, Bob Nichol, Saurabh Jha, Benjamin Dilday) and Don Schneider and Richard Kessler for reviewing the manuscript and providing useful comments that improved the manuscript. N.Y. acknowledges support by the JSPhisto- 

core-to-core program “International Research Network for Dark 

Energy” and the JSPhoUSA bilateral programme; M.F. is 
supported by a Grant-in-Aid of the Ministry of Education in Japan, and Ambrose Monell Foundation at Princeton.

Funding for the SDSS and SDSS-II has been provided by the Alfred P. Sloan Foundation, the Participating Institutions, the National Science Foundation, the U.S. Department of Energy, the National Aeronautics and Space Administration, the Japanese Monbukagakusho, the Max Planck Society, and the Higher Education Funding Council for England. The SDSS Web Site is http://www.sdss.org/.

The SDSS is managed by the Astrophysical Research Consortium for the Participating Institutions. The Participating Institutions are the American Museum of Natural History, Astrophysical Institute Potsdam, University of Basel, University of Cambridge, Case Western Reserve University, University of Chicago, Drexel University, Fermilab, the Institute for Advanced Study, the Japan Participation Group, Johns Hopkins University, the Joint Institute for Nuclear Astrophysics, the Kavli Institute for Particle Astrophysics and Cosmology, the Korean Scientist Group, the Chinese Academy of Sciences (LAMOST), Los Alamos National Laboratory, the Max-Planck-Institute for Astronomy (MPIA), the Max-Planck-Institute for Astrophysics (MPA), New Mexico State University, Ohio State University, University of Pittsburgh, University of Portsmouth, Princeton University, the United States Naval Observatory, and the University of Washington.

APPENDIX

VERIFICATION OF THE ANALYSIS METHOD

In order to examine the validity of our analysis, we have made simulations for our observation creating a set of light curves using the SALT2 model in combination with SNANA. 

The distribution of the $x_1$ parameter is assumed to be a Gaussian with the mean 0 and the dispersion 0.90. The distribution of $c$ is a Gaussian with the mean 0.063 and the dispersion 0.11. We assume the relation involving absolute magnitude $M$, $x_1$, and $c$, 

\[ M = M_0 - \alpha x_1 + \beta c \]  

with $\alpha = 0.10$, $\beta = 2.45$, and $M_0 = -19.375$ as defaults. These input parameters are selected not far from the reality. The intrinsic dispersion of 0.11 mag was applied for every passbands and

produced around SNe Ia. We observe only seven SNe Ia which show color excess more than $E(B-V) > 0.3$ mag among our 137 SN Ia samples. In addition, we do not observe a systematic dependence of the $x_1$ parameter on the distance from the center of galaxies; see Figure 17(b).

6. CONCLUSION

The SNe Ia sample acquired in the SDSS II, containing 137 low-redshift SNe from $z = 0.05$ to 0.3, indicates that the occurrence of SNe Ia is primarily proportional to luminosity of galaxies. The LF of SN Ia host galaxies matches very well with that of field galaxies multiplied by luminosity, and color of SN Ia host galaxies does not differ from that of field galaxies. Our low-redshift sample does not indicate an active signature that the occurrence of SNe Ia follows star formation activity, except that possible enhancement in the SN Ia rate is noted in late-type galaxies ($\lesssim 31 \pm 35\%$) compared with the rate in elliptical galaxies. Our low-redshift sample is compatible with the two-component model where the effect of the prompt component is modest (10%–30% of the total rate), as in the current models. We are not able to differentiate SN rates among late-type galaxies. Our sample contains eight SNe Ia, whose host galaxies were not identified. It is shown, however, that they are consistent with them occurred in low-luminous galaxies beyond the survey limit for galaxies. Luminosity of SNe Ia does not appear to depend upon luminosity or color of host galaxies.

The LF of SNe Ia is Gaussian with $M_B = -19.42$ and $\sigma = 0.24$ mag (FWHM is a factor 1.4 in luminosity), if the color variation is interpreted as reddening obeying the extinction law of the Milky Way with the standard value $R_V = 3.1$. This Gaussian distribution is further tightened and $\sigma = 0.14$ mag if the extinction to reddening ratio $R_V$ is reduced to $\approx 2$ and if the correlation is taken into account between maximum brightness and the decline rate. Reddening of SN Ia, inferred from the color variation, is 0.1 mag in $A_V$, which is lower approximately by a factor of 5 than that for star-forming field galaxies measured from Hα and Hβ emission.

The variation of colors of SN Ia is constant and does not depend on the distance from the center of galaxies unlike what is expected for the supernova that would happen in the galactic disc. The color variation of SNe Ia may likely be ascribed to intrinsic color variation or immediate neighborhood of SNe rather than extinction in host galaxies. The total column density of dust should not give extinction more than 0.2 mag in the V passband.

We also showed that the distribution of the SN Ia occurrence that extends to a few tens of kpc agrees with the general light distribution of galaxies that have bulges with the de Vaucouleurs type profiles. The variation of colors of SN Ia is constant and does not depend on the distance from the center of galaxies unlike what is expected for the supernova that would happen in the galactic disc. The color variation of SNe Ia may likely be ascribed to intrinsic color variation or immediate neighborhood of SNe rather than extinction in host galaxies. The total column density of dust should not give extinction more than 0.2 mag in the V passband.
every epochs. The epochs of observation are fixed to actual date, and the signal-to-noise ratio of each observing point was calculated by using real observational condition of our observation including seeing and sky brightness. The detection efficiency searching for variable objects is also included in our simulation, and the SN Ia rate of $r_{\text{SN Ia}} = 2.2 \times 10^{-5} (1+z)^{1.5} \text{Mpc}^{-3} \text{yr}^{-1}$ is assumed. The cut based on the quality of light curves as described in the text is also applied to the simulation, and the spectroscopic incompleteness of Equation (1) is applied with $z_c = 0.15$ and $\Delta z = 0.3$. The simulation generated 210 SNe light curves. We make 50 sets of simulated light curves and apply the method, the same as that applied to the observed data set. The incompleteness correction $\epsilon(z)$ in Equation (3) is calculated separately for each simulation set.

Figure 18 shows the comparison of the input and the output SN LFs after correcting for the effect of $c$ parameter. The solid curve represents the expected LF taking into account the error of the determination of $m_B$ and $c$ from light-curve fits. Histograms and error bars are the mean and dispersion in each magnitude bin calculated from the 50 simulated light-curve sets. A good agreement is seen between solid curves and histograms.

We also make a simulation incorporating the completeness as a function of apparent peak $r$-passband brightness of SNe like $\epsilon(m) = 1.0 - (r_{\text{max}} - 19.75)/3.15$ to see the effect when the completeness is not a function of redshift but of apparent peak magnitude. The completeness $\epsilon(c)$ in Equation (3) is estimated as a function of redshift. The result is shown in Figure 19. We see again a good agreement between the input and output SNe. This is due to the fact that the redshift and apparent peak magnitude are well correlated.

Figure 19. Same as Figure 18 but completeness is assumed to be a function of apparent peak $r$-band magnitude.

Figure 20. Same as Figure 18 but for the distribution of the $E(B-V) = c$ parameter (left) and the $s_1$ parameter (right).
Good agreements are seen between input and output SN LFs for both cases where incompleteness is a function of redshift or apparent magnitude, respectively. These numbers are compared with the input value of $3 \times 10^{-5}$ (Mpc$^{-3}$ yr$^{-1}$). The volumetric SN rate is properly recovered within the error.

We also examine the distribution of the $c$ and $x_1$ parameters of SALT2 light-curve fit in Figure 20. SNe with the high $c$ value or low $x_1$ value will be faint and such SNe may suffer from incompleteness. The analysis is similar to that of the LF. In Equation (3), we replace the absolute magnitude $M$ with the color parameter $c$ or the shape parameter $x_1$. We do not see any systematic effect for the recovered distribution.

We then modify the distributions of the $c$ and $x_1$ parameters and the relation between $M_0$, $a$, and $b$, so that the resulting distribution should be closer to the observed data set. Namely, based on an input distribution and the relation we simulate the SNe and calculate the completeness of the observed sample and estimate the “true” distribution and the relation from observed data as an output. These output data are used as an input for simulation in the next step and obtain the output distribution, and the relation again. We repeated this procedure until the input and the output did not change (one or two iterations are sufficient). The final values are $x_1 = 0 \pm 0.96$, $c = 0.039 \pm 0.093$, and $M = M_0 - 0.12x_1 + 2.52c$. With these final values, the redshift distribution of the true and the observed SNe distributions, and therefore the selection function, are obtained, as presented in Figure 21, which indicates that the estimated observed distribution reproduces that actually observed.

We conclude that the method we used in this paper works well even for the data with incompleteness after the proper account taken for incompleteness. We do not expect any specific biases in the analysis.

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