Analysis of oil droplet deformation model in oil-water dynamic hydrocyclone

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Abstract. The flow field in the rotating drum of dynamic oil-water hydrocyclone separator is a high turbulent shear flow field. Oil droplets are easily deformed, broken and even emulsified when the oil droplets enter the drum. Because oil droplets are affected by high turbulent shear flow field. If oil-water emulsification occurs, the separation performance of dynamic oil-water hydrocyclone will be dramatically reduced. Therefore, it is an important subject to study dynamic oil-water hydrocyclone separator and improve its separation performance to carry out oil drop deformation model research, which can reduce oil drop deformation and fragmentation and formulate measures to avoid oil-water emulsification.

1. Introduction
Dynamic oil-water hydrocyclone separator is a new type of oil-water separator developed on the basis of static oil-water hydrocyclone separator. Dynamic oil-water hydrocyclone separator has a wide range of advantages, such as wide processing range and strong separation capacity. It has been widely used in mining, petrochemical industry, environmental protection and other fields. The dynamic Liquid-liquid Hydrocyclone is based on the principle that the centrifugal force is different when the density of two or more phases is different in rotating motion. The centrifugal force field needed to realize separation is established by external force to realize the separation of two or more phases.

2. Dynamic oil-water hydrocyclone separation process
From the sketch of total dynamic liquid-liquid hydrocyclone as shown in figure 1, high-speed rotating motion of moving parts (drum, grille, etc.) is driven by external power (motor). The oil-water mixture is transported to the inlet of the dynamic hydrocyclone under the action of dynamic pressure (pump). Under certain pressure, the liquid flow at the inlet enters the central hole of the rotating grate along the axial inlet and flows into its runner, then flows through the guide cone at the end of the rotating grate. The liquid flow passing through the rotating grate is actually a process of diversion and pre-rotating acceleration of the liquid flow. The liquid flow rotates at high speed and then enters the swirl chamber in the rotary drum. The larger and stronger swirl velocity field is formed by the friction between the liquid flow and the inner wall of the rotary drum. Because of the density difference between the two phases, the light dispersed phase is compressed to the center of the cylinder under centrifugal force; the heavy continuous phase moves to the inner wall of the cylinder, and the liquid flow moves to the other end of the swirl chamber under the action of axial force along the same direction of the cylinder. At this time, the oil in the central part is collected by the overflow nozzle and discharged through the outlet, while the water around the cylinder is discharged through the outlet. Finally, the two-phase media can be separated stably.
3. Analysis of oil drop deformation model

During the operation of the dynamic oil-water hydrocyclone separator, the rotary drum rotates at a high speed driven by the motor, and the oil-water two-phase turbulent shear flow field is formed in the rotary drum. Serious oil-water emulsification may occur in the cylinder, which results in separation failure. This is mainly due to the deformation and fragmentation of oil droplets under the force of the cylinder. In order to reduce the decrease of separation efficiency caused by oil-water emulsification, the stress and deformation of a single oil drop are analyzed, and the critical conditions of oil drop deformation and fragmentation are discussed. Then it is applied to the turbulent shear flow field of a dynamic oil-water hydrocyclone separator with low or medium concentration. For the deformation of single dispersed phase oil droplets in shear flow field, affine deformation [1], shear deformation [2] and three-dimensional deformation [3] have been deeply studied. When the oil droplets undergo affine deformation, the oil droplets deform into ellipsoids with the same two short axes, while when the oil droplets undergo shear deformation, the oil droplets deform into ellipsoids with different two short axes, and the three-dimensional deformation model shows that the three main axes deform.

3.1. Affine deformation

In the turbulent shear flow field of a dynamic oil-water hydrocyclone separator, assuming that the dispersed phase oil droplets undergo affine deformation under viscous shear force, the oil droplets will deform along the OX axis under viscous shear force \( f \), the deformation is \( \delta x \).

According to the axisymmetry of Taylor's theoretical deformation, there will be deformation in the other two directions \( OY \) and \( OZ \), the deformation is \( -k\delta x \). \( k \) is the shrinkage coefficient of oil drop deformation, \( k \in (0,1) \). According to the incompressibility of fluid and the volume invariance of oil droplets. The following relations can be obtained:

\[
\frac{4}{3}pR^3 = \frac{4}{3}p(R + dx)(R - kdx)^2
\]

(1)

According to the solution of quadratic equation:

\[
k = \frac{R - R \sqrt{\frac{1}{R + dx}}} {dx} = \frac{1}{e} \left( 1 - \frac{1}{1 + e} \right) = \frac{1}{1 + e + \sqrt{1 + e}}
\]

(2)

Where, \( e \) is the elongation of oil droplet deformation, \( e = \delta x / R \). The expression of affine deformation of a single oil drop is as follows:
\[
\frac{x^2}{(R + \delta x)^2} + \frac{y^2}{(R - k \delta x)^2} + \frac{z^2}{(R - k \delta x)^2} = 1
\]

(3)

Where, \( k \) is the coefficient of deformation and shrinkage of oil droplets.

From equation (2), it can be seen that \( k \) decreases with the increase of \( \varepsilon \), the relationship curve was shown in figure 2. Through experiments Taylor found that small deformation of oil droplets occurs when \( \varepsilon < 0.2 \). If \( \delta x \ll R \), then \( \varepsilon \rightarrow 0 \), \( k = 0.5 \), If, \( \varepsilon = 10 \) then \( k = 0.07 \), At this time, the oil droplets will undergo large deformation, showing a slender filament.

![Figure 2. Shrinkage coefficient and stretch ratio of oil droplets deformation.](image)

### 3.2. Shear deformation

The rotary drum of the dynamic oil-water hydrocyclone separator rotates at high speed driven by an electric motor. Assuming that the force and deformation of oil droplets in the turbulent shear flow field inside the rotary drum are shear deformation, under the action of viscous shear force \( f \), the oil droplets have a deformation along the OX axis direction and the deformation is \( \delta x \).

According to the characteristics of shear deformation, the deformation along the direction of \( OY \) is \(-k\delta x\), where \( k \) is the coefficient of deformation and contraction of oil droplets, \( k \in (0, 1) \) and there is no deformation along the direction of \( OZ \). According to the incompressibility of fluid and the volume invariance of oil droplets, the following relations can be obtained:

\[
\frac{4}{3} p R^3 = \frac{4}{3} p (R + dx)(R - k dx)R
\]

(4)

Thus the following formula can be obtained.

\[
k = \frac{1}{1 + \varepsilon}
\]

(5)

Where, \( \varepsilon \) is the elongation of oil drop deformation, \( \varepsilon = \delta x / R \).

The expression of oil droplets after shear deformation is as follows:

\[
\frac{x^2}{(R + dx)^2} + \frac{y^2}{(R - k dx)^2} + \frac{z^2}{R^2} = 1
\]

(6)
3.3. 3D deformation

Assuming that the interfacial tension $\sigma$ is constant and the initial state of oil droplets is spherical, ellipsoidal deformation occurs in the turbulent shear flow field in the rotating drum of a dynamic oil-water hydrocyclone separator in a Newtonian oil-water mixture system, as shown in figure 3.

The relationship between point A, point B and point C after oil droplet deformation is as follows:

point A: $p - \tau = \sigma (a/b^2 + a/c^2)$;

point B: $p = \sigma (b/a^2 + b/c^2)$;

point C: $p + \tau = \sigma (c/a^2 + c/b^2)$.

![Figure 3. Three-dimensional deformation of single droplet.](image)

In order to further study the deformation of oil droplets under shear conditions, it is assumed that the deformation of oil droplets in $OX$, $OY$ and $OZ$ directions is $R + \delta x$, $R - k_1\delta x$ and $R - k_2\delta x$, respectively. Because of the incompressibility of fluid, and considering that the pressure at point B does not change, it can be obtained that:

$$R^3 = (R + \delta x)(R - k_1\delta x)(R - k_2\delta x)$$

$$2/R = (R - k_2\delta x)/(R - k_1\delta x)^2 + (R - k_1\delta x)/(R + \delta x)^2$$

Simultaneous equations (7) and (8), from which we can get:

$$k_1^3[-2\delta x^2(R + \delta x)^3] + k_2^2(6R \delta x^4 + 18R^2 \delta x^3 + 18R^3 \delta x^2 + 5R^4 \delta x)$$

$$+ k_1(-6R^2 \delta x^3 - 18R^3 \delta x^2 - 18R^4 \delta x - 5R^5) + R^3(2\delta x^2 + 5R \delta x + 4R^2) = 0$$

If $R \gg \delta x$, the deformation of oil droplets is small and can be approximately obtained:

$$k_1 = (9\varepsilon - 2\sqrt{1 + 4\varepsilon + 14\varepsilon^2} + 2)/5\varepsilon$$

where, $\varepsilon$ is the elongation of oil drop deformation, $\varepsilon = \delta x/R$, $k_1$ and $k_2$ are shrinkage coefficients of oil drop deformation, or shrinkage coefficients.

If the deformation of oil droplets is large, it can be calculated accurately:

$$k_1 = \frac{1}{6\varepsilon(\varepsilon + 1)} \left[ \sqrt[3]{-54(\varepsilon + 1)^8 - 1 + 3\sqrt{6}(\varepsilon + 1)^4 \sqrt{54(\varepsilon + 1)^8 + 2}} + 2 + \sqrt[3]{-54(\varepsilon + 1)^8 - 1 - 3\sqrt{6}(\varepsilon + 1)^4 \sqrt{54(\varepsilon + 1)^8 + 2}} + 6(\varepsilon + 1)^3 - 1 \right]^{-1}$$
From equation (7), it can be concluded that:

\[ k_2 = [k_1(\varepsilon + 1) - 1]/[(\varepsilon + 1)(k_1\varepsilon - 1)] \] (12)

Assuming that the initial state of oil droplets is spherical and elliptical deformation occurs, the relationship between the tensile rate of oil droplets and the shear stress can be calculated.

\[ \tau = (\sigma/R)[(\varepsilon + 1)/(1 - k_2\varepsilon)^2 + (\varepsilon + 1)/(1 - k_2\varepsilon)^2 - 2] \] (13)

After affine deformation of oil droplets, the expression of their shape is as follows:

\[ \frac{x^2}{(R + \delta x)^2} + \frac{y^2}{(R - k_2\delta x)^2} + \frac{z^2}{(R - k_2\delta x)^2} = 1 \] (14)

4. Model selection

In the oil drop deformation model established by Taylor [4,5] and Cox [6], it is assumed that the oil drop deforms into an ellipsoid with circular cross section, i.e. affine deformation, and the prediction formulas of deformation degree and orientation angle are given as follows.

\[ \frac{L - B}{L + B} = Ca \cdot \frac{19\lambda + 16}{2(16\lambda + 16)} \cdot \frac{1}{\sqrt{(19\lambda Ca/40)^2 + 1}} \] (15)

\[ \alpha = \frac{\pi}{4} + \frac{1}{2} \tan^{-1}19\lambda Ca/40 \]

Choi S J [7] predicts the maximum cross-sectional radius of a single slender oil drop, with longitudinal and semi-axial ratios of longitudinal to transverse, respectively:

\[ b_2 = 0.0578 \frac{\tau}{\sigma\eta_c} Ca^{3/4}, \quad a_2 = 3.45 \frac{\tau}{\sigma\eta_c} Ca^{2/3}, \quad p = \frac{a_2}{b_2} \geq 6Ca^{3/4} \] (16)

It can be obtained that \( b_2^2a_2 = 1.15r^3 \). Therefore, the predicted oil droplets are affine deformation, which coincides with the theoretical geometric deformation.

5. Conclusion

In this study, the deformation, fracture and coalescence of oil droplets in the rotating drum of a dynamic oil-water hydrocyclone separator are analyzed. The deformation of oil droplets in the shear flow field is assumed to be affine. For a certain concentration of oil-water mixture, the dispersed phase of oil droplets may exhibit deformation, fracture and aggregation in the flow field, but the deformation model should be consistent with that of a single oil droplet.

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