Abstract

**Background/Objectives:** An Algorithm, the Particle filter, is proposed for implementing the bearings only Torpedo Motion Analysis (TMA). The required density of the state vector is represented as a set of random samples, which are updated and propagated by the algorithm. The method is not restricted by assumptions of linearity or Gaussian noise.

**Methods/Statistical analysis:** The particle filter is combined with Modified Gain Bearings Only Extended Kalman Filter and the results are compared with that of Extended Kalman Filter or Unscented Kalman Filters. **Findings:** Almost similar performance is obtained. The algorithm is applied to track a torpedo using measurements available from towed array.

**Application/Improvements:** The results in simulation mode and with sea trial data are presented.

** Keywords:** Algorithm, Estimation, Gaussianity, Kalman Filter, Linearity, Simulation, Towed Array

1. **Introduction**

Target Motion Analysis (TMA) refers to the process of estimating the dynamics of target from the noisy corrupt Line of Sight (LOS) angle or bearing measurements. As range measurement is not available and the bearing measurement is not linearly related to the target states, the whole process becomes nonlinear. Added to this, since bearing measurements are extracted from single passive sonar, the process remains unobservable until observer executes a proper manoeuvre due to non-linearity nature of the estimation problem.

Non-linear filtering estimates and tracks the state of a nonlinear stochastic system from non-Gaussian noisy bearing measurements. The Extended Kalman Filter (EKF) has been the standard technique usually applied and this requires the computation of the Jacobian matrix of the state. But, for severe nonlinearities, the EKF can be very unstable and performs poorly. Recently, several new approaches to recursive nonlinear filtering have appeared in the literature. These include grid based methods, Unscented Kalman Filter (UKF), Particle Filter methods. The Particle Filter is the suitable candidate for this application.

The authors have developed modified gain bearings-only particle filter for passive bearings only target tracking using towed array measurements. Particle filter is the point mass (particle) representation of the probability density function. Particle filter provides an approximate solution to the discrete Bayesian estimation problem by recursively updating the posterior filtering density. Approximation of the density by a large set of N samples (particles) where each particle has an assigned relative weight, chosen so that all weights sum to unity. The location and weight of each particle reflect the value of the density in that region of the state space. The particle filter updates the particle location and the corresponding weights recursively with each new observed bearing measurement.

The Particle filter algorithm is developed on simulation platform. This algorithm is also tried out against sea trial data and the results are encouraging. Section 2 describes mathematical modeling of measurements, formulation of PF and initialization of state vector and its covariance. Section 3 is about simulation and results and the paper is concluded in Section 4.

2. **Mathematical Modelling**

2.1 **State and Measurement Equations**

The alternative derivation of the modified gain function of
Song and Speyer’s extended Kalman filter is slightly modified and the corresponding mathematical modeling and filter equations can be discussed in rich literature^{2,3,4}.

2.2 Particle Filter

The particle filter is a statistical, brute-force approach and referred by many other names, including sequential importance sampling, bootstrap filtering, the condensation algorithm, interacting particle approximations, Monte Carlo filtering and Sequential Monte Carlo (SMC) filtering.

The particle filter is a completely nonlinear state estimator with greater computational effort. However these days’ processors with high operating frequency and with high computation in less time are available. So, hardware is not a problem to adapt Particle filter and hence particle filter can be used for tracking targets safely.

The particle filter is derived from the Bayesian estimator. Initially, \( N \) state vectors based on the initial known pdf \( P (X_0 (0)) \) are randomly generated. These state vectors are called particles and are denoted as \( X_{s} (k / k) \) \((k = 1, 2, ...., N)\). At each time step we propagate the particles to the next time step using the process equation \( f(\cdot) \):

\[
X_{s} (k + 1/k) = f(X_{s} (k - 1/k), w (k + 1)) \quad (k = 1, 2,....,N) \quad (1)
\]

Where, each \( w (k + 1) \) noise vector. That is, we evaluate the pdf \( P (Z (k) | X_{s} (k + 1/k)) \). This can be done if we know the nonlinear measurement equation and the pdf of the measurement noise. For example, if an \( m \)-dimensional measurement equation is given as \( Z (k) = h (X_{s} (k) + v (k)) \) and \( v (k) \sim N(0, R) \) then a relative likelihood \( q (k) \), that the measurement is equal to a specific measurement \( Z \) given the premise that \( X_{s} (k) \) is equal to the particle \( X_{s} (k + 1/k) \) can be computed as follows.

\[
q (k) = P [Z (k) = Z' | X_{s} (k) = X_{s} (k + 1/k)] = P [v (k) = Z' - h (X_{s} (k + 1/k))]
\]

\[
= \frac{1}{\sqrt{(2\pi)^m | P |}} \exp \left( -\frac{1}{2} \| Z' - h(X_{s} (k + 1/k)) \| ^2 \right) \quad (2)
\]

If this equation is used for all the particles, \( X_{s} (k + 1/k) \) \((i = 1, 2,....,N)\), then the relative likelihoods that the state is equal to each particle will be correct. Now we normalize the relative likelihoods as follows.

\[
q_{k} = \frac{q (k)}{\sum q (k)} \quad (3)
\]

2.3 Particle Filtering Combined with Other Filters

In this approach, each particle is updated at the measurement time using the EKF, UKF or MGBEKF, and then resampling (if required) is performed using the measurement. This is like running a bank of \( N \) Kalman filters (one for each particle) and then adding a resampling step after each measurement. After \( X_{s} (k + 1/k) \) is obtained, it can be refined using the EKF, UKF or MGBEKF measurement-update equations. In this paper Particle filter is combined with the MGBEKF. The measurement is obtained at time \( k, X_{s} (k + 1/k) \) is updated to \( X_{s} (k + 1 + k) \) according to the following MGBEKF equations.

\[
P(k + 1/k) = \phi(k + 1/k), P(k/k), \phi^T (k + 1/k), + \Gamma Q(k + 1)/\Gamma^T \quad (4)
\]

\[
G(k + 1) = P(k + 1/k), H(k + 1), \sigma^2 + H(k + 1), P(k + 1/k), H^T (k + 1) \quad (5)
\]

\[
X_{s}(k + 1/K + 1) = X_{s}(k + 1/k) + G(k + 1), [R_{s}(k + 1/k) - h(k + 1, X_{s}(k + 1/k))] \quad (6)
\]

\[
P(k + 1/k + 1) = [I - G(k + 1), g(B_{s}(k + 1/k), X_{s}(k + 1/k))] + [P(k + 1/k)], [I - G(k + 1), g(B_{s}(k + 1/k), X_{s}(k + 1/k)),] \quad (7)
\]

where, \( G(k + 1) \) is Kalman gain and \( P(k + 1/k) \) is a priori estimation error covariance for the \( i \)-th particle.

\( g(.) \) is modified gain function and it is given by

\[
g = \left[ \begin{array}{c}
0 \\
0 \\
\cos B_{s}/(\bar{R}_{s} \sin B_{s} + \bar{R}_{s} \cos B_{s}) \\
- \sin B_{s}/(\bar{R}_{s} \sin B_{s} + \bar{R}_{s} \cos B_{s})
\end{array} \right]
\]

Since true bearing is not available in practice, it is replaced by the measured bearing to compute the function \( g(.) \).

3. Simulation and Results

The task is to estimate the target motion parameters of the torpedo, while observer is in attack by a torpedo. The observer safety maneuver is based on 70° relative bearing method to escape from the torpedo. Range should decrease to get more bearing rate with increase in time. In general, the observer tries to increase the speed after turning to the required course. This is required for the observer to escape from the target as early as possible. The scenarios considered in this paper is present in Table 1.
The measurement is available every 1.28 seconds. TA detection range limit is 10000 meters. For TA it is assumed that there is degradation in the bearing accuracy during and settling time (assumed to be 1 minute) after observer maneuver. (SNR reduces by -5 dB.) Towed array is operating at 2.5 K Hz (Center frequency). The errors in the bearing measurement are taken as per NPOI's MVDR data. In underwater applications the acceptable errors in estimated range, course and speed are less than or equal to 20 %, 5 degrees and 20 % knots respectively.

Table 1.

| Parameter       | Scenario 1 |
|-----------------|------------|
| Initial Range   | 4500       |
| Initial Bearing | 280        |
| Target Speed(Knots) | 20       |
| Target Course(degrees) | 52        |
| Observer Speed(Knots)  | 15        |
| Observer Course(degrees) | 0        |

3.1 Implementation

The particle filter approximates the density by a large set of particles (filters). The mean and weight of each particle (filter) reflect the value of the density in that region of state space. The particle filter updates the particle mean recursively with each new bearing measurement. Using the particles (filters) and corresponding weights Bayesian equations can be approximately solved by means of combined mean and covariance.

3.2 Initialization State Vector

The basic idea is to parallel operation of numerous independent MGBEKFs trackers with a different initial estimate. To do so, the parameterization of range, course and speed is incorporated. Let us suppose the range, course and speed intervals of interest is (range_min, range_max), (course_min, course_max) and (speed_min, speed_max) respectively with range, course, speed subintervals. The subintervals are 100 meters, 1 degree, 1 m/sec in range, course, and speed respectively. Let

\[ i = \frac{\text{range}_{\text{max}} - \text{range}_{\text{min}}}{100} \]
\[ j = \frac{\text{course}_{\text{max}} - \text{course}_{\text{min}}}{1} \]
\[ k = \frac{\text{speed}_{\text{max}} - \text{speed}_{\text{min}}}{1} \]

So number of filters, \( N \) is given by \( i \times j \times k \). The state vector is initialized in such away that, it will have all different combinations of range, course and speed in order to cover the entire state vector search space. So, the initial target state vectors are in the following form

\[
\begin{bmatrix}
0 & 0 & \cos B_m/(\hat{R}, \sin B_m + \hat{R}, \cos B_m) & -\sin B_m/(\hat{R}, \sin B_m + \hat{R}, \cos B_m)
\end{bmatrix}
\]

Where, \( B_m(0) \) is the initial bearing measurement.

3.3 Initialization Covariance Matrix

It is assumed that the state vector \( X(0/0) \) is uniformly distributed. Then the element of initial covariance matrix is a diagonal matrix and can be written as

\[
P(0/0) = \text{Diag} \begin{bmatrix} 4 \cdot \mathbf{x'}(0/0)_i & 4 \cdot \mathbf{y'}(0/0)_i & 4 \cdot \mathbf{r'}(0/0)_i & 4 \cdot \mathbf{r}^2(0/0)_i \end{bmatrix}
\]

The weight of each MGBEKFs solution is given by

\[
q(k) = \frac{1}{(2\pi)^{N/2}|P|^1/2}\exp\left\{-\frac{1}{2}(B_m(k) - \tilde{B}(k+1/k))' R^{-1}(B_m(k) - \tilde{B}(k+1/k))\right\}
\]

where, \( R = \sigma^2 + H(k + 1) P(k + 1 | k) H^T(k + 1) \)

Here initialization of target state vector is carried out in such away that resampling of the process is not required. Resampling is required when the effective sample size, \( N_{\text{eff}} \) is less than \( N / 3 \).

\[
N_{\text{eff}} = \frac{1}{\sum q_i^2}
\]

Here resampling is a major bottleneck in particle filter. There are many methods available in literature and each one has its own merits and demerits. All these methods are adhoc and these are to be used accordingly to the situation. This problem is avoided by using larger state vector search space with all possible combinations of range, course, and speed. The maximum filters required are around 9000, which is found through evaluation of several tactical scenarios and \( N_{\text{eff}} \) is around 6000. So resampling is not required. Here subinterval size is chosen in such away that no of filters are not too high and at the same the accuracy of the solution is obtained with in the required time.
4. Conclusion and Outlook

Particle filter is the advanced estimation technique for target motion applications. The simulation results for the scenarios also agree with the above inference. The 70 degree relative bearing method is used to escape from the incoming torpedo. The escape as well as estimation of the torpedo is simultaneously featured in naval methodology.

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