Strategies for The Determination of $\phi_3$ in $B^- \rightarrow D^0K^-$

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Direct CP violation in decays such as $B^- \rightarrow D^0K^-$ is sensitive to the CKM angle $\phi_3$, because these decays allow the interference of $b$-quark to $c$-quark with $b$-quark to $u$-quark transitions. Indeed, $\phi_3$ may be determined if one can infer the strong phase of the $B$ and subsequent $D^0$ decays from experimental data. In this talk, I will discuss how this can be carried out using either a single decay mode of the $D^0$ by combining data from a number of $D^0$ decay modes as well as the use of other, analogous decays and the prospects of implementing such methods at various $B$-factories. Since the properties of the $D^0$ decays are crucial to these methods, it is possible that $D^0$-bar $D^0$ mixing at the 1% level will contaminate the results. I will therefore discuss various methods to remove such confounding effects so that $\phi_3$ may be determined even if such mixing is present.

1 Introduction

The asymmetric B experiments BaBar and BELLE have already obtained preliminary measurements of the angle $\phi_1$ of the unitarity triangle of the Cabibbo Kobayashi-Maskawa (CKM) matrix through the “gold-plated” mode $\psi K$. Using $B^0\bar{B}^0$ oscillation it may also be possible to extract $\phi_2$ via modes such as $\pi\pi$ and $\pi\rho$. However to extract $\phi_3$ using oscillations requires $B_s$-mesons and so is inaccessible to $\Upsilon(4S)$ machines.

Although experiments to extract $\phi_3$ via $B_s$ oscillations may be performed at hadronic B-facilities it is also possible to measure $\phi_3$ through direct CP violation in the $B$ system. Thus, the complete set of unitarity angles may, in principle, be accessible at $\Upsilon(4S)$ machines. Specifying as many parameters of the unitarity triangle as possible is, of course an important check of the Standard Model (SM). In addition, if $\phi_3$ is measured via direct CP violation the comparison to the measurement through indirect CP-violation in the $B_s$ system provides another non-trivial check of the SM.

The idea behind the measurement of $\phi_3$ through direct CP violation is to consider a process which allows interference of the quark level processes $b \rightarrow u\bar{c}s$ and $b \rightarrow u\bar{c}s$. This may be accomplished if both processes ultimately hadronize to a common final state. In particular $b \rightarrow u\bar{c}s$ can drive the decay $B^- \rightarrow D^0K^-$ while $b \rightarrow u\bar{c}s$ can drive the decay $B^- \rightarrow \bar{D}^0K^-$ which will thus interfere provided that $D^0$ and $\bar{D}^0$ are detected through decays into a common final state.

In this talk, I will discuss various strategies for the determination of $\phi_3$ in such decays. The most crucial element is the selection of the $D^0$ decay modes which are to be used. In the simplest case is where CP violation is seen in a single mode we will see that there is not enough information to precisely determine $\phi_3$. However, in Section 2 I will show if the CP violation is large, significant bounds may be placed on $\phi_3$ thus modes where CP violation may be large are especially significant. More generally, as discussed in Section 3, $\phi_3$ may be extracted if two or more modes are measured. A single three body mode may also give equivalent information because each point on the $D$-decay Dalitz plot may be regarded as a separate “mode”. These methods are subject to possible contamination from $D\bar{D}$ oscillation if it is on the order of 1%, I will discuss the impact of this possibility and methods to deal with it in Section 4 and in Section 5 I will give my conclusions.

2 One $D^0$ Decay Mode

Consider the case where $D^0$ and $\bar{D}^0$ can decay to a common final state $X$. This may either be CP eigenstate (e.g. $K^+K^-$) or a state such as $K^+\pi^-$ which is a Cabibbo allowed (CA) decay of the $D^0$ but a doubly Cabibbo suppressed (DCS) decay of the $\bar{D}^0$. In either case, one can determine $\phi_3$ if one can measure the rates $d(X) = Br(B^- \rightarrow K^-[X])$ and $\bar{d}(X) = Br(B^+ \rightarrow K^+\bar{X})$ (here $[X]$ means a decay to $X$ via $D^0$ mixed with the $\bar{D}^0$ channel) provided one also knows the branching ratios $a = Br(B^- \rightarrow K^-D^0)$; $b = Br(B^- \rightarrow K^-\bar{D}^0)$; $c(X) = Br(D^0 \rightarrow X)$ and $\bar{c}(X) = Br(\bar{D}^0 \rightarrow X)$.

This information allows us to solve (up to an eight fold ambiguity) for the weak phase $\phi_3$ as well as the total strong phase difference $\xi$.

In practice, however, $b$ is not easy to determined directly because it is difficult to find a prominent tag for $\bar{D}^0$. For instance, a leptonic tag has a large background from leptonic decay of $B^-$ while one tries to tag it through decays such as $D^0 \rightarrow K^+\pi^-$ the signal is subject to interference effects from $D^0 \rightarrow K^+\pi^-$. In fact it is the existence of such interference effects we wish to exploit as a source of CP violation that make the direct determination of $b$ via a hadronic tag impossible.
Of course if CP violation were seen (i.e. \( d \neq \bar{d} \)) then within the SM it must be the case that \( \phi_3 \neq 0 \). As might therefore be expected, in the absence of \( b \) we can use the rest of the information to establish a lower bound on \( \phi_3 \).

In particular, if we define \( Q \equiv \sin^2 \phi_3 \) then we obtain the following bound on \( Q \) and incidentally \( b \):

\[
Q \geq Q_{\text{min}} = (1+z) \left( 1 - \frac{\sqrt{1-y/(1+z)}}{2} \right)
\]

\[
-\sqrt{1+z+y/|y|} \leq \sqrt{u-1} \leq \sqrt{1+z-|y|}
\]  (1)

where \( 1+z = (d(X) + \bar{d}(\bar{X}))/\sqrt{2}ac(X) \), \( y = (d(X) - \bar{d}(\bar{X}))/\sqrt{2}ac(X) \) and \( u = \bar{b}c(X)/a(X) \).

In the case where \( z \) is negative, an upper bound can also be placed on \( Q \):

\[
Q \leq 1 + z
\]  (2)

which is similar in form to the upper bound on \( Q \) obtained from tree-penguin interference in \( B \to K\pi \).

Note that it can apply even if CP violation is absent however \( z \) may only be negative if it happens that \( u \leq 4 \).

Motivation for these bounds can be found in Fig. 1 where we plot the relation between \( u \) and \( \phi_3 \) for the experimental inputs \( z = 1.5 \) and \( y = 0, 1 \) and 2 where the larger values of \( y \) correspond to the smaller “lazy eight” curves. The boxes indicate the bounds obtained by the inequality eqn. (1). Since \( y \) is proportional to CP violation, it is clear that the most stringent inequality bounds obtain where CP violation is large. A useful property of these curves is that even though the strong phase difference is not explicitly given, it may be read off (up to a four fold ambiguity) since for a given value of \( (\phi_3, u) \) on the curve, the horizontal line through that point also intersects the curve at \( (\xi', u) \) where one of \( \{\xi', \pi - \xi', \pi + \xi', \xi'\} \) is the strong phase difference.

Large CP violation is only possible when the two interfering amplitudes are similar in magnitude. Such a situation may happen if we consider a final state \( X \) where \( D^0 \to X \) is DCS (e.g. \( K^+\pi^- \)). Thus, while \( a \) is about two orders of magnitude greater than \( b \) due to color suppression, this if offset by \( c(X) \) being about two orders of magnitude greater than \( c(x) \) so \( b\bar{c}(X) \sim ac(X) \). It is obviously advantageous to experimentally study all modes of this kind in order to find the one which gives the largest lower bound on \( Q \).

Of course we would like to determine \( \phi_3 \) exactly rather than just establishing a bound for it. There are three possible approaches to obtaining this quantity if \( b \) cannot be experimentally measured: First of all, one could use a theoretical model to estimate \( b \). Second, one could use an analogous decay \( B^0 \to K^0[\bar{D} \to K^+\pi^-] \) or \( \Lambda_b \to \Lambda[D^0 \to K^+\pi^-] \) where the cross channel is not color enhanced and interference are \( \sim 30\% \) (we will discuss this more below) or third of all, one can consider multiple \( D \) decay modes each of which may decay with a different strong phase. In this last case I also include \( D \) decays to a multi-body final state where each point in phase space may be considered as a separate “mode”.

3 Two Modes and Three Body Decays of \( D^0 \)

If \( d \) and \( \bar{d} \) are measured for exactly two modes, then there are the same number of equations as unknowns and \( Q \) may be determined up to some discrete ambiguity. Graphically, two general curves such as in Fig. 2 will intersect at up to 4 points so in this case there may be a 4-fold ambiguity in \( Q \) therefore a 16-fold ambiguity in \( \phi_3 \). If three or more modes are considered then the curves in the \( b-\phi_3 \) plane should only intersect at one point in the first quadrant of \( \phi_3 \) so \( \phi_3 \) has a 4-fold ambiguity.

In Fig. 2 we show the results of a sample calculation from \( \Phi_3 \) where the modes: \( K^+\pi^- \), \( K_S\pi^0 \), \( K^+\rho^- \), \( K^+\rho^0 \), \( K^+\rho^0 \), \( K^+\pi^- \) were considered. For the parameters of that calculation, the inner edge of the shaded region indicates the 68\% CL and the outer region indicates the 95\% CL given \( N = \text{(number of } B^+\text{ acceptance) = } 10^8 \). In this example it was found that with \( N = 10^8 \) statistical errors in \( \phi_3 \) were \( \sim 5^\circ \text{ to } 10^\circ \) for a variety of initial values of \( \phi_3 \) and strong phases.

In order to determine \( \phi_3 \) is it therefore advantageous to consider a number of modes. In addition to the different final states such as those considered above, we can also replace \( K \to K^* \) and \( D \to D^* \). Of course if we do both at once so that both sides have \( J \neq 0 \) then we need to do a more complicated angular analysis as considered by \( \Phi_3 \). Note that here \( \phi_3 \) is common but each case
Figure 2: The curves in the $\phi_3$ – $b$ plane using the parameters considered in $^{10}$ are shown for the modes $K^+\pi^-$ (solid curve); $K_S\pi^0$ (short dashed curve); $K^+\rho^-$ (long dashed curve); $K^+a_1^-$ (dash-dot curve); $K_{S}\rho^0$ (dash-dot-dot curve) and $K^{+}\pi^-$ (dash-dash-dot curve). The inner edge of the shaded region corresponds to the 68% CL for $N = 10^8$ while the outer edge corresponds to the 95% CL.

has a separate $b$ – axis. To Drive up additional modes we can also consider analogous decays where we replace the spectator with a $d$ (i.e. $B^0 \rightarrow D^0K^0$) or a $ud$ (i.e. $\Lambda_b \rightarrow D^0\Lambda$). The point here is that we may be justified in putting these cases on a common $b$ axis.

Because the main point of combining multiple modes is to overcome the lack of knowledge of $b$, $B^0 \rightarrow K^0D$ where the $D$ is decays to a state such as $K^+\pi^-$ may be a particularly important mode to use in this way since the dominant contribution is proportional to $b$ while a is color suppressed in this channel $^4$. As emphasized by $^4$ the complimentary case where $B^- \rightarrow K^-D^0$ and $D^0$ decays to a CP eigenstate (e.g. $\pi^+\pi^-$) and so the a channel is much larger than the $b$ channel also gives the same kind of information since the amount of interference evident in the system determines $b$ without a strong dependence on $\phi_3$.

An additional source of constraints that can be helpful may be obtained from a charm factory data which can constrain the strong phase differences between $D^0$ and $\bar{D}^0$ decays as well as give definitive information concerning $D\bar{D}$ oscillations $^4$.

Note that a number of the modes we consider are instances of three body final states. For a single three body mode (e.g. $K^+\pi^-\pi^0$) we can consider each of the points on the dalitz plot as having a separate strong phase so clearly in principle there is enough information to determine $\phi_3$. One can thus fit the data to a resonance model as in $^2$ together with the overall strong and weak phase differences.

In this talk I would like to emphasize a different method of analysis based on the saturation of eq. (1). Regarding each point of the Dalitz plot as a separate mode, one may find the value of $Q_{min}$ in Eqn. (2) for each point of the Dalitz plot. Just as in the case of a number of discrete modes, the true value of $Q$ must exceed all lower bounds. In this case, however, because the strong phases due to resonances vary across the Dalitz plot, it is likely that the greatest value of $Q_{min}$ is in fact equal to $Q$.

In Fig. 3 I show a map “magic” points where $Q = Q_{min}$ on the Dalitz plot for the case of $D^0 \rightarrow K^+\pi^-\pi^0$ using the model of $^4$ that uses the data from E687 $^4$ as input together with SU(3) to give the DCS channel. Here I have taken $\phi_3 = 60^\circ$ with an overall strong phase difference of $0^\circ$ for the solid curve and $60^\circ$ for the dashed curve.

4 The Influence of $D\bar{D}$ mixing

In the discussion so far we have explicitly assumed that $D^0\bar{D}^0$ was negligible. In particular, since we often take advantage of interferences involving DCS decays which are $O(1\%)$ of the interfering CA decay, the total probability of mixing must be less than $O(1\%)$ for this case to remain valid. In fact, it has been suggested that the standard model may cause $D\bar{D}$ mixing at about this level $^4$. In fact, it can be shown $^4$ that the changes to $d$ and $\bar{d}$ from such mixing will be $O(10\%)$ which leads to an inherent error in the determination of $\phi_3$ of $\sim 10^\circ - 15^\circ$ $^2$. 

Figure 3: The locus of points on a dalitz plot for the final state $K^+\pi^-\pi^0$ where $Q_{min} = Q$ for $\phi_3 = 60^\circ$ and an overall strong phase difference of $0^\circ$ (solid line) and $\zeta = 60^\circ$ (dashed line). Here $s = (p_{\pi^-} + p_{K^+})^2$ and $t = (p_{\pi^-} + p_{d\pi^-})^2$. 

Figure 4: The Influence of $D\bar{D}$ mixing
In order to overcome this possible systematic error, there are two approaches:

1. Using information on the the time between the $B^-$ decay and the subsequent $D^0$ decay, then the effects of possible mixing can be eliminated.

2. If the parameters of $D\bar{D}$ mixing are known independently, then they can be taken into account in interpreting the time integrated data

Indeed, if the mixing parameters and time dependent data is available, then one can in principle extract $\phi_3$ from just one mode though most likely, the time dependence in the decay is too weak to make this a useful method.

Here I will emphasise the fact that if we have time dependent data, it is particularly simple to separate out the contributions of mixing and thus proceed with the analysis as in the absence of mixing at some cost in statistics.

The key point is that for $D^0$ the decay time is much shorter than the oscillation time and therefore it is valid to write

$$\frac{d}{d\tau}d(X) \approx (d_0(X) + d_1(X)\tau)e^{-\tau}$$

$$\frac{d}{d\tau}d(\bar{X}) \approx (\bar{d}_0(\bar{X}) + \bar{d}_1(\bar{X})\tau)e^{-\tau}$$

where $\tau = t\Gamma_D$. Thus $d_0(X)$ and $\bar{d}_0(\bar{X})$ would be the branching ratios absent mixing so if we extract them from the time dependence we may proceed as if there were no mixing.

This can be accomplished through weighting the data with $w_0(\tau) = 2 - \tau$ so that

$$d_0 = \int_0^\infty [d(\tau)w_0(\tau)]d\tau; \quad \bar{d}_0 = \int_0^\infty [\bar{d}(\tau)w_0(\tau)]d\tau$$

Using this method more data would be required to obtain the same statistical results as in the unmixed case. In the unmixed case where $d_1 = 0$ one could obtain $d_0$ more effectively by taking the time integrated rate. Thus in the unmixed case if a measurement of $d_0$ is based on $n$ events, the uncertainty in $d_0$ is given by: $(\Delta d_0)^2/d_0^2 = 1/n$. In the mixed case, using eqn. (4) the uncertainty is $(\Delta d_0)^2/d_0^2 = (2/n)(1 + d_1/d_0)^2$. Since $d_1 \sim O(d_0/10)$, this means that roughly twice the data is needed to have the same statistical power as in the unmixed case. In order to gauge the precision of time measurement required, we can smear out the distribution in eq. (4) with a Gaussian resolution function of the form $r(\tau, \tau') \propto e^{-\frac{(\tau - \tau')^2}{2\sigma^2}}$ where $\tau$ is the actual time of the decay, $\tau'$ is the measured time of the decay and $\sigma$ is the resolution (all in units of $1/\Gamma_D$). Since $r$ is symmetric under $\tau \leftrightarrow \tau'$, the fact that $w$ is linear in $\tau$ implies eq. (4) will still be true for $\tau'$ but now the error is:

$$(\Delta d_0)^2/d_0^2 = (2/n)(1 + \sigma^2)(1 + d_1/d_0)^2$$

As can be seen, the number of events required is not adversely effected if $\sigma \leq 1/\Gamma_D$ but will be significantly degraded otherwise.

5 Conclusions

In conclusion, in the $B^\pm$ system direct CP violation in the decay $D^0 K^\pm$ may provide a means of determining $\phi_3$ with $10^8 - 10^9$ $B$ mesons. The key is to observe the correct $D^0$ or combination of $D^0$ decay modes. If large CP violation is seen in any one mode, it may establish a lower bound on $\sin^2\phi_3$ while data from multiple modes or three body modes can be used to determine $\sin^2\phi_3$.

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