Black Hole as a Wormhole Factory

Sung-Won Kim* and Mu-In Park†

Department of Science Education and Research Institute of Curriculum Instruction,
Ewha Womans University,
Seoul, 120-750, Korea

Abstract

There have been lots of debates about the final fate of an evaporating black hole and the singularity hidden by an event horizon in quantum gravity. However, on general grounds, one may argue that a black hole stops radiation at the Planck mass \( (\hbar c/G)^{1/2} \sim 10^{-5} g \), where the radiated energy is comparable to the black hole’s mass. And also, it has been argued that there would be a wormhole-like structure, known as “space-time foam”, due to large fluctuations below the Planck length \( (\hbar G/c^3)^{1/2} \sim 10^{-33} cm \). In this paper, we show that there is actually an exact classical solution which represents nicely those two properties in a recently proposed quantum gravity model based on different scaling dimensions between space and time coordinates. The solution, called “Black Wormhole”, consists of two different states, depending on its mass parameter \( M \) and an IR parameter \( \omega \): For the black hole state (with \( \omega M^2 > 1/2 \)), a wormhole occupies the interior region of the black hole around the singularity at the origin, whereas for the wormhole state (with \( \omega M^2 < 1/2 \)), the interior wormhole is exposed to an outside observer as the black hole horizon is disappeared from evaporation. The black hole state becomes thermodynamically stable as it approaches to the merge point where the interior wormhole throat and the black hole horizon merges, and the Hawking temperature vanishes at the exact merge point (with \( \omega M^2 = 1/2 \)). This solution suggests the “Generalized Cosmic Censorship” by the existence of a wormhole-like structure which protects the naked singularity even after the black hole evaporation. One could understand the would-be wormholes inside the black hole horizon as the results of microscopic wormholes created by negative energy quanta which have entered the black hole horizon in Hawking radiation processes: \textit{The quantum black hole could be a wormhole factory!} It is found that this picture may be consistent with the recent “\( ER = EPR \)” proposal for resolving the recent black hole entanglement debates.

PACS numbers: 04.20.Jb, 04.20.Dw, 04.20.Gz, 04.60.-m

* E-mail address: sungwon@ewha.ac.kr
† E-mail address: muinpark@gmail.com
It is widely accepted that general relativity (GR) would not be appropriate for describing the small scale structure of space-time. For example, GR, when combined with quantum mechanics, provides a length scale $l_P = (\hbar G/c^3)^{1/2} \sim 10^{-33} \text{cm}$, which may provide an absolute limitation for the measurements of space-time distances. Actually, this is the length scale on which quantum fluctuations of the space-time are expected to be of order of unity. On the other hand, the singularity theorem, stating the necessary existence of singularities, where the classical concept of space and time breaks down, at certain space-time domains with some reasonable assumptions in GR, may be regarded as an indication of the incompleteness of GR.

These circumstances may provide strong motivations to find the quantum theory of gravity which can treat the above mentioned problems of GR. Actually, the necessity of quantizing the gravity has been argued in order to have a consistent interaction with a quantum system. Moreover, it has been also shown that even small quantum gravitational effects dramatically change the characteristic features of a black hole so that it can emit radiations though the causal structures of the classical geometry is unchanged in the semiclassical treatment.

However, as the black hole becomes smaller and smaller by losing its mass from emitting particles, the semiclassical treatment becomes inaccurate and one can not ignore the back reactions of the emitted particles on the metric and the quantum fluctuations on the metric itself anymore. Actually, regarding the back reaction effects, one can argue that a black hole stops radiations at the Planck mass $m_P = (hc/G)^{1/2} \sim 10^{-5} \text{g}$, where the radiated energy is comparable to the black hole’s mass, since a black hole can not radiate more energy than it has, via the pair creation process near the black hole horizon. This implies that the black hole should become thermodynamically stable as it becomes smaller and finally has the vanishing Hawking temperature at the smallest black hole mass. It seems that this should be one of verifiable predictions that any theory of quantum gravity make. Moreover, according to large fluctuations of metric below the Planck length, the wormhole-like structure, known as “space-time foam”, has been proposed by Wheeler. This may be another verifiable prediction of the quantum gravity also.

The purpose of this paper is to show that there is actually an exact classical solution which represents nicely those two properties in a recently proposed quantum gravity model, known as Hořava gravity, based on different scaling dimensions between space and time coordinates. The solution, called “Black Wormhole”, consists of two different states depending on its mass parameter $M$ and an IR (infrared) parameter $\omega$: For the black hole state (with $\omega M^2 > 1/2$), a wormhole occupies the interior region of the black hole around the singularity at the origin, whereas for the wormhole state (with $\omega M^2 < 1/2$), the interior wormhole is exposed to an outside observer as the black hole horizon is disappeared from evaporation. The black hole state becomes thermodynamically stable as it approaches to the merge point where the interior wormhole throat and the black hole horizon merges, and the Hawking temperature vanishes at the exact merge point (with $\omega M^2 = 1/2$).

The solution suggests that, in quantum gravity, the ‘conventional’ cosmic censorship can be generalized even after black hole evaporation by forming a wormhole throat around the used-to-be singularity. In GR, black hole and wormhole are quite distinct objects due to their completely different causal structures. But the claimed “Generalized Cosmic Censorship” suggests that the end state of a black hole is a wormhole, not a naked singularity. This may correspond to a foam nature of space-time at short length scales.

In the conventional approach for (traversable) wormholes, there are basically two unsat-
isfactory features \cite{6,7}. First, we do not know much about the mechanism for wormhole formation. This is in contrast to the black hole case, where the gravitational collapse of ordinary matters, like stars, can form a black hole, if its mass is enough. Second, the usual steps of ‘constructing’ wormholes are too artificial as summarized by the following three steps: (1). Prepare two or several universes; (2). Connect the universes by cut and paste of their throats; (3). Put the needed “exotic” matters, which violate the energy conditions, to the throats so that Einstein’s equations are satisfied. In this paper, we will see how these problems can be resolved following the recent construction of wormholes by BottaCantcheff-Grandi-Sturla (BGS) \cite{8}.

To see how this can occur explicitly, we consider the Ho\v{r}ava gravity which has been proposed as a four-dimensional, renormalizable, higher-derivative quantum gravity without ghost problems, by adopting different scaling dimensions for space and time coordinates in UV (ultraviolet) energy regime, [$t$] = $-1$, [$x$] = $-z$ with the dynamical critical exponents $z \geq 3$, at the expense of Lorentz invariance \cite{9}. We formally define the quantum gravity by a path integral

$$Z = \int [\mathcal{D}g_{ij}][\mathcal{D}N_i][\mathcal{D}N]e^{iS/\hbar}$$

with the proposed action ($z = 3$ is considered, for simplicity), up to the surface terms,

$$S = \int dt d^3x \sqrt{|g|} N \left[ \frac{2}{\kappa^2} \left( K_{ij} K^{ij} - \lambda R^{ij} \right) - \frac{\kappa^2}{2 \mu^4} C_{ij} C^{ij} + \frac{\kappa^2 \mu^2}{2 \nu^2} \epsilon^{ijk} R^{(3)} \nabla_j R^{(3)} \right] + \frac{\kappa^2 \mu^2}{8 (3 \lambda - 1)} R^{(3)}$$

and with the ADM decomposition of the metric,

$$ds^2 = -N^2 c^2 dt^2 + g_{ij} \left( dx^i + N^i dt \right) \left( dx^j + N^j dt \right),$$

the extrinsic curvature,

$$K_{ij} = \frac{1}{2N} \left( g_{ij} - \nabla_i N_j - \nabla_j N_i \right),$$

the Cotton tensor,

$$C^{ij} = \epsilon^{ik\ell} \nabla_k \left( R^{(3)} \delta^j_{\ell} - \frac{1}{4} R^{(3)} \delta^j_{\ell} \right),$$

and coupling constants, $\kappa, \lambda, \nu, \mu, \Lambda_W, \omega$. The proposed action is not the most general form for a power-counting renormalizable gravity, compatible with the assumed foliation preserving $\text{Diff}$ but it is general enough to contain all the known GR solutions, and the qualitative features of the solutions are expected to be similar \cite{10–13}. Here, originally, the non-relativistic higher-derivative deformations were introduced from the technical reason of the necessity of renormalizable interactions without the ghost problem which exists in relativistic higher-derivative theories \cite{8}. But the (UV) Lorentz violation might have more fundamental reason in our quantum gravity set-up since this may be consistent with the existence of the absolute minimum length $l_P$ which does not depend on the reference frames. The last term in the action \cite{2} represents a “soft” violation, with the IR parameter $\omega$, of
FIG. 1: Plots of $f(r)$ in Hořava gravity for varying $M$ with a fixed $\omega$ (left) and for varying $\omega$ with a fixed $M$ (right). In particular, in the left we consider $M = 0, 0.25, 0.5, 1$ (top to bottom solid lines) with $\omega = 2$, and in the right $\omega = 0.25, 0.5, 2$ (top to bottom solid lines) with $M = 1$, in contrast with Schwarzschild solution for $M = 1$ in GR (doted line).

the “detailed balance” condition in [9] and this modifies the IR behaviors so that Newtonian gravity limit exists [11–13].

For the simplest case of static non-rotating uncharged black holes, where only the last three terms in the action (2) are relevant, the exact solutions have been found completely for arbitrary values of coupling constants, $\lambda, \Lambda, W$, and $\omega$ [10–13]. However, for the present purpose we only consider a simple example of $\lambda = 1, \Lambda = 0, W$,

$$
\text{with}$$

$$
N^2 = f = 1 + \omega r^2 - \sqrt{r [\omega^2 r^3 + 4 \omega M]}$$

so that the standard Einstein-Hilbert action and the asymptotically flat, Schwarzschild black hole solution are recovered in the IR limit, i.e., $N^2 = f = 1 - 2M/r + O(r^{-4})$ with $c^2 = \kappa^4 \mu^2 \omega/32$, $G = \kappa^2 c^2/32\pi$ [11, 12]. Here $M$ denotes the $(G/c^2) \times$ ADM mass and the positive IR parameter $\omega$ controls the strength of higher-derivative corrections so that the limit $\omega \to \infty, \mu \to 0$ with $\mu^2 \omega = \text{fixed}$ corresponds to GR limit. One remarkable property of the solution is that there is an inner horizon $r_-$ as well as the outer horizon $r_+$, which solves $f(r_\pm) = 0$, at

$$
r_\pm = M \left(1 \pm \sqrt{1 - \frac{1}{2\omega M^2}}\right)
$$

as the result of higher (spatial) derivatives (Fig. 1): The higher derivative terms act like some (non-relativistic) effective matters in the conventional Einstein equation so that there is some “repulsive” interaction at short distances. Moreover, even though the metric converges to the Minkowski’s flat space-time at the origin $r = 0$, its derivative is not continuous so that there is a curvature singularity at $r = 0$, which may be captured by the singularity of $R \sim r^{-3/2}$, $R^\mu{}_{\nu\alpha\beta} R_{\mu\nu\alpha\beta} \sim r^{-3}$. Even though the singularity at $r = 0$ is a time-like line (i.e.,
time-like singularity) and is milder than that of Schwarzschild black hole, \( R^\mu_\alpha\beta R_{\mu\alpha\beta} \sim r^{-6} \) (and also Reissner-Nordstrom’s \( R^\mu_\alpha\beta R_{\mu\alpha\beta} \sim r^{-4} \)) and surrounded by the (two) horizons provided

\[
\omega M^2 \geq \frac{1}{2},
\]

this might indicate that the proposed gravity does not completely resolve the singularity problem of GR still, classically.

This circumstance looks inconsistent with the cosmology solution, where the initial singularity does not exist whence there exist the higher-derivative effects, \( i.e., \) non-flat universes \([11, 12]\). Moreover, the black hole singularity becomes naked for \( \omega M^2 < 1/2 \) so that the cosmic censorship might not work in this edge of solution space.

But according to recent BGS’s construction of wormholes, it seems that there is another possibility for resolving the unsatisfactory circumstance \([8]\). What they found was that there exits a wormhole solution also, in addition to the naked singularity solution for \( \omega M^2 < 1/2 \), without introducing additional (exotic) matters at the throat. Their obtained wormhole solution

\[
ds^2 = -N_\pm(r)^2 c^2 dt^2 + \frac{dr^2}{f_\pm(r)} + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right)
\]

with

\[
N^2_\pm = f_\pm = 1 + \omega r^2 - \sqrt{r[\omega^2 r^3 + 4\omega M_\pm]}
\]

is made of two coordinate patches, each one covering the range \([r_0, +\infty)\) in one universe and the two patches joining at the wormhole throat \( r_0 \), which is defined as the minimum of the radial coordinate \( r \).

In order that the solution are smoothly joined at the throat, so that the additional compensating (singular or non-singular) matters are not needed, the metric and its derivatives should be continuous at the throat. But, if we consider the reflection symmetric two universes, \( i.e., \) \( f_+(r) = f_-(r), N^2_+(r) = N^2_-(r) \), with \( \omega_+ = \omega_- \equiv \omega, M_+ = M_- \equiv M \), the only possible way of smooth patching at the throat is

\[
\left. \frac{df_\pm}{dr} \right|_{r_0} = 0
\]

and the throat radius is obtained as

\[
r_0 = \left( \frac{M}{2\omega} \right)^{1/3}.
\]

Note that the throat is located always inside the black hole horizon, \( i.e., \) \( r_- < r_0 < r_+ \) for \( \omega M^2 > 1/2 \) so that it is unobservable to an outer observer, whereas the wormhole

---

1 In BGS’s paper \([8]\), it is claimed that the junction condition \([12]\) is not always necessary for \( \lambda = 1 \) but more general class of wormhole solutions could be possible. But, this is only for the case of singular, \( \delta \)-function discontinuities in the equations of motions. Whereas, for the non-singular discontinuities at the throat, which can not be properly treated in the BGS’s analysis, the condition \([12]\) is still essential for our wormhole construction.
throat emerges for $\omega M^2 < 1/2$, instead of a naked singularity (Fig. 2). For a fixed $\omega$, the throat radius, after emerging from the coincidence with the extremal black hole radius, $r_0 = r_+ = r_- = M$, decreases monotonically as $M$ decreases and finally vanishes for $M = 0$, i.e., Minkowski vacuum (Fig. 2, left). But, for a fixed $M$ and varying $\omega$, one find that the throat radius increases again indefinitely as $\omega$ decreases. This means that the wormhole size can be quite large when the coupling constant $\omega$, which could flow under renormalization group, becomes smaller at quantum gravity regime, like the Planck size black hole or wormhole, or the primordial black hole or wormhole (Fig. 2, right). But in the actual quantum mechanical process, like black hole evaporations, $M$ as well as $\omega$ can vary so that we need to consider some combinations of situations of the left and right in Fig. 2.

So, we have obtained the wormhole solution when the black hole horizon disappear for $\omega M^2 < 1/2$. Whereas, the solution for $\omega M^2 > 1/2$ with the throat inside the horizon is not the “traversable” wormhole since the throat will be still inside the black hole horizon of the “mirror” black hole in another universe. This is similar to the Einstein-Rosen bridge [14] but the difference is that the throat may not coincide with the black hole horizon generally in our case. Moreover, in our case the throat is located at the fixed “time” $r_0$ so that any time-like trajectories should meet the throat if exits (Fig. 3). This means that the black hole solution (10), (11) with the “time-like” throat for $\omega M^2 > 1/2$ should be considered as a physically distinct object from the black hole solution (6), (7) having the (hidden) singularity at $r = 0$. And in order to avoid a rather strange situation that the wormhole throat “suddenly” emerge from the extremal black hole which has a singularity at $r = 0$ still, it would be natural to consider the time-like throat in (10), (11) for $\omega M^2 > 1/2$ as the “would-be” wormhole throat. In order to be distinguished from the usual black hole solution (6), (7), we may call the solution (10), (11) as the “Black Wormhole” solution.

Now, we have a completely regular vacuum solution, without the curvature singularities, which interpolates between the black hole state for $\omega M^2 > 1/2$ and wormhole state for $\omega M^2 < 1/2$. This seems to support Wheeler’s foam picture of the quantum space-time in quantum gravity but as a real static, not as a virtual time-dependent, wormhole. And in our quantum gravity theory, where the concept of horizon emerges for low energies, escaping the horizons is not impossible for high energy particles with Lorentz-violating dispersion relations. But for low energy point of view, without probing the interior structure of real black holes, the observable consequences of the black wormhole solution would be expressed...
FIG. 3: A black hole pair which are connected by a wormhole throat $r_0$ inside the black hole horizons $r_+$. The arrows represent the time-like geodesics inside the horizons.

in the form of a “Generalized Cosmic Censorship”, suggesting that “the naked singularity does not appear still by forming wormholes even after the horizons disappear in quantum gravity” though, before evaporation, there would be no naked singularity by the existence of horizons as usual.

Another important implication of the black wormhole solution to low energy observers is that there would be transformations between black holes and wormholes $^2$, which is known to be impossible in GR, due to the no-go theorem for topology change $^{[16]}$. In other words, once a (primordial) wormhole is formed in the quantum gravity regime, due to quantum fluctuation in the early universe, it may evolve into a black hole state by the combinations of Fig. 2 (right) from renormalization group flows to GR and Fig. 2 (left) from accretion of matters. It would be interesting to see whether this can be a mechanism for the primordial black holes and supermassive black holes, which are believed to be formed very early in the universe and distinguished from the stellar-mass black holes which are generated from collapsing stars.

On the other hand, once a black hole is formed in quantum gravity, it always has the would-be wormhole inside the horizon but this inside wormhole is exposed to outer observers when the horizon disappears after the complete evaporation. But we know that the inside

$^2$ Hayward has suggested a similar black hole-wormhole transformation on the grounds of “trapping” horizons which may describe black holes and wormholes unifiedly $^{[15]}$. But in his framework, assuming the bifurcating black hole horizons is essential and it is not clear how to extend his framework to our case of non-bifurcating horizons of extremal state where wormhole throat and extremal black hole horizon coincides. Moreover, he did not suggest that interior structure of a black hole might be changed after the transformation from a wormhole so that the singularity might not occur in the black hole state also.
FIG. 4: The microscopic would-be wormholes created by negative energy quanta which have entered the black hole horizon with its Hawking radiated, positive energy, partner quanta outside the horizon.

A wormhole is absent in the GR limit, as can be seen in Fig. 2 (right). Then, where does the inside wormhole come from in quantum gravity regime? It seems that the only possible answer to this question can be found in the Hawking radiation process, which involves virtual pairs of particles near the event horizon, one of the pair enters into the black hole while the other escapes. The escaped particle is observed as a real particle with positive energy with respect to an observer at infinity and then, the absorbed particle must have a negative in order that the energy is conserved. This implies that the negative energy particle that fall into black hole could be a source for a wormhole formation inside the horizon: The quantum black hole could be a wormhole factory! Interestingly, this may provide a physical origin of the Einstein-Rosen bridge in the recent “ER = EPR” proposal for resolving the issue of entanglement in a black hole spacetime claiming that “the black hole and its Hawking radiation are entangled via Einstein-Rosen bridges” (Fig. 4). Following their proposal, one could understand the would-be wormholes inside the black hole horizon as the results of microscopic wormholes created by negative energy quanta which have entered the black hole horizon but entangled with its Hawking radiated, positive energy, partner quanta, which now lives in a single universe, widely separated from the mother black hole. This may be consistent with our reflection symmetric configuration but it not clear how to extend this picture to more general configurations which can not be interpreted as a configuration in a

---

Previously, the negative energy particle has been thought to result in just reducing the black hole mass by some cancelation process with positive energy sources, which may exist inside the horizon, for the black hole mass. But this does not necessarily mean that all the negative energy particles are disappeared by some annihilation process inside the black hole horizon.
single universe. Moreover, it is not clear either how the throats of Einstein-Rosen’s bridges evolve into our would-be wormhole in detail.

In conclusion, we have obtained a vacuum and static black wormhole solution which is regular, i.e., singularity-free, and interpolates between the black hole state for $\omega M^2 > 1/2$ and wormhole state for $\omega M^2 < 1/2$ through the coincidence state of an extremal black hole and a Einstein-Rosen bridge for $\omega M^2 = 1/2$. From this, we have suggested the transformation between the black hole and wormhole states, and its resulting generalized cosmic censorship.

But then, “Can the transformation really occur dynamically?” In order to answer to this question, we first consider the transformation from a black hole state to a wormhole state. Of course, this has some obstacle in GR since the black hole should reach the extremal black hole to become a wormhole state which has vanishing Hawking radiation and temperature so that the extremal black hole is the ground state in the black hole states. But this obstacle may not be quite realistic in our case since it is known that the above thermodynamic description would break down in the extreme limit [18] and moreover, the concept of Hawking temperature makes sense only in IR for our quantum gravity model with UV Lorentz violations.

Now then, let us consider the transformation from a wormhole state to a black hole state. In this case, there does not seem to exist any thermodynamic obstacle against this transformation since the entropy would increase in this process, which resembles the collapse of stars made of ordinary matter to make a black hole. Actually, in the conventional wormhole context with exotic matters in GR, the collapse of the Morris-Thorne wormhole was observed from non-linear instability under ordinary matter perturbations [19]. We expect the similar non-linear instability, though linearly stable, exits for our wormhole state so that it transforms to the extremal black hole state, a seed of large black holes.

Acknowledgments

This work was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (2010-0013054) (SWK), (2-2013-4569-001-1) (MIP).

[1] H. Salecker and E. P. Wigner, Phys. Rev. 109, 571 (1958); C. A. Mead, Phys. Rev. 143, 990 (1966).
[2] R. Penrose, Phys. Rev. Lett. 14, 57 (1965); S. W. Hawking, Proc. R. Soc. A 300, 182 (1967); S. W. Hawking and R. Penrose, Proc. R. Soc. A 314, 529 (1970).
[3] K. Eppley and E. Hannah, Found. Phys. 7, 51 (1977).
[4] S. W. Hawking, Nature 248, 30 (1974); Commun. Math. Phys. 43, 199 (1975).

---

Even though we have obtained the solution in a particular quantum gravity model which is power-counting renormalizable without ghost problems, the features of the small scale structure seems to be quite generic if the vanishing Hawking temperature for a Planck mass black hole, which implying the existence of the wormhole throat $r_0$ satisfying $df/dr|_{r_0} = 0$, is considered as a verifiable prediction that any theory of quantum gravity makes.
[5] J. A. Wheeler, Ann. Phys. (N.Y.) 2, 604 (1957).
[6] M. S. Morris and K. S. Thorne, Am. J. Phys. 56, 395 (1988).
[7] M. Visser, Nucl. Phys. B328, 203 (1989); Lorenzian wormholes (AIP Press, 1995).
[8] M. Botta-Cantcheff, N. Grandi and M. Sturla, Phys. Rev. D 82, 124034 (2010) [arXiv:0906.0582 [hep-th]].
[9] P. Hořava, Phys. Rev. D 79, 084008 (2009) [arXiv:0901.3775 [hep-th]].
[10] H. Lu, J. Mei and C. N. Pope, Phys. Rev. Lett. 103, 091301 (2009) [arXiv:0904.1595 [hep-th]].
[11] A. Kehagias and K. Sfetsos, Phys. Lett. B 678, 123 (2009) [arXiv:0905.0477 [hep-th]].
[12] M. -I. Park, JHEP 0909, 123 (2009) [arXiv:0905.4480 [hep-th]].
[13] E. B. Kiritsis and G. Kofinas, JHEP 1001, 122 (2010) [arXiv:0910.5487 [hep-th]].
[14] A. Einstein and N. Rosen, Phys. Rev. 48, 73 (1935).
[15] S. A. Hayward, Int. J. Mod. Phys. D 8, 373 (1999) [gr-qc/9805019].
[16] R. Geroch, J. Math. Phys. 8, 782 (1967); F. Tipler, Ann. Phys. (N.Y.), 108, 1 (1977); J. L. Friedman, K. Schleich and D. M. Will, Phys. Rev. Lett, 71, 1486 (1993).
[17] J. Maldacena and L. Susskind, Fortsch. Phys. 61, 781 (2013) [arXiv:1306.0533 [hep-th]].
[18] J. Preskill, P. Schwarz, A. D. Shapere, S. Trivedi and F. Wilczek, Mod. Phys. Lett. A 6, 2353 (1991).
[19] H. A. Shinkai and S. A. Hayward, Phys. Rev. D 66, 044005 (2002) [gr-qc/0205041].