A consensus opinion model based on the evolutionary game

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Abstract – We propose a consensus opinion model based on the evolutionary game. In our model, both of the two connected agents receive a benefit if they have the same opinion, otherwise they both pay a cost. Agents update their opinions by comparing payoffs with neighbors. The opinion of an agent with higher payoff is more likely to be imitated. We apply this model in scale-free networks with tunable degree distribution. Interestingly, we find that there exists an optimal ratio of cost to benefit, leading to the shortest consensus time. Qualitative analysis is obtained by examining the evolution of the opinion clusters. Moreover, we find that the consensus time decreases as the average degree of the network increases, but increases with the noise introduced to permit irrational choices. The dependence of the consensus time on the network size is found to be a power-law form. For small or larger ratio of cost to benefit, the consensus time decreases as the degree exponent increases. However, for moderate ratio of cost to benefit, the consensus time increases with the degree exponent. Our results may provide new insights into opinion dynamics driven by the evolutionary game theory.

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Introduction. – The dynamics of opinion sharing and competing and the emergence of consensus have become an active topic of recent research in statistical and nonlinear physics [1]. One of the most successful methodologies used in opinion dynamics is agent-based modeling. The idea is to construct the computational devices (known as agents with some properties) and then simulate them in parallel to model the real phenomena. In physics this technique can be traced back to Monte Carlo simulations. A number of agent-based models have been proposed, which include the voter model [2], the majority rule model [3–5], the bounded-confidence model [6] and the social impact model [7]. Some models display a disorder-order transition [8–13], from a regime in which opinions are arbitrarily diverse to one in which most individuals hold the same opinion. Other models focus the emergence of a global consensus [14–19], in which all agents finally share the same opinion.

If we consider individuals with different opinions as players taking different strategies, then the process of opinion formation can be combined with evolutionary game theory. Evolutionary game theory as a powerful mathematical framework, has been widely used to understand cooperative behavior [20,21], traffic flow [22,23], epidemic spreading [24,25] and so on. Recently, the impact of evolutionary games on the opinion formation has attracted increasing attention. Di Mare and Latora made a first attempt to implement a strategic game theoretical approach to simulate interactions between agents’ opinions [26]. In Di Mare and Latora’s (DL) model, interaction strategies are change, keep and agree, which mean change into the other player’s opinion, maintain one’s own opinion and change into an intermediate opinion, respectively. When the two individuals take different strategies, they receive different payoffs. Ding et al. introduced a new “not participate” strategy into the DL model and obtained abundant dynamical regimes [27]. Ding et al. also proposed an evolutionary game theory model of binary opinion formation which can successfully restore the majority-voter model [28].

The consensus time is an important physical quantity in opinion dynamics [2,4]. However, how to accelerate consensus in opinion models based on game theory remains unclear. In this letter, we study how the payoff parameter and the network structure affect the consensus time in an evolutionary game theory model of binary opinion formation. We assume that both of the two connected individuals receive a benefit if they have the same opinion, otherwise they both pay a cost. Here the benefit in opinion dynamics represents the feeling of belonging to a

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Fig. 1: (Color online) (a) The consensus time $T_c$ as a function of the cost $c$ for different values of the average degree of the network $\langle k \rangle$. The network size $N = 5000$, the noise $\kappa = 1$ and the degree exponent $\gamma = 3$. (b) The consensus time $T_c$ as a function of the cost $c$ for different values of the network size $N$. The average degree of the network $\langle k \rangle = 6$, the noise $\kappa = 1$ and the degree exponent $\gamma = 3$. (c) The consensus time $T_c$ as a function of the cost $c$ for different values of the noise $\kappa$. The network size $N = 5000$, the average degree of the network $\langle k \rangle = 6$ and the degree exponent $\gamma = 3$. (d) The consensus time $T_c$ as a function of the cost $c$ for different values of the degree exponent $\gamma$. The network size $N = 5000$, the noise $\kappa = 1$ and the average degree of the network $\langle k \rangle = 6$.

group and the cost reflects a peer pressure of disagreeing with others [29,30]. The above assumption has been demonstrated by many psychological experiments in which dissent often leads to punishment either psychologically or financially, or both, as human individuals attempt to attain social conformity [31,32]. An individual updates its opinion by comparing payoffs with a randomly selected neighbor. The more payoff the chosen neighbor has, the higher the probability that its opinion will be imitated. We apply our model in scale-free networks with tunable degree distribution. Interestingly, we find that there exists an optimal ratio of cost to benefit, leading to the shortest consensus time.

Model. – Our model is described as follows. For a given network of any topology, each node represents an agent. Initially the two opinions denoted by the values $\pm 1$ are randomly assigned to agents with equal probability. Both of the two connected agents receive a benefit $b$ if they have the same opinion, otherwise they both pay a cost $c$. Thus, the total payoff of agent $x$ can be calculated as

$$P_x = \frac{(b - c)k_x}{2} + \frac{(b + c)\sum_{i\epsilon \Omega_x} S_xS_i}{2},$$

where $k_x$ is the degree of agent $x$, $S_x$ is the opinion of agent $x$, and the sum runs over the nearest-neighbor set $\Omega_x$ of agent $x$.

Agents asynchronously update their opinions in a random sequential order. At each time step, we randomly select an agent $x$ who obtains the payoff $P_x$ according to eq. (1). Then we choose one of agent $x$’s nearest neighbors at random, and the chosen agent $y$ also acquires its payoff $P_y$ by the same rule. In the evolutionary games such as the prisoner’s dilemma game and the public goods game, the Fermi function has been widely used to study the evolution of cooperative and defective behaviors [33–36]. In this letter, we use the Fermi function to study the evolution of different opinions. We suppose that the probability that agent $x$ adopts agent $y$’s opinion is given by the Fermi function:

$$W(S_x \leftarrow S_y) = \frac{1}{1 + \exp[(P_x - P_y)/\kappa]},$$

where $\kappa$ characterizes the noise introduced to permit irrational choices.

In the Fermi updating rule, the opinion of an agent with higher payoff is more likely to be imitated. For $P_y > P_x$ ($P_y < P_x$), the probability that agent $x$ adopts agent $y$’s opinion is larger (smaller) than 0.5.

Scale-free networks with tunable degree distribution. – Empirical studies have shown that the degree distribution of some social networks exhibits a power-law form [37]. In this letter, we adopt the algorithm proposed by Dorogovtsev et al. [38] to generate the scale-free networks with tunable degree distribution.

Initially, there are $m$ fully connected nodes. At each time, a newly added node makes $m$ links to $m$ different nodes already present in the network. The probability $\Pi_i$ that the new node will be connected to an old node $i$ is

$$\Pi_i = \frac{k_i + Am}{\sum_j(k_j + Am)},$$

where $k_i$ is the degree of node $i$, the sum runs over all old nodes, and $A$ is a tunable parameter ($A > 1$). After a long evolution time, this algorithm generates a scale-free network following the power-law degree distribution $P(k) \sim k^{-\gamma}$ with the degree exponent $\gamma = 3 + A$. Particularly, this algorithm produces the Barabási-Albert network model [39] when $A = 0$. The average degree of the network is $\langle k \rangle = 2m$. 

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Fig. 2: (a) The value of $c_{\text{opt}}$ as a function of the average degree of the network $\langle k \rangle$. The network size $N = 5000$, the noise $\kappa = 1$ and the degree exponent $\gamma = 3$. (b) The value of $c_{\text{opt}}$ as a function of the network size $N$. The average degree of the network $\langle k \rangle = 6$, the noise $\kappa = 1$ and the degree exponent $\gamma = 3$. (c) The value of $c_{\text{opt}}$ as a function of the noise $\kappa$. The network size $N = 5000$, the average degree of the network $\langle k \rangle = 6$ and the degree exponent $\gamma = 3$. (d) The value of $c_{\text{opt}}$ as a function of the degree exponent $\gamma$. The network size $N = 5000$, the noise $\kappa = 1$ and the average degree of the network $\langle k \rangle = 6$.

Results. – Each data point is based on 100 realizations of the network and 10 realizations on each network. To simplify, we set the benefit from the common opinion as $b = 1$.

We define the consensus time $T_c$ as the time steps required to reach the global consensus where all agents in a network share the same opinion. Each time step consists of an average one opinion-updating event for all agents. Figure 1 shows $T_c$ as a function of the cost $c$ for different values of the average degree of the network $\langle k \rangle$, the network size $N$, the noise $\kappa$ and the degree exponent $\gamma$. From fig. 1, one can see that for given values of other parameters, there exists an optimal value of $c$, hereafter denoted by $c_{\text{opt}}$, resulting in the shortest consensus time $T_c$.

The value of $c_{\text{opt}}$ changes with the average degree of the network $\langle k \rangle$, the network size $N$, the noise $\kappa$ and the degree exponent $\gamma$. For given values of other parameters, $c_{\text{opt}}$ decreases from 1.4 to 1.1 as $\langle k \rangle$ increases from 4 to 14 (see fig. 2(a)). The value of $c_{\text{opt}}$ decreases from 1.3 to 1.2 as $N$ increases from 1000 to 20000 (see fig. 2(b)), and $c_{\text{opt}}$ increases from 1.2 to 1.6 as $\kappa$ increases from 0.2 to 20 (see fig. 2(c)). The value of $c_{\text{opt}}$ increases from 1.1 to 1.3 as the degree exponent $\gamma$ increases from 2.2 to 6 (see fig. 2(d)). From fig. 2, one can also observe that the value of $c_{\text{opt}}$ is more than the benefit which is previously set to be 1.

In scale-free networks a very few nodes have high degrees (usually called hubs). The influence of hubs increases as their degrees increase, but decrease as the noise increases. In our network model, the degrees of hubs increase as the average degree of the network or the network size increases, but decrease as the degree exponent increases. It is interesting to find that there is a negative correlation between the optimal cost and the influence of hubs. From figs. 2(a) and (b), one can see that $c_{\text{opt}}$ decreases as the degrees of hubs increase. On the contrary, $c_{\text{opt}}$ increases as the degrees of hubs decrease (fig. 2(d)) or the noise increases (fig. 2(c)).

Initially, the two competing opinions are randomly distributed among the population with equal probability. Thus, according to mean-field theory, the initial payoff of agent $x$ can be approximatively expressed as $P_x(0) = (b-c)k_x/2$. For $b > c$, the initial payoff of agent $x$ increases with its degree. However, for $b < c$, $P_x(0)$ decreases as the agent’s degree increases.

It has been shown that the opinion dynamics is proceeded by formation of some large opinion clusters centered at hubs [40–43]. An opinion cluster is a connected component (subgraph) fully occupied by nodes holding the same opinion. Through the competition of different opinion clusters, one cluster will invade the others and finally dominate the system with a global consensus. For very large values of the cost $c$, initially the payoffs of high-degree agents are much lower than that of low-degree agents.
agents and the opinions of hubs have a very small probability to be imitated. Thus it becomes difficult for agents to form large opinion clusters centered at hubs. On the other hand, for very small values of $c$, a hub has so strong influence on its low-degree neighbors that the cluster formed by this hub is extremely stable. As a result, the merging of different clusters becomes very difficult for too large $c$, leading to a longer consensus time. Taken together, we can expect that the shortest consensus time would be realized at the moderate value of $c$.

To verify the above analysis, we study the number of opinion clusters $N_{cl}$ as the time $t$ evolves for different values of the cost $c$. From fig. 3, one can see that initially there exist hundreds of opinion clusters in the network. After a few time steps, the system forms dozens of big opinion clusters centered at hubs. Once a hub with an opinion ($e.g., +1$ opinion) successfully invades the other hub with the opposite opinion ($e.g., -1$ opinion), the $-1$ big opinion cluster decomposes and gradually merges into the $+1$ big opinion cluster. The decomposition of some big opinion clusters will cause a temporary increase of $N_{cl}$. As the integration of different big opinion clusters continually proceeds, $N_{cl}$ decreases to 1. For $c = 5$, initially $N_{cl}$ decreases much more slowly, compared with $c = 0.1$ and 1.2, indicating that it is hard to form big opinion clusters when $c$ is large. For $c = 0.1$, $N_{cl}$ decreases faster than the cases of $c = 1.2$ and $c = 5$ in the early stage ($t < 100$). However, when only a few clusters remain in the system, for example, $N_{cl} < 40$, $N_{cl}$ decreases very slowly for $c = 0.1$, indicating that the competition among big opinion clusters becomes furious when $c$ is too small.

Finally, we investigate the effects of the average degree of the network $\langle k \rangle$, the network size $N$, the noise $\kappa$ and the degree exponent $\gamma$ on the consensus time. From fig. 4(a), we see that for a given value of the cost $c$, the consensus time $T_c$ decreases as $\langle k \rangle$ increases. The consensus time $T_c$ scales as $N^3\beta$ with the exponent $\beta$ depending on the value of $c$ (see fig. 4(b)). The exponent $\beta = 0.19, 0.13, 0.15$, corresponds to $c = 0.2, 1.2, 3$, respectively. In particular, the optimal $c = 1.2$ results in the lowest value of $\beta$. From fig. 4(c), we observe that the consensus time $T_c$ increases as the noise $\kappa$ increases. This phenomenon indicates that the more rational choice (following the opinion of the agent with higher payoff) will accelerate the formation of consensus. From fig. 4(d), one can find that the consensus time $T_c$ decreases as the degree exponent $\gamma$ increases when the cost is small ($e.g., c = 0.2$) or large ($e.g., c = 3$). However, $T_c$ increases with $\gamma$ for the moderate value of the cost ($e.g., c = 1.2$).

Conclusions and discussions. – In conclusion, we have proposed a consensus opinion model based on the evolutionary game. An agent receives a benefit if it has the same opinion with a neighbor, otherwise it pays a cost. An agent randomly selects a neighbor as a reference. The agent has a higher (lower) probability to imitate the opinion of the chosen neighbor if its payoff is lower (larger) than that of the selected neighbor. The results in scale-free networks show that the shortest consensus time can be obtained when the cost of conflicting opinions is a little larger than the benefit of common opinions. For very high ratio of cost to benefit, initially hubs have so low payoffs that their opinions are seldom imitated, which prevents the formation of large opinion clusters. On the other hand, too low ratio of cost to benefit makes the merging of different clusters very difficult. Thus the shortest consensus time must be realized at the moderate ratio of cost to benefit.

The interplay between the opinion dynamics and the evolutionary games is a very interesting topic. Previous
studies have shown that the introduction of opinion dynamics can greatly affect the evolution of cooperation [44–48]. For example, Szolnoki and Perc discovered that the spatial selection for cooperation is enhanced if an appropriate fraction of the population chooses the most common rather than the most profitable strategy within the interaction range [44]. Yang et al. found that cooperation is promoted by punishing neighbors with the opposite strategy [45]. Previous and our works, together offer an underlying connection between the evolutionary games and the opinion dynamics.

Our findings presented here raise a number of questions, answers to which could further deepen our understanding of the opinion formation based on game theory. For example, in the present model we assume that dissent leads to punishment. But what happens if contrarians can also receive payoffs? Another significant question is how do other network structures and updating rules affect the consensus time? The above questions deserve to be further investigated.

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REFERENCES

[1] Castellano C., Fortunato S. and Loreto V., Rev. Mod. Phys., 81 (2009) 591.
[2] Sood V. and Redner S., Phys. Rev. Lett., 94 (2005) 178701.
[3] Galam S., Eur. Phys. J. B, 25 (2002) 403.
[4] Krapivsky P. L. and Redner S., Phys. Rev. Lett., 90 (2003) 238701.
[5] Borgesi C. and Galam S., Phys. Rev. E, 73 (2006) 066118.
[6] Duffuant G., Ambillard F., Weisbuch G. and Faure T., J. Artif. Soc. Soc. Simul., 5 (2002) 4.
[7] Nowak A., Kuś M., Urbaniaik J. and Zarycki T., Physica A, 287 (2000) 613.
[8] Sánchez A. D., López J. M. and Rodríguez M. A., Phys. Rev. Lett., 88 (2002) 048701.
[9] Mobilia M. and Redner S., Phys. Rev. E, 68 (2003) 046106.
[10] Galam S., Physica A, 333 (2004) 453.
[11] Barabási A.-L. and Albert R., Science, 286 (1999) 509.
[12] Tang C.-L., Liu B.-Y., Wang W.-X., Hu M.-B. and Wang B.-H., Phys. Rev. E, 75 (2007) 027101.
[13] Shao J., Havlin S. and Stanley H. E., Phys. Rev. Lett., 103 (2009) 018701.
[14] Yang H.-X., Wang W.-X., Liu Y.-C. and Wang B.-H., Phys. Rev. E, 80 (2009) 046108.
[15] Yang H.-X., Wang W.-X., Li Y.-C. and Wang B.-H., Phys. Lett. A, 376 (2012) 282.
[16] Szolnoki A. and Perc M., J. R. Soc. Interface, 12 (2015) 20141299.
[17] Yang H.-X., Wang Z.-X., Rong Z. and Lai Y.-C., Phys. Rev. E, 91 (2015) 022121.
[18] Yang H.-X. and Wang Z., EPL, 111 (2015) 60003.
[19] Szolnoki A. and Perc M., Sci. Rep., 6 (2016) 23633.
[20] Szolnoki A. and Perc M., EPL, 113 (2016) 58004.