A study of gravitational collapse with decaying of the vacuum energy

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Abstract. We study the gravitational collapse of a dust dark matter, in a Λ-background. We consider two distinct cases: First we do not have a dark matter and dark energy coupling; second, we consider that Λ decay in dark particles. The approach adopted assumes a modified matter expansion rate and we have formation of a black hole, since that, we have the formation of an apparent horizon. Finally, a brief comparison of the process of the matter condensation using the gravitational collapse approach and the linear scalar perturbation theory is considered.

PACS numbers: 04.70.Bw
A study of gravitational collapse with decaying of the vacuum energy

1. Introduction

In the course of the eighteen century, astronomers were discussing the internal structure of the individual nebulae rather than the wider cosmological problem. Probably, one of the obstacle to study the wider context is related to the question: Is the universe finite?

On the other hand, the infinitely extended universe considered by the Newton’s theory of gravitation was one of the reasons for the believe that Newton’s theory was cosmologically unacceptable.

The advent of the special and general theory of relativity by A. Einstein furnish a new basis to discuss the wider cosmological problem. We mention two important discoveries about the universe that emerge from the general relativity. First, follow Poisson and Israel [1], “black hole theory is without any doubt one the major triumphs of classical general relativity”. Although, based in the existence of a spacelike singularity that destroys quantum information and in the presence of an infinite red-shift surface, it was speculate that black holes may not exit at all [2]. Second, one of the recent important discoveries about the cosmos is the accelerating expansion of the cosmic fluid. In order to obtain a universe with an accelerated expansion we must provide a new component for the energy distribution, that is usually referred as dark energy. One of the candidates responsible for this process is the cosmological constant, frequently denoted by $\Lambda$.

Only few years after Albert Einstein introduced the fundamentals of the general relativity theory, himself includes in the field equations of the general relativity theory the cosmological constant. Moreover, in the light of the experimental evidence of the universal expansion, Einstein considered the inclusion of $\Lambda$ “...the biggest blunder of my life.” [3].

Before the results from supernova of type IA observations appear in the literature, that indicates an accelerated expansion of the universe, L. Krauss and M. Turner call our attention that “The Cosmological Constant is Back.” They cited the age of the universe, the formation of the large scale structure and the matter content of the universe as the data that indicates the insertion of the cosmological constant [4]. The cosmic microwave background radiation anisotropy and large scale structure, also indicates this acceleration process of the universe [5], [6].

The mechanism that triggered the acceleration of the universe has not been identified, and the simplest explanation for the this process is the inclusion of a non null cosmological constant. However, the inclusion of the cosmological constant creates new problems. Some of them are old, as the discrepancy among the observed value for the energy density of the vacuum, and the large value suggested by the particle physics models [7], [8]. In spite of of the problems caused by the inclusion of $\Lambda$, the cosmological scenario with $\Lambda$ has a good agreement in respect to the age of the universe estimate, the anisotropy of microwave background radiation and the supernova experiments. Beyond this, making several assumptions concerning with the spectrum of fluctuations in the early universe and the formation of galaxies, G. Efstathiou suggests that the small value
A study of gravitational collapse with decaying of the vacuum energy

of the cosmological constant may be explained by the anthropic principle \[9\].

The observational data also suggest that the universe is flat. The flatness is consequence of the cosmic content and indicates that our universe consists of \(\frac{2}{3}\) of dark energy and \(\frac{1}{3}\) of dark matter, approximately. The dark matter component dominates at small scales (< 500 Mpc) and the dark energy component dominates at large scale (> 1000 Mpc).

The evidence that these new components of the universe are different substances has been considered in literature \[7\]. The component at small scale, generally, is considered as weakly interacting massive particles and the component at large scale is associated to some form of a scalar field. A link between both components to a scalar field is study by Padmanabhan and Choudury \[11\]. Consequently both components have a scale-dependent state equation, but, today, both components are unknown in respect to the your nature.

The presence of the dark matter was first discussed by Zwiecky \[12\] in your study of the dynamics of galaxy clusters. At the present time the presence of the dark matter is indicate by the study of galaxy rotation curves, the structure of the galaxy groups and clusters, large scale cosmic flows and gravitational lensing. This last, a phenomenon proposed by Zwiecky himself as an astronomical tool \[13\].

An alternative model that furnish a negative pressure in the cosmic fluid and results in an accelerated expansion of the universe is known as open system cosmology \[14\]. In this model the particles number of the universe do not conserve and the energy-momentum tensor is reinterpreted in the Einstein’s equations. This extra pressure is known as creation pressure and is negative \[15,16\]. The creation process is due to the expenses of the gravitational field and is an irreversible process. One of the attractive features of the hypothesis of particle production is the relation between the large scale properties of the universe and the atomic phenomena \[17\].

If the final state of a gravitational collapse results a black hole or a naked singularity, it is necessary to look for the development of apparent horizons and to examine if there are any families of outgoing non-spacelike trajectories, which terminates in the past at the singularity. The apparent horizon, that is formed within the star, characterise a trapped surface region of the spacetime. The singularity evolves to a naked singularity if the neighbourhood of the star center do not gets trapped earlier than the singularity. In this case the non space-like future trajectories escaping from the center of the star to outside observers \[18\]. Otherwise, the black hole is formed if we have a region around the neighbourhood the star center gets trapped earlier than the singularity.

The natural question, how the dark energy effects the process of the gravitational collapse was studied by several authors, \[19,20,21,22\]. Cai and Wang consider the collapse of two component fluid constituted by a dust cloud dark matter coupled with a dark energy component. They assume that the interaction between the two components is given by:

\[
\frac{\rho_\Lambda}{\rho_{dm}} = AR^{3n},
\]

(1)
A study of gravitational collapse with decaying of the vacuum energy

where $A$ and $n$ are constants. They conclude that, in presence of dark energy the gravitational collapse of the dark matter cloud can occurs. Although, the junction of the star to the spacetime outside is not considered by the authors and, in this work, we assume identical approach.

Resuming, in this work our intention is study the gravitational collapse of a cloud of dark matter in a background with a cosmological constant, where we have particle production of dark matter particles at the expense of the dark energy backgound.

2. The collapse of a dark matter cloud

We can divide the spacetime of the star into three regions. One correspondent to the interior of the star, one to the exterior and one relative to a spherical surface, that divide the interior of the exterior. The motion of this surface is described by a timelike three dimensional space $\Sigma$, given by $r_{\Sigma} = constant$. The metric on $\Sigma$ can be written as $ds_{\Sigma}^2 = d\tau^2 - R(\tau)^2 d\Omega^2$, where we use the intrinsic coordinates $\xi^a \equiv (\tau, \theta, \phi)$, $a(\tau) = r_{\Sigma}R(\tau)$ and $\tau = t$. Once the spacetime inside the surface is fixed, whether a thin shell appears on $\Sigma$ or not is completely determined by the spacetime outside the star [19].

Our interest in the interior of the star is due to a coupled of reasons. First, as discussed by Cai and Wang [19], we do not have a unique matching of exterior and the interior spacetimes. Second, the discussion about the formation of black holes, reduces to the one whether apparent horizons develop inside the star.

We assume that the spacetime inside the star is homogeneous and isotropic. Consequently, the interior of the star is described by

$$ds^2 = dt^2 - R(t)^2(d\tau^2 + r^2d\Omega^2),$$

where $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$.

The formation of a black hole creates an apparent horizon, hence

$$a_{,\alpha}a_{,\beta}g^{-\alpha\beta} = [r\dot{R}(t)]^2 - 1 = 0,$$

where $a(r,t) = rR(t)$ is the geometrical radius of the two-sphere $t, r = constant$ and the dot means the time derivative.

Cahill and Mac Vittie [23] using curvature coordinates and considering the metric tensor continuous across the surface of discontinuity, which separates a spherical distribution of material from the surrounded empty space, find a function that may be defined as the total mass-energy entrapped inside the radius $r$ at the time $t$, namely

$$m(t,r) = \frac{1}{2}a(1 + a_{,\alpha}a_{,\beta}g^{\alpha\beta}) = \frac{1}{2}r^3\dot{R}^2.$$ (4)

If we consider that the all star collapses inside the apparent horizon, using (3) we have

$$\dot{a}^2(\tau_c) |_{\Sigma} = 1,$$ (5)

where $t_c$ is the time necessary to occur the collapse.
A study of gravitational collapse with decaying of the vacuum energy

Therefore, on the surface of the star, eq. (4) furnish the total mass of the collapsed star, namely
\[ M = \frac{1}{2} a(\tau) \hat{a}(\tau)^2, \] (6)
and the mass of the black hole formed is \( M_{BH} = M(\tau_c) \).

The energy momentum tensor inside the star is given by
\[ T_{\mu\nu} = (\rho_\Lambda + \rho_{dm} + P_\Lambda) u_\mu u_\nu, \] (7)
where \( \rho_\Lambda \) and \( P_\Lambda \) are the density and the pressure of dark energy and \( \rho_{dm} \) is the dark matter density, while \( u_\mu \) is their four velocities.

Taking into account that the reference system is just the matter filling the space, the Einstein field equations are:
\[ \frac{\dot{R}}{R^2} = -\frac{1}{6} \kappa (\rho_\Lambda + \rho_{dm} + 3P_\Lambda), \] (8)
\[ \frac{\ddot{R}}{R^2} = \frac{1}{3} \kappa (\rho_\Lambda + \rho_{dm}). \] (9)

The interaction between the dust matter and dark energy follows from the conservation law
\[ T_{\mu\nu;\lambda} g^{\nu\lambda}, \] (10)
which in the present study, taking into account the decay of \( \Lambda \) into dark particles, assumes the form
\[ \dot{\rho}_{dm} + 3 \frac{\dot{R}}{R} \rho_{dm} = -\dot{\rho}_\Lambda. \] (11)
In the absence of the coupling between dark matter and dark energy we have \( \dot{\rho}_{dm} + 3 \frac{\dot{R}}{R} \rho_{dm} = 0 \), which integration results \( \rho_{dm} = \rho_{dm0} R^{-3} \), where \( \rho_{dm0} \) is an integration constant.

Considering that the dark energy decay into cold dark matter, this will dilute in a smaller rate compared with their usual relation proportional to \( R^{-3} \) [24]. Hence
\[ \rho_{dm} = \rho_{dm0} R^{\epsilon - 3}, \] (12)
where \( \epsilon \) is a positive constant, that indicates the deviation from the absence of the coupling between the dark energy and dark matter.

We can write equation (11) as
\[ \frac{d\rho_{dm}}{dR} + 3 \frac{\rho_{dm}}{R} = -\frac{d\rho_\Lambda}{dR}. \] (13)

Substituting (12) into (13), we find the expression for the dark energy density, namely
\[ \rho_\Lambda = \rho_{\Lambda0} - \epsilon \rho_{dm0} \frac{R^{\epsilon - 3}}{3 - \epsilon}. \] (14)

Our intent is study the gravitational collapse, then the condition \( \dot{R} < 0 \) must be considered.
A study of gravitational collapse with decaying of the vacuum energy

Note that, in the collapse process the scale factor diminishes, hence the density of the dark matter grows and energy density of Λ-component diminishes. In addition, for $R \to 0$, we have $\rho_\Lambda \to -\infty$.

The validity interval for $\epsilon$ is $\epsilon \leq 1$ in conformity with SNe IA observation. Otherwise, we have an accelerated universe in a matter dominated era. On the other hand, the SNe IA data indicates that we have an decelerated expansion before redshift $z \approx 0.46 \pm 0.13$ [25]. The evidence for cosmic deceleration that precede the accelerated expansion, on another words, a strong evidence for a cosmic jerk [26], is inconsistent with very rapid evolution of dark energy. There isn’t any anomalous cold dark matter expansion rate observable. So, we expect that $\epsilon \ll 1$.

With the auxilious of equations (9), (12) and (14) we find the equation

$$\dot{R}^2 - K_I R^2 - K_{II} R^{\epsilon - 1} = 0,$$

where $K_I = \frac{8\pi G}{3} \rho_\Lambda_0$ and $K_{II} = \frac{8\pi G}{3} (1 - \frac{\epsilon}{3\epsilon}) \rho_{dm0}$.

Although we have a vacuum decaying, the state equation $w = \frac{P_\Lambda}{\rho_\Lambda}$ is still constant.

2.1. Gravitational collapse of a dust cloud without decay of Λ

In this section we consider the collapse of a dust cloud in the presence of a cosmological constant without creation of dark particles. Therefore, $\epsilon = 0$ and equation (15) can be written as

$$\dot{R}^2 - Q_I R^2 - Q_{II} R^{-1} = 0,$$

where $Q_I = \frac{8\pi G}{3} \rho_\Lambda_0$ and $Q_{II} = \frac{8\pi G \rho_{dm0}}{3}$.

The integration of last equation results:

$$R(t) = R_0 \left\{ \sinh \frac{3 \sqrt{Q_I} (t_0 - t)}{2} \right\}^{\frac{2}{3}},$$

where $t_0$ is an integration constant and $R_0 = \left( \frac{Q_{II}}{Q_I} \right)^{1/4}$.

Consequently, the expressions for the dark matter and dark energy densities are, respectively:

$$\rho_{dm} = \rho_{dm0} \frac{R_0^3}{R^5} \left\{ \sinh \frac{3 \sqrt{Q_I} (t_0 - t)}{2} \right\}^{-2},$$

$$\rho_\Lambda = \rho_\Lambda_0. \quad (19)$$

The other relevant pertinent quantities are:

$$\dot{a}(t) = - a_0 \sqrt{Q_I} \left\{ \sinh \frac{3 \sqrt{Q_I} (t_0 - t)}{2} \right\}^{-\frac{3}{4}} \cosh \frac{3 \sqrt{Q_I} (t_0 - t)}{2}, \quad (20)$$

$$M(\tau) = \frac{1}{2} a_0^3 Q_I \left\{ \cosh \frac{3 \sqrt{Q_I} (t_0 - \tau)}{2} \right\}^2, \quad (21)$$

where $a_0 = r R_0$.

In the limit $t \to t_0$, the equations (17)-(21) reduces to

$$R(t) = \frac{3}{2} R_0 \sqrt{Q_I (t_0 - t)}^{\frac{2}{3}} \quad (22).$$
A study of gravitational collapse with decaying of the vacuum energy

\[ \rho_{dm} = \rho_{dm0} R_0^3 (t_0 - t)^{-2} \]  
\[ \rho_{\Lambda} = \rho_{\Lambda_0} \]  
\[ \dot{a}(t) = -a_0 \sqrt{Q_I} (t_0 - t)^{-\frac{1}{3}} \]  
\[ M(t) = \frac{1}{2} a_0^3 Q_I, \]

that are identical to Oppenheimer-Synyder solution, considered as the first study of the gravitational collapse [27].

Comparing the formation of the apparent horizon in the presence of \( \Lambda \) with Oppenheimer-Synyder solution, note that for the first case the apparent horizon appear in a time anterior that the Oppenheimer-Synyder case, vide Fig. 1.

![Graph](image_url)

**Figure 1.** Evolution for \( \dot{R}^2 \) considering the Oppenheimer-Synyder solution (OS) and the solution in the presence of \( \Lambda \).

With respect to the total mass collapsed, we have a constant value for the Oppenheimer-Synyder solution, given by the Schwarzschild black hole mass.

The spacetime singularity is formed at \( \tau = \tau_0 \) and the apparent horizon is formed at

\[ \tau = \tau_0 - \left( \frac{1}{Q_I a_0^3} \right)^{-\frac{1}{3}}. \]  

Hence, the black hole is formed since that the apparent horizon is anterior to the reach of the singularity and the star is not initially trapped.

2.2. Gravitational collapse of a dust cloud with decay of \( \Lambda \)

Now, we consider the creation of dark particles at the expenses of the \( \Lambda \) decay. Consequently, the integration of eq.(15) results

\[ R(t) = R_0 \{ \sinh \frac{\sqrt{K_I} (3 - \epsilon) (t_0 - t)}{2} \}^{\frac{3}{3-\epsilon}}, \]  

where \( t_0 \) is an integration constant and \( R_0 = \left( \frac{K_I}{K_{II}} \right)^{\frac{1}{3}} \). Naturally this solution decay in (22) for \( \epsilon = 0 \).

Considering (28) the physically important quantities are:

\[ \rho_{dm} = \rho_{dm0} R_0^{-3} \{ \sinh \frac{\sqrt{K_I} (3 - \epsilon) (t_0 - t)}{2} \}^{-2}, \]  

(29)
A study of gravitational collapse with decaying of the vacuum energy

\[ \rho_\Lambda = \rho_{\Lambda_0} + \frac{\epsilon}{\epsilon - 3} \rho_{dm0} \sinh \left( \frac{\sqrt{K_I} (3 - \epsilon)(t_0 - t)}{2} \right)^{-2} \]  

(30)

\[ \dot{a}(t) = -a_0 \sqrt{K_I} \sinh \left( \frac{\sqrt{K_I} (3 - \epsilon)(t_0 - t)}{2} \right)^{-\frac{1}{2}} \]  

(31)

\[ \left[ \cosh \left( \frac{\sqrt{K_I} (3 - \epsilon)(t_0 - t)}{2} \right) \right] \]  

\[ M(\tau) = \frac{K_I}{2} \rho_{\Lambda_0} \sinh \left( \frac{\sqrt{K_I} (3 - \epsilon)(\tau_0 - \tau)}{2} \right)^{\frac{1}{2}} \]  

\[ \left[ \cosh \left( \frac{\sqrt{K_I} (3 - \epsilon)(\tau_0 - \tau)}{2} \right)^2 \right] \]  

(32)

The profile of dark energy density appear in the Figure 2.

\[ \text{Figure 2. Evolution of the dark energy density for } \epsilon = 0.1. \]

It is interesting look to the change of \( \Lambda \) signal. The background change to an anti-deSitter type spacetime in a time anterior to the reach of the singularity that is given by:

\[ t = t_0 - \arcsinh \left\{ \left( \frac{\epsilon \rho_{dm0}}{(3 - \epsilon)\rho_{\Lambda_0}} \right)^\frac{1}{2} \right\} . \]  

(33)

Physically, a negative \( \Lambda \) corresponds in the Newtonian limit to an extra attractive term in the gravitational force. The anti-de Sitter spacetime play an important role in the superstring theory. Probably, a scenario with superior dimensions is more adequate to study this change of signal of \( \Lambda \) and the consequences.

The evolution of the \( \dot{R}^2 \) is better analysed making a graph. The profile for two different values for \( \epsilon \) appear in the Fig.3. Note that, with the increase of \( \epsilon \), \( \dot{R}^2 \) reaches the unity in an epoch anterior than smaller values for \( \epsilon \). Using other words, small values for \( \epsilon \) favour the formation of the apparent horizon. In this case, identically to the anterior case, we have the formation of a black hole

In respect to the mass inside the star we obtain an identical behaviour. Hence, \( M_{\epsilon=0.2} < M_{\epsilon=0.01} \). Consequently, we hope that the increase of \( \epsilon \) is harder for the process of matter condensation.

A similar conclusion can be obtained studying the evolution of the scalar perturbations. The equation that governs the evolution of the scalar linear perturbations in a matter dominated phase is \[ 28 \]

\[ R^2 \frac{d^2 \delta}{dR^2} + \frac{3}{2} (1 + \frac{\Lambda}{3H^2}) R \frac{d\delta}{dR} + \left( \frac{\Lambda}{2H^2} - \frac{3}{2} \right) \delta = 0, \]  

(34)
A study of gravitational collapse with decaying of the vacuum energy

Figure 3. Evolution of the $\dot{R}^2$ in the presence of the $\Lambda$ decay into dark particles for two different values for $\epsilon$.

that assumes the form:

$$R^2 \frac{d^2 \delta}{dR^2} + \frac{3 + \epsilon}{2} R \frac{d\delta}{dR} + \frac{\epsilon - 3}{2} \delta = 0.$$  \hspace{1cm} (35)

The last equation was founded using eqs. (9), (12) and (14). In addition, is also necessary assume that $\frac{\rho_{dm}}{\rho_{\Lambda}} = \frac{3-\epsilon}{\epsilon}$, that is equivalent to $\rho_{\Lambda} << 0$. This assumption with the auxilious of eqs. (9), (12) and (14), results $\Lambda = \epsilon H^2$.

When the $\Lambda$ terms are ignored in eq. (34) we obtain the usual solution $\delta \propto R$

Integrating the eq. (35) we obtain

$$\delta \propto R \sqrt{\frac{-6 + 25 - \epsilon}{4}},$$ \hspace{1cm} (36)

which evolve slower than the usual solution for the density contrast proportional to $R$.

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A study of gravitational collapse with decaying of the vacuum energy

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